Study on intelligent control of yaw stability of electric vehicle with in-wheel motors

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Abstract: In order to improve yaw stability of electric vehicles with in-wheel motors, a yaw moment fuzzy control method based on yaw rate and side-slip angle is designed. A hierarchical control structure was set up. The upper layer is motion tracking layer, which is composed of yaw angle and side-slip angle. When the side-slip angle is small, the heading angle is mainly determined by yaw angle. At this time, the actual yaw rate can determine the vehicle's stability; but when the side-slip angle is large, the yaw angle can no longer represent the vehicle's track. The side-slip angle can better reflect the yaw stability of the vehicle.

1 Introduction

An electric vehicle with four independently driven in-wheel motors has the torque and driving/braking of its four wheels controllable independently, at the same time, the torque response of the wheel motor is prompt and accurate, so to bring new development possibilities for upgrading the vehicle stability [1].

Comparing with traditional vehicles, the quantity of actuator is increased. An electric vehicle with four independently driven in-wheel motors is an actuator redundant control system [2]. Additional yaw moment is obtained by feed-forward control based on side-slip angle and error feedback control based on yaw rate and side-slip angle. Then a simple average distribution method is used to distribute the moment, i.e. the two sides moments have the same magnitude and opposite direction. The hierarchical control structure of yaw moment is determined in [3]. Based on the fuzzy control theory, a decision-making controller of additional yaw moment is designed, and the average distribution method in [2] is used to distribute the four-wheel driving moment. In [4], the torque distribution theory is transformed the required yaw moment into an optimum problem, and obtained the longitudinal force of four wheels under actuator and adhesion coefficient constraints, so to control vehicle stability. However, under extreme conditions, due to the limitation of motor performance, the theoretical optimal allocation may not guarantee the optimal stability control, which makes the study of stability control under motor–hydraulic composite action of great significance.

For this, this paper proposes a strategy of stability moment allocation with combined motor/hydraulic system, which can rationally optimise the allocation of additional yaw moment and improve the stability of vehicles under extreme conditions.

2 Structure of integrated control system

Vehicle stability control mainly involves track maintenance and stability control. Generally, the track problem can be described by the side-slip angle, while the stability is described by the yaw rate. These two control variables are mutually coupled [5]. At the same time, the vehicle track is determined by the vehicle's heading angle, which is composed of yaw angle and side-slip angle. When the side-slip angle is small, the heading angle is mainly determined by yaw angle. At this time, the actual yaw rate can determine the vehicle's stability; but when the side-slip angle is large, the yaw angle can no longer represent the vehicle's track. The side-slip angle can better reflect the yaw stability of the vehicle.

In order to improve the vehicle stability, yaw rate and side-slip angle are selected as control variables, for multi-objective control, and design of hierarchical control structure. As shown in Fig. 1, the upper layer is the motion tracking layer. Based on the control algorithm of reference model tracking, according to the current state feedback and the ideal state of the reference model, the additional yaw moment needed for control is calculated by using the fuzzy control theory. The lower layer is the moment distribution control layer, which takes full account of various constraints and distributes the additional yaw moment to each wheel to realise the active yaw moment control of the vehicle.

3 Motion tracking layer

3.1 Reference model

When the vehicle steers on the well-adhered road surface with a small lateral acceleration (generally considered <0.4 g), the vehicle operating characteristics can be approximately described by a linear two-degree-of-freedom (TDF) vehicle model [6]. In this paper, a linear TDF monorail vehicle model is selected as the reference model. In order to ensure the performance of the control system, zero side-slip angle is the driver’s desired state, so we take zero as nominal side-slip angle, while steady yaw rate is taken as the desired state, considering the restrictions of road adhesion conditions. The nominal side-slip angle and nominal yaw rate are obtained [7]:

\[
\dot{\beta}_y = 0
\]

\[
\alpha_{wd} = \min \left[ \frac{v}{L(1 + K_v)} \cdot \left| \frac{0.85 \delta f}{v} \right| \cdot \text{sgn}(\delta_f) \right]
\]

\[
K = \left( \frac{m}{L} \right) \left( \frac{a}{v^2} - \frac{b}{v} \right)
\]

Among which, \(\dot{\beta}_y\) is the nominal side-slip angle, \(\alpha_{wd}\) is the nominal yaw rate, \(v\) is the vehicle velocity, \(L\) is the wheel base, \(\delta_f\) is the
The design of fuzzy controller includes fuzzification, fuzzy reasoning and anti-fuzzification [8]. The amount of input and output are described by seven linguistic variables: negative big (NB), negative medium (NM), negative small (NS), zero (ZO), positive small (PS), positive medium (PM), and positive big (PB). The fuzzy subset of input variables is {NB, NS, ZO, PS, PB}, and the fuzzy subset of output variables is {NB, NM, NS, ZO, PS, PM, PB}. The quantisation domains are [−1, +1]. The membership functions are all triangular functions, as shown in Figs. 2 and 3.

Considering the influence of yaw moment on yaw rate and side-slip angle, the fuzzy control rules shown in Table 1 are obtained. The fuzzy controller uses the Mamdani method in the form of rules IF–THEN for fuzzy reasoning, and uses the gravity center method to de-fuzzify, so as to obtain the additional yaw moment needed to control vehicle stability ΔM.

### Table 1 Fuzzy rules table

| Δβ     | NB | NS | Δωr  | ZO | PS | PB |
|--------|----|----|------|----|----|----|
| NB     | PB | PB | NB   | NS | NM | PB |
| NS     | PB | PM | NB   | NM | PB | PB |
| ZO     | PM | PS | NB   | NM | NS | PB |
| PS     | PB | PM | NB   | NS | NM | PB |
| PB     | PB | PS | PB   | NS | PB | PB |

3.2 Motion controller

In order to obtain the desired vehicle dynamic performance, a yaw moment decision-making fuzzy controller is set up based on the fuzzy control theory. The input is the difference between the actual yaw rate and the expected value Δωr, the difference between the actual and the expected value of the side-slip angle Δβ, and the output is the additional yaw moment to keep the vehicle running steadily ΔM.

4 Torque distribution layer

The advantage of wheel-motor-driven electric vehicle is that the torque of each wheel is independently controllable and its response is fast. The disadvantage is that the peak power of the existing wheel motor at high speed is limited, and it may not meet the torque requirements of stability control under extreme conditions [9]. Therefore, under the premise of fully considering actuator constraints and tire longitudinal and lateral force coupling, the torque allocation controller adopts the WLS to optimise the generalised force requested by vehicle stability control via the combination of hydraulic system and motor system [10], so as to ensure that the potential of each wheel is fully exploited and the handling stability and active safety under extreme conditions are guaranteed.

4.1 Weighted least square method

Quadratic programming is widely applied in mathematical programming. The sequential least square is the evolution of standard quadratic programming. To solve the complexity of the calculation of sequential least square, weighted least square is generated. The mathematical expression of the WLS is as follows:

\[
\begin{align*}
    u &= \arg \min_{\bar{u}} \left( \| W_d (\bar{u} - u_d) \|^2 + \gamma \| W_f (Bu - v) \|^2 \right) \\
    \text{s.t.} & \quad \left\{ \begin{array}{l} \\
    Bu = v \\
    C\bar{u} \geq U
    \end{array} \right.
\end{align*}
\]

The WLS problem with constraints is often solved by the efficient set algorithm. The standard form of the effective set is

\[
\min_u \quad J = \| Au - b \|^2 \\
\text{s.t.} \quad \left\{ \begin{array}{l} \\
    Bu = v \\
    C\bar{u} \geq U
    \end{array} \right.
\]

Among which, \( \gamma \) and \( \bar{F} \) are lower and upper bounds, \( W_d \) and \( W_f \) are non-singular weighting matrices, \( \gamma \) is the weighting factor, \( u \) is the control vector, \( u_d \) is the preferred control vector. Other parameters are described in Section 4.3.

4.2 Optimising objectives and constraints

In order to improve the vehicle stability, the tire utilisation ratio should be optimised to obtain the optimal tire force distribution when the additional yaw moment is optimised [11]. Since the tire lateral force is hard to control, and according to the coupling relationship between the longitudinal force and lateral force of the friction circle, the longitudinal force of the tire is minimised to maintain a lower tire utilisation rate, thereby improving the stability margin of the tire and the vehicle. Therefore, the
optimisation objective function is established by introducing the minimum longitudinal tire force utilisation ratio of the whole vehicle

\[
\min J = \sum_i \frac{F_{zi}^2}{(\mu_i F_{zfr})^2}, \quad i = ff, fl, rf, rr
\]  

(5)

where \( \mu_i \) is the adhesion coefficient, \( F_{zi} \) is the longitudinal force of the wheel, and \( F_{zfr} \) is the vertical force of the wheel.

Considering the restriction of the road adhesion condition, the restriction conditions of the longitudinal force of each wheel are obtained

\[
|F_{zi}| \leq \sqrt{\mu_i F_{zfr}^2 - F_{zi}^2} \leq \mu_i F_{zfr}, \quad i = ff, fl, rf, rr
\]  

(6)

where \( F_{zfr} \) is the lateral force of the wheel.

If the motor system alone is used as the actuator, the constraints of the wheel motor execution capability are considered:

\[
T_{bmul, r} \leq F_{zi} \leq T_{dmul, r}, \quad i = ff, fl, rf, rr
\]  

(7)

where \( T_{bmul} \) and \( T_{dmul} \) represent the maximum braking moment and maximum driving moment of the motor at the current speed, respectively, and \( r \) is the rolling radius of the wheel.

If the motor system and the hydraulic system act as actuators at the same time, neglecting the braking energy recovery and considering only the stability as the goal, the restriction condition of the wheel motor's execution ability is canceled at this time, and the corresponding hydraulic logic relationship is established to assist the compensation of the hydraulic braking system.

Taking the left front wheel as an example, if the longitudinal force allocated to the wheel is the driving force, which is greater than the motor's execution ability, the right front wheel is added with hydraulic braking, thus forming the equivalent yaw moment

\[
\text{if } F_{fl} > T_{dmul, r} \Rightarrow F_{fl} = T_{dmul, r} \quad F_{fl} = F_{fl} - (F_{fl} - T_{dmul, r})
\]

\[
\text{else if } F_{fl} = F_{fl, l} \quad F_{fl} = F_{fl}
\]

where \( T_{dmul} \) is the longitudinal driving force allocated to the left front wheel, \( F_{fl} - T_{dmul, r} \) is hydraulic compensating braking force, \( F_{fl} \) is the final longitudinal force of the left front wheel and \( F_{fl} \) is the final longitudinal force of the right front wheel.

Taking the left front wheel as an example, if the longitudinal force allocated to the wheel is the braking force, which is greater than the motor's execution ability, added hydraulic braking to this wheel to compensate. The detailed hydraulic logic relationship is as follows:

\[
\text{if } F_{fl} > T_{bmul, r} \quad F_{fl} = T_{bmul, r} + (F_{fl} - T_{bmul, r}) \quad F_{fl} = F_{fl}
\]

\[
\text{else if } F_{fl} = F_{fl, l} \quad F_{fl} = F_{fl}
\]

where \( F_{fl, l} \) is the longitudinal braking force distributed to the left front wheel and \( F_{fl, l} - T_{bmul, r} \) is hydraulic compensated braking force.

4.3 Torque optimal distribution algorithm

According to (5), the optimal objective considers only the distribution of longitudinal force, and the simplified force analysis of stability control of an electric vehicle with four independently driven in-wheel motors is shown in Fig. 4.

According to the vehicle dynamics equation, we can get

\[
\begin{align*}
F_{zfr} & = F_{xffl, r} \cos \delta_f + F_{xfrl, r} \cos \delta_f + F_{xfrl, r} + F_{xfrl, r} \\
\Delta M & = \left( -\frac{d}{2} \cos \delta_f + \sin \delta_f \right) F_{xffl, r} + \left( \frac{d}{2} \cos \delta_f \right. \\
& \quad + \sin \delta_f ) F_{xfrl, r} - \frac{d}{2} F_{xfrl, r} + \frac{d}{2} F_{xfrl, r}
\end{align*}
\]  

(8)

where \( F_{zfr} \) is the total longitudinal force required for maintain vehicle movement, obtained from the actual vehicle speed and the expected vehicle speed through the proportional integral controller [12].

Carry out static allocation, ignoring the dynamic response of the actuator, together with (8), we get

\[
v = Bu
\]  

(9)

\[
u = [F_{zfr}, \Delta M]^T
\]

\[
B = \begin{bmatrix}
\cos \delta_f & -\frac{d}{2} \cos \delta_f + \sin \delta_f \\
\cos \delta_f & \frac{d}{2} \cos \delta_f + \sin \delta_f \\
1 & -\frac{d}{2} \\
1 & \frac{d}{2}
\end{bmatrix}
\]

\[
u = [F_{xffl, r}, F_{xfrl, r}]^T
\]

According to the above optimisation objectives and constraints, the moment allocation problem is transformed into a WLS problem. In order to facilitate the use of efficient set algorithm to solve the problem, the standard form of effective set is obtained by simple transformation

\[
u = \arg \min_u \left( \| W_u (u - u_d) \|_2^2 + \gamma \| W_v (Bu - v) \|_2^2 \right)
\]

(10)

where

\[
W_u = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & F_{zfl, r} & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
u_d = (0, 0)
\]

\[
W_v = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]
In order to verify the effectiveness of the proposed control method, Simulink, a non-linear seven-degree-of-freedom vehicle model combined with magic formula tire model is set up. This vehicle real-time simulation system. In the environment of MATLAB/ Simulink, a non-linear seven-degree-of-freedom vehicle model combined with magic formula tire model is set up. This vehicle real-time simulation system. In the environment of MATLAB/Simulink, a non-linear seven-degree-of-freedom vehicle model combined with magic formula tire model is set up. This vehicle real-time simulation system.

**5 Simulation experiment and result analysis**

In order to verify the effectiveness of the proposed control method, a rapid control prototype test bench is set up based on the dSPACE real-time simulation system. In the environment of MATLAB/Simulink, a non-linear seven-degree-of-freedom vehicle model combined with magic formula tire model is set up. This vehicle model includes the longitudinal, lateral and yaw motion of the vehicle body, and the rotation motion of four wheels. The control algorithm is designed and compiled by both real-time interface and real-time work-shop and downloaded to the dSPACE real-time processor DS1005 to run. Finally, the experimental process and online debugging are monitored and managed by Control Desk software.

![Fig. 5 Input signal of front wheel steering angle](image)

![Fig. 6 Yaw rate simulation result](image)

![Fig. 7 Side-slip angle simulation result](image)

![Fig. 8 Phase diagram of side-slip angle and yaw rate](image)

$W_u$ is the controlled input weight matrix, $u_d$ is expected control input vector, $W_r$ is distribution demand weight matrix, $\gamma$ is weight coefficient, $u$ is upper limit of the control input vector, $u$ is lower limit of the control input vector.

In order to better track the total longitudinal force and additional yaw moment required for control, it usually takes a larger value.

**Table 2** Partial parameters of vehicle model

| Parameters                        | Value   |
|-----------------------------------|---------|
| vehicle mass m/kg                 | 1580    |
| distance between the center of mass to the front axle a/m | 1.237   |
| distance between the center of mass to the rear axle b/m | 1.303   |
| yaw rotational inertia $I_x/(kg \cdot m^2)$ | 2360    |
| centroid height h/m               | 0.552   |
| effective rotate radius of wheel R/m | 0.317   |
| wheel base d/m                    | 1.42    |
| cornering stiffness of the front axle $k_f/(kN/\text{rad})$ | $-50,000$ |
| cornering stiffness of the rear axle $k_r/(kN/\text{rad})$ | $-60,000$ |

**Table 3** Status parameters comparison

| Item                              | Maximum yaw rate deviation, rad/s | Maximum side-slip angle deviation, rad |
|-----------------------------------|----------------------------------|---------------------------------------|
| even distribution                 | 0.22454                          | 0.06694                               |
| optimal distribution with motor only | 0.22370                          | 0.05034                               |
| optimal distribution with hydraulic assistance | 0.09235                          | 0.01200                               |

**Simulation conditions:** Adhesion coefficient is 0.8, vehicle velocity is 80 km/h, and steering wheel sinusoidal angle input is as shown in Fig. 5. Some parameters used in the vehicle model are shown in Table 2.

The simulation results are shown in Figs. 6–8, which shows the control of vehicle under different torque distributions: vehicle response characteristics under even torque distribution, under optimal torque distribution involving motor only and under optimal torque distribution with combined hydraulic brake system auxiliary control.

Figs. 6 and 7 are response curves of yaw rate and side-slip angle under different control modes, respectively. Table 3 shows the comparative analysis results of maximum deviation under three torque allocation modes. We can see, compared with the average distribution of torque, these two optimal distribution methods can better track the expected value, restrain the instability of the vehicle, maintain the driving stability of the vehicle, thus reducing the driver's operation burden. At the same time, compared with the motor/hydraulic auxiliary control methods, it is found that the control results of motor only will have slight variation. The maximum deviation value of yaw rate is 0.2237 rad/s and the maximum deviation of side-slip angle is 0.05034 rad. Under combined hydraulic auxiliary control, the maximum deviation of yaw rate can be reduced by ~59% and the maximum deviation of side-slip angle can be reduced by ~76% in.

Fig. 8 is the phase diagram of side-slip angle and yaw rate. Compared with the even distribution of torque, the optimal distribution of torque converges eventually and is a stable process. At the same time, the convergence area of the optimal distribution of combined hydraulic brake system assistance is smaller and more stable than that of the optimal distribution of motor only, which shows that the control effect of the combined hydraulic brake system aided control mode is stable and faster. It can maximise the yaw stability of the vehicle under high-speed steering control.

**6 Conclusion**

In order to improving yaw stability of an electric vehicle with four independently driven in-wheel motors, the author has studied the fuzzy control method for yaw moment controlled jointly by yaw rate and side-slip angle.

This paper introduces a hierarchical control structure, designs a fuzzy controller with additional yaw moment decision-making and an optimal torque allocation control algorithm, and adds a hydraulic brake system for auxiliary control.
Based on the dSPACE real-time simulation system, a rapid control prototype test bench was set up. The simulation experiments were carried out under sinusoidal steering conditions under different torque distribution control modes. The results show that the optimal yaw moment distribution control strategy assisted by the combined hydraulic brake system can better track the expected values and improve the vehicle handling stability, taking full account of various constraints.

7 Acknowledgments

This project is Supported by Scientific and Technological Research Program of Chongqing Municipal Education Commission (Grant no. KJ1600911).

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