Modified dispersion relations in extra dimensions

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Abstract

It has recently been shown that the thermodynamics of a FRW universe can be fully derived using the generalized uncertainty principle (GUP) in extra dimensions as a primary input. There is a phenomenologically close relation between the GUP and Modified Dispersion Relations (MDR). However, the form of the MDR in theories with extra dimensions is as yet not known. The purpose of this letter is to derive the MDR in extra dimensional scenarios. To achieve this goal, we focus our attention on the thermodynamics of a FRW universe within a proposed MDR in an extra dimensional model universe. We then compare our results with the well-known results for the thermodynamics of a FRW universe in an extra dimensional GUP setup. The result shows that the entropy functionals calculated in these two approaches are the same, pointing to a possible conclusion that these approaches are equivalent. In this way, we derive the MDR form in a model universe with extra dimensions that would have interesting implications on the construction of the ultimate quantum gravity scenario.

1 Introduction

A common feature of all promising candidates for quantum gravity is the existence of a minimal observable length \cite{1, 2, 3, 4, 5}. Modified dispersion relations (MDR) and the generalized uncertainty principle (GUP) are two approaches to phenomenologically incorporate this finite resolution of the space-time points within the theoretical framework of the standard model. In fact, MDR and GUP are common features to all candidates of quantum gravity models. In particular, in the study of loop quantum gravity and models based on non-commutative geometry, there has been strong interest in some modifications of the energy-momentum dispersion relation \cite{6, 7, 8, 9, 10}. On the other hand, the generalized uncertainty principle has been considered in string theory and in the models based on noncommutative geometry \cite{1, 2, 3, 4, 5}. The MDR and GUP essentially affect the thermodynamics of physical systems at energy scales within the realm of quantum gravity. Thus the exact form of the GUP and MDR could essentially lead one to a deeper understanding of the ultimate quantum gravity proposal.

Our goal here is to deduce the form of the MDR in a model universe with extra dimensions through the application of the machinery of black hole thermodynamics to the universe as a unique physical system. In this respect, we look at the status of this connection in two main frameworks.
Firstly, it has been shown recently that one can generalize the well-known approach to black hole thermodynamics to study thermodynamics of the Universe as a unique physical system through its apparent horizon. In this way, it has been revealed that there is a deep connection between thermodynamics and gravity through the laws of black hole thermodynamics [11, 12, 13]. This connection has also been realized for a FRW universe [14]. In a FRW universe, which is the subject of the present study, one would replace the event horizon of a black hole by the apparent horizon of a FRW spacetime. One then assumes that the apparent horizon has an associated entropy $S$ defined as $S = \frac{A}{4G}$ and a temperature $T$ defined as $T = \frac{1}{2\pi \tilde{r}_A}$, where $G$ is the gravitational constant, $A$ is the area of the apparent horizon and $\tilde{r}_A$ is the radius of the apparent horizon. In this viewpoint, the first law of thermodynamics, that is $dE = T dS$, can be translated into the language of the Friedmann equation. The first law of thermodynamics plays a crucial role in different theories such as Einstein gravity, Gauss-Bonnet gravity, Lovelock gravity and various braneworld scenarios [15, 16, 17]. Hence one can infer a deep connection between gravity and thermodynamics in this viewpoint [18].

Secondly, the Hawking radiation at the vicinity of a black hole event horizon [12, 19] or at the apparent horizon of a FRW spacetime [20] is another important issue worth discussing. The Hawking radiation is a quantum mechanical effect in the classical background of a black hole spacetime or FRW geometry. Therefore, quantum mechanics, gravitational theory and thermodynamics meet each other when the physics of black holes and FRW spacetime are concerned. A thorough study of black holes with a size comparable to the Planck scale or a FRW universe in the Planck era needs quantum gravity considerations. In other words, a complete quantum gravity scenario is required [18] to handle these important issues. Since GUP and MDRs are common features to all quantum gravity approaches, one may apply them to obtain thermodynamics of a FRW universe or black hole in the small length scale, or equivalently in the high energy regime. In the past few years, the GUP and MDR have been applied to modify the Hawking radiation and Bekenstein-Hawking entropy of black holes [21, 22, 23, 24, 25, 26, 27, 29]. Thermodynamical properties of a FRW universe as another important example of a quantum gravity regime has been studied within the GUP formalism in arbitrary dimensions [18]. The thermodynamics of the FRW universe has also been considered within the MDR formalism in a 4-dimensional spacetime [28].

The study of thermodynamics when extra dimensions are present is an interesting aspect of the discussions above, on which we shall concentrate in this work. One may consider the role played by extra dimensions in the study of thermodynamics of the FRW universe within the MDR framework and compare the results with that when the GUP is used. We therefore start by deriving the FRW universe thermodynamics using the well-known form of the generalized uncertainty principle in extra dimensions. Then, motivated by the form of the GUP in an extra dimensional scenario, we suggest a modified dispersion relation. We then use this form of the MDR to find the FRW universe entropy. Since MDR and GUP are different manifestations of the same concept (existence of a minimal length scale or a maximum momentum) in the quantum gravity theory, we expect that the entropy calculated in these two frameworks should be the same. In fact, we expect the results of application of these two approaches to the issue of apparent horizon thermodynamics should be the same at least in their functional form (and not necessarily in their numerical coefficients). The assumption behind this expectation is that GUP and MDR are phenomenologically two (though seemingly different) faces of an underlying quantum gravity proposal. By comparing the results deduced from these two approaches, we fix the functional form of the MDR suggested for a higher dimensional spacetime. In other words, consistency of thermodynamics in these two approaches constrains the form of the MDR suggested for a higher dimensional spacetime. Since we know the exact form of the extra dimensional GUP from various works (see [18, 29] for instance), we deduce the form of MDR in a spacetime with extra dimensions. Such a study has been lacking in the literature and would be useful in the construction of a successful theory of quantum gravity.
2 Entropy of a FRW universe

We consider a \((n + 1)\)-dimensional FRW universe with the following line element

\[ ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega_{n-1}^2 \right), \]

(1)

where \(d\Omega_{n-1}^2\) denotes the line element of a \((n - 1)\)-dimensional unit sphere, \(a(t)\) is the scale factor of our universe and \(k\) is the spatial curvature constant. Using the notation \(\tilde{r} = ar\), the radius of the apparent horizon can be written as

\[ \tilde{r}_A = \frac{1}{\sqrt{H^2 + k/a^2}}, \]

(2)

where \(H \equiv \frac{\dot{a}}{a} = \frac{da}{dt} a\) is the Hubble parameter. We assume that the apparent horizon has an associated entropy \(S\) and temperature \(T\) defined as

\[ S = \frac{A}{4G}, \quad T = \frac{1}{2\pi \tilde{r}_A}, \]

(3)

respectively. Here, \(A\) is the apparent horizon area and \(G\) is the gravitational constant. It has been shown that the first law of thermodynamics

\[ dE = TdS, \]

(4)

reproduces the Friedmann equation in this framework [18]. With these preliminaries, we are now ready to obtain the entropy of a FRW universe in the frameworks of the GUP and MDR models in a universe with extra dimensions. We then compare our results obtained in these two approaches to constrain the form of the proposed MDR in extra dimensional scenarios.

2.1 The GUP

We consider the following GUP

\[ \delta x \delta p \geq 1 + \alpha^2 l_p^2 \delta p^2, \]

(5)

where \(l_p\) is the Planck length depending on the dimensionality and fundamental energy scale of the extra dimensional model universe we are interested in and \(\alpha\) is a dimensionless real constant [18, 29]. It is easy to show that

\[ \delta p \geq \frac{1}{\delta x} \left[ \frac{\delta x^2}{2\alpha^2 l_p^2} - \frac{\delta x^2}{\delta x^2} \sqrt{1 - \frac{4\alpha^2 l_p^2}{\delta x^2}} \right] = \frac{1}{\delta x} \Psi (\delta x^2), \]

(6)

where

\[ \Psi (\delta x^2) = \frac{\delta x^2}{2\alpha^2 l_p^2} - \frac{\delta x^2}{\delta x^2} \sqrt{1 - \frac{4\alpha^2 l_p^2}{\delta x^2}}, \]

(7)

shows the departure of the GUP from the standard uncertainty principle. Here we assume that a particle with energy \(dE\) is absorbed or radiated via the apparent horizon. The energy of this particle may be identified by \(dE \simeq \delta p\) (with \(c = 1\)) [22]. Within the Heisenberg uncertainty principle, \(\delta p \geq 1/\delta x\), one may find from equations (3) and (4) that

\[ dA = \frac{4G}{T} dE \simeq \frac{4G}{T} \frac{1}{\delta x}. \]

(8)
Incorporating the effect of the GUP via the inclusion of $\Psi$, one finds

$$dA_\Psi = \frac{4G}{T}dE \simeq \frac{4G}{T} \frac{1}{\delta x} \Psi \left(\delta x^2\right). \quad (9)$$

Using equation (8), we find

$$dA_\Psi \simeq \Psi(\delta x^2)dA. \quad (10)$$

The position uncertainty $\delta x$ of the particle crossing through the apparent horizon can be chosen as its Compton wavelength which is proportional to the inverse of the Hawking temperature. Hence one can write [23, 29]

$$\delta x \simeq 2\tilde{x}_A = 2 \left(\frac{A}{n\Omega_n}\right)^{\frac{1}{n-1}}, \quad (11)$$

where $\Omega_n$ is the volume of an $n$-dimensional unit sphere. Now one obtains $\Psi(\delta x^2)$ as a function of the area of the apparent horizon as follows

$$\Psi(A) = \frac{2}{\alpha^2 l_p^2} \left(\frac{A}{n\Omega_n}\right)^{\frac{2}{n-1}} \left(1 - \sqrt{1 - \alpha^2 l_p^2 \left(\frac{n\Omega_n}{A}\right)^{\frac{2}{n-1}}}ight). \quad (12)$$

At $\alpha = 0$, one may use the Taylor expansion to obtain

$$\Psi(A) = 1 + \frac{\alpha^2 l_p^2}{4} \left(\frac{n\Omega_n}{A}\right)^{\frac{2}{n-1}} + \frac{\alpha^4 l_p^4}{8} \left(\frac{n\Omega_n}{A}\right)^{\frac{4}{n-1}} + \frac{15\alpha^6 l_p^6}{192} \left(\frac{n\Omega_n}{A}\right)^{\frac{6}{n-1}} + \cdots. \quad (13)$$

Considering only the terms up to the sixth power of the Planck length (without loss of generality in conclusion), one finds

$$dA_\Psi \simeq \left[1 + \frac{\alpha^2 l_p^2}{4} \left(\frac{n\Omega_n}{A}\right)^{\frac{2}{n-1}} + \frac{\alpha^4 l_p^4}{8} \left(\frac{n\Omega_n}{A}\right)^{\frac{4}{n-1}} + \frac{15\alpha^6 l_p^6}{192} \left(\frac{n\Omega_n}{A}\right)^{\frac{6}{n-1}}\right] dA. \quad (14)$$

Integrating equation (14) gives the modified area of the apparent horizon, $A_\Psi$. One may then substitute $A_\Psi$ in $S_\Psi = \frac{A_\Psi}{4G}$ to find the GUP-corrected entropy. For $n = 3$, the GUP-corrected entropy of the FRW universe will be

$$S_\Psi \simeq \frac{A}{4G} + \frac{1}{4} \alpha^2 l_p^2 \left(\frac{3\Omega_4}{4G}\right) \ln \frac{A}{4G} - \frac{1}{8} \alpha^4 l_p^4 \left(\frac{3\Omega_4}{4G}\right)^2 \frac{4G}{A} - \frac{15}{384} \alpha^6 l_p^6 \left(\frac{3\Omega_4}{4G}\right)^3 \left(\frac{4G}{A}\right)^2. \quad (15)$$

For $n = 4$ the corrected entropy is

$$S_\Psi \simeq \frac{A}{4G} + \frac{3}{4} \alpha^2 l_p^2 \left(\frac{4\Omega_4}{4G}\right) \left(\frac{A}{4G}\right)^{\frac{3}{2}} - \frac{3}{8} \alpha^4 l_p^4 \left(\frac{4\Omega_4}{4G}\right)^{\frac{3}{2}} \left(\frac{4G}{A}\right)^{\frac{3}{2}} - \frac{15}{192} \alpha^6 l_p^6 \left(\frac{4\Omega_4}{4G}\right)^{\frac{3}{2}} \left(\frac{4G}{A}\right)^{\frac{3}{2}}. \quad (16)$$

Similarly, for $n = 5$ the GUP-corrected entropy is

$$S_\Psi \simeq \frac{A}{4G} + \frac{1}{2} \alpha^2 l_p^2 \left(\frac{5\Omega_5}{4G}\right)^{\frac{1}{2}} \left(\frac{A}{4G}\right)^{\frac{1}{2}} + \frac{1}{8} \alpha^4 l_p^4 \left(\frac{5\Omega_5}{4G}\right)^{\frac{1}{2}} \left(\frac{5\Omega_5}{4G}\right) \ln \frac{A}{4G} - \frac{30}{192} \alpha^6 l_p^6 \left(\frac{5\Omega_5}{4G}\right)^{\frac{1}{2}} \left(\frac{4G}{A}\right)^{\frac{1}{2}}. \quad (17)$$

As can be seen from the above relations, the logarithmic correction term only appears in a FRW spacetime with even number of dimensions. In other words, only for odd $n$, the GUP-corrected entropy contains a logarithmic correction term. The impact of the generalized uncertainty principle on the black hole entropy was previously considered in [29] where the emergence of the logarithmic correction term was restricted to the even-dimensional spacetimes. If the mysterious existence of the logarithmic correction term in the black hole or FRW universe entropy is proven rigorously in future, it will impose stringent constraints on the number of the spacetime dimensions.
2.2 The MDR

As was indicated previously, the functional form of a typical MDR in a model universe with extra dimensions has not been studied yet. Ordinarily, an understanding of the concept of MDR in a \((n + 1)\)-dimensional spacetime may require one to expect that its fundamental length scale, \(L_p\), should be different from that of the 4-dimensional case in order to account for the existence of the extra dimensions. What we propose to do in this regard is to postulate that the modified dispersion relation in a model universe with extra dimensions can be written in the same way as in 4-dimensions

\[
\left( \frac{\delta p}{p} \right)^2 = f(E, m; L_p) \simeq E^2 - \mu^2 + \alpha L_p^2 E^4 + \alpha L_p^4 E^6 + \alpha L_p^6 E^8 + \mathcal{O} \left( L_p^8 E^{10} \right),
\]

(18)

where \(L_p\) is the Planck length which depends on the dimensions of the spacetime we are interested in and \(f\) is the function that gives the exact dispersion relation. On the right hand side we have assumed the applicability of a Taylor-series expansion for \(E \ll \frac{1}{L_p}\). The coefficients \(\alpha_i\) may take different values in different quantum gravity proposals. Note that \(m\) is the rest energy of the particle and the mass parameter \(\mu\) on the right hand side is directly related to the rest energy. However, \(\mu \neq m\) if \(\alpha_i\)'s do not all vanish. As we have emphasized previously, to incorporate quantum gravitational effects, thermodynamics of the FRW universe should be modified. Of course MDR may provide a perturbation framework for this modification. Using equation (18) and applying a Taylor expansion, we find

\[
d\mu \simeq dE \left[ 1 + 3 \alpha L_p^2 E^2 + \left( \frac{5}{2} \alpha' - \frac{5}{8} \alpha^2 \right) L_p^4 E^4 + \left( \frac{7}{2} \alpha'' - \frac{7}{4} \alpha \alpha' + \frac{21}{48} \alpha^3 \right) L_p^6 E^6 \right],
\]

(19)

where we have kept only the terms up to the sixth power of the Planck length, without loss of generality in conclusion. Some manipulations will then lead to

\[
dE \simeq d\mu \left[ 1 - \frac{3}{2} \alpha L_p^2 E^2 + \left( -\frac{5}{2} \alpha' + \frac{23}{8} \alpha^2 \right) L_p^4 E^4 + \left( -\frac{7}{2} \alpha'' + \frac{37}{4} \alpha \alpha' - \frac{273}{48} \alpha^3 \right) L_p^6 E^6 \right].
\]

(20)

To first order, assuming \(E \sim \delta E\), we may apply the standard uncertainty formulae, \(\delta E \geq \frac{1}{\delta x}\) and \(\delta p \geq \frac{1}{\delta x}\) to obtain

\[
dE \geq \frac{1}{\delta x} \left[ 1 - \frac{3}{2} \alpha L_p^2 \frac{1}{\delta x^2} + \left( -\frac{5}{2} \alpha' + \frac{23}{8} \alpha^2 \right) L_p^4 \frac{1}{\delta x^4} + \left( -\frac{7}{2} \alpha'' + \frac{37}{4} \alpha \alpha' - \frac{273}{48} \alpha^3 \right) L_p^6 \frac{1}{\delta x^6} \right].
\]

(21)

This relation can be rewritten as

\[
dE \geq \frac{1}{\delta x} \Phi \left( \delta x^2 \right),
\]

(22)

where by definition

\[
\Phi(\delta x^2) = 1 - \frac{3}{2} \alpha L_p^2 \frac{1}{\delta x^2} + \left( -\frac{5}{2} \alpha' + \frac{23}{8} \alpha^2 \right) L_p^4 \frac{1}{\delta x^4} + \left( -\frac{7}{2} \alpha'' + \frac{37}{4} \alpha \alpha' - \frac{273}{48} \alpha^3 \right) L_p^6 \frac{1}{\delta x^6}.
\]

(23)

The corresponding relation in the standard framework is given by equation (8). We note that in these equations the trace of extra dimensions is encoded in \(L_p\) which depends on the dimensionality of spacetime manifold. Taking into account the effect of MDR, we find

\[
dA_\Phi = \frac{4G}{T} dE \simeq \frac{4G}{T} \frac{1}{\delta x} \Phi \left( \delta x^2 \right).
\]

(24)

This means that

\[
dA_\Phi \simeq \Phi(\delta x^2) dA.
\]

(25)
Now, using equation (11), we can write
\[ dA_\Phi \sim \Phi(A) dA, \] (26)
where
\[
\Phi(A) = 1 - \frac{3}{8} \alpha L_p^2 \left( \frac{n n_{m}}{A} \right)^{\frac{2}{n-1}} + \left( -\frac{5}{32} \alpha' + \frac{23}{128} \alpha^2 \right) L_p^4 \left( \frac{n n_{m}}{A} \right)^{\frac{4}{n-1}} + \\
+ \left( -\frac{7}{128} \alpha'' + \frac{37}{256} \alpha \alpha' - \frac{273}{3072} \alpha^3 \right) L_p^6 \left( \frac{n n_{m}}{A} \right)^{\frac{6}{n-1}}.
\] (27)

Integrating equation (26) and substituting the result in equation \( S_\Phi = \frac{48}{4G} \), we derive the entropy of the FRW universe for a model universe with extra dimensions. For \( n = 3, 4, 5 \) the entropy of the FRW universe becomes
\[
S_\Phi \sim \frac{A}{4G} - \frac{3}{8} \alpha L_p^2 \left( \frac{3\Omega_3}{4G} \right)^{\frac{1}{2}} \left( \frac{A}{4G} \right)^{\frac{1}{2}} - \left( -\frac{5}{32} \alpha' + \frac{23}{128} \alpha^2 \right) L_p^4 \left( \frac{3\Omega_3}{4G} \right)^{\frac{1}{2}} \left( \frac{4G}{A} \right) \frac{A}{4G} \\
- \frac{1}{2} \left( -\frac{7}{128} \alpha'' + \frac{37}{256} \alpha \alpha' - \frac{273}{3072} \alpha^3 \right) L_p^6 \left( \frac{3\Omega_3}{4G} \right)^{\frac{1}{2}} \left( \frac{4G}{A} \right)^{\frac{1}{2}}
\] (28)
\[
S_\Phi \sim \frac{A}{4G} - \frac{9}{2} \alpha L_p^2 \left( \frac{4\Omega_4}{4G} \right)^{\frac{1}{2}} \left( \frac{A}{4G} \right)^{\frac{1}{2}} - \left( -\frac{5}{32} \alpha' + \frac{23}{128} \alpha^2 \right) L_p^4 \left( \frac{4\Omega_4}{4G} \right)^{\frac{1}{2}} \left( \frac{4G}{A} \right) \frac{A}{4G} \\
- \left( -\frac{7}{128} \alpha'' + \frac{37}{256} \alpha \alpha' - \frac{273}{3072} \alpha^3 \right) L_p^6 \left( \frac{4\Omega_4}{4G} \right)^{\frac{1}{2}} \left( \frac{4G}{A} \right)^{\frac{1}{2}}
\] (29)
\[
S_\Phi \sim \frac{A}{4G} - \frac{6}{8} \alpha L_p^2 \left( \frac{5\Omega_5}{4G} \right)^{\frac{1}{2}} \left( \frac{A}{4G} \right)^{\frac{1}{2}} + \left( -\frac{5}{32} \alpha' + \frac{23}{128} \alpha^2 \right) L_p^4 \left( \frac{5\Omega_5}{4G} \right)^{\frac{1}{2}} \left( \frac{4G}{A} \right) \frac{A}{4G} \\
- \left( -\frac{7}{128} \alpha'' + \frac{37}{256} \alpha \alpha' - \frac{273}{3072} \alpha^3 \right) L_p^6 \left( \frac{5\Omega_5}{4G} \right)^{\frac{1}{2}} \left( \frac{4G}{A} \right)^{\frac{1}{2}}
\] (30)
respectively. The entropy for a FRW universe with other dimensionality can be similarly derived. It is seen that the logarithmic correction term appears only for odd \( n \) (even spacetime dimensionality). Now, one can compare the entropy of the FRW spacetime calculated in the GUP and MDR frameworks by asking if they are functionally equivalent.

Comparison with equations (15-17) shows that the results of extra dimensional form of the GUP and MDR as functions of the apparent horizon and planck length are functionally the same, apart from numerical coefficients. Since GUP and MDR have essentially the same phenomenology, having the results with similar functional form is reasonable. Therefore, one may conclude that our suggested form of the MDR for a model universe with extra dimensions is in fact reasonably satisfactory. It is also necessary to point out that the Planck length in the extra dimensional form of the MDR depends on the dimensionality of the spacetime. In addition, the Planck length appearing in the expressions for GUP and MDR in 4-dimensions are the same [30]. The Functional consistency between the results of extra dimensional form of GUP and MDR for the variables \( A \) and Planck length and the necessity of having the same result in each approach points to the possibility that the Planck length in the extra dimensional forms of the GUP and MDR is also the same, that is \( l_p \) of section 2.1 is equivalent to \( L_p \) in 2.2.

As another point, we can mention the logarithmic correction term in the 4-dimensional spacetime. The emergence of a positive logarithmic correction term within the MDR and GUP in 4-dimensional FRW spacetime is interesting to note. As in our earlier work [30], the parameter \( \alpha \) in MDR is a negative quantity of order one (see also [31]). There, we compared the results of two approaches.
the generalized uncertainty principle and modified dispersion relation within the context of black hole thermodynamics with that of the string theory and Loop quantum gravity. Demanding the same results in all approaches and considering string theory and loop quantum gravity as more comprehensive, we put some constraints on the form of GUP and MDR. Also, we found that GUP and MDR are not independent concepts. In fact, they could be equivalent in an ultimate quantum gravity theory. The existence of a positive minimal observable length necessitates a positive value for the model dependent parameter $\alpha$ in the form of GUP. Since we know the relation between the model dependent parameters in GUP and MDR in [30], we set the parameter $\alpha$ as a negative value for MDR in this paper. Now, it is easy to see that the logarithmic correction term in a 4-dimensional FRW spacetime is positive in both approaches. It is interesting to note that the existence of a logarithmic term in the entropy-area relation is restricted to the even dimensionality of the spacetime within both GUP and MDR. This result may be helpful in paving the way for a better understanding of quantum gravity.

3 Conclusions

In this work, we obtained the entropy of a FRW universe for different dimensions of spacetime via the well-known extra dimensional form of the generalized uncertainty principle. We also suggested a form for the modified dispersion relation in a model universe with extra dimensions in the same way as that in 4-dimensions, except that the Planck length would now depend on the dimensionality of the spacetime. This is actually the case since one expects that the fundamental length scale in MDR, $L_p$, would be different from that in 4-dimensions in order to account for the the existence of the extra dimensions. By this assumption, we obtained the entropy of a FRW spacetime within the extra dimensional modified dispersion relation formalism. Since GUP and MDR have essentially the same phenomenology, they must lead to results exhibiting the same functional form, but not necessarily the same numerical coefficients. We found that the results have equivalent functional forms in terms of $\frac{A}{4\pi}$. In other words, the entropy of the FRW universe calculated based on the GUP and our suggested MDR are essentially the same. Since the form of the GUP for a model universe with extra dimensions is known, this equivalency shows that our suggested form of the MDR is conceivable. The GUP and MDR being equivalent may therefore be looked upon as the common feature of all quantum gravity theories. Having a functionally equivalent entropy in these two models, leads to having a relationship between model dependent parameters in GUP and MDR which may be considered as another interesting outcome of our study. One may also assume that the Planck length expression in extra dimensional form of the GUP and MDR is the same. On the other hand, we have found that the logarithmic correction term, whose existence is still somewhat mysterious, may only emerge in the even-dimensional FRW universe in both GUP and MDR models. Therefore, the existence of the logarithmic correction term in the entropy formulae depends on the dimensions of the spacetime. If one insists on the existence of the logarithmic correction term in the entropy of a FRW universe, the spacetime dimensions is restricted to be even. This fact provides a constraint on any viable quantum gravity theory.

Another point that should be stressed here is the connection between MDR and the spacetime non-commutativity. In fact, MDR is a manifestation of the spacetime non-commutativity in quantum gravity. Therefore, our knowledge of the MDR in spacetimes with extra dimensions provides a background to study non-commutativity in extra dimensions. It seems that a complete knowledge of the form of the MDR in model universes with extra dimensions opens new directions in quantum gravity and noncommutative geometry.

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