Chiral transition and mesonic excitations for quarks with thermal masses

Yoshimasa Hidaka\textsuperscript{1} and Masakiyo Kitazawa\textsuperscript{1}\textsuperscript{*}

\textit{RIKEN-BNL Research Center, Brookhaven National Laboratory, Bldg.510A, Upton, 11973, NY, USA}
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We study the effect of a thermal quark mass, $m_T$, on the chiral phase transition and mesonic excitations in the light quark sector at finite temperature in a simple chirally-symmetric model. We show that while nonzero $m_T$ lowers the chiral condensate, the chiral transition remains of second order. It is argued that the mesonic excitations have large decay rate at energies below $2m_T$, owing to the Landau damping of the quarks and the van Hove singularities of the collective modes.

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\section{Introduction}
Quantum chromodynamics (QCD) at nonzero temperature exhibits rich physics, and is expected to undergo chiral and deconfinement transitions at a critical temperature $T_c$. The RHIC experiments revealed interesting features of a quark-gluon plasma (QGP) phase above $T_c$ \cite{RHIC}. One of the historical problems is to explore the existence and the nature of the mesonic excitations in the QGP phase \cite{mesonicexcitations1,mesonicexcitations2,mesonicexcitations3,mesonicexcitations4,mesonicexcitations5}. Recent lattice QCD simulations suggest the existence of such excitations in the heavy quark sector \cite{mesonicexcitations2}. In the light quark sector, it is theoretically predicted that there exist light collective modes in the sigma- and pi-channels, as the soft modes of the chiral phase transition \cite{mesonicexcitations2}.

It is also known that the quarks show a peculiar behavior in the QGP phase. At extremely high temperatures, the hard thermal-loop (HTL) approximation \cite{HTL} can be adopted, and the quark propagator has two collective excitations, normal quasiparticles and plasmino modes \cite{HTL,plasminos1,plasminos2,plasminos3}. The excitation spectra of both collective modes have a mass gap termed the “thermal mass” $m_T \sim gT$, where $g$ and $T$ are the gauge coupling and the temperature, respectively. Unlike the Dirac mass, the thermal mass does not break the chiral symmetry. Although the validity of the HTL approximation may be violated around $T_c$, lattice simulations suggest a large quark mass near but above $T_c$ \cite{mesonicexcitations2}. If the quarks have large thermal masses near $T_c$, they can affect the properties of mesonic excitations, especially in the light quark sector \cite{mesonicexcitations2}, and also the chiral transition at finite temperature. We notice that the quark spectrum can have a thermal mass in the vicinity of $T_c$ as a result of the soft mode of the chiral transition \cite{mesonicexcitations2}.

In the present work, we explore the effects of the thermal mass on the chiral phase transition and mesonic excitations at finite temperature in a simple model. We employ the propagator obtained in the HTL approximation for the quarks, with the thermal mass $m_T$ introduced as a parameter. We also adopt a chirally-symmetric four-Fermi interaction in the scalar and pseudoscalar channels. Using this model, we show that the order of the chiral transition does not change, while the value of the chiral condensate is suppressed, when $m_T \neq 0$ is included. It will be shown that even at finite $m_T$, there are soft modes of the chiral transition in the scalar and pseudoscalar channels \cite{mesonicexcitations2}. Using these modes, we study the effect of $m_T$ on the mesonic excitations above $T_c$.

How does the thermal mass of the quark affect the properties of mesonic excitations? Naively, a mesonic excitation below the expected threshold energy, $\omega_{\text{thr}} = 2m_T$, is tightly bounded, since the decay process into quark and anti-quark is forbidden below $\omega_{\text{thr}}$. We show in the present work, however, that the effect of the thermal mass is completely contrary to this statement; if the quark has the spectrum calculated in the HTL approximation with a thermal mass $m_T$, the decay rate of the mesonic excitations is enhanced below the energy $2m_T$. It is discussed that these decay rates mainly originate from the continuum in the quark spectrum which physically corresponds to the Landau damping of the quasi-quarks.

This paper is organized as follows. In the next Section, we present the model employed in this work. In Section III, we calculate the chiral condensate in the mean-field approximation and demonstrate the chiral phase transition in our model. We then calculate the spectral function in the scalar and pseudoscalar channels above $T_c$ in Section IV. Section V is devoted to discussions and a summary.

\section{Model}
Before we present the model employed, let us first briefly review the property of the quark propagator in the high temperature limit of QCD, in order to see the emergence of the gapped collective excitations in the quark spectra. In this limit, the retarded quark propagator is calculated by the HTL approximation as

\begin{equation}
S_{\text{HTL}}(\omega, \mathbf{p}) = \left[ (\omega + i\eta)^2 - \mathbf{p} \cdot \mathbf{\gamma} - \Sigma_{\text{HTL}}(\omega + i\eta, \mathbf{p}) \right]^{-1},
\end{equation}

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\end{equation}
where $p = |p|$ and
\[ \Sigma^{HTL}(\omega, p) = \frac{m^2}{p} Q_0 \left( \frac{\omega}{p} \right) \gamma^0 + \frac{m^2}{p} (1 - \frac{\omega}{p} Q_0 \left( \frac{\omega}{p} \right)) \gamma \cdot \hat{p}, \]
(2)
is the quark self-energy in the one-loop level with $m^2 = (1/6) g^2 T^2$ and $Q_0 = (1/2) \ln(x + 1)/(x - 1)$ \[8\]. Notice that the quark propagator $S^{HTL}(\omega, p)$ is chirally symmetric as one can easily check that it anti-commutes with $\gamma^5$.
The quark propagator Eq. (1) can be decomposed as
\[ S^{HTL}(\omega, p) = (S^{HTL}_+ (\omega, p) P_+ (p) + S^{HTL}_- (\omega, p) P_- (p)) \gamma^0, \]
(3)
with projection operators $P_\pm (p) = (1 \pm \gamma_0 \gamma \cdot \hat{p})/2$. The spectral functions corresponding to $S^{HTL}_\pm (\omega, p)$ are found to be,
\[ \rho^{HTL}_\pm (\omega, p) \equiv -2 \text{Im} S^{HTL}_\pm (\omega, p) = 2 \pi [Z_\pm (p) \delta (\omega - \omega_\pm (p)) + Z_\pm (p) \delta (\omega + \omega_\mp (p)))] + \rho_5^0 (\omega, p). \]
(4)
In this limit, $\rho^{HTL}_+ (\omega, p)$ and $\rho^{HTL}_- (\omega, p)$ have two poles at $\omega = \pm \omega_\pm (p)$ and $\pm \omega_\mp (p)$, respectively, with $\omega_\pm (p) > 0$ and the residues $Z_\pm (p) = (\omega_\pm (p) - p^2)/(2m^2)$. Since the excitation spectra $\omega_\pm (p)$ becomes $m_T$ at $p = 0$, $m_T$ is called the thermal mass. The spectral functions Eq. (4) have the continuum $\rho_5^0 (\omega, p)$ in the space-like region, which originates from the Landau damping of the quark $\Sigma$. Since the quark propagator Eq. (1) is a function of $\omega$, $p$ and $m_T$, in the following we write $S^{HTL}_\pm (\omega, p) \equiv S^{HTL}_\pm (\omega, p)$ to show $m_T$ dependence explicitly.

While the HTL and one-loop approximations used in the above discussion may be invalid near $T_c$, numerical results from lattice QCD obtained large masses near $T_c$ \[9\]. It is therefore plausible that the quarks have large thermal masses in the non-perturbative region near $T_c$. The non-perturbative gauge interactions also produce an attractive interaction between quarks and antiquarks, and generate the chiral symmetry breaking. In order to model this possibility, we adopt Eq. (1) for the quark propagator near $T_c$ and introduce a four-Fermi quark-antiquark interaction. We then arrive at the following model;
\[ \mathcal{L} = \bar{\psi} [S^{HTL}_{m_T} (\omega, k)]^{-1} \psi + G_S [\bar{\psi} \psi]^2 + (\bar{\psi} i \gamma_5 \psi)^2, \]
(5)
where $\psi$ is the quark field in the chiral limit, and $G_S$ is the scalar coupling. The thermal mass $m_T$ is introduced as a parameter to be varied by hand. We introduce the three-dimensional cutoff $\Lambda$ to eliminate the ultraviolet divergence. This model with $m_T = 0$ is equivalent to the Nambu–Jona-Lasinio model \[11\].

### III. CHIRAL PHASE TRANSITION

Let us first explore the chiral phase transition at finite temperature in the model of Eq. (5). In the mean-field approximation, the chiral condensate $\sigma \equiv -2 G_S \langle \bar{\psi} \psi \rangle$ for $m_T/\Lambda = 0.1, 0.2$ and $0.3$ with fixed $G_S A^2 = 13.01$. The chiral transition is of second order irrespective of $m_T$. For larger $m_T$, $\sigma$ becomes smaller.

![Figure 1: Order parameters for the chiral phase transition $\sigma \equiv -2 G_S \langle \bar{\psi} \psi \rangle$ for $m_T/\Lambda = 0, 0.1, 0.2$ and $0.3$ with fixed $G_S A^2 = 13.01$. The chiral transition is of second order irrespective of $m_T$. For larger $m_T$, $\sigma$ becomes smaller.](image.png)

\[ V(\sigma) = \frac{\sigma^2}{4 G_S} - T \sum_n \left[ \int \frac{d^3 k}{(2\pi)^3} \text{tr} \text{Im} \ln\left( \frac{\mathcal{S}(i\omega_n, \kappa; m_T, \sigma)}{\sigma} \right) \right]^{-1} \]
(6)
where $\omega_n = (2n + 1)\pi T$ is the Matsubara frequency for fermions and
\[ \tilde{S}(i\omega_n, \kappa; m_T, \sigma) = \left( \frac{S^{HTL}_{m_T}(i\omega_n, \kappa)}{\sigma} - 1 \right)^{-1}, \]
(7)
with $\tilde{S}_{m_T}$ being the Matsubara propagator of the quark in the HTL approximation; $S^{HTL}_{m_T}(i\omega_n, \kappa) = (S^{HTL}(\omega, k)|_{\omega \rightarrow i\omega_n})$. The retarded propagator $S^R$ in the right hand side of Eq. (6) is defined by the analytic continuation $S^R(\omega) = S(i\omega_n)|_{\omega \rightarrow \omega + i\eta}$. The global minimum of $\text{Eq. (6)}$ satisfies the following stationary condition (gap equation);
\[ -\frac{1}{2 G_S} = \int \frac{d^3 k}{(2\pi)^3} \int \frac{d\omega}{\pi} \text{tanh} \frac{\omega}{2 T} \text{tr} \text{Im} S^R(\omega, \kappa; m_T, \sigma). \]
(8)
In general, Eq. (8) has several solutions corresponding to local extrema of Eq. (6). One must, thus, refer to Eq. (6) to find the global minimum. In our case, however, we have checked that Eq. (8) always has only one solution and it is certainly the minimum of Eq. (6). Therefore, we determine the order parameter only with Eq. (8).

In Fig. 1 we show the chiral condensate $\sigma$ as a function of $T$ with several values of $m_T$; although $m_T$ may change with a variation of $T$, we fix it to study the effect of $m_T$. The scalar coupling $G_S A^2 = 13.01$ is fixed so that $T_c/\Lambda = 0.3$ at $m_T = 0$. From the Figure, we see...
that the chiral phase transition is of second order for any \( m_T \). This result is natural in the sense of the symmetry argument of the phase transitions, since the symmetry of the system does not change with the incorporation of \( m_T \). It is notable, however, that the order parameter \( \sigma \) no longer has the meaning of the mass gap of the quark spectra with finite \( m_T \).

One sees that, as \( m_T \) becomes larger, the chiral condensate decreases and similarly the critical temperature does. This behavior is physically interpreted as the effect of the excitation energy \( m_T \) in the quark spectra: Because the chiral symmetry breaking is induced by the condensate between the quark and anti-quark, the increase of the excitation energy of these particles suppresses the condensate. Here, we remark that if we incorporate the current quark mass \( m_c \) in our model, the value of the chiral condensate becomes larger \([11]\), although \( m_c \) explicitly breaks the chiral symmetry and works as an external field; in terms of the effective potential \( V(\sigma) \), \( m_c \) gives rise to a linear term proportional to \( m_c \sigma \).

**IV. MESONIC EXCITATIONS**

Since the chiral transition is of second order in our model, there should appear soft modes associated with the transition \([3]\). In this Section, we show that this is indeed the case. Using these soft modes, we then discuss the transition \([3]\). In this Section, we show that this is indeed the case. Using these soft modes, we then discuss the effect of the thermal mass on the mesonic excitations above \( T_c \). In the following, we limit our attention in the chirally symmetric phase above \( T_c \), where the excitation spectra in the scalar and pseudoscalar channels are degenerate.

In the random phase approximation, the retarded meson propagators in these channels \( D^R_\sigma(\omega, \mathbf{p}) \) is given by

\[
D^R_\sigma(\omega, \mathbf{p}) = \frac{-1}{G^{-1}_S + \Pi^R_\sigma(\omega, \mathbf{p})},
\]

where the polarization function \( \Pi^R_\sigma(\omega, \mathbf{p}) \) is determined by the analytic continuation \( \Pi^R_\sigma(\omega, \mathbf{p}) = \tilde{\Pi}_\sigma(i\nu_n, \mathbf{p})\big|_{\nu_n = -\omega + i\eta} \) with

\[
\tilde{\Pi}_\sigma(i\nu_n, \mathbf{p}) = 2T \sum_m \int \frac{d^3k}{(2\pi)^3} \text{tr}[S^\text{HTL}_{m_T}(i\omega_m, \mathbf{k}) \times S^\text{HTL}_{m_T}(i\nu_n + i\omega_m, \mathbf{p} + \mathbf{k})]
\]

\[
= 2 \sum_{s, t = \pm} \int \frac{d^3k}{(2\pi)^3} \text{tr}[P_s(\mathbf{k}) \gamma^0 P_t(\mathbf{p} + \mathbf{k}) \gamma^0]
\]

\[
\times \int d\omega_1 d\omega_2 \omega^{-1}(\omega_1 - \omega_2) \int \frac{d\omega_n}{(2\pi)^2} \rho^\text{HTL}_s(\omega_1, \mathbf{k}) \rho^\text{HTL}_t(\omega_2, \mathbf{p} + \mathbf{k})
\]

and the Matsubara frequency for bosons \( \nu_n = 2n\pi T \). In Fig. 2 we show the diagrammatic representation of our approximation.

**FIG. 2:** Diagrammatic representation for the meson propagator \( D^R_\sigma(\omega, \mathbf{p}) \) and the polarization function \( \Pi^R_\sigma(\omega, \mathbf{p}) \). The thick-solid line denotes the quark propagator in the HTL approximation Eq. (11), while the thin-solid line represents the free quark propagator.

The imaginary part of \( \Pi^R_\sigma(\omega, \mathbf{p}) \) is proportional to the difference between the decay and creation rates of the mesonic excitations. For \( \mathbf{p} = 0 \), it is calculated to be

\[
\text{Im} \Pi^R_\sigma(\omega, 0) = -\frac{2}{\pi}(I_{PP}(\omega) + I_{PC}(\omega) + I_{CC}(\omega)),
\]

with

\[
I_{PP}(\omega) = -\frac{p^2 Z_+^2}{2|\omega_+|^2} [1 - f_+]|_{\omega = -2\omega_+} + \frac{p^2 Z_-^2}{2|\omega_-|^2} [1 - f_-]|_{\omega = 2\omega_-} - \frac{2p^2 Z_+ Z_-}{|\omega_+ - \omega_-|^2} [f_+ - f_-]|_{\omega = -\omega_+ + \omega_-} - (\omega \leftrightarrow -\omega),
\]

\[
I_{PC}(\omega) = 2\pi \int \frac{d^3k}{(2\pi)^3} Z_+ [f(\omega_+) - f(\omega + \omega_+)] \rho^L_+(\omega + \omega_+, \mathbf{k}) + 2\pi \int \frac{d^3k}{(2\pi)^3} Z_- [f(\omega_- - \omega) - f(\omega_-)] \rho^L_+(\omega_- - \omega, \mathbf{k}) - (\omega \leftrightarrow -\omega),
\]

\[
I_{CC}(\omega) = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \int dE [f(E) - f(\omega + E)] \rho^L_+(E, \mathbf{k}) \rho^L_+(\omega + E, \mathbf{k}) - (\omega \leftrightarrow -\omega),
\]

\[
\text{with } f(E) = [\exp(E/T) + 1]^{-1}, f_\pm = f(\omega_\pm), \omega_\pm = d\omega_\pm(p)/dp, \text{ and each } p \text{ in Eq. (12) denotes the momentum of normal quasiquarks or plasminos which satisfies } \omega = -2\omega_+, \omega = 2\omega_- \text{ and } \omega = \omega_- - \omega_+, \text{ respectively.}
In Fig. 5, we show $\rho_{12, 13, 14}$. While momenta, and interpreted as the van Hove singularities of the energy spectra $2\omega, \omega_1, \omega_2$, respectively: Notice that the Landau damping is the scattering process into the normal quasi-quark and quasi-antiquark (the plasmino and anti-plasmino), and the third term is the Landau damping of the quark. We show the diagrammatical interpretation for the decay processes included in $I_{PC}(\omega)$ and $I_{CC}(\omega)$ in Fig. 3(a) and (b), respectively: Notice that the Landau damping is the scattering processes accompanied with a thermally excited particle.

In Fig. 4 we show each component of $\text{Im} \Pi^R(\omega, p = 0) = I_{PP}(\omega) + I_{PC}(\omega) + I_{CC}(\omega)$ at $T/\Lambda = m_T/\Lambda = 0.3$. $I_{PP}(\omega)$, $I_{PC}(\omega)$ and $I_{CC}(\omega)$ include the decay processes into two quasiparticle poles, a quasiparticle pole and continuum, and two continuums, respectively.

The first (second) term in Eq. (12) includes the decay process into the normal quasi-quark and quasi-antiquark (the plasmino and anti-plasmino), and the third term is the Landau damping of the mesons between the normal quasiparticle and the plasmino. The decay processes corresponding to $I_{PC}(\omega)$ and $I_{CC}(\omega)$ include the continuum $\rho^b(\omega, p)$, i.e. the Landau damping of the quark. We show the diagrammatical interpretation of decay processes included in $I_{PC}(\omega)$ and $I_{CC}(\omega)$ in Fig. 3(a) and (b), respectively: Notice that the Landau damping is the scattering processes accompanied with a thermally excited particle.

In Fig. 3 we show each component of $\text{Im} \Pi^R$ for a typical parameter set $T/\Lambda = m_T/\Lambda = 0.3$. From the Figure, one sees that $I_{PP}(\omega)$ has divergences at two energies $\omega_1$ and $\omega_2$, with $\omega_1/m_T \simeq 0.4$ and $\omega_2/m_T \simeq 1.8$. These singularities are due to the divergence of the density of states of the energy spectra $2\omega, \omega_1 - \omega_2$ at finite momenta, and interpreted as the van Hove singularities $12, 13, 14$. While $I_{PP}(\omega)$ vanishes at $\omega_1 < \omega < \omega_2$ due to the kinematics, $I_{PC}(\omega)$ and $I_{CC}(\omega)$ take finite values in the whole range of energy. In particular, $I_{PC}(\omega)$ has a large contribution at $\omega \lesssim 3m_T$.

The excitation spectrum of the mesons is given by the spectral function

$$\rho_\sigma(\omega, p) = -2\text{Im} D^R(\omega, p).$$

In Fig. 5, we show $\rho_\sigma(\omega, p)$ for $m_T/\Lambda = 0$ and $0.3$ at $p = 0$ in the upper and lower panels, respectively; the temperature dependence of $m_T$ is not considered as before. In these Figures, the scalar coupling $G_S$ is determined so that $T_c/\Lambda = 0.3$ for each $m_T$. In the upper panel, we see that there appears a peak in $\rho_\sigma(\omega, p)$, and the peak becomes sharper and moves toward the origin as $T$ approaches $T_c$ from high temperature. This is the soft mode of the chiral phase transition $\rho_\sigma(\omega, p)$ behaves in a more complicated way. First, $\rho_\sigma(\omega, p)$ becomes zero at two energies $\omega_1$ and $\omega_2$ corresponding to van Hove singularities in $I_{PP}(\omega)$. Second, although there appears a sharp peak in $\rho_\sigma(\omega, p)$ as a soft mode of the chiral transition, the heights of the peaks in $\rho_\sigma(\omega, p)$ are depressed compared from the upper panel with $m_T = 0$. This result contradicts the naive expectation that the mesonic excitation at $\omega < 2m_T$ becomes more tightly bound and thereby have a small width in this range.

In order to understand the origin of the large decay width of the mesonic excitations with $m_T > 0$, we show $\text{Im} \Pi_\sigma(\omega, p = 0)$ for $m_T/\Lambda = 0$ and $0.3$ in Fig. 6. One sees that the imaginary part with $m_T/\Lambda = 0.3$ takes larger values at $\omega < 2m_T$ than that with $m_T = 0$, indeed. As discussed previously, the decay processes corresponding to $I_{PC}(\omega)$ and the van Hove singularities in $I_{PP}(\omega)$ has a large contribution in this range.

V. SUMMARY AND DISCUSSIONS

In this work, we explored the effect of the thermal mass of the quark on the chiral transition and the mesonic
excitations at finite temperature using a simple model. It is shown that the order of the chiral transition does not change by incorporating a thermal mass. We also found that the thermal mass reduces the chiral condensate. By studying the spectral function of the mesonic excitations in the scalar and pseudoscalar channels, we found that the peak of excitations in the spectral function is strongly suppressed below $2m_T$. We showed that the continuum in the quark spectrum, and the van Hove singularities, play a significant role to enhance the decay rate of the mesonic excitations below $2m_T$.

The decay processes included in $I_{PC}(\omega)$ and $I_{CC}(\omega)$ contain the effect of the Landau damping of the quarks, i.e. the scattering process of the quasiparticles by a thermally excited particle as shown in Fig. 3. These processes are specific to nonzero $T$ and it is natural that such processes are obtained in our calculation with $S_{HTL}^{\omega}(\omega, p)$. Since the contribution of these terms, as well as the van Hove singularities, may not depend on the channels of the excitation qualitatively, our calculation strongly suggests that any mesonic excitations in the light quark sector are easily destroyed by the thermal effect in the QGP phase. It is notable that there can nevertheless appear sharp peaks in the mesonic spectra as the soft modes of the chiral transition near $T_c$ [3].

Although we employed the fixed $m_T$ to show the numerical results, $m_T \sim gT$ may vary as a function of $T$. The energies $\omega_1$ and $\omega_2$, i.e. zeros of $\rho_\sigma(\omega, p)$, thus, change with respect to $T$. Moreover, the width of the collective excitation of the quark can depress the van Hove singularities. Therefore, the depression of the meson spectra is not, unfortunately, an experimental signature.

While we employed the quark propagator Eq. (1) in this work, the quark spectrum near $T_c$ may have the large decay width and a complicated dispersion [10, 13]. It is interesting to explore mesonic excitations with such quark spectra instead of $S_{HTL}^{\omega}$. This is beyond the scope of this work.

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