Mass Loss from Globular Clusters

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Abstract. We consider both observational and theoretical issues related to dynamical mass loss from the old globular star clusters of the Galaxy. On the observational side the evidence includes tidal tails and extratidal extensions, kinematic effects, effects on the mass function, and influences on the statistical properties of surviving objects. Even for isolated clusters, the theoretical issues are not fully understood. The effects of a steady tide (i.e. for a cluster in a circular orbit) include the imposition of a tidal boundary, and lowering of the escape energy. Less familiar, however, are the effects of induced mass loss. Even the definition of an “escaper” is not straightforward. When mass loss is driven by relaxation, as in N-body models, the rate of loss of mass does not scale in a simple way with the relaxation time. Reasons for this include the very long time scales on which stars escape even with energies above the escape threshold. For the realistic case of unsteady tides it is still unclear under what circumstances mass loss is dominated by relaxation or tidal heating.

1. Introduction

Among the reasons for studying this problem are the following:

1. There is a growing amount of direct observational evidence showing that globular clusters lose mass (cf. Sec.2 and the paper by Meylan in these Proceedings, and several of the poster papers).

2. Preferential loss of stars of low mass causes the mass function to evolve from its primordial form (cf. Sec.2).

3. It affects the statistical distribution of cluster properties in the galaxy, causing the death of clusters of low mass or large radius (Sec.2).

4. It has much in common with the problem of disruption of satellite galaxies, in which there has been much interest recently (e.g. Kroupa 1997, Ibata et al. 1997).

5. Decayed globular clusters may be a substantial contribution to the stellar halo of the Galaxy (cf. Ashman & Zepf 1998).

6. On the theoretical side, understanding mass loss turns out to be a vital issue in establishing the right boundary conditions to use in simplified
2. Observational Issues

2.1. Tidal tails and extensions

Two surveys (Grillmair et al. 1995; Meylan et al. 1999, Leon, Meylan & Combes 2000, and Meylan, these Proceedings) and some data on individual objects (Zaggia et al. 1997 for M55, Testa et al. 2000 for M92) have shown the presence of stars beyond the conventional tidal radius. In the case of the surveys the shapes tend to resemble those of tidal tails observed in simulations (cf. Fig.1), while the studies of M55 and M92 revealed roughly spherical extensions.

These studies raise a number of theoretical questions. In Grillmair et al. the cluster with the strongest tidal tail is M2 (=NGC7089), and modelling based on this data (Johnston et al. 1998) suggests that its lifetime is of order 1-3Gyr. Theoretical estimates, however, imply that this object is losing mass at one of the very slowest rates of all Galactic globular clusters, on a time of order 10–20

Figure 1. Tidal extensions in an N-body model. Here the tide is steady, caused by the host galaxy far away along the vertical axis, and mass loss is caused by relaxation, and “induced” mass loss (Sec.3). As with most simulations, however, near the cluster the tidal extension is in the direction of the perturbing field, while further away it is parallel to the direction of orbital motion. Initial model: King, with $W_0 = 3$, $N = 10^4$; time: 1 galactic orbital period; units: N-body units.

models, such as Fokker-Planck models (Takahashi & Portegies Zwart 1998, 1999; Spurzem & Takahashi 2000).

7. Also on theory, mass loss turns out to be a key to understanding how to scale N-body models with N (cf. Sec.3).

The emphasis in this review is on theoretical problems, but it begins with a summary of the observational position.
Hubble times (Gnedin & Ostriker 1997, Aguilar et al. 1988). The direction of the tail is not, at first sight, consistent with the most plausible cause (shocking during a recent passage through the Galactic disk). Meylan et al. themselves draw attention to two clusters in their sample with very similar orbits (and hence strength of tidal effects) but tails of very different strength. Furthermore the dependence on radius is flatter than in any of their simulations. The presence of a nearly spherical extra-tidal extension, as in the case of M92, raises the question of what the real value of the tidal radius is.

2.2. Kinematic Evidence

By studying the radial velocities of large numbers of stars in M15, Drukier et al (1998) found a slight rise in the velocity dispersion at large radii (but still well within the tidal radius). This was interpreted at first as evidence of tidal heating, but it was then realised, using N-body modelling, that this effect can occur even in clusters on circular galactic orbits. (This can also be seen in Giersz & Heggie 1997.) In this case the tide is steady (in a rotating frame), and its effect cannot be described as “heating” (Sec.3).

In two individual clusters, stars are observed with radial velocity exceeding the escape velocity (Gunn & Griffin 1979; Meylan, Dubath & Mayor 1991). As with extratidal extensions, the significance of these observations depends on the reliability of the models which yield the value of the escape speed. If taken at face value, however, such observations are usually interpreted as ejecta from three-body interactions (e.g. Davies 1992).

An alternative interpretation is suggested by recent N-body simulations (Fig.2). They show that the number of stars inside the tidal radius with energies above the energy of escape (“potential escapers”) is surprisingly large. Furthermore, the maximum mass of this population, $M_{pe}$, decreases surprisingly slowly with increasing $N$: $M_{pe}/M(0) \sim 0.1(N/3 \times 10^4)^{-0.23}$, where $M(0)$ is the initial cluster mass. Therefore even for a real cluster the proportion could be as large as $\sim 5\%$, and a considerably larger proportion of the current cluster mass. These results apply to models on circular galactic orbits, however.

2.3. Evidence from Mass Functions

At one time there seemed very clear evidence that the slope of the mass function in observed clusters is correlated with galactocentric radius and $z$, consistent with an interpretation in terms of preferential loss of low-mass stars (Capaccioli et al. 1993). The best that can be said now (Piotto & Zoccali 1999) is that, while there seems to be a trend, assessing its statistical significance is hard.

One cluster in which the effect of mass loss may be most pronounced is NGC 6712, which definitely appears to have a flatter mass function than any other comparable cluster (de Marchi et al. 1999), and has received considerable attention. Takahashi & Portegies Zwart (1999) gave an example of an N-body model which, after losing 99% of its mass, resembles this cluster. This is not implausible for a cluster whose present mass is estimated at $10^5 M_\odot$, and a tentative reconstruction of its orbit (Dauphole et al. 1996) suggests a very small galactocentric radius and therefore heavy tidal mass loss. A large mass loss rate for this object was also estimated by Gnedlin & Ostriker (1997), though there is nothing about the appearance of this cluster to suggest that it is close to death.
2.4. Evolution of the Galactic Globular Cluster system

There have been many studies, especially in recent years, of the effects of mass loss and other dynamical processes on the statistical properties of the system of Galactic globular clusters. A few (e.g. Aguilar et al. 1988, Gnedin & Ostriker 1997) have given estimates for all individual clusters about which enough is known. In particular, Gnedin & Ostriker (following Fall & Rees 1977) plot a survival diagram of half-mass radius and cluster mass to show that the disruptive effects of relaxation, tidal shocking and dynamical friction roughly account for the present-day distribution of the Galactic globular clusters. Even at a statistical level the agreement is imperfect – there are too many low-mass clusters – which suggests that either initial conditions also have a role, or the dynamical processes are not yet sufficiently understood quantitatively.

3. Theoretical Issues

Several processes which lead to mass loss from star clusters have been suggested. In roughly the historical order in which they have come to prominence, they are

(i) two-body relaxation, which was the main process considered until the 1960s;
(ii) tidal effects;
(iii) mass loss from stellar evolution, which began to be allowed for routinely in this subject only after Applegate (1986); and
(iv) massive dark halo objects, a relatively speculative mechanism about which no more will be said here.
3.1. Isolated Systems and Systems with a Tidal Cutoff

Though highly idealised, the simplest problem of all would seem to be mass loss from equal-mass systems with no external field. In fact this problem has been more-or-less abandoned without being completely understood. The essential issue is whether the time scale of mass loss follows the relaxation time scale $t_\text{r} \sim t_{\text{cr}} N / \ln N$ or the “strong encounter” time scale $t_\text{r}^* \sim t_{\text{cr}} N$, where $t_{\text{cr}}$ is the crossing time. It is not inconsistent to suppose that internal processes like core collapse proceed on the time scale $t_\text{r}$ while the time scale of escape, which may depend on energetic encounters, is more like $t_\text{r}^*$. Long ago Hénon (1960, 1969) gave formulae for the escape rate involving essentially $t_\text{r}^*$, and this prediction is confirmed by $N$-body results (Giersz & Heggie 1994). Only slightly more recently, however, Spitzer & Shapiro (1972) gave a theory for isolated systems which gives escape on a time scale $\propto t_\text{r}$.

These issues are not irrelevant when we turn to tidally limited systems, at least if the tide is represented as a cutoff. There is no reason why Hénon’s beautiful but little-known general formula for the escape rate (Hénon 1960), i.e.

$$\dot{N} = -\frac{256}{3} \sqrt{2\pi^4 G^2 m^2} \int_0^{r_{\text{max}}} r^2 dr \int \int \frac{f(E) f'(E') (E + E' - \phi)^{3/2}}{E^2} dEdE',$$

which leads to an escape time scale $\propto t_\text{r}^*$, is inapplicable, and yet virtually all theorists would adopt a time scale based on the usual relaxation time $t_\text{r}$. One reason for this is that so much work is based on the Fokker-Planck equation, which has no other time scale. Among theorists one exception is Wielen (e.g. 1988), who argues on the basis of $N$-body data that strong encounters greatly dominate for clusters with a few hundred stars. Since the Coulomb logarithm varies so slowly with $N$ he concludes that they still dominate for systems of the size of globular clusters.

3.2. Steady Tides

Even in a steady tide, e.g. a cluster in a circular galactic orbit, it is useful to distinguish three effects: (i) the finite boundary, usually taken at the radius of the Lagrangian point (cf. Spitzer 1987); (ii) the reduced escape energy; and (iii) “induced” mass loss, which is described further below.

Definition of Escape Several escape criteria are in common use, and all are wrong. Unless the potential of the cluster changes, a star may have an energy above the escape energy and yet remain within the tidal radius, $r_t$ indefinitely (Fukushige & Heggie 2000 and references therein). Within the usual tidal approximation it is possible for a star to recede to an arbitrary distance and eventually return to the cluster (Ross, Mennim & Heggie 1997), and so a simple distance or apocentre criterion fails. Ross et al. give a rigorous criterion, but in practice what is usually done in $N$-body simulations is to measure the mass within $r_t$, and then the escape criterion for simplified models (gas or Fokker-Planck), though based on physical ideas, is adjusted for reasonable agreement (e.g. Portegies Zwart & Takahashi 1999).

Induced Mass Loss This non-standard term is used here to describe the following fact. If a cluster in a steady tide loses mass for any reason (e.g. mass loss
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Figure 3. The “ski-jump” problem: mass against time for models dominated by mass-loss through stellar evolution (large $N$), and those dominated by relaxation (small $N$), from Fukushige & Heggie (1995), where all details are given. Imagine a skier descending along each curve!

of stellar evolution), then the tidal radius decreases and the escape threshold changes, so that still more mass is lost as the cluster readjusts. This additional mass loss is referred to as “induced mass loss”. If the cluster has little mass near the tidal radius or near the escape energy then induced mass loss is small. In King models it is a few percent of the initial mass loss. In Woolley models, however, in which the velocity distribution is a Maxwellian truncated at the escape speed, it is a much large factor, and in fact models with a Woolley concentration parameter $W_0 \lesssim 4$ are unstable to induced mass loss.

Consider a cluster in which the tide is treated as a cutoff, the cluster loses mass by stellar evolution, and relaxation is very slow. (Weinberg (1993) gives results of such models.) In this situation even an initial King-like distribution evolves to one more like a Woolley model. It seems likely that the above instability is responsible for the rapid loss of mass which such models appear to exhibit (Chernoff & Weinberg 1990, Fukushige & Heggie 1995) at the end of their life. Clusters in which mass loss is dominated by relaxation do not show this behaviour. The distinction between these two kinds of evolution is sometimes referred to as the “ski-jump” problem (Fig.3), and was highlighted by Portegies Zwart et al. (1998).

The Rate of Mass Loss The traditional picture (in the absence of mass loss by stellar evolution) is that stars gain sufficient energy to escape on the relaxation time scale, $t_r$, while escape itself takes place on a crossing time scale. As the latter is much shorter when $N$ is large, it follows that the time scale of mass loss is proportional to $t_r$. $N$-body simulations give results that are sufficiently different from this prediction (Fig.4) that extrapolation to, say, $N \sim 10^6$, which
would be required for scaling these models to a typical globular cluster, would yield results differing by a factor of order two.

Much work has been done on this so-called “scaling” problem since its discovery. The problem is reduced somewhat if self-consistent initial conditions are used, i.e. a model which is in equilibrium within the tidal field, unlike a standard King model (cf. Fukushige & Heggie 2000). The main problem, however, is the time scale on which stars escape, and the resulting population of “potential escapers” (Fig.2). It has been known for a long time (King, pers. comm.) that stars with energy above the escape energy may remain in a cluster for many crossing times before, if ever, finding a way out, but it is only recently that this aspect has been quantified (Fukushige & Heggie 2000) as a function of energy. These authors also determined the fraction of stars with energy above the escape energy which would never escape (if the potential remained fixed). How this new understanding resolves the scaling problem is a story which has been taken up by Baumgardt (these proceedings, and Baumgardt 2000).

3.3. Unsteady Tides: Bulge Shocking

Unsteady tides usually considered are of two kinds: (i) bulge shocking, (ii) disk shocking. The dynamical problems they pose are similar, and the remainder of this review concentrates heavily on bulge shocking. It happens only to clusters on elliptic orbits, however, whereas disk shocking would affect any cluster except one lying strictly in the galactic plane.
Theory of Bulge Shocking  This mechanism is often referred to as bulge heating, and indeed considerable emphasis is placed on its effect on the energy of stars and clusters. The effect of the heating, however, is to accelerate the expansion of the halo of a cluster across the tidal boundary, and so this mechanism is a legitimate subject of interest in this review.

Spitzer (1987) treated the problem impulsively, with qualitative discussion of the so-called adiabatic correction (for the fact that the orbital period of a star within the cluster need not be much greater than the time scale of variation in the tide). The argument was quantified and elaborated by Aguilar, Hut & Ostriker (1988), using an exponential form of adiabatic correction due to Spitzer in another context. The next interesting development was Weinberg's realisation (Weinberg 1994a,b,c) that adiabatic invariants do not protect resonant combinations of frequencies in multi-dimensional systems (such as three-dimensional motion in a fixed cluster potential). (Some of the dynamical issues are ably expounded also in Henrard 1982 and Sridhar & Touma 1996.) The effect of this is a significant change from an exponential adiabatic correction to a power law, as has been checked (with some empirical improvements) in simulations (Gnedin & Ostriker 1999).

One application of these results is their incorporation into Fokker-Planck models. This has been accomplished by Weinberg (1994c) and by Gnedin & Ostriker (1997), and applied to the evolution of clusters by Murali & Weinberg (1997) and Gnedin, Lee & Ostriker (1999). Some results must be treated with caution, however, as the formulation of Murali & Weinberg results in bulge heating even in the case of circular orbits, where the tide is steady and cannot change the energy of a star.

Such work can give a first answer to the question whether it is tidal shocking or two-body relaxation which dominates the mass loss of galactic clusters. One answer, based on the work of Gnedin & Ostriker (1997), is given in Fig.5. When we come to study simulations in the next sub-subsection, this is a question on which we shall concentrate.

Simulations of Bulge Shocking  Much can be learned from the motion of test particles in a fixed cluster-like potential orbiting in a galaxy-like potential. Keenan & Innanen (1975) used a King-like cluster model and both point-mass and axisymmetric galaxy potentials, while Keenan (1981ab) used point masses for both. This case is the “elliptic restricted three-body problem”, which has experienced a modest renaissance recently in other contexts (e.g. Benest 1998 and references therein).

Relaxation can be added to this simple model by incorporating diffusion (Oh, Lin & Aarseth 1992), and in this way Oh & Lin (1992) concluded that the presence of a tidal field could suppress escape. Perhaps this result, which at face value contradicts all other wisdom on this question, may be explained by the increase in the escape time scale in a tidal field, as already discussed.

Self-consistent calculations of tidal effects on clusters on elliptical orbits were carried out by Chernoff & Weinberg (1991), though the initial models considerably exceeded their tidal radii, and therefore perhaps relate better to young clusters than to the old Galactic globular clusters. Much the same may be said of the later studies by Johnston, Sigurdsson and Hernquist (1999), though they took care to quantify the effects of relaxation. This was also done in the
Figure 5. Time scale of rate of destruction by shocking (bulge and disk) against the destruction time scale for relaxation, from Gnedin & Ostriker (1997). Each point represents one Galactic globular cluster from their sample, using the “OC Isotropic” data in their Table 3. Units are per Hubble Time, and the line corresponds to equality of the two destruction rates. Note that both the most rapidly and the least rapidly dissolving cluster are relaxation-dominated (below the line).

Simulations by Combes et al. (1999), though the choice of initial conditions is too patchy to arrive at general conclusions on the relative importance of bulge heating and relaxation.

An independent attack on this question was begun by Baumgardt (1997), who computed some small $N$-body models of clusters on elliptical orbits about a point mass galaxy with initial conditions similar to those of Murali & Weinberg (1997), i.e. the cluster is started at apocentre with a limiting radius equal to the tidal radius at pericentre.

One attraction of these initial conditions is that they correspond to one of the conclusions of Oh & Lin (1992), which was that the limiting radius of clusters is determined at pericentre and maintained all along the orbit. Another attraction is that they allow a systematic exploration of the relative importance of tides and relaxation. Collaborative work with H. Baumgardt is currently under way at Edinburgh on this problem (Fig.6).

One of the interesting questions which even this research will leave unanswered is the behaviour of clusters in axisymmetric potentials. The essential difference is that the perigalactic distance varies considerably from one bulge passage to the next. It may be that mass loss through bulge shocking is much less regular and more infrequent than is suggested by Fig.6.
Figure 6. Mass loss from a cluster on an elliptical orbit ($e = 0.5$) about a point-mass galaxy. The initial model is a King model with scaled central potential $W_0 = 3$, and other initial conditions are given in the text. $N = 1K$ to $64K$ in steps of a factor of 2, from bottom to top. The potential is softened and all stars have equal mass. For low $N$ the rate of mass loss is approximately proportional to $1/N$, as would be expected for relaxation-dominated mass loss; for large $N$ the rate appears to level off, as would be expected for mass loss dominated by bulge shocking. The two processes appear to be roughly comparable for the largest $N$ here.

4. Summary and Conclusions

Observation of tidal tails of Galactic globular clusters now seem well established and consistent, but the detailed modelling and understanding of various individual objects remains almost unexplored. Kinematic data also suggests that tidal effects may impose an observable signature, while it is suggested in this paper that stars apparently observed above the escape energy may be genuine cluster members trapped for long periods in the combined field of the cluster and galaxy. It seems certain that mass loss has altered both the stellar mass function of each cluster and the distribution of cluster properties throughout the galaxy. The expected correlations among the stellar mass functions are a little elusive, however, and it is only in extreme cases that the effects of mass loss seem clear cut.

There are still loose ends to be tied up in the theory of escape from a cluster with a tidal cutoff: is the escape time scale proportional to the relaxation time, with the Coulomb logarithm, or is it dominated by single energetic scatterings, as in the formula of Hénon? The “induced” loss of mass may help to understand the distinction between “skiing” and “jumping” models, but a quantitative understanding of this distinction is still missing. When the tide is represented by a proper external field, the effect of the large population of “potential escapers”
complicates the time scale for mass loss, and a theory of this problem has recently been developed by Baumgardt. In the more realistic case of an elliptic galactic orbit, an interesting focus for current research is the question of which mass loss process dominates: relaxation, or bulge shocking? An interesting future development will be to understand the effect of the varying perigalactic distance in axisymmetric potentials.

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References

Aguilar L., Hut P., Ostriker J.P., 1988, ApJ, 335, 720
Applegate J.H., 1986, ApJ, 301, 132
Ashman K.M., Zepf S.E., 1998, \textit{Globular Cluster Systems} (Cambridge: Cambridge University Press)
Baumgardt H., 1997, PhD thesis, University of Heidelberg
Baumgardt H., 2000, preprint
Benest D., 1998, A&A, 332, 1147
Capaccioli M., Piotto G., Stiavelli M., 1993, MNRAS, 261, 819
Chernoff D.F., Weinberg M., 1990, ApJ, 351, 121
Chernoff D.F., Weinberg M., 1991, in ASP Conf. Ser. Vol. 13, The Formation and Evolution of Star Clusters, ed. K. Janes (San Francisco: ASP), 373
Combes F., Leon S., Meylan G., 1999, A&A, 352, 149
Dauphole B., Geffert M., Colin J., Ducourant C., Odenkirchen M., Tucholke H.-J., 1996, A&A, 313, 119
Davies M.B., 1992, PhD thesis, Harvard University
de Marchi G., Leibundgut B., Paresce F., Pulone L., 1999, A&A, 343, L9
Drukier G., Slavin S.D., Cohn H.N., Lugger P.M., Berrington R.C., Murphy B.W., Seitzer P.O., 1998, AJ, 115, 708
Fall S.M., Rees M.J., 1977, MNRAS, 181, 37P
Fukushige T., Heggie D.C., 1995, MNRAS, 276, 206
Fukushige T., Heggie D.C., 2000, MNRAS, submitted
Giersz M., Heggie D.C., 1994, MNRAS, 268, 257
Giersz M., Heggie D.C., 1997, MNRAS, 286, 709
Gnedin O.Y., Lee H.M., Ostriker J.P., 1999, ApJ, 522, 935
Gnedin O.Y., Ostriker J.P., 1997, ApJ, 474, 223
Gnedin O.Y., Ostriker J.P., 1999, ApJ, 513, 626
Grillmair C.J., Freeman K.C., Irwin M., Quinn P.J., 1995, AJ, 109, 2553
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Gunn J.E., Griffin R.F., 1979, AJ, 84, 752
Heggie D.C., Giersz M., Spurzem R., Takahashi K., 1998, in Highlights of Astronomy, vol.11A, ed J. Andersen (Kluwer, Dordrecht), p.591
Heggie D.C., Ramamani N., 1995, MNRAS, 272, 317
Hénon M., 1960, Ann. d’Astr., 23, 668
Hénon M., 1969, A&A, 2, 151
Henrard J., 1982, in NATO ASI Ser. C, 82, Applications of Modern Dynamics to Celestial Mechanics and Astrodynamics, ed. V. Szebehely (Dordrecht: Reidel), 153
Ibata R.A., Wyse R.F.G., Gilmore G., Irwin M.J., Suntzeff N.B., 1997, AJ, 113, 634
Johnston K.V., Sigurdsson S., Hernquist L., 1999, MNRAS, 302, 771
Keenan D.W., 1981ab, A&A, 95, 334 and 340
Keenan D.W., Innanen K.A., 1975, AJ, 80, 290
Kroupa P., 1997, New Astron., 2, 139
Leon S., Meylan G., Combes F., 2000, astro-ph/0006106
Meylan G., Dubath P., Mayor M., 1991, ApJ, 383, 587
Meylan G., Leon S., Combes F., 1999, astro-ph/9912041
Murali C., Weinberg M., 1997, MNRAS, 288, 749
Oh K.S., Lin D.N.C., Aarseth S.J., 1992, ApJ, 386, 506
Oh K.S., Lin D.N.C., 1992, ApJ, 386, 519
Piotto G., Zoccali M., 1999, A&A, 345, 485
Portegies Zwart S.F., Hut P., Makino J., McMillan S.L.W., 1998, A&A, 337, 363
Portegies Zwart S.F., Takahashi K., 1999, CeMDA, 73, 179
Ross D.J., Mennim A., Heggie D.C., 1997, MNRAS, 284, 811
Spitzer L., Jr, 1987, Dynamical Evolution of Globular Clusters. Princeton U.P., Princeton.
Spitzer L., Jr, Shapiro S.L., 1972, ApJ, 173, 529
Spurzem R., Takahashi K., 2000, preprint
Sridhar S., Touma J., 1996, MNRAS, 279, 1263
Takahashi K., Portegies Zwart, 1998, ApJ, 503L, 49
Takahashi K., Portegies Zwart, 1999, astro-ph 9903366
Testa V., Zaggia S.R., Andreon S., Longo G., Scaramella R., Djorgovski S.G., de Carvalho R., 2000, A&A, 356, 127
Weinberg M.D., 1993, in ASP Conf. Ser. Vol. 48, The Globular Cluster-Galaxy Connection, ed. G.H. Smith, J.P. Brodie (San Francisco: ASP), 689
Weinberg M.D., 1994abc, AJ, 108, 1398, 1403 and 1414
Wielen R., 1988, in IAU Symp.126., The Harlow-Shapley Symposium on Globular Cluster Systems in Galaxies, ed. J.E. Grindlay, A.G.D. Philip (Dordrecht: D. Reidel), p.393
Zaggia S.R., Piotto G., Capaccioli M., 1997, A&A, 327, 1004