Properties of waste-distilled engine oil and biodiesel ternary blends

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ABSTRACT

This study aims to improve the fuel properties limitations of biodiesel which affect the engine performance characteristics in diesel engines. A ternary mixture simplex axial design model was used to determine the fuel properties of ternary blend mixture of waste distilled engine oil, waste cooking oil biodiesel, and petroleum diesel, and comparing it with existing physical properties models. The fuel properties namely: heating value, flash point, cetane number, density, and viscosity were determined by changing the composition in the ternary mixture design. Furthermore, the experimental data of the mixture model was fitted with existing viscosity, density, heating value, and flash point models. The viscosities were fitted with the Cragoe, Bingham, Arrhenius, and Kendall–Monroe viscosity models at 40°C respectively. The best fit of the experimental data occurred in the following descending order: Arrhenius, Kendall–Monroe, Bingham, and Cragoe with \( R^2 \) values of 0.9771, 0.9529, 0.9508, and 0.6096, respectively. The density at 20°C, heating value, flash point, and cetane number were fitted with Kay’s model based on the mixing empirical equation. The results showed that these properties were well predicted by Kay’s model mixing rule empirical model due to high values of \( R^2 \) of 0.9880, 0.978, 0.9929, and 0.961 respectively. The viscosity, density, heating value, and flash point of the ternary blend mixtures are within the American Society for Testing and Materials (ASTM) D 6751 and ASTM D 975 specifications range.

1. Introduction

Fossil fuel reserves, the vast majority of the World’s energy resources, have been used for a very long time and due to high global energy requirements and increase in air pollution and harmful emissions resulting from the large-scale production and consumption of these fuels, there is the need to develop new alternative fuels that would satisfy the demands of the transportation sector, which is the largest consumer. One of such which has been widely used is non-conventional fuel such as biodiesel. Another, which is gaining popularity is a diesel-like fuel distilled from waste engine oil. Recent studies have shown that the energy demand worldwide is expected to increase by approximately 30% by the year 2030 (Imadul et al., 2015). The use of diesel from fossil fuels has caused lots of public outcries, especially from environmentalists. Some of the shortcomings associated with petroleum diesel derived from fossil fuel include negative environmental effects, price fluctuations, and diminishing well reservoirs. This calls for the development of innovative ways to enhance long-term energy security soon. Recent research has shown that energy security can be improved by using fuel produced from renewable sources such as biodiesel and recycling fossil fuels such as engine oil. The energy produced from biodiesel can contribute to the socio-economic development of societies at a global level, hence provide an alternative source of fuel to aid the depleting fossil fuel reserves (Debbarma et al., 2020).

Biodiesel is a renewable energy source which is an excellent proposal that fits perfectly as a solution to the numerous emission problems caused by fossil fuel use to the environment. Biodiesel is produced in presence of a catalyst via the transesterification process using alcohol.

Waste distilled engine oil is an environmental pollutant that has been on the rise due to the ever-increasing number of automobiles. Waste distilled engine oil has the potential as an additive that can be used to improve biodiesel and diesel properties hence mitigate environmental pollution problems caused by its disposal. Usually, the amount of engine oil that is used in the engine before it is drained is about 20% while 80%...
is taken out as waste (Beg et al., 2010). Waste engine oil can be distilled to a diesel-like fuel at temperatures of about 300 °C and thereafter it can be blended with biodiesel and petroleum diesel.

Some of the properties associated with biodiesel include high viscosity, high density, high pour point, and a higher Cetane number (CN) as compared to petroleum diesel produced from fossil fuel. Despite biodiesel being a preferred alternative fuel source, some of its shortcomings include; lower engine performance, higher density and viscosity due to the presence of long-chain fatty acids, and nitrogen oxides (NOx) emission which is due to low heating value (Mathew et al., 2019). These shortcomings have led to problems such as choking of the injector which impacts greatly on engine efficiency and its durability (Dey and Ray, 2020). Studies done in the last decade have shown that blending biodiesel with other fuels depicts a significant improvement in its properties which aids in biodiesel use as a fuel (Martin et al., 2017).

The density, viscosity, heating values, and flash point of binary blends of biodiesel from different oils and diesel have been investigated and generalized equations, as well as models, have been used to estimate the fuels blend properties to modify the physical characteristics of biodiesel (Yahagi et al., 2019; Samuel et al., 2020).

Design of Experiment (DOE) is a statistical and mathematical method used for the systematic execution of the experiments and the efficient analysis of results. The technique allows the use of a small number of experiments, in which many experimental parameters are routinely and simultaneously varied to obtain adequate valuable information for analysis (Montgomery, 2017).

Most research studies found in literature are based on binary blends and there is little information on the usage of ternary mixture design to predict the fuel properties of ternary blend mixture and to compare it with the existing physical properties models. Atmanli et al. (2015) determined the fuel properties of a diesel-vegetable oil-n-butanol ternary blend by using basic properties of pure fuels and following Kay’s mixing rule for density, lower heating value, and cetane number calculations. It appears that no studies have been done on the possibility of using this ternary mixture of waste distilled engine oil-biodiesel-diesel fuel. The independent variables in a mixture experiment are ratios of various components in a blend. Models such as simplex-lattice, simplex-axial, and simplex-centroid designs models have been applied in most mixture experimental designs (Cornell, 2002). Ternary blend mixtures have been optimized using response surface methodology to determine engine operating parameters.

For the waste distilled engine oil fuel to be used safely and advantageously in diesel engines both from the viewpoints of combustion characteristics and exhaust emissions, there is the need to close the gap concerning the basic properties of the blends. Hence, the main objective of the present work is to determine the properties of the fuel blend mixture of biodiesel produced from waste cooking, waste distilled engine oil, and petroleum diesel using a ternary mixture design based on a simplex-axial design.

2. Methods and analysis approach

2.1. Materials

Spent engine oil and diesel were obtained from a local gas service station whereas waste cooking oil was sourced from nearby restaurants. 98% concentrated sulphuric acid, tetra ethylene pentamine, sodium silicate, potassium hydroxide, methanol, and isopropyl were obtained from a local chemical supplier.

2.2. Biodiesel production

The waste cooking oil was filtered to remove foreign unwanted particles. Free fatty acids (FFA) content was determined by using the method developed by (Yesilyurt, 2019). The determined FFA content was less than 1% hence, a single step transesterification process was used.

The reaction was carried out under the following conditions: (i) methanol to oil mass ratio (30 wt.%); (ii) reaction time (1 h); (iii) temperature (60 °C) and (iv) catalyst to oil ratio (1.4 wt.%) according to Modiba et al. (2014). The reaction was done using a flat bottom conical flask fitted with a condenser at a stirring speed of 250 rpm. The separation of glycerol, catalyst, and biodiesel mixture was done using a decanter. Gas chromatography was used to quantify the amount of free fatty acid methyl esters. This was done in replicates of three and the average was determined using Eq. (1).

\[
\text{Biodiesel \% yield} = \frac{\text{Mass of biodiesel} \times \% \text{ FAMEs}}{\text{Mass of oil}} \times 100
\]

2.3. Distilled waste engine oil production

The production of waste distilled engine oil was done according to Sharma and Soni (2013). The used engine oil was first pre-treated and thereafter distilled. The pre-treatment process involves filtering the waste engine oil through 20 μm sizes of mesh filter papers to remove dust, carbon soot, metal, and other particles. After filtration, the filtered engine oil undergoes acid treatment and neutralization by addition of 98% concentrated sulphuric acid and NaOH, respectively. 90 g of used engine oil was reacted with 30 g of sulphuric acid. The mixture was then allowed to decant for 24 h. The top layer was then heated to a temperature between 40 - 50 °C for about 1 h with continuous stirring. Thereafter 2 g of tetra ethylene pentamine and sodium silicate was added slowly until a clear yellow oil solution was formed. The mixture was then mixed with activated clay and filtered using a vacuum pump. The oil was distilled at 330 °C in atmospheric conditions. The density at 20 °C, viscosity at 40 °C, heating value, flash point, and cetane number were determined according to ASTM D941, D445, D2015, D975, and D613 respectively. This was done three times and the average value was recorded.

2.4. Statistical mixture design (simplex lattice design)

Figure 1 depicts the design of experiments of a simplex axial 3-component mixture design of a waste distilled engine oil – diesel – biodiesel blend. The experimental values of the fuel properties measured are shown in Table 1.
2.5. Viscosity models

Viscosity is an important physical property parameter in diesel engine fuel. For atomization to occur, diesel is sprayed near the nozzle exit into compressed air (Tat and van Gerpen, 1999). This is affected by viscosity quality in terms of the size of fuel drop and penetration (Nogami and Yagi, 2004). Alptekin and Canakci (2008) blended biodiesel with diesel using a binary mixture design and the resultant viscosity values were fitted with Arrhenius (Arrhenius, 1887), Bingham (Bingham, 1914), Kendall-Monroe (Kendall and Monroe, 1917), and Cragoe (Cragoe, 1933). There is no recent work that has been done by fitting the experimental values of viscosity of a ternary blend of a mixture design with the existing viscosity models and therefore the Arrhenius, Bingham, Kendall and Monroe and Cragoe viscosity models can be modified into Eqs. (2), (3), (4), and (5) respectively.

\[
\ln(\mu_{TBM}) = Y_A \ln(\mu_A) + Y_B \ln(\mu_B) + Y_C \ln(\mu_C)
\]

(2)

\[
\frac{1}{\mu_{TBM}} = \frac{Y_A}{\mu_A} + \frac{Y_B}{\mu_B} + \frac{Y_C}{\mu_C}
\]

(3)

\[
\mu_{TBM}^{1/3} = Y_A \mu_A^{1/3} + Y_B \mu_B^{1/3} + Y_C \mu_C^{1/3}
\]

(4)

\[
\frac{1}{\ln(2000\mu_{TBM})} = \frac{Y_A}{\ln(2000\mu_A)} + \frac{Y_B}{\ln(2000\mu_B)} + \frac{Y_C}{\ln(2000\mu_C)}
\]

(5)

The blend viscosity is associated with the volume fraction of the various components, where \(\mu_{TBM}\) is the ternary blend mixture viscosity, \(\mu_A\) and \(\mu_B\) and \(\mu_C\) is the diesel viscosity and its ratio respectively, and \(\mu_C\) and \(Y_C\) are the biodiesel viscosity and its fraction respectively.

2.6. Density, flash point, and heating value fuel properties fitted with kay’s mixing rule model

There have been several studies conducted to establish analytical models for measuring and predicting density, flashpoint, and heating value of biodiesel-diesel blends. To predict the fuel properties several studies have used kay’s mixing rule. The rule assumes that the fuel property of a mixture is determined by pure-component average property, is determined by pure-component average (Benmekki and Mansoori, 1989).

The model assumes negligible excess volumes where physical and chemical properties are closely related. The experimental data from individual components can be used to determine the density, flash point, and heating value of a ternary blend of waste distilled engine oil-biodiesel-diesel mixture using Equation (6), Equation (7) and Equation (8) and Equation (9) respectively.

\[
\rho_{TBM} = Y_A \rho_A + Y_B \rho_B + Y_C \rho_C
\]

(6)

\[
F_{TBM} = Y_A F_A + Y_B F_B + Y_C F_C
\]

(7)

\[
H_{TBM} = Y_A H_A + Y_B H_B + Y_C H_C
\]

(8)

\[
C_{TBM} = Y_A C_A + Y_B C_B + Y_C C_C
\]

(9)

where \(\rho_A\), \(\rho_B\), and \(\rho_C\) are the volume fractions of waste distilled engine oil, diesel, and biodiesel respectively; \(\rho_A\), \(\rho_B\), and \(\rho_C\) is the density of waste distilled engine oil, diesel, and biodiesel respectively; \(F_A\), \(F_B\), and \(F_C\) are the flash point of waste distilled engine oil, diesel, and biodiesel respectively; \(H_A\), \(H_B\), and \(H_C\) are the heating values of waste distilled engine oil, diesel, and biodiesel respectively while \(\rho_{TBM}\), \(F_{TBM}\), \(H_{TBM}\), and \(C_{TBM}\) are density, flash point heating value and cetane number of the ternary blend mixture, respectively.

3. Results and discussion

3.1. Characterisation of fuel properties and standards specifications

The density, kinematic viscosity, flash point, heating value, and cetane number of biodiesel produced from waste cooking oil and pure diesel are in the range of the values specified in the ASTM D-6751 and ASTM D 975, respectively. The fuel properties of pure distilled engine oil are within the ASTM D975 of diesel as compared to the ASTM D-6751 fuel property specification of biodiesel. As shown in Table 1 the viscosity of pure biodiesel, pure diesel, and waste distilled engine oil are 2.5991 mm²/s, 2.1392 mm²/s, and 1.9934 mm²/s respectively. This shows that when waste engine oil is cleaned and distilled there is a reduction in the viscosity which can improve the atomization, penetration, and flow characteristics in the engine and reduce exhaust smoke, engine deposit, and emissions. Upon performing a ternary blend of an equal mixture of waste distilled engine oil, biodiesel, and diesel a viscosity of 2.1945 mm²/s was achieved.

As it can be seen in Table 1, the flash point of pure biodiesel, pure diesel, and waste distilled engine oil is 146 °C, 54 °C, and 58 °C, respectively. Biodiesel has a higher flash point which is an important aspect of safety issues. As depicted in Table 1 a flash point of 84 °C is achieved upon mixing biodiesel, pure diesel, and distilled waste engine oil in equal proportions. Since the obtained mixture flash point is greater than the diesel flash point, the safety aspects are achieved and overcome the limitation of using diesel only. The engine combustion properties and the brake thermal efficiency is affected fuel calorific value. The heating value of pure biodiesel, pure diesel, and waste distilled engine oil is 39.61 MJ/kg, 42.63 MJ/kg, and 41.9 MJ/kg, respectively. This shows that the heating value of biodiesel is lower than that of the normal diesel fuel.

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Table 1. Mixture ternary fuel blends for experimental fuel properties according to ASTM standards.

| Run | WDEO Petroleum diesel | Biodiesel | Density at 20 °C (g/cm³) | Viscosity at 40 °C (mm²/s) | Heating value (MJ/kg) | Flash point (°C) | Cetane number |
|-----|-----------------------|-----------|--------------------------|---------------------------|----------------------|----------------|--------------|
| 1   | 0.00                  | 0.00      | 1.00                     | 0.8812                    | 2.5991               | 39.61          | 146          | 51           |
| 2   | 1.00                  | 0.00      | 0.00                     | 0.8220                    | 1.9934               | 41.90          | 58           | 43           |
| 3   | 0.00                  | 1.00      | 0.00                     | 0.8362                    | 2.1633               | 42.63          | 54           | 41           |
| 4   | 0.50                  | 0.00      | 0.50                     | 0.8301                    | 1.9926               | 42.60          | 60           | 42           |
| 5   | 0.50                  | 0.50      | 0.00                     | 0.8491                    | 2.2955               | 41.03          | 103          | 47           |
| 6   | 0.00                  | 0.50      | 0.50                     | 0.8584                    | 2.3655               | 41.13          | 101          | 46           |
| 7   | 0.66                  | 0.17      | 0.17                     | 0.8433                    | 2.1392               | 42.46          | 77           | 44           |
| 8   | 0.17                  | 0.66      | 0.17                     | 0.8488                    | 2.1823               | 42.56          | 73           | 43           |
| 9   | 0.33                  | 0.34      | 0.33                     | 0.8434                    | 2.1945               | 41.12          | 84           | 44           |
| 10  | 0.17                  | 0.17      | 0.66                     | 0.8707                    | 2.4156               | 40.99          | 124          | 47           |
thus, when used alone it lowers the engine performance. Upon performing a ternary blend of an equal mixture of waste distilled engine oil, biodiesel, and diesel a heating value of 41.12 MJ/kg is achieved.

The performance of diesel fuel is measured using cetane rating or also well-known as cetane number. The greater the value, the enhanced the rate at which a fuel burns in an engine of an automobile. The cetane value of pure biodiesel, pure diesel, and waste distilled engine oil is 51, 41, and 43, respectively. This shows that the cetane number of biodiesel is higher than that of the normal diesel fuel thus, when used alone it improves the burning properties in the engine. Upon performing a ternary blend of an equal mixture of waste distilled engine oil, biodiesel, and diesel cetane number value of 44 is achieved.

As it is depicted in Table 1, the densities of pure biodiesel, pure diesel, and waste distilled engine oil are 0.8812 g/cm³, 0.8362 g/cm³, and 0.8220 g/cm³, respectively. This shows that the density of biodiesel is higher than that of the normal diesel fuel, thus when biodiesel is used

| Run | WDEO | Petroleum diesel | Biodiesel | Experimental Value | Predicted Arrhenius Model | Predicted Kendal-Monroe Model | Predicted Bingham Model | Predicted Cragoe Model | Absolute Error Arrhenius Model | Absolute Error Kendal-Monroe Model | Absolute Error Bingham Model | Absolute Error Cragoe Model |
|-----|------|------------------|-----------|-------------------|---------------------------|-----------------------------|------------------------|------------------------|--------------------------|-------------------------------|-------------------------------|-------------------------------|
| 1   | 0.00 | 0.00             | 1.00      | 2.5990            | 2.5990                    | 2.5990                      | 2.5990                 | 2.5990                 | 0.0000                   | 0.0000                        | 0.0000                        | 0.0000                        |
| 2   | 1.00 | 0.00             | 0.00      | 1.9934            | 1.9934                    | 1.9934                      | 1.9934                 | 1.9934                 | 0.0000                   | 0.0000                        | 0.0000                        | 0.0000                        |
| 3   | 0.00 | 1.00             | 0.00      | 2.1633            | 2.1633                    | 2.1633                      | 2.1633                 | 2.1633                 | 0.0000                   | 0.0000                        | 0.0000                        | 0.0000                        |
| 4   | 0.50 | 0.00             | 0.05      | 1.9926            | 2.0766                    | 2.0771                      | 2.0748                 | 2.0761                 | 0.0840                   | 0.0845                        | 0.0822                        | 0.0835                        |
| 5   | 0.50 | 0.50             | 0.00      | 2.2955            | 2.2761                    | 2.2860                      | 2.2562                 | 2.2713                 | 0.0194                   | 0.0095                        | 0.0393                        | 0.0242                        |
| 6   | 0.00 | 0.50             | 0.50      | 2.3655            | 2.3711                    | 2.3744                      | 2.3612                 | 2.3688                 | 0.0056                   | 0.0089                        | 0.0043                        | 0.0033                        |
| 7   | 0.66 | 0.17             | 0.17      | 2.1392            | 2.1291                    | 2.1808                      | 2.0828                 | 1.9434                 | 0.0101                   | 0.0415                        | 0.0564                        | 0.1958                        |
| 8   | 0.17 | 0.66             | 0.17      | 2.1823            | 2.2180                    | 2.2698                      | 2.1720                 | 2.0237                 | 0.0357                   | 0.0875                        | 0.0103                        | 0.1586                        |
| 9   | 0.33 | 0.34             | 0.33      | 2.1945            | 2.2199                    | 2.1759                      | 2.2468                 | 2.4326                 | 0.0254                   | 0.0186                        | 0.0523                        | 0.0281                        |
| 10  | 0.17 | 0.17             | 0.66      | 2.4156            | 2.4312                    | 2.4877                      | 2.3715                 | 2.2127                 | 0.0156                   | 0.0721                        | 0.0441                        | 0.2029                        |

Figure 2. Experimental viscosity data of waste engine distilled oil-biodiesel-diesel ternary blend mixture fitted with Arrhenius (A), Kendal-Monroe (B), Bingham (C) and Cragoe model (D) at 40 °C.
alone it may stick on the injector nozzles. Further, as it can be seen from Table 1, upon performing a ternary blend of an equal mixture of waste distilled engine oil, biodiesel, and diesel a lower density value of 0.8434 g/cm³ was achieved which overcomes the limitation of using biodiesel only (see Table 2).

### 3.2. Viscosity models

Table 3 shows the measured, calculated, and absolute errors values according to the Arrhenius, Kendall–Monroe, Bingham, and Cragoe viscosity models. Figure 2A, B, C, D depicts the plots of experimental values against predicted values for determination of coefficients (R²) using the Arrhenius, Kendall–Monroe, Bingham, and Cragoe viscosity models, respectively. As shown in Table 3, commonly, the Arrhenius model predicted viscosity values are greater than the experimental values. The maximum absolute error between the predicted and experimental values are 0.0840 with R² of value 0.9751 as shown in Figure 2A. From Table 3, Kendal-Monroe model predicted viscosity values are typically higher than the experimental values, and the absolute error between the predicted value and the experimental value is 0.0875 with R² of value 0.9507 as shown in Figure 2B. Similarly, the majority of the experimental values are lower than the predicted values according to the Bingham model. The absolute error between the predicted value and the experimental value is 0.08 with R² of value 0.9508 depicted in Figure 2A. The viscosity values determined using the Cragoe model are significantly lower than those obtained by the experiment. The maximum absolute error between the values calculated and expected for the Cragoe is 0.2029. As shown in Figure 2D, the R² of value 0.6009 shows a poor fit.

![Figure 3](image.png)

**Figure 3.** Experimental values of density at 20 °C (A), flash point (B), heating value (C) and cetane number (D) of the waste distilled engine-biodiesel-diesel ternary blend mixture fitted with the Kay mixture model.

| Run | WDEO | Petroleum diesel | Biodiesel | Density at 20 °C (g/cm³) | Heating value (MJ/kg) | Flash point (°C) | Cetane number | Absolute Error Density | Absolute Error Heating value | Absolute Error Flash point | Absolute Error Cetane number |
|-----|------|------------------|-----------|--------------------------|-----------------------|-------------------|---------------|-----------------------|-----------------------------|-----------------------------|-----------------------------|
| 1   | 0.00 | 0.00             | 1.00      | 0.8812                   | 39.61                 | 146               | 51            | 0.0000                | 0.00                        | 0                           | 0                           |
| 2   | 1.00 | 0.00             | 0.00      | 0.8220                   | 41.90                 | 58                | 43            | 0.0000                | 0.00                        | 0                           | 0                           |
| 3   | 0.00 | 1.00             | 0.00      | 0.8362                   | 42.63                 | 54                | 41            | 0.0000                | 0.00                        | 0                           | 0                           |
| 4   | 0.50 | 0.50             | 0.00      | 0.8291                   | 42.27                 | 56                | 42            | 0.0010                | 0.33                        | 4                           | 0                           |
| 5   | 0.50 | 0.00             | 0.50      | 0.8516                   | 40.76                 | 102               | 47            | 0.0025                | 0.27                        | 1                           | 0                           |
| 6   | 0.00 | 0.50             | 0.50      | 0.8587                   | 41.12                 | 100               | 46            | 0.0003                | 0.01                        | 1                           | 0                           |
| 7   | 0.67 | 0.17             | 0.17      | 0.8427                   | 42.05                 | 74                | 43.8          | 0.0006                | 0.41                        | 3                           | 0.22                        |
| 8   | 0.17 | 0.67             | 0.17      | 0.8498                   | 42.42                 | 71                | 43.9          | 0.0010                | 0.14                        | 2                           | 0.98                        |
| 9   | 0.33 | 0.33             | 0.33      | 0.8381                   | 40.97                 | 85                | 44.2          | 0.0053                | 0.15                        | 1                           | 0.22                        |
| 10  | 0.17 | 0.17             | 0.67      | 0.8723                   | 40.91                 | 116               | 48.8          | 0.0016                | 0.08                        | 8                           | 1.78                        |
correlation between the model predicted values and the experimental values. From the absolute error the Arrhenius, Kendall-Monroe, and Bingham models showed that experimental and predicted values are similar and low. This indicates that the measurements are closer to the true values. The Cragoe model had a higher absolute value and therefore there is an indication of the uncertainty in the measurement of viscosity. In summary, the best fit of the viscosity experimental data to the existing model occurred in the following descending order: Arrhenius, Kendall-Monroe, Bingham, Cragoe with \( R^2 \) values of 0.9771, 0.9529, 0.9508, and 0.6096, respectively. The Arrhenius model had the highest \( R^2 \) and can, therefore, be used to calculate the viscosity of a ternary mixture of the biodiesel-diesel-waste engine oil blend. This is excellent with the agreement with results obtained by Alptekin and Canakci (2008) who predicted the viscosity of a binary mixture of biodiesel and diesel blend.

### 3.3. Density model

From Figure 3, the density values of the blend did not differ much. The density values were in the range of 0.8220 g/cm\(^3\) to 0.8812 g/cm\(^3\). Figure 3A shows the density comparison of measured and calculated values using Kay’s mixing rule. Eq. (6) was used to calculate the densities of the ternary blend mixture as shown in Table 4. There is the best fit between the experimental and the predicted values from Kay’s mixing rule model. The minimum \( R^2 \) and the maximum absolute error is of the fuel blends were 0.9880 and 0.0053 respectively.

### 3.4. Flash point model

Table 3 shows the experimental values and the predicted data using Eq. (7). As it can be seen there is no significant change between the experimental and predicted flashpoint values based on Kay’s rule model for a ternary blend. The flash point values varied from 54 °C to 146 °C. Figure 3B shows that the measured flash point values for the fuel blend matched the calculated values. The experiment and the calculated values based on Kay’s rule model were in excellent agreement. The minimum \( R^2 \) and the maximum absolute error were 0.9929 and 8 °C respectively.

### 3.5. Heating value model

The heating values of the ternary mixture blends were calculated using Kay’s model mixing rule model using Eq. (8). It was found that the experimental and the calculated heating value did not differ much as depicted in Table 3 and Figure 3 (C). The heating values ranged from 39.61 MJ/kg to 42.63 MJ/kg. Figure 3 (C) shows the measured heating values for the fuel blend matched to the calculated values. Table 3 shows the heating value obtained from Eq. (8). The minimum \( R^2 \) and the maximum absolute error between the experimental and the calculated values were 0.9784 and 0.33 MJ/kg respectively.

### 3.6. Cetane number

The cetane values of the ternary mixture blends were calculated using Kay’s model mixing rule model using Eq. (9). It was found that the experimental and the calculated cetane number did not differ much as depicted in Table 3 and Figure 3 (D). The cetane values ranged from 41 to 53. Figure 3 (D) shows the measured cetane number for the fuel blend matched to the calculated values. The minimum \( R^2 \) and the maximum absolute error between the experimental and the calculated values were 0.961 and 1.78 respectively.

### 4. Conclusion

In this study, the basic properties of ternary blends of waste distilled engine oil, biodiesel, and diesel were determined for use in a diesel engine as an alternative for diesel fuel. Results showed that the viscosities and densities of biodiesel were lowered using the ternary mixture based on simplex axial experimental design and thus, the diesel engine and combustion characteristics can be improved. Upon mixing biodiesel, pure diesel, and distilled waste engine oil in equal proportions, the combustion characteristics can be improved. Upon mixing biodiesel, pure diesel, and distilled waste engine oil in equal proportions, the safety aspects can be accomplished and hence, overcome the limitations of using diesel only. A higher heating value is obtained by conducting a ternary blend of the same mixture of waste distilled engine oil, biodiesel, and diesel, which overcomes the constraint of using only biodiesel and hence, improving the brake thermal efficiency and combustion characteristics of an engine. The best fit of the viscosity experimental data to the existing model occurred in the following descending order: Arrhenius, Kendall-Monroe, Bingham, Cragoe with \( R^2 \) values of 0.9771, 0.9529, 0.9508, and 0.6096, respectively. The density at 20 °C, heating value, and flash point was fitted with the Kay model based on the mixing empirical equation. Results showed that density, heating value, flash point, and cetane number were found to be represented well by the Kay mixing rule empirical model due to high values of \( R^2 \) of 0.9880, 0.9784, 0.9929, and 0.961 respectively.

The viscosity, density, heating value, cetane number, and flash point of the blend produced in this study are within the ASTM D 6751 and ASTM D 975 specifications range.

### Declarations

**Author contribution statement**

Dennis Kipkorir: Analyzed and interpreted the data; Wrote the paper. Francis Nturanabo: Conceived and designed the experiments. Robert Tewo & Christopher Enweremadu: Analyzed and interpreted the data.

Hilary Ruto: Conceived and designed the experiments; Performed the experiments; Contributed reagents, materials, analysis tools or data; Wrote the paper.

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**Data availability statement**

Data included in article/supp. material/referenced in article.

**Declaration of interests statement**

The authors declare no conflict of interest.

**Additional information**

No additional information is available for this paper.

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