M5-branes in $\text{AdS}_4 \times Q^{1,1,1}$ spacetime

De-Sheng Li$^1$, Zheng-Wen Liu$^2$, Jun-Bao Wu$^{1,6}$ and Bin Chen$^{3,4,5,6}$

$^1$Institute of High Energy Physics, and Theoretical Physics Center for Science Facilities, Chinese Academy of Sciences, 19B Yuquan Road, Beijing 100049, P. R. China
$^2$Department of Physics, Renmin University of China, Beijing 100872, P. R. China
$^3$Department of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, No. 5 Yiheyuan Rd, Beijing 100871, P. R. China
$^4$Center for High Energy Physics, Peking University, No. 5 Yiheyuan Rd, Beijing 100871, P. R. China
$^5$Beijing Center for Mathematics and Information Interdisciplinary Sciences, 105 W 3rd Ring Rd N, Beijing 100048, P. R. China
$^6$Kavli Institute for Theoretical Physics China, CAS, Beijing 100190, P. R. China

Abstract

In this paper, we study the M5-brane configurations in $\text{AdS}_4 \times Q^{1,1,1}$ spacetime. We consider the configurations with an $\text{AdS}_2$ factor embedding into $\text{AdS}_4$, and manage to construct two solutions, which could be dual to line defects in the boundary gauge theory. Moreover we discuss their BPS nature and find that neither of them is supersymmetric. We show that the M5-brane with a $\text{R}_2$ or an $\text{AdS}_3$ factor found before is half-BPS.
1 Introduction

Great progress on understanding of the low energy effective action of $N$ M2-branes at large $N$ limit has been made since the construction of ABJM theory [1] which was in part inspired by [2–6] among with other works. ABJM theory is a three-dimensional $\mathcal{N} = 6$ super-Chern-Simons theory with gauge group $U(N)_k \times U(N)_{-k}$. This theory is dual to M-theory on $AdS_4 \times S^7/Z_k$ or type IIA theory on $AdS_4 \times \mathbb{CP}^3$. By solving the ABJM matrix model obtained via supersymmetric localization [7], people finally had a satisfying understanding [8] of the scaling behavior $N^{3/2}$ for the counting of the degrees of freedom for $N$ M2-branes, first obtained through the computations in the gravity side [9]. Many examples of $AdS_4/CFT_3$ correspondence with less supersymmetries has been studied as well. In the gravity side, the correspondence involves M-theory on $AdS_4 \times Y^7$ with $Y^7$ being certain manifolds or orbifolds. The dual field theory could be three-dimensional Chern-Simons-matter theory with $\mathcal{N} = 1 (2,3)$ supersymmetries when $Y^7$ is a weak $G_2$ (Sasaki-Einstein, 3-Sasaki) manifold (or its orbifold preserving the same amount of supersymmetries) [10,11]. The three-dimensional Chern-Simons-matter field theories with $\mathcal{N} = 4, 5, 6$ supersymmetries, corresponding to certain orbifolds $Y^7$, have also been studied in [12–16]. Among them, the study of M-theory on $AdS_4 \times Q^{1,1,1}$ is of particular interest, because the metric of Sasaki-Einstein manifold $Q^{1,1,1}$ is quite simple.

In M-theory, there are two kinds of nonperturbative objects: M2-brane and M5-brane. Their roles in $AdS_4/CFT_3$ correspondence are not completely clear. For example, the dimension reduction of M2-brane to ten dimension may give us fundamental string, which could be dual to the Wilson loop in the field theory [23,24]. However, though the simplest embedded of F-strings inside the dual IIA string theory background $AdS_4 \times \mathbb{CP}^3$ is half BPS [25,26], the field theory construction of BPS Wilson loop operator is highly nontrivial [25–31]. Less supersymmetric Wilson loops in ABJM theory were studied in [32–35]. General studies on Wilson loops in $\mathcal{N} = 2$ super-Chern-Simons theory were performed in [36]. Very recently, the BPS M2-branes in $AdS_4 \times Q^{1,1,1}$ dual to BPS Wilson loops and vertex loops were studied in [37] based on explicit expressions of Killing spinors. Other types of membranes in $AdS_4 \times Q^{1,1,1}$ were studied in [38–40].

Besides M2-branes, there are also M5-branes in M-theory. In the context of $AdS/CFT$ correspondence, M5-brane could be dual to the baryonic operator or the defects including the line defect and the domain wall in the field theory. It may also appear due to the Myers’ polarization effect of multiple M2-branes [42,43]. It is not easy to find the M5-brane configuration in curved spacetime because its equations of motion are hard to solve. In the case of $AdS_4 \times Q^{1,1,1}$, the M5-branes with a $R_t$ and $AdS_3$ factor have been studied in [39,45]. The first M5-brane is dual to certain baryonic operator while the second one is dual to domain wall. An unanswered question on these solutions is whether or not they are supersymmetric.

The main topics of this note is to study M5-brane solutions and their BPS nature in $AdS_4 \times Q^{1,1,1}$ spacetime. We pay special attention to M5-branes whose worldvolume includes an $AdS_2$ factor. These M5-branes should dual to certain one-dimensional defects in the dual gauge theory, though may not dual to the Wilson loop operator or the vertex
loop operator [41]. With the projection condition on the Killing spinor in mind, we make two kinds of ansatz which have the potential to be supersymmetric. However after solving the M5-brane equations of motion and studying the BPS conditions for M5-branes, we find that none of them is BPS. This shows that it is quite hard to find BPS M5-branes with an $AdS_2$ factor in background with such less supersymmetries. Besides, we check the supersymmetries preserved by the previously-mentioned M5-brane with a $R_t$ or $AdS_3$ factor and find that both of them are half-BPS.

In the next section, we will briefly review M5-brane equations of motion and the projection condition for the supersymmetries preserved by the probe M5-brane. In section 3, we introduce the background fields and the Killing spinors of M-theory on $AdS_4 \times Q^{1,1,1}$. In section 4, we present two M5-brane solutions whose worldvolumes involve an $AdS_2$ factor. In section 5, we discuss the supersymmetries preserved by M5-brane with a $R_t$ or $AdS_3$ factor. We conclude this paper with some brief discussions. We gather in the appendix the explicit form of the connection coefficients used in the main text.

## 2 M5-brane equations of motion

Various proposals and aspects of M5-brane actions have been studied in [46–51] (for a review of M-theory branes, see [52]). In this section we briefly review the covariant equations of motion for M5-branes [46] and the supersymmetric conditions for the probe M5-brane.

The massless bosonic fields of 11-dimensional M-theory include the metric

$$ds_{11}^2 = g_{mn} dx^m dx^n,$$

and the 4-form field strength

$$H_4 = H_{m_1 \ldots m_4} dx^{m_1} \wedge \ldots \wedge dx^{m_4}.$$  

We also need the target space vielbein $E_{\mathbf{a}}^m$ satisfying

$$E_{\mathbf{a}}^m E_{\mathbf{b}}^n \eta_{\mathbf{a} \mathbf{b}} = g_{mn},$$

and the Hodge dual of $H_4$ denoted as $H_7$ whose components are $H_{m_1 \ldots m_7}$.

The probe M5-brane solution is described in terms of the embedding $x^m(\xi^m)$ and a self-dual 3-form field $h_{mnp}$ on the M5-brane worldvolume. Here $\xi^m, m = 0, \ldots, 5$ are coordinates of the worldvolume. From the embedding, we can define the induced metric

$$g_{mn} = E_{\mathbf{a}}^m E_{\mathbf{b}}^n \eta_{\mathbf{a} \mathbf{b}},$$

with

$$E_{\mathbf{a}}^m = \partial_m x^\mathbf{a} E_{\mathbf{a}}^\mathbf{b} \eta_{\mathbf{b}}.$$  

---

*Our notation is as follows: indices from the beginning(middle) of the alphabet refer to frame(coordinate) indices, and the underlined indices refer to target space ones.*
Starting with $h_{mn}$ which is self dual with respect to this induced metric, we define the following list of quantities

\begin{align*}
    k^m_n &= h_{pq}h^{npq}, \\
    Q &= 1 - \frac{2}{3}k^m_nh_m, \\
    m_p^q &= \delta_p^q - 2k_p^q, \\
    H_{mn} &= 4Q^{-1}(1 + 2k)^m_nh_{np}, \\
    G^{mn} &= \left(1 + \frac{2}{3}k^2\right)g^{mn} - 4k^{mn}, \\
    P_a^c &= \delta_a^c - \epsilon_a^m\epsilon_n^c, \\
    Y_{mn} &= (4\ast H - 2(m\ast H + \ast H) + m\ast H)_{mn},
\end{align*}

where

\begin{equation}
    \ast H^{mn} = \frac{1}{4!\sqrt{-g}}\epsilon^{mnprqs}H_{prqs}.
\end{equation}

The covariant derivative $\nabla_m e^c_n$ is defined as

\begin{equation}
    \nabla_m e^c_n = \partial_m e^c_n - \Gamma^p_{mn}e^c_p + \epsilon^a_m e^b_n\omega^c_{ab},
\end{equation}

where $\Gamma^p_{mn}$ is the Christoffel symbol with respect to the induced metric on the world-volume and $\omega^c_{ab}$ is the spin connection of the background spacetime.

After defining these quantities, the equations of motion of a M5-brane include three parts:

- **Bianchi identity**
  \[
  dH_3 = -P[H_4],
  \]
  where $P[H_4]$ is the pull-back of the target space 4-form flux.

- **Scalar equation**
  \[
  G^{mn}\nabla_m e^c_n = \frac{Q}{\sqrt{-g}}(m_1\cdots m_6)\left(\frac{1}{6!}H^2_{m_1\cdots m_6} + \frac{1}{(3!)^2}H^4_{m_1m_2m_3Hm_4m_5m_6}\right)P_a^c,
  \]

- **Tensor equation**
  \[
  G^{mn}\nabla_m H_{npq} = Q^{-1}(4Y - 2(mY + Ym) + mYm)_{pq}.
  \]

To study the supersymmetries preserved by the probe M5-brane, we need to solve the kappa symmetry projection condition

\begin{equation}
    \Gamma_{M5}\eta = \eta,
\end{equation}

where $\eta$ is solution of Killing spinor equation of the M-theory background

\begin{equation}
    \nabla_m \eta + \frac{1}{576}(3\Gamma_{npq}\Gamma_m - \Gamma_m\Gamma_{npq})H^{npq}\eta = 0,
\end{equation}

4
and $\Gamma_{M5}$ is determined by the embedding of M5-brane and the flux on it \[46\]

$$
\Gamma_{M5} = \frac{1}{6!\sqrt{-g}} \epsilon^{i_1 \ldots i_6} \left( \Gamma_{j_1 \ldots j_6} + 40 \Gamma_{j_1 j_2 j_3} h_{j_4 j_5 j_6} \right).
$$

Here $g$ is the determinant of the induced worldvolume metric component, $h_{j_1 j_2 j_3}$ is the self-dual 3-form on the M5-brane. And $\Gamma_{j_1 \ldots j_n}$ is defined as

$$
\Gamma_{j_1 \ldots j_n} = e_{i_1} \ldots e_{i_n} \Gamma_{a_1 \ldots a_n},
$$

where $\Gamma_{a_1 \ldots a_n}$ is the antisymmetrized product of the Gamma matrices in the orthonormal frame.

### 3 The background fields and the Killing spinors

The metric on $AdS_4 \times Q^{1,1,1}$ is

$$
ds^2 = R^2 (ds_4^2 + ds_7^2),
$$

$$
ds_4^2 = \frac{1}{4} (\cos^2 u (- \cos^2 \rho dt^2 + d\rho^2) + du^2 + \sin^2 u d\phi^2),
$$

$$
ds_7^2 = \frac{1}{8} \sum_{i=1}^{3} (d\theta_i^2 + \sin^2 \theta_i d\phi_i^2) + \frac{1}{16} \left( d\psi + \sum_{i=1}^{3} \cos \theta_i d\phi_i \right)^2,
$$

with $\theta_i \in [0, \pi], \phi_i \in [0, 2\pi]$ ($i = 1, 2, 3$), $\psi \in [0, 4\pi]$. The four-form field strength on this background is

$$
H_4 = \frac{3R^3}{8} \cosh^2 u \sinh u \cosh \rho dt \wedge d\rho \wedge du \wedge d\phi.
$$

The vielbeins of the eleven-dimensional metric are

$$
e_0 = \frac{R}{2} \cosh u \cosh \rho dt,
$$

$$
e_1 = \frac{R}{2} \cosh \rho d\rho,
$$

$$
e_2 = \frac{R}{2} du,
$$

$$
e_3 = \frac{R}{2} \sinh u d\phi,
$$

$$
e_4 = \frac{R}{2\sqrt{2}} d\theta_1,
$$

$$
e_5 = \frac{R}{2\sqrt{2}} \sin \theta_1 d\phi_1,
$$

$$
e_6 = \frac{R}{2\sqrt{2}} d\theta_2,
$$

$$
e_7 = \frac{R}{2\sqrt{2}} \sin \theta_2 d\phi_2,
$$

$$
e_8 = \frac{R}{2\sqrt{2}} d\theta_3,
$$

$$
e_9 = \frac{R}{2\sqrt{2}} \sin \theta_3 d\phi_3,
$$

$$
e_{10} = \frac{R}{4} \left( d\psi + \sum_{i=1}^{3} \cos \theta_i d\phi_i \right),
$$

such that $H_4$ can now be written as

$$
H_4 = \frac{6}{R} e_0 \wedge e_1 \wedge e_2 \wedge e_3.
$$

5
As mentioned in the previous section, the Killing spinors of $AdS_4 \times Q^{1,1,1}$ satisfy the following equation

$$\nabla_m \eta + \frac{1}{576} \left( 3\Gamma_{mpq} \Gamma_m - \Gamma_m \Gamma_{mpq} \right) H^{pq} \eta = 0. \quad (28)$$

Our convention about the product of the eleven $\Gamma$ matrices is

$$\Gamma_{0123456789} = 1. \quad (29)$$

Using the vielbeins given above and the spin connections given in the appendix, we find that the solution to the above equation is

$$\eta = e^{\frac{1}{2} \Gamma_2 \hat{\Gamma} e^{\frac{1}{2} \Gamma_2 \hat{\Gamma} e^{\frac{1}{2} \Gamma_3 \hat{\Gamma} e^{\frac{1}{2} \Gamma_4 \hat{\Gamma} e^{-\frac{1}{2} \Gamma_5 \hat{\Gamma} \eta_0}}}}. \quad (30)$$

where $\eta_0$ is independent of all the coordinates and satisfies the projection conditions

$$\Gamma^{45} \eta_0 = \Gamma^{67} \eta_0 = \Gamma^{89} \eta_0, \quad (31)$$

and $\hat{\Gamma}$ is defined as

$$\hat{\Gamma} \equiv \Gamma_{0123}. \quad (32)$$

We will also need the metric of $AdS_4$ in the Poincare coordinates

$$ds^2 = \frac{1}{4} \left( -dt^2 + dx_1^2 + dx_2^2 + dy^2 \right) \quad (33)$$

now the vielbeins in the $AdS_4$ part are

$$e^0 = \frac{R}{2} \frac{dt}{y}, \quad e^1 = \frac{R}{2} \frac{dy}{y},$$

$$e^2 = \frac{R}{2} \frac{dx_1}{y}, \quad e^3 = \frac{R}{2} \frac{dx_2}{y}, \quad (34)$$

and the corresponding spin connections are

$$\omega^{03} = -\frac{2}{R} e^0, \quad \omega^{43} = -\frac{2}{R} e^1, \quad \omega^{23} = -\frac{2}{R} e^2. \quad (35)$$

In Poincare coordinates, the solutions to the Killing spinor equations are

$$\eta = y^{1/2} \eta_+ + y^{-1/2} (\eta_- + x^{12} \Gamma_{12} \eta_+) \quad (36)$$

Here $\eta_\pm = \exp \left( -\frac{1}{2} \Gamma_{45} \eta_{12}^0 \right)$, and $\eta_{12}^0$ satisfies

$$\Gamma_3 \Gamma_4 \eta_{12}^0 = \pm \eta_{12}^0, \quad \Gamma^{45} \eta_{12}^0 = \Gamma^{67} \eta_{12}^0 = \Gamma^{89} \eta_{12}^0. \quad (37)$$

2Killing spinor in a slightly different moving frame was given in [37]. The Killing spinors of $Q^{1,1,1}$ were also studied previously in [53-54]. The Killing spinors of $AdS_4$ were given in this coordinate system in [25].
4 M5-branes dual to line defects

In this section we find two M5-brane solutions dual to line defects in the boundary gauge theory. The first solution has an $AdS$ factor in the $AdS_4$ part of the background geometry, while the second solution has an $AdS_2 \times S^1$ factor in the $AdS_4$ part.

4.1 The first solution

For this solution, the topology of the worldvolume of M5-brane is $AdS_2 \times M^4$ with $AdS_2 \subset AdS_4$ and $M^4 \subset Q^{1,1,1}$. The embedding of this M5-brane is

$$\xi^0 = t, \quad \xi^1 = \rho,$$

$$\xi^2 = \theta_1, \quad \xi^3 = \phi_1, \quad \xi^4 = \theta_2, \quad \xi^5 = \phi_2,$$

with other coordinates fixed. We choose the 3-form $h_3$ to be zero.

Now the induced metric is

$$\tilde{s}^2 = R^2 \left( \frac{1}{4} \cosh^2 u_0 (\cosh^2 \rho dt^2 + d\rho^2) + \frac{1}{8} \sum_{i=1}^2 (d\theta_i^2 + \sin^2 \theta_i d\phi_i^2) \right)$$

$$+ \frac{1}{16} \left( \sum_{i=1}^2 \cos \theta_i d\phi_i \right)^2.$$

The nonzero components of $\mathcal{E}^a_m$ are

$$\mathcal{E}^0_\rho = \frac{R}{2} \cosh u_0 \cosh \rho, \quad \mathcal{E}^1_\rho = \frac{R}{2} \cosh u_0,$$

$$\mathcal{E}^2_{\theta_1} = \mathcal{E}^2_{\phi_1} = \frac{R}{\sqrt{8}} \sin \theta_1, \quad \mathcal{E}^2_{\phi_2} = \frac{R}{\sqrt{8}} \sin \theta_2,$$

$$\mathcal{E}^4_{\phi_1} = \frac{R}{4} \cos \theta_1, \quad \mathcal{E}^4_{\phi_2} = \frac{R}{4} \cos \theta_2.$$

From $h_3 = 0$, it is easy to obtain that $H_3 = 0$ and $G_{mn} = g_{mn}$. Then the Bianchi identity and the tensor equations are satisfied trivially. And after some computations, we find that the scalar equations give the constraint that $u_0 = 0$.

The Killing spinor on the worldvolume of this M5-brane is

$$\eta = e^{\frac{2}{\sqrt{2}} \Gamma^A_1} e^{\frac{1}{\sqrt{2}} \Gamma^A_2} e^{\frac{\sqrt{2}}{2} \Gamma^{14567}} e^{-\frac{\sqrt{2}}{2} \Gamma^{01234567}} \eta_0.$$  

In this case $\Gamma_{M5}$ is

$$\Gamma_{M5} = \frac{\sqrt{2} \sin \theta_1 \sin \theta_2 \Gamma_{014567} + \sin \theta_1 \cos \theta_2 \Gamma_{014567} + \cos \theta_1 \sin \theta_2 \Gamma_{014567}}{\sqrt{\sin^2 \theta_1 + \sin^2 \theta_2}}.$$  

Note especially that $u$ takes a fixed value $u_0$. 

7
Considering the points in the submanifold \( t = \rho = \theta_2 = 0, \theta_1 = \pi/2 \) on the worldvolume, \( \Gamma_{M5}\eta = \eta \) becomes
\[
\Gamma_{0145670} = \eta_0. \tag{46}
\]
However this projection condition is not compatible with the projection condition \( \Gamma_{4567}\eta_0 = -\eta_0 \) from the Killing spinor equations. This leads to the conclusion that this M5-brane is not supersymmetric.

### 4.2 The second solution

The topology of the worldvolume of this M5-brane is \( \text{AdS}_2 \times S^1 \times M_3 \) with \( \text{AdS}_2 \times S^1 \subset \text{AdS}_4 \) and \( M_3 \subset Q^{1,1,1} \). The embedding is
\[
\xi^0 = t, \quad \xi^1 = \rho, \quad \xi^2 = \phi, \quad (47)
\]
\[
\xi^3 = \theta_1, \quad \xi^4 = \phi_1, \quad \xi^5 = \psi, \quad (48)
\]
with other coordinates being fixed. The 3-form field \( h_3 \) is chosen to be
\[
h_3 = a(\xi) \left( \frac{R^3}{8} \cosh^2 u_0 \sinh u_0 \cosh \rho dt \wedge d\rho \wedge d\phi - \frac{R^3}{32} \sin \theta_1 d\theta_1 \wedge d\phi_1 \wedge d\psi \right), \tag{49}
\]
satisfying the condition that \( h_3 = *h_3 \).

The induced metric is
\[
ds^2 = R^2 \left( \frac{1}{4} \cosh^2 u_0 (-\cosh^2 \rho dt^2 + d\rho^2) + \frac{1}{4} \sinh^2 u_0 d\phi^2 \\
+ \frac{1}{8} (d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2) + \frac{1}{16} (d\psi + \cos \theta_1 d\phi_1)^2 \right). \tag{50}
\]

The nonzero components of \( E^a_m \) are
\[
E^0_t = \frac{R}{2} \cosh u_0 \cosh \rho, \quad E^1_\rho = \frac{R}{2} \cosh u_0, \quad E^2_\phi = \frac{R}{2} \sinh u_0, \quad (51)
\]
\[
E^3_{\theta_1} = \frac{R}{\sqrt{8}}, \quad E^4_{\phi_1} = \frac{R}{\sqrt{8}} \sin \theta_1, \quad E^5_\psi = \frac{R}{4} \cos \theta_1, \quad E^{\#}_\psi = \frac{R}{4}. \tag{52}
\]

We list some important quantities for this solution here
\[
k_{nm}^m = \begin{pmatrix}
-2a^2 I_{3\times3} & 0 \\
0 & 2a^2 I_{3\times3}
\end{pmatrix}, \tag{53}
\]
\[
Q = 1 - \frac{2}{3} \text{Tr} k^2 = 1 - 16a^4. \tag{54}
\]
The nonzero components of \( G_{mn} \) are
\[
G_{mn} = (1 + 4a^2)^2 g_{mn}, \tag{55}
\]
when \( n, m \in \{ t, \rho, \phi \} \), and
\[
G_{mn} = (1 - 4a^2)^2 g_{mn}, \tag{56}
\]
when \( n, m \in \{ \theta_1, \phi_1, \psi \} \). And
\[
H_3 = \frac{aR^3 \cosh^2 u_0 \sinh u_0 \cosh \rho}{2(1 + 4a^2)} dt \wedge d\rho \wedge d\phi - \frac{aR^3 \sin \theta_1}{8(1 - 4a^2)} d\theta_1 \wedge d\phi_1 \wedge d\psi. \tag{57}
\]
The Bianchi identity gives that
\[
dH_3 = 0 \tag{58}
\]
which lead to the fact that \( a \) is a constant. The tensor equations are automatically satisfied under this condition. By some computations, we find that scalar equations give the following relation between \( a \) and \( u_0 \)
\[
2 \tanh u_0 + \coth u_0 = \frac{12a}{1 + 4a^2}. \tag{59}
\]
On the worldvolume of this M5-brane, the Killing spinor reads
\[
\eta = e^{\frac{3}{2} \Gamma_0} e^{\frac{1}{2} \Gamma_0} e^{\frac{1}{2} \Gamma_3} e^{\frac{1}{2} \Gamma_4} e^{-\frac{1}{2} \Gamma_5} \eta_0. \tag{60}
\]
And \( \Gamma_{M5} \) now becomes
\[
\Gamma_{M5} = \Gamma_{013452} - 2a(\Gamma_{013} + \Gamma_{452}). \tag{61}
\]
By studying the special cases with \( \rho = t = \phi = \psi = 0 \) and \( \rho = t = \psi = 0, \phi = \pi/2 \), we find that this M5-brane is non-BPS.

5 Supersymmetric M5-branes

The M5-branes with an AdS_2 factor we found in the last section are not supersymmetric. In this section, we discuss the BPS nature of two other M5-brane configurations proposed in the literature and find that both of them keep half of the supersymmetries.

5.1 M5-brane with a R_t factor

The M5-brane with a \( R_t \) factor in \( AdS_4 \) and with other five directions in \( Q^{1,1,1} \) was studied in [39, 45]. Now we show explicitly that this brane configuration satisfy M5-brane equations of motion and preserve half of the supersymmetries of the \( AdS_4 \times Q^{1,1,1} \) background. The embedding of this M5-brane is
\[
\xi^0 = t, \quad \xi^1 = \theta_1, \quad \xi^2 = \phi_1, \quad \xi^3 = \theta_2, \quad \xi^4 = \phi_2, \quad \xi^5 = \psi, \tag{62}
\]
with \( \theta_3, \phi_3, u, \rho, \phi \) being fixed on the worldvolume, and the 3-form field \( h_3 \) is chosen to be zero. The induced metric on the worldvolume is
\[
d\tilde{s}^2 = R^2 \left( -\frac{1}{4} \cosh^2 u \cosh^2 \rho dt^2 + \frac{1}{8} \sum_{i=1}^{2} (d\theta_i^2 + \sin^2 \theta_i d\phi_i^2) \right.
\]
\[
+ \frac{1}{16} \left( d\psi + \sum_{i=1}^{2} \cos \theta_i d\phi_i \right)^2 \right). \tag{63}
\]
The nonzero components of $E_a^m$ are

$$E^0_t = \frac{R}{2} \cosh u \cosh \rho, \quad (64)$$

$$E^a_{\theta_1} = E^a_{\theta_2} = \frac{R}{\sqrt{8}}, \quad E^5_{\phi_1} = \frac{R}{\sqrt{8}} \sin \theta_1, \quad E^7_{\phi_2} = \frac{R}{\sqrt{8}} \sin \theta_2, \quad (65)$$

$$E^z_{\phi_1} = \frac{R}{4} \cos \theta_1, \quad E^z_{\phi_2} = \frac{R}{4} \cos \theta_2, \quad E^z_\psi = \frac{R}{4}. \quad (66)$$

After some short computations, we obtain

$$Q = 1, \quad H_3 = 0, \quad G_{mn} = g_{mn}. \quad (67)$$

Now the Bianchi identity and the tensor equation are satisfied automatically and the scalar equations give the constraint that

$$u = \rho = 0. \quad (68)$$

Now we can easily get that

$$\Gamma_{M5} = \Gamma_{045672}. \quad (69)$$

Using eqs. (60) and (61), we find that the supersymmetric condition

$$\Gamma_{M5} \eta = \eta, \quad (70)$$

is equivalent to the project condition

$$\Gamma_{02} \eta_0 = -\eta_0. \quad (71)$$

Since this condition is compatible with the project conditions in Eq. (61), we arrive at the conclusion that this M5-brane is half-BPS.

### 5.2 M5-brane with an $AdS_3$ factor

The M5-brane with an $AdS_3$ factor was studied in [45] and was argued there to be dual to a domain wall in the field theory. Here we show explicitly that this configuration does satisfy the equations of motion for probe M5-brane and moreover is half-BPS. We will also make contact with general discussions on BPS M5-branes in $AdS_4 \times Y^7$ background in [55].

Now we use the Ponicaré coordinates of $AdS_4$. The embedding of this M5-brane is

$$\xi^0 = t, \quad \xi^1 = x_1, \quad \xi^2 = y, \quad \xi^3 = \theta_1, \quad \xi^4 = \phi_1, \quad \xi^5 = \psi. \quad (72)$$

The 3-form field $h_3$ is chosen to be

$$h_3 = a(\xi) \left( \frac{R^3}{8} \sqrt{1 + f^2} dy \wedge dt \wedge dx_1 - \frac{R^3}{32} \sin \theta_1 \wedge d\phi_1 \wedge d\psi \right), \quad (74)$$
satisfying the condition that \( h_3 = h_3 \). The topology of the worldvolume of this M5-brane is \( AdS_3 \times M_3 \). And notice that the \( M_3 \) part is the same as the one in subsection 4.2.

The induced metric reads

\[
ds^2 = \frac{R^2}{4y^2} \left( -dt^2 + dx_1^2 + (1 + f'^2)dy^2 \right) + \frac{R^2}{8} \left( d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2 \right) + \frac{R^2}{16} \left( d\psi + \cos \theta_1 d\phi_1 \right)^2.
\]

(75)

The nonzero components of \( \mathcal{E}_{ab}^m \) are

\[
\begin{align*}
\mathcal{E}_{t}^0 &= \frac{R}{2y}, & \mathcal{E}_{x_1}^{1} &= \frac{R}{2y}, & \mathcal{E}_{y}^{2} &= \frac{Rf'}{2y}, & \mathcal{E}_{\theta_1}^{3} &= \frac{R}{2y}, \\
\mathcal{E}_{\phi_1}^{4} &= \frac{R}{2\sqrt{2}}, & \mathcal{E}_{\psi}^{5} &= \frac{R}{2\sqrt{2}}, & \mathcal{E}_{\phi_1}^{6} &= \frac{R}{2\sqrt{2}}, & \mathcal{E}_{\psi}^{7} &= \frac{R}{2\sqrt{2}}, \quad (76)
\end{align*}
\]

Now we have

\[
k^m_n = \begin{pmatrix}
-2a^2I_{3\times 3} & 0 \\
0 & 2a^2I_{3\times 3}
\end{pmatrix},
\]

(78)

\[
Q = 1 - \frac{2}{3} \text{Tr} k^2 = 1 - 16a^4.
\]

(79)

The nonzero components of \( G_{mn} \) are

\[
G_{mn} = (1 + 4a^2)^2 g_{mn}, \quad (80)
\]

when \( m, n \in \{t, x_1, y\} \), and

\[
G_{mn} = (1 - 4a^2)^2 g_{mn}, \quad (81)
\]

when \( m, n \in \{\theta_1, \phi_1, \psi\} \). The nonzero components of \( P_{\alpha}^\mathcal{C} \) are

\[
\begin{align*}
P_{2}^{2} &= \frac{1}{1 + f'^2}, \\
P_{3}^{2} &= \frac{f'^2}{1 + f'^2}, \\
P_{2}^{3} &= \frac{P_{2}^{2}}{f'^2} = \frac{-f'}{1 + f'^2}, \\
P_{5}^{5} &= P_{2}^{2} = P_{8}^{8} = P_{9}^{9} = 1.
\end{align*}
\]

(82)

(83)

(84)

(85)

The 3-form field \( H_3 \) is

\[
H_3 = aR^3 \sqrt{\frac{1 + f'^2}{2y^3(1 + 4a^2)}} dy \wedge dt \wedge dx_1 - \frac{aR^3 \sin \theta_1}{8(1 - 4a^2)} d\theta_1 \wedge d\phi_1 \wedge d\psi.
\]

(86)

The Bianchi identity implies that \( a \) should be a constant. Under this condition, the tensor equations are satisfied and the only non-trivial condition given by the scalar equations is

\[
\frac{y}{\sqrt{1 + f'^2}} \left( -\frac{3f'}{y} + \frac{f''}{1 + f'^2} \right) = \frac{12a}{1 + 4a^2}.
\]

(87)
For the special case, \( f(y) = \kappa y \) with \( \kappa \) a constant, we get
\[
\frac{-\kappa}{\sqrt{1 + \kappa^2}} = \frac{4a}{1 + 4a^2},
\] (88)
when \( \kappa = 0 \), it gives \( a = 0 \). When \( \kappa \neq 0 \), we have
\[
a = \frac{\pm 1 - \sqrt{1 + \kappa^2}}{2\kappa}.
\] (89)
We also notice that when we choose the plus sign in the above equation, the limit of \( \kappa \to 0 \) gives \( a \to 0 \). We now discuss the BPS condition in the special case when \( f = \kappa y \).

Now \( \Gamma_{M5} \) becomes
\[
\Gamma_{M5} = \frac{1}{\sqrt{1 + \kappa^2}} \left( \kappa \Gamma_{01245} + \Gamma_{01345} - 2a(\kappa \Gamma_{012} + \Gamma_{013}) \right) - 2a \Gamma_{452}.
\] (90)
After some computation using Eq. (36) and the projection conditions Eq. (37), we find that \( \Gamma_{M5} \eta = \eta \) is equivalent to the projection conditions
\[
\begin{align*}
\Gamma_2 \eta_+ &= \mp \eta_+, & \Gamma_2 \eta_- &= \mp \eta_-.
\end{align*}
\] (91)
The signs of the right hand side of above two equations follow the choice of the sign in Eq. (89). Since these projection conditions are compatible with the projection conditions in Eq. (37). This M5-brane solution is half-BPS.

In [55], M5-brane with worldvolume \( AdS_3 \times M_3 \) embedded in \( AdS_4 \times M_7 \) has been shown to be half-BPS provided that \( M_7 \) is a weak \( G_2 \) manifold and \( M_3 \) is an associate submanifold. Consider the three form
\[
\Phi = \frac{1}{32} \left( d\psi + \sum_{i=1}^{3} \cos \theta_i d\phi_i \right) \wedge \sum_{i=1}^{3} \left( d\theta_i \wedge \sin \theta_i d\phi_i \right)
\] (92)
in \( Q^{1,1,1} \), one can easily show that
\[
d\Phi = -4 \ast \Phi,
\] (93)
and
\[
\Phi|_{M_3} = dvol_{M_3}.
\] (94)
This shows explicitly that \( Q^{1,1,1} \) is a weak \( G_2 \) manifold as Sasaki-Einstein manifolds are special cases of \( G_2 \) manifolds and \( M_3 \) used here is in fact an associate submanifold. So our results are consistent with the ones in [55]. We also notice that similar BPS M5-brane with worldvolume \( AdS_3 \times S^3 \) in \( AdS_4 \times S^7 \) was studied in [44, 56].
6 Discussions

In this work, we studied some solutions of the complicated M5-brane equations of motion in the M-theory background $AdS_4 \times Q^{1,1,1}$. For the two M5-brane solutions whose worldvolme has factor $AdS_2$, we found that both of them are non-BPS by studying the projection conditions. Our experiences indicate that there seems no BPS M5-branes with such $AdS_2$ factor. It would be interesting to establish such general no-go results for $AdS_4 \times Q^{1,1,1}$ and more general background with 8 supercharges. It is also interesting to study such M5-branes with $AdS_2$ factor in $AdS_4 \times Y^7$ with $Y^7$ a 3-Sasakian manifolds (M5-brane with an $AdS_3$ factor in $AdS_4 \times N(1,1)$ was studied in [57]).

The M5-branes with an $AdS_3$ factor [44, 45, 55–57] are believed to dual to some domain walls in the field theory. It will be interesting to give more concrete description of these domain walls since now we have known much more about the dual superconformal field theory. The BPS nature of these M5-branes would allow us to establish the detailed correspondence between the computations in the bulk theory and in the boundary field theory.

Acknowledgments

Z. L. would like to thank prof. Chuan-Jie Zhu for his generous support, guidance and encouragement. He would also like to thank the ICTP for financial support for participating in ‘Spring School on Superstring Theory and Related Topics’. J. W. would like to thank Meng-Qi Zhu for collaborations on related topics and ICTS-USTC for warm hospitality during a recent visit. This work was in part supported by NSFC Grant No. 11275010(B. C.), No. 11335012(B. C.), No. 11325522(B. C.), No. 11105154(D. L. and J. W.), No. 11222549(D. L. and J. W.) and No. 11135006(Z. L.). J. W. gratefully acknowledges the support of K. C. Wong Education Foundation and Youth Innovation Promotion Association, CAS as well.
Appendix: connection coefficients

The spin connections with respect to the vielbeins (26) are

\[ \omega_{01}^1 = \frac{2}{R} \tanh \frac{\rho}{\cosh u}, \quad \omega_{01}^2 = \frac{2}{R} \tanh \frac{u e}{\cosh u}, \quad \omega_{02}^1 = \frac{2}{R} \tanh \frac{\rho}{e}, \quad \omega_{02}^2 = \frac{2}{R} \coth \frac{ue}{\cosh u}, \]

\[ (95) \]

\[ \omega_{11}^2 = \frac{1}{R} (-2 \sqrt{2} \cot \theta_1 e^2 + e^1), \quad \omega_{12}^2 = \frac{1}{R} (-2 \sqrt{2} \cot \theta_2 e^2 + e^1), \]

\[ (96) \]

The Levi-Civita connection coefficients of the induced metric (40) are

\[ \Gamma^t_{tt} = \tanh \rho, \quad \Gamma^u_{tt} = \sinh \rho \cosh u, \]

\[ (101) \]

\[ \Gamma^\theta_{\phi_1 \phi_1} = -\frac{1}{2} \sin \theta_1 \cos \theta_1, \quad \Gamma^\theta_{\phi_1 \phi_2} = \frac{1}{4} \sin \theta_1 \cos \theta_2, \]

\[ (102) \]

\[ \Gamma^\theta_{\phi_2 \phi_1} = -\frac{1}{2} \sin \theta_2 \cos \theta_1, \quad \Gamma^\theta_{\phi_2 \phi_2} = \frac{1}{4} \sin \theta_2 \cos \theta_2, \]

\[ (103) \]

\[ \Gamma^\phi_{\theta_1 \phi_1} = \frac{\sin 2 \theta_1 (\cos 2 \theta_2 - 7)}{8(\cos 2 \theta_1 + \cos 2 \theta_2 - 2)}, \]

\[ (104) \]

\[ \Gamma^\phi_{\theta_1 \phi_2} = \frac{\cos \theta_1 \sin \theta_2 (\cos 2 \theta_2 + 5)}{4(\cos 2 \theta_1 + \cos 2 \theta_2 - 2)}, \quad \Gamma^\phi_{\theta_2 \phi_1} = \frac{\cos \theta_1 \sin \theta_2 (\cos 2 \theta_2 + 5)}{4(\cos 2 \theta_1 + \cos 2 \theta_2 - 2)}, \]

\[ (105) \]

\[ \Gamma^\phi_{\theta_2 \phi_2} = \frac{\sin 2 \theta_2 (\cos 2 \theta_2 - 7)}{8(\cos 2 \theta_1 + \cos 2 \theta_2 - 2)}, \quad \Gamma^\phi_{\theta_1 \phi_2} = \frac{\cos \theta_2 \sin \theta_1 (\cos 2 \theta_1 + 5)}{4(\cos 2 \theta_1 + \cos 2 \theta_2 - 2)} \]

\[ (106) \]
The Levi-Civita connection coefficients of the induced metric (50) are

\[ \Gamma^t_{\rho \theta} = \tanh \rho, \quad \Gamma^{\theta}_{tt} = \cosh \rho \sinh \rho, \]  
\[ \Gamma^h_{\phi_1 \phi_1} = -\frac{1}{2} \sin \theta_1 \cos \theta_1, \quad \Gamma^{\phi_1}_{\phi_1 \psi} = \frac{1}{4} \sin \theta_1, \]  
\[ \Gamma^{\psi}_{\phi_1 \theta_1} = \frac{3}{4} \cot \theta_1, \quad \Gamma^{\psi}_{\theta_1 \psi} = -\frac{1}{4 \sin \theta_1}, \]  
\[ \Gamma^{\psi}_{\phi_1 \theta_1} = -\frac{3}{4} \cot \theta_1 \cos \theta_1 - \frac{1}{2} \sin \theta_1, \quad \Gamma^{\psi}_{\theta_1 \psi} = \frac{1}{4} \cot \theta_1. \]

The nonzero components of the Christoffel symbol for the metric (63) are

\[ \Gamma^{\phi_1}_{\phi_1 \phi_1} = -\frac{1}{2} \sin \theta_1 \cos \theta_1, \quad \Gamma^{\phi_1}_{\phi_1 \psi} = \frac{1}{4} \sin \theta_1, \]  
\[ \Gamma^{\phi_1}_{\phi_1 \phi_1} = \frac{3}{4} \cot \theta_1, \quad \Gamma^{\phi_1}_{\theta_1 \psi} = -\frac{1}{4 \sin \theta_1}, \]  
\[ \Gamma^{\psi}_{\phi_1 \theta_1} = -\frac{3}{4} \cot \theta_1 \cos \theta_1 - \frac{1}{2} \sin \theta_1, \quad \Gamma^{\psi}_{\theta_1 \psi} = \frac{1}{4} \cot \theta_1. \]

The Christoffel symbol of the reduced metric (75) are

\[ \Gamma^{\phi_1}_{\phi_1 \phi_1} = -\frac{1}{2} \sin \theta_1 \cos \theta_1, \quad \Gamma^{\phi_1}_{\phi_1 \psi} = \frac{1}{4} \sin \theta_1, \]  
\[ \Gamma^{\phi_1}_{\phi_1 \phi_1} = \frac{3}{4} \cot \theta_1, \quad \Gamma^{\phi_1}_{\theta_1 \psi} = -\frac{1}{4 \sin \theta_1}, \]  
\[ \Gamma^{\psi}_{\phi_1 \theta_1} = -\frac{3}{4} \cot \theta_1 \cos \theta_1 - \frac{1}{2} \sin \theta_1, \quad \Gamma^{\psi}_{\theta_1 \psi} = \frac{1}{4} \cot \theta_1. \]
References

[1] O. Aharony, O. Bergman, D. L. Jafferis and J. Maldacena, “$\mathcal{N} = 6$ superconformal Chern-Simons-matter theories, M2-branes and their gravity duals”, JHEP 0810 091 (2008) [arXiv:0806.1218].

[2] J. Bagger and N. Lambert, “Modeling multiple M2’s”, Phys. Rev. D 75, 045020 (2007) [hep-th/0611108].

[3] J. Bagger and N. Lambert, “Gauge Symmetry and Supersymmetry of Multiple M2-Branes”, Phys. Rev. D 77, 065008 (2008) [arXiv:0711.0955].

[4] J. Bagger and N. Lambert, “Comments on Multiple M2-branes”, JHEP 0802, 105 (2008) [arXiv:0712.3738].

[5] A. Gustavsson, “Algebraic structures on parallel M2-branes”, Nucl. Phys. B 811 66 (2009) [arXiv:0709.1260].

[6] A. Gustavsson, “One-loop corrections to Bagger-Lambert theory”, Nucl. Phys. B807 315 (2009) [arXiv:0805.4443].

[7] A. Kapustin, B. Willett and I. Yaakov, “Exact Results for Wilson Loops in Superconformal Chern-Simons Theories with Matter”, JHEP 1003, 089 (2010) [arXiv:0909.4559].

[8] N. Drukker, M. Marino and P.Putrov, “From weak to strong coupling in ABJM theory”, Commun. Math. Phys. 306, 511 (2011) [arXiv:1007.3837].

[9] I. R. Klebanov and A. A. Tseytlin, “Entropy of near extremal black p-branes”, Nucl. Phys. B 475, 164 (1996) [hep-th/9604089].

[10] B. S. Acharya, J. M. Figueroa-O’Farrill, C. M. Hull and B. J. Spence, “Branes at conical singularities and holography”, Adv. Theor. Math. Phys. 2, 1249 (1999) [hep-th/9808014].

[11] D. R. Morrison and M. R. Plesser, “Nonspherical horizons. I”, Adv. Theor. Math. Phys. 3, 1 (1999) [hep-th/9810201].

[12] K. Hosomichi, K. -M. Lee, S. Lee, S. Lee and J. Park, “$\mathcal{N} = 5,6$ Superconformal Chern-Simons Theories and M2-branes on Orbifolds”, JHEP 0809, 002 (2008) [arXiv:0806.4977].

[13] O. Aharony, O. Bergman and D. L. Jafferis, “Fractional M2-branes”, JHEP 0811, 043 (2008) [arXiv:0807.4924].

[14] Y. Imamura and K. Kimura, “On the moduli space of elliptic Maxwell-Chern-Simons theories”, Prog. Theor. Phys. 120, 509 (2008) [arXiv:0806.3727].
[15] S. Terashima and F. Yagi, “Orbifolding the Membrane Action”, JHEP 0812, 041 (2008) [arXiv:0807.0368].

[16] M. Benna, I. Klebanov, T. Klose and M. Smedback, “Superconformal Chern-Simons Theories and AdS4/CFT3 Correspondence”, JHEP 0809, 072 (2008) [arXiv:0806.1519].

[17] D. Fabbri, P. Fré, L. Gualtieri, C. Reina, A. Tomasiello, A. Zaffaroni and A. Zampa, “3D superconformal theories from Sasakian seven manifolds: New nontrivial evidences for AdS4/CFT3”, Nucl. Phys. B 577, 547 (2000) [hep-th/9907219].

[18] S. Franco, A. Hanany, J. Park and D. Rodriguez-Gomez, “Towards M2-brane Theories for Generic Toric Singularities”, JHEP 0812, 110 (2008) [arXiv: 0809.3237].

[19] S. Franco, I. R. Klebanov and D. Rodriguez-Gomez, “M2-branes on Orbifolds of the Cone over Q1,1”, JHEP 0908, 033(2009) [arXiv: 0903.3231].

[20] M. Aganagic, “A Stringy Origin of M2 Brane Chern-Simons Theories”, Nucl. Phys. B 835, 1 (2010) [arXiv:0905.3415].

[21] F. Benini, C. Closet and S. Cremonesi, “Chiral flavors and M2-branes at toric CY4 singularities”, JHEP 1002, 036 (2010) [arXiv: 0911.4127].

[22] D. L. Jafferis, “Quantum corrections to $\mathcal{N}=2$ Chern-Simons theories with flavor and their AdS4 duals”, JHEP 1308 046 (2013) [arXiv: 0911.4324].

[23] S.-J. Rey and J.-T. Yee, “Macrosopic strings as heavy quarks in large $\mathcal{N}$ gauge theory and anti-de Sitter supergravity”, Eur. Phys. J. C22 (2001) 379 [hep-th/9803001].

[24] J. M. Maldacena, “Wilson loops in large $\mathcal{N}$ field theories”, Phys. Rev. Lett. 80 (1998) 4859 [hep-th/9803002].

[25] N. Drukker, J. Plefka and D. Young, “Wilson loops in 3-dimensional $\mathcal{N}=6$ supersymmetric Chern-Simons Theory and their string theory duals”, JHEP 0811, 019 (2008) [arXiv:0809.2787].

[26] S. -J. Rey, T. Suyama and S. Yamaguchi, “Wilson Loops in Superconformal Chern-Simons Theory and Fundamental Strings in Anti-de Sitter Supergravity Dual”, JHEP 0903, 127 (2009) [arXiv: 0809.3786].

[27] D. Gaiotto and X. Yin, “Notes on superconformal Chern-Simons-Matter theories”, JHEP 0708, 056 (2007) [arXiv:0704.3740].

[28] D. Berenstein and D. Trancanelli, “Three-dimensional $\mathcal{N}=6$ SCFT’s and their membrane dynamics”, Phys. Rev. D 78, 106009 (2008) [arXiv:0808.2503].

[29] B. Chen and J. -B. Wu, “Supersymmetric Wilson Loops in $\mathcal{N}=6$ Super Chern-Simons-matter theory,” Nucl. Phys. B 825, 38 (2010) [arXiv:0809.2863].
[30] N. Drukker and D. Trancanelli, “A Supermatrix model for $N = 6$ super Chern-Simons-matter theory”, JHEP 1002, 058 (2010) [arXiv:0912.3006].

[31] K.-M. Lee and S. Lee, “1/2-BPS Wilson Loops and Vortices in ABJM Model”, JHEP 1009, 004 (2010) [arXiv:1006.5589].

[32] L. Griguolo, D. Marmirol, G. Martelloni and D. Seminara, “The generalized cusp in $ABJ(M) N = 6$ Super Chern-Simons theories”, JHEP 1305, 113 (2013) [arXiv:1208.5766].

[33] V. Cardinali, L. Griguolo, G. Martelloni and D. Seminara, “New supersymmetric Wilson loops in $ABJ(M)$ theories”, Phys. Lett. B 718, 615 (2012) [arXiv:1209.4032].

[34] N. Kim, “Supersymmetric Wilson loops with general contours in $ABJM$ theory”, Mod. Phys. Lett. A 28, 1350150 (2013) [arXiv:1304.7660].

[35] D. H. Correa, J. Aguiler-Damia and G. A. Silva, “Strings in $AdS_4 \times CP^3$, Wilson loops in $N=6$ super Chern-Simons-matter and Bremsstrahlung functions”, [arXiv:1405.1396].

[36] D. Farquet and J. Sparks, “Wilson loops and the geometry of matrix models in $AdS_4/CFT_3$”, JHEP 1401 (2014) 083 [arXiv:1304.0784].

[37] J.-B. Wu and M.-Q. Zhu, “BPS $M_2$-branes in $AdS_4 \times Q^{1,1,1}$ Dual to Loop Operators”, [arXiv:1312.3030].

[38] I. R. Klebanov, S. S. Pufu and T. Tesileanu, “Membranes with Topological Charge and $AdS_4/CFT_3$ Correspondence”, Phys. Rev. D 81, 125011 (2010) [arXiv:1004.0413].

[39] N. Benishti, D. Rodriguez-Gomez and J. Sparks, “Baryonic symmetries and $M5$-branes in the $AdS_4/CFT_3$ correspondence”, JHEP 1007, 024 (2010) [arXiv:1004.2045].

[40] N. Kim and J. H. Lee, “Multispin membrane solutions in $AdS_4 \times Q^{1,1,1}$”, Int. J. Mod. Phys. A 26, 1019 (2011).

[41] N. Drukker, J. Gomis and D. Young, “Vortex Loop Operators, $M2$-branes and Holography”, JHEP 0903, 004 (2009) [arXiv:0810.4344].

[42] R. C. Myers, “Dielectric-Branes”, JHEP 9912, 022 (1999) [hep-th/9910053].

[43] B. Chen, W. He, J.-B. Wu and L. Zhang, “$M5$-branes and Wilson Surfaces”, JHEP 0708, 067 (2007) [arXiv:0707.3978].

[44] O. Lunin, “1/2-BPS states in $M$ theory and defects in the dual CFTs”, JHEP 0710, 014 (2007) [arXiv:0704.3442].
[45] C.-h. Ahn, “$\mathcal{N} = 2$ SCFT and M theory on $AdS_4 \times Q^{1,1,1}$”, Phys. Lett. B 466, 171 (1999) [hep-th/9908162].

[46] P. S. Howe, E. Sezgin and P. C. West, “Covariant field equations of the M-theory five-brane”, Phys. Lett. B 399 (1997) 49 [hep-th/9702008].

[47] E. Sezgin and P. Sundell, “Aspects of the M$5$-brane”, [hep-th/9902171].

[48] P. S. Howe and E. Sezgin, “Superbranes”, Phys. Lett. B 390 (1997) 133 [hep-th/9607227]; “$D = 11, p = 5$”, Phys. Lett. B 394 (1997) 62 [hep-th/9611008].

[49] C. S. Chu and E. Sezgin, “M-Fivebrane from the open supermembrane”, JHEP 9712, 001 (1997) [hep-th/9710223].

[50] M. Cederwall, B. E. W. Nilsson and P. Sundell, “An action for the super-5-brane in $D = 11$ supergravity”, JHEP 9804 (1998) 007 [hep-th/9712059].

[51] I. Bandos, K. Lechner, A. Nurmagambetov, P. Pasti, D. Sorokin and M. Tonin, “On the equivalence of different formulations of the M Theory Five-brane”, Phys. Lett. B 408 135 [hep-th/9703127].

[52] D. S. Berman, “M-theory branes and their interactions”, Phys. Rept. 456, 89 (2008) [arXiv:0710.1707].

[53] P. Hoxha, R. R. Martinez-Acosta and C. N. Pope, “Kaluza-Klein consistency, Killing vectors and Kahler spaces”, Class. Quant. Grav. 17, 4207 (2000) [hep-th/0005172]

[54] A. Donos, and J. P. Gauntlett, “Supersymmetric quantum criticality supported by baronic charges”, JHEP 1210, 120 (2012) [arXiv: 1208.1494].

[55] S. Yamaguchi, “AdS branes corresponding to superconformal defects”, JHEP 0306, 002 (2003) [hep-th/0305007].

[56] B. Chen, “The Self-dual String Soliton in $AdS_4 \times S^7$ spacetime”, Eur. Phys. J. C 54, 489 (2008) [arXiv:0710.2593].

[57] M. Fujita, “M5-brane defect and quantum Hall effect in $AdS_4 \times N(1,1)/N = 3$ superconformal field theory”, Phys. Rev. D 83, 105016 (2011) [arXiv:1011.0154].