Even performed pre-measurements have no results

Marek Żukowski and Marcin Markiewicz

1International Centre for Theory of Quantum Technologies (ICTQT), University of Gdansk, 80-308 Gdansk, Poland
(Dated: March 20, 2020)

The title of our work is a paraphrase of the title of Asher Peres’ paper Unperformed experiments have no results. We show what are the lessons to be learned from the gedankenexperiments presented by Frauchiger and Renner (claim that quantum theory cannot consistently describe the use of itself), and Brukner (a no-go theorem for observer independent facts). One has to remember Bohr’s remark “the unambiguous account of proper quantum phenomena must, in principle, include a description of all relevant features of experimental arrangement”, which specifically to the gedankenexperiments means: in all your quantum mechanical thinking about measurements, think in terms of the full quantum measurement theory. The theory sees measurement as composed of two stages: pre-measurement (entanglement, i.e. quantum correlation, of the measured system with the pointer variable), and next decoherence via interaction with an environment, which leaves a record of the result. The nature of the second stage is that the environment is beyond our control, thus the decoherence cannot be undone, neither by the original observer nor by someone else. If one uses in the description of measurement only the pre-measurement stage, there are no recorded results, and the process can be undone. We can have a quantum-marker-quantum-eraser situation. The gedankenexperiments are effectively scenarios of this type. The process described by Frauchiger and Renner is possible only if the Friends of the Wigners constrain themselves to pre-measurements. We also present a version of Bell’s inequality, which holds for hypothetical ‘outcomes’ obtained by the Friends during their (pre-)measurements. Inclusion of this notion is shown to be at odds with quantum mechanical predictions. Thus, it cannot be used in any argumentation about a quantum process which involves pre-measurements. As all that is related to the Heisenberg cut, we give its operational definition, applicable to interference experiments. We also note that E.P. Wigner introduced his Friend as a conscious agent who makes measurements and not just pre-measurements.

I. INTRODUCTION

The notion of Wigner’s Friend is back in the current discussion about foundations of quantum mechanics. It is used in the Frauchiger-Renner gedankenexperiment [1], which according to the authors shows that “quantum theory cannot consistently describe the use of itself”. We show that if Friends perform proper measurements, the gedankenexperiment presented in [1], when universal validity of the quantum description is taken into account, never leads to those correlations of results which are the basis of the paradox (contradiction) presented in [1]. Universal validity of quantum description must take into account measurement devices and the measurement process. Usually one can skip this in some cases, but it is essential, at least in a sketchy way, in the case of sequential measurements. The contradiction shown in [1] is a consequence of equating, in the case of Friends, of pre-measurement with measurement. The related gedankenexperiment of Brukner [2] according to our analysis rules out ‘results’ of pre-measurements as a well defined notion, and shows that they can be erased. The real message of the two gedankenexperiments is that pre-measurements have no results, and therefore do not generate ‘facts’, while they themselves are facts. We present a reasoning showing that a Friend who does only a pre-measurement cannot ascribe ‘a single outcome’ to a pre-measurement, as this leads to a Bell-type contradiction.

The basis of our analysis will be the quantum theory of measurement. It shows a peaceful coexistence between the collapse postulate (which will not be explicitly used here) and unitary dynamics, see e.g. [3], or the introduction in [4]. The most comprehensive and definitive exposure of it is in the trailblazing works of Zurek [5, 6].

The crucial stages (elements of the process) of measurement interaction are:

- Pre-measurement: the measured system $s$ in a state $|\Psi_s\rangle = \sum_i \alpha_i |\Psi_i\rangle_s$, where $|\Psi_i\rangle$ are eigenstates of the measured observable, gets entangled (correlated) with the pointer observable of the measuring device (states of the preferred pointer basis of it are denoted here as $|P_i\rangle_p$). Consequently we have an evolution leading from $\sum_i \alpha_i |\Psi_i\rangle_s |P_{\text{initial}}\rangle_p$ to $\sum_i \alpha_i |\Psi_i\rangle_s |P_i\rangle_p$.
- Decoherence: the measuring device is a macroscopic object, thus it has its own internal and external uncontrollable environments, which can be thought of as all other (zillions of) microscopic degrees of freedom which we usually ignore, as we are interested only in the position of the pointer. Interaction of the pointer variable with these environments leads to decoherence, which is irreversible [5, 6] (because we cannot control the environment), and to a broadcast of the results, see e.g. [7, 8]. After the decoherence the effective system-pointer state is an incoherent mixture of states $|\Psi_i\rangle_s |P_i\rangle_p$ with respective probabilities given by $|\alpha_i|^2$.

Without the second part of the measurement process, we have only a pre-measurement, which is in principle
reversible (see the excellent illustrations of this in the famous Lectures of Feynman [9]). For example within a Mach-Zehnder interferometer with an entry polarizing beamsplitter one can correlate the paths of a photon with its polarization, but then this can be undone with suitable wave plates, see e.g. [10]. As a matter of fact such ideas are behind the quantum-marker quantum eraser experiments, see [11–13]. In this case the path-polarization correlation was erased by a suitable filtering of polarization in a complementary basis (with respect to the marking basis), to recover proper (conditional) Mach-Zehnder type interference. This was done behind the exit polarization-neutral beamsplitter of the interferometer.

We shall show below that what the authors of [1] call measurement (by Friends) is in fact a pre-measurement. For proper measurement the paradox is void. Also the description in [1] of Wigner’s Friend is departing from the one by E.P. Wigner [14] (see the final note of this section). Moreover, we present a gedanken experiment (related with the one of [2]), which shows that if Wigner’s Friends make only pre-measurements, this is logically inconsistent with Friends being aware of concrete ‘results’ (or in the context of [2] and [15, 16], that pre-measurements do not generate facts).

There are many manuscripts and papers which discuss consequences of the Frauchiger-Renner paradox for various interpretations of quantum mechanics. Such a discussion is also in [1]. Some authors indicate that the source of the paradox is hidden in referring to incompatible experimental contexts [17–19], others state that decoherence must be involved in the discussion of a measurement process [20, 21]. Additional hidden assumptions in the Frauchiger-Renner paradox have also been suggested. In Ref. [22] such a hidden assumption is a kind of non-invasiveness of a measurement, called Intervention Insensitivity, whereas in Ref. [23] as much as five additional assumptions concerning the measurement process and quantum dynamics are discussed. There were also comments suggesting that the reason for the paradox is the indefinitness of the Heisenberg cut [24, 25], and insufficient objectivity of outcomes of a measurement [26, 27].

We agree with most of these comments. Here we present an analysis based solely on the formalism of quantum mechanics, without imposing additional assumptions, including the ones of [1]. The first part of our analysis will be concurrent with the critique by [20, 21] concerning the omission of decoherence in the analysis of Ref. [1].

Our analysis is interpretation neutral. Interpretations usually involve a specific understanding of the notion of the quantum state. For us a quantum state is a theory-specific description (in terms of, in general, density operators) of a statistical ensemble of equivalently prepared systems, which allows for statistical (probabilistic) predictions of future measurements, via the Born rule. The state describes an individual system only as a member of such an ensemble. The theory itself is ”a set of rules for calculating probabilities for macroscopic detection events, upon taking into account any previous experimental information”, [28]. Or if you like, one can use E. P. Wigner’s statement “the wave function is only a suitable language for describing the body of knowledge - gained by observations - which is relevant for predicting the future behaviour of the system” [14]. Note that all internally consistent interpretations (not modifications) of quantum mechanics agree with the above. They only add some other properties to the quantum state, or to individual members of the ensemble (systems), without any modification of the calculational rules of quantum theory (based on the statistical ensemble approach).

Note. We shall use the word Wigner to symbolize the final observer in the gedankenexperiments, whereas when referring to the Nobel Prize winning scientist, we shall use E.P. Wigner.

II. ANALYSIS OF FRAUCHIGER-RENNER GEDANKEN EXPERIMENT

A. Simple description of underlying process: no Wigners, no Friends

We shall show the gist of the process used by Frauchiger and Renner by mapping it into an interferometric process. The notation which we shall use will differ from the one in [1], as we aim at its simplicity.

Imagine two entangled qubits, $g$ and $s$. The letters are to remind us about the quantum random number generator and the system used in [1]. Assume that they are in the following initial state

$$|\text{init}\rangle_{gs} = \sqrt{\frac{1}{3}}|H\rangle_g |\!\!\!1\rangle_s + \sqrt{\frac{2}{3}}|T\rangle_g |\!\!\!1\rangle_s ,$$

(1)

where $|H\rangle_g$ and $|T\rangle_g$, and states $|\!\!\!1\rangle_s$ form orthonormal bases, and $|\pm\rangle_s = \sqrt{\frac{1}{2}}(|\!\!\!1\rangle_s \pm |\!\!\!1\rangle_s$). The state can be put in an alternative way as

$$|\text{init}\rangle_{gs} = \sqrt{\frac{1}{3}}[(|H\rangle_g + |T\rangle_g)|\!\!\!1\rangle_s + |T\rangle_g |\!\!\!1\rangle_s ]$$

(2)

A glance at the formulas (1) and (2) shows us that:

- **Situation 1**: A measurement of $g$ in a basis consisting of $|\pm\rangle_g = \sqrt{\frac{1}{2}}(|H\rangle_g \pm |T\rangle_g)$, and a measurement of $s$ in basis $|\pm\rangle_s$, according to (2), gives ”−” for $g$ and ”−” for $s$ with with a non-zero probability.

- **Situation 2**: A fully complementary measurement in basis $\{ |H\rangle_g, |T\rangle_g \}$, and a measurement of $s$ in basis $|\pm\rangle_s$, according to (1), cannot lead to $T$ for $g$ and ”−” for $s$. That is, the joint probability of $T$ for $g$, and ”−” for $s$ is zero.
Only the first itemized situation, as we shall see further on, allows \( \bar{w} = \bar{w} \) and \( w = w \) ‘halting condition’ of \([1]\) (i.e. results \"\(-\)\" for both \( q \) and \( s \)), which is the one on which the authors of \([1]\) base their paradox. In the second, complementary, situation \( T \) and \( w = \bar{w} \) is impossible.

**B. Reworded original description involving pre-measurements by Friends**

Let us now introduce agents in the form of (super) Wigners \( W_g \) and \( W_s \), and their respective (virtual) Friends (see footnote \([33]\)) \( F_g \) and \( F_s \), as well as their devices \( D_g \) and \( D_s \). All this is with the obvious relation to the two laboratories of the Friends: \( L_g \), which supposedly generates inside a random output of a quantum coin \( H = \text{heads} \) and \( T = \text{tails} \), and \( L_s \), the one which operates on the system \( s \) emitted by \( L_g \) in correlation with \( H \) or \( T \) (pre-measurement) ‘result’.

The initial state of the ‘coin’ of the generator is 
\[
|\text{init}\rangle_{L_g} = \sqrt{\frac{T}{3}} |H\rangle_{g} |H\rangle_{D_g} |H\rangle_{F_s} |−⟩_s + \sqrt{\frac{T}{3}} |T\rangle_{L_g} |+⟩_s ,
\]
where in our notation \(|X\rangle_{L_g} = |X\rangle_{g} |X\rangle_{D_g} |X\rangle_{F_s} \) and \( X = H, T \). The kets related to the subsystems \( g, D_g \) and \( F_g \) are these which are correlated with each other in the pre-measurement process, i.e. in fact we have:

\[
|\text{init}\rangle_{L_g} = \sqrt{\frac{T}{3}} |H\rangle_{g} |H\rangle_{D_g} |H\rangle_{F_s} |−⟩_s \\
+ \sqrt{\frac{T}{3}} |T\rangle_{g} |T\rangle_{D_g} |T\rangle_{F_s} |+⟩_s .
\]

The system \( s \) is sent to lab \( L_s \). As the result of a pre-measurement by \( F_s \), with respect to the basis \(|±⟩\)\(_s\), one gets the following overall state:

\[
|\text{init}\rangle_{L_g L_s} = \sqrt{\frac{T}{3}} [(|H\rangle_{L_g} |T\rangle_{L_g} |−⟩_L_s + |T\rangle_{L_g} |+⟩L_s)],
\]
where in our notation \(|Y\rangle_{L_s} = |Y\rangle_s |Y\rangle_{D_s} |Y\rangle_{F_s} \) with \( Y = ±1 \). Please note that it is isomorphic with \([2]\). Equivalently:

\[
|\text{init}\rangle_{L_g L_s} = \sqrt{\frac{T}{3}} |H\rangle_{L_g} |−⟩_L_s + \sqrt{\frac{T}{3}} |T\rangle_{L_g} |+⟩_L_s ,
\]
where \(|±⟩_L_s = \sqrt{2}(|−⟩_L_s ± |+⟩_L_s \). This state is isomorphic with \([1]\). Isomorphisms of this kind would pop up further down.

Now enter the (super) Wigners, who make full proper measurements on the respective labs. \( W_g \) measures in a basis containing \(|±⟩_{L_g} = \sqrt{\frac{T}{3}}(|H\rangle_{L_g} ± |T\rangle_{L_g} \), whereas \( W_s \) in a basis containing \(|±⟩_L_s \). Whenever \( W_g \) obtains \(|−⟩_{L_g} \) there is probability \( 1/2 \) that \( W_s \) obtains \(|−⟩_L_s \). This is the \( ok \) and \( \bar{w} \) situation of \([1]\).

Despite the fact that within \( L_g \) only a (reversible) pre-measurement takes place, it is assumed in \([1]\) that somehow \( F_g \) may think that a real measurement took place with a definite result \( H \) or \( T \) (this is shown further on here to be an internally inconsistent idea). When one considers \( T \) as a possible state of mind of \( F_g \), then to \( F_g \) ‘ok’ by \( W_s \) is (seems?) impossible (see the previous section, situation 2, and recall the isomorphism). This is because in such a situation system \( s \) is sent to \( L_s \) in state \(|+⟩_s \) which after the pre-measurement interaction in \( L_s \) leads to \(|+⟩_L_s \), while ‘ok’ of \( W_s \) is associated with \(|−⟩_L_s \). The purported paradox is that, according to the Authors, one can have \( T \) for \( F_g \) as her ‘outcome’, while both Wigners obtain outcomes “\(-\)” (ok and \( \bar{w} \)).

Note that the initial state \([3]\) is not stated openly in \([1]\) as the crux of the protocol, but only mentioned in passing, during the discussion. When the main protocol is described, it is said that whenever \( F_g \) sees \( H \), system \( s \) in state \(|−⟩_g \) is sent, whereas for \( T \) it is sent in \(|+⟩_s \). Still the description using \([3]\) is the only one consistent with the situation resulting with \( w = \bar{w} = ok \) (which is isomorphic with situation 1). An incoherent mixture of \(|H⟩_g |H⟩_{D_g} |H⟩_{F_s} |−⟩_s \) and \(|T⟩_g |T⟩_{D_g} |T⟩_{F_s} |+⟩_s \) in the case of \( F_g \) knowing that \( T \) happened leads to an isomorphism with situation 2, and one cannot have \( w = \bar{w} = ok \).

**C. Description involving full proper quantum measurements by Friends**

Here we shall consider the consequences of using the full quantum measurement theory, which involves decoherence and irreversibility of recorded results. Note that this means that we assume that quantum theory has a ‘universal validity’ which encompasses the full measurement process. We assume here that both Friends and Wigners perform full proper measurements.

Consider the stages of the measurement. Let us take lab \( L_g \) which is slightly more important here, but a similar description can be used in the case of the other lab. Assume the system \( g \) is in a superposition \( \alpha |H⟩_g + \beta |T⟩_g \). In the pre-measurement stage it gets entangled with the pointer variable of device \( D_g \). The states of the pointer basis are denoted here as above by \(|X⟩_{D_g} \). We get as a result \( \alpha |H⟩_{D_g} |H⟩_{D_s} + \beta |T⟩_{D_g} |T⟩_{D_s} \).

Now enters the decoherence. Of course the environment might be in a mixed initial state, but for simplicity of presentation we shall write it down as a pure one. Within the considered scenario the decoherence takes place due to interaction with the environment constrained to lab \( L_g \). The two distinguishable states of the environment related with distinct states of the pointer-device \( X = H, T \) will be denoted as \(|X⟩_{E_g} \). Thus if one
given by:
\[\begin{align*}
\alpha |H\rangle_g |H\rangle_{D_g} |H\rangle_{F_g} |H\rangle_{E_g} | -1 \rangle \\
+ \beta |T\rangle_g |T\rangle_{D_g} |T\rangle_{F_g} |T\rangle_{E_g} | +1 \rangle .
\end{align*}\] 

The trouble is that \(W_g\) upon his measurement, which perhaps could involve the imaginary conscious Friend \(F_g\), cannot undo the irreversible interaction with the environment (and thus cannot undo the records). His power is limited to measurements of \(g\), maybe even \(g \otimes D_g\), and if he is cruel and super-sophisticated to \(g \otimes D_g \otimes F_g\). But in all such cases because of the entanglement with environment \(E_g\), effectively, he deals with a reduced density matrix of the subsystems, which is a classically correlated state (for \(\alpha \beta \neq 0\)): \(|H\rangle_g |H\rangle_{D_g} |H\rangle_{F_g} + |T\rangle_g |T\rangle_{D_g} |T\rangle_{F_g}\), with probability \(|\alpha|^2\) and \(|\beta|^2\), respectively.

Thus a consistent use of the full measurement theory nullifies the paradox. \(F_g\) records either \(H\) or \(T\). If it is \(T\), there is an isomorphism with situation 2, and there is no way to formulate the paradox of [1].

Therefore we come back to the old dictum by Asher Peres: *unperformed experiments have no results* [29]. One should here add: pre-measurements are not performed measurements as they can be undone. Every completed full measurement leaves traces in the environment, which are correlated with the results (pointer positions). As a state vector (or a density operator) is a mathematical description of a statistical ensemble of equivalently prepared systems, giving probabilistic predictions, we get a classical probability distribution of states of the environment correlated with pointer states.

However, all that leaves us with the task of analyzing the situation of the previous subsection in which Friends perform only pre-measurements. We shall show that in such a case the Friends cannot know ‘outcomes’. This also nullifies the paradox.

III. BELL THEOREM FOR PRE-MEASUREMENTS

We shall present here a related gedanken experiment which shows that “pre-measurements have no results”. We shall test the consistency of the idea that an agent who performs only a pre-measurement may know its result (which is a tacit assumption in [1], as the authors do not differentiate between pre and full measurements).

A full operational description of such an experiment, involving measurements of polarization of photons will be presented. On the way we shall show how one can realize operationally the situation in which Wigner knows that his Friend preformed a pre-measurement, without knowing what was the setting of her device, and what was the result. Additionally we shall show an operational arrangement in which Wigner may face one of two complementary pre-measurement situations prepared by Friend, upon her whim. We shall test whether one can endow Friends with ability to know ‘results’ of their pre-measurements, without actually performing a full measurement (as it is effectively the case in [1]). Obviously, we are here inspired by [2], still our approach and aim is different.

A. The gedankenexperiment. Operational blueprint

Assume that we use a type-II parametric down conversion source. Consider an emission of a single pair. One photon goes to lab \(L_a\) and the other one to a spatially separated lab \(L_b\). The (partially sealed) labs are operated from inside by Friends \(F_a\) and \(F_b\). The photons are polarization entangled: \[\frac{1}{\sqrt{2}}(|h\rangle_a |v\rangle_b - |v\rangle_a |h\rangle_b).\] Here \(h\) stands for the horizontal polarization, and \(v\) for vertical. Each lab is equipped with a universal polarization beamsplitter. It directs two orthogonal polarizations of light into two different exit beams. An operational method of obtaining such a beamsplitter is to use a standard \(h\) and \(v\) polarization separating beamsplitter, and to place in front of its (used) input port a suitable polarization rotator, and an additional tunable/replacable wave-plate which changes the relative phase of \(h\) and \(v\) polarizations. Such a device is capable to perform any \(U(2)\) transformation on polarization states. If one sets transformation \(U_h(\alpha, \phi)\) to be such that it transforms \(|h\rangle\) into \(\cos \alpha |h\rangle + e^{i\phi} \sin \alpha |v\rangle\), and \(|v\rangle\) into \(\cos \alpha |v\rangle + e^{i\phi} \sin \alpha |h\rangle\), the device is such that photons of elliptic polarization \(U_h(\alpha, \phi) |h\rangle\) exit by output \(h\) (with polarization \(h\)) and those of polarization \(U_h(\alpha, \phi) |v\rangle\) via exit \(v\) (with polarization \(v\)).

Additionally let us assume that both exit beams of the universal polarizer are at an angle with the original beam. Thus when there is no polarizer the photons travel along the original path (this can be done in a more sophisticated way than by removing the polarizer - by putting in front of it a Mach-Zehnder interferometer, and allowing in it two internal phase differences, one that directs all light to an exit port which leads to the polarizer and another one which directs the light to the other exit port, which does not lead to the polarizer). Finally, let us place behind the \(v\) exit port of the polarizer a polarization rotator which rotates \(v\) polarization to \(h\).

Let us now (partially) seal the lab, in such a way that respective Wigner, \(W_a\) or \(W_b\), has no knowledge which two elliptic polarizations are split by the beamsplitter (simply, he does not know the local \(U(\alpha, \phi)\)). Still we would allow the lab to have three exit optical fibers, two with their inputs behind the beamsplitter, and one for the ”no-measurement” channel. If Wigner registers a photon in the last channel, then this signals that no pre-measurement was done. Registration in the other two channels signals that a pre-measurement was done within the lab (for simplicity we assume a perfect detec-
tion efficiency). As no matter what is the device setting by Friend (the choice of $U(\alpha, \phi)$), the final result of this is a detection of an $h$ polarized photon by Wigner in one of the "measurement" exits. Until he unseals the lab to look inside at the setting of the device, he does not know which pre-measurement was done.

**B. Bell Theorem for values of pre-measurements**

Assume now that Friend has a mind, and its state is determined by the pre-measurement which she does, and that she somehow "knows" the "result/outcome" (as it is done in [1]). We can give to these states of mind a numerical value $S_F(x) = \pm 1$, where $x = 1, 2$ numbers one of two complementary settings of her polarizers. The value depends on by which exit, according to Friend, the photon left the polarizer. Note that values $S_F(x)$, in the case of pre-measurements are not a part of quantum formalism.

We consider a pair of Wigners $W_a, W_b$ and a pair of their respective Friends $F_a, F_b$. The pairs $W_a, F_a$ and $W_b, F_b$ are spatially separated. The states of Friend’s minds are denoted by $S_{F_a}(x)$ and $S_{F_b}(y)$, respectively. We allow each of the Friends to choose freely and perfectly randomly between two settings, indexed respectively $x = 1, 2$ and $y = 1, 2$. Wigners open the respective local labs after each run, to, effectively, read out the states of minds of Friends (note that they do this after the photon is detected). Seeing the setting of the apparatus they determine the local setting $x$, and this allows them to determine which value of $S_F(x)$ was in the mind of the local Friend (as Wigners know where the photon was detected). Note that this was only in Friends’ minds. Only local Wigners see which detector fired as the detectors are outside of Friends labs.

As obviously:

$$S_{F_a}(1)S_{F_b}(1) + S_{F_a}(1)S_{F_b}(2) + S_{F_a}(2)S_{F_b}(1) - S_{F_a}(2)S_{F_b}(2) = \pm 2, \quad (8)$$

we can derive a CHSH Bell inequality for states of mind of Friends, and obviously find optimal measurement settings for the described quantum process to violate the inequality by a factor of $\sqrt{2}$. I.e., states of mind of Friends concerning “results” of their pre-measurements form an ill defined notion (just like local hidden variables). Note, that if one assumes that Friends must have minds which give them awareness of pre-measurement "results", this version of Bell’s theorem shows that they do not exist (such Friends, or if you like such ‘results’). There is no need to show any details concerning the derivation of the violation of a Bell inequality for pre-measurement results. This is because what we have here is a standard Bell type experiment with a dissection of the measurement process into the pre-measurement by Friends, and the read-out by Wigners.

Here come three remarks: Note that we do not demand that Wigners perform some super measurements like those in a basis in which Friends (plus perhaps local devices) and the photon are entangled, e.g. states $|\pm\rangle_{L_S}$. What they do is just checking what is the macroscopic situation inside the given local lab. Effectively, they always measure in the same basis in which the Friends pre-measured.

Note further that, in all early discussions about Wigner’s Friend, including the original one by E. P. Wigner [14], it was always assumed that whenever Wigner ‘measures’, then it is in a basis which is in concurrence with the measurement of Friend, he just confirms Friend’s result.

- E.P. Wigner: "my friend’ has the same types of impressions and sensations as I - in particular he is not in that state of suspended animation which corresponds to the wave function $\alpha(\Psi_1 \times X_1) + \beta(\Psi_2 \times X_2)$, [14]. In E.P. Wigner’s notation $\Psi_i$ are (mutually orthogonal wavefunctions of the measured system, and $X_i$ are mutually orthogonal states of the Friend or her consciousness.

Finally, please notice that we have made above a tacit assumption which is now revealed: states of mind of Friends depend only on their local settings. That is why we have $S_{F_a}(x)$ and not $S_{F_a}(x, y)$, etc. This is the usual locality assumption, which here could be interpreted as an exclusion of superluminal telepathic communication between the minds of the Friends. Anyone allowing for this can reject our theorem.

**C. Gedanken-experiment of brukner vs. ours**

In our interpretation brukner’s gedanken experiment [2] confronts the ‘results’ of fixed-setting pre-measurements done by Friends, that are followed by, either, a final full ‘confirming’ measurement, or, by final measurements by Wigners which are complementary to the confirming ones. The latter ones erase the ‘earlier results’ by Friends. The impossibility of coexistence of ‘results’ by Friends and results by Wigners for the latter case is revealed by showing the non-existence of their joint probability distribution. In our experiment non-existence of pre-measurement results is shown without requiring that Wigners have powers to make measurements on the entire labs of Friends, which are in a basis which eradicates the ‘results’ of the pre-measurements (see further). They just read out what happened in the labs of Friends. Thus our experiment is not gedanken at all, and effectively is a kind of ‘delayed setting knowledge’ Bell experiment in which the local observer first learns which detector fired, and only later what was the random setting of the local measurement device (and the settings are decided randomly by some process which is independent of the observer).
IV. DISCUSSION

As the process which is at the crux of argumentation in [1] does not take place, once one requires real measurements by Friends, there is no need to analyze which of the assumptions given in the work is violated in the case of full measurement theoretic description of the gedanken experiment. Still it is worthwhile to point that the full quantum description is consistent with all three “natural sounding assumptions” of [1]: (Q): universal validity of quantum mechanics, (C) consistency of predictions of all agents, (S) for an agent who carries out a particular measurement, this measurement has only one outcome. Note that the last assumption is not valid at the stage of pre-measurement, see our Bell experiment. It holds only for full measurements. This hidden assumption that pre-measurements are on equal footing with (completed) measurements is the root of the (apparent) Frauchiger-Renner paradox. The assumption is unfounded, because of the basics of measurement theory, and also because one can show a Bell-type no-go theorem which shows such an assumption to be internally inconsistent. Therefore, the conclusion is that the paradox must be given a different interpretation than the one in [1]: pre-measurements have no definite results, and therefore Friends who perform just pre-measurements are not agents who satisfy (S).

Quantum mechanics consistently describes its use, provided we follow the following dictum by Bohr, [30], “the unambiguous account of proper quantum phenomena must, in principle, include a description of all relevant features of experimental arrangement,” [34]. “Experimental arrangement” obviously includes the measuring devices, and the process of the read-out of the outcomes, if these are relevant. When paradoxes pop up, it may mean that our quantum description misses some important element. This consistency can be show without resorting to some specific interpretation of quantum mechanics, like e.g. the Bohmian one (compare [31]).

A. Super-Wigners and virtual-Friends, and the Heisenberg Cut

We use these strange names in order to stress the difference between E.P. Wigner’s original idea [14], and the ‘agents’ used in both Refs. [1] and [2].

Here we formulate definitions of such agents, written in the context of the gedanken-experiment of Ref. [1].

- Definition of a Super-Wigner $W_g$ (with the reformulated wording of [1]): an agent who is capable to perform measurement in a basis containing $|\pm\rangle_{L_g} = \frac{1}{\sqrt{2}}(|H\rangle_g |H\rangle_{D_g} |H\rangle_{F_g} |\pm\rangle_{T_g} |T\rangle_{D_g} |T\rangle_{F_g})$. Super-Wigners perform full (irreversible) measurements.

Comment: this is daunting task, as especially the device, $D_g$, and the friend $F_g$ may be complex objects endowed with many degrees of freedom. Still as long as all these degrees of freedom are controllable, a Super-Wigner might be able to perform such an experiment. Obviously, if we have in lab $L_g$ an uncontrollable environment interacting with $D_g$ or $F_g$, a measurement in such a basis is impossible.

- Definition of virtual-Friend $F_g$: an agent who is capable to perform an action which is a reversible pre-measurement. That is, she does not interact with an uncontrollable environment, and is able to steer herself and the lab into a superposition $|\alpha H\rangle_g |H\rangle_{D_g} |H\rangle_{F_g} + \beta |T\rangle_g |T\rangle_{D_g} |T\rangle_{F_g}$, in response to the quantum system $g$ being initially in superposition $|\alpha H\rangle_g + \beta |T\rangle_g$.

Virtual Friends do not perform measurements, but only pre-measurements, which could be called virtual-measurements, as they can be undone, e.g. by a suitable measurement by a Super-Wigner.

- Super Wigner $W_g$ and virtual Friend $F_g$ are defined in an analogical way.

Obviously, we deal exactly with such agents in [1] and [2]. Still ‘virtuality” of the Friends is additionally amplified by if one additionally assumes that they ‘know’ the ‘result’ of their pre-measurement (in concurrence with [2] we have shown the inconsistency of this notion).

Moreover, Super-Wigners erase the ‘result’ by their Friends, if their measurement bases are like in [1], or more generally not concurrent with the pre-measurement of their Friend. Say, $W_g$ registers result which is concurrent with $|-\rangle_{L_g} = \frac{1}{\sqrt{2}}(|H\rangle_g |H\rangle_{D_g} |H\rangle_{F_g} - |T\rangle_g |T\rangle_{D_g} |T\rangle_{F_g})$. Then, obviously, all that he knows is that the ‘result’ of $F_g$ is either $H$ or $T$ with probability 1/2. I.e., he knows nothing. Also there is no way to recover the ‘result’ of $F_g$ as the measurement of $W_g$ is final (in [1] it is the results of Super-Wigners which halt the ‘experimental procedure’, or restart it from scratch, thus they are accessible to an experimenter or a macroscopic automaton which controls the experiment).

I. Operationally defined Heisenberg Cut

Super-Wigners cannot exist without virtual-Friends. The division between them is related to the Heisenberg Cut. The Cut is movable, as with technological progress Super-Wigners may be able to control more and more of degrees of freedom associated with ever more complex Friends. The cut is an objective division between the controllable and uncontrollable part of the full measurement process involving $F_g$ and $W_g$. Still it is defined by the technology used, it is dependent on ‘all relevant features of the experimental arrangement’.

Let us attempt to give an operational method to define such a Cut in the experiment considered here. Obviously sealing in a laboratory of our friend (say, Harald) and
expecting him to get entangled in a pre-measurement superposition like \( \pm \langle L_\theta | = \frac{1}{\sqrt{2}} (|H\rangle_g |H\rangle_{D_g} |H\rangle_{F_g} \pm |T\rangle_g |T\rangle_{D_g} |T\rangle_{F_g} ) \), seems out of question. This is because states \( |Y\rangle_{F_g} \) are in fact states of very many, say \( 10^{40} \) or something, degrees of freedom. Thus we can start our experiments with a qubit Friend, \( F_1 \), of states spanned by \( |X\rangle_{F_1} \). Control and observation by Wigner of superpositions \( \frac{1}{\sqrt{2}} (|H\rangle_g |H\rangle_{D_g} |H\rangle_{F_1} + e^{i\phi} |T\rangle_g |T\rangle_{D_g} |T\rangle_{F_1} ) \) is very easy, as it is enough to impose a phase shift in basis \( |\pm\rangle \) by making measurements in the basis \( |\pm\rangle \). The threshold \( m_0 \) signifies the Heisenberg cut. Above it, the state of the system is very easy, as it is enough to impose a phase shift on system \( g \). Here we assume that \( D_g \), the pointer variable, is also a qubit. However Wigner may try a more complicated system, with an artificial Friend composed of \( m \) qubits. Thus we replace \( |Y\rangle_{F} \) by \( |Y\rangle_{F} = \otimes_{n=1}^{m} |Y_n\rangle_{F_n} \), and require that \( \phi \langle T|H\rangle_F = 0 \). To get interference it is again enough to phase shift the qubit \( g \) to get \( \frac{1}{\sqrt{2}} (|H\rangle_g |H\rangle_{D_g} |H\rangle_F + e^{i\phi} |T\rangle_g |T\rangle_{D_g} |T\rangle_F ) \) and then for example to unitarily transform \( |T\rangle_g |D_g\rangle |T\rangle_F \) into \( |T\rangle_g |H\rangle_{D_g} |H\rangle_F \), to get \( \frac{1}{\sqrt{2}} (|H\rangle_g + e^{i\phi} |T\rangle_g ) |H\rangle_{D_g} |H\rangle_F \). Finally Wigner makes his own measurement of \( g \) in basis \( \frac{1}{\sqrt{2}} (|H\rangle_g \pm |T\rangle_g ) \). This will lead to a perfect interference.

Our experimenter may be ambitious, and would aim at the biggest possible \( m \). At a certain \( m \) his ability to transform \( |T\rangle_F \) into \( |H\rangle_F \), i.e. to control the microscopic state of Friend will be lost. Possible reasons may be the following: too high \( m \) to handle, or Friend’s interaction with the environment (uncontrollable by definition).

Note that the environment can be thought of as a part of the Friend, hence this case is reduced to a question of a too high \( m \) for a quantum control of the Friend’s degrees of freedom. The threshold \( m_0 \) signifies the Heisenberg cut. Above it, the state of the system, qubit, and Friend is a probabilistic mixture of \( |H\rangle_g |H\rangle_{D_g} |H\rangle_F \) and \( |T\rangle_g |D_g\rangle |T\rangle_F \). This signifies that Friend effectively makes a full measurement (an effective decoherence steps in), and a classical description of the Friend is permitted.

To put it short, if Wigner is able to obtain a \( \phi \) dependent interference in an experiment on system, plus pointer, plus Friend in state \( |\phi\rangle_{L_\phi} = \frac{1}{\sqrt{2}} (|H\rangle_g |H\rangle_{D_g} |H\rangle_{F_g} + e^{i\phi} |T\rangle_g |T\rangle_{D_g} |T\rangle_{F_g} ) \) by making measurements in the basis \( |\pm\rangle \) of \( L_\phi \), then his Friend performs only a pre-measurement. System, plus pointer plus Friend can and should be described using quantum mechanics. If such interference is impossible, quantum mechanical description is still possible, however it is redundant. We have passed the Cut.

Note, that there is a widespread view that the irreversibility of outcomes assured by the decoherence works only For All Practical Purposes - FAPP, and therefore is not fundamentally justified. However, as pointed out by R. F. Streater [32], if we accept this viewpoint, we have the same problem in classical physics in the theory of phase transitions. Formally phase transitions occur in the thermodynamic limit, which is the limit of an infinite size of the considered system. In analogy, one can argue, that formally the quantum measurement is irreversible only in the limit of infinite size of the device (or its environment). In our opinion both of these processes are fundamentally objective, however their justification needs an idealization in the mathematical description. Therefore the problem is not with the physical reality, but rather with the understanding that the formalism itself is only a tool to describe it, and as such some idealizations within the formalism are unavoidable.

* 

ACKNOWLEDGMENTS

Acknowledgments.—The work is part of the ICTQT IRAP (MAB) project of FNP, co-financed by structural funds of EU. MZ dedicates this work to the memory of prof. dr Fritz Haake.

V. APPENDIX: QUANTUM MARKER - QUANTUM ERASER EXPERIMENTS

A beautiful example of an experiment, which undoes a pre-measurement, can be found in [11–13]. Here we shall present just the basic theory of such experiments, which is not a direct description of the experiments in [11–13].

Imagine a Mach Zehnder interferometer with the front beam-splitter replaced by a polarizing beamsplitter which splits polarizations \( h \), horizontal and \( v \), vertical. A photon of polarization \( |d\rangle = \frac{1}{\sqrt{2}} (|h\rangle + |v\rangle) \) enters the beamsplitter, and leaves it in a state \( \frac{1}{\sqrt{2}} (|h\rangle + |v\rangle) \), where \( |j\rangle \), with \( j = 1, 2 \), denote the photon being in the upper, 1, or lower, 2, arm of the interferometer. Let us place a phase shifter in the upper arm. After that the state becomes \( \frac{1}{\sqrt{2}} (e^{i\phi}|h\rangle|1\rangle + |v\rangle|2\rangle) \). Let the exit beamsplitter be, as it is usually the case, a symmetric polarization neutral one. Thus it transforms \( |1\rangle \) into \( \frac{1}{\sqrt{2}} (|1\rangle + i|2\rangle) \) and 2 into \( \frac{1}{\sqrt{2}} (|2\rangle + i|1\rangle) \). The final state is thus:

\[
\frac{1}{2} (e^{i\phi}|h\rangle|1\rangle + ie^{i\phi}|h\rangle|2\rangle + |v\rangle|2\rangle + i|v\rangle|1\rangle).
\] (9)

We have a superposition of four orthogonal states. The probability of finding the photon in exit beam 1 is 1/2. We have no interference (no dependence on \( \phi \)). This is because we do not have the required indistinguishability of paths ‘taken’ by the photon within the interferometer. The polarizations mark the paths, or if you like (pre) measure them (well, it is the polarization beamsplitter that does this). However this can be undone, by placing in front of the detector a polarization filter letting through only polarization \( |d\rangle = \frac{1}{\sqrt{2}} (|h\rangle + |v\rangle) \). Then the amplitude for the photon to reach the detector in say
beam 1 would be \(\left(\frac{1}{2}\right)^3(e^{i\phi} + i)\), which gives an interference of visibility equal to 1. The paths are indistinguishable beyond the polarization filter. Obviously if we place some photon-non-destroying detection devices in the internal paths of the interferometer recovering of the interference would be impossible.