Hyperfine splitting of heavy quarkonium hybrids

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JS, Sandra Tomàs Valls, Phys. Rev. D 108, 014025 (2023)
Heavy Hadrons

- Heavy quarks: $Q = c, b, t$, $m_Q \gg \Lambda_{QCD}$
- Heavy hadrons: hadrons containing at least a heavy quark: $Q = b, c$
- In the hadron rest frame the heavy quarks move slowly $\Rightarrow$ use a non-relativistic approximation
- A universal way to encode it together with relativistic correction is using Effective Field Theories
- NRQCD/HQET are the suitable ones
- They imply heavy quark spin symmetry at leading order.
NRQCD

W.E. Caswell and G.P. Lepage, Phys. Lett. 167B, 437 (1986)
G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D 51 (1995) 1125

\[ m_Q \gg m_Q v, \quad m_Q v^2, \quad \Lambda_{QCD} \]

\[ \mathcal{L}_\psi = \psi^\dagger \left\{ iD_0 + \frac{1}{2m_Q} D^2 + \frac{1}{8m_Q^3} D^4 + \frac{c_F}{2m_Q} \sigma \cdot gB + \right. \]

\[ + \frac{c_D}{8m_Q^2} (D \cdot gE - gE \cdot D) + \left. i \frac{c_S}{8m_Q^2} \sigma \cdot (D \times gE - gE \times D) \right\} \psi \]

\( c_F, c_D \) and \( c_S \) are short distance matching coefficients calculable from QCD in powers of \( \alpha_s \). They depend on \( m_Q \) and \( \mu \) (factorization scale) but not on the lower energy scales.
Exotic Hadrons

- Hadrons beyond mesons $q\bar{q}$ and baryons $qqq$
- QCD: any color singlet state made out of quarks and gluons may become a hadron
- I will restrict myself to discuss hadrons containing two heavy quarks
- The starting point can then be NRQCD
Hadrons with two heavy quarks

\[ Q = b, c, \quad q = u, d, s \]

- **QQ**+ light quarks and gluons
  - Double Heavy Baryons: \( QQq \)
  - Tetraquarks: \( QQ\bar{q}\bar{q} \)
  - Pentaquarks: \( QQqq\bar{q} \)
  - Hybrids: \( QQqg \)
  - ... 

- **Q\bar{Q}**+ light quarks and gluons
  - Heavy Quarkonium: \( Q\bar{Q} \)
  - Hybrids: \( Q\bar{Q}g \)
  - Tetraquarks: \( Q\bar{Q}q\bar{q} \)
  - Pentaquarks: \( QQqqq \)
  - ...
**Heavy Quarkonium**

$Q\bar{Q}$ bound state, $m_Q \gg \Lambda_{QCD}$, $\alpha_s(m_Q) \ll 1$

- Heavy quarks move slowly $v \ll 1$
- Non-relativistic system $\rightarrow$ multiscale problem
  - $m_Q \gg m_Q v$ (Relative momentum)
  - $m_Q v \gg m_Q v^2$ (Binding energy)
  - $m_Q \gg \Lambda_{QCD}$

- EFTs are useful (N. Brambilla, A. Pineda, JS and A. Vairo, Rev. Mod. Phys. 77, 1423 (2005))
  - NRQCD: $m_Q \gg m_Q v, m_Q v^2, \Lambda_{QCD}$ (W.E. Caswell and G.P. Lepage, Phys. Lett. 167B, 437 (1986))
  - pNRQCD (weak coupling): $m_Q v \gg m_Q v^2, \Lambda_{QCD}$ (A. Pineda, JS, Nucl.Phys.Proc.Suppl.64:428-432,1998)
  - pNRQCD (strong coupling): $m_Q v, \Lambda_{QCD} \gg m_Q v^2$ (N. Brambilla, A. Pineda, JS, A. Vairo, Nucl.Phys.B566:275,2000)
How does the hadron look like?

\[ m_Q v \sim \frac{1}{r} \gg m_Q v^2 \gtrsim \Lambda_{QCD} \]

weak coupling pNRQCD

\[ m_Q v \sim \frac{1}{r} \gtrsim \Lambda_{QCD} \gg m_Q v^2 \]

strong coupling pNRQCD

Born-Oppenheimer EFT

Figures: Najjar, Bali, 2009
QQ/Q\bar{Q} + light quarks and gluons \((m_Q v, \Lambda_{QCD} \gg m_Q v^2)\)

(JS, J. Tarrús Castellà, 20)

\[
\mathcal{L}_{\text{HEH}} = \sum_{\kappa, p} \psi_{\kappa, p}^\dagger \left[ i \partial_t - h_{\kappa, p} \right] \psi_{\kappa, p}
\]

\[
h_{\kappa, p} = \frac{p^2}{m_Q} + \frac{P^2}{4m_Q} + V_{\kappa, p}^{(0)}(r) + \frac{1}{m_Q} V_{\kappa, p}^{(1)}(r, p) + O\left(\frac{1}{m_Q^2}\right)
\]

- LDF\(\equiv\) light quarks + gluons, characterized by their quantum numbers \((\kappa, p \ldots)\)
  - \(\kappa\) \equiv total angular momentum, \(p\) \equiv parity \((P/CP)\)
  - Quantum numbers not explicitly displayed: baryon number \((B)\), isospin \((I)\), strangeness \((S)\), principal quantum number

- \(V_{\kappa, p}^{(0)}, V_{\kappa, p}^{(1)}, \ldots\) must be calculated non-perturbatively

- A truncation of \(\mathcal{L}_{\text{HEH}}\) needed for practical calculations \(\implies\) keep a limited number of lower lying \(\kappa, p\)
- $V_{\kappa p}^{(0)}$ is a $(2\kappa + 1) \times (2\kappa + 1) \times \mathbb{I}_2 Q_1 \times \mathbb{I}_2 Q_2$ matrix, which can be decomposed into irreducible representations of $D_{\infty h}$, the symmetry group of a diatomic molecule.

\[
V_{\kappa p}^{(0)}(r) = \sum_{\Lambda} V_{\kappa p \Lambda}^{(0)}(r) \mathcal{P}_{\kappa \Lambda}
\]

$\mathcal{P}_{\kappa \Lambda}$ projects onto LDF angular momenta $\pm \Lambda$ in the direction joining the two heavy quarks, $\Lambda = \kappa, \kappa - 1, \ldots, \kappa - [\kappa]$

\[
\begin{align*}
\mathcal{P}_{\frac{1}{2} \frac{1}{2}} &= \mathbb{I}_{2}^{lq} \\
\mathcal{P}_{\frac{3}{2} \frac{1}{2}} &= \frac{9}{8} \mathbb{I}_{4}^{lq} - \frac{1}{2} \left( \hat{r} \cdot S_{3/2} \right)^2 \\
\mathcal{P}_{\frac{3}{2} \frac{3}{2}} &= -\frac{1}{8} \mathbb{I}_{4}^{lq} + \frac{1}{2} \left( \hat{r} \cdot S_{3/2} \right)^2 \\
\mathcal{P}_{10} &= \mathbb{I}_{3}^{lq} - (\hat{r} \cdot S_1)^2 \\
\mathcal{P}_{11} &= (\hat{r} \cdot S_1)^2 \\
\cdots
\end{align*}
\]
\[ V_{\kappa^p}^{(1)} = V_{\kappa^pSI}^{(1)} + V_{\kappa^pSD}^{(1)} \]

- \( V_{\kappa^pSI}^{(1)} \) does not depend on the spin or orbital angular momentum of the heavy quarks \( \Rightarrow \) admits the same decomposition as \( V_{\kappa^p}^{(0)} \)

- \( V_{\kappa^pSD}^{(1)} \) depends on the spin and orbital angular momentum of the heavy quarks

\[
V_{\kappa^pSD}^{(1)}(r) = \sum_{\Lambda\Lambda'} \mathcal{P}_{\kappa\Lambda} \left[ V_{\kappa^p\Lambda\Lambda'}^{sa}(r) \mathbf{S}_{QQ} \cdot (\mathbf{P}_{10} \cdot \mathbf{S}_\kappa) + V_{\kappa^p\Lambda\Lambda'}^{sb}(r) \mathbf{S}_{QQ} \cdot (\mathbf{P}_{11} \cdot \mathbf{S}_\kappa) \right.

\left. + V_{\kappa^p\Lambda\Lambda'}^{l}(r) (\mathbf{L}_{QQ} \cdot \mathbf{S}_\kappa) \right] \mathcal{P}_{\kappa\Lambda'}
\]

\[
2\mathbf{S}_{QQ} = \sigma_{QQ} = \sigma_{Q_1} \times \mathbb{I}_2 \mathbf{Q}_2 + \mathbb{I}_2 \mathbf{Q}_1 \times \sigma_{Q_2} \quad , \quad \mathcal{P}_{10}^{ij} = \hat{r}^i \hat{r}^j \quad , \quad \mathcal{P}_{11}^{ij} = \delta_{ij} - \hat{r}^i \hat{r}^j
\]
Matching to NRQCD

- Build an NRQCD operator with the quantum numbers of $\Psi_{KP}$

$$
O_{KP}^{Q\bar{Q}}(t, r, R) = \chi_c^\top(t, x_2)\phi(t, x_2, R)Q_{KQ}\bar{Q}_{KP}(t, R)\phi(t, R, x_1)\psi(t, x_1)
$$

$$
O_{KP}^{QQ}(t, r, R) = \psi^\top(t, x_2)\phi^\top(t, R, x_2)Q_{QQK}\bar{Q}_{KP}(t, R)\phi(t, R, x_1)\psi(t, x_1)
$$

- Examples:
  - Hybrid
    $$
    Q_{1^+}(t, x) = (e_\alpha^\dagger \cdot B(t, x))
    $$
  - $Q\bar{Q}q\bar{q}$ tetraquark
    $$
    Q_{0^{++}}(t, x) = [\bar{q}(t, x) T^a q(t, x)] T^a
    $$
  - Doubly heavy baryons
    $$
    Q_{(1/2)^+}(t, x) = T^l \left[ P_+ q^l(t, x) \right]^\alpha
    $$
  - $QQ\bar{q}\bar{q}$ tetraquark
    $$
    Q_{0^-}(t, x) = \left[ \bar{q}(t, x) T^l \gamma^2 q^*(t, x) \right] T^l
    $$
Matching to NRQCD

- Impose \( \mathcal{O}^{h}_{\kappa p}(t, r, R) = \sqrt{Z_{h\kappa p}} \psi_{h\kappa p}(t, r, R), \quad h = QQ, Q\bar{Q}. \)

\[
\langle 0 | T \{ \mathcal{O}^{h}_{\kappa p}(t/2)\mathcal{O}^{h\dagger}_{\kappa p}(-t/2) \} | 0 \rangle = \sqrt{Z_{h\kappa p}} \langle 0 | T \{ \psi_{h\kappa p}(t/2)\psi^{\dagger}_{h\kappa p}(-t/2) \} | 0 \rangle \sqrt{Z_{h\kappa p}^{\dagger}}
\]

- Then at \( \mathcal{O}(1) \)

\[
V^{(0)}_{h\kappa p\Lambda}(r) = \lim_{t \to \infty} \frac{i}{t} \log \left( \text{Tr} \left[ \mathcal{P}_{\kappa\Lambda} \langle 1 \rangle_{h\kappa p} \right] \right)
\]
\[ V_{\kappa p \Lambda \Lambda'}^{sb} = -c_F \lim_{t \to \infty} \sqrt{\frac{\text{Tr} [\mathcal{P}_\kappa \Lambda] \text{Tr} [\mathcal{P}_\kappa \Lambda']} {\text{Tr} [\mathcal{P}_\kappa \Lambda \langle 1 \rangle^{h_{\kappa p}}] \text{Tr} [\mathcal{P}_\kappa \Lambda' \langle 1 \rangle^{h_{\kappa p}}]}} \]

\[ \times \ln \left( \frac{\text{Tr} [\mathcal{P}_\kappa \Lambda \langle 1 \rangle^{h_{\kappa p}}] \text{Tr} [\mathcal{P}_\kappa \Lambda']} {\text{Tr} [\mathcal{P}_\kappa \Lambda' \langle 1 \rangle^{h_{\kappa p}}] \text{Tr} [\mathcal{P}_\kappa \Lambda]} \right) \]

\[ \times 2t \sinh \left( \ln \sqrt{\frac{\text{Tr} [\mathcal{P}_\kappa \Lambda \langle 1 \rangle^{h_{\kappa p}}] \text{Tr} [\mathcal{P}_\kappa \Lambda']} {\text{Tr} [\mathcal{P}_\kappa \Lambda' \langle 1 \rangle^{h_{\kappa p}}] \text{Tr} [\mathcal{P}_\kappa \Lambda]}} \right) \]

\[ \times \int_{-t/2}^{t/2} dt' \frac{\text{Tr} \left[ (S_\kappa \cdot \mathcal{P}_{11}) \cdot (\mathcal{P}_\kappa \Lambda \langle gB(t', x_1) \rangle^{h_{\kappa p}} \mathcal{P}_{\kappa \Lambda'}) \right]} {\text{Tr} \left[ (S_\kappa \cdot \mathcal{P}_{11}^{c.r.}) \cdot (\mathcal{P}_\kappa \Lambda S_\kappa \mathcal{P}_{\kappa \Lambda'}) \right]} \]
Applications

- Doubly Heavy Baryons: $QQq$ (JS, Tarrús Castellà, 20, 21)
- Hyperfine splittings of Heavy Quarkonium Hybrids: $Q\bar{Q}g$ (JS, Tomàs Valls, 23)

Disclaimer:

- Interactions with heavy-light meson/baryon pairs neglected
- They have been addressed in the BOEFT for heavy quarkonium (Tarrús Castellà, 22; Bruschini, 23)
- It has been recently generalized to double heavy exotics (Tarrús Castellà, 24)
Heavy Quarkonium Hybrids

- **Spin average spectrum** (Braaten, Langmack, Hudson Smith, 2014; Berwin, Brambilla, Tarrús Castellà, Vairo, 15; Oncala, Soto, 17)
  - Based on lattice data (Juge, Kuti, Morningstar, 02; Bali, Pineda, 03)
  - More recent and accurate lattice data available (Capitani, Philipsen, Reisinger, Riehl, Wagner, 18; Schlosser, Wagner, 21; Höllwieser, Knechtli, Korzec, Peardon, Urrea-Niño, 23)

- **Inclusive decay width to heavy quarkonium** (Oncala, JS, 17)
  - Revisited in (Brambilla, Lai, Mohapatra, Vairo, 22)
    - Improved the $\Delta S = 0$ transitions $\mathcal{O}(1/m_Q^0)$
    - Calculated the $\Delta S = 1$ transitions $\mathcal{O}(1/m_Q^2)$

- **Mixing with heavy quarkonium** (Oncala, JS, 17)
  - Important effects when a quarkonium state and a hybrid state with the same quantum numbers have similar masses
  - Leads to violations of spin conservation
  - Likely to increase the estimates of the $\Delta S = 1$ transitions above

- **Selection rules for exclusive decays** (Braaten, Langmack, Hudson Smith, 14; Bruschini, 23)
The hyperfine splitting of heavy quarkonium hybrids

- The lower lying hybrid potentials correspond to $\kappa^P = 1^+$
- This leads to two spin projections on the direction $Q-\bar{Q}$, $\Lambda = 0, 1$
- The general formulas above imply that there are two independent potentials: $V_{1+11}^{sa}(r)$, $V_{1+10}^{sb}(r)$ (Oncala, JS, 17; Brambilla, Lai, Segovia, Tarrús Castellà, Vairo, 18, 19)
- No lattice calculation available for them. How to estimate them?
  - Brambilla et al. used weak coupling pNRQCD short distance expressions to estimate them which hold for $r \ll 1/\Lambda_{QCD}$. The $1/m_Q^2$ spin dependent potentials were also included.
  - We (JS, Tomàs Valls, 23) use an interpolation between the short distance expressions and long distance ones calculated in the QCD effective string theory (Pérez-Nadal, JS, 08; Brambilla, Groher, Martinez, Vairo, 14).

- Typical values: $\langle 1/r \rangle \sim 0.17 - 0.42$ GeV for $c\bar{c}g$, $\langle 1/r \rangle \sim 0.22 - 0.53$ GeV for $b\bar{b}g$ (Berwein, Brambilla, Tarrús Castellà, Vairo, 14)
Hyperfine Splittings

They appear at $\mathcal{O}(1/m_Q)$ ($\mathcal{O}(1/m_Q^2)$) in hybrids (quarkonium).

They lead to the following mass formulae:

$$\frac{M_{1J+1} - M_{0J}}{M_{1J} - M_{0J}} = -J$$
$$\frac{M_{1J-1} - M_{0J}}{M_{1J} - M_{0J}} = J + 1$$

1. $(s/d)_1: M_{2--} + M_{0--} = M_{1--} + M_{1- -}$
2. $p_1: M_{2++} + M_{0++} = M_{1+-} + M_{1++}$
3. $(p/f)_2: M_{3+ -} + M_{1+ -} = M_{2+ -} + M_{2++}$
4. $d_2: M_{3- -} + M_{1- -} = M_{2- -} + M_{2--}$

- Consistent with the values of the lattice HSC
- Induces mixing between hybrid states in different multiplets
The short distance potentials

- The two independent potentials are rearranged in

\[
V_{hf}(r) = \frac{1}{6} V_{1+11}^{sa}(r) - \frac{1}{3} V_{1+10}^{sa}(r) \quad (\text{spin} - \text{spin})
\]

\[
V_{hf2}(r) = -\frac{1}{2} \left( V_{1+11}^{sa}(r) + V_{1+10}^{sb}(r) \right) \quad (\text{tensor})
\]

- At short distances:

\[
\frac{V_{hf}(r)}{m_Q} = A + \mathcal{O}(r^2) \quad , \quad A \sim c_F \Lambda_{QCD}^2/m_Q
\]

\[
\frac{V_{hf2}(r)}{m_Q} = Br^2 + \mathcal{O}(r^4) \quad , \quad B \sim c_F \Lambda_{QCD}^4/m_Q
\]

The short distance potentials depend on two unknown non-perturbative parameters.
The long distance potentials

- At long distances:

\[
\frac{V_{1+11}^{sa}(r)}{m_Q} = -\frac{2c_F\pi^2g\Lambda'''}{m_Q\kappa r^3} \equiv V_{ld}^{sa}(r)
\]

\[
\frac{V_{1+10}^{sb}(r)}{m_Q} = \mp \frac{c_Fg\Lambda'\pi^2}{m_Q\sqrt{\pi\kappa}} \frac{1}{r^2} \equiv V_{ld}^{sb}(r)
\]

- \(\kappa\) is the string tension \(\sim \Lambda_{QCD}^2\)
- \(g\Lambda', g\Lambda''' \sim \Lambda_{QCD}\) also enter the spin dependent potentials for heavy quarkonium
- They can be extracted from lattice calculations of those potentials (Koma, Koma, 09; Eichberg, Wagner, 23)

\[g\Lambda' \sim -59 \text{ MeV} \quad ; \quad g\Lambda''' \sim \pm 230 \text{ MeV}\]

(Oncala, JS, 17)
The interpolating potentials

- We use the following interpolation

\[
\frac{V_{hf}(r)}{m_Q} = A + \left( \frac{r}{r_0} \right)^2 \left( \frac{1}{6} V_{ld}^{sa}(r_0) - \frac{r}{3r_0} V_{ld}^{sb}(r_0) \right) \frac{1}{1 + \left( \frac{r}{r_0} \right)^5}
\]

\[
\frac{V_{hf2}(r)}{m_Q} = B r^2 - \left( \frac{r}{r_0} \right)^5 \left( \frac{r_0}{2r} V_{ld}^{sa}(r_0) + \frac{1}{2} V_{ld}^{sb}(r_0) \right) \frac{1}{1 + \left( \frac{r}{r_0} \right)^7}
\]

- \( r_0 \sim 3.96 \text{ GeV}^{-1} \sim 1/\Lambda_{QCD} \)
Charmonium Hybrids HFS

- We use lattice data of the HSC for charmonium to fix $A$ and $B$ (relativistic charm, $m_\pi \sim 240$ MeV, Cheung, O’Hara, Moir, Peardon, Ryan, Thomas, Tims, 16)
- We focus on hyperfine splittings not on spin averages
- Same strategy as Brambilla et al., 19
  - We have a 2 parameter fit and get $A = 0.115 \pm 0.034$ GeV, $B = 0.0038 \pm 0.0154$ GeV$^3$ with a $\chi^2$/dof $\sim 0.64$
  - Brambilla et al., 19 have an 8 parameter fit with a $\chi^2$/dof $\sim 0.99$
  - Including long distance information from the QCD string improves the description of lattice data
- Once $A$ and $B$ are fixed, we can predict the bottomonium hyperfine splittings
Charmonium Hybrids HFS

Figure: The spectrum of the lower-lying $n(s/d)_1 (H_1)$, $np_1 (H_2)$, $n(p/f)_2 (H_4)$ and $np_0 (H_3)$ charmonium hybrids
Figure: The spectrum of the lower-lying $n(s/d)_1 (H_1)$, $n p_1 (H_2)$, $n(p/f)_2 (H_4)$ and $n p_0 (H_3)$ bottomonium hybrids
Error budget

- Input long distance parameters $\Lambda'$, $\Lambda'''$: negligible
- Interpolation (moving $r_0$) $\sim 3$ MeV in average, eventually neglected
- $\bar{c}cg$
  - Fit error in the short distance parameters ($A$ and $B$): $5 - 45$ MeV
  - Higher order terms $\sim 30$ MeV
- $\bar{b}bg$
  - Error in the short distance parameters ($A'$ and $B'$): $< 4$ MeV
  - Higher order terms $\sim 3$ MeV
- Unaccounted systematic errors from input lattice data
  - Static potentials, $n_f = 0$, fixed $a$ (Juge, Kuti, Morningstar, 02), continuum limit (Bali, Pineda, 03)
  - $\Lambda'$, $\Lambda'''$, $n_f = 0$, several $a$ (Koma, Koma, 09)
  - Spectrum, $n_f = 2 + 1 + 1$ ($m_\pi = 240$ MeV), fix $a$, (Cheung, O’Hara, Moir, Peardon, Ryan, Thomas, Tims, 16)
Conclusions

- The BOEFT provides a QCD based framework to address doubly-heavy exotics systematically.
- It requires non-perturbative potentials as an input.
- When those potentials are not available, a Cornell-like approach of interpolating between the short distance QCD (pNRQCD) calculation and a long distance string calculation appears to be promising.
String breaking

Bali, Neff, Duessel, Lippert, Schilling, 2005
String breaking

Bulava, Hörz, Knechtli, Koch, Moir, Morningstar, Peardon, 2019
| Resonance   | $J^{PC}$ | Assignement | Mass (MeV) | Observations                                                                 |
|------------|----------|-------------|------------|------------------------------------------------------------------------------|
| X(3823)    | 2$^{--}$ | 1d          | 3792       |                                                                              |
| X(3860)    | 0 or 2$^{++}$ | 2p       | 3968       |                                                                              |
| X(3872)    | 1$^{++}$ | 2p          | 3967       |                                                                              |
| X(3915)    | 0 or 2$^{++}$ | 2p       | 3968       |                                                                              |
| X(3940)    | ??       | 2p          | 3968       |                                                                              |
| Y(4008)    | 1$^{--}$ | 1(s/d)$_1$  | 4004       |                                                                              |
| X(4140)    | 1$^{++}$ | ??          | ??         |                                                                              |
| X(4160)    | ??       | 1$p_1$      | 4146       |                                                                              |
| Y(4220)    | 1$^{--}$ | 2$d$        | 4180       |                                                                              |
| X(4230)    | 1$^{--}$ | 2$d$        | 4180       |                                                                              |
| X(4350)    | ?$^{++}$ | 2(s/d)$_1$  | 4355 or 4369|                                                                 |
| Y(4320)    | 1$^{--}$ | 2(s/d)$_1$  | 4366       |                                                                              |
| Y(4360)    | 1$^{--}$ | 2(s/d)$_1$  | 4366       |                                                                              |
| X(4390)    | 1$^{--}$ | 2(s/d)$_1$  | 4366       |                                                                              |
| X(4500)    | 0$^{++}$ | 1$p_0$      | 4566       |                                                                              |
| Y(4630)    | 1$^{--}$ | 3$d$        | 4559       |                                                                              |
| Y(4660)    | 1$^{--}$ | 3(s/d)$_1$  | 4711       |                                                                              |
| X(4700)    | 0$^{++}$ | 4$p$        | 4703       |                                                                              |
| Ξ(10860)   | 1$^{--}$ | 5$s$        | 10881      |                                                                              |
| $Y_b(10890)$ | 1$^{--}$ | 2(s/d)$_1$  | 10890      |                                                                              |
| Ξ(11020)   | 1$^{--}$ | 4$d$        | 10942      |                                                                              |
| Resonance     | $J^{PC}$ | Assignement | Mass (MeV) | Observations       |
|--------------|----------|-------------|-----------|-------------------|
| $\psi_2(3823)$ | 2$--$   | $1d$        | 3792      |                   |
| $\psi_3(3842)$ | 3$--$   | $1d$        | 3792      |                   |
| $\chi_{c0}(3860)$ | 0$^{++}$ | ??          | ??        |                   |
| $\chi_{c1}(3872)$ | 1$^{++}$ | ??          | ??        |                   |
| $\chi_{c0}(3915)$ | 0$^{++}$ | $2p$        | 3968      |                   |
| $\chi_{c2}(3930)$ | 2$^{++}$ | $2p$        | 3968      |                   |
| X(3940)       | ??       | $2p$        | 3968      |                   |
| $\psi(4040)$  | 1$--$   | $1(s/d)_1$  | 4004      | mixing            |
| $\chi_{c1}(4140)$ | 1$^{++}$ | $1p_1$      | 4146      | not enough mixing |
| X(4160)       | ??       | $1p_1$      | 4146      |                   |
| $\psi(4230)$  | 1$--$   | $2d$        | 4180      | mixing            |
| $\chi_{c1}(4272)$ | 1$^{++}$ | $3p$        | 4369      |                   |
| X(4350)       | ??$^+$  | $2(s/d)_1$  | 4355 or 4369 |                   |
| $\psi(4360)$  | 1$--$   | $2(s/d)_1$  | 4366      | mixing            |
| $\psi(4415)$  | 1$--$   | $4s$        | 4513      |                   |
| $\chi_{c0}(4500)$ | 0$^{++}$ | $1p_0$      | 4566      | not enough mixing |
| X(4630)       | 1$--$   | $3d$        | 4559      |                   |
| $\psi(4660)$  | 1$--$   | $3(s/d)_1$  | 4711      | mixing            |
| $\chi_{c1}(4685)$ | 1$^{++}$ | $4p$        | 4727      |                   |
| $\chi_{c0}(4700)$ | 0$^{++}$ | $4p$        | 4703      |                   |
| $\Upsilon(10753)$ | 1$--$  | $3d$        | 10712     | mixing            |
| $\Upsilon(10860)$ | 1$--$  | $5s$        | 10881     | mixing            |
| $\Upsilon(11020)$ | 1$--$  | $4d$        | 10942     |                   |
### $1^{--}$ bottomonium spectrum $\sim 2019$

| $NL_J$    | $\lambda = 0.6$ | Hybrid % | PDG       |
|-----------|------------------|----------|-----------|
| $1s$      | 9.441            | 0        | $\Upsilon(1S)$ |
| $2s$      | 10.000           | 2        | $\Upsilon(2S)$ |
| $1d$      | 10.133           | 2        | $\Upsilon(1D)$ |
| $3s$      | 10.352           | 0        | $\Upsilon(3S)$ |
| $2d$      | 10.440           | 2        | $\Upsilon(10520)$ ? (Belle, 19) |
| $4s$      | 10.635           | 1        | $\Upsilon(4S)$ |
| $1(s/d)_1$ | 10.688          | 79       | ??         |
| $3d$      | 10.713           | 56       | $\Upsilon(10750)$ (Belle, 19) |
| $5s$      | 10.881           | 17       | $\Upsilon(10860)$ |
| $2(s/d)_1$ | 10.886          | 75       | $\Upsilon(11020)$ |
| $4d$      | 10.942           | 11       | $\Upsilon(10890)$ |

- The 17% hybrid component of $\Upsilon(10860)$ may explain the observed spin symmetry violating decays to $h_b$
The 17% hybrid component of $\Upsilon(10860)$ may explain the observed spin symmetry violating decays to $h_b$. 

| $NL_J$ | $\lambda = 0.6$ | Hybrid % | PDG       |
|-------|-----------------|-----------|-----------|
| 1$s$  | 9.441           | 0         | $\Upsilon(1S)$ |
| 2$s$  | 10.000          | 2         | $\Upsilon(2S)$ |
| 1$d$  | 10.133          | 2         | $\Upsilon(1D)$ |
| 3$s$  | 10.352          | 0         | $\Upsilon(3S)$ |
| 2$d$  | 10.440          | 2         | ??         |
| 4$s$  | 10.635          | 1         | $\Upsilon(4S)$ |
| 1$(s/d)_1$ | 10.688 | 79        | ??         |
| 3$d$  | 10.713          | 56        | $\Upsilon(10750)$ |
| 5$s$  | 10.881          | 17        | $\Upsilon(10860)$ |
| 2$(s/d)_1$ | 10.886 | 75        | ??         |
| 4$d$  | 10.942          | 11        | $\Upsilon(11020)$ |