Non-Perturbative Determination of $c_{SW}$ in Three-flavor Dynamical QCD

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We present a fully non-perturbative determination of the $O(a)$ improvement coefficient $c_{SW}$ in three-flavor dynamical QCD for the RG improved as well as the plaquette gauge actions, using the Schrödinger functional scheme. Results are compared with one-loop estimates at weak gauge coupling.

1. Introduction

Realistic simulation of QCD requires treating the light up, down and strange quarks dynamically. Incorporating a degenerate pair of up and down quarks have become almost standard by now, and a first attempt toward the continuum extrapolation has shown that the deviation of the quenched hadron mass spectrum from experiment [1] is sizably reduced[2]. Adding a dynamical strange quark is the next step, which has become possible with the recent algorithmic development for odd number of fermions[3].

The CP-PACS and JLQCD Collaborations have jointly started a 2+1 flavor dynamical QCD, employing the polynomial HMC (PHMC) algorithm for strange quark and the HMC algorithm for up and down quarks. We choose the renormalization-group (RG) improved action for the gauge fields, in order to avoid the lattice artifact present for the plaquette action[4]. We wish to use a fully $O(a)$-improved action for quarks to control lattice spacing errors. Here we report on a non-perturbative determination of $c_{SW}$ for three-flavor QCD by the Schrödinger functional scheme both for the plaquette and RG-improved gauge actions.

2. Method and Simulations

For the determination of $c_{SW}$, we basically follow the method of ref.[5], except for the choice B for the boundary weight of the RG-improved gauge action [6]. We refer to ref. [5] and references therein for notations in this report.

We mainly use an $8^3 \times 16$ lattice in our determination of $c_{SW}$ for the RG-improved as well as the plaquette action with $N_f = 3$ dynamical quarks at several values of $\beta$. Simulations with $N_f = 4, 2, 0$ are also made for comparison.
We measure the modified PCAC quark masses, \(M\) and \(M'\), and their difference \(\Delta M = M - M'\), at several values of \(c_{SW}\) and \(K\). We have taken these parameters to realize \(M = 0\) by an interpolation, except at strong couplings for the case of \(N_f = 3\), where an extrapolation to \(M = 0\) is necessary as shown in Fig. 1.

From the linear fit of \(\Delta M\) as a function of \(M\) and \(c_{SW}\):

\[
\delta M = a_0 + a_1 M + a_2 c_{SW},
\]

we obtain the \(O(\alpha)\) improvement coefficient \(c_{SW} = (\Delta M^{(0)} - a_0)/a_2\), where \(\Delta M^{(0)} = 0.000277\), marked by the horizontal dotted line in Fig. 1, is the tree-level value of \(\Delta M\) on a \(8^3 \times 16\) lattice. Note that the dependence of \(\Delta M\) on \(c_{SW}\) becomes weaker at stronger couplings, so that the determination of \(c_{SW}\) is more difficult, and hence the error is larger, at stronger couplings.

3. Results

In the upper plot of Fig. 2 we show the non-perturbative value of \(c_{SW}\) as a function of the bare gauge coupling \(g_0^2\) for the RG-improved gauge action with \(N_f = 3\) (circles), 2 (diamonds) and 0 (squares), together with the one-loop estimate (solid line) and the mean-field (MF) estimate (dashed line) used in ref. [2]. Similarly, results for the plaquette action with \(N_f = 3\) (circles) and 0 (squares) are given in the lower plot of Fig. 2, together with the one-loop estimate (solid line) and the non-perturbative values by the Alpha collaboration with \(N_f = 2\) (dotted line) [5] and 0 (long-dashed line) [7].

In both cases, the non-perturbative values of \(c_{SW}\) are almost \(N_f\) independent at weak coupling while they become larger for smaller \(N_f\) at strong coupling. This tendency can be clearly seen in Fig. 3, where \(c_{SW}\) is plotted as a function of \(N_f\) for the RG action (open symbols) and the plaquette action (solid circles).

4. Comparison with perturbative estimates

At first sight, the non-perturbative \(c_{SW}\) seems to undershoot the one-loop estimate at weak coupling for the RG action, while it converges smoothly from above for the plaquette action.

We have found that the discrepancy seen for the RG action is caused by the one-loop \(O(a/L)\) error in \(c_{SW}\), which becomes leading after the \(O(a/L)\) error at tree level is removed by requiring \(\Delta M = \Delta M^0\). In Fig. 4, the non-perturbative \(c_{SW}\) is compared with the one-loop estimate we have calculated on the same lattice size employed in the simulation, \(8^3 \times 16\). As seen from the figure the non-perturbative value agrees with the one-loop estimate much better on the \(8^3 \times 16\) lattice than in the infinite box. Note that the \(O(g_0^2 a/L)\) contribution to \(c_{SW}\) slightly depends on \(N_f\) through the fermion tadpole in the presence of the background gauge field of the Schrödinger functional scheme. Such an \(N_f\) dependence is inherent in the \(O(a)\) improvement.
5. Discussion

We have determined the non-perturbative value of \( c_{SW} \) for the RG action at several gauge couplings with \( N_f = 3, 2, 0 \). In order to obtain an interpolation formula of \( c_{SW} \) as a function of \( g_0^2 \), we have to eliminate large \( O(g_0^2 a/L) \) corrections to \( c_{SW} \) present for the RG action. We are currently investigating this problem.

We are also measuring the hadron spectrum for the RG action at \( \beta \equiv 2 \) with \( N_f = 3 \) using the preliminary value of \( c_{SW} \), in order to determine the corresponding lattice spacing.

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