Drag and diffusion coefficients of $B$ mesons in hot hadronic matter

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The drag and diffusion coefficients of a hot hadronic medium consisting of pions, kaons and eta using open beauty mesons as a probe have been evaluated. The interaction of the probe with the hadronic matter has been treated in the framework of chiral perturbation theory. It is observed that the magnitude of both the transport coefficients are significant, indicating substantial amount of interaction of the heavy mesons with the thermal bath. The results may have significant impact on the experimental observables like the suppression of single electron spectra originating from the decays of heavy mesons produced in nuclear collisions at RHIC and LHC energies.

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I. INTRODUCTION

The suppression of the transverse momentum ($p_T$) distribution of hadrons produced in nucleon-nucleus relative to (binary scaled) proton-proton interactions at the Relativistic Heavy Ion Collider (RHIC) has been used as a tool to understand the properties of matter formed in such collisions. The large value of the elliptic flow of hadrons measured at RHIC along with the suppression of the high $p_T$ hadrons mentioned above indicate that the matter might have been formed in the partonic phase with liquid like properties characterized by low value of shear viscosity ($\eta$) to entropy density ($s$) ratio, $\eta/s$ with a lower bound of $\eta/s \sim 1/4\pi$.

In addition to elliptic flow ($v_2$) and nuclear suppression ($R_{AA}$) of light hadrons these quantities have also been measured for the single electron spectra originating from the decays of the open charm and beauty mesons produced at RHIC collisions. The advantages with heavy mesons are two-fold. Firstly, they contain either a charm or a beauty quark which is produced very early and hence can witness the evolution of the partonic matter since its inception until it reverts to hadronic matter through phase transition/cross over and secondly, the heavy quarks do not decide the bulk properties of the latter. Therefore, charm and beauty quarks are considered to be efficient probes for the characterization of the partonic phase. In most of the earlier works aimed at extracting the properties of quark gluon plasma (QGP) by analyzing the $R_{AA}$ and $v_2$ of heavy flavours the role of the hadronic matter was ignored. However, for the characterization of QGP the interactions of heavy flavours with hadronic matter should be taken into consideration and the effects of hadrons must be subtracted out from the observables. Though a large amount of work has been done on the diffusion of heavy quarks in the QGP the diffusion of heavy mesons in hadronic matter has received much less attention so far. Recently the diffusion coefficient of $D$ meson has been calculated using heavy meson chiral perturbation theory and also by using the empirical elastic scattering amplitudes of $D$ mesons with thermal hadrons. The $D$-hadron interactions also have been evaluated using Born amplitudes and unitarized chiral effective $D\pi$ interactions. It has been found that the contributions of $B$ meson to the single electron spectra dominate over those from $D$ meson for large transverse momentum, $p_T > 5$ GeV (see also [24]). Moreover, the future experiments are progressing toward precision measurement over a wide range of kinematical variables. In view of this the use of $B$ meson as a probe to extract the properties of matter at high temperature assumes importance.

In the next section we discuss the formalism adopted to evaluate the drag and diffusion coefficients of the heavy flavoured mesons in a hadronic matter consists of pions, kaons and eta. Results are presented in section III and section IV is dedicated to summary and discussions.

II. FORMALISM

In the present work the drag and diffusion coefficients of the $B$ meson propagating through a hot hadronic matter are evaluated within the ambit of Heavy Meson Chiral Perturbation Theory ($HM\chi PT$) in LO, NLO and NNLO approximations. We also revisit the transport coefficients of $D$ meson in a similar theoretical framework. We consider the elastic interaction of the $B$ meson with thermal pions, kaons and etas in the temperature ($T$) range 100 – 170 MeV. Detailed analysis of the experimental data on the hadronic yield in heavy ion collisions show that the value of the temperature for the chemical freeze-out of the system produced at RHIC energies is about 170 MeV (see [25] for a review). This indicates that the inelastic interactions which are responsible for the change in the number of hadrons become rarer for $T$ below $\sim$ 170 MeV. Thus the con-
tributions of the inelastic collisions in evaluating the
drag and diffusion coefficients of the hadronic matter
probed by the heavy flavoured mesons can be ignored.

The drag \( \gamma \) and diffusion \( B_0 \) coefficients of the heavy
mesons are evaluated using elastic interaction
with the thermal hadrons. For the (generic) process,
\( B(p) + h(q) \rightarrow B(p') + h(q') \) (h stands for pion, kaon
and eta), the drag \( \gamma \) can be calculated by using the
following expression \( ^{26} \):

\[
\gamma = p_i A_i / p^2
\]

where \( A_i \) is given by

\[
A_i = \frac{1}{2E_p} \int \frac{d^3q}{(2\pi)^3 \xi^q} \int \frac{d^3p'}{(2\pi)^3 E'_p} \int \frac{d^3q'}{(2\pi)^3 E'_q} \times \frac{1}{g_B} \sum |M|^2 (2\pi)^4 \delta^4(p+q-p'-q') f(q) (1+f(q')) \langle |(p-p')| \rangle
\]

\( g_B \) being the statistical degeneracy of the \( B \) meson.
The factor \( f(q) \) denotes the thermal phase space factor for
the particle in the incident channel and \( 1+f(q') \) is the Bose
enhanced final state phase space factor. From Eq. \( 2 \) it is clear
that the drag coefficient is a measure of the thermal average of
the momentum transfer, \( p-p' \), weighted by the interaction
through the square of the invariant amplitude,
\(|M|^2 |.

The diffusion coefficient, \( B_0 \) can be defined as:

\[
B_0 = \frac{1}{4} \left[ \langle (p^2) \rangle - \frac{\langle (p.p)^2 \rangle}{p^2} \right]
\]

Both the drag and diffusion coefficients can be evaluated from a single expression:

\[
\langle (p^2) \rangle = \frac{1}{512\pi^4 E_p} \int_0^\infty \int_0^1 d(cos \theta_{cm})
\times \int_0^{2\pi} d\phi_{cm} \frac{q^2 dq d(cos \chi)}{E_q} (1+f(q'))
\times \bar{f}(q) \frac{\sqrt{s} (m_p^2, m_q^2, m_{q'}^2)}{\sqrt{s}} \sum |M|^2 T(p')
\]

with an appropriate choice of \( T(p') \). In Eq. \( 4 \)
\( \lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx \), is the
triangular function.

We start our discussion on the determination of
the scattering amplitudes with the Lagrangian of
Covariant Chiral Perturbation Theory \( (C\chi PT) \)
involving the heavy \( B \) (or \( D \)) mesons \( ^{27} \) given by

\[
\mathcal{L}_{C\chi PT} = \langle D_\mu P\eta P^\mu P^1 \rangle - m_B^2 (Pp^1)
\]

\[
-\langle D_\mu P^\mu D^\nu P_{\nu}^1 \rangle + m_B^2 (P^\mu P^\nu P_{\nu}^1)
\]

\[
+ig (P_{\mu} P_{\nu} P_{\nu}^1 - P_{\mu} P_{\nu} P_{\nu}^1) + \ldots
\]

where the heavy-light pseudoscalar meson triplet
\( P = (B^0, B^+, B_s^+) \), heavy-light vector meson triplet
\( P^\mu = (B^0, B^+, B_s^+) \) and \( \ldots \) denotes trace in flavor
space. The covariant derivatives are defined as
\( D_\mu P_a = \partial_\mu P_a - B_\mu \Gamma^a_{\mu a} \) and \( D^\mu P^1_i = \partial^\mu P^1_a + \Gamma^a_{\mu ab} P^1_b \)
with \( a, b \) are the \( SU(3) \) flavor indices.

The vector and axial-vector currents are respectively
given by \( \Gamma_\mu = \frac{1}{2} (u^1 \partial_\mu u + u^2 \partial_\mu ) \) and \( u_\mu = i (u^1 \partial_\mu u - u^2 \partial_\mu u) \) where \( u = \exp(\frac{\eta}{\sqrt{6}}) \).
The unitary matrix \( \Phi \) collects the Goldstone boson fields and is given by

\[
\Phi = \sqrt{2} \left( \begin{array}{ccc} \pi^0 & \pi^- & K^- \\ -\sqrt{2}/\sqrt{6} & 0 & \frac{\sqrt{2}}{\sqrt{6}} K^0 \\ -\sqrt{2}/\sqrt{6} & \sqrt{2}/\sqrt{6} & K^0 \end{array} \right)
\]

To lowest order in \( \Phi \) the vector and axial-vector currents are:

\[
\Gamma_\mu = \frac{1}{8F_0^2} [\Phi, \partial_\mu \Phi], \quad u_\mu = -\frac{1}{F_0} \partial_\mu \Phi \quad .
\]

From the first term (or kinetic part of the \( P \)-
fields) of \( \mathcal{L}_{C\chi PT} \), the matrix elements for contact
diagram in terms of Mandelstam variables \( (s, t, u) \) are obtained as

\[
M_{B^+\pi^\pm} = -M_{B^-\pi^\pm} = -\frac{1}{4F_\pi^2} (s-u)
\]

\[
M_{B^0\pi^0} = M_{B^+\pi^-} = M_{B^-\pi^+} = 0
\]

\[
M_{B^\pm K^\mp} = -M_{B^\mp K^\pm} = -\frac{1}{4F_K^2} (s-u)
\]

These can be represented in the isospin basis as

\[
M_{B^\pm s}^{(3/2)} = -\frac{1}{4F_\pi^2} (s-u) \quad M_{B^\pm s}^{(1/2)} = \frac{1}{2F_\pi^2} (s-u)
\]

\[
M_{B^0}^{(1)} = 0 \quad M_{B^0}^{(0)} = -\frac{1}{4F_K^2} (s-u)
\]

\[
M_{B^K}^{(1)} = -M_{B^\bar{K}}^{(1)} = \frac{1}{2F_\pi^2} (s-u) \quad M_{B^\bar{K}}^{(0)} = -\frac{1}{4F_K^2} (s-u)
\]

where the isospin of the \( B\Phi \) system appears in the
superscript. Denoting the threshold matrix elements by \( T \), these are obtained from \( ^{3} \) and are given by

\[
T_{B^\pm s}^{(3/2)} = -\frac{m_B m_\pi}{F_\pi^2} \quad T_{B^\pm s}^{(1/2)} = \frac{2m_B m_\pi}{F_\pi^2}
\]

\[
T_{B^0}^{(1)} = 0 \quad T_{B^0}^{(0)} = -\frac{m_B m_K}{F_K^2}
\]

\[
T_{B^K}^{(1)} = -T_{B^\bar{K}}^{(1)} = -\frac{m_B m_K}{F_K^2}
\]

One can reproduce these \( T \)-matrix elements in the
isospin basis using the lowest order \( H M \chi PT \) Lagrangian
for heavy mesons containing a heavy quark
\[ Q \text{ and a light antiquark of flavor } a \text{ as given below}\]

\[ \mathcal{L}_{\text{HM}\chi PT} = -i \text{ tr}_D(\tilde{H}_a^Q \partial_\mu \gamma^\mu H_b^Q) - i \text{ tr}_D(\tilde{H}_a^Q \partial_\mu \gamma^\mu H_b^Q) + \frac{g}{2} \text{ tr}_D(\tilde{H}_a^Q \gamma^\mu \gamma^5 u^{ab}_a H_b^Q) + \ldots \]  

where \( H_a^Q = \frac{1+\gamma^0}{2} \left( P_a^\mu \gamma^\mu + i P_a^\mu \gamma^5 \right) \) and \( \tilde{H}_a^Q = \left( P_a^\mu \gamma^\mu + i P_a^\mu \gamma^5 \right) \frac{1+\gamma^0}{2} \) and \( \text{tr}_D \) denotes trace in Dirac space. In this formalism, since the factor \( \sqrt{m_P} \) and \( \sqrt{m_{P'}} \) have been absorbed into the \( P_a^\mu \) and \( P_{a'}^\mu \) fields, the threshold \( \mathcal{T} \)-matrix element \( (\tilde{T}_{th}^{P'}) \) now has the dimension of scattering length \( a_P \) whereas in \( C\chi PT \), we get a dimensionless \( \mathcal{T} \)-matrix element \( (T_{th}^{P'}) \). The relation between these two \( \mathcal{T} \)-matrix elements and the scattering length \( a_P \) is given by

\[ T_{th}^{P'} = m_P \tilde{T}_{th}^{P'} = 8\pi (m_\Phi + m_P) a_P \]  

The square of the isospin averaged \( \mathcal{T} \)-matrix element is given by

\[ \sum |T_{th}^{P'}|^2 = |T_{th}^{B+}|^2 + |T_{th}^{BK}|^2 + |T_{th}^{B-}|^2 \]  

where

\[ |T_{th}^{B+}|^2 = \frac{1}{(2\pi)^4} \left( 2|T_{B+}^{(1/2)}|^2 + 4|T_{B+}^{(3/2)}|^2 \right) \]

and

\[ |T_{th}^{B+}\tilde{K}|^2 = \frac{1}{(2\pi)^4} \left( |T_{B+}\tilde{K}|^2 + 3|T_{B+}\tilde{K}|^2 \right) \]

**III. RESULTS**

We evaluate the drag coefficients of the \( B \)-meson by using the momentum dependent and momentum independent matrix elements given by Eqs. 8 and 9 respectively. The results are shown by the dot dashed and dashed lines in Fig. 1. Inspired by the fact that the results for the two scenarios are not drastically different in the LO we proceed to evaluate the drag coefficient of heavy mesons by replacing \( \sum |M|^2 \) by \( \sum |T|^2 \) in NLO and NNLO also where the \( \mathcal{T} \)-matrix elements will be obtained from the scattering lengths.

Liu et al. [29] have obtained the \( B\Phi \) scattering lengths (see also [30]) up to NNLO in \( HM\chi PT \) by using the coupling constant from recent unquenched lattice results [31]. Using these NLO and NNLO results we estimate the isospin averaged drag coefficients of \( B \) mesons. The results are depicted in Fig. 1. The drag coefficient evaluated with NNLO matrix elements increases by 22% compared to the NLO result at \( T = 170 \text{ MeV} \).

We now focus on the temperature dependence of the drag coefficient of \( B \)-mesons as shown in Fig. 1. As mentioned before, \( \gamma \) is the thermal average of the square of the momentum exchanged between the heavy mesons and the bath particle weighted by the interaction strength through the invariant amplitude of the process. Therefore, with the increase in temperature of the thermal bath the kinetic energy of the hadrons increases. Hence the hadrons gain the ability to transfer larger momentum during their interaction with the \( B \) mesons - resulting in the increase of the drag coefficient. This tendency is observed in Fig. 1 quite clearly. The increase of drag with temperature is characteristic of a gaseous system. In case of a liquid the drag diminishes with \( T \) (except for very few cases). In this case a significant part of the thermal energy goes into making the attraction between the interacting particles weaker and once this happens the constituents move more freely resulting in a smaller drag force. Therefore, the variation of the drag with temperature can be used to understand the nature of interaction of the fluid.

Since the diffusion coefficient involves the square of the momentum transfer it is also expected to increase with \( T \). This is seen in Fig. 2. The drag and the diffusion coefficients are related through the Einstein relation as:

\[ B_0 = M_B \gamma T. \]  

where \( M_B \) is the mass of the \( B \)-meson. The temperature dependence of the diffusion coefficient evaluated by using Eqs. 4 and the [33] are displayed in Fig. 2. The difference between the results obtained from Eq. 4 and the Einstein’s relation is about \( 6 - 7\% \) at \( T = 170 \text{ MeV} \) for the \( B \) meson momentum, \( p = 100 \text{ MeV} \). This small difference illustrates the validity of
the Einstein relation in the low momentum (non-relativistic) domain.

The energy loss of a $B$ meson moving through a hadronic system may be estimated from the relation

$$-\frac{dE}{dx} = \gamma p . \tag{14}$$

The magnitude of $\gamma$ obtained in the present calculation reveals that the $B$ mesons dissipate significant amount of energy in the medium. This might have crucial consequences on quantities such as the nuclear suppression factor of single electrons originating from the decays of heavy mesons.

We also evaluate the $D$ meson drag and diffusion coefficients using the interactions of $D$ mesons with thermal hadrons discussed in Refs. [27, 29] in LO, NLO and NNLO approximations. The results are displayed in Fig. 3. In the LO approximation the drag is similar for both the cases. However, for NLO and NNLO, the drag co-efficient evaluated using the $T$-matrix elements obtained from the scattering lengths of Ref. [29] is slightly higher than that obtained from Ref. [27].

The drag of $D$ mesons in hot hadronic matter has recently been studied by using different approaches. While empirical scattering cross sections were used in Ref. [20], the authors of Ref. [22] used unitarized chiral effective $D\pi$ interactions to evaluate the drag. We observe that the magnitude of the drag of $D$ meson obtained in the present work is similar to that obtained in Refs. [22, 20]. The smaller value in the present case is due to the lower values of the $D$ meson-hadron cross sections.

**IV. SUMMARY AND DISCUSSIONS**

In summary we have evaluated the drag and diffusion coefficients of open beauty mesons interacting with a hadronic background composed of pions, kaons and eta. It is found that the values of both the transport coefficients increase with temperature. The magnitude of the drag coefficient of the $B$ meson indicates that while evaluating the suppression of the high $p_T$ single electrons originating from the decays of $B$ mesons the effects of hadrons should be taken into account. Within the same formalism, the transport coefficients of the $D$ meson has been calculated. The $D$ meson drag coefficient is found to be lower than the values obtained in Refs. [20, 22].

Some comments on the effects of the exclusions of inelastic channels and non-perturbative processes on the drag and diffusion coefficients are in order here. The inelastic channels do contribute to the scattering matrix through coupled channels via loops in the unitarisation procedure. However, in a thermal background such as in a heavy ion collision the scattering amplitude is weighted by phase space factors which essentially control the rate of reactions. The fact that heavy ion collisions undergo chemical freeze-out at about 170 MeV means that number changing reactions will certainly be inhibited below this temperature. The presence of resonances makes the evaluation of scattering amplitudes a non-perturbative problem. Unitarisation of scattering amplitudes, preferably with loops evaluated with thermal propagators will certainly improve the reli-

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**FIG. 2:** Variation of diffusion co-efficient as a function of temperature. The solid line indicates the variation of the diffusion coefficient with temperature obtained from Eqs. [3] and [1]. The momentum of the $B$ meson is taken as 100 MeV. The dashed line stands for the diffusion coefficient obtained from the Einstein relation (Eq. [13]).

**FIG. 3:** The variation of drag coefficients of $D$ mesons with temperature due to interaction with thermal pions, kaons and eta in LO, NLO and NNLO approximations for interactions of $D$ with thermal hadrons taken from Ref. [27] (left panel) and [29] (right panel).
ability of our results at higher energies. However, in the absence of any information about B* mesons so far, this exercise will however be far less constrained than the charm sector where the masses and widths of the excited states are known.

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