

\section*{1 \(\gamma\)-ray strength function}

The \(\gamma\)-ray strength function (\(\gamma\)SF) \cite{1,2,3} is a nuclear statistical quantity of describing the nuclear electromagnetic response that is employed in the Hauser-Feshbach (HF) model \cite{4} of the compound nuclear reaction.

\subsection*{1.1 downward \(\gamma\)-ray strength function}

The \(\gamma\)SF in the de-excitation mode which we refer to as downward \(\gamma\)SF is a key quantity in the HF model calculation of radiative neutron capture cross sections. The downward \(\gamma\)SF for dipole radiation with a given energy \(\epsilon\) is defined \cite{1,5} by

\[
\epsilon_{\gamma} f_{X1}(\epsilon_{\gamma}) = \frac{\langle \Gamma_{X1}(\epsilon_{\gamma})/\epsilon_{\gamma} \rangle}{D_\ell}.
\]

Here \(X\) is either electric (\(E\)) or magnetic (\(M\)), \(\Gamma_{X1}(\epsilon_{\gamma})\) is a partial radiation width, the symbol \(\langle \rangle\) stands for unweighted averaging over included resonances, and \(D_\ell\) is the average level spacing for s-wave (\(\ell=0\)) or p-wave (\(\ell=1\)) neutron resonances.

\subsection*{1.2 upward \(\gamma\)-ray strength function}

In contrast, the \(\gamma\)SF in the excitation mode which we refer to as upward \(\gamma\)SF is defined \cite{1,5} by the average cross section for \(E1/M1\) photoabsorption \((\sigma_{X1}(\epsilon_{\gamma}))\) to the final states with all possible spins and parities \cite{2}:}

\[
\overrightarrow{\epsilon_{\gamma} f_{X1}(\epsilon_{\gamma})} = \frac{\epsilon_{\gamma}^{-1}(\sigma_{X1}(\epsilon_{\gamma}))}{3(\pi\hbar c)^2}.
\]

\section*{2 Brink-Axel hypothesis \(A\), \(B\), and \(C\)}

It is convenient to define the Brink-Axel hypothesis \cite{6,7} in three versions \(A\), \(B\) and \(C\) on the upward and downward \(\gamma\)SF.

\subsection*{2.1 Brink-Axel hypothesis \(A\)}

The version \(A\) is the equality of the upward \(\gamma\)SF built on the ground state and excited states. The photoabsorption cross section and thus the photoneutron cross section for GDR were assumed to be of Lorentzian shape. Historically this hypothesis has led to the experimental investigation of nuclear properties of hot nuclei \cite{8,9}, which was triggered by the observation of radiations from GDR built on highly excited states \cite{10}.

\subsection*{2.2 Brink-Axel hypothesis \(B\)}

The equality similar to the version \(A\) may apply to photodeexcitation as well. This version is backed by the detailed balance theorem \cite{11} which links photo-emission and absorption between given initial and final states. Recently, it was experimentally shown that the equality of \(\gamma\)SF in photodeexcitation (downward \(\gamma\)SF) from initial states at different excitation energies \cite{12} and to different final states \((2^+\text{ and }4^+)\) holds under the presence of M1 upbend \cite{13}.
2.3 Brink-Axel hypothesis C

The version C is concerned with the equality of upward and downward γSFs. A low-energy enhancement called M1 upbend was experimentally observed in downward γSF [14–16] and theoretically supported by the shell-model calculation [17–23]. A recent systematic study across the chart of nuclei has formulated the low-energy enhancement as zero-limit E1 and M1 strengths in the analytical form based on the shell-model calculation [24]. The presence of the zero-limit strength which corresponds to γ-ray transitions between high-lying states is unique to the downward γSF, showing that the Brink-Axel hypothesis C is violated.

3 Systematic study of (γ, n) and (n, γ) cross sections

We present here a systematic investigation of the (n, γ) and (γ, n) cross section within the γ-ray strength function method [25, 26] in the context of astrophysical applications for Ni isotopes including 63Ni, a branching point nucleus along the weak s-process path and Ba isotopes in the vicinity of the neutron magic number 82 along the main s-process path.

3.1 Ni isotopes

Figure 1 shows downward γSFs, $f_{\text{E1}}(E_{\gamma})$, for Ni isotopes constructed in the present study [25]. The present experimental (γ, n) cross sections for 60Ni, 61Ni, and 64Ni were used to constrain the γSF from the Hartree-Fock-Bogolyubov plus quasiparticle-random phase approximation based on the Gogny D1M interaction for E1 and M1 components (hereafter denoted as D1M+QRPA). Phenomenological corrections include a broadening the QRPA strength to take the neglected damping of GDR into account and a shift of the strength to lower energies due to the contribution beyond one-particle-one-hole excitations and the coupling between the single-particle and low-lying collective phonon degrees of freedom (see Ref. [24, 25] for more details). The phenomenological correction was systematically applied throughout the Ni isotopic chain including 59Ni and 60Ni. The Oslo data whenever available are shown in Fig. 1. We follow the same prescriptions as used in Ref. [24], i.e., the final E1 and M1 strengths, referred to as D1M+QRPA+0lim, include the QRPA as well as the zero-limit contributions and are expressed as

$$f_{\text{E1}}(E_{\gamma}) = f^\text{QRPA}_{\text{E1}}(E_{\gamma}) + f_{\text{0lim}} U[1 + e^{(E_{\gamma}-E_0)/f}]$$

(3)

$$f_{\text{M1}}(E_{\gamma}) = f^\text{QRPA}_{\text{M1}}(E_{\gamma}) + C e^{-\eta E_{\gamma}}$$

(4)

where an M1 zero limit $C = 10^{-8}$ MeV$^{-3}$ derived from shell-model calculations [24] was found to provide a rather good systematic description of available photoneutron data, average resonance capture data, Oslo γSF as well as averaged radiative widths. Larger values could be envisioned from previous Oslo measurements [27, 29]. For this reason, two different values are adopted in the present analysis, namely $C = 3 \cdot 10^{-8}$ and $10^{-7}$ MeV$^{-3}$. The D1M+QRPA calculation is in relatively good agreement with the photoneutron data, even in the 10 MeV region, where one can see extra M1 strength on top of the E1 component, as seen in 64Ni.

Figure 2 shows (n, γ) cross sections predicted with the TALYS code [37] based on the downward γSF shown in Fig. 1 in comparison with experimental data. In addition to the γSF, the radiative neutron capture is rather sensitive to the nuclear level densities. For this reason, five
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Enhancement as zero-limit $E_1$ and $M_1$ strengths in the an-
proximation based on the Gogny D1M interaction for 
For this reason, two different nuclear level density models have been considered [38–41], all of them being adjusted to experimental low-lying states as well as $s$-wave resonance spacings whenever available experimentally [42]. The hashed areas shown in Fig. 2 represent the prediction uncertainties associated with different nuclear level density models. Radiative neutron capture cross sections are reasonably reproduced by the experimentally constrained downward $\gamma$SF with the zero-limit strength though it remains difficult to reconcile $\gamma$SF and cross section data in some cases.

3.2 Ba isotopes

Figure 3 shows upward $\gamma$SFs, $f_{\gamma 1}(E_\gamma)$, for $^{137}\text{Ba}$ and $^{138}\text{Ba}$ [47]. Two relatively different models of $\gamma$SF, the semi-microscopic D1M+QRPA and phenomenological Simple Modified Lorentzian (SMLO) models, are employed. Similarly to Ni isotopes, the phenomenological correction is systematically applied to the Ba isotopic chain. In addition, a specific correction that is an energy shift of 0.5 MeV of the overall $E_1$ strength, is required in the case of $^{138}\text{Ba}$.

Hauser-Feshbach model calculations of $\langle n, \gamma \rangle$ cross sections and the Maxwellian-averaged cross sections (MACS) were performed with the TALYS code. The upward $\gamma$SF shown in Fig. 3 supplemented with the zero-limit $E_1$ and $M_1$ components was used as the downward $\gamma$SF for $^{137}\text{Ba}$ and $^{138}\text{Ba}$ in the TALYS calculation. Re-
sults of a systematic study of the MACS over the Ba isotopic chain, including those for $^{131}\text{Ba}(n,\gamma)^{132}\text{Ba}$ and $^{133}\text{Ba}(n,\gamma)^{134}\text{Ba}$ reactions, are shown in Fig. 4 in comparison with experimental data [48].

4 Summary

There is a growing research interest in the study of the $\gamma$-ray strength function which governs photo-emission and absorption processes in nuclear physics and astrophysics. The Brink hypothesis in three versions has been a navigator of the experimental study of the $\gamma$-ray strength function in a variety of nuclear reactions such as radiative neutron capture, photonucleon, nuclear resonance fluorescence, inelastic and transfer reactions. We have systematically performed TALYS Hauser-Feshbach model calculation of $(n,\gamma)$ cross sections over the Ni and Ba isotopic chain along the s-process nucleosynthesis path based on the $\gamma$-ray strength function method, where $(\gamma, n)$ cross sections were used as experimental constraints on the upward D1M+QRPA $\gamma$-ray strength function and the downward $\gamma$-ray strength function was constructed by supplementing the upward $\gamma$-ray strength function with the zero-limit $M1$ and $E1$ strengths. The calculated $(n,\gamma)$ cross sections are in rather good agreement with experimental data.

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