Gravity and the stability of the Higgs vacuum

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We discuss the effect of gravitational interactions on the lifetime of the Higgs vacuum where generic quantum gravity corrections are taken into account. Using a “thin-wall” approximation, we provide a proof of principle that small black holes can act as seeds for vacuum decay, spontaneously nucleating a new Higgs phase centered on the black hole with a lifetime measured in millions of Planck times rather than billions of years. The corresponding parameter space constraints are extremely stringent however, therefore we also present numerical evidence suggesting that with thick walls, the parameter space may open up. Implications for collider black holes are discussed.

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With the discovery of the Higgs boson at the LHC \cite{1,2}, and the measurement of its mass, it seems we live in interesting times: the running of the coupling of the Higgs quite possibly means that our vacuum is only metastable, and the true Higgs vacuum in fact lies at large expectation values of the Higgs and negative vacuum energy. Although this metastability at first might seem alarming, in order for the vacuum to decay, it must tunnel through a sizeable energy barrier, and the probability for this typically has an exponential factor,

$$\Gamma \sim A e^{-B/\hbar}$$

(1)

where \(B\) is the action of a solution to the Euclidean field equations, “the bounce”, which interpolates between the metastable (false) and true vacua; the prefactor \(A\) is determined from fluctuations around the bounce. Since the action \(B\) is usually large (we will set \(\hbar\) to 1 for the rest of this discussion) the probability of vacuum decay is very low. For the decay of a false vacuum, the process was understood and the probability computed in a series of papers by Coleman, Callan and de Luccia \cite{3,4,5}. This ‘gold standard’ calculation is now used ubiquitously to estimate decay rates and the half life of a false vacuum state in field theory, and for the Higgs vacuum predicts a lifetime well in excess of the age of the universe.

The Coleman et al. picture of vacuum decay is however very idealized, in that an exactly homogeneous and isotropic false vacuum decays into a very nearly as symmetric configuration: a completely spherical bubble of true vacuum which expands outwards with uniform acceleration. In everyday physics however, first order phase transitions are far from clean, and often proceed not via some perfect nucleation process but rather by impurities acting as sites for the condensation of a new phase. Recently in \cite{6} we investigated the impact of gravitational impurities, in the guise of black holes, on the usual Coleman de Luccia picture (see also \cite{7,8} for early investigations of this issue) and found a significant enhancement of vacuum decay in a wide range of cases – the intuition of an impurity seeding nucleation of a true vacuum bubble was entirely borne out by the analysis.

To recall Coleman’s original intuition: the nucleation of a bubble costs energy because a wall with energy and tension is formed as a barrier between the false and true vacua, but to counter that, energy is gained by the volume of space inside the bubble having lower energy by virtue of having transitioned to the true vacuum. A bubble of just the right size then optimises this energy pay-off and once formed, grows. The picture with a gravitational inhomogeneity is similar; the bubble forms around the (euclidean) black hole but because of the distortion of space the payoff between volume inside a bubble and its surface area is changed, and bubbles form at a smaller radius and hence the ‘cost’ of forming them is lower: the instanton has a smaller action and the decay process can be significantly enhanced. Alternatively, in terms of the original energy argument of Coleman and de Luccia, the addition of a seed black hole which is eliminated or reduced by the bubble can change the energy balance dramatically.

Can this process affect the lifetime of the Higgs vacuum? We will show that it can, although only if small black holes nucleate the decay. Such black holes could result from gradual evaporation of primordial black holes formed in the early universe \cite{9}; alternatively, if there are “large” extra dimensions \cite{10,11} responsible for producing a hierarchically large Planck scale in our universe, then small black holes can be produced at the LHC \cite{12}. Depending on the tension of the bubble wall, which is directly related to parameters in the Higgs potential, the enhancement of vacuum decay can be large.

To briefly review the Higgs potential, note that at large values of the Higgs field, we can pick any component \(\phi\) and approximate the potential using an effective coupling constant \(\lambda_{\text{eff}}\),

$$V(\phi) = \frac{1}{4} \lambda_{\text{eff}}(\phi) \phi^4.$$  

(2)

The effective coupling is obtained by combining the running of \(\lambda\) under the renormalisation group with the low-energy particle physics parameters. Two-loop calculations of the running coupling \cite{13}, including contributions...
this has been looked at by Branchina et al. [15], who take the CDL \[ \text{CDL} \] to include a black hole. We find that within \( \lambda \) and take a negative \( \lambda \) where the potential is determined predominantly by \( \lambda \) and \( \lambda \) fixed with the Planck scale. If the coefficients \( \lambda \) etc. are similar in magnitude, then the small size of \( \lambda_{\text{eff}} \) at the Planck scale means that the interesting physics occurs where the potential is determined predominantly by \( \lambda_{\text{eff}} \) and \( \Lambda_6 \). In figure [1] we illustrate the effect of these corrections on the standard model potential with \( \lambda_6 = -0.01 \).

Quantum tunnelling in a corrected potential such as this has been looked at by Branchina et al. [14], who take \( \lambda_6 \sim -0.1 \), where the potential barrier occurs at \( \phi \ll M_p \), and take a negative \( \Lambda_6 = -2 \). They argue the existence of a greatly enhanced tunnelling rate, however, their discussion entirely neglected gravitational back-reaction of the instanton on the geometry.

In order to explore the impact of a gravitational impurity we will extend the method of Coleman and de Luccia (CDL) [5], to include a black hole. We find that within the CDL “thin-wall” description, the tunneling amplitude can be significantly enhanced by a small black hole, albeit within a small region of parameter space. This provides a ‘proof of principle’ and motivates a numerical analysis of Higgs instantons, which confirm the presence of strongly enhanced decay in the presence of black holes.

The equations of motion for a wall bounding two different regions of spacetime with different cosmological constants and black hole masses can be expressed in the form \( R^2 + V(R) = 0 \), where \( R \) is the bubble radius (as a function of Euclidean time), and \( V(R) \) is an effective potential involving the wall tension. For the decay of the Higgs vacuum, we assume the standard model has \( \Lambda_6 = 0 \), and write the true vacuum cosmological constant as \( \Lambda_\pm = -3/\ell^2 \), then the potential \( V \) depends on \( \ell \), the black hole masses \( M_\pm \) and the surface tension of the bubble wall. (See [10] for explicit forms of this potential.)

To recapitulate the results of [6], the action of a general instanton with a black hole was found to be

\[
B = \frac{A^+_{\text{h}}}{4G} - \frac{A^-_{\text{h}}}{4G} + \frac{1}{4G} \oint d\lambda \{(2R - 6GM_+)\dot{\tau}_+ - (2R - 6GM_-)\dot{\tau}_- \}
\]

where \( R(\tau_\pm) \) is the solution for the bubble wall, and \( A^\pm_{\text{h}} \) are the black hole horizon areas corresponding to \( M_\pm \). This result includes a careful treatment of the conical deficits which can arise in the Euclidean section when the periodicity of the bubble solution is not the same as that of the black hole, and although the specifics of computing actions in vacuum and AdS vary from that of dS, the essence of the calculation remains the same as the presentation in [6], and the result, [6], identical in form.

This bounce action feeds directly into the exponent in [1], and following Callan and Coleman [4], we estimate the prefactor by taking a factor of \( (B/2\pi)^{1/2} \) for the single time-translational zero mode of the instanton but use the light crossing time of the black hole, \( (GM_+)^{-1} \), as a rough estimate of the remaining determinant of fluctuations giving

\[
\Gamma_D \approx \left( \frac{B}{2\pi GM_+} \right)^{1/2} e^{-B}.
\]

Typically, the CDL action is of order \( O(10^{3-6}) \) for the Higgs potentials, leading to a huge exponential suppression of the decay rate, and to the conclusion that gravitational tunneling is irrelevant.

However, the effect of a black hole, [6], on the tunneling action can be very significant for low tension bubble walls and small mass black holes. As the seed black hole mass \( M_+ \) is switched on, the instanton action drops rapidly, and the bubble initially nucleates by removing the black hole. However, as the seed black hole mass continues to increase, a critical mass \( M_C \) is reached at which the potential \( V(B) \) has a single point at which \( V = V' = 0 \), and there exists a static bubble wall solution. In this case, an unstable static bubble nucleates.
which will either recollapse or expand with roughly equal probability. As the seed black hole mass increases further, the nucleated bubble now has a black hole remnant in the bubble interior, with the action now rising with increasing seed mass. The quantitative values of this critical mass, and the maximal suppression of the bounce action at $M_C$ depend on the wall tension parameter $\sigma$, and the true vacuum energy, however, unless the combination $\sigma \ell$ is Planck scale, this suppression is several orders of magnitude at $M_C$, thus changing the exponential factor in \( \phi \) from an irrelevant $10^6$ to a potentially extremely relevant $10^{9-2}$.

Whether or not this enhancement is relevant depends on its magnitude relative to other physical decay processes, specifically, black hole evaporation. The key indicator is therefore the branching ratio of the static tunneling decay rate to the Hawking evaporation rate,

$$ \Gamma_H \approx 3.6 \times 10^{-4} (G^2 M_3^3)^{-1} \quad [17]; $$

$$ \Gamma_D / \Gamma_H \approx 44 (M_+^3 / M_p^3) B^{1/2} e^{-B}. \quad (7) $$

For our thin wall instantons, there is indeed a range of $M_+$ (small, though still above the Planck mass), for which we have very strong enhancement of bubble tunneling.

The main wrinkle in this argument is that the condition for the thin wall approximation requires that the energy at the potential minimum is smaller than the potential barrier height, and scanning through parameter space we find that requiring a thin wall is very constraining: the range of $\lambda_6$ for which this occurs is very small, and occurs for large values of the parameter $\lambda_6 \gtrsim 10^3 - 10^5$, depending on $\lambda_*$. On the other hand, computing the branching ratio, \( \phi \), for these models shows that tunneling does indeed dominate. Thus, while our pseudo-analytic discussion is limited in the sense of parameter space, it has provided a proof of principle that black holes could potentially seed vacuum decay.

In order to decide whether this effect is restricted to a niche of parameter space, or is potentially relevant, a full exploration of instantons outside of the thin wall approximation is necessary. Motivated by our thin wall results, in which the enhanced tunneling takes place with the static instanton (as $M_+ > M_C$, which is typically less than the Planck mass), we have made a preliminary numerical investigation of static instantons, taking $\lambda_*= -0.01$, and $b = 10^{-4}$ as representative values for the Higgs potential.

Static bounce solutions to the Einstein-scalar equations with rotational symmetry on a black hole AdS background can be found using a spherically symmetric metric ansatz

$$ ds^2 = f(r)e^{2\lambda(r)} dt^2 + \frac{dr^2}{f(r)} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (8) $$

where

$$ f = 1 - \frac{2GM(r)}{r}. \quad (9) $$

The solutions are obtained using a shooting technique, varying the value of the scalar field at the black hole horizon and aiming for $\phi \to 0$ as $r \to 0$. In Ref. [6], it was shown that the action is given by the area terms in \( \phi \), as in the thin wall case. The resulting values of the action for a selection of Higgs models is shown in figure 2. Note that the semi-classical bubble nucleation argument only applies when the action $B > 1$.

Computing the branching ratio now with these “thick wall” solutions gives figure 3. Although black holes produced in the early universe start out with relatively high masses, their temperature is nonetheless above that of the microwave background, and they evaporate down into the range plotted in figure 3. At this point, the mass hits a range in which vacuum decay is more probable, i.e. the tunneling half life becomes smaller than the (instantaneous) Hawking lifetime of the black hole. Note that this range is well above the Planck mass, where we have some confidence in the validity of the vacuum decay calculation. Given that this evaporation timescale is $\sim 10^{-28}$s for a $10^5 M_p$ mass black hole, it is clear that once a primordial black hole (BBH) forms, it will seed vacuum decay in these models. Hence with these Higgs potentials, the presence of any primordial black holes will eventually trigger a catastrophic phase transition from our standard model vacuum thus ruling out potentials with parameters in these ranges.

Since our results show that it is precisely for small black holes that the risk of seeded tunneling is greatest, a natural question is what happens with collider black holes. These can be produced if the fundamental (higher dimensional) Planck scale is near the TeV scale [12]. These black holes have features inherited from their higher dimensional nature, and while there are no known exact solutions, evaporation rates have been computed assuming a higher dimensional Myers-Perry solution [13].
FIG. 3. The branching ratio of the false vacuum nucleation rate to the Hawking evaporation rate as a function of the seed mass for a selection of Higgs models. Each plot corresponds to a different value of $\lambda_6$ in (4), with $\lambda_n = -0.01$.

with emission cross sections appropriate to a braneworld scenario 19.

Black hole seeded tunneling is now a more involved process, as it should involve a bubble forming around the higher dimensional black hole triggered by the Higgs field transitioning on the brane, the bubble then expanding out to fill the extra dimensions before finally becoming effectively four-dimensional and seeding true decay of our universe. While this process is beyond the reach of the analytic approximations we have used here, we can estimate the effect by modelling the instanton with a higher dimensional counterpart of the solutions described above. In this case, the form of the potential $V(R)$ for the bubble motion is modified, but of a remarkably similar form, essentially replacing $R \rightarrow R^{n+1}$, where $n$ is the number of extra dimensions. Assuming the static bubble we can then calculate the horizon radius and area: the action will be the difference in seed and remnant black hole horizon areas. It turns out this calculation is relatively insensitive to the number of extra dimensions (the horizon areas $A \propto M^{(n+2)/(n+1)}$ whereas the evaporation rate of black holes is enhanced, in part because of the increased Hawking temperature, $T \propto M^{-1/(n+1)}$, and in part because of grey-body factors. The branching ratio tends to be suppressed with extra dimensions, making collider black holes less risky for vacuum decay, however black holes produced by particle collisions could still cause vacuum decay in certain regions of parameter space. Fortunately, we have some reassurance about the safety of the LHC from the fact that cosmic ray collisions have occurred at energies higher than those reached at the collider 20!

To sum up: We have shown that the Coleman de Luccia result for the lifetime of our universe in Higgs potentials with metastability seems crucially dependent on the absence of inhomogeneities - the presence of primordial black holes can dramatically reduce the barrier to vacuum decay, and seed nucleation to a universe with a very different “standard model”. Such a conclusion of course depends on the existence of said small black holes – by no means a certainty – and a detailed numerical study of parameter space. However, these results are suggestive that the issue of metastability of our universe may not be as simple as initially was thought.

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