COMPARATIVE MAXIMUM POWER DENSITY ANALYSIS OF A SUPERCritical CO\textsubscript{2} BRAYTON POWER CYCLE

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ABSTRACT
The supercritical CO\textsubscript{2} (s-CO\textsubscript{2}) power cycle has been taking into account as one of the most effective alternatives for energy conversion because of its higher efficiency and smaller compressor and turbine sizes for many years. A plenty number of parametric and experimental studies for the different type of s-CO\textsubscript{2} cycles have been accomplished in the literature. In this paper, a performance analysis based on a power density criterion has been carried out for a simple s-CO\textsubscript{2} Brayton power cycle. The parameters which are obtained from analyzes were compared with those of a power performance criterion that is shown that design parameters at maximum power density give a chance to smaller cycle components and more efficient s-CO\textsubscript{2} Brayton power cycle. Due to loses in the cycle, the power and thermal efficiency will reduce by a certain amount, however, the maximum power density conditions will still give a better performance than at the maximum power output conditions. The analysis exemplified in this paper may provide a reference for the finding of optimal operating conditions and the design parameters for real s-CO\textsubscript{2} Brayton power cycles.

Keywords: Power Density, Supercritical CO\textsubscript{2} Cycle, Brayton Cycle, Thermodynamics

INTRODUCTION
The Brayton cycle based on supercritical carbon dioxide (s-CO\textsubscript{2}) as the working fluid is an innovative concept for converting thermal energy to electrical energy. There is a sufficiently long history of s-CO\textsubscript{2} cycles. When open sources literature were examined the first application noticed, seems to be the patched half-condensation CO\textsubscript{2}-Brayton cycle which belongs to the Sulzer brothers [1]. Although studies on this topic have not been continued adequately after this date, the main works that draw attention to these cycles have been studies of supercritical thermodynamic power cycles made by Feher in 1962 [2] and in 1968 [3]. It is widely known that, supercritical phase is a phase when an element properties between liquid and gas at critical temperature (T\textsubscript{c}) and critical pressure (P\textsubscript{c}). Fluids in the supercritical phase have liquid-like densities and act as a liquid solvent. As it is known, CO\textsubscript{2} has high heat capacity with low viscosity and mass transfer property. Surface tension coefficients, viscosities are low and therefore the pumping energy is low. CO\textsubscript{2} gas is not corrosive in the dry environment. It is not flammable, explosive or toxic and also is not harmful to the environment. The critical temperature for CO\textsubscript{2} is 304.3 K and the critical pressure is 7.38 MPa [4]. Benefits that will be reveal from successful research and development of the s-CO\textsubscript{2} power conversion cycle will include, several heat sources including fossil fuel, nuclear and renewables such as nuclear energy, solar energy, geothermal energy, waste heat recovery and coal power plants [5–10]. Also, this technology will lead to lower capital costs and reduced water usage and most importantly lower emissions. Briefly, system is comprehensively feasible and worthy. The technology readiness can be divided into three parts. Mature components, less-mature components and system integration. Mature components contain electrical generation subsystems, control units and instrumentation. Less mature components contain turbine, heater etc. Lastly, system integration is needed to optimize the operating and design parameters and also systems start-up, shut-down, transient and part load operations of the system.

Numerous studies have been carried out in different fields on s-CO\textsubscript{2} power cycles. These studies consist of: thermodynamic cycle models and s-CO\textsubscript{2} cycles on commercial or research-based tests [11–16]. In addition, studies on the real-time response of s-CO\textsubscript{2} power cycles and the development of cycle control strategies [17–19], furthermore, research on turbo machines specially designed for s-CO\textsubscript{2} flow and on air bearings and seals with turbo machine subcomponents [20–23], the work consists of studies on high speed electric motor technologies which is essential component for the s-CO\textsubscript{2} cycles to be compact [24–28] and material investigations on the interaction of different materials with s-CO\textsubscript{2} fluid under high temperature and pressure [29–32]. Apart from these studies, there is no doubt that one of the parameters that must be taken into consideration is the maximum power...
density \( MPD \) when performing cycle optimization. There are many studies in the literature that have been done with \( MPD \) in cycle analysis [33–36]. Şahin et al. [37] defined the power density as the ratio of power to the specific volume in the cycle and obliquely added the effects of engine sizes to the analysis which leads to smaller and more efficient engines. Likewise, Gonca et al. [38] studied comprehensive analyses and comparisons for irreversible cycle engines and they examined effects of design parameters on \( MPD \). Likewise, Chen et al. [39] investigated advantages and disadvantages of \( MPD \) design analysis to observe effects of some design parameters. And they concluded that \( MPD \) leads to a better efficiency at cycle analysis. Apart from prior studies, in this study, comprehensive comparison parameters which are obtained from analyzes were compared with those of a power performance criterion. It is shown that design parameters at maximum power density give a chance to smaller cycle components and more efficient s-CO\textsubscript{2} Brayton power cycle.

**THERMODYNAMIC MODEL**

Schematic and T-s diagrams of closed loop ideal Brayton cycle are shown in figure 1. An isentropic compression occurs between 1 and 2 in a compressor, constant-pressure heat addition between 2 and 3 then isentropic expansion occurs between 3 and 4 in a turbine then it follows with a constant pressure heat rejection between 4 and 1 which finish the whole cycle.

![Figure 1. Schematic and T-s diagrams of a Brayton power cycle [4]](image)

The energy balance for a steady-state flow process of ideal Brayton cycle can be expressed as below:

\[
(q_{23} - q_{41}) + (w_{in} - w_{out}) = (h_{out} - h_{in})
\]  

(1)

As known, heat transfer occurs to the working fluid and likewise from the working fluid are, respectively,

\[
q_{23} = (h_3 - h_2) = c_p (T_3 - T_2)
\]  

(2a)

and

\[
q_{41} = (h_4 - h_1) = c_p (T_4 - T_1)
\]  

(2b)

With regards to the cold air standard assumptions the thermal efficiency of the ideal Brayton cycle becomes:
\[ \eta_\alpha = \frac{W_{net}}{q_{23}} = 1 - \frac{q_{23}}{q_{23}} = 1 - \frac{c_p (T_4 - T_1)}{c_p (T_3 - T_1)} = 1 - \frac{T_4}{T_3} \left( \frac{T_1}{T_4} - 1 \right) \]  

(3)

While processes of 1-2 and 3-4 are isentropic, \( P_2 = P_3 \) and \( P_4 = P_1 \). Thus,

\[ \frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = \left( \frac{P_4}{P_3} \right)^{\frac{k-1}{k}} = \frac{T_4}{T_3} \]  

(4)

Substituting equation (4) into the equation (3) and then re-arranging becomes:

\[ \eta_{th} = 1 - \frac{1}{r_p^{\frac{k-1}{k}}} \]  

(5)

where \( r_p \) is the pressure ratio and \( k \) is the specific heat ratio. Apart from above equations, \( MPD \) can be expressed as below and it can be used as work density instead of power density:

\[ MPD = \frac{W_{net}}{V_{max}} \]  

(6)

**RESULTS AND DISCUSSION**

In the calculations, the constants are considered as, ambient temperature and pressure are 298 K and 100 kPa, respectively. Compressor and turbine isentropic efficiencies are 0.90 and pressure drop at heat exchangers (\( \Delta P \)) is 0.03 bar. Generally, increasing the pressure ratio of a Brayton cycle is the most effective way that increases the overall thermal efficiency which cause the cycle to approach the Carnot cycle. When Figure 2 is examined, it can be seen that \( MPD \) increases to a certain value with increasing \( r_p \) and then begins to decrease when approximately \( r_p \) is 9.286. Likewise, \( \eta_{th} \) and \( r_{bw} \) of the \( s \)-CO2 cycle is also seen to increase with increasing \( r_p \). When considering \( \alpha \) is 2.5 and constant as figure 2, the cycle thermal efficiency is 24.83\%, 25.28\% and 25.47\%, respectively, by increasing \( r_p \) from 8.673 to 9.898 and 11.

![Figure 2](image-url)

*Figure 2. Variation of \( \alpha \) on \( \eta_{th}, f_{bw} \) and \( MPD \) with respect to \( r_p \).*
The turbine inlet temperature (TIT) is known to be limited by the thermal resistance limits of the turbine materials can withstand. Therefore, the temperature ratio must be within a certain range. When the temperature ratio is optimized, the efficiency of the cycle must also be considered. Figure 3 shows that when the pressure ratio is 7, there is an increase in MPD and thermal efficiency with increasing the temperature ratio from 2.5 to 3.5. In contrast to this situation, the rate of back work ratio decrease with the increase in the temperature ratio. While $r_p$ is 7 and constant, increasing $\alpha$ from 2.908 to 3.5 value, MPD increases 14.89%.

Figure 3. Effects of temperature ratio $\alpha$ on $\eta_{th}$, $p_{bw}$ and MPD

It is seen that increasing the pressure ratio and temperature ratio increases the net work in the system if it is optimized by considering the temperature resistances of the turbine blades. Referring to figure 4, the pressure ratio is increased from 5 to 11 and also the temperature ratio increased from 2.5 to 3.5. While the temperature ratio is 2.5, the MPD is increased first by increasing the pressure ratio, and then decreased by a certain amount. Therefore, the pressure ratio should be optimized by taking into account the MPD. While the temperature ratios are 3 and 3.5, it is seen that the MPD increases by increasing the pressure ratio between 5 and 11. These increases seem to lead increases on $w_{net}$ and MPD.

Figure 4. Variation of MPD and $w_{net}$ with respect to various $\alpha$ and $r_p$
When optimizing the system, the pressure and temperature ratios need to be monitored in order to maximize the net work output and thermal efficiency. Figure 5 shows effect of increasing the temperature ratio and the pressure ratio on the thermal efficiency and MPD. An increase in thermal efficiency and an increase in MPD was observed with increasing temperature ratio. Similarly, thermal efficiency and MPD increase with increasing pressure ratio. The MPD reaches the highest value at the maximum pressure and maximum temperature ratios. The highest MPD will directly lead to lower machine dimensions which means low costs.

![Figure 5. Variation of MPD and $\eta_{th}$ with respect to various $\alpha$ and $r_p$](image)

As it is known, the ratio of the work to the turbine work used to operate the compressor is called the back work. The increase in compressor work reduces the net work of the system. Thus, one can say; the less the back work, the higher the system efficiency. Figure 6 shows a decrease in the back work ratio with increasing the temperature ratio. However, an increase in MPD was observed with increasing temperature ratio from 2.5 to 3.5. Similarly, increasing the pressure ratio has led to an increase in the MPD and $r_{bw}$.

![Figure 6. Variation of MPD and $r_{bw}$ with respect to various $\alpha$ and $r_p$](image)
When Figure 7 is examined, an increase in $w_{net}$ is observed with increasing temperature ratio. As the temperature ratio increases, the thermal efficiency also increases at the same pressure ratio line which is an expected result. In addition, increasing the pressure ratio has also increased the thermal efficiency. When the figure is examined in detail, it is seen that the thermal efficiency increases with increasing pressure ratio for temperature ratio 2.5 and it increases with increasing pressure ratios at other temperature ratios but there is an optimum compression ratio that makes the net work maximum.

**Figure 7.** Variation of $\eta_{th}$ and $w_{net}$ respect to various $\alpha$ and $r_p$

**CONCLUSION**

The parameters which are obtained from analyzes were compared with those of a power performance criterion. And it is shown that design parameters at maximum power density give a chance to smaller cycle components and more efficient s-CO$_2$ Brayton power cycle. Major challenges of s-CO$_2$ system can be expressed as, materials strength to improve reliability and cycle efficiency, identifying entire system design, developing oxy-combustors for direct fired system, model control strategies and lastly integration of fossil energy heat sources to the cycles. Due to loses in the cycle, the power and thermal efficiency are reduced by a certain amount, however, the maximum power density condition still gives a better performance than the maximum power output conditions. Effect of increasing the temperature ratio and the pressure ratio on the thermal efficiency and MPD have been examined. It can be concluded that MPD increases to a certain value with increasing $r_p$ and then begins to decrease approximately when $r_p$ is 9.286. An increase in thermal efficiency and an increase in MPD is observed with increasing $\alpha$. Similarly, thermal efficiency and MPD increases with increasing pressure ratio. In addition, while $r_p$ is 7 and constant, increasing $\alpha$ from 2.908 to 3.5 value, MPD increases 14.89%. To sum up, the results show that when system design optimizations are made, thermodynamic analyzes must also include MPD analysis in order to operate the system with smaller components.

**NOMENCLATURE**

- $c_p$: Specific heat capacity, kJ/kg.K
- $h$: Specific enthalpy, kJ/kg
- $k$: Isentropic coefficient
- $MPD$: Maximum power density, kJ/m$^3$
- $TIT$: Turbine inlet temperature, K
- $P$: Pressure, kPa
- $q$: Heat transfer, kJ/kg
- $r_p$: Pressure ratio
- $r_{bw}$: Back work ratio
- $T$: Temperature, K
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Greek Letters

\( w \) Specific work, kJ/kg

\( \eta \) Thermal Efficiency

\( \alpha \) Temperature Ratio
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