Gauge invariant formulation of massive totally symmetric fermionic fields in (A)dS space

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Abstract

Massive arbitrary spin totally symmetric free fermionic fields propagating in $d$-dimensional (Anti)-de Sitter space-time are investigated. Gauge invariant action and the corresponding gauge transformations for such fields are proposed. The results are formulated in terms of various mass parameters used in the literature as well as the lowest eigenvalues of the energy operator. We apply our results to a study of partial masslessness of fermionic fields in $(A)dS_d$, and in the case of $d = 4$ confirm the conjecture made in the earlier literature.

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1 Introduction

Conjectured duality \[1\] of conformal \(\mathcal{N} = 4\) SYM theory and superstring theory in \(AdS_5 \times S^5\) Ramond-Ramond background has led to intensive study of field (string) dynamics in AdS space. By now it is clear that in order to understand the conjectured duality better it is necessary to develop powerful approaches to study of field (string) dynamics in AdS space. Light-cone approach is one of the promising approaches which might be helpful to understand AdS/CFT duality better. As is well known, quantization of Green-Schwarz superstrings propagating in flat space is straightforward only in the light-cone gauge. Since, by analogy with flat space, we expect that quantization of the Green-Schwarz \(AdS\) superstring with Ramond - Ramond flux \[2\] will be straightforward only in a light-cone gauge \[3\] we believe that from the stringy perspective of AdS/CFT correspondence the light-cone approach to field dynamics in \(AdS\) is a fruitful direction to go. Light-cone approach to dynamics of massive fields in \(AdS\) space was developed in \[4, 5\] and a complete description of massive arbitrary spin bosonic and fermionic fields in \(AdS_5\) was obtained in \[6\]. Unfortunately, this is not enough for a complete study of the AdS/CFT correspondence because in order to apply the light-cone approach to study of superstring in \(AdS\) space we need a light-cone formulation of field dynamics in \(AdS_5 \times S^5\) Ramond-Ramond background. Practically useful and self-contained way to give a light-cone gauge description is to start with a Lorentz covariant and gauge invariant description of field dynamics in \(AdS_5 \times S^5\) Ramond-Ramond background and then to impose the light-cone gauge. Our experience led us to conclusion that the most simple way to develop light-cone approach in \(AdS_5 \times S^5\) space is to start with gauge invariant description of fermionic fields. It turns out, however, that gauge invariant description of massive fermionic fields (with fixed but arbitrary spin) even in \(AdS_5\) is still not available in the literature. In this letter we develop Lagrangian Lorentz covariant and gauge invariant formulation\[1\] for massive totally symmetric arbitrary spin fermionic fields in \((A)dS_d\) space. We believe that our results will be helpful to find a gauge invariant description of arbitrary spin fields in \(AdS_5 \times S^5\) case. Our approach allows us to study fermionic fields in \(AdS_d\) space and \(dS_d\) space on an equal footing. In this letter we apply our results to study of partial masslessness of fermionic fields in \((A)dS_d\). For \(d = 4\) our results confirm the conjecture made in Ref.\[7\].

Before proceeding to the main theme of this letter let us mention briefly the approaches which could be used to discuss gauge invariant action for fields in \((A)dS\). Since the works \[8]-[11]\ devoted to massless fields in \(AdS_d\) various descriptions of massive and massless arbitrary spin fields in \((A)dS\) have been developed. In particular, an ambient space formulation was discussed in \[12]-[13]\ and various BRST formulations were studied in \[14]-[17]\. The frame-like formulations of free fields which seems to be the most suitable for formulation of the theory of interacting fields in \((A)dS\) was developed in \[18-19]\. Other interesting formulations of higher spin theories were also discussed recently in \[20]-[23]\. In this paper we adopt the approach of Ref.\[24\] devoted to the bosonic fields in \((A)dS\). This approach turns out to be the most useful for our purposes.

2 Gauge invariant action of massive fermionic field in \((A)dS\)

In \(d\)-dimensional \((A)dS_d\) space the massive totally symmetric arbitrary spin fermionic field is labelled by one mass parameter and by one half-integer spin label \(s + \frac{1}{2}\), where \(s > 0\) is an integer number. To discuss Lorentz covariant and gauge invariant formulation of such field we introduce Dirac complex-valued tensor-spinor spin \(s' + \frac{1}{2}\) fields of the \(so(d − 1, 1)\) Lorentz algebra \(\psi^{A_1...A_{s'}\alpha}\),

\[\text{Sometimes, a gauge invariant approach to massive fields is referred to as a Stueckelberg approach.}\]
$s' = 0, 1, \ldots, s$ (where $A = 0, 1, \ldots, d - 1$ are flat vector indices of the $so(d - 1, 1)$ algebra), i.e. we start with a collection of the tensor-spinor fields

$$
\sum_{s'=0}^{s} \oplus \psi^{A_1 \ldots A_{s'}} \alpha.
$$

(2.1)

In order to obtain the gauge invariant description of a massive field in an easy–to–use form, let us introduce a set of the creation and annihilation operators $\alpha^A$, $\zeta$ and $\bar{\alpha}^A$, $\bar{\zeta}$ defined by the relations

$$
[\bar{\alpha}^A, \alpha^B] = \eta^{AB}, \quad [\bar{\zeta}, \zeta] = 1, \quad \bar{\alpha}^A |0\rangle = 0, \quad \bar{\zeta} |0\rangle = 0,
$$

(2.2)

where $\eta^{AB}$ is the mostly positive flat metric tensor. The oscillators $\alpha^A$, $\bar{\alpha}^A$ and $\zeta$, $\bar{\zeta}$ transform in the respective vector and scalar representations of the $so(d - 1, 1)$ Lorentz algebra. The tensor-spinor fields (2.1) can be collected into a ket-vector $|\psi\rangle$ defined by

$$
|\psi\rangle \equiv \sum_{s'=0}^{s} \zeta^{s-s'} |\psi_{s'}\rangle ,
$$

(2.3)

$$
|\psi_{s'}\rangle \equiv \alpha^{A_1} \ldots \alpha^{A_{s'}} \psi^{A_1 \ldots A_{s'}} (x) |0\rangle .
$$

(2.4)

Here and below spinor indices are implicit. The ket-vector $|\psi_{s'}\rangle$ (2.4) satisfies the constraint

$$
(\alpha^A \bar{\alpha}^A - s') |\psi_{s'}\rangle = 0 , \quad s' = 0, 1, \ldots, s ,
$$

(2.5)

which tells us that $|\psi_{s'}\rangle$ is a degree $s'$ homogeneous polynomial in the oscillator $\alpha^A$. The tensor-spinor field $|\psi_{s'}\rangle$ is subjected the basic algebraic constraint

$$
\gamma \bar{\alpha} \bar{\alpha}^2 |\psi_{s'}\rangle = 0 , \quad s' = 0, 1, \ldots, s , \quad \gamma \bar{\alpha} \equiv \gamma^A \bar{\alpha}^A , \quad \bar{\alpha}^2 \equiv \bar{\alpha}^A \bar{\alpha}^A ,
$$

(2.6)

which tells us that $|\psi_{s'}\rangle$ is a reducible representation of the Lorentz algebra $so(d - 1, 1)$\footnote{2 We use oscillator formulation \cite{10,25} to handle the many indices appearing for arbitrary spin fields. It can also be reformulated as an algebra acting on the symmetric-spinor bundle on the manifold $M$ \cite{21}. Note that the scalar oscillators $\zeta$, $\bar{\zeta}$ arise naturally by a dimensional reduction \cite{15} from flat space. Our oscillators $\alpha^A$, $\bar{\alpha}^A$, $\zeta$, $\bar{\zeta}$ are respective analogs of $dx^\mu$, $\partial^\mu$, $du$, $\partial_u$ of Ref. \cite{21} dealing, among other thing, with massless fermionic fields in $(A)dS$. We thank A.Waldron for pointing this to us.}. Note that for $s' = 0, 1, 2$ the constraint (2.6) is satisfied automatically. In terms of the ket-vector $|\psi\rangle$ (2.3) the algebraic constraints (2.5), (2.6) take the form

$$
(N_\alpha + N_\zeta - s) |\psi\rangle = 0 ,
$$

(2.7)

$$
\gamma \bar{\alpha} \bar{\alpha}^2 |\psi\rangle = 0 ,
$$

(2.8)

$$
N_\alpha \equiv \alpha^A \bar{\alpha}^A , \quad N_\zeta \equiv \zeta \bar{\zeta} .
$$

(2.9)

Equation (2.7) tells us that $|\psi\rangle$ is a degree $s$ homogeneous polynomial in the oscillators $\alpha^A$, $\zeta$.

Lagrangian for the massive fermionic field in $(A)dS_d$ space we found takes the form

$$
\mathcal{L} = \mathcal{L}_{der} + \mathcal{L}_m .
$$

(2.10)
where $L_{\text{der}}$ stands for a derivative depending part of $\mathcal{L}$, while $L_m$ stands for a mass part of $\mathcal{L}$,

\[ ie^{-1} L_{\text{der}} = \langle \psi | L | \psi \rangle, \tag{2.11} \]
\[ ie^{-1} L_m = \langle \psi | M | \psi \rangle. \tag{2.12} \]

The standard first-derivative differential operator $L$ which enters $L_{\text{der}}$ (2.11) is given by

\[ L \equiv \partial - \alpha D \gamma \bar{\alpha} - \gamma \alpha \bar{\alpha} D + \gamma \alpha \partial \gamma \bar{\alpha} + \frac{1}{2} \gamma \alpha \alpha D \bar{\alpha}^2 + \frac{1}{2} \alpha^2 \gamma \bar{\alpha} \bar{\alpha} D - \frac{1}{4} \alpha^2 \partial^2 \bar{\alpha}^2, \tag{2.13} \]

where we use the notation

\[ \begin{align*}
\gamma \alpha &\equiv \gamma^A \alpha^A, \\
\gamma \bar{\alpha} &\equiv \gamma^A \bar{\alpha}^A, \\
\alpha^2 &\equiv \alpha^A \alpha^A, \\
\bar{\alpha}^2 &\equiv \bar{\alpha}^A \bar{\alpha}^A,
\end{align*} \tag{2.14} \]

and $\partial$ stands for the Lorentz covariant derivative

\[ \partial \equiv \gamma^A D_A, \quad \alpha D \equiv \alpha^A D^A, \quad \bar{\alpha} D \equiv \bar{\alpha}^A D^A, \quad D_A \equiv e^\mu_A D_\mu, \tag{2.15} \]

where $M$ is given by

\[ M_{AB} = M_b^{AB} + \frac{1}{2} \gamma^{AB}, \quad M_b^{AB} \equiv \alpha^A \bar{\alpha}^B - \alpha^B \bar{\alpha}^A, \quad \gamma^{AB} \equiv \frac{1}{2} (\gamma^A \gamma^B - \gamma^B \gamma^A). \tag{2.16} \]

The $\omega^{AB}_\mu$ is the Lorentz connection of $(A)dS_d$ space, while a spin operator $M^{AB}$ forms a representation of the Lorentz algebra $so(d - 1, 1)$:

\[ M^{AB} = M_b^{AB} + \frac{1}{2} \gamma^{AB}, \quad M_b^{AB} \equiv \alpha^A \bar{\alpha}^B - \alpha^B \bar{\alpha}^A, \quad \gamma^{AB} \equiv \frac{1}{2} (\gamma^A \gamma^B - \gamma^B \gamma^A). \tag{2.17} \]

We note that our derivative depending part of the Lagrangian $L_{\text{der}}$ (2.11) is nothing but a sum of the Lagrangians of Ref. [9] for the tensor-spinor fields (2.1). We now proceed with discussion of the mass operator $M$ (2.12). The operator $M$ is given by

\[ \mathcal{M} = (1 - \gamma \alpha \gamma \bar{\alpha} - \frac{1}{4} \alpha^2 \bar{\alpha}^2) \bar{m}_1 + \bar{m}_4 (\gamma \alpha \bar{\zeta} - \frac{1}{2} \alpha^2 \bar{\zeta} \gamma \bar{\alpha}) \]
\[ - (\zeta \gamma \bar{\alpha} - \frac{1}{2} \gamma \alpha \zeta \bar{\alpha}) \bar{m}_4, \tag{2.18} \]

where operators $\bar{m}_1$, $\bar{m}_4$ do not depend on the $\gamma$-matrices and $\alpha$-oscillators, and take the form

\[ \bar{m}_1 = \frac{2s + d - 2}{2s + d - 2 - 2N_\zeta} m, \tag{2.19} \]
\[ \bar{m}_4 = \left( \frac{2s + d - 3 - N_\zeta}{2s + d - 4 - 2N_\zeta} F(m, s, N_\zeta) \right)^{1/2}. \tag{2.20} \]

Function $F(m, s, N_\zeta)$ depends on a mass parameter $m$, spin $s$ and operator $N_\zeta$, and is given by

\[ F(m, s, N_\zeta) = m^2 + \rho \left( s + \frac{d - 4}{2} - N_\zeta \right)^2. \tag{2.21} \]

$F$ is restricted to be positive and throughout this paper, unless otherwise specified, we use the convention$^3$

\[ \rho = \begin{cases} 
-1 & \text{for AdS space}, \\
0 & \text{for flat space}, \\
+1 & \text{for dS space}. \end{cases} \tag{2.22} \]

\[^4\langle \psi \rangle \text{ is defined according the rule } \langle \psi \rangle = \langle (\psi) \rangle^1 \gamma^0. \text{ We use } e \equiv \text{det } e^A_\mu, \text{ where } e^A_\mu \text{ is vielbein of } (A)dS_4 \text{ space.} \]

$^3$Thus our Lagrangian gives description of massive fermionic fields in $(A)dS$ space and flat space on an equal footing. Discussion of massive fermionic fields in flat space in framework of BRST approach may be found in [26, 31].
The mass parameter $m$ is a freedom of our solution, i.e. gauge invariance allows us to find Lagrangian completely by module of mass parameter as it should be for the case of massive fields.

Now we discuss gauge symmetries of the action

$$S = \int d^d x \, \mathcal{L}.$$  \hspace{1cm} (2.23)

To this end we introduce parameters of gauge transformations $\epsilon^{A_1 \ldots A_{s'} \alpha}$, $s' = 0, 1, \ldots, s - 1$ which are $\gamma$-traceless (for $s' > 0$) Dirac complex-valued tensor-spinor spin $s' + \frac{1}{2}$ fields of the $so(d-1,1)$ Lorentz algebra, i.e. we start with a collection of the tensor-spinor fields

$$\sum_{s' = 0}^{s-1} \oplus \epsilon^{A_1 \ldots A_{s'} \alpha}, \quad \gamma^A e^{A_2 \ldots A_{s'} \alpha} = 0, \quad \text{for} \ s' > 0.$$  \hspace{1cm} (2.24)

As before to simplify our expressions we use the ket-vector of gauge transformations parameter

$$\langle \epsilon \rangle \equiv \sum_{s' = 0}^{s-1} \zeta^{s-1-s'} |\epsilon_{s'} \rangle,$$  \hspace{1cm} (2.25)

$$|\epsilon_{s'} \rangle \equiv \alpha^{A_1} \ldots \alpha^{A_{s'}} \epsilon^{A_1 \ldots A_{s'} \alpha}(x)|0 \rangle.$$  \hspace{1cm} (2.26)

The ket-vector $|\epsilon \rangle$ satisfies the algebraic constraints

$$(N_{\alpha} + N_{\zeta} - s + 1)|\epsilon \rangle = 0,$$  \hspace{1cm} (2.27)

$$\gamma \bar{a}|\epsilon \rangle = 0.$$  \hspace{1cm} (2.28)

The constraint (2.27) tells us that the ket-vector $|\epsilon \rangle$ is a degree $s - 1$ homogeneous polynomial in the oscillators $\alpha^{A}, \zeta$, while the constraint (2.28) respects the $\gamma$-tracelessness of $|\epsilon \rangle$.

Now the gauge transformations under which the action (2.23) is invariant take the form

$$\delta|\psi \rangle = (\alpha D + \mathcal{F})|\epsilon \rangle,$$  \hspace{1cm} (2.29)

$$\mathcal{F} \equiv \zeta \tilde{f}_1 + \gamma \alpha \tilde{f}_2 + \alpha^2 \tilde{f}_3 \zeta,$$  \hspace{1cm} (2.30)

where operators $\tilde{f}_1, \tilde{f}_2, \tilde{f}_3$ do not depend on the $\gamma$-matrices and $\alpha$-oscillators, and take the form

$$\tilde{f}_1 = \left( \frac{2s + d - 3 - N_{\zeta}}{2s + d - 4 - 2N_{\zeta}} F(m, s, N_{\zeta}) \right)^{1/2},$$  \hspace{1cm} (2.31)

$$\tilde{f}_2 = \frac{2s + d - 2}{(2s + d - 2N_{\zeta})(2s + d - 4 - 2N_{\zeta})^m},$$  \hspace{1cm} (2.32)

$$\tilde{f}_3 = -\left( \frac{2s + d - 3 - N_{\zeta}}{(2s + d - 4 - 2N_{\zeta})^3} F(m, s, N_{\zeta}) \right)^{1/2},$$  \hspace{1cm} (2.33)

and $F$ is defined in (2.21). Thus we expressed our results in terms of the mass parameter $m$. Since there is no commonly accepted definition of mass in $(A)dS$ we relate our mass parameter $m$ with various mass parameters used in the literature. One of the most-used definitions of mass, which we denote by $m_D$, is obtained from the following expansion of mass part of the Lagrangian:

$$ie^{-1} \mathcal{L}_m = \langle \psi_s | m_D | \psi_s \rangle + \ldots,$$  \hspace{1cm} (2.34)
where dots stand for terms involving $|\psi_{s'}\rangle$, $s' < s$, and for contribution which vanishes while imposing the constraint $\gamma \bar{\alpha} |\psi\rangle$. Comparing (2.34) with (2.18), (2.19) leads then to the identification

$$m = m_D.$$  

(2.35)

Another definition of mass parameter for fermionic fields in $AdS_d$ [5], denoted by $m$, can be obtained by requiring that the value of $m = 0$ corresponds to the massless fields. For the case of spin $s + \frac{1}{2}$ field in $AdS_d$ the mass parameter $m$ is related with $m_D$ as

$$m_D = m + s + \frac{d - 4}{2} \quad \text{for} \quad AdS_d,$$  

(2.36)

where $m > 0$ corresponds to massive unitary irreps of the $so(d-1,2)$ algebra [12, 5]. Below we demonstrate that natural generalization of (2.36) which is valid for both $AdS$ and $dS$ spaces is given by

$$m_D = m + \sqrt{-\rho(s + \frac{d - 4}{2})} \quad \text{for} \quad (A)dS_d.$$  

(2.37)

Since sometimes in the case of $AdS$ the formulation in terms of the lowest eigenvalue of energy operator $E_0$ is preferable we now express our results in terms of $E_0$. To this end we use the relation found in [5]:

$$m = E_0 - s - d + \frac{5}{2} \quad \text{for} \quad AdS_d.$$  

(2.38)

Making use then (2.35), (2.36) we get for the case of $AdS_d$ the desired relations

$$m = E_0 - \frac{d - 1}{2},$$  

(2.39)

$$F = (E_0 - s - d + \frac{5}{2} + N\zeta)(E_0 + s - \frac{3}{2} - N\zeta).$$  

(2.40)

### 3 Limit of massless fields in $(A)dS_d$

In previous section we presented the action for the massive field. In limit as the mass parameter $m$ tends to zero our Lagrangian leads to the Lagrangian for massless field in $(A)dS_d$. Let us discuss the massless limit in detail. To realize limit of massless field in $(A)dS_d$ we take (see (2.35), (2.37))

$$m \to \sqrt{-\rho(s + \frac{d - 4}{2})} \quad \text{and} \quad m \to 0.$$  

(3.1)

We now demonstrate that this limit leads to appearance of the invariant subspace in $|\psi\rangle$ (2.3) and this invariant subspace, denoted by $|\psi^{m=0}\rangle$, is given by the leading $(s' = s)$ term in (2.3):

$$|\psi^{m=0}\rangle = |\psi_s\rangle.$$  

(3.2)

All that is required is to demonstrate that in the limit (3.1), the ket-vector $|\psi^{m=0}\rangle$ satisfies the following requirements: i) $|\psi^{m=0}\rangle$ is invariant under action of the mass operator $M$; ii) the gauge transformation of the ket-vector $|\psi^{m=0}\rangle$ becomes the standard gauge transformation of massless field. To this end we note that an action of the mass operator $M$ on $|\psi_s\rangle$ and the gauge transformation of $|\psi_s\rangle$ take the form

$$M|\psi_s\rangle = (1 - \gamma \alpha \bar{\alpha} - \frac{1}{4} \alpha^2 \bar{\alpha}^2) \tilde{m}_1(0)|\psi_s\rangle + (\gamma \alpha - \frac{1}{2} \alpha^2 \gamma \bar{\alpha}) \tilde{m}_4(0)|\psi_{s-1}\rangle,$$  

(3.3)

$$\delta|\psi_s\rangle = (\alpha D + \gamma \alpha \tilde{f}_2(0))|\epsilon_{s-1}\rangle + \alpha^2 \tilde{f}_3(0)|\epsilon_{s-2}\rangle.$$  

(3.4)
where \( \tilde{m}_{1,4}(0) \) (3.3) and \( \tilde{f}_{2,3}(0) \) (3.4) stand for \( \tilde{m}_{1,4} (2.19), (2.20) \) and \( \tilde{f}_{2,3} (2.32), (2.33) \) in which we set \( N_c = 0 \). Taking into account

\[
\lim_{m \to 0} \tilde{m}_4(0) = 0, \quad \lim_{m \to 0} \tilde{f}_3(0) = 0, \tag{3.5}
\]

we see that if \( m = 0 \) then the ket-vector \( |\psi_s\rangle \) is indeed invariant under action of the mass operator \( \mathcal{M} \) and a realization of the mass operator on the ket-vector \( |\psi_s\rangle \) takes the form

\[
\mathcal{M}^{m=0} = \sqrt{-\rho}(s + d - 4) - \frac{1}{2}(1 - \gamma \alpha \gamma \bar{\alpha} - \frac{1}{4} \alpha^2 \bar{\alpha}^2), \tag{3.6}
\]

while the relations (3.4), (3.5) lead to the gauge transformation

\[
\delta |\psi^{m=0}\rangle = (\alpha D + \sqrt{-\rho} \gamma \alpha) |\epsilon_{s-1}\rangle, \tag{3.7}
\]

which is noting but the standard gauge transformation of massless field in \((A)dS\) space. Thus the Lagrangian for massless spin \( s + \frac{1}{2} \) fermionic field in \((A)dS_d\) space takes the form\(^6\)

\[
ie^{-1} \mathcal{L} = \langle \psi^{m=0}|L + \mathcal{M}^{m=0}|\psi^{m=0}\rangle, \tag{3.8}
\]

where \( |\psi^{m=0}\rangle \) is given by (3.2), (2.4), while the operators \( L \) and \( \mathcal{M}^{m=0} \) are defined by (2.13) and (3.6) respectively. The remaining ket-vectors \( |\psi_{s-1}\rangle, \ldots, |\psi_0\rangle \) in (2.3) decouple in the massless limit and they describe spin \( s - \frac{1}{2} \) massive field, i.e. in the massless limit the generic field \( |\psi\rangle \) is decomposed into two decoupling systems - one massless spin \( s + \frac{1}{2} \) field and one massive spin \( s - \frac{1}{2} \) field. Adopting (2.34) for spin \( s - \frac{1}{2} \) field we find mass of the massive spin \( s - \frac{1}{2} \) field: \( m = \sqrt{-\rho}(s + (d - 2)/2) \).

\section{Partial masslessness of fermionic fields in \((A)dS_d\)}

Here we apply our results to study of partial masslessness\(^7\) of fermionic fields in \((A)dS_d\). We confirm conjecture of Ref.[7] for \( d = 4 \) and obtain an generalization to the case of arbitrary \( d > 4 \). In this section we assume that the \( \rho \) (2.22) takes the values \( \pm 1 \). We start our discussion of partial masslessness of fermionic fields with simplest case of (see also Ref.[34]).

**Massive spin \( 5/2 \) field.** Such field is described by ket-vectors \( |\psi_2\rangle, |\psi_1\rangle, |\psi_0\rangle \) (see (2.3)). For spin \( 5/2 \) field there is one critical value of \( m \) which leads to appearance of partial massless field. For this critical value of \( m \) the generic field \( |\psi\rangle \) is decomposed into one partial massless field and one massive spin \( \frac{5}{2} \) field. To demonstrate this we consider the gauge transformations (2.29),

\[
\delta |\psi_2\rangle = \alpha D |\epsilon_1\rangle + \gamma \alpha \tilde{f}_2(0) |\epsilon_1\rangle + \alpha^2 \tilde{f}_3(0) |\epsilon_0\rangle, \tag{4.1}
\]

\[
\delta |\psi_1\rangle = \alpha D |\epsilon_0\rangle + \tilde{f}_1(0) |\epsilon_1\rangle + \gamma \alpha \tilde{f}_2(1) |\epsilon_0\rangle, \tag{4.2}
\]

\[
\delta |\psi_0\rangle = \tilde{f}_1(1) |\epsilon_0\rangle, \tag{4.3}
\]

\(^6\)Our Lagrangian (3.8) is a generalization to \( d \)-dimensions of the Lagrangian of Ref.[9] for massless field in \((A)dS_4\). Alternative Lagrangian descriptions of massless fermionic fields in \((A)dS_d\) may be found in [11, 21].

\(^7\) Partial masslessness was discovered in [32]. Recent discussion of this theme and to some extent complete list of references may be found in [33].
where \( \tilde{f}_{1,2,3}(n) \) are given in (2.31)-(2.33) in which we set \( s = 2 \) and argument \( n \) stands for an eigenvalue of the operator \( N_c \). The critical value of \( m \) is obtained from the requirement of decoupling of the field \( |\psi_0\rangle \). This requirement amounts to the equation \( \tilde{f}_1(1) = 0 \) which leads to the critical value
\[
m_{(0)}^2 = -\rho \left( 1 + \frac{d - 4}{2} \right)^2. \tag{4.4}
\]
For this value of \( m \) the generic field \( |\psi\rangle \) is decomposed into two decoupling systems - one partial massless field described by \( |\psi_2\rangle, |\psi_1\rangle \) and one massive spin \( \frac{1}{2} \) field described by \( |\psi_0\rangle \). We proceed with discussion of partial masslessness for

**Massive spin 7/2 field.** Spin 7/2 field \( |\psi\rangle \) is described by ket-vectors \( |\psi_3\rangle, |\psi_2\rangle, |\psi_1\rangle, |\psi_0\rangle \) (see (2.3)). The gauge transformations (2.29) for these ket-vectors take the form
\[
\delta|\psi_3\rangle = \alpha D|\epsilon_2\rangle + \gamma \alpha \tilde{f}_2(0)|\epsilon_2\rangle + \alpha^2 \tilde{f}_3(0)|\epsilon_1\rangle, \tag{4.5}
\]
\[
\delta|\psi_2\rangle = \alpha D|\epsilon_1\rangle + \tilde{f}_1(0)|\epsilon_2\rangle + \gamma \alpha \tilde{f}_2(1)|\epsilon_1\rangle + 2\alpha^2 \tilde{f}_3(1)|\epsilon_0\rangle, \tag{4.6}
\]
\[
\delta|\psi_1\rangle = \alpha D|\epsilon_0\rangle + \tilde{f}_1(1)|\epsilon_1\rangle + \gamma \alpha \tilde{f}_2(2)|\epsilon_0\rangle, \tag{4.7}
\]
\[
\delta|\psi_0\rangle = \tilde{f}_1(2)|\epsilon_0\rangle, \tag{4.8}
\]
where expressions for \( \tilde{f}_{1,2,3}(n) \) are given in (2.31)-(2.33) in which we set \( s = 3 \) and argument \( n \) stands for an eigenvalue of the operator \( N_c \). For the spin 7/2 field there are two critical values of \( m \). For each critical value of \( m \) the generic field \( |\psi\rangle \) is decomposed into one partial massless field and one massive field. We consider these critical values in turn.

First critical value of \( m \) is obtained from the requirement of decoupling of the field \( |\psi_0\rangle \) (see (4.8)). This requirement amounts to the equation \( \tilde{f}_1(2) = 0 \) which leads to the critical value
\[
m_{(0)}^2 = -\rho \left( 1 + \frac{d - 4}{2} \right)^2. \tag{4.9}
\]
For this value of \( m \) the generic field \( |\psi\rangle \) is decomposed into two decoupling systems - one partial massless field described by \( |\psi_3\rangle, |\psi_2\rangle, |\psi_1\rangle \) and one massive spin \( \frac{1}{2} \) field \( |\psi_0\rangle \).

The second critical value of \( m \) is obtained from the requirement of decoupling of the fields \( |\psi_1\rangle, |\psi_0\rangle \) (see (4.6),(4.7)). This requirement amounts to the equations \( \tilde{f}_1(1) = 0, \tilde{f}_3(1) = 0 \) which lead to the critical value
\[
m_{(1)}^2 = -\rho \left( 2 + \frac{d - 4}{2} \right)^2. \tag{4.10}
\]
For this \( m \) the generic field \( |\psi\rangle \) is decomposed into one partial massless field described by \( |\psi_3\rangle, |\psi_2\rangle \) and one massive spin \( \frac{1}{2} \) field described by \( |\psi_1\rangle, |\psi_0\rangle \). We finish with partial masslessness for

**Massive arbitrary spin \( s + \frac{1}{2} \) field.** Such field is described by ket-vectors \( |\psi_{s'}\rangle, s' = 0, 1, \ldots s \). Gauge transformations (2.29) for these ket-vectors take the form
\[
\delta|\psi_{s'}\rangle = \alpha D|\epsilon_{s'-1}\rangle + \tilde{f}_1(s - s' - 1)|\epsilon_{s'}\rangle + \gamma \alpha \tilde{f}_2(s - s')|\epsilon_{s'-1}\rangle + (s - s' + 1)\alpha^2 \tilde{f}_3(s - s')|\epsilon_{s'-2}\rangle, \tag{4.11}
\]
where \( \tilde{f}_{1,2,3}(n) \) are given in (2.31)-(2.33) and argument \( n \) stands for an eigenvalue of the operator \( N_c \). For values \( s' = 0, 1, s \) (4.11) we use the convention \( |\epsilon_{-2}\rangle = |\epsilon_{-1}\rangle = |\epsilon_{s}\rangle = 0 \). For the spin \( s + \frac{1}{2} \) field there are \( s - 1 \) critical values of \( m \), denoted by \( m_{(n)}, n = 0, 1, \ldots, s - 2 \) (the case of
\( n = s - 1 \) leads to massless field and was considered in Section 3). For each \( m_{(n)} \) the generic field \( |\psi\rangle \) is decomposed into one partial massless field and one spin \( n + \frac{1}{2} \) massive field. To find \( m_{(n)} \) we note that the requirement of decoupling of the fields \( |\psi_n\rangle, \ldots, |\psi_0\rangle, n = 0, \ldots, s - 2 \) amounts to equations \( \tilde{f}_1(s - n - 1) = 0, \, \tilde{f}_5(s - n - 1) = 0 \). Solution to these equations is

\[
m_{(n)}^2 = -\rho \left( n + 1 + \frac{d - 4}{2} \right)^2,
\]

(4.12)
is in agreement with conjecture made in Ref.\([7]\) for the case of \( d = 4 \). Thus we confirmed conjecture of Ref.\([7]\) and obtained \( m_{(n)} \) for \( d > 4 \). For each \( m_{(n)} \) the generic field \( |\psi\rangle \) is decomposed into two decoupling systems - one partial massless field \( |\psi_{par}^{(s)}\rangle \) described by \( |\psi_s\rangle, \ldots, |\psi_{n+1}\rangle \), and one massive spin \( n + \frac{1}{2} \) field \( |\psi_{msv}^{(n)}\rangle \) described by \( |\psi_n\rangle, \ldots, |\psi_0\rangle \). This is to say that by decomposing \( |\psi\rangle \) (2.3) into the respective ket-vectors

\[
|\psi_{par}^{(s)}\rangle \equiv \sum_{s'=n+1}^{s} \zeta^{s-s'} |\psi_{s'}\rangle, \quad |\psi_{msv}^{(n)}\rangle \equiv \sum_{s'=0}^{n} \zeta^{s-s'} |\psi_{s'}\rangle,
\]

(4.13)
one can make sure that if \( m = m_{(n)} \) then the mass part of the Lagrangian (2.12) is factorized

\[
ie^{-1} \mathcal{L}_m = \langle \psi_{par}^{(s)} | \mathcal{M} | \psi_{par}^{(s)} \rangle + \langle \psi_{msv}^{(n)} | \mathcal{M} | \psi_{msv}^{(n)} \rangle.
\]

(4.14)

For values \( m = m_{(n)} \) the gauge transformations (2.29) are also factorized, while \( \mathcal{L}_{der} \) (2.11) is factorized for arbitrary \( m \).8

## 5 Uniqueness of Lagrangian for massive fermionic field

We now demonstrate that the Lagrangian and gauge transformations are uniquely determined by requiring that the action be gauge invariant. We formulate our statement. Suppose the derivative depending part of the Lagrangian is given by (2.12), while the derivative depending part of gauge transformations (2.29) is governed by \( \alpha D\text{-term} \). Suppose the gauge field |\psi\rangle (2.3) and the gauge transformations parameter |\epsilon\rangle (2.25) satisfy the respective constraints (2.8, 2.28). Then we state that the mass operator \( \mathcal{M} \) given in (2.18) and the operator \( \mathcal{F} \) which enters gauge transformations (2.29) are uniquely determined by the following requirements: i) the action be gauge invariant; ii) there are no invariant subspaces in |\psi\rangle under action of gauge transformations. Here we outline prove of this statement. We start with general form of the mass operator \( \mathcal{M} \) and the operator \( \mathcal{F} \):

\[
\mathcal{M} = m_1 + \gamma \alpha m_2 \bar{\alpha} + \alpha^2 m_3 \bar{\alpha}^2 + \gamma \alpha m_4 + \alpha^2 m_5 \gamma \bar{\alpha}
- m_4 \gamma \bar{\alpha} - \gamma \alpha m_5 \bar{\alpha}^2 + \alpha^2 m_6 + m_6 \bar{\alpha}^2,
\]

(5.1)

\[
\mathcal{F} = f_1 + \gamma \alpha f_2 + \alpha^2 f_3,
\]

(5.2)

8 We note that the key point is not positivity or even reality of the mass part of action (2.12), but rather stability of the energy and unitarity of the underlying physical representations. For bosons a negative mass term is allowed in AdS (the Breitenlohner-Freedman bound, \([35]\)), while partially massless fermions even have an imaginary mass term in their actions but are still stable and unitary in dS. Partially massless fermions are not unitary in AdS (see Refs.\([7]\)[34][36]).

9 m- and m- masses of the field |\psi_{par}^{(s)}\rangle are given by: \( m = \sqrt{-\rho(n + 1 + (d - 4)/2)} \), \( m = \sqrt{-\rho(n + 1 - s)} \), while for the field |\psi_{msv}^{(n)}\rangle we get \( m = \sqrt{-\rho(s + (d - 2)/2)} \), \( m = \sqrt{-\rho(s - n + 1)} \).
where \( m_{1,\ldots,6} \) and \( f_{1,2,3} \) do depend on \( \gamma \)-matrices and \( \alpha \)-oscillators, and are given by
\[
\begin{align*}
    m_1 &= \tilde{m}_1, & m_2 &= \tilde{m}_2, & m_3 &= \tilde{m}_3, & m_4 &= \tilde{m}_4 \gamma, & m_5^\dagger &= \zeta \tilde{m}_5^\dagger; \\
    m_5 &= \tilde{m}_5 \gamma, & m_5^\dagger &= \zeta \tilde{m}_5^\dagger, & m_6 &= \tilde{m}_6 \gamma^2, & m_6^\dagger &= \zeta^2 \tilde{m}_6^\dagger,
\end{align*}
\]  
(5.3)
(5.4)

Operators \( \tilde{m}_{1,\ldots,6} \) and \( \tilde{f}_{1,2,3} \) depend only on \( N_\zeta \) (2.9). \( \tilde{m}_{1,\ldots,6}^\dagger \) stand for hermitian conjugated of \( \tilde{m}_{1,\ldots,6} \). Since \( \tilde{m}_{1,2,3} \) are hermitian, \( \tilde{m}_{1,2,3}^\dagger = \tilde{m}_{1,2,3} \), and depend only on the hermitian operator \( N_\zeta \) the operators \( \tilde{m}_{1,2,3} \) are real-valued functions of \( N_\zeta \) from the very beginning. These properties of \( \tilde{m}_{1,\ldots,6} \) and \( \tilde{f}_{1,2,3} \) and expressions for \( \mathcal{M} \) (5.1), \( \mathcal{F} \) (5.2) are obtained by requiring that:

i) \( \mathcal{M} \) and \( \mathcal{F} \) commute with the spin operator of the Lorentz algebra \( M^{AB} \) (2.17) and satisfy the commutators \([N_\alpha + N_\zeta, \mathcal{M}] = 0, [N_\alpha + N_\zeta, \mathcal{F}] = \mathcal{F} \);

ii) \( \mathcal{M} \) does not involve terms like \( \alpha^2 \gamma \alpha p_1 \) and \( p_2 \alpha^2 \gamma \alpha \), where \( p_{1,2} \) are polynomial in the oscillators (such terms in view of (2.8) do not contribute to \( \mathcal{L}_m \));

iii) \( \mathcal{F} \) does not involve terms like \( \alpha^2 \gamma \alpha p_3 \), \( p_4 \gamma \alpha \), where \( p_{3,4} \) are polynomial in the oscillators (the \( p_3 \)-terms lead to violation of constraint (2.8) for gauge transformed field (2.29), while the \( p_4 \)-terms in view of (2.28) do not contribute to \( \delta \psi \) (2.29));

iv) \( \mathcal{M} \) and hermitian conjugated of \( \mathcal{M} \) satisfy the relation \( \mathcal{M}^\dagger = -\gamma^0 \mathcal{M} \gamma^0 \).

Thus all that is required is to find dependence of the operators \( \tilde{m}_{1,\ldots,6} \) and \( \tilde{f}_{1,2,3} \) on \( N_\zeta \). We now demonstrate that this dependence can be determined by requiring that the action be gauge invariant. We evaluate the variation of the action (2.10), (2.23) under gauge transformations (2.29), (5.2),
\[
\delta S = -i \int d^4 x \epsilon \langle \psi | \left( \mathcal{D} X_1 + \gamma \alpha \bar{\alpha} DX_2 + \alpha DX_3 + \gamma \alpha \mathcal{D} X_4 + \alpha^2 \bar{\alpha} DX_5 \\
+ \alpha^2 \mathcal{D} X_6 + \alpha D \gamma \alpha X_7 + \alpha^2 \alpha DX_8 + \bar{\alpha} DX_9 \\
+ Y^{(0)} + \gamma \alpha (Y^{(1)} + Y^{(1)}_{(A)ds}) + \alpha^2 Y^{(2)} \right) |\psi \rangle + h.c.,
\]  
(5.6)

where we use the notation
\[
\begin{align*}
    X_1 &= f_1 - m_4^\dagger, & X_2 &= -f_1 - 2m_5^\dagger, & X_3 &= -(2s + d - 4 - 2N_\zeta) f_2 + m_1, & X_4 &= (2s + d - 4 - 2N_\zeta) f_2 + m_2, \\
    X_5 &= \frac{1}{2} (2s + d - 4 - 2N_\zeta) f_2 + 2m_3, & X_6 &= -\frac{1}{2} (2s + d - 4 - 2N_\zeta) f_3 + m_5, & X_7 &= (2s + d - 4 - 2N_\zeta) f_3 + m_4, \\
    X_8 &= m_6, & X_9 &= 2m_6^\dagger, \\
    Y^{(0)} &= m_1 f_1 - (2s + d - 2N_\zeta) m_4^\dagger f_2,
\end{align*}
\]  
(5.7)
(5.8)
(5.9)
(5.10)
(5.11)
(5.12)
(5.13)
(5.14)
\[ Y^{(1)} = m_1 f_2 + (2s + d - 2 - 2N_\zeta) m_2 f_2 + m_4 f_1 \]
\[ - 2m_4^t f_3 - 2(2s + d - 2 - 2N_\zeta) m_5^t f_3 , \tag{5.15} \]
\[ Y^{(2)} = m_1 f_3 + 2m_2 f_3 + 2(2s + d - 4 - 2N_\zeta) m_3 f_3 \]
\[ + m_4 f_2 + (2s + d - 4 - 2N_\zeta) m_5 f_2 , \tag{5.16} \]
\[ Y^{(1)}_{(A)_{ds}} = -\frac{i}{4} (2s + d - 3 - 2N_\zeta)(2s + d - 4 - 2N_\zeta) . \tag{5.17} \]

Requiring this variation to vanish gives the equations\(^\text{10}\)

\[ X_a = 0 , \quad a = 1, \ldots , 9 , \tag{5.18} \]
\[ Y^{(0)} = 0 , \tag{5.19} \]
\[ Y^{(1)} + Y^{(1)}_{(A)_{ds}} = 0 , \tag{5.20} \]
\[ Y^{(2)} = 0 . \tag{5.21} \]

Solution to Eqs. (5.18) is easily found to be

\[ m_1 = (2s + d - 4 - 2N_\zeta) f_2 , \quad m_4 = -(2s + d - 4 - 2N_\zeta) f_3 , \tag{5.22} \]
\[ m_2 = - m_1 , \quad m_3 = - \frac{1}{4} m_1 , \quad m_5 = - \frac{1}{2} m_4 , \tag{5.23} \]
\[ m_4^t = f_1 , \quad m_5^t = - \frac{1}{2} f_1 , \quad m_6 = m_6^t = 0 , \tag{5.24} \]

i.e. Eqs. (5.18) allow us to express the operators \( m_a , a = 1, \ldots , 5 \), entirely in terms of the operators \( f_2 , f_3 \) which enter the gauge transformations. Moreover, the expressions for \( m_4 \) (5.22) and \( m_4^t \) (5.24) imply the relation

\[ f_1^t = -(2s + d - 4 - 2N_\zeta) f_3 . \tag{5.25} \]

We now analyze (5.19). Inserting \( m_1 \) (5.22) and \( m_4 \) (5.24) in (5.19) we cast (5.19) in the form

\[ f_1 \left( (2s + d - 6 - 2N_\zeta) f_2^t - (2s + d - 2 - 2N_\zeta) f_2 \right) = 0 , \tag{5.26} \]

where \( f_2 \equiv f_2(N_\zeta) \) and \( f_2^t \equiv f_2(N_\zeta + 1) \). Solution to (5.26) \( f_1 = 0 \) for all \( N_\zeta \) (or \( f_1 = 0 \) for some particular eigenvalues of \( N_\zeta \)) leads to massless fields (or partial massless fields), i.e. such solution leads to appearance of invariant subspaces in \( |\psi\rangle \). We are not interested in such solution and assume \( f_1 \neq 0 \) for all \( N_\zeta \). Equation (5.26) allows us then to find dependence of the operator \( f_2 \) on \( N_\zeta \):

\[ f_2 = \frac{f_{2(0)}}{(2s + d - 2 - 2N_\zeta)(2s + d - 4 - 2N_\zeta)} , \tag{5.27} \]

where \( f_{2(0)} \) is dimensionfull parameter not depending on \( N_\zeta \). Inserting \( f_2 \) (5.27) in (5.21) one can make sure that (5.21) is satisfied automatically. All that remains then is to solve Eq. (5.20). Making

\(^\text{10}\) Expressions for \( Y^{(1)} \)'s given in (5.14), (5.17) are valid provided that the equations \( X_8 = X_9 = 0 \) are satisfied.
use of (5.22)-(5.25) and (5.27) one can make sure that (5.20) amounts to the equation

\[ Z(N_\xi) - Z(N_\xi - 1) - \frac{(2s + d - 3 - 2N_\xi)(f_2(0))^2}{(2s + d - 2 - 2N_\xi)^2(2s + d - 4 - 2N_\xi)^2} = \frac{\rho}{4}(2s + d - 3 - 2N_\xi) = 0, \]  

where we use the notation

\[ Z(N_\xi) \equiv (2s + d - 4 - 2N_\xi)(N_\xi + 1)(\tilde{f}_3)^2. \]  

The relation (5.29) implies a condition \( Z(-1) = 0 \). This condition and (5.28) lead to the initial condition

\[ Z(0) = \frac{(2s + d - 3)(f_2(0))^2}{(2s + d - 2)^2(2s + d - 4)^2} + \frac{\rho}{4}(2s + d - 3). \]  

Equation (5.28) and the initial condition (5.30) allow us to find \( Z(N_\xi) \) uniquely and taking into account (5.29) we obtain

\[ (\tilde{f}_3)^2 = \frac{2s + d - 3 - N_\xi}{2s + d - 4 - 2N_\xi} \left( \frac{(f_2(0))^2}{(2s + d - 2)^2(2s + d - 4 - 2N_\xi)^2} + \frac{\rho}{4} \right). \]  

Thus we satisfied all equations imposed on \( M \) and \( F \) by the requirement of gauge invariance of the action and the expressions (5.22)-(5.25), (5.27), (5.31) determine \( M \) and \( F \) uniquely. In view of the first relation in (5.22) and (5.27) the \( f_2(0) \) is real-valued and introducing the mass parameter \( m \) (which is assumed to be positive) by relation

\[ f_2(0) = (2s + d - 2)m, \]  

we arrive at the expressions for \( M \) and \( F \) given in Section 2.

To summarize, we found the gauge invariant action for the fermionic fields in \((A)\text{dS}_d\). All that remains to construct action for fermionic fields in \( AdS_5 \times S^5 \) Ramond-Ramond background is to add appropriate dependence of \( S^5 \)-coordinates and take into account contribution of Ramond-Ramond background fields\(^2\). The result will be reported elsewhere.

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### Appendix A  Notation and commutators of oscillators and covariant derivative

We use \( 2^{[d/2]} \times 2^{[d/2]} \) Dirac gamma matrices \( \gamma^A \) in \( d \)-dimensions, \( \{ \gamma^A, \gamma^B \} = 2\eta^{AB}, \gamma^A \dagger = \gamma^0 \gamma^A \gamma^0 \), where \( \eta^{AB} \) is mostly positive flat metric tensor and flat vectors indices of the \( so(d - 1, 1) \) algebra

\(^{11}\) It is easy to demonstrate that making use of field redefinitions, the phase factors of \( \tilde{f}_3 \) can be normalized to be equal to \(-1\). Therefore in (5.28) and below \( \tilde{f}_3 \) is assumed to be real-valued and negative. Relations (5.3) and Eq. (5.25) imply then that \( f_1 \) is real-valued and positive.

\(^{12}\) Study of some leading contributions of Ramond-Ramond background fields to mass operator of the bosonic fields in \( AdS_5 \times S^5 \) Ramond-Ramond background may be found in [37]. Precise form of mass operator for bosonic fields is still to understood.
take the values $A, B = 0, 1, \ldots, d - 1$. To simplify our expressions we drop $\eta_{AB}$ in scalar products, i.e. we use $X^A Y^A \equiv \eta_{AB} X^A Y^B$. Indices $\mu, \nu = 0, 1, \ldots d - 1$ stand for indices of space-time base manifold.

We use the algebra of commutators for operators that can be constructed out the oscillators $\alpha^A$, $\bar{\alpha}^A$ (2.2) and derivative $D^A$ (2.15), (2.16) (see also Appendix A in Ref. [4]). Starting with

$$[\hat{\partial}_A, \hat{\partial}_B] = \Omega_{AB}^C \hat{\partial}_C, \quad \Omega^{ABC} \equiv -\omega^{ABC} + \omega^{BAC}, \quad \omega^{ABC}_A \equiv \epsilon^A_\mu \omega^{\mu BC},$$

(A.1)

where $\hat{\partial}_A \equiv \epsilon^\mu_A \partial_\mu$, $\partial_\mu \equiv \partial/\partial x^\mu$, and $\Omega^{ABC}$ is a contorsion tensor we get the basic commutator

$$[D^A, D^B] = \Omega^{ABC} D^C + \frac{1}{2} R^{ABCD} M^{CD},$$

(A.2)

and $R^{ABCD}$ is a Riemann tensor which for $(A)dS_d$ geometry takes the form

$$R^{ABCD} = \rho(\eta^{AC} \eta^{BD} - \eta^{AD} \eta^{BC}).$$

(A.3)

The spin operator $M^{AB}$ is given in (2.17). For flexibility in (A.2) and below we present our relations for a space of arbitrary geometry and for $(A)dS_d$ space. Using (A.2) and the commutators

$$[D^A, \alpha^B] = -\omega^{ABC} \alpha^C, \quad [D^A, \bar{\alpha}^B] = -\omega^{ABC} \bar{\alpha}^C, \quad [D^A, \gamma^B] = -\omega^{ABC} \gamma^C,$$

(A.4)

we find straightforwardly

$$[D^A, \alpha^2] = 0, \quad [\bar{\alpha}^2, D^A] = 0, \quad [D^A, \gamma \alpha] = 0,$$

(A.5)

$$[\bar{\alpha}^2, \alpha D] = 2\bar{\alpha} D, \quad [\gamma \bar{\alpha}, \alpha D] = \ell \phi, \quad \{\ell \phi, \gamma \alpha\} = 2\alpha D,$$

(A.6)

$$\ell \phi^2 = D^A D^A + \omega^{AAB} D^B + \frac{1}{4} \gamma^{AB} R^{ABCD} M^{CD} = D^A D^A + \omega^{AAB} D^B + \frac{\rho}{2} \gamma^{AB} M^{AB},$$

(A.7)

$$[\bar{\alpha} D, \alpha D] = D^AD^A + \omega^{AAB} D^B - \frac{1}{4} R^{ABCD} M_{b}^{AB} M^{CD} = D^A D^A + \omega^{AAB} D^B - \frac{\rho}{2} M_{b}^{AB} M^{AB},$$

(A.8)

$$[\ell \phi, \alpha D] = \frac{1}{2} \gamma^A \alpha^B R^{ABCD} M^{CD} = \rho \left( \gamma \alpha (\alpha \bar{\alpha} + \frac{d - 1}{2}) - \alpha^2 \gamma \bar{\alpha} \right),$$

(A.9)

$$[\bar{\alpha} D, \ell \phi] = -\frac{1}{2} \gamma^A \bar{\alpha}^B R^{ABCD} M^{CD} = \rho \left( (\alpha \bar{\alpha} + \frac{d - 1}{2}) \gamma \bar{\alpha} - \gamma \alpha \bar{\alpha}^2 \right).$$

(A.10)
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