Probing CPT symmetry in transitions with entangled neutral kaons

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Abstract. A new kind of CPT test for transitions in the neutral kaon system is presented, where the exchange of in and out states (and CP conjugation), required for a direct and genuine CPT test, is performed exploiting the entanglement of the kaon pair produced at a φ-factory. Using this method it would be possible for the first time to directly test the CPT symmetry in transition processes between meson states, rather than comparing masses, lifetimes, or other intrinsic properties of particle and anti-particle states. The proposed test in the neutral kaon system is very clean and fully robust, and might shed light on possible new CPT violating mechanisms, or further improve the precision of the present experimental limits. It could be implemented at the DAΦNE facility in Frascati, where the KLOE-2 experiment can reach a statistical sensitivity of $O(10^{-3})$ on the newly proposed observable quantities.

1. Introduction

CPT symmetry, i.e. the symmetry under the combination of charge conjugation (C), parity (P), and time reversal (T) transformations, at present appears to be the only discrete symmetry of Quantum Mechanics respected in Nature. This result has a very solid theoretical foundation in the well known CPT theorem [1, 2, 3, 4], ensuring exact CPT invariance for any quantum field theory formulated on flat space-time assuming (1) Lorentz invariance, (2) Locality, and (3) Unitarity (i.e. conservation of probability).

A violation of the CPT symmetry would have a dramatic impact on our present theoretical picture and would definitely constitute an unambiguous signal of a New Physics framework, thus strongly motivating both experimental searches and theoretical studies on this subject.

CPT invariance has been confirmed by all present experimental tests, particularly in the neutral kaon system where strong limits have been set to a variety of possible CPT violation effects which might arise in a quantum gravity scenario [5, 6, 7, 8, 9, 10]. The best limits on the $\delta$ parameter expressing CPT violation in the $K^0-\bar{K}^0$ mixing matrix, i.e. in the standard Weisskopf-Wigner approach [11] are obtained in the CPLEAR experiment for $\Re \delta$ [12], and using the Bell-Steinberger relation for $\Im \delta$ [13, 14], yielding a stringent limit on the difference of mass terms for $K^0$ and $\bar{K}^0$: $|m_{K^0} - m_{\bar{K}^0}| < 4 \times 10^{-19}$ GeV at 95% c.l. [5].

The CPT violating probe has been, however, often limited to a difference of masses (and other intrinsic properties) for a particle and its anti-particle, i.e. to diagonal mass terms. In many physical phenomena the perturbing effect does not appear at first order in perturbation theory: it would be sufficient that the perturbation breaks a symmetry of the non-perturbed states. This vanishing effect at first order for the diagonal elements, like e.g. the case of the...
Here a new kind of CPT test for transitions in the neutral kaon system is discussed where the exchange of in and out states (and CP conjugation), required for a direct and genuine CPT test, is performed exploiting the entanglement of the kaon pair produced at a φ-factory (see Ref. [16] for a more detailed discussion). This methodology has been proposed for a direct test of the T symmetry in the same context [17], similarly to the one adopted for the performed test in the B meson system at B-factories [18, 19, 20, 21, 15]. The decay is not an essential ingredient for a non-vanishing effect and it is only used for filtering the appropriate initial and final states of the neutral kaon transition [22]. Explicitly, in the standard Weisskopf-Wigner approach to this system, our CPT-violating effects can be connected to the η parameter, a genuine CPT-violating effect independent of ΔΓ = ΓS − ΓL, with ΓS and ΓL the widths of the physical states.

2. Direct test of CPT symmetry with neutral kaons

In order to implement a direct test of the CPT symmetry, the Einstein-Podolsky-Rosen (EPR) entanglement of neutral mesons produced at a φ-factory is exploited. In fact in this case the initial state of the neutral kaon pair produced in φ-factories -ππ decay can be rewritten in terms of any pair of orthogonal states |K+⟩ and |K−⟩:

\[ |i⟩ = \frac{1}{\sqrt{2}} \{ |K^0⟩|\bar{K}^0⟩ − |\bar{K}^0⟩|K^0⟩ \} = \frac{1}{\sqrt{2}} \{ |K_+⟩|K_−⟩ − |K_−⟩|K_+⟩ \} . \tag{1} \]

Here one can consider the states |K+⟩, |K−⟩ defined as follows: |K+⟩ is the state filtered by the decay into ππ (π+π− or π0π0), a pure CP = +1 state; analogously |K−⟩ is the state filtered by the decay into 3π0, a pure CP = −1 state. Their orthogonal states correspond to the states which cannot decay into ππ or 3π0, defined, respectively, as

\[ |\tilde{K}_−⟩ \propto [ |K_L⟩ − η_{ππ} |K_S⟩] \]
\[ |\tilde{K}_+⟩ \propto [ |K_S⟩ − η_{3π0} |K_L⟩] , \tag{2} \]

with ηππ = ⟨ππ|T|K_L⟩/⟨ππ|T|K_S⟩ and η3π0 = ⟨3π0|T|K_S⟩/(3π0|T|K_L). With these definitions of states, it can be shown that the condition of orthogonality ⟨K−|K+⟩ = 0, (i.e. |K_+⟩ ⊥ |K_−⟩) and |K_−⟩ ⊥ |\tilde{K}_−⟩) corresponds to assume negligible direct CP (or CPT) violation contributions, assumption quite well satisfied for neutral kaons [16]. The validity of the ΔS = ΔQ rule is also assumed, so that the two flavor orthogonal eigenstates |K0⟩ and |\bar{K}0⟩ are identified by the charge of the lepton in semileptonic decays, i.e. a |K0⟩ can decay into π−ℓ+ν and not into π+ℓ−ν, and vice-versa for a |\bar{K}0⟩.1

Thus, exploiting the perfect anticorrelation of the states implied by Eq. (1), it is possible to have a “flavor-tag” or a “CP-tag”, i.e. to infer the flavor (K0 or \bar{K}0) or the CP (K_+ or K_−) state of the still alive kaon by observing a specific flavor decay (π+ℓ−ν or π−ℓ+ν) or CP decay (ππ or π0π0) of the other (and first decaying) kaon in the pair.

In this way one can experimentally access – for instance – the transition K0 → K+ or K0 → K− transitions, i.e. the T, CP and CPT conjugated transitions, respectively. Specifically for the CPT symmetry test, one can directly compare the probabilities for the reference transition and the CPT conjugated one through the

1 It is important to underline that both these assumptions can be relaxed for some specific observables, as discussed below.
following ratios of probabilities:

\[ R_{1,CPT}(\Delta t) = P[K_+(0) \rightarrow \bar{K}^0(\Delta t)] / P[K^0(0) \rightarrow K_+(\Delta t)] \]
\[ R_{2,CPT}(\Delta t) = P[K^0(0) \rightarrow K_-(\Delta t)] / P[K_-(0) \rightarrow \bar{K}^0(\Delta t)] \]
\[ R_{3,CPT}(\Delta t) = P[K_+(0) \rightarrow K^0(\Delta t)] / P[\bar{K}^0(0) \rightarrow K_+(\Delta t)] \]
\[ R_{4,CPT}(\Delta t) = P[\bar{K}^0(0) \rightarrow K_-(\Delta t)] / P[K_-(0) \rightarrow K^0(\Delta t)] . \]  

(3)

The measurement of any deviation from the prediction \( R_{i,CPT}(\Delta t) = 1 \) imposed by CPT invariance is a signal of CPT violation.

It is worth noting that for \( \Delta t = 0 \):

\[ R_{1,CPT}(0) = R_{2,CPT}(0) = R_{3,CPT}(0) = R_{4,CPT}(0) = 1 , \]  

(4)

i.e. the CPT-violating effect is built in the time evolution of the system, and it is absent at \( \Delta t = 0 \), within our approximations.

For \( \Delta t \gg \tau_S \), assuming the presence of CPT violation only in the mass matrix \( (\delta \neq 0) \) and nothing else, one gets:

\[ R_{2,CPT}(\Delta t \gg \tau_S) \simeq 1 - 4\Re \delta \]  

(5)

\[ R_{4,CPT}(\Delta t \gg \tau_S) \simeq 1 + 4\Re \delta , \]  

(6)

i.e. the CPT-violating effect built in the time evolution reaches a “plateau” regime and dominates in this limit. It is a genuine effect because \( \Re \delta \) does not depend on \( \Delta \Gamma \) as an essential ingredient [15, 22].

At a \( \phi \)-factory one can define two observable ratios:

\[ R_{2,CPT}^{\exp}(\Delta t) = \frac{I(\ell^-,3\pi^0;\Delta t)}{I(\pi\pi,\ell^-;\Delta t)} \]  

(7)

\[ R_{4,CPT}^{\exp}(\Delta t) = \frac{I(\ell^+,3\pi^0;\Delta t)}{I(\pi\pi,\ell^+;\Delta t)} , \]  

(8)

where \( I(f_1,f_2;\Delta t) \) are the double decay rates into decay products \( f_1 \) and \( f_2 \) as a function of the difference of kaon decay times \( \Delta t [17, 23] \), with \( f_1 \) occurring before \( f_2 \) decay for \( \Delta t > 0 \), and viceversa for \( \Delta t < 0 \).

They are related to the \( R_{i,CPT}(\Delta t) \) ratios defined in eqs (3) as follows, for \( \Delta t \geq 0 \):

\[ R_{2,CPT}^{\exp}(\Delta t) = R_{2,CPT}(\Delta t) \times D_{CPT} \]
\[ R_{4,CPT}^{\exp}(\Delta t) = R_{4,CPT}(\Delta t) \times D_{CPT} \]  

(9)

whereas for \( \Delta t < 0 \) one has:

\[ R_{2,CPT}^{\exp}(\Delta t) = R_{1,CPT}(\Delta t) \times D_{CPT} \]
\[ R_{4,CPT}^{\exp}(\Delta t) = R_{3,CPT}(\Delta t) \times D_{CPT} , \]  

(10)

with

\[ D_{CPT} = \frac{|\langle 3\pi^0|T|K^- \rangle|^2}{|\langle \pi\pi|T|K^+ \rangle|^2} = \frac{\text{BR} (K_L \rightarrow 3\pi^0) \Gamma_L}{\text{BR} (K_S \rightarrow \pi\pi) \Gamma_S} , \]  

(11)
where the last r.h.s. equality holds with a high degree of accuracy, at least $O(10^{-7})$. The value of $D_{CPT}$ can be therefore directly evaluated from branching ratios and lifetimes.

The explicit expressions of ratios (7) and (8) (neglecting higher order terms in small parameters and for not too large negative $\Delta t$) are:

$$R_{2,CPT}^{exp}(\Delta t) = \frac{P[K^0(0) \rightarrow K^-(\Delta t)]}{P[K^-(0) \rightarrow K^0(\Delta t)]} \times D_{CPT}$$

$$\simeq |1 - 2\delta|^2 \left| 1 + 2\delta e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2 \times D_{CPT}, \quad (12)$$

$$R_{4,CPT}^{exp}(\Delta t) = \frac{P[K^0(0) \rightarrow K^-(\Delta t)]}{P[K^-(0) \rightarrow K^0(\Delta t)]} \times D_{CPT}$$

$$\simeq |1 + 2\delta|^2 \left| 1 - 2\delta e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2 \times D_{CPT}. \quad (13)$$

The expected behavior of the observables $R_{2,CPT}^{exp}(\Delta t)$ and $R_{4,CPT}^{exp}(\Delta t)$ as a function of $\Delta t$, and without the approximations of eqs. (12) and (13), is shown in figure 1, where – for visualization purposes – the probabilities involved have been evaluated fixing the CPT violating parameters $\Re \delta$ and $\Im \delta$ to a value different from zero, and equal to their present uncertainties [5], i.e. $\Re \delta = 3.3 \times 10^{-4}$ and $\Im \delta = 1.6 \times 10^{-5}$. In figure 2 a zoom of the $\Delta t > 0$ region, where the “plateau” regimes (5) and (6) dominate, is shown. Experimentally, this is the most interesting and statistically most populated region, where the best sensitivity to CPT violation effects can be reached by the KLOE-2 experiment [16, 17, 24].

The presence of direct CP violation contributions in the decay amplitudes, even though in principle could mimic CPT violation effects, turns out to be totally irrelevant for the plateau region $\Delta t \gg \tau_S$ (see detailed description in Ref. [16]). The effect of a possible violation of the $\Delta S = \Delta Q$ rule is also not affecting the CPT test in the same region with the double ratio defined as:

$$\frac{R_{2,CPT}^{exp}(\Delta t \gg \tau_S)}{R_{4,CPT}^{exp}(\Delta t \gg \tau_S)} = 1 - 8\Re \delta - 8\Re x_-. \quad (14)$$

with $x_-$ describing CPT violation in the $\Delta S \neq \Delta Q$ semileptonic decay amplitudes. Therefore the double ratio (14) constitutes one of the most robust observables for our proposed CPT test. It is independent of $D_{CPT}$, and in the limit $\Delta t \gg \tau_S$ it exhibits a pure and genuine CPT violating effect, even without the assumptions of the validity of the $\Delta S = \Delta Q$ rule and of negligible contaminations from direct CP violation.

The KLOE-2 experiment at the DAΦNE facility aims at reaching a total integrated luminosity $L \geq 5$ fb$^{-1}$ in the next couple of years, and could make a precise measurement of the two observable ratios $R_{2,CPT}^{exp}(\Delta t)$ and $R_{4,CPT}^{exp}(\Delta t)$. By considering a large $\Delta t$ interval in the statistically most populated region, e.g. $0 \leq \Delta t \leq 300 \tau_S$, a statistical sensitivity on the double ratio (14) of $(3.0 \times 10^{-5})$ and $(2.1 \times 10^{-5})$ is obtained for $L = 5$ and 10 fb$^{-1}$, respectively [16]. Once translated into an uncertainty on $\Re \delta$, these results will improve its present limit.

3. Conclusions

A novel CPT test has been studied in the neutral kaon system based on the direct comparison of a transition probability with its CPT reverse transition. The appropriate preparation and detection of in and out states in both the reference and the reverse processes is made by exploiting
Figure 1. The ratios $R_{2,\text{CPT}}^{\text{exp}}(\Delta t)$ and $R_{4,\text{CPT}}^{\text{exp}}(\Delta t)$ as a function of $\Delta t$. For visualization purposes the CPT violating parameters have been fixed to the values $\Re \delta = 3.3 \times 10^{-4}$ and $\Im \delta = 1.6 \times 10^{-5}$.

Figure 2. A zoom of the plots shown in Fig.1 in the region $0 \leq \Delta t \leq 20\tau_S$. 
the EPR entanglement of neutral kaons produced in a $\phi$-factory and using their decays as filtering measurements of the kaon states only.

Possible spurious effects induced by CP violation in the decay and/or a violation of the $\Delta S = \Delta Q$ rule have been shown to be either negligible or disentangled by the dependence with the time evolution. The proposed measurement is thus fully robust leading to definite conclusions on CPT Violation.

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