Security guaranteed wireless communication over common noise attack under spatial assumption

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Abstract
We assume that Eve and Bob are connected to Alice with additive white Gaussian noise channel, and their noises are correlated. We allow Eve to optimize the correlation between her noise generated outside her detector, which is called the common noise attack. We give a necessarily and sufficient condition to generate the agreed secure key between Alice and Bob over the common noise attack under a spatial condition for Eve. Further, we give a concrete protocol with the reverse reconciliation to generate secure final keys whose leaked information is rigorously and quantitatively guaranteed even with finite block-length code. Based on the method of post selection, we discuss the security when Eve inserts artificial noise. We also consider how Bob can improve their key generation rate by inserting artificial noise to Eve’s observation.

Index Terms
secret key generation, common noise attack, reverse reconciliation, post selection, spatial assumption, artificial noise

I. INTRODUCTION

Recently, secure wireless communication attracts much attention as a practical method to realize physical layer security [1], [2], [3], [4], [5], [6], [7], [8]. In particular, wire-tap channel model [9], [10], [11], [12] is considered as a typical model for physical layer security. In the wire-tap channel model, the authorized sender, Alice is willing to transmit her message to the authorized receiver, Bob without any information leakage to the adversary, Eve. In this case, we usually assume that the noise in the channel to Eve is larger than that in the channel to Bob. However, it is not easy to guarantee this assumption under the real wireless communication. In cryptography, it is usual to consider that the adversary, Eve is more powerful than the authorized users, Alice and Bob in some sense like RSA cryptography [13]. However, the above wire-tap channel requires the opposite assumption. So, it does not necessarily have sufficient powers of conviction to assume the above wire-tap channel in real wireless communication.

In stead of wire-tap channel model, we often employ secure key agreement, in which, Alice and Bob generate the agreed secure key from their own correlated random variables [14], [15]. This problem has a similar problem because to generate secure keys via one-way communication from Alice to Bob, they need to assume that the mutual information between Alice and Bob is larger than that between Alice and Eve. Further, although there exist proposals to generate secure key from wireless communication [16], [17], [18], [19], they do not give a quantitative and rigorous security evaluation for the final keys under a reasonable assumption advantageous to Eve. Indeed, several papers [20], [21], [22], [23] considered the case when Bob controls the noise in Eve’s observation. However, they do not discuss the possibility that the noise in Bob’s observation is controlled by Eve. Also, there is a possibility that a part of the noise in Bob’s observation is commonly contained in Eve’s observation in the optimal way under the natural restriction, which is called the common noise attack. Hence, for a practical realization of secure wireless communication, it is needed to give a method satisfying the following conditions.

1. The assumption is physically reasonable, and allows Eve to have several choices including the common noise attack. Further, Eve is allowed to control a part of the noise in Bob’s observation.

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(2) The security of final keys is guaranteed quantitatively and rigorously based on acceptable criterion even for cryptography community (e.g. the variational distance criterion \cite{24} or the mutual information criterion) even though Eve takes the optimal strategy under the above assumption. Additionally, the formula to derive the security evaluation has sufficiently small calculation complexity.

(3) The calculation complexity of the whole protocol is sufficiently small.

One might consider that we need a useful formula for the evaluation of the correctness of final keys. However, this requirement is not necessarily because the correctness can be checked by random sampling of the generated keys, which is called error verification. Since the security cannot be evaluated directly from the random sampling of final keys, we need its theoretical formula as the above.

In wireless communication, the input signal is given as a continuous random variable $X_1$ and we often assume that the output random variable of Bob is given as $a_B X_1 + e_B Y$ by using the additive normalized white noise $Y$, i.e., the random variable $Y$ is subject to normalized Gaussian distribution with average $0$ \cite{1}. Similarly, the output random variable of Eve is given as $a_E X_1 + e_E Z$ by using another additive normalized white noise $Z$. Hence, when the signal-noise ratio $\frac{a_B^2}{e_B^2}$ of the communication from Alice to Bob is smaller than the ratio $\frac{a_E^2}{e_E^2}$ from Alice to Eve, simple application of wire-tap channel model cannot realize secure communication.

When the additive white noise $e_B Y$ in the channel from Alice to Bob is independent of the additive white noise $e_E Z$ in the channel from Alice to Eve, secure key generation is possible by the reverse reconciliation from Bob to Alice followed by privacy amplification. However, when the noise $e_E Z$ is correlated to the noise $e_B Y$, our problem is more complicated. When Eve controls the noise in Bob’s observation, Eve can choose the common noise in the optimal way. We call this type attack the common noise attack, and give a solution for this attack.

In this paper, we focus on the fact that both noises can be divided into two parts, the noise generated outside of the detector and the noise generated inside of the detector. The latter noises are independent of other noises. Further, due to spatial assumption for Eve, we assume a reasonable upper bound of $a_E^2$. Using the existence of this kind of noise even in Eve’s detector and the post selection, under this spatial assumption, we guarantee the security whatever information for the noise generated outside Bob’s detector Eve has.

In this paper, we mathematically formulate this assumption, and clarify the requirement to generate secure key for secure wireless communication when our channel is quasi static \cite[Section 5.4.1]{25}. Further, under this requirement, combining existing results with finite block-length analysis \cite{26, 27, 29, 28}, we give a concrete method to generate a secure key whose leaked information can be quantitatively guaranteed.

This paper is organized as follows. Section II gives the mathematical formulation for our model for the common noise attack, and optimizes Eve’s strategy. Then, we derive the maximum of the information leaked to Eve as the main theorems. Section III derives an efficient protocol to generate secure keys whose security is rigorously and quantitatively guaranteed. Section IV discusses two additional Eve’s attacks in our model. One is increasing the number of Eve’s antennas and the other is Eve’s control of Bob’s noise. For the latter attack, we propose a method to reduce it to the common noise attack by using post selection. Unfortunately, there is a possibility that Alice and Bob have a difficulty to satisfy the required condition. In Section V we propose a method to resolve this problem by adding artificial noise. Section V gives the proofs of the main theorems and several important statements. In Section VII we discuss the relations between our result and related topics.

II. SECURITY OVER COMMON NOISE ATTACK

Now, we give a formulation with reverse reconciliation when our channel is quasi static. We assume that Alice generates the signal $X_1$ subject to the standard Gaussian distribution in the initial transmission
from Alice to Bob. Eve also receives this information. When the noises are subject to the white Gaussian distribution, the signals Bob and Eve receive can be generally written as

\[ B := a_B X_1 + b_B X_2 + c_B X_3, \]
\[ E := a_E X_1 + b_E X_2 + c_E X_4, \]

where \( a_B, b_B, c_B, a_E, b_E, \) and \( c_E \) are constants and the random variables \( X_i \) are subject to the standard Gaussian distribution independently because our channel is quasi static. Eve can eavesdrop the information by using the correlation in the noise. Since Alice and Bob cannot estimate the coefficient \( c_E \) at all, we need to consider the worst case, i.e., the case with the optimal coefficient for Eve. We call this optimal choice for Eve the \textit{common noise attack}. The following analysis covers the security analysis over the common noise attack. This description does not directly cover the case when Eve knows the value of the random variable \( X_2 \), as shown in Section IV, such a case can be reduced to the current analysis. Hence, our analysis covers the case when the noise is artificially generated by Eve when the artificial noise is subject to the white Gaussian distribution.

Without loss of generality, we can assume that \( b_E \) is positive. Under this model, the second terms express the common noise and the third terms expresses individual noise. Indeed, there is a possibility that \( B \) has the coefficients of \( X_2 \) with the opposite sign of that of \( E \). However, such a case is not advantageous to Eve, we consider only the case when they have the same sign.

When the signal-noise ratio of Eve is not smaller than that of Bob, i.e., \( \frac{\sigma_B^2}{b_B^2 + c_B^2} \geq \frac{\sigma_E^2}{b_E^2 + c_E^2} \), secure communication with forward reconciliation is impossible. However, there is a possibility of secure communication with reverse reconciliation. To discuss this issue, we compare the correlation coefficient \( \rho_E(b_E) \) between \( B \) and \( E \) and the correlation coefficient \( \rho_A \) between \( B \) and \( X_1 \), which are defined as the ratio between the covariance and the product of squares of the variances and calculated as follows.

\[ \rho^2_A = \frac{a_B^2}{a_B^2 + b_B^2 + c_B^2}, \]
\[ \rho^2_E(b_E) = \frac{(a_B a_E + b_B b_E)^2}{(a_B^2 + b_B^2 + c_B^2)(a_E^2 + b_E^2 + c_E^2)}. \]

Now, we obtain the following theorem.

\textbf{Theorem 1:} The maximum value of the correlation coefficient \( \rho_E(b_E) \) are characterized as follows.

\[ \rho^2_{E,\text{max}} := \max_{b_E} \rho^2_E(b_E) = \frac{1}{a_B^2 + b_B^2 + c_B^2} \left( \frac{a_B^2 a_E^2}{a_B^2 + c_B^2} + b_B^2 \right). \]

This maximum is realized when \( b_E = b_{E,o} := \frac{b_B(a_B^2 + c_B^2)}{a_B a_E}. \)

\textbf{Lemma 2:} The inequality \( \frac{1}{a_B^2 + b_B^2 + c_B^2} \left( \frac{a_B^2 a_E^2}{a_B^2 + c_B^2} + b_B^2 \right) < \frac{a_E^2}{a_B^2 + b_B^2 + c_B^2} \) holds if and only if

\[ \frac{a_B^2}{b_B^2} > \frac{a_E^2}{c_B^2} + 1. \]
conclude that the random variable $c_E X_4$ contains the noise generated inside Eve’s detector. That is, $c_E^2$ is greater than the intensity of the noise generated inside Eve’s detector. Hence, we can estimate a lower bound of $c_E^2$ from the possible performance of Eve’s detector. Further, Alice can estimate the lower bound of the parameter $a_E^2$ from the following spatial assumption. The intensity $a_E^2$ behaves as $Cd^\alpha$ with positive constants $C$ and $\alpha$ when the distance from Alice’s transmitting antenna is $d$. For example, the free space with no obstacle has the constant $\alpha = 2$ [25]. It is natural that Eve’s detector is sufficiently far from Alice’s transmitting antenna, that is, $d \geq d_0$ with a certain constant $d_0$, when Alice can check no suspicious receiving antenna within visual confirmation. We call this assumption the spatial assumption. When we accept the spatial assumption, we can guarantee that $a_E^2 \leq Cd_0^\alpha$.

![Fig. 1. Breakdown of Bob’s and Eve’s noises](image)

Unfortunately, in this scenario, it is impossible to estimate the parameter $b_E$ at all. However, without the estimation of the parameter $b_E$, we can upper bound the correlation coefficient $\rho_E(b_E)$ by Theorem 1. That is, when the condition (6) holds. Now, we assume that Bob and Eve use a detector with the same performance. We also assume that $a_E^2 \leq a_B^2$. In this case, when the relation $a_E^2 > \frac{a_B^2}{10} + 1$ hold, we have the condition (6). This condition requires that the intensity of the noise generated outside Bob’s detector is sufficiently smaller than the noise generated inside Bob’s detector. Maybe, this condition does not necessarily hold even in the usual setting because the coefficient $b_B$ fluctuates dependently of the time. Here, we should remark that the coefficients $a_B, b_B, c_B, a_E, b_E$, and $c_E$ can be regarded as constants within short time span and they behave as random variable with long time span because our channel is quasi static [25, Section 5.4.1]. Fortunately, Alice and Bob can estimate the parameter $b_B$ adaptively by the sampling in the above way. Hence, they can select only the case when the condition (6) holds. That is, they discard the random variables when the condition (6) does not hold. This method is often called post selection.

### III. Efficient Protocol to Generate Secure Keys

Although there exist several methods to asymptotically attain the optimal one way key distillation rate from Gaussian random variables by using suitable discretization [30], [31], [32], [33], there is no protocol to distill secure keys from Gaussian random variables satisfying the following conditions.

1. The whole calculation complexity is not so large.
2. A rigorous security evaluation of the final key is available with finite block-length.

Here, we propose a protocol satisfying the above conditions by a very simple idea as follows. Firstly, Alice and Bob prepare the amount of $c_B, c_E,$ and $a_E$. Next, for each round, Alice and Bob choose the
sampling pulses for $X_1$ and $B$ as mentioned the above. Then, they estimate $a_B$ and $b_B$. Based on these estimates, they apply key distillation as follows.

Since the difficulty of its efficient construction is caused by the continuity, Bob applies very simple discretization, i.e., he converts his random variable to $1$ or $-1$ by taking the sign of $B$, and obtains the new bit random variable $B'$ in $\mathbb{F}_2$ as $(-1)^{B'} = \text{sgn} B$. When another Gaussian random variable $X$ has the correlation coefficient $\rho$ with the original Gaussian random variable $B$, the conditional entropy $H(B'|X)$ is given by the function $g(\rho)$ as

$$g(\rho) := \int_{-\infty}^{\infty} P_{B',X}(0, x) \log \frac{P_{B',X}(0, x) + P_{B',X}(1, x)}{P_{B',X}(0, x)} dx + \int_{-\infty}^{\infty} P_{B',X}(1, x) \log \frac{P_{B',X}(0, x) + P_{B',X}(1, x)}{P_{B',X}(1, x)} dx$$

$$= -\int_{-\infty}^{\infty} \sqrt{\frac{2}{\pi}} e^{-\frac{x^2}{2}} Q(\rho, x) \log Q(\rho, x) dx,$$

where $P_{B',X}$ expresses the joint probability density function of $B'$ and $X$ and $Q(\rho, x)$ is defined to be

$$\frac{1}{\sqrt{2\pi}} \int_{-\rho}^{\rho} e^{-\frac{y^2}{2}} dy.$$

In this case, we introduce the function $\phi_\rho(t)$ as

$$\phi_\rho(t) := \log \frac{1}{2\pi} \int_{-\infty}^{\infty} (P_{B',X}(0, x) \frac{1}{1-t} + P_{B',X}(1, x) \frac{1}{1-t})^{1-t} dx$$

$$= \log \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (Q(\rho, x) \frac{1}{1-t} + (1 - Q(\rho, x)) \frac{1}{1-t})^{1-t} e^{-\frac{x^2}{2}} dx.$$

The proofs of (8) and (9) are given in Subsection VI-B.

Here, we have $-\frac{d}{ds} \phi_\rho(s)|_{s=0} = g(\rho)$. The function $\phi_\rho(t)$ satisfies the following properties.

**Lemma 3:** (1) The function $\phi_\rho(t)$ is monotone increasing for $\rho$ with any $t \in (0, 1)$. (2) The function $\phi_\rho(t)$ is convex for $t \in (0, 1)$.

This lemma is shown in Subsection VI-C. Since the limit $\lim_{t \to 0} -\frac{\phi_\rho(t)}{t}$ equals the conditional entropy $g(\rho)$, the function $g(\rho)$ is monotone decreasing for $\rho$. The mutual information between $B'$ and $X_1$ is $\log 2 - g(\rho_A)$ and the mutual information between $B'$ and $E$ is $\log 2 - g(\rho_E(b_E))$.

Now, we give a concrete protocol for Alice and Bob. First, they fix the rate $R_1$ of error correction, which is less than $\log 2 - g(\rho_A)$. Then, they choose the sacrifice rate $R_2$, which is larger than $\log 2 - g(\rho_E(b_E))$. So, the key generation rate is $R_1 - R_2$. Alice and Bob fix an error correcting code $C \subset \mathbb{F}_2^n$, where $n$ is the block-length. For error correction, Bob computes the syndrome as an element $[B'^n]$ of the coset space $\mathbb{F}_2^n/C$ from his bit sequence $B'^n$, and sends it to Alice. Alice applies the error correction to recover $B'^n$.

Next, they apply a randomized function $f_H$ to $B'^n$ that maps $C$ to $K$, where $H$ is the random variable identifying the function $f_H$ and $K$ is the set of final keys. Here, to keep the uniformity of the final key, we assume the following condition;

$$|f_H^{-1}(k)| = \frac{|C|}{|K|}$$

for any $k \in K$ and any $h \in H$, where the random variable $H$ takes values in the set $H$. This kind of function is called a randomized hash function. In this protocol, Alice and Bob need to prepare random seeds $H$ to identify the function $f_H$. The seeds $H$ is allowed to be leaked to Eve. Hence, Alice (or Bob) generates it locally and can send it to Bob (or Alice) via public channel. The randomized hash function $f_H$ is called a universal2 hash function when the collision probability satisfies the inequality

$$\Pr\{f_H(c) = f_H(c')\} \leq \frac{1}{|K|}$$
for any distinct elements $c \neq c' \in C$ [34], [35]. In the above equation, $Pr$ expresses the probability with respect to the choice of $H$. In the following discussion, we assume that our randomized hash function $f_H$ is a universal 2 hash function. A typical example of a universal 2 hash function is given by using Toeplitz matrix. Its detail construction and the evaluation of the complexity of its construction are summarized in the recent paper [36].

To evaluate the leaked information, as the security measures, we adopt the conditional mutual information $I(K : E|H)$ between Bob and Eve and the variational distance measure $d(K : E|H)$ conditioned with $H$ as

$$I(K : E|H) := \sum_{m,h} P_H(h) P_{K|H=h}(k) D(P_{E|K=k,H=h}\|P_{E|H=h})$$

$$d(K : E|H) := \sum_{m,h} P_H(h) P_{K|H=h}(k) d(P_{E|K=k,H=h}, P_{E|H=h}),$$

where $P_K$ is the distribution for the final key, $P_{E|K=k}$ is the conditional distribution for Eve’s information when the key is $k$, $P_E$ is the marginal distribution for Eve’s information. It is known that the latter satisfies the universal composable property [24]. Further, $D(P\|Q)$ is the relative entropy defined as $\sum_x P(x)(\log P(x) - \log Q(x))$ and $d(P, Q)$ is the variational distance defined as $\sum_x |P(x) - Q(x)|$. From the discussions in [26], [27], [29], [28], we have

$$I(K : E|H) \leq \inf_{s \in (0,1)} \frac{1}{s} e^{n(s(\log 2 - R_2) + \phi_{PE_E}(s))} \leq \inf_{s \in (0,1)} \frac{1}{s} e^{n(s(\log 2 - R_2) + \phi_{PE_E}(s))},$$

$$d(K : E|H) \leq 3 \min_{t \in [0,\frac{1}{2}]} e^{n(t(\log 2 - R_2) + \phi_{PE_E}(t))} \leq 3 \min_{t \in [0,\frac{1}{2}]} e^{n(t(\log 2 - R_2) + \phi_{PE_E}(t))}. \quad (15)$$

The detail derivation is available in Subsection VI-D. Since the function $t \mapsto (\log 2 - R_2) + \phi_{PE_E}(t)$ is convex (Lemma 3), the minimum $\min_{t \in [0,\frac{1}{2}]} t(\log 2 - R_2) + \phi_{PE_E}(t)$ is computable by the bisection method [37, Algorithm 4.1], which gives the RHS of (15). Since $s \mapsto - \log s$ is convex, the function $s \mapsto n(s(\log 2 - R_2) + \phi_{PE_E}(s)) - \log s$ is convex. So, the infimum $\inf_{t \in [0,1]} t(\log 2 - R_2) + \phi_{PE_E}(t)$ is computable in the same way. That is, we can calculate the RHS of (14). When we cannot perfectly identify the parameters $a_E$ and $c_E$ to decide the coefficient $\rho_{E,\max}$, it is enough to replace the constant $\rho_{E,\max}$ by the maximum of $\rho_{E,\max}$ with respect to $a_E$ and $c_E$ among the possible range of $a_E$ and $c_E$.

Indeed, our condition for the random hash function $f_H$ can be relaxed to $c$-almost universal dual hash function [38], whose survey with non-quantum terminology is available in [29]. The latter class allows more efficient random hash functions with less random seeds [36]. Even when the random seeds $H$ is not uniform random number, we have similar evaluations by attaching the discussion in [36]. Indeed, it is possible to apply left over hashing lemma [39], [40] and smoothing to the min entropy [41] to our analysis. However, as is discussed in [26], [29], our evaluation is better than such a combination even in the asymptotic limit.

IV. Eve’s attacks and solutions

Now, we consider two possible attacks by Eve. Firstly, Eve can prepare $k$ antennas. When Eve prepares $k$ antennas and receives signals $E_j$ ($j = 1, \ldots, k$), these signals can be transformed to $E' := \sum_{j=1}^k \frac{1}{k} E_j$ and its orthogonal components via orthogonal transformation. Since these orthogonal components are independent of $B$ and $E'$, we can assume that Eve receives only $E'$ without loss of generality. When we replace $E'$ by $E := \frac{1}{k} E'$, we can apply the above analysis with replacement of $c_E$ by $\frac{c_E}{k}$. Hence, when there is a possibility that Eve prepares plural antennas, it is enough to set the constant $c_E$ to be a sufficiently small number. When Eve prepares infinitely many antennas, Alice and Bob cannot disable Eve to access their secret information. However, considering the constraint for Eve’s budget, Alice and Bob can assume a reasonable value for the constant $c_E$.

As another attack, Eve generates an artificial noise $X_E$ known to Eve and inserts it to the signal $B$ received by Bob as Fig. 2 Then, the correlation coefficient between $E$ and $B$ becomes larger than their
expectation. However, this attack can be detected by estimating the noise between Alice and Bob as follows. When Eve inserts a large artificial noise, the condition (6) does not hold. So, Alice and Bob discard their own random number in this round.

![Diagram](image)

**Fig. 2.** Eve inserts artificial noise to Bob’s observation

However, there exists a possibility that Eve inserts the artificial noise in the level satisfying the condition (6) and the artificial noise is subject to the white Gaussian distribution. That is, we need to discuss the case when Eve knows $X_2$ as well as $E$. In this case, the security analysis falls in the common noise attack, i.e., the case when $b_E$ is chosen to be the optimal value $b_{E,o}$ as follows. For this analysis, we introduce two random variables $E' := a_E X_1 + c_E X_4 + b_{E,o} X_2$ and $E'' := \frac{b_{E,o}}{\sqrt{a_E^2 + c_E^2}} (a_E X_1 + c_E X_4) - \sqrt{a_E^2 + c_E^2} X_2$. We see that $E'$ is independent of $E''$ because the covariance is $\frac{b_{E,o}}{a_E^2 + c_E^2} (a_E^2 + c_E^2) - b_{E,o} \sqrt{a_E^2 + c_E^2} = 0$. Since the covariance between $B$ and $E''$ is $\frac{b_{E,o}}{a_E^2 + c_E^2} a_E a_B - \sqrt{a_E^2 + c_E^2} b_B = \frac{b_{E,o}}{a_E a_B} a_E a_B - \sqrt{a_E^2 + c_E^2} b_B = 0$, $E''$ is independent of $B$. So, since the pair of $E'$ and $E''$ has a one-to-one correspondence with the pair of $E$ and $X_2$, we can consider that Eve knows only the random variable $E'$ without loss of generality. This case has been already discussed in Theorem 1 as the optimal case.

If Eve can change the artificial noise dependently of the pulse, Eve can insert the large artificial noise only to the pulses that Alice and Bob use to the sampling for $X_1$ and $B$. The amount of artificial noise does not satisfy the condition (6). Then, Eve can succeed in eavesdropping without detection by Alice and Bob. Currently, we might not have such a technology, however, we cannot deny such an eavesdropping in future. Fortunately, there is a solution for this attack as follows. Alice and Bob do not fix the sampling pulse priorly. They choose the sample pulse after the transmission from Alice to Bob as the random sampling. Then, Eve cannot selectively insert the artificial noise. In this solution, the post selection by Alice and Bob works effectively. This idea is essentially the same as the idea of BB84 protocol of quantum key distribution [42].

Further, there is a possibility that Eve generates a small artificial noise and the noise is not subject to the white Gaussian distribution. In such a case, the random variable $X_1$ is not subject to the white Gaussian distribution. Hence, due to the above sampling, Alice and Bob can detect this difference. That is, when the observed $X_1$ in their sampling is not subject to the white Gaussian distribution, Alice and Bob consider that there exists Eve’s attack and have to discard the obtained random variables. For example, if the average of $X_1$ is far from 0, they consider that the noise is not white Gaussian noise. Further, if the third cumulant of $X_1$ is far from 0, they consider that the noise is not Gaussian noise because the random variable $X_1$ is subject to the Gaussian distribution if and only if its third cumulant is zero.

Rigorously, in this protocol, Alice and Bob need to authenticate each other. In this case, Alice and
Bob prepare several common secret keys to authentication. However, the length of the keys for the authentication is smaller than the length of generated keys. This problem was well studied in the literatures [34], [35], [43], [43]. Indeed, the authentication can be done in the same way as error verification as explained in [45] Section VIII. In this protocol, we can check the non-existence of disagreement of keys between Alice and Bob by error verification. The concrete protocol for authentication and error verification is available in [45] Section VIII. So, this protocol well works totally.

V. ARTIFICIAL NOISE BY BOB

When the noise generated outside Bob’s detector is too strong and/or Eve has so good detector and/or so many detectors, it is quite difficult to select the case when the condition (6) holds. In this case, Alice and Bob can realize the case when the condition (6) holds, in the following way. As discussed in the literature [20], Bob can generate an artificial noise that does not effect Bob’s observation almost. This operation does not change \( b_B \) and increases \( c_E^2 \). Hence, even when the intensity of the artificial noise is not so strong as the original Alice’s signal, Alice and Bob can make the condition (6) valid by generating an artificial noise in this way.

However, it might be difficult to realize that the additional noise does not effect Bob’s observation at all [21]. So, we discuss the case when the effect to Bob’s observation exists but is much smaller than the effect to Eve’s observation. Now, we denote the effects to Bob’s and Eve’s observations by \( d_B X_5 \) and \( d_E X_5 \), respectively. That is, we have the model:

\[
B := a_B X_1 + b_B X_2 + d_B X_5 + c_B X_3, \quad (16)
\]
\[
E := a_E X_1 + b_E X_2 + d_E X_5 + c_E X_4, \quad (17)
\]

where \( X_5 \) is subject to the standard Gaussian distribution independently of \( X_i \) for \( i = 1, 2, 3, 4 \). That is, Bob generates the noise \( X_5 \) subject to the standard Gaussian distribution.

Next, we consider the case when Bob knows the value \( X_5 \). In this case, since our channel is quasi static, Alice and Bob can estimate the value \( a_B, b_B^2 + c_B^2 \), and \( d_B \) by the sampling for \( X_1, B, \) and \( X_5 \) because the covariance of \( X_1 \) and \( B \) is \( a_B \), the covariance of \( X_5 \) and \( B \) is \( d_B \), and the variance of \( X_1 \) is \( a_B^2 + b_B^2 + c_B^2 + d_B^2 \). Since they priorly know \( c_B \), they can estimate \( b_B \). Hence, Bob obtains the random variable

\[
B' := B - d_B X_5 = a_B X_1 + b_B X_2 + c_B X_3. \quad (18)
\]

Then, Alice and Bob can apply the protocol given in Section III to \( B' \). In this case, we can apply the above discussion by replacing \( c_E \) by \( \sqrt{c_E^2 + d_E^2} \) in the same way as the case when \( d_B = 0 \).

Now, we consider a more difficult case, i.e., the case when Bob cannot estimate the noise \( X_5 \). Then, the correlation coefficients \( \rho_A \) and \( \rho_E(b_E) \) are calculated as

\[
\rho_A^2 = \frac{a_B^2}{a_B^2 + b_B^2 + c_B^2 + d_B^2}, \quad (19)
\]
\[
\rho_E(b_E)^2 = \frac{(a_B a_E + d_B b_E + d_B d_E)^2}{(a_B^2 + b_B^2 + c_B^2 + d_B^2)(a_E^2 + b_E^2 + c_E^2 + d_E^2)}. \quad (20)
\]

Now, we obtain the following theorem.

**Theorem 4:** The maximum value of the correlation coefficient \( \rho_E(b_E) \) are characterized as follows.

\[
\max_{b_E} \rho_E(b_E)^2 = \frac{1}{a_B^2 + b_B^2 + c_B^2 + d_B^2} \left( \frac{(a_B a_E + d_B d_E)^2}{a_E^2 + c_E^2 + d_E^2} + b_B^2 \right). \quad (21)
\]

This maximum is realized when \( b_E = b_{E,o} := \frac{d_B (a_E^2 + c_E^2 + d_E^2)^2}{a_B a_E + d_B d_E} \).

Since the RHS of (21) is monotone decreasing for \( |d_E| \) when \( d_B^2 \) is sufficiently small, we need a lower bound of \( |d_E| \). To receive Alice’s signal, Eve’s detector is not so far from Alice’s transmitting antenna.
Since Bob is not so far from Alice’s transmitting antenna, Eve’s detector is not so far from Bob. Based on this assumption, we can derive a lower bound of $|d_E|$. That is, we can substitute the lower bound into $|d_E|$.

**Lemma 5:** The inequality \( \frac{1}{a_B^2 + b_B^2 + d_E} + \frac{a_B a_E + b_B d_E}{a_B^2 + b_B^2 + d_E} + \frac{b_B^2}{a_B^2 + b_B^2 + d_E} \) holds if and only if \( d_B + \frac{d_E}{2a_B a_E} d_B^2 < \frac{a_B^2}{2d_E a_B a_E} \left(1 - \frac{\sqrt{c_E^2 + d_E^2}}{a_B^2 + b_B^2 + d_E^2} \right)^2 \). (22)

Now, we consider the above ideal case with \( d_B = 0 \). It is enough to replace \( c_E \) by \( \sqrt{c_E^2 + d_E^2} \). So, the condition (6) is equivalent to the positivity of the RHS of (22), i.e., the positivity of \( 1 - \frac{\sqrt{c_E^2 + d_E^2}}{a_B^2 + b_B^2 + d_E^2} \).

In our situation, \( |d_B| \) is not zero but is sufficiently small in comparison to \( |d_E| \). So, the condition (22) can be simplified to the condition that \( d_B \) is smaller than the RHS of (22). That is, to generate the key, Bob needs to control the coefficient \( d_B \) to satisfy this condition.

When Alice and Bob cannot estimate the exact values of \( d_B \) but can estimate its range \( D \). In this case, they cannot exactly estimate the value \( b_B \) as well. They can estimate only the value \( \sqrt{b_B^2 + d_B^2} \) by the sampling for \( X_1 \) and \( B \). Hence, they obtain only the range \( \hat{D} \) of the two parameters \((b_B, d_B)\). In this case, they need to consider the maximum value of the RHS of (22) with the range \( \hat{D} \).

Now, we consider the case when Eve knows the value \( X_2 \) as well as \( E \). Similar to the discussion in the end of Section [IV], we introduce two random variables \( E' := a_E X_1 + c_E X_4 + d_E X_5 + b_E X_2 \) and \( E'' := \sqrt{c_E^2 + d_E^2} (a_E X_1 + c_E X_4 + d_E X_5) - \sqrt{a_E^2 + d_E^2} X_2 \). We can show that \( E' \) is independent of \( E'' \) and \( B \) in the same way. Hence, the security analysis falls in the case when \( b_E \) is chosen to be the optimal value \( b_{E,o} \).

Indeed, even might detect the existence of the artificial noise \( X_5 \) because the total intensity becomes larger than the natural case when the artificial noise \( X_5 \) exists. However, even though Eve knows it, Eve cannot discard such a case disadvantageous to Eve. Overall, Bob can employ the artificial noise effectively but Eve cannot employ it effectively because the post selection can be done only by the pair of Alice and Bob. In these scenarios, Eve can insert an artificial noise to \( B \) only with an amount satisfying the condition (6) while Bob can insert an artificial noise to \( E \) with a larger amount as Fig. 3. In practice, the artificial noise generated by Bob has some restriction due to a legal constraint. However, this restriction is much weaker than that for the artificial noise generated by Eve. In this way, Bob can make the situation advantageous to Alice and Bob.

![Battle between Bob and Eve with artificial noises](image-url)
VI. PROOFS

A. Proofs of Theorems 1 and 4 and Lemmas 2 and 5

For our proofs of Theorems 1 and 4, we prepare the following lemma.

Lemma 6: For positive real numbers \( \alpha \) and \( \gamma \) and a real number \( \beta \), we have

\[
\max_x C \left( \frac{\alpha x + \beta^2}{x^2 + \gamma} \right) = C \left( \frac{\beta^2}{\gamma} + \alpha^2 \right). 
\]  

The maximum is attained when \( x = \frac{\alpha \gamma}{\beta^2} \).

Proof: Define the function \( f(x) := \frac{(\alpha x + \beta^2)}{x^2 + \gamma} \). Then, the first derivative is \( f'(x) = \frac{-2\alpha x^2 + 2(\alpha^2 \beta^2 - \beta^2) x + 2\alpha \beta \gamma^2}{(x^2 + \gamma)^2} \).

The solution of \( f'(x) = 0 \) is \( x = \frac{\alpha \gamma}{\beta^2}, -\frac{\beta}{\alpha} \). So, the first derivative test shows that the maximum is \( C f \left( \frac{\alpha \gamma}{\beta^2} \right) = C \left( \frac{\beta^2}{\gamma} + \alpha^2 \right) \) in spite of the sign of \( \beta \).

Therefore, applying Lemma 6 to the case with \( \alpha = b_B, \beta = a_B a_E, \gamma = a_E^2 + c_{2, E}^2, C = \frac{1}{a_B b_B + c_{2, B}^2 + d_B^2} \), and \( x = b_E \), we obtain Theorem 1. Similarly, applying Lemma 6 to the case with \( \alpha = b_B, \beta = a_B a_E + d_B d_E, \gamma = a_E^2 + c_{2, E}^2 + d_{E}^2, C = \frac{1}{a_B b_B + c_{2, B}^2 + d_B^2} \), and \( x = b_E \), we obtain Theorem 4.

Now, we show Lemma 2. The condition \( \frac{1}{a_B b_B + c_{2, B}^2 + d_B^2} \left( \frac{a_B a_E + d_B d_E}{a_B^2 + c_{2, E}^2 + d_{E}^2} \right)^2 + b_B^2 < \frac{a_B^2}{a_B b_B + c_{2, B}^2 + d_B^2} \) is equivalent to \( 0 < a_E^2 - \left( \frac{a_B^2 a_E + d_B d_E}{a_B^2 + c_{2, E}^2 + d_{E}^2} \right)^2 + b_B^2 \).

This condition is equivalent to

\[
0 < a_E^2 - \left( \frac{a_B a_E d_E d_B + d_{E}^2 d_{B}^2}{a_B^2 c_{2, E}^2 + d_{E}^2 c_{2, B}^2 + d_{E}^2 d_{B}^2} \right) < a_B b_B + c_{2, B}^2 + d_B^2.
\]

This condition is equivalent to

\[
1 - \frac{2 a_B a_E d_E d_B + d_{E}^2 d_{B}^2}{a_B^2 c_{2, E}^2 + d_{E}^2 c_{2, B}^2 + d_{E}^2 d_{B}^2} \geq \frac{b_B^2}{a_B^2} \left( 1 + \frac{a_E^2}{c_{2, E}^2 + d_{E}^2} \right).
\]

So, we have

\[
\frac{2 a_B a_E d_E d_B + d_{E}^2 d_{B}^2}{a_B^2 c_{2, E}^2 + d_{E}^2 c_{2, B}^2 + d_{E}^2 d_{B}^2} \leq 1 - \frac{b_B^2}{a_B^2} \left( 1 + \frac{a_E^2}{c_{2, E}^2 + d_{E}^2} \right).
\]

Dividing both sides by \( \frac{a_B a_E d_E d_B + d_{E}^2 d_{B}^2}{a_B^2 c_{2, E}^2 + d_{E}^2 c_{2, B}^2 + d_{E}^2 d_{B}^2} \), we obtain Lemma 5.

B. Proof of (8) and (9)

In the calculation of \( g(\rho) \) and \( \phi_p(t) \), it is enough to consider the case when \( X \) and \( B \) are subject to the standard Gaussian distribution. Then, the joint probability density function of \( X \) and \( B \) is given as

\[
P_{X, B}(x, b) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2(\pi\rho^2)}(x^2 - 2\rho x b + b^2)}
\]

\[
= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2(\pi\rho^2)}(b - \rho x)^2 - \frac{1}{2}x^2}.
\]

The joint probability density function \( P_{B, X}(0, x) \) is calculated as

\[
P_{B, X}(0, x) = \int_0^\infty P_{X, B}(x, b) db = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2x^2}} \int_0^\infty \frac{1}{\sqrt{2\pi(1 - \rho^2)}} e^{-\frac{1}{2(\pi\rho^2)}(b - \rho x)^2} db = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} Q(\rho, x)
\]  

(24)
where \( y = \frac{b-\rho x}{\sqrt{1-\rho^2}} \). Similarly, we have
\[
P_{B',X}(1, x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} (1 - Q(\rho, x)).
\] (25)

Since \( P_{B',X}(0, x) + P_{B',X}(1, x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \), we have
\[
\log \frac{P_{B',X}(0, x) + P_{B',X}(1, x)}{P_{B',X}(0, x)} = -\log Q(\rho, x).
\] (26)

Thus,
\[
\int_{-\infty}^{\infty} P_{B',X}(0, x) \log \frac{P_{B',X}(0, x) + P_{B',X}(1, x)}{P_{B',X}(0, x)} dx = -\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} Q(\rho, x) \log Q(\rho, x) dx.
\] (27)

Since the second term of (7) is the same as the first term, we obtain (8). Substituting (24) and (25) into the definition of \( \phi_{\rho}(t) \), we obtain (9).

**C. Proof of Lemma 3**

The function \( \rho \mapsto \rho \frac{1}{1-t} + (1 - \rho) \frac{1}{1-t} \) is monotone decreasing in \([0, \frac{1}{2}]\) and is monotone increasing in \([\frac{1}{2}, 1]\). The function \( \rho \mapsto Q(x, \rho) \) is monotone increasing in \([0, 1]\) for \( x > 0 \) and is monotone decreasing in \([0, 1]\) for \( x < 0 \). Since \( Q(0, \rho) = \frac{1}{2} \), we conclude that the function \( \rho \mapsto (Q(x, \rho) \frac{1}{1-t} + (1 - Q(x, \rho)) \frac{1}{1-t}) \) is monotone increasing in \([0, 1]\) for any \( x \). So, \( \phi_{\rho}(t) \) is monotone increasing for \( \rho \).

Now, we introduce Gallager function
\[
E_{0}(t|P_{B'|X}, P_{B'}) := \log \int_{-\infty}^{\infty} \left( \sum_{b'} P_{B'}(b) P_{X|B'}(x|b') \right) \rho, x (1-t) dx,
\]
which is known to be convex for \( t \) [46]. Since \( e^{\phi_{\rho}(t)} = e^{E_{0}(s|P_{B'|X}, P_{B'})} \left( \frac{1}{2} \right)^{1-t} (1-t) \), we have \( \phi_{\rho}(t) = E_{0}(t|P_{B'|X}, P_{B'}) + t \log 2 \), which shows that \( \phi_{\rho}(t) \) is convex for \( t \).

**D. Proofs of (14) and (15)**

Now, we show (14) and (15). For this purpose, we introduce a function for a joint distribution \( P_{X,Y} \) as \( \phi(t|X|Y|P_{X,Y}) := \int_{Y} \sum_{x} P_{X,Y}(x, y) \rho, x (1-t) dy \), which is denoted by \( -\bar{H}^{G}_{1}(X|Y|P_{X,Y}) \) in [29] or \( -\bar{H}^{Gl}_{1}(X|Y|P_{X,Y}) \) in [28]. The function \( \phi(t|X|Y|P_{X,Y}) \) is a generalization of \( \phi_{\rho}(t) \). Applying [26] (67) and [28] (21), we have
\[
d(K : E|H) \leq 3 \min_{t \in [0, \frac{1}{2}]} e^{tn(R_{1} - R_{2}) + \phi(tB''|E^{|n}|P_{B'|n,E^n})} \tag{28}
\]
\[
\leq 3 \min_{t \in [0, \frac{1}{2}]} e^{tn(R_{1} - R_{2}) + tn(\log 2 - R_{1}) e^{\phi(tB''|E^{|n}|P_{B'|E^n,E^n})} \tag{29}
\]
\[
= 3 \min_{t \in [0, \frac{1}{2}]} e^{tn(\log 2 - R_{2})} e^{n(\phi(tB'|E|P_{B'|E}) \tag{30}
\]
\[
= 3 \min_{t \in [0, \frac{1}{2}]} e^{tn(\log 2 - R_{2})} e^{n(\phi_{E|B}(t)) \tag{31}
\]
where (a) and (b) follows from [26] (67) and [28] (21), respectively. So, we obtain (15). When we replace the role of [26] (67) by [29] (54) and Lemma 22, we obtain a similar evaluation as (15) for \( \epsilon \)-almost universal dual hash function.
Now, we introduce another function for a joint distribution $P_{X,Y}$ as

$$H_{1+s}(X|Y|P_{X,Y}) := -\frac{1}{s} \log \int_Y \left( \sum_x P_{X|Y=y}(x)^{1+s} \right) P_Y(y)dy.$$  

We denote $sH_{1+s}(X|Y|P_{X,Y})$ by $H_{1+s}(X|Y|P_{X,Y})$ in [27] or $sH_{1+s}^L(X|Y|P_{X,Y})$ in [29]. Applying [27 (3)], [29, Lemma 5], and [28 (21)], we have

$$I(K : E|H) \leq \inf_{s \in (0,1)} \frac{1}{s} e^{sn(R_1-R_2)} \leq \inf_{s \in (0,1)} \frac{1}{s} e^{sn(R_1-R_2)+sn} e^{\phi(s)|E_n|P_{E_n|P_{E_n}}(s)} \leq \inf_{s \in (0,1)} \frac{1}{s} e^{sn(R_1-R_2)+sn} e^{\phi(s)|E_n|P_{E_n|P_{E_n}}(s)}$$

where $(a)$, $(b)$, and $(c)$ follow from [27 (3)], [29, Lemma 5], and [28 (21)], respectively. So, we obtain [14]. When we need evaluation with $\epsilon$-almost universal dual hash function, it is sufficient to replace the role of [27 (3)] by [29 (56) and Theorem 23].

VII. DISCUSSION

Under our model (1) and (2) allowing the common noise attack, in Theorem 1 and Lemma 2, we have derived a necessary and sufficient condition (6) of the coefficients $a_B, b_B, e_B, a_E, c_E$ for realizing greater the correlation coefficient, i.e., mutual information between Alice and Bob than that between Bob and Eve under a spatial condition for Eve. We have also derived rigorous and exact security evaluation for finite block-length final keys quantitatively when we employ the reverse information reconciliation. Since the calculation complexity of the upper bounds (14) and (15) do not depend on the block length $n$, this bound can be applied to any size of final keys. Further, since the calculation complexity of realization of required random hash function is only $O(n \log n)$ [36], the proposed protocol is implementable.

Further, we have discussed two Eve’s attacks in our model, increasing the number of Eve’s antennas and Eve’s controlling Bob’s noise. Then, we have proposed the solutions for these two attacks. The post selection by Alice and Bob works effectively as the solution for the latter attack. For an implementation of our protocol in a real secure wireless communication, we need to care about the precision of the estimation of channel parameters $a_B, b_B$ (and $d_B$) dependently of the coherent time. To cover the possibility of Eve’s controlling Bob’s noise, we need to attach the precise testing method to guarantee that the random variable $B$ is subject to the white Gaussian distribution. Combining these statistical procedure is remained to a future research.

There is a possibility to realize the condition (6). For our solution for this issue, we have proposed the method that Bob inserts an artificial noise. Under this situation, we have derived a necessary and sufficient condition (22) for realizing greater the correlation coefficient, i.e., mutual information between Alice and Bob than that between Bob and Eve. In this scenario, Bob can insert any amount of artificial noise but Eve can insert only small amount of artificial noise satisfying the condition because only Alice and Bob can decide which pulses are used for key generation by their post selection.

For a realization of secure wireless communication, we need to investigate whether the obtained necessary and sufficient condition for secure communication holds with realizable communication. Since a larger minimum distance between Alice and Eve satisfies the condition, we need to derive the threshold for the distance. To clarify the threshold, we need numerical analysis based on real wireless communication...
model as the next step. As another future study, we need to improve our model to reflect various effects overlooked in this model and optimize Eve’s strategy in the improved model because the proposed model might be too simple. Improving our model, we can enhance the quality of the security of our the secure wireless communication. Also, we need to extend our result to multiple-input and multiple-output (MIMO) case.

Indeed, secure wireless communication brings us an information-theoretic security only with a reasonable assumption for Eve, which is contrastive with quantum key distribution that provides unconditional security. Improving our model, we can clarify the assumption for Eve. Such a clarification promotes the introduction of secure wireless communication because the customers can understand what kind of eavesdropper cannot eavesdrop their communication. Since the intended purpose of secure wireless communication is different from that of quantum key distribution, secure wireless communication might be used in the mobile phone of ordinary people.

ACKNOWLEDGMENTS

The author is very grateful to Professor Hideichi Sasaoka and Professor Hisato Iwai for helpful discussions. He is also grateful to Dr. Shun Watanabe, Dr. Himanshu Tyagi, and Dr. Toyohiro Tsurumaru for helpful comments. The works reported here were supported in part by the Okawa Reserach Grant and Kayamori Foundation of Informational Science Advancement.

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