Supersymmetric Electrovens In Gauged Supergravities

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ABSTRACT

We show that the D=6 SU(2) gauged supergravity of van Nieuwenhuizen et al, obtained by dimensional reduction of the D=7 topologically massive gauged supergravity and previously thought not to be dimensionally reducible, can be further reduced to five and four dimensions. On reduction to D=4 one recovers the special case of the SU(2)×SU(2) gauged supergravity of Freedman and Schwarz for which one of the SU(2) coupling constants vanishes. Previously known supersymmetric electrovens of this model then imply new ground states in 7-D. We construct a supersymmetric electrovac solution of N=2 SU(2) gauged supergravity in 7-D. We also investigate the domain wall solutions of these theories and show they preserve a half of the supersymmetry.
1. Introduction

There are many supergravity theories with a scalar potential without critical points and hence no obvious ground state. In many simple cases the scalar potential takes the form

\[ V(\phi) \sim e^{-a\phi} \] (1.1)

where \( a \) is a constant which is related to a very useful quantity \( \Delta \), by the equation

\[ a^2 = \Delta + \frac{2(D - 1)}{(D - 2)} \] (1.2)

where \( D \) is the spacetime dimension.† It is convenient to reexpress the parameter \( a \) in terms of \( \Delta \) because \( \Delta \) remains unchanged after reduction on \( S^1 \) [1]. Moreover, \( \Delta \) also allows us to distinguish between massive and gauged supergravities with potentials of the above form. Massive supergravities are theories that can contain topological mass terms as well as explicit mass terms for some of the antisymmetric tensor potentials in the model. The value of \( \Delta \) for these theories is 4. A classic example of such a supergravity is the D=10 massive IIA supergravity [2], in which a second rank antisymmetric tensor acquires a mass in a Higgs type mechanism. Gauged supergravities on the other hand, are distinct from massive supergravities in that the automorphism group (or one of its subgroups) of the supersymmetry algebra is gauged, the vector fields of the supergravity multiplet playing the role of the gauge fields. The subclass of gauged supergravities with potentials like (1.1) have \( \Delta \) negative in all known cases. Well known examples are the D=7 N=2 SU(2) gauged supergravity [3], D=6 N=4 SU(2) gauged supergravity [4] and the D=4 N=4 SU(2)×SU(2) model of Freedman and Schwarz (FS) [5]. The value of \( \Delta \) for all these SU(2) gauged supergravities is -2. As \( \Delta \) is unchanged under both Kaluza-Klein (KK) and Scherk Schwarz reduction on \( S^1 \), one might suspect that

† we assume the dilaton is canonically normalised
these theories are related. This is in fact true but until recently it was believed these supergravities couldn’t be dimensionally reduced. The reason why it is now believed that they can be will be explained later.

At this point we note that there also exist supergravities whose potentials are sums of functions of the type given in (1.1). Supergravities which are both gauged and possessing topological mass terms are examples of these types of theories.

There are two important questions to be asked of supergravities with potentials as in (1.1). The first concerns the nature of the ground state of these theories and the second concerns whether these theories are consistent truncations of higher dimensional theories. Due to the lack of a Minkowski vacuum preserving all of the supersymmetry, it is natural to look for solutions preserving some fraction of the supersymmetry. These supersymmetric solutions would of course be stable and therefore presumably important to understanding whether these theories make sense. The FS model and it’s SU(2)×U(1)³ ‘half gauged’ version, obtained by setting one of the SU(2) coupling constants to zero, are known to have supersymmetric electrovac solutions [6]. These supersymmetric electrovac solutions have the property that the dilaton is zero and a two form gauge field strength is covariantly constant. The D=10 massive type IIA supergravity and the D=7 N=2 SU(2) gauged supergravity possess domain wall ((D-2) brane) solutions preserving a half of the supersymmetry [7,8,9]. These solutions differ from the much studied p-brane solutions [10]† in that the dilaton is the only non-constant non-vanishing field and so also have a quite different geometry to the supersymmetric electrovacs. A further question of interest then is whether the FS model has supersymmetric domain wall solutions and whether the D=7 N=2 SU(2) supergravity has supersymmetric electrovac solutions. This will be answered by looking at the question of dimensional reduction.

Because of the form of V(φ), these supergravities admit no S¹ × M_{D-1} direct product vacuum solutions so it might seem that dimensional reduction is not

† p ≠ D-2
possible. However, it was argued in [9] that one can always perform a consistent dimensional reduction, regardless of the solution space, by simply implementing the standard KK ansatz on the fields of the higher dimensional theory (substitution of the standard KK ansatz for the fields into the lagrangian gives a lower dimensional lagrangian whose field equations are the same as those obtained by direct substitution of the KK ansatz into the higher dimensional field equations). However, one is not guaranteed solutions of the lower dimensional theory unless, in the higher dimension, there exists a solution with a U(1) isometry, in which case it will map into a solution of the lower dimensional theory. The dimensional reduction of massive supergravities was studied in [11]. In this paper we focus on the gauged supergravities with SU(2) gauge groups. Now the FS model and the D=7 SU(2) gauged model have no known higher dimensional origin. It was believed these supergravities couldn’t be dimensionally reduced, for reasons given above, and hence were unrelated. However van Nieuwenhuizen et al [13] argued that with the inclusion of a topological mass term, the scalar potential of N=2 SU(2) gauged 7-D supergravity depends on two parameters and does possess a stable minimum, so therefore can be compactified to 6-D. They showed that if the parameter in front of the topological mass term is non-zero, the resulting 6-D theory is irreducible. They also showed that if this parameter is allowed to tend to zero in 6-D, the theory describes the reducible coupling of an SU(2) gauged pure N=4 supergravity multiplet to an N=4 vector supermultiplet. This D=6 N=4 SU(2) gauged supergravity was a special case of the massive gauged supergravities presented in [4].

One of the purposes of this paper therefore is to continue this reduction to four dimensions, making contact with the SU(2)×SU(2) gauged model of Freedman and Schwarz. Given we know the D=7 SU(2) gauged supergravity to have

§ At the time of writing a paper has appeared [12] showing D=10 type I supergravity on $S^3 \times S^3$ yields the Freedman Schwarz model. The implications of this result will be explored in future publications.
¶ Following [9], we note that the D=6 N=4 SU(2) gauged supergravity obtained in [13] can also be obtained by dimensional reduction of D=7 N=2 SU(2) gauged supergravity without a topological mass term.
a domain wall solution with an R-isometry*, the reduction will be consistent and we are guaranteed solutions in the lower dimension. We will show that D=6 N=4 SU(2) gauged supergravity can be reduced on T^2 to yield, after appropriate truncations, a version of the SU(2)×SU(2) FS model for which one of the SU(2) gauge coupling constants vanishes. This relation between the D=7 and D=4 theories has implications for their solutions. In particular we must be able to lift the supersymmetric electrovac of the ‘half gauged’ FS model from 4-D to 7-D. We use this fact to construct a previously unknown and supersymmetric electrovac in 7-D. We are also able to double dimensionally reduce the 7-D supersymmetric domain wall to give a supersymmetric domain wall of the FS model.

The layout of the paper is as follows. In section 2 we dimensionally reduce the bosonic sector of the 6-D supergravity of [13] to 5-D giving details of how to truncate out the bosonic fields of a vector supermultiplet. In section 3 we continue this reduction to 4-D in less detail and make contact with the FS model. In section 4 we solve the Killing spinor equations of SU(2) gauged 7-D supergravity to obtain a supersymmetric electrovac preserving a half of the supersymmetry. In section 5 we double dimensionally reduce the 1/2 supersymmetric domain wall solution of 7-D gauged supergravity to obtain a domain wall in 4-D (which could have been found directly but this method demonstrates the consistency of the reduction). In section 6 we show that this domain wall preserves a half of the supersymmetry.

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* R-isometries are as good as U(1) isometries for the purposes of this argument.
2. Dimensional Reduction Of D=6 Gauged Supergravity

We now turn to the reduction of the N=4 SU(2) gauged supergravity in 6-D. We do not consider the reduction of the fermionic sector of the lagrangian or the supersymmetry transformations. However, it can be argued that under dimensional reduction (i.e. assuming all fields are independent of a particular coordinate) any symmetry of the higher dimensional lagrangian remains a symmetry of the lower dimensional lagrangian (this is not always true at the level of solutions though). Hence supersymmetry will be preserved when we reduce to five and four dimensions and our bosonic lagrangians are guaranteed supersymmetric extensions. We therefore consider the reduction of the bosonic sector only, which is [13]*

\[
L = e^\frac{\sigma}{\sqrt{6}} R_6 - \frac{1}{2} (\partial_{[\mu} \hat{\phi}^j_{\nu]}^i)^2 - \frac{1}{4} e^{-\frac{2\hat{\phi}}{\sqrt{6}}} [(\hat{F}_{\mu\nu}^j)^2 + (\hat{G}_{\mu\nu})^2] - \frac{1}{12} e^{\frac{2\hat{\phi}}{\sqrt{6}}} (\hat{F}_{\mu\nu\rho}^j)^2 + 4\alpha^2 e^{\frac{\hat{\phi}}{\sqrt{6}}}
\]

\[
- \frac{1}{24} e^{\mu\rho\sigma\lambda\delta} \hat{F}_{\mu\nu\rho}^j [\hat{G}_{\sigma\lambda} \hat{B}_\delta + Tr (\hat{F}_{\sigma\lambda} \hat{A}_\delta - \frac{2i\alpha}{3} \hat{A}_\sigma \hat{A}_\lambda \hat{A}_\delta)]
\]

(2.1)

where \( \hat{F}_{\mu\nu} = 3\partial_{[\mu} \hat{A}_{\nu]} \), \( \hat{G}_{\mu\nu} = 2\partial_{[\mu} \hat{B}_{\nu]} \) and \( \hat{F}_{\mu\nu}^j = 2(\partial_{[\mu} \hat{A}_{\nu]}^j + i\alpha \hat{A}_{[\mu}^{|i} \hat{A}_{\nu]}^j |k \)). \( \hat{F}_{\mu\nu}^j \) is a 2×2 symmetric matrix with i,j = 1,2. To perform the reduction to five dimensions, the ansatz for the fields are :

\[
e^\frac{\sigma}{\sqrt{6}} e^\frac{\sigma}{\sqrt{6}} e^\frac{a}{e} m \left( \begin{array}{cc} e^\sigma & 0 \\ e^{\frac{\sigma}{\sqrt{6}}} A_m & e^{-\frac{\sigma}{\sqrt{6}}} \end{array} \right)
\]

(2.2)

Where hats refer to D=6. \( \hat{a} = (a, z) \) are local Lorenz indices. \( \hat{m} = (m, z) \) are world indices.

\[
d\hat{S}_6^2 = e^{\frac{\sigma}{\sqrt{6}}} dS_5^2 + e^{\frac{\sigma}{\sqrt{6}}} (dz + \hat{A})^2
\]

(2.3)

Where \( \hat{A} = \hat{A}_m dx^m \) and \( f_2 = d\hat{A} \).

\[
\hat{B}_1 = B_1^{(1)} + \rho dz
\]

(2.4)

* We use the mostly plus metric convention.
\[ \hat{A}_2 = A_2 + C_1 dz \] (2.5)

\[ \hat{A}_{1i}^j = A_{1i}^j + A_i^j dz \] (2.6)

\[ \hat{\phi}(x^m, z) = \phi(x^m) \] (2.7)

Since we are reducing a non-maximal supergravity theory, in 5-D one necessarily obtains another non-maximal supergravity coupled to other supermultiplets, in this case a vector supermultiplet. We are interested only in the supergravity multiplet so we need to truncate out the vector supermultiplet. For the truncation to be consistent all one requires is for the full field equations to allow the fields to be truncated to be set to zero. For the truncation to preserve supersymmetry we require the variation of the truncated fermions to vanish (i.e. to be at least linear in the fields that are set to zero in the truncation). We have not explicitly shown this but we add that the final results suggest that supersymmetry is in fact preserved.

An N=4 vector supermultiplet in 5-D contains one vector and five scalar bosonic degrees of freedom. Now in [13], it was shown that in order to consistently truncate out the N=4 vector supermultiplet it is necessary to linearly combine the dualisation of the antisymmetric potential \( A_{\mu
u\rho} \) with the KK vector and it is necessary to linearly combine the dilaton (from the dilaton in 7-D) with the KK scalar. This suggests similar tasks must be undertaken in 5-D and 4-D to consistently truncate out the vector supermultiplets. Proceeding in the same spirit we retain the scalars \( \sigma \) and \( \phi \) which we will later linearly combine using an SO(2) rotation. We need to set four of the scalars to zero, we choose these to be \( A_i^j \) and \( \rho \). The
resulting action in 5-D is

\[ S_5 = \int d^5 x e \left\{ R_5 - \frac{1}{2} |d\sigma|^2 - \frac{1}{4} e^{-\frac{4\sigma}{\sqrt{6}}} |f_2|^2 - \frac{1}{2} |d\phi|^2 - \frac{1}{4} e^{-\frac{4\sigma}{\sqrt{2}}} |F_2|^2 + |G_2^{(1)}|^2 \right\} \\
- \frac{1}{12} e^{-\frac{2\sigma}{\sqrt{2}}} |F_3|^2 - \frac{1}{4} e^{-\frac{2\sigma}{\sqrt{2}}} |C_2|^2 + 4\alpha^2 e^{-\sqrt{2} + \frac{2\sigma}{\sqrt{6}}} \right\} \\
+ \frac{1}{2} \int C_2 [G_2^{(1)} B_1^{(1)} + Tr(F_2 A_1 - \frac{i\alpha}{3} A_1 A_1 A_1)] \]  

(2.8)

where \( F_2 = dA_1 + i\alpha A_1 A_1 \), \( G_2^{(1)} = dB_1^{(1)} \), \( F_3 = dA_2 - dC_1 \tilde{A} \), \( C_2 = dC_1 \) and wedge products are implied in the Chern-Simons term.

We still need to truncate a vector and a scalar. As a first step towards truncating a vector we dualise \( A_{\mu \nu} \) to a vector \( A'_\mu \) and linearly combine with the KK vector. The relevant part of the action is

\[ S'_5 = -\frac{1}{12} \int d^5 x e^{-\frac{2\sigma}{\sqrt{2}} - \frac{2\sigma}{\sqrt{6}}} (F_{\mu \nu \rho} F^{\mu \nu \rho}) = \int d^5 x L \]  

(2.9)

where \( F_{\mu \nu \rho} = 3(\partial_{[\mu} A_{\nu \rho]} - 2(\partial_{[\mu} C_{\nu]} \tilde{A}_{\rho]}). \)

We replace \( 3\partial_{[\mu} A_{\nu]} \) by a independent field \( a_{\mu \nu \rho} \). Let \( X_{\mu \nu \rho} = -6\partial_{[\mu} C_{\nu]} \tilde{A}_{\rho]} \) and add to the Lagrangian ,

\[ \Delta L = a e^{\mu \nu \rho \sigma \gamma} a_{\mu \nu \rho} F'_{\sigma \gamma} \]  

(2.10)

where \( F'_{\sigma \gamma} = 2\partial_{[\sigma} A'_{\gamma]} \) and \( a \) is a constant.

\[ L + \Delta L = -\frac{e^{-\frac{2\sigma}{\sqrt{2}} - \frac{2\sigma}{\sqrt{6}}}}{12} [a_{\mu \nu \rho} + X_{\mu \nu \rho}][a_{\mu \nu \rho} + X_{\mu \nu \rho}] + ae^{\mu \nu \rho \sigma \gamma} a_{\mu \nu \rho} F'_{\sigma \gamma} \]  

(2.11)

Variation w.r.t. \( A'_{\mu} \) gives \( a_{\mu \nu \rho} = 3\partial_{[\mu} A_{\nu \rho]} \), and the original Lagrangian, \( L \), can be recovered up to a total derivative after substitution.

Variation w.r.t \( a_{\mu \nu \rho} \) leads to

\[ a_{\mu \nu \rho} = 6a ee^{-\frac{2\sigma}{\sqrt{2}} + \frac{2\sigma}{\sqrt{6}}} e_{\mu \nu \rho \sigma \gamma} F'_{\sigma \gamma} - X_{\mu \nu \rho} \]  

(2.12)

which upon substitution back in the intermediate Lagrangian, \( L + \Delta L \), and choosing
the constant \( a = +\frac{1}{12} \) for correct normalisation, we obtain the dual Lagrangian \( L_D \),

\[
L_D = -e^{-\frac{2\phi}{\sqrt{2}}} e^{\frac{2\sigma}{\sqrt{6}}} |F_2'|^2 - \frac{1}{12} e^{\mu\nu\rho\sigma\gamma} X_{\mu\nu\rho} F'_{\sigma\gamma}
\]

(2.13)

Next we need to linearly combine the dilaton \( \phi \) with the KK scalar \( \sigma \) so that

\[
\frac{\phi}{\sqrt{2}} + \frac{\sigma}{\sqrt{6}} = \text{const} \psi.
\]

The constant is chosen so that \( U \epsilon \text{SO}(2) \) where

\[
U = \frac{1}{\text{const}} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{pmatrix}
\]

(2.14)

This requires the \( \text{const} = \sqrt{\frac{2}{3}} \), i.e.

\[
\begin{pmatrix} \psi \\ \psi_\perp \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{3}{2}} \\ -\frac{1}{2} \sqrt{\frac{3}{2}} \end{pmatrix} \begin{pmatrix} \phi \\ \sigma \end{pmatrix}
\]

(2.15)

Truncating out the orthogonal combination \( \psi_\perp = 0 \), we have

\[
\sigma = \frac{\psi}{2} \text{ and } \phi = \frac{\sqrt{3}}{2} \psi.
\]

The resulting action is

\[
S_5 = \int d^5x e \{ R_5 - \frac{1}{2} |d\psi|^2 - \frac{1}{4} e^{-\frac{2\phi}{\sqrt{2}}} |F_2|^2 + |G_2^{(1)}|^2 + |f_2|^2 + |F_2'|^2 - \frac{1}{4} e^{\frac{2\phi}{\sqrt{6}}} |C_2|^2
\]

\[
+ 4\alpha^2 e^{\frac{2\phi}{\sqrt{2}}} \} + \frac{1}{2} \int C_2 [G_2^{(1)} B_1^{(1)} + Tr(F_2 A_1 - \frac{i\alpha}{3} A_1 A_1 A_1) + 2F_2' \tilde{A}]
\]

(2.16)

We still need to remove a vector to obtain a pure N=4 supergravity multiplet. But we have the choice of truncating out any of the 2-form field strengths \( f_2, F_2', G_2, F_2, C_2 \) or any linear combination of them. Since we ultimately want to reduce the model to 4-D and compare with the 'half gauged' FS model [5], we don’t touch \( F_2 \) or \( G_2 \) as they already have the required Chern-Simons terms. Since \( C_2 \)’s kinetic term has a different dilatonic prefactor to those of the other 2-forms we also leave \( C_2 \) alone. This suggests that we should linearly combine \( f_2 \) and \( F_2' \) and
truncate the orthogonal linear combination. This procedure, i.e. linearly combining the dualisation of an antisymmetric tensor potential (which is a remnant of the 3-form potential $A_{\mu\nu\rho}$ from 7-D) with the KK vector, was actually performed in 6-D in [13] and will also have to be performed in 4-D to obtain the 'half gauged' FS model. This is done by another SO(2) rotation, $F'_2 = G^{(2)}_2 + H_2/\sqrt{2}$ and $f_2 = G^{(2)}_2 - H_2/\sqrt{2}$ where $G^{(2)}_2 = dB^{(2)}_1$ and $H_2 = dh_1$. We can then consistently truncate a vector, $H_2 = 0$.

After an integration by parts the resulting SU(2)×U(1)$^2$ gauged bosonic action in 5-D is:

$$S_5 = \int d^5x e\{R_5 - \frac{1}{2} |d\psi|^2 - \frac{1}{4} e^{-\frac{2\sigma}{\sqrt{3}}} |F_2|^2 + |G^{(1)}_2|^2 + |G^{(2)}_2|^2 - \frac{1}{4} e^{-\sigma} |C_2|^2$$

$$+ 4\alpha^2 e^{-\frac{2\sigma}{\sqrt{3}}}) - \frac{1}{2} \int C_1 [G^{(1)}_2 G^{(1)}_2 + G^{(2)}_2 G^{(2)}_2 + Tr(F_2 F_2)] \}
$$

(2.17)

where $G^{(p)}_2 = dB^{(p)}_1$ (p=1,2), $C_2 = dC_1$ and

$$F_{\mu\nu}^j = 2(\partial_{[\mu} A_{\nu]}^j + i\alpha A_{[\mu|j|}^k A_{\nu]}^k j) (i,j=1,2).$$

3. Dimensional Reduction To D=4

We proceed as in the previous case. In the following $\tilde{A}$ and $\sigma$ are now new KK fields. Since they don’t appear in (2.17) we will use the symbols again. The ansatz for the fields are:

$$\dot{e}_{\tilde{m}}^a = \begin{pmatrix} e^{\frac{\sigma}{\sqrt{3}} e_m^a} & 0 \\ e^{-\frac{\sigma}{\sqrt{3}} \tilde{A}_m} & e^{-\frac{\sigma}{\sqrt{3}}} \end{pmatrix} \quad (3.1)$$

Where hats refer to D=5.

$$d\tilde{S}_5^2 = e^{\frac{\sigma}{\sqrt{3}}} dS_4^2 + e^{-\frac{2\sigma}{\sqrt{3}}} (dz + \tilde{A})^2 \quad (3.2)$$

where $\tilde{A} = \tilde{A}_m dx^m$ and $f_2 = d\tilde{A}$.

$$\dot{A}_{1i}^j = A_{1i}^j + A_i^j dz \quad (3.3)$$
\[ \hat{C}_1 = C_1 + Bdz \]  
(3.4)

\[ \hat{B}^{(p)}_1 = B^{(p)}_1 + \rho^{(p)}dz \]  
(3.5)

\[ \hat{\psi}(x^m, z) = \psi(x^m) \]  
(3.6)

where \((p) = 1, 2\) and \(i, j = 1, 2\).

The resulting action in 4-D is (after setting \(A^{ij}_i = \rho^{(p)} = 0\) )

\[
S_4 = \int d^4x \left\{ R_4 - \frac{1}{2} |d\sigma|^2 - \frac{1}{2} |d\psi|^2 - \frac{1}{4} e^{-\frac{2\psi}{\sqrt{3}}} |f_2|^2 - \frac{1}{4} e^{\frac{4\psi}{\sqrt{6}}} \sqrt{3} |C_2|^2 ight.
\]

\[
- \frac{1}{2} e^{\frac{4\psi}{\sqrt{6}}} \sqrt{3} |dB|^2 - \frac{1}{4} e^{-\frac{2\psi}{\sqrt{3}}} \sqrt{3} \left[ |F_2|^2 + |G_2^{(1)}|^2 + |G_2^{(2)}|^2 \right] + 4\alpha^2 e^{\frac{2\psi}{\sqrt{6}} + \frac{4\psi}{\sqrt{3}}} \left\{ \right. 
\]

\[
- \frac{1}{2} \int B[G_2^{(1)} G_2^{(1)} + G_2^{(2)} G_2^{(2)} + Tr(F_2 F_2)] \}
\]

where \(f_2 = dA_2\), \(C_2 = dC_1 - dB\tilde{A}\), \(G_2^{(p)} = dB_1^{(p)}\), and \(F_{\mu\nu\lambda} = 2(\partial_{[\mu} A_{\nu\lambda]} + i\alpha A_{[\mu|k} A_{\nu]k}^\lambda)\).

As above, we need to linearly combine \(\psi\) with the KK scalar \(\sigma\). The procedure is the same as that described for 5-D except

\[ U = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}} \end{pmatrix} \]  
(3.8)

This leads to \(\sigma = \frac{1}{\sqrt{3}} \phi\) and \(\psi = \sqrt{\frac{2}{3}} \phi\).

In order to fully decouple an N=4 D=4 vector supermultiplet (1 vector, 6 scalars) we need to dualise \(C_\mu\) (the remnant in 4-D of the 7-D field \(A_{\mu\nu\rho}\)) to \(C'_\mu\), linearly combine with \(\tilde{A}\) and truncate the orthogonal combination. This is suggested by what has already been done in 6-D and 5-D. The procedure as described...
\[
L = -\frac{1}{4} e^{\frac{4\phi}{\sqrt{\alpha}}} |C_2|^2
\]  
(3.9)

with

\[
L_D = -\frac{1}{4} e^{-\frac{4\phi}{\sqrt{\alpha}}} |C'_2| + \frac{1}{2} e^{\mu\nu\rho\sigma} \partial_\mu B \tilde{A}_\nu C'_\rho C'_\sigma
\]  
(3.10)

where \( C'_2 = dC'_1 \). Then \( C'_1 \) is linearly combined with \( \tilde{A} \) using the same SO(2) matrix as in 5-D, i.e.

\[
\begin{pmatrix}
  f_2 \\
  C'_2
\end{pmatrix} = \begin{pmatrix}
  \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
  \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{pmatrix} \begin{pmatrix}
  G^{(3)}_2 \\
  H_2
\end{pmatrix}.
\]  
(3.11)

After truncating the abelian field strength \( H_2 \), the contribution to the Chern-Simons term from (3.10) is

\[
-\frac{1}{2} \int B G^{(3)}_2 G^{(3)}_2.
\]  
(3.12)

So then the N=4 vector supermultiplet decouples from the SU(2) gauged N=4 supergravity multiplet and the resulting action is

\[
S_4 = \int d^4x E \left\{ R_4 - \frac{1}{2} |d\phi|^2 - \frac{1}{2} e^{2\phi} |dB|^2 + 4\alpha^2 e^\phi \\
-\frac{1}{4} e^{-\phi} (|F_2|^2 + |G^{(1)}_2|^2 + |G^{(2)}_2|^2 + |G^{(3)}_2|^2) \right\}
\]  
(3.13)

where \( G^{(p)}_2 = dB^{(p)}_1 \) (p=1,2,3) and \( F_{\mu\nu}^i j = 2(\partial_{[\mu} A_{\nu]} i j + i\alpha A_{[\mu} i |^k A_{\nu]} k j) \) (i,j=1,2).

With the identification \( e_A = \sqrt{2}\alpha \), we recognise this action as the bosonic sector of the FS model [5] with one of the SU(2) coupling constants equal to zero.

The model contains both an abelian and a non-abelian sector. The abelian sector has three U(1) field strengths \( G^{(p)}_2 \), whose origins lie with the four form field strength in seven dimensions. The non-abelian sector contains a triplet of SU(2) gauge fields with field strengths \( F_{\mu\nu}^i j \) coming straight down from the corresponding SU(2) triplet in seven dimensions.
4. A Supersymmetric Electrovac In 7-D

So far the 7-D SU(2) gauged supergravity [3] has been reduced from seven to four dimensions on $T^3$. In four dimensions the reduction results in a version of the SU(2)$\times$SU(2) gauged N=4 supergravity of Freedman and Schwarz in which one of the gauge coupling constants is zero.

In recent years there has been much interest in p-brane solutions of supergravity theories. We will see later that these non-maximal SU(2) gauged supergravities posses ground state solutions which have the interpretation of a domain wall (D-2 brane) preserving a half of the supersymmetry. The domain wall in 7-D being the lift of the domain wall solution of the FS theory.

Gibbons and Freedman [6] however, had previously found a different type of ground state solution of the 4-D SU(2)$_A \times$SU(2)$_B$ gauged theory, namely a supersymmetric electrovac solution. Now that we have shown the $e_B = 0$ version of the FS model to have a seven dimensional origin, the existence of a supersymmetric electrovac solution in four dimensions means that there must exist a similar solution preserving a half of the supersymmetry in 7-D (and of course in 5 and 6-D) which is precisely the lift to 7-D of this 4-D supersymmetric electrovac.

In this section therefore we will concentrate on constructing directly in 7-D an electrovac solution of the N=2 SU(2) gauged supergravity preserving a half of the supersymmetry. Setting $e_B$ to zero, the Freedman/Gibbons electrovac (constant electric and magnetic fields) preserves no supersymmetry if there are both electric and magnetic fields present, and a half of the supersymmetry if it involves only electric fields. The spacetime background being in the former case (AdS)$_2 \times$S$^2$ and in the latter case (AdS)$_2 \times$R$^2$. The gauge fields of the non-abelian sector are zero, as is the pseudoscalar B. The dilaton is constant and the constant electric and magnetic fields arise from the abelian sector of the model. This indicates that the solution we are seeking in 7-D only involves the fourth rank antisymmetric tensor (the abelian sector) and the dilaton (which is constant). Our strategy will be to find solutions of the Killing spinor equations.
We note at this point that there exist non-supersymmetric electrovacs (involving magnetic fields) in 4-D which could also be lifted to 7-D to give solutions. These solutions would be distinct from the 7-D non-supersymmetric electrovacs found in [14]. We shall be concerned only with the supersymmetric electrovacs.

Consider first the background spacetime. One might have supposed it would be \((\text{AdS})_2 \times \mathbb{R}^2 \times T^3\). However, the metric is not simply diagonal and this is crucial to finding the supersymmetric electrovac solution in 7-D. In [14] Quevedo used a diagonal metric ansatz and the resulting electrovacs were non-supersymmetric. The reason that the metric contains off diagonal terms is because the U(1) gauge potentials in 4-D responsible for the constant electric fields of the Gibbons/Freedman solution are linear combinations of vector fields arising from the four form in 7-D and the KK vectors from the metric i.e. the KK vectors are not zero in the electrovac solution. Thus there is a mixing between the vector fields from the metric with those from the four form at each step of the reduction from 7-D to 4-D.

Having already performed the reduction we can now write the seven dimensional metric in terms of the non-zero fields appearing in the Gibbons/Freedman 4-D electrovac.

\[
dS_7^2 = e^{\frac{2}{5}\phi}dS_4^2 + e^{-\frac{2}{5}\phi}(dZ_5 + \frac{B_1^{(3)}}{\sqrt{2}})^2 + e^{-\frac{2}{5}\phi}(dZ_6 + \frac{B_1^{(2)}}{\sqrt{2}})^2 + e^{-\frac{2}{5}\phi}(dZ_7 + \frac{B_1^{(1)}}{\sqrt{2}})^2
\]

In the coordinates of [6] \(dS_4^2\), the line element of \((\text{AdS})_2 \times \mathbb{R}^2\), is

\[
dS_4^2 = \frac{1}{K \cos^2 \rho}(-dt^2 + d\rho^2) + (dX^2)^2 + (dX^3)^2
\]

where \(K\) is the Gaussian curvature given by \(K = 2e_A E\), with \(E\) a constant to be identified as the electric field later. In order to find expressions for \(B_1^{(i)}\) we need to consider the 4-D solution. The relevant part of the FS action is

\[
I_{FS} = \int d^4x \sqrt{-g} [R - \frac{1}{2} (\partial \phi)^2 + e^{-\phi} 4 \sum |G_2^{(i)}|^2 - 2e_A^2 e^\phi]
\]
where the summation is over (i) from one to three. The $\phi$ equation of motion is

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} \partial^\mu \phi) = \frac{1}{4} e^{-\phi} \sum |G_2^{(i)}|^2 + 2e_A^2 e^{\phi} = V'_{\text{eff}}(\phi)$$  \hspace{1cm} (4.4)$$

For $V_{\text{eff}}(\phi)$ to have a minimum we require

$$\sum G^{(i)}_{\mu\nu} G^{\mu\nu(i)} = -8E^2$$  \hspace{1cm} (4.5)$$

then $V'_{\text{eff}}(\phi) = 0$ when $e^{\phi} = \frac{E}{e_A}$ hence $E$ is identified as the electric field. Therefore for simplicity and without loss of generality we can choose

$$B_1^{(2)} = B_1^{(3)} = 0 \quad \text{and} \quad G_{\mu\nu}^{(1)} G^{\mu\nu(1)} = -8E^2$$  \hspace{1cm} (4.6)$$

A suitable potential is

$$B_t^{(1)} = \frac{2Etan\rho}{K}$$  \hspace{1cm} (4.7)$$

all other components vanishing (remember, in the reduction of $B_1^{(1)}$ we set the scalars $\rho^{(1)}$ to zero).

Putting this all together with a relabelling $X^4 = Z_5$, $X^5 = Z_6$ and $X^6 = Z_7$ we have:

$$ds^2 = \left( -\frac{e^{3\phi}}{K \cos^2 \rho} + e^{-\frac{2\phi}{3}} 2E^2 tan^2 \rho \right) dt^2 + \frac{e^{3\phi}}{K \cos^2 \rho} d\rho^2 + e^{\frac{3\phi}{2}} (dX^2)^2 + e^{\frac{3\phi}{2}} (dX^3)^2$$

$$+ e^{-\frac{2\phi}{3}} (dX^4)^2 + e^{-\frac{2\phi}{3}} (dX^5)^2 + e^{-\frac{2\phi}{3}} (dX^6)^2 + e^{-\frac{2\phi}{3}} \frac{2\sqrt{2E}tan\rho}{K} dt dX^6$$  \hspace{1cm} (4.8)$$

Choosing the vielbein frame as

$$e^4 = \frac{e^{\frac{3\phi}{10}}}{\sqrt{K \cos \rho}} dt \quad e^5 = \frac{e^{\frac{3\phi}{10}}}{\sqrt{K \cos \rho}} d\rho$$

$$e^2 = e^{\frac{3\phi}{10}} dX^2 \quad e^3 = e^{\frac{3\phi}{10}} dX^3$$

$$e^4 = e^{-\frac{\phi}{5}} dX^4 \quad e^5 = e^{-\frac{\phi}{5}} dX^5$$

$$e^6 = e^{-\frac{\phi}{5}} \frac{\sqrt{2E} tan\rho}{K} dt + e^{-\frac{\phi}{5}} dX^6$$  \hspace{1cm} (4.9)$$
the only non-zero components of the spin connection are computed to be

\[
\begin{align*}
\omega_{tt}^\rho &= -\tan(1 - \frac{E^2}{K} e^{-\phi}) \\
\omega_{t6}^\rho &= \frac{E}{\sqrt{2} K e^{-\phi}} \\
\omega_{p6}^\rho &= \frac{E}{\sqrt{2} K \cos \rho} e^{-\phi} \\
\omega_{t6}^\rho &= \frac{E}{\sqrt{2} K \cos \rho} e^{-\phi} 
\end{align*}
\]

(4.10)

where we have denoted 'flat space' Lorentz indices by underlining.

Next we need to understand which components of \( F_{\mu\nu\rho\sigma} \) are non-vanishing. In the reduction from 7-D to 6-D \( F_4 \) reduced as follows

\[
F_{\mu\nu\rho\sigma} = F'_{\mu\nu\rho\sigma} + F_{\mu\nu\rho} dX^6 
\]

(4.11)

\( F'_{\mu\nu\rho\sigma} \) was then dualised to a two form which, after linearly combining with the KK vector, became \( G^{(1)}_{\mu\nu} \). Hence \( F'_{\mu\nu\rho\sigma} \) is given by

\[
F'_{\mu\nu\rho\sigma} = -\frac{e^2}{2\sqrt{2}} e^{-\frac{\psi}{2\sqrt{2}}} \epsilon_{\mu\nu\rho\sigma\lambda\tau} G^{\lambda\tau(1)}
\]

(4.12)

where \( \psi \) and \( G^{(1)}_2 \) are 6-d fields and \( \mu = 0, \ldots, 5 \). The further reduction of \( \psi \) and \( G^{(1)}_2 \) to 4-D is straightforward.

In the reduction from 6-D to 5-D \( F_{\mu\nu\rho} \) was reduced as

\[
F_{\mu\nu\rho} = F'_{\mu\nu\rho} + C_{\mu\nu} dX^5 = F_{\mu\nu\rho 6}
\]

(4.13)

\( F'_{\mu\nu\rho} \) was then dualised to a two form and linearly combined with a KK vector so that

\[
F'_{\mu\nu\rho} = -\frac{e^2}{2\sqrt{2}} e^{-\frac{\psi'}{2\sqrt{2}}} \epsilon_{\mu\nu\rho\sigma\lambda} G^{\sigma\lambda(2)}
\]

(4.14)

where \( \psi' \) and \( G^{(2)}_2 \) are 5-d fields and \( \mu = 0, \ldots, 4 \). The further reduction of \( G^{(2)}_2 \) to 4-D is straightforward.
Similarly after the reduction from 5-D to 4-D $C_{\mu\nu}$ becomes

$$C_{\mu\nu} = C'_{\mu\nu} + \partial_{\mu} B dX^4 = F_{\mu\nu 65}$$  \hspace{1cm} (4.15)

with

$$C'_{\mu\nu} = \frac{e}{2\sqrt{2}} e^{-\phi} \epsilon_{\mu\nu\rho\sigma} G^{0\rho\sigma(3)} \quad \mu = 0, \ldots, 3 \hspace{1cm} (4.16)$$

Now since $G_2^{(2)} = G_2^{(3)} = B = 0$ in the 4-D electrovac and since the only non-zero component of $G_2^{(1)}$ is $G^{(1)01}$ this implies the only non-zero component of $F_4$ is

$$F_{2345} \propto \epsilon_{012345} G^{(1)01}$$  \hspace{1cm} (4.17)

We can now attempt to solve the 7-D Killing spinor equations. They are

$$\delta \lambda_i = \frac{1}{2\sqrt{2}} \Gamma^M D_M \Phi \epsilon_i - \frac{i\sigma_0}{2\sqrt{20}} \Gamma^{MN} F_{MNI}^j \epsilon_j + \frac{\sigma_0^{-2}}{24\sqrt{20}} \Gamma^{NMPQ} F_{MNPQ} \epsilon_i + \frac{\alpha \sigma_0^{-1}}{\sqrt{5}} \epsilon_i$$

$$\delta \psi_{Mi} = D_M \epsilon_i + \frac{i\sigma_0}{20 (\Gamma^NP_M - 8 \delta_M^N \Gamma^P) F_{NPI}^j \epsilon_j + \frac{i\alpha}{\sqrt{2}} A_M^j \epsilon_j}$$

\begin{equation}
\frac{\sigma_0^{-2}}{160} (\Gamma^{NPQR}_M - \frac{8}{3} \delta_M^N \Gamma^{PQR}) F_{NPRQ} \epsilon_i - \frac{\alpha \sigma_0^{-1}}{5} \Gamma_M \epsilon_i \hspace{1cm} (4.18)
\end{equation}

where $\sigma_0 = e^{-\frac{\Phi}{\sqrt{m}}}$.

With $F_{MNI}^j = 0$, $\Phi =$constant and using the lemma

$$\Gamma_M^{NPQR} = \Gamma_M^{NPQR} - \delta_M^N \Gamma^{PQR} + \delta_M^P \Gamma^{QRN} - \delta_M^Q \Gamma^{RNP} + \delta_M^R \Gamma^{NPQ} \hspace{1cm} (4.19)$$

they become

$$\delta \lambda_i = \Gamma^{2345}_{M} F_{2345} \epsilon_i + 2\alpha \sigma_0 \epsilon_i$$
\[ \delta \psi_{Mi} = \partial_M \epsilon_i + \frac{1}{4} \omega_{Ma} \Gamma^a \epsilon_i + \frac{\sigma_0}{96} [\Gamma_M \Gamma^N - 4 \delta_M^N] \Gamma^{PQR} F_{NPQR} \epsilon_i \] \tag{4.20}

Since in 7-D the product of all the gamma matrices is 1, a useful expression is

\[ \Gamma^{2345} = -\Gamma^2 \Gamma^4 \Gamma^6 \] \tag{4.21}

The 7-D dilaton is related to the 4-D dilaton by a scaling, \( \Phi = \sqrt{\frac{2}{5}} \phi \) hence \( \sigma_0 = e^{-\frac{\Phi}{5}} \). Therefore for \( \delta \lambda_i = 0 \), \( F_{2345} \) must satisfy

\[ \Gamma^{2345} F_{2345} \epsilon_i = -2 \alpha e^{-\frac{\Phi}{5}} \epsilon_i \] \tag{4.22}

Consider the \( \delta \psi_{ti} \) equation. Using (4.10) we have

\[
\delta \psi_{ti} = \partial_t \epsilon_i - \frac{\tan \rho}{2} (1 - e^{-\phi} \frac{E^2}{K}) \Gamma^t \epsilon_i + e^{-\frac{\phi}{2}} \frac{E}{2\sqrt{2}K \cos \rho} \Gamma^6 \epsilon_i \\
- \frac{e^{\frac{\phi}{2}}}{4} (c_7^t \Gamma^t + c_7^6 \Gamma_6) \Gamma^{2345} F_{2345} \epsilon_i
\] \tag{4.23}

Substituting the expressions for the vielbein components (4.9), the equation becomes

\[
\delta \psi_{ti} = \partial_t \epsilon_i - \frac{\tan \rho}{2} (1 - e^{-\phi} \frac{E^2}{K}) \Gamma^t \epsilon_i + e^{-\frac{\phi}{2}} \frac{E}{2\sqrt{2}K \cos \rho} \Gamma^6 \epsilon_i \\
- \frac{e^{\frac{\phi}{2}}}{4} \frac{\Gamma^t}{\sqrt{K} \cos \rho} \Gamma^{2345} F_{2345} \epsilon_i + e^{\frac{\phi}{2}} \frac{E \tan \rho}{2\sqrt{2}K} \Gamma^6 \Gamma^{2345} F_{2345} \epsilon_i
\] \tag{4.24}

Using (4.21) in the last term, (4.22) in the fourth term and both (4.21) and (4.22) in the third term we have

\[
\delta \psi_{ti} = \partial_t \epsilon_i + \frac{\Gamma^t}{2\cos \rho} \left[ - e^{-\frac{\phi}{10}} \frac{E}{\sqrt{2}K \alpha} F_{2345} + e^{-\frac{\phi}{5}} \frac{\alpha}{\sqrt{K}} \right] \epsilon_i \\
+ \frac{\tan \rho}{2} \left[ - 1 + e^{-\phi} \frac{E^2}{K} + e^{\phi} \frac{E}{\sqrt{2}K} F_{2345} \right] \Gamma^4 \epsilon_i
\] \tag{4.25}
Next consider $\delta \psi_{\rho i}$. Using (4.10) we have

$$\delta \psi_{\rho i} = \partial_\rho \epsilon_i + e^{-\phi} \frac{E}{2\sqrt{2K}\cos\rho} \Gamma^{245}\epsilon_i + \frac{e^{2\phi}}{4} \epsilon_i \Gamma_6 \Gamma^{2345} F_{2345}\epsilon_i$$  \hspace{1cm} (4.26)

Using the expression for $e_6^\rho$ and equations (4.21) and (4.22) we get

$$\delta \psi_{\rho i} = \partial_\rho \epsilon_i - \frac{\Gamma^2}{2\cos\rho} \left[ e^{\phi} \frac{\alpha}{\sqrt{\alpha}} - e^{-\frac{2\phi}{3}} \frac{E}{2\sqrt{2K}\alpha} \Gamma^{2345} F_{2345} \right] \epsilon_i$$ \hspace{1cm} (4.27)

We will return to $\delta \psi_{\rho i}$ and $\delta \psi_{ti}$ later. Next consider $\delta \psi_{2i}$, $\delta \psi_{3i}$, $\delta \psi_{4i}$, and $\delta \psi_{5i}$. Assuming $\epsilon_i$ depends only on $\rho$ and $t$ we have

$$\delta \psi_{2i} = \frac{e^{2\phi}}{96} \left[ 4! \Gamma^2 \Gamma^{2345} F_{2345} - 4 \frac{2\phi}{3} \Gamma^{345} F_{2345} (3!) \right] \epsilon_i = 0$$ \hspace{1cm} (4.28)

The expressions for $\delta \psi_{3i}$, $\delta \psi_{4i}$ and $\delta \psi_{5i}$ vanish similarly. Finally consider $\delta \psi_{6i}$.

$$\delta \psi_{6i} = e^{-\phi} \frac{E}{2\sqrt{2}} \Gamma^6 \epsilon_i + \frac{e^{2\phi}}{4} \epsilon_i \Gamma_6 \Gamma^{2345} F_{2345}\epsilon_i$$ \hspace{1cm} (4.29)

again, substituting the expression for $e_6^\rho$ and the equations (4.21),(4.22) we have,

$$\delta \psi_{6i} = \left[ - e^{-\frac{2\phi}{3}} \frac{E}{4\sqrt{2}\alpha} F_{2345} - \frac{\alpha}{2} \right] \epsilon_i$$ \hspace{1cm} (4.30)

For $\delta \psi_{6i} = 0$ we require

$$F_{2345} = -e^{\frac{2\phi}{3}} \frac{2\sqrt{2}\alpha^2}{E}$$ \hspace{1cm} (4.31)

Using this result in (4.22) it becomes

$$(\Gamma^{2345} - e^{-\phi} \frac{E}{\sqrt{2}\alpha}) \epsilon_i = 0$$ \hspace{1cm} (4.32)

Now we know

$$e^\phi = \frac{E}{e_A} \hspace{1cm} e_A = \sqrt{2}\alpha$$ \hspace{1cm} (4.33)

hence $\epsilon_i$ must be of the form

$$\epsilon_i = \frac{1}{2} (\Gamma^{2345} + 1) \xi_i$$ \hspace{1cm} (4.34)

where $\xi_i$ is a constant spinor.
Considering again the $\delta\psi_{ti}$ and $\delta\psi_{\rho i}$ equations and using (4.31), (4.33) and $K = 2e_A E$, they become:

$$\delta\psi_{ti} = \partial_t \epsilon_i + \frac{\Gamma^t}{2cos\rho} \epsilon_i - \frac{tan\rho}{2}\Gamma^\rho \epsilon_i$$  (4.35)

$$\delta\psi_{\rho i} = \partial_\rho \epsilon_i - \frac{\Gamma^\rho}{2cos\rho} \epsilon_i$$  (4.36)

By comparison with the Freedman/Gibbons electrovac in 4-D we see that if $\epsilon_i$ is of the form

$$\epsilon_i = S(t, \rho)\frac{1}{2}(1 + \Gamma^{2345})\xi_i$$  (4.37)

with $\xi_i$ a constant spinor and the function $S(t, \rho)$ given by

$$S(t, \rho) = \frac{1}{(cos\rho)^{1/2}} [cos \frac{\rho}{2} + \Gamma^\rho sin \frac{\rho}{2}] [cos \frac{t}{2} - \Gamma^t sin \frac{t}{2}]$$  (4.38)

then $\delta\psi_{ti} = \delta\psi_{\rho i} = 0$ and a half of the supersymmetry remains unbroken.

To summarise the 1/2 supersymmetric 7-D electrovac solution we choose $E = e_A$ so that $\phi = 0$. Then the metric and four form field strength take the forms:

$$F_{2345} = -24\sqrt{2}e_A$$  (4.39)

$$dS_t^2 = \frac{1}{2e_A^2cos^2\rho} [-dt^2 + d\rho^2] + [dX^6 + \frac{tan\rho}{\sqrt{2}e_A} dt]^2 + dS^2(\mathbb{E}^4).$$  (4.40)

where $\mathbb{E}^4$ is a 4-d Euclidean space containing the 2,3,4 and 5 spacial directions and all other fields and components are vanishing. We have shown that this metric is $(\text{ADS})_3 \times \mathbb{E}^4$. 

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5. Reduction Of D=7 Domain Wall

In this section, we turn to a consideration of the (D-2)-brane solutions that arise in gauged supergravities.

As was shown in [8,9], supergravities with bosonic sectors of the form

\[ L = e^R - \frac{1}{2} (\partial \phi)^2 + \frac{m^2}{2} e^{-a\phi} \]  \hspace{1cm} (5.1)

have domain wall solutions of the form

\[ ds^2 = H^{\frac{4}{2(D-2)}} dx^\mu dx^\nu \eta_{\mu\nu} + H^{\frac{4(D-1)}{2(D-2)}} dy^2 \]  \hspace{1cm} (5.2)

\[ e^\phi = H^{\frac{2a}{\Delta}} \]  \hspace{1cm} (5.3)

where D is the spacetime dimension and \( \mu, \nu = 1,..,D - 1 \). \( H \) is a harmonic function on the 1-dimensional transverse space with coordinate \( y \), of the general form \( H = c \pm My \), where \( c \) is an arbitrary constant, and \( M = \frac{1}{2} m\sqrt{-\Delta} \). See [9] for details of the spacetime structure of \( \Delta = -2 \) domain walls.

D=7 N=2 SU(2) gauged supergravity has the 5-brane solution (in the Einstein frame) [8]:

\[ ds_7^2 = H^{-\frac{2}{5}} dx^\mu dx^\nu \eta_{\mu\nu} + H^{-\frac{12}{5}} dy^2 \]  \hspace{1cm} (5.4)

\[ e^{\phi_7} = H^{\frac{2}{\Delta}} \]  \hspace{1cm} (5.5)

where \( \mu, \nu = 0,..,5 \), for which \( \Delta = -2 \). Notice that unlike the domain walls of massive supergravities [11] where \( \Delta \) is always positive, and equal to 4 when only one dilaton is involved in the solution, the value of \( \Delta \) for these gauged supergravities is negative. This implies that these gauged theories cannot be obtained by ordinary KK or Scherk-Schwarz reductions on \( T^n \) of eleven dimensional supergravity or massive IIA supergravity [11,15,2].
We now double dimensionally reduce this 5-brane to obtain a membrane solution of the FS model in 4-D. Since $\Delta$ is unchanged under dimensional reduction it also has $\Delta = -2$. This implies that $a=1$ which is indeed the case for the FS model, demonstrating the consistency of the reduction.

The ansatz for the reduction from $D=7$ to $D=6$ is

$$d\hat{S}_7^2 = e^{\frac{a}{\sqrt{10}}} dS_6^2 + e^{-\frac{4a}{\sqrt{10}}} dz^2$$

(5.6)

All fields are assumed to be independent of the compactification coordinate $z$. Therefore $z$ must be identified with a world volume coordinate $x_5$ implying

$$e^{-\frac{4a}{\sqrt{10}}} = H^{-\frac{2}{5}}$$

(5.7)

This then leads very simply to the $D=6$ 4-brane [8] :

$$ds_6^2 = H^{-\frac{1}{2}} dx^\mu dx^\nu \eta_{\mu\nu} + H^{-\frac{2}{5}} dy^2$$

(5.8)

$$e^{\phi_6} = H^{\sqrt{\frac{T}{2}}}$$

(5.9)

where $\mu, \nu = 0, \ldots, 4$.

In the same way, using the ansatz (2.3) and (3.2) (with $\tilde{A} = 0$) one can double dimensionally reduce this $D=6$ 4-brane to a $D=5$ 3-brane :

$$ds_5^2 = H^{-\frac{2}{3}} dx^\mu dx^\nu \eta_{\mu\nu} + H^{-\frac{2}{5}} dy^2$$

(5.10)

$$e^{\phi_5} = H^{\sqrt{\frac{T}{3}}}$$

(5.11)

where $\mu, \nu = 0, \ldots, 3$

and a $D=4$ 2-brane :

$$ds_4^2 = H^{-1} dx^\mu dx^\nu \eta_{\mu\nu} + H^{-3} dy^2$$

(5.12)

$$e^{\phi_4} = H$$

(5.13)

where $\mu, \nu = 0, \ldots, 2$.
6. Supersymmetric Domain Walls
Of The Freedman/Schwarz Model

In this section we consider the supersymmetry properties of the membrane solution of the FS model. As in [5] we let $e_A$ and $e_B$ denote the two gauge coupling constants and we define

$$e^2 = e_A^2 + e_B^2$$

(6.1)

The potential of this model has the form $V = -2e^2\Phi$, so $\Delta = -2$. This model therefore has a domain wall solution of the type given in (5.12),(5.13). i.e.

$$ds^2 = H^{-1}(y)dx^\mu dx^\nu \eta_{\mu\nu} + H^{-3}(y)dy^2$$
$$\Phi = H(y)$$

(6.2)

where $H = m|y| + \text{constant}$ and $m^2 = 2e^2$. Denoting ‘flat space’ Lorentz indices by underlining, the only non-zero components of the spin connection $\omega_{\mu\underline{\nu}}$ are

$$\omega_{\underline{tt}y} = -\omega_{\underline{xx}y} = -\omega_{\underline{zz}y} = \frac{m}{2}$$

(6.3)

To check for supersymmetry, all that is needed here are the supersymmetry transformations of the four Majorana gravitini $\psi$ and spin $\frac{1}{2}$ fields $\chi$ in a bosonic background for which all Yang-Mills field strengths vanish. In our ‘mostly plus’ metric convention the D=4 Dirac matrices can be assumed to be real. We define $\gamma_5$ to be the product of all four Dirac matrices satisfying $\gamma_5^2 = -1$. In these conventions the required supersymmetry transformations of the FS model are

$$\delta \psi_\mu = 2[D_\mu \epsilon - \frac{\sqrt{2}}{4}e^2(\epsilon e_A - \gamma_5 e_B)\Gamma_\mu \epsilon]$$
$$\delta \chi = \frac{1}{\sqrt{2}}[\partial_\mu \phi \Gamma^\mu + \sqrt{2}e^2(\epsilon e_A + \gamma_5 e_B)] \epsilon$$

(6.4)

The amount of supersymmetry preserved by the domain wall solution is the number of independent solutions for $\epsilon$ of the conditions $\delta \psi_\mu = 0$ and $\delta \chi = 0$ in the domain
wall background. Using $\epsilon = H^{-\frac{1}{4}}(y)e_0 = e^{\frac{A(y)}{2}}\epsilon_0$, these conditions are

$$\delta \psi_\mu = -\frac{1}{2}e^{\frac{A}{2}}[-mH(y)\frac{1}{2}\Gamma^y + \sqrt{2}e^{\frac{\phi}{2}}(e_A - \gamma_5e_B)]\Gamma_\mu \epsilon_0 = 0 \quad \mu = x, z$$

$$\delta \psi_t = -\frac{1}{2}e^{\frac{A}{2}}[mH(y)\frac{1}{2}\Gamma^y + \sqrt{2}e^{\frac{\phi}{2}}(e_A - \gamma_5e_B)]\Gamma_t \epsilon_0 = 0$$

$$\delta \psi_y = e^{\frac{A}{2}}[\partial_y A(y) - \frac{\sqrt{2}}{2}e^{\frac{\phi}{2}}(e_A - \gamma_5e_B)\Gamma^y] \epsilon_0 = 0$$

$$\delta \psi_y = e^{\frac{A}{2}}[\partial_y A(y) + \frac{\sqrt{2}}{2}e^{\frac{\phi}{2}}(e_A + \gamma_5e_B)\Gamma^y] \Gamma^y \epsilon_0 = 0 \ .$$

They become

$$\delta \psi_\mu = -\frac{1}{2}e^{A} \Gamma^y (e_A + \gamma_5e_B)[\pm \frac{m}{e}H(y)\frac{1}{2} + \sqrt{2}e^{\frac{\phi}{2}}] \Gamma_\mu \epsilon_0 = 0 \quad \mu = x, z, t$$

$$\delta \psi_y = e^{\frac{A}{2}}[\partial_y A(y) + \frac{\sqrt{2}}{2}e^{\phi}(e_A + \gamma_5e_B)\Gamma^y] \epsilon_0 = 0$$

$$\delta \psi_y = e^{\frac{A}{2}}[\partial_y A(y) - \frac{\sqrt{2}}{2}e^{\phi}(e_A - \gamma_5e_B)\Gamma^y] \Gamma^y \epsilon_0 = 0$$

(6.5)

provided the constant spinor $\epsilon_0$ satisfies

$$e^{-1}(e_A - \gamma_5e_B)\frac{1}{2} \epsilon_0 = \pm \epsilon_0 \ .$$

(6.6)

The conditions on $\phi(y)$ and $A(y)$ for preservation of supersymmetry are the same as demanded by the domain wall solution. The condition (6.7) implies that half the supersymmetry is preserved.

In the case $e_B = 0$ the above supersymmetry transformation laws are much simplified and the domain wall preserves a half of the supersymmetry provided the constant spinor $\epsilon_0$ satisfies

$$\Gamma^y \epsilon_0 = \pm \epsilon_0 \ .$$

(6.8)

It was shown in [8] that the killing spinors of the 7-D 5-brane and 6-D 4-brane only depend on the transverse coordinates but are independent of the compactification coordinate. Therefore in reducing from 6-D to 5-D, there remains the same number of killing spinors, hence the 5-D 3-brane also preserves half of the supersymmetry.
We note that unlike the electrovac groundstates where a gauge field is non-vanishing and the dilaton is zero, domain walls involve no gauge fields and the dilaton is non-vanishing.

7. Conclusion

Certain gauged supergravity theories contain dilatonic potentials of the form $e^{-a\phi}$ and hence possess domain wall solutions. Despite the absence of an $S^1 \times M_{D-1}$ ground state these theories can be consistently reduced [9] to yield other gauged supergravities in lower dimensions.

The $SU(2) \times SU(2)$ gauged Freedman Schwarz model [5] has previously been identified as part of the effective D=4 field theory for the heterotic string in an $S^3 \times S^3$ vacuum [16]. The ‘no-go’ theorem of Gibbons, Freedman and West [17] is avoided due to that fact that the D=4 dilaton is not presumed to be constant. However, at the supergravity level it is not known which theory upon compactification yields the FS model. In this paper we have learnt that the $SU(2) \times U(1)^3$ gauged version can be obtained by dimensional reduction of an SU(2) gauged supergravity in 7-D [3]. This connection then implies the existence of a supersymmetric electrovac in 7-D which we have found. Currently there is no known link between this 7-D gauged non-maximal supergravity and 11-D supergravity. However, dimensional reduction of 11-D supergravity on $S^4$ yields an SO(5) gauged 7-D supergravity [18]. It is unclear as yet whether the 7-D non-maximal theory is a truncation of this SO(5) gauged model. If it is or if it is linked to D=11 via compactification on a different space, then these supersymmetric electrovac ground states and domain walls would be interesting new solutions of M-theory and may have an interpretation in terms of branes or intersecting configurations of branes.

In performing the reduction we have obtained the bosonic sector of an $SU(2) \times U(1)^2$ 5-D gauged theory which presumably has a supersymmetric extension. It is interesting to contrast this D=5 model with the D=5 $N=4$ $SU(2) \times U(1)$
gauged supergravity of Romans [19]. The bosonic sector of the D=5 model presented in section 2 includes a pair of vectors, $B_{\mu}^{(i)}$, which form a doublet under a global SO(2) symmetry. The bosonic sector of the D=5 model of Romans differs from this in that the doublet of vectors is replaced by a doublet of second rank antisymmetric tensor potentials allowing the SO(2) (hence SU(2)×U(1)) symmetry to be gauged. This results in an additional term in the potential proportional to the SO(2) coupling constant $g_1$. The gauging is effected using ‘odd dimensional self duality’ (ODSD) [20] but the ungauged limit can be recovered by eliminating one of the second rank antisymmetric tensors via its ODSD equation. This leads to a Lagrangian of the usual form for a single massive second rank antisymmetric tensor from which the ungauged limit can be obtained. This is similar to the situation arising in D=7 where the SO(5) gauged model of [18], containing a 5-plet of massive, self dual, third rank antisymmetric potentials, is understood in principle to be related, using ODSD, to the ungauged maximal D=7 supergravity of [21] which contains a 5-plet of second rank potentials. However, in this case the ungauged limit is not recoverable.

It is also interesting to compare the D=5 supersymmetric magnetovac of [19] with the lift to D=5 of the 1/2 supersymmetric electrovac of the ‘half gauged’ FS theory. This will be referred to as the D=5 Gibbons Freedman (GF) magnetovac. In the electrovac of the ‘half gauged’ FS model, the non-abelian sector and the scalars were zero. The electric field was due to one of the three remaining U(1) potentials non-vanishing. It preserved a 1/2 of the supersymmetry and the background was $\text{ADS}_2 \times \mathbb{R}^2$. The relevant part of the action was,

$$S_4 = \int d^4x \left\{ R_4 + 4\alpha^2 - \frac{1}{4}(|G_2^{(1)}|^2 + |G_2^{(2)}|^2 + |G_2^{(3)}|^2) \right\}.$$  

Without loss of generality one could choose $G_2^{(3)} \neq 0$ and $G_2^{(1)} = G_2^{(2)} = 0$. Being an electrovac, the non-zero components of $G_2^{(3)}$ were $G_0^{(3)}$. But we know that in lifting this electrovac up to D=5 $G_2^{(3)}$ came from $C_2$. Hence, as a dualisation was
involved, $C_2$ has only space-space components and so the lift of this supersymmetric electrovac is in fact a supersymmetric magnetovac. By the same token, lifting up to six would result in a supersymmetric ‘electrovac’ and in D=7 we would get a supersymmetric ‘magnetovac’. As the only non-zero component of $F_4$ supporting the D=7 ‘electrovac’ of section 4 was $F_{2345}$, perhaps this solution should have been called a ‘magnetovac’. In the 5-D ‘magnetovac’ of [19] the second rank antisymmetric tensor potentials are zero but the SU(2) and U(1) gauge fields are non-vanishing. Hence, apart from the extra term in the scalar potential proportional to $g_1$, the sectors of the model probed by this solution are the same as the sectors of the D=5 model (2.17) probed by the D=5 GF magnetovac in which $G_2^{(1)} = G_2^{(2)} = 0$. However, these two D=5 magnetovacs cannot be identified as in the latter solution, the SU(2) gauge fields are zero. One might suppose that a limit can be taken in which the SU(2) gauge fields of Romans’ magnetovac can be turned off. This is only possible if $g_1$ can be sent to zero. However, the field equations for the second rank potential are identically satisfied by choosing these fields to vanish. In the remaining field equations $g_1$ can be sent to zero allowing the identification with the corresponding remaining field equations of (2.17) and of the supersymmetric magnetovac solutions to be made.

We have also shown that the 5-brane of the 7-D gauged theory [8] can be double dimensionally reduced to yield a domain wall in 4-D which is a solution of the full SU(2)×SU(2) gauged model preserving a half of the supersymmetry.

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