Production of the $\eta_1(1855)$ and its possible C-parity partner via kaon induced reactions

Xiao-Yun Wang,$^{1,2,*}$ Fan-Cong Zeng,$^1$ and Xiang Liu$^{3,4,2,5,†}$

$^1$Department of physics, Lanzhou University of Technology, Lanzhou 730050, China
$^2$Lanzhou Center for Theoretical Physics, Key Laboratory of Theoretical Physics of Gansu Province, Lanzhou University, Lanzhou, Gansu 730000, China
$^3$School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China
$^4$Research Center for Hadron and CSR Physics, Lanzhou University and Institute of Modern Physics of CAS, Lanzhou 730000, China
$^5$Frontier Science Center for Rare Isotopes, Lanzhou University, Lanzhou 730000, China

By the reaction of kaon interacting with a proton, we investigate the production of the newly observed $\eta_1(1855)$ and its C-parity partner $\eta_1'(1855)$ predicted in the picture of the $KK_1(1400)$ molecular state. The total and differential cross sections for the concrete $K^-p \rightarrow \eta_1(1855)\Lambda$ and $\eta_1'(1855)\Lambda$ reactions are obtained. At the center of mass energies about 3.4–3.6 GeV, the minimum cross-section of $\eta_1(1855)$ and $\eta_1'(1855)$ production via the $K^-p$ reaction are about 0.59 nb and 19.2 nb, respectively. The differential cross sections for both reactions at the different center of mass energies are also available. Furthermore, we present the Dalitz processes of $2 \rightarrow 3$ and $2 \rightarrow 4$, and initially discuss the feasibility of finding out the $\eta_1(1855)$ and $\eta_1'(1855)$ in experiments like J-PARC.

I. INTRODUCTION

Until now, the BESIII experiment has collected the world largest data sample of $J/\psi$, which provide good platform to carry out the study of light hadron spectrum [1]. Recently, by analyzing the partial wave of the $J/\psi \rightarrow \eta \eta'$ decay, the BESIII Collaboration [2, 3] reported the observation of the $\eta_1(1855)$ in the $\eta \eta'$ invariant mass spectrum with a significance 19$\sigma$, which has quantum number $I^G(J^{PC}) = 0^+(1^{++})$. And its mass and width are

$$m = 1855 \pm 9^{+6}_{-0} \text{ MeV}, \quad \Gamma = 188 \pm 18^{+6}_{-0} \text{ MeV}, \quad (1)$$

respectively. Obviously, the observed $\eta_1(1855)$ cannot be grouped into the conventional hadron, which is a good candidate of exotic states as indicated by some theoretical groups, where the $\eta_1(1855)$ was interpreted as a hybrid state [4–6], a molecular state [7, 8], and a tetraquark state [9].

At present, the $\eta_1(1855)$ was only observed in the radiative decay $J/\psi \rightarrow \gamma \eta \eta'$, confirming this observation by others processes is an interesting experimental topic, which will be helpful to establish this exotic state in experiment and may provide some valuable information to reflect its inner structure. Under different assignments to the $\eta_1(1855)$ [4, 7], the strong decays of the $\eta_1(1855)$ were studied, where the $\eta_1(1855)$ has strong interaction with the $K^{(*)}K^{(*)}$ channels. This result inspires us to propose that the reaction of Kaon and proton can be applied to study the production of the $\eta_1(1855)$.

If checking the Particle Data Group [10], we may find that the reaction of kaon and proton is an ideal process to explore light hadron spectrum. Taking the $\phi(1020)$ as example, the $\phi(1020)$ was firstly found in the $K^-p \rightarrow \Lambda K\bar{K}$ reaction [11]. When investigating the production problem of the $\eta_1(1855)$, we naturally pay attention to the $Kp$ reaction, which is an effective approach among these possible production processes. There are available experiments at OKA@U-70 [12] and SPS@CERN [13], and J-PARC [14, 15]. For answering whether the $\eta_1(1855)$ can be accessible at these facilities, we should have a realistic study of the production of the $\eta_1(1855)$ via the $K^-p$ reaction.

In this work, we calculate the production of $K^-p \rightarrow \eta_1(1855)\Lambda$ with a $t$-channel $K/K^*$ exchange, where the effective Lagrangian approach and Regge model are adopted. In the following section, the details will be given. Besides studying the production of the $\eta_1(1855)$, we also explore the production of the predicted $\eta_1'(1855)$ through $K^-p \rightarrow \eta_1'(1855)\Lambda$. In the picture of the $KK_1(1400)$ molecular state [7], as the C-parity partner of the $\eta_1(1855)$, the $\eta_1'(1855)$ with $J^{PC} = 1^{--}$ was predicted. Thus, searching for the predicted $\eta_1'(1855)$ can be as a test of molecule explanation to the $\eta_1(1855)$ [7].

This paper is organized as follows. After introduction, we present formalism including Lagrangians and amplitudes of the $\eta_1(1855)$ and $\eta_1'(1855)$ production in Sec. II. The numerical results are discussed in Sec. III, followed by a brief summary in Sec. IV.

II. THE PRODUCTION OF THE $\eta_1(1855)$ AND $\eta_1'(1855)$ VIA THE $Kp$ REACTION

The exotic states $\eta_1(1855)$ ($\equiv \eta_1$) and $\eta_1'(1855)$ ($\equiv \eta_1'$) can be produced via the $Kp$ reaction, where the diagram is shown in Fig. 1 when the $t$-channel exchange of the $K/K^*$ mesons is considered. In the present work, the contribution from $s$-channel with nucleon pole is not considered since such contribution is negligibly small [16]. In addition, since the coupling strength of $\eta_1(NN)$ is unknown, and the Reggeized $u$-channel with nucleon exchange usually has a small contribution to the cross section [16], therefore, the contributions from the $u$-channel with nucleon exchange is also not considered in this work.

For kaon induced production of the $\eta_1(1855)$ and its C-parity partner $\eta_1'(1855)$, the relevant Lagrangians for the $t$-
channel read as below [17–20]

\[
\begin{align*}
\mathcal{L}_{\eta^i KK} &= -ig_{\eta^i KK}[(\bar{q}q)K - (\bar{p}p)K] \eta^{i(\mu)}, \\
\mathcal{L}_{K\Lambda} &= i g_{K\Lambda} \bar{N} \gamma_5 \Lambda K + \text{H.c.}, \\
\mathcal{L}_{\eta^i K*K} &= \frac{g_{\eta^i K*K}}{m_{\eta^i}} \varepsilon_{\mu\nu\rho\sigma} \bar{q}^{\mu} \gamma^\rho K^\nu \eta^{i(\sigma)}, \\
\mathcal{L}_{\Lambda K^*\eta^i} &= -g_{\Lambda K^*\eta^i} \bar{N} \left( K^* - \frac{m_{\Lambda K^*}}{2m_N} \sigma_{\mu\nu} \gamma^\nu \gamma^\rho \eta^{i(\rho)} \right) + \text{H.c.},
\end{align*}
\]

where \( \varepsilon_{\mu\nu\rho\sigma} \) is the Levi-Civita tensor. \( \eta, \eta', K, K^*, N \) and \( \Lambda \) stand for the fields of the \( \eta(1585) \), \( \eta'(1585) \), \( K, K^* \) meson, nucleon and \( \Lambda \), respectively. The coupling constant \( g_{K\Lambda} = -13.24 \) can be determined [17] by the SU(3) flavor symmetry relation [21, 22]. Moreover, in the Nijmegen potential [23], the calculated results show that the values of the coupling constants \( g_{K^*\eta^i} \) and \( \kappa_{K^*\Lambda} \) are -4.26 and 2.66, respectively. The partial widths of the \( \eta(1585) \) and \( \eta'(1585) \) decays to \( KK \) and \( K^*K \) are given in Ref. [7], by which one can calculate the corresponding coupling constants \( g_{\eta KK} \approx 0 \), \( g_{\eta' KK} \approx 0.26 \), \( g_{\eta K*K} \approx 0.73 \) and \( g_{\eta' K*K} \approx 0.77 \).

### Table I. The partial decay widths of the \( \eta(1585) \) under the hybrid [4] and molecular [7] pictures, and the partial decay widths of the predicted \( \eta'(1585) \) in the molecular explanation [7]. Here, the unit is MeV for the listed partial widths.

| Channels | \( \eta(i_{PC} = 1^{++}) \) | \( \eta'(i_{PC} = 1^{++}) \) |
|----------|----------------|----------------|
| \( K^*K \) | 26.3 | 38.1 |
| \( KK \) | 0 | 0.5 |
| \( K^*K \) | 98.1 | 0.9 | 1.0 |
| \( a_1 \pi \) | 9.2 | 0 |
| \( f_1 \eta \) | 0.2 | 0 |
| \( \eta' \) | 26.9 | 0 |
| \( J^* \omega \) | 0 | 0.2 |
| \( \rho \) | 0.04 | 0 |
| \( \pi \) | 0 | 6.4 |
| \( \eta \) | 0 | 0.4 |
| \( \omega \) | 0.01 | 0 |
| \( \omega' \) | 0.4 | 0 |
| \( KK^* \pi \) | 105.0 | 130.0 |

With the above Lagrangians, the amplitude of the \( \eta(1585) \) and \( \eta'(1585) \) production via \( t \)-channel \( K \) or \( K^* \) exchange in the \( K^*-p \) scatterings can be written as

\[
i M_K = g_{\eta KK}^i g_{K\Lambda} F(q^2) \frac{m_N}{2m_N} \frac{\Gamma_\eta(2\eta)}{m_N^2} \frac{1}{t - m_K^2} \times (k_{1\mu} + q_\mu) u_N(p_1), \tag{6}
\]

\[
i M_{K^*} = g_{\eta' K*K}^i g_{K^*\Lambda} F(q^2) \frac{m_N}{2m_N} \frac{\Gamma_{\eta'}(2\eta')}{m_N^2} \frac{1}{t - m_{K^*}^2} \times \varepsilon_{\eta' K} \cdot (k_{1\mu} + q_\mu) u_N(p_1), \tag{7}
\]

where \( \varepsilon_{\eta' K} \) is the polarization vector of the \( \eta(1585) \) and \( \eta'(1585) \), and \( \bar{u}_N \) or \( m_N \) is the Dirac spinor of nucleon or \( \Lambda \) baryon. For the \( r \) channel meson exchange [19], the form factor \( F(q^2) = (\Lambda^2 - m^2)/(\Lambda^2 - q^2) \) is adopted. Here, \( t = q^2 = (k_1 - k_2)^2 \) is the Mandelstam variables. The cutoff \( \Lambda \) in the form factor is the only free parameter. In Sec. III, we discuss how to fix it when presenting the results.

Usually, the Regge trajectory model is successfully applied to analyze the hadron production at high energy [16, 17, 24–28]. The Reggeization can be done by replacing the \( t \)-channel propagator in the Feynman amplitudes (Eqs. (6)-(7)) with the Regge propagator

\[
\frac{1}{t - m_K^2} \rightarrow \left( \frac{s}{s_{scale}} \right)^{\alpha_K(t)} \frac{\pi \alpha_K}{\Gamma(1 + \alpha_K(t)) \sin[\pi\alpha_K(t)]}, \tag{9}
\]

\[
\frac{1}{t - m_{K^*}^2} \rightarrow \left( \frac{s}{s_{scale}} \right)^{\alpha_{K^*}(t)-1} \frac{\pi \alpha_{K^*}}{\Gamma[\pi\alpha_{K^*}(t)] \sin[\pi\alpha_{K^*}(t)]}, \tag{10}
\]

The scale factor \( s_{scale} \) is fixed at 1 GeV. In addition, the Regge trajectories \( \alpha_K(t) \) and \( \alpha_{K^*}(t) \) read as [27]

\[
\alpha_K(t) = 0.70(t - m_K^2), \quad \alpha_{K^*}(t) = 1 + 0.85(t - m_{K^*}^2). \tag{11}
\]

It is necessary to note that no additional parameter is introduced after applying the Reggeized treatment.

### III. Numerical Results

#### A. Cross section

With the preparation shown in the previous section, the cross section of the \( K^- p \rightarrow \eta(1585) \Lambda \) and \( K^- p \rightarrow \eta'(1585) \Lambda \) reactions can be calculated. The differential cross section in the center of mass (c.m.) frame is written as

\[
\frac{d\sigma}{d\cos\theta} = \frac{1}{32\pi s} \frac{1}{|t_{c.m.}|} \left| \sum |M|^2 \right| \left( \frac{1}{t_{c.m.}} \right), \tag{12}
\]

where \( s = (k_1 + p_1)^2 \) is defined, and \( \theta \) denotes the angle of the outgoing \( \eta(1585) \) meson relative to the \( K \) beam direction in the
c.m. frame, \( \vec{p}_{1}^{\text{c.m.}} \) and \( \vec{p}_{2}^{\text{c.m.}} \) are the three-momenta of initial \( K \) beam and final \( \eta_{1}^{(r)} \), respectively.

Since there does not exist the experimental data for the \( K^{-}p \rightarrow \eta_{1}(1855)\Lambda \) and \( K^{-}p \rightarrow \eta_{1}(1855)\Lambda \) reaction, here we give the prediction of the cross section of these two reactions as presented in Figures 2-3. In these calculations, the cutoff parameter involved in the form factor is the only free parameter. In Ref. [20], by fitting the experimental data of the \( \pi^{+}p \rightarrow K^{+}\Sigma^{+} \) process, the \( \Lambda_{10} = 1.67 \pm 0.04 \) GeV corresponding to the contributions of Reggeized \( t \)-channel \( K^{(s)} \) exchange is obtained. Moreover, in Ref. [27], for the Reggeized \( t \)-channel with the \( K \) and \( K^{*} \) exchanges, the experimental data can be reproduced well by taking cutoff \( \Lambda_{t} = 1.55 \) GeV. Also in Ref. [29], the result indicate that \( \Lambda_{t} = 1.60 \) GeV is a reasonable value for the \( K^{-}p \rightarrow f_{1}(1420)\Lambda \) process through Reggeized \( t \)-channel \( K^{*} \) exchange. Thus, in this work one intends to take the value of cutoff for the \( t \)-channel \( K^{(s)} \) exchange to be \( 1.6 \pm 0.3 \) GeV.

In Fig. 2, we present the total cross section of the \( K^{-}p \rightarrow \eta_{1}(1855)\Lambda \) and \( K^{-}p \rightarrow \eta_{1}^{(r)}(1855)\Lambda \) reactions within the Regge trajectory model by taking \( \Lambda_{t} = 1.6 \pm 0.3 \) GeV, respectively. It is found that the line shape of the total cross sections of the \( K^{-}p \rightarrow \eta_{1}(1855)\Lambda \) and \( K^{-}p \rightarrow \eta_{1}^{(r)}(1855)\Lambda \) processes goes up very rapidly and has a peak around \( W = 3.4 \sim 3.6 \) GeV. The peak of the total cross section of the \( \eta_{1}(1855) \) and \( \eta_{1}^{(r)}(1855) \) production in the \( K \) induced reaction can reach up to about 2.5 \( nb \) and 35 \( nb \) at \( W = 3.4 \sim 3.6 \) GeV, respectively, which indicates that the \( W \in (3.4 \text{ GeV}, 3.6 \text{ GeV}) \) is the best energy window for searching for the \( \eta_{1}(1855) \) or \( \eta_{1}^{(r)}(1855) \) via the kaon induced reaction. Even if we take into account the error of the cutoff value, at the best energy window, the minimum cross section of the \( \eta_{1}(1855) \) or \( \eta_{1}^{(r)}(1855) \) production via the \( K^{-}p \) reaction are about 0.59 \( nb \) and 19.2 \( nb \), respectively. In addition, the partial width of the decay of \( \eta_{1} \) to \( KK^{*} \) given in the Ref. [4] is 98.1 MeV, and the coupling constant \( g_{\eta_{1}KK^{*}} \approx 7.6 \) is determined by this partial width, which is about an order of magnitude higher than the coupling constant obtained by using the partial width in the Ref. [7]. The corresponding cross section is also an order of magnitude higher than the current cross section of \( K^{-}p \rightarrow \eta_{1}(1855)\Lambda \) in Fig. 2. Of course, we still try to consider the small cross section value in the subsequent discussion, which is helpful for us to examine the feasibility of the experimental measurement.

In Fig. 3, one presents the prediction of the differential cross section of the \( K^{-}p \rightarrow \eta_{1}(1855)\Lambda \) and \( K^{-}p \rightarrow \eta_{1}^{(r)}(1855)\Lambda \) reactions within the Regge trajectory model by taking a cutoff \( \Lambda_{t} = 1.6 \pm 0.3 \) GeV. It can be seen that the differential cross sections of these two reactions are very sensitive to the \( \theta \) angle and show strong forward-scattering enhancements especially at higher energies. Thus, the measurement at forward angles is suggested, which can be used to check the validity of the Reggeized treatment.
and Fig. 4 shows the change of II

FIG. 4. (color online). The t-distribution for the K−p → η′(1855)Λ and K−p → η′(1855)Λ reactions at different c.m. energies W = 3.5 GeV, 3.6 GeV and 4 GeV. Here, the notations are as in Fig. 2.

FIG. 5. (color online). Same as Fig. 4 except that the c.m. energy is expanded to 6 GeV, 8 GeV and 10 GeV.

FIG. 6. (color online). The limiting value of |t|_{min} as a function of W.

son, it is found that the difference of this shape is more obvious at high energy, which is mainly caused by the limitation of |t|_{min}. Figure 6 shows the change of |t|_{min} value with energy. When the center of mass energy is 3.5 GeV and 6 GeV, the corresponding |t|_{min} values are 0.298 GeV² and 0.046 GeV², respectively. It means that at low energies, neither experiment nor theory can give values of t distributions in the region of t ~ 0.

B. Dalitz process

Note that both η′(1855) and η′(1855) are the intermediate states usually reconstructed by their decays. And from Table I, the decay widths of the η′(1855) and η′(1855) to different final states are different. For example, the partial widths of η′(1855) and η′(1855) decays to the ρπΛ and the ratio of σ(K−p → η′(1855)Λ → ρπΛ)/σ(K−p → ρπΛ), which may provide a useful reference for future experiments. Usually, the invariant mass distribution for the Dalitz process K−p → η′(1855)Λ → ρπΛ can be defined with the two-body process [30]

\[ \frac{dσ_{K^-p \rightarrow η_1(1855)Λ \rightarrow ρπΛ}}{dM_{ρπ}} \approx \frac{2M_{η_1′}M_{ππ}}{π} \frac{σ_{K^-p \rightarrow η_1′Λ}(Γ_{η_1′} \rightarrow ρπ)}{(M_{ρπ}^2 - M_{η_1′}^2)^2 + M_{ππ}^2Γ_{η_1′}^2}, \]

Simple to K−p → η′(1855)Λ → ρπΛ process, the Dalitz process K−p → η′(1855)Λ → η(548)η(958)Λ can be written as [30]

\[ \frac{dσ_{K^-p \rightarrow η_1(1855)Λ \rightarrow ρπΛ}}{dM_{ηη}} \approx \frac{2M_{η_1′}M_{ππ}}{π} \frac{σ_{K^-p \rightarrow η_1′Λ}(Γ_{η_1′} \rightarrow ρπ)}{(M_{ηη}^2 - M_{η_1′}^2)^2 + M_{ππ}^2Γ_{η_1′}^2}, \]

where the full width Γ_{η_1′} and Γ_{η_1′} are both taken as 188 MeV. For the partial width Γ_{η_1′}→η_1 and Γ_{η_1′}→η_1, one take the theoretical prediction value predicted in Ref. [7] as Γ_{η_1′}→η_1 and Γ_{η_1′}→η_1 are taken as 26.9 MeV and 6.4 MeV, respectively, as listed in Table I. Thus the invariant-mass distribution dσ_{K^-p \rightarrow η_1(1855)Λ \rightarrow ρπΛ}/dM_{ηπ} and dσ_{K^-p \rightarrow η_1′(1855)Λ \rightarrow ρπΛ}/dM_{ηπ} for W = 3.5 – 8 GeV are calculated, as shown in Fig. 7 and Fig. 8. It is seen that there exists an obvious peak at M_{ηπ} and M_{ηη} near 1.86 GeV. These results will inform future experimental measurements.

As mentioned above, the minimum cross section of η′(1855) production via the K−p reaction is above 19.2 nb at W = 3.5 GeV. With the branching ratio BR(η′ → KK′π) = (Γ_{η′→KK′π}/Γ_{η′} × 100%) ≈ 69.1%, one obtains the total cross section σ_{K^-p \rightarrow η_1(1855)Λ → KK′πΛ} ≈ 13.3 nb at W = 3.5 GeV. There are several experimental data available for the cross section of the K−p → ΛπKK’ reaction [31], which are listed in Table II.

In the K−p → ΛπKK’ process, we estimate the total cross section is 13.8 μb near the c.m. energy W = 3.5 GeV according to the average cross-section of these energies. Therefore, we get the the ratio at W = 3.5 GeV as

\[ \frac{σ(K^-p \rightarrow η_1′Λ → KK′πΛ)}{σ(K^-p \rightarrow KK′πΛ)} ≈ 0.096%. \]
TABLE II. The cross section for the $K^- p \rightarrow \Lambda\pi^0 K^*$ reaction at different beam momentum from several experiments [31].

| Reaction            | $W$ (GeV) | Cross-section (μb) |
|---------------------|-----------|--------------------|
| $K^- p \rightarrow \Lambda\pi K^0 K^*$ | 3.52      | 13 ± 6             |
| $K^- p \rightarrow \Lambda\pi^0 K^0 K^*$ | 3.52      | 12 ± 9             |
| $K^- p \rightarrow \Lambda\pi^0 K^* K^0$ | 3.52      | 16.5 ± 7.5         |

According to the minimum cross section of the $\eta_1(1855)$ production is above 0.59 nb near the $W = 3.5$ GeV and the branching ratio $BR(\eta_1 \rightarrow KK\pi) = 55.9\%$, we get the the cross-section ratio at $W = 3.5$ GeV as

$$\frac{\sigma(K^- p \rightarrow \eta_1 \Lambda \rightarrow K\bar{K}^*\pi\Lambda)}{\sigma(K^- p \rightarrow K\bar{K}^*\pi\Lambda)} \approx 0.0024\%. \quad(14)$$

When checking the present experimental status, we notice that the J-PARC experiment can provide Kaon meson beams [14, 15], which an ideal facility for probing the $\eta_1(1855)$ and $\eta_1^\prime(1855)$ production by the $Kp$ reaction. Assuming that the acceptance of the $K^- p \rightarrow K\bar{K}^*\pi$ reaction on J-PARC can reach 50% and taking the branching ratios of the $K^*$ decay chain to be as 0.67, it can be expected to detect about $0.9 \times 10^7$ events for the $K\bar{K}^*\pi\Lambda$ production per day, in which about dozens of events are related to the $\eta_1(1855)$ and $\eta_1^\prime(1855)$.

### IV. SUMMARY

Recently, the BESIII Collaboration observed $\eta_1(1855)$ in the $J/\psi \rightarrow \gamma \eta\eta'$ decay [2]. Since the $\eta_1(1855)$ is an isoscalar resonance with exotic $J^{PC} = 1^{-+}$ quantum numbers, different explanations of exotic states like hybrid [4–6], the $K\bar{K}_1(1400)$ molecular state [7, 8], and tetraquark state [9] were proposed, which shows that this observation attracted theorist’s attention in the past months. Besides decoding the nature of the observed $\eta_1(1855)$, establishing the $\eta_1(1855)$ in experiment is also crucial. How to confirm the observation of the $\eta_1(1855)$ by other experiments becomes a central issue.

In this work, we propose that the $Kp$ reaction can be as one way to further explore the $\eta_1(1855)$, which is stimulated by the study of its strong decays [7]. Our calculation provides the information of the total and differential cross sections of the $K^- p \rightarrow \eta_1(1855)\Lambda$ reaction. We also notice the possible $C$-parity partner $\eta_1^\prime(1855)$, which was predicted in the picture of the $K\bar{K}_1(1400)$ molecular state [7]. Thus, in this work, the production of the predicted $\eta_1^\prime(1855)$ via the $Kp$ reaction is discussed, where the $K^- p \rightarrow \eta_1^\prime(1855)\Lambda$ process is focused. For telling our experimental colleague more information, we also further analyze the relevant Dalitz processes, which is also valuable to future experimental search for them.

In summary, we suggest these available experiments at OKA@U-70 [12] and SPS@CERN [13], and J-PARC [14, 15] to explore the newly reported $\eta_1(1855)$ and its possible partner $\eta_1^\prime(1855)$ due to considerable events as estimated in the present work.
V. ACKNOWLEDGMENTS

This project is supported by the National Natural Science Foundation of China under Grant Nos. 12065014 and 12047501, and by the West Light Foundation of The Chinese Academy of Sciences, Grant No. 21JR7RA201. X.L. is also supported by the China National Funds for Distinguished Young Scientists under Grant No. 11825503, National Key Research and Development Program of China under Contract No. 2020YFA0406400, and the 111 Project under Grant No. B20063, and by the Fundamental Research Funds for the Central Universities.

[1] M. Ablikim et al. [BESIII], “Future Physics Programme of BESIII,” Chin. Phys. C 44, no.4, 040001 (2020).
[2] M. Ablikim et al. [BESIII], “Observation of an isoscalar resonance with exotic $J^{PC} = 1^{++}$ quantum numbers in $J/\psi \to \gamma \eta'$,” arXiv:2202.00621.
[3] M. Ablikim et al. [BESIII], “Partial wave analysis of $J/\psi \to \gamma \eta'$,” arXiv:2202.00623.
[4] H. X. Chen, N. Su and S. L. Zhu, “QCD axial anomaly enhances the $\eta'$ decay of the hybrid candidate $\eta_1(1855)$,” arXiv:2202.04918.
[5] L. Qiu and Q. Zhao, “Towards the establishment of the $J^{PC} = 1^{++}$ hybrid nonet,” arXiv:2202.09904.
[6] V. Shastry, C. S. Fischer and F. Giacosa, “The phenomenology of the exotic hybrid nonet with $\pi_1(1600)$ and $\eta_1(1855)$,” arXiv:2203.04327.
[7] X. K. Dong, Y. H. Lin and B. S. Zou, “Interpretation of the $\eta_1(1855)$ as a $KK_0(1400)$ c.c. molecule,” arXiv:2202.00863.
[8] F. Yang and Y. Huang, “Analysis of the $\eta_1(1855)$ as a $KK_0(1400)$ molecular state,” arXiv:2203.06934.
[9] B. D. Wan, S. Q. Zhang and C. F. Qiao, “A possible structure of newly found exotic state $\eta_1(1855)$,” arXiv:2203.14014.
[10] P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020) and 2021 update.
[11] P. E. Schlein, W. E. Slater, L. T. Smith, D. H. Stork and H. K. Ticho, “Quantum Numbers of a 1020 MeV K Kbar Resonance,” Phys. Rev. Lett. 10, 368 (1963).
[12] V. Obraztsov [OKA Collaboration], “High statistics measurement of the $K^+ \to \pi^0 e^+ v(Ke3)$ decay formfactors,” Nucl. Part. Phys. Proc. 273-275, 1330 (2016).
[13] B. Velghe [NA62-RK and NA48/2 Collaborations], “$K^+ \to \pi^0 \gamma \gamma$ Studies at NA48/2 and NA62-RK Experiments at CERN,” Nucl. Part. Phys. Proc. 273-275 (2016) 2720.
[14] T. Nagae, “The J-PARC project,” Nucl. Phys. A 805, 486 (2008).
[15] J-PARC Loi proposal, “$\Xi$ Baryon Spectroscopy with High-momentum Secondary Beam”, May, 2014: http://www.jp-parc.jp/researcher/Hadron/en/Proposal_e.html#1301.
[16] X. Y. Wang and J. He, “Investigation of pion-induced $f_1(1285)$ production off a nucleon target within an interpolating Reggeized approach,” Phys. Rev. D 96, 034017 (2017).
[17] X. Y. Wang and J. He, “$K^*\Lambda$ photoproduction off a neutron,” Phys. Rev. C 93, 035202 (2016).
[18] H. Y. Ryu, A. I. Titov, A. Hosaka and H. C. Kim, “$\phi$ photoproduction with coupled-channel effects,” PTEP 2014, 023D03 (2014).
[19] X. H. Liu, Q. Zhao and F. E. Close, “Search for tetraquark candidate $Z(4430)$ in meson photoproduction,” Phys. Rev. D 77, 094005 (2008).
[20] J. Xiang, X. Y. Wang, H. Xu and J. He, “Pion-induced $K^*$ production with $\Sigma^*$ baryon off proton target,” Commun. Theor. Phys. 72, 115303 (2020).
[21] Y. Oh and H. Kim, “$K^*$ photoproduction off the nucleon: $\gamma N \to K^*\Lambda$,” Phys. Rev. C 73, 065202 (2006).
[22] Y. Oh and H. Kim, “Scalar kappa meson in $K^*$ photoproduction,” Phys. Rev. C 74, 015208 (2006).
[23] V. G. J. Stoks and T. A. Rijken, “Soft core baryon baryon potentials for the complete baryon octet,” Phys. Rev. C 59, 3009 (1999).
[24] X. Y. Wang and X. R. Chen, “Production of the superheavy baryon $\Lambda^*_c(4209)$ in kaon-induced reaction,” Eur. Phys. J. A 51, 85 (2015).
[25] H. Haberzettl, X. Y. Wang and J. He, “Preserving Local Gauge Invariance with t-Channel Regge Exchange,” Phys. Rev. C 92, 055503 (2015).
[26] Y. X. Wang, J. He and H. Haberzettl, “Analysis of recent CLAS data on $\Sigma^*(1385)$ photoproduction off a neutron target,” Phys. Rev. C 93, 045204 (2016).
[27] S. Ozaki, H. Nagahiro and A. Hosaka, “Charged $K^*$ Photoproduction in a Regge model,” Phys. Rev. C 81, 035206 (2010).
[28] X. Y. Wang and J. He, “Analysis of recent CLAS data on $f_1(1285)$ photoproduction,” Phys. Rev. D 95, 094005 (2017).
[29] X. Y. Wang, J. He, Q. Wang and H. Xu, “Productions of $f_1(1420)$ in pion and kaon induced reactions,” Phys. Rev. D 99, 014020 (2019).
[30] S. H. Kim, S. I. Nam, D. Jido and H. C. Kim, “Photoproduction of $\Lambda(1405)$ with the $N^*$ and $t$-channel Regge contributions,” Phys. Rev. D 96, 014003 (2017).
[31] A. Baldini, V. Flamino, W.G. Moorhead and D.R.O. Morris, Landolt-Börnstein, Numerical Data and Functional Relationships in Science an Technology, vol. 12, ed. by H. Schopper, Springer-Verlag(1988), Total Cross Sections of High Energy Particles.