Route optimization for multi-rotor aerial vehicles

Arzamastsev A A

Tambov State University, Department of Mathematical Modeling and Information Technologies, 392000, Tambov, International str., 33, Russia.
Voronezh State University, Department of Mathematical and Applied Analysis, 394018, Voronezh, University Square 1, Russia.

E-mail: arz_sci@mail.ru

Abstract. The mathematical statement of the problem for route optimization of multi-rotor aerial vehicles has been developed. In this formulation, the indicated problem is a Traveling Salesman Problem (TSP) of an asymmetric closed type. As a target functions of optimization the total energy consumption of MRAV motion along the broken line passing through n given points in the order given by the permutation (problem 1) and the total time of flying around the route along a broken line passing through n given points in the order given by the permutation (problem 2) have been proposed. It is shown that when a MRAV DJI Inspire 1 flies around the route points which have different altitudes, the differences between the “intuitive” and the optimal variants can be up to 43%. The more is difference in altitudes of the points to be flied around, the more is the difference between the “intuitive” and the optimal route variants. From practical point of view, the solution to problem 1 can be used in case there is a need to save MRAV power sources for solving some other problems. The solution to problem 1 is needed in case when there is a need for faster flying around the route. On average, the benefit of using optimization for an aerial vehicle of this type is 17-18%.

1. Introduction

In our papers [1-3], there were proposed several techniques for engineering calculations of multi-rotor aerial vehicles (MRAVs): formulas for calculating static propulsion of airscrews, flight duration of aerial vehicles, optimization of battery capacity, etc. For the first time, there were also formulated MRAVs routing problems which were further solved by us on the basis of the available experimental data in [2]. For this purpose we used branch-and-bound algorithm supplemented by a program for forming the initial data matrix based on MRAVs technical characteristics.

This paper presents the latest results which have been recently obtained at the Department of Mathematical Modeling and Information Technology and in the laboratory of unmanned robotic systems (URS) at Tambov State University on the basis of the tests conducted on a DJI Inspire 1 quadcopter.

It should be noted that some models of up-to-date MRAVs can be programmed to fly around a closed route over the points having three predetermined coordinates, two of which are the coordinates on a map, and the third one is the altitude. It is possible for DJI quadcopters and others. The MRAV energy consumption for its moving horizontally, vertically up, vertically down and hovering differs...
significantly [2, 3]. Since MRAV movement upwards is the most energy consuming, then the optimal (in terms of energy consumption) plan to fly round route points may differ significantly from the intuitively proposed one.

Since the MRAV information system usually works in such a way that the coordinates of the points and the parameters of their flying around are input manually, it is necessary to develop a program that could determine the optimal route for flying round the specified points with the aim to save the energy (charge) of the battery most economically (problem 1) or fly around the route for the minimum time (problem 2). In so doing, each point of the route must be flew around only once.

In this formulation, the indicated problem is a classical Traveling Salesman Problem (TSP) of an asymmetric closed type.

This problem is known to be referred to transcomputational ones. The statement of optimization problem is NP-hard.

2. Mathematical statement of the problem

A finite set $X$ and an objective function $U(x)$ on this set are given. It is necessary to find such $x^* \in X$, that $U(x^*)$ is the smallest, that is, $U(x^*) \leq U(x)$ for all $x \in X$. In this case, we take $S_n$ - a set of permutations of the $n$-element set as set $X$, and as a target function $U(x)$ - the total energy consumption of MRAV motion along the broken line passing through $n$ given points in the order given by the permutation $x \in X$ (problem 1) and the total time of flying around the route along a broken line passing through $n$ given points in the order given by the permutation $x \in X$ (problem 2).

These problems are referred to combinatorial optimization problems. For their solution we used branch-and-bound method, elastic network method, annealing simulation method, and others. For small $n$ (for example, $n=5-10$), the problem can be solved in a limited period of time on a “ordinary” computer using the exhaustive search. Since, during the MRAV flight, it is often necessary to fly around 5-7 route points, the solution of the traveling salesman problem by one of these methods can lead to significant energy savings of the power source or reducing the time of flying around the route points.

Thus, the main idea of this work is to use a closed-type asymmetric traveling salesman problem solution to optimize the route of a MRAV flying around the points. This problem, with reference to the subject area, is called by us “the MRAV routing problem” [2].

For some MRAVs, their technical characteristics have been determined, including energy consumption for: vehicle hovering, which greatly depends on its mass; ascending and descending, depending on both the mass of the vehicle and the vertical speed; horizontal flight, also depending both on the mass of the vehicle and the horizontal speed. However, for other MRAVs these parameters are unknown.

Therefore, this article deals not only with the solution of optimization problems for MRAVs with known parameters, but also with the methodology for determining these parameters.

In general, energy consumption is significantly affected by: 1) the technical characteristics of the vehicle, such as the aerodynamic parameters of the MRAV itself, the airscrews used and their rotation frequency; 2) air characteristics: temperature, pressure, air humidity, wind speed and various types of atmospheric turbulence, as well as some other factors.

The study of the effectiveness of a MRAV flight, depending on all of these factors, is a complex scientific and technical problem. Here we confine ourselves to the description of the method for determining the optimal route only for a vehicle of a certain construction, and with certain constant parameters of air environment. In a lot of practically important cases, for example, when calculating the optimal route of flying around several points located at different distances from the start-finish point and different altitudes, such a calculation would be quite sufficient if MRAV energy consumption in different modes is determined at average environmental parameters.

The structure of the article is such that its first part is devoted to the necessary experimental studies, and the second one presents the solution to the stated optimization problems.
3. Experimental studies
Experiments on the study of the energy consumed for MRAV hovering, horizontal flight, ascending and descending were performed on a DJI Inspire 1 quadcopter in the URS laboratory of Tambov State University. All the studies were carried out at an air temperature of 14-20°C using batteries with a capacity of 4500 and 5700 mAh. The average wind speed in all the experiments was about 3 m/s. The total mass of the vehicle in these cases was 2.97 and 3.04 kg, respectively. After the vehicle take-off, by means of the DJI GO program telemetry there were measured the percentage of residual battery charge at different points in time, vertical and horizontal flight speeds, altitude and distance of the vehicle from the starting point, which served as the basis for subsequent calculations.

Determine the total power consumption of engines during aerial vehicle hovering. After the vehicle take-off and its fixation at an altitude of 4 m with automatic retention of the altitude and coordinates. The percentage of residual battery charge was measured at different points in time. The results of the experiments are given in tables 1 and 2.

Power consumption while vehicle hovering and its horizontal flight was estimated according to the formula:

\[
N_i = \frac{3600 U_i (C_i - C_{i+1})}{(t_{i+1} - t_i)},
\]

where, \(i = 1, \ldots, n-1\) is the number of the time interval for which the power consumption is estimated; \(N_i\) is power estimate on the \(i\)-th time interval, \(W\); \(C_i, C_{i+1}\) is the residual capacity of the battery at the beginning (\(C_i\)) and at the end (\(C_{i+1}\)) of the \(i\)-th time interval, \(Ah\); \(U_i\) is averaged voltage of the battery, \(V\) (for a battery with a capacity of 4700 mAh, according to the nameplate data, this value is 22.2 V, and for a battery with a capacity of 5700 mAh it is 22.8 V); 3600 is the conversion factor \(s \cdot h^{-1}\); \(t_i, t_{i+1}\) are the beginning and end times of the \(i\)-th interval, \(s\).

**Table 1.** Experiment 1. Estimation of power consumed during aerial vehicle hovering with a battery capacity of 4500 mAh.

| Time, s | Percentage (%) of residual battery capacity | Residual capacity of the battery, Ah | Estimation of the total power of engines, W |
|--------|--------------------------------------------|------------------------------------|------------------------------------------|
| 0      | 100                                       | 4.50                               | 389.61                                   |
| 120    | 87                                        | 3.92                               | 479.52                                   |
| 180    | 79                                        | 3.56                               | 419.58                                   |
| 420    | 51                                        | 2.30                               | 419.58                                   |
| 480    | 44                                        | 1.98                               | 539.46                                   |
| 540    | 35                                        | 1.58                               | 299.7                                    |
| 600    | 30                                        | 1.35                               | -                                        |

The mean value: 424.6
The mean value for the entire time period: 419.6

Thus, it can be asserted that power consumed by MRAVs when hovering is about 420 W with a 4500 mAh battery (the total mass of the vehicle in this case is 2.97 kg) and it equals 546 W with a 5700 mAh battery (the total mass of the vehicle in this case is 3.04 kg).

When using the indicated values of power consumption for these batteries, the total MRAV hovering time should be about 10 minutes, taking into account the fact that in our case the residual capacity of the battery is 30%. These values correspond to real time measurements.

Determine the total power consumption of engines during a horizontal flight of the aerial vehicle. After the vehicle take-off, its fixation at an altitude of 30 m by automatically holding the altitude, the flight was carried out according to a box-pattern with average speeds of 22 and 45 km/h. The results of the experiments are given in tables 3 and 4. Power consumption during horizontal flight of the vehicle was estimated according to formula (1).
Table 2. Experiment 2. Estimation of power consumed during aerial vehicle hovering with a battery capacity of 5700 mAh.

| Time, s | Percentage (%) of residual battery capacity | Residual capacity of the battery, Ah | Estimation of the total power of engines, W |
|---------|------------------------------------------|--------------------------------------|------------------------------------------|
| 0       | 100                                      | 5.70                                 | 682.29                                   |
| 240     | 65                                       | 3.71                                 | 389.88                                   |
| 300     | 60                                       | 3.42                                 | 467.86                                   |
| 360     | 54                                       | 3.08                                 | 467.86                                   |
| 420     | 48                                       | 2.74                                 | 467.86                                   |
| 480     | 42                                       | 2.39                                 | 561.43                                   |
| 540     | 36                                       | 2.05                                 | 401.02                                   |
| 600     | 30                                       | 1.71                                 | -                                        |

The mean value: 491.2
The mean value for the entire time period: 545.8

Thus, it can be asserted that MRAV power consumption during horizontal flight is 431 W with a 4500 mAh battery and an average speed of 22 km/h and it equals 486 W with a 5700 mAh battery and an average speed of 45 km/h.

Determining the total power consumption of engines during the ascending and descending of the aerial vehicle. The vehicle was ascended from an initial altitude of 10 m to an altitude of 100 m and 200 m. The vehicle was descended from an altitude of 200 m and 100 m to an initial altitude of 10 m. The ascending speed according to telemetry was 3-3.2 m/s, the descending speed was 3 m/s. The results of the calculations are given in tables 5 and 6.

To estimate the power consumption during the ascending and descending of the vehicle, the following formula was used:

$$ N_i = \frac{3600 \cdot \rho \cdot v \cdot (h_i - h_{i+1})}{|h_{i+1} - h_i|}, \quad (2) $$

where, $i = 1, ..., n-1$ is the number of the time interval for which the power consumption is estimated, $h_i$, $h_{i+1}$ is the altitudes of the vehicle at the beginning and at the end of the $i$-th time interval, $m$; $v$ is the speed of the ascending or descending of the vehicle, m/s.

Table 3. Experiment 3. Evaluation of power consumed during the horizontal flight of the aerial vehicle with an average speed of 22 km/h (6.1 m/s) with a battery capacity of 4500 mAh.

| Time, s | Percentage (%) of residual battery capacity | Residual capacity of the battery, Ah | Estimation of the total power of engines, W |
|---------|------------------------------------------|--------------------------------------|------------------------------------------|
| 0       | 95                                       | 4.28                                 | 599.40                                   |
| 30      | 90                                       | 4.05                                 | 599.40                                   |
| 60      | 85                                       | 3.83                                 | 359.64                                   |
| 90      | 82                                       | 3.69                                 | 359.64                                   |
| 120     | 79                                       | 3.56                                 | 479.52                                   |
| 150     | 75                                       | 3.38                                 | 239.76                                   |
| 180     | 73                                       | 3.29                                 | 479.52                                   |
| 240     | 65                                       | 2.93                                 | 359.64                                   |
| 300     | 59                                       | 2.66                                 | 419.58                                   |
| 360     | 52                                       | 2.34                                 | 419.58                                   |
| 42      | 45                                       | 2.03                                 | 419.58                                   |
| 480     | 38                                       | 1.71                                 | 359.64                                   |
| 540     | 32                                       | 1.44                                 | -                                        |

The mean value: 424.6
The mean value for the entire time period: 430.9
Table 4. Experiment 4. Estimation of power consumed during the horizontal flight of the aerial vehicle with an average speed of 45 km/h (12.5 m/s) with a battery capacity of 5700 mAh.

| Time, s | Percentage (%) of residual battery capacity | Residual capacity of the battery, Ah | Estimation of the total power of engines, W |
|---------|---------------------------------------------|-------------------------------------|------------------------------------------|
| 0       | 91                                          | 5.19                                | 545.83                                   |
| 60      | 84                                          | 4.79                                | 467.86                                   |
| 120     | 78                                          | 4.45                                | 311.90                                   |
| 150     | 76                                          | 4.33                                | 779.76                                   |
| 180     | 71                                          | 4.05                                | 311.90                                   |
| 210     | 69                                          | 3.93                                | 623.81                                   |
| 240     | 65                                          | 3.71                                | 311.90                                   |
| 270     | 63                                          | 3.59                                | 467.86                                   |
| 300     | 60                                          | 3.42                                | 623.81                                   |
| 330     | 56                                          | 3.19                                | 467.86                                   |
| 360     | 53                                          | 3.02                                | 311.90                                   |
| 390     | 51                                          | 2.91                                | 623.81                                   |
| 420     | 47                                          | 2.68                                | 632.81                                   |
| 450     | 43                                          | 2.45                                | 467.86                                   |
| 480     | 40                                          | 2.28                                | 311.90                                   |
| 500     | 38                                          | 2.17                                | -                                        |

The mean value: 431.5
The mean value for the entire time period: 486.2

Table 5. Experiment 5. Estimation of power consumed during vehicle ascending with an average speed of 11.2 km/h (3.1 m/s) with a battery capacity of 5700 mAh.

| The initial altitude of the vehicle, m | The final altitude of the vehicle, m | Battery capacity consumption ΔC, Ah | Estimated power consumption of engines, W |
|---------------------------------------|--------------------------------------|-------------------------------------|------------------------------------------|
| 10                                    | 200                                  | 0.57                                | 763.34                                   |
| 10                                    | 100                                  | 0.40                                | 1128.05                                  |
| 10                                    | 100                                  | 0.29                                | 805.75                                   |

The mean value: 899.04

Table 6. Experiment 6. Estimation of power consumed during vehicle descending with an average speed of 10.8 km/h (3 m/s) with a battery capacity of 5700 mAh.

| The initial altitude of the vehicle, m | The final altitude of the vehicle, m | Battery capacity consumption ΔC, Ah | Estimated power consumption of engines, W |
|---------------------------------------|--------------------------------------|-------------------------------------|------------------------------------------|
| 200                                   | 10                                   | 0.114                               | 147.74                                   |
| 100                                   | 10                                   | 0.171                               | 467.86                                   |
| 100                                   | 10                                   | 0.114                               | 311.90                                   |

The mean value: 309.17

Thus, it can be stated that energy consumption and average speeds of DJI Inspire 1 vehicle in different flight modes correspond to those given in Table 7. These values will be used in further calculations.
Table 7. Power consumption and average speeds of MRAVs in various flight modes with a battery capacity of 5700 mAh.

| Flight mode    | Speed, m/s | Power consumption, W |
|----------------|------------|----------------------|
| Horizontal     | 12.5       | 486                  |
| Hovering       | 0          | 546                  |
| Ascending      | 3.1        | 899                  |
| Descending     | 3.0        | 309                  |

It should be noted that earlier, in paper [2], we made estimates of the same parameters for a Black Bird quadcopter with a weight of 1.74 kg, assembled in the URS laboratory. For this vehicle there were obtained: horizontal flight – speed 8 m/s, power consumption 257 W; hovering – power consumption 160 W; ascending – speed 3.5 m/s, power consumption – 340 W; descending – speed 2.4 m/s, power consumption 50 W. Power calculation during ascending according to simple formula N = mg/ht for DJI Inspire 1 gives – 92.4 W, for Black Bird – 59.7 W. Thus, it is possible to make estimates of the efficiencies of propulsion systems of these quadcopters during ascending. For DJI Inspire 1 – 10.3%, for Black Bird – 17.6%. The obtained values are very low that may be caused by the imperfection of the aerodynamic layout of the vehicles during their ascending due to a large number of supplementary equipment, the imperfection of the rotor-motor group, as well as significant losses associated with heating of engines and batteries.

4. Solving the problems of optimal routes for DJI Inspire 1

The optimal route was developed according to the same scheme as in [2], but using the data in table 7. Suppose there are n points (in our example n = 6), which a MRAV should once fly around and land at the starting point. All the points are concentrated on the site 0 ≤ x ≤ 500 m, 0 ≤ y ≤ 500 m (figures 1-3). For each of the n points, the altitude at which a MRAV should perform its actions is also known (in figures 1-3 this is the third coordinate of each point). A characteristic feature of the problem is that the altitudes differ very significantly 10 ≤ z ≤ 350 m.

When a MRAV moves horizontally and vertically at the same time, the energy consumption will be considered as the sum of energies for moving in these directions. Herein, the travel time is calculated by the formula:

\[ t_{i,j} = \max \left\{ \begin{array}{c}
  z_i \geq z_j, \frac{(z_i - z_j)}{v_{dec}}, \\
  z_i < z_j, \frac{(z_j - z_i)}{v_{asc}}, \\
  \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} / v_h
\end{array} \right\}, i = 1, \ldots, 6 \text{ and } i \neq j \quad (3) \]

where, \( v_{dec} \) is the speed of the vehicle descending, m/s; \( v_{asc} \) is the speed of the vehicle ascending, m/s; \( v_h \) is the speed of horizontal movement of the vehicle, m/s.

At each point of the route, a MRAV must solve a specific problem, for which it must stay at this point in a hovering state time \( \tau \). In this case, additional time should be added to our calculations (for example, \( \tau \approx 10 \) s), and a constant value of 5460 J should be added to the total energy consumption for each route point or its equivalent for a six-cell battery of 0.067 Ah, which corresponds to 10 s of hovering.

Problems 1 and 2 were solved by exhaustion method and branch-and-bound method. Table 8 presents a matrix of MRAV energy consumption and the times of its movement from one point of the route to another. The matrix is not symmetric. For example, to fly from point 2 to point 3 of the route, a MRAV should spend 47.3 kJ, while for flying from point 3 to point 2 of the route it should spend 101.2 kJ.
Table 8. Energy consumption (kJ) is the first line; the used capacity of a six-cell LiPo battery (Ah) is the second line; flight time (s) of a MRAV from the initial point to the destination point is the third line.

| Destination point | Initial point | 1   | 2   | 3   | 4   | 5   | 6   |
|-------------------|--------------|-----|-----|-----|-----|-----|-----|
|                   |              | 1   | 44.1| 25.8| 65.2| 23.4| 44.0|
|                   |              | 2   | 0.54| 0.31| 0.79| 0.29| 0.54|
|                   |              | 3   | 100.0| 39.4| 116.7| 34.0| 66.7|
|                   |              | 4   | 101.2| 28.8| 101.7| 52.0|     |
|                   |              | 5   | 1.22| 1.23| 0.35| 1.24| 0.63|
|                   |              | 6   | 96.8| 92.9| 37.5| 92.3| 36.1|
|                   |              | 1   | 28.1| 47.3| 48.1| 15.7| 41.5|
|                   |              | 2   | 0.34| 0.58| 0.59| 0.19| 0.51|
|                   |              | 3   | 39.4| 96.0| 112.7| 20.6| 62.7|
|                   |              | 4   | 38.2| 111.3| 111.7| 63.1|     |
|                   |              | 5   | 1.59| 0.47| 1.36| -   | 0.77|
|                   |              | 6   | 112.9| 37.5| 109.0| 108.4| 48.4|
|                   |              | 1   | 26.0| 48.2| 16.1| 48.8| 31.3|
|                   |              | 2   | 0.32| 0.59| 0.20| 0.60| -   | 0.38|
|                   |              | 3   | 34.0| 95.3| 20.6| 112.0| 62.0|
|                   |              | 4   | 81.4| 33.3| 76.7| 35.0| 66.1|
|                   |              | 5   | 0.99| 0.41| 0.93| 0.43| 0.81| -   |
|                   |              | 6   | 64.5| 36.1| 60.6| 50.0| 60.0|     |

The intuitive solution to the routing problem. Initially, the problem of flying around the route was solved intuitively, in such a way that most of the MRAV pilots would do it. A diagram of such a flight around the route is shown in figure 1.

At that, the total energy consumption of batteries and flight time were:

Energy consumption (kJ) = 100.2 + 47.3 + 111.3 + 48.8 + 66.1 + 44.0 = 417.7 kJ

Battery energy consumption (Ah) = 1.22 + 0.58 + 1.36 + 0.60 + 0.81 + 0.54 = 5.11 Ah

Route flight time (s) = 96.8 + 96.0 + 109.0 + 112.0 + 60.0 + 66.7 = 540.5 s.

Figure 1. Intuitive flying around destination points.
Problem 1. Minimizing energy consumption while flying around the route. At the second stage, the problem of flying around the route was solved as an optimization one. The objective function in this case was the total energy consumption during the route. We chose such a route which corresponded to the minimum value of the objective function on the set of all possible closed trajectories passing through each point once. In this case, the route had to start and end at point 1. The route shown in figure 2 turned out to be optimal.

The total energy consumption of the batteries and the flight time were:

Energy consumption (kJ) = 100.2 + 38.2 + 35.0 + 31.3 + 15.7 + 25.8 = 246.2 kJ
Battery energy consumption (Ah) = 1.22 + 0.47 + 0.43 + 0.38 + 0.19 + 0.31 = 3.0 Ah
Route flight time (s) = 96.8 + 37.5 + 50.0 + 62.0 + 20.6 + 39.4 = 306.3 s.

Problem 2. Minimizing the time of flying around the route. At the third stage, the problem of flying around the route was also solved as an optimization one. However, the objective function in this case was the time of flying around the route. We chose such a route which corresponded to the minimum value of the objective function on the set of all possible closed trajectories passing through each point once. In this case, the route had to start and end at point 1. The route shown in figure 3 turned out to be optimal.

The total energy consumption of the batteries and the flight time were:

Energy consumption (kJ) = 81.4 + 63.1 + 28.8 + 47.3 + 16.1 + 23.4 = 260.1 kJ
Battery energy consumption (Ah) = 0.99 + 0.77 + 0.35 + 0.58 + 0.20 + 0.29 = 3.18 Ah
Route flight time (s) = 64.5 + 48.4 + 37.5 + 96.0 + 20.6 + 34.0 = 301 s.

Figure 2. Optimal flying around destination points providing minimum energy consumption.
The results of all the calculations are integrated in table 9. From its analysis we can draw the following conclusions.

**Table 9.** The results of the optimal MRAV routing in comparison with the “intuitive” version of flying around the route.

|                                | “Intuitive” flying around the route | Problem 1. Minimizing energy consumption while flying around the route | Problem 2. Minimizing the time of flying around the route |
|--------------------------------|------------------------------------|---------------------------------------------------------------------|----------------------------------------------------------|
| Energy consumption, $kJ$       | 417.7                              | 246.3                                                               | 260.1                                                    |
| Battery energy consumption, Ah | 5.11                               | 3.0                                                                 | 3.18                                                     |
| Route flight time, $s$         | 540.5                              | 306.3                                                               | 301                                                      |

If a quadcopter DJI Inspire 1 uses the routes, obtained by solving the problems of minimizing the energy consumption and flight time, then a saving of about 40% of energy resources of MRAVs is achieved and the flight time of flying around the route is reduced by 43% compared to the “intuitive” variant. These figures significantly exceed the "benefit" that can be achieved for the Black Bird apparatus of our design (about 25%).

To solve these routing problems by means of branch-and-bound and exhaustion methods, we developed a special program [4].

5. Solution to the route optimization problems for DJI Inspire 1 "on average"

In sections 2-3 of this article, we showed that, depending on the kinematic scheme of the MRAV used, its mass, the mass of the carried load and the efficiency factor, the benefit in terms of energy consumption and time can reach 25-50% when flying around the route shown in figures 1-3. Obviously, this route was chosen in such a way that the difference in the altitudes of the route points was very significant. Therefore, the resulting effect from solving the problems of optimal route search is also very significant. This effect will be greatly reduced if all the points of the flight have equal or similar altitudes.
It should be noted that a consumer, buying a particular type of a MRAV for solving some practical tasks, would like to know what possible benefit can be obtained on average for this type of the vehicle, if optimization of its route is used.

In this section, the problems of minimizing energy consumption and time will be solved on average using simulation techniques.

For the purpose of simulation we used the data on energy consumption and average speeds of a MRAV in various flight modes given in table 7. The coordinates of the six points of the route were randomly generated using a uniform distribution, so that $0 \leq x \leq 500 \text{ m}, 0 \leq y \leq 500 \text{ m}, 0 \leq z \leq 500 \text{ m}$. The intuitive flying round the route chosen for comparison was thought as flying around these points in a clockwise direction and on the outer contour.

The computational experiment was carried out as follows:

1) six triples of coordinates $x_i, y_i, z_i, i=1,...,6$ for each of the six points of the route were generated;
2) for the generated coordinates of the points, an outer contour was chosen, which was taken as the “intuitive” flying around the route by a pilot;
3) the data from table 7 was used to solve the optimization problem by means of the exhaustion method; to develop the optimal route, the value of the objective function (energy consumption or time) was calculated;
4) the value of the objective function obtained in the optimal solution was compared with its value in the “intuitive” flight and the benefit was determined in this variant;
5) if the number of studied variants corresponded to the representativeness of the average calculated value of the benefit, the iterations were stopped, otherwise, we proceeded to point 1.

Similar steps were taken to determine the average benefit for energy consumption and time of the flight route.

Figure 4 shows that the used random number generator allowed us to obtain arrays of three coordinates - $x, y,$ and $z$ which correspond well to a uniform distribution on the $[0, 500]$ segment. These coordinates were used in computational experiments.

Figures 5-6 show the results of solving the problem of minimizing energy consumption on average. Figures 7-8 present the results of solving time minimization problem on average.

From figure 5 a) it is clear that in approximately 20% of cases (tests) the optimal route coincides with the intuitive one. Excluding these coincidences, the average value of the benefit obtained from route optimization in solving the problem of minimizing energy consumption is about 20% (figure 5 b), while in general such a benefit is about 17% (figure 6). From the same figure, it is clear that with a small number of computational experiments, there are significant fluctuations in the average benefit from 17 to 23%. With an increase in the number of computational experiments, the average value of the obtained benefit converges to a value of 17%. In this case, to obtain a representative result, one would confine himself to 400-500 experiments.
Figure 4. Histograms of the distribution density of coordinates generated for computational experiment.

Figure 5. Histograms of the empirical distribution of MRAV energy consumption benefit, obtained as a result of solving the optimization problem: a) – full; b) - except the cases when the optimal route coincides with the intuitive one.
Figure 6. To the determination of the required number of computational experiments and the average MRAV energy consumption benefit as a result of route optimization.

When solving on average the problem of minimizing the time of flying around the route, similar results were obtained. Thus, from figure 7 a) it is clear that in about 14% of cases (tests) the optimal route coincides with the intuitive one. Excluding these coincidences, the average value of the benefits obtained from optimizing the route when solving the problem of minimizing time is about 22% (figure 7 b), while in general this benefit is about 18% (figure 8). From the same figure, it is clear that with a small number of computational experiments, there are significant variations in the average benefit from 20 to 25%. As the number of computational experiments increases, the average value of the benefit converges to a value of 18%. At the same time, in order to obtain a representative result, one would confine himself to 350-400 experiments.

Figure 7. Histograms of the empirical distribution of the MRAV flight time benefit obtained as a result of solving the optimization problem: a) - full; b) - except the cases when the optimal route coincides with the intuitive one.
Figure 8. To the determination of the required number of computational experiments and the average time benefit of the MRAV flight as a result of route optimization.

Thus, solving the problems of optimizing the routes of a MRAV DJI Inspire 1 on average allows us to state that the benefit obtained from this is 17-18%. These values are significant enough to reflect on the integration of information systems designed to solve these problems in the unified MRAV management system.

6. Conclusion
Thus, it is shown that when a MRAV DJI Inspire 1 flies around the route points which have different altitudes, the differences between the “intuitive” and the optimal variants can be up to 43%. The more is difference in altitudes of the points to be flew around, the more is the difference between the “intuitive” and the optimal route variants.

When a MRAV flies around the route points which have the same altitudes, the problem becomes symmetrical, that is, solutions to problem 1 and problem 2 should coincide.

From practical point of view, the solution to problem 1 can be used in case there is a need to save MRAV power sources for solving some other problems. The solution to problem 1 is needed in case when there is a need for faster flying around the route.

On average, the benefit of using optimization for an aerial vehicle of this type is 17-18%.

This work can be continued in the following directions. It is necessary:
- to study the influence of external factors, such as wind speed and direction, temperature, air pressure and humidity, on the solution of the routing problem;
- to take into account the dynamic characteristics of MRAVs while determining the speeds and times of flying around the route sections (acceleration and deceleration), as well as the dynamics of changes in currents consumed by engines and the corresponding energy consumption for these purposes;
- to investigate how the “benefit” depends on the technical characteristics of MRAVs;
- to combine the solution to route (routing) optimization problems with preflight programming.

Acknowledgements
The author is grateful to the Director of the Institute of Mathematics, Natural Science and Information Technology of Tambov State University named after G.R. Derzhavin Professor Yemelyanov A.V. for
the opportunity to study a MRAV DJL Inspire 1, to Bogdanov M.N., a laboratory engineer of unmanned robotic systems for his assistance in conducting full-scale experiments, to Associate Professor of Mathematical Modeling and Information Technology Department Sletkov D.V. for his help in conducting computational experiments, to undergraduates Belov Ya. D. and Pronin Ya. P. for their assistance in developing the programs.

References
[1] Arzamastsev A.A. and Kryuchkov 2014 A.A. Mathematical models for engineering calculations of multi-rotor aerial vehicles (Part 1) TSU Reports 19 (6) 1821-1828 (in Russian)
[2] Arzamastsev A.A. 2015 Mathematical models for engineering calculations of multi-rotor aerial vehicles (Part 2). Routing Problems TSU Reports 20 (2) 465-468 (in Russian)
[3] Arzamastsev A.A. and Obraztsov D.V. 2016 Study of the main characteristics of the flight of a hexacopter TSU Reports 21 (2) 665-667 (in Russian)
[4] Arzamastsev A.A. and Belov Ya D. 2017 The program for developing the optimal flight trajectory of a multi-rotor aerial vehicle Certificate of registration of the computer program 2017619656 Russian Federation Applicant and copyright holder Federal State Budgetary Educational Institution of Higher Education “Tambov State University named after G.R. Derzhavin 2017616526