Dynamical wear prediction along meshing path in mixed lubrication of spiral bevel gears

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Abstract
Surfaces of gears under combined rolling and sliding motions may suffer a complicated wear process due to the transient time-varying effect along the meshing path. In this paper, a methodology for predicting the wear of tooth surfaces is developed for the spiral bevel gears. In the wear model, the machined surface roughness, mixed lubrication, friction, flash temperature and the dynamic behavior of gears are all considered. Tooth-Contact-Analysis (TCA) method is used to get the time-varying parameters of meshing points along the meshing path. By simulating real movement process, the material is removed according to the Arrhenius equation. First, the distribution of pressure and film thickness is obtained by solving the mixed EHL model. After that, the flash temperature can be computed by the point heat source integration method with the obtained pressure, film thickness and velocity vector. The material removal is based on surface temperature and sliding distance. The numerical results are compared to the ball-on-disk experiments to demonstrate the reasonableness of the present wear model. And it shows that the angle difference between velocity vectors has strong influences on the wear profile. Furthermore, the mechanism of surface wear evolution is investigated systematically in spiral bevel gears. The difference of the wear track between the pinion and gear surfaces is observed. Besides, in the meshing process of tooth surface, the wear along the meshing path is uneven, which appears to be much greater at the engaging-in and engaging-out areas. There is a position with maximum wear rate in the meshing process, and the position is affected by the load and speed.

Keywords
Spiral bevel gear, wear, mixed lubrication, surface topography

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Introduction
Surface wear is a common failure mode in gear transmission system. Spiral bevel gears, as key components in the transmission of energy and kinematics between two crisscross axes, widely existed in helicopters, heavy truck, etc., mostly operate in mixed lubrication condition.¹ In mixed lubrication, the lubricant oil does not completely separate the rubbing gear surfaces, indicating that asperity contacts happen. Local asperity contact can cause adhesive wear and abrasive wear due to significant sliding velocity in spiral bevel gears. Wear changes the morphology of the tooth surfaces, which may cause a serious impact on the accuracy and efficiency of the transmission. Therefore, the mechanism investigation of surface wear evolution caused by the sliding velocity is very important for transmission
Finite element method (FEM), based on the assumption of dry contact, were used in the early studies of spiral bevel gears. For example, Park and Kahraman,\(^3\) Kahraman et al.,\(^4\) ever combined the FEM with Archard wear theory to predict the wear profile of spiral bevel gear tooth. As the FEM is very time-consuming, some quick prediction methods for wear are proposed. In Schlecht’s work,\(^5\) and also in Park et al.,\(^6\) the lapping process was discretized with time, the spiral bevel gear meshing point was considered as a two-cylindrical smooth Hertz contact, and the load tooth contact simulation was executed using BECAL program\(^7\) at each time step to predict surface wear. Similarly, based on smooth and dry contact assumptions, Huang et al.\(^8\) proposed an integrated model, which combined the Archard’s wear equation with a conjugate approach.

Lubrication and surface roughness play an important role in surface wear in the meshing of gears. However, only a few literatures have considered the effects of surface roughness and lubrication in the wear prediction model. Early studies mainly focused on the lubrication and wear in different meshing positions by separating the meshing process of gears to obtain a steady-state solution.\(^9,10\) Most of these works are focused on spur or helical gear, but few researches focus on the wear of spiral bevel gear due to its complicated tooth surface geometry. Morales-Espejel et al.\(^11\) ever built a micro pitting fatigue model to predict the wear of spur gears. In their publication, the film thickness is calculated through Grubin’s experience formula\(^12\) and the asperity contact area is calculated by a statistical equation with a conjugate approach.

Huang et al.\(^8\) proposed an integrated model, which combined the Archard’s wear equation with a conjugate approach.

Dynamic wear model for spiral bevel gears

As the tooth surface geometry of spiral bevel gears are very complex, the parameters such as the radius of curvature and surface velocity vector along the meshing path, cannot be expressed by analytical expression directly. The tooth surface parameters of the spiral bevel gears are determined by tooth cutting operation. To obtain these parameters in contact position of the coupling gears, it is necessary to take the two gear parameters into the same coordinate system. The assembly relationship of the pinion and gear is shown in Figure 1.

In Figure 1, \(p_p\) and \(p_\theta\) are the unit vectors along the axes direction of pinion and gear, \(\theta_e\) is the angle between the entraining speed and the short axis of the contact ellipse, and \(\theta_s\) is the angle between the sliding speed and the short axis of the contact ellipse.

Basic equations

The relative sliding speed and entraining speed between pinion and gear can be obtained by the following formula:
\[
\begin{align*}
\{ u_s & = \omega_p (p_p \times R_{bg}) - \frac{Z_p}{Z_g} \omega_p (p_g \times R_{bg}) \\
\{ u_c & = \frac{1}{2} \left[ \omega_p (p_p \times R_{bg}) + \frac{Z_p}{Z_g} \omega_p (p_g \times R_{bg}) \right]
\end{align*}
\]

The velocities of the two surfaces in the direction of the elliptical axis can be expressed as:

\[
\begin{align*}
u_{x1} & = |u_e| \cos(\theta_e) - 0.5|u_e| \cos(\theta_e) \\
u_{y1} & = |u_e| \sin(\theta_e) - 0.5|u_e| \sin(\theta_e) \\
u_{x2} & = |u_e| \cos(\theta_e) + 0.5|u_e| \cos(\theta_e) \\
u_{y2} & = |u_e| \sin(\theta_e) + 0.5|u_e| \sin(\theta_e)
\end{align*}
\]

The following deformation coordination equation and load balance equation are used to get the load distribution.

\[
\begin{align*}
\Delta \phi_i & = \frac{\Delta \xi_i}{|R_{bg} \times p_{bg}|} \cdot \left( R_{bg} \times p_{bg} \right) \cdot n_{bg}' + \Delta \phi_{\text{whg}} \\
\Delta \phi_i & = \Delta \phi_1 \cdots = \Delta \phi_{N_c} \\
T_g & = \sum_{i=1}^{N_c} \left| (R_{bg} \times p_{bg}) \cdot n_{bg}' \right| F_i
\end{align*}
\]

Where, \(N_c\) is the number of meshing teeth, \(\Delta \xi_i\) is the component of displacement in the direction of normal vector \(n_{bg}'\), \(F_i\) represents the engagement force of the \(i\)-th tooth, the total torque of each tooth is the torque of the gear \(T_g\). Detailed information of the dynamics process solving equation (1–3).21,28

Although the Archard’s wear formula has been widely recognized, with the development of nano-tribology, more and more experiments have proved that the wear is related to energy. Arrhenius-based wear formula has been further developed, such as Bernd and Mark,27 Risken and Frank.29 They found that the wear rate can be obtained by the following formula.

\[
W_{\text{abr,adh}} = e^{\frac{-E_a}{k_B T}}
\]

The subscripts and indicate the abrasion and abrasive wear, respectively, \(E_a\) is the activation energy, \(k_B\) is the Boltzmann constant, and \(T\) is the absolute temperature (in kelvins). Assuming a wear coefficient \(C_k\), the wear formula is as follows

\[
W(x, y, t) = C_k \int u_e(t)e^{\frac{-E_a}{k_B T(x, y)}} dt
\]

Some simplified assumptions are set to calculate wear, (1) Wear occurs where the gap is 0, (2) the wear rate is in accordance with equation (5) and (3) The abrasive particles generated by wear do not affect subsequent wear, and it is will be discharged quickly with relative movement.

\(E_a\) decreases with increasing shear force. In the simplest form, the reduction is proportional to the shear stress.27

\[
E_a = E_{\alpha 0} - V_{\alpha} \tau
\]

\(E_{\alpha 0}\) is the initial activation energy, and it is taken as 0.983 eV in the simulation. \(V_{\alpha}\) is empirical material constant, and its value is \(10^{-29} - 1.5 \times 10^{-28} m^3\).

In order to calculate the wear between the surfaces, parameters such as film thickness, pressure, shear stress and flash temperature need to be obtained by solving the Reynolds equation and temperature equation. For spiral bevel gears, the velocity vector is not parallel to the short axis of the contact ellipse, and the entraining speed can be decomposed into \(x\) and \(y\) directions (\(u_{ex}\) and \(u_{ey}\)). The Reynolds equation considering the direction of velocity vectors under isothermal conditions can be expressed as,

\[
\frac{\partial}{\partial x} \left( \frac{\rho h^3}{12 \eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\rho h^3}{12 \eta} \frac{\partial p}{\partial y} \right) = u_{ex} \frac{\partial (\rho h)}{\partial x} + u_{ey} \frac{\partial (\rho h)}{\partial y} + \frac{\partial (\rho h)}{\partial t}
\]

Figure 1. Contact relationship in spiral bevel gears.
The film thickness (or gap) equation is as follows,
\[
\begin{align*}
    h &= h_0(t) + \frac{x^2}{2R_x} + \frac{y^2}{2R_y} + v_e(x,y,t) + s(x,y,t) \\
    s(x,y,t) &= \delta_1(x,y,t) + \delta_2(x,y,t) + W_1(x,y,t) + W_2(x,y,t)
\end{align*}
\] (8)

Where \( h_0 \) is the normal approximation, which is determined by the load balance equation, \( R_x \) and \( R_y \) are the equivalent curvature radii on the ellipse axis \( x \) and \( y \), respectively, \( v_e \) is the elastic deformation, \( s \) is the composite surface topography, \( \delta \) represents the asperity height, \( W \) represents the wear height, subscripts 1 and 2 represent the two contact bodies.

The elastic deformation is as follows:
\[
v_e(x,y,t) = \frac{2}{\pi E} \int_{\Omega} \frac{p(\xi,\zeta)}{\sqrt{(x-\xi)^2 + (y-\zeta)^2}} d\xi d\zeta
\] (9)

Isothermal viscosity equation is used in solving Reynolds equation,
\[
\eta = \eta_0 e^{\alpha p}
\] (10)

The density is also solved using an isothermal pressure density formula
\[
\rho / \rho_0 = 1 + \frac{Ap}{1 + Bp}
\] (11)

In the formula above, \( \rho_0 \) is the lubricant density at normal temperature and pressure, \( A \) and \( B \) are the experimental constants. \( A = 0.6 \times 10^{-9} \text{m}^2/\text{N}, \quad B = 1.7 \times 10^{-9} \text{m}^2/\text{N}. \)

The load balance equation is as follows
\[
w(t) = \int_{\Omega} p(x,y,t) dx dy
\] (12)

Detail solutions of the Reynolds equations can refer to Pu et al.\(^23\) and Hu and Zhu.\(^30\) The temperature and shear forces can be solved using the pressure and film thickness calculated by Reynolds equation.

In the EHL region, the Bair-Winer’s rheological model is used,
\[
\dot{\gamma} = \frac{\tau}{G_s} - \frac{\tau_L}{\eta} \ln(1 - \frac{\tau}{\tau_L})
\] (13)

\( G_s \) and \( \tau_L \) are the limiting shear elastic modulus and limiting shear stress of the lubricant, respectively, which are functions of pressure and temperature, and they can be calculated by the Dyson’s empirical formula, as follows,
\[
\begin{align*}
    G_s(p, T) &= \frac{1.2p}{2.52 + 0.024T} - 10^{-9} \\
    \tau_L(p, T) &= 0.25G_s
\end{align*}
\] (14)

In EHL region, the shear rate is generally considered to be inversely proportional to film thickness. In boundary lubrication region, the friction coefficient is usually set to be a constant (take 0.07–0.15).

A point heat source integration method proposed by Carslaw and Jaeger,\(^31\) which ignores the influence of thermal convection has been widely accepted. The following fast heat source model\(^32–34\) is used,
\[
\begin{align*}
    \bar{T}_{1}(\xi) &= T_{b1} + \left( \frac{1}{\pi \rho_e C_{AH1} k_A} \right)^{0.5} \int_{-\infty}^{\xi} \frac{k_f}{T_{2}(\lambda)} \left[ T_{2}(\lambda) - T_{b1} \right] + \frac{q(\lambda)}{2} \times \frac{d(\lambda)}{(\xi - \lambda)^{0.5}} \\
    \bar{T}_{2}(\xi) &= T_{b2} + \left( \frac{1}{\pi \rho_e C_{BH2} k_B} \right)^{0.5} \int_{-\infty}^{\xi} \frac{k_f}{T_{1}(\lambda)} \left[ T_{1}(\lambda) - T_{b2} \right] + \frac{q(\lambda)}{2} \times \frac{d(\lambda)}{(\xi - \lambda)^{0.5}}
\end{align*}
\] (15)

The two surfaces in equation (15) are assumed to be 1, 2, represented by the subscript. So, \( \bar{T} \) is the surface temperature, \( T_b \) is the bulk temperature, \( \rho, C \) and \( k \) are the density, specific heat and thermal conductivity. The generation of \( q \) is based on Francis.\(^35\) The distribution of heat flow is according to the Plint.\(^36\)

The friction calculation formula can be expressed in the following form:
\[
\begin{align*}
    F_c &= F_{lc} + F_{cc} \\
    f &= F_{c}/w
\end{align*}
\] (16)

\( F_{lc} \) and \( F_{cc} \) represent the frictional force in the fluid lubrication area and the frictional force in the boundary lubrication area, respectively.

**Numerical simulation process**

In the present study, in order to consider the transient squeezing actions along the meshing path caused by the variation of speed, contact load and radius of curvature, the wear process in a mesh cycle for one pair of teeth is calculated in a continuous way. The Figure 2 shows the wear simulation process. First, the contact path from engaging-in to engaging-out is divided into \( n_{\text{path}} \) discrete points \((n_1, n_2, \ldots, n_{\text{path}})\), the reason will be discussed later in Chapter 4.1). At the initial point (engaging-in point \( n_1 \)), the film thickness,
pressure distribution and flash temperature can be obtained based on these basic equations (7–16) under steady condition (Figure 3).

After that, the lubrication behavior and surface wear at the following meshing point $n_2$ can be calculated by considering the transient squeezing action from the previous meshing point $n_1$. Repeat this process, until the wear at the $n_{\text{path}}$ points (engaging-out point) is obtained. As a result, the wear in a mesh cycle for one pair of teeth is computed.

The contact point of spiral bevel gears is an elliptic point contact in the simulation. Three-layer meshes are used in the model, the first one is the discrete grid for Reynolds equation, and the other two meshes are the discrete roughness grids for the two surfaces. Because the speed directions of the two contact bodies do not coincide, in order to save the number of grids and to ensure the continuity of the grid movement under continuous motion, independent coordinate systems are introduced for the three kinds of meshes. The mesh numbers are as follows.

As shown in the figure above, the contact center is chosen as the zero point for the three coordinate systems. For the solution of Reynolds equation, the short
axis of contact ellipse is chosen as the x-axis. For the coordinate systems of contact bodies, the speed vector direction of the surface is along the x direction. When the speed directions are fixed, the simulation wear track can be fixed. Meanwhile, the length of the surfaces in the surface coordinate system only needs to be larger than $2b \cos(\theta)$, to meet the simulation needs. Even if the direction of the speed is changed, this method can meet the simulation requirements with a small number of meshes. It is assumed that the subscript $re$ represents the coordinate system of the Reynolds equation, and $sf$ represents the coordinate system of the asperity surface height, the angle between the two coordinate systems is $\theta$. The coordinate transformation relationship is as follows.

$$\begin{bmatrix} x_{re}, y_{re} \end{bmatrix} \cdot \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \Rightarrow \begin{bmatrix} x_{sf}, y_{sf} \end{bmatrix}$$

(17)

It should be noted that the mesh of the surface should be denser than that of the Reynolds solving region, and different meshes can interact with each other through interpolation and mapping.

**Model validation**

Because it is difficult to measure the surface wear of spiral bevel gears, and the key of the present wear model is considering the entrainment speed angle, a ball-on-disk wear experiment with a speed angle, 90°, is conducted to verify the present wear model.

**Experiment introduction**

The surfaces of ball and disk used in the experiments are machined, and the ball has been polished. Diameter of the ball is 38 mm. Materials of the ball and disk are GCr15 bearing steel, and the ball has been hardened after heat treatment. Root mean square roughness of the ball and disk are 0.05 $\mu$m and 0.30 $\mu$m respectively. The load is set to 400 N, the speeds of the disk and ball are 0.12 m/s and 0.08 m/s, respectively. The angel between the two speeds is fixed at 90°. The lubricant oil is applied to the surface of the disk, as shown in Figure 4. Base oil without additives is used, the viscosity is 0.023 Pa·s, and the pressure-viscosity coefficient is 1.25E-8 Pa$^{-1}$. The experiment is conducted at room temperature (27°C).

The wear experiments are performed for 64 min and 128 min, respectively. The topography of disk surface is scanned using an optical profiler. Since the hardness of the ball is greater, it is assumed that the ball is not worn. The wear coefficient of ball is also set to be 0 in the simulation. Meanwhile, the change of the friction coefficient during the wear process is recorded. More information about the laboratory device.

**Surface evolution of the disk over time**

256 × 256 grids are used in the simulation. The solving domain is $-2.0 \, a$ to $2.0 \, a$ in x direction, and $-2.0 \, b$ to $2.0 \, b$ in y direction, for the Reynolds equation. The verification logic is as follows, the acceleration wear coefficient $k_1$ is obtained by fitting the result of 64 min wear. It is assumed that the wear volume or wear profile obtained after $n_c$ cycles are close to the result of 64 min wear. If the wear volume or wear profile obtained after 2$n_c$ cycles are close to the experiment results of 128 min wear, indicating the prediction result of the present model is reasonable.

The test results are shown in A Figure 5. The 64-minutes wear volume is used to fit the wear coefficient and the wear simulation lasts for 3000 cycles. The simulation cycles corresponding to 64 min is 1000 cycles. The simulation results are shown as follows.

It can be seen from Figure 6 that there are scratches inclined to the trajectory direction, which is similar to the experiment results. With the increase of wear simulation cycles, the wear gradually increases, and the inclined scratches become obvious. Because the wear track is shallow, the y-z section is greatly affected by
the roughness, so the average cross section is used to show the wear. Average wear profiles in \( y-z \) section are shown in Figure 7.

It appears from Figure 7 that the wear track is not symmetrical, which is different from the wear track under traditional rolling and sliding contact.\(^{37}\) And the wear on the left side is greater because of its closer to the exit area. At 1000 wear cycles, the shape of the average wear track is not in good agreement with the experimental results. This may be caused by the random roughness error. Meanwhile, the simplified assumptions made by the model also have a certain impact, but with the increase of wear cycles, the simulation results are getting closer to the experimental tracks. Overall, the wear track predicted by the simulation show good agreement with the experimental results.

Figure 8 shows the evolution of film thickness and surface flash temperature distributions during the wear simulation process. The white area in the film indicates the asperity contact, and the white area in flash temperature indicates that the temperature rise is higher than 65°C. It can be seen that with the influence of wear, the asperity peaks on the disk are worn away, the film thickness will increase, and the flash temperature will decrease. After 3000 cycles of wear, the contact area is still partly in boundary lubrication, and the boundary lubrication area is mostly concentrated on the left entrance area. According to the typical point EHL film distribution for which the velocity in \( y \) direction is 0, the film thickness is symmetrical along the \( x \) direction, and there is a constriction at the outlet which causes the film at the outlet is thinner than that at the

Figure 5. Wear tracks after different running times (speed angle 90°).

Figure 6. Wear simulation results at different cycles (speed angle 90°).
inlet area. When the velocity in \( y \) directions is not 0, the distribution of film is no longer symmetrical along the \( x \) direction. Because the film thickness at the outlet is thinner than that at the inlet, the wear on the side of the exit direction are more severe. The distributions of flash temperature are also asymmetrical. Due to the influence of velocity angle, the temperature distribution of disk will be offset relative to the movement direction of ball.

Figure 9 shows the change of friction coefficient during the wear process. The simulation friction coefficient is very close to the measure ones when the wear is not happened, and the friction coefficient decreases during the wear process. The difference is that the simulation results underestimate the friction coefficient after wear.

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**Figure 7.** Average wear tracks: (a) Experiment and (b) Simulation.

**Figure 8.** Film thickness and flash temperature as a function of time.

**Figure 9.** Friction coefficient as a function of time.
This may be that the simulation environment is very ideal, and the influences of wear debris are ignored. In the present model, wear is an added term of the film thickness equation, so the film thickness may be overestimated when calculating friction after wear. But in general, the simulation results appear to reasonable.

## Wear analysis of spiral bevel gears

### Simulation parameters

The gear parameters for the simulation are shown in Table 1.

Taking the torque of 100 N·m and 25 rpm (gear speed) as an example, the dynamic parameters along the meshing path are shown in Figure 10.

It can be seen from Figure 10(a) that along the meshing path, the speeds of the pinion and gear are almost the same in the direction of the major axis of the contact ellipse (y direction), but the speed changes greatly in x directions. In x direction, the velocity of pinion increases with the rotation of gear, while the velocity of the gear decreases. Figure 10(b) and (c) shows the evolution of radii and load along the meshing path. Based on the load and radii evolution, the contact half-width distribution on the meshing path can be obtained (Figure 10(d)).

According to Figure 10(d), the contact width may be more than double compared to the engaging-in point during the meshing process, if the solving region is not long enough, a large error may occur. Therefore, in the simulation, the solving range is $-4.3a_1-2.7b_1$ in $x$ direction, and $-3.5b_1-3.5b_1$ in $y$ direction, where $a_1, b_1$ are the contact half widths when gear angle is 0°. The total solution domain is divided into $256 \times 256$ grids. When the torque is 100 N·m, $a_1 = 0.3655nm$, $b_1 = 1.081mm$. The grid gap for Reynolds equation is 0.01 mm ($dx$) and 0.03 mm ($dy$).

According to TCA results, the meshing path is about 9.6mm. During the single-tooth meshing process, the gear rotates about 0.19 rad. So, dt can be calculated by discretizing the path into 2001 points, and its value is about 0.363 ms. The length of the surfaces should be greater than the sum of the contact path and the half-contact width at both ends. The optical profiler cannot scan such a long path, directly. In present study,
a 13 mm × 4.2 mm surface is generated by rotation and array of a partial scan. The surface data is composed of 6501 × 3001 points, the distances between two neighbor points are 0.002 mm (longitudinal direction) and 0.0014 mm (width direction), which is much smaller than the grid gap for the Reynolds equation. Because the gears are processed in the same way, same surface roughness is used for pinion and gear, and root mean square roughness is set to be 0.32 μm. Gears and the scanned surface are shown in Figure 11.

The speed effect
To study the effect of speed on wear, the torque is fixed at 100 N·m in the simulation, and the rotation speed of gear is set to 25, 100, 400 and 1600 rpm, respectively. The gear meshing at these four speeds is simulated, and the wear coefficient is set as 0.005 in the simulation. A set of cases without wear is also simulated for comparison. Figure 12 shows the change of film thickness ratio along the contact path during three engagement processes at different rotation speeds.

In the simulation, the first 500 cycles are used to iterate to convergence, and the next 2000 steps are used to simulate the actual contact process. Therefore, the 500–2500 cycles, 3000–5000 cycles, and 5500–7500 cycles represent the three-meshing process because the contact path from engaging-in to engaging-out is divided into 2000 discrete points as shown in section 2.2. After the wear is introduced, the changing trends of film thickness during each meshing process are basically the same. Figure 13 shows the changes of film thickness during a single meshing process (500–2500 cycles). The film thickness ratio in this research is the ratio of film

![Figure 11. Pinion, gear and the scanned surface.](image)

![Figure 12. Film thickness ratio as a function of simulation cycles with wear.](image)
thickness to the root mean square roughness after deformation.

It is clear in Figure 13 that the film thickness ratio increases with the increase of rotation speed in the fixed meshing position. During the meshing process, the film thickness ratio initially decreases and then increases, when the gear rotates to about 0.15 rad, due to the sudden change in load (see Figure 10), the film thickness ratio increases rapidly. Compared to the film thickness ratio curve before wear with that after wear, it can be found that the film thickness ratio has increased after the introduction of wear, especially at the low speed of 25 rpm. But the introduction of wear will not change the evolution trend of film thickness ratio along the meshing path.

As can be seen in Figure 14, it is obvious that the friction coefficient decreases with the increase of speed. During the meshing process, the friction coefficient increases first and then decreases. After the introduction of wear, the overall friction coefficient has decreased. The friction coefficient changes significantly along the meshing path before and after the introduction of wear. Under low-speed conditions, the oil film is thin, and asperity contact shear stress is the main source of friction. When wear is introduced, the lubrication film is enhanced, and asperity contact decreases. So, the trend at low speed becomes closer to the trend at high speed. Similar friction evolution trend can be seen in Cao et al.38

Figure 15 shows the distribution of film thickness and flash temperature at different rotation angles of the gear during the first meshing process (rotation speed = 100 rpm). The evolutions of film and flash temperatures provide further support for friction and wear mechanism along the meshing path. Obviously, the contact width is affected by the load, and the average film thickness is affected by the combination of load and speed. In the transmission process, when the gear angle is between 0.0 and 0.10 rad, the relative sliding speed shows a trend of decreasing first and then increasing (the turning point is at 0.065 rad), and the load first increase and then remains stable (the turning point is at 0.05 rad). Combining the dynamic parameters with the above figure, it can be concluded that during the meshing-in period (about 0–0.065 rad), due to the continuous increase of the load, the film becomes thinner, and the flash temperature decreases due to the decrease of relative sliding speed. Although the increase
of load and the thinning of film may increase the flash
temperature, the effect of sliding speed may be greater.
During the meshing-out period (about 0.05–0.14 rad),
due to the increase of the total speed and sliding speed,
the film becomes thicker, and the flash temperature
increases. The gear angle at 0.15 rad is the position in
multi-tooth engagement, the other tooth takes away
some torque, resulting in a sudden reduction in load,
the film becomes thicker and flash temperature
decreases.

Figure 16 is the wear rate of gear and pinion with
the rotation of gear during the first meshing cycle (500–
2500 cycles).

It should be pointed out that the materials and wear
coefficient are the same for pinion and gear in the
simulation, so the wear rates are almost identical,
which may be different from that in engineering. It can
be seen from Figure 16 that the speed has a great influ-
ence on wear rate. In a single mesh, the higher the
speed, the smaller the wear rate. Generally, the film
thickness increases with the increase of speed, thereby
reducing asperity contact. It can be found that the wear
rate along the gear angle is very uneven. During the
meshing-in period (0–0.065 rad), due to the continuous
increase of the load and the continuous decrease of
relative sliding speed, the wear rate first increase and
then decrease. During the meshing-out period (about
0.065–0.14 rad), the load does not change much, both
the total speed and relative sliding speed show an
increasing trend. The increase in speed will improve the
lubrication performance and reduce wear, while the
increase in relative sliding speed will increase wear. So,
the wear rate also shows a trend that first increase and
then decrease. At low speeds, the wear rate does not
increase significantly during the meshing-in period.
Comparing the wear rate at the four speeds, it can be
found that the maximum position of wear rate changes
with the speed. At the speed of 25 rpm, the maximum
wear rate occurs in the meshing-out period. At the speed of 1600 rpm, the maximum wear rate occurs in the meshing-in period. And at 400 rpm, the maximum wear rate is almost the same in the meshing-in and meshing-out period.

The morphological evolution of the surfaces with increasing meshing cycles (rotation speed = 100 rpm, load = 100 N/m) reveals the mechanism of spiral bevel gear wear (Figure 17). Generally, for spur gears, the wear at the tooth tip and root is greater than that at the pitch. A similar effect is achieved in our simulation for the spiral bevel gears. The wear tracks for pinion and gear are obviously different. The track of pinion is more curved. This may be attributed to that the speed of pinion is higher, and its speed direction changes more with gear angle.

The load effect
To compare the effect of load on wear, the speed is fixed at 200 rpm in the simulation, and the load is set to 12 N-m, 50 N-m and 100 N-m, respectively. The gear meshing at these three loads is simulated. When the gear rotates to 0.145 rad, there will be a sudden decrease in load, resulting in very low wear rate. These little wears may have limited effect on the gear performance. Therefore, the gear angle of 0–0.145 rad during a mesh cycle is analyzed in the subsequent analysis.

Figure 18 shows the changes of film thickness during the first meshing process for different loads. Under light load, the change of film thickness ratio along the meshing path is not obvious, but under heavy load, the film thickness ratio has a downward trend at the meshing-in period. After the wear is introduced, the film thickness ratio under heavy load has increased, while the film thickness under light load remains invariable. This may be because wear is related to the temperature, the smaller the load, the less the shear heat.

As can be seen from Figure 19(a), there is a rising phase for friction coefficient in the meshing-in process, and the friction coefficient increases with the increase of load. After the wear is introduced (Figure 19(b)), the increase of friction coefficient is scarcely observed under the load of 12 N-m. This may be because that the
friction coefficient decreases after the introduction of wear, which offsets the increase trend of friction. It is obvious that large load would cause a greater friction coefficient.

It is evident in Figure 20 that load has a great influence on wear. Increasing load results in an increase in wear rate. The evolution trends of wear rate under different loads are basically the same in Figure 20. The location of maximum wear rate also varies under different loads, compared to the cases under different speeds.

Conclusions

A wear model of spiral bevel gears has been introduced in this paper. The model considers the time-varying load, geometry, speeds, lubricant, elastic deformation, real machined surfaces and other factors during the meshing process. And it is verified through ball-disk experiments. Numerical wear simulations are executed to reveal the variations of gap and flash temperature distribution along the meshing path. The influence of rotation speed and load on wear and the evolution of surfaces have been studied. The following conclusions can be drawn.

1. Under mixed lubrication, when the velocity directions of the two contact bodies are staggered, the distribution of wear tracks will be asymmetric, and the wear are more severe on the side close to the outlet area.
2. Speed and load have a great influence on wear. Increasing the speed or reducing the load can effectively reduce wear.
3. Whether it is in the process of meshing-in or meshing-out, the wear rate increases first and then decreases.
4. There is a position with maximum wear rate in the meshing process, and the position is affected by the load and speed.

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**Appendix**

**Nomenclature**

\( p_p, p_g \) = the unit vectors along the axe’s direction of pinion and gear

\( \theta_e \) = the angle between the entraining speed and the short axis of the contact ellipse

\( \theta_s \) = the angle between the sliding speed and the short axis of the contact ellipse

\( u_1, u_2 \) = the surface velocity of the two contact bodies

\( a \) = semi-axis of the Hertzian contact ellipse in the \( x \)-direction

\( b \) = semi-axis of the Hertzian contact ellipse in the \( y \)-direction

\( E' \) = effective Young’s modulus

\( h, H \) = dimensional and dimensionless local film thickness (or gap), \( H = h/a \)

\( p, P \) = dimensional and dimensionless pressure, \( P = p/P_h \)

\( P_h \) = nominal maximum Hertzian contact pressure

\( t \) = time

\( V \) = surface elastic deformation

\( \alpha \) = pressure-viscosity exponent

\( w \) = load

\( E_a \) = activation energy

\( \tau \) = the shear stress

\( v_e \) = the elastic deformation

\( \delta_1, \delta_2 \) = asperity height of Surface 1 and Surface 2, respectively

\( \eta, \eta_0 \) = viscosity, and viscosity under ambient condition

\( \rho_0 \) = density under ambient condition

\( T_b, T_{b1}, T_{b2} \) = bulk temperature on Surface 1 and Surface 2, respectively

\( T_1, T_2 \) = local temperature on Surface 1 and Surface 2, respectively

\( \mu_1, \mu_2 \) = Poisson’s ratios of body 1 and body 2, respectively

\( W_1, W_2 \) = wear depth of body 1 and body 2, respectively