Decay widths of lowest massive Regge excitations of open strings

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Abstract

With the advent of the LHC there is widespread interest in the discovery potential for physics beyond the standard model. In TeV-scale open string theory, the new physics can be manifest in the excitation and decay of new resonant structures, corresponding to Regge recurrences of standard model particles. An essential input for the prediction of invariant mass spectra of the decay products (which could serve to identify the resonance as a string excitation) are the partial and total widths of the decay products. We present a parameter-free calculation of these widths for the first Regge recurrence of the $SU(3)$ gluon octet, of the $U(1)$ gauge boson which accompanies gluons in D-brane constructions, and of the quark triplet.
Particles created by vibrations of relativistic strings populate Regge trajectories relating their spins and masses. The threshold mass $M$ for the production of on-shell string excitations is related to the Regge slope $\alpha'$: $M = 1/\sqrt{\alpha'}$. Although it is generally expected that $M$ is of order of the Planck mass, much lower values can be envisaged, even $M \sim 1$ TeV, provided that spacetime extends into large extra dimensions [1]. The low string mass scenarios require the presence of open strings stretching between D-branes. In particular, gluons come from strings with both ends attached to a single stack of coinciding D-branes while quarks from strings stretching between branes at angles [2].

In proton collisions at the LHC, Regge states will be produced as soon as the energies of some partonic subprocesses cross the threshold at $\hat{s} > M^2$. But even below the threshold, virtual Regge excitations will contribute. In either case, the computations of the corresponding cross sections involve decay rates of massive string excitations. Actually, many discovery signals are sensitive to decay widths, especially those involving particles produced at large transverse momenta [3, 4]. In this work, we discuss decays of the first excited string level.\(^1\)

In open superstring\(^2\) theory bosons and fermions originate from the ten-dimensional Neveu-Schwarz (NS) and Ramond \(^7\) sectors, respectively. The massless level of open strings associated to a single stack of $N$ D-branes contains gauge bosons, gauginos and scalars in the adjoint representation of $U(N)$. The numbers of gauginos and scalars depend on the dimensionality of D-branes and details of compactification from ten to four dimensions, specifically on the extent to which supersymmetry is preserved in four dimensions, i.e. on the number of conserved supercharges. The string vertex operators creating these particles involve “internal” fields of superconformal field theory (SCFT) describing string propagation on the (model-dependent) compact space. Throughout this work, we adopt the term “model-dependent” to characterize such particles, in contrast to “universal” particles like gauge bosons with the vertices depending on the SCFT fields describing four spacetime coordinates only (and their SCFT superpartners). A realistic supersymmetry breaking mechanism must at the end generate masses for all gauginos and adjoint scalars.

The first excited level of open strings ending on a single stack of D-branes can contain as many as 128 bosonic and 128 fermionic degrees of freedom (d.o.f.) \(^8\) in the adjoint representation of $U(N)$. In the maximal case, they form the full massive spin 2 supermultiplet of $\mathcal{N} = 4$ supersymmetry. However, at the leading order of string perturbation theory (i.e. disk world-sheets), only a few of these particles couple, and consequently decay, to four-dimensional gauge bosons \(^10\). In the bosonic (NS) sector, only one massive spin $J = 2$ particle (5 d.o.f.) and one spin $J = 0$ particle (1 d.o.f.) can decay into two gauge bosons. These massive bosons are universal, in the sense explained above. They have model-dependent fermionic superpartners, which decay into gauginos and gluons. However, due to supersymmetry breaking, gauginos acquire masses, therefore such decays are kinematically suppressed. There also exist model-dependent $J = 1$ massive vector bosons, inaccessible in gluon scattering, which appear as resonances in some gluino fusion channels.\(^3\) The number of these states depends on supersymmetry: $\mathcal{N} = 1$ requires only one but $\mathcal{N} = 4$ as many as 27 massive vector bosons to complete a massive spin 2 supermultiplet \(^11\). The fate of

\(^1\) See Refs. [5, 6] for previous discussions of some decay modes of string resonances.

\(^2\) We do not discuss bosonic strings because they contain unstable (tachyon) excitations.

\(^3\) There can be also as many as 41 additional (model-dependent) scalars. Since they do not couple to gluons, they are excluded from our discussion and should not be confused with the universal $J = 0$ state described below in more detail. They can also appear in certain gaugino fusion channels.
these particles upon supersymmetry breaking is unclear; however they are phenomenologically interesting because they can appear as resonances in quark-antiquark annihilation at exactly the same center of mass energies as the universal $J = 2$ and $J = 0$.\footnote{We are grateful to Michael Peskin for pointing this out.}

The part of the spectrum originating from strings stretching between branes at angles, including quarks, is model-dependent. Nevertheless, it exhibits some universal features. Quarks have $J = 1/2$ and $J = 3/2$ Regge excitations which are accessible in $gq \to gq$ parton scattering. Since the corresponding amplitudes\footnote{By convention, all helicities refer to incoming particles.} do not depend on the compactification details, the decay rates of these Regge excitations into $gq$ are common to all intersecting brane models.

We will extract the decay rates by factorizing the polarized four-parton scattering amplitudes on the kinematic poles (at $\hat{s} = M^2$) due to Regge excitations propagating in the intermediate one-particle channels. This is particularly simple because at the disk level, all amplitudes with four external gauge bosons can be extracted from the so-called MHV amplitude describing just one, maximally helicity violating configuration ($- - + +$)\footnote{We use the standard notation of \cite{17,18}, although the gauge group generators are normalized here in a different way, according to $\text{Tr}(T^a T^b) = \frac{1}{2} g_{ab}$.} Decay rates into quarks will be extracted in a similar way.

The $J = 0$ particle is a quantum of a completely antisymmetric three-index tensor field (three-form) $E^{\mu \nu \rho}$ subject to the constraint $\partial_\mu E^{\mu \nu \rho} = 0$, which is dual to a massive vector field $V_\mu$ with a vanishing field strength\footnote{For simplicity, we drop carets for the parton subprocesses.}. Its single d.o.f. is quite peculiar because it disappears in the formal $M \to 0$ ($\alpha' \to \infty$) limit, hence it cannot be associated to a standard real scalar field. Due to angular momentum conservation, this particle decays into two gauge bosons of same helicity, $(++)$ or $(- -)$. It seems that already at the tree-level, it could virtually propagate between pairs of gauge bosons and contribute to “all-plus” $(++++)$ or “all-minus” $(---)$ amplitudes which are known to vanish. A possible explanation of this puzzle is that this single d.o.f. is akin to a “real chiral” scalar existing in two non-interacting species coupled to self-dual and anti-self-dual sectors of Yang-Mills theory, respectively\footnote{For simplicity, we drop carets for the parton subprocesses.} One of them is created by and decays to $(++)$ while the other is created by and decays to $(- -)$. Note that the massive $J = 2$ particle is always created by and decays to two gauge bosons with opposite helicities.

The starting point for computing the decay rates of $J = 0$ and $J = 2$ states into two gauge bosons is the MHV amplitude\footnote{We are grateful to Michael Peskin for pointing this out.}:

\begin{equation}
\mathcal{M}(g_1^-, g_2^-, g_3^+, g_4^+) = 4 g^2 \langle 12 \rangle^4 \left[ V_t \langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle \text{Tr}(T^{a_1} T^{a_2} T^{a_3} T^{a_4} + T^{a_2} T^{a_1} T^{a_4} T^{a_3}) \\
+ V_u \langle 13 \rangle \langle 34 \rangle \langle 42 \rangle \langle 21 \rangle \text{Tr}(T^{a_2} T^{a_1} T^{a_3} T^{a_4} + T^{a_1} T^{a_2} T^{a_4} T^{a_3}) \\
+ V_s \langle 14 \rangle \langle 42 \rangle \langle 23 \rangle \langle 31 \rangle \text{Tr}(T^{a_1} T^{a_3} T^{a_2} T^{a_4} + T^{a_3} T^{a_1} T^{a_4} T^{a_2}) \right],
\end{equation}

where the string “formfactor” functions of the Mandelstam variables $s, t, u$ ($s + t + u = 0$) are defined as

\begin{equation}
V_t = V(s, t, u), \quad V_u = V(t, u, s), \quad V_s = V(u, s, t),
\end{equation}

4 We are grateful to Michael Peskin for pointing this out.

5 By convention, all helicities refer to incoming particles.

6 We use the standard notation of \cite{17,18}, although the gauge group generators are normalized here in a different way, according to $\text{Tr}(T^a T^b) = \frac{1}{2} g_{ab}$.

7 For simplicity, we drop carets for the parton subprocesses.
In order to factorize amplitudes on the poles due to lowest massive string states, it is sufficient to consider $s = M^2$. In this limit, $V_s$ is regular while

$$V_i = \frac{u}{s - M^2}, \quad V_u = \frac{t}{s - M^2}. \quad \text{(4)}$$

Thus the s-channel pole term of the amplitude [1], relevant to $(---)$ decays of intermediate states, is

$$\mathcal{M}(g_1^-, g_2^-, g_3^+, g_4^+) \rightarrow 4 g^2 \text{Tr}(\{T^{a_1}, T^{a_2}\} \{T^{a_3}, T^{a_4}\}) \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} \frac{u}{s - M^2}. \quad \text{(5)}$$

The amplitude with the s-channel pole relevant to $(++-)$ decays is

$$\mathcal{M}(g_1^-, g_2^+, g_3^+, g_4^-) \rightarrow 4 g^2 \text{Tr}(\{T^{a_1}, T^{a_2}\} \{T^{a_3}, T^{a_4}\}) \frac{\langle 14 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} \frac{u}{s - M^2}. \quad \text{(6)}$$

In order to extract decay rates into quark-antiquark pairs, we also need the amplitude [12]

$$\mathcal{M}(q_1^-, \bar{q}_2^+, g_3^+, g_4^+) = 2 g^2 \frac{\langle 13 \rangle^2}{\langle 14 \rangle \langle 24 \rangle} \left[ \frac{t}{s} V_i(T^{a_3} T^{a_4})_{a_1a_2} + \frac{u}{s} V_u(T^{a_4} T^{a_3})_{a_1a_2} \right]. \quad \text{(7)}$$

Its s-channel singularity is

$$\mathcal{M}(q_1^-, \bar{q}_2^+, g_3^-, g_4^+) \rightarrow 2 g^2 \{T^{a_3}, T^{a_4}\}_{a_1a_2} \frac{\langle 13 \rangle^2}{\langle 14 \rangle \langle 24 \rangle} \frac{tu}{M^2(s - M^2)}. \quad \text{(8)}$$

In order to use Eqs. (5), (6) and (8), a softening of the pole term to a Breit-Wigner form, $[s - M^2]^{-1} \rightarrow [(s - M^2) + i\Gamma M]^{-1}$, is necessary. Here, $\Gamma$ is the total width of the excitation. To obtain the partial widths we follow a standard procedure. Consider a particle with mass $M$, spin $J$ and gauge group index $a$ decaying at rest [with momentum $P = (M, \vec{0})$] into particles 3 and 4. (We will reserve the numbers 1 and 2 for incoming particles in a scattering situation.) Let the initial spin component along the z axis be $J_z = \Lambda$. The final state may be specified by the angle $\theta$ of the decay axis $z'$ w.r.t. the $z$ axis, the c.m. momenta $\vec{p}_3, \vec{p}_4 = -\vec{p}_3$, the helicities $\lambda_3, \lambda_4$ and gauge indices $a_3, a_4$ of the final state particles. (See Fig. [1])

The $S$-matrix element for the decay into particular helicity states is

$$S = i(2\pi)^4 \delta^4(P - p_3 - p_4) \langle \bar{p}_3 \lambda_3 a_3; \bar{p}_4 \lambda_4 a_4| \mathcal{L}| 0, \Lambda, a \rangle, \quad \text{(9)}$$

which in a standard manner gives for the decay width into these states

$$\Gamma_{\lambda_3 \lambda_4; a_3 a_4}^{aJ} = \frac{1}{2M} \frac{(2\pi)^4}{(2\pi)^6} \int d^4p_3 \int d^4p_4 \delta^4(P - p_3 - p_4) \delta^+(p_3^2 - m_3^2) \delta^+(p_4^2 - m_4^2) \times ||\bar{p}_3 \lambda_3 a_3; \bar{p}_4 \lambda_4 a_4| \mathcal{L}| 0, \Lambda, a \rangle|^2$$

$$= \frac{p^*}{32\pi^2 M^2} \int d\Omega_3 \ |\langle \bar{p}_3 \lambda_3 a_3; -\bar{p}_3 \lambda_4 a_4| \mathcal{L}| 0, \Lambda, a \rangle|^2, \quad \text{(10)}$$
where $\mathcal{L}$ is an interaction Lagrangian, and $p^* = |\vec{p}_3| = |\vec{p}_4| = M/2$ for relativistic particles in the final state. We now implement the Wigner expansion into spin eigenstates of the decaying particle along $z'$ [19], the decay axis:

$$|0, \Lambda\rangle = \sum_{\Lambda'} |0, \Lambda'\rangle \langle 0, \Lambda' | 0, \Lambda\rangle$$

(11)

$$= \sum_{\Lambda'} d^{\Lambda}_{\Lambda' \Lambda} (\theta) |0, \Lambda'\rangle ,$$

(12)

where by rotational invariance $\Lambda' = \lambda_3 - \lambda_4$. Thus,

$$\Gamma^{aJ}_{\lambda_3 \lambda_4; a_3 a_4} = \frac{1}{64\pi^2 M} \int d\Omega |F^{aJ}_{\lambda_3 \lambda_4; a_3 a_4}|^2 |d^{\Lambda}_{\Lambda' \Lambda} (\theta)|^2 ,$$

(13)

where the collinear amplitudes $F^{aJ}$ are matrix elements for the decay of a particle with $J_{z'} = \lambda_3 - \lambda_4$ into particles 3, 4 with momenta along the $\pm z'$ axis. Since

$$\int d\Omega \ |d^{\Lambda}_{\Lambda', \lambda_3 - \lambda_4} (\theta)|^2 = \frac{4\pi}{2J + 1}$$

(14)

for any $\Lambda$ and helicities, the width of the decaying particle into a particular helicity pair is

$$\Gamma^{aJ}_{\lambda_3 \lambda_4; a_3 a_4} = \frac{1}{16(2J + 1)\pi M} |F^{aJ}_{\lambda_3 \lambda_4; a_3 a_4}|^2 ,$$

(15)

independent of the initial spin component $\Lambda$ of the decaying particle. Our goal is to extract $F^{aJ}$ from the center-of-mass resonant scattering amplitude:

$$\langle 34; \theta |\mathcal{M}|12; 0\rangle = \sum_{a, J} \langle 34; \theta |\mathcal{M}^{aJ}|12; 0\rangle ,$$

(16)
where
\[ \langle 34; \theta | M^{\alpha J} | 12; 0 \rangle = (s - M^2)^{-1} F^{\alpha J}_{\lambda_3 \lambda_4; a_3 a_4} F^{\alpha J}_{\lambda_1 \lambda_2; a_1 a_2} d_{\lambda_1 - \lambda_2; \lambda_3 - \lambda_4}^J (\theta). \] (17)

For our purposes, it is convenient to split the \( U(N) \) generators \( T^a = (T^0, T^A) \) into \( SU(N) \) “color” generators \( T^A, A = 1, \ldots, N^2 - 1 \) and the \( U(1) \) generator \( T^0 = \mathbb{I}_N / \sqrt{2N} \). The gauge bosons \( g^a = (C^0, G^A) \) include the color singlet \( C^0 \) in addition to \( SU(N) \) gluons \( G^A \). In what follows, we also refer to the first Regge excitations as \( g^{a*} = (C^{0*}, G^{A*}) \). The group-theoretical factors of amplitudes (5), (6), and (8) contain anticommutators
\[ \{ T^{a_1}, T^{a_2} \} = 4 \sum_a d^{a_1 a_2 a} T^a, \] (18)
where the completely symmetric \( d \) symbol denotes the symmetrized trace
\[ d^{a_1 a_2 a_3} = \text{STr}(T^{a_1} T^{a_2} T^{a_3}). \] (19)

Note that, in particular,
\[ d^{000} = \frac{1}{\sqrt{8N}}, \quad d^{00A} = 0, \quad d^{0AB} = \frac{1}{\sqrt{8N}} \delta^{AB}. \] (20)

We shall first consider the case where all four particles are \( U(N) \) gauge bosons \( g^a, a = 0, \ldots, N^2 - 1 \). The group-theoretical factor of the resonant amplitudes (5) and (6) is
\[ \text{Tr}(\{T^{a_1}, T^{a_2}\} \{T^{a_3}, T^{a_4}\}) = 8 \sum_a d^{a_1 a_2 a} d^{a_3 a_4 a} \equiv Q^{a_3 a_4}_{a_1 a_2}. \] (21)

From Eqs. (5) and (6) it follows that
\[ \sum_a \langle \pm 1 \pm 1; \theta | M^{a J=0} | \pm 1 \pm 1, 0 \rangle = Q^{a_3 a_4}_{a_1 a_2} \frac{(2gM)^2}{s - M^2}, \] (22)
\[ \sum_a \langle \pm 1 \mp 1; \theta | M^{a J=2} | \pm 1 \mp 1, 0 \rangle = Q^{a_3 a_4}_{a_1 a_2} \frac{(2gM)^2}{s - M^2} d^2_{\pm \pm \mp \mp}(\theta), \] (23)
\[ \sum_a \langle \mp 1 \pm 1; \theta | M^{a J=2} | \mp 1 \pm 1, 0 \rangle = Q^{a_3 a_4}_{a_1 a_2} \frac{(2gM)^2}{s - M^2} d^2_{\pm \pm \mp \mp}(\theta). \] (24)

Thus, comparing Eqs. (22), (23), and (24) with Eq. (17) we obtain the collinear amplitudes (up to a phase):
\[ F^{a J=0}_{+ + a_3 a_4} = F^{a J=0}_{- - a_3 a_4} = F^{a J=2}_{+ + a_3 a_4} = F^{a J=2}_{- - a_3 a_4} = 4\sqrt{2gM} d^{a_3 a_4 a}. \] (25)

By using Eq. (15), we obtain the width for the decay of a resonance \( g^{a*} \), with \( J = 0 \) or \( J = 2 \), into two gauge bosons:
\[ \Gamma^J_{g^{a*} \to gg} = \frac{1}{2} \times \frac{1}{16(2J + 1)\pi M} \sum_{\lambda_3, \lambda_4} \sum_{a_3, a_4} |F^{a J}_{\lambda_3 \lambda_4; a_3 a_4}|^2 \] (26)
\[ = \begin{cases} \frac{g^2 M}{(2J + 1)\pi} \sum_{a_3, a_4} d^{a_3 a_4 a} d^{a_3 a_4 a} & J = 0 \\ \frac{g^2 M}{(2J + 1)\pi} \sum_{a_3, a_4} d^{a_3 a_4 a} d^{a_3 a_4 a} & J = 2 \end{cases}. \]
where a factor of $1/2$ have been introduced to take account of either double counting or identical particles in the final state. Note that for $J = 2$ there are two possible helicity configurations for the gluons in the final state, see Eq. (25). For $J = 0$, the situation is different because as discussed above, in order to prevent the excitations of the resonance through a $(- -)$ initial state and its subsequent decay into a $(++)$ final state, one must in effect consider two degenerate resonances with non-trivial chiral properties, one of which decays into $(++)$ and the other one into $(- -)$.

The color sum in Eq. (26) is evaluated for the varying possibilities of the $SU(N)$ and $U(1)$ assignments:

- For the decay of $G^* \to GG$,
  \[
  \sum_{B,C=1}^{N^2-1} d^{ABC} d^{ABC} = \frac{N^2 - 4}{16N}, \tag{27}
  \]
  for any initial $A = 1, \ldots, N^2 - 1$.

- For $G^* \to GC^0$,
  \[
  2 \sum_{B=1}^{N^2-1} d^{AB0} d^{AB0} = \frac{1}{4N}, \tag{28}
  \]
  for any initial $A = 1, \ldots N^2 - 1$.

- Similarly, for $C^{0*} \to GG$
  \[
  \sum_{B,C=1}^{N^2-1} d^{BC0} d^{BC0} = \frac{N^2 - 1}{8N}; \tag{29}
  \]

- and for $C^{0*} \to C^{0}C^0$,
  \[
  d^{000} d^{000} = \frac{1}{8N}. \tag{30}
  \]

Turning now to fermions, we first consider the process $q_{\alpha_1} \bar{q}_{\alpha_2} \to g^{a_3} g^{a_4}$, where $\alpha_1, \alpha_2$ are group indices in the fundamental and anti-fundamental representations of $U(N)$, respectively. In this case, the group factor in the respective amplitude (8) reads

\[
\{T^{a_3}, T^{a_4}\}_{\alpha_1 \alpha_2} = 4 \sum_a d^{[a_3 a_4 a} T^a_{\alpha_1 \alpha_2} \equiv Q^{a_3 a_4}_{\alpha_1 \alpha_2}. \tag{31}
\]

The non-zero resonant amplitudes are

\[
\sum_a \langle \pm \frac{1}{2}, \pm \frac{1}{2}; \theta | M^{a,J=2} = 1 \pm 1, 0 \rangle = Q^{a_3 a_4}_{\alpha_1 \alpha_2 \frac{k}{s}} \frac{(gM)^2}{s-M^2} d^2_{\pm \frac{1}{2}, \pm 1}(\theta), \tag{32}
\]

\[
\sum_a \langle \mp \frac{1}{2}, \mp \frac{1}{2}; \theta | M^{a,J=2} = 1 \mp 1, 0 \rangle = Q^{a_3 a_4}_{\alpha_1 \alpha_2 \frac{k}{s}} \frac{(gM)^2}{s-M^2} d^2_{\pm \frac{1}{2}, \mp 1}(\theta). \tag{33}
\]
The amplitude vanishes for \( J = 0 \). Since the collinear vertex function for decay of the resonance \( g^a \to g^a_1 g^a_2 \) is \( 4 \sqrt{2} g M d^{a_3 a_4} \), we may identify the decay vertex \( g^a \to q_{\alpha_1} \bar{q}_{\alpha_2} \) from Eqs. (31), (32) and (33) using factorization:

\[
F^{a,J=2}_{\frac{1}{2}+\frac{1}{2} a_1 a_2} = \frac{1}{\sqrt{2}} T^a_{a_1 a_2} g M \tag{34}
\]

From Eqs. (34) and (15), the width for decay into \( q \bar{q} \) is

\[
\Gamma_{g^\ast \to q \bar{q}} = \frac{2}{16(2J+1)\pi M} \left( \frac{g M}{\sqrt{2}} \right)^2 \text{Tr}(T^a T^a) \tag{35}
\]

\[
= \frac{g^2}{160\pi} M \quad \text{per flavor,} \quad J = 2 \tag{36}
\]

Note that there are two helicity configurations in the \( q \bar{q} \) final state. It is worthwhile to note that this value for the width is the same for all \( N^2 \) gauge bosons, and is independent of \( N \).

In order to compute the decay rate of \( J = 1 \) gluonic excitation into a quark-antiquark pair, we proceed in two steps. First, since this resonance is inaccessible through gluon-gluon scattering, we identify it as an intermediate state in the four-gluino amplitude. To that end, we use the supersymmetric Ward identity [21]:

\[
\mathcal{M}(\Lambda_1^-, \Lambda_2^+, \Lambda_3^+, \Lambda_4^-) = \langle \text{23} \rangle \langle 14 \rangle \mathcal{M}(g_1^-, g_2^+, g_3^+, g_4^-), \tag{37}
\]

which implies that the two amplitudes are equal up to a phase factor. Now the \( s \)-channel singularity (6), previously attributed to pure \( J = 2 \) propagation, needs reinterpretation appropriate to \((\pm \frac{1}{2} \pm \frac{1}{2})\) external states:

\[
d_{2,2,2}(\theta) = \frac{1}{4} d_{1,1,1}(\theta) + \frac{3}{4} d_{1,1,1}(\theta), \tag{38}
\]

which exhibits both \( J = 2 \) and \( J = 1 \) propagation. By using Eqs. (25) and (37), we obtain the collinear amplitudes

\[
F^{a,J=1}_{\frac{1}{2}+\frac{1}{2} a_3 a_4} = 4 \sqrt{\frac{3}{2}} g M d^{a_3 a_4 a}. \tag{39}
\]

In the second step, we consider the the four-fermion amplitude with two gauginos and two quarks. Here again, we use the identity [21]

\[
\mathcal{M}(q_1^-, \bar{q}_2^+, \Lambda_3^-, \Lambda_4^+) = \frac{[23]}{[24]} \mathcal{M}(q_1^-, \bar{q}_2^+, g_3^+, g_4^+), \tag{40}
\]

and extract the pole term by using Eq. (8):

\[
\mathcal{M}(q_1^-, \bar{q}_2^+, \Lambda_3^-, \Lambda_4^+) \to 8 g^2 \sum_a d^{a_3 a_4 a} T^a_{a_1 a_2} \langle 13 \rangle \langle 24 \rangle \frac{tu}{M^2(s-M^2)}. \tag{41}
\]

In this case, both \( J = 1 \) and \( J = 2 \) propagate in the \( s \)-channel:

\[
tu = \frac{1}{4} M^2 \left[ d_{1,-1,-1}^2(\theta) + d_{1,1,-1}^1(\theta) \right] \quad \text{at} \quad s = M^2. \tag{42}
\]

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8 See Ref. [22] for a formal proof of supersymmetric Ward identities in full-fledged superstring theory.
After extracting the part of \([11]\) describing \(q\bar{q} \to g^*(J = 1) \to \Lambda\Lambda\) and factorizing out the \(g^*(J = 1) \to \Lambda\Lambda\) vertex given in Eq.\([39]\), we obtain the following collinear amplitude for the decay of the \(J = 1\) resonance into a quark-antiquark pair:

\[
F_{\alpha J=1}^{1/2} = \frac{1}{\sqrt{6}} T_{\alpha_1\alpha_2}^a g M. \tag{43}
\]

The corresponding decay width is

\[
\Gamma_{g \to q\bar{q}} = \frac{g^2}{288\pi} M \quad \text{per flavor,} \quad J = 1. \tag{44}
\]

It is clear from the use of \(N = 1\) supersymmetry that this particular \(J = 1\) state is the vector boson component of the massive spin 2, \(N = 1\) supermultiplet. As mentioned before, it is a model-dependent particle, hence its properties may be affected by the supersymmetry breaking mechanism. It is worth mentioning that Eqs.\([38]\) and \([12]\) can also be used to recheck the \(J = 2\) case \([36]\).

Finally, we turn to the first Regge recurrences of quarks, color triplets [or more generally, in the fundamental representation of \(SU(N)\)] \(q^*_\alpha\) with mass \(M\) and spins \(J = 1/2, 3/2\). They appear as resonances in quark-gluon scattering. The corresponding amplitudes can be obtained by crossing from Eq.\([7]\). The relevant \(s\)-channel pole terms are

\[
\mathcal{M}(q_1^-, g_2^+, q_3^+, g_4^+) \to 2 g^2 \left( \sum_{\alpha} T_{\alpha_1\alpha_2}^{a_2} T_{\alpha_3\alpha_4}^{a_4} \right) \frac{(12)^2}{(14)(34)} \frac{u}{M^2(s-M^2)} \quad \text{for } J = 1/2, \tag{45}
\]

\[
\mathcal{M}(q_1^-, g_2^+, q_3^+, g_4^+) \to 2 g^2 \left( \sum_{\alpha} T_{\alpha_1\alpha_2}^{a_2} T_{\alpha_3\alpha_4}^{a_4} \right) \frac{(14)^2}{(12)(23)} \frac{u}{M^2(s-M^2)} \quad \text{for } J = 3/2. \tag{46}
\]

After repeating the same steps as for other resonances, we obtain the following collinear amplitudes:

\[
F_{\alpha J=1/2}^{\frac{1}{2}+1 \lambda_3 \lambda_4} = F_{\alpha J=1/2}^{\frac{1}{2}-1 \lambda_3 \lambda_4} = F_{\alpha J=3/2}^{\frac{1}{2}+1 \lambda_3 \lambda_4} = F_{\alpha J=3/2}^{\frac{1}{2}-1 \lambda_3 \lambda_4} = \sqrt{2} g M T_{\alpha_3 \alpha}. \tag{47}
\]

By using the above result, we obtain the width for the decay of a quark resonance \(q^*_\alpha\), with \(J = 1/2\) or \(J = 3/2\), into a quark and gluon:

\[
\Gamma_{q \to qg}^J = \frac{1}{16(2J+1)\pi M} \sum_{\lambda_3, \lambda_4} \sum_{\alpha_3, \alpha_4} |F_{\lambda_3 \lambda_4 \alpha_3 \alpha_4}^\alpha|^2 \tag{48}
\]

\[
= \begin{cases} 
\frac{g^2 M}{8(2J+1)\pi} \sum_{\alpha_3, \alpha_4} T_{\alpha \alpha_3}^{a_3} T_{\alpha_3 \alpha}^{a_4} & J = 1/2 \\
\frac{g^2 M}{4(2J+1)\pi} \sum_{\alpha_3, \alpha_4} T_{\alpha_3 \alpha}^{a_3} T_{\alpha \alpha_4}^{a_4} & J = 3/2 
\end{cases}
\]

Note that for \(J = 3/2\) there are two possible helicity configurations in the final state, see Eq.\([47]\). For \(J = 1/2\), the situation is different because similarly to the case of gluonic \(J = 0\), in order to prevent the excitations of the resonance through a \((-\frac{1}{2} - 1)\) initial state and its subsequent decay into a \((+\frac{1}{2} + 1)\) final state, one must in effect consider two degenerate resonances with non-trivial chiral properties, one of which decays into \((+\frac{1}{2} + 1)\) and the other one into \((-\frac{1}{2} - 1)\). The color sum in Eq.\([48]\) depends whether the final vector boson is the proper gluon in the adjoint representation of \(SU(N)\) or a color singlet.
For the decay of $q^* \to q G$,

$$
\sum_{a=1}^{N^2-1} \sum_{\beta=1}^{N} T_{\alpha\beta}^a T_{\beta\alpha}^a = \frac{N^2 - 1}{2N},
$$

for any initial $\alpha = 1, \ldots, N$.

Similarly, for $q^* \to q C^0$,

$$
\sum_{\beta=1}^{N} T_{\alpha\beta}^0 T_{\beta\alpha}^0 = \frac{1}{2N}.
$$

Our results are summarized in tables III, IV, V, and VI. In table I partial and total widths are given for any $N$ and number of flavors $N_f$. The entries involving $C^{0*}$ and $C^0$ require some discussion. The singlet gauge field $C^0_\mu$ is in reality a linear combination of the electroweak gauge boson $Y_\mu$ coupled to hypercharge, and an orthogonal set of $U(1)$ gauge bosons. Any vector boson $Z'_\mu$, orthogonal to the hypercharge, must grow a mass $M_{Z'}$ in order to avoid long range forces between baryons other than gravity and Coulomb forces. The widths given in tables VI and III are premised on the assumption that corrections of order $(M_{Z'}/M)^2$ are negligible, both in obtaining matrix elements and in calculating phase space. In table III we give numerical values for the widths in the case of $N = 3$ and $N_f = 6$. In tables IV and V we give the corresponding partial and total widths for the lowest Regge excitation of the quark triplet.

### TABLE I: Partial and total widths of the lowest Regge excitation of the $U(N)$ gauge bosons. All quantities are to be multiplied by $(g^2/4\pi) M \simeq 100$ GeV (M/TeV).

| channel | $J = 0$ | $J = 1$ | $J = 2$ |
|---------|---------|---------|---------|
| $GG$    | $\frac{N^2-4}{4N}$ | $\frac{N^2-1}{2N}$ | $-\frac{N^2-4}{10N}$ | $\frac{N^2-1}{5N}$ |
| $GC^0$  | $\frac{1}{N}$ | $-\frac{1}{2N}$ | $-\frac{2}{5N}$ | $-\frac{1}{5N}$ |
| $C^0C^0$| $-\frac{1}{2N}$ | $-\frac{1}{2N}$ | $-\frac{1}{2N}$ | $-\frac{1}{2N}$ |
| $q\bar{q}$ | $0$ | $\frac{N_f}{72}$ | $\frac{N_f}{72}$ | $\frac{N_f}{40}$ | $\frac{N_f}{40}$ |
| all     | $\frac{N}{4}$ | $\frac{N}{2}$ | $\frac{N_f}{72}$ | $\frac{N_f}{72}$ | $\frac{N_f}{10} + \frac{N_f}{40}$ | $\frac{N}{5} + \frac{N_f}{40}$ |

\[9\] The anomalous mass growth allows the survival of global baryon number conservation, preventing fast proton decay [23].
TABLE II: Partial and total widths, in GeV, of the lowest Regge excitation of the $U(3)$ gauge bosons. (We have taken $N_f = 6$.) All quantities are to be multiplied by $M/{\text{TeV}}$.

| channel | $J = 0$ | $J = 1$ | $J = 2$ |
|---------|---------|---------|---------|
| $GG$    | $G^*$   | $C^0*$  | $G^*$   | $C^0*$  |
|         | 41.6    | 133.3   | –       | –       |
| $GC^0$  | 33.3    | –       | –       | –       |
| $C^0C^0$| –       | 16.7    | –       | –       |
| $q\bar{q}$ | 0      | 0       | 8.3     | 8.3     |
| all     | 75.0    | 150.0   | 8.3     | 8.3     |

TABLE III: Partial and total widths of the lowest Regge excitation $q^*$ of the quark in the fundamental representation of $SU(N)$. All quantities are to be multiplied by $(g^2/4\pi) M \simeq 100$ GeV ($M/{\text{TeV}}$).

| channel | $J = 1/2$ | $J = 3/2$ |
|---------|---------|---------|
| $qG$    | $\frac{N^2 - 1}{8N}$ | $\frac{N^2 - 1}{8N}$ |
| $qC^0$  | $\frac{1}{8N}$     | $\frac{1}{8N}$     |
| all     | $\frac{N}{8}$      | $\frac{N}{8}$      |

TABLE IV: Partial and total widths, in GeV, of the lowest Regge excitation $q^*$ of the $SU(3)$ quark triplet. (We have taken $N_f = 6$.) All quantities are to be multiplied by $M/{\text{TeV}}$.

| channel | $J = 1/2$ | $J = 3/2$ |
|---------|---------|---------|
| $qG$    | 33.3    | 33.3    |
| $qC^0$  | 4.2     | 4.2     |
| all     | 37.5    | 37.5    |

In summary, within the framework of TeV-scale open string theory we have presented model independent partial and total decay widths for the first Regge excitations of the gluon octet, the accompanying $U(1)$ color singlet, and the quark triplet. This is of most immediate interest for the LHC, as correct values of the widths are critical in predicting $\gamma + \text{jet}$ and dijet invariant mass spectra.

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