PREDICTING A THIRD PLANET IN THE KEPLER-47 CIRCUMBINARY SYSTEM

Tobias C. Hinse\textsuperscript{1,2}, Nader Haghighipour\textsuperscript{3}, Veselin B. Kostov\textsuperscript{4}, and Krzysztof Goździewski\textsuperscript{5}

\textsuperscript{1} Korea Astronomy \& Space Science Institute, Korea; tchinse@gmail.com
\textsuperscript{2} Armagh Observatory, College Hill, BT61 9DG, UK
\textsuperscript{3} Institute for Astronomy, University of Hawaii-Manoa, Honolulu, HI, USA
\textsuperscript{4} Johns Hopkins University, Baltimore, MD, USA
\textsuperscript{5} Toruń Centre for Astronomy of the Nicolaus Copernicus University, Grudziadzka 5, Poland

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ABSTRACT

We have studied the possibility that a third circumbinary planet in the Kepler-47 planetary system is the source of the single unexplained transiting event reported during the discovery of these planets. We applied the MEGNO technique to identify regions in the phase space where a third planet can maintain quasi-periodic orbits, and assessed the long-term stability of the three-planet system by integrating the entire five bodies (binary + planets) for 10 Myr. We identified several stable regions between the two known planets as well as a region beyond the orbit of Kepler-47c where the orbit of the third planet could be stable. To constrain the orbit of this planet, we used the measured duration of the unexplained transit event (∼4.15 hr) and compared that with the transit duration of the third planet in an ensemble of stable orbits. To remove the degeneracy among the orbits with similar transit durations, we considered the planet to be in a circular orbit and calculated its period analytically. The latter places an upper limit of 424 days on the orbital period of the third planet. Our analysis suggests that if the unexplained transit event detected during the discovery of the Kepler-47 circumbinary system is due to a planetary object, this planet will be in a low eccentricity orbit with a semi-major axis smaller than 1.24 AU. Further constraining of the mass and orbital elements of this planet requires a re-analysis of the entire currently available data, including those obtained post-announcement of the discovery of this system. We present details of our methodology and discuss the implication of the results.

Key words: binaries: eclipsing – celestial mechanics – chaos – planets and satellites: dynamical evolution and stability – stars: individual (Kepler-47)

1. INTRODUCTION

During the past few years, the Kepler telescope has discovered several planets in circumbinary orbits. All these planets have been detected photometrically, exhibiting transit signatures when passing in front of the stars of the binary. The first of these circumbinary planets (CBPs) was Kepler-16b, discovered by Doyle et al. (2011). Since then several more Kepler CBPs have been discovered, namely Kepler-38b (Orosz et al. 2012a), Kepler-34b and Kepler-35b (Welsh et al. 2012), Kepler-47b\textsuperscript{c} (Orosz et al. 2012b), Kepler-64b (Kostov et al. 2013; Schwamb et al. 2013), Kepler-413b (KIC 12351927b) (Kostov et al. 2014), and KIC 9632895b (Welsh et al. 2014).

Among the currently known Kepler circumbinary planetary systems, Kepler-47 presents an interesting case. The fact that this system harbors two planets is a strong indication that, similar to planet formation around single stars, circumbinary planets can also form in multiples. The latter implies that more planets may exist in any of the currently known circumbinary systems. In the Kepler-47 system, the existence of a third planet was speculated in its 2012 discovery paper as a way to account for an unexplained feature observed in the light curve of this binary (Orosz et al. 2012b). As reported by these authors, a single, 0.2% deep transit event was detected that could not be explained by the two known transiting planets.

In this paper, we test the above-mentioned hypothesis. Our approach is to study the dynamics of the three-planet system, and use long-term stability to identify the viable regions where the orbit of the third planet would be stable. Using dynamical stability to predict additional planets has been presented in several other studies (Barnes & Raymond 2004; Raymond & Barnes 2005; Fang & Margot 2012). In cases where stable regions are identified, we use transit timing and transit duration variations to constrain the orbit of the third planet.

The dynamics and orbital stability of planets in circumbinary orbits have been the subject of studies for nearly three decades. Dvorak (1986), Rabl & Dvorak (1988), and Holman & Wiegert (1999) carried out long-term orbital integrations of test-particles aiming at exploring a large area of the system’s parameter space. In particular, for P-type orbits, the authors established stability criteria for a planet’s semi-major axis as a function of the binary orbital parameters and mass ratio. Musielak et al. (2005) and Eberle et al. (2008) also studied the stability of planetary orbits in P-type systems and presented criteria for stable, marginally stable, and unstable circular planetary orbits. Recent analytic analysis of the dynamics of circumbinary planets has also been presented by Doolin & Blundell (2011) and Leung & Lee (2013).

This paper is structured as follows. In Section 2, we briefly review the Kepler-47 system as described by Orosz et al. (2012b). In Section 3, we describe our numerical techniques, and in Section 4, we present the results of our stability analysis using the chaos indicator MEGNO (Mean Exponential Growth factor of Nearby Orbits). In Section 5, we calculate the transit durations of Kepler-47b and the candidate third planet, and compare them with their measured value as reported by Orosz et al. (2012b) to constrain the orbit of the third planet. Finally, in Section 6, we conclude this study by presenting a summary and discussing the implications of the results.

2. THE KEPLER-47 SYSTEM

Kepler-47 is a single-lined spectroscopic binary with a 1.043 $M_\odot$ primary and a secondary with a mass of 0.362 $M_\odot$. 

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Table 1
Orbital Parameters of the Kepler-47 System and their 1σ Uncertainties (Orosz et al. 2012b)

| Parameter                  | Kepler-47 (Star B) | Kepler-47b | Kepler-47c |
|----------------------------|--------------------|------------|------------|
| Semi-major axis (AU)       | 0.0836 ± 0.0014    | 0.2956 ± 0.0047 | 0.989 ± 0.016 |
| Eccentricity               | 0.0234 ± 0.001     | 0.034      | 0.41       |
| Inclination (deg)          | 89.34 ± 0.12       | 89.59 ± 0.50 | 89.826 ± 0.010 |
| Argument of pericenter (deg)| 212.3 ± 4.4      | 0.0 (fixed) | 0.0 (fixed) |
| Longitude of node (deg)    | 0.0 (fixed)        | 0.0 (fixed) | 0.0 (fixed) |
| Mean anomaly (deg)         | 0.0 (fixed)        | 0.0 (fixed) | 0.0 (fixed) |
| Orbital period (days)      | ~7.5               | ~49.5      | ~303.2     |
| Mass                       | 0.362 M☉          | 10 M☉      | 23 M☉      |
| Radius                     | 0.3506 R☉ ± 0.0063 | 2.98 R☉ ± 0.12 | 4.61 R☉ ± 0.20 |

Notes. The inclination of the third planet was chosen to be equal to the inclination of Kepler-47c, and the remaining angles were set to zero. Orbital elements indicated by “(fixed)” have been undetermined from observations. The mass parameter \( \mu = k^2(M_1 + M_2 + m_i) \) has been used for a given planet with mass \( m_i \) when transforming elements. Symbols \( R_* \) and \( M_* \) denote the radius and mass of Earth. The mass of the primary star (Star A) is 1.043 M☉.

The period of this binary is 7.5 days. In 2012, Orosz et al. announced the detection of two planets in circumbinary orbits around this system. These authors analyzed long- and short-cadence photometric data from the Kepler space telescope, spanning 1050.5 days from Quarter 1 to 12, and identified 18 transit events of the inner planet (Kepler-47b) and three for the outer planet (Kepler-47c). Table 1 shows the published (oscillating) orbital parameters of Kepler-47 and its two planets. We note that because of observational degeneracies, not all orbital parameters can be determined from the photodynamical\(^6\) model as described in Orosz et al. (2012b). We also note that all orbital elements are in the (geometric) Jacobian coordinate system.

The inner planet, Kepler-47b, has a period of 50 days and is the smaller of the two, with a radius of ~3 Earth-radii. Orosz et al. (2012b) estimated that the mass of this planet is ~7–10 Earth-masses. Due to the nondetection of ETVs, an upper limit of two Jupiter-masses can firmly be established for this object. The outer planet, Kepler-47c, has an orbital period of ~303 days with a ~4.6 Earth-radii. The authors estimated a plausible mass in the range 16–23 Earth-masses. The upper limit for the mass of this planet was determined to be 28 Jupiter-masses.

In their analysis of the light curve of Kepler-47 system, Orosz et al. (2012b) detected a single transit event that could not be explained by the transits of the two planets. With a formal significance of 10.5σ, this transit event occurred at BJD 2,455,977.363 ± 0.004, approximately 12 hr after the last transit of Kepler-47b in the Q12 data set. The duration of this transit was observed to be ~4.15 hr. Orosz et al. (2012b) suggested that if this transit event was due to a third planet, given its depth of 0.2%, the planet must have a radius ~4.5 Earth-radii.

The rest of this paper is devoted to examining this hypothesis using dynamical considerations. Integrating the five-body system of the binary and three planets, we will determine the ranges of the parameters for which the orbit of a hypothetical third planet will be stable. Using the properties of the above-mentioned single transit event, we will also identify the most probable regions around the binary where the orbit of this planet may exist.

3. METHODOLOGY AND NUMERICAL TECHNIQUES

We note that results presented in this work have been obtained from direct numerical integration of the equations of motion using the initial conditions shown in Table 1. The orbit of the binary system is fully resolved and the three planets are treated as massless as well as massive objects.

We adopted two different algorithms for solving equations of motion: the IDL implementation of the Livermore Solver (LSODE), which is an adaptive algorithm with a step-size control, and an accurate extrapolation method implemented as the Gragg–Bulirsch–Stoer (GBS) algorithm (the ODEX code; Hairer et al. 1993). The latter is frequently used in celestial mechanics and orbit calculations (see Gożdziewski et al. 2012, 2013; Gożdziewski & Migaszewski 2014 and references therein). Both algorithms use a relative and absolute error tolerance parameter to control the integration accuracy. We set these parameters to one part in 10\(^{15}\). When integrating several test orbits of the five-body problem, we obtained identical results using both algorithms.

We would like to note that, when transforming orbital elements, we use Jacobi-like coordinates with a mass-parameter \( \mu = k^2(M_1 + M_2 + m_i) \) for each planet. Here \( m_i \) is the mass of the \( i \)th planet, \( M_1 \) and \( M_2 \) represent the masses of the binary stars, and \( k \) is the Gauss gravitational constant. The transformed orbital elements are then given relative to the center of mass of the binary system. This approach differs from the usual definition of Jacobi elements, where the position and velocity of a planet are given relative to the center of mass of all remaining massive bodies within its orbit.

Our stability analysis employs the well-established fast chaos indicator MEGNO technique (Cincotta & Simó 1999, 2000; Cincotta et al. 2003), which enables us to explore the phase space topology of the system. The MEGNO technique has found widespread applications within dynamical astronomy (Gożdziewski et al. 2001, 2008; Gożdziewski 2003; Hinse et al. 2010; Kostov et al. 2013) and is closely related to the Fast Lyapunov Indicator (FLI; Mestre et al. 2011). In brief, MEGNO, shown by \( \langle Y \rangle \) here and throughout the paper, has the following properties. For initial conditions resulting in quasi-periodic orbits, \( \langle Y \rangle \rightarrow 2.0 \) for \( t \rightarrow \infty \). For chaotic orbits, \( \langle Y \rangle \simeq 2\lambda/t \) for \( t \rightarrow \infty \) where \( \lambda \) is the Maximum Lyapunov Exponent (MLE). In the simulations presented in this paper, we chose to stop a given integration when \( \langle Y \rangle > 5 \). Orbits with quasi-periodic time evolution usually assume values of \( |\langle Y \rangle - 2| < 0.001 \) at the end of the numerical integration. We used the MEGNO implementation within the MECHANIC package (Slonina et al. 2015), and considered the integration time for each orbit to be \( 4.7 \times 10^4 \) binary periods.

\(^6\) https://github.com/dfm/photodynam
In a multi-planet system, different types of perturbation affect the dynamical evolution of the system on different timescales. In general, there is a timescale associated with the effect of short-term MMRs, a timescale due to secular resonances (slow variation of orbital elements), and a longer evolutionary timescale due to the tidal effects, mass-loss, and other weak perturbations. When using a fast stability indicator such as MEGNO, the stability of a particular orbital configuration must be presented in the context of these timescales because the properties of a given solution in the phase space must be determined on a small length of the orbit. In a compact configuration, for instance when the third planet is between Kepler-47b and c, the dynamics of the system are mainly affected by the short-term two- and three-body MMRs. Similar numerical experiments, such as those by Goździewski & Migaszewski (2014) and references therein, suggest that in a study like the one presented in this paper, when the value of MEGNO converges to 2 (indicating a quasi-periodic orbit), the stability of the system (the orbit-crossing time) is guaranteed for a time 2–3 orders of magnitude longer than the MEGNO integration timespan. In the case of the Kepler-47 system, the crossing time is longer than at least $\sim 5 \times 10^7$ binary period. The protecting effect of MMRs implies that in some regions, such as a number of orbits in the islands of quasi-periodicity at $a_d < 1.5$ AU (Figure 1), the entire five-body system will be stable for even longer times (e.g., as long as the lifetime of the system’s stars).

It is important to note that when longer-period, multi-body, mean-motion or secular resonances exist, the convergence of MEGNO for relatively short integration times does not necessarily imply stability for the lifetime of the binary. The perturbations from these resonances may result in dynamical instability over tens of millions or billions of years (e.g., Laskar & Gastineau 2009). This means that either the MEGNO integrations must cover a significant fraction of the secular timescales, or direct numerical integrations have to be carried out in order to ensure that the system is stable over the timespan of interest.

Despite the above-mentioned shortcoming, MEGNO integrations can still provide accurate and reliable characterization of the phase space with a very small CPU overhead. They are also very useful in obtaining rough stability limits. In Figure 1, we plotted such limits (black curves) around the orbit-crossing curves of the third planet with Kepler-47b and c (red curves) following the semi-empirical stability criterion presented by Giuppone et al. (2013). The stability criterion by these authors was derived from the Wisdom’s stability criterion for the restricted three-body problem. Unfortunately, this criterion does not seem to properly determine the limits of stable regions in the Kepler-47 system. For instance, as will be shown later, the interesting region of $0.5(AU) \leq a_d \leq 1.5(AU)$ is considered to be unstable when using the stability criterion by Giuppone et al. (2013). However, as our MEGNO maps show (see also Figure 2), this region contains a set of MMRs where the entire five-body system can be stable. A similar argument applies to the region immediately beyond the orbit of Kepler-47c. As indicated by our calculations, a few islands of stability exist in this area that correspond to two-body MMRs (e.g., 3d:4c, 2d:3c and 4d:7c), whereas according to the stability criterion by Giuppone et al. (2013), this region is unstable.

In Figure 2, we show these regions in more detail. In each panel, we label each quasi-periodic island with its associated two-body, mean-motion resonance. Tables 2 and 3 give the values of the inner and outer semi-major axes of the third planet, indicating the width of each resonance valid for the third planet on a circular orbit.

From all the orbits shown in Figure 2, we chose 11 (labeled IC1 to IC11, standing for Initial Conditions 1 to 11) and studied their long-term dynamical evolution by integrating them for $10^7$ yr. Results for IC1 and IC2 are shown in Figure 3. In order to highlight the details of the evolution of these orbits, we only show the results of the first 10,000 yr of integrations. As shown here, both orbits have identical initial eccentricities (0.01). However, their initial semi-major axes are different by $\Delta a = 0.0127$ AU. Results indicated that despite such a small difference in initial conditions, the evolutions of these two orbits are profoundly different. A chaotic characteristic is clearly
visible for IC2, which exhibits a random walk in semi-major axis and eccentricity. This random walk is an indication of a stochastic process over time, slowly destabilizing the orbit by driving the orbital eccentricity to unity. We find the third planet to be eventually ejected from the system after some 190,000 yr. For IC1, the third planet follows a bound stable quasi-periodic orbit over at least $10^7$ yr, and shows no sign of chaotic diffusion. The orbital evolution of the two known planets of the system, Kepler-47b and Kepler-47c (both with their assigned masses), show similar dynamical behavior with their orbits bound between a minimum and maximum value for their orbital elements.

### Table 2

| Resonance   | $a_{\text{inner}}$ (AU) | $a_{\text{outer}}$ (AU) |
|-------------|-------------------------|--------------------------|
| 4c:7d       | 0.677                   | 0.680                    |
| 3c:5d       | 0.700                   | 0.704                    |
| 5c:8d       | 0.720                   | 0.723                    |
| 2c:3d       | 0.740                   | 0.757                    |
| 5c:7d       | 0.786                   | 0.791                    |
| 3c:4d       | 0.811                   | 0.818                    |
| 7c:9d       | 0.833                   | 0.835                    |
| 4c:5d       | 0.847                   | 0.853                    |
| 5c:6d       | 0.871                   | 0.875                    |
| 6c:7d       | 0.888                   | 0.893                    |

**Notes.** Quantities $a_{\text{inner}}$ and $a_{\text{outer}}$ denote the inner and the outer boundary of the quasi-periodic regions shown in Figure 2 for a circular orbit.

### Table 3

| Resonance   | $a_{\text{inner}}$ (AU) | $a_{\text{outer}}$ (AU) |
|-------------|-------------------------|--------------------------|
| 3d:4c       | 1.189                   | 1.199                    |
| 2d:3c       | 1.279                   | 1.303                    |
| 5d:8c       | 1.346                   | 1.352                    |
| 3d:5c       | 1.385                   | 1.386                    |
| 4d:7c       | 1.424                   | 1.437                    |
| 5d:9c       | 1.471                   | 1.478                    |
| 1d:2c       | 1.538                   | 1.595                    |

**Notes.** Mean-motion resonances with semi-major axes larger than 1.6 AU have not been included.

### 4.1. Identification of Two-Body Resonances

In Figure 2, we labeled the islands of quasi-periodic orbits with their associated mean-motion resonances. To illustrate the true resonant character of these orbits, we calculated their critical arguments (resonant angle) using

$$\theta_{md:lc} = l\lambda_c - m\lambda_d - n\varpi_c - p\varpi_d.$$  \hspace{1cm} (1)

In this equation, $\lambda$ and $\varpi$ denote the mean longitude and longitude of pericenter, respectively, and coefficients $(l, m, n, p)$ are integer numbers satisfying d’Alambert rule. We found that the resonant angle corresponding to each quasi-periodic island exhibits a clear librational behavior around zero degrees. We show the time evolution of the resonant angle for a few selected initial conditions (IC1, IC3, IC4 in the region between Kepler-47b and c, and IC8 beyond the orbit of Kepler-47c) in Figure 4. As shown in this figure, the variation of the resonance angle for IC8 has larger amplitudes. The MMR-locking is deep in all cases and supports the interpretation that the islands of quasi-periodic orbits in the MEGNO dynamical maps are associated with mean-motion resonances. These results also have encouraging implications for the possible detection of a third planet, since MMRs can lead to an amplified TTV signal compared to those of the orbits in non-resonant configurations (Agol et al. 2005, 2007; Haghighipour & Kirste 2011).

### 4.2. Predicting the Mass of the Third Planet

The results shown in the previous section enabled us to identify regions of the parameter space where a test particle could maintain a stable orbit alongside the two massive planets,
Figure 3. Time evolution of (Jacobian) orbital element of a third massless companion considering the initial conditions IC1 (left) and IC2 (right). Both orbits have initial eccentricity of 0.01. However, for IC2, the initial semi-major axis of the planet is slightly larger at 0.7657 AU.

Kepler-47b and c. In reality, however, the third planet is also massive and its interaction with other planets can alter the orbital architecture of the system. In the following, we explore the dynamical characteristics of the system by considering the third planet to be massive as well.

As stated by Orosz et al. (2012b), the mass of the putative third planet is practically unconstrained. However, the depth of its single transit points toward a planet with a radius of $\simeq 5$ Earth-radii. Because of the lack of a precise model for the density and interior of such planets, we consider a range of values for the mass of the third planet from 0 to 150 Earth-masses.

Figures 5 and 6 show the results of the calculation of MEGNO for the entire five-body system, considering the third planet to have a mass of 0, 10, 23, and 100 Earth-masses. We probed the semi-major axis of the third planet in the interval of 0.1 AU to 1 AU in Figure 5 and between 1 AU and 3 AU in Figure 6. In all maps, we considered the orbital eccentricity of the third planet to vary in the range of 0–0.5. As shown in these figures, there are regions between the planets Kepler-47b and c where the third planet could have a stable orbit. These figures also show that the change in the mass of the third planet does not seem to have a significant impact on the overall dynamical structure of the
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Figure 4. Time evolution of the resonant angle ($\theta$) for a selection of initial conditions (IC1, IC3, IC4 and IC8). In all cases we find the resonant argument to librate around zero. This finding supports the results obtained from MEGNO confirming the correct identification of quasi-periodic MMRs in the semi-major axis–eccentricity space of the third planet.

We also noticed that some of the mean-motion resonances around 0.8 AU vanished when considering the third planet to be of 100 Earth-masses. For such a high value of the mass, the locations of quasi-periodic mean-motion resonances between the third planet and Kepler-47c appear to have shifted to smaller semi-major axes. The effect of changing the values of the mass of a planet on the location of mean-motion resonances has been illustrated in detail by Goździewski et al. (2013) in their study of the system of \( \nu \) Octantis. In general, resonances may become wider, narrower, or even split if the masses of the planets change. Diminished quasi-periodic regions exterior to the orbit of Kepler-47c were also observed when the third planet was taken to be 100 Earth-masses. It seems that low-eccentricity, co-orbital, quasi-periodic orbits are more likely when the third planet has a higher mass.

Results shown in Figures 5 and 6 suggest that a third planet in a low-eccentricity orbit can be stable in the region between Kepler-47b and c, as well as for semi-major axes larger than 1.75 AU. The main requirement for long-term orbital stability is set by the third planet’s pericenter distance. In order for the planet to be stable, this distance has to be well-separated from Kepler-47b and c (hence its low-eccentricity orbit), so that the gravitational perturbation of these bodies cannot alter the dynamics of the planet.

To examine the long-term stability of the third planet, we carried out two single-orbit integrations for 10^7 yr. We considered the third planet to be 50 Earth-masses and chose its initial semi-major axis and eccentricity the same as in IC3 in Figure 2 (the third planet being interior to Kepler-47c) and IC6 in Figure 2 (the third planet being exterior to Kepler-47c). Figure 7 shows the result. As shown here, no sign of chaotic diffusion is observed for any values of the orbital elements of the third planet. This suggests that a third planet with a mass of 30 Earth-masses or smaller can have a stable orbit either between the two known planets (IC3) or in an orbit exterior to Kepler-47c (IC6). The orbits of planets b and c exhibit a very weak signature of chaotic dynamics. In both simulations, the orbits of these planets remained bound and did not show any sign of a random walk.

We carried out similar simulations for higher values of the mass of the third planet. We found that a simulation with initial conditions IC3 results in chaotic and unstable orbits when the mass of the third planet is as high as 150 Earth-masses. However, when starting the third planet with initial conditions IC6, we found that larger masses are allowed, rendering an overall stable configuration.

5. ANALYSIS OF TRANSIT TIMING AND TRANSIT DURATION VARIATIONS

As shown in the previous section, dynamical considerations point to regions in the Kepler-47 two-planet system where a third body can have a stable orbit. However, this orbit is unconstrained. In this section, we use the measured time and duration of transits in the Kepler-47 system to constrain the orbit of the third planet.

To calculate the values of TTV and TDV of a planet, we use the same numerical integration algorithm that was used in generating MEGNO maps. We integrate the full five-body system using the values of the masses and radii of the stars.
Figure 5. Dynamical MEGNO maps of the third planet for different values of its mass. Integrations were carried out considering the full five-body system. The two planets, Kepler-47b and c, are shown by black circles. Initial orbital elements of these two planets were taken from Table 1. Color-coding is the same as in Figure 1.

Figure 6. Same as Figure 5, but probing the region exterior to the orbit of Kepler-47c. Color-coding is the same as in Figure 1.
Figure 7. Time evolution of the (Jacobian) orbital element of the third planet considering initial conditions IC3 (left; third planet between Kepler-47b and c) and IC6 (right; third planet exterior to Kepler-47c). In both cases, the mass of the third planet is 50 Earth-masses.

and planets, as well as the orbital elements of the two known planets as given in Table 1. During the integration, we monitor the on-sky projected position of the planet \( r_{sky} \) relative to the primary star. Once the planet has crossed the north–south axis passing through the center of the star, we determine the time of ingress \( t_1 \) from a series of back- and forth-integrations. At each integration step, the quantity \( r_{sky} \) is compared to the sum of the star and planet radii \( R_A + R_{pl} \). In each iteration, our algorithm decreases the time-step by a third in order to ensure convergence. The time of ingress is defined when \( r_{sky} - (R_A + R_{pl}) < 10^{-15} \).

By reversing the velocities of all bodies, the egress time \( t_2 \) is determined. The mid-transit time is then calculated from \( t_1 + (t_2 - t_1)/2 \). We use linear regression to calculate the variations in the transit times (TTV). The transit duration (TDV) is calculated using \( t_2 - t_1 \). We tested the calculation of TTV by reproducing the results given in Nesvorný & Morbidelli (2008). Figure 8 shows the results for Kepler-47b. We considered the third planet to have one Earth-mass and started it at IC6. The top three panels in Figure 8 show the resulting variations in the semi-major axis, eccentricity, and inclination. The bottom two
panels show the values of the TTV and TDV. Here we connected consecutive transit events with a solid line. The vertical lines show the cycle numbers for which the first or last transit is detected while being part of a consecutive transit series. An example of a consecutive transit series is the transits 59–105. As shown in the bottom two panels, there are two gaps between transits 22 and 59, and between transits 105 and 143 (where there is missing data). In these gaps, isolated timing measurements (shown by plus signs) represent non-consecutive transit events. In other words, for those TTV/TDV data points that are not connected, the planet did not transit the star before and after the given data point. For instance, we see from Figure 8 that there are four isolated near-miss transits after the transit cycle 22. There are missing transits in all gaps, which have not been plotted for the obvious reason. Such missed transit events were also observed in the light curve of Kepler-413b (Kostov et al. 2014). The only explanation for such single-transit or near-miss events is the short-term variation in the planet’s orbital elements. As shown in Figure 8, the orbital elements of Kepler-47b do indeed undergo significant changes from one transit to another, suggesting that the gaps are due to the low orbital inclination of Kepler-47b relative to the plane of the sky. We recall that the closer the inclination is to 90°, the closer the planet will be transiting along the star’s equator. Therefore, for the specific geometry of the Kepler-47 system, the projected orbital plane of Kepler-47b on the plane of the sky will be outside the star’s disk ($r_{\text{sky}} > R_{\text{pl}}$), and therefore, no timing measurements can be computed. As a result, such events do not appear in Figure 8. However, for a given value of the orbital inclination, we do obtain single transit events and alternating missing transits when Kepler-47b is close to the edge of the star.

Interestingly, we see no near-miss or isolated transit prior to transit 59. A detailed examination of the light curve at around transit 59 shows that the transit-to-transit variations in the semi-major axis and eccentricity of Kepler-47b are minimal. We conclude that the short-term orbital variations are significant and capable of shifting the planet’s projected disk in and out of the stellar disk. This is an interesting result as missing transit events can be used to further constrain the results of photodynamical models in future discoveries of transiting circumbinary planets.

5.1. Transit Duration of the Third Planet

In this section, we calculate the durations of the transits of the third planet and compare them with the duration of the single, unexplained transit event of the system to constrain the orbit of this body. It is important to note that because of the mutual interactions between planets, the duration of the transits of the third planet will vary from one transit cycle to another. Figure 9 shows this for different values of the mass and initial orbital configuration of the third planet. In making this figure, we only considered initial conditions for which our MEGNO calculations indicated quasi-periodic orbits. In particular, we considered the initial conditions IC1 to IC11 shown in Figure 2 (except for IC2 for which the orbit of the planet was found to be highly chaotic). We also considered two cases for the mass of the third planet: A massless object shown on the left panels, and a 10 Earth-mass planet depicted on the right. The methodology for calculating transit duration is similar to the procedure described in the previous section. In each panel, the horizontal line at 4.15 hr corresponds to the duration of the single transit event identified by Orosz et al. (2012b). As shown in Figure 9, in general, the mass of the planet does not play a significant role in the duration of its transit. We found that in spite of its variations,
there are several transit cycles in which the duration of the transit of the third planet is comparable to that of the observed single transit (∼4.15 hr). For instance, in the top-left panel of Figure 9, where the planet is massless and starts at IC1, the transit number 83 appeared to have a duration of 4.16 hr. Transit durations of around 4 hr were also found for IC3 and IC4. We recall that IC1, IC3, and IC4 are initial conditions where the third planet starts between the two planets Kepler-47b and c. We also found that when the planet is massive, transit durations of around 4 hr are possible for IC6, where the orbit of the third planet is exterior to Kepler-47c.

The results mentioned above point to a strong degeneracy. It seems impossible to determine the correct orbit of the third planet using its transit duration. This degeneracy can, however, be broken assuming the third planet is in a circular orbit. As shown by Kostov et al. (2013), in this case, the orbital period of

Figure 9. Graphs of the transit duration of the third planet for various initial conditions. The left column corresponds to a massless object and the right column is for a 10 Earth-mass planet. The horizontal line marks a transit duration of 4.15 hr. We considered various initial mean longitudes of the third planet. Other initial mean longitudes were also tried and resulted in similar results. As shown here, the duration of the transit does not have strong dependence on the initial phase of the third planet. In addition, no significant differences between transit durations of a test mass and a massive planet were observed. Symbols that are not connected with a line have a missed trailing or leading transit, and represent isolated transit events. Note the increase in the period of transit durations for longer periods of the transiting third planet.
the third planet can be determined using

\[ P_d = P_{bin}(0.74\sqrt{1-b^2} + 0.26)^{-3}, \]

where \( b \) is the (a priori unknown) impact parameter. As \( b \) varies only between 0 and 1, we can use Equation (1) to constrain the orbital period of the third planet and break the degeneracy. Specifically, if \( b = 0 \) (i.e., a central transit) then \( P_d = P_{bin} \), representing a third body with the same orbit as that of the binary. This is consistent with the measured transit duration of 4.15 hr being comparable to the duration of the stellar eclipse. On the contrary, if \( b \approx 1 \) (i.e., a grazing transit), Equation (1) gives \( P_d = 57 \times P_{bin} \), suggesting an upper limit for the period of the third planet of \( \sim 424 \) days. For a binary mass of 1.4 \( M_\odot \), this corresponds to a semi-major axis of \( \sim 1.24 \) AU, effectively ruling out IC6 through IC11, as well as orbits with progressively larger semi-major axis. Thus the third planet is either on a stable orbit in the vicinity of Kepler-47b, between the two known planets Kepler-47b and c, or started along IC5.

6. DISCUSSION AND CONCLUSIONS

In this study, we used dynamical considerations to examine the possibility that the single, unexplained transit in the Kepler-47 system as reported by Orosz et al. (2012b) was due to a third planet. Using the MEGNO technique, we identified regions in the phase-space where the third planet could follow quasi-periodic orbits considering the five-body problem. We determined several of such quasi-periodic regions between the two known planets Kepler-47b and Kepler-47c, where they also include orbital mean-motion resonances with either of the two bodies.

Using accurate single orbit integrations, we examined the long-term orbital stability of the third planet within the framework of a five-body problem. Results identified ranges of semi-major axis and eccentricity that would render the third planet stable over a period of 10 million years. To examine the extent of the dependence of the results on the mass of this planet, we carried out integrations for different values of its mass, and showed that a third planet with a mass as high as 50 Earth-masses can still maintain stable orbits either between Kepler-47b and Kepler-47c, or beyond the orbit of Kepler-47c. For higher masses of the third planet, the quasi-periodic stable regions in the vicinity of Kepler-47b cease. For a selection of initial conditions within quasi-periodic islands we established a clear association of these islands with two-body mean-motion resonances by demonstrating a librating (around zero) critical argument.

To constrain the orbit of the third planet, we calculated its transit durations as well as the TTV and TDV of Kepler-47b for various initial conditions. We found that transit duration and transit timing variations are affected by short-term changes in the orbit that have been caused by perturbations due to other bodies in the system. This implies that it is imperative to consider gravitational interactions when studying multi-body systems where mutual perturbations can be significant.

Also, when the line-of-sight inclination of the transiting planet becomes small (the planet approaches the limb of the star), short-term perturbation on timescales on the order of the orbital period will have the effect of shifting the on-sky position of the planet. This can result in missing transits from one orbit to the other. These transit-missing events can be used to further constrain best-fit photodynamical models. In this study, for the first time, we have correlated the cause of missing transits with the short-term (transit-to-transit) variations of orbital elements. We also confirmed the possibility of long-duration transits. For instance in Figure 9, considering the case of IC6 for a massless third planet, the duration of a single transit can last for nearly 60 hr.

When calculating transit durations of the third planet, we found that the results suffer from a large degeneracy. Several orbits produced similar transit durations as that of the single transit, 4.15 hr. We were able to break this degeneracy for circular orbits and determined an upper limit of 424 days for the orbit of the third planet.

In a recent study by Kratter & Shannon (2014), a period of 186 days was conjectured for the third planet in Kepler-47. Using Kepler’s third law, we obtain a semi-major axis of 0.714 AU (relative to the binary center of mass) for the third planet, placing it close to the stable 5c:8d mean-motion resonance or nearby resonances (see Figure 2) with Kepler-47c. Our work predicts several islands of quasi-periodic orbits in the neighborhood of the 186 days orbit. Kratter & Shannon (2014) also carried out long-term numerical integrations of the five-body system. Their results support the finding that all planets maintain stable orbits over at least \( 2 \times 10^6 \) yr.

In the present analysis we suggest that a third planet could in fact be the cause of the single, unexplained transit event reported by Orosz et al. (2012b). This planet will have a low eccentricity orbit either (1) in the vicinity of Kepler-47b (for low masses), (2) between planets Kepler-47b and Kepler-47c, or (3) exterior to planet c with a semi-major axis smaller than 1.24 AU.

The detection of a third planet could follow along the route of measuring TTVs or TDVs of Kepler-47b and/or Kepler-47c as caused by this planet. However, the process of finding additional bodies using timing variations is highly degenerate and can lead to various dynamical configurations that produce the same timing signal (Nesvorný 2009; Nesvorný & Beaugé 2010). We therefore suggest that the entire currently available Kepler photometric data on Kepler-47b be analyzed and searched for additional transit signatures that cannot be explained by the two known planets Kepler-47b and Kepler-47c.

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