Abstract

In this work we present a dual mapping between the Kalb-Ramond and antisymmetric tensor matter (ATM) field actions. Our procedure shows that the correlation functions associated with both the Noether current and the topological current are equivalent.

Key words: Kalb Ramond, matter field, dual mapping

PACS: 11.30.-j, 11.90.+t

Kalb-Ramond fields first appeared as a tensorial generalization of vector gauge fields. These allow the construction of topological invariants in $D$-dimensional manifolds \cite{123} and have an important role in dualization \cite{450}. Such antisymmetric tensor fields are the key to generate mass for the vector gauge fields through the topological mass mechanism \cite{8910}. More recently one finds some applications in the so-called physics of extra dimension. It is also noteworthy that the Kalb-Ramond fields appear in effective theories of specific low energy superstring models, and may describe axion physics or torsion of a Riemannian manifold \cite{11}.

Different types of (second rank) antisymmetric tensor fields have been introduced by Avdeev and Chizhov in \cite{12}. There, on the other hand, the action was constructed with a matter field rather than a gauge field exhibiting several interesting features. As showed later by Lemes et al \cite{13}, in a BRST framework, some specific model which have antisymmetric tensor matter (ATM) fields is

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renormalizable to arbitrary perturbative orders. In a subsequent paper they further realized that the ATM field is a real component of a complex tensor field that satisfies a complex self-dual condition [14]. As shown by the authors this condition makes the ATM field massless.

In a more recent paper [16] we proposed a mechanism to generate mass to the ATM field which preserves the U(1) symmetry. By this mechanism, a topological term is introduced via a complex vector field. In [17] we also analyzed the possibility to give mass to a ATM field through the Higgs mechanism. There, a scalar field is coupled to the ATM field and requiring parity conservation it is described as a doublet where one of its components is a pseudo-scalar. We also showed that a topological term for the ATM field can be also generated by spontaneous symmetry breaking.

In spite of a large variety of papers thereafter published about these two types of tensors, no one has reported a possible connection between them. In the present paper we will show the existence of a dual mapping between the Kalb-Ramond and ATM fields. We apply the dual mapping method developed by Fosco et al. [18] by making use of the complex self-dual condition and the $U(1)$ symmetry.

Let us consider the action for the ATM field written in terms of the complex self-dual tensor [14]

$$S(\varphi, \varphi^\dagger) = \int d^4x \partial_\mu \varphi^{\mu\nu} \partial^\rho \varphi^\dagger_{\rho\nu},$$

(1)

where $\varphi_{\mu\nu} = T_{\mu\nu} + i \tilde{T}_{\mu\nu}$ is the complex anti-symmetric tensor field satisfying the complex self-dual condition $\varphi_{\mu\nu} = i \tilde{\varphi}_{\mu\nu}$ [14]. In order to show the dual mapping procedure we first linearize the derivatives of the kinetic term with the introduction of a complex field $b_\mu = c_\mu + id_\mu$. We can now write the action (1) in the form

$$S(\varphi, b) = \int d^4x \left( b^\dagger_\mu \partial_\rho \varphi^{\rho\mu} + b_\nu \partial_\rho \varphi^{\rho\nu \dagger} - b_\mu b^\dagger_\mu \right).$$

(2)

The on-shell equivalence of (1) and (2) can be obtained by eliminating $b_\mu$ through the equation of motion

$$\frac{\delta S}{\delta b_\mu} = \partial^\rho \varphi_{\rho\mu} - b_\mu = 0.$$

(3)

The Noether current related to the global $U(1)$ invariance of Eq. (2) is obtained as usual by replacing the normal derivative of the tensor field $\varphi_{\mu\nu}$ by its covariant derivative $D_\mu \varphi_{\rho\sigma} = \partial_\mu \varphi_{\rho\sigma} + is_\mu \varphi_{\rho\sigma}$, namely

$$J^\mu = i \left( b^\dagger_\mu \varphi^{\mu\nu} - b_\nu \varphi^{\nu\mu \dagger} \right).$$

(4)
where $s_\mu$ is an external source.

The generating functional $Z[s_\mu]$ for current-current correlation is

$$Z[s_\mu] = \int [D\varphi Db] e^{-i(S(\varphi,b) + \int d^4x s_\mu J^\mu)},$$

(5)

where the invariant functional measure can be written as

$$[D\varphi Db] = \delta(\varphi_{\mu\nu} - i\bar{\varphi}_\mu \varphi_\nu) D\varphi_\mu D\varphi^\dagger_\mu Db_\mu Db^\dagger_\mu.$$  

(6)

From gauge invariance of (5), it follows that

$$Z[s_\mu] = \int [D\varphi Db] D\eta_\mu \delta[f_{\mu\nu}(\eta)] e^{-i(S(\varphi,b) + \int d^4x (s_\mu + \eta_\mu)J^\mu)},$$

(7)

Here we have defined $f_{\mu\nu} = \varepsilon_{\mu\nu\rho\sigma} \partial_{\rho} \eta_{\sigma}$, $\eta_\mu = \partial_\mu \alpha$, and $\alpha$ is a scalar field.

With the following representation of the Dirac’s delta functional

$$\delta[f_{\mu\nu}(\eta)] = \int DB_{\mu\nu} e^{-i \int d^4x B_{\mu\nu} \varepsilon_{\mu\nu\rho\sigma} \partial_\rho \eta_\sigma},$$

(8)

where $B_{\mu\nu}$ is an anti-symmetric tensor field, Eq. (7) can be written as

$$Z[s_\mu] = \int [D\varphi Db] DB_{\mu\nu} D\eta_\mu e^{-i(S(\varphi,b) + \int d^4x (s_\mu + \eta_\mu)J^\mu + \int d^4xB_{\mu\nu} \varepsilon_{\mu\nu\rho\sigma} \partial_\rho \eta_\sigma)}.$$

(9)

Redefining $\eta_\mu$ by $\eta_\mu = \nu_\mu - s_\mu$, the external source $s_\mu$ decouples from the matter field, and we get

$$Z[s_\mu] = \int [D\varphi Db] DB_{\mu\nu} D\nu_\mu e^{-i(S(\varphi,b) + \int d^4x \nu_\mu J^\mu + \int d^4xB_{\mu\nu} \varepsilon_{\mu\nu\rho\sigma} \partial_\rho (\nu_\sigma - s_\sigma))}.$$  

(10)

We can now define a dual action for $B_{\mu\nu}$ as follows

$$e^{-iS_{\text{dual}}[B_{\mu\nu}]} = \int [D\varphi Db] D\nu_\mu e^{-i(S(\varphi,b) + \int d^4x \nu_\mu J^\mu + \int d^4xB_{\mu\nu} \varepsilon_{\mu\nu\rho\sigma} \partial_\rho \nu_\sigma)}.$$

(11)

such that

$$Z[s_\mu] = \int DB_{\mu\nu} e^{-i(S_{\text{dual}}[B_{\mu\nu}]-\int d^4x s_\mu \varepsilon_{\mu\nu\rho\sigma} \partial_\rho \omega_\sigma)}.$$  

(12)

As we can see from Eq. (11), $B_{\mu\nu}$ is in fact a true gauge field with a gauge invariant action

$$S_{\text{dual}}[B_{\mu\nu}] = S_{\text{dual}}[B_{\mu\nu} + \partial_\mu \omega_\nu].$$

(13)

The $U(1)$ global symmetry for the fields and the complex self-dual condition are the key ingredients to implement our proposed duality. Actually, it is a
well-known fact that the existence of a global symmetry is crucial to obtain dualities.

Let us underline here that the same generating functional given by the equations \((5)\) and \((12)\) written in terms of \(\varphi_{\mu\nu}\) or \(B_{\mu\nu}\) gives the same correlation function for the \(U(1)\) current and the topological current, \(i.e.,\)

\[
\langle J_{\mu_1}(x_1)J_{\mu_2}(x_2) \ldots J_{\mu_n}(x_n) \rangle_{\varphi_{\mu\nu}} = \langle j^T_{\mu_1}(x_1)j^T_{\mu_2}(x_2) \ldots j^T_{\mu_n}(x_n) \rangle_{B_{\mu\nu}},
\]

where

\[
j^T_{\mu} = \epsilon^{\mu\nu\rho\sigma} \partial_\nu B^{\rho\sigma},
\]

and

\[
J_{\mu} = i \left( \partial_\rho \varphi^{\rho\nu} \varphi_{\mu\nu} - \partial_\rho \varphi^{\mu\nu} \varphi_{\rho\nu} \right) = 2(\partial_\rho \tilde{T}^{\rho\mu\nu} T_{\mu\nu} - \partial_\rho T^{\rho\mu\nu} \tilde{T}_{\mu\nu}).
\]

Note that both \(j^T_{\mu}\) and \(J_{\mu}\) are axial currents. As a consequence, parity is preserved in this duality. It is worth mentioning that the dual mapping developed in \([18]\) breaks the parity symmetry for that scalar model in three dimension.

In conclusion, in this paper we have obtained a dual mapping between the Kalb-Ramond and the ATM field actions. This kind of dualization was implemented by the requirement of complex self-duality. As we saw, the same correlation function for the \(U(1)\) current and the topological current was obtained. As a consequence parity is preserved and it is free of axial anomalies.

**Acknowledgment**

We wish to thank H.R Christiansen for a critical reading of the manuscript. The Conselho Nacional de Desenvolvimento Científico e tecnológico-CNPq is gratefully acknowledged for financial support.

**Dedicatory**

R. R. Landim - This paper is dedicated to the memory of my wife, Isabel Mara.
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