Visualization of intermittency of low-Reynolds-number anisotropic steady incompressible turbulence using implicit/SGS-model-based large-eddy simulation

Mayuka Oshibuchi1, Hiroki Suzuki2, 3 and Shinsuke Mochizuki1

1 Graduate School of Sciences and Technology for Innovation, Yamaguchi University, 2-16-1 Tokiwadai, Ube-shi, Yamaguchi 755-8611, Japan
2 Graduate School of Natural Science and Technology, Okayama University, 3-1-1 Tsushima-naka, Kita-ku, Okayama-shi, Okayama 700-8530, Japan

3 E-mail: h.suzuki@okayama-u.ac.jp

Abstract. The present study validates a large-eddy simulation from the viewpoint of the intermittency of a turbulence field for low-Reynolds-number anisotropic steady turbulence. Analyses based on implicit large-eddy simulation and the Vreman-model are validated. The results of the two analyses are compared with results obtained through direct numerical simulation. The mean value of the global turbulent kinetic energy obtained through the analysis based on the large-eddy simulation is consistent with that obtained through the present direct simulation. The frequency of large-scale vortex structures found in instantaneous fields and higher-order turbulence statistics are examined. The intermittency of the turbulent fields obtained through the implicit large-eddy simulation is higher than that obtained through the other analysis.

1. Introduction
Turbulence is widely found in equipment related to fluids engineering [1]. In addition, turbulent diffusion is often used when considering turbulent mixing of heat and mass (e.g., [2-3]). Numerical analysis is widely used to analyze this kind of turbulence field. Large-eddy simulation (LES) is commonly used as a numerical method for analyzing turbulence fields. In LES, the number of grid points required can be reduced by modeling a small-scale turbulence field. The present study considers that the most widely used LES model is the Smagorinsky model [1]. This model is applied to turbulent fields in which the Reynolds number is sufficiently high. The Vreman model [4-6] is often used when the Reynolds number is low, as found in the turbulent transition from laminar to turbulent flow. This model is found in a review on the statistical characteristics of LES models [7] and is also applied in a recent numerical work [8]. The implicit LES is also used as a technique that does not require a subgrid scale (SGS) model [9-11].

Low-Reynolds-number turbulence can often be found in turbulent fields. For example, the Reynolds number of turbulence is low in the inner layer of wall turbulence. Such turbulent fields are often anisotropic. The LES model has been used and validated for the analysis of wall turbulence fields (e.g., [12-14]). The turbulence statistics of anisotropic steady turbulence fields with low Reynolds numbers can be predicted with sufficient accuracy by the LES model. On the other hand, the turbulent field near a wall surface is intermittent. Few previous studies have validated the LES model.
with a focus on intermittency. Therefore, the present study focuses on validating the LES model for a low-Reynolds-number anisotropic turbulence field in the presence of intermittency.

The purpose of the present study is to validate the results of implicit LES and LES based on an SGS model in low-Reynolds-number steady turbulence. Here, an idealized low-Reynolds-number anisotropic steady turbulence is used. In this way, the LES analysis results can be examined in more detail; the Vreman model [4] is used as the SGS model. In contrast to the Smagorinsky model, adjustment of the model constants is not required when analyzing low-Reynolds-number turbulence. In addition, in the present study, implicit LES and LES based on the SGS model are examined from the perspective of intermittency. In order to validate the results, visualization of the instantaneous turbulence structure (e.g., [15]) and the turbulence statistics that quantify the intermittency are used. The results obtained through this LES are compared with those obtained through direct numerical simulation (DNS). This study considers that the innovation of this paper is to clearly examine the accuracy of LES analysis from the perspective of intermittency.

2. Numerical methods

The flow targeted by the present study is incompressible. The governing equations for incompressible flows are the continuity equation and the Navier-Stokes equation with forcing terms. Here, the coordinate system comprises the streamwise, transverse, and spanwise directions, i.e., the $x_1$, $x_2$, and $x_3$ directions. The equations are made dimensionless using the bulk Reynolds number. The governing equations contain an external forcing term, which is used to keep the turbulent flow field steady. This external force $F_i$ ($i = 1, 2, 3$) is expressed based on a linear forcing scheme. Based on the linear forcing scheme (e.g., [16]) and a previous study [17], the external forcing term is given as follows:

$$F_i = Qu^i,$$

where $u^1 = -\cos(x_1) \sin(x_2)$, $u^2 = \sin(x_1) \cos(x_2)$, and $u^3 = 0$. Here, the constant $Q$ has a value of unity. The use of an anisotropic forcing term is based on Taylor's analytical solution.

Two methods are used for the LES analysis: implicit LES and LES based on the Vreman model. Implicit LES has been used in previous studies [4-6] and provides a small-scale turbulent field without using a sub-grid scale (SGS) model. The Vreman model [4] has also been often used in previous studies (e.g., [6]). In contrast to the Smagorinsky model, the Vreman model can be used to calculate laminar-turbulent transitions. Since in the present study, low-Reynolds-number steady anisotropic turbulence is investigated, is used. A value of a model constant included in the Vreman model is set to 0.025 based on a previous study [4].

A periodic cubic computational domain is used, with each side being $2\pi$. The number of grid points for LES analysis is $32^3$. On the other hand, grid points of $64^3$ is set in DNS analysis as a finer spatial resolution. The governing equations are discretized using a fourth-order central difference scheme [18], for which the kinetic energy of the turbulence is within the inviscid limit [19]. Here the convective terms are discretized using the skew-symmetric form. The governing equations are integrated over time using the fractional step method based on the fourth-order Runge-Kutta scheme. Poisson's equation is solved at each step using a fast Fourier transform. This numerical code is developed and verified by the authors. Also, the present numerical code is constructed by modifying that of a previous work of the present research group [12]. The calculation method of ILES in this study is the same as that of DNS. Since the Reynolds number of this study is low, this study confirms that the numerical vibration caused by the low spatial resolution is not affected by the visualization results. The bulk Reynolds number is set to 40, so that the turbulence Reynolds number based on the Taylor microscale is approximately 40. A value of turbulent Reynolds number does not necessarily correspond to that of the bulk Reynolds number. In the present turbulence, the value of the turbulent Reynolds number agrees fortuitously with that of the bulk Reynolds number. Not only the value of the bulk Reynolds number but also the value of turbulent Reynolds number are sufficiently similar between LES analysis and DNS. This study considers this agreement to be more important to derive the present findings.
3. Results and discussion

The temporal intermittency of the turbulence kinetic energy, which was spatially averaged in the computational domain, was examined. This is referred to as the global turbulence kinetic energy. Figure 1(a) shows time series of the global turbulence kinetic energy obtained using DNS, implicit LES (ILES), and standard LES (SLES). Here, VLES is often used as an abbreviation for very large-eddy simulation. In this study, SLES is used rather than VLES in order to avoid confusing LES based on the Vreman model with the analysis method. The amplitude of the global turbulence kinetic energy is similar for all methods. The temporal change is not similar to a sinusoidal function, and exhibits intermittency. This is also true for the ILES and SLES results. Figure 1(b) shows the time-averaged turbulence kinetic energy values for DNS, ILES, and SLES. The mean values for ILES and SLES roughly agree with those for DNS, with a difference of less than 8%.

The above results validate the accuracy of ILES and SLES visualization of the instantaneous anisotropic steady turbulence field [15]. Figure 2 shows the instantaneous turbulence field at $t = 150, 200, 250, 300, 350$, and $400$, where $t$ is nondimensional time. These visualizations are based on the isosurfaces of negative values of the instantaneous pressure, which allow large-scale vortices to be seen. As shown in the figure, a vortex tube structure occurs intermittently, and is different each time. In the DNS results, a two-dimensional vortex structure with an axis in the span direction near the
Figure 2. DNS results for instantaneous turbulence field visualized using the negative value of static pressure at $t = 150, 200, 250, 300, 350,$ and $400$. The red arrows indicate a large-scale turbulence vortex structure around the center of the computational domain. The center of the computational domain is found at times $t = 200$ and 250. The velocity fluctuation in the span direction deviates from that in the two orthogonal directions. This large-scale vortex structure is found at two of the six times visualized, which indicates that it is temporally intermittent. At other times, smaller vortex tube structures are found in the turbulence field.

Figure 3 shows ILES visualization results for the instantaneous turbulent field. Here, the isosurface of the negative static pressure fluctuations is also used. Moreover, the six times that the turbulent field is visualized are the same as those obtained by DNS. As shown in the figure, the ILES turbulence field also has a large vortex structure, and the direction of vorticity is along the $x_3$-axis, as in the case of the DNS results; it also has a similar shape. Using ILES, this vortex structure is found three out of six times, whereas for DNS, it is found just two out of six times.

Figure 4 shows the visualization results for the turbulence field reproduced by SLES based on the Vreman model. Here, the isosurface of the negative static pressure fluctuations is also used. A large vortex structure is seen only at $t = 400$, and its shape is similar to that found in the DNS turbulence field at time $t = 200$, although it has a slightly larger radius. However, since the vortex structure fluctuates slightly with time, this difference is not considered to be significant. To perform a statistical analysis, the skewness and flatness were used. The skewness $S(k)$ and the flatness $F(k)$ for a fluctuation component with a kinetic energy $k$ are expressed as:

$$S(k) = \langle k^3 \rangle / \langle k^2 \rangle^{3/2} \quad \text{and} \quad F(k) = \langle k^4 \rangle / \langle (k^2)^2 \rangle.$$

(2)

The fluctuation of the global turbulence kinetic energy $k$ is given as follows: $K = \langle K \rangle + k$, where $\langle \rangle$ denotes the ensemble average. The skewness and flatness can be used to quantify the intermittency of a time-series signal. If the probability density function for a time-series signal is Gaussian, then the skewness and flatness factors are theoretically zero and three, respectively. If the time-series signal is intermittent, the skewness and flatness deviate from these values, and so can be used as a measure of intermittency.
Figure 3. For instants of ILES, instantaneous turbulence fields visualized by using the negative value of static pressure. Here turbulence fields are visualized at $t = 150, 200, 250, 300, 350, \text{ and } 400$. The red arrows indicate the large-scale turbulence vortical structure.

Figure 4. Instantaneous turbulence fields determined by SLES using the negative value of static pressure at $t = 150, 200, 250, 300, 350, \text{ and } 400$. The red arrow indicates a large-scale turbulence vortex structure.
Figure 5. Skewness and flatness for fluctuation of turbulence kinetic energy using ILES, SLES and DNS. Here the magnitude of ±8% uncertainty is also indicated in the figures as the same with that in Figure 1.

Figures 5(a) and 5(b) show the skewness and flatness for the global turbulence kinetic energy for the DNS, ILES, and SLES results. It can be seen that the skewness and flatness obtained using DNS are larger than zero and three, respectively, indicating intermittency. Similar results were found using ILES and SLES. The skewness and flatness obtained through SLES are comparable to those obtained through DNS with an uncertainty smaller than 8%. This indicates that the intermittency of the turbulence field obtained through SLES is comparable to that obtained through DNS. On the other hand, the skewness and flatness obtained through ILES are larger than those obtained through DNS, particularly for the skewness, which indicates higher intermittency. As discussed earlier, the frequency of large-scale vortex structures obtained by ILES is higher than that using DNS and SLES. In the turbulence field obtained through ILES, the present study considers that the intermittency of the turbulence fields increases because this large-scale vortex structure exists as a long-term phenomenon.

As shown in the present study, the mean turbulent kinetic energy value of ILES is similar to those of DNS and SLES. In particular, the difference in the mean values between ILES and SLES is a few percent. The study considers that this difference is not statistically significant because of the small value of the percent. On the other hand, the skewness and flatness values of ILES are significantly higher than those of DNS and SLES. In particular, the difference between the values of these factors is clearer between ILES and SLES. The skewness and flatness of turbulent kinetic energy are non-dimensional and characterize the shape of its probability density function. This agreement between the values of the two factors implies that the structures of the flow field reproduced in ILES are different from that of DNS and SLES. As can be seen from the results of this study, under the condition of low Reynolds number, ILES can predict the value of turbulence kinetic energy with comparable accuracy as DNS and SLES. On the other hand, as shown by the visualization results and the results of skewness and flatness factors, the present results suggest that, when the structure of the turbulence field also needs to be analyzed, the analysis accuracy of ILES is insufficient, and the use of SLES should be considered.

4. Conclusions
The present study examined the turbulence field reproduced by the implicit LES and the LES based on the Vreman model from the viewpoint of intermittency. The results for anisotropic steady turbulence obtained using both types of LES analyses were compared with those obtained using DNS. The visualization results and turbulence were used to confirm the intermittency of the turbulence field obtained using both types of LES analysis. The governing equations were discretized using the fourth-
order finite difference method, for which the kinetic energy of the turbulence is within the inviscid limit.

The mean value obtained using both LES analyses agreed with that obtained using DNS. Large-scale vortex structures were intermittently found in the turbulent field obtained using DNS, but were more frequent using ILES. The intermittency determined using SLES was found to be similar to that obtained using DNS. However, the intermittency obtained using ILES was the highest, which may be due to the long-term presence of large vortices. This study has shown that, despite ILES can analyze the mean value of turbulent energy with sufficient accuracy at the low Reynolds number, this cannot predict the magnitude of intermittency of the turbulence field with sufficient accuracy. It was also shown that this decrease in accuracy for the magnitude of intermittency found in ILES is not found in SLES.

In the future, a more realistic turbulence field, such as a turbulent mixed layer, should be considered. A steady turbulence field was used for the LES analysis. However, if for example, a mean freestream fluid acceleration existed, then the turbulence field could be considered to be unsteady (e.g., [20-21]). Taking into account the influence of the mean fluid acceleration, validation using this unsteady turbulence field will also be an area for future research. Also, the magnitude of eddy viscosity used for the SGS modeling could depend on spatial resolution. Therefore, the influence of changing spatial resolution could also be a future study.

Acknowledgements
The present study was supported in part by the Japanese Ministry of Education, Culture, Sports, Science and Technology through Grants-in-Aid (Nos. 18H01369, 20H02069, and 21K03859).

References
[1] Pope S B 2000 Turbulent Flows Cambridge University Press
[2] Suzuki H, Nagata K, Sakai Y and Hasegawa Y 2013 Phys. Ser. 2013(T155) 014062
[3] Suzuki H, Nagata K and Sakai Y 2012 J. Vis. 15 109-117
[4] Vreman A W 2004 Phys. Fluids 16 3670-3681
[5] Park N, Lee S, Lee J and Choi H 2006 Phys. Fluids 18 125109
[6] Sayadi T and Moin P 2012 Phys. Fluids 24 114103
[7] Boukharfane R, Parsani M and Bodart J 2020 Sci. Rep. 10 1-19
[8] Moser R D, Haering S W and Yalla G R 2020 Ann. Rev. Fluid Mech. 53 in press
[9] Uranga A, Persson P O, Drela M and Peraire J 2011 Int. J. Numer. Meth. Eng. 87 232-261
[10] Garmann D J, Visbal M R and Orkvis P D 2013 Int. J. Numer. Meth. Fluids 71 1546-1565
[11] Grinstein F F, Margolin L G and Rider W J. (Eds.) 2007 Implicit Large Eddy Simulation Cambridge University Press
[12] Suzuki H, Nagata K, Sakai Y, Hayase T, Hasegawa Y and Ushijima T 2013 Int. J. Numer. Meth. Fluids 73 509-522
[13] Moser R D, Kim J and Mansour N N 1999 Phys. Fluids 11 943-945
[14] Lee M and Moser R D 2015 J. Fluid Mech. 774 395-415
[15] Watanabe T, Sakai Y, Nagata K, Terashima O, Suzuki H, Hayase T and Ito Y 2013 Int. J. Mod. Sim. Sci. Comput. 4 1341001
[16] Carroll P L and Blanquart G 2013 Phys. Fluids 25 105114
[17] Goto S and Vassilicos J C 2015 Phys. Lett. A 379 1144-1148
[18] Suzuki H, Nagata K, Sakai Y, Hayase T, Hasegawa Y and Ushijima T 2013 Fluid Dyn. Res. 45 061409
[19] Morinishi Y, Lund T S, Vasilyev O V and Moin P 1998 J. Comput. Phys. 143 90-124
[20] Suzuki H, Mochizuki S and Hasegawa Y 2018 Flow Meas. Instrum. 62 1-8
[21] Suzuki H, Mochizuki S and Hasegawa Y 2020 Adv. Mech. Eng. 12 1-13