Boson Stars: Alternatives to primordial black holes?

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The present surge for the astrophysical relevance of boson stars stems from the speculative possibility that these compact objects could provide a considerable fraction of the non-baryonic part of dark matter within the halo of galaxies. For a very light ‘universal’ axion of effective string models, their total gravitational mass will be in the most likely range of \( \sim 0.5 \, M_\odot \) of MACHOs. According to this framework, gravitational microlensing is indirectly “weighing” the axion mass, resulting in \( \sim 10^{-10} \, eV/c^2 \). This conclusion is not changing much, if we use a dilaton type self-interaction for the bosons. Moreover, we review their formation, rotation and stability as likely candidates of astrophysical importance.

Keywords: Boson stars, Axions, Effective string models, Dark matter, MACHOs

I. INTRODUCTION

A. Dark matter — Issue of missing mass

The rotation velocities of spiral galaxies can be accurately measured from the Doppler effect. At large radii where the stellar surface brightness is falling exponentially, velocities are obtained for clouds of neutral hydrogen using the 21 cm hyperfine line. The resulting ‘rotation curves’ are found to be roughly flat out to the maximum observed radii of about 50 kpc, which implies an enclosed mass increasing linearly with radius. This mass profile is much more extended than the distribution of starlight, which typically converges within \( \sim 10 \, \text{kpc} \); thus, the galaxies are presumed to be surrounded by extended halos of dark matter.

Perhaps the most compelling evidence for dark matter comes from clusters of galaxies. These are structures of about 1 Mpc size containing more than 100 galaxies, representing an overdensity of about a factor 1000 relative to the mean galaxy density. It is assumed that they are gravitationally bound since the time for galaxies to cross the cluster lasts only about 10% of the age of the Universe. The cluster masses are determined in several independent ways: First, the virial theorem uses the radial velocities of individual galaxies as ‘test particles’. Second, observations of hot gas at about \( 10^7 \, K \) contained in the clusters, which is observed in X-rays via thermal bremsstrahlung. The gas temperature is derived from the X-ray spectrum, and the density profile from the map of the X-ray surface brightness. Assuming the gas is pressure-supported against the gravitational potential leads to a mass profile for the cluster. The third method is gravitational lensing of background objects by the cluster potential. There are two regimes: the ‘strong lensing’ regime at small radii, which leads to arcs and multiple images, and the weak lensing regime at large radii, which causes background galaxies to be preferentially stretched in the tangential direction. All three methods yield estimates for cluster masses which show that visible stars contribute only a few percent of the observed mass, and the hot X-ray gas only about 10–20%, hence, clusters are dominated by dark matter.

On the largest scales, there is further evidence for dark matter: ‘streaming motions’ of galaxies (e.g., towards nearby superclusters such as the “Great Attractor”) can be compared to maps of the galaxy density from redshift surveys to yield estimates of \( \Omega \). Here the theory is more straightforward since the density perturbations are still in the linear regime, but the observations are less secure. A similar estimate may be derived by comparing our Galaxy’s 600 km/s motion towards the Virgo cluster relative to the cosmic rest frame, confirmed by the observed temperature dipole in the cosmic microwave background (CMB).

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Furthermore, it is possible to connect the observed large-scale structure in the galaxy distribution with the results of the CMB anisotropies if the universe is dominated by non-baryonic dark matter. Commonly, the present matter/energy density $\Omega_0 = \Omega_M + \Omega_\Lambda$ of the Universe is decomposed into two components. There is accumulating evidence for $\Omega_0 = 1 \pm 0.2$ and (total) matter density $\Omega_M = 0.4 \pm 0.1$ which implies a vacuum energy or dimensionless cosmological constant of $\Omega_\Lambda = 0.85 \pm 0.2$. Theories of inflation prefer a flat Universe with $\Omega_0 = 1$ as its most ‘natural’ value; this also requires non-baryonic dark matter.

### B. Dark matter — Candidates

What are realistic candidates for dark matter? Hot gas appears to be excluded by limits on the Compton distortion of the blackbody CMB spectrum; atomic hydrogen due to 21 cm observations; and ordinary stars by faint star counts. Asteroids are very unlikely since stars do not process hydrogen into heavy elements very efficiently and hydrogen ‘snowballs’ should evaporate or lead to excessive cratering on the Moon. Black holes more massive than $\sim 10^5 \, M_\odot$ would destroy small globular clusters by tidal effects.

Today’s most viable dark matter candidates fall into two broad classes: astrophysical size objects called MASSive Compact Halo Objects (MACHOs), and so-called Weakly Interacting Massive Particles (WIMPs). Actually these classes could possibly be interrelated, as we are going to show.

Several different objects belong to the first class: Jupiter-size brown dwarfs consisting of hydrogen and helium less massive than $0.08 \, M_\odot$ are the most prominent possibility. Below this limit, the central temperature is not sufficient in order to ignite hydrogen fusion, so these objects just radiate very weakly in the infrared due to gravitational contraction. Other MACHO candidates include stellar remnants such as cool white dwarfs, neutron stars, and primordial black holes or as a result of a supernova.

The WIMP candidates are the invisible axion (hypothesized to solve the strong CP problem or reemerging ‘universally’ in effective string Lagrangians), one of the neutrinos (provided it has a mass of about 10 eV), and the lightest supersymmetric particle, the neutralino, which is expected to be stable. All these have to have a very weak interaction, so that they could not be detected so far.

### C. Gravitational microlensing

The conclusion of gravitational microlensing of stars within the Large Magellanic Cloud (LMC) is that MACHOs in the planetary mass range $10^{-8}$ to $0.05 \, M_\odot$ do not contribute a substantial fraction of the Galactic dark halo. In the two-year data of the LMC events of the MACHO group 8 events could be detected which are well in excess of the predicted background of approximately 1.1 events arising from known stellar populations. Hence, MACHOs in a dark halo appear to be a natural explanation.

A statistical analysis of the galactic halo via microlensing suggests that MACHOs account for a significant part ($> 20\%$) of the total halo mass of our galaxy. Their most likely mass range seems to be in the range $0.3 - 0.8 \, M_\odot$, with an average mass of $0.5 \, M_\odot$, cf. [10]. If the bulge is more massive than the standard halo model assumes, the average MACHO mass will be somewhat lower at $\sim 0.1 \, M_\odot$.

This can be viewed as an indication that MACHOs form an distinct large class of old objects that cannot be easily extrapolated from any familiar stellar population, such as brown or white dwarfs.

However, there are some astrophysical difficulties with this interpretation, mainly arising from the estimated mass $\sim 0.5 \, M_\odot$ for the lenses. These cannot be hydrogen-burning stars in the halo since such objects are limited to less than 3% of the halo mass by deep star counts. Modifying the halo model to slow down the lens velocities can reduce the implied lens mass somewhat, but probably not below the substellar limit $0.08 \, M_\odot$. Old white dwarfs have about the right mass and can evade the direct-detection constraints, but it is difficult to form them with high efficiency, and there may be problems with overproduction of metals and overproduction of light at high redshifts from the luminous stars which were the progenitors of the white dwarfs. Primordial black holes are a viable possibility, though one has to appeal to a coincidence to have them in a stellar mass range.

Due to these difficulties of getting MACHOs in the inferred mass range without violating other constraints, there have been a number of suggestions for explaining the LMC events without recourse to a dark population: most of these suggestions construct some non-standard distribution of ‘ordinary’ stars along the LMC line of sight. However, these proposals appear somewhat contrived, but can be tested observationally in the near future.
D. Boson stars or axion stars as alternatives?

It has been recently suggested that MACHOs could rather be primordial black holes formed during the early QCD epoch in the inflationary scenario. For cosmological dark matter, bound states of gravitational waves, so-called ‘gravitational geons’ built from spin–2 bosons, are also considered recently [4].

Since the standard model of elementary particles as well as their superstring extensions involve also Higgs type scalar fields, we will analyze here the alternative possibility that primordial boson stars account for this non-baryonic part of dark matter [38–40]. Boson stars are descendants of the so-called geons of Wheeler [17–19], except that they are built from scalar particles (spin–0) instead of electromagnetic fields, i.e. spin–1 bosons. If scalar fields exist in nature, such localized soliton-type configurations kept together by their self-generated gravitational field can form within Einstein’s general relativity.

In the case of complex scalar fields, an absolutely stable branch of such non-topological solitons with conserved particle number exists. In the spherically symmetric case, we have shown via catastrophe theory [20,21] that these boson stars have a stable branch with a wide range of masses and radii.

Kaup’s first investigation [22] of the spherically symmetric boson star (BS) was based on massive scalar particles. Lateron, a nonlinear $U(|\Phi|^2)$ potential was introduced by Mielke and Scherzer [23], where also solutions with nodes, i.e. “principal quantum number” $n > 1$, were found. In building macroscopic boson stars, a nonlinear Higgs type self-interaction potential $U(|\Phi|^2)$ was later considered [24] as an additional repulsive interaction. Thereby the Kaup limit for boson stars can even exceed the limiting mass of $3.23\,M_\odot$ for neutron stars [25].

Three surveys [26–28] summarize the present status of the non-rotating case, a more recent survey including the rotating BS can be found in [29].

Recently, we construct for the first time the corresponding localized differentially rotating configurations via numerical integration of the coupled Einstein–Klein–Gordon equations. Due to gravito–magnetic effect, the ratio of conserved angular momentum and particle number turns out to be an integer $a$, the azimuthal quantum number of our soliton–type stars. The resulting axisymmetric metric, the energy density and the Tolman mass are completely regular.

E. Are fundamental scalar fields part of nature?

The physical nature of the spin–0–particles out of which the boson star (BS) consists is still an open issue. Until now, no fundamental elementary scalar particle has been found in accelerator experiments which could serve as the main constituent of the boson star. In the electroweak theory of Glashow, Weinberg, and Salam, a Higgs boson dublett $(\Phi^+, \Phi^0)$ and its anti-dublett $(\Phi^-, \bar{\Phi}^0)$ are necessary ingredients in order to generate masses for the $W^\pm$ and $Z^0$ gauge vector bosons [28]. After symmetry breaking, only one scalar particle, the Higgs particle $h := (\Phi^0 + \bar{\Phi}^0)/\sqrt{2}$, remains free and occurs as a state in a constant scalar field background. Nowadays, calculations of the two–loop electroweak effects enhanced by powers of the mass of the rather heavy top quark [33] has lead to an indirect determination of the Higgs mass, cf. [34]. For $M_t = 173.8 \pm 5$ GeV/c$^2$, one finds $m_h = 104^{+92}_{-45}$ GeV/c$^2$. So far, the experimental constraints are weak; even for the unrealistic case of a Higgs mass up to $1000$ GeV/c$^2$, the discrepancies for, e.g., the mass of the $W$ boson would be small. Above 1.2 TeV/c$^2$, however, the self–interaction $U(\Phi)$ of the Higgs field is so large that the perturbative approach of the standard model becomes unreliable. Therefore a conformal extension of the standard model with gravity included could be necessary, see [35]. Fermilab’s upgraded tevatron [36] has a mass reach of $135 < m_h < 186$ GeV/c$^2$, while the high–energy experiments at the LHC at Cern will ultimately reveal if these Higgs particles really exist in nature.

As free particles, the Higgs boson is unstable with respect to the decays $h \rightarrow W^+ + W^-$ and $h \rightarrow Z^0 + Z^0$. In an hypothetical compact object like the BS, these decay channels are expected to be in equilibrium with the inverse process $Z^0 + Z^0 \rightarrow h$, for instance. Presumably, this is in full analogy with the neutron star [37] or quark star [38] where one finds an equilibrium of $\beta^–$ and inverse $\beta^–$-decay of the neutrons or quarks and thus stability of the macroscopic star with respect to radioactive decay. Such Higgs sector nontopological solitons [41] may also be candidates for cold dark matter. Nishimura and Yamaguchi [42] constructed a neutron star using an equation of state of an isotropic fluid built from Higgs bosons.

Nowadays there are many attempts of unifying the standard model with gravity on the quantum level, like string theory [43]. Commonly, the four–dimensional effective models make the prediction [44] that the tensor field $g_{\mu\nu}$ of gravity is accompanied by one or several scalar fields.

In string effective supergravities [4], the mass of the dilaton $\varphi$ can be related to the supersymmetry breaking scale $m_{\rm SUSY}$ by $m_\varphi \simeq 10^{-3}(m_{\rm SUSY}/\text{TeV/c}^2)^2$ eV/c$^2$ with interesting astrophysical implications [40], but this is not the only possibility.
II. BOSON STARS

In a 1968 perspective paper, Kaup [22] has studied for the first time the full generally relativistic coupling of linear Klein–Gordon fields to gravity in a localized configuration. This ‘Klein–Gordon geon’ is nowadays christened mini–boson star and can be regarded as a macroscopic quantum state. It was already realized that no Schwarzschild type event horizon occurs in such numerical solutions. Moreover, the problem of the stability of the resulting scalar ‘geons’ with respect to radial perturbations is treated. It is shown that such objects are, below a well-defined critical mass, resistant to gravitational collapse. These considerations are on a semiclassical level, since the Klein–Gordon field is treated. It is shown that such objects are, below a well-defined critical mass, resistant to gravitational collapse. These considerations are on a semiclassical level, since the Klein–Gordon field is treated as a classical field.

The Lagrangian density of gravitationally coupled complex scalar field $\Phi$ reads

$$\mathcal{L}_{\text{BS}} = \frac{\sqrt{g}}{2\kappa} \left[ R + \kappa [g^{\mu\nu}(\partial_\mu \Phi^*)(\partial_\nu \Phi) - U(|\Phi|^2)] \right] ,$$

where $\kappa = 8\pi G$ is the gravitational constant. Using the principle of variation, one finds the coupled Einstein–Klein–Gordon equations

$$G_{\mu\nu} := R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\kappa T_{\mu\nu}(\Phi) ,$$

$$\left( \square + \frac{dU}{d|\Phi|^2} \right) \Phi = 0 ,$$

where

$$T_{\mu\nu}(\Phi) = \frac{1}{2} [(\partial_\mu \Phi^*)(\partial_\nu \Phi) + (\partial_\nu \Phi)(\partial_\mu \Phi^*)] - g_{\mu\nu} \mathcal{L}(\Phi)/\sqrt{|g|}$$

is the stress–energy tensor and $\square := \left( 1/\sqrt{|g|} \right) \partial_\mu \left( \sqrt{|g|} g^{\mu\nu} \partial_\nu \right)$ the generally covariant d’Alembertian.

A. Spherically symmetric solutions

The stationarity ansatz

$$\Phi(r, t) = P(r)e^{-i\omega t}$$

describes a spherically symmetric bound state of the scalar field with frequency $\omega$.

In the case of spherical symmetry and isotropic coordinates, the line-element reads

$$ds^2 = e^{\nu(r)} dt^2 - e^{\lambda(r)} \left[ dr^2 + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right] ,$$

in which the functions $\nu = \nu(r)$ and $\lambda = \lambda(r)$ depend on the radial coordinate $r := \sqrt{x^2 + y^2 + z^2}$.

In a nut–shell, a boson star is a stationary solution of a (non-linear) Klein–Gordon equation in its own gravitational field; cf. [54,55]. As in the case of a prescribed Schwarzschild background [56], the spacetime curvature affects the resulting radial Schrödinger equation

$$[\partial_{rr} + V_{\text{eff}}(r) + \omega^2 - m^2] P = 0$$

for the Kalb–Ramond three form $H := e^{\phi} \ast d\sigma$. (There are some speculations [17,43] to identify it with the the axial part of a possible torsion of spacetime). From the Hubble scale $H_{\text{eq}} \sim 10^{-27} \text{eV}/c^2$ of matter–radiation equilibrium and the temperature $T_m \sim 100 \text{MeV}$ of mass generation at the epoch of chiral symmetry breaking, one can derive [19,50] the condition $m_\sigma > (T_m/\text{eV})^2 H_{\text{eq}}$. This allows a very light axion mass $m_\sigma = 7.4 \times (10^7 \text{GeV}/f_a) \text{eV}/c^2 > 10^{-11} \text{eV}/c^2$ with decay constant $f_a$ close to the inverse Planck time, thus a prime candidate for dark matter. (This should not be confused with the Goldstone boson $a$ of the Pececi–Quinn symmetry [54] of standard QCD, for which a recent experiment [52] has excluded the range of $m_a \sim 10^{-6} \text{eV}/c^2$. From cooling neutron stars, there can be inferred [53] an upper limit of $m_a < 0.06 - 0.3 \text{eV}/c^2$, depending on the equation of state of the nucleon fluid.
for the radial function \( P(r) \) essentially via an effective gravitational potential \( V_{\text{eff}}(r) \), when written in terms of the tortoise coordinate \( dr^* := e^{(\lambda - \nu)/2} dr \). Then it can be easily realized that localized solutions fall off asymptotically as \( P(r) \sim (1/r) \exp(\pm \sqrt{m^2 - \lambda^2} r) \) in a Schwarzschild-type asymptotic background.

The energy–momentum tensor becomes diagonal, i.e. \( T_\mu^\nu(\Phi) = \delta(\rho, -p_r, -p_\perp, -p_\perp) \) with

\[
\rho = \frac{1}{2}(\omega^2 P^2 e^{-\nu} + P^2 e^{-\lambda} + U),
\]

\[
p_r = \rho - U,
\]

\[
p_\perp = p_r - P^2 e^{-\lambda}.
\]

This form is familiar from fluids, except that the radial and tangential pressure generated by the scalar field are in general different, i.e. \( p_r \neq p_\perp \), due to the different sign of \( (P^\nu)^2 \) in these expressions.

In general, the resulting system of three coupled nonlinear equations for the radial parts of the scalar and the (strong) gravitational tensor field has to be solved numerically. (Exact solution of massless scalar fields \[22\] or of the coupled Maxwell–Einstein–Klein–Gordon equation \[23\] tend to be plagued with singularities.)

In order to specify the starting values for the ensuing numerical analysis, asymptotic solutions at the origin and at spatial infinity are instrumental. The resulting configuration turns out to be completely regular and does not exhibit an apparent event horizon, cf. \[31\].

The stress–energy tensor of a BS, unlike a classical fluid, is in general anisotropic \[22\]. In contrast to neutron stars \[20\] where the ideal fluid approximation demands an isotropic symmetry for the pressure, for spherically symmetric boson stars there are different stresses \( p_r \) and \( p_\perp \) in radial or tangential directions, respectively. The fractional anisotropy \( \alpha := (p_r - p_\perp)/p_r = P^2 e^{-\lambda}/(\rho - U) \) depends essentially on the self-interaction; cf. Ref. \[21\].

So, the perfect fluid approximation is inadequate for boson stars.

There exists a decisive difference between self-gravitating objects made of fermions or bosons: For a many fermion system the Pauli exclusion principle forces the typical fermion into a state with very high quantum number, whereas symmetric boson stars there are different stresses \( p \) for fermions \[62\] and \( j \) for massive bosons without self-interaction.

Cold mixed boson–fermion stars have been studied by Henrique et al. \[23\] and Jetzer \[26\].

### B. Critical masses of boson stars

Since boson stars are macroscopic quantum states, below a certain critical mass \( M_{\text{crit}} \) they are prevented from complete gravitational collapse by the Heisenberg uncertainty principle \( \Delta r \Delta p \sim \pi \hbar \), cf. Ref. \[14\]. This provides us also with crude mass estimates: For a boson to be confined within the star of effective radius \( R_{\text{eff}} := (1/N) \int_0^\infty j^0 dr \), the Compton wavelength of the collective boson has to satisfy \( \lambda_\Phi = (2\pi \hbar/mc) \leq 2R_{\text{eff}} \). On the other hand, the star’s radius should be of the order of the last stable Kepler orbit \( 3R_S \) around a black hole of Schwarzschild radius \( R_S := 2GM/c^2 \) in order to avoid an instability against complete gravitational collapse.

For a mini-boson star, i.e. a massive boson model with merely the mass term \( U(|\Phi|) = m^2|\Phi|^2 \) as self-interaction, an effective radius \( R_{\text{eff}} \approx (\pi/2)^2 R_S \) close to the last stable Kepler orbit of a black hole, one obtains the estimate

\[
M_{\text{crit}} \approx (2/\pi)M_{Pl}/m \geq 0.633 M_{Pl}^2/m = M_{\text{Kaup}},
\]

cf. Ref. \[23\]. This provides us with a rather good upper bound on the so-called Kaup limit. The correct value in the second expression was found numerically \[22\] as a limit of the maximal or critical mass of a stable mini–BS. Here \( M_{Pl} := \sqrt{\hbar c/G} \) is the Planck mass and \( m \) the mass of a bosonic particle.

For the likely mass \( m_h = 100 \text{ GeV}/c^2 \) of the Higgs particle, e.g., one can estimate the total mass of this mini-boson star to be \( M \approx 10^{10} \text{ kg} \) and its radius \( R_{\text{eff}} \approx 10^{-18} \text{ m} \) yielding a density \( 10^{45} \text{ times that of a neutron star} \).

A boson star is an extremely dense object, since non-interacting scalar matter is very “soft”. However, these properties are changed considerably by considering a repulsive self-interaction

\[
U(|\Phi|) = m^2|\Phi|^2 \left( 1 + \frac{1}{8} \Lambda(|\Phi|^2) \right) = m^2_{\text{ren}}|\Phi_{\text{ren}}|^2.
\]
where $\Lambda(\Phi^2)$ denotes an arbitrary nonlinear self-interaction. The choice $\Lambda(\Phi^2) = (4\lambda/m^2)|\Phi|^2$ would lead us back to the quartic self-interaction of Colpi et al. [24]. If we adopt the value $|\Phi_0| \simeq M_{Pl}/\sqrt{16\pi}$ inside the boson star, one finds for the energy density

$$\rho \simeq m^2|\Phi_0|^2 (1 + \Lambda/8) \, ,$$

where $\Lambda := \Lambda(M_{Pl}^2/16\pi)$ is a dimensionless coupling constant such that we would recover $\Lambda := (\lambda M_{Pl}^2/4\pi m^2)$ for the quartic interaction. The self-interaction becomes dominating for $\Lambda \geq 8$ or $\lambda \geq 32\pi(m/M_{Pl})^2$. Thus, even a rather tiny coupling constant $\lambda$ could have drastic effects on a BS.

Formally, this corresponds to a star formed from non-interacting bosons $\Phi_{\text{ren}} = \Phi(1+\Lambda/8)$ with a lower renormalized mass $m \rightarrow m_{\text{ren}} := m/\sqrt{1+\Lambda/8}$ but larger Compton wave length $\lambda_{\text{ren}}$ and, consequently, a larger radius $R_{\text{eff}}$. (A reverse rescaling of the mass, as was presumed in a recent preprint [23], leads to a smaller Compton wave length and other inconsistencies.) Consequently, we can apply again (2.1) for the Kaup limit and find that the maximal mass of a stable BS scales approximately as

$$M_{\text{crit}} \simeq (2/\pi)M_{Pl}^3/m_{\text{ren}} = \frac{2}{\pi}\sqrt{1 + \Lambda/8}M_{Pl}^3/m \rightarrow \frac{1}{\pi}\sqrt{2\Lambda}M_{Pl}^3/m \quad \text{for } \Lambda \rightarrow \infty .$$

For the quartic self-interaction, this accounts rather well for the numerical results of Colpi et al. [24]. Our formula not only reproduces their asymptotic mass formula (11) for $\Lambda \rightarrow \infty$, but, by construction, interpolates as well with the Kaup limit (2.9) for $\Lambda = 0$.

| Compact Object | Critical mass | Particle Number |
|----------------|--------------|-----------------|
| Fermion Star:  | $M_{\text{Ch}} := M_{Pl}^3/m^2$ | $\sim (M_{Pl}/m)^3$ |
| Mini–BS:       | $M_{K\text{aup}} = 0.633 M_{Pl}^3/m$ | $0.653 (M_{Pl}/m)^2$ |
| Boson Star:     | $(1/2\pi)^3\sqrt{\lambda} M_{\text{Ch}}$ | $\sim (M_{Pl}/m)^3$ |
| Soliton Star: [63,27] | $10^{-2}(M_{Pl}^3/m\Phi_{\text{ren}}^2)$ | $2 \times 10^{-3}(M_{Pl}^3/m^2\Phi_{\text{ren}}^3)$ |

The Chandrasekhar limit is $M_{\text{Ch}} := M_{Pl}^3/m^2 \simeq 1.5(\text{GeV}/\text{mc}^2)^2 M_{\odot}$, where $M_{\odot}$ denotes the solar mass. In astrophysical terms, the maximal BS mass is $M_{\text{crit}} \cong 0.06\sqrt{\lambda} M_{Pl}^3/m^2 = 0.1\sqrt{\lambda} (\text{GeV}/\text{mc}^2)^2 M_{\odot}$ which for $\lambda = 1$ and proton mass $m \simeq 1 \text{ GeV}/\text{c}^2$ is in the interesting mass range $\sim 0.1 M_{\odot}$ of MACHO’s.

In a scale-invariant theory built from nonlinearly coupled dilatons $\varphi$, there arise a conserved dilaton charge via Noether’s theorem from Weyl rescaling and thus will ensure the stability of the configuration. For a dilaton star with quadratic self-interaction [67], the same formula (2.12) applies, but the coupling constant $\Lambda := (\lambda M/4\pi \omega)^2$ will be $\omega$ dependent. For a very light dilaton $\varphi$ of mass $m_{\text{dil}} = 10^{-11} \text{ eV}/\text{c}^2$, resembling a misaligned ‘universal’ axion at its lower mass bound, Gradwohl and Käbelmann [68] found

$$M_{\text{crit}} = 7\sqrt{\lambda} M_{\odot} \, , \quad R_{\text{crit}} = 40\sqrt{\lambda} \text{ km} \, ,$$

where $\lambda := (\lambda M/\omega)^2$ is the rescaled coupling constant of the $\varphi^4$ interaction.

To repeat, in building macroscopic boson a Higgs-type self-interaction $U(|\Phi|)$ is crucial for accommodating a repulsive force besides gravity. This repulsion between the constituents is instrumental to blow up the boson star so that much more particles will have room in the confined region. Thus the maximal mass of a BS can reach or even extend the limiting mass of 3.23 $M_{\odot}$ for neutron stars [23,63,10] with realistic equations of state $p = p(\rho)$ for which the (phase) velocity of sound is $v_s = \sqrt{\partial p/\partial \rho} \leq c$. However, this fact depends on the strength of the self–interaction.

Therefore, if scalar fields would exist in nature, such compact objects could even question the observational AGN black hole paradigm in astrophysics.

### III. EXCITED BOSON STARS

#### A. Gravitational atoms as boson stars

Ruffini and Bonazzola [61,72] used the formalism of second quantization for the complex Klein–Gordon field and noticed an important feature: If all scalar particles are within the same ground state $|\Phi\rangle = (N,n,l,a) = (N,0,0,0)$,
which is possible because of Bose–Einstein statistics, then the semi–classical Klein–Gordon equation of Kaup can be recovered in the Hartree–Fock approximation for the second quantized two–body problem. In contrast to the Newtonian approximation, the full relativistic treatment avoids an unlimited increase of the particle number $N$ and negative energies, but induces critical masses and particle numbers with a global maximum.

Due to this Hartree–Fock approximation and while also neglecting effects of the quantized gravitational field, the same coupled Einstein–Klein–Gordon equations (2.2,2.3) apply. Therefore, a boson star is also called a gravitational atom [72]. Since a free Klein–Gordon equation for a complex scalar field is a relativistic generalization of the Schrödinger equation, we consider for the ground state a generalization of the wave function

$$|N,n,l,a⟩ : \Phi = R^n(r) Y^a_l(θ, ϕ)e^{−i(Eₙ/h)t}$$

$$= \frac{1}{\sqrt{4π}} R^n(r) P^a_l(\cos θ) e^{iaϕ} e^{−i(Eₙ/h)t} (3.1)$$

of the hydrogen atom. Here $R^n(r)$ is the radial distribution, $Y^a_l(θ, ϕ)$ the spherical harmonics, $P^a_l(\cos θ)$ are the normalized Legendre polynomials, and $|a| \leq l$ are the quantum numbers of azimuthal and angular momentum.

Due to their inherent ‘gravitational confinement’ gravitational atoms represent coherent quantum states, which nevertheless can have macroscopic size and large masses. The gravitational field is self-generated via the energy–momentum tensor, but remains completely classical, whereas the complex scalar fields are treated to some extent as Schrödinger wave functions, which in quantum field theory are referred to as semi-classical.

Moreover, Feinblum and McKinley [73] found eigensolutions with nodes corresponding to the principal quantum number $n$ of the H–atom. Motivated by Heisenberg’s non-linear spinor equation [74] additional self–interacting terms describing the interaction between the bosonic particles in a “geon” type configuration were first considered by Mielke and Scherzer [23], where also solutions with nodes, i.e. “principal quantum number” $n > 1$ and non-vanishing angular momentum $l ≠ 0$ for a ’t Hooft type monopole [75] ansatz $Φ^{l=a} ∼ R(r) P^a_l(\cos θ)$ were found. These highly interesting instances of a possible fine structure in the energy levels of gravitational atoms poses the question if quantum geons [18,19] are capable of internal excitations? Recently, without reference to these earlier works, such excited boson stars” were recovered [4,74] and their stability properties corroborated in some numerical details. Moreover, Rosen [78] reviewed his old idea of an elementary particle built out of scalar fields within the framework of the Klein–Gordon geon (or the mini–boson star, as they are christened today).

Several surveys [26,27,79,29] summarize the present status of the non-rotating case.

### B. Rotating boson stars

In the framework of Newtonian theory, boson stars with axisymmetry have been constructed by several groups. Static axisymmetric boson stars, in the Newtonian limit [80] and in GR [81], show that one can distinguish two classes of boson stars by their parity transformation at the equator. In both approaches only the negative parity solutions reveal axisymmetry, while those with positive parity merely converged to solutions with spherical symmetry. The metric potentials and the components of the energy–momentum tensor are equatorially symmetric despite of the antisymmetry of the scalar field. In the Newtonian description, Silveira and de Sousa [82] followed the approach of Ref. [72] and constructed solutions which have no equatorial symmetry at all. Hence, in GR, we have to separate solutions with and without equator symmetry.

Kobayashi et al. [83] tried to find slowly rotating states (near the spherically symmetric ones) of general relativistic boson stars, but they failed. The reason for that is a quantization of the total angular momentum [84]

$$J = \int T^0_3 \sqrt{|g|} d^3x = aN \quad a = 0, ±1, ±2, \cdots (3.2)$$

of boson stars which is proportional to the particle number $N$ and vanishes if $a = 0$. This relation between angular momentum and particle number was first derived by Mielke and Schunck [1].

In recent papers [20,55], we proved numerically that rapidly rotating boson stars with $a ≠ 0$ exist in general relativity. Because of the finite velocity of light and the infinite range of the scalar matter within the boson star, our localized configuration can rotate only differentially, but not uniformly. This new axisymmetric solution of the coupled Einstein–Klein–Gordon equations represent the field-theoretical pendant of rotating neutron stars which have been studied numerically for various equations of state and different approximation schemes [19,70,80,57] as well as for differentially rotating superfluids [88] as a model for (millisecond) pulsars.

On the basis of Ref. [83], it has erroneously been claimed [81] that “rapidly rotating boson stars cannot exist”. However, more recently the same Japanese group (as well as [10]) followed exactly our Ansatz and could verify
all our earlier results \[31\]\[31\], albeit of some extension to stronger gravitational fields, due to better computational facilities. Due to the anisotropy of the stress–energy tensor, our configuration is \textit{differentially rotating}, see \[31\]\[31\] for more details.

Moreover, the energy density of our rotating boson star is concentrated in an effective \textit{mass torus} \[85\]. Thus this first \textit{nonsingular} model of a \textit{rotating body} in GR realizes to some extent the suggestion of Newman et al. \[71\] to fill in the Kerr metric, in view of its ring singularity, with a \textit{toroidal} rather than a spherical \textit{source}. Toroidal structure occurs also in relativistic star systems with an accretion disk \[12\].

Since rotating BS have a toroidal structure, there seems to exist the more speculative possibility of \textit{knotted} vortex like excitations, cf. Ref. \[93\]. For an \(O(3)\) Skyrme model, their existence has recently been demonstrated numerically \[94\],\[95\].

C. Formation of (primordial) boson stars

The possible abundance of solitonic stars with astrophysical mass but microscopic size could have interesting implications for galaxy formation, the microwave background, and formation of protostars. The formation of non-gravitating non-topological solutions was already studied by Frieman et al. \[96\].) In comparison with primordial black holes, it is therefore an important question if boson stars can actually form from a primordial bosonic “cloud” \[97\].

Collisionless star systems are known to settle to a centrally denser system by sending some of their members to larger radius. Likewise, a bosonic cloud will settle to a unique boson star by ejecting part of the scalar matter. Since there is no viscous term in the KG equation \(2.3\), the ‘radiation of the scalar field is the only dissipationless relaxation process called \textit{gravitational cooling}\. Seidel and Suen \[98\],\[99\] demonstrated this numerically by starting with a spherically symmetric configuration with \(M_{\text{initial}} \geq M_{\text{Kaup}}\), i.e. which is more massive than the Kaup limit. Actually such oscillating and pulsating branches have been predicted earlier in the stability analysis of Kusmartsev, Mielke, and Schunck \[100\],\[21\],\[101\] by using \textit{catastrophe theory}. Oscillating soliton stars were constructed by using real scalar fields which are periodic in time \[102\]. Without spherical symmetry, i.e. for \(\Phi \sim R_a(r)Y^a(\theta, \varphi)\), the emission of gravitational waves would also be necessary.

For the formation of \textit{primordial} BSs, an important issue is the breaking of unified gauge \((\text{super–})\)symmetry at high temperature in order to yield a scalar-antisymmetric \textit{symmetry} \(\epsilon_\gamma = N_\gamma / N_\gamma\), as in the case of baryon-antibaryon asymmetry, where \(\epsilon_B = N_B / N_\gamma \sim 10^{-10}\). Here, we recall the situation of collapsing homogeneous mini-BS clouds in the early Universe, cf. \[103\],\[28\]. Because the Jeans scale at decoupling time is greater than the horizon scale, a bosonic mass of \(M_{\text{Pl}}^3 / m^2\) immediately collapses and since this is a factor \(M_{\text{Pl}} / m\) higher than the maximal mass allowed within the mini-BS model, only black holes can form. For an asymmetry factor of order \(\epsilon_\gamma \sim m / M_{\text{Pl}}\), however, the total mass remaining within the horizon is \(M_{\text{Pl}}^2 / m\), hence, BSs could form, avoiding the final state of a black hole.

For a real \((\text{pseudo–})\)scalar field, like the axion \(a\) of the broken Peccei–Quinn symmetry \[51\] in QCD, the outcome is quite different. The axion has the tendency to form compact objects (oscillatons) in a short time scale. Due to its intrinsic oscillations it would be unstable, contrary to a BS. Since the field disperses to infinity, finite non-singular self–gravitating solitonic objects cannot be formed with a massless Klein–Gordon field \[103\],\[105\] in Ref. \[106\]. a different mechanism for forming \textit{axion miniclusters} and starlike configurations was proposed. The self–coupling relaxation time \[97\] is compatible or larger as the age of the Universe. For fermionic soliton stars, there is a temperature dependence \[107\] in the forming of cold configurations.

D. Gravitational waves

In the last stages of boson star formation, one expects that first a highly excited configuration forms in which the quantum numbers \(n, l\) and \(a\) of the gravitational atom, i.e. the number \(n - 1\) of nodes, the angular momentum and the azimuthal angular dependence \(e^{ia\varphi}\) are non-zero.

In a simplified picture of BS formation, all initially high modes have eventually to decay into the ground state \(n = l = a = 0\) by a \textit{combined emission} of scalar radiation and gravitational radiation.

In a Newtonian approximation of Ferrell and Gleiser \[72\], the energy released by scalar radiation from states with zero quadrupole moment can be estimated by

\[
E_{\text{rad}} \sim (n - 1)M_{\text{Pl}}^3 / m .
\]  

This is accompanied by a loss of boson particles with the rate \(\Delta N \sim (n - 1)(M_{\text{Pl}} / m)^2\). For investigating the gravitational radiation of macroscopic boson stars with large self-interaction, a reduction of the differential equations can be taken into account \[100\].
The lowest BS mode which has quadrupole moment and therefore can radiate gravitational waves is the 3d state with \( n = 3 \) and \( l = 2 \). For \( \Delta J = 2 \) transitions, it will decay into the 1s ground state with \( n = 1 \) and \( l = 0 \) while preserving the particle number \( N \). The radiated energy is quite large, i.e., \( E_{\text{rad}} = 2.9 \times 10^{22} \) (GeV/me²) Ws. Thus the final phase of the BS formation would terminate in an outburst of gravitational radiation despite the smallness of the object.

E. Gravitational evolution

There would occur an evolution of boson stars if the external gravitational constant \( \kappa \) changes its value with time \([103, 112]\). This can be outlined within the theory of Jordan–Brans–Dicke or a more general scalar tensor theory. The results show that the mass of the boson star decreases due to a space-depending gravitational constant, given through the Brans–Dicke scalar. The mass of a boson star with constant central density is influenced by a changing gravitational constant. Moreover, the possibility of a gravitational memory of boson stars or a formation effect upon their surrounding has been analyzed as well \([111]\).

IV. ARE MACHOS AXION STARS?

Direct observation of boson stars seems to be impossible also in the far future. We propose here several effects which could possibly give indirect evidence \([113]\). In the asymptotic region, the rotation velocity of baryonic objects surrounding the boson star can reveal the star’s mass. Assuming that the scalar matter of the BS interacts mainly through the Brans–Dicke scalar. The mass of a boson star with constant central density is influenced by a changing gravitational potential. For further investigations of rotation curves, cf. Ref. \([114, 115]\), and of boson stars as gravitational lenses, cf. \([116]\).

Solutions with an infinite range can be found where the mass increases linearly \([111, 114]\). In the context of the dark matter hypothesis, it may be speculated if such boson halos as well as excited BS states \([115, 114]\) can be used to fit the observed rotation curves for dwarf and spiral galaxies \([115, 114]\). Boson halos have a finite radius if a positive cosmological constant exists as most recent results from supernovae reveal \([114]\).

Moreover, BSs could be the solution for the MACHO problem, as we are going to analyze in more detail. In effective string theories, the dilaton \( \varphi \), another moduli field \( \beta \), and the ‘universal’ invisible axion \( \sigma \) are predicted \([44]\). This can be read off from the effective string Lagrangian

\[
\mathcal{L}_{\text{eff}} = \sqrt{g} |e^{-\varphi}| \left[ R + g_{\mu\nu} \left( \partial_\mu \varphi \partial_\nu \varphi - 6 \partial_\mu \beta \partial_\nu \beta - \frac{1}{2} e^{2\varphi} \partial_\mu \sigma \partial_\nu \sigma \right) - 2\Lambda \right].
\]

This corresponds to Eq. (11) with \( \eta = 2 \) of Ref. \([117]\) and allows to combine \([118]\) the axion and the dilaton into a single complex scalar field \( \Phi := \sigma + ie^{-\varphi} \), the axidilaton. In the conformally related Einstein frame \( g_{\mu\nu} \rightarrow g_{\mu\nu} := e^{-\varphi} g_{\mu\nu} \) and for constant modulus \( \beta \), our results on BSs can easily be transferred to this axidilaton content of strings.

A. Mass range of axion stars

As macroscopic quantum states, BSs are quite generally prevented from complete gravitational collapse below a critical total mass \( M_{\text{crit}} \) which, typically, depends inversely on the particle mass, see Eq. (2.9).

The numerical results are shown in Fig. 1. The left figure exhibits the dependence of the mass \( M \) and the particle number \( N \) (rest mass) on the central density \( \rho_0 \). Stable axionic BSs exist at central densities lower than the maximum mass. The critical values are: \( M_{\text{crit}} = 0.846 \, M_\odot \), \( mN_{\text{crit}} = 0.873 \, M_\odot \) and \( \rho_c = 9.1 \times 10^{17} \) kg/m³ is the average density of nuclei. Since non-interacting bosons are very “soft”, BSs are extremely dense objects with a critical density higher than for neutron or strange stars \([10]\). The figure on the right hand side gives the mass depending on the radius (measured in km). For the mass–radius diagram, we have chosen as radius 99.9% of the total mass. This ensures that the exponentially decreasing ‘atmosphere’ of the BS has almost no influence on the asymptotic Schwarzschild spacetime.

The stable BSs or axion stars (ASs) have radii larger than the minimum at 20.5 km and a mass of 0.846 \( M_\odot \). In order to derive these values, we have assumed that the mass of the scalar field is \( 10^{-8} \) eV close to the lower bound of axions, leading to an asymmetry factor of \( \epsilon_s \sim 10^{-38} \), and that no self-interaction exists. We stress that the total mass of these relativistic ASs is just in the observed range of 0.3 to 0.8 \( M_\odot \) for MACHOs. One could also turn
this argument around: By identifying the MACHOs with known gravitational mass of about 0.5 \(M_\odot\) with ASs, we are essentially "weighing", via \(M_{Kaup}/N_{\text{crit}} \approx m\), the axion mass to \(m_\sigma \sim 10^{-10}\) eV/c². It is gratifying to note that such a low value is perfectly compatible with the constraints on the mass range of the Kalb–Ramond axion seeding the large-scale CMB anisotropy, cf. the recent results of Gasperini and Veneziano [49,50] within low-energy string cosmology.

For the other option of dilaton \(\varphi\) being stabilized [19] through the axion, a much smaller dilaton mass of \(m_\varphi \sim 10^{-6}\) m\(_\odot\) could be generated non-perturbatively, such that the dilaton behaves very similar to misalignment produced Peccei–Quinn axion \(a\). Our conclusion also with respect to the mass range of an axidilaton star will not changed much, if we use the full Brans-Dicke type interaction [8] for the combined axilladions.

Thus, for cosmologically relevant (invisible) axions as cosmological dark matter also an AS [7,113,106] with a rather large mass of would be possible and stable.

Therefore, if such–string inspired scalar fields would exist in Nature, axions could not only solve the non–baryonic dark matter problem [6], but their gravitationally confined mini–clusters, the axion stars, would also represent the observed MACHOs in our Galaxy.

V. OUTLOOK: GRAVITATIONALLY CONFINED HAWKING RADIATION?

Commonly for the Bekenstein–Hawking radiation the spacetime geometry is treated as a given fixed background, e.g. the Schwarzschild solution. However, due to the universality of gravitational interaction, the evaporating quantum field, say a scalar field \(\hat{\Phi}\), may have a “back-reaction” upon the spacetime geometry via the semi–classical Einstein equation

\[
G_{\mu\nu} = -\kappa \langle 0 | T_{\mu\nu}(\hat{\Phi}) | 0 \rangle. \tag{5.1}
\]

For instance, a ‘bouncing shell’ model [121] with retarded time \(u\) leads to \(\langle 0 | T_{uu} | 0 \rangle \to \kappa^2/48\pi\), the standard Hawking result. The situation becomes, however, much more complicated by the fact that the vacuum expectation value \(\langle 0 | 0 \rangle\) of the energy–momentum tensor \(T_{\mu\nu}\), for instance defined by the point-splitting prescription, is not unique. One ambiguity in \(\langle 0 | T_{\mu\nu}(\hat{\Phi}) | 0 \rangle\) is of the type \(m^2G_{\mu\nu}\), i.e. linear in the curvature, and can be readily absorbed in a redefinition of the ‘bare’ gravitational constant \(\kappa\). However, the next order corrections are quadratic in the curvature and therefore of the same one–loop order arising from the notorious nonrenormalizability of perturbative quantum gravity, cf. Ref. [21], p. 90. To some extent, the finite part of such higher order curvature counterterms in the Lagrangian can be simulated by a self-interaction potential \(U(\hat{\Phi})\), cf. [122].

Already on the semiclassical level one could ask the question what happens to the (massive) particles associated with the second quantized field \(\hat{\Phi}\) in a patch of some strong gravitational background field? Could the particles created by the Unruh effect instead of evaporating to infinity rather form a bound state within their self-consistently generated gravitational field? Moreover, could it be possible that the full back-reaction on the geometry is strong enough lead to a curved spacetime without horizon and singularities, similarly as in some exact solvable (2+1)-dimensional models? Actually some aspects of this issue were already answered by Ruffini and Bonazzola [5] for a spherically symmetric self-gravitating configuration of N particles in a Hartree–Fock approximation. Thus the back–reaction (5.1) may lead us back exactly to some stable branch of boson stars where the particles are treated on the first quantization level. These type of stars have an exponentially decreasing energy density of the scalar field, an ‘exosphere’ of particles in the stable state of equilibrium of particle creation and annihilation. Moreover, for these type of compact objects with an effective radius close to the last stable Kepler orbit an event horizon is suppressed due to the back-reaction [11].

Below the Kaup limit, we have seen that such macroscopic quantum states are absolutely stable, at higher central densities the configuration becomes more and more unstable, and undergoes complete gravitational collapse.

So could it be that the picture of an evaporating black hole is just a first order semi-classical approximation; rather, below some mass limit, we may end up in a self-consistent state of a boson or fermion star with a gravitationally confined Hawking radiation, a quantum geon?

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[1] R.G. Carlberg et al., in “Large-Scale Structure: Tracks and Traces”, 12th Potsdam Cosmology Workshop, ed. V. Mueller (World Scientific, Singapore 1998).
[2] R.D. Blandford and R. Narayan, Annual Rev. Astron. Astrophys. 30, 311 (1992).
[3] M.A. Strauss and J.A. Willick, Phys. Rep. 261, 271 (1995).
[4] S. Perlmutter et al., Nature 391 (1998) 51.
[5] A.G. Riess et al., “Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant”, accepted by Astron.J. (1999).
[6] M.S. Turner, in: The Galactic Halo, The Third Stromlo Symposium, B.K. Gibson, T.S. Axelrod, and M.E. Putman, eds. APS Conference Series, Vol. 666 (1999).
[7] B. Carr, Annual Rev. Astron. Astrophys. 32, 531 (1994).
[8] B. Paczynski, Astrophys. J. 304 (1986) 1; Annual . Rev. Astron. Astrophys. 34 (1996) 419.
[9] W. Sutherland, Rev. Mod. Phys. 71 (1999) 421.
[10] Ph. Jetzer, Helv. Phys. Acta 69 (1996) 179.
[11] A. Gould, J.N. Bahcall and C. Flynn, Ap.J. 482, 913 (1997).
[12] S. Charlot and J. Silk, Astrophys. J. 445, 124 (1995).
[13] K. Jedamzik, Phys. Rev. D55 (1997) R5871; Phys. Rep. 307, 155 (1998).
[14] D.E. Holz, W.A. Miller, M. Wakano, and J. A. Wheeler: “Coalescence of primal gravity waves to make cosmological mass without matter”, in: Directions in general relativity (papers in honor of Dieter Brill), B.L. Hu and T.A. Jacobson, eds. (Cambridge University Press, Cambridge 1994), p. 339.
[15] F.E. Schunck, “Massless scalar field models rotation curves of galaxies”, in: Aspects of Dark Matter in Astro- and Particle Physics, H.V. Klapdor-Kleingrothaus and Y. Ramachers, eds. (World Scientific, Singapore 1997), p. 403-408; “Boson Halo”, in: Proceedings of the UC Santa Cruz Workshop on Galactic Halos, D. Zaritsky, ed. (AAS, 1998), p. 403-405.
[16] F.E. Schunck, A scalar field matter model for dark halos of galaxies and gravitational redshift, Report no. astro-ph/9802258.
[17] J.A. Wheeler, Phys. Rev. 97 (1955) 511.
[18] J.A. Wheeler, Rev. Mod. Phys. 33 (1961) 63.
[19] J.A. Wheeler (with Kenneth Ford) : Geons, Black Holes, and Quantum Foam — A Life in Physics (1998).
[20] F. V. Kusmartsev, E. W. Mielke, and F. E. Schunck: “Stability of neutron and boson stars: A new approach based on catastrophe theory”, Phys. Lett. A 157 (1991) 465. (winning 1991 ‘Honorable Mention’ of the Gravity Research Foundation).
[21] F.E. Schunck, F.V. Kusmartsev, and E.W. Mielke: “Stability of charged boson stars and catastrophe theory”, in: Approaches to Numerical Relativity, R. d’Inverno, ed. (Cambridge Univ. Press, Cambridge, 1992), pp. 130–140.
[22] D.J. Kaup, Phys. Rev. 172 (1968) 1331; N.N. Rao and D.J. Kaup, J. Phys. A: Math. Gen. 24, L993 (1991).
[23] E.W. Mielke and R. Scherzer, Phys. Rev. D24 (1981) 2111.
[24] M. Colpi, S. L. Shapiro, and I. Wasserman, Phys. Rev. Lett. 57 (1986) 2485.
[25] C.E. Rhoades and R. Ruffini, Phys. Rev. Lett. 32, 324 (1974).
[26] Ph. Jetzer, Phys. Rep. 220 (1992) 163.
[27] T.D. Lee and Y. Pang, Phys. Rep. 221, 251 (1992).
[28] A.R. Liddle and M.S. Madsen, Int. J. Mod. Phys. 1, 191 (1992).
[29] E.W. Mielke and E. Mielke: “Boson Stars: Early history and recent prospects”, 8th Marcel-Grossmann-Meeting, T. Piran, ed. (World Scientific, Singapore 1999, to appear).
[30] F.E. Schunck and E.W. Mielke: “Rotating boson stars”, in Proceedings of the Bad Honnef Workshop Relativity and Scientific Computing: Computer Algebra, Numerics, Visualization, eds: F.W. Hehl, R.A. Punigam, and H. Ruder (Springer–Verlag, Berlin 1996), pp. 8 (color plates), 138.
[31] E.W. Mielke and F.E. Schunck: “Rotating boson stars” in: Gravity, Particles and Space–Time, ed. by P. Pronin and G. Sardanashvily (World Scientific, Singapore 1996), pp. 391–420.
[32] C. Quigg: Gauge Theories of the Strong, Weak, and Electromagnetic Interactions (Benjamin/Cummings Publishing, London 1983).
[33] F. Abe et al. (CDF Collaboration), Phys. Rev. Lett. 74 (1995) 2626.
[34] P. Gambino: “Precision test of the standard model: Higher order corrections and the Higgs mass”, RADCORE 98, Int. Symposium on Radiative Corrections, Barcelona, Sept. 1998.
[35] F.W. Hehl, J.D. McCrea, E.W. Mielke, and Y. Ne’eman, Phys. Rep. 258 (1995) 1.
[36] T. Han and R.-J. Zhang, Phys. Rev. Lett. 82 (1999) 25.
[87] S. Bonazzola, E. Gourgion, M. Salgado, and J.A. Marck, Astron. Astrophys. 278, 421 (1993).
[88] D. Langlois, D.M. Sedrakian, and B. Carter, Mon. Not. R. Astron. Soc. 297 (1998) 1189.
[89] S. Yoshida and Y. Eriguchi, Phys. Rev. D56 (1997) 762.
[90] F.D. Ryan, Phys. Rev. D 55 (1997) 6081.
[91] E.T. Newman and A.I. Janis, J. Math. Phys. 6, 915 (1965).
[92] S. Nishida, Y. Eriguchi, and A. Lanza, Astrophys. J. 401 (1992) 618.
[93] E.W. Mielke, Gen. Rel. Grav. 8 (1977) 175 – 196. [reprinted in Knots and Applications, L.H. Kauffman, ed. (World Scientific, Singapore 1995), p. 229 – 250].
[94] L. Faddeev and A.J. Niemi, Nature 387 (1997) 58.
[95] R.A. Battye and P.M. Sutcliffe, Phys. Rev. Lett. 81 (1998) 4798.
[96] J.A. Frieman, G.B. Gelmini, M. Gleiser and E.W. Kolb, Phys. Rev. Lett. 60 (1988) 2101.
[97] I.I. Tkachev, Phys. Lett. B261, 289 (1991).
[98] E. Seidel and W.–M. Suen, Phys. Rev. D 42 (1990) 384.
[99] E. Seidel and W.–M. Suen, Phys. Rev. Lett. 72 (1994) 2516.
[100] F.V. Kusmartsev, E.W. Mielke, and F.E. Schunck, Phys. Rev. D 43 (1991) 3895.
[101] F.V. Kusmartsev and F.E. Schunck, Physica B 178 (1992) 24.
[102] E. Seidel and W.–M. Suen, Phys. Rev. Lett. 66 (1991) 1659.
[103] M.Yu. Khlopov, B.A. Malomed, and Ya.B. Zeldovich, Mon. Not. R. Astron. Soc. 215, 575 (1985).
[104] D. Christodoulou, Commun. Math. Phys. 105 (1986) 337.
[105] T. Piran, Phys. Rev. D36 (1987) 3575.
[106] E.W. Kolb and I.I. Tkachev, Phys. Rev. Lett 71 (1993) 3051.
[107] W.N. Cottingham and R. Vinh Mau, Phys. Lett. B 261 (1991) 93.
[108] M.A. Gunderson and L.G. Jensen, Phys. Rev. D 48 (1993) 5628.
[109] D.F. Torres, Phys. Rev. D56 (1997) 3478.
[110] G.L. Comer and H. Shinkai, Class. Quantum Grav. 15 (1998) 669.
[111] D.F. Torres, A.R. Liddle, and F.E. Schunck, Phys. Rev. D57, 4821 (1998).
[112] D.F. Torres, F.E. Schunck, and A.R. Liddle, Class. Quantum Grav. 15, 3701 (1998).
[113] F.E. Schunck and A.R. Liddle, Phys. Lett. B404 (1997) 25.
[114] S.J. Sin, Phys. Rev. D 50, (1995) 3650; S.U. Ji and S.J. Sin, Phys. Rev. D 50, (1995) 3655.
[115] J.W. Lee and I.G. Koh, Phys. Rev. D 53, (1996) 2236.
[116] M.P. Dąbrowski and F.E. Schunck: Boson stars as gravitational lenses, Report no. astro-ph/9807039.
[117] T. Dereli, M. Önder, and R.W. Tucker, Class. Quantum Grav. 12 (1995) L25.
[118] A. Sen, Mod. Phys. Lett. A8 (1993) 2023.
[119] R. Dick, Mod. Phys. Lett. A12 (1997) 47; Fortschr. Phys. 45 (1997) 537.
[120] C.R. Stephens, G. ’t Hooft and B.F. Whiting, Class. Quantum Grav. 11 (1994) 621.
[121] R.M. Wald: Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics (University of Chicago Press, Chicago 1994).
[122] J. Benítez, J., A. Macías, E.W. Mielke, O. Ohregón, and V.M. Villanueva, Int. J. Mod. Phys. 12 (1997) 2835.
FIG. 1. Left: Mass $M$ and particle number $N$ (or rest mass $mN$ at infinity) of a BS depending on the central density $\rho$. Right: Mass–radius dependence of an axionic BS.