The Dynamical Behaviors in (2+1)-Dimensional Gross-Neveu Model with a Thirring Interaction

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Abstract

We analyze (2+1)-dimensional Gross-Neveu model with a Thirring interaction, where a vector-vector type four-fermi interaction is on equal terms with a scalar-scalar type one. The Dyson-Schwinger equation for fermion self-energy function is constructed up to next-to-leading order in 1/N expansion. We determine the critical surface which is the boundary between a broken phase and an unbroken one in \((\alpha_c, \beta_c, N_c)\) space. It is observed that the critical behavior is mainly controlled by Gross-Neveu coupling \(\alpha_c\) and the region of the broken phase is separated into two parts by the line \(\alpha_c = \alpha_c^* (= \frac{8}{\pi^2})\). The mass function is strongly dependent upon the flavor
number $N$ for $\alpha > \alpha_c^*$, while weakly for $\alpha < \alpha_c^*$. For $\alpha > \alpha_c^*$, the critical flavor number $N_c$ increases as Thirring coupling $\beta$ decreases. By driving the CJT effective potential, we show that the broken phase is energetically preferred to the symmetric one. We discuss the gauge dependence of the mass function and the ultra-violet property of the composite operators.
1 Introduction

Dynamical symmetry-breaking (DSB) plays an important role in applying gauge field theories to particle physics. The structure of chiral symmetry has been studied extensively for variety of gauge models [1 - 4]. Since the work of Nambu and Jona-Lasinio [1], DSB has been investigated as the mechanism of generating fermion masses in elementary particle physics for almost one generation [2 - 4]. Appelquist et al. [4] showed that (2+1)-dimensional QED exhibits a critical behavior as the flavor number N approaches $N_c = \frac{32}{\pi^2}$ in the framework of the 1/N expansion. Such a behavior was also confirmed by the lattice simulation [5].

The search for the novel solution of Dyson-Schwinger (DS) equation for a fermion propagator was initiated by Johnson, Baker, and Willey [2]. The same equation can be derived by extremizing the effective potential of Cornwall, Jackiw, and Tomboulis (CJT) [6]. The CJT effective potential enables one to check the stability of non-trivial solutions of DS equation, that is, whether the solutions are energetically preferred to the trivial solution or not.

(2+1)-dimensional four-fermi interaction models were shown to be renormalizable in the framework of the 1/N expansion despite its non-renormalizability in ordinary weak coupling expansion [7]. Such a dramatic transmutation is due to the fact that the composite operator acquires the large anomalous dimension in strongly-correlated region. (2+1)-dimensional Gross-Neveu (GN)
model possesses a non-trivial ultra-violet (UV) fixed point at the leading order of $1/N$ expansion, which survives beyond the leading order [8]. It provides a ground to study DSB. $(2+1)$-dimensional Thirring model has also been studied in the context of the gauge structure and a dynamical mass generation of the fermions in the $1/N$ expansion [9]. It is reported that Thirring model shows the phase transition with the strong dependence of the critical flavor number on Thirring coupling constant, and possesses non-trivial UV fixed point albeit at non-perturbative order in $1/N$ [9]. While the gap equation at the leading order is local in GN model, the first non-trivial gap equation is non-local in Thirring model. It turns out that the gap equation is a relation to fine-tune the coupling as in GN model rather than the value of the fermion mass as in $(2+1)$-dimensional QED. The dynamically generated fermion mass then becomes a physical parameter which is much smaller than the natural cut-off of the theory.

Thirring interaction is another independent composite operator which is relevant to GN interaction in strongly coupled regime [8, 9]. Accordingly, it is natural for us to consider whether the above four-fermi interactions co-operate dynamically in the generation of fermion mass.

In this paper, we consider $(2+1)$-dimensional Gross-Neveu model with a Thirring interaction, and investigate how Thirring interaction plays a part on the critical behavior of GN model. In Sec. 2, we introduce our model and derive the dressed propagators whose UV behavior improved by adding the gauge-fixing-like term to the Lagrangian. In Sec. 3, the DS equation is
constructed up to $1/N$-order. We determine the critical surface which is the boundary between a broken phase and an unbroken one in $(\alpha, \beta, N)$ space, and also present the fermion mass function in an appropriate approximation scheme. The critical behavior is mainly controlled by the Gross-Neveu coupling $\alpha_c$ and the region of the broken phase is separated into two parts by the line $\alpha_c = \alpha_c^*(= \frac{8}{n\pi})$. The gauge dependence of the mass function is discussed. In Sec. 4, we show that our nontrivial solution is energetically preferred to the trivial one by deriving the CJT effective potential. In Sec. 5, we discuss the consequences of results and the UV property of the composite operators.

2 The Model

Our model is given in the Euclidean version by the Lagrangian

$$L = \bar{\psi}(i\gamma \cdot \partial)\psi + \frac{1}{2N} \left( g^2(\bar{\psi}\psi)^2 + h^2(\bar{\psi}\gamma_\mu\psi)^2 \right),$$

where $\psi$ are the four-component Dirac spinors whose flavor indices are suppressed and $N$ is the fermion flavor number. As one can see, scalar-scalar and vector-vector type four-fermion interactions are parallelly introduced in the above equation. The $\gamma$ matrices are chosen to be anti-hermitean as follows,

$$\gamma_0 = \begin{pmatrix} 0 & iI \\ iI & 0 \end{pmatrix}, \quad \gamma_j = \begin{pmatrix} 0 & -\sigma_j \\ \sigma_j & 0 \end{pmatrix},$$

where $I$ is $2 \times 2$ identity matrix, $\sigma_j$’s are the Pauli matrices and $j = 1, 2$.

Introducing the auxiliary scalar field $\sigma$ and the auxiliary vector field $A_\mu$
to facilitate $1/N$ expansion [7, 9], we can rewrite Eq. (1) as

$$L^{eq} = \bar{\psi}(i\gamma \cdot \partial)\psi - \frac{1}{2h^2}A^2_\mu - \frac{1}{2g^2}\sigma^2 + \frac{1}{\sqrt{N}}A_\mu(\bar{\psi}\gamma_\mu\psi) + \frac{1}{\sqrt{N}}\sigma(\bar{\psi}\psi). \quad (3)$$

It is easy to see the equivalence between $L$ and $L^{eq}$ as follows

$$L = -\ln \left\{ \int D\sigma D\bar{A}_\mu \exp\left[-\int L^{eq}d^3x\right] \right\}. \quad (4)$$

If we do not want to worry about what quantities should be required to be renormalizable, we can improve the UV behavior of all the Green’s functions by adding to Eq. (3) the term $(\partial_\mu A_\mu)^2/(2\xi)$ and postulating that the observables are those quantities which are independent of the gauge parameter $\xi$.

The new theory also has a restricted gauge symmetry [9].

The tree propagators, after the introduction of the gauge-fixing term, are

$$\Delta^{(0)}_{\mu\nu}(p) = h^2(\delta_{\mu\nu} - p_\mu p_\nu/p^2) + \frac{\xi}{p^2 + \xi/h^2(p_\mu p_\nu/p^2)}, \quad (5)$$

$$G^{(0)}(p) = g^2. \quad (6)$$

To find the $1/N$ leading propagator, we evaluate the three kinds of one-loop two-point functions by the method of dimensional regularization as follows:

$$\Pi_{\mu\nu}(p) = -\text{tr} \int \frac{d^3k}{(2\pi)^3} \gamma_\mu \frac{1}{\gamma \cdot k + m} \gamma_\nu \frac{1}{\gamma \cdot (k-p) + m}$$

$$= -(\delta_{\mu\nu} - p_\mu p_\nu/p^2)F(p), \quad (7)$$

$$\Pi(p) = -\text{tr} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\gamma \cdot k + m} \frac{1}{\gamma \cdot (k-p) + m}$$

$$= -2F(p), \quad (8)$$

$$\Pi_\mu(p) = -\text{tr} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\gamma \cdot k + m} \gamma_\mu \frac{1}{\gamma \cdot (k-p) + m}$$

$$= 0. \quad (9)$$
where the trace is acting on a spinor space, \( m \) is the fermion mass generated dynamically through 1/N corrections and \( F(p) \) can be expressed without introducing the regulating parameter as follows [3, 4]

\[
F(p) = \frac{1}{\pi} \left[ \frac{m}{2} + \frac{p^2 - 4m^2}{4\sqrt{p^2}} \arcsin\left( \sqrt{\frac{p^2}{p^2 + 4m^2}} \right) \right].
\] (10)

We adapt the approximation \( m^2 \ll p^2 \) in the following analysis and in that case the \( F(p) \) goes to \( \sqrt{p^2}/8 \) for the evaluation of DS gap equation in Sec. 3.

There is no mixing between the scalar field \( \sigma \) and the vector field \( A_{\mu} \), since the scalar-vector one-loop diagram vanishes (Eq. (9)).

We can evaluate the dressed propagators by summing the chain diagrams as follows:

\[
\Delta_{\mu\nu}(p) = \frac{1}{1/h^2 + F(p)} (\delta_{\mu\nu} - p_{\mu}p_{\nu}/p^2) + \frac{\xi}{p^2 + \xi/h^2} (p_{\mu}p_{\nu}/p^2),
\] (11)

\[
G(p) = \frac{1}{1/g^2 + 2F(p)}.
\] (12)

3 The Dyson-Schwinger Equation

We start with the theory with a UV cut-off as was usually done in QED\(_{3+1}\) [10]. By using the dressed propagators, we can construct the following DS gap equation [2 - 4] up to 1/N-order,

\[
S^{-1}(p) = S^{(0)}^{-1}(p) + \sqrt{N}G(0) \int_{0}^{\Lambda} \frac{d^3k}{(2\pi)^3} \Gamma(k) \text{tr}[S(k)]
\]

\[-\frac{1}{N} \int_{0}^{\Lambda} \frac{d^3k}{(2\pi)^3} \bar{\gamma}_{\mu} S(k) \gamma_{\nu} \Delta_{\mu\nu}(p - k) - \frac{1}{N} \int_{0}^{\Lambda} \frac{d^3k}{(2\pi)^3} S(k)G(p - k),
\] (13)
\[ S^{-1}(p) = A(p) \gamma \cdot p + \Sigma(p), \quad (14) \]
\[ S^{(0)}^{-1}(p) = \gamma \cdot p, \quad (15) \]
\[ \Gamma(p) = \frac{1}{\sqrt{N}} \left( \frac{\Lambda^2}{p^2} \right)^{\gamma}, \quad \gamma = \frac{3}{\pi^2 N}, \quad (16) \]
\[ A(p) = \left( \frac{\Lambda^2}{p^2} \right)^{\gamma_{\psi}}, \quad \gamma_{\psi} = \frac{-1}{\pi^2 N}. \quad (17) \]

\( \Sigma(p) \) is the fermion self-energy function generated dynamically and \( \Lambda \) is the natural cut-off of the theory. \( A(p) \) and \( \Gamma(p) \) denote the renormalization constants of the wave function and the vertex, respectively. They can be calculated perturbatively from the bare Lagrangian \( \mathcal{L}^{eq} \) and their detailed derivations are presented in the Appendix. The second term in the right hand side of Eq. (13) denotes the tadpole diagram. We can not neglect the corrections of \( 1/N \)-order which stems from \( A(p) \) and \( \Gamma(p) \), since we want to consider the theory up to that order. The extra two terms denote the self-energy diagrams by vector field \( A_\mu \) and scalar field \( \sigma \). The crossing diagram between \( \Delta_{\mu\nu}(p) \) and \( G(p) \) correspond to the order of \( 1/N^2 \), therefore, it is excluded from the above equations.

Taking the trace over the gamma matrices in Eq. (13), we get the following DS gap equation

\[
\Sigma(p) = 4g^2 \int_0^\Lambda \frac{d^3k}{(2\pi^3)} \frac{\Sigma(k)}{k^2 + \Sigma^2(k)} \ln(\Lambda^2/k^2) \left[ \gamma - 2\gamma_{\psi} \frac{k^2}{k^2 + \Sigma^2(k)} \right] 
\]
\[ + \frac{2}{N} \int_{0}^{\Lambda} \frac{d^3 k}{(2\pi^3) k^2 + \Sigma^2(k)} \frac{1}{|p - k|/8 + 1/h^2} \]
\[ - \frac{1}{N} \int_{0}^{\Lambda} \frac{d^3 k}{(2\pi^3) k^2 + \Sigma^2(k)} \frac{1}{|p - k|/4 + 1/g^2} \]
\[ + \frac{\xi}{N} \int \frac{d^3 k}{(2\pi^3) k^2 + \Sigma^2(k)} \frac{1}{(k - p)^2 + \xi/h^2}, \quad (18) \]

where \(|p - k| = \sqrt{(p - k)^2}.

In the case of \(N \leq 1\) it is meaningless for us to analyze Eq. (18) since the
\(1/N\) expansion is failed in that region. We intend to present the comparable
feature with the previous studies \([4, 5]\), in which it is reported that the
fermion mass is vanished when the fermion flavor \(N\) goes to \(N_c = 32/\pi^2\). We
plot the following figures in the case of \(N=1\) for illustrative purposes.

Since the last term in Eq. (18) has the gauge parameter \(\xi\), \(\Sigma(p)\) is a
gauge-dependent quantity. However, the fact that it is not identically zero has
physical consequence (i.e., chiral-symmetry breaking). Accordingly, whether
the fermion mass is generated or not is the gauge invariant statement \([9]\).

### 3.1 Critical surface in \((\alpha_c, \beta_c, N_c)\) space

Let us approximate \(\Sigma(k)\) in Eq. (18) to \(m \approx \Sigma(k)|_{k=0}\). Taking the limit
\(m \to 0\), we can access the critical region as follows:

\[(g^2, h^2, N) \xrightarrow{m \to 0} (g_c^2, h_c^2, N_c). \quad (19)\]

In that case, the critical surface is defined by

\[1 = \frac{8}{\alpha_c \pi^2} \left(1 + \frac{10}{N_c \pi^2}\right) + \frac{8}{N_c \pi^2} \ln \left(\frac{1 + \beta_c}{\beta_c}\right) - \frac{2}{N_c \pi^2} \ln \left(\frac{1 + \alpha_c}{\alpha_c}\right)\]
\[ + \frac{4}{N_c \pi^2} \sqrt{\frac{\xi}{\beta_c}} \arctan \left( \frac{1}{\sqrt{\xi \beta_c}} \right), \quad (20) \]

where the dimensionless quantities are given by,

\[ \alpha_c = \frac{4}{\Lambda g_c^2}, \quad \beta_c = \frac{8}{\Lambda h_c^2}, \quad \tilde{\xi} = \frac{\xi}{8\Lambda}. \quad (21) \]

From Eq. (20), regardless of \( \beta_c \), there exists the critical flavor \( N_c \) only in the region of \( \alpha_c > \alpha^*_c \) and \( \alpha_c \) goes to \( \alpha^*_c (= \frac{8}{\pi}) \) as \( N_c \) goes to infinity. Eq. (20) is plotted in Fig. 1 and Fig. 2 for two different gauge conditions, which are \( \xi = 0 \) (Landau gauge) and \( \xi = 1 \) (Feynman gauge), respectively. Each curve in Fig. 1 and Fig. 2 corresponds to the boundary between a broken phase and an unbroken one. One can see that the curves of \( \beta_c = 1.0 \times 10^4 \) are not deformed in two figures. The other curves in Fig. 2 are shifted to the right for the corresponding curves in Fig. 1.

### 3.2 Fermion mass function

After taking the integrations in Eq. (18), we obtain the following equation:

\[ N = \frac{\frac{8}{\alpha \pi^2} B(M) + \frac{1}{\pi^2} C(M) + \frac{4}{\pi^2} \tilde{\xi} D(M)}{1 - \frac{8}{\alpha \pi^2} A(M)}, \quad (22) \]

where

\[ M = \frac{m}{\Lambda} \quad (23) \]
\[ A(M) = 1 - M \arctan(1/M) \quad (24) \]
\[ B(M) = \frac{1}{\pi^2} \left[ (-10a - 5b - \frac{10}{3}c - 2bM^2 + 14cM^2) \right. \\
\left. +(12a - 16cM^2)M \arctan(1/M) \right] \]
\begin{align}
C(M) &= 8F(\beta, M) - 2F(\alpha, M), \\
D(M) &= \frac{4 \bar{\xi}}{\pi^2} M \arctan(1/M) - \sqrt{\xi \beta} \arctan(1/\sqrt{\xi \beta}), \\
F(\alpha, M) &= \frac{1}{M^2 + \alpha^2} \left[ M^2 \ln \left( \frac{1 + \alpha}{M^2} \right) - M \alpha \arctan(1/M) + \alpha^2 \ln \left( \frac{1 + \alpha}{\alpha} \right) \right],
\end{align}

where we have taken the following interpolation:

\[ x \ln x \approx ax + bx^2 + cx^3, \quad \text{for } x \in [0, 1] \tag{29} \]

with \( a = -2.5, \ b = 4.0, \ c = -1.5. \)

The fermion mass functions in Landau gauge are presented for \( \alpha < \alpha_c^* \) and \( \alpha > \alpha_c^* \) in Fig. 3 and Fig. 4, respectively. The fermion mass functions in Feynman gauge are presented for \( \alpha < \alpha_c^* \) and \( \alpha > \alpha_c^* \) in Fig. 5 and Fig. 6, respectively. The mass functions in Fig. 3 and Fig. 5 are strongly dependent on the flavor number \( N \), while those in Fig. 4 and Fig. 6 weakly. One can easily confirm that the Eq. (22) is equivalent to Eq. (20) in the limit of \( M \to 0 \).

\section{4 The Vacuum Stability}

The DS equation is obtained by extremizing the CJT \([5]\) effective potential with respect to fermion full propagator \( \bar{S}(p) \). The CJT effective potential for
our model is given by

\[ V^{CJT}(\bar{S}) = \int \frac{d^3q}{(2\pi)^3} \text{tr} \left[ \ln(S^{(0)^{-1}}(q)\bar{S}(q)) - S^{(0)^{-1}}(q)\bar{S}(q) + 1 \right] \]
\[ -\frac{\sqrt{N}}{2} G(0) \left( \int \frac{d^3q}{(2\pi)^3} \Gamma(q)\text{tr}[\bar{S}(q)] \right)^2 \]
\[ + \frac{1}{2N} \int \frac{d^3q d^3k}{(2\pi)^6} \text{tr} \left[ \gamma_\mu \bar{S}(q)\gamma_\nu \bar{S}(q+k)\Delta_{\mu\nu}(k) + \bar{S}(q)\bar{S}(q+k)G(k) \right], \tag{30} \]

where the first term in the right hand side of Eq. (30) denotes the fermion one-loop contribution and the extra terms denote the two-loop contributions.

We consider the extremal condition in the CJT effective potential as follows

\[ \frac{\delta V^{CJT}(\bar{S})}{\delta \bar{S}(p)} |_{\bar{S}=S} = 0. \tag{31} \]

In that case, we meet the following equation:

\[ S(p)^{-1} = S^{(0)^{-1}}(p) + \Xi(p), \tag{32} \]

where,

\[ \Xi(p) = \sqrt{N}G(0) \int \frac{d^3k}{(2\pi)^3} \Gamma(k)\text{tr}[S(k)] \]
\[ -\frac{1}{N} \int \frac{d^3k}{(2\pi)^3} \gamma_\mu S(k)\gamma_\nu \Delta_{\mu\nu}(p-k) - \frac{1}{N} \int \frac{d^3k}{(2\pi)^3} S(k)G(p-k). \tag{33} \]

One see that Eq. (32) with Eq. (33) coincides with the previous DS equation presented in Eq. (13). By inserting the Eq. (32) with Eq. (33) into the Eq.
(30), we obtain the CJT effective potential at the extremal propagator \( S(p) \) as follows

\[
V^{CJT}(S) = -2 \int \frac{d^3p}{(2\pi)^3} \left[ \ln(1 + \frac{\Sigma(p)^2}{p^2}) - \frac{\Sigma^2(p)}{p^2 + \Sigma^2(p)} \right].
\]  

(34)

Since the function \( \ln(1 + x) - x/(1 + x) \) is positive for all positive \( x \), then \( V^{CJT}(S) \) is less than or equal to zero. Hence, the energy of any nontrivial solutions is lower than that of the trivial (perturbative) solution \( \Sigma(p) = 0 \), therefore, the broken phase is always energetically preferred to the symmetric one.

5 Discussion

We studied (2+1)-dimensional Gross-Neveu model with a Thirring interaction, where a vector-vector type four-fermi interaction is on equal terms with a scalar-scalar type one. To solve the DS gap equation up to 1/\( N \)-order, the renormalization constants (i.e., \( A(p), \Gamma(p) \)) were calculated perturbatively up to that order and the fermion self-energy function \( \Sigma(p) \) was approximated to the constant \( m(\approx \Sigma(p = 0)) \). We expect that these approximations do not change the qualitative feature of the phase structure of our model.

Our gap equation (Eq. (18)) contains a gauge parameter \( \xi \), and thus it is gauge dependent. The position \( A(p)^2p^2 = -\Sigma(p)^2 \) of the pole of fermion propagator is a physical quantity that must be independent upon the parameter. We, unfortunately, have ignored the momentum dependence of \( \Sigma(p) \) in our analysis, we could not present the physical mass of fermion. The numer-
ical evaluation of Eq. (18) will allow one to investigate the physical mass of fermion. However, as was discussed in Ref. [9], the fact that the fermion mass is not identically zero has physical consequence (i.e., chiral-symmetry breaking). Accordingly, whether the fermion mass is generated or not is a gauge invariant statement. The critical surfaces for $\tilde{\xi} = 0.0$ (Landau gauge) and $\tilde{\xi} = 1.0$ (Feynman gauge) are presented in the contour shape in Fig. 1 and Fig.2, respectively. The curves of $\beta_c = 1.0 \times 10^4$ are not deformed in two gauge conditions, since in that case our model is equivalent to GN model in the bare Lagrangian level. The other curves in Fig. 2 are shifted to the right from the corresponding curves in Fig. 1. Such a feature is natural since it means that the vector bosons are more correlated than the scalar bosons in the neighborhood of the phase boundary.

The critical line $\alpha_c = \alpha_c^* (= \frac{8}{\pi^2})$ in the Gross-Neveu model of the leading order turns into a critical surface in $(\alpha_c, \beta_c, N_c)$ space in our model (see Fig. 1 and Fig. 2). The fermion mass function in Landau gauge are presented in the region of $\alpha < \alpha_c^*$ and $\alpha > \alpha_c^*$ in Fig. 3 and Fig. 4. They show that the fermion mass is generated dynamically in both regions. The fermion mass is weakly dependent on the fermion flavor number $N$ in the former, while strongly in the region of the latter. Such features can be reasonably understood. For $\alpha < \alpha_c^*$, the critical behaviors are dominated by the tadpole diagram, then the theory always shows the mass generation as to all of $N$. However, for $\alpha > \alpha_c^*$, the tadpole term does not contribute to the mass generation, therefore the critical behaviors are controlled by the terms of the
order of 1/N and they are QED$_{2+1}$-like or Thirring-like. As argued in Ref. [9], the effective infrared coupling weakens like 1/N as N increase due to the screening effect of fermions. Therefore, the infrared coupling becomes so weak that the fermion condensates cannot occur in the case of $N > N_c$.

All of the curves in Fig. 1 approaches to the line $\alpha^*_c$ as N increase, since our model in the large N limit is the GN model of the leading order (Eq. (18) and Eq. (20)). The fermion mass function in Landau gauge are also presented in the region of $\alpha < \alpha^*_c$ and $\alpha > \alpha^*_c$ in Fig. 5 and Fig. 6.

It is interesting to discuss the UV property [7] of composite operators (i.e., $(\bar{\psi}\psi)^2$, and $(\bar{\psi}\gamma_\mu\psi)^2$) in the strongly correlated region. From the dressed propagators (Eq. (11) and Eq. (12)) we can see the asymptotic behavior in UV region (i.e., $p \to \Lambda$) as follows:

$$\Delta_{\mu\nu}(p) \sim \frac{1}{p} (\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}),$$

$$G(p) \sim \frac{1}{p},$$

thus the mass dimensions of the fields in that region are $[\sigma]_{UV} = 1$, and $[A_\mu]_{UV} = 1$. Noting the equivalence $\sigma \sim \bar{\psi}\psi$ and $A_\mu \sim \bar{\psi}\gamma_\mu\psi$ from the Lagrangian (Eq. (3)), we see the following facts:

$$\left[(\bar{\psi}\psi)^2\right]_{UV} = 2,$$

$$\left[(\bar{\psi}\gamma_\mu\psi)^2\right]_{UV} = 2.$$

Accordingly, the renormalizability in that region is guaranteed [7].

Our model contains two of the most fundamental types of four-fermi interactions, and they co-operate dynamically in the generation of fermion
mass. In the region of $\alpha < \alpha^*_c$, the general features of mass generation is not disturbed by the higher $1/N$-order corrections by the dominant tadpole term. In that of $\alpha > \alpha^*_c$, Thirring interaction plays important roles not only in generating the fermion mass dynamically but also in making its strong dependence on $N$. In the latter, the critical flavor number $N_c$ is increases as $\beta$ decreases. Our model, therefore, is probably a good ground for studying the fundamental phase transitions of the Nature. It is important to evaluate Eq. (18) numerically to check the stability [9] of the results. Authors are in process for analyzing it.

This work is supported in part by KOSEF (Korea Science and Engineering Foundation).
Appendix

We derive the renormalization constants of the wave function and the vertex up to 1/N-order. The former can be calculated from the self energy diagrams by the vector field and the scalar field,

\[
\mathcal{D}_1 + \mathcal{D}_2 = \frac{1}{N} \int \frac{d^3k}{(2\pi)^3} \gamma_\mu \Delta(k)_{\mu\nu} \gamma_\nu S^{(0)}(p - k) + \frac{1}{N} \int \frac{d^3k}{(2\pi)^3} G(k) S^{(0)}(p - k) = \frac{4}{3N\pi^2} (\gamma \cdot p) \ln(\Lambda^2/p^2) + \text{finite terms.} \tag{35}
\]

Thus the wave function renormalization constant is given by

\[
A(p) = 1 - \frac{1}{N\pi^2} \ln(\frac{\Lambda^2}{p^2}). \tag{36}
\]

The latter can be calculated by the vertex diagrams by the vector field and the scalar field,

\[
\mathcal{V}_1 + \mathcal{V}_2 = \frac{1}{N^{3/2}} \int \frac{d^3k}{(2\pi)^3} \gamma_\mu \Delta(k)_{\mu\nu} \gamma_\nu S^{(0)}(p_1 + k) S^{(0)}(p_2 + k) + \frac{1}{N^{3/2}} \int \frac{d^3k}{(2\pi)^3} G(k) S^{(0)}(p_1 + k) S^{(0)}(p_2 + k) = \frac{4}{N^{3/2}\pi^2} \ln(\Lambda^2/p_{\text{max}}^2) + \text{finite terms}, \tag{37}
\]

where \(p_{\text{max}}^2\) is the largest of \(p_1^2\) and \(p_2^2\) with \(p_1\) and \(p_2\) the incoming and outgoing fermion momenta, respectively. The vertex renormalization constant for tadpole diagram is given by

\[
\Gamma(p) = \frac{1}{\sqrt{N}} + \lim_{p_1, p_2 \to p} \frac{3}{N^{3/2}\pi^2} \ln(\Lambda^2/p_{\text{max}}^2) = \frac{1}{\sqrt{N}}(1 + \frac{3}{N\pi^2} \ln(\Lambda^2/p^2)).
\]

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Figure Captions

Fig. 1: The projection of the critical surface on \((N_c, \alpha_c)\) plane for various \(\beta_c\). \(\tilde{\xi}\) is chosen to be zero (Landau gauge).

Fig. 2: The projection of the critical surface on \((N_c, \alpha_c)\) plane for various \(\beta_c\). \(\tilde{\xi}\) is chosen to be one (Feynman gauge).

Fig. 3: The fermion mass \(M\) as a function of the flavor number \(N\) for various \(\alpha\) (\(\alpha < \alpha^*_c\)) and \(\beta\) in \(\tilde{\xi} = 0\).

Fig. 4: The fermion mass \(M\) as a function of the flavor number \(N\) for various \(\alpha\) (\(\alpha > \alpha^*_c\)) and \(\beta\) in \(\tilde{\xi} = 0\).

Fig. 5: The fermion mass \(M\) as a function of the flavor number \(N\) for various \(\alpha\) (\(\alpha < \alpha^*_c\)) and \(\beta\) in \(\tilde{\xi} = 1\).

Fig. 6: The fermion mass \(M\) as a function of the flavor number \(N\) for various \(\alpha\) (\(\alpha > \alpha^*_c\)) and \(\beta\) in \(\tilde{\xi} = 1\).