Decomposition of nano $\alpha$-I-open sets

V Inthumathi, M Parveen Banu and R Abinprakash
Department of Mathematics, Nallamuthu Gounder Mahalingam College, Pollachi, Coimbatore, Tamilnadu-642001, India.
E-mail: inthumathi65@gmail.com

Abstract. In this paper we decomposed the notion of $NaIO$-sets by introducing the notion of $NPIO$-sets. Inter relational properties of the above sets are discussed.

Keywords. $NIO$-sets, $NPIO$-sets, $NSIO$-sets, $NaIO$-sets, nano ideal topological spaces.

2010 Subject Classification: 54A05, 54A10, 54B05

1. Introduction
The concepts of $\alpha$-open, semi-open and pre-open sets were introduced by Njasted[10], Levine[8] and Mashhour et.al.[9] respectively. In 1990, Jankovic and Hamlett[4] introduced the notion of ideal topological spaces. Later, many authors introduced several generalized open sets in ideal topological spaces such as pre I-open sets[2], semi I-open sets [3], $\alpha$-I-open sets[3], $\alpha$ g-I-open sets [11] and gp-I-open sets [11] and obtained decompositions of continuity and some weaker forms of continuity.

In 2013, Lellis Thivagar and Carmel Richard[5] established the field of nano topological spaces. Many researchers like [1],[14] obtained several generalizations of nano open sets. Further, in 2016, they have defined nano local function and explore the field of nano ideal topological spaces. It has an excellent potential for application in several fields such as medical diagnosis, Food analysis and in decision making problems.

In 2018, M.Parimala and Jafari [12] introduced the notion of nano I-open sets and studied several properties. Recently V.Rajendran et.al.[15] have introduced the notion of $NI_\theta$ -closed sets in nano ideal topological spaces.

In this paper we introduce the notion of nano pre I-open sets to obtain a decomposition of nano $\alpha$-I-open sets in nano ideal topological spaces.

2. Preliminaries
Definition 2.1. [6] For a non-empty finite set $U$ (universe set) with the equivalence relation $R$ on $U$, the lower approximation $L_R(X)$ and the upper approximation $U_R(X)$ of the subset $X \subseteq U$ is defined as
$L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where $R(x)$ is the equivalence class of $x$.
$U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$.
The boundary region $B_R(X) = U_R(X) - L_R(X)$. 
Now the collection $\tau_R(X) = \{\phi, U, L_R(X), U_R(X), B_R(X)\}$ is a topology called the nano topology. The pair $(U, \tau_R(X))$ represents the nano topological space.

**Definition 2.2.** [6] A subset $A$ of $(U, \tau_R(X))$ is called as nano $\alpha$-open (resp. nano pre open and nano semi open) if $A \subseteq NInt(NCl(NInt(A)))$ (resp. $A \subseteq NInt(NCl(A)$) and $A \subseteq NCl(NInt(A))$).

The complement of the above defined nano open sets are called as their respective closed sets.

**Definition 2.3.** [7] A nano topological space $(U, \tau_R(X))$ with an ideal $I$ [4] on $U$ is called a nano ideal topological space and denoted as $(U, \tau_R(X), I)$.

For convenience we use the symbols $\mathcal{U}$ and $\mathcal{U}_I$ to represent the notions $(U, \tau_R(X))$ and $(U, \tau_R(X), I)$ respectively.

**Definition 2.4.** [7] The nano local function is defined as $(A)^N = \{x \in U : U \cap A \notin I\}$; for every $U \in \tau_R(X)$. $NCl^*(A) = A \cup (A)^N$ is the nano closure operater of $A$.

**Definition 2.5.** Any subset $A$ of $\mathcal{U}_I$ is called as nano I-open[12] (resp.nano $\alpha$-I-open [7] and nano semi I-open [7]) if $A \subseteq NInt((A)^N)$ (resp.$A \subseteq NInt[NCl^*[NInt(A)]$ and $A \subseteq NCl^*[NInt(A)]$).

The complement of the above defined open sets are called as their respective closed sets.

**Theorem 3.5.** [7] For any subset $A$ of $\mathcal{U}_I$, if $A \subseteq A^N$, then $A^N = NCl((A)^N) = NCl(A) = NCl^*(A)$.

3. Nano pre I-open sets

**Definition 3.1.** A subset $S$ of $\mathcal{U}_I$ is defined as nano pre I-open $(NPIO)$ if $S \subseteq NInt[NCl^*(S)]$. The complement of a $NPIO$-set is called as nano pre I-closed $(NPIC)$.

$NPInt(S)$ is the union of all $NPIO$-sets contained in $S$.

$NPICd(S)$ is the intersection of all $NPIC$-sets containing $S$.

Through out this paper we use the notations $NO$, $NOO$, $NSO$, $NPO$, $NIO$, $NOO$ and $NSIO$ to represent the nano open, nano $\alpha$-open, nano semi open, nano pre open, nano $\alpha$-open and nano semi pre open sets respectively.

**Theorem 3.2.** Union of two $NPIO$-sets is $NPIO$-set.

**Proof:** Let $S$ and $T$ are two $NPIO$-sets of $\mathcal{U}_I$. Then $S \subseteq NInt[NCl^*(S)]$ and $T \subseteq NInt[NCl^*(T)]$.

Now, $S \cup T \subseteq NInt[NCl^*(S)] \cup NInt[NCl^*(T)] \subseteq NInt[NCl^*(S \cup T)] = NInt[NCl^*(S \cup T)]$.

Thus $S \cup T$ is a $NPIO$-set.

**Remark 3.3.** The intersection of two $NPIO$-sets need not be $NPIO$.

**Example 3.4.** Let $Q = \{x_1, x_2, x_3, x_4\}$ be the universe, $X = \{x_2, x_4\} \subseteq Q$, $Q/R = \{\{x_1\}, \{x_2\}, \{x_3, x_4\}\}$, $\tau_R(X) = \{\phi, Q, \{x_2\}, \{x_3, x_4\}, \{x_2, x_3, x_4\}\}$ and the ideal $I = \{\phi, \{x_1\}\}$. Then the sets $S = \{x_1, x_2, x_3\}$ and $T = \{x_1, x_2, x_4\}$ are $NPIO$-sets, but $S \cap T = \{x_1, x_2\}$ is not $NPIO$.

**Theorem 3.5.** If $S$ and $T$ are two $NPIC$-sets of $\mathcal{U}_I$, then $S \cap T$ is also $NPIC$.

**Proof:** Let $S$ and $T$ are two $NPIC$-sets of $\mathcal{U}_I$. Since $S$ and $T$ are $NPIC$, $S^c$ and $T^c$ are $NPIO$-sets. Then by the theorem 3.2, $S^c \cup T^c = (S \cap T)^c$ is $NPIO$. Therefore $S \cap T$ is $NPIC$. 

---

2
Example 3.6. Let \( Q = \{ x_1, x_2, x_3, x_4 \} \) be the universe, \( X = \{ x_2, x_4 \} \subseteq Q \), \( Q/R = \{ \{ x_1 \}, \{ x_2 \}, \{ x_3, x_4 \} \} \), \( \tau_I(X) = \{ \phi, Q, \{ x_2 \}, \{ x_3, x_4 \}, \{ x_2, x_3, x_4 \} \} \) and the ideal \( I = \{ \phi, \{ x_1 \} \} \). Then the sets \( S = \{ x_3 \} \) and \( T = \{ x_4 \} \) are NPIC-sets, but its union \( S \cup T = \{ x_3, x_4 \} \) is not NPIC.

Theorem 3.7. The union of NIO-sets and NαIO-sets of \( U_I \) is NPIO.

Proof: Let \( S \) be a NIO-set and \( T \) be a NαIO-set of \( U_I \). Then \( S \subseteq NInt((S)^N) \) and \( T \subseteq NInt(NCl^*(NInt(T))) \). Now, \( S \cup T \subseteq NInt((S)^N) \cup NInt(NCl^*(NInt(T))) \)
\[ \subseteq NInt(NCl^*(S)) \cup (NCl^*(NInt(T))) \]
\[ \subseteq NInt((NCl^*(S)) \cup (NCl^*(T))) = NInt[NCl^*(S \cup T)] \]
Thus \( S \cup T \) is NPIO.

Theorem 3.8. The union of NαIO-sets and NPIO-sets of \( U_I \) is NPIO.

Proof: Let \( S \) be a NαIO-set and \( T \) be a NPIO-set of \( U_I \). Then \( S \subseteq NInt(NCl^*(NInt(S))) \) and \( T \subseteq NInt(NCl^*(NInt(T))) \). Now, \( S \cup T \subseteq NInt(NCl^*(NInt(S))) \cup NInt(NCl^*(NInt(T))) \)
\[ \subseteq NInt(NCl^*(NInt(S))) \cup (NCl^*(NInt(T))) \]
\[ \subseteq NInt((NCl^*(S)) \cup (NCl^*(T))) = NInt[NCl^*(S \cup T)] \]
Thus \( S \cup T \) is NPIO.

Theorem 3.9. If \( S \) is both NO and NPIO of \( U_I \), then \( S \) is NαIO.

Proof: Let \( S \) be NO in \( U_I \), then \( S = NInt(S) \) and \( NCl^*(S) = NCl^*(NInt(S)) \). Now, \( NInt(NCl^*(S)) = NInt(NCl^*(NInt(S))) \). Since \( S \) is NPIO, we have \( S \subseteq NInt(NCl^*(S)) = NInt(NCl^*(NInt(S))) \). Thus \( S \) is NαIO.

Theorem 3.10. If \( S \subseteq (S)^N \) and \( S \) is NPIO in \( U_I \), then \( S \) is NIO.

Proof: Let \( S \) be a NPIO-set in \( U_I \). Then \( S \subseteq NInt(NCl^*(S)) \). Since \( S \subseteq (S)^N \), we have \( (S)^N = NCl^*(S) \). Then \( S \subseteq NInt(S)^N \). Thus \( S \) is NIO.

4. Decomposition of nano α-I-open sets

Theorem 4.1. For any NO subset \( S \) of \( U_I \) the following are equivalent.

(i) \( S \) is NSIO
(ii) \( S \) is NPIO.

Proof: Let \( S \) be a NO-set in \( U_I \).
Assume \( S \) is a NSIO-set. Then \( S \subseteq NCl^*(NInt(S)) \)
\( NInt(S) \subseteq NInt(NCl^*(S)) \)
\( S \subseteq NInt(NCl^*(S)) \). Thus \( S \) is NPIO.
Conversely, assume \( S \) is a NPIO-set. Then \( S \subseteq NInt(NCl^*(S)) \subseteq NInt(NCl^*(NInt(S))) \subseteq NCl^*(NInt(S)) \). Thus \( S \) is NSIO.

Theorem 4.2. Let \( S \) be a subset of \( U_I \). If \( I = \{ \phi \} \) then the following holds:

(i) NPO, NIO and NPIO are equivalent.
(ii) \( S \) is NSO if and only if \( S \) is NSIO.
Proof: Let $I = \{ \phi \}$. Then for any subset $S$ of $U$ we have $S^N = NCl(S)$ hence $S \cup (S)^N = NCl(S) = NCl^*(S)$. Therefore, $S^N = NCl(S) = NCl^*(S)$
Thus, (i) and (ii) follows immediately.

The following figure shows how $NPIO$-sets are related to some similar types of generalized $NIO$-sets.

![Figure 1 Relation between weaker forms of nano ideal open sets.](image)

We denote the family of all $NO$-sets (resp. $NOO$, $NPO$, $NSO$, $NIO$, $NaIO$, $NPIO$ and $NSIO$) as $NO(Q,X)$ (resp. $NOO(Q,X)$, $NPO(Q,X)$, $NSO(Q,X)$, $NIO(Q,X)$, $NaIO(Q,X)$, $NPIO(Q,X)$ and $NSIO(Q,X)$).

Example 4.3. Let $Q = \{ x_1, x_2, x_3, x_4 \}$ be the universe, $X = \{ x_2, x_4 \} \subseteq Q, Q/R = \{ \{ x_1, x_4 \}, \{ x_2 \}, \{ x_3 \} \}$, $\tau_R(X) = \{ \phi, Q, \{ x_2 \}, \{ x_1, x_4 \}, \{ x_1, x_2, x_4 \} \}$ and the ideal $I = \{ \phi, \{ x_1 \}, \{ x_2 \}, \{ x_1, x_2 \} \}$. Then
(i) $\{ x_2 \} \in NPIO(Q,X)$ but $\{ x_2 \} \notin NIO(Q,X)$.
(ii) $\{ x_1 \} \in NPO(Q,X)$ but $\{ x_1 \} \notin NPIO(Q,X)$.
(iii) $\{ x_2, x_3 \} \in NSO(Q,X)$ but $\{ x_2, x_3 \} \notin NSIO(Q,X)$.
(iv) $\{ x_2, x_4 \} \in NPO(Q,X)$ but $\{ x_2, x_4 \} \notin NOO(Q,X)$ and $\{ x_2, x_4 \} \notin NSO(Q,X)$.
(v) $\{ x_1, x_3, x_4 \} \in NSO(Q,X)$ but $\{ x_1, x_3, x_4 \} \notin NOO(Q,X)$ and $\{ x_1, x_3, x_4 \} \notin NPO(Q,X)$.
(vi) $\{ x_1, x_2, x_4 \} \in NO(Q,X)$ but $\{ x_1, x_2, x_4 \} \notin NIO(Q,X)$ and $\{ x_4 \} \in NIO(Q,X)$ but $\{ x_4 \} \notin NO(Q,X)$.

Example 4.4. Let $Q = \{ x_1, x_2, x_3, x_4 \}$ be the universe, $X = \{ x_1, x_4 \} \subseteq Q, Q/R = \{ \{ x_1 \}, \{ x_4 \}, \{ x_2, x_3 \} \}$, $\tau_R(X) = \{ \phi, Q, \{ x_1, x_4 \} \}$ and the ideal $I = \{ \phi, \{ x_1 \} \}$. Then $\{ x_1, x_2, x_4 \} \in NOO(Q,X)$ but $\{ x_1, x_2, x_4 \} \notin NO(Q,X)$.

Example 4.5. Let $Q = \{ x_1, x_2, x_3, x_4 \}$ be the universe, $X = \{ x_1 \} \subseteq Q, Q/R = \{ \{ x_1 \}, \{ x_2, x_3 \}, \{ x_3 \} \}$, $\tau_R(X) = \{ \phi, Q, \{ x_1 \} \}$ and the ideal $I = \{ \phi, \{ x_1 \} \}$. Then $\{ x_1, x_2, x_3 \} \in NOO(Q,X)$ but $\{ x_1, x_2, x_3 \} \notin NOIO(Q,X)$.

Example 4.6. Let $Q = \{ x_1, x_2, x_3, x_4 \}$ be the universe, $X = \{ x_2, x_4 \} \subseteq Q, Q/R = \{ \{ x_1 \}, \{ x_2 \}, \{ x_3, x_4 \} \}$, $\tau_R(X) = \{ \phi, Q, \{ x_2 \}, \{ x_3, x_4 \}, \{ x_2, x_3, x_4 \} \}$ and the ideal $I = \{ \phi, \{ x_1 \} \}$. Then
(i) The set $\{ x_1, x_2, x_4 \} \in NPIO(Q,X)$ but $\{ x_1, x_2, x_4 \} \notin NOIO(Q,X)$ and $\{ x_1, x_2, x_4 \} \notin NSIO(Q,X)$. 

4
(ii) The set \( \{x_1, x_2\} \in NSIO(Q, X) \) but \( \{x_1, x_2\} \notin N\alpha IO(Q, X) \) and \( \{x_1, x_2\} \notin NPIO(Q, X) \).

**Theorem 4.7.** For a subset \( S \) of \( U \), the following are equivalent.

(i) \( S \in N\alpha IO(Q, X) \),
(ii) \( S \in NSIO(Q, X) \cap NPIO(Q, X) \).

**Proof:** (i) \( \Rightarrow \) (ii) is straightforward from the fact that, every \( N\alpha IO \)-set is \( NSIO \) and \( NPIO \).

Conversely, assume that \( S \in NSIO(Q, X) \cap NPIO(Q, X) \).

Then \( S \subseteq NInt[NCl^*(S)] \subseteq NInt[NCl^*(NCl^*(NInt(S)))] = NInt[NCl^*(NInt(S))] \)

Hence \( S \in N\alpha IO(Q, X) \).

**References**

[1] Dhanis Arul Mary A and Arokiarani I 2014 On semi pre closed sets in nano topological spaces *Mathematical Sciences International Research Journal* 3(2) pp 771-773

[2] Dontchev J 1999 Idealization of Ganster-Reilly decomposition theorems Math. GN/9901017 (Internet)

[3] Hatir E and Noiri T 2002 On decomposition of continuity via Idealization *Acta Math.Hungar* 96 (4) pp 341-349

[4] Jancovic D and Hamlett T R 1990 New topologies from old via ideals *Amer. Math. Monthly* 97 pp 295-310

[5] Kuratowski K 1966 Topology 1 Academic Press New York

[6] LellisThivagar M and Carmel Richard 2013 On Nano forms of weakly open sets *International Journal of Mathematics and Statistics Invention* 1(1) pp 31-37

[7] Lellis Thivagar M and Sutha Devi V 2016 New sort of operator in nano ideal topology *Ultra Scientist* 28(1)A pp 51-64

[8] Levine N 1963 Semi-open sets and semi continuity in topological spaces *Amer. Math. Monthly* 70 pp 36-41

[9] Mashhour A S, Abd El-Monsef M E and El-Deep S N 1982 On precontinuous and weak precontinuous mappings *Proc. Math. Phys. Soc. Egypt* 53 pp 47-53

[10] Njasted O 1965 On some classes of nearly open sets *Pacific J. Math.* 15 pp 961-970

[11] Noiri T, Rajumani M and Inthumathi V 2007 On decomposition of g-continuity via idealization *Bull.Cal.Math.Soc* 99 (4) pp 415-424

[12] Parimala M and Jafari S 2018 On Some New notions in nano ideal topological spaces *International Balkan Journal of mathematics* 1 (3) pp 85-92

[13] Parimala M, Jafari S and Murali S 2016 Nano ideal generalized closed sets in nano ideal topological spaces (Communicate).

[14] Parvathy C R and Praveena S 2017 On nano generalized pre regular closed sets in nano topological spaces *IOSR Journal of Mathematics* 13 (2) pp 56-60

[15] Rajendran V, Sathishmohan P and Lavanya K 2018 On NIg-closed sets in nnao ideal topological spaces *Int.J.Math And Appl.* 6 (2-A) pp 193-199