Breakpoint phenomenon in layered superconductors

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Abstract. We study theoretically the multiple branch structure in the IV-characteristics of intrinsic Josephson junctions in HTSC and investigate in detailed its outermost branch at different values of the dissipation parameter. A different character of the IV-characteristics in the different intervals of the dissipation parameter $\beta$ was observed. This feature follows from the fact of the creation of the longitudinal plasma wave with different wave number $k$. The possibility to observe experimentally the change of the wave vector of the longitudinal plasma wave by changing the temperature is analyzed.

1. Introduction

A simulation of the IV-characteristics (IVC) of intrinsic Josephson junctions (IJJ) at different values of the model parameters such as the coupling $\alpha$ and dissipation $\beta$ parameters is a way to predict the properties of such systems. McCumber and Steward have investigated the IVC and the return current as a function of the dissipation parameter in a single Josephson junction a long time ago.[1] In the case of the system of junctions, the situation is cardinally different. The IVC of IJJ is characterized by a multiple branch structure and branches have a breakpoint regions with their breakpoint currents (BPC) $I_{bp}$ and transition current $I_j$ to another branch.[2, 3] The BPC is determined by the creation of the longitudinal plasma waves (LPW) with a definite wave number $k$, which depends on the parameters $\alpha$ and $\beta$, the number of junctions in the stack, and boundary conditions.

In Ref.[4] we generalized the McCumber-Steward dependence of the return current for the case of IJJ in the HTSC. We investigated the BPC $I_{bp}$ of the outermost branch as a function of the coupling $\alpha$ and dissipation $\beta$ parameters for the stacks with a different number of IJJ and demonstrated a plateau with the BPC oscillation. Based on the idea of the parametric resonance in the stack of IJJ, a modeling of the $\alpha\beta$-dependence of the BPC was done, and good qualitative agreement with the results of simulation was obtained. We showed that the $\alpha\beta$-dependence of the $I_{bp}$ is an instrument to determine the mode of LPW which is created at the breakpoint. We predicted the different commensurability effects in the IVC of stacks with a different number of IJJ.[5, 4]

In this paper we simulate the IVC of the stacks of IJJ and investigate the outermost branch in the IVC at different values of the dissipation parameter. We observe the different character of the IVC in the different intervals of the dissipation parameter $\beta$, which follows from the fact of creation of the LPWs with different wave numbers $k$. The IVC of intrinsic Josephson
junctions are studied at different temperatures and demonstrated that temperature dependence of the breakpoint current is changed with the coupling parameter and boundary conditions. We discuss the possibility to observe experimentally the change of the wave vector of created at the breakpoint LPW by changing the temperature.

2. IVC at different value of the dissipation parameter $\beta$

In the capacitively coupled Josephson junctions model with diffusion current (CCJJ+DC model)[6, 2] the stacks with $N$ intrinsic Josephson junctions is described by a system of dynamical equations

$$\frac{d}{dt}V_i = I - \sin \varphi_i - \beta \frac{d\varphi_i}{dt}$$

$$\frac{d}{dt}\varphi_i = V_i - \alpha(V_{i+1} + V_{i-1} - 2V_i)$$

for the gauge-invariant phase differences $\varphi_i(t) = \theta_{i+1}(t) - \theta_i(t) - \frac{2e}{\hbar} \int_{l_i}^{l_i+1} dz A_z(z, t)$ between superconducting layers (S-layers). Here $\theta_l$ is the phase of the order parameter in S-layer $l$, $A_z$ is the vector potential in the barrier. Time $t$ is normalized to the plasma frequency $\omega_p$ ($\omega_p^2 = 2eI_c/\hbar C$), the voltage - to the value $V_0 = \hbar \omega_p/2e$, the current - to the critical current $I_c$.

This system of equations might be written in the form

$$\frac{d^2}{dt^2}\varphi_i = (I - \sin \varphi_i - \beta \frac{d\varphi_i}{dt}) + \alpha(\sin \varphi_{i+1} + \sin \varphi_{i-1} - 2\sin \varphi_i) + \alpha\beta(\frac{d\varphi_{i+1}}{dt} + \frac{d\varphi_{i-1}}{dt} - 2\frac{d\varphi_i}{dt})$$

which demonstrates the difference between the CCJJ+DC and CCJJ models: the last term in this equation, which is proportional to the $\alpha\beta$ is absent in CCJJ model.

We solve this system of equations for stacks with different numbers $N$ of intrinsic Josephson junctions. The numerical procedure has been done as follows. For a given set of model parameters $N, \alpha, \beta, \gamma$ we simulate the IVC of the system. A change in the parameters $N, \alpha, \beta, \gamma$ changes the branch structure in the IVC essentially. Their influence on the IVC in the CCJJ and CCJJ+DC models was discussed in Refs.[2, 7, 8]. To calculate the voltages $V_i(I)$ in each point of the IVC (for each value of $I$), we simulate the dynamics of the phases $\varphi_i(t)$ using the fourth-order Runge-Kutta method. After simulation of the phase dynamics we calculate the voltages on each junction from the generalized Josephson relation

$$\frac{\partial \varphi_i}{\partial t} = \sum_{i'} A_{ii'}V_{i'}$$

with the matrix $A$ given in Ref.[5]

The average voltage $\bar{V_i}$ is given by

$$\bar{V_i} = \frac{1}{T_{max} - T_{min}} \int_{T_{min}}^{T_{max}} V_i dt$$

where $T_{min}$ and $T_{max}$ determine the interval for the averaging. After completing the voltage averaging for current $I$, the current $I$ is increased or decreased by a small amount $\delta I$ to calculate the voltages at the next point of the IVC. We use the distribution of phases and their derivatives achieved in the previous point of the IVC as the initial distribution for the current point. Finally we can obtain the total dc voltage $V$ of the stack by
\[ V = \sum_{l=1}^{N} \bar{V}_l \] (5)

At some current \( I \) some junction (or junctions) switches to the nonzero voltage state and it gives some branch of the IVC. The details concerning the numerical procedure are given in Refs. [8, 9].

Fig. 1 presents the simulated outermost branch in the IVC of the stack with eleven IJJ at \( \alpha = 1 \), periodic boundary conditions and different values of the dissipation parameter \( \beta \). Fig. 1a demonstrates the outermost branch in the IVC for \( \beta \) in the interval \((0.105, 0.45)\). We can see the continuous variation of the branch slope, the appearance and disappearance the breakpoint region in this interval of \( \beta \). The value of the BPC \( I_{bp} \) is increasing with \( \beta \), approaching its maximum. Fig. 1b corresponds to the interval of \( \beta \) \((0.46, 0.51)\). In this interval the slope of the branches does not change practically, but \( I_{bp} \) demonstrates maximum with increase in \( \beta \). The IVC do not manifest the BPR in this interval, practically. The breakpoint current \( I_{bp} \) in the interval \((0.52-0.6)\) demonstrates the maximum as well (see Fig. 1c) As we can see, there is very specific increase in the width \( w_{bp} \) of the BPR here. Fig. 1d shows the outermost branch at \( \beta = 0.64; 0.65; 0.66 \). The breakpoint current is continuously increasing and the IVC have not the BPR in this interval of \( \beta \). The region \( \beta > 0.667 \) corresponds to the non hysteretic behavior.
3. \(\beta\) and \(\alpha\) dependence of the BPC \(I_{bp}\)

The investigation of the breakpoint phenomenon in the stack of IJJ leads to very interesting features of the breakpoint current dependence on coupling and dissipation parameters.[5, 4] The coupling parameter \(\alpha\) is considered as a constant for given material and the dependence of the \(I_{bp}\) on \(\alpha\) cannot be measured in a single sample. But in principal, the dependence \(I_{bp}\) on \(\beta\) can be measured directly due to the temperature dependence of the McCumber parameter.

Fig. 2a shows the dependence of the \(I_{bp}\) as a function of the \(\beta\). Analysis of the time dependence of the charge in the superconducting layers and use of the ”\(k - \alpha \beta\)”- methods allow us to determine the values of the wave vector in the different intervals of \(\beta\). We find that in the first interval \((0.105, 0.45)\) of \(\beta\) the LPW with \(k = 10\pi/11\) is created, in the second interval \((0.46, 0.51)\) wave vector is equal to \(k = 8\pi/11\), in third interval \((0.52-0.6)\) we observe \(k = 6\pi/11\), and in the fourth interval \((0.64, 0.66)\) the LPW with \(k = 4\pi/11\) is created. So, the different character of the IVC in shown intervals of \(\beta\) related to the different value of the LPW wave vector.

The general discussion of the effect of coupling on the dissipation dependence of the breakpoint current have been done in Ref.[4]. At \(\alpha = 0\), the IVC does not manifest the multibranch structure, and the value of the breakpoint current coincides with the return current. The \(\alpha\) dependence of the \(I_{bp}\) at small \(\beta\) is monotonic and \(I_{bp}\) is increasing with \(\alpha\). At some \(\beta\) the oscillations of \(I_{bp}\) are appeared, which reflect the change of the wave number of the created longitudinal plasma wave. With increase in \(\beta\) the transition to the non-hysteretic regime is observed at smaller \(\alpha\). The increase in \(\beta\) leads to the fact that the value of the \(I_{bp}\) changes strongly at small \(\alpha\). Fig. 2b demonstrates this behavior, where the \(\alpha\) dependence of the \(I_{bp}\) at \(\beta = 0.1, 0.2, 0.3, 0.5\) and 0.6 is shown. The inset shows the details of the curves in the interval of \(I_{bp}\) \((0.8, 0.85)\).

We demonstrated also in Ref.[4], that the curves of the \(\beta\) dependence of the \(I_{bp}\) at \(\alpha \neq 0\) have new features in comparison with the case without coupling. Particularly, with increase in \(\alpha\) they show a stronger increase of the \(I_{bp}\) at small \(\beta\), a plateau on this \(\beta\)-dependence and the oscillation of the \(I_{bp}\) on this plateau, and a transition to the non-hysteretic regime (second
4. Temperature dependence of the BPC

The temperature dependence of the breakpoint current is determined by the temperature dependence of the critical current, McCumber parameter, number of junctions in the stack, coupling between junctions and boundary conditions. The temperature dependence of the McCumber parameter $\beta_c$ in its turn depends on the temperature dependence of the critical current $I_c(T)$, junction resistance $R_J(T)$ and capacitance $C_J(T)$. In present paper we consider the case that the capacitance of the junctions does not depend on temperature and it is geometrical capacitance $C_J = \varepsilon_r \varepsilon_0 S/D$, where $S$ is the area of the junctions, $\varepsilon_r$ is dielectric constant, $\varepsilon_0$ is electric constant, and $D$ is the thickness of the insulating layer.

In the simple parallel resistance model,[10] a single junction resistivity $\rho_J(T)$ at subgap voltage region is given by

$$\rho_J^{-1}(T) = \rho_{sg}^{-1} + \rho_C^{-1}(T)$$

(6)

where $\rho_{sg}$ is the temperature independent tunnel resistivity of the junction, and

$$\rho_C(T) = a \exp(b/T) + cT + d$$

(7)

is the empirical Heine formula of the c-axis resistivity[11] with $a, b, c, d$ as fitting parameters. At low temperature c-axis resistance freezes out, and the junction resistance is dominated by the temperature independent term $\rho_{sg}$. 

Figure 3. (Color online) The temperature dependence of the: (a)- junction resistance $R_J(T)$ according to (8) and c-axis resistivity according to the empirical Heine formula (7); (b)- normalized critical current $I_c(T)/I_{c0}$ according to (9) and dissipation parameter $\beta$ according to (10) plateua) at smaller $\beta$. In materials with a larger coupling the LPW with a smaller wave number is created at the breakpoint.

The dissipation parameter $\beta$ depends on the values of the critical current $I_c$ and the junction resistance $R_J$ as $\beta^2 = 1/\beta_c = h/2eC_J R_N^2 I_C$, where $\beta_c$ is the McCumber parameter and $C_J$ is the junction capacitance. Because $I_c$ and $R_N$ depend on the temperature, the investigation of the $\beta$-dependence of the BPC $I_{bp}$ will allow us to predict its temperature dependence.

The simulated IVC has a breakpoint on its outermost branch. We have calculated the $\beta$-dependence of the BPC $I_{bp}$ at fixed value of $\alpha$. The result of the calculation is presented in Fig. 2. It demonstrates the $\beta$-dependence of the $I_{bp}$ for the stacks with 11 IJJ at $\alpha = 1$ and periodic boundary conditions. As we can see, there are four $\beta$ regions in this dependence, which correspond to the creation of the LPW with different $k$.[4]
Figure 4. (Color online) The simulated T-dependence of the \( I_{bp} \) for the stacks with 11 IJJ at different values of the coupling parameter \( \alpha \).

Estimating the tunnel resistivity by \( \rho_{sg} = \Delta(0)S/eD I_c(0) \), the energy gap \( \Delta \) from the expression \( 2\Delta(0)/kT_c = 6 \) and using (7), we can find the temperature dependence of the \( R_J \)

\[
R_J = \frac{\rho_{sg}\rho_c}{(\rho_{sg} + \rho_c)S} D
\]

Using the Ambegaokar and Baratoff\[12\] temperature dependence of the critical current \( I_c = \frac{\pi \Delta(T)}{2\pi R_J} \tanh \frac{\Delta(T)}{2T} \) modeled as

\[
I_c(T) = I_c(0)\sqrt{\cos \frac{\pi}{2} \left( \frac{T}{T_c} \right)^2} \tanh (0.88\sqrt{\cos \frac{\pi}{2} \left( \frac{T}{T_c} \right)^2})
\]

we can find the temperature dependence of the dissipation parameter by formula

\[
\beta^2 = 1/\beta_c = \hbar/2eC_J R_J^2 I_c
\]

In our simulations we chose \( S = 2.32*10^{-10} m \) for the area, \( T_c = 90 K \) for the critical temperature, \( j_c(0) = 9*10^6 A/m^2 \) at \( T = 0 \). In Ref.[13] the fitting parameters were chosen as \( a = 6*10^{-4} \Omega m \) for the density of critical current, and \( b = 273 K \), \( c = 24*10^{-6} \Omega m/K \), \( d = 1.23*10^{-2} \Omega m \) as a parameters values. The temperature dependence of the junction resistance \( R_J(T) \) according to (8), the c-axis resistivity according to the empirical Heine formula (7), the normalized critical current \( I_c(T)/I_c(0) \) according to (9) and the dissipation parameter \( \beta \) according to (10) is demonstrated in Fig. 3.

The knowledge of the temperature dependence of \( \beta \) allow us to simulate the temperature dependence of the breakpoint current \( I_{bp} \). Fig. 4 shows the results of such simulations for the
stacks with 11 IJJ at different values of coupling parameter $\alpha$. In the inset to this figure we show the oscillations related to the change of the wave number $k$. As we can see, the increase in the value of the coupling parameter makes this effect more pronounced on the IVC of the stack of IJJ. We would like to stress the main fact: the LPW with another $k$ can be realized in the stack by changing the temperature. So, in the materials with different value of the coupling parameter and different number of junctions in the stack, the LPW with different wave vector can be created just by the temperature variation.

As a summary, we demonstrate the different character of the IV-characteristics in the different intervals of the dissipation parameter which related to the creation of the longitudinal plasma wave with different wave number. We predict the corresponding features on the temperature dependence of the breakpoint current, which might be observed experimentally. An interesting question appears, concerning the influence of the different kind of the temperature dependence of the dissipation parameter on this phenomenon.

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