The mathematical model and assessment of the possibility of implementing the specified movements of a ground-effect vehicle in the vertical plane

Yu F Vshivkov and S M Krivel

Engineering and Mathematical Center, Institute of Mathematics and Computer Technologies, Irkutsk State University, 1 K. Marx St., 664003 Irkutsk, Russia

E-mail: krivel66@mail.ru

Abstract. Based on the analysis of the tasks of the initial stages of the ground-effect-vehicle design, this work proposes a complex of mathematical models, united by a single research goal. A simplified, partially linearized multi-parameter mathematical model of the aerodynamic characteristics of a ground-effect vehicle was developed and substantiated. The authors propose a method of pre-assessment of the stability of the ground-effect vehicle and calculation of the range of acceptable positions of the vehicle’s center of mass based on the requirements for the device stability and balancing characteristics. The method of calculating balancing parameters of the ground-effect vehicle layout is based on the definition of a particular solution of the differential equation system of the vertical movement of the ground-effect vehicle using elements of automatic control theory. The solution is formed by automatic regulation of the specified (controllable) parameters through changing the control factors of the differential equation system over time using PID controllers (a virtual “automatic control system”). The required level of control factors allows us to draw a conclusion about the validity of the specified position of the center of mass and the possibility of performing a specified movement of the ground-effect vehicle. The paper provides some modeling results.

1. Introduction

The main flight mode of the ground-effect vehicle is to fly at low height near the underlying surface. The main objective of aerodynamic design of the ground-effect vehicle is to ensure high aerodynamic quality in the cruise mode, while providing acceptable stability and controllability characteristics in the envelope of flight parameters. This is greatly complicated by the presence of the ground effect and nonlinear dependencies of aerodynamic characteristics from kinematic flight parameters. Besides, the set of kinematic parameters of the ground-effect vehicle is much wider than that of the aircraft. This is due to the fact that aerodynamic characteristics are determined only by the shape of the vehicle (the deflection angles of control surfaces and other ones), its orientation relative to the velocity vector and rotation around the center of mass. The most important are the position of the vehicle relative to the underlying surface and the nature of the change in this position, as well as the presence of an air cushion (dynamic and static). Moreover, additional slight pressure in the air cushion can be created by blowing air jets from the power plant elements.
2. The method of forming a mathematical model of aerodynamic characteristics

The aerodynamic characteristics necessary to form a mathematical model of the ground-effect vehicle were obtained using a computational experiment in the ANSYS CFD simulation software suite. The reliability of the results obtained is confirmed by comparative assessments of experimental and calculated data [1,2].

The method of forming a mathematical model is to present the process of designing a ground-effect vehicle as a series of successive stages. In turn, the stages are defined as solving certain design problems in a manner defined by the goals and complexity of solving these problems (figure 1).

![Diagram](image)

**Figure 1.** Structural diagram of the method of sequential formation of the mathematical model of the ground-effect vehicle aerodynamics in longitudinal motion.

The first stage of the design regards the horizontal rectilinear flight of the ground-effect vehicle at a constant speed. The model is an approximating algorithm for determining the frontal drag coefficients $c_{xa}$, the lifting force $c_{ya}$ and the pitching moment $m_z$ depending on the angle of attack $\alpha$, the relative distance of the center of mass of the ground-effect vehicle from the underlying surface $\tilde{h} = \frac{h}{b_a}$, the relative position of the center of mass $\tilde{x}_G = \frac{x_G}{b_a}$, the relative speed of the jet behind the air propeller plane $\tilde{V}_c = \frac{V_c}{V_\infty}$.

Here $h$ is the distance from the center of mass to the underlying surface; $b_a$ is the mean aerodynamic chord; $x_G$ is the coordinate of the center of mass; $V_c$ is the jet speed behind the air propeller rotation plane; $V_\infty$ is the vehicle air speed.

It should be noted that the position of the center of mass $x_G$ is considered an independent parameter, as it is the height of the position of the center of mass above the underlying surface that is considered the determining parameter of the flight mode. In order to reduce the amount of aerodynamic calculations in the work, it is proposed to determine the position of the pressure center $x_p$, that is, the point of application of the full aerodynamic force and based on this, to recalculate aerodynamic coefficients for other positions of the center of mass. By specifying the envelope of the angles of attack $\alpha$ and relative distances $\tilde{h}$, we calculate the aerodynamic coefficients relative to the point $O$ and determine the relative position of the pressure center $\tilde{x}_p = \frac{x_p}{b_a}$ (figure 2). Then we set the position of the center of gravity $x_G$, for which we need to find aerodynamic coefficients and its relative distance $\tilde{h}$. According to the known value of $\tilde{h}_0$ (relative distance of the origin of coordinates)
by interpolation and approximation, we determine aerodynamic coefficients of lifting force and frontal drag. Then we calculate the pitching moment coefficient by the formula:

\[ m_z = -(\bar{x}_G - \bar{x}_P) (c_{y_a} \sin \alpha + c_{x_a} \cos \alpha). \]

The formation of a mathematical model at the first stage can be presented as a diagram shown in figure 2.

![Diagram showing method of recalculating aerodynamic coefficients according to different positions of the center of mass.](image)

**Figure 2.** Method of recalculating aerodynamic coefficients according to different positions of the center of mass.

If the studied layout corresponds a high aerodynamic quality requirement, we can move on to the second stage of designing a ground-effect vehicle.

The second stage of the design calculates aerodynamic characteristics and determines the positions of the center of mass, at which the ground-effect vehicle will be stable, and the effectiveness of the control devices is sufficient for balancing at the specified flight parameters.

The mathematical model of the aerodynamics of the ground-effect vehicle should take into account the influence of the deflection angles of the longitudinal control devices and wing mechanization on the aerodynamic coefficients. Consider a case when the ground-effect vehicle is equipped with three groups of control surfaces that affect the characteristics in longitudinal motion. Their deflection angles are \( \delta_1, \delta_2, \delta_3 \).

If the range of ground-effect vehicle attack angles \( \alpha \) is insignificant with a high degree of certainty, it can be considered that the increment of the corresponding aerodynamic coefficient when the control surface is deflected does not depend on the angle of the attack (remains constant when it changes). However, the increments of aerodynamic coefficients \( \Delta c_{x_k} (\bar{h}, \delta_k, \vec{V}_c), \Delta c_{y_k} (\bar{h}, \delta_k, \vec{V}_c), \Delta m_z (\bar{h}, \delta_k, \bar{x}_G, \vec{V}_c), k = 1...3 \) are non-linear dependencies on these parameters. Thus, aerodynamic coefficients are determined by expressions.

\[ c_{x_k} = c_{x_k} (\alpha, \bar{h}, \bar{x}_G, \vec{V}_c) + \Delta c_{x_{k1}} (\bar{h}, \delta_1, \vec{V}_c) + \Delta c_{x_{k2}} (\bar{h}, \delta_2, \vec{V}_c) + \Delta c_{x_{k3}} (\bar{h}, \delta_3, \vec{V}_c); \]
\[ c_{y_k} = c_{y_k} (\alpha, \bar{h}, \bar{x}_G, \vec{V}_c) + \Delta c_{y_{k1}} (\bar{h}, \delta_1, \vec{V}_c) + \Delta c_{y_{k2}} (\bar{h}, \delta_2, \vec{V}_c) + \Delta c_{y_{k3}} (\bar{h}, \delta_3, \vec{V}_c); \]
\[ m_z = m_z (\alpha, \bar{h}, \bar{x}_G, \vec{V}_c) + \Delta m_{z1} (\bar{h}, \delta_1, \bar{x}_G, \vec{V}_c) + \Delta m_{z2} (\bar{h}, \delta_2, \bar{x}_G, \vec{V}_c) + \Delta m_{z3} (\bar{h}, \delta_3, \bar{x}_G, \vec{V}_c). \]
When determining aerodynamic increments, the ground-effect vehicle attack angle is set to zero. In this case the increments of the frontal drag coefficients $\Delta c_{x_{ai}}(h, \delta, V_c)$ and the lifting force $\Delta c_{y_{ai}}(h, \delta, V_c)$ do not depend on the ground-effect vehicle’s position of the center of mass. However, the definition of an increment of the coefficient associated with the wing mechanization $\Delta m_{ai}(h, \delta, x_G, V_c)$ requires calculations for the various relative positions of the center of mass.

3. Method of assessing the longitudinal stability of the ground-effect vehicle

A concept of aircraft static stability by parameter is widely used in aircraft design theory [3]. This concept makes it possible to significantly simplify the preliminary assessment of the aircraft stability during the design stage. The paper proposes to perform the stability assessment using the methodology set forth in [4]. The screenplan is considered to be statically stable in longitudinal motion while the following conditions are simultaneously fulfilled:

$$\frac{dc_{x_{ai}}}{dh} = \frac{\hat{c}_{x_{ai}}}{dh} + \frac{\hat{c}_{x_{ai}}}{h \alpha} \frac{d\alpha}{dh} < 0,$$

$$\frac{dm_{ai}}{d\alpha} = -c_{x_{ai}}(\frac{\hat{c}_{x_{ai}}}{dh} + \frac{\hat{c}_{x_{ai}}}{h \alpha} \frac{d\alpha}{dh}) - (\frac{\hat{c}_{x_{ai}}}{dh} + \frac{\hat{c}_{x_{ai}}}{h \alpha} \frac{d\alpha}{dh})(\bar{x}_G + \frac{\hat{c}_{x_{ai}}}{dh} + \frac{\hat{c}_{x_{ai}}}{h \alpha} \frac{d\alpha}{dh} < 0).$$

In fact, this method involves the control of the pressure center $\bar{x}_p$ relative to the center of mass $x_G$ in the area of specified operational relative distances $\bar{h}$, angles of attack $\alpha$, relative positions of the center of mass $x_G$. With that, a number of additional requirements for the control of signs and the values of other components of these expressions are imposed. This method allows one to assess the static stability of the ground-effect vehicle in the first approximation and determine the extreme aft center-of-gravity position.

The following approach is based on the ground-effect vehicle longitudinal stability assessment by balancing characteristics. This paper examines the generally accepted system of differential equations and kinematic ratios (the last two equations) when the aircraft moves in the vertical plane:

$$\frac{dV}{dt} = \frac{P - X_a - mg \sin \theta}{m}$$

$$\frac{d\theta}{dt} = \frac{Y_a - mg \cos \theta}{mV}$$

$$\frac{d\alpha}{dt} = \frac{M_z}{I_z}$$

$$\frac{dX_{\bar{g}}}{dt} = V \cos \theta$$

$$\frac{dh}{dt} = V \sin \theta$$

Here $V$ is the speed of the vehicle’s center of mass; $P$ is the power plant thrust force; $X_a = c_{x_{ai}} q S$ is the frontal drag force; $q$ is the velocity head; $S$ is the characteristic area; $m$ is the vehicle’s weight; $g$ is the free fall acceleration; $\Theta$ is the flight-path angle; $Y_a = c_{y_{ai}} q S$ is the lifting force; $\alpha_{\bar{z}}$ is the angular pitching velocity; $M_z = m_c q S b_a$ is the pitching aerodynamic moment; $I_z$ is the vehicle’s moment of inertia; $X_{\bar{g}}$ is the distance travelled by the center of mass relative to the Earth's coordinate system.
Based on the second-stage mathematical model of the aerodynamics of the ground-effect vehicle in longitudinal motion and the reduced system of differential equations, we calculate the balancing characteristics of the ground-effect vehicle using computer simulation.

Obtaining balancing characteristics and, particularly, the ground-effect vehicle balancing process is carried out by a virtual system of automatic control based on PID controllers [5]. Thus, for the first equation of the differential equation system in question, which allows one to determine the required thrust for a horizontal direct flight, the solution algorithm can be presented by the following diagram (figure 3). The PID controller maintains constant flight speed by controlling the engine thrust. That is, the PID controller ensures a transient process from the initial speed to the specified one. The power plant thrust, under which this condition will be fulfilled, is a required thrust for a horizontal flight at a constant speed in a specified mode.

![Figure 3. Thrust balancing diagram of the flight speed of the ground-effect vehicle in the vertical plane.](image)

An algorithm of balancing the vehicle at a specified height $h$ with $\frac{d\theta}{dt} = 0$ is built similarly. The control factor in this case is the angle of attack $\alpha$, which determines the magnitude of the lifting force $Y_a$. The vehicle is also balanced by the deflection of the appropriate control surface in order to provide $\omega = 0$.

Solving the system of differential equations allows one to obtain a number of practically significant results. For example, it is possible to build Zhukovsky's curves, with which one can assess the response of the ground-effect vehicle to the controlling action, to draw a conclusion on the allowability of the specified position of the center of mass and the possibility of performing a specified movement (figure 4). Of great interest is information about the change in the control surface deflection angles of the ground-effect vehicle when it performs a vertical maneuver with specified flight parameters. Particular evaluation of maneuvering characteristics is reduced to comparing the required deflection angles of control surfaces with available ones, that is, with those that are maximally possible under the conditions of the assigned task. As a result, a conclusion is formulated about the possibility of implementing the specified movements of the considered aerodynamic layout of the ground-effect vehicle.

The model of the dynamics of the ground-effect vehicle movement in the vertical plane and the proposed method for calculating the balancing characteristics are implemented in the MATLAB Simulink dynamic modeling environment.

4. Simulation results

Let us consider some study results of the ground-effect vehicle layout of the tandem scheme (figure 5) [6]. Figure 6 shows the results of evaluating the static longitudinal stability of the ground-effect vehicle using the proposed method.

For the specified operational envelope of relative distance $\bar{h}$ and angles of attack $\alpha$, calculations of aerodynamic characteristics and the position of the pressure center for a number of positions of the center of mass are performed. With the help of the presented graphical and analytical method, the fact of the presence of static longitudinal stability for the specified center of gravity positions in the angles
of attack and relative distances is estimated and the maximum aft center-of-gravity position of the vehicle is determined, which ensures its stability at the specified \( \bar{h} \) and \( \alpha \).

Analysis of the pressure center position \( x_p \) at different angles of attack \( \alpha \) and relative distances \( \bar{h} \) for different positions of the center of mass \( \bar{x}_r \) shows the following (figure 7). With the relative position of the center of mass \( \bar{x}_G = 0.9840 \), except for the extremely small distances \( \bar{h} \) and the angles of attack \( \alpha > 2^0 \), the ground-effect vehicle is unstable. With the relative position of the center of mass \( \bar{x}_G = 1.2299 \), the stability area of the ground-effect vehicle increases. With the relative position of the center of mass \( \bar{x}_G = 1.4757 \), the ground-effect vehicle is stable within the entire considered range of parameters.

The calculated balancing characteristics demonstrate that the required elevator deflection angles at the specified positions of the center of mass and flight parameters do not exceed the available values (figure 8). So for the studied layout, the maximum elevator deflection angle of the elevator is \( \delta_{3\text{max}} = \pm 30^0 \). Figure 8 shows that for a specified center of mass \( \bar{x}_G = 1.2299 \), the elevator efficiency will be sufficient so as to the vehicle is balanced, a straight horizontal flight is established, and the remaining deflection angle margin is sufficient for control.

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**Figure 4.** Balancing curves for a specified position of the center of mass.

**Figure 5.** Geometric model of an experimental ground-effect vehicle (a) and its computational scheme (b).
Figure 7. The position of the pressure center depending on the relative distance $\tilde{h}$ and the angle of attack $\alpha$ at different positions of the center of mass (a - $\bar{x}_G = 0.9840$, b - $\bar{x}_G = 1.2299$, c - $\bar{x}_G = 1.4757$).

5. Conclusions

The proposed method makes it possible at the initial design stage, under conditions of a limited set of aerodynamic characteristics, to qualitatively assess the longitudinal stability of the ground-effect vehicle, to determine the range of center-of-gravity positioning, to evaluate the ground-effect vehicle controllability, to determine the aerodynamic and flight performance characteristics of the ground-effect vehicle in a balanced flight with the specified parameters. The method allows reducing the volume of preliminary aerodynamic studies of the layout and allows increasing the reliability of ground-effect vehicle properties assessment at the design stage.

Figure 8. Balancing characteristics of the studied layout of the ground-effect vehicle for the position of the center of mass $\bar{x}_G = 1.2299$. 
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