Properties of localized protons in neutron star matter for realistic nuclear models *

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We study the localization of protons in the core of neutron stars for ten realistic nuclear models that share a common behaviour of nuclear symmetry energy which saturates and eventually decreases at high densities. This results in the low proton fraction of beta-stable neutron star matter. Protons form a small admixture in the neutron star core, which is localized at sufficiently high densities. For every model we calculate the density $n_{\text{loc}}$ above which the localization effect is present. Our results indicate that localization occurs at densities above $0.5 - 1.0 \, fm^{-3}$. The phase with localized protons occupies a spherical shell or a core region inside neutron stars which contains significant fraction of all nucleons. Proton localization is of great importance for astrophysical properties of neutron stars as it strongly affects transport coefficients of neutron star matter and can produce spontaneous magnetization in neutron stars.

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Observational parameters of neutron stars shed light on some properties of nucleon interactions. These interactions determine the structure and properties of neutron stars. The transport and magnetic properties of neutron stars depend strongly on the structure of the so called liquid core. Particularly important is the fraction and structure of the proton component.

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Nuclear symmetry energy plays a crucial role for composition and other important properties of dense matter in neutron star. The interaction part of the nuclear symmetry energy $E_S(n)$ determines the composition of dense nuclear matter \[1\]. Vanishing of the nuclear symmetry energy implies proton-neutron separation instability in dense nuclear matter \[1\]. Negative values of the symmetry energy result in disappearance of protons at high densities. Realistic nuclear matter calculations \[2, 3\] show that the symmetry energy becomes negative at high densities, and in consequence this leads to the vanishing of protons.

There are two mechanisms of separation of protons and neutrons in neutron star matter: a bulk separation of protons and neutrons and localization of individual protons in neutron matter. We consider in this paper the latter possibility. A bulk separation means that pure neutron matter coexists with nucleon matter containing some proton fraction $x_c$.

Protons which form admixture tend to be localized in potential wells corresponding to neutron matter inhomogeneities created by the protons in the neutron medium. To compare the energy of a normal phase of uniform density and a phase with localized protons we apply the Wigner-Seitz approximation and divide the system into cells, each of them enclosing a single localized proton \[4, 5\]. The neutron background is treated in the Thomas-Fermi approximation and the localized proton is described by the Gaussian wave function. The neutron density profile is obtained by solving the appropriate variational equation \[6\]. We calculate the difference of the energy per proton in the two phases and look for a critical density for localization at which the localized phase becomes the ground state.

The localization effect is a result of the interaction of protons with small density oscillations of the neutron background \[5\]. The protons behave as localized polarons which form a periodic lattice at high densities \[7\]. The aim of this paper is to study the proton localization for a number of realistic nuclear models with selfconsisted variational method.

The amount of protons present in the neutron star matter, which is charge neutral and $\beta$-stable, is crucial for the cooling rate of neutron stars and also plays an important role for magnetic and transport properties of neutron star matter. Nuclear models do not uniquely predict the proton fraction of the neutron star matter at high densities. This controversy is discussed in details in Ref. \[1, 8\] where the discrepancy of the proton fraction in various models is shown to reflect the uncertainty of the nuclear symmetry energy at high densities. In this paper we consider a class of nuclear interaction models for which the proton fraction is of the order of a few (for five of them of above ten) percent and decreases at high densities. For the calculations we have chosen ten realistic nuclear interaction models. These are interactions derived by Myers and Swiatecki \[9\] (MS), the Skyrme
model with parameters \((S_{I}', S_{II}', S_{III}', S_L, S_{ka}, S_{KM}, S_{GII}, R_{ATP}, T_6)\) from Ref. \([10, 11]\), the Friedman and Pandharipande interactions \([12]\) (as parametrized by Ravenhall in Ref. \([10]\)) (FPR), three models, \(UV_{14}+T_{NI}\), \(AV_{14}+UV_{II}\) and \(UV_{14}+UV_{II}\), from Ref. \([2]\) by Wiringa et al. and four modern APR models \([13]\), which provides a fit to the nucleon-nucleon scattering data in the Nijmegen data base. Fig. 1 shows the proton fraction of beta-stable and charge neutral matter containing neutrons, protons, electrons and muons for all the interactions.

![Proton fraction of neutron star matter for indicated interaction models.](image)

**Fig. 1.** Proton fraction of neutron star matter for indicated interaction models.

In calculations we apply selfconsisted method described in our paper \([6]\). The threshold densities (in \(fm^{-3}\)) as well as rms proton radii (in \(fm\)), when the localization effect becomes energetically favorable are listed in Table 1. We also present the minimum masses of neutron stars \(M_{\text{min}}\), when central density is equal to the threshold density of localization of protons. For every model we calculated also the maximum mass of neutron stars \(M_{\text{max}}\) (in solar units). The localization effect is present above the threshold barion number density \(n_{\text{loc}}\). For some models protons vanish from the system above some density \(n_{v}\), so the localization effect is present in the spherical shape region of neutron stars in the density range \(\langle n_{\text{loc}}; n_{v}\rangle\) Fig. 2. For Ska parametrization of the Skyrme model and the \(A_{18}+\delta v\) model the threshold density for proton localization is also the central density of predicted maximum mass configuration of neutron star, so there is no localization of proton inside the observed neutron stars in these two cases.

Localization of protons influences the global properties of neutron star,
so it is of importance to calculate the fraction of nucleons inside the region with localized protons in the neutron star. We show this fraction in Fig. 3.

Results of our calculations for nuclear interactions we use indicate that the proton impurity in neutron star matter becomes localized at densities above $0.5 - 1.0 \text{ fm}^{-3}$. This has important consequences for neutron stars as densities in this range correspond to the inner core of neutron stars with masses exceeding one solar mass, $M > 1M_\odot$.

| Potential | $n_{\text{loc}}$ | $R_{P}^{\text{loc}}$ | $M_{\text{min}}$ | $M_{\text{max}}$ |
|-----------|-----------------|---------------------|------------------|------------------|
| MS        | 1.030           | 0.905               | 2.019            | 2.038            |
| SI        | 0.351           | 1.519               | 0.876            | 2.242            |
| SIT       | 0.361           | 1.688               | 0.829            | 1.966            |
| SIII'     | 0.337           | 1.552               | 0.898            | 2.266            |
| SL        | 0.964           | 1.384               | 1.480            | 1.631            |
| Ska       | 1.016           | 0.804               | 2.241            | 2.241            |
| SKM       | 0.979           | 1.330               | 1.570            | 1.677            |
| SGII      | 0.899           | 1.367               | 1.432            | 1.661            |
| RATP      | 0.709           | 1.397               | 1.225            | 1.726            |
| T6        | 0.530           | 1.985               | 0.600            | 1.429            |
| FPR       | 0.721           | 1.262               | 1.435            | 1.800            |
| UV14+TNI  | 0.731           | 1.209               | 1.489            | 1.827            |
| AV14+UVII | 0.789           | 0.971               | 1.793            | 2.123            |
| UV14+UVII | 0.766           | 0.913               | 1.986            | 2.207            |
| A18       | 1.493           | 1.136               | 1.647            | 1.673            |
| A18+$\delta v$ | 1.627 | 0.915 | 1.805 | 1.805 |
| A18+UIX   | 0.645           | 0.911               | 2.095            | 2.386            |
| A18+$\delta v+\mathrm{UIX}^\ast$ | 0.819 | 0.878 | 2.052 | 2.213 |

According to the Ref. [14] significant number of measured neutron star masses are inside the range $\langle M_{\text{min}}; M_{\text{max}} \rangle$. It indicates that localization of protons is an universal state of dense nuclear matter in neutron stars.

Strongly asymmetric nuclear matter is unstable with respect to proton localization [15, 4]. A uniform proton distribution and a periodic proton arrangement result in very different properties [1]. The presence of the localized protons inside neutron star cores would have profound astrophysical consequences. In particular, the cooling proceeds in a quite different
Fig. 2. The spherical shell of neutron star, where the localization effect occurs. The region corresponds to density above the threshold and below the density for which the protons vanish. Neutron star density profiles are labelled by logarithm of the central density.

Fig. 3. The fraction of nucleons in the region with localized protons of the neutron star versus neutron star mass.

way. Recent analysis [16] shows that the presence of such localized proton phase results in more satisfactory fits of temperatures of observed neutron
stars. Also, spin ordering of localized protons could strongly affect magnetic properties of the system [15][17]. The spin ordered phase can contribute significantly to the observed magnetic moments of neutron stars [18][19].

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