Scaling the localisation lengths for two interacting particles in one-dimensional random potentials

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Abstract

Using a numerical decimation method, we compute the localisation length $\lambda_2$ for two onsite interacting particles (TIP) in a one-dimensional random potential. We show that an interaction $U > 0$ does lead to $\lambda_2(U) > \lambda_2(0)$ for not too large $U$ and test the validity of various proposed fit functions for $\lambda_2(U)$. Finite-size scaling allows us to obtain infinite sample size estimates $\xi_2(U)$ and we find that $\xi_2(U) \sim \xi_2(0)^{\alpha(U)}$ with $\alpha(U)$ varying between $\alpha(0) \approx 1$ and $\alpha(1) \approx 1.5$. We observe that all $\xi_2(U)$ data can be made to coalesce onto a single scaling curve. We also present results for the problem of TIP in two different random potentials corresponding to interacting electron-hole pairs.

In two recent articles [1,2], we studied as a simple and tractable approach to the problem of interacting electrons in disordered materials the case of only two interacting particles (TIP) in 1D random potentials. Previous considerations [3] had led to the idea that attractive as well as repulsive interactions between TIP give rise to the formation of particle pairs whose localisation length $\lambda_2$ is much larger than the single-particle (SP) localisation length $\lambda_1 \approx 105/W^2$,

$$\lambda_2 \sim U^2 \lambda_1^2$$  \tag{1}

at two-particle energy $E = 0$, with $U$ the Hubbard interaction strength. Although many papers have numerically investigated the TIP effect [3–9], an unambiguous reproduction of Eq. (1) is still lacking. However, it appears well established that some TIP delocalisation such as $\lambda_2 > \lambda_1$ does indeed exist due to the interaction. Recently, a duality in the spectral statistics for $U$ and $\sqrt{24}/U$ has been proposed [11] for small and very large $|U|$. 

Preprint submitted to Elsevier Preprint  Revision : 1.4; compiled 24 March 2022
In Refs. [1,2], we have employed a numerical decimation method [10], i.e., we replaced the full Hamiltonian by an effective Hamiltonian for the doubly-occupied sites only. In [1], we considered the case of TIP with \( n, m \) corresponding to the positions of each particle on a chain of length \( M \) and random potentials \( \epsilon_1^n = \epsilon_2^n \in [-W/2, W/2] \). In [2], we studied the case where \( \epsilon_1^k \) and \( \epsilon_2^k \) are chosen independently from the interval \([-W/2, W/2]\), which may be viewed as corresponding to an electron and a hole on the same chain (IEH). Via a simple inversion, we then obtained the Green function matrix elements \( \langle 1, 1 | G_2 | M, M \rangle \) between doubly-occupied sites \((1,1)\) and \((M,M)\) and focused on the localisation length \( \lambda_2 \) obtained from the decay of the transmission probability from one end of the system to the other, i.e.,

\[
\frac{1}{\lambda_2} = -\frac{1}{|M - 1|} \ln |\langle 1, 1 | G_2 | M, M \rangle|.
\]  

In Fig. 1 we present data for \( \lambda_2(U) \) obtained for three different disorders for system sizes \( M = 201 \) at \( E = 0 \). In agreement with the previous arguments and calculations [6,7,11], we find that the enhancement is symmetric in \( U \) and decreases for large \( |U| \). In [11] it has been argued that at least for \( \lambda_1 \approx M \), there exists a critical \( U_c = 24^{1/4} \approx 2.21 \), which should be independent of \( W \), at which the enhancement is maximal. We find that in the present case with \( \lambda_1 < M \) the maximum of \( \lambda_2(U) \) depends somewhat on the specific value of disorder used. The data in Fig. 1 may be compatible with the duality of Ref. [11], but only for the large disorder \( W = 5 \). For the smaller disorders and for the range of interactions shown, we do not observe the duality. We emphasize that the duality observed in [11] is for spectral statistics and need not apply to quantities such as the localisation length \( \lambda_2 \).
In order to reduce the possible influence of the finiteness of the chain length, we constructed finite-size-scaling (FSS) curves for 11 interaction values $U = 0, 0.1, \ldots, 1$ from the $\lambda_2$ data for 26 disorder values $W$ between 0.5 and 9, for 24 system sizes $M$ between 51 and 251, averaging over 1000 samples in each case. In Fig. 2 we show the infinite-size localisation lengths (scaling parameters) $\xi_2$ obtained from these 11 FSS curves. A simple power-law fit $\xi_2 \propto W^{-2\alpha}$ in the disorder range $W \in [1, 5]$ yields an exponent $\alpha$ which increases with increasing $U$ as shown in the inset of Fig. 2, e.g., $\alpha = 1.55$ for $U = 1$ and $\alpha = 1.1$ for $U = 0$. Because of the latter, in the following we will compare $\xi_2(U \neq 0)$ with $\xi_2(0)$ when trying to identify an enhancement of the localisation lengths due to interaction.

Song and Kim [5] suggested that the TIP localisation data may be described by a scaling form $\xi_2 = W^{-\alpha_0} g(|U|/W^\Delta)$ with $g$ a scaling function. They obtain $\Delta = 4$ by fitting the data. Our data can be best described when $\alpha_0$ is related to the disorder dependence of $\xi_2$ as $(\alpha - \alpha_0)/\Delta \approx 1/4$. As shown in Fig. 2, the scaling is only good for $W \in [1, 5]$ and $U \geq 0.3$. We note that assuming an interaction dependent exponent $\alpha(U)$, we still do not obtain a good fit to the scaling function with the data for all $U$.

In Fig. 3, we show that a much better scaling can be obtained when plotting

$$\xi_2(U) - \xi_2(0) = \tilde{g} [f(U)\xi_2(0)]$$

with $f(U)$ determined by FSS. Now the scaling is valid for all $U$ and $W \in [0.6, 9]$. As indicated by the straight lines, we observe a crossover from a slope

![Graph showing TIP localisation lengths after FSS.](image)

**Fig. 2.** Left panel: TIP localisation lengths $\xi_2$ after FSS. The dashed lines represent power-law fits. Inset: Exponent $\alpha$ obtained by the power-law fits. Right panel: Scaling plot according to [5] with TIP localisation lengths $\xi_2(U)$ for $W \in [1, 5]$. The solid line indicates a slope of $1/4$, the dashed line the value of $\xi_2 W^{2.1}$ in the limit $U = 0$. 
2 to a slope $3/2$. There are some deviations from scaling, but these occur for large and very small values of $\xi_2(U)$ and are most likely due to numerical inaccuracy [1]. In the inset of Fig. 3, we show the behavior of $f(U)$. For $U \geq 0.3$ a linear behavior $f(U) \propto U$ appears to be valid which translates into a $U^2$ ($U^{3/2}$) dependence of $\xi_2(U) - \xi_2(0)$ in the regions of Fig. 3 with slope 2 ($3/2$). For $U \leq 0.5$, we have $f(U) \propto \sqrt{U}$ which yields $\xi_2(U) - \xi_2(0) \propto U (U^{3/4})$.

Thus in summary it appears that our data cannot be described by a simple power-law behavior with a single exponent as in Eq. (1) neither as function of $W$, nor as function of $\xi_2(0)$ [1], nor after scaling the data onto a single scaling curve.

As for TIP we computed [2] the IEH localisation lengths by the DM along the diagonal using 100 realizations for each $(U, M, W)$. We find that the data for IEH are very similar to the case of TIP. We again perform FSS and observe that the infinite-size estimates $\xi_2(U)$ are well characterized by an exponent $\alpha(U)$. We can again scale the $\xi(U)$ data for IEH onto a single curve as shown in Fig. 3. However, here the crossover from slope 2 to $3/2$ is much less prominent and the data can be described reasonably well by a single slope of 1.61. Also, the crossover behavior in $f(U)$ is suppressed. We remark that these differences may be due to the smaller number of samples used for IEH.

In conclusion, we observe an enhancement of the two-particle localisation length due to onsite interaction both for TIP and IEH. This enhancement persists, unlike for TMM [6,8,9], in the limit of large system size and after constructing infinite-sample-size estimates from the FSS curves. We remark that the IEH case is of relevance for a proposed experimental test of the TIP effect [12].
Acknowledgements

R.A.R. gratefully acknowledges support by the Deutsche Forschungsgemein-
schaft (SFB 393).

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