"Form of analyzing power and the determination of the basic parameters of hadron scattering amplitude"

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Abstract

The determination of magnitudes of basic parameters of the high energy elastic scattering amplitude are examined at small momentum transfers with taking account of the Coulomb-hadron interference effects.

An actual problem of the modern elementary particles physics, the research of strong interaction processes at large distances and high energies, is considered in the framework of different approaches by using various models of the structure of hadrons and the dynamics of their interactions. The diffraction scattering cannot yet be described quantitatively in the framework of the perturbative QCD. Therefore, it is necessary to apply different models which can describe the hadron-hadron interaction at large distances. The research of elastic scattering requires the knowledge of properties of the pomeron, the object determining the interaction of hadrons in this range. In this case the study of the structure and spin properties of both the hadron and the pomeron acquires a special role.

Now we recognize that the research of the pomeron exchange requires not only a pure elastic process but also many physical processes involving electroweak boson exchanges. There are two approaches to the pomeron, the "soft" pomeron built of multiperipheral hadron exchanges and a more current perturbative-QCD "hard" pomeron built of the gluon-ladder. The "soft" pomeron dominates in high energy hadron-hadron diffractive reactions while the "hard" Pomeron dominates in high energy $\Upsilon - \Upsilon$ scattering \cite{1} and determines the small x-behaviour of deep inelastic structure functions and spin-averaged gluon distributions.

The "corner stone" for many models of the Pomeron is the power of the total cross sections growth. The "soft" pomeron of the standard form with $\alpha_{pom}(0) = 1 + \epsilon$ was introduced in \cite{2}. The observed growth of inelastic cross sections and the multiplicity match this idea. The perturbative QCD leading log calculation of the gluon ladder diagrams gives the following result \cite{4}: $\epsilon = 12 \frac{\alpha_s}{\pi} \ln 2 \sim 0.5$. This "hard" pomeron is
not yet observed experimentally. Really, the new global QCD analysis of data for various hard scattering processes leads to the small x behaviour of the gluon structure function determined by the "hard" pomeron contribution \[ g(x) \sim 1/x^{1+\epsilon} \] with \( \epsilon = 0.3 \).

There exist many discussions about the energy dependence of the elastic and total cross section in hadron-hadron scattering [6, 7]. Essential uncertainty in the determination of the values of \( \sigma_{\text{tot}} \) has been shown in [8]. Now we have a large discussion about the value of \( \sigma_{\text{tot}} \) at \( \sqrt{s} = 1.8 \) TeV. In [9] it has been found that, at \( \sqrt{s} = 1800 \) GeV, \( \sigma_{\text{tot}} = 72.2 \) mb. Recent results of the CDF Collaboration [10] are \( (1 + \rho^2)\sigma_{\text{tot}} = 62.64 \pm 0.95 \) (mb) at \( \sqrt{s} = 546 \) GeV, \( (1 + \rho^2)\sigma_{\text{tot}} = 81.83 \pm 2.29 \) (mb) at \( \sqrt{s} = 1.8 \) TeV.

Let us examine the future \( pp \rightarrow pp \) experiment at \( \sqrt{s} = 500 \) GeV, as an example. The differential cross section and spin parameters \( A_N \) are defined as

\[
\frac{d\sigma}{dt} = \frac{2\pi}{s^2}(|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 + 4|\phi_5|^2),
\]

(1)

\[
A_N \frac{d\sigma}{dt} = -\frac{4\pi}{s^2} \text{Im}[(\phi_1 + \phi_2 + \phi_3 - \phi_4)\phi_5^*],
\]

(2)
in the framework of the usual helicity representation.

Elastic differential cross section will be regarded as having 2% statistical errors. Now, in the fitting procedure we take into account the standard supposition for high energy elastic hadron scattering at small angles: the simple exponential behavior with slope \( B \) of imaginary and real parts of the scattering amplitude; the hadron spin-flip amplitude does not exceed 10% of the hadron spin-non-flip amplitude.

The differential cross section was calculated by using [10] \( \sigma_{\text{tot}} = 63.5 \) mb; \( \rho = 0.15; B = 15.5 \) GeV\(^{-2}\); at 150 points, from \( t_0 = 0.00075 \) with \( \Delta t = 0.00025 \) GeV\(^2\)), and then it was put through a special random process by using 2% errors.

After that the obtained "experimental" data are fitted (see Fig. 1). The systematical errors are taken into account as free parameters \( n \), by which the fitting curve is multiplied. The most important value, which influences the error of the magnitude of \( \sigma_{\text{tot}} \), is the normalization coefficient of the differential cross section. Its small errors lead to significant errors in \( \sigma_{\text{tot}} \).

So, for the fixed \( n = 1 \) we obtain \( \sigma_{\text{tot}} = 63.54 \pm 0.12 \) mb and with free \( n = 1.04 \pm 0.04 \) \( \sigma_{\text{tot}} = 63.6 \pm 1.3 \) mb, and we see that the normalization of experimental data is the most important problem for the definition of \( \sigma_{\text{tot}} \). That factor reduces polarization effects as it is represented as the ratio of polarization and unpolarization experimental data. So, maybe, these data help us to determine the total cross section. In any case, these experiments
are very important for the determination of the hadron spin-flip amplitude also. Now the
diffraction processes play significant role in the researches of modern accelerators. There
are some models which predict nondecreasing spin effects at superhigh energies in the
diffraction reactions. The unknown magnitude of hadron spin-flip is a serious bound to
use the Coulomb-nuclear effect for the determination of the beam polarization at future
colliders.

One important task of the \( pp2pp \) experiment at RHIC is to measure the analyzing
power, \( A_N(t) \), for elastic \( pp \)-scattering in the CNI region. To study this, it was looked at
the performance of the apparatus to reconstruct an input \( A_N(t) \). The collision energy was
taken as \( \sqrt{s} = 500 \text{ GeV} \), the beam polarization was set to 70\%, the running luminosity
is assumed to be \( 2 \times 10^{30} \text{ cm}^{-2} \times \text{s}^{-1} \). For simulation of the "left-right" analyzing power,
the simple form of \( A_N(t) \) an given by equation (2) was applied with the usual high energy
supposition at small momenta transfer for hadron spin-flip amplitudes with the slope of
the hadron spin-flip amplitude equal to the slope of the hadron spin-non-flip amplitude
without the kinematic parameter \( \sqrt{|t|} \).

We use the conventional helicity amplitudes \( \phi_i, i = 1, \ldots, 5 \) as defined in [11] and assume
the addition of the hadronic and electromagnetic amplitudes \( \Phi_i = \phi_i^h + e^{i\delta} \phi_i^e, i = 1 \ldots 5 \)
with the Coulomb phase-shift \( \delta_i \) and make the usual approximations \( \phi_1^h = \phi_3^h \) and \( \Phi_2, \Phi_4 \)
being negligible at high energies and small \( t \). Our model calculations for \( A_N \) are shown
in Fig.2.

Then we check the presence of the hadron spin flip amplitude on our fitting procedure
of \( ds/dt \). We find that the hadron spin-flip amplitude that does not exceed 10\% of the
hadron non-spin-flip amplitude, without kinematical factor of $t$, does not change the result of fit of $ds/dt$.

As usual, we assume that the hadron spin-flip amplitude is a slowly varying function of $t$ apart the kinetic factor and, following [15], we parametrize it as

\[
\phi_h^5 = \sqrt{|t|} m (\rho k_2 + ik_1) \text{Im} \phi_h^1.
\]

where $\rho, k_1, k_2$ are slowly changing functions of $s$. The coefficients $k_1$ and $k_2$ are the ratio of the real and imaginary parts of the spin-flip to spin-non-flip amplitudes without the kinematic factor $\sqrt{|t|}$. They are related to $R$ and $I$ in paper [14] as

\[
I = \text{Im} \phi_h^5 / (\sqrt{|t|} \text{Im} \phi_h^1) = k_1; \quad R = \text{Re} \phi_h^5 / (\sqrt{|t|} \text{Im} \phi_h^1) = \rho k_2
\]

As a result, $A_N$ can be written as:

\[
- \frac{A_N}{8\pi P_B} \frac{d\sigma}{dt} = -\text{Im} \phi_h^5 \frac{\alpha}{m\sqrt{|t|}} (\mu - 1) - k_1 + \frac{\sqrt{|t|}}{m} \rho |\text{Im} \phi_h^1|^2 (k_2 - k_1)
\]

For example, when the phase between the hadron spin-non-flip and spin-flip amplitudes are equal at small transfer momenta, then $k_1 = k_2$, and there is no term in the polarization depending only on the hadron amplitudes. In that case we clearly see that we have an additional contribution to the analyzing power coming from the imaginary part of the hadron spin-flip amplitude and, which is most important, having the same form as the basic term of the Coulomb-nucleon interference. This contribution has the same effect
as the error of the beam polarization, i.e. if the imaginary part of the hadron spin-flip amplitude (without the kinematic parameter) is only 5% of the imaginary part of the hadron non-flip amplitude, we have the 5% error in the definition of the beam polarization from the Coulomb-nucleon interference effect.

This fit in case $A_N$ is presented in Fig.2 by a solid line. The dashed line in Fig.2 shows the CNI effect without the hadron spin-flip amplitude.

Let us made the fit for both the data on the differential cross section and analyzing power. The new fit gives a slight decrease in the error of $\sigma_{tot}$ approximately by 10%. But the determination of the magnitude of real and imaginary parts of the hadron spin-flip amplitude becomes three time more accurate.

As the contribution of that hadron spin-flip amplitude to the maximum of CNI effect is 8%, it gives the uncertainty in the definition of maximum $A_N$ in the region of CNI effect with 2% errors.

The dependence of the additional contribution to CNI effect on the imaginary and real parts of the hadron-spin flip amplitude at $\sqrt{s} = 500$ GeV is shown in Fig. 3. It is clear that this contribution has the same sign for both parts. The reason is in fact that the basic contribution $k_1$ gives in first term of $A_N$ which have practically the same form as usual CNI effect. But at other points of $t$, this contribution is different in sign for imaginary and real parts of the hadron spin-flip amplitude. This is reflected in the second term of (2) that contains $(k_2 - k_1)$.

Now we turn to the discussion of difficulties which can be encountered in applications of

| $N$ | $\sigma_T \text{mb}$ | $\rho$ | $k_1$ | $k_2$ | $n_2$ |
|-----|---------------------|-------|-------|-------|-------|
| $A_1$ | 63.5 ± 3.4 | 0.14 ± 0.08 | −0.005 ± 0.07 | 0. ± 0.05 | 1. -fix |
| $A_2$ | 63.46 ± 3.8 | 0.14 ± 0.15 | 0.1 ± 0.06 | 0.1 ± 0.1 | 1.13 ± 2.4 |
| $B_1$ | 63.5 ± 3.8 | 0.14 ± 0.09 | 0.095 ± 0.07 | 0.14 ± 0.11 | 1. -fix |
| $B_2$ | 62.7 ± 4. | 0.13 ± 0.3 | 0.05 ± 6.3 | 0.1 ± 5.6 | 0.93 ± 3.7 |
| $C_1$ | 63.4 ± 3.6 | 0.14 ± 0.09 | 0.095 ± 0.07 | −0.14 ± 0.11 | 1. -fix |
| $C_2$ | 63.5 -fix | 0.15-f | 0.1 ± 0.015 | −0.14 ± 0.011 | 1. - fix |
| $C_3$ | 63.9 ± 1.83 | 0.01 ± 0.05 | 0.06 ± 0.037 | −0.05 ± 0.035 | 1. - fix |
Fig. 3 $A_N$ at the point of maximum of CNI at $\sqrt{s} = 500$ GeV.
(solid and dashed lines - the contributions from $ReF^\pm_h$ and $ImF^\pm_h$)

Fig. 4 The position of the maximum of CNI at $\sqrt{s} = 500$ GeV.
(solid and dashed lines - the contributions from $ReF^\pm_h$ and $ImF^\pm_h$)

a new scheme to experiments. Mostly they are systematic errors in the beam polarization.
If we take into account the systematic errors of the beam polarization as $n_2$ in variants
$B$ and $C$, by which we multiply our fitting curve, we find that the coefficients $k_1$ and
$k_2$ cannot be found separately. In this case we can find only some ratio of the real to
imaginary parts of the hadron spin-flip amplitude [15]. But the point of maximum $A_N$ is
independent of the systematic errors.

This value is tightly connected with the magnitude of $\sigma_T$ and other parameters. The
dependence of $t_{\text{max}}$ on $k_1$ and $k_2$ is shown in Fig. 4. It can be seen that this depend-
ence is different in sign for real and imaginary parts of the hadron spin-flip amplitude.
Our 10% hadron spin-flip amplitude leads to a small exchange of the point of maximum $A_N$. We obtain for the clear ($k_1 = k_2 = 0$) CNI effect at $\sqrt{s} = 500$ GeV that
$-t_{\text{max}} = 1.17910^{-3}$ GeV$^2$. For the case when $k_1 = 0.1$ and $k_2 = -0.15$, we obtain
$-t_{\text{max}} = 1.8610^{-3}$ GeV$^2$

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