Recent developments on Oka manifolds

Abstract: The notion of an Oka manifold developed from the classical Oka-Grauert-Gromov theory and was formally introduced in 2009. Oka manifolds are characterized by the property that they admit many holomorphic maps from Stein manifolds, that is, closed complex submanifolds of complex Euclidean spaces. In particular, the Runge-Oka-Weil approximation theorem and the Oka-Cartan extension theorem hold for such maps in the absence of topological obstructions. I will discuss recent developments in this field and mention several new applications to holomorphic factorization problems in complex geometry. I will also discuss the new notion of an Oka-1 manifold; these are complex manifolds with many open holomorphic curves. It turns out that every Kummer surface, every elliptic K3 surface, and every compact rationally connected manifold is an Oka-1 manifold, but it is an open problem whether these manifolds are Oka.