PROJECTIVE STRUCTURES AND EXACT VARIATIONAL FORMULA
OF MONODROMY GROUP OF THE LINEAR DIFFERENTIAL
EQUATIONS ON COMPACT RIEMANN SURFACE.

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The monodromy groups of linearly polymorphic functions on compact Riemann surface have appeared of the 19-th century in works by F. Klein, H. Poincare, E. Picard [see 1-3], in connection with a uniformization problem compact Riemann surface. In 70 years of 20-th century monodromy groups have appeared again in works by C. Earle [4], I. ra [5; 6], B. Maskit [7], D.A. Hejhal [1-3] and R. Gunning [8], in connection with general uniformization problem and with the theory Teichmueller spaces. In 1985 year P.G. Zograf, L. chajain [9] solved the problem of the accessory parameters of the linear differential equation of the second order (so-called Fuchsian type) on compact Riemann surface. B. Venkov [10] has found the explicit formulas for these parameters in terms monodromy groups, which are a Fuchsian group.

1. Uniformization and linearly polymorphic functions.

In this section 1 we are investigated the monodromy group for linearly polymorphic functions on compact Riemann surface of genus \( g \geq 2 \), in connection with standard uniformization of these surfaces by Kleinian groups. Uniformization \((\Delta, G)\) for compact Riemann surface \( F \) is called to be standard, if group \( G \) have not an elliptic elements and have not accidental parabolic elements, or natural projection \( \pi : \Delta \rightarrow F \) is planar regular not ramified covering [7]. Here we are shown, how the algebraic description of monodromy group is connected with standard uniformization of compact Riemann surface. Also we find a neccessary and sufficients conditions, that a linearly polymorphic function on compact Riemann surface gave a standard uniformization of this surface. These conditions have simple topological sense.

Let \( F \) be compact Riemann surface of genus \( g \geq 2 \), \( \pi_1(F, O) \) - fundamental group surface \( F \) with basic point \( O \). A complex projective structure on \( F \) is a maximal atlas from cards on \( F \) such that all transition maps belong to the group \( PSL(2, \mathbb{C}) \) [8].

Definition 1.1. A locally meromorphic (multivalued) function \( z \) on \( F \), which transforms linear fractionally under action group \( \pi_1(F, O) \), is called linearly polymorphic functions on \( F \).

Further we are considered only locally schlcht linearly polymorphic function on \( F \). They play a very important role in modern uniformization theory and in theory of the Teichmueller spaces.

Let \((U, \pi)\) be the universal covering for \( F \), where \( \pi : U \rightarrow F \) - natural projection, \( U \) - unit disc with the center in the origin of coordinates on the extended complex plane \( \overline{\mathbb{C}} \), and \( \Gamma \) is a Fuchsian group such that \( F = U/\Gamma \).
The linearly polymorphic function $z$ on $F$ can be lifted to a meromorphic (single-valued) function $z = z(t)$ on $U$ in such a way that
\[ z(At) = \tilde{A}z(t), t \in U, A \in \Gamma, \]
where $\rho(A) = \tilde{A} \in \text{PSL}(2, \mathbb{C})$ is group of linear fractionally transformations. Therefore a monodromy homomorphism $\rho : \Gamma \to \text{PSL}(2, \mathbb{C})$ is determined. The image $\rho(\Gamma)$ for $\Gamma$ on $\rho$ is called monodromy group $\mathcal{M}[z]$ for $z$ on $F$. I. Kra [6] has named this function to be $(\Gamma, \rho)$—deformation of Fuchsian group $\Gamma$ in $U$, nd R. Gunning [8](see also E. povich [11]) is called it a developing map of this unramified complex projective structure on $F$.

For the theory of functions more suitable is the term linearly polymorphic function, accepted in work D., Hejhal. This is according to historical tradition, coming from H. Poincare, P. ppell and E. Coursat [12]. Let us give two classical examples of linearly polymorphic function, which are connected with uniformization compact Riemann surfaces.

Let $\pi : U \to F$ be universal covering mapping, for which $\Gamma$ is group of covering transformations. Then multivalued inverse function $w = \pi^{-1}(\xi)$ will be locally schlicht linearly polymorphic function on $F$ with monodromy group $\Gamma$.

Let $D \subset \mathbb{C}$ and $\pi_S : D \to F$ is a Schottky covering, where the group of covering transformations of $\pi_S$ is the Schottky group $\Gamma_S$, hen function $w = \pi_S^{-1}(\xi)$ will be locally single-valued linearly polymorphic function on $F$ with the monodromy group $\Gamma_S$.

Let us note, that there are monodromy groups, which algebraically are arranged as a Fuchsian group $\Gamma$ or as a Schottky group $\Gamma_S$, but not connected with uniformization compact Riemann surface [1].

The function
\[ q(t) = \{z, t\} = \left(\frac{z''(t)}{z'(t)}\right)' - \frac{1}{2} \left(\frac{z''(t)}{z'(t)}\right)^2 \]
is a Schwartz derivative for function $z = z(t)$ on $U$. This function is holomorphic on $U$ and satisfies the condition
\[ q(At)(dAt)^2 = q(t)dt^2, t \in U, A \in \Gamma. \]
Hence, $q = q(t)dt^2$ is the holomorphic (abelian) quadratic differential on $U/\Gamma = F$.

Such linearly polymorphic functions on $F$ can be received, as the quotient of two the linearly independent solutions of the linear differential equation of the second order Fuchsian type on $F$ [1].

Let $B_2(U, \Gamma)$ be a complex vector space of holomorphic quadratic differentials $q = q(t)dt^2$ with respect to $\Gamma$ on $U$ with norm
\[ \|q\| = sup_{t \in U} \{(1 - |t|^2)^2|q(t)|\} < \infty. \]
Classical method to construct of linearly polymorphic function on quadratic differential consist in the following: Let $q \in B_2(U, \Gamma)$, then there exists the unique locally schlicht meromorphic function $z = z(t)$ on $U$ such that $\{z, t\} = q(t)$ and $z(t) = \frac{1}{t} + O(|t|)$ in neighborhood point $t = 0$. Hence for any $A \in \Gamma$ there is unique element $\rho_z(A) \in \text{PSL}(2, \mathbb{C})$ such that $z(At) = \rho_z(A)z(t), t \in U$ [1].

Let $\pi_0 = [F, \{a_k, b_k\}_{k=1}^g]$ be a marked compact Riemann surface of genus $g \geq 2$. We will choose and fixed a point $t_0 \in U$ and set $O = \pi(t_0)$. Then there exists a natural isomorphism between $\pi_1(F, O)$ and marked Fuchsian group
\[ \Gamma = \{A_1, ..., A_g, B_1, ..., B_g : [A_1, B_1]...[A_g, B_g] = 1\} \]
of the first kind on $U$, defined by $a_j \to A_j, b_j \to B_j, j = 1, ..., g$. 

From the theory of Teichmueller spaces $T_g$ it is known that there is a homeomorphism translating $\tau = [F_\tau, \{a_k(\tau), b_k(\tau)\}_{k=1}^g] \in T_g$ in group

$$\Gamma_\tau = \{A_1(\tau), ..., B_g(\tau) : \prod_{j=1}^g [A_j(\tau), B_j(\tau)] = 1\}$$

from space of normalized marked Fuchsian groups on $U$ [13]. Let $z$ be a linearly polymorphic function on $F_\tau = U/\Gamma_\tau$, then for meromorphic function $z = z(t)$ on $U$ we have $z(At) = \tilde{A}z(t)$, $A \in \Gamma_\tau$, $\tilde{A} \in PSL(2, \mathbb{C})$. The mapping $A \to \tilde{A}$ is called the monodromy homomorphism. It determines the marked monodromy group

$$\mathcal{M}[z] = \{\tilde{A}_1(\tau), ..., \tilde{B}_g(\tau) : [\tilde{A}_1(\tau), \tilde{B}_1(\tau)]...[\tilde{A}_g(\tau), \tilde{B}_g(\tau)] = 1\},$$

i.e. $\mathcal{M}[z]$ is a point in $[PSL(2, \mathbb{C})]^g$.

Let $\mathcal{F}_\tau$ be a fundamental polygon for group $\Gamma_\tau$ in $U$, whose border

$$\partial \mathcal{F}_\tau = a_1^+(\tau)b_1^+(\tau)b_1^-\tau a_g^+(\tau)b_g^-\tau$$

is lifting of commutator path

$$[a_1(\tau), b_1(\tau)]...[a_g(\tau), b_g(\tau)]$$

from $t_0 \in U$; the sides of $\mathcal{F}_\tau$ are paired identified by transformations $A_k(\tau) : a_k^-\tau \to a_k^+, B_k(\tau) : b_k^-\tau \to b_k^+, k = 1, ..., g$.

According to F.Klein, we define the fundamental membrane $R_z$ for function $z = z(t)$ as the Riemannian (multivalued) image $z(\mathcal{F}_\tau)$. Its is simply connected and unramified, but it is possible for mapping $z : F_\tau \to z(\mathcal{F}_\tau)$ to be $n$-valued, $n \geq 2$. The sides of membrane are paired identified by transformations $\tilde{A}_1(\tau), ..., \tilde{B}_g(\tau)$. There is an equivalence, connecting linearly polymorphic function $z$ and its fundamental membrane. So, given a simply connected unramified domain $R$ with indicated properties, there exists a locally schlicht linearly polymorphic function $z$ on some marked compact Riemann surface $F_\tau$ such that $R_z = R$ [1].

Explicit construction of such a function $z$ on $R$ is indicated in [1], and there is its generalizations, where instead of $(U, F_\tau)$ it one take the standard uniformization $(\Delta, G)$ for compact Riemann surface of genus $g$. Indeed, let $z = z(t)$ be a locally schlicht linearly polymorphic function on $\tau \in T_g$ such that

$$\mathcal{M}[z] = \{T_1, ..., T_h, 1, ..., 1, U_1, ..., U_{g-h}, V_1, ..., V_{g-h} : [U_1, V_1] = ... = [U_s, V_s] = \prod_{j=1}^{i_1}[U_{s+j}, V_{s+j}] = ... = \prod_{j=1}^{i_m}[U_{g-i_m+j}, V_{g-i_m+j}] = 1\}.$$  

(*)

Suppose that $w : U \to D_\tau$ is a Koebe uniformization of signature $\sigma = (h,s; i_1, ..., i_m) \neq (0,2;0,0,0,0)$, $|\sigma| = h + s + i_1 + ... + i_m = g$, $i_j \neq 1$, $j = 1, ..., m$, for marked compact Riemann surface $\tau$ and

$$G_\tau = \{T'_1, ..., T'_h, U'_1, ..., U'_{g-h}, V'_1, ..., V'_{g-h} : [U'_1, V'_1] = ... = [U'_s, V'_s] = ... = \prod_{j=1}^{i_1}[U'_{s+j}, V'_{s+j}] = ... = \prod_{j=1}^{i_m}[U'_{g-i_m+j}, V'_{g-i_m+j}] = 1\}$$

is the corresponding marked Koebe group of signature $\sigma$. Then the function $Z = zw^{-1}$ is locally schlicht linearly polymorphic function on $D_\tau$ (invariant component of group $G_\tau$). It satisfies the following relations:

$$ZT_k(\tilde{t}) = T_kZ(\tilde{t}), k = 1, ..., h, ZU'_j(\tilde{t}) = U'_jZ(\tilde{t}),$$
\[ ZV_j(t) = V_jZ(t), \quad j = 1, \ldots, g - h. \]

Let \( K_\tau \) be a standard fundamental \((2h + s + m)\)-connected domain for group \( G_\tau \) which sides are identified (in pairs) by standard generators of group \( G_\tau \), changing orientation of determining curves.

We list the basic properties of a fundamental membrane \( Z(K_\tau) \):

1) \( Z(K_\tau) \) is planar (i.e. there is conformal mapping its on planar domain) and \((2h + s + m)\)-connected,
2) \( Z(K_\tau) \) is unramified,
3) the sides of \( Z(K_\tau) \) are identified, by linear fractionally transformation

\[ T_1, \ldots, T_h, 1, \ldots, 1, U_1, \ldots, U_{g-h}, V_1, \ldots, V_{g-h}, \]

with relations (*). Repeating topological and analytical construction by D. Hejhal [1, c. 28-29], we have receive that for any membrane \( R \) with properties 1) - 3) exists linearly polymorphic functions \( z = z(t) \) on \( U \) and \( Z(t) \) on \( D_\tau \) for some \( \tau \in T_g \), which satisfy to the equation

\[ z = Zw, \]

where \( R \) has properties 1) - 3).

**Theorem** (Kra - Gunning) [8; 5; 6]. Let \( z \) be a locally schlicht linearly polymorphic function on compact Riemann surface \( U/\Gamma = F \) of genus \( g \geq 2 \). Then the following conditions are equivalent:

1) \( \mathcal{M}[z] \) acts on \( z(U) \) discontinuously,
2) \( z : U \rightarrow z(U) \) is a (topological) covering mapping,
3) \( z(U) \neq \overline{C} \).

**Lemma 1.1.** Let \( w = w(t) \) be a locally schlicht linearly polymorphic function on compact Riemann surface \( F \) of genus \( g \geq 2 \) and \( w(U) \neq \overline{C} \). Then \( \mathcal{M}[w] \) is nonelementary, finite generated Kleinian group with invariant component \( w(U) \).

Proof. Nonelementarity follows from theorem 7 [8] since \( w(U) \) has a hyperbolic covering. Discontinity of \( \mathcal{M}[z] \) or that \( \mathcal{M}[z] \) is Kleinian group, follows from the theorem Kra-Gunning. As a consequence from this theorem we have that \( \mathcal{M}[z] \) can not act discontinuously in greater domain than \( w(U) \), i.e. the domain \( w(U) \) is invariant component for group \( \mathcal{M}[z] \). Lemma is proved.

The main result in this section 1 is the following

**Theorem 1.2.** Let \( w = w(t) \) be a locally schlicht linearly polymorphic function on compact Riemann surface \( F \) of genus \( g \geq 2 \). Then \( w = w(t) \) is an uniformization of \( F \) if and only if the following conditions are carried out:

1) \( w(U) \neq \overline{C} \),
2) \( w(U)/\mathcal{M}[w] \) is a compact surface of genus \( g \).

Proof. If \( w = w(t) \) be an uniformization of \( F \), then, by definition, pair \((w(U), \mathcal{M}[w])\) is those that \( w(U)/\mathcal{M}[w] \) is conformal equivalent to \( F \), and \( w(U) \) is the covering surface for \( F \). Hence, universal covering surface for \( w(U) \) will be a disc and so \( w(U) \neq \overline{C} \).

Conversely, from the condition \( w(U) \neq \overline{C} \) it follows that \( w : U \rightarrow w(U) \) is a topological covering. Consider the commutative diagram of mapping given below. Here \( \pi \) \( \tilde{\pi} \) are natural projections and \( \tilde{w} \) is proper holomorphic covering mapping from \( F \) on compact Riemann surface \( F_1 \) [16]. We notice that \( \tilde{\pi} \) is ramified if and only if \( \tilde{w} \) is ramified.
From the second condition of the theorem and Riemann-Hurwitz formula \[17; 16\] we are received that \( \tilde{w} \) be one-to-one and \( \tilde{w} \) is a conformal homeomorphism \( F \) on \( F_1 \). Theorem is proved.

Remark 1.1. It is possible to prove that from condition 2) of the theorem 1.2 implies the condition 1). Inverse is not correct, see for example \([6]\).

From the theorem 1.2 and the theorem 3 \([22]\) we obtain following

**Corollary 1.3.** Let \( w = w(t) \) is a locally schlicht linearly polymorphic function on marked compact Riemann surface \( F \) of genus \( g \geq 2 \) such that \( \mathcal{M}[w] \) is a marked Koebe group of signature \( \sigma = (h, s; i_1, \ldots, i_m) \neq (0, 2; 0, \ldots, 0), |\sigma| = g, i_j \neq 1, j = 1, \ldots, m. \) If \( w(U) \neq \overline{C} \), then \( w = w(t) \) is a uniformization of \( F \) by the Koebe group of signature \( \sigma \).

Remark 1.2. If \( w = w(t) \) is such as in the corollary 1.3, but \( w(U) = \overline{C} \), then statement of the corollary 1.3 is not truly. For the proof it is enough to construct a fundamental membrane \( R_w \) for \( w \) such that \( \mathcal{M}[w] \) is the same as in corollary 1.3, but the mapping \( w : \mathcal{F} \to R_w = w(\mathcal{F}) \) is \( n \)-valued, \( n \geq 2 \). Here \( \mathcal{F} \) is an fundamental polygon of Fuchsian group \( \Gamma_w \), which uniformize \( F \) in disc \( U \). For signature \( \sigma = (h, s; i_1, \ldots, i_m) \neq (0, 2; 0, \ldots, 0), |\sigma| = g \geq 2 \), we will consider the following cases:

1) if \( h \geq 1 \), then \( R_w \) can be construct as in the theorem 6 \([1]\) in the form of two-sheeted covering of a ring on ambient surface \( \{w^2 = z\} \), and from the top sheet it is necessary to remove domains which are bounded by: \( 2(h - 1) \) by the closed curves, \( s \) curvilinear quadrangles and \( m \) curvilinear polygons with \( 4i_1, \ldots, 4i_m \) sides;

2) if \( h = 0, s \geq 2 \), then \( R_w \) can be construct as two-sheeted ring domains, which are bounded two curvilinear quadrangles, and further - as in 1);

3) case \( h = 0, s = 1, i_1 = \ldots = i_m = 0 \) does not meet in to kind that \( g \geq 2 \);

4) if \( h = 0, s = 0, 1 \), then exists \( k \) such that \( i_k \geq 2 \), and \( R_w \) can be construct as in the theorem 4 \([1]\), as two-sheeted covering on ambient surface \( \{w^2 = (z - c_1)(z - c_2), c_1, c_2 \in C, c_1 \neq c_2 \} \), and further - as in 1).

Remark 1.3. The corollary 1.3 and remark 1.2 show, that, as well as in classical problem of a choise of the accessory parameters for uniformization by Fuchsian groups, problem of a choise of accessory parameters \([1]\) for any standard uniformization compact Riemann surface by Koebe groups has unique solution, up to linear fractionally transformation, if linearly polymorphic function \( w \) has the limited image of a disc, i.e. \( w(U) \neq \overline{C} \).

Remark 1.4. The corollary 1.3 include, in particular, the theorem 3 and 5 by D.A. Hejhal \([1]\) for the Fuchsian group and the Schottky group respectively. Under the remark 1.2 we receive examples of monodromy groups, which are algebraically arranged as the marked Koebe group, but they are act non discontinuously on \( w(U) \), though \( \mathcal{M}[w] \) is group of conformal homeomorphisms many-sheeted Riemannian domain \( w(U) \) on itself.

2. The monodromy mapping
In this section we are investigated in the monodromy mapping \( p : T_gQ \rightarrow \mathcal{M} \), where \( T_gQ \) is a vector bundle of holomorphic quadratic abelian differentials over the Teichmüller space of compact Riemann surfaces of genus \( g \), and \( \mathcal{M} \) is a space of monodromy groups for of genus \( g \). D.A. Hejhal in [1] has shown that mapping \( p \) is local homeomorphism, but has not by the lifting of path property over \( \mathcal{M} \). But over a part \( \mathcal{M}_q \), consisting of quasifuchsian uniformizations, it already has this property. Naturally it is interesting to find parts of space \( \mathcal{M} \), admitting this property. In this section is proved that over any space, which consist of quasiconformal deformations by Koebe group of signature \( \sigma = (h, s; i_1, ..., i_m) \), connected with standard uniformization compact Riemann surface of genus \( g = |\sigma| \), this mapping \( p \) has the lifting of path property.

Two ordered collections \( X_1, ..., X_g, Y_1, ..., Y_g \) and \( X'_1, ..., X'_g, Y'_1, ..., Y'_g \) are \( PSL(2, \mathbb{C}) \)–equivalent, if exists \( B \in PSL(2, \mathbb{C}) \) such that \( X'_k = BX_kB^{-1}, Y'_k = BY_kB^{-1} \) for all \( k = 1, ..., g \). Let \( \widetilde{\mathcal{M}} \) be the set of marked monodromy groups \( \mathcal{M}[z] \) for all \( \tau \in T_g \).

Set \( \mathcal{M} = \mathcal{M} \text{mod} PSL(2, \mathbb{C}) \). This space we will named as space of the marked monodromy groups the fixed of genus \( g \). In \( \mathcal{M} \) and \( \mathcal{M} \) can be defined the topology of convergence on generators and the factor-topology respectively. R. Gunning [8] and D.A. Hejhal [1] have proved that \( \mathcal{M} \subset \mathcal{N} \subset [PSL(2, \mathbb{C})]^g \text{mod} PSL(2, \mathbb{C}) \), where \( \mathcal{N} \) be complex-analytic manifold of complex dimension \( 6g - 6 \) and \( \mathcal{N} \) Hausdorff locally metrizable space and \( \mathcal{M} \) be subdomain in \( \mathcal{N} \).

For \( \tau \in T_g \) we will denote through \( Q(\tau) \) the complex vector space of holomorphic quadratic differentials \( q = q(t, \tau)dt^2 \) on \( U/\Gamma_{\tau} \). The solution \( z = z(t) \) of the Schwartz equation \( \{z, t\} = q(t, \tau) \) is determined up to transformation from \( PSL(2, \mathbb{C}) \). Such a way, the monodromy mapping is well defined by

\[
p(\tau, q(t, \tau)dt^2) = \mathcal{M}[z] \text{mod} PSL(2, \mathbb{C}).
\]

It is well known that: 1) \( dim_{\mathbb{C}}Q(\tau) = 3g - 3 \), 2) it is possible to enter basis \( q_k(t, \tau)dt^2, k = 1, ..., 3g-3 \), in \( Q(\tau) \), which is globally complex-analytic depending from \( \tau \) on \( T_g \) [18]. Let \( T_gQ \) be holomorphic vector bundle of holomorphic quadratic differentials over complex-analytic manifold \( T_g \). Hence, the monodromy mapping is determined

\[
p : T_gQ \rightarrow \mathcal{M}.
\]

It is easy to see that \( dim_{\mathbb{C}}T_gQ = 6g - 6 \). Indeed by the theorem of H. Grauert [19], by virtue of simply connectivity of \( T_g \), the bundle is analytically equivalent to the trivial vector bundle of rank \( 3g - 3 \) over \( T_g \).

A Beltrami differential with respect to a Kleinian group \( G \) is the form \( \mu = \mu(z)dz/d\bar{z} \), where

1) \( \mu(z) \in L_{\infty}(\mathbb{C}) \),
2) \( \mu(Az)A'(z)/A'(z) = \mu(z), A \in G \),
3) \( \mu|\Lambda(G) = 0 \), where \( \Lambda(G) \) be limit set of group \( G \). We will denote through \( M(\Delta, G) = \{ \mu : \text{supp} \mu \subset \Delta \} \) a complex Banach space with the norm \( ||\mu||_\infty \) and \( M_0(\Delta, G) = \{ \mu \in M(\Delta, G) : ||\mu||_\infty < 1 \} \). Set that \( 0, 1, \infty \in \Lambda(G) \). We will consider a quasiconformal automorphisms \( w = f^\mu(z) \) on plane, where \( \mu \in M_0(\Omega(G), G) \), i.e. quasiconformal deformations of group \( G \). Every automorphisms \( f^\mu \) generate a Kleinian group \( G_\mu = f^\mu G(f^\mu)^{-1} \) and isomorphism \( \chi_\mu : G \rightarrow G_\mu \). Such two automorphisms \( f^{\mu_1} \) and \( f^{\mu_2} \), and also Beltrami differentials \( \mu_1 \) and \( \mu_2 \) appropriate to them from \( M_0(\Omega(G), G) \), we will name (strongly) quasiconformal equivalent, if

1) they are homotopic on each surface \( F_j \subset \Omega(G)/G \),
2) \( f^{\mu_1} = f^{\mu_2} \) on \( \Lambda(G) \) [20].
It is clear that under these conditions $G_{\mu_1} = G_{\mu_2}$ and $\chi_{\mu_1} = \chi_{\mu_2}$. Set of classes of quasiconformal equivalent Beltrami differentials $\mu \in M_0(\Omega(G), G)$ is called the space $\hat{T}(\Omega(G), G)$ of quasiconformal deformations of group $G$. Let $\Delta$ be an $G$-invariant union connected components from $\Omega(G)$. We will name $\mu_1$ and $\mu_2$ from $M_0(\Delta, G)$ to be of weakly quasiconformal equivalent, if $f^{\mu_1} = f^{\mu_2}$ on $\Lambda(G)$ (after suitable linear fractionally normalization). Factor-space $T(\Delta, G)$ space $M_0(\Delta, G)$ under this relation of equivalence we will name the space of weak quasiconformal deformations of group $G$ with supports in $\Delta$. Note that if all components from $\Delta$ will be simply connected, then $T(\Delta, G) = \hat{T}(\Delta, G)$ [20].

Denote through $T(G_\sigma) \equiv T(\Omega(G_\sigma), G_\sigma)$ the space of weak quasiconformal deformations of marked Koebe group $G_\sigma$ of signature $\sigma$, which give standard uniformization in invariant component $\Delta_\sigma$ of marked compact Riemann surface $F_\sigma$ of genus $g \geq 2$, where $|\sigma| = g$.

**Definition 2.1.** Appropriate to pair $(\tau, q) \in T_gQ$ the solution $z : U \to \overline{C}$ of the Schwartz equation $\{z, t\} = q(t, \tau)$ is defined normalized quasiconformal deformation $f$, $[\mu_f] \in T(G_\sigma)$, if exists $A \in PSL(2, \mathbb{C})$ such that:

1) $z(F_\tau) = Af(F_{\Delta_\sigma})$, where $F_\tau$ and $F_{\Delta_\sigma}$ be standard fundamental domains for uniformizations for $\tau$ and $F_\sigma$ by Fuchsian group $\Gamma_\tau$ in $U$ and by $F_\sigma$ in $\Delta_\sigma$ respectively,

2) $\mathcal{M}[z] = AfG_\sigma f^{-1}A^{-1}$ (equality the marked groups).

This definition is correct as if exists $f_1, f_2$ appropriate to pair $(\tau, q)$ and $z$, then from 2) follows $A_1f_1 \equiv A_2f_2$ on $\Lambda(G_\sigma)$ for some $A_1, A_2 \in PSL(2, \mathbb{C})$, i.e. $\mu_{f_1} \in [\mu_{f_2}]$ in $T(G_\sigma)$.

We will identify $T(G_\sigma)$ with his image in $\mathcal{M}$ on mapping

$$T(G_\sigma) \ni [\mu] \to (\chi_\mu(A_1'), \chi_\mu(B_1'), \ldots, \chi_\mu(A_g'), \chi_\mu(B_g')) \in \mathcal{M},$$

where $G_\sigma = \{A_1', B_1', \ldots, A_g', B_g'\}$ be fixed marked Koebe group $G_\sigma$ of signature $\sigma = (h, s; i_1, \ldots, i_m)$, $g = |\sigma|$, and its has $h$ generators equal 1, for example, $A_1' = \ldots = A_h' = 1$. Without restriction of a generality we assume that $0, 1, \infty \in \Lambda(G_\sigma)$. Here $\chi_\mu(A) = w^\mu A(w^{-1})^{-1}$, $A \in G_\sigma$, $\mu$ is the Beltrami differential for $G_\sigma$ on $\Omega(G_\sigma)$ and $w^\mu$ is a quasiconformal automorphism on $\overline{C}$, $w^\mu(0) = 0$, $w^\mu(1) = 1$, $w^\mu(\infty) = \infty$, being the solution of the Schwartz equation with coefficient $\mu$ [20].

We note that for the $\sigma = (h, s; i_1, \ldots, i_m)$ the space $T(G_\sigma)$ is complex-analytic submanifold of complex dimension $3g - 3 + 3(i_1 + \ldots + i_m - m)$ in complex-analytic manifold $\mathcal{M}$ of complex dimension $6g - 6$ [20; 15].

Remind the following classical results

**Theorem** (H. Poincare [12]). If $z = z(t)$ and $w = w(t)$ are locally schlicht linearly polymorphic function on the same marked compact Riemann surface $\tau \in T_gQ$, and $\mathcal{M}[z] = \mathcal{M}[w]$, then $z(t) = w(t)$ on $U$.

The main results of this section is

**Theorem 2.1.** The monodromy mapping $p : T_gQ \to \mathcal{M}$ has the lifting of path property over any space $T(\Omega(G_\sigma), G_\sigma) \subset \mathcal{M}$, where the marked Koebe group $G_\sigma$ standardly uniformize a marked compact Riemann surface of genus $g \geq 2$, in the invariant component $\Delta_\sigma$, $\sigma = (h, s; i_1, \ldots, i_m) \neq (0, 2; 0, \ldots, 0)$, $i_j \neq 1, j = 1, \ldots, m$ and $|\sigma| = g$.

**Proof.** Let a signature $\sigma \neq (0, 0; 0)$, i.e. $G_\sigma$ is not a Fuchsian group, since this case is considered in [1]. Denote by $B_\sigma$ the set of pairs $(\tau, q(t, \tau)dt^2) \in T_gQ$ such that any solution of the Schwartz equation $\{z, t\} = q(t, \tau)$ is defined by normalized quasiconformal deformation of group $G_\sigma$. By the theorem 1[1] the mapping $p : B_\sigma \to p(B_\sigma) \equiv T(G_\sigma)$ is continuous and is a local homeomorphism. But mapping $p : B_\sigma \to T(G_\sigma)$ is not one-to-one for $h \neq 0$. Indeed, let $(\tau_1, q_1)$ and $(\tau_2, q_2)$ be distinct elements from $B_\sigma$ such that $\mathcal{M}[z_1] = \mathcal{M}[z_2]$ (equality marked monodromy groups). Hence $z_1(U) \neq \overline{C}$ and $z_2(U) \neq \overline{C}$. By the Maskit’s theorem [21] there exists a conformal similarity of these groups on the marked Koebe group.
\( \tilde{G}_\sigma \) same signature \( \sigma \). Hence, under the theorem 3 [22] we have \( \tau_1 = [F, \{a_k, b_k\}_{k=1}^g] \) and \( \tau_2 = [F, \{\psi(a_k), \psi(b_k)\}_{k=1}^g] \), where \( F \) - compact Riemann surface of genus \( g \), \( \psi(O) = O \) and \( \psi \) belongs to not trivial group \( \Theta_\sigma \) (see [22]) of homeomorphisms surface \( F \) on itself. If \( \tau_1 = \tau_2 \) in \( T_\sigma \), then under the theorem Poincare we will receive \( q_1 = q_2 \). From here \( \tau_1 \neq \tau_2 \) in \( T_\sigma \), i.e. \( \psi \) is not homotopically identical mapping on \( F \) and distinct points \( (\tau_1, q_1), (\tau_2, q_2) \) of \( B_\sigma \) under mapping \( p \) pass in the same point \( M[z_1] = M[z_2] \) in \( M \).

By the Poincare theorem the mapping \( p \) on fiber \( B_\sigma(\tau_0) \) of set \( B_\sigma \) is homeomorphism for any \( \tau_0 \in T_\sigma \). It is possible to identify the fiber \( B_\sigma(\tau_0) \) with space \( T(\overline{C} \setminus (G_{0,\sigma}), G_{0,\sigma}) \) by a weak quasiconformal deformations of marked Koebe group \( G_{0,\sigma} \), with supports on \( \overline{C} \setminus (G_{0,\sigma}) \), where group \( G_{0,\sigma} \) is of signature \( \sigma \) and standardly uniformize \( \tau_0 \) in invariant component \( \Delta(G_{0,\sigma}) \). Since, all non invariant component of \( G_{0,\sigma} \) are simply connected, we have

\[
T(\overline{C} \setminus (G_{0,\sigma}), G_{0,\sigma}) = \hat{T}(\overline{C} \setminus (G_{0,\sigma}), G_{0,\sigma}),
\]

where \( \hat{T}(\overline{C} \setminus (G_{0,\sigma}), G_{0,\sigma}) \) is the space of quasicontinormal deformations of group \( G_{0,\sigma} \) with supports in \( \overline{C} \setminus (G_{0,\sigma}) \) [20]. Therefore \( T(\overline{C} \setminus (G_{0,\sigma}), G_{0,\sigma}) \), as well \( B_\sigma(\tau_0) \), is simply connected. Since fiber \( B_\sigma(\tau_0) \) are contractible, \( B_\sigma \) is also contractible and simply connected[1].

Now we show that any continuous path \( \gamma = \{M(\xi) : 0 \leq \xi \leq 1\} \) in \( T(\sigma) \) is lifted to continuous path \( \tilde{\gamma} \) in \( T_\sigma Q \) from any point laying over \( M(0) \). Let \( (\tau_0, q_0) \in B_\sigma \) lays over \( M(0) = M[z_0] \), where \( z_0, t_0 = q_0(t, \tau_0) \). Denote by \( \Gamma_{\tau_0} \) a Fuchshian group, which uniformize \( \tau_0 \) with fundamental polygon \( F_{\tau_0} \) in disc \( U \). We will consider a compact Riemann surface \( z_0(U)/M(0) \) of genus \( g \) with marking type \( (h, g - h) \), induced by image \( z_0(\partial F_{\tau_0}) \). In case \( h \neq 0 \) this making we will add to homotopical basis, i.e. \( z_0(U)/M(0) = [F_0, \{a_k, b_k\}_{k=1}^g] \), \( a_k \cap b_k = O \in F_0 \). By the theorem 3 [22] we obtain that \( \tau_0 = [F_0, \{\tilde{\psi}(a_k), \tilde{\psi}(b_k)\}_{k=1}^g] \), where \( \tilde{\psi} \) is some homeomorphism \( F_0 \) on itself, \( \psi(0) = O \).

For continuous family of groups \( M(\xi) = \{A_{1,\xi}, ..., B_{g,\xi}\} \) there exists a continuous family of quasicontinormal mappings \( f_\xi \) of plane \( C \) onto itself such that \( M(\xi) = f_\xi M(0)f_\xi^{-1} \), \( \xi \in [0, 1] \). Hence, there is a continuous family of quasicontinormal homeomorphisms \( \tilde{f}_\xi \) from surface \( \tau_0 \) on \( \tau_\xi = \{\tilde{f}_\xi(F_0), \{\psi_k \tilde{f}_\xi(a_k), \psi_k \tilde{f}_\xi(b_k)\}_{k=1}^g\} \), \( \psi_\xi = \tilde{f}_\xi \psi_0(\tilde{f}_\xi)^{-1} \) is homeomorphism \( \tilde{f}_\xi(F_0) \) onto itself and \( \tilde{f}_\xi \) map \( z_0(F_{\tau_0})/M(0) \) on \( f_\xi(z_0(F_{\tau_0}))/M(\xi) \). Let \( \tau_\xi = [F_\xi, \tilde{f}_\xi] \), where \( [F_0, \tilde{f}_0] = [F_0, \{\psi_0(a_k), \psi_0(b_k)\}_{k=1}^g] = \tau_0 \) and \( \tilde{f}_0 = 1 \) (identical mapping of \( F_0 \)). Notice that automorphism of group \( \pi_1(\tilde{f}_\xi(F_0)) \) with generators \( \{\tilde{f}_\xi(a_k), \tilde{f}_\xi(b_k)\}_{k=1}^g \), induced by \( \psi_\xi \), coincides with automorphism of group \( \pi_1(F_0) \) with generators \( \{a_k, b_k\}_{k=1}^g \), induced by \( \psi_0 \). Quasiconformal mapping \( \tilde{f}_\xi \) is lifted up to quasicontinormal mapping \( \varphi_\xi : U \to U \). So, as \( \tilde{f}_\xi \) is determined up to isotopy and \( M(\xi) \) is continuous on \( \xi \), the mapping \( \varphi_\xi(t) \) is continuous on \( U \times [0, 1] \), and appropriate normalized Fuchshian group \( \Gamma_\xi = \{A_{1,\xi}, ..., B_{g,\xi}\} \) has fundamental polygon \( F_{\tau_\xi} = \varphi_\xi(F_{\tau_0}) \), \( 0 \leq \xi \leq 1 \). Moreover \( \varphi_0 \) is identical mapping of disc \( U \). We received family of mappings

\[
z_\xi = f_\xi z_0 \varphi_\xi^{-1} : U \to z_\xi(U), 0 \leq \xi \leq 1,
\]

such that \( z_\xi(F_{\tau_\xi}) = f_\xi(z_0(F_{\tau_0})) \). Hence, we have a continuous family of fundamental membranes, sides of whose are paired identified by generators of the group \( M(\xi) \). Hence, \( \{z_\xi\} \) is a continuous family locally schlicht linearly polymorphic function on continuous family of marked compact Riemann surfaces \( [F_\xi, \{\psi_k \tilde{f}_\xi(a_k), \psi_k \tilde{f}_\xi(b_k)\}_{k=1}^g] \), \( \xi \in [0, 1] \), respectively. Moreover, \( f_\xi(z_0(F_{\tau_0})) \) defines a fundamental membrane for locally schlicht analytic linearly polymorphic function \( z_\xi \) on \( U \) with the monodromy group \( M(\xi) \) [1]. Thus, we have constructed continuous path \( \tilde{\gamma}_0 = \{(\tau_0, q_\xi = \{z_\xi, t\} : \xi \in [0, 1]\} from \( (\tau_0, q_0) \) in \( B_\sigma \) over \{M(\xi) : \xi \in [0, 1]\}. 

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Let now $\mathcal{M}[w] = \mathcal{M}(0)$ and $w = w(s)$ corresponds $([F_w, h_w], q_w) \in T_0 \mathcal{Q} \setminus B_0$ with Fuchsian collection

$$\{F_w, F_w, \Gamma_w = \{A_{k,w}, B_{k,w}\}_{k=1}^g\},$$

where $\Gamma_w$ be Fuchsian group with fundamental polygon $F_w$, which uniformize $F_w$ in disc $U$. By the theorem of Kra-Gunning we have $w(U) = \overline{T}$, i.e. $w$ is not uniformization for $[F_w, h_w]$. By the theorem 2 [1] it is possible to achieve that appropriate vertices of membranes $w(F_w)$ and $z_0(F_0)$ would have identical coordinates. A fundamental membrane $f_\xi(w(F_w))$ defines at $\xi \in [0, 1]$ (not analytic) locally schlicht linearly polymorphic function $f_\xi w$ on $U$ such that $f_\xi w(A_{k,w}) = \tilde{A}_{k,\xi}f_\xi w$, $f_\xi w(B_{k,w}) = \tilde{B}_{k,\xi}f_\xi w$, $k = 1, ..., g$. Continuously deforming the complex-analytic structure on these membranes with the help quasiconformal mapping [1], [6, c.348], we receive that $f_\xi w(F_w)$ defines a fundamental membrane for analytic locally schlicht linearly polymorphic function on $U$ with the monodromy group $\mathcal{M}(\xi)$ for every $\xi \in [0, 1]$.

Since $\gamma$ is continuously of $\xi$, by the theorem 1[1], using found the fundamental membranes for linearly polymorphic function on $U$, we received a continuous path $\tilde{\gamma}_0$ from $(\tau_0, q_0) \in B_0$ and $\tilde{\gamma}_w$ from $([F_w, h_w], q_w) \in T_0 \mathcal{Q} \setminus B_0$. Theorem is proved.

Remark 2.1. By the theorem 1[1] and the theorem 2.1 it follows that:

1) mapping $p$ defined over space $T(G_\sigma) = T(\Omega(G_\sigma), G_\sigma)$, as in the theorem, is a topological covering;

2) since space $B_\sigma$ is simply connected it is universal covering space for $T(G_\sigma)$ and $p : B_\sigma \to T(G_\sigma)$ is the universal covering mapping.

3. The exact variational formula for monodromy group of the linear differential equation of the second order and for the solution of the nonlinear Schwartz equation on compact Riemann surface.

In work [2] D.A. Hejhal have started the research of monodromy group for linearly polymorphic function on compact Riemann surface with the help of variational methods. He has found the first variation for monodromy group. Then C. Earle [4] has deduced the formula the first variation with the help quasiconformal mapping [1], [6, c.348]. In this section we will received an exact variational formula for monodromy group of the linear differential equation of the second order and the first variation for solution of the Schwartz equation on compact Riemann surface.

Let $F$ be a compact Riemann surface of genus $g, g \geq 2$; $\pi_1(F, O)$ is the fundamental group for $F$ with basic point $O$, and $\Gamma$ is the group of covering transformations for universal covering $(U, \pi)$ over $F$. Here $U = \{t \in C : |t| < 1\}$, $\pi : U \to F$ is natural projection and $U/\Gamma = F$. Fix a point $t_0 \in U$, laying over $O$, and we will construct natural isomorphism of group $\pi_1(F, O)$ on Fuchsian group $\Gamma$ of first kind.

A multivalued locally meromorphic function $z$ on $F$, which transforms linear fractionally under action group $\pi_1(F, O)$, is called linearly polymorphic function on $F$. Lifted it on $(U, \pi)$, we received meromorphic single-valued function $z = z(t)$ on $U$, which satisfies the condition

$$z(\tilde{L} t) = \tilde{L} z(t), \tilde{L} \in PSL(2, C),$$

for $L \in \Gamma, t \in U$. Mapping $L \mapsto \tilde{L}$ gives a homomorphism of group $\Gamma$ in group $PSL(2, C)$.

Group, which consist of mappings $\tilde{L}$, when $L$ runs $\Gamma$, is called by monodromy group for function $z = z(t)$. Function

$$2q(t) = \{z, t\} = \left(z''/z'\right)' - \frac{1}{2}\left(z''/z'\right)^2$$
satisfies the relation
\[ q(t) = q(Lt)L'(t)^2 \]
for \( L \in \Gamma, t \in U \). Hence, \( q(t) \) defines the quadratic differential on \( F = U/\Gamma \).

A locally schlicht linearly polymorphic function \( z = z(t) \) satisfies on \( U \) of the Schwartz equation
\[ \{z, t\} = 2q(t), \quad (2) \]
where \( q(t) \) be holomorphic function on \( U \). Putting
\[ z(t) = v(t)/u(t), v(t) = z(t)/\sqrt{z'(t)}, u(t) = 1/\sqrt{z'(t)}, \]
we receive that \( v = v(t), u = u(t) \) satisfy of the linear differential equation of the second order
\[ u''(t) + q(t)u(t) = 0 \quad (4) \]
on \( F = U/\Gamma \). We denote by \( Q(F) \) the vector space holomorphic quadratic (abelian) differentials on \( F = U/\Gamma \). For equation \((2)\) we will consider only the normalized solutions \( z = z(t) \) such that
\[ z(t) = (t - t_0) + O((t - t_0)^3), \sqrt{z'(t)} = 1 + O((t - t_0)^2), t \to t_0. \quad (5) \]
We obtain that \( v = v(t) \) and \( u = u(t) \) are two linearly independent the solution of the equation \((4)\) with conditions
\[ u(t_0) = 1, u'(t_0) = 0, v(t_0) = 0, v'(t_0) = 1, \quad (6) \]
for any \( q = q(t)dt^2 \in Q(F) \). It is well known that the conditions \((5)\) and \((6)\) define the unique solutions of the equation \((2)\) and \((4)\) respectively.

In work \([2]\) was found the following relation
\[ \begin{pmatrix} v(Lt) \\ u(Lt) \end{pmatrix} = \xi_L(t) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v(t) \\ u(t) \end{pmatrix}, \quad (7) \]
where
\[ z(Lt) = \tilde{L}z(t) = (\alpha z(t) + \beta)(\gamma z(t) + \delta)^{-1}, \xi_L(t) = \sqrt{L'(t)}, \]
and for any \( L, K \in \Gamma \) valid a relations \( \xi_{LK}(t) = \xi_L(Kt)\xi_K(t) \). There is a special choise of a sign at \( \xi_L(t) \) and at a matrix \( \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \) (see also \([11, \ 163]\)).

Choose arbitrary \( r = r(t)dt^2, q = q(t)dt^2 \) from \( Q(F) \). Consider normalized by \((5)\) a solutions \( z(t, h) \) of the Schwartz equation
\[ \{z, t\} = 2[r(t) + hq(t)] \quad (8) \]
and normalized by \((6)\) a solutions \( v(t, h) \) and \( u(t, h) \) of the linear equation
\[ u''(t) + [r(t) + hq(t)]u(t) = 0, \quad (9) \]
where \( h \in \mathbb{C}, |h| < \varepsilon, \varepsilon \) is sufficiently small a positive number. Here \( z(t, h) = v(t, h)/u(t, h) \). Applying the Poincare theorem about small parameter \([23]\), the Caushy-Kovalevski theorem, we receive Hartog’s series \([24]\)
\[ u(t, h) = u(t) + u_1(t)h + u_2(t)h^2 + \ldots + u_n(t)h^n + \ldots, \]
\[ v(t, h) = v(t) + v_1(t)h + v_2(t)h^2 + \ldots, \quad (10) \]
which are uniformly converged on any compact in \( U \times \{h : |h| < \varepsilon\} \), where \( u(t), v(t), u_i(t), v_i(t) \) are holomorphic functions on \( U \). From normalization \((6)\) it follows that
\[ u(t_0) = 1, u'(t_0) = 0, v(t_0) = 0, v'(t_0) = 1, \]
\[ u_i(t_0) = v_i(t_0) = u'_i(t_0) = v'_i(t_0) = 0, i \geq 1. \quad (11) \]
Substituting the series (10) in (9), we receive a infinite system of pairs of equations

\[
\begin{align*}
\{ & u''(t) + r(t)u(t) = 0 \\
& v''(t) + r(t)v(t) = 0 \\
& u''(t) + r(t)u_1(t) = -q(t)u(t) \\
& v''(t) + r(t)v_1(t) = -q(t)v(t) \\
& \vdots \\
& u''(t) + r(t)u_{n-1}(t) = -q(t)u_{n-1}(t) \\
& v''(t) + r(t)v_{n-1}(t) = -q(t)v_{n-1}(t)
\end{align*}
\]

Solving the Cauchy problem with zero initial conditions in point \( t_0 \) for system with pair \((u_1(t), v_1(t))\) on a method of an elementary variation of parameters, we find that

\[
\begin{pmatrix} v_1(t) \\ u_1(t) \end{pmatrix} = \int_{t_0}^{t} \begin{pmatrix} -qu & quv \\ -quu & quv \end{pmatrix} ds \begin{pmatrix} v(t) \\ u(t) \end{pmatrix},
\]

where \( u = u(t), v = v(t) \) are the solutions the Cauchy problem with initial conditions \( u(t_0) = v'(t_0) = 1, u'(t_0) = v(t_0) = 0 \) for first pairs of equations. By induction for any \( n, n \geq 1 \), we will receive a relations

\[
\begin{pmatrix} v_n(t) \\ u_n(t) \end{pmatrix} = \int_{t_0}^{t} \begin{pmatrix} -quv_{n-1} & qvv_{n-1} \\ -quu_{n-1} & quv_{n-1} \end{pmatrix} ds \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} = \equiv A_{n-1}(t) \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} = \int_{t_0}^{t} A_{n-2}(s)A(s)ds \begin{pmatrix} v(t) \\ u(t) \end{pmatrix},
\]

where

\[
A(s) = q(s) \begin{pmatrix} -u(s)v(s) & v^2(s) \\ -u^2(s) & u(s)v(s) \end{pmatrix},
\]

\( A_0(t) = \int_{t_0}^{t} A(s)ds \).

It is easy to see that formula (13) gives two solutions of the Cauchy problem with zero initial conditions in point \( t_0 \) for system with pair \((u_n(t), v_n(t))\).

Hence we receive the exact variational formula for solutions of the linear equation (9)

\[
\begin{pmatrix} v(t, h) \\ u(t, h) \end{pmatrix} = \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} + h \begin{pmatrix} v_1(t) \\ u_1(t) \end{pmatrix} + h^2 \begin{pmatrix} v_2(t) \\ u_2(t) \end{pmatrix} + \cdots + h^n \begin{pmatrix} v_n(t) \\ u_n(t) \end{pmatrix} + \cdots
\]

\[
= \left[ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] + h \int_{t_0}^{t} A(s)ds + h^2 \int_{t_0}^{t} A_0(s)A(s)ds + \cdots + h^n \int_{t_0}^{t} A_{n-2}(s)A(s)ds + \cdots.
\]

The expression in square brackets is called the matrizont of \( hA(s) \).

For finding of the exact variational formula of the monodromy group for function \( z(t, h) \) it is necessary deduce some relations. By analogy with (7) we write for any \( h, |h| < \varepsilon \),

\[
z(Lt, h) = \tilde{L}_h z(t, h) = (\alpha_L(h)z(t, h) + \beta_L(h))(\gamma_L(h)z(t, h) + \delta_L(h))^{-1},
\]

\[
\begin{pmatrix} v(Lt, h) \\ u(Lt, h) \end{pmatrix} = \xi_L(t) \begin{pmatrix} \alpha_L(h) & \beta_L(h) \\ \gamma_L(h) & \delta_L(h) \end{pmatrix} \begin{pmatrix} v(t, h) \\ u(t, h) \end{pmatrix}.
\]

By elementary transformations, using (7), we obtain the following relations

\[
\int_{L_{t_0}}^{Lt} A(x) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} dx = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \int_{t_0}^{t} A(s)ds,
\]

\[
\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} \alpha_L(0) & \beta_L(0) \\ \gamma_L(0) & \delta_L(0) \end{pmatrix}.
\]
From the formulas (12), (7), (15) we have the equality

\[
\begin{pmatrix}
v_1(Lt) \\ u_1(Lt)
\end{pmatrix} = \int_{t_0}^{Lt} A(s)ds \cdot \begin{pmatrix}
v(Lt) \\ u(Lt)
\end{pmatrix} = \int_{t_0}^{Lt} A(s)ds \cdot \xi_L(t) \begin{pmatrix}
\alpha & \beta \\ \gamma & \delta
\end{pmatrix} \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} = \\
= \xi_L(t) \int_{t_0}^{Lt} A(s) \begin{pmatrix}
\alpha & \beta \\ \gamma & \delta
\end{pmatrix} ds \cdot \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} + \\
+ \xi_L(t) \begin{pmatrix}
\alpha & \beta \\ \gamma & \delta
\end{pmatrix} \int_{t_0}^{t} A(s)ds \cdot \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} = \\
= \xi_L(t) \int_{t_0}^{Lt} A(s)ds \cdot \begin{pmatrix}
\alpha & \beta \\ \gamma & \delta
\end{pmatrix} \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} + \xi_L(t) \begin{pmatrix}
\alpha & \beta \\ \gamma & \delta
\end{pmatrix} \begin{pmatrix} v_1(t) \\ u_1(t) \end{pmatrix}.
\] (16)

Next, the equality (13), (7), (15) give us

\[
\begin{pmatrix}
v_2(Lt) \\ u_2(Lt)
\end{pmatrix} = \xi_L(t) \int_{t_0}^{Lt} (\int_{t_0}^{x} A(s_1)ds_1)A(x) \begin{pmatrix}
\alpha & \beta \\ \gamma & \delta
\end{pmatrix} dx \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} = \\
= \xi_L(t)A_1(Lt_0) \begin{pmatrix}
\alpha & \beta \\ \gamma & \delta
\end{pmatrix} \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} + \xi_L(t)A_0(Lt_0) \begin{pmatrix}
\alpha & \beta \\ \gamma & \delta
\end{pmatrix} \begin{pmatrix} v_1(t) \\ u_1(t) \end{pmatrix} +
\] (17)

since

\[
\int_{Lt_0}^{Lt} (\int_{t_0}^{x} A(s_1)ds_1)A(x) \begin{pmatrix}
\alpha & \beta \\ \gamma & \delta
\end{pmatrix} dx = \\
= \int_{t_0}^{t} \left( \int_{t_0}^{L(s)} A(s_1)ds_1 \right)A(Ls) \begin{pmatrix}
\alpha & \beta \\ \gamma & \delta
\end{pmatrix} dL(s) = \\
= \int_{t_0}^{t} \left( \int_{t_0}^{L(s)} A(s_1)ds_1 \right) \begin{pmatrix}
\alpha & \beta \\ \gamma & \delta
\end{pmatrix} A(s)ds = \\
= \left( \int_{t_0}^{Lt_0} A(s_1)ds_1 \right) \begin{pmatrix}
\alpha & \beta \\ \gamma & \delta
\end{pmatrix} \int_{t_0}^{t} A(s)ds + \\
+ \int_{t_0}^{t} \int_{t_0}^{s} A(Lx_1) \begin{pmatrix}
\alpha & \beta \\ \gamma & \delta
\end{pmatrix} dL(x_1)A(s)ds = \\
= A_0(Lt_0) \begin{pmatrix}
\alpha & \beta \\ \gamma & \delta
\end{pmatrix} A_0(t) + \begin{pmatrix}
\alpha & \beta \\ \gamma & \delta
\end{pmatrix} A_1(t).
\]

By induction on $n, n \geq 2$, we find the relations

\[
\begin{pmatrix}
v_n(Lt) \\ u_n(Lt)
\end{pmatrix} = \int_{t_0}^{Lt} q(s) \begin{pmatrix}
v_{n-1}(s) \\ u_{n-1}(s)
\end{pmatrix} \begin{pmatrix}
-u(s) & v(s) \\ 0 & 0
\end{pmatrix} ds \begin{pmatrix} v(Lt) \\ u(Lt) \end{pmatrix} = \\
= \xi_L(t)A_{n-1}(Lt_0) \begin{pmatrix}
\alpha & \beta \\ \gamma & \delta
\end{pmatrix} \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} +
\] + $\xi_L(t) \int_{Lt_0}^{Lt} q(x) \begin{pmatrix}
v_{n-1}(x) \\ u_{n-1}(x)
\end{pmatrix} \begin{pmatrix}
-u(x) & v(x) \\ 0 & 0
\end{pmatrix} \begin{pmatrix}
\alpha & \beta \\ \gamma & \delta
\end{pmatrix} dx \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} = \\
= [\xi_L(t)A_{n-1}(Lt_0) \begin{pmatrix}
\alpha & \beta \\ \gamma & \delta
\end{pmatrix} + 
\]
\[ + \xi_L(t) \int_0^t \frac{q(s)\xi_L(s)}{L'(s)} \left(\begin{array}{c} v_{n-1}(Ls) \\ u_{n-1}(Ls) \end{array}\right) \left(\begin{array}{c} -u(s) \\ v(s) \end{array}\right) ds \left(\begin{array}{c} v(t) \\ u(t) \end{array}\right) = \]

\[ = \xi_L(t)A_{n-1}(Lt_0) \left(\begin{array}{c} \alpha \\ \beta \\ \gamma \\ \delta \end{array}\right) \left(\begin{array}{c} v(t) \\ u(t) \end{array}\right) + \]

\[ + \xi_L(t) \int_0^t \frac{q(s)\xi_L^2(s)}{L'(s)} [A_{n-2}(Lt_0) \left(\begin{array}{c} \alpha \\ \beta \\ \gamma \\ \delta \end{array}\right) \left(\begin{array}{c} v(s) \\ u(s) \end{array}\right) + \]

\[ + \sum_{j=1}^{n-2} A_{n-2-j}(Lt_0) \left(\begin{array}{c} \alpha \\ \beta \\ \gamma \\ \delta \end{array}\right) \left(\begin{array}{c} v_j(s) \\ u_j(s) \end{array}\right) \]

\[ + \left(\begin{array}{c} \alpha \\ \beta \\ \gamma \\ \delta \end{array}\right) \left(\begin{array}{c} v_{n-1}(s) \\ u_{n-1}(s) \end{array}\right) \right] \left(\begin{array}{c} -u(s) \\ v(s) \end{array}\right) ds \left(\begin{array}{c} v(t) \\ u(t) \end{array}\right) = \]

\[ = \xi_L(t)[A_{n-1}(Lt_0) \left(\begin{array}{c} \alpha \\ \beta \\ \gamma \\ \delta \end{array}\right) \left(\begin{array}{c} v(t) \\ u(t) \end{array}\right) + \]

\[ + \sum_{j=1}^{n-1} A_{n-1-j}(Lt_0) \left(\begin{array}{c} \alpha \\ \beta \\ \gamma \\ \delta \end{array}\right) \left(\begin{array}{c} v_j(t) \\ u_j(t) \end{array}\right) + \left(\begin{array}{c} \alpha \\ \beta \\ \gamma \\ \delta \end{array}\right) \left(\begin{array}{c} v_n(t) \\ u_n(t) \end{array}\right) \].

Using the relations (14) and (16) - (18), we receive

\[ \xi_L(t) \left(\begin{array}{c} \alpha_L(h) \\ \beta_L(h) \\ \gamma_L(h) \\ \delta_L(h) \end{array}\right) \left(\begin{array}{c} v(t, h) \\ u(t, h) \end{array}\right) = \left(\begin{array}{c} v(Lt) \\ u(Lt) \end{array}\right) + h \left(\begin{array}{c} v_1(Lt) \\ u_1(Lt) \end{array}\right) + h^2 \left(\begin{array}{c} v_2(Lt) \\ u_2(Lt) \end{array}\right) + \ldots + h^n \left(\begin{array}{c} v_n(Lt) \\ u_n(Lt) \end{array}\right) + \ldots = \]

\[ = \xi_L(t) \left(\begin{array}{c} \alpha \\ \beta \\ \gamma \\ \delta \end{array}\right) \left(\begin{array}{c} v(t) \\ u(t) \end{array}\right) + hA_0(Lt_0) \left(\begin{array}{c} \alpha \\ \beta \\ \gamma \\ \delta \end{array}\right) \left(\begin{array}{c} v(t) \\ u(t) \end{array}\right) + \]

\[ + h \left(\begin{array}{c} \alpha \\ \beta \\ \gamma \\ \delta \end{array}\right) \left(\begin{array}{c} v_1(t) \\ u_1(t) \end{array}\right) + h^2 A_1(Lt_0) \left(\begin{array}{c} \alpha \\ \beta \\ \gamma \\ \delta \end{array}\right) \left(\begin{array}{c} v(t) \\ u(t) \end{array}\right) + \]

\[ + h^2 A_0(Lt_0) \left(\begin{array}{c} \alpha \\ \beta \\ \gamma \\ \delta \end{array}\right) \left(\begin{array}{c} v_1(t) \\ u_1(t) \end{array}\right) + h^2 \left(\begin{array}{c} \alpha \\ \beta \\ \gamma \\ \delta \end{array}\right) \left(\begin{array}{c} v_2(t) \\ u_2(t) \end{array}\right) + \ldots + \]

\[ + h^n A_{n-1}(Lt_0) \left(\begin{array}{c} \alpha \\ \beta \\ \gamma \\ \delta \end{array}\right) \left(\begin{array}{c} v(t) \\ u(t) \end{array}\right) + h^n \sum_{j=1}^{n-1} A_{n-1-j}(Lt_0) \left(\begin{array}{c} \alpha \\ \beta \\ \gamma \\ \delta \end{array}\right) \left(\begin{array}{c} v_j(t) \\ u_j(t) \end{array}\right) + \]

\[ + h^n \left(\begin{array}{c} \alpha \\ \beta \\ \gamma \\ \delta \end{array}\right) \left(\begin{array}{c} v_n(t) \\ u_n(t) \end{array}\right) + \ldots \]

\[ = \xi_L(t) \left(\begin{array}{c} \alpha \\ \beta \\ \gamma \\ \delta \end{array}\right) \left(\begin{array}{c} v(t, h) \\ u(t, h) \end{array}\right) + hA_0(Lt_0) \left(\begin{array}{c} \alpha \\ \beta \\ \gamma \\ \delta \end{array}\right) \left(\begin{array}{c} v(t, h) \\ u(t, h) \end{array}\right) + \]

\[ + h^2 A_1(Lt_0) \left(\begin{array}{c} \alpha \\ \beta \\ \gamma \\ \delta \end{array}\right) \left(\begin{array}{c} v(t, h) \\ u(t, h) \end{array}\right) + \ldots + h^n A_{n-1}(Lt_0) \left(\begin{array}{c} \alpha \\ \beta \\ \gamma \\ \delta \end{array}\right) \left(\begin{array}{c} v(t, h) \\ u(t, h) \end{array}\right) + \ldots . \]

Hence, we have the exact variational formula for elements of monodromy group of function \( z(t, h) \)

\[ \left(\begin{array}{c} \alpha_L(h) \\ \beta_L(h) \\ \gamma_L(h) \\ \delta_L(h) \end{array}\right) = \]

\[ = \left[ \begin{array}{c} 1 \\ 0 \\ 0 \\ 1 \end{array} \right] + hA_0(Lt_0) + h^2 A_1(Lt_0) + \ldots + h^n A_{n-1}(Lt_0) + \ldots \right] \left(\begin{array}{c} \alpha_L(0) \\ \beta_L(0) \\ \gamma_L(0) \\ \delta_L(0) \end{array}\right) \equiv \left[ \Omega_{t_0}^{Lt_0} hA(s) ds \right] \left(\begin{array}{c} \alpha_L(0) \\ \beta_L(0) \\ \gamma_L(0) \\ \delta_L(0) \end{array}\right) . \]
\[ A_0(x) = \int_{t_0}^{x} A(s) ds, A(s) = q(s) \begin{pmatrix} -u(s)v(s) & v^2(s) \\ -u^2(s) & u(s)v(s) \end{pmatrix}, \]

\[ A_n(x) = \int_{t_0}^{x} A_{n-1}(s) A(s) ds, n \geq 1, u(s) = u(s,0), v(s) = v(s,0). \]

Now we will deduce the variational formula for the solution of the Schwartz equation (8). From the formula (10) we have

\[ z(t, h) = \frac{v(t) + v_1(t)h + v_2(t)h^2 + \ldots}{u(t) + u_1(t)h + u_2(t)h^2 + \ldots} = \]

\[ = \frac{v(t)}{u(t)} + h \frac{v_1(t)u(t) - v(t)u_1(t)}{u^2(t)} + o(h), h \to 0. \]

Using (12), we receive the equality

\[ \frac{v_1(t)u(t) - v(t)u_1(t)}{u^2(t)} = \]

\[ = \frac{1}{u^2(t)}[u(t)(v(t) \int_{t_0}^{t} (-quv) ds + u(t) \int_{t_0}^{t} qv^2 ds) - v(t)(v(t) \int_{t_0}^{t} (-qu^2) ds + \]

\[ + u(t) \int_{t_0}^{t} qvuds)] = \]

\[ = \frac{1}{u^2(t)}[u^2(t) \int_{t_0}^{t} qv^2 ds - 2u(t)v(t) \int_{t_0}^{t} quvds + v^2(t) \int_{t_0}^{t} qu^2 ds] = \]

\[ = \int_{t_0}^{t} q(s)[v(s) - z(t,0)u(s)]^2 ds, \]

where \( u = u(s,0), v = v(s,0). \) Hence, we have the variational formula

\[ z(t, h) = z(t, 0) + h \int_{t_0}^{t} q(s)[v(s) - z(t, 0)u(s)]^2 ds + o(h), h \to 0. \]

By applying the standard formulas for coefficients can be received any variational term for \( z(t, h). \)

Consider the Schwartz equation

\[ \{ z, t \} = 2[r(t) + \sum_{j=1}^{d} h_jq_j(t)] \tag{19} \]

and the linear equation

\[ u''(t) + [r(t) + \sum_{j=1}^{d} h_jq_j(t)]u(t) = 0 \tag{20} \]

on \( F = U/T, \) where \( q_1(t)dt^2, \ldots, q_d(t)dt^2 \) is a basis in space \( Q(F), \) \( h = (h_1, \ldots, h_d) \in C^d, \) \( d = 3g - 3. \) Again we will consider only normalized solutions \( z(t, h) \) and \( v(t, h), u(t, h), \) with conditions (5) and (6) respectively, for any \( h \) such that \( |h| = \max\{1 \leq j \leq d\} |h_j| < \varepsilon, \varepsilon \) is sufficiently small a positive number.

Now we deduce the variational formulas for the differential equations (19) and (20). By the Poincare theorem about small parameter [23], using decomposition for power series on
homogeneous polynomials in polydisk \( \{ t \in \mathbb{C} : |t| < \delta \} \times \{ h \in \mathbb{C}^{3g-3} : |h| < \varepsilon \} \) [24, p. 52] and the analytical continuation, we will receive the series

\[
u(t, h) = \sum_{|k|=0}^{\infty} u_{|k|:(k_1,\ldots,k_d)}(t)h_1^{k_1}\cdots h_d^{k_d} \equiv \sum_{|k|=0}^{\infty} u_{|k|}(t)h^k,
\]

\[
v(t, h) = \sum_{|k|=0}^{\infty} v_{|k|}(t)h^k = v(t) + \sum_{|k|=1}^{\infty} v_{1,k}(t)h^k + \sum_{|k|=2}^{\infty} v_{2,k}(t)h^k + \cdots
\]

which are uniformly converged on any compact in polydisk \( \{ t : |t| < 1 \} \times \{ h : |h| < \varepsilon \} \). Here \( k = (k_1,\ldots,k_d) \) is the vector with integer nonnegative coordinates, and \( |k| = k_1 + \cdots + k_d \).

From normalization

\[
u(t_0, h) = 1 = v'(t_0, h), u'(t_0, h) = 0 = v(t_0, h)
\]

for any \( h \), we receive

\[
u_{n,k}(t_0) = u'_{n,k}(t_0) = v_{n,k}(t_0) = v'_{n,k}(t_0) = 0, n \geq 1, |k| = n.
\]

Substituting the series (21) in the equation (20), we receive a infinite system of pairs of linear differential equations

\[
\begin{cases}
u''(t) + r(t)\nu(t) = 0
\vspace{0.5cm}
u''(t) + r(t)v(t) = 0,
\end{cases}
\]

\[
\begin{cases}
u''_{1,k}(t) + r(t)\nu_{1,k}(t) = -q_j(t)\nu(t)
\vspace{0.5cm}
u''_{1,k}(t) + r(t)v_{1,k}(t) = -q_j(t)v(t),
\end{cases}
\]

for \( k = e_j = (0,\ldots,0,1,0,\ldots,0) \), where 1 stands on the \( j \)-th place, \( j = 1,\ldots,d; \)

\[
\begin{cases}
u''_{n,k}(t) + r(t)\nu_{n,k}(t) = -\sum_{(j,k_j\neq 0)} q_j(t)\nu_{n-1,k-e_j}(t)
\vspace{0.5cm}
u''_{n,k}(t) + r(t)v_{n,k}(t) = -\sum_{(j,k_j\neq 0)} q_j(t)v_{n-1,k-e_j}(t),
\end{cases}
\]

for \( k = (k_1,\ldots,k_d), |k| = n; \)

... Hence, by method of elementary variation of parameters we find

\[
\begin{pmatrix} v_{1,k}(t) \\ u_{1,k}(t) \end{pmatrix} = \int_{t_0}^{t} \begin{pmatrix} -q_j uv & q_j vu \\ -q_j uu & q_j uv \end{pmatrix} ds \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} =
\]

\[
= \int_{t_0}^{t} M_j(s)ds \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} \equiv A_{0,k}(t) \begin{pmatrix} v(t) \\ u(t) \end{pmatrix},
\]

for \( k = e_j \) and \( M_j = q_j \begin{pmatrix} -uv & vv \\ -uu & uv \end{pmatrix}, j = 1,\ldots,d. \)

Then for \( k = (k_1,\ldots,k_d), |k| = 2, \) we have

\[
\begin{pmatrix} v_{2,k}(t) \\ u_{2,k}(t) \end{pmatrix} = \sum_{(j,k_j\neq 0)} \int_{t_0}^{t} \begin{pmatrix} -q_j uv_{1,k-e_j} & q_j vu_{1,k-e_j} \\ -q_j uu_{1,k-e_j} & q_j vu_{1,k-e_j} \end{pmatrix} ds \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} =
\]

\[
= \sum_{(j,k_j\neq 0)} \int_{t_0}^{t} A_{0,k-e_j}(s)M_j(s)ds \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} \equiv A_{1,k}(t) \begin{pmatrix} v(t) \\ u(t) \end{pmatrix}.
\]
Similarly for $k = (k_1, ..., k_d), |k| = n$, we will receive equality
\[
\begin{pmatrix}
v_n;k(t) \\
u_n;k(t)
\end{pmatrix} = \sum_{\{j: k_j \neq 0\}} \int_t^t \begin{pmatrix}
-q_j u v_{n-1;k-e_j} & q_j v v_{n-1;k-e_j} \\
-q_j u v_{n-1;k-e_j} & q_j u v_{n-1;k-e_j}
\end{pmatrix} ds \begin{pmatrix}
v(t) \\
u(t)
\end{pmatrix} = \sum_{\{j: k_j \neq 0\}} \int_t^t A_{n-2;k-e_j}(s) M_j(s) ds \begin{pmatrix}
v(t) \\
u(t)
\end{pmatrix} = A_{n-1;k}(t) \begin{pmatrix}
v(t) \\
u(t)
\end{pmatrix}.
\]

Thus, we have proved the following

**Theorem 3.1.** For pair of independent solutions of the equation (20) with normalization (6) valid the exact variational formula
\[
\begin{pmatrix}
v(t) \\
u(t)
\end{pmatrix} = \begin{pmatrix}
v(t) \\
u(t)
\end{pmatrix} + \sum_{|k|=1} \begin{pmatrix}
v_{1;k}(t) \\
u_{1;k}(t)
\end{pmatrix} h^k + \sum_{|k|=2} \begin{pmatrix}
v_{2;k}(t) \\
u_{2;k}(t)
\end{pmatrix} h^k + ... + \sum_{|k|=n} \begin{pmatrix}
v_{n;k}(t) \\
u_{n;k}(t)
\end{pmatrix} h^k + ... = \left[ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] + \sum_{|k|=1} A_{0;k}(t) h^k + \sum_{|k|=2} A_{1;k}(t) h^k + ... + \sum_{|k|=n} A_{n-1;k}(t) h^k + ... \begin{pmatrix}
v(t) \\
u(t)
\end{pmatrix},
\]

where $A_{0;k}(t) = \int_t^t M_j(s) ds$ for $k = e_j$, $M_j(s) = q_j(s) \begin{pmatrix} -u(s)v(s) & v(s)v(s) \\
u(s)v(s) & u(s)v(s) \end{pmatrix}$, $j = 1, ..., d$;

\[A_{n;(k_1, ..., k_d)}(t) = \int_t^t \sum_{\{j: k_j \neq 0\}} A_{n-1;k-e_j}(s) M_j(s) ds,\]

$n \geq 1, |k| \geq 2, d = 3g - 3, |h| < \varepsilon$.

Derive the first variation for the solution of the Schwartz equation (19):
\[
z(t, h) = \frac{v(t, h)}{u(t, h)} = \frac{v(t) + \sum_{i=1}^d v_i(t) h_i + ...}{u(t) + \sum_{i=1}^d u_i(t) h_i + ...} = (v(t) + \sum_{i=1}^d v_i(t) h_i + ...)(\frac{1}{u(t)} + \sum_{i=1}^d h_i(-\frac{u_i(t)}{u^2(t)}) + ...) = z(t, 0) + \sum_{i=1}^d h_i \int_0^t q_i(s)(v(s) - z(t,0)u(s))^2 ds + ... = z(t, 0) + \sum_{i=1}^d h_i \int_0^t q_i(s)(v(s) - z(t,0)u(s))^2 ds + o(|h|), |h| \to 0.
\]

Thus we have proved the following

**Theorem 3.2.** Let $z(t, h)$ be the solution of the Schwartz equation (19) with normalization (5). Then for $h = (h_1, ..., h_d), |h| < \varepsilon$, valid the variational formula
\[
z(t, h) = z(t, 0) + \sum_{i=1}^d h_i \int_0^t q_i(s)(v(s) - z(t,0)u(s))^2 ds + o(|h|), |h| \to 0.
\]

Remark 3.1. Applying the formulas for coefficients quotient power series can be receive also any variational term for the solution $z(t, h)$ of the Schwartz equation (19) on fixed compact Riemann surface $F = U/\Gamma$. 
To conclude of the variational formulas for elements of monodromy groups it is necessary to find some additional relations connected with group $\Gamma$. First of all, for $k = (k_1, \ldots, k_\mathbf{d}) = e_j, |k| = 1, j = 1, \ldots, d$, we find

$$
\begin{pmatrix}
 v_{1;k}(L_t) \\
 u_{1;k}(L_t)
\end{pmatrix}
= \int_{t_0}^{L_t} M_j(s) ds
\begin{pmatrix}
 v(L_t) \\
 u(L_t)
\end{pmatrix} = \\
= \xi_L(t) \int_{t_0}^{L_{t_0}} M_j(s) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} ds \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} + \\
+ \xi_L(t) \int_{L_{t_0}}^{L_t} M_j(s) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} ds \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} = \\
= \xi_L(t) A_{0;k}(L t_0) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} + \xi_L(t) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v_{1;k}(t) \\ u_{1;k}(t) \end{pmatrix}.
$$

For $k = (k_1, \ldots, k_\mathbf{d}), |k| = 2$, we get

$$
\begin{pmatrix}
 v_{2;k}(L_t) \\
 u_{2;k}(L_t)
\end{pmatrix}
= \xi_L(t) A_{1;k}(L_t) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} = \\
= \xi_L(t) A_{0;k}(L t_0) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} + \\
+ \xi_L(t) \int_{L_{t_0}}^{L(t)} \sum_{\{j:k_j \neq 0\}} A_{0;k-e_j}(s) M_j(s) ds \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} = \\
= \xi_L(t) A_{0;k}(L t_0) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} + \\
+ \xi_L(t) \sum_{\{j:k_j \neq 0\}} A_{0;k-e_j}(L t_0) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} A_{0;e_j}(t) \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} + \\
+ \xi_L(t) \left( \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \right) \sum_{\{j:k_j \neq 0\}} \int_{t_0}^{t} A_{0;k-e_j}(s) M_j(s) ds \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} = \\
= \xi_L(t) A_{1;k}(L t_0) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} + \\
+ \xi_L(t) \sum_{\{j:k_j \neq 0\}} A_{0;k-e_j}(L t_0) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v_{1;e_j}(t) \\ u_{1;e_j}(t) \end{pmatrix} + \\
+ \xi_L(t) \left( \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \right) \begin{pmatrix} v_{2;k}(t) \\ u_{2;k}(t) \end{pmatrix}.
$$

For $k = (k_1, \ldots, k_\mathbf{d}), |k| = 3$, we find

$$
\begin{pmatrix}
 v_{3;k}(L_t) \\
 u_{3;k}(L_t)
\end{pmatrix}
= \xi_L(t) A_{2;k}(L t_0) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} + \\
+ \xi_L(t) \int_{L_{t_0}}^{L(t)} \sum_{\{j:k_j \neq 0\}} A_{1;k-e_j}(s) M_j(s) ds \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} = \\
= \xi_L(t) A_{2;k}(L t_0) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v(t) \\ u(t) \end{pmatrix} + \\
+ \xi_L(t) \sum_{\{j:k_j \neq 0\}} A_{1;k-e_j}(L t_0) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v_{1;e_j}(t) \\ u_{1;e_j}(t) \end{pmatrix} + \\
+ \xi_L(t) \left( \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \right) \begin{pmatrix} v_{3;k}(t) \\ u_{3;k}(t) \end{pmatrix}.
$$
\[+\xi_L(t) \sum_{\{j,k\neq 0\}} A_{1; k-e_j} (L_{t_0}) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v_{1; e_j} (t) \\ u_{1; e_j} (t) \end{pmatrix} +
\]

\[+\xi_L(t) \sum_{\{j,k\neq 0, (k-e_j) \neq 0\}} A_{0; k-e_j-e_s} (L_{t_0}) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v_{2; e_j + e_s} (t) \\ u_{2; e_j + e_s} (t) \end{pmatrix} +
\]

\[+\xi_L(t) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v_{3; k} (t) \\ u_{3; k} (t) \end{pmatrix}.\]

By induction for any \(n > 1\), we receive the equality

\[
\begin{pmatrix} v_{n+1; k} (L_t) \\ u_{n+1; k} (L_t) \end{pmatrix} =
\]

\[= \sum_{\{j,k\neq 0\}} \int_{t_0}^{L_t} q_j (s) \begin{pmatrix} v_{n; k-e_j} (s) u_{n; k-e_j} (s) \\ v_{n; k-e_j} (s) u_{n; k-e_j} (s) \end{pmatrix} ds \begin{pmatrix} v (L_t) \\ u (L_t) \end{pmatrix} =
\]

\[= \xi_L(t) A_{n; k} (L_{t_0}) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v (t) \\ u (t) \end{pmatrix} +
\]

\[+\xi_L(t) \sum_{\{j,k\neq 0\}} \int_{t_0}^{L_t} q_j (x) \begin{pmatrix} v_{n; k-e_j} (x) u_{n; k-e_j} (x) \\ v_{n; k-e_j} (x) u_{n; k-e_j} (x) \end{pmatrix} dx \begin{pmatrix} v (t) \\ u (t) \end{pmatrix} =
\]

\[= \xi_L(t) A_{n; k} (L_{t_0}) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v (t) \\ u (t) \end{pmatrix} +
\]

\[+\xi_L(t) \sum_{\{j,k\neq 0\}} \int_{t_0}^{t} q_j (s) \xi_L(s) \begin{pmatrix} v_{n; k-e_j} (L_s) u_{n; k-e_j} (L_s) \\ v_{n; k-e_j} (L_s) u_{n; k-e_j} (L_s) \end{pmatrix} ds \begin{pmatrix} v (t) \\ u (t) \end{pmatrix} =
\]

\[= \xi_L(t) A_{n; k} (L_{t_0}) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v (t) \\ u (t) \end{pmatrix} +
\]

\[+\xi_L(t) \sum_{\{j,k\neq 0\}} \int_{t_0}^{t} q_j (s) [A_{n-1; k-e_j} (L_{t_0})] \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v (s) \\ u (s) \end{pmatrix} +
\]

\[+ \sum_{\{j_1: (k-e_j) \neq 0\}} A_{n-2; k-e_j-e_{j_1}} (L_{t_0}) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v_{1; e_{j_1}} (s) \\ u_{1; e_{j_1}} (s) \end{pmatrix} +
\]

\[+ \sum_{\{j_1: (k-e_j) \neq 0\}} \sum_{\{j_2: (k-e_j-e_{j_1}) \neq 0\}} A_{n-3; k-e_j-e_{j_1}-e_{j_2}} (L_{t_0}) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v_{2; e_{j_1} + e_{j_2}} (s) \\ u_{2; e_{j_1} + e_{j_2}} (s) \end{pmatrix} +
\]

\[+ \sum_{\{j_1: (k-e_j) \neq 0\}} \sum_{\{j_2: (k-e_j-e_{j_1}) \neq 0\}} \sum_{\{j_3: (k-e_j-e_{j_1}-e_{j_2}) \neq 0\}} \ldots + \sum_{\{j_n: (k-e_j-e_{j_1}-
\]

\[\ldots - e_{j_{n-1}}) \neq 0\}} A_{0; k-e_j-e_{j_1}-e_{j_2}-
\]

\[\ldots - e_{j_{n-1}} (L_{t_0}) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v_{n-1; e_{j_1} + e_{j_2} + \ldots + e_{j_{n-1}}} (s) \\ u_{n-1; e_{j_1} + e_{j_2} + \ldots + e_{j_{n-1}}} (s) \end{pmatrix} +
\]

\[+ \xi_L(t) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v_{1; e_j} (t) \\ u_{1; e_j} (t) \end{pmatrix} +
\]

\[+\xi_L(t) \sum_{\{j,k\neq 0\}} A_{n-1; k-e_j} (L_{t_0}) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v_{1; e_j} (t) \\ u_{1; e_j} (t) \end{pmatrix} +
\]

\[+\xi_L(t) \sum_{\{j,k\neq 0\}} A_{n-2; k-e_j-e_{j_1}} (L_{t_0}) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v_{2; e_{j_1}+e_{j_2}} (s) \\ u_{2; e_{j_1}+e_{j_2}} (s) \end{pmatrix} +
\]

\[+\xi_L(t) \sum_{\{j,k\neq 0\}} A_{n-3; k-e_j-e_{j_1}-e_{j_2}} (L_{t_0}) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v_{2; e_{j_1}+e_{j_2}} (s) \\ u_{2; e_{j_1}+e_{j_2}} (s) \end{pmatrix} +
\]

\[+\xi_L(t) \sum_{\{j,k\neq 0\}} A_{n-4; k-e_j-e_{j_1}-e_{j_2}-e_{j_3}} (L_{t_0}) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} v_{3; k} (t) \\ u_{3; k} (t) \end{pmatrix}.
\]
Taking into account the previous equality we receive that the following equality is valid

\[ \sum_{\{j:k_j \neq 0\} \{j_1:(k-e_j)_{j_1} \neq 0\}} A_{n-2;k-e_j-e_{j_1}}(L_{t_0}) \left( \frac{\alpha}{\gamma} \frac{\beta}{\delta} \right) \left( \frac{v_{2;e_j+e_{j_1}}(t)}{u_{2;e_j+e_{j_1}}(t)} \right) + \]

\[ + \sum_{\{j:k_j \neq 0\} \{j_1:(k-e_j)_{j_1} \neq 0\} \{j_2:(k-e_j-e_{j_1})_{j_2} \neq 0\}} A_{n-3;k-e_j-e_{j_1}-e_{j_2}}(L_{t_0}) \left( \frac{\alpha}{\gamma} \frac{\beta}{\delta} \right) \left( \frac{v_{3;e_j+e_{j_1}+e_{j_2}}(t)}{u_{3;e_j+e_{j_1}+e_{j_2}}(t)} \right) + \ldots + \sum_{\{j:k_j \neq 0\} \{j_1:(k-e_j)_{j_1} \neq 0\} \{j_2:(k-e_j-e_{j_1})_{j_2} \neq 0\}} \sum_{\{j_3:k_j \neq 0\} \{j_1:(k-e_j)_{j_1} \neq 0\} \{j_2:(k-e_j-e_{j_1})_{j_2} \neq 0\}} \ldots + \sum_{\{j_{n-1}:k-e_j-e_{j_1}-\ldots-e_{j_{n-2}} \neq 0\}} A_{0;k-e_j-e_{j_1}-\ldots-e_{j_{n-1}}}(L_{t_0}) \left( \frac{\alpha}{\gamma} \frac{\beta}{\delta} \right) \left( \frac{v_{n+1;k}(t)}{u_{n+1;k}(t)} \right) \]

Taking into account the previous equality we receive that the following equality is valid

\[ \xi_L(t) \left( \frac{\alpha_L(h)}{\gamma_L(h)} \frac{\beta_L(h)}{\delta_L(h)} \right) \left( \frac{v(t, h)}{u(t, h)} \right) = \left( \frac{v(L_t)}{u(L_t)} \right) + \sum_{|k|=1} h^k \left( \frac{v_{1;k}(L_t)}{u_{1;k}(L_t)} \right) + \sum_{|k|=2} h^k \left( \frac{v_{2;k}(L_t)}{u_{2;k}(L_t)} \right) + \ldots + \sum_{|k|=n} h^k \left( \frac{v_{n;k}(L_t)}{u_{n;k}(L_t)} \right) + \ldots = \]

\[ = \xi_L(t) \left[ \left( \frac{\alpha}{\gamma} \frac{\beta}{\delta} \right) \left( \frac{v(t)}{u(t)} \right) + \sum_{|k|=1} h^k A_{0;k}(L_{t_0}) \left( \frac{\alpha}{\gamma} \frac{\beta}{\delta} \right) \left( \frac{v(t)}{u(t)} \right) + \sum_{|k|=2} h^k \left( \frac{v_{1;k}(L_t)}{u_{1;k}(L_t)} \right) + \sum_{|k|=3} h^k A_{1;k}(L_{t_0}) \left( \frac{\alpha}{\gamma} \frac{\beta}{\delta} \right) \left( \frac{v(t)}{u(t)} \right) + \right. \]

\[ \left. + \sum_{|k|=3} h^k \sum_{\{j:k_j \neq 0\}} A_{0;0-k-e_j}(L_{t_0}) \left( \frac{\alpha}{\gamma} \frac{\beta}{\delta} \right) \left( \frac{v_{1;e_j}(t)}{u_{1;e_j}(t)} \right) + \sum_{|k|=2} h^k \left( \frac{v_{2;k}(L_t)}{u_{2;k}(L_t)} \right) + \sum_{|k|=3} h^k A_{2;k}(L_{t_0}) \left( \frac{\alpha}{\gamma} \frac{\beta}{\delta} \right) \left( \frac{v(t)}{u(t)} \right) + \right. \]

\[ \left. + \sum_{|k|=3} h^k \sum_{\{j:k_j \neq 0\}} A_{1;1-k-e_j}(L_{t_0}) \left( \frac{\alpha}{\gamma} \frac{\beta}{\delta} \right) \left( \frac{v_{1;e_j}(t)}{u_{1;e_j}(t)} \right) + \right. \]

\[ \left. + \sum_{|k|=3} h^k \sum_{\{j:k_j \neq 0\}} \sum_{\{(k-e)_j \neq 0\}} A_{0;0-k-e_j-e_{j_1}}(L_{t_0}) \left( \frac{\alpha}{\gamma} \frac{\beta}{\delta} \right) \left( \frac{v_{2;e_j+e_{j_1}}(t)}{u_{2;e_j+e_{j_1}}(t)} \right) + \right. \]

\[ \left. + \sum_{|k|=3} h^k \left( \frac{v_{3;k}(L_t)}{u_{3;k}(L_t)} \right) + \ldots + \sum_{|k|=n} h^k A_{n-1;k}(L_{t_0}) \left( \frac{\alpha}{\gamma} \frac{\beta}{\delta} \right) \left( \frac{v(t)}{u(t)} \right) + \right. \]

\[ \left. + \sum_{|k|=n} h^k \sum_{\{j:k_j \neq 0\}} A_{n-2;k-e_j}(L_{t_0}) \left( \frac{\alpha}{\gamma} \frac{\beta}{\delta} \right) \left( \frac{v_{1;e_j}(t)}{u_{1;e_j}(t)} \right) + \right. \]

\[ \left. + \sum_{|k|=n} h^k \sum_{\{j:k_j \neq 0\}} \sum_{\{j_1:(k-e_j)_{j_1} \neq 0\}} A_{n-3;k-e_j-e_{j_1}}(L_{t_0}) \left( \frac{\alpha}{\gamma} \frac{\beta}{\delta} \right) \left( \frac{v_{2;e_j+e_{j_1}}(t)}{u_{2;e_j+e_{j_1}}(t)} \right) + \right. \]

\[ \left. + \ldots \right] \]

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Thus, we received main result

**Theorem 3.3.** Let \( M[z(t, h)] \) be the monodromy group for the solution \( z(t, h) \) of the Schwartz equation (19) with normalization (5) on compact Riemann surface \( F = U/\Gamma \). Then valid the exact variational formulas for \( h = (h_1, ..., h_d), |h| < \varepsilon \),

\[
\begin{pmatrix}
\alpha_L(h) & \beta_L(h) \\
\gamma_L(h) & \delta_L(h)
\end{pmatrix} = \left[I + \sum_{|k|=1} A_{0,k}(L_0)h^k + \sum_{|k|=2} A_{1,k}(L_0)h^k + \ldots \right] \begin{pmatrix}
\alpha_L(0) & \beta_L(0) \\
\gamma_L(0) & \delta_L(0)
\end{pmatrix},
\]

where \( I \) be unit matrix of order 2, \( L \in \Gamma \), \( A_{0,k}(t) = \int_{t_0}^t M_j(s)ds \) for \( k = e_j, M_j(s) = q_j(s) \begin{pmatrix}
-u(s)v(s) & v(s)v(s) \\
-u(s)u(s) & u(s)v(s)
\end{pmatrix}, \quad j = 1, ..., d; \]

\[
A_{n:(k_1, ..., k_d)}(t) = \int_{t_0}^t \sum_{|k|=n} A_{n-1,k-e_j}(s)M_j(s)ds,
\]

\( n \geq 1, |k| \geq 2, d = 3g - 3, |h| < \varepsilon \).

Remark 3.2. The variational formulas show us how the monodromy group and the solution of the Schwartz equation depend of accessory parameters \( (h_1, ..., h_d) \). In particular, they give the exact variational formula for generators any quasifuchian group and any Koebe group which uniformize a compact Riemann surface of genus \( g \geq 2 \).

Remark 3.3. From a relation

\[
dA_0(Lt) = \begin{pmatrix}
\alpha_L(0) & \beta_L(0) \\
\gamma_L(0) & \delta_L(0)
\end{pmatrix} dA_0(t) \begin{pmatrix}
\alpha_L(0) & \beta_L(0) \\
\gamma_L(0) & \delta_L(0)
\end{pmatrix}^{-1}, L \in \Gamma, t \in U,
\]

follows that the matrix differential form \( dA_0(t) \) is a Prym differentials in sense of Gunning [25, 160, 224], with respect to characters, equal the monodromy homomorphism for Fuchsian group \( \Gamma \) with values in \( PSL(2, \mathbb{C}) \). Thus, the variational theory of monodromy groups for linearly polymorphic functions (complex projective structures) on compact Riemann surface depends on the periods such Prym differentials.

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