Determinations of $|V_{cb}|$ and $|V_{ub}|$ from baryonic $\Lambda_b$ decays

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Abstract

We extract the Cabibbo-Kobayashi-Maskawa matrix element $V_{cb}$ from the exclusive decays of $\Lambda_b \to \Lambda_c \ell \bar{\nu}_\ell$ and $\Lambda_b \to \Lambda_c M(c)$ with $M = (\pi^-, K^-)$ and $M_c = (D^-, D_s^-)$, where the branching ratios of $\Lambda_b \to \Lambda M(c)$ measured with high precisions have not been used in the previous studies. Explicitly, we find $|V_{cb}| = (44.0 \pm 3.5) \times 10^{-3}$, which agrees with the value of $(42.11 \pm 0.74) \times 10^{-3}$ from the inclusive $B \to X_c \ell \bar{\nu}_\ell$ decays. Furthermore, based on the most recent ratio of $|V_{ub}|/|V_{cb}|$ from the exclusive $\Lambda_b$ decays, we obtain $|V_{ub}| = (4.2 \pm 0.4) \times 10^{-3}$, which is close to the value of $(4.49 \pm 0.24) \times 10^{-3}$ from the inclusive $B \to X_u \ell \bar{\nu}_\ell$ decays. We conclude that our determinations of $|V_{cb}|$ and $|V_{ub}|$ from the exclusive $\Lambda_b$ decays favor the inclusive extractions in the $B$ decays.
I. INTRODUCTION

In the Standard Model (SM), the unitary $3 \times 3$ Cabibbo-Kobayashi-Maskawa (CKM) matrix elements present the coupling strengths of quark decays, with the unique physical weak phase for CP violation. Being unpredictable by the theory, the matrix elements as the free parameters need the extractions from the experimental data. Nonetheless, there exists a long-standing discrepancy between the determinations of $|V_{cb}|$ based on the exclusive $B \to D^{(*)}\ell\bar{\nu}_\ell$ and inclusive $B \to X_c\ell\bar{\nu}_\ell$ decays, given by [1–3]

$$|V_{cb}| = (39.18 \pm 0.99) \times 10^{-3} \quad (B \to D\ell\bar{\nu}_\ell),$$

$$|V_{cb}| = (38.71 \pm 0.75) \times 10^{-3} \quad (B \to D^{*}\ell\bar{\nu}_\ell),$$

$$|V_{cb}| = (42.11 \pm 0.74) \times 10^{-3} \quad (B \to X_c\ell\bar{\nu}_\ell).$$

From the data in Eq. (1), we see that the deviations between the central values of the inclusive and exclusive decays are around $(2-3)\sigma$. For the resolution, the analysis in Ref. [4] suggests that the $B \to D^{*}$ transition form factors developed by Caprini, Lellouch and Neubert (CLN) [5] may underestimate the uncertainty that associates with the extraction of $|V_{cb}|$. Moreover, it has been recently pointed out that the theoretical parameterizations of the $B \to D^{(*)}$ transitions given by Boyd, Grinstein and Lebed (BGL) [6] are more flexible to reconcile the difference [7, 8]. Similar to the data for $|V_{cb}|$ in Eq. (1), there also exists a tension for the determination of $|V_{ub}|$ between the exclusive and inclusive $B$ decays, which has drawn a lot of theoretical attentions to search for the solutions in the SM and beyond [9–14].

On the other hand, the baryonic $\Lambda_b$ decays could provide some different theoretical inputs for the CKM matrix elements, which are able to ease the tensions between the exclusive and inclusive determinations. Indeed, to have an accurate determination of $|V_{ub}|/|V_{cb}|$ the LHCb Collaboration has carefully analyzed the ratio of [15]

$$\mathcal{R}_{ub} \equiv \frac{B(\Lambda_b \to p\mu\bar{\nu}_\mu)_{q^2>15 \text{ GeV}^2}}{B(\Lambda_b \to \Lambda_c^+\mu\bar{\nu}_\mu)_{q^2>7 \text{ GeV}^2}} = \frac{|V_{ub}|^2/|V_{cb}|^2}{R_{FF}},$$

where $B$ denotes the branching fraction and $q$ is the certain range of the integrated energies for the data collection. In Eq. (2), $\mathcal{R}_{ub}$ by relating $B(\Lambda_b \to p\mu\bar{\nu}_\mu)$ to $B(\Lambda_b \to \Lambda_c\mu\bar{\nu}_\mu)$ reduces the experimental uncertainties, while $R_{FF}$ is a ratio of the $\Lambda_b \to \Lambda_c$ and $\Lambda_b \to p$ transition form factors, calculated by the lattice QCD (LQCD) model [16] with a less theoretical uncertainty.
In this work, we would like to first explore the possibility to determine $|V_{cb}|$ from the baryonic decays. In particular, we use the observed branching ratios of $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell \ell \bar{\nu}_\ell$ and $\Lambda_b \rightarrow \Lambda_c M_{(c)}$ with $\ell = e^-$ or $\mu^-$, $M = (\pi^-, K^-)$ and $M_{c} = (D^-, D_s^-)$, which have never been used in the previous studies. The full energy-range measurements of the semileptonic decays are given by $[17]$:

$$B(\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell) = (6.2^{+1.4}_{-1.3}) \times 10^{-2},$$

$$R_{cb} \equiv \frac{B(\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell)}{B(\Lambda_c \rightarrow \Lambda \ell \bar{\nu}_\ell)} = 1.7 \pm 0.4,$$  \(3\)

where $R_{cb}$ combines the data of $B(\Lambda_b \rightarrow \Lambda_c^+ \ell \bar{\nu}_\ell)$ and $B(\Lambda_c^+ \rightarrow \Lambda \ell \bar{\nu}_\ell)$ to eliminate the uncertainties, similar to $R_{ub}$ in Eq. (2). The decay branching ratios of $\Lambda_b \rightarrow \Lambda_c^+ M_{(c)}$ are observed as $[17]$

$$B(\Lambda_b \rightarrow \Lambda_c^+ \pi^-) = (4.9 \pm 0.4) \times 10^{-3},$$

$$B(\Lambda_b \rightarrow \Lambda_c^+ K^-) = (3.59 \pm 0.30) \times 10^{-4},$$

$$B(\Lambda_b \rightarrow \Lambda_c^+ D^-) = (4.6 \pm 0.6) \times 10^{-4},$$

$$B(\Lambda_b \rightarrow \Lambda_c^+ D_s^-) = (1.10 \pm 0.10) \times 10^{-2}. \quad \text{(4)}$$

The above modes in Eq. (4) can be regarded to proceed through the $\Lambda_b \rightarrow \Lambda_c$ transition together with the recoiled mesons, such that the theoretical estimations give

$$\frac{B(\Lambda_b \rightarrow \Lambda_c^+ \pi^-)}{B(\Lambda_b \rightarrow \Lambda_c^+ K^-)} \simeq R(M) \left( \frac{V_{ud}}{V_{us}} \right)^2 \left( \frac{f_\pi}{f_K} \right)^2 = 13.2,$$

$$\frac{B(\Lambda_b \rightarrow \Lambda_c^+ D^-)}{B(\Lambda_b \rightarrow \Lambda_c^+ D_s^-)} \simeq R(M_{c}) \left( \frac{V_{cs}}{V_{cd}} \right)^2 \left( \frac{f_{D_s}}{f_D} \right)^2 = 25.1,$$  \(5\)

where $f_{M_{(c)}}$ are the meson decay constants and $R(M_{(c)})$ are the rates to account for the mass differences from the phase spaces. Note that the ratios in Eq. (5) remarkably agree with $(13.6 \pm 1.6, 24.0 \pm 3.8)$ from the data in Eq. (4), respectively. This implies that the theoretical calculations of $B(\Lambda_b \rightarrow \Lambda_c M_{(c)})$ can be reliable to be involved in the fitting of $|V_{cb}|$. Particularly, the data in Eq. (4) have the significances of $(8-12)\sigma$, which apparently benefit the precise determination of $|V_{cb}|$. As a result, the extraction of $|V_{cb}|$ from the data in Eqs. (3) and (4) can be an independent one besides those from the $B \rightarrow D^{(*)} \ell \bar{\nu}_\ell$ and $B \rightarrow X_c \ell \bar{\nu}_\ell$ decays. With the newly extracted $|V_{cb}|$ value, we will be then able to determine $|V_{ub}|$. 

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II. FORMALISM

As seen in Fig. 1 in terms of the effective Hamiltonian at quark level for the semileptonic $b \to c\ell\bar{\nu}_\ell$ and non-leptonic $b \to c\bar{c}\beta$ $(\bar{c} = \bar{u}(c)$ and $\beta = q = d, s)$ transitions by the $W$-boson external emissions, the amplitudes of the $\Lambda_b \to \Lambda_c\ell\bar{\nu}_\ell$ and $\Lambda_b \to \Lambda_cM_{(c)}$ decays are found to be \[16,18\]

$$A(\Lambda_b \to \Lambda_c^+\ell\bar{\nu}_\ell) = \frac{G_F}{\sqrt{2}} V_{cb} \langle \Lambda_{c}^+ | \bar{c} \gamma_{\mu} (1 - \gamma_{5}) b | \Lambda_b \rangle \bar{\epsilon}^{\gamma}_{\mu} (1 - \gamma_{5}) \nu_{\ell},$$

$$A(\Lambda_b \to \Lambda_c^+M_{(c)}) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs} a_{1}^{M_{(c)}} i f_{M_{(c)}} q^{\mu} \langle \Lambda_{c}^+ | \bar{c} \gamma_{\mu} (1 - \gamma_{5}) b | \Lambda_b \rangle,$$  \hspace{1cm} (6)

where $G_F$ is the Fermi constant, $V_{\alpha\beta} = V_{u(c)q}$ $(q = d, s)$ for $M_{(c)} = \pi^{-}(D^{-}), K^{-}(D_s^{-})$, and the matrix elements of $\langle M_{(c)} | \beta \gamma_{\mu} (1 - \gamma_{5}) \alpha | 0 \rangle = i f_{M_{(c)}} q^{\mu}$ have been used for the meson productions. Note that the amplitude of the $\Lambda_c \to \Lambda_c \ell\bar{\nu}_\ell$ decay through $c \to s\ell\bar{\nu}_\ell$ can be given by replacing $(b,c)$ with $(c,s)$ in $A(\Lambda_b \to \Lambda_c^+\ell\bar{\nu}_\ell)$ of Eq. (6). The parameters $a_{1}^{M_{(c)}} = c_{1}^{eff} + c_{2}^{eff}/N_{c}^{eff}$ are derived by the generalized factorization approach with the effective Wilson coefficients $c_{1,2}^{eff}$ and color number $N_{c}^{eff}$ \[19\].

In the helicity-based definition, the matrix elements of the $\Lambda_b \to \Lambda_c$ transition are given by \[16\]

$$\langle \Lambda_{c} | \bar{c} \gamma_{\mu} b | \Lambda_b \rangle = \bar{u}_{\Lambda_{c}}(p',s') \left[ f_{0}(q^{2})(m_{\Lambda_b} - m_{\Lambda_c}) \frac{q^{\mu}}{q^{2}} + f_{+}(q^{2}) \frac{m_{\Lambda_b} + m_{\Lambda_c}}{s_{+}} \right] u_{\Lambda_b}(p, s),$$

$$\langle \Lambda_{c} | \bar{c} \gamma_{\mu} \gamma_{5} b | \Lambda_b \rangle = -\bar{u}_{\Lambda_{c}}(p',s') \gamma_{5} \left[ g_{0}(q^{2})(m_{\Lambda_b} + m_{\Lambda_c}) \frac{q^{\mu}}{q^{2}} + g_{+}(q^{2}) \frac{m_{\Lambda_b} - m_{\Lambda_c}}{s_{-}} \right] u_{\Lambda_b}(p, s),$$ \hspace{1cm} (7)
where \( q = p - p' \), \( s_{\pm} = (m_{\Lambda_b} \pm m_{\Lambda_c})^2 - q^2 \), and \( (f_0, f_+, f_{\perp}) \) and \( (g_0, g_+, g_{\perp}) \) are form factors. The momentum dependences of \( f = f_j \) and \( g_j \) \( (j = 0, +, \perp) \) are written as \[16\]

\[
f(t) = \frac{1}{1 - t/(m_{\text{pole}}^f)^2} \sum_{n=0}^{n_{\text{max}}} a_n^f \left[ \frac{\sqrt{t_+ - t_0} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}} \right]^n,
\]

where \( (n_{\text{max}}, t_+, t_0) = (1, (m_{\text{pole}}^f)^2, (m_{\Lambda_b} - m_{\Lambda_c})^2) \) with \( m_{\text{pole}}^f \) representing the corresponding pole masses. Note that the form factors for the \( \Lambda_c \to \Lambda \) transition have similar forms as in Eqs. \[7\] and \[8\], given in Ref. \[20\]. In terms of the equations in Ref. \[17\], one is able to integrate over the variables of the phase spaces in the two-body and three-body decays for the decay widths.

### III. NUMERICAL RESULTS AND DISCUSSIONS

For the numerical analysis, we perform the minimum \( \chi^2 \) fit with \( |V_{cb}| \) being a free parameter to be determined. The parameters \( a_1^{M(c)} \) are able to accommodate the non-factorizable effects, provided that \( N_{\text{eff}}^c \) is taken as the effective color number to range from 2 to \( \infty \) in accordance with the generalized factorization \[19\], leading to the initial inputs of \( a_1^{M(c)} = 1.0 \pm 0.2 \). Note that \( a_1^{M(c)} \approx O(1.0) \) has presented the insensitivity to the non-factorizable effects in the \( b \)-hadron decays. Besides, the data in Eq. \[14\] enable the accurate determination of \( a_1^{M(c)} \), instead of using sub-leading calculations like the QCD factorization, which are not available yet in \( \Lambda_b \to \Lambda_c M(c) \). The theoretical inputs for the CKM matrix elements and

| branching ratios \( \Lambda_b \to \Lambda_c \ell \bar{\nu}_\ell \) | experimental data \[17\] |
|-----------------------------------------------|--------------------------|
| \( 10^2 \mathcal{B}(\Lambda_b \to \Lambda_c \ell \bar{\nu}_\ell) \) | \( 6.2^{+1.4}_{-1.3} \) |
| \( \mathcal{R}_{cb} \equiv \frac{\mathcal{B}(\Lambda_b \to \Lambda_c^+ \ell \bar{\nu}_\ell)}{\mathcal{B}(\Lambda_b \to \Lambda_{c\tau} \ell \bar{\nu}_\tau)} \) | \( 1.7 \pm 0.4 \) |
| \( 10^2 \mathcal{B}(\Lambda_b \to \Lambda_c^+ \pi^-) \) | \( 4.9 \pm 0.4 \) |
| \( 10^4 \mathcal{B}(\Lambda_b \to \Lambda_c^+ K^-) \) | \( 3.6 \pm 0.3 \) |
| \( 10^4 \mathcal{B}(\Lambda_b \to \Lambda_c^+ D^-) \) | \( 4.6 \pm 0.6 \) |
| \( 10^2 \mathcal{B}(\Lambda_b \to \Lambda_c^+ D^-) \) | \( 1.1 \pm 0.1 \) |
decay constants are given by

\[(|V_{cd}|, |V_{cs}|) = (0.220 \pm 0.005, 0.995 \pm 0.016),\]
\[(|V_{ud}|, |V_{us}|) = (0.97417 \pm 0.00021, 0.2248 \pm 0.0006),\]
\[(f_\pi, f_K) = (130.2 \pm 1.7, 155.6 \pm 0.4) \text{ MeV},\]
\[(f_D, f_{D_s}) = (203.7 \pm 4.7, 257.8 \pm 4.1) \text{ MeV},\]

while the experimental inputs in Eqs. (3) and (4) are accounted to be 6 data points, listed in Table I. Note that the information of the \(\Lambda_b \rightarrow \Lambda_c\) and \(\Lambda_c \rightarrow \Lambda\) form factors in Eq. (8) are adopted from Refs. [16, 20]. Subsequently, we obtain

\[|V_{cb}| = (44.0 \pm 3.5) \times 10^{-3},\]

with \(\chi^2/d.o.f = 5.5/5 = 1.1\) and \((a_1^M, a_1^{M_{(c)}}) = (1.0 \pm 0.1, 0.8 \pm 0.1)\), where d.o.f denotes the degrees of freedom. Note that our fit with \(\chi^2/d.o.f \sim 1\) indicates a very good fit, while the value in Eq. (10) clearly agrees with the inclusive result in Eq. (1) from \(B \rightarrow X_c \ell \bar{\nu}_\ell\). With the improved ratio of \(|V_{ub}|/|V_{cb}| = 0.095 \pm 0.005\) in Ref. [17] from the exclusive \(\Lambda_b\) decays, along with the new extraction of \(|V_{cb}|\), we get

\[|V_{ub}| = (4.2 \pm 0.4) \times 10^{-3},\]

which is consistent with the inclusive result of \((4.49 \pm 0.24) \times 10^{-3}\) from \(B \rightarrow X_u \ell \bar{\nu}_\ell\) [17] but different from the exclusive one of \((3.72 \pm 0.19) \times 10^{-3}\) from \(B \rightarrow \pi \ell \bar{\nu}_\ell\) [17]. To compare our fitting results with different data inputs, we set 4 scenarios:

\[(S0)\quad B(\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell) + R_{cb} + B(\Lambda_b \rightarrow \Lambda_c M_{(c)}),\]
\[(S1)\quad B(\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell) + B(\Lambda_b \rightarrow \Lambda_c M_{(c)}),\]
\[(S2)\quad B(\Lambda_b \rightarrow \Lambda_c M_{(c)}),\]
\[(S3)\quad B(\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell) + R_{cb},\]

where \(S0\) corresponds to the fitting shown in Eqs. (10) and (11), which gives the lowest uncertainty for \(|V_{cb}|\) along with the best value of \(\chi^2/d.o.f\). In Table III we summarize our results as well as the data from the \(B\) decays. As seen from Table III S0 and S3 give similar results, but the value of \(\chi^2/d.o.f = 0.1\) for \(S3\) is too low to be trustworthy.

Finally, we remark that if we take the \(\Lambda_b \rightarrow \Lambda_c\) and \(\Lambda_c \rightarrow \Lambda\) transition form factors in the forms of \(f(q^2) = f(0)/[1 - a(q^2/m_{\Lambda_b}) + b(q^2/m_{\Lambda_b})^2]\), adopted from Refs. [21, 22], we
obtain a lower value of $|V_{cb}| = (34.9 \pm 2.8) \times 10^{-3}$ with $\chi^2/d.o.f = 0.7$ by keeping the 6 data points in Table I in the fitting. In this case, less flexible inputs for the form factors with only central values for $(f(0), a, b)$ are used, leading to the result similar to the extraction from the exclusive $B \to D^{(*)}\ell\bar{\nu}_\ell$ decays with the CLN parameterization for the $B \to D^{(*)}$ transitions [5].

IV. CONCLUSIONS

In sum, since the extractions of $|V_{cb}|$ showed the $(2 - 3)\sigma$ deviations between the exclusive $B \to D^{(*)}\ell\bar{\nu}_\ell$ and inclusive $B \to X_c\ell\bar{\nu}_\ell$ decays, we have performed an independent determination from the exclusive $\Lambda_b \to \Lambda_c\ell\bar{\nu}_\ell$ and $\Lambda_b \to \Lambda_cM_{(c)}$ decays. We have obtained $|V_{cb}| = (44.0 \pm 3.5) \times 10^{-3}$ to agree with the extraction in $B \to X_c\ell\bar{\nu}_\ell$. With the improved ratio of $|V_{ub}|/|V_{cb}|$ from the LHCb and PDG, we have derived $|V_{ub}| = (4.2 \pm 0.4) \times 10^{-3}$ which is close to the result from the inclusive decays of $B \to X_u\ell\bar{\nu}_\ell$. Consequently, we have demonstrated that our extractions of $|V_{cb}|$ and $|V_{ub}|$ from the exclusive $\Lambda_b$ decays support those from the inclusive $B$ decays. Clearly, the reliabilities for the determinations of $|V_{cb,ub}|$ from the exclusive $B$ decays should be reexamined.

| TABLE II. The fitting results for the different scenarios in comparison with the experimental data. |
|-----------------|--------|-----------------|-----------------|
| $\chi^2/d.o.f$  | $|V_{cb}| \times 10^3$ | $|V_{ub}| \times 10^3$ |
|-----------------|-----------------|-----------------|-----------------|
| $S0$            | 1.1             | $44.0 \pm 3.5$  | $4.2 \pm 0.4$   |
| $S1$            | 1.3             | $42.8 \pm 4.3$  | $4.1 \pm 0.5$   |
| $S2$            | 1.7             | $40.0 \pm 6.5$  | $3.8 \pm 0.6$   |
| $S3$            | 0.1             | $45.0 \pm 3.6$  | $4.3 \pm 0.4$   |
| $B \to D\ell\bar{\nu}_\ell$ [1] | 39.18 ± 0.99    |                 |
| $B \to D^{*}\ell\bar{\nu}_\ell$ [1] | 38.71 ± 0.75    |                 |
| $B \to X_c\ell\bar{\nu}_\ell$ [2] | 42.11 ± 0.74    |                 |
| $B \to \pi\ell\nu_\ell$ [17] | 3.72 ± 0.19     |                 |
| $B \to X_u\ell\bar{\nu}_\ell$ [17] | 4.49 ± 0.24     |                 |
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