Mitigating Memorization in Sample Selection for Learning with Noisy Labels

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Abstract

Because deep learning is vulnerable to noisy labels, sample selection techniques, which train networks with only clean labeled data, have attracted a great attention. However, if the labels are dominantly corrupted by few classes, these noisy samples are called dominant-noisy-labeled samples, the network also learns dominant-noisy-labeled samples rapidly via content-aware optimization. In this study, we propose a compelling criteria to penalize dominant-noisy-labeled samples intensively through class-wise penalty labels. By averaging prediction confidences for the each observed label, we obtain suitable penalty labels that have high values if the labels are largely corrupted by some classes. Experiments were performed using benchmarks (CIFAR-10, CIFAR-100, Tiny-ImageNet) and real-world datasets (ANIMAL-10N, Clothing1M) to evaluate the proposed criteria in various scenarios with different noise rates. Using the proposed sample selection, the learning process of the network becomes significantly robust to noisy labels compared to existing methods in several noise types.

1. Introduction

For training deep neural networks (DNNs) using a supervised learning method, the labeled data are predominantly obtained from web queries (Liu et al., 2011), crowdsourcing (Welinder et al., 2010; Han et al., 2018a; Yan et al., 2014), e-commerce (Xiao et al., 2015) and social-network (Cha & Cho, 2012). However, several noisy labeled samples (in this study, we omit “labeled” for brevity) that are annotated with erroneous or noisy labels are present in the data. Zhang et al. (2016) confirmed that a DNN is significantly vulnerable to noisy samples because it can adequately learn (memorize) their patterns. To robustly train a network regardless of noisy samples, learning with noisy labels has been studied actively. These studies can be divided into two categories based on the technique employed: loss correction and sample selection.

In the loss correction category, the loss (or label) of a mini-batch is modified using a noise transition matrix (Goldberger & Ben-Reuven, 2016; Patrini et al., 2017; Han et al., 2018b; Hendrycks et al., 2018) or prediction confidences of samples (Reed et al., 2014; Chang et al., 2017; Tanaka et al., 2018; Ren et al., 2018; Wang et al., 2019b). However, if the number of classes or noise rate is large, the prediction confidences and noise transition matrix can be inaccurate (Han et al., 2018c; Song et al., 2019), leading to false correction and error accumulation in the training stage.

Recently, to overcome this limitation, many studies have been based on the sample selection method. In the sample selection category, only clean samples, whose labels are accurate, are selected from the mini-batch and thereafter used to train the network (Han et al., 2018c; Malach & Shalev-Shwartz, 2017; Wang et al., 2019a; Yu et al., 2019; Shen & Sanghavi, 2019; Song et al., 2019; Kong et al., 2019). They are based on “content-aware optimization” (also called “memorization effect”), wherein it initially learns simple patterns shared by multiple training samples across the data regardless of their labels (Arpit et al., 2017). Using this characteristic, sample selection based algorithms (Fig. 1a) assume that the DNN initially learns clean samples more rapidly than noisy ones. Therefore, the algorithms first select samples which have high scores (low losses) using observed erroneous or noisy labels are present in the data. Zhang et al. (2016) confirmed that a DNN is significantly vulnerable to noisy samples because it can adequately learn (memorize) their patterns. To robustly train a network regardless of noisy samples, learning with noisy labels has been studied actively. These studies can be divided into two categories based on the technique employed: loss correction and sample selection.

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However, the performances of the sample selection algorithms vary depending on types of noise. If the labels are dominantly corrupted by some classes, noisy samples generally have shared patterns and can also be learned rapidly by content-aware optimization (we call these noisy samples dominant noisy samples). Thus, dominant noisy samples have high score with observed labels, thus selection algorithms can not exclude these samples. In Fig. 1(a), many dominant noisy samples are selected for training DNN be-
cause dominant noisy samples have higher scores than the other noisy samples and some clean samples. They cause severe memorization of DNN, so it is important to penalize dominant noisy samples in sample selection.

In this study, we introduce Criteria\_ALL, which subtracts new Criteria with the proposed Penalty Label (Criteria\_PL) from the conventional Criteria\_OL with the observed label for penalizing dominant noisy samples (Fig. 1b). In the proposed Criteria\_PL with penalty label, dominant noisy samples have higher scores than the non-dominant ones and clean samples. Therefore, Criteria\_ALL is effective for penalizing dominant noisy samples (Fig. 1b; red samples). To estimate the penalty label, we use the average-prediction confidence of samples that belong to identical observed label after normalizing the result (Section 2.2; Fig. 2). In summary, the main contributions of this study are the following:

- We observe that dominant noisy samples cause memorization (reduce generalization) of DNN through empirical evidence.

- We propose a new Criteria\_PL that have high criteria scores for dominant noisy samples, and can penalize dominant noisy samples by subtracting Criteria\_PL from the conventional Criteria\_OL (Criteria\_ALL).

2. Main methodology: penalizing dominant noisy samples in sample selection

In this section, we describe our sample selection method to achieve robust learning in detail.

2.1. Sample selection with conventional Criteria\_OL

In the \( K \)-class classification problem, let \( D = \{ (x_i, \hat{y}_i) \}_{i=1}^{n} \) be training dataset, where \( x_i \) is the \( i \)th sample and \( \hat{y}_i \) is the \( i \)th observed label that includes corrupted label and is in the label space \( \{ e^k : k \in \{ 1, 2, \ldots, K \} \} \), i.e., \( e^k \)-th one-hot vector such as \( \{ 0, \ldots, 0, 1, 0, \ldots, 0 \} \). The objective is to train the DNN parameter \( \theta \) through the maximum likelihood estimation

\[
\arg\max_{\theta} \sum_{i=1}^{n} P (\hat{y}_i | x_i; \theta)
\]

Herein, the prediction confidence in \( j \)th class, which indicates the probability of being assigned to \( j \)th class (probability for the label \( e^j \)), is obtained by applying a softmax function to the DNN \( f \):

\[
P (y = e^j | x_i; \theta) = \frac{\exp \{ f (x_i)_j \}}{\sum_{l=1}^{K} \exp \{ f (x_i)_l \}}, \forall j = 1, \cdots, K,
\]

where \( f (\cdot)_j \) is \( j \)th component of \( f (\cdot) \). The loss function is generally defined by the cross-entropy scheme with the observed label \( \hat{y}_i \) and prediction \( P (y = e^j | x_i; \theta) \):

\[
\mathcal{L} (x_i, \hat{y}_i; \theta) = - \log P (y = \hat{y}_i | x_i; \theta).
\]

DNN is trained through a stochastic optimization method, which updates the parameter \( \theta \) using the mini-batch proce-
where $\theta^t$ is a parameter in the $t$th iteration, $\eta$ is the learning rate, and $B$ is the mini-batch fetched from $D$.}

Recently, several algorithms based on sample selection method (Han et al., 2018c; Jiang et al., 2018; Wang et al., 2019a; Yu et al., 2019) have been developed to robustly train the DNN for noisy labels. These methods assume that clean samples are trained more easily compared with the noisy samples, because DNN is trained by content-aware optimization wherein it initially learns simple patterns shared among multiple samples (Arpit et al., 2017). Therefore, in a mini-batch manner, sample selection methods select top $R\%$ of sorted samples in descending order relative to probability for the observed label (Fig. 1a). Then the network parameter is updated using only selected samples as

$$\theta^{t+1} = \theta^t - \eta \nabla \left( \frac{1}{|S|} \sum_{(x_i, \tilde{y}_i) \in S} L (x_i, \tilde{y}_i; \theta^t) \right),$$

where

$$S = \text{SELECT}^{\text{TOP } R\%}_{(x_i, \tilde{y}_i) \in B} K \sum_{j=1}^{K} \tilde{y}_i (j) \ P (y = e^j \mid x_i; \theta^t),$$

where TOP $R\%$ means top $R\%$ of sorted samples in descending order, $\tilde{y}_i (\cdot)$ is an element of one-hot vector $\tilde{y}_i$, $R\% = 100 - \varepsilon$, and $\varepsilon$ is the percentage of noise label. It is based on selection criteria, Criteria_{OL}, where "sample should have high prediction confidence for observed label" as follows:

$$\text{Criteria}_{OL} (x_i, \tilde{y}_i; \theta^t) = \sum_{j=1}^{K} \tilde{y}_i (j) \ P (y = e^j \mid x_i; \theta^t).$$

However, if the labels are largely corrupted by some classes, these samples have high probability of having several shared patterns among themselves, like clean labels data. Therefore, dominant noisy samples can also be included in selected samples, causing memorization (reducing generalization) of DNN (empirical evidence is provided in Section 3.1).

2.2. Criteria\_ALL: combination of proposed Criteria\_PL and Criteria\_OL

To penalize dominant noisy samples, we propose the criteria, Criteria\_PL, which exploits proposed penalty label. In Criteria\_PL, dominant noisy samples have higher values than the non-dominants. This is because the penalty label is set to have high values in the classes where dominant noisy samples originate; moreover, (dominant) noisy samples generally have high prediction confidences in their true class. Therefore, to penalize dominant noisy samples, we introduce a selection criteria (Criteria\_ALL) by subtracting Criteria\_PL "sample should have high prediction confidence for proposed penalty label $\tilde{y}$" from Criteria\_OL as follows:

$$\text{Criteria\_ALL} = \text{Criteria\_OL} - \lambda \text{Criteria\_PL},$$

where

$$\text{Criteria\_OL} (x_i, \tilde{y}_i; \theta^t) = \sum_{j=1}^{K} \tilde{y}_i (j) \ P (y = e^j \mid x_i; \theta^t),$$

$$\text{Criteria\_PL} (x_i, \tilde{y}_i; \theta^t) = \sum_{j=1}^{K} \tilde{y}_i (j) \ P (y = e^j \mid x_i; \theta^t),$$

where $\lambda$ is hyperparameter (user design parameter) and $\tilde{y}_i (\cdot)$ is an element of $i$th penalty label. The effectiveness of Criteria\_PL is described in detail with empirical evidence in Section 3.1.

To penalize dominant noisy samples through the proposed Criteria\_ALL, it is necessary to estimate the penalty label properly. The process to obtain class-wise penalty label $\tilde{y}$ is shown in Fig. 2. First, we average the prediction confidences among samples that have same observed label $C^k = \{(x_i, \tilde{y}_i) : \tilde{y}_i = e^k\}$ to measure the extent of dominance of the noisy samples. This method is reasonable because the percentage of noisy samples that are dominant is high; the dominant noisy samples have high prediction confidence in their true class (this will be shown in Section 3.1). Finally, to ensure the same scale with observed labels, the compensated average-prediction confidence is normalized except the class that the observed label $\tilde{y} = e^k$ indicates (labeled class $k$). This is because samples in labeled class $k$ are not needed to be penalized. The penalty label $\tilde{y}_i \in \{ m^k : k \in \{1, 2, \cdots, K\}\}$ is de-
Algorithm 1 Sample selection with Criteria\_ALL.

1: for \( t = 1 : \text{num}\_epochs \) do
2: \hspace{1em} for \( l = 1 : \text{num}\_iterations \) do
3: \hspace{2em} Predict mini-batch \( B \);
4: \hspace{2em} Stack prediction confidences \( P \);
5: \hspace{2em} Select samples \( S_{\text{prop}} \) from mini-batch \( B \) within top \( R\% \) using Criteria\_ALL;
6: \hspace{2em} Update network using selected samples \( S_{\text{prop}} \);
7: \hspace{1em} end for
8: \hspace{1em} Update penalty label \( \tilde{y}^t \);
9: end for

Defined by

\[
m^k (j) = \begin{cases} \frac{1}{A} \bar{P}_{C^k} (y = e^j | \theta^t), & \text{if } j \neq k, \\ 0, & \text{if } j = k, \end{cases}
\]

where

\[
A = \sum_{l \neq k} \bar{P}_{C^k} (y = e^l | \theta^t),
\]

\[
\bar{P}_{C^k} (y = e^j | \theta^t) = \sum_{(x_i, \bar{y}_i) \in C^k} P (y = e^j | x_i; \theta^t),
\]

where \( C^k \) is \( \{(x_i, \bar{y}_i) : \bar{y}_i = e^k\} \).

Finally, the network is trained with samples selected using Criteria\_ALL by

\[
S_{\text{prop}} = \text{SELECT}^{\text{TOP} R\%}(x_i, \bar{y}_i, \tilde{y}_i; \theta^t).\]

Algorithm 1 describes the overall procedure of sample selection with Criteria\_ALL. The penalty label is updated at the end of each epoch (in step 9 of Algorithm 1).

3. Deep understanding for Criteria\_ALL

In this section, we mainly describe the effectiveness of Criteria\_ALL based on empirical evidence on CIFAR-10 dataset (Krizhevsky et al., 2009).

3.1. Why Criteria\_OL and Criteria\_PL are necessary?

We explain the validity of Criteria\_ALL with respect to the following two points.

1. Which noisy samples should be penalized in sample selection with Criteria\_OL?
2. How can Criteria\_PL penalize noisy samples using the proposed penalty label?

First, when the labels are significantly corrupted by certain classes (pair > mixed > symmetry in Fig. 4), dominant noisy samples generally have shared patterns with themselves, and thus can also be learned rapidly by content-aware optimization. In pair flipping, because noisy samples are corrupted by a single class (all noisy samples are dominant), several noisy samples are included in the selected samples (Fig. 3a top panel; red circles on the right-hand side of the black dashed line). In mixed flipping, noisy samples originate in several classes and can be dominant or non-dominant depending on their proportion. Because dominant noisy samples have considerable shared patterns compared to the non-dominants, the dominants are trained more rapidly than the non-dominants by content-aware opti-
mization; consequently, some noisy samples among dominant noisy samples are included in selected samples (Fig. 3b top panel; red solid circles on the right-hand side of the black dashed line). Therefore, dominant noisy samples cause memorization (reduce generalization) of DNN, and it is important to penalize them in sample selection.

Thereafter, we focus on how Criteria\textsubscript{PL} can penalize dominant noisy samples using penalty labels. For general noisy label distributions (e.g., mixed flipping case), because dominant noisy samples cause significant memorization than non-dominant ones, we propose a novel class-wise penalty label that reflects extent of dominance for penalizing noisy samples differently. When the penalty label and prediction confidence of each sample are multiplied by inner product, then the dominant noisy samples will have higher values than that of the non-dominants. This is because (dominant) noisy samples generally have high prediction confidence in their true class, and the penalty label is set to have relatively higher value in the class, which dominant noisy samples originate, than the other classes. Therefore, we can distinguish dominant noisy sample using the proposed criteria, Criteria\textsubscript{PL}, defined by inner product of penalty label and prediction confidence of each sample. In Fig. 3b, because penalty label is set to have a relatively high value (e.g., sixth value in penalty label: 0.60) in class 6 where dominant noisy samples originate, and prediction confidences of dominant noisy samples also have the highest value (e.g., sixth value in last prediction confidence: 0.46) in class 6 (its true class), most dominant noisy samples have higher criterion scores than clean and non-dominant samples in Criteria\textsubscript{PL} (middle panel). Finally, we subtract the Criteria\textsubscript{PL} from Criteria\textsubscript{OL} (defined as Criteria\textsubscript{ALL}) for penalizing dominant noisy samples largely. Using Criteria\textsubscript{ALL}, penalized dominant and non-dominant noisy samples have smaller criteria scores compared to most clean samples and have similar criteria scores with each other (Fig. 3b bottom panel). Lastly, in symmetry flipping case, Criteria\textsubscript{PL} does not affect the selection of clean samples because of no dominant noisy samples (proofed in Appendix C).

4. Experiments

We evaluate the effect of the proposed Criteria\textsubscript{ALL} with regard to two perspectives. First, we evaluate the performance using three benchmark datasets by corrupting clean labels using three different type of noisy label with various noise rates. Second, we evaluate the performance using two real-world datasets (Appendix B.5). Three independent trials were performed and average and standard error are used to represent the performance of the proposed, and conventional algorithms. Dataset are described in (Appendix B.2).

4.1. Algorithms

We compared Criteria\textsubscript{ALL} with several algorithms in the loss correction and sample selection categories. Default (baseline) was network that was trained without processing for noisy labels. Active Bias (Chang et al., 2017) (loss correction) trained DNN by assigning large weights to uncertain samples that exhibit high variance. MentorNet (Jiang et al., 2018) (sample selection) used Criteria\textsubscript{OL} to select clean samples and trains DNN using only selected samples. Co-teaching (Han et al., 2018c) (sample selection) used two networks and exchanged information of sample selection to enhance the performance. Co-teaching+ (sample selection) applied Co-teaching to the samples with different prediction confidence. SL (Wang et al., 2019b) (loss correction) combined reverse cross entropy term with cross entropy loss to enhance cross entropy’s learning on hard classes and tolerance to noisy labels. SELFIE (Song et al., 2019) (hybrid) corrected loss of samples with high precision while conducting the conventional sample selection. MentorNet–Prop is a basic sample selection method that uses Criteria\textsubscript{ALL} rather than Criteria\textsubscript{OL}. Because Criteria\textsubscript{ALL} can be applied flexibly to other algorithms, we also conducted experiments using SL–Prop and SELFIE–Prop by combining Criteria\textsubscript{ALL} with SL and SELFIE (Appendix A.2).

4.2. Experimental results on benchmark datasets

To examine the effect of the proposed Criteria\textsubscript{ALL} in the sample selection category, we first compared the test errors of Default (without Criteria), MentorNet (with Criteria\textsubscript{OL}), and MentorNet–Prop (with Criteria\textsubscript{ALL}). In pair flipping, MentorNet–Prop presented the lowest test errors at all noise rates and for all datasets (Fig. 5), particularly when the noise rate was high. For mixed flipping, we conducted experiments with noise rate 40% (Table 2). Similar to pair flipping, MentorNet–Prop recorded the lowest test error in three datasets.

Thereafter, we compared MentorNet–Prop, SL–Prop, and SELFIE–Prop with conventional algorithms in the loss correction and sample selection categories with various noise rates. In Table 1 pair flipping, MentorNet–Prop displayed the overall lower test error compared to other conventional algorithms. Furthermore, SL–Prop and SELFIE–Prop...
achieved lowest test error. When the proposed Criteria_ALL is applied to SL, the test errors obtained were 0.9–6.4% lower than those of SL. This means selecting sample using Criteria_ALL is also effective in loss correction category. SELFIE–Prop achieved test errors 0.1–6.0% lower than those of SELFIE. In conclusion, when labels are corrupted as pair flipping, proposed Criteria_ALL is powerful.

The performances of the algorithms at various noise rates are also compared for the symmetry case in Table 1. MentorNet–Prop achieved lower or similar test errors as the conventional algorithms. Particularly, when the proposed Criteria_ALL is applied to SL, the test errors were 0.9–6.4% lower than in SL. Experimental results show that Criteria_ALL is also effective in the loss correction category in symmetry flipping case. For mixed flipping case, MentorNet–Prop achieved lower or similar test errors to the conventional algorithms. Particularly, SELFIE–Prop had the lowest test errors.

5. Conclusion

To address learning with noisy labels, we introduced a novel criteria, Criteria_ALL that penalizing dominant noisy samples effectively by subtracting the proposed Criteria_PL...
from the conventional Criteria\textsubscript{OL}. Because the penalty label reflects the extent of dominance of noisy samples, dominant noisy samples have higher criteria scores compared to non-dominant noisy samples and clean samples in Criteria\textsubscript{PL}. Therefore, Criteria\textsubscript{ALL} is effective for penalizing dominant noisy samples by subtracting Criteria\textsubscript{PL}. From our experiments, sample selection using the Criteria\textsubscript{ALL} was effective for several benchmark and real-world datasets.

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A. Additional details for improving class-wise penalty label and compatibility of Criteria_ALL

With Criteria_ALL, dominant noisy samples are effectively penalized in sample selection. In this section, we describe additional consideration when Criteria_ALL is applied to the noisy label fields.

A.1. Improvement of class-wise penalty label

In Sections A.1.1, the effectiveness of the update method is described in detail.

A.1.1. Updating penalty label using temporal ensembling

Algorithm 2 Update penalty label using prediction confidences at the end of an epoch

1: for $t = 1 : num\_epochs$ do
2:   for $l = 1 : num\_iterations$ do
3:     Predict mini-batch $B$;
4:     Select samples $S_{Prop}$ from mini-batch $B$ within top $R\%$ using Criteria_ALL;
5:     Update network using selected samples $S_{Prop}$;
6:   end for
7:  Re-predict all samples $P$;
8:  Update penalty label $\hat{y}^t$ through prediction confidences;
9: end for

Algorithm 3 Update penalty label using prediction confidences for every iteration

1: for $t = 1 : num\_epochs$ do
2:   for $l = 1 : num\_iterations$ do
3:     Predict mini-batch $B$;
4:     Stack prediction confidences $P$;
5:     Select samples $S_{Prop}$ from mini-batch $B$ within top $R\%$ using Criteria_ALL;
6:     Update network using selected samples $S_{Prop}$;
7:   end for
8:  Update penalty label $\hat{y}^t$ through stacked prediction confidences;
9: end for

When a DNN is robustly trained (considering noisy label), accurate average-prediction confidence and penalty label estimation are likely to occur. Therefore, penalty label should be updated for each epoch. Two types of methods are available for estimating the penalty label during the training stage.

In single model method (Algorithm 2), new prediction confidences for all the training data are estimated at the end of each epoch. This results in a new penalty label as well, which is used in the next epoch. However, as the parameters of the DNN are updated iteratively by a stochastic optimization method, it can approach bad local minima with respect to generalization (Huang et al., 2017a). When the DNN falls into the local minima at the end of an epoch in the initial training, the penalty label is inaccurately estimated. This can adversely affect the learning of the DNN in the next epoch.

In our ensemble method (Algorithm 3; we re-describe Algorithm 1 as Algorithm 3 for clear comparison), the prediction confidences for the mini-batch are stored per iteration, and the average at the end of the present epoch is used to obtain the penalty label. From (Huang et al., 2017a; Laine & Aila, 2016), because a DNN passes several local minima as training progresses, the prediction confidences are as diverse as those estimated using different networks. This results in a temporal ensemble effect. If we store the prediction confidences per iteration, diverse prediction confidences are obtained, and the penalty label can be estimated stably. Moreover, because the prediction confidences per iteration are already obtained during the forward pass of the training, only they need to be stored. In contrast, single model method requires additional computation to re-predict the training data at the end of each epoch. Therefore, our ensemble method is significantly stable and efficient compared to the single model. Detailed experimental results are presented in Section B.6.

A.2. Collaboration with recent methods

Criteria_ALL can be applied to various algorithms for learning with noisy labels. Among them, we apply Criteria_ALL to Symmetric cross entropy Learning (SL; a recent algorithm in loss correction category) and SElectively reFurBlish unCLear samples (SELFIE; a recent algorithm in hybrid category).

A.2.1. SL WITH Criteria_ALL

To improve Cross Entropy’s (CE) learning on hard classes and tolerant to noisy label, SL (Wang et al., 2019b) introduces $\mathcal{L}_{SL}$ which combines new term, namely Reverse Cross Entropy (RCE), with CE as follows

$$
\mathcal{L}_{SL} (x_i, \hat{y}_i; \theta) = \alpha \mathcal{L}_{CE} (x_i, \hat{y}_i; \theta) + \beta \mathcal{L}_{RCE} (x_i, \hat{y}_i; \theta),
$$

where

$$
\mathcal{L}_{CE} (x_i, \hat{y}_i; \theta) = - \sum_j \hat{y}_i (j) \log P (y = e^j | x_i; \theta),
$$

$$
\mathcal{L}_{RCE} (x_i, \hat{y}_i; \theta) = - \sum_j P (y = e^j | x_i; \theta) \log \hat{y}_i (j),
$$

$\alpha$, $\beta$ are the hyperparameters (user design parameters).
In SL, \( \mathcal{L}_{SL} \) is applied to entire training data. However, when the sample selection with Criteria_ALL is used, we can distinguish the clean and noisy samples. Therefore, to increase robustness of DNN for noisy labels, we apply \( \mathcal{L}_{SL} \) only to the selected clean samples. In Section 4.2, the effectiveness of SL with Criteria_ALL is demonstrated.

A.2.2. SELFIE with Criteria_ALL

Because hybrid methods are also based on sample selection, Criteria_ALL can be applied to the SELFIE (Song et al., 2019) scheme, which achieved the highest performance in its category. In this method, two types of samples (clean and corrected samples) are selected independently at different usage. Clean samples are selected using the conventional sample selection (Criteria OL) and corrected samples are selected using the predictive uncertainty that is low if stacked prediction confidences are consistent. Thereafter, DNN is trained using clean samples with the observed label and corrected samples with the estimated label (corrected label) that indicates class of highest prediction confidence. When clean and corrected samples overlap, SELFIE trains DNN using corrected labels (Fig. 6a) because clean samples, selected with Criteria OL, have relatively larger error than corrected samples. In addition, to achieve high performance, SELFIE retrains the network several times while preserving the corrected information.

In contrast, clean and noisy samples can be accurately distinguished using SELFIE with Criteria_ALL compared to using SELFIE with Criteria OL. Therefore, we can modify SELFIE to consider the clean samples before the corrected ones when Criteria_ALL is used (Fig. 6b), i.e., overlapping samples are trained with observed labels, not corrected labels. In Section 4.2, we demonstrate the effectiveness of SELFIE with Criteria_ALL.

B. Additional details for experiments

B.1. Evaluation metric

To evaluate the performance of the experiments, we used the following test error and precision definitions:

\[
\text{Test error} = 1 - \frac{\# \text{ of correct predictions in test data}}{\# \text{ of test data}},
\]

\[
\text{Precision} = \frac{\# \text{ of clean samples in selected samples}}{\# \text{ of selected samples in training data}}.
\]

B.2. Dataset

We used CIFAR-10 (Krizhevsky et al., 2009), CIFAR-100 (Krizhevsky et al., 2009), and Tiny-ImageNet (Tin), which are widely used benchmark datasets in the noisy label fields (Han et al., 2018c; Song et al., 2019; Yu et al., 2019; Kong et al., 2019; Arazo et al., 2019; Chen et al., 2019; Kim et al., 2019). CIFAR-10 and CIFAR-100 consist of 10 and 100 classes, respectively, of \( 32 \times 32 \) color images. Each class is composed of subsets with 80 million categorical images. The numbers of their training and test data are 50,000 and 10,000, respectively. Tiny-ImageNet, a subset of ImageNet, consists of 200 classes. Its numbers of training and test data are 100,000 and 10,000, respectively. The experiment was conducted by resizing the images from \( 64 \times 64 \) to \( 32 \times 32 \). These benchmark datasets consist only of clean samples. Therefore, we artificially corrupted their labels using three commonly used types of noise transition matrices: pair flipping (Fig. 4a), symmetry flipping (Fig. 4b), and mixed flipping (Fig. 4c) (Tanaka et al., 2018; Han et al., 2018c; Song et al., 2019; Yu et al., 2019; Kong et al., 2019).

From (Han et al., 2018c), pair flipping is a more general case compared to symmetry flipping in the real world. In addition, we conducted experiments based on various noise rates: \( \varepsilon = \{10\%, 20\%, 30\%, 40\%\} \) for pair flipping and \( \varepsilon = \{0\%, 20\%, 40\%, 60\%\} \) for symmetry flipping. Finally, we conducted experiments for mixed flipping with a noise rate 40% where the labels are corrupted by labels of a class as \( \varepsilon_1 = 30\% \) and the other classes as \( \varepsilon_2 = 10\% \) (Fig. 4c).

Thereafter, the ANIMAL-10N dataset (Song et al., 2019) and Clothing1M (Xiao et al., 2015), whose data are collected from the real world, were employed to examine the performance of Criteria_ALL. ANIMAL-10N dataset, whose labels were corrupted by human errors, consists of images from 10 animals of similar appearance. The numbers of its training and test data are 50,000 and 5,000, respectively, and its noise rate is 8%. Furthermore, an experiment was conducted with \( 64 \times 64 \) color images without data augmen-
ALL is effective for real-world datasets. If \( \varepsilon \) is unknown in advance, its value can be inferred from the validation set (Song et al., 2019; Liu & Tao, 2015; Yu et al., 2018). In SL and SL–Prop, respectively. Because the noise rate was set depending on the dataset (CIFAR-10: 0.08, CIFAR-100: 0.3, Tiny-ImageNet: 0.3, ANIMAL-10N: 0.01, Clothing1M: 0.08). In SELFIE, we set the uncertainty threshold to 0.05, history length to 15, and restart to 2 (totally three runs), as in (Song et al., 2019). In SELFIE–Prop, samples can be corrected precisely owing to the capability of Criteria\_ALL that distinguishes clean and noisy samples regardless of noise type. Therefore, we set the hyperparameter tightly (uncertainty threshold = 0.005 and history length = 25) for SELFIE–Prop. Moreover, we set \( \lambda \) to 1 by the experiments in Section B.7 and \( q \) is assumed to be uniform in the benchmark and real-world datasets.

B.4. Experimental result on symmetry flipping case

In the symmetry flipping case, because no dominant noisy samples were present, MentorNet–Prop should have exhibited similar performance as MentorNet, as indicated in Theorem 1 (Fig. 7).

B.5. Experimental result on a real-world dataset

We applied the algorithms on the ANIMAL-10N and Clothing1M to examine the effectiveness of Criteria\_ALL in real-world datasets. From (Song et al., 2019), the ANIMAL-10N dataset was influenced by human errors during the image labeling process, because the classes consist of five pairs of “confusing” animals. The noise rate (\( \varepsilon = 8\% \)) was estimated by cross-validation with grid search (Song et al., 2019). The experimental results indicate that the test error of MentorNet–Prop was 0.5% lower than that of SELFIE, which exhibited the lowest test error among the conventional algorithms (Table 3). Furthermore, SL–Prop and SELFIE–Prop achieved a test error 1.0% and 0.9% lower than that of SL and SELFIE, respectively. Because the noise rate was small (8%), the difference in performance was small. However, the experimental results indicate that Criteria\_ALL is still effective.

Experiments for Clothing1M were conducted using pre-trained ResNet-50 as (Xu et al., 2019). We fine-tuned the network for 10 epochs where the learning rate was 1.0 \( \times \) 10\(^{-6}\) for the first 5 epochs and 0.5 \( \times \) 10\(^{-6}\) in the next 5 epochs. Several methods in loss correction (Patrini et al., 2017; Xu et al., 2019; Miyato et al., 2018; Zhang & Sabuncu, 2018; Yao et al., 2019) were compared with the proposed method, and we used the results reported by (Xu et al., 2019). Experimental results indicate that our MentorNet–Prop had lower test errors than all conventional algorithms (Table 4). When the loss of MentorNet is changed to the loss of SL (SL–Prop), we can achieve 0.46% lower test error than MentorNet–Prop. Consequently, we can show that the Criteria\_ALL is effective for real-world datasets.

![Figure 7. Best test error (%) based on various noise rates with symmetry flipping; (a) CIFAR-10; (b) CIFAR-100; (c) Tiny-ImageNet.](image-url)
Table 3. Best test error (%) in ANIMAL-10N (8% noise)

| Algorithm                  | Default | Active Bias | Co-teaching | Co-teaching+ | MentorNet | SL | SELFIE | MentorNet-Prop | SL-Prop | SELFIE-Prop |
|----------------------------|---------|-------------|-------------|--------------|-----------|----|--------|----------------|---------|-------------|
| DenseNet (L=25, k=12)      | 17.9    | ±0.2        | 17.6        | ±0.17        | 17.5      | ±0.17 | 17.3   | ±0.02          | 17.0    | ±0.17       |
|                           | 17.6    | ±0.14       | 17.5        | ±0.20        | 18.0      | ±0.15 | 17.3   | ±0.05          | 16.3    | ±0.19       |
|                           | 17.5    | ±0.17       | 17.3        | ±0.17        | 16.5      | ±0.19 | 16.3   | ±0.04          | 16.1    | ±0.13       |

Table 4. Test error (%) in Clothing1M

| Algorithm                  | Default | VAY (Miyato et al., 2018) | F-correction (Patrini et al., 2017) | GCE (Zhang & Sabuncu, 2018) | LCCN (Yao et al., 2019) | DMI (Xu et al., 2019) | MentorNet | SL | MentorNet-Prop | SL-Prop |
|----------------------------|---------|---------------------------|-------------------------------------|-----------------------------|-------------------------|-----------------------|------------|----|----------------|---------|
| ResNet-50                  | 31.06   | 30.83                     | 29.17                               | 30.91                       | 28.33                   | 27.54                  | 30.39      | 29.40 | 27.44          |         |
|                           | 26.98   |                           |                                     |                             |                         |                       |            |      |                |         |

B.6. Effect of update method for penalty label

It is important to estimate a penalty label accurately when Criteria_ALL is used. To increase the accuracy of the penalty label, we propose the temporal ensemble method (Section A.1.1; Algorithm 3). With regard to the update method, Algorithm 3 exhibited a lower test error than that of Algorithm 2 for pair and symmetry flipping (Table 5). Because the predictions were stored per iteration, diverse predictions were obtained, and thus the penalty label can be estimated stably using the ensemble effect. Furthermore, the difference between these two update methods was apparent for pair flipping because penalty label is significantly effective for that noise type.

Table 5. Best test error (%) of MentorNet–Prop depending on update methods for penalty label and considering noise rate of 40%

| Dataset       | Method | Pair flipping | Symmetry flipping |
|---------------|--------|---------------|-------------------|
| CIFAR-10      | Algorithm 2 | 30.8          | 14.5              |
|               | Algorithm 3 | 12.2          | 14.4              |
| CIFAR-100     | Algorithm 2 | 42.9          | 39.4              |
|               | Algorithm 3 | 36.6          | 38.4              |
| Tiny-ImageNet | Algorithm 2 | 73.0          | 62.1              |
|               | Algorithm 3 | 62.5          | 61.0              |

B.7. Experiments on the hyperparameter

Figure 8. Experiments on hyperparameter at noise rate 40% in CIFAR-100.

The Criteria_ALL exhibits only one hyperparameter \( \lambda \) that adjusts the influence between Criteria_OL and Criteria_PL. To analyze the effect of \( \lambda \), experiments were conducted with several values of \( \lambda = \{0.0, 0.5, 1.0, 1.5, 2.0\} \), at a noise rate of 40% in CIFAR-100 for both pair and symmetry flipping (Fig. 8). In pair flipping, the test error decreased steeply when \( \lambda \) was lower than one owing to the high effectiveness of the penalty label. In symmetry flipping, the test error slightly increased with increasing \( \lambda \), owing to the incorrect penalty label. Therefore, we empirically set \( \lambda = 1 \) to consider the trade-off between pair and symmetry flipping.

B.8. Discussion

In this section, we discuss the effect of Criteria_OL, Criteria_PL, and Criteria_ALL on sample selection, and the difference between penalty label and noise transition matrix for pair and symmetry flipping cases.

B.8.1. Comparison of precisions of the criteria

The precisions of Criteria_OL, Criteria_PL, and Criteria_ALL were compared to examine the performance in selecting clean samples for each epoch (Fig. 9). Each criteria was applied to MentorNet from the 25th epoch (warm-up). In addition, the learning rate was reduced by 0.2 times at the 50th and 75th epochs. In the pair flipping case, owing to the shared patterns of dominant noisy samples, severe memorization occurred. This results in a precision of approximately 70% for Criteria_OL. In contrast, owing to the penalty applied to the dominant noisy samples, the precision of Criteria_PL was approximately 15% higher than that of Criteria_OL. Furthermore, Criteria_ALL, which uses the two complementary criteria simultaneously, exhibited the highest precision (approximately 92%), because Criteria_OL and Criteria_PL evaluate samples through different patterns caused by observed label and penalty label.

In symmetry flipping, because there were no dominant noisy samples, Criteria_OL exhibited a high precision (approxi-
Table 6. Best test error (%) of Default, loss correction with noise transition matrix (F–correction), and MentorNet–Prop with noise rate of 40% in Tiny-ImageNet

| Algorithm         | Default | F–correction | MentorNet–Prop |
|-------------------|---------|--------------|----------------|
| Pair flipping     | 69.9    | 72.8         | 62.5           |
| Symmetry flipping | 64.7    | 75.3         | 61.0           |

approximately 92%), whereas Criteria_PL did not affect the selection of clean samples (approximately 60% precision). Initially, Criteria_ALL exhibited lower precision than Criteria_OL. However, as the training stage progressed, the accuracy of estimation of the penalty labels increased steadily, and the precision was similar to that of Criteria_OL.

B.8.2. Comparison between Criteria_ALL and Noise Transition Matrix

The average-prediction confidence of each observed label (which is used to estimate the penalty label) can also be used to estimate the noise transition matrix. Therefore, the performance of the sample selection using Criteria_ALL (MentorNet–Prop) was compared with that of F–correction (Patrini et al., 2017), a representative method of loss correction with a noise transition matrix. In this case, the noise transition matrix was replaced by the average-prediction confidence used to estimate proposed penalty label in Criteria_PL. From (Han et al., 2018c; Song et al., 2019; Jiang et al., 2018), it is challenging to estimate the noise transition matrix when the number of classes is large. Consequently, the noise transition matrix, estimated by the average-prediction confidence of each observed label in Tiny-ImageNet (200 classes), was inaccurate. Furthermore, F–correction does not distinguish clean and noisy samples, unlike MentorNet–Prop. Therefore, in Table 6, F–correction achieved a test error even 2.9–10.6% higher than that of the Default. In contrast, because in Criteria_ALL, only relative average-prediction confidence is needed to significantly penalize the dominant noisy samples unlike noise transition matrix, MentorNet–Prop exhibited remarkable performance regardless of the number of classes (Tables 1, 6).
C. Special case: Symmetry flipping

If the label of a true class is corrupted by the labels of the other classes with the same noise rate (symmetry flipping), Criteria\_PL should not affect the selection of clean samples because there are no dominant noisy samples and penalty label has same value. This property is stated in the following Theorem.

**Theorem 1** Suppose that the noise type is symmetry flipping with ideal penalty label \( m^k = \frac{1}{K-1} \), the order of samples with Criteria\_ALL is identical to that with Criteria\_OL as

\[
\text{ORDER}^{\text{Descent}}_{(x_i, \hat{y}_i) \in B} \text{Criteria\_ALL} (x_i, \hat{y}_i, \tilde{y}_i; \theta) = \text{ORDER}^{\text{Descent}}_{(x_i, \hat{y}_i) \in B} \text{Criteria\_OL} (x_i, \hat{y}_i; \theta) .
\]

Proof.

Criteria\_ALL \( (x_i, \hat{y}_i, \tilde{y}_i; \theta) \) = \[ \sum_j \hat{y}_i (j) P (y = e^j | x_i; \theta) - \lambda \sum_j \hat{y}_i (j) P (y = e^j | x_i; \theta) \]

\[ = \sum_j e^k (j) P (y = e^j | x_i; \theta) - \lambda \sum_j m^k (j) P (y = e^j | x_i; \theta) \quad (\because \hat{y}_i = e^k, \tilde{y}_i = m^k) \]

\[ = \sum_{j=k} P (y = e^j | x_i; \theta) - \frac{\lambda}{K-1} \sum_{j \neq k} P (y = e^j | x_i; \theta) \]

\[ \quad (\because \text{In ideal case, } m^k (j) = \frac{1}{K-1} \text{ for } j \neq k) \]

\[ = \sum_{j=k} P (y = e^j | x_i; \theta) - \frac{\lambda}{K-1} \left( 1 - \sum_{j=k} P (y = e^j | x_i; \theta) \right) \]

\[ \quad (\because \sum_j P (y = e^j | x_i; \theta) = 1) \]

\[ = \left( 1 + \frac{\lambda}{K-1} \right) \sum_{j=k} P (y = e^j | x_i; \theta) - \frac{\lambda}{K-1} \]

\[ = \left( 1 + \frac{\lambda}{K-1} \right) \sum_j \hat{y}_i (j) P (y = e^j | x_i; \theta) - \frac{\lambda}{K-1} \]

\[ = \left( 1 + \frac{\lambda}{K-1} \right) \text{Criteria\_OL} (x_i, \hat{y}_i; \theta) - \frac{\lambda}{K-1} . \]

Because the scaling and bias do not alter the order, the order of samples with both the criteria is identical. \( \square \)