Theory of Second Order Optical Processes from A Luttinger Liquid

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Abstract

We develop a theory for the total optical secondary emission from a 1D interacting electron system modelled as a Tomonaga-Luttinger liquid. We separate the emission into two parts which may originate in hot luminescence (HL) and Raman Scattering (RS) respectively when we neglect the interference effect. We find a peak around $\Delta \omega = v_f |q|$ in the RS part which does not come from a structure factor peak. In general the total emission cannot be separated into HL and RS. However at resonance, and taking into account the $k$ dependence of the optical matrix element, a part of the RS is proportional to the structure factor $S(q_1 - q_2, \omega_1 - \omega_2)$.

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Recent advances in fabrication of semiconductors permit the construction of systems, such as the quantum wire, with effective geometrical dimensionality of one (1D). This has permitted study of the transport properties of 1D systems. The optical properties can also be studied, so as to compare and contrast with electronic models (Fermi Liquid or Luttinger Liquid) for 1D electronic and optical response. It is well known that the Tomonaga-Luttinger (TL) model [1] is necessary to describe a pure 1D interacting electron system [2]. However, in an actual semiconductor quantum wire, the scattering and finite size effects may serve to damp out the low-energy plasmons and restore the Fermi surface [3]. It is not yet clear whether the TL liquid or the normal Fermi liquid theory is more appropriate to describe properties of semiconductor quantum wires. As a matter of fact, both TL liquid and Fermi liquid theory have been used to study the transport and optical properties of 1D semiconductor electron systems [3, 8, 4, 7].

Optical methods are powerful tools for the investigation of the collective and single particle excitations. For example, the Fermi edge singularities (FES) [9, 10] in the photoemission and soft-x-ray emission and absorption provide information about the existence of the Fermi surface and the low energy excitations. The conditions for FES have been studied in both normal Fermi liquid theory [6] and TL liquid theory [7, 8]. Although Raman Scattering (RS) experiments have been carried out in these 1D quantum wires [11], there are only a few theoretical studies using normal Fermi liquid theory [12] which aim to describe the RS spectra of these systems. However, the calculation of the RS efficiency from the collective excitations in 1D systems needs special caution, especially in the resonance region, as we discuss later. The objective of this paper is (1) to give a theory of RS in a TL liquid; (2) to reveal that generally the second order RS efficiency is not proportional to the dynamical structure factor in the resonance region contrary to a widely used approximation; (3) to discuss the difference between RS from TL and that from FL. We also remark on the possibility of separating hot luminescence (HL) and Raman scattering (RS). Since TL is an exactly solvable model, our calculation may serve as a special prototype for the discussion of various aspects of RS, such as threshold behavior.
Let us first consider the general RS which is due to the $\vec{A} \cdot \vec{P}$ interaction. Our discussion here will follow the standard literature on the RS from collective excitations [13,14]. If we here neglect the "final state interaction" of the core holes, then in second order perturbation theory, the transition rate can be written as:

\[
W(q_1, q_2; \omega_1, \omega_2) = \sum_{k, k'} M_{k' - q_1} M_{k - q_2} M_{k'' - q_-2} M_{k' - q_2} \int_{-\infty}^{+\infty} d\tau \int_{-\infty}^{0} dt' \int_{0}^{0} dt \times \langle 0|c_{k' - q_1}^{\dagger}(t' + \tau)c_{k'' - q_2}(\tau)c_{k' - q_2}(0)c_{k - q_2}(t)|0\rangle e^{-i(\omega_1 + \epsilon_d)(t - t') - i(\omega_2 - \omega_1)\tau}
\]  

(1)

Here $q_1, \omega_1 (q_2, \omega_2)$ are the wave vector and frequency of the incident (scattered) photons, $c_k$ ($c_k^{\dagger}$) is the annihilation (creation) operator of the conduction band electron, $\epsilon_d$ is the eigenenergy of the core hole, and $M_k$ is the optical matrix element. The quasi-particle approximation can be used to simplify the calculation as well as to "pick-up" only the virtual RS process contribution. In this approximation, we substitute $c_{k' - q_1}^{\dagger}(t' + \tau) = e^{i\epsilon_{k' - q_1}t'}c_{k' - q_1}(\tau)$ and $c_{k - q_2}(t) = e^{-i\epsilon_{k - q_2}t}c_{k - q_2}(0)$ [13] in Eq.[1], and we obtain

\[
W(q_1, q_2; \omega_1, \omega_2) = \sum_{k, k'} \frac{M_{k' - q_1} M_{k - q_2} M_{k'' - q_2} M_{k' - q_2}}{(\epsilon_{k - q_1} - \epsilon_d)(\epsilon_{k' - q_1} - \epsilon_d)} \times \int_{-\infty}^{+\infty} d\tau \langle 0|c_{k - q_1}^{\dagger}(\tau)c_{k - q_2}(\tau)c_{k - q_2}(0)c_{k - q_2}(0)|0\rangle e^{-i(\omega_2 - \omega_1)\tau}
\]  

(2)

If we treat $M_g$ as a constant, and assume the RS is near but not at resonance, \textit{i.e.} incident photon frequency $\omega_1 \sim \Delta_g$ but $|\omega_1 - (\epsilon_{k - q_1} - \epsilon_d)| >> \epsilon_{k - q_1}$, where $\Delta_g$ is the semiconductor band gap, then we obtain the celebrated form [13,14] \( W(q_1, q_2; \omega_1, \omega_2) = \frac{|M_g|^4}{(\omega_1 - \Delta_g)^2} S(q_1 - q_2, \omega_1 - \omega_2) \) and the Random-Phase Approximation (RPA) can be used to calculate the RS efficiency from collective excitations. In this case, the RS efficiency is proportional to the dynamical structure factor $S(q_1 - q_2, \omega_1 - \omega_2)$ with an enhancement prefactor. Although this is a very useful result, its applicability in the resonance region is invalid, especially for the 1D electron gas.

Since the 1D TL model is exactly solvable even including the core hole effect, it would be of interest to study RS from the TL liquid. Before including the core hole effect, we show that taking into account the $k$-dependence of $M_k$ there is a contribution in Eq.[4]
which is proportional to the structure factor \( S(q_1 - q_2, \omega_1 - \omega_2) \) for RS from the TL liquid. Expand \( M_k \) around some point \( k_0 \) \([13]\) and \( \epsilon_k \) around some point \( k_f' \) \((k_f' \sim \text{Fermi momentum } k_f)\). Since in resonance RS, we can choose the appropriate \( k_0 \) and \( k_f' \), then \( \frac{M_{k_0}}{\omega_1 - (\epsilon_{k_0} - \epsilon_d)} \sim \frac{|M|^2}{\omega_1 - (\epsilon_{k_0} - \epsilon_d)} - \frac{2Re(M'M^*)}{v_f} \). Here \( v_f \) is the Fermi velocity. It is easy to see that the 2nd term will give a contribution proportional to the structure factor \( S(q_1 - q_2, \omega_1 - \omega_2) \) in Eq\([1]\). The relative amplitude of this contribution depends on detailed information about the system.

Let us now calculate the RS efficiency for the spinless TL liquid with constant matrix elements \( M_k = M \). Experimentally the spinless model can be realized by applying a strong magnetic field that polarizes electron spins. Generalization to the spin-1/2 case is straightforward but the calculation is more complicated. In the standard bosonization \([2]\) of the 1D electron gas, the Hamiltonian of the coupled TL bosons and localized core hole is \([16]\):

\[
H_0 = \sum_k v_{TL}|k|b_k^\dagger b_k + \sum_x \epsilon_d d_x^\dagger d_x + \frac{1}{\sqrt{L}} \sum_{k,x} V_k e^{ikx} d_x^\dagger b_k + h.c. \tag{3}
\]

The electron-radiation interaction can be written as:

\[
H_{int} = M \sum_{x,q,\alpha} \psi^\dagger_{\alpha}(x)d_x A_q e^{iqx} + h.c. \tag{4}
\]

where \( \psi^\dagger_{\alpha}(x) \) is the fermion operator of right moving \((\alpha = +)\) and left moving \((\beta = -)\) electrons, and \( b_k^\dagger \) and \( b_k \) are the operators of the TL boson whose velocity is \( v_{TL} = [(v_f + V_{1k})^2 - (V_{2k})^2]^{1/2} \). \( d_x \) and \( d_x^\dagger \) are operators of the localized core hole with energy \( \epsilon_d \).

Here we only include the forward scattering of the electron by a core hole in Eq\([3]\), the effects of backscattering of electrons will be discussed in the end. \( V_k \) is the renormalized interaction between the TL boson and the core hole \( V_k = V_{3k} (|k| 2\pi)^{1/2} \cosh \phi_k - \sinh \phi_k \). Here we use the notation as in \([3]\) for \( V_{1k}, V_{2k} \) and \( \phi_k \). \( V_{1k} \) and \( V_{2k} \) are for the forward Fermion scattering interaction, so in "g-ology", \( V_{1k} \equiv g_4 \) and \( V_{2k} \equiv g_2 \). \( V_{3k} \) is the interaction between electron and core hole. In the further calculation, we will use the short-range \( \delta \)-potentials for \( V_i(x) \). The effect of \( V_1 \) is a shift of the Fermi velocity, so we will define \( \tilde{v}_f = v_f + V_1 \). For the \( \delta \)-potentials, the effective Fermion-Fermion coupling constant \( g \equiv \sinh^2(\phi_k) = \frac{1}{2} [1/\sqrt{1 - v_{f2}^2/\tilde{v}_f^2} - 1] \). To calculate the transition rate \( W(q_1, q_2; \omega_1, \omega_2) \), we need to calculate an 8-point correlation function if we include the core hole effect:
Using a unitary transform \[17\], the 8-point correlation function decouples to two 4-point correlation functions, which can be calculated easily. The transition rate is calculated as:

\[
W^{\alpha\beta}(q_1, q_2; \omega_1, \omega_2) = \text{const} \int dy e^{iVTL(q_1-q_2)y} \int du \int dv e^{-i(\omega_2u+\omega_1v)} f(u, v, y)
\]

\[
\times \left[\frac{[\sqrt{g+1} - \delta(\sqrt{g+1} - \sqrt{g})]^2}{[4x^2 - (u-v+2i\epsilon)^2]} \right]^{g_{\alpha\beta}^+} \left[\frac{[\sqrt{g} - \delta(\sqrt{g+1} - \sqrt{g})]^2}{[4x^2 - (u-v-2i\epsilon)^2]} \right]^{g_{\alpha\beta}^-}
\]

With

\[
f(u, v, y) = \int_\frac{|u+v|}{2}^\infty dx \frac{[4x^2 - (u-v+2y-2i\epsilon)^2]^{g_{\alpha\beta}^+}}{[4x^2 - (u-v-2y-2i\epsilon)^2]} \\
\times \left[\frac{[\sqrt{g} - \delta(\sqrt{g+1} - \sqrt{g})]^2}{[4x^2 - (u+v+2i\epsilon)^2]} \right]^{g_{\alpha\beta}^-}
\]

Here \( \epsilon \) is the inverse of the ultraviolet momentum cutoff, and we have renormalized the photon frequencies by \( \omega_i \rightarrow \omega_i - (\epsilon_f - \epsilon_d) \). The \( g \)'s are defined as:

- \( g_+ = g_{++} = g_- = [\sqrt{g+1} - \delta(\sqrt{g+1} - \sqrt{g})]^2 \)
- \( g_- = g_{++} = g_+ = [\sqrt{g} - \delta(\sqrt{g+1} - \sqrt{g})]^2 \)
- \( g_{\alpha\beta}^\pm = \sqrt{g_+g_-} \) \( (\alpha \neq \beta) \). A useful combination is \( \eta = g_+ + g_- \). Here \( \delta = V_3/v_{TL} \) is the phase shift of noninteracting electrons scattered by \( V_3 \). \[17\].

Our exact results Eq.[6-7] are similar to that in Ref. \[18\], where Nozières and Abrahams discussed the threshold behavior of RS from noninteracting electron systems. But they did not discuss the interference effects. If we neglect the space-dependence and the electron-electron interaction effects, we will get the same results as in Ref. \[18\]. Note however the exponents we calculated here are different because of the coupling of the core hole to both branches (\( k \) and \( -k \)) of the electrons.

Now let us calculate the transition rate \( W(Q, \omega_1, \omega_2) \) with \( Q = q_1 - q_2 \). We drop the superscript \( \alpha = \beta = + \) used in Eq.[3] and write it later. We can obtain the result for \( \alpha = \beta = - \) by changing \( Q \rightarrow -Q \) in \( W(Q, \omega_1, \omega_2) \). The calculation for \( \alpha \neq \beta \) is similar and we will only discuss the results. It’s easy to see there is a divergence in Eq.[7], and this divergence can be cured by introducing the core hole lifetime \( \tau \). In addition to removing the unphysical divergence, we introduce the core hole lifetime \( \tau \), as in Ref. \[18\] in order to be able to separate ”hot luminescence” (HL) and resonance Raman scattering (RRS) in the
total secondary emission given by Eq. [5]. Following Ref. [18], \( f(u, v, y) \) can be split into two parts: 
\[
f(u, v, y) = \tau + \frac{1}{2} |u + v| \Psi \left( \frac{u - v - 2i \varepsilon}{|u + v|}, \frac{\varepsilon}{|u + v|} \right)
\]

\[
\Psi(z, y) = \int_1^\infty dx \left\{ \frac{x^2 - (z + y)^2}{x^2 - 1} \right\}^{g_+} \left\{ \frac{x^2 - (z - y)^2}{x^2 - 1} \right\}^{g_-} - 1 \right\} - 1 \quad (8)
\]

After substituting Eq. [7], the first term in Eq. [6] which is proportional to \( \tau \) will give the hot luminescence (HL) contribution, and the second term will give the Raman Scattering (RS) contribution. Define \( \omega_\pm = \omega_1 - \omega_2 \pm Q \nu_{TL} \), for \( \eta > 1 \), then the HL contribution to the transition rate is [19]:
\[
W_F(Q, \omega_1, \omega_2) = \text{const} \Theta(\omega_1) \Theta(\omega_2) \eta^{-1} e^{-\varepsilon(\omega_1 - \omega_2)} \int_0^1 ds s^{g_- - 1} (1 - s)^{g_+ - 1} (2 \omega_2 s + \omega_-)^{g_- - 1} (2 \nu_1 - 2 \omega_2 s - \omega_-)^{g_+ - 1} \Theta(2 \omega_2 s + \omega_-) \Theta(2 \omega_1 - 2 \omega_2 s - \omega_-) \quad (9)
\]

The numerical calculation of \( W_F \) is shown in Fig. (1). The calculation shows that there is singular behavior around \( \omega_1 - \omega_2 = |Q \nu_{TL}| \). Since \( \omega_2 < 0 \), so from \( \Theta(2 \omega_2 s + \omega_-) \) and \( \Theta(2 \nu_1 - 2 \omega_2 s - \omega_-) \) we have \( W_F(Q, \omega_1, \omega_2) \propto \Theta(\omega_1 - \omega_2 - |Q \nu_{TL}|) \). The other two peaks are at \( \omega_1 + \omega_2 = \pm Q \nu_{TL} \). The asymptotic behavior of \( W_F \) can be given at these singular peaks using the following integral
\[
\int_0^c x^a(c - x)^b(x + a)^w = c^{1+a+v} a^w \frac{\Gamma(1 + u) \Gamma(1 + v)}{\Gamma(2 + u + v)} 2 F_1(-w, 1 + u, 2 + u + v, -c/a) \quad (10)
\]

Where \( 2 F_1(-w, 1 + u, 2 + u + v, -c/a) \) is the Hypergeometric function. Without losing generality, we assume here \( Q \nu_{TL} \geq 0 \). Define \( c = (\omega_1 - \omega_2 - Q \nu_{TL})/(2 \omega_2) \) and \( \varepsilon = (\omega_1 + \omega_2 + Q \nu_{TL})/(2 \omega_2) \), then \( \varepsilon = c + (Q \nu_{TL}/\omega_2 - 1) \). Now we discuss various limits.

(i) at \( c \gtrsim 0 \), and \( 0 < \varepsilon < Q \nu_{TL}/(\omega_2) \) [as in Fig. (1)]
\[
W_F \sim (\omega_2)^{-\eta-3} \int_0^c ds s^{g_- - 1} (c - s)^{g_- - 1} (s + \varepsilon)^{g_+ - 1} 
\sim c^{2g_- - 1} \varepsilon^{g_+ - 1} 2 F_1(1 - g_+, g_-, 2g_-, -c/\varepsilon)
\]

If \( c \ll \varepsilon \), then \( W_F \sim c^{2g_- - 1} \); If \( c \gg \varepsilon \), then \( W_F \sim C_1 c^{2g_- - 2g_+} \varepsilon^{2(g_+ - 1)} + C_2 c^{3g_- - 1} \varepsilon^{g_+ - 1 - g_-} \), where \( C_1 \) is constant. If \( c \sim \varepsilon \), then \( W_F \sim c^{g_- + \eta - 2} \).

(ii) at \( c \sim 0 \), and \( \varepsilon < 0 \)
\[ W_F \sim (-\omega)^{-\eta-3} \int_{-\varepsilon}^{c} ds s^{g-1}(c-s)^{g-1}(s+\varepsilon)^{g+1} \]
\[ \sim \Theta(c-|\varepsilon|)(c-|\varepsilon|)^{g-1}|\varepsilon|^2 F_1(1-g_-, g_+, 2\eta, 1-c/|\varepsilon|) \] (11)

If \( c \gg |\varepsilon| \), then \( W_F \sim C_1 c^g |\varepsilon|^{2g-1} + C_2 c^{g-2g+1} |\varepsilon|^{g-1} \). If \( c \sim |\varepsilon| \), then \( W_F \sim (c-|\varepsilon|)^{g-1} |\varepsilon|^{g-1} \).

(iii) at \( \varepsilon \sim Q v_{TL}/(-\omega_2) \) then \( c \sim 1 \). Define \( \Delta = c - 1 \), then

\[ W_F \sim (-\omega)^{-\eta-3} \int_{0}^{1} ds s^{g-1}(s+\Delta)^{g-1}\Theta(s+\Delta) \]
\[ \sim \begin{cases} 
C_1 \Delta^{2g-2} + C_2 \Delta^{g-1}, & \text{if } \Delta \geq 0; \\
C_1 |\Delta|^{2g-2} + C_2 |\Delta|^{g-1} \Delta^{g-1}, & \text{if } \Delta < 0 
\end{cases} \] (12)

(iv) In the above cases, we have assumed \( \omega_2 \neq 0 \). If \( |\omega_2| \ll \omega_- \) and \( |\omega_1 + \omega_2 \pm Q v_{TL}| \gg |\omega_2| \), then

\[ W_F \sim (-\omega)^{-\eta-1}(\omega_-)^{g-1}(\omega_1 + \omega_2 + Q v_{TL})^{g+1} \] (13)

For \( \eta < 1 \), the HL contribution will diverge. We believe this divergence comes from the interference effect which we included. The calculation of the RS part contribution for \( \eta < 1 \) is an extremely tedious task. However since in this case the HL part will diverge and the integral of the RS converges well, we will not discuss the calculation in this region. For \( \eta > 1 \), the integration in Eq. (8) can be estimated around the lower limit of the integration. The RS part can be calculated:

\[ W_R(Q, \omega_1, \omega_2) = \text{const} (\omega_1 + \omega_2)^{\eta-1} \Theta(\omega_1 - \omega_2)[\delta(\omega_-) + O(\varepsilon)\tilde{f}_-(\omega_-)] + \cdots \] (14)

Here \( \tilde{f}_-(\omega_-) \) and \( \tilde{f}_+(\omega_+) \) are smooth functions of \( \omega_- \) and \( \omega_+ \) respectively.

We can see that there is a peak around \( \omega_1 - \omega_2 = |Q v_{TL}| \). The prefactor \( (\omega_1 + \omega_2)^{\eta-1} \) clearly shows that this peak has a different origin from the peak of the structure factor.

In the first order optical process such as photoemission and absorption, the competition between electron-electron interaction and electron-hole coupling in the TL is manifest in the FES exponent; the condition for the critical point of positive and negative exponents is \( \eta = 1 \)
In secondary optical processes, because of the interference of light from recombination of holes and electrons at different sites, the spectrum is more complicated, the competition of electron-electron interaction and electron-hole coupling will be manifest in the RS part as well as HL part. But the critical point is the same (\(\eta = 1\)).

Now we turn to discuss \(W^{\alpha\beta}\) with \(\alpha \neq \beta\), which is related to the processes which describes the situation where photon excited electron and the recombination electron are in different branches. For \(\eta > 1\), the RS contribution is the same as Eq.[14]. The HL contribution is proportional to \(\tau^{1-(\sqrt{g_--\sqrt{g_+}})^2} << \tau\). So for \(\sqrt{g_--g_+} > 1/2\), the HL contribution can be neglected. If we assume the core hole lifetime \(\tau\) is larger than the characteristic time scale, which will cure the divergence when \(\eta < 1\), the HL contribution for \(\sqrt{g_--g_+} < 1/2\) is also negligible compared to the \(\alpha = \beta\) case.

It is tempting to discuss the threshold behavior of RS from a higher-dimension (2D or 3D) electron system as Schotte and Schotte did for FES in emission (absorption) [17]. However, the possibility of extending our result to 3D depends essentially on the scattering phase shift being pure s-wave, which is not true in our case, since we do include interference between different core hole states. Technically this prevents carrying through a partial wave expansion which is needed for the 1D \(\rightarrow\) 3D analogy to be valid from TL liquid absorption/emission to threshold optical behavior of the 3D Fermi gas. But we can still use our result to discuss the interference effects and interaction effects in Fermi liquid in high dimensions. Following the recent arguments in renormalization group theory of Fermi liquid [20], this is especially true in the threshold region of the secondary optical processes. When neglecting the interference effects, similar to the absorption (emission) case, the threshold behavior of RS in a high dimension (2D or 3D) non-interacting Fermion gas is similar to that of the TL liquid [18]. From our above results, one effect of the interference which seems to be true in a high dimensional Fermion gas is that \(W_F\) can not be factorized into absorption and emission any more. Similar effects have been found in RS from polaritons [21]. In Fig.[1], there are features around \(\omega_1 - \omega_2 = |Qv_{TL}|\) which suggest \(W_F\) looks more like Raman Scattering than luminescence.
A more striking effect in the TL liquid is that the transition rate is very sensitive to the coupling strength of TL bosons to the core hole, and the value of $g$. We show in Fig.[2] the range of $V_2$ and $V_3$ that gives the divergent and convergent HL $W_F$. We can see from Fig.[2] that large $g$ (or $V_2$) and small $V_3$ is needed to have convergent $W_F$. A interesting result is that the $W_F$ diverges for a 1D Fermi Liquid ($g = 0$). So the HL part of contribution will dominate the secondary optical processes. The coupling of electron-hole ($V_3$) will increase this effect. When the electron-electron interaction is strong enough, the HL part $W_F$ will converge and the RS $W_R$ will dominate the total secondary emission. Since the HL part and RS part have different peak (singular) structure, the theory does show different secondary optical properties between TL and FL.

Strictly speaking, the HL part and RS part can not be called "Hot Luminescence" and "Raman Scattering" separately, since $W_F$ can not be factorized into absorption and emission in the usual sense, and there is singular structure around $\omega_1 - \omega_2 = |Qv_{TL}|$ in $W_F$. But we think of them as having originated from "Hot Luminescence" and "Raman Scattering" respectively although the interference effects make them non-separable.

Our main objective in this Letter has been to give an exact model of secondary optical process in the resonance region. For clarity and simplicity, we used a model of a localized hole coupling with the TL without backscattering. Also the electron-electron interaction is taken as short-range ($\delta$-potential). The effects of finite hole mass and unscreened long range electron-electron interaction need further study. The most important effect is that we omitted the backscattering of the electron by core hole in TL liquid. In the point potential, [22,23] each correlation function in Eq.(6) can be written as product of a correlation function due to forward and backscattering potential $G = G_fG_b$. So there are additional contributions to both $W_F$ and $W_R$. It was shown by a RG calculation in Ref. [4] that a TL liquid with repulsive interaction will be decoupled into two separate TL liquids by even a weak impurity if the backscattering is taken into consideration. There are recent studies of backscattering effects on the FES in photoemission and photoabsorption [22,24]. In the weak interaction limit, Kane et al. showed that there is a crossover energy between strong
and weak backscattering for $\epsilon^* \sim (V_b/\hbar v_f)^{1/(1-g)}$. At $\epsilon < \epsilon^*$, $G_b$ has power-law behavior, and the exponents due to backscattering are positive. We suggest this result is generally true even for strong electron-electron interactions \cite{22,24}. Since the divergence of the HL is due to the low energy excitations, the backscattering effect will make the HL converge better. If the core hole lifetime is smaller than $\epsilon^*$, the HL contribution $W_F$ will be changed mainly due to the increase of $g_{\pm}$. If the core hole inverse lifetime is larger than the backscattering energy interaction of electron and core hole, the backscattering effect on $W_F$ will be less dramatic, and we suggest here the main feature of $W_F$ will not be changed. The RS is a virtual process and independent of the core hole lifetime. From Eq.(14), we think the backscattering effect will only change the prefactors of the resonance peak at $\omega_1 - \omega_2 = |Qv_T L|$.

In conclusion, we have studied the RS from a TL liquid. If we treat the electron-photon matrix element as constant i.e. independent of wave-vector, the RS efficiency can not be simplified to be proportional to the structure factor $S(q_1 - q_2, \omega_1 - \omega_2)$. Generally it has some complicated structure which is sensitive both to the coupling between the electron and core hole and the electron-electron interaction strength of the TL liquid. If we take into account the $k$-dependence of $M_k$, in the resonance region, the RS efficiency does have some part of the contribution which is proportional to the structure factor $S(q_1 - q_2, \omega_1 - \omega_2)$. We found the total secondary emission can not be rigorously separated into Hot Luminescence and Raman Scattering.

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**FIGURE CAPTIONS**

**Fig.1** Total HL transition rate $W^+_F + W^-_F$ as a function of $\omega_1$ (incident photon frequency) and $\omega_2$ (scattered photon frequency) at $g_+ = 0.8$, $g_- = 0.4$. The unit for the $x$, $y$ axes is $|Q|v_{uf}$. The positions of the two peaks are at $\omega_1 + \omega_2 = \pm Qv_{TL}$. The singular line is at $\omega_1 - \omega_2 = |Q|v_{TL}$. Inset (a): The cross section at $\omega_1 - \omega_2 = |Q|v_{TL} + 0^+$; Inset (b): The cross section at $\omega_1 = \sqrt{2}|Q|v_{TL}$.

**Fig.2** The divergent region and convergent region of $W_F$. 