Time-Varying Stiffness Calculation of Spiral Bevel Gears Based On SIMPACK

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Abstract. Aiming at the difficulty in calculating the time-varying stiffness of spiral bevel gears, the time-varying stiffness acquisition and calculation of spiral bevel gears under the actual coincidence degree based on SIMPACK is proposed. Taking a pair of spiral bevel gears as an example, the process of modeling and simulation with SIMPACK is described in detail. The numerical results are compared with the ISO standard calculation method, which verifies its correctness and rationality. Through the analysis of the relationship between meshing stiffness and load and damping under different working conditions, it is shown that meshing stiffness increases nonlinearly with the increase of load, and load damping makes the change of stiffness curve gentler.

1. Introduction
In the transmission process of spiral bevel gears, tooth contact is a complex non-linear process. There are some uncertainties in the load area and pressure distribution between tooth surfaces. At the same time, the overlap degree of gear teeth varies with the load. Therefore, the meshing stiffness of gears is time-varying, and time-varying meshing stiffness is one of the important factors affecting the dynamic characteristics of gears, and it is also an important basic work to study gear modification. Therefore, obtaining the time-varying meshing stiffness of gears under actual coincidence degree is of great significance to engineering practice [1, 2, and 3].

At present, there are mainly three methods for calculating the time-varying stiffness of gears: ISO empirical formula method, material mechanics method, finite element method and so on. Zuobing Chen and Zhishan Liu [4] analyzed the calculating methods of the gear stiffness in ISO standard and international standard, and gave the empirical formula and drawing according to the analysis results. At the same time, in order to consider the wheel stiffness in engineering, the dimension coefficient of the wheel was introduced. However, due to the complexity of tooth profile, the deformation and stiffness of gear teeth are only an approximate calculation, sometimes the calculation results are quite different. Yapeng Li [5] proposed an improved Ishikawa formula for calculating time-varying meshing stiffness, aiming at the low efficiency of calculating time-varying meshing stiffness mainly by finite element method, and that the analytical Ishikawa formula only considers the stiffness of gear teeth and does not consider the stiffness of gear wheels. On the calculation of meshing stiffness of spiral bevel gears, Jinyuan Tang and Taiping Pu [6] constructed a pair of five-tooth spiral bevel gears model based on finite element analysis software ABAQUS. The meshing stiffness calculation method of loaded gears based on finite element method was proposed and the accuracy of calculation was verified.
However, this model neglected that the direction of meshing force during the meshing process of spiral bevel gears changed with the meshing position of gears. The effect of change. Yanzhong Wang [7] established a time-varying meshing stiffness numerical model based on tooth contact analysis (TCA) and tooth loading contact analysis (LTCA) by calculating the tooth deformation flexibility of instantaneous contact points, and analyzed the influence of time-varying meshing stiffness on the parameter vibration stability of spiral bevel gears.

In summary, there are many methods to calculate the meshing stiffness of spur gears, while the meshing stiffness of spiral bevel gears is mainly calculated and verified by finite element method. The finite element method has high accuracy, but it has low computational efficiency and cannot simulate the continuous meshing process. In addition, there is no unified method for determining contact stiffness in finite element analysis. In the analysis of contact problems, the normal contact stiffness factor FKN has a great influence on the calculation results [8]. SIMPACK provides a powerful complete recursive algorithm and gear contact analysis method, which has the characteristics of fast solving speed, high precision and strong stability. In this paper, a pair of contact models of spiral bevel gears are established by SIMPACK software. This model can overcome the shortcomings of finite element method, complete the simulation analysis of continuous meshing of gear teeth, and quickly obtain the time-varying stiffness of gear teeth under corresponding working conditions, which adds a new way to the calculation of time-varying stiffness.

2. Establishment of Contact Simulation Model for Spiral Bevel Gear Pairs

2.1. Parameters of Spiral Bevel Gears

Material: 20CrMnTi;

| Material: 20CrMnTi; Modulus of elasticity: $E = 2.0675 \times 10^5$ Mpa; Poisson ratio: $\mu = 0.3$; density: $\rho = 7.85 \times 10^3$ kg/m$^3$. 

According to ISO 23509:2006 [9], the geometric parameters of gear pairs are shown in Table 1.

| Table. 1 Gear pair geometric parameters |
|-----------------------------------------|
| Number of teeth | 14 | 39 |
| Module/mm | 4.536 |
| Pressure angle/° | 20 |
| Spiral angle/° | 35 |
| Modification coefficient | 0.505 | -0.505 |
| Tooth side clearance | 0.127 | 0.127 |
| Tooth width/mm | 25.4 |
| Outer cone distance/mm | 93.973 |
| Pitch angle/° | 19.747 | 70.253 |
| Cutter radius/mm | 114.3 |

2.2. Modeling of Spiral Bevel Gears

In this paper, the three-dimensional contact model of spiral bevel gears is established by inputting the geometric parameters of small and large wheels into SIMPACK full parameter gear modeling function, as shown in Figure 1. Unlike the finite element contact simulation analysis, SIMPACK provides a powerful complete recursive algorithm for meshing analysis of complete gears. It does not need to reduce the computational cost in order to facilitate meshing and reduce the computational load of the model, only some gears (3 or 5 pairs of teeth) are contacted. In this way, on the one hand, the stiffness distribution of the whole model is more reasonable, on the other hand, it is more convenient to load and impose constraints according to the actual situation.
2.3. Define boundary Conditions and Loads

In the process of gear dynamic meshing, the spiral bevel gear acts as a primary input. Its power is applied to the small wheel by coupling and shaft, and then driven by contact between the tooth surfaces to rotate around the central axis of the driven wheel. Finally, the force balance of the driven wheel is achieved under the action of input torque and load resistance moment. Therefore, as shown in the two-dimensional topological structure of Fig. 2, Joint pair’s No. 27 and No. 3 are inserted into the main and driven wheels respectively. In addition, in order to reflect the actual working conditions of the large wheels more truly, spring damping force element No. 12 is added. The tooth surface contact is set to 225 force element, which is based on DIN3990-1-1987 [10] standard. As shown in Fig. 3, the force element can not only consider the factors such as backlash, contact damping and Coulomb friction, but also calculate the meshing force between teeth according to the penetration of the tooth surface and the stiffness parameters of the contact line.
Figure 3 Contact parameters of spiral bevel gears

3. Analysis of Simulation Results of Meshing Process

3.1. Contact Ratio of Spiral Bevel Gears

In order to calculate the position of contact points and the deformation of tooth surface, and reduce the search complexity of contact points, SIMPACK uses the geometric relationship of gear theory to judge whether there is contact between tooth surfaces by comparing the relationship between the center angle $\Delta \phi$ and circumferential clearance $\phi$. If the central angle is less than the circumferential clearance, there is no tooth contact [11].

$$\Delta \phi = \phi_1 + \frac{n_1}{n_2} \phi_2$$

(1)

SIMPACK can automatically calculate the moment of inertia according to the tooth shape and working condition. Firstly, before the time domain meshing simulation of spiral bevel gears, without considering the influence of load damping, load is added to the gear pair: the angular speed of the driving wheel rotating around the central axis is 100 rad/s, and the load moment of the driven wheel is 100 N/m. The time domain meshing simulation is carried out to obtain the logarithmic variation law of the engagement gear as shown in Fig. 4. When the gear teeth are running under load, as long as the gear teeth enter meshing, stress will inevitably occur under the interaction between the tooth surfaces. Within the elastic range, the strain will disappear as soon as the gear teeth withdraw from meshing. The gear coincidence degree [3]4-5 is calculated by comparing the meshing time of single tooth and the time between adjacent teeth.

$$\varepsilon = \frac{\Delta T}{\Delta t}$$

(2)

Through the function of SIMPACK slice, the load change of tooth surface from meshing to meshing is obtained as shown in Figure 5. According to formula (2) calculating method of gear coincidence degree, the coincidence degree of spiral bevel gears under this condition is obtained by
calculating the time from the beginning of meshing to the meshing of gear A and the time difference between the two adjacent teeth entering into meshing.

![Figure 4 Variation rule of mesh tooth surface](image)

**Figure 4** Variation rule of mesh tooth surface

![Fig. 5 Load variation on tooth surface](image)

**Fig. 5** Load variation on tooth surface

According to the standard of gear transmission design manual [12], the formula for calculating gear coincidence degree is as follows:

\[
\varepsilon = \sqrt{\varepsilon_a^2 + \varepsilon_\beta^2}
\]

\[
\varepsilon_a = \frac{1}{2\pi} \left[ \frac{z_1}{\cos \delta_1} (\tan \tau_1 - \tan \alpha_{1s}) + \frac{z_2}{\cos \delta_2} (\tan \tau_2 - \tan \alpha_{2s}) \right]
\]

\[
\varepsilon_\beta = \frac{b \tan \beta_s}{(1 - 0.5 \phi_s) \pi m}
\]
Through the calculation formula of coincidence degree, the end surface coincidence degree \( \varepsilon_a = 1.388 \), tooth line coincidence degree and theoretical coincidence degree of the pair of spiral bevel gears are obtained. The error of coincidence degree obtained by comparing the two methods is 18.7%, which indicates that there is a certain deviation between the coincidence degree and the theoretical coincidence degree under actual load.

### 3.2. Calculation of Gear Meshing Stiffness

Tooth stiffness is the load required to generate a disturbance of 1 \( \mu \text{m} \) on the tooth width of one or more pairs of precise teeth meshing at the same time. The main factors affecting the stiffness of gear teeth are: gear parameters; wheel structure; load per tooth width in normal section, etc. The time-varying stiffness of SIMPACK is expressed by stiffness ratio parameters. According to the established gear model and input stiffness ratio \( \phi \), the relative angular torsional stiffness of gear can be calculated.

\[
C_{\text{max}} = C^1 \times C_R
\]  
(6)

\[
S_R = C_{\text{min}} \div C_{\text{max}}
\]  
(7)

\[
C(\phi) = C_{\text{min}} \left[ 1 - (1 - S_R)^{\frac{S(\phi)}{\max( S_l, S_s) \times 2}} \right]
\]  
(8)

where, the meshing stiffness, stiffness ratio of single pair of teeth, the meshing contact path of relative rotation angle, the length of meshing path entering and leaving the meshing path.

As shown in Fig. 6, the time-varying stiffness curve corresponding to coincidence degree is obtained. The graph shows that the time-varying stiffness of spiral bevel gears varies periodically. The maximum meshing stiffness of spiral bevel gears is \( 5.331 \times 10^3 \) N/m and the minimum stiffness is \( 4.858 \times 10^3 \) N/m.

Equivalent transformation is needed to calculate the stiffness of spiral bevel gears, which is transformed into equivalent cylindrical gears at the midpoint of tooth width. In ISO6336-1:2006 [14], the meshing stiffness formula of gears is as follows:
\[ K = c' b \delta \]  
\[ c' = c_m c_t c \cos \beta \]  

Where, \( b \) tooth width, \( \delta \) deformation, \( c_m \) single pair tooth stiffness, \( c_t \) theoretical correction coefficient, \( c \) F wheel blank structural coefficient, \( c \) G basic tooth profile coefficient.

Because the root height of the large and small spiral bevel gears is different, height modification is adopted at the same time. The formula for calculating tooth profile coefficient is as follows:

\[ c_n = [1 + 0.5(1.25 - \frac{h_n}{m_a})][1 - 0.02(20^\circ - \alpha_n)] \]  
\[ C_n = 0.5(C_{n1} + C_{n2}) \]

The maximum meshing stiffness of spiral bevel gears is \( 5.166 \times 10^6 \) \( N/m \) and the minimum meshing stiffness is \( 4.005 \times 10^6 \) \( N/m \). Comparing the two methods, the error of the maximum meshing stiffness is less than 6%. It shows that the calculated stiffness of SIMPACK in this paper meets the requirements.

3.3. Effect of Load Change on Time-varying Stiffness

In order to reflect the process of gear transmission more truly, three groups of load dampers, 1Ns/m, 3Ns/m and 5Ns/m, are selected to analyze the influence of load dampers on time-varying stiffness without considering the torsional stiffness of large axles. Using force element No. 12 to exert multiple sets of loads on large wheels, the formula for calculating the torque is as follows:

\[ T = C(\varphi - \varphi_n) + D \dot{\varphi} + T_o \]

Where, \( C \) torsional stiffness, \( \varphi \) initial angle, \( D \) load damping, \( T_o \) load torque.

Through SIMPACK time domain simulation calculation, according to the time-varying stiffness curve, the maximum and minimum stiffness values of gear teeth under different working conditions can be quickly obtained. After calculating the average values, the average meshing stiffness curves under different working conditions can be obtained as shown in Figure 7.
Fig. 7 Average Stiffness Change Curve under Different Working Conditions

As can be seen from Fig. 7, the meshing stiffness of gear teeth increases nonlinearly with the increase of load torque, but with the further increase of load, the increasing trend of gear stiffness gradually slows down. At the same time, with the increase of the damping value, the stiffness curve becomes smoother. For example, when the damping value is 5, the load increases by 2.597% from 10N·m to 400N·m and 1.675% from 400N·m to 800N·m. In the case of no damping, the load increases by 5.237% from 10N·m to 400N·m and 1.984% from 400N·m to 800N·m.

4. Conclusion

(1) A three-dimensional contact analysis model of Spiral Bevel Gears Considering clearance and load damping is established based on the force element and algorithm of gear pairs provided by SIMPACK software. Through the time domain simulation analysis of the load applied on the model, the calculation and acquisition of the coincidence degree and the time-varying stiffness curve of spiral bevel gears under working conditions are proposed, which adds a new way to the calculation of the gear stiffness.

(2) By applying different load damping and load torque to the model, the change curve of the average meshing stiffness of the spiral bevel gears under different working conditions is obtained, which shows that the meshing stiffness increases nonlinearly with the increase of load, and the load damping makes the change of the stiffness curve gentler.

References

[1] Runfang Li. Rigidity Analysis and Repair Method of Gear Drive [M]. Chongqing University Press, 1998.1-20.

[2] Xiaozhong Deng, Ming Fan, Hongbin Yang. The relationship between the coincidence of spiral bevel gears and load capacity [J]. China Mechanical Engineering, 2001 (08): 38-40+4.

[3] Xiaozhong Deng, Yinqing Shu, Guiming Liang. Coincidence of spiral bevel gears under different loads [J]. Journal of Luoyang Institute of Technology, 1994 (02): 47-52.

[4] Zuobing Chen, Zhishan Liu. Stiffness analysis and calculation of spur gears [J]. Gear, 1987 (01): 11-14+26+10.

[5] Yapeng Li, Wei Sun, Jing Wei, Tao Chen. An improved method for calculating the time-varying meshing stiffness of gears [J]. Mechanical transmission, 2010, 34 (05): 22-26.

[6] Jinyuan Tang, Taiping Pu. Meshing stiffness calculation of spiral bevel gears based on finite element method [J]. Journal of Mechanical Engineering, 2011, 47 (11): 23-29.
[7] Yanzhong Wang, Yuanzi Zhou, Guoquan Li. Study on meshing stiffness and parametric vibration stability of spiral bevel gears [J]. Aviation Dynamics Daily, 2010, 25 (7): 1664-1669.

[8] Chunfeng Su, Yanting Ai, Xiaobao Lou. Study on the method of selecting contact stiffness factor in contact non-linear simulation [J]. Journal of Shenyang Institute of Aeronautical Technology, 2009, 26 (3): 5-9.

[9] International Organization for Standardization 23509:2006(E) Bevel and hypoid gear geometry [M]. Geneva: International Organization for Standardization, 2007.04.01:85-96.

[10] German Standards Committees. Calculation of load capacity of cylindrical gears: Introduction and general influence factors (DIN3990-1-1987). Technical Report. Berlin: German Institute for Standardisation, 1987:50-55.

[11] Ebrahimi S, Eberhard P. Rigid-elastic modeling of meshing gear wheels in multibody systems [J]. Multibody System Dynamics, 2006, 16(1):55-71.

[12] Datong Qin, Liyang Xie. Mechanical transmission design [M]. Beijing: Chemical Industry Press, 2013.3:348.

[13] Xiaoci Huang. Dynamic characteristics analysis of multi-stage gear transmission mechanism based on SIMPACK [J]. Mechanical transmission, 2014, 38(09): 148-150+177.

[14] International Organization for Standardization 6336-1:2006(2007) Calculation of load capacity of spur and helical gears-Part1:Basic principles, introduction and general influence factor [M]. Geneva: International Organization for Standardization, 2007.4.1:80-81.