Transport signatures of spatially modulated electronic nematic phases

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Electronic nematic phases are broadly characterized by spontaneously broken rotational symmetry. Although they have been widely recognized in the context of high temperature cuprates, bilayer ruthenates, and iron-based superconductors, the focus so far has been exclusively on the uniform nematic phase. Recently, however, it was proposed that on a square lattice a nematic instability in the d-wave charge channel could lead to a spatially modulated nematic state, where the modulation vector $\mathbf{q}$ is determined by the relative location of the Fermi level to the van Hove singularity. [1] Interestingly, this finite-$q$ nematic phase has also been identified as an additional leading instability that is as strong as the superconducting instability near the onset of spin density wave order. [2] Here we study the electrical conductivity tensor in the modulated nematic phase for a general modulation vector. Our results can be used to identify modulated nematic phases in correlated materials.

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Introduction - Identifying the genuine ground states of doped Mott insulators has been one of the key issues in the field of strongly correlated electron systems. It was proposed that in the ground states of doped Mott insulators, electrons arrange themselves in certain charge and/or spin density patterns due to strong electron interactions and quantum fluctuations introduced by electron or hole doping. [3] These self-organized phases are classified by different broken symmetries. Among them is the electronic nematic phase, which is characterized by a spontaneous Fermi surface (FS) deformation that reduces the rotational symmetry of the underlying lattice. On the square lattice the nematic order parameter has the form

$$\Delta_{n,\sigma} = \sum_{\mathbf{k}} (\cos k_x - \cos k_y) (c_{\mathbf{k},\sigma}^{\dagger} c_{\mathbf{k},\sigma})$$

where $c_{\mathbf{k},\sigma}^{\dagger}$ ($c_{\mathbf{k},\sigma}$) creates (annihilates) an electron with momentum $\mathbf{k}$ and spin $\sigma$. For a charge nematic phase one has $\Delta_n,\uparrow \equiv \Delta_n,\downarrow \equiv \Delta_n$. Note that a finite $\Delta_n$ represents 90 degree rotational symmetry breaking, as $\Delta_n$ changes sign under such a lattice rotation.

Combined with the tight binding model for a square lattice, a simple nematic mean field Hamiltonian is then given by

$$H_{MF} = \sum_{\mathbf{k},\sigma} [\epsilon_{\mathbf{k}} - \mu + \Delta_n (\cos k_x - \cos k_y)] c_{\mathbf{k},\sigma}^{\dagger} c_{\mathbf{k},\sigma}$$

where $\epsilon_{\mathbf{k}} = -2t(\cos k_x + \cos k_y) - 4t' \cos k_x \cos k_y - 2t''(\cos (2k_x) + \cos (2k_y))$ and $\mu$ is the chemical potential. The parameters $t$, $t'$, and $t''$ denote nearest, next-nearest, and third-nearest neighbor hoppings, respectively. The uniform nematic phase can hence be associated with distinct nearest-neighbor hopping integrals along $x$- and $y$-directions. During the last few years, theoretical studies have focussed on the microscopic route to such an effective Hamiltonian, the experimental consequences, and the nature of the isotropic-nematic phase transition. [4–15] Fascinating experimental results, which support the existence of the electronic nematic phase, [16] were also reported in the high temperature cuprates, [17–20] bilayer ruthenates, [21–24] and iron-pnictides. [25–28]

![FIG. 1: General nematic modulation (see main text for details).](image-url)
the filling relative to the van Hove filling. An extended Hubbard model was furthermore studied to analyze its renormalization group flow. [29] The spatially modulated nematic order suggested in these studies can be described by the following bond order parameter operator,

\[ \hat{\Delta}_n(\mathbf{q}) = \sum_{\mathbf{k},\sigma} (\cos k_x - \cos k_y)c_{\mathbf{k}+\mathbf{q}/2,\sigma}^\dagger c_{\mathbf{k}-\mathbf{q}/2,\sigma}, \] (3)

where a finite \( \mathbf{q} \) determines the periodicity and direction of the modulation and the form factor \( (\cos k_x - \cos k_y) \equiv d_{\mathbf{k}} \) represents the nematic character. A typical example is given in Fig. 1 where the red and blue colors on the bonds indicate the nematicity, and the modulation of the bonds gives rise to a striped pattern tilted at an angle \( \theta \) relative to the x-axis.

In this paper, we study the transport behavior of such a modulated nematic phase using the effective medium theory. [29,32] Our results can be used to pinpoint the existence of the modulated nematic phase in correlated materials in general. Before we proceed to the study of transport, let us check what factors determine the direction of the modulation vector \( \mathbf{q} \), as it leads to a qualitative difference in the resistivities which will be presented later.

Direction of the modulation vector - In Ref. [11], the modulation vector \( \mathbf{q} \) is determined by the shape of the FS close to van Hove filling, assuming that the nematic interaction has only a weak momentum-dependence around this regime of FS. It was found that for fillings above the van Hove singularity (vHS) the modulation vector is oriented diagonal and below parallel to the crystal axes. Here we study a tight binding model with a different FS shape and find opposite behavior where \( \mathbf{q} \) is parallel to the x- or y-axis above the vHS and lies diagonally below.

Let us consider an effective Hamiltonian of the form

\[ H_{\text{eff}} = \sum_{\mathbf{k},\sigma} (\epsilon_{\mathbf{k}} - \mu) c_{\mathbf{k},\sigma}^\dagger c_{\mathbf{k},\sigma} + \frac{1}{N} \sum_{\mathbf{q}} g(\mathbf{q}) \hat{\Delta}_n(\mathbf{q}) \hat{\Delta}_n(-\mathbf{q}), \]

(4)

where \( g(\mathbf{q}) \) denotes an effective attractive interaction favoring a nematic instability and \( N \) is the number of lattice sites.

Assuming \( g(\mathbf{q}) \) does not have a strong \( \mathbf{q} \)-dependence, the nematic susceptibility is determined by the d-wave static (\( \omega = 0 \)) bare polarization function (the d-waviness originates from \( d_{\mathbf{k}} \)) given by

\[ \Pi_d(\mathbf{q}, \omega = 0) = \frac{1}{N} \sum_{\mathbf{k}} d_{\mathbf{k}}^2 \frac{f(\xi_{\mathbf{k}+\mathbf{q}/2}) - f(\xi_{\mathbf{k}-\mathbf{q}/2})}{\xi_{\mathbf{k}+\mathbf{q}/2} - \xi_{\mathbf{k}-\mathbf{q}/2}}, \]

(5)

where \( f(\xi_{\mathbf{k}}) \) is the Fermi function and \( \xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu \). Similarly the s-wave polarization function can be obtained by setting \( d_{\mathbf{k}} = 1 \).

Setting the tight binding parameters to \( t = 1, t' = 0.375 \) and \( t'' = 0 \), Fig. 2 (a) shows the FSs for several chemical potentials \( \mu \). The values \( \mu = 1.45, 1.5 \) and 1.52 lie below (blue), at (black) and above (red) the vHS, respectively. The d-wave (solid lines) and s-wave (dashed lines) static polarization functions for each \( \mu \) along \( \Gamma - X - M - \Gamma \) are plotted in (b). In all cases, the d-wave polarization is more singular than the s-wave case. At van Hove filling, the logarithmic singularity occurs at \( \mathbf{q} = 0 \) leading to the pure nematic phase. However, for \( \mu \) below the vHS (blue solid line), the d-wave polarization function is singular at a finite \( \mathbf{q} \) along the \( \Gamma - M \) direction in \( \mathbf{q} \) space. On the other hand, above the vHS (red solid line), it is most singular for \( \mathbf{q} \parallel \Gamma - X \). The panels (c) and (d) show the d-wave and s-wave bare polarization functions in the two-dimensional \( \mathbf{q} \) space.

Qualitatively, the results are the same as those obtained in Ref. [11]. The main quantitative difference though is the relation between electron filling and the direction of \( \mathbf{q} \). However, this difference is simply due to the relative meaning of filled and empty states. For electron-filling [the grey area in (a)] the blue FS is below the van Hove points, but for hole-filling it is above. Here as well as in the study [11] the preferred orientation

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FIG. 2: [color online] (a) Fermi surfaces for different chemical potentials \( \mu = 1.45 \) (blue), 1.5 \( \) (black) and 1.52 \( \) (red). The shaded area indicates the occupied states. (b) The bare d-wave (solid lines) polarization function at zero frequency for the same \( \mu \)-values as in (a). The bare s-wave function (dashed lines) is also shown for comparison. (c) The bare d-wave polarization function in the two-dimensional \( \mathbf{q} \)-space for the three cases below, at and above van Hove filling with \( 2k_F \) lines (dotted lines) partially superimposed. The \( 2k_F \)-lines are defined by \( \xi_{\mathbf{q}+\mathbf{G}/2} = 0 \), where \( \mathbf{G} \) belongs to the set of reciprocal lattice vectors. [11] (d) The bare s-wave polarization for the same parameters as in (c) for comparison.
of the modulation vector for the blue FS is along the diagonal direction. Thus the direction of the modulation vector depends on the FS shape near the van Hove points/saddle points rather than on the position of the Fermi level with respect to the vHS. Now let us present how the transport in the modulated nematic phase is affected, in particular, when the leading nematic instability occurs for \( q \) parallel or diagonal to the crystal axes.

**Conductivity Tensor** - The spatially modulated nematic phase can be viewed as a collection of uniform nematic states with varying bond strengths and periodicity \( 2\pi/|q| \). As mentioned before, an example of the modulated nematic phase with \( q = (-\frac{2\pi}{24}, \frac{2\pi}{15}) \) is shown in Fig. 1 where the red/blue bond has a higher/lower hopping integral. For a given \( y \)-position, the red bond changes to blue and vice versa along the \( x \)-direction. Moving from row to row, the modulation along the \( x \)-direction is shifted by a phase factor. The finite-\( q \) nematic phase can also be viewed as a collection of parallel stripes of alternating, approximately uniform nematic domains running at an angle \( \theta \) relative to the crystal \( x \)-axis, since the nematic phase possesses Ising symmetry.

The principal components of the conductivity tensor of each nematic domain are denoted by \( \sigma_{xx} \) and \( \sigma_{yy} \) forming a diagonal 2 \( \times \) 2 conductivity tensor. The principal axes are in the same direction as the crystal axes, as the directions are pinned by the lattice potential. Based on the Ising symmetry, there are two types of nematic domains. One type has higher \( \sigma_{xx} \), while the other has lower \( \sigma_{xx} \) (and vice versa for \( \sigma_{yy} \)). We denote these two types of nematic domains \( A \) and \( B \), and assume an area fraction of \( p \) for type \( A \) and \( 1 - p \) for type \( B \). Thus the conductivity tensor for each domain is given by

\[
\sigma_{A/B} = \begin{pmatrix} \sigma_{xx}^{A/B} & 0 \\ 0 & \sigma_{yy}^{A/B} \end{pmatrix},
\]

where \( \sigma_{xx}^A = \sigma_{yy}^B \) and \( \sigma_{xx}^B = \sigma_{yy}^A \). Since the stripes make an angle of \( \theta \) relative to the \( x \)-axis, we introduce the rotated coordinate system \( x' - y' \) such that the \( x' \)-direction runs parallel and the \( y' \)-direction runs perpendicular to the stripes (see Fig. 1). Our goal is to get the effective conductivity tensor in the \( x - y \) coordinate system.

The system under consideration is an example of a material whose conductivity is a function of position, i.e. \( \sigma = \sigma(r) \), where in the present case \( r \) is a two-component position vector. In such a material, the macroscopic properties can be described by an effective conductivity \( \sigma_e \), where \( \sigma_e \) is position-independent. If the material of interest is additionally anisotropic, as in the present case, then \( \sigma(r) \) is a d-dimensional tensor (i.e., a \( d \times d \) matrix, where \( d \) is the spatial dimension). In this case, \( \sigma_e \) remains position-independent but will also become a d-dimensional tensor, defined by the relation (see, e.g., Refs. [30], [31] and [32])

\[
\langle J \rangle = \sigma_e \langle E \rangle.
\]

Here the triangular brackets \( \langle ... \rangle \) denote a space-average, and \( J \) and \( E \) are vectors denoting the position-dependent current density and electric field, respectively. Thus, for example,

\[
\langle J \rangle = \frac{1}{V} \int d^d r J(r)
\]

in \( d \) dimensions, where \( V \) is the \( (d \times d) \)-dimensional volume of the system. A similar equation holds for \( \langle E \rangle \).

In general, if \( \sigma(r) \) is a random function of position, \( \sigma_e \) can only be calculated approximately. However, if \( \sigma(r) \) is a non-random function of position, one can often calculate the needed space-averaged fields exactly, by solving the relevant electrostatic equations \( \nabla \cdot J = 0, \nabla \times E = 0 \), \( J(r) = \sigma(r)E(r) \). This is the situation for the case of periodic stripes considered in the present paper.

To get the effective conductivity tensor, we then proceed with the following steps. First, we transform the conductivity tensors \( \sigma_A \) and \( \sigma_B \) into the rotated coordinate system \( x' - y' \). Next, we calculate the effective conductivity tensor in the rotated coordinate system. This is straightforward by imposing the boundary conditions that the component of electric field parallel to the stripes, and the component of the current density perpendicular to the stripes, should be continuous. Finally, we transform the resulting conductivity tensor back to the \( x - y \) coordinate system, in which experiments measure the conductivity (or resistivity).

First, the conductivities \( \sigma_A' \) and \( \sigma_B' \) in the rotated coordinates have the following relation to the conductivities in the original coordinates \( \sigma_A \) and \( \sigma_B \),

\[
\begin{align*}
\sigma_A' &= R^{-1} \sigma_A R \\
\sigma_B' &= R^{-1} \sigma_B R,
\end{align*}
\]

where \( R \) is a 2 \( \times \) 2 rotation matrix with elements

\[
R = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}.
\]

Carrying out this matrix multiplication, we obtain

\[
\begin{align*}
\sigma_{i'j'} &= \sigma_{i'j'}^{xx} \cos^2 \theta + \sigma_{i'j'}^{yy} \sin^2 \theta \\
\sigma_{i'j'} &= \sigma_{i'j'}^{xx} \sin^2 \theta + \sigma_{i'j'}^{yy} \cos^2 \theta \\
\sigma_{i'j'} &= \sigma_{i'j'}^{xx} \cos \theta \sin \theta,
\end{align*}
\]

where \( i = A \) or \( B \).

Now, let us study the components of the effective conductivity matrix in the rotated system. Note that the electric field has components in both the \( x' \)- and \( y' \)-direction, while the corresponding components of the current density are

\[
\begin{align*}
\langle J \rangle_{x'} &= \sigma_e^{x'x'} \langle E \rangle_{x'} + \sigma_e^{x'y'} \langle E \rangle_{y'} \\
\langle J \rangle_{y'} &= \sigma_e^{y'x'} \langle E \rangle_{x'} + \sigma_e^{y'y'} \langle E \rangle_{y'}.
\end{align*}
\]
the two possible nematic orientations, as well as the pa-
sor in the original crystal lattice. This is done by writing

\[ \rho = \rho^p \text{ and } \rho^\sigma \]  

insets display the typical real space patterns of the modulated
nematic phases. For diagonal modulation (a) both \( \rho^p \) (red) and \( \rho^\sigma \) (black) are larger than in the isotropic phase. On
the other hand, for parallel modulation (b), \( \rho^p \) is close to \( \rho_0 \), while \( \rho^\sigma \) is larger than \( \rho_0 \). See the main text
for further discussion.

Imposing the boundary conditions mentioned above, we
obtain the effective conductivity matrix \( \sigma_e \) as follows,

\[ \sigma_e^{x'x'} = p \sigma_A^{x'x'} + (1-p) \frac{\sigma_B^{y'y'} (\sigma_A^{y'y'} - \sigma_B^{y'y'})}{\sigma_B^{y'y'}} \\
+ \left( p \sigma_A^{y'y'} + (1-p) \frac{\sigma_B^{x'x'} (\sigma_A^{x'x'} - \sigma_B^{x'x'})}{\sigma_B^{x'x'}} \right) \left( 1-p \right) \frac{\sigma_B^{y'y'} (\sigma_A^{y'y'} - \sigma_B^{y'y'})}{\sigma_B^{y'y'}} \\ \\
\sigma_e^{y'y'} = \frac{p \sigma_A^{x'x'} (\sigma_A^{x'x'} - \sigma_B^{x'x'})}{p \sigma_B^{y'y'} + (1-p) \sigma_B^{y'y'}} = \sigma_e^{y'y'} \\
\sigma_e^{y'y'} = \frac{1}{p \sigma_A^{y'y'} + (1-p) \sigma_B^{y'y'}}. \tag{13} \]

The final step is to transform the effective conductivity
tensor back into the original coordinate system, which
coincides with the crystal lattice. This is done by writing

\[ \sigma_e = R \sigma'_e R^{-1}, \tag{14} \]

where \( \sigma'_e \) is the matrix whose elements are given in Eq.
[13]. The final result gives the effective conductivity
tensor in the original \( x - y \) coordinate system, which
will depend on the elements of the conductivity tensors
of the two possible nematic orientations, as well as the pa-
rameters \( p \) and \( \theta \). Note that \( \sigma_e \), unlike the conductivity
tensors of the uniform nematic phase, has nonzero off-
diagonal elements in the \( x - y \) coordinate system. This
apparently arises because of the asymmetrical geometry
produced by the stripes, which are at a finite angle to the
\( x \)-axis.

As we have shown above, the modulation vector \( \mathbf{q} \) lies
either diagonal (\( \theta = \pi/4 \)) or parallel (\( \theta = 0 \)) to the crystal
axes. In both cases, the modulation is determined by a
single wave vector, so that \( p=1/2 \) due to the equal
population of \( A \) and \( B \) type. This leads to \( \sigma_e^{x'x'} = \sigma_e^{y'y'} \),

\[ \sigma_e^{x'x'} = \sigma_A^{x'x'} \text{ and } \sigma_e^{y'y'} = -\sigma_e^{x'x'}, \]

and simplify the above equations to

\[ \sigma_e^{x'x'} = \frac{1}{2} (\sigma_A^{x'x'} + \sigma_B^{y'y'}) - 2 (\sigma_A^{x'x'} - \sigma_B^{y'y'})^2 \tag{15} \]

\[ \sigma_e^{y'y'} = \frac{2(\sigma_A^{x'x'} + \sigma_B^{y'y'})}{\sigma_A^{x'x'} + \sigma_B^{y'y'}}, \quad \sigma_e^{x'y'} = \frac{(\sigma_A^{x'x'} - \sigma_B^{y'y'})}{\sigma_A^{x'x'} + \sigma_B^{y'y'}}. \]

For \( \theta = \pi/4 \), the effective resistivity tensor using \( \rho_e = \sigma_e^{-1} \) is then given by

\[ \rho_e^{xx} = \rho_e^{yy} = \frac{1}{2(\sigma_A^{x'x'} + \sigma_B^{y'y'})} \left( \frac{\sigma_B^{x'x'} + \sigma_B^{y'y'}}{2(\sigma_A^{x'x'} + \sigma_B^{y'y'})^2} \right) \]

\[ \rho_e^{xy} = \frac{1}{4(\sigma_A^{x'x'} + \sigma_B^{y'y'})^2} (\sigma_A^{x'x'} - \sigma_B^{y'y'}). \tag{16} \]

A schematic plot of \( \rho_e \) is displayed in Fig. 3 (a), where
the effective resistivities in both directions \( \rho_e^{xx} \) and \( \rho_e^{yy} \)
are higher than the isotropic resistivity \( \rho_0 \). The absolute
magnitude of \( \rho_e^{xx} \) depends on \( \sigma_A^{x'x'} \) and \( \sigma_B^{y'y'} \). For example,
if \( \sigma_A^{x'x'} = 0.5 \sigma_0 \) and \( \sigma_B^{y'y'} = 1.5 \sigma_0 \) where \( \sigma_0 \) is the
conductivity in the isotropic phase, one obtains \( \rho_e^{xx} = \rho_0^{yy} = 2.3 \rho_0 \). There is a finite \( \rho_e^{xy} \) as well due to the
stripe diagonal orientation as discussed earlier.

On the other hand, for \( \theta = 0 \) as shown in Fig. 3 (b),
the effective resistivity tensor is given by

\[ \rho_e^{xx} = \frac{2}{(\sigma_A^{x'x'} + \sigma_B^{y'y'})}, \quad \rho_e^{yy} = \frac{\sigma_A^{x'x'} + \sigma_B^{y'y'}}{2(\sigma_A^{x'x'} + \sigma_B^{y'y'})}, \quad \rho_e^{xy} = 0. \tag{17} \]

Note that the effective resistivity parallel to \( \mathbf{q} \), i.e., \( \rho_e^{yy} \)
in this case, is given by the average of two resistivities
\( \rho_e^{yy} = \rho_0^{yy} + \rho_0^{yy} = \sigma_A^{x'x'} + \sigma_B^{y'y'} \), since the resistiv-
ities add in series. On the other hand, \( \rho_e^{xx} = 1/\sigma_A^{x'x'} \), \( \rho_e^{yy} = 2/(\sigma_A^{x'x'} + \sigma_B^{y'y'}) \) is obtained from the average of the con-
ductivities. It is striking that for the parallel modulated
nematic state only \( \rho_e^{yy} \) becomes larger than \( \rho_0 \), whereas
\( \rho_e^{xx} \) remains largely unchanged from the isotropic value.
This is in contrast to the pure nematic phase, where the
conductivity along one lattice direction is smaller than
among the other lattice direction, while the average of
both is close to the isotropic conductivity, 11[12 34]
resulting in one principal resistivity component smaller
and the other larger than the isotropic resistivity \( \rho_0 \).

Discussion and summary - The residual resistivities in
the spatially modulated nematic state are qualitatively
distinct from the uniform nematic phase. In the uni-
form nematic phase, the Fermi velocities for momenta
in \( x \) - and \( y \)-direction differ (in magnitude) due to the
deformation of the FS, which then leads to different
longitudinal resistivities \( \rho^{xx} \neq \rho^{yy} \). However, in gen-
eral the resistivity in one direction is higher and that
in the other direction is lower than the resistivity in the
isotropic phase \( \rho_0 \).

In the modulated nematic phase the resistivities de-
depend on the angle \( \theta \) between the single ordering wave vec-
tor \( \mathbf{q} \) and the crystal \( x \)-axis, and the strength of the ne-
ematicity given by \( \sigma_A^{x'x'} - \sigma_B^{y'y'} \) in a nematic domain. When \( \mathbf{q} \)
is along the diagonal direction of the square lattice, the effective resistivities along x- and y-axes are equal and higher than in the isotropic phase. This behavior resembles the results obtained in Ref. [23], where the coupling between nematic ordering and a specific phonon mode results in diagonal nematic domains, which then enhances the resistivities $\rho^{xx}$ and $\rho^{yy}$. On the other hand, when the ordering wave vector is parallel to the crystal axes, the direction parallel to $\mathbf{q}$ is more resistive, while the direction perpendicular to $\mathbf{q}$ exhibits a resistivity close to that of the isotropic phase.

Our results show a surprising similarity to the longitudinal, magnetic field tuned resistivities in the bilayer ruthenates Sr$_3$Ru$_2$O$_7$. When the magnetic field is applied along the c-axis, the longitudinal resistivities in a- and b-direction are higher than in the isotropic phase, similar to Fig. 3 (a), within a magnetic field window, which is bounded by meta-magnetic transitions. As the magnetic field is tilted away from the c-axis, the resistivity parallel to the in-plane field component remains more resistive, but the resistivity along the perpendicular direction becomes similar to the isotropic resistivity, similar to Fig. 3 (b). Thus it is tempting to suggest that the magnetic field acts by effectively tuning the angle $\theta$.

A further support for a modulated nematic phase with a diagonal ordering pattern may be gained from considering the bare tight binding FS of Sr$_3$Ru$_2$O$_7$ in the presence of a magnetic field as displayed in Ref. [5]. Focussing on the $\gamma_2$ FS sheet (which harbors a vHS near the Fermi level), one spin component of the $\gamma_2$ sheets opens up near momenta $(0, \pm \pi)$ and $(\pm \pi, 0)$ for a large field along the c-axis, giving rise to a cross-shaped hole pocket as shown in Fig. 2 (a) in Ref. [5]. The corresponding $2k_F$-lines have a similar shape as the contour of this $\gamma_2$ pocket and resemble the $2k_F$-lines displayed in Fig. 2 (c) for $\mu = 1.52$ but rotated by $\pi/4$. The leading instability for a modulated nematic state therefore could involve momenta $\mathbf{q}$ lying in the diagonal direction, yielding resistivities consistent with the transport measurements. When the magnetic field is tilted away from the c-axis, acquiring a finite a- or b-axis component, the diagonal hot-spots disappear, and parallel nematic stripes emerge instead.

However, the limitation of the present theory also deserves some discussion. As discussed throughout, the FS shape is important in determining the direction of $\mathbf{q}$. The FS of the bilayer ruthenate is complex (further complicated by the presence of an external field) and displays more than one vHS near the Fermi level. Thus the competition between different $\mathbf{q}$ instabilities needs to be investigated in addition to the origin of microscopic interaction causing such instabilities. Furthermore, the present effective medium theory does not contain magnetic field effects such as cyclotron motion. While the microscopic origin of a modulated nematic instability is still under investigation, the current study provides a way to search for such a phase via the transport properties of correlated materials.

In summary, based on effective medium theory we studied the electrical resistivity when nematic order is spatially modulated in a stripe-like pattern. The modulation wave vector $\mathbf{q}$ is determined by the FS shape near the van Hove points. Assuming that the nematic interaction has only weak momentum dependence near $\mathbf{q} = 0$, it was found that the modulation wave vector is either parallel or diagonal to the crystal axes. When $\mathbf{q}$ is parallel to one of the crystal axes, the resistivity along the parallel direction is higher than in the isotropic phase, while the resistivity in the perpendicular direction remains roughly unchanged. On the other hand, when $\mathbf{q}$ is diagonal to the crystal axes, both $\rho^{xx}$ and $\rho^{yy}$ are larger than in the isotropic phase. The present study suggests that it is worthwhile to look for modulated nematic instabilities starting from microscopic Hamiltonians, and motivates further experimental exploration in correlated materials.

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