Weighting Temporary Change Outlier by Modified Huber Function with Monte Carlo Simulations

I M Md Ghani¹ and H A Rahim¹

¹Faculty of Ocean Engineering Technology and Informatics, Universiti Malaysia Terengganu, 21030 Kuala Nerus, Terengganu Darul Iman

Abstract. Robust method is a popular approach to dealing the existence of outliers in the data. Many researchers have applied Huber weight function. The aim of this paper is to evaluate the performance of the Huber weight function and the modification of the Huber weight function on the temporary change (TC) outliers. The data used in this paper were generated as ARMA(1,0)-GARCH(1,2) model via the Monte Carlo simulation. There are three major situations in this simulations: without weight (WW), with Huber weight (WH) and with a modified Huber weight (WMH). Three different TC contamination (0%, 10% and 20%) and three different time series length (100, 500 and 1000) were tested. The performance of the three situations was compared on the basis of AIC, SIC, HQIC, MAE, MSE and RMSE. The results of the numerical simulations show that the performance in the WMH situation is better than the WH situation in the presence of TC outliers.

1. Introduction

Most time series data are influenced by the existence of outliers. Some literature uses other outlier terms such as noise, anomalies, abnormalities and discordant. The existence of outliers may be due to several factors of causes, such as measurement errors, economic crises, incidents of conflict, extreme conditions, or sudden political events. [1] has started to recognize the problem of outliers in its Autoregressive (AR) model over the past four decades. [2] and [3] studies selected the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model to predict volatility and analyse the outlier effect. However, the outliers phenomenon can affect GARCH parameter estimation [4-6], GARCH model identification and estimation [7], and forecasting [2, 6-7]. Therefore, several alternative ways to deal with outliers were created, which is called robust methods.

The robust method has been one of the most interesting research topics due to its ability to overcome the impact of outliers. In statistics, the determination of robust methods is based on four characteristics, namely the breakdown point, the influence function, the efficiency, and equivariance. More than a decade ago, robust methods were widely used in many fields such as chemometrics [8], mechanical systems [9], dam changes in water levels [10], power systems [11] and electricity prices [12]. Although there are many types of robust methods, the majority of researchers prefer M-estimator compared the others.

The M-estimator was introduced by [13] which is the result of the development of the maximum likelihood method. The selection of M-estimator is because based on the weight function. [10] noted that the weight is given to each observation point depends on how far the point is from the fitted line. A maximum weighting is given to points near the line, while points farther away from the line are given a decreased weight. Several studies have tested and compared the weight functions of M-estimator. In 2000, [8] tested four types of weighting functions (Bisquare, Cauchy, Fair, and Huber)
when examining the use of partial least squares (PLS) for multiple outlier detection in infrared spectroscopic applications. The [9] study evaluated the performance of six types of M-estimators (Huber, modify Huber, Fair, Cauchy, Welsch, and Tukey) with respect to mechanical applications. In another study, [11] has chosen Huber and Hampel functions as a robust approach in power systems. Recently, [12] compared Huber’s, the polynomial and the Tukey’s weight function to obtain the best weighting function for Generalized M-estimators of SETAR process.

In summary, many interesting results indicating the M-estimator’s potential have been reported. However, most of the open literature studies used the existing method where the weights depend upon the residual [8-12, 14]. In addition, there is still a lack of effect on the modification of Huber’s weight function. Thus, this paper will evaluate the performance of Huber weight function and the modification of the Huber weight function, especially on temporary change outlier.

The following section of this paper is arranged as follows. In Section 2 the ARMA\((g,h)\) model, GARCH\((i,j)\) model, robust method and temporary change outliers are briefly described. The algorithm for Monte Carlo simulations of the Huber weight function and the modification of the Huber weight function on the temporary change outliers performed in Section 3. The result and discussion of ARMA\((1,0)\)-GARCH\((1,2)\) model based on Monte Carlo simulations reported in Section 4. Finally, the conclusion is summarized in Section 5.

2. Methodology

Consider a stationary time series model, ARMA\((g,h)\) model which can expressed in sigma notation as [15],

\[
Y_t = F + \sum_{a=1}^{g} \theta_a Y_{t-a} + \sum_{b=1}^{h} \phi_b \varepsilon_{t-b} + \varepsilon_t
\]  

(1)

where \(F\) is a fixed value, \(\theta_a\) are the parameter of the autoregressive component of order \(g\), \(\phi_b\) are the parameters of the moving average component of order \(h\), and \(\varepsilon_t\) is the error term at time \(t\). The order \(g\) and \(h\) are non-negative integers.

Consider a simple linear GARCH\((i,j)\) model which developed by [16] with the conditional mean equation \((\tau_t)\) and conditional variance equation \((\sigma_t^2)\) can be defined in equation (2) and equation (4), respectively.

\[
\tau_t = F + \varepsilon_t  
\]  

(2)

\[
\varepsilon_t = Z_t \sigma_t, \ Z_t \sim N(0,1) 
\]  

(3)

\[
\sigma_t^2 = \sigma_0 + \sum_{a=1}^{i} \alpha_a \sigma_{t-a}^2 + \sum_{b=1}^{j} \beta_b \varepsilon_{t-b}^2 
\]  

(4)

where \(Z_t\) is the standardized residual, \(\sigma_t^2\) is the conditional variance at time \(t\), \(\sigma_0\) is the fixed parameter, \(\sigma_t^2\) is the forecast variance for the current period (GARCH term) and \(\varepsilon_t^2\) is the new information about volatility observed in the current period (ARCH term) with conditions \(\alpha_0 > 0, \alpha_a \geq 0; \sigma_0 > 0; i = 1, \ldots, i\) and \(\beta_b \geq 0; j = 1, \ldots, j\).

In order to determine the persistence of GARCH\((i,j)\) model, the non-negative constants can calculate as \(\Sigma_{a=1}^{i} \alpha_a + \Sigma_{b=1}^{j} \beta_b\). When the \(\Sigma_{a=1}^{i} \alpha_a + \Sigma_{b=1}^{j} \beta_b < 1\), hence GARCH\((i,j)\) model is called covariance stationary. If the constant parameter, \(\alpha_0\) combined with \(\Sigma_{a=1}^{i} \alpha_a + \Sigma_{b=1}^{j} \beta_b\), the level of unconditional variance (known as long term volatility) can be determined by
In this paper, the hybrid of ARMA(1,0)-GARCH(1,2) model was selected in order to achieve the objective. From the general form of ARMA\((g,h)\) model and GARCH\((i,j)\) model in equation (1) and equation (4), respectively, the ARMA(1,0)-GARCH(1,2) model can be written as follows:

\[
Y_t = F + \theta_1 Y_{t-1}
\]

\[
\sigma_t^2 = \alpha_0 + \alpha_1 \sigma_{t-1}^2 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 \varepsilon_{t-2}^2
\]

where \(\sigma_{t-1}^2\) is the forecast variance in the preceding period (GARCH term), \(\varepsilon_{t-1}^2\) is the information about volatility observed in the preceding period (ARCH term) and \(\varepsilon_{t-2}^2\) is the information about volatility observed in the two preceding period (ARCH term) with condition \(\alpha_0 > 0, \alpha_1 \geq 0\) and \(\beta_1, \beta_2 \geq 0\).

### 2.1 Temporary Change Outlier

Temporary change (TC) outlier is a condition in which the data point causes the values of subsequent observations to change unexpectedly in sequence, but varies from the current level change in that this change gradually dies out over time [17]. [18] also stated that TC is an impact change that declines exponentially. The dynamic pattern of TC effect is given by

\[
\frac{1}{1 - \delta A}
\]

where \(\delta\) is the exponential decay parameter such that \((0 < \delta < 1)\). If \(\delta\) tends to 0, the TC reduces to an additive outlier, whereas if \(\delta\) tends to 1, the TC reduces to a level change outlier [18].

From equation (4), GARCH\((i,j)\) model can be written as an ARMA\((g,h)\) model for \(\varepsilon_t^2\) [16] as follows:

\[
\varepsilon_t^2 = \alpha_0 + \sum_{b=1}^{g} (\beta_b + \alpha_a) \varepsilon_{t-a}^2 + \eta_t - \sum_{a=1}^{h} \alpha_a \eta_{t-a}
\]

with \(g = \max\{i,j\}\) and \(\eta_t = \varepsilon_t^2 - \sigma_t^2; t = 1, 2, ..., n\) where \(\varepsilon_t^2\) known as outlier free time series while \(\eta_t\) known as outlier-free residuals. Equation (8) can be written

\[
\varepsilon_t^2 = \frac{\alpha_0}{1 - \beta(A) - \alpha(A)} + \frac{1 - \alpha(A)}{1 - \beta(A) - \alpha(A)} \eta_t
\]

\[
= \frac{\alpha_0}{1 - \beta(A) - \alpha(A)} + \pi^{-1}(A) \eta_t
\]

with \(\alpha(A) = \sum_{a=1}^{i} \alpha_a A^a, \beta(A) = \sum_{b=1}^{j} \beta_b A^b\) and \(\pi(A) = \frac{1 - \beta(A) - \alpha(A)}{1 - \alpha(A)}\).

According to [19], when TC presence in GARCH model becomes

\[
\varepsilon_t^2 = \omega_{TC} \xi_{TC}(A) l_t(T) + \varepsilon_t^2
\]
From equation (10) can be interpreted as a regression model for $e_t^2$ and rewrite as

$$e_t^2 = \omega_{TC}x_t + \varepsilon_t^2$$ (11)

where

$e_t^2$ is an observed series $\varepsilon_t^2$,

$\omega_{TC}$ is the magnitude effect of TC, which is $\omega_{TC}(T) = \frac{\sum_{t=1}^{n} \varepsilon_t^2 x_t}{\sum_{t=1}^{n} (x_t)^2}$

$x_t$ represent as $x_t = \begin{cases} 0 & t < T \\ 1 & t = T \\ \delta^k - \sum_{b=1}^{k-1} \delta^{k-b} \pi_b - \pi_k & t = T + k, k \geq 1 \end{cases}$

$\xi_{TC}(A)$ is the dynamic pattern of TC effect, which is $\xi_{TC}(A) = \frac{1}{1-\delta^A}$

$I_t(T)$ is the indicator function which can explain the effect of outliers as

$I_t(T) = \begin{cases} 1 & t = T \\ 0 & \text{otherwise} \end{cases}$, where $T$ is the location of TC occurred.

### 2.2 Robust Method

Generally, there are seven types of robust methods such as least absolute deviation [20], M-estimator [13], R-estimators [21], least median squares [22], least trimmed squares [22], S-estimators [23] and MM-estimators [24]. However, the focus in this paper is M-estimator.

Based on the robustreg procedure in [25], there are ten types of weight functions that can be obtained in M-estimators: Andrews, Bisquare, Cauchy, Fair, Hampel, Huber, Logistic, Median, Talworth and Welsch. [26] were categorized eight from these ten weight functions to three groups namely hard redescenders (Andrews, Bisquare and Talworth), soft redescenders (Cauchy and Welsch) and monotone ($\delta$) functions (Huber, Logistic and Fair). However, this paper focuses only on the Huber’s weight function. The class of M-estimators for location and regression models was introduced by [13].

#### 2.3 Huber weight Function

The Huber weight function can be expressed as

$$w(r) = \begin{cases} 1 & \text{if } |r| \leq c \\ \frac{1}{c} |r| & \text{if } |r| > c \end{cases}$$

where $c$ is the tuning constant ($c = 1.345$) that produce 95% efficiency for normally distributed, $\varepsilon_t$ and $r$ is the standardized residuals.

The standardized residual is calculated as equation (12)

$$r = \frac{e_t}{s}$$ (12)

where $s$ is the robust measure of the standard deviation. The standard deviation in the Huber weight function is replaced by the Median Absolute Deviation (MAD) given by $s = \text{MAD}/0.6745$ (as mentioned in [10] and [13]).

#### 2.4 Modified Huber weight Function
In contrast to what has recently been done in the literature, the $r$ value in the equation (12) refers to the contamination data. Suppose that

$$z_1 = \frac{x - f(\mu)}{g(\sigma)}$$

where $f(\mu)$ is any central tendency measurement and $g(\sigma)$ is any dispersion measurement. In this paper, the $f(\mu)$ modified to median instead of mean. While the $g(\sigma)$ modified to interquartile range (IQR/3) instead of MAD.

### 2.5 Criteria of the Best Model

The performance of the Huber function on the ARMA(1,0)-GARCH(1,2) model was compared to the modification of the Huber function. There are three model selection criteria: Akaike Information Criteria (AIC) [27], Schwarz’s Bayesian Information Criterion (SIC) [28] and the Hannan-Quinn Information Criterion (HQIC) [29]. The AIC, SIC and HQIC can be computed as

$$\text{AIC} = -2\ln(L) + 2k , \quad \text{SIC} = -2\ln(L) + \ln(N)k , \quad \text{HQIC} = -2\ln(L) + 2\ln(\ln(N))k$$

where $L$ is the value of the likelihood function evaluated at the parameter estimates, $N$ is the number of observations, and $k$ is the number of estimated parameters. The minimum value of AIC, SIC and HQIC was selected as the better model.

While the performance of forecasting models is evaluated using three measures: Mean Absolute Error (MAE), Mean Square Error (MSE) and Root Mean Square Error (RMSE).

$$\text{MAE} = \frac{1}{T} \sum_{t=T_1}^{T} |\tilde{\sigma}^2_t - \hat{\sigma}^2_t| , \quad \text{MSE} = \frac{1}{T} \sum_{t=T_1}^{T} (\sigma^2_t - \tilde{\sigma}^2_t)^2 , \quad \text{RMSE} = \sqrt{\frac{1}{T} \sum_{t=T_1}^{T} (\sigma^2_t - \tilde{\sigma}^2_t)^2}$$

where $T$ is the number of total observations and $T_1$ is the first observation. The $\sigma^2_t$ and $\tilde{\sigma}^2_t$ is the actual and predicted conditional variance at time $t$, respectively. The smallest value of MAE, MSE and RMSE, the best accurate forecast model.

### 3. Monte Carlo Simulations

The selecting of the best time series model is one of the factors that make the estimation more precise. Nevertheless, before the resulting model is applied to the actual data, a process needs to be done first in order to test the performance of the developed model. This process is known as the Monte Carlo method. According to [30] state that Monte Carlo method as “representing the solution of a problem as a parameter of a hypothetical population, and using a random sequence of numbers to construct a sample of the population, from which statistical estimates of the parameter can be obtained”.

There are three types of TC percentage contamination that will be examined: 0%, 10% and 20%. Each percentage of contamination by TC has been divided into three situations which are without weight (WW), with Huber weight (WH) and with modified Huber weight (WMH). The three situations will be compared on the basis of AIC, SIC, HQIC, MAE, MSE and RMSE.

The simulation was generated using version 3.5.3 of the statistical package R software that developed by [31]. Throughout this phase, the ARMA(1,0)-GARCH(1,2) model used the tseries package [32] and fGarch package [33] consisting of garchSpec, garchSim and garchFit in R code. The general algorithm conducted as follows:
Step 1: The ARMA(1,0)-GARCH(1,2) model specified using `garchSpec` function with set the true value of parameters: $F = 0.043, \theta_1 = -0.312, \alpha_0 = 0.011, \alpha_1 = 0.913, \beta_1 = 0.224$ and $\beta_2 = -0.136$.

Step 2: The GARCH process simulated 100 observations with mean=0 and standard deviation=1 using `garchSim`.

Step 3: About 10% from sample size contaminated as TC. The number of points that have to contaminate after the first contamination was set with 5. The location of TC was identified. The magnitude for each contaminated point were calculated by using normal distribution where mean=0 and standard deviation=16.

Step 4: The Huber weight function was calculated to 10% contamination of TC. The new data was determined based on the weighting Huber function that given to the 10% contamination of TC.

Step 5: The modification of Huber weight function was calculated to 10% contamination of TC. The new data was determined based on the weighting modify Huber function that given to the 10% contamination of TC.

Step 6: Step 3 to 5 then repeated with increased the contamination of TC to 20%.

Step 7: The parameters of the ARMA(1,0)-GARCH(1,2) model for three situations fitted using `garchFit` function in normal error distribution.

Step 8: The performance of ARMA(1,0)-GARCH(1,2) model for three situations was evaluated.

Step 9: Step 1 to 8 then repeated for different time series length, T=500 and T=1000.

4. Results and Discussion
The plot of returns with 10% and 20% TC with Huber and with a modified Huber weight for T=100 were shown in figure 1. From the figure 1 (left panel), when contaminate with 10% TC there are large negative and positive returns values especially on time 29 and 76 which are -17.1926 and 16.5536, respectively. While the observation on 56 plotted in figure 1 (right panel) clearly show large negative returns value ($T_{s0}=24.2305$) during contaminated with 20% TC.
Figure 1. Plot of simulation returns with Huber weight and modified Huber weight with 10% TC (left panel) and 20% TC (right panel) for T=100.

When time series length increased to 500, the largest positive ($T_{131}=22.9485$) and negative values ($T_{406}=-27.2835$) of returns during contaminate with 10% TC are presented in figure 2 (left panel). While there are about three points from 20 contaminated observations exhibited the returns more than 20 ($T_3=25.0996$, $T_{131}=22.9485$, $T_{255}=25.5726$), noticeable in figure 2 (right panel).

Figure 2. Plot of simulation returns with Huber weight and modified Huber weight with 10% TC (left panel) and 20% TC (right panel) for T=500.

As shown in figure 3 (left panel), out of 20 contaminated results, there are about five points showing the returns over 15. There are $T_{213}=18.6065$, $T_{452}=15.7728$, $T_{615}=18.4910$, $T_{730}=15.2679$ and $T_{853}=15.6562$. The most positive and negative return values were 26.5080 and -32.1647, respectively located on time 555 and 352, during contamination with 20% TC, see figure 3 (right panel).
Figure 3. Plot of simulation returns with Huber weight and modified Huber weight with 10% TC (left panel) and 20% TC (right panel) for T=1000.

The comparison of selection criteria for contaminated 0%, 10% and 20% with Huber weight and with a modified Huber weight for three different time series length are presented in Table 1. The value of AIC, SIC and HQIC in T=100 showed in range 3.0 to 3.1 during contaminated 0% TC. The AIC, SIC and HQIC reported as 5.0019, 4.9952 and 5.0651, respectively when the level of TC contamination increased to 10%. This shows the rise of 66.1%, 66.2% and 64.7% for all three criteria.

When the Huber weight is added to 10% of TC, the AIC, SIC and HQIC values are 3.0256, 3.0190 and 3.0889, respectively. This indicates that the three criteria in the WH situation for 10% TC are lower than the WW situation (AIC_{10\%\text{WH}} = 3.0256 < AIC_{10\%\text{WW}} = 5.0019; SIC_{10\%\text{WH}} = 3.0190 < SIC_{10\%\text{WW}} = 4.9952; HQIC_{10\%\text{WH}} = 3.0889 < HQIC_{10\%\text{WW}} = 5.0651). There was a strong downward trend of 39.5%, 39.6% and 39.0% for the AIC, SIC and HQIC criteria. However, the AIC, SIC and HQIC values reported 2.0313, 2.0246 and 2.0946, respectively during WMH situation. This shows a decrease of 59.4%, 59.5% and 58.6% for all three criteria compared to the WW situation.

Table 1. The selection criteria for contaminated 0%, 10% and 20% TC with Huber weight and with a modified Huber weight for T=100, 500 and 1000.

| T   | Percentage contamination | AIC        | SIC        | HQIC       |
|-----|--------------------------|------------|------------|------------|
| 100 | 0%                       | 3.0114     | 3.0048     | 3.0747     |
|     | 10% WW situation         | 5.0019     | 4.9952     | 5.0651     |
|     | 10% WH situation         | 3.0256     | 3.0190     | 3.0889     |
|     | 10% WMH situation        | 2.0313     | 2.0246     | 2.0946     |
|     | 20% WW situation         | 5.5806     | 5.5739     | 5.6439     |
|     | 20% WH situation         | 3.0900     | 3.0833     | 3.1532     |
|     | 20% WMH situation        | 2.1305     | 2.1238     | 2.1937     |
| 500 | 0%                       | 2.7754     | 2.7751     | 2.7953     |
|     | 10% WW situation         | 4.6078     | 4.6075     | 4.6276     |
|     | 10% WH situation         | 2.6751     | 2.6748     | 2.6949     |
|     | 10% WMH situation        | 1.6481     | 1.6478     | 1.6679     |
|     | 20% WW situation         | 5.1874     | 5.1871     | 5.2072     |
|     | 20% WH situation         | 2.8217     | 2.8214     | 2.8415     |
|     | 20% WMH situation        | 1.8172     | 1.8169     | 1.8370     |
| 1000| 0%                       | 2.8440     | 2.8439     | 2.8552     |
In addition, there was an improvement to 5.5806, 5.5739 and 5.6439 for AIC, SIC and HQIC criteria in the 20% TC contamination situation. This indicates that the AIC and SIC criteria were raised by 11.6% while HQIC criteria were raised by 11.4% compared to 10% contaminated. During WH situations the AIC, SIC and HQIC values were 3.0900, 3.0833 and 3.1532, respectively. In contrast, the AIC, SIC and HQIC criteria in the WMH situation are smaller than the WH situation (AIC_{20\%\text{ WMH}} = 2.1305 < AIC_{20\%\text{ WH}} = 3.0900; SIC_{20\%\text{ WMH}} = 2.1238 < SIC_{20\%\text{ WH}} = 3.0833; HQIC_{20\%\text{ WMH}} = 2.1937 < HQIC_{20\%\text{ WH}} = 3.1532). This showed a decrease of 61.8%, 61.9% and 61.1% for all three criteria during the WMH situation compared to the WW situation. While, the percentage of decrease for AIC, SIC and HQIC during WH situation were 44.6%, 44.7% and 44.1%, respectively. This consistent with the increase in time series length to 500 and 1000.

As shown in table 2, based on model evaluation, the contamination of 0%, 10% and 20% TC with three major situations are compared. In T=100, the MAE, MSE and RMSE values provides 0.8217, 1.0550 and 1.0271, respectively. However, during contaminated with 10% TC, the three measures shows that MAE=1.3969, MSE=7.7405 and RMSE=2.7822.

The values of MAE, MSE and RMSE are 0.8662, 1.0702 and 1.0345 when the Huber function is applied to 10% of TC. This shows that the three measures for 10% of TC in the WH situation are lower than the WW situation (MAE_{10\%\text{ WH}} = 0.8662 < MAE_{10\%\text{ WW}} = 1.3969; MSE_{10\%\text{ WH}} = 1.0702 < MSE_{10\%\text{ WW}} = 7.7405; RMSE_{10\%\text{ WH}} = 1.0345 < RMSE_{10\%\text{ WW}} = 2.7822). For the MAE, MSE and RMSE measures, there has been a clear downward trend of 38.0%, 86.2% and 62.8%. Nevertheless, during the WMH situation, the values of MAE, MSE and RMSE were 0.5742, 0.3961 and 0.6294. It indicates a decrease of 58.9%, 94.9% and 77.4% relative to the WW situation for all three measures.

Table 2. The model evaluation for contaminated 0%, 10% and 20% TC with Huber weight and with a modified Huber weight for T=100, 500 and 1000.

| T | Percentage contamination | MAE     | MSE     | RMSE    |
|---|---------------------------|---------|---------|---------|
| 0% |                           | 0.8217  | 1.0550  | 1.0271  |
| 10% WW situation | 1.3969 | 7.7405 | 2.7822 |
| 10% WH situation | 0.8662 | 1.0702 | 1.0345 |
| 10% WMH situation | 0.5742 | 0.3961 | 0.6294 |
| 20% WW situation | 1.7149 | 13.8361 | 3.7197 |
| 20% WH situation | 0.9113 | 1.1412 | 1.0683 |
| 20% WMH situation | 0.5996 | 0.4373 | 0.6613 |
| 0% |                           | 0.7678  | 0.9191  | 0.9587  |
| 10% WW situation | 1.2181 | 6.1696 | 2.4839 |
| 10% WH situation | 0.7737 | 0.8306 | 0.9114 |
| 10% WMH situation | 0.5016 | 0.2975 | 0.5454 |
| 20% WW situation | 1.4481 | 10.2815 | 3.2065 |
| 20% WH situation | 0.8361 | 0.9610 | 0.9803 |
| 20% WMH situation | 0.5414 | 0.3524 | 0.5936 |
| 0% |                           | 0.7957  | 0.9953  | 0.9977  |
| 10% WW situation | 1.3190 | 7.8615 | 2.8038 |
| 10% WH situation | 0.7849 | 0.8533 | 0.9238 |
As the level of TC contamination rose to 20%, the MAE, MSE and RMSE measure increased to 1.7149, 13.8361 and 3.7197, respectively. This explained that the increase in the percentage of the three measures were 22.8%, 78.7% and 33.7%, compared to the 10% contamination of TC. However, during WH situations the MAE, MSE and RMSE values decreases to 0.9113, 1.1412 and 1.0683, respectively. When the modified Huber weight is added to 20% of TC, the MAE, MSE and RMSE measure provides lower than in the WH situation (MAE_{20\%}\text{, WH} = 0.5996 < \text{MAE}_{20\%}\text{, WMH} = 0.9113; \\
\text{MSE}_{20\%}\text{, WH} = 0.4373 < \text{MSE}_{20\%}\text{, WMH} = 1.1412; \text{RMSE}_{20\%}\text{, WH} = 0.6613 < \text{RMSE}_{20\%}\text{, WMH} = 1.0683).

During the WMH situation, which shows a decline of 65.0%, 96.8% and 82.2% relative to the WW situation for all three measures. Whereas the percentage of decrease for MAE, MSE and RMSE was 46.9%, 91.8% and 71.3% respectively during the WH situation. This reflects the increase in the length of the time series to 500 and 1000.

5. Conclusions
This paper presented results of numerical simulation of time series data. The following conclusions were obtained. 1) In the WH situation, the performance of the ARMA(1,0)-GARCH(1,2) model is better than that of the WW situation in the presence of TC outlier contamination. 2) The performance of the ARMA(1,0)-GARCH(1,2) model in the WMH situation is better than the WH situation in the presence of TC outlier contamination. The findings of this paper make a contribution to the current literature. The use of IQR/3 in the Huber weight function has produced a better result than the MAD. Future research needs to be done to assess the efficiency of the modified Huber weight function for other types of outliers (additive, innovative and level shift).

References
[1] Fox A J 1972 Journal of the Royal Statistical Society. Series B (Methodological) 34(3) 350–363
[2] Franses P H and Ghysels H 1999 International Journal of Forecasting 15(1) 1–9
[3] Charles A and Darné O 2005 Economics Letters 86 347–352
[4] Sakata S and White H 1998 Econometrica 66(3) 529–567
[5] Melo Mendes B V D 2000 Journal of Statistical Computation and Simulation 67(4) 359–376
[6] Charles A 2008 Journal of Forecasting 27 551–565
[7] Carnero M A, Peña D and Ruiz E 2007 Journal of Time Series Analysis 28(4) 471–497
[8] Pell R J 2000 Chemometrics and Intelligent Laboratory Systems 52(1) 87–104
[9] Pennacchi P 2008 Journal of Sound and Vibration 310(4–5) 923–946
[10] Erdoğan H 2012 Experimental Techniques
[11] FarrokhiFard M, Hatami M and Parniani M 2015 Electric Power Systems Research 124 74–84
[12] Grossi L and Nan F 2019 Technological Forecasting and Social Change 141(May 2018) 305-318
[13] Huber P J 1964 The Annals of Mathematical Statistics 35 73–101
[14] Moller S F, Von Frese J and Bro R 2005 Journal of Chemometrics 19(10) 549–563
[15] Box G E P, Jenkins G M, Reinsel G C and Ljung G M 2015 Time Series Analysis: Forecasting and Control (New Jersey: John Wiley and Sons)
[16] Bollerslev T 1986 Journal of Econometrics 31 307–327
[17] Aguinis H, Gottfredson R K and Joo H 2013 Organizational Research Methods 16(2) 270–301
[18] Becker C, Fried R and Kuhnt S 2013 Robustness and complex data structures: Festschrift in honour of Ursula Gather. Robustness and Complex Data Structures: Festschrift in Honour of
Ursula Gather (London: Springer) p 248
[19] Chen C and Liu L-M 1993 Journal of the American Statistical Association 88(421) 284–297
[20] Edgeworth F Y 1887 Hermathena 6(13) 279–285
[21] Jaeckel L A 1972 The Annals of Mathematical Statistics 43(5) 1449–1458
[22] Rousseeuw P J 1984 Journal of the American Statistical Association 79(388) 871–880
[23] Rousseeuw P and Yohai V 1984 Robust regression by means of S-estimators In In Franke J., Härdle W., Martin D. (eds) Robust and Nonlinear Time Series Analysis, Lecture Notes in Statistics vol 26 (Springer)
[24] Yohai V J 1987 The Annals of Statistics 15(20) 642–656
[25] SAS Institute Inc 2008 SAS/STAT 9.2 User’s Guide Cary, NC: SAS Institute Inc
[26] Holland P W and Welsch R E 1977 Communications in Statistics - Theory and Methods 6(9)
[27] Akaike H 1974 A New Look at the Statistical Model Identification. In Selected Papers of Hirotugu Akaike ed. Springer Series in Statistics (Perspectives in Statistics) pp 215–222 (Springer)
[28] Schwarz G 1978 The Annals of Statistics 6(2) 461–464
[29] Hannan E J and Quinn B G 1979 Journal of the Royal Statistical Society. Series B (Methodological) 41(2) 190–195
[30] Halton J H 1970 SIAM Review 12(1) 1–63
[31] R Core Team 2019 R: A language and environment for statistical computing Vienna, Austria: R Foundation for Statistical Computing
[32] Trapletti A and Hornik K 2018 ts: Time Series Analysis and Computational Finance
[33] Wuertz D, Setz T, Chalabi Y, Boudt C, Chausse P and Miklavc M 2017 fGarch: Rmetrics Autoregressive Conditional Heteroskedastic Modelling