We discuss the main myths related to the vacuum energy and cosmological constant, such as: “unbearable lightness of space-time”; the dominating contribution of zero point energy of quantum fields to the vacuum energy; non-zero vacuum energy of the false vacuum; dependence of the vacuum energy on the overall shift of energy; the absolute value of energy only has significance for gravity; the vacuum energy depends on the vacuum content; cosmological constant changes after the phase transition; zero-point energy of the vacuum between the plates in Casimir effect must gravitate, that is why the zero-point energy in the vacuum outside the plates must also gravitate; etc. All these and some other conjectures appear to be wrong when one considers the thermodynamics of the ground state of the quantum many-body system, which mimics macroscopic thermodynamics of quantum vacuum. In particular, in spite of the ultraviolet divergence of the zero-point energy, the natural value of the vacuum energy is comparable with the observed dark energy. That is why the vacuum energy is the plausible candidate for the dark energy.

Keywords: cosmological constant, dark energy, zero point energy

1. Introduction: Old and new myths

Quantum mechanics killed the old myth that the vacuum is an empty space. Now everybody agrees that the vacuum is filled with zero-point fluctuations of relativistic quantum fields (see reviews [1,2,3]). However, this new point created new myths.

The main myth is referred to as the main cosmological constant problem or the “unbearable lightness of space-time” [4]. It is based on assumption that the natural value of the energy density of the quantum vacuum and thus of the cosmological constant is determined by the (microscopic) Planck energy scale:

$$\Lambda_{\text{natural}} \sim \frac{E^4_{\text{Planck}}}{\hbar^2 c^3},$$

(1)

This value is fantastically too big compared to the observational upper limit for the cosmological constant, which forces us to think that our Universe is dramatically unnatural [5].

The related myth is that the zero-point energy of bosonic quantum fields and the negative energy of Dirac sea of the fermionic quantum fields are the dominating
sources of vacuum energy. The summation over all the modes till the Planck energy scale gives the estimate in Eq. (1). The possible supersymmetry between bosons and fermion may reduce the cut-off, but the result is still unbearably huge.

But does the equation (1) really represent the natural value of the vacuum energy? In Sec. 2 we find that the natural value of the vacuum energy is determined by macroscopic physics, and it is by many orders of magnitude smaller than it follows from the naive summation of modes. And in Sec. 5 we discuss how the zero-point energy and/or the negative energy of Dirac vacuum are compensated by the microscopic (trans-Planckian) degrees of freedom without any fine-tuning.

The discussion is based on our knowledge of the many-body systems, where the ground state mimics the quantum vacuum in many respects, and even the similar problems related to the vacuum energy also arise. We know the microscopic (atomic=trans-Planckian) structure of the vacuum in the many-body system, and thus are able to see how these problems are solved there. We find that the vacuum energy is the macroscopic characteristic of the quantum vacuum, and its behavior is generic and does not depend on the details of the microscopic physics. This encourages us to extend the found properties of the vacuum energy to the quantum vacuum of our Universe whose microscopic structure still remains unknown, and unveil different myths related to the vacuum energy.

One of the myths is that the magnitude of the vacuum energy is only important when the gravity is present, otherwise it can be removed by the shift of the energy. This is not so, the consideration of the many-body system in Sec. 3 demonstrates that the properly determined vacuum energy density is well defined in any system (relativistic or non-relativistic, with gravity or without gravity) and it does not depend on the choice of zero (Sec. 4). The macroscopic properties of the quantum vacuum do not depend on whether the gravity emerges in the system or not. If so, the problems related to the vacuum energy can be considered in the more general context without only referring to the systems with gravity.

Of course, there exist the many-body systems where the dynamic metric field emerges as one of the collective modes of the quantum vacuum. Moreover, we have examples where gravity emerges together with the ingredients of the Standard Model – chiral fermions and gauge fields. These are the Fermi systems with the topologically stable Fermi points in the fermionic spectrum, where the relativistic quantum fields and gravity emerge due to the so-called Atiyah-Bott-Shapiro construction. However, this encouraging fact of the existence of the class of many-body systems which almost perfectly mimic the vacuum of the Standard Model is not important for our discussion of the more general properties of the quantum vacuum related to vacuum energy.

The next myth is about the energy of the false vacuum. The false vacuum is the state corresponding to the local minimum of energy. It has higher energy than the true vacuum which corresponds to absolute minimum. That is why, it is always assumed that if $\Lambda = 0$ in the true vacuum, it must be positive and big in the false vacuum. This is important for the phenomenon of inflation – the
exponential super-luminal expansion of the Universe. The related myth is the effect of the symmetry breaking phase transition, such as electroweak phase transition or chiral phase transition in strong interactions, on the vacuum energy. Both lead to extremely large values of vacuum energy relative to what is allowed. In Sec. 7 we discuss how the condensed matter systems treat these problems.

The other related myth is that the vacuum energy depends on the vacuum content. Indeed, at first glance this is correct: the vacuum is complicated, there are many contributions to vacuum energy from different quantum fields. There are many species of fermions each with its own mass. Thus the vacuum energy must depend on the fermion masses, on Higgs field, etc. (see e.g. [5]). We discuss this problem in Sec. 6.

As follows from the condensed matter experience, the short answer to the above problems is that the natural value of the vacuum energy is zero, and thus does not depend on the vacuum content, on the divergence of the zero-point energy, on whether the vacuum is false or true, on the phase transition in the vacuum, etc.

If the natural value of the vacuum energy is zero should we return to the old myth that the vacuum does not gravitate? We should not, since the non-zero cosmological constant has been observed producing again the new myth that this observation has catalyzed the crisis in physics [5]. It is easier to assume that the vacuum energy is zero than to explain the reduction by 120 orders of magnitude. We discuss this point in Sec. 6 and demonstrate how the deviations of the vacuum from the perfectness naturally induce the non-zero vacuum energy comparable with observations.

One of the perturbations of the vacuum which lead to the non-zero vacuum energy is the boundary conditions. This perturbation produces the Casimir effect. In Sec. 6.3 we discuss: why the zero-point energy between the plates in the Casimir effect must gravitate, while the zero-point energy in the vacuum outside the plates does not gravitate; and the related question posed in Ref. [5] – why the zero-point energy must gravitate in the environment of the atom (Lamb shift) and not in vacuum.

2. What is natural value of vacuum energy?

Let us start with the main cosmological constant problem, which is based on the myth that the natural value of the energy density of the quantum vacuum and thus of the cosmological constant is determined by the Planck energy scale. So, what is the natural value of the vacuum energy?

2.1. Natural microscopic value

To get some insight into this problem, let us consider whether this assumption true or not in quantum many-body systems, such as quantum liquid or solid. At first glance this is true. The microscopic (atomic) physics determines the characteristic scales for the quantities describing liquids and solids. The role of the Planck length scale is played by the inter-atomic distance $a$. Correspondingly the role of the Planck
energy scale is played either by the Debye temperature, or by melting temperature, or by the transition temperature to the superfluid state. As a result we have the following estimates for the characteristic temperature, energy density and pressure:

\[ T_{\text{micro}} \sim E_{\text{Planck}} \sim \hbar c/a \quad , \quad \epsilon_{\text{micro}} \sim p_{\text{micro}} \sim \frac{E_{\text{Planck}}^4}{\hbar^3 c^3}. \]  

Here \( c \) is the speed of sound waves or of other relevant bosonic or fermionic excitations, which plays the role of the maximum attainable speed (the speed of light).

2.2. Natural macroscopic value

In spite of the similarity between Eq.(1) and Eq.(2), the conjecture that the microscopic (Planck) scales naturally enter the vacuum energy is not correct. This is because the real temperature and real pressure of liquids and solids are macroscopic variables, which are determined not by the internal microscopic physics but by the environment. In particular, if there is no interaction with the environment, the many-body system will be cooled down to zero temperature by radiation and/or evaporation, while its pressure in the final equilibrium state will be zero, because the external pressure is absent:

\[ T_{\text{macro}} = 0 \quad , \quad p_{\text{macro}} = 0. \]  

This demonstrates the huge difference between the parameters of the microscopic and macroscopic physics. The many-body system as a whole obeys the macroscopic physics which does not depend on the details of the microscopic physics. Since this is the general property of any system, it would be strange if the vacuum does not obey the same rule. The vacuum pressure belongs to the macroscopic physics, and thus the natural value of the vacuum energy density must be zero:

\[ \Lambda_{\text{natural}} = \epsilon_{\text{vac}} = -p_{\text{vac}} = 0, \]  

if we believe that our Universe does not interact with the environment.

3. The absolute value of energy only has significance for gravity

One may argue that the vacuum energy is only important for gravity, i.e. only gravity is sensitive to vacuum energy, otherwise the vacuum energy can be removed by the shift of the zero level. That is why the analogy with condensed matter makes no sense (unless gravity emerges in condensed matter). This is another myth.

First of all, we are interested not in the absolute value of the total vacuum energy. We are interested in the cosmological constant, which represents the density of the vacuum energy, and this density is related to the vacuum pressure by the equation of state

\[ \epsilon_{\text{vac}} = -p_{\text{vac}}, \]  

Irrespective of whether the gravity is present or absent, the vacuum is the substance with this equation of state, and this does not depend on the choice of the zero.
Let us consider how the equation of state (5) emerges for the vacuum (ground state) of condensed matter.

### 3.1. Quantum field theory of condensed matter

The rules of the game are simple: the ground state of condensed matter is the analogue of the vacuum. Particle-like excitations above the ground state – quasiparticles – serve as analogue of matter: quasiparticles do not scatter on the underlying atoms of the liquid or solid if the liquid or solid is in its ground state. That is why for quasiparticles, the homogeneous ground state is the perfect vacuum.

First, one must specify what thermodynamic potential in the quantum condensed matter plays the role of the vacuum energy density. Let us start with the "Theory of Everything" for condensed matter system. The underlying microscopic physics of a liquid or solid formed by a system of $N$ atoms obeys the conventional quantum mechanics and is described by the $N$-body Schrödinger wave function $\Psi(r_1, r_2, \ldots, r_i, \ldots, r_N)$. The corresponding many-body Hamiltonian is

$$H = -\frac{\hbar^2}{2m} \sum_{i=1}^{N} \frac{\partial^2}{\partial r_i^2} + \sum_{i=1}^{N} \sum_{j=i+1}^{N} U(r_i - r_j),$$

where $m$ is the bare mass of the atom, and $U(r_i - r_j)$ is the pair interaction of the bare atoms $i$ and $j$.

In the thermodynamic limit where the volume of the system $V \to \infty$ and $N$ is macroscopically large, there emerges an equivalent description of the system in terms of quantum fields, in a procedure known as second quantization. The quantum field is presented by the bosonic or fermionic annihilation operator for atoms $\psi(x)$. The Schrödinger many-body Hamiltonian (6) is transformed to the Hamiltonian of the quantum field theory (QFT)

$$\hat{H}_{\text{QFT}} = \hat{H} - \mu \hat{N} = \int dx \psi^\dagger(x) \left[ -\frac{\nabla^2}{2m} - \mu \right] \psi(x)$$

$$+ \frac{1}{2} \int dx dy U(x - y) \psi^\dagger(x) \psi^\dagger(y) \psi(y) \psi(x).$$

Here $\hat{N} = \int d^3x \ \psi^\dagger(x)\psi(x)$ is the operator of the particle number (number of atoms); $\mu$ is the chemical potential – the Lagrange multiplier introduced to take into account the conservation of the number of atoms.

One can see that in condensed matter, the emerging quantum field theory is governed not by the Hamiltonian $\hat{H}$ but by the Hamiltonian $\hat{H}_{\text{QFT}} = \hat{H} - \mu \hat{N}$ which allows us to avoid the constraint imposed on the quantum field $\psi$ by the conservation of particle number. The Hamiltonian (7) serves as a starting point for the construction of the effective QFT for quasiparticles living at low energy, and it is responsible for their vacuum. In the QFT description, the energy density of the vacuum is given by the thermodynamic potential $E - \mu N$, which is the vacuum
expectation value of the Hamiltonian $\hat{H}_{\text{QFT}}$:

$$\epsilon_{\text{vac}} = \frac{1}{V} \langle \hat{H}_{\text{QFT}} \rangle_{\text{vac}}.$$  \hspace{1cm} (8)

3.2. **Equation of state for vacuum in condensed matter**

One can also check that the energy density (8) is the right choice for the vacuum energy density by using the Gibbs-Duhem relation of thermodynamics, which is valid in the thermodynamic limit $N \to \infty$. It states that if the condensed matter is in equilibrium it obeys the following relation between the energy $E = \langle \hat{H} \rangle$, and the other thermodynamic variables – the temperature $T$, the entropy $S$, the particle numbers $N = \langle \hat{N} \rangle$, the chemical potentials $\mu$, and the pressure $p$:

$$E - TS - \mu N = -pV.$$  \hspace{1cm} (9)

Applying this relation to the equilibrium vacuum, i.e. to the equilibrium state at $T = 0$, one obtains the relation (5) between the vacuum pressure and vacuum energy density:

$$\epsilon_{\text{vac}} = \frac{1}{V} \langle \hat{H}_{\text{QFT}} \rangle_{\text{vac}} = \frac{1}{V} \langle \hat{H} \rangle_{\text{vac}} - \frac{1}{V} \mu \langle \hat{N} \rangle_{\text{vac}} = -p_{\text{vac}}.$$  \hspace{1cm} (10)

3.3. **Thermodynamics of vacuum**

This thermodynamic analysis does not depend on the microscopic structure of the vacuum and thus can be applied to any quantum vacuum. It follows from the simple analysis of the equilibrium of the piston between the quantum vacuum and the empty space (see Fig. 1). The force acting on the piston is $F = -dE_{\text{vac}}/dx = -AdE_{\text{vac}}/dV = -A\epsilon_{\text{vac}}$, where $A$ is the area of the piston. On the other hand the force divided by area $A$ must be equal to the vacuum pressure. This gives the equation of state (5) for the vacuum.

It is the general property, which follows from thermodynamics, that the vacuum behaves as a medium with the equation of state (5). Thus it is not surprising that the equation of state is applicable also to the particular case of the vacuum of the relativistic quantum field theory (RQFT). This demonstrates that the problem of the vacuum energy can be considered from a more general perspective not constrained by the relativistic Hamiltonians. Moreover, it is not important whether gravity emerges or not in the system, i.e. whether the vacuum is gravitating or not.

The vacuum energy plays an important role even in the absence of gravity. Let us imagine the world without gravity, i.e. the world where the metric field does not emerge in the low-energy corner, or the world where the Newton constant $G = 0$. In such a world the matter would expand due to the matter pressure. However, comparing this situation with what occurs in condensed matter, where the role of the vacuum is played by the ground state, and the role of matter is played by quasiparticles, we shall see in the Sec. 8.1 that the Universe in such world can be stabilized by the vacuum pressure which compensates the pressure of matter.
**Vacuum Energy: Myths and Reality**

3.4. **What is \( \mu \) in quantum vacuum?**

The frequently asked question is what is the analog of the chemical potential \( \mu \) in the underlying microscopic physics of the quantum vacuum in our Universe. In all the known quantum condensed matter system one has one or several conservation laws for different atoms or even for different atomic states of the same atoms. The general Hamiltonian for \( n \) quantum fields \( \psi_i (i = 1, \ldots, n) \) for \( n \) species of atoms is

\[
\hat{H}_{QFT} = \hat{H} - \sum_{i=1}^{n} \mu_i \hat{N}_i.
\]  

(11)

Application of the Gibbs-Duhem relation to this system,

\[
E - TS - \sum_{i=1}^{n} \mu_i N_i = -pV,
\]  

(12)
G. E. Volovik

again gives the equation of state for the quantum vacuum (the equilibrium state at $T = 0$)

$$
\epsilon_{\text{vac}} = \frac{1}{V}\langle \hat{H}_{\text{QFT}} \rangle_{\text{vac}} = -p_{\text{vac}} .
$$

This equation of state does not depend on the number of the conservation laws, which means that the underlying physics of quantum vacuum may have any number $l$ of chemical potentials, including $l = 0$. If for our Universe one has $l = 0$, then the question asked in the title of this sub-section makes no sense.

However, there is an opinion that, in order for the condensed-matter approach to work for the quantum vacuum, there should exist at least one chemical potential, and it is the challenge of the approach to find out its origin. This is true that among the quantum condensed matter systems we do not have examples of a stable system with $l = 0$. But this does not mean that the quantum vacuum of our Universe must have nonzero $l$: there is no reason for the structure of the quantum vacuum to mimic exactly the condensed matter. In principle, the non-zero $l$ is not excluded, and the candidate for the conserved quantity can be, for example, the fermion number. In Appendix we use the model of the quantum vacuum with $l = 1$ for the discussion of the cosmological constant emerging in Einstein Universe.

4. Does vacuum energy depend on overall shift?

Now we can turn to another myth, that the vacuum energy depends on the overall energy shift.

This is not so, because the vacuum pressure is the intensive thermodynamic variable. It is invariant under the overall energy shift: under transformation $E \rightarrow E + E_0$ the pressure does not change, $p = -dE/dV \rightarrow p$. The vacuum energy density is also the intensive thermodynamic variable, and thus it must be also invariant under the overall energy shift. This is seen from the equation of state for the vacuum, which relates the vacuum energy density to pressure.

The other frequent comment related to the shift of the energy is that the energy per particle in Eq. for condensed matter is not defined in unique way. For example, one may add the rest energy $mc^2$ to the energy of each particle, $p^2/2m \rightarrow mc^2 + p^2/m$ or add the constant potential $p^2/2m \rightarrow p^2/m + U_0$. This leads to the overall energy shift with $E_0 = Nmc^2$ or $E_0 = NU_0$ which now depends on the volume $V$. However, the thermodynamic quantities do not depend on such transformation. In particular, the properly defined vacuum energy density in Eq. is invariant under this transformation, since the corresponding transformation of the chemical potential, $\mu \rightarrow \mu + mc^2$ or $\mu \rightarrow \mu + U_0$ correspondingly, compensates the change in $E$ leaving $\epsilon_{\text{vac}}$ invariant.
5. Is zero-point energy dominating in vacuum energy?

5.1. Zero-point contribution

From Sec. 2.2 it follows that the natural value of the vacuum pressure of the system in the absence of the environment is \( p_{\text{vac}} = 0 \). Then from the equation of state for the vacuum [5] it follows that the natural value of the vacuum energy density must be also zero. But this poses the problem: what to do with the zero-point energy. The zero value of the vacuum energy is in huge contradiction with our next myth: the positive zero-point energy of bosonic quantum fields and the negative energy of Dirac vacuum of the fermionic quantum fields are the dominating sources of vacuum energy, each comprising the Planck scale estimate (1) for the vacuum energy and cosmological constant. The naive summation over the \( \nu_b \) bosonic and \( \nu_f \) fermionic modes of the quantum fields gives

\[
\epsilon_{\text{vac}} = \frac{1}{V} \left( \frac{1}{2} \sum_b \sum_p c_p - \sum_f \sum_p c_p \right) \sim \frac{1}{c^3} \left( \frac{1}{2} \nu_b - \nu_f \right) E_{\text{Planck}}. \tag{14}
\]

This contradiction between the zero value, which follows from the macroscopic physics, and the Planck scale value following from the microscopic (Planckian) physics exists also in the many-body system. Observers living in such a system would also make the same estimation, which is based on their knowledge of the low-energy bosonic and fermionic fields with corresponding parameters \( c, E_{\text{Planck}}, \nu_f \) and \( \nu_b \) appropriate for a given system. For example, if they live in solids, their life is based on the “relativistic” bosonic fields of phonons. They are aware that there is the Planck energy scale (played by Debye temperature) where the “Lorentz invariance” is violated, i.e. the spectrum of phonons starts to deviate from the linear “relativistic” spectrum. That is why these inner observers will be able to estimate theoretically the vacuum energy as zero-point energy of the phonon field, and they will be also puzzled by the disparity of many orders of magnitude between the estimates and observations.

5.2. Automatic compensation of zero-point energy

But we know the microscopic physics of solids and liquids, and we can easily explain to the inner observers where their theory goes wrong. Actually there is nothing wrong with their estimation, it is simply incomplete. The equation (14) takes into account only the modes below the “Planck” energy cut-off, i.e. those degrees of freedom which are described by an effective theory (theory of elasticity in solids or hydrodynamics in liquids). At higher energies, which correspond to distances of order of inter-atomic spacing \( a \), the microscopic structure of solids or liquids in terms of the interacting atoms in Eq. (4) must be taken into account. They cannot be expressed in terms of the relativistic quantum fields of phonons. When one sums up all the contributions to the vacuum energy, sub-Planckian (phonons) and trans-Planckian (all other degrees of freedom), one obtains the zero result (micro-
scopic calculations of the ground state energy of the system of $N$ atoms obeying
the ‘Theory of Everything in Eq. (6) can be found in Ref. 13, one can check that
\[
\langle \hat{H} \rangle_{\text{vac}} = \mu \langle \hat{N} \rangle_{\text{vac}}
\]
and thus $\epsilon_{\text{vac}} = 0)$. The exact nullification occurs without any
special fine-tuning, it is provided by the macroscopic physics – thermodynamics –
applied to the whole equilibrium vacuum.

The main lesson from us to inner observers (or from condensed matter to par-
ticle physics), which they may or may not accept, is this: the energy density of the
homogeneous equilibrium state of the quantum vacuum is zero in the absence of
an external environment. The higher-energy (trans-Planckian, microscopic) degrees
of freedom of the quantum vacuum, whatever they are, perfectly cancel the huge
positive contribution of the zero-point fluctuations of the quantum fields as well as
the huge negative contribution of the Dirac vacuum. This is the consequence of the
macroscopic physics which forces the microscopic degrees of freedom to automati-
cally fine-tune.

This effect of the automatic compensation without fine tuning can be found
also in some relativistic theories. Example is provided by a model, in which our
world is represented by the (3 + 1)-dimensional membrane embedded in the (4 + 1)-
dimensional anti-de Sitter space. In the equilibrium vacuum, the huge contributions
to the cosmological constant coming from different sources cancel each other without
fine-tuning 12.

5.3. Myth in condensed matter community

It happens, that the myth on the zero-point energy is still alive even in condensed
matter community. Some people erroneously add the zero-point energy of the quanti-
zized phonon field to the energy of the liquid or solid. This leads, however, to the
double counting. Phonons are quanta of the classical sound waves propagating on
the background of the liquid or solid. Their spectrum is $E(p) = cp$, where the
speed of sound $c$ is determined by the energy density of the system as a function
of the mass density: $c^2 = \rho d^2 \epsilon/d\rho^2$. The energy $\epsilon(\rho)$ of the background on which
the phonons are propagating takes into account already all the quantum degrees of
freedom of underlying liquid, including those which can be expressed via phonons
– the quantized field of small oscillations of liquid or solid.

In other words, sound waves represent the classical output of the quantum sys-
tem, in which phonons are already quantized from the very beginning. By quantizing
phonons again one only reproduces the low-energy part of already existing quan-
tum states of the system. That is why one makes mistake if one adds the zero-point
energy of phonons to the overall ground state energy of the system.

6. Does vacuum energy depend on vacuum content?

The next myth is that the vacuum energy depends on the vacuum content. Indeed,
at first glance this is correct: the vacuum is complicated, there are many contribu-
tions to vacuum energy from different quantum fields 8. There are many species
of fermions each with its own mass. Thus the vacuum energy must depend on the fermion masses, on Higgs field, etc. (see e.g. 5). For example, the general form of the contribution to the vacuum energy density from the electron-positron degrees of freedom of the quantum vacuum is

$$\epsilon = a_4 M^4 + a_2 M^2 m^2 + a_0 m^4 \ln \frac{M^2}{m^2},$$

(15)

where $m$ is the electron mass, and $M$ is the ultraviolet energy cut-off. If the cut-off is provided by the Planck scale, the estimate (1) is obtained (we use units $\hbar = c = 1$).

Different regularization schemes were suggested in order to obtain the dimensionless parameters $a_i$. However, this is not relevant for the estimation of the total vacuum energy density. From condensed matter experience we know that condensed matter may contain different species of atoms and zero-point energy of different effective bosonic fields (phonons, magnons, etc.). Nevertheless, the vacuum energy density relevant for quantum field theory is the macroscopic parameter: it does not depend on the microscopic physics and thus on the vacuum content. It remains zero in a full equilibrium.

The particular contributions to the vacuum energy becomes important, when we consider the coexistence of two vacua with slightly different vacuum content, for example with the slightly different $m$.

7. Myth on energy of false vacuum

Let us turn to the myth on the energy of the false vacuum. The false vacuum corresponds to the local minimum of energy, and it has higher energy than the true vacuum which corresponds to absolute minimum. That is why, if $\Lambda = 0$ in the true vacuum (as is typically assumed), it must be nonzero (positive) in the false vacuum; or the other way round: if $\Lambda = 0$ in a false vacuum, it must be nonzero (negative) in the true vacuum.

This is important for the phenomenon of inflation – the exponential super-luminal expansion of the Universe. In some theories, the inflation is caused by a false vacuum. It is usually assumed that the energy of the true vacuum is zero, and thus the energy of the false vacuum must be positive. Though the false vacuum can be locally stable at the beginning, $\Lambda$ in this vacuum must be big and positive constant, which causes the exponential de-Sitter expansion. This scenario is wrong, because as we know the cofally stable quantum vacuum does not gravitate.

The related myth is the effect of the symmetry breaking phase transition, such as electroweak phase transition or chiral phase transition in strong interactions, on the vacuum energy. Both lead to extremely large values of vacuum energy relative to what is allowed.

The related question also, why the cosmological constant is (approximately) zero only for that vacuum in which we leave? Our vacuum has no special preference compared to the other possible minima of the effective potential. To avoid this
problem, the idea of the Multiple Point Principle has been introduced, which states that the vacuum energy must be (approximately) zero in all of the vacua.\textsuperscript{16}

Let us look at all these problems related to vacua corresponding to local minima using our knowledge of the general thermodynamic properties of the quantum vacuum.

7.1. Cosmological constant in false and true vacua

Analyzing the Gibbs-Duhem relation we find that in our derivation of the vacuum energy, we never use the fact that our system is in the true ground state. We only assume that our system is in the thermodynamic equilibrium at $T = 0$. But this is applicable to the metastable state too if we neglect the tiny, exponentially small probability of the transition between the false and true vacua, such as quantum tunneling and thermal activation. Thus we come to the following, at first glance paradoxical, conclusion: the cosmological constant in all homogeneous vacua in equilibrium is zero, irrespective of whether the vacuum is true or false. This poses constraints on some scenarios of inflation.

Fig. 2. The condensed-matter scenario of the evolution of the energy density $\epsilon_{\text{vac}}$ of the quantum vacuum in the process of the first order phase transition from the equilibrium false vacuum to the equilibrium true vacuum. Before the phase transition, i.e. in the false but equilibrium vacuum, one has $\epsilon_{\text{vac}} = 0$. During the transient period the microscopic parameters of the vacuum readjust themselves to new equilibrium state, where the equilibrium condition $\epsilon_{\text{vac}} = 0$ is restored.
7.2. How the phase transition occurs

The related myth is the effect of the symmetry breaking phase transition on the vacuum energy. One may ask, how and why the transition between the vacua occurs if they have the same energy density. Again the answer comes from the analysis of the phase transition in condensed matter.\textsuperscript{15}

Let us consider the typical example of the first-order phase transition which occurs between the metastable quantum liquid $^3\text{He}$-A and the stable quantum liquid $^3\text{He}$-B, Fig. 2. Initially the vacuum is in the state A. Since this state is the local minimum, its thermodynamic potential relevant for the vacuum energy is zero, $E_A - \mu_A N = 0$. Let us assume that this vacuum A is metastable, i.e. there exists the vacuum B for which the energy calculated at the same fixed chemical potential $\mu = \mu_A$ is negative, $E_B - \mu_A N < 0$. Thus the liquid prefers the phase transition from the “false” vacuum A to the “true” vacuum B. When the transition to the B-phase occurs, the vacuum energy becomes negative. However, this corresponds to the non-equilibrium state, since we know that in equilibrium the vacuum energy must be zero. What happens? The microscopic degrees of freedom start to rearrange themselves to the new vacuum B. On the microscopic level this means that the inter-atomic spacing changes in such a way that the chemical potential $\mu_B$ in the new vacuum starts to satisfy the equilibrium condition, $E_B - \mu_B N = 0$. This means that after some transient period of relaxation the vacuum energy density $\epsilon_{\text{vac}}$ in the true vacuum B also becomes zero.

We can readily apply this consideration to the quantum vacuum in our Universe. This condensed-matter example suggests that the cosmological constant is zero before the cosmological phase transition. During the non-equilibrium transient period of time, the microscopic (Planckian) parameters of our vacuum are adjusted to a new equilibrium state in a new vacuum, and after that the cosmological constant becomes zero again. Thus vacuum energy in each vacuum is zero, but this only occurs when the system is in equilibrium within a given vacuum. This demonstrates that to have the zero cosmological constant in each vacuum we do not need the Multiple Point Principle suggested in Ref. \textsuperscript{16} which states that the effective potential in Fig. 2 should have degenerate minima. This does not happen in condensed matter: minima are typically non-degenerate (if there is no special symmetry). They will be degenerate only in the special case when two (or several) vacua coexist in space across the interface\textsuperscript{17}.

We do not know what are the microscopic degrees of freedom in our Universe and how the microscopic parameters relax in the new vacuum to establish the new equilibrium after phase transition. This already depends on the details of the system and cannot be extracted from the analogy with quantum vacua in liquids. However, using our experience with quantum liquids we can try to estimate the range of change of the microscopic parameters after the transition.
Let us consider, for example, the electroweak phase transition, assuming that it is
of the first order and thus can occur at low temperature, so that we can discuss
the transition in terms of the vacuum energy. In this transition, the vacuum energy
density changes from zero in the initially equilibrium false vacuum to the negative
value on the order of

$$\delta \epsilon_{\text{vac}}^{\text{ew}} \sim -E_{\text{ew}}^4$$

(16)
in the true vacuum, where $E_{\text{ew}}$ is the electroweak energy scale. To restore the
equilibrium in the new vacuum, this negative energy must be compensated by the
adjustment of the microscopic (trans-Planckian) parameters. As such a parameter
we can use the value of Planck energy $E_{\text{Planck}}$, since it determines the characteristic
microscopic energy scale. We know that the contribution of the modes close to or
above the Planck energy to the vacuum energy is huge, since in equilibrium it must
compensate the $E_{\text{Planck}}^4$ contribution from the sub-Planckian modes. The variation
of this microscopic energy due to the change of the parameter $E_{\text{Planck}}$ is

$$\delta \epsilon_{\text{vac}}^{\text{Planck}} \sim E_{\text{Planck}}^3 \delta E_{\text{Planck}}$$

(17)

In a new equilibrium vacuum, the density of the vacuum energy must be zero

$$\delta \epsilon_{\text{vac}}^{\text{ew}} + \delta \epsilon_{\text{vac}}^{\text{Planck}} = 0$$

(18)

and thus the relative change of the microscopic parameter $E_{\text{Planck}}$ which com-
pen-sates the change of the electroweak energy after the transition is

$$\frac{\delta E_{\text{Planck}}}{E_{\text{Planck}}} \sim \frac{E_{\text{ew}}^4}{E_{\text{Planck}}^4}$$

(19)

This, for example, leads to the tiny change of the Newton constant, which is deter-
mined by the Planck scale, $G \sim 1/E_{\text{Planck}}^2$:

$$\frac{\delta G}{G} \sim \frac{E_{\text{ew}}^4}{E_{\text{Planck}}^4} \sim 10^{-65}$$

(20)

The response of the deep vacuum to the electroweak transition appears to be ex-
tremely small: the energy at the microscopic Planck scale is so high that a tiny
variation of the microscopic parameters is enough to restore the equilibrium viol-
ated by the cosmological transition.

The same actually occurs at the first-order phase transition between $^3\text{He}$-$\text{A}$
and $^3\text{He}$-$\text{B}$: the change in the energy of the superfluid vacuum after the transition
is compensated by a tiny change of the microscopic parameter, the inter-atomic
distance, $\delta a/a \sim 10^{-6}$.

This remarkable fact of the response of the deep vacuum to the cosmological
transition may have some consequences for the dynamics of the cosmological con-
stant after the phase transition. Probably this would imply that vacuum energy
relaxes rapidly, with the characteristic time determined by the trans-Planckian
physics. However, at the moment we have no reliable theory describing the processes of relaxation of $\Lambda$.

8. Myth on non-gravitating vacuum

The natural value of the vacuum energy is zero. Should we return to the old myth that the vacuum does not gravitate? Of course, not, though it is easier to assume that the vacuum energy is zero than to explain the reduction by 120 orders of magnitude. In particular, there exists a rather broad belief that the problem of the vacuum energy can be avoided simply by the proper choice of the ordering of the QFT operators $\psi$ and $\psi^\dagger$. Let us consider how the quantum many-body systems help us to solve the cosmological constant problem #2 – why the vacuum energy is nonzero.

For quantum liquids or solids, the zero result has been obtained using the original pre-QFT microscopic theory – the Schrödinger quantum mechanics of interacting atoms, from which the QFT emerges as a secondary (second-quantized) theory. In this approach the problem of the ordering of the operators in the emergent QFT is resolved on the microscopic level, and it has nothing to do with the calculations of the vacuum energy density determined by the macroscopic physics.

The zero result for the vacuum energy has been obtained for the perfect equilibrium non-perturbed vacuum in the absence of the interaction with the environment. It was assumed that the system satisfies the following conditions: (i) it is in complete thermodynamic equilibrium; (ii) static or stationary; (iii) homogeneous in space, (iv) with zero curvature; (v) does not contain excitations (which play the role of matter); (vi) does not interact with environment; (vii) is at $T = 0$; (viii) has no boundaries. What happens, when any of the above conditions is violated? Then from the same thermodynamic analysis one finds that almost all the deviations from the perfectness lead to non-zero value of the local vacuum energy density, which is proportional to perturbations of the perfect vacuum.

8.1. Vacuum response to matter

Let us start with the Universe without gravity, i.e. suppose that we live in the world obeying the laws of special relativity without general relativity. Is the static Universe possible under this condition? Of course, it is possible if there is no matter. If matter is present, we have a problem: for $T \neq 0$ matter has positive pressure and thus will expand. It happens that the Universe with matter can be stabilized by the pressure of the vacuum, and this provides an example of the importance of the vacuum energy even in the absence of gravity.

For the Universe to be static, the pressure in the equilibrium Universe must be zero if there is no external environment, and thus the vacuum pressure must compensate the pressure of matter:

$$p_{\text{total}} = p_{\text{matter}} + p_{\text{vac}} = 0 .$$  \hspace{1cm} (21)
This gives the relation between the energy density of matter and the energy density of the vacuum in the static world without gravity:

\[ \epsilon_{\text{vac}} = -p_{\text{vac}} = p_{\text{matter}} = w_{\text{matter}} \epsilon_{\text{matter}}, \tag{22} \]

where \( p_{\text{matter}} = w_{\text{matter}} \epsilon_{\text{matter}} \) is the equation of state for matter. The response of the vacuum energy to matter does not depend on the details of the microscopic (trans-Planckian) physics.

Exactly the same relation

\[ \epsilon_{\text{vac}} = w_{\text{matter}} \epsilon_{\text{matter}}, \tag{23} \]

takes place in condensed matter systems. Let us consider a droplet of a quantum liquid or a piece of a solid in space, i.e. in the absence of the environment, but now at non-zero temperature. At \( T \neq 0 \), one has in addition to the vacuum the gas of quasiparticles – phonons – which play the role of matter. The gas of ‘relativistic’ phonons has its own partial pressure:

\[ p_{\text{matter}} = \frac{\pi^2}{90c^3} T^4. \tag{24} \]

This is equivalent to the pressure of radiation which has the same equation of state \( w_{\text{matter}} = \frac{1}{3} \), since the equation of state for the gas of massless “relativistic” (quasi)particles does not depend on whether \( c \) is the speed of light or the speed of sound. Since there is no external pressure, the sum of partial pressures of the condensed-matter vacuum and quasiparticles (matter) must be zero, and one obtains the non-zero vacuum energy obeying the equation \( (24) \) with \( w_{\text{matter}} = \frac{1}{3} \).

This demonstrates that the vacuum response to matter is universal. Moreover, for the hot Universe, where \( w_{\text{matter}} = \frac{1}{3} \), the vacuum energy density appears to be comparable to the energy density of matter. But we still did not introduce gravity.

8.2. Vacuum energy in Einstein Universe

In the presence of gravity, i.e. for \( G \neq 0 \), we can apply thermodynamic consideration to the stationary Universes, which allows us to obtain the result for the vacuum energy density without solving Einstein equations. In particular, it can be done for the static closed Einstein Universe and the Einstein result is reproduced \(^{15} \):

\[ \epsilon_{\text{vac}} = \frac{1}{2} \epsilon_{\text{matter}}(1 + 3w_{\text{matter}}). \tag{25} \]

Now, in the presence of gravity, even in the cold Universe, where \( w_{\text{matter}} = 0 \), the vacuum energy density is comparable to the energy density of matter. This is due to the response of the vacuum energy to the gravitational field. The discussion of the vacuum energy and cosmological constant in the Einstein Universe is in Appendix.
8.3. **No response to local perturbation**

There can be other perturbations of the quantum vacuum, which lead to the non-zero vacuum energy, such as boundaries which provide the Casimir effect. We are almost sure that the Casimir energy emerging between the neutral perfectly conducting plates is gravitating, though it is still too far from the experimental check. If so, then one may ask, why the zero-point energy in the space between the plates gravitates and not the zero-point energy outside the plates. The related question is why the zero-point energy gravitates in the environment of the atom (Lamb shift) and not in vacuum. The answer is simple, the local perturbation of the vacuum (by an atom or by plates) does not change the pressure at infinity, and thus the cosmological constant is not perturbed by local perturbations.

Actually the answer to these questions has been given by Einstein. In Ref. 18, Einstein noted that the Λ-term must be added to his equations if the density of matter in the Universe is non-zero in average. In particular, this means that Λ = 0 if matter in the Universe is so inhomogeneous that its average over big volumes V tends to zero. We just discussed the special case when this condition is satisfied, i.e. when the perturbation occupies the finite region of the infinite Universe. This is one more example of remarkable Einstein intuition.

8.4. **Vacuum perturbations in our Universe**

In the above simple cases of static perturbations of the vacuum we found that the vacuum energy density is on the order of the matter density. This gives some hint on how to solve the coincidence problem in cosmology.

In our expanding Universe there are several factors which lead to non-zero contributions to vacuum energy: expansion with the Hubble parameter $H$; gravitating matter with the energy density $\epsilon_{\text{matter}}$; temperature $T$; possibly the curvature $1/R^2$ (experiments demonstrate that our Universe is most probably flat); etc. The effect of the static perturbations (from curvature, temperature and matter) can be easily calculated, using the macroscopic thermodynamic analysis. For the time-dependent perturbations there is no simple recipes, because the purely thermodynamic consideration fails to work in the non-equilibrium situation, while the dynamics of the vacuum energy and thus of the cosmological constant is not known and most probably is not universal, i.e. it may depend on the details of microscopic physics. We are only able to make the order of magnitude estimations.

Then one finds that the above perturbations of the vacuum lead to the non-zero vacuum energy expressed through the macroscopic parameters $G, T, R, H, \dot{H}$. The corresponding contributions to the vacuum energy density are proportional to $H^2/G, \epsilon_{\text{matter}}, T^4, \dot{H}/G, 1/GR^2$, etc. These are the natural macroscopic scales for the cosmological constant, which are always extremely small compared to the naive estimation of the vacuum energy $E_{\text{Planck}}^4$ expressed in terms of the microscopic Planck scale. How the natural value of the cosmological constant emerges in the Einstein static Universe is discussed in Appendix.
The dynamics of the cosmological constant is a real problem. But there is nothing unnatural with this problem, and there are no big puzzles related to cosmological constant: The present cosmological constant is small because all the perturbations of the vacuum are small, and it has the right order of magnitude.

9. Conclusion

Unbearable lightness of space-time, i.e. that the cosmological term is too small compared to “its natural value”, is the typical myth. A simple exercises with the vacuum energy of the many-body quantum system considered here demonstrate that the natural value of the energy density of the vacuum is determined not by the microscopic (Planck or other) energy scale cut-off, $E_{\text{Planck}}^4/c^3$, but by the macroscopic parameters of the system.

There are several ways of how to calculate the vacuum energy in condensed matter: (i) using consideration in terms zero-point energy of the effective bosonic quantum fields, or in terms of the energy of Dirac vacuum of the effective fermionic quantum fields (practically any quantum condensed matter, if its temperature is sufficiently low, is described by bosonic and/or fermionic quantum fields); (ii) by application of the macroscopic laws of thermodynamics; (iii) by exact calculations of the energy density using the microscopic (atomic) physics.

The naive consideration (i) gives the big vacuum energy, $\epsilon_{\text{vac}} \sim +E_{\text{Planck}}^4/c^3$ or $\epsilon_{\text{vac}} \sim -E_{\text{Planck}}^4/c^3$, depending on the vacuum content. In crystals and in superfluid $^4\text{He}$, the role of the Planck energy scale is played by the Debye energy; $c$ is the speed of sound; and the sign is $+$, since the energy is obtained by the summation over the zero-point energies of the bosonic fields (phonons). In superfluid $^3\text{He}$, $c$ is the (anisotropic) speed of fermions; the sign is $-$, since the contribution comes from the occupied negative energy levels of the Dirac sea.

The consideration (ii) gives exactly zero energy density, $\epsilon_{\text{vac}} = 0$, if the condensed matter system satisfies the following conditions: it is in complete thermodynamic equilibrium; stationary; homogeneous, does not contain excitations (quasiparticles, which play the role of matter); does not interact with environment; is at $T = 0$; the effects of boundaries are neglected.

This follows from the very simple thermodynamic argument, known as Gibbs-Duhem relation. If all these conditions (except for the requirement of non-interaction with the environment) are satisfied, one obtains the universal equation of state, $\epsilon_{\text{vac}} = -p_{\text{vac}}$. It is valid both for the vacuum of relativistic quantum fields and for the ground state of non-relativistic quantum condensed matter systems.

If in addition there is no interaction with environment, the external pressure is zero and thus the vacuum energy density $\epsilon_{\text{vac}} = 0$.

There is a clear contradiction between the results of semi-microscopic consideration (i) and macroscopic consideration (ii).

The pure microscopic consideration (iii) confirms the zero result of the macroscopic approach (ii) under the same conditions as it was obtained from the macro-
scopic physics. The microscopic physics also shows how the contradiction between the naive approach (i) and the macroscopic physics (ii) is resolved. In addition to the phonon degrees of freedom, the condensed matter system contains the high-energy degrees of freedom which cannot be described in terms of the long-wavelength acoustic fields (acoustic phonon fields are only determined for the wavelength bigger that the inter-atomic distance). It appears that the high-energy atomic degrees of freedom automatically adjust themselves in such a way, that the vacuum as a whole obeys the macroscopic thermodynamic relations. In the perfect vacuum (satisfying the above conditions), the contribution of the high-energy degrees of freedom to the vacuum energy density automatically compensates the contribution of the sub-Planckian effective quantum fields to satisfy the zero pressure condition. Thus the ultraviolet divergence of the contribution of the quantum fields is absolutely irrelevant for the vacuum energy: the macroscopic effect of compensation does not depend on the cut-off.

Since the macroscopic thermodynamic consideration does not depend on details of the microscopic (trans-Planckian) physics, the same reasoning should be applicable to the vacuum of relativistic quantum fields. The only assumption made is the mere existence of the microscopic (trans-Planckian) degrees of freedom. If they exist, they must obey the macroscopic laws and thus they will be automatically arranged to satisfy the zero pressure condition in the absence of the interaction with the environment.

If the vacuum is perturbed, the microscopic degrees of freedom are automatically rearranged to satisfy the new equilibrium. If the perturbation is caused by the phase transition from one vacuum to another (say, from false to true vacuum), the microscopic degrees of freedom will be slightly deformed to adjust to the new vacuum state. As a result, after the equilibrium is reached, the zero value of the vacuum energy density is restored in the new vacuum.

One can also find that the readjustment of the microscopic parameters after the transition is extremely small. For example, in superfluid $^3$He, the relative change of the inter-atomic distance $a$ (Planck length) after the superfluid transition is of order $\delta a/a \sim T_c^2/E_F^2 \sim 10^{-6}$, where $E_F$ is the microscopic energy (Fermi energy) and $T_c$ is the transition temperature. In a similar way, after the electroweak transition, the relative change of the Planck scale and thus the relative change of the Newton constant $G \propto 1/E_{\text{Planck}}^2$ are of order $\delta G/G \sim M_W^4/E_{\text{Planck}}^4$, where $M_W$ is the gauge boson mass.

If the perturbations are caused by matter, boundaries, curvature, temperature, expansion, etc., the non-zero vacuum energy emerges. One can easily calculate the response of the vacuum energy density to the static perturbations. Then one finds that vacuum energy density is proportional to perturbations of the vacuum, and is determined by macroscopic parameters, rather than microscopic. The same occurs for the time-dependent perturbations. Though in this case we are not able to calculate the vacuum response exactly, since we do not know the dynamics of the vacuum energy, we can expect that the order of magnitude estimation in terms of
the macroscopic parameters remains correct.

In our expanding Universe there are different perturbations of the quantum vacuum: expansion with the Hubble parameter $H$; gravitating matter with the energy density $\epsilon_{\text{matter}}$; temperature $T$; possibly a small curvature $1/R^2$; etc. One can estimate that these perturbations lead to the non-zero vacuum energy expressed through the macroscopic parameters $G, T, R, H, \dot{H}$. The corresponding contributions to the vacuum energy density are proportional to $H^2/G$, $\epsilon_{\text{matter}}$, $T^4$, $\dot{H}/G$, $1/GR^2$, etc. These are the natural macroscopic scales for the cosmological constant, which are always extremely small compared to the naive estimation of the vacuum energy $E_{\text{Planck}}^4$ expressed in terms of the microscopic Planck scale (see also Appendix).

Concerning the absolute value of vacuum energy, some people think that it only has significance for gravity, otherwise it can be shifted by the redefinition of the zero energy. From the condensed matter examples one can find that the properly defined energy density, which is relevant for the vacuum of quantum fields, does not depend on the choice of the zero level. The overall shift of the energy does not change $\epsilon_{\text{vac}}$: the vacuum energy density (as well as pressure) is the intensive thermodynamic quantity (not extensive). It is the real thermodynamic quantity well defined in any condensed matter independent of whether the effective gravity emerges or not in the systems.

The vacuum response to the time dependent perturbations and the dynamics of the vacuum energy and cosmological constant is the real problems. Even in the many-body systems, where we know all the microscopic physics, the non-equilibrium dynamics is the complicated subject. But there is nothing unnatural with the problem of the dynamical adjustment of the vacuum energy to perturbations of the vacuum. Let us look at the general features of the adjustment mechanism discussed in Ref. [19]:

1. Some compensating agent must exist.
   In our approach the compensating agent is the reservoir of the trans-Planckian degrees of freedom of quantum vacuum, with thermodynamic compensation occurring without fine tuning.

2. Quite natural to expect that the compensation is not complete and the resulting vacuum energy density is close to the critical energy density.
   This is also naturally fulfilled in our approach, where the compensation is not complete if the vacuum is not perfect.

3. A realistic model is needed.
   In our approach this must be the phenomenological theory which describes the dynamics of the trans-Planckian degrees of freedom. In condensed matter it is the dynamics of the density of the underlying atoms, $n \propto E_{\text{Planck}}^3$, which is responsible for the relaxation of the vacuum energy density to its equilibrium value. The dynamics of the 3D or 4D density $n$ of the underlying ‘atoms’ of the vacuum, which is proportional to $E_{\text{Planck}}^3$ or $E_{\text{Planck}}^4$ correspondingly, can be probably constructed as the dynamic extension of the equations discussed in Appendix. This will allow us
to consider the decay of our Universe to the perfect equilibrium vacuum state with Minkowski space-time, zero $\Lambda$ and no matter.

In conclusion, there are no unbearable contradictions between the theoretical estimate for the vacuum energy and the observed dark energy. In spite of the ultraviolet divergence of the zero-point energy, the natural value of the vacuum energy is comparable with the observed dark energy. That is why the vacuum energy is the plausible candidate for the dark energy.

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Appendix. Cosmological constant in Einstein Universe

Let us illustrate how the natural macroscopic value of the vacuum energy and cosmological constant emerges in the Einstein static closed Universe. We start with the action:

$$ S = S_E + S_\Lambda + S_{\text{matter}}. $$

Here $S_{\text{matter}}$ is the matter action; $S_E$ is the Einstein curvature action; and $S_\Lambda$ is the action which gives rise to the cosmological term. We shall treat the vacuum energy in the same way as in condensed matter, i.e. by describing the microscopic (Planckian) degrees of freedom of the quantum vacuum in terms of the ground state of the system of some constituent particles with particle number density $n$ and the chemical potential $\mu$ – the Lagrange multiplier responsible for the conservation of the particle number (though the physical meaning of quantities $n$ and $\mu$ may be different, see below). This approach assumes that the microscopic physics contains the principal collective mode, which bears the information on the vacuum energy. The mass of this mode has the Planck energy scale.

Following the condensed matter experience, we choose the following phenomenological action for the real scalar field $n$:

$$ S_\Lambda = - \int d^4 x \sqrt{-g} \left( \epsilon(n) - \mu n + \gamma(n) g^{\mu\nu} \partial_\mu n \partial_\nu n \right). $$

In the Einstein curvature action the Newton constant $G$ depends on $n$:

$$ S_E = - \frac{1}{16\pi} \int d^4 x \frac{1}{G(n)} \sqrt{-g} R. $$

The total action is similar to the scalar-tensor theory of gravity with spin-0 scalar field $n$ (see e.g. [23]). The only difference is that the potential for the $n$-field contains the Lagrange multiplier $\mu$. 

The introduced quantities have the natural Planck scales, for example: \( \mu \sim E_{\text{Planck}} \); \( \epsilon \sim E_{\text{Planck}}^4 \); \( n \sim E_{\text{Planck}}^3 \); \( \gamma \sim E_{\text{Planck}}^{-4} \); \( G^{-1} \sim E_{\text{Planck}}^2 \); and \( M \sim E_{\text{Planck}} \) where \( M \) is the mass of the \( n \) field. In this presentation, the quantity \( n \sim E_{\text{Planck}}^3 \) plays the role of the number density of the quantum modes in the vacuum. In condensed matter, the number of modes per volume is finite (it is \( 3 \times \) number density of atoms): the momentum and energy of each atom is not constrained in the excited state, but the energy, momentum and number of the modes which form the ground state of, say, crystal, are limited. Then \( \mu \) serves as the Lagrange multiplier responsible for the conservation of the number of modes in the quantum vacuum.

In principle, the physical meaning of \( \mu \) and \( n \) and their dimensions can be different. For example, one can choose the following sets, which are more compatible with Lorentz invariance: (1) \( \mu \sim \epsilon \sim E_{\text{Planck}}^4 \); \( n \sim E_{\text{Planck}}^0 \); \( \gamma \sim E_{\text{Planck}}^{-6} \); \( M \sim E_{\text{Planck}} \); and (2) \( n \sim \epsilon \sim E_{\text{Planck}}^4 \); \( \mu \sim E_{\text{Planck}}^0 \); \( \gamma \sim E_{\text{Planck}}^{-2} \); \( M \sim E_{\text{Planck}} \). In the first set, \( \mu \) has dimension of vacuum energy and cosmological constant. In the latter set it is the 4D + 4D phase space which is conserved, with dimensionless Lagrange multiplier \( \mu \). Also for generality, the equation must be added which allows \( \mu \) to relax. But here it is not important since we are looking for the stationary solutions corresponding to Einstein Universes.

The variation over the metric \( g^{\mu \nu} \) gives the Einstein equations:

\[
- \frac{1}{8\pi G} \left( R_{\mu \nu} - \frac{1}{2} R g_{\mu \nu} \right) + (\epsilon(n) - \mu n) g_{\mu \nu} + T_{\mu \nu}^M = 0 ,
\]

where \( T_{\mu \nu}^M \) is the energy-momentum tensor for matter; and \( \epsilon(n) - \mu n \) plays the role of the cosmological constant \( \Lambda \). We do not take into account the gradients of \( n \) because of the big Planck-size mass of the \( n \)-field, while we are looking for the homogeneous solution of equations. The variation over \( n \) gives

\[
\frac{d\epsilon}{dn} - \mu + \frac{1}{16\pi} \frac{dG^{-1}}{dn} R = 0 .
\]

For the Einstein Universe the solution of the Einstein equations gives

\[
\Lambda \equiv \epsilon(n) - \mu n = \frac{1}{2} \rho_{\text{matter}}(1 + 3w_{\text{matter}}) ,
\]

\[
\frac{1}{4\pi GR^2} = \rho_{\text{matter}}(1 + w_{\text{matter}}) ,
\]

where \( R \) is the radius of the Universe, and \( P_{\text{matter}} = w_{\text{matter}} \rho_{\text{matter}} \) is the equation of state for matter. Equation (29) with \( R = -6/G \), and \( dG^{-1}/dn = \beta G^{-1}/n \) with \( \beta \) of order unity gives

\[
\frac{d\epsilon}{dn} - \mu = \frac{3\beta}{8\pi G n R^2} .
\]

These 3 equations express \( n, \mu \) and \( R \) in terms of matter density.

Let \( n_0 \) and \( \mu_0 \) be the values at \( R = \infty \) when \( \rho_{\text{matter}} = 0 \), which corresponds to the perfect vacuum state. These quantities obey equations

\[
(\frac{d\epsilon}{dn})|_{n_0} = \mu_0 , \quad \epsilon(n_0) = \mu_0 n_0 .
\]
These are exactly the equations for the ground state of condensed matter system in the absence of environment, i.e. at zero external pressure (see Sec. 3.3 in Ref. 6). The cosmological constant and vacuum energy in this pure and non-disturbed vacuum are zero as expected from the thermodynamic arguments:

$$\Lambda(n_0) = \epsilon(n_0) - \mu_0 n_0 = 0 .$$  \hspace{1cm} (35)

Gravity and matter perturb the vacuum energy and $\Lambda$. These perturbations are obtained by expanding $\epsilon(n) - \mu n$ in the vicinity of the values $n_0$ and $\mu_0$ in the perfect vacuum:

$$\epsilon(n) - \mu n \approx \frac{1}{2} \epsilon''(\delta n)^2 - n_0 \delta \mu - \delta n \delta \mu .$$  \hspace{1cm} (36)

Then the first-order corrections are:

$$\Lambda \equiv -n_0 \delta \mu = \frac{1}{2} \rho_{\text{matter}}(1 + 3w_{\text{matter}}) ,$$  \hspace{1cm} (37)

$$n_0 \epsilon'' \delta n - n_0 \delta \mu = \rho_{\text{matter}}(1 + w_{\text{matter}}) ,$$  \hspace{1cm} (38)

or

$$\frac{\Lambda}{\Lambda_{\text{Planck}}} \sim \frac{\delta \mu}{\mu_0} = - \frac{\rho_{\text{matter}}(1 + 3w_{\text{matter}})}{2n_0^2 \epsilon''} ,$$  \hspace{1cm} (39)

$$\delta G = -\beta \frac{\delta n}{n_0} \approx \rho_{\text{matter}}(1 - w_{\text{matter}}) \sim \frac{\Lambda}{\Lambda_{\text{Planck}}} .$$  \hspace{1cm} (40)

Here $\Lambda_{\text{Planck}} \sim E_{\text{Planck}}^4$ (for any choice of the field $n$, one has $n_0^2 \epsilon'' \sim E_{\text{Planck}}^4$); it is the “natural” value of the cosmological constant, as it follows from the naive estimation of the vacuum energy. The true cosmological constant $\Lambda$ in Eq.(37), which naturally emerges from the macroscopic physics, is much smaller. It does not depend on the details of the microscopic (trans-Planckian) physics.

On the other hand, the value of the Newton constant $G$ is almost completely determined by the microscopic physics of the vacuum: the corrections to $G$ caused by matter and gravity in Eq.(40) are extremely small. This rules out any time dependence of $G$ during the observational cosmological time.

Equation (39) demonstrates that, though the microscopic (Planck) degrees of freedom are described by the natural Planck scales, the macroscopic physics is governed by the macroscopic scales. In particular, in the Einstein Universe the natural value of $\Lambda$ is determined by the energy density of matter $\rho_{\text{matter}}$ and the radius $R$ of the Universe, rather than by the Planck energy scale.

What should be done next? First the dynamical equations must be extended to describe the relaxation of the parameter $\mu$ (and thus of the cosmological constant $\Lambda$), including the readjustment of these parameters to new equilibrium after the phase transition. Then the principal mechanism of relaxation must be found which drives the non-equilibrium Universe to the final equilibrium state with zero $\Lambda$ and without matter (maybe after several cycles). In condensed matter, one of the possible scenarios of the decay of the non-equilibrium vacuum is the Suhl instability. \[2122\]
It is the parametric instability of the precessing magnetization (analog of the time-dependent vacuum state) with respect to excitations of pairs of spin waves (analog of radiation of pairs of photons or/and gravitons in the process of reheating).

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