Addendum to “HTN Acting: A Formalism and an Algorithm”

Lavindra de Silva
Department of Engineering,
University of Cambridge, UK
Lavindra.deSilva@eng.cam.ac.uk

Abstract
Hierarchical Task Network (HTN) planning is a practical and efficient approach to planning when the ‘standard operating procedures’ for a domain are available. HTN planning performs hierarchical and context-based refinement of goals into subgoals and basic actions. However, while HTN planners ‘lookahead’ over the consequences of choosing one refinement over another, BDI agents interleave refinement with acting. There has been renewed interest in making HTN planners behave more like BDI agent systems, e.g. to have a unified representation for acting and planning. However, past work on the subject has remained informal or implementation-focused. This paper is a formal account of ‘HTN acting’, which supports interleaved deliberation, acting, and failure recovery. We use the syntax of the most general HTN planning formalism and build on its core semantics, and we provide an algorithm which combines our new formalism with the processing of exogenous events. We also study the properties of HTN acting and its relation to HTN planning.

1 Introduction
Hierarchical Task Network (HTN) planning is a practical and efficient approach to planning when the ‘standard operating procedures’ for a domain are available. HTN planning is similar to Belief-Desire-Intention (BDI) agent reasoning, HTN planning performs hierarchical and context-based refinement of goals into subgoals and basic actions. However, while HTN planners ‘lookahead’ over the consequences of choosing one refinement over another before suggesting an action, BDI agents interleave refinement with acting in the environment. Thus, while the former approach has remained informal or implementation-focused, the latter approach is able to quickly respond to environmental changes and exogenous events, and recover from failure. This paper presents a formal semantics that builds on the core HTN semantics in order to enable such response and recovery.

One motivation for our work is a recent drive toward adapting the languages and algorithms used in Automated Planning to build a framework for ‘refinement acting’ [14], i.e., deciding how to carry out a chosen recipe of action to achieve some objective, while dealing with environmental changes and failures. To this end, [14] proposes the Refinement Acting Engine (RAE), an HTN-like framework with continual online processing and recipe repair in the case of runtime failure. A key consideration in the RAE is a unified hierarchical representation and a core semantics that suits the needs of both acting and lookahead. We are also motivated by recent work [3] which suggests that a fragment of the recipe language of HTN planning does not have a direct (nor known) translation to the recipe languages of typical BDI agent programming languages such as AgentSpeak [22] and CAN [30]. For example, HTNs allow a flexible specification of how steps in a recipe should be interleaved, whereas steps in CAN recipes must be sequential or interleaved in a ‘series-parallel’ manner.

There have already been some efforts toward adapting HTN planning systems to make them behave more like BDI agent systems. Perhaps the first of these efforts was the RETSINA architecture [27], which used an HTN language and semantics for representing recipes and refining tasks, but also interleaved task refinement with acting in the environment. RETSINA is an implemented architecture which has been used in a range of real-world applications. In [7], the JSHOP [20] HTN planner is modified in two ways: (i) to execute a solution (comprising a sequence of actions) found via lookahead, and then re-plan if the solution is no longer viable in the real world (due to a change in the environment), and (ii) to immediately execute the chosen refinement for a task, instead of first performing lookahead to check whether the refinement will accomplish the task. The latter modification made JSHOP as effective as the industry-strength JACK BDI agent framework [29], in terms of responsiveness to environmental changes.

However, both RETSINA and the JSHOP variant lack a formalism, making it difficult to study the properties (e.g., correctness) of their semantics, and to compare them to other similar systems. The same applies to the algorithms and abstract syntax of the RAE framework, which are presented only in pseudocode.

There is also some work on making BDI-like agent systems behave more like HTN planning systems. In particular, both the REAP algorithm in [14] and the CANPlan [24, 25] BDI agent programming language (and its extensions such as [3, 6]) can make informed decisions about refinement choices by using a lookahead capability. Similarly, there are agent programming languages and systems that support some form of planning (though not HTN-style planning) [19], such as the PRS [13] based Propice-Plan [8] system and the situation-calculus based IndiGolog [4] system. Finally, there are also some interesting extensions to HTN and HTN-like planning [12, 11, 17, 51, 26, 2], e.g. approaches that combine classical and HTN planning. In contrast, our work is not concerned with lookahead or planning, but with adapting the HTN planning semantics to enable BDI-style behaviour.

Thus, our contribution is a formal account of HTN acting, which supports interleaved deliberation, acting, and recovery from fail-
ure, e.g., due to environmental changes. To this end, we use the syntax of the most general HTN planning formalism $[11][9]$, and we build on its core semantics by developing three main definitions: execution via reduction, action, and replacement. We then provide an algorithm for HTN acting which combines our new formalism with the processing of exogenous events. We also study the properties of HTN acting, particularly in relation to HTN planning.

2 Background: HTN Planning

In this section we provide the necessary background material on HTN planning. Some definitions are given only informally; we refer the reader to $[11][9]$ for the formal definitions.

An HTN planning problem is a tuple $(d, I, D)$ comprising a task network $d$, an initial state $I$, which is a set of ground atoms, and a domain $D = (Op, Me)$, where $Me$ is a set of reduction methods and $Op$ is a set of STRIPS-like operators. HTN planning involves iteratively decomposing/reducing the ‘abstract tasks’ occurring in $d$ and the resulting task networks by using methods in $Me$, until only STRIPS-like actions remain that can be ordered and executed from $I$ relative to $Op$.

A task network $d$ is a couple $[S, \phi]$, where $\phi$ is a constraint formula, and $S$ is a non-empty set of labelled tasks, i.e., constructs of the form $(n : t)$; element $n$ is a task label, which is a 0-ary task-label symbol (in FOL) that is unique in $d$ and $D$, and $t$ is a non-primitive or primitive task, which is an n-ary task symbol whose arguments are function-free terms. The constraint formula $\phi$ is a Boolean formula built from negation, disjunction, conjunction, and constraints, each of which is either: an ordering constraint of the form $(n < n')$, which requires the task (corresponding to label $n$) to precede task $n'$; a before (resp. an after) state-constraint of the form $(l, n)$ resp. $(n, l)$, which requires literal $l$ to hold in the state just before (resp. after) doing $n$; a between state-constraint of the form $(n, l, n')$, which requires $l$ to hold in all states between doing $n$ and $n'$; or a variable binding constraint of the form $(o = o')$, which requires $o$ and $o'$ to be equal, each of which is a variable or constant. We ignore variable binding constraints as they can be specified as state-constraints, using the binary logical symbol ‘=’.

Instead of specifying a task label, a constraint may also refer, using expression $fst[S]$ or $lst[S]$, to the action that is eventually ordered to occur first or last (respectively) among those that are yielded by the set of task labels $S$. While these expressions can be ‘inserted’ into a constraint when a task is reduced, we assume that they do not occur in methods.

A primitive task, or action, $t$, has exactly one relevant operator in $Op$, i.e., one operator associated with a primitive task $t'$ that has the same task symbol and arity as $t$; any variable appearing in the operator also appears in $t'$ and its precondition. Given a primitive task $t$, we denote its precondition, add-list and delete-list relative to $Op$ as pre$(t, Op)$, $add(t, Op)$ and $del(t, Op)$, respectively. A non-primitive task can have one or more relevant methods in $Me$. A method is a couple $[(v), d]$, where $f(v)$ is a non-primitive task, the arguments $v$ are distinct variables and $d$ is a task network.

Given an HTN planning problem $(d = [S_d, \phi_d], I, (Op, Me))$, the core planning steps involve selecting a relevant method $m = [(v_m), d_m] \in Me$ for some non-primitive task $(n : t) \in S_d$ and then reducing the task to yield a ‘less abstract’ task network. Reducing $(n : t)$ with $m$ involves replacing $(n : t)$ with the tasks in $S_m$ (where $d_m = [S_m, \phi_m]$) and updating $\phi_d$, e.g. to include the constraints $\phi_m$; formal definitions for method relevance and reduction are given in Section 3. The set of reductions of $d$ is denoted red$(d, (Op, Me))$.

If all non-primitive tasks in the initial and subsequent task networks have been reduced, a completion is obtained from the resulting ‘primitive’ task network. Informally, $\sigma$ is a completion of a primitive task network $d = [S, \phi]$ at a state $I$, denoted $\sigma \in comp(d, I, D)$, if $\sigma$ is a total ordering of a ground instance of $d$ that satisfies $\phi$; if $d$ mentions a non-primitive task, then $comp(d, I, D) = \emptyset$.

Finally, the set of all HTN solutions is defined as $sol(d, I, D) = \bigcup_{n<\omega} sol_n(d, I, D)$, where $sol_n(d, I, D)$ is defined inductively as

\[
\begin{align*}
sol_1(d, I, D) &= comp(d, I, D), \\
sol_{n+1}(d, I, D) &= sol_n(d, I, D) \cup \bigcup_{d' \in red^*(d, D)} sol_n(d', I, D).
\end{align*}
\]

In words, the HTN solutions for a given planning problem is the set of all completions of all primitive task networks that can be obtained via zero or more reductions of the initial task network.

A Running Example

Let us consider the example of a rover agent exploring the surface of mars. A part of the rover’s HTN domain is illustrated in Figure 11 (with braces omitted in $fst[]$ and $lst[]$ expressions). The top-level non-primitive task is to transfer, to the lander, previously gathered soil analysis data from a location $X$, and if possible to also deliver the soil sample for further analysis inside the lander.

The top-level task is achieved using either method $m_1$ or $m_2$, both of which require the data and sample from $X$ to be available (i.e., for $didExp(X)$ to hold). If the rover is low on battery charge ($lowBat$, $m_1$ is used. This transmits the soil data but it does not deliver the soil sample, which may result in losing it if it is later discarded to make room for other samples. Method $m_1$ prescribes establishing radio communication with the lander, sending it the data by first including metadata, and then breaking the connection, while checking continuously that the connection is not lost between the first and last tasks (including those of $m_2$). If the rover is not low on battery charge, $m_2$ is used to achieve the top-level task; $m_2$ prescribes navigating to a lander $L$ and then uploading and depositing the soil data and sample, respectively.

Navigation is performed using $m_4$ or $m_5$. Method $m_4$ prescribes calibrating the onboard instruments, moving the cameras to point straight (which asserts $camMoved$), and moving to the lander; while the first two actions can happen in any order, the third must happen last. The method requires that the instruments are not currently calibrated (~cal) and the battery charge is not low. Method $m_5$ is similar except that it is used only if the instruments are already calibrated, for example due to a recent calibration to achieve another task.

Action $mv$ requires ~lowBat to hold, and it consumes a significant amount of charge, i.e., it asserts lowBat$^2$. Action procLim (not shown) requires raw and ~lowBat to hold and asserts ~raw and lowBat; the action processes and compresses new raw images

\footnote{For simplicity, we assume ‘low charge’ is less than or equal to 50% of the maximum charge, and an action requiring a ‘significant’ amount of charge consumes 50%. We also consider it unsafe for the charge to reach 0%.}
In this section we formally define the notion of reduction, and we state the remaining assumptions.

First, we separate the notion of method relevance from the notion of reduction in \([11]\). In what follows, we use the standard notion of substitution \([18]\), and of applying a substitution \(\theta\) to an expression \(E\), which we denote by \(E[\theta]\).

**Definition 1** (Relevant Method). Let \(D = (Op, Me)\) be a domain, \(t\) a non-primitive task, and \([t', d]\) \(\in Me\) a method. If \(t = t'\theta\) for some substitution \(\theta\), then \(d\theta\) is a relevant method-body for \(t\) relative to \(D\). The set of all such method-bodies is denoted by \(rel(t, D)\).

In the definition of reduction below, and in the rest of the paper, we denote by \(lab(S)\) the set of all task labels appearing in a given set of labelled tasks \(S\).
4 A Formalism for HTN Acting

We now develop a formalism for HTN acting by defining, in particular, three notions of execution: via reduction, action, and replacement. The first notion is based on task reduction; the second notion defines what it means to execute an action in the HTN setting, in particular, the gathering and evaluating of constraints relevant to the action; and the third notion represents failure handling, i.e., the replacement of ‘blocked’ tasks by alternative ones.

We only allow a task occurring in a task network to be executed via action or reduction if it is a primary task in the network, i.e., there are no other tasks that must precede it. Formally, given a task network \( d = [S, \phi] \), we first define the following sets of tasks:

\[
S_1 = \{ \langle n: t \rangle \in S \mid (x \prec x') \in \phi, n \text{ occurs in } x' \},
\]

\[
S_2 = \{ \langle n: t \rangle \in S \mid t \text{ is an action and either } (n \prec x) \in \phi \text{ or } -(\text{lst}[\{n\}] \prec x) \in \phi \}.
\]

That is, \( S_1 \) and \( S_2 \) contain the tasks that cannot be primary ones; the above action \( n \) occurring in a negated ordering constraint cannot be a primary task because one or more tasks (represented by \( x \) above) must precede \( n \). Then, we define the set of primary tasks of task network \( d \) as primary(\( d \)) = \( S \setminus (S_1 \cup S_2) \).

For example, given task network \( d_1 \) in method \( m_1 \) in Figure 1

\[
\text{primary}(d_1) = \{ \},
\]

and given task network \( d_4 \) in method \( m_4 \),

\[
\text{primary}(d_4) = \{ 8, 9 \}.
\]

We can now define our first notion, an execution via reduction of a task network, as the reduction of an arbitrary primary non-primitive task via a relevant method. To enable trying alternative reductions for the task if the one that was selected fails or is not applicable, we maintain the set of all relevant methods for the task, and update the set as alternative methods are tried. We use the term reduction couple to refer to a couple comprising two sets: (i) the set representing the reductions being pursued for a task (and its subtasks), and (ii) the set of current alternative method-bodies for the task. We use \( R \) to denote the set of reduction couples corresponding to the tasks reduced so far, where each couple is of the form \( \langle S, \phi \rangle \), with \( S \) being a set of labelled tasks, and \( \phi \) a set of task networks. While the initial value of \( R \) and how it can ‘evolve’ will be made concrete via formal definitions, we shall for now illustrate these with an example.

Let us consider the task network \([S, \phi = \text{true}]\), where the set \( S = \{(A: \text{transDS(loc1)}), (B: \text{procImg})\}\); the initial state \( I = \{(\text{raw, cal}, \text{didExp(loc1)}, \text{landr}(\text{lan1}))\} \); the ‘initial’ set of reduction couples \( R = \{(S, \emptyset)\} \); and the domain \( \emptyset \) as depicted in Figure 1. An execution via reduction of the task network from \( I \) relative to \( R \) and \( \emptyset \) is the tuple \( \langle \{S', \phi'\}, I, R', \emptyset, D \rangle \), where \( S' = \{6, 7, B\} \), formula \( \phi' = \phi_2 \) in Figure 1 with variable \( X \) substituted with \( \text{loc1} \), and the resulting set of reduction couples \( R' = \{(S', \emptyset), (\{6, 7\}, \{d_1\})\} \), where \( d_1 \) is the alternative method-body for \( A \). Moreover, an execution via reduction of \( S' \) is the tuple \( \langle \{S'', \phi''\}, I, R'', \emptyset, D \rangle \), where \( S'' = S'''' \cup \{7, B\} \), set \( S'''' = \{11, 12\} \), formula \( \phi'' = \phi_3 \) and \( \phi' \) updated to account for the reduction, and set \( R'' = \{(S'', \emptyset), (S'' \cup \{7\}, \{d_1\}), (S''', \emptyset)\} \).

We call a 4-tuple of the form \( \langle d, I, R, D \rangle \), as in the example above, a configuration. (For brevity, we omit the fifth element \( \theta \).

---

5This is provided none of the actions associated with \( x \) have already been executed. As we show later, in our semantics, such an execution will result in the (then ‘realised’) constraint being removed.

6To account for a negated between state-constraint \( \neg (n_1, l, n_2) \), we check in every state between \( n_1 \) and \( n_2 \) whether \( \neg l \) holds. If so, we remove the constraint from the formula. If \( \neg (n_1, l, n_2) \) exists when the first action of \( n_2 \) is executed, \( \neg l \) is then relevant for it.
before executing other actions, are removed from the network’s constraint formula. The constraints that do need to be re-evaluated are the between-state-constraints that require literals to hold from the end of an action that was executed earlier, up to an action that is yet to be executed. Formally, given a task network \( d = [S, \phi] \) and an action \((n : t) \in S\), we denote by \( C_1 \) the realised ordering constraints upon executing \( n \) relative to \( d \), i.e., the set

\[
\{ (x \prec x') \in \phi \mid \text{for some } x' \text{ and } x \in \{n, \text{lst}([n])\}\} \cup \{ -(x' \prec x) \in \phi \mid \text{for some } x' \text{ and } x \in \{n, \text{fst}([n, \ldots])\}\},
\]

where \( x' \) represents an action(s) that is yet to be executed. Notice that a negated ordering constraint is realised only if one or more (or all) of the actions corresponding to \( x' \) are executed after the first (or only) one corresponding to \( x \). Next, we denote by \( C_2 \) the realised state constraints upon executing \( n \), i.e., the set obtained from \( \text{bel}(n, d) \) by removing any between state-constraint \((x, l, x')\) when \( x' \neq n \) and \( x' \neq \text{fst}([n, \ldots]) \). Then, we can define the set of realised constraints upon executing \( n \) relative to \( d \) as \( \text{fin}(n, d) = C_1 \cup C_2 \), and the result of executing an action as follows.

**Definition 7** (Action Result). Let \( Op \) be a set of operators, \( I \) a state, \( d \) a task network, \( R \) a set of reduction couples, \( (n : t) \in \text{primary}(d) \) an action, and \( \theta \) a substitution. The result \( d \) of executing \( n \) from \( I \) relative to \( \theta \) is the tuple \( \langle [S', \phi'], I', R \rangle \theta \), where

- \( S' = S \setminus \{ (n : t) \} \), where \( d = [S, \phi] \);
- \( I' = (I \setminus \text{del}(t, \theta, Op)) \cup \text{add}(t, \theta, Op) \); and
- \( \phi' \) is obtained from \( \phi \setminus \text{fin}(n, d) \) by removing all occurrences of \( n \) within \( \text{lst}() \) expressions.\(^7\)

Notice that the only possible update to \( R \) is a substitution of one or more variables (we do not remove executed actions from reduction couples). Finally, we define an execution via action of a task network as the execution of (a ground instance of) an applicable primary action in it.

**Definition 8** (Execution via Action). Let \( \mathbb{D} = \langle Op, Me \rangle \) be a domain, \( I \) a state, \( R \) a set of reduction couples, and \( d = [S, \phi] \) a task network such that \( I \models \psi \) for some \( \psi \) and action \( (n : t) \in \text{primary}(d) \), where \( \psi = \Phi(n, d, Op) \theta \) is a ground formula. An execution via action of \( d \) from \( I \) relative to \( R \) and \( \mathbb{D} \) is the configuration \( \langle d', I', R', \mathbb{D} \rangle \), where \( \langle d', I', R' \rangle = \text{res}(n, I, d, \theta, R, Op) \).

Continuing with our running example, let \( \langle [S, \phi], I, R, \mathbb{D} \rangle \), with \( S = \{11, 12, 7, B\} \), be the configuration resulting from the two reductions from before. Then, an execution via action of \( d \) from \( I \) relative to \( R \) and \( \mathbb{D} \) is the configuration \( \langle [S', \phi'], I', R', \mathbb{D} \rangle \), where \( I' = I \cup \{ \text{camMoved} \} \); set \( S' = \{12 : \text{mv}(\text{lan}(11)), 7, B\} \); formula \( \phi' \) is obtained from \( \phi \) by removing all constraints except for \( \text{lst}[11, 12] \prec 7 \), which is updated to \( \text{lst}[12] \prec 7 \); and \( R' \) is obtained from \( R \) by applying substitution \( \{\text{L/lan}\} \).

Observe that the applicability of a method (relative to the current state) is not checked at the point that it is chosen to reduce a task, but immediately before executing (for the first time) an associated primary action—which may be after performing further reductions and unordered actions. On the other hand, BDI agent programming languages such as AgentSpeak and CAN check the applicability of a relevant recipe at some point before (not necessarily just before) executing an associated primary action. Thus, in cases where the environment changes between checking the recipe’s applicability and executing an associated primary action (for the first time), and makes the recipe no longer applicable, the action will still be executed (provided, of course, the action itself is applicable). Such behaviour is not permitted by our semantics.

We now define the final notion of execution: execution via replacement, i.e., replacing the reductions being pursued for a task if they have become blocked. Intuitively, this happens when none of the primary actions in the pursued reductions are applicable, and none of the primary non-primitive tasks have a relevant method.

Formally, let \( \mathbb{D} = \{Op, Me\} \) be a domain, \( I \) a state, \( d \) a task network, and \( \langle S, D \rangle \) a reduction couple with \( S \cap \text{primary}(d) \neq \emptyset \). Then, set \( S \) is blocked in \( d \) if \( I \) relative to \( \mathbb{D} \), denoted \( \text{blocked}(S, I, D) \), if for all \( (n : t) \in S \cap \text{primary}(d) \), either \( t \) is an action and \( I \not\models \Phi(n, d, Op) \), or \( t \) is a non-primitive task and \( \text{rel}(t, D) = \emptyset \). Recall that \( S \) represents the reductions that are being pursued for a particular task (and its subtasks).

When such pursued reductions are blocked, they are replaced by an alternative relevant method-body for the task. In the definition below, we use the \( \text{fst}() \) and \( \text{lst}() \) constructs (if any) ‘inserted’ into the constraint formula by the first reduction of the task (Definition 11). Recall that these constructs represent the ‘inheritance’ of the task’s associated constraints by its descendant tasks.

**Definition 9** (Replacement). Let \( d = [S_d, \phi_d] \) be a task network, \( \langle S, D \rangle \) a reduction couple, and \( d_{\text{new}} = [S_{\text{new}}, \phi_{\text{new}}] \in D \). The replacement of (the elements of) \( S \) in \( S_d \) with \( S_{\text{new}} \) relative to \( d_{\text{new}} \) and \( d \), denoted \( \text{rep}(S, d_{\text{new}}, d) \), is the task network \([\{S_d \setminus S'\} \cup S_{\text{new}}, \psi \land \phi_{\text{new}}] \), where \( S' = S \cap S_d \), and \( \psi \) is obtained from \( \phi_d \) by (i) replacing any occurrence of (all) the task labels in \( \text{lab}(S') \)—within a \( \text{lst}() \) or a \( \text{fst}() \) expression—with the labels in \( \text{lab}(S_{\text{new}}) \), and then (ii) removing any element mentioning a task label in \( \text{lab}(S) \).

After a replacement, we need to update the set of reduction couples accordingly, by doing the same replacement in all relevant reduction couples. In the definition below, the set \( S' \) and task network \( d_{\text{new}} \) are as above.

**Definition 10** (Update). Let \( R \) be a set of reduction couples with \( \langle S, D \rangle \in R \), let \( S' \subseteq S \), and \( d_{\text{new}} \in D \). The update of \( S' \) in \( S \) with \( S_{\text{new}} \) relative to \( d_{\text{new}} \) and \( R \), denoted \( \text{upd}(S', S, d_{\text{new}}, R) \), is the set obtained from \( R' = (R \setminus \{\langle S, D \rangle\}) \cup \{\langle S', D\rangle \} \}) \) by replacing any couple \( \langle S'', D'' \rangle \) with \( \langle (S' \cup S''), D'' \rangle \cup S_{\text{new}}, D'' \rangle \), and then removing any couple that still mentions an element in \( S' \).

Finally, we combine the two definitions above to define the configuration that results from an execution via replacement. While we provide a general definition, for replacing any task’s blocked (pursued) reductions, one might instead want to, as in depth-first search, first replace a least abstract task’s blocked reductions. That is, one might want to first consider the smallest replaceable reduction couples. Formally, given a set of reduction couples \( R \), a couple \( \langle S, D \rangle \in R \) is a smallest replaceable one in \( R \), denoted \( \langle S, D \rangle \in \text{smallest}(R) \), if \( D \neq \emptyset \) and for each couple \( \langle S', D' \rangle \in R \), either (a) \( S' \supseteq S \); (b) \( S' \subset S \) and \( D' = \emptyset \); or (c) \( S \cap S' = \emptyset \).

**Definition 11** (Execution via Replacement). Let \( \mathbb{D} = \{Op, Me\} \) be a domain, \( I \) a state, \( d = [S_d, \phi_d] \) a task network, and \( R \) a set of reduction couples with \( r = [S, \{d_{\text{new}}, \ldots\}] \in R \) such that

\(^7\)We also remove from \( \phi' \) any (remaining) constraint of the form \((x, l, x')\) such that \( n \) occurs in \( x' \), i.e., a between state-constraint that holds trivially.
blocked\((S, d, I, D)\) holds. An execution via replacement of \(d\) from \(I\) relative to \(R\) and \(D\) is the configuration 
\[
(rep(S, d_{new}, d), I, upd(S \cap S_d, S, d_{new}, R), D);
\]
the replacement is complete if \(S \subseteq S_d\) and partial otherwise, and a jump if \(r \notin \text{smallest}(R)\).

A complete-replacement represents the BDI-style searching of an achievement-goal’s (i.e., a task’s) set of relevant recipes in order to find one that is applicable, and a partial-replacement represents BDI-style recovery from the failure to execute (or successfully execute) an action, e.g. due to an environmental change. We illustrate these notions of replacement with the following examples.

Continuing with our running example, let \(\langle [S, \phi], I, R, D \rangle\) be the configuration resulting from the two reductions from before. Suppose however that the rover’s instruments were not calibrated, i.e., \(I \neq \text{cal}\). Then, action (11 : \text{mvC}) is not applicable, and an execution via complete-replacement is performed on tasks in \([S, \phi]\) to obtain configuration \(\langle [S', \phi'], I, R', D\rangle\), where \(S' = S'' \cup \{7, B\}\); set \(S'' = \{8, 9, 10\}\); formula \(\phi'\) is the conjunction of \(\phi_d\) and \(\phi\) updated by, e.g. removing the constraints that were copied from \(\phi_b\) and replacing constraint \((\text{landr}(L), \text{fat}[11, 12])\) with \((\text{landr}(L), \text{fat}[8, 9, 10])\), and the set of couples \(R' = \{\langle S', \emptyset \rangle, \langle S'' \cup \{7, \{d_1\}\} \rangle, \langle S', \emptyset \rangle\}\).

Suppose we now perform two executions via action to obtain configuration \(\langle [S'''', \phi'''], I', R'', D\rangle\), with \(S''' = \{10 : \text{mv(land1)}\}, 7, B\) and formula \(\phi''\) (resp. set \(R''\)) being \(\phi'\) (resp. \(R'\)) updated to account for the executions. Finally, suppose that the battery level drops due to the execution of top-level image processing action \((B : \text{proclmg})\), which makes \(\text{mv(land1)}\) no longer applicable. We shall show later how \text{proclmg} could instead be absent in the initial task network and arrive ‘dynamically’ from the environment.) Then, an execution via partial-replacement will be performed on tasks in \([S'''', \{\langle B : \text{proclmg}\rangle, \phi''\}\) to obtain configuration \(\langle \{1, 2, 3\}, \phi'''', I'', R''', D\rangle\), where \(\phi'''\) (resp. \(I''\)) is the updated \(\phi''\) (resp. \(I''\)) and the set \(R''' = \{\langle 8, 9, 1, 2, 3, B\rangle, \emptyset\}, \{\langle 8, 9, 1, 2, 3\rangle, \emptyset\}\).

5 Properties of the Formalism

In this section, we discuss the properties of our formalism, and in particular how it relates to HTN planning.

The properties are based on the definition of an execution trace, which formalises the consecutive execution of a configuration—via reduction, replacement, or action—as in our running example. In what follows, we use \(\tau \in \text{exec}(d, I, R, D)\) to denote that a configuration \(\tau\) is an execution via reduction, action, or replacement of a task network \(d\) from a state \(I\) relative to a set of reduction couples \(R\) and a domain \(D\).

Definition 12 (Execution Trace). Let \(d = [S_d, \phi_d]\) be a task network, \(I\) a state, and \(D\) a domain. An execution trace \(\tau\) of \(d\) from \(I\) relative to \(D\) is any sequence of configurations \(\tau_1, \ldots, \tau_k\), with each \(\tau_i = \langle d_i, I_i, R_i, D\rangle\), such that \(d_1 = d\), \(I_1 = I\); \(R_1 = \{\langle S_d, \emptyset\rangle\}\); and \(\tau_{i+1} \in \text{exec}(d_i, I_i, R_i, D)\) for all \(i \in [1, k-1]\).

We also need some auxiliary definitions related to execution traces. Consider configuration \(\tau_k\) above. First, if \(S_k = \emptyset\) (where \(d_k = [S_k, \phi_k]\)), then the trace is successful. Second, if for all couples \((S, D) \in R_k\) we have that \(S \cap \text{primary}(d_k) \neq \emptyset\) entails both

\[\text{blocked}(S, d_k, I_k, D)\] holds. Then, the trace is blocked. The following theorem states that if a trace is successful or blocked as we have “syntactically” defined, then there is no way to ‘extend’ the trace further, and vice versa.

Proposition 1. Let \(\tau\) be an execution trace of a task network \(d\) from a state \(I\) relative to a domain \(D\). There exists an execution trace \(\tau = \tau_1 \cdot \ldots \cdot \tau_k\), with \(k > 0\), of \(d\) from \(I\) relative to \(D\) if and only if \(\tau\) is neither successful nor blocked. The inverse also holds.

Proof. If there exists a trace \(\tau = \tau_1 \cdot \ldots \cdot \tau_k\) with \(k > 0\) then \(\tau\) cannot be successful as its final task network \(d_k = [S_k, \phi_k]\) would then not mention any tasks, and thus we cannot ‘extend’ it to \(\tau_1\). The fact that \(\tau\) cannot be blocked follows from the fact that an execution via replacement, action, or reduction of \(d_k\) is possible. Conversely, if \(\tau\) is neither successful nor blocked, then the only reason it would not be possible to ‘extend’ it is if \(S_k \neq \emptyset\) but \(\text{primary}(d_k) = \emptyset\). However, this is only possible if a method-body exists where its constraint formula contains inconsistent (possibly negated) ordering constraints. Such method-bodies are not allowed due to our assumption in Section 3. The inverse of the theorem is similar.

\[\square\]

The next three properties rely on traces that are free from certain kinds of execution. A trace \(\tau = \tau_1 \cdot \ldots \cdot \tau_k\) is complete-replacement free if there does not exist an index \(i \in [1, k-1]\) such that \(\tau_{i+1}\) is an execution via complete-replacement of \(d_i\) from \(I_i\) relative to \(R_i\) and \(D\). We define partial-replacement free and jump-free traces similarly.

Given any execution trace, the next theorem states that there is an equivalent one—in terms of actions performed—that is complete-replacement free. Intuitively, this is because, either with some ‘lookahead’ mechanism or ‘luck’, a complete-replacement can be avoided by choosing a different (or ‘correct’) relevant method-body for a task. We define the actions performed by a trace \(\tau = \tau_1 \cdot \ldots \cdot \tau_k\) (or the pursued ‘solution’), denoted \(\text{act}(\tau)\), as follows. Given an index \(i \in [1, k-1]\), we first define \(\text{act}(i) = t\) if \(S_i \setminus S_{i+1} = \{(n : t)\}\) and \(\tau_{i+1}\) is an execution via action of \(d_i\) from \(I_i\) relative to \(R_i\) and \(D\); otherwise, we define \(\text{act}(i) = \emptyset\). Then, \(\text{act}(\tau) = \text{act}(1) \cdot \ldots \cdot \text{act}(k-1)\) with substitution \(\theta\) of configuration \(\tau_\theta\) applied to the sequence.

Theorem 1. Let \(\tau\) be an execution trace of a task network \(d\) from a state \(I\) relative to a domain \(D\). There exists a complete-replacement free execution trace \(\tau'\) of \(d\) from \(I\) relative to \(D\) such that \(\text{act}(\tau) = \text{act}(\tau')\) and \(|\tau'| \leq |\tau|\).

Proof. Without loss of generality, we use a slightly modified version of Definition 2 that stores also the unique task label that was reduced, i.e., we add tuple \(\langle n, S_n, D \setminus \{d_n\} \rangle\) to \(R'\) instead of the one that is currently added in the definition. Then, given a tuple \(\tau_i = \langle d_i, I_i, R_i, D\rangle\) occurring in the above execution trace \(\tau = \tau_1 \cdots \tau_k\), with a tuple \(\langle n, S, D \rangle \in R_i\), we say that the set \(S'\) is an evolution of \(n\) (relative to the trace and \(i\)), denoted \(S' \in \text{evol}(n, \tau_1 \cdots \tau_k, i)\), if \((n, S', D') \in R_i\) for some \(D'\) and \(i \leq j < k\).

Consider the smallest \(0 < m < k\) such that \(\tau_{m+1}\) (with each \(\tau_i = \langle d_i, I_i, R_i, D\rangle\) and \(D = \emptyset\)) is an execution via complete replacement of task network \(d_m\) from \(I_m\) relative to \(R_m\) and \(D\). If there is no such \(\tau_m\) then the theorem holds trivially; otherwise, \(\tau_{m+1} = \text{rep}(S, d', d_m)\) for some \(\langle S, d', D \rangle \in R_m\) and \(d' \in D\).

Consider prefix \(\tau_1 \cdots \tau_i\) with the smallest \(i < m\) such that the ‘incorrect’ reduction was performed at \(i\), i.e., where \(d_{i+1} =
\([S_{i+1}, \phi_{i+1}]\) is an execution via reduction of \(d_i\) from \(I_i\) relative to \(R_i\) and \(D\), and \((\hat{w}, S_{I+1}^R, D) \in R_{i+1}\) (for some \(S_{I+1}^R\) and \(D\)) but \((\hat{w}, S_i^R, D) \notin R_i\) (for any \(S_i^R\)), where \(S_{I+1}^R\) is a set of ‘ancestors’ of \(S\), i.e., \(S_{I+1}^R \in \text{evol}(\hat{w}, T, i+1)\).

Let \(d_{i+1} = \text{red}(d_i, m, \hat{d})\) for some \((m, \hat{d}) \in S_i\) and \(d \in \text{rel}(t, D)\), i.e., \(d\) is the ‘incorrect’ reduction. Suppose instead that the ‘correct’ one was performed on \(d_i\), i.e., let tuple \(\tau'_{i+1} = \langle d_{i+1}^\prime, T_{i+1}, R_{i+1}', \hat{d} \rangle\) be an execution via reduction of \(d_i\) from \(I_i\) relative to \(R_i\) and \(D\) such that \(d_{i+1}^\prime = \text{red}(d_i, m, \hat{d}')\) where \(d'\) is from earlier. We now show that all executions performed from \(\tau_{i+1}\) up to \(\tau_m\) (which do not involve complete-replacements) can also be performed from \(\tau'_{i+1}\).

Suppose that there is at least one such execution, i.e., \(i+1 < m\). Then \(\tau_{i+1}\) is an execution via reduction, partial-replacement or action of \(d_{i+1}\) from \(I_{i+1}\) relative to \(R_{i+1}\) and \(D\). Let \(\text{lab}(S_{i+1}^R) \subset \text{lab}(S_{i+1})\) be the task that was executed or reduced, or the tasks that were replaced, i.e., the largest set such that \(\text{lab}(S_{i+1}^R) \cap \text{lab}(S_{i+1}) = \emptyset\); in the case of an execution via reduction or partial-replacement, let \(\text{lab}(S_{i+1}^R) \subset \text{lab}(S_{i+1})\) be the new tasks, i.e., the largest set such that \(\text{lab}(S_{i+1}^R) \cap \text{lab}(S_{i+1}) = \emptyset\). If \(\text{lab}(S_{i+1}^R) \subset \text{lab}(S_{i+1})\), i.e., the execution is ‘relevant’ to \(d'_{i+1}\) and the execution is not a reduction of some ‘descendant’ of \(S_{i+1}\), we then show that there exists also a corresponding tuple \(\tau'_{i+1}\) that is an execution via reduction, partial-replacement or action of \(d_{i+1}'\) from \(I_{i+1}\) relative to \(R_{i+1}\) and \(D\), such that \(\text{lab}(S_{i+1}^R) \cap \text{lab}(S_{i+1}') = \emptyset\), and in the case of an execution via reduction or partial-replacement, \(\text{lab}(S_{i+1}^R) \subset \text{lab}(S_{i+1})\) and \(\text{lab}(S_{i+1}) \cap \text{lab}(S_{i+1}) = \emptyset\).

There are two main cases to consider: an execution via action and partial-replacement.

In the case of an execution via partial-replacement, observe from Definition [1] that all tasks in \(S_{i+1}\) are blocked in \(d_{i+1}\) from \(I_{i+1}\) relative to \(D\). The same applies to \(S_{i+1}^R\) in \(d_{i+1}^R\) from \(I_{i+1}\) relative to \(D\) for the following two reasons. Consider any primitive task \((n' : t') \in S_{i+1}\) (which is not applicable in \(I_{i+1}\) relative to \(d_{i+1}\)). First, observe from Definition [2] that \(\phi_{i+1}\) and \(\phi_{i+1}'\) are identical except for the tasks and constraints that were introduced by the two different reductions of \(d_i\) above. Second, observe from Definition [5] that any constraint occurring in \(\phi_{i+1}\) and \(\phi_{i+1}'\) containing expression \(\text{fst}[n', \ldots]\) is relevant to \(n'\) irrespective of the other task labels that occur in the expression. Similarly, any constraint occurring in \(\phi_{i+1}\) and \(\phi_{i+1}'\) containing expression \(\text{lst}[n', \ldots]\) is not relevant to \(n'\) irrespective of the other task labels that occur in the expression. The same applies when \(n'\) does not occur within such expressions.

In the case of an execution via action, i.e., \(S_{i+1} = \{(n' : t')\}\) for some \((n' : t')\), \(n'\) is applicable in \(I_{i+1} = I_{i+1}\) relative to both \(d_{i+1}\) and \(d_{i+1}^R\) for the same reasons as above.

It is not difficult to see that the remaining relevant executions performed from \(d_{i+2}\) to \(d_m\) (if any)—which do not involve complete-replacements—can also be performed from \(d_{i+2}'\). Let \(T' = \tau_1 \cdots \tau_i \cdots \tau_i' \cdots \tau_{m+1}'\) be the trace corresponding to those remaining executions from \(d_{i+1}'\), appended to the prefix that ends just before the ‘incorrect’ reduction. Since the set \(S\) above represents the descendants of \(n\) (with all actions left in tact), it follows from Definitions [2] and [9] that \(\tau_i' \equiv \tau_m + 1\).

If \(m + 1 = k\), then \(T'\) is a complete-replacement free execution trace of \(d\) from \(I\) relative to \(D\), and the theorem holds. Otherwise, we first create an ‘adjusted’ copy of the suffix from index \(m + 2\) of the original trace by inserting the ‘incorrect’ task network \(d'\) (which \(T'\) did not use). Let the trace \(\tau_1 \cdots \tau_x = \tau_{m+2} \cdots \tau_k\). Then, we replace each tuple \((\hat{w}, S, D)\) occurring in \(\tau_1 \cdots \tau_x\) with tuple \((\hat{w}, S, D \cup \{d'\})\) to obtain the new trace \(T'' = T' \ast \tau_1 \cdots \tau_x\).

Finally, we can now remove the first execution via complete-replacement from \(T''\) as we did before with trace \(T\), and then continue this process (a finite number of times) with the resulting traces, until one is obtained where there is no execution via complete-replacement.

An equivalent complete-replacement free trace may, however, unavoidably specify one or more replacements that are jumps—where the smallest replaceable reduction couples were skipped. To see why this holds, consider once again our running example, but suppose that the constraints associated with \(\sim\text{lowBit}\) do not exist in \(\phi_4\) and \(\phi_5\) in Figure [1]. Suppose also that after the first reduction (of task A), task 6 is reduced using method \(m_4\) instead of \(m_5\), which means that the complete-replacement in the previous example will not occur. The resulting set of reduction couples will then contain the couple \((S, \{d_3\})\), with \(S = \{8, 9, 10\}\), instead of the couple \((S, \emptyset)\) in the previous example (after the complete-replacement was performed). Thus, after the two executions via actions of tasks 8 and 9 as before, the subsequent partial-replacement must ‘skip’ couple \((S, \{d_3\})\), which is the smallest replaceable one, and ‘jump’ to couple \((S \cup \{7\}, \{d_1\})\) in order to avoid performing \((11 : m_5C)\). Intuitively, the jump is needed to ‘mimic’ the actions yielded by the trace depicted by the previous example, which considered \(d_5\) but then removed it (via the complete-replacement) because it was not applicable. This observation is stated formally below.

**Proposition 2.** There exists a domain \(D\), state \(I\), task network \(d\), and an execution trace \(T\) of \(d\) from \(I\) relative to \(D\) such that any complete-replacement free execution trace \(T'\) of \(d\) from \(I\) relative to \(D\) is not jump free when \(\text{act}(T) = \text{act}(T')\).

**Proof.** This follows from the example above.

The next result makes the link concrete between our HTN acting formalism and HTN planning. It states that the solution yielded by any execution trace that is successful and free from partial-replacements can also be yielded via HTN planning. Conversely, given any HTN planning solution, there exists such an execution trace that yields it. The trace must be free from partial-replacements because such behaviour is specific to BDI-style recovery from runtime failure.

**Theorem 2.** Let \(D\) be a domain, \(I\) a state, and \(d\) a task network. Then, \(\sigma \in \text{sol}(d, I, D)\) if and only if there exists a partial-replacement free and successful execution trace \(T\) of \(d\) from \(I\) relative to \(D\) such that \(\sigma = \text{act}(T)\).

**Proof.** This proof relies on some auxiliary functions. First, given a task label \(n\) that appears in a sequence of task networks \(d\), we define \(f(n, d)\) (denoted \(f(n)\) when \(d\) is obvious from context) as the index \(i > 1\) in \(d\) such that \(n \in \text{lab}(S_i)\) but \(n \notin \text{lab}(S_{i-1})\), where \(S_j, \phi_j\) denotes the element at index \(j\) in \(d\); if there is no such index \(i\), take \(i = 1\).

---

8The execution cannot be via complete- or partial-replacement of a descendant of \(S_{I+1}\) either, as the first execution via complete-replacement of \(S\) happens at index \(m\).
Second, given a task label \( n \) that appears in a trace \( T \), we define \( g(n, T) \) (denoted \( g(n) \) when \( T \) is obvious from context) as the index \( i > 0 \) in \( T \) such that \( d_{i+1} \) is an execution via action of \( d_i \) from \( I_i \) relative to \( R_i \) and \( D_i \), and \( n \in \text{lab}(S_i) \) but \( n \not\in \text{lab}(S_{i+1}) \), where each \( \tau_j \in T \) is of the form \( (d_j = [S_j, \phi_j], I_j, R_j, D_j) \).

Finally, we sometimes assume that functions \( \text{sol} \) and \( \text{act} \) do not remove task labels, i.e., the latter, or an element of the former, can be of the form \( (n_1 : t_1) \ldots (n_m : t_m) \).

We shall now prove each direction of the theorem.

\( \implies \) Let us assume that \( \sigma \in \text{sol}(d, I, D) \). Then, from the definition of an HTN solution, there exists a sequence \( d = d_1 \ldots d_m \), with \( d = d_1 \), such that (i) for all \( i \in [1, m - 1] \), \( d_{i+1} = \text{red}(d_i = [S_i, \phi_i], n_i, d_i) \) for some \( d_i \in \text{rel}(t, D) \) and \( (n : t) \in S_i \); (ii) \( \sigma \in \text{comp}(d_1, I, D) \); (iii) \( d_1 \ldots d_f(n_1), \) with \( (n_1 : t_1) \in \sigma \) for each \( (n_i : t_i) \) denotes the element at index \( j \) in \( \sigma \), is the shortest possible sequence of reductions that yields \( n_1 \); and similarly, (iv) for any pair \( (n_i : t_i), (n_{i+1} : t_{i+1}) \in \sigma \), the sequence \( d_1 \ldots d_y \) is the shortest possible one such that \( x = f(n_i) \) and \( y = f(n_{i+1}) \), unless \( f(n_{i+1}) > f(n_i) \) (i.e., \( n_{i+1} \) is yielded in the process of yielding \( n_i \)), in which case \( y = f(n_i) \). Since the order of reductions does not matter \(^{10}\), we can always obtain such a sequence \( d \) by ‘re-ordering’ the reductions in a given sequence.

We now show that \( d \) also has a corresponding trace \( T \) as above. To this end, we first prove the following weaker theorem: there exists a partial-replacement free extraction trace of \( d \) from \( I \) relative to \( D \) such that \( \sigma = \text{act}(T) \).

We prove this by induction on the length of the prefixes of \( \sigma \). For the base case, we consider only the first action \( (q_1 : L) \in \sigma \). Let \( \tau_1 \ldots \tau_k \), with each \( \tau_i = (d_i = [S_i, \phi_i], I_i, R_i, D_i) \), be the trace of \( d_1 \) from \( I \) relative to \( R_i \) and \( D_i \), and the action of \( d_i \) on \( I_i \). Then, we can take a trace \( T = \tau_1 \ldots \tau_k \) of \( \langle S', \phi', I', R', D' \rangle \), where the last tuple in the trace is an extraction via action of \( d_k \) from \( I \) (relative to \( R_k \) and \( D_k \)) such that \( I = \phi(\phi_k, n, \text{Op}) \) also holds (Definition\(^{10}\)). Any such constraints will also occur in \( \phi_k \) (though a constraint in \( \phi_k \) mentioning a \( \text{fsr}[N] \) expression with \( n \in N \) might have fewer elements in \( N \) than the constraint’s ‘evolution’ in \( \phi_m \) and no more before state-constraints can occur in \( \phi_k \) that are relevant to \( n \)). The same applies for ordering constraints associated with \( q_1 \). Then, we can take a trace \( T = \tau_1 \ldots \tau_k \) of \( \langle S'(q_1 : L), \phi', I', R', D' \rangle \), where the last tuple in the trace is an extraction via action of \( d_k \) from \( I \) (relative to \( R_k \) and \( D_k \)) such that \( \phi_k \in \text{lab}(S_k) \) but \( n \not\in \text{lab}(S') \). Thus, the weaker theorem above holds in the base case.

For the induction hypothesis, we assume that the weaker theorem holds for any prefix of \( \sigma \) of length up to \( \ell < |\sigma| \).

We now show that the weaker theorem also holds for the prefix of \( \sigma \) of length \( \ell + 1 \). Let \( (n^\ell : t^\ell) \) and \( (n^{\ell+1} : t^{\ell+1}) \) be the actions at indices \( \ell \) and \( \ell + 1 \) in \( \sigma \), respectively. Let \( j = f(n^\ell) \), and \( k = f(n^{\ell+1}) \) if \( f(n^{\ell+1}) > f(n^\ell) \), and \( k = j \) otherwise.

From the induction hypothesis, we know that there exists a trace \( T^\ell \) of \( d \) from \( I \) relative to \( R_1 = \{S_1, \emptyset\} \) and \( D \) for the prefix of \( \sigma \) of length \( \ell \). Let \( (d^\ell : t^\ell, R^\ell, D^\ell) \) be the last configuration in \( T^\ell \), and let \( x = |T^\ell| - \ell \), i.e., \( x \) is the index in \( d \) where the next reduction—immediately after yielding \( n^\ell \)—is performed. Moreover, if \( k = j \), then \( T^{\ell+1} = T^\ell \); otherwise (if \( k > j \)), let \( T^{\ell+1} = T^\ell \cdot \tau_1 \ldots \tau_{k-j} \), where for each \( i \in [1, k - j - 1] \): (i) \( \tau_i \) is of the form \( (d'_i = [S'_i, \phi'_i], R'_i, D'_i) \); (ii) \( \tau_{k-j} \) is an extraction via reduction of \( d'_j \) from \( I^\ell \) relative to \( R^\ell \) and \( D^\ell \), and in particular, \( d'_{i+1} = \text{red}(d'_i, n_{x+i}, d_{x+i}) \) and (iii) \( \tau_j \) is an extraction via reduction of \( d'_j \) from \( I^\ell \) relative to \( R^\ell \) and \( D^\ell \), and in particular, \( d'_i = \text{red}(d'_i, n_{x+i}, d_{x+i}) \).

Finally, we claim that we can take the trace \( T = T^{\ell+1} \).

\( \Longleftarrow \) Let us assume that there exists a partial-replacement free and successful execution trace \( T \) of \( d \) from \( I \) relative to \( D \). We show that \( \text{act}(T) \in \text{sol}(d, I, D) \) also holds, that is, there exists a sequence \( d_1 \ldots d_f \) such that \( d = d_1 \), \( \text{act}(T) \in \text{comp}(d_1, I, D) \) and \( d_{i+1} = \text{red}(d_i = [S_i, \phi_i], n, d) \) for some \( (n : t) \in S_i \), and \( d \in \text{rel}(I, D) \) for all \( i \in [1, j - 1] \). In other words, we need to show that \( \text{act}(T) \) is a permutation of a ground instance of \( S_i \) (with task labels removed), and that the following holds for any prefix of \( \text{act}(T) \), where for a given prefix of \( \text{act}(T) \):

\begin{enumerate}
  \item[(ii)] the prefix is executable in \( I \), i.e., the first action in the prefix is executable in \( I \), and any other action in the prefix is executable in the state resulting from executing the previous action; and
  \item[(iii)] any constraint in \( \phi_k \) that is relevant to an action or a pair of actions in the prefix is satisfied relative to the prefix and \( I \).
\end{enumerate}

Let \( \tau_1 \ldots \tau_m \) be the complete-replacement free extraction of \( T \) (Theorem\(^{11}\)). Observe that for each \( i \in [1, m - 1] \), \( \tau_{i+1} \) is an execution via action of \( d_i \) from \( I_i \) relative to \( R_i \) and \( D_i \), or \( d_{i+1} = \text{red}(d_i, n_i, d_i) \) for some \( d_i \in \text{rel}(t_i, D_i) \) and \( (n_i : t_i) \in S_i \), where \( \tau_i = (d_i = [S_i, \phi_i], I_i, R_i, D_i) \). We prove parts (ii) and (iii) by induction on the lengths \( k \) of the prefixes of the trace where \( k = g(n) \) for some \( n \).

For the base case, consider the smallest such prefix \( k = g(n) \) for some \( n \), and the sequence of task networks \( d_1 \ldots d_k \) corresponding to the trace. Then, since \( (q : L) \in \text{primary}(d_k) \) for some \( L \), and by Definition\(^{8}\), the precondition of \( \phi_k \) holds in \( I \), and any (possibly negated) before state-constraint in \( \phi_k \) that is associated

\(^{10}\)Substitution \( \theta \) must be a subset of the one used to compute \( \text{comp}(d_m, I, D) \).

\(^{11}\)Substitution \( \theta' \) must be applied to \( d_{x+i} \) to account for executions via action, where given \( (n_{x+i} : t_{x+i}) \) from before, \( t' = t_{x+i} \theta' \) and \( (n_{x+i} : t') \in S_i' \).
with \( n \) is also satisfied in \( I \), parts (ii) and (iii) above hold for prefix \( (\nu : \lambda) \) of \( \text{act}(T) \), similarly to the inductive case from before.

For the induction hypothesis, let \( T^\ell \) (resp. \( T^{\ell+1} \)) be any prefix of \( T \) of length up to \( \ell = g(\nu^c) \) (resp. \( \ell + 1 \)) for some \( \nu^c \), with \( k \leq \ell < m \). (The step that yields \( \tau_m \) must be an execution via action, as \( T \) is successful.) Let \( \sigma^\ell \) be the corresponding subplan, i.e., \( \sigma^\ell = \text{act}(T^{\ell+1}) \). Then, we assume that parts (ii) and (iii) above hold for prefix \( \sigma^\ell \).

Let \( c > 0 \) be the smallest number such that \( \ell + c = g(\nu^{c+e}) \) for some \( \nu^{c+e} \). We now show that parts (ii) and (iii) also hold for subplan \( \sigma^{\ell+e} = \text{act}(T^{\ell+e+1}) \), where \( T^{\ell+e+1} \) is the prefix of \( T \) of length \( \ell + c + 1 \).

Let \( d^\ell \) be the sequence of task networks corresponding to \( T^\ell \) and let \( d^{\ell+e} \) be the last task network in \( d^\ell \). If \( c = 1 \), then let \( d^{\ell+e} = d^\ell \). Otherwise, consider the sequence of ‘reductions’ \((n_{\ell+1}, d_{\ell+1}) \cdot \ldots \cdot (n_{\ell+c-1}, d_{\ell+c-1})\) performed in the trace \( \tau_1 \cdot \ldots \cdot \tau_m \) above. Let \( d^{\ell+e} = d^\ell \cdot d_1^{\ell+1} \cdot \ldots \cdot d_{\ell+c-1}^{\ell+1} \) where \( d_i^{\ell+1} = \text{red}(d^\ell, n_i, d_i+1, d_i+1) \) and \( d_{i+1}^{\ell+1} = \text{red}(d^{\ell+e}, n_{i+1}, d_i+1) \) for all \( i \in [1, c - 2] \). That is, we append a new sequence of task networks to the one corresponding to the induction hypothesis. Observe that parts (ii) and (iii) above hold for subplan \( \sigma^{\ell+e} = \sigma^\ell \cdot (n_{\ell+e+1}, d_{\ell+1}^{\ell+e+1}) \) for the following main reasons. First, any (possibly negated) constraint in the constraint formula \( T^{\ell+e+1} \) is that \( d^{\ell+e} \) and satisfied relative to \( \sigma^\ell \) and \( I \) is still a constraint occurring in \( d_{\ell+c-1}^{\ell+1} \) (though a possibly ‘evolved’ one—e.g. with variables replaced by constants or more elements added to the set of task labels in expressions of the form \( \text{fst}(\nu^d, \ldots) \)) that is relevant to \( d^\ell \) and satisfied relative to \( \sigma^\ell \) and \( I \). Second, \( d_{\ell+1}^{\ell+e+1} \) is executable (in the state resulting from applying \( \sigma^\ell \) to \( I \)) by Definition 8 and any before state-contraints relevant to \( \nu^{e+1} \), and after state-contraints relevant to \( \nu^d \) are satisfied for the reasons discussed in the previous inductive case. The case of between state-contraints is proved similarly.

Thus, there exists a sequence of task networks \( d_1 \cdot \ldots \cdot d_n \) as above, such that (ii) and (iii) hold for \( \text{act}(T) \). Finally, point (i) above also holds for \( \text{act}(T) \) due to \( T \) being successful.

If a trace is not free from partial-replacements, it may not be possible to obtain its solution via HTN planning (given the same inputs). A similar property exists in the CANPlan semantics: BDI-style recovery from failure enables solutions that cannot be found using CANPlan’s built-in HTN planning construct.

**Theorem 3.** There exists a domain \( D \), state \( I \), task network \( d \), and successful execution trace \( T \) of \( d \) from \( I \) relative to \( D \) such that \( \text{act}(T) \notin \text{sol}(d, I, D) \).

**Proof.** Consider the trace from our running example, up to the point where an execution via partial-replacement is performed using method \( m_1 \). If the resulting task network is successfully executed, we get the solution corresponding to the sequence of action labels \( 8 \cdot 9 \cdot 0 \cdot B \cdot 1 \cdot 4 \cdot 5 \cdot 3 \), which is not an HTN solution: for example, an HTN solution cannot contain the actions (corresponding to) both 8 and 1.

6 An Algorithm for HTN Acting

In this section we present the Sense-Reason-Act algorithm for HTN acting, which combines our formalism with the processing of exogenous events. In the algorithm we use \( S_{\text{nop}} \) to denote the initial set of tasks \( \{ (0 : \text{nop}) \} \), and \( \text{top}(R) \) to denote the (unique) set \( S \) of tasks in the ‘top level’ reduction couple, given a set of reduction couples \( R \), i.e., the couple \( (S, \text{top}(\{0\})) \). The algorithm takes the current state and HTN domain as input and continuously performs two main steps as follows.

**Step 1.** The algorithm ‘processes’ newly observed (external) tasks (if any) and inserts them as top-level tasks to a copy of the current configuration’s task network \( d \) and set of reduction couples \( R \) (lines 12 to 13), which are used to create the ‘next’ configuration.

Such tasks could be the initial requests, for example to transfer the soil data and sample and then recharge, or requests that arrive later, possibly while other tasks are being achieved. For example, task \text{proclmg} \ could be a newly observed task in the iteration following the execution of the actions corresponding to task labels 8 and 9 in method \( m_4 \) (as opposed to \text{proclmg} being an initial request). A newly observed task could also represent an exogenous event triggered by a change in the environment: for example, the arrival of primitive task \text{stormy} \ could represent the event that it has just become stormy, and it could have the add-list \{\text{isStormy}\}, which will be applied to the agent’s state when the task is executed. Given a domain \( D = (\text{Op}, \text{Me}) \), we stipulate that any newly observed task \( t \) is such that \( \text{pre}(t, \text{Op}) = \text{true} \) if \( t \) is primitive, and \( \text{rel}(t, D) \neq \emptyset \) otherwise.

**Step 2.** If one or more new tasks were indeed observed, the corresponding ‘next’ configuration is appended (line 10) to the current ‘dynamic’ execution trace, or \( d \)-trace \( T \). A \( d \)-trace is slightly different to an execution trace (Definition 12) in that the former may include tasks that are not just obtained by reduction but also dynamically from the environment. If an execution via reduction, action, or replacement is possible from the last configuration in the \( d \)-trace (line 12), the execution is then performed and the resulting configuration is appended to the trace (lines 13 and 14).

The following theorem states that any \( d \)-trace produced by the algorithm is sound, i.e., any such \( d \)-trace, which may include new tasks observed over a number of iterations, is equivalent to some (standard) execution trace such that all of those tasks are present in the first configuration, but their execution is ‘postponed’.

**Theorem 4.** Let state \( I_{\text{ini}} \) and domain \( D \) be the inputs of algorithm Sense-Reason-Act. Let \( T \) and \( T \) be the values of variables \( T \) and \( T \), respectively, on reaching line 16 in the algorithm (after one or more iterations). Then, \( \text{act}(T) = \text{act}(T) \) for some ex-
The execution trace $\mathcal{T}$ of task network $[\mathcal{T}]$ found from $\mathcal{I}_{in}$ relative to $\mathcal{D}$.

Proof. D-trace $\mathcal{T}$ in the algorithm, which is incrementally built, is similar to an execution trace, except for (i) the initial ‘empty’ task network of $\mathcal{T}$; and (ii) the task networks appended in line 10 to account for newly observed tasks. We obtain an execution trace $\mathcal{T}^{(11)}$ as follows: take the last element $\tau_i \in \mathcal{T}^{(11)}$ (each $\tau_k = (\langle S_0, R_k, D_k \rangle)$ such that $S_j \subset S_{j+1}$, i.e., there are newly observed tasks in $S_{j+1}$; remove $\tau_{j+1}$ from $\mathcal{T}^{(11)}$ and add the elements in $S_{j+1} \setminus S_j$ to each $S_i$ and $top(R_i)$, for $i \in [1, j]$; and repeat these steps on the resulting d-traces until an execution trace is obtained.

To see why ‘propagating’ tasks up a d-trace as above does not make the latter invalid, consider a tuple $\tau_j$ (with $j > 0$) in the original $\mathcal{T}^{(11)}$ such that $S_j \subset S_{j+1}$. Let us now add any task $(n : t) \in S_{j+1} \setminus S_j$ to $S_i$ and $top(R_i)$ for some $i < j$. Since no constraints are added to $\phi$ and no existing ones in $\phi$ are modified, any other task $(n' : t') \in S_i$ (with $n' \neq n$) that can (resp. cannot) be executed (from $\mathcal{I}_i$ relative to $R_i$ and $\mathcal{D}$) when $(n : t)$ is not in $S_i$, still can (resp. cannot) be executed when $(n : t)$ is in $S_i$.

\section{Discussion and Future Work}

While some implementations of HTN acting frameworks do exist in the literature, this paper has, for the first time, provided a formal framework, by using the most general HTN planning syntax and building on the core of its semantics. In doing so, we have carried over some of the advantages of the HTN planning formalism, such as the ability to flexibly interleave the actions associated with a method \cite{5}, and to check a method’s applicability immediately before first executing an action. We have also compared HTN acting to HTN planning, and to a BDI agent programming language.

We could now explore adding a ‘controlled’ and ‘local’ account of HTN planning into HTN acting. The result should be a similar semantics to CANPlan, which allows a BDI agent to perform HTN planning but only from user-specified points in a hierarchy. One approach might be, given a ground non-primitive task $t$, to use the construct Plan($t$) to indicate that HTN planning (as opposed to an arbitrary reduction) must be performed on $t$, and to define the new notion ‘execution via HTN planning’. Given a current configuration $\langle d, \mathcal{I}, R, \mathcal{D} \rangle$ with task network $d = \{\{(n : Plan(t)), \ldots, \phi\}$, the definition would, for example, check whether there exists a ground instance $d'_{i}$ of a method-body $d_i \in \text{rol}(t, \mathcal{D})$ such that $\text{sol}(d'_{i}, \mathcal{I}, \mathcal{D}) \neq \emptyset$ holds (defined in Section 2).

We could also investigate an improved semantics where a ‘tried’ method-body is re-tried to achieve a task. Recall that when a relevant method-body is selected to reduce a task, and the body turns out to be ‘non-applicable’ (i.e., it is unable to execute any of its tasks) in the current state, we consider the body to have been ‘tried’, in the same way that we consider a body to have been tried if it fails (becomes blocked) during execution, e.g. due to an environmental change. To enable re-trying a body that was not applicable in an earlier state, we should at the least be able to check whether that state is different to the current one (both of which are sets of ground atoms). Ideally, however, we should also be able to quickly check (in polynomial time) whether the conditions that differ between the two states are likely to make the method-body applicable. To enable re-trying a method-body that had failed, we could explore techniques for analysing the conditions responsible for the failure in order to check that they no longer hold, as suggested in \cite{10}.

\begin{thebibliography}{10}

\bibitem[1]{1} Ron Alford, Pascal Bercher, and David W. Aha. Tight bounds for HTN planning with task insertion. In \textit{Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI)}, pages 1502–1508, 2015.

\bibitem[2]{2} Ron Alford, Vikas Shivashankar, Mark Roberts, Jeremy Frank, and David W. Aha. Hierarchical planning: Relating task and goal decomposition with task sharing. In \textit{Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI)}, pages 3022–3029, 2016.

\bibitem[3]{3} Kim Bauters, Weiru Liu, Jun Hong, Carles Sierra, and Lluís Godo. CAN(PLAN)+: extending the operational semantics of the BDI architecture to deal with uncertain information. In \textit{Proceedings of the Conference on Uncertainty in Artificial Intelligence (UAI)}, pages 52–61, 2014.

\bibitem[4]{4} Giuseppe Di Giacomo, Yves Lespérance, Hector J. Levesque, and Sebastian Sardina. IndiGolog: A high-level programming language for embedded reasoning agents. In Rafael H. Bordini, Mehdi Dastani, Jürgen Dix, and Amal El Fallah-Seghrouchni, editors, \textit{Multi-Agent Programming: Languages, Platforms and Applications}, chapter 2, pages 31–72. Springer, 2009.

\bibitem[5]{5} Lavindra de Silva. BDI agent reasoning with guidance from HTN recipes. In \textit{Proceedings of the International Conference on Autonomous Agents and Multi-Agent Systems (AAMAS)}, pages 759–767, 2017.

\bibitem[6]{6} Lavindra de Silva, Anthony Dekker, and James Harland. Planning with time limits in BDI agent programming languages. In \textit{Proceedings of the Australasian Symposium on Theory of Computing (CATS)}, pages 131–139, 2007.

\bibitem[7]{7} Lavindra de Silva and Lin Padgham. A comparison of BDI based real-time reasoning and HTN based planning. In \textit{Proceedings of the Australian Joint Conference on AI (AI)}, pages 1167–1173, December 2004.

\bibitem[8]{8} Olivier Despouys and François Felix Ingrand. Propice-Plan: Toward a unified framework for planning and execution. In \textit{Proceedings of the European Conference on Planning (ECP)}, volume 1809 of \textit{Lecture Notes in Computer Science (LNCS)}, pages 278–293. Springer, 1999.

\bibitem[9]{9} Kutluhan Erol, James Hendler, and Dana S. Nau. HTN planning: Complexity and expressivity. In \textit{Proceedings of the National Conference on Artificial Intelligence (AAAI)}, pages 1123–1128, 1994.

\bibitem[10]{10} Kutluhan Erol, James A. Hendler, and Dana S. Nau. Semantics for hierarchical task-network planning. Technical Report CS-TR-3239, UMIACS-TR-94-31, Computer Science Dept., University of Maryland, College Park, MD, USA, 1994.

\bibitem[11]{11} Kutluhan Erol, James A. Hendler, and Dana S. Nau. Complexity results for HTN planning. \textit{Annals of Mathematics and Artificial Intelligence}, 18(1):69–93, 1996.

\end{thebibliography}
[12] Thomas Geier and Pascal Bercher. On the decidability of HTN planning with task insertion. In Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI), pages 1955–1961, 2011.

[13] Michael P. Georgeff and Francois Felix Ingrand. Decision making in an embedded reasoning system. In Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI), pages 972–978, 1989.

[14] Malik Ghallab, Dana Nau, and Paolo Traverso. Automated Planning and Acting. Cambridge University Press, New York, NY, USA, 1st edition, 2016.

[15] Malik Ghallab, Dana S. Nau, and Paolo Traverso. Automated Planning: Theory and Practice. Morgan Kaufmann Publishers Inc., 2004.

[16] Koen V. Hindriks, Frank S. De Boer, Wiebe Van der Hoek, and John-Jules Ch. Meyer. Agent Programming in 3APL. Autonomous Agents and Multi-Agent Systems, 2(4):357–401, 1999.

[17] Subbarao Kambhampati, Amol Dattatraya Mali, and Biplov Srivastava. Hybrid planning for partially hierarchical domains. In Proceedings of the National Conference on Artificial Intelligence (AAAI), pages 882–888, 1998.

[18] John W. Lloyd. Foundations of Logic Programming. Springer, second edition, 1987.

[19] Felipe Meneguzzi and Lavindra de Silva. Planning in BDI agents: A survey of the integration of planning algorithms and agent reasoning. Knowledge Engineering Review, 30(1):1–44, 2015.

[20] Dana S. Nau, Yue Cao, Amnon Lotem, and Héctor Muñoz-Avila. SHOP: Simple hierarchical ordered planner. In Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI), pages 968–973, 1999.

[21] Dana S. Nau, Héctor Muñoz-Avila, Yue Cao, Amnon Lotem, and Steven Mitchell. Total-order planning with partially ordered subtasks. In Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI), pages 425–430, 2001.

[22] Anand S. Rao. AgentSpeak(L): BDI agents speak out in a logical computable language. In Proceedings of the European Workshop on Modeling Autonomous Agents in a Multi-Agent World (Agents Breaking Away), volume 1038 of Lecture Notes in Computer Science (LNCS), pages 42–55. Springer, 1996.

[23] Anand S. Rao and Michael P. Georgeff. Modeling rational agents within a BDI-architecture. In Proceedings of the International Conference on Principles of Knowledge Representation and Reasoning (KR), pages 473–484, 1991.

[24] Sebastian Sardina, Lavindra de Silva, and Lin Padgham. Hierarchical planning in BDI agent programming languages: A formal approach. In Proceedings of the International Conference on Autonomous Agents and Multi-Agent Systems (AAMAS), pages 1001–1008, May 2006.

[25] Sebastian Sardina and Lin Padgham. A BDI agent programming language with failure recovery, declarative goals, and planning. Autonomous Agents and Multi-Agent Systems, 23(1):18–70, 2011.

[26] Vikas Shivashankar, Ron Alford, Ugur Kuter, and Dana Nau. The GoDeL planning system: A more perfect union of domain-independent and hierarchical planning. In Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI), pages 2380–2386, 2013.

[27] Katia Sycara, Massimo Paolucci, Martin Van Velsen, and Joseph Giampapa. The RETSINA MAS infrastructure. Autonomous Agents and Multi-Agent Systems, 7(1-2):29–48, 2003.

[28] Jacobo Valdes, Robert E. Tarjan, and Eugene L. Lawler. The recognition of series parallel digraphs. In Proceedings of the Annual ACM Symposium on Theory of Computing, pages 1–12, 1979.

[29] Michael Winikoff. Jack™ intelligent agents: An industrial strength platform. In Rafael H. Bordini, Mehdi Dastani, Jürgen Dix, and Amal El Fallah Seghrouchni, editors, Multi-Agent Programming: Languages, Platforms and Applications, pages 175–193. Springer US, 2005.

[30] Michael Winikoff, Lin Padgham, James Harland, and John Thangarajah. Declarative & procedural goals in intelligent agent systems. In Proceedings of the International Conference on Principles of Knowledge Representation and Reasoning (KR), pages 470–481, 2002.

[31] Zhanhao Xiao, Andreas Herzig, Laurent Perrussel, Hai Wan, and Xiaoheng Su. Hierarchical task network planning with task insertion and state constraints. In Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI), pages 4463–4469, 2017.