Analysis and determination of first ply failure of the composite laminate using different failure criteria

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Abstract. This work relates to the first ply failure load investigation of composite laminate plates for different failure criteria under in-plane loading condition in two-dimensional spaces. In this context, first ply failure strengths investigated for different angle ply and number of the lamina. Writing a programme of FEM (finite element method) in MATLAB and calculating failure load for each lamina of the composite laminate by using various failure criteria such as max stress failure criterion, max strain failure criterion, Tsai-Hill, Tsai-Wu and Hoffman failure theory. Failure load calculated for three-ply composite laminate with [45°/-45°/45°] ply angle on the basis of these five failure theories shows very good conformity with the previous paper results. Kevlar/epoxy is taken for the calculation of first ply failure load with different ply orientation.

Keywords: Composite laminate, First ply failure, Ply orientation, Failure Theories, Failure load

1. Introduction

Present industries extensively use the composite laminate materials to a made structural component which has high stiffness and strength, small maintenance cost, small specific gravity and good flexibility. The analysis of the failure of composite laminates is difficult in nature because of different phases and the property of the matrix. The composition of two or more material in the microscopic scale is called composite material but having chemically distinct phases. This shows the homogeneous property at the macroscopic scale and heterogeneous property at the microscopic scale. A composite laminate is the combination of the different or the same lamina with different ply orientation. The layers of the laminate are commonly bounded by a thin layer of resin or by the material of matrix which is same in the individual lamina. This thin layer transfers the load and displacement from one layer to another adjacent layer. Laminate can be made up of the plate of the same materials or different materials. The number of plates in the laminate is depending on the desired thickness of the laminate. The aim of lamination is to increase the strength and stiffness in particular direction of a composite material related to the loading environment of the element.

2. Literature Review

N.B.V. Lakshmi Kumari et al.[1] have performed the analysis on CFRP (carbon fibre reinforced polymer) laminate having ten plies with the orientation of [90°/0°/0°/-45°/+45°], subjected to in-plane loading state. In which mathematical analysis for finding the failure condition is prepared by stress
program writing on MATLAB and by using ANSYS. Carbon/epoxy is taken for the analysis and the stresses obtained by using MATLAB and ANSYS shows the component is safe and reliable under working condition.

J.Y. Ang et al.[2] developed the ANN (artificial neural network) model for forecasting of the failure of composite pipes made by glass/epoxy subjected to multiaxial loadings. This model can forecast the first ply failure for the composite pipe using a different ratio of biaxial stresses. ANN model can be used for other modes of failure of laminated composite pipes subjected by various stress condition. This process may be used for pipe rating as per standard ASTM qualification process. In this study for checking the accuracy of this model, compare the result of this model with previous experimental data and this comparison shows the range of 95% to 99.66% accuracy.

J.L. Liu et al.[3] were reported that small inter-ply angles ranging from 5° to 10° in helicoidal laminates provide good resistant to delamination. This condition avoids catastrophic failure because of transverse crack not propagate from the surface of the laminate. This work gives the result of the peak transverse loading condition for the laminate of helicoidal having small inter-ply angle up to 73% higher than cross-ply laminate of same thickness.

Yanan Yuan et al.[4] done experiments, theoretical model and FEM (finite element method) to predict the failure modes and tensile strength of lean ply CFRP (carbon fibre-reinforced polymer) with different ply orientation and areal weights. The experiment shows tensile strengths decreases with reducing the fibre weight with respect to the area. Theoretical model and FEM (finite element method) used to analyse the failure mode and strength of CFRP (carbon fibre-reinforced polymer) laminates for different ply thickness and fibre volume fraction.

Ramesh Talreja[5] has performed the comparison of the phenomenon of theories for the failure of the composite material by analysing some assumptions of the failure criterion and check their validity with test data. He founded that Tsai-Wu and Tsai-Hill do not consider the physical nature of mechanisms so this theory modified by Hashin and Puck.

Marek Romanowicz[6] presented a technique for calculating in situ strength for fibre-reinforced composite laminas subjected by three types of loading in transverse direction compression load, tension load and shear load. The nonlinear hardening behaviour of the matrix affects more the in situ transverse strength of composite lamina as compare to the perfectly plastic behaviour of the matrix. Experimental data is also taken for verification of forecasting of the in situ transverse strength of cross-ply composite laminate subjected by tension in a uniaxial direction.

Y.V. Satish Kumar et al.[7] work on first ply failure load prediction of cross-ply stiffened plates subjected to sinusoidal load and uniformly distributed load according to different failure criteria such as max stress failure criteria, max strain failure criteria, Hoffman criteria, Tsai-Hill criteria, Tsai-Wu criteria, Hoffman and Yeh-Stratton failure criteria. For this study Cross-section of the plate is I and hat section with different fibre angles ranging from 15° to 75°. An element in this study has eighteen degrees of freedom. I-section has higher failure loads when fibre angles less than 45°.

B.G. Prusty et al.[8] developed a FEM (finite element model) to predict the first ply failure load of the bare plate and shell panels subjected by various loading conditions. Analysis of stiffened panel by three noded curved beam elements for the stiffener and eight noded isoparametric quadratic elements for the shell are taken. Using iterative procedure different failure theories such as Max stress criteria, Max strain criteria, Hoffman criteria, Tsai-Wu and Yeh-Stratton criteria used to the prediction of first ply failure load.
3. Theory and Problem Formulation

3.1 Finite Element Formulation

In present Finite element method is taken for the analysis and determination of the plate element of the laminated composite. In which 8 noded isoparametric quadratic plate elements is considered. The shape functions are given in the natural coordinates $\zeta, \eta$, which show the relation between displacements within the element at any point and values of the displacements at the nodes. Displacement function in co-ordinate $\zeta$ and $\eta$ can be given as

$$u(\zeta, \eta) = b_0 + b_1 \zeta + b_2 \eta + b_3 \zeta^2 + b_4 \zeta \eta + b_5 \eta^2 + b_6 \zeta^2 \eta + b_7 \zeta \eta^2$$

(3.1)

Where $b_0, b_1, b_2, \ldots, b_7$ are the constant of the polynomial equations

The shape functions of interpolation polynomial are given as[9]

$$N_j = \frac{1}{4}(1 + \zeta J_j)(1 + \eta J_j)(\zeta J_j + \eta J_j - 1) \quad \ldots \ldots j = 1, 2, 3, 4$$

$$N_j = \frac{1}{2}(1 + \eta J_j)(1 - \zeta^2) \quad \ldots \ldots j = 5, 7$$

$$N_j = \frac{1}{2}(1 + \zeta J_j)(1 - \eta^2) \quad \ldots \ldots \quad j = 6, 8$$

(3.2)

Where $\zeta$ and $\eta$ are local natural co-ordinates and $\zeta_j = +1$ for node number 2, 3 and 6, $\zeta_j = -1$ for node number 1, 4 and 8, $\eta_j = +1$ for node number 3, 4 and 7 and $\eta_j = -1$ for node number 1, 2 and 5 as shown in figure below. The shape functions are correct or not can be checked by the following relation

$$\sum_{j=1}^{8} N_j = 1, \sum_{j=1}^{8} \frac{\partial N_j}{\partial \zeta} = 0 \quad \text{and} \quad \sum_{j=1}^{8} \frac{\partial N_j}{\partial \eta} = 0$$

(3.3)

The co-ordinate in terms of x and y within an eight noded element at any point can be found as

$$x = \sum_{j=1}^{8} N_j x_j \quad \text{and} \quad y = \sum_{j=1}^{8} N_j y_j$$

(3.4)

Displacement at any point in co-ordinates $\zeta, \eta$ are expressed as

$$u = \sum_{j=1}^{8} N_j u_j, \quad v = \sum_{j=1}^{8} N_j v_j, \quad w = \sum_{j=1}^{8} N_j w_j, \quad \theta_x = \sum_{j=1}^{8} N_j \theta_{xj}, \quad \theta_y = \sum_{j=1}^{8} N_j \theta_{yj}$$

(3.5)

and

$$\begin{bmatrix} N_{j,x} \\ N_{j,y} \end{bmatrix} = [J]^{-1} \begin{bmatrix} N_{j,\zeta} \\ N_{j,\eta} \end{bmatrix}$$

(3.6)
\begin{align}
\mathbf{J} &= \begin{bmatrix} x_\zeta & y_\zeta \\
               x_\eta & y_\eta \end{bmatrix} \text{ show the jacobian matrix.}
\end{align}

Strain displacement matrix \( \mathbf{[B]} \) is given as

\begin{equation}
\mathbf{[B]} = \begin{bmatrix}
J_{11}^* & J_{12}^* & 0 & 0 \\
0 & 0 & J_{21}^* & J_{22}^* \\
J_{21}^* & J_{22}^* & J_{11}^* & J_{12}^* \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\frac{\partial N_1}{\partial \zeta} & 0 & \frac{\partial N_2}{\partial \zeta} & 0 & \frac{\partial N_3}{\partial \zeta} & 0 & \frac{\partial N_4}{\partial \zeta} & 0 \\
0 & \frac{\partial N_1}{\partial \eta} & 0 & \frac{\partial N_2}{\partial \eta} & 0 & \frac{\partial N_3}{\partial \eta} & 0 & \frac{\partial N_4}{\partial \eta} \\
0 & 0 & \frac{\partial N_1}{\partial \zeta} & 0 & \frac{\partial N_2}{\partial \zeta} & 0 & \frac{\partial N_3}{\partial \zeta} & 0 & \frac{\partial N_4}{\partial \zeta} \\
0 & 0 & 0 & \frac{\partial N_1}{\partial \eta} & 0 & \frac{\partial N_2}{\partial \eta} & 0 & \frac{\partial N_3}{\partial \eta} & 0 & \frac{\partial N_4}{\partial \eta}
\end{bmatrix}
\end{equation}

Where \( J_{11}, J_{12}, J_{21} \) and \( J_{22}^* \) are an element of the Jacobian inverse matrix.

Then the element stiffness matrix \( \mathbf{[K]} \) is given as

\begin{equation}
\mathbf{[K]} = \int \mathbf{[B]}^T \mathbf{[D]} \mathbf{[B]} dV
\end{equation}

\( \mathbf{[D]} \) is the rigidity matrix and \( dV \) is the volume of the element.

3.2 Different Failure criteria

In this study, the laminated composite is taken with a combination of some layer of laminas. The structure of each lamina contains continuous and parallel fibres surrounded in the material of the matrix. The highly frequent criteria for analysis of composite materials is the criteria given by Tsai criteria and Tsai and Wu criteria[9]. Once stress in each lamina is known than the failure criterion used to find out the failure load carrying capacity of composite material. There is some failure criterion-
3.2.1 Maximum Stress Failure Criteria

Max stress failure theory is an addition of max normal stress theory given by Rankine and max stress theory given by Tresca for isotropic and homogeneous materials. In this failure criterion, all the stresses in the composite material must be less than the individual strength to avoid failure of the material.

The condition for tensile stresses,
\[ \sigma_1 \leq \sigma_t \quad \sigma_2 \leq \sigma_t \quad \gamma_{12} \leq \gamma_t \]
(3.9)

The condition for compressive stresses,
\[ \sigma_1 \geq \sigma_c \quad \sigma_2 \geq \sigma_c \]
(3.10)

Also,
\[ |\tau_{12}| \leq S \]
(3.11)

where \( \sigma_1, \sigma_2 \) and \( \tau_{12} \) are the normal stress and shear component in 1 and 2 direction; \( \sigma_t, \sigma_c \) are tensile and compressive strength of the composite lamina.

3.2.2 Maximum Strain Failure Criteria

Max strain failure theory is an expansion of max normal strain theory given by St. Venant and max shear stress theory given by Tresca for materials in isotropic nature. This failure criterion is quite similar to the max stress failure criterion. In which all the strain in the composite material must be less than the individual strength to avoid failure of the material.

\[ \varepsilon_1 \leq \varepsilon_t \quad \varepsilon_2 \leq \varepsilon_t \quad |\gamma_{12}| \leq S_e \quad \varepsilon_1 \geq \varepsilon_c \quad \varepsilon_2 \geq \varepsilon_c \]
(3.12)

where \( \varepsilon_1, \varepsilon_2, \gamma_{12} \) are strains in principal material coordinates, \( \varepsilon_t, \varepsilon_c \) are maximum tensile strain and compressive normal strain in first and second directions, \( S_e \) is a maximum shear strain in the first-second co-ordinates.

3.2.3 Tsai-Hill Failure Criteria

Tsai-Hill failure criteria are a modification of distortional energy yield criterion of Von-Mises for range from isotropic materials to anisotropic materials and unidirectional lamina. According to this failure criterion lamina has failed if

\[ (B+C)\sigma_1^2 + (A+C)\sigma_2^2 + (A+B)\sigma_3^2 - 2C\sigma_1\sigma_2 - 2B\sigma_1\sigma_3 - 2A\sigma_2\sigma_3 + 2D\tau_{23}^2 + 2E\tau_{13}^2 + 2F\tau_{12}^2 \geq 1 \]
(3.13)

is violated. Hill’s yield stresses A, B, C, D, E and F are the failure strengths of the lamina.

3.2.4 Hoffman Failure Criteria

Hoffman extended Hill’s equation for different tensile strength and compressive strength by adding linear terms. According to Hoffman failure criteria composite lamina has failed if
\[
A_1 (\sigma_2 - \sigma_3)^2 + A_2 (\sigma_3 - \sigma_1)^2 + A_3 (\sigma_1 - \sigma_2)^2 + A_4 \sigma_1 + A_5 \sigma_2 + A_6 \sigma_3 + A_7 \tau_{12}^2 \\
+ A_8 \tau_{31}^2 + A_9 \tau_{12}^2 \geq 1
\]

(3.14)
is violated. Where \(A_i\) are calculated from the strengths of the lamina in principal material coordinate.

### 3.2.5 Tsai-Wu Tensor Failure Criteria

Tsai-Wu failure criteria are related to the total strain energy failure theory. Tsai and Wu derived that a failure surface exists in the form in six-dimensional stress space

\[
F_a \sigma_b + F_{ab} \sigma_a \sigma_b \geq 1 \\
a, b = 1, \ldots, 6
\]

(3.15)

here \(F_a\) is strength tensors of the second rank and \(F_{ab}\) is strength tensors of the fourth rank and

\[
F_1 = \frac{1}{x} + \frac{1}{x_i} \\
F_{11} = \frac{1}{x_i x_c} \\
F_2 = \frac{1}{y} + \frac{1}{y_i} \\
F_{22} = \frac{1}{y_i y_c} \\
F_6 = 0 \\
F_{66} = \frac{1}{S^2}
\]

(3.16)\,(3.17)\,(3.18)

Tsai-Wu failure criterion is a more common nature than the Tsai-Hill or Hoffman failure criterion.

### 4. Results and discussion

In this part, the result arrived for the first ply failure analysis (failure load) of the composite laminate by using various failure criteria. The different failure criteria taken in this study are Max stress failure criteria, Max strain failure criteria, Tsai-Hill failure criteria, Hoffman failure criteria and Tsai-Wu tensor failure criteria. This study carried out to analysis and determination of first ply failure load related to different ply angle, the degree of orthotropy, number of plies and ply thickness of each lamina. Kevlar/epoxy angle ply laminated composite material is taken for study. The length of the composite laminate, the width of composite laminate and thickness of composite laminate are 9 in., 5 in. and 0.005 in/ply respectively. Geometry and boundary condition of a composite laminate is subjected to in-plane loading in x and y coordinate as shown in figure 2.
WHEREIN THE THREE-LAYERED COMPOSITE LAMINATE WITH PLY ORIENTATION [45°/-45°/45°] FIRST PLY FAILURE LOAD RESULTS ARE SHOWN AND THE RESULTS ARE VALIDATED WITH REDDY AND PANDEY[10] FOR FIVE DIFFERENT FAILURE CRITERIA. THE FAILURE LOADS CALCULATED USING THE PRESENT FINITE ELEMENT CODING IN MATLAB AND THE FAILURE LOADS OBTAINED IN PREVIOUS WORK BY REDDY AND PANDEY ARE ESTABLISHED TO BE IN GOOD CONFORMABILITY AS SHOWN IN TABLE 1. VALIDATION IS ALSO PRESENTED FOR FAILURE LOADS CALCULATED IN PRESENT FEM (FINITE ELEMENT METHOD) VERSUS FAILURE LOADS OBTAINED BY REDDY AND PANDEY.

**Table 1** First ply failure load results of composite laminate [45°/-45°/45°] for various failure criteria and validation.

| Failure criteria | Failure Load (N) Reddy and Pandey[10] | Failure Load (N) Present FEM |
|------------------|----------------------------------------|-------------------------------|
| Max. stress      | 2854.5                                 | 2653                          |
| Max. strain      | 2947.68                                | 2950                          |
| Tsai-Hill        | 2308.32                                | 2229.7                        |
| Tsai-Wu          | 2282.24                                | 2210.5                        |
| Hoffman          | 2259.68                                | 2215.16                       |

A RECTANGULAR COMPOSITE LAMINATE OF KEVLAR/EPoxy IS TAKEN FOR THE ANALYSIS OF FIRST PLY FAILURE (FAILURE LOAD) WITH PLY ORIENTATION OF CROSS-PLY [0°/90°/90°/0°]T, ANTISYMMETRIC ANGLE PLY [45°/-45°/45°/-45°]T AND SYMMETRIC ANGLE PLY [0°/45°/-45°/45°]. MATERIAL PROPERTY OF KEVLAR/EPoxy COMPOSITE IS CONSIDERED AS E1=11×10⁶ psi, E2=0.8×10⁶ psi, v12=0.34, G12=0.3×10⁶ psi, X₁=200×10³ psi, Y₁=4×10³ psi, X₃=40×10³ psi, Y₃=20×10³ psi and S=6.4×10³ psi.[11] THE FIRST PLY FAILURE LOADS OF THREE TYPES OF PLY ORIENTATION AND FOR FIVE FAILURE CRITERIA SHOWN IN TABLE 2.

**Table 2** First ply failure load of composite laminate for different ply orientation and for different failure criteria subjected to in-plane loading condition.

| Failure criteria | Failure load (N) [0°/90°/0°/90°]T | Failure load (N) [45°/-45°/45°/-45°]T | Failure load (N) [0°/45°/-45°/45°]T |
|------------------|-------------------------------------|----------------------------------------|--------------------------------------|
| Maximum stress   | 2565.1                              | 2961.2                                 | 4485.6                               |
| Maximum strain   | 2650.8                              | 2870.3                                 | 4367.5                               |
| Tsai - Hill      | 2547.4                              | 2344.7                                 | 4397.4                               |
| Tsai - Wu        | 2540.5                              | 2477.5                                 | 4370.7                               |
| Hoffman          | 2537.3                              | 2478.2                                 | 4367.6                               |
5. Conclusion

The different number of failure criteria such as max stress failure criteria, max strain failure criteria, Tsai-Hill failure criteria, Tsai-Wu failure criteria and Hoffman failure criteria used to find the first ply failure load in cross-ply, antisymmetric ply and symmetric ply for in-plane loading condition. The boundary conditions for analysis are simply supported at the edges. A finite element programme is written in MATLAB for the analysis. Failure load calculated for three plies composite laminate with [45°/-45°/45°] ply angle on the basis of these five failure theories shows very good agreement with the reference paper. From the present assessment for different failure criteria, it may be considered that all failure criteria are similar in analysing the failure when composite laminates are subjected to in-plane loading condition.

6. References

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