Problem of Calculating Strength of Beams of Corrosion-Resistant Cross-section during Bending with Constrained Torsion

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Abstract. Corrosion damage to steel in beam structures is one of the reasons for the loss of their performance and reduced durability. Acceleration of corrosion processes can be the accumulation of dust on the horizontal elements of the structure, especially in the case of wetting. In addition to corrosive damage, the load applied to the beam structures can act with eccentricity. The displacement of the load leads to the development of additional stresses from the constrained torsion. Consideration of changes in the geometric characteristics of the beam cross section because of corrosion, especially in the presence of constrained torsion, is necessary when calculating and designing such structures. In this paper, we analyzed the changes in the geometric parameters of the beam as a result of corrosion of the lower flange, investigated the beam of corrosion-resistant section from sheets and pipes, proposed a method for calculating the flexural-torsional characteristic, developed a method for determining the stress under constrained torsion to check the section strength.

1. Introduction
The calculation of beam structures under the joint action of bending and constrained torsion is a very relevant topic for research at present [1–4]. Many steel building structures are subjected to corrosion. Depending on the degree of aggressive impact, these damages can be so significant that they can lead to a complete loss of the carrying capacity. Corrosion of beam structures, as a rule, is not uniform. The lower flange, on which dust accumulates, corrodes more. In order to reduce the accumulation of dust deposits in design the pipe cross section of the lower flanges used. A beam made of sheets and pipe has slightly different flexural and torsional characteristics than the I-beam with the same flanges. In addition to bending, beam structures can be loaded with a load applied with eccentricity. This causes cramped torsion and the formation of additional stresses. Assessment of changes in geometric, flexural and torsional characteristics by reducing the thickness of the elements, as well as analysing of changes in the magnitudes and nature of the distribution of stresses during strength testing is a very relevant task.

The conducted studies allowed to form an idea about the effect of corrosion damage on the geometric parameters of sections of the beam from pipe and sheets and to develop a proposal for determining the stresses from the constrained torsion of such structures.
2. Materials and methods

The strength of the bent elements depends on the geometrical parameters of the cross section and the internal forces resulting from the application of an external load.

2.1. The effect of corrosion on the change in geometric characteristics

As a result of a long period of operation in the presence of corrosive factors, a reduction in the thickness of the elements constituting the cross section may occur. This leads to a change in the geometric characteristics, as well as the position of the center of gravity and the center of torsion. All these changes lead to a redistribution of internal stresses and can lead to the exceeding of the carrying capacity. Analysing the effects of corrosion on I-beams, we can conclude that the lower flange is most susceptible to corrosion. This is because areas of increased dust deposition are formed on it. In the practice of design, to reduce the effect of corrosion damage due to dust deposits, sections that are less prone to these processes are used. For example, the replacement of the lower flange from a plate to an angle or pipe leads to a decrease in dust accumulations on it and, as a result, to a decrease in corrosive wear. So, the corrosion rate for the lower flange of an I-beam is: from a plate $v = 0.142$ mm/year, from an angle $v = 0.127$ mm/year, from a pipe $v = 0.112$ mm/year. That is, the use of pipes for the lower flange of the beam, in the presence of corrosive factors, is more rational.

2.2. Calculation of the strength of beams with bending with constrained torsion

Consider the issue of testing the strength of bent elements in terms of design standards. As an example, consider a single-span simple supported beam. The beam section was initially selected for the maximum bending load (figure 1). During operation for 10 years, the lower flange can get corrosive, which will lead to a change in the geometric characteristics of the beam section, the position of the center of gravity and the center of torsion (figure 2).

![Figure 1. Beam design](image1.png)

![Figure 2. Beam section geometry](image2.png)
With an estimated design life of 10 years, the reduction in thickness will be: for the lower flange of the I-beam - 1.4 mm, for the pipe - 1.1 mm. Analysis of the change in the position of the centre of gravity and the centre of torsion for a given case showed that the centres of gravity and torsion shift towards the upper flange.

Testing the strength of beams as bended elements of continuous section under the action of the moment in the same plane and the presence of bi-moment is made according to the formula:

\[
\frac{M_x}{J_x R_y \gamma_c} y \pm \frac{B \cdot \omega}{J_\omega R_y \gamma_c} \leq 1, \tag{1}
\]

here:
- \(M_x\), \(B\) – internal force factors, bending moment and bimoment;
- \(J_x\), \(J_\omega\) – geometric characteristics, moment of inertia of the section and moment of inertia with constrained torsion;
- \(\omega\) – sector coordinate;
- \(R_y\), \(\gamma_c\) – design resistance of steel for yield strength and coefficient of working condition, respectively.

In formula (1), the first term is a test of the strength at normal bending stresses, and the second term is a test of strength under the action of normal stresses from constrained torsion.

To plot the plots of normal stresses in the most loaded section, a calculation was made by the finite element method of the models of the beams under consideration. The calculation results are presented in figures 3-4 and in table 1.

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Figure 3. Plots of normal stresses in MPa for beam B1-K
a) - bending; b) - when bending with constrained torsion; c) - from constrained torsion.
Figure 4. Plots of normal stresses in MPa for B2-K beam.
   a) - bending; b) - when bending with constrained torsion; c) - from constrained torsion.

Table 1. Stresses and strains for points of average section of beams

| Indicators                                      | B1-0                          | B1-K                          | B2-0                          | B2-K                          |
|------------------------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| Total stresses in the lower flange, $\sigma_{\text{max}}$, MPa | 343.8                         | 403.3                         | 232.2                         | 272.6                         |
| Total stresses in the upper flange, $\sigma_{\text{max}}$, MPa | -343.9                        | -352.4                        | 207.6                         | 213.4                         |
| Sectorial tension in the lower flange, $\sigma_{\text{sn}}$, MPa | 113.3                         | 141.9                         | 14.8                          | 17.9                          |
| Sectorial stresses in the upper flange, $\sigma_{\text{sn}}$, MPa | -114.1                        | -117.2                        | 18.1                          | 17.6                          |
| % sectorial stresses from full for the lower flange | 33.2                          | 35.2                          | 6.4                           | 6.6                           |
| % sectorial stresses from full for upper flange | 33                            | 33.3                          | 8.7                           | 8.2                           |
| Maximum deflection, mm | 39                             | 43                             | 23                            | 25                            |
| The angle of twist of the cross section in the center of torsion, degrees | 9.1                            | 10.4                           | 2                             | 2.2                           |

The results show the following:
- reduce the thickness of the lower flange, the displacement of the centres of gravity and torsion towards the upper flange leads to an increase in stresses in the lower flange;
- for an I-section, the stresses in the lower flange reach the tensile strength, which indicates a possible development of plastic deformations at the points of the cross section;
- cross-section of the beam with a flange from the pipe is more resistant to twisting;
- the twist angle of a beam with a flange made of a pipe is significantly less than an I-beam, the share of sectorial stresses of the constrained torsion is 3 times less than the I-beam section.

Corrosion processes significantly reduce the bearing capacity of the structure, and taking into account the random eccentricity and the increase in sectorial stresses can lead to the exhaustion of the bearing capacity of strength.

A corrosion-resistant beam cross section has proven to be more effective in the case of an eccentricity load. However, a comparison of the results obtained in the calculation by the finite element method with the results of a theoretical calculation using formula (1) shows a significant discrepancy. The theoretical values are obtained more than the values by the finite element method. Moreover, the discrepancy is given by the second term of formula (1) associated with constrained torsion. In this regard, it is considered relevant to develop a method for calculating the strength of beams with a lower flange made from a pipe in case of constrained torsion.

It is based on the method for calculating constrained torsion stresses of I-beams developed by professors V.Z. Vlasov [5].

\[
\sigma_{\omega} = \frac{B_{\omega}}{I_{\omega}}, \quad (2)
\]

To analyze the influence of geometric characteristics, calculations were made by the finite element method of beams with different parameters of the sections of the elements. Such indicators as width and thickness of sheets, diameter and thickness of pipes varied. According to the obtained stresses \(\sigma_{\omega}\), the value of the bimoment was determined for different points of the cross section, figure 5, formula (3).

\[
B = \frac{\sigma_{\omega} I_{\omega}}{\omega}. \quad (3)
\]

Figure 5. Points at which stresses \(\sigma_{\omega}\) were determined by finite element method

Based on the V.Z. Vlasov theory of the bi-moment value for a single-span hinged beam loaded with a uniformly distributed load, the maximum bi-moment value can be determined from the formula:

\[
B_{\text{max}} = mp l^2, \quad (4)
\]

where:
- \(l\) - is the span of the beam;
- \(m = q \cdot e\) - is the torsion moment;
- \(q\) - load;
- \(e\) - is the eccentricity of the applied load.

Then:

\[
p = \frac{B_{\text{max}}}{ml^2}. \quad (5)
\]
Based on the theory [5], it is possible to determine the value of the coefficient $p$ by the formula:

$$ p = \frac{ch^{kl-1}}{k^2 \sqrt{2ch^{kl}}} \quad (6) $$

The value of the flexural-torsional characteristic $k$ for the beam in question:

$$ k = \sqrt{\frac{G-J_{\omega}}{EJ_{\omega}}} \quad (7) $$

We write the value of the moment of inertia of torsion $J_{\omega}$ in the form:

$$ J_{\omega} = \alpha \cdot \sum \frac{bt^3}{3} + \frac{\pi d^3 s}{4} \quad (8) $$

Then:

$$ k = \sqrt{\frac{G \cdot (\alpha \cdot (\sum \frac{bt^3}{3} + \frac{\pi d^3 s}{4}) \cdot \pi \cdot \alpha \cdot \omega \cdot \sum \frac{bt^3}{3} + \frac{\pi d^3 s}{4})}{EJ_{\omega}}} \quad (9) $$

where:
- $G$ - is the shear modulus;
- $E$ - is the modulus of elasticity;
- $J_{\omega}$ - is the sectorial moment of inertia;
- $b$, $t$ - is the width and thickness of the sheets;
- $d$, $s$ - is the average diameter $m$ of the pipe wall thickness;
- $\alpha$ - correction factor.

From here:

$$ \alpha = \frac{k^2 \cdot J_{\omega} \cdot E}{G \cdot (\sum \frac{bt^3}{3} + \frac{\pi d^3 s}{4})} \quad (10) $$

To determine the correction coefficient $\alpha$, beams were calculated with various geometrical parameters of section elements. Based on the calculations built dependence presented in Figure 6.

**Figure 6.** Graph of correction coefficient $\alpha$ versus relative index of geometric parameters $(b \cdot t \cdot a)/J_{\omega}$.

- $b$, $t$ - is the width and thickness of the upper flange;
- $a$ - is the distance from the centre of gravity of the flange to the centre of gravity of the pipe;
- $J_{\omega}$ - is the sectorial moment of inertia of the section.

The graph shows a virtually linear dependence of the coefficient $\alpha$. 
3. Conclusions
The work done to study the effect of corrosion and constrained torsion on the work of beam structures allows us to conclude the following:

- Uneven corrosion of steel structure elements leads to a change in the position of the centers of gravity and torsion, as well as a change in the flexural-torsional characteristics.
- Compared with the I-beam, the cross-section of the beam of sheets and pipe is more resistant to the perception of constrained torsion.
- Determination of constrained torsion stresses for sheet and pipe beams can be performed using the constrained torsion theory of I-beams using the correction factor $\alpha$, determined according to the proposed method.

References
[1] A.R. Tusnin, Calculation of I-beams under the action of torsional loads, Scientific and Technical Bulletin of the Volga region. 2012. № 5. S. 335-339.
[2] A.V. Solovyov, A.O. Lukin, Evaluation of the effect of constrained torsion on the work of a beam with a corrugated wall, Izvestiya Vysshikh Uchebnykh Zavedenii. Building. 2012. № 6 (642). Pp. 112-118.
[3] E.O. Fetisov, Experimental and numerical study of the deformed state of a thin-walled open profile beam, Bulletin of the Chelyabinsk State Agroengineering Academy. 2014. T. 68. P. 88-93.
[4] V.N. Kiselev, Yu.V. Popkov, V.I. Atrahimovich, Work of thin-walled metal rods with constrained torsion, Bulletin of Polotsk State University. Series F: Construction. Applied Science. 2011. No. 8. P. 33-36.
[5] V.Z. Vlasov, Thin-walled elastic rods, Fizmatgiz, Moscow, 1956.