This manual describes new functionality in FAST 8 that simulates the addition of tuned mass dampers (TMDs) in the nacelle for structural control. For application studies of these systems, refer to [1–6]. The TMDs are two independent, 1 DOF, linear mass spring damping elements that act in the fore-aft ($x$) and side-side ($y$) directions. We first present the theoretical background and then describe the code changes.

1 Theoretical Background

1.1 Definitions

- $O$: origin point of global inertial reference frame
- $P$: origin point of non-inertial reference frame fixed to nacelle where TMDs are at rest
- $TMD$: origin point of a TMD
- $G$: axis orientation of global reference frame
- $N$: axis orientation of nacelle reference frame with unit vectors $i, j, k$

\[
\vec{r}_{TMD/OG} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{TMD/OG} \quad : \text{position of a TMD with respect to (w.r.t.) } O \text{ with orientation } G
\]

\[
\vec{r}_{TMD/PN} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{TMD/PN} \quad : \text{position of a TMD w.r.t. } P_N
\]

\[
\vec{r}_{TMDX} \quad : \text{position vector for } TMD_X
\]
\[ \vec{r}_{TMD_Y} \] : position vector for TMD \( Y \)

\[ \vec{r}_{P/OG} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{P/OG} \] : position vector of nacelle w.r.t. \( O_G \)

\( R_{N/G} \): 3 x 3 rotation matrix transforming orientation \( G \) to \( N \)

\( R_{G/N} = R_{N/G}^T \) : transformation from \( N \) to \( G \)

\[ \vec{\omega}_{N/O_N} = \begin{bmatrix} \dot{\theta} \\ \phi \\ \psi \end{bmatrix}_{N/O_N} \] : angular velocity of nacelle in orientation \( N \); defined likewise for \( G \)

\[ \ddot{\vec{r}}_{N/O_N} = \ddot{\vec{a}}_{N/O_N} : \text{angular acceleration of nacelle} \]

\[ \vec{a}_{G/O_G} = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}_{/O_G} \] : gravitational acceleration in global coordinates

\[ \ddot{\vec{a}}_{G/O_N} = R_{N/G} \ddot{\vec{a}}_{G/O_G} = \begin{bmatrix} a_{GX} \\ a_{GY} \\ a_{GZ} \end{bmatrix}_{/O_N} \] : gravity w.r.t. \( O_N \)

### 1.2 Equations of motion

The position vectors of the TMDs in the two reference frames \( O \) and \( P \) are related by

\[ \vec{r}_{TMD/OG} = \vec{r}_{P/OG} + \vec{r}_{TMD/PG} \]

Expressed in orientation \( N \),

\[ \vec{r}_{TMD/ON} = \vec{r}_{P/ON} + \vec{r}_{TMD/PN} \]

\[ \Rightarrow \vec{r}_{TMD/PN} = \vec{r}_{TMD/ON} - \vec{r}_{P/ON} \]

Differentiating,

\[ \dot{\vec{r}}_{TMD/PN} = \dot{\vec{r}}_{TMD/ON} - \dot{\vec{r}}_{P/ON} - \vec{\omega}_{N/O_N} \times \vec{r}_{TMD/PN} \]

differentiating again gives the acceleration of the TMD w.r.t. \( P \) (the nacelle position), oriented with \( N \):

\[ \dddot{\vec{r}}_{TMD/PN} = \dddot{\vec{r}}_{TMD/ON} - \dddot{\vec{r}}_{P/ON} - \vec{\omega}_{N/O_N} \times \dddot{\vec{r}}_{TMD/PN} - \vec{a}_{N/O_N} \times \vec{r}_{TMD/PN} \]

\[ -2 \vec{\omega}_{N/O_N} \times \vec{a}_{TMD/PN} \]

The right-hand side contains the following terms:

\[ ^1 \text{Note that } (R \vec{a}) \times (R \vec{b}) = R(\vec{a} \times \vec{b}). \]
\( \ddot{r}_{TMD/ON} \): acceleration of the TMD in the inertial frame \( O_N \)

\( \ddot{r}_{P/ON} = R_{N/O} \ddot{r}_{P/OG} \): acceleration of the Nacelle origin \( P \) w.r.t. \( O_N \)

\( \omega_{N/ON} = R_{N/O} \omega_{N/OG} \): angular velocity of nacelle w.r.t. \( O_N \)

\( \omega_{N/ON} \times (\omega_{N/ON} \times \ddot{r}_{TMD/PN}) \): Centrifugal force

\( \alpha_{N/ON} \times \ddot{r}_{TMD/PN} \): Euler force

\( 2\dot{\omega}_{N/ON} \times \dot{r}_{TMD/PN} \): Coriolis force

The acceleration in the inertial frame \( \ddot{r}_{TMD/ON} \) can be replaced with a force balance

\[
\ddot{r}_{TMD/ON} = \frac{1}{m} \sum \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}_{TMD/ON} = \frac{1}{m} \ddot{F}_{TMD/ON}
\]

Substituting the force balance into Equation 1 gives the general equation of motion for a TMD:

\[
\ddot{r}_{TMD/PN} = \frac{1}{m} \ddot{F}_{TMD/ON} - \ddot{r}_{P/ON} - \omega_{N/ON} \times (\omega_{N/ON} \times \ddot{r}_{TMD/PN}) - \alpha_{N/ON} \times \ddot{r}_{TMD/PN} - 2\dot{\omega}_{N/ON} \times \dot{r}_{TMD/PN}
\]

We will now solve the equations of motion for \( TMD_X \) and \( TMD_Y \).

**TMD\_X**: The external forces \( \ddot{F}_{TMDX/ON} \) are given by

\[
\ddot{F}_{TMDX/ON} = \begin{bmatrix} -c_x \ddot{x}_{TMDX/PN} - k_x x_{TMDX/PN} + m_x a_{GX/O_N} + F_{ext_x} + F_{StopFrcX} \\ F_{TY_{TMDX/ON}} + m_x a_{GY/O_N} \\ F_{TZ_{TMDX/ON}} + m_x a_{GZ/O_N} \end{bmatrix}
\]

\( TMD_X \) is fixed to frame \( N \) in the \( y \) and \( z \) directions so that

\[
r_{TMDX/PN} = \begin{bmatrix} x_{TMDX/PN} \\ 0 \\ 0 \end{bmatrix}
\]

The other components of Eqn. 2 are:

\[
\omega_{N/ON} \times (\omega_{N/ON} \times \ddot{r}_{TMDX/PN}) = x_{TMDX/PN} \begin{bmatrix} -\dot{\phi}_{N/ON}^2 - \psi_{N/ON}^2 \\ -\dot{\theta}_{N/ON} \dot{\phi}_{N/ON} \\ -\dot{\theta}_{N/ON} \dot{\psi}_{N/ON} \end{bmatrix}
\]

\[
2\dot{\omega}_{N/ON} \times \dot{r}_{TMDX/PN} = \dot{\ddot{x}}_{TMDX/PN} \begin{bmatrix} 0 \\ 2\psi_{N/ON} \\ -2\dot{\phi}_{N/ON} \end{bmatrix}
\]
\[ \vec{a}_{N/ON} \times \vec{r}_{TMDY/PN} = \vec{x}_{TMDY/PN} \begin{bmatrix} 0 \\ \ddot{\psi}_{N/ON} \\ -\ddot{\phi}_{N/ON} \end{bmatrix} \]

Therefore \( \vec{x}_{TMDY/PN} \) is governed by the equations

\[ \ddot{x}_{TMDY/PN} = \left( \dot{\phi}_{N/ON}^2 + \dot{\psi}_{N/ON}^2 - \frac{k}{m_x} \right)x_{TMDY/PN} - \left( \frac{c_x}{m_x} \right)\dot{x}_{TMDY/PN} - \ddot{x}_{P/ON} + a_{Gx/ON} \]  

\[ + \frac{1}{m_x}(F_{extx} + F_{StopFrC}) \]  

(3)

The forces \( F_{x/TMDY/ON} \) and \( F_{z/TMDY/ON} \) are solved noting \( \ddot{y}_{TMDY/PN} = \ddot{z}_{TMDY/PN} = 0 \):

\[ F_{x/TMDY/ON} = m_x \left( -a_{Gy/ON} + \ddot{y}_{P/ON} + (\ddot{\psi}_{N/ON} + \dot{\theta}_{N/ON} \dot{\phi}_{N/ON})x_{TMDY/PN} + 2\dot{\psi}_{N/ON} \ddot{x}_{TMDY/PN} \right) \]  

(4)

\[ F_{z/TMDY/ON} = m_x \left( -a_{Gz/ON} + \ddot{z}_{P/ON} - (\ddot{\phi}_{N/ON} - \dot{\theta}_{N/ON} \dot{\psi}_{N/ON})x_{TMDY/PN} - 2\dot{\phi}_{N/ON} \ddot{x}_{TMDY/PN} \right) \]  

(5)

**TMD_Y**: The external forces \( \vec{F}_{TMDY/PN} \) on TMD_Y are given by

\[ \vec{F}_{TMDY/PN} = \begin{bmatrix} F_{x/TMDY/ON} + m_y a_{Gx/ON} \\ -c_y \ddot{y}_{TMDY/PN} - k_y y_{TMDY/PN} + m_y a_{Gy/ON} + F_{exty} + F_{StopFrcy} \\ F_{z/TMDY/ON} + m_y a_{Gz/ON} \end{bmatrix} \]

TMD_Y is fixed to frame N in the x and z directions so that

\[ r_{TMDY/PN} = \begin{bmatrix} 0 \\ y_{TMDY/PN} \end{bmatrix} \]

The other components of Eqn. 2 are:

\[ \vec{\omega}_{N/ON} \times (\vec{\omega}_{N/ON} \times \vec{r}_{TMDY/PN}) = \dddot{y}_{TMDY/PN} \begin{bmatrix} \dddot{\phi}_{N/ON} \\ \dddot{\psi}_{N/ON} \\ \dddot{\theta}_{N/ON} \end{bmatrix} \]

\[ 2\dddot{\psi}_{N/ON} \times \vec{r}_{TMDY/PN} = \dddot{y}_{TMDY/PN} \begin{bmatrix} -2\dot{\psi}_{N/ON} \\ 0 \\ 2\dot{\phi}_{N/ON} \end{bmatrix} \]

\[ \dddot{\phi}_{N/ON} \times \vec{r}_{TMDY/PN} = \dddot{y}_{TMDY/PN} \begin{bmatrix} -\dot{\psi}_{N/ON} \\ 0 \\ \dddot{\theta}_{N/ON} \end{bmatrix} \]

Therefore \( \dddot{y}_{TMDY/PN} \) is governed by the equations

\[ \dddot{y}_{TMDY/PN} = \left( \dot{\phi}_{N/ON}^2 + \dot{\psi}_{N/ON}^2 - \frac{k_y}{m_y} \right)y_{TMDY/PN} - \left( \frac{c_y}{m_y} \right)\dddot{y}_{TMDY/PN} - \dddot{y}_{P/ON} + a_{Gy/ON} \]  

\[ + \frac{1}{m_y}(F_{exty} + F_{StopFrcy}) \]  

(6)
The forces $F_{X_{TMDY/ON}}$ and $F_{Z_{TMDY/ON}}$ are solved noting $\ddot{x}_{TMDY/PN} = \ddot{z}_{TMDY/PN} = 0$:

$$F_{X_{TMDY/ON}} = m_y \left(-a_{GX/ON} + \ddot{x}_{P/ON} - (\ddot{\psi}_{N/ON} - \dot{\phi}_{N/ON})y_{TMDY/PN} - 2\dot{\psi}_{N/ON} \dot{y}_{TMDY/PN} \right)$$ (7)

$$F_{Z_{TMDY/ON}} = m_y \left(-a_{GZ/ON} + \ddot{z}_{P/ON} + (\ddot{\theta}_{N/ON} + \dot{\phi}_{N/ON})y_{TMDY/PN} + 2\dot{\theta}_{N/ON} \dot{y}_{TMDY/PN} \right)$$ (8)

### 1.3 State Equations

**Inputs:** The inputs are the nacelle linear acceleration and angular position, velocity and acceleration:

$$\ddot{\bar{u}} = \begin{bmatrix} \ddot{\nu}_{P/OG} \\ \ddot{\alpha}_{N/G} \\ \ddot{\dot{\phi}}_{P/OG} \end{bmatrix} \Rightarrow \begin{bmatrix} \ddot{v}_{P/ON} \\ \ddot{\omega}_{N/OG} \\ \ddot{\alpha}_{P/OG} \end{bmatrix} = \begin{bmatrix} \ddot{R}_{N/G} \dot{\nu}_{P/OG} \\ \ddot{R}_{N/G} \ddot{\omega}_{N/OG} \\ \ddot{R}_{N/G} \ddot{\alpha}_{P/OG} \end{bmatrix}$$

**States:** The states are the position and velocity of the TMDs along their respective DOFS in the Nacelle reference frame:

$$\ddot{R}_{TMD/PN} = \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix}_{TMD/PN} = \begin{bmatrix} x_{TMDX/PN} \\ \dot{x}_{TMDX/PN} \\ x_{TMDDY/PN} \\ \dot{y}_{TMDDY/PN} \end{bmatrix}$$

The equations of motion can be re-written as a system of non-linear first-order equations of the form

$$\ddot{R}_{TMD} = A\ddot{R}_{TMD} + B$$

where

$$A(\bar{u}) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \ddot{\phi}_{P/ON}^2 + \ddot{\psi}_{P/ON}^2 - \frac{\ddot{x}}{m_x} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddot{\theta}_{P/ON}^2 + \ddot{\psi}_{P/ON}^2 - \frac{\ddot{y}}{m_y} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$$B(\bar{u}) = \begin{bmatrix} -\ddot{x}_{P/ON} + a_{GX/ON} + \frac{1}{m_x} (F_{extX} + F_{StopFrcX}) \\ -\ddot{y}_{P/ON} + a_{GY/ON} + \frac{1}{m_y} (F_{extY} + F_{StopFrcY}) \end{bmatrix}$$

The inputs are coupled to the state variables, resulting in A and B as $f(\bar{u})$. 
1.4 Outputs

The output vector $\vec{Y}$ is

$$\vec{Y} = \begin{bmatrix} \vec{F}_{PG} \\ \vec{M}_{PG} \end{bmatrix}$$

The output includes reaction forces corresponding to $F_{Y_{TMDX/O_N}}$, $F_{Z_{TMDX/O_N}}$, $F_{X_{TMDY/O_N}}$, and $F_{Z_{TMDY/O_N}}$ from Eqns. 4, 5, 7, and 8. The resulting forces $\vec{F}_{PG}$ and moments $\vec{M}_{PG}$ acting on the nacelle are

$$\vec{F}_{PG} = R^T_{N/G} \begin{bmatrix} k_x x_{TMD/PN} + c_x \dot{x}_{TMD/PN} - F_{\text{StopFrcX}} - F_{\text{extx}} - F_{X_{TMDY/O_N}} \\ k_y y_{TMD/PN} + c_y \dot{y}_{TMD/PN} - F_{\text{StopFrcY}} - F_{\text{exty}} - F_{Y_{TMDX/O_N}} \\ -F_{Z_{TMDX/O_N}} - F_{Z_{TMDY/O_N}} \end{bmatrix}$$

and

$$\vec{M}_{PG} = R^T_{N/G} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = R^T_{N/G} \begin{bmatrix} -(F_{Z_{TMDY/O_N}}) y_{TMD/PN} \\ (F_{Z_{TMDY/O_N}}) x_{TMD/PN} \\ (F_{X_{TMDY/O_N}}) y_{TMD/PN} \end{bmatrix}$$

Stop Forces The extra forces $F_{\text{StopFrcX}}$ and $F_{\text{StopFrcY}}$ are added to output forces in the case that the movement of TMD$X$ or TMD$Y$ exceeds the maximum track length for the mass. Otherwise, they equal zero. The track length has limits on the upwind (UW) and downwind (DW) ends in the $x$ direction ($X_{UWSP}$ and $X_{DWSP}$), and the positive and negative lateral ends in the $y$ direction ($Y_{PLSP}$ and $Y_{NLSP}$). If we define a general maximum and minimum displacements as $x_{max}$ and $x_{min}$, respectively, the stop forces have the form

$$F_{\text{StopFrc}} = \begin{cases} k_S \Delta x & : (x > x_{max} \land \dot{x} <= 0) \lor (x < x_{min} \land \dot{x} >= 0) \\ k_S \Delta x + c_S \dot{x} & : (x > x_{max} \land \dot{x} > 0) \lor (x < x_{min} \land \dot{x} < 0) \\ 0 & : \text{otherwise} \end{cases}$$

where $\Delta x$ is the distance the mass has traveled beyond the stop position and $k_S$ and $c_S$ are large stiffness and damping constants.

2 Code Modifications

The TMD function is submodule called in ServoDyn. In addition to references in ServoDyn.f90 and ServoDyn.txt, new files that contain the TMD module are listed below.

2.1 New Files

- TMD.f90 : TMD module
- TMD.txt : registry file
  include files, inputs, states, parameters, and outputs shown in Tables 1 and 2
- TMD\_Types.f90 : automatically generated
Table 1: Summary of field definitions in the TMD registry. Note that state vector $tmd_x$ corresponds to $\vec{R}_{TMD/PN}$, and that the outputs $\vec{F}_{PG}$ and $\vec{M}_{PG}$ are contained in the MeshType object (y.Mesh). $X_{DSP}$ and $Y_{DSP}$ are initial displacements of the TMDs.

### 2.2 Variables

The input, parameter, state and output definitions are summarized in Table 1. The inputs from file are listed in Table 2.

### 3 Acknowledgements

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### References

[1] Matthew A. Lackner and Mario A. Rotea. Passive structural control of offshore wind turbines. *Wind energy*, 14(3):373–388, 2011.

[2] Matthew A. Lackner and Mario A. Rotea. Structural control of floating wind turbines. *Mechatronics*, 21(4):704–719, 2011.

[3] Hazim Namik, M. A. Rotea, and Matthew Lackner. Active structural control with actuator dynamics on a floating wind turbine. In Proceedings of the 51st AIAA Aerospace Sciences Meeting, pages 7–10, 2013.
| Field Name   | Field Type | Description                                      |
|--------------|------------|--------------------------------------------------|
| TMD_CMODE    | int        | Control Mode (1:passive, 2:active)               |
| TMD_X_DOF    | logical    | DOF on or off                                   |
| TMD_Y_DOF    | logical    | DOF on or off                                   |
| TMD_X_DSP    | real       | TMD_X initial displacement                       |
| TMD_Y_DSP    | real       | TMD_Y initial displacement                       |
| TMD_X_M      | real       | TMD mass                                         |
| TMD_X_K      | real       | TMD stiffness                                    |
| TMD_X_C      | real       | TMD damping                                      |
| TMD_Y_M      | real       | TMD mass                                         |
| TMD_Y_K      | real       | TMD stiffness                                    |
| TMD_Y_C      | real       | TMD damping                                      |
| TMD_X_DWS    | real       | DW stop position (maximum X mass displacement)  |
| TMD_X_UWS    | real       | UW stop position (minimum X mass displacement)   |
| TMD_X_KS     | real       | stop spring stiffness                            |
| TMD_X_CS     | real       | stop spring damping                              |
| TMD_Y_PLS    | real       | positive lateral stop position (maximum Y mass displacement) |
| TMD_Y-NLS    | real       | negative lateral stop position (minimum Y mass displacement) |
| TMD_Y_KS     | real       | stop spring stiffness                            |
| TMD_Y_CS     | real       | stop spring damping                              |
| TMD_P_X      | real       | x origin of P in nacelle coordinate system       |
| TMD_P_Y      | real       | y origin of P in nacelle coordinate system       |
| TMD_P_Z      | real       | z origin of P in nacelle coordinate system       |

Table 2: Data read in from TMDInputFile.
[4] G. Stewart and M. A. Lackner. Optimization of a passive tuned mass damper for reducing loads in offshore wind turbines. *IEEE Transactions on Control Systems Technology*, 21(4):1090–1104, 2013.

[5] Gordon M. Stewart and Matthew A. Lackner. The effect of actuator dynamics on active structural control of offshore wind turbines. *Engineering Structures*, 33(5):1807–1816, 2011.

[6] Gordon M. Stewart and Matthew A. Lackner. The impact of passive tuned mass dampers and windwave misalignment on offshore wind turbine loads. *Engineering Structures*, 73:54–61, 2014.