Topological planning for autonomous MMGs: an ordered binary decision diagram-based approach

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Abstract: This paper finds its motivation from the perspective of utility planners, where the flexibility of choice while reconfiguring a distribution system into Multi-Microgrids (MMGs) is critical. To address the topology planning problem for MMGs from this dimension, a holistic algorithm incorporating graph theory, ordered binary decision diagrams, and a modified Gauss Siedel power flow algorithm is proposed. This multi-indexed, multi-tier scheme guarantees self-adequacy and reliability of individual microgrids (MGs) by effectively solving a modified balanced partitioning problem, and ensures validation of inequality constraints through power flow studies. The available solutions are ranked according to two system-level performance indices to provide a hierarchically feasible solution list to system planner. This algorithm is presented as an alternate tool for heuristic techniques to ensure a complete search of solution space and to increase the number of available topology planning solutions for the system operator. The flexibility of choice offered by this planning algorithm, tendency to be employed as a resource assessment tool, and incorporation of power flow analysis for autonomous MGs renders it to be practically superior to its counterparts. The proposed algorithm is tested to furnish its advantages and applicability in practical utility systems.

Nomenclature

- \(A\): boolean expression representing all paths between node \(i\) and node \(j\)
- \(A^k\): boolean expression representing a path of length \(k\) between node \(i\) and node \(j\)
- \(d\): active power tolerance threshold in each microgrid
- \(E\): set of edges of the graph
- \(e\): reactive power tolerance threshold in each microgrid
- \(G(V, E, W)\): graph function
- \(I_B\): set of vertices containing energy storage devices
- \(I_{x,y}\): transmission line joining bus \(x\) and bus \(y\)
- \(m_p\): frequency droop coefficient
- \(N\): total number of nodes in the graph
- \(n_q\): voltage droop coefficient
- \(P_{k,i}\): active power generation on the \(k\)th node of the graph
- \(P_{k,i}\): active power load on the \(k\)th node of the graph
- \(Q_{k,i}\): reactive power generation on the \(k\)th node of the graph
- \(Q_{k,i}\): reactive power load on the \(k\)th node of the graph
- \(V\): set of vertices of the graph
- \(W\): set of edge weights of the graph
- \(W_p\): set of active power weights associated with vertices
- \(W_q\): set of reactive power weights associated with vertices
- \(\otimes\): logical AND operator
- \(\oplus\): logical XOR operator

1 Introduction

Microgrids (MGs) form the backbone of the futuristic idea of smart grids. The concept of MGs has gained widespread popularity during the last decade due to an exponential increase in the penetration of distributed energy resources (DERs) in the conventional distribution system [1]. The advantages posed by DERs include an increase in the qualitative reliability and economic viability of the system have opened new research streams in active distribution systems (ADSs) [2], their operation as MGs, and more recently, the planning, operation, and economics associated with multi-MG (MMG)-based ADSs. According to IEEE Standard 1547.4, MGs can be regarded as the building blocks of future ADSs [3]. These building blocks would join together to constitute a complete power network, hence forming an MMG structure.

1.1 Motivation and incitement

The effective and efficient realisation of MGs is only possible if the ADS as a whole has been meticulously planned. Conventional distribution system planning techniques including distribution system expansion planning and distribution network reconfiguration cannot be applied without modifications to ADSs [4, 5]. Hence, a set of comprehensive planning tools is imperative to design and operate MMG-based ADS that has the tendency to work in both grid-connected and islanded modes of operations. This paper formulates and implements the MMG topology planning problem from the perspective of utilities trying to extract the most benefit from the distribution system while simultaneously ensuring the supply security and service reliability to customers. Partitioning a distribution system into individual MGs and operating them in an MMG environment is instrumental for both the utility and the customers. By optimising the number of components in the designed MGs, reliability of the overall system increases concerning annual average repair duration [6]. Sectionalisation strategies not only decrease the latency in communication network but also render the system to be more suitable for decentralised control structures [7]. This decreases the number of control actions required for the system to achieve self-healing characteristics in a post-fault or post-disturbance period. One of the major advantages acquired in the perspective of system self-healing is the improved efficiency of the associated automatic fault detection algorithms [8]. If done effectively, partitioning into self-sufficient and reliable MGs can reduce the overall system losses and can hence decrease the amount of unavoidable load shedding in case of contingencies. This decrease would manifest...
itself in the form of improved security of supply and customer satisfaction [1]. With high proliferation of DERs in the current distribution systems, and exponential increase expected henceforth, the ownership and location of DERs in power networks can play a pivotal role in the reconfigured MMG topology and overall efficiency of the system. Partitioning a distribution system can serve as an effective resource assessment tool to demarcate the areas requiring further DER installations. To gain all the aforementioned critical benefits and to operate in an MMG environment, the utility can decide to draft policies that incentivise the addition of independent power producer (IPP)-owned DERs in certain areas of its footprint. This would not only streamline the incoming penetration of DERs but would also help the utility in envisaging the concept of zonal pricing for IPP-owned DERs during autonomous operation of MGs.

From the above discussion, the importance of formulating a comprehensive algorithm for sectionalisation a distribution system into MGs working in an MMG configuration is evident. This paper aims to bridge this research gap and to contribute toward the realisation of smart grid vision.

1.2 Literature review

MG planning has been studied in the literature using various approaches. Optimal placement of DERs, distributed reactive sources, and distributed energy storage resources have been one of the intensively studied areas of MG planning [8]. This research area has delved deeply into the reliability enhancement domain of MGs at the planning stage using $N-1$ contingency studies [9], mixed-integer linear optimisation models for optimal capacity and location planning of renewable and non-renewable DERs in MGs [10], and multi-node modelling approach for multi-energy MGs employing electrical power flow and heat transfer equations [11]. However, MG topology planning has recently attracted prime research flux due to its significant effect on the efficient realisation and stable operation of MGM structures. Supply adequacy is one of the basic considerations for MG topology planning and refers to such a distribution system reconfiguration, whereby each of the autonomous MGs is self-sufficient in terms of power generation. This self-sufficiency can also be modelled as an objective of minimising power flows among individual MGs and has been recently analysed using multiple techniques. For example, a graph partitioning approach has been presented in [8] to partition a distribution system into loop-based MGs. The proposed optimal topology planning method, however, results in infeasible and sub-optimal MG topologies. The design approach in [12] takes these deficiencies into account and presents a model graph partitioning algorithm for clustering a loop-based ADS into individual MGs connected via separate tie lines to the main grid. The criterion of self-adequacy for individual MGs has been incorporated to evaluate the MG boundaries. A tabu search-based algorithm is employed in [1] to optimally evaluate the position and status of switches for the objective of reconfiguring a system into multiple autonomous MGs. Kali vecti et al. [13] propose a new heuristic tool for designing the electrical systems for military forward operating bases. Owing to their inherent nature, these distribution systems are a classic example of MGs. The design phase combines graph theoretic approaches and clustering methods to partition the network into a set of stable MGs. Similarly, the concept of supply adequacy is investigated in [14] for minimising the resistive power losses within MG feeders. The structural modification of MGs through tie switches and mobile storage capability of electric vehicles is used to reconfigure MGs using social spider optimisation optimally. Another MG planning algorithm is presented in [15] to maintain a cumulative supply adequacy within each MMG structure. To this end, the optimal power routing problem is solved for exchange of power between power-excessive and power-deficient MGs within a cluster of MGs. The proposed strategy is an ideal manifestation of MMG-based ADSs.

In addition to guaranteeing supply adequacy of individual MGs in an MMG structure, the aspect of system reliability also plays an instrumental role in MG planning. Traditionally, this measure is introduced at the operational stage to evaluate the performance of utilities and power networks. However, with the introduction of reliability-based performance regulations in some countries, this metric has lately drawn major research efforts from utilities [16]. Arefifar et al. [6] devised an algorithm that not only takes into account the supply adequacy but also the system reliability aspects in a long-term MG planning problem by the application of multi-objectivity index. The reliability index comprises average interruption frequency index (SAIFI), system average interruption duration index (SAIDI), and momentary average interruption frequency index (MAIFI). To incorporate the idea of self-healing in MG planning, a method for sectionalisation a distribution system into MGs is proposed in [17]. This method, however, is a contingency planning tool since it takes into account only the faulted area of the distribution system. A unified design methodology for planning hybrid MGs is presented in [18]. This algorithm optimises the reliable power flow and amalgamates the topology planning and capacity/location planning of DERs in MGs. It is, however, worth noting that the definition of MMG topology planning is different in [18] as compared with that in this paper. The perspective of qualitative reliability in MMGs, Gazijahani and Salehi [19] propose a multi-objective particle swarm optimisation (PSO)-based algorithm for MG planning under uncertainty. The objective of reliability, modelled as minimisation of ‘energy not supplied’, along with the minimisation of various costs, constitutes the cornerstone of this work. Gazijahani and Salehi [20] propose a reconfiguration algorithm for ADSs with the objective of minimising various uncertainties in MGs and maximising the robustness of MMG structures in various scenarios. Grey Wolf optimisation is employed to discover the optimally economic and reliable topology of MG-based distribution system. Shen et al. [21], Khodaei et al. [22], and Khayatian et al. [23] detail some other approaches regarding MG planning.

A detailed literature survey shows that only the basic considerations such as supply adequacy and reliability have been accounted for in this planning problem. Even for the studies available, the planning algorithms result in a single topology for autonomous MGs, rendering the choice of topology in various operating conditions to be inflexible for the utility. Since most of these approaches do not consider the economic viability of reconfiguration, load criticality in some parts of distribution systems, physical constraints associated with switching some feeders, and various ownership profiles of DERs, a single theoretical topology planning result may not be the best practical solution in most cases. Also, the inclusion of power flow stage for autonomously operating MGs, which is critically different from its grid-connected counterpart, is found to be lacking in the reported literature. As these MGs are designed to be self-adequate and reliable, and are expected to operate in both grid-connected and autonomous (islanded) modes of operation, considering the autonomous mode power flow analysis is of critical importance for the validation of voltage and frequency constraints and calculation of MG system losses. The MMG topology planning approaches reported in the literatures [1, 6, 8] sectionalise the ADS into autonomously operating MGs; however, they employ conventional grid-connected power flow studies for the validation of the aforementioned constraints. The problems associated with this methodology along with the need to incorporate specialised power flow algorithms for autonomously operating MGs will be discussed in Section 4 of this paper.

An exhaustive literature survey identifies some key research gaps in the MMG topology planning problem that are individually accounted for in Section 2.3 of this paper. The identified research gaps from the literature are summarised as follows:

- The reported algorithms for MMG topology planning are heuristic in nature and hence generate one topology for this reconfiguration problem. Owing to those above technical and practical constraints, this single topology may not be well-suited from the perspective of utilities for system reconfiguration in all the operating scenarios. Therefore, a planning methodology is required that generates a set of MMG topologies for particular
ADS, such that all these topologies conform to the requirements of self-adequacy and reliability of the MMG structure.

- The present research on MMG topology planning does not cater to the essential need for incorporation of power flow algorithms for autonomously operating MGs. This can jeopardise the acceptable voltage and frequency envelopes of individual MGs in the real-time operation of MMGs; therefore, raising concerns about the practical validity of such studies.

This paper addresses these two major research gaps and furnishes its contributions by effectively proposing a feasible planning framework for the practical implementation of MMG topologies in ADSs.

1.3 Contribution and paper organisation

This paper presents a holistic MMG topology planning algorithm that incorporates graph theory, ordered binary decision diagrams (OBDDs), and the modified Gauss Siedel (MGS) power flow algorithm, presented in [24], for autonomous MGs. This multi-stage, multi-indexed method addresses the critical planning considerations required for the successful operation of MGs in grid-connected and autonomous modes of operation. The main contributions of this paper are listed as follows:

- The planning algorithm designed in this paper employs OBDDs and furnishes a set of MMG topologies at its culmination, instead of a single MMG topology. Each member of this set completely satisfies the criteria of self-adequacy and reliability of MMG structure, hence providing the flexibility of choice to the system operator at the time of reconfiguration.
- Autonomous mode MGs power flow algorithm is incorporated in the planning process to validate the voltage and frequency constraints of individual MGs in the MMG structure. This inclusion renders the proposed algorithm to be superior to its published counterparts in terms of applicability to practical ADSs.
- Two system-level indices – total system loss and system voltage index (SVI) – are developed for MMGs so that the resultant MMG topologies can be ranked in hierarchical order for ease of choice in any particular operating scenario.
- The effectiveness of the proposed algorithm as an effective resource assessment tool for utilities is furnished using additional studies employing variations in DER locations and capacities, and loads.

The remainder of this paper is organised as follows. Section 3 addresses some preliminaries regarding OBDD and system representation. Section 4 outlines the methodology used for solving the MMG topology planning problem along with a brief background on incorporated algorithms. Test system case study is presented in Section 5, which also includes the associated numerical results and their validation. Section 6 discusses applicability of the proposed algorithm as a resource assessment tool for utilities and summarises the sensitivity analyses on the acquired results. Section 7 concludes this paper.

2 Preliminaries and OBDD-based system representation

2.1 MMG topology planning problem

This work defines the MMG topology planning problem in the context of finding sectionalisation strategies for partitioning a distribution system into multiple MGs. These MGs would have the tendency to work either in grid-connected or in an autonomous mode of operation. Since the latter is critical to ensure successful operation of any MMG structure and hence the complete distribution system, the partitioning algorithm devised in this paper focuses primarily on the autonomous operation of individual MGs. A comprehensive system splitting strategy would require each partition to have minimum mismatch in active and reactive powers, the presence of at least one storage device in each MG, and satisfaction of bus voltage and frequency constraints. The crucial nature of these constraints and their impact on system partitioning is described in detail in [25]. However, most of the existing literatures does not take into account the mandatory presence of storage devices in MGs. This critical feature allows not only a fast response ability toward system dynamics but also provides a mechanism to cater for sudden variations in loads for autonomously working MGs. These constraints are summarised as follows:

- **Storage device constraint (SDC):** Each partitioned MG should contain at least one storage device.
- **Active power balance constraint (APBC):** Each of the sectionalised MGs should have active power mismatch within the specified tolerance limit.
- **Reactive power balance constraint (RPBC):** Each of the sectionalised MGs should have reactive power mismatch within the specified tolerance limit.
- **Inequality constraints (ICs):** Voltage profile of all buses along with the individual operating frequency of each MG should be within conventional operating limits.

A ‘Balanced Partition Problem’ as defined in [26] is solved by taking into account all the binaries above, equality, and ICs. Graph theory representation of power system is employed to initiate the algorithm for solving the MMG topology planning problem. A detailed overview of the designed solution algorithm is presented in Section 4.

2.2 OBDD-based system representation

Various methods are available for representation of boolean expressions. These methods include truth tables, binary decision trees, binary diagrams, and OBDDs. OBDD is a graphical depiction of boolean expressions and is characterised by its directed acyclic nature [27]. This representation is superior to its counterparts because of the associated compactness in comparison with decision trees or truth tables. This feature, however, depends heavily on the ordering of variables used while constructing an OBDD. The evaluation of the well known satisfiability checking problem (which is a famous NP-complete problem [28]) for boolean expressions using OBDDs can be performed in polynomial time. The problem of dividing a graph into balanced partitions is an NP-complete problem and owes its complexity to the number of variables and associated strategy space explosion. In case of distribution system balanced partitioning problem, strategy space explosion is unavoidable since a large number of transmission lines present in the system is equal to the number of variables in the satisfiability checking problem [29]. Hence, a compact and efficient representation of boolean functions in distribution system partitioning problem is critical to the solution of their associated satisfiability checking problem in a practically viable period. OBDD has been used in various applications for balanced partitioning and satisfiability checking problems including the verification of digital circuits and other finite-state systems [30]. Some of the major advantages of application of OBDD-based algorithm for MMG topology planning are mentioned below:

- The satisfaction of active and reactive power mismatch constraints in MMG-based distribution systems constitutes a typical satisfiability checking problem. OBDD has proved to be one of the most efficient and easily applicable tools for this problem.
- Most heuristic and stochastic algorithms have a tendency to generate local optimal solutions with no guarantee of not missing a globally optimal solution in a search space. OBDD, on the contrary, sorts the complete strategy space for all the possible solutions of a resolvable problem in polynomial time interval [26].
- The inherent nature of MMG topology planning problem is primarily dependent on the initial system topology. OBDD exploits this feature and provides an opportunity for the system planner to implement a variable ordering that favours the...
This section details the various components incorporated in the topology planning employs one of the graph theory approaches to

• The flexibility to divide a problem into multiple sub-problems, whose individual OBDDs can be created independently, offers a huge advantage for MMG topology planning problem. This characteristic attracts the system planner to distinguish between online and offline tasks, formulate associated individual OBDDs, and reduce the computational complexity of the overall system. The option to implement such algorithms in the system operational domain, with recomputation of only the online tasks in real time, renders an additional benefit to OBDD-based algorithms in MMG topology planning problems [27].

The above discussion shows that OBDD-based algorithms are well-suited for topology planning problems and offer a wide range of advantages over other conventionally adopted techniques.

3 MMG topology planning methodology

This section details the various components incorporated in the propagating MMG topology planning. The designed algorithm operates in three successive stages integrating graph theory, OBDD, and MGS power flow method for planning the final topology of an MMG structure. This section provides a detailed insight into these individual stages and their analogous application to the MMG topology planning problem.

3.1 Graph theory for MMG topology planning

Sectionalisation of a distribution system into individual autonomous MGs operating in an MMG structure can be regarded as a ‘System Splitting’ problem as described in [27], with regards to its objective and some of the constraints. As discussed in Section 3, the combinatorial explosion of such problems is a major concern in their solution and is unavoidable in most of the cases due to a large number of transmission lines present in the system. This limitation not only constraints the search of complete strategy space but also introduces excessive time complexity in obtaining the solution of such problems.

To address this problem, the designed algorithm for MMG topology planning employs one of the graph theory approaches to simplify the initial distribution system into a smaller representative system. For the application of graph theory, the power network has to be transformed into a representative graph given by \( G(V, E, W) \), where \( G \) denotes the graph function, \( V \) represents the set of vertices of the graph, \( E \) is the set of edges, and \( W \) is the set of weights associated with each edge of the power network graph. In this formulation, system buses are denoted by vertices, transmission lines are represented as edges, while the weight for the \( i \)th edge is given by the following equation:

\[
w_i = Z_i \times \alpha \tag{1}
\]

where \( Z_i \) depicts the impedance of the \( i \)th line and \( \alpha \) can be chosen as a reasonable number to reflect the length of the transmission line from its associated impedance. This approach finds its base in the notion that a proportionality holds between the line length and the line impedance and, hence, given a proportionality constant, one parameter can be transformed into another.

Once the graphical representation of the distribution system is complete, a modified heavy edge matching (HEM) [31] algorithm is used for the coarsening phase. HEM is extensively used in communication networks as a coarsening tool such that an iteratively chosen node from the graph is selected to be combined with one of its neighbouring nodes, depending on the criterion of minimum/maximum edge weights. The problem of MMG topology planning, however, requires a modification to this standard algorithm since the variables of interest, in this case, are edges instead of vertices of the graph. Also, the notion of edge weight in this problem, as already defined, is the length of the graph edge. This variable is chosen for system coarsening to minimise system losses in the autonomous operation of MGs by ensuring that the long transmission lines form the candidate sets of system partitioning in the next phase, hence eliminating the possibility of short lines being considered as inter-MG connections.

To address the requirements of the problem under discussion, HEM is modified to fixed shortest edge matching (FSEM) algorithm in this planning strategy. As evident from the name, this algorithm ranks the edge lengths in ascending or descending order and joins the associated vertices of the shortest \( n \) edges iteratively. The number \( n \) is user-defined and depends on the trade-off between extremely high accuracy and time complexity associated with the combinatorial explosion of search space in system splitting problem in the succeeding stages. An illustrative graph explaining FSEM is shown in Fig. 1. The randomisation in the process is eliminated so that for a fixed system topology, a reasonable coarsened graph can be obtained as an offline task without sacrificing the accuracy of the overall algorithm in real time. Once the representative coarsened graph is obtained, OBDD is applied to this graph to generate all the possible splitting strategies to divide a distribution system into MGs in an MMG structure.

3.2 OBDD-based MMG topology planning

3.2.1 System representation for OBDD application: As discussed in Section 3, MMG topology planning is a system splitting problem and a balanced partitioning formulation, since the latter is a subset of the former in terms of requisite constraints and objectives. Most of the balanced partitioning problems are formulated as satisfiability checking problems and are tested against a set of predefined conditions for satisfiability. MMG topology planning can be formulated as satisfiability checking module by defining a semi-certain boolean variable matrix or semi-certain adjacency matrix (SAM) for the graphical representation of coarsened distribution system obtained in the preceding phase. The SAM for an undirected, connected, and \( n \)-vertex graph is given by the matrix \( M \)

\[
M = \begin{bmatrix}
0 & a_{12} & a_{13} & \cdots & a_{1n} \\
a_{21} & 0 & a_{23} & \cdots & a_{2n} \\
a_{31} & a_{32} & 0 & \cdots & a_{3n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
a_{n1} & a_{n2} & a_{n3} & \cdots & 0
\end{bmatrix}
\]

As evident from the above matrix, SAM is symmetric and an element of \( M \) \( a_{ij} \) would have a certain value if vertices \( v_i \) and \( v_j \) are connected in the coarsened graph. All the non-zero elements of SAM are considered as binary decision variables in the OBDD implementation and their determined values define the opening or closing of circuit breaker associated with the transmission line \( a_{ij} \). The solution of SAM for determining the values of all decision variables partitions the distribution system into an MMG structure.

It is worth noting that the elements of SAM are different from the elements of a normal adjacency matrix developed for any
graph. This is because sectionalisation of a distribution system into various MGs would not only require opening the circuit breaker between two nodes but would also demand to open all circuit breakers of indirect lines joining the two vertices, to terminate the power flow between the vertices of interest from all possible paths. Therefore, an element of SAM defined henceforth as $A_{ij}$ can be redefined in the following expression [25]:

$$A_{ij} = I \oplus A_{ij} \oplus A_{ij} \oplus \ldots \oplus A_{ij}^m$$

(2)

where $A_{ij}$ is a boolean expression representing all paths between node $i$ and node $j$, $I$ is the identity matrix, $\oplus$ is the logical OR operator, and $A_{ij}^k$ denotes the path of length $k$ between the mentioned nodes. The above equation assumes that $m$ is the longest continuous path that joins any two nodes in the assumed graph.

3.2.2 OBDD-based MMG topology problem formulation: Section 3 details the objectives and constraints under consideration for planning the topology of an MMG structure. The formulation of binary expressions for each of these objectives is summarised in the following discussion.

SDC demands that each of the partitioned MG should have at least one storage device of specified minimum capacity. This objective can be realised by ranking the installed storage devices in terms of their capacity and choosing the top $n$ devices to be part of different MGs. This would ensure a minimum available storage capacity in all the sectionalised MGs and would also determine the number of MGs the system needs to be partitioned into, which is $n$ in this case. Assuming a set of nodes that contains the top-ranked energy storage devices in the system, given by $I_k = \{v_1, v_2, \ldots, v_n\}$, where $I_k \subseteq I$ can be defined as $I_k = \{v_i\}; k = 1, 2, \ldots, n$. Hence, SDC can be represented by the following boolean expression:

$$SDC = \bigotimes_{v_i \in I_k, v \in V} [(A_{ij}),_{ij} \bigodot \ldots \bigodot (A_{ij}),_{ij}]$$

(3)

where $\bigotimes$ refers to logical AND and $\bigodot$ refers to exclusive OR operator.

APBC can be formulated as a boolean expression in the following format:

$$APBC = \bigotimes_{w_i, \gamma \in Y} \left| (A_{ij}),_{ij} \times W_{ij} \right| \leq d$$

(4)

where $W_{ij} = \{w_1, w_2, \ldots, w_n\}$ is a set of weights associated with all the nodes (total number of nodes = $N$) in the graph and each element of this set is defined in this phase as $w_k = P_{ik} - Q_{ik}$, $k = 1, 2, \ldots, N$, which represents the difference between the active load power ($P_{ik}$) and active power generation ($P_{ik}$) on each bus of the power network. APBC can be simplified to get the following expression:

$$APBC = \prod_{i = 1, \ldots, n} \prod_{x_i \in I_k} \left| (A_{ij}),_{ij} \times W_{ij} \right| \leq d$$

(5)

A similar formulation can also be applied for RPBC with the modification of node weight matrix $W$, referred to in this case as $W_{ij}$ and given by (6) and (7). It is worth noting that the tolerance limits ($d$ and $e$) specified in (5) and (7) are different in this case but depend primarily on the objectives and planning priorities of the system planners.

$$RPBC = \bigotimes_{w_i, \gamma \in Y} \left| (A_{ij}),_{ij} \times W_{ij} \right| \leq e$$

(6)

$$RPBC = \prod_{i = 1, \ldots, n} \prod_{x_i \in I_k} \left| (A_{ij}),_{ij} \times W_{ij} \right| \leq e$$

(7)

where $W_{ij} = \{r_i, r_2, \ldots, r_n\}$ and $r_i = Q_{i1}$ is the difference between the reactive load power ($Q_{i1}$) and reactive power generation ($Q_{i1}$).

Equations (3), (5), and (7) can be used to generate three different types of OBDDs for the three objectives of MMG topology planning. Since individual OBDDs of sub-tasks can be combined using logical operators, these three OBDDs would be merged using logical AND operator to get the final OBDD of the problem. The final results of OBDD would provide all the possible combinations for splitting a distribution system into $n$ MGs such that each MG satisfies SDC, APBC, and RPBC.

3.3 Power flow analysis for autonomous MGs

In this paper, ICs are incorporated to ensure the successful operation of all MGs working in the islanded mode of operation after the distribution system has been partitioned. The MGs power flow algorithm presented in [24] is used to perform power flow studies for each of the newly designed MGs. This method has been incorporated since these MGs are no longer connected to the main grid, and therefore conventional power flow algorithms cannot be used in this case.

In the autonomous mode of operation, the frequency of the MG is not constant due to droop characteristics of DERs, and also, no slack bus exists in the system. Some power flow algorithms classify droop buses as photovoltaic buses but since in most cases the size of DER is extremely small as compared with the grid, the DER cannot act as a constant source of power for maintenance of frequency in its footprint [32]. To cater to all these distinctions, MGs power flow algorithm is used to analyse the frequency and voltage profiles of buses in each islanded MG. All the DERs are assumed to be droop based and are governed by the following equations:

$$\omega = \omega_0 - m_{\omega}P_{G0}$$

(8)

$$V = V_0 - n_{V}Q_{G0}$$

(9)

where $P_{G0}$ and $Q_{G0}$ are the generated active and reactive powers, respectively; $m_{\omega}$ and $n_{V}$ are the frequency and voltage droop coefficients, respectively.

Updating the frequency in each iteration of the power flow requires an associated modification of the $Y$ bus, which is followed by the categorisation of buses into droop or non-droop buses. Equations (8) and (9) are used to update the active and reactive power generation setpoints for DERs, and the bus voltages are calculated accordingly. The algorithm converges when the mismatch in voltage and frequency falls below a predefined tolerance threshold (a threshold value of 0.001 is used for this case).

Power flow analysis of all the constituent MGs of the partitioning combinations generated by OBDD screens out the partitioning strategies that do not satisfy ICs. The remaining strategies after this sorting form the final results of the MMG topology planning problem.

3.4 Formulation of SVI for MMGs

Total system losses and a newly defined SVI are employed to rank the various sets of MMG topologies such that the operator has a hierarchical list of available reconfiguration strategies for ease of operation and decision making. The SVI takes into account the voltage of each bus in the MMG and compares it with a reference voltage (1 pu is used in this case). The VI is calculated for each of the individual MG in an MMG structure, and the overall system performance is then compared among different available options. The index under consideration is defined as
VI = \frac{1}{N} \sum_{i=1}^{N} \left( v_{ref} - v_{bi} \right) \quad (10)

SVI = \frac{1}{M} \sum_{j=1}^{M} VI_j \quad (11)

where \( VI \) is the voltage index for a single autonomously operating MG containing \( N \) number of buses, each characterised by voltage \( v_{bi} \), and a reference voltage value of \( v_{ref} \). Similarly, to define a voltage metric for the complete MMG system, the voltage metrics of all the constituent MGs are added together with the normalisation factor of \( M \), which corresponds to the total number of MGs in the MMG structure. It is evident that a lower value of SVI would correspond to a better voltage profile for the MMG topology under consideration.

A flowchart explaining all the stages of the devised planning algorithm is shown in Fig. 2.

4 Test system and simulation studies

The well known PG&E 69-bus distribution system [33] is utilised to evaluate the proposed MMG topology planning algorithm. The test system is modified by adding DERs of fixed capacities at specific buses in the system, given by Table 1. The DER placed on bus 58 operates at a power factor of 0.9 lagging while all other DERs operate at 0.8 lagging power factor. Energy storage devices are allocated at buses 18, 33, 44, 52, 58, and 67, and the final test system is shown in Fig. 3.

Using graph theory, the test system is represented as a node weighted graph, as shown in Fig. 4a, whereas the resulting coarsened graph acquired using the FSEM algorithm is shown in Fig. 4b. To reduce the computational time of OBDD and to avoid search space explosion, the representative coarsened system is used to generate individual OBDDs for SDC, APBC, and RPBC. The values of \( d \) and \( e \) for APBC and RPBC in each MG are set to 35 kW and 75 kVAR, respectively. The sets of combinations that individually satisfy these equality constraints are attained through individual OBDDs of SDC, APBC, and RPBC. The active and reactive power mismatch for all MGs reported in this table are within the specified tolerance thresholds. To validate the ICs about voltage profiles on buses and operating frequency, all the combinations provided in Table 2 are checked for autonomous power flow of MGs and are found to satisfy the required constraints. The MGS power flow results for all MGs from the first
inherent characteristic to scan each point of the search space for the MMG topology planning problem. In case only one solution is possible candidature of the final solution. Unlike heuristic algorithms, it, therefore, generates all the possible solutions for the MMG topology planning problem. In case only one solution is obtained, it may or may not be physically possible to implement that particular solution due to various operational constraints including load criticality, system maintenance schedules etc. This is an open access article published by the IET under the Creative Commons Attribution License (http://creativecommons.org/licenses/by/3.0/)
the objective of minimum total system loss, PSO generates the optimal MMG topology which is same as the first topology reported in Table 2, and shown in Fig. 5. However, when SVI is used as the minimisation objective using PSO, topology 4 reported in Table 2 and shown in Fig. 6 is acquired as the optimal MMG topology.

A comparative analysis of the proposed algorithm with the conventional PSO algorithm validates the results and novelty of the proposed scheme. As discussed previously, the OBDD-based methodology generates a set of MMG topologies as compared with a single topology attained by its heuristic counterparts. It is worth noting that both the topologies generated by two different PSO studies are part of the final MMG topology planning results generated by OBDD. However, with SVI as the objective function, PSO does not generate the globally optimum result, which is topology 1 reported in Table 2, as is evident by the values of SVI in Table 4. This result validates the use of the proposed algorithm and lends credibility to its claim of complete search of solution space for all possible solutions of MMG topology planning problem.

5 Resource assessment and sensitivity analysis

Several studies are conducted to observe the effects of size and location of DERs along with changes in loads, on MMG topology planning studies. These studies are intended to serve as sensitivity analyses and resource assessment tools to provide an insight for the system planner and operator in terms of future generation expansion, streamlining the incoming penetration of DERs, and also the robustness of the proposed algorithm for changes on DERs’ locations and sizes. The size of each DER is varied independently to observe the effect on MMG topology, and the results are shown in Fig. 7. This figure also depicts the allowable increase in load on each bus of the test system, such that the acquired MMG topology results remain the same. It can be observed that buses 30–35 have the highest robustness to variation in DER capacity, which can be increased by 43 kVA without affecting the attained MMG topology results. On the contrary, buses 1–14 are highly sensitive to the additional generation capacity, whereby an increase of generation capacity by 1 kVA would render it impossible to partition the ADS into an MMG structure containing six self-sufficient and reliable MGs. Hence, the individual sensitivity of these buses offers critical benefit to the system planner for determining the locations for additional DERs and incentivise policies in this regard. Additionally, to gauge the effects and hence to streamline the additional increase in system load, maximum threshold for allowable increase in load is defined for each bus of the system. These values would be instrumental in defining thresholds for additional load on each bus of the system and the associated designing and implementation of corresponding policies. From Fig. 7, buses 64–69 have the highest tolerance to additional loads, whereas buses 57 and 58 offer the least flexibility for increase in load. These results are a direct implication of active and reactive power mismatch in different MGs of the MMG structure. Similarly, the effect of changing the location of DERs is observed on MMG topology results obtained by the proposed algorithm. It is evident that changing the location of DERs such that the DER contained in a particular MG is reallocated to any other bus in the same MG does not change the MMG topologies. This is a rational protraction of the fact that this reallocation will not affect $\Delta P$ and $\Delta Q$ of the constituent MGs, hence not affecting the overall MMG topology. The results of one of these studies are shown in Fig. 8, where positions of three of the installed DERs are varied to observe the effects on MMG topology planning results. In line with the presented discussion, the boundaries of MGs in the MMG configuration do not change as is evident in Fig. 8.

These studies show that the proposed algorithm is robust in terms of generated MMG topology planning results since the reallocation and sizing of DERs in particular ranges do not significantly change the MG boundaries in the MMG configuration. In addition to the robustness of acquired results, the effectiveness of this algorithm as a resource assessment tool is also furnished using these analyses.
This paper presents a novel multi-indexed, multi-stage MMG topology planning algorithm incorporating graph theory, OBDD, and MGS power flow for MGs operating in an autonomous mode of operation. The proposed algorithm addresses the aspect of MG self-sufficiency in terms of active and reactive powers and ensures validation of operational ICs by employing MGS power flow analysis. A modified distribution system containing DERs and storage devices is sectionalised employing the proposed hybrid planning methodology in order to gain multiple benefits from the hence obtained MMG structure including but not limited to the self-healing capabilities of the system and easier control. The novelty of the designed algorithm is furnished by its following characteristics:

- The attainment of multiple partitioning strategies at the culmination of planning procedure and their associated ranking based on developed system indices, rendering it a flexible methodology to be adopted by planning engineers in real-time system planning and operation.
- Furnishing the proposed algorithm as an effective resource assessment tool for DER capacity/location planning, hence streamlining the penetration of DERs in distribution systems and incentivising policy measures in this domain.
- Incorporation of MGS power flow for islanded MGs adds another previously ignored dimension to this research ensuring successful operation of each MG, in general, and of the MMG structure in particular.

Considering the benefits of the proposed algorithm in comparison with its heuristic counterparts, this approach can serve as a practical procedure for transforming an existing distribution system into an MMG structure, which would prove to be a milestone achieved for the realisation of smart grids. The adoption of the proposed planning methodology would also open new avenues for enticing research in the domain of proposing control and communication mechanisms for such MMGs, development of new market structures based on the proposed design, and formulation of comprehensive techno-economic planning frameworks for MMG-based ADSs.

7 References

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