Progressive Failure Analysis of Composite Laminates Including Shear Nonlinearity

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Abstract. A progressive damage model including the non-linear shear behavior was developed to investigate the failure mechanisms of composite laminates. The non-linear shear response of the T300/Cycom970 lamina was derived by cyclic loading-unloading tensile tests on [+45/-45]_2s laminate coupons. The strain-based Hashin type failure criteria and ply discount method were employed to predict damage initiation and propagation. The model was implemented in commercial finite element software ABAQUS by a UMAT subroutine. The tensile behavior of open-hole tension specimen was predicted with this model. A reasonable good agreement was achieved between numerical predictions and experimental results.

1. Introduction
Composite laminates have been increasingly used in aerospace structures due to their excellent properties. It is well known that fiber reinforced composite laminates exhibits significant non-linear shear behavior. It is necessary to develop a damage model considering shear nonlinearity for composite laminates in designing composite structures.

To date, the most widely used non-linear shear model in failure analysis of composite materials was the Hahn-Tsai model [1]. However, the shear strain was expressed as a cubic function of shear stress in this model. It’s more convenient to be implemented in finite element analysis when the stress is expressed as a function of strain. It is also found by McCarthy et al. [2] that no acceptable fit to experimental data for carbon/epoxy laminates can be achieved with Hahn-Tsai non-linear shear model. To overcome the limitations of the Hahn-Tsai model, a novel approach involved the use of cubic spline interpolation was proposed to implement the three-dimensional progressive damage analysis for composite laminates. The open hole compressive strength and fatigue strength of bolted joint structures were predicted with the approach using cubic spline interpolation [3,4]. For failure analysis, unloading of the bulk material will occur around the failure zone, so a proper description of the unloading behavior is of importance. Van Paepegem et al. [5] proposed a phenomenological model includes both damage and plasticity for shear nonlinearity. It can be fitted with respect to observed loading/unloading behavior with both stiffness degradation and permanent strain. Nikbakht et al. [6] developed a non-linear progressive damage model considering in-plane and out-of-plane shear stresses to predict the failure mechanisms of composite laminates.

In order to accurately predict the non-linear behavior of composite laminates, a progressive damage model considering the loading-unloading shear behavior is proposed. The strain-based Hashin type failure criterion and ply discount method were employed to predict fiber and matrix damage. The model was implemented in ABAQUS finite element code with the user defined material subroutine UMAT. The capability of the non-linear progressive damage model is validated with predicting the tensile behavior of notched composite laminates.
2. Progressive damage model

2.1. Failure criteria

In order to predict the damage initiation and propagation of each intralaminar failure of the material, strain-based Hashin type failure criterion was employed to determine the longitudinal fiber failure and transverse matrix failure, it is shown as follows:

(1) Longitudinal fiber failure:

\[ f_1 = \begin{cases} \left( \frac{\epsilon_{11}^{f,T}}{\epsilon_{11}^{f,T}} \right)^2 + \left( \frac{\gamma_{12}}{\gamma_{12}^f} \right)^2, & \epsilon_{11} \geq 0 \\ \left( \frac{\epsilon_{11}}{\epsilon_{11}^{f,C}} \right)^2, & \epsilon_{11} < 0 \end{cases} \]

(2) Transverse matrix failure:

\[ f_m = \begin{cases} \left( \frac{\epsilon_{22}^{f,T}}{\epsilon_{22}^{f,T}} \right)^2 + \left( \frac{\gamma_{12}}{\gamma_{12}^f} \right)^2, & \epsilon_{22} \geq 0 \\ \left( \frac{\epsilon_{22}}{\epsilon_{22}^{f,C}} \right)^2 + \left( \frac{\gamma_{12}}{\gamma_{12}^f} \right)^2, & \epsilon_{22} < 0 \end{cases} \]

where \( \epsilon_{11}^{f,T} = X_T / E_{11} \), \( \epsilon_{11}^{f,C} = X_C / E_{11} \), \( \epsilon_{22}^{f,T} = Y_T / E_{22} \), \( \epsilon_{22}^{f,C} = Y_C / E_{22} \), \( \gamma_{12}^f = S_l / G_{12} \). \( \epsilon_{11}^{f,T} \) and \( \epsilon_{11}^{f,C} \) are the tensile and compressive failure strain in fiber direction of composite lamina, respectively. \( \epsilon_{22}^{f,T} \) and \( \epsilon_{22}^{f,C} \) are the tensile and compressive failure strain perpendicular to the fiber direction, respectively. \( \gamma_{12}^f \) is the in-plane shear failure strain. \( X_T \) and \( X_C \) are tensile strength and compressive strength in fiber direction of composite lamina, \( Y_T \) and \( Y_C \) are the tensile strength and compressive strength perpendicular to the fiber direction, \( S_l \) is the in-plane shear strength. \( E_{11} \) and \( E_{22} \) are the elastic modulus in fiber direction and perpendicular to the fiber direction, \( G_{12} \) is the in-plane shear modulus. Damage will occur when \( f_1 \) or \( f_m \) exceeds 1.

2.2. Non-linear shear model

The material responses of T300/Cycom970 composite laminates in fiber direction and transverse direction were determined from tension tests on [0]_8 and [90]_16 coupons, respectively, in accordance with the ASTM standard D3039 [7]. Stress-strain curves for longitudinal and transverse tension are shown in figure 1. The fiber direction response is essentially linear to failure, while the transverse stress-strain curve is quasi-linear to failure.

The in-plane shear response was determined from tensile tests on [±45]_2S coupons in accordance with the ASTM standard D3518 [8]. The cyclic loading/unloading stress-strain curve is shown in figure 2, showing clearly that the response is non-linear. The shear strength value was determined as shear stress at a shear strain of 5%. The degradation of unloading shear modulus is shown in figure 3.
Figure 1. Typical stress-strain curves of T300/Cycom970 composites (a) [0]_8 ; (b) [90]_{16}

Figure 2. In-plane shear stress-strain curve of T300/Cycom970 composites

Figure 3. In-plane shear modulus degradation of T300/Cycom970 laminates

Polynomial function and exponential function are introduced respectively to describe the non-linear shear behavior and degradation law of unloading modulus for T300/Cycom970 laminates as in equation (3) and equation (4). \(G_{12}^0\) and \(G_{12}^u\) are the initial shear modulus and unloading shear modulus, \(d_{12}\) indicates the reduction of shear modulus under cyclic loading. The fitting parameters are shown in Table 1.

\[
\tau_{12} = c_1\gamma_{12}^0 + c_2\gamma_{12}^1 + c_3\gamma_{12}^2 + c_4\gamma_{12}^3 + c_5\gamma_{12}^4
\]

\[
G_{12}^u = G_{12}^0(\alpha + \beta e^{\alpha \gamma_{12}})
\]
\[ G_{12}^0 = G_{12}^{\text{tan}} \bigg|_{\gamma_{12}=0} = \frac{d\tau}{d\gamma} \bigg|_{\gamma_{12}=0} = c_1 \]  
\[ d_{12} = 1 - \frac{G_{12}^0}{G_{12}} = 1 - \alpha - \beta e^{\alpha \gamma_{12}} \]  

Table 1. The fitting parameters of loading-unloading shear strain-stress curve

| Parameters | \( c_1 \) | \( c_2 \) | \( c_3 \) | \( c_4 \) | \( c_5 \) | \( \alpha \) | \( \beta \) | \( m \) |
|------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Value      | 5737.94   | -2.40\times10^5 | 6.06\times10^6 | -7.96\times10^7 | 4.18\times10^8 | 0.3995   | 0.60      | -63.84    |

2.3. Damage evolution

The composite damage analysis plays an important role in the analysis of the laminate structures. The effect of different damage modes on the material properties can be realized by reducing the stiffness matrix of the material. The material properties of damaged material are updated with a ply discount method proposed by Camanho [9]. The stiffness matrix of the material in the progressive failure model is shown as follows:

\[
C_4 = \begin{bmatrix}
\alpha C_{11} & \alpha \beta C_{12} & \alpha C_{13} & 0 & 0 & 0 \\
\beta C_{22} & \beta C_{23} & 0 & 0 & 0 & 0 \\
C_{33} & \alpha \beta (1-d_{12})C_{44} & 0 & 0 \\
\text{sym} & C_{55} & 0 & C_{66}
\end{bmatrix}
\]  

where \( C_{ij} \) are the components of undamaged stiffness matrix. The parameters \( \alpha \) and \( \beta \) are defined as equation (8), \( d_1 \) and \( d_2 \) are damage variables corresponding to fiber failure and matrix failure, respectively.

\[ \alpha = (1-d_1), \quad \beta = (1-d_2) \]  

3. Applications

3.1. Loading-unloading test of \([\pm45]_{2S} \) laminates

In order to test the capability of the numerical model in shear, a cyclic loading–unloading shear test was carried out and compared with experimental results. The stress–strain curve predicted by the numerical model is shown in figure 4. It is not possible to exactly replicate the test cycle with the simulation cycle. Nevertheless, the numerical results agreed well with the experimental results, which verify the validity of the non-linear shear model.
3.2. Open hole tensile test of [±30]_{2S} laminates

For further verifying the capability of the non-linear progressive damage model, the open hole tensile behavior of [±30]_{2S} laminates was simulated. The material properties of T300/Cycom970 lamina are shown in Table 2. The Hashin damage model and Linde damage model built in abaqus without considering the shear nonlinearity were also employed to simulate the open hole tensile behavior of [±30]_{2S} laminates. The reduction of in-plane shear modulus of [±30]_{2S} laminates at the ultimate load simulated with the non-linear model is shown in figure 5. The numerical and experimental results are shown in figure 6. It is shown that the results predicted with the non-linear shear model agreed well with the experimental results.

Table 2. The material properties of T300/Cycom970 composites

| E_{11} (GPa) | E_{22} = E_{33} (GPa) | G_{12} = G_{13} (GPa) | G_{23} (GPa) | v_{12} = v_{13} | v_{23} | X_T (MPa) | X_C (MPa) | Y_T (MPa) | Y_C (MPa) | S_L (MPa) |
|--------------|-----------------------|-----------------------|--------------|-----------------|--------|------------|------------|------------|------------|-----------|
| 125          | 8.59                  | 5.73                  | 3.0          | 0.33            | 0.45   | 1927       | 1400       | 55.5       | 140        | 76        |

Figure 5. Degradation of in-plane shear modulus of [±30]_{2S} laminates at the ultimate load
4. Conclusions
A progressive damage model considering the non-linear shear behavior of composite laminates was developed. Compared to the results simulated with the linear models built in abaqus, the results simulated with the non-linear shear model agreed well with the experimental results. The result shows that it is necessary to consider the shear nonlinearity in failure analysis of composite laminates.

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6. References
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