Rho Mesons in Asymmetric Nuclear Matter:
A Renormalization Group Approach

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Abstract

The running of the $\rho\bar{N}N$ couplings in presence of nuclear matter is studied, and the relevant $\beta$-functions are shown to be different for $\rho^0$ and $\rho^+$, when the nuclear medium is asymmetric.
The properties of dense nuclear matter and of particles propagating in it have been seriously investigated in recent times \cite{1,2}. Such studies are important from the viewpoint of nuclear astrophysics as well as heavy ion collisions in the upcoming hadronic colliders \cite{3,4}. A proper development of the theory will undoubtedly help one to analyze the signals obtained in these collisions and thus to determine the equation of state of the system. Here we focus only on $\rho$ mesons inside a nuclear medium, as its significant role in heavy ion collisions in connection with the dilepton spectroscopy has been pointed out earlier \cite{5}. We show that the $\rho N\bar{N}$ coupling is not constant; rather, it is “running” in nature and as one increases the four-momentum $q$ of the $\rho$ meson, the value of this coupling increases, analogous to what one sees in QED. The case of asymmetric nuclear medium is more interesting: the $\rho^0 N\bar{N}$ and the $\rho^+ N\bar{N}$ couplings run in a different way, and if they are same at some low value of $q^2$, they are bound to be different at a higher $q^2$ ($q$ always denotes the four-momentum of the $\rho$ meson). This effect vanishes for a symmetric medium, as expected.

We start with the Lagrangian \cite{2}

\[ \mathcal{L}_{int} = g_v \bar{N} \gamma^\mu \tau_i N \rho^i_\mu, \]  

(1)

where $\tau_i$s ($i = 1$ to 3) are the usual $2 \times 2$ Pauli matrices, and $N$ is the two-component ($p$ and $n$) nucleon spinor. The Lagrangian in \cite{2} contains a tensor interaction term too; however, we have not taken that into our subsequent discussions for simplicity. Effects of that term upon our results are discussed later. For our later convenience, we write eq. (1) as

\[ \mathcal{L}_{int} = g_v^0 \bar{p} \gamma^\mu p - \bar{n} \gamma^\mu n |\rho^0_\mu + \sqrt{2} g_v^+ \bar{p} \gamma^\mu n \rho^+_\mu + \text{h.c.} \]  

(2)

with $\rho^0 \equiv \rho^3$ and $\rho^+ \equiv (\rho^1 \mp i \rho^2)/\sqrt{2}$. The superscripts on the coupling

2
constants are inserted merely to keep track of the neutral and the charged meson interactions, which, though same in the Lagrangian, are anticipated to run differently in asymmetric nuclear matter.

In nuclear medium, the nucleon propagator takes the form

\[ G(k) = G_F(k) + G_D(k) \]  

(3)

where

\[ G_F(k) = \frac{i(\not{k} + M^*)}{k^2 - M^{*2} + i\epsilon} \]  

(4)

and

\[ G_D(k) = -(\not{k} + M^*) \left[ \frac{\pi}{E^*(k)} \delta(k_0 - E^*(k)) \theta(k_F - |\vec{k}|) \right] \]  

(5)

with \( M^* \) and \( E^* \) denoting, respectively, the effective mass and total energy of the nucleon in the medium[6]. The second term, eq. (5), comes from the Pauli blocking of nucleon states and the effective mass of the nucleon is different from its mass in vacuum due to the presence of this Fermi sphere.

Before one embarks on the one-loop calculations, it is worthwhile to note that the integrations having one or more \( G_D \) are all finite; this is because the momentum integration has a finite upper limit dictated by the \( \theta \)-function. Thus, only those terms which contain vacuum propagator for nucleon have divergences and are relevant for performing renormalization. The rest of the paper will solely focus on these infinite terms.

One finds that the \( \rho \bar{N}N \) vertex renormalization \( Z_1 \) and the nucleon wavefunction renormalization \( Z_2 \) are equal, and using dimensional regularization and setting the renormalization scale at \( \mu_R \), one obtains

\[ Z_1 = Z_2 = 1 - \frac{1}{8\pi^2} \frac{1}{\epsilon} \left( \frac{\mu}{\mu_R} \right)^\epsilon \left[ (g_0^0)^2 + 2(g_v^0)^2 \right] \]  

(6)

where \( \epsilon = 4 - d \), \( d \) being the dimension in which the integrations are performed. The \( \rho^+ \bar{p}n \) vertex contains wavefunction renormalizations for both
proton and neutron; though the mass renormalizations for these two fermion fields are different in an asymmetric nuclear medium, the wavefunction renormalizations are still same, which we denote as $Z_2$. A pertinent question is the choice of $\mu_R$: for $Z_2$, we take $\mu_R^2 = k^2$, $k$ being the four-momentum of the nucleon, and for $Z_1$, we again take $\mu_R = k$, this time $k$ being the four-momentum of the nucleon coming in the vertex. It is clear that this procedure is exactly analogous to QED; the only difference is that for QED, $Z_1$ is evaluated setting the four-momentum of the photon to zero. The renormalized coupling contains $Z_1^{-1}Z_2$, and so these two terms do not appear in the final formulae.

Next we proceed to find $Z_3$, the $\rho$-wavefunction renormalization. The renormalized coupling $g^{ren}_\nu$ is related to the unrenormalized coupling $g^{un}_\nu$ by

$$g^{ren}_\nu = Z_1^{-1}Z_2Z_3^{1/2} \frac{g^{un}_\nu}{Z_3^{1/2}}. \quad (7)$$

The two Feynman diagrams needed to compute $Z_3$ for $\rho^0$ and $\rho^+$ are shown in figs. 1(a) and 1(b) respectively. For $\rho^0$, one gets

$$i\Pi^\mu\nu(q^2) = i\frac{(g^0_\nu)^2}{3\pi^2} \left( \frac{\mu}{\mu_R}\right)^{\epsilon} \frac{1}{\epsilon} (q^\mu q^\nu - q^2 g^{\mu\nu}) \quad (8)$$

and for $\rho^+$,

$$i\Pi^\mu\nu(q^2) = i\frac{(g^+_\nu)^2}{3\pi^2} \left( \frac{\mu}{\mu_R}\right)^{\epsilon} \frac{1}{\epsilon} (q^\mu q^\nu - q^2 g^{\mu\nu})$$

$$+ i\frac{(g^+_\nu)^2}{2\pi^2} \left( \frac{\mu}{\mu_R}\right)^{\epsilon} \frac{1}{\epsilon} \left( M^*_p - M^*_n \right)^2 g^{\mu\nu}. \quad (9)$$

Note that $\Pi^{\mu\nu}$ in eq. (8) has the same Lorentz structure as that of a bare propagator in Landau gauge. Borrowing the terminology of QED, we can say that it is “gauge invariant”, though we stress that the symmetry discussed here is not a gauged one. This is because $\rho^0$ is blind to the isospin
projection of the nucleon in so far as the Π-function is concerned, as the square of its coupling strength is same for both proton and nucleon, and both of them contribute equally. It does not matter whether the nuclear medium is asymmetric or not, since only the free part \((G_F)\) comes into play.

This is, however, not true for \(\rho^+\) (and \(\rho^-\)). As is evident from eq. (9), an extra piece proportional to \(g^{\mu\nu}\) gets added to the usual Π-function (we again emphasize that only the divergent parts are being focussed upon). This part comes from the breaking of isospin symmetry in the nucleon sector \((M_p^* \neq M_n^*)\), and from pure dimensional arguments, is proportional to \((M_p^* - M_n^*)^2\). Obviously, for symmetric matter, this part will vanish, and both \(\rho^0\) and \(\rho^\pm\) will have same Π-functions.

One can easily extract \(\text{Z}_3\) from eq. (8); it is

\[
\text{Z}_3^{\rho^0} = 1 - \frac{(g^0_\mu g^0_\nu)^2}{3\pi^2} \left( \frac{\mu}{\mu_R} \right) \frac{1}{\epsilon}. \tag{10}
\]

Extraction of \(\text{Z}_3\) from eq. (9) is a little tricky, because the coefficients of \(g^{\mu\nu}\) and \(-q^\mu q^\nu / q^2\) are not equal. Consider the analogous procedure for a gauge theory where the Π-function happens to be not gauge invariant. One then has, in the action term \(F_{\mu\nu}F^{\mu\nu}\) for the gauge field, a part which is proportional to \(g^{\mu\nu}\) and only have the field renormalization \(\text{Z}_3\) in it (coming from \(A^\mu \Box A^\nu\), where \(A^\mu\) is the gauge field and \(\Box \equiv \partial^2 / \partial t^2 - \partial^2 / \partial \vec{x}^2\)). The second part, which is proportional to \(q^\mu q^\nu\) (stemming from \((\partial^\mu A_\mu)^2\)), contains both \(\text{Z}_3\) as well as the renormalization of the gauge-fixing term (the gauge-fixing term needs renormalization only because the Π-function is not gauge invariant).

Analogous to the above procedure, we collect the coefficient of \(g^{\mu\nu}\) in eq. (9) and have

\[
\text{Z}_3^{\rho^+} = 1 - \frac{(g^+_\mu g^+_\nu)^2}{3\pi^2} \left( \frac{\mu}{\mu_R} \right) \frac{1}{\epsilon} \left[ 1 - \frac{3(M_p^* - M_n^*)^2}{2q^2} \right]. \tag{11}
\]

5
To remove the unphysical parameter $\mu$, we compare the values of $g_v$ at two different renormalization scales, viz., $\mu_R$ and $\mu_R'$, which gives

\[
g_v^0(\mu_R) = g_v^0(\mu_R') - \frac{(g_v^0)^3}{6\pi^2} \left( \frac{\mu_R'}{\mu_R} \right)^{\epsilon - 1},
\]

(12)

\[
g_v^+(\mu_R) = g_v^+(\mu_R') - \frac{(g_v^+)^3}{6\pi^2} \left( \frac{\mu_R'}{\mu_R} \right)^{\epsilon - 1} \left[ 1 - \frac{3(M_p^* - M_n^*)^2}{2q^2} \right].
\]

(13)

From eqs. (12) and (13), we find the $\beta$-functions for $g_v^0$ and $g_v^+$ to be

\[
\beta(g_v^0) \equiv \frac{\partial g_v^0}{\partial \ln \mu_R} = \frac{(g_v^0)^3}{6\pi^2},
\]

(14)

\[
\beta(g_v^+) \equiv \frac{\partial g_v^+}{\partial \ln \mu_R} = \frac{(g_v^+)^3}{6\pi^2} \left[ 1 - \frac{3(M_p^* - M_n^*)^2}{2q^2} \right].
\]

(15)

Writing $\alpha_v^0 \equiv (g_v^0)^2/4\pi$ and $\alpha_v^+ \equiv (g_v^+)^2/4\pi$, one obtains

\[
\frac{1}{\alpha_v^0(Q^2)} = \frac{1}{\alpha_v^0(\mu^2)} - \frac{2}{3\pi} \ln \frac{Q^2}{\mu^2},
\]

(16)

\[
\frac{1}{\alpha_v^+(Q^2)} = \frac{1}{\alpha_v^+(\mu^2)} - \frac{2}{3\pi} \left[ 1 - \frac{3(M_p^* - M_n^*)^2}{2q^2} \right] \ln \frac{Q^2}{\mu^2}.
\]

(17)

Eqs. (16) and (17) give us the required running of $g_v$ with $q^2$. Of course, one needs to know the values of $\alpha_v^0$ and $\alpha_v^+$ at a certain scale $\mu$, which need not be the same for $\rho^0$ and $\rho^+$, as we know [7] that in an asymmetric nuclear medium, the mass degeneracy of the $\rho$-isotriplet is lifted.

Eq. (16) is quite analogous to QED in the sense that $\alpha_v^0$ increases with $q^2$, and the dependence, apart from numerical factors, has the same functional form. Taking as per the Bonn group estimate [8] $g_v^0 = 2.67$ at $\mu = 0.77$ GeV, the mass of $\rho^0$ in vacuum, we see from eq. (16) that this model with $\rho^0$ and nucleon starts behaving like a free field theory from $q^2 \approx 50$ GeV, which is the well-known Landau pole. On the other hand, the behaviour of $\alpha_v^+$ is interesting, more so because the coefficient of the logarithmic running term is a function of $q^2$. The essential features are similar for both the cases; an infrared (IR) fixed point exists at $q^2 = 0$, and with increasing $q^2$, $\alpha_v$ hits the
Landau pole. However, the dominating terms for small $q^2$ are different and hence the slope near the IR fixed point also differs.

We, however, note that in the stable collective excitation region, the Landau pole is never reached, and only a very limited portion of $\alpha_v(q^2)$ can be studied. Nevertheless, we think that this study, pursued in greater detail, can throw much light on the dynamics of low-energy hadronic systems.

One may ask what happens if we add a tensor interaction to the vector interaction taken in the present model. In that case, a “gauge invariant” piece, i.e., a piece proportional to $q^\mu q^\nu - q^2 g^{\mu\nu}$, gets added to the $\Pi$-functions for $\rho^0$ and $\rho^+$. The magnitude of this term is, in general, quite small, and thus hardly any qualitative change in the $\beta$-functions takes place.

To summarize, we have shown that the $\rho\bar{N}N$ couplings change with $q^2$ in a way similar to QED, and have an IR fixed point at the origin. In asymmetric matter, the Lorentz structure of the $\Pi$-functions for $\rho^0$ and $\rho^+$ — even the divergent parts only — differ, due to the breaking of nuclear isospin symmetry. This is a typical matter-induced feature and is absent for field theories in vacuum. The model is shown to have a Landau pole, and like QED, this pole is quite outside the physically interesting region.

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Figure captions

1(a). Vacuum polarization diagram for $\rho^0$. Both $p\bar{p}$ and $n\bar{n}$ loops contribute. The loop momentum is $q$.

1(b). Vacuum polarization diagram for $\rho^+$. 

Fig 1(b)