Vacuum Alignment in “Composite Technicolor” Models

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Abstract

We consider the question of vacuum alignment in the recently proposed “composite technicolor” (CTC) models. In these models, explicit chiral symmetry breaking due to masses of electroweak-singlet “ultrafermions” is communicated to the quarks and leptons by a chiral condensate produced by strong “ultracolor” gauge interactions. In order for these models to work, the ultrafermion condensate must align in a particular way, driven by the competition between ultrafermion masses and “flavor” gauge boson exchange. We show that for ultrafermion masses large enough to explain the top quark mass, order-of-magnitude estimates are sufficient to establish that the required vacuum alignment cannot occur for perturbative values of the flavor gauge couplings. If the top quark gets its mass from some other mechanism, a detailed calculation is required. Using spectral function sum rules and a vector-meson saturation approximation which is known to work well in QCD, we determine the correct vacuum for this case in a limit where the flavor gauge bosons are weakly coupled and the number of ultracolors \( N \) is large. We again find that the vacuum required by CTC models does not occur for perturbative values of the flavor gauge couplings. We conclude that there is no evidence that the vacuum aligns as required in CTC models.

This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.
1. Introduction

By far the most difficult problem facing technicolor theories is how to generate fermion masses and mixings without at the same time giving rise to flavor-changing neutral currents at unacceptable levels. In ref. [1], it was pointed out that a symmetry which guarantees a Glashow–Iliopoulos–Maiani (GIM) mechanism [2] could in principle be imposed on the low-energy effective theory arising from a technicolor model. In ref. [3], the generic problems in implementing this idea in models without elementary scalars were identified, and “composite technicolor” (CTC) models were proposed which are supposed to solve these problems. (See also ref. [4].) These models rely crucially on the assumption that the vacuum aligns in a particular way, driven by the competition between ultrafermion mass terms and “flavor” gauge boson exchange. In the desired vacuum, the chiral symmetry breaking due to ultrafermion masses is communicated to the ordinary quarks through massive flavor gauge boson exchange. It has been conjectured [4][5] that the desired vacuum is obtained for flavor couplings that are large, but not large enough so that they themselves break chiral symmetry.

In this paper, we consider the effective potential for this model. We show that in the case where the ultrafermion masses are sufficiently large so that this mechanism can give rise to the top quark mass, order-of-magnitude estimates are sufficient to establish that the required vacuum alignment cannot occur for perturbative values of the flavor gauge couplings.

We can also imagine models where this mechanism gives rise to the masses of the first two generations only, and the ultrafermion masses are small. In this case, a calculation is required to settle the alignment question. We compute the effective potential for this case in the limit that the flavor gauge coupling is weak and the number of “ultracolors” $N$ is large. In this limit, the vacuum alignment depends only on a current–current correlation function which can be estimated using a vector-meson saturation approximation which is known to work well in QCD. We find that the lowest-order perturbative effective potential predicts that the vacuum required by the CTC models occurs only for flavor gauge couplings well outside the range of validity of the calculation. We therefore conclude that there is no evidence that the vacuum aligns as required by the CTC models.

The plan of this paper is as follows: In section 2, we will introduce the model whose vacuum alignment we wish to determine, and discuss the form of the relevant effective potential. We argue that the required vacuum cannot occur in models where the top quark mass is given by the CTC mechanism. In section 3, we briefly review the formalism for addressing the question of vacuum alignment in QCD-like theories and compute the effective potential for the case where only light fermion masses occur through the CTC.
mechanism. Section 4 contains our conclusions.

2. The Model

The model we will consider has a gauge group

\[ SU(N)_{UC} \times SU(K)_L \times SU(K)_R, \]  

where \( SU(N)_{UC} \) is a strongly-coupled “ultracolor” group and \( SU(K)_L \times SU(K)_R \) is a “flavor” gauge group. The model contains the fermions

\[
\begin{align*}
Q_L &\sim (1, K, 1) \times N, \\
Q_R &\sim (1, 1, K) \times N, \\
\psi_L &\sim (N, K, 1), \\
\psi_R &\sim (N, 1, K), \\
\chi_L &\sim (N, 1, 1) \times K', \\
\chi_R &\sim (N, 1, 1) \times K'.
\end{align*}
\]

Note that some of the representations are repeated. All gauge anomalies are easily seen to cancel. In addition to the gauge interactions, the \( \chi \) fermions are assumed to have Dirac masses of the form

\[ \chi_L m_\chi \chi_R^c + \text{h.c.} \]  

(We work in a basis where \( m_\chi \) is diagonal and positive.) This model is not intended to be realistic by itself, but it is a crucial building block for the realistic models of refs. [3][4]. (See also ref. [6].) In order for these realistic models to work, the model above must choose a particular vacuum. This vacuum alignment question is the subject of this paper.

Suppose that the \( SU(N)_{UC} \) gauge coupling becomes strong at a scale \( \Lambda_{UC} \). If we neglect \( m_\chi \) and the \( SU(K)_L \times SU(K)_R \) gauge couplings, the theory undergoes spontaneous symmetry breaking in the pattern

\[ SU(K + K')_L \times SU(K + K')_R \rightarrow SU(K + K')_{L+R}. \]

The \( \chi \) masses and \( SU(K)_L \times SU(K)_R \) gauge couplings break the symmetry explicitly down to

\[ SU(K)_L \times SU(K)_R \times [U(1)]^{K'}. \]

(Upon diagonalization, the mass matrix \( m_\chi \) preserves \( K' \) separate \( U(1) \) “\( \chi \)-number” symmetries.) If \( m_\chi \ll \Lambda_{UC} \) and the \( SU(K)_L \times SU(K)_R \) couplings are small, then we can treat the explicit breaking effects perturbatively.
When the $SU(N)_{UC}$ gauge group becomes strong, it will give rise to the condensate

$$\langle \Psi_{Lj} \Psi_{Rk}^c \rangle = U_{jk} A^3_{UC},$$

(6)

where

$$\Psi \equiv \left(\begin{array}{c} \psi \\ \chi \end{array}\right),$$

(7)

and $U$ is a $SU(K + K')$ matrix. In the limit where $SU(K + K')_L \times SU(K + K')_R$ is an exact symmetry, $U$ has no physical significance, since we can perform a chiral rotation to set $U = 1$. However, in the presence of explicit breaking, $U$ determines the orientation of the exact symmetry group (5) inside the unbroken $SU(K + K')_{L+R}$ group.

The physics below the scale $\Lambda_{UC}$ can be summarized by an effective lagrangian containing the Nambu–Goldstone bosons (NGB's) and the $SU(K)_L \times SU(K)_R$ gauge fields [8]. This lagrangian should be viewed as a systematic simultaneous expansion in derivatives and the symmetry-breaking parameters. The NGB fields $\pi_A$ are collected into a $SU(K + K')$ matrix

$$\Sigma = e^{i\pi_A L_A / f}$$

(8)

transforming under $SU(K + K')_L \times SU(K + K')_R$ as

$$\Sigma \mapsto U_L \Sigma U_R^\dagger.$$

(9)

(Here $L_A$ are $SU(K + K')$ generators normalized so that $\text{tr} L_A L_B = \delta_{AB}$.) The gauge covariant derivative acting on $\Sigma$ is

$$D_\mu \Sigma = \partial_\mu \Sigma + ig A_{L\mu a} T_{La} \Sigma - ig A_{R\mu a} \Sigma T_{Ra},$$

(10)

where

$$T_{La} = T_{Ra} = \begin{pmatrix} t_a & 0 \\ 0 & 0 \end{pmatrix}$$

(11)

in the basis defined by eq. (7), and we have set the left- and right-handed flavor gauge couplings equal for simplicity. We normalize the gauge generators such that $\text{tr} t_a t_b = \delta_{ab}$.

The effective lagrangian contains the derivative terms

$$\mathcal{L} = \frac{f^2}{2} \text{tr} \left[ (D^\mu \Sigma)^\dagger D_\mu \Sigma \right] + \cdots.$$  

(12)

In addition, the theory contains non-derivative terms which give rise to a tree-level potential term for the NGB’s:

$$V(\Sigma) = a f^3 \text{tr} \left( \Sigma^\dagger M_\chi + \text{h.c.} \right) + O(M_\chi^2)$$

$$+ \frac{b_1 g^4}{16\pi^2} f^4 \text{tr} \left( T_{La} \Sigma T_{Rb} \Sigma^\dagger \right) \text{tr} \left( T_{La} \Sigma T_{Rb} \Sigma^\dagger \right)$$

$$+ \frac{b_2 g^4}{16\pi^2} f^4 \text{tr} \left( T_{La} \Sigma T_{Rb} \Sigma^\dagger T_{La} \Sigma T_{Rb} \Sigma^\dagger \right) + O(g^6),$$

(13)
where

\[ M_\chi \equiv \begin{pmatrix} 0 & 0 \\ 0 & m_\chi \end{pmatrix}. \]  

(14)

Based on the ideas of “naive dimensional analysis” [9], we expect that \(|b_1|, |b_2| \sim 1\). This is confirmed by the computations of the next section.

The full effective potential includes loop corrections. We write

\[ V_{\text{eff}}(\Sigma) = V(\Sigma) + \delta V(\Sigma), \]  

(15)

where \(\delta V\) denotes the loop corrections. These can be computed by standard methods [7]. The contribution from one gauge boson loop gives

\[ \delta V(\Sigma) = \frac{3}{64\pi^2} \text{tr} M^4 \ln \frac{M^2}{\mu^2}, \]  

(16)

where

\[ M^2 \equiv g^2 f^2 \begin{pmatrix} 1 & -t \\ -t & 1 \end{pmatrix}, \quad t_{ab} \equiv \text{tr} (T_{La} \Sigma T_{Rb} \Sigma^\dagger). \]  

(17)

\(M^2\) is the tree-level mass matrix of the \(SU(K)_L \times SU(K)_R\) gauge bosons in the background field described by \(\Sigma\), and \(\mu\) is an arbitrary renormalization scale. Eq. (16) is the leading correction for small \(g\). Note that the nonanalytic correction term are \(a\ priori\) of the same order of magnitude as the tree-level term.

Minimizing the full effective potential gives a vacuum expectation value for the NGB fields which specifies the alignment of the vacuum. In fact, comparing with eq. (6), we see that

\[ \langle \Sigma \rangle \equiv e^{i(\pi_A) L_A/f} = U. \]  

(18)

The parameters \(a, b_1, \) and \(b_2\) must be determined by matching this effective theory onto the underlying \(SU(N)_{UC}\) gauge theory, which requires solving a nonperturbative problem. The parameter \(a\) can be determined by using QCD as an analog computer, since a similar term gives rise to the \(\pi\) and \(K\) masses. It turns out that \(a \simeq -30\), which tends to stabilize the vacuum \(U = 1\). In CTC models \(K = K'\), and in order for these models to work, one requires that the potential be minimized by

\[ U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \]  

(19)

This can only be the correct vacuum if the contribution to the effective potential from flavor gauge boson exchange stabilizes this vacuum. This requires

\[ g^4 \left[ b_1 - \frac{b_2}{K} + 3 \ln 2 + \frac{3}{2} \ln \frac{g^2 f^2}{\mu^2} \right] > -\frac{32\pi^2}{K^2 - 1} \frac{a}{f} \text{tr} m_\chi \simeq \frac{10^3}{f} \text{tr} m_\chi. \]  

(20)
If at least one of the eigenvalues of $m_\chi$ is of order $f$, as required by the models of refs. [3][4][6], we find that even if assume that the left-hand-side is positive, the inequality is satisfied only for flavor gauge couplings $\alpha \gtrsim 5$ if we believe the NDA estimate that $|b_1|, |b_2| \sim 1$ for $\mu \sim 4\pi f$. This critical value for $\alpha$ is well outside the perturbative regime. (For comparison, studies of the QCD Schwinger–Dyson equations in ladder approximation typically show that chiral symmetry breaking occurs at a critical coupling $\alpha_s \sim 0.8$.) We conclude that there is no reason to expect the CTC models to choose the vacuum of eq. (19).

If all of the eigenvalues of $m_\chi$ are small, then the vacuum alignment is a more delicate question, and it is clear that a detailed calculation is required. This situation might arise in models similar to the CTC models in the literature, but where the top mass arises through some other mechanism. This would give rise to a GIM mechanism for the first two generations. However, the computation of the next section shows that in a limit where the vacuum alignment question can be settled with reasonable certainty, $U = 1$ is still the preferred vacuum for perturbative values of the flavor gauge coupling.

Before presenting the formalism of the next section, it may be worthwhile to ask if there is a simple physical picture which suggests the answer to the vacuum alignment question. In the case where the effective potential induced by weak gauge boson exchange is of order $g^2$, we can view the the effective potential as arising from the Coulomb force between fermions. We then expect the condensate to form in the most attractive channel, an expectation which is born out by detailed calculations. In the present case, there is no classical force between $\Psi_L$ and $\Psi_R$, since they transform under independent gauge groups. A force between $\Psi_L$ and $\Psi_R$ develops only as a result of spontaneous symmetry breaking, and this force itself depends on the alignment. This makes it difficult to conceive of a convincing intuitive picture, and we must therefore rely on the more formal considerations of the next section.

3. Computation of the Effective Potential

We wish to evaluate the vacuum energy as a function of the orientation of the condensate. We will neglect the contribution from the $\chi$ mass terms and focus on the effects of flavor gauge boson exchange. The relevant formalism is due to many authors [10].

To study the vacuum alignment question, we choose a canonical vacuum state $|0\rangle$ appropriate for the condensate (6) with $U = 1$. We then consider the energy of the trial state

$$|U\rangle \equiv \hat{U}|0\rangle,$$

(21)
where \( \hat{U} \) is the operator representing a \( SU(K)_L \times SU(K)_R \) transformation with \( U_L U_R^{\dagger} = U \) which specifies the vacuum alignment. We can evaluate the effective potential which determines the vacuum alignment by evaluating

\[
V_{\text{eff}}(U) = \langle U | \mathcal{H} | U \rangle = \langle 0 | \hat{U}^{\dagger} \mathcal{H} \hat{U} | 0 \rangle,
\]

(22)

where \( \mathcal{H} \) is the Hamiltonian. The calculation is done in two steps. First, we perform the functional integral over the \( SU(N)_{UC} \) gauge fields. Since the \( SU(N)_{UC} \) couplings preserve the full \( SU(K + K')_L \times SU(K + K')_R \) global symmetry, this computation is independent of \( U \). We then perform the functional integral over the \( SU(K)_L \times SU(K)_R \) gauge fields perturbatively, using the the Euclidean space interaction

\[
\mathcal{L}_{\text{int}} = g A_{\mu a} J_{\mu a}^{(U)} + g A_{R \mu a} J_{\mu a}^{(U)},
\]

(23)

where

\[
J_{\mu a}^{(U)} = \overline{\Psi}_L i \gamma_\mu U_L^{\dagger} T_a U_L \Psi_L,
\]

(24)

etc. The vacuum energy is then given by minus the sum of connected Euclidean vacuum–vacuum graphs.

The graphs which contribute to the effective potential at order \( g^4 \) are shown in fig. 1. At order \( g^4 \), the contribution of fig. 1b is independent of \( U \). The order \( g^4 \) contribution of fig 1c comes from the four-point function \( \langle J_L J_L J_R J_R \rangle \). This contribution is easily seen to be subleading in the large-\( N \) limit. Thus, we need only evaluate fig. 1a in the large-\( N \) limit.

Fig. 1a gives rise to the effective potential [7]

\[
V_{\text{eff}}(U) = \frac{3}{2} \int \frac{d^4k}{(2\pi)^4} \text{tr} \ln \Pi^{(U)}(k^2),
\]

(25)

where the trace is over the adjoint representation of \( SU(K)_L \times SU(K)_R \), and the gauge boson vacuum polarization has been written

\[
\Pi_{\mu \nu}^{(U)}(k) = \left( \delta_{\mu \nu} - k_\mu k_\nu / k^2 \right) \Pi^{(U)}(k^2).
\]

(26)

Working to order \( g^2 \), we can write

\[
\Pi^{(U)}(k^2) = \Pi_0(k^2) \cdot 1 + \delta M^2(k^2),
\]

(27)

\( \dagger \) We work in Euclidean space in order to simplify the job of keeping track of signs. Our conventions are that \( A_{\mu a} \) and \( J_{\mu a} \) are hermitian operators, so that their two-point functions are positive-definite.

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where $\Pi_0$ is the diagonal and $U$-independent part of the vacuum polarization and
\begin{equation}
\delta M^2(k^2) \equiv \begin{pmatrix} \delta m^2(k^2) \\ 0 \delta m^2(k^2) \\ 0 \end{pmatrix}, \quad (28)
\end{equation}
\begin{equation}
\delta m^2_{ab}(k^2) \equiv -g^2 \text{tr} \left( T_{La}U_T R_{b}U^\dagger \right) \langle J_L(k)J_R(-k) \rangle, \quad (29)
\end{equation}
where we have defined
\begin{equation}
\langle J_L\mu A(k)J_R\nu B(-k) \rangle \equiv (\delta_{\mu\nu} - k_\mu k_\nu/k^2) \delta_{AB} \langle J_L(k)J_R(-k) \rangle. \quad (30)
\end{equation}

To estimate $\delta m^2$, we note that the operator product expansion (OPE) tells us that
\begin{equation}
\langle J_L(k)J_R(-k) \rangle \sim \frac{1}{k^4} \quad \text{for } k^2 \to \infty. \quad (31)
\end{equation}
This will guarantee that the $U$-dependent part of eq. (25) converges. Imposing eq. (31) and assuming that $\langle J_LJ_R \rangle$ is dominated by the lowest-lying vector and axial vector resonances, we obtain [11]
\begin{equation}
\langle J_L(k)J_R(-k) \rangle \simeq f^2 m_a^2 m_a^2 \frac{1}{k^2 + m^2} \frac{1}{k^2 + m_a^2}. \quad (32)
\end{equation}
Here, $\rho$ is the lowest-lying vector resonance, and $a$ is the lowest-lying axial resonance. We will estimate their masses by scaling up QCD. The estimate eq. (32) is supported by $e^+e^-$ annihilation and $\tau$ decay data [12], as well as the classic calculation of the $\pi^+\pi^0$ mass difference [13] and by the success of the resulting relations among the resonance parameters [11]. (See also refs. [10].)

When we use the estimate eq. (32), we see that the $U$-dependent part of $V$ is sensitive only to $\Pi_0(k^2)$ at low momenta. We therefore approximate
\begin{equation}
\Pi_0(k^2) \simeq (k^2 + g^2 f^2) \cdot 1. \quad (33)
\end{equation}
We expect that the corrections to this approximation become important for momenta $k^2 \gtrsim m^2\rho$, and so corrections to $V$ will be suppressed by $\sim g^2 f^2/m^2\rho$.

Our task is therefore to calculate
\begin{equation}
V(U) = \frac{3}{2} \int \frac{d^4k}{(2\pi)^4} \text{tr} \ln \left[ 1 + \delta M^2(k^2) \frac{1}{k^2 + g^2 f^2} \right], \quad (34)
\end{equation}
where we have added an irrelevant $U$-independent constant. This may be expanded as an infinite series in $\delta M^2$. In all but the first two terms in this series, the integral is dominated by low momenta, and we can approximate $\delta M^2(k^2) \simeq \delta M^2(0)$. In this way, we find that
\begin{equation}
V(U) = -\frac{3g^4 f^4}{128\pi^2} \left[ 1 - \frac{2m^2_a}{m^2_a - m^2\rho} \ln \frac{m^2_a}{m^2\rho} \right] \text{tr} \left( T_{La}U_T R_{b}U^\dagger \right) \text{tr} \left( T_{La}U_T R_{b}U^\dagger \right) + \frac{3}{64\pi^2} \text{tr} M^4 \ln \frac{M^2}{m^2\rho}. \quad (35)
\end{equation}
Comparing to the form of the low-energy effective lagrangian, we can read off \( b_1 \simeq +0.8 \) for \( \mu = m_\rho \).

Even in the absence of \( \chi \) mass terms, eq. (35) predicts that the vacuum eq. (19) required by CTC models occurs only for flavor gauge couplings \( \alpha \gtrsim 2 \). We conclude that there is no reason to expect that the vacuum required by CTC models will occur.

4. Conclusions

We have argued that the recently-proposed “composite technicolor” models favor a phenomenologically unacceptable vacuum as long as the “flavor” gauge coupling can be treated perturbatively. If the top quark gets its mass from some other mechanism, the result was established only in the limit where the number of ultracolors \( N \) is large. While it is still possible that the \( 1/N \) corrections or non-perturbative effects for large values of the flavor gauge coupling change the preferred vacuum, there seems to be no evidence to support such an assertion.

5. Acknowledgements

I would like to thank R. Sundrum for many noisy discussions on the subject of this paper. This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.
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Figure Captions

Fig. 1. Contributions to the vacuum energy in the CTC model to order $g^4$. The solid lines represent flavor gauge boson propagators, and the shaded blobs represent the gauge boson vertex functions.