Nonpoisson queueing analysis of patas bus on the west and east line at Tirtonadi Surakarta bus station

M Asri¹, Sugito¹, A Hoyyi¹ and A R Hakim¹
¹Department of Statistics, Diponegoro University
Jl. Prof. Soedharto, SH, Tembalang, Semarang 50275, Indonesia
E-mail: pratiwimaharani5@gmail.com

Abstract. Queue system is a group of customer, service and some rules to regulate arrival customers. Queue happen if customers which need a serve more than service capacity. Phenomenon of queue will find easily in public facility, one of them bus queue at Tirtonadi Surakarta main bus station. Bus lines observed are Solo-Yogyakarta, Solo-Semarang, Surabaya-Solo-Semarang, Solo-Jakarta and Solo-Surabaya routes. The discipline of the queue is FIFO (First In First Out). Based on the analysis result, the steady state condition of the five tracks is eligible. The model chosen is a model that has 3 types of distribution that are Lognormal, Weibull and Beta distributions. From the five lines, there are 3 service system model in Tirtonadi main bus station (G/Weib/1):((GD/∞/∞) for Solo-Yogyakarta, (Logn/G/1):(GD/∞/∞) for Solo-Semarang, Surabaya-Solo-Semarang and Solo-Jakarta, and (M/Beta/1): (GD/∞/∞) for Solo-Surabaya a by showing good system performance measure.

1. Introduction
Queue theory can be used to evaluate queue phenomena from the point of view of customers and service providers, so that optimal solutions will be generated. Service providers still make a profit and customers do not feel complained about long queues. The purpose of the queue system is to minimize total costs which is the sum of the costs incurred due to waiting and the costs incurred due to adding service facilities.

The queue phenomenon appears to be easily found in public service facilities, one of which is seen in the queue of bus lines west and east on Tirtonadi Bus Station in Surakarta. The Tirtonadi Surakarta Bus station was inaugurated as a type A bus station. However, the management was still handed over to the Surakarta City Government. Furthermore, the busiest bus station is the largest bus station in Surakarta City which was inaugurated as a type A bus station which was used as a national pilot. The Tirtonadi Surakarta Bus station operates 24 hours because it is a route that connects bus transportation from East Java and West Java[1]. Every day the number of buses going in and out of the bus station is very large, both buses that serve routes in the city and out of town. The increase in the number of intercity bus routes causes a queue of buses on the departure lane.

One way to reduce the problem that occurs in a queue is to apply the queue theory to the service system at the bus station. The steps that need to be done is to conduct a research on the place which in the end will be made a decision whether the path being studied is good or not.

2. Literature review
2.1. General description of Tirtonadi Surakarta bus station
The Tirtonadi Surakarta bus station was inaugurated as a type A bus station. However, the management was still handed over to the Surakarta City Government. Furthermore, the busiest bus
station in Central Java will be used as a national pilot so that regional bus stations in Indonesia can be built and managed to imitate Tirtonadi, so that the bus transportation mode is again in demand by the public. At Tirtonadi Bus station there is also a sky bridge, which connects Tirtonadi with the Solo Balapan Railway Station along 438 meters.

2.2. Steady State
Steady state is a condition when the properties of the system do not change with time (constant). According to Taha (1996)[2], for example, \( \lambda \) is the average customer arrival to service place per time unit, \( \mu \) is the average customer that has been served per unit of time, and \( c \) is the number of service facilities (server), then \( \rho \) is defined as the ratio between average customer arrivals (\( \lambda \)) with the average customer served per time unit (\( \mu \)), or can be written as follows:

\[
\rho = \frac{\lambda}{c\mu}
\]

2.3. Poisson Process and Exponential Distribution
Generally the queue process is assumed that the time between arrivals and service times follows the Exponential distribution, or equal to the number of arrivals and the number of services following the Poisson distribution. The number of events stated \( \{N(t), t \geq 0\} \) will be said as a summation process if \( N(t) \) shows the number of arrival numbers that occur until time \( t \), with \( N(0) = 0 \) and will be declared as a Poisson process if it meets three assumptions namely regularity, homogeneity in time, and independence.

2.4. Lognormal Distribution
Variables in a system sometimes follow an exponential relationship as \( x = \exp(w) \). If the exponent is a random variable \( W \), then \( X = \exp(W) \) is a random variable with a distribution of interest. An important special case occurs when \( W \) has a normal distribution. In that case, the distribution of \( X \) is called a lognormal distribution. The name follows from the transformation \( \ln(X) = W \). That is, the natural logarithm of \( X \) is normally distributed. Probabilities for \( X \) are obtained from the transform of the normal distribution. The range of \( X \) is \((0, \theta)\). Let \( W \) have a normal distribution with mean \( \theta \) and variance \( \omega^2 \); then \( X = \exp(W) \) is a lognormal random variable with probability density function :

\[
f(x) = \begin{cases} 
\frac{1}{x\omega\sqrt{2\pi}} e^{-\frac{1}{2\left(\ln x - \theta\right)^2}}, & x > 0 \\
0, & x \leq 0
\end{cases}
\]

The mean and variance of \( X \) are:

\[
E(x) = e^{\theta + \frac{1}{2}\omega^2}
\]

\[
V(x) = e^{2\theta + \omega^2}\left(e^{\omega^2} - 1\right)
\]

2.5. Weibull Distribution
Modern technology has enabled engineers to design many complicated systems whose operation and safety depend on the reliability of the various components making up the systems. For example, a fuse may burn out, a steel column may buckle, or a heat-sensing device may fail. Identical components subjected to identical environmental conditions will fail at different and unpredictable times. We have seen the role that the gamma and exponential distributions play in these types of problems. Another distribution that has been used extensively in recent years to deal with such problems is the Weibull distribution, introduced by the Swedish physicist Waloddi Weibull in 1939.

The function of the Weibull distribution density with the parameters \( \alpha \) and \( \beta \) is
\[ f(x) = \alpha \beta x^{\beta-1} e^{-\alpha x^\beta}, \quad x > 0 \]

The mean and variance of parameters \( \alpha \) and \( \beta \) are:

\[
E(x) = \alpha^{-\beta} \Gamma \left( 1 + \frac{1}{\beta} \right) \\
V(x) = \alpha^{-\beta} \left\{ \Gamma \left( 1 + \frac{2}{\beta} \right) - \left[ \Gamma \left( 1 + \frac{1}{\beta} \right) \right]^2 \right\}
\]

2.6. Beta Distribution

A continuous distribution that is flexible but bounded over a finite range is useful for probability models. The proportion of solar radiation absorbed by a material or the proportion (of the maximum time) required to complete a task in a project are examples of continuous random variables over the interval \([0, 1]\).

The function of the Weibull distribution density with the parameters \( \alpha \) and \( \beta \) is

\[
f(x) = \begin{cases} 
\frac{1}{B(\alpha, \beta)} x^{\alpha-1} (1-x)^{\beta-1}, & 0 < x < 1 \\
0, & \text{others}
\end{cases}
\]

The mean and variance of parameters \( \alpha \) and \( \beta \) are:

\[
E(x) = \frac{\alpha}{\alpha + \beta} \\
V(x) = \frac{\alpha \beta}{(\alpha + \beta)^2 (\alpha + \beta + 1)}
\]

2.7. Queue System Model

2.7.1 Queue Model (M/M/1) : (GD/\infty/\infty) According to Taha (1996)\[2\] this service model is a single service model with no limits on the capacity of the system or the capacity of the calling source. The following is a formula for obtaining all basic measures of service model performance (M/M/1):(GD/\infty/\infty):

1. Number of customers expected in line:

\[
L_q = L_s - \frac{\lambda}{\mu} = \frac{\rho^2}{1-\rho}
\]

2. Number of customers estimated in the system:

\[
L_s = E(n) = \frac{\rho}{1-\rho}
\]

3. The expected waiting time in line:

\[
W_q = \frac{L_q}{\lambda} = \frac{\rho}{\mu(1-\rho)}
\]

4. The estimated waiting time in the system:

\[
W_s = \frac{L_s}{\lambda} = \frac{1}{\mu(1-\rho)}
\]
2.7.2 Queue Model (M/G/1): (GD/∞/∞) or Formula Pollazck-Khintchine. Pollazck-Khintchine formula is often called a formula (P-K). Kakiay (2004) [3] states that this formula is described through a single service with a situation based on the following three assumptions:

a. The arrival distribution follows a Poisson process with an average level of $\lambda$.
b. General service time distribution with expectations (average) $E[t] = 1 / \mu$ and var variant $[t]$ 
c. The state of steady state is expressed by $\rho = \frac{\lambda}{\mu} < 1$

The following is a formula for obtaining all basic measures of service model performance (M/G/1):(GD/∞/∞):

1. Number of customers expected in line:
   $$L_q = \frac{\lambda}{\mu}$$
2. Number of customers estimated in the system:
   $$L_s = \rho + \frac{\rho^2 + \lambda^2 \text{var}(t)}{2(1 - \rho)}$$
3. The expected waiting time in line:
   $$W_q = \frac{L_q}{\lambda}$$
4. The expected waiting time in the system:
   $$W_s = \frac{L_s}{\lambda}$$

2.7.3 Queue Model (G/G/c): (GD/∞/∞). According to Gross and Harris (1998)[4], the queue model (G/G/c) : (GD/∞/∞) is a queue model with a general distribution (General) arrival pattern, general distribution arrival pattern, with $c$ service facilities as many as service. The queue discipline used in this model is general FIFO (First In First Out), the maximum capacity allowed in the system is $\infty$. System performance measures in this General model follow the performance measures on the M/M/c model, except for the calculation of the number of customers estimated in line (Lq) are as follows:

1. Number of customers expected in line:
   $$L_q = \frac{\lambda}{\mu}$$
2. Number of customers estimated in the system
   $$L_s = L_q + \frac{\mu^2 v(t) + v(t') \lambda^2}{2}$$
3. The expected waiting time in line
   $$W_q = \frac{L_q}{\lambda}$$
4. The expected waiting time in the system
3. Research methodology

The research was carried out at Tirtonadi bus station with the implementation time on 18 to 23 December 2017 for the west lane and 25 to 30 December 2017 for the east lane. The study was conducted from 08.00 to 17.00 WIB every day. The data used is bus arrival data and bus service time data.

3.1 Analysis Step

The steps in carrying out research and data analysis are as follows:

1. Determine the place of research and the method to be used.
2. Conducting direct research on the Patas bus in Tirtonadi bus station.
3. Data obtained must meet steady-state conditions ($\rho < 1$).
4. Test the suitability of distribution for arrivals and visitors served by using the Kolmogorov Smirnov and Chi-Square tests.
5. Determine the appropriate queue model. Judging from the distribution of arrivals, distribution of services, number of servers (number of services), queue discipline used (FIFO, LIFO, SIRO or Priority), capacity in the system and source of calling.
6. Determine system performance, namely the number of customers estimated in the system ($L_s$), the number of customers estimated in the queue ($L_q$), waiting time in the system ($W_s$), and waiting time in the queue ($W_q$).
7. Make results and discussions obtained from system performance measures. With this system performance measurement can be obtained an optimal model.
8. To draw conclusions about the service at the Tirtonadi bus station in Surakarta as a whole.

4. Results and discussions

4.1 General description of queue system in Tirtonadi bus station

The Tirtonadi Surakarta bus station is the largest bus station in the city of Surakarta. This bus station is located in Banjarsari District, Surakarta. This bus station operates 24 hours because it connects bus transportation from West Java and East Java. At Tirtonadi bus station there are 2 lines that operate, namely the west and east lines. The queue model at the Tirtonadi Surakarta bus station has 3 (three) types of service, namely service at the arrival post, service at the passenger service post, and service at the departure post. At each service post each bus will be grouped according to its route. There are 5 lines for Patas buses, namely Solo - Yogyakarta, Solo - Semarang, Surabaya - Solo - Semarang, Solo - Jakarta, and Solo - Surabaya.

4.2 Descriptive analysis of traffic

Table 1. Total number of patas buses in the west and east line at Tirtonadi bus station

| Days       | Solo – Yogyakarta | Solo – Semarang | Solo – Jakarta | Solo – Surabaya | Total |
|------------|-------------------|-----------------|---------------|-----------------|-------|
| Monday     | 95                | 61              | 57            | 49              | 31    | 293   |
| Tuesday    | 67                | 46              | 30            | 19              | 25    | 187   |
| Wednesday  | 72                | 27              | 41            | 34              | 35    | 209   |
| Thursday   | 79                | 27              | 32            | 24              | 36    | 198   |
| Friday     | 69                | 34              | 17            | 12              | 38    | 170   |
| Saturday   | 62                | 50              | 17            | 11              | 39    | 179   |
| Total      | 444               | 245             | 194           | 149             | 204   | 1236  |

4.3 Steady state
From the research data obtained the steady-state values of the five majors as follows:

Table 2. Steady state

| Data       | Line                           | $\rho$  |
|------------|--------------------------------|---------|
| Solo – Yogyakarta |                                | 0.34259 |
| Solo – Semarang  |                                | 0.51924 |
| Total – Time | Surabaya – Solo – Semarang    | 0.06287 |
| Solo – Jakarta   |                                | 0.04599 |
| Solo – Surabaya  |                                | 0.17490 |

The usability value is less than one, which means that the steady-state condition is fulfilled, meaning that the average bus arrival does not exceed the average service time. So that the bus service system in the five majors is good.

4.4 Test of Suitability of Distribution

The distribution suitability test used to test data on the number of arrivals and bus service times for the five fast bus majors is the Kolmogorov-Smirnov test. With the Kolmogorov-Smirnov test, it will be determined whether the data of the number of arrivals is Poisson distribution and the bus service time is exponential distribution. The following is the result of the Kolmogorov-Smirnov test for arrival data which will be determined whether the data is Poisson distributed or not.

Table 3. Test of suitability for data distribution of arrival amounts

| No. | Line                           | $D$  | $N$  | $D^*$ | $D<D^*$ | Distribution | Decision     |
|-----|--------------------------------|------|------|-------|---------|--------------|--------------|
| 1   | Solo-Yogyakarta               | 0.213| 54   | 0.185 | No      | Nonpoisson   | Ho rejected  |
| 2   | Solo-Semarang                 | 0.156| 162  | 0.107 | No      | Nonpoisson   | Ho rejected  |
| 3   | Surabaya-Solo-Semarang        | 0.337| 54   | 0.185 | No      | Nonpoisson   | Ho rejected  |
| 4   | Solo-Jakarta                  | 0.314| 54   | 0.185 | No      | Nonpoisson   | Ho rejected  |
| 5   | Solo-Surabaya                 | 0.099| 162  | 0.107 | Yes     | Poisson      | Ho accepted  |

The following is the result of the Kolmogorov-Smirnov test for service time data that will be determined whether the data has an exponential distribution or not.

Table 4. Compatibility test of service time data distribution

| No | Line                           | $D$   | $N$  | $D^*$ | $D<D^*$ | Distribution | Decision     |
|----|--------------------------------|-------|------|-------|---------|--------------|--------------|
| 1  | Solo-Yogyakarta                | 0.383 | 54   | 0.185 | No      | Non exponential | Ho rejected  |
| 2  | Solo-Semarang                  | 0.133 | 109  | 0.130 | No      | Non exponential | Ho rejected  |
| 3  | Surabaya-Solo-Semarang         | 0.253 | 41   | 0.212 | No      | Non exponential | Ho rejected  |
| 4  | Solo-Jakarta                   | 0.249 | 36   | 0.227 | No      | Non exponential | Ho rejected  |
| 5  | Solo-Surabaya                  | 0.272 | 130  | 0.119 | No      | Non exponential | Ho rejected  |

From the table 3 and tables 4 obtained the result that the queue model in the five majors is $(G/G/c):(GD/\infty/\infty)$ except in the Solo-Surabaya department which still contains the M (Poisson) model.
4.5 Queue System Models
To find out the actual distribution of the number of vehicles and the number of bus services that have a General distribution, a distribution test is conducted based on the Arena output.

| Line                  | Data         | Distribution | Output  | Decision |
|-----------------------|--------------|--------------|---------|----------|
| Solo – Yogyakarta     | Number of arrivals | General     | 0,0127  | H_0 rejected |
|                       | Service time | Weibull      | 0,0853  | H_0 accepted |
| Solo – Semarang       | Number of arrivals | Lognormal  | 0,265   | H_0 accepted |
|                       | Service time | General      | <0,005  | H_0 rejected |
| Surabaya – Solo – Semarang | Number of arrivals | Lognormal | 0,241   | H_0 accepted |
|                       | Service time | General      | <0,005  | H_0 rejected |
| Solo – Jakarta        | Number of arrivals | Lognormal  | 0,128   | H_0 accepted |
|                       | Service time | General      | <0,005  | H_0 rejected |
| Solo – Surabaya       | Number of arrivals | Poisson    |         |           |
|                       | Service time | Beta        | 0,0828  | H_0 accepted |

Table 5. Distribution match test based on total-time arena output

Based on Table 5, obtained the final model for the queue model from five lines, obtained 3 service system models at Tirtonadi Bus Station, namely (G/Weib/1):(GD/∞/∞) for Solo-Yogyakarta, (Logn/G/1):(GD/∞/∞) for Solo-Semarang, Surabaya-Solo-Semarang and Solo-Jakarta, and models (M / Beta / 1):(GD/∞/∞) for Solo-Surabaya.

4.6 System Performance Measures
Based on the output, bus queue system performance measures are obtained for the following five majors:

| Line                  | C     | \( \Lambda \) | \( \mu \) | \( L_q \) | \( L_s \) | \( W_q \) | \( W_s \) |
|-----------------------|-------|---------------|-----------|----------|----------|----------|----------|
| Solo – Yogyakarta     | 1     | 0,137         | 0,4       | 0,1785   | 0,5211   | 1,3028   | 3,8028   |
| Solo – Semarang       | 1     | 0,076         | 0,1456    | 0,5608   | 1,0800   | 7,4164   | 14,2831  |
| Surabaya – Solo – Semarang | 1     | 0,0599       | 0,9523    | 0,0042   | 0,0670   | 0,0704   | 1,1204   |
| Solo – Jakarta        | 1     | 0,0459        | 1         | 0,0022   | 0,0482   | 0,0482   | 1,0482   |
| Solo – Surabaya       | 1     | 0,0209        | 0,12      | 0,0370   | 0,2119   | 1,7664   | 10,0997  |

Table 6. System performance measures

5. Conclusions
From the results of the analysis of the research that has been carried out, it can be concluded several things. Obtained the final model for the queue model from five lines, obtained 3 service system models at Tirtonadi Bus Station, namely (G/Weib/1):(GD/∞/∞) for Solo-Yogyakarta, (Logn/G/1):(GD/∞/∞) for Solo-Semarang, Surabaya-Solo-Semarang and Solo-Jakarta, and models (M / Beta / 1):(GD/∞/∞) for Solo-Surabaya.

Overall the service in all of the Tirtonadi Bus Station lines is good, because there are no buses waiting to serve passengers. This is evidenced by the value of \( L_q \) in all majors, namely 0. Therefore, it can be said that the service is good.

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