EQUILIBRATION IN HEAVY ION COLLISIONS
AT LHC AND AT RHIC

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Abstract

We consider the evolution of a parton plasma created in Au+Au collisions at LHC and at RHIC energies. Using Boltzmann equation, relaxation time approximation and perturbative QCD, we show the physics of both thermal and chemical equilibration in a transparent manner. In particular, we show inelastic processes are, contrary to common assumption, more important than elastic processes, the state of equilibration of the system can compensate for some powers of $\alpha_s$ for the purpose of equilibration, the two-stage equilibration scenario is, barring any unknown non-perturbative effects, inevitable and is an intrinsic feature of perturbative QCD, and gluon multiplication is the leading process for entropy generation.

1 Introduction

Essential questions to ask in heavy ion collisions, especially at RHIC and at LHC, assuming a gas of weakly interacting partons can be formed in the central region, is how fast will this quark and gluon system approaches equilibrium if the expansion is not too rapid for the interactions. If indeed this parton gas can approach equilibrium, will it be able to end up as a quark-gluon plasma? And what is the degree of equilibration can one reasonably expect when the phase transition sets in? In short, is equilibration fast and how fast?

A number of previous works have already attempted at providing answers to these and other related questions of equilibration. However, due to the fact that non-equilibrium problems are difficult, they addressed either only thermalization [1, 2, 3, 4] or only parton chemical equilibration [5, 6]. Furthermore, previous attempts at the thermalization problem have only been done in a heuristic manner without really using QCD interactions. With the exception of the parton cascade model (PCM) [7], which is based on present knowledge of perturbative QCD and some very involved computations, only then it is able to consider both thermalization and chemical equilibration simultaneously as it should be and as it happens in the collisions. In this talk, we present a relatively simple way to do this and hence keeping things simple and clear so that the physics becomes transparent.

A much used assumption and starting point in the studies of the physics of heavy ion collisions is kinetic equilibration is rapid $\lesssim 1.0$ fm/$c$ and hydrodynamics expansion is well underway. PCM has shown that such short rapid thermalization is too optimistic. In the following, we will consider the evolution of a parton plasma and show that complete kinetic and chemical equilibration are slow, although interactions can indeed dominate over the expansion, and full equilibration cannot be attained before the phase transition assumed to be at $T_c \sim 200$ MeV. We also show that inelastic processes are more important than elastic ones in equilibration contradicting the common untested assumption of the contrary.

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2 To Determine the Evolution of a QCD Plasma

To study the evolution of a plasma of quarks and gluons, it is sufficient to know the particle distributions. For this purpose, we use the set of rather standard assumptions for relativistic heavy ion collisions and Baym’s form of the Boltzmann equation \[1\]

\[
\left( \frac{\partial f(p_{\perp}, p_z, \tau)}{\partial \tau} \right)_{p_{\perp}, p_z, \tau} = C(p_{\perp}, p_z, \tau) .
\]

where \( \tau = \sqrt{t^2 - z^2} \), with the collision terms on the right hand side approximated by the relaxation time approximation

\[
C(p, \tau) = -\frac{f(p, \tau) - f_{eq}(p, \tau)}{\theta(\tau)}
\]

where \( f_{eq} \) is the equilibrium distribution \( f_{eq} = \frac{1}{\exp(p/T_{eq}) \mp 1} \) and \( \theta(\tau) \) is the collision time. Whether the plasma equilibrates or not and how fast does it equilibrate depends very much on \( \theta(\tau) \) \[4, 8\].

With the combination of Eq. (1) and Eq. (2), one can already write down a solution to Eq. (1), which depends, however, on the two numerical parameters \( T_{eq} \) and \( \theta \) that need to be determined from QCD. We use the simplest QCD interactions at the tree level for the collision terms for this purpose. They are

\[
\begin{align*}
gg & \leftrightarrow ggg , \quad gg \leftrightarrow gg , \\
gg & \leftrightarrow q\bar{q} , \quad gg \leftrightarrow gq , \quad g\bar{q} \leftrightarrow g\bar{q} , \\
q\bar{q} & \leftrightarrow q\bar{q} , \quad qq \leftrightarrow qq , \quad \bar{q}\bar{q} \leftrightarrow \bar{q}\bar{q} .
\end{align*}
\]

Here to keep things simple and for our purpose, it is sufficient to include the simplest two leading inelastic processes\footnote{Here we assign a single chemical potential, \( \mu_q \), to all \( n_f \) flavours of fermions so flavour changing interactions in the first interactions of Eq. (5) is not considered as inelastic.} the first term of Eqs. (3) and (4) and all the binary elastic processes. Quarks and gluons will be treated as different particle species and not as generic partons so that their respective distributions, \( f_g \) and \( f_q \), are governed by different Boltzmann equations as they should be and depend on different collision times, \( \theta_g \) and \( \theta_q \), and different equilibrium temperatures, \( T_{eq,g} \) and \( T_{eq,q} \).

Using perturbative QCD and suitably infrared regularized the matrix elements by medium effects, one can construct the collision terms for gluons \( C_g \) and for quarks \( C_q \) semi-classically in the usual way \[8\]. For soft gluon emissions, Landau-Pomeranchuk-Migdal effect \[9\] has to be incorporated due to multiple scatterings in the medium \[5, 8\].

With the real collision terms from QCD, one can construct two equations for each particle species to solve for the two time-dependent unknowns in the particle distribution. We choose the following rate equations.

1) Energy density rate in an one-dimensional expanding system

\[
\frac{d\epsilon_i}{d\tau} + \frac{\epsilon_i + p_{L,i}}{\tau} = -\frac{\epsilon_i - \epsilon_{eq,i}}{\theta_i} = \nu_i \int \frac{d^3p}{(2\pi)^3} \ p \ C_i(p_{\perp}, p_z, \tau) ,
\]
2) Collision entropy density rate

\[
\frac{ds_i}{d\tau} = -\nu_i \int \frac{d^3p}{(2\pi)^3} C_i(p_\perp, p_z, \tau) \ln \left( \frac{f_i}{1 \pm f_i} \right) = \nu_i \int \frac{d^3p}{(2\pi)^3} \frac{f_i - f_{eq,i}}{\theta_i} \ln \left( \frac{f_i}{1 \pm f_i} \right),
\]

where \( i = g, q, \bar{q}, \) \( p_L \) is the longitudinal pressure defined later in Eq. (8) and \( \nu_i \) is the multiplicity, for gluons \( n_g = 2 \times 8 \) and for quarks \( n_q = 2 \times 3 \times n_f \). These equations are constructed from Eqs. (1) and (2) and \( C_i \)'s are now understood to be the real QCD collision terms. From them, \( \theta_i \)'s and \( T_{eq,i} \)'s can be determined. The collision entropy equations, in fact, allow one to break down the entropy generation process due to collisions into each of its contributing elements and find out which processes are more important for equilibration. We will show this later on in Sect. 3.

3 The Approach to Equilibrium

We take the initial conditions at the isotropic moment \( \tau_0 \) from HIJING [10] results for Au+Au collisions at RHIC and at LHC [5, 8]. The various equations described in Sect. 2 are solved numerically and the various collective variables can be calculated. The numerical details and parameters for solving the equations can be found in [8]. Here we concentrate on the results and the physics of equilibration.

As mentioned in the introduction, there are two types of equilibration: chemical and thermal, which happens simultaneously in heavy ion collisions. To check the parton composition in the plasma, it is common to use the concept of fugacity defined as \( l_i = \exp(\mu_i/T) \) so that the kinetically equilibrated distributions can be written as \( f_i = 1/(\exp(p/T_i)l_i^{-1} \mp 1) \). Since chemical equilibration is essentially \( \mu_i \to 0 \) starting from some initial negative values so that in terms of fugacity, \( l_i \) approaches 1.0. Clearly \( l_i \)'s as well as \( T_i \)'s exist only when local kinetic equilibrium has been achieved which is not the case most of the time during the evolution of the parton gas. Nevertheless, one can estimate these quantities from the energy densities \( \epsilon_i \)'s and number densities \( n_i \)'s in the usual way [8].

These are shown in Fig. 1. The solid lines are for gluons and the dashed lines are for quarks. The evolution is stopped when the temperature estimates all fall below 200 MeV, the assumed phase transition temperature. As can be seen, the gluon fugacities approach 1.0 much more rapidly than those of the quarks both at RHIC and at LHC. Gluon fugacities are close to 1.0 near the end as a result but not those of the quarks as in agreement with previous studies [5, 6, 7].

To check for kinetic equilibration, there is not the equivalent quantity of \( l_i \) as for chemical equilibration so instead one checks the isotropy of momentum distribution by comparing the ratio of the longitudinal to transverse pressure \( p_L/p_T \) and the ratio of a third of the energy density \( \epsilon/3p_T \) to the transverse pressure. These pressures are defined by

\[
p_{L,T,i}(\tau) = \nu_i \int \frac{d^3p}{(2\pi)^3} \frac{p_{\perp}^2}{p} f_i(p_\perp, p_z, \tau).
\]

These quantities are related of course by \( \epsilon = 2p_T + p_L \) to the energy density, so isotropy means \( p_T = p_L = \epsilon/3 \). These ratios should approach 1.0 in an equilibrating parton gas. We have plotted these ratios in Fig. 2 for gluons and for quarks.
Figure 1: The evolution of the species temperatures $T_g$ and $T_q$ and the fugacities $l_g$ and $l_q$ at LHC and at RHIC. The solid (dashed) lines are for gluons (quarks). The rising (falling) curves are the fugacity (temperature) estimates.

Figure 2: The plots of the evolution of the ratios of the longitudinal pressure (solid line) and one-third of the energy density (dashed line) to the transverse pressure, $p_L/p_T$ and $\epsilon/3p_T$ respectively, for gluons and for quarks at LHC and at RHIC.

The ratios are 1.0 initially because we have started from an isotropic momentum configuration. As the system expands, the distributions become anisotropic and reach maximum anisotropy quickly. The subsequent return towards isotropy is only progressive and at the end, complete isotropy is not fully recovered. For the gluons, one can perhaps argue an approximate isotropy has been achieved near the end especially at LHC. For the quarks, they have not yet passed the half-way mark. To check that, although slow, these are indeed thermalization behaviours, we can compare with the case that the system ends up in free streaming. In that case, interactions are not important and can be simulated by letting $\theta_g$ and $\theta_q$ to $\infty$ and the particle distributions are described by $f_0$. The corresponding pressures and ratios work out, as $\tau \to \infty$, to be

$$p_L/p_T \to 2 \tau_0^2/\tau^2 \to 0 \quad \text{and} \quad \epsilon/3p_T \to 2/3$$

which are clearly not the behaviours in Fig. 2. This provides clear evidence that the
interactions indeed dominate over the expansion.

Finally, we show the dominant processes in the equilibration of the plasma. It is common to assume that thermalization is driven by elastic processes in the studies of the various physics in heavy ion collisions. Inelastic processes are relegated to the minor role of essentially only for chemical equilibration. To show that this is not true, we break down the gluon and quark collision entropy density rate, \( (ds_g/d\tau)_{\text{coll}} \) and \( (ds_q/d\tau)_{\text{coll}} \) respectively, to their contributing elements and plot their ratios. The ratios plotted in Fig. 3 are each process to gluon multiplication for gluon and to gluon-gluon conversion into quark-antiquark pair for quark.

Initially, all the ratios are below 1.0 in the top figures i.e. gluon multiplication is dominant, at some point around 2.0 fm/c at LHC and 4.0 fm/c at RHIC. \( gg \leftrightarrow q\bar{q} \) (solid line) and \( gg \leftrightarrow gg \) or \( g\bar{q} \leftrightarrow g\bar{q} \) (dot-dashed) to \( gg \rightarrow ggg \) for gluons (top figures) and \( gq \leftrightarrow gq \) or \( g\bar{q} \leftrightarrow g\bar{q} \) (solid) and the sum of all fermion elastic scatterings to \( gg \rightarrow q\bar{q} \) for quarks (bottom figures).

Figure 3: The evolution of the collision entropy density ratios of \( gg \leftrightarrow gg \) (dashed), \( gg \leftrightarrow q\bar{q} \) (solid), and \( gq \leftrightarrow gq \) or \( q\bar{q} \leftrightarrow q\bar{q} \) (dot-dashed) to \( gq \leftrightarrow ggg \) for gluons (top figures) and \( gq \leftrightarrow gq \) or \( g\bar{q} \leftrightarrow g\bar{q} \) (solid) and the sum of all fermion elastic scatterings to \( gg \rightarrow q\bar{q} \) for quarks (bottom figures).
direction, it is not simply the forward or backward reaction that enters the collision terms of the Boltzmann equation Eq. (1) but the difference of the two. So it is not the sizes of the interaction cross-sections which determine what processes should be or should not be more important in the equilibration of a many body system.

To summarize, in this talk, we have shown that strictly speaking equilibration cannot be completed in heavy ion collisions at RHIC and at LHC. One can at best consider the system as a fluid mixture of an approximately thermalized and chemical equilibrated gluon plasma and a still far from equilibrated quark and antiquark plasma. The usual belief that elastic processes are responsible for thermalization is flawed. As we have shown, inelastic processes are even more important. One has to be very careful in comparing interactions in a medium, if it is done by relying on the sizes of the scattering cross-sections alone, one is prone to getting the wrong answer.

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References

[1] G. Baym, Phys. Lett. B 138, 18 (1984).

[2] S. Gavin, Nucl. Phys. B 351, 561 (1991); K. Kajantie and T. Matsui, Phys. Lett. B 164, 373 (1985).

[3] E.V. Shuryak, Phys. Rev. Lett. 68, 3270 (1992).

[4] H. Heiselberg and X.N. Wang, Nucl. Phys. B 462, 389 (1996), Phys. Rev. C 53, 1892 (1996).

[5] T.S. Biró, E. van Doorn, B. Müller, M.H. Thoma and X.N. Wang, Phys. Rev. C 48, 1275 (1993); P. Lévai, B. Müller and X.N. Wang, Phys. Rev. C 51, 3326 (1995); X.N. Wang, Nucl. Phys. A 590, 47 (1995).

[6] E.V. Shuryak and L. Xiong, Phys. Rev. C 49, 2203 (1994).

[7] K. Geiger and B. Müller, Nucl. Phys. B 369, 600 (1991); K. Geiger, Phys. Rev. D 46, 4965, 4986 (1992); K. Geiger and J.I. Kapusta, Phys. Rev. D 47, 4905 (1993).

[8] S.M.H. Wong, Nucl. Phys. A 607, 442 (1996); Phys. Rev. C 54, 2588 (1996).

[9] M. Gyulassy and X.N. Wang, Nucl. Phys. B 420, 583 (1994); M. Gyulassy, M. Plümer and X.N. Wang, Phys. Rev. D 51, 3436 (1995); R. Baier, Yu.L. Dokshitzer, S. Peigné and D. Schiff, Phys. Lett. B 345, 277 (1995).

[10] M. Gyulassy and X.N. Wang, Phys. Rev. D 44, 3501 (1991).

[11] G. Baym, H. Heiselberg, C.J. Pethick and J. Popp, Nucl. Phys. A 544, 569c (1992).