I recall some points on the present status of QCD results in the short distance regions and illustrate the case of event shape in QCD radiation

1 Hard QCD studies

QCD studies have produced successful results, specially for observables involving short distance regions, for over a quarter of century. High accuracy calculations of hard distributions are possible and often one is able to make quantitative predictions which are strongly constrained. Calculations are often difficult but there are constant improvements in techniques and this is always associated to a better understanding of QCD features. A consequence of these successful results is that QCD is often considered an established and understood theory. Actually, there are a variety of unsolved theoretical and experimental questions in QCD. They range from the two extreme regions. On one side we still need to understand how quark and gluon elementary fields change into hadrons at large distances. Since we think we almost dominate the short distance regions by perturbative (PT) methods, there are attempts to extend these methods and even the language to the study of non-perturbative (NP) corrections. This is an approach to large distances from the “short distance window”. Going in the other extreme region, the standard understanding is that in the ultraviolet region the theory becomes free and no structures are present. But, again, this is based on PT studies. Actually, it has been argued that short distance structures could be present in QCD.

Hard QCD is systematically and analytically studied by resummation of the PT expansion. This procedure starts from free quarks and gluons and reconstructs their interaction by using the point-like vertices in the QCD Lagrangian. It is reassuring to discover afterwards that quarks and gluons cannot be free at large distances, although this region it is not accessible to PT methods. In order to resumm PT expansions for physical observables one needs in general to know multi-parton distributions. Of course we need approximations. I recall that the present PT status of the art for multiparton distributions concerns: collinear singularities, soft singularities, fixed order exact matrix elements.

1. Collinear singularities. In general, all leading and next to leading
contributions to the multi-parton distributions are obtained by generalizing of the DGLAP evolution equation \(^2\) (for a generalization to “exclusive” multi-parton distributions see \(^3\)). They are given in terms of the anomalous dimensions which are known to leading and next-to-leading order. Recently there are attempts to compute NNNL contributions \(^4\). Singular contributions to multi-parton distributions require a collinear cutoff. Universality of collinear singularities allows a unified description of hard distributions for all processes such as lepton-lepton, lepton-hadron and hadron-hadron collisions.

2. Soft singularities. Also in this case all leading (and some next-to-leading) contributions to the multi-parton distributions are known. They are given in terms of factorized soft radiation from hard emitters \(^3\). In the case of two or three emitters it is possible to factorize the soft radiation at two-loop \(^5\). For more emitters the two soft parton factorization requires the planar approximation (leading order in \(1/N_c\)). I recall the most important results and features of QCD we learned in the 80th and 90th:

- coherence of soft radiation;
- colour connection and angular ordering;
- gluon multiplication and hadron multiplicity;
- hump-backed plateau;
- jet-finding algorithms in \(e^+e^−\), DIS and hadron collisions;
- LHPD and colour preconfinement;
- heavy quark distribution and radiation;
- jet-shape distribution.

All these results, together with the known leading (and next-to-leading) contributions allows one to formulate multi-parton distributions in terms of a coherent-branching algorithms. This is the bases of most of the present Monte Carlo simulations for hard QCD (and beyond QCD) processes, see \(^6\).

An other area in which soft physics is involved is the small-\(x\) region in DIS and hadron collisions \(^7\). Here soft “exchanged” gluon are involved rather than the soft emitted ones, as in the previous item. Enormous progresses have been made recently. These results are however less constrained by PT analysis since NP effects enter at all hard scales \(^8\). Further progresses in this area definitively requires further developments in our QCD understanding beyond PT expansions.
3. Exact matrix element results. Large technical progresses have been made both in analytical calculations and in numerical programs. These results are essential in order to make quantitative predictions away from the soft or collinear regions of phase space. This is achieved by matching these exact fixed order results with the ones from resummations described in the previous two items. Similar matching procedure are now developed on a systematically bases in Monte Carlo simulations.

In the following I will illustrate the QCD calculations of jet-shape observables which are collinear and infrared safe (CIS). They are expressed as linear combinations of the momenta of emitted hadrons so that their values do not change if two momenta are replaced by their sum when the two are collinear or when one is soft. For these CIS distributions one reaches the highest accuracy available in PT calculations. This is due to the fact that the cutoff of collinear and infrared singularities cancels and all PT terms are finite. Near the phase space boundary the cancellation are partial and logarithmically enhanced contributions enter the PT terms which needs to be resummed. The particular interest of these distributions is not only that they can be computed at the highest PT accuracy, but that they have large $1/Q$-power corrections which are of NP origin. These corrections are phenomenologically important. Once we have a reliable PT estimate, with the help of data we can then constrain the size and hopefully the nature of these NP corrections.

After a brief review of the results for event shapes in $e^+e^-$ annihilation, I discuss how these results can be extended to jet shape distributions in hadron-hadron and lepton-hadron collisions. This will open the way to the study of further multi-jet observables.

2 Hadron emission in $e^+e^-$ annihilation

Two-jet event shapes (such as thrust $T$, broadening $B$, heavy-jet mass $M_H$ and $C$-parameter) have been intensively studied both theoretically and experimentally. The state-of-the-art level of PT analysis of two-jet observables consists of: resummation of all double- (DL) and single-logarithmic (SL) enhanced contributions (due to multiple radiation of soft and collinear partons); matching of the resummed result with the exact fixed order result. To reach such an accuracy one needs: soft matrix elements at two-loop order and reconstruction of the running coupling at the proper scale; factorization and exponentiation of observable and kinematical constraints; proper treatment of hard intra-jet and soft inter-jet radiation.

Besides the pure PT contributions, experimental data (see for instance) have revealed the presence of power suppressed $1/Q$ corrections ($Q$ is the
$e^+e^-$ center-of-mass energy), which emerge as a shift both to mean values and differential distributions. It is widely believed that such contributions are of NP origin and arise from the running of the coupling into the infrared domain.

A systematic way to deal with power corrections is provided by the dispersive approach, in which a prescription is given to extend the QCD coupling into the confinement region. In terms of this dispersive coupling, the typical large distance contribution to an event shape $V$ can be factorized in an observable dependent coefficient $c_V$ and a universal NP parameter $\alpha_0$. More precisely, in any ‘linear’ shape variable $V$ the dependence on the rapidity $\eta_i$ and transverse momentum $k_{ti}$ of emitted hadron $i$ factorizes,

$$V = \sum_i k_{ti} f_V(\eta_i),$$

so that the NP contribution to $V$ becomes

$$\delta V = C_F c_V \lambda^{\mathrm{NP}}, \quad c_V = \int d\eta f_V(\eta), \quad \lambda^{\mathrm{NP}} = \mu_I \frac{4M}{\pi} \left( \alpha_0(\mu_I) + O(\alpha_s(Q)) \right), \quad \alpha_0(\mu_I) = \frac{1}{\mu_I} \int_0^{\mu_I} dk \alpha_s(k).$$

Here $M$ is a known coefficient, the Milan factor, which takes into account the non complete inclusiveness of the observable. The NP parameter $\alpha_0$ is the average of the running coupling in the region $k \leq \mu_I$ and measures the interaction strength in the confinement region. The $O(\alpha_s(Q))$ piece in $\lambda^{\mathrm{NP}}$ is needed in order to merge PT and NP results in a renormalon free manner and ensures that the final answer is independent of the infra-red matching scale $\mu_I$. The NP parameter $\alpha_0(2\text{GeV})$ has been measured and appears to be consistent with the universality hypothesis within a 10-20% accuracy, see

Only very recently these techniques have been extended to the case of three-jet observables in the near-to-planar region. In particular the mean values and distributions of the thrust minor $T_m$ and the $D$-parameter have been computed. Both of them are a measure of QCD radiation out of the event plane. This study has revealed an unexpectedly rich geometry and colour dependence both of the PT result and of the NP corrections. The crucial point is the rapidity dependence of the observables. Actually, $T_m$ is independent of rapidity, while $D$ is exponentially damped. On the PT side, this implies that hard parton recoil affects the observable in the $T_m$ case, while it does not contribute to $D$. Therefore the $T_m$ distribution is sensitive not only to the underlying hard event geometry (the angles between the jets), but also on its colour configuration through the kinematical constraints which define the event plane.
As far as NP corrections are concerned, one finds that the shift to the $D$ distribution is simply geometry dependent, while the $T_{m}$ distribution is also squeezed, since the shift depends logarithmically on $T_{m}$. This behaviour (already encountered for the $B$ in two-jet events) results from a complicated interplay between PT and NP effects.

The same physical features appearing in the NP shift to the $T_{m}$ distribution are also present in hadron-hadron and lepton-hadron collisions, as we are going to discuss in the following.

3 Out-of-plane radiation in hadron collisions

We consider hadron collisions and study an observable similar to $T_{m}$\(^{2}\). We select events in which a $Z_{0}$ is produced with large transverse momentum. We can then define an ‘event plane’ as the one containing the $Z_{0}$ momentum and the beam axis and introduce a measure of out-of-plane radiation

$$K_{\text{out}} = \sum_{h} |p_{h}^{\text{out}}| .$$

Here $p_{h}^{\text{out}}$ is the out-of-plane momentum of the hadron $h$ and the sum extends to all hadrons with rapidity in the range $|\eta_{h}| < \eta_{0}$, in order to avoid measurements in the beam region. At Born level one has two incoming partons $p_{1}$ and $p_{2}$ and an additional hard parton $p_{3}$ recoiling against the vector boson $q$. The event-plane definition gives rise to the conservation law

$$p_{3}^{\text{out}} + \sum_{i} k_{i}^{\text{out}} = 0 ,$$

so that only $p_{3}$ can take an out-of-plane recoil, while the remaining hard momenta are fixed in the event plane.

The main difference between hadronic and $e^{+}e^{-}$ collisions is the presence of initial state radiation. Due to coherence of QCD radiation, its contribution can be factorized giving rise to the standard parton density function for the incoming partons $p_{2},p_{3}$. However, while in the total cross section the hard scale is the $Z_{0}$ hardness, here the scale is $K_{\text{out}}$. The PT distribution is essentially given by the product of the initial state parton distributions $P_{\text{in}}(K_{\text{out}})$ and a CIS “radiation factor”

$$\Sigma(K_{\text{out}}) \sim P_{\text{in}}(K_{\text{out}}) \cdot e^{-R(K_{\text{out}})} \cdot S(R') .$$

The radiator $R$ is the same as that of $T_{m}$ in $e^{+}e^{-}$, it resums all DL contributions and accounts for SL effects due to hard intra-jet and soft inter-jet
radiation. It is given in terms of three hard parton ‘antennae’, each one proportional to the colour charge $C_a$ of emitting parton $p_a$ ($C_a$ equals $C_F$ for a quark and $C_A$ for a gluon)

$$R(K_{\text{out}}) = \sum_{a=1}^{3} C_a \int_{K_{\text{out}}}^{Q_a} \frac{dk}{k} \frac{2\alpha_s(2k)}{\pi} \ln \frac{Q_a}{2k} \simeq \sum_{a} C_a \frac{\alpha_s}{\pi} \ln^2 \frac{Q_a}{2K_{\text{out}}}.$$  \hspace{1cm} (6)

The hard scales $Q_a$ are determined essentially by soft radiation at large angles, which is a characteristic of multi-jet observables. In particular, apart from a factor due to hard collinear splitting, the scale for the quarks is the invariant mass of the $q\bar{q}$ system, for the gluon is its invariant transverse momentum with respect to the $q\bar{q}$ pair. The SL function $S$ depends on the logarithmic derivative of the radiator $R'$ and accounts for all effects coming from multiple secondary radiation.

The NP correction is proportional to the same parameter $\lambda_{\text{NP}}$ which enters the power corrections to $e^+e^-$ event shapes in  and is given by

$$\delta K_{\text{out}} = \frac{2}{\pi} \lambda_{\text{NP}} \left( C_1 (\eta_0 - \eta_3) + C_2 (\eta_0 + \eta_3) + C_3 \ln \frac{Q_t}{|p_{3\text{out}}^\perp|} \right),$$  \hspace{1cm} (7)

where $Q_t$ is the transverse momentum of the emitted boson. Such a shift arises from integration over the rapidity of emitted gluons (see eq. 3). Since $K_{\text{out}}$ is independent of rapidity, one has to carefully consider the effective rapidity cutoff. For radiation from the incoming partons $p_1$ and $p_2$ it is given by the distance between $\eta_3$ (the rapidity of $p_3$) and the experimental resolution $\eta_0$. On the contrary, for a NP gluon emitted from $p_3$, it is the hard parton recoil momentum $p_{3\text{out}}^\perp$ which provides the needed cutoff. Moreover, since $p_3$ always takes recoil, one has $p_{3\text{out}}^\perp \sim K_{\text{out}}$. This interplay between PT and NP emissions makes the shift logarithmically dependent on $K_{\text{out}}$.

Unfortunately, this is not the end of the story. In hadronic collisions one has to add a soft contribution due to the beam remnant interactions. This is the so-called ‘soft underlying event’, which was systematically studied and introduced for the first time in the analysis of the ‘pedestal height’ in hadronic jet production 12. Therefore the analysis of $K_{\text{out}}$ is also important to understand the physics of soft collisions.

4 Out-of-plane radiation DIS

Even in DIS one can define an event plane and measure the out-of-plane radiation. In the Breit frame, we define the thrust major $T_M$ in analogy with
\( e^+e^- \) annihilation

\[
T_M Q = \max_{\vec{n}_M \cdot \vec{n} = 0} \sum_h |\vec{p}_h \cdot \vec{n}_M|,
\]

(8)

where \( \vec{n} \) and \( Q^2 = -q^2 \) are the Breit axis and the virtuality of the exchanged boson \( q \) respectively. As in the previous case, the sum extends to all hadrons not in the beam direction (with \( \eta < \eta_0 \)). At Born level, we have one incoming parton \( p_1 \) which is struck by a vector boson \( q \) and produces two hard large angle partons \( p_2 \) and \( p_3 \) recoiling one against each other. We define \( K_{\text{out}} \) again as the cumulative out-of-event-plane momentum, where the event plane is the one containing \( \vec{n} \) and the \( T_M \) axis. The event-plane definition implies the following kinematical constraint

\[
p_{\text{out}2}^2 + \sum_{i \in U} k_{\text{out}i}^2 = p_{\text{out}3}^2 + \sum_{i \in D} k_{\text{out}i}^2,
\]

(9)

with \( U \) (\( D \)) the region containing parton \( p_2 \) (\( p_3 \)).

The PT \( K_{\text{out}} \) distribution has the same form as \( 5 \), with \( P_{\text{inc}}(K_{\text{out}}) \) the structure function of the incoming parton, again at the scale \( K_{\text{out}} \). The radiator turns out to be the same as in \( e^+e^- \) and in hadron-hadron collisions, due to universality of soft and collinear radiation. However, the exact functional form of the SL function \( S \) differs between these, due to different event-plane kinematics.

The NP shift is similar to the one in \( 7 \)

\[
\delta K_{\text{out}} = \frac{2}{\pi} \Lambda_{\text{NP}} \left( C_1 \ln \frac{Q_{1,\text{NP}} e^{\eta_0}}{Q} + C_2 \ln \frac{Q_{2,\text{NP}}}{|p_{\text{out}2}|} + C_3 \ln \frac{Q_{3,\text{NP}}}{|p_{\text{out}3}|} \right),
\]

(10)

with \( Q_{a,\text{NP}} \) the proper NP hard scales, which are proportional to the ones entering the PT radiator. The crucial difference is that here both \( p_2 \) and \( p_3 \) are not fixed in the event plane. This implies that the way the NP correction in \( 10 \) affects the PT distribution is very different according to which phase space region is considered. Namely one has the two regimes

- \( \alpha_s \ln^2 K_{\text{out}}/Q \gg 1 \): one has well developed QCD radiation, so that all \( p_a (a = 2, 3) \) take recoil and \( p_{\text{out}a} \sim K_{\text{out}} \). This gives rise to a logarithmically enhanced shift

\[
\delta K_{\text{out}} \sim C_1 \ln \frac{Q_{1,\text{NP}} e^{\eta_0}}{Q} + C_2 \ln \frac{Q_{2,\text{NP}}}{K_{\text{out}}} + C_3 \ln \frac{Q_{3,\text{NP}}}{K_{\text{out}}},
\]

(11)

- \( \alpha_s \ln^2 K_{\text{out}}/Q \ll 1 \): radiation from one hard parton dominates. According to the event-plane kinematics in \( 9 \), when PT radiation comes from the
U region (which happens with probability \((\frac{1}{2}C_1 + C_2)/(2C_F + C_A)\)), only \(p_2\) takes recoil, so that that \(p_2^{\text{out}} \sim K_{\text{out}}\), while \(p_3^{\text{out}} \ll p_2^{\text{out}}\). The part of the shift proportional to \(C_2\) gets logarithmically enhanced as in the previous case, while the one proportional to \(C_3\) gives rise to a very singular contribution proportional to \(1/\sqrt{\alpha_s}\), coming from the average of \(\ln Q/p_3^{\text{out}}\) over its DL Sudakov form factor \(\exp\{-(\frac{1}{2}C_1 + C_3)\frac{\alpha_s}{\pi}\ln^2 Q/p_3^{\text{out}}\}\). This gives

\[
\delta K_{\text{out}} \sim C_1 \ln \frac{Q_1^{\text{NP}} e^{\eta_0}}{Q} + C_2 \ln \frac{Q_2^{\text{NP}}}{K_{\text{out}}} + C_3 \frac{\pi}{2\sqrt{(\frac{1}{2}C_1 + C_3)\alpha_s}}. \quad (12)
\]

The converse argument holds for an emission in the D region.

In conclusion, we have now a promising theoretical method to deal with multi-jet observables. Not only are we able to identify all sources of SL contributions, but we can also relate the NP \(1/Q\) power-suppressed corrections with the ones which affect \(e^+e^-\) two-jet shapes. A next step will be the extension of the above results to event shapes involving any number of jets. We hope that this analysis will greatly improve the understanding of confinement physics, especially at hadron colliders.

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