Effective viscosity and elasticity of dynamically jammed region and their role for the hopping motion on dense suspensions

Pradipto and Hisao Hayakawa

Yukawa Institute for Theoretical Physics, Kyoto University, Kitashirak awaoiwake-cho, Sakyo-ku, Kyoto 606-8502, Japan

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We investigate the dynamically jammed region (DJR) induced by an impact in dense suspensions and quantify its effective viscosity and elasticity. We propose a phenomenological model, called the DJR model, that contains the contributions from the effective viscosity and elasticity. We confirm good agreements between the results of the simulation and the DJR model for a free-falling impactor. We also discuss the impact of a foot-spring-body system in dense suspensions to mimic the running on the top of dense suspensions. The foot undergoes multiple rebounds and also hops multiple times due to the spring force and the rigidity of the suspensions. We then apply the DJR model to the foot-spring-body system and reproduces the results of the simulations. We also check the parameter dependences of the hopping motion and found that hopping and multiple rebounds are suppressed as the spring stiffness increases.

I. INTRODUCTION

The phenomenon of being able to run on suspension has attracted the interest of scientists as well as the general public [1, 2]. Such impact-induced hardening of dense suspensions is often chosen as an example of discontinuous shear thickening (DST) [1], but it has been shown earlier that the underlying mechanism of the impact-induced hardening differs from that of DST [3]. Indeed, the impact-induced hardening is a transient process in which the only normal stress becomes large and the system is heterogeneous, while DST is a steady process in which both the shear and normal stresses become large and the system is homogeneous.

Most of physical studies for impact-induced hardening use a free-falling impactor or a constant speed penetrating intruder. Using a free-falling impactor, Ref. [4] reported the existence of a localized rigid region beneath the impactor called the dynamically jammed region (DJR). Since such a DJR grows in size, Ref. [4] proposed the added-mass model, which treats the impact as an inelastic collision between the impactor and the DJR that evolves with time. Then, Ref. [5] visualized the flow field inside dense suspensions around penetrating intruder and found that the strain rate is peaked on the boundary of the DJR. Inspired by this observation, Ref. [6] proposed a model based on the viscous force acting on the boundary of the DJR. Yet, none of the above models can explain the existence of elastic response of dense suspensions under impacts such as fractures [7], high stress near boundary [8], and rebound of the impactor [9]. Reference [8] proposed a constitutive model and measured the elastic modulus once the DJR spans from the impactor to the boundary. Then, the viscoelastic response of dense suspensions under the impact is captured using the floating + force chains model [10], where percolated force chains of contacting suspended particles are necessary to recover the elastic motion. However, such a picture neglects the fact that even if the force chains are not touching the boundary, the DJR beneath the impactor might have effective elasticity. Therefore, the rigidity of such a DJR needs to be thoroughly investigated.

The motion of a running or walking person is more complicated than the free-falling impactor or penetrating intruder. One particular approach to directly investigate the running motion on the suspensions has been carried out in Ref. [11]. They discussed the maximum penetration depth of a foot for various impact velocities corresponding to walking, jogging, and running, and fit the data using a constitutive model with the aid of the elastic modulus obtained in Ref. [8]. They also showed that the added mass model is not sufficient to recover the response of the suspensions under running motion. Nevertheless, little is still known about the dynamics of multiple impacts on dense suspensions, which are important for running and walking motions. Some studies have tried to reproduce mechanical models for legged animal locomotions. One of the simplest and most celebrated models is the spring-mass model inspired by biomechanical observations [12]. Such a model has been realized as one-legged hopping robot [13]. In the spring-mass model, the human leg is represented by a spring and the human body is simply represented by a point mass. Inspired by such a model, this paper studies a foot-spring-body system in dense suspensions.

The structure of this paper is as follows. In Sec. 2, we describe the coarse-graining method to characterize the elasticity and viscosity of the DJR. We compare the result of the model including the effective elasticity and that of the floating model without elasticity [10] to clarify the role of elasticity of the DJR when force chains are not connected with the bottom boundaries. We then extend the previous floating + force chains model to include the effective elasticity. In Sec. 3, we describe the simulation setup for the foot-spring-body model. By using the coarse-graining method, we discuss the hopping

*pradipto@yukawa.kyoto-u.ac.jp
motion of such a system to clarify the criterion for the hopping motion. In Sec. 4, we conclude our findings and discuss the future prospect of this study. In Appendix A, we describe the details of the coupled lattice Boltzmann discrete element method (LB-DEM) used in our simulation.

II. DYNAMICAL JAMMED REGION MODEL WITH EFFECTIVE VISCOSITY AND ELASTICITY

A. Setup for a free-falling impactor simulation

We adopt the coupled LBM-DEM simulation as in Refs. 3, 10. Details of the simulation setup are written in Appendix A. The simulation setup is as follows 10. A suspension with N particles and volume fraction φ is contained in a box sized $W \times H \times D$. Throughout this paper, we have adopted the perfect density matching between the solvent and suspended particles, where the densities of particles and solvent satisfy the relation $\rho_p = \rho_f$, where $\rho_p$ and $\rho_f$ are the densities of a suspended particle and solvent fluid, respectively. In this section, a spherical impactor with the diameter $D_I$ and density $\rho_I$, is released from the height $H_0$ which corresponds to the impact speed $u_0 = \sqrt{2gH_0}$ with the gravitational acceleration $g$. In our simulation $\rho_I$ and $D_I$ satisfy $\rho_I = 4\rho_f$ and $D_I = 6\sigma_{\text{min}}$, respectively, where $\sigma_{\text{min}}$ is the radius of smallest suspended particle. We also introduce the time scale $t_g = \sqrt{\sigma_{\text{min}}/2g}$, speed scale $u^* = \sqrt{2g\sigma_{\text{min}}}$, and force scale $F_g = \frac{\pi}{3}\rho_f(D_I/2)^2g$. We evaluate the impactor motions in deep and shallow containers. Here we use $\phi = 0.53$, $H = 3D_I$, $W = D = 4D_I$, and $N = 960$ for the deep container case and $\phi = 0.53$, $H = 2D_I$, $W = D = 4D_I$, and $N = 670$ for the shallow case.

B. Coarse-graining method and delineating the dynamically jammed region

The DJR can be defined once we obtain the strain rate, strain, and stress field inside the suspensions. The approximate description of such fields from discrete particles data can be carried out using the coarse-grain method that has been used in granular materials 14, 15. As an example, the discrete particle data for microscopic mass density $\rho^\text{dis}$ at position $r$ and time $t$ can be expressed as

$$\rho^\text{dis}(r, t) = \sum_i m_i \delta(r - r_i(t)), \quad (1)$$

where $r_i$ and $m_i$ are the position and mass of particle $i$, respectively. For smoothed and coarse-grained density $\rho$, the delta function in Eq. (1) is replaced with a coarse-graining function $\Phi(r)$ as

$$\rho(r, t) = \sum_i m_i \Phi(r - r_i(t)). \quad (2)$$

Here, we adopt

$$\Phi(r - r_i) = \frac{1}{(w\sqrt{2\pi})^3} \exp\left[-\frac{(r - r_i)^2}{2w^2}\right], \quad (3)$$

where $r = (x, y, z)$ and $r_i = (x_i, y_i, z_i)$ are the field position and particle position, respectively. Similarly, the coarse-grained momentum density $p(r, t)$ can be introduced as

$$p(r, t) = \sum_i m_i u_i(t) \Phi(r - r_i(t)), \quad (4)$$

where $u_i$ is the velocity of particle $i$. The velocity field $u(r, t)$ is defined by $u(r, t) = p(r, t)/\rho(r, t)$. The stress tensor $\overrightarrow{\sigma}$ contains the contributions from the contact $\overrightarrow{\sigma}^c$ and hydrodynamics $\overrightarrow{\sigma}^h$

$$\overrightarrow{\sigma} = \overrightarrow{\sigma}^h + \overrightarrow{\sigma}^c. \quad (5)$$
FIG. 2. Time evolutions of the impactor velocity $u_z/u^*$ under various situations: (a) Solution of Eq. (18) is plotted alongside simulation results and the floating model [10] without rebound and percolating force chains (impact velocity $u_0 = 1.8u^*$, deep container with $H = 3D_I$). (b) Solution of Eq. (18) is plotted alongside simulation results and the floating + force chains model [10] for the case with rebound and percolating force chains (impact velocity $u_0 = 4.2u^*$, shallow container with $H = 2D_I$). (c) Solution of Eq. (18) is plotted alongside simulation results and the floating model [10] for the case without percolating force chains but with a small elastic response (impact velocity $u_0 = 4.2u^*$, deep container with $H = 3D_I$). The inset of (c) shows the magnified impactor velocity around the elastic response.

The contact contribution is calculated from the pairwise contact force

$$\hat{\sigma}^c(r) = \frac{1}{2} \sum_{i,j} F_{ij}^c \otimes r_{ij} \Phi(r - r_i)$$

(6)

where $F_{ij}^c$ and $r_{ij}$ are the pairwise contact force and the interparticle distance between particles $i$ and $j$, respectively. Here $\otimes$ denotes the tensor product. Meanwhile, the hydrodynamic contribution is given as

$$\hat{\sigma}^h(r) = \sum_i \hat{\sigma}_i^h \Phi(r - r_i),$$

(7)

where $\hat{\sigma}_i^h$ is the hydrodynamic stress tensor on each particle, obtained from the LBM and lubrication stresslet [3]. The vector field of particle overlaps $\delta_n$ (which represents the deformation) can be obtained from the contact overlap on each particle $\delta_i^c$

$$\delta_n = \sum_i \delta_i^c \Phi(r - r_i).$$

(8)

Note that we ignore the contributions from rattlers in this paper.

Once the flow field is obtained, one can get the symmetric part of the strain rate tensor $\hat{\varepsilon}$

$$\hat{\varepsilon} = \sqrt{\frac{2}{2} \hat{\sigma} : \hat{\sigma}}.$$

(9)

The field of $\hat{\varepsilon}$ can be seen in Fig. 1(a). Then, the effective viscous stress $\sigma^{(vis)}$ is simply given by [16, 17]

$$\sigma^{(vis)} = \eta \hat{\varepsilon}.$$  

(10)

Note that $\sigma^{(vis)}$ contains both the contributions from the normal and shear parts, though, as shown in Ref. [3], it is dominated by the normal part. Finally, similar to the strain rate, the scalar strain fields $\varepsilon$ is defined as

$$\varepsilon = \sqrt{2 \hat{\varepsilon} : \hat{\varepsilon}}.$$  

(11)

Similar to Eq. (11) one can introduce the rigidity as

$$G = \frac{1}{2} \hat{\sigma} : \hat{\varepsilon} : \hat{\varepsilon}.$$  

(12)

Thus, the effective elastic stress is given by

$$\sigma^{(el)} = G \varepsilon.$$  

(13)

C. The dynamically jammed region

The DJR can be defined by the following two ways. First, it can be delineated from the strain rate field since the front of the DJR corresponds to the peak in the strain rate field [3]. As shown in Fig. 1(a) the $z$-position of the peak of $\hat{\varepsilon}$ is denoted as $z_{front}$. The height of the DJR $H_{djr}$ is defined as the distance between the front and the deepest point of the impactor. Thus, one can approximate the DJR as a hemi-sphere with radius $H_{djr}$ (the
red shaded region in Fig. 1(b). We denote this approach the \( \dot{\varepsilon} \)-based delineation of the DJR. Alternatively, we can also define the DJR as the region that has non-zero rigidity \( G \) (yellow shaded region in Fig. 1(b)). We denote this approach the \( G \)-based delineation of the DJR. Such \( G \)-based delineation of the DJR essentially represents the region formed by contacting suspended particles. As one can see in Fig. 1(b) both delineations yield qualitatively similar regions.

Once the DJR is delineated, one can obtain the effective quantities by integrating the viscosity \( \eta \) and rigidity \( G \) fields over the region shown in Fig. 1(b) as

\[
\eta_{\text{eff}} = \eta_0 + \frac{1}{V_{\text{djr}}} \int d \eta dV, \quad G_{\text{eff}} = \frac{1}{V_{\text{djr}}} \int d G dV, \quad (17)
\]

where \( d \) represents the region enclosed by the yellow or red surface in Fig. 1(b) and \( V_{\text{djr}} \) is its volume. Here, \( \eta_0 \) is the apparent viscosity of the suspensions multiplied by the solvent viscosity before the impact. In Figs. 1(c) and 1(d) we plot the time evolutions of the effective viscosity \( \eta_{\text{eff}} \) and rigidity \( G_{\text{eff}} \) obtained from the \( \dot{\varepsilon} \)-based delineation of the DJR as well as the \( G \)-based delineation of the DJR. For the viscosity, one can see that \( \eta_{\text{eff}} \) increases with time. This confirms the enhancement of the effective viscosity as the result of the impact as indicated in our previous paper [10]. We also observe non-zero rigidity even when the DJR does not touch the bottom boundary. One can see that the \( \dot{\varepsilon} \)-based and the \( G \)-based delineations yield a qualitatively similar effective viscosity \( \eta_{\text{eff}} \) and rigidity \( G_{\text{eff}} \). However, quantitative differences exist since the \( \dot{\varepsilon} \)-based delineations approximate the region as a hemi-sphere. From the next section onwards, we solely use the \( G \)-based delineation of the DJR to obtain the effective quantities.

D. Updated viscoelastic model with effective viscosity and elasticity

In the floating + force chains model in Ref. [10], the elastic response only exists when the force chains percolate. Now, we include the effect of elastic response through the elastic modulus of the DJR even if the force chains are not percolated in the present paper. Thus, we propose the dynamically-jammed-region (DJR) model as

\[
m_1 \ddot{z}_I = -m_1 g - 3\pi \eta_{\text{eff}}(t) |z| \dot{z}_I + C k_{\text{eff}}(t) |z|, \quad (18)
\]

where \( m_1 \) is the mass of impacter, \( z \) is the deepest point of the impacter, \( z_I \) is the vertical position of the impacter, \( \dot{z}_I := \frac{dz_I}{dt} \), \( k_{\text{eff}} \), \( \ddot{z}_I := \frac{d^2 z_I}{dt^2} \), \( k_{\text{eff}} \) is the effective spring constant defined as \( k_{\text{eff}} = G_{\text{eff}}(t) A_D/H_D \), with \( A_D \) and \( H_D \) are the top surface area and height of the dynamically jammed region, respectively. We also introduce a fitting parameter \( C \) which is the order of unity.

First, we check the validity of the DJR model for the case without rebound and percolating force chains (impact velocity \( u_0 = 1.8u^* \), deep container with \( H = 3D_1 \)). In Fig. 2(a) we plot the solution of Eq. (18) for both the solution of the DJR model with \( C = 1.0 \) and the solution from the floating model with a fitting parameter \( \eta_{\text{eff}} \). One can see that both solutions agree well with the results of simulation while they have subtle differences. Then, we check Eq. (18) for the case of rebound and percolating force chains (impact velocity \( u_0 = 4.2u^* \), shallow container with \( H = 2D_1 \)). In Fig. 2(b) we plot the solution of Eq. (18) for the DJR model with \( C = 0.9 \) alongside with the solution from the floating + force chains model with fitting parameters \( \eta_{\text{eff}} \) and \( n(t) \) as in Ref. [10]. In this case, the DJR percolates and is spanned throughout the system thus enhancing the elastic response. Although the floating + force chains model reproduces the result of the simulation almost perfectly, the results of...
DJRs seems to have slight deviations for relatively short time regime. Finally, we examine Eq. (18) for the case without percolating force chains but with a small elastic response (impact velocity $\bar{u}_0 = 4.2u^*$, deep container with $H = 3D_1$). In Fig. 2(c) we plot the solution of Eq. (18) for the DJR model with $C = 0.8$, alongside with the solution from the floating model with a fitting parameter $\tilde{u}_{\text{eff}}$. The floating model cannot capture the elastic rebound, while the DJR model can (see the inset of Fig. 2(c)), though the solution of DJR model slightly deviate from the result of the simulation around $t/t_g = 0.1$. Thus, the ability to describe the elastic response without percolating force chains is the advantage of the DJR model.

**III. FOOT-SPRING-BODY DYNAMICS IN DENSE SUSPENSIONS**

![Graph](image)

**FIG. 4.** Time evolutions of the foot position in $z$-direction $z_p$ with (a) various spring stiffness $k_s$ at $u_0 = 4u^*$ and (b) various initial velocity $u_0$ at $k_s = 100m_0/(a_{\text{min}}t_g^2)$.

Now let us introduce the foot-spring-body model as a toy model to express a bouncing motion on suspension liquids. The foot in the foot-spring-body model is expressed as a rectangular plate impactor with volume $V_p = W_p \times H_p \times D_p$ and mass $m_p = V_p \rho_p$, where $\rho_p$ is the density of the foot plate and we adopt $\rho_p = 1.2\rho_f$. The body is represented by a sphere with diameter $D_b$ and mass $m_b$. We adopt the density of the body $\rho_b$ as $\rho_b = 2\rho_p = 2.4\rho_f$. The body and the foot are then connected with a massless spring with the stiffness $k_s$ and natural length $L_0$. Here we use $\phi = 0.51$, $H = 2D_b$, $W = D = 4D_b$, and $N = 618$. Note that we are only interested in the vertical ($z$ direction) motion of the system. We then modify Eq. (18) to be the set of equations

\[
\begin{align*}
    m_b \ddot{z}_b &= - m_b g - k_s (z_b - z_p - L_0) - \zeta_s \dot{z}_b \\
    m_p \ddot{z}_p &= - m_p g - 3\pi \eta_{\text{eff}}(t) |z_p| + C k_{\text{eff}}(t) |z| + k_s (z_b - z_p - L_0) - \zeta_s \dot{z}_p,
\end{align*}
\]

where $m_b$ and $z_b$ are the mass and the vertical position of the body, respectively, $z_p$ is the vertical position of the plate impactor, $\zeta_s$ is the damping constant, $\zeta_s = \sqrt{k_s (m_p + m_b)}/2$. With the parallel procedure to the free-falling case, one can draw the DJR induced by the impact between the foot and the suspensions. The illustration of this setup and the DJR induced by the impact can be seen in Fig. 3(a). A typical motion of the foot-spring-body system can be seen in Figs. 3(b) and 3(c). Here, we adopt $C = 0.9$. One can see that the solution of Eq. (19) agrees well with the results of the simulation. Initially, the foot experiences a strong deceleration as in the free-falling impactor due to the interaction between the foot and suspensions. Meanwhile, the body still accelerates due to gravity. Then, the system exhibits a damped oscillation. Thanks to the spring force and the rigidity of the suspensions, the foot undergoes multiple rebounds ($u_p < 0$), and also hops ($z_p > 0$) multiple times. This result suggests that composite materials including elastic springs inside the body can maintain its position for a while. This is the first step to realize running process on a suspension.

We then evaluate the multiple rebound motion of the foot in detail. First, we check how the motion of the foot depends on the stiffness of the spring $k_s$. The simulation results for various $k_s$ are presented in Fig. 4(a). Here one can see a lower tendency to have multiple rebounds or hops for higher $k_s$. Moreover, for a rigid system ($k_s \to \infty$), the foot only rebound once and then sink afterwards. This is similar to the prediction by the added-mass model in Ref. [11], where they suggested that running on top of suspensions is impossible for a perfectly stiff leg. Then, we check the initial velocity ($u_0$) dependences in Fig. 4(b). As one might expect, the foot and the foot sink and does not hop for low $u_0$ since the impact-induced hardening is stronger for high $u_0$ [3, 4, 6, 9].
IV. DISCUSSIONS AND CONCLUSIONS

With the aid of a coarse-graining method of the simulation results, we quantified the DJR by the impact on dense suspensions and measured its effective viscosity and elasticity. As a benchmark, we used the obtained viscosity and modulus and extend the model in Ref. [10] so that such a model can capture the elastic motion even when the DJR does not touch the bottom plate. We found good agreements between the simulation results and the DJR model. To mimic the hopping motion, we discussed the impact of the foot-spring-body system on the top of dense suspensions. Our model agrees well with the simulation results, and we found a lower tendency for viscosity and modulus and extend the model in Ref. [10] and elasticity. As a benchmark, we used the obtained dense suspensions and measured its effective viscosity by the impact on itself. The model for the growth of DJR in the cases of free-falling impactor, hopping foot, penetrating intruder has been proposed [5, 19]. However, such sinking process is beyond the scope of this paper. In this paper, the time-dependent effective viscosity is calculated from the time evolution of the discrete field is calculated from the time evolution of the discrete

\[ m_i \frac{d \mathbf{u}_i}{dt} = F^c_i + F_i^b + F_{i}^{\text{lub}} + F_i^q, \]  
\[ I_i \frac{d \mathbf{w}_i}{dt} = T^c_i + T_i^{\text{lub}} + T_i^b. \]  

Here, \( \mathbf{u}_i, \mathbf{w}_i, m_i, \) and \( I_i = (2/5)m_i a_i^2 \) with \( a_i \) the radius of particle \( i \), are the translational velocity, angular velocity, mass, and the moment of inertia of particle \( i \), respectively. \( F^q_i = -m_i g \hat{z} \) is the gravitational force acting on the suspended particle \( i \), where \( \hat{z} \) is the unit vector in the vertical direction.

Note that our LBM accounts for both the short-range lubrication force \( F_{i}^{\text{lub}} \) and torque \( T_{i}^{\text{lub}} \), as well as the long-range hydrodynamic force \( F^h_i \) and torque \( T^h_i \) as in Ref. [22, 26]. The long-range parts \( (F^h_i \text{ and } T^h_i) \) are calculated using the direct forcing method [3, 25], while the lubrication force \( F_{i}^{\text{lub}} \) and torque \( T_{i}^{\text{lub}} \) are expressed by pairwise interactions as \( F_{i}^{\text{lub}} = \sum_{j \neq i} F_{ij}^{\text{lub}} \) and \( T_{i}^{\text{lub}} = \sum_{j \neq i} T_{ij}^{\text{lub}} \), respectively [22, 26, 28]. The explicit expressions of \( F_{ij}^{\text{lub}} \) and \( T_{ij}^{\text{lub}} \) can be found in Ref. [26].

We adopt the linear spring-dashpot version of the DEM [26] for the contact interaction between particles, which involves both the normal and the tangential contact forces. Note that we omit the dissipative part for the tangential contact force. For the particle \( i \), the contact force \( F^c_i \) and torque \( T^c_i \) are, respectively, written as \( F^c_i = \sum_{j \neq i} (F^n_{ij} + F^\text{tan}_{ij}) \) and \( T^c_i = \sum_{j \neq i} a_i \mathbf{n}_{ij} \times F^\text{tan}_{ij} \), where \( a_i \) is the radius of particle \( i \). The normal force is explicitly expressed as

\[ F^n_{ij} = (k_n \delta^n_{ij} - \zeta(n) u^n_{ij}) \mathbf{n}_{ij}, \]  

where \( k_n \) is the spring constant, \( \delta^n_{ij} \) is the normal overlap, \( \mathbf{n}_{ij} \) is the normal unit vector between particles, \( u^n_{ij} \) is the normal velocity difference of the contact point, \( u^n_{ij} = u_{ij}^{(n)} - \dot{u}_i^{(n)} \), and \( \zeta(n) = \sqrt{m_0 k_n} \) is the damping constant, where \( m_0 \) is the average mass of the suspended particles. If the tangential contact force is smaller than a slip criterion, tangential contact force is represented as

\[ F^\text{tan}_{ij} = k_t \delta^t_{ij} T_{ij}, \]  

where \( k_t \), assumed to be 0.2\( k_n \), is the tangential spring constant, \( \delta^t_{ij} \) is the tangential compression and \( T_{ij} \) is the tangential unit vector at the contact point between particles \( i \) and \( j \). We adopt the Coulomb friction rules as

\[ |F^\text{tan}_{ij}| = \mu |F^n_{ij}| \quad \text{if } |F^\text{tan}_{ij}| \geq |F^n_{ij}| \]  
\[ |F^\text{tan}_{ij}| = |\dot{F}^\text{tan}_{ij}| \quad \text{if } |\dot{F}^\text{tan}_{ij}| \leq |F^n_{ij}|. \]  

In addition, to simulate the free surface of the fluid, it is necessary to introduce interface nodes between the fluid and gas nodes [3, 23, 25]. Equations of motion and the torque balance of particle \( i \) are, respectively, given by
where \( \delta_{ij} \) is updated each time with relative tangential velocity \([24]\).

Finally, \( F' \) is the electrostatic repulsive force, also expressed by pairwise interactions as \( F' = \sum_{j \neq i} F'_{ij} \). The explicit expression of \( F'_{ij} \) is expressed by the Derjaguin-Landau-Verwey-Overbeek (DLVO) theory \([30, 32]\) for the double layer electrostatic force as

\[
F'_{ij} = F_0 \exp(-h/\lambda) n_{ij},
\]

where \( F_0 = k_B T \lambda_B \frac{Z^2}{e}(a_{\text{min}}/\lambda^2)(1 + a_{\text{min}}/\lambda)^2/\lambda^2 \) with the charge number \( Z \), the Bjerrum length \( \lambda_B \) and the Debye-Hückel length \( \lambda \). Note that \( \lambda_B \) can be expressed as \( \lambda_B = e^2/(4\pi\epsilon_0 e_c k_B T) \) where \( e \), \( e_0 \), \( e_c \), and \( k_B \) are the elementary charge, the vacuum permittivity, the dielectric constant, and the Boltzmann constant, respectively \([32]\). Here, we adopt the Debye length \( \lambda = 0.02a_{\text{min}} \).

Our simulation ignores the Brownian force. Thus, the electrostatic repulsion force is important to prevent the suspended particles from clustering \([26, 28]\).

For the impactor, the contact force \( F^{I,c} \) and torque \( T^{I,c} \), which arise from the interactions with the suspended particles, are also calculated by the DEM. The lubrication force \( F^{I,\text{lub}} \) and torque \( T^{I,\text{lub}} \) are also calculated in a similar manner as used in suspended particles. The long-range hydrodynamic force \( F^{I,h} \) and torque \( T^{I,h} \) are calculated using the bounce-back rule which satisfies the no-slip boundary condition between the fluid and the surface of the impactor \([20, 21]\). In the bounce-back rule, the LBM discrete distribution function that streams from fluid nodes to the boundary nodes is reflected. Then, the hydrodynamic force on each node is calculated from the momentum transferred in this reflection process. In our implementation, the bounce-back rule is implemented by treating the surface of the impactor as boundary nodes.

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