Chapter 2

Metal enrichment and reionization constraints on early star formation

The period of transition of the IGM from a completely neutral to a completely ionized state is known as the epoch of reionization [EoR; Loeb & Barkana, 2001]. The study of EoR has been an active area of research in recent years. Theoretical ideas about the reionization history have been constrained by a variety of observations [Choudhury & Ferrara, 2006; Fan, Carilli & Keating, 2006]. For example, observations of Gunn-Peterson troughs in AGN spectra at $z \sim 6$ [Becker et al., 2001; Fan et al., 2006] indicate that the process of reionization was nearly complete by that redshift. Bounds on luminosity function of Ly$\alpha$ galaxies at high redshifts [Malhotra & Rhoads, 2004; Stern et al., 2005; Bouwens et al., 2008, 2010a] also constrain the EoR. These bounds are consistent with the conclusion that the IGM was completely ionized by $z \sim 6$. Furthermore, Thomson scattering by free electrons in the IGM of the anisotropic photon distribution that constitutes the CMB leaves a signature in its temperature and polarization anisotropy. The optical depth due to this scattering [Spergel et al., 2003; Dunkley et al., 2009] is another probe of EoR. WMAP five-year data indicate a value of $\tau \sim 0.084 \pm 0.016$ that corresponds to $z \sim 10$ as the redshift for instantaneous reionization. Realistic scenarios, however, predict a protracted EoR where the process of ionization starts around $z \sim 20$ and ends by $z \sim 6$.

In this chapter, we consider the question of the sources of reionization. In the currently-favoured $\Lambda$CDM cosmological model, large scale structures in the universe like galaxies and clusters of galaxies are believed to have formed by gravitational amplification of small perturbations [Peebles, 1980; Padmanabhan, 2002; Bernardeau et al., 2002]. Much of the matter in galaxies and clusters of galaxies is the so called dark matter that is believed to be weakly interacting and non-relativistic [Trimble, 1987; Komatsu et al., 2009]. Dark matter responds mainly to gravitational forces, and by virtue of larger density than baryonic matter, assembly of matter into haloes and large scale
structure is driven by gravitational instability of initial perturbations. Galaxies are believed to form when gas in highly over-dense haloes cools and collapses to form stars in significant numbers [Hoyle, 1953; Rees & Ostriker, 1977; Silk, 1977; Binney, 1977]. The formation of first stars [McKee & Ostriker, 2007; Zinnecker & Yorke, 2007; Bromm & Larson, 2004] in turn leads to emission of UV radiation into the IGM. Due to this UV radiation, early star-forming galaxies are the most-favoured candidates as the sources of IGM reionization.

Although stellar sources are believed to be the most plausible candidates, many other sources of ionizing radiation have also been considered in the literature [Yan & Windhorst, 2004; Schneider et al., 2006; Choudhury & Ferrara, 2007]. With their hard spectra, active galactic nuclei (AGNs) of high redshift galaxies can be very effective in ionizing large regions of the IGM. However, the AGN density goes down more rapidly for \( z > 3 \) than the density of star-forming galaxies [Miralda-Escude & Ostriker, 1990; Madau, Haardt & Rees, 1999; Haehnelt et al., 2001] and therefore AGNs are not expected to contribute significantly to the ionizing radiation. X-rays from low mass quasars, X-ray binaries and supernova remnants are constrained by the soft X-ray background observed today [Dijkstra, Haiman & Loeb, 2004]. Particle decays can also play only a minor role in reionization [Bharadwaj & Sethi, 1998; Mapelli & Ferrara, 2005]. In this chapter, we assume that it was radiation from early stars that ionized the IGM and ignore other possibilities.

The total photon emissivity of early stars is poorly known. Studies of reionization typically use observations with semi-analytic models of heating and ionization of the IGM where efficiency of star formation, evolution of star formation rates, the number of ionizing photons emitted per baryon in stars, etc. are parameterized in some manner [Chiu & Ostriker, 2000; Choudhury & Ferrara, 2005]. Given the complexity of most of these approaches, and the number of parameters, it is often impractical to scan the parameter space. The main lesson we learn from these studies is that we may not require extraordinary physical processes in order to satisfy available observational constraints of reionization. In the approach that we take here, we make an attempt to simplify modeling of star formation and other astrophysical aspects of the problem. This allows us to reduce the number of free parameters in this sector while retaining many significant astrophysical relationships. Statements can then be made about quantities like the escape fraction of UV photons, \( f_{\text{esc},\gamma} \), and their correlations with the cosmological parameters.

Large uncertainties exist in parameters related to early star formation, e.g. the escape fraction of UV photons, \( f_{\text{esc},\gamma} \), the stellar initial mass function (IMF) and the efficiency of star formation, \( f_* \) [Bunker et al., 2004]. It has been suggested in the literature that a top-heavy IMF with very massive stars is not necessarily favored to satisfy the reionization and metal enrichment constraints [Daigne et al., 2006; Venkatesan & Truran, 2003].

In this work we assume that early star formation happens predominantly during
Table 2.1: Various IMFs used in this study are summarized here. $M_{\text{low}}$ and $M_{\text{high}}$ are the lower and upper mass cut-offs for the IMFs respectively. $Z_{\text{input}}$ denotes the metallicity of the gas from which stars are formed. $N_{\gamma}$ is the number of ionizing photons produced per baryon in stars and $p$ is the metal yield per baryon in them. $N_{\gamma}$ and $p$ are obtained using population synthesis models. The last column lists the value of $N_{\gamma}f_{\text{esc},\gamma}f_{\star}$ for the WMAP5 best-fit model with the corresponding IMF. This quantity is proportional to the number of photons escaping into the IGM for every baryon inside a collapse halo.

| IMF  | $M_{\text{low}}/M_{\odot}$ | $M_{\text{high}}/M_{\odot}$ | $Z_{\text{input}}$ | $N_{\gamma}$ | $p$ | $N_{\gamma}f_{\text{esc},\gamma}f_{\star}$ |
|------|----------------------------|-----------------------------|-------------------|-------------|-----|-----------------|
| 1. Kroupa | 0.1 | 100 | 0.0004 | 6804 | 0.0123 | 50.0 |
| 2. Kroupa | 0.5 | 100 | 0.0004 | 9280 | 0.0167 | 50.0 |
| 3. Kroupa | 0.1 | 100 | 0.001 | 6297 | 0.0159 | 50.0 |
| 4. Salpeter | 1  | 100 | 0.001 | 11237 | 0.0283 | 50.0 |
| 5. Kroupa | 0.1 | 100 | 0.02 | 3996 | 0.0261 | 50.2 |

formation and major mergers of haloes, and occurs as a short lived burst. Photon emissivities and metal yields can then be calculated using population synthesis models for different IMFs [Leitherer et al., 1999; Bruzual & Charlot, 2003]. We then test if these scenarios generate enough photons to ionize the universe by $z \sim 6$ by comparing with the observed Thomson scattering optical depth. We also require the models to satisfy constraints arising from the observations of the metal content of the IGM. This constrains the amount of processed elements that escape from the ISM to the IGM. We can use models for outflows as a guide and put constraints on the efficiency of star formation, or use “reasonable” values of the efficiency of star formation to constrain the metal escape fraction. We can also combine the two constraints to scale out efficiency of star formation. As a result, we are able to constrain both the evolution of the universal stellar IMF during the EoR and its slope at the high mass end.

In summary, in this chapter we combine constraints of enrichment of the intergalactic medium with observations of reionization, and check whether the extra information can provide constraints on the initial mass function during early star formation. We also ask if the combined constraints be used to make useful statements with regard to other potential sources of ionization. Finally, we study any correlations between the parameters that describe star formation and cosmological parameters.

### 2.1 Observations

The IGM occupies most of the space and a substantial amount of matter in the Universe. Indeed, it is believed that at least half of the baryons in the universe are in the IGM. Thus it is not surprising that the observations of IGM dominate when we discuss constraints
on models of reionization. We introduce observations that are used for constraining models in this chapter.

Observational constraints that we use are essentially two: metallicity of the IGM as seen in quasar absorption systems, and the Thomson scattering optical depth from the cosmic microwave background. A third observation that can potentially be used is that of the cosmic stellar matter density, which we model in a manner explained below. We do not discuss constraints arising from this type of observations as the observational bounds on models are not very strong at present. It is not possible to use other observational constraints in the global averaged model discussed here [Choudhury & Ferrara, 2007].

2.1.1 Metallicity of the IGM

Observations of absorption systems in quasar spectra have been used to put constraints on the average metallicity of the IGM [Cowie et al., 1995; Songaila, 1997; Ellison et al., 2000; Simcoe, Sargent & Rauch, 2004; Becker, Rauch & Sargent, 2009; Ryan-Weber et al., 2009]. These observations indicate that the amount of C IV in the IGM does not evolve significantly between $2 \leq z \leq 5.5$ [Songaila, 2001; Becker, Rauch & Sargent, 2009; Ryan-Weber et al., 2009]. It is not yet clear whether the IGM has been contaminated by metals throughout, or if the enriched regions of the IGM are restricted to the neighborhood of galaxies and filaments. There are also issues related with understanding the ionization state of metals to map the absorption by a particular species to average metallicity [Schaye et al., 2003]. Observations also support a correlation between density and metallicity of the IGM, indicating that regions in proximity of galaxies are enriched to higher level than regions of IGM far away from galaxies [Schaye et al., 2003; Pieri, Schaye & Aguirre, 2006; Scannapieco et al., 2006]. We assume that the IGM is uniformly enriched at the level indicated by Songaila [2001] at $z \sim 5.5$.

Winds and outflows are expected to be the dominant processes that lead to ejection of metals from the inter-stellar medium (ISM) of galaxies. There is considerable evidence in favor of this mechanism as observations have detected outflows around almost all galaxies at high redshifts [Pettini et al., 2001; Frye, Broadhurst & Benítez, 2002]. The fraction of processed metals that can be deposited from the ISM to the IGM without disturbing the IGM in an observable manner is not known. Several authors often assume that around 1% of metals produced in galaxies can be ejected and deposited in the IGM. This may also be computed from first principles in detailed models [Daigle et al., 2004, 2006; Samui, Subramanian & Srianand, 2008]. In these models supernova-driven outflows are responsible for the IGM enrichment. The efficiency of these outflows depends on the star formation efficiency, the IMF and the efficiency of winds [Madau, Ferrara & Rees, 2001; Scannapieco, Ferrara & Madau, 2002; Scannapieco, 2005; Furlanetto & Loeb, 2003]. For a star formation efficiency of 10% the volume filling factor of the ejecta can be 20–30% and at $z \sim 3$ the IGM metallicity could be around [-3] as detected
2.1.2 CMB constraints on IGM reionization

Finally, observations of the temperature and polarization anisotropies in the CMB provide a constraint on the EoR. Free electrons produced during reionization scatter the CMB photons and suppress temperature anisotropies on scales smaller than the Hubble radius at that time. The suppression is of the form $C_{l}^T = e^{-2\tau} C_{l}^T$ where

$$\tau = \int n_e(t) \sigma_T c \, dt$$  \hspace{1cm} (2.1)

is the Thomson scattering optical depth. Here $n_e(t)$ is the number density of free electrons and $\sigma_T$ is the Thomson scattering cross-section. This damping is degenerate with the amplitude of the primordial power spectrum. The degeneracy is broken by detection of a polarization anisotropy for scales greater than the Hubble radius at the reionization redshift, an effect that dominates at scale of the Hubble radius at EoR and has an amplitude proportional to $\tau$. Any model of reionization must reproduce the observed value of optical depth\(^1\).

2.2 Analytical Model

The reionization history depends on the star formation history of the universe, which in the simplified models is closely related to the halo formation history. The IMF of stars and the escape fraction for ionizing photons then give us the number of ionizing photons that are available as a function of time. These can then be used to compute the evolution of the neutral or ionized fraction of gas in the universe. Here, we assume that star formation is triggered during formation of haloes. As most time scales of interest are longer than the dynamical time scale over which the bulk of star formation takes place, we assume star formation to be instantaneous in our model.

Observations of galaxies at high redshifts can be used to infer the density of matter in stars, $\rho_*$, at those redshifts. One way of estimating this quantity is to use the luminosity function of galaxies in various wavelength passbands and combine these with population synthesis models and an assumed IMF [Madau et al., 1996; Lilly et al., 1996; Bouwens et al., 2007]. We do not use these observations here as current observations do not provide sufficiently strong constraints. This is expected to change in coming years with better observations.

\(^1\)For a forecast on anticipated improvements of observational estimation of $\tau$, please see Colombo & Pierpaoli [2009].
Instead, we assume that stars form only in virialised haloes. We take the minimum mass of star-forming haloes to be $10^8 \, M_\odot$, this is a proxy for haloes with a virial temperature of $10^4 \, \text{K}$. We do not take into account star formation in haloes of around $10^6 \, M_\odot$ that is aided by molecular cooling [Tegmark et al., 1997]. We also ignore the effects of feedback that raise the Jeans mass to around $10^9 \, M_\odot$. We can then obtain metal and photon yields by using population synthesis models: we use the STARBURST99 code [Leitherer et al., 1999; Vázquez & Leitherer, 2005]. This assumption allows us to connect the rate of change of stellar mass to the rate of change of the total mass contained in massive haloes. We obtain this rate from the Press-Schechter formalism for a Gaussian PDF as

$$
\dot{F}(m, z) = -\sqrt{\frac{2}{\pi}} \frac{\dot{z}}{\sigma(m) d_+(z)} \exp \left( \frac{-\delta_c^2}{2\sigma^2(m)} \right) \frac{\partial \log d_+}{\partial \log a},
$$

where an overdot denotes derivative with respect to the cosmic time, a prime denotes derivative with respect to the redshift, and $F$ is the fraction of haloes with mass greater than $m$ [Press & Schechter, 1974]. The critical density for spherical collapse is symbolized by $\delta_c$, and $\sigma^2(m)$ is the variance in the initial density fluctuation field when smoothed with a top-hat filter of a scale corresponding to mass $m$. The rate of growth for perturbations in the linear theory is denoted by $d_+(z)$.

The total amount of baryons added to haloes of mass greater than $10^8 \, M_\odot$ is taken to be the amount of gas available for star formation. Gas already present in haloes is not considered for star formation. We do not consider the contribution of minihaloes as these do not contribute significantly to the total star formation due to radiative feedback [Trenti & Stiavelli, 2009]. We define the efficiency of star formation, $f_\ast$ as the fraction of this gas that is converted into stars. The rate of change of stellar mass can now be written as

$$
\dot{\rho}_\ast(z) = f_\ast \Omega_b \rho_c \dot{F}(10^8 \, M_\odot, z),
$$

where $\rho_c$ is the critical density and $\Omega_b$ is the density parameter for baryons. Here we have ignored mass lost by stars through winds, outflows, and supernovae. This can be taken into account using, for example, population synthesis models. Equation (2.3) illustrates the small number of parameters and approximations that go into estimating $\dot{\rho}_\ast$ in our formulation.

### 2.2.1 Reionization

We consider a globally averaged evolution of ionized fraction instead of following evolution of HII regions around haloes, the approach used in most studies [Chiu & Ostriker, 2002]. We have checked that including the effects of radiative feedback increases the required star formation efficiency by around 20%. A starburst with a Kroupa IMF and an initial metallicity of 0.02 loses about 10% of its mass to the ISM through these effects under normal assumptions.
Figure 2.1: The products $f_s f_{\text{esc},Z}$ and $f_s f_{\text{esc},\gamma}$ against cosmological parameters $n_s$ and $\tau$ for our fiducial model — model 1 of Table (2.1). The star symbol denotes the WMAP5 best fit model. These points also represent lower bounds on $f_{\text{esc},Z}$ and $f_{\text{esc},\gamma}$. Please see text for details.

Figure 2.2: The product $f_s f_{\text{esc},Z}$ against cosmological parameters $\tau$ and $\sigma_8$ for model 1 of Table (2.1). The star symbol denotes the WMAP5 best fit model.
Figure 2.3: The product $f_* f_{\text{esc}, \gamma}$ against cosmological parameters $n_s$ and $\sigma_8$ for model 1 of Table (2.1). The star symbol denotes the WMAP5 best fit model.

2000; Sethi, 2005]. Further, we assume that during reionization, a region is either neutral or completely ionized. With these assumptions, the evolution of the ionized fraction evolves as:

\[
\dot{x} = -\alpha_B C^2 n_H x + \sigma_p y n_H c (1 - x), \tag{2.4}
\]

\[
\dot{y} = -\sigma_p y n_H c (1 - x) + m_p f_* f_{\text{esc}, \gamma} F N_{\gamma}, \tag{2.5}
\]

where $x$ is the fractional volume that is ionized, and $y$ is the number of ionizing photons per baryon. $\sigma_p$ denotes the effective cross-section of photoionization, $\alpha_B$ is the recombination coefficient for all levels except the ground state of neutral hydrogen, and $m_p$ denotes the mass of a proton.

The first term on the right hand side of equation (2.4) describes recombination. $C$ is the clumping factor defined as $C^2 = \langle n_H^2 \rangle / \langle n_H \rangle^2$. This term usually involves square of the ionized fraction but in our model we assume that the ionized fraction is either unity or zero. This, when used in volume averaging over the universe with an additional assumption that the clumping is the same in ionized and neutral regions, leads to a linear dependence. In the process of averaging, the meaning of $x$ changes from the ionized fraction to the volume filling fraction of the ionized regions. We can express this in terms of equations:

\[
\frac{1}{V} \int n_H^2 x^2 dV = \frac{\langle n_H^2 \rangle}{V} \int x^2 dV = \frac{\langle n_H^2 \rangle}{V} \int x dV = \langle n_H^2 \rangle x. \tag{2.6}
\]

We have assumed that the clumping factor is the same in all parts of the universe, this allows us to take $\langle n_H^2 \rangle$ outside the integral. The third equality in equation (2.6) follows...
from the definition of $x$ as a filling fraction. We also expect $C$ to change with redshift due to the evolution of clustering. We take this dependence to be of the form

$$C^2 \simeq 26.2917 \exp \left[ -0.1822z + 0.003505z^2 \right], \quad (2.7)$$

as obtained from high-resolution simulations by Iliev et al. [2007].

Sources of ionizing radiation are represented in the last term of equation (2.5), $\dot{F}$ being related to the formation rate of collapsed haloes. This is obtained from the Press-Schechter formalism as described above. $N_\gamma$ denotes the number of photons produced per unit mass of star formation. Ionization of neutral hydrogen is described in the last term on the right hand side of equation (2.4). This term occurs in both equations. We neglect the contribution of collisional ionization.

### 2.2.2 Metal Enrichment

The amount of metals produced per baryon in stars can also be computed once we fix the initial mass function (IMF) of stars and metallicity of the star-forming gas [Leitherer et al., 1999; Vázquez & Leitherer, 2005]. We can write

$$n_Z = f_{esc,Z} \Omega_b \frac{\rho_c}{m_p} \dot{F}(10^8 M_\odot, z) p \quad (2.8)$$

for the number density of metals that reaches the IGM. Here $f_{esc,Z}$ is the fraction of total metals produced that is deposited in the IGM and $p$ is the metal yield of the stars per baryon. Note that we assume the same escape fraction for metals from galaxies of different masses, whereas it is far more likely that low mass galaxies lose nearly all the metals from the ISM and more massive galaxies lose very little [Dekel & Silk, 1986].

Having modeled the ionization and metal enrichment of the IGM, we solve these equations numerically for different cosmological models. The system of equations (2.4) and (2.5) is “stiff,” since $(x = 0, y = 0)$ is a stable point and time scales for evolution of $x$ and $y$ are very different. Further, $x$ is bounded from above (by unity) while $y$ is not. Thus the usual forward differencing methods do not give accurate solutions easily. We bypass this problem by noting that during the process of reionization almost every ionizing photon will be immediately absorbed by the medium\(^4\). This means that the two terms in the right hand side of the second equation are of the same order till $x$ becomes nearly equal to 1, whereas the left hand side is much smaller and may be assumed to be zero. This reduces the system of equations to a single equation, which can now be solved using forward differencing methods. Note that this approximation is not valid

\(^4\)There are two approximations being discussed here: one approximation is that all UV photons are available for ionizing atoms. This is not really true as ionized regions can be expected to host an ionizing background. The other approximation is in solving the equations that we arrive at with the first approximation.
Figure 2.4: The products $f_{\star}f_{\text{esc},Z}$ and $f_{\star}f_{\text{esc},\gamma}$ for model 2 of Table (2.1). Corresponding plots from Figure 2.1) are superimposed in grey. Filled stars denote values for the WMAP5 best-fit models.

Figure 2.5: The products $f_{\star}f_{\text{esc},Z}$ and $f_{\star}f_{\text{esc},\gamma}$ for model 3 of Table (2.1). Corresponding plots from Figure (2.1) are superimposed in grey. Filled stars denote values for the WMAP5 best-fit models.
when \( x \) approaches 1, although in practice the approximate solution is fairly accurate up to \( x \sim 0.9 \). Indeed, if we use the approximation up to \( x = 1.0 \) then we make an error in estimation of \( \tau \) of less than 5\%.

We take \( \alpha_B = 1.0 \times 10^{-13} \text{ cm}^3 \text{ sec}^{-1} \), ignoring its dependence on temperature. This dependence is fairly weak at temperatures of interest. We use \( \sigma_p = 6.30 \times 10^{-18} \text{ cm}^2 \).

We thus assume that most of the ionizing radiation is around the Lyman limit. The number of ionizing photons released per baryon of stars formed, denoted by \( N_\gamma \), depends on the initial mass function (IMF) of the stars. We obtain this number from the STARBURST99 stellar population synthesis code \(^5\) [Leitherer et al., 1999; Vázquez & Leitherer, 2005].

### 2.3 Results

Figure 2.7 shows evolution of ionization fraction in our model for two values of \((f_*, f_{\text{esc}})\) when a constant clumping factor is used. Left panel shows result for \((f_*, f_{\text{esc}}) = (0.1, 0.1)\) and \((f_*, f_{\text{esc}}) = (0.3, 0.5)\). Higher values of these parameters increase the emissivity of ionizing radiation, thereby causing early reionization. Three curves in each panel correspond to three different values of the clumping factor, given by \( C = 1 \)

\(^5\)We consider instantaneous starbursts with a fixed total stellar mass of \(10^6 \text{ M}_\odot\). We use the Geneva evolutionary tracks for models with metallicity 0.001 and the Padova tracks with AGB stars for models with metallicity 0.0004.
Figure 2.7: Result of our model for two values of \((f_s, f_{esc})\) when a constant clumping factor is used. Left panel shows result for \((f_s, f_{esc}) = (0.1, 0.1)\) and \((f_s, f_{esc}) = (0.3, 0.5)\). Higher values of these parameters increase the emissivity of ionizing radiation, thereby causing early reionization. Three curves in each panel correspond to three different values of the clumping factor, given by \(C = 1\) (red), 5 (green), 10 (blue).

Our aim now is to study a variety of models with varying cosmological parameters as well as parameters related to star formation and enrichment. We consider a random subset of flat \(\Lambda\)CDM models allowed by WMAP 5-year data [Komatsu et al., 2009; Dunkley et al., 2009]. We do not consider models with massive neutrinos or a non-vanishing tensor component, or models where the primordial power spectrum deviates from a pure power law. We use only WMAP constraints for limiting cosmological parameters. We used the MCMC chains made available by the WMAP team. We considered a random subset of all models allowed with a confidence level of 68% from the MCMC chains. We studied a handful of models for parameters related to star formation; details of these are given in Table (2.1). The table lists the IMFs used in our study\(^6\). We have also listed the amount of ionizing photons produced per baryon in stars, and the total metal yield per baryon for these IMFs. These numbers also depend on the metallicity of gas from which stars form and this is listed in the table as well. It is interesting to note that for a given IMF, as the metallicity increases, the production of ionizing photons per

\(^6\)We should note that there is considerable uncertainty in the shape of the IMF in the local neighborhood [Kroupa, 2002; Conroy, Gunn & White, 2009]. This can easily have a significant impact on our conclusions. The uncertainties introduced by other assumptions and approximations should be seen with the uncertainty in the IMF as the reference.
baryon comes down but the amount of enriched material returned to the ISM increases. Gas that forms the first stars is likely to have primordial abundance [Olive, Steigman & Walker, 2000]. In our analysis of all the models, we keep metallicity fixed and hence it is appropriate to use low values of input metallicity.

Metallicity of the IGM constrains the product \( f_* f_{esc,Z} \) for a given model. Similarly we constrain the product \( f_* f_{esc,\gamma} \) with the optical depth due to reionization for the CMB. Figures 2.1, 2.2 and 2.3 shows these products for all cosmological models studied here, when the star formation parameters for model 1 in Table (2.1) are used. Given that the efficiency of star formation can at best be 100\%, i.e., \( f_* \leq 1 \), the points in these figures also represent lower bounds on \( f_{esc,Z} \) and \( f_{esc,\gamma} \). These are shown as a function of the slope of the primordial power spectrum (\( n_s \)), optical depth due to reionization (\( \tau_{CMB} \)), and, amplitude of clustering at the scale of 8 h\(^{-1}\)Mpc (\( \sigma_8 \)). The best fit WMAP5 model is marked in each panel as a star. We find that there is some correlation between the lower bound on \( f_* f_{esc,\gamma} \) and \( \tau_{CMB} \), and also between \( f_* f_{esc,Z} \) and \( n_s \) as highlighted in Figure 2.1. There are weak correlations with other cosmological parameters, as seen in Figures 2.2 and 2.3 but nothing as remarkable as the two mentioned above.

### 2.3.1 Constraints on IMF evolution

The efficiency of star formation is likely to be much less than unity in any realistic scenario. Indeed, if we try to keep the different efficiencies and escape fractions at the same order then we require these to be around 0.1–0.15, or 10–15\%. While the escape fraction for ionizing photons and star formation efficiency we have obtained are comparable to those found in other studies, these are higher than the values seen in local galaxies. In particular, it is not clear if it is possible to expel 10\% of the metals from the ISM to distant parts of the IGM using known physical mechanisms. Before commenting on the numbers, let us consider the sensitivity of the result to our assumptions by constraining the evolution of the IMF.

If the IMF has a low mass cutoff that is higher than the 0.1 M\(_\odot\) used for model 1 from Table 2.1, then a larger fraction of mass goes into high mass stars that produce the ionizing photon flux and the enriched material. This can lower the required \( f_{esc,Z} \) by a significant amount. The product \( f_* f_{esc,Z} \) for model 2 from Table 2.1 is shown in Figure 2.4. There is some evidence that there are more intermediate mass stars in the population of metal poor stars in the halo of the Galaxy as compared to metal rich stars, if we normalize the two distributions at low stellar masses [Tumlinson, 2007; Komiya et al., 2007]. Thus a higher cutoff for \( M_{low} \) may be required for explaining other observations. Also, our analysis assumes that the metallicity of gas that forms stars is fixed. If we do a self-consistent analysis where this is allowed to evolve, the gas metallicity gradually increases. It is then clear from Table 2.1 that later generations of stars will enrich the ISM faster. As an illustration, we can see the results of analysis with higher fixed input metallicity for models 2–4 in Figures 2.5 and 2.6. Our estimates show
that \( f_{\text{esc},Z} \) required to satisfy observations can come down by a few tens of percents due to this. There is a corresponding increase in \( f_{\text{esc},\gamma} \) due to stars with higher metallicity producing fewer ionizing photons.

Apart from the IMF, we have assumed that haloes with mass above \( 10^8 \, \text{M}_\odot \) can form stars. The standard approach is to assume that radiative feedback during the EoR increase this by about an order of magnitude in regions that have been photo-ionized [Efstathiou, 1992]. We do not take this into account as it has been pointed out that the actual effect may only be to reduce the efficiency of star formation in lower mass haloes [Mesinger & Dijkstra, 2008]. If we do consider the effect of radiative feedback as disabling star formation then it leads to a reduction in total gas available for star formation, and hence requires slightly higher efficiencies and escape fractions. We find that this effect requires an increase in the two products by about 20%. Furthermore, we have assumed that the universe is enriched uniformly. It may very well happen that the enrichment process is effective only in the vicinity of galaxies. In such a case only overdense regions are enriched. The escape fraction of metals required can be lowered by as much as a factor of two if this is the case.

Thus we may require only around 5% of the ISM to be ejected to the IGM on an average in models with \( f_e \simeq 0.2 \) This is comparable with semi-analytic \( \text{ab initio} \) models of early star formation, outflows and IGM enrichment that have been studied in the literature. We have also assumed the same loss fraction for ISM for galaxies over the entire range of masses. This, of course, is not true. We expect that the low mass galaxies can potentially disperse a large fraction of the ISM in supernova explosions but larger galaxies can retain most of their ISM [Larson, 1974; Dekel & Silk, 1986]. If most of the IGM enrichment is done by metals that form in dwarf galaxies then the constraint is not very stringent. In most models, the fraction of mass in galaxies with a halo mass of less than \( 10^{10} \, \text{M}_\odot \) is larger than 10% even at \( z \simeq 6 \). If these galaxies lose a significant fraction of the ISM on an average and heavier galaxies lose very little mass then we can comfortably satisfy the constraints from enrichment of the IGM.

### 2.3.2 Constraints on high mass star formation

We now turn our attention to the ratio of escape fractions for photons and metals in order to draw constraints on high mass star formation. Figure 2.8 shows the ratio \( f_{\text{esc},Z}/f_{\text{esc},\gamma} \) for the first IMF listed in Table 2.1. It is interesting to note that the ratio \( f_{\text{esc},Z}/f_{\text{esc},\gamma} \) is of order unity, differing from unity by at most a factor of a few. Thus the fraction of ionizing photons that escape galaxies is broadly of the same order as the fraction of metals that must leak into the IGM in order to explain the observed enrichment of the IGM. We have not plotted this ratio for other IMFs in Table 2.1 as the expected change can be seen from other figures. Indeed, the change in the ratio is less than a factor two as we consider IMFs listed in rows 2–4 of the Table 2.1. The last row in Table 2.1 is more appropriate for late time star formation and we need not discuss that here.
Most studies of reionization have tended to focus on the escape of ionizing photons. In order to satisfy observations of $\tau$ or the luminosity function of high redshift galaxies, these often invoke a top heavy IMF for the first generation of stars [Cen, 2003; Haiman & Holder, 2003; Wyithe & Loeb, 2003c; Bromm, 2004]. Other sources of ionizing radiation like AGNs, magnetic fields and decaying dark matter particles have also been studied [Pierpaoli, 2004; Schleicher, Banerjee & Klessen, 2008]. It is interesting to note that all such modifications lead to an enhanced production of ionizing photons without affecting the production of metals in a significant manner. This is because the very massive stars are expected to implode and do not enrich the ISM with the products of nuclear fusion that takes place in the core. The ratio $f_{\text{esc},Z}/f_{\text{esc},\gamma}$ changes on addition of extra sources of ionizing radiation. In view of the arguments presented above, all modifications that have been discussed so far lead to a smaller $f_{\text{esc},\gamma}$. In other words, the ratio plotted in Figure 2.8 should be thought of as a lower bound.

An important implication of this is that the constraints from enrichment of the IGM require certain amount of star formation, and this requirement needs to be satisfied even when we invoke other sources of ionizing radiation during the epoch of reionization. That is, adding new potential sources of ionizing radiation can be helpful only in lowering the escape fraction of ionizing photons and not in lowering the amount of star formation\(^7\). We may even end up with a scenario where a much larger fraction of processed elements need to be transferred from the ISM to the IGM as compared to the fraction of UV photons escaping from galaxies.

\(^7\)An exception is the scenario where the universe is not enriched throughout. In such a case even $f_{\text{esc},Z}$ can be reduced by a significant amount.
2.4 Conclusions

In this chapter, we have compared simple models for star formation in the early universe with two observational constraints. The simplicity of the model allows us to consider variation in cosmological parameters as well. We present a summary of our results here:

- The product of star formation efficiency and escape fraction of ionizing photons, \( f_* f_{\text{esc,}\gamma} \), is correlated with the optical depth due to reionization.

- The product of star formation efficiency and escape fraction of ISM, \( f_* f_{\text{esc,}Z} \), is anti-correlated with the index of the primordial power spectrum.

- These are weak correlations, in the sense that the values for the two products do not change strongly for small changes in the cosmological parameter in question.

- We do not find any other correlation amongst parameters of star formation and cosmological parameters.

- We are able to satisfy observational constraints with the standard initial mass function for stars observed in the local universe [Kroupa, 2002], and with reasonable values for star formation efficiency and escape fractions for photons and ISM. Given that the local IMF itself is somewhat ill constrained, this implies that we do not require a significant evolution of the IMF in order to explain observations considered here.

- Small variations in the IMF, indicated by observations of metal poor stars in the Galaxy, reduce the efficiency of star formation and the escape fractions required for the standard IMF.

- Approximations used by us in the model do not change the overall numbers by more than 10–20%. Indeed, different approximations change numbers in different directions so we can consider the overall results to be fairly robust.

- Our model allows us to estimate the ratio of the two escape fractions. We find that the two escape fractions are of the same order.

- If we consider other potential sources of ionizing photons then the required escape fraction for photons can come down, however the escape fraction for processed elements does not change. Indirectly, the required amount of star formation is required to remain the same unless there is some very efficient mechanism for transporting processed elements into the IGM while keeping the escape fraction of ionizing photons low.
The most important conclusion of this chapter is that star formation without a significant evolution of the IMF is sufficient for satisfying the two constraints considered here. The escape fractions, and/or the star formation efficiency is required to be higher than we see in local galaxies. One can consider other sources of ionizing radiation, indeed at least some of these must be present. But as we have pointed out, these help in reducing only the escape fraction for ionizing radiation as none of the other potential sources help in transporting enriched material from the inter-stellar medium to the inter-galactic medium. This highlights the significance of the constraint arising from enrichment of the IGM for epoch of reionization studies.

Finally, we have neglected the presence of other sources of reionization, e.g., metal-free stars, minihaloes, and so on. It is expected that these sources would be too faint to affect the luminosity function in the ranges we are considering. Recently, such structures have also been shown to supersonic coherent flows of baryons relative to the underlying potential wells created by the dark matter [Tseliakhovich & Hirata, 2010]. However, these sources may affect the thermal history of the medium, e.g., the metal-free stars would produce higher temperatures because of harder spectra. In such cases, it is most likely that feedback would occur at magnitude brighter than what we have indicated and hence would possibly be easier to detect.
