ESTIMATING INFLATION RISK PREMIA USING INFLATION-LINKED BONDS: A REVIEW

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Abstract. This paper provides an overview of studies that estimate the inflation risk premium using inflation-linked bond (ILB) yields. I categorize existing studies, outline their research designs and compare their estimates for the inflation risk premium. Furthermore, the importance of accounting for ILB illiquidity and an overview of existing ILB liquidity proxies are demonstrated. A discussion of current literature developments, such as the zero lower bound, and an outline for future research directions conclude the paper.

Keywords. Break-even inflation rate; Inflation risk premium; Inflation-linked bonds; Treasury inflation-protected securities

1. Introduction

With nominal bonds, investors have transparency on yields in nominal terms. Since inflation would diminish their returns, investors look for compensation on expected inflation in nominal bond yields. Loosely speaking, as expected inflation is uncertain, investors demand an additional premium for this uncertainty (i.e. the inflation risk premium). Obtaining precise information on the inflation risk premium is crucial for several reasons: First, it is possible to extract inflation expectations from nominal bonds and inflation-linked bonds (ILBs). This measure, however, is biased by several factors, including the inflation risk premium. Knowledge of the inflation risk premium’s magnitude thus enables more accurate information on inflation expectations derived from market-based measures. Second, sovereign debt management is also interested in the size of the inflation risk premium as they pay this premium when issuing nominal bonds. A positive inflation risk premium and simultaneous absence of any other premium leads to lower costs for the Treasury when issuing ILB (relative to their nominal counterparts). Finally and more generally, the sign of the inflation risk premium offers some intuition about the state of the economy (i.e. supply-side shocks vs. demand-side shocks). While a strictly positive inflation risk premium is commonly assumed,1 recent research incorporates the possibility of the measure taking on negative values. These emerging discrepancies and current developments in literature evoke the need for a precise measure of the inflation risk premium.

The current study will demonstrate that the estimation of the inflation risk premium is not as easy a task as it may seem on first glance. First, different sources of data can be used to estimate the inflation risk premium. A considerable body of research literature uses survey inflation forecasts and nominal bond yields to estimate the inflation risk premium (see the beginning of Section 3 for a short discussion).

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The availability of ILB offered new possibilities to evaluate the premium. With now about 20 years of experience with and a vast amount of research on ILB (mainly in the USA), a comprehensive overview of the different approaches applied, data used and assumptions made is required. This article provides the aforementioned review, focusing on differences in research designs and approaches as well as highlighting limitations of existing studies, thus striving to answer the following research questions:

- What are advantages and limitations of using ILB data to estimate the inflation risk premium?
- How does existing research cope with these obstacles?
- What are the estimates for the inflation risk premium and how do they differ depending on whether studies do or do not use ILB data?

Several limitations are relevant when the inflation risk premium is estimated using ILB: illiquidity, an indexation lag and an embedded deflation option. These are considered and taken into account to various degrees in different studies, as will be shown in this paper. Thus, this review focuses on how studies have evolved to control and adjust for these limitations and how they have added additional data such as survey inflation forecasts or macro-economic variables to improve their estimation. However, in-depth technical issues, such as parameter restriction or estimation issues, are beyond the scope of this paper.

In this survey, I present and compare existing studies’ research designs and data used and finally show their estimates for the inflation risk premium, thus building on the work of Bekaert and Wang (2010) and D’Amico et al. (2018), who already compare (their own estimates with) different estimates for the inflation risk premium. A discussion of ILB liquidity and current issues like the zero lower bound complete the summary of existing research designs, liquidity proxies and state of the literature, thus ensuring that the current paper can be used as a convenient starting point for interested reader and future research. This survey is related to an excellent article by Bekaert and Wang (2010) that reviews inflation risk worldwide. While the authors focus on the concept of inflation hedging, they also survey studies that estimate the inflation risk premium by using ILB. In the present review, I (i) update and extend Bekaert and Wang’s (2010) review with respect to research design, (ii) provide a detailed overview on the obstacles to using ILB (Bekaert and Wang, 2010, already discussed the problem of ILB illiquidity) and (iii) continue the comparison of different estimates for the inflation risk premium in a comprehensive table.

The remainder of this paper is structured as follows: Section 2 provides an overview of the most relevant concepts discussed: the inflation risk premium (Section 2.1) and the link between ILB and nominal bonds (Section 2.2). Section 3 reviews studies that estimate the inflation risk premium without a term structure model (Section 3.1) and with a term structure model (Section 3.2). Section 3.3 includes an evaluation and comparison of the various estimates for the inflation risk premium. In Section 4, I discuss the impact of liquidity and present a list of liquidity proxies used in previous research. Sections 5 and 6 examine issues in current literature and conclude the paper, respectively.

### 2. Overview of Relevant Concepts

#### 2.1 Inflation Risk Premium

To illustrate the origin of the inflation risk premium more formally, one can start by defining a nominal stochastic discount factor \( M_t^N \) and a real stochastic discount factor \( M_t^R \). In an arbitrage-free setting (ensuring consistent pricing), bond prices are defined as follows: The price of a nominal bond that pays one dollar at time \( \tau \) is given by

\[
P_t^N(\tau) = E_t \left[ \frac{M^N_{t+\tau}}{M_t^N} \right]
\]

(1)
similarly, the price of a real (ILB) bond that pays one unit of the consumption basket (i.e. in real terms) at time $\tau$ is

$$P_t^R(\tau) = \mathbb{E}_t \left[ \frac{M_t^R}{M_t^{R_t}} \right]$$  \hspace{1cm} (2)

Given both bond prices, they must be consistent to each other implying the following price level:

$$Q_t = \frac{M_t^R}{M_t^N}$$  \hspace{1cm} (3)

Further, decomposition of the nominal bond’s price delivers

$$P_t^N(\tau) = \mathbb{E}_t \left[ \frac{M_t^N}{M_t^R} \right] = \mathbb{E}_t \left[ \frac{M_t^R}{M_t^{R_t}} \frac{Q_t}{Q_{t+\tau}} \right] = \mathbb{E}_t \left[ \frac{M_t^R}{M_t^{R_t}} \right] \cdot \frac{Q_t}{Q_{t+\tau}} + \text{Cov}_t \left[ \frac{M_t^R}{M_t^{R_t}}, \frac{Q_t}{Q_{t+\tau}} \right]$$  \hspace{1cm} (4)

This and all other expressions can be easily converted to yield-to-maturity expressions (see, e.g. Christensen et al., 2010). More importantly, the decomposition of nominal bond prices shows that they basically consist of three terms: The real stochastic discount factor $\mathbb{E}_t [\frac{M_t^R}{M_t^{R_t}}]$, expected inflation $\mathbb{E}_t [\frac{Q_t}{Q_{t+\tau}}]$ and the covariance between both terms $\text{Cov}_t [\frac{M_t^R}{M_t^{R_t}}, \frac{Q_t}{Q_{t+\tau}}]$. As usual, the sign of the covariance between an asset’s return and investors’ wealth or consumption defines the sign and the magnitude of the risk premium. In this particular case, the covariance term which defines the riskiness of a nominal bond is known as the inflation risk premium. A negative covariance between the real stochastic discount factor and expected inflation implies a positive inflation risk premium. This happens in times when the real stochastic discount factor is high and purchasing power is low (i.e. high inflation). From a consumption-based view, marginal utility (with low expected consumption growth) occurs at times of high inflation. Since nominal bonds are a poor investment in that case, investors will demand a higher (inflation) risk premium for these assets. On the other hand, if there is a positive correlation between output growth and inflation, nominal bonds are a good hedge implying a negative (or at least lower) inflation risk premium. In a simple sketch, Chen et al. (2016) recently estimated the correlation between forward consumption growth and long-run inflation. Similar to the above, a negative correlation indicates a positive inflation risk premium, as could be observed between 1975 and 2009. Nowadays, however, correlation is positive, implying a potentially negative inflation risk premium. The authors trace the change of inflation risk premium’s sign back to various types of shocks. More specifically, while formerly shocks have been used to be restricted to the supply side (moving inflation and real growth in opposite directions), they are now increasingly emerging from the demand side (moving inflation and real growth equally). While this exercise by Chen et al. (2016) solely serves as an indicator about the sign of the inflation risk premium, a large body of research concentrates on estimating the inflation risk premium. As the focus of this review is on studies using and adding ILB data to achieve this objective, the next section outlines basic knowledge on ILB itself and on the relation between ILB and nominal bonds.

2.2 Inflation-Linked Bonds and the Break-Even Inflation Rate

ILBs are considered a popular alternative to nominal sovereign debt and are characterized as debt securities where the bond’s principal is adjusted to the changes of a pre-specified (consumer) price index. By linking the principal value to realized inflation, coupon payments as a percentage component of the principal are similarly adjusted. Standard wise, a specific country’s consumer price index (CPI), is used as the ILB’s
underlying. The following features are important to note: First, the indexation exhibits a lag of three months (or even eight months, as used to be the case in the United Kingdom). Hence, inflation protection (in terms of CPI inflation) is not perfect, and a small part of inflation uncertainty remains in ILB. Studies by Evans (1998) and Grishchenko and Huang (2013) explicitly account for the indexation lag and report that the ‘indexation premium’ is rather small. Second, if CPI growth is negative during the maturity of an ILB (i.e. deflation), the principal is commonly not adjusted. Thus, ILBs contain a deflation option. Christensen et al. (2016) and Grishchenko et al. (2016) analyse the value of the deflation option in ILB and find that it exhibits considerable time variation with peaks during the financial crisis.

Countries that have issued ILB include the USA, the United Kingdom, Canada, Germany and France. While the USA, the United Kingdom and Canada link their ILB to their respective CPI (or the retail price index in the case of the United Kingdom), German ILBs are linked to the harmonized price index for the euro area. France, on the other hand, offers both: ILB linked to the French CPI and ILB linked to the harmonized price index for the euro area. Regarding the size of the different sovereign ILB markets, of course, the US market naturally represents the largest one in absolute terms ($550 billion of ILB outstanding in the USA in 2017). In relation to total government debt outstanding, on the other hand, the share of ILB is about 25% in the United Kingdom, 12% in France, 8% in the US and 6% in Germany. Comprehensive overviews of the development in the US ILB market and in ILB markets worldwide can be found in Fleming and Krishnan (2012) or Campbell et al. (2009) and Deacon et al. (2004), respectively.

It is important to discuss the link between nominal bonds and ILB in more detail. Converting the bond prices from Section 2.1 to yield-to-maturity expressions, writing expected inflation as $E_t[\pi]$ and taking bonds with the same maturity (for readability, I suppress $\tau$), one can show their relation to each other. This link between nominal bond yields $y_t^N$ and (real) ILB yields $y_t^R$ is called Fisher equation and reads

$$y_t^N = y_t^R + E_t[\pi]$$  \hspace{1cm} (5)$$

where $y_t^N$ and $y_t^R$ are the yield-to-maturity expression of a nominal and ILB, respectively. To extract a market-implied measure of inflation expectations (of maturity $\tau$), one simply solves Equation (5) for $E_t[\pi]$ and obtains

$$E_t[\pi] = y_t^N - y_t^R$$  \hspace{1cm} (6)$$

In fact, subtracting ILB yields from nominal bond yields is often referred to as the break-even inflation rate. However, because expected inflation might deviate from realized inflation, investors could also request compensation for this uncertainty (see Section 2.1). The extended Fisher equation accounts for this compensation and adds an inflation risk premium. For simplicity, I write the inflation risk premia as $\phi^{IRP}$. Equation (5) is extended to

$$y_t^N = y_t^R + E_t[\pi] + \phi^{IRP}$$  \hspace{1cm} (7)$$

If one now calculates the break-even inflation rate by subtracting (real) ILB yields from nominal yields, we have

$$y_t^N - y_t^R = y_t^R + E_t[\pi] + \phi^{IRP} - y_t^R = E_t[\pi] + \phi^{IRP}$$  \hspace{1cm} (8)$$

which means that the break-even inflation rate includes not only expected inflation but also the inflation risk premium. Finally, if nominal bonds and ILB differ in liquidity, investors might request a liquidity premium for the illiquid asset. Since (at least US) nominal bonds are known to be highly liquid, a liquidity premium in less liquid ILB might be possible. Given that investors request a liquidity premium for ILB, observed ILB yields $y_t^{ILB}$ are given by

$$y_t^{ILB} = y_t^R + \phi^{LIQ}$$  \hspace{1cm} (9)$$
where $\phi^{LQ}$ is the liquidity premium. This add-on again implies that the break-even inflation rate is ‘biased’ by not only a potential inflation risk premium but also a potential liquidity premium:

$$\gamma_i^N - \gamma_i^{ILB} = \gamma_i^R + \mathbb{E}_t[\pi] + \phi^{IRP} - \gamma_i^R - \phi^{LQ} = \mathbb{E}_t[\pi] + \phi^{IRP} - \phi^{LQ}$$

(10)

Since only nominal bond yields and ILB yields are directly observable, existing research faces a challenge in decomposing these yields in their respective components. For the break-even inflation rate, in particular, it implies potential biases by both, the inflation risk premium and the liquidity premium. Only in the case of no risk premia (i.e. $\phi^{IRP} = \phi^{LQ} = 0$) or if both risk premia would cancel each other out (i.e. $\phi^{IRP} = \phi^{LQ}$), break-even inflation rate would properly represent inflation expectations. On the other hand, positive and differing risk premia bias the break-even inflation rate. To sum up, drawing implications from solely observing the break-even inflation rate (and neglecting the indexation lag and the value of the deflation option) leads to:

- an underestimation of expected inflation if $\phi^{IRP} < \phi^{LQ}$,
- a correct estimation of expected inflation if $\phi^{IRP} = \phi^{LQ}$ or
- an overestimation of expected inflation if $\phi^{IRP} > \phi^{LQ}$.

Two important implications follow: First, the presence of a potential liquidity premium in ILB yields complicates exact estimation of the inflation risk premium via nominal and real (ILB) bond data. Second, obtaining information on both the liquidity premium and the inflation premium is essential before using the break-even inflation rate as a market-implied measure for inflation expectations. The next section surveys articles that focus on the determination of the inflation risk premium with the use of ILB yields. Studies differ in various aspects, such as the way in which ILB liquidity or additional inflation data are accounted for.8

3. Estimating the Inflation Risk Premium with ILB Yields

This section surveys articles that include ILB yields in their analysis to estimate the inflation risk premium in US nominal bond yields.9 A considerable body of the literature has studied the inflation risk premium without using ILB. Because of the plentiful availability of nominal bond data, these studies typically draw on much longer time periods than the studies presented in this section. Among others, Buraschi and Jiltsov (2005) derive a structural model for a (nominal) bond pricing solution that is based on the monetary supply and inflation and that allow the inflation risk premium to be extracted. Ang et al. (2008) develop a regime-switching arbitrage-free term structure model for nominal bond yields and inflation and thereby account for the regime-switching properties of inflation and interest rates. Their model also allows the inflation risk premium to be estimated. A detailed discussion of these models, however, is beyond the scope of this survey, but I include their estimates for the inflation risk premium in my evaluation below.

Recently, information from survey inflation forecasts is incorporated in term structure models (see, e.g. Chernov and Mueller, 2012). It has been shown that surveys in general are a useful source of information: For survey inflation forecasts, for instance, Ang et al. (2007) show that they outperform a variety of other measures for inflation expectations when predicting inflation. For interest rate surveys, on the other hand, Kim and Orphanides (2012) illustrate that they help to overcome small-sample problems due to persistency of interest rates. Some of studies discussed below that use ILB yields to estimate the inflation risk premium incorporate survey inflation forecasts as well. I firstly consider studies that estimate the inflation risk premium without a complete term structure model (Section 3.1). Section 3.2 then outlines term structure models that estimate the inflation risk premium; in Section 3.3, I provide an aggregate evaluation of studies from both sub-sections.
3.1 Regression-Based Approaches

The first study that uses observed ILB yields in the USA to analyse the inflation risk premium is by Roll (2004). However, he addresses the inflation risk premium only indirectly by evaluating yield curves’ steepness of nominal bonds and ILB. The fact that yield curves for nominal bonds are steeper than those for ILB might indicate a positive inflation risk premium. The main focus of Roll’s (2004) paper is more on diversification effects with ILB than on the inflation risk premium.

Shen (2006) subtracts the (10-year) survey inflation forecast from the observed (10-year) break-even inflation rate. The resulting series can potentially be biased by liquidity and is therefore regressed on two liquidity proxies. Since the regression is estimated in changes rather than in levels, the author calculates cumulative changes in the estimated liquidity risk premium (based on the model’s fitted values), which are declining between 1999 and 2006 for 5- and 10-year maturities. While the approach does not allow decomposition of inflation compensation into expected inflation and inflation risk premium, a negative value of survey inflation minus break-even inflation indicates that the liquidity premium is larger than the inflation risk premium (see also Figure 1 above for the relation between liquidity premium, inflation risk premium and break-even inflation rate).

Similarly, Söderlind (2011) calculates so-called break-even deviations for which survey inflation expectations are subtracted from break-even inflation rates for 3- and 10-year maturities. Deviations might mainly be driven by the inflation risk premium and/or by a liquidity premium. To further investigate the origin of these deviation, the author regresses break-even deviations on a set of liquidity proxies as well as measures for inflation uncertainty (i.e. cross-sectional forecast dispersion for the Survey of Professional Forecasters (SPF) and disagreement of inflation forecasts from Michigan survey). For both maturities, liquidity proxies and one measure for inflation uncertainty (SPF forecaster dispersion) have significant

Figure 1. Illustration of Studies’ Research Designs Discussed in Section 3.1.

Notes: The figure summarizes the studies’ research designs of this subsection. Note that Roll (2004) and Shen (2006) are not included as they do not explicitly estimate the inflation risk premium. Studies can be identified by their unique symbols and shadings. Note that Söderlind (2011) stops at calculating the liquidity-adjusted break-even inflation rate and does not derive the inflation risk premium.

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coefficients, with signs consistent with intuition. The author presents a premium-adjusted break-even inflation rate but does not decompose the proportion of the inflation risk premium and liquidity premium.

The paper by Auckenthaler et al. (2015) does not use the break-even inflation rate but rather builds on the work of Campbell and Shiller (1996) by calculating hypothetical ILB yields. These synthetic ILB yields are compared with their observed counterpart, and in the case of no premia at all, the difference between hypothetical and observed yields should be zero. Since this is not the case for 10-year ILB yields, the authors regress this series (i.e. hypothetical minus observed ILB yields) on a set of liquidity proxies and show that some of the series’ variation is due to ILB illiquidity. Applying Gürkaynak et al.’s (2010) approach (see below) to extract the liquidity premium (i.e. normalizing the liquidity premium to zero during a period of high ILB liquidity), the authors derive the liquidity premium over time, which is on average 56 basis points from 2001 to 2011. The authors further calculate the liquidity-adjusted break-even inflation rate and subtract SPF survey inflation expectations from it. The resulting series is fully assigned to the inflation risk premium, which amounts to 22 basis points on average during their same sample period. Before the financial crisis and after 2009 again, the inflation risk premium ranges between 0 and 100 basis points, but during the financial crisis, it drops to ~200 basis points.

The paper by Pflueger and Viceira (2016) focuses on return predictability. The presence of time-varying premia should make bond returns predictable (see Campbell et al., 2013, for the predictability of nominal bonds). The authors test for return predictability of ILB and the break-even inflation rate. While predictability in ILB returns is attributed to a time-varying liquidity premium, predictability in the break-even inflation rate is assigned to a time-varying inflation risk premium. Pflueger and Viceira (2016) estimate the liquidity premium in the break-even inflation rate by regressing the series on three liquidity proxies. The authors derive a liquidity-adjusted break-even inflation rate and find that it is likewise predictable. This result indicates the presence of an inflation risk premium. The authors assume that the expected liquidity excess return and expected liquidity-adjusted break-even return are fully assigned to the liquidity and inflation risk premium, respectively. Decomposing bond risk premia leads to an average liquidity premium of 92 basis points and to an average inflation risk premium of 163 basis points for the period from 1999 to 2014.

The next three studies apply some structural form (in an arbitrage-free setting) but do not establish a complete term structure model. First, Gürkaynak et al. (2010) regress the 5- and 10-year break-even inflation rate on a collection of liquidity proxies to extract the liquidity premium. As already outlined above, the regression’s fitted values (i.e. the estimated liquidity premium) are normalized to zero during a period where the liquidity premium is expected to be very low or absent. Using a state space model that includes the liquidity-adjusted break-even inflation rate as well as survey inflation data, inflation expectations are estimated with the use of the Kalman filter. The authors do not derive an inflation risk premium but mention that it could be extracted in the same vein. However, this approach is limited to some extent because of the normalization of the liquidity premium. The paper by Gürkaynak et al. (2010) is furthermore important from another perspective since the authors calculate the ILB yield curve for the USA and provide it in a periodically updated fashion on the Federal Reserve’s website (in an earlier paper, Gürkaynak et al., 2007, also provide the US yield curve for nominal bonds).

Second, Kajuth and Watzka (2011) use a state space model to estimate the liquidity premium and the inflation risk premium. More specifically, they base their state space model on the (extended) Fisher equation and include proxies for inflation risk and ILB illiquidity. As measures of inflation risk, the authors use (i) the estimated conditional standard deviation of inflation from a GARCH model, (ii) the SPF forecaster dispersion and (iii) the moving standard deviation of inflation. As proxies for liquidity risk, the usual set of measures is used (see Section 4). The model is estimated without information on expected inflation from survey forecasts. Expected inflation is estimated with the Kalman filter and is corrected for the liquidity premium and inflation risk premium. The model-implied estimates for the inflation risk premium are below 60 basis points up to 2006 and increase up to 200 basis points during the crisis. The
liquidity premium, on the other hand, is low between 2000 and 2007 (below 50 basis points) and high during the financial crisis (above 100 basis points).

Finally, Grishchenko and Huang (2013) present a ‘model-free’ approach that does not impose any term structure model (see Evans, 1998, for an antecedent). They take yields as given and estimate (under consideration of the three-month indexation lag and within an arbitrage-free setting) real yields. These estimated real yields are used to calculate break-even inflation rates that are subsequently used to derive the inflation risk premium by subtracting expected inflation. To do so, the authors use three different proxies for expected inflation (i.e. historical inflation average, VAR(1) estimates and survey inflation forecasts). Finally, a liquidity adjustment (13 basis points on average) is added to the inflation risk premium to control for ILB illiquidity. The liquidity adjustment is obtained by the fitted values of regressing difference between break-even and survey inflation rate on a set of liquidity proxies. For the 2000–2008 sample period, the liquidity-adjusted inflation risk premium ranges from -9 to 4 basis points.

While the basic idea of how to extract the inflation risk premium is rather similar among the studies, their ‘starting points’ differ. Figure 1 provides an overview of how they vary in using different input data. Furthermore, Table 1 in Section 3.3 gives a complete summary of estimated inflation risk premia among the studies discussed in this and the next section. With additional information on expected inflation, one can estimate the inflation risk premium. However, as discussed in Section 1, the inflation risk premium emerges from the covariance between the real stochastic discount factor and inflation. Consequently, for a model-consistent decomposition of the break-even inflation rate in expected inflation, inflation risk premium and a potential liquidity premium, a term structure model is required. The use of a pricing model further allows us to include information from the cross section of both nominal bond yields and ILB yields. Finally, these models also contain a no-arbitrage condition, meaning that there are no opportunities for arbitrage over time and across maturities (i.e. arbitrage-free term structure models).

3.2 Term Structure Models

The arbitrage-free term structure literature started with the founding models of Vasicek (1977) and Cox et al. (1985). All models building on these initial models specify (risk-neutral) dynamics of underlying yield curve factors and risk premia under no-arbitrage conditions over time and across bond maturities. While the original models consist of only one factor (i.e. the short rate), more recent models have at least three factors. With Duffie and Kan (1996), affine arbitrage-free models have become popular, since yields are linear (i.e. affine) functions of the underlying factors. While the risk-neutral Q-world is sufficient for pricing issues, forecasting and decomposing the term structure requires real P-world dynamics. To link both worlds, a functional form for market prices of risk is required. Completely affine risk premia (Dai and Singleton, 2000), essentially affine risk premia (Duffee, 2002) or extended affine risk premia (Cheridito et al., 2007) are most extensively used in the literature. Dai and Singleton (2000) also introduce the notation of $A_m(n)$-models to classify affine term structure models, where $m$ is the number of square-root processes and $n$ is the number of factors in the model. Excellent textbook information on term structure models in general can be found in Singleton (2006) and Piazzesi (2010). Grishchenko and Huang (2012) provide a compact survey on term structure models that focuses on expected inflation, and Rebonato (2016) present a thought-provoking review of current term structure models.

3.2.1 Term Structure Models without Inflation Data

Chen et al. (2010) are among the first to develop a term structure model for the USA that includes both nominal and ILB yields. They develop a two-factor correlated Cox et al. (1985) model in which the first factor represents the real rate and the second factor represents the inflation rate. Hence, the combination of both factors should map the nominal rate. The authors provide a closed-form solution to the nominal
and real term structure, which enables a (two step) estimation of the parameters in a state space form. Because of the non-linear measurement equation, the unscented Kalman filter is applied (see the paper for a discussion regarding why the unscented Kalman filter instead of the extended Kalman filter is applied). Market prices of inflation risk for each maturity are estimated within the model’s setting. As the difference between nominal and real yields is assumed to be captured by only one factor (i.e. inflation), no further decomposition is possible. Thus, the inflation risk premium is estimated by subtracting the market price of a bond with real (observed) risk parameters from a bond with risk-neutral (model) risk parameters. Chen et al. (2010) show that the inflation risk premium’s term structure is positive and averages 0.24 basis points for a maturity of three months and 77.24 basis points for a maturity of 20 years during their sample period 1998–2007. However, the authors do not account for ILB illiquidity, which might distort their estimates for the inflation risk premium at least during the first years of the sample period.

In contrast to the two-factor model above, Adrian and Wu (2009) apply a five-factor affine term structure model: two factors explain the real pricing kernel (level and slope) and two factors model inflation expectations. The fifth factor is a variance factor that governs the state variables’ variances and covariances dynamics. While the second moments are estimated within a GARCH model, the term structure model is estimated as usual with maximum likelihood, and state variables are obtained with a Kalman filter. The inflation risk premium is then derived by the difference between the (observed) break-even inflation rate and the model-implied estimates for the expected inflation. The inflation risk premium for bonds with a maturity of 10 years varies between 0 and 100 basis points during 2003–2008 and rises sharply during the financial crisis, up to 170 basis points (Adrian and Wu, 2009, figure 5).

The next three studies are all based on the workhorse model by Christensen et al. (2011) (CDR hereafter), which is an arbitrage-free version of the classical model by Nelson and Siegel (1987). CDR show that their model fits in-sample and out-of-sample forecasts quite well. Since the model has affine state and measurement equations and Gaussian errors, maximum likelihood estimation with a Kalman filter can be applied. The feature of their workhorse model is that it includes the benefits of arbitrage-free models and the Nelson–Siegel world in terms of its dynamic extension by Diebold and Li (2006). In essentially affine models, it is common to restrict parameters with low t-statistics in pre-estimations to zero in order to overcome problematic model over-parameterization. Such restrictions, however, often lack of theoretical interpretation. The CDR model exhibits (due to the Nelson–Siegel base) a more parsimonious structure and is therefore less problematic to estimate, facilitates model’s tractability and allows for factor interpretation (see the textbook by Diebold and Rudebusch, 2013, for a further excellent reference on the CDR model).

To start with, the paper by Christensen et al. (2010) extends the CDR model for joint pricing nominal bond yields and ILB yields. According to the authors, the following four factors are adequate to map the joint term structure: two-level factors (one nominal and one ILB), a common slope and curvature factor. To extract the inflation risk premium, model-implied break-even inflation rate is compared with the observed break-even inflation rate. The difference between both series is fully assigned to the inflation risk premium, which can thus have both positive and negative values. The estimates for the premium (for 5- and 10-years’ maturity) range between –50 and 50 basis points. Since the sample period starts only four years after the first ILB issuance (in 2003), liquidity concerns are less relevant during this period.
than during the first years of ILB history. However, as the sample ends in 2008 at the beginning of the financial crisis, ILB illiquidity will play an important role at this time and potentially affects the estimates for the inflation risk premium.

In a follow-up paper, Christensen and Gillan (2012) extend the joint model to allow stochastic volatility and correct ILB yields for the liquidity premium. Since the liquidity premium is not directly observable, the authors use a ‘model-independent range’ for the ILB liquidity premium: the difference between the break-even inflation rate and the inflation swap rate is calculated. In a frictionless world, both rates should be equal. If these rates are unequal, risk premia are present. By assuming that the spread between both rates is fully attributed to either swap illiquidity or bond illiquidity, the authors derive a range for the potential ILB liquidity premium. Minimum and maximum liquidity-adjusted ILB yields are then used as the input variable for the term structure model. Given that the liquidity premium is assumed to be the maximum, the resulting (minimum) inflation risk premium amounts to 5 and 10 basis points on average between 2005 and 2011 for 5 and 10 years maturity, respectively.

A recent working paper by Andreasen et al. (2017) further extends the CDR model by including ILB (as Christensen et al., 2010), by accounting for the value of deflation protection (as Christensen et al., 2016) and by explicitly including a latent liquidity factor. By comparing the primary dealer transaction volume for nominal bonds and ILB and by observing that the level is quite different but that the pattern of the time series’ is rather similar, the authors conclude that one factor that accounts for relative ILB illiquidity is adequate. Their model further allows security-specific liquidity factor sensitivities by including raw prices of ILB (with time since issuance and time to maturity). Andreasen et al.’s (2017) model is the only one that accounts for security-specific liquidity risk. They estimate an average liquidity premium of 38 basis points (from 1997 to 2013) with peaks in 2002 (100 basis points) and in 2009 (300 basis points) that includes all issued ILB (Andreasen et al., 2017, figure 5). Alternatively, the liquidity premium for on-the-run ILB is extracted as well: for a 10-year maturity, the average liquidity premium is 30 basis points, with a maximum value of 100 basis points. The authors attribute this dramatic difference in liquidity premia for on-the-run and off-the-run ILB to buy-and-hold investors who ‘lock up’ the outstanding amount of ILB and limit the amount of ILB available for trading. Thus, security-specific liquidity factor sensitivities are vital to account for this characteristic. Finally, break-even inflation rates are decomposed into expected inflation, inflation risk premium and liquidity premium (note that the preferred model by the Bayesian information criterion lacks deflation option adjustment). The inflation risk premium (for a 10-year maturity) is positive on average and varies between −100 and 100 basis points. However, it is only negative during 1998 and 2002, as well as for a short period during the financial crisis (Andreasen et al., 2017, figure 13). While the negative inflation risk premium during the financial crisis is in line with the existing literature, the negative premium in 1998 is typically found only by papers that do not account for ILB illiquidity.

3.2.2 Term Structure Models Including Inflation Data

In a similar vein as above, Abrahams et al. (2016) develop a joint term structure model for nominal and ILB yield curves. Their model is based not on CDR but rather on the more general class of essentially affine term structure models introduced by Duffee (2002). Thus, market prices of risk are less restricted (i.e. they do not have to be consistent with the Nelson-Siegel yield curve). However, for parsimony, their model consists of three nominal pricing factors, two ILB pricing factors and one ILB liquidity factor. While the three nominal pricing factors originate from a principal component analysis, the two ILB pricing factors are derived as follows: First, ILB yields are regressed on the principal components for the nominal yields curve. Second, two principal components are derived from the residuals of the step 1 regression exercise. While this procedure reduces collinearity among the factors, an economic interpretation is impossible. The sixth (liquidity) factor comprises the equally weighted average of two liquidity proxies. The estimation is done with maximum likelihood but uses excess returns rather than zero coupon yields (see Adrian et al., 2013, for a discussion of why returns are favourable against yields).
Note that the calculation of nominal bond and ILB excess returns requires both the risk-free short rate and realized inflation. The model allows the break-even inflation rate to be decomposed into expected inflation, the liquidity premium and the inflation risk premium. The authors report the decomposition for the 10-year and 5–10-year forward break-even inflation rate. While the inflation risk premium for the 10-year break-even inflation rate varies between 0 and 50 basis points (with a short peak during the financial crisis), the inflation risk premium for the forward break-even rate is more volatile: It ranges from about 0–100 basis points, again with a peak during the financial crisis (Abrahams et al., 2016, figure 1).

Similarly, the essentially affine term structure model by D’Amico et al. (2018) is a joint model of nominal yields, ILB yields and inflation and consists of four factors. As the standard three-factor models exhibit a poor fit of ILB yields (ILB yields are treated as model-implied real yields corrected for the indexation lag), the authors add a fourth factor to their model, which they model as an ILB-specific factor. To test whether this latent factor can be related to ILB illiquidity issues, the authors regress it on a variety of liquidity measures. They also check whether the series is related to certain technical factors, such as the embedded deflation option, seasonality, flight-to-safety effects or Federal Reserve Purchases. When all these controls are included, the effect of ILB illiquidity remains observable implying that the ILB-specific factor actually captures an ILB liquidity premium. Their four-factor model allows the observed break-even inflation rate to be decomposed into its different components. The estimated inflation risk premium is positive and rather stable (even during the financial crisis) at about 30 basis points. Most of the variation during the crisis is covered in the liquidity risk premium, which increases to 200 basis points in 2008 and 2009. Note also that inflation data and survey inflation forecasts are included in their state-space model, which covers the sample period from 1990 to 2013 (prior to the availability of ILB, where ILB yield data are treated as missing data).

3.2.3 Macro-Finance Term Structure Models

Hördahl and Tristani (2014) develop a joint model of term structure and macro-economic dynamics in the vein of Ang and Piazzesi (2003). The advantage of jointly modelling inflation, monetary policy, output and term structure movements is given by the fact that the impact of macro-economic variables on the inflation risk premium can easily be analysed. Their model is rather based on the framework developed by Hördahl et al. (2006), which allows an additional feedback effect from short-term rates to macro-economic variables. As usual in essentially affine models, risk premia can be linked to the model’s state variables. In this case, since the model’s state variables are macro-economic variables, the estimated inflation risk premium can be linked to movements in these variables. The authors estimate their model from 1990 to 2013, and ILB yields are included in the estimation as soon as they become available. Hördahl and Tristani (2014) explicitly correct for illiquidity by regressing ILB yields on a set of liquidity proxies before they are included in the estimation. The model-implied inflation risk premium is obtained by subtracting estimated expected inflation and (liquidity-adjusted) real yields from nominal yields. During 1999 and 2007, the 10-year inflation risk premium ranges between −40 and 50 basis points. After 2007, the inflation risk premium is more volatile for maturities with values between −80 and 100 basis points for 10-year maturity (Hördahl and Tristani, 2014, figures 5 and 6).

Finally, one specification of Chernov and Mueller’s (2012) macro-finance term structure model additionally includes ILB yield data. Their (preferred) five-factor term structure model uses data from nominal bond yields, ILB yields, output, inflation and survey inflation forecasts. The authors compare different model specifications and observe that the inclusion of ILB yields increases (decreases) the average model-implied real yields (inflation risk premium). Similarly, volatility of model-implied real yields (inflation risk premium) increases (decreases) as well but model’s inflation expectations remain rather stable. Their sample period ranges from 1971 to 2008 but ILB yields are only included from 2003 onwards and are not adjusted for a potential liquidity premium. The estimated inflation risk premium averages 67 basis points without ILB yields and 157 basis points with ILB yields.

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3.2.4 The Use of Inflation Swap Rates Instead of ILB

As shown above, the paper by Christensen and Gillan (2012) uses inflation swap rates to derive a range for the ILB liquidity premium. Inflation swap rates can also be used to substitute ILB yields, as done in the studies by Haubrich et al. (2012) or Roussellet (2016). Haubrich et al. (2012) develop a completely affine term structure model with seven factors and four stochastic drivers. Three of the factors are the real interest rate, expected inflation and ‘central tendency’ of inflation. While these factors determine the cross section of bond yields, they have no direct impact on risk premia. Rather, risk premia depend on the other four factors (i.e. volatility state variables). Such a separation of factors (i.e. one part determines the cross section, the other part determines the risk premia) enables a highly flexible model. The model’s input variables are nominal bond yields, survey inflation forecasts and inflation swap rates as soon as they become available in 2004. Thus, it is possible to ‘create’ inflation-indexed yields without using ILB yields by subtracting inflation swap rates from nominal bond yields with the same maturity.\textsuperscript{12} While the model allows nominal bond yields to be decomposed (during their full sample from 1982 to 2009, the 10-year inflation risk premium amounts to 45 basis points on average), the authors also compare their synthetic real yields with ILB yields and confirm the studies discussed above that ILBs were highly mispriced (potentially because of a liquidity premium) in the first years of issuance and during the financial crisis. In line with Haubrich et al. (2012), Fleckenstein et al. (2014) similarly use inflation swap rates but focus on the mispricing of ILB and the resulting costs rather than on the inflation risk premium (see Appendix B for a discussion). More recently, Roussellet (2016) proposes an affine (four-factor) term structure model that includes macro-economic variables, uses real interest rates from inflation swaps and is consistent with the zero lower bound (see Section 5 for a discussion). The latter is achieved by modelling a Gamma-distributed short rate that accounts for non-negativity but allows the possibility to stay at the zero lower bound. In an empirical implementation, Roussellet (2016) extracts the inflation risk premium by decomposing nominal bond yields and ILB yields. The inflation risk premium in nominal bond yields with a maturity of one year is very volatile and negative over long periods. For a 10-year maturity, however, the premium is considerably less volatile and only negative during the financial crisis and in the 1990s. However, it is important to note that both studies by Haubrich et al. (2012) and Roussellet (2016) do not use ILB data.

Turning back to ILB, it is obvious that accounting for their illiquidity is essential to obtain unbiased measures for the inflation risk premium and break-even inflation rate. The models discussed in this section apply various strategies in considering liquidity effects. They also differ with respect to the use of additional data such as inflation, survey inflation forecasts or other macro-economic data. Similar to Figure 1 in the section before, Figure 2 now provides an overview of the studies discussed in this section.

3.3 Evaluating the Use of ILB to Estimate the Inflation Risk Premium

Both the subsections and figures above show that strategies to estimate the inflation risk premium vary considerable throughout the literature. Not only the approaches differ (i.e. regression-based vs. term-structure model) but also the ways to consider ILB illiquidity, the embedded deflation option or the indexation lag. It is interesting to consider the research’s development over time: Studies started with ‘simple’ term structure models that jointly price the nominal and ILB yield curve. As one observed that ILB yields were too low compared to nominal bond yields, there was the need to consider ILB illiquidity: Studies by Christensen and Gillan (2012) or Hörda and Tristani (2014) adjusted ILB yields for the liquidity premium and include these liquidity-adjusted yields in their models. Recent papers now include directly a liquidity factor in the term structure model to obtain model-implied estimates for the liquidity premium (see also Figure 2). Among these studies, latent liquidity factors (Andreasen et al., 2017; D’Amico et al., 2018) or liquidity-proxy factors (Abrahams et al., 2016) are used to account for ILB illiquidity.
### Table 1. Overview of Studies' Estimates For the Inflation Risk Premium and the Liquidity Premium.

| Paper | Period covered | Model | Min IRP | Max IRP | Min LIQ | Max LIQ | Corrected for |
|-------|----------------|-------|---------|---------|---------|---------|---------------|
| **ILB-based estimates** | | | | | | | |
| Auckenthaler et al. (2015) | 2001–2011 | Regression-based approach | −210 (2008) | 100 (2004) | 0 (2008) | 100 (2002) | x |
| Kajuth and Watzka (2011) | 1997–2009 | State space model | 10 (1999) | 220 (2009) | 20 (2006) | 130 (2009) | x |
| Grishchenko and Huang (2013) | 2000–2008 | ‘Model-free’ approach | −13 (00-04) | 8 (00-04) | −7 (04-08) | 33 (00-04) | x x |
| Chen et al. (2010) | 1998–2007 | Two-factor TS model | 45 (2000) | 70 (2006) | | | |
| Adrian and Wu (2009) | 2003–2009 | Five-factor TS model | 10 (2006) | 170 (2008) | (x) | | |
| Christensen et al. (2010) | 2003–2008 | Four-factor TS model | −60 (2003) | 60 (2004) | | | |
| Christensen and Gillan (2012) | 2005–2011 | Four-factor TS model | −150 (2009) | 100 (2008) | 0 (2008) | 120 (2009) | x |
| Andreasen et al. (2017) | 1997–2013 | Five-factor TS model | −90 (1998) | 100 (2004) | −20 (2000) | 300 (2009) | x (x) |
| Abrahams et al. (2016) | 1999–2014 | Six-factor TS model | −40 (2001) | 100 (2008) | 0 (2005) | 210 (2008) | x (x) |
| D’Amico et al. (2018) | 1999–2013 | Four-factor TS model | 0 (2008) | 50 (2002) | 0 (2005) | 220 (2008) | x x x |
| Hördahl and Tristani (2014) | 1999–2013 | Macro-finance TS model | −80 (2008) | 100 (2012) | 0 (2007) | 140 (2009) | x |
| Chernov and Mueller (2012, w ILB) | 1971–2008 | Macro-finance TS model | −30 (2008) | 250 (1983) | | | |
| **Inflation swap-based estimates** | | | | | | | |
| Haubrich et al. (2012) | 1982–2010 | Seven-factor TS model | 25 (2009) | 59 (1984) | | | |
| Roussellet (2016) | 1990–2010 | Four-factor TS model | 80 (2012) | −100 (2008) | | | |

### Notes: All estimates are given in basis points and refer to a maturity of (unless noted otherwise) 10 years. Estimates are extracted from respective papers’ figures and preferred specification. IRP and LIQ refer to inflation risk premium and liquidity premium, respectively. Minimum and maximum values for both premia are given with the year of realization in brackets. The last three columns indicate whether there is an adjustment for ILB illiquidity (Liq.), the embedded deflation option (Defl.) and the indexation lag (Lag). A checked option in brackets (i.e. (x)) indicates that a correction is not included in the final specification but part of robustness checks or alternative specifications. Studies are listed as they appear in the main text above. Shen (2006), Gürkaynak et al. (2010) and Söderlind (2011) do not estimate the inflation risk premium. Pfueger and Viceira (2016) derive annualized excess returns rather than yield premia; to avoid confusion, I do not report their (excess return) estimates. Auckenthaler et al.’s (2015) minimum liquidity premium refers to 2008Q1 before the bankruptcy of Lehman Brothers. Kajuth and Watzka’s (2011) estimates refer to their model 1. Grishchenko and Huang (2013) only provide summary statistics for their estimates; therefore, minimum and maximum values for both premia can only be listed within a range of four years. For Christensen and Gillan (2012), maximum liquidity premia and minimum inflation risk premium are listed. D’Amico et al.’s (2018) estimates only refer to the period in which ILB yields are included. The estimates from Chernov and Mueller (2012) are based on their model AOT5 (with ILB) and A05 (without ILB) over their entire sample period. Haubrich et al.’s (2012) and Roussellet’s (2016) estimates for the inflation risk premium are over the study’s entire sample period. Further, Haubrich et al. (2012) do not explicitly derive a liquidity premium but rather compare swap-based real yields with ILB yields and attributing the difference to a potential liquidity premium.
Other macro data

Nominal bond yields  
Inflation-linked bond yields

CPI inflation

Survey inflation

Other macro data

Liquidity correction

Term structure model

Liquidity correction

Liquidity factor

Inflation risk premium

Expected inflation
(Liquidity premium)

Adrian & Wu (2009)

Hördahl & Tristani (2014)

Chen et al. (2010)

Andreasen et al. (2016)

Christensen et al. (2010)

Abrahams et al. (2016)

Christensen & Gillan (2012)

D’Amico et al. (n.d.)

Figure 2. Illustration of Studies’ Research Designs Discussed in Section 3.2.

Notes: The figure summarizes the studies’ research designs of this subsection. Studies can be identified by their unique symbols and shadings.

The indexation lag, another ‘feature’ of ILB, has shown to be less relevant: Grishchenko and Huang (2013) take explicitly the three-month indexation lag into account and derive an indexation lag premium which ranges between 0.03 basis points (one-year maturity) and 4.2 basis points (10-year maturity). Adrian and Wu (2009) also control for the indexation lag and find no considerable differences. Thus, carry adjustment of yields due to the indexation lag is not very relevant. Moreover, mainly short maturities are affected by the carry effect and the frequently used Gürkaynak et al.’s (2010) ILB yield curve data are not available for maturities below two years. The third ‘feature’ of ILB, which potentially complicates estimations, is the embedded deflation option. Recently, Christensen et al. (2016) and Grishchenko et al. (2016) analyse the value of this option in ILB yields and find the value to be rather low except during the financial crisis. The increased option value during the financial crisis is rational, given the ‘deflation fears’ at this time. However, apart from such times of financial turmoil, the embedded deflation option seems to be negligible and studies, like Andreasen et al. (2017) for example, show that model fit is better without an option adjustment.

Not only research designs differ, but studies’ estimates for the inflation risk premium vary as well. Table 1 gives an overview of the studies discussed and presents their minimum and maximum values for the inflation risk premium. Unless noted otherwise, the values refer to a maturity of 10 years. Moreover, information on the size of the potential liquidity premium and an indication whether the studies control for the above-mentioned features is given. For comparison, I add three major studies (Buraschi and Jltsov, 2005, for an equilibrium macro-finance term structure model; Ang et al., 2008, for a reduced-form spanned macro-finance term structure model and Chernov and Mueller, 2012, for a reduced-form unspanned macro-finance term structure model) that estimate the inflation risk premium in the USA without using ILB yields.13 At first glance, minimum and maximum values for the (10-year) inflation risk premium vary considerably throughout the studies. By having a closer look at term structure models,
differences across these studies are less massive: Studies that adjust (at least) for ILB illiquidity exhibit similar maximum values for the inflation risk premium (around 100 basis points), but which occur, however, at different points in time. Most of these term structure models also report a negative inflation risk premium, at least once during their sample period. It is noteworthy that term structure models which differ quite considerably in their research design still end up with similar estimates for the inflation risk premium. For instance, as mentioned above, Andreasen et al. (2017) do not include any information on inflation or survey inflation forecasts. Abrahams et al. (2016) include additionally inflation data and D’Amico et al. (2018) use both inflation as well as survey inflation forecast data.

The inflation risk premium’s development over time, however, varies considerably across the studies: Some studies’ estimates exhibit a peak during the financial crisis (Adrian and Wu, 2009; Kajuth and Watzka, 2011; Abrahams et al., 2016; Pfleuger and Viceira, 2016), whereas other studies report a drop of the inflation risk premium during this time (Hördahl and Tristani, 2014; Auckenthaler et al., 2015; Andreasen et al., 2017). And yet others find the premium to be permanently rather stable (D’Amico et al., 2018). Consequently, also the premium’s second moment varies considerable throughout these studies. Interestingly, Abrahams et al.’s (2016) and Andreasen et al.’s (2017) estimates appear to include a similar slight seasonal pattern but a different shape during the financial crisis. Some of the authors kindly provided their estimates for the inflation risk premia. The corresponding figure and a short discussion is included in Appendix C.

The estimated liquidity premium, on the other hand, ranges unanimously between 0 and 300 basis points. More remarkable, minima and maxima liquidity premia occur at similar points in time (see Section 4 for a full discussion). Note, however, that minima and maxima values for the inflation risk premium are rather similar for studies that consider ILB illiquidity and for studies that do not account for it. This would indicate that considering the liquidity premium is more relevant for the extraction of an unbiased break-even inflation rate than for the estimation of the inflation risk premium.

A clear-cut comparison with studies not using ILB yields is difficult due to different sample periods. The studies by Buraschi and Jiltsov (2005), Ang et al. (2008) or Chernov and Mueller (2012) have already started in 1960, 1952 and 1971, including periods of high inflation in the 1970s and 1980s. However, estimates at the end of their sample periods are in a similar range as studies using ILB yields. The basic idea of adding ILB yields is to improve the estimation of the inflation risk premium. An improvement can emerge due to more available data (cross sectional in terms of different maturities and time series in terms of frequency) as well as due to a very close proxy for real rates. An explicit study which applies different model specifications including and excluding information from ILB, inflation swap rates and survey inflation forecasts to compare differences between the estimates would be the next step for future research. Furthermore, a recent paper by García and Werner (2010) which analyses euro area term structure with ILB yields (see endnote 6 for an outline) establishes the link between the inflation risk premium and the uncertainty in survey inflation forecasts. Whether there is a link between the survey inflation forecast uncertainty and the inflation risk premium (estimated without the use of survey information at best) for the USA might be subject for future research.

One crucial obstacle concerning the use of ILB yields to estimate the inflation risk premium is liquidity. Since the liquidity premium plays such an important role for an unbiased estimation, the next section is dedicated to discuss this issue.

4. Impact of Liquidity

Although the impact of liquidity is less relevant for nominal bonds, which include this survey’s object of interest (i.e. the inflation risk premium), it is important to know the impact of liquidity on ILB yields when these yields are included in the estimation process. From the section above, it is obvious that a potential ILB liquidity premium has to be considered. It is also evident that especially two ‘events’ have a strong impact on the ILB liquidity premium: the first years of ILB issuance and the financial crisis.

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First, ILB illiquidity during the first years of issuance often refers to the fact that ILBs have been a rather new financial instrument. Market participants were unfamiliar with ILB since they have not traded (USA) ILB before (see, e.g. Roll, 2004). As data on ILB yields increase, it might be worthwhile to exclude these years in an analysis, as ‘unfamiliarness’ is rather difficult to measure with an appropriate proxy and might not be perfectly captured with traditional liquidity proxies. Second, periods of financial crises are crucial because during such a time, it is challenging to disentangle effects on nominal bond yields or break-even inflation rates. Suppose, for instance, that a drop in break-even inflation rate could be either due to an increased liquidity premium or due to a change in inflation expectations. On the other hand, an increasing break-even inflation rate could be the consequence either of rising inflation expectations or of an increase in the inflation risk premium. Thus, controlling for a potential ILB liquidity premium is vital to estimate at least the liquidity-adjusted break-even inflation rate.

Figures 1 and 2 from the sections above show that studies account quite differently for ILB illiquidity in their research designs. In addition, a variety of proxies for liquidity have been used. Some proxies have been used frequently, whereas other others have been used only once. Proxies can broadly be categorized into (i) ILB-specific liquidity proxies, (ii) relative ILB liquidity proxies, (iii) Treasury liquidity proxies and (iv) proxies for uncertainty. Table 2 below provides an overview of used liquidity proxies, including the articles in which they are applied.

The application of these proxies depends on the research approach. For regression-based approaches or term structure models that use liquidity-adjusted ILB yields as input data, these proxies are used in a regression to estimate the liquidity premium (see, e.g. Hördahl and Tristani, 2014, or Auckenthaler et al., 2015). Term structure models that explicitly incorporate liquidity uses these proxies in a principal component analysis to extract one (or more) liquidity factors (see Abrahams et al., 2016). And still others model a latent factor that is assigned to a liquidity premium. To test this attribution, the latent factor is regressed on a set of liquidity proxies (see D’Amico et al., 2018).

More recent research has started focusing on the ILB liquidity premium itself and how it is affected by Federal Reserve’s activities. Both Coroneo (2016) and Christensen and Gillan (2017) analyse how large-scale asset purchases by the central bank—known as quantitative easing—have an effect on the US ILB market. Christensen and Gillan (2017) combine, analogous to Christensen and Gillan (2012), inflation swaps with ILB yields to construct their measure of liquidity premium and apply an event study methodology to analyse the effect of the quantitative easing programme. Without going into detail, the authors find that asset purchases have temporarily lowered the liquidity premium in ILB yields and inflation swaps. Coroneo (2016), on the other hand, identifies an ILB liquidity premium within a state-space model for nominal bond yields and ILB yields. Coroneo’s (2016) approach is different to existing studies because the ILB liquidity premium (as a component of ILB yields) is unspanned by nominal bond yields. Thus, it does neither require any liquidity proxy nor the specification of a full-term structure model. While the author does not focus on the inflation risk premium, the extracted liquidity premium still resembles other studies’ liquidity premia. Based on a counterfactual analysis (i.e. constructing ILB yields without the asset purchase programme), Coroneo (2016) shows that the quantitative easing programme has only slightly impacted the ILB liquidity premium. Given the size of the ILB liquidity premium and the consequential costs for the Treasury, further in-depth analyses on how the ILB liquidity premium can be lowered represent a promising avenue for future research.

5. Current Developments

5.1 Inflation Risk Premium and Monetary Policy

Analysing the impact of ‘surprising’ monetary policy events is possible without the use of ILB by looking at nominal bond yields or survey inflation forecasts. With a term structure model that is able to
Table 2. Overview of Applied Liquidity Proxies.

| Proxy                                | Frequency | Source                          | Modifications (studies used in)                                                                 |
|---------------------------------------|-----------|---------------------------------|-------------------------------------------------------------------------------------------------|
| **ILB-specific liquidity proxies**    |           |                                 | calculations as outlined in Hu et al. (2013) (used by Abrahams et al., 2016; D’Amico et al., 2018; Grishchenko and Huang, 2013) |
| ILB fitting errors                    | Daily     | CRSP US Treasury Database       | no modification (used by D’Amico et al., 2018)                                                |
| ILB bid-ask spread                    | Daily     | TradeWeb (ThomsonReuters)       |                                                                                                 |
| **Relative ILB liquidity proxies**    |           |                                 |                                                                                                 |
| Outstanding ILB                      | Monthly   | TreasuryDirect                  | * relative to all nominal debt outstanding with comparable maturity (used by Kajuth and Watzka, 2011; Shen, 2006) |
|                                       |           |                                 | * relative to all nominal debt outstanding (used by Auckenthaler et al., 2015)                    |
| ILB Primary Dealer Transactions      | Weekly    | Federal Reserve Bank of New York| * logarithm of ILB transaction volume (used by Shen, 2006)                                      |
|                                       |           |                                 | * relative to volume of ILB outstanding (used by Grishchenko and Huang, 2013; Söderlind, 2011)  |
|                                       |           |                                 | * relative to all nominal transactions (used by Auckenthaler et al., 2015; Gürkaynak et al., 2010; Hördahl and Tristani, 2014) |
|                                       |           |                                 | * relative to all nominal transactions with comparable maturity (used by Kajuth and Watzka, 2011) |
|                                       |           |                                 | * relative to all transactions and 13-week moving average (used by Abrahams et al., 2016; D’Amico et al., 2018) |
|                                       |           |                                 | * relative (log) to all long-term nominal transactions and 3-month moving average (used by Pfueger and Viceira, 2016) |

(Continued)
Table 2. continued.

| Proxy                                                                 | Frequency | Source            | Modifications (studies used in)                                                                 |
|----------------------------------------------------------------------|-----------|-------------------|------------------------------------------------------------------------------------------------|
| ILB asset swap spread—nominal off-the-run asset swap spread         | Daily     | Barclays          | no modification (used by D’Amico et al., 2018)                                                |
| Inflation swap rate—break-even inflation rate                        | Daily     | Bloomberg         | no modification (used by Christensen and Gillan, 2012; D’Amico et al., 2018; Pflueger and Viceira, 2016) |
| Treasury liquidity proxies                                           |           |                   |                                                                                                |
| On-the-run/off-the-run spread                                       | Daily     | Bloomberg or CRSP | no modification (used by Grishchenko and Huang, 2013; Kajuth and Watzka, 2011; Pflueger and Viceira, 2016; Shen, 2006; Söderlind, 2011) |
| Spread between Refcorp strips and nominal Treasury strips           | Daily     | Bloomberg         | no modification (used by Gürkaynak et al., 2010 and Hördahl and Tristani, 2014)               |
| Proxies for uncertainty                                             |           |                   |                                                                                                |
| Implied volatility of S&P500 (VIX)                                  | Daily     | Chicago Board Options Exchange | no modification (used by Auckenthaler et al., 2015; Söderlind, 2011)                        |
| Implied volatility of 10-year future notes                          | Daily     | Bloomberg         | no modification (used by Auckenthaler et al., 2015)                                            |
| Spread between interbank rate and federal funds rate                | Daily     | Bloomberg         | no modification (used by Auckenthaler et al., 2015)                                            |

Notes: This table summarizes all ILB liquidity proxies that are used in the studies above. The last column informs about potential modifications and lists studies that use this proxy. Stated frequency refers to the data frequency in general and not to the one used in the respective studies.
decompose yield curves into its single components, however, more detailed interferences can be drawn: By using solely nominal bond yields, for instance, one cannot differentiate between effects on inflation expectations and effects on the inflation risk premium. With survey inflation forecasts, on the other hand, their low frequency does not allow analysing immediate effects. By decomposing the yield curve, both monetary policy effects as well as macro-economic effects, in general, can be analysed very effectively and implications about the origin of the inflation risk premium can be drawn more precisely.

Hördahl and Tristani (2014) and Abrahams et al. (2016) are among the studies that link the components of the term structure with monetary policy. In a basic analysis, Abrahams et al. (2016) show the relation between the inflation risk premium and, among others, the survey inflation forecast disagreement (positive correlation) or the consumer confidence (negative correlation). Similarly, also, Hördahl and Tristani (2014) find a countercyclical inflation risk premium until the financial crisis in 2008. During the financial crisis, the premium has a rather pro-cyclical development which is, however, in accordance with the discussion by Chen et al. (2016) about the inflation risk premium’s sign (see Section 2.1).

Regarding the effects of monetary policy, Abrahams et al. (2016) analyse both the impact of conventional as well as unconventional monetary policy on the yield curve components. Concerning conventional monetary policy, the authors analyse the response of yield curve components during Federal Open Market Committee (FOMC) announcements between 1999 and 2008. While the effect on inflation expectations is negligible, the response of the inflation risk premium is negative and statistically significant. The response is negative over all maturities (from 2 to 10 years) but shows a U-shaped pattern, implying that inflation risk is less relevant in the medium term. This is again in line with the discussion in Section 2.1: Monetary shocks, in this case emerged by FOMC announcements, increase the covariance between inflation and consumption growth leading to a reduction of the inflation risk premium or even to a change in the premium’s sign. On the contrary, note that existing studies, such as Hanson and Stein (2015), are unable to explicitly decompose the yield curve and to provide implications for the single curve’s components.

5.2 Estimation Issues

It is obvious that fully fledged term structure models should be favoured to more simple approaches because of their model-implied (and therefore consistent) estimates for expected inflation and risk premia. However, the more complex (and thus also more flexible) models become, the more relevant estimation issues are. While all studies in Section 3.2 use a maximum likelihood estimation with the Kalman filter (or a modification of it), there might arise estimation problems with canonical term structure models. Loosely speaking, the latent factor specification could lead to multiple maxima during the estimation (because factors can rotate). While the CDR model builds on a theory (i.e. dynamic Nelson-Siegel model), this might be a weak spot of latent-factor models (see, e.g. Kim and Orphanides, 2012, for a discussion). New developments in estimation techniques have emerged and comprise, for instance, a three-step linear regression approach proposed by Adrian et al. (2013). In an earlier version of the Abrahams et al. (2016) paper, this approach is applied to estimate the model (see Abrahams et al., 2015). Joslin et al. (2011) or Hamilton and Wu (2012) are among the studies that present alternative estimation methods. However, as the focus of this survey is more on studies’ conceptual structure than on technical or estimation issues, the interested reader should refer to the original studies. Thus, one can say at the bottom line that (i) parsimonious models are needed to make over-parameterization less relevant and (ii) the more flexible in terms of factors and correlation structures term structure models become, the more caution is need during the estimation.

5.3 Zero Lower Bound

Recent times with (very) low interest rates cause existing term structure models for nominal interest rates to be revised. The point is that nominal bond yields are expected to approach a lower bound which
is zero or slightly negative. The majority of term structure models, however, allow interest rates to be negative which can lead to implausible estimation results. In other words, during times of very low interest rates, standard term structure models face difficulties to account for the non-linearity at the zero lower bound (see, e.g. Kim and Singleton, 2012 for a discussion). Research started to consider this problem by introducing a so-called shadow rate proposed by Black (1995). Including such a shadow rate in a term structure model causes severe estimation issues due to its non-linear structure. Initial studies that include the shadow rate in their term structure models are limited to a maximum of two factors because of estimation complexity. More precisely, if no closed-form solutions are available, one would have to draw on computation-intensive numerical evaluations (see, e.g. Gorovoi and Linetsky, 2004 or Kim and Singleton, 2012 for a discussion). New simulation or approximation techniques to estimate these models are proposed by Ichiue and Ueno (2013), Kim and Priebsch (2013) and Wu and Xia (2016). Recently, however, Monfort et al. (2017) propose an affine term structure model which accommodates current challenges and provides closed-form pricing formulas. More specifically, their model includes autoregressive gamma-zero processes which are consistent with non-negative yields and compatible with a short-term rate that stays at the zero lower bound. Monfort et al. (2017) demonstrate the model’s features for Japanese government bonds.

An alternative way to overcome these computational burdens is proposed by Krippner (2013) with an option-based approach for estimating a shadow rate term structure model. This general procedure allows to estimation of higher dimensional term structure models and mainly serves as a workhorse model for succeeding studies that account for the zero lower bound. The basic idea of this option-based approach is that an observed bond, with a yield constrained by the zero lower bound, may be considered as a combination of a shadow bond that can take on negative yields and a call option that represents the value of the option to hold physical currency rather than being obliged to receive negative shadow rates if they occurred. Christensen and Rudebusch (2015) incorporate this approach into the CDR model and apply it to the Japanese yield curve. Similarly, Christensen et al. (2016) apply the shadow rate CDR model to the USA and document a better model fit compared to standard model during times of low interest rates.

To account for the zero lower bond is also important with regard to the estimation of the inflation risk premium since the nominal yield curve is used as input data. Note that the zero lower bond is, however, less relevant for ILB yields since investors accept negative real yields due to the additional inflation compensation. Thus, jointly pricing nominal bond and ILB yields curves in a low interest rate situation requires to account for the zero lower bound as well because break-even inflation rate will be (even further) distorted. More specifically, distortion arises from the option effect in nominal bond yields described above (representing the value to hold physical currency rather than having to invest at negative shadow rates). To the best of the author’s knowledge, Carriero et al. (2016) is the only study that jointly prices the nominal and ILB yield curve and accounts for the zero lower bound. The authors develop their joint shadow rate model by merging Christensen et al.’s (2010) model with the shadow rate model by Christensen and Rudebusch (2015). Carriero et al. (2016) apply their model to the UK yield curve and show that the consideration of the zero lower bond is essential in the current low-interest situation. Given the relevance of this topic and the scarcity of existing studies, this is an important issue for future research.

6. Conclusion

Information about the inflation risk premium is crucial for policy makers, investors and sovereign debt management alike. While various possibilities exist to estimate the inflation risk premium in nominal bond yields, this survey focused on studies that additionally use ILB yields. The benefit of using ILB yields are, among others, a high frequency of data and the availability of different maturities. These features are not given for survey inflation forecasts. While this on first glance implies the possibility of improved
estimates for the inflation risk premium when ILB yields are taken into account, current research shows an added complexity to this apparently obvious result.

I show that existing studies considerably differ in both their models and their input data. While the categorization of approaches in regression-based and term-structure models can be easily achieved, further classification of the input data proves to be more complex. In addition, a liquidity premium in ILB yields further complicates estimations. This review outlines how existing studies differ in the degree to which they account for the liquidity premium, and presents a list of existing ILB liquidity proxies. Both research designs and studies’ estimates for the inflation risk premium are compared. The estimates differ as well considerably among the studies of which some explicitly consider a liquidity premium and others do not. As one main result of the review, the possibility of finding and applying one single and universal approach of IRP estimation can be neglected: factors like estimation and specification uncertainty or how one accounts for the ILB liquidity premium drive the discrepancies between published estimates. Nonetheless, one observes a tendency towards term structure models that account for the liquidity premium in ILB yields. In fact, most recent papers include the liquidity correction within their term structure models.

Current developments in the research area have been outlined in this paper, like the connection of term structure models with monetary policy, developments in term structure estimation strategies and the consideration of the zero lower bound, to give an overview of the state of the art. The survey also presents potential directions for future research like, for instance, term structure models that account for the zero lower bound in nominal yields but allow for negative ILB yields. Similarly, in-depth analyses of the ILB liquidity premium (like Coroneo, 2016 or Christensen and Gillan, 2017, for instance) and its determinations are still necessary because the premium’s elimination would allow Treasuries to save an enormous amount of money by issuing ILB. Finally, topics like recent finance-based approaches (rather than macro-economic models) to measure the equilibrium real rate (see, e.g. Christensen and Rudebusch, 2017) or two-(or multi-)country term structure models that additionally include ILB yields represent further important directions for future research.

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Notes

1. Differences in the steepness of the nominal and real term structure, in fact, suggest (in the absence of any other risk premia) a positive inflation risk premium. See also Bekaert and Wang (2010) for a discussion.
2. Since most of the studies use US data, I focus on the USA in the main text. However, Endnote 9 gives an overview of studies using data from the United Kingdom (UK) and the euro area.
3. This derivation is closely related to Christensen et al. (2010). Alternatively, one arrives at the same conclusion when using only a real stochastic discount factor with a price deflator (see, e.g. Geiger, 2011).
4. Data are derived from the website of the respective debt management office.
5. Note that in the USA, these securities are called Treasury inflation-protected securities (TIPS). Before and after the first years of issuance, these bonds were typically known as Treasury inflation-indexed Securities (TIIS). For the remainder of the paper, I simply refer to ILB.
6. In fact and more precisely, there is also the real risk premium because, put simply, expected real returns might deviate from realized real returns. The real risk premium is relevant for both, nominal bond yields and ILB yields and cancels out when the break-even inflation rate is calculated. For the sake of readability, I suppress the real risk premium in Equations (5)–(10). If I explicitly introduce a real risk premium \( \phi^{RRP} \). Equations (7) and (9) would then read as \( y^N_t = E_t[r] + \phi^{RRP} + E_t[\pi] + \phi^{RL} \) and \( y^{ILB}_t = E_t[r] + \phi^{RRP} + \phi^{LIQ} \), respectively, where \( E_t[r] \) is the expected real return and \( \phi^{RRP} \) is the real risk premium. Calculating \( y^N_t - y^{ILB}_t \) results in \( E_t[\pi] + \phi^{RRP} - \phi^{LIQ} \) which is the same as in Equation (10).

7. More precisely, the Fisher equation is given by \( (1 + y^N_t) = (1 + y^R_t) \cdot (1 + E_t[\pi]) \). Multiplying out leads to \( 1 + y^N_t = 1 + y^R_t + E_t[\pi] + E_t[\pi] \cdot y^R_t \). Since both \( E_t[\pi] \) and \( y^R_t \) are typically rather small values, the cross product gets even smaller and is neglected in the approximation. The approximation is frequently written as equality: \( y^N_t = y^R_t + E_t[\pi] \).

8. Note that for the case that ILB would be as liquid as their nominal counterpart and that there exists a positive inflation risk premium in nominal bond yields, borrowing money would be cheaper with ILB than with nominal bonds for a sovereign. This is one advantage of issuing ILB among others. An overview of ILB benefits is given in Appendix A, and Appendix B discusses whether the US Treasury actually has saved money by issuing ILB.

9. Note that the United Kingdom started to issue ILB already in 1981. As a consequence, studies using ILB yields evolve earlier than in the USA. However, studies by Arak and Kreicher (1985), Wilcox (1985) or Woodward (1990) assume the inflation risk premium to be zero or to be constant. Chu et al. (1995) apply a different approach by analysing the correlation between inflation forecast errors and nominal returns. Similarly, Shen (1998) extracts the inflation risk premium for the United Kingdom by subtracting the average expected inflation (based on survey data) from the break-even inflation rate. This computation results in an inflation risk premium sized between 130 (for a maturity of 10 years) to 160 (for a maturity of 25 years) basis points over a two year period (i.e. 1996–1997). Söderlind (2011), Auckenthaler et al. (2015) and Pflueger and Viceira (2016) apply regression-based approaches for the USA (see main text) as well as for the United Kingdom and/or the euro area. Kanas (2014) presents a slightly different approach by additionally using bond futures. The author estimates the covariance between both nominal bond yields and bond futures as well as the covariance between ILB yields and bond futures. The difference in covariances is fully attributed to inflation risk and averages 87 basis points between 1985 and 2012. Term structure models have been developed by Risa (2001), Evans (2003), Joyce et al. (2010) and most recently, by Carriero et al. (2016). Both Risa (2001) and Joyce et al. (2010) present essentially affine term structure model that jointly price the nominal and ILB term structure for the United Kingdom. Evans (2003) develops a term structure model that includes a regime-switching property. This feature allows for a higher flexibility but estimation difficulties at the same time if too many states are included. Carriero et al. (2016) extend the term structure model developed by Christensen and Rudebusch (2015) which accounts for the zero lower bound by joint pricing nominal bonds and ILB. For a closer discussion of the zero lower bound issue, see Section 5 in the main text. A rather small research body focuses on the euro area which is mainly due to the limited availability of ILB (France, Germany and Sweden have issued ILB in a considerable amount; Italy and Spain re)started recently issuing ILB). The paper by Hördahl and Tristani (2014) (outlined in Section 3.2 of the main text) also analyses euro area data. García and Werner (2010) solely focus on euro area and develop a three-factor term structure model for the nominal and real yield curve which includes (among nominal bond yields and ILB yields) inflation and survey inflation forecast data. Most notably, the authors derive proxies for inflation risk (i.e. inflation uncertainty and asymmetry in inflation risks) from survey inflation forecasts and relate these proxies to their estimate of the inflation risk premium. None of these studies has focused on ILB mispricing due to illiquidity indicating that a potential liquidity premium seems to be less relevant in the United Kingdom and the euro area. This could, however, also be driven by
a lower availability of liquidity proxies implying that a potential liquidity premium is more difficult to ‘detect’. The development of reasonable instrument-specific liquidity proxies (especially for the euro area) describes an avenue for future research.

10. For simplicity and readability, I do not state the specific liquidity proxies throughout this section but list them altogether in Table 2 (see Section 4).

11. See Bekaert and Wang (2010, p. 780) for a typical technique to develop a term structure model.

12. So-called hypothetical ILB yields are also estimated by Campbell and Shiller (1996). However, they estimate ex ante real yields with a VAR system that includes three-month nominal T-bill rates and inflation rates. By forecasting ex ante real yields to the respective maturity, the authors derive hypothetical ILB yields. This procedure, however, requires that the expectations hypothesis holds and that the real term premium is zero (see Auckenthaler et al., 2015, for a discussion).

13. See Bauer and Rudebusch (2017) for an excellent review of macro–finance term structure models.

14. The rationale for the zero lower bound is, loosely speaking, because investors can hold cash. However, investors will, in principle, accept mildly negative yields for the convenience and safety value of electronic balances relative to physical currency. In fact, we observe that several central banks have set slightly negative policy interest rates.

15. Note that practically, also CIR models have to feature of non-zero interest rates due to the square-root process. In these models, however, the zero interest rate rather acts as a reflecting barrier which conflicts with empirical evidence.

16. Note that the paper by Roussellet (2016) discussed in Section 3.2 is similar but uses inflation swap rates rather than ILB data.

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Appendix A: Overview of the Advantages of ILB

The reasons for a sovereign state to issue ILB are twofold: First, it can be related to credibly signalling that moderates inflation rates that are put into consideration. This rationale was used, for instance, by Israel in 1955 or the United Kingdom in 1981, while they suffered from high inflation rates in the past.
and tried to lower these rates in the future. The issuance of ILB reduces the incentive of governments to ease the real value of sovereign debt via higher inflation rates and therefore serves as a credible signalling device. In this context, Margaret Thatcher called these securities a ‘sleeping policeman’ that helps monitor inflation, and more recently, Ewald and Geissler (2015) present a monetary policy framework that reduces the central banks inflationary bias owing to the availability of inflation-linked debt (see also García and van Rixtel, 2007, who generally analyse ILB from a central banks point of view).

The second argument for issuing ILB is more recent and refers to a cost-saving strategy of sovereigns debt management. As discussed in this survey, nominal bond yields include, besides the real interest rate and expected inflation, an inflation risk premium. If this premium is positive and if ILBs do not include any premium, issuing ILB instead of nominal bonds would lower sovereigns financing costs. These potential savings for the Treasury have been used as an argument for the ILB issuance in, among other countries, the USA or Canada. However, the magnitude of the potential savings is unknown ex ante for two reasons: First, the inflation risk premium cannot directly be observed; second, ILB should be free of any premia. As shown in this paper, both a (most of the time positive) inflation risk premium and a liquidity premium in ILB yields exist. A clear-cut evaluation of whether the Treasury has saved money by issuing ILB is not possible. Appendix B discusses this issue.

While these features are beneficial only for sovereigns, there also exist advantages for policy makers and investors. The most obvious benefit for investors is inflation protection, which is, for instance, not fully given in nominal bonds (in the case of considerable differences between the expected and realized inflation rate). Furthermore, the presence of inflation-linked securities allows a shift in asset allocation and could lead to better diversification of investors portfolios (see, for instance, Bodie, 1990; Chen and Terrien, 2001; or Kothari and Shanken, 2004). Finally, as discussed extensively in the paper, ILBs are useful for policymakers since their yields can be used to extract market-implied inflation expectations. However, as shown, break-even inflation rates are biased by inflation risk premium and liquidity premium, and adjusting for these premia is essential before the break-even inflation rate can be interpreted.

Appendix B: Have ILB Issuances Led to Costs or Savings for the Treasury?

The presence of an inflation risk premium in nominal bond yields and a liquidity premium in ILB yields—as outlined in this paper—raises the question of whether the Treasury has actually saved money by issuing ILB, as was the main argument for the ILB issuance at least in the USA. This appendix focuses on studies that analyse the costs and savings of ILB issuances. Basically, every paper that estimates the (liquidity-adjusted) inflation risk premium can be used to proxy the potential savings for the Treasury.

Based on the calculations of the inflation-risk premium, only Christensen and Gillan (2012) further compute the potential savings for the Treasury owing to the saving of the inflation-risk premium. The authors’ so-called ‘minimum liquidity-adjusted inflation risk premium’ is positive on average and therefore implies savings for the Treasury. Christensen and Gillan (2012) furthermore argue that an expansion of the ILB programme in the USA would save the Treasury about $25 billion over a period of 10 years for a constant debt level and an increase of outstanding ILB to 25% of all US government debt outstanding. On the other hand, if the US debt level is increasing constantly (as in the last 10 years) and if the ratio of outstanding ILB would be again 25%, potential savings would amount to about $51 billion. These figures, however, are rather conservative since the calculations are based on the authors’ maximum liquidity premium (see also Section 3.2). Given an expansion of the ILB programme, the liquidity premium in ILB yields will likely disappear or at least decrease, implying that the liquidity-adjusted inflation risk premium will increase in the same way.

The paper by Fleckenstein et al. (2014) also investigates the ILB market in the USA by using inflation swap rates. More specifically, the authors rebuild the identical cash flows of a nominal bond by combining the cash flows of an ILB with an inflation swap. Each ILB outstanding is matched with a nominal
bond of the same maturity and a similar issue date, and then, by combining ILB with inflation swaps, the mispricing for each bond pair is calculated. Fleckenstein et al. (2014) calculate the average ILB mispricing from 2004 to 2010, which exhibits about 40 basis points until 2008. Doing the time around the Lehman Brothers bankruptcy, however, mispricing increased to about 170 basis points (see Fleckenstein et al., 2014, figure 2). Regarding the costs that emerge from this mispricing, the authors discuss the following option: The Treasury could buy back all outstanding ILB, issue new nominal debt with the same maturity and hedge the inflation risk with inflation swaps. This buy back could save the Treasury about $11 billion at the end of the authors’ sample period. At the peak of the mispricing in end-2008, the savings with this procedure would have amounted to about $56 billion. An alternative approach would be to calculate the total costs of the Treasury for issuing ILB instead of nominal bonds, which would amount to about $10 billion during the authors’ sample period. Based on these ex post calculations, Fleckenstein et al. (2014) conclude that the issuance of ILB was rather expensive for the Treasury.

Similar to Sack and Elsasser (2004), Fleckenstein et al. (2014) also calculate the ex post costs of the ILB issuance for the Treasury. Sack and Elsasser’s (2004) sample period lasts from 1997 to 2003 and therefore mainly covers the early years of ILB issuance, which were characterized by a high liquidity premium. The approach to calculate the costs is different from that of Fleckenstein et al. (2014) and basically compares observed, unadjusted break-even inflation rates with either realized inflation or survey inflation forecasts. The idea is to use realized and survey inflation as a reliable measure for inflation, and the difference between break-even inflation rate and their inflation proxy determines the costs or savings of issuing ILB. Thus, Sack and Elsasser’s (2004) approach yields current costs (in 2004) in the amount of about $3 billion (by using realized inflation) and total (expected) costs of about $12 billion (by using survey inflation forecasts). Nonetheless, the authors argue that these costs can be the consequence of different, ILB-specific characteristics such as the fact that ILBs constitute a novel asset class in the USA, ILBs are very attractive for buy and hold investors and ILB might be rather illiquid owing to the low amount issued thus far. In this context, Fleming (2003) documents a smaller trading volume, a longer turnaround time and a wider bid-ask spread for US ILB in comparison with their nominal counterparts.

In a similar vein, the paper by Roush (2008) calculates the savings/costs of the US ILB programme until 2007 and derives a range for the current costs (in 2007) of $5 to $8 billion. Roush (2008) also derives hypothetical savings/costs assuming that ILB would be as liquid as in 2007 over the entire sample period (i.e. 1997–2007). In this case, ILB issuances would have led to savings (in 2007) that amount to about $14 to $17 billion. Therefore, as the author concludes, proving a liquid market is of key importance for the Treasury in order to shift costs of ILB issuances into savings (see also, for instance, Shen, 2009). Similarly, Dudley et al. (2009) investigate the costs and savings of ILB issuance in the USA and apply the same method as Roush (2008) by comparing the auction break-even inflation rate with survey inflation forecasts. Dudley et al. (2009) confirm the findings of the previous study and show for the most recent ILB issuances (in the beginning of 2008, before the Lehman Brothers bankruptcy) that the costs for issuing a nominal bond are ex ante virtually equal to the costs of issuing an ILB. However, as shown in, for instance, Fleckenstein et al. (2014), the costs of ILB considerably increased again during the financial crisis. This period of financial turmoil, however, is not included in the authors’ analysis owing to the paper’s publication date (i.e. 2009).

Only a few studies provide international evidence on the costs and savings of ILB issuances. Most importantly, in this context, the follow-up paper by Fleckenstein (2013) calculates the ILB mispricing in the vein of Fleckenstein et al. (2014) for the USA and six other countries (i.e. Canada, France, Germany, Italy, Japan and United Kingdom). Mispricing is shown to be rather low on average before the financial crisis but increasing during the crisis for all countries. Interestingly, the mispricing peak around the Lehman Brothers bankruptcy is highest for the USA, followed by the United Kingdom. For the United Kingdom, a publication by the Debt Management Office reveals that ILBs ‘have led to a significant reduction in the cost of funding’ (UK Debt Management Office, 2001, p. 39) and similarly for France where a publication by the Minister of the Economy, Finances and Industry summarizes savings owing to
ILB issuances up to €120 million for the period 1998–2004 (see Coeuré and Sagnes, 2005). In addition, Reschreiter (2004) studies different risk compensations of nominal bonds and ILB for the United Kingdom and finds that they are different from each other. While the author concludes that ILB could significantly reduce funding costs for the Treasury, there is no analysis of the real (ex post) savings or costs of ILB issuance in the United Kingdom (see also Reschreiter, 2010a and Reschreiter, 2010b).

Appendix C: Comparison of Estimates for the Inflation Risk Premium

The following figure shows estimated inflation risk premia of the following studies: Christensen et al. (2010), Kajuth and Watzka (2011), Haubrich et al. (2012), Hördahl and Tristani (2014), Auckenthaler et al. (2015), Abrahams et al. (2016), and Andreasen et al. (2017). Time periods (and thus minimum/maximum values as well) may vary to the values in Table 1 since some authors provided updated time series. It is quite impressive that even during the period 2003–2007, the inflation risk premium varies between −50 basis points and +100 basis points. During the financial turmoil in 2008 and 2009, on the other hand, the estimates fluctuate between −200 basis points and +200 basis points. Recalling the interaction between inflation risk premium in nominal bond yields and liquidity premium in ILB yields, Christensen et al.’s (2010) low values for the inflation risk premium in 2008 can be attributed to the fact that the paper does not account for ILB illiquidity. Comparing the most recent studies that provided the data for this survey, namely Abrahams et al. (2016) and Andreasen et al. (2017), one can observe a parallel movements with, however, a level difference of about 100 basis points.

**Figure C1.** Estimated Inflation Risk Premia of Selected Studies Discussed in the Survey.

*Notes:* The figure presents estimated inflation risk premia of the following studies: Christensen et al. (2010), Kajuth and Watzka (2011), Haubrich et al. (2012), Hördahl and Tristani (2014), Auckenthaler et al. (2015), Abrahams et al. (2016) and Andreasen et al. (2017). For inflation risk premia with a daily frequency, monthly averages are calculated (i.e. Christensen et al., 2010 and Abrahams et al., 2016). For inflation risk premia with a quarterly frequency, linear interpolation is applied (i.e. Auckenthaler et al., 2015). Time periods may vary to the time periods described in Table 1 since some authors provided updated series.