Abstract. Orbital parameters of planets are fitted directly to an appropriate set of observations. It is shown how to use the rigorous Deming method combined with a numerical integration of gravitation equations. In all, 65 parameters of the nine planets (masses and initial positions and velocities at J2000) are listed. The complete set of their standard errors, and the associated variance-covariance matrix are presented for the first time. Derived parameters as the Solar gravitational quantity $G \times Mo$, the Astronomical distance $AU$, and the light time $\tau_A$ (for one $AU$) are re-evaluated. It is demonstrated that a direct fit using time units overcomes the very high correlation (99.99996 %) between the gravitational constant $G$ and the Solar mass $Mo$. Much more accurate values for these fundamental quantities are obtained:
1. Introduction

A great debate has been started about the future climate of the Earth. In this context, astronomy should provide long-term predictions for the Earth orbit (for instance values of the perihelion and aphelion).

To undertake these extrapolations, we need first a complete set of orbital parameters for planets (including all masses and initial positions and velocities) in order to perform a numerical integration of gravitation equations. Also, we need the variance-covariance matrix associated with these orbital parameters, because we must calculate the statistical errors propagated on these predictions from the variance-covariance matrix.

Unfortunately, these starting data are not all available in regular astronomical literature. Of course, recent astronomical compilations of Yoder (1995) and Simon et al (1998) provide good values for masses of planets. However, initial positions and velocities of planets (at J2000) are not published in this literature. Moreover, the usual standard errors connected with these parameters, are missing in these compilations, as well as their variance-covariance matrix.

So, the aim of this paper is to provide a complete and self-consistent set of orbital parameters for planets, including all standard errors, and also the associated variance-covariance matrix.

| parameter | value             | units | standard-deviation |
|-----------|-------------------|-------|--------------------|
| $G \times Mo$ | $1.327731601427 \times 10^{+20}$ | m$^3$/s$^2$ | $2.2 \times 10^{-09}$ |
| $AU$      | $1.496206824595 \times 10^{+11}$ | m     | $2.4$              |
| $\tau_A$ | $499.0808756753$  | s     | $8.1 \times 10^{-09}$ |
As pointed out by many authors belonging to different disciplines (Wentworth 1965a,b, Zare et al. 1973), the reduction of observed quantities should satisfy the two following requirements: \( (i) \) The fitting procedure must be a direct least-squares approach yielding the minimum-variance unbiased estimates; \( (ii) \) The method should maintain the physical or mechanical meaning of parameters.

In the case of measurements of planet positions, we are dealing with three variables (time, right ascension, and declination), and with their three corresponding uncertainties. So, fitting methods similar to the one of Oesterwinter and Cohen (1972), where errors with time are ignored, are not very rigorous (time is not an exact variable as a quantum number). Also, in view of the second requirement \( (ii) \), fitting methods using time polynomial expansions are not appropriate for a good numerical treatment of these astronomical data. First, their higher order coefficients have no physical meaning. In addition, these polynomial expressions cannot be used for extrapolations, because the relevant quantities are becoming infinite with time. So, we believe that the rigorous fitting procedure of Deming (1964, see also Wentworth’s papers (1965a,b)), combined with a numerical integration of gravitation equations satisfies these two requirements.

Thus, our plan of campaign is quite clear: \( (1) \) First, we shall begin by performing a coherent data-file of observations for the Sun and all planets. \( (2) \) Then, carry out the fitting procedure connected with these data. \( (3) \) Finally, extend astronomical data from these results.

2. Description of data
An input data-file containing 3549 lines was carried out as indicated in Table I: First, Saturn + Uranus + Neptune + Pluto observations performed at the La Palma Observatory between 1984 and 1993 were extracted from the Carlsberg Meridian Catalogue. In addition, 61 observations of Saturn obtained between 1970 and 1978 at the Paris and San Fernando Astrolabes (Chollet et al, 1973, Debarbat et al, 1975, Sanchez et al, 1992) were also included; as well as 26 observations of Pluto recorded at the Moscow and Mt Maidanak Observatories (Dolganova et al, 1993).

In order to fit simultaneously all the orbital elements of planets, these observations were extended between year 1930 (year 1900 for Pluto) and year 2000, by selecting periodic data from the “Connaissance des Temps” ephemeris for all planets, and also for the Sun (Berthier et al., 1999). Table I shows the interval (in days) between successive data for each object.

Thus, we get in all a consistent data set of 3549 observations, allowing to fit correctly the orbital parameters for planets.

3. Calculation of planet positions

Cartesian coordinates of planet positions are first computed in the standard “J2000.0” heliocentric system. For all the relevant time values, these quantities have been determined from a numerical integration of the usual following gravitation equations:

\[
\ddot{\mathbf{r}}_j + \frac{G(M_o + M_j)}{r_j^3} \mathbf{r}_j = \sum_{k \neq j} G M_k \left( \frac{\mathbf{r}_k - \mathbf{r}_j}{|\mathbf{r}_k|^3} - \frac{\mathbf{r}_k}{|\mathbf{r}_j|^3} \right)
\]

*Equation (3.1)*
Where \( \mathbf{r}_j \) represents the heliocentric position of the planet \( j \) having a mass \( M_j \), \( G \) is the gravitational constant, and \( M_0 \) the Solar mass at the standard time J2000.0. We put \( r_j = | \mathbf{r}_j | \), and so on.

Remember that refined effects such as the advance of the perihelion of Mercury could be explained by General Relativity corrections. As discussed in the Möller’s text-book (Möller, 1972, p. 493), it is quite simple to introduce the following \( \gamma \) term in eqs. (3.1):

\[
\gamma = \left( 1 - \frac{2G M_0 r_j}{c^2} - \left( \frac{u_j}{c} \right)^2 \right)^{1/2}
\]

where \( u_j \) designates the heliocentric velocity of the planet, and \( c \) the usual light speed.

In our case (with nine planets), equations (3.1) yield 54 first order differential equations. These coupled equations have been integrated, numerically, from the 10th degree Fehlberg’s algorithm (Fehlberg, 1969) with a step of integration equal to \( h = 3 \times 10^4 \) sec (\( \approx 0.347 \) day).

Then, heliocentric coordinates of the center of the Earth are extracted from positions of the barycentric Earth-Moon system, by using Lunar data of Chapront-Touzé et al (1988).
The apparent geocentric direction of the planet (or the Sun) at time $t_i$ is then obtained from the geometric positions of the Earth and the object at the retarded time $t_i - \tau_i$ (backward integration), where $\tau_i$ is the time of flight of the photon (Danjon, 1959).

Finally, the apparent angular coordinates $\alpha_i$ (right ascension) and $\delta_i$ (declination) are calculated, and corrected for precession from data of Williams (1994), and for nutation via the routine of Kinoshita et al (1990).

4. The fitting procedure

The orbital parameters of planets have been simultaneously fitted to the set of data described in section 2 by using the fitting procedure of Deming (1964), applied to physical sciences by Wentworth (1965a,b).

Note that this method allows to minimize the sum of the weighted squares of residuals:

$$S = \sum_{i=1}^{n} \left[ W_t (t_i - \bar{t}_i)^2 + W_\alpha (\alpha_i - \bar{\alpha}_i)^2 + W_\delta (\delta_i - \bar{\delta}_i)^2 \right]$$

equation (4.1a)

where $t_i, \alpha_i, \delta_i$ and $\bar{t}_i, \bar{\alpha}_i, \bar{\delta}_i$ represent the observed and calculated quantities, respectively. The statistical weights are given by:

$$W_t = \frac{\sigma^2}{\sigma(t)_i^2}; \quad W_\alpha = \frac{\sigma^2}{\sigma(\alpha_i)^2}; \quad W_\delta = \frac{\sigma^2}{\sigma(\delta_i)^2}$$

equation (4.1b)
where $\sigma(t)$, $\sigma(\alpha)$, $\sigma(\delta)$ are the corresponding uncertainties, and $\sigma_0^2$ the arbitrary variance of unit weight.

Note that the goodness of the fit is given by the reduced variance

$$\sigma = \left( \frac{S}{(n - p)} \right)^{1/2} \quad (4.2)$$

where $n$ is the total number of observations, and $p$ the number of fitted parameters. In our case of three variables, it is convenient to put $\sigma_0^2 = 1/3$, because a perfect fit corresponds to $\sigma \simeq 1$. Of fundamental importance is the variance-covariance matrix

$$V = \sigma^2 N^{-1}$$

where $N$ is the normal equation matrix whose elements are given in appendix A. Remember that diagonal elements of $V$ represent the squares of standard errors of the fitted parameters. $V$ is also essential for calculations of the errors propagated to astronomical quantities.

5. Results

Our final fit yields a reduced variance $\sigma = 0.6$. This rather good value indicates that all modulus of residuals are nearly smaller than their corresponding uncertainties. We note that it is the case of those of Mercury, suggesting that, as expected, the General Relativity correction $\gamma$ (eq. 3.2) is convenient. In addition, 65 orbital parameters have been simultaneously fitted to the preceding 3549 input data. They are collected, for each planet, along with their statistical errors, in Tables II-III.

Because of the very high correlation coefficient (99.99996 %) between the gravitational constant $G$ and the Solar mass $M_0$, time units (TU), with $G = 1$ and $c = 1$, are the most
appropriate ones for these calculations (Synge, 1966). These results appear on the left part of Tables II-III. Physical units (PU) have been also used, allowing to fit the gravitational constant $G$. The corresponding quantities are displayed on right part of these tables. Some recent data of Yoder (1995) and of Simon et al (1998) are also reported for comparison. We emphasize that all these fitted orbital parameters (positions and velocities) are those for the standard epoch J2000.0: January 1st 2000 at 12h (Julian day 2 451 545.0).

Table II indicates that the solar mass $M_o$ is determined with a very good accuracy of about $2 \times 10^{-11}$ by using time units, whereas this accuracy is only $2 \times 10^{-4}$ with physical units, because of the high correlation coefficient between $G$ and $M_o$. Hence, the accuracy of the gravitational constant is also of about $2 \times 10^{-4}$.

We note that our determinations of $G$ and $M_o$ (with physical units) are in agreement with previous data of Yoder and Simon et al. Of fundamental importance is also the product $G \times M_o$, which appears at the bottom of Table II. Note that our value of $G \times M_o$ has the same accuracy ($2 \times 10^{-11}$) as our $TU$ value of $M_o$, because masses in $PU$ and in $TU$ are connected by relation (Synge, 1966):

$$M(\text{PU}) = M(TU) \times \frac{c^3}{G}$$

(5.1)

So, $G \times M_o$ in $PU$ is merely the product of the cube of the speed of light (which is an exact value) by $M_o$ in $TU$. In other words, this important parameter $G \times M_o$ is fitted directly to the data, if $TU$ are used.

Table II shows that there is a strong disagreement between our value and the one reported by Yoder (the relative difference is of about $1.5 \times 10^{-4}$). First, we remark that their value was not fitted directly to their data, but calculated via the Kepler’s third law $G \times M_o = k^2(AU)^3d^{-2}$. So, as the Gaussian constant $k$ and the Julian day $d$ are exact quantities, we believe that it is their $AU$ value which is not as accurate as they wrote. This is due to the
fact that parameters involving both $G$ and $M_o$ could not be fitted simultaneously with a very good accuracy, because of the very important correlation coefficient between $G$ and $M_o$. In other words, previous data for $G \times M_o$, $AU$, and $\tau_A$ (light time for one $AU$) reported in literature contain ineluctably this “correlation” error of about $2 \times 10^{-4}$ (that we have obtained with $PU$). So, the most accurate way for obtaining these fundamental quantities is: (1) Fit the data by using $TU$; (2) calculate $G \times M_o$ (with eq. 5.1); (3) compute $AU$, and then $\tau_A$ from the Kepler’s third law. Our revised values for these parameters appear in Table II. Their precision is also of about $2 \times 10^{-11}$ (as for $M_o$ with $TU$).

Note also that we have carried out an attempt to detect the time-dependence of the Solar mass $M_s$ via the relation (at time $t_i$) $M_s=M_0 \times \exp(-\rho \times t_i)$. As expected (because our data-file contains only observations for one century), the determination of this new parameter $\rho$ was not statistically significant. Thus, this coefficient was fixed to zero in all our fits.

From Table III, consider now masses of planets. For bigger ones, their masses are determined with an accuracy of about 0.02 % for Jupiter and Saturn, 0.7 % for Uranus, and 1 % for Neptune. For small planets, the precision is 0.2 % for masses of Venus and for the Earth-Moon system, 0.6 % for Mars, and only 4% for Mercury. In the case of Pluto, our accuracy was so bad (a standard error as great as the parameter) that this mass was fixed in our fits to the value reported by Simon et al. (1998).

Determinations of masses are very difficult because derivatives $\frac{\partial \alpha}{\partial M}$ and $\frac{\partial \delta}{\partial M}$ are very small. Note also that masses in $PU$ and $TU$ are connected by relation (5.1). Except for Pluto (which has a fixed value), this relationship is satisfied for all planets, within the combined standard errors.

From results collected in Table III, we have also calculated in Table IV the accuracy $\frac{\Delta r_o}{r_o}$ and $\frac{\Delta u_o}{u_o}$ of the initial values of positions and velocities of planets. We note that these precisions are very good: about $7 \times 10^{-8}$ for the barycentre of the Earth-Moon system in the best case, few units of $10^{-7}$ for Jupiter, Mars and Venus, and of the
order of $10^{-6}$ for the other planets, even for Pluto. In other words, orbits of planets depend strongly on the initial positions and velocities, but only a little on their masses. The case of Mercury and Pluto illustrates this remark: they have nearly similar masses, but very different trajectories.

Finally, our results are compared (in Table III) with those previously obtained by Yoder (1995) and Simon et al (1998) for masses, even if errors are rarely reported by these authors. Nevertheless, there is a rather good agreement between all these masses.

6. Conclusion

This paper provides a self-consistent set of orbital parameters, including their standard errors, which have been simultaneously fitted to appropriate data. By combining these parameters with the relevant variance-covariance matrix, it will be possible to perform long-term predictions of astronomical data, along with their propagated errors. Concerning parameters, we have shown that it is possible to overcome the very high correlation between the gravitational constant $G$ and the Solar mass $M_0$. So, we have proposed values for the fundamental quantities $G \times M_0$, $AU$, and $\tau_A$, which are much more accurate than those previously reported in literature.

In a near future we plan, as far as possible, to add new observations in our data-file (for instance those of Mars and Jupiter recorded at the San Fernando Observatory, when these raw data will be available). However, we believe that these forthcoming extended fits will bring only minor variations to parameters listed in the present work. (preceding data-files are available on request from the author).

Appendix A: Elements of the normal equation matrix
This section collects, without demonstrations, exact relationships allowing to code computing programs for fitting orbital parameters for planets. It follows the original work of Deming (1964), revisited by Wentworth (1965 a,b). The case of observations of positions of planets corresponds to the method detailed in Deming’s book (p. 50), for three variables ($t, \alpha, \delta$) connected by two “condition equations” for each object, and with many parameters. Essentially, the method of Lagrange multipliers allows to minimize the sum of the weighted squares of residuals (see eq. 4.1a).

Recall that it is an iterative method. Thus, the correction-vector $\Delta A$ (with components $\Delta a_j$) to the initial approximate parameter vector $A^o$ (with components $a_j^o$) is given by:

$$\Delta A = N^{-1} \Delta W$$

So, corrected parameters are ($p$ is the total number of parameters):

$$a_j = a_j^o - \Delta a_j \quad (j = 1, 2, 3, \ldots, p)$$

Elements of the normal equation matrix $N$ (square-symmetric of dimensions $p \times p$) are given by the following equations:

$$N(j, k) = \sum_i \left( \frac{\partial \alpha}{\partial a_k} \times u_i(j) + \frac{\partial \delta}{\partial a_k} \times v_i(j) \right)$$

Remark that dimensions of the normal equation matrix $N$ ($p \times p$) are quite small. We do not need to handle “enormous” matrix as reported in calculations of Oesterwinter and Cohen (1972).

The $p$ elements of the error-vector $\Delta W$ are:

$$\Delta W(j) = \sum_i \left( V \alpha^o \cdot u_i(j) + V \delta^o \cdot v_i(j) \right)$$
where \( V\alpha^o \) and \( V\delta^o \) represent the initial residuals of right ascension and declination. Summations are extended over all the \( n \) observations.

Furthermore, the \( u_i(j) \) and \( v_i(j) \) terms are computed as follows:

\[
    u_i(j) = s_i \left( q_i \frac{\partial \alpha}{\partial a_i} - r_i \frac{\partial \delta}{\partial a_i} \right)
\]

\[
    v_i(j) = s_i \left( p_i \frac{\partial \delta}{\partial a_i} - r_i \frac{\partial \alpha}{\partial a_i} \right)
\]

with (see also eq. 4.1b)

\[
    p_i = \left( \frac{1}{W\alpha} \right) + \left( \frac{\partial \alpha}{\partial t_i} \right)^2 / Wt_i
\]

\[
    q_i = \left( \frac{1}{W\delta} \right) + \left( \frac{\partial \delta}{\partial t_i} \right)^2 / Wt_i
\]

\[
    r_i = \left( \frac{\partial \alpha}{\partial t_i} \frac{\partial \delta}{\partial t_i} \right) / Wt_i
\]

\[
    s_i = \left( p_i q_i - r_i^2 \right)^{-1}
\]

Finally, residuals of the three variables \((Vt_i, V\alpha, V\delta)\) could be calculated via the Lagrange multipliers \( \lambda_i \) and \( \mu_i \):
\[ V\alpha = \lambda / W\alpha ; \quad V\delta = \mu / W\delta ; \quad V_t = (\lambda + \mu) / W_t, \]

with

\[ \lambda_i = s_i (d_r - b_q) \]
\[ \mu = s_i (b_r - d_q) \]

\[ b_i = V\alpha^0 - \sum_{j=1}^{p} \frac{\partial \alpha_i}{\partial a_j} \Delta a_j \]
\[ d_i = V\delta^0 - \sum_{j=1}^{p} \frac{\partial \delta_i}{\partial a_j} \Delta a_j \]

Then, we can calculate the reduced variance \( \sigma \) and the variance-covariance matrix \( V \). All preceding relationships have been used (and coded) in our work. Remark that all derivatives appearing in preceding equations have been calculated numerically.

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Table I. Description of data.

| object   | number of observations | number of predictions | period    | interval (days) | number of data |
|----------|------------------------|-----------------------|-----------|-----------------|----------------|
| Pluto    | 220 a + 26 b           | 122                   | 1900-2000 | 300             | 368            |
| Neptune  | 813 a                  | 77                    | 1930-2000 | 300             | 890            |
| Uranus   | 712 a                  | 78                    | id.       | 300             | 790            |
| Saturn   | 147 a + 61 c           | 158                   | id.       | 300             | 366            |
| Jupiter  | 183                    | id.                   | 250       |                 | 183            |
| Mars     | 200                    | id.                   | 200       |                 | 200            |
| Venus    | 231                    | id.                   | 150       |                 | 231            |
| Mercury  | 294                    | id.                   | 100       |                 | 294            |
| Sun      | 227                    | id.                   | 200       |                 | 227            |

total input data = 3549

a): data from the Carlsberg Meridian Catalogue. b): data of Dolganova et al.
c): data from Paris-Astrolabe. d): interval (in days) between predictions of Connaissance des Temps (only for data before 1980).
Table II. Gravitational constant, Solar mass, and Astronomical Quantities.

| parameter | time units (Gsec) | physical units (Gsec,Gm,GKg) | ref. |
|-----------|------------------|-----------------------------|------|
| value     | std. err.        | value                       | std. err. |
| $G$       | 1                | 6.6721168E-11               | 9.6E-15 | a |
|           |                  | 6.67259 E-11                | 8.4E-15 | b |
| $M$       | 4.927744515681E-15 | 1.9899702E+21              | 2.9E+17 | a |
|           | 8.0E-26          | 1.9891 E+21                 |         | d |
|           | 4.920            | 1.9889 E+21                 |         | b |

$G^*M$ (SI units: m$^3$/s$^2$)

| $G^*M$   | (SI units: m$^3$/s$^2$) | 1.327731601427E+20 | 2.2E+09 | a |
|          |                         | 1.3271243994 E+20  | 5.0E+10 | b |
|          |                         | 1.32712440 E+20    |         | c |

$AU$ (m)

| $AU$     | (m)            | 1.496206824595E+11 | 2.4E+00 | a |
|          |                | 1.4959787066 E+11  | 5.0E+01 | b |
|          |                | 1.4959787061 E+11  |         | c |

$\tau_A$ (s)

| $\tau_A$ | (s)             | 499.0808756753     | 8.1E-09 | a |
|          |                 | 499.00478370       |         | b |
|          |                 | 499.00478353       |         | c |

Values for J2000.0 (JD 2451545.0)

Note: a) present work; b) from Yoder(1995); c) data of Simon et al (1998); d) value of Synge(1966).
Table III. Orbital Parameters for Planets

| Parameter | Time Units | Physical Units |
|-----------|------------|----------------|
|           | (Gsec)     | (Gkg, Gm, Gsec) |
| value     | std. err.  | value          | std. err. |
| M         | 8.5633E-22 | 2.9E-23        | 3.4440E+14 | 1.2E+13 |
|           |            |                | 3.302E+14  | b        |
|           |            |                | 3.3018E+14 | c        |
| X         | -6.49314738E-08 | 1.4E-13 | -1.94658046E+01 | 4.3E-05 |
| Y         | -2.23231262E-07 | 1.0E-13 | -6.69230937E+01 | 3.1E-05 |
| Z         | -1.22762797E-08 | 2.0E-13 | -3.68033898E+00 | 6.3E-05 |
| Ux        | 1.23420487E-04 | 8.8E-11 | 3.70005539E+04  | 2.7E-02 |
| Uy        | -3.72477728E-05 | 7.3E-11 | -1.11665193E+04 | 2.3E-02 |
| Uz        | -1.43710226E-05 | 1.1E-10 | -4.30831412E+03 | 3.4E-02 |

Venus

| Parameter | Time Units | Physical Units |
|-----------|------------|----------------|
|           | (Gsec)     | (Gkg, Gm, Gsec) |
| value     | std. err.  | value          | std. err. |
| M         | 1.20865E-20 | 2.0E-23        | 4.88325E+15 | 8.4E+12 |
|           |            |                | 4.8685E+15  | b        |
|           |            |                | 4.8685E+15  | c        |
| X         | -3.58490929E-07 | 4.5E-14 | -1.07472864E+02 | 1.4E-05 |
| Y         | -1.62967002E-08 | 6.1E-14 | -4.88562247E+00 | 1.9E-05 |
| Z         | 2.04694016E-08  | 6.5E-14 | 6.13657094E+00  | 2.0E-05 |
| Ux        | 4.61015017E-06  | 1.6E-11 | 1.38208667E+03  | 4.8E-03 |
| Uy        | -1.17233301E-04 | 1.5E-11 | -3.51456604E+04 | 4.7E-03 |
| Uz        | -1.86840660E-06 | 2.1E-11 | -5.60133527E+02 | 6.6E-03 |

Barycentric Earth-Moon system.

| Parameter | Time Units | Physical Units |
|-----------|------------|----------------|
|           | (Gsec)     | (Gkg, Gm, Gsec) |
| value     | std. err.  | value          | std. err. |
| M         | 1.49946E-20 | 2.3E-23        | 6.05494E+15 | 9.9E+12 |
|           |            |                | 6.0471E+15  | b        |
|           |            |                | 6.0471E+15  | c        |
| X         | -8.84165321E-08 | 1.7E-14 | -2.65065886E+01 | 5.4E-06 |
| Y         | 4.82720728E-07  | 3.1E-14 | 1.44716027E+02  | 9.7E-06 |
| Z         | -5.71869163E-13 | 5.1E-14 | -1.76554413E-04 | 1.6E-05 |
| Ux        | -9.93720161E-05 | 6.3E-12 | -2.97909808E+04 | 2.0E-03 |
| Uy        | -1.82759998E-05 | 4.4E-12 | -5.47900403E+03 | 1.4E-03 |
### Table III (continued)

#### Mars

|   | 1.60070E-21 | 8.2E-24 | 6.47358E+14 | 3.5E+12 |
|---|-------------|---------|-------------|---------|
| M | 6.94079815E-07 | 6.9E-14 | 2.08079888E+02 | 2.2E-05 |
| X | 1.46658954E-06 | 4.0E-12 | 9.83006415E+02 | 1.3E-03 |
| Z | -1.84188989E-07 | 6.0E-13 | -5.52184725E+01 | 1.9E-04 |
| Ux | -2.47942585E-05 | 4.2E-11 | -7.43313111E+03 | 1.3E-02 |
| Uy | 2.24720635E-05 | 2.0E-11 | 6.73695495E+03 | 6.3E-03 |
| Uz | 5.94615340E-07 | 6.5E-12 | 1.78261186E+02 | 2.0E-03 |

#### Jupiter

|   | 4.708566E-18 | 7.5E-22 | 1.901473E+18 | 4.2E+14 |
|---|-------------|---------|-------------|---------|
| X | 1.99691012E-06 | 4.5E-13 | 5.98658565E+02 | 1.4E-04 |
| Y | 1.46658954E-06 | 4.0E-12 | 9.83006415E+02 | 1.3E-03 |
| Z | -1.84188989E-07 | 6.0E-13 | -5.52184725E+01 | 1.9E-04 |
| Ux | -2.47942585E-05 | 4.2E-11 | -7.43313111E+03 | 1.3E-02 |
| Uy | 2.24720635E-05 | 2.0E-11 | 6.73695495E+03 | 6.3E-03 |
| Uz | 5.94615340E-07 | 6.5E-12 | 1.78261186E+02 | 2.0E-03 |

#### Saturn

|   | 1.408626E-18 | 2.7E-22 | 5.688479E+17 | 1.4E+14 |
|---|-------------|---------|-------------|---------|
| X | 3.19733321E-06 | 7.1E-12 | 9.58536398E+02 | 2.2E-03 |
| Y | 3.27895665E-06 | 4.0E-12 | 9.83006415E+02 | 1.3E-03 |
| Z | -1.84188989E-07 | 6.0E-13 | -5.52184725E+01 | 1.9E-04 |
| Ux | -2.47942585E-05 | 4.2E-11 | -7.43313111E+03 | 1.3E-02 |
| Uy | 2.24720635E-05 | 2.0E-11 | 6.73695495E+03 | 6.3E-03 |
| Uz | 5.94615340E-07 | 6.5E-12 | 1.78261186E+02 | 2.0E-03 |
Table III (continued)

|     | Uranus     | Neptune     | Pluto        |
|-----|------------|-------------|--------------|
| M   | 2.16327E-19 | 2.51337E-19 | 3.5460E-23   |
|     | 1.4E-21    | 2.6E-21     | 1.4320E+13   |
|     | 8.73649E+16| 1.01637E+17 |              |
|     | 6.0E+14 a  | 1.1E+15 a   |              |
|     | 8.6634 E+16| 1.0280 E+17 |              |
|     | b          | 8.6840 E+16 |              |
|     | 2.15934020E+03| 2.51542922E+03|              |
|     | 2.1E-03 a  | 1.2E-03 a   |              |
|     | 8.6840 E+16| 1.0246 E+17 |              |
| X   | 7.2026288E-06| 8.39056779E-06|              |
|     | 6.0E-12    | 4.0E-12     |              |
|     | 2.1E-03 a  | 1.2E-03 a   |              |
| Y   | -6.5450368E-06| -1.24728664E-05|              |
|     | 1.6E-11    | 1.0E-11     |              |
|     | 5.5E-03 a  | 3.1E-03 a   |              |
| Z   | -1.18848871E-07| 6.34943840E-08|              |
|     | 1.5E-12    | 2.1E-12     |              |
|     | 4.7E-04 a  | 6.7E-04 a   |              |
| Ux  | 1.54697452E-05| 1.48988775E-05|              |
|     | 3.6E-11    | 1.6E-11     |              |
|     | 1.1E-02 a  | 4.9E-03 a   |              |
| Uy  | 1.54386933E-05| 1.02641971E-05|              |
|     | 7.2E-11    | 3.9E-11     |              |
|     | 2.2E-02 a  | 1.2E-02 a   |              |
| Uz  | -1.43079191E-07| -5.54000997E-07|              |
|     | 3.6E-12    | 6.0E-12     |              |
|     | -1.2E-03 a | -1.9E-03 a  |              |

(Values for J2000.0 (JD 2451545.0))

a) present work; b) from Yoder (1995); c) data of Simon et al (1998).
Table IV. Errors for initial positions and velocities.

| planet          | error for initial position | error for initial velocity |
|-----------------|---------------------------|---------------------------|
| Mercury         | 6.2E-07                   | 9.0E-07                   |
| Venus           | 1.4E-07                   | 1.4E-07                   |
| Earth-Moon      | 6.8E-08                   | 6.9E-08                   |
| Mars            | 1.1E-07                   | 1.1E-07                   |
| Jupiter         | 2.5E-07                   | 2.3E-07                   |
| Saturn          | 1.7E-06                   | 1.3E-06                   |
| Uranus          | 1.7E-06                   | 3.5E-06                   |
| Neptune         | 7.0E-07                   | 1.9E-06                   |
| Pluto           | 1.6E-06                   | 1.9E-06                   |

Values for J2000.0 (JD 2451545.0)