The 4d effective action of 5d gauged supergravity with boundaries

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Abstract

We consider gauged five dimensional supergravity with boundaries and vector multiplets in the bulk. We analyse the zero modes of the BPS configurations preserving $N = 1$ supergravity at low energy. We find the 4d low energy effective action involving the moduli associated to the BPS zero modes. In particular, we derive the Kähler potential on the moduli space corresponding to the low energy 4d $N = 1$ effective action.

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1 Introduction

Supergravity in five dimensions provides the framework to build a host of interesting extra-dimensional models. Two particularly important examples are the compactified version of M–theory on a Calabi–Yau manifold [1] and the supersymmetric Randall–Sundrum model [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]. In both cases, the fifth dimension is an interval corresponding to a $Z_2$ orbifold with two fixed points. The four dimensional end–points of the interval are two branes where matter can be confined. In particular, these branes have a tension leading to a warped gravitational background in the bulk. Hence the type of extra–dimensions present in these supergravity models differs drastically from the usual flat embedding of branes in Minkowski space. The warping of the fifth dimension has led to some interesting phenomenological developments concerning the hierarchy problem [15, 16] and the cosmological constant problem [17].

Such brane models have two typical regimes. In the high energy regime above the brane tension, peculiar phenomena, such as quadratic terms in the matter density contributing to the Friedmann equation, appear [18]. At low energy below the brane tension, the physics can be described by a four dimensional effective action obtained by integration over the extra–dimension. The dynamics are encoded in the moduli corresponding to supersymmetric flat directions of the five dimensional models. For BPS configurations preserving one half of the original supersymmetry, the low energy action is a 4d $N = 1$ action entirely specified by its Kähler potential expressed in terms of the moduli.

In the present paper, we will focus on supergravity in singular spaces as formulated by Bergshoeff, Kallosh and Van Proeyen [4]. It is defined as gauged supergravity in 5d with $n$ vector multiplets in the bulk coupled to two boundary branes. The moduli space is $2(n + 1)$ dimensional. In section 2 we recall some properties of the moduli space both for the real moduli and the associated axion fields. In section 3, we analyse the low energy effective action and give a closed expression for the Kähler potential in terms of the moduli.

2 Supergravity with Boundary Branes

The bulk theory is $N = 2$ pure supergravity [19] coupled to arbitrary vector multiplets [20]. We will not treat the most general case with hypermultiplets and tensor multiplets, which were coupled in [21]. Gauged supergravity with boundary branes in five dimensions has been elegantly constructed when vector multiplets live in the bulk [4, 5]. The supergravity multiplet comprises the metric tensor $g_{ab}$, $a, b = 1 \ldots 5$, the gravitini $\psi^A_a$ where $A = 1, 2$ is an $SU(2)_R$ index and the graviphoton field $A_a$. The $N=2$ vector multiplets in the bulk possess one vector field, a $SU(2)_R$ doublet of symplectic Majorana spinors and one real scalar. When considering $n$ vectors multiplets, it is convenient to denote by $A^I_a$, $I = 1 \ldots n + 1$, the $(n + 1)$ vector fields including the graviphoton.

The vector multiplets comprise scalar fields $\phi^i$ parametrising the manifold $M$

$$C_{IJK}h^I(\phi)h^J(\phi)h^K(\phi) = 1$$

with the functions $h^I(\phi), I = 1 \ldots n + 1$ playing the role of auxiliary variables. The
manifold $M$ has dimension $n$. Defining the metric

$$G_{IJ} \equiv -2C_{IJK}h^K + 3h_I h_J$$  \hspace{1cm} (2.2)

where $h_I \equiv C_{IJK}h^J h^K$, the bosonic part of the Lagrangian (vector fields not included) reads

$$S_{\text{bulk}} = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g_5} \left( R - \frac{3}{4} (g_{ij}\partial_\mu \phi^i \partial^\mu \phi^j + V) \right)$$  \hspace{1cm} (2.3)

where the sigma-model metric $g_{ij}$ is

$$g_{ij} = 2G_{IJ} \frac{\partial h^I}{\partial \phi^i} \frac{\partial h^J}{\partial \phi^j}$$  \hspace{1cm} (2.4)

and the potential is given by

$$V = U_i U^i - U^2$$  \hspace{1cm} (2.5)

where $U_i = \frac{\partial U}{\partial \phi^i}$ and indices are raised using the sigma-model metric $g^{ij}$. Notice that the metric can be written as

$$G_{IJ} = -\frac{1}{3} \frac{\partial^2}{\partial h^I \partial h^J} \ln(C_{PQR}h^P h^Q h^R) |_M$$  \hspace{1cm} (2.6)

where the constraint defining $M$ is only used after the two derivatives have been computed. The superpotential $U$ defines the dynamics of the theory. It is given by

$$U = 4\sqrt{\frac{2}{3}} gh^i q_i$$  \hspace{1cm} (2.7)

where $g$ is a gauge coupling constant and the $q_i$'s are real numbers such that the $U(1)$ gauge field is $A_0 q_i$.

The boundary action depends on two fields. There is a supersymmetry singlet $G$ and a four form $A_{\mu\nu\rho\sigma}$ [4]. One also modifies the bulk action by replacing $g \rightarrow G$ and adding a direct coupling

$$S_A = \frac{2}{4!\kappa_5^2} \int d^5x \epsilon^{abcde} A_{abcd} \partial_\varepsilon G.$$  \hspace{1cm} (2.8)

The boundary action is taken as

$$S_{\text{bound}} = -\frac{1}{\kappa_5^2} \int d^5x (\delta x_5 - \delta_{x_5-R})(\sqrt{-g_4} \frac{3}{2} U + \frac{2g}{4!} \epsilon^{\mu\nu\rho\sigma} A_{\mu\nu\rho\sigma}).$$  \hspace{1cm} (2.9)

where $\mu, \nu, \rho, \sigma$ are four-dimensional indices on the branes. Notice that the four-form $A_{abcd}$ is not dynamical.

The supersymmetry algebra closes on shell where

$$G(x) = g \epsilon(x_5),$$  \hspace{1cm} (2.10)

and $\epsilon(x_5)$ jumps from -1 to 1 at the origin of the fifth dimension. On shell the bosonic Lagrangian reduces to the bulk Lagrangian coupled to the boundaries as,

$$S_{\text{bound}} = -\frac{3}{2\kappa_5^2} \int d^5x (\delta x_5 - \delta_{x_5-R})\sqrt{-g_4} U;$$  \hspace{1cm} (2.11)
Crucially, the boundary branes couple directly to the bulk superpotential. Notice that the two branes have opposite (field-dependent) tensions

$$\lambda_\pm = \pm \frac{3}{2\kappa_5^2} U$$

(2.12)

where the first brane has positive tension.

The gauge fields in the bulk have a kinetic term parametrised by the metric $G_{IJ}$

$$S_{gauge} = -\frac{1}{4\kappa_5^2} \int d^5x \sqrt{-g_5} G_{IJ} F_I^{ab} F_J^{ab}$$

(2.13)

The dimensional reduction of this term to 4d will lead to the axion fields at low energy.

We will study BPS configurations preserving one half of supersymmetry. These BPS configurations are associated to a complex $(n+1)$-dimensional moduli space with a Kählerian structure. The low energy action of 5d supergravity with boundaries reduces to a supergravity theory in 4d whose structure depends only on the Kähler potential on the moduli space. The moduli space comprises $(n+1)$ real directions associated to $n$ scalar fields corresponding to tangent directions to the manifold $M$ and one scalar mode of gravitational origin, the radion, which can either be seen as the distance between the branes or the $55$ component of the bulk metric. On the moduli space, there is no potential and therefore no superpotential as the moduli correspond to supersymmetric flat directions.

### 3 The Moduli Space

The previous theory admits flat directions where the original supersymmetry is broken to $N = 1$ in 4d. These BPS configurations are obtained by requiring that the gravitino and gaugino variations vanish. The BPS backgrounds are determined by first order differential equations for which the boundary conditions at the branes are automatically satisfied. The moduli space of the theory is parametrised by the constants of integrations of the BPS equations, corresponding to BPS zero modes. For $n$ vector multiplets, there are $(n+1)$ moduli. These moduli are associated to as many axion fields, the whole moduli space becoming a Kähler manifold whose Kähler potential will be determined later.

The BPS equations corresponding to the presence of Killing spinors are given by [4]

$$\frac{d\phi^x}{dz} = g^{xy} \frac{\partial W}{\partial \phi^y}, \quad \frac{d\ln \tilde{a}}{dz} = -\frac{W}{4}$$

(3.1)

where the bulk metric has been written

$$ds^2 = dz^2 + \tilde{a}^2(z) \eta_{\mu\nu} dx^\mu dx^\nu$$

(3.2)

It is convenient to define

$$dy = \tilde{a}^2(z) dz$$

(3.3)

so that the metric (with $a(y) = \tilde{a}(z)$)

$$ds^2 = \frac{dy^2}{a^4(y)} + a^2(y) \eta_{\mu\nu} dx^\mu dx^\nu$$

(3.4)
yields an effective action in the Einstein frame when integrating over the fifth dimension. From this one gets that
\[ g_{xy} \frac{d\phi^y}{dy} = 2G_{IJ} \frac{\partial h^I}{\partial \phi^x} \frac{\partial h^J}{\partial y} \] (3.5)

Using the relation \( \frac{\partial h^I}{\partial y} = -G_{IJ} \frac{\partial h^J}{\partial y} \), we find that
\[ h^I_y \left( \frac{d(a^2 h^I)}{dy} + \epsilon(y) \frac{q_I}{2} \right) = 0 \] (3.6)
where we have used \( h^I_y h^I = 0 \). Similarly we get that
\[ h^I \left( \frac{d(a^2 h^I)}{dy} + \epsilon(y) \frac{q_I}{2} \right) = 0 \] (3.7)
Notice that the \( h^I \)'s form a basis of vectors corresponding to the tangent space \( TM \) and the normal space to \( M \) is parametrised by \( h^I \). Being orthogonal to a basis of the \( (n+1) \)-dimensional space where the moduli space is embedded, we deduce that
\[ \frac{d(a^2 h^I)}{dy} + \epsilon(y) \frac{q_I}{2} = 0 \] (3.8)
leading to
\[ \tilde{h}_I \equiv a^2 h^I = t_I - \frac{1}{2} q_I |y| \] (3.9)
where \( t_I \) is an integration constant. There are thus \( (n+1) \) integration constants. These \( (n+1) \) integration constants parametrise the real part of the moduli space of the theory.

The scale factor can be obtained via
\[ a^2 = \tilde{h}_I h^I \] (3.10)
or equivalently
\[ a^3 = C_{IJK} \tilde{h}^I \tilde{h}^J \tilde{h}^K \] (3.11)
where \( \tilde{h}^I = ah^I \). These variables are solutions of
\[ \tilde{h}_I = C_{IJK} \tilde{h}^J \tilde{h}^K \] (3.12)
in such a way that \( \tilde{h}^I \) is a function of \( \tilde{h}_I \). Note that according to (3.11), the \( (n+1) \) variables \( h^I \) defined as \( \tilde{h}^I(\tilde{h}_K)/a(\tilde{h}_K) \) automatically belong to the \( n \)-dimensional manifold \( M \).

Let us now find the imaginary parts associated to the real moduli. These axion fields are associated with the fifth components of the bulk gauge fields. Let us assume that \( A^5_\mu \) is the only non-vanishing component ; equivalently, we choose the \( A^5_\mu \) field even under the orbifold parity, while \( A^\mu_\mu \) is odd. The effective theory will consequently contain no \( N = 1 \) vector multiplet. The equations of motion read
\[ \partial_a (\sqrt{-g} g^{ac} g^{bd} G_{IJ} F^{Jcd}) = 0 \] (3.13)
We find that for \( a = 5 \)
\[ F^{I}_{\mu 5} = \frac{1}{a^4} G^{IJ} \partial_\mu b_J(x) \] (3.14)
The \( a = \mu \) case reduces to
\[ \Box^{(4)} b_I = 0 \] (3.15)
Hence we find \( (n+1) \) axion zero modes.

On the whole the moduli space comprises \( 2(n+1) \) scalar fields \( t_I \) and \( b_I \).
4 The Kähler Potential

At low energy the bulk metric can be parametrised using

\[ ds^2 = \frac{dy^2}{a^4(y, x)} + a^2(y, x)g_{\mu\nu}(x)dx^\mu dx^\nu \]  

(4.1)

where \( g_{\mu\nu} \) is a metric corresponding to the graviton zero mode, and the warp factor depends on all coordinates implicitly through \( \tilde{h}_I(x, y) = t_I(x) - \frac{1}{2} y_I y \). Note that the branes are straight in this coordinate system. It leads to 4d gravity at low energy as the 5d Einstein–Hilbert term leads to

\[ \frac{1}{2\kappa_5^2} \int d^4x dy \sqrt{-g_5} R \supset \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} R^{(4)} \]  

(4.2)

where \( R^{(4)} \) is the Ricci scalar associated to \( g_{\mu\nu} \) and

\[ \frac{1}{\kappa_4^2} = \frac{2d}{\kappa_5^2} \]  

(4.3)

with \( d = \int_{y^-}^{y^+} dy = \frac{1}{2} \int dy \) evaluated between the two branes located at constant \( y = y_\pm \). Hence the ansatz leads to 4d gravity in the Einstein frame. We will see later that the implicit dependence of \( a(y, x) \) on \( x \) due to the dependence on \( t_I(x) \) leads to a contribution to the moduli kinetic terms.

We now promote the axions to be fields \( b_I(x) \) and compute the kinetic terms resulting from the gauge field kinetic term in the Lagrangian. We obtain that the axions have kinetic terms

\[ S_{\text{axion}} = -\frac{1}{4\kappa_5^2} \int d^4x \sqrt{-g} \left( \int dy \frac{1}{a^4} G^{IJ} \right) g^{\mu\nu} \partial_\mu b_I \partial_\nu b_J \]  

(4.4)

It has the form of a non-linear sigma model with a metric

\[ K^{IJ} = \frac{1}{\kappa_5^2} \int dy \frac{1}{a^4} G^{IJ} \]  

(4.5)

We will see later that this metric derives from a Kähler potential.

The kinetic terms for the real moduli follow from

\[ g_{xy} \partial_\mu \phi^x \partial_\mu \phi^y = \frac{(G^{IJ} - h^I h^J)}{a^4} g^{\mu\nu} \partial_\mu t_I \partial_\nu t_J \]  

(4.6)

where we have used the fact that \( \tilde{h}_I \) depends on \( x \) only via \( t_I \). Notice too that we have used the fact that the kinetic terms coming from the scalar fields are such that

\[ (G^{IJ} - h^I h^J) \partial_\mu h_I \partial^\mu h_J = G^{IJ} \partial_\mu h_I \partial^\mu h_J \]  

(4.7)

corresponding to the projection of the metric \( G^{IJ} \) to the tangent space of \( M \). Hence the kinetic terms coming from the scalar fields in 5d only involve \( n \) moduli. Note also that

\[ (G^{IJ} - h^I h^J) \partial_\mu h_I = \frac{(G^{IJ} - h^I h^J)}{a^2} \partial_\mu t_J \]  

(4.8)
and therefore the previous result (4.6).

The Einstein-Hilbert term in 5d can be evaluated and leads to a contribution to the moduli kinetic terms

$$\int d^5x \sqrt{-g_5} R \supset -\frac{3}{2} \int d^4x dy g^{\mu \nu} \frac{\partial_\mu a^2 \partial_\nu a^2}{a^4} \tag{4.9}$$

where

$$\partial_\mu a^2 = h^I \partial_\mu t_I \tag{4.10}$$

implying that the Einstein-Hilbert term contributes as

$$-\frac{3}{2} \int d^4x dy g^{\mu \nu} h^I h^J \frac{\partial_\mu t_I \partial_\nu t_J}{a^4} \tag{4.11}$$

This term corresponds to a projection of the moduli kinetic terms on the normal to \(M\). As can be seen from its origin, i.e. the dependence of the scale factor \(a\) on \(x\), it is associated to the variations of the 55 component of the bulk metric, i.e. the radion.

Collecting the factor from the scalar field kinetic terms and the Einstein-Hilbert we obtain that the kinetic terms of the scalar fields lead to

$$-\frac{3}{4\kappa_5^2} \int d^4x \sqrt{-g} (\int dy \frac{G^{IJ}}{a^3}) g^{\mu \nu} \partial_\mu t_I \partial_\nu t_J \tag{4.12}$$

Defining the complex moduli

$$T_I = \sqrt{\frac{3}{2}} t_I + i \frac{b_I}{\sqrt{2}} \tag{4.13}$$

and using complex variable notations we find that the kinetic terms read

$$S_{\text{moduli}} = -\frac{1}{2\kappa_5^2} \int d^4x \sqrt{-g} (\int dy \frac{G^{IJ}}{a^3}) g^{\mu \nu} \partial_\mu T_I \partial_\nu T_J \tag{4.14}$$

To complete our description of the moduli space, we need to show that

$$K^{IJ} = \frac{1}{\kappa_5^2} \int dy \frac{G^{IJ}}{a^4} \tag{4.15}$$

is a second derivative. Let us define

$$F(t_I) = C^{IJK} \tilde{h}_I \tilde{h}_J \tilde{h}_K \tag{4.16}$$

where \(C^{IJK} \equiv G^{IL} G^{JM} G^{KP} C_{LMP}\). Using the fact that

$$h_I h_J \partial C^{IJK} = 0 \tag{4.17}$$

where the derivative \(\partial\) is taken with respect to \(\tilde{h}_I\), one finds that

$$\frac{\partial^2 \ln F}{\partial h_I \partial h_J} = -\frac{3}{a^4} G^{IJ} \tag{4.18}$$
We have used $h_I \partial h^I = 0$ and $G^{IJ} \partial h_J = - \partial h^I$ as $h^I(\tilde{h}_J)$ is always on $M$. One obtains that

$$K^{IJ} = - \frac{1}{3\kappa_5^2} \partial^2_{t_I} \partial^2_{t_J} \int dy \ln(F)$$  \hfill (4.19)

This is the expected result implying that the Kähler potential is given by

$$K = - \frac{4}{3\kappa_5^2} \int dy \ln(F) \left( \frac{T_I + \bar{T}_I}{2} \right)$$ \hfill (4.20)

as a function of $t_I = (T_I + \bar{T}_I)/2$, or equivalently

$$K = - \frac{4}{3\kappa_5^2} \int dy \ln \left[ C^{IJK} \left( \frac{T_I + \bar{T}_I}{2} - \frac{q_I}{2} y \right) \left( \frac{T_J + \bar{T}_J}{2} - \frac{q_J}{2} y \right) \left( \frac{T_K + \bar{T}_K}{2} - \frac{q_K}{2} y \right) \right]$$ \hfill (4.21)

which depends only on the real moduli $t_I$ only.

Let us illustrate this general result with two examples. In the case of non-gauged supergravity the Kähler potential reads ($q_I = 0$)

$$K = - \frac{8d_3}{3\kappa_5^2} \ln \left( C^{IJK} \left( \frac{T_I + \bar{T}_I}{2} \right) \left( \frac{T_J + \bar{T}_J}{2} \right) \left( \frac{T_K + \bar{T}_K}{2} \right) \right)$$ \hfill (4.22)

As the metric $G_{IJ}$ does not depend on $y$, we can define

$$T^I = G^{IJ} T_J$$ \hfill (4.23)

leading to

$$K = - \frac{8d_3}{3\kappa_5^2} \ln \left( C_{IJK} \left( \frac{T_I + \bar{T}_I}{2} \right) \left( \frac{T_J + \bar{T}_J}{2} \right) \left( \frac{T_K + \bar{T}_K}{2} \right) \right)$$ \hfill (4.24)

where $d = \int_{y^-}^{y^+} dy = \frac{1}{2} \int dy$. It coincides with the known result in linearised M-theory [1] and the recent analysis [22].

The simplest gauged case is given by a one vector multiplet model ($n = 1$) with only non-zero coefficient $C_{112} = 1$ (and permutations thereof). The defining relation for the field manifold M is then $3(h^2)^2 = 1$. Solving the constraint

$$\tilde{h}_I = C_{IJK} \tilde{h}_J \tilde{h}_K$$ \hfill (4.25)

we find

$$\tilde{h}^1 = \sqrt{\tilde{h}_2}, \quad \tilde{h}^2 = \frac{\tilde{h}_1}{2\sqrt{\tilde{h}_2}}$$ \hfill (4.26)

and the warp factor

$$a(y, x) = \left( \frac{3}{2} \right)_{1/3} \left( \frac{\tilde{h}_1}{h_2} \right)^{1/3}$$ \hfill (4.27)

The metric is now diagonal

$$G_{11} = \left( \frac{2}{3} \right)^{1/3} \left( \frac{\tilde{h}_1}{h_2} \right)^{2/3}$$

$$G_{22} = 2 \left( \frac{2}{3} \right)^{1/3} \left( \frac{\tilde{h}_2}{h_1} \right)^{4/3}$$

$$G_{12} = 0$$ \hfill (4.28)
and the only non-zero entry is
\[ C^{112} = (G_{11})^{-2}(G_{22})^{-1}C_{112} = 3/4 \] (4.29)
and permutations thereof. The Kähler potential is finally
\[
K = -\frac{32d}{3q_1\kappa_4^2} \left[ (t_1 - \frac{q_1}{2}y_+) \ln(t_1 - \frac{q_1}{2}y_+) - (t_1 - \frac{q_1}{2}y_-) \ln(t_1 - \frac{q_1}{2}y_-) \right] \\
-\frac{16d}{3q_2\kappa_4^2} \left[ (t_2 - \frac{q_2}{2}y_+) \ln(t_2 - \frac{q_2}{2}y_+) - (t_2 - \frac{q_2}{2}y_-) \ln(t_2 - \frac{q_2}{2}y_-) \right] \\
+\frac{16d}{3\kappa_4^2} \left( \frac{3}{2} - \ln \frac{3}{2} \right) \] (4.30)
such that the Kähler metric \( K^{I\bar{J}} = \frac{\partial^2 K}{\partial T_I \partial \bar{T}_J} \) is diagonal
\[
K^{1\bar{1}} = \frac{4d^2}{3\kappa_4^2} \left( t_1 - \frac{q_1}{2}y_+ \right) \left( t_1 - \frac{q_1}{2}y_- \right), \\
K^{2\bar{2}} = \frac{2d^2}{3\kappa_4^2} \left( t_2 - \frac{q_2}{2}y_+ \right) \left( t_2 - \frac{q_2}{2}y_- \right) \] (4.31)
In the ungauged case \( q_I = 0 \) this simplifies to
\[
K = -\frac{16d}{3\kappa_4^2} \ln(\frac{T_1 + \bar{T}_1}{2}) - \frac{8d}{3\kappa_4^2} \ln(\frac{T_2 + \bar{T}_2}{2}) - \frac{16d}{3\kappa_4^2} \ln \frac{3}{2} \] (4.32)

Let us now introduce matter on the boundary branes. We couple the matter fields to the induced metric on the ith–brane leading to an action for the matter scalar field \( s \) coupled to the moduli
\[
\int d^4x \sqrt{-g} \left( a^2(t_I) \left( \partial s \partial \bar{s} \right) + a^4(t_I) \left| \frac{\partial w(s)}{\partial s} \right|^2 \right) \] (4.33)
up to derivative terms in the \( t_I \)’s. We have denoted by \( w \) the superpotential of the supersymmetric theory on the brane. As we are supersymmetrising the matter action only at zeroth order in \( \kappa_4 \), we have suppressed the non-renormalizable terms in the matter fields for fixed moduli, hence the globally supersymmetric form of the potential. Such an action can be supersymmetrised
\[
- \int d^4xd^4\theta E^{-1} a^2(T_I + \bar{T}_I) \Sigma \Sigma \] (4.34)
where \( \Sigma = s + \ldots \) is the chiral superfield of matter on the brane. Similarly the potential on the brane follows from
\[
\int d^4xd^2\theta \Phi^3 W(T_I, \Sigma) \] (4.35)
where
\[
W(T_I, S) = a^3(T_I)w(\Sigma) \] (4.36)
and \( \Phi \) is the chiral compensator whose \( F \)–term is the gravitational scalar auxiliary field. At low energy this leads to a direct coupling between matter fields and the moduli. When
matter is on both branes, a sum over the branes contributions evaluated at the brane positions is understood.

A particularly interesting case corresponds to constant superpotentials on the branes as a function of the moduli. In that case, the tensions of the branes are shifted from their BPS values. When only the superpotential on the second brane does not vanish, this is a hidden brane scenario of supersymmetry breaking, as was studied in [23] in the case of a single bulk vector multiplet. The analysis of the corresponding physics is left for future work.

5 Conclusion

We have studied 5d gauged supergravity with an arbitrary number of vector multiplets and boundaries. In particular we have focused on the low energy effective action parametrised by the moduli of BPS configurations preserving $N = 1$ supersymmetry in 4d. The 4d effective action is determined by the Kähler potential expressed in terms of the moduli. We have given a closed expression for the Kähler potential in terms of the warping of the 5d metric and the cubic polynomial defining the real–special geometry of 5d supergravity with vector multiplets.

The coupling of the moduli to matter on the branes has also been made explicit. In particular, this may be useful in analysing the way supersymmetry breaking may be generated in 5d brane models.

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