Acoustic Wave Based Time-Domain Full Waveform Inversion and Its Application

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Abstract. In this paper, the acoustic wave-based time-domain full waveform inversion (FWI) is investigated. The FWI is a typical nonlinear ill-posed problem. In order to overcome its ill-posedness, the strategy of stepwise inversion is adopted. The inversion is carried out from low frequency to high frequency, and the inverse result at low frequency is chosen as the initial model for the high frequency inversion. This strategy effectively strengthens the robustness of inversion. The forward simulation is implemented efficiently with the explicit finite difference scheme while the inverse iteration is based on the LBFGS algorithm. The gradient of the objection function is computed by the wavefield backward technique. Numerical computations are not only completed for the benchmark Overthrust model but also for a real data set and good inversion results are yielded.

Introduction

The FWI is a method for interfering the media parameters underground by using the data observed on surface or in cross-well. It has the advantage of high accuracy. However, it is a typical ill-posed problem and the solution is nonunique. The FWI can be implemented in the time domain [1-3] or in the frequency domain [4,5]. An overview of FWI can be found in the reference [6]. The time-domain method does not need Fourier transform for data and the forward can be solved efficiently in the time domain. The frequency-domain method requires to make Fourier transform for data and the forward requires to solve a large-scale system and the cost is relatively high.

The FWI is an optimization iterative process of minimizing the residuals between the simulated data and observed data. It is very sensitive to the initial model because of the phenomenon of multi-extreme values of the objective function. In order to improve the inverse robustness, several methods have been developed, for example, the multiscale method [2,5], the regularization method [7-9] and the preconditioned method [10]. Among of them, the frequency multiscale method is an effective regularization strategy, which has been applied to both time-domain FWI and frequency-domain FWI. This method starts inversion from low frequency data. And a relatively good background model can be estimated because the inversion with low frequency data is not easy to fall into local extreme points. Then the inversion is continued for high frequency data step by step. Thus, the inversion may yield more accurate result. It is a regularization strategy essentially. In this paper, we focus on the development of time-domain inversion method and its application to real data.

Methodology

Forward Simulation

In the time domain, the two-dimensional (2D) acoustic wave equation can be written as

$$\frac{1}{v^2} \frac{\partial^2 u}{\partial t^2} = \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right) + s(t)\delta(x-x_s)\delta(z-z_s)$$  (1)
where \( x \) is the surface coordinate, \( z \) is the depth coordinate, \( u(x, z, t) \) is the pressure, \( v(x, z) \) is the media velocity, \( s(t) \) is the source or shot function and \( (x_s, z_s) \) is its location. Eq. 1 and Eq. 2 can be solved numerically by many methods. Here we solve it with the explicit finite difference scheme because of its high efficiency. Since the computational domain is limited, the absorbing boundary conditions are required to eliminate boundary reflections [11-13]. In this paper the paraxial approximation [11] is adopted. This method does not require additional layer around the domain.

**Inverse Method**

The FWI is an iterative process of minimizing the residuals between the synthesized data \( u_{cal} \) and observed data \( u_{obs} \). Suppose the shot number is \( N_s \), the receiver number is \( N_r \) and the number of time points is \( N_t \), then the inversion for velocity \( v(x, z) \) is minimizing the following problem

\[
\min_v f(v) = \frac{1}{2} \sum_{n=1}^{N_s} \sum_{m=1}^{N_r} \sum_{i=1}^{N_t} [u_{obs}(n_i, m_i, n) - u_{cal}(v; n_i, m_i, n)]^2
\]

where both \( u_{cal} \) and \( u_{obs} \) are the \( N_s \times N_r \times N_t \) dimensional vectors. Let the number of discrete points in space be \( M = N_s N_r M \). Obviously, it is also the dimensions of velocity vector \( v \) we want to inverse. To start the optimal iteration, we choose an initial velocity model \( v_0 \) and update it according to the following formula to ensure the decreasing of the objection function \( f(v) \)

\[
v_{k+1} = v_k + \alpha_k p_k
\]

where \( k \geq 0 \) is the iterative index, \( p \) is the descent direction and \( \alpha \) is the steplength at the \( k \)-th iteration. This process can be implemented by the typical optimization methods such as the steepest descent method, the conjugate gradient method and so on. In this paper, the descent direction \( p \) is found by the LBFGS algorithm and the steplength \( \alpha \) is searched by the Wolfe line search method [14]. The iteration is stopped if the one of the three rules is satisfied: (1) The gradient of the objection satisfies \( \| g \| \leq \varepsilon_t \), that is to say, the iteration is stopped when the gradient becomes a small quantity. (2) The descending rate of the objective function satisfies \( \frac{f_p - f}{f} \leq \varepsilon_z \). Here \( f \) is the present objective function and \( f_p \) is that at some previous iterations. This condition means that the objective function decreases small at recent iterations and the solution is near the minimum point. (3) If the iteration number is larger than the maximal iteration number \( k_{max} \), then the iteration is stopped.

In our computations, we set \( \varepsilon_t = 10^{-8}, \varepsilon_z = 10^{-7} \), \( f_p \) is the objective function at previous four iterations.

In the LBFGS algorithm, the gradient is required. Since the nonlinear property between wavefield \( u \) and velocity \( v \), the computational efficiency of gradient is very important for the inversion is a large-scale computational problem. We use the adjoint method to compute the gradient [1]. First, we solve a backward problem numerically like the forward simulation but the source term in Eq.1 is replaced by the residuals. Then the gradient \( g \) is obtained based on the following formula [5]

\[
g = -\sum_{shot} \int_0^T w \frac{2}{v^3} \frac{\partial^2 u}{\partial t^2} dt
\]

where \( T \) is the terminal time, \( w \) is the backward wavefield and \( u \) is the synthesized wavefield with present velocity model. In fact, it is the cross-correlation between the forward wavefield excited by the source and the backward wavefield inspired by the residuals. This approach avoids to compute the gradient directly by the perturbation of model parameters and save computational cost greatly.
Numerical Computations

Overthrust Model

The Overthrust model is an international 2D benchmark model and its exact velocity is shown in Fig.1. The spatial steps are both 12m in $x$ and $z$ directions with discrete points $N_x = 801$ and $N_z = 187$. The sources and receivers are placed at the depth of 12m and we set $N_s = 300$ and $N_r = 180$. Fig.2 is the initial velocity for inversion. Fig.3, Fig.4, Fig.5 and Fig.6 are the inverse results with the data in four different frequency bands, i.e., 0-2.5Hz, 0-5Hz, 0-15Hz and 0-25Hz, respectively. The stepwise inverse strategy is applied to ensure the inversion convergences gradually to the exact model. The structures such as the faults in final result Fig.6 are inversed clearly although they are very vague in the initial model. In this inversion, the inverse unknown number is 149787 and the parallel computations are implemented. The total CPU time is 17h with 240 processors.

Figure 1. Overthrust exact velocity model.
Figure 2. Initial velocity for Overthrust model inversion.
Figure 3. Inverse result with the data in the first stage frequency band 0-2.5Hz.
Figure 4. Inverse result with the data in the second stage frequency band 0-5Hz.
Real Data Inversion

We now consider the velocity inversion for a real data. There are 100 shots in the data set in all. Each shot has 480 receivers placed on both sides of the shot. The time step is 0.001s. The shot space is 20m and the receiver space is 10m. Fig.7 is one of shot gather. From Fig.7, we can see moderate regular and irregular noises in the data. Fig.8 is the inverse result. On both sides of Fig.8, the result is vague because of insufficient shot gather data in this area. The target layer is around \((x, z) = (6000\, m, 1500\, m)\) and we can see clearly a high velocity layer there. We also can see clearly the media velocity is high near the bottom of the model in Fig.8 which means the bed rock there.

Conclusions

The FWI is an optimization iterative process of minimizing the residuals between the synthesized data and observed data. In order to improve inverse robustness, the regularization strategy of stepwise inversion is developed. The inversion is carried out from low frequency band to high frequency band.
step by step and the high frequency band contains the low frequency band. Moreover, the inverse result at present stage is chosen as the initial model for the next inverse stage. Numerical computations for the benchmark Overthrust mode and a real data set based on parallel computations are implemented. The results show the effectiveness and good accuracy of the time-domain FWI method in this paper. The Hessian information of the objective function in the LBFGS algorithm is utilized, which is helpful to improve inverse accuracy. The present method can be expected to process real data further in the future.

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