A Squeezed Vacuum State Laser with Zero Diffusion

F. de Oliveira Neto\(^1\), G. D. de Moraes Neto\(^2\)\(^,*\), and M. H. Y. Moussa\(^1\)
\(^1\)Instituto de Física de São Carlos, Universidade de São Paulo, Caixa Postal 369, 13560-970, São Carlos, São Paulo, Brazil
\(^2\)Department of Physics, Zhejiang Normal University, Jinhua 321004, People’s Republic of China

We propose a method for building a squeezed vacuum state laser with zero diffusion, which results from the introduction of the reservoir engineering technique into the laser theory. As well as the reservoir engineering, our squeezed vacuum laser demands the construction of an effective atom-field interaction. And by building an isomorphism between the cavity field operators in the effective and the Jaynes-Cummings Hamiltonians, we derive the equations of our effective laser directly from the conventional laser theory. Our method, which is less susceptible to errors than reservoir engineering, can be extended for the construction of other nonclassical state lasers, and our squeezed vacuum laser can contribute to the newly emerging field of gravitational interferometry.

I. INTRODUCTION

The laser theory is one of the central developments in the physics of radiation-matter interaction. Based on the theoretical framework provided by Townes and Schawlow [1], the first laser, built by Maiman [2], date back to the 1960s, and has since played a major role both in basic and applied physics, with applications in many technical aspects of modern society. The quantum theory of the laser was built basically from contributions led by H. Haken [3], W. E. Lamb [4], and M. Lax [5], from which are derived the more realistic models where a transmitting window [6], and the pumping statistics of the lasing atoms [7] are included.

Among many others, we mention the uses of lasers for cooling and trapping atoms [8], for Bose-Einstein condensation in dilute gases of alkali atoms [9], for the development of optical tweezers and their application in biological and physical sciences [10], and for generating ultrashort high-intensity laser pulses extensively used across physics and chemistry [11]. These achievements draw a broader picture of the unique progress that quantum optics has undergone since the 1980s. In addition to this picture we mention the generations of squeezed states of the radiation field [12], essential for enhancing interferometric sensitivity [13], and today a critical challenge for the development of gravitational wave interferometry [14,15]. We also mention the applications of squeezed states in optical waveguide tap [16], quantum nondemolition measurements [17], quantum information processing [18], and quantum metrology [19]. There is also the development of different sources for generating entangled photon states, used for investigating fundamentals of quantum mechanics [20].

Parallel to the developments of quantum optics, we witnessed the emergence of quantum communication and computation, which resulted in the new and promising field of quantum information theory [21]. The need for implementation of quantum logic operations demanded new techniques for engineering nonclassical states [22], effective interactions [23,24] and reservoirs [25,26] for phase coherence control. These demands have pushed the physics of the radiation-matter interaction to a new level through platforms such as cavity quantum electrodynamics [27], trapped ions [28], circuit quantum electrodynamics [29], and all related topics. Regarding coherence control, a key issue for the construction of a squeezed vacuum laser with zero diffusion, many methods have been designed [30]; however, the reservoir engineering [29] — which seems inspired by the laser theory — is of particular interest here. Its basic idea is to submit the system of interest, say a dissipative cavity mode (described by the creation and annihilation operators \(a^\dagger\) and \(a\), and whose particular state \(|\Psi\rangle\) we intend to protect from the action of the environment), to an interaction with an auxiliary strongly dissipative system as, for example, a two-level atom (described by the Pauli raising and lowering operators \(\sigma_+\) and \(\sigma_-\)). This interaction must then be engineered so that it takes the bilinear form

\[
\chi \left( A S_+ + A^\dagger S_- \right),
\]

with \(\chi\) being an effective atom-field coupling and \(A\), \(A^\dagger\) \((S_+, S_-)\) defined by a canonical transformation on the original operators for the cavity mode: \(a^\dagger\), \(a\) (auxiliary atom: \(\sigma_+, \sigma_-\)), with the central requirement \(A |\Psi\rangle = 0\). The master equation for the cavity mode, coming from reservoir engineering, is given by

\[
\dot{\rho} = (\Gamma/2) \left( 2A \rho A^\dagger - A^\dagger A \rho - \rho A^\dagger A \right) + \left( \tilde{\Gamma}/2 \right) \left( 2a \rho a^\dagger - a^\dagger a \rho - \rho a^\dagger a \right),
\]

with the assumption \(\Gamma \propto \chi \gg \tilde{\Gamma}\). It is therefore clear that the Lindbladian for \(a^\dagger\), \(a\) acts as a perturbation over that for \(A^\dagger\), \(A\), causing the fidelity of the protected state, necessarily an eigenstate of \(A\) with null eigenvalue [25] \((A |\Psi\rangle = 0)\), to be slightly less than unity, since \(F \propto 1 - \Gamma/\tilde{\Gamma}\).

Compared to the method of state protection through engineered reservoir, the conventional laser mechanism
is far more generous since the stringent requirement $\mathcal{A} |\Psi\rangle = 0$ is relaxed, with the laser coherent steady state $|\alpha\rangle$ being an eigenstate of $a$ with non null eigenvalue, $a |\alpha\rangle = \alpha |\alpha\rangle$. Nonetheless, this relaxed condition implies the coherence loss of the laser stationary state due to phase diffusion. Our squeezed vacuum laser, however, despite based on the requirement $\mathcal{A} |\Psi\rangle = 0$, is also more generous than the engineered reservoir method, since the Lindbladian for $a^\dagger a$ — which acts in the engineered reservoirs to prevent the fidelity from being equal to the unit — is absent from the laser master equation. We must return to this interesting point later.

Our strategy here is to bring the reservoir engineering method into the laser mechanism, aiming to produce a steady squeezed vacuum state preserving phase coherence. From the reservoir engineering we must implement the bilinear interaction of the cavity mode with the auxiliary system, now the laser active medium. By its turn, from the laser theory we subject the active medium to a linear amplification process which, due to its interaction with the cavity mode, feeds it by stimulated emission concomitantly producing a saturation which helps constructing the far-from-equilibrium steady state. Then, by adding together the reservoir engineering technique to the laser theory, we should be able to build a laser with zero diffusion or zero line width, and therefore a decoherence-free state of the cavity field.

As it should be clear in the next section, the effective bilinear Hamiltonian required for building up the squeezed vacuum laser is a particular form of that in Eq. (1), given by

$$\chi \left( A \sigma_+ + A^\dagger \sigma_- \right),$$

which suggests an isomorphism between this interaction and the Jaynes-Cummings model, a map between the field operators $a \leftrightarrow A$ and $a^\dagger \leftrightarrow A^\dagger$. This isomorphism would allow us to derive the equations of our effective laser directly from those of the conventional laser theory. For the construction of such an isomorphism relation we must derive a vector basis for the cavity field, $\{|n\rangle, A\}$, on which the action of our generalized operators $(A^\dagger, A)$ emulate that of the usual creation and annihilation operators $(a^\dagger, a)$ in the Fock space $\{|n\rangle\}$. With this, we immediately derive the generalized master equation that describes, in the above-threshold regime, the construction of the steady squeezed vacuum state. In short, all we have to do is to establish the isomorphism between the cavity field operators in our squeezed vacuum laser and in the conventional coherent state laser; given the isomorphism, the equations for our laser are automatically settled.

We stress here that the constructed isomorphism between the field operators in the effective and the Jaynes-Cummings Hamiltonians automatically results in the engineered Lindbladian $(\Gamma/2) \left( 2 A \rho A^\dagger - A^\dagger A \rho - \rho A^\dagger A \right)$, differently from what happens in the original reservoir engineering protocol [25], in which a set of approximations is required to obtain the desired Lindbladian. In other words, the engineered Lindbladian comes as a gift from the constructed isomorphism, avoiding the set of approximation imposed by the original reservoir engineering method. Furthermore, in the derivation of the engineered Lindbladian through the constructed isomorphism, we have the advantage of eliminating the unwanted Lindbladian for $a^\dagger a$ which, as observed above, acts as a perturbation over that for $A^\dagger A$ (causing the fidelity of a protected state to be slightly less than unity). Although the method we have used is not exactly that of reservoir engineering as presented in [25], all the main ingredients of the latter method are actually present in our construction: the engineering of the effective atom-field interaction and, consequently, of the associated Lindbladian.

A laser with zero linewidth is most useful for a variety of application, among which we mention optical sensing, metrology, higher order coherent communication, high-precision detection, and laser spectroscopy. Therefore, the method here presented can contribute to or inspire the design of lasers with exceedingly small diffusion and linewidth, with broad technological applications.

Squeezed states are most efficiently generated from optical parametric down-conversion in a non-linear $\chi^{(2)}$ crystal [32, 33]. We also mention the generation of squeezed states by four wave mixing in an optical cavity [34]. However, our purpose here is to demonstrate the possibility of generating squeezed state of light through the laser mechanism, with the required nonlinearity being constructed through the atom-field interaction itself. The squeezing of cavity-field states through their effective interaction with atoms have been systematically pursued in cavity quantum electrodynamics [35].

Our paper is organized as follows: In Section II we present a scheme, based on the adiabatic elimination of fast variables, for the construction of the effective Hamiltonian required for the operation of the squeezed vacuum laser. In Section III we construct the isomorphism between the cavity field operators in the effective and the Jaynes-Cummings interactions, and in Section IV we present the master equation for the squeezed vacuum laser and the numerical analysis demonstrating the effectiveness of our method. Finally, in Section V we present our conclusions.

II. THE EFFECTIVE ATOM-FIELD INTERACTION

The first step for achieving our goal is to engineer the atom-field interaction through which we implement the amplification-saturation mechanism building up and sustaining our squeezed vacuum state. The effective interaction follows from considering the transitions induced by quantum and classical fields in a three-level Lambda-type configuration, as depicted in Fig. 1. The intermediate (more-excited) atomic level $|\tilde{e}\rangle$ must be considered, apart from the lasing levels $|g\rangle$ and $|e\rangle$. The cavity
good approximation, we obtain the second-order effective Hamiltonian in the interaction picture, the non-diagonal Hamiltonian terms describing the process is given by

\[
H_{\text{eff}} = \frac{\hbar}{2} \left( \sum_{\ell} (e^{i\omega_{\ell}t} \Omega_{\ell} \sigma_{1g} + e^{-i\omega_{\ell}t} \Omega_{\ell} \sigma_{1e}) + h.c. \right),
\]

(4b)

the generalized operators read

\[
A = \frac{a + \alpha a^\dagger}{\sqrt{1 - \kappa^2}}, \quad A^\dagger = \frac{a^\dagger + \alpha a}{\sqrt{1 - \kappa^2}},
\]

(6)

with \([A, A^\dagger] = [a, a^\dagger] = 1\).

In order to verify the validity of the approximations leading from \(V(t)\) to \(H_{\text{eff}}\), we plot in Figs. 2(a) and (b) the variances of the quadratures \(X_1 = (a^\dagger + a)/2\) and \(X_2 = (a - a^\dagger)/2i\) for the field states generated by both the full and the effective Hamiltonians in Eqs. (4) and (5), against \(gt\), i.e. the number of cycles of the effective coupling. We start with the atom in the excited state \(|e\rangle\) and the cavity mode in the vacuum state \(|0\rangle\), considering, in units of the Rabi frequency \(\lambda\), the parameters \(\delta_{g1} = 10^4\), \(\delta_{e1} = 6 \times 10^3\), \(\Omega_{g1} = 40\), and \(g = 0.05\), such that \(\kappa = 0.6\). In Figs. 2(a) and (b) the straight and dotted lines refer to the effective and full Hamiltonians, respectively, showing a good agreement between both curves up to \(gt \approx 7\). We have also plotted in Figs. 2(c) and (d) the excitations \(\langle a^\dagger a\rangle (t)\) and \(\langle \sigma_{ee} \rangle (t)\) against \(gt\), and we again see a very good agreement between the curves generated by the full and the effective Hamiltonians until the same \(gt \approx 7\).

Regarding the engineering of the effective atom-field interaction [5], a detailed account on Raman transition in cavity quantum electrodynamics can be found in Ref. [8]. We note that the atomic level configuration we have used to engineer the required interaction is certainly not unique; it can be engineered from other level configuration using more or less classical fields. Finally, we stress that in engineering the effective Hamiltonian [5] we have not take into account the usually small amplitude and phase fluctuations of the required laser beams, which would indeed result in some phase diffusion of our squeezed vacuum laser. At this point we mention that another proposal to achieve squeezed lasing has been presented in which the cavity is parametric driven with the help of a non-linear \(\chi^{(2)}\) crystal inside the cavity [39]. Our engineered atom-field interaction thus replaces the parametric driven process in Ref. [39], dispensing the non-linear crystal inside the cavity and the coherent drive of the cavity mode. However, since our laser requires the effective atom-field interaction, it present an operating timescale after which it must be restarted.

III. THE ISOMORPHISM BETWEEN THE A\(^\dagger\), A AND a\(^\dagger\), a ALGEBRAS

Having engineered the required interaction [5], we now start to construct the vector basis \(|\{n\}_A\rangle\) for the cavity field, in whose states \(|n\>_A\) the action of operators \(A, A^\dagger\) and \(A\) must lead to the same relations as those resulting from the actions of \(a^\dagger a\), \(a^\dagger\) and \(a\) on the Fock basis \(|\{n\}\rangle\),

![Diagram](image-url)
i.e.:  
\[ A^\dagger A |n\rangle_A = n |n\rangle_A, \]
\[ A^\dagger |n\rangle_A = \sqrt{n+1} |n+1\rangle_A, \]
\[ A |n\rangle_A = \sqrt{n} |n-1\rangle_A. \]

All the basis states \(|n\rangle_A\) are constructed from the vacuum state \(|0\rangle_A\), starting from the relation
\[ A |0\rangle_A = 0, \]
which enables us to determine the probability amplitudes \(c_n\) defining the superposition \(|0\rangle_A = \sum_n c_n |n\rangle\). Considering the operator \(A\) as given by Eq. \((6)\), we first compute the vacuum state from Eq. \((8)\) and then, using the relation \((A^\dagger)^n |0\rangle_A / \sqrt{n!}\), we derive all the even and odd generalized excitations, given by
\[
|2m\rangle_A = \frac{(1 - \kappa^2)^{1/4}}{\sqrt{2m!}} \sum_{n=0}^{\infty} (-\kappa)^{n-m} \sqrt{(2n-1)!!} (2n)!! (2n+1)!! |2n\rangle_A, \\
|2m+1\rangle_A = \frac{(1 - \kappa^2)^{3/4}}{\sqrt{(2m+1)!}} \sum_{n=0}^{\infty} (-\kappa)^{n-m} \sqrt{(2n+1)!!} (2n)!! (2n+1)!! |2n+1\rangle_A.
\]

The basis defined by the even and odd number states given by Eqs. \((9)\), together with the laser model establishes the isomorphism between the laser field generated by the interaction from Eq. \(3\), written in the new basis, and the usual laser field, in the Fock basis, since their master equations are similar, containing each the Liouville-Von Neumann term and the cavity dissipative term in the Lindblad form, referring to the loss of an Harmonic Oscillator (HO) to the environment. The Liouville-Von Neumann term would be equal in both lasers, once we use the Hamiltonian in the new basis, as in Eq. \(3\), and since now we have a system guided by an interaction in the form of Jaynes-Cummings, as describing an HO, naturally we can say that the cavity dissipates in the Lindblad form. Knowing that the steady state of the conventional laser is the coherent state \(|\alpha\rangle\) (owing to the Jaynes-Cummings atom-field interaction), it is then automatic to derive the steady state of our laser, which results from the effective Hamiltonian \([5]\). Once the isomorphism is established, all we have to do is to describe the coherent state in the vector basis \(|\{n\rangle_A\}\), i.e., \(|\alpha\rangle_A = D_A(\alpha) |0\rangle_A = \exp(\alpha A^\dagger - \alpha^* A) |0\rangle_A = e^{-|\beta|^2/2} e^{\beta A^\dagger} e^{-\beta^* A} |0\rangle_A\), with \(\beta = (\alpha - \kappa \alpha^*) / \sqrt{1 - \kappa^2}\). We obtain, expanded in the usual Fock basis \(|\{n\rangle\}\), the state
\[ |\alpha\rangle_A = (1 - \kappa^2)^{1/4} e^{(\kappa \alpha^2 - |\alpha|^2)^2 / 2} \sum_{n=0}^{\infty} \sqrt{n!} (\sum_{\ell=0}^{n} \frac{\kappa}{2} \frac{\ell/2}{(n-\ell)!} H_\ell(x) ) |n\rangle, \]
where we note that for \(\kappa = 0\) we immediately recover the usual Fock basis states from Eq. \((9)\) and the usual coherent state from Eq. \((10)\).

IV. THE MASTER EQUATION FOR THE SQUEEZED VACUUM LASER

From the isomorphism we have established, and following the footsteps of the conventional laser theory, we can derive the master equation describing the dynamics of the cavity field when interacting with a pumped atomic sample, through the effective Hamiltonian \([5]\) and the environment. This master equation is given by
\[ \dot{\rho} = \mathcal{L}_A \rho + \mathcal{L}_B \rho + \mathcal{L}_C \rho, \]
where the Lindbladians accounting for gain, saturation and cavity loss, obey the expressions
\[
\mathcal{L}_A \rho = \frac{A}{2} (2A^\dagger \rho A - AA^\dagger \rho - \rho AA^\dagger), \\
\mathcal{L}_B \rho = \frac{B}{2} \left( \frac{1}{4} AA^\dagger (AA^\dagger \rho + 3\rho AA^\dagger) \right. \\
+ \frac{1}{4} (\rho AA^\dagger + 3AA^\dagger \rho) AA^\dagger \\
\left. - A^\dagger (AA^\dagger \rho + \rho AA^\dagger) A \right) + ..., \\
\mathcal{L}_C \rho = \frac{C}{2} (2A \rho A^\dagger - A^\dagger A \rho - \rho A^\dagger A). 
\]
The coefficients for gain \(A = 2R(g/\gamma)^2\), saturation \(B = 4A(g/\gamma)^2\), and loss \(C = \omega / Q\) are defined from the atomic pumping rate \(R = K \rho\), \(K\) being the total atomic
FIG. 2: Validity of the Effective interaction. The the variances (a), $\langle \Delta X_1^2 \rangle (t)$ and (b), $\langle \Delta X_2^2 \rangle (t)$, and the excitations (c), $\langle a^\dagger a \rangle (t)$ and (d), $\langle \sigma_{ee} \rangle (t)$ in function of $gt$, for the field states generated by the effective and full Hamiltonians (straight and dotted lines, respectively). We have started with the atom and the cavity mode in the excited and the vacuum state, $|e\rangle |0\rangle$, considering, in units of the Rabi frequency $\lambda$, the parameters $\delta_{g1} = 10^3$, $\delta_{c1} = 6 \times 10^2$, $\Omega_{g1} = 40$, and $g = 0.05$, such that $\kappa = 0.6$.

To further support our results coming from the isomorphism —that the laser resulting from the atom-field interaction described by the effective Hamiltonian (5) still applies collectively, since it has being developed individually. The first atom finds the cavity field steady state. To do that, we numerically simulate the passage of a dense flux of atoms through an excitation region where each atom has a probability $p$ of being excited to the level $|e\rangle$ before entering the cavity. Considering that this flux has a regular pump rate, $K$, we can say that the number of atoms passing through a time $\Delta t$ is equal to $K \Delta t$, such as the average number of excited atoms that reach the cavity is $N = k_0 = k_0 K$. The atoms arrive successively in the cavity so that the atom-field interaction described by (6) still applies collectively, since it has being developed individually. The first atom finds the cavity field in its initial vacuum state $\rho(0)$, leading it to the state $\rho(t)$, with $t = 1/k$. After its passage through the cavity, we compute the reduced density operator for the field state by tracing over the atomic degrees of freedom. The second atom then finds the cavity in this reduced state, leading it to another reduced state at time $2t$, and so on until the time $Nt$, each step being described by the equation

$$
\dot{\rho}_{i+1}(t) = k [\rho(t_{i+1}) - \rho(t_i)] - i [H_{eff}(t_i), \rho(t_i)]
+ (C/2) [2A\rho(t_i)A^\dagger - A^\dagger A\rho(t_i) - \rho(t_i)A^\dagger A]
+ (\gamma/2) (2\sigma_-\rho(t_i)\sigma_+ - \sigma_+\sigma_-\rho(t_i) - \rho(t_i)\sigma_-\sigma_+). \tag{14}
$$

We next analyze the laser state derived from Eq. (14), by comparing it with the squeezed vacuum state defined
FIG. 3: The photon number distribution $P_n$ and the Wigner function. (a), generated by the effective Hamiltonian for $\alpha = 0.18$ and $r = 0.69$. (b), ideal squeezed vacuum $S(r = 0.69) \vert 0 \rangle$.

FIG. 4: The photon number. The mean occupation number $\langle a^\dagger a \rangle(t)$, considering the same parameters used in Fig. 2, together with $C/g = 0.35$, $r/g = 92$ and $\gamma/g = 0.5$, leading to $A/g = 736$. We consider the cavity initially in the vacuum state and the atoms prepared in their excited states.

FIG. 5: Squeezed vacuum laser. (a), The photon number distribution $P_n$ and the phase space projection of the Wigner function of the produced laser state for $gt = 4$, and (b) the fidelity of the evolved laser state against $gt$. We use the same parameters of Fig. 4.

FIG. 6: The variances, $(\Delta X_1)^2(t)$ (solid line) and $(\Delta X_2)^2(t)$ (dotted line) of the generated laser state against $gt$, using the same parameters of Fig. 4.

In Eq. (10), which by its turn comes from Eq. (12). All the following figures consider the same parameters used in Eq. (5), together with the choices $C/g = 0.35$, $r/g = 92$ and $\gamma/g = 0.5$, which lead to the rate $A/g = 736$.

In Fig. 4 we plot the mean occupation number $\langle a^\dagger a \rangle(t)$ of the laser state resulting from Eq. (14). We verify that the mean occupation number reaches the steady value $\langle a^\dagger a \rangle(t) = 0.62$ for $gt \approx 3$, long before the validity of the effective Hamiltonian is compromised, for $gt \approx 7$. This steady excitation is in good agreement with that predicted by Eq. (10), given by $\langle a^\dagger a \rangle = 0.56$, about 10% less than the numerical simulation. The value $gt \approx 3$ follows after the passage of 1734 atoms through the cavity.

In Fig. 5(a) we present the photon number distribution $P_n$ and the phase space projection of the Wigner function of the laser state following from Eq. (14), for $gt = 4$. We again verify a good agreement with the ideal squeezed vacuum state as it becomes clear from Fig. 5(b).
We have presented a method to produce a squeezed vacuum laser with zero diffusion. This method is based on merging together the reservoir engineering technique with the laser theory. The reservoir engineering demands us to build up an effective interaction between the system whose state we want to protect (the cavity field) and an auxiliary system (the laser active medium), of the form of Eq. (3), \( \chi (A\sigma_+ + A^\dagger\sigma_-) \), with the laser steady state being an eigenstate of \( A \) with null eigenvalue: \( A |\Psi\rangle = 0 \). The effective interaction must enable the construction of an isomorphism between the field operators in the effective \( (A^\dagger, A) \) and the Jaynes-Cummings \( (a^\dagger, a) \) Hamiltonians. The isomorphism is carried out by building a basis state \( \{|n\}_A \) for the operators \( A^\dagger, A \) similar to the Fock basis state \( \{|n\}\) for \( a^\dagger, a \). The laser theory, by its turn, provides the mechanism by which the cavity mode is fed by the stimulated emission of the active medium when subjected to linear amplification, in addition to inducing a saturation which results in a far-from-equilibrium steady state.

Our method has an advantage over the reservoir engineering, in which we cannot, evidently, eliminate the environment (described by the Lindbladian for \( a^\dagger, a \) in Eq. (2)) that acts to introduce errors in the action of the artificially constructed reservoir (described by the Lindbladian for \( A^\dagger, A \)). In our method, this does not occur, the only source of errors being the protocol for building up the effective interaction, based on the adiabatic elimination of fast variables, which is also present in reservoir engineering. This is a very unique aspect of our method, which shows that the association of reservoir engineering with the laser mechanism results in both a more robust protocol than the reservoir engineering (since the Lindbladian for \( a^\dagger, a \) is absent) and also an unconventional laser field described by a nonclassical coherence-preserving state.

In addition to the many applications we have already listed for a zero linewidth laser, we observe that a squeezed vacuum laser may be useful for high-resolution interferometry, a timely topic due to the newly emerging field of gravitational-wave interferometry. Moreover, the present method challenges us to design effective interactions leading to other nonclassical laser states, as for example steady superposition states. This challenge can lead us to a new chapter regarding the preparation of nonclassical steady states, which could be useful for a variety of fundamental and technological applications.

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