Evidence For SUSY From GUTS? 
Evidence For GUTS From SUSY!

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Abstract

We review the theoretical and experimental status of minimal grand unified theories (GUTS), contrasting the failure of minimal non-supersymmetric $SU(5)$ with the success of the minimal supersymmetric $SU(5)$ and minimal supersymmetric Flipped $SU(5) \times U(1)$ models. We show that a reasonable value for the universal soft supersymmetry-breaking gaugino mass, $45 \text{ GeV} < m_{1/2} < 1 \text{ TeV}$, and a $1 - \sigma$ range of the other inputs constrains the strong coupling, $\alpha_3(m_Z) > .114$. We define the supersymmetric standard model (SSM), the minimal supersymmetric extension of the standard model with gauge coupling unification and universal soft supersymmetry-breaking at the unification scale, as a baseline model for unified theories. We review the structure of the allowed parameter space of the SSM and suggest sparticle spectroscopy as the experimental means to determine the parameters of the SSM and search for departures from the baseline SSM.

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1. Overview

In addition to the major theoretical problem of the gauge hierarchy, minimal non-supersymmetric $SU(5)$ \cite{1} flunks the test of proton decay and $\sin^2 \theta_W(m_Z)$. Although supersymmetry technically solves the gauge hierarchy problem, a fine tuning problem re-emerges in minimal supersymmetric $SU(5)$ \cite{2} in splitting the triplet and doublet components of the five dimensional matter superfields in the model: the triplets must have GUT scale masses, while the doublets must remain light to yield electroweak scale symmetry breaking.

The prediction of minimal supersymmetric $SU(5)$ for $\sin^2 \theta_W(m_Z)$ matches so closely the experimental value \cite{3} that one may hope to constrain the spectrum of the model through the threshold contributions to this prediction \cite{4}\cite{5}. This shall be the main focus of this work.

Supersymmetrizing the minimal $SU(5)$ model increases the GUT scale, thus ensuring that the dimension six proton decay operators, which doomed the minimal non-supersymmetric theory, are no problem in the supersymmetric theory. However, dimension five operators give proton decay very near the experimental limit and sensitive to the details of the supersymmetric spectrum. This allows constraints to be placed on the supersymmetric spectrum of the model from the non-observation of proton decay \cite{6}. One study shows that the combined constraints of proton decay, naturalness, and a neutralino relic density smaller than the closure density severely constrains the model \cite{7}. However, another study finds that experimental values of the low-energy couplings allow a larger higgs triplet mass than had been previously considered, which considerably relaxes the constraints from proton decay \cite{8}. Similarly, as will be emphasized, the constraints from coupling constant unification are extremely sensitive to the experimental inputs and subtleties of the actual calculation. The two questions of proton decay and coupling constant are interdependent as both the light and heavy spectum of the model enter each prediction.

Besides the doublet-triplet splitting problem of minimal supersymmetric $SU(5)$, the model faces another challenge when one tries to reconcile it with the only available consistent theory of quantum gravity, the string. The adjoint representation needed to break the $SU(5)$ symmetry is unavailable in string theories at Kac-Moody level $k=1$\cite{9}. Though someday, realistic string models with higher Kac-Moody levels may be possible, there is a simpler and more elegant GUT whose particle spectrum is available in the string at level $k = 1$. This minimal supersymmetric Flipped $SU(5) \times U(1)$ GUT \cite{10} has a natural
doublet-triplet splitting mechanism which eliminates the fine-tuning problem and reduces
the effect of the dimension five proton-decay operators well below that of the dimension six
operators. The prediction for $\sin^2 \theta_W (m_Z)$ for supersymmetric $SU(5)$ becomes an upper
bound for the prediction of $\sin^2 \theta_W (m_Z)$ in minimal supersymmetric Flipped $SU(5) \times U(1)$
because of the extra scale. Thus, the constraints from proton decay and coupling constant
unification in the Flipped model are considerably less stringent than in the minimal su-
persymmetric $SU(5)$ model.

Although, coupling constant unification and proton decay rule out the minimal non-
supersymmetric $SU(5)$ GUT, this only constitutes circumstantial evidence for supersym-
metry. All of the problems of minimal non-supersymmetric $SU(5)$ could probably be fixed,
except perhaps the hierarchy problem, by appropriate non-supersymmetric extensions of
the minimal model. The real verification of supersymmetry will be direct observation of
sparticles.

To simplify extracting low-energy predictions of unified models, it is useful to eliminate
the model-dependent GUT structure and consider a minimal supersymmetric extension of
the standard model with coupling constant unification and universal soft supersymmetry
breaking at the unification scale. We refer to this model as the supersymmetric standard
model (SSM), which has been extensively studied (for a recent review see [11]). The low
energy-predictions of the SSM are very near those of supersymmetric $SU(5)$ and super-
symmetric Flipped $SU(5) \times U(1)$ and can be used as a baseline to search for departures
indicating a particular unified theory. The SSM has five unmeasured parameters, the
top mass $m_t$, the ratio of higgs vevs $\tan \beta$, and the three soft susy-breaking parameters
$m_{1/2}$, $m_0$, and $A$. The entire spectrum and S-matrix of this model can be calculated for
any point in this five-dimensional parameter space. Electroweak breaking and experiment-
al constraints then give a boundary between allowed and disallowed points in this five
dimensional parameter space.

The spectrum of sparticles corresponding to the two light generations of fermions has
a particularly simple structure which depends only on $\tan \beta$, $m_{1/2}$, and $m_0$. Measurement
of three sparticle masses determines the values of $\tan \beta$, $m_{1/2}$, and $m_0$ with a fractional
uncertainty for $m_{1/2}$ and $m_0$ comparable to that of the mass measurements [12]. The
discussion in the SDC talk at this conference of a 10% resolution for the gluino mass gives
a first indication that this program of sparticle spectroscopy may be feasible. Sooner or
later, we must face the necessity of verifying GUT scale physics with precision low-energy
experiments. In addition to looking for proton-decay, lepton flavour violation, and other
rare decays, the sparticle spectrum’s sensitivity to almost every detail of a theory makes it an ideal place to look for evidence for GUTS from SUSY.

The unification scale of the class of minimal supersymmetric theories like $SU(5)$ and $SU(5) \times U(1)$, whose low-energy predictions are close to that of the SSM, turns out to be about $10^{16}$ GeV. The string gives a coupling constant unification scale of about $10^{18}$ GeV [13]. One way of reconciling the discrepancy between the unification scale predicted by the string and that predicted by the SSM is to look for string models where moduli dependent corrections bring the string unification scale down to $10^{16}$ GeV [14]. Alternatively, adding massive vector representations to the SSM can bring its unification scale up to $10^{18}$ GeV. The minimal set of massive representations with standard-model-like quantum numbers which could accomplish this are a vector pair of quark doublets with mass of about $10^{13}$ GeV and a vector pair of right-handed down quarks with mass of about $10^5$ GeV. This model has been named the String Inspired Standard Model (SISM) [15]. Varying the mass of the two vector pairs allows $\sin^2 \theta_W(m_Z)$ and the unification scale to match respectively the experimental value and the string unification scale. The SISM represents the minimal particle content needed to do so and many other extra-vector models exist with more than this minimal content. The SISM depends on the same five parameters as the SSM and the SISM low energy predictions have been found to be qualitatively similar but quantitatively different than the predictions of the SSM [15].

Table 1. summarizes the various tests of the models discussed in this section.

2. What Is The Strong Coupling?

Coupling constant unification and the GUT scale depend on the value of the low energy couplings. All these inputs will be taken at $m_Z$ in the $\overline{MS}$ renormalization scheme. The $1 - \sigma$ values of the electromagnetic coupling and $\sin^2 \theta_W(m_Z)$ we use are:

$$\sin^2 \theta_W(m_Z) = 0.2328 \pm 0.0009 \quad [16]$$

$$\alpha_{em}(m_Z) = \frac{1}{127.9 \pm 0.2} \quad [17][18].$$

However, a glance at Table 2, which summarizes different determinations of the strong coupling, [19] indicates a problem with specifying the value of $\alpha_3(m_Z)$: although the individual measurements have fairly small errors, different determinations of the strong
coupling give very different results. These results fall into two main classes: the first five LEP measurements at \( m_Z \) which average to \( \alpha_3(m_Z) = .122 \), and the last three low energy measurements extrapolated to \( m_Z \) which average to \( \alpha_3(m_Z) = .109 \). One suggestion is that higher order QCD corrections to jet shapes reduce the high LEP measurements [20]. Another suggestion is that the gluino mass is in the swiftly shrinking light gluino window. If this were the case, the QCD beta function would include gluino contributions when running the low energy measurements up to \( m_Z \). Doing this brings the low-energy measurements of \( \alpha_3(m_Z) \) into amazing agreement with the LEP measurements [21]. Whatever the details of the explanation, it seems certain that the proper inclusion of the different radiative corrections to each type of measurement of the strong coupling is the key to resolving this problem. Because of this uncertainty, the results of this talk will be presented as bounds on the strong coupling.

3. Dimension Six Proton Decay

Table 3 summarizes the bounds from dimension six proton decay on the mass of the superheavy gauge bosons and the strong coupling using a limit on the partial lifetime of the proton

\[ \tau(p \to e^+\pi^0) < 5.5 \times 10^{32} \text{yr} \quad [22]. \]

The two values in Table 3 for each model correspond to the extremes of an order-of-magnitude uncertainty in the hadronic matrix element \( .003 < \alpha < .03 \). The minimal non-supersymmetric models are ruled out, while the minimal supersymmetric models have no trouble with dimension six proton decay. Note however that the parameter space of minimal supersymmetric \( SU(5) \) is severely constrained by dimension five proton decay [6]. In supersymmetric Flipped \( SU(5) \times U(1) \), the unification scale depends on both \( \alpha_3(m_Z) \) and \( \sin^2 \theta_W(m_Z) \) so the results must be presented in the \( \sin^2 \theta_W(m_Z), \alpha_3(m_Z) \) plane.

4. Constraints in the \( \sin^2 \theta_W(m_Z), \alpha_3(m_Z) \) Plane Without Thresholds

Figure 1 shows constraints in the \( \sin^2 \theta_W(m_Z), \alpha_3(m_Z) \) plane from proton decay and coupling constant unification without threshold effects. The two ellipses give \( 1 - \sigma \) experimental areas for the average \( \alpha_3(m_Z) = .122 \) of the first five high values of \( \alpha_3(m_Z) \) and the average \( \alpha_3(m_Z) = .109 \) of the last three low values of \( \alpha_3(m_Z) \) in Table 2. To be conservative, the errors of the average high and low \( \alpha_3(m_Z) \) ellipses have been taken as the
smallest error of the individual experiments, ±.005. The dark line shows the prediction for \( \sin^2 \theta_W(m_Z) \) from supersymmetric SU(5) without threshold effects. The width of the line corresponds to the uncertainty in the \( \alpha_{em}(m_Z) \). This solid line is an upper bound on \( \sin^2 \theta_W(m_Z) \) in the Flipped model. The two dotted lines represent a lower bound on \( \sin^2 \theta_W(m_Z) \) in the Flipped model from proton decay, with the top dotted line using a hadronic matrix element of \( \alpha = .03 \), and the bottom dotted line using a hadronic matrix element of \( \alpha = .003 \). The two lighter solid lines represent the prediction for \( \sin^2 \theta_W(m_Z) \) from non-supersymmetric SU(5), with the bottom line corresponding to a one higgs doublet model, and the top line corresponding to a two higgs doublet model. Both minimal non-supersymmetric SU(5) models are many \( \sigma \) off in their prediction for \( \sin^2 \theta_W(m_Z) \), while the two supersymmetric models are right on the money.

5. Threshold Corrections in the Minimal Supersymmetric SU(5) GUT

The prediction for \( \sin^2 \theta_W(m_Z) \) in minimal supersymmetric SU(5) may be written as

\[
\sin^2 \theta_W(m_Z) = 0.2 + \frac{7\alpha_{em}(m_Z)}{15\alpha_3(m_Z)} + 0.0029 + \delta_s(light) + \delta_s(heavy) + \delta_s(\text{conv}) \tag{5.1}
\]

where 0.0029 corrects the analytic one-loop calculation to two-loop accuracy, \( \delta_s(light) \) gives the correction from light thresholds, and \( \delta_s(heavy) \) gives the correction from heavy thresholds. The scheme conversion term \( \delta_s(\text{conv}) \) is negligible.

If, for a moment, we assume \( \delta_s(heavy) = 0 \) and simply parameterize the light fields by \( m_t \), half the higgs degrees of freedom at or below \( m_Z \), and the rest of the fields beyond the Standard Model in the SSM degenerate at a scale \( m_{SUSY} \), \( \delta_s(light) \) becomes

\[
\delta_s(light) = \frac{\alpha_{em}(m_Z)}{20\pi} \left[ -3\ln\left(\frac{m_t}{m_Z}\right) - \frac{19}{3} \ln\left(\frac{m_{SUSY}}{m_Z}\right) \right], \tag{5.2}
\]

and (5.1) can be solved for \( m_{SUSY} \) in terms of the other inputs.

Using the range of \( \sin^2 \theta_W(m_Z) \) and \( \alpha_{em}(m_Z) \) given in (2.1) and (2.2), and a 1 − \( \sigma \) global fit, 92 GeV < \( m_t < 147 \) GeV, from \( \sin^2 \theta_W(m_Z) \) [16] gives a range of allowed values for \( m_{SUSY} \) as a function of \( \alpha_3(m_Z) \) shown as the band between the two lines in Figure 2. Since the spectrum is not actually degenerate, what this really means is that there must be at least one field with mass in this range! This gives the remarkable conclusion that for \( \alpha_3(m_Z) > .118 \), there must be at least one new field with mass less than about 1 TeV.
With a general GUT structure, the sign and magnitude of \( \delta_s(\text{heavy}) \) is uncertain, and washes out this conclusion \[24\]. But, the minimal supersymmetric \( SU(5) \) GUT has a very simple heavy threshold contribution:

\[
\delta_s(\text{heavy}) = \frac{\alpha_{em}(m_Z)}{20\pi} \left[-6 \ln \left( \frac{M_{\text{GUT}}}{M_{Dc}} \right) + 4 \ln \left( \frac{M_{\text{GUT}}}{M_V} \right) + 2 \ln \left( \frac{M_{\text{GUT}}}{M_\Sigma} \right) \right], \tag{5.3}
\]

where \( M_{Dc}, M_V, M_\Sigma \) are the masses of the proton-decay mediating higgs triplet matter superfields, super-heavy gauge superfields, the uneaten remnants of the \( SU(5) \) adjoint matter superfield, and \( M_{\text{GUT}} = \text{max}(M_{Dc}, M_V, M_\Sigma) \) is the scale where the couplings become equal. Note that the only possibility of a negative contribution from \( \delta_s(\text{heavy}) \) is if \( M_{Dc} < M_{\text{GUT}} \). Since for reasonable values of \( \alpha_3(m_Z) \), \( M_{\text{GUT}} \) cannot be much greater than \( 10^{16} \) GeV, and \( M_{Dc} = 10^{16} \) GeV already gives substantial constraints on the SUSY spectrum, it is likely that \( M_{Dc} = M_{\text{GUT}} \), especially for small \( \alpha_3(m_Z) \) where \( M_{\text{GUT}} \) is even lower. Since the regions that will be eventually constrained are \( \alpha_3(m_Z) < .114 \), it is safe to take \( \delta_s(\text{heavy}) > 0 \). To effectively explore the possibility that \( M_{Dc} < M_{\text{GUT}} \) would require an analysis of the combined constraints on the supersymmetric spectrum from both proton decay and coupling constant unification.

The requirement that \( \delta_s(\text{heavy}) > 0 \) leaves only the lower bound on \( m_{\text{SUSY}} \) which translates into at least one field with mass above the lower line in Figure 2. For \( \alpha_3(m_Z) \) less than about 0.11, the graph shows that there must be at least one field with mass greater than 1 TeV. If one’s naturalness criteria forbids supersymmetric fields greater than 1 TeV, then \( \alpha_3(m_Z) < .11 \) is excluded in the minimal supersymmetric \( SU(5) \). This type of analysis \[4\] reveals the essential physics, and the following sections derive even tighter bounds on \( \alpha_3(m_Z) \) in the minimal supersymmetric \( SU(5) \) model by using a more detailed parameterization of the light thresholds.

6. An Explicit Parameterization of the Light Thresholds

The contribution to \( \sin^2\theta_W(m_Z) \) from light particle thresholds was derived in \[23\]:

\[
\delta_s(\text{light}) = \frac{\alpha_{em}(m_Z)}{20\pi} \left[-3 \ln \left( \frac{m_t}{m_Z} \right) + \frac{28}{3} \ln \left( \frac{c_\tilde{g} m_{1/2}}{m_Z} \right) - \frac{32}{3} \ln \left( \frac{c_\tilde{w} m_{1/2}}{m_Z} \right) - \ln \left( \frac{m_h}{m_Z} \right) - 4 \ln \left( \frac{\mu}{m_Z} \right) + \frac{4}{3} f(y, w) \right] \tag{6.1}
\]
where
\[ f(y, w) = \frac{15}{8} \ln(\sqrt{c_q + y}) - \frac{9}{4} \ln(\sqrt{c_{\bar{q}} + y}) + \frac{3}{2} \ln(\sqrt{c_{\bar{e}_t} + y}) - \frac{19}{48} \ln(\sqrt{c_{\bar{q}} + y + w}) - \frac{35}{48} \ln(\sqrt{c_{\bar{q}} + y - w}) \] (6.2)

and \( y \equiv (m_0/m_{1/2})^2 \), \( m_0 \) is a universal primordial supersymmetry-breaking spin-zero mass, \( w \) was defined in [23] and the logarithms should be set to zero if the threshold is below \( m_Z \).

With this parameterization for \( \delta_s(\text{light}) \) and taking \( \delta_s(\text{heavy}) > 0 \), in the region where \( m_{\tilde{w}} < m_Z < m_{\tilde{g}} \), (5.1) can be manipulated to yield
\[ \ln\left(\frac{m_{1/2}}{m_Z}\right) < \frac{1}{7} X - \frac{\pi}{\alpha_3} - \ln(c_{\tilde{g}}) \] (6.3)

where
\[ X = \frac{15\pi}{\alpha_{em}(m_Z)}(\sin^2 \theta_W(m_Z) - 0.209) + \frac{9}{4} \ln\left(\frac{m_t}{m_Z}\right) + 3 \ln\left(\frac{\mu}{m_Z}\right) + \frac{3}{4} \ln\left(\frac{m_h}{m_Z}\right) - f(y, w) \] (6.4)

Now consider the region where both the gluinos and the winos are heavier than \( m_Z \), in which case (5.1) gives:
\[ \ln\left(\frac{m_{1/2}}{m_Z}\right) > -X + \frac{7\pi}{\alpha_3} + 7 \ln(c_{\tilde{g}}) - 8 \ln(c_{\tilde{w}}) \] (6.5)

The most generous bounds from (6.3) and (6.5), result from maximizing \( X \), minimizing \( c_{\tilde{g}} \), and maximizing \( c_{\tilde{w}} \).

For physically-relevant values of \( w \) (those which give positive squared masses for the stop squarks), \( f(y, w) \) is minimized at \( w = -8(c_q + y)/27 \). With this value of \( w \), \( f(y, w) \) has one extremum, a maximum, at \( y = (c_{\bar{t}} c_{\bar{l}_r} + 2c_{\bar{l}_l} c_q - c_{\bar{t}_r} c_q)/(3c_{\bar{t}_l} - 2c_{\bar{l}_r} - c_q) \), and approaches -0.025 as \( y \) becomes very large. Since the values of the \( c' \)s that we encounter satisfy \( c_q > 1 > c_{\bar{t}_l}, c_{\bar{l}_r} \), the minimum of \( f(y, w) \) is indeed -0.025 for values of \( y > 0 \). We have numerically searched the region where the scalars are lighter than \( m_Z \) to verify that \( f(y, w) \) does not take smaller values in this region. This minimum value of \( f(y, w) = -0.025 \) maximizes \( X \).

Taking the extreme \( 1 - \sigma \) values \( \sin^2 \theta_W(m_Z) = 0.2337 \), \( m_t = 92 \text{GeV} \), \( \alpha_{em}(m_Z) = 1/128.1 \) and using a naturalness bound of 1 TeV for \( \mu \) and \( m_h \) gives a maximum numerical
value of $X = 195.0$. Note that the extreme values of $\sin^2 \theta_W(m_Z)$ and $m_\ell$ are correlated, and that the contribution of the squarks and sleptons represented by $f(y, w)$ is negligible. Table 4 shows the sensitivity of $X$ to the inputs: central values for $\sin^2 \theta_W(m_Z)$, $m_\ell$, and $\alpha_{em}(m_Z)$ give $X = 189.8$, and extreme $1 - \sigma$ values of the inputs give a minimum $X = 184.6$. However, because minimum values of $\delta_s(heavy)$ and $f(y, w)$ have been used to maximize $X$, only the upper bound $X < 195.0$ really matters.

The ratios of the gaugino masses to the universal soft supersymmetry-breaking gaugino mass are

$$c_\tilde{g} = \frac{\alpha_3(m_\tilde{g})}{\alpha(M_{GUT})} \quad c_\tilde{w} = \frac{\alpha_2(m_\tilde{w})}{\alpha(M_{GUT})}. \quad (6.6)$$

Approximating $c_\tilde{g}$ and $c_\tilde{w}$ at $m_Z$ without including threshold effects gives

$$c_\tilde{g} \approx 2.7 \quad c_\tilde{w} \approx 0.79, \quad (6.7)$$

which turns out to be a very bad assumption [25].

Putting all this together gives a bound on $m_{1/2}$ as a function of $\alpha_3(m_Z)$ excluding the parameter space in the minimal supersymmetric $SU(5)$ to the left of the line in Figure 3. To have a reasonable soft supersymmetry breaking scale $45 \text{ GeV} < m_{1/2} < 1 \text{ TeV}$, the strong coupling is bounded by $\alpha_3(m_Z) > .114$. The next section shows how a correct treatment of the gaugino masses effects this bound.

7. The EGM Effect

Since the gaugino masses should be computed using $c_\tilde{g}$ and $c_\tilde{w}$ evaluated at the gaugino mass, the numerical values used for the $c'$s in the previous section (6.7) become increasingly inaccurate for higher gaugino masses. This evolution of the gaugino mass (EGM) effect [25] and several other subtle points in the computation of gauge coupling unification have been extensively studied [26]. Since the gaugino masses where the $c'$s should be evaluated depends on the value of $m_{1/2}$, (3.3) and (3.5) must be evaluated iteratively, recomputing the $c'$s at each iteration. This can be simplified by realizing the bounds on $m_{1/2}$ remain rigorous using a minimum for $c_\tilde{g}$ and a maximum for $c_\tilde{w}$.

From the one-loop expression for renormalizing a coupling from $m_Z$ up to its gaugino threshold,

$$\alpha(m_{gaugino}) = \frac{\alpha(m_Z)}{1 - \frac{b}{2\pi} \ln\left(\frac{\alpha(m_{gaugino})m_{1/2}}{m_Za_U}\right)}, \quad (7.1)$$

8
we see that, for $b < 0$, $\alpha(m_{\text{gaugino}})$ increases with $b$. In order to minimize $c_{\tilde{g}}$, we want to use the minimum value of $b_3 = -7$ possible in the SSM below the gluino threshold. Similarly, to maximize $c_{\tilde{w}}$ we use the maximum value of $b_2 = -1/3$ possible below the wino threshold.

Fitting the results of an analytic one-loop calculation to a numeric two-loop calculations for central values gives [23]:

$$\frac{1}{\alpha_U} = \frac{3}{20\alpha_{em}(m_Z)} + \frac{3}{5\alpha_3(m_Z)} - 0.7 + \frac{1}{5\pi} \left[ 3\ln\left(\frac{m_{\tilde{g}}}{m_Z}\right) + \frac{1}{8}\ln\left(\frac{m_h}{m_Z}\right) + \frac{3}{8}\ln\left(\frac{m_{\tilde{t}}}{m_Z}\right) \right]$$

$$+ \frac{3}{8}\ln\left(\frac{m_{\tilde{t}}}{m_Z}\right) + \frac{17}{4}\ln\left(\frac{m_{\tilde{w}}}{m_Z}\right) + \frac{83}{48}\ln\left(\frac{m_{\tilde{d}}}{m_Z}\right) + \frac{1}{2}\ln\left(\frac{m_{\tilde{d}}}{m_Z}\right) + \frac{1}{2}\ln\left(\frac{\mu}{m_Z}\right)$$

where the stop squarks have been taken degenerate with the other squarks. Thus, $\alpha_U$ decreases with the thresholds. Taking upper bounds of 147 GeV on the top mass and 3 TeV on the other thresholds gives the range

$$\frac{3}{20\alpha_{em}(m_Z)} + \frac{3}{5\alpha_3(m_Z)} - 0.7 < \frac{1}{\alpha_U} < \frac{3}{20\alpha_{em}(m_Z)} + \frac{3}{5\alpha_3(m_Z)} + 1.4$$

for the coupling at the unification scale. Numerically, we find a slight variation of the solutions of (6.3) and (6.5) over this range of $\alpha_U$, with both values increasing with $\alpha_U$. Therefore, we use the maximum value in (6.3) and the minimum value in (7.3).

The results of this calculation are shown in Figure 4. The region to the left of the solid line is excluded in the minimal supersymmetric $SU(5)$. Bounds for $X_{\text{central}}$ and $X_{\text{max}}$ are shown as dashed and dotted lines for reference. To have a reasonable range for the universal soft supersymmetry-breaking gaugino mass, $45\text{GeV} < m_{1/2} < 1\text{TeV}$, between the horizontal dashed lines in Figure 4, the strong coupling is constrained by $\alpha_3(m_Z) > .114$. Note that the EGM effect modifies the slope of the bound for $m_{\tilde{w}} > m_Z$. However, this has little effect on the overall bound for $\alpha_3(m_Z)$ which comes from low $m_{1/2}$ regions where the EGM effect is small.

8. The Parameter Space of the SSM and Sparticle Spectroscopy

By cleverly using the constraints from electroweak symmetry breaking, the SSM can be described by five unknown parameters; $m_t$, $\tan\beta$, $m_{1/2}$, $m_0$, and $A$ [27]. The last two parameters are more conveniently expressed in terms of the dimensionless parameters $\xi_0 = m_0/m_{1/2}$ and $\xi_A = A/m_{1/2}$. The parameter space splits into two sections, one
for each sign of the higgs mixing parameter $\mu$, its magnitude determined by radiative breaking in terms of the other parameters. The light quark masses, KM elements, and gauge couplings, despite experimental errors, are considered as known. Furthermore, only the bottom and tau mass and the gauge couplings effect the supersymmetric spectrum significantly. This amazing simplification over a generic supersymmetric extension of the Standard Model results from the assumptions of universal soft supersymmetry-breaking at the unification scale in the SSM. Because the whole supersymmetric spectrum results from just five parameters, of which $m_t$ should soon be known, the model is extremely predictive.

For the yukawa couplings to remain perturbative up to the unification scale, $m_t$ must be less than about 190 GeV and $tan\beta$ less than about 50. Since radiative breaking requires $tan\beta > 1$ and experiment gives $m_t > 90$ GeV, the parameter space is completely bounded in $m_t$ and $tan\beta$. The values of the soft supersymmetry-breaking parameters can be bounded from above by naturalness [28], but the exact bounds remain somewhat a matter of taste.

To get a feeling for this parameter space, Figure 5 shows some representative slices [27]. In these figures, the solid line corresponds to $\xi_A = 0$, the dotted lines to $\xi_A = 1$, and the dashed lines to $\xi_A = -1$. Computer visualization can be used to show a three-dimensional slice of the parameter space, and even a four-dimensional slice as a movie. Some first attempts in this direction were seen at this conference.

The allowed parameter space of the SSM is huge. However, knowing a few sparticle masses would very quickly narrow it. The sparticles corresponding to the two light generations have a very simple dependence on only three of the SSM parameters.

$$m_{\tilde{p}}^2 = m_0^2 + c_{\tilde{p}}(m_{\tilde{p}})m_{1/2}^2 + 2\left[T_{3L} - \frac{3}{5}Y\tilde{p}tan^2\theta_W\right]m_W^2$$

Measurements of three sparticle masses can be translated into a determination of $tan\beta$, $m_{1/2}$, and $m_0$ with fractional uncertainties in the determination of $m_{1/2}$, and $m_0$ comparable to the fractional uncertainties of the sparticle masses [12].

Sufficiently accurate determination of more sparticle masses could be used to discriminate between different extensions of the SSM such as extensions of the Standard Model gauge group, additional Yukawas, generational-dependent extra heavy gauge bosons, and non-universal supersymmetry-breaking which all leave distinct imprints on the sparticle spectrum.
9. Conclusions

The success of minimal supersymmetric GUTS compared to the failure of minimal non-supersymmetric GUTS gives strong circumstantial evidence that a viable GUT should be supersymmetric. The interplay between naturalness, proton decay, and coupling constant unification provides a tool to constrain the parameter space of supersymmetric GUTS. These constraints begin to rule out areas in the minimal supersymmetric $SU(5)$ model. In particular, to have a reasonable range for the universal soft supersymmetry-breaking gaugino mass $45 \text{ GeV} < m_{1/2} < 1 \text{ TeV}$, the strong coupling is constrained by $\alpha_3(m_Z) > .114$.

The SSM, the minimal supersymmetric extension of the Standard Model with coupling constant unification and universal soft supersymmetry-breaking at the unification scale, closely reproduces the low-energy structure of supersymmetric GUTS. Five unknown parameters $m_t, \tan \beta, m_{1/2}, m_0,$ and $A$ specify the entire spectrum and S-matrix of the model. Electroweak symmetry breaking and experimental constraints can be used to produce a boundary between allowed and disallowed regions of parameter space in the model.

Measuring the masses of three sparticles corresponding to the two light generations could be used to experimentally extract the SSM values of $\tan \beta$, $m_{1/2}$, and $m_0$ with fractional uncertainty of $m_{1/2}$, and $m_0$ comparable to that of the mass determinations of the sparticles. Other spartner masses could be use to determine $A$ and check the consistency of the SSM for corrections due to a GUT structure which leave a distinct imprint on the sparticle spectrum regardless of the scale of extra physics.

Now that GUTS have revealed the need for SUSY, it is time to test GUTS by measuring SUSY!

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| Test                        | Non-Susy $SU(5)$ | Susy $SU(5)$ | Susy $SU(5) \times U(1)$ | SSM | SISM |
|-----------------------------|------------------|--------------|--------------------------|-----|------|
| $\sin^2 \theta_W(m_Z)$     | X                | ?            | √                        | ?   | √    |
| proton decay                | X                | ?            | √                        | √   | √    |
| $m_b/m_{\tau}$              | √                | √            | √                        | √   | √    |
| fine-tuning                 | X                | √            | √                        | √   | √    |
| doublet-triplet splitting   | -                | X            | √                        | -   | -    |
| $k=1$ string                | X                | X            | √                        | √   | √    |
| $M_U$                       | $\approx 10^{14}$| $\approx 10^{16}$| $\approx 10^{16}$       | $\approx 10^{16}$| $\approx 10^{18}$ |

*Table 1 - Comparison of minimal unified models.*
| Experiment     | Central Value | Error  |
|---------------|--------------|--------|
| ALEPH jets    | 0.125        | ±0.005 |
| DELPHI jets   | 0.113        | ±0.007 |
| L3 jets       | 0.125        | ±0.009 |
| OPAL jets     | 0.122        | ±0.006 |
| OPAL $\tau$   | 0.123        | ±0.007 |
| $J/\Psi$      | 0.108        | ±0.005 |
| $\Upsilon$    | 0.109        | ±0.005 |
| Deep Inelastic| 0.109        | ±0.005 |
| Average       | 0.116        | ±0.005 |

*Table 2 - Experimental Values of $\alpha_3(m_Z)$.***
| Model                     | $M_X : \alpha = 0.03$ | $M_X : \alpha = 0.03$ | $\alpha_3 : \alpha = 0.03$ | $\alpha_3 : \alpha = 0.03$ |
|--------------------------|-----------------------|-----------------------|-----------------------------|-----------------------------|
| Non-Susy $SU(5) : N_h = 1$ | $M_X > 1.1 \times 10^{15} GeV$ | $M_X > 3.6 \times 10^{15} GeV$ | $\alpha_3 > 0.140$ | $\alpha_3 > 0.180$ |
| Non-Susy $SU(5) : N_h = 2$ | $M_X > 1.1 \times 10^{15} GeV$ | $M_X > 3.6 \times 10^{15} GeV$ | $\alpha_3 > 0.153$ | $\alpha_3 > 0.203$ |
| Susy $SU(5)$             | $M_X > 1.5 \times 10^{15} GeV$ | $M_X > 4.7 \times 10^{15} GeV$ | $\alpha_3 > 0.088$ | $\alpha_3 > 0.100$ |
| Susy $SU(5) \times U(1)$ | $M_X > 1.0 \times 10^{15} GeV$ | $M_X > 3.2 \times 10^{15} GeV$ | see graph                 | see graph                   |

*Table 3 - Limits from dimension six proton decay: $\tau(p \rightarrow e^+ \pi^0) > 5.5 \times 10^{32} yr$*
### Table 4 - Sensitivity of $X$ to various inputs:

$\mu, m_h = 1$ TeV unless otherwise stated.
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Figure Captions

Fig. 1. Figure 1 shows constraints in the $\sin^2 \theta_W(m_Z)$, $\alpha_3(m_Z)$ plane from proton decay and coupling constant unification without threshold effects. The two ellipses give $1 - \sigma$ experimental areas for the average $\alpha_3(m_Z) = .122$ of the first five high values of $\alpha_3(m_Z)$ and the average $\alpha_3(m_Z) = .109$ of the last three low values of $\alpha_3(m_Z)$ given in Table 2. The dark line shows the prediction for $\sin^2 \theta_W(m_Z)$ from supersymmetric $SU(5)$ without threshold effects. The width of the line corresponds to the uncertainty in $\alpha_{em}(m_Z)$. This dark line is an upper bound on $\sin^2 \theta_W(m_Z)$ in the Flipped model. The two dotted lines represent a lower bound on $\sin^2 \theta_W(m_Z)$ in the Flipped model from proton decay, with the top dotted line using a hadronic matrix element of $\alpha = .03$, and the bottom dotted line using a hadronic matrix element of $\alpha = .003$. The two lighter solid lines represent the prediction from non-supersymmetric $SU(5)$, with the bottom line corresponding to a one higgs doublet model, and the top line corresponding to a two higgs doublet model.

Fig. 2. The prediction for the average supersymmetric mass scale as a function of $\alpha_3(m_Z)$ assuming $\delta_s(heavy) = 0$ is shown as the area between the two lines. At least one field must have mass in this range. For $\delta_s(heavy) > 0$, only the lower bound remains. To ensure that no field has mass above the 1 TeV dashed line, $\alpha_3(m_Z)$ must be above about 0.11.

Fig. 3. An explicit parameterization of the light thresholds neglecting the EGM effect excludes the parameter space to the left of the solid line which gives the prediction for $m_{1/2}$ as a function of $\alpha_3(m_Z)$ for the $X_{max}$ given in Table 4. A reasonable value for the soft supersymmetry-breaking gaugino mass represented by the dashed lines, $45\text{ GeV} < m_{1/2} < 1\text{ TeV}$, and a $1 - \sigma$ range of the other inputs constrains the strong coupling, $\alpha_3(m_Z) > .114$.

Fig. 4. Same as Figure 3, but including the EGM effect. The main effect is to modify the slope of the boundary for high values of $m_{1/2}$ loosening the bound in those regions. Bounds for other values of $X$ summarized in Table 4 are shown as dashed and dotted lines for reference.

Fig. 5. One-loop boundaries for the SSM with all consistency and phenomenological cuts imposed for $m_{1/2} = 150, 250\text{ GeV}$, both signs of $\mu$ and the following $(\xi_0, \xi_A)$ values: (a) $(0,-1)(\text{dashed})$, $(0,0)(\text{solid})$, $(0,1)(\text{dotted})$; (b) $(1,-1)(\text{dashed})$, $(1,0)(\text{solid})$, $(1,1)(\text{dotted})$. The figures show the progression of the left boundary to higher values of $m_t$ due to the unit variations of $\xi_0$ and $\xi_A$.