Design of membraneless gas-evolving flow-through porous electrodes

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HIGHLIGHTS

- Membraneless cells with lower resistance than cells with diaphragm are demonstrated.
- A minimum electrolyte velocity prevents gas from entering the interelectrode gap.
- An analytical multiphase porous electrode model predicts the minimum required velocity.
- Significant total energy savings are possible by using flow instead of a membrane.
- Simple expressions are provided for design parameters that minimize the energy losses.

ABSTRACT

Flow-through electrolyzers, with flow parallel to the current, are used in a wide range of industrial applications. The presence of flow avoids concentration gradients but can also be used to separate evolved gases, allowing membrane-less operation. In this work, we propose a simple multiphase flow-through electrode model. We derive and experimentally validate an analytical expression for the minimum velocity required to ensure effective gas separation. From this relation, we analytically derive design parameters that show that significant energy savings can be made using flow, compared to a physical separator.

1. Introduction

Porous flow-through electrodes have been studied for a long time due to their superior mass transfer [1,2]. Flow-through porous electrodes are traditionally used in electrosynthesis [3,4], redox flow batteries [5-7], metals removal from waste streams [8-10], electrochemical destruction of cyanide [11,12], and water electrolysis [13-17].

The flow in a porous electrode can be parallel to the current as in Fig. 1, normal, or a combination as in the interdigitated configuration [18]. A flow-through electrode with flow parallel to the current has traditionally in the literature often been referred to as flow-by electrodes [19,20]; a term nowadays, particularly in the flow battery literature, more commonly reserved for flow that largely bypasses the electrodes in a separate channel next to the electrodes, see e.g. Refs. [6,21]. Here we will consider the flow configuration of Fig. 1 in which the flow enters in between and flows out through the electrodes. This configuration has similarities with both cross-flow and dead-end separators used for filtration.

In many electrochemical processes, the gases are evolved in a liquid electrolyte; including the chlor-alkali process used for producing chlorine, the electrosynthesis of chlorate [22], or fluorine [23], the Hall-Héroult process for smelting aluminium, electrowinning of metals,
Various forms of CO₂ reduction [24], and water electrolysis.

Typically a membrane or separator is used to avoid gases crossing over to the opposing electrode where they may react back and cause inefficiencies, cause product impurity, or safety concerns as in the case of oxygen and hydrogen for example [25–27]. Various membraneless designs have also been proposed, for example based on selective coating [28] or lift forces on bubbles [29]. Since a membrane or separator adds costs, resistance, and potential degradation, there seems significant potential for membraneless designs. The design under consideration here uses outward flow to separate the produced gases, as depicted schematically in Fig. 1. This configuration was already studied for water electrolysis in, e.g., Refs. [13,30,31] but only recently its potential for membraneless operation has been highlighted in Refs. [14,15,32,33].

Many studies have been devoted to modelling flow-through electrodes [19,20,34,35] focusing mostly on mass transfer [31,36,37] while far fewer include the effect of gas evolution [38–42]. All these models assume that distinct bubbles move with a velocity proportional to the liquid velocity. This assumption is likely invalidated when the pores become smaller than the 100 μm [22] that is typical for the diameter of electrolysically generated bubbles. For the small pores typically present in rocks or the diffusion layers of PEM fuel cells [43–45] and electrolyzers [46–48], often the assumption is made that gas forms continuous pathways rather than bubbles [49]. This can be likened to the annular flow regime in pipe flow. In this case, Darcy’s law, which supposes a linear relationship between pressure gradient and velocity, can be assumed to hold approximately. Although this is a strong simplification with various limitations [50], we will discuss here the conclusions that follow from such a model and contrast it with the hitherto used bubble flow assumption.

A largely unanswered question is: what values for the various geometrical design parameters in Fig. 1, like electrode thickness, height, gap thickness, and pore size, are optimal from the perspective of energy efficiency. The answer crucially depends on both the flow direction and the minimum required flow rate. When a flow is used for mass transfer, various useful references exist [19,20,35,51,52]. However, when a flow is used to allow membrane-less operation of a gas-evolving electrolyzer very little is known. Reference [16] achieved, in a membraneless alkaline water electrolyzer, a current density of roughly 2 A/cm² at a cell voltage of 2.5 V and 4 A/cm² at 3.3 V. Compared to, typically much thinner, separators the used 2.5 mm gap seems to leave significant room for improvement. Smaller gaps, however, also require smaller pores, to ensure good distribution of flow, hence our interest in models beyond the assumption of bubbly flow. Recently [17,53] an extremely high current density of 25 A/cm² was obtained at 3.5 V, for more than 100 h, in an alkaline water electrolyzer with flow through a microfibre nickel felt perpendicular to the current. These results were obtained using an extremely thin polyethersulfone 0.1–0.15 μm thickness membrane, which showed immediate failure due to heat at even higher current densities.

The likely reasons that high multi-A/cm² current densities have been achieved in alkaline systems only in the presence of flow is likely due to flow-enhanced mass transfer of dissolved gases, electrolyte, and bubbles, reducing the Nernstian open-circuit potential or concentration polarization [54,55], bubble overpotentials [56–58] and increase the limiting current density [27,59], respectively.

Our primary goal here will be to determine the optimal geometrical and operational conditions for membrane-less flow-through electrolysis, to ascertain whether this can be competitive with conventional operation with a separator. A question that, however, first has to be answered is: what is the required flow velocity, as a function of current density, to allow membraneless operation.

The remainder of the paper is organized as follows. The proposed multiphase model to determine the minimum liquid velocity is discussed in Section 2. Experimental verification and observations with an alkaline water electrolyzer setup are discussed in Section 3. In Section 4, we

Nomenclature

\( \eta \) Effectiveness factor, [-]
\( m \) Exponent in effective conductivity \( s^m \), [-]
\( q \) Exponent in the relative permeability \( s^q \), [-]
\( s \) Liquid saturation, [-]
\( e \) Porosity, [-]
\( \eta \) Activation overpotential, [V]
\( \gamma \) Surface tension, [N/m]
\( \kappa \) Effective electrode conductivity \( \approx \kappa_0 e^{1.5} \), [-]
\( \lambda \) Pore size distribution index
\( \mu \) Dynamic viscosity, [Pa s]
\( c_1 \) Material constant \( K/c_1 d_{\text{max}} \), [-]
\( r \) Derivative \( d/dx \)
\( 0 \) Subscript indicating electrode entrance \( x = 0 \)
\( \Delta \) Total pressure drop \( \Delta p + \Delta p_{\text{gap}} \), [Pa]
\( A \) Frontal electrode area \( hw \), [m²]
\( b \) Tafel slope, [V]
\( C \) Reactant inlet concentration, [mol/m³]

\( d_{\text{max}} \) Maximum pore diameter, [m]
\( F \) Faraday constant 96485.3329, [C/mol]
\( h \) Electrode/gap height, [m]
\( i \) Local electronic current density, [A/m²]
\( j \) Current density magnitude, [A/m²]
\( K \) Electrode permeability, [m²]
\( L \) Electrode thickness, [m]
\( l \) Gap width, [m]
\( N \) Ratio \( \Delta p/\Delta p_{\text{gap}} \), [Pa]
\( n \) Electrons per reactant molecule
\( p \) Pressure, [Pa]
\( P_{\text{pump}} \) Power lost to friction, [W]
\( p_t \) Threshold pressure \( 4/d_{\text{max}} \), [Pa]
\( R \) Gas constant 8.31446, [J/mol/K]
\( T \) Temperature, [K]
\( u \) Superficial velocity in \( x \)-direction, [m/s]
\( V_n \) Molar volume, [m³/mol]
\( w \) Electrode/gap depth, [m]
\( x \) Streamwise coordinate in electrode, [m]
2. Multiphase flow-through electrode model

2.1. Governing equations

In the model previously used in Refs. [38–42] and considered in Appendix A.2, gas bubbles always move in the direction of the flow so that any non-zero velocity allows effective gas separation. Obviously, especially at large current densities, gas bubbles will also enter the gap between the electrodes, unless an unknown minimum flow-through liquid velocity is used. Here we consider a simple multiphase electrode model to obtain explicit analytical expressions for this velocity. We consider pores that are small compared to the electrode thickness so that an average description over many pores can be used. The gas will be assumed to form continuous pathways so that the extended Darcy equation can be used.

For a hydrophilic electrode we may use the following differential equation, discussed in more detail in Ref. [49], to describe the liquid fraction or saturation $s$,

$$\frac{dp_s}{dx} = \frac{\mu_l u_i} {K(1-s)} + \mu_g u_i \frac{ds}{dx}$$  \hspace{1cm} (1)

The left-hand side shows the derivative of the capillary pressure, the gas pressure minus the liquid pressure, with respect to the $x$-coordinate through the electrode; with $x = 0$ upstream, at the gap, and $x = L$ downstream, at the rear. According to Darcy’s law, the gas and liquid pressure gradients are proportional to the gas and liquid superficial velocities $u_g$ and $u_l$ and viscosities $\mu_g$ and $\mu_l$. The capillary pressure $p_s s^{-1/4}$ is related to the saturation through the Brooks-Corey relation, with $i$ the pore size distribution index - a smaller value corresponding to a more narrow distribution. The threshold pressure $p_t$ is the capillary pressure at $s = 1$, so corresponding to the largest pore. It is routinely measured in the analytical porosimetry technique as the ‘bubble point pressure’, the minimum pressure that has to be applied to the gas phase on one side of a porous medium to see bubbles appearing on the other side.

The permeability $K$ for the liquid is reduced by the presence of the gas through the ‘relative permeability’ $d^i$, and similarly $(1-d)^i$ for the gas permeability. Although actually only consistent with the Brooks-Corey relation for $q = 3$ in the limit $\lambda \rightarrow \infty$ [49,60], often a power-law fits well to experiments, see for example Ref. [61]. We note that in the following analysis, approximations near $s = 1$ are used.

Approximating the pores as cylinders with a maximum diameter $d_{\text{max}}$ we can write

$$p_t = \frac{4\mu_g}{d_{\text{max}}} K = c_1 e^{d_{\text{max}}^2},$$  \hspace{1cm} (2)

Here $\epsilon$ is the porosity and $c_1$ a material-dependent parameter [49,60,62,63].

If both the gas and the electronic current move to the right through the porous electrode, the gas velocity $u_g$ will be proportional to local electronic current density $i$ by Faraday’s law as

$$u_g = \frac{V_m i}{nF}$$  \hspace{1cm} (3)

Here the gas molar volume of an ideal gas $V_m = RT/p$, $n$ is the number of electrons per gas molecule, and $F$ is Faraday’s constant giving the charge of a mole of electrons.

At the gap side $x = 0$ both $u_g$ and $i$ vanish, and at the rear both are at their maximum and $i(x=L) = j$ is equal to the current density magnitude. In general we can write $i_0 \equiv \frac{\mu_l}{\rho} |u| = \frac{1}{\rho} \int u \, ds$ in terms of the electrode effectiveness factor $\epsilon^r$ as illustrated in Fig. 2. Using the approximation of Eq. (A.12) this gives

![Fig. 2. A possible profile of the electronic current $i$ throughout the electrode, with most of the current generated in the shaded region within a distance $\varepsilon L$ to the gap, minimizing the ohmic drop that ions have to travel.](image)

$$i_0 \approx \frac{j}{L} \left( 1 - \frac{jL}{2\kappa_b} \right), \quad \kappa \approx e^{3/2} \kappa_0.$$  \hspace{1cm} (4)

The effective conductivity of the porous electrode is often approximated by the Bruggemans relation $\kappa \approx e^{3/2} \kappa_0$ in terms of the electrolyte conductivity $\kappa_0$. Here we neglect the influence of the gas fraction, which

![Fig. 3. The saturation profile throughout the electrode from solving Eqs. (1) and (3) with a linearly current density profile, or $\frac{\mu_g}{\rho} |u| = \frac{3(\lambda/2)}{\pi^2 + \pi^2} \frac{1}{\rho}$ with $q = 3$, $\lambda = 4 \int j \frac{dV_g}{d\sigma} |i_0| = 3 \times 10^{-4}$, and $\frac{\mu_g}{\rho} = \frac{\mu_l}{\rho} = 1.0$ and $0.3$. At the rear side, we somewhat arbitrarily assumed $s(L) = 0.999$ but the exact value can be seen to impact only the solution near $x = L$. As the liquid velocity is decreased, $i(x = 0)$ decreases, which could result in an accumulation of gas bubbles near the entrance. Overall most of the domain the right-hand side of Eq. (1) approximately vanishes so that $s \approx 1/(1 + [j(x/L)/\Pi_1])^{1/3}$, a solution that is indicated by the dotted lines. Equating the derivative of this solution to that of the solution near $x = 0$, indicated by the dashed lines, we obtain $x_0 \approx 1 - \sqrt{j/q_1 \Pi_1} = 0.99$ and 0.97 for $\Pi_1 = 1$ and $\Pi_1 = 0.3$, respectively, in agreement with the numerical result. This expression for $s_0$ is used to derive Eq. (5).](image)
we show in Appendix A to be modest.

2.2. Numerical solutions

Fig. 3 shows the solution to Eqs. (1) and (3) for a linearly varying current density \( i = (x/L)j \) and a boundary condition \( s(x = L) = 0.999. \) The actual value used is not of much influence for the solution near \( x = 0. \) The produced gases hydrogen and oxygen are created in dissolved form first and advected with the flow, before a sufficiently high super-saturation causes nucleation of gas bubbles that grow and coalesce. We therefore assume that a minimum saturation \( s(x = 0) = s_0, \) which can be slightly below unity that will allow effective membrane-less operation without bubbles entering the gap. Next, we aim to find an analytical expression for the associated minimum liquid velocity.

2.3. Analytical solution

The numerical solutions reveal two distinctive features that can be used to find an analytical approximation for the required liquid velocity. First, the second term in Eq. (1), representing the frictional pressure gradient due to liquid flow, dominates near \( x = 0 \) when \( s_0 \approx 1. \) Locally, this allows the equation to be approximated by \(-\frac{1}{2}p_s = \frac{x}{s_0^2}.\)

A second observation from the numerical solutions of Fig. 3 is that, in the rest of the domain, the two terms in Eq. (1) approximately cancel so that \( q(1 - s_0)^{-1} - \frac{1}{2} - s_0^{-1} \approx \frac{\mu_k n F}{\mu m}, \) where \( u_0' = \frac{d\mu}{dx|x=0}.\) Matching the two foregoing approximations for \( s_0 \) gives \( u_0'(1 - s_0)^{-1} - s_0^{-1} = \mu_l n F u_0' / q\mu_m.\) This solution for \( s_0 \) was compared with the numerical result and found to give excellent agreement as long as \( s_0 \geq 0.9. \) From Eqs. (3) and (4) \( u_0' = \frac{v_s}{l} \left( 1 + \frac{jL}{2cb} \right) \) so that with Eq. (2) we obtain

\[
u_l = \sqrt{\frac{c_2 j \mu_{\max}^2}{L} \left( 1 + \frac{jL}{2cb} \right)}.
\]

with \( c_2 = \frac{4 \mu_l n F}{\mu m (1 - s_0)^3} \frac{\mu_l n F}{\mu m} = [\text{m}^3/\text{C/s}]. \) If we assume that the minimum saturation \( s_0 \) required to avoid bubbles in the gap is approximately constant, Eq. (5) gives an expression for the minimally required liquid velocity. This velocity is predicted to increase with \( \sqrt{j/L} \) or \( j \) for electrodes that are thin or thick relative to \( x \) and \( j \), respectively. The associated value of \( s_0 \) in this model is a constant that has to be obtained from experiments.

3. Experiments

3.1. Materials and methods

To validate the above model predictions we consider a typical “filter press”-like [64, 65] zero-gap [66] alkaline water electrolyzer setup, consisting of a stacking of several transparent perspex plates, see Fig. 4. A 30% w aqueous KOH electrolyte (from Honeywell Fluka with 85% purity) is pumped through the set-up using a Heidolph peristaltic pump at ambient conditions, and back through an ultrasonic cleaner to degas [67]. A BK Precision 9151 power source is used to supply current and measure the cell voltage.

We use two different fibrous nickel electrodes shown in Fig. 4, one felt and one plain weave mesh, and also two different plain weave nylon meshes as spacers. The geometrical features of these materials are summarized in Table 1.

Fig. 4. Top: Images of electrode morphologies using a Scanning Electron Microscope (SEM). (a) The SEM image on the left is the nickel felt electrodes with a ‘filter size’ of \( d_{\text{max}} = 8 \) μm and the image on the right is the mesh electrodes with an aperture of \( d_{\text{ap}} = 300 \) μm. (b) An exploded view of various PMMA layers (grey), electrodes (brown rectangles), and the rubber gasket (black) and (c) showing the various compression bolts, electrolyte inlet (top right) and outlet (bottom right). (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)
The first plain-weave nylon mesh consists of fibres of a specified 31 μm diameter giving, with some compression of the fibres, a reported thickness 53 μm. These are pressed to the electrodes in configuration 1 of Fig. 5 by the flow and on the circumference by a rubber gasket, see Fig. 4. The second, much coarser, mesh of Table 1 is used as a spacer to fill the entire gap, in configuration 2 of Fig. 5.

3.2. Pressure drop

An estimate of the pressure drop Δpadv through the Ni-felt electrodes was obtained using the reported pump curve, a magnetically driven pump (HTP-B, March May Ltd.). Correcting for an estimate of pressure drop in the tubes, we found a pressure drop through the electrodes \( Δp_{adv} \approx 0.2 \) bar at a superficial electrolyte velocity \( u_l = 4.25 \) mm/s. As shown in Appendix A.3 we can neglect the effect of the produced gas on the pressure drop, so that \( \frac{\Delta p}{\mu L} \approx \frac{\kappa g}{K} \). This gives \( K \approx 1.3 \times 10^{-13} \) m². Using this in Eq. (2), with \( \epsilon = 0.75 \) and \( d_{max} = 8 \) μm gives \( c_1 \approx 5 \times 10^{-3} \).

3.3. Cell voltage

Fig. 7 shows the recorded time-average (≈ 200 s) steady-state (after ≈ 40 s) cell voltage for configuration 1 of Fig. 5 with the Ni-felt electrode, as a function of liquid flow rate, for three different current densities. The cell voltage for the largest used flow rate was subtracted. The cell voltage can still be significantly reduced by applying a catalytic coating, reducing the gap width, and using an elevated temperature to increase the electrolyte conductivity. However, here we are interested in the increase in cell potential that is observed when the velocity drops below a few mm/s. The dramatic increase of almost 1.5 V for \( j = 4000 \) A/m², shown in Fig. 7, can only be explained by an additional ohmic drop \( \beta \) introduced by gas entering the gap. In the presence of a nylon mesh, as in configuration 1, the additional voltage disappears again upon applying a larger flow rate showing that bubbles can be pushed into the electrodes. Approximating the electrolyte conductivity as \( \kappa_0 (1 - \epsilon) \) [68], with \( l = 1.1 \) mm and \( \kappa_0 \approx 60 \) S/m [69], we find that a very large gas fraction \( \epsilon_g \approx 0.86 \) is required to explain the observed overpotential. Note that this is beyond the maximum packing of uniform spherical bubbles, requiring the gap to be almost completely filled with large gas slugs. To investigate this further, we made a dedicated visually easily accessible set-up, as detailed in the next section.

3.4. Visualization inside porous electrode

We designed and carried out an experiment to visualize what happens inside a gas-evolving porous electrode [70]. The mesh electrode of Table 1 is sandwiched horizontally between two perspex layers, see Fig. 6(a). Using a syringe pump, the electrolyte flows from right to left through the mesh electrode. Images from a camera mounted above the set-up are shown in Fig. 6(b) for two different flow rates, \( Q \approx 0.5 \) and \( \approx 9 \) ml/min. With a width of 1 cm and a height of 0.3 mm, this amounts to superficial velocities of 2.8 mm/s and 50 mm/s, respectively. At the highest flow rate, primarily very small bubbles can be seen, most of which move along with the liquid flow. For the lowest flow rate, however, very large gas slugs form, both inside the electrode, and in front of the electrode. Of course, this configuration is substantially different from that studied in the previous section. The much larger pores and the

![Fig. 5.](image-url) In Configuration 1, nylon meshes of thickness 53 μm, aperture \( d_{max} \approx 35 \)μm, and dimensions slightly larger than the electrodes, are pressed to the electrodes at their circumference by a 1 mm thick rubber gasket. In Configuration 2, a nylon mesh of thickness 1.1 mm and aperture \( d_{max} = 1.93 \) mm is used as a spacer between the electrodes.

![Fig. 6.](image-url) a) A schematic side-view of the visualization setup in which electrolyte flows from right to left through the mesh electrode while a camera records from the top b) Top-view images of a slug flow regime at an electrolyte velocity \( u_l \approx 3 \) mm/s, and a bubble flow regime at higher electrolyte flow rates of \( u_l \approx 50 \) mm/s.

| filter size | \( L \) (μm) | \( d_{max} \) (μm) | \( d_f \) (μm) | \( \epsilon \) [-] | \( a_0 \) (m⁻¹) |
|------------|-----------|----------------|---------|--------|-----------|
| Ni-felt    | 300       | 180            | 10      | 0.75   | 10⁵       |
| Ni-mesh    | 300       | 120            | 10      | 0.6    | 1.3×10⁴   |
| Nylon 1    | 53        | 35             | 31      | –      | –         |
| Nylon 2    | 1100      | 1930           | 550     | –      | –         |
The solid lines in Fig. 8 indicate Eq. (5) with a coefficient $c_2 = 5 \times 10^{-8}$ m$^4$/C/s, showing reasonable agreement for both $L = 300 \mu$m and 600 $\mu$m. With the parameters of Tables 1 and 2 this gives $s_0 \approx 0.97$, well within the range in which Eq. (5) was successfully verified against numerical simulations of Eq. (1).

We also performed similar experiments for configuration 2 of Fig. 5, showing that in this case a larger liquid velocity is required. The obtained value of $c_2 = 10^{-7}$ m$^4$/Cs corresponds with the values in Tables 1 and 2 with $s_0 \approx 0.98$. It seems likely that the hydrophilic nylon 1 with its relatively small aperture size of 35 $\mu$m of nylon 1, not too dissimilar from the pore sizes of the Ni felt shown in Fig. 4, avoids bubbles to enter the gap. Comparing with tests without a nylon spacer, an added advantage seems to be that bubbles trapped in the gap can be again removed upon increasing the flow rate. The aperture size of nylon 2 is however too large, both compared to the felt pore sizes as well as typical bubble sizes, to have a strong positive influence.

Finally, we also did an experiment with the Ni mesh in configuration 1. Here we found that for not too large current densities, bubbles in the gap could be largely avoided even without liquid flow. This shows that the hydrophilic nylon 1 for low current densities can act as a coarse separator for bubbles crossing over. Since electrolytically generated bubbles are typically of the order of 100 $\mu$m, this can be understood by the principle that such bubbles require additional surface deformation energy to cross into the gap.

4. Optimal design of membraneless flow-through electrolyzer

The experimental results from the previous section quantified what flow velocities allow operation without a real separator. Such a membraneless flow-through type of electrolyzer can operate with a significantly lower ohmic drop. However, thinner the gap, higher are the pumping losses. Therefore, a legitimate question is whether an overall benefit can be gained at all from using liquid flow and, if so, what the associated optimal parameters are. Using the experimental results of the previous section and simple well-established engineering relations, we will derive the parameters for such an optimal configuration. We define optimal here purely from the perspective of energy efficiency, leaving additional potential benefits in terms of hydrogen purity outside of our analysis.

3.5. Model validation

To compare the cell voltage measurements of Fig. 7 with the multi-phase flow-through electrode model of section 2 we somewhat arbitrarily define the minimum required liquid velocity as that which keeps the bubble-induced losses below 0.1 V, as shown in Fig. 7. The associated minimum liquid velocities are plotted in Fig. 8.

With the values from Table 2 we have $2a_{\delta}/L \approx 1.3$ A/cm$^2$ so that, in the range of current densities of Fig. 8, Eq. (5) predicts that the minimum liquid velocity scales approximately with $\sqrt{j/L}$. This proportionality is indeed approximately observed in the data of Fig. 8, where the required liquid velocity increases sub-linearly with current density. As an additional test, we also checked the dependence on electrode thickness by stacking two Ni felts together to give $L = 600 \mu$m. As predicted by Eq. (5) a lower liquid velocity is indeed required in this case.

Table 2

| Parameter | Value |
|-----------|-------|
| $\mu_i$  | 2 mPa s |
| $\mu_2$  | 8.4 $\mu$Pa s |
| $K$      | 40 S/m |
| $B$      | 50 mV |
| $\gamma_f$ | 25 L/mol |
| $\Gamma$ | 73 mN/m |
| $N$      | 2 – |
| Ni felt - conf. 1 | |
| $\rho_1$ | 0.37 bar |
| $K_1$    | 10$^{-33}$ m$^2$ |
| $\Lambda$| 4/3 – |
| $c_1$    | 5.10$^{-3}$ – |
| $c_2$    | 5.10$^{-8}$ m$^4$/Cs |
| $z_0$    | 0.97 – |

The presence of the perspex plates significantly influence the bubble shapes. Nevertheless, these observations show a plausible mechanism by which the measured voltage increase can be interpreted - due to large gas slugs accumulating in front of and inside the electrodes.

The measured minimum required liquid velocity $u_l$ to keep the bubble associated overpotential below 0.1, as outlined in Fig. 7. For configuration 1 (diamonds) we varied both the electrode thickness (circles) and the aperture size (crosses). The continuous lines show a comparison with Eq. (5) using $c_2 = 5 \times 10^{-8}$ m$^4$/Cs. For configuration 2 (squares) a larger value $c_2 = 1 \times 10^{-7}$ m$^4$/Cs is required to obtain a reasonable agreement, while for the Ni-mesh we do expect a bubbly flow regime in which the theory behind the equation does not hold.

Fig. 7. The relative cell voltage measured in configuration 1 with nickel felt electrodes, as a function of the superficial liquid velocity, for three different current densities. We subtracted the cell voltage electrodes, as a function of the superficial liquid velocity, for three different current densities, respectively. The horizontal solid line is the threshold of 100 mV used in Fig. 8.

Fig. 8. The measured minimum required liquid velocity $u_l$ to keep the bubble associated overpotential below 0.1, as outlined in Fig. 7. For configuration 1 (diamonds) we varied both the electrode thickness (circles) and the aperture size (crosses). The continuous lines show a comparison with Eq. (5) using $c_2 = 5 \times 10^{-8}$ m$^4$/Cs. For configuration 2 (squares) a larger value $c_2 = 1 \times 10^{-7}$ m$^4$/Cs is required to obtain a reasonable agreement, while for the Ni-mesh we do expect a bubbly flow regime in which the theory behind the equation does not hold.
4.1. Cell model

We use a simple model for the cell voltage $E_{cell} = E_{oc} + \eta + j/\kappa_0$ consisting of the open-circuit potential $E_{oc} = -\Delta G/F$, with $\Delta G$ the change in Gibbs free energy per mole of electrons involved, the activation overpotential $\eta_0$ at $x = 0$ given by Eq. (A.13) and ohmic drop over the gap $j/\kappa_0$. Of the total power used per unit area per cross-sectional gap area $A = hw$, $E_{cell}$, only $E_{oc}$ can theoretically be recovered as electrical power in an ideal fuel cell, so we may define an energy efficiency as

$$\eta = \frac{jE_{oc}}{jE_{oc} + \eta_0 + j/\kappa_0} + P_{pump}/A$$

where $P_{pump}/A$ is power used to pump the electrolyte, per unit electrode area $A$. We will assume that the total pressure drop

$$\Delta p = N\Delta p_{tot}, \quad \text{with } N > 1,$$

so that the pressure drop through the electrodes is much larger than that through the gap, so that the flow will distribute itself more or less homogeneously over the electrode surface. When the liquid, entering with a velocity $u_0 = 2m/\ell$, leaves homogeneously distributed along the height $h$ of the electrodes under laminar conditions, the frictional pressure drop over the gap from bottom to top is $\Delta p_{gap} = 6\nu h u_0 / \ell^2$ [71, 72], or half the Hagen-Poiseuille pressure drop in case all the fluid would leave from the top.

The minimally required pumping power is given by the pressure drop times volumetric flow rate so that

$$\frac{P_{pump}}{A} = 2u_0 \Delta p, \quad \Delta p = \frac{12N\mu_h r^3}{\nu} \frac{\Delta h}{\ell}.$$  

(8)

Note that due to Eq. (7), in order to avoid flow maldistributions, the pressure drop is independent of electrode properties and depends only on properties of the gap.

4.2. Optimization

Equation. (5) shows that the minimum liquid velocity to avoid gas in the gap is minimized when $L = 2e \kappa b/\ell$, in which case

$$u_l = j \sqrt{\frac{12\Delta h_{max}}{2b}}$$

(9)

independent of the electrode and gap thicknesses $L$ and $l$, respectively.

The pumping losses of Eq. (8) may be minimized by using as small as possible height $h$, which will however be limited by design practicalities. Alternatively, increasing the gap width $l$ decreases the pumping losses, but increases the ohmic dissipation. The optimum is found by solving $\phi = 0$ or $dE_{el}/dl = 0$ for $l$, which gives with Eqs. (8) and (9)

$$l_{opt} = \left( \frac{72\mu h N}{14} \right)^{1/4} \sqrt{\frac{\Delta h}{\ell}}$$

(10)

where $36N\mu h/N = 48N\mu h^2/\ell$, $\Delta h_{max}$ [m]. The first result in Eq. (10) becomes that of Ref. [73] for a parallel plate electrolyzer, upon replacing $2Nh^2$ with $\ell^2$, reflecting the different flow outlet conditions. Using the values from Table 2, along with $\Delta p_{tot}/\Delta p_{pump} = 10$ so that $N = 11$, and $h = 0.1$ m gives $l_{opt} \approx 0.5$ mm, similar to a typical commercial separator thickness [27,74], but with no solid material in the way giving a significantly lower ohmic resistance.

4.3. Pressure drop

Inserting Eq. (10) we find $\nu = P_{pump}/A = \Delta p l / \nu$ so that the pumping contribution to the total losses only 1/3-rd of the ohmic losses. With Eqs. (8)-(10)

$$\Delta p = \left( \frac{N\mu h}{18\ell^4} \right)^{1/4} \sqrt{\frac{\Delta h}{\ell}}.$$  

(11)

With $N = 11$, $h = 0.1$ m, $j = 0.3$ A/cm$^2$ and the properties from Table 1 and the Ni-felt of Table 1 this gives a modest pressure drop $\Delta p = 0.1$ bar. With $\Delta p = \Delta p_{el} + \Delta p_{pump}$ Eq. (7) implies that $\Delta p_{el} = \frac{K_f}{\ell} \Delta p$, which with Eq. (A.18) and (8) gives

$$K_f = \frac{\nu L}{12(N - 1) h^2} \ell.$$  

(12)

This criterion is required to give the electrodes sufficient hydraulic resistance to ensure a homogeneous flow distribution. With the previously obtained optimal value $l = 0.5$ mm, $N = 11$, $h = 0.1$ m, and say $L = 1$ mm, this gives $K_f = 10^{-13}$ m$^2$, equal to that of the Ni-felt of Tables 1 and 2.

4.4. Mass transfer-limited

Finally, we consider the case where no gaseous products are evolved, but we instead want to use the flow to aid diffusive mass transfer in supplying reactants. In this case we replace Eq. (9) with $u_l = j/nFc$, the minimum liquid velocity required to supply enough reactants with inlet concentration $C$. To avoid back-flow of reaction products requires the Péclet number $Pe = u_L/\ell = jL/nFCD = 1$, with $D$ the effective diffusivity of the product. This is often satisfied. Using the expression for the conductivity of a dilute electrolyte [59], we see that usually $2k_2 g_{2FCD}$ when the concentration of the electrolyte exceeds that of the reactants. In this case we automatically have $Pe = 1$ when $jL/2k_2b_2 = 1$, a criterion preferable to satisfy in order to avoid thermodynamic overpotential of Eq. (A.13). Since the minimum velocity is independent of $l$ we can use Eq. (10) to give

$$l_{opt} = \left( \frac{72\mu h N}{14} \right)^{1/4} \sqrt{\frac{\Delta h}{\ell}}.$$  

(13)

As an example we consider CO$_2$-reduction [24,42,75,76] to a non-gaseous product like formate. In a not too strong aqueous electrolyte, the CO$_2$ solubility at ambient pressure is around $C = 30$ mM, so that with $N = 11$, $\mu = 1$ mPas, $h = 0.1$ m, $n = 2$, and $k = 10$ S/m we obtain $l_{opt} < 0.2$ mm, giving a conductance that is competitive with most anion exchange membranes [77].

Inserting Eq. (13) and $u_l = j/nFc$ in Eq. (8) gives $\Delta p = \left( N\mu /18c^2 \right)^{1/4} \sqrt{\nu FCD}$. Using the same numbers as before, for a current density of $0.3$ A/cm$^2$ we require a liquid velocity $u_l \approx 0.5$ mm/s, requiring roughly 0.6 bar of pressure drop.

5. Discussion

We have made various simplifications in the analysis that warrant further discussion. First of all, our assumption of a flow regime in which the gas forms a continuous phase rather than dispersed bubbles is likely valid for small enough pores, but the exact conditions under which Darcy’s law holds are still under investigation [50]. For definiteness, we assumed power-law dependences on the liquid saturation of the relative permeabilities and capillary pressure in Eq. (1), but these can equally be replaced by different forms in the derivation of Eq. (5). Note that, since the derivation considers high liquid saturations, the model parameters are ideally obtained from empirical fits in this same range.

The free parameter $s_0$, representing the liquid saturation at the gap-electrode interface, is influenced by flow impacting bubble nucleation and release through drag and transport of dissolved reaction products. We found that $s_0$ could be reduced by adding a nylon spacer. The hydrophilic nature and relatively small aperture size likely gives it some of
the characteristics of a separator, albeit of much lower thickness and cost.

For simplicity, we did not include the impact of a spacer on the pressure drop in the gap. The factor \( N \) may be multiplied by a factor denoting deviation from the used Hagen-Poiseuille equation, and additionally divided by the pump efficiency. Given the very weak dependence of the optimal gap thickness of Eq. (10) on \( N \), including these effects will only result in relatively small corrections. Spacers have been studied intensively for filtration and reverse osmosis applications, where improving mass transport is important. Increased mass transport also tends to lead to increased pressure drop, and typically a supra-linear dependence of pressure drop on velocity is observed due to inertial effects \([76]\). Spacers may be designed for low tortuosity by avoiding direction changes using, for example, primarily flow-aligned fibres.

We considered the porous anode and cathode to have similar properties and focussed on hydrogen evolution in application to water electrolysis. We note that the dynamic viscosity \( \mu_g \) of oxygen is roughly twice that of hydrogen, but also \( n = 4 \) is double that for hydrogen evolution so that the relevant ratio \( \mu_g/n \) in \( C_2 \) is similar.

Besides using smaller pores or thicker electrodes, a smaller electrode permeability \( K \), if required by Eq. (12), may also be obtained by deploying an additional layer behind the electrodes with sufficient hydraulic resistance. The use of a smaller pore size however, counter-intuitively, has the advantage of reducing the overall pressure drop according to Eq. (11). The reason is of course that, by Eq. (5), smaller pores are predicted to require lower velocities to avoid bubbles entering the gap.

6. Conclusion

We studied here the potential of flow-through electrodes for membraneless operation in case of gas-evolving reactions. To minimize ohmic dissipation, a small gap between the electrodes is preferred, increasing pumping losses. Therefore, an important parameter is the minimally required flow velocity that allows effective membraneless operation without bubbles entering the gap. Previously studied bubble models cannot give insight into this parameter, nor are they suitable to describe electrodes with small pores. Small pores are preferred for their high reactive surface area and sufficient pressure drop to allow a homogeneous flow distribution. Therefore, we studied a multiphase Darcy-flow model from which we derived a simple analytical relation, Eq. (5), for the required liquid velocity. This relation was successfully verified with respect to numerical simulations and validated against experimental data.

A balance between ohmic and pumping losses lead to an explicit expression, Eq. (10), for the optimal gap thickness, showing that a significant reduction in resistance is possible compared to operation with a membrane. The optimal gap thickness depends most sensitively on the desired electrode height, dictated by practical design considerations, and only very weakly on material parameters of the electrolyte, electrode, and gap spacers.

We found that, up to modest current densities, a coarse Ni-mesh allowed operating without any liquid flow, when a finer simple nylon spacer was placed directly adjacent to the electrodes inside the gap. However, it seems likely that significant gas cross-over takes place in this case, while cross-over can be strongly reduced in the presence of liquid flow compared to operation with a membrane \([27]\). Also the use of coarse mesh electrodes, as in Refs. \([14,15,79]\), requires relatively large flow velocities of many centimetres per seconds at higher current densities, and relatively large gap widths of a millimetre or more, even for small electrode dimensions of a few centimetres, to avoid flow maldistribution. We have shown that effective membrane-less operation with much smaller pores can be obtained at much smaller velocities, gap widths, with reasonable pressure drops. These flow velocities are nonetheless high enough to avoid mass transport limitations associated with dilute reactants and removal of products, strongly reducing the concentration polarization typically present in systems with a membrane, and potentially improving the product purity.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Activation overpotential

Appendix A.1. General equations

The superficial gas velocity is given by Faraday’s law as

\[
 u_s = \frac{V_a}{\epsilon F},
\]

where \( i \) is the electronic current density, which varies from \( i = 0 \) at \( x = 0 \) to \( i = \) j at \( x = L \). Assuming the electrode conductivity is much higher than the electrolyte conductivity and assuming negligible concentration gradients of electrolyte and reactants, Ohm’s law and the Tafel equation can be written as \([80]\).

\[
 \frac{di}{dx} = i - \frac{d\eta}{dx},
\]

\[
 \frac{d\eta}{dx} = j_a e^{\xi b}.
\]

Here the surface overpotential \( \eta \) is equal to a constant minus the electrolyte potential, \( a \) is the active volumetric surface area, and \( b \) is the Tafel slope. We assumed the effective electrode conductivity can be written as \( s x^m \) with Archie’s exponent \( m \), often taken to be 1.5 or 2, to account for the presence of gas \([81,82]\). Equation A.3 gives the overpotential at \( x = 0 \) as
\[ \eta_0 = b \ln \left( \frac{j}{aLj_e \nu} \right), \]  
(A.4)

where the effectiveness factor \( \zeta = \frac{j}{j_e} \).

### Appendix A.2. Bubble flow regime

For completeness and comparison we provide here the analysis for larger pore sizes in which a bubble flow regime may be assumed. The average interstitial gas and liquid velocities, \( \nu_g \) and \( \nu_l \), are related to superficial velocities, \( u_g \) and \( u_l \), as

\[ \nu_s = \frac{u_s}{e(1 - s)} \quad \nu_l = \frac{u_l}{eS} \]  
(A.5)

Assuming the gas bubbles move with a fraction \( f \equiv \nu_g/\nu_l \) of the liquid velocity, Eqs. (A.5) and (A.1) combine to give

\[ s = \frac{1}{1 + ri/(b)} \quad r \equiv \frac{jV_m}{fa \nu_l F} \]  
(A.6)

with \( rf \) the ratio \( u_g/u_l \) at \( x = L \). Taking a derivative of Eq. (A.3) with respect to \( x \) and inserting Eqs. (A.2) and (A.6) gives

\[ \frac{d^2i}{dx^2} = \left( 1 + \frac{r}{b} \right)^m \frac{di}{dx} \]  
(A.7)

With \( z \equiv \frac{r}{b} \), we have \( \frac{d^2i}{dx^2} = \frac{d^2i}{dz^2} \) so that Eq. (A.7), using Eq. (A.3), can be written as

\[ \frac{d}{dz} \left( j_s \nu_l^{m+1} \right) = \left( 1 + \frac{r}{b} \right)^{m+1} \frac{di}{dz} \]  
(A.8)

For constant parameters, we can integrate with respect to \( i \) between \( i(0) = 0 \) and \( i(L) = j \) to give

\[ e^{\eta_{b/b}} - e^{\eta_{l/b}} = \frac{J^2}{2Ja \nu_l F} \]  
(A.9)

where

\[ \kappa_{\text{eff}} = \frac{2}{r(1 + m)} \left( \frac{(r + 1)^{m+1} - 1}{r(2 + m)} - 1 \right). \]  
(A.10)

In the presence of bubbles, the effective conductivity \( \kappa_{\text{eff}} \) decreases below \( \kappa \). With, for example \( m = 1, 2 \) or 3, Eq. (A.10) simplifies to

\[ \kappa_{\text{eff}} = 1 + r \times \left\{ \begin{array}{ll} \frac{1}{3} & m = 1 \\ \frac{2 + r}{3m} & m = 2 \\ \frac{1 + r + \frac{r^2}{10}}{m} & m = 3. \end{array} \right. \]  
(A.11)

For high current densities we can neglect the second term on the left-hand side of Eq. (A.9) with respect to the first, to obtain Eq. (A.4) with the electrochemical Thiele modulus \( 1/\kappa_{\text{eff}} \nu_l = \frac{JL}{2\kappa_{\text{eff}}b} \). This is equal to the ohmic drop of ions reacting halfway the electrode at this effective conductivity [80,83,84]. Using the approximation of Ref. [80] we may write

\[ \zeta \approx \frac{1}{1 + JL/2\kappa_{\text{eff}}b} \]  
(A.12)

With this, a general expression

\[ \eta_0 \approx b \ln \left( \frac{j}{j_s \nu_{l} + \frac{J^2}{2Ja \nu_l F}} \right) \]  
(A.13)

is obtained. To explain the measured overpotentials of 0.1–0.5 V in Ref. [42] required unrealistically high gas fractions \( \epsilon_g \approx 0.85 \) associated with a very small slip factor \( f = \nu_g/\nu_l \approx 0.0016 \). Visual observations from our set-up of Fig. 6 rather showed \( f \approx 1.8 \) [70].

### Appendix A.3. Darcy flow regime

As mentioned in the main text, numerical simulations with \( s_0 \approx 1 \) show that over most of the domain the two terms of Eq. (1) approximately balance so that

\[ s \approx \frac{1}{1 + \left( \frac{aL}{s_0} \right)^{1/q}} \]  
(A.14)

Comparing with Eq. (A.6) we can redo the above analysis replacing \( ri/j \) with \( R(ri/j)^{1/q} \) in Eq. (A.8), where
\[ R^\prime = \frac{\mu_k}{\mu_l} \frac{V_m}{nF u_l} = \frac{\mu_p}{\mu_l} \frac{V_m}{nF} \sqrt{\frac{2 \kappa_2}{cG_{\text{max}}}} \]  
\hspace{1cm} \text{(A.15)}

is the ratio between gas and liquid velocities at \( x = L \) times the ratio \( \mu_k/\mu_l \). In the second equality of Eq. (A.15) we inserted Eq. (9). Integrating Eq. (A.8) with \( R (i/j)^{1/q} \) instead of \( r/j \) gives, for example with \( q = 3 \)

\[ \frac{\kappa}{\kappa_{\text{eff}}} = 1 + \frac{R}{\mu_k} \left[ \frac{9}{14} + \frac{9R}{20} \right] m = 1 \]
\[ \frac{\kappa}{\kappa_{\text{eff}}} = 1 + \frac{R}{\mu_k} \left[ \frac{27}{14} + \frac{27R}{20} + \frac{R^2}{3} \right] m = 2 \]
\[ \frac{\kappa}{\kappa_{\text{eff}}} = 1 + \frac{R}{\mu_k} \left[ \frac{9k^2}{4} + \frac{k^3}{2} \right] \]
\hspace{1cm} \text{(A.16)}

With the values from Tables 1 and 2 for Ni-felt and, say, \( C = 30 \text{ mM} \), we obtain \( R = 0.1 \). From Eq. (A.16) this gives \( \frac{\kappa}{\kappa_{\text{eff}}} \leq 1.2 \) when \( m \leq 3 \), so only a small correction.

We conclude with providing an approximation for the pressure drop over the electrode in the presence of gas. Assuming \( \varepsilon \approx 1 \) we have \( \varepsilon = j x/L \) so that Eq. (3) gives \( u_k \approx \frac{u_0}{\sqrt{\mu_k}} \). Using this in Eq. (A.14) and integrating the liquid phase pressure gradient, the second term in Eq. (1), gives with for example \( q = 3 \)

\[ \frac{\kappa_{\text{eff}}}{\kappa} \approx \int_0^k \left( 1 + \frac{R}{\mu_k} \left[ \frac{9k^2}{4} + \frac{k^3}{2} \right] \right) \]  
\[ \approx 1 + \frac{9k^2}{4} + \frac{k^3}{2} \]  
\hspace{1cm} \text{(A.17)}

Here \( k^3 = \frac{u_0}{\sqrt{\mu_k}} \frac{m}{\sqrt{\mu_k}} \frac{m}{\sqrt{\mu_k}} \) where in the second equality we used Eq. (5) assuming \( \varepsilon \approx 1 \). So we see from Eq. (A.17) that, as long as \( k \ll 1 \), deviations from the single-phase pressure drop:

\[ \Delta P_{\text{el}} = \frac{\mu_k j x}{\kappa_l} L (k \ll 1), \]
\hspace{1cm} \text{(A.18)}

will be small. With the values from Tables 1 and 2 for nickel felt, we obtain with \( j = 0.5 \text{ A/cm}^2 \) a value \( k \approx 0.1 \), so that we can approximately neglect the effect of gas on the pressure drop.

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