Simple Relations for two body B Decays to Charmonium and tests for $\eta - \eta'$ mixing.

Alakabha Datta\textsuperscript{a}, Harry J. Lipkin\textsuperscript{b} and Patrick. J. O’Donnell\textsuperscript{c}

\textsuperscript{a} Laboratoire René J.-A. Lévesque, Université de Montréal, C.P. 6128, succ. centre-ville, Montréal, QC, Canada H3C 3J7

\textsuperscript{b} Department of Particle Physics, Weizmann Institute, Rehovot 76100, Israel and School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences Tel-Aviv University, Tel-Aviv, Israel and High Energy Physics Division Argonne National Laboratory Argonne, IL 60439-4815, USA

\textsuperscript{c} Department of Physics and Astronomy, University of Toronto, Toronto, Canada.

Abstract

\textsuperscript{1}email: datta@lps.umontreal.ca

\textsuperscript{2}email: harry.lipkin@weizmann.ac.il

\textsuperscript{3}email: pat@medb.physics.utoronto.ca
The two body decays of $B_d$ and $B_s$ decays into $J/\psi M$, where $M$ is a light meson, is studied under the very simple assumptions that the spectator quark does not play a role in the decay of the weak heavy quark or antiquark. This hypothesis leads to interesting relations between decay amplitudes. The assumption of $SU(3)$ symmetry leads to additional relations between the decay amplitudes and in particular, the eight CP eigenstates $J/\psi K_S$, $J/\psi \eta$, $J/\psi \eta'$ and $J/\psi \pi^o$ are all given in terms of three parameters. If agreement with experiment validates these assumptions the parameters over determined by the results will give information about the ratio of penguin to tree contributions to the "golden channel" $B^o \rightarrow J/\psi K_S$ decay and will provide tests for the standard $\eta - \eta'$ mixing, which assumes that this mixing is determined by a single mixing angle, as well as determine the value of the mixing angle. We also present tests of the standard $\eta - \eta'$ mixing involving semileptonic $D$ decays.
1 Introduction - The inactive spectator approach

New data will soon accumulate on both $B^0$ and $B_s$ decays into final CP eigenstates containing charmonium. We present a simple method to facilitate their analysis and the extraction of parameters relevant to CP violation. These states are of particular interest both because they provide “golden channels” like $J/\psi K_S$ important for CP violation, and because the presence of a bound $c\bar{c}$ pair greatly simplifies the analysis of large groups of different decays related by symmetries. The $c\bar{c}$ pair is a singlet under color, isospin and flavor SU(3) and is an eigenstate of $C$ and $P$. The color coupling of the two-meson final state is unique and the $c\bar{c}$ pair is inert under the various symmetries which act only on the light quark pair.

The dominant tree and penguin diagrams describing nonleptonic $B_d$ and $B_s$ decays to charmonium and a meson can all be described as a $b$ decay in which the spectator quark does not participate in a flavor-changing interaction and later combines with a light antiquark to make the final light meson. We now apply this “inactive spectator” approach to all such decays and first note a selection rule that forbids all decays in which the spectator quark does not appear in the final state:

\[ A[B^0 \to J/\psi M(\bar{q}s)] = 0 \]
\[ A[B_s \to J/\psi M(\bar{q}d)] = 0 \]  

(1)

where $M(\bar{q}s)$ and $M(\bar{q}d)$ denote respectively any $\bar{q}q$ meson meson with the constituents $\bar{q}s$ and $\bar{q}d$. This selection rule can immediately be tested in many ways to check the validity of our basic assumption when data are available; e.g.

\[ A_L(B^0 \to J/\psi \rho^0) = A_L(B^0 \to J/\psi \omega) \]  

(2)

\[ A_L(B_s \to J/\psi \rho^0) = A_L(B_s \to J/\psi \omega) = A_L(B^0 \to J/\psi \phi) = 0 \]  

(3)

where $L$ denotes any partial wave for the vector-vector final state in any basis; e.g. in the s, p, d orbital angular momentum basis, the helicity basis or the transversity basis.

Eq. (2) is a particularly robust test for violations of our approach by the presence of contributions in which the spectator quark is annihilated. The $\rho^0$ and $\omega$ have opposite relative signs in the forbidden $u\bar{u}$ and allowed $d\bar{d}$ components of their wavefunctions. The branching ratios of $B_d$ to $J/\psi \rho^0$ and $J/\psi \omega$ can differ by a factor of 2 if the forbidden $u\bar{u}$ amplitude is 20% of the allowed $d\bar{d}$ amplitude, which means only a 4% ratio of the direct forbidden to direct allowed contributions. Such forbidden contributions can arise from diagrams expected to be
small; e.g. $W$ exchange OZI violating diagrams of the type $\bar{b} + d = \bar{u} + u + 3G = \bar{u} + u + J/\psi$. There can also be contributions of the type $\bar{b} + d = \bar{c} + c + 3G = J/\psi + V$ which violate Eq. [3]. Whether these contributions are indeed small can now be checked by experimental tests of eqs. (2-3) and sensitive upper limits on their magnitudes can be given if they are not observed.

If the absence of these transitions is confirmed, the remaining decays are all describable by the two transitions:

\[ B(\bar{b}q) \rightarrow J/\psi \bar{dq} \rightarrow J/\psi M(\bar{dq}) \]
\[ B(\bar{b}q) \rightarrow J/\psi \bar{s}q \rightarrow J/\psi M(\bar{s}q) \] (4)

where $B(\bar{b}q)$ denotes a $B$ meson with the quark constituents $\bar{b}q$, the spectator quark $q$ can be $s$, $u$ or $d$, and $M(\bar{dq})$ and $M(\bar{s}q)$ denote the final mesons.

The decay amplitudes are then described as the product of a $\bar{b}$ decay amplitude and a hadronization function $h$ describing the combination of a quark-antiquark pair to make the final meson.

\[ A[B^0 \rightarrow J/\psi M^0(\bar{s}d)] = A(\bar{b} \rightarrow J/\psi \bar{s}) \cdot h[\bar{s}d \rightarrow M^0(\bar{s}d)] \]
\[ A[B_s \rightarrow J/\psi M^0(\bar{d}s)] = A(\bar{b} \rightarrow J/\psi \bar{d}) \cdot h[\bar{d}s \rightarrow M^0(\bar{d}s)] \] (5)

\[ A[B_d^0 \rightarrow J/\psi M^0(\bar{d}d)] = A(\bar{b} \rightarrow J/\psi \bar{d}) \cdot h[\bar{d}d \rightarrow M^0(\bar{d}d)] \]
\[ A[B_s \rightarrow J/\psi M^0(\bar{s}s)] = A(\bar{b} \rightarrow J/\psi \bar{s}) \cdot h[\bar{s}s \rightarrow M^0(\bar{s}s)] \] (6)

\[ A[B^+ \rightarrow J/\psi M^+(\bar{s}u)] = A(\bar{b} \rightarrow J/\psi \bar{s}) \cdot h[\bar{s}d \rightarrow M^+(\bar{s}u)] \]
\[ A[B^+ \rightarrow J/\psi M^+(\bar{d}u)] = A(\bar{b} \rightarrow J/\psi \bar{d}) \cdot h[\bar{d}d \rightarrow M^+(\bar{d}u)] \] (7)

where the relations apply for any charmonium state as well as $J/\psi$. The charged $B^+$ decays \( \bar{b} \) are uniquely related by isospin symmetry to the corresponding $B^0$ decays and are not considered further.

The pairs of decays \( \bar{b} \) into charge-conjugate strange final states are related by charge conjugation invariance which is valid for all strong interactions. They differ only by the weak interaction vertex which violates $C$ and by possible kinematic and form factor differences induced by the $B_d - B_s$ mass difference. For instance, the hadronization functions for these decays, which have the same functional dependence because of charge conjugation, depend on the magnitude of the relative momentum of the $s$ and the $d$ quark which are
the same in both decays up to corrections from the $B_s - B_d$ mass difference. Hence the values of the hadronization functions for the two decays are the same up to corrections from the $B_s - B_d$ mass difference. Note also that there is no $SU(3)$ breaking in final state interactions for the above final states because of charge conjugation symmetry[1]. This is true for elastic as well as inelastic rescattering where an example of the latter are the processes $B_d \to D^- D^+_s \to J/\psi K^{*0}$ and $B_s \to D^+ D^- \to J/\psi K^{*0}$. Note that $SU(3)$ breaking may arise because of differences in the production of the $D^- D^+_s$ and $D^+ D^-_s$ intermediate states but the transition from the intermediate state to the final state are equal in the two decays because of charge conjugation symmetry.

The decays into strange (5) and nonstrange (6) final states are related by $SU(3)$ symmetry which in this model only affects the hadronization functions which differ by interchanging $s$ and $d$ flavors and possible kinematic and form factor differences induced by the mass differences.

2 B decays into charmonium and a vector meson

We first consider the decays $A(B \to J/\psi V)$ where $V$ is a vector meson and immediately obtain the selection rules (2-3). Next we note that charge conjugation invariance, as already discussed above, requires that for a given partial wave [1]

$$A(B_s \to J/\psi \bar{K}^{*0})_L = F_{CKM}^L \cdot A(B_d \to J/\psi K^{*0})_L$$

(8)

where $F_{CKM}^L$ is a factor depending on the ratios of the $CKM$ matrix elements and the ratio of various weak interaction diagrams; e.g. penguin and tree, contributing to the $B_d$ and $B_s$ decays.

$$F_{CKM}^L = \frac{A_L(\bar{b} \to J/\psi d)}{A_L(\bar{b} \to J/\psi s)}$$

(9)

For the dominant tree diagram and penguin diagram contributions with a charmed quark loop the weak transition is $\bar{b} \to c + W^+ \to \bar{c} + c + q$ where $q$ is $d$ or $s$. For this case $F_{CKM}^L = V_{cd}/V_{cs}$.

Finally, the additional assumption of $SU(3)$ symmetry leads to the full set of predictions

$$A_L(B_d^0 \to J/\psi \rho^0) = A_L(B_d^0 \to J/\psi \omega) = (1/\sqrt{2}) \cdot A_L(B_s \to J/\psi \bar{K}^{*0})$$

$$A_L(B_s \to J/\psi \phi) = A_L(B^0 \to J/\psi K^{*0})$$

$$A_L(B_s \to J/\psi \rho^0) = A_L(B_s^0 \to J/\psi \omega) = A_L(B^0 \to J/\psi \phi) = 0$$

(10)
3 \( \eta - \eta' \) mixing and B decays into charmonium and a pseudoscalar meson

We now note that Eq. 6 leads to relations between amplitudes involving the \( \eta \) or the \( \eta' \) in the final state. These decays are interesting as recent experimental data for \( B \) decays into such final states have so far remained unexplained by the standard treatments of these decays [2].

The most general description of \( \eta - \eta' \) system involves four different radial wave functions and cannot be described by diagonalizing a simple \( 2 \times 2 \) matrix with a single mixing angle. One can therefore write the normalized \( \eta - \eta' \) wavefunctions as

\[
|\eta\rangle = \cos \phi |N\rangle - \sin \phi |S\rangle \\
|\eta'\rangle = \sin \phi' |N'\rangle + \cos \phi' |S'\rangle
\]

(11)

where \( |N\rangle, |N'\rangle, |S\rangle \) and \( |S'\rangle \) are respectively arbitrary isoscalar nonstrange and strange quark-antiquark wavefunctions. In the traditional picture, where the \( \eta - \eta' \) mixing is described by a single mixing angle,

\[
|N\rangle = |N'\rangle \\
|S\rangle = |S'\rangle \\
\phi = \phi'
\]

(12)

A particular example of the general \( \eta - \eta' \) mixing can be found in Ref[3] where we considered the possibility that the \( \eta \) and \( \eta' \) wavefunctions are mixtures of ground state and radially excited \( q\bar{q} \) systems.

Note that in the \( B_d \) decays the \( \eta \) and the \( \eta' \) are produced via their nonstrange components, \( B_d \rightarrow J/\psi N(N') \rightarrow J/\psi \eta(\eta') \) while in \( B_s \) decays the \( \eta \) and the \( \eta' \) are produced via their strange components, \( B_s \rightarrow J/\psi S(S') \rightarrow J/\psi \eta(\eta') \). With the mixing in Eq. 11 and Eq. 6 we get the following predictions.

\[
A(B_d \rightarrow J/\psi \eta) = A(B_d \rightarrow J/\psi N) \cos \phi \\
A(B_d \rightarrow J/\psi \eta') = A(B_d \rightarrow J/\psi N') \sin \phi' \\
A(B_s \rightarrow J/\psi \eta) = -A(B_s \rightarrow J/\psi S) \sin \phi \\
A(B_s \rightarrow J/\psi \eta') = A(B_s \rightarrow J/\psi S') \cos \phi'
\]

(13)

With the the standard mixing in Eq. 12 we find

\[
A(B_d \rightarrow J/\psi \eta) = \cot \phi \cdot A(B_d \rightarrow J/\psi \eta') \\
A(B_s \rightarrow J/\psi \eta) = -\tan \phi \cdot A(B_s \rightarrow J/\psi \eta')
\]

(14)
These relations were obtained in Ref.[1] including only tree diagrams. However, these relations continue to be true even when a penguin contribution is included as long as the model of nonleptonic decays in Eq. 3 is valid.

We thus see that the mixing angle with standard mixing can be obtained from experiment in two different ways. We can then test standard mixing by seeing that both ways give the same result. Allowing for the \( \eta - \eta' \) mass difference and including phase space factors we can construct ratios of experimentally measured quantities,

\[
\begin{align*}
    r_d & \equiv \frac{p_\eta^3 \Gamma(\bar{B}^0 \rightarrow J/\psi \eta)}{p_\eta^3 \Gamma(\bar{B}^0 \rightarrow J/\psi \eta')} = \cot^2 \phi \\
    r_s & \equiv \frac{p_\eta^3 \Gamma(B_s^0 \rightarrow J/\psi \eta)}{p_\eta^3 \Gamma(B_s^0 \rightarrow J/\psi \eta')} = \tan^2 \phi
\end{align*}
\]

We then have the prediction

\[
r = \sqrt{r_d r_s} = 1
\]

Any large deviation of \( r \) from 1 would indicate evidence of non standard \( \eta - \eta' \) mixing. To see what we might expect for the values of \( r \) with non standard mixing we consider the example of non standard mixing considered in Ref.[3] where we find that the ratio \( r \) can be in the range \( r = 0.82 - 0.2 \). Hence, in general, deviation of \( r \) from unity by a factor of 2 or more would be an unambiguous signal for nonstandard \( \eta - \eta' \) mixing. In light of our earlier discussion we note that the relations in Eq. 13 will be violated in \( W \) exchange OZI violating diagrams in \( B_s \) decay of the type \( \bar{b} + s = \bar{u} + u + 3G = \bar{u} + u + J/\psi \). One can also have OZI violating diagrams due to the anomaly, gluon couplings to the flavor singlet component of the \( \eta' \) or the intrinsic charm content of the \( \eta(\eta') \) leading to the processes \( \bar{b} + s(d) = \bar{c} + c + 2G = J/\psi + \eta' \) and \( \bar{b} + s(d) = \bar{c} + c + 3G = J/\psi + \eta(\eta') \) that will violate the relations in Eq. 13. Hence the violation of the prediction in Eq. 13 and Eq. 17 would indicate evidence of non standard \( \eta - \eta' \) mixing and/or the presence of OZI violating contributions.

Unfortunately, the large \( \eta - \eta' \) mass differences may introduce other corrections beyond simple phase space. However, we can extend our tests of standard mixing by noting that the assumptions that the states \( |N⟩ \) and \( |S⟩ \) have the same radial wave functions implies an SU(3) symmetry that also includes the kaon wave functions. We can therefore include the
transitions to final states with kaons produced by the same $\bar{b} \to J/\psi + \bar{d}$ or $\bar{b} \to J/\psi + \bar{s}$ decays and differing only by the flavor of the spectator quark. We then have the predictions,

$$\sqrt{2} \cdot A(B_d \to J/\psi N) = A(B_s \to J/\psi K^0) = \sqrt{2} A(B_d \to J/\psi \pi^0)$$  \hspace{1cm} (18)

$$A(B_s \to J/\psi S) = A(B_d \to J/\psi K^0)$$  \hspace{1cm} (19)

These can be combined with Eq. 13 to give sum rules independent of the mixing angle for standard mixing,

$$|A(B_d \to J/\psi \eta)|^2 + |A(B_d \to J/\psi \eta')|^2 = (1/2) \cdot |A(B_s \to J/\psi K^0)|^2$$

$$|A(B_s \to J/\psi \eta)|^2 + |A(B_s \to J/\psi \eta')|^2 = |A(B_d \to J/\psi K^0)|^2$$  \hspace{1cm} (20)

As in Eq. 8 charge conjugation requires

$$A(B_s \to J/\psi \bar{K}^0) = F_{CKM} \cdot A(B_d \to J/\psi K^0)$$  \hspace{1cm} (21)

where $F_{CKM}$ is defined as in Eq. 9 but for the non spin flip transition needed to form a spin zero meson in the final state.

$$F_{CKM} = \frac{A(\bar{b} \to J/\psi \bar{d})}{A(b \to J/\psi \bar{s})}$$  \hspace{1cm} (22)

Thus to obtain a different point of view we can define the ratios of $B_d$ and $B_s$ decays to $J/\psi \eta$ and $J/\psi \eta'$.

$$r_\eta = \frac{p_{B_d \eta}^3 \Gamma(B_d \to J/\psi \eta)}{p_{B_d \eta}^3 \Gamma(B_s \to J/\psi \eta)} = (F_{CKM})^2 \cdot \cot^2 \phi$$  \hspace{1cm} (23)

$$r'_{\eta} = \frac{p_{B_s \eta'}^3 \Gamma(\bar{B}^0 \to J/\psi \eta')}{p_{B_s \eta'}^3 \Gamma(B_s \to J/\psi \eta')} = (F_{CKM})^2 \cdot \tan^2 \phi$$  \hspace{1cm} (24)

We then have the prediction

$$r_B = \sqrt{r_\eta r'_{\eta}} = (F_{CKM})^2$$  \hspace{1cm} (25)

We can refine these ratios and overcome kinematic factors by normalizing them to the kaon modes,

$$R_\eta = \frac{p_{B_s \eta}^3 \Gamma(B^0 \to J/\psi \eta)}{p_{B_d \eta}^3 \Gamma(B_s \to J/\psi \eta)} \cdot \frac{p_{B_d K}^3 \Gamma(B_s \to J/\psi K^0)}{p_{B_s K}^3 \Gamma(B_d \to J/\psi K^0)} = (F_{CKM})^4 \cdot \cot^2 \phi$$  \hspace{1cm} (26)
\[ R'_\eta = \frac{p_{B_{d,0}}^3}{p_{B_s,0}^3} \frac{\Gamma(B^0 \to J/\psi \eta')}{\Gamma(B^0_s \to J/\psi \eta')} \cdot \frac{p_{B_{d,0}}^3}{p_{B_s,0}^3} \frac{\Gamma(B^0 \to J/\psi K^0)}{\Gamma(B^0_s \to J/\psi K^0)} = (F_{CKM})^4 \cdot \tan^2 \phi \] (27)

We then have the prediction

\[ R_B = \sqrt{R_\eta R'_\eta} = (F_{CKM})^4 \] (28)

We now note that under the assumptions of standard mixing, SU(3) symmetry and the non-participation of the spectator quark in the weak transition we have described the branching ratios for eight transitions in terms of three parameters, \( F_{CKM}, \phi \) and an overall normalization. If these relations hold experimentally, the standard mixing and the value of the mixing angle will be confirmed and established, the validity of SU(3) symmetry for these transitions will be confirmed, and the value of \( F_{CKM} \) will determine the ratio of the penguin to tree contributions to the decay \( B_d \to J/\psi K_S \) which is the "golden channel" for CP violation experiments. If experimental violations of this description are observed, they will indicate the breakdown of particular assumptions and perhaps give clues to new physics.

4 \( \eta - \eta' \) mixing in charmed meson decays

One could, in principle, construct similar ratios with \( D(D_s) \to \eta(\eta')P \) where \( P = \pi, \rho, K \). However, non nonleptonic \( D \) decays are are not very well understood [8] and therefore these decays are not very useful to test non standard \( \eta - \eta' \) mixing. On the other hand semileptonic \( D(D_s) \) decays of the type \( D(D_s) \to \eta(\eta')l\bar{\nu} \) can provide clean tests for the \( \eta - \eta' \) mixing.

Here the lepton pair is a singlet like charmonium under all strong interaction symmetries. But the lepton pair mass and final momentum have continuous spectra, and effects of the \( \eta - \eta' \) mass difference can be large. We therefore must consider the decay dynamics in more detail.

The Lagrangian for the semileptonic \( D(D_s) \) decays involving the transitions \( c \to ql\bar{\nu} \), where \( q = s, d \) and \( l = \mu, e \), has the standard current-current form after the \( W \) boson is integrated out in the effective theory.

\[ H_W = \frac{G_F}{2\sqrt{2}} V_{cq} q \bar{q} \gamma_\nu (1 - \gamma_5) c \bar{\nu} \gamma^\mu (1 - \gamma_5) l \] (29)

The differential decay distribution, neglecting the lepton masses, is then given by,

\[ \frac{d\Gamma}{dq^2}(D_q \to Pl\bar{\nu}) = \frac{G_F^2 |V_{cq}|^2}{24\pi^3} \frac{p}{p_F} |F_1(q^2)|^2 \]
where $P = \eta(\eta')$ and $p$ and $E$ are the magnitude of the momentum and energy of the pseudoscalar meson $P$. The form factor $F_1$ is defined as

$$F_1(q^2) = \frac{m_D^2 - m_P^2}{q^2} q_\mu F_0(q^2)$$

Let us now define the two ratios

$$r_d = \frac{\Gamma(D \rightarrow \eta\nu)}{\Gamma(D \rightarrow \eta'\nu)}$$
$$r_{s} = \frac{\Gamma(D_s \rightarrow \eta\nu)}{\Gamma(D_s \rightarrow \eta'\nu)}$$

It then follows from the mixing in Eq. (32) that

$$r_d = \frac{\int_0^{(m_D - m_\eta)^2} p_\eta^3 |F_i^d(q^2)|^2 dq^2 \cot^2 \phi}{\int_0^{(m_D - m_\eta')^2} p_\eta'^3 |F_i^d(q^2)|^2 dq^2}$$
$$r_{s} = \frac{\int_0^{(m_{D_s} - m_\eta)^2} p_\eta^3 |F_i^s(q^2)|^2 dq^2 \tan^2 \phi}{\int_0^{(m_{D_s} - m_\eta')^2} p_\eta'^3 |F_i^s(q^2)|^2 dq^2}$$

To calculate $r_{d,s}$ we have to model the $q^2$ dependence of the form factors $F_1^{d,s}(q^2)$ and no simple observable - independent of the form factors- can be constructed that can test for nonstandard mixing even in the $SU(3)$ limit.

It is more useful to define the two ratios

$$r_\eta = \frac{\Gamma(D \rightarrow \eta\nu)}{\Gamma(D \rightarrow \eta\nu)}$$
$$r_{\eta'} = \frac{\Gamma(D \rightarrow \eta'\nu)}{\Gamma(D \rightarrow \eta'\nu)}$$

It then follows from the standard mixing in Eq. (32) that

$$r_\eta = \frac{\int_0^{(m_D - m_\eta)^2} p_\eta^3 |F_i^d(q^2)|^2 dq^2 \cot^2 \phi}{\int_0^{(m_D - m_\eta')^2} p_\eta'^3 |F_i^d(q^2)|^2 dq^2}$$
$$r_{\eta'} = \frac{\int_0^{(m_{D_s} - m_\eta)^2} p_\eta^3 |F_i^s(q^2)|^2 dq^2 \tan^2 \phi}{\int_0^{(m_{D_s} - m_\eta')^2} p_\eta'^3 |F_i^s(q^2)|^2 dq^2}$$

(33)
In the U spin limit we then have the prediction

\[ r_D = \sqrt{r_\eta r_{\eta'}} = 1 \]  \hspace{1cm} (36)

Any large deviation of \( r_D \) from 1 by a factor of 2 would indicate evidence of non standard \( \eta - \eta' \) mixing as they are unlikely to originate from U spin breaking.

With more experimental data one could devise tests of the \( \eta - \eta' \) mixing by looking at the various decay distributions. Let us define the two ratios

\[
R_d(q^2) = \frac{p_{\eta'}^3 \frac{d\Gamma}{dq^2}(D_d \to \eta l\nu)}{p_\eta^3 \frac{d\Gamma}{dq^2}(D_d \to \eta' l\nu)}
\]

\[
R_s(q^2) = \frac{p_{\eta'}^3 \frac{d\Gamma}{dq^2}(D_s \to \eta l\nu)}{p_\eta^3 \frac{d\Gamma}{dq^2}(D_s \to \eta' l\nu)}
\]  \hspace{1cm} (37)

It then follows for the standard mixing in Eq. 12 that

\[
R_d(q^2) = \cot^2 \phi \\
R_s(q^2) = \tan^2 \phi
\]  \hspace{1cm} (38)

We see from the above equations that measurement of \( R_{d,s} \) allows us to calculate the mixing angle \( \phi \). What is also interesting is that the the ratios \( R_{d,s}(q^2) \) are independent of \( q^2 \) and we have the prediction

\[
R = R_d(q^2)R_s(q^2) = 1
\]  \hspace{1cm} (39)

for any value of \( q^2 \). A deviation of \( R \) from unity would indicate evidence of non standard mixing. In particular, with the mixing in Ref[3] one would predict a value of \( R \) different from unity and a \( q^2 \) dependence for \( R, R_d \) and \( R_s \).

## 5 Conclusions

In summary we studied the two body decays of \( B_d \) and \( B_s \) decays into \( J/\psi M \), where \( M \) is a light meson, under the very simple assumptions that the spectator quark does not play a role in the decay of the weak heavy quark or antiquark. Using this model we derived interesting relations between decay amplitudes and tests for the standard \( \eta - \eta' \) mixing. With the further assumption of \( SU(3) \) symmetry we derived additional relations between decay amplitudes and in particular the eight CP eigenstates \( J/\psi K_S, J/\psi \eta, J/\psi \eta' \) and \( J/\psi \pi^0 \) were given in
terms of three parameters which, when determined from experiments, will give information about the ratio of penguin to tree contributions to the ”golden channel” \( B^0 \rightarrow J/\psi K_S \) decay and will give the value of the \( \eta - \eta' \) mixing angle. We also presented tests of the standard \( \eta - \eta' \) mixing involving semileptonic \( D \) decays.

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