Abstract - This study proposed the mathematical model to obtain the appropriate storage methods for raw material at a case study company. Three classes of raw material as A, B, and C were stored using three types of storage equipment: pallet, shelf, and roller rack. The optimal solution showed that 94.60% of the storage area was used for current volume of raw material. After that, the amount of raw material was increased by 20% due to the expected customer demand. The optimal solution cannot be obtained because the storage area was not sufficient. The optimal solution indicated that the current storage area can handle up to 13.6% increment. Then, the sensitivity analyses were carried out. The results from the tests showed that to handle 20% increment as forecast, an additional storage area of increment 128.55 m² should be added to the current.

Keywords - Mathematical Model, Warehousing, Case Study

I. INTRODUCTION

Nowadays, we are in the fourth industrial revolution (Industry 4.0). Instead of a traditional warehousing, smart warehouses are a warehousing concept of Industry 4.0 at which a variety of technologies, i.e. Radio-Frequency IDentification (RFID), NarrowBand IoT (NB-IoT), robotics, etc., should be introduced [1]. The initial step of smart warehousing is to prepare the current warehousing to be systematic and optimized.

Warehousing is one of the activities in logistics and supply chain management that has an effect on a company’s total cost. Warehouse is a place that inventories, i.e. finished products, raw material, supplies, are stocked. To manage any warehouse, the area utilization is one of the main objectives that many companies are considered.

Warehouse design was addressed in [2] that the key of warehouse design is to maintain a high percentage of the usable of storing product space. Not only storage space, but aisles are needed for operators to travel, pick, or place the products, as well. More aisles deduce a usable storing space in warehouse. To balance between the number of aisles and the usable storage spaces should be concerned during the process of warehouse design.

After layout of storage area is completed, storage assignment is the following task. Addressed in [3], to assign products in the storage locations which the aim was to satisfy the requirement of the measuring performance, i.e. total transportation distance, total transfer time, storage utilization, etc. There are many policies used in storage assignment. The class-based storage policy is mentioned in [4, 5, 6, and 7]. This policy distributes products based on their classes and reserved an area within the storage area for each class. This method is widely used in many research works because it is easy to understand.

To assign the storing product, there are two groups of general techniques. The first technique is exact algorithm that works on mathematical model formulation. The aim of this technique is to find the optimal solution as in [3, 7, and 8]. The other groups is heuristics algorithm, especially meta-heuristics that widely applied in many research works. For example, the particle swarm optimization (PSO) applied in [2] and the differential evolution (DE) applied in [9].

Currently, the company’s studying case found that the sale trends are increasing. Thus, the company needs to increase the production volume that has an effect on increasing the volume of raw material stock keeping. With the limited warehouse area, the company should plan to optimize the use of the storage area.

This study aims to propose the mathematical model for minimizing the use of the storage area in the company’s case study. The results from this model are the appropriate method for storing each raw material item that helps in minimizing the use of the current area. In addition, the sensitivity analyses are conducted to find the maximum stock that can be handled by the current area and to obtain the solution when the volume of raw material stock is increased to 20% as the company’s forecasting.

II. CASE STUDY

The company case study owns an internal warehouse for storing 31 groups of raw material including 205 items. There are 4 basic operations here including material receiving, incoming inspection, stocking in, and stocking out. Raw materials are classified as 3 classes as Class-A, Class-B, and Class-C based on main products of the company (the example list of raw material item was presented as Table I). The storage area and operation flows were presented as in Fig 1.
Currently, there are three types of storing equipment as pallets, shelf, and roller rack. Different storing equipment occupies the different area (as Fig.2).

Fig. 1. Storage area and operations flow

Fig. 2. Storing Equipment.

**III. PROPOSED METHOD AND RESULTS**

**A. Redesign Storage Area**

The first task of this research was to redesign the storage area. Current area as 1,600 m², 20% of this area was assigned to aisle and the remaining 80% was assigned to storage area. Within storage area, the area was allocated based on the proportion of each raw material class as 21%, 52%, and 27% for A, B, and C, respectively. Thus, 195, 490, and 255 m² were assigned to A, B, and C, respectively.

The equipment was assigned to be used in each area of class by maximizing the total area used. The mathematical model was developed as follows.

**First Model:**

Indices: \( i \) = Storing equipment (\( i = 1, 2, 3 \ldots m \))

Parameter:
- \( P_i \) = Area used per unit of equipment \( i \).
- \( W_i \) = Area used including aisle per unit of equipment \( i \).
- \( K \) = Available Area
- \( U_i \) = Maximum unit of equipment \( i \) that can be used at the area
- \( L_i \) = Minimum number of equipment \( i \) that should be used.

Decision Variable:
- \( X_i \) = Assigned number of equipment \( i \) used

Objective Function:

\[
\text{Max } Z (\text{Total Area}) = \sum_{i=1}^{m} P_i X_i \quad (1.1)
\]

\[
\sum_{i=1}^{m} W_i X_i \leq K \quad (1.2)
\]

\[
X_i \leq U_i \quad \forall i \quad (1.3)
\]

\[
X_i \geq L_i \quad \forall i \quad (1.4)
\]

\( X_i \) are non-negativity \( (1.5) \)

The mathematical model was developed to maximize the area occupied by all of the storing equipment as equation (1.1). Index \( i \) represents equipment type. The data parameters are the area used, and the total area used including aisle per unit of equipment used; available area; maximum unit of equipment that can be used; and minimum unit of each storing equipment type that should be used. The decision variables were the number of equipment that should be used. Equation (1.2) was to limit the area used. Equation (1.3) was to assign the number of equipment used to be less than or equal to the maximum number of equipment that can be used. Equation (1.4) was to use the number of equipment to be greater than or equal to the lower limit. Equation (1.5) was the non-negativity constraint.

The optimal results obtained by LINGO showed that 162.79, 536.39, and 209.30 m² were the occupied area for setting storage equipment at each class area. Then, the results were validated by drawing as the layout in Fig.3.

Fig. 3. Warehouse area allocation.

Fig. 3 showed the area used when the storage equipment and aisles were located. The area occupied for each class were 167.40, 537.66, and 210.65 m² for A, B
and C, respectively. Table II presented the number of storing equipment used and the capacity of each class area for storing raw material.

| Area of Class | Pallet | Shelf | Roller | Storage Area (m²) |
|---------------|--------|-------|--------|-------------------|
| A             | 8      | 18    | 18     | 198.32            |
| B             | 9      | 24    | 124    | 662.41            |
| C             | 20     | 24    | 20     | 259.72            |
| Total Capacity|        |       |        | 1,120.45          |

### B. Assigning Raw Material

The mathematical model was developed to assign storing methods for each raw material item leading to minimizing the use of storage area. The assumptions of this problem were addressed as follows.

- Considering the storage area was limited.
- Storing policy is a free location system with commodity system.
- All parameters are deterministic.
- Each raw material item can be assigned to only one type of equipment.

Second Model:

Indices: $i = \text{Raw material item (} i = 1, 2, 3 \ldots n\text{)}$

$j = \text{Storing equipment (} j = 1, 2, 3 \ldots m\text{)}$

Parameter:

$D_i = \text{Stocking volume of item } i$

$A_j = \text{Unit of area occupied when } j \text{ is adopted per unit of } j$

$Q_{ij} = \text{Unit of item } i \text{ when storing by } j \text{ per unit of } j$

$L_j = \text{Area available for each storing equipment type } j$

Decision Variable:

$$X_{ij} = \begin{cases} 1 & \text{when } i \text{ was assigned to store by } j \text{ or } \\ 0 & \text{otherwise.} \end{cases}$$

Objective Function:

$$\text{Min } Z \text{(Total Area)} = \sum_{j=1}^{m} \sum_{i=1}^{n} \frac{D_i A_j}{Q_{ij}} x_{ij}$$  \hspace{1cm} (2.1)

s.t.

$$\sum_{i=1}^{n} \frac{D_i A_j}{Q_{ij}} x_{ij} \leq L_j \quad \forall j; j = 1, \ldots, m$$  \hspace{1cm} (2.2)

$$\sum_{j=1}^{m} X_{ij} = 1 \quad \forall i; i = 1, \ldots, n$$  \hspace{1cm} (2.3)

$$X_{ij} \text{ are binary}$$  \hspace{1cm} (2.4)

The mathematical model was developed here. There were two indices as $i$ and $j$ representing raw material item and equipment type. The data parameters were stocking volume of each raw material; used area per unit of each equipment type; unit of each item that can be handled by each type of equipment; and area available for each storing equipment type. The decision variables were binary which means that a raw material item was assigned or not assigned to be stored by this equipment. The objective function was to minimize the storing area as equation (2.1) referred to summation of area used for all items using all storing equipment as objective value or $Z$. Equation (2.2) and (2.3) were constraints of this problem. Equation (2.2) was to assign the area used by each equipment type that should be less than or equal to limited area of each equipment type. Equation (2.3) was to assign only one type of storing equipment to each raw material item. Equation (2.4) was the binary constraint.

The optimal solutions from LINGO were presented as Table III.

| No. of Item | Raw Mat. Class | Total |
|-------------|----------------|-------|
|             | A              | 17    | 163  | 25   | 205  |
| Storage     | Method         | Pallet| Shelf| Roller Rack |
|             |                | 8     | 9    | 20 | 37   |
| Used Storage Area (m²) |          | 150.56 | 651.61 | 257.72 | 1059.89 |

From Table III, the optimal solution showed that 1059.89 m² was occupied when the area capacity was 1,120.45 m² or approximately 94.60% utilization.

### IV. SENSITIVITY ANALYSIS

The company forecasted that the demand of products is most likely increase 20%. This increasing has an effect on the increasing for class B and C as 37% and 15%, respectively. Applied the first model form part A, the optimal number of equipment used and the occupied areas were presented as Table IV.

| Area of Class | Pallet | Shelf | Roller | Area Size (m²) | Storage Area (m²) |
|---------------|--------|-------|--------|---------------|-------------------|
|               | 8      | 18    | 18     | 167.40        | 198.32            |
| B             | 9      | 108   | 13     | 577.80        | 762.73            |
| C             | 19     | 28    | 28     | 244.44        | 305.54            |
| Total         | 27     | 154   | 59     | 989.64        | 1266.59           |

*No change

Then, the second model from part B was applied to find the optimal solution for this case. Unfortunately, in B area, the solution was infeasible solution. It indicated that there was not enough area for raw material class B. On the other hand, the optimal solution of C was obtained as 305.54 m² that referred to maximum utilization of this area.

The results for the current situation showed that 162.79, 536.39, and 209.30 m² were area occupied for setting storage equipment at each class area. The total area used by the proposed method was 908.48 m². Comparing with the current area occupied as 1036 m², the proposed method gave 12.31% improvement.
solutions showed that the new layout owns 1,120.45 m² for the raw material storage area, while 1059.89 m² currently used as approximately 94.60% utilization.

As mentioned earlier, 94.60% utilization indicated that storage area was almost fully used. Thus, the company foresees that the volume of raw material is almost likely increase about 20% and the increasing have effects on class B and C. The sensitivity analysis showed that the occupied area for C can cover the increasing volume of C. In contrast, it is not possible to handle the increasing of B. Then, the sensitivity analyses were conducted as two aspects.

A. To Identify the Capability of Storage Area of A and C

At this step, the capability of assigned area for A and C classes were identified. The tests were conducted by increasing the volume of A and C from 15% to 35%. The results were presented at Table V.

| Class | Area Used when % Increasing in Volume (m²) |
|-------|------------------------------------------|
| A     | 172.7 176.9 189.7 196.2 N/A             |
| C     | 305.54 N/A                               |

The results from Table V showed that the A’s storage area can cover the increasing volume until 30%, but the C’s area can cover until 15% increasing.

B. To Identify the Capability of Storage Area of B

For raw material B, the previous test revealed that the assigned area cannot handle the 37% increment of B which implied to 20% increasing of the total raw material volume. Thus, at this step, the volume of B was reduced from 37% by 10%, 15%, 20%, and 25% until the optimal solution can be obtained. The results were presented as Table VI.

| Area Used when % Decreasing in Volume (m²) |
|------------------------------------------|
| B                                      |
| N/A                                    |
| 747.61                                 |

From Table VI, the results showed that the area of class B can cover the increasing volume until 12%.

Thus, to provide the solution for 20% increment, the sensitivity analysis of the B’s area was conducted. At this step, the area of B was increased from 10% until the optimal solution were found. The results were presented as Table VII.

As mentioned previously, there was 577.80 m² allocated for B. When, the volume of raw material increased, 128.55 m² was required to handle this increasing.

IV. CONCLUSION

This study presented the application of mathematical models in warehouse management. The first model was applied to find the optimal number of storage equipment used that maximizes the use of storage area. Then, the second model was used to assign storage equipment to each raw material item for minimizing the storage area. Considering the company’s case study, there are three classes of raw material as A, B and C. The storage area was allocated as three zones for each class. At the current situation, the optimal solution showed that the use of storage area was 94.60% utilization. Then, the sensitivity analysis was conducted due to 20% increasing of raw material stock. The optimal solution showed that the current area can handle up to 13.6% increment. To handle 20% increment, the additional area should be provided at B-area as 128.55 m².

In addition, sensitivity analysis showed that 30%, 15%, and 12% increment of volume of A, B, and C can be covered by the current area.

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