Probing fundamental aspects of quantum mechanics with quantum computers

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IBM quantum computers are used to perform a recently-proposed experiment testing the necessity of complex numbers in the standard formulation of quantum mechanics. While the noisier devices are incapable of delivering definitive results, it is shown that certain devices possess sufficiently small error rates to yield convincing evidence that a faithful description of quantum phenomena must involve complex numbers. The results are consistent with previous experiments and robust against daily calibration of these devices. This work demonstrates the feasibility of using freely-available, noisy, intermediate-scale quantum devices to test foundational features of quantum mechanics.

In recent years, the field of quantum information science has experienced tremendous growth as cloud-based, noisy-intermediate scale quantum (NISQ)\textsuperscript{1} hardware has become widely available from several providers including IBM\textsuperscript{2}, Rigetti\textsuperscript{3}, and IonQ\textsuperscript{4}. In particular, a number of IBM Quantum devices are currently available freely, with additional devices and features available to members of the IBM Quantum network. The accessibility of IBM’s cloud-based, NISQ hardware presents the incredible opportunity for essentially anyone to perform legitimate quantum experiments\textsuperscript{5} by making use of the Qiskit\textsuperscript{6} software development kit.

Long before the availability of functional NISQ technology, it was hypothesized that quantum computers would be capable of simulating systems which would be effectively impossible to simulate classically\textsuperscript{7}. Later it was realized that quantum computers can provide a highly efficient scheme for factoring products of two prime numbers\textsuperscript{8}. As one of the most widely-employed cryptographic frameworks is based on the impracticality of quickly factoring composite numbers\textsuperscript{9}, this particularly mathematical application of quantum computers is of great interest.

Recent investigations of many-body dynamics\textsuperscript{10,11} and cryptography\textsuperscript{12,13} using currently-available systems have demonstrated the impressive development of controllable quantum hardware. But it remains unclear when advances in hardware design will yield devices capable of demonstrating a clear “quantum advantage” compared to classical approaches to these interesting questions\textsuperscript{14}. The noisy devices currently available are limited by unavoidable errors which accumulate considerably for even modest computations. Though some simple methods exist for correcting basic measurement errors\textsuperscript{15}, devising more elaborate and scalable error-correcting schemes for large systems is a highly nontrivial task\textsuperscript{16,17}.

A natural question in this interesting time of rapid development is: what can these noisy quantum devices do now? That is, are there any tasks for which today’s imperfect devices are particularly well suited to demonstrate definitive value? The aim of this paper is to demonstrate that one viable application of current quantum hardware is to simulate small systems, capable of testing fundamental aspects of quantum mechanics. Previous work has shown that Bell’s theorem\textsuperscript{18} and its extensions provide numerous opportunities\textsuperscript{19–22} for these noisy devices to clearly demonstrate the validity of the traditional framework of quantum mechanics.

Recently, a Bell-like test was conceived\textsuperscript{23} for testing the necessity of complex numbers in the traditional formulation of quantum mechanics. At its core, this test computes a score $\Gamma$ which, due to a specific Clauser-Horne-Shimony-Holt (CHSH) inequality\textsuperscript{24}, cannot exceed $\Gamma_{\text{crit}} \approx 7.66$ unless complex numbers are employed in the mathematical description of the basic postulates of quantum theory. The standard, complex-based quantum framework yields a total score of $\Gamma = 6\sqrt{2} \approx 8.49$. Two impressive experiments\textsuperscript{25,26} have since shown convincing evidence that the traditional formulation of quantum theory does require complex-valued quantities with experimentally obtained values of $\Gamma$ exceeding $\Gamma_{\text{crit}}$ significantly. The main focus of the remainder of this paper is to demonstrate how the experimental setup in Ref.\textsuperscript{23} can be performed on IBM quantum hardware. It is shown that consistent experimental results can be obtained on these freely-available, cloud-based devices. Using Qiskit to assemble quantum circuits for this experiment has the benefit of not requiring the user to possess significant expertise in the low-level operation of the hardware (e.g., gate construction via microwave pulses and calibration protocols), thus providing a highly accessible pathway for using quantum technology to perform legitimate experiments which can probe fundamental aspects of quantum theory.

A quantum circuit for testing the relevant CHSH inequality is described below. Results from executing this circuit on several IBM Quantum devices are shown. To support the claim that IBM Quantum devices are useful for performing quantum experiments such as Bell tests which are accessible with small systems, particular attention is paid to demonstrating the robustness of the results. One tradeoff of accessing cloud-based, programmable quantum hardware with Qiskit compared to specially-fabricated devices\textsuperscript{25} is that one generally loses at least some control over the precise calibration of the device. Evidence is presented to demonstrate the results obtained are robust with respect to IBM’s daily calibrations.

The experimental system considered is described in detail
in Ref. [23], and a schematic of this experiment is depicted in Fig. 1. One considers two independent sources, each of which creates an entangled Bell state $|\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$. From each of these two entangled states, one qubit is sent to observer Bob (B). Bob performs a measurement in the Bell-state basis from which he concludes that the two-qubit system is in one of the following states:

$$
|\Phi^\pm\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2},
|\Psi^\pm\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2},
$$

(1)

With the remaining two qubits, Alice performs a measurement of $x$ with $x \in \{\hat{\sigma}^x, \hat{\sigma}^y, \hat{\sigma}^z\}$, while Charlie performs a measurement of $z$ with $z \in \{\hat{\sigma}^z, (\hat{\sigma}^z \pm \hat{\sigma}^y)/\sqrt{2}, (\hat{\sigma}^z \pm \hat{\sigma}^x)/\sqrt{2}\}$. Alice’s and Charlie’s measurement operators are chosen randomly. The outcomes of measurements made by Alice ($a = \pm 1$) and Charlie ($c = \pm 1$) are dependent upon the choice of operators ($xz$) and also (due to residual entanglement) dependent upon the result of Bob’s measurement $y$, which maps the states $|\Phi^\pm\rangle$, $|\Psi^\pm\rangle$ to the binary values $b = 00, 01, 10, 11$, respectively.

Defining the joint conditional probability $P(abc|xz)$ as the probability that Alice obtains $a$, Bob obtains $b$, and Charlie obtains $c$ given a choice $xz$ for Alice’s and Charlie’s operators, a particularly interesting correlation function is [23]

$$
\Gamma = \sum_{abc,xz} w_{abc,xz} P(abc|xz),
$$

(2)

where $w_{abc,xz} = \pm 1$ are weights [23, 25, 27], $xz \in \{11, 12, 21, 22, 13, 14, 33, 34, 25, 26, 35, 36\}$, and $abc \in \{0000, 0001, \ldots, 1110, 1111\}$ so that the sum over $abc$ runs over the entire four-qubit computational basis. It has been demonstrated [23] that $\Gamma \leq 7.66$ for a standard formulation of quantum mechanics which relies upon only real quantities. The main goal of this work is to measure $\Gamma$ experimentally using IBM quantum devices. A quantum circuit which realizes this experiment is shown in Fig. 2. Details of the implementation are relegated to supplemental material [27].

The circuit in Fig. 2 can be used to reconstruct the joint conditional probability distribution $P(abc|xz)$ needed to evaluate the score $\Gamma$ in Eq. (2) by randomly choosing measurements $xz$ and repeatedly executing the circuit. It is possible to sample $xz$ randomly and send a set of circuits to the IBM devices for single-shot experiments. However, one is limited by the maximum number of circuits which can be executed within a particular job, so the maximum number of single-shot runs is $O(10^2)$. Since one may execute the same circuit for at least 20,000 times (“shots”), it is computationally advantageous to execute each of the 12 possible cases of $xz$ for some equal and large number of shots. In what follows, the random sampling over $xz$ will be approximated by an equal sampling over all 12 configurations for $xz$. The reconstructed probability distributions are shown in Fig. 4.
FIG. 4: Final circuit results, $P(abc|xz)$, where the 16 possible four-qubit states are indexed vertically and the horizontal axis depicts one of the 12 possible Alice/Charlie measurement combinations $xz$. Theoretical prediction (upper left) follows from standard quantum mechanics calculation of the gate operations; QASM simulator refers to IBM’s quantum assembly language (QASM) simulator. The label “e.m.” on lower plots indicates that basic readout error mitigation as described in the text has been performed.

To minimize errors in the execution of the circuit in Fig. 2, it is essential to ensure the physical qubits employed are connected in such a way that the CNOT gates can be implemented directly. The currently available IBM Quantum devices possess several types of qubit topology. Figure 3 depicts which subset of qubits is chosen for each device used. While multiple options are typically available for each device, the particular choices used in this work are dictated by trying to minimize CNOT and measurement readout errors. Detailed calibration information is included in the supplemental material [27].

The circuit in Fig. 2 has been sent to several IBM Quantum devices: ibmq_lima v1.0.35, a Falcon r4T processor; ibmq_manila v1.0.29, a Falcon r5.11L processor; and ibm_lagos v1.0.27, a Falcon r5.11H processor. The first two machines are five-qubit devices which are freely available, while ibm_lagos is a seven-qubit device currently available to members of the IBM Quantum Researchers Program [28]. For the five-qubit devices, 20,000 executions (“shots”) per choice of $xz$ are employed in a single job, resulting in a statistical uncertainty estimate of $\pm0.007$ for $\Gamma$. The seven-qubit device allowed 32,000 shots per choice of $xz$ with a corresponding statistical uncertainty estimate of $\pm0.006$. Each job yielded a list of counts for each of the 16 possible four-qubit states. The conditional probabilities $P(abc|xz)$ can then be reconstructed, and $\Gamma$ can be computed from Eq. (2). Results of these computations—the main results of this work—are summarized in Table I and depicted graphically in Fig. 5.

### Table I: Measured scores for the QASM simulator and IBM devices with and without readout error mitigation across several days during which multiple device calibrations occur between computations.

| Device       | Date       | Shots per $xz$ | $\Gamma$ (raw) | $\Gamma$ (r.e.m.) |
|--------------|------------|----------------|----------------|-------------------|
| QASM simulator | 2022-04-12 | 32,000         | 8.488          | n/a               |
| ibmq_lima    | 2022-04-14 | 20,000         | 5.844          | 7.631             |
| ibmq_lima    | 2022-04-19 | 20,000         | 5.902          | 7.731             |
| ibmq_lima    | 2022-04-24 | 20,000         | 5.860          | 7.651             |
| ibmq_manila  | 2022-04-15 | 20,000         | 6.263          | 7.817             |
| ibmq_manila  | 2022-04-19 | 20,000         | 6.500          | 7.946             |
| ibmq_manila  | 2022-04-24 | 20,000         | 6.369          | 7.745             |
| ibm_lagos    | 2022-04-12 | 32,000         | 7.166          | 7.970             |
| ibm_lagos    | 2022-04-19 | 32,000         | 7.202          | 8.030             |
| ibm_lagos    | 2022-04-24 | 32,000         | 7.427          | 8.117             |

Results in Table I depict raw output as well as a computation
of $\Gamma$ corrected using basic, readout-error mitigation \cite{25, 27}. Upon implementing readout-error mitigation \cite{27}, the noisiest device, \texttt{ibmq\_lima}, yielded values of $\Gamma$ which hover around the upper limit for real-number-based quantum mechanics \cite{23}. The devices \texttt{ibmq\_manila} and \texttt{ibm\_lagos} give values for $\Gamma$ which comfortably exceed this bound of 7.66 when statistical error bars are considered. The readout error mitigation applied in this work consists of basic inversion of the measurable calibration matrix and least-squares optimization, which is not necessarily scalable to large systems \cite{17}.

A potentially confounding aspect of using cloud-based devices to perform experiments via Qiskit is that the user has little control over the calibration and tuning of the devices. Strictly speaking, the Qiskit Pulse programming kit \cite{29} does afford users the ability to specify pulse-level timing dynamics in the hardware. Such low-level control could potentially improve the quality of results obtained from these devices. The intent of the present work is to demonstrate that certain devices (e.g., \texttt{ibm\_lagos}) can deliver reliable results for Bell-test-like experiments even without such low-level control. Only a basic working knowledge of quantum circuits with the simplest form of readout-error mitigation is necessary to obtain the results presented.

IBM devices are calibrated daily, and the relevant parameters are easily accessible to users. The calibrations affect the tuning of microwave pulses used to manipulate the qubits, and these daily calibrations tend to cause the results to “drift” significantly compared to differences attributable to statistical fluctuations for results collected within the same day. Data have been collected across several days during which several calibrations cycles have occurred. Detailed calibration information is collected in the supplemental material \cite{27}. The data collected from \texttt{ibm\_lagos} give results which are consistently well above the real/complex threshold. The results from \texttt{ibmq\_lima} and \texttt{ibmq\_manila} drift substantially, making firm conclusions difficult to construct.

Given that the results from \texttt{ibm\_lagos} are robust with $\Gamma \geq 7.97 \pm 0.006$, it is notable that Ref. \cite{25} found $\Gamma = 8.09(1)$ using a custom-designed quantum processor. The present work shows that the freely-available IBM processors are capable of obtaining decisive results for this same experiment while only requiring the user to arrange a set of basic quantum circuit elements. No device yields impressive results without readout-error mitigation, but all hardware considered yields experimental values of $\Gamma$ which approach or exceed the threshold for the necessity of complex numbers in a traditional formulation of quantum mechanics. Drift in results due to the daily calibration of IBM devices makes definitive results difficult to extract from the five-qubit devices in this particular experiment, but the results from \texttt{ibm\_lagos} appear insensitive to this drift and in firm agreement with previous experimental results \cite{25, 26}. Taken together, the results confirm the fairly primitive nature of currently available hardware but also demonstrate that some hardware is capable of performing delicate experiments consistently. The availability of such technology in a way that does not contain excessive barriers for non-specialists creates substantial opportunities for using NISQ devices as scientific tools.

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\[ \text{Ref. [25], [26], [27], [28]} \]

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Supplementary Material

Circuit representation

This section contains a brief summary of how basic operations on qubits are represented by quantum gates using Qiskit [6]. In the computational basis, the two qubit basis states can be represented as two-component vectors

\[ |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \]  

(3)

The single-qubit Hadamard gate \( \hat{H} \) acts according to \( \hat{H}|0\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \), \( \hat{H}|1\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \) and takes the matrix representation

\[ \hat{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \]  

(4)

The controlled NOT (CNOT) gate flips a target qubit only if the control qubit is in the state \( |1\rangle \), so its matrix representation is

\[ \text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \]  

(5)

IBM hardware initializes all qubits in the quantum register to the \( |0\rangle \) state. In order to create the entangled states \( |\Phi^+\rangle \) required for the experiment, one may apply a combination of \( \hat{H} \) and CNOT to the state \( |00\rangle \) to obtain

\[ \text{CNOT}_{i,j}[\hat{H} \otimes \hat{I}][|0\rangle_i \otimes |0\rangle_j] = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle). \]  

(6)

With two copies of \( |\Phi^+\rangle \), Bob must perform a Bell state measurement (BSM) with one qubit from each pair. It can be shown that the application of \( \hat{H} \) followed by CNOT converts

\[ |\Phi^+\rangle \rightarrow |00\rangle, \]

\[ |\Phi^-\rangle \rightarrow |10\rangle, \]

\[ |\Psi^+\rangle \rightarrow |01\rangle, \]

\[ |\Psi^-\rangle \rightarrow |11\rangle. \]  

(7)

Thus, Bob’s actual result of ‘00’, ‘01’, ‘10’, or ‘11’ allows an unambiguous determination of the two-qubit state in the Bell state basis prior to the gate transformations. For Alice and Charlie measure Pauli operators \( \hat{\sigma}^j \) (or combinations thereof), each must rotate the qubit to bring the measurement basis into the computational basis. Given an axis of rotation \( P \), the rotation gate for angle \( \theta \) is given by

\[ \hat{R}_P(\theta) = \cos \frac{\theta}{2} \hat{i} - i \sin \frac{\theta}{2} \hat{n}. \]  

(8)

The basic idea is that one can measure \( \hat{\sigma}^\theta \) where \( \hat{n} = \cos \phi \sin \theta \hat{x} + \sin \phi \sin \theta \hat{y} + \cos \theta \hat{z} \) by first rotating the qubit by \( -\phi \) about the \( z \)-axis (applying \( R_z(-\phi) \)) and then rotating by \( -\theta \) about the \( y \)-axis (applying \( R_y(-\theta) \)) to bring the relevant measurement into the computational basis. Given Alice’s choice of \( x \in \{\hat{\sigma}^x, \hat{\sigma}^y, \hat{\sigma}^z\} \) and Charlie’s choice \( z \in \{\hat{\sigma}^x, \hat{\sigma}^y, \hat{\sigma}^z\} \), there are a total of 18 individual configurations \( xz \). Of these, 12 are used in the computation of \( \Gamma \), so we restrict attention to \( xz \in \{11, 12, 21, 22, 13, 14, 33, 34, 25, 26, 35, 36\} \). For example, \( xz = 36 \) corresponds to Alice measuring \( \hat{\sigma}^x \) and Charlie measuring \( \hat{\sigma}^z \). To rotate each qubit appropriately so that a computational-basis measurement returns each of these spin projections, Alice must rotate her qubit by \( -\phi_1 = -\frac{\pi}{2} \) about the \( z \)-axis followed by a rotation of \( -\theta_1 = -\frac{\pi}{2} \) (with \( \phi_1 = 0 \)) about the \( y \)-axis. Charlie must rotate his qubit by \( -\phi_2 = \frac{\pi}{4} \) about the \( z \)-axis and then by \( -\theta_2 = -\frac{\pi}{2} \) about the \( y \)-axis. A complete list of rotation angles \( \phi_{1,2}, \theta_{1,2} \) for each choice of \( xz \) is given below in Table[S1].
| xz | $\phi_1$ | $\theta_1$ | $\phi_2$ | $\theta_2$ |
|----|--------|--------|--------|--------|
| 11 | 0      | 0      | 0      | $\pi/4$ |
| 12 | 0      | 0      | 0      | $-\pi/4$ |
| 21 | $\pi/2$ | 0      | $\pi/4$ |
| 22 | $\pi/2$ | 0      | $-\pi/4$ |
| 13 | 0      | 0      | $\pi/2$ | $\pi/4$ |
| 14 | 0      | 0      | $-\pi/2$ | $\pi/4$ |
| 33 | $\pi/2$ | $\pi/2$ | $\pi/4$ |
| 34 | $\pi/2$ | $-\pi/2$ | $\pi/4$ |
| 25 | $\pi/2$ | $\pi/4$ | $\pi/2$ |
| 26 | $\pi/2$ | $-\pi/4$ | $\pi/2$ |
| 35 | $\pi/2$ | $\pi/2$ | $\pi/4$ |
| 36 | $\pi/2$ | $-\pi/4$ | $\pi/2$ |

TABLE S1: Rotation angles used for all 12 cases of xz.

Error mitigation

To motivate the readout error mitigation, note that any job returns a set of “counts” as its result. As a simplified example, consider a two-qubit system in which both qubits are measured. The final result will be a set of counts corresponding to the number of times each of the four possible two-qubit states is measured. In the simplest possible two-qubit circuit, one creates a state (say $|00\rangle$) and measures both qubits. Supposing this circuit is run $N = 1024$ times, the existence of errors means that measurements will not always correspond to the state $|00\rangle$. As an example, one might find 900 instances of $|00\rangle$, 61 instances of $|01\rangle$, 54 instances of $|10\rangle$, and 9 instances of $|11\rangle$. Repeating this process for all two-qubit states $|b_0b_1\rangle$ with $b_{0,1} \in \{0,1\}$, one may build a calibration matrix $C$ for which each of these calibration runs yields a row,

$$m_{\text{exp}} = Cm_{\text{ideal}}.$$

(9)

Here $m_{\text{exp}}$ are the obtained counts (in the basis $|00\rangle, \ldots, |11\rangle$, and $m_{\text{ideal}}$ are the expected results. This matrix $C$ obtained from basic state measurement on a particular device can then be inverted and interpreted as an average correction transformation for results obtained for any circuit which should mitigate any errors due solely to errors in the readout (measurement) process.

One potentially problematic aspect of this procedure is that the corrected counts can attain negative values, which is clearly unphysical. It is still possible to compute reasonable expectation values from these corrected counts, interpreting the (potentially negative) probabilities as quasiprobabilities. In the context of our results, these negative values only occurs in the corrected counts for results obtained on $\text{ibmq\_lima}$, and the magnitudes of negative counts are fairly small fractions of the total counts. An alternative procedure is to perform a constrained least-squares optimization to solve the system

$$y = Ax$$

(10)

for $x = m_{\text{ideal}}$, given $A = C$ and $y = m_{\text{exp}}$. The constraint $x_i > 0$ is applied to yield a non-negative probability distribution $x$, where normalization is imposed so that $|x|_2^2 = 1$. The procedure can be implemented using the SciPy optimization routine nnls(A,y) or the MATLAB function lsqlin(A,y). Results obtained with constrained optimization are identical to those obtained by directly inverting the calibration matrix except when applied to data collected from $\text{ibmq\_lima}$ in which matrix inversion yields negative counts. While the effective difference in computation of $\Gamma$ is fairly small, the reported values for $\Gamma$ in the main text correspond the the smaller score obtained from constrained optimization where any difference occurs. For all results presented in the main text, we apply this procedure as basic readout error mitigation. For each set of circuits, an additional $2^4 = 16$ “calibration” circuits are attached to the job so that the computations described above can be performed to perform the error mitigation.

Weights

Specifically, one defines
\[ T_b = (-1)^{b_2} \left( S_{11}^b + S_{12}^b \right) + (-1)^{b_1} \left( S_{21}^b - S_{22}^b \right) \\
+ (-1)^{b_2} \left( S_{13}^b + S_{14}^b \right) - (-1)^{b_1+b_2} \left( S_{33}^b - S_{34}^b \right) \\
+ (-1)^{b_1} \left( S_{25}^b + S_{26}^b \right) - (-1)^{b_1+b_2} \left( S_{35}^b - S_{36}^b \right), \] (11)

where

\[ S_{xz}^b = \sum_{a,c=\pm 1} P(abc|xz), \] (12)

and \( b \in \{00, 01, 10, 11\} \). The connection between \( \Gamma \) and \( T_b \) is

\[ \Gamma = \sum_b T_b = 6\sqrt{2} \approx 8.49, \] (13)

where the approximate value represents an ideal score for a quantum description based on complex numbers.

**Calibration data**

Tables S2–S19 depict calibration metrics for the devices used at the time these devices were used to generate the data depicted in the main text.

As the IBM devices are calibrated on an approximately daily basis, the actual results obtained from the hardware can drift substantially from day to day. Calibration metrics affect the precise manner in which circuit gates implemented in Qiskit are translated into microwave pulses used to manipulate the physical qubits. Each quantum circuit used in this work has been executed at least 20,000 times, resulting in a fairly small statistical uncertainty which is generally smaller than the “drift” in results that occurs as a consequence of daily calibrations.

### IBMq_lima

| Qubit | \( T_1 \) (\( \mu s \)) | \( T_2 \) (\( \mu s \)) | freq. (GHz) | Anharmonicity (GHz) | readout assignment error |
|-------|----------------|----------------|-------------|---------------------|------------------------|
| 0     | 93.46          | 118.47         | 5.030       | -0.33574            | 1.78 \times 10^{-2}    |
| 1     | 122.62         | 131.38         | 5.128       | -0.31835            | 1.67 \times 10^{-2}    |
| 2     | 117.13         | 131.29         | 5.247       | -0.33360            | 2.22 \times 10^{-2}    |
| 3     | 135.20         | 106.71         | 5.303       | -0.33124            | 2.68 \times 10^{-2}    |
| 4     | 21.41          | 21.57          | 5.092       | -0.33447            | 5.10 \times 10^{-2}    |

TABLE S2: Calibration data for qubits in IBMq_lima taken 2022-04-14.

| Connection | error rate | gate time (ns) |
|------------|------------|----------------|
| 0-1        | 5.290 \times 10^{-3} | 305.777 |
| 1-2        | 2.632 \times 10^{-2}  | 334.222 |
| 1-3        | 1.263 \times 10^{-2}  | 497.777 |
| 3-4        | 1.680 \times 10^{-2}  | 519.111 |

TABLE S3: Calibration data for CNOT gates in IBMq_lima taken 2022-04-14.
### TABLE S4: Calibration data for qubits in ibmq_lima taken 2022-04-19.

| Qubit | $T_1$ ($\mu$s) | $T_2$ ($\mu$s) | freq. (GHz) | Anharmonicity (GHz) | readout assignment error |
|-------|----------------|----------------|-------------|----------------------|-------------------------|
| 0     | 71.55          | 97.69          | 5.030       | -0.33574             | 2.27 × 10^{-2}          |
| 1     | 123.87         | 107.28         | 5.128       | -0.31835             | 1.50 × 10^{-2}          |
| 2     | 123.98         | 134.58         | 5.247       | -0.33360             | 2.30 × 10^{-2}          |
| 3     | 96.41          | 96.81          | 5.303       | -0.33124             | 2.92 × 10^{-2}          |
| 4     | 22.11          | 22.55          | 5.092       | -0.33447             | 6.20 × 10^{-2}          |

### TABLE S5: Calibration data for CNOT gates in ibmq_lima taken 2022-04-19.

| Connection | error rate | gate time (ns) |
|------------|------------|----------------|
| 0-1        | 5.247 × 10^{-3} | 305.777       |
| 1-2        | 7.181 × 10^{-2} | 334.222       |
| 1-3        | 1.171 × 10^{-2} | 497.777       |
| 3-4        | 1.707 × 10^{-2} | 519.111       |

### TABLE S6: Calibration data for qubits in ibmq_lima taken 2022-04-24.

| Qubit | $T_1$ ($\mu$s) | $T_2$ ($\mu$s) | freq. (GHz) | Anharmonicity (GHz) | readout assignment error |
|-------|----------------|----------------|-------------|----------------------|-------------------------|
| 0     | 169.65         | 188.84         | 5.030       | -0.33574             | 2.30 × 10^{-2}          |
| 1     | 125.59         | 117.96         | 5.128       | -0.31835             | 1.47 × 10^{-2}          |
| 2     | 78.30          | 96.82          | 5.247       | -0.33360             | 2.22 × 10^{-2}          |
| 3     | 101.35         | 69.09          | 5.303       | -0.33124             | 3.31 × 10^{-2}          |
| 4     | 23.48          | 22.98          | 5.092       | -0.33447             | 4.82 × 10^{-2}          |

### TABLE S7: Calibration data for CNOT gates in ibmq_lima taken 2022-04-24.

| Connection | error rate | gate time (ns) |
|------------|------------|----------------|
| 0-1        | 4.707 × 10^{-3} | 305.777       |
| 1-2        | 7.186 × 10^{-2} | 334.222       |
| 1-3        | 1.184 × 10^{-2} | 497.777       |
| 3-4        | 1.905 × 10^{-2} | 519.111       |

### ibmq_manila

### TABLE S8: Calibration data for qubits in ibmq_manila taken 2022-04-15.

| Qubit | $T_1$ ($\mu$s) | $T_2$ ($\mu$s) | freq. (GHz) | Anharmonicity (GHz) | readout assignment error |
|-------|----------------|----------------|-------------|----------------------|-------------------------|
| 0     | 107.80         | 66.96          | 4.962       | -0.34335             | 2.70 × 10^{-2}          |
| 1     | 135.11         | 74.09          | 4.838       | -0.34621             | 6.37 × 10^{-2}          |
| 2     | 157.35         | 22.65          | 5.037       | -0.34366             | 4.16 × 10^{-2}          |
| 3     | 197.38         | 65.29          | 4.951       | -0.34355             | 2.50 × 10^{-2}          |
| 4     | 138.50         | 45.29          | 5.065       | -0.34211             | 2.97 × 10^{-2}          |
## TABLE S9: Calibration data for CNOT gates in 「bmq_manila」 taken 2022-04-15.

| Connection | Error Rate | Gate Time (ns) |
|------------|------------|----------------|
| 0-1        | 8.959 × 10^{-3} | 277.333 |
| 1-2        | 1.114 × 10^{-2}  | 469.333 |
| 2-3        | 7.623 × 10^{-3}  | 355.556 |
| 3-4        | 5.750 × 10^{-3}  | 334.222 |

## TABLE S10: Calibration data for qubits in 「bmq_manila」 taken 2022-04-19.

| Qubit | T1 (μs) | T2 (μs) | f (GHz) | Anharmonicity (GHz) | Readout Assignment Error |
|-------|---------|---------|---------|----------------------|--------------------------|
| 0     | 131.94  | 70.25   | 4.962   | -0.34335             | 3.38 × 10^{-2}           |
| 1     | 130.10  | 76.29   | 4.838   | -0.34621             | 2.92 × 10^{-2}           |
| 2     | 142.48  | 25.95   | 5.037   | -0.34366             | 2.44 × 10^{-2}           |
| 3     | 157.93  | 60.11   | 4.951   | -0.34355             | 1.90 × 10^{-2}           |
| 4     | 170.45  | 41.78   | 5.065   | -0.34211             | 2.59 × 10^{-2}           |

## TABLE S11: Calibration data for CNOT gates in 「bmq_manila」 taken 2022-04-19.

| Connection | Error Rate | Gate Time (ns) |
|------------|------------|----------------|
| 0-1        | 7.651 × 10^{-3}  | 277.333 |
| 1-2        | 9.658 × 10^{-3}  | 469.333 |
| 2-3        | 6.794 × 10^{-3}  | 355.556 |
| 3-4        | 6.265 × 10^{-3}  | 334.222 |

## TABLE S12: Calibration data for qubits in 「bmq_manila」 taken 2022-04-24.

| Qubit | T1 (μs) | T2 (μs) | f (GHz) | Anharmonicity (GHz) | Readout Assignment Error |
|-------|---------|---------|---------|----------------------|--------------------------|
| 0     | 199.20  | 99.76   | 4.962   | -0.34335             | 3.2 × 10^{-2}            |
| 1     | 195.62  | 72.03   | 4.838   | -0.34621             | 3.18 × 10^{-2}           |
| 2     | 158.43  | 24.61   | 5.037   | -0.34366             | 4.64 × 10^{-2}           |
| 3     | 159.02  | 40.88   | 4.951   | -0.34355             | 3.23 × 10^{-2}           |
| 4     | 130.71  | 40.62   | 5.065   | -0.34211             | 2.29 × 10^{-2}           |

## TABLE S13: Calibration data for CNOT gates in 「bmq_manila」 taken 2022-04-24.

| Connection | Error Rate | Gate Time (ns) |
|------------|------------|----------------|
| 0-1        | 5.223 × 10^{-3}  | 277.333 |
| 1-2        | 9.977 × 10^{-3}  | 469.333 |
| 2-3        | 9.831 × 10^{-3}  | 355.556 |
| 3-4        | 6.666 × 10^{-3}  | 334.222 |

「bmq_manila」

「bmq_lagos」
| Qubit | $T_1$ ($\mu$s) | $T_2$ ($\mu$s) | freq. (GHz) | Anharmonicity (GHz) | readout assignment error |
|-------|----------------|----------------|-------------|---------------------|-------------------------|
| 0     | 110.77         | 46.41          | 5.235       | $-0.33987$          | $7.30 \times 10^{-3}$    |
| 1     | 130.34         | 84.81          | 5.100       | $-0.34325$          | $1.14 \times 10^{-2}$    |
| 2     | 123.18         | 112.70         | 5.188       | $-0.34193$          | $5.40 \times 10^{-3}$    |
| 3     | 207.35         | 143.35         | 4.987       | $-0.34529$          | $1.97 \times 10^{-2}$    |
| 4     | 161.92         | 46.82          | 5.285       | $-0.33923$          | $1.20 \times 10^{-2}$    |
| 5     | 133.28         | 88.03          | 5.176       | $-0.34079$          | $1.18 \times 10^{-2}$    |
| 6     | 137.78         | 178.13         | 5.064       | $-0.34276$          | $5.30 \times 10^{-3}$    |

**TABLE S14:** Calibration data for qubits in *i*bm_lagos taken 2022-04-12.

| Connection | error rate | gate time (ns) |
|------------|------------|----------------|
| 0-1        | $6.544 \times 10^{-3}$ | 305.777 |
| 1-2        | $6.707 \times 10^{-3}$ | 327.111 |
| 1-3        | $9.804 \times 10^{-3}$ | 334.222 |
| 3-5        | $1.061 \times 10^{-2}$ | 334.222 |
| 4-5        | $5.090 \times 10^{-3}$ | 362.667 |
| 5-6        | $3.599 \times 10^{-3}$ | 256.000 |

**TABLE S15:** Calibration data for CNOT gates in *i*bm_lagos taken 2022-04-12.

| Qubit | $T_1$ ($\mu$s) | $T_2$ ($\mu$s) | freq. (GHz) | Anharmonicity (GHz) | readout assignment error |
|-------|----------------|----------------|-------------|---------------------|-------------------------|
| 0     | 131.57         | 49.25          | 5.235       | $-0.33987$          | $7.60 \times 10^{-3}$    |
| 1     | 138.38         | 116.49         | 5.100       | $-0.34325$          | $1.11 \times 10^{-2}$    |
| 2     | 112.36         | 142.46         | 5.188       | $-0.34193$          | $4.90 \times 10^{-3}$    |
| 3     | 105.22         | 93.65          | 4.987       | $-0.34529$          | $1.99 \times 10^{-2}$    |
| 4     | 107.02         | 43.79          | 5.285       | $-0.33923$          | $1.37 \times 10^{-2}$    |
| 5     | 129.64         | 83.89          | 5.176       | $-0.34079$          | $1.28 \times 10^{-2}$    |
| 6     | 180.21         | 109.66         | 5.064       | $-0.34276$          | $7.50 \times 10^{-3}$    |

**TABLE S16:** Calibration data for qubits in *i*bm_lagos taken 2022-04-19.

| Connection | error rate | gate time (ns) |
|------------|------------|----------------|
| 0-1        | $7.808 \times 10^{-3}$ | 305.777 |
| 1-2        | $5.743 \times 10^{-3}$ | 327.111 |
| 1-3        | $8.710 \times 10^{-3}$ | 334.222 |
| 3-5        | $1.513 \times 10^{-2}$ | 334.222 |
| 4-5        | $5.895 \times 10^{-3}$ | 362.667 |
| 5-6        | $4.501 \times 10^{-3}$ | 256.000 |

**TABLE S17:** Calibration data for CNOT gates in *i*bm_lagos taken 2022-04-19.

| Qubit | $T_1$ ($\mu$s) | $T_2$ ($\mu$s) | freq. (GHz) | Anharmonicity (GHz) | readout assignment error |
|-------|----------------|----------------|-------------|---------------------|-------------------------|
| 0     | 187.41         | 49.35          | 5.235       | $-0.33987$          | $8.70 \times 10^{-3}$    |
| 1     | 117.38         | 118.40         | 5.100       | $-0.34325$          | $8.90 \times 10^{-3}$    |
| 2     | 64.03          | 119.95         | 5.188       | $-0.34193$          | $6.80 \times 10^{-3}$    |
| 3     | 145.37         | 140.26         | 4.987       | $-0.34529$          | $1.66 \times 10^{-2}$    |
| 4     | 167.96         | 51.35          | 5.285       | $-0.33923$          | $1.41 \times 10^{-2}$    |
| 5     | 91.06          | 77.16          | 5.176       | $-0.34079$          | $1.18 \times 10^{-2}$    |
| 6     | 127.67         | 146.32         | 5.064       | $-0.34276$          | $7.60 \times 10^{-3}$    |

**TABLE S18:** Calibration data for qubits in *i*bm_lagos taken 2022-04-24.
| Connection | error rate       | gate time (ns) |
|------------|-----------------|----------------|
| 0-1        | $7.570 \times 10^{-3}$ | 305.777        |
| 1-2        | $5.387 \times 10^{-3}$ | 327.111        |
| 1-3        | $5.706 \times 10^{-3}$ | 334.222        |
| 3-5        | $8.753 \times 10^{-2}$ | 334.222        |
| 4-5        | $4.983 \times 10^{-3}$ | 362.667        |
| 5-6        | $5.735 \times 10^{-3}$ | 256.000        |

TABLE S19: Calibration data for CNOT gates in ibm_lagos taken 2022-04-24.