Transport properties of proximitized double quantum dots

G. Görski1,∗ and K. Kucab1,†

1Institute of Physics, College of Natural Sciences, University of Rzeszów, ul. Pigonia 1, PL-35-310 Rzeszów, Poland

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We study the sub-gap spectrum and the transport properties of a double quantum dot coupled to metallic and superconducting leads. The coupling of both quantum dots to the superconducting lead induces a non-local pairing in both quantum dots by the Andreev reflection processes. Additionally, we obtain two channels of Cooper pair tunneling into a superconducting lead. In such a system, the direct tunneling process (by one of two dots) or the crossed tunneling process (by both quantum dots at the same time) is possible. We consider the dependence of the Andreev transmittance on an inter-dot tunneling amplitude and the coupling between a quantum dot and the superconducting lead. We also consider the occurrence of interferometric Fano-type line shapes in the linear Andreev conductance spectra.

Keywords: double quantum dots, Andreev scattering, superconducting proximity effect, Fano effect

I. INTRODUCTION

The rapid development of electronics results in research of the transport properties of different types of heterostructures consisting of nano-objects such as the quantum dots or nanowires. One of the directions of interest is the system consisting of double quantum dots (DQD) placed between superconducting, magnetic or metallic leads [1–30].

For a system with a DQD coupled to two metallic or ferromagnetic electrodes [1–3, 5, 6, 10, 14], we observe the coexistence of Kondo [31] and Fano [32] effects. The connection of one quantum dot (QD1) and metallic electrodes leads to the widening of a dot level. When the second dot (QD2) is side coupled to the first one, there is a possibility to obtain the Fano-like asymmetric line shapes in the linear conductance [3–7, 14] which are obtained as a result of the interference between discrete QD2 level with a broad band of QD1. Additionally, for interacting dots, one obtains a two-stage Kondo effect [2, 3, 14, 29, 30]. The Fano destructive interference partially suppresses the Kondo resonance.

For the case of one quantum dot attached to one superconducting (SC) and one normal metallic (N) contact (N-QD-SC system), the propagation of a Cooper pair into SC lead and a hole reflection into a metallic lead (Andreev reflection process) occurs in the system [33, 34]. The connection of second QD into the N-QD-SC system causes the competition between the Andreev and Fano effect [1, 8, 15, 20, 28]. Other options of DQD with SC and N leads connection are also considered, e.g. connection of the first dot to two metallic leads and the second dot to the SC lead [11, 12, 19, 23, 25]. In such systems, there are possible both the normal electron transfers (when the single electron transfers between both normal metallic electrodes) and direct (DAR) and crossed (CAR) Andreev reflections (when electrons of the Cooper pair tunnel into SC lead and the holes tunnel to the same (DAR) or the second (CAR) metallic lead). The hybrid DQD structure can be used as a Cooper pair splitter if each QD is connected with separate metallic leads [20, 21, 24].

FIG. 1. Schematic representation of the double quantum dot system. The metallic lead is coupled to the first quantum dot (QD1), the superconductor substrate is attached to both quantum dots (QD1 and QD2).

In this paper we consider a system consisted of two quantum dots embedded in a superconducting substrate (see Fig. 1). We assume that a metallic lead is connected with one of these dots. Such a system can be realized experimentally by the use of scanning tunneling microscopy (STM) measurements of metallic atoms (e.g. Fe atoms) embedded in the superconducting substrate (e.g. Pb substrate) by the use of metallic tip. The STM-based single-atom manipulations technique is currently widely used for the detection of Majorana bound states in metallic chains [35–39]. This method allows for precise positioning of atoms on the substrate and for the local determination of spectral and transport properties of individual atoms [39]. In the system considered by...
us, the neighborhood of SC substrate with QDs, by the proximity effect, generates the Andreev states both on QD₁ and on QD₂. The coupling of both QDs with SC lead causes that the Cooper pair can tunnel to the SC lead via one of two dots (direct tunneling) or via both dots at the same time (crossed tunneling). Our aim is to analyze the spectral and transport properties of the quantum dot QD₁ depending on its coupling with proximitized QD₂. These results are important in the context of the distinction between the coupling of QD with trivial Andreev bound states and topological Majorana bound states [40].

This work is structured as follows. In Sec. II we introduce the microscopic model which describes the system considered by us. In this section, we introduce the relations describing the transport properties of the system. In Sec. III we present the numerical results for the local characteristics of the system. We assume that the QDs are connected to the same SC leads, so the difference of the superconducting phases does not occur and the Josephson current is not observed. The total Hamiltonian of our setup has the following form:

\[
H = \sum_{i\sigma} \varepsilon_i d_i^\dagger d_i + \sum_{\sigma} t_{12}(d_{1\sigma}^\dagger d_{2\sigma} + h.c.) + \sum_{k\sigma\beta} \xi_{k\beta} c_{k\sigma\beta}^\dagger c_{-k\sigma\beta} - \sum_k (\Delta_c^t_{k1S} c_{k1S}^\dagger + h.c.) + \sum_{k\sigma} (V_{kN} d_{1\sigma}^\dagger c_{k\sigma N} + h.c.) + \sum_{k\sigma i\sigma} (V_{kSi} d_{i\sigma}^\dagger c_{k\sigma i} + h.c.),
\]

where \(d_{i\sigma}^\dagger(d_{i\sigma})\) are the creation (annihilation) operators of an electron with spin \(\sigma\) at QD₁ \((i = 1, 2)\), \(\varepsilon_i\) is the energy level of QD₁, \(t_{12}\) is the inter-dot tunneling amplitude, \(c_{k\sigma\beta}^\dagger(c_{k\sigma\beta})\) denote the creation (annihilation) operators of an electron with momentum \(k\) and spin \(\sigma\) in the metallic tip \((\beta = N)\) or in the superconducting substrate \((\beta = S)\), \(\xi_{k\beta} = \varepsilon_{k\beta} - \mu_\beta\) is an energy dispersion of the lead \(\beta\) measured with respect to the electrochemical potentials \(\mu_\beta\). We assume that \(\mu_\beta = 0\) and \(\mu_N = eV\). \(V_{kN}\) is the tunneling amplitude between the QD₁ and the metallic tip, and \(V_{kSi}\) is the tunneling amplitude between the \(i\)-dot and superconducting substrate. \(\Delta\) is the superconducting energy gap.

The coupling between QDs and an SC substrate leads to the Andreev reflection processes [33], where taking an electron from a metallic lead causes the injection of a Cooper pair into the SC lead and the reflection of a hole into a metallic lead. In the considered system, for \(V_{kS2} \neq 0\), the direct tunneling is possible when the injection of the Cooper pair occurs from one of two QDs, or crossed tunneling is possible when the Cooper pair creates one electron from each dot.

We focus on the Andreev transport regime, so we use the \(\Delta \rightarrow \infty\) limit [20, 41, 42]. In a wide-bandwidth limit, we introduce the coupling constant between QD₁ and a metallic lead \(\Gamma_N = 2\pi \sum |V_{kN}|^2 \delta(\omega - \xi_{kN})\), and the coupling constant between QD₁ and superconducting substrate \(\Gamma_{S1} = 2\pi \sum |V_{kSi}|^2 \delta(\omega - \xi_{kS})\). The effective Hamiltonian takes on the following form:

\[
H_{\text{eff}} = \sum_{i\sigma} \varepsilon_i d_{i\sigma}^\dagger d_{i\sigma} + \sum_{\sigma} t_{12}(d_{1\sigma}^\dagger d_{2\sigma} + h.c.) + \sum_{k\sigma\beta} \xi_{k\beta} c_{k\sigma\beta}^\dagger c_{-k\sigma\beta} - \sum_k (\Delta_c^t_{k1S} c_{k1S}^\dagger + h.c.) + \sum_{i\sigma} (V_{kN} d_{1\sigma}^\dagger c_{k\sigma N} + h.c.) - \sum_{i\sigma} \frac{\Gamma_{S1}}{2}(d_{i\sigma}^\dagger d_{i\sigma}^\dagger + h.c.) + \sum_{i\sigma} \frac{\Gamma_{S2}}{2}(d_{i\sigma}^\dagger d_{i\sigma} + h.c.),
\]

where \(i = 2\) for \(i = 1\) and \(i = 1\) for \(i = 2\), \(\Gamma_{S2}(\Gamma_{S1})\) is the direct (cross) coupling between \(i\)-dot and the SC substrate. We assume that \(\Gamma_{S12} = \Gamma_{S21} = \sqrt{\Gamma_{S1}\Gamma_{S2}}\) [20, 25, 42].

Using the equation of motion method, we obtain the matrix of Green’s functions \(G(\omega) = \langle \langle \Psi; \Psi \rangle \rangle\), where \(\Psi = (d_{11\uparrow}, d_{11\downarrow}, d_{21\uparrow}, d_{21\downarrow})\), in the following notation

\[
G^{-1}(\omega) = \begin{pmatrix}
\omega - \varepsilon_1 + \frac{\Gamma_{S1}}{2} & \frac{\Gamma_{S1}}{2} & -t_{12} & -t_{12} \\
-t_{12} & \omega + \varepsilon_1 + d_{21} & \frac{-\Gamma_{S1}}{2} & \frac{t_{12}}{2} \\
\frac{-\Gamma_{S1}}{2} & \frac{\Gamma_{S1}}{2} & \omega - \varepsilon_2 & t_{12} \\
t_{12} & -t_{12} & \frac{t_{12}}{2} & \omega + \varepsilon_2
\end{pmatrix}.
\]

In the SC atomic limit, \(\Gamma_N \rightarrow 0\), the Green’s functions are characterized by four poles

\[
\begin{align*}
\varepsilon_{A1} &= 1/\sqrt{2}\sqrt{A + \sqrt{A^2 - 4B}}, \\
\varepsilon_{A2} &= -1/\sqrt{2}\sqrt{A + \sqrt{A^2 - 4B}}, \\
\varepsilon_{A3} &= 1/\sqrt{2}\sqrt{A - \sqrt{A^2 - 4B}}, \\
\varepsilon_{A4} &= -1/\sqrt{2}\sqrt{A - \sqrt{A^2 - 4B}}.
\end{align*}
\]
where $A = \epsilon_1^2 + \epsilon_2^2 + (\Gamma_{S1} + \Gamma_{S2})^2/2 + 2t_{12}^2$ and $B = (\epsilon_1 \epsilon_2 - t_{12}^2)^2 + (\epsilon_1 \Gamma_{S2} + \epsilon_2 \Gamma_{S1} + \Gamma_{S1} t_{12})^2/2$. These poles correspond to four Andreev resonances. The non-zero value of $\Gamma_N$ causes the broadening of these resonances. The generation of four Andreev states is related to the fact that the proximity effect generates the Andreev states on both quantum dots. The generation of Andreev states on QD$_1$ is related to the direct coupling of the QD$_1$ with the SC lead. For this quantum dot the $\epsilon_{A1}$ and $\epsilon_{A2}$ states are dominant. For QD$_2$ we have two methods of generation of the Andreev states, (i) the direct one for $\Gamma_{S2} \neq 0$; (ii) the indirect one (via QD$_1$) for $\Gamma_{S2} = 0$ and $t_{12} \neq 0$. For this quantum dot $\epsilon_{A3}$ and $\epsilon_{A4}$ states dominate.

The properties of nanoscopic systems can be analyzed experimentally using the current characteristics, especially the zero-bias differential conductance. The current flowing from the N lead can be calculated using the following equation [43-45]:

$$I_N = -\frac{e}{h} \int \frac{d \omega}{N} \sum_{k\sigma} \epsilon_k N \langle \sigma \rangle N \rangle \quad (5)$$

where $G^r$ ($G^c$) are the retarded (lesser) Green’s functions, respectively, and $f(\omega)$ is the Fermi distribution function.

The total current is the sum of the normal and Andreev currents. At low temperatures, the normal current, when an electron moves from N lead to SC lead, is realized for $eV \geq \Delta$. For $eV \leq \Delta$ in the N-QD-SC system, we observe the Andreev current arising when an electron from N lead pairs with a second electron with opposite spin and as the Cooper pair they are tunneling to the SC lead and simultaneously, a hole with opposite spin is reflected back to the N lead [33, 46]. In our N-DQD-SC system, with $\Gamma_{S1} \neq 0$ and $\Gamma_{S2} \neq 0$, the Cooper pair can tunnel to the SC lead via one of two dots (direct tunneling) or via both dots at the same time (crossed tunneling).

For $\Delta \to \infty$ in our system there occurs the Andreev current only, so the relation describing the current $I_N$ has the following form [33]:

$$I_N = \frac{e}{h} \int T_A(\omega) [f(\omega - \mu_N) - f(\omega + \mu_N)] d\omega \quad (6)$$

where

$$T_A(\omega) = 2\Gamma_N^2 |G_{12}^r(\omega)|^2 \quad (7)$$

is the total Andreev transmittance. The maximum of $T_A$ value is equal to 2. The Andreev transmittance is always symmetric, $T_A(\omega) = T_A(-\omega)$, because the anomalous Andreev scattering involves both the particle and hole degrees of freedom.

The knowledge of an Andreev transmittance allows us to calculate the zero-bias differential conductance as

$$G_A(V=0) = \frac{\partial I_N}{\partial V}|_{V \to 0} = \frac{2e^2}{h} \int T_A(\omega) \left[ -\frac{\partial f(\omega)}{\partial \omega} \right] d\omega. \quad (8)$$

At low temperatures, this equation can be simplified as follows:

$$G_A(V=0) = \frac{2e^2}{h} T_A(0). \quad (9)$$

III. RESULTS

In this section, we present the numerical results for the spectral density of QDs and the Andreev transmittance. As a unit of energy we assume the coupling parameter between QD$_1$ and SC substrate ($\Gamma_{S1} = 1$). The computations were carried out at $T = 0$.

A. Spectral density

The normalized spectral density of a quantum dot is defined as:

$$A_i(\omega) = \frac{\Gamma_N}{2} \text{Im} \langle d_i^\dagger d_i^\dagger \rangle \omega. \quad (10)$$

With such defined normalized spectral density, the maximum value of $A_1(\omega)$ is equal to 1.

In Fig. 2 we present the normalized spectral density of QD$_1$ (top panel) and QD$_2$ (bottom panel) as a function of the inter-dot tunneling amplitude ($t_{12}$) and for different values of the coupling parameter $\Gamma_{S2}$. For the computations, we assumed an equal energy level for QDs ($\epsilon_1 = \epsilon_2 = 0$), which is consistent with a chemical level of SC lead.

For $\Gamma_{S2} = 0$ (left panel) we obtain the system with a side-coupled QD$_2$, which is not directly coupled to the leads (metallic or superconducting) [4] [8] [10] [13] [15] [16]. In this case, the dependence of spectral density for QD$_1$ and QD$_2$ is symmetric ($A_i(\omega) = A_i(-\omega)$). We also observe the symmetry of spectral density as a function of the inter-dot tunneling amplitude with respect to $t_{12} = 0$. As Barański and Domański shown [8], in this T-shape configuration the Fano-type resonances and antiresonances localized near $\pm \epsilon_2$ are obtained. The Fano resonances are characterized by a typical asymmetric line shape and are obtained if a broad spectrum interferes with a discrete level. In our system, a broad QD$_1$ spectrum, resulting from coupling QD$_1$ with metallic lead, interferes with discrete QD$_2$ level. As a result of Fano type quantum interference, for $\omega = \epsilon_2$ we obtain the spectral density $A_1(\omega = \epsilon_2) = 0$ for all values of $t_{12} \neq 0$.

The location of this antiresonance also does not depend on $\epsilon_1$. At strong inter-dot tunneling ($t_{12} > \Gamma_N$), both for $A_1(\omega)$ and $A_2(\omega)$, one can observe four resonance states
The non-zero value of $\Gamma$ (see Eq. 4), which come from a direct Andreev effect for QD$_1$ and from an indirect Andreev effect for QD$_2$ [8]. For $\varepsilon_1 = \varepsilon_2 = 0$ two outer states (localized near $\varepsilon_{A1}$ and $\varepsilon_{A2}$) dominate for $A_1(\omega)$, while two inner states (localized near $\varepsilon_{A3}$ and $\varepsilon_{A4}$) dominate for $A_2(\omega)$.

The finite value of coupling between QD$_2$ and SC lead (middle and right panel of Fig. 2) causes a direct induction of pairing on QD$_2$. In this case, despite the dots energies which are equal to 0, the symmetry breaking of $A_i(\omega)$ is observed, whereas one can observe the $A_i(\omega, t_{12}) = A_i(-\omega, -t_{12})$ dependence. The outer Andreev states are still highly visible, and their location depends both on $\Gamma_{S1}$ and $\Gamma_{S2}$. For the inner states, we obtain a strong peak near $\varepsilon_{A3}$ and very weak peak near $\varepsilon_{A4}$. The non-zero value of $\Gamma_{S2}$ causes that we do not observe the Fano-type line shape of $A_1(\omega)$ ($A_1(\omega = \varepsilon_2) \neq 0$). For identical quantum dots with $\varepsilon_1 = \varepsilon_2$, which are characterized by an identical coupling with SC substrate $\Gamma_{S1} = \Gamma_{S2}$, we obtain three-center structure with one pair of Andreev resonances localized near $\varepsilon_{A1}$ and $\varepsilon_{A2}$, and with strong resonance near $\varepsilon_2 + t_{12}$ (see right panel of Fig. 2).

### B. Andreev transmittance

The dependence of Andreev transmittance (Eq. 7) as a function of inter-dot tunneling amplitude $t_{12}$ is shown in Fig. 3. In the case of side-coupled QD$_2$ ($\Gamma_{S2} = 0$ and $t_{12} \neq 0$), two pairs of resonance states are visible in Andreev transmittance (see Fig. 3a) near $\omega = \varepsilon_{A1}$. The

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**FIG. 2.** The normalized spectral density of quantum dots (for QD$_1$ - top panel, and for QD$_2$ - bottom panel) as a function of the inter-dot tunneling amplitude $t_{12}$ and for different values of the coupling parameter $\Gamma_{S2}$. Other parameters are $\varepsilon_1 = \varepsilon_2 = 0$, $\Gamma_{S1} = 1$ and $\Gamma_N = 0.25$.

**FIG. 3.** The Andreev transmittance as a function of the inter-dot tunneling amplitude $t_{12}$, and for different values of the coupling parameter $\Gamma_{S2}$. Other parameter are $\varepsilon_1 = \varepsilon_2 = 0$, $\Gamma_{S1} = 1$ and $\Gamma_N = 0.25$. 
FIG. 4. The Andreev transmittance as a function of the coupling parameter $\Gamma_{S2}$ and for different values of the inter-dot tunneling amplitude $t_{12}$ and $\varepsilon_2$. Other parameter are $\varepsilon_1 = 0$, $\Gamma_{S1} = 1$ and $\Gamma_N = 0.25$.

broad resonances are obtained for $\omega = \varepsilon_{A1}$ and $\omega = \varepsilon_{A2}$ and the narrow resonances are obtained for $\omega = \varepsilon_{A3}$ and $\omega = \varepsilon_{A4}$. Taking into account the coupling between QD2 and a superconducting lead ($\Gamma_{S2} \neq 0$), one obtains the extinction of inner Andreev transmittance resonances (see Figs 3(b) and (c)). The increase of $\Gamma_{S2}$ causes the shift of Andreev transmittance resonances localized near $\varepsilon_{A1}$ and $\varepsilon_{A2}$ towards higher energies. Additionally, these resonances become narrower. For the identical coupling of quantum dots with SC substrate, $\Gamma_{S1} = \Gamma_{S2}$, we obtain the total extinction of inner Andreev transmittance resonances (see Fig 3(c)).

Now we will discuss the influence of the hybridization parameter $\Gamma_{S2}$ on the Andreev transmittance (Fig. 4). In our analysis we consider the system of two QDs with equal energy $\varepsilon_1 = \varepsilon_2 = 0$ (top panel) and with different energies $\varepsilon_1 = 0$ and $\varepsilon_2 = 0.3$ (bottom panel). For $\Gamma_{S2} = 0$ (solid black line) we obtain the double QDs coupled in a T-shape configuration with metallic and superconducting lead [4, 8, 15]. As Barański and Domański [8] shown, in this configuration, for small $t_{12} \ll \Gamma_N$ one can obtain the Fano-type line shapes of Andreev transmittance. At high values of $t_{12}$, the Fano-type features disappear, evolving into the new quasi-particle peaks. Additional peaks can be interpreted as the Andreev peaks being a consequence of the indirect proximity effect induced by QD1 on the side-attached QD2. In this configuration, generally, for all values of $t_{12} \neq 0$, one obtains the $T_A(\varepsilon_2) = T_A(-\varepsilon_2) = 0$ dependence.

For a double quantum dot system, the Fano resonance is melded by the coupling of a narrow level related to QD2, and a broad level related to QD1. In the case of direct coupling of QD2 with SC lead $\Gamma_{S2} \neq 0$, as a result of proximity effect, there are created the narrow resonance Andreev levels on QD2. In this case, we obtain the zero-value of Andreev transmittance (Fano dip) for energy value equal to

$$\varepsilon_F = \pm \sqrt{\varepsilon_2^2 + 2t_{12}\Gamma_{S1}\varepsilon_2 + \Gamma_{S2}t_{12}^2} / \Gamma_{S1}. \quad (11)$$

As we have shown in Fig. 4(a) and (d), the zeroing of Andreev transmittance ($T_A(-\varepsilon_F) = T_A(\varepsilon_F) = 0$) does not require $t_{12} \neq 0$. At $t_{12} = 0$ and $\Gamma_{S2} \neq 0$, the zero value of Andreev transmittance is obtained for $\varepsilon_F = \pm \varepsilon_2$, while for $t_{12} \neq 0$ we obtain that $\varepsilon_F$ is not constant for a given $\varepsilon_2$ but it also depends on $\Gamma_{S1}$, $\Gamma_{S2}$, $\Gamma_{S12}$ and $t_{12}$ (see Fig. 4(b), (c), (e) and (f)).

For QDs which do not interact directly ($t_{12} = 0$), the increase of $\Gamma_{S2}$ causes the broadening of an effective value of the coupling parameter $\Gamma_{S_{eff}} = \Gamma_{S1} + \Gamma_{S2}$, and as the effect, it causes the shift of Andreev resonances localized near $\omega = \varepsilon_{A1}$ and $\omega = \varepsilon_{A2}$, towards higher energy levels (see Fig. 4(a) and (d)). In this case, the maximum of Andreev transmittance is close to 2. For $t_{12} \neq 0$ the increase of $\Gamma_{S2}$ causes the decreasing of the maximum value of Andreev transmittance ($T_A(\varepsilon_{A1}) < 2$ and $T_A(\varepsilon_{A2}) < 2$).

In Fig. 5 we present the dependence of Andreev transmittance as a function of QD2 energy for different values of inter-dot tunneling amplitude $t_{12}$ and for coupling parameter $\Gamma_{S2}$. For $t_{12} = 0$ and $\Gamma_{S2} = 0$ (Fig. 5(a)) the transmittance is independent of $\varepsilon_2$. The non-zero value of
FIG. 5. The Andreev transmittance as a function of the QD energy and for different values of the inter-dot tunneling amplitude $t_{12}$ and coupling parameter $\Gamma_{S2}$. Other parameters are $\varepsilon_1 = 0$, $\Gamma_{S1} = 1$ and $\Gamma_N = 0.25$.

$t_{12}$ or $\Gamma_{S2}$ (Figs 5(b)-(i)) causes that $T_A$ is $\varepsilon_2$ dependent. For almost all values of $\varepsilon_2$ we obtain four resonances of Andreev transmittance.

For $t_{12} \neq 0$ and $\Gamma_{S2} = 0$ (i.e. for a T-shape configuration (4) (8)) or for $t_{12} = 0$ and $\Gamma_{S2} \neq 0$ (i.e. without direct coupling between QDs) we obtain $T_A(\omega, \varepsilon_2) = T_A(\omega, -\varepsilon_2)$ (see Figs 5(b)-(c) and Figs 5(d),(g), respectively). In the case of $t_{12} \neq 0$ and $\Gamma_{S2} = 0$ the zero value of Andreev transmittance is obtained for $\omega = \pm \varepsilon_2$ ($T_A(\varepsilon_2) = T_A(-\varepsilon_2) = 0$). One can see that the clear four resonances of Andreev transmittance are visible for any value of $\varepsilon_2$ (Figs 5(b)-(c)). For $t_{12} \neq 0$ and $\Gamma_{S2} \neq 0$ (Figs 5(e)-(f),(h)-(i)) the dependence $T_A(\omega, \varepsilon_2)$ is not symmetrical with respect to $\varepsilon_2 = 0$. For small values of $t_{12}$, the inner Andreev resonances come close to each other and, in effect, one obtains with properly chosen $\varepsilon_2 < 0$, one very strong peak of transmittance, close to 2, for $\omega = 0$.

The formation of a strong peak for $T_A(\omega = 0, \varepsilon_2)$ is of great importance for the zero-bias Andreev conductance $G_A = \partial I_N / \partial V|_{V \to 0}$. In Fig. 6 we show the dependence of $G_A$ as a function of QD energy for different values of inter-dot tunneling amplitude $t_{12}$. We have used the symmetric coupling of QDs with SC lead, $\Gamma_{S1} = \Gamma_{S2} = 1$ and $\Gamma_N = 0.25$.

For $\varepsilon_2 < 0$ and properly chosen value of $t_{12}$, we obtain the maximum value of $G_A = 4\varepsilon^2/h$. For $\varepsilon_2 = -t_{12}$ we obtain $G_A = 0$.

These results can be compared to the zero-bias An-
C. The influence of Coulomb interaction

The Coulomb interaction is very important, taking into account the spectral and transport properties of a quantum dot connected to the SC and metallic leads [31, 45, 49, 53]. For the systems with a weak coupling of QD with metallic lead, the increase of Coulomb interaction causes the quantum phase transition between the (spin-less) BCS-like singlet and the (spin-full) dou

FIG. 7. The Andreev transmittance as a function of energy ω and the QD1 Coulomb interaction U1 for different values of the inter-dot tunneling amplitude t12. Other parameters are ε1 = −U1/2, ε2 = 0, ΓS1 = 1, ΓS2 = 0 and ΓN = 0.25. The black dashed line marks U1cr - see the text.

FIG. 8. The Andreev transmittance as a function of energy ω and the Coulomb interaction U1 = U2 = U for different values of the inter-dot tunneling amplitude t12. Other parameters are ε1 = −U/2, ε2 = −U/2, ΓS1 = 1, ΓS2 = 0 and ΓN = 0.25.

The Coulomb interaction also modifies the Andreev transmittance of the N-QD-SC system. For U < ΓS we obtain two transmittance peaks. The increase of U values close to ΓS causes the connection of these peaks to one central peak. For U > ΓS the disappearance of transmittance’s central peak occurs.

Now, we will analyze the influence of the Coulomb interaction on the transport properties of a N-QD-SC system using the second-order perturbation theory [32]. In our analysis we will consider two cases: (i) the Coulomb interaction exists only for QD1 electrons (U1 ≠ 0 and U2 = 0); (ii) the Coulomb interaction exists for both quantum dots (U1 = U2 = U ≠ 0). Additionally, we will focus on the particle-hole symmetry case, i.e. when ε1 = −U1/2 and ε2 = −U2/2. In the first case, we will assume that the N-QD1-SC system, consisted of correlated QD1 with U1 ≠ 0, is additionally connected to the uncorrelated QD2. In fig. 7 we show the dependence
of \( T_A(\omega) \) as a function of \( U_1 \) for different values of the inter-dot tunneling amplitude \( t_{12} \). For all values of \( U_1 \) we obtain \( T_A(0) = 0 \). This result shows that a competition between the Fano and Kondo effects does not allow for the formation of the central Kondo peak for large values of \( U_1 \) interaction. The increase of \( U_1 \) interaction causes that the Andreev peaks \( \varepsilon_{A1} \) and \( \varepsilon_{A2} \) are getting closer but they will never connect each other. The location of \( \varepsilon_{A3} \) and \( \varepsilon_{A4} \) Andreev peaks, related to the proximity effect on QD2, slightly shifts when the \( U_1 \) interaction increases. With properly selected value of \( U_1 = U_{1cr} \) interaction, the overlap of \( \varepsilon_{A1} \) and \( \varepsilon_{A3} \), and also \( \varepsilon_{A2} \) and \( \varepsilon_{A4} \) peaks occurs, and as a result, for \( U_1 > U_{1cr} \), we observe two narrowing peaks. The value of \( U_{1cr} \) increases as the \( t_{12} \) increases. In this case, the Fano-type resonance plays the dominant role.

Now, we will show the influence of the Coulomb interaction on the Andreev transmission in a \( U_1 = U_2 = U \) case (see Fig. 8). In this case, the increase of the Coulomb interaction causes that \( T_A(0) \neq 0 \) for all values of \( U \neq 0 \). Additionally, one can see that the Andreev peaks are getting closer to each other. For large values of \( t_{12} \) and \( U \) interactions, the Andreev peaks localized near \( \varepsilon_{A1} \) and \( \varepsilon_{A2} \) disappear, while the peaks localized near \( \varepsilon_{A3} \) and \( \varepsilon_{A4} \) are still visible.

IV. CONCLUSIONS

We have analyzed the spectral density and the transport properties of a double quantum dot. Both dots were coupled to a superconducting lead, while only one of them (QD1) was coupled to the metallic lead. The coupling of both quantum dots with SC lead allows for direct transport of the Cooper pair to the SC lead through one of two quantum dots and for a crossed transport via both quantum dots simultaneously.

The shape of spectral densities of quantum dots strongly depends on the coupling between QD2 and SC lead. For \( \Gamma_{S2} = 0 \), the spectral densities of quantum dots show the existence of two pairs of Andreev resonances. Additionally, we obtain the Fano dip near \( \omega = \varepsilon_2 \).

The non-zero coupling of QD2 with SC lead causes the extinction of the inner Andreev resonances. Additionally, \( \Gamma_{S2} \neq 0 \) causes the vanishing of the Fano dip. In the case of \( \Gamma_{S2} = \Gamma_{S1} \) and \( \varepsilon_1 = \varepsilon_2 \), we obtain a three-center structure of spectral density consisting of one pair of Andreev resonances and a strong resonance peak near \( \varepsilon_2 + t_{12} \).

For \( \Gamma_{S2} = 0 \), the Andreev transmittance shows one pair of broad peaks localized near \( \varepsilon_{A1} \) and \( \varepsilon_{A2} \), and narrow resonances near \( \varepsilon_{A3} \) and \( \varepsilon_{A4} \). In this case, the Andreev transmittance shows the Fano dip near \( \pm \varepsilon_2 \).

The non-zero value of \( \Gamma_{S2} \) causes the extinction of the inner Andreev transmittance resonances. In this case, the Fano dip are still visible, but its location depends on \( \varepsilon_2 \), \( \Gamma_{S1} \), \( \Gamma_{S2} \) and \( t_{12} \). The level of QD2 and \( t_{12} \) allows for strong modification of the zero-bias Andreev conductance value, \( G_A \). For \( \varepsilon_2 + t_{12} = 0 \) we obtain the total reduction of \( G_A \), while for properly selected values of \( \varepsilon_2 \) and \( t_{12} \) we can obtain very high values of \( G_A \) near \( 4e^2/h \).

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