Analytic continuation in two-color QCD: new results on the critical line

P. Cea\textsuperscript{a} L. Cosmai\textsuperscript{b} M. D’Elia\textsuperscript{c} A.Papa\textsuperscript{d}

\textsuperscript{a}Dipartimento di Fisica Univ. Bari \& INFN - Bari - Italy
\textsuperscript{b}INFN - Bari - Italy
\textsuperscript{c}Dipartimento di Fisica Univ. Genova \& INFN - Genova - Italy
\textsuperscript{d}Dipartimento di Fisica Univ. Calabria \& INFN - Cosenza - Italy

Abstract

We test the method of analytic continuation from imaginary to real chemical potential in two-color QCD, which is free from the sign problem. In particular, we consider the analytic continuation of the critical line to real values of the chemical potential.

Key words: QCD at finite density, analytic continuation

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1. Introduction

Understanding the phase diagram of QCD on the temperature-chemical potential \((T, \mu)\) has many important implications in cosmology, in astrophysics and in the phenomenology of heavy ion collisions. It is known that the fermion determinant in QCD becomes complex in presence of a non-zero chemical potential, thus preventing us from performing standard Monte Carlo simulations. Various techniques have been exploited to circumvent this problem. Among these techniques we consider here the use of an imaginary chemical potential for analytic continuation [1–16]. It is very important to have some control on the method of analytic continuation and a direct test of this method in simpler models can be very useful. To this purpose we explored the case of 2-color QCD where, for any value of the chemical potential, the fermion matrix is real. In ref. [9] we found, by looking at the behavior in both real and imaginary chemical potential of the Polyakov loop, the chiral condensate and the fermion density, that the method gives reliable results, within appropriate ranges of the chemical potential, and that a considerable improvement can be achieved if ratio of polynomials are used to interpolate data with imaginary chemical potential [9, 10].
On the other hand one of the most important applications of analytic continuation is the determination of the critical line or of critical surfaces for small values of $\mu$ [1–6,11,12]. The theoretical basis in this case is not as straightforward as for physical observables and relies on the assumption that susceptibilities, whose peaks signal the presence of the transition, be analytic functions of the parameters on a finite volume [1,2]. Direct tests of the method are even more important in this case, therefore we extended [17] our analysis of two-color QCD to the study of the (pseudo)critical line. As in usual QCD simulations, we will determine locations of the critical line for $\mu^2 < 0$ and interpolate them by suitable functions to be continued to $\mu^2 > 0$. The prediction obtained at real $\mu$ will then be compared with direct determinations of the transition line.

2. Lattice simulations

Numerical simulations have been performed using hybrid Monte Carlo with $dt = 0.01$. Typical statistics collected corresponds to 20k trajectories. Simulations have been done using resources of the INFN apeNEXT computing center.

Our aims are finding the best interterpolation of critical line at imaginary chemical potential and testing the analytic continuation of the critical line from imaginary to real chemical potential. We considered SU(2) gauge theory with $N_f = 8$ staggered fermions with mass $am = 0.07$ on a $16^3 \times 4$ lattice [17]. In Fig. 1 we present results for the extrapolation of the critical line to $\mu^2 > 0$ compared with the direct determination of the critical coupling $\beta_c$. As one can see there is a discrepancy between the extrapolated...
critical line and the direct determination of $\beta_c(\mu^2)$. One can infer that or the critical line is not analytic in the whole range of $\mu^2$ considered here or that the interpolation at $\mu^2 \leq 0$ is not accurate enough to correctly reproduce the behavior at $\mu^2 > 0$. A common fit to all available data with a third order polynomial in $\mu^2$ suggests the second possibility may be the correct one. In order to go deep inside this question we have started new simulations using a different mass ($am = 0.2$) on a $16^3 \times 4$ lattice and mass $am = 0.07$ on a $16^3 \times 6$ lattice. We have also considered the test of the analytical continuation of the critical line in the case of SU(3) with finite isospin and $N_f = 8$ staggered fermions with mass $am = 0.1$ on a $8^3 \times 4$ lattice. Fig. 2 displays our preliminary results for the analytical continuation of the critical line in the case of SU(3) at finite isospin density. As one can see the discrepancy between the extrapolated critical line and the direct determination of $\beta_c(\mu^2)$ is less severe.

3. Conclusions

We have tested in two-color QCD the analytic continuation of the critical line in the $(T, \mu)$ plane from imaginary to real chemical potential. We have found that the critical line around $\mu = 0$ can be described by an analytic function. Indeed, a third order polynomial in $\mu^2$ fits all the available data for the critical coupling. However, when trying to infer the behavior of the critical line at real $\mu$ from the extrapolation of its behavior at imaginary $\mu$, a very large precision would be needed to get the correct result. In the case of polynomial interpolations there is a clear indication that high-order terms play
a relevant role at $\mu^2 > 0$ but are less visible at $\mu^2 < 0$, this calling for an accurate knowledge of the critical line in all the first Roberge-Weiss sector. This scenario could be peculiar of two-color QCD. If confirmed in other theories free of the sign problem, such as QCD at finite isospin density, then one should seriously reconsider the analytic continuation of the critical line in the physically relevant case of QCD at finite baryon density.

For the reason above we have started a systematic program of investigations both in two-color QCD and in SU(3) QCD at finite isospin. Our preliminary results suggest that it may be useful to revisit the interpolations used in QCD (first order polynomial in $\mu^2$) and make the effort of determining accurately at least one more term in the polynomial fit.

References

[1] P. de Forcrand, O. Philipsen, The QCD phase diagram for small densities from imaginary chemical potential, Nucl. Phys. B642 (2002) 290–306. arXiv:hep-lat/0205010, doi:10.1016/S0550-3213(02)00626-0.

[2] P. de Forcrand, O. Philipsen, The QCD phase diagram for three degenerate flavors and small baryon density, Nucl. Phys. B673 (2003) 170–186. arXiv:hep-lat/0307020.

[3] M. D’Elia, M.-P. Lombardo, Finite density QCD via imaginary chemical potential, Phys. Rev. D67 (2003) 014505. arXiv:hep-lat/0209146.

[4] M. D’Elia, M. Lombardo, QCD thermodynamics from an imaginary $\mu(B)$: Results on the four flavor lattice model, Phys. Rev. D70 (2004) 074509. arXiv:hep-lat/0406012.

[5] V. Azcoiti, G. Di Carlo, A. Galante, V. Laliena, Phase diagram of QCD with four quark flavors at finite temperature and baryon density, Nucl. Phys. B723 (2005) 77–90. arXiv:hep-lat/0503010.

[6] H.-S. Chen, X.-Q. Luo, Phase diagram of QCD at finite temperature and chemical potential from lattice simulations with dynamical Wilson quarks, Phys. Rev. D72 (2005) 034504. arXiv:hep-lat/0411023.

[7] P. Giudice, A. Papa, Real and imaginary chemical potential in 2-color QCD, Phys. Rev. D69 (2004) 094509. arXiv:hep-lat/0401024.

[8] S. Kim, P. de Forcrand, S. Kratochvila, T. Takaishi, The 3-state Potts model as a heavy quark finite density laboratory, PoS LAT2005 (2006) 166. arXiv:hep-lat/0510069.

[9] P. Cea, L. Cosmai, M. D’Elia, A. Papa, Analytic continuation from imaginary to real chemical potential in two-color QCD, JHEP 02 (2007) 066. arXiv:hep-lat/0612018.

[10] A. Papa, P. Cea, L. Cosmai, M. D’Elia, Analytical continuation from imaginary to real chemical potential in 2-color QCD under scrutiny, PoS LAT2006 (2006) 143. arXiv:hep-lat/0610088.

[11] P. de Forcrand, O. Philipsen, The chiral critical line of $N_f = 2+1$ QCD at zero and non-zero baryon density, JHEP 01 (2007) 077. arXiv:hep-lat/0607017.

[12] P. de Forcrand, S. Kim, O. Philipsen, A QCD chiral critical point at small chemical potential: is it there or not?, PoS LAT2007 (2007) 178. arXiv:0711.0929.

[13] M. D’Elia, F. Di Renzo, M. P. Lombardo, The strongly interacting Quark Gluon Plasma, and the critical behaviour of QCD at imaginary chemical potential, Phys. Rev. D76 (2007) 114509. arXiv:0705.3914, doi:10.1103/PhysRevD.76.114509.

[14] S. Conradi, M. D’Elia, Imaginary chemical potentials and the phase of the fermionic determinant, Phys. Rev. D76 (2007) 074501. arXiv:0707.1987, doi:10.1103/PhysRevD.76.074501.

[15] P. Cea, L. Cosmai, M. D’Elia, A. Papa, Analytic continuation of the critical line in 2-color QCD at nonzero temperature and density, PoS LAT2007 (2007) 214. arXiv:0710.2068.

[16] F. Karbstein, M. Thies, How to get from imaginary to real chemical potential, Phys. Rev. D75 (2007) 025003. arXiv:hep-th/0610243, doi:10.1103/PhysRevD.75.025003.

[17] P. Cea, L. Cosmai, M. D’Elia, A. Papa, The critical line from imaginary to real baryonic chemical potentials in two-color QCD, Phys. Rev. D77 (2008) 051501. arXiv:0712.3755, doi:10.1103/PhysRevD.77.051501.