Geometry of the pore space and dynamic pore and cracked media deforming

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Abstract. In this paper presents the elements of blocked media deforming theory. It means that these media have a specific surface and (related with it) average distance from one crack (pore) to another one. This way requires the creation a new model of continuum, which is contains the integral geometry of pore space. The equations of motion and equilibrium are equations of the infinite order due to infinite numbers of freedom degrees. Along the usual seismic waves, these equations describe very slow waves, not bounded below and, besides of it, they predict the instable solutions, due to parametric resonances in structures bodies. The number of instable solutions corresponds to seismological Gutenberg-Richter law. The dispersion of an average size of structure produces both the fast catastrophes (small dispersion) and slow catastrophes (high dispersion).

1. Introduction

The characteristic size of structure leads to the fact that the average distance between one of the crack to another one or one of the pore to another one given by specific surface of sample. Fig.1 shows an element of the volume of a structured body, where $l_0$ is the average distance between one pore to another. There is a theorem of integral geometry, which relates the specific surface $\sigma_0$ and $l_0$ with porosity $f$ \cite{1, 2}.

$$\sigma_0 l_0 = 4(1 - f) \quad (1)$$
Figure 1. There is an element of a structured body with an average distance $l_0$ between pores. There is a problem of creation of equilibrium equation into arbitrary element of discrete medium. The volume bounded of a surface C is in equilibrium state, while the volume, which bounded of a surface D, may not.

Therefore, if there is a specific surface of sample, the average range of microstructure $l_0$ automatically defined. It is evident that the minimal distance, which gives us a structure, cannot be less than the distance from one particle to its nearest neighbour for granular medium. The same we can tell about average distance from any crack to its nearest neighbour for cracked medium. Actually, the representative size of structure related with statistical characteristics of pore space. The distinction of the classic and the structured continuum looks clear on the Fig.1. In the volume bounded by a surface C, there is equilibrium state, because there is a compensation of all internal surface forces. There is no equilibrium in the volume, which bounded by a surface D because all forces are concentrated on one side of a surface of grain while another side is free from forces. We can construct new model of a medium as follows. Let us consider the finite element of volume, which bounded by the sphere of radius $l_0$. Surface forces are acting on the surface of the element. The forces of inertia will attach to its centre and in our case, and there is no possibility to put the elementary volume to zero, like in classical continuum. Therefore, we must consider namely a finite volume as representative volume of the body. Physically based equations of motion of such elementary volumes will create, if we apply the operator of translation of the surface forces from the centre of this structure. So there appears a possibility to use usual laws of conservation for the forces transferred from the centre of structure. In other words, operators of translation will transform the real medium to its continuous image, where all space filled by a field of forces. Some results of a new model of the structured continuum published earlier (Siibiryakov, Prioulos 2007). The one-dimensional operator of the field translation from the point $x$ into the point $x \pm l_0$ specified by the symbolic formula [3].

$$u(x \pm l_0) = u(x) \exp(\pm l_0 \frac{\partial}{\partial x}); \quad D_x = \frac{\partial}{\partial x} \quad (2)$$

The formal Taylor series expansion in (2) gives the finite increment of the field as a series of the infinite number of all order derivatives with different powers of the value $l_0$.

The similar operator of translation in three-dimension space for some sphere given by expression:

$$P(D_{x}, D_{y}, D_{z}; l_0) = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^\pi \exp[l_0(D_{x} \sin \theta \cos \phi + D_{y} \sin \theta \sin \phi + D_{z} \cos \theta)] \sin \theta d\theta d\phi =$$

$$\frac{\sinh(l_0 \Delta)}{l_0 \sqrt{\Delta}} = E + \frac{l_0^2}{3!} \Delta + \frac{l_0^4}{5!} \Delta \Delta + \cdots \quad (3)$$

The integral (3) may calculate according to Poisson formula [4]:

$$\int_0^{2\pi} \int_0^\pi f(\alpha \cos \theta + \beta \sin \theta \cos \phi + \gamma \sin \theta \sin \phi) \sin \theta d\theta d\phi = 2\pi \int_0^\pi f(\sqrt{\alpha^2 + \beta^2 + \gamma^2}) \sin \phi d\phi \quad (4)$$

And the operator in (3) may rewritten as follows:

$$P(D_{x}, D_{y}, D_{z}; l_0) = \frac{1}{2} \frac{1}{-1} \int_{-1}^{1} \exp(l_0 \sqrt{\Delta} t) dt = \frac{\sinh(l_0 \sqrt{\Delta})}{l_0 \sqrt{\Delta}} = E + \frac{l_0^2}{3!} \Delta + \frac{l_0^4}{5!} \Delta \Delta + \cdots \quad (5)$$

It is interesting that a $P$ operator in (5) as a function of symbolic variables

$$p_1 = \frac{\partial}{\partial x}; \quad p_2 = \frac{\partial}{\partial y}; \quad p_3 = \frac{\partial}{\partial z}$$

satisfies to equation kind of Helmholtz with pure image frequency:

$$\frac{\partial^2 P}{\partial p_1^2} + \frac{\partial^2 P}{\partial p_2^2} + \frac{\partial^2 P}{\partial p_3^2} = l_0^2 P \quad (6)$$

One of solutions (6) which tends to unit at $l_0 \to 0$ takes a form:
2. The equation of motion in blocked media

By using the operator, $P$ we can write the equation of motion of micro-inhomogeneous body because for an average stresses in structure the law of impulse conservation takes a usual form, namely

$$\frac{\partial}{\partial x_k} [P(\sigma_{ik})] = \rho \ddot{u}_i$$

(8)

The stress tensor is $\sigma_{ik}$. The displacement vector is $u_i$, the density is $\rho$. The equation (8) may rewritten in detailed form as follows

$$\frac{\partial}{\partial x_k} \left[ \left( E + \frac{l_0^2 \Delta}{3!} + \frac{l_0^4 \Delta \Delta}{5!} + \cdots \right) (\sigma_{ik}) \right] = \rho \ddot{u}_i$$

(9)

$E$ means the unit operator. It the average size of structure tends to zero the operator $P \to E$, and the equation (8) coincides to classic equation of motion for continuous medium. Cauchy and Poisson classic continuous model corresponds to operator equality $P = E$. In particular, if we take into account only the first and second terms in this equation, we obtain the fourth order equation of motion. For one dimension case, equation (9) takes more simple expression

$$u_{xx} \left( E + \frac{l_0^2 \Delta}{3!} + \frac{l_0^4 \Delta \Delta}{5!} + \cdots \right) + k_s^2 u = 0$$

(10)

This equation by substitution of $u = A exp(ikx)$ into (10) gives us the dispersion equation for unknown wave number $k$

$$\sin(kl_0) = \frac{k_s^2}{k^2}$$

(11)

In (11) $k$ is unknown wavelength some stationary vibration, and $k_s = \sqrt{\frac{\rho \omega^2}{k + 2 \mu}}$ or $\sqrt{\frac{\rho \omega^2}{\mu}}$ are usual wavelength for $P$ or $S$ waves in classic continuum. Alternatively, for unknown wave velocity, which depends on range $l_0$ of structure or specific surface $\sigma_0$ of sample, according to the formula (1). In case of, $l_0 \to 0$ the wave number $k \to k_s$, i.e. the wave velocity is equal to $V_p$ or $V_s$ elastic wave velocities. However, if $l_0$ is not very small value, the wave velocity decreases up to a zero when $kl_0 \to m\pi$, where $m$ is integer number. Hence, this model describes along with usual seismic waves many waves of very small velocities, which unbounded from below. Such effect is more substantial for the $S$ waves than for the $P$ ones. Thus, if the Poisson's ratio measured on the samples using $V_p$ and $V_s$ velocities, we have the growth of ratio $V_s/V_p$ with growth of $l_0$, and this effect can produce abnormally small Poisson's ratio up to its negative values. Set of velocities, described by (11) closely relates with infinite degrees of freedom of structured media. The same approach for case of infinite degrees of freedom published in the paper (Sibiryakov, Prilous 2007). Another feature of equation (11) is the existence of complex roots representing the unstable solutions, describing phenomenon of so-called parametric resonance.

3. Seismological law of Gutenberg–Richter

Equation (11) gives us an infinite number of roots both real and complex ones. The real roots correspond to stable solutions, while complex ones correspond to instabilities. The number of complex roots is growing at decreasing of dimensionless specific surface of cracks, which represents by expression

$$\frac{1}{\varepsilon} = \frac{\sigma_0 \lambda_S}{8 \pi (1 - f)}$$

(12)

The symbol $\lambda_S$ in (12) is the wavelength of usual shear wave, while $\varepsilon = k_s l_0$. Theoretical dependence of the number of complex roots versus specific surface on the log-log scale plot (Fig.2) represents something closed to linear dependence. It is evident that the energy of cracked body is proportional to
surface of cracks. The deficit of potential energy owing to occurrence of cracks is equal to product of stresses before cracking process and displacement of cracks surface, namely

\[ \bar{E} = \oint_S P_i u_i dS \]  \hspace{1cm} (13)

Value \( \bar{E} \) in (13) is equal to kinetic energy of waves due to law of energy conservation. It means that we can compare the experimental seismological relation (number of earthquakes versus energy) with the theoretical relationship (number of instable solutions versus specific surface). On the Fig. 2-3 there are theoretical and experimental diagrams. The Fig. 2 shows series of straight horizontal segments, representing complex roots, corresponding to detached number of events. The experimental diagram energy versus event number of earthquakes was made Riznichenko [5]. The tangent of the angle with vertical axes is changing on the experimental plot from 0.5 to 0.52 and depends on different data processing (Fig.3). On the theoretical plot (Fig.2) this tangent is closed to 0.5. It is interesting that for large energy there is set of non-uniqueness. One line segment of the vertical scale corresponds a lot of surfaces (energy). The energy is proportional to specific surface.

Such non-uniqueness in experimental data usually interpreted as a deficiency of number of great events and pure statistic estimations of such events. Proposed theoretical approach explains a principal non-uniqueness of such processes. Besides, of it, these processes have shear character without of sufficient crack opening. At sufficient crack opening the potential energy would be proportional not to surface, but to volume of cracks. Hence, the theoretical plot on Fig.2 shows, that law Gutenberg–Richter caused mainly shear mechanisms of earthquakes.

![Figure 2](image1.png)  
**Figure 2.** Theoretical dependence of an instable number of events from the specific surface of cracks. Tangent of the angle \( \gamma = 0.5 \). It is clearly visible non-uniqueness of solutions with the great energies.

![Figure 3](image2.png)  
**Figure 3.** The experimental dependence seismic number of events versus energy (class of earthquakes), which is proportional to the cracks specific surface. Tangent of the angle \( \gamma = 0.5 - 0.52 \).

### 4. The long wave approximation. Stable and instable solitons

For small values \( l_0 \) in comparison with wavelength, there is a possibility to reduce the equation of motion of an infinite order to the equation of the fourth order, neglecting the members containing values \( l_0^4 \) and above. In this case, we can consider some nonlinear relations between stresses and strains. Let us assume that we have a nonlinear loading and linear unloading. (Fig.4). For rocks and grounds, the
The reduction of stresses takes place at the increase in deformations. It means, that in such media, where shock waves are absent, while nonlinear waves represented by Riemann waves. The reduced equation of motion (9) takes a form

\[ \frac{\partial}{\partial x_k} \left[ (E + \frac{l_0^2}{3!}) \sigma_{ik} \right] = \rho \ddot{u}_i \]  

(14)

We suppose quadratic relations between stresses and strains of loading process in the form \( \sigma_{xx} = (\lambda + 2\mu) (u_x - bu_x^2) \) and the linear behavior of unloading process.

\[ \sigma_{xx} = \frac{1}{2} \left( \frac{E}{\sqrt{\Delta}} \right) \Delta x \]  

(15)

This equation (16) is equation kind of Korteweg-de-Vries. The solution in the waveform is

\[ u = u(z) = c_0 TF \left( \frac{t - x}{c_0 \xi} \right) \]  

(17)

\( \xi = \text{const} \) is the defect of velocity, and it is some medium parameter. It may to be in common case different for P and S waves. For usual elastic medium \( \xi = 1 \). In (17) \( c_0 T = L \) is the effective wavelength, \( T \) is a characteristic time of the pulse. \( F \) is the dimensionless function with respect to dimensionless argument \( \frac{t - x}{c_0 \xi} \), \( c_0^2 = \frac{(\lambda + 2\mu)}{\rho} \), \( b \) is the nonlinearity factor, \( z = \frac{t - x}{c_0 \xi} \). Substituting the expression (17) into (16) we can write

\[ F_{xx}(1 - \xi^2) + \epsilon^2 F_{xxxx} + \frac{b}{\xi} (F_x^2)_{xx} = 0; \quad \epsilon^2 = \frac{l_0^2}{3! L^2}; \]  

(18)

Substitution \( \theta = F_z \) gives
\[
\theta(1 - \xi^2) + \frac{b}{\xi} \theta^2 \text{sign}\theta + \varepsilon^2 \theta_{zz} = 0;
\]  \hspace{1cm} (19)

For \( S \) wave there is an analogous equation
\[
\theta(1 - \xi^2) + \frac{b}{\xi} \theta^2 \text{sign}\theta + \varepsilon^2 \theta_{zz} = 0;
\]  \hspace{1cm} (20)

The equation (19) at the monotone growing of \( u_x \) i.e.
\[
u_{xx}(1 - 2b u_x) = \frac{1}{c_0^2} \left( \ddot{u} - \frac{l_0^2}{3!} \ddot{u}_{xx} \right)
\]  \hspace{1cm} (21)

has the soliton solution kind of
\[
\theta = A \cosh^{-2}(\beta z) \]  \hspace{1cm} (22)

where \( \beta \) is a constant, while
\[
\xi^2 = 1 + 4\beta^2 \varepsilon^2; \quad A = \frac{6\xi}{b} \beta^2 \varepsilon^2 \]  \hspace{1cm} (23)

From (23) follows that soliton solutions are at \( \xi > 1 \). It means solitons are stable if the wave velocity growth with amplitude. Many rocks containing gas have the opposite properties, and there are no solitons in them. Nevertheless, rocks, saturated with liquid, can contain solitons.

5. Small vibrations and multiple frequencies

In the case \( \xi < 1 \), there are oscillating solutions of nonlinear equation (19). One of them is
\[
\int_0^\theta \frac{dG}{\sqrt{D - G^2 (1 - \xi^2) + 2G \frac{G^3}{3 \varepsilon^2}}} = \frac{t - \frac{x}{c_0 \xi}}{T} \]  \hspace{1cm} (24)

In (23) \( D \) is an arbitrary constant. We can put that the ratio \( \frac{1 - \xi^2}{\varepsilon^2} \) is order to unit. The third term in the left hand of integral \( \frac{2G^3}{3 \varepsilon^2} \) may be very small, but in denominator, there is too very small factor \( \varepsilon^2 \). It means that the dispersion is growing due to non-linearity and this effect much more, than usual quadratic effect. On the Fig.5 shown a damping of sinusoidal pulse due to nonlinearity plus dispersion at deformation about \( 10^{-6} \). Attenuation of sinusoidal pulse with distance due to nonlinearity displayed on Fig.5.

![Spectral amplitude](image1)

Figure 6. Signal spectrum of a shear wave at frequencies \( f_1 \) and \( f_2 \) on the right and left faces of specimen. (Mashinsky and Egorov 2011).

![Spectral amplitude](image2)

Figure 7. Spectra of combination frequencies between right and left faces of specimen. (Mashinsky and Egorov, 2011).

The attenuation of sinusoidal impulse depends on distance. The nonlinear parameter is 0.1. The logarithmic decrement is almost constant value. The parameter of nonlinearity is not very small due
to large factor $\frac{1}{\varepsilon^2}$. On this figure shown, how to appear the attenuation for sinusoidal impulse. When the nonlinear parameter equal to zero, attenuation is absent. If it is equal to 0.1 there is some small attenuation with almost constant logarithmic decrement. For larger nonlinear parameter equal to 0.5 there is a sufficient attenuation without of constant decrement. Equation (24) also contains growing solutions. However, in this section, we shall deal only with damping vibrations. According to experimental observations, the sample of artificial sandstone (length-1m, diameter-0.76m, porosity-0.3 and density 2g/cm$^3$) undergoes of excitation simultaneously by two vibrators with frequencies 6100 Hz and 7720 Hz. The spectra of signals on the plane of cylindrical sample, where the source is located, given on the Fig.6 and Fig.7. The receiver registers different frequency 1620 Hz on the distance 75 cm from the source. It is interesting that the amplitude of difference frequency extremely high, i.e. reached the order of a several percent of initial signal. Classic approach, connected with second-order equations of motion predicts the effect, which is proportional to quadrate of strains. The dispersion phenomena in porous media sharply increases nonlinear processes even for weak oscillations [6].

6. Solutions corresponding to equal wavelength for $P$ and $S$ waves

The latent nonlinearity produces some asymptotic solutions with the same wavelength of $P$ and $S$ waves, not equal frequencies. Classic elastic solutions gives us the same frequencies, but not equal wavelength. Nevertheless, the equation (19) in the asymptotic case, $b\xi G \ll 1$, there are two independent parameters $\xi^2$ and $\varepsilon^2$. For plane $P$ and $S$ waves, the equations (19-20) take a form

$$\theta(1 - \xi^2) + \varepsilon^2\theta_{zz} = 0; H(1 - \xi^2) + \frac{\varepsilon^2}{\gamma^2} H_{zz}; \gamma = \frac{V_S}{V_P}$$

For equal velocity defect $\xi^2_P = \xi^2_S$ there is usual situation with equal frequencies and different wavelength like in usual elastic medium

$$G_{zz} + 1 - \xi^2_P G = 0; H_{zz} + \gamma^2 \frac{1 - \xi^2_P}{\varepsilon^2} H = 0$$ (26).

Choosing $1 - \xi^2_P = \frac{1 - \xi^2\gamma^2}{\varepsilon^2}$ we can take the same equations for plane $P$ and $S$ waves with the same wavelength but not the same frequencies

$$\theta_{zz} + \frac{1 - \xi^2_P}{\varepsilon^2}\theta = 0; H_{zz} + \frac{1 - \xi^2_P}{\varepsilon^2} H = 0$$ (27)

These relations are in the many experiments, in spite of classic solutions. The terms $\xi^2_P$ and $\xi^2_S$ are very closed to unit with accuracy to $\varepsilon^2$. The arbitrary value $\xi^2_P$ and $\xi^2_S$ give some interval for different value of seismic $P$ and $S$ waves.

7. Intermediate states between statics and dynamics, and some catastrophic scenarios in random structural media

In present, there is an approach to describe the instabilities and catastrophes using the classical model of space (Cauchy and Poisson continuum) with a sufficiently complicate equation of state [7, 8]. This approach allows us, in principal, to explain many phenomena with high parameter values, which may be interpreted as the critical state or destroying of a body. The self-organized critical dynamic is a very interesting process, which produces destruction according to the Gutenberg-Richter law and other laws with linear relations between the number of catastrophes and the energy of them. It is interesting that the more organized the structures are, the more the velocity and the number of catastrophes. It may suggested that the chaos prevents catastrophes. However, there is another approach for description of these phenomena. This approach based on a suggestion that the medium is not a continuous body, but represented as a structured subject with its own geometry, which described with integral geometry of pore or cracked space [9]. The main parameters of this inner geometry are porosity, specific surface, the average and Gaussian curvatures. As to the state equation, it is chosen extremely simple, namely, the linear relation between the average stresses and the average strains. However, the average parameters
and the parameters in any point of the medium may differ considerably. Besides, the equations of motion and equilibrium for the structured media are equations of the infinite order due to infinite degrees of freedom in the blocked media. This circumstance gives a hope to describe the transfer from statics in one scale to dynamic in another scale. The scaling defined by specific surface of the body, or by average distance from one crack to another one in this body.

As a rule, statics and dynamics distinguished by presence or absence of inertial forces. However, in the structured media the equilibrium at large scales coexists with vibrations at small scales. It means that statics and dynamics are not absolute matters, and there are intermediate states between them. If the model of medium does not comprise any scales, every point is the representative one for the material, and the medium has all properties of the small volume. In this case, there is no possibility to translate inertial forces from one scale to another one. It means there are no any intermediate states between statics and dynamics. If we can create a continual image of the real medium due to translation of field operator \( P \), and every point (including pores and cracks) saturated by the translated forces, in this case the differential equation of motion of the infinite order appears, because in the structured media there are infinite degrees of freedom.

Classic Cauchy and Poisson continuum means that mentioned \( P \) operator is equal to the unit operator \( E \), i.e. the real medium and its continuum model are identical.

8. Gamma distribution of the structure sizes and the structure of continuity operator

The gamma distribution for random value \( x \) takes a form [10]

\[
F(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^x z^{\alpha-1} e^{-\beta z} dz
\]  

(28)

This integral has a sense at \( \alpha > 0 \), the physical sense requires the zero value of probability for infinite small cracks and we need to use more hard require (11). The average value of some quantity \( z \) takes a form

\[
<z> = \frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^x z^{\alpha-1} e^{-\beta z} dz = \frac{\Gamma(\alpha + 1)}{\Gamma(\alpha)\beta} = 1; \beta = \alpha
\]

(29)

The requirement of average distance between one crack to another leads the equality of two parameters in the formula (29). The deviation at mentioned average distance equal to unit has a form

\[
\sigma^2 = \frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^x (z - 1)^2 z^{\alpha-1} e^{-\beta z} dz = \frac{\beta^\alpha}{\Gamma(\alpha)} \left[ \frac{\Gamma(\alpha + 2)}{\beta^{\alpha+2}} - 2 \frac{\Gamma(\alpha + 1)}{\beta^{\alpha+1}} + \frac{\Gamma(\alpha)}{\beta^\alpha} \right] = 1
\]

(30)

Hence in this case a parameter \( \alpha \) is the inverse deviation. A structure of operator \( P \) gives by expression [1]

\[
<P(D_x, D_y, D_z; \alpha, l_0; x)>) = \frac{1}{4\pi} \int \int_0^\infty [\xi^{\alpha-2} \exp(-\alpha \xi) d\xi] \exp[l_0 \xi(D_x n_i)] d\omega
\]

\[
P(D_x, D_y, D_z; \alpha, l_0) = \frac{(1 - \sqrt{\Delta} l_0 / \alpha)^{-\alpha+1} - (1 + \sqrt{\Delta} l_0 / \alpha)^{-\alpha+1}}{2 l_0 \sqrt{\Delta}}
\]

If \( \alpha = \frac{1}{\sigma^2} \to \infty \), this operator tends to the previous form (5), namely

\[
P(D_x, D_y, D_z; \infty, l_0) = \frac{\sinh(l_0 \sqrt{\Delta})}{l_0 \sqrt{\Delta}}
\]

(31)

Thus for blocked media with random distance between cracks and pores with gamma distribution we have dispersion equation for stationary vibrations

\[
\frac{1}{2} \int_{-1}^1 \left( \frac{\alpha}{\alpha - ik l_0 t} \right)^\alpha dt = \frac{k_s^2}{k^2}
\]

(32)

At \( \alpha \to \infty \), equation (22) takes a form, which published earlier [1]

\[
\frac{\sin(k l_0)}{k l_0} = \frac{k_s^2}{k^2}
\]

(33)
9. The latent dynamics

The latent dynamics means that inertial forces tend to zero on the large scale, but not zero on the small scale. This can be described by applying the $P-E$ operator to the inertial forces. If a medium is the classic continuum, $P-E=0$ and we have equation of equilibrium. In the opposite case

$$P - E = \frac{l_0^2\Delta}{3!} + \frac{l_0^4\Delta\Delta}{5!} + \frac{l_0^6\Delta\Delta\Delta}{7!} + \cdots$$

and latent dynamics terms are in the higher derivatives of field. It means that the equation of latent dynamic in blocked media takes form

$$P \left( \frac{\partial \sigma_{ik}}{\partial x_k} \right) = (P - E) \rho \dddot{u}_i$$

For one dimensional situations and stationary vibration the corresponding dispersion equation written as

$$\frac{1}{2} \int_{-1}^{1} \left( \frac{\alpha}{\alpha - ik_0t} \right)^{\alpha} dt = \frac{k_0^2}{k^2} \left( \frac{1}{2} \int_{-1}^{1} \left( \frac{\alpha}{\alpha - ik_0t} \right)^{\alpha} dt - 1 \right)$$

(34)

The values $k$ may be both real and complex ones. The real roots of dispersion equation (34) correspond to vibration with different wavelength and to instable solutions for complex values of them. Parameter $\alpha$ changes the distribution of the dispersion equation roots. Decreasing $\alpha$ reduces the number of instable solutions and velocity of catastrophes. The value $\alpha = 1$ corresponds to exponential distribution instead of gamma distribution. It means that there is a set of extremely small particles like in classical continuum Cauchy and Poisson. These media not contain unstable solutions due to set of extremely small particles. The roots of the dispersion equations form the discrete set and this corresponds to the infinite number of scenarios of vibrations or instable states (for complex roots). Every root corresponds to a certain scenario. There is a question about transition of vibration from statics to the vibration in usual sense. It is evident that at $l_0 \to 0$ the root is $k = 0$ which corresponds to statics. On the Fig. 8 the real part of value $kl_0$ is along $x$ axis, and the imaginary part of it is along $y$-axis. The inverse dispersion $\alpha = 300$ is sufficiently large. The vertical axis corresponds to the known value $k_S l_0$. If the values $Re l_0 \approx k_S l_0$, it means that the velocity of the wave, corresponding to wavenumber $k$ at fixed frequency, is equal to usual $S$ seismic wave. The case $Re kl_0 \ll k_S l_0$ corresponds to very small wave velocity. We can see, that for $Re kl_0 < 2.5$ there is no vibration; it is area of pure statics. For $Re kl_0 > 2.5$ there is area of small wave velocities (small points) ($Re kl_0 < 6$). After it there are catastrophes (black color) and damping scenarios (grey color), i.e. $Re kl_0 \approx k_S l_0 \approx 6$. This figure demonstrates fast catastrophes (velocity close to $S$ seismic waves).

![Figure 8](image)

Figure 8 The $x$ axe corresponds to real part of $kl_0$, the $y$ axe corresponds to imaginary part of it, and $z$ axis corresponds to $k_S l_0$. The value $\alpha = 300$ is sufficiently large. Small vertical coordinate compare to horizontal one (x) means the small wave velocity compare to usual seismic shear wave. White field ($0 < x < 2.6$) corresponds to statics. Area ($2.6 < x < 6.2$) corresponds to waves with velocities from zero to the shear wave one ($x=5.1$). Black colour corresponds to fast catastrophes with velocities close to the shear wave one.
Figure 9. The $x$ axis corresponds to real part of $k_l\phi$, the $y$ axis corresponds to imaginary part of it, and $z$ axis corresponds to $k_l\phi$. The value $\alpha = 300$ is sufficiently large. Small vertical coordinate compared to horizontal one ($x$) means the small wave velocity compared to usual seismic shear wave. White field ($0 < x < 2.6$) corresponds to statics. Area ($2.6 < x < 6.2$) corresponds to waves with velocities from zero to the shear wave one ($x = 5.1$). Black color corresponds to fast catastrophes with velocities close to the shear wave one. Axes are the same as on the figure 8. The value $\alpha = 5$ is very small. It means that the size of structure dispersion is sufficiently large. Notations are the same as on the Fig. 8. However, the diapason of $k_l\phi$ on the $z$ axis is from 0 to 0.08 (On the fig. 8 from 0 to 6). It means that the catastrophes are moving with velocities 75 times less than shear seismic waves.

Figure 10. This figure corresponds to the value $\alpha = 1$. It means that the size of structure dispersion is the maximum large. There is no catastrophic scenarios. Axes are the same as on the figure 8. The value $\alpha = 1$ is extremely small for Gamma-distribution. It corresponds to probability of 0.5 for zero length of cracks. It means that with probability equal to 0.5 the body has infinite specific surface, in the other words, this body partially destroyed.

10. Conclusions
1. The model of the structured continuum with an account of specific surface of blocked medium or average size of structure, gives us the differential equations of motion of the infinite order. This model includes collective properties of pore space like the porosity, specific surface and predicts besides of usual elastic waves many unusual waves with very small velocities.
2. Dynamic equation of structural continuum gives unstable solutions, which correspond to complex roots of dispersion equation. The number of complex roots depending of specific surface of cracks represents as a straight line in the logarithmic scale if the specific surface and the distance between two cracks is a constant. This straight line corresponds to well-known Gutenberg-Richter Law.
3. The long wave approach gives us equation of motion kind of Korteweg-de-Vries, so called as Boussinesq equation. The soliton solutions exist in skeletons, which saturated of liquid. The solutions with multiply frequencies correspond to porous space, containing gas. The solitons are stable in the media, which there are stable shock waves.

4. For usual continuous medium nonlinear effects in weak waves are negligible small. Nevertheless in structured media this effect is not small, it is real phenomenon. The dispersion term sharply energizes nonlinear effects in micro-inhomogeneous media.

5. Latent nonlinearity and dispersion forming pulses with the same wavelength for $P$ and $S$ waves. Usual elastic theory gives us opposite effect. The same frequencies and different wavelength. The experimental data about different frequencies of $P$ and $S$ waves can explain due to mentioned effects.

6. For random sizes of structure, with gamma distribution, the velocity of catastrophes is different. The large velocity exists for small dispersions of structure sizes (order of seismic shear wave velocity) and small velocities (two orders less) exist for large dispersions of them. For maximum dispersion ($\alpha = 1$), catastrophes are impossible.

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