Giant magnon-like solution in $Sch_5 \times S^5$

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Abstract

In this paper we have found a classical giant magnon-like solution with both infinite and finite angular momentum moving in $Sch_5 \times S^5$ with B-field, which is believed to be dual to dipole-deformed $\mathcal{N} = 4$ super Yang-Mills theory. This string state propagates as a point particle in non-trivial subspace of the $Sch_5$ space but shows a giant magnon-like property in the $S^2$ subspace. We derive the energy-momentum dispersion relations and their finite-size correction for the case of finite but large angular momentum.

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1 Introduction

Integrability is a key feature in AdS/CFT correspondence to find exact solutions of both string theory in AdS space and gauge theory on its boundary. Classical string solutions, such as giant magnon, are related to asymptotic particle states which describe certain non-BPS operators of dual conformal field theory (CFT). This special property, discovered in original formulation of the AdS/CFT, has also appeared in various deformations of the correspondence.

Dipole deformation of the $\mathcal{N} = 4$ super-Yang-Mills (SYM) theory is one of recent developments in this direction. With minimal non-locality, imposed by dipole deformation in a null light-cone direction, the theory maintains non-relativistic conformal symmetry in three-dimensional perpendicular directions \cite{1, 2, 3, 4}. Furthermore, the dual supergravity background has been worked out and identified with geometry known as “Schrödinger” space-time. It is an interesting issue if this duality can be established in the context of integrability, as the original AdS/CFT duality has shown.

Dipole deformation is a special case of integrable Yang-Baxter deformations of the $AdS_5 \times S^5$ string \cite{5} as shown in \cite{6, 7}. More recently, this issue is addressed again \cite{8} as a Drinfeld twist \cite{9} of the $\mathcal{N} = 4$ SYM as is the case of other $*$-product deformations \cite{10}. A unified point of view on these integrability structures has been provided in \cite{11}. Furthermore, in \cite{8} a weak coupling limit in spin chain $sl(2)$ sector has been studied and semiclassical solutions such as spinning BMN-like strings have been also derived (see also \cite{12}).

Full quantum integrability of the deformed theory can be encoded in the world-sheet $S$-matrix, which is conjectured to be Drinfeld-Reshetikhin twist of the undeformed one \cite{8}. The giant magnons appear as fundamental asymptotic particles not only in the undeformed theory but also in several related theories such as $\beta$-deformed theory \cite{13} and $\eta$-deformed $\mathcal{N} = 4$ SYM \cite{5}. Therefore, it is very important to find giant magnon-like solutions in the $Sch_5 \times S^5$ string target space along with exact energy-momentum dispersion relation to understand full non-perturbative AdS/CFT correspondence of the deformed theory. For this purpose, we consider a classical string configuration which lives both in the Schrödinger space and sphere, whose dispersion relation have a similar form as the original giant magnon solution of the undeformed theory. In the supergravity limit, our solution should reproduce the supersymmetric BMN-like solution.

Another topic we address in this paper is the finite-size correction to the energy-momentum
dispersion relation derived from the exact classical string solution with finite angular momentum. This information can provide valuable information on the $S$-matrix between the particles, hence some insight into non-perturbative aspects of the dipole-deformed theory.

This paper is organized as follows. In section 2 we introduce the metric of the “Schrödinger” space-time and impose conformal gauge in the Polyakov string action along with Virasoro constraints. In section 3 we obtain the giant magnon-like state and its dispersion relation in the decompactified limit where the angular momentum in $S^2$ gets infinite. Interesting result on the finite-size correction when the angular momentum gets large but finite is explained in section 4. We conclude the paper in section 5 with some comments and future research directions.

## 2 The string Lagrangian and Virasoro constraints

According to [8], the metric on $\text{Sch}_5 \times S^5$ in global coordinates can be written as

$$\frac{ds^2}{l^2} = -\left(1 + \frac{\mu^2}{Z^4}\right) dt^2 + \frac{2 dt dV - \vec{X}^2 dt^2 + d\vec{X}^2 + dZ^2}{Z^2} + (d\psi + P)^2 + ds^2_{CP^2}, \quad (2.1)$$

where the metric on $S^5$ is given as $U(1)$ Hopf fibre over $CP^2$. The parameter $\mu$ is deforming the background from the usual AdS space. The $B$-field is given by

$$\alpha' B = \frac{l^2}{Z^2} dt \wedge (d\psi + P). \quad (2.2)$$

The metric of $CP^2$ and Hopf fibre over it can be expressed by [12]

$$ds^2_{CP^2} = d\theta_1^2 + \frac{1}{4} \sin^2 \theta_1 \left[ \cos^2 \theta_1 (d\phi_1 + \cos \theta_2 d\phi_2)^2 + d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2 \right], \quad (2.3)$$

$$P = \frac{1}{2} \sin^2 \theta_1 (d\phi_1 + \cos \theta_2 d\phi_2). \quad (2.4)$$

We focus on a $S^2$ subspace of $S^5$ by choosing

$$\phi_1 = 0, \quad \theta_2 = \frac{\pi}{2} \quad (2.5)$$

which lead to

$$P = 0; \quad ds^2_{CP^2} \rightarrow ds^2 = d\theta_1^2 + \frac{1}{4} \sin^2 \theta_1 d\phi_2^2. \quad (2.6)$$
We further consider string solutions in a subspace of $Sch_5$ obtained by $\vec{X} = 0$, $Z = Z_0$. Also we set $l = 1, \alpha' = 1$ from now on. The resulting metric is then simplified to $(\theta \equiv \theta_1, \phi \equiv \phi_2/2)$

$$ds^2 = g_{MN} dX^M dX^N = - \left(1 + \frac{\mu^2}{Z_0^2}\right) dt^2 + \frac{2}{Z_0^2} dt dV + d\psi^2 + d\theta^2 + \sin^2 \theta d\phi^2, \quad (2.7)$$

$$B = b_{MN} dX^M dX^N = \frac{\mu}{Z_0^2} dt \wedge d\psi.$$

In our considerations we will use conformal gauge in the Polyakov string action, in which the string Lagrangian and Virasoro constraints have the form

$$\mathcal{L}_s = \frac{T}{2} (G_{00} - G_{11} + 2B_{01}) \quad (2.8)$$

$$G_{00} + G_{11} = 0, \quad (2.9)$$

$$G_{01} = 0, \quad (2.10)$$

where the induced metric and $B$ fields are given by

$$G_{mn} = g_{MN} \partial_m X^M \partial_n X^N, \quad B_{mn} = b_{MN} \partial_m X^M \partial_n X^N, \quad M, N = (0, 1, \ldots, 9),$$

with the derivatives w.r.t. the world-sheet coordinates $\eta^0 = \tau, \eta^1 = \sigma$.

We consider the following string embedding in the background, as given in (2.7),

$$t = \kappa \tau, \quad V = \mu^2 m \tau, \quad \psi = \omega_1 \tau, \quad \phi = \omega \tau + f(\xi), \quad \theta = \theta(\xi), \quad \xi = \sigma - \nu \tau. \quad (2.11)$$

This choice implies that the string behaves like a point particle in the subspace of $Sch_5$ while the stringy behaviour appears only in the subspace $S^2$. In a particular limit where the string image disappears in the $S^2$, the configuration is reduced to the spinning BMN-like solution considered in [8]. This condition fixes the constant coordinate $Z_0$ to

$$Z = Z_0 = \sqrt{\frac{\kappa}{m}}. \quad (2.12)$$

Replacing (2.11) into (2.8) one finds that the string Lagrangian is reduced to an effective one-dimensional one (prime is used for $d/d\xi$)

$$L = -\frac{T}{2} \left\{ (1 - v^2)\theta'^2 + \kappa^2 - \beta^2 \omega^2 + [(1 + v)f' - \omega] [(1 - v)f' + \omega] \sin^2 \theta \right\}, \quad (2.13)$$

where the deformation parameter $\beta$ is defined by

$$\beta^2 \equiv \frac{\omega^2 + \mu^2 m^2}{\omega^2}. \quad (2.14)$$
The equation of motion from (2.13) gives a solution for $f$

$$f'(\xi) = \frac{1}{1 - v^2} \left( \frac{C}{\sin^2 \theta} - \nu \omega \right),$$

(2.15)

where $C$ is an arbitrary integration constant.

From (2.11) and (2.15), the Virasoro constraints (2.9), (2.10) can be written as

$$\theta'^2 = \frac{1}{1 + v^2} \left[ \kappa^2 - \beta^2 \omega^2 + \frac{4C\nu\omega}{(1 - v^2)^2} - \frac{(1 + v^2)(C^2 \csc^2 \theta + \omega^2 \sin^2 \theta)}{(1 - v^2)^2} \right],$$

(2.16)

$$C\omega = \nu(\kappa^2 - \beta^2 \omega^2).$$

(2.17)

The first Virasoro constraint (2.16) is equivalent to the first integral of the equation of motion for $\theta$ as shown in general case [14]. The second constraint determines the integration constant $C$. After replacing it in (2.16) one finds that the non-isometric coordinate $\theta$ satisfies a first-order ordinary differential equation:

$$\theta'^2 = \frac{(\kappa^2 - \beta^2 \omega^2 - \omega^2 \sin^2 \theta)}{(1 - v^2)^2 \omega^2 \sin^2 \theta} \left( \omega^2 \sin^2 \theta - v^2(\kappa^2 - \beta^2 \omega^2) \right).$$

(2.18)

3 The giant magnon-like solution in infinite volume

3.1 Solution

The giant magnon is introduced in [15] as a string image on $S^2$, which is dual to the magnon excitation of the SYM spin chains. The geometric meaning of the momentum carried by the magnons is a deficit angle of $\phi$ of the infinite-size string on the equator $\theta = \pi/2$ of the $S^2$ space. Therefore, we impose the following condition on a giant magnon-like solution

$$\theta'^2 = 0 \quad \text{for} \quad \theta = \pi/2,$$

(3.1)

only when we consider the giant magnon in infinite volume. For the case of finite volume, this condition should be relaxed as we will see in next section.

As can be seen from (2.18), this condition reduces to

$$\left( \kappa^2 - \beta^2 \omega^2 - \omega^2 \right) \left( \omega^2 - v^2(\kappa^2 - \beta^2 \omega^2) \right) = 0.$$
Among two possibilities, we choose for the giant magnon

$$\kappa^2 = (1 + \beta^2)\omega^2. \quad (3.3)$$

This is consistent with the undeformed case $$\beta = 0$$ ($$\omega_1 = 0$$, $$\mu = 0$$) which leads to $$\kappa = \omega$$. Along with Eq. (2.17), this also fixes $$C = v\omega$$.

With this choice, Eq. (2.18) simplifies to

$$\theta^2 = \frac{\omega^2 \cos^2 \theta (\sin^2 \theta - v^2)}{(1 - v^2)^2 \sin^2 \theta}, \quad f'(\xi) = \frac{v\omega}{1 - v^2} \left(\frac{1}{\sin^2 \theta} - 1\right). \quad (3.4)$$

The solution of this equation is given by

$$\cos \theta(\xi) = \sqrt{1 - v^2} \text{sech} \left(\frac{\omega}{\sqrt{1 - v^2}} \xi\right). \quad (3.5)$$

This is exactly the Hofman-Maldacena solution for the infinite-size giant magnon [15]. Replacing (3.5) into (2.15), one can find the solution for the isometric coordinate $$\phi$$ on $$S^2$$.

### 3.2 The energy-charge relation

For the case under consideration the conserved quantities corresponding to isometric coordinates $$t, V, \psi$$ and $$\phi$$ are the string energy $$E_s$$, spin $$M$$ and two angular momenta - $$J_1$$ and $$J$$. In the limit of decompactified string, with $$L \to \infty$$, these conserved charges are given by

$$E_s = \int_{-L/2}^{L/2} d\sigma \frac{\partial L_s}{\partial (\partial_0 t)} = T\kappa \int_{-L/2}^{L/2} d\sigma = T\kappa L, \quad (3.6)$$

$$M = \int_{-L/2}^{L/2} d\sigma \frac{\partial L_s}{\partial (\partial_0 V)} = Tm \int_{-L/2}^{L/2} d\sigma = TmL,$$

$$J_1 = \int_{-L/2}^{L/2} d\sigma \frac{\partial L_s}{\partial (\partial_0 \psi)} = T\omega_1 \int_{-L/2}^{L/2} d\sigma = T\omega_1 L,$$

$$J = \int_{-L/2}^{L/2} d\sigma \frac{\partial L_s}{\partial (\partial_0 \phi)} = T\omega \left[\int_{-L/2}^{L/2} d\sigma - \frac{1}{1 - v^2} \int_{-L/2}^{L/2} \cos^2 \theta d\sigma\right] = T \left[\omega L - 2\sqrt{1 - v^2}\right].$$
While each of these quantities diverges, a finite combination is possible if we consider

$$E_s - \sqrt{\mu^2 M^2 + J_1^2 + J^2}$$

(3.7)

$$= T\kappa L - \sqrt{(\mu T m L)^2 + (T\omega_1 L)^2 + (T L \omega)^2 \left[1 - \frac{2\sqrt{1 - v^2}}{L\omega}\right]^2}$$

$$= T L \left\{ \kappa - \sqrt{1 + \beta^2} \omega + \frac{2}{\sqrt{1 + \beta^2}} \sqrt{1 - v^2} \right\}$$

$$= \frac{2T}{\sqrt{1 + \beta^2}} \sqrt{1 - v^2},$$

where we used Eq. (3.3) at the last line.

To establish correspondence with the dual dipole-deformed SYM theory, we compute the angle deficit

$$\Delta \phi = \int_{-\infty}^{\infty} f'(\xi) d\xi = \arccos v \quad \rightarrow \quad v = \cos \frac{\Delta \phi}{2}. \quad (3.8)$$

Identifying $\Delta \phi$ with the momentum $p$ of the magnon excitation, we can establish the energy-momentum dispersion relation as follows:

$$E_s - \sqrt{\mu^2 M^2 + J_1^2 + J^2} = \frac{2T}{\sqrt{1 + \beta^2}} \sin \frac{p}{2}. \quad (3.9)$$

This result shows how the deformation affects on the energy-momentum dispersion relation of the giant magnon-like string state in $S\text{ch}_5 \times S^5$. In the undeformed limit $\beta = 0$ ($\omega_1 = 0$, $\mu = 0$), this reduces to that of ordinary giant magnon. In the point-particle limit ($p = 0$, $J = 0$), this relation reduces to that of spinning BMN-like strings considered in [8].

4 The giant magnon-like solution in finite volume

4.1 Solution

Introducing new variables

$$\chi = \cos^2 \theta, \quad W = \frac{\kappa^2}{\omega^2} - \beta^2, \quad (4.1)$$

we can rewrite Eq. (2.18) as

$$\chi'^2 = \frac{4\omega^2}{(1 - v^2)^2} \chi (\chi_p - \chi)(\chi - \chi_m), \quad (4.2)$$
where \( \chi_m \) and \( \chi_p \) are given by
\[
\chi_p = 1 - v^2 W, \quad \chi_m = 1 - W.
\] (4.3)

The solution of Eq. (4.2) is then given by
\[
\xi(\chi) = \frac{1 - v^2 \omega}{2 \omega} \int \frac{d\chi}{\sqrt{\chi(\chi_p - \chi)(\chi - \chi_m)}}.
\] (4.4)
\[
= \frac{1 - v^2}{\omega} \frac{1}{\sqrt{\chi_p}} F\left(\arcsin \sqrt{\frac{\chi_p - \chi}{\chi_p - \chi_m}}, 1 - \frac{\chi_m}{\chi_p}\right),
\]
where \( F \) is the incomplete elliptic integral of first kind. Replacing (4.4) into (2.15), one can find the solution for the isometric coordinate \( \phi \) on \( S^2 \).

### 4.2 The conserved quantities

For the case at hand, the conserved quantities corresponding to isometric coordinates \( t, V, \psi \) and \( \phi \) are the string energy \( E_s \), spin \( M \) and two angular momenta - \( J_1 \) and \( J \) as introduced in (3.6). By changing the integration variable from \( d\xi \) to \( d\chi/\chi' \) for the finite size \( L \), we can express the charges by
\[
E_s = 2T \frac{(1 - v^2)k}{\omega \sqrt{\chi_p}} K(1 - \epsilon),
\] (4.5)
\[
M = 2T \frac{(1 - v^2)m}{\omega \sqrt{\chi_p}} K(1 - \epsilon),
\] (4.6)
\[
J_1 = 2T \frac{(1 - v^2)\omega_1}{\omega \sqrt{\chi_p}} K(1 - \epsilon),
\] (4.7)
\[
J = 2T \sqrt{\chi_p} [K(1 - \epsilon) - E(1 - \epsilon)].
\] (4.8)

While these charges diverge as \( K \to \infty \) in large volume limit, the ratios between them remain finite.

The angular difference \( \Delta \phi \) can be also obtained as
\[
\Delta \phi = \frac{2v}{\sqrt{\chi_p}} \left[ \frac{1}{v^2} \Pi \left( -\frac{\chi_p}{1 - \chi_p} (1 - \epsilon), 1 - \epsilon \right) - K(1 - \epsilon) \right].
\] (4.9)

Here \( K(1 - \epsilon), E(1 - \epsilon) \) and \( \Pi \left( -\frac{\chi_p}{1 - \chi_p} (1 - \epsilon), 1 - \epsilon \right) \) are the complete elliptic integrals of first, second and third kind, and
\[
\epsilon = \frac{\chi_m}{\chi_p}.
\] (4.10)
4.3 The energy-charge relation

From the explicit expressions for the charges, the energy dispersion relation of the giant magnon given in (3.9) can be expressed by

\[
E_s - \sqrt{\mu^2 M^2 + J_1^2 + J_2^2} = 2T \frac{(1 - v^2)}{\sqrt{1 - v^2 W}} K(1 - \epsilon) \left[ \sqrt{\beta^2 + W} - \sqrt{\beta^2 + \left( \frac{1 - v^2 W}{1 - v^2} \left( 1 - \frac{E(1 - \epsilon)}{K(1 - \epsilon)} \right) \right)^2} \right].
\]

(4.11)

Now we consider the energy correction for large but finite \( L \gg T \). Since the ratio \( E/K \) is very small in this limit, we expand first

\[
E_s - \sqrt{\mu^2 M^2 + J_1^2 + J_2^2} \approx 2T \frac{(1 - v^2)}{\sqrt{1 - v^2 W}} \times \left\{ \sqrt{\beta^2 + W} - \sqrt{\beta^2 + \left( \frac{1 - v^2 W}{1 - v^2} \right)^2} \left[ K(1 - \epsilon) + \frac{(1 - v^2 W)}{1 - v^2} E(1 - \epsilon) \right] \right\}.
\]

(4.12)

Now we assume that the parameters behave as follows for small \( \epsilon \):

\[
v = v_0 + (v_1 + v_2 \log \epsilon) \epsilon, \quad (4.13)
\]

\[
W = W_0 + (W_1 + W_2 \log \epsilon) \epsilon. \quad (4.14)
\]

From conditions \( \Delta \phi = p \) in (4.9) and (4.10) with (4.3), one can find the coefficients as

\[
v_0 = \cos \frac{p}{2}, \quad v_1 = \frac{1 - \log 16}{4} \cos \frac{p}{2} \sin^2 \frac{p}{2}, \quad v_2 = \frac{1}{4} \cos \frac{p}{2} \sin^2 \frac{p}{2}, \quad (4.15)
\]

\[
W_0 = 1, \quad W_1 = -\sin^2 \frac{p}{2}, \quad W_2 = 0. \quad (4.16)
\]

The coefficient of \( K(1 - \epsilon) \) in (4.12) is as small as \( \mathcal{O}(\epsilon) \) so that the logarithmic divergent term disappears.

With these and expansions of the elliptic functions, we find

\[
E_s - \sqrt{\mu^2 M^2 + J_1^2 + J_2^2} = \frac{2T \sin \frac{p}{2}}{\sqrt{1 + \beta^2}} \left[ 1 - \frac{\sin^2 \frac{p}{2} + \beta^2 \left( 1 - 5 \cos^2 \frac{p}{2} \right)}{4(1 + \beta^2)} \epsilon + \mathcal{O}(\epsilon^2) \right].
\]

(4.17)

The series expansion of \( J \) for small \( \epsilon \) is

\[
J \approx T \sin \frac{p}{2} \left( -2 - \log \frac{\epsilon}{16} \right), \quad (4.18)
\]
from which the expression for $\epsilon$ for $J \gg T$ can be found as

$$\epsilon = 16 \exp \left( -\frac{J}{T \sin \frac{\nu}{2}} - 2 \right). \tag{4.19}$$

Combining together, we obtain the leading finite-size correction of the energy-charge dispersion relation to be

$$E_s - \sqrt{\mu^2 M^2 + J_1^2 + J^2} = \frac{2T \sin \frac{\nu}{2}}{\sqrt{1 + \beta^2}} \left[ 1 - 4 \sin^2 \frac{\nu}{2} + \beta^2 \left( 1 - 5 \cos^2 \frac{\nu}{2} \right) e^{- \frac{J}{T \sin \frac{\nu}{2}} - 2} \right], \tag{4.20}$$

where the deformation parameter $\beta$ is defined in Eq.(3.9). The leading term is the energy dispersion relation in infinite volume as we have obtained in the previous section. The second term is the finite-size correction to the energy and is our main result in this paper. The coefficient in front of the exponential factor and its dependence on the deformation parameter $\beta$ defined in (2.14) contains important information on the interaction between the giant magnon states.

For the undeformed case of $\beta \to 0$, this result reduces to

$$E_s - J = 2T \sin \frac{\nu}{2} \left[ 1 - 4 \sin^2 \frac{P}{2} e^{- \frac{J}{T \sin \frac{\nu}{2}} - 2} \right], \tag{4.21}$$

which was obtained previous in [16].

5 Concluding remarks

In this paper, we have computed classical giant magnon-like solutions moving in the $Sch_5 \times S^5$ target space. We have considered the string configuration similar to conventional giant magnons, namely, point-like in the $Sch_5$ space and extended string-like in the $S^2$ subspace of $S^5$. We have obtained results for both infinite angular momentum and large but finite one. The conserved charges and the corresponding energy-charge relations are expressed in terms of elliptic integrals. We have confirmed that these results are consistent with previously known results in point-like and undeformed limits. A possible generalization is to consider dyonic giant magnon solution in $Sch_5 \times S^5$. This solution can live in $S^3$ where additional finite angular momentum is introduced. Another direction is to deform the sphere $S^5$ in addition to the dipole deformation of the $Sch_5$. It will be important to see if two or more deformations can maintain (classical) integrability. We hope to report on these in near future.
Note added: After the first version of this paper was posted in the arXiv, new paper has appeared where classical string solutions have derived [17]. Main difference from ours is that it considers string-like solutions even in the $Sch_5$ space.

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