The Conformal Window and Walking Technicolor

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I discuss recent progress in the uncovering of the phase diagram of non-supersymmetric gauge theories. The nature of the conformal window for higher dimensional representations suggests a possible way to construct realistic technicolor models. Then explicitly provide two such theories. One of these models also has a natural cold dark matter candidate.

THE PHASE DIAGRAM

Let us first set the notation by denoting the generators of the gauge group in the representation $r$ by $T^a_r$, $a = 1 \ldots N^2 - 1$. They are normalized according to $\text{Tr}[T^a_rT^b_r] = T(r)\delta^{ab}$ while the quadratic Casimir $C_2(r)$ is given by $T^a_rT^a_r = C_2(r)I$. The trace normalization factor $T(r)$ and the quadratic Casimir are connection via $C_2(r)d(r) = T(r)d(G)$ where $d(r)$ is the dimension of the representation $r$. The adjoint representation is denoted by $G$.

Let us first consider an $SU(N)$ gauge theory with $N_f(r_i)$ Dirac fermions in the representation $r_i$, $i = 1, \ldots, k$ of the gauge group. To estimate the conformal window we shall employ the recently conjectured all-orders beta function for non-supersymmetric theories \cite{8}

\[ \beta(g) = -\frac{g^3}{(4\pi)^2} \beta_0 - \frac{2}{3} \sum_{i=1}^k T(r_i)N_f(r_i)\gamma_i, \]  

(1)

with

\[ \beta_0 = \frac{11}{3} C_2(G) - \sum_{i=1}^k T(r_i)N_f(r_i), \]  

(2)

\[ \gamma_i(g^2) = \frac{3}{2} C_2(r_i)\frac{g^2}{4\pi^2} + O(g^4). \]  

(4)

Here $g$ is the gauge coupling, $\beta_0$ is the first coefficient of the beta function and $\gamma_i(g^2)$ is the anomalous dimension of the fermion mass. One should note that for small coupling the beta function reduces correctly to the two loop beta function.

First the loss of asymptotic freedom is determined by the change of sign in the first coefficient of the beta function

\[ \sum_{i=1}^k \frac{4}{11} T(r_i)N_f(r_i) = C_2(G). \]  

(5)

Second we note that at the zero of the beta function we have

\[ \sum_{i=1}^k \frac{2}{11} T(r_i)N_f(r_i)(2 + \gamma_i) = C_2(G). \]  

(6)

Having reached the zero of the beta function the theory is conformal in the infrared and hence the dimension of the chiral condensate must be larger than one in order not to contain negative norm states \cite{13}. Since the dimension of the chiral condensate is $3 - \gamma_i$ we see that $\gamma_i = 2$ for all representations $r_i$ yields the maximum possible bound of the conformal window

\[ \sum_{i=1}^k \frac{8}{11} T(r_i)N_f(r_i) = C_2(G). \]  

(7)
Hence in the case of a single representation the bound is
\[
\frac{11 \, C_2(G)}{8 \, T(r)} < N_f(r) < \frac{11 \, C_2(G)}{4 \, T(r)}.
\] (8)

In Fig. 1 we plot the conformal window for various representations. One should note the remarkable feature that only a low number of flavors for the adjoint and two-indexed symmetric representation is needed in order to be near the conformal window. This has two important implications

- Such (near) conformal theories are easily accessible on the lattice.
- They are perfect candidates for walking technicolor theories able to break the electroweak symmetry.

We stress that the above prediction of the conformal window is in agreement with all of the recent lattice calculations [14].

**MINIMAL WALKING TECHNICOLOR**

From Fig. 1 it is clear that the simplest model able to possess walking dynamics is an SU(2) gauge theory with two Dirac flavors in the adjoint representation. To couple it to the SM we arrange the left handed fields into three doublets of the SU(2)l weak interactions while the right handed fields are singlets under the SM gauge group. They are denoted by \( Q^a_L = (U^a, D^a)_L \), \( U^a_R \), \( D^a_R \), \( a = 1, 2, 3 \).

The model so far suffers from the Witten topological anomaly [16]. This is easily accommodated for by adding a new fermionic doublet charged under the electroweak symmetry and neutral under the technicolor interactions \( L_I = (N, E)_L \), \( N_R, E_R \). The gauge anomalies cancel for the following generic choice of hypercharge
\[
Y(Q_L) = \frac{y}{2}, \quad Y(U_R, D_R) = \left( \frac{y + 1/2}{2}, \frac{y - 1}{2} \right),
\] (9)
\[
Y(L_L) = \frac{3y}{2}, \quad Y(N_R, E_R) = \left( -\frac{3y + 1}{2}, -\frac{3y - 1}{2} \right),
\] (10)
where \( y \) can be any real number. The above model is called the Minimal Walking Technicolor (MWT) model [5]. Once the condensate \((\bar{U}_R U_L + \bar{D}_R D_L)\) forms the electroweak symmetry breaking provides masses for the associated gauge bosons.

Since the techniquarks belong to the adjoint representation of the gauge group the global symmetry is enhanced to SU(4). Assuming the standard breaking to the maximal diagonal subgroup SU(4) breaks to SO(4). This leaves nine Goldstone bosons. Three of these become the longitudinal degrees of freedom of the massive weak gauge bosons while the remaining six Goldstone bosons acquire mass through yet unspecified extended technicolor interactions.

**ULTRA MINIMAL WALKING TECHNICOLOR**

Another possibility of constructing realistic walking technicolor models is to consider fermions transforming according to two distinct representations of the gauge group. First we are interested in having the smallest possible naive S parameter. This is achieved by choosing two technicolors and two Dirac fermions in the fundamental representation. We charge these fermions under the electroweak symmetry in the standard way as done above for MWT. Second we are interested in obtaining walking dynamics. One solution is to add the remaining fundamental fermions uncharged under the electroweak symmetry needed to be near the conformal window. Such models have been termed partially gauged technicolor [6].

However a more economic alternative is to let the remaining fermions belong to the adjoint representation of the gauge group. Then according to the prediction of the conformal window, Eq. 7, the critical number of adjoint Dirac flavors needed to enter the conformal window is \( \sim 1 \) depending on the critical value of the anomalous dimension. Hence our candidate theory consists of an SU(2) gauge group with two Dirac flavors in the fundamental representation and charged under the electroweak symmetry together with one Dirac flavor in the adjoint representation uncharged under the electroweak symmetry. This model has been termed the Ultra Minimal Walking Technicolor (UMT) model [9].

Due to the fact that the fermions belong to pseudo-real and real representations the global symmetry is enhanced to SU(4) \( \times SU(2) \times U(1) \). All the fermions...
are charged under the abelian $U(1)$ symmetry which is anomaly free. Again assuming the standard breaking to the maximal diagonal subgroup the global symmetry breaks to $Sp(4) \times SO(2) \times Z_2$ leaving $5+2+1 = 8$ Goldstone bosons. Except for the triplet of Goldstone bosons which will be eaten by the massive gauge bosons the rest of the states are electroweak singlets. Specifically one of these states is a natural cold Dark Matter candidate whose mass can be very low [9].

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