Definition and Quantification of Shock/Impact/Transient Vibrations

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Abstract

Vibration injury in the hand-arm system from hand-held machines is one of the most common occupational health injuries and causes severe and often chronic nerve and vascular injury to the operator. Machines emitting shock vibrations, e.g., impact wrenches have since long been identified as a special risk factor. In legislative and standard texts the terms shock, impact, and transient vibration are frequently used to underline the special risk associated with these kinds of vibrations. In spite of this, there is no mathematically stringent definition what a shock vibration is or how the amplitude of the shock is defined.

This lack of definitions is the subject of this article. This document discusses a number of candidate definitions for a vibration shock index (VSI) that quantifies different vibration signals in terms of how localized they are in the time domain. The VSI is intended to be used to classify and compare different vibration sources.

The VSI is independent of the vibration level, i.e., it is unchanged if the vibration signal is rescaled. The traditional root mean square method to determine the vibration level will not produce a value representative for the shocks occurring in a signal with high VSI. Thus, there is a need for a complementing quantification method for the localized signal parts. Possible definitions for such a vibration shock level (VSL) are suggested.

A problem formulation is first stated together with a description of the approach used for designing the VSI and the VSL. After this, model signals are defined, which are used to discuss and evaluate the different candidate definitions. Then, a number of candidate definitions are discussed, leading up to a conclusion on which candidate definitions that are promising for experimental evaluation.
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1 Introduction

Humans are sensitive to mechanical vibrations, but the health effects depend on many parameters, including the vibration amplitude/frequency and the exposure time. One further aspect, which traditionally has not been given sufficient attention, is the fact that some vibrating equipment generates potentially harmful vibrations outside the frequency interval considered by the ISO 5349-1 standard [1, 2, 3]. These may lead to, e.g., nerve and blood vessel injuries in fingers. In analogy with the term ultrasound, these vibrations have been named ultravibrations and is a central concept of this report.

Ultravibrations can be generated either by tools working at high frequency such as, e.g., dental drills, or by tools that generate short pulses. One example of the latter is impact wrenches, which generate shocks with a wide frequency spectrum and periodically delivers power during short time intervals [4]. It is clear that for extremely powerful pulses, even a single pulse can have a quantifiable effect on human health. Thus, the total health effect could be estimated from this health effect per pulse multiplied by the number of pulses for which the user is exposed. If, on the other hand, a simple time average is used, significant underestimation of the deleterious health effects may result as this would fail to take the localized nature of the vibration into account. The question is then when a vibration signal should be considered to be pulsed or not.

This document has two main goals. The first is to suggest a method that answers to what extent a given vibration signal should be considered to contain shocks. This is done by introducing and discussing a number of candidate methods that quantify how strongly localized a given acceleration signal is in the time domain. This parameter is here referred to as the vibration shock index (VSI) of the vibration signal. The second goal is to quantify the vibration level in the shocks of the signal, which will act as a complement to the traditionally used root mean square (RMS) value. This is referred to as the vibration shock level (VSL).

It is pointed out that these two goals are related but distinct. The VSI answers the question to what extent the signal contains localized features and this is a dimensionless quantity. The VSL assigns a representative vibration level value to the localized features. The dimensions of this quantity is the same as for the signal. It should also be noted that quantifying the health effects resulting from a certain vibration source is a separate topic, which is outside the scope of this report. The VSI and VSL do not say anything direct about health effects, but is of potential use as parameters in a method for quantifying this.

The organization of this document is as follows. First, the problem is given a clear formulation and a simplified physical model is used to introduce the physical quantities the VSI and VSL will be based on. Then, model signals are defined, which are used to evaluate the candidate VSI and VSL algorithms. After this, a number of candidate definitions are given, together with a discussion of their relative merits. The work is then concluded.

2 VSI and VSL Definition Process

The aim of this section is to give all background information needed to discuss possible VSI/VSL candidate definitions. First, a number of required properties for the VSI/VSL are specified, with the intention that these properties will act as guiding principles during the development process. A simple physical model is then introduced together with a problem formulation. Simple model signals that are used to evaluate the candidate definitions are then introduced. It is of central importance to later test the candidate definitions using a large number of experimentally obtained vibration signals from different tools in order to decide to what extent the VSI/VSL are valuable concepts in practice, but this is outside the scope of the current document.
2.1 VSI and VSL Selection Criteria

A VSI/VSL definition that is useful in practice is expected to have a number of properties, which are listed in the following.

- **Dimensions:** The VSI quantifies a property that is dimensionless. The VSL has the same dimensions as the vibration signal.

- **Intuitive:** The definitions should give results that are intuitively understandable, implying that, e.g., the following is true.
  - If a vibration signal consists of a train of clearly visible pulses, then the VSI should be high. If this signal is compared with one consisting of identical pulses with larger time separation, then the latter should have an even higher VSI.
  - The VSL should be a value between zero and the maximum amplitude of the vibration signal. When there are shocks in the signal, the VSL should be a value representative for the shocks.

- **Relevant:** The VSI/VSL should be based only on physical quantities that are clearly relevant for the health effects related to vibrating equipment. In order to be compatible with standardized vibration measurements, it can only take the acceleration signal as input.

- **Unambiguous:** The definitions should be completely clear and to remove all subjectivity in the calculation process, the definitions must be given in terms of an algorithm that take the measured acceleration as input and produces the VSI/VSL as output.

- **Robust:** The VSI/VSL should show robust numerical/statistical properties. This includes but is not limited to the following.
  - The VSI/VSL should show a small sensitivity to noise and other imperfections, such as outliers, in the measured data.
  - The VSI/VSL should converge in the sense that using a longer signal sequence should lead to more accurately estimated values.
  - The difference in the VSI/VSL as estimated from two different measurements on the same vibration source should be small, given that both measurements are of good quality and sufficiently long.

- **Parsimonious:** The definitions should contain as few arbitrary parameters and use as few assumptions as possible and be applicable to the widest possible range of experimentally obtained acceleration signals. When there are multiple suggestions with similar performance, preference should be given to the simplest approach.

It is also pointed out that the VSI is a classification of the shape of the waveform and should therefore not change if time or acceleration is rescaled.

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1. [https://en.wikipedia.org/wiki/Occam%27s_razor](https://en.wikipedia.org/wiki/Occam%27s_razor)
2.2 Physical Model

It is assumed that the vibrating system (including the human operator and the material being machined) can be modeled as a driven harmonic oscillator. This is very simplified since, e.g., (i) the real system is distributed and not necessarily possible to describe using a simple lumped system, (ii) the coefficients are here modeled to be independent of frequency, and (iii) there may be additional terms, including nonlinear terms, in a model equation describing the real system. Nevertheless, since it is not possible to know or account for the differences between systems, this approach seems to be a reasonable compromise. As is shown in Appendix A, the dissipated power is then proportional to the square of the velocity of the vibration. Since the dissipated energy can be obtained by integrating the power with respect to time, it may seem natural to base the VSI on this signal. However, there are a number of arguments against this approach.

1. Although the velocity can, in principle, be obtained by integrating the measured acceleration, this may not be straightforward in practice for a number of reasons.

   • The accelerometer noise will be integrated into a random walk and the variance for such a process increases linearly with time.
   • In addition to noise, the accelerometer signal may be affected by bias drifts, nonlinearity etc. Integrating these types of errors may lead to a diverging velocity, i.e., a velocity error that grows with time.
   • In addition to the tool acceleration, the accelerometer also measures gravity, which may be integrated into a diverging velocity.

   Even though these problems can be mitigated by high-pass filtering of the acceleration, there is no guarantee that it is robust to base the VSI on the velocity.

2. A more fundamental problem is that is not clear that the dissipated energy is the physical quantity that correlates best with the health effects from vibrations. It is, e.g., known that heating is not what causes damage to biological tissue from vibrations. It may be that the force is more relevant, in which case the VSI should be based on the acceleration, and some results even indicate that it may be the time derivative of the force that is important.

2.3 Problem Formulation

If it would be assumed that (i) the described physical model is relevant and (ii) the dissipated energy is the central concept, the previous loose discussion can be replaced with a clear and intuitive criterion for a high VSI: In a vibration signal that is “localized” it is possible to find short time intervals that contribute a large fraction of the total dissipated energy.

It is unfortunate that it is not known which is the most relevant physical quantity to base the VSI on. We choose to select the “power signal” to be equal to the square of the acceleration instead of the square of the velocity. It should be noted that it is the square root of the mean value of this quantity that is the primary quantity of standardized measurements [5, Section 4.4], implying that this choice brings the approach here closer to the existing standard.

With this modified definition of the power signal, the remainder of this report will be based on the criterion above. Obviously, this must be made quantitative in order to specify an algorithm, but this problem formulation will serve as a guiding principle.

\[^2\]Citation marks are used here to indicate that there is no direct physical correspondence to the dissipated power. These citation marks will be dropped later in this document.
2.4 Model Signals

Two model signals are used to produce example results from the candidate VSI/VSL definitions. In practice, it will always be the case that the vibrating equipment is stationary over long times, i.e., when the acceleration is integrated two times, then the obtained position should oscillate around an approximately fixed point. The easiest way to ensure this is to specify the position and obtain the acceleration by a double differentiation with respect to time and this is the method used here to define one “continuous signal” and one “pulsed signal”. In addition to these, white Gaussian noise (WGN) with RMS value of one will be used as a model signal. A summary of the selected model signals is given here and the calculations are found in Appendix B.

As a model for a continuous signal, a harmonic oscillation is used according to

\[ x_c(t) = A_c \cos(2\pi f_c t + \phi_c), \]  

where \( A_c \) is the amplitude, \( f_c \) is the frequency, and \( \phi_c \) is the phase, which is set to zero. As a convenient example value, choosing \( A_c = 3.58 \mu m \) and \( f_c = 100 \) Hz, the RMS value is one and the signals in Fig. 1 are obtained. It is seen that the peak acceleration \( \hat{a}_c = A_c(2\pi f_c)^2 \approx 1.4 \) m/s².

As a model for a pulsed signal, a train of Gaussian pulses is used according to

\[ x_p(t) = \sum_{k=-\infty}^{\infty} A_p \exp \left[ -\frac{(t - kT_p)^2}{2t_p^2} \right], \]

where \( A_p \) is the amplitude, \( t_p \) is the pulse width, and \( T_p \) is the time separation between pulses. The pulse width \( t_p \) is selected such that the Fourier transform of \( a_p \) has its maximum at \( f_c \). The value for \( T_p \) is left as a free parameter, but for a signal with a high VSI the pulse width should be much smaller than the pulse separation, i.e., \( t_p \ll T_p \). The value for \( A_p \) is selected such that the root mean square (RMS) values are equal for the two acceleration signals. Setting \( T_p = 0.1 \) s, the signals in Fig. 2 are obtained. In this case, the peak acceleration, \( \hat{a}_p = A_p/t_p^2 \approx 5.8 \) m/s², i.e., about four times the peak value of the continuous signal.

3 VSI and VSL Candidate Definitions

In the following, a number of candidate definitions for how the VSI and VSL could be selected are given. Initially, the focus is on the VSI. All these suggestions are not promising, but the context provided by this description is useful for discussing the relative merits of the different candidate definitions.

3.1 Using Pulse Identification

From Figs. 1 and 2 it is obvious that there are large differences in the visual appearance of the model signals. This would suggest that, e.g., identifying the pulses and using their duty cycle could be a useful approach. However, this parameter, which should be proportional to \( t_p/T_p \), would require unambiguous identification of the pulses and their widths. While this is easy to do for the pulsed model signal, it becomes significantly harder or even impossible if, e.g., there is a considerable amount of noise, if the pulses are close to each other or have multiple peaks, if the pulses have varying widths, and/or the pulse train is not periodic. Thus, it seems difficult to find an algorithm with general validity that identifies the pulses in the time domain.

\(^3\text{Note that a rescaling of the signal amplitude should not affect the VSI and this condition is introduced simply in order to define the amplitude.}\)
Figure 1: The continuous model signal.

Figure 2: The pulsed model signal.
In the frequency domain, the continuous model signal has a very narrow spectrum and it may seem straightforward to define the VSI in terms of the wide spectra that arise for narrow time domain events like shocks. However, spectra for periodic trains of pulses often have a complicated form, giving rise to similar difficulties as using the pulse shapes in the time domain. Furthermore, it should be noted that there are signals that have extremely broad spectra without showing any pulsed behavior in the time domain. One example is WGN, which has a flat spectrum that stretches over all frequencies.

From this, it is expected that while it is possible to describe some signals directly from the shape of the time and/or frequency domain signals, it seems difficult to formulate a general VSI definition in this way.

### 3.2 Using the Excess Kurtosis

One idea is to use a statistical approach to quantify the distributions of samples in the power signals. In order to discuss this, histograms for the two model power signals, \(a_c^2\) and \(a_p^2\), are found in Figs. 3 and 4. Although these may look counterintuitive, the properties of the figures are straightforward to understand from \(a_c(t)\) and \(a_p(t)\).

- In the case of \(a_c\), squaring the harmonic signal doubles the frequency and the oscillation occurs between zero and the maximum value. The most likely signal values are at the extreme points simply due to the fact that the signal is “moving quickly” through all other values. At the turning points, however, the signal has zero slope and this leads to a large number of samples close to these values.

- In the case of \(a_p\), on the other hand, the histogram shows that it is very likely that a randomly selected sample is close to zero and this observation is readily explained from the fact that \(a_p(t)\) has the same fall-off rate as a Gaussian pulse, \(\sim \exp(-t^2)\). Nevertheless, there are some samples with very high amplitude, although they are barely visible.

The discussion about a potential VSI definition can start from the definitions of the standardized moments of a probability distribution.\(^4\)

- The first standardized moment, \(\tilde{\mu}_1\), is the first moment of the probability distribution function (PDF) around the mean, which is identically zero.

- The second standardized moment, \(\tilde{\mu}_2\), is one because it is the second moment about the mean (i.e., the variance) normalized to the variance, \(\sigma^2\).

- The third standardized moment is a measure of skewness and this quantity does not seem to be of use in this case.

- The fourth standardized moment, defined according to \(\tilde{\mu}_4 = \mu_4/\sigma^4\), is promising and is further described below.

The fourth standardized moment is known as the kurtosis and is related to the asymptotic fall-off rate of a PDF. The (univariate) normal distribution has a kurtosis of 3 and it is common to use the excess kurtosis, which is the kurtosis minus 3. A distribution with negative excess kurtosis is called platykurtic and the most extreme example of such a PDF is the Bernoulli distribution with \(p = 1/2\). This is the discrete PDF for a random variable that takes the values 0 and 1 with equal probability and has an excess kurtosis of \(-2\). A continuous uniform distribution,

\(^4https://en.wikipedia.org/wiki/Standardized_moment
where the PDF is constant for a certain interval and zero outside this interval, has an excess kurtosis of $-6/5$. The numerically obtained kurtosis for the continuous signal is $-1.5$, which is not surprising as its PDF can (in a vague sense) be said to be somewhere between the two distributions discussed above.

A distribution with positive excess kurtosis, on the other hand, is called *leptokurtic*. The excess kurtosis is high when there are many samples that occur far away from the mean value, when the distance is measured in standard deviations of the PDF. One example of a leptokurtic distribution is the Laplace distribution, which falls off as $\sim \exp(-|x|)$ as compared to $\sim \exp(-x^2)$ for the normal distribution. The excess kurtosis for the Laplace distribution is 3, but this is very low compared to the value obtained for the pulsed model signal, which is 35.6. The reason for this is that the standard deviation is small due to the accumulation of samples close to zero, but there are some samples with high values and these increase the excess kurtosis significantly.

It is noted that WGN will be classified in a reasonable way as the excess kurtosis of squared WGN is about 12, i.e., in between the values for a continuous and a pulsed signal. Furthermore, if $T_p$ is decreased, then the excess kurtosis becomes comparable to the value for the continuous model signal. For example, if the pulse width is set to $T_p = 0.01\, s$, which makes the continuous and pulsed model signals quite similar in appearance, then the excess kurtosis of $a_p^2$ becomes $-0.23$. If, on the other hand, $T_p$ is made larger than 0.1 s, then also the excess kurtosis is increased.

This suggests that the excess kurtosis could be used for defining the VSI. A drawback of this, however, is that although the excess kurtosis is a well-known statistical concept that is used in many contexts, it may still be unknown to a large fraction of engineers and technicians. It is also fair to say that even with knowledge about the kurtosis, it is still somewhat hard to develop an intuitive understanding for the VSI when defined in this way. Furthermore, it is not obvious how the corresponding VSL could be defined if the VSI is based on kurtosis.

### 3.3 Using Cumulative Energy—“Energy Steps”

Instead trying to build more directly on the argument from Section 2.3, the VSI should be high if few samples in the signal carry a large part of the signal energy. This means that for a signal that contains powerful pulses, the energy signal, defined

$$ W(t) = \int_{-\infty}^{t} P(\tau) d\tau = \int_{-\infty}^{t} a^2(\tau) d\tau, \quad (3) $$

is expected to be shaped approximately like a staircase. To exemplify this, $W(t)$ has been plotted for the continuous and the pulsed model signals in Figs. 5 and 6. It is clear that in the case of the pulsed model signal, $W(t)$ is close to being step-shaped.

To discuss this case in detail, an ideal step-shaped function is defined according to

$$ W(t) = k \Delta W, \quad |t - k \Delta t| < \Delta t/2, \quad \forall k, \quad (4) $$

where $\Delta W$ is the “energy step” and $\Delta t$ is the time duration of each step. For an ideal continuous signal, the energy function would instead be linear, $W(t) \propto t$. The best linear approximation to both types of curves would be

$$ \tilde{W}(t) = \frac{\Delta W}{\Delta t} t, \quad (5) $$

where $\Delta W/\Delta t$ in both cases can be obtained by fitting a straight line. The error can then be defined as the difference between the energy signal and its linear fit according to

$$ \epsilon = W(t) - \tilde{W}(t) = W(t) - \frac{\Delta W}{\Delta t} t \quad (6) $$
Figure 3: The histogram for the continuous power signal.

Figure 4: The histogram for the pulsed power signal. Note the barely visible samples with sample values above 30.
and the mean square error (MSE) can be analytically calculated for the step-shaped function according to

\[ \langle \epsilon^2 \rangle = \frac{\Delta W^2}{12}. \]  

(7)

This implies that it is possible to find the effective energy step and for a signal that is not step-shaped, this quantity is expected to be small.

While this may seem like a promising way to define the VSI, the calculated quantity, \( \Delta W \), is not dimensionless, which was one of the criteria demanded for the VSI above. In this case, \( \Delta W \) will change, e.g., if the acceleration signal is rescaled, which is not surprising as \( \Delta W \) is the pulse energy (in arbitrary units). This was not what was intended to be quantified. A possible measure for how localized the signal is could potentially be found by relating the pulse energy to the signal mean energy, but this must then be interpreted as mean energy per pulse period and this, consequently, raises the question what the pulse period is, which may not have an unambiguous answer and for a non-pulsed signal (such as WGN) the concept pulse period may not even be meaningful. Thus, it is concluded that while the concept of calculating the “energy steps” is interesting, it seems difficult to base the VSI definition on it.

3.4 Using the Power Signal Distribution

Instead trying to apply the argument from Section 2.3 to the power signal histograms discussed in Section 3.2, there should, in a relative sense, be few samples with a large value in the pulsed model signal. On the other hand, as seen in Fig. 4, there should be many samples with an amplitude close to zero. Then, by selecting a threshold and counting the number of samples below, \( n_{\text{low}} \), and above, \( n_{\text{high}} \), this threshold, the VSI could be defined

\[ \text{VSI} \equiv \frac{n_{\text{low}}}{n_{\text{high}}}. \]

(8)

If the threshold is set to 50 % of the maximum \( a^2 \) value, it is obtained that VSI = 1 for a harmonic oscillation and VSI \( \approx 47 \) for the pulsed model signal. Thus, this parameter may seem promising, but one problem is that it is very sensitive to outliers, which can be exemplified by using WGN. It is then found that the VSI is very high (\( \gtrsim 1000 \)), although WGN is not localized in time. Even worse, the VSI calculated for WGN is varying significantly between simulations and the result may be exceptionally large when an unlikely high maximum value happens to have been drawn. As a matter of fact, the normal distribution has a finite probability for any value of the amplitude and this implies that any parameter based on the maximum signal value has undesirable statistical properties. Thus, the requirement for a robust definition of the VSI excludes any use of the maximum of the power signal.

3.5 Using Cumulative Energy from the Power Signal Distribution

To handle the problem with outliers encountered in Section 3.4, the approach can be modified to be based on cumulative energy instead of power samples according to the following.

1. The power signal samples are calculated and sorted.

2. The sorted power signal is integrated into the cumulative energy using a cumulative sum, i.e., a sequence of partial sums.

3. Using a selected threshold, \( W_{\text{th}} \), for the cumulative energy, the number of samples below and above the threshold are counted.
Figure 5: The $W(t)$ function for the continuous model signal.

Figure 6: The $W(t)$ function for the pulsed model signal.
4. The VSI is calculated according to

\[ \text{VSI} = \frac{n_{\text{low}}}{n_{\text{high}}} \]  

(9)

A strict definition of this approach is found in Appendix C.1 and a Matlab implementation is found in Appendix C.2. Figs. 7 and 8 show the cumulative sums using normalized x- and y-axes. It is clearly seen that for the pulsed model signal, few samples contribute a large fraction of the total signal energy. One advantage with this definition of the VSI is that it is in direct correspondence with the criterion described in Section 2.3.\(^5\) Outliers should not affect this quantity as a negligible part of the total energy should be contributed by these.

It remains to select the threshold for the cumulative energy. One obvious suggestion is to set the level at 50 \% of the total energy, \( W_{\text{tot}} \). This should work well but one alternative is to select the threshold such that the VSI for a harmonic oscillation is one. The calculations for this are found in Appendix D.1 and D.2 and the result is that by selecting the threshold value according to

\[ \frac{W_{\text{th}}}{W_{\text{tot}}} = \frac{1}{2} - \frac{1}{\pi} \approx 0.18, \]  

(10)

then VSI = 1 for a harmonic oscillation. For the pulsed model signal, it is obtained that VSI \( \approx 17.7 \) and, crucially, for WGN it is obtained that VSI \( \approx 2.0 \). Thus, compared to a harmonic oscillation WGN gives a somewhat higher but comparable value and the value for the pulsed model signal is much larger.

It should be noted that the cumulative energy plots makes it possible to discuss the signal in other intuitive ways. For example, in Fig. 7 it is seen that the curve passes very close to the point (0.9, 0.8). The interpretation of this is that 10 \% of the signal samples carry 20 \% of the energy. In Fig. 8, on the other hand, the curve reaches the 80 \% energy level approximately at the x-coordinate 0.994, i.e., 20 \% of the signal energy are carried by 0.6 \% of the samples.

Having found a candidate VSI definition, a method for calculating VSL is needed. The intention for this parameter is not to produce the peak values for the shock events in the vibration signal. Such a parameter would be sensitive to outliers and even in the absence of outliers, the value would be unreasonably high if there is a single event in the signal with very high amplitude. Instead, a characteristic value for the shocks is sought and this can be obtained from the VSI procedure described above.

One method is simply to define the VSL as the square root of the sorted power signal at the threshold energy level. Using the same definition of \( W_{\text{th}} \) as above the VSL value for the continuous signal is 1 m/s\(^2\), i.e., the RMS value. The VSL value for the pulsed signal is 2.4 m/s\(^2\). The VSL will for both signals be more comparable to the peak values if the \( W_{\text{th}} \) is selected at the 50 \% level. The ratio of the peak amplitudes is 4.1. For WGN, the VSL is very similar to the RMS value.

3.6 Using a Weighted Mean Square Value

A different approach to using the criterion from Section 2.3 is to ask if the parts of the signal that carry a large part of the signal energy can be identified and isolated in some way. The aim of this section is to use weighted mean squares to perform this. When doing this, it is more natural to first seek a definition of the VSL and then the VSI can be based on the VSL.

\(^5\)The essential difference from the “energy step” method is that the signal is sorted before it is integrated.
Figure 7: The cumulative sum of the sorted power of the continuous model signal.

Figure 8: The cumulative sum of the sorted power of the pulsed model signal.
The averaging method used in ISO 5349 does not yield a result that is representative for the shock parts in a signal with high VSI. A different method is therefore needed but as weighted mean squares can be viewed as a generalization of the method used in the standard, a brief overview of the standardized method will first be given.

### 3.6.1 Quantification Method in ISO 5349

It is here convenient to think about an idealized situation where the acceleration is measured with a perfect instrument without any bandwidth limitation. The quantification of a vibration measurement according to ISO 5349 is then performed in two steps. First, the signal is filtered. Then, the ISO value, $a_{hv}$, is calculated using the RMS values for the three orthogonal axes according to ISO 5349-1 [5, Section 4.5],

$$a_{hv} = \sqrt{a_{hwx}^2 + a_{hwy}^2 + a_{hwz}^2}. \quad (11)$$

The filtering operation can be written

$$\mathcal{F}^{-1}[\mathcal{F}[] H(f)], \quad (12)$$

where the Fourier transformation is denoted by $\mathcal{F}[]$ and $H(f)$ is the transfer function defined in ISO 5349-1 [5, Annex A]. In order to quantify the impact of ultravibrations this filter will need to be modified as the high frequency content otherwise is lost, but the discussion about how to select this is outside the scope of this report.

### 3.6.2 Weighted Vibration Signal Averaging

From the definition of the root mean square concept, the mean square (MS) of a vibration signal can be written

$$\text{MS} = \frac{\int a^2(t) \, dt}{\int dt} = \frac{1}{T} \int a^2(t) \, dt, \quad (13)$$

where the integration is performed over a representative measurement of the signal of duration $T$. If the signal is pulsed, then the RMS value has no direct connection to the signal values occurring within the pulses. In fact, by moving the pulses further and further apart, the RMS value would approach zero. The question is then how a value that represents the pulses can be calculated. One approach is to modify the RMS value into a weighted value, where the weighting puts special emphasis on the pulses. In general, a weighted time average of a signal $u(t)$ is calculated according to

$$\frac{\int u(t)w(t) \, dt}{\int w(t) \, dt}, \quad (14)$$

where $w(t)$ is a suitably chosen weight function. It is seen that the mean square calculation of ISO 5349 uses $w(t) = 1$ and $u(t) = a^2(t)$, which corresponds to weighting all parts of the signal equally. The weighted mean square (WMS) concept is introduced in a general way in Appendix E, where also a number of examples of well-known applications are given.

In order to quantify the shocks, the power signal should be averaged, i.e., $u(t) = P(t) = a^2(t)$. Emphasis should be put on the parts of the signal where the power is high and this suggests that the weight function should be a monotonous function that is increasing with the power level. There is an infinite number of such functions and it is not obvious how to make a selection. The
guiding principle here is that the weighting function should be as simple as possible but still produce reasonable numerical values for the pulse amplitude.

The simplest possible function is to select $w(t)$ to a constant value, but in order to average the high-power parts of the signal, a piecewise constant function is needed. By selecting $w(t)$ to be one for the high-power parts and zero elsewhere, the weighting function becomes a windowing function that removes the low-power parts completely. One obvious problem with this is how the threshold value for the power signal should be selected. In fact, if such a value can be selected, the definition of the VSL could very well be based on it, leading to a circular argument.

Using monomials is another very simple choice of weighting functions, giving

$$w(t) = P^K(t) = a^{2K},$$

where $K$ is not necessarily an integer. The MS definition in the standard corresponds to selecting $K = 0$, but selecting a larger value will make the weight function narrower, thereby selecting a smaller, more intense part of the shock. Increasing $K$ to a large value would lead to extreme weighting that will, in effect, result in the maximum of the amplitude. As this is undesirable, it seems reasonable to limit $K$ to a small value. For example, selecting $K = 2$, the WMS becomes

$$WMS = \frac{\int a^2(t)P^2(t) \, dt}{\int P^2(t) \, dt} = \frac{\int a^6(t) \, dt}{\int a^4(t) \, dt}$$

and the VSL is then defined as a root weighted mean square according to

$$VSL = RWMS = \sqrt{WMS}.$$  

(17)

A straightforward way to define the VSI is then

$$VSI = RWMS/RMS,$$

(18)

where the RMS value is obtained by using $w(t) = 1$.

Selecting $K = 2$, the VSI $\approx 1.29$ for a harmonic oscillation. For the pulsed model signal, it is obtained that VSI $\approx 5.1$ and for WGN it is obtained that VSI $\approx 2.2$. Compared to the cumulative energy method, the separation of the VSI values is smaller, but in a relative sense, both methods give fully reasonable results. The VSL value for the continuous signal is 1.3 m/s$^2$. The VSL value for the pulsed signal is 5.1 m/s$^2$, i.e., 3.9 times higher. The ratio of the peak amplitudes is 4.1. For WGN, the VSL is 2.2 m/s$^2$. Also these values follow what is intuitively expected.

4 Conclusion

The aim of this document has been to suggest a definition for the vibration shock index and the vibration shock level parameters. These quantify to what extent a given vibration signal is to be considered to contain shocks/impacts/transients and what a characteristic vibration level of these events within the signal is, respectively. Among the investigated candidate definitions, the ones based on the excess kurtosis, the cumulative energy from the power signal distribution, and the weighted mean squares, seem to fulfill the criteria stated in Section 2.1. The candidate definition based on excess kurtosis seems less intuitive than the alternatives and also seems to make a definition of the VSL difficult. The candidate definitions based on cumulative energy and weighted mean squares are both well worth evaluating on experimental signals.
A Dissipated Energy from a Harmonic Oscillator

The model equation for a driven harmonic oscillator is

$$m\dddot{x} + c\dot{x} + kx = F(t), \quad (19)$$

where $m$ is the mass, $c$ is the damping, $k$ is the spring constant, and $F$ is the drive force. The instantaneous power delivered by the external force is $Fv$, i.e.,

$$m\dddot{x} + c\dot{x}^2 + k\dot{x}x = F(t)\dot{x} = P(t), \quad (20)$$

which is rewritten to

$$m\frac{d\dot{x}^2}{dt} + c\dot{x}^2 + k\frac{dx^2}{dt} = P(t). \quad (21)$$

Integrating $\int_{t'}^{t} dt$ gives

$$\frac{m}{2}\ddot{x}^2\big|_{t'}^{t} + c\int_{t'}^{t} \dot{x}^2 dt + \frac{k}{2}\ddot{x}^2\big|_{t'}^{t} = \int_{t'}^{t} P(t') dt' = W_{\text{tot}}(t) \quad (22)$$

and assuming that $x \to 0$ and $\dot{x} \to 0$ as $t \to -\infty$, it is obtained that

$$W_k(t) + c\int_{-\infty}^{t} \dot{x}^2 dt + W_p(t) = W_{\text{tot}}(t). \quad (23)$$

Systems of the kind considered here operate a long time and the kinetic energy, $W_k$, and the potential energy, $W_p$, are bounded and oscillatory. This implies that after the initial transient, all energy delivered by the driving force will be dissipated by the damping term. It is true that the energy dissipation mainly occurs in the material being machined and in the vibrating equipment itself, but it is reasonable to assume that the energy dissipated in the operator is also proportional to the squared velocity, i.e.,

$$P_{\text{hand}} \propto \dot{x}^2 \quad (24)$$

with the corresponding dissipated energy

$$W_{\text{hand}} \propto \int \dot{x}^2 dt. \quad (25)$$

References

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B  Model Signal Calculations

The continuous signal

\[ x_c(t) = A_c \cos(2\pi f_c t + \phi_c), \]
\[ v_c(t) = -A_c(2\pi f_c) \sin(2\pi f_c t + \phi_c), \]
\[ a_c(t) = -A_c(2\pi f_c)^2 \cos(2\pi f_c t + \phi_c). \] (26)

The pulsed signal

\[ x_p(t) = \sum_{k=-\infty}^{\infty} A_p \exp \left[ -\frac{(t-kT_p)^2}{2t_p^2} \right], \]
\[ v_p(t) = -\sum_{k=-\infty}^{\infty} A_p \frac{(t-kT_p)}{t_p^2} \exp \left[ -\frac{(t-kT_p)^2}{2t_p^2} \right], \]
\[ a_p(t) = \sum_{k=-\infty}^{\infty} A_p \frac{(t-kT_p+t_p)(t-kT_p-t_p)}{t_p^4} \exp \left[ -\frac{(t-kT_p)^2}{2t_p^2} \right]. \] (27)

The pulse width \( t_p \) is selected such that the Fourier transform of \( a_p \) has its maximum at \( f_c \), which gives

\[ t_p = \frac{1}{\sqrt{2\pi f_c}}. \] (28)

The RMS value for the continuous signal is

\[ a_{c,RMS}^2 = \frac{1}{T_c} \int_{-T_c/2}^{T_c/2} a_c^2(t) \, dt = 8\pi^4 f_c^4 A_c^2. \] (29)

Assuming that \( t_p \ll T_p \), the corresponding expression for the pulsed signal is

\[ a_{p,RMS}^2 = \frac{1}{T_p} \int_{-\infty}^{\infty} a_p^2(t) \, dt = \frac{3\sqrt{\pi} A_p^2}{4T_p t_p^3}. \] (30)

Setting these equal, it is found that

\[ A_p = 4 \sqrt{\frac{f_c T_p}{3}} \sqrt{\frac{\pi}{2}} A_c. \] (31)

C  Algorithm based on Cumulative Energy

C.1 Algorithm Definition

The purpose of this section is to give an unambiguous definition of the algorithm introduced in Section 3.5. Assume that the measurement of the acceleration has resulted in the samples \( a_n, n = 1, 2, \ldots, N \). The power signal samples are then \( P_n = a_n^2 \) and the sorting results in a reordered set of power signal samples denoted \( \tilde{P}_n, n = 1, 2, \ldots, N \). The cumulative energy is calculated in terms of the \( N \) partial sums

\[ W_M \equiv \Delta t \sum_{n=1}^{M} \tilde{P}_n, \quad M = 1, 2, \ldots, N, \] (32)
where $\Delta t$ is the sampling interval. The largest value for $M$ for which $W_M < W_{th}$ is found and the VSI is calculated according to

$$VSI = \frac{M}{N - M}. \quad (33)$$

This can be expressed simpler using the total energy, which is the $N$th partial sum

$$W_{tot} = \Delta t \sum_{n=1}^{N} \tilde{P}_n. \quad (34)$$

Introducing normalized partial sums

$$\tilde{W}_M \equiv \frac{W_M}{W_{tot}} = \frac{\sum_{n=1}^{M} \tilde{P}_n}{\sum_{n=1}^{N} \tilde{P}_n}, \quad M = 1, 2, \ldots, N, \quad (35)$$

the largest value for $M$ for which $\tilde{W}_M < W_{th}/W_{tot}$ should instead be found. It should be noted that $\tilde{W}_M$ is the normalized cumulative energy plotted in Figs. 7 and 8 as a function of the normalized sample index vector, i.e., the vector $1/N, 2/N, \ldots, 1$.

### C.2 Matlab Implementation

```matlab
function [vsi, vsl, n_norm, W_norm] = algorithmCumulEnergy(a)
    n = 1:length(a); % Sample index vector
    n_norm = n/length(n);
    P = a.^2; % Power signal
    P_sort = sort(P);
    W = cumsum(P_sort); % Cumulative energy
    W_norm = W/W(end); % W_th = 0.5;
    W_th = 1/2 - 1/pi;
    n_low = sum(W_norm < W_th); % Number of samples below threshold
    n_high = sum(W_norm > W_th); % Number of samples above threshold
    vsi = n_low/n_high;
    idx = find(W_norm > W_th); % Samples above threshold
    vsl = sqrt(P_sort(idx(1)));
end
```

### D Harmonic Signal

#### D.1 Sample Distribution

If the signal

$$y = \sin t \quad (36)$$

is sampled at a random time, the sample will be drawn from a certain distribution. To find this distribution, the function is first restricted to the interval $t \in [-\pi/2, \pi/2]$, where the function is
monotonous. The sample distribution, \( f(y) \), by definition fulfills

\[
\Pr[y_1 \leq y \leq y_2] = \int_{y_1}^{y_2} f(y) \, dy.
\]

(37)

The sampling time is assumed to be drawn from a uniform random distribution, implying that the probability that the sample is drawn in a small interval \( dt \) is \( dt/\pi \). The \( y \)-value will then be in the interval \([y, y + dy]\), for which the probability by definition is \( f(y) \, dy \). Setting these two probabilities equal gives

\[
f(y) = \frac{1}{\pi} \frac{dt}{dy} = \frac{1}{\pi} \frac{d}{dy} \arcsin(y) = \frac{1}{\pi \sqrt{1 - y^2}}.
\]

(38)

D.2 Cumulative Energy

For a power signal that is a harmonic oscillation, it is possible to calculate the cumulative energy analytically. First, it is noted that the energy signal, which is the square of the power signal is also harmonic, although it is oscillating from zero to a maximum value. Thus, the PDF for the power signal has the shape derived in Appendix D.1, although the limits for the distribution should be shifted from \([-1, 1]\) to \([0, P_{\text{max}}]\). Instead of shifting the distribution, the power corresponding to the original PDF is introduced as \((P + 1)\), i.e., a linearly increasing function starting at zero. The energy is then obtained by integration according to

\[
W = \int (P + 1) f(P) \, dP
\]

\[
= \int (P + 1) \frac{1}{\pi} \frac{1}{\sqrt{1 - P^2}} \, dP
\]

\[
= \frac{1}{\pi} \left( \int \frac{1}{\sqrt{1 - P^2}} \, dP + \int \frac{P}{\sqrt{1 - P^2}} \, dP \right)
\]

\[
= \frac{1}{\pi} \left( \arcsin(P) - \sqrt{1 - P^2} \right).
\]

(39)

Integrating over the entire interval, it is obtained that

\[
W_{\text{tot}} = \int_{-1}^{1} (P + 1) f(P) \, dP = 1.
\]

(40)

Instead, integrating over half of the samples, the result is

\[
W = \int_{-1}^{0} (P + 1) f(P) \, dP
\]

\[
= \frac{1}{\pi} \left[ (\arcsin(0) - \sqrt{1 - 0}) - (\arcsin(-1) - \sqrt{1 - 1}) \right]
\]

\[
= \frac{1}{\pi} \left( \frac{\pi}{2} - 1 \right) = \frac{1}{2} - \frac{1}{\pi} \approx 0.18.
\]

(41)

This means that if the signal is harmonic and the threshold for the cumulative energy is set to \( W_{\text{th}}/W_{\text{tot}} = 1/2 - 1/\pi \), then there will be an equal number of samples above and below the threshold.
## E Weighted Averaging

The aim of this section is to give an introduction to weighted averaging and provide an intuitive explanation of the expression

$$\frac{\int u(t)w(t)\, dt}{\int w(t)\, dt},$$  \hspace{1cm} (42)

which was used as a starting point for the discussion about the WMS value above.

The arithmetic mean value of a set of samples $x_i$, $i = 1, 2, \ldots, N$ is defined

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i = \frac{\sum_{i=1}^{N} x_i \times 1}{\sum_{i=1}^{N} 1},$$  \hspace{1cm} (43)

where the second expression is written on a form that is more similar to the weighted expression in (42). In this case, it is $x_i$ that is being averaged and all samples are weighted equally. If the values among the samples $x_i$ occur $p_i$ times (instead of a single time), then the summation can be performed over the (different) values instead of over the samples. One example is a die which can land on only one of the six sides. The averaging operation is then modified to a weighted form according to

$$\bar{x} = \frac{\sum_{j=1}^{M} x_j p_j}{\sum_{j=1}^{M} p_j},$$  \hspace{1cm} (44)

where $M$ is the number of different values. This is a discrete version of the WMS expression in (42). Continuous WMS expressions, which are exactly on the form of (42) are also common in statistics and physics. One example is the expectation of a random variable $X$, which is defined

$$E[X] = \frac{\int_{-\infty}^{\infty} x f(x)\, dx}{\int_{-\infty}^{\infty} f(x)\, dx},$$  \hspace{1cm} (45)

where $f(x)$ is the probability density function (PDF) for $X$. Usually, the denominator is not included in the definition, but this is because the PDF is assumed to be normalized, i.e., the denominator is one. A very similar expression is the definition for the center of mass, which is

$$R = \frac{\iiint r \rho\, dV}{\iiint \rho\, dV} = \frac{1}{M} \iiint r \rho\, dV,$$  \hspace{1cm} (46)

where $\rho$ is the density and the integration is over a continuous body. An expression exactly on the form of (42) is obtained in the one-dimensional version

$$R_x = \frac{\int x \rho_x\, dx}{\int \rho_x\, dx},$$  \hspace{1cm} (47)

where $\rho_x$ is the mass per length unit, not per volume unit. The interpretation of this is that the center of mass is a weighted average where the density acts as weighting function.

Similar to the center of mass definition, it is suggested in this report to use a WMS value, where the weighting function emphasizes the parts with high power level, to quantify the vibration level within the shocks. However, to further draw parallels to the definition of the WMS
expression above, it is actually possible to also view the traditional mean square value (used to calculate the RMS value) as a WMS value. To see this, the mean square value of $a^2$ is written

$$\text{MS} = \frac{1}{T} \int_0^T a^2(t) \, dt = \frac{\int_0^T a^2(t) \, dt}{\int_0^T dt} = \frac{\int_{-\infty}^{\infty} a^2(t) w(t) \, dt}{\int_{-\infty}^{\infty} w(t) \, dt}. \quad (48)$$

In order to obtain equality in the last step, the weighting function is chosen to be one in the interval $t \in [0, T]$ and zero otherwise. In this case, it would be better to call $w(t)$ a windowing function, but from the expression it is clear that it is a WMS on the form above.

Many more examples of WMS expressions are found in the scientific literature, but those given here should be a sufficient introduction to the calculation of the shock amplitude.

F  Algorithm based on Weighted Mean Squares

F.1 Algorithm Definition

The purpose of this section is to give an unambiguous definition of the algorithm introduced in Section 3.6. Assume that the measurement of the acceleration has resulted in the samples $a_n$, $n = 1, 2, \ldots, N$. The power signal samples are then $P_n = a_n^2$ and and the WMS is defined

$$\text{WMS}(K) = \sum_{n=1}^{N} P_n P_n^K / \sum_{n=1}^{N} P_n^K, \quad (49)$$

where $K$ is the exponent in the weighting monomial function. The MS value is obtained from

$$\text{MS} = \text{WMS}(0) \quad (50)$$

and the VSL is defined

$$\text{VSL} = \text{RWMS} = \sqrt{\text{WMS}}. \quad (51)$$

The VSI is defined

$$\text{VSI} = \frac{\text{RWMS}}{\text{RMS}} = \sqrt{\frac{\text{WMS}}{\text{MS}}}. \quad (52)$$

F.2 Matlab Implementation

```matlab
function [vsi, vsl] = algorithmWeightedMeanSquare(a)
    P = a.^2;  \% Power signal
    K = 2;  \% Monomial exponent for RWMS
    RWMS = sqrt(sum(P .* P.^K)/sum(P.^K));
    K = 0;  \% Monomial exponent for RMS
    RMS = sqrt(sum(P .* P.^K)/sum(P.^K));
    vsl = RWMS;
    vsi = RWMS/RMS;
end
```