Universe with the linear law of evolution

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Abstract

The model of the homogenous and isotropic universe is considered in which the coordinate system of reference is not defined by the matter but is a priori specified. The scale factor of the universe changes following the linear law. The scale of mass changes proportional to the scale factor. The temperature of the relativistic matter changes inversly proportional to the root of the scale factor. The model under consideration avoids the flatness and horizon problems. The predictions of the model are fitted to the observational constraints: Hubble parameter, age of the universe, magnitude-redshift relation of the high-redshift supernovae, and primordial nucleosynthesis.

1 Introduction

As known [1, 2], the Friedmann model of the universe has fundamental difficulties such as the flatness and horizon problems due to the slow growth of the scale factor of the universe. In the Friedmann universe, the coordinate system of reference is associated with the matter of the universe. The evolution of the scale factor of the universe is given by

\[ a \sim t^{1/2}, \quad a \sim t^{2/3} \] (1)

where the first equation corresponds to the matter as a relativistic gas, and the second, to the dust-like matter. Growth of the scale factor of the universe governed by the power law with the exponent less than unity is slower than growth of the horizon of the universe \( h \sim t \) that causes the flatness and horizon problems.

To resolve these problems an inflationary episode in the early universe is introduced [1, 2]. However there is another way of resolving the problems. This is based on the premise that the coordinate system of reference is not defined by the matter but is a priori specified.

2 Theory

Let us consider the model of the homogenous and isotropic universe. Let us assume that the coordinate system of reference is not defined by the matter but is a priori specified. Let the coordinate system of reference be the Euclidean space with the spatial metric \( dl \) and absolute time \( t \)

\[ dl^2 = a(t)^2(dx^2 + dy^2 + dz^2), \quad t. \] (2)

That is the coordinate system of reference is the space and time of the Newton mechanics, with the scale factor of the universe is a function of time. Since the metric (2) is not defined
by the matter, we can a priori specify the evolution law of the scale factor of the universe. Let us take the linear law when the scale factor of the universe grows with the velocity of light

$$a = ct.$$  \hfill (3)

In the Friedmann universe, the law (3) corresponds to the Milne model \[3\] which is derived from the condition that the density of the matter tends to zero $\rho \to 0$. Here Eq. (3) describes the universe in which the evolution of the scale factor do not depend on the presence of the matter. Hence the density of the matter is not equal to zero $\rho \neq 0$. The total mass of the universe relative to the background space includes the mass of the matter and the energy of its gravity. Let us adopt that the total mass of the universe is equal to zero, that is the mass of the matter is equal to the energy of its gravity

$$c^2 = \frac{Gm}{a}.$$ \hfill (4)

Allowing for Eq. (3), from Eq. (4) it follows that the mass of the matter changes with time as

$$m = \frac{c^2a}{G} = \frac{c^3t}{G},$$ \hfill (5)

and the density of the matter, as

$$\rho = \frac{3c^2}{4\pi Ga^2} = \frac{3}{4\pi Gt^2}. \hfill (6)$$

Hence the model under consideration yields the change of the scale of mass proportional to the scale factor of the universe. At the Planck time $t_{Pl} = (\hbar G/c^5)^{1/2}$, the mass of the matter is equal to the Planck mass $m_{Pl} = (\hbar c/G)^{1/2}$. At present, the mass of the matter is of order of the modern value $m_0 = c^2a_0/G$.

Let us study time dependence of the temperature of the relativistic matter. Density of the relativistic matter is defined by its temperature as

$$\rho \sim T^4.$$ \hfill (7)

From Eq. (6) it follows that the temperature of the relativistic matter changes with time as

$$T \sim a^{-1/2} \sim t^{-1/2}. \hfill (8)$$

Let us consider the flatness and horizon problems within the framework of the model under consideration. Remind \[1, 2\] that the horizon problem in the Friedmann universe is caused by that the universe observable at present consisted of the causally unconnected regions in the past that is inconsistent with the high isotropy of the background radiation. In the universe under consideration, the size of the universe (the scale factor of the universe) coincides with the size of the horizon during all the evolution of the universe. Hence the presented model avoids the horizon problem.

Remind \[1, 2\] that the essence of the flatness problem in the Friedmann universe is connected with impossibility to gain the modern density of the matter at present starting from the Planck density of the matter at the Planck time. In the presented theory, the density of the matter of the universe changes from the Planckian value at the Planck time to the modern value at the modern time. Hence the flatness problem is absent in the presented theory.
3 Predictions

Write down some relations describing the universe via the cosmological redshift given by

\[ z = \frac{T}{T_0} - 1 \tag{9} \]

where \( T_0 = 2.73 \) K is the modern temperature of the cosmic microwave background. In view of Eq. (8), the scale factor of the universe at the redshift \( z \) is given by

\[ a(z) = \frac{a_0}{(z + 1)^2}, \tag{10} \]

and the age of the universe at the redshift \( z \) is given by

\[ t(z) = \frac{t_0}{(z + 1)^2}. \tag{11} \]

Angular diameter distance at the redshift \( z \) is given by

\[ d(z) = a_0 \left( 1 - \frac{a}{a_0} \right) = a_0 \left( 1 - \frac{1}{(z + 1)^2} \right). \tag{12} \]

Let us estimate the modern age of the universe from Eq. (8). Taking into account change of the electromagnetic constant \( \alpha \), we obtain

\[ t_0 = t_{\text{Pl}} \frac{\alpha_0}{\alpha_{\text{Pl}}} \left( \frac{T_{\text{Pl}}}{T_0} \right)^2. \tag{13} \]

Allowing for that \( \alpha_0 = 1/137, \alpha_{\text{Pl}} = 1, t_{\text{Pl}} = 5.39 \cdot 10^{-44} \) s, \( T_{\text{Pl}} = 1.42 \cdot 10^{32} \) K, the modern age of the universe is equal to \( t_0 = 1.06 \cdot 10^{18} \) s = 33.7 Gyr. In view of Eq. (3), the relation between the Hubble parameter and the age of the universe is given by

\[ H = \frac{1}{a} \frac{da}{dt} = \frac{1}{t}. \tag{14} \]

Then the modern value of the Hubble parameter is \( H_0 = 0.94 \cdot 10^{-18} \) s\(^{-1} = 29 \) km/s/Mpc.

The observed Hubble parameter is derived from the relation for the angular diameter distance. In the Friedmann model, the angular diameter distance at low redshifts is given by

\[ d(z) \sim z \quad z \ll 1. \tag{15} \]

In the presented model, the angular diameter distance at low redshifts is given by

\[ d(z) \sim 2z \quad z \ll 1. \tag{16} \]

From this we must divide the observed Hubble parameter obtained in the Friedmann model by the factor 2. If we adopt the observed Hubble parameter obtained in the the Friedmann model as \( H_0 = 60 \pm 10 \) km/s/Mpc [1], then in the presented model we obtain \( H_0 = 30 \) km/s/Mpc which is in agreement with the above prediction.
Authors of [5] investigate constraints on power-law models of the universe from the present age of the universe, from the magnitude-redshift relation of the high-redshift supernovae, and from primordial nucleosynthesis. Constraints from the current age of the universe [6] and from the high-redshift supernovae data [7] require \(a \sim t\), while constraints from primordial nucleosynthesis [3] require \(T^{-1} \leq t^{0.58}\). Since in the Friedmann universe \(T^{-1} \sim a\), the above constraints are inconsistent within the framework of the Friedmann universe. The presented theory predicts \(a \sim t\) and \(T^{-1} \sim t^{0.5}\) which are in agreement with the above relations.

The age of the universe can be estimated by the observed age of the oldest globular clusters. Age of the globular cluster depends on the distance to the cluster such that a revision of 0.1 mag in the distance scale changes the age of the cluster by 10 \%. Since the angular diameter distance in the presented model is greater than that in the Friedmann model by the factor 2, the distance scale increases by \(5 \log 2 = 1.5\) mag, and the age of the oldest globular clusters in the presented model is 2.5 times greater than that in the Friedmann model. If we adopt the observed age of the oldest globular clusters obtained in the Friedmann model as \(t_{GC} = 14 \pm 2\) Gyr [6], then in the presented model, this yields \(t_{GC} = 14 \times 2.5 = 35\) Gyr that is in agreement with the age of the universe predicted by the presented model.

Deceleration parameter \(q_0\) is defined via the relation of luminosity distances at different redshifts. The high-redshift supernovae data [7] favour \(q_0 = 0\). In this case, the Friedmann and presented models give the same relation of luminosity distances at different redshifts. So the estimate \(q_0 = 0\) obtained from the high-redshift supernovae data within the framework of the Friedmann model is also valid for the presented model.

Inequality \(T^{-1} \leq t^{0.58}\) [5] is obtained from the condition that at the beginning of nucleosynthesis when \(T \sim 80\) keV the age of the universe should be less than the lifetime of neutron \(t \leq 887\) s. The inferred primordial abundances of helium-4 [4] and deuterium [9] require \(T^{-1} \sim t^{0.55}\). When \(T^{-1} < t^{0.55}\), deuterium is overproduced relative to helium-4. Relation \(T^{-1} \sim t^{0.55}\) corresponds to the modern age of the universe \(t_0 = 14\) Gyr. Increase of the modern age of the universe up to \(t_0 = 33.7\) Gyr gives \(T^{-1} \sim t^{0.54}\). From this relation \(T^{-1} \sim t^{0.5}\) predicted by the presented theory is inconsistent with the inferred primordial abundances of helium-4 and deuterium.

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