Modeling of composite coupling technology for oil-gas pipeline section resource-saving repair

Irina Donkova¹, Yuriy Yakubovskiy² and Mikhail Kruglov²

¹Tyumen State University, Volodarskogo str., 6, Tyumen, 625003, Russia
²Industrial University of Tyumen, Volodarskogo str., 38, Tyumen, 625000, Russia

E-mail: irina_donkova@mail.ru

Abstract. The article presents a variant of modeling and calculation of a main pipeline repair section with a composite coupling installation. This section is presented in a shape of a composite cylindrical shell. The aim of this work is mathematical modeling and study of main pipeline reconstruction section stress-strain state (SSS). There has been given a description of a structure deformation mathematical model. Based on physical relations of elasticity, integral characteristics of rigidity for each layer of a two-layer pipe section have been obtained. With the help of the systems of forces and moments which affect the layers differential equations for the first and second layer (pipeline and coupling) have been obtained. The study of the SSS has been conducted using the statements and hypotheses of the composite structures deformation theory with consideration of interlayer joint stresses. The relations to describe the work of the joint have been stated. Boundary conditions for each layer have been formulated. To describe the deformation of the composite couple with consideration of the composite cylindrical shells theory a mathematical model in the form of a system of differential equations in displacements and boundary conditions has been obtained. Calculation of a two-layer cylindrical shell under the action of an axisymmetric load has been accomplished.

1. Introduction

The growth of fuel and energy resource scarcity and environmental concerns make energy-saving technologies one of the priorities of the oil industry. Main pipeline transport is the most common and efficient type of petroleum products transportation. Pipelines long-term operation leads to a decrease in bearing capacity and increased risk of accidents and failures. To improve the reliability of pipelines a selective repair of defective sections is carried out. Among the existing technologies of main and technological pipeline repair an installation of reinforcing couplings (coupling technology) is one of the promising methods [1, 2]. In contrast with the overhaul repair methods use of repair structures in the form of a series of couplings is a resource-saving and environmentally friendly method, as it is performed without stopping oil pumping and pipe replacement. For example, the use of such couplings as composite and clamp weld-on ones allow restoring a pipeline defective section bearing capacity to a failsafe level during further operation. During the repair coupling installation, the SSS of the recovered pipeline section significantly changes which causes the need for strength calculations.

A pipeline section which is repaired with coupling installation can be regarded as a composite structure, consisting of two layers in the form of cylindrical shells and the joint. Connection between the layers provides consistency of the layers work. The connection of layers depends on a method of fixing the coupling. Paper [1] states that when repairing pipelines they use the following couplings: welded reinforcing and sealing ones. Reinforcing couplings cover the defect area without sealing. For
sealing couplings sealing the ends with "girth side fillet weld" [1] is provided for. The boundary conditions which meet the specified conditions of fixing the couplings can be considered as a hinged support for reinforcing couplings and as rigid fixing for sealing ones.

2. Materials and Methods

To describe the mathematical model and to study the SSS of a composite coupling which is used to restore a pipeline defect section the composite structure theory statements are applied [2 – 5]. Each shell is considered as the \( i - \text{th} \) layer (\( i = 1, 2 \)) of thickness \( h^{(i)} \) (the thickness of pipe walls and couplings). The SSS of each layer of the shell is described by a system of forces, moments, strains, and displacements, which is applied in the classical theory using Kirchhoff–Love’s hypotheses (L. I. Balabuha – I. V. Novozhilov variant). To describe the structure deformation the curvilinear, orthogonal right-handed coordinate system is used: \( x, \varphi, z^{(i)} \). The middle layer surface of the \( i - \text{th} \) layer is a coordinate surface of the \( z^{(i)} = 0 \) type. Coordinate lines \( x, \varphi \) coincide with the middle surface main curvature lines of each \( i - \text{th} \) layer, axis \( z^{(i)} \) is straight. Lame parameters \( A_1^{(i)}, A_2^{(i)} \) are calculated with coordinates \( x \) – the distance along the generatrix \( (0 \leq x \leq L) \varphi \) is the angle along directing line \( (0 \leq \varphi \leq 2\pi) \).

The shell thickness is measured from the middle surface of the \( i - \text{th} \) layer in the direction of coordinate \( z^{(i)} \). The distances from the middle surface of the outer layer (coupling) and inner layer (the very pipe) to the joint are marked as \( a, b \). The distance between the median surfaces can be presented as \( C = a + b \).

In the adopted coordinate system, the displacement of points on the middle surface of the \( i - \text{th} \) layer (\( i = 1, 2 \)) has components \( u^{(i)}, v^{(i)}, w \) along axes \( u^{(i)}, v^{(i)}, w \). Deflection \( w \), according to [2, 3] is adopted as similar for each layer.

For a circular cylindrical shell we consider the case when boundary and surface loads do not depend on angle \( \varphi \). Then the deformation of the shell will not depend on this angle, i.e. will be axisymmetric. The calculation, when the shell undergoes deformation \( q = \{q_1, 0, q_3\} \) in the absence of component \( q_2 \), is of practical interest. Then, the elements of the structure are only affected by normal forces \( N_1^{(i)}, N_2^{(i)} \) and bending moments \( M_1^{(i)}, M_2^{(i)} \); and shearing forces \( S^{(i)} \) and torques \( H^{(i)} \) are missing.

Under the action of axisymmetric load deformations in the surfaces, which are located at distance \( z^{(i)} \) from the middle surface of the \( i - \text{th} \) layer (\( i = 1, 2 \)), are determined by formulas [1]:

\[
\varepsilon^{(i)}_{1(z)} = \varepsilon^{(i)}_1 + z^{(i)} \kappa_1; \varepsilon^{(i)}_{2(z)} = \varepsilon^{(i)}_2; \gamma^{(i)}_{12(z)} = 0; \chi^{(i)}_{12(z)} = 0. \tag{1}
\]

Here tangential deformation parameters \( \varepsilon^{(i)}_1, \varepsilon^{(i)}_2, \gamma^{(i)}_{12}, \chi^{(i)}_{12} \), parameters of changing curvatures \( \kappa_1^{(i)}, \kappa_2^{(i)} \) and torsion parameter \( \chi^{(i)}_{12} \) are equal to [3, 4]:
\[ \varepsilon_1^{(i)} = \frac{du^{(i)}}{dx}, \quad \varepsilon_2^{(i)} = \frac{w}{r} \cdot \kappa_1^{(i)} = -\frac{d^2w}{dx^2} \cdot \kappa_2^{(i)} = 0, \quad \gamma_{12}^{(i)} = 0, \quad \chi_{12}^{(i)} = 0. \]  \tag{2}

Physical correlations of the structure material are defined according to generalized Hooke’s law. Thus, for the stresses components in layers \((i = 1, 2)\) ratios [3, 4] are used:

\[ \sigma_{1(z)}^{(i)} = b_{11}^{(i)} \varepsilon_1^{(i)} + b_{12}^{(i)} \varepsilon_2^{(i)} + b_{11}^{(i)} \kappa_1^{(i)}; \quad \sigma_{2(z)}^{(i)} = b_{21}^{(i)} \varepsilon_1^{(i)} + b_{22}^{(i)} \varepsilon_2^{(i)}; \quad \tau_{12(z)}^{(i)} = 0. \]  \tag{3}

Here the coefficients with account of material orthotropy of the structure layers are determined by formulas [1, 2].

\[ b_{11}^{(i)} = \frac{E_{1}^{(i)}}{1 - \nu_1^{(i)} \nu_2^{(i)}}, \quad b_{12}^{(i)} = b_{21}^{(i)} = \frac{E_{2}^{(i)}}{1 - \nu_1^{(i)} \nu_2^{(i)}}, \quad b_{22}^{(i)} = \frac{E_{3}^{(i)}}{1 - \nu_1^{(i)} \nu_2^{(i)}}. \]  \tag{4}

For the case of isotropic properties of the layers material the coefficients are equal to [1 ÷ 4].

\[ b_{11}^{(i)} = b_{22}^{(i)} = \frac{E^{(i)}}{1 - \nu^{(i)^2}}, \quad b_{12}^{(i)} = b_{21}^{(i)} = \frac{E^{(i)}}{1 - \nu^{(i)^2}}; \quad b_{66}^{(i)} = \frac{E^{(i)}}{2(1 + \nu^{(i)})}. \]  \tag{5}

We can go to line forces that act at the level of the middle surface of the \(i-th\) layer from the stresses at an arbitrary point of the \(i-th\) layer by means of integrating by thickness:

\[ N_1^{(i)} = \int_{-h^{(i)/2}}^{h^{(i)/2}} \sigma_{1(z)}^{(i)} dz^{(i)}; \quad N_2^{(i)} = \int_{-h^{(i)/2}}^{h^{(i)/2}} \sigma_{2(z)}^{(i)} dz^{(i)}; \quad M_1^{(i)} = \int_{-h^{(i)/2}}^{h^{(i)/2}} \sigma_{1(z)}^{(i)} dz^{(i)}; \]  \tag{6}

where \(N_1^{(i)}, N_2^{(i)}\) are forces, \(M_1^{(i)}\) are bending moments.

To calculate the line forces under axisymmetric load, it is necessary to integrate the expressions for strains at an arbitrary point of this layer. After the integration, we obtain the following elasticity relations:

\[ N_1^{(i)} = B_{11}^{(i)} \varepsilon_1^{(i)} + B_{12}^{(i)} \varepsilon_2^{(i)}; \quad N_2^{(i)} = B_{21}^{(i)} \varepsilon_1^{(i)} + B_{22}^{(i)} \varepsilon_2^{(i)}; \quad M_1^{(i)} = D_{11}^{(i)} k_1. \]  \tag{7}

Here the rigidity parameters \(B_{11}^{(i)}, B_{12}^{(i)}, B_{21}^{(i)}, B_{22}^{(i)}, D_{11}^{(i)}\) are calculated by means of the following integral expressions:
Differential equations of equilibrium for the \( i-th \) layer subject to shear stresses in interlayer joint \( \tau \) will be presented in accordance with the methodology [1 – 5]:

\[
\frac{dN_1^{(i)}}{dx} + \tau + q_1^{(i)} = 0, \quad \frac{dQ_1^{(i)}}{dx} \cdot \frac{N_2^{(i)}}{r} + q_3^{(i)} = 0, \quad \frac{dM_1^{(i)}}{dx} + \tau a - Q_1^{(i)} = 0.
\]

where \( q_1^{(i)}, q_3^{(i)} \) are the components of the external load, which are distributed along the median surface of the \( i-th \) layer \((i = 1, 2)\).

The shear stress along the generatrix in the joint is determined by the formula: \( \tau = \eta \Delta u \) (10)

Here, \( \eta \) is the stiffness coefficient of connection of the joint shear between the main pipe of the pipeline and the coupling. \( \Delta u \) is the difference between the longitudinal displacements on both sides of the adjacent layers joint in the direction of coordinate \( X \), which is determined by the formula:

\[
\Delta u = u^{(2)}(X) - u^{(1)}(X) + C \frac{dW}{dx}.
\]

Here, \( u^{(2)}, u^{(1)} \) are the displacements in the middle surfaces of the shells (pipeline and coupling), \( W \) are lateral shifts of the points.

Differential equations of equilibrium in displacements for the \( i-th \) structure layer are presented in papers [1 – 5]. The order of the system of differential equations determines the number of boundary conditions.

3. Results

Composite structures in the form of a circular cylindrical shell have been regarded. Along coordinate \( X \) on each of the two transverse layers boundary conditions have been specified. Along coordinate \( \varphi \) periodicity conditions of the required function must be met.

In the case of axisymmetric deformation of a two-layer cylindrical shell, the system of differential equations in displacements will be written as [3, 5]:

\[
-D_1 \frac{d^2 W}{dx^2} + C \eta \frac{d^2 W}{dx^2} \frac{d^2 W}{dx^2} - B_{11}^{(2)} \frac{dW}{dx} + \left( C_1^{(2)} \right) \frac{dW}{dx} - \left( C_1 \eta + B_{11}^{(1)} \right) \frac{dW}{dx} = -q_1;
\]

\[
B_{11}^{(0)} \frac{dW}{dx} + \left( B_{11}^{(0)} + C \eta \right) \frac{dW}{dx} - \eta u_1^{(0)} + \eta u_2^{(0)} = 0.
\]
where $C$ is the distance between middle surfaces of the layers. $D_{ij} = \sum_{i=1}^{2} D_{ij}^{(i)}$ is cylindrical rigidity. According to the general technical theory: $q_3(q_3^{(1)} + q_3^{(2)})$.

The variants of boundary conditions for axisymmetric loading of a composite cylindrical shell for each $i-th$ layer ($i = 1, 2$) are given with $x = 0, x = L$.

1. If the structural contour is free (unloaded), then the static conditions are met:

$$N_1^{(i)} = 0, M_1^{(i)} = 0, q_1^{(i)} = 0.$$  

2. With the hinged contour mixed boundary conditions are valid:

$$w = 0, M_1 = 0, u^{(i)} = 0.$$  

Boundary conditions in displacements are: $w = 0, u^{(i)} = 0, \frac{d^2 w}{dx^2} = 0$.

3. For the hinge-supported contour free in normal direction the following conditions must be met:

$$u^{(i)} = 0, M_1^{(i)} = 0, q_1^{(i)} = 0.$$  

4. For the hinge-supported contour free in tangential direction mixed boundary conditions are met:

$$N_1^{(i)} = 0, M_1^{(i)} = 0, w = 0.$$  

Boundary conditions in displacements are: $w = 0, \frac{d^2 w}{dx^2} - \frac{du^{(i)}}{dx} = 0$.

5. Boundary conditions in displacements for the fixed fast contour are:

$$u^{(i)} = 0, w^{(i)} = 0, \vartheta^{(i)} = 0.$$  

Here, $\vartheta^{(i)}$ are the rotation angles of the normal to the middle surface of the $i-th$ layer, which are determined by equations:

$$\vartheta^{(i)} = - \frac{dw}{dx} + \frac{u^{(i)}}{r}.$$  

Technical description and diagrams that correspond to the boundary conditions are given in [1].

Thus, the mathematical model of the SSS of the recovered section of a main pipeline under axisymmetric load is a system of differential equations (12) and boundary conditions. For example, if the boundary conditions correspond to hinged support of the ends (case 4), then the system of differential equations can be solved by decomposition of the displacement functions and load vector in trigonometric series [7]:

$$u^{(1)}(x) = \sum_{m=1}^{\infty} U_{m}^{(1)} \cos \left( \frac{m \pi x}{L} \right), u^{(2)}(x) = \sum_{m=1}^{\infty} U_{m}^{(2)} \cos \left( \frac{m \pi x}{L} \right), w(x) = \sum_{m=1}^{\infty} W_{m} \sin \left( \frac{m \pi x}{L} \right), q_{3}(x) = \sum_{m=1}^{\infty} q_{m} \sin \left( \frac{m \pi x}{L} \right).$$  

(13)
Under the uniformly distributed load the value of the coefficients in the trigonometric series for the load vector is defined by expression \( q_m = \frac{4q}{m\pi} \), where \( m = 1, 3, 5, \ldots \). We can move to the system of equations from the system of equilibrium equations of a two-layer composite cylindrical shell [5]:

\[
D_1 \sum_{m=1}^{\infty} W_m \left( \frac{m\pi}{L} \right)^{4} \sin \left( \frac{m\pi x}{L} \right) + C^2 \eta \sum_{m=1}^{\infty} W_m \left( \frac{m\pi}{L} \right)^{2} \sin \left( \frac{m\pi x}{L} \right) + \\
+ \left( \frac{B_{22}^{(1)} + B_{22}^{(2)}}{r^2} \right) \sum_{m=1}^{\infty} W_m \sin \left( \frac{m\pi}{L} \right) - \left( \frac{B_{21}^{(1)} + C\eta}{r} \right) \sum_{m=1}^{\infty} U_m^{(1)} \left( \frac{m\pi}{L} \right) \sin \left( \frac{m\pi x}{L} \right) + \\
+ \left( C\eta - \frac{B_{21}^{(2)}}{r} \right) \sum_{m=1}^{\infty} U_m^{(2)} \left( \frac{m\pi}{L} \right) \sin \left( \frac{m\pi x}{L} \right) = \sum_{m=1}^{\infty} q_m \sin \left( \frac{m\pi x}{L} \right);
\]

\[
\left( \frac{B_{21}^{(2)}}{r} + C\eta \right) \sum_{m=1}^{\infty} W_m \left( \frac{m\pi}{L} \right) \cos \left( \frac{m\pi x}{L} \right) - B_{11}^{(1)} \sum_{m=1}^{\infty} U_m^{(1)} \left( \frac{m\pi}{L} \right)^{2} \cos \left( \frac{m\pi x}{L} \right) - \\
- \eta \sum_{m=1}^{\infty} U_m^{(1)} \cos \left( \frac{m\pi x}{L} \right) + \eta \sum_{m=1}^{\infty} U_m^{(2)} \cos \left( \frac{m\pi x}{L} \right) = 0;
\]

\[
\left( \frac{B_{12}^{(2)}}{r} - C\eta \right) \sum_{m=1}^{\infty} W_m \left( \frac{m\pi}{L} \right) \cos \left( \frac{m\pi x}{L} \right) - B_{11}^{(2)} \sum_{m=1}^{\infty} U_m^{(2)} \left( \frac{m\pi}{L} \right)^{2} \cos \left( \frac{m\pi x}{L} \right) + \\
+ \eta \sum_{m=1}^{\infty} U_m^{(1)} \cos \left( \frac{m\pi x}{L} \right) - \eta \sum_{m=1}^{\infty} U_m^{(2)} \cos \left( \frac{m\pi x}{L} \right) = 0.
\]

The values of coefficients \( W_m, U_m^{(1)}, U_m^{(2)} \) of the required functions \( W, U^{(1)}, U^{(2)} \) can be found from the solution of a system of linear algebraic equations if the coefficients of the corresponding trigonometric functions are equated.

During the study of the interlayer connection stiffness on the SSS of a structure under axisymmetric loading there was made a calculation of a two-layer pipe section with the following geometric parameters: the length \( L = 500 \text{ mm} \), the internal diameter \( 500 \text{ mm} \), the thickness of the outer layer (composite coupling) \( h_1 = 12 \text{ mm} \), the thickness of the inner layer (the very pipe) \( h_1^{(2)} = 12 \text{ mm} \). During the calculation the following mechanical properties of isotropic layers are adopted: the modulus of longitudinal elasticity of the layers material \( E_1^{(1)} = E_1^{(2)} = 2.1 \times 10^4 \text{ MPa} \); Young's modulus \( v_1^{(1)} = v_1^{(2)} = 0.3 \); the joint stiffness varies within \( \eta = 0 \div 10^3 \text{ N/mm}^2 \). Zero stiffness of the interlayer connection (coefficient \( \eta = 0 \text{ N/mm}^2 \)) corresponds to the study of the structure with installation of a crimp coupling through the elastic layer [1]. The variant of calculation with unlimited increase of the stiffness coefficient is used for welded couplings in the absence of shear between the layers. The calculation is made for the internal pipeline pressure under a uniformly distributed load with the intensity of \( q = 0.5 \div 1 \text{ MPa} \) and under axisymmetric local load with
intensity $q = 1 \text{MPa}$ distributed in the central area of the shell. The boundary conditions meet hinged ends (case 4).

The calculations are accomplished on the external surfaces of the shell layers under the evenly distributed load. Normal stresses $\sigma_1^{(1)}$ of the outer layer are calculated at $z = -\frac{h}{2}$, and values $\sigma_1^{(2)}$ of the inner layer are calculated at $z = \frac{h}{2}$. Figure 1 represents the diagrams of distributions of bending moments $M_1^{(1)}$ in the first layer along the generatrix under the evenly distributed load.

According to the obtained results we can conclude that with variation of the stiffness coefficient within $\eta = 0 \div 500 \frac{N}{mm}$, the nature of functions distribution for deflections $W$ and bending moments $M_1^{(1)}$ changes. Deflection $W$ in the middle part of the cylindrical shell increased by 10% with $\eta = 0 \div 10^5 \frac{N}{mm}$. The maximum bending moments changed twice.

![Figure 1](image.png)

Figure 1. Diagrams of distribution of bending moments $M_1^{(1)}$ of the outer layer along the generatrix under the evenly distributed load. Values of interlayer connection stiffness

1: $100 \frac{N}{mm}$, 2: $0.1 \frac{N}{mm}$, 3: $500 \frac{N}{mm}$.

4. Discussion
The solution of the problem of bending of a closed two-layer cylindrical shell (composite coupling) with steel layers under axisymmetric load showed that with stiffness exceeding $10^6 \frac{N}{mm}$, the structure can be regarded as single, without taking into account slippage between layers. The load distribution variation for a closed composite cylindrical shell showed that the interlayer connection stiffness under evenly distributed load significantly affects the stress – strain state at the near-anchorage area, and under local load at the central area. To assess the authenticity of the calculation
results exact solutions for a single-layer structure under axisymmetric loading are used when the stiffness factor is equal to zero and rises unlimitedly. For example, if the interlayer connection stiffness of a two-layer shell is equal to zero, the bending rigidity is defined as a sum of rigidities of each individual layer $D_{11} = \sum_{i=1}^{2} D_{11}^{(i)}$. Here, $D_{11}^{(i)}$ is a rigidity of a separate layer. Provided that geometrical and mechanical characteristics coincide, i.e. $D_{11}^{(1)} = D_{11}^{(2)}$, the differential equation coincides with the equation of cylindrical shell axisymmetric deformation [3]:

$$\frac{d^4 w}{dx^4} + 4 \beta^4 = \frac{q_3}{D_{11}} \quad \text{where} \quad \beta^4 = \frac{3(1-\nu^2)}{(rh)^2} \quad D_{11} = \frac{Eh^3}{12(1-\nu^2)}. \quad (14)$$

Here, $w$ is a deflection function, $x$ is the distance along the generatrix, $\nu$ is Young’s modulus, $r$ is the cylindrical shell radius of curvature, $h$ is the cylindrical shell thickness. General solution of differential equation (14) is defined by formula:

$$w = C_1 e^{-\beta x} \cos \beta x + C_2 e^{-\beta x} \sin \beta x + C_3 e^{\beta x} \cos \beta x + C_4 e^{\beta x} \sin \beta x + w_{ch},$$

where constant $C_i$ and particular solution $w_{ch}$ are determined from the boundary conditions.

5. Conclusions

Thus, a mathematical model has been obtained in the form of differential equations which allows calculating the values of deformations, stresses and forces in layers under different methods of coupling installation, assessing the structural reliability of the pipeline, and determining the influence of geometrical and mechanical structure parameters on the strength of the repair pipeline section. Reconstruction of damaged pipeline sections using coupling technology leads to a decrease in stress and unloading the main pipe, which improves resistance to damage and increases the service life of pipelines due to an additional layer. Modeling and calculation of an operating pipeline without stopping oil pumping allow using energy resources efficiently and environmentally friendly; therefore, are a promising direction for further scientific and technical studies.

References

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