Extended mathematical derivations for the decentralized loss minimization algorithm with the use of inverters

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Abstract—This document contains extended mathematical derivations for the communication- and model-free loss minimization algorithm. The algorithm is applied in the distribution grids and exploits the capabilities of the inverters to control the reactive power output.

Index Terms—Loss minimization, model-free approach, communication-free approach, inverter

I. INTRODUCTION

MODERN distribution grids are characterized by great inclusion of renewable energy sources (RESs), especially rooftop photovoltaic solar panels (PVs), which are connected to the grid via inverters. Power generation settings of the PVs and inverters are available resources for minimizing active power losses in the grid. To minimize active power losses, we control the reactive generation settings of the inverters. In this document, we extend some mathematical derivations provided in the main paper, we use original numbers of the equations and derivations for the communication- and model-free loss minimization algorithm with the use of inverters. In Section II, we mathematically assess the performance of the approaches proposed in [1] and provide full derivations for the additional cases.

II. DETAILED MODEL OF THE RADIAL DISTRIBUTION SYSTEM

The total active power losses in the radial distribution system can be calculated by (1). At each bus $i$, the nodal current is computed as:

$$I_i = \left( \frac{S_i}{V_i} \right)^* , \quad \forall i = 1, ..., N_{bus} \quad (1A)$$

where $S_i$ and $V_i$ are complex nodal injection and voltage phasor in bus $i$, respectively; $N_{bus}$ is the number of buses in the system. The complex nodal injection at bus $i$ is defined as:

$$S_i = P_i^L - P_i^G + j(Q_i^L - Q_i^G) \quad (2A)$$

where $P_i^L$ and $P_i^G$ are active load and generation in bus $i$, respectively; $Q_i^L$ and $Q_i^G$ are reactive load and generation in bus $i$, respectively. The voltage phasor in bus $i$ is given as following:

$$V_i = |V_i|e^{-j\phi_i} \quad (3A)$$

where $|V_i|$ and $\phi_i$ are voltage magnitude and voltage angle in bus $i$, respectively. Branch currents can be calculated in an upstream manner:

$$I_{br}^k = I_{br}^{k+1} + I_{k+1}^b , \quad \forall k = 1, ..., N_{br} \quad (4A)$$

where for the leaf node $m$, the branch current $I_{br}^m = 0$, as a leaf node does not have downstream branches.

III. EXTENDED ASSESSMENT OF THE HEURISTIC AND “NO-ACTION” APPROACHES

In the main paper, the case when both $m$ and $m - 1$ are sender nodes is considered. In this section, we derive the other cases and prove that the approach described in [1] always provides a better solution than the “no-control” strategy.

A. Both $m$ and $m - 1$ are recipient nodes

First, we consider the case, when both $m$ and $m - 1$ are recipient nodes, which implies that $Q_m^G = Q_{m-1}^G = Q_m^L$. By substituting $Q_m^G$ and $Q_{m-1}^G$ into (13), the following is obtained:

$$P_{loss_{tot}}^{loss_{tot}} = R_{br}^2 \frac{2 \left( c_m (Q_m^G - Q_m^L)^2 \right)}{|V_m^G|^2} \quad (5A)$$

where $|V_m^G|$ denotes the voltage magnitude in node $m$ while setting the reactive generation of the inverter to the maximum value. Further, comparing (15) and (5A), the following can be concluded:

$$P_{loss_{tot}}^{loss_{tot}} > P_{loss_{tot}}^{loss_{tot}} \quad (6A)$$

The comparison of only the first components is conducted, which is sufficient for estimation of the whole equations (15), (5A). It is proven in the main paper, that higher $Q_i^G$ leads to higher $|V_i|$. Thus, similarly to (21) the following is valid:

$$|V_i^{G1}| < |V_i^{G2}| \quad (7A)$$

This work is supported by the ID-EDGE project, funded by Innovation Fund Denmark, Grant Agreement No. 8127-00017B.

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Due to (7A) the left side of (8A) is positive. To analyze the right side of (8A), we note that for equation (5A), $\overline{Q}_m^G < Q_m^L$, which follows from the definition of recipient nodes. Then it becomes obvious that $(Q_m^G)^2 > (Q_m^L - Q_m^L)^2$, as the expression in the square on the right side is decreased by the value of $\overline{Q}_m^G$. In addition, $|V_m^G|^2 < |V_m^L|^2$, which follows from (7A). Considering both of these observations, it becomes obvious that the right side expression is negative, while the sign of the left side is positive. By this, the sign in (6A) is justified. And (9A), (10A) allow to see the equivalence of the left and right sides of (8A) to the first components of (15), (5A), respectively.

\[
\begin{align*}
&c_m^2 \left( |V_m^G|^2 - |V_m^L|^2 \right) \\
&> (\overline{Q}_m^G - Q_m^L)^2|V_m^G|^2 - (Q_m^L)^2|V_m^G|^2 \leftrightarrow \quad (8A) \\
&\left| V_m^G \right|^2 \left( c_m^2 + (Q_m^L)^2 \right) \\
&> \left| V_m^G \right|^2 \left( c_m^2 + (\overline{Q}_m^G - Q_m^L)^2 \right) \leftrightarrow \quad (9A) \\
&2 \left( c_m^2 + (Q_m^L)^2 \right) > 2 \left( c_m^2 + (\overline{Q}_m^G - Q_m^L)^2 \right) \\
&\frac{|V_m^L|^2}{|V_m^G|^2} \\
&\frac{2 \left( c_m^2 + (\overline{Q}_m^G - Q_m^L)^2 \right)}{\left| V_m^G \right|^2} \\
&\left(10A\right)
\end{align*}
\]

### B. Only m is a recipient node

Next, let’s consider the case, when $m$ is a recipient node and $m - 1$ is a sender node, which implies that $Q_m^G = \overline{Q}_m^G$, $Q_{m-1}^G = Q_{m-1}^L$. By substituting $Q_m^G$ and $Q_{m-1}^L$ into (13), the following is obtained:

\[
\begin{align*}
P_{\text{loss,}\overline{Q}_m^G}^{\text{tot}} &= R_{\text{br}} \frac{2 \left( c_m^2 + (\overline{Q}_m^G - Q_m^L)^2 \right)}{|V_m^G|^2} \quad (11A) \\
&+ R_{\text{br}} \frac{2c_m c_{m-1}}{|V_m^G| \left| V_m^{L_{m-1}} \right|} + R_{\text{br}} \frac{c_m^2}{|V_m^G|^2}
\end{align*}
\]

By component-wise comparison of (11A) and (5A) it becomes obvious that:

\[
P_{\text{loss,}\overline{Q}_m^G}^{\text{tot}} < P_{\text{loss,}Q_0^G}^{\text{tot}} \quad (12A)
\]

Taking into account (6A), we can conclude that:

\[
P_{\text{loss,}\overline{Q}_m^G}^{\text{tot}} < P_{\text{loss,}Q_0^G}^{\text{tot}} \quad (13A)
\]

### C. Only $m - 1$ is a recipient node

Finally, we consider the case, when $m - 1$ is a recipient node and $m$ is a sender node, which implies that $Q_{m-1}^G = \overline{Q}_{m-1}^G$, $Q_m^G = Q_m^L$. By substituting $Q_m^G$, and $Q_{m-1}^L$ into (13), the following is obtained:

\[
\begin{align*}
P_{\text{loss,}Q_m^G}^{\text{tot}} &= R_{\text{br}} \frac{c_m^2}{|V_m^{L_{m-1}}|^2} + \frac{\left( Q_m^{L_{m-1}} - Q_m^L \right)^2}{|V_m^{L_{m-1}}|^2} \quad (14A) \\
&+ R_{\text{br}} \frac{2c_m c_{m-1}}{|V_m^G||V_m^{L_{m-1}}|} + R_{\text{br}} \frac{c_m^2}{|V_m^G|^2}
\end{align*}
\]

From the component-wise comparison of (14A) and (5A) we can easily conclude the following:

\[
P_{\text{loss,}Q_m^G}^{\text{tot}} < P_{\text{loss,}Q_0^G}^{\text{tot}} \quad (15A)
\]

**REFERENCES**

[1] K. Turitsyn, P. Åaulc, S. Backhaus, and M. Chertkov, “Distributed control of reactive power flow in a radial distribution circuit with high photovoltaic penetration,” IEEE PES General Meeting, 2010, 2010.