Low-Scale Closed Strings and their Duals

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We study large dimensions and low string scale in four-dimensional compactifications of type II theories of closed strings at weak coupling. We find that the fundamental string scale, together with all compact dimensions, can be at the TeV, while the smallness of the string coupling accounts for the weakness of gravitational interactions. This is in contrast to the situation recently studied in type I theories, where the string scale can be lowered only at the expense of introducing large transverse dimensions felt by gravity only. As a result, in these type II strings, there are no strong gravity effects at the TeV, and the main experimental signature is the production of Kaluza-Klein excitations with gauge interactions. In the context of type IIB theories, we find a new possibility providing a first instance of large non-transverse dimensions at weak coupling: two of the internal dimensions seen by gauge interactions can be at the TeV, with the string scale and all other dimensions at intermediate energies of the order of $10^{11}$ GeV, where gravity becomes also strong. Finally, using duality, we provide a perturbative description for the generic case of large dimensions in the heterotic string. In particular, we show that the two type II theories above describe the cases of one and two heterotic large dimensions. A new M-theory derivation of heterotic-type II duality is instrumental for this discussion.

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1 Introduction

Large dimensions are of particular interest in string theory because of their possible use to explain outstanding physical problems, such as the mechanism of supersymmetry breaking [1], the gauge hierarchy [2, 3] and the unification of fundamental interactions [4, 5, 6].

In the context of perturbative heterotic string theory, large TeV dimensions are motivated by supersymmetry breaking which identifies their size to the breaking scale. Although the full theory is strongly coupled in ten dimensions, there are many quantities that can be studied perturbatively, such as gauge couplings that are often protected by non renormalisation theorems [1, 7], as well as all soft breaking terms because of the extreme softness of the supersymmetry breaking mechanism, in close analogy with the situation at finite temperature [1, 8]. An obvious question is whether there is some dual theory that provides a perturbative description of the above models.

More recently, it was proposed that the observed gauge hierarchy between the Planck and electroweak scales may be due to the existence of extra large (transverse) dimensions, seen only by gravity which becomes strong at the TeV [2]. This scenario can be realised [9, 10] in the context of a weakly coupled type I' string theory with a string scale at the TeV [11] and the Standard Model living on D-branes, transverse to $p$ large dimensions of size ranging from (sub)millimeter (for $p = 2$) to a fermi (for $p = 6$). As we will discuss below, these models are dual to the heterotic ones with $n = 4$, 5 or 6 large dimensions. More precisely, the cases $n = 4$ and $n = 6$ correspond to $p = 2$ and $p = 6$, respectively, while the description of $n = 5$ uses an anisotropic type I' theory with 5 large and one extra large dimension.

In this work, we study large dimensions in the context of weakly coupled type II string compactifications. One of the main characteristics of these theories is that non-abelian gauge symmetries appear non-perturbatively, even at very weak coupling, when the compactification manifold is singular [12, 13]. In particular, on the type IIA (IIB) side, they
can be obtained from even (odd) D-branes wrapped around even (odd) collapsing cycles. As we will show in Section 2, we find two novel possibilities which cannot be realised perturbatively either in the heterotic or in type I constructions.

The first case consists of type IIA four-dimensional (4d) compactifications with all internal radii of the order of the string length, that can be as large as TeV$^{-1}$ due to a superweak string coupling. Despite this, Standard Model gauge couplings remain of order unity because their magnitude is determined by the geometry of the internal manifold and not by the value of the string coupling\footnote{This situation can also be realised in type I string theory but only in six dimensions \cite{11,9}.}. This scenario offers an alternative to the brane framework for solving the gauge hierarchy, with very different experimental signals. There are no missing energy events from gravitons escaping into the bulk \cite{2,8,14}, while string excitations interact with Planck scale suppressed couplings. As a result, the production of Kaluza-Klein (KK) excitations with gauge interactions remain the only experimental probe \cite{1,15}. Furthermore, the problem of proton decay and flavor violation becomes in principle much easier to solve than in low energy quantum gravity models. This model will be shown in Section 3 to be dual to the heterotic string with $n = 2$ large dimensions.

The second case consists of type IIB theory with two large dimensions at the TeV and the string tension at an intermediate scale of the order of $10^{11}$ GeV. The string coupling is of order unity (but perturbative), while the largeness of the 4d Planck mass is attributed to the large TeV dimensions compared to the string length. Gravity becomes strong at the intermediate scale and the main experimental signal is again the production of KK gauge modes. However, the gauge theory above the compactification scale is very different than in the previous type IIA case. In fact, this model offers the first instance of large radius along non-transverse directions and contains an energy domain where its effective theory becomes six-dimensional below the string scale. This limit corresponds to a non-trivial infrared conformal point described by a tensionless self-dual string \cite{16}. This model is
again dual to the heterotic string with a single \((n = 1)\) large dimension at the TeV.

Of course, in the above two examples, one can increase the type IIA string coupling or lower the type IIB string scale by introducing some extra large dimensions transverse to the 5-brane where gauge interactions are localised. In particular, we will show that the heterotic string with \(n = 3\) large TeV dimensions is described by a type IIA compactification with a string scale and two longitudinal dimensions at the TeV, four transverse dimensions at the fermi scale, and order 1 string coupling.

The paper is organised as follows. In Section 2, we study TeV strings and TeV dimensions in type II theories and we describe the first two examples mentioned above. In Section 3, we review briefly the string dualities among heterotic, type I and type II theories, and give the basic relations we use in the sequel; in particular, we give a simple (yet heuristic) derivation of heterotic–type IIA duality in the framework of M-theory. In Section 4, we discuss large dimensions in heterotic compactifications and provide a perturbative description of all cases using heterotic – type I or heterotic – type II dualities. For completeness, in Section 5, we examine large dimensions in type II theories, and show how the heterotic theories of Section 4 are recovered. Finally, Section 6 contains some concluding remarks.

2 Type II theories with low string scale

The Standard Model gauge interactions can be in principle embedded within three types of four-dimensional string theories, obtained by compactification of the ten-dimensional heterotic, type II and type I theories. On the heterotic side, gauge interactions appear in the perturbative spectrum and, like the gravitational interactions, are controlled by the string coupling \(g_H\). In type I theories, gauge interactions are described by open strings and confined on D-branes, whereas gravity propagates in the bulk; both interactions are controlled again by the string coupling \(g_I\), although gauge forces are enhanced by a factor...
In type II theories, the matching condition forbids the existence of non-abelian vector particles in the perturbative spectrum; however, gauge interactions do arise non-perturbatively at singularities of the $K_3$-fibered Calabi-Yau manifold, from D2-branes (in type IIA) or D3-branes (in type IIB) wrapping the vanishing cycles. The gauge symmetry is dictated by the intersection matrix of the vanishing two-cycles in the $K_3$ fiber, whereas extra matter arises from vanishing cycles localised at particular points on the base. As a result, gauge interactions are localised on 5-branes at the singularities with a gauge coupling given by a geometric modulus (the size of the base in type IIA), whereas gravitational effects are still controlled by the string coupling $g_{II}$.

More precisely, the gauge and gravitational kinetic terms in the effective type IIA four-dimensional field theory are, in a self-explanatory notation:

$$S_{IIA} = \int d^4x \sqrt{-g} \left( \frac{1}{g_{6IIA}^2} \frac{R_5 R_6}{l_{II}^2} \mathcal{R} + \frac{R_5 R_6}{l_{II}^2} F^2 \right),$$

where $l_{II}$ is the type II string length, $g_{6IIA}$ is the six-dimensional string coupling, and $R_5 R_6$ is the two-dimensional volume of the base, along the 5-brane where gauge fields are localised. For simplicity, we consider here the base to be a product of two circles with radii $R_5$ and $R_6$. In eq. and henceforth we set all numerical factors to one, although we take them into account in the numerical examples. Identifying the coefficient of $\mathcal{R}$ with the inverse Planck length $l_P^2$ and the coefficient of $F^2$ with the inverse gauge coupling $g_{YM}^2$, one gets:

$$\frac{1}{g_{YM}^2} = \frac{R_5 R_6}{l_{II}^2} \quad \text{and} \quad g_{6IIA} = \frac{1}{g_{YM} l_{II}^2}. \quad (2)$$

Keeping the Yang-Mills coupling of order unity implies that $R_5 R_6$ is of order $l_{II}^2$, while $g_{6IIA}$ is a free parameter that allows to move the string length away from the Planck scale. As a result, one can take the type II string scale to be at the TeV, keeping all compactification radii to be at the same order of $\text{TeV}^{-1}$, by introducing a tiny string coupling of the order of $10^{-6}$. Here we take as an example the well-understood case of $N = 2$ supersymmetric compactifications, since it already exhibits the main features of interest. Our discussion carries over trivially to $N = 1$ models obtained for instance by freely acting orbifolds.

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10^{-14}. We stress again that this situation cannot be realised in heterotic or type I theories at weak coupling.

This scenario is very different from the TeV strings arising in type I theory, where the string coupling is fixed by the 4d gauge coupling, while the type I string scale is lowered by introducing large transverse dimensions implying that gravity becomes strong at the TeV scale. Here, all internal dimensions have string size (TeV^{-1}) and gravitational/string interactions are extremely suppressed by the 4d Planck mass. As a result, Regge excitations cannot be detected in particle accelerators and the main experimental signal is the production of KK excitations of gauge particles, along the (5, 6) directions parallel to the 5-brane where gauge interactions are localised, due to the singular character of the compactification manifold; furthermore, these excitations come in multiplets of $N = 4$ supersymmetry, which is recovered in the six-dimensional limit. Quarks and leptons, on the other hand, do not in general have KK excitations since matter fields are localised at particular points of the base \[18\]; they look similar to the twisted fields in heterotic orbifold compactifications. Notice the similarity of these predictions with those of heterotic string with large dimensions, despite its strong 10d coupling \[1\]. The requirement of $N = 4$ excitations was, there, a way to keep the running of gauge couplings logarithmic above the compactification scale. In fact, as we will show in Section 4, the above type II models are dual to heterotic compactifications with two large TeV dimensions.

An obvious advantage of this scenario is that several potential phenomenological problems, such as proton decay and flavor violations, appear much less dangerous than in type I TeV strings, since they are restricted to the structure of KK gauge modes only, and one does not have to worry about string excitations. Model building, on the other hand, becomes more involved since it requires a deeper understanding of the gauge theory on the 5-brane localised at the singular points of $K_3$; due to the weakness of the string coupling, the dynamics of the gauge theory is determined by classical string theory in a strongly
curved background, which can be analysed, for instance, in the framework of geometric engineering [18].

A related question is the one of gauge hierarchy, which in the present context consists of understanding why the type IIA string coupling is so small. The technical aspect of this problem is whether string interactions decouple from the effective gauge theory on the 5-brane, in the limit of vanishing coupling. Although naively such a decoupling seems obvious, there may be subtleties related to the non-perturbative origin of non-abelian gauge symmetries due to the singular character of the compactification manifold. In fact, we would like to argue that there are in general logarithmic singularities similar to the case of having an effective transverse dimensionality $d_\perp = 2$ in the D-brane/type I scenario of TeV strings [3].

The argument is based on threshold corrections to gauge and gravitational couplings, that take the form:

$$\Delta = b \ln(\mu a) + \Delta_{\text{reg}}(t_i),$$

where the first term corresponds to an infrared (IR) divergent contribution, regularised by an IR momentum cutoff $\mu$, $b$ is a numerical $\beta$-function coefficient, and $\Delta_{\text{reg}}$ is a function of the moduli $t_i$, which in $N = 2$ compactifications belong to vector supermultiplets. For dimensional reasons, we have also introduced an ultraviolet (UV) scale $a$, which naively should be identified with the type II string length $l_{\text{II}}$. However because of the relation (2), in supergravity units, $\Delta$ would acquire a dependence on the 4d string coupling, which is impossible because it belongs to a neutral hypermultiplet that cannot mix with vector multiplets in the low energy theory. This suggests that $a$ should be identified with $l_P$ and thus, in string units, there should be an additional contribution depending logarithmically on the string coupling. This can also be understood either as a result of integration over the massive charged states which have non perturbative origin, implying a UV cutoff proportional to $(l_{\text{II}} g_{\text{II}})^{-1}$, or from heterotic–type II duality that we will discuss in Sections 3
and 4. Such a logarithmic dependence on the string coupling has also been observed in gravitational thresholds [19].

The logarithmic sensitivity on $g_{\text{II}}$ is very welcome because it allows in principle the possible dynamical determination of the hierarchy by minimising the effective potential. Note that the one-loop vacuum energy in non-supersymmetric type II vacua behaves as $\Lambda \sim l_{\text{II}}^{-4}$ and thus is different from a quadratically divergent contribution that would go as $(l_p l_{\text{str}})^{-2}$. This should be contrasted to the generic case of softly broken supersymmetry, as well as to TeV type I strings with large transverse dimensions, where the cancellation of this quadratic divergence implies a condition on the bulk energy density [9].

Above, we discussed the simplest case of type II compactifications with string scale at the TeV and all internal radii having the string size. In principle, one can allow some of the $K_3$ transverse directions to be large. From eq. (2), it follows that the string coupling $g_{\text{II}} = g_{6\text{IIA}}(V_{K_3}/l_{\text{II}}^4)^{1/2}$, with $V_{K_3}$ the volume of $K_3$, increases making gravity strong at distances $l_p(V_{K_3}/l_{\text{II}}^4)^{1/2}$ larger than the Planck length. In particular, it can become strong at the TeV when $g_{\text{II}}$ is of order unity. This corresponds to $V_{K_3}/l_{\text{II}}^4 \sim 10^{26}$. It follows that in the isotropic case there are 4 transverse dimensions at a fermi, while in the anisotropic case $V_{K_3} \sim R^\ell l_{\text{II}}^{4-\ell}$ the size of the transverse radii $R$ increases with $\ell$ and reaches a micron for $\ell = 2$. A more detailed analysis will be given in Section 5.

We now turn on type IIB. As in type IIA, non-abelian gauge symmetries arise at singularities of $K_3$ from D3-branes wrapping around vanishing 2-cycles times a 1-cycle of the base. Therefore, at the level of six dimensions, they correspond to tensionless strings [16]. The gauge and gravitational kinetic terms in the effective type IIB four-dimensional action can be obtained from eq. (2) by T-duality with respect, for instance, to the 6th direction:

$$R_6 \rightarrow \frac{l_{\text{II}}^2}{R_6} \quad g_{6\text{IIA}} \rightarrow g_{6\text{IIB}} = g_{6\text{IIA}} \frac{l_{\text{II}}}{R_6}.$$ (4)
One obtains

$$S_{\text{IIB}} = \int d^4 x \sqrt{-g} \left( \frac{1}{g_{6\text{IIB}}^2} \frac{R_5 R_6}{l_{II}^4} R + \frac{R_5}{R_6} F^2 \right),$$

which leads to the identifications:

$$\frac{1}{g_{YM}^2} = \frac{R_5}{R_6}, \quad g_{6\text{IIB}} = \frac{l_P}{l_{II}} \sqrt{\frac{R_5 R_6}{l_{II}^2}} = g_{YM} \frac{R l_P}{l_{II}^2}. \quad (6)$$

Keeping the Yang-Mills coupling of order unity, now implies that $R_5$ and $R_6$ are of the same order, $R_5 = R_6/g_{YM}^2 \equiv R$, while $g_{6\text{IIB}}$ is a free parameter. Obviously, in order to get a situation different from type IIA, $R$ should be larger than the string length $l_{II}^m$. This corresponds to a type IIA string with large $R_5$ and small $R_6$ so that $R_5 R_6 \sim l_{II}^2$.

Imposing the condition of weak coupling $g_{II} = g_{6\text{IIB}}(V_{K_3}/l_{II}^4)^{1/2} < 1$, we find

$$l_{II} \geq \sqrt{g_{YM} R l_P}, \quad (7)$$

when all $K_3$ radii have string scale size. As a result, the type IIB string scale can be at intermediate energies $10^{11}$ GeV when the compactification scale $R^{-1}$ is at the TeV. The string coupling is then of order unity and gravity becomes strong at the intermediate string scale. This brings back some worry on proton stability although in a much milder form than in TeV scale quantum gravity models. Of course, the string scale can be lowered by decreasing the string coupling or by introducing large transverse dimensions in the $K_3$ part. In the latter case, gravitational interactions become strong at lower energies.

This result provides the first instance of a weakly coupled string theory with large longitudinal dimensions seen by gauge interactions. In fact, as we will show in Section 4, this theory describes heterotic compactifications with a single large dimension. The existence of such dimension is motivated by supersymmetry breaking in the process of compactification [1]. Note that the physics above the compactification scale but below the type IIB string scale is described by an effective six-dimensional theory of a tensionless

\[\text{\footnote{Since } R \text{ corresponding to a longitudinal direction, one can always choose } R > l_{II} \text{ by T-duality.}}\]
This theory possesses a non-trivial infrared dynamics at a fixed point of the renormalisation group. It would be interesting to study the dynamics of such theories in detail from the viewpoint of the reduced four-dimensional gauge theory.

3 Heterotic–type I–type II triality

Here, we review briefly the basic ingredients of the dualities between the heterotic, type I and type II string theories, that we will use in our subsequent analysis. The reader with less string theoretical background may skip most of equations in this part with the exception of the duality relations (8) and (18).

As is now well known, these three string theories are related by non-perturbative dualities, which take the simplest form in the $N = 4$ supersymmetric case of $(E_8 \times E_8$ or $SO(32))$ heterotic theory compactified on $T^6$, type II on $K_3 \times T^2$ and type I on $T^6$. We will restrict our attention to this case, since it already exhibits the main features we want to discuss. Heterotic–type I duality relates the two ten-dimensional string theories with $SO(32)$ gauge group, upon identifying

$$l_I = g_{H}^{1/2} l_H, \quad g_I = \frac{1}{g_H},$$

(8)

where $l_I$ and $l_H$ denote the type I and heterotic string length. The heterotic–type I duality itself holds in lower dimensions as well, and does not affect the physical shape or size of the compactification manifold.

The $E_8 \times E_8$ heterotic theory can in turn be perturbatively related to the $SO(32)$ heterotic theory upon compactifying to nine dimensions on a circle, since a $SO(1,17)$ boost transforms the two even self-dual lattices into one another. More precisely, the two theories are related by T-duality after breaking the gauge symmetry to $SO(16) \times SO(16)$

A similar conclusion was obtained by studying the ultraviolet behavior of the effective gauge theory

[2].
on both sides by Wilson lines \[23\]‡

\[ R_{H'} = \frac{l_H^2}{R_H}, \quad g_{H'} = g_H \frac{l_H}{R_H}. \quad (9) \]

where the prime refers to the $E_8 \times E_8$ theory. On the type I side, T-duality maps the theory with 32 D9-branes to a theory with 32 lower D$p$-branes, referred to in this work as type I$'_p$; the momentum states along the D9-branes are mapped to winding states in the directions transverse to the D$p$-branes, and the action on the radii and coupling constant is the standard T-duality relation $R \rightarrow l_I^2/R$, $g_I \rightarrow g_I l_I/R$.

On the other hand, heterotic–type IIA duality only arises in 6 dimensions or lower, and identifies again two theories with inverse six-dimensional couplings $g_{6H} = g_H (l_H^4/V_4)^{1/2}$ and $g_{6IIA} = g_{II} (l_{II}^4/V_{K3})^{1/2}$, with $V_4$ and $V_{K3}$ the volumes of $T^4$ and $K_3$, respectively \[26, 12\]:

\[ l_{II} = g_{6IIA} l_{H}, \quad g_{6IIA} = \frac{1}{g_{6H}}. \quad (10) \]

The identification of the scalar fields other than the dilaton can be obtained by decomposing the moduli space as

\[
\frac{SO(4,20)}{SO(4) \times SO(20)} \supset \left[ \mathbb{R}^+ \times \frac{SO(3,19)}{SO(3) \times SO(19)} \right]. \quad (11)
\]

On the heterotic side, this involves choosing a preferred direction of radius $R_1$ in $T^4$, parametrising the $\mathbb{R}^+$ factor, whereas on the type IIA side, the two factors occur naturally as the overall volume and complex structure of $K_3$, respectively. The volume of $K_3$ in type IIA units is thus related to the radius $R_1$ in heterotic units as

\[
\left( \frac{R_1}{l_H} \right)^2 = \frac{V_{K3}}{l_{II}^4}. \quad (12)
\]

Relating the other moduli fields requires a more precise understanding of the geometry of $K_3$ \[24\], but in the following we will be able to obtain a partial identification from the M-theory point of view.

\[\text{‡}5\text{This is in contrast to the unbroken } SO(32) \text{ or } E_8 \times E_8 \text{ phase, where each theory is self-dual under T-duality.}\]
Indeed, the above dualities can be understood in the M-theory description\(^\text{\textdagger}\), which subsumes the heterotic and type II descriptions at strong coupling. The type IIA string theory then appears as M-theory compactified on a vanishingly small circle of radius \(R_s\) given by

\[
R_s = g_\text{H} l_\text{H}, \quad l_\text{M}^3 = g_\text{H} l_\text{H}^3
\]

where \(l_\text{M}\) denotes the eleven-dimensional Planck length \([12, 27]\); the \(E_8 \times E_8\) heterotic string is obtained upon compactifying on a segment \(I\) of length \(R_I\) given by analogous formulae:

\[
R_I = g_\text{H} l_\text{H}, \quad l_\text{M}^3 = g_\text{H} l_\text{H}^3.
\]

The two nine-branes at the end of the segment support the non-abelian gauge-fields, whereas gravity propagates in the bulk \([28]\). In this framework, a double T-duality symmetry \((R_1, R_2) \rightarrow (l_\text{H}^2/R_2, l_\text{H}^2/R_1)\) of type IIA acts as \([23, 25]\)

\[
T_{ijk} : \tilde{R}_i = \frac{l_\text{M}^3}{R_j R_k}, \quad \tilde{R}_j = \frac{l_\text{M}^3}{R_i R_k}, \quad \tilde{R}_k = \frac{l_\text{M}^3}{R_i R_j}, \quad \tilde{l}_\text{M}^3 = \frac{l_\text{H}^6}{R_i R_j R_k},
\]

where one of the radii \(i, j, k\) corresponds to the eleventh dimension; by eleven-dimensional general covariance, this symmetry still holds for any choice of the three radii. As for the heterotic T-duality \(R_1 \rightarrow l_\text{H}^2/R_1\), it translates into a symmetry

\[
T_{II} : \tilde{R}_i = \frac{l_\text{M}^3}{R_i R_i}, \quad \tilde{R}_i = \frac{R_1^{1/2} l_\text{M}^{3/2}}{R_i}, \quad \tilde{l}_\text{M}^3 = \frac{l_\text{H}^{9/2}}{R_1^{1/2} R_1},
\]

where \(R_i\) denotes the radius of any circular dimension and \(R_I\) the length of the (single) segment direction.

As depicted in Figure 1, we can now obtain the heterotic–type I–type II relationships by interpreting the compactification of M-theory on a manifold \(S_1(R_1) \times I(R_I) \times T^3(R_2, R_3, R_4)\) in various ways. (i) Considering \(I(R_I)\) as the eleventh dimension simply gives the \(E_8 \times E_8\) heterotic string on \(T^4(R_1, R_2, R_3, R_4)\) with string length and coupling given by eq. (14). (ii)\(^\text{\textdagger}\)See for instance \([25]\) for a review.
Figure 1: Heterotic–Type I–Type II triality from M-theory

Considering $S_1(R_1)$ as the eleventh dimension gives type IIA on $I \times T^3$, or more properly type $I'_8$ on $I \times T^3$: (iii) By T-duality $T_{II}$ along the segment $I$, the theory (i) translates into heterotic $SO(32)$, whereas the theory (ii) turns into type I: it is straightforward to check that these two are related by the duality relations (8) [28]. (iv) if we perform a $T_{234}$ duality on the torus $T^3$ before identifying $S_1(R_1)$ with the eleventh dimension, we obtain a type IIA theory compactified on a four-dimensional manifold $I(R_I) \times T^3(\tilde{R}_2, \tilde{R}_3, \tilde{R}_4)$, which by (i) is the same as the $E_8 \times E_8$ heterotic string on $T^4(R_1, R_2, R_3, R_4)$. It is easy to check that these two theories are related by the heterotic–type II duality relations (10,12). It is therefore tempting to identify

$$K_3 = I(R_I) \times T^3(\tilde{R}_2, \tilde{R}_3, \tilde{R}_4), \quad (17)$$
where the type II parameters are related to the heterotic ones as

\[ l_{\text{II}} = \frac{g_{\text{II}} l_{\text{H}}^3}{\sqrt{R_1 R_2 R_3 R_4}} , \quad g_{\text{II}} = \frac{\sqrt{R_1^3 R_2 R_3 R_4}}{g_{\text{H}} l_{\text{H}}^3} , \quad (18a) \]

\[ R_I = g_{\text{H}} l_{\text{H}} , \quad \tilde{R}_i = \frac{g_{\text{II}} l_{\text{H}}^3}{R_j R_k} , \quad i, j, k = 2, 3, 4 . \quad (18b) \]

In units of the respective string length, this is\(^7\)

\[ \left( \frac{R_I}{l_{\text{II}}} \right)^2 = \frac{R_1 R_2 R_3 R_4}{l_{\text{H}}^4} , \quad (19a) \]

\[ \left( \frac{\tilde{R}_2}{l_{\text{II}}} \right)^2 = \frac{R_1 R_2}{R_3 R_4} , \quad \left( \frac{\tilde{R}_3}{l_{\text{II}}} \right)^2 = \frac{R_1 R_3}{R_2 R_4} , \quad \left( \frac{\tilde{R}_4}{l_{\text{II}}} \right)^2 = \frac{R_1 R_4}{R_2 R_3} , \quad (19b) \]

where we recognize a triality transformation in the \([SO(4) \times SO(4)] \setminus SO(4, 4)\) subspace of the moduli space. Even though (17) is not a proper \(K_3\) surface (for one thing it is not simply connected), it still is a bona fide compactification manifold, albeit singular. Indeed, it has been argued that such a “squashed” shape arises in the decompactification limit of the heterotic torus \(T^4\) at the \(E_8 \times E_8\) enhanced symmetry point \([31]\). This representation of \(K_3\) will turn out to be very convenient in the discussion of large radius behaviour of heterotic and type II theories in the sequel.

4 Large dimensions in heterotic string

Here we consider the heterotic string compactified in four dimensions with a certain number \(n\) of large internal dimensions. Keeping the four-dimensional gauge coupling \(g_{\text{YM}}\) of order unity, the heterotic theory is strongly coupled with a ten-dimensional string coupling and four-dimensional Planck length

\[ g_{\text{H}} = g_{\text{YM}} \left( \frac{R}{l_{\text{H}}} \right)^{n/2} \gg 1 , \quad l_P = g_{\text{YM}} l_{\text{H}} \quad (20) \]

\(^7\) This mapping was independently obtained by Polchinski as referred to in \([30]\).
where $R$ is the common radius of the large dimensions, while the remaining $6 - n$ are assumed to be of the order of the string length $l_H$. For $n < 6$, the distinction between the $SO(32)$ and $E_8 \times E_8$ heterotic theories is irrelevant, since a T-duality (3) along a heterotic-size direction converts one into another; we will therefore omit this distinction until we discuss the $n = 6$ case, where such a dualisation is no longer innocuous.

In order to obtain a perturbative description of this theory, we consider first its type I dual obtained through the relations (8). The physical radii of the internal manifold are unaffected by this duality. In particular, there are still $6 - n$ dimensions of size $l_H$, and $n$ of size $R$. We therefore have the following type I string length and coupling

$$l_I = g_{YM}^{1/2} R^n l_H^{1/2}, \quad g_I = \frac{1}{g_{YM}^2} \left( \frac{l_H}{R} \right)^{\frac{n}{2}},$$

or more explicitly

\[
\text{Type I:} \begin{cases} 
  n = 1, 2, 3, 4 : & l_H < l_I < R \\
  n = 5, 6 : & l_H < R < l_I
\end{cases}
\]

In both cases, there are dimensions $(6 - n$ or 6 respectively) with size smaller than the type I string length, which should be T-dualised in order to trade light winding modes for Kaluza-Klein (KK) field-theory states. In so doing, we move to a type I' description where the gauge interactions are localised on D-branes (extended in $3 + n$ or 3 spatial directions, respectively). Using the standard ($\hat{R} = l_I^2/R$, $\hat{g}_I = g_I l_I/R$) T-duality relations, we obtain the dual radii

$$\hat{l}_H = g_{YM} R^n l_H^{\frac{1-n}{2}}, \quad \hat{R} = g_{YM} R^{\frac{n-2}{2}} l_H^{\frac{2-n}{2}}$$

and the hierarchies

\[
\text{Type I' :} \begin{cases} 
  n = 1, 2 : & g_V = g_{YM} \frac{R}{l_H} \left( \frac{R}{l_H} \right)^{\frac{n(4-n)}{4}}, \quad l_I < \hat{l}_H < R \\
  n = 3, 4 : & g_V = g_{YM} \frac{R}{l_H} \left( \frac{R}{l_H} \right)^{\frac{n(4-n)}{4}}, \quad l_I < R < \hat{l}_H \\
  n = 5, 6 : & g_V = g_{YM}^2, \quad l_I < \hat{R} < \hat{l}_H
\end{cases}
\]
where the T-dualised hatted radii correspond to transverse dimensions.

In the cases \( n = 1, 2, 3 \), type I' theory is also strongly coupled, as seen from eqs. (24), which is a consequence of the fact that the internal longitudinal directions of the D-branes (of size \( R \)) are larger than the type I' string length. In the cases \( n = 5, 6 \) however, the type I' theory does offer a perturbative description of the theory of interest, where the gauge interactions are confined on D3-branes with large transverse dimensions of size \( \hat{R} \) (\( n \) of them) and \( \hat{l}_H \) (6 – \( n \) of them). This is depicted in the following diagrams:

\[
\begin{align*}
\text{Het } n &= 5 \\
V_3, g_{\nu} &= g_{YM}^2 \\
l_H, R_6 &\quad 1 \\
R \quad l_1 \quad \hat{R}_{1,2,3,4,5} \quad \hat{R}_6
\end{align*}
\]

\[
\begin{align*}
\text{Het SO(32) } n &= 6 \\
V_3, g_{\nu} &= g_{YM}^2 \\
l_H &\quad 1 \\
R \quad g_{YM}^{1/2}R_{5/4} \quad g_{YM}R_{3/2} \quad g_{YM}R_{5/2}
\end{align*}
\]

In particular, the \( SO(32) \) heterotic string with \( n = 6 \) large dimensions, say at 10\(^8\) GeV, is dual to a type I' with string tension at the TeV and six transverse dimensions at 0.1 fermi. This is one of the examples that were treated recently in the context of TeV strings [9]. The type I threshold appears in the strongly coupled heterotic theory below the KK scale of 10\(^8\) GeV [32], which is then identified with the scale of the (superheavy) type I' winding states around the fermi-size transverse dimensions.

In the \( n = 4 \) case, two distinct type I' perturbative descriptions are possible, due to the proximity of the radius \( R \) of the four large dimensions with the type I string scale \( l_I = g_{YM}^{1/2}R \). For \( g_{YM} < 1 \), it is sufficient to T-dualise the two directions of size \( l_H \), resulting in a D7-brane type I' description with string coupling unity, which can be lowered by increasing for instance the size of the two small dimensions slightly above the heterotic length. For \( g_{YM} > 1 \) on the other hand, one should T-dualise also the remaining four directions, resulting in a D3-brane type I' description as above. In both cases, the type I' scale is close to the size of the four heterotic large dimensions, say at the TeV scale. This
provides another example of type I' TeV strings [9]. The gauge interactions are confined on D-branes transverse to two large dimensions of (sub)millimeter-size. The type I threshold now appears at the same order as the KK scale (at the TeV) [33], while the heterotic scale – which is also the KK scale of the remaining two dimensions – is identified with the mass of the type I' winding modes around the two millimeter-size dimensions. This model is of particular interest, because it offers a possibility to keep the apparent unification of gauge couplings close to the heterotic scale, due to the logarithmic sensitivity of the gauge theory on the brane with respect to the size of the two-dimensional transverse space [3, 3].

\[
\begin{align*}
\text{Het } n = 4 & \quad \downarrow \\
V', g_V = 1 & \quad 1 \quad g_{YM}^{1/2} R \quad R \quad g_{YM} R^2 \\
& \quad \quad l_H \quad l_1 \quad R_{1,2,3,4} \quad \tilde{R}_{5,6}
\end{align*}
\]

\[
\begin{align*}
\text{Het } n = 4 & \quad \downarrow \\
V', g_V = g_{YM}^2 > 1 & \quad 1 \quad R \quad g_{YM}^{1/2} R_{YM} \quad g_{YM} R^2 \quad g_{YM} R^2 \\
& \quad \quad l_H \quad l_1 \quad \tilde{R}_{1,2,3,4} \quad \tilde{R}_{5,6}
\end{align*}
\]

In order to obtain a perturbative description for the cases \( n = 1, 2, 3 \), we now consider the type IIA dual of the original heterotic theory. As described in Section 3, the heterotic–type IIA duality selects four preferred dimensions of radii \( R_{1,2,3,4} \) on the heterotic side, while at the \( E_8 \times E_8 \) enhanced symmetry point the compactification manifold for the type IIA string takes the simplified form

\[
K_3 \times T^2 = \left[ I(R_I) \times T^3(\tilde{R}_2, \tilde{R}_3, \tilde{R}_4) \right] \times T^2(R_5, R_6) .
\]

The remaining two-torus of radii \( R_5, R_6 \) is common to both descriptions, which also have the same four-dimensional gauge coupling and Planck mass

\[
\frac{1}{g_{YM}^2} = \frac{R_1 R_2 R_3 R_4 R_5 R_6}{g_{H\text{H}}^2 l_H^2} = \frac{R_5 R_6}{l_{II}^2} ,
\]

\[
\frac{1}{l_P^2} = \frac{R_1 R_2 R_3 R_4 R_5 R_6}{g_{H\text{H}}^2 l_H^2} = \frac{R_I \tilde{R}_2 \tilde{R}_3 \tilde{R}_4 R_5 R_6}{g_{H\text{H}}^2 l_{II}^2} .
\]
In terms of these quantities, the duality map (18) takes the form

\[ l_{\text{II}} = g_{\text{YM}} \sqrt{R_5 R_6} \, , \quad g_{\text{II}} = \frac{1}{g_{\text{YM}}} \frac{R_1}{\sqrt{R_5 R_6}} \]  

(31a)

\[ R_I = g_H l_{\text{H}} \, , \quad \tilde{R}_i = \frac{g_H l_{\text{H}}^3}{R_j R_k} \, , \quad i, j, k = 2, 3, 4 \]  

(31b)

As we mentioned in the previous Sections, the four \( K_3 \) directions corresponding to \( R_I \) and \( \tilde{R}_i \) are transverse to the 5-brane where gauge interactions are localised.

In order to obtain a weakly coupled type II description, we therefore need to carefully arrange the choice of the \( n \) large dimensions on the heterotic side. For instance, in the \( n = 1 \) case, choosing \( R_1 \) as the large radius results in a strongly coupled type II theory with \( g_{\text{II}} \sim R \) in units of the heterotic string length; choosing \( R_2 \) (or \( R_3, R_4 \)) as the large dimensions gives a type II dual with moderate coupling \( g_{\text{II}} \sim 1/g_{\text{YM}} \), but with radii \( \tilde{R}_3, \tilde{R}_4 \sim 1/\sqrt{R} \) much smaller than the string length; after T-dualisation along these directions, the theory becomes strongly coupled. The last option is to take \( R_5 \) (or \( R_6 \)) as the large radius, which yields a weakly coupled type II dual string with \( g_{\text{II}} \sim 1/g_{\text{YM}} \sqrt{R} \) and \( l_{\text{II}} \sim g_{\text{YM}} \sqrt{R} \); after T-dualizing the heterotic-size direction \( R_6 \), we obtain a weakly coupled type IIB description with hierarchy

\[
\begin{array}{cccc}
\text{Het } n = 1 & 1 & g_{\text{YM}} R^{1/2} & g_{\text{YM}}^2 R \ R \\
\uparrow & & & \\
\text{IIB } g_{\text{II}} = 1 & l_{\text{H}} & l_{\text{II}}, R_{1,2,3,4} & \tilde{R}_6, R_5 \\
\end{array}
\]  

(32)

where we denoted by \( \tilde{R}_6 \) the radius of the T-dual sixth dimension. This is one of the models discussed in Section 2 with two radii at the TeV and a string scale at intermediate energies \( 10^{11} \) GeV.

In the \( n = 2 \) case, the same reasoning leads to choosing the radii \( R_5, R_6 \sim R \) as the large heterotic dimensions, and gives a weakly coupled type IIA description

\[
\begin{array}{cccc}
\text{Het } n = 2 & 1 & g_{\text{YM}} R & R \\
\uparrow & & & \\
\text{IIA } g_{\text{II}} = \frac{1}{g_{\text{YM}} R} & l_{\text{H}} & l_{\text{II}}, R_{1,2,3,4} & R_{5,6} \\
\end{array}
\]  

(33)
without need of T-dualizing any direction. This is the other type II model discussed in Section 2, with string scale and all internal dimensions at the TeV, and with an infinitesimal string coupling $10^{-14}$ accounting for the largeness of the four-dimensional Planck mass.

In the cases $n = 3, 4, 5, 6$, we choose the directions of radii $R_1, R_5, R_6 \sim R$ as three of the large heterotic dimensions, and for $n > 3$ also switch on $n - 3$ large dimensions in the heterotic $T^3(R_2, R_3, R_4)$ torus. The type II dual has string length $l_{\Pi} = g_{YM} R$ and coupling $g_{\Pi} = 1/g_{YM}$, while the $K_3$ manifold has size (in heterotic units):

| $n$ | $R_I$  | $\tilde{R}_2$  | $\tilde{R}_3$  | $\tilde{R}_4$ |
|-----|------|------|------|------|
| 3   | $g_{YM} R^{3/2}$ | $g_{YM} R^{3/2}$ | $g_{YM} R^{3/2}$ | $g_{YM} R^{3/2}$ |
| 4   | $g_{YM} R^2$     | $g_{YM} R^2$    | $g_{YM} R$      | $g_{YM} R$      |
| 5   | $g_{YM} R^{5/2}$ | $g_{YM} R^{3/2}$ | $g_{YM} R^{3/2}$ | $g_{YM} R^{1/2}$ |
| 6   | $g_{YM} R^3$     | $g_{YM} R$      | $g_{YM} R$      | $g_{YM} R$      |

Except for the $n = 5$, where the existence of the small radius $\tilde{R}_4$ implies strong coupling after T-duality, all these cases correspond to a weakly coupled type II dual. In the $n = 3$ case, the type II dual provides a perturbative description of the heterotic theory that could not be reached on the type I side:

$$
\begin{array}{c}
\text{Het } n = 3 \\
\text{IIA } g_{\Pi} = \frac{1}{g_{YM}}
\end{array}
\begin{array}{cccc}
\downarrow & \uparrow & \downarrow & \uparrow \\
1 & 1 & 1 & 1 \\
l_H & l_{\Pi} & R_{5,6} & R_{1,2,3,4}
\end{array}

(34)

This is the type II model discussed in Section 2 with string scale and two longitudinal dimensions at the TeV, and an isotropic $K_3$ with 4 transverse directions at a fermi.

In the $n = 4$ case, the type II dual theory provides a perturbative description, alternative to the type I'. The type II dual string has the same scale hierarchy as the type I, up to factors of $g_{YM}$:
These two models should provide equivalent perturbative descriptions of the same theory.

In the \( n = 6 \) case, we now obtain a weakly coupled description of the \( E_8 \times E_8 \) heterotic string with \( n = 6 \) large radii as a type IIA string with string length \( l_{II} = g_{YM}R \):

\[
\begin{array}{cccccc}
\text{Het} & E_8 \times E_8, n = 6 & 1 & g_{YM}R & R & g_{YM}R^3 \\
\text{IIA} & g_{II} = \frac{1}{g_{YM}} & l_H & l_{II}, R_{2,3,4} & R_{5,6} & R_I
\end{array}
\]  

(36)

Due to the occurrence of gravitational KK states\(^{18} \) at the scale \( R_I \), the type II string tension as well as the heterotic compactification scale cannot be lower than \( 10^8 \) GeV, corresponding to the bound \( R_I \lesssim 1 \) mm. This situation should be contrasted with the case of the \( SO(32) \) heterotic string with \( n = 6 \) large radii, which admits a perturbative description \((26)\) as a type I string with length \( l_I = g_{YM}R^{3/2} \). The bound on \( R \) still applies, corresponding now to a type I string scale at a TeV, and six transverse dimensions at 0.1 fermi. Note that the difference between the type I and type II string scales does not lead to any inconsistency, since the two perturbative descriptions are not simultaneously possible.

## 5 Large dimensions in type II theories and their duals

Having discussed the large radius behaviour of the dual heterotic theory, we now reconsider the type IIA models we introduced in Section 2, and discuss their dual descriptions. We therefore consider type IIA theory, compactified on the simplified model \((29)\) of \( K_3 \times T^2 \), with a weak string coupling \( g_{II} \ll 1 \), two string-size directions \( R_5, R_6 \sim l_{II} \) and possibly \( \ell \)

\(^{18}\)Note that these excitations are not stable due to the lack of momentum conservation along the interval \( I(R_I) \).
large transverse directions of size \( R \gg l \) within \( K_3 \). This theory is then identified to a strongly coupled heterotic string compactified on \( T^6 \), with parameters simply obtained by inverting eq. (18):

\[
\begin{align*}
    l_H &= \frac{g_{\text{II}} l_{\text{II}}^3}{\sqrt{R_1 R_2 R_3 R_4}}, \quad g_H = \frac{\sqrt{R_1^3 R_2 R_3 R_4}}{g_{\text{II}} l_{\text{II}}^3}, \\
    R_1 &= g_{\text{II}} l_{\text{II}}, \quad R_i = \frac{g_{\text{II}} l_{\text{II}}^3}{R_j R_k}, \quad i,j,k = 2,3,4.
\end{align*}
\]  

(37a)

while the torus \( T^2(R_5, R_6) \), common to both sides, is still at the type II string scale.

In the \( \ell = 0 \) case, the dual heterotic string has a scale \( l_H = g_{\text{II}} \) in type II units, of the same order as the radii \( R_{1,2,3,4}; R_{5,6} \) on the other hand still have a type II string scale, and are much larger (at weak type II coupling) than the heterotic scale. This situation is therefore identical to the heterotic \( n = 2 \) case in (33), which did not admit a perturbative type I dual.

For \( \ell \geq 1 \), T-duality on the \( K_3 \) manifold allows us to choose one of the large directions as the interval of length \( R_I = R \gg l_{\text{II}} \). We therefore consider a regime where

\[
\begin{align*}
    R_I &= R, \quad R_2 R_3 R_4 = R^{\ell-1}, \quad R_5 = R_6 = l_{\text{II}}, \quad g_{\text{II}} \ll 1
\end{align*}
\]  

(38)

in units of the type IIA string length \( l_{\text{II}} \). The parameters for the dual heterotic string therefore scale as

\[
\begin{align*}
    l_H &= g_{\text{II}} R^{-\ell/2}, \quad g_H = \frac{R^{2+\ell}}{g_{\text{II}}}, \quad R_1 = g_{\text{II}}, \quad R_i = \frac{g_{\text{II}}}{R_j R_k}.
\end{align*}
\]  

(39)

A simple case by case study shows that the \( \ell = 1, 2, 4 \) cases are identical to the \( n = 6, 4, 3 \) heterotic cases, up to powers of \( g_{\text{II}} \) which we now consider of order 1. The \( \ell = 3 \) case on the other hand is new, since it involves, after T-duality along the direction \( R_4 \), three large directions of size \( R_{2,3,4} \sim l_H (R/l_{\text{II}})^{1/2} \), and three extra-large ones of size \( R_{1,5,6} \sim l_H (R/l_{\text{II}})^{3/2} \). It does not yield, however, any perturbative description on the type I side. Again we see that the \( n = 5 \) heterotic case does not appear, since it corresponds to a strongly coupled type II theory.
We now turn to the type IIB theory, again compactified on the model (29) of $K_3 \times T^2$ for simplicity. Since T-duality on one of the circles $R_{5,6}$ identifies the type IIA and IIB theories, it is sufficient to restrict our attention to the case where both circles are much larger than the type II string length, but still of comparable size in order to maintain a small gauge coupling $g_{YM}^2 = R_5/R_6$. The type IIB theory is then equivalent to a strongly coupled heterotic theory with parameters

$$l_H = \frac{g_{\text{IIB}}^3 l_{\text{II}}}{R_6 \sqrt{R_{1i} R_{2j} R_{3k}}}, \quad g_H = \frac{R_6 \sqrt{R_{1i} R_{2j} R_{3k}}}{g_{\text{IIB}}^2 l_{\text{II}}},$$

(40a)

$$R_1 = \frac{g_{\text{IIB}} l_{\text{II}}}{R_6}, \quad \tilde{R}_6 = \frac{l_{\text{II}}}{R_6}, \quad \tilde{R}_i = \frac{g_{\text{IIB}}^3 l_{\text{II}}}{R_{ij} R_{jk}}, \quad i, j, k = 2, 3, 4,$$

(40b)

where now the l.h.s. refers to type IIB variables. For $\ell = 0$, the dual heterotic theory has one large dimension of radius $R_5 = R$ and five heterotic-string–size dimensions, up to factors of $g_{\text{IIB}} \sim 1$, which corresponds to the situation in (32). For $\ell \geq 1$, we obtain again a heterotic theory with more than two scales, and heterotic–type I duality does not yield any valuable perturbative description.

6 Concluding remarks

In this paper, we studied new scenarios of TeV strings or large dimensions in weakly coupled type II theories and related them by duality to heterotic string compactifications with large dimensions. In particular, we described a type IIA theory with all compactification and string scales at the TeV, but with a tiny string coupling which explains the weakness of gravitational interactions. We also described a type IIB theory with two large non-transverse dimensions at the TeV and a fundamental string scale at $10^{11}$ GeV. The main features of our discussion are summarised in Figure 2.

As a result, the heterotic string with $n \leq 4$ large dimensions at the TeV has a weakly coupled description in terms of type II or type I theory, as indicated in the table. When
| $n$ | $R_H^{-1}$ | Dual | $l_{\text{Dual}}^{-1}$ | Radii | QG Scale |
|-----|------------|------|-----------------------|-------|-----------|
| 1   | TeV        | IIB  | $10^{11}$ GeV         | 2 at TeV$^{-1}$, 4 at $l_{\text{Dual}}$ | $10^{11}$ GeV |
| 2   | TeV        | IIA  | TeV                   | all at TeV$^{-1}$               | $10^{18}$ GeV |
| 3   | TeV        | IIA  | TeV                   | 2 at TeV$^{-1}$, 4 transv. at fm | TeV |
| 4   | TeV        | I or IIA | TeV                  | 4 at TeV$^{-1}$, 2 transv. at 0.1 mm | TeV |
| 5   | $> 10^6$ GeV | I    | TeV                   | 1 transv. at mm, 5 transv. at GeV$^{-1}$ | TeV |
| 6   | $> 10^8$ GeV | I    | TeV                   | 6 transv. at 0.1 fm             | TeV |
| 6'  | $> 10^8$ GeV | IIA  | $10^8$ GeV            | 1 transv. at mm, 5 at $l_{\text{Dual}}$ | $10^8$ GeV |

Figure 2: Weakly coupled dual descriptions of heterotic string with $n$ large dimensions or radius $R_H$. The two last columns list the size of the internal radii in the dual theory and the scale at which string interactions and quantum gravity become relevant.

If the number of large dimensions is $n = 5$ or 6, there is an upper bound for the compactification scale because the string threshold of the weakly coupled dual theory appears in lower energies \[^{32}\]. The entries in the last three rows correspond to a saturation of this bound. Moreover, the case $n = 5$ is generally forbidden since the dual type I theory has an anisotropic transverse space with one dimension very large compared to the others; this invalidates the decoupling of the gauge theory on the brane unless local tadpole cancellation is imposed \[^{3}\].

In particular, we showed that the first two simple type II examples above describe the heterotic string with one or two large TeV dimensions. In fact, these are the only two cases that have been previously considered seriously in the context of the heterotic theory before knowing its strong coupling behavior \[^{4}, \[^{15}\]. Our analysis here showed that many of the properties and predictions of the heterotic string for these two cases remain valid, despite its strong ten-dimensional coupling. More precisely: (i) the existence of KK excitations for all Standard Model gauge bosons in $N = 4$ supermultiplets, and their absence for quarks
and leptons; (ii) the absence of visible quantum gravity effects at the TeV scale, above
which there is a genuine six-dimensional gauge theory, regulated by the underlying type
IIA or IIB theory; (iii) the possible relation of the TeV dimensions with the mechanism
of supersymmetry breaking by the process of compactification. All soft breaking terms
can then be studied reliably in the effective field theory due to the extreme softness of the
breaking above the compactification scale \cite{8}; (iv) the possibility that the unification of low
energy gauge couplings remains at the experimentally inferred GUT scale, which is much
higher than the fundamental string scale of the weakly coupled type II theory.

Many questions and open problems remain of course to be done. Certainly, the possi-
bilities discussed here give new “viable” directions of how string theory may be possibly
connected with the description of our observed low energy world. We note however that
the strong coupling regime of the heterotic string is traded for a strong (singular) curvature
situation in the type II framework, which is only partially accounted for in the geometric
engineering field theory approach.

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