Research on Well Trajectory Deduction Method Based on Pythagorean-Hodograph Quintic Space Curves

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Abstract. Considering the influence of key parameters such as bore angle, azimuth and sounding of wellbore, based on the spatial five-time PH curve with continuous unit tangent and directional curvature, no need for iteration, using Bessel polynomial interpolation method, optimization the spatial model and the curve description can accurately deduct the spatial parameters of the fifth point according to the determined four points of the space, and then establish the wellbore trajectory analytical equation to generate the well trajectory curve. Taking a directional well as an example, the wellbore trajectories of the wells from 403.9 meters to 976 meters were fitted by PH, cubic spline and linear interpolation method respectively, and compared with the real measuring points. The results show that the wells fitted by PH method, the differences of the bevel angle and the measured point, the azimuth angle and the measured point are 0.910° and 5.7°. The differences of the cubic spline and the measured point, the straight line method and the measured point are 4.240°, 5.86° and 6.54°, 9.09°. For the PH method, the trajectory inclination and azimuth differences are the smallest, which are 3.51% and 3.17%.

1. Introduction

The finite number of discrete sounding points obtained by logging and the corresponding well angle and azimuth angle describe the actual drilling trajectory, which is important for us to determine the column mechanics, buckling analysis, friction analysis, casing wear analysis and stress intensity analysis accuracy. One ring [1-2]. However, the traditional cubic spline interpolation method has some defects in the three-dimensional description of the actual drilling trajectory. The limitation of the iterative algorithm makes it impossible to perform local modification, and the wellbore trajectory fitting effect of the straight line segment is not good [3-4].

Du Chun proposed a cubic spline interpolation method based on the characteristics of the cubic spline, fitting the well trajectory curve to obtain more points to draw a continuous smooth well trajectory curve, and using this method to describe the well. The spatial position of any point was actually close to the measured value [5]. Li Zhaodong proposed to use the successive displacement method to calculate the wellbore trajectory interpolation. It was assumed that the well segment is a cylindrical spiral, and the interpolation hole trajectory parameters were obtained through an iterative process through multiple calculations [6]. Gao Deli and Liu Qingyou also used the cubic spline interpolation method to fit the well trajectory. Based on this, the axial load mechanical model of the injection column was established to reasonably describe the axial load distribution along the axis of the wellbore [7-8]. The wellbore
trajectory curve constructed by cubic spline interpolation method is quadratic continuous, and the result
is highly realistic. It is the most commonly used method to fit the well trajectory, but it also has certain
limitations[9]: (1) Local modification It involves the recalculation of the overall curve; (2) it cannot
solve the problem of vertical tangent, and the fitting effect is not good when the well trajectory has a
straight line segment. However, the space five-time PH curve can overcome the difficulties of solving
the closed solution by the spatial curve constructed by the natural parameter form, and avoid the iterative
algorithm to solve the path length, which shows great in terms of controllability and calculation rate and
accuracy. The advantage, and it has bending points and provides sufficient flexibility, with continuous
unit tangential and directional curvature. In 2002, Farouki gave the C1 interpolation algorithm for the
five-time PH curve in space. In 2004, the G1 continuous interpolation algorithm for the spatial three-
time PH curve was given [10-11].

In summary, considering the geometric characteristics of the borehole trajectory space curve,
considering the influence of the well inclination angle, azimuth angle and sounding depth, based on the
spatial five-time PH curve with continuous unit tangent and directional curvature, no need for iterative
calculation, using Bézier polynomial interpolation the calculation model and algorithm can be optimized
according to the determined four points of space, and the spatial parameters in the middle can be
calculated accurately. Based on this, the well trajectory analytic function is established to generate the
well trajectory curve.

2. Theoretical analysis of spatial curve using PH Quintic interpolation
In actual working conditions, the wellbore trajectory is a smooth and continuous three-dimensional
space curve, which can be represented by the spatial PH curve \( \vec{r}(t) = \left[ x(t), y(t), z(t) \right] \) with \( t \) as the
parameter. There is a polynomial \( \sigma(t) \) such that [10]:

\[
\sigma(t) = \frac{x'(t)^2 + y'(t)^2 + z'(t)^2}{x'(t) + y'(t) + z'(t)}
\]

(1)

For real polynomial \( \sigma(t) \), analogous two-dimensional PH curve polynomial construction method,
we introduce three-dimensional form [11]:

\[
\begin{align*}
x'(t) &= 2h(t)u(t)v(t) \\
y'(t) &= 2h(t)u(t)w(t) \\
z'(t) &= h(t)\left[u(t)^2 - v(t)^2 - w(t)^2\right]
\end{align*}
\]

(2)

where \( u(t), v(t), w(t), h(t) \)—all is Polynomial, \( \gcd(u(t), v(t), w(t)) = \text{constant} \).

We usually choose \( h(t) = 1 \), otherwise the corresponding PH curve is not conventionally parameterized,
\( \left| \mathbf{r}'(t) \right| \) is not non-zero everywhere. The parameter speed of the curve defined by (2) is simply

\[
\left| \mathbf{r}'(t) \right| = \sigma(t) = u^2(t) + v^2(t) + w^2(t)
\]

The Bézier polynomial curve has the advantages of simple calculation method and high stability, and
has good compatibility with the spatial PH curve. Therefore, the five-time PH curve can be transformed
into a Bézier quadratic polynomial for description. The calculation formula is:

\[
\tilde{r}(t) = \sum_{j=0}^{5} \sum_{i=0}^{n} p_i \frac{5!}{j!(5-j)!} t^j (1-t)^{5-j}
\]

(3)

where \( t \in [0,1] \), \( p_i \) is the curve control point, \( p_i = (x_i, y_i, z_i) \), \( (i = 1, 2, \ldots, n) \).

We substitute the Bézier quadratic polynomial \( u(t), v(t), w(t) \) into the formula (2), and integrate it to
obtain the PH curve of the fifth-order space curve with the form (2), where \( h(t) = 1 \), its control points can
only be expressed as:
\[
p_i = p_0 + \frac{1}{5}(2u_0v_0, 2u_0w_0, u_0^2 - v_0^2 - w_0^2) \\
p_2 = p_1 + \frac{1}{5}(u_0v_1 + u_0v_2 + u_0w_1 + u_0w_2, u_0u_1 - v_0v_1 - w_0w_1) \\
p_3 = p_2 + \frac{2}{15}(2u_1v_1, 2u_1w_1, u_1^2 - v_1^2 - w_1^2) + \frac{1}{15}(u_1v_0 + u_1v_2, u_1w_0 + u_1w_2, u_1u_2 - v_1v_2 - w_1w_2) \\
p_4 = p_3 + \frac{1}{5}(u_2v_2 + u_2v_3 + u_2w_1 + u_2w_2, u_2u_2 + u_2v_1 - v_2v_2 - w_2w_2) \\
p_5 = p_4 + \frac{1}{5}(2u_2v_2, 2u_2w_2, u_2^2 - v_2^2 - w_2^2) \\
\]

When the main control points \( p_0 \) and \( p_5 \), and from the points to be controlled \( p_1 \) and \( p_4 \) are determined, the polynomial coefficients can be determined:

\[
\begin{align*}
(u_x, v_x, w_x) &= \pm \sqrt{\frac{5}{2}} \left[ \sqrt{\Delta z_0 + \Delta \rho_0} \frac{\Delta x_0}{\sqrt{\Delta z_0 + \Delta \rho_0}}, \sqrt{\Delta z_1 + \Delta \rho_1} \frac{\Delta y_1}{\sqrt{\Delta z_1 + \Delta \rho_1}}, \frac{\Delta z_1}{\sqrt{\Delta z_1 + \Delta \rho_1}} \right] \\
(u_y, v_y, w_y) &= \pm \sqrt{\frac{5}{2}} \left[ \sqrt{\Delta z_1 + \Delta \rho_1} \frac{\Delta y_1}{\sqrt{\Delta z_1 + \Delta \rho_1}}, \sqrt{\Delta z_2 + \Delta \rho_2} \frac{\Delta x_2}{\sqrt{\Delta z_2 + \Delta \rho_2}}, \frac{\Delta z_2}{\sqrt{\Delta z_2 + \Delta \rho_2}} \right] \\
(u_1, v_1, w_1) &= \frac{3}{4}(u_0 + u_1, v_0 + v_1, w_0 + w_1) \pm \sqrt{\frac{11}{2}} \left[ \sqrt{c + d} \frac{a}{\sqrt{c + d}} + \frac{b}{\sqrt{c + d}} \right]
\end{align*}
\]

where \( \Delta p_0 = p_1 - p_0 \), \( \Delta p_4 = p_5 - p_4 \),

\[
a = \frac{15}{2}(x_4 - x_1) + \frac{9}{8}(u_4v_0 + u_2v_2) + \frac{5}{8}(u_4w_0 + u_2w_2) \quad b = \frac{15}{2}(y_4 - y_1) + \frac{9}{8}(u_4w_0 + u_2w_2) + \frac{5}{8}(u_4w_0 + u_2w_2) \\
c = \frac{15}{2}(z_4 - z_1) + \frac{9}{16}(u_0^2 - v_0^2 - w_0^2 + u_2^2 - v_2^2 - w_2^2) + \frac{5}{8}(u_4w_2 - v_4v_2 - w_4w_2) \\
d = \sqrt{a^2 + b^2 + c^2}
\]

However, since the values of the polynomial coefficients are different, the four feasible solutions can be finally obtained after combination, and the smaller \( \|p_2 - p_1\| + \|p_3 - p_4\| \) is, the closer it is to the Bézier control polygon, and the more consistent with the actual curve. In summary, according to the PH curve algorithm, the other control points of the Pézier control polygon of the PH curve between each control point are calculated, and the optimal intermediate control point is determined according to the discriminating condition, and the corresponding curve can be obtained.

3. Quintic PH interpolation method for depicting spatial curve example analysis

Taking a directional well in Sichuan as an example, as shown in Table 1, the basic parameter sounding, well angle and azimuth angle of the actual drilling trajectory obtained from the well logging from 403.9m to 976m. In order to verify the reliability and accuracy of the directional wellbore trajectory simulated by the five-time PH curve in space, according to the analysis process of formulas (1)-(6), the Matlab software program can be written, and the measured well trajectory data is input into the computer, the interpolation will be performed. The fitting obtained the inclination angle and azimuth data every 30 m, and the results are shown in Table 2.

| Deep Well/m | Well angle/° | Azimuth/° |
|-------------|--------------|-----------|
| 403.9       | 4.33         | 167       |
| 461.3       | 6.83         | 170.5     |
| 518.6       | 8.59         | 176       |
| 576         | 13           | 180       |
| 633.2       | 19           | 178.5     |
| 690.2       | 23.91        | 178.5     |
Table 2. Interpolation fitting results of PH quintic curve in space

| Deep Well/m | Well angle /° | Azimuth /° | Deep Well/m | Well angle /° | Azimuth /° |
|-------------|---------------|------------|-------------|---------------|------------|
| 430         | 5.19          | 168.44     | 700         | 12.72         | 176.94     |
| 460         | 6.06          | 170.07     | 730         | 13.72         | 177.42     |
| 490         | 6.88          | 171.42     | 760         | 14.80         | 177.85     |
| 520         | 7.67          | 172.56     | 790         | 15.98         | 178.24     |
| 550         | 8.46          | 173.55     | 820         | 17.31         | 178.60     |
| 580         | 9.25          | 174.40     | 850         | 18.82         | 178.92     |
| 610         | 10.06         | 175.16     | 880         | 20.58         | 179.21     |
| 640         | 10.91         | 175.82     | 910         | 22.72         | 179.48     |
| 670         | 11.79         | 176.41     | 940         | 25.47         | 179.73     |

4. Comparative Analysis of PH Curve, Cubic Spline and Linear Interpolation Method

Taking a directional well in Sichuan as an example, we can take four points of 403.9m, 461.3m, 918.3m and 976m in Table 1, the Matlab software is used to fit the wellbore trajectory with five PH curves, cubic spline curves and linear interpolation. The inclination angle and azimuth angle of the other measuring points are obtained, and the error analysis of the inclination angle and azimuth angle of the original measuring point is performed to verify the calculation accuracy and reliability of the three methods, as shown in Figure 1.

It can be seen from Figure 1 that the PH curve is the smoothest and more in line with the actual working conditions in the well trajectory fitted by the PH curve, the cubic spline and the linear interpolation. It can be seen from Figure 2 that the maximum difference of the inclination angle data obtained by the five-time PH curve fitting wellbore trajectory is 0.910°, and the relative error is 3.5%, and the maximum difference between the cubic spline and the linear interpolation method respectively 4.240° and 5.86°, the relative errors are 17.73% and 24.51%, respectively. It can be seen from Fig.3 that the maximum difference of the azimuth data obtained by the five-time PH curve fitting wellbore trajectory is 5.70°, the relative error is 3.17%, and the maximum difference between the data obtained by the cubic spline and the linear interpolation method. The relative errors of 6.54° and 9.09° are 3.64% and 5.05%.

Figure 1. Three-method simulated wellbore trajectory
5. Conclusion

(1) Considering the influence of key parameters such as the inclination angle, azimuth and sounding of the well trajectory, based on the five-time PH curve of space, it has the advantages of continuous unit tangential and directional curvature, no need for iteration, etc., we can use Bézier polynomial interpolation method. The space model and the curve description are optimized, and the spatial parameters of the fifth point can be accurately deduced according to the determined four points of the space, and the well path trajectory analytical equation is established according to the method to generate the well trajectory curve.

(2) In order to make the trajectory of the wellbore smoother, the result is more realistic, local adjustment of individual points is needed. However, the iterative algorithm of cubic spline has the effect of modifying the coordinates of one point to change the shape of the overall curve. the PH quintic curve can be modified locally.

(3) The wells fitted by PH method, the differences of the bevel angle and the measured point, the azimuth angle and the measured point are 0.910° and 5.7°. The differences of the cubic spline and the measured point, the straight line method and the measured point are 4.240°, 5.86° and 6.54°, 9.09°.

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