Dynamic behaviour of stiffened orthotropic plates subjected to Friedlander blast load

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Abstract. The increase in the number of terrorist attacks have shown that the effect of blast loads, such as the Friedlander blast load on elements of a building structure is a serious matter. Their effects on the structure should be taken into consideration in the design of the structure’s element. The objective of this study is to numerically analyze the blasting effect on stiffened orthotropic concrete plates within the elastic range of its material. The effects of the stiffener configuration, the location of the localized blast load as well as the influence of the thickness to the vertical deflection of the plate are solved numerically by using the two auxiliary equations in the x and y-directions. The analysis yields the vertical deflection versus time relationship which can be used to determine the stress distributions and the maximum stress value of the plate. This paper introduces the effect of the blast load on the orthotropic plates and essential techniques used to increase the capacity of a structure’s element to provide protection against explosive effects.

Keywords: blast load, Friedlander blast load, stiffened orthotropic plate, auxiliary equation, stress distribution.

1. Introduction
Blast loading is a very rapid release of stored energy which imposes extreme loading on the floor of the building structure. In addition, it produces lots of high velocity airborne sharp fragments, the cause of a large percentage of all non-fatal injuries. The pressure due to the localized blast on an orthotropic plate of the building can cause bodily harm to occupants and is also the source of extensive property damage. The blast design of a structure has become important not only for federal and military buildings but also for other high-risk buildings such as school, hospitals, public buildings, and stiffened plates. If the structure is properly designed for this loading, the damaging effect can be minimized. In the past few decades, methods to predict blast loading, structural response and the consequent design to withstand these loads have been developed [1, 2, 3].

Blast loading research has gained interest in the last two decades. Though previous research has focused on empirical methods, modern computer and finite element method have facilitated numerical studies in recent years [4]. Jacob et al studied the fixed supported quadrilateral steel plates with different thickness and varying length to width ratios using experimental results and numerical predictions [5].
In his study, the effects of loading conditions and plate geometries on the deflection of the plates to various localized blast loads were considered. In 2008, Kadid [6] investigated the effect of time duration of blast pressure on other parameters, such as strain rate, and showed that including strain-rate effects resulted in a much stiffer response. Having investigated the steel stiffened plates, Alisjahbana and Wangsadinata [7] concluded that the duration of blast loading is one of the most important parameters which greatly influences other parameters including bending distribution and maximum dynamic deflection. In 2014, Tavakoli and Kikakojouri studied the nonlinear dynamic response of square steel stiffened plates using numerical methods and included parameters such as stiffener configurations, boundary conditions, fixing details and strain rate [4]. The mid-span vertical deflections and energy models of the plate were also examined in detail.

In addition, Alisjahbana et al [8] investigated the effects of positive and negative phases of blast loading and found out that negative phases of the localized blast load influenced the vertical displacement of the plate.

To further study the behavior of the concrete plates subjected to the Friedlander blast load done by Alisjahbana et al in 2018, this paper aims to consider other parameters that may affect the dynamic response of the plates subjected to Friedlander blast load. In particular, this paper considers stiffener configurations, load location, and plate thickness. Special emphasis is focused on the evaluation of mid-span deflection and maximum stress distribution within the plate region. The objective of this paper is to also propose a new model to overcome the shortcoming of the previous researchers.

2. Description of the orthotropic plate
The dynamic responses of an orthotropic concrete plate with semi rigid condition along its edges under the Friedlander blast load are highly dependent on the material properties of the concrete and reinforcement of the stiffeners [7]. In the following numerical analysis, the assumption that bonding between reinforcing bars and concrete is assumed to be perfect.

The size of the concrete plate studied in this paper is 8.0 x 5.0 m with rectangular stiffeners parallel to x axes and a thickness of 0.23 m and 0.25 m. Figure 1 shows the rectangular orthotropic stiffened concrete plate with semi rigid boundary conditions along its edges subjected to the Friedlander blast load \( f(x,y,t) \).

![Figure 1. Rectangular orthotropic concrete plate subjected to the Friedlander blast load at \((x_0,y_0)\).](image)

Using the theory of thin plates, the equilibrium equation of an elastic orthotropic stiffened plate can be expressed as:

\[
D_x \frac{\partial^4 U}{\partial x^4} + 2B \frac{\partial^3 U}{\partial x^3 \partial y} + D_y \frac{\partial^3 U}{\partial y^3} + \xi h \frac{\partial^3 U}{\partial t^3} + \rho h \frac{\partial^3 U}{\partial t^3} = f(x, y, t) \tag{1}
\]

where \( U = X(x)Y(y) \) is the spatial function consisting of the multiplication of the \( X(x) \) and the \( Y(y) \) functions, \( T(t) \) is the temporal function,
$$D_x = \frac{E' h^3}{12} + \frac{E' h^3}{12} \left[ h_x - \left( e_x - \frac{h}{2} \right) \right]^2 \left( 2 h_x + e_x + h \right) - \left( e_x - \frac{h}{2} \right) \left( e_x + h \right)$$ is the flexural rigidity in the $x$-direction, $D_y = \frac{E' h^3}{12}$ is the flexural rigidity in the $y$-direction, $B = \sqrt{D_x D_y}$ is the torsional rigidity and $f(x,y,t)$ is the Friedlander blast load [8].

According to Hamedani et al., the values of natural frequencies of the plate with only one stiffener is not affected by the eccentricity of the free vibration of stiffened plates. Only as the number of stiffeners increases will the effect of eccentricities become significant [9]. Since this paper only takes into consideration 4 configurations of stiffeners and thickness (configuration 1-without stiffener, $h=0.23$ m; configuration 2-without stiffener, $h=0.25$ m; configuration 3-1 stiffener, $h=0.23$ m, and configuration 4-2 stiffeners, $h=0.23$ m), the effect of the eccentricity is ignored in determining the dynamic response of the system.

3. Localized Friedlander blast loading

A localized Friedlander decaying function, $F(t)$, located at an arbitrary position $(x_0, y_0)$ within the plate region [10], [8] can be written in the form:

$$F_p(t) = f_{r,\text{max}} \left( 1 - \frac{t}{t_p} \right) e^{-\frac{t}{t_p}} \quad \text{for } 0 \leq t \leq t_p$$

$$F_n(t) = -f_{r,\text{min}} \left( \frac{6.75(t-t_p)}{t_n} \right) \left( 1 - \frac{t-t_p}{t_n} \right)^2 \quad \text{for } t_p \leq t \leq (t_p + t_n)$$

![Figure 2. Idealized pressure-time profile for a Friedlander blast load.](image)

The Friedlander blast load $f(x,y,t)$ according to equation (1) and figure 2 is the piecewise function given by [8]:

for $0 \leq t \leq t_p$:

$$f(x,y,t) = F_p(t) \delta[x-x_0] \delta[y-y_0]$$ (4a)

for $t_p \leq t \leq (t_p + t_n)$:

$$f(x,y,t) = F_n(t) \delta[x-x_0] \delta[y-y_0]$$ (4b)
4. Dynamic analysis

The dynamic analysis of the plate was solved using the separation of variables method. By assuming that the temporal function \( T(t) \) is harmonic, the free vibration response of the system can be written in the form:

\[
u(\mathbf{x}, y, t) = U(\mathbf{x}, y) \sin \omega t = X(x)Y(y)\sin \omega t
\]

(5)

where \( \omega \) is the natural frequency of the system, the functions \( X(x) \) and \( Y(y) \) can be determined from the boundary conditions. The natural frequencies of the system for the semi rigid boundary conditions along the edges of the plate can be obtained from the two auxiliary equations as follow [2]:

\[
\omega_{pq} = \sqrt{\frac{\pi^4}{\rho h} \left[ D_x \frac{p^4}{a^4} + 2B \frac{p^2 q^2}{a^2 b^2} + D_y \frac{q^4}{b^4} \right]}
\]

(6)

In equation (6), \( p \) and \( q \) are the dynamic modes of the system which are real numbers.

A modal analysis was also conducted for analyzing the dynamic response of the stiffened orthotropic plates subjected to the Friedlander blast load. The total vertical displacement of the plate is the multiplication of the spatial function \( U(x,y) \) and temporal function \( T(t) \) resulting from the Duhamel integration method as follows:

\[
u(\mathbf{x}, y, t) = \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} X(x)Y(y) \int_0^{L_x} \int_0^{L_y} \frac{X(x)dx}{\rho h Q \omega_d} \int_0^t \int_0^t f(x, y, \tau) e^{-\omega_d (t-\tau)} \sin \omega_d (t-\tau) d\tau dy dx
\]

(7)

where \( \omega_d \) is the damped frequency of the system and \( Q \) is a normalization factor that can be expressed as:

\[
Q = \int_0^{L_x} X(x)X(x)dx \int_0^{L_y} Y(y)Y(y)dy
\]

(8)

5. Numerical results and discussions

In this numerical approach, four configurations of the concrete plate were modelled. By considering the convergence of the Eigen vectors, the number of dynamics modes (\( p \) and \( q \)) used in calculating the response of the plate is equal to 6. To calculate the vertical deflection and the distribution of stress in the system, the geometries and the mechanical properties of material are introduced in table 1 [11].

| Notation | Unit |
|----------|------|
| \( L_x \) | Plate length in \( x \)-direction | 8.0 m |
| \( L_y \) | Plate length in \( y \)-direction | 4.0 m |
| \( h \) | Plate thickness | 0.23, 0.25 m |
| \( \rho \) | Mass density of concrete | \( 2.4 \times 10^3 \) kg/m\(^3\) |
| \( \nu_x \) | Poisson’s ratio in the \( x \)-direction | 0.2 |
| \( \nu_y \) | Poisson’s ratio in the \( y \)-direction | 0.15 |
| \( E_x \) | Young’s modulus in the \( x \)-direction | \( 23.5 \times 10^6 \) N/m\(^2\) |
| \( E_y \) | Young’s modulus in the \( y \)-direction | \( 22.045 \times 10^6 \) N/m\(^2\) |
| \( k_1 \) | Rotational spring constant in the \( x \)-direction | \( 2.5 \times 10^7 \) N/m/rad |
| \( k_2 \) | Rotational spring constant in the \( y \)-direction | \( 2.5 \times 10^7 \) N/m/rad |
| \( a_x \) | Distance between stiffener | 4.2, 67 m |
| \( e_x \) | Eccentricity of the stiffener | 0.25 m |
5.1. Configuration effects of stiffeners
For the 4 different configurations, the absolute maximum vertical deflection was computed. The mid-
point displacement was decreased significantly with the addition of stiffeners; the mid-point displacement for configuration 1 (without stiffeners) is 0.8899 mm occurs at $t = 0.0082$ s, while for configuration 3 (1 stiffener) and configuration 4 (2 stiffeners) are 0.8130 mm at $t = 0.0080$ s and 0.7614 mm at $t = 0.0078$ s respectively. All configurations of the stiffeners were computed with a damping ratio of 5%. It is also shown that by the introduction of stiffeners to the plate, the time at which maximum dynamic deflection occurs on the plate is earlier than that of the plate without stiffeners, as shown in figure 3. The existence of the stiffeners within the plate region also affects the maximum stress distribution. Therefore, as shown in figure 4, the response of the stiffened orthotropic plate is significantly influenced by the configuration of the stiffeners.

5.2. Effect of the plate thickness
For configuration 1, increasing the floor thickness from 0.23 m to 0.25 m for the damped system with $\xi=5\%$ decreased the mid-span displacement by 16.73%. Increasing the thickness of the concrete also resulted in a decrease of the maximum compression stress within the plate region, as shown in figure 4.
Figure 4. The maximum bending stress distribution within the plate region subjected to the Friedlander blast load at the mid-span computed at \( t=0.008 \) s.

Figure 5. 3D stress distribution within the plate region computed at \( t=0.008 \) s.

Figure 5 shows the 3D stress distribution for two different plate thickness. The stress distribution along the boundaries for both cases need to be examined carefully, particularly for the design of bars.
along the plate edges. It is now possible to conclude that the stress distribution depends on the plate thickness.

5.3. Effect of the blast load position to the dynamic response

Table 2 shows the maximum dynamic deflection for different positions of the load in x-direction. It can be seen that the maximum vertical deflection at the mid-span of the plate increases as the position of the blast load nears the middle.

The effect of the blast load position to the dynamic deflection as a function of time for configuration 1 is shown in figure 6. The maximum vertical deflection of configuration 1 is reduced by 19.15% if the position of the blast load moved further away a/8 from the middle point.

Table 2. The maximum vertical deflection of concrete plate subjected to the Friedlander load for different configurations.

| Position of the load | Configuration 1 | Configuration 2 | Configuration 3 | Configuration 4 |
|----------------------|----------------|----------------|----------------|----------------|
| h (cm)               | m             | m             | 1             | 2             |
| (a/8, b/2)           | 0.4291        | 0.3491        | 0.3935        | 0.3960        |
| (a/4, b/2)           | 0.6599        | 0.5343        | 0.6315        | 0.6308        |
| (3a/8, b/2)          | 0.7195        | 0.5898        | 0.6781        | 0.6660        |
| (a/2, b/2)           | 0.8899        | 0.7411        | 0.8130        | 0.7614        |

Figure 6. The dynamic deflection of orthotropic damped plate (configuration 1) subjected to the Friedlander blast load for different load position.
Conclusions
The dynamic analysis of the vertical deflections and the flexural stresses in orthotropic concrete plate subjected to the Friedlander blast load was performed using the modified Bolotin method. The following conclusions can be drawn:

- The vertical deflection at the mid-span of the plate are reduced significantly as the number of the stiffeners and the plate thickness are increased.
- The maximum flexural stresses within the plate region are reduced when the plate thickness is increased.
- The location of the blast load effects the maximum vertical deflection of the plate. As the location of the blast load approaches the mid-span of the plate, the maximum vertical deflection is increased.

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