Hawking radiation and the Bloom–Gilman duality

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Abstract

The decay widths of the quantum black hole precursors, determined from the poles of the resummed graviton propagator, are matched to the expected lifetime given by the Hawking decay. In this way, we impose a kind of duality between a perturbative description and an essentially non-perturbative description, bearing some similarity with the Bloom–Gilman duality for the strong interactions. General relations are then obtained for the widths and masses of the poles in terms of the number of particle species and the renormalisation scale of gravity. A lower bound on the lifetime of the quantum black holes is also obtained.

Keywords: black holes, duality, graviton propagator

1. Introduction

Black holes are non-perturbative objects that arise in general relativity, and describe a strong regime of gravity in which all signals are classically confined within their horizon. Like for other bound states, one can hope to find hints of their existence already at the perturbative level of quantum field theory. In fact, the resummed one-loop propagator of the graviton interacting with matter fields obtained in [1, 2] contains non-trivial poles, which may be interpreted as precursors of black holes [3, 4].

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The complete analytic structure of this propagator was further studied in [5], where a broad spectrum of such resonance-like states was found. In particular, the resummed graviton propagator has the following form:

$$iD^{\alpha\beta}(p^2) = i \left( L^{\alpha\mu}L^{\beta\nu} + L^{\alpha\nu}L^{\beta\mu} - L^{\alpha\beta}L^{\mu\nu} \right) G(p^2),$$  

(1)

where

$$L^{\mu\nu}(p) = \eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2},$$  

(2)

and

$$G^{-1}(p^2) = 2p^2 \left[ 1 - \frac{Np^2}{120 \pi m_P^2} \ln \left( \frac{p^2}{\mu^2} \right) \right].$$  

(3)

Here, $m_P$ denotes the Planck mass, $\mu$ is the renormalization scale, $N = N_s + 3N_f + 12N_V$, where $N_s$, $N_f$, $N_V$ are the number of scalar, fermion and vector fields, respectively. In the standard model of particle physics, $N_s = 4$, $N_f = 45$, $N_V = 12$ and $N = 283$. The propagator (1) has a standard pole at $p^2 = 0$ and an infinite number of other poles, which are the zeros of the expression (3). The masses of these resonance-like states are given by [5]

$$m_n = m_p \sqrt{\frac{120 \pi \sin \theta_n}{N}} \sin \left( \frac{\theta_n}{2} \right),$$  

(4)

and the corresponding widths by

$$\Gamma_n = m_p \sqrt{\frac{120 \pi \sin \theta_n}{N}} \frac{\sin \theta_n}{|\sin(\theta_n/2)|},$$  

(5)

where the integer $n \geq 0$ labels the $n$th Riemann sheet. Their ratio is thus

$$\frac{\Gamma_n}{m_n} = 2 \cot \left( \frac{\theta_n}{2} \right),$$  

(6)

where the angle $\theta_n$ is the solution of the equation

$$\frac{\theta_n}{\sin \theta_n} \exp \left( - \frac{\theta_n}{\tan \theta_n} \right) = \frac{120 \pi m_P^2}{N \mu^2},$$  

(7)

defining the phase of the resonance on the $n$th Riemann sheet. It is important to recall that in [5] we showed that the phase $\theta$ should belong to one of the intervals

$$2\pi n \leq \theta_n \leq (2n + 1) \pi,$$  

(8)

for these solutions to represent proper resonances.

One may nevertheless ask why such an essentially non-perturbative object as a quantum black hole can be described by means of the perturbation theory. A similar problem inevitably arises in quantum chromodynamics (QCD), where, due to the confinement property, the observed objects are non-perturbatively formed hadrons, whereas the actual calculations can be performed at the level of quarks and gluons. The central role here is played by various types of quark-hadron duality. The first one was probably the famous Bloom–Gilman duality [9] between parton distributions and hadronic resonances, which was marked by Feynman as a manifestation of Bohr’s complementarity [10]. The quark-hadron duality is crucial in the applications of QCD sum rules [11], where the non-perturbative vector-meson coupling
can be related to the simple quark loop. The quark-hadron duality is also related to such a fundamental quantity as the axial anomaly [12]. Recently, a similar duality was observed in relativistic hydrodynamics as an effective theory [13].

In the present paper, we conjecture that a quantum black hole is related to perturbative matter loops like, for example, the $\rho$-meson is related to quark loops. To do so, we confront our previous results [5], based on perturbative calculations [1, 2], with the standard description of the Hawking evaporation effect, based on the consideration of quantum particles on the classical Schwarzschild background [6]. Namely, we postulate that the widths (5) determined by the poles of the graviton propagator (1) equal the inverse of the decay times following from the canonical Hawking evaporation. This assumption leads to general relations between the parameters that describe the black hole precursors.

2. Duality for black holes

A black hole with mass of the order of the Planck mass is most likely a quantum object and, without an established theory of quantum gravity, one can only conjecture how the Hawking evaporation will affect such a quantum black hole. As a working hypothesis, we shall assume that the usual expression of the Hawking radiation [6] provides an estimate of the black hole lifetime $\tau$, from which the decay width reads

$$\Gamma_H = \frac{\tau P m P}{\tau} = \frac{m_p^4}{\alpha m^3},$$

(9)

where $\tau_P$ is the Planck time\(^7\) and $\alpha$ is a positive dimensionless parameter, which depends on the unknown details of the Hawking emission from such extreme black holes. For example, in the case of the standard thermodynamic approach exploiting the Stefan–Boltzmann law (see e.g. [7, 8]) this coefficient is rather large, that is

$$\alpha_{SB} = 5120 \pi.$$

(10)

Now, we make the strong assumption that this width should coincide with the one given in equation (5),

$$\Gamma_H = \Gamma_n.$$  (11)

Substituting (5) and (9) into (11), with $m = m_n$ from (4), yields

$$F(\theta) \equiv \frac{\sin^3(\theta) \sin^3(\theta/2)}{\theta^2} = A,$$

(12)

where $\theta = \theta_n$ must satisfy the condition (8) and

$$A \equiv \frac{N^2}{(120 \pi)^2 \alpha} \simeq 7.0 \cdot 10^{-6} \frac{N^2}{\alpha},$$

(13)

which gives $A \simeq 0.56/\alpha$ for $N = 283$. This is the value of $N$ we will mostly refer to, but we already notice that $A$ is very sensitive to the actual particle content of the theory.

The parameter $\alpha$ plays an especially important role for quantum black holes, whose evaporation process is essentially modified [14]. Indeed, the width $\Gamma_H$ for macroscopic black holes is ensured to be small (and the lifetime $\tau \sim 1/\Gamma_H$ large) by the very large ratio $m/m_P$, whereas the relative stability for microscopic black holes strongly depends on the value of $\alpha$ in $\Gamma_n$.

\(^7\)The Planck time equals the Planck length $l_P$ in our units with $c = 1$. We also recall that the Newton constant $G_N = \ell_P/m_P$ and the Planck constant $\hbar = \ell_P m_P$. 

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The first interesting thing to note is that there exists a minimum value of $\alpha$ for which equation (12) admits a solution given a fixed number $N$ of matter particles. In fact, the function $F$ inside the allowed intervals (8) is a bell-shaped curve like the one shown in figure 1 for the Riemann sheet $n=0$. For larger values of $n$, the shape is similar, except the maximum value of $F$ simply decreases like $\theta^{-2} \sim n^{-2}$. The absolute maximum which occurs for $\theta$ in the $n=0$ sheet can be easily found to be $F_{\text{max}} \equiv F(\theta_{\text{max}} \simeq 1.48) \simeq 0.21$. This implies that one must have $A \lesssim F_{\text{max}}$ or

$$
\alpha \gtrsim 4.9 \frac{(120 \pi)^2}{N^2} \simeq 2.7,
$$

(14)
for a solution $\theta_0$ to exist. We consider this bound as one of the important results of this paper. For $A < F_{\text{max}}$, one will have two such solutions, say $\theta_0^\pm$, which degenerate to one for $A = F_{\text{max}}$, that is $\theta_0^\pm = \theta_{\text{max}}$. This case corresponds to $m_0 \simeq 0.64 m_P$, $\Gamma_0 \simeq 1.4 m_P$, (15) and, from equation (7), a renormalisation scale $\mu_0 \simeq 0.3 m_P$.

For sufficiently small $A$ (equivalently, for sufficiently large $\alpha$), one can find solutions in Riemann sheets with $n > 0$, and the number of sheets with solutions increases for increasing $\alpha$. For instance, for $\alpha = 100$ one has two solutions $\theta_0^\pm$ in the first Riemann sheet, and two more solutions $\theta_1^\pm$ in the second Riemann sheet, as can be seen from figure 2. The corresponding relevant physical quantities are displayed in table 1. Likewise, for $\alpha = \alpha_{\text{SB}}$ in equation (10), one can see from figure 3 that solutions $\theta_n^\pm$ exist up to $n = 19$, and we display a few in table 2.

We can now present some general considerations. It is easy to see from equation (6) that the ratio $\Gamma_n/m_n$ is minimised for $\theta_n^+ \simeq (2n + 1)\pi$ inside the allowed intervals (8). For any given value of $A$ that allows for the existence of resonances, $\theta_0^+$ is always the largest phase (modulo $2\pi$), and will therefore correspond to the (relatively) most stable resonance. On the contrary, since $\theta_0^-$ is the closest to 0 (modulo $2\pi$), the corresponding resonance will always be the (relatively) most unstable. Moreover, the larger $\alpha$ (and thus the larger the number of sheets with solutions), the larger is $\theta_0^+$ (modulo $2\pi$), which makes it more stable (conversely, $\theta_0^-$ is more unstable). We also notice that $\Gamma_n/m_n \simeq 2$ for $\theta_n \simeq \pi/2$ (modulo $2\pi$), which can be a solution only provided if $A$ is close to the maximum of $F$. This will occur if $n$ is the largest integer that admits solutions, so that $\theta_0^- \simeq \theta_0^+$. Such properties are clearly displayed by the cases we considered explicitly above.

| $\alpha = 100$ | $m$ | $\Gamma$ | $\Gamma/m$ | $\mu$ |
|---------------|-----|---------|-----------|------|
| $\theta_0^-$ = 0.29 | 0.16 | 2.3 | 14 | 0.6 |
| $\theta_0^+$ = 2.8 | 0.41 | 0.15 | 0.4 | $3 \cdot 10^{-3}$ |
| $\theta_1^-$ = 7.6 | 0.25 | 0.66 | 2.6 | 0.4 |
| $\theta_1^+$ = 8.5 | 0.32 | 0.32 | 1 | $5 \cdot 10^{-3}$ |

Table 1. Phases, and corresponding masses and widths (in units of $m_P$), for $\alpha = 100$.

Figure 3. The function $F(\theta) - A$ for $\alpha = \alpha_{\text{SB}}$, in the interval $0 \leq \theta \leq 41\pi$ (or $n = 0, 1, \ldots, 20$). Phases $\theta_n^\pm$ are given by intersections with the $\theta$ axis and exist up to $n = 19$ included.
3. Conclusions

In this work, we have explored the opportunity to apply ideas similar to the well-known quark-hadron duality to the poles of the resummed graviton propagator (1), which are interpreted as describing the smallest possible black holes.

Assuming the standard Hawking formula (9) for the decay time of a macroscopic black hole still provides an estimate of the decay width for a quantum black hole, we found the bound in equation (14) for the unknown parameter $\alpha$. This constraint can in turn be interpreted as an upper bound for the Hawking decay width (equivalently, a lower bound for the black hole lifetime), if $\alpha$ itself does not change significantly while the mass $m$ of the radiating black hole is larger than the minimum pole mass $m_0 \approx m_P$. In fact, one approximately has

$$\tau \approx \alpha \ell_P \left( \frac{m_0}{m_P} \right)^3 \gtrsim 0.7 \tau_P, \quad (16)$$

where the values in equations (14) and (15) were used. This bound is one of the main results of this work. In this respect, it is interesting to note that this estimate of the black hole decay time leads to a very slow growth with $\alpha$, as the decrease in the pole masses tends to compensate for the increase in $\alpha$. For instance, for $\alpha = 100$, one can consider the poles with phases $\theta_1^+ = 5.2 \cdot 10^{-2}$ and $\theta_1^- = 3.1 \cdot 10^{-2}$ in table 1, which correspond to a width $\Gamma \lesssim m$, and find $\tau \approx 0.7 \tau_P$. Likewise, for $\alpha = \alpha_{SB}$ one obtains $\tau \approx 83 \tau_P$ for the cases in table 2 with $\Gamma \lesssim m$. In particular, the decay time computed from $\theta_0^+$ as a function of $\alpha$ is shown in figure 4, from which one can estimate

| $\alpha = \alpha_{SB}$ | $m$         | $\Gamma$   | $\Gamma/m$ | $\mu$ |
|------------------------|-------------|------------|------------|-------|
| $\theta_0^+$ = 5.2 \cdot 10^{-2} | 3.0 \cdot 10^{-2} | 2.3        | 77         | 0.6   |
| $\theta_0^+$ = 3.1     | 1.7 \cdot 10^{-1}  | 1.2 \cdot 10^{-2} | 7.0 \cdot 10^{-2} | 1 \cdot 10^{-11} |
| $\theta_{19}^+$ = 121.0 | 7.5 \cdot 10^{-2}  | 1.5 \cdot 10^{-1} | 1.9        | 7 \cdot 10^{-3} |
| $\theta_{19}^+$ = 121.4 | 8.4 \cdot 10^{-2}  | 1.0 \cdot 10^{-1} | 1.2        | 2 \cdot 10^{-15} |

Figure 4. Numerical values of the decay time $\tau \sim 1/\Gamma$, in Planck units, as a function of $\alpha$ (dots joined by dotted line) and the analytical approximation (17) (solid line).
\[ \tau \simeq \frac{2}{3} \alpha_{\text{f}}^{1/2} \tau_p. \]  

(17)

Based on this consideration about the decay time, one could therefore conjecture that the description of quantum black holes is subject to a large degeneracy, mathematically represented by the (potentially infinite) number of Riemann sheets where the poles of the propagator (1) are found.

There is some analogy between the perturbative and non-perturbative physics in QCD and gravity. The contribution of quark loops to the photon propagator [11] allows one to describe the properties of vector mesons in QCD. At the same time, the contribution of matter loops to the graviton propagator might be related to properties of black holes.

Note that perturbative QCD is not sufficient to describe the form of the spectral function. Indeed, in order to establish the duality quantitatively and its domain of validity, the notion of vacuum condensates is also required [11]. At the same time, the perturbative loop contributions are already sufficient to identify the structures which may be interpreted as Breit–Wigner peaks.

The origin of duality is connected with unitarity and the possibility to have different choices for a full set of states. In QCD, for example, one can use either the fundamental (quark and gluon) states or the physical (hadron) states as such full sets, since they are complementary to each other and not additive (for the notions of duality and additivity for various types of QCD factorisation, see [15]). The duality for black holes would mean that the matter states and the states of quantum black holes are also complementary and one should not need to consider both of them together. This would also mean that the well-known degeneracy of the black hole spectrum [16] may provide the large amount of states required to form a complete basis.

To conclude, the emerging analytical structure of the graviton propagator provides a rich set of black hole states. It is possible that they indeed form a full set of states and provide a complementary view to the quantum theory. In this respect, it would be very interesting to repeat our analysis starting from alternative descriptions of (quantum) black hole evaporation (see, e.g. [17–21]), which lead to a departure from the Hawking expression (9) for the decay time.

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