Testing general relativity with cosmological large scale structure

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Abstract
In this paper I investigate the possibility to test Einstein’s equations with observations of cosmological large scale structure. I first show that we have not tested the equations in observations concerning only the homogeneous and isotropic Universe. I then show with several examples how we can do better when considering the fluctuations of both, the energy momentum tensor and the metric. This is illustrated with galaxy number counts, intensity mapping and cosmic shear, three examples that are by no means exhaustive.

Keywords Cosmology · General Relativity · Large Scale Structure · Cosmological Surveys

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This article belongs to a Topical Collection: In Memory of Professor T Padmanabhan.

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1 Introduction

This contribution is written to honour my dear colleague Thanu Padmanabhan (Paddy) who passed away on September 17, 2021. I have great admiration for Paddy, especially for his crystal clear arguments which are found in all his papers and which are especially present in his books. His three volumes on Astrophysics [1–3] and his book on General Relativity [4] are master pieces of textbooks in our field.

Paddy’s excellence in teaching is a consequence of his deep understanding of physics. This is also reflected in his choice of research topics which are in most cases an attempt to address a really deep open question in theoretical physics, like e.g. the riddle of dark energy.

In this article written for him I also would like to discuss an important question, namely:

Can we test the non-vacuum sector of General Relativity?

The vacuum equations,

\[ R_{\mu\nu} = 0, \]  

have been tested in many ways: First via light deflection which made Einstein famous, but also via the perihelion advance of Mercury. Both these observations test the Schwarzschild exterior solution of stars. Later, especially binary pulsars allowed very detailed tests of the most important vacuum solutions, the Schwarzschild and Kerr spacetimes. We have also measured the Lense-Thirring effect and, finally gravitational waves from binary black holes and neutron stars have been discovered. Even though the interior of a neutron star is not a vacuum solution, this has so far not been tested with significant precision. The same is true for the interior of other stellar objects where relativistic effects are less relevant than in neutron stars.

But can we also test the equations in the presence of matter?

\[ G_{\mu\nu} = 8\pi G T_{\mu\nu}. \]  

Here we include a possible cosmological constant \( \Lambda \) as vacuum energy density given by \( \rho_v = \Lambda/(8\pi G) = -P_v \) in the energy momentum tensor. Since there is no experiment that can distinguish between vacuum energy and a cosmological constant, we should not do so also in our theories. Therefore we use the terms ‘cosmological constant’ and ‘vacuum energy’ as synonyms. To test Einstein’s equations we must measure both, the geometry of spacetime that determines the Einstein tensor \( G_{\mu\nu} \) and the energy momentum distribution in the Universe that determines \( T_{\mu\nu} \). In this article, I shall argue that we can do this in the near future in cosmology. The cosmological metric is probably the most relevant and best measured non-vacuum solution of General Relativity (GR).

In the present paper I shall not develop and discuss any new tests, but rather outline/sketch and summarize some of the main ideas how to test GR with present and especially near future observations of cosmological large scale structure. These are part of a program that is being carried out by the Geneva cosmology group and other
groups working on large scale structure observations and that will bear its fruits in the coming decade.

The remainder of this article is structured as follows. In the next section I discuss homogeneous cosmology and argue that we have not tested Einstein’s equations in observations of the background metric. In Sect. 3 we discuss the observations of cosmological large scale structure and I propose several ways to test Einstein’s equations with these observations. Is Sect. 4 I summarize and conclude.

2 Homogeneous cosmology

The most relevant known non-vacuum solution which is being measured with more and more accurate observations is the cosmological solution. At very large scales, we suppose that the geometry of the Universe is well approximated as spatially homogeneous and isotropic, a so called Friedmann–Lemaître (FL) Universe [5–7],

\[ ds^2 = a^2(t)\left(-dt^2 + \gamma_{ij}dx^i dx^j\right). \]  

Here \( t \) is conformal time, \( \gamma \) is the metric of a 3-space of constant curvature \( K \) and \( a(t) \) is the scale factor. For such a highly symmetric spacetime also the energy momentum tensor must obey the same symmetries and is given by an energy density \( \rho(t) = -T^0_0 \) and a pressure \( P(t) = T^i_i \) (no sum). Einstein’s equations become the so called Friedmann equations, where we separate the matter and radiation energy and pressure and the cosmological constant for convenience,

\[ H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = -\frac{K}{a^2} + \frac{8\pi G}{3} \rho + \frac{\Lambda}{3}, \]  

\[ \frac{1}{a^2} \frac{d}{dt} \left(\frac{\dot{a}}{a}\right) = -\frac{4\pi G}{3} (\rho + 3P) + \frac{\Lambda}{3}. \]  

Have we tested these equations in cosmology?

We have measured the luminosity distance out to many Cepheids and especially to supernovae of type Ia up to redshift \( z \sim 2.2 \), see [8] for a recent analysis. In a FL Universe, the luminosity distance out to redshift \( z \) is given by

\[ d_L(z) = (1 + z)\chi_K \left(\int_0^z \frac{dz}{H(z)}\right), \]  

where

\[ \chi_K(r) = \begin{cases} 
  r & \text{if } K = 0 \\
  \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}r) & \text{if } K > 0 \\
  \frac{1}{\sqrt{-K}} \sinh(\sqrt{-K}r) & \text{if } K < 0.
\end{cases} \]  

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Normalizing the scale factor to 1 today, we have \( z + 1 = 1/a \) and measuring \( d_L(z) \) precisely for many redshifts \( z \), allows us in principle to determine \( K \) and \( H(z) \). In order to test Eq. (4), we need an independent measurement of \( \rho \) and \( \Lambda \). But there is the crux of observational cosmology: If we simply count galaxies and clusters in a large volume and assign them a mass which we roughly infer from their content of stars and gas, we obtain a matter density \( \rho \) which is much too small to fit the observed distance \( d_L(z) \). Furthermore, we have absolutely no handle on the cosmological constant. Observers actually proceed in the opposite way. They assume that GR is valid and set

\[
H(z) = H_0 \sqrt{\Omega_K (1 + z)^2 + \Omega_m (1 + z)^3 + \Omega_\Lambda}
\]

where

\[
\Omega_K = -\frac{K}{H_0^2}, \quad \Omega_m = \frac{8\pi G \rho_0}{3H_0^2}, \quad \Omega_\Lambda = \frac{\Lambda}{3H_0^2},
\]

where \( \rho_0 \) is the present matter density. The first Friedmann equation requires \( \Omega_K + \Omega_m + \Omega_\Lambda = 1 \). Observers then fit the three free parameters \( \Omega_K \), \( \Omega_m \) and \( H_0 \) to the data. A reasonably good fit to present data can be achieved with \( \Omega_K = 0 \), \( \Omega_m = 0.3 \) and \( \Omega_\Lambda = 0.7 \), see [8] for the latest analysis with more than 1500 supernovae Type Ia. A nice (but not the most recent) fit is shown in Fig. 1. There, the observed luminosity distance is compared to the one of an empty universe given by negative curvature only, i.e. with \( \Omega_K = 1 \) and \( \Omega_m = \Omega_\Lambda = 0 \). This is actually simply a part of Minkowski spacetime in accelerated coordinates. It is easy to check that its Riemann tensor vanishes. It is called the Milne universe. Even if it is not overwhelming, the fit is certainly ok and the data cannot be fitted with \( \Omega_\Lambda = 0 \) (the green range in the figure).

**Fig. 1** The distance-redshift relation is shown. The variable plotted in the vertical axis is \( \Delta = 2.5 \log_{10} [d_L(z)/d_L^{\text{Milne}}(z)] \), where \( d_L^{\text{Milne}}(z) \) is the luminosity distance in an empty universe which only contains negative curvature, a so called ’Milne universe’. Figure from [9]
Cepheids and Supernovae are so called (modified) standard candles, i.e. objects of which we know the intrinsic luminosity (or we can infer it from other observables). In cosmology we also have ‘standard rulers’, i.e. objects of which we know the size. The most accurate is the ‘sound horizon’, \( \lambda_s(z) \), i.e., the distance a sound wave in the primordial photon-electron-baryon plasma can travel from the big bang up to redshift \( z \). This scale manifests itself in the acoustic oscillations of the fluctuation spectrum of the cosmic microwave background (CMB) but also in the galaxy correlation function. A standard ruler of size \( L \) can be used to determine the angular diameter distance out to redshift \( z \). If we see our ruler at redshift \( z \) under an angle \( \alpha \) in the sky, the angular diameter distance to it is defined by \( d_A(z) = L/\alpha \). It is related to the luminosity distance via \( d_A = d_L/(1+z)^2 \). The three red points in Fig. 1 actually are from the angular diameter distance out to the corresponding redshift as inferred from the acoustic peak in the galaxy correlation function. A much more precise value can be inferred from the CMB which yields \( d_A(z_{CMB}) \) with \( z_{CMB} \simeq 1080 \). All data together lead to the cosmological parameters \([10]\)

\[
\Omega_K = 0.0007 \pm 0.0019, \quad \Omega_m = 0.348 \pm 0.03, \quad \Omega_\Lambda = 1 - \Omega_K - \Omega_m. \tag{10}
\]

Clearly, this data does not test the validity of GR. It assumes GR and infers the matter content of the Universe by fitting the distance redshift relation to a FL Universe with matter and a cosmological constant. The surprising result is that the present expansion rate of the Universe, \( H(z) \) for \( z < 0.5 \), is dominated by a cosmological constant or more generally by a substance with strong negative pressure, \( P < -\rho/3 \). We call this substance ‘dark energy’. Admitting an arbitrary equation of state, \( P = w\rho \), the data \([10]\) require \( w = -1.04 \pm 0.1 \). This is close to the equation of state of a cosmological constant which has \( w = -1 \). However, allowing also for curvature which only enters the Friedmann constraint equation, but does not ’curve’ the spatial sections of the Universe, degrades this accuracy and leads to a degeneracy, see \([11]\).

So far we have used standard candles and standard rulers to measure the geometry of the background Universe, and we have used Einstein’s equations to infer the matter content of the Universe. Doing so, we have found that the Universe is presently dominated by a component of ‘dark energy’ which might be a cosmological constant and the matter content is dominated by ‘dark matter’ which cannot be the ordinary matter which makes up the stars and gas in galaxies and clusters, but must be something else which interacts much less with photons and ordinary matter and is relatively cold, we termed it cold dark matter (CDM). This has led to a so called cosmological standard model \( \Lambda \)CDM with the present content of the Universe given by \( \Omega_\Lambda \sim 0.68 \), \( \Omega_m \simeq 0.32 \) of which the baryons contribute\(^1\) only \( \Omega_b \simeq 0.05 \) and the rest is CDM.

Hence only 5% of the content of the Universe consists of a substance that we are familiar with and that we have seen also via interactions other than gravity. The other 95%, dark matter and dark energy are inferred by assuming gravity to be governed

\(^1\) The baryon density is ’measured’ in the CMB fluctuations with very high precision. Another way to infer it is primordial nucleosynthesis, i.e. the process by which Helium and Deuterium are formed in the early Universe. Both methods agree well. Counting the baryons in stars and hot gas which make up the galaxies and clusters of galaxies one obtains a smaller value. There are therefore also ‘dark baryons’ probably in the diffuse intergalactic medium where hydrogen is mainly ionized and cannot be observed.
by the equations of GR. Dark matter is also seen in the velocity dispersion of galaxy clusters as well as in rotation curves of galaxies and even dwarf galaxies which are determined by Newtonian gravity only, but dark energy is inferred solely by cosmological distance measurements and by assuming the Friedmann equations to hold.

It is therefore quite natural to ask: Might it be that on large cosmological scales GR is no longer valid? That instead of a cosmological constant we actually see a modification of general relativity? In the next section we propose that this question can be addressed when studying not only the homogeneous and isotropic background but also its fluctuations.

3 Large scale structure observations

Even if at very large scales, the Universe is close to homogeneous and isotropic, this is not the case on small and intermediate scales. There are galaxies, which are very high local over densities of matter and which are often arranged in groups or clusters connected by filaments which surround large voids. The hypothesis is that the cosmological large scale structure of the matter distribution (LSS) grew out of small inhomogeneities by gravitational instability. This hypothesis is supported by the fact that the fluctuations of the CMB which represent fluctuations at \( z \approx 10^{80} \) are very small, of order \( 10^{-5} \). Furthermore, they have precisely the spectrum predicted by a very early inflationary phase which amplifies quantum fluctuations and stretches them into classical fluctuations with a nearly scale invariant spectrum of scalar fluctuations as is seen in the CMB.

At early times and on large enough scales we can therefore use linear cosmological perturbation theory to study the evolution of fluctuations. At late times, especially on small scales, perturbations grow large and must be studied with numerical N-body simulations. In this paper I shall not discuss this important topic, but let me just mention that after a long and exhaustive investigation of Newtonian N-body simulations, see e.g. [12, 13], also relativistic N-body simulations are presently under study [14]. These can be used to calculate the observables, which we discuss below within linear perturbations theory, also on smaller, non-linear scales, see [15, 16] for first examples.

Relativistic fluctuation variables are usually gauge dependent, i.e., they depend on the coordinate system used to describe the background universe. But of course observations are independent of coordinates, i.e., gauge invariant. Therefore it is always possible to describe observables in terms of gauge-invariant quantities.

For simplicity, we restrict ourselves to the spatially flat case in this section. The case of non-vanishing curvature is also discussed in the literature, see e.g., Ref. [17] for the number counts.

3.1 Number counts

Let us consider a survey observing galaxies and measuring their directions and redshifts. A possibility to quantify the fluctuations in the matter distribution is to count the number of galaxies seen in a small solid angle \( d\Omega \) around a direction \( \mathbf{n} \) in the sky and in a
The magnification bias

\[ f_{\text{evo}}(z) \equiv \mathcal{H}^{-1} \frac{d \ln (a^3 \bar{N})}{dt} = -(1 + z) \frac{d}{dz} \ln \left( \frac{\bar{N}}{(1 + z)^3} \right). \]  

(12)

The magnification bias \( s(z) \) comes from the fact that a galaxy survey is usually flux limited. If a galaxy is sufficiently amplified by lensing due to foreground matter, it may make it into our survey even if its unlensed flux would be below the limit of the survey. This quantity depends on the flux limit of the survey considered and on the logarithmic derivative of the mean galaxy number density at this limit. If we see all the galaxies (of a given type), \( s(z) = 0 \). Denoting the limiting apparent magnitude of the survey by \( m_\ast \), the magnification bias is given by

\[ s(z) = \frac{\partial \log_{10} \bar{N}(z, m < m_\ast)}{\partial m_\ast}. \]  

(13)

Note also that in \( \Delta(n, z) \) the perturbations variables \( D \), \( V \), \( \Psi \) and \( \Phi \) which depend on position \( \mathbf{x} \) and time \( t \) have to be evaluated on the background lightcone, \( \mathbf{x} = r(z)\mathbf{n} \), \( t = t(z) = t_0 - r(z) \). Also the integrals \( dr \) are along the background lightcone, i.e. the
integrand is evaluated at \((x, t) = (r_n, t_0 - r)\). In [23] the magnification and evolution biases for some near future surveys are estimated.

The most interesting aspect of Eq. (11) is that it contains not only the density fluctuations \(D\) and the velocity field \(V\) but also the Bardeen potentials \(\Phi\) and \(\Psi\). In longitudinal gauge these are simply the metric perturbations,

\[
d s^2 = a^2(t) \left[ -(1 + 2\Psi) dt^2 + (1 - 2\Phi) \delta_{ij} dx^i dx^j \right]. \tag{14}
\]

Hence if we could isolate the different terms, we could in fact measure both, the perturbations of the energy momentum tensor and of the metric. In the derivation of Eq. (11) the only assumption on ‘matter’ that enters is that galaxies and light follow geodesics. Einstein’s equations are not used. Hence (11) is valid for any metric theory of gravity.

The burning question is now : does the measurement of \(\Delta(n, z)\) for many different redshifts and directions allow us to isolate its different contributions from \(D, V, \Phi\) and \(\Psi\)? The short answer is NO. But let us study in somewhat more detail what we can do.

First we need to stress that our theories of the generation and evolution of cosmological perturbations do not predict the values of any perturbation variable at given positions and times, e.g., \(\Psi(x, t)\), but they only provide statistical ensemble averages like, e.g., power spectra or correlation functions. By definition, the mean of \(\Delta(n, z)\) vanishes, but its correlation or power spectra do not. At fixed redshift, \(z\), \(\Delta(n, z)\) is a function on the sphere which we can expand in spherical harmonics,

\[
\Delta(n, z) = \sum a_{\ell m}(z) Y_{\ell m}(n). \tag{15}
\]

If we require statistical isotropy, the expectation values of the \(a_{\ell m}\)'s vanish and

\[
\langle a_{\ell m}(z) a_{\ell m'}^*(z') \rangle = \delta_{\ell \ell'} \delta_{m m'} C_\ell(z, z'). \tag{16}
\]

The \(C_\ell(z, z')\)'s are the angular power spectra of \(\Delta(n, z)\). The correlation function is then given by

\[
\langle \Delta(n, z) \Delta(n', z') \rangle = \frac{1}{4\pi} \sum_\ell (2\ell + 1) C_\ell(z, z') P_\ell(n \cdot n'). \tag{17}
\]

where \(P_\ell\) is the Legendre polynomial of order \(\ell\). The fact that \(\langle \Delta(n, z) \Delta(n', z') \rangle\) depends on directions only via \(n \cdot n'\) is again a consequence of statistical isotropy.

Of course we can not really measure an ensemble average, but each measured \(a_{\ell m}(z) a_{\ell m'}^*(z')\) provides an independent estimator of \(C_\ell(z, z')\). If perturbations are Gaussian, which is a good approximation on linear scales, the power spectra contain all the statistical information. The error due to the fact that there are at best \(2\ell + 1\) different \(a_{\ell m}(z)\)'s is \(\sqrt{2/(2\ell + 1)} f_{\text{sky}}\), where \(f_{\text{sky}}\) denotes the observed sky fraction. This minimal error is called ‘cosmic variance’. Hence the low multipoles always have
very large errors due to the fact that we have only very few estimators of them in the observable Universe.

Studying the relation of metric perturbations and density perturbations via Einstein’s equations one finds that $\Psi, \Phi \sim (\mathcal{H}/k)^2 D$. Hence on subhorizon scales, $k \gg \mathcal{H}$, metric perturbations are substantially suppressed. Therefore, the terms in Eq. (11) containing the gravitational potential are typically relevant only on very large scales, i.e. very low $\ell$’s for which cosmic variance is large. There is one exception to this which is the lensing term given by the Newtonian convergence\(^2\) $\kappa^{(N)}$, as

$$2 - 5s \frac{1}{2} \int_0^{r(z)} dr \frac{r(z) - r}{r(z)r} \Delta \Omega (\Phi + \Psi) = (2 - 5s) \kappa^{(N)} (n, z).$$ (18)

For a given harmonic $\ell$ the angular Laplacian $\Delta \Omega$ yields a factor $-\ell(\ell + 1)$ which is of the order $(kr(z))^2 \sim (k/\mathcal{H}(z))^2$, for not too small redshifts. However, as it is an integrated term, cancellations happen and reduce this term typically to the order of $(k/\mathcal{H}(z)) \Psi$. What is more interesting, as this is an integrated term, it contributes significantly to $C_{\ell}(z, z')$ for $z \neq z'$, while the non-integrated terms are very small for well separated redshift bins. It has been argued that this term will be measured well in future survey, especially in the relatively wide redshift bins of photometric galaxy surveys [26–28]. It has already been measured in the past by correlating foreground galaxies e.g. with background quasars, see, e.g., [29].

In the auto correlations, $z = z'$, one mainly measures the Newtonian contributions,

$$\Delta^{(N)} (n, z) = bD + \frac{1}{\mathcal{H}} \partial_r (V \cdot n)$$ (19)

which are well known [30] and have been measured very precisely in the power spectrum in redshift space. The density term prominently shows the so called baryon acoustic oscillations (BAO) which are a remnant from the coupled baryon photon plasma which performed acoustic oscillations before decoupling. Since the acoustic horizon is well known, the angular scale of these oscillations yields an excellent measure of the angular diameter distance, $d_A(z)$. The three red points in Fig. 1 are from such measurements with the BOSS [31] survey. In the future it will also be possible to measure the radial redshift difference corresponding to this scale,

$$\lambda_s (z) = r(z + \Delta z/2) - r(z - \Delta z/2) \simeq \Delta z/\mathcal{H}(z).$$ (20)

This will provide a measure of the Hubble parameter at redshift $z$.

The velocity term in (19) is called the redshift space distortion (RSD). It comes from the fact that galaxies with radial peculiar velocities are seen at slightly higher, respectively lower redshift. This leads to a radial volume distortion, the RSD. It can be measured in the correlation function. When observing a small patch of sky which

\(^2\) I call this the Newtonian convergence since the full convergence which appears in the Jacobi map is given by all the terms which multiply the factor $-5s$, see, e.g. [24]. In a quasi-Newtonian study of lensing, however, one only obtains $\kappa^{(N)}$, see, e.g., [25].
can be treated within the flat sky approximation, \( \frac{1}{H_0} \partial_t (\mathbf{V} \cdot \mathbf{n}) = \mu^2 f(z) D \), where \( \mu \) is the cosine of the angle between the observation direction and the velocity. Here

\[
 f(z) = \frac{d \ln D_1}{d \ln a} \simeq (\Omega_m(z))^{0.55}
\]

is the linear growth rate of density fluctuations, and we have used that within linear perturbation theory the continuity equation relates density fluctuations to the velocity potential via \( V = \mathcal{H}(z) f(z) D \). This growth rate is very sensitive to the theory of gravity and measuring it is an excellent test of GR. Present measurements of the growth rate are, however, degenerate with the amplitude of perturbations cast in \( \sigma_8 \). Here \( \sigma_8^2 \) is the variance of density fluctuations inside a ball of radius 8Mpc.

In a small survey which can be treated with the flat sky approximation, the quadrupole and hexadecapole in the correlation function are excellent measures of the growth rate, while the monopole is used to identify the BAO feature. Several recent surveys give rather accurate determinations of \( \sigma_8 f(z) \), see [32–41]. Many recent measurements are shown in Fig. 2.

With cosmological galaxy number counts we can therefore measure density fluctuations, the growth rate or, more generically, velocity perturbations and the lensing potential. Knowing the lensing potential out to many different redshifts we can use these measurements to infer the so called Weyl potential,

\[
 \Phi_W = \frac{1}{2}(\Phi + \Psi) .
\]

In \( \Lambda \)CDM cosmology, where anisotropic stresses (nearly) vanish at late time,

\[
 \Phi \simeq \Psi \simeq \Phi_W ,
\]
with relative differences of the order of 0.1\%. Furthermore, within a metric theory of gravity, where galaxies move on geodesics, \( d(a \nabla)/dt = -a \nabla \Psi \). Hence measuring the velocity via redshift space distortions at many different redshifts, allows in principle also to isolate \( \Psi \) and so to test (22). A way to measure the velocity via the 'Doppler term', \( \mathbf{V} \cdot \mathbf{n} \) in Eq. (11) has been suggested in [42, 43]. Since this term is odd in \( \mathbf{n} \), it can be isolated as the antisymmetric part in the correlation function of two different tracers, e.g. red and blue or luminous and faint galaxies. A violation of (22) could either indicate a violations of the equivalence principle for dark matter, see also [44], or a modification of the theory of gravity which usually implies \( \Phi \neq \Psi \). Together with the growth rate this already provides two excellent tests of general relativity.

Very recently, the idea to weigh cosmological number counts, e.g., with the redshift has been explored [45, 46], and it has been shown that this can lead to interesting complementary cosmological information. It will be important to investigate these new observables in more detail, also in view of the their power to test GR.

### 3.2 Intensity mapping

Mapping out individual galaxies in a survey and measuring their redshift is very costly. A 'cheaper' alternative to galaxy number counts is the so called intensity mapping. In this technique we do not count galaxies, but just measure the intensity of the sky brightness in a specific, strong atomic or molecular line, e.g., the 21cm line from the hyperfine spin-flip transition of neutral hydrogen [47, 48]. This yields the surface density of hydrogen in the sky at a fixed redshift. The difficulty here is the foreground removal, as usually the foreground is many orders of magnitude larger than the signal. Foreground removal can be achieved by requesting a very strong intensity change as a function of wavelength as it is only present in lines.

At high redshift, before reionization, \( z > 6.5 \), intensity mapping of the 21cm line is interesting by itself as it can measure the evolution of small, linear density fluctuations. Around \( z \sim 6 \) one can investigate the process of reionization by this method. After reionization, \( z < 6 \), neutral hydrogen is mainly present in galaxies and proto-galaxies, and 21cm intensity mapping is actually a coarse grained mapping of large scale structure similar to the galaxy distribution, see [49] for an overview.

So far, most measurements have correlated 21cm emission with existing galaxy surveys [50–52]. A very surprising result [53] is a measurement of a 21cm signal at \( z \simeq 17 \) which has more than twice the expected intensity. This result has, however, not been confirmed (nor refuted) until now. Several experiments to directly detect large scale structure with intensity mapping are presently being prepared and shall take data in the near future [54–56]. A first result has very recently been put on the arXiv [57]. Forecasts for the prospects of intensity mappings are detailed in several papers-[58–60]

The HI intensity is usually cast into an equivalent temperature \( T_{\text{HI}} \) and its fluctuation is nearly proportional to the fluctuation in the number of galaxies. The main difference is that intensity is determined per surface area and, as is well known, gravitational lensing conserves surface brightness. Therefore, at first order in perturbations theory, like for the CMB, there are no lensing terms in intensity mapping. The rela-
tive fluctuations in intensity are therefore given exactly by Eq. (11) when setting the
magnifications bias $s \equiv 2/5$. This result is derived in detail in [61]. Therefore, while
intensity mapping cannot be used to determine the convergence $\kappa$, it will be very useful
for density and RSD. Of course also the bias will be the one of hydrogen, $b_{\text{HI}}$ which
is usually somewhat different from the galaxy bias $b$.

The cross-correlations of intensity maps and galaxy number counts will prove
especially useful for velocity measurements [62] and for identifying $\kappa^{(N)}$ in the number
count surveys [63].

Also other lines, not only the $21\text{cm}$ line of hydrogen, are being used for intensity
mapping, see [64] for an overview.

3.3 Shear measurements

The foreground gravitational field acts like a medium with spatially dependent index
of refraction on light rays which come to us from far away sources. The images are
stretched, sheared and can also be rotated. For small images this is described with the
so called Jacobi map [65]. To first order in cosmological perturbation theory there is
no rotation [24]. The shear from scalar perturbations is determined fully by the lensing
potential

$$\phi = \int_0^{r(z)} dr \frac{r(z) - r}{r(z)r}(\Phi + \Psi). \quad (23)$$

The shear is usually cast as a complex helicity-2 variable defined as

$$\gamma = \left( \nabla_1^2 - \nabla_2^2 + i \nabla_1 \nabla_2 \right) \phi, \quad (24)$$

where $\nabla_i$ is the (covariant) derivative in direction $e_i$ and $(e_1, e_2)$ form an orthonormal
basis on the sphere. Even though $\gamma$ depends on our choice of basis, once we determine
the correlation function $\langle \gamma(n, z)\gamma(n', z') \rangle$, we can choose $e_1$ as the vector pointing
from the point $n$ to $n'$ on the sphere (more precisely as the tangent vector to the
geodesic from $n$ to $n'$), which defines the basis intrinsically.

The ellipticity of background galaxies is affected by the foreground shear, see [25,
66, 67] for reviews on the subject. Several recent measurements of the shear correlation
function and the shear angular power spectrum in different redshift bins have been
performed. Some recent results are found in Refs. [68–73]. These have usually been
analysed with the assumptions of purely scalar perturbations in $\Lambda$CDM, or one of its
variants like $w$CDM, where the cosmological scalar perturbations in $\Lambda$CDM, or one of its
variants like $w$CDM, where the cosmological constant is replaced by a dark energy
component with the equation of state $P = w\rho$. In these studies the authors have set
(in Fourier space) $k^2 \Phi = k^2 \Psi = -(3/2)(H_0/k)^2 \Omega_m (1 + z)D$ and used their results
to constrain a variable closely related to $\Omega_m D$, more precisely they measured

$$S_8 = \sigma_8 \Omega_m^{1/2}. \quad (25)$$
Their results are somewhat in tension with the value of $S_8$ inferred from Planck. They obtain a best fit value for $S_8$ that is typically about 2 standard deviation below the Planck result.

However, before jumping to conclusions, it must be noted that cosmic shear measurements are very difficult. They have been proposed for the first time in the 60ties [74] and first serious attempts to measure shear were made in the 80ties, see, e.g., [75]. However, measurements of the shear correlation function have presented first results only a couple years ago. The reason for this is mainly intrinsic alignment: If the ellipticity of two nearby galaxies is correlated this can have two reasons. Either the fact that we see them through the same foreground shear field or that they have been formed from the same large scale over density and are intrinsically aligned. Present shear analysis is always combined with a model for intrinsic alignment which has several free parameters that have to be fitted simultaneously with the cosmological parameters. Nevertheless, it is not clear whether the intrinsic alignment model is sufficiently detailed to capture all the relevant physics, or whether it may lead to a systematic error in our estimate of $S_8$.

Recently, a new method to measure cosmic shear via its rotation of the principal axis of a galaxy has been proposed [76]. For this, however, one needs another direction which keeps the information of the original orientation of the galaxy in the source plane. This can be provided by the polarization of light which is parallel transported along the light ray. The polarization direction, e.g., of radio galaxies is correlated with their semi-minor axis. Simultaneous determination of the semi-minor axis in the image and of the polarization direction can then be used to determine the rotation of this axis due to shear. This measure is not plagued by intrinsic alignment. It will have to be investigated in detail what its systematic uncertainties are and what its potential is to improve shear measurements.

If the shear is measured it can be used as an independent measurement of the Weyl potential, but it can also be used to test its relation with $\kappa^{(N)}$ measured on galaxy number counts. For purely scalar perturbations their power spectra should be related as, see e.g. [77]

$$C^{(N)}_{\ell}(z, z') = \frac{\ell(\ell + 1)}{(\ell + 2)(\ell - 1)} C'_{\ell}(z, z').$$

It has been shown, see e.g., [24] that at very large scales, low $\ell \lesssim 20$ this relation does not hold for the full convergence in the Jacobi map, however at higher values of $\ell$ it is very accurate.

Testing this relation will be very important. If it does not hold this means that either vector or tensor fluctuations are relevant for which this relation is not valid, see [24, 78] or then that photons do not move on geodesics but are subject to some ‘fifth force’ or similar. The second would be a very significant deviation from GR, where not Einstein’s equations are put in question, these are not used for relation (26), but the ‘equivalence principle’ for photons. But also the first would be interesting and might hint, e.g., to a stochastic gravitational wave background.
4 Conclusion

In this paper we have shown with several examples that measurements of cosmological perturbations or more precisely their correlation functions and power spectra can be used to measure both, components of the matter energy momentum tensor and components of the metric. In this way they open the route to test General Relativity in the non-vacuum case and on cosmological scales. We have only mentioned the best known examples here but there are several more. Furthermore, apart from the two point correlation function and the power spectrum, there are other observables. For example the higher order statistical quantities, \( N \)-point correlation functions and the corresponding \( N \)-point spectra, as well as other expectation values for which we can develop estimators, like, e.g., Minkowski functionals, see [79, 80], or wavelets. \( N \)-point correlation functions are especially relevant in the non-linear regime where deviations from Gaussian statistics become very significant and contain independent information. In particular the three-point function or the corresponding power spectrum, the bi-spectrum, see e.g. [81–83], have very specific properties for example in the squeezed limit where the wavelength of one density fluctuation is much larger than the other two. These properties are a simple consequence of the equivalence principle and can therefore be used to test it. Another interesting observable is the correlation between CMB anisotropies and galaxy number counts which are due to lensing of the CMB by foreground galaxies, see e.g. [84].

In the past decades we used cosmological observations mainly to determine cosmological parameters with good precision. The excellent CMB data has allowed us to come up with a simple cosmological standard model, flat \( \Lambda \)CDM, that fits the CMB data very well and requires only 6 parameters. Even though discrepancies of local measurements of the Hubble constant [85] might hint to shortcomings of the model, there is also the proverbial 'elephant in the room': The \( \Lambda \)CDM model contains about 95% 'dark content', dark energy in the form of the cosmological constant \( \Lambda \) and cold dark matter, which have not been detected in any other way than via their gravitational interaction; this is certainly unsatisfactory. It is mainly this fact that motivates us to use present and future cosmological observations not only to improve on the precision of the cosmological model parameters, but much more to test the basic underpinning of \( \Lambda \)CDM, Einstein’s theory of General Relativity.

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Data Availability In this article no data is being used.

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