ON THE RELATIONS BETWEEN TWO-PHOTON AND LEPTONIC WIDTHS OF LOW-LYING S-WAVE STATES OF CHARMONIUM

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Abstract

The relation between the ratio \( \Gamma_{ee}(\psi')/\Gamma_{ee}(J/\psi) \) and \( \Gamma_{2\gamma}(\eta_c')/\Gamma_{2\gamma}(\eta_c) \), expressed in terms of the configuration mixing amplitudes induced by the contact spin-spin interaction of quarks in the ground and radial excitation states, is shown to give, after the inclusion of the newly derived relativistic corrections, the radiative \( \eta_c' \)- and \( \eta_c \)-width ratio in fair accord with recent experiments. The dynamical model is proposed to derive the ratio of relative probability of the ground (\( \eta_c(2980) \))- and first radial excitation (\( \eta_c'(3640) \))-state formation in \( \gamma\gamma \)-collisions followed by their decay into the \( \bar{K}K\pi \) and \( p\bar{p} \) channels.

1. Two-photon decays of heavy quarkonia provide valuable information on heavy quark dynamics and have been under consideration in recent experimental [1, 2, 3, 4, 5, 6, 7, 8, 9] and theoretical [12, 13, 14, 15, 16, 17, 18] studies. It seems reasonable to assume, at least as a first approximation, that there is no mixing between light and heavy quark sectors, and that one can consider the needed transition amplitudes separately for mesons constructed of the u-, d-, s-, and heavy c- and b-quarks.

In this work, we concentrate on the charmed quark sector and our main concern in this problem will be the question of the degree of model (in)dependence of the S-wave \( Q\bar{Q} \) annihilation rates, or their ratios, with respect to the role of short-range spin-dependent forces. This is an important theoretical question because in many approximate relativistic approaches to description of the annihilation of bound antiquark-quark S-wave states that are subjected to strong short-range interactions one needs to introduce the cut-off procedures to get rid of singular behaviour of matrix elements considered, the cut-off parameters or the "smearing" procedures being introduced basically on the phenomenological grounds.

2. We start with just postulating for a mass operator of the heavy quark \( (Q = c, b) \) S-wave systems the simplest spin-dependent matrix

\[
\langle n' | \hat{M} | n \rangle = M_n \delta_{nn'} + C \cdot \psi_{ns}(0) \psi_{n's}(0) \cdot \langle \sigma_\sigma \rangle
\]

and write down simple perturbation theory expressions to have an estimation for the mutual change of the ground and first radial excited states due to switching on the contact spin-spin potential

\[
\Psi_n(0) = \psi_n(0) + \sum_m |m\rangle \langle m| \hat{V} |n\rangle / (E_n^{(0)} - E_m^{(0)}),
\]
where \(|i\rangle \equiv \psi_i\), and \(E_i^{(0)}\) are the wave functions and energies of the unperturbed hamiltonian and \(V\) is the perturbation potential that we identify with the contact spin-spin interaction of quarks.

Applying Eq. (2) successively to \(\psi_{nS}(J)\), where \(n = 1, 2\) are the quantum numbers of the S-wave radial excitations of the \(\bar{c}c\) quarkonia with the spin \(J = 0, 1\), using the specific (i.e., \(|\psi_{nS}(J)\rangle\) everywhere the terms of the first order in \(\mathcal{O}(\hat{V})\) we get the relations

\[
R_{2S/1S}^J|_{hfs} \approx \frac{\Psi_{2S}^{(0)}(0)^2}{\Psi_{1S}^{(0)}(0)^2}|_{hfs}
\]

\[
\approx \frac{\psi_{2S}(0)^2}{\psi_{1S}(0)^2} \cdot \frac{[(1 + V_{21}^{(0)}(m_1^0 - m_0^0)^{-1} + V_{32}^{(0)}(m_2^0 - m_3^0)^{-1} + \ldots)]^2}{[(1 + V_{22}^{(0)}(m_1^0 - m_2^0)^{-1} + V_{33}^{(0)}(m_2^0 - m_3^0)^{-1} + \ldots)]^2}
\]

\[
\approx \frac{\psi_{2S}(0)^2}{\psi_{1S}(0)^2} \cdot [1 + 2(s_2^J - s_1^J)],
\]

where \(s_i^J, J = 0, 1\) is the sum of terms of the order \(\mathcal{O}(\hat{V})\) entering into the denominator and numerator of (3). Using the evident relation \(V^{J=0} = -3V^{J=1}\) we obtain

\[
R_{2S/1S}^{J=0}|_{hfs} + 3 \cdot R_{2S/1S}^{J=1}|_{hfs} \approx 4 \cdot \frac{\psi_{2S}(0)^2}{\psi_{1S}(0)^2},
\]

We make an estimation for the ratio \(\Gamma(\eta_c^\prime \to \gamma\gamma)/\Gamma(\eta_c \to \gamma\gamma)\) on the basis of experimental data of leptonic charmonium decays, approximate validity of the lowest order perturbation theory for the color-hyperfine splitting interaction as well as on our anew obtained form of the relativistic corrections to the considered decays.

We remind first the known results [10, 11] for the lowest order QCD corrections

\[
\frac{m_{\eta_c}^2 \Gamma(\eta_c \to \gamma\gamma)}{m_{J/\psi}^2 \Gamma(J/\psi \to e^+e^-)} = \frac{4}{3}(1 + 1.96 \frac{\alpha_s}{\pi}) \frac{|\Psi_{\eta_c}(0)|^2}{|\Psi_{J/\psi}(0)|^2},
\]

where \(\alpha_s\) should be evaluated at the charm scale [16].

In certain chosen ratios such as

\[
\frac{\Gamma_{\eta_c}^{\prime \gamma\gamma} \Gamma_{J/\psi}^{J/\psi}}{\Gamma_{\eta_c}^{\prime \eta_c} \Gamma_{\gamma\gamma}^{\gamma\gamma}} = \left(\frac{m_{\eta_c} m_{\eta_c^\prime}}{m_{J/\psi} m_{J/\psi}}\right)^2 \frac{R_{2S/1S}^{J=0}}{R_{2S/1S}^{J=1}}
\]

the ratios of \(\psi_{nS}(0)\)’s are cancelled and also the QCD radiative corrections are assumed to be mutually compensated not only in the next-to-leading order (NLO), but also in the higher orders of the perturbation theory, which, as seen from refs. [15, 19, 20], are not negligible. We assume further on that each type of studied corrections can be approximately represented in factorized form

\[
\Psi_{nS}^{(0)}(0) \equiv |\psi_{nS}(0)|^2(1 + \delta^J(rad) + \delta^J_{nS}(rel) + \delta^J_{nS}(hfs)) \simeq |\psi_{nS}(0)|^2(1 + \delta^J(rad))(1 + \delta^J_{nS}(rel))(1 + \delta^J_{nS}(hfs)).
\]

That means that the \(1 + \delta^J(rad)\) factor coincides in the lowest order radiative correction with \(1 + (a^J/\pi)\alpha_s(m_c)\), where \((a^J=1 = -5.34)\) and \((a^J=0 = -3.38)\), and it is assumed to
be cancelled, together with higher order terms, in both the leptonic \( R_{2S/1S}^{J=1} \) and 2-photon \( R_{2S/1S}^{J=0} \) "single ratios".

The notation \( \delta_{nS}^{J}(hfs) \) refers to the correction factor due to the spin-spin potential-induced factors in \[4\] and this type of corrections is not cancelled in the double ratio \[5\] unlike the mentioned linear combination of the "single ratios" \[4].

The relativistic correction factor \( 1 + \delta_{nS}^{J}(rel) \) is defined in the following manner. The static approximation for both \( \Gamma_{ee} \) and \( \Gamma_{\gamma\gamma} \), resulting in their proportionality to the respective wave function value "at origin", \( \psi_{nS}(0) \), follows from neglecting of the dependence of the bound quark annihilation amplitudes on their internal motion momenta

\[
| \int d\vec{p} \ A(c\bar{c}(\vec{p}) \rightarrow ee(\gamma\gamma)) \ \phi_{nS}(\vec{p})|^2 \sim |\psi_{nS}(0)|^2|A_{thr}(c\bar{c} \rightarrow ee(\gamma\gamma))|^2. \tag{8}
\]

We have found that the appearance of the \( \psi_{nS}(0) \) together with the relativistic correction factor \( 1 + \delta_{nS}^{J}(rel) \) can be justified from more general expression that includes the non-static annihilation amplitudes depending on the internal quark momenta taken into account.

\[
2\sqrt{1 + \delta_{nS}^{J=0}(rel)} \simeq \frac{m_c^2}{\varepsilon_{nS}} \int \frac{d^3p}{(2\pi)^3} \phi_{nS}(p) \frac{\varepsilon_{nS}}{p} \log(\frac{\varepsilon_{nS} + p}{\varepsilon_{nS} - p}) / \int \frac{d^3p}{(2\pi)^3} \phi_{nS}(p), \tag{9}
\]

\[
\sqrt{1 + \delta_{nS}^{J=1}(rel)} \simeq \int \frac{d^3p}{(2\pi)^3} \phi_{nS}(p)(1 - \frac{\varepsilon_{nS} - m_c}{3\varepsilon_{nS}}) / \int \frac{d^3p}{(2\pi)^3} \phi_{nS}(p), \tag{10}
\]

where \( \phi_{nS}(p) \) is the \( nS \)-state wave function in the momentum representation, and \( \varepsilon_{nS} \) is the quark energy in the \( nS \)-state. The momentum-dependent factors of the \((c\bar{c})_{nS} \rightarrow e^+e^- (\gamma\gamma) \) amplitudes are given, \( e.g., \) in \[17\], but instead of taking \( \varepsilon(p) = (m_c^2 + p^2)^{1/2} \) we prefer to define the continuation of the \( c\bar{c} \)-annihilation amplitudes to the bound state kinematics following the so-called "on-energy-shell / off-mass-shell" prescription when \( \varepsilon(p) \rightarrow \varepsilon_{nS} = m_{nS}/2 \) now remains independent of the internal motion momentum \( \vec{p} \) while being averaged with the wave functions \( \phi_{nS}(\vec{p}) \) in \[10\]. This picture of the bound state dynamics underlies the derivation of the relativistic Schrödinger-type wave equations such as the quasipotential equation suggested by Todorov \[23\] or different variants thereof, \( e.g., \) \[24\]. In this presentation, the relativistic correction factor for the electron-positron annihilation of vector charmonia is especially simple because there are no additional momentum-dependent factors in the integrand, while \( \delta_{nS}^{J=0}(rel) \) is derived directly from the relation

\[
2[1 + \delta_{nS}^{J=0}(rel)]^{1/2} = \frac{m_c^2}{\varepsilon_{nS}^2} \int \frac{d^3p}{(2\pi)^3} \phi_{nS}(p) \frac{\varepsilon_{nS}}{p} \log(\frac{\varepsilon_{nS} + p}{\varepsilon_{nS} - p}) / \int \frac{d^3p}{(2\pi)^3} \phi_{nS}(p)
\]

\[
= 2 \frac{m_c^2}{\varepsilon_{nS}} \int_0^\infty dr \psi_{nS}(r) \sin(\varepsilon_{nS} r) / \psi_{nS}(0) \tag{11}
\]

\[
\simeq 2 \frac{m_c^2}{\varepsilon_{nS}^2} \cdot \left[ \psi_{nS}(0) - \frac{1}{\varepsilon_{nS}^2} \psi_{nS}''(0) + \ldots \right]/\psi_{nS}(0), \tag{12}
\]

where masses \( m_{nS} \) correspond to masses calculated without spin-dependent corrections: \( m_{1S} \simeq (3m_{J/\psi} + m_{\psi})/4, m_{2S} \simeq (3m_{\psi'} + m_{\psi})/4 \).

Keeping only the first term in asymptotic series for the Fourier integral \[12\], we obtain, in the accord with the Riemann-Lebesgue lemma, the simple approximate relation

\[
[1 + \delta_{nS}^{J=0}(rel)]^{1/2} \simeq \frac{m_c^2}{\varepsilon_{nS}^2} \tag{13}
\]
demonstrating more strong dependence on the relativistic corrections of the two-photon decay amplitude as compared to the leptonic one

\[ [1 + \delta_{nS}^{J=1}(rel)]^{1/2} = 1 - \frac{\epsilon_{nS} - m_c}{3\epsilon_{nS}}. \]  

(14)

Besides the relativistic corrections to be taken explicitly, we have observed a significant contribution to our relation for ratios \( [\ref{1}] \) due to inclusion of the short-ranged spin-dependent interaction which modifies the vector and pseudoscalar wave functions "at zero" quite asymmetrically. However, for the linear combination \( [\ref{4}] \) of two ratios the "hyperfine" corrections are compensated up to terms of the order \( \mathcal{O}(V^2) \).

Collecting now all found corrections we get the resulting relation between the widths of the lowest lying states of charmonia

\[
\frac{\Gamma_{\eta'\gamma\gamma}}{\Gamma_{\gamma\gamma}^{\psi}} \left( \frac{m_{\eta'} m_{2S}^2}{m_{\eta} m_{1S}^2} \right)^2 + 3 \frac{\Gamma_{ee}^{\psi}}{\Gamma_{ee}^{J/\psi}} \left( \frac{m_{\psi} m_{2S}(m_{1S} + m_c)}{m_{J/\psi} m_{1S}(m_{2S} + m_c)} \right)^2 = 4 \left( \frac{\psi_{2S}(0)}{\psi_{1S}(0)} \right)^2. \]  

(15)

Hence, it follows that

\[
\frac{\Gamma_{\eta'\gamma\gamma}}{\Gamma_{\gamma\gamma}^{\eta}} = 0.21 \pm 0.06, \]  

(16)

if we take \( (\psi_{2S}(0)/\psi_{1S}(0))^2 = 0.653 \) and \( m_c = 1.48 \text{ GeV} \) according to \([21, 22]\), \( \Gamma_{\gamma\gamma}^{\eta'\eta} \), masses and leptonic widths from \([1]\). A rather large uncertainty of the ratio obtained is largely due to experimental errors of the measured leptonic widths. It should be noted that unlike the very \( \psi_{nS}(0) \)'s their calculated ratios are much less model-dependent. In particular, the ratio entering into \( [15] \) calculated with the Cornell (i.e., the "linear+Coulomb") type potential is close to that calculated with the "running" \( \alpha_s(r) \), which makes the one-gluon-exchange potential softer at small distances, although the very \( \psi_{1S}(0) \)'s differs about two times from each other \([22]\). We believe that mild smearing or regularization of the short-range quark interactions providing formal finiteness of \( \psi_{nS}(0) \)'s following from the relativistic equations will also leave their ratios relatively intact.

As has been mentioned, the ratios of \( \psi_{nS}(0) \) are dropped in \( [\ref{0}] \) and one can obtain a kind of the lower bound for the double width’s ratio leaving only one, beyond and next to unity, term in every infinite sum of the perturbation corrections to \( \Psi_{nS}^{J}(0) \) due to the contact spin-spin potential of charmed quarks treated as a first order term of perturbation theory. Including then already fixed relativistic corrections and assuming, as earlier, the cancellation of the (static!) radiative corrections, we obtain

\[
\frac{\Gamma_{\gamma\gamma}^{\eta'}}{\Gamma_{\gamma\gamma}^{\eta}} \geq \frac{\Gamma_{\gamma\gamma}^{J/\psi}}{\Gamma_{\gamma\gamma}^{ee}} \left( \frac{m_{\eta'} m_{2S} m_{J/\psi}(m_{2S} + m_c)}{m_{\eta} m_{1S} m_{\psi}(m_{1S} + m_c)} \right)^2 \geq \left( 1 + \frac{V_{J=0}^{11} + V_{J=0}^{22}}{m_{2S} - m_{1S}} \right)^2 / \left( 1 + \frac{V_{J=1}^{11} + V_{J=1}^{22}}{m_{2S} - m_{1S}} \right)^2 \simeq 1 - 8 \frac{V_{J=1}^{11} + V_{J=1}^{22}}{m_{2S} - m_{1S}} \simeq 1 - 8 \frac{\psi_{J=1}^{11} + \psi_{J=1}^{22}}{m_{2S} - m_{1S}}, \]  

(17)

from where a new constraint follows

\[
\frac{\Gamma_{\gamma\gamma}^{\eta'}}{\Gamma_{\gamma\gamma}^{\eta}} \geq 0.1. \]  

(18)
and where the numerical values $\Gamma_{J/\psi}$ for $m_{J/\psi}^{I=1}$, $m_{J/\psi}^{I=2}$, defined earlier, and $V_{11}^{I=1} = (m_{J/\psi} - m_{J/\psi}^1)/4$, $V_{22}^{I=1} = (m_{J/\psi} - m_{J/\psi}^1)/4$, were used. The comparison of $\Gamma_{J/\psi}$ and $\Gamma_{J/\psi}^0$ tells about a significant role of sums $< nS|\bar{V}|nS >$ over $n \geq 1, 2$ in the definition of the individual ratios of $\Gamma_{J/\psi}^{ee}/\Gamma_{J/\psi}^{ee}$ and especially of $\Gamma_{J/\psi}^0/\Gamma_{J/\psi}^0$.

To make contact with the available experimental data for radiative widths of charmonia it is necessary to estimate also their relative branchings referring to the studied hadronic decay channels.

The main assumptions underlying our estimations of the branching ratios $\mathcal{B}_{\eta c}$ and $\mathcal{B}_{h c}$ entering into the experimentally measured processes of the two-photon fusion producing $\eta_c$ and $\eta_c'$ and subsequent hadronic decays $\eta_c(\eta_c') \rightarrow h$, where $h \equiv K \bar{K} \pi$ [3] or $h \equiv p \bar{p}$ [3], are the following. We assume a simple kinematic structure of the respective decay amplitudes and an approximate dynamical assumption for the ratio of relevant couplings or, rather, complex form-factors in the considered vertices

\begin{equation}
A(\eta_c(\eta_c') \rightarrow K \bar{K} \pi) = g(\eta_c(\eta_c') \rightarrow K \bar{K} \pi) \frac{1}{2} \varphi_{\eta_c}^* \varphi_{\eta_c} \varphi_{\eta_c} \varphi_{\eta_c},
\end{equation}

\begin{equation}
A(\eta_c(\eta_c') \rightarrow p \bar{p}) = g(\eta_c(\eta_c') \rightarrow p \bar{p}) F_c(m_{\eta_c}^2) \bar{u}(P_p) \gamma_5 v(P_p),
\end{equation}

\begin{equation}
A(\eta_c(\eta_c') \rightarrow G_m G_m) = g(\eta_c(\eta_c') \rightarrow G_m G_m) \varepsilon_{\mu \rho \sigma \varepsilon_1}^{\mu} \varepsilon_2^{\rho} \varepsilon_2^{\sigma}.
\end{equation}

where only isospin structure of the $K \bar{K} \pi$ decay channel is indicated in the first line and the generalization to the $SU(3)$-symmetry can easily be written down. We include the form-factor $F_c(Q^2)$ in (20) to mention about its possible variation depending on the time-like momentum transferred in the interval $m_{\eta_c}^2 \leq Q^2 \leq m_{\eta_c}'$. We note that unlike the vector charmonia $J/\psi$– and $\psi'$-decays, the pseudoscalar decay branchings $\mathcal{B}(\eta_c(\eta_c') \rightarrow (h, \gamma)(c\bar{c}))$ containing the charmed quarks in the final state, e.g., the decay $\eta_c(\eta_c') \rightarrow 2 \pi \eta_c$, are much less significant due to smallness of the coupling constant $\alpha_s(m_c) \simeq 0.3$, as compared to the branching ratios $\mathcal{B}(\eta_c(\eta_c') \rightarrow h)$, where $h$ denotes hadron states composed of the light ($u, d, s$) quarks. Following a usual practice (or, alternatively, the quark-hadron duality hypothesis for inclusive processes), we identify the total width of these processes with the width of the bound $c\bar{c}$-quark annihilation to pair of ”free” gluons $\Gamma(\eta_c(\eta_c')) \rightarrow G_m G_m$. Further, we attribute to gluons finite ”effective” (or dynamical) mass $m_G$ of the order 0.7 GeV, which was advocated in ref. [25] and in some earlier works cited therein, on the basis of the detailed study of the experimental photon spectrum in the inclusive reaction $J/\psi \rightarrow \gamma G_m G_m \rightarrow \gamma X$. The evaluation of the important transition probabilities $\eta_c(\eta_c') \rightarrow K \bar{K} \pi$, studied in several experiments, see, e.g., [3] and further references therein, has been performed with the help of the integral relation for invariant 3-body phase space

\begin{equation}
\rho(m_0 \rightarrow m_1 + m_2 + m_3) = \frac{1}{128 \pi m_0^2} \int_{s_2}^{s_3} \frac{ds}{s} F(s, s_1, s_2, s_3, s_4),
\end{equation}

\begin{equation}
F(s, s_1, s_2, s_3, s_4) = [(s - s_1)(s - s_2)(s_3 - s)(s_4 - s)]^{0.5}
\end{equation}

where $s_{2,1} = (m_1 \pm m_2)^2$, $s_{3,4} = (m_0 \pm m_3)^2$ and we choose $m_0 = m_{\eta_c(\eta_c')}$, $m_1 = m_2 = m_K$, $m_3 = m_\pi$. After that, the integral which is the essential part of the ratio $\mathcal{B}_{\eta c}^{0}/\mathcal{B}_{\eta c}^{\eta c'}$ can easily be calculated numerically.
Including standard relativistic normalization factors of the initial states, summing and averaging over spin degrees of freedom of the listed amplitudes squared and using the assumed relation (22), we obtain

\[
\frac{B_{h}^{c}B_{h}^{c'}}{B_{h}^{c}B_{h}^{c}} = 0.83, \text{ for } h = K\bar{K}\pi, \\
0.65 \cdot \left(\frac{F_{c}(m_{\eta_{c}}^{2})}{F_{c}(m_{\eta_{c}}^{2})}\right)^{2}, \text{ for } h = p\bar{p}.
\] (24)

Hence, our results are presented in Table 1, where the upper bound for the \(p\bar{p}\)-decay channel refers to the ratio of the form-factors put equal to unity and where we have included also some recent theoretical predictions and experimental results for the widths of the \(\eta_{c}(\eta'_{c})\)-to-\(\gamma\gamma\) decays and their ratios.

**Table 1: Recent theoretical and experimental results of the \(\eta_{c}\) and \(\eta'_{c}\) two-photon decay width** (the notations: \(h \equiv K\bar{K}\pi\) for ref. [3] and \(h \equiv p\bar{p}\) for ref. [7, 6], the entry with superscript \((x)\) is our estimate using other works)

|                      | \(\Gamma_{2\gamma}^{\eta_{c}}\) KeV | \(\Gamma_{2\gamma}^{\eta'_{c}}\) KeV | \(\Gamma_{2\gamma}^{\eta'_{c}}/\Gamma_{2\gamma}^{\eta_{c}}\) | \(B_{h}^{c}B_{h}^{c}/B_{h}^{c}B_{h}^{c}\) |
|----------------------|-------------------------------------|-------------------------------------|-----------------------------------------------|-----------------------------------------------|
| PDG [1]              | 7.5 ± 0.8                           |                                    |                                               |                                               |
| CLEO [2]             | 7.6(0.8)(2.3)                       |                                    | 0.18(0.05)(0.02)                              |                                               |
| E760/E835 [7, 6]     | 6.7±2.4                             | (2.3)                              |                                               |                                               |
| L3 [8]               | 6.9(1.7)(2.1)                       |                                    | 0.16                                          |                                               |
| DELPHI [9]           |                                    |                                    |                                               |                                               |
| Gupta [12]           | 10.94                               |                                    |                                               |                                               |
| Münz [13]            | 3.50±0.40                           | 1.38±0.30                          | 0.39±0.10\(^{(x)}\)                           |                                               |
| Chao [14]            | 6-7                                 | 2                                  | 0.28−0.33\(^{(x)}\)                          |                                               |
| Fabiano [16]         | 8.18(0.57)(0.04)                    |                                    |                                               |                                               |
| Ebert [17]           | 5.5                                 | 1.8                                | 0.33\(^{(x)}\)                               |                                               |
| Kim [18]             | 7.14±0.95                           | 4.44±0.48                          | 0.62±0.10\(^{(x)}\)                          |                                               |
| This work            | 1.6±0.5\(^{(x)}\)                  | 0.21±0.06                          | 0.18, \(h \equiv K\bar{K}\pi\) ≤ 0.15, \(h \equiv p\bar{p}\) |

The behaviour of the unitary-singlet, pseudoscalar form-factors in the hadronic transition vertices is of considerable interest for the understanding of mechanisms of sequential processes \(\eta_{c}(\eta'_{c}) \rightarrow G_{m}G_{m} \rightarrow \text{light hadrons}\). The closeness of our estimated \(R(\eta'_{c}/\eta_{c})\) decay-ratio in the \(K\bar{K}\pi\) channel, with no additional form-factors included, to the CLEO data looks intriguing and needs more investigation to be understood. At any rate, in the \(p\bar{p}\) channel one should expect a stronger dependence of the result on the \((m_{\eta_{c}}^{2}/m_{\eta'_{c}}^{2})^{n}\) ratio, where the effective power \(n\) is expected to be two units larger in the decay amplitude for two-baryon final state as compared to the transitions into the two-meson states, according the quark-counting rules. The two-gluon state mediating the \(c\bar{c}\)-
and $q^n\bar{q}^n$ - states, where $q = u, d, s$, would provide a new testing ground for checking the generalized parton approach or the diquark model which were successful in the description of the two-photon annihilation processes, like $\gamma\gamma \to p\bar{p}$, etc. Therefore, a further study of the reactions $p\bar{p} \to \eta_c(\eta'_c) \to \gamma\gamma$ (or $K\bar{K}\pi$) with better statistics and accuracy is of interest, e.g., via the ongoing experiments at FNAL or at a planned antiproton storage ring at GSI.

3. Finally, we note that relation (15) can be applied to any pairs of the ”hyperfine-split” radial-excited states. In particular, using it for pairs of ratios $R_{3S/1S}$ and $R_{3S/2S}$ with the needed input values $m_\psi = 4039$ MeV, $\Gamma_\psi = .89\pm .08$ keV $|\psi_{3S}(0)/\psi_{1S(2S)}(0)|^2 = .56(86)$ $|22|$ we obtain the estimation $m_{\eta'_c} \simeq 4003$ MeV and $\Gamma_{\eta'_c}/\Gamma_{\eta_c} \simeq .22$ of the ”naked” (i.e., without possible hadronic corrections due to virtual, open-charm intermediate ($D\bar{D}^* + \bar{D}D^*$) - states) parameters of the still to be observed $\eta'_c$-resonance.

Our main results, relation (15) and the numerical entries in Table 1, demonstrate a considerable suppressing effect of the relativistic and ”hyperfine” spin-dependent corrections on the recently observed two-photon decay of the $\eta'_c(3640)$-resonance. If it is true, this effect should also display itself in the total width $\Gamma_{\eta'_c}^{\text{tot}}$ represented by the decay into two gluons, either massless or effectively massive. Using the average value $\Gamma_{\eta'_c}^{\text{tot}} = 32.3 \pm 2.2$ MeV of the CLEO $[3]$ and BaBar $[4]$ results, we have got the estimate of the total width of the $\eta'_c(3640)$-resonance

$$\Gamma_{\eta'_c}^{\text{tot}} \simeq (32.3 \pm 2.2) \cdot (2.1 \pm .06) \simeq 6.8 \pm 2.0 \text{ MeV}, \text{ for } m_G = 0,$$

$$\simeq 7.8 \pm 2.3 \text{ MeV, for } m_G \simeq .7 \text{ GeV}.$$ 

A more precise measurement of this important parameter, as compared to the published $[3]$ result $\Gamma_{\eta'_c}^{\text{tot}} = 6.3^{+12.4}_{-4.0}$ MeV, would be very desirable.

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