Supercapacitor Modeling for Conversion Systems and Power Supply in Electric Vehicles

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Abstract. The main obstacle in an Electric Vehicle (EV) is the large torque requirement at the start. And in this case, it must be overcome with a battery that is bigger and heavier. However, a large battery will become a burden when moving and braking. The use of supercapacitors can make the vehicle move more powerful at the beginning of the movement because of its very fast energy release. This paper presents a supercapacitor modeling to support battery performance in an energy storage system. This research is focused on modeling the supercapacitor connected to the drive converter circuit by looking for a mathematical model in the form of a state-space equation, the results of which are simulated using the C Mex S function in Matlab / Simulink. From the modeling carried out, it is obtained a system that has a system response that is built and shows the system is working properly, the output voltage can follow the reference results.

1. Introduction
The development of technology will affect the development of energy storage systems, especially in electric vehicles. However, the problem that is often faced with electric vehicles is the need for large torque at the start. Thus, a storage system that can support a system such as batteries, batteries, and supercapacitors is needed [1].

Supercapacitors, also known as ultracapacitors or electric double-layer capacitors (EDLC), have unique characteristics that can be used as energy storage in various applications, one of which is a hybrid system [2]. On a life cycle basis, the supercapacitor has a longer life cycle than the battery. Besides that, another advantage of the supercapacitor is the very fast filling and discharging process, so it is very suitable for use in circuits that require a fast release of energy with a large amount of energy. The theory on which the supercapacitor is based was first described by Hermann von Helmholtz in 1853. He argued that the interaction between the conductor and the electrolyte in the capacitor is determined by the electrostatic relationship and that no chemical reaction is involved in the process [3]. One of the problems with the use of supercapacitors is how accurate the State of Charge (SOC) of the Energy storage system [4].

This paper presents two aspects. The first is supercapacitor modeling which is based on parameter values that vary according to the State of Charge (SOC). The second presents the Buck-Boost Converter model which is connected to the supercapacitor. Modeling the two proposed models, namely by looking for a mathematical model into the form of state space and simulated using CMex S-Function in Simulink Matlab [5]. This paper presents a supercapacitor modeling connected to a buck-boost converter, by looking for a mathematical model into state space and simulating it using the cmex s-function in Matlab [2]. From the results obtained, it is known that modeling using cmex has a good response and the model built has characteristics that resemble the original.
2. Method
Supercapacitor and Buck-Boost Converter are modeled by determining the state space equation model. State-space modeling is an analytical method for a complex control system [6]. This method is used to analyze a control system with multiple inputs and multiple outputs or what is called Multiple Input and Multiple Output (MIMO). The State Space equation model is expressed in the form of a matrix (1) (2) where A, B, C, and D are state-space matrices, \( \dot{x} \) is the derivative matrix of the x matrix, \( u \) is the input matrix and \( y \) is the output matrix.

\[
\dot{x} = Ax + Bu \quad (1)
\]

\[
y = Cx + Du \quad (2)
\]

2.1. Supercapacitor State Space Model
The Supercapacitor model described in the literature is usually based on an equivalent electrical circuit that can represent a voltage or current profile. The supercapacitor model is shown in Figure 1. In the supercapacitor circuit, the voltage across the main capacitance is OCV (Open Circuit Voltage), which corresponds to the voltage that appears at the supercapacitor terminal after the transient remains. Because the transients simulated by the R-C block have a time constant.

From the supercapacitor model that has been selected, the derivative equations (3) and (4) can be obtained. From the derivative equation, it can be written into a state-space model, with the state variable in the model determined, namely \( V_c = x_1 \) and \( V_{c1} = x_2 \), with \( I_o = u \). Then it can be expressed in the form of a matrix as defined in equations (1) and (2), the state space equations (5) and (6) are obtained.

\[
\frac{dV_c}{dt} = - \frac{I_o}{C} \quad (3)
\]

\[
\frac{dV_{c1}}{dt} = - \frac{V_{c1}}{R_1 * C_1} - \frac{I_o}{C_1} \quad (4)
\]

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix}
= 
\begin{bmatrix}
0 & 0 \\
0 & - \frac{1}{R_1 * C_1}
\end{bmatrix}
\begin{bmatrix}
V_c \\
V_{c1}
\end{bmatrix}
+ 
\begin{bmatrix}
\frac{1}{C} \\
0
\end{bmatrix}
I_o 
\]

\[
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix}
= 
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
V_c \\
V_{c1}
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
0
\end{bmatrix}
I_o 
\]

One of the most common techniques for estimating SOC is called the OCV-SOC correlation. This technique consists of a simple ampere-clock calculation, with voltage-based compensation. The supercapacitor SOC equation (7) to obtain the parameters from the SOC output value table 1. From the selected supercapacitor model Figure 1 with the characteristics of the supercapacitor table 2.
\[ \text{SOC} = \frac{Q}{Q_{\text{max}}} = \frac{\int i(t)\,dt}{CV_{\text{max}}} = \frac{V_{oc}}{V_{\text{max}}} \] (7)

An experimental test has also evaluated the dynamics of the model response of the element values R0, R1, C1 at different SOC [5].

Table 1. Identification of Model Parameters as a function of SOC

| SOC  | \( V_{oc}(V) \) | \( R_0 \) (mΩ) | \( R_1 \) (mΩ) | \( R_1 C_1(s) \) |
|------|-----------------|----------------|----------------|-----------------|
| 0.52 | 1.43            | 0.51           | 0.11           | 4.2             |
| 0.76 | 2.04            | 0.50           | 0.10           | 11              |
| 1    | 2.85            | 0.48           | 0.09           | 16              |

Table 2. Characteristics of the Supercapacitor cell

|                         | Nominal Capacitance (F) | Maximum Voltage (V) | Nominal Voltage (V) | Max Continuous Current (A) ΔT 15°C | Max Continuous Current (A) ΔT 40°C | Max Peak Current (A) | Internal resistance (mΩ) | Leakage resistance Current* (mA) | Mass (kg) |
|-------------------------|-------------------------|---------------------|--------------------|-------------------------------------|-------------------------------------|----------------------|---------------------------|----------------------------------|-----------|
|                         | 3000                    | 2.85                | 2.7                | 130                                 | 210                                 | 2000                 | 0.29                      | 5.2                              | 0.5       |

* after 72 Hours at nominal voltage, the initial leakage current can be higher

2.2. Buck Boost Converter State Space Model

The buck-boost converter is a type of DC-DC converter that is capable of producing DC voltage variations that are smaller or lower than the input voltage according to the switching frequency [7]. The components of the buck-boost converter consist of several constituent components, namely the transistor, inductor (L), diode (D), and capacitor (C) which are arranged as shown in Figure 2.

Figure 2. Buck Boost Converter Fundamental, ‘On’ and ‘Off’ state equivalent

The fundamental buck converter circuit and is On’ and ‘Off’ state equivalent. On condition ‘On’, the inductor gets input from \( V_{sc} \). For current flow to the capacitor \( I_L = 0 \). Equations derived from the state
and (9) with state variables namely $V_c$ and $I_L$. It can be written into a matrix of state space $A$ and $B$ as defined in equation (10).

$$\frac{di_L}{dt} = \frac{V_{sc}}{L} - \frac{I_L \cdot R_{on}}{L} \quad (8)$$

$$\frac{dV_c}{dt} = \frac{V_c}{C \cdot R_c} \quad (9)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{R_{on}}{L} \\ \frac{1}{R \cdot R_c} & 0 \end{bmatrix} \begin{bmatrix} V_c \\ I_L \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \quad (10)$$

In the ‘Off’ state, the voltage source is zero. Then the derivative equation obtained (11) and (12)

$$\frac{di_L}{dt} = \frac{I_L \cdot R_{on}}{L} \quad (11)$$

$$\frac{dV_c}{dt} = -\frac{V_c}{(R_c + R)C} + \frac{I_L \cdot R}{(R_c + R)C} \quad (12)$$

From equations (11) and (12), the state variable $V_c = x_1$ and $I_L = x_2$ then we can write equation (13) state space model for the ‘Off’ condition of the Buck Boost Converter circuit.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{(R_c + R)C} & \frac{R}{(R_c + R)C} \\ 0 & \frac{R_{on}}{L} \end{bmatrix} \begin{bmatrix} V_c \\ I_L \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \quad (13)$$

Matrix $C$ and the matrix $D$ in the state space model can be written in equation (14).

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} V_c + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u \quad (14)$$

After obtaining the A and B matrices in the 'On' and 'Off' state in the buck boost converter circuit. A matrix of the mean space equations (15) and (16) is required. The equilibrium state is the superposition of the state space matrix of the two conditions, taking into account the duty cycle ($d$), thus obtained (18) and (19).

$$\dot{x} = Ax + Bu \quad (15)$$

$$y = C^T x \quad (16)$$

Where,

$$A = dA_{ON} + (1 - d)A_{OFF} \quad (17)$$
\[ A = d \begin{bmatrix} 0 & -\frac{R_{on}}{L} \\ \frac{1}{R \cdot R_c} & 0 \end{bmatrix} + (1 - d) \begin{bmatrix} \frac{1}{(R_c + R)C} & \frac{R}{(R_c + R)C} \\ 0 & \frac{R_{on}}{L} \end{bmatrix} \begin{bmatrix} \frac{(1 - d)}{(R_c + R)C} & \frac{R}{(R_c + R)C} \\ \frac{d}{R \cdot R_c} & \frac{R_{on}(1 - D)}{L} \end{bmatrix} \] (18)

\[ B = dB_{ON} + (1 - d)B_{OFF} \] (19)

\[ B = d \begin{bmatrix} 1 \\ 0 \end{bmatrix} + (1 - d) \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} d \\ L \end{bmatrix} \] (20)

To complete the buck boost converter model, the average state space matrix, can be substituted for equations (1) and (2), then the state space equation of the buck boost converter system becomes (18).

\[ \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{(1 - d)}{(R_c + R)C} & \frac{R}{(R_c + R)C} \\ \frac{d}{R \cdot R_c} & \frac{R_{on}(1 - D)}{L} \end{bmatrix} \begin{bmatrix} V_c \\ I_L \end{bmatrix} + \begin{bmatrix} d \\ L \end{bmatrix} u \] (21)

### 3. Results and Discussion

From the supercapacitor model and the buck boost converter model, the state space equation for each condition has been obtained. These equations are simulated using the S-Function block in Matlab Simulink [5]. The block diagram of the simulation system is shown in Figure 3. It consists of a Supercapacitor block, a Buck Boost Converter block, PID controller, and PWM generator.

![Simulated Block Diagram with Cmex S-Function in Matlab Simulink](image)

The Pulse Width Modulator (PWM) block converts the output from the compensator to the duty ratio. The output voltage of the compensator \( V_c \) is compared to a triangular wave with an amplitude of \( V_p \). The output of the PWM block is the high value when \( V_c \) is greater than the triangle wave and is zero when \( V_c \) is smaller than the triangle wave. If the output voltage \( V \) is smaller than the output voltage \( V_{ref} \), the error value will increase so that the \( V_c \) value increases and the duty ratio increases. Conversely, if the large output voltage \( V \) increases, it will reduce the duty ratio. This PWM pulse will then be used to open and close the switches on the switching converter.
The parameter values of the supercapacitor are obtained based on SOC (Table 1) and the boost converter is simulated with the parameter values $R = 1.5 \, \Omega, C = 10^{-3} \, F, L = 10.3 \times 10^{-3} \, H$, the switching frequency is 10 kHz, the duty cycle is 25%, the buck boost converter voltage source is obtained from the output voltage of the supercapacitor model of 2.851 V. The output is voltage and current, where the output current is used as the input of the supercapacitor, the simulation results are shown in Figure 4. From the whole system, the output value of the $V_c$ converter is 0.007114 V, the error value is obtained from the comparison of the set point with the $V_c$ output value of -0.007115 V, the duty cycle of 0.03729 which is located in the control section with a value of $K_p = 10^{-1}, T_i = 4e^{-3}$ and $T_d = 0$, by simulating the reference voltage is set from 1 V then at 5 seconds it drops to 0.25 V Figure 4. The output of the duty cycle is inputted to drive the PWM. Looks like the system is working properly. Where the output voltage can follow the reference.

![Simulation output results from the Buck Boost Converter circuit](image)

**Figure 4.** Simulation output results from the Buck Boost Converter circuit

4. **Conclusion**

From the simulation that has been done with Cmex S-Function on Simulink/Matlab. Then the final result shows that the system built is in accordance with its characteristics and has a response that resembles the original and the system can work well. Where the output voltage can follow the reference.

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