The Velocity Dispersion – Temperature Correlation from a Limited Cluster Sample

Christina M. Bird
Department of Physics and Astronomy, University of Kansas, Lawrence, KS 66045
tbird@kula.phsx.ukans.edu

Richard F. Mushotzky
NASA/Goddard Space Flight Center, Laboratory for High Energy Astrophysics,
Code 666, Greenbelt, MD 20771
richard@xray-5.gsfc.nasa.gov

and

Christopher A. Metzler
Department of Physics, University of Michigan, Ann Arbor, MI 48109
metzler@pablo.physics.lsa.umich.edu

Submitted to the Astrophysical Journal

Received ________________; accepted _______________
Most studies of correlations between X-ray and optical properties of galaxy clusters have used the largest samples of data available, regardless of the morphological types of clusters included. Given the increasing evidence that morphology is related to a cluster’s degree of dynamical evolution, we approach the study of X-ray and optical correlations differently. We evaluate the relationship between velocity dispersion and temperature for a limited set of galaxy clusters taken from Bird (1994), which all possess dominant central galaxies and which have been explicitly corrected for the presence of substructure. We find that $\sigma_r \propto T^{0.61\pm0.13}$. We use a Monte Carlo computer routine to estimate the significance of this deviation from the $\sigma_r \propto T^{0.5}$ relationship predicted by the virial theorem. We find that the simulated correlation is steeper than the observed value only 4% of the time, suggesting that the deviation is significant. The combination of protogalactic winds and dynamical friction reproduces nearly exactly the observed relationship between $\sigma_r$ and $T$. 
1. Introduction

Galaxy clusters occupy a unique position in the dynamical evolution of the universe. Unlike lower-mass systems such as galaxies, which for the most part retain little dynamical information about their formation, clusters of galaxies are within one or two crossing times of their formation. This suggests that they may retain valuable clues to their initial conditions (as well as hints about the collapse and formation of structure in the early universe). The effect of the dense cluster environment on galaxy evolution, as well as other trends in the physical properties of clusters (see, for instance, Dressler 1984; Giovanelli & Haynes 1985; Edge & Stewart 1991), suggests that they are gravitationally bound and that their galaxies no longer participate in the Hubble flow. This distinguishes clusters from superclusters and other large-scale structures. The study of galaxy clusters thus provides a unique opportunity to explore gravitational interactions and dynamical evolution in the universe.

Clusters of galaxies contain two luminous components, hot gas and galaxies. If a cluster is sufficiently old and unperturbed, these tracer particles will have equilibrated within the cluster gravitational potential. This enables use of the equations of hydrostatic and dynamical equilibrium to explore the physical properties of these systems. For a hot gas in equilibrium with a spherical gravitational potential, the equation of hydrostatic equilibrium may be written

\[ M_X(r) = -\frac{kT r}{G \bar{m}} \left( \frac{d \ln n_{gas}}{d \ln r} + \frac{d \ln T}{d \ln r} \right) \]  

(e.g. Fabricant, Lecar & Gorenstein 1981), where \( M_X \) is the X-ray determined virial mass, \( T \) is the temperature of the X-ray emitting gas, \( n_{gas} \) is the gas density and \( \bar{m} \) is the average mass per gas particle. Similarly, the Jeans equation relates the kinetic energy of the galaxies
to the virial mass of the cluster:

$$\frac{- G n_{gal} M_{opt}(r)}{r^2} = \frac{d(n_{gal}\sigma_r^2)}{dr} + \frac{2n_{gal}}{r\sigma_t^2}(1 - \frac{\sigma_r^2}{\sigma_t^2})$$

(Merritt 1987), where $M_{opt}$ is the optically-determined virial mass, $r$ is the clustercentric radius, $n_{gal}$ is the galaxy density, $\sigma_r$ and $\sigma_t$ are the radial and tangential velocity dispersions respectively, and $A$ is the anisotropy parameter describing the distribution of galaxy orbits.

For an isothermal cluster in dynamical equilibrium, with no source of energy other than gravity, the masses as determined by the galaxies and by the gas are expected to be equal. As shown by Bahcall & Lubin (1994) among others, the ratio of the kinetic energies of the galaxies and gas is then equal to the ratio of the logarithmic slope of the gas density profile to that of the galaxies:

$$\frac{\sigma_r^2}{\sqrt{m}} = \frac{d \ln n_{gas}/d \ln r}{d \ln n_{gal}/d \ln r + 2A}$$

(where $A = 0$ for an isotropic distribution of galaxy orbits). Therefore, using the assumptions that the gas and galaxies are both in equilibrium with the cluster gravitational potential, and that gravity is the only source of energy, allows us to predict that the velocity dispersion (as measured from galaxy velocities) and the temperature of the intrachuster medium (as determined from X-ray spectra) should be correlated, with $\sigma_r \propto T^{0.5}$. The ratio of the kinetic energies is called $\beta_{spec}$. The ratio of the logarithmic slopes of the density profiles is $\beta_{fit}$.

Despite the many difficulties in accurately measuring cluster temperatures and velocity dispersions, studies of X-ray and optical cluster samples reveal a well-behaved correlation between these quantities (Mushotzky 1984; Edge & Stewart 1991, hereafter ES91; Lubin
& Bahcall 1993, hereafter LB93). The relationship between $\sigma_r$ and $T$ expected from virial considerations is consistent with the data, although there is a large scatter about the $\sigma_r \propto T^{0.5}$ line. This scatter has been attributed to incomplete gas thermalization, cooling flows, velocity anisotropies in the galaxy orbits, foreground/background contamination, and substructure in the clusters (cf. ES91; LB93 and references therein).

It is important to remember, however, that the predicted $\sigma_r - T$ correlation derives from the virial theorem, and that in order to test it one must consider the dynamical state of the clusters in the dataset (cf. Gerbal et al. 1994). The high frequency of substructure in clusters of all morphologies, as determined by both X-ray and optical studies (see, e.g. Davis & Mushotzky 1993; Mohr, Fabricant & Geller 1993; Beers et al. 1991; Bird 1993, 1994), is generally believed to indicate that clusters are dynamically-young. If clusters are only within a few crossing times of formation, then in many cases virial equilibrium has not been established. This certainly influences the broad distribution of clusters about the canonical $\sigma_r \propto T^{0.5}$ relation.

In this paper we will quantify the effects of morphology and substructure on the velocity dispersion-temperature correlation for clusters. In Section 2 we present the limited cluster sample, in which the morphological type of the cluster sample has been restricted and the effects of substructure have been minimized. We have supplemented the available published X-ray temperature data with new, more accurate temperatures from ASCA and Ginga. In Section 3 we present the regressions between the velocity dispersion and temperature. Section 4 summarizes proposed mechanisms for modifying the slope of the $\sigma_r - T$ correlation. In Section 5 we present a summary.
2. The Limited Cluster Sample

The morphology of a cluster may be described by its gas and/or galaxy distribution. As our observations of clusters have improved, it has become clear that morphology is related to the dynamical age of a cluster. Irregular clusters are dynamically young, and tend to be spiral-rich and gas-poor. They tend to have non-Gaussian velocity distributions and kinematically-distinct subconcentrations of galaxies. Regular clusters are dominated by ellipticals, have Gaussian velocity distributions and tend to be luminous X-ray emitters (cf. Sarazin 1988 and references therein; Bird 1993, 1994).

Bird (1994) presents a detailed analysis of the dynamics of nearby clusters \( (z < 0.1) \) with central galaxies. These clusters tend to have smooth morphologies and X-ray cooling flows, and in the past it has been assumed that they represent the most relaxed, dynamically-evolved clusters in the universe. However, Bird (1994) shows that these clusters also possess significant substructure. An objective partitioning algorithm called KMM (McLachlan & Basford 1988; Ashman, Bird & Zepf 1994) is used to remove galaxies belonging to subsystems in the clusters, and the dynamical properties of the “cleaned” (i.e., substructure corrected) cluster datasets are presented. It is the 25 clusters in this “cD database” which form the optical sample of the present analysis.

Of the 25 clusters used in Bird (1994), 21 have accurate X-ray temperature measurements. These clusters, which will be referred to as the limited cluster sample, are listed in Table 1. Table 1 includes the following information: column (1), the cluster name; (2), the 1-D velocity dispersion of the cluster (estimated using the robust biweight estimator \( S_{BI} \); Beers, Flynn & Gebhardt 1991) without substructure correction; (3), the velocity dispersion corrected for substructure; (4), the X-ray temperature, (5) the source code for the X-ray measurement. The optical redshifts are taken from the literature, with sources given in Bird (1994). In addition we have added the Centaurus Cluster (A3526),
which was excluded from the cD study because of its proximity. The X-ray temperatures are taken from single-temperature models to ASCA or Ginga spectra where available, and then from EXOSAT and the Einstein MPC. For the clusters A1736 and A3558, the GINGA observations are best-fit by a two-temperature model (Day et al. 1991), in contradiction to both the Einstein and ROSAT spectra. Because the data are inconclusive, we have included both temperatures in Table 1 for these two clusters, and we will consider them both in the statistical analysis.

Note that the velocity dispersion presented here is measured only along our line of sight to the cluster. We assume for the moment that any velocity anisotropy in these clusters is small and therefore \( \sigma_{\text{LOS}} \) is comparable to \( \sigma_r \) (we will explore this assumption in more detail below).

In Table 2 we present the individual values of \( \beta_{\text{spec}} \) for the limited cluster sample, both with and without substructure correction. With no substructure correction, the mean value of \( \beta \) is \( 1.20^{+0.30}_{-0.18} \), with an rms scatter of 0.66 (GINGA: \( 0.99^{+0.24}_{-0.17} \), rms 0.43). The high mean value and large scatter are due to the inclusion of A2052 in the dataset. The uncorrected velocity dispersion of this cluster is extremely high, 1404 km s\(^{-1}\), with corresponding \( \beta_{\text{spec}} = 3.51 \). If this datapoint is excluded from the list, the mean drops to \( 1.09^{+0.15}_{-0.15} \) with rms scatter 0.43 (GINGA: \( 0.97^{+0.24}_{-0.17} \), rms 0.42). Including the substructure correction to the velocity dispersion (and retaining A2052, which is no longer anomalous), \( \langle \beta_{\text{spec}} \rangle = 0.90^{+0.10}_{-0.15} \) with an rms scatter of 0.37 (where the confidence intervals are the 90% bootstrapped estimates) (GINGA: \( 0.87^{+0.12}_{-0.17} \), rms 0.38).

To demonstrate the effect of morphology on \( \beta_{\text{spec}} \), these numbers should be compared to the values from the LB93 study. Lubin & Bahcall use 41 clusters of widely varying morphology. Their mean value of \( \beta_{\text{spec}} \) is \( 1.14^{+0.08}_{-0.08} \) with an rms scatter of 0.57. The ES91 sample, being based on an X-ray flux-limited catalog of clusters, is biased toward X-ray
luminous systems, which are less likely to be affected by major substructure. This sample yields \( \langle \beta_{\text{spec}} \rangle = 0.91^{+0.11}_{-0.13} \) with an rms scatter of 0.38. It is clear that when examining correlations between temperature and velocity dispersion, uncertainty may be introduced by neglecting the effects of morphology and substructure in the dataset.

3. The Velocity Dispersion – Temperature Correlation

In Figure 1, we present the velocity dispersion and temperature data for the 22 clusters in the limited sample. The velocity dispersions are corrected for substructure. The dashed lines are the correlations predicted by the virial theorem, for \( \beta_{\text{spec}} = 1 \) and for \( \beta_{\text{spec}} = 0.67 \). Recall that for these data \( \langle \beta_{\text{spec}} \rangle = 0.90 \). The solid line is the best fit to the data using the lower temperatures for A1736 and A3558:

\[
\sigma_r = 10^{2.50 \pm 0.09} T^{-0.61 \pm 0.13}
\]

Similarly, we find that

\[
T = 10^{-3.15 \pm 0.60} \sigma_r^{1.31 \pm 0.21}
\]

For the higher GINGA temperatures for these two clusters, we find that

\[
\sigma_r = 10^{2.39 \pm 0.09} T^{-0.76 \pm 0.11}
\]

and

\[
T = 10^{-3.21 \pm 0.61} \sigma_r^{1.34 \pm 0.21}
\]

In both equations the uncertainties quoted are the bootstrapped 1-\( \sigma \) values. This fit includes the errors in the measurements, using a linear fitting technique developed by Akritas, Bershady & Bird (1995, in preparation). This algorithm, based on the ordinary least-squares bisector first defined by Isobe et al. (1990), explicitly includes both intrinsic
scatter in the relation and uncorrelated measurement errors. The bisector method assumes that neither variable is dependent on the other, which is probably appropriate for the current physical situation. The velocity dispersion and X-ray temperature are both determined by the depth of the gravitational potential (and perhaps other physical effects), and are therefore independent of each other.

This subtlety in the application of linear regression algorithms has been previously noted by astrophysicists for other applications, such as the Tully-Fisher effect (see Isobe et al. 1990 for a detailed discussion), but not yet applied to the problem of X-ray and optical correlations. The use of an inappropriate or biased regression technique can have a significant effect on the coefficients of the linear fit, as we demonstrate in Table 3. To simplify this discussion, in Table 3 we present the following:

- the published linear regressions given in ES91 and LB93
- the linear regressions determined from an ordinary least squares fit, without measurement errors
- the linear regressions from the bisector lines, with and without measurement errors

for the ES91 and LB93 datasets, as well as similar regressions for our limited cluster dataset. The uncertainties in the linear coefficients are the 1-σ values, determined using a bootstrap method which is the preferred estimator for small datasets.

First of all, we see that the published linear regressions are recovered for both the ES91 and the LB93 datasets using the ordinary least squares (OLS) regressions, without errors. For these fits, the velocity dispersion is assumed to be dependent on the temperature, which as discussed above does not seem like a physically well-motivated assumption. In addition, simulations suggest that the OLS regressions are severely biased for such small sample sizes. The bisector slopes for all three datasets are much steeper than the OLS slopes, varying
from 0.61 for our limited cluster dataset and the *Einstein* data to 0.87 for the LB93 dataset. The regression for our limited cluster dataset is marginally consistent with the slope of 0.5 predicted by the virial theorem. For the ES91 and LB93 datasets, the fitted slopes are at least $3\sigma$ away from the canonical value of 0.5.

Given the large dispersions between the individual linear regressions, as well as the coefficients of the regressions for the three datasets, how significant is this difference? To estimate the significance of the observed deviation, we utilize a Monte Carlo computer routine. This code simulates 22 cluster temperatures between 2.0 and 10.0 keV and generates velocity dispersions using the virial relation and a $\beta$ value of 1. It then includes a velocity term for the intrinsic scatter in the relationship (which is generated by choosing a velocity perturbation from a uniform distribution of width 150 km s$^{-1}$) as well as measurement errors in both velocity and temperature (these are modelled as Gaussians; the dispersion in velocities is 150 km s$^{-1}$ and in temperature is 0.5 keV). For 1000 simulations, only 40 of the random datasets had measured bisector slopes greater than 0.61, the lowest value obtained for the limited cluster dataset. The average value for the 1000 runs was $0.55 \pm 0.03$. The highest value of the slope obtained for any of the simulated datasets is 0.64, which is comparable to the value obtained for the ES91 dataset but still strongly inconsistent with the LB93 regression and the limited cluster dataset (with the high temperatures for A1736 and A3558).

These simulations suggest that while the deviation between the observed correlation between velocity dispersion and temperature and that predicted by the virial theorem is small, it is significant. Clearly larger individual cluster datasets, higher-quality X-ray spectra, and a larger dataset of clusters will be vital to improving our understanding of this fundamental correlation.

The deviation of the $\sigma_T - T$ relationship from that predicted by the equilibrium
model described in Section 1 implies that $\beta$ is a function of the depth of the gravitational potential, as estimated by either the temperature or the velocity dispersion. In this case, defining an average (unweighted) value of $\beta_{\text{spec}}$ for a cluster sample which covers a wide range of physical parameters yields a quantity which is poorly defined. The dependence of $\beta$ on temperature and/or velocity dispersion is no doubt partially responsible for the high scatter about the $\sigma_r - T$ relation, which remains even after elimination of the effects of substructure from the optical dataset.

We have seen in Section 2 that consideration of morphology and substructure significantly reduces the scatter in the values of $\beta_{\text{spec}}$ for the individual clusters. Examination of Table 3 reveals that the same effect does not hold true for the determination of the $\sigma_r - T$ correlation. Inclusion of the substructure correction actually raises the scatter in the parameters of the fit slightly, although it remains comparable to the values obtained by both ES91 and LB93. It is clear that although substructure influences the scatter in the relationship, other physical effects must also be significant (see also Gerbal et al. 1994).

Previous authors have claimed that their data was consistent with the canonical virial theorem dependence of velocity dispersion on temperature, $\sigma_r \propto T^{0.5}$ (ES91, LB93). We have seen that this “consistency” is due to the inaccurate use of the least squares linear regression, and that none of the three datasets are consistent with the canonical prediction. Correction for substructure has very little effect on the slope of the $\sigma_r - T$ correlation. The scatter to high velocity dispersions implied by the “steeper than virial” relation has been noted by all previous studies and generally attributed to velocity substructure. However, we demonstrate that correction for substructure has little effect on the correlation.

4. Mechanisms for Explaining the Discrepancy
The virial theorem prediction of the relationship between galaxy velocity dispersion and gas temperature is based on three assumptions: that the galaxy orbits are isotropic, that the gas and the galaxies occupy the same potential well, and that gravity is the only source of energy for either the gas or the galaxies. Any process which may contribute to the deviation of the slope from the virial value must operate to a different degree in hot, high-\(\sigma_v\) clusters than in cooler, low-\(\sigma_v\) systems, to skew the relationship in the observed fashion (although the effect need not be large). Mechanisms which have been proposed include anisotropy in the distribution of galaxy orbits, incomplete thermalization of the gas, pressure support of the ICM from magnetic fields, biasing and protogalactic winds.

4.1. Anisotropy and Magnetic Pressure Support

The anisotropy parameter \(A\) is not well-determined for more than one or two clusters. Recall that \(A = 1 - \sigma_r^2/\sigma_t^2\). For radial orbits, with \(\sigma_r > \sigma_t\), \(A < 0\) and \(\beta\) is increased (relative to the value determined by profile fitting; see eqn. 3). For circularized orbits, \(\sigma_r < \sigma_t\), \(A > 0\) and \(\beta\) is decreased. To reproduce the observed trend in the \(\sigma_r - T\) relation, we estimate that hot clusters require \(A \leq -0.1\) (slightly radial orbits), and cool clusters require \(A \sim 0.6\) (moderately circular orbits). Such an extreme variation in galaxy anisotropy is not predicted by any current theory of cluster formation. Kauffmann & White (1993) do find some evidence for a dependence of formation history on mass, but this variation is negligible over the range of masses included in the limited cluster sample (5 \(\times\) 10\(^{13}\) – 1 \(\times\) 10\(^{15}\) \(M_\odot\); S. White, 1994, private communication).

In most observations, the temperature profile of the ICM is flat out to the radius where the background dominates the cluster spectrum (Mushotzky 1994). Nonetheless, simulations by Evrard (1990) suggest that the cluster gas will not be completely thermalized after only
one crossing time. This effect is evident in more detailed calculations by Metzler & Evrard (1995, in preparation), who find that the degree of thermalization is not systematically dependent on temperature. Incomplete thermalization clearly affects the distribution of temperatures measured for the limited cluster sample, but does not affect the slope of the $\sigma_r - T$ relationship in the required direction.

In an attempt to resolve the discrepancy between cluster masses determined by gravitational lensing and those determined from X-rays (Miralda-Escudé & Babul 1994), Loeb & Mao (1994) propose magnetic pressure support of the intracluster medium, at least in the cores of cooling flows. To be dynamically significant, tangled magnetic fields must contribute a similar amount of potential energy to the ICM as the gravitational potential. The required field strength (on the order of $50 \mu G$) is large, but Loeb & Mao argue that such fields may be generated within cooling flows, where gas and magnetic field lines are confined and compressed.

Comparison of the limited cluster sample with Table 1 of Edge, Stewart & Fabian 1992 reveals that the majority of the limited cluster sample possesses cooling flows (as determined from deprojection analysis) and therefore may benefit from magnetic pressure support. Remember, however, that the Loeb & Mao (1994) analysis is restricted to the inner $120h^{-1}$ kpc of A2218 (inside the radius of the cooling flow), whereas our temperatures and velocity dispersions are determined for the entire cluster (again assuming that the cluster ICM temperature profiles are flat outside the cooling radius, as ASCA data suggest). It is unclear whether the variation in $\beta$ deriving from magnetic pressure support would be detected in our analysis of the X-ray and optical data.

4.2. Protogalactic Winds
Protogalactic winds provide an additional source of heating of the ICM. Yahil & Ostriker (1973), Larson & Dinerstein (1975) and White (1991) discuss ram pressure stripping and protogalactic winds as mechanisms for the metal enrichment of the ICM. In the winds scenario, the specific energy of the ICM is affected by the initial collapse of the cluster, the relative motions of galaxies in the cluster, and winds from supernova explosions during the formation of elliptical galaxies at early times. Of these three physical processes, White (1991) demonstrates that only protogalactic winds can boost the energy of the gas above the value determined through the virial theorem. In addition he shows that the energy contribution due to winds will be larger in cool clusters than in hot ones.

Using White’s Equation 2, we generated a distribution of temperatures for velocity dispersions ranging from 350-1200 km sec$^{-1}$ (taking his values for the fraction of intracluster gas coming from winds ($w = 0.5$) and the typical wind velocity in terms of the galactic velocity dispersion ($f_w = 3$)). Fitting these simulated data, we find that the protogalactic winds model predicts a correlation between the velocity dispersion and the temperature of a cluster:

$$\sigma_r \propto T^{0.68}$$

(8)

This depends slightly on the choice of $w$ and $f_w$; for $f_w = 2$ we find that $\sigma_r \propto T^{0.62}$. The protogalactic wind model reproduces nearly exactly the dependence of velocity dispersion on ICM temperature that we find in the limited cluster sample (and which is consistent with the slopes found by earlier studies).

### 4.3. Winds and Biasing

Another effect which may produce the steepness of the $\sigma_r - T$ relationship is a velocity bias between cluster galaxies and the background dark matter, which is driven by dynamical
friction (Carlberg 1994; Carlberg & Dubinski 1991). Simple virial analysis predicts $\sigma_r \propto T^{0.5}$ if the collisionless component has experienced no cooling or heating. If $\sigma_{DM}$ and $\sigma_{gal}$ refer to the background dark matter and galaxy velocity dispersions respectively, and assuming the virial equilibrium holds for the dark matter, then we can write

$$\sigma_{gal} = \sigma_{DM} \frac{\sigma_{gal}}{\sigma_{DM}} \propto \frac{\sigma_{gal}}{\sigma_{DM}} T^{0.5} \tag{9}$$

If the ratio of velocity dispersions is temperature–dependent, then this will modify the observed $\sigma - T$ relation.

For the purposes of illustration, we take the distribution of background dark matter velocities to be Maxwellian,

$$f(v) = \frac{n(r)}{(2\pi \sigma^2)^{3/2}} \exp \left(-v^2/2\sigma^2\right), \tag{10}$$

in which case the Chandrasekhar dynamical friction formula for a galaxy of mass $M$ in a dark matter potential well with density $\rho$ can be written as

$$\frac{dv_M}{dt} = -\frac{4\pi \ln G^2 \rho M}{v_M^3} \left[\text{erf} \left(X\right) - \frac{2X}{\sqrt{\pi}} \exp \left(-X^2\right)\right] v_M, \tag{11}$$

with $X = v_M/\sqrt{2}\sigma$ (Binney and Tremaine 1987). This can be rearranged for a characteristic timescale, and writing the bias for the individual galaxy of mass $M$, $b = v_M/\sigma$, we have

$$t_{fric} = \frac{b^3 \sigma^3}{4\pi \ln G^2 \rho M} \left[\text{erf} \left(b/\sqrt{2}\right) - b \sqrt{\frac{2}{\pi}} \exp \left(-b^2/2\right)\right]^{-1} \tag{12}.$$
Simulations provide an ideal mechanism to test these ideas. Metzler & Evrard (1995) have conducted an ensemble of N–body + hydrodynamic simulations of the formation and evolution of individual clusters, explicitly including galaxies and galactic winds. These simulated clusters are compared to a ensemble drawn from the same initial conditions — but without galaxies and winds — to isolate the effects of winds on clusters. The method is explained in Metzler & Evrard (1994).

Figure 2 shows velocity dispersion – temperature data drawn from their models. A “virial radius” is identified for each simulated cluster as the radius with a mean interior overdensity of 170. The temperatures used are mass–averaged over all gas within the virial radius; the velocity dispersions are averages drawn from the full 3D velocity information for all dark matter or galaxies within $r_{\text{vir}}$. A solid line corresponding to $\beta_{\text{spec}} = 1$ has also been placed on the plots.

Comparing the dark matter velocity dispersion to the average interior temperature shows that in the simple two–fluid models, the simulated clusters are well–fit by the virial relation $\sigma \propto T^{0.5}$. This is sensible; there is no physics in these models beyond that used to derive the expected relation. Note that the values of $\beta_{\text{spec}}$ are consistently larger than one; this is a signature of the incomplete gas thermalization previously seen in other studies. It is not clear whether this is physical or numerical in origin; a series of runs with different resolution would clarify this.

The models including galaxies and winds show different behavior. Here, the inclusion of energetic winds, plus dynamical friction of the galaxy component, provide the necessary physics to deviate from the virial $\sigma - T$ relation. For the dark matter, the temperature dependence is steeper than 0.5, a result of the inclusion of energetic winds. When galaxies are used to calculate the velocity dispersion, however, the relation steepens to $\sigma \propto T^{0.65}$, comparable to our observed result. The simulations thus provide evidence for a
temperature–dependent velocity bias, \( \sigma_{gal}/\sigma_{DM} \propto T^{0.1} \). Both this bias and the increase in gas temperatures due to energetic winds are responsible for the final correlation.

It should be noted, of course, that the agreement between the simulated ensemble and our real clusters is to some degree fortuitous. The wind model used in the simulations of Metzler & Evrard is intentionally of much greater wind luminosity than expected for real early–type galaxies, and the dynamical accuracy of modelling galaxies by heavy collisionless particles in the cluster potential is unclear (Frenk et al. 1995). Nonetheless, this corroborates the theoretical expectation that both energetic winds and velocity bias can result in the observed \( \sigma - T \) relation.

5. Discussion

Although Lubin & Bahcall (1993) found that the correlation between cluster velocity dispersion and temperature was somewhat steeper than that predicted by the virial theorem, the scatter in their dataset was too broad for them to rule out consistency with the hydrostatic isothermal model. We show that for our limited dataset, \( \sigma_r \propto T^{0.61 \pm 0.13} \) (GINGA: \( \sigma_r \propto T^{0.76} \)), slightly but significantly (at 96% confidence) steeper than that predicted by the virial theorem. For the ES91 and LB93 datasets, this discrepancy is significant at the > 99% level. It seems improbable that this is an artifact of the substructure correction algorithm. The mixture modelling technique used to remove substructure from the cluster datasets does not preferentially raise the velocity dispersion of high-\( \sigma_r \) clusters and lower that in low-\( \sigma_r \) systems, as examination of Table 1 reveals.

The protogalactic winds model of White (1991), in addition to possible velocity bias due to dynamical friction acting on the cluster galaxies, quantitatively reproduces the observed variation in the \( \sigma_r - T \) relationship. Preliminary measurements of cluster emission
line diagnostics from ASCA show metal abundances typical of Type II supernovae, also supporting the protogalactic winds model (Mushotzky 1994). (Contrary to the model, however, there is as yet no conclusive evidence that low-temperature clusters have higher global abundances than hot systems.) It seems plausible that other physical mechanisms, such as velocity anisotropy, incomplete thermalization of the gas and/or the galaxies, and magnetic pressure support in cluster cores (which are all likely to be present in some unknown and variable degree in clusters) are responsible for the large scatter about the best-fit $\sigma_r - T$ line. This scatter is apparent even after morphology and substructure are considered in the determination of cluster parameters.

Finally we can relate our revised determination of $\beta_{spec}$ to the long-standing $\beta$-discrepancy. Early studies of cluster X-ray spectroscopy and imaging revealed an important inconsistency: $\langle \beta_{spec} \rangle = 1.2$ (Mushotzky 1984) but $\langle \beta_{fit} \rangle = 0.7$ (Jones & Forman 1984). We have seen that the corrections for morphology and substructure bring $\langle \beta_{spec} \rangle$ down to 0.9, only marginally consistent with $\langle \beta_{fit} \rangle$ (but confirming the earlier results of ES91). For many individual clusters, $\langle \beta_{spec} \rangle$ and $\langle \beta_{fit} \rangle$ are completely different. Perseus (A426) is the most obvious example, with $\beta_{spec} = 1.53$ and $\beta_{fit} = 0.57$. So what is the current status of the $\beta$-discrepancy?

First of all, we can compare current data on the distribution of gas and galaxies in clusters. Schombert (1988) summarizes the data on cluster density profiles determined from a variety of tracer particles:

$$\rho_{gal} \propto r^{-2.6\pm0.3}$$
$$\rho_{gas} \propto r^{-2.1\pm0.2}$$

(13)

In the hydrostatic isothermal model,

$$\rho_{gas} \propto \rho_{gal}^{\beta_{fit}}$$

$$\beta_{fit} = \beta_{spec}$$

(14)
For our value of $\beta_{\text{spec}}$, $\rho_{\text{gas}} \propto r^{-2.3}$, which is at best only marginally consistent with the dependence $\rho_{\text{gas}} \propto r^{-2.1}$ determined by Jones & Forman (1984).

As Gerbal et al. (1994) point out in their theoretical analysis of the $\beta$-discrepancy, however, in order to test the consistency of the gas and galaxy scale lengths one must simultaneously observe their radial dependence independently, not fitting them together as Jones & Forman did. In the next stage of this project (Bird & Mushotzky 1995), we present non-parametric determinations of the galaxy and gas density profiles based on the MAPEL package (Merritt & Tremblay 1994). MAPEL, a constrained maximum likelihood algorithm, allows us to determine the best-fit model to the surface density profiles without assuming a King-model (or other isothermal) fit to the data (Merritt & Tremblay 1994). This is important because there is growing evidence from gravitational lensing experiments and computer simulations that the King model fit is not a good description of the gravitational potential of a galaxy cluster (Navarro, Frenk & White 1994; see also Beers & Tonry 1986). These profiles will allow us to test on a cluster-by-cluster basis whether the galaxy and gas profiles differ – a comparison which in the past has only been possible in a statistical sense (cf. Bahcall & Lubin 1994).

Note also that in the time since White (1991) appeared, ROSAT PSPC and ASCA surface density profiles of cool clusters have become publicly available. These clusters will be included in the continuation of this project (velocity data are published in Beers et al. 1994). The protogalactic winds model predicts that cool clusters will have a larger scale length of gas density than hot clusters (again, because the relative energy contribution of winds to the ICM is greater in cool systems). Use of the expanded dataset for these clusters will allow us to directly test this prediction and to probe the effects of protogalactic winds on $\beta_{\text{fit}}$.

We would like to thank Lori Lubin, Neta Bahcall, Ray White III, Bill Forman, Christine
Jones and the other attendees of the Aspen Summer Workshop for their contributions to this project. Claude Canizares, Keith Ashman and Alistair Edge also provided useful conversations during the course of this work. Andy Fabian’s critical reading of the manuscript greatly improved our statistical analysis. We are grateful to Simon White for clarification of issues relating to cluster evolution and parametrization of cluster density profiles. This research was supported in part by NSF EPSCoR grant No. OSR-9255223 to the University of Kansas.
| Cluster  | $S_{BI}$ (uncorr) km s$^{-1}$ | $S_{BI}$ (corr) km s$^{-1}$ | $T_X$ (keV) | Source Code |
|----------|-------------------------------|-------------------------------|-------------|-------------|
| A85      | $810^{+76}_{-80}$             | $810^{+76}_{-80}$             | $6.6^{+1.8}_{-1.4}$ | E91         |
| A119     | $862^{+165}_{-140}$           | $1036^{+214}_{-221}$          | $5.1^{+1.0}_{-0.8}$ | E91         |
| A193     | $726^{+130}_{-108}$           | $515^{+176}_{-153}$           | $4.2^{+1.6}_{-0.9}$ | E91         |
| A194     | $530^{+149}_{-107}$           | $470^{+98}_{-78}$             | $2.0^{+1.0}_{-1.0}$ | JF84        |
| A399     | $1183^{+126}_{-108}$          | $1224^{+131}_{-116}$          | $6.0^{+2.1}_{-1.5}$ | E91         |
| A401     | $1141^{+132}_{-101}$          | $785^{+111}_{-81}$            | $8.6^{+1.4}_{-1.6}$ | E91         |
| A426     | $1262^{+171}_{-132}$          | $1262^{+171}_{-132}$          | $6.3^{+0.2}_{-0.2}$ | D93         |
| A496     | $741^{+96}_{-83}$             | $533^{+86}_{-76}$             | $4.0^{+0.06}_{-0.06}$ | W94         |
| A754     | $719^{+143}_{-110}$           | $1079^{+234}_{-243}$          | $8.7^{+1.8}_{-1.6}$ | E91         |
| A1060    | $630^{+66}_{-56}$             | $710^{+78}_{-78}$             | $3.3^{+0.2}_{-0.2}$ | Ikebe 1994 ASCA |
| A1644    | $919^{+156}_{-114}$           | $921^{+168}_{-141}$           | $4.1^{+1.4}_{-0.6}$ | E91         |
| A1736†   | $955^{+107}_{-114}$           | $528^{+136}_{-87}$            | $4.6^{+0.7}_{-0.6}$ | D93         |
|          |                               |                               |             | 6.2$^{+0.7}_{-0.7}$ | DFER |
| A1795    | $834^{+142}_{-119}$           | $912^{+192}_{-129}$           | $5.6^{+0.1}_{-0.1}$ | W94        |
| A2052    | $1404^{+401}_{-348}$          | $714^{+143}_{-148}$           | $3.4^{+0.6}_{-0.5}$ | E91         |
| A2063    | $827^{+148}_{-119}$           | $706^{+117}_{-109}$           | $3.4^{+0.3}_{-0.3}$ | Yamashita 1992 |
| A2107    | $684^{+126}_{-104}$           | $577^{+177}_{-127}$           | $4.2^{+4.4}_{-1.6}$ | D93         |
| A2199    | $829^{+124}_{-118}$           | $829^{+124}_{-118}$           | $4.5^{+0.07}_{-0.07}$ | W94        |
| A2634    | $1077^{+212}_{-152}$          | $824^{+142}_{-133}$           | $3.4^{+0.2}_{-0.2}$ | D93         |
| A2670    | $1037^{+109}_{-81}$           | $786^{+203}_{-239}$           | $3.9^{+1.6}_{-0.9}$ | D93         |
| A3526    | $1033^{+118}_{-79}$           | $780^{+100}_{-100}$           | $3.8^{+0.3}_{-0.3}$ | F94         |
| A3558†   | $923^{+120}_{-101}$           | $781^{+111}_{-98}$            | $3.8^{+2.0}_{-2.0}$ | D93         |
| DC1842-63| $522^{+98}_{-82}$             | $565^{+138}_{-117}$           | $1.4^{+0.5}_{-0.4}$ | D93         |
| Cluster       | $\beta_{\text{spec}}$(uncorr) | $\beta_{\text{spec}}$(corr) |
|--------------|-------------------------------|------------------------------|
| A85          | 0.60                          | 0.60                         |
| A119         | 0.88                          | 1.27                         |
| A193         | 0.76                          | 0.38                         |
| A194         | 0.85                          | 0.67                         |
| A399         | 1.41                          | 1.51                         |
| A401         | 0.92                          | 0.43                         |
| A426         | 1.53                          | 1.53                         |
| A496         | 0.83                          | 0.43                         |
| A754         | 0.36                          | 0.81                         |
| A1060        | 0.73                          | 0.92                         |
| A1644        | 1.25                          | 1.25                         |
| A1736        | 1.20                          | 0.37                         |
|              | Einstein                      | Einstein                     |
|              | 0.89                          | 0.27                         |
|              | GINGA                         | GINGA                        |
| A1795        | 0.75                          | 0.90                         |
| A2052        | 3.51                          | 0.91                         |
| A2063        | 1.22                          | 0.89                         |
| A2107        | 0.67                          | 0.48                         |
| A2199        | 0.92                          | 0.92                         |
| A2634        | 2.07                          | 1.21                         |
| A2670        | 1.67                          | 0.96                         |
| A3526        | 1.70                          | 0.97                         |
| A3558        | 1.36                          | 0.97                         |
|              | Einstein                      | Einstein                     |
|              | 0.83                          | 0.60                         |
| DC1842-63    | 1.18                          | 1.38                         |
Table 3: Fitting the $\sigma_r - T$ Correlation

| Source                           | Best Fit                                      |
|----------------------------------|------------------------------------------------|
| Edge & Stewart 1991              | $\sigma_r = 10^{2.60 \pm 0.08} T^{0.46 \pm 0.12}$ |
| $N_{clus} = 23$ (pub)            | $T = 10^{-3.22 \pm 0.77} \sigma_r^{1.35 \pm 0.27}$ |
| Ordinary least squares (no errors)| $\sigma_r = 10^{2.61 \pm 0.06} T^{0.45 \pm 0.09}$ |
| Bisector (no errors)             | $\sigma_r = 10^{2.46 \pm 0.06} T^{0.68 \pm 0.10}$ |
| Bisector (errors)                | $\sigma_r = 10^{2.41 \pm 0.51} T^{0.75 \pm 0.08}$ |
| Lubin & Bahcall 1993             | $\sigma_r = 10^{2.53 \pm 0.06} T^{0.62 \pm 0.09}$ (unweighted) |
| $N_{clus} = 41$ (pub)            | $\sigma_r = 10^{2.52 \pm 0.07} T^{0.60 \pm 0.11}$ (weighted†) |
| Ordinary least squares (no errors)| $\sigma_r = 10^{2.54 \pm 0.06} T^{0.61 \pm 0.09}$ |
| Bisector (no errors)             | $\sigma_r = 10^{2.38 \pm 0.05} T^{0.84 \pm 0.08}$ |
| Bisector (errors)                | $\sigma_r = 10^{2.36 \pm 0.05} T^{0.87 \pm 0.08}$ |
| This paper, no substructure correction | $\sigma_r = 10^{2.48 \pm 0.25} T^{0.73 \pm 0.38}$ |
| $N_{clus} = 22$ (bisector with errors) | $T = 10^{-2.79 \pm 1.54} \sigma_r^{1.16 \pm 0.52}$ |
| Ordinary least squares (no errors)| $\sigma_r = 10^{2.75 \pm 0.08} T^{0.31 \pm 0.13}$ |
| Bisector (no errors)             | $\sigma_r = 10^{2.51 \pm 0.07} T^{0.69 \pm 0.12}$ |
| This paper, substructure correction†† | $\sigma_r = 10^{2.50 \pm 0.09} T^{0.61 \pm 0.13}$ |
| $N_{clus} = 22$ (bisector with errors) | $T = 10^{-3.15 \pm 0.60} \sigma_r^{1.31 \pm 0.21}$ |
| Ordinary least squares (no errors)| $\sigma_r = 10^{2.62 \pm 0.07} T^{0.42 \pm 0.11}$ |
| Bisector (no errors)             | $\sigma_r = 10^{2.45 \pm 0.09} T^{0.69 \pm 0.13}$ |
| This paper, substructure correction†† | $\sigma_r = 10^{2.39 \pm 0.09} T^{0.76 \pm 0.11}$ |
| $N_{clus} = 22$ (bisector with errors) | $T = 10^{-3.21 \pm 0.61} \sigma_r^{1.32 \pm 0.21}$ |
REFERENCES

Ashman, K.M. & Carr, B.J. 1988, MNRAS, 234, 219
Ashman, K.M., Bird, C.M. & Zepf, S.E. 1994, AJ, 108, 2348
Bahcall, N.A. & Lubin, L.M. 1994, to appear ApJ
Beers, T.C., Forman, W., Huchra, J.P., Jones, C. & Gebhardt, K. 1991, AJ, 102, 1581
Beers, T.C., Kriessler, J., Bird, C.M. & Huchra, J.P. 1995, AJ, 109, to appear March
Beers, T.C. & Tonry, J.L. 1986, ApJ, 300, 557
Binney, J. & Tremaine, S. 1987, *Galactic Dynamics* (Princeton: Princeton University Press)
Bird, C.M. 1993, Ph.D thesis, University of Minnesota and Michigan State University
Bird, C.M. 1994, AJ, 107, 1637
Bird, C.M. & Mushotzky, R.F. 1995, in preparation
Buote, D.A. & Canizares, C.R. 1992, ApJ, 400, 385
Carlberg, R.G. 1994, ApJ, 433, 468
Carlberg, R.G. & Dubinski, J. 1991, ApJ, 369, 13
Davis, D.S. & Mushotzky, R. F. 1993, AJ, 105, 491
David, L.P., Slyz, A., Jones, C., Forman, W., Vrtilek, S.D. & Arnaud, K. 1993, ApJ, 412, 479
Day, C.S.R., Fabian, A.C., Edge, A.C. & Raychaudhury, S. 1991, MNRAS, 252, 394
Dressler, A. 1984, ARA&A, 22, 185
Edge, A.C. 1991, MNRAS, 250, 103
Edge, A.C. & Stewart, G.C. 1991, MNRAS, 252, 428
Edge, A.C., Stewart, G.C. & Fabian, A.C. 1992, MNRAS, 258, 177
Evrard, A.E. 1990, in *Clusters of Galaxies*, eds. Oegerle, W.R., Fitchett, M.J. & Danly, L., (New York: Cambridge University Press), 287
Fabricant, D.M., Lecar, M. & Gorenstein, P. 1980, ApJ, 241, 552
Frenk, C.S., Evrard, A.E., White, S.D.M., & Summers, F.J. 1995, ApJ, submitted
Fukuzawa, Y., Ohashi, T., Fabian, A.C., Canizares, C.R., Ikebe, Y., Makishima, K., Mushotzky, R.F. & Yamashita, K. 1994, PASJ, 46, L55
Gerbal, D., Durrett, F. & Lacièze-Rey, M. 1994, å, 288, 746
Giovanelli, R. & Haynes, M. 1985, ApJ, 292, 404
Jones, C. & Forman, W. 1984, ApJ, 276, 38
Kauffmann, G. & White, S.D.M. 1993, MNRAS, 261, 921
Kent, S.M. & Gunn, J.E. 1982, 87, 945
Kent, S.M. & Sargent, W.L.W. 1983, 88, 697
Larson, R.B. & Dinerstein, H.L. 1975, PASP, 87, 911
Loeb, A. & Mao, S. 1994, ApJ, 435, L109
Lubin, L.M. & Bahcall, N.A. 1993, ApJ, 415, L17
McLachlan, G.J. & Basford, K.E. 1988, *Mixture Models*, (New York: Marcel Dekker)
Merritt, D. 1987, ApJ, 313, 121
Merritt, D. & Tremblay, B. 1994, AJ, 108, 514
Metzler, C.A. & Evrard, A.E. 1994, ApJ, 437, 564
Metzler, C.A. & Evrard, A.E. 1995, ApJ, in preparation
Miralda-Escudé, J. & Babul, A. 1994, preprint
Mohr, J., Fabricant, D. & Geller, M. 1993, CfA preprint

Mushotzky, R.F. 1984, Phys. Scripta, T7, L157

Mushotzky, R.F. 1994, *New Horizons*, (Tokyo: publisher unknown), in press

Navarro, J., Frenk, C. & White, S.D.M. 1994, MNRAS, in press

Sarazin, C.L. 1988, *X-ray emissions from clusters of galaxies*, (Cambridge: Cambridge University Press)

Schombert, J.M. 1988, ApJ, 328, 475

Sharples, R.M., Ellis, R.S. & Gray, P.M. 1988, MNRAS, 231, 479

White, R.E. III. 1991, ApJ, 367, 69

White, R.E. III, Day, C.S.R., Hatsukade, I. & Hughes, J.P. 1994, ApJ, 433, 583

Yahil, A. & Ostriker, J.P. 1973, ApJ, 185, 787

Yamashita, K. 1992, *Frontiers of X-ray Astronomy*, (Tokyo: publisher unknown), 473
Velocity dispersions are the average for all galaxies or dark-matter particles within an overdensity of 170. Temperatures are the mass-average temperature for all gas within an overdensity of 170. The upper panel shows the results for the ensemble of two-fluid simulations (without galaxies or energetic winds); here \( \sigma_{DM} \propto T^{0.50} \). The lower panel shows the results from the ensemble including galaxies and winds; the crosses show the \( \sigma - T \) relation for the dark matter in these runs, while the boxes use cluster galaxies. The results, \( \sigma_{DM} \propto T^{0.55} \) and \( \sigma_{gal} \propto T^{0.65} \), are steeper than the simple virial relation.

NOTES TO TABLES

Table 1. Source code: E91 = Edge 1991, JF84 = Jones & Forman 1984, DFER = Day et al. 1991, D93 = David et al. 1993, W94 = White et al. 1994, F94 = Fukuzawa et al. 1994; † Two-temperature spectral models based on GINGA observations (Day et al. 1991) suggest that these clusters may have higher temperatures than the Einstein data suggest. We have performed our statistical analysis for both sets of temperatures.

Table 2. †: LB93 did not published a regression for temperature on velocity dispersion. The first regression of velocity dispersion on temperature does not include weighting by the measurement errors; the second regression is weighted following a \( \chi^2 \) algorithm. ††: The first set of regressions uses the lower temperatures for A1736 and A3558; the second set uses the higher temperatures.