Disordered Type-II Superconductors: A Universal Phase Diagram for Low-\(T_c\) Systems

S. S. Banerjee\(^1\), A. K. Grover\(^1\), M.J. Higgins\(^2\), G. I. Menon\(^3\)*, P. K. Mishra\(^4\), D. Pal\(^1\), S. Ramakrishnan\(^1\), T.V. C. Rao\(^4\), G. Ravikumar\(^4\), V. C. Sahni\(^4\), S. Sarkar\(^1\)† and C. V. Tomy\(^5\)

\(^1\) Department of Condensed Matter Physics and Materials Science, Tata Institute of Fundamental Research, Mumbai-400005, India
\(^2\) NEC Research Institute, 4 Independence Way, Princeton, New Jersey 08540, U.S.A
\(^3\) The Institute of Mathematical Sciences, C.I.T Campus, Taramani, Chennai 600 113, India

\(^4\) Technical Physics and Prototype Engineering Division, Bhabha Atomic Research Centre, Mumbai-400085, India

\(^5\) Department of Physics, Indian Institute of Technology, Powai, Mumbai- 400076, India

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Abstract

A universal phase diagram for weakly pinned low-\(T_c\) type-II superconductors is revisited and extended with new proposals. The low-temperature “Bragg glass” phase is argued to transform first into a disordered, glassy phase upon heating. This glassy phase, a continuation of the high-field equilibrium vortex glass phase, then melts at higher temperatures into a liquid. This proposal provides an explanation for the anomalies observed in the peak effect regime of 2H-NbSe\(_2\) and several other low-\(T_c\) materials which is independent of the

*Corresponding Author, Email:menon@imsc.ernet.in

†Corresponding Author, Email:shampa@mailhost.tifr.res.in
microscopic mechanisms of superconductivity in these systems.

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I. INTRODUCTION

Random pinning destroys the long-ranged translational order of the Abrikosov flux-line lattice (FLL) [1,2]. This phase is believed to be replaced by a new thermodynamic phase, the “Bragg glass” phase, in which translational correlations decay as power laws [3]. Experiments see a first-order, temperature-driven melting transition out of the Bragg glass in weakly disordered single crystals of the cuprate superconductors Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (BSCCO) and YBa$_2$Cu$_3$O$_{7-\delta}$ (YBCO), for fields $H$ less than a critical value $H_{cr}$ [4–7]. For BSCCO, $H_{cr} \simeq 500$ G; for YBCO $H_{cr} \simeq 10$ T.

The Bragg glass (BG) transforms discontinuously [7] into a highly disordered solid phase as $H$ is increased at fixed low temperature $T$ [3,5,6,8]. Such a disordered phase, in which translational correlations decay exponentially [9], is also encountered on temperature ($T$) scans at $H > H_{cr}$; the data suggest a continuous transition into this low-$T$ phase from the equilibrium disordered liquid at high temperatures. This low-temperature phase may be a thermodynamic “vortex glass” (VG) phase, analogous to a spin glass phase, separated from the disordered liquid (DL) by a line of continuous phase transitions [10]. (Alternatively, it could be a “frozen” liquid, similar to a structural glass [11] or even a highly entangled phase [12,13]). Consensus phase diagrams for disordered high-$T_c$ materials which incorporate these phases are based principally on studies of the cuprates. Such phase diagrams show the continuous VG-DL transition line meeting the first-order BG-DL and BG-VG lines at a multicritical point $(T_{cr},H_{cr})$ [4–7].

For the cuprates, properties such as intrinsic layering, anisotropy, small coherence lengths and large penetration depths amplify the effects of thermal fluctuations. As a consequence, disordered phases such as the flux liquid phase occupy much of the phase diagram; the associated phase boundaries
The Ginzburg number $G_i$, \((= (k_B T_c/H_c^2 \epsilon \xi^3)^2 / 2)\), measures the importance of thermal fluctuations: $\xi$ is the coherence length, $\epsilon$ the mass anisotropy, $T_c$ the superconducting transition temperature and $H_c$ the thermodynamic critical field \([2]\). In most conventional low-$T_c$ materials $G_i \sim 10^{-8}$, whereas $G_i \sim 10^{-2}$ for BSCCO and YBCO. For superconductors with a small $G_i$, phase boundaries such as the melting line lie very close to $H_{c2}$ and are thus hard to assign without ambiguity.

In the low-$T_c$ system 2H-NbSe$_2$ ($T_c \simeq 7.2$K), the effects of thermal fluctuations are enhanced for reasons similar to those in the cuprates. For this material, $G_i \sim 10^{-4}$ \([16]\). For the C15 Laves-phase superconductor CeRu$_2$ ($T_c \simeq 6.1$K), $G_i \sim 10^{-5}$, while for the ternary rare-earth stannides Ca$_3$Rh$_4$Sn$_{13}$, ($T_c \simeq 8$K) and Yb$_3$Rh$_4$Sn$_{13}$, ($T_c \simeq 7.6$K) , $G_i \sim 10^{-7}$. For the quarternary borocarbide YNi$_2$B$_2$C ($T_c \simeq 15.5$K), $G_i \sim 10^{-6}$. Such relatively large values of $G_i$ imply that the phase diagram of relatively clean, low-$T_c$ type-II superconductors can be studied using such compounds, in regimes where the phases and phase transitions discussed above in the context of the cuprates are experimentally accessible.

These systems all exhibit a sharp “Peak Effect” (PE) in the critical current density $j_c$, a measure of the force required to depin the lattice \([17,18]\). The PE is the anomalous increase in $j_c$ seen close to the $H_{c2}(T)$; this increase terminates in a peak, as $H$ (or $T$) is increased, before $j_c$ collapses rapidly in the vicinity of $H_{c2}$. One mechanism for the peak effect \([19]\) is the softening of the shear elastic modulus of the vortex solid as $H$ approaches $H_{c2}(T)$. Lattice (shear) distortions which maximize the energy gain due to the pinning thus cost less elastic energy. As a consequence, $j_c$ increases since the flux-line lattice can adapt better to its pinning environment. In an approximation believed to be valid for weakly pinned vortex line systems, $j_c B = (\frac{n_p\langle f_p^2 \rangle}{\xi})^{\frac{1}{2}}$ \([20]\). Here $B$ is the magnetic induction, $n_p$ the density of pins, $f_p$ the elementary pinning
interaction and $V_c$ the correlation volume of a Larkin domain. $V_c$ and $f_p$ are both suppressed as $H \to H_{c2}$. A peak in $j_c$ thus implies a more rapid reduction in $V_c$ as compared to $<f_p^2>$. Above the peak, the rapid collapse in the shielding response [21] is governed entirely by the suppression of pinning on approaching $H_{c2}$.

A peak effect can also occur across a melting transition, when a shear modulus vanishes abruptly [16]; early studies of the peak effect in 2H-NbSe$_2$ suggested such a mechanism for the peak effect in this material. Evidence in favour of this correlation between melting phenomena and (some but not all) sharp peak effects has mounted in recent years.

The locus of points in $(H, T)$ space from which $j_c$ begins to increase to its peak value defines a line called $T_{pl}(H)$; the locus of maxima of $j_c$ is $T_p(H)$ [11]. The regime of fields and temperatures between $T_{pl}$ and $T_p$ is the peak regime. This regime is dynamically anomalous. Its properties (the PE anomalies) include the following: (i) A profound history dependence of static and dynamic response [16, 22, 28], (ii) large noise signals in electrical transport and ac susceptibility measurements [16, 25, 29, 30], (iii) substantial memory effects [16, 24, 31, 32], (iv) “switching” phenomena by which the system can transit between $j_c$ values characteristic of a field cooled (FC) or zero-field-cooled (ZFC) state on applying $T$ or ac amplitude pulses [27] and (v) “open hysteresis loops” in thermal cycling [25]. Why these anomalies arise is still incompletely understood; an explanation is outlined in this paper.

This paper presents a phase diagram (Fig. 1) for weakly pinned low-$T_c$ type-II superconductors with point pinning disorder [33], which incorporates and extends earlier related proposals [34]. We propose the following: The equilibrium high-field VG phase survives in the intermediate field regime, $\Phi_0/\lambda^2 < H < H_{cr}$ [37] as a sliver intervening between quasi-ordered and liquid phases, as shown in Fig. 1. This sliver is precisely the regime in
which PE anomalies – non-trivial relaxation behaviour, memory effects and substantial history dependence in measured properties – are seen. We propose that such properties are generic and arise due to the continuation of the high field glassy behaviour into a regime where sensitivity to external perturbations is enhanced and relaxation times become accessible. The inset to Fig. 1 expands the regime of fields and temperatures shown in the boxed region of the main figure and depicts the expected phase behaviour at low fields (\( H_{c1} < H < \Phi_0/\lambda^2 \)), where the BG-VG phase boundary shows reentrant behaviour (see below).

Our phase diagram differs from the conventionally accepted one; it contains no multicritical point and the BG phase never melts directly into the liquid. We believe that this is due to the substantial thermal renormalization of disorder relevant to the high-\( T_c \) cuprates close to the melting transition; such effects suppress the irreversibility line \(^{[6]}\) to below the melting line and can effectively render the sliver phase unresolvable \(^{[38-40]}\).

Our proposals are based on a study of the systematics of the peak effect in a variety of low \( T_c \) systems via ac susceptibility and dc magnetization measurements on relatively pure single crystals \(^{[25,26,35,41,42]}\). AC susceptibility measurements access \( j_c \) via the real part of the ac susceptibility \( \chi' \) in the following way: Once the applied ac field penetrates the sample fully, \( \chi' \sim -\beta \frac{j_c}{h_{ac}} \), from the Bean’s critical state model \(^{[36,13]}\). Here \( \beta \) is a geometry and size dependent factor, while \( h_{ac} \) is the magnitude of the applied ac field. At very low temperatures, \( \chi' \approx -1 \), indicating perfect screening of applied fields. Non-monotonic behaviour in \( \chi' \) reflects non-monotonicity in \( j_c(H,T) \); the minimum in \( \chi' \) corresponds to a peak in \( j_c \).
II. EXPERIMENTAL RESULTS

Results of typical measurements of $\chi'$ performed on 2H-NbSe$_2$ [25], CeRu$_2$ [25], Ca$_3$Rh$_4$Sn$_{13}$ [28], Yb$_3$Rh$_4$Sn$_{13}$ [41] and YNi$_2$B$_2$C [42] are shown in Figs. 2(a)-(e) for field values as indicated in the figures and at temperatures close to $T_c(H)$. The two curves shown in each figure refer to different thermomagnetic histories – zero-field-cooled (ZFC) and field-cooled (FC). The structure in $\chi'$ is broadly similar for both ZFC and FC histories, although FC histories yield far less dramatic peak effects. Field cooling in these samples generically leads to higher $j_c$ values (and thus smaller correlation volumes [20]) than cooling in zero field. This is in contrast to the phenomenology of spin glasses. Such behaviour is also opposite to that observed for high-$T_c$ materials. An explanation for this anomalous behaviour will be presented in what follows.

Note the existence of abrupt discontinuities in $\chi'$ in Figs. 2(a)-(e). These data show discontinuities at $T_p$ and $T_{pl}$, which have the following properties: (i) their locations are independent of the ac field amplitude and the frequency over a substantial range, (ii) two discontinuities associated with the onset and the peak positions of the PE are generically seen, although more complicated intermediate structure is manifest in some samples (iii) their locations correlate precisely with $T_p$ and $T_{pl}$ measured in transport measurements – such measurements show that the locations of such features are independent of the magnitude of the driving current and are hence properties of the sample and not of the measurement technique and (iv) they are seen both in ZFC and FC data and at identical locations. These properties indicate that it is thermodynamic behaviour which is being reflected and that these phenomena are independent of the measurement technique.

As $j_c$ collapses from its peak value above $T_p$, the diamagnetic $\chi'$ response becomes a paramagnetic one across the irreversibility temperature $T_{irr}$ [20].
Above $T_{irr}$, $j_c \approx 0$ and the mixed state of the superconductor has reversible magnetic properties. In the reversible phase, differential magnetic response is positive (i.e., diamagnetic dc magnetization decreases as $H$ or $T$ is increased). Such a differential paramagnetic effect identifies the depinned ($j_c = 0$) state. A $\chi'(T)$ measurement at fixed $H$ thus yields three characteristic temperatures, denoted as $T_{pl}$, $T_p$, and $T_{irr}$ [26].

Fig. 3(a) shows the peak onset at $(H_{pl}, T_{pl})$, the location of the peak at $(H_p, T_p)$, and the apparent irreversibility line $(H_{irr}, T_{irr})$, in 2H-NbSe$_2$; the behaviour in the intermediate field regime is qualitatively the same for CeRu$_2$, Ca$_3$Rh$_4$Sn$_{13}$, Yb$_3$Rh$_4$Sn$_{13}$ and YNi$_2$B$_2$C (cf. Figs. 3(b) to 3(e)). The $T_p$ and $T_{pl}$ lines begin to separate at high values of the field; at low field values, a similar broadening of the peak regime also occurs. The peak effect is sharpest at intermediate field values. At high and low $H$, the discontinuities become weaker. In some circumstances, the discontinuities evolve into a broad dip in $\chi'(T)$ and resistive response [45]. For sufficiently low fields in 2H-NbSe$_2$, the $T_p$ line curves backwards, moving to lower field values as the temperature is decreased. We have argued elsewhere that this behaviour signals the reentrant character of the low-field melting transition in the pure system [44].

The nature of the phase boundaries at low values of the field can also be studied through $j_c(H)/j_c(0)$ vs $H/H_{c2}$ plots at fixed $T$ [46]. Fig. 4 shows such plots for a single crystal of 2H-NbSe$_2$ at different values of $t$ (≈$T/T_c$). At $t=0.75$, the power law behaviour in $j_c(H)$ marks the collectively pinned regime, which we identify as a Bragg glass. This power law regime terminates for large $H/H_{c2}$, when $j_c(H)$ begins to vary anomalously, signalling the PE regime. For low fields, another anomalous variation in $j_c(H)$ (see Fig. 4) marks the transition from the collectively pinned power-law regime to another regime, which some of us have identified in previous work as a “small bundle pinning” regime [46]. This anomalous variation has been termed a “Plateau
Effect” by some of us [46].

Recent transport measurements of Paltiel et al [47] in single crystals of 2H-NbSe$_2$ confirm that the onset of each of these anomalies in $j_c(H)$ (both at low and high field ends) occurs abruptly. The anomalous variation in $j_c$ thus exhibits reentrant behaviour [26, 44–46] as $H$ is varied. Fig. 5 shows the boundary which separates the collectively pinned power-law regime from the low and high field regimes in which $j_c(H)$ varies anomalously. The power law region shrinks as the temperature increases. Above a characteristic temperature (which correlates with the quenched random disorder in the crystal [36,46]), this boundary is not encountered in an isothermal scan. This is reflected in the monotonic variation of $j_c(H)$ at $t=0.983$ shown in Fig. 4.

III. DISCUSSION

An interpretation of the above mentioned phenomena is now proposed; it builds on the work of Refs. [25], [28] and [46] by emphasizing the relation to the equilibrium phase diagram of Fig. 1. The presence of two discontinuities in $\chi'$ vs. $T$ in Figs. 2(a)-(e) implies that three different phases are encountered in $T$ scans which commence from within the Bragg glass at intermediate $H$. If the second discontinuity is associated to the transition into a uniform fluid, the first must be interpreted as a transition between a relatively ordered phase and a more disordered phase. The melting of the Bragg glass on a temperature scan is thus argued to be a two-stage process, with the transformation into the liquid preceded by a transformation into an intermediate state with a larger $j_c$ [27]. The intermediate state defines a small “sliver” in the phase diagram; its width in temperatures is bounded by $T_p$ and $T_{pl}$.

What is the fate of the sliver (or PE regime) at high fields?. We argue here that the sliver regime must broaden out into the equilibrium vortex glass
phase expected at low temperatures and high fields; these phases are smoothly connected, as shown in Fig. 1. This phenomenology, as well as the absence of a multicritical point, is suggested both by the increasing separation of the $T_p$ and $T_{pl}$ lines at high fields and the strongly anomalous behaviour seen in the peak regime. The phase diagram of Fig. 1 immediately suggests a simple physical origin of PE anomalies: the peak regime is anomalous precisely because it is glassy. In terms of Fig. 1, a scan in $T$ first encounters the BG-VG phase boundary. On further increasing $T$, the VG-DL phase boundary is encountered, whereupon the second transition occurs. Experimentally, nowhere in the intermediate field regime do the two discontinuities appear to merge. Thus, the “sliver” of intermediate phase appears to intrude smoothly between the high-field and the low field end, as corroborated by Fig. 5. At low fields, the data suggest the phase boundaries shown in the inset of Fig. 1, in particular the reentrant nature of the BG-VG phase boundary [51].

Of the two transitions out of the Bragg glass phase at intermediate fields, the second – between VG and DL phases – can be argued to be the true remnant of the underlying first-order melting transition in the pure flux line lattice system (see below). Thermodynamic signatures of the underlying (discontinuous) melting transition in the pure system should thus be strongest across this boundary; this is consistent with magnetization experiments on untwinned and twinned single crystals of YBCO [38, 48, 50]. However, the two stage nature of the transition implies that some part of the difference in density between ordered and liquid phases can be accommodated across the width of the sliver regime [52].

We argue that the variation in $\chi'$ between the ZFC and the FC histories simply reflects the fact that the system, on field cooling, first encounters the irreversibility line and then the highly disordered glassy sliver phase. Since it is in the irreversible region, it is trapped into a metastable state easily and
only drastic mechanical perturbations, such as a “shaking” of the crystal via large amplitude ac fields, enable it to lower its free energy \[27\]. In contrast, zero-field cooling can ensure that the thermodynamically stable Bragg glass phase is the preferred state, since the intermediate irreversible glassy state is not encountered.

The anomalies in the noise spectrum measured in the PE regime are explained in the following way: If the VG phase is a genuine glass (in the sense that any appropriately coarse-grained free energy landscape has many metastable minima, \textit{i.e.}, it is \textit{complex}), transitions between such minima should yield non-trivial signals in the noise spectrum. This is consistent with the suggestive observations of Merithew \textit{et al.} \[30\], who find that the noise arises from \textit{rearrangements} of the \textit{pinned} condensate in the peak regime. This implies that the noise originates in the static properties of the underlying phase and not exclusively from its flow properties \[53\]. We emphasize that the central idea which differentiates our proposals from those of others is our connection of these dynamical anomalies to the static phase diagram proposed in Fig. 1.

Paltiel \textit{et al.} \[54\] have proposed that many features of the PE regime can be understood in terms of the surface barriers to vortex entry. In an imaginative scenario, these authors suggest that vortices enter in a disordered state at the boundaries and then anneal in the (ordered) bulk. It is argued that the disordered phase invades the bulk from the boundaries as the peak regime is entered, leading to slow relaxation and memory effects.

The ideas presented here substantiate and extend these proposals by linking them to the underlying thermodynamic behaviour. We have suggested that the bulk is highly disordered in the peak regime as a consequence of the connection of this regime to the VG phase. This link is embodied in the \textit{static} phase diagram of Fig. 1 and is \textit{independent} of the details of the dynamics. We
can thus argue that vortices injected from the boundaries in the PE regime equilibrate slowly because the structure to which they are equilibrating is a glass with many metastable states. This extension of the picture of Paltiel et al. [54] has the following appealing features: (i) it rationalizes the observation of discontinuous changes in $\chi'$ vs. $T$; such discontinuous behaviour cannot be otherwise obtained, (ii) it demonstrates unambiguously why such behaviour is unique to the PE regime, and (iii) it clarifies the connection of PE behaviour with the underlying bulk thermodynamic melting transition in the pure system.

Recent simulations also support the ideas presented here. A. van Otterlo et al. [55] remark that the existence of a sliver of the VG phase preempting a direct BG-DL transition is a possibility consistent with the simulation results. Other simulations suggest that the experimental observation of a split fishtail peak in a relatively pure untwinned YBCO sample [56] reflects a sequence of two transitions out of the Bragg glass phase [57]. These proposals have their precise counterpart in Fig. 1; as argued here and elsewhere [25,28,35,58], the peak effect in 2H-NbSe$_2$ and related materials actually comprises two separate transitions – the BG-VG and the VG-DL transitions. The phase diagram of Fig. 1 illustrates how the peak effect in $T$ scans at intermediate values of the field should evolve smoothly and continuously into behaviour characteristic of the BG-VG field driven transition both at low and high fields. This is a simple consequence of the absence of an intervening multicritical point in the phase diagram of Fig. 1. In other words, the robustness of the two-step character of the PE anomaly which occurs as a precursor to $H_{c2}$ rules out the possibility of a multicritical point.

We emphasize that the putative multicritical point and the single sharp thermal melting transition featured in theoretical models of the phase behaviour of high-$T_c$ materials [4,14] are not mandated by theory. Indeed, our
proposal is simpler and possibly more generic. The instability of the Bragg glass phase to a spontaneous proliferation of a high density of dislocations could well be preempted by a transition into a phase with an intermediate density of dislocations; nothing in the theory forbids this.

What might be the structure of the sliver phase? We suggest that it is a “multi domain” structure, with fairly well-ordered crystalline domains and local amorphous or liquid-like regions. Translational (and orientational) correlations would then decay strongly beyond the typical domain size. In this picture, the second jump in $\chi'$ is related to the melting transition of the multi domain solid \[59,60\], while the first represents the “fracturing” \[25\] of the BG phase into such a multi domain structure. Provided the typical size of domains is much larger than the typical correlation length at the melting transition, one expects sharp signals of melting. These signals would progressively decrease in amplitude as the system became more disordered, or, given the expected correlation between $H$ and disorder, as $H$ is increased.

The above interpretation is consistent with the intermediate jumps in $\chi'$ seen in some measurements of the peak effect (cf. Fig. 2(e) for Yb$_3$Rh$_4$Sn$_{13}$). These jumps could then be taken as signalling the melting of particularly large domains or perhaps a percolation of liquid-like or amorphous regions. Such “precursor” melting could arise due to disorder-induced variations in melting temperatures in different regions of the sample. This physical picture agrees with the proposals of Paltiel et al. \[54\] who argue that the PE regime reflects the dynamic coexistence of ordered and disordered phases. (We argue here that such behaviour in the peak regime actually reflects the complex nature of the statics). The observation of open hysteresis loops in experiments on 2H-NbSe$_2$ and other low $T_c$ systems and their rationalization in terms of a fracturing of the disordered PE state \[25,28\] point to this interpretation. Such fracturing would then be a consequence of large free energy barriers separating
such a multi domain or “microcrystalline” state, with its relatively short-ranged translational correlations, from the BG phase, where such correlations are infinite ranged. These ideas also correlate well with the observation of a critical point of the vortex glass-liquid transition in the experiments of Nishizaki et al. [48] on YBCO, beyond which the distinction between these phases vanishes.

Voltage noise measured in the flowing state created just above the depinning threshold shows a marked increase in the peak regime of 2H-NbSe$_2$. Marley et al. [29] have probed such low frequency, broadband noise through transport measurements and found strong dependences on the current and $H$ in the power, spectral shape and non-gaussian character of the noise. Related enhancements in the $\chi'$ noise have been observed by Banerjee et al. [25]. Merithew et al. [30] have measured the higher order fluctuators of this noise to extract the number of independent channels or contributors to the noise spectrum. A fairly small value for the number of correlators is derived, justifying its non-gaussian character [31]. Such a small number of correlators emerges naturally in terms of a picture of large, correlated solid-like “chunks”, as in a multi domain arrangement, with each chunk fluctuating collectively. Recent simulations are also consistent with this possibility [31]. Direct tests of this proposal, through spatially resolved Hall probe measurements, Bitter decoration or neutron scattering would be welcome.

In conclusion, this paper proposes a general phase diagram for weakly pinned type-II superconductors based on a study of the peak effect systematics in a variety of low-$T_c$ superconductors. The arguments presented here do not rely on the particular microscopic mechanism of superconductivity in these materials and are hence general. We have argued that the sensitivity to external perturbations, novel history effects and switching phenomena exhibited in the peak regime derive from the static complexity of the vortex
glass state. Such complexity is envisaged in theories of the low-temperature properties of structural and spin glasses. The PE regime may provide a remarkable new testing ground for these theories. Further work is clearly called for to test these ideas. More results, in particular on issues related to the phase diagram of high-$T_c$ materials will be published elsewhere [38,39].

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Phase diagrams closely related to the one exhibited in Fig. 1 have been proposed earlier; see e.g., Ref. \[26,35,36\]. Fig. 1 differs from these in the following respects: (i) the different phases (Bragg Glass, Vortex Glass and Disordered Liquid) are completely enclosed from each other by transition boundaries as shown; Fig. 1 explicitly rules out a multicritical point (or critical end point) whereas earlier phase diagrams left this possibility open, (ii) it includes only those phases and phase transition lines which we believe can be justified in equilibrium and (iii) the word “plastic glass” which carries a dynamic connotation and was used in earlier work to describe the PE regime, is replaced by “vortex glass” to indicate our understanding that the measurements, though dynamic in nature, reveal information about the static phases.

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For low-$T_c$ systems, $H_{cr}$ and $T_{cr}$ notionally represent the point in the H-T phase diagram where broadening of the narrow sliver regime at high fields becomes significant.

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For very pure samples of low-$T_c$ materials (such as sample A of 2H-NbSe$_2$ referred to in Ref. 25), the $T_{pl}$ and $T_p$ lines cannot be resolved separately, although an extremely narrow peak effect (its width is smaller than the width of the zero-field superconducting phase transition) is seen. In more disordered samples such as the one discussed here, the two-step nature of the transition is clearly evident.

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FIGURES

FIG. 1. Proposed phase diagram for a low-T\textsubscript{c} Type-II superconductor incorporating the effects of thermal fluctuations and quenched random point pinning. For a description of the phases, see the text. Note that the vortex glass phase intrudes between Bragg glass and disordered liquid phases everywhere in the phase diagram. The disordered liquid may have irreversible or reversible magnetic properties. In the intermediate field range, the glass phase is confined to a slim sliver but broadens out again for sufficiently low fields. The irreversibility line (dotted) lies above the VG-DL phase boundary (see text). The inset to the figure expands the boxed region shown in the main panel at low fields and temperature values close to T\textsubscript{c}(0) and illustrates the reentrant nature of the BG-VG phase boundary at low fields.

FIG. 2. Temperature dependence of $\chi'$ for vortex arrays created in zero field cooled (ZFC) and field cooled (FC) modes at the fields indicated in single crystals of (a) 2H-NbSe\textsubscript{2}, (b) CeRu\textsubscript{2}, (c) Ca\textsubscript{3}Rh\textsubscript{4}Sn\textsubscript{13}, (d) YNi\textsubscript{2}B\textsubscript{2}C and (e) Yb\textsubscript{3}Rh\textsubscript{4}Sn\textsubscript{13}. Note the two sharp changes in $\chi'$ response at the onset temperature $T_{pl}$ and peak temperature $T_p$ in the ZFC mode. The difference in $\chi'$ behavior between ZFC and FC histories disappears above the peak temperature.

FIG. 3. Vortex phase diagrams (for $H > 1$ kOe) of (a) 2H-NbSe\textsubscript{2}, (b) CeRu\textsubscript{2}, (c) Ca\textsubscript{3}Rh\textsubscript{4}Sn\textsubscript{13}, (d) YNi\textsubscript{2}B\textsubscript{2}C and (e) Yb\textsubscript{3}Rh\textsubscript{4}Sn\textsubscript{13}. Note that the lines marking the onset of the PE ($H_{pl}$), the onset of the reversibility ($H_{irr}$) and the upper critical field ($H_{c2}$) can be distinctly identified. For a description of the different phases shown in this figure, see text.

FIG. 4. Plot of $\log(j_c/j_c(0))$ vs $\log(H/H_{c2})$ in a crystal of 2H-NbSe\textsubscript{2} at three different temperatures. The power law region at $t=0.75$ has been marked and the two regions of anomalous variation in $j_c(H)$ have been identified as the Peak Effect and the Plateau Effect, following Ref. 46 (see text for details).
FIG. 5. The (H,T) phase diagram showing the demarcation of the collectively pinned region (corresponding to the power law behaviour in $j_c(H)$) as distinct from the regions corresponding to anomalous variation (i.e., the Peak Effect and the Plateau Effect) in $j_c(H)$ in 2H-NbSe$_2$. Note that the collective pinned region is sandwiched between region of anomalous variation in $j_c(H)$ at high as well as at low field ends.
``H_{c2}(T)''

NORMAL

VORTEX GLASS

DISORDERED LIQUID

BRAGG GLASS

MEISSNER

IRREVERSIBILITY LINE

TEMPERATURE (K)

MAGNETIC FIELD (T)
Figure 2(a) and 2(b)  S.S. Banerjee et al

\[ \chi' \text{ (normalised)} \]

\[ 2H-NbSe_2 \]

\[ T_{pl} \]

\[ H//c \]

\[ ZFC \]

\[ FCW \]

\[ T_p \]

\[ H_{dc} = 7.5 \text{ kOe} \]

\[ CeRu_2 \]

\[ T_p \]

\[ T_{pl} \]

\[ H_{dc} = 13.5 \text{ kOe} \]

Reduced Temperature \((t=T/T_c(0))\)
Figure 2(c) and 2(d) S.S. Banerjee et al
Figure 2(e) S. S. Banerjee et al
Figure 3(a) and 3(b)  S.S. Banerjee et al
Figure 3(c) and 3(d) S.S. Banerjee et al
Figure 3(e) S. S. Banerjee et al
Figure 4 S. S. Banerjee et al
Reduced temperature \( t = \frac{T}{T_c(0)} \)

| 0.75 | 0.80 | 0.85 | 0.90 | 0.95 | 1.00 |
|------|------|------|------|------|------|

\( H_{pl} \)

Onset of Peak Effect

Power law region (Bragg Glass)

Onset of Plateau effect

"small bundle pinning"

Figure 5 of S.S. Banerjee et al

2H-NbSe\(_2\)

\( T_c(0) \approx 6.0 \text{ K} \)

\( H \parallel c \)

Anomaly in \( J_c \) (PE)

\( H_{c2} \)

Normal