D-Brane Solutions from New Isometries of pp-Waves

Mohsen Alishahiha\textsuperscript{a} and Alok Kumar\textsuperscript{b}

\textsuperscript{a} Institute for Studies in Theoretical Physics and Mathematics (IPM)
P.O. Box 19395-5531, Tehran, Iran

\textsuperscript{b} Institute of Physics
Bhubaneswar 751 005, India

Abstract

We use recently proposed translations isometries of pp-waves to construct $D_4$ and $D_3$ brane solutions, using $T$-duality transformations, in exactly solvable pp-wave background originating from $\text{AdS}_3 \times S^3$ geometry. A unique property of the new brane solutions is the breaking of $SO(10-p-1)$ symmetry in the transverse direction of the branes due to the presence of constant $NS-NS$ and $R-R$ background fluxes. We verify that the our ‘localized’ solutions satisfy the field equations and explicitly present the corresponding Killing spinors. We also show the connection of our results to certain M5-branes in pp-wave geometry.

\textsuperscript{1}alishah@theory.ipm.ac.ir
\textsuperscript{2}kumar@iopb.res.in
1 Introduction

A good understanding of $D$-brane solutions is of great importance in understanding the strong coupling aspects of string theory, as well as that of gauge theories through a string theory/gauge theory duality. In this connection, pp-wave backgrounds [1, 2, 3, 4, 5] have been of much interest, since they provide examples of duality [6] within the context of exactly solvable string theories [7, 8, 9, 10]. This intriguing proposal has been studied by many authors [11]-[46].

In view of these developments, we investigate the D-brane solutions [47, 48, 49] (see also [50, 51] in pp-wave background further, using the new isometries of pp-waves proposed in [52], and obtain new D-brane solutions. In the case of pp-waves originating from the $AdS_3 \times S^3$ geometry in a Penrose limit [48], we use the isometries of pp-waves proposed in [52], to construct $D3$ and $D4$ brane solutions, starting from the $D5$ solution of [48]. A unique property of these solutions is the breaking of $SO(10 - p - 1)$ symmetry along the transverse directions of the branes due to the presence of constant $NS$ $- NS$ and $R$ $- R$ fluxes. Due to this, the construction of the localized solution from the delocalized one, obtained from the $T$-duality transformations on the $D5$-brane solution is not immediately obvious. We however explicitly verify that our localized solutions indeed satisfy the field equations. We also show the stability of these solutions by explicitly finding the Killing spinors.

The plan of this letter is as following: In section-2 starting from D5-brane solution in the PP-wave background, we will obtain the supergravity solution of other D-branes by making use of T-duality. In section-3 we shall study the supersymmetric properties of the solutions. Section-4 is devoted to discussion and comments.

2 Supergravity solution of branes in PP-wave background

Consider a system of $N$ D5-branes solution in the PP-wave of type IIB string theory [48]

$$
\begin{align*}
    ds^2 &= f^{-\frac{1}{2}} \left( 2dx^+ dx^- - \mu^2 x_i^2 (dx^+)^2 + dx_i^2 \right) + f^{\frac{1}{2}} \left( dr^2 + r^2 d\Omega_3^2 \right), \\
    e^{2\phi} &= f^{-1}, \quad F_{12} = F_{34} = 2\mu, \\
    F_{mnl} &= \epsilon_{mnp} \partial_p f, \quad f = 1 + \frac{Ng_s l_s^2}{r^2},
\end{align*}
$$

(1)

where $F$’s are RR 3-forms and $i = 1, 2, 3, 4$. This solution has $SO(2) \times SO(2) \times SO(4)$ symmetry.

In order to find the Dp-branes solution for $p < 5$, we apply T-duality on the D5-brane
solution (1). To do this, it is useful to make the following change of coordinates:

\begin{align*}
x^+ &= \hat{x}^+, \\
x^- &= \hat{x}^- - \mu \hat{x}_1 \hat{x}_2, \\
x_I &= \hat{x}_I, \quad \text{for } I = 3, 4 \\
x_1 &= \hat{x}_1 \cos(\mu \hat{x}^+) - \hat{x}_2 \sin(\mu \hat{x}^+), \\
x_2 &= \hat{x}_1 \sin(\mu \hat{x}^+) + \hat{x}_2 \cos(\mu \hat{x}^+).
\end{align*}

Then the D5-brane solution (1) takes the form:

\begin{align*}
ds^2 &= f^{-\frac{1}{2}} \left(2d\hat{x}^+ d\hat{x}^- - \mu^2 \hat{x}_1^2 (d\hat{x}^+)^2 - 4\mu \hat{x}_2 d\hat{x}_1 d\hat{x}^+ + d\hat{x}_1^2 + \hat{x}_1^2 \right) + f^\frac{1}{2} \left(dr^2 + r^2 d\Omega_3^2 \right), \\
e^{2\phi} &= f^{-1}, \\
F_{+12} &= F_{+34} = 2\mu, \\
F_{mnlp} &= \epsilon_{lmnp} \partial_p f, \quad f = 1 + \frac{Ng_s l_s^2}{r^2}. \quad (3)
\end{align*}

In this coordinate system, \( \frac{\partial}{\partial \hat{x}_1} \) is a manifest isometry. This is the direction one then makes compact and performs T-duality along it. By making use of the T-duality rules (5), the T-dual background of the above solution reads:

\begin{align*}
ds^2 &= f^{-\frac{1}{2}} \left(2d\hat{x}^+ d\hat{x}^- - \mu^2 \hat{x}_1^2 (d\hat{x}^+)^2 + \hat{x}_2^2 d\hat{x}_1 d\hat{x}^+ + d\hat{x}_1^2 + \hat{x}_1^2 \right) + f^\frac{1}{2} \left(dr^2 + r^2 d\Omega_3^2 \right), \\
e^{2\phi} &= f^{-\frac{1}{2}}, \\
F_{+2} &= F_{+34} = 2\mu, \\
F_{+1} &= 2\mu \hat{x}_2, \\
F_{1mnlp} &= \epsilon_{1lmnp} \partial_p f, \quad f = 1 + \frac{Ng_s l_s^2}{r^2}. \quad (4)
\end{align*}

This should corresponds to the smeared D4-brane in the type IIA string theory on the PP-wave. The localized D4-brane solution can be easily found as following:

\begin{align*}
ds^2 &= f^{-\frac{1}{2}} \left(2d\hat{x}^+ d\hat{x}^- - \mu^2 \hat{x}_1^2 (d\hat{x}^+)^2 + \hat{x}_2^2 d\hat{x}_1 d\hat{x}^+ + d\hat{x}_1^2 + \hat{x}_1^2 \right) + f^\frac{1}{2} \left(dr^2 + r^2 d\Omega_4^2 \right), \\
f &= 1 + \frac{c_4 Ng_s l_s^3}{r^3}, \\
F_{mnlp} &= \epsilon_{mnlpq} \partial_q f, \quad (5)
\end{align*}

other fields remain the same as the one in smeared solution of (4), with \( f \) given in (5). We have explicitly verified that the solution presented in eqn. (5) satisfies the type IIA field equations. In this context, we point out that \( \mu \) dependence appears nontrivially only in the graviton field equations with \((++)\) components. For example, in the expression for the Ricci tensor \( R_{++} \), one obtains a constant contribution of \( 6\mu^2 \), which is canceled by equal contributions coming from the background \( R - R \) 2-form and 4-form fluxes, as well as \( NS - NS \) background 3-form flux of the above pp-wave solution. Remaining nonconstant terms in \( R_{++} \) are canceled by terms coming from the dilaton contribution, as well as the 4-form field strength coupling to the D4-brane, since \( f \) satisfies the Greens function equation in the five dimensional transverse space. The above solution has \( SO(2) \times SO(5) \)
symmetry for the metric, which is then broken by the constant field strengths associated with various p-form fields.

We can now proceed to generate a D3-brane solution in a pp-wave background. One then again makes the change of coordinates similar to eqn. (2), but in the 3 and 4 directions, i.e.

\[ \hat{x}^+ = y^+, \quad \hat{x}^- = y^- - \mu y_3 y_4, \quad \hat{x}_2 = y_2, \]
\[ \hat{x}_3 = y_3 \cos(\mu y^+) - y_2 \sin(\mu y^+), \quad \hat{x}_4 = y_3 \sin(\mu y^+) + y_4 \cos(\mu y^+). \]  

(6)

In this system of coordinates, the metric in (5) reads:

\[ ds^2 = f^{-\frac{1}{2}} \left( 2dy^+ dy^- - 4\mu^2 y_2^2(dy^+)^2 - 4\mu y_4 dy_3 dy^+ + dy_2^2 + dy_1^2 \right) + f^{\frac{1}{2}} \left( dr^2 + r^2 d\Omega_4^2 \right). \]  

(7)

In this coordinate system, \( \frac{\partial}{\partial y^3} \) is a manifest isometry. One can now compactify this direction and perform T-duality. Doing so one finds:

\[ ds^2 = f^{-\frac{1}{2}} \left( 2dy^+ dy^- - 4\mu^2 [y_2^2 + y_4^2](dy^+)^2 + dy_2^2 + dy_1^2 \right) + f^{\frac{1}{2}} \left( dr^2 + r^2 d\Omega_4^2 \right), \]
\[ F_{+32} = F_{+14} = 2 \mu, \quad B_{+1} = 2 \mu y_2, \quad B_{+3} = 2 \mu y_4, \]
\[ F_{3mntp} = \epsilon_{3mntpq} \partial_q f, \quad f = 1 + \frac{c_4 Ng_s l_s^3}{r^3}. \]  

(8)

with a constant dilaton. This is the smeared D3-brane solution in the PP-wave background. The localized solution is given by

\[ ds^2 = f^{-\frac{1}{2}} \left( 2dy^+ dy^- - 4\mu^2 [y_2^2 + y_4^2](dy^+)^2 + dy_2^2 + dy_1^2 \right) + f^{\frac{1}{2}} \left( dr^2 + r^2 d\Omega_4^2 \right), \]
\[ F_{+32} = F_{+14} = 2 \mu, \quad B_{+1} = 2 \mu y_2, \quad B_{+3} = 2 \mu y_4, \]
\[ F_{mntpq} = \epsilon_{mntpq} \partial_s f, \quad f = 1 + \frac{c_3 Ng_s l_s^4}{r^4}. \]  

(9)

Once again, we have explicitly verified that the field equations are satisfied by the solution in eqn. (3). For example, in this case, the constant \( \mu \) dependent part in \( R_{++} \) is equal to \( 8\mu^2 \), which is canceled by equal contribution from the NS–NS and \( R–R \) field strengths. The non-constant part is canceled by the 5-form field strength of the D3-brane with \( f \) satisfying the Greens function equation in six dimensional transverse space: \( x^1, x^3, x^5, ..., x^8 \).

As another example, we can also find the M5-brane solution in the PP-wave background of M-theory. To do this one can liftup the D4-brane solution (5) to M-theory. Using the relation between 11 and 10 dimensional metric as

\[ ds^2_{11} = e^{-\frac{2\phi}{3}} ds^2_{10} + e^{\frac{4\phi}{3}} (dx_{11} + A_\mu dx^\mu)^2, \]  

(10)
where $A_\mu$ is R-R one form, one can find the M5-brane solution from D4-brane solution (5). The M5-brane metric is given by

$$
\begin{align*}
\text{ds}^2 &= f^{-\frac{1}{3}} \left( 2dx^+dx^- - \mu^2 (x_1^2 + 4x_2^2)(dx^+)^2 + dx_2^2 + dx_i^2 + (dx_{11} - 2\mu x_2 dx^+) \right) \\
&+ f^{\frac{2}{3}} \left( dr^2 + r^2 d\Omega_4^2 \right), \\
&= f^{-\frac{1}{3}} \left( 2dx^+dx^- - \mu^2 x_1^2 (dx^+)^2 - 4\mu x_2 dx_{11} dx^+ + dx_2^2 + dx_i^2 + dx_{11}^2 \right) \\
&+ f^{\frac{2}{3}} \left( dr^2 + r^2 d\Omega_4^2 \right),
\end{align*}
$$

where $f = 1 + \frac{NL_5}{l_p}$, with $l_p$ being the 11-dimensional Plank length. Using a change of coordinate as inverse to the once in (2), one finds the following result for M5-brane solution:

$$
\begin{align*}
\text{ds}^2 &= f^{-\frac{1}{3}} \left( 2dx^+dx^- - \mu^2 (x_1^2 + x_2^2 + x_{11}^2)(dx^+)^2 + dx_2^2 + dx_i^2 + dx_{11}^2 \right) \\
&+ f^{\frac{2}{3}} \left( dr^2 + r^2 d\Omega_4^2 \right), \\
C^{(4)} &= 2\mu(dx^+ dx_1 dx_3 dx_4 + dx^+ dx_1 dx_2 dx_{11}) + F_4,
\end{align*}
$$

where $x_i$ is the a transverse direction to the brane, and $F_4$ comes from the R-R 4-form in (5). This is the same solution as the one obtained in [54]. One can also proceed to find the NS5-brane solution. To do this we should first delocalize the M5-brane solution in a direction, say $x_l$ and then compactify the solution in this direction. Doing so one finds following type IIA NS5-brane solution in the PP-wave background:

$$
\begin{align*}
\text{ds}^2 &= 2dx^+dx^- - \mu^2 x_1^2 (dx^+)^2 + dx_2^2 + f (dr^2 + r^2 d\Omega_3^2), \\
e^{2\phi} &= f, \quad H_{12} = H_{34} = 2\mu, \quad H_{mnp} = \epsilon_{mnpq} \partial_q f,
\end{align*}
$$

where $i = 1, 2, 3, 4$ and $f = 1 + \frac{NL_5}{l_p}$ and $H$’s are NS-NS 3-forms. As expected, this NS5-brane solution matches with the NS5-brane of type IIB in pp-wave background [48]. These results have interesting implications for the supersymmetry of our solutions and we discuss them in the next section.

### 3 Supersymmetric properties of the brane solution in PP-wave background

We now first discuss the supersymmetry properties of the above solutions on general grounds and then go on to solve the Killing spinor equations. It was already shown in
the previous section that our \( D4 \)-brane solution in eqn.(3) can be lifted to an \( M5 \)-brane solution of \[54\]. Also, the full \( M5 \)-brane solution preserves \( 3/8 \) supersymmetry, with brane preserving \( 1/2 \) of the background pp-wave supersymmetry. We therefore expect that our \( D4 \)-brane also preserves same amount of supersymmetries. Moreover, it was also shown in the previous section that the \( M5 \)-brane solution can be reduced to an \( NS5 \)-brane of type IIA theory. Now, since this solution is also identical to the \( NS5 \)-brane of type IIB, we expect that \( NS5 \)-brane also preserves the same supersymmetry in IIB string theory. In fact, the identical pp-wave backgrounds for the above IIA \( NS5 \) brane and the one in IIB theory \[48\] are related by a simple \( T \)-duality along one of the transverse directions, as can be seen by setting the source term: \( f = 1 \) in the two solutions.

We now first analyze the Killing spinor equations of the \( D5 \)-brane solution given in eqn.(1) and then go over to other brane solutions. To begin, we write the relevant supersymmetry transformations for dilatino and gravitino fields in type IIB supergravity in ten dimensions \[55\] in string frame metric:

\[
\delta \lambda_\pm = \frac{1}{2} \left( \Gamma^\mu \partial_\mu \phi + \frac{1}{12} \Gamma^{\mu \nu \rho} H_{\mu \nu \rho} \right) \epsilon_\pm + \frac{1}{2} e^\phi \left( \pm \Gamma^M F_M^{(1)} + \frac{1}{12} \Gamma^{\mu \nu \rho} F_{\mu \nu \rho}^{(3)} \right) \epsilon_\mp, \tag{14}
\]

\[
\delta \Psi_\mu^\pm = \left[ \partial_\mu + \frac{1}{4} (\omega_{\mu \hat{a} \hat{b}} + \frac{1}{2} H_{\mu \hat{a} \hat{b}}) \Gamma^{\hat{a} \hat{b}} \right] \epsilon_\pm + \frac{1}{8} e^\phi \left[ \mp \Gamma^\mu F_\mu^{(1)} - \frac{1}{3!} \Gamma^{\mu \nu \rho} F_{\mu \nu \rho}^{(3)} + \frac{1}{2 \cdot 5!} \Gamma^{\mu \nu \rho \sigma \alpha \beta} F_{\mu \nu \rho \sigma \alpha \beta}^{(5)} \right] \Gamma_\mu \epsilon_\mp. \tag{15}
\]

Using indices: \((+, -, i, a)\) to denote the ten dimensional coordinates, with \( i = 1, \ldots, 4 \) and \( a = 5, \ldots, 8 \), we get the following condition from the dilatino equation for the \( D5 \) solution given in eqn.(1) (hats denoting the corresponding tangent space coordinates):

\[
\Gamma^\hat{a} \epsilon_\pm + \frac{1}{3!} \epsilon_{\hat{a} \hat{b} \hat{c} \hat{d}} \Gamma^{\hat{b} \hat{c} \hat{d}} \epsilon_\mp = 0, \tag{16}
\]

\[
\Gamma^+ \left( \Gamma^{i2} + \Gamma^{34} \right) \epsilon_\pm = 0. \tag{17}
\]

Equation (16) is the standard supersymmetry condition for a \( D5 \)-brane even in the flat space:

\[
\epsilon_\pm = \Gamma^{5678} \epsilon_\mp, \tag{18}
\]

and reduces the supersymmetry to \( 1/2 \) of the maximal one. Both the equations (17) and (18) are in fact necessary, in order that dilatino variation vanishes. These conditions are also seen to emerge from the gravitino variations that we analyze below.

Using the \( D5 \)-brane supersymmetry condition (16), the gravitino variation equations reduce to:

\[
\delta \Psi_\pm^\mp \equiv \partial_\pm \epsilon_\pm - \frac{\mu^2 x_i}{2} \Gamma^{\hat{i}} \epsilon_\pm - \frac{\mu}{4} (\Gamma^{\hat{i}2} + \Gamma^{34}) \epsilon_\mp = 0, \tag{19}
\]
\[ \delta \Psi^\pm \equiv \partial_\pm \epsilon^\pm = 0, \quad (20) \]
\[ \delta \Psi^\pm_i \equiv \partial_i \epsilon^\pm - \frac{\mu}{4} \left( \Gamma^\pm_{12} + \Gamma^\pm_{34} \right) \delta_{ai} \Gamma^i \epsilon^\pm = 0, \quad (21) \]
\[ \delta \Psi^\pm_a \equiv \partial_a \epsilon^\pm - \frac{\mu}{4} \left( \Gamma^\pm_{a} \Gamma^\pm_{12} + \Gamma^\pm_{a} \Gamma^\pm_{34} \right) \delta_{aa} \Gamma^a \epsilon^\pm = 0. \quad (22) \]

We now use eqn.(17) to simplify eqns. (19) - (22) further. First, assuming \( \Gamma^+ \epsilon^\pm \neq 0 \) gives:
\[ \left( 1 + \Gamma^\pm \Gamma^\pm \right) \epsilon^\pm = 0. \quad (23) \]

In writing this equation, as well as the ones give below, we have made use of the identities: \( \Gamma^+ \Gamma^+ = -\Gamma^+, \Gamma^+ \Gamma^- = -\Gamma^- \) etc.. Now, by defining 32-component real spinors:
\[ \eta \equiv \begin{pmatrix} \epsilon^+ \\ \epsilon^- \end{pmatrix}, \quad (24) \]
and using eqn.(23), eight spinor equations in (19)-(22), following from \( \delta \Psi^\pm \mu = 0 \) can be written as:
\[ \partial_+ \eta - \frac{\mu^2 \Gamma^\pm_i \Gamma^\pm_i \eta}{2} - \frac{\mu}{2} \Gamma^\pm_{12} \Gamma^\pm \otimes \sigma_1 \eta = 0, \quad (25) \]
\[ \partial_- \eta = 0, \quad (26) \]
\[ \partial_i \eta - \frac{\mu}{2} \left( \Gamma^\pm_{a} \Gamma^\pm_{12} \delta_{ai} \Gamma^i \right) \otimes \sigma_1 \eta = 0, \quad (27) \]
\[ \partial_a \eta = 0, \quad (28) \]
where \( \sigma_1 \) is the Pauli matrix, mixing the two components \( \epsilon^+, \epsilon^- \) of \( \eta \). Moreover, using eqns. (16), (18), (23) and the fact that both \( \epsilon^\pm \) are spinors of same space-time helicity, one has: \( \eta = \sigma_1 \eta \), or both components of \( \eta \) are to be equal. However, one should take into account that such a condition is not independent with respect to what has been written earlier in eqns.(16) and (23).

The solution of the above Killing spinor equation is then found in a similar way as in \[ 8, 56 \] and is given as:
\[ \eta = \left( 1 + \frac{\mu^2 \Gamma^\pm_i \delta_{ai} \Gamma^i}{2} \right) \xi(u), \quad (29) \]
\[ \xi(u) = \exp \left( \frac{\mu}{2} \Gamma^\pm_{12} \Gamma^- u \right) \xi_0, \quad (30) \]
with \( \xi_0 \) being a constant spinor. The Killing spinors found in eqns. (29), (30) are constrained by the conditions (23) and (14). The fact that the solutions for both these equations, (18), (23), are also given by (29) and (30), is due to the fact that matrices \( \Gamma^{5678} \), as well as \( \Gamma^\pm_{1234} \), both commute with the product of Gamma matrices appearing in eqns. (29) and (30). Before doing the final count of the number of supersymmetries that are preserved by our background fields, we now examine the possibility of having more
solutions of the Killing spinor equations. Later on, we also discuss the connection of our
Killing spinors with the ‘normal’ and ‘supernumerary’ Killing spinors in [57, 54].

Additional solutions of both the dilatino and the gravitino variations are possible by
using projections:

\[ \Gamma^\hat{+} \epsilon_\pm = 0. \]  
(31)

The condition (17) is now trivially satisfied and eqns. (19)-(22) are replaced by:

\[ \partial_+ \epsilon_\pm = \frac{\mu}{2} (\Gamma^{12} + \Gamma^{34}) \epsilon_\mp = 0, \]  
\[ \partial_- \epsilon_\pm = 0, \quad \partial_\ell \epsilon_\pm = 0, \quad \partial_a \epsilon_\pm = 0. \]  
(32)

(33)

However, before solving these equations, we notice by using

\[ \Gamma^\hat{+} = \frac{1}{2} \Gamma^\hat{+} (1 - \Gamma^\hat{-}), \]  
(34)

that a part of the solutions of eqn. (31) already coincide with the solutions of the type in
eqns. (29), (30): further constrained by the projection condition: (23). As a result, only
those spinors in (23) are relevant which satisfy:

\[ \Gamma^{1234} \epsilon_\pm = \epsilon_\pm. \]  
(35)

In any case, the other projection:

\[ \Gamma^{1234} \epsilon_\pm = -\epsilon_\pm, \]  
(36)

is inconsistent with eqn. (23), as one will then have \( \Gamma^\hat{+} \epsilon_\pm = 0 \), using eqn. (34). Total
number of supersymmetries preserved by the background pp-wave will be 3/4 of the
maximal ones, since eqn. (22) can be solved easily after imposing either (37) or (36). In
addition, the brane breaks another 1/2 supersymmetry given by eqns. (16), (18), so that
total solution (11) preserves 3/8 of the maximal supersymmetry.

We now comment on the relation of the Killing spinor solutions discussed above,
with the ones in [8], [54]. It is now apparent that the solutions of the Killing equations
with projection (31) are the ‘normal’ Killing spinors of [54], whereas the ones given by
solution of the Killing spinor equations, with condition (23) are the ‘supernumerary’
Killing spinors, since independent solutions of this projection condition is also a solution
of another projection condition given by (35).

To summarize, combining results presented in eqns. (16), (18), (23), (29), (30) and
(31), (35), we conclude that the D5-brane solution in eqn. (11) preserves 1/2 of the ‘back-
ground’ supersymmetry. The pp-wave background itself preserves 3/4 of the maximal
supersymmetries. We have also examined the pp-wave ‘background’ geometry of the IIB
NS5-brane. The analogy with the Killing equations of [54] holds there as well and once
again gives 3/4 supersymmetry.
The supersymmetry analysis for the D3-brane solution presented in eqn. (9) of this paper is very similar to the one given above. The dilatino variation now gives:

$$\delta \lambda_\pm \equiv \mp \frac{\mu}{2} f^{\frac{1}{2}} \left( \Gamma^{+12} + \Gamma^{+34} \right) \epsilon_\pm + \frac{\mu}{2} f^{\frac{1}{2}} \left( \Gamma^{+32} + \Gamma^{+14} \right) \epsilon_\mp = 0,$$

and is solved by imposing condition:

$$\Gamma^+ \epsilon_\pm = 0.$$

The need to impose condition (38) for the D3-brane solution becomes apparent while writing the gravitino variation equations for components \((i = 2, 4)\) below, as the NS−NS 3-form flux contributes a term which can be consistently set to zero by imposing the above condition. More explicitly, we have (using eqn.(38)):

$$\partial_+ \epsilon_\pm - \frac{1}{8} f_{\hat{a}} \Gamma^{\hat{a} \hat{b}} \epsilon_\pm + \frac{\mu}{2} \left( \Gamma^{12} + \Gamma^{34} \right) \epsilon_\pm - \frac{\mu}{4} \left( \Gamma^{32} + \Gamma^{14} \right) \Gamma^{-} \epsilon_\mp = 0,$$

$$\partial_- \epsilon_\pm = 0, \quad \partial_i \epsilon_\pm = 0, \quad (i = 2, 4),$$

$$\partial_a \epsilon_\pm + \frac{1}{8} f_{\hat{c}} \delta_{\hat{a} \hat{b}} \Gamma^{\hat{b} \hat{c}} \epsilon_\pm + \frac{1}{8} \Gamma^{a1...a8} \epsilon_{\bar{a}1...\bar{a}8} \frac{f_{\hat{a}}}{f^2} \Gamma^{-} \epsilon_\mp = 0, \quad (a = 1, 3, 5, \ldots, 8).$$

Then the D3-brane supersymmetry condition:

$$\epsilon_+ = \Gamma^{135...8} \epsilon_-$$

implies that all the Killing spinor equations are satisfied, provided one also imposes

$$(\Gamma^{32} + \Gamma^{14}) \epsilon_\pm = 0.$$  

or

$$(\Gamma^{13} + \Gamma^{34}) \epsilon_\pm = 0.$$  

Equations (38), (43) (or (44)) give the Killing spinor conditions on the background pp-wave and the condition in eqn.(42) is the supersymmetry breaking due to the presence of D3-brane. Imposing all these conditions, one finally observes using (43), that \(\epsilon_\pm\) are given as:

$$\epsilon_\pm = exp \left( \frac{\mu}{2} \Gamma^{12} u \right) \epsilon_\pm^0.$$  

This is a valid solution, as the projection matrices \(\Gamma^+\) as well as \(\Gamma^{1234}\), both commute with the term in the exponential in eqn.(43). Combining these with another set of spinors coming from (44), we have 1/2 supersymmetry for the pp-wave background metric in eqn.(9). In addition, the brane once again breaks another 1/2 supersymmetry, leading to
1/4 supersymmetry for the full solution. As an observation, we mention that additional Killing spinors are possible, when one of $NS - NS$ background fields has an opposite signature than the ones in (4), as the dilatino equation (37) can be satisfied by imposing a projection different from the one in (38): $\Gamma^{32\overline{24}} \epsilon = \epsilon$. This condition also simplifies the gravitino equations, leading now to 3/4 supersymmetry for the background. Finally, the above exercise can be repeated for the $D4$-brane solution in eqn.(5) as well and we once again expect the solution to be stable.

4 Discussions

In this paper we have found the supergravity solution of different branes in the PP-wave background using toroidal isometries of pp-waves, following Michelson[52]. To find these solutions, we have started from the known D5-brane solution and then used S- and T-duality. We have also studied the supersymmetric properties of these branes where we showed that they preserve half of the background supersymmetry.

Having supergravity solution of different branes one could proceed to study the world-volume theory of them. For D7-branes this has been studied in [47] where it has been shown that the CS-term plays an important role in order to get the correct mass for gauge field. To study other branes we note, however, that it is also crucial to consider the Myers term[59] as well. In particular, for D3-brane case it seems that it is the Myers term which provides the correct mass for the adjoint scalars. We however leave this as a future exercise.

Another interesting feature of these supergravity solutions would be to extend the original Maldacena’s conjecture for the string theory to the one in the PP-wave background. Namely one might expect that the theory on the worldvolume of D3-brane in the PP-wave background decouples from the bulk gravity, providing a dual description of the string theory on the near-horizon limit of (9).

Note added: While we were preparing our paper for submission we received the paper [58] where the supergravity solution of branes have also been studied.

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