Theoretical Study of Harmonic Drive Flexible Bearing Durability

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Abstract. The research object is a harmonic drive with a cam wave generator. Harmonic drive operation is limited by flexible gear (FG) fatigue resistance, flexible bearing (FB) durability and teeth sides wear resistance. The article describes a refined method for determining flexible bearing durability, taking into account the spatial nature of elastic interaction between a rigid gear, a flexible gear and a flexible bearing. According to the suggested method, the impact of the maximum flexible gear deformation and length on the durability of a flexible bearing of a cup-type harmonic drive with a flexible gear has been studied. The dependences of the life of the flexible bearing on the length of the flexible wheel and the maximum deformation of the flexible wheel at the nominal torque and various forms of its deformation are obtained. It is shown that small values of the flexible wheel length can significantly reduce the service life of the flexible bearing. The obtained dependencies expand the knowledge about the impact of various factors on the durability of flexible bearings of harmonic drives.

1. Introduction

The load bearing capacity of harmonic drives (HD) is conditioned by flexible gear fatigue resistance and flexible bearing durability [1-5]. With a low teeth hardness and a high gear ratio, the drive performance can be limited by teeth side wear. With the $U \geq 100...120$ gear ratio, the critical HD performance factor is wave generator durability.

Harmonic drives can have cam, disc and roller wave generators [1,4]. Cam generators, which better keep the flexible gear deformation shape are most widespread. A cam wave generator is a ball bearing with flexible gears. Flexible gears basically break down due to fatigue flaking of the inner ring (IRFB) raceway in the major deformation axis area. In this area the ball-raceway interaction force reaches its maximum.

By checking calculation of roller bearing dynamic load rating the bearing service life is determined basing on the surface fatigue flaking [6, 7]. Flexible bearing calculation features are described in [3,5,6].

2. Problem Description

Flexible bearing durability depends largely on the contact stresses impacting the raceways. These stresses depend on the radial load $F_r$ and its distribution among the balls. In conventional calculations, uneven force distribution among the balls is accounted for in the bearing dynamic load rating $C$. For single-row ball bearings, the load rating $C$ depends only on the raceway shape and ball diameter and number [6-8].
When determining the flexible bearing dynamic load rating $C$ and radial force $F_r$ impacting the cam, the impact of the change in the basic harmonic drive parameters on force distributing among the balls is not accounted for.

The purpose of this study is developing a refined method for flexible bearing durability determination taking into account the impact of the change in the basic harmonic drive.

3. Mathematical Model
A harmonic drive with a cam wave generator and a fixed rigid gear is shown in figure 1. To determine the forces impacting the balls, a spatial mathematical model of the harmonic drive was used [9]. The model accounts for the spatial nature of the interaction of its elements: a flexible gear (FG), a rigid gear (RG) and an outer ring of the flexible gear (ORFB). All of the mentioned links are taken by elastic elements deformed in accordance with Hooke's law. The calculation model accounts for the interaction between the FG and RG teeth sides, the inner surface of the FG and the outer surface of the ORFB, the inner surface of the ORFB and the balls:

![Figure 1. HD structure:](image)

1 — flexible gear; 2 — rigid gear; 3 — flexible bearing; 4 — cam

The interacting surfaces were divided with mutually perpendicular lines. Each grid point was associated with a basic function of a hexagonal pyramid form. The distributed forces of interaction between the FG and the RG teeth sides, the inner surface of the FG and the outer surface of the ORFB were substituted with linear combinations of the basic.

When composing the resolving system of equations, the following notation was used: drive element interaction grid point force vector $F = [F^{(1)T}, F^{(2)T}, F^{(3)T}]^T$; grid point gap vector $\delta = [\delta^{(1)T}, \delta^{(2)T}, \delta^{(3)T}]^T$; element displacement as rigid bodies $a = [a^{(1)T}, a^{(2)T}, a^{(3)T}]$.

Here $F^{(1)}$, $F^{(2)}$, $F^{(3)}$ are vectors of RG - FG, FG - ORFB, ORFB –balls interaction forces, respectively; $\delta^{(1)}$, $\delta^{(2)}$, $\delta^{(3)}$ are vectors of grid points gaps between the same elements; $\delta_0^{(1)}$, $\delta_0^{(2)}$, $\delta_0^{(3)}$ are initial gap (undeformed system gap) vectors; $a^{(1)} = [\Delta x^{(1)}, \Delta y^{(1)}, \Delta \varphi_z^{(1)}]^T$, $a^{(2)} = [\Delta x^{(2)}, \Delta y^{(2)}]^T$, $a^{(3)} = [\Delta x^{(3)}, \Delta y^{(3)}, \Delta \varphi_z^{(3)}, \Delta \varphi_x^{(3)}]^T$, $e = [e_x, e_y]^T$ are RG, FG, ORFB and cam displacement vectors, respectively.

The resolving system of equations is as follows
where \( D^{(1)} \), \( D^{(2)} \), \( D^{(3)} \), \( D^{(21)} \), \( D^{(23)} \), \( D^{(32)} \), \( D^{(33)} \) are grid point compliance matrices; \( G^{(1)} \), \( G^{(12)} \), \( G^{(22)} \), \( G^{(23)} \), \( G^{(33)} \), \( G^{(34)} \) are matrices binding grid point gap increments to the \( \mathbf{a}^{(1)} \), \( \mathbf{a}^{(2)} \), \( \mathbf{a}^{(3)} \), \( \mathbf{e} \) displacement vectors; \( C^{(1)} \), \( C^{(22)} \) are RG and FG shaft rigidity matrices; \( \mathbf{B} \) is an external load vector, 
\[
\mathbf{B} = \begin{bmatrix} 0 \times 0^{T} \end{bmatrix} \quad (M_{c} \text{ is modulus of resistance applied to RG}).
\]

System (1) consists of two groups of equations and inequations. Equations of the first group (lines 1-3) were obtained using the Bubnov-Galerkin method [10], i.e. by scalar multiplication of surface mutual non-penetration equations by the basic functions. Equations of the second group (lines 4–6) are body equilibrium equations. The last two inequations and the equation express unilateral surface interaction. The resolving system of equations was solved by introducing restoring forces [11].

To determine flexible bearing durability, a contact fatigue curve equation was used [8]

\[
\frac{[\sigma_{H}]}{\sigma_{H}} = \frac{N}{N_{0}}
\]

where \( \sigma_{H} \) is the maximum value of contact stress variables; \( N \) is the number of load cycles to contact surface destruction; \( m = 9 \) is the contact fatigue curve degree; \( N_{0} = 10^{7} \) is the basic number of contact stress cycles.

The allowable contact stresses for radial ball bearings are \( [\sigma_{H}] \approx 4200 \text{MPa} \) [15].

Flexible gears break down due to fatigue flaking of the inner ring (IRFB) raceway. The maximum contact stresses caused by balls-raceway interaction are located in the cam major axis area. To determine the maximum force of interaction between the balls and raceway of the IRFB \( P_{H \text{max}} \) elastic interaction of HD elements was calculated using the method described in [9]. The calculations were carried out at various ball locations with regard to the cam major axis. As a result, the maximum force \( P_{H \text{max}} \) impacting the balls was determined.

The maximum value of contact stress variables was calculated using the following formula [6, p. 153-154]

\[
\sigma_{H} = A : \sqrt[3]{P_{H \text{max}}},
\]

where \( A \) is a factor depending on the elastic properties of the balls and the raceway, ball diameter, and IRFB raceway and groove radii.

The flexible bearing inner ring experiences the maximum contact stresses near the cam major axis. Therefore, the number of FB inner ring load cycles was determined using the following formula

\[
N = n_{c}^* \cdot 60 \cdot z \cdot L_{a},
\]

where \( n_{c}^* \) is separator rpm with regard to the inner ring of FB; \( z \) is the number of FB balls; \( L_{a} \) is drive operation hours.

Taking into accounts expressions (2) and (4), a formula for flexible bearing durability determination was obtained

\[
L = \frac{N_{0}}{10^{6} \cdot z} \cdot \frac{n}{n_{c}^*} \left( \frac{\sigma_{H}}{\sigma_{H}} \right)^{w}, \text{ mln rpm},
\]
where \( n \) is cam rpm; \( L \) is millions of wave generator rpm.

Figure 2 shows directions of angular velocities of the cam \( \omega_h \), flexible gear \( \omega_f \), separator \( \omega_k \), as well as linear velocities of the balls contact points with the inner \( \bar{v}_1 \) and outer \( \bar{v}_2 \) flexible bearing rings. On the assumption that the balls roll along the raceways without slipping, we obtained the following formula for determination of \( n^* \) and \( n \) frequency ratio

\[
\frac{n^*}{n} = \frac{\omega_h - \omega_k}{\omega_f} = \left( 1 - \frac{r_k - R_k}{2r_0} \right), \quad (5)
\]

where \( R_k \), \( r_k \) are the radii of the FB inner and outer ring raceways, respectively; \( r_0 = \frac{R_k + r_k}{2} \) is the average bearing diameter; \( u^b_{wh} \) is harmonic drive gear ratio.

**Figure 2.** Velocity distributing in a roller bearing:
1 – outer ring; 2 – inner ring; 3– ball; 4 – separator

### 4. Study Results

For a theoretical study of flexible gear durability, a harmonic drive with the following parameters was taken: RG teeth number \( z_b = 152 \), FG teeth number \( z_g = 150 \), RG shift factor \( x_b = 2.95 \), FG shift factor \( x_g = 2.85 \), module \( m = 0.8 \mathrm{\text{mm}} \), FG shell thickness \( h_0 = 1 \mathrm{\text{mm}} \), FG thickness under tooth wheel rim \( h_t = 1.2 \mathrm{\text{mm}} \), FG length \( L = 114 \mathrm{\text{mm}} \), tooth wheel rim width \( b_w = 27 \mathrm{\text{mm}} \), the number of flexible bearing (FB) balls \( n = 23 \), FB outer diameter \( D_B = 120 \mathrm{\text{mm}} \), FB width \( B = 18 \mathrm{\text{mm}} \), FG maximum deformation \( w_0 = m \), FB thickness \( a_t = 2.4 \mathrm{\text{mm}} \), nominal moment \( M_n = 440 \mathrm{\text{Nm}} \). The other harmonic drive dimensions were taken according to GOST 30078.2-93, flexible bearing dimensions – according to GOST 2379-78 and recommendations in [1].

The impact of various parameters on flexible bearing durability was studied under the nominal load. The article considers the following types of deformation: 1) according to the law \( w = w_0 \cdot \cos(2\phi) \); 2) according to the law of the ring deformed by four forces acting symmetrically to the major axis with \( \beta = 25^\circ, 30^\circ \) and \( 35^\circ \).

Figure 3 shows dependencies of FB durability on the maximum flexible gear deformation at the nominal moment and various types of deformation. With \( W_0/m = 1.3 \) FB durability is weakly dependent on the type of deformation. Decrease of \( w_0 \) results in increased durability. The maximum increase is observed at the second deformation type and \( \beta = 35^\circ \): decrease of \( w_0/m \) from 1.3 to 0.9 increases durability almost 9 times. Durability decrease with the increase of \( w_0 \) is conditioned by the narrowing of the area of engagement in a loaded drive, resulting in the increase of forces impacting the balls.
Figure 3. Dependency of FB durability on FG maximum radial deformation:

\[ 1- \beta = 35^\circ; 2- \beta = 30^\circ; 3- \beta = 25^\circ; 4- w = w_o \cdot \cos(2\varphi) \]

Figure 4 shows flexible bearing dependency on the flexible gear length \( L \) at \( W_0/m = 1 \). Curve 1 corresponds to the second deformation type at \( \beta = 35^\circ \), curve 2 – to the first deformation type. Durability decrease at a low \( L \) value is conditioned by the fact that the teeth start interacting, when the FG tooth starts penetrating the RG recess from the inner side of the FG tooth wheel ring.

Figure 4. Dependency of FB durability on the FG length: 1 – \( \beta = 35^\circ \); 2 – \( w = w_o \cdot \cos(2\varphi) \)

Conclusion
1. A method for calculating harmonic drive flexible bearing durability accounting for the impact of various drive parameters has been developed.
2. The cam shape has a significant impact on flexible bearing durability. Deformation according to the law of the ring deformed by four forces at \( \beta = 35^\circ \) provides for a higher FB durability.
3. A small flexible gear length can considerably decrease flexible bearing durability.

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