Behavior of the Hadron Potential at Large Distances and Properties of the Hadron Spin-flip Amplitude

E. Predazzi\textsuperscript{1}, O.V. Selyugin\textsuperscript{2}

\textsuperscript{1}Dartimento di Fisica Teorica - Universit\`a di Torino and Sezione INFN di Torino, Italy
\textsuperscript{2}BLTPh, JINR, Dubna, Russia

Abstract

The impact of the form of the hadron potential at large distances on the behaviour of the hadron spin-flip amplitude at small angles is examined. The $t$-dependence of the spin-flip amplitude of high energy hadron elastic scattering is analyzed under different assumptions on the hadron interaction. It is shown that the long tail of the non-Gaussian form of the hadron potential of the hadron interaction in the impact parameter representation leads to a large value of the slope of the spin-flip amplitude (without the kinematical factor $\sqrt{|t|}$) as compared with the slope of the spin-non-flip amplitude. This effect can explain the form of the differential cross section and the analyzing power at small transfer momenta. The methods for the definition of the spin-dependent part of the hadron scattering amplitude are presented. A possibility to investigate the structure of the hadron spin-flip amplitude from the accurate measure of the differential cross section and the spin correlation parameters is shown.

1 Introduction

Spin phenomena provide a powerful tool for analyzing the properties of hadronic interaction. The spin structure of the pomeron is an important question concerning the diffractive scattering of polarized particles. There were many observations of spin effects at high energies and at fixed momentum transfers. Several attempts to extract the spin-flip amplitude from the experimental data show that the ratio of spin-flip to spin-nonflip
amplitudes can be non-negligible and may be only slightly dependent of energy \( n \). Thus, the diffractive polarized experiments at HERA and RHIC allow one to study spin properties of quark-Pomeron and proton-Pomeron vertices and to search for a possible odderon contribution.

This provides an important test of the spin properties of QCD at large distances. In all of these cases, pomeron exchange is expected to contribute the observed spin effects at some level \( n \).

In the framework of perturbative QCD, it was shown that the analyzing power of hadron-hadron scattering can be large and proportional to the hadron mass \( n \). Hence, one could expect a large analyzing power for moderate \( p_t^2 \) where the spin-flip amplitudes are presumably relevant for diffractive processes. The case, when large distance contributions are considered, leads to a more complicated spin structure of the pomeron coupling. For example, the spin-flip amplitude has been estimated in the QCD Born approximation by using the nonrelativistic quark model for the nucleon wave function \( n \) in the case where the nucleon contains a dynamically enhanced component with a compact diquark. The spin-flip part of the scattering amplitude can be determined by the hadron wave function for the pomeron-hadron couplings or by the gluon-loop corrections for the quark-pomeron coupling \( n \). As a result, spin asymmetries appear which have a weak energy dependence as \( s \rightarrow \infty \). Additional spin-flip contributions to the quark-pomeron vertex may also have their origins in instantons (see e.g. \( n \)).

The procedure of how to separate the various parts of the scattering amplitude is the model-dependent. These questions include also the determination of the odderon contribution to different exclusive reactions and to pseudoscalar meson production, the study of the structure of high energy elastic hadron-hadron scattering amplitude at small angles and the problems related to the extraction of \( \sigma_{tot} \) from the experimental data, the study of the behavior of the parameter \( \rho \), - the ratio the real to imaginary part of scattering amplitude \( n \) :  

Including in the analysis the experimental data of spin correlation parameters does not simplify the task. In the general case, the form of the analyzing power, \( A_N \), for
example, and the position of the maximum of $A_N$ depends on the parameters of the elastic scattering amplitude $\sigma_{tot}$, $\rho(s, t)$, the Coulomb-nucleon interference phase $\varphi_{cn}(s, t)$ and the elastic slope $B(s, t)$. For the definition of new effects at small angles and especially in the region of the diffraction minimum one must know the effects of the Coulomb-hadron interference with sufficiently high accuracy. The Coulomb-hadron phase was calculated in the entire diffraction domain taking into account the form factors of the nucleons \cite{11}. Some polarization effects connected with the Coulomb hadron interference, including some possible odderon contribution, were also calculated \cite{12}.

The dependence of the hadron spin-flip amplitude on the momentum transfer at small angles is tightly connected with the basic structure of the hadrons at large distances. We show that the slope of the so-called “reduced” hadron spin-flip amplitude (the hadron spin-flip amplitude without the kinematic factor $\sqrt{|t|}$) can be larger than the slope of the hadron spin-non-flip amplitude as was observed long ago \cite{13, 14}. Its behavior can be defined by small effects in the differential hadron cross section and the real part of hadron non-flip amplitude.

2 Definition of the physical quantities.

The total helicity amplitudes can be written as $\Phi_i(s, t) = \phi_i^h(s, t) + \phi_i^{em}(t) \exp[i\alpha_{em}\varphi_{cn}(s, t)]$, where $\phi_i^h(s, t)$ is the pure strong interaction of hadrons, $\phi_i^{em}(t)$ is the electromagnetic interaction of hadrons, $\alpha_{em} = 1/137$ is the electromagnetic constant, and $\varphi_{cn}(s, t)$ is the electromagnetic-hadron interference phase factor. So, to determine the hadron spin-flip amplitude at small angles one should take into account all electromagnetic and interference electromagnetic-hadrons effects. A recent reanalysis of the data is found in \cite{13}.

The differential cross section and the spin parameters $A_N$ and $A_{NN}$ are defined as

$$\frac{d\sigma}{dt} = \frac{2\pi}{s^2}(|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 + 4|\phi_5|^2), \quad (1)$$

$$A_N \frac{d\sigma}{dt} = -\frac{4\pi}{s^2} \text{Im}[(\phi_1 + \phi_2 + \phi_3 - \phi_4)\phi_5^*)], \quad (2)$$
and
\[ A_{NN} \frac{d\sigma}{dt} = \frac{4\pi}{s^2} \left[ \text{Re}(\phi_1^* \phi_2 - \phi_3^* \phi_4) + 2|\phi_5|^2 \right], \] (3)
in terms of the usual helicity amplitudes.

In this paper we define the hadronic spin-nonflip amplitude as
\[ F_{nf}^h(s, t) = (\phi_1^h(s, t) + \phi_3^h(s, t))/(2s) \]
and
\[ F_{nf}^c(s, t) = (\phi_1^em(s, t) + \phi_3^em(s, t))/(2s) \]
Taken into account the Coulomb-nuclear phase \( \varphi_{cn} \) we define \( \text{Im} F_{nf}^c = \alpha_{em} \varphi_{cn} F_{nf}^c \).

The “reduced” spin-flip amplitudes are denoted as
\[ \tilde{F}_{sf}^h(s, t) = \tilde{\phi}_5^h(s, t)/(s\sqrt{|t|}) \]
and
\[ \tilde{F}_{sf}^c(s, t) = \tilde{\phi}_5^em(s, t)/(s\sqrt{|t|}). \]

### 3 The slope of the hadron amplitudes

As it is not possible to calculate exactly the hadronic amplitudes from first principles, we have to resort to some assumptions for what concerns their form (\( s \) and \( t \) dependence). Let us define the slope of the scattering amplitude as the derivative of the logarithm of the amplitudes with respect to \( t \). For an exponential form of the amplitudes this coincides with the usual slope of the differential cross sections divided by 2.

If we define the forms of the separate hadron scattering amplitude as:
\[ \text{Im} F_{nf}(s, t) \sim \exp(B_1^+ t), \quad \text{Re} F_{nf}(s, t) \sim \exp(B_2^+ t), \] (4)
\[ \text{Im} \tilde{F}_{sf}(s, t) = \frac{1}{\sqrt{|t|}} \text{Im} \phi_5^h(s, t) \sim \exp(B_1^- t), \quad \text{Re} \tilde{F}_{sf}(s, t) \sim \exp(B_2^- t), \] (5)
then, at small \( t \) (\( \sim 0 \div 0.1 \text{ GeV}^2 \)), practically all semiphenomenological analyses assume:
\[ B_1^+ \approx B_2^+ \approx B_1^- \approx B_2^- . \]

Actually, if we take the eikonal representation for the scattering amplitude
\[ \phi^h(s, t) = \frac{1}{2it\pi} \int d^2 p e^{i \cdot \tilde{\Delta}} \left[ e^{i\sigma_0 + i[\tilde{n} \times \tilde{\sigma}]_1} - 1 \right], \] (6)
and use
\[ J_0(x) = \frac{1}{2\pi} \int_0^{2\pi} d\vartheta e^{ix\cos \vartheta}, \quad J_1(x) = -\frac{1}{2\pi} \int_0^{2\pi} d\vartheta e^{ix\cos \vartheta} \sin \vartheta, \]
we obtain
\[ \phi^h_1(s, t) = -ip \int_0^\infty \rho \, d\rho \, J_0(\rho \Delta) (e^{\chi_0(s, \rho)} - 1), \quad (7) \]
\[ \phi^h_5(s, t) = -ip \int_0^\infty \rho \, d\rho \, J_1(\rho \Delta) \chi_1(s, \rho) \, e^{\chi_0(s, \rho)}. \quad (8) \]
where
\[ \chi_0(s, \rho) = \frac{1}{2ip} \int_{-\infty}^{\infty} dz \, V_0(\vec{\rho}, z); \quad (9) \]
\[ \chi_1(s, \rho) = \frac{\rho}{2} \int_{-\infty}^{\infty} dz \, V_1(\vec{\rho}, z). \quad (10) \]

If the potentials \( V_0 \) and \( V_1 \) are assumed to have a Gaussian form
\[ V_{0,1}(\rho, z) \sim \int_{-\infty}^{\infty} e^{B \, r^2} \, dz = \frac{\sqrt{\pi}}{\sqrt{B}} e^{-B \, \rho^2} \]
in the first Born approximation \( \phi^h_1 \) and \( \phi^h_5 \) will have the same form
\[ \phi^h_1(s, t) \sim \int_0^\infty \rho \, d\rho \, J_0(\rho \Delta) e^{-B \, \rho^2} = e^{-B \Delta^2}, \quad (11) \]
\[ \phi^h_5(s, t) \sim \int_0^\infty \rho^2 \, d\rho \, J_1(\rho \Delta) \, e^{\chi_0(s, \rho)} e^{-B \, \rho^2} = q \, B \, e^{-B \Delta^2}. \quad (12) \]

In this special case, therefore, the slopes of the spin-flip and “residual” spin-non-flip amplitudes are indeed the same.

At best, however, a Gaussian form of the potential is adequate to represent the central part of the hadronic interaction. This form cuts off the Bessel function and the contributions at large distances. As a consequence we can confine ourselves to the two first terms of the small-\( x \) expansion of the Bessel functions
\[ J_0(x) \simeq 1 - (x/2)^2; \quad and \quad 2 \, J_1/x = (1 - 0.5 \, (x/2)^2), \quad (13) \]
and we recover the previous result, i.e., the integral representation for spin-flip and spin-non flip amplitudes will be the same as in [11,12]. If, however, the potential (or the corresponding eikonal) has a long tail (exponential or power) in the impact parameter, the approximation [13] for the Bessel functions does not lead to a correct results and one has to perform the full integration.

The first observation that the slopes don’t coincide was made in [13]. It was found from the analysis of the $\pi^{\pm}p \rightarrow \pi^{\pm}p$ and $pp \rightarrow pp$ reactions at $p_L = 20 \div 30$ GeV/c that the slope of the “residual” spin-flip amplitude is about twice as large as the slope of the spin-non flip amplitude. This conclusion can also be obtained from the phenomenological analysis carried out in [14] for spin correlation parameters of the elastic proton-proton scattering at $p_L = 6$ GeV/c.

The model-dependent analysis based on all the existing experimental data of the spin-correlation parameters above $p_L \geq 6$ GeV allows us to determine the structure of the hadron spin-flip amplitude at high energies and to predict its behavior at super-high energies [16]. This analysis shows that the ratios $Re \phi^h_5(s,t)/(\sqrt{|t|} Re \phi^h_1(s,t))$ and $Im \phi^h_5(s,t)/(\sqrt{|t|} Im \phi^h_1(s,t))$ depend on $s$ and $t$. At small momentum transfers, it was found that the slope of the “residual” spin-flip amplitudes is approximately twice the slope of the spin-non flip amplitude.

Let us see what we obtain in the case of an exponential tail for the potentials. If we take

$$\chi_i(b,s) \sim H e^{-a \rho},$$

and use the standard integral representation

$$\int_0^\infty x^{\alpha-1} e^{-p x} J\nu(cx) \, dx = I^\alpha\nu,$$

with

$$I^{\nu+2} = 2p (2c)^\nu \Gamma(\nu + 3/2)1/[\sqrt{\pi}(p^2 + c^2)^{3/2}],$$

we obtain

$$F_{nf}(s,t) = \int \rho \, d\rho \, e^{-a \rho} J_0(\rho q)$$
\[
\frac{a}{(a^2 + q^2)^{3/2}} \approx \frac{1}{a\sqrt{a^2 + q^2}} e^{-Bq^2}
\]
with \(B = 1/a^2\), where we have used the
\[
\frac{1}{1+x} \sim (1-x) \sim \exp(-x).
\]
For the “residual” spin-flip amplitude, on the other hand, we obtain
\[
\sqrt{|t|} \tilde{F}_{sf}(s, t) = \int \rho^2 \, d\rho \, e^{-a\rho} J_1(\rho q)
= \frac{3 a q}{(a^2 + q^2)^{5/2}} \approx \frac{3 aq B^2}{\sqrt{a^2 + q^2}} e^{-2Bq^2}
\]
In this case, therefore, the slope of the “residual” spin-flip amplitude exceeds the slope of the spin-non-flip amplitudes by a factor of two.

If, further, we take the first Born term of an inverse power form factor
\[
\chi(s, \rho) = \int q \, dq \, J_0(\rho q) \frac{\Lambda^{2n}}{(\Lambda^2 + q^2)^n} = \frac{\Lambda^{2n}}{48}(\Lambda\rho)^{n-1}K_{n-1}(\Lambda\rho),
\]
the spin-flip amplitude is given by
\[
\sqrt{|t|} \tilde{F}_{sf}(s, t) = \frac{\Lambda^2}{48} \int \rho^2 \, d\rho \, J_1(\rho q)K_{n-1}(\Lambda\rho)
= \frac{8 q}{\Lambda^2} \frac{1}{(1 + q^2/\Lambda^2)^{n+1}}
\]
Hence, a long tail hadron potentials implies a significant difference of the slopes of the “residual” spin-flip and of the spin-nonflip amplitudes. Note also that the procedure of eikonalization will lead to a further increase of the difference of these two slopes.

4 The determination of the structure of the hadron spin-flip amplitude

How one can find the value of the slope of the spin-flip amplitude? First, note that if the “reduced” spin-flip amplitude is not small, the impact of a large \(B\) will reflect in the behavior of the differential cross section at small angles [17]. Of course, this method gives
only the absolute value of the coefficient of the spin-flip amplitude. The imaginary and real parts of the spin-flip amplitude can be found only from the measurements of the spin correlation coefficient.

Now let us examine the form of the analyzing power $A_N$ at small transfer momenta with different values of the slope of the hadron spin-flip amplitude. To this aim, we take the spin nonflip amplitude in the standard exponential form with definite parameters: slope $B^+$, $\sigma_{tot}$ and $\rho^+$. For the “residual” spin-flip amplitude, on the other hand, we consider two possibilities: equal slopes $B^- = B^+$ and $B^- = 2B^+$. For example we take at $\sqrt{s} = 50 \text{ GeV}$

$$\sigma_{tot} = 42 \text{ mb}; \quad \rho = 0.075; \quad B^+ = \frac{13}{2}.$$

The results of these two different calculations are shown in Fig.1. It is clear that around the maximum of the Coulomb-hadron interference, the difference between the two variants is very small. But when $|t| > 0.01 \text{ GeV}^2$, this difference grows. So, if we try to find the contribution of the pomeron spin-flip, we should take into account this effect.

![Fig. 1](image)

The calculation of $A_N$ at $\sqrt{s} = 50 \text{ GeV}$ (the solid line is with the slope $B_1^+$ of $F_{sf}$ equal to the slope $B_1^+$ of $F_{nf}$); the dashed line is with the $B_1^- = 2B_1^+$. 

As the value of $A_N$ depends on the determination of the beam polarization, let us calculate the derivative of $A_N$ with respect to $t$, for example, at $\sqrt{s} = 500 \text{ GeV}$. In this
case we take

\[ \sigma_{tot} = 62 \text{ mb}; \quad \rho = 0.15; \quad B^+ = 15.5/2. \]

The results of these two calculations are shown in Fig.2.

Fig. 2. The calculation of \(-\delta A_N/\delta t\) at \(\sqrt{s} = 500 \text{ GeV}\): the solid line refers to the case when the slope \(B^-\) of \(\tilde{F}_{sf}\) is equal to the slope \(B^+\) of \(F_{nf}\); the dashed line when \(B^- = 2 B^+\).

If we know the parameters of the hadron spin non-flip amplitude, the measurement of the analyzing power at small transfer momenta helps us to find the structure of the hadron spin-flip amplitude. Let us examine the behavior of the analyzing power (12), which can be rewritten as

\[
A_N \frac{d\sigma}{dt} = 2 \left( \text{Im} F_{nf} \text{Re} F_{sf} - \text{Re} F_{nf} \text{Im} F_{sf} \right) = 2 \left[ \text{Im} F_{nf}^h \text{Re} F_{sf}^c + \text{Im} F_{nf}^c \text{Re} F_{sf}^h - \text{Re} F_{nf}^h \text{Im} F_{sf}^c - \text{Re} F_{nf}^c \text{Im} F_{sf}^h \right]
\]

in two specific regions.

We begin from the point \(t_{im}\) where the absolute value of the real part of the Coulomb amplitude is equal to the imaginary part of the hadron spin-non-flip amplitude \(|\text{Re} F_{nf}^c| = \ldots\).
\[ |Im F_{nf}^h| \]. Let us denote by \( P' \) the part of the analyzing power \([17]\) which can be calculated, if we know \( \sigma_{tot}, B, \rho \) for the non-flip amplitude, and by \( \Delta P \) the part of \( A_N \) which depends on the hadron spin-flip amplitude.

\[
\Delta P = A_N^{\text{exper.}} - P'.
\]  

Here \( A_N^{\text{exper.}} \) is the measured value of the analyzing power. Let us factor out the term \( |F_{nf}^c| \) from the numerator, the term \( |F_{nf}^c|^2 \) from the denominator and multiply \( \Delta P \) by \( |F_{nf}^c| \):

\[
A'_N = (\Delta P)|F_{nf}^c| \approx 2 \frac{-Im F_{sf}^h + Re F_{sf}^h(Im F_{nf}^h/|F_{nf}^c|)}{1 + (Im F_{nf}^h/|F_{nf}^c|)^2}.
\]  

At the point \( t_{re} \) where \( Im F_{nf}^h = -Re F_{nf}^c \) for proton-proton scattering at high energy we obtain

\[
A'_N = -Re F_{sf}^h - Im F_{sf}^h.
\]  

And, as \( t \to 0 \), due to the growth of \( |F_{nf}^c| \) we have the result

\[
A'_N \to -2Im F_{sf}^h.
\]  

There is another specific point of the differential cross sections and of \( A_N \) on the axis of the momentum transfer, - \( t_{re} \), where the absolute value of the real part of the Coulomb amplitude equals the absolute value of the real part of the hadron spin-non flip amplitude. This point \( t_{re} \) can be found from the measurement of the differential cross sections \([18]\).

At high energies and small angles the analyzing power \([17]\) can rewritten in form

\[
-A_N \frac{d\sigma}{dt}/2 = Im F_{nf}^h(Re F_{sf}^c + Re F_{sf}^h) + Im F_{nf}^c(Re F_{sf}^c + Re F_{sf}^h)
\]

\[
-Im F_{sf}^c(Re F_{nf}^c + Re F_{nf}^h) - Im F_{sf}^h(Re F_{nf}^c + Re F_{nf}^h).
\]  

We obtain for proton-proton scattering at high energies at the point \( t_{re} \) where \( Re F_{nf}^c = -Re F_{nf}^c \)

\[
Re F_{sf}^h(s, t) = \frac{-1}{2(Im F_{nf}^h(s, t) + Im F_{nf}^c(t))} A_N(s, t) \frac{d\sigma}{dt} - Re F_{sf}^c(t).
\]
The form of $Re(F^{sf})$: solid and long-dashed lines are calculations by (14); short-dashed and dottes lines are model amplitudes with the slopes $B_1^- = B_1^+$ and $B_1^- = 2 B_1^+$. We can again take the hadron spin-nonflip and spin-flip amplitudes with definite parameters and calculate the magnitude of $A_N$ by the usual complete form (17) while the real part of the hadron spin-flip amplitude is given by (22). Our calculation by this formula and the input real part of the spin-flip amplitude are shown in fig. 3. At the point $t_{re}$ both curve coincides. So if we obtain from the accurate measurement of the differential cross sections the value of $t_{re}$, we can find from $A_N$ the value of the real part of the hadron spin-flip amplitude at the same point of momentum transfer.

5 Conclusion

By accurate measurements of the analyzing power in the Coulomb-hadron interference region we can find the structure of the hadron spin-flip amplitude, and this gives us further information about the behavior of the hadron interaction potential at large distances. Such contribution can be taken into account in the peripheral dynamic model [19]. In fact, the model takes into account the contribution of the hadron interaction at large
distances, and the calculated hadron spin flip amplitude leads to the ratio of the slopes of the “residual” spin-flip and spin-nonflip amplitudes at small momenta transfer. The model gives also the large spin effects in the diffraction dip domain [20]. We should note that all our consideration are based on the usual supposition that the imaginary part of the high energy scattering amplitude has an exponential behaviour. The other possibility, that the slope changes slightly with $t \to 0$, requires a more refined discussion that will be the subject of a subsequent paper.

Acknowledgements. One of us (OVS) would like to thank the Department of Theoretical Physics of the University of Torino for its hospitality and the INFN for its financial support.

References

[1] N. Akchurin, N. H. Buttimore and A. Penzo, Phys. Rev. D \textbf{51}, 3944 (1995).

[2] O. V. Selyugin, Phys. Lett. \textbf{B333}, 245 (1993).

[3] A.F. Martini and E. Predazzi, [hep-ph/0104061].

[4] A. V. Efremov and O. V. Teryaev, Phys. Lett. \textbf{B150}, 383 (1985).

[5] B. Kopeliovich, M. Zaharov, Phys.Lett. \textbf{B 226} 156 (1989).

[6] S. V. Goloskokov, Phys. Lett. \textbf{B315}, 459 (1993).

[7] M. Anselmino and S. Forte, Phys. Rev. Lett. \textbf{71}, 223 (1993).

[8] A. E. Dorokhov, N. I. Kochelev and Yu. A. Zubov, Int. Jour. Mod. Phys. \textbf{A8}, 603 (1993).

[9] P.Gauron, B.Nicolescu, O.V.Selyugin, Phys.Lett \textbf{B390} (1997) 405.

[10] P.Gauron, B.Nicolescu, O.V.Selyugin, Phys.Lett. \textbf{397} (1997) 305.
[11] O.V. Selyugin, Phys. Rev. D 60 (1999) 074028.

[12] O.V. Selyugin, in Proc. “New Trends in High Energy Physics”, Crimea, 2000, ed. P. Bogolyubov, L. Jenkovszky, Kiev, 2000.

[13] E. Predazzi, G. Soliani, Nuovo Cim. A 2 (1967) 427; K. Hinotani, H.A. Neal, E. Predazzi and G. Walters, Nuovo Cim., A 52 (1979) 363.

[14] M. Sawamoto, S. Wakaizumi, Progr. of Theor. Physics, 62 (1979) 1293.

[15] V. Petrov, E. Predazzi, A. Prokudin, in print (2001).

[16] O.V. Selyugin, Phys. of Atomic Nuclei 62 (1999) 333.

[17] O.V. Selyugin, Mod. Phys. Lett., A 9 (1994) 1207.

[18] O.V. Selyugin, Nucl. Phys. B (Proc. Suppl.) 99A (2001) 60.

[19] Goloskokov S.V., Kuleshov S.P., Selyugin O.V., Z. Phys. C 50, 455 (1991).

[20] N. Akchurin, S.V. Goloskokov, O.V. Selyugin, Int. J. of Mod. Phys. A 14 (1999) 253.