Influence of Loading and Unloading Effect on Bearing Capacity of Pile Tip

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Abstract: The upper pile of the O-cell test pile has an unloading effect on the soil around the pile, resulting in the pile tip resistance being lower than the top-loaded pile. In order to solve this problem, the Mindlin formula is used to calculate the additional stress caused by the lateral friction resistance at the pile tip. Then introduce this additional stress into the Meyerhof ultimate tip resistance formula and obtain an improved formula for calculating the pile tip resistance. The improved formula can fully consider the influence of different loading directions on the pile tip resistance. The top-loaded pile has a loading effect on the pile tip soil, and the bottom-supported pile has the unloading effect on the pile tip. The example verification shows that the pile tip resistance of top-loaded pile and O-cell test pile calculated by the modified Meyerhof tip resistance formula are close to the measured values. The improved formula is of great significance for further study of the effect of the loading and unloading effect on the pile tip resistance.

1. Preface

Pile foundation test is an important measure to ensure the reliability of pile foundation. The traditional in-situ static loading test pile is the most direct and accurate method to test the bearing capacity of pile foundation. However, this method has difficulties in using for large tonnage loading and using on complex sites (such as rivers, lakes and sloping fields). In order to solve this problem, Osterberg developed the O-cell test pile method in 1985.

The O-cell test pile is embedded a load box at the pile body or pile tip and the load box is filled with a high pressure oil pump. The bottom of the upper pile is subjected to the upward support force, and the form of force is called the bottom-supported pile. And the top of the lower pile is subjected to downward pressure. The mechanical characteristics of this pile test method is quite different from that of the traditional in-situ static loading test pile (hereinafter referred to as the top-loaded pile)¹².

The existing research shows that the negative lateral frictional resistance of the upper pile of the O-cell test pile is lower than the positive lateral frictional resistance of the top-loaded pile. “Technical specification for self-balance test of pile bearing capacity” (DB/T291-1999) and “Static loading test of foundation pile-Self-balanced method” (JT/T 738-2009) assume that the force of the lower pile of the
O-cell test pile is the same as that of the top-loaded pile under the same depth, and the effect of the upward force generated by the upper pile on the lower pile is not considered.

The field test data shows that the mechanical characteristics of O-cell test pile is different from that of the top-loaded pile\cite{1-4}. In the construction of Hangzhou Qianjiang New City, Xu Changjie\cite{5} found that the ultimate tip resistance of three top-loaded piles was about 10,492 kPa, while the ultimate tip resistance of three O-cell test piles was 9,295 kPa. The ultimate tip resistance of the O-cell test pile was lower than the top-loaded pile.

The upper pile of the O-cell test pile displaces the surrounding soil upwards, and generates an upward additional stress on the bottom plane of the pile, causing the soil below the load box (equilibrium point) to be unloaded and the tip resistance to be decreased. While the upper pile of the top-loaded pile generates a downward additional stress, which has a loading effect on the bottom plane of the pile and increases the tip resistance. The opposite loading effect leads to the difference in the tip resistance between the O-cell test pile and the top-loaded pile.

In 1936, Mindlin\cite{6} proposed a basic solution of stress and displacement at any point in space when a concentrated force is applied in a uniform, continuous, isotropic elastic half-space. In 1966, Geddes\cite{7} simplified the force of the pile on the soil into the lateral frictional resistance of the triangular distribution, the lateral frictional resistance of the uniform distribution, and the pile tip resistance in the form of concentrated forces. And derived the soil stress formulas under the three forces respectively based on the basic solution of Mindlin. Since the Mindlin solution is very complicated, in order to avoid the situation that multiple integrals cannot be solved, Novak and El Sharnobyy\cite{8} proposed using point loads instead of lateral friction resistance and tip resistance in 1985.

In the current literature, such as the literature\cite{9}, the displacement superposition is usually performed by means of the Mindlin displacement solution\cite{10} to consider the effect of additional stress on the tip resistance of the O-cell test pile, which can’t directly reflect the difference of the tip resistance of the two test piles. Tan Xianjiang\cite{11} introduced the Mindlin stress solution into the pile tip displacement, and obtained the pile tip stiffness considering the additional stress. However, it also only tells the difference of pile tip displacement of the two kinds of test piles, not the difference of the tip resistance of them. It needs to establish a stiffness matrix which is complex to solution. Fei Qinfa, Ma Hailong\cite{12,13} (1995, 1997) used the Mindlin stress solution to analyze the interaction between piles and soils in single piles and group piles by using the stratified sum method.

On the basis of analyzing the mechanical characteristics of O-cell test pile and top-loaded pile, this paper use the additional stress to explain the difference of the tip resistance of the two kinds of test piles and use Mindlin stress solution and Meyerhof tip resistance bearing formula to modify the tip resistance of the O-cell test pile.

### 2. Principle of loading and unloading effect of the top-loaded pile and the O-cell test pile

Assuming that when the test pile is completed, the pile body does not generate extra lateral frictional resistance due to factors such as under-consolidation of the soil. After the top-loaded pile and the O-cell test pile being loaded, the stress of the soil is shown in Figure 1 a–c, and Figure 2 a–c.

According to the Mindlin stress solution, the lateral friction resistance will generate additional stress at any point in the soil. The additional stress can be regarded as the external load. For the plane of the pile tip, the larger the overlying load, the larger the resistance that the soil shear damage needs to overcome, and the greater the bearing capacity of the pile tip. If the overlying load becomes smaller, the bearing capacity of the pile tip also decreases.

As shown in the Figure 1 c, for the top-loaded pile, the soil of the pile side is subjected to a downward force\cite{14}, and a downward additional stress $\sigma_c$ is generated at the pile tip, and the total load at the plane of the pile tip is $\gamma H + \sigma_c$. 


As shown in the Figure 2 e, for the pile tip soil of the O-cell test pile, since the soil on the side of the upper pile is subjected to an upward force, an additional stress in the upward direction is generated at the pile tip, which is equivalent to the self-weight stress of the overlying soil is reduced. While additional stress in the downward direction in generated at the pile tip of the lower pile, which has the loading effect on the pile tip soil. The sum of the additional stress of the upper pile and the lower pile can be expressed by $\sigma'_z$. Then the load acting at the plane of the pile tip is $\gamma H + \sigma'_z$. When the unloading effect of the upper pile is greater than that of the lower pile, $\sigma'_z$ is a negative value. Due to the existence of the upper pile of the O-cell test pile, the tip resistance of the O-cell test pile is inevitably smaller than that of the top-loaded pile.

3. Theoretical model and calculation of loading and unloading effect

3.1. Basic assumptions

(1) The soil is a continuous, uniform, isotropic semi-infinite space elastic body;
(2) ignoring the influence of the radial force on the pile side;
(3) The pile body is segmented, and the lateral friction resistance of each pile body is approximately regarded as the concentrated force.
3.2. Pile-soil unit division
Divide the pile and soil into n units equally.

The load box of the O-cell test pile applies upper and lower pressure to the pile top. The pile unit is subjected to lateral friction resistance $\tau_i$ ($i=1, 2, \ldots, n$), as well as the soil unit. But the direction of friction resistance of pile unit and soil unit is opposite, and the pile-soil unit division is shown in Figure 3. The unit division of the top-loaded pile is the same as that of Figure 3. In order to save space, the schematic diagram is not given.

![Figure 3. Unit division of the O-cell test piles](image)

a. The pile unit  

b. The soil unit

3.3. Matrix equation of soil unit interaction
Regard the soil as a semi-infinite space elastic body, and the forces of each soil unit will affect each other due to the continuity of the soil. As shown in Figure 4, a concentrated force $F$ acts on the soil depth $h$, and an additional stress is generated at any point $M$ in the soil.

![Figure 4. The internal force caused by the vertical concentrated force in the elastic semi-infinite body](image)

Let the resultant force of the lateral friction resistance of the $i$-th soil unit be $\pi R \tau_i \Delta h$, and insert the resultant force as the concentrated force into the Mindlin formula, then obtain the vertical downward additional stress $\sigma_i$ generated by resultant force acting at anywhere of the pile tip plane:

$$\sigma_i = \frac{\pi R \tau_i \Delta h}{h^2} I_b$$  \hspace{1cm} (1)

In the formula:
- $\Delta h$ —— length of each pile-soil unit, $\Delta h = H/n$;
- $h$ —— the distance between the pile-soil unit and the pile top;
- $R$ —— pile diameter;
- $I_b$ —— Vertical stress coefficient.
\[ I_i = \frac{1}{8\pi(1-\mu)} \left[ \frac{(1-2\mu)(m-1)}{A^3} + \frac{(1-2\mu)(m-1)}{B^3} - \frac{3(m-1)^3}{A^4} - \frac{3(3-4\mu)m(m+1)^2 - 3(m+1)(5m-1)}{B^5} - \frac{30m(m+1)^3}{B^7} \right] \] (2)

In the formula:
\( A^2 = n^2 + (m-1)^2 \);
\( B^2 = n^2 + (m+1)^2 \);
\( n = r_i / h_i \);
\( m = z/h_i \) (\( z \) is the distance from any point in the soil to the ground; \( r_i \) is the distance between each point in the soil and the center point of the pile.).

In the horizontal direction, the additional stress is near the point of application of the concentrated force. The further the distance is, the smaller the additional stress is, which can be negligible. The additional stress effective influence range \( L \) can be defined as the distance from the intersection of the pile tip plane and the slip line to the pile side. The average additional stress generated by the concentrated force in \( L \) is:
\[ \sigma_i = \frac{\int_0^L \pi R^2 \Delta h_i I_i dr}{L} \] (3)

The direction of the additional stress \( \sigma_i \) is the same as the lateral friction resistance \( \tau_i \), then the sum of the additional stress generated by the lateral frictional resistance of \( n \) unit on pile tip plane is:
\[ \sigma_s = \sum_{i=1}^n \sigma_i \] (4)

3.4. Ultimate tip resistance considering the effect of loading and unloading

In terms of the pile foundation, Meyerhof proposed a formula for calculating the bearing capacity of the pile tip considering the friction between the surrounding fill and the pile sidewall. Due to the deep depth of the pile foundation, the slip surface of the soil cannot reach the ground, so it curls upward and forms a pear-shaped fracture surface\(^{[16]} \), as shown in Figure 5. The improved formula in this paper is based on the Meyerhof pile tip ultimate bearing capacity formula.

![Figure 5. Sliding soil partition of pile foundation]

The Meyerhof pile tip ultimate bearing capacity calculation formula is:
\[ f_u = cN \gamma S_d + qN \gamma S_q d_q + 0.5 \gamma R N \gamma S_d \] (5)

After considering the additional stress at the pile tip, the improved pile tip ultimate bearing capacity calculation formula is:
\[ f_u = cN_cS_c + (q + \sigma_0)N_qS_qd_q + 0.5\gamma RN_S_d \]  \hfill (6)

In the formula:
- \(q\) — overload above the pile tip plane;
- \(c, \varphi, \gamma\) — soil cohesion, internal friction angle, severity;
- \(N_c, N_q, N_I\) — bearing capacity coefficient;
- \(S_c, S_q, S_I\) — shape factor;
- \(d_c, d_q, d_I\) — depth coefficient.

The empirical coefficient is shown in Table 1[17].

| Coefficient | Expression | Coefficient | Expression |
|-------------|------------|-------------|------------|
| \(N_c\)    | \((N_c - 1)\cot \varphi\) | \(N_q\)    | \(c^{(45^\circ + \varphi/\omega)}\) |
| \(S_c\)    | \((N_c - 1)\tan(1.4\varphi)\) | \(S_q\)    | \(1 + 0.2K_n\) |
| \(d_c\)    | \(1 + 0.2\sqrt{K_n\frac{D}{R}}\) | \(d_q\)    | \(\varphi \geq 10^\circ; d_q = d_I = 1\) |
| \(d_I\)    | \(\varphi \leq 10^\circ; d_q = d_I = 1 + 0.1\sqrt{K_n\frac{D}{R}}\) |

\(K_n = \tan^2(45^\circ + \varphi/\omega), D\) is the depth of foundation.

4. Example verification

4.1. Basic information

The project site is located in Hangzhou Qianjiang New City[5]. In the project, the bearing capacity test of the pile was carried out at the same site. The test was divided into two groups, and there are three test piles in each group. The top-loaded piles are numbered SA1, SA2, and SA3. The O-cell test piles are numbered SA4, SA5, and SA6. The test adopts the slow loading method. The test pile length is 48.1 m, the pile diameter is 1 m, and the pile tip supporting layer is 5-2 round gravel layer. This case is a special case where the load box is placed at the pile tip, but the effect of additional stress on the pile bearing capacity can also be revealed. Q-s curves and lateral friction resistance can be found in reference [5].

The three top-loaded test piles were stopped to be loaded when the load reached 16,000 kN. And at this time, the settlement of SA1, SA2 and SA3 are 42.50mm, 45.22mm and 40.23mm. After subtracting the lateral friction resistance, the pile tip resistance is 10,392 kPa, 10,516 kPa, and 10,568 kPa respectively. The lateral frictional resistance reaches the limit value (no longer increases with the increase of the external load) when the load reached 14,400kN, so the lateral frictional resistance at 16000kN is still the same as that at 14,400kN. When the three O-cell test piles are loaded at 8,030kN, the displacement of the pile top and the pile tip is more than 5 times that of the previous stage. Therefore, the lateral friction resistance of the piles reaches the limit value, and 7,300kN is taken as the ultimate loading value. Since the load box is placed at the pile tip, the pressure of the load box can be considered as the bearing capacity of the pile tip. So the tip resistance of the O-cell test pile is 9,295 kPa.
4.2. Comparative analysis of ultimate bearing capacity between the top-loaded pile and the O-cell test pile

Calculate the additional stress $\sigma_z$ generated by the ultimate lateral friction resistance on the pile tip plane according to the formulas (1) to (4), and calculate the pile tip bearing capacity according to the formula (6). The results are shown in Table 2.

| Number | Type of test pile | Theoretical value of the tip resistance without considering $\sigma_z$ /kPa | Theoretical value of the improved ultimate tip resistance /kPa | Measured value of the ultimate tip resistance /kPa |
|--------|------------------|---------------------------------------------------------------|-------------------------------------------------------------|--------------------------------------------------|
| SA1    | Top-loaded pile  | 9808                                                          | 49.22                                                       | 10333                                            |
| SA2    | Top-loaded pile  | 9808                                                          | 48.66                                                       | 10327                                            |
| SA3    | Top-loaded pile  | 9808                                                          | 48.69                                                       | 10327                                            |
| SA4    | O-cell test pile | 9808                                                          | -42.49                                                      | 9355                                             |
| SA5    | O-cell test pile | 9808                                                          | -42.65                                                      | 9354                                             |
| SA6    | O-cell test pile | 9808                                                          | -42.40                                                      | 9356                                             |

It can be seen from the measured results that the tip resistance of the O-cell test pile is about 9.84% smaller than that of the top-loaded pile. The ultimate tip resistance of these two groups of test piles calculated by the Meyerhof formula are the same, indicating that it does not reflect the actual loading and unloading effect of the project. However, if the improved ultimate bearing capacity formula is adopted, the calculation result is very close to the measured value. The calculation result is that the tip resistance of the O-cell test pile is 9.06% smaller than that of the top-loaded pile, which proves that it is more reasonable to calculate the pile tip bearing capacity by using the ultimate bearing capacity calculation formula with the additional stress.

5. Conclusion

Based on the Mindlin stress solution of the elastic theory, considering the continuity of the soil, the matrix equations of the interactions between the soil elements are derived, and then the additional stress of the pile tip plane is obtained. Put the additional stress into the Meyerhof tip resistance calculation formula, and then obtain the improved tip resistance calculation formula of top-loaded pile and O-cell test pile. The conclusions are as follows:

1. The tip resistance of the O-cell test pile is lower than that of the top-loaded pile. If the influence of the additional stress at the pile tip is not considered, the ultimate bearing capacity of the O-cell pile will be less than that of the top-loaded pile. As a result, the bearing capacity of the test pile is underestimated. In this case, the tip resistance of the O-cell test pile is underestimated by about 9%.

2. The tip resistance of the top-loaded pile and the O-cell test pile calculated by the improved Meyerhof tip resistance formula is close to the measured tip resistance of the field, which verifies the accuracy of the improved tip resistance calculation formula.
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