Determining Spin through Quantum Azimuthal-Angle Correlations

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Abstract

Determining the spin of new particles is critical in identifying the true theory among various extensions of the Standard Model at the next generation of colliders. Quantum interference between different helicity amplitudes was shown to be effective when the final state is fully reconstructible. However, many interesting new physics processes allow only for partial reconstruction. In this paper, we show how the interference effect can be unambiguously extracted even in processes that have two-fold ambiguity, by considering the correlation between two decay planes in $e^+e^-$ collisions.

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The Large Hadron Collider (LHC) will soon usher us into the arena of the electroweak symmetry breaking scale and beyond. Once the TeV regime is explored, it is highly anticipated that the true theory for the origin and stability of the electroweak scale [1] will be revealed.

One possible result from the LHC, generically predicted in new most models, is the presence of new particles partnered with some or all of the SM particles. For instance, every SM particle in the minimal supersymmetric standard model (MSSM) [2, 3] has a heavier partner whose spin differs by 1/2. Alternatively, in the minimal universal extra dimension (UED) model [4] with a compactified electroweak-scale extra space dimension, each SM particle is paired with a tower of Kaluza-Klein (KK) states with identical spin. Thus, model-independent spin measurements are crucial in discriminating among many extensions of the SM.

There have been several proposals for measuring spin at both the LHC and the prospective $e^+e^-$ International Linear Collider (ILC) [5]. Threshold scans in $e^+e^-$ collisions can be used to distinguish scalars from spinors at the ILC, as the scalar production cross section increases slowly $\sim \beta^3$ while the spinor cross section increases steeply $\sim \beta^5$ [6]. [Such a method cannot be used at the LHC as the center of mass (c.m.) energy at the parton level is not fixed.] The production angle can give insight on spin as well. The polar-angle distribution of a $s$-channel pair produced scalars is proportional to $\sin^2 \Theta$, while for spinors it approaches $1 + \cos^2 \Theta$ asymptotically at high energies. The presence of $t/u$-channel exchanges may render the production-angle measurement of spin more demanding [6], although it is feasible in some cases [7, 8]. The polar-angle dependence in decays at the ILC [8] and the invariant-mass distributions in sufficiently long decay chains at the LHC [9] can also be used for spin measurements. However, these techniques rely strongly on the final state spins and the chiral structure of couplings.

In this report we study the fully-correlated azimuthal-angle distributions in the production of a new particle-antiparticle pair in $e^+e^-$ collisions and both of their sequential decays [10]. [This work is a natural extension to the previous works [11, 12] where the azimuthal-angle correlation of the production and only one of the decays has been investigated.] These distributions develop through quantum interference between the different helicity states in a coherent sum. By extracting this angular dependence, we can determine which helicity states contribute to the sum, and thus the spin of the decaying particle in a model-independent way. To be specific, we restrict ourselves to production of an electrically-charged particle-antiparticle ($F^+F^-$) pair in $e^+e^-$ collisions and decay of each produced particle $F^\pm$ to a charged particle $f^\pm$ and an invisible particle $\chi$,

$$e^+e^- \rightarrow F^+F^- \rightarrow (f^+\chi)(f^-\chi) \rightarrow f^+f^- E.$$  (1)

As suggested by the WIMP solution to the cold dark matter puzzle, large missing energy signatures are considered likely at the TeV scale and are generic in many extensions to the SM [14].

For example, the lightest supersymmetric particle (LSP), typically the lightest neutralino $\tilde{\chi}^0_1$, in supersymmetric (SUSY) models with $R$-parity; and the lightest KK odd particle (LKP), typically the lightest KK gauge boson $\gamma_1$, in UED models with KK parity are stable and escape detection. Prototype processes in the SUSY and UED models with the same event topologies as the process (1)

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*The SM processes $e^+e^- \rightarrow \tau^+\tau^- \rightarrow (\tau^+\nu_\tau)(\tau^-\bar{\nu}_\tau)$ and $e^+e^- \rightarrow W^+W^- \rightarrow (\ell^+\nu_\ell)(\ell^-\bar{\nu}_\ell)$ with $\ell = e$ and $\mu$ also have a large missing energy signature carried away by invisible neutrinos.
are

\[ e^+e^- \rightarrow \mu_R^+ \tilde{\mu}_R^- \rightarrow (\mu^+ \chi_1^0)(\mu^- \chi_1^0) \rightarrow \mu^+\mu^- \mathcal{E}, \]
\[ e^+e^- \rightarrow \mu_R^+\mu_R^- \rightarrow (\mu^+\gamma_1)(\mu^-\gamma_1) \rightarrow \mu^+\mu^- \mathcal{E}. \]

Both generate the same experimental signatures \( \mu^+\mu^- \mathcal{E} \) with large missing energy.

**Figure 1:** The definition of the polar angles \( \theta^\pm_\pm \) of the visible particle \( f^\pm \) momentum in the rest frame of the decaying particle \( F^\pm \) and of the correlated azimuthal angle \( \phi \) between two decay planes formed in the correlated production-decay process \( X \rightarrow F^-F^+ \rightarrow (f^-\chi)(f^+\chi) \) in the rest frame of the \( X = \{e^+e^-\} \) system, corresponding to the \( e^+e^- \) c.m. frame for the processes considered in this report. Here, \( X = \{e^+e^-\} \) denotes any single- or multiple-particle intermediate state formed in \( e^+e^- \) annihilation. Note that \( \phi \) is invariant under the Lorentz boost along the \( F^\pm \) flight direction.

The characteristic observables for measuring spin of the particles \( F^\pm \) through the process (1) are the angular distributions of the final-state particles \( f^\pm \) in the \( F^\pm \) decays, encoding the helicities of the \( F^\pm \) states. We denote the polar angles of the particles \( f^\pm \) in the rest frames of the \( F^\pm \) particles by \( \theta^\pm_\pm \), and the azimuthal angles by \( \phi^\pm_\pm \) with respect to the production plane defined by the \( e^- \) and \( F^- \) momentum directions, respectively. Then the angle \( \phi \) with its range \( [0, 2\pi] \) between the two decay planes (see Fig. 1) is the azimuthal angle defined by the angle difference \( \phi \equiv \phi^+_\pm - \phi^-_\pm \pmod{2\pi} \) invariant under any Lorentz boost along the \( F^\pm \) flight direction.

If we label the \( F^\pm \) helicities by \( \lambda^\pm_\pm \) and \( \lambda'^\pm_\pm \), the joint production-decay distribution reads:

\[
W(E_{\text{cm}}; \Theta; \theta^\pm_\pm, \phi^\pm_\pm) = \sum_{\lambda^\pm_\pm, \lambda'^\pm_\pm = -j}^{j} \mathcal{P}^{\lambda^\pm_\pm, \lambda'^\pm_\pm}_{\lambda^\pm_\pm, \lambda'^\pm_\pm}(E_{\text{cm}}, \Theta) \mathcal{D}^+_{\lambda^\pm_\pm, \lambda'^\pm_\pm}(\theta^+_\pm, \phi^+_\pm) \mathcal{D}^-_{\lambda^\pm_\pm, \lambda'^\pm_\pm}(\theta^-_\pm, \phi^-_\pm),
\]

where \( E_{\text{cm}} \) is the \( e^+e^- \) c.m. energy and \( \Theta \) is the production angle of \( F^- \) with respect to the \( e^- \) direction, and \( j \) is the spin of the particle \( F^\pm \). The production density matrix \( \mathcal{P} \) is defined in terms of the helicity amplitudes \( T \) of the process \( e^+e^- \rightarrow F^+F^- \) for unpolarized beams by

\[
\mathcal{P}^{\lambda^\pm_\pm, \lambda'^\pm_\pm}_{\lambda^\pm_\pm, \lambda'^\pm_\pm} = \sum_{\sigma = \pm 1/2} T_{\sigma - \sigma^+; \lambda^\pm_\pm, \lambda'^\pm_\pm} T^*_{\sigma^+ - \sigma; \lambda^\pm_\pm, \lambda'^\pm_\pm},
\]

where \( \sigma \) is the \( e^\pm \) helicity, and each \( F^\pm \) decay density matrix \( \mathcal{D}^\pm \) has a simple azimuthal-angle dependence of a pure kinematical origin as

\[
\mathcal{D}^\pm_{\lambda^\pm_\pm, \lambda'^\pm_\pm}(\theta^+_\pm, \phi^+_\pm) = D^\pm_{\lambda^\pm_\pm, \lambda'^\pm_\pm}(\theta^+_\pm) e^{\mp i(\lambda^\pm_\pm - \lambda'^\pm_\pm)\phi^+_\pm},
\]
reflecting an overall rigid rotation of the decay plane around the parent particle momentum.

Integrating the joint production-decay distribution $W$ in Eq. (4) over the production angle $\Theta$, the decay angles $\theta^*_{\pm}$ and $\phi^*_{\pm}$ with the azimuthal angle $\phi$ fixed, we can derive the correlated azimuthal-angle distribution between the two decay planes as

$$\frac{dC}{d\phi} = \int W(E_{cm}; \Theta; \theta^*_{\pm}, \phi^*_{\pm}) \ d\Theta \ d\cos \theta^* \ d\cos \theta^*_{\pm} d\phi^*_{\pm}. \quad (7)$$

We note from Eqs. (4) and (6) that the dependence on $\phi^*_{\pm}$ is of the form $\exp[-i \phi^*_{\pm}(\lambda_\mu - \lambda'_1 - \lambda_1 + \lambda'_2)]$ so that the integral over $\phi^*_{\pm}$ leaves only those terms in Eq. (4) satisfying the relation $\lambda_1 - \lambda'_1 = \lambda_\mu - \lambda'_2 \equiv \Lambda$ in the range $[-2j, 2j]$. If the distribution is further integrated over the angle $\phi$, only the incoherent terms with $\lambda_{\pm} = \lambda'_{\pm}$ survive. Thus, any non-trivial azimuthal-angle distribution indicates the presence of quantum interference between the different helicity amplitudes.

In weakly-interacting and CP-invariant theories with negligible particle-width and loop effects, the general form of the normalized azimuthal-angle correlation for the production and decay of a spin-$j$ particle pair is

$$\frac{1}{C} \frac{dC}{d\phi} = \frac{1}{2\pi} \left[1 + A_1 \cos(\phi) + \cdots + A_{2j} \cos(2j\phi)\right]. \quad (8)$$

Each coefficient can be worked out from the standard rules of constructing matrix elements. However, it is guaranteed on a general footing that the highest non-vanishing coefficient is always $A_{2j}$. This is because the production of a charged pair $F^\pm$ in $e^+e^-$ collisions gets a non-zero spin-1 photon-exchange contribution to the production amplitudes, $T_{\sigma-\sigma^\prime;\pm;j,j;j}$. However this term tends to be suppressed by $\sim m^2_{F^\pm}/E^2_{cm}$ at high energies because of a final-state helicity flip. Thus, the spin $j$ can be determined by identifying the highest $\cos(2j\phi)$ mode at a c.m. energy not far away from the production threshold, if the production amplitudes contributing to the coefficient $A_{2j}$ are not so suppressed and the decays $F^\pm \rightarrow f^\pm \chi$ do not have too small polarization analyzing powers.

The correlated azimuthal angle distribution for the smuon-pair process (2) is flat, as it must be, and the distribution for the KK muon-pair process (3) is given by

$$\frac{1}{C} \frac{dC}{d\phi} \left[\mu^-_{R1}\mu^+_{R1}\right] = \frac{1}{2\pi} \left[1 - \frac{\pi^2 m^2_{\mu^+_{R1}}}{8(E^2_{cm} + m^2_{\mu^+_{R1}})} \left(1 - \frac{2m^2_{\gamma_1}/m^2_{\mu^+_{R1}}}{1 + 2m^2_{\gamma_1}/m^2_{\mu^+_{R1}}}\right)^2 \cos \phi\right]. \quad (9)$$

It is apparent from the expression (9) that (a) the coefficient of the highest $\cos \phi$ mode is maximal in magnitude at the production threshold and it decreases rapidly with increasing energy in conformity to the general rule as outlined above, and (b) it is very sensitive to the values of the $\mu_{R1}$ and $\gamma_1$ masses, leading to the restriction that the magnitude of the coefficient $A_1$ of the highest $\cos \phi$ mode cannot be larger than $\pi^2/48 \simeq 0.206$.

As well known [7, 8], there exists a two-fold discrete ambiguity in completely reconstructing the $F^\pm$ four-momenta and thus the azimuthal angles $\phi_{\pm}$ in the process in the laboratory frame,

$^1$Even in the CP-noninvariant case, all the sine terms are washed out by taking the average over two possible azimuthal angles, which is unavoidable due to a two-fold ambiguity in reconstructing the $F^\pm$ momentum as described in the following.
even if all particle masses are known. In [11] it was shown that this ambiguity could obscure the helicity information in $\phi^\pm$, curtail their use as measurements of spin. Nevertheless, the cosine of the azimuthal angle $\phi$ is unambiguously determined by measuring the $f^\pm$ four-momenta event by event. To prove this important point analytically, let the pair produced particles $F^\pm$ and the invisible particle $\chi$ have mass $m_\pm$ and $m_0$ and denote the $f^\pm$ flight direction in the laboratory frame by a unit vector $\hat{n}_\pm$, respectively. Then, the opening angles $\theta_\pm$ between the visible $f^\pm$ tracks and the parent $F^\pm$ particles in the laboratory frame can be determined from the relation

$$m_\pm^2 - m_0^2 = E_{\text{cm}} E_{f^\pm} \left(1 - \sqrt{1 - 4m_\pm^2/E_{\text{cm}}^2 \cos \theta_\pm}\right),$$

(10)

defining two cones about the $f^+$ and $f^-$ axes which intersect in two lines - the true $F^\pm$ flight direction and a false direction. True and false solutions are mirrored on the plane spanned by the $f^+$ and $f^-$ flight directions, leading to the relation $\phi_T = 2\pi - \phi_F = \phi$ between the true and false values, $\phi_T$ and $\phi_F$, of the azimuthal angle $\phi$ (see Fig. 2). Therefore, the cosine of the azimuthal angle is uniquely determined and its expression is given by the simple expression:

$$\cos \phi = (\hat{n}_+ \cdot \hat{n}_- + \cos \theta_+ \cos \theta_-)/\sin \theta_+ \sin \theta_-,$$

(11)

expressed in terms of the unit vectors, $\hat{n}_\pm$, and the opening angles, $\theta_\pm$.

Two cosines, $\cos(n_a \phi)$ and $\cos(n_b \phi)$, for any integers $n_a$ and $n_b$, are functions of $\cos \phi$ and orthogonal to each other for $n_a \neq n_b$. Therefore, we can project out all the coefficients $A_k$ ($k = 1, \ldots, 2j$) by fitting the expression of $(1/C) dC/d\phi$ in Eq. (8) to the distribution measured experimentally.

For a numerical demonstration of this spin-determination method, we compare the correlated azimuthal-angle $\phi$ distribution of the SUSY process (2) with that of the UED process (3) in a specific scenario, which we will simply call “BCMM” for convenience in the following, with the particle mass spectrum,

$$\text{BCMM: } m_\pm = m_{\tilde{\mu}_R} = m_{\tilde{\mu}_L} = 200 \text{ GeV} \quad \text{and} \quad m_0 = m_{\tilde{\chi}_1^0} = m_{\gamma_1} = 50 \text{ GeV}. \quad (12)$$
We stress that the mass spectrum is chosen only as a simple illustrative example for SUSY and UED models with different spins but similar final states and so the spin-determination method demonstrated here can, in principle, be exploited equally for any other scenarios beyond as well as within the SM.

\[ e^+ e^- \rightarrow \tilde{\mu}_R^+ \tilde{\mu}_R^- \]
\[ e^+ e^- \rightarrow \mu_R^+ \mu_R^- \]
\[ m_{\tilde{\mu}} = m_{\mu} = 200 \text{ GeV} \]

\[ 4 \times \sigma(\mu_R^+ \mu_R^-) \]

**Figure 3:** (a) The c.m. energy dependence of the production cross sections for smuons and KK muons; and (b) the normalized azimuthal-angle distribution \((2\pi/C)\text{d}C/\text{d}\phi\) in the BCMM scenario with \(E_{\text{cm}} = 450\) GeV. The solid (dashed) line is the distribution without (with) the rapidity cut on the \(\mu^\pm\) directions and the total missing momentum, \(|\eta| < 2.5\).

The total c.m. energy at the ILC is expected to reach up to 1 TeV, and an integrated luminosity of 500 fb\(^{-1}\) is not unrealistic. For the mass spectra chosen, we expect several thousand to several hundreds of thousands of events available as shown in Fig. 3(a). [At high energies we note that the \(\mu_R^\pm\) scalar cross section scales in the same way as the \(\tilde{\mu}_R^\pm\) spinor cross section, but with a coefficient 4 times as large due to differences in the number of spin degrees of freedom, as familiar from QED processes.]

To simulate the effects of experimental cuts which are unavoidable due to the geometry of the detector, we place cuts on the pseudo-rapidity of the \(\mu^\pm\) directions and the total missing momentum: \(|\eta| < 2.5\), as otherwise the leptons would vanish unseen down the beam and the missing momentum is not guaranteed to be carried away by the invisible \(\chi\) particles. Two representative distributions for scalar and spinors (both with and without rapidity cuts) are shown in Fig. 3(b). The SUSY distribution is flat and the UED distribution has a clear \(\cos \phi\) dependence above a flat distribution, as expected. Furthermore, the rapidity cut reduces the total number of events by about 2% but hardly modifies the correlated azimuthal-angle distribution. It is therefore apparent even at this level of analysis that the non-trivial correlated azimuthal-angle distribution contains the spin-1/2 information of the KK muon, \(\mu_R^{\pm}\).

Using the least-square method we fit the generated distributions to \((1 + A_1 \cos \phi + A_2 \cos 2\phi)/2\pi\) after placing the cut on the pseudo-rapidities of the \(\ell^\pm\) and the total missing momentum. Only the coefficient \(A_1\) for the scalar \(\tilde{\mu}_R\) and spinor \(\mu_R^{\pm}\) are shown in Fig. 4 because the coefficient...
\( A_2 \) is found to be extremely small \(< 0.1\% \). This is consistent with the statistical errors introduced from the event generation and fitting procedure and confirms our general argument that \( A_i = 0 \) for \( i > 1 \). As can be seen, in the BCMM spectrum the values of \( A_1 \) for the scalar muons, \( \tilde{\mu}_R \), are consistent with zero for all energies, as expected. For the KK muons, \( \mu_{R1}^\pm \), the coefficient \( A_1 \) is manifestly non-zero, allowing us to clearly distinguish the spinor field from spin-0 scalar states.

![Graph showing \( A_1 \) values for BCMM spectrum](image.png)

**Figure 4**: Coefficient \( A_1 \) for the BCMM point as a function of the \( e^+e^- \) c.m. energy for both the SUSY \( \tilde{\mu}_R^\pm \) and UED \( \mu_{R1}^\pm \) pair production with an integrated luminosity of 500 \( fb^{-1} \). Error bars obtained with \( Br(\tilde{\mu}_R^\pm \rightarrow \mu_1^\pm \tilde{\chi}_1^0) = Br(\mu_{R1}^\pm \rightarrow \mu^\pm \gamma_1) = 1 \) correspond to the 1-\( \sigma \) uncertainty range.

Due to the large \( A_1 \) signal for the \( \mu_{R1}^\pm \) process on the order of 10\% and the negligible rapidity-cut effect on the normalized azimuthal-angle distribution, the ILC with a large integrated luminosity can discern that the particle is not a spin-0 but a spin-1/2 particle. However it is not always guaranteed that, in the case of higher spins, the highest mode is sufficiently large to be discerned. As a result, the question still remains whether this spin-determination method may be practically applied to discriminate spinors from vectors and vice versa in general cases. It is worthwhile to study the case of pair production of massive spin-1/2 fermions in SUSY contrasted with vector production in UED.

In particular, we consider the processes - the production and two-body decays of a lighter chargino pair and those of a first KK W-boson pair:

\[
e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^- \rightarrow (\ell^+ \bar{\nu}_\ell)(\ell^- \bar{\nu}_\ell^*),
\]

\[
e^+e^- \rightarrow W_1^+W_1^- \rightarrow (\ell^+ \nu_\ell)(\ell^- \bar{\nu}_\ell^*),
\]

where the charged leptons \( \ell^\pm \) can be either a muon or an electron type. An explicit calculation shows that, when CP is preserved and all the absorptive parts are negligible, the correlated azimuthal-angle distributions \( (2\pi/C) \partial C/\partial \phi \) are given in terms of the production matrices by

\[
\tilde{\chi}_1^+ \tilde{\chi}_1^- : 1 + \frac{\pi^2}{8} \text{Re}(\rho_{\pm\pm}) \cos(\phi),
\]
where the super/sub-scripts ± stand for the helicities ±1/2 and ±1 of $\tilde{\chi}_1^\pm$ and $W_1^\pm$, respectively. The integrated production density matrix $\rho$ is defined as:

$$
\rho_{\lambda_-,\lambda_+} = \int \mathcal{P}_{\lambda_-,\lambda_+}(E_{cm}, \Theta) \, d\cos \Theta / \sum_{\lambda_\pm} \int \mathcal{P}_{\lambda_-\lambda_+}(E_{cm}, \Theta) \, d\cos \Theta.
$$

We note that the $\tilde{\chi}_1^\pm$ and $W_1^\pm$ two-body decays do not suppress the $\cos \phi$ terms, while the highest $\cos(2\phi)$ mode in the $W_1^\pm$ case may be strongly suppressed due to a small polarization analyzing power of the $W_1^\pm$ decay. In particular, the coefficient becomes vanishing if two states, $W_1^\pm$ and $\nu_1$, are degenerate.

For our numerical demonstration in the $\tilde{\chi}_1^\pm$ and $W_1^\pm$ cases, we take the masses of $\tilde{\chi}_1^\pm/W_1^\pm$ and $\tilde{\ell}/\nu_{\ell1}$ in our BCMM scenario to be

$$
\text{BCMM: } m_{\pm} = m_{\tilde{\chi}_1^\pm} = m_{W_1^\pm} = 300 \text{ GeV and } m_0 = m_{\tilde{\ell}} = m_{\nu_{\ell1}} = 200 \text{ GeV.}
$$

Even though the $\nu_{\ell1}/\tilde{\ell}$ are not the LSP/LKP, they decay to neutrinos and the LSP/LKP, neither of which is visible in the detector.

We perform fits of the azimuthal angle distributions obtained with an integrated luminosity of 500 fb$^{-1}$ to $(1 + A_1 \cos \phi + A_2 \cos 2\phi)/2\pi$. The results for the BCMM mass spectrum are displayed in Fig. 4. Error bars corresponding to the 1-$\sigma$ uncertainty range are obtained with $\text{Br}(\tilde{\chi}_1^\pm \to \tilde{\ell}^\pm \nu_{\ell}) = \text{Br}(W_1^\pm \to \tilde{\ell}^\pm \nu_{\ell1}) = 0.4$. First, we note that the coefficient $A_2$ is too small (less than 0.5%) in magnitude to distinguish the spin-1 $W_1^\pm$ state from the spin-1/2 $\tilde{\chi}_1^\pm$ state. This strong suppression in the $W_1^\pm$ case is not only due to the small analyzing power of the $W_1^\pm$ two-body decays in the BCMM scenario but also due to the cancellation of the corresponding production helicity amplitudes, $\sim m_{W_1^\pm}^2/E_{cm}^2$, that is forced by electroweak gauge invariance to save the unitarity [23]. On the contrary, the coefficient $A_1$ in both the $\tilde{\chi}_1^\pm$ and the $W_1^\pm$ case is sufficiently large so that they can clearly be distinguished from the spin-0 case.

To conclude. The fully-correlated azimuthal-angle correlations encoding quantum interference between different helicity states can provide a powerful method of spin measurements at the ILC. We have found that, if all the particle masses are known, all the cosines of the azimuthal angle between two decay planes in the process $\mathbb{11}$ are unambiguously determined despite the inherent 2-fold discrete ambiguity in determining the four-momenta of the decaying particles.

Quantitatively, we have shown in a specific scenario that the spin-0 smuons can be distinguished from spin-1/2 KK muons or higher-spin states. However, it turned out to be difficult to establish

\footnote{A detailed derivation of the explicit forms of the production density matrices will be presented in a separate publication.}
Figure 5: Coefficients $A_1$ and $A_2$ as a function of $E_{cm}$ for the SUSY $\tilde{\chi}_1^\pm$ and the UED $W_1^\pm$ production in the BCMM spectrum with an integrated luminosity of 500 $fb^{-1}$. Error bars obtained with $Br(\tilde{\chi}_1^\pm \to \ell^\pm \tilde{\nu}_\ell) = Br(W_1^\pm \to \ell^\pm \nu_\ell) = 0.4$ correspond to the 1-$\sigma$ uncertainty range.

the spin-1 nature of the KK $W$-boson due to the strong suppression of the highest $\cos 2\phi$ mode, requiring other methods such as the decay polar-angle distributions \cite{8} and the singly-correlated azimuthal-angle distributions \cite{11}. However, the latter suffers from the two-fold ambiguities in the reconstruction.

Before closing, we emphasize once more that, although applied only to the SUSY and UED processes explicitly in this report, the proposed spin-determination method through quantum azimuthal-angle correlations can be used for any process with a generic event topology similar to that in the process \cite{11} in the SM and beyond.

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