Nonextremal black holes are BPS

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Abstract

Extremal charged black holes are BPS solutions. It is commonly thought that their nonextremal counterparts are not. Further, experience with BPS solutions in flat spacetime suggests that all BPS solutions are supersymmetric; i.e. that they are invariant under some supersymmetry charges of either the original field theory or an appropriately extended version thereof. Using nonextremal Reissner-Nordström black holes as counterexamples, we show that neither of these expectations is universally valid. These black holes correspond to a one-parameter family of BPS solutions. By showing that, subject to one very plausible assumption, no generalized Killing spinor can be constructed for these, we show that there is no supergravity theory for which these BPS solutions preserve a fraction of the supersymmetry, nor is there an associated Witten-Nester positive energy bound.

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1 Introduction

It is a well known property of supersymmetric theories that solutions to the equations of motion that preserve a fraction of the supersymmetries are BPS; i.e., they can be obtained by solving a set of equations that are first order in the fields. It is easy to see why this is so. Invariance of the solution under a supersymmetry implies that the supersymmetry variations of the fermions must vanish for the solution, and this condition gives the first-order equations. In all known examples in flat spacetime field theory, the converse is also true: If a field theory admits BPS solutions, then it can be extended to a supersymmetric theory, with the BPS solutions invariant under some of the supersymmetries [1, 2]. However, there is no demonstration that this need be so, and one might well ask whether there can be counterexamples.

We will address this question in this paper. We will work in the context of a field theory coupled to gravity. Recall that for nongravitational field theories with BPS solutions the energy can be written in the form

\[ E = \int d^3x \sum_a G_a(\phi_j, \nabla \phi_j)^2 + \int_{|x| \to \infty} d^2S \, H(\phi_j, \nabla \phi_j). \]  

(1.1)

Here the \( G_a \) are linear in \( \nabla \phi_j \) and the second integral, over the sphere at spatial infinity, is completely specified by the values of the global charges of the theory. For fixed values of these charges, the energy is thus bounded from below and is minimized by the BPS configurations obeying the first-order equations \( G_a = 0 \). This is the BPS bound. In the supersymmetric extension of the theory, the \( G_a \) are the supersymmetric variations of the fermion fields, while the surface integral can be expressed in terms of the central charges of the supersymmetry algebra.

When gravity is included, static BPS solutions are naturally described in terms of the action, rather than the energy. One still has a structure similar to that in Eq. (1.1), but with the energy replaced by the action, and with the crucial difference that some of the \( G_a^2 \) terms enter with an overall minus sign. While it is still true that the vanishing of all the \( G_a \) produces a solution of the full field equations of the theory, such a solution is no longer automatically associated with the saturation of any bound. In this paper we will refer to solutions as being BPS if they can be obtained from first-order equations, whether or not they saturate an associated BPS bound.

However, there is a set of (spherically symmetric) black hole solutions that are both BPS and associated with a bound. The extremal solutions whose inner and outer horizons coincide are known to be supersymmetric and to saturate the bound imposed by the positive energy theorem. Witten and Nester’s proof of the positive energy theorem in fact relates these properties [3, 4]. In a supergravity theory one of the BPS equations is the Killing spinor equation (the vanishing of the variation of the gravitino), and the existence of a Killing spinor means that the positive energy bound is saturated. Because the nonextremal solutions do not appear to saturate an energy bound, it has often been thought that they are not BPS. We will show in this paper that this is not necessarily so.

Our interest in this problem was first aroused by trying to construct nonextremal D3-D7-brane solutions to describe thermal gauge theories with light flavors. Since our results are easily extended to arbitrary static spherically symmetric \( p \)-branes, we will work instead with a simpler example, that of charged black holes in four spacetime dimensions. It is well known that the extremal Reissner-Nordström black hole is BPS and supersymmetric. We will show that despite expectations, the nonextremal Reissner-Nordström solutions are also BPS, and that they can be derived from a one-parameter family of first-order equations with the parameter describing the deviation from extremality. The extremal solution is a limiting case. We will see that the underlying mechanism
that allows for a family of first-order solutions is the fact, alluded to above, that gravitational actions are unbounded from below.

Given that all known nongravitational BPS solutions preserve a fraction of some supersymmetry, it is natural to ask whether there is a supersymmetric extension of the theory such that the nonextremal black hole solutions preserve part of the supersymmetry. In particular, can we reobtain the first-order BPS equations from some supersymmetry variation of the fermion fields? A necessary consequence of such a preserved supersymmetry would be the existence of a (generalized) Killing spinor corresponding to the variation of the gravitino. We will show that, subject to one mild assumption, no such spinor exists for the for the nonextremal solutions. This immediately tells us that there is no supergravity extension for which the nonextremal BPS solutions preserve a subset of the supersymmetries.

The remainder of this paper is organized as follows. In Sec. 2 we consider charged black hole solutions. After reducing the problem to an equivalent (1+1)-dimensional effective theory, we show that not only the extremal, but also the nonextremal, Reissner-Nordström black hole solutions are BPS. We then go on to explain how the existence of a family of BPS solutions can be understood as a consequence of the nonpositivity of the gravitational action. Next, in Sec. 3, we turn to the question of supersymmetry. We show that the effective theory of the previous section can be extended to a supersymmetric theory in two spacetime dimensions, and that the BPS solutions — both extremal and nonextremal — preserve half of the supersymmetry. However, we then show that, within our assumption, there is no lift of this supersymmetry to four dimensions under which the nonextremal solutions are supersymmetric. We make some concluding remarks and discuss some implications for the saturation of energy bounds in Sec. 4.

2 Nonextremal BPS black holes

The example we will use to show that nonextremal solutions can be BPS is the $d = 4$ Reissner-Nordström black hole. The approach below is easily generalized to generic static charged spherically symmetric $p$-branes in dilaton gravity.

The action is the Einstein-Hilbert gravitational action with a coupling to electromagnetism; with units chosen so that $4\pi G = 1$,

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{4} R - \frac{1}{4} F_{\mu\nu}^2 \right].$$

(2.1)

The nonextremal black hole solution we seek is SO(3) symmetric and $t$-independent. In a parameterization that will prove convenient, the most general metric respecting these symmetries can be written as

$$ds_{RN}^2 = -e^{2n_4} dt^2 + e^{-2n_4} \left[ e^{2n_3} (d\Omega_2^2 + e^{2n_1} du^2) \right],$$

(2.2)

where $d\Omega_2^2$ is the metric on the unit 2-sphere and the $n_j$ are functions of only $u$. Anticipating our results, we note that there will be a horizon, at a zero of $e^{n_4}$. We will also require that the spacetime be asymptotically flat. In this asymptotic region, $e^{n_4}$ must tend to a constant which, by a rescaling of coordinates, can be chosen to be unity.$^1$ For later reference, note that these coordinates can be converted to the more usual ones by making use of the fact that the circumference of the two-sphere is $2\pi$ times

$$r(u) = e^{n_3-n_4}.$$

(2.3)

$^1$When this metric is generalized to $p$-branes, one must also impose the condition that the horizon be finite and nonzero.
For the electromagnetic field strength we make a standard ansatz consistent with the rotational symmetry,
\begin{align*}
F_{tu} &= e^{n_1-n_3+2n_4}Q(u), \\
F_{\theta\phi} &= 0.
\end{align*}
\tag{2.4}

The electromagnetic field equation reduces to the requirement that \(Q\) be constant; integrating the flux over the sphere at infinity, we see that \(4\pi Q\) is the total charge carried by the black hole.\(^2\)

To find the solution, we will use the approach pioneered in \([5]\); see also \([6, 7]\). Substituting Eqs. (2.2) and (2.4) into the four-dimensional action of Eq. (2.1) yields (after factoring out an overall \(\pi\)) an equivalent (1+1)-dimensional action,
\begin{equation}
S_{\text{eff}} = \int dt L_{\text{eff}}
\end{equation}
\tag{2.5}
where the time-independent effective Lagrangian is\(^3\)
\begin{equation}
L_{\text{eff}} = \int du e^{n_3-n_1} \left[ 2(n_3')^2 - 2(n_4')^2 + 2e^{2n_1} - 2Q^2 e^{2(n_1-n_3+n_4)} \right] + \int du \frac{d}{du} \left[ e^{n_3-n_1} (2n_4' - 4n_3') \right].
\end{equation}
\tag{2.6}

Note that \(n_1'\) does not enter here, reflecting the fact that \(n_1\) can be eliminated by a coordinate transformation. The field equation for \(n_1\) is then simply an algebraic constraint equation. Writing the integrand of the Lagrangian as \(e^{-n_1}T - e^{n_1}V\), this constraint becomes the “zero energy condition” \(e^{-n_1}T + e^{n_1}V = 0\).

We will henceforth assume that the coordinates have been chosen so that \(n_1 = n_3\). Our effective Lagrangian then takes the form
\begin{equation}
L_{\text{eff}} = \int du \left[ 2(n_3')^2 - 2(n_4')^2 + 2e^{2n_3} - 2Q^2 e^{2n_4} \right] + \int du \frac{d}{du} (2n_4' - 4n_3')
\end{equation}
\tag{2.7}

The total derivative term can be ignored in deriving the solution, but it will be important later.

### 2.1 The extremal BPS solution

This dimensionally reduced effective Lagrangian is easily written as a sum of squares plus a total derivative term,
\begin{equation}
L_{\text{eff}} = \int du \left[ 2(n_3')^2 - 2(n_4')^2 + 2e^{2n_3} - 2Q^2 e^{2n_4} \right] - 2Z_{\text{ext}},
\end{equation}
\tag{2.8}
where
\begin{equation}
Z_{\text{ext}} = \int du \frac{d}{du} \left[ 2e^{n_3} - 2Qe^{n_4} + 2n_3' - n_4' \right]
\end{equation}
\tag{2.9}
with the first two terms coming from completing the squares and the last two carried over from Eq. (2.7).

Requiring that each of the squares separately vanish yields a pair of first-order equations. These separate and are solved by
\begin{align*}
e^{-n_3} &= u + c_3 = u, \\
e^{-n_4} &= Qu + c_4 = Qu + 1.
\end{align*}
\tag{2.10}

\(^2\)The generalization of the argument below to that of a magnetically charged black hole is straightforward.

\(^3\)Note that we have not included a Gibbons-Hawking term.
In the second equalities we have set $c_3 = 0$ by a choice of the origin of $u$, while the boundary condition from asymptotic flatness fixes $c_4 = 1$. Referring to Eq. (2.3), we find that

$$r = \frac{1}{u} + Q ,$$

so that $u = 0$ corresponds to asymptotically flat spatial infinity, while at the horizon, $u = \infty$, we have $r(\infty) = Q$. Converting from $u$ to $r$, we obtain the standard extremal Reissner-Nordström solution

$$ds^2 = -H dt^2 + H^{-1}dr^2 + r^2d\Omega_2^2 ,$$

$$H(r) = \left( 1 - \frac{Q}{r} \right)^2 .$$

Finally, we note that substitution of our solution into Eq. (2.9) yields

$$Z_{\text{ext}} = \int_0^\infty du \frac{d}{du} [-Qe^{n_3}] = Q .$$

The limits on the integral correspond to an integration over the region outside the horizon.

### 2.2 The nonextremal BPS solutions

The conventional wisdom has it that only the extremal solution above is BPS, and that to obtain nonextremal solutions one would have to solve the second-order field equations. However, with our choice of coordinates the second-order equations separate and can each be reduced by quadratures to a first-order equation. This led us to surmise that one could recognize these more general first-order equations in the Lagrangian as well. It takes little effort to confirm that there is indeed a more general way to write the $L_{\text{eff}}$ in BPS form, namely

$$L_{\text{eff}} = \int du \left[ 2 \left( n_3' + \sqrt{e^{2n_3} + \beta^2} \right)^2 - 2 \left( n_4' + \sqrt{Q^2e^{2n_4} + \beta^2} \right)^2 \right] - 2Z_{\text{non-ext}}$$

with

$$Z_{\text{non-ext}} = \int du \frac{d}{du} \left[ 2n_3' - n_4' - 2\sqrt{Q^2e^{2n_4} + \beta^2} + 2\sqrt{e^{2n_3} + \beta^2} + 2\beta \arcsinh \left( \frac{\beta}{Q}e^{-n_3} \right) - 2\beta \arcsinh \left( \beta e^{-n_3} \right) \right] .$$

As before, requiring that the two squares both vanish gives a pair of first-order equations that separate. Their solution is

$$e^{-n_3} = \frac{1}{\beta} \sinh \beta u ,$$

$$e^{-n_4} = \frac{Q}{\beta} \sinh(\beta u + c_4) .$$

Here we have again absorbed the integration constant in the $n_3$ equation by shifting the origin of $u$. The requirement of asymptotic flatness gives $c_4 = \arcsinh(\beta/Q)$. The transformation to standard coordinates is now given by

$$r(u) = e^{n_3-n_4} = Q \frac{\sinh(\beta u + c_4)}{\sinh \beta u} ,$$

$$Z_{\text{non-ext}} = \int_0^\infty du \frac{d}{du} [-Qe^{n_3}] = Q .$$
so that at the horizon, $u = \infty$,

$$r = r_+ = Qe^{\nu} = \sqrt{Q^2 + \beta^2 + \beta}.$$  \hfill (2.18)

If we now define $M$ by

$$\beta = \sqrt{M^2 - Q^2},$$  \hfill (2.19)

substitute the warp factors into the metric of Eq. (2.2), and change coordinates from $du$ to $dr$, we recover the form of Eq. (2.12), but with $H(r)$ taking the nonextremal form

$$H(r) = \left(1 - \frac{r_+}{r}\right) \left(1 - \frac{r_-}{r}\right)$$  \hfill (2.20)

with

$$r_\pm = M \pm \sqrt{M^2 - Q^2}.$$  \hfill (2.21)

Thus, we have established that, contrary to conventional wisdom, the nonextremal Reissner-Nordstr¨om solution is BPS.

With the metric in this standard form, we recognize $M$ as the Arnowitt-Deser-Misner (ADM) mass, and see that $\beta$ is a measure of the nonextremality of the black hole. In the case of the extremal black hole, we found that the Lagrangian of the BPS solution, which is just equal to $2Z_{\text{ext}}$, was precisely twice the ADM mass $M = Q$. This result does not carry over to the nonextremal case. Instead,

$$Z_{\text{non-ext}} = -\int_0^\infty du \frac{d}{du} \left[ \sqrt{Q^2 e^{2m} + \beta^2} - 2\beta c_4 \right] = \left[ -\beta + \sqrt{Q^2 + \beta^2} \right] = \left[ M - \sqrt{M^2 - Q^2} \right].$$  \hfill (2.22)

### 2.3 Families of BPS solutions

How is it that this system admits a family of BPS solutions? Consider a theory involving scalar fields $\phi_i$ in one spatial dimension whose static solutions are governed by an effective Lagrangian of the form

$$L_{\text{eff}} = -\int dz G^{ij} \left[ \nabla \phi_i \nabla \phi_j + f_i(\phi) f_j(\phi) \right] + 2 \int dz \nabla H ,$$  \hfill (2.23)

with $G^{ij}$ possibly a function of $\phi$. If the $f_i$ are such that

$$f^i = G^{ij} f_j = \frac{\partial W}{\partial \phi_i}$$  \hfill (2.24)

for some function $W(\phi)$, then we can rewrite $L_{\text{eff}}$ in the BPS form

$$L_{\text{eff}} = -\int dz G^{ij} \left( \nabla \phi_i - f_i \right) \left( \nabla \phi_j - f_j \right) - 2 \int dz \left( \nabla W - \nabla H \right).$$  \hfill (2.25)

We want to know whether it is possible to rewrite this action with the functions $f_i$ replaced by a second set, $\tilde{f}_i$, that are the derivatives of a different function, $\tilde{W}(\phi)$. Clearly, this can only be done if

$$G^{ij} \left( f_i f_j - \tilde{f}_i \tilde{f}_j \right) = 0.$$  \hfill (2.26)

If $G_{ij}$ has only positive eigenvalues, as would be the case for standard scalar field theories, this has the obvious solutions $\tilde{f}_i = \pm f_i$. In some cases there may also be isolated nontrivial solutions that
lead to a distinct $\tilde{W}$. The new feature in the gravitational case, where $G_{ij}$ is not positive definite, is the existence of null vectors that yield continuous families of solutions to Eq. (2.26). If $\beta$ is such a null vector, then Eq. (2.26) is satisfied if

$$\tilde{f}_i \tilde{f}_j - f_i f_j = \pm \beta_i \beta_j . \quad (2.27)$$

In a basis in which $G^{ij}$ is diagonal, this implies that

$$\tilde{f}_i = \sqrt{f_i^2 \pm \beta_i^2} . \quad (2.28)$$

In addition, the requirement that $\tilde{f}^i = \partial \tilde{W}/\partial \phi_i$ implies that for any $i \neq j$ (and still in the basis that diagonalizes $G^{ij}$)

$$\frac{\partial \tilde{f}_i}{\partial \phi_j} = \frac{\partial \tilde{f}_j}{\partial \phi_i} . \quad (2.29)$$

If the $\beta_j$ are constants (i.e., independent of the $\phi_j$), it follows that

$$0 = \frac{f_i}{\sqrt{f_i^2 \pm \beta_i^2}} \left( \frac{\partial f_i}{\partial \phi_j} \right) - \frac{f_j}{\sqrt{f_j^2 \pm \beta_j^2}} \left( \frac{\partial f_j}{\partial \phi_i} \right)$$

$$= \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \left[ \left( 1 \pm \frac{\beta_i^2}{f_i^2} \right)^{-1/2} - \left( 1 \pm \frac{\beta_j^2}{f_j^2} \right)^{-1/2} \right] . \quad (2.30)$$

The quantity in brackets in the last line only vanishes if $\beta_j^2/f_j^2$ is the same for all values of $j$. This would imply that $f$, like $\beta$, would be a null vector and so would not contribute to the action at all. The only alternative, then, is that the mixed second derivatives of $W$ all vanish; i.e., that each of the $f_i$ be a function of only the corresponding $\phi_i$. Indeed, a quick glance at Eq. (2.14) shows this is precisely the case for our Reissner-Nordström black holes.

### 2.4 Extremality and the BPS bound

In the nongravitational examples, the BPS solutions saturate a lower bound on the energy. Although there is no such BPS bound for the Lagrangian in our gravitational example, these systems do respect a physically inspired bound. To avoid a naked singularity, charged black holes must obey the positive energy theorem, which requires that their ADM energy be greater than or equal to their charge; the work of Witten and Nester showed that this bound is the analogue of the BPS bound for gravitational systems [3, 4, 8]. This bound is, by definition, saturated by extremal black holes with ADM energy $M$ equal to the charge $Q$.

In our BPS solutions the Lagrangian was equal to twice a quantity $Z$, whose relation to a central charge we will discuss in the next section. For the present, we just restate that while in the extremal case we have $Z_{\text{ext}} = M$, the corresponding equality does not hold for the nonextremal solutions, where $Z_{\text{non-ext}} < M$.

### 3 Supersymmetry and the BPS bound

The intimate connection between BPS first-order solutions and solutions preserving a fraction of supersymmetries raises the question of whether the full one-parameter family of nonextremal solutions preserves some supersymmetry — whose form might depend on the value of the nonextremality parameter $\beta$ — or whether the extremal solution is special in that regard. We will show in this section
that the full one-parameter family of black hole solutions in the previous section can preserve half the supersymmetries in an appropriate (nongravitational) supersymmetric extension. The Lagrangian of Eq. (2.7) defines a (1+1)-dimensional linear sigma model with a complicated potential, but all first-order solutions in an \( \mathcal{N} = 2 \) extension of such a theory preserve half the supersymmetries.

This need not imply, however, that the nonextremal solutions preserve some of the supersymmetries of a supersymmetric extension of the original \( d = 4 \) Einstein-Maxwell gravity. There could be an obstruction to lifting the preserved supersymmetries in the lower-dimensional effective theory back to four dimensions. In fact, we know beforehand that this extension cannot be trivial \( \mathcal{N} = 2, d = 4 \) supergravity. The BPS states of that theory have long since been known and are limited to the extremal ones \cite{9} (see also \cite{10, 11}). Any putative supergravity for which nonextremal solutions preserve a subset of the symmetries must therefore be a novel one.

Rather than searching directly for this novel supergravity, we address the issue by looking for a generalized Killing spinor. Any preserved supersymmetry in a supergravity theory must yield such a spinor, from the variation of the gravitino. (The converse, however, need not be true; the existence of a Killing spinor does not imply supersymmetry. We will return to this point below.) We will show, modulo a mild assumption that we explain below, that only the extremal solution allows such a Killing spinor. Hence, the nonextremal solutions cannot be supersymmetric.

### 3.1 Preserved supersymmetries in the lower-dimensional effective theory

The effective Lagrangian of Eq. (2.7) describes dynamics in one spatial dimension. The extremal and nonextremal solutions to the field equations have a nontrivial profile only along this dimension, and they are therefore equivalent to static solitons in a (1+1)-dimensional field theory. The appropriate effective supersymmetric extension to consider is therefore an \( \mathcal{N} = 2, (1+1) \)-dimensional theory. This theory is the reduction of a minimally supersymmetric \( \mathcal{N} = 1 \) theory in 3+1 dimensions. Its dynamics are well known and in component language the most general action is\(^4\)

\[
S_{1+1}^{\mathcal{N}=2} = \int dx \, dt \left[ -G_{ab} \left( \nabla_\mu \phi^a \nabla^\mu \phi^b + \bar{\psi}^a \nabla \psi^b - F^a F^b \right) + F^a \partial_b W + F^b \partial_a W \right]
\]

\[
- \frac{1}{2} \left( \bar{\psi}^a \psi^b \partial_a \partial_b W + (\bar{\psi})^a (\psi^b) \partial_a \partial_b W \right) .
\]

(3.1)

Here \( W(\phi^a) \) is the holomorphic superpotential of the fields and the metric \( G_{ab} = \partial_a \partial_b K(\phi, \bar{\phi}) \) is the mixed second derivative of the real Kahler potential; to avoid confusion between derivatives with respect to fields and spacetime derivatives, we have denoted the latter by \( \nabla_\mu \). By construction the action is invariant under the supersymmetry transformations

\[
\delta \phi^a = \epsilon\psi^a, \quad \delta \psi^a = \nabla \phi^a \epsilon + F^a \epsilon^+, \quad \delta F^a = \epsilon \nabla \psi^a .
\]

(3.2)

Because \( \epsilon \) is a two-component complex spinor, there are four supercharges.

As is well known, integrating out the auxiliary scalar field \( F^a = -G^{ab} \partial_b \bar{W} \) yields a scalar field potential \( G^{ab} \partial_a W \partial_b \bar{W} \) that, for real fields \( \Phi = \frac{1}{2}(\phi + \bar{\phi}) \), is precisely of the form that appears in Eq. (2.23). For solutions of the corresponding BPS equation, \( \nabla_1 \Phi^a + G^{ab} \partial_1 W(\Phi) = 0 \), the effect

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\(^4\) Our conventions are such that \( \bar{\phi}^a = (\phi^a)^* \) is the complex conjugate of the scalar field \( \phi^a \); \( \bar{\psi}^a = (\psi^a)^{\dagger} i\gamma^0 \) is the Dirac conjugate, and \( \bar{\psi}^a = (\psi^a)^\dagger i\gamma^0 = (\psi^a)^\dagger C \) the Majorana conjugate of the two-component complex spinor \( \psi^a \); and \( \{ \gamma^\mu, \gamma^\nu \} = 2\eta^{\mu\nu} \) with \( \gamma^0 \) anti-Hermitian and \( \gamma^1 \) Hermitian. Recall that in 1+1 dimensions all supersymmetry multiplets are dual to the chiral multiplet.
of a supersymmetry transformation on the fermion,
\[ \delta \psi^\alpha = \gamma^1 \nabla_1 \Phi^a \epsilon - G^{ab} \partial_b W(\Phi) \epsilon^* \]
\[ = \nabla_1 \Phi^a \left( \frac{1 + \gamma_1}{2} \right) \epsilon - \nabla_1 \Phi^a \left( \frac{1 - \gamma_1}{2} \right) \epsilon^* , \]
vanishes if \( \frac{1}{2} (1 + \gamma_1) \text{Im} (\epsilon) = \frac{1}{2} (1 - \gamma_1) \text{Re} (\epsilon) = 0. \) Thus, the BPS solutions preserve half of the supersymmetry. Furthermore, it is easy to verify, by steps completely analogous to those in Eqs. (2.23)-(2.25), that for the BPS solutions the one-dimensional effective Lagrangian is just equal to twice the central charge,
\[ Z_{\text{SUSY}} = \int_{x_{\min}}^{x_{\max}} \nabla W = W(x_{\max}) - W(x_{\min}) . \]

Applying this to the nonextremal black hole example of the previous section, we see that \( W \) differs from the quantity appearing in the integrand of \( Z_{\text{non-ext}} \) in Eq. (2.15) by the surface terms inherited from Eq. (2.7); i.e.,
\[ W = -2 \sqrt{Q^2 e^{2n_4} + \beta^2} + 2 \sqrt{e^{2n_3} + \beta^2} + 2 \beta \arcsinh \left( \frac{\beta}{Q} e^{-n_4} \right) - 2 \beta \arcsinh \left( \beta e^{-n_3} \right) . \]

Substituting the BPS solution into this yields
\[ W(u) = -2 \beta \frac{\cosh(\beta u + c_4)}{\sinh(\beta u + c_4)} + 2 \beta \frac{\cosh(\beta u)}{\sinh(\beta u)} + 2 \beta^2 (u + c_4) - 2 \beta^2 u , \]
which diverges as \( u \to 0 \) (i.e., as \( r \to \infty \)). Hence, \( Z_{\text{SUSY}} = W(\infty) - W(0) \) will be infinite. In Sec. 2.2 this divergence was cancelled by the inclusion in \( Z_{\text{ext}} \) and \( Z_{\text{non-ext}} \) of the additional surface term appearing in Eq. (2.15). This can be interpreted as the subtraction of the (also infinite) central charge of Minkowski space.

### 3.2 Is there a Killing spinor?

If nonextremal black holes are also supersymmetric in some four-dimensional theory, there must be Killing spinors corresponding to the preserved supersymmetries for which the variations of the gravitinos \( \psi_{\mu j} \) and the spin-1/2 fields \( \chi_I \) vanish. Thus, there must be spinors \( \epsilon_j \) such that
\[ 0 = \delta \psi_{\mu i} = (\hat{\nabla}_\mu)_i^j \epsilon_j , \]
\[ 0 = \delta \chi_I = (R_I^j) \epsilon_j . \]

Here the generalized covariant derivative \( (\hat{\nabla}_\mu)_i^j \) and the functional \( R_I^j \) can in principle depend on any of the bosonic fields of the theory. Naturally the generalized covariant derivative must contain the standard spin connection; its most general form is therefore
\[ (\hat{\nabla}_\mu)_i^j \epsilon_j = (\nabla_\mu \delta^{ij} + Y^{ij}) \epsilon_j \]
\[ = \left[ \partial_\mu \delta^{ij} + \omega_{ab} \frac{1}{4} (\gamma_{ab})_{\beta} \delta^{ij} + (Y^{ij})_{\beta} \right] \epsilon_j^\beta . \]

In the second line we have explicitly written out the spinor indices \( \alpha, \beta \). All (possibly nonlinear) dependence on the nongravitational bosonic fields of the theory is contained in \( (Y^{ij})_{\alpha} \) and \( R_I^j \). This dependence must be such that the commutators of the supersymmetry variations are consistent
with the supersymmetry algebra. We will assume that this is possible only if these quantities vanish whenever the nongravitational fields all vanish. For our solutions, this means that \( Y_\mu \) must vanish if \( F_{\mu\nu} = 0 \).

If such a Killing spinor existed, it would lead to a generalized BPS bound through the positive energy theorem. However, the relation of this bound to supersymmetry is not manifest. Even though a Killing spinor which implies a saturation of the positive energy bound may exist, it is not evident that such Killing spinor arises as the gravitino variation of a known supergravity theory. In fact recent research has shown that quite a number of spacetimes exist which are BPS, saturate a positive energy bound, have a Killing spinor, but do not preserve any of the supersymmetries of any known supergravity theory [12, 13, 14, 15, 16]. Thus, even though demonstrating the existence of a Killing spinor would establish that the nonextremal solutions saturate a modified energy bound, it would only be a first step toward showing that they preserve some supersymmetry.

On the other hand, if it can be shown that there is no Killing spinor, then there is no preserved supersymmetry and, presumably, no saturated bound associated with these solutions. We will now proceed to show that this is the case. We will do this by showing that there is no choice for \( Y_\mu \) that (1) when evaluated on the nonextremal solution admits a nonzero spinor satisfying

\[
(\nabla_\mu + Y_\mu)\epsilon = 0 .
\]

and (2) vanishes when all of the nongravitational fields are set equal to zero.

We start by substituting the nonextremal Reissner-Nordström metric into the Killing equation to determine what form \( Y_\mu \) can take. In doing this, it will be helpful to perform some conformal transformations on the metric. We first recall that under a conformal rescaling \( \tilde{g}_{\mu\nu} = e^{2A}g_{\mu\nu} \), the covariant derivative \( \nabla_\mu \epsilon \) of a spinor is transformed to

\[
\tilde{\nabla}_\mu \epsilon = \left[ \nabla_\mu + \frac{1}{2} \gamma_\mu^\nu \nabla_\nu A \right] \epsilon ,
\]

(3.10)

where \( \gamma^{\mu\nu} = \frac{1}{2} [\gamma^\mu, \gamma^\nu] \).

Now let us define the metrics

\[
\begin{align*}
ds^2_{(4)} &= e^{-2n_4}d\tilde{s}^2 = -dt^2 + e^{2n_3-4n_4} \left( d\Omega_2^2 + e^{2n_3}du^2 \right) , \\
ds^2_{(3)} &= -dt^2 + \left( d\Omega_2^2 + e^{2n_3}du^2 \right) , \\
ds^2_{(1)} &= -dt^2 + d\Omega_2^2 + du^2 .
\end{align*}
\]

(3.11)

We will use superscripts (4) and (3) to denote the covariant derivatives and gamma matrices corresponding to \( ds^2_{(4)} \) and \( ds^2_{(3)} \); the gamma matrices corresponding to \( ds^2_{(1)} \) will be indicated by hats.

The first of these metrics, \( ds^2_{(4)} \), is just an overall conformal rescaling of our metric \( ds^2 \). Hence, using Eq. (3.10), we obtain

\[
\nabla_\mu \epsilon = \left[ \nabla_\mu^{(4)} + \frac{1}{2} \gamma_\mu^{(4)\nu} \nabla_\nu n_4 \right] \epsilon
\]

\[
= \left[ \nabla_\mu^{(4)} + \frac{1}{2} \gamma_\mu^{(4)u} \nabla_u n_4 \right] \epsilon .
\]

(3.12)

In the last step we have used the fact that \( n_4 \) only depends on the coordinate \( u \).
We next note that the metric $ds^2_{(4)}$ is a direct product between $\mathbb{R}$ (time) and a three-dimensional space spanned by $u$, $\theta$, and $\phi$. We can therefore decompose the spinor covariant derivative into two equations and perform a second conformal rescaling, just on the directions $m = u, \theta, \phi$. As a result the connections and gamma matrices are those for the metric $ds^2_{(3)}$. Again using Eq. (3.10), we find that

$$
\nabla_t \epsilon = \left\{ \frac{\partial}{\partial t} + \frac{1}{2} \gamma^{(3)} \gamma_s^{(u)} \nabla u n_4 \right\} \epsilon = \left\{ \frac{\partial}{\partial t} + \frac{1}{2} \gamma^{(3)} \gamma_s^{(u)} e^{-n_3 + 2n_4} \nabla u n_4 \right\} \epsilon ,
$$

$$
\nabla_m \epsilon = \left\{ \nabla^{(3)} + \frac{1}{2} \gamma^{(3)} \gamma_s^{(u)} \nabla u n_4 \right\} \epsilon = \left\{ \nabla^{(3)} + \frac{1}{2} \gamma^{(3)} \gamma_s^{(u)} [\nabla u (n_3 - 2n_4)] + \frac{1}{2} \gamma^{(3)} \nabla u n_4 \right\} \epsilon .
$$

Implicit here is a simultaneous decomposition of the $(3+1)$-dimensional spinor and Dirac matrices into one- and three-dimensional ones; we will make this explicit shortly.

By construction the spatial part of $ds^2_{(3)}$ is again a direct product and so we can, by a similar procedure, transform to $ds^2_{(1)}$ via a conformal transformation on the one-dimensional radial part of the metric, leading to

$$
\nabla_t \epsilon = \left\{ \frac{\partial}{\partial t} + \frac{1}{2} \gamma^{(3)} \gamma_s^{(u)} e^{-n_3 + 2n_4} \nabla u n_4 \right\} \epsilon = \left\{ \frac{\partial}{\partial t} + \frac{1}{2} \gamma^{(3)} \gamma_s^{(u)} e^{2(n_4 - n_3)} \nabla u n_4 \right\} \epsilon ,
$$

$$
\nabla_\theta \epsilon = \left\{ \nabla^{(3)} + \frac{1}{2} \gamma^{(3)} \gamma_s^{(u)} [\nabla u (n_3 - n_4)] \right\} \epsilon = \left\{ \nabla^{(3)} + \frac{1}{2} \gamma^{(3)} \gamma_s^{(u)} [\nabla u (n_3 - n_4)] \right\} \epsilon ,
$$

$$
\nabla_\phi \epsilon = \left\{ \nabla^{(3)} + \frac{1}{2} \gamma^{(3)} \gamma_s^{(u)} [\nabla u (n_3 - n_4)] \right\} \epsilon = \left\{ \nabla^{(3)} + \frac{1}{2} \gamma^{(3)} \gamma_s^{(u)} [\nabla u (n_3 - n_4)] \right\} \epsilon ,
$$

$$
\nabla_u \epsilon = \left\{ \nabla^{(3)} + \frac{1}{2} \gamma^{(3)} \gamma_s^{(u)} [\nabla u (n_3 - n_4)] \right\} \epsilon = \frac{\partial}{\partial u} \epsilon .
$$

This makes the full $u$-dependence of the covariant derivative explicit.

Inserting now the appropriately rescaled generalized connection $Y_\mu = e^a_\mu \hat{Y}_a$ with $e^a_\mu$ equal to $\delta^a_\mu$ times the appropriate conformal factor as described in Eq. (3.11), we find that the conditions for the existence of a generalized Killing spinor are

$$
0 = \left\{ \frac{\partial}{\partial t} + \frac{1}{2} \gamma^{(3)} \gamma_s^{(u)} e^{2(n_4 - n_3)} \nabla u n_4 + e^{n_4} \hat{Y}_t \right\} \epsilon ,
$$

$$
0 = \left\{ \nabla^{(S^2)}_\theta + \frac{1}{2} \gamma^{(3)} \gamma_s^{(u)} e^{-n_3} [\nabla u (n_3 - n_4)] + e^{n_3 - n_4} \hat{Y}_\theta \right\} \epsilon ,
$$

$$
0 = \left\{ \nabla^{(S^2)}_\phi + \frac{1}{2} \gamma^{(3)} \gamma_s^{(u)} e^{-n_3} [\nabla u (n_3 - n_4)] + e^{n_3 - n_4} \hat{Y}_\phi \right\} \epsilon ,
$$

$$
0 = \left\{ \frac{\partial}{\partial u} + e^{2n_3 - n_4} \hat{Y}_u \right\} \epsilon .
$$

The decomposition of the spacetimes $ds^2_{(i)}$ into direct products, which we used to implement the conformal rescalings, implies that the $(3+1)$-dimensional spinor $\epsilon$ can be written in the form

$$
\epsilon(t, \theta, \phi, u) = \eta(t, u) \otimes \zeta(\theta, \phi) .
$$

The time-independence of all the Killing vectors of the metric of Eq. (2.2) demands that the Killing spinor $\epsilon$ must be time-independent. Then the first equation of (3.15) implies that $\hat{Y}_t$ must be such
that on the Reissner-Nordström background
\[ e^{n_4 - 2n_3} \gamma^u \nabla_u n_4 \epsilon = - \gamma^t \tilde{Y}_t \epsilon \]  
(3.17)
where here (and in similar cases below) there is no sum over \( u \) or \( t \). Substituting this background expression for \( \nabla_u n_4 \) into the second and third equations of (3.15), we obtain
\[ \left[ \nabla^{(S^2)}_\theta + \frac{1}{2} \gamma \gamma_u e^{-n_3} \nabla_u n_3 + e^{n_3 - n_4} \gamma_\theta \left( \gamma^t \tilde{Y}_t + \gamma^\theta \tilde{Y}_\theta \right) \right] \epsilon = 0 , \]
\[ \left[ \nabla^{(S^2)}_\phi + \frac{1}{2} \gamma \gamma_u e^{-n_3} \nabla_u n_3 + e^{n_3 - n_4} \gamma_\phi \left( \gamma^t \tilde{Y}_t + \gamma^\phi \tilde{Y}_\phi \right) \right] \epsilon = 0 . \]  
(3.18)
Compatibility with the decomposition of the Killing spinor of Eq. (3.16) demands that we find a spinor \( \zeta \) on the unit two-sphere that obeys
\[ \nabla^{(S^2)}_\theta \zeta = \sigma_3^{(S^2)} \sigma_\theta^{(S^2)} A \zeta , \quad \nabla^{(S^2)}_\phi \zeta = \sigma_3^{(S^2)} \sigma_\phi^{(S^2)} A \zeta . \]  
(3.19)
with \( A \) independent of \( \theta, \phi \). The choice to include an additional factor of \( \sigma_3^{(S^2)} \) is made to ensure that \( A \) is real and that there exists a Killing spinor for the extremal case; see below. Acting twice with Eq. (3.19) gives the integrability constraint
\[ \frac{1}{4} R^{(S^2)\alpha\beta\sigma} \sigma_{\alpha\beta}^{(S^2)} \zeta = \left( \nabla^{(S^2)}_\theta \nabla^{(S^2)}_\phi - \nabla^{(S^2)}_\phi \nabla^{(S^2)}_\theta \right) \zeta = 2 \sigma_3^{(S^2)} A^2 \zeta . \]  
(3.20)
As \( S^2 \) is the unit two-sphere, it follows that \( A = \pm \frac{1}{2} \). If we embed the \( S^2 \) Dirac matrices \( \sigma_\theta, \sigma_\phi \) into the four-dimensional ones via
\[ \begin{align*}
\gamma^t &= i \sigma_2 \otimes \sigma_3 , \\
\gamma^\theta &= \sigma_3 \otimes \sigma_\theta = 1 \otimes \sigma_1 , \\
\gamma^\phi &= \sigma_3 \otimes \sigma_\phi = 1 \otimes \sigma_2 , \\
\gamma^u &= \sigma_3 \otimes \sigma_3 ,
\end{align*} \]  
(3.21)
Eq. (3.18) reduces to the requirement that
\[ \left[ \frac{1}{2} \gamma_\mu \left( e^{-n_3} \nabla_u n_3 \pm 1 \right) + e^{n_3 - n_4} \left( \gamma^t \tilde{Y}_t + \gamma^\theta \tilde{Y}_\theta \right) \right] \epsilon = 0 , \]
\[ \left[ \frac{1}{2} \gamma_\mu \left( e^{-n_3} \nabla_u n_3 \pm 1 \right) + e^{n_3 - n_4} \left( \gamma^t \tilde{Y}_t + \gamma^\phi \tilde{Y}_\phi \right) \right] \epsilon = 0 . \]  
(3.22)
We next decompose \( Y_\mu \) as
\[ Y_\mu = \gamma_\mu Y + C_{\rho\sigma} \gamma^{\rho\sigma} \gamma_\mu , \]  
(3.23)
where \( Y \), but not \( C_{\rho\sigma} \), is a matrix in spinor space. Because Lorentz transformations must act in the standard way, we can do so without loss of generality.\(^5\) The spherical symmetry of the background

\(^5\)The need to separate out \( C_{\rho\sigma} \) is due to the \( d = 4 \) identity \( \gamma^{\mu_1\mu_2\mu_3\mu_4} \gamma_{\mu_1\mu_2\mu_3\mu_4} = 0 \). To prove that \( Y_\mu \) can always be written as \( Y_\mu = \gamma_\mu Y + \gamma^{\rho\sigma} C_{\rho\sigma} \gamma_\mu \), note that \( Y_\mu - \gamma^{\rho\sigma} C_{\rho\sigma} \gamma_\mu = \sum_{n=0}^{d} Z_{\alpha_1...\alpha_n} \gamma^{\alpha_1...\alpha_n} \) can be uniquely reconstructed from \( Y = \sum_{n=0}^{d} X_{\alpha_1...\alpha_n} \gamma^{\alpha_1...\alpha_n} \) through \( Z_{\alpha_1...\alpha_n} = \frac{1}{n!} \text{Tr} (\gamma^{\alpha_1...\alpha_n} Y) \). Using that \( \gamma_\alpha...\gamma_\mu = \gamma_\alpha...\gamma_\mu + \frac{1}{(\alpha-1)!} \gamma_\alpha...\gamma_{\alpha-1} \eta_\alpha \gamma_\mu \), one finds that \( Z_{\alpha_1...\alpha_n} = X_{\alpha_1...\alpha_n} \).
implies that the only nonzero components of $C_{ab}$ are $C_{tu}$ and $C_{\theta \phi}$. As a consequence of the Dirac matrix identities

$$C_{tu} \left( \gamma^t \gamma^u + \gamma^u \gamma^t + \gamma^\theta \gamma^\phi \gamma^t + \gamma^\phi \gamma^\theta \gamma^t \right) = 0,$$

$$C_{\theta \phi} \left( \gamma^\theta \gamma^\phi \gamma^t + \gamma^\phi \gamma^\theta \gamma^t \right) = 0,$$

(3.24)

the $C_{ab}$ parts vanish identically in Eq. (3.22), leaving the single equation

$$\left[ \frac{1}{2} \hat{\gamma}_u \left( e^{-n_3} \nabla_u n_3 \pm 1 \right) + e^{n_3-n_4} 2Y \right] \epsilon = 0.$$ 

(3.25)

For the extremal solution, this is solved by taking the upper sign in the first term and setting $Y = 0$. It is then straightforward to show that the remaining equations can be solved, and hence that there is a Killing spinor, if $C_{\mu \nu} = F_{\mu \nu}$.

For the nonextremal solutions, we would need a $Y$ constructed out of the background bosonic fields that was such that

$$4Y \epsilon = -e^{-n_3+n_4} \gamma^u (e^{-n_3} \nabla_u n_3 \mp 1) \epsilon.$$ 

(3.26)

We could proceed by evaluating the quantity on the right-hand side and then trying to express it in terms of the background fields. However, there is no need to do so. We are assuming that any choice of $Y_\mu$ that gives a gravitino variation consistent with the superalgebra must vanish when the nongravitational fields are set equal to zero. In the present case, the only nongravitational field is $F_{\mu \nu}$, which clearly vanishes when $Q = 0$. However, substituting the $Q = 0$ nonextremal solution yields

$$e^{-n_3} \nabla_3 \mp 1 = -\frac{1}{2} \sqrt{\left(1 - M/r\right) + \frac{1}{1 - M/r}} + 2 \mp 1 \neq 0$$

(3.27)

Except in the trivial $M = 0$ case, Eq. (3.26) then requires a nonzero $Y$, in contradiction to our assumption. Hence, the nonextremal solutions do not admit Killing spinors.

### 4 Discussion and Conclusion

We have shown that static nonextremal black hole solutions are BPS; i.e., they are solutions to first-order equations obtained by writing the Lagrangian in terms of a sum of squares. As we explained in the introduction, all known BPS solutions in flat spacetime field theory preserve a fraction of supersymmetries in some extension of the theory. However, we have seen that this empirical result cannot extend to our gravitational theory.

For the dimensionally reduced (1+1)-dimensional theory it is a straightforward matter to solve the Killing spinor equations and find a preserved supersymmetry. The difficulty is with the lift to (3+1) dimensions. If we make the plausible assumption that consistency with the supersymmetry algebra requires that any additional terms in the generalized connection must vanish when the nongravitational fields are all zero, we then find that the (3+1) nonextremal solutions admit no Killing spinors. Hence, there cannot be a supergravity extension of the theory where the nonextremal BPS solutions preserve a fraction of the supersymmetries.

We can thus distinguish four types of gravitational solutions: (i) first-order solutions that preserve a fraction of supersymmetries in a known supergravity theory, (ii) first-order solutions that saturate a bound following from the existence of a generalized Killing spinor, but that do not correspond to a known gravitino variation, (iii) first-order solutions that do not appear to saturate
a bound, and (iv) standard second-order solutions. In this note we have shown that nonextremal black branes belong to category (iii) and why they do so.

Finally, we wish to point out that our results share a connection with Sen’s recent construction of an entropy function for extremal black holes [17, 18, 19]. For extremal black holes with near-horizon \( AdS_2 \times S^2 \) geometry this entropy function is the Legendre transform with respect to the electric charges of the Lagrangian density integrated over the \( S^2 \). This two-sphere is conformal to the one over which we reduced to construct our \( (1+1) \)-dimensional effective action. Hence, the \( (1+1) \)-dimensional Lagrangian density for which both extremal and nonextremal black holes are BPS solutions is related to Sen’s entropy function. It would be interesting to investigate whether this entropy function can be used for nonextremal black holes as well. A naive extension of Sen’s entropy function to nonextremal black holes does not work, as it relies crucially on the \( AdS_2 \times S^2 \) near horizon geometry of extremal black holes rather than the BPS structure [20].

Added note: After this work was completed, we learned of Ref. [21], where it was noted that some nonextremal anti-de Sitter black holes can be obtained from first-order equations that integrate to a superpotential. The methods we use in Sec. 3 are readily generalized to this case, and show that these nonextremal black holes are not supersymmetric.

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