THE RADIAL EXTENT AND WARP OF THE IONIZED GALACTIC DISK. II. A LIKELIHOOD ANALYSIS OF RADIO-WAVE SCATTERING TOWARD THE ANTICENTER

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ABSTRACT

We use radio-wave scattering data for extragalactic sources and pulsars to constrain the distribution of ionized gas in the outer Galaxy. Like previous models, our model for the H ii disk includes parameters for the radial scale length and scale height of the ionized gas. In addition, we have used the known H i distribution in the outer Galaxy in constructing our model, and we allow the H ii disk to warp and flare. We also include the Perseus arm in our model. We use a likelihood analysis of 18 anticenter sources with measured scattering observables: 11 extragalactic sources and 7 pulsars. We find that the strength of scattering in the Perseus arm is no more than 60% of the level contributed by spiral arms in the inner Galaxy and is equivalent to a scattering diameter of 1.5 mas at 1 GHz. Our analysis favors an unwarped, nonflaring disk with a scale height of 1 kpc, though this may reflect the nonuniform and coarse coverage of the anticenter provided by the available data. One extragalactic source has a size a factor of 2 smaller than predicted by our model, possibly indicating the existence of holes in the scattering material. The lack of a warp in the scattering material indicates that VLBI observations near 1 GHz with an orbiting station having baseline lengths of a few Earth diameters will not be affected by interstellar scattering at moderate Galactic latitudes, \(|b| \approx 15^\circ\). The radial scale length is 15–20 kpc, but the data cannot distinguish between a gradual decrease in the electron density and a truncated distribution. We favor a truncated one because we associate the scattering with massive star formation, which is also truncated near 20 kpc. A radial extent of 20 kpc is also comparable to the radial extent of Hx emission observed for nearby spiral galaxies. We find that the distribution of electron density turbulence must decrease more rapidly with Galactocentric distance than does the distribution of hydrogen. Alternate ionizing and turbulent agents—the intergalactic ionizing flux and the passage of satellite galaxies through the disk—are unlikely to contribute significant amounts to scattering in the anticenter. We cannot exclude the possibility that a largely ionized but quiescent disk, similar to that inferred for some Lyx absorbers, extends to \(\gtrsim 100\) kpc.

Subject headings: dust, extinction — Galaxy: structure — H ii regions — scattering — surveys — turbulence

1. INTRODUCTION

Early investigations of the Galaxy’s H i emission revealed that it extends well past the solar circle and that, in the outer Galaxy, the emission is warped systematically from its midplane in the inner Galaxy (Burke 1957; Kerr 1957; Westerhout 1957; Oort, Kerr, & Westerhout 1958). More recent stellar (Djorgovski & Sosin 1989), infrared (Sodroski et al. 1987; Freudenberg et al. 1994), and molecular (Wouterloot et al. 1990) observations have shown that these disk constituents also extend well past the solar circle and are warped similarly to the H i layer.

Ionized gas occupies potentially 10% or more of the volume of the interstellar medium (ISM) near the solar circle and is probably a dynamically important constituent (see Reynolds 1977; Kulkarni & Heiles 1987), but its radial extent is poorly constrained. Hx measurements are limited to distances of a few kiloparsecs by interstellar absorption (Reynolds 1983). The frequency at which the Galactic plane becomes optically thick because of free-free absorption can indicate the extent of the disk, but for plausible disk sizes (see below), this frequency is less than 10 MHz and so is difficult to observe.

Few pulsars are known in the anticenter direction. Fewer than 10 have dispersion-measure–independent distance estimates (DM = \(\int n_e \, ds\)), and the estimated distances to these are less than 2 kpc (Frail & Weisberg 1990); the remainder have dispersion measures of 30–125 pc cm \(^{-3}\), consistent with distances of a few kiloparsecs (Taylor, Manchester, & Lyne 1993; Zepka et al. 1996). Fluctuations in the ionized gas produce radio-wave scattering that manifests itself as angular broadening of compact sources (see Rickett 1990 for a review of the full variety of interstellar radio wave propagation effects). Scattering measurements have been biased toward the inner Galaxy, even for surveys of angular broadening of extragalactic sources (see, e.g., Fey, Spangler, & Mutel 1989; Fey, Spangler, & Cordes 1991). Only one angular broadening survey has been conducted toward the outer Galaxy (Dennison et al. 1984), and, as we illustrate below, most of the sources in that survey had Galactic latitudes too large to provide effective constraints on the radial extent of the ionized Galactic disk. Measurements of interplanetary scintillation determine source diameters indirectly but in general do not have sufficient resolution to provide stringent constraints.

Though the radial extent of the ionized gas is poorly constrained, a number of lines of evidence suggest that its radial extent may equal or exceed that of H i:

1. Savage, Sembach, & Lu (1995) find C iv absorption

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along the line of sight to H 1821 + 643 (l = 94°, b = 27°). Among the velocity components contributing to this absorption is low-density (n ~ 5.6 × 10^{-3} cm^{-3}), warm (T ~ 10^4 K) gas at a velocity of -120 km s^{-1}, corresponding to a kinematic Galactocentric distance of 25 kpc.

2. The H I disks of nearby galaxies are truncated at radii of order 25–50 kpc, at which the surface density drops to \( N_{\text{HI}} \lesssim 2 \times 10^{19} \text{ cm}^{-2} \) (Corbelli, Schneider, & Salpeter 1989; van Gorkom 1991; Bland-Hawthorn 1997). This truncation is observed even for galaxies without nearby companions and likely occurs where the disks become optically thin to the intergalactic ionizing flux (Sunyaev 1969; Corbelli & Salpeter 1993). Bland-Hawthorn, Freeman, & Quinn (1997) have reported the detection of ionized gas beyond the observed H I disk of NGC 253. Charlton, Salpeter, & Hogan (1993) have proposed that at least some of the low-redshift Lyα clouds seen in quasar spectra may be caused by residual H I in extended, nearly fully ionized disks of normal spiral galaxies. Our Galaxy would then be a prototypical z = 0 absorber.

3. Material blown out of the Galactic disk by the action of clustered supernovae may account for a fraction of high-velocity clouds and later return to the disk forming a Galactic fountain (Shapiro & Field 1976; Bregman 1980; Houck & Bregman 1990; Spitzer 1990; Kahn 1991). Models of high-velocity clouds often require the material to be supported by gas pressure at large Galactocentric radii, \( R \gtrsim 25 \) kpc (see, e.g., Bregman 1980).

Taylor & Cordes (1993; hereafter TC93) modeled the Galactic distribution of ionized gas with three global components:

1. An extended component with a scale height of approximately 1 kpc and a \( 1/e \) radial scale length of order 20 kpc.
2. An inner Galaxy component with a scale height of 0.15 kpc and a radial scale length of 3.5 kpc.
3. Spiral arms, the number and shape of which were determined by appeal to radio and optical observations of H II regions and radio observations of H I and nonthermal emission.

The data available to constrain the model parameters consisted of 74 pulsars with distances independent of dispersion measure (DM), 223 scattering measurements toward pulsars, masers, and extragalactic sources, and the Galactic longitude distribution for 553 pulsar DMs.

Because of the paucity of measurements summarized above, TC93 could place only a lower bound on the scale length of the extended component, \( A_1 \). They adopted \( A_1 = 20 \) kpc, though \( A_1 \approx 50 \) kpc produced comparable fits to the data. Moreover, they modeled the extended component as planar; if the ionized disk does extend to 20 kpc or more, it is likely to be warped similarly to the other outer Galaxy disk constituents.

Figure 1 demonstrates that angular broadening measurements of extremely low-latitude, \( |b| < 1° \), extragalactic sources toward the Galactic anticenter have the potential to constrain \( A_1 \). The line of nodes of the H I disk is fairly constant with Galactocentric radius and is near a Galactic longitude of 170°, so sources toward the anticenter probably provide the longest path length through the ionized disk. Extremely low-latitude sources are required because the scale height of the extended component near the solar circle is 0.88 kpc and only for sources with \( |b| < 1° \) does the line of sight remain within one scale height for path lengths of 50 kpc or more. Only three of the sources in the Dennison et al. (1984) survey meet this criterion; of these, one may show excessive scattering because of an H II region along the line of sight and another shows complex structure, making it difficult to estimate a scattering diameter.

We have carried out a program of multifrequency Very Long Baseline Array (VLBA) observations of 12 anticenter sources, seven of which have \( |b| < 0.5° \) (Lazio & Cordes 1998; hereafter Paper I). We detected all but one of the sources at one or more of our observation frequencies—0.3, 1.6, and 5 GHz. As Figure 1 illustrates, the nominal resolutions of the VLBA are such that 18 cm observations are sensitive to scale lengths of \( A_1 \gtrsim 100 \) kpc and 90 cm observations should detect scattering even if \( A_1 \lesssim 10 \) kpc.

Figure 1 also shows the nominal resolution of the space VLBI satellite HALCA and thereby illustrates another important aspect of improving our knowledge of the Galactic distribution of scattering. At low Galactic latitudes, interstellar scattering will determine the limiting resolution for baselines in excess of the Earth’s diameter at frequencies near 1 GHz. If the H II disk flares or warps similarly to the H I disk, interstellar angular broadening could be non-negligible at much higher latitudes (e.g., \( |b| \approx 30° \)).

In this paper we combine the sources from our survey with other radio-wave scattering measurements from the literature and use a likelihood analysis to constrain the distribution of ionized gas in the outer Galaxy. In § 2 we describe our model for the distribution of ionized gas in the outer Galaxy, in § 3 we extract scattering diameters from our measured angular diameters and develop a likelihood analysis of scattering in the outer Galaxy, and in § 4 we discuss our results and present our conclusions.
2. A MODEL OF THE IONIZED DISK IN THE OUTER GALAXY

In this section, we develop a model for the distribution of free electrons in the outer Galaxy. Based on the close correspondence between the HI disk and other outer Galaxy constituents (see § 1), we shall use the HI distribution in the outer Galaxy as a basis for modifying the TC93 model. We begin with a discussion of the connection between the observed scattering angle, \( \theta_s \), and the modeled electron density, \( n_e \).

2.1. Electron Density Fluctuations and Angular Broadening

The density fluctuations responsible for angular broadening are commonly parameterized with a power-law spectrum,

\[
P_{ns} = C_n^2 q^{-\alpha} ,
\]

over a range of spatial wavenumbers, \( q_0 \ll q \ll q_1 \), where \( l_0 = 2\pi/q_0 \) and \( l_1 = 2\pi/q_1 \) are the outer and inner scales, respectively, to the spectrum. The quantity \( C_n^2 \) sets the amplitude of the density fluctuations and varies spatially. Throughout we adopt a spectral index of \( \alpha = 11/3 \), the Kolmogorov value, as suggested by a number of observations (Rickett 1990).

The scattering angle for plane-wave radiation propagating a distance \( D \) through a medium filled with such a spectrum of density fluctuations is (Cordes et al. 1991; Cordes & Lazio 1991)

\[
\theta_s = 128 \text{ mas SM}^{3/5} v_{\text{GHz}}^{-11/5} .
\]

The quantity SM is the line-of-sight integral of \( C_n^2 \),

\[
\text{SM} = \int_0^D ds C_n^2(s) ,
\]

and \( v_{\text{GHz}} \) is the frequency in GHz.

Cordes et al. (1991) demonstrated that the level of scattering, as measured by SM, correlates with the dispersion measure (DM) for nearby pulsars; for pulsars toward the inner Galaxy, the level of scattering increases faster with distance than DM. This correlation suggests that the electrons responsible for dispersion are also responsible for scattering. Cordes et al. (1991) and TC93 adopted

\[
dSM = C_u F n_e^2 ds .
\]

Here \( n_e \) is the electron density in \( \text{cm}^{-3} \), \( F \) is the fluctuation parameter and is a measure of how effectively density fluctuations are produced and maintained, \( ds \) is a path length interval in kpc, and \( C_u \) is a constant responsible for producing SM in the typical units of kpc \( m^{-2/3} \), \( C_u = 3.4(2\pi)^{-1/3} \text{ m}^{-2/3} \text{ cm}^6 \). With a model for \( n_e \) in the outer Galaxy, we can integrate equation (4) along the line of sight to a source to find SM and \( \theta_s \).

2.2. Free Electron Density Model

Of the four components in the TC93 model, only the spiral arms and the extended component are relevant to our study of the outer Galaxy. The inner Galaxy component has a scale length of 3.5 kpc, and, at the solar circle, its contribution to the electron density has decreased to 0.2% of that from the extended component. The Gum Nebula, which was also included in the model because of its proximity to the Sun, affects only lines of sight within about 20° of \((\ell, b) = (260°, 0°)\), well removed from the directions to the sources considered here.

We retain the spiral arm component because one of the spiral arms, the Perseus arm, is outside the solar circle over the longitude range of interest. Lines of sight with \( |b| \leq 10° \) pass within one scale height of the center of this spiral arm. In the TC93 model, this arm contributes a scattering measure of \( \text{SM} \sim 0.01 \text{ kpc m}^{-20/3} \text{ equivalent to a 1 GHz} \)

scattering angle of \( \theta_s \sim 10 \text{ mas} \) for the line of sight \((\ell, b) = (180°, 0°)\).

The remaining component is the extended component. In the TC93 model, this component consisted of a flat disk centered on the Galaxy’s midplane with a scale height \( h_l = 0.88 \text{ kpc} \) and scale length \( A_s = 20 \text{ kpc} \). We use HI observations toward the outer Galaxy as a guide for constructing our model for three reasons. First, in the inner Galaxy, the mean and rms electron density distributions generally follow the distribution of massive stars, and, toward the outer Galaxy, sites of massive star formation follow the HI distribution (Wouterloot et al. 1990). Second, Savage et al. (1995) detected ionized gas that is spatially coincident with warped HI gas. Finally, models of low-redshift Lyx clouds, in which the outer extent of the Galaxy is nearly fully ionized by the intergalactic ionizing flux (Charlton et al. 1993), predict that the HI disk is ionized to form the HI disk. The HI structure in the outer disk has been reviewed by Burton (1992). Here we shall summarize only salient details as we construct our model for the outer HI disk.

In the outer Galaxy, the distribution of \( n_e \) is the sum of the extended component and the Perseus arm,

\[
n_e(x, y, z) = n_s g_1(R) \text{sech}^2 \left[ \frac{Z(R)}{h_1(R)} \right] + f_s n_s \text{sech}^2 \left( \frac{r}{h_1} \right) ,
\]

The nominal densities of the two components are \( n_{s,1} \), their radial dependences are given by the functions \( g_1, g_2 \), and their scale heights are given by \( h_{1,2} \), respectively. The midplane of the extended component is given by \( Z(R) \), and the Perseus arm has a fine tuning parameter of \( f_s \). The following sections explain the various quantities in more detail. As in TC93, the coordinate system has the \( x \)-axis directed parallel to \( \ell = 90° \), the \( y \)-axis directed parallel to \( \ell = 180° \), and the \( z \)-axis directed toward \( b = 0° \), and the Galactocentric radius is \( R = (x^2 + y^2)^{1/2} \). The minimum distance between the position \((x, y)\) and a point on the Perseus arm is denoted by \( d \) (see TC93 for a full description of the spiral arms). Following TC93 we take the radial and \( z \)-dependences to be separable.

2.2.1. Radial Dependence of the Extended Component

We consider two functional forms for the radial dependence of the extended component, \( g_1(R) \). The first is a \( \text{sech}^2 \) dependence,

\[
g_1^{(1)}(R) = \text{sech}^2 \left( \frac{R}{A_1^{(1)}} \right) / \text{sech}^2 \left( 8.5 \text{ kpc} / A_1^{(1)} \right) .
\]

We shall refer to this as a \( \text{sech}^2 \) disk. This functional form exhibits a gradual decrease in the electron density with \( R \); for \( R > A_1^{(1)} \), \( g_1^{(1)}(R) \propto \exp \left( -2R/A_1^{(1)} \right) \). The second form is

\[
g_1^{(2)}(R) = \begin{cases} 
\cos \left( \pi R / 2A_2^{(2)} \right) / \cos \left( \pi R / 2A_1^{(2)} \right) , & R \leq A_2^{(2)} \\
0 , & R > A_2^{(2)}
\end{cases}
\]

We shall refer to this as a truncated disk because it is zero for \( R > A_2^{(2)} \).

Figure 2 compares these functional forms with each other.
and with the H I density (Gordon & Burton 1976). Our choice for these particular functional forms is motivated by a number of considerations. The sech$^2$ dependence is that assumed by TC93 and so our results can be compared directly to theirs. When compared to the H I density, the sech$^2$ disk has a slower radial fall off while the truncated disk has a faster radial fall off. These two functional forms should bracket the actual $A_1$, if the electron density distribution follows that of the H I. The truncated disk also allows us to model a disk with variable scattering properties. At $R > A_1^{(2)}$, there is no additional scattering. As we have written in equation (7), this truncation occurs because $n_e = 0$ cm$^{-3}$ for $R > A_1^{(2)}$. An equivalent model is one in which there is ionized gas but $F_1 = 0$ for $R > A_1^{(2)}$. Such a truncation could occur if the distribution of scattering agents decreased more rapidly with $R$ than does $n_e$. Finally, our estimates of $A_1$ are model dependent. Comparison of the two forms allows us to assess the sensitivity of our estimates of $A_1$ to the assumed models. Both functions are normalized so that $g_1(R_0) = 1$. Henceforth, we shall drop the superscripts (1) and (2) as it will be clear from the context which $A_1$ parameter we are describing.

2.2.2. z-Dependence of the Extended Component and the Galactic Warp

The $z$-dependence of the outer Galaxy density distribution is allowed to differ from that of the TC93 model by two effects. First, the H I disk is observed to flare to larger scale heights as $R$ increases. We model this effect by allowing the scale height to vary with Galactocentric distance, $h_z(R)$. The second is that the H I layer is warped. The actual shape of the H I warp is complex, with differences between the northern and southern Galactic hemispheres and with an amplitude that is radically dependent (though other tracers of the outer disk are more symmetric than H I; Djorgovski & Sosin 1989; Wouterloot et al. 1990). Because the longitude range of our data, $150^\circ < \ell < 210^\circ$, is significantly less than the $180^\circ$ longitude range over which the north/south asymmetry is important, we shall ignore the asymmetry in the warp.

The scattering measure is an integrated quantity, while H I and CO measurements yield velocity information.

We therefore model the outer ionized disk as a single tilted ring or torus. Our approximation to the disk in the outer Galaxy is illustrated in Figure 3.

The H I disk begins to warp and flare significantly at the same radius, $R \approx 10.5$ kpc. The line of nodes of the H I disk varies with $R$ but is approximately centered on $\ell = 170^\circ$. The onset of the H II warp is assumed to occur at the same radius as the H I warp does. At $R < 10.5$ kpc, our model agrees with the TC93 model, with a scale height, $h_{1,in}$, and fluctuation parameter, $F_{1,in}$. At $R > 10.5$ kpc, the H II disk is tilted by an angle $\Psi$ with respect to $b = 0^\circ$, and there is a line of nodes $\ell'_0$, a scale height $h_{1,out}$, and fluctuation parameter $F_{1,out}$. The radial dependences in equations (6) and (7) are continuous through $R = 10.5$ kpc.

In the warped disk, the $z$-dependence and scale height in equation (5) are relative to the midplane of the gas, not $b = 0^\circ$ (see Fig. 3). Interior to the warp, the distance above the plane is $Z = s \sin b$ where $s$ is a distance along the line of sight. Within the warp, a source with Galactic coordinates $(\ell, b)$ has a latitude $b'$ relative to the torus’ midplane. The corresponding $z$-height is $Z = s \sin b'$, where $b'$ is given by

$$
\sin b' = \sin b \cos \Psi + \cos b \sin (\ell - \ell'_0) \sin \Psi.
$$

The warped portion of the disk is assumed to be axisymmetric, as is the unwarped extended component.

The division between $F_{1,in}$ and $F_{1,out}$ is to allow for the possibility that the mechanism for generating or maintaining density fluctuations in the far outer Galaxy may differ from that in the inner Galaxy. The radius at which the transition from $F_{1,in} \rightarrow F_{1,out}$ occurs was chosen to be that at which the disk begins to flare and warp significantly. The key assumption utilized here is that the flaring and warping may be symptomatic of other processes that could result in a change from $F_{1,in} \rightarrow F_{1,out}$.

2.2.3. Spiral Arm Component

The functional dependences for the spiral arm component are unaltered from the TC93 model. In particular,
Determination of Scattering Diameters

For each source, we have a measurement of its apparent angular diameter at one to three frequencies, \( \theta_{\text{app}}(v) \). We assume the apparent diameter is a quadrature sum of the intrinsic and scattering diameters and model it as

\[
\theta_{\text{app}}^2(v) = \frac{\theta_i^2}{\nu^2} + \frac{\theta_s^2}{\nu^4}.
\]

Here \( \theta_i \) and \( \theta_s \) are the intrinsic and scattering diameters at 1 GHz, respectively. For the scattering diameter, we have used the \( \nu^{-2.2} \) dependence, as is appropriate for moderately strong scattering. For a homogeneous source with a peak brightness temperature \( T_B \), \( \propto \nu^2 \) (Kellermann & Owen 1988).

We have considered a number of ways of using equation (10) to solve for \( \theta_s \):

1. **Ignore intrinsic size.**—Setting \( \theta_i = 0 \) mas and \( \theta_s = \theta_{\text{app}}(v)/\nu^2 \). Since this method assumes that scattering dominates the apparent diameter, the scattering diameter so derived is an upper limit.

2. **Dual frequency measurements.**—For sources with angular diameters measured at 18 and 90 cm, if we set \( \propto = 1 \), we have two equations in two unknowns and can solve for \( \theta_i \) and \( \theta_s \).

3. **Ignore scattering at high frequencies and assume homogeneous sources.**—At 6 cm, scattering should be unimportant for lines of sight to the outer Galaxy. For those sources detected at 6 cm, we take the 6 cm diameter to be the intrinsic diameter and scale it to 1 GHz, with \( \propto = 1 \). We use the observed frequency dependence to find the 1 GHz parameter for the Perseus arm, was set to unity; we shall allow it to vary.

Our justification for allowing only \( f_s \) to vary is that it is the only spiral arm parameter that can be modified without affecting the model in the inner Galaxy. While the nine sources from our survey in Paper I are a substantial fraction of the number of available scattering measurements toward the anticenter (see Table 2 and Fig. 4), the total number of scattering measurements toward the anticenter is only approximately 20. This number is a small fraction of the nearly 300 DM and SM measurements that TC93 used to constrain the model parameters. Thus, incorporating our additional measurements from Paper I into the 300 used by TC93 and repeating their analysis would not lead to any substantial change of the model in the inner Galaxy.

With the above model we can now integrate \( dS_{\text{M}} \), equation (4), along the line of sight toward a source at \((\ell, b)\) to form SM. Since the two components have unequal fluctuation parameters, the contribution from each component is determined separately, then summed to form the total scattering measure. The predicted angular diameter is given by equation (2), where the modeled SM is a function with the following parameters \( n_l, f_s, A_1, h_{1,\text{in}}, h_{1,\text{out}}, F_{1,\text{in}}, F_{1,\text{out}}, c, \Psi, \ell, b; R_{\text{warp}} \).

3. ANALYSIS

In this section, we describe how we have used the measured angular diameters (Paper I) to obtain scattering diameters, discuss additional scattering measurements we have used in our analysis, develop the likelihood functions we will use to constrain the model parameters, and present the results of this likelihood analysis.

3.1. Determination of Scattering Diameters

The source 87GB 0558 + 2325 has a measured diameter of approximately 4 mas at 1 GHz (Paper I), a factor of 2 less than that predicted by the Perseus arm’s contribution alone (TC93). In order that our model not overpredict scattering diameters, we must modify the spiral arm component as well. Of the nine describing the spiral parameters diameters, we must modify the spiral arm component as indicated by where the modeled SM is a function with equation (2), where the modeled SM is a function with the following parameters \( n_l, f_s, A_1, h_{1,\text{in}}, h_{1,\text{out}}, F_{1,\text{in}}, F_{1,\text{out}}, c, \Psi, \ell, b; R_{\text{warp}} \).

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\[
\begin{align*}
\theta_{\text{app}}^2(v) &= \frac{\theta_i^2}{\nu^2} + \frac{\theta_s^2}{\nu^4}.
\end{align*}
\]

Here \( \theta_i \) and \( \theta_s \) are the intrinsic and scattering diameters at 1 GHz, respectively. For the scattering diameter, we have used the \( \nu^{-2.2} \) dependence, as is appropriate for moderately strong scattering. For a homogeneous source with a peak brightness temperature \( T_B \), \( \propto \nu^2 \) (Kellermann & Owen 1988).

We have considered a number of ways of using equation (10) to solve for \( \theta_s \):

1. **Ignore intrinsic size.**—Setting \( \theta_i = 0 \) mas and \( \theta_s = \theta_{\text{app}}(v)/\nu^2 \). Since this method assumes that scattering dominates the apparent diameter, the scattering diameter so derived is an upper limit.

2. **Dual frequency measurements.**—For sources with angular diameters measured at 18 and 90 cm, if we set \( \propto = 1 \), we have two equations in two unknowns and can solve for \( \theta_i \) and \( \theta_s \).

3. **Ignore scattering at high frequencies and assume homogeneous sources.**—At 6 cm, scattering should be unimportant for lines of sight to the outer Galaxy. For those sources detected at 6 cm, we take the 6 cm diameter to be the intrinsic diameter and scale it to 1 GHz, with \( \propto = 1 \). We use the observed frequency dependence to find the 1 GHz

3. Preliminary Constraints on Model Parameters

The source 87GB 0558 + 2325 has a measured diameter of approximately 4 mas at 1 GHz (Paper I), a factor of 2 less than that predicted by the Perseus arm’s contribution alone (TC93). In order that our model not overpredict scattering diameters, we must modify the spiral arm component as well. Of the nine describing the spiral parameters diameters—\( n_l, h_a, F_a, w_a, A_a \)—and four fine-tuning parameters \( f_s \)—TC93 used DM and scattering data to fit for the first three. They appealed to other radio and optical observations in fixing \( w_a = 0.3 \) kpc and \( A_a = 8.5 \) kpc. In the inner Galaxy, TC93 used \( f_s \) to obtain better agreement between the model and the observations toward the tangent points of certain spiral arms. In the TC93 model, \( f_s \), the fine tuning

3 Strictly speaking, the shapes of the arms are described by fiducial points that are also parameters. However, they have been determined largely from radio and optical observations of H II regions and radio observations of thermal and H I emission. We have not altered the fiducial points.
apparent diameter from the 18 and 90 cm diameters. The scattering diameter is then found by subtracting in quadrature the scaled intrinsic diameter from the apparent diameter.

4. Ignore scattering at high frequencies.—The final method also utilizes 6 cm diameters. Rather than assuming $\alpha = 1$ in scaling the intrinsic diameter to 1 GHz, we solve for $\alpha$ using the 6 and 18 cm diameters, assuming $\theta_s = 0$ mas. Then, using the 18 and 90 cm diameters to solve for the 1 GHz apparent diameter, we again subtract in quadrature the scaled intrinsic diameter from the apparent diameter.

Clearly not all of these methods can be used for all sources. For those sources for which multiple methods can be used, we utilize as many of the methods as possible and then adopt the estimate that places the most stringent limit on $\theta_s$. In general, method 2 and method 1 produce estimates of $\theta_s$ that are the same within the errors. We note that using method 1 tends to bias us toward larger disk scale lengths because this method assumes that the intrinsic size makes no contribution to the measured diameter. We find 1 GHz scattering diameters or limits in the range 1.5–48 mas; these are tabulated in Table 1.

### 3.2. Available Data

We augment the scattering measurements from our survey (Paper I) with others from the literature within the same longitude range, $150^\circ \leq \ell \leq 210^\circ$. These are summarized in Table 2, scaled to 1 GHz. Two kinds of scattering measurements were found in the literature, angular broadening measurements similar to those reported here and scintillation bandwidth, $\Delta v_s$, measurements of pulsars (Cordes 1986).

The resulting data set consists of three classes of sources having the following observables:

1. Extragalactic sources having measured scattering diameters or upper limits on the scattering diameter, $\theta_s$.—There are 11 such sources.

2. Pulsars with DM-independent distance estimates.—This class has only one member, the Crab pulsar, for which a DM and scattering diameter have been measured.

3. Pulsars without DM-independent distance estimates.—There are 10 such sources with measured DMs and $\Delta v_s$.

All of these sources are shown in Figure 4.

Two of the pulsars that have a measured scintillation bandwidth will not be included in our analysis. The lines of sight to the pulsars PSR B 0823 +26 and PSR B 1112 +50 are likely to be dominated by local scattering material. Both pulsars are closer than 0.5 kpc, closer than the inner edge of the Perseus arm.

Our intention is to constrain the distribution of scattering material in the outer Galaxy. In selecting extragalactic sources to include in our analysis, we have focused on sources for which scattering makes a measurable contribution to the observed diameter. Such sources are marked by visibility functions displaying a Gaussian-like profile with increasing baseline length or by images containing only simple structures, typically a single Gaussian component. An alternate approach would be include in our analysis all extragalactic sources in the anticenter. Since the observed angular size is a convolution of the intrinsic size with the scattering diameter, we can always derive an upper limit to the scattering diameter for any source (see Method 1 in § 3.1). However, these upper limits are usually larger by factors of at least 5–10 than the scattering diameters predicted by the TC93 model. Sources for which scattering appears to dominate the observed angular diameters suggest that the level of scattering toward the anticenter is actually less than that predicted by the TC93 model. The upper limits for the scattering diameters of most sources therefore place no meaningful constraint on the scattering toward the anticenter (see also § 3.5). We illustrate the lack of constraints provided by most sources with four sources

### Table 2

**Scattering Measurements from the Literature**

| Name          | $\ell$ (deg) | $b$ (deg) | $\Delta v_s$ (MHz) | $\theta_s$ (mas) | $D$ (kpc) | Reference |
|---------------|--------------|-----------|--------------------|------------------|-----------|-----------|
| PSR B 0301 +19 | 161.14       | -33.27    | 9.55 ± 2.4         | 0.29 ± 0.097     | 0.94      | 1         |
| PSR B 0320 +39 | 152.18       | -14.33    | 2.92 ± 0.77        | 0.46 ± 0.15      | 1.47      | 1         |
| PSR B 0450 +55 | 152.62       | 7.54      | 9.55 ± 2.4         | 0.29 ± 0.097     | 0.78      | 1         |
| PSR B 0525 +21 | 183.86       | -6.89     | 0.62 ± 0.2         | 0.72 ± 0.24      | 2.27      | 1         |
| PSR B 0531 +21 | 184.60       | -5.80     | ...                | 0.50 ± 0.05      | 2.0       | 2         |
| PSR B 0540 +23 | 184.36       | -3.31     | 0.11 ± 0.03        | 1.4 ± 0.47       | 3.53      | 1         |
| PSR B 0611 +22 | 188.79       | 2.39      | 0.04 ± 0.01        | 2.0 ± 0.67       | 4.72      | 1         |
| PSR B 0626 +24 | 188.82       | 6.22      | 0.08 ± 0.02        | 1.4 ± 0.47       | 4.69      | 1         |
| PSR B 0656 +14 | 201.11       | 8.25      | 8.51 ± 2.1         | 0.34 ± 0.11      | 0.76      | 1         |
| PSR B 0823 +26 | 196.96       | 31.74     | 9.55 ± 2.4         | 0.29 ± 0.097     | 0.37      | 1         |
| PSR B 1112 +50 | 154.41       | 60.36     | 19.50 ± 6.4        | 0.26 ± 0.087     | 0.54      | 1         |
| 0053 +467 ....  | 161.00       | 3.70      | ...                | 25 ± 1.4         | ...       | 4         |
| 0629 +109 ....  | 201.50       | 0.50      | ...                | ...              | ...       | 4         |

**Note:** All quantities have been scaled to 1 GHz. The scattering angle for the pulsars depends upon both the scintillation bandwidth and the adopted distance, $\theta_s = 0.85$ mas/($D_v_s, \Delta v_s$)$^{1/2}$.

**References:** (1) Cordes 1986; (2) Gwinn et al. 1993; (3) Spangler et al. 1986; (4) Dennison et al. 1984.
originally included in the TC93 analysis but not included in this analysis.

Four extragalactic sources included in the TC93 analysis are not included here because a reanalysis of the existing observations suggests that no scattering diameter has been measured. The four sources are CTA 21 (0316 + 162), 0611 + 131, 4C 14.18 (0622 + 147), and 3C 190 (0758 + 143). In the TC93 analysis, CTA 21 and 3C 190 were taken to have scattering diameters of approximately 200 mas at 74 MHz, based on a single-baseline VLBI experiment (Resch 1974). A later multistation VLBI experiment at 609 MHz showed CTA 21 to have a core-halo structure, with the halo having a size \( \theta \geq 130 \) mas (Wilkinson et al. 1979). Wilkinson et al. (1979) suggest that the halo is responsible for the interplanetary scintillation this source exhibits at 81 MHz. The halo is also likely to be the component responsible for the aforementioned 74 MHz angular diameter. The core itself appears to be a blend of two components. A characteristic size of these blended components is about 10 mas, equivalent to an upper limit on the scattering diameter of 3.4 mas at 1 GHz; this upper limit is more than a factor of 5 larger than the predicted TC93 scattering diameter. For 3C 190, a later multistation VLBI experiment at 609 MHz showed it to have three components of comparable flux and similar, steep spectra with diameters of approximately 100 mas (Rendong et al. 1991). It is not clear if one of these components dominates at 74 MHz and hence is responsible for the aforementioned angular diameter measurement, or if the observations of Resch (1974) sample a complex visibility function and his results do not represent an actual diameter at all. The equivalent upper limit on the 1 GHz scattering diameter is approximately 40 mas, nearly a factor of 100 larger than the predicted model diameter. The source 4C 14.18 has a steep spectrum and a non-Gaussian visibility function (Dennison et al. 1984). By fitting a Gaussian to the visibility data, they place an upper limit of 22 mas on the 1 GHz scattering diameter, more than a factor of 5 larger than that predicted by the TC93 model. The source 0611 + 131 was taken to have a scattering diameter of 40 mas at 408 MHz (Dennison et al. 1984) in the TC93 analysis. Our new observations at 1.6 and 5 GHz show this diameter to result from the blending of at least two source components (Paper I). We place an upper limit of 30 mas on the 1 GHz scattering diameter, a factor of 10 larger than the predicted value.

Our initial attempts to form the likelihood function included the Crab pulsar (PSR B 0531 + 21). However, we found that it ended up dominating the resulting likelihood functions, in some cases contributing as much as 50% of the log likelihood. Because the Crab is relatively nearby \((D \approx 2 \text{ kpc})\), we do not believe it should be the dominant source in the likelihood function. The results presented here do not include the Crab.

The reason for the Crab’s large contribution to the likelihood is a large discrepancy between the observed and modeled quantities: the observable quantities are \( \text{DM} = 56.8 \text{ pc cm}^{-3} \) and \( \theta_j = 0.5 \pm 0.05 \) mas, while typical values for these quantities in our models are \( \hat{\text{DM}} \approx 30 \text{ pc cm}^{-3} \) and \( \hat{\theta}_j \approx 0.7 \) mas. Our modeled values are comparable to those in the TC93 model. The models overpredict the scattering angle while underpredicting the DM. In particular, the discrepancy in the observed versus modeled DM results in a significant contribution to the likelihood function.

The Crab nebula is unlikely to be the source of the discrepancies. Its contribution to the DM is probably no more than 1% (Isaacman 1977). Its contribution to the scattering diameter is deleveraged by a factor of order \( L/D \sim 10^{-3} \), where \( L \) is the diameter of the nebula. Evidence supporting a small nebular contribution to the scattering diameter comes from comparing the measured scattering diameter with that inferred from pulse broadening for the pulsar. The pulse broadening has a variable and a constant component; the constant contribution arises from the ISM distributed between the pulsar and the Earth. The scattering diameter estimated from the constant pulse broadening agrees well with the observed scattering diameter (Vandenbergh 1976; Isaacman & Rankin 1977; Gwinn, Bartel, & Cordes 1993). We conclude that the ionized gas along the line of sight to the Crab has a relatively high electron density but is not strongly turbulent.

We are left with a total of 18 sources, 11 extragalactic sources and 7 pulsars.

3.3. Likelihood Functions for Scattering in the Outer Galaxy

We shall use a likelihood analysis to constrain the parameters of the ionized disk model presented in § 2.2. For the \( i \)th line of sight, the probability of obtaining the observable \( x (\theta_i, \text{DM}, \text{or} \Delta v_i) \) is

\[
p(x_i | \hat{x}_i) \approx f(x_i | \hat{x}_i) \delta x_i = \frac{1}{\sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{x_i - \hat{x}_i}{\delta x_i} \right)^2 \right],
\]

where \( \hat{x}_i \) is the value predicted for that line of sight and \( \delta x_i \) is the uncertainty associated with the measured value of the observable. For many of the extragalactic sources, we have only an upper limit to a scattering diameter because the scattering diameter has been estimated from a single frequency. The probability that a scattering observable \( x \) is less than an upper limit \( X \) is

\[
p(x_i \leq X_i | \hat{x}_i) = \frac{1}{\delta x_i} \int_{0}^{X_i} f(x' | \hat{x}_i) \, dx' \]

\[
= \frac{1}{2} \left[ \text{erf} \left( \frac{X_i - \hat{x}_i}{\delta x_i \sqrt{2}} \right) + \text{erf} \left( \frac{\hat{x}_i}{\delta x_i \sqrt{2}} \right) \right],
\]

where \( \text{erf}(x) \) is the error function.

The global likelihood function for all sources is

\[
P = \prod_{i=1}^{N} p_i.
\]

The modeled quantities and measurement uncertainties for the various classes of sources are as follows:

1. The predicted extragalactic scattering diameters are found by integrating \( ds \) along the line of sight and using equation (4). Most of the extragalactic source scattering diameters in the literature are single frequency measurements; we scale the errors for these to 1 GHz assuming a \( J^{-2.2} \) dependence. For the scattering diameters we report, the errors are estimated from the formal statistical errors on the fits to the data and then scaled (single-frequency measurement) or propagated (multiple-frequency determination) to 1 GHz. The uncertainties are 10%–25%.

2. For the pulsars, we integrate \( n_s \, ds \) and a weighted form of equation (4) until the modeled DM equals the measured
DM. The appropriate weighting factor for the scintillation bandwidth is $w(s) = (s/D)(D - s)/D$, where $D$ is the distance to the source and $s$ is the distance along the line of sight. This weighting factor accounts for the fact that $\Delta \nu_q$ is a measure of the excess time delay taken by scattered lines of sight. The scintillation bandwidth is then calculated as

$$\Delta \nu_q = 145 \text{ Hz} \ D^{-1}_{\text{kpc}} (\text{SM})^{-6/5},$$

with SM in the conventional units of kpc m$^{-20/3}$ and $D_{\text{kpc}}$ the distance in kpc. The measurement uncertainties for $\Delta \nu_q$ range from 20% to 70%; we adopt $\delta(\Delta \nu_q)/\Delta \nu_q = 0.33$.

### 3.4. Results

We have searched for the maximum of the global likelihood, equation (13), using an iterative grid search procedure. Initial ranges for the various parameters, described in more detail below, were estimated based on the TC93 model results and the structure of the H I disk. Then the ranges and the grid search resolutions were refined to locate the maximum.

The model in §2.2 contains nine parameters: $f_s$, $n_l$, $A_I$, $h_{1,\text{in}}$, $h_{1,\text{out}}$, $F_{1,\text{in}}$, $F_{1,\text{out}}$, $\ell_0$, and $\Psi$. We held $n_l$ and $h_{1,\text{in}}$ fixed at their values in the TC93 model, $n_l$ = 0.0165 kpc cm$^{-3}$ and $h_{1,\text{in}}$ = 0.88 kpc (n$_l$ = 0.0188 cm$^{-3}$), respectively. These quantities are constrained by the DMs of high-latitude pulsars, particularly those in globular clusters. There is only one high-latitude pulsar, B 0301+19 (b = 33°) in our sample, and it is not in a globular cluster. Further, $n_l$ and $h_{1,\text{in}}$ describe the ionized medium near the solar circle and were constrained (along with nine other parameters) by a fit to nearly 300 DM and SM measurements. As we discuss at the end of §2.2, we do not expect our more limited set of measurements to change appreciably those parameters that describe the electron density distribution in the inner Galaxy or even near the solar circle.

We show that $F_{1,\text{in}}$ may have a value different than that adopted in the TC93 model. Because $F_{1,\text{in}}$ also affects sources in the solar neighborhood that we do not include in our data sample, we shall not conduct a grid search over it but illustrate its effects by adopting one of two fiducial values.

The existence and amplitude of the warp are modeled by $\Psi$ and $\ell_0$. From Figure 4, it is apparent that the current complement of anticenter scintillation measurements sample the outer Galaxy both coarsely and far from uniformly. In particular, the sources are in three groups, with $\ell \approx 160^\circ$, $180^\circ$–$190^\circ$, and $200^\circ$. The number of sources ($\approx 20$) available for constraining the shape of the outer H I disk is also far smaller than the number used to describe the H I warp ($\sim 40,000$ telescope beams; Burton & Hekker 1986), the H$_2$ warp ($\sim 1300$ IRAS sources; Wouterloot et al. 1990), or the stellar warp ($\sim 20,000$–$90,000$ stars; Djorgovski & Sosin 1989; Carney & Seitzer 1993). Our survey is more restricted in longitude than these other surveys, but, even so, the above surveys have 10–100 times as many lines of sight. We shall therefore not conduct a search over $\Psi$ or $\ell_0$ but shall choose two fiducial pairs and compare the resulting maximum likelihood values. One pair will be for an unwarped disk, $\Psi = 0^\circ$ ($\ell_0$ is, of course, undefined for an unwarped disk), and the second pair will be characteristic of the warp in the H I disk, ($\Psi$, $\ell_0$) = ($20^\circ$, $170^\circ$).

In summary, we never attempted to fit for more than three parameters, $A_I$, $h_{1,\text{out}}$, and $F_{1,\text{out}}$, at a time. In the rest of this section, we place preliminary constraints on $A_I$, assess the importance of the Perseus arm, and then reevaluate our constraints on $A_I$.

#### 3.4.1. Preliminary Constraints on $A_I$

Figure 5 shows the likelihood as a function of $A_I$ for a model with an unwarped, nonflaring disk and in which the Perseus arm does not contribute to the scattering, i.e., $f_A = 0$.

We have excluded PSR B 0611+22 and the extragalactic source 0629+109 from this fit. The pulsar we exclude because its DM and scintillation bandwidth are likely to be affected by the Strömgren spheres of stars in the I Gem association (Weisberg, Rankin, & Boriakoff 1990). The extragalactic source we exclude because it has a 1 GHz diameter of 25 mas (Dennison et al. 1984), approximately a factor of 5 larger than that for extragalactic sources within a few degrees. The line of sight for this source passes close to the edge of the H II region Sharpless 273, ($\ell$, $b$) $\approx (202^\circ$, $2^\circ$). This H II region probably enhances the scattering for this line of sight. We retain these sources in the fits that include the spiral arm component (below) because the spiral arms have been incorporated in the TC93 model specifically to account for enhanced dispersion and scattering such as would occur from OB associations and H II regions.

The maximum likelihood occurs at $A_I \approx 17$ kpc for a sech$^2$ disk while $A_I \approx 25$ kpc for a truncated disk. The truncated disk is larger because there is no scattering material outside $A_I$ while, for the sech$^2$ dependence, $A_I$ is approximately the half-power point and there is a non-negligible amount of ionized gas at $2A_I$ (10%). The likelihood favors the sech$^2$ disk, but by a factor less than 2.

Provided that $F_{1,\text{out}}$ is not substantially smaller than $F_{1,\text{in}}$, these likelihood results place an upper limit on $A_I$. Any scattering contributed by the Perseus arm would reduce the estimate of $A_I$ (as the lower panel of Fig. 5}

![Fig. 5.—Likelihood estimates of $A_I$ for an unwarped, nonflaring disk. The solid line shows the likelihood results for a disk with a radial sech$^2$ dependence described by eq. (6), the dashed line for a truncated disk described by eq. (7). Top: the likelihood function if the Perseus spiral arm is ignored. Bottom: the likelihood function if the Perseus spiral arm has a strength of 25% that of the inner Galaxy spiral arms.](image-url)
illustrates and as we discuss below). Similarly, allowing the disk to flare or warp or both produces, on average, larger scattering diameters for high-latitude sources. To reproduce a given scattering diameter in the absence of flaring or warping, the scattering material must extend to a large enough distance, i.e., $A_1$ must be large enough, to compensate for the $z$-dependent fall-off of the scattering material. As we demonstrate below, however, if $F_{1,\text{out}}$ is quite small as compared to $F_{1,\text{in}}$, a much larger radial extent would be favored.

On the basis of Figure 5, we conclude that the large $A_1$ values that were allowed by the TC93 analysis, e.g., $A_1 \approx 50$ kpc, are now shown to be disfavored.

### 3.4.2. Scattering in the Perseus Arm

The nominal Perseus arm in the TC93 model contributes enough scattering that some scattering diameters are overpredicted by a factor of 2. Thus, we must decrease the amount of scattering contributed by this arm.

In the TC93 model, this arm has a fluctuation parameter equal to that of all other arms, $F_a = 6$, and a fine tuning parameter $f_a = 1$. Our model allows $f_a$ to vary. Since the Perseus arm scattering measure is $SM \propto F_a f_a^2$, adjusting $f_a$ is equivalent to allowing the various arms to have different fluctuation parameters.

If we set $n_0 = 0 \text{ cm}^{-3}$, thereby “turning off” the scattering in the disk, we determine how large $f_a$ must be to account for all of the scattering required by the observed scattering diameters. We have performed two separate fits for $f_a$, one using all of the sources, the second using just the extragalactic sources. Since the extragalactic sources are clearly well beyond all of the Galaxy’s scattering material, the extragalactic sources should provide an upper limit to $f_a$. In both cases, we find $f_a \approx 0.6$. Allowing the disk to contribute to the scattering will require an even smaller $f_a$.

The estimate for $A_1$ in the previous section assumed $f_a = 0$. Figure 5 also shows the estimate of $A_1$ obtained for $f_a = 0.25$, with PSR B 0611+22 and 0629+109 included in the fitting. Because of the additional scattering contributed by the arm, the estimates of $A_1$ are smaller, $A_1 \lesssim 15$ kpc for a sech$^2$ dependence and $A_1 \lesssim 22$ kpc for a truncated disk. The likelihood ratio between the sech$^2$ and truncated disk again favors the sech$^2$ disk by a factor of less than 2.

In the following, we reconsider our limit on $A_1$ while allowing the disk to flare and warp. Rather than conduct a search over the range $0 \leq f_a \leq 0.6$, we shall evaluate the likelihood function at a fiducial value. Comparing the magnitudes of the likelihood functions for $f_a = 0$ and $f_a = 0.6$, the value $f_a = 0$ is favored by more than an order of magnitude. We adopt $f_a = 0.25$. With this value of $f_a$, the Perseus arm contributes an SM $\sim 6.25 \times 10^{-4}$ kpc m$^{-20/3}$, equivalent to a 1 GHz scattering diameter of 1.5 mas. Figure 5 indicates how a different choice for $f_a$ would affect the maximum likelihood value of $A_1$.

### 3.4.3. The Outer Ionized Disk

In addition to $A_1$, we shall be fitting for $h_{1,\text{out}}$ and $F_{1,\text{out}}$. Our initial ranges were centered approximately on the values in the TC93 model of 0.88 kpc and 0.4, respectively. Our initial range for $h_{1,\text{out}}$ was 0.5 kpc $\leq h_{1,\text{out}} \leq 5$ kpc. The upper limit is comparable to the scale height for H I (Burton & te Lintel Hekkert 1986); the lower limit is approximately one-half the inner scale height, $h_{1,\text{in}} = 0.88$ kpc. Further, the TC93 model value for the scale height of the material in the spiral arms is $h_s = 0.3$ kpc. Values of $h_{1,\text{in}} \lesssim 0.5$ kpc could indicate an underestimate of the scattering in the Perseus arm; i.e., $f_A$ is too low. Our initial range for $F_{1,\text{out}}$ was 0 $\leq F_{1,\text{out}} \leq 2$. The lower limit describes a quiescent, i.e., non-scattering, outer disk. The fluctuation parameter for the spiral arms is $F_a = 6$, so fitted values of $F_{1,\text{out}} \gtrsim 2$ could also indicate that the scattering contribution from the Perseus arm was underestimated.

Figure 6 compares the likelihoods for a sech$^2$ radial dependence versus a truncated disk for an unwarped disk. The primary difference between these two models is that $A_1 \approx 22$ kpc for a truncated disk whereas $A_1 \approx 15$ kpc for the sech$^2$ disk. These differences are comparable to those found in the more simple model shown in Figure 5.

The likelihood results favor $h_{1,\text{out}} \approx 0.6$ kpc, as compared to a value of 0.88 kpc for the inner Galaxy from the TC93 model. Above we identify an underestimate of the level of scattering in the Perseus arm as one means of producing $h_{1,\text{out}} \lesssim h_{1,\text{in}}$. We now identify three additional possibilities: (1) misclassification of a source as extragalactic rather than Galactic; (2) less scattering near the solar circle than predicted by the nominal TC93 model; or (3) a patchy distribution of scattering material.

If a source is classified as extragalactic, the model value of SM is calculated from an integral along the entire line of sight. If the source were instead classified as Galactic, the integral would extend out only to the source’s estimated distance. Not only would a shorter path length result in a smaller SM and smaller predicted diameter, but the path would sample less of the material at large $z$. The scale height would be accordingly less constrained.

The diameter of the source 87GB 0600+2957 ($b = 4\deg 0$) is 1.5 mas. The predicted diameter of the source in the unwarped disk models described above is approximately 2.5 mas. The predicted diameter from the TC93 model is nearly 6 mas. If we repeat the above fitting, excluding 87GB 0600+2957 from the sample of sources, the maximum likelihood estimate of $h_{1,\text{out}}$, nearly doubles while the estimates of $A_1$ and $F_{1,\text{out}}$ remain essentially unchanged. However, as we discuss in Paper I, we can find no compelling reason to classify this source as Galactic.

The second possibility is that the TC93 model may overestimate the level of scattering toward the anticenter. TC93 estimated the fluctuation parameter in the extended component to be $F_{1,\text{in}} = 0.36 \pm 0.10$ and adopted a nominal value of $F_{1,\text{in}} = 0.4$. We have repeated the analysis above with $F_{1,\text{in}} = 0.3$. The likelihood results are essentially unchanged, as we might expect. This lower value of $F_{1,\text{in}}$ is a reduction of only 25%, while the discrepancy between the observed and modeled diameters for 87GB 0600+2957 is nearly a factor of 2. A lower value of $F_{1,\text{in}}$ can reduce, but not eliminate, this discrepancy. We therefore conclude that the anticenter distribution of scattering material may contain holes or gaps—a source shining through one of these gaps would have an anomalously small scattering diameter.

Harrison & Lyne (1993) compared the velocities as determined by proper motions and interstellar scintillation pattern velocities for a number of high-latitude pulsars. They concluded that the scale height of the scattering gas and of the ionized gas were markedly different, approximately 0.1 kpc for the former and 1 kpc for the latter. We assume that the scattering traces the distribution of ionized gas; however, our analysis considers only the scattering
data. Regardless of the correct explanation for the small angular diameter of 87GB 0600 +2957, we strongly disfavor a scale height as small as 0.1 kpc and find the scale height to be 5–10 times larger.

Figure 7 (left-hand panels) shows the likelihood function for a warped disk, constructed using the observables from all 18 sources. The estimates of $A_1$ change only slightly from the unwarped disk. However, the estimates of $h_{1,\text{out}}$ increase by more than a factor of 5. Larger values of $h_{1,\text{out}}$ are allowed primarily because of our coarse angular sampling. Scattering observables for sources near $\ell \approx 180^\circ$ are little affected by a warp in the disk. The warp is such that the maximum electron column density shifts to positive latitudes in the second quadrant and negative latitudes in the third. Examination of Figure 4 shows that the warp generally increases the angular distance between sources and the midplane of the disk. Thus, larger scale heights are allowed, indeed required, in order to reproduce the observed source diameters.

In Figure 7 (right-hand panels), we assess the influence of the pulsars on the likelihood functions. The pulsars’ contribution to the likelihood is constructed from the observed and modeled scintillation bandwidth, $\Delta V_{s}$ and $\Delta V_{d}$, respectively. In turn, $\Delta V_{s}$ is a function of both $\text{SM}$ and $\text{DM}$, equation (14). The DM dependence occurs because $\Delta V_{d}$ and the weighting function for $\text{SM}$ both depend upon distance, which is estimated by integrating $n_e ds$ until it equals the observed DM. In contrast, the scattering diameter for an extragalactic source is a function of only $\text{SM}$, equation (2). Hence systematic errors in the model have a greater impact on the pulsars’ contribution to the likelihood.

The regions of maximum likelihood in the left-hand and right-hand panels of Figure 7 overlap, though the allowed regions in the right-hand panels are somewhat larger. The larger regions are to be expected since fewer sources were used to constrain the model parameters. Comparing the left-hand and right-hand panels of Figure 7, the most significant difference is in the maximum likelihood estimates (MLEs) for $A_1$ and $F_{1,\text{out}}$. The MLE for $A_1$ is smaller and the MLE for $F_{1,\text{out}}$ is larger when only extragalactic sources are used as compared to all sources. This difference is caused primarily by the presence of 0629 + 109 in the sample. Its scattering diameter is large enough that considerable scattering, i.e., large $F_{1,\text{out}}$, is needed. However, the smaller diameters of the other extragalactic sources then drive $A_1$ to smaller values in order that their scattering diameters not be overestimated. We have also repeated this analysis excluding 0629 + 109 from the sample. In this case, the likelihood functions including and excluding the pulsars are nearly indistinguishable, indicating that the contribution of the extragalactic sources to the likelihood function dominates that of the pulsars.
3.5. Preferred Model

Our likelihood results favor the unwarped disk over the warped disk by a factor of 5–10. In both models, the sech² and truncated radial dependences are nearly equally likely. We favor the truncated disk because its radial extent is in good agreement with the radial extent of sites of massive star formation, § 4.1.2 (see Fig. 9).

The nominal set of parameters can be obtained largely by inspection of Figure 6 and is summarized in Table 3. We adopt $A_1 = 20$ kpc and $F_{1,\text{out}} = 0.4$; the latter value is slightly less than what Figure 6 suggests but is in good agreement with the value that TC93 derive for the inner Galaxy. Thus, the extended scattering component has a continuous fluctuation parameter through the onset of the warp. For the scale height $h_{1,\text{out}}$ we adopt 1 kpc. This is larger than what Figure 6 suggests, but, as we discuss above, the estimate of $h_{1,\text{out}}$ is influenced by 87GB 0600 + 2957. A value of 1 kpc is intermediate between that value derived using 87GB 0600 + 2957 in the fitting and that value derived with 87GB 0600 + 2957 omitted from the fitting.

In Figure 8, we show the contribution of the individual sources to the total likelihood of Figure 6 (see also Fig. 5). This figure can be used to assess how well our model reproduces the observed scattering diameter. The fact that we have been able to limit $A_1$ has largely been because of the addition of two sources, 87GB 0600 + 2957 and 87GB 0621 + 1219 (Paper I). The remaining sources place either a lower limit on $A_1$ or do not constrain it very well at all. One source not shown on this figure is 0629 + 109. Its contribution to the likelihood is significantly less than that of the other sources. Since the line of sight to 0629 + 109 passes close to the H II region S 273, this low likelihood could indicate either that our adopted value for $f_4$ is too small or that the assumption that spiral arms are smooth structures is beginning to break down.

This figure also demonstrates why using upper limits to the scattering diameters of all extragalactic sources in the

![Fig. 7](image-url)

Fig. 7.—As for Fig. 6. The model disk is warped with $\Psi = 20^\circ$ and $\zeta_0 = 170^\circ$ and has a sech² dependence. Left-hand panels: likelihood function constructed using all data; right-hand panels: likelihood function constructed using only extragalactic sources.

### Table 3

| Preferred Model Parameters | Adopted Value |
|----------------------------|---------------|
| Radial form ...            | Truncated disk |
| $n_1$ (cm⁻³)               | 0.0188        |
| $h_{1,\text{in}}$ (kpc)    | 0.88          |
| $F_{1,\text{in}}$          | 0.4           |
| $r_4$                      | 0.25          |
| $A_1$ (kpc)                | 20            |
| $h_{1,\text{out}}$ (kpc)   | 1             |
| $F_{1,\text{out}}$         | 0.4           |
| $\Psi$ (deg)               | 0             |

* This parameter was not varied in our likelihood analysis; its value is taken from the Taylor-Cordes model.
4. DISCUSSION AND CONCLUSIONS

4.1. Free Electrons and Turbulence in the Outer Galaxy

Our likelihood results indicate that \( A_1 \approx 15 \text{ kpc} \) if the fluctuating part of the electron density distribution exhibits a gradual decrease in the outer Galaxy or \( A_1 \approx 20 \text{ kpc} \) if the \( \text{H} \, \text{II} \) disk is truncated. The apparent decrease in the rms \( n_e \), could result if sites of turbulence became less numerous while the mean \( n_e \) remained constant or it could result from decreases in the mean \( n_e \) itself. In this section, we assess the extent to which our inferred value for \( A_1 \) can distinguish between these possibilities.

To discuss variations in the electron density, we recast equation (4) in terms of the \( \text{H} \, \text{I} \) density and ionization and integrate over a path length \( D \),

\[
\text{SM} = \int_0^D C_y F X_i^2 n_{H_2} \, ds .
\]

In this form, it is clear that there are three means by which the amount of scattering in the outer Galaxy could be limited:

1. \( n_{H_2} \).—It is well known that the distribution of \( \text{H} \, \text{I} \) decreases at large Galactocentric radii (Burton 1992). Outer Galaxy scattering could be limited because there is simply not enough gas to be ionized and produce scattering.

2. \( fX_i^2 \).—This factor is the product of the fractional ionization, \( X_i \), and the volume filling factor, \( f \), of the ionized gas. If the number density of ionization sources, e.g., \( \text{H} \, \text{II} \) regions and supernovae, decreases faster than does \( n_{H_2} \), the scattering would be limited by a lack of ionized gas, even though there would be sufficient amounts of neutral gas. We treat the product \( fX_i^2 \) rather than \( f \) and \( X_i \) separately because our measurements cannot distinguish between the two. Changes in \( f \) can be balanced by changes in \( X_i^2 \) so as to keep the product constant.
3. \( F \) — This factor is a measure of the level of turbulence. Since the rate of star formation does fall off toward the outer Galaxy, unless alternate or additional sources of ionization are present in the outer Galaxy, e.g., the intergalactic ionizing flux, the scattering could be limited by a lack of turbulent energy input into the medium rather than the ionization.

Our motivation for assuming that SM is separable in this manner is to consider ionization and energy input mechanisms from sources not generally thought to be operative in the inner Galaxy. We consider each of these factors in turn.

4.1.1. Hydrogen Distribution

Outside the solar circle, atomic hydrogen dominates molecular hydrogen (Gordon & Burton 1976) and we take \( n_H = n_{HI} \). Comparison of the emission measure \( \text{EM} = \int n_i^2 \, ds \) and DM toward high-latitude pulsars suggest \( f \gtrsim 0.1 \) and \( X_i \approx 1 \) (Reynolds 1977). Because we are assuming that the strength of scattering in the outer Galaxy is dominated by the decrease in the H I density, we take \( f, X_i \), and \( F \) to be constant as functions of \( R \).

We estimate the quantity \( \int n_{HI} \, ds \) toward the anticenter using the mass models of Dehnen & Binney (1998). We are required to use a model to estimate this quantity because the self-opacity of H I toward the anticenter means that the H I column density is not an observable (Burton & TeLintel Hekkert 1986).

Our estimate is \( \int n_{HI} \, ds \approx 2.3 \, \text{cm}^{-6} \) kpc. Assuming that \( F = 0.4 \), the resulting SM is \( \log (\text{SM}) = -0.8 \). This SM produces a 1 GHz scattering diameter of 40 mas, approximately 2–5 times larger than what we observe. We conclude that the distribution of electron density turbulence must decrease more rapidly with Galactocentric distance than does the distribution of hydrogen. This decrease may be caused by an overall decrease in ionized gas or by a diminution of turbulence in the ionized gas.

4.1.2. Sources of Ionization: Stellar versus Intergalactic

Interior to \( R \approx 25 \) kpc, the H I surface density is greater than \( 10^{-19} \) cm\(^{-2}\) (Burton 1992; Dehnen & Binney 1998) and the disk is optically thick to the intergalactic ionizing flux. Consequently, \( fX_i^2 \) should increase with \( R \) as the H I surface density and the disk’s optical depth decrease. Patchiness in the outer Galaxy H I distribution would also increase \( fX_i^2 \).

As the previous section showed, if \( fX_i^2 \approx 0.1 \) and is constant as a function of \( R \), the H I distribution overpredicts the scattering diameters. Allowing \( fX_i^2 \) to increase with \( R \) would increase the size of this discrepancy. Further, the scale length of the scattering is smaller than the Galactocentric distance at which the disk becomes optically thin, indicating that the intergalactic ionizing flux does not play a significant role in the scattering in the outer Galaxy.

We favor star formation in the outer Galaxy as the more likely source of ionization. Wouterloot et al. (1990) showed that the distribution of molecular clouds with embedded massive star formation terminates at 20 kpc. This truncated distribution has an extent comparable to what our likelihood functions imply for the radial extent of scattering. The limited radial extent of the molecular clouds is also the reason we favor the truncated disk model to describe the radial dependence of scattering. Figure 9 illustrates schematically the spatially coincident distributions of molecular clouds and turbulent gas in the outer Galaxy.

4.1.3. Sources of Turbulence: Stellar versus Galactic

The factoring of the scattering measure into separate ionization and turbulent contributions, equation (15), ignores possible correlations between these factors: many of the same sources responsible for the ionization of the gas can also serve to produce turbulence, e.g., \( \text{H II} \) regions and supernovae. We utilize this factoring in order to consider an alternate source of turbulence not associated with massive star formation. We conclude that turbulence is in fact associated with star formation.

The orbits of the Magellanic clouds cause them to cross the midplane of the disk. As these and other satellite galaxies cross the disk, they generate mixing layers and wakes. The eddy turnover time, which is related to the energy dissipation rate, is \( t \sim l_0 / u \) (Tennekes & Lumley 1972), where \( l_0 \) is the outer scale of the turbulence and \( u \) is a characteristic velocity. For \( l_0 \sim 100 \) pc (Rickett 1990; Spangler 1991; and references within) and \( u \sim 100 \) km s\(^{-1}\), \( t \sim 10^9 \) yr. Since the orbital period of a typical satellite galaxy is of order \( 10^9 \) yr, the passage of a satellite galaxy through the outer disk will provide only a transitory source of turbulence.

The distribution of molecular clouds with embedded massive star formation extends to 20 kpc (Wouterloot et al. 1990), comparable to the extent of the extended ionized component. The stars embedded in these molecular clouds have early B spectral types (Wouterloot, Brand, & Henkel 1988). These stars are probably sufficiently powerful to produce turbulence: the pulsar B 0611 + 22 exhibits strong interstellar scintillation and the line of sight to it passes near several late O and early B stars (Weisberg et al. 1980).

4.2. Comparison with External Galaxies

Diffuse ionized gas in external galaxies has been detected in H\( \alpha \) emission (Rand 1996, and references within). This diffuse gas is presumably the equivalent of the Galaxy’s warm ionized medium (Kulkarni & Heiles 1987). Spangler & Reynolds (1990) showed that the scattering diameters for extragalactic sources are correlated with H\( \alpha \) emission, suggesting that the same (warm ionized) gas responsible for the H\( \alpha \) emission is also responsible for the scattering. In this section, we compare the radial extent of the Galaxy as inferred in our likelihood analysis with that determined for other galaxies.

Table 4 presents a subset of H\( \alpha \) measurements extracted from the literature. We restrict the list to those galaxies for which the published observations include images large enough that radial extents can be estimated reliably. The last entry in Table 4 is our estimate, based on the likelihood results, for the radial extent of the Galaxy as it would appear to an external observer. We have obtained this estimate in the following manner. The \((1/\sigma)\) sensitivities for the H\( \alpha \) observations are typically \( \delta(\text{EM}) \approx 3 \, \text{cm}^{-6} \) pc. We integrated \( n_i^2 \, ds \) to produce the modeled EM, \( \tilde{E} \), along a line of sight appropriate for an external observer seeing the Galaxy edge on, i.e., along the path parallel to the x-axis (the discussion following eq. [5] describes the coordinate system). Trial and error was sufficient to determine that, for a truncated disk, \( \tilde{E} \approx \delta(\text{EM}) \) at \( R \approx A_J / 2 \). The radial extent of the Galaxy is comparable to the radial extent of these other galaxies.

A measure of the star formation rate of the galaxy is provided by \( L_{\text{FIR}} / D_{25}^2 \), the far-infrared luminosity within the optical isophotal diameter at 25th magnitude. This is an imperfect measure of the star formation rate, however, as
low-mass stars heating "cirrus" clouds can contribute to the far infrared luminosity (Rand 1996) and the optical and infrared luminosities may have different extents.

Although the star formation rate, as measured by \( \frac{L_{\text{FIR}}}{D_{25}^2} \), is a good predictor of the amount of extraplanar gas (Rand 1996), it is not well correlated with the radial extent of Hz. For instance, the radial extent of NGC 4217 is only 25% (4 kpc) smaller than that of UGC 10288 even though the star formation rate is at least a factor of 3 higher in UGC 10288; NGC 5746 has a radial extent larger than that of the Galaxy even though its star formation rate is an order of magnitude less than the Galaxy's. More likely, the star formation rate is determined by a quantity like the H I or H2 surface density.

Further support for our proposal that our measurements trace the extent of the turbulent ionized disk is found by comparing the Hz emission in external galaxies (Rand 1996, Figs. 1–9) with the molecular cloud distribution in the inner Galaxy (Wouterloot et al. 1990, Fig. 4). The Hz emission is concentrated toward the galaxies' centers with a gradient to large radial distances. Near the edge of the Hz disk, the emission becomes patchy. The molecular cloud distribution in the outer Galaxy has a similar appearance—it displays a strong Galactocentric gradient, and, for \( R \approx 15–20 \text{kpc} \), the distribution is patchy.

High-latitude structure in the extended component could influence our estimates for the various model parameters, in particular \( h_{1,\text{out}} \). The morphology of the extraplanar diffuse gas in external galaxies shows considerable variety: NGC 891 shows diffuse gas up to 4 kpc off the plane with many vertical filaments (Rand, Kulkarni, & Hester 1992); UGC 10288 shows vertical filaments but no diffuse gas (Rand 1996); NGC 4278 shows patchy emission (Rand 1996); and NGC 4565 shows little halo diffuse gas (Rand et al. 1992). Hz observations of the diffuse gas in the solar neighborhood show filamentary structure (Ogden & Reynolds 1985), and the vertical morphology of the extended component could be quite complex.

4.3. Conclusions

We have modified the outer Galaxy portion of the Taylor-Cordes model for the global distribution of ionized gas. Our modifications are motivated by the observed warping and flaring of the H I, H2, and stellar constituents of the outer Galaxy. The data available to constrain the model consist of 18 sources, 9 extragalactic sources from a survey we conducted (Paper I) and 7 pulsars and 2 extragalactic sources extracted from the literature (Table 2). We used a likelihood analysis to constrain the model parameters. The two most important parameters are the radial scale length of the ionized disk \( A_R \), and the strength of scattering in the Perseus arm. The adopted model parameters are summarized in Table 3.

The scattering in the Perseus arm is, at most, 60% of the level seen in the inner Galaxy spiral arms. This upper limit assumes that all of the scattering for sources toward \( l \sim 180^\circ \) is caused by the Perseus arm. Our analysis favors a level of scattering less than this upper limit. We adopt a value 25% of that in the inner Galaxy; the equivalent scattering diameter is 1.5 mas at 1 GHz.

We considered two different radial dependences for the electron density, a smooth decrease of the electron density with Galactocentric distance, equation (6), and a truncated distribution, equation (7). The current data cannot distinguish between these two forms. The radial scale length for the ionized disk is \( A_R \approx 15–20 \text{kpc} \), comparable to the extent that TC93 adopted with fewer anticenter measurements. We favor the truncated disk because the radial extent inferred for sites of massive star formation also appears truncated at approximately 20 kpc, as indicated schematically in Figure 9. Hz observations of external galaxies show that they have radial extents comparable to that which we infer for the Galaxy.

Our analysis favors an unwarped, nonflaring disk with a scale height of 1 kpc, though this may reflect the nonuniform and coarse coverage of the anticenter provided by the available data.

The observed scattering diameter of one extragalactic source (87GB 0600 + 2957) is a factor of 2 smaller than the modeled scattering diameter, suggesting the possibilities of holes in the scattering material. The Hz emission in the outer portions of the disks of these external galaxies also appears patchy, similar to the distribution of molecular clouds in the outer Galaxy. A patchy distribution of massive star formation sites would allow the possibility of holes in the scattering material.

We conclude that scattering in the outer Galaxy traces star formation, as it does in the inner Galaxy. The inter-

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**Table 4**

| Name          | R (kpc) | \( \frac{L_{\text{FIR}}}{D_{25}^2} \) \( \times 10^{40} \) ergs s\(^{-1}\) kpc\(^{-2}\) | \( \delta(\text{EM}) \) (pc cm\(^{-6}\)) | Reference |
|---------------|---------|-------------------------------------------------|---------------------------------|-----------|
| NGC 891       | 15      | 2.2                                             | 6.5                             | 1         |
| NGC 3079      | 11      | 8.9                                             | 90                              | 2         |
| NGC 4013      | 9       | 2.6                                             | 2.5                             | 3         |
| NGC 4217      | 12      | <0.12                                           | 3.2                             | 3         |
| NGC 4302      | 10      | <2.3                                            | 2.1                             | 3         |
| NGC 4565      | 17      | 0.3                                             | 2.3                             | 4         |
| NGC 4631      | 14      | 1.8                                             | 3.2                             | 4         |
| NGC 4762      | 6       | <0.15                                           | 2.4                             | 3         |
| NGC 5023      | 4       | <0.09                                           | 2.3                             | 3         |
| NGC 5746      | 26      | 0.2                                             | 3.7                             | 3         |
| NGC 5907      | 22      | 0.8                                             | 3.2                             | 3         |
| UGC 4278      | 8       | <0.04                                           | 2.7                             | 3         |
| UGC 10288     | 16      | 0.4                                             | 4.4                             | 3         |
| Galaxy        | 10      | 3.0                                             | ...                             | 5         |

**References**

- (1) Rand, Kulkarni, & Hester 1990;
- (2) Veilleux, Cecil, & Bland-Hawthorn 1995;
- (3) Rand 1996;
- (4) Rand et al. 1992;
- (5) the present paper.
galactic ionizing flux and turbulence generated by satellite galaxies passing through the disk contribute little to the scattering. However, the ionized disk of the Galaxy could extend to much larger radii (R \approx 100 \text{kpc}), comparable to that inferred from Ly\alpha absorption systems (see, e.g., Charlton et al. 1993), if the extreme outer disk is quiescent and contributes little scattering.

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