An Azimuth Frequency Domain NCS Algorithm for Missile-borne SAR

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Abstract. SAR data in highly squint mode features large range walk and small range curvature. Therefore, an advanced Frequency Phase Filtering Algorithm based on line range walk correction is proposed in this paper. Firstly, range walk is removed in range frequency and azimuth time domain. Then, a chirp scaling (CS) algorithm is adopted to correct range curvature. Based on the above operation, a novel high-order of phase filtering factor is introduced into the azimuth time and frequency domain in order to correct the azimuth-dependence. Finally, the signal is focused in the Doppler domain by SPECtral Analysis technique. Simulation results illustrate that it satisfies the imaging quality of SAR in high squint mode and large scene swath.

1. Introduction

Synthetic aperture radar (SAR), becoming an increasingly prevalent research area, is a tool that wrought a technological revolution aimed at improving radar’s information acquisition ability to obtain high-resolution microwave images of the observed scene, regardless of meteorological conditions and solar illumination\cite{1}. For missile-borne SAR working in highly squinted mode, the echo features large range walk and small range curvature. With the increase of squint angle and scene, the two-dimensional coupling between range dimension and azimuth dimension is enhanced. In order to eliminate the coupling, a series of algorithms have been proposed in recent years. In reference \cite{2}, Keystone transformation is adopted to remove the range walk, and then an ANCS algorithm is used to eliminate the space variance of doppler center and azimuth frequency modulation in the frequency domain. However, Keystone transformation is an interpolation operation, which burdens the operation rate. Chen\cite{3} proposed a highly squinted CS algorithm based on FrFT, however, the effect of range walk to the azimuth space variance is not taken into consideration, bearing small imaging scene. For sub-aperture algorithms\cite{4}\cite{5}, the aperture is divided into several sub-apertures to expand the focused imaging area, theses algorithms need to improve the overlap rate of data processing to improve the azimuth imaging effect, and thus the imaging efficiency is limited. Because of the complex and inefficient interpolation, the computation efficiency of Omega-K algorithms\cite{6}\cite{7} is limited.

Therefore, an azimuth frequency domain NCS (AFNCS) algorithm for Missile-borne SAR is proposed in this paper. In range dimension, the accurate range cell migration correction of all targets is achieved by chirp scaling algorithm. In azimuth direction, a novel method of azimuth space variance correction is proposed. Firstly, in order to increase the freedom of the system, a higher filtering factor is introduced in azimuth time domain, and then a new filtering factor is introduced in azimuth frequency domain to correct the azimuth frequency modulation and cubic cavitation. Finally, the focus
of the signal is completed in azimuth time domain. The algorithm proposed in this paper only includes complex multiplication operation and FFT operation, decreasing the computation. Simulation results are presented to validate the proposed method.

2. Geometric model and signal model

In this paper, we consider the highly squinted missile-borne SAR geometry, as shown in Fig.1. The mathematical symbols and their definitions used in this paper are given as below.

| Symbol | Definition |
|--------|------------|
| $\theta_0$ | Squint angle. |
| $V$ | Velocity of the platform. |
| $t_r, \eta$ | Range and azimuth time variables. |
| $R_B$ | Closest ranges from the antenna to the target $P$. |
| $R_s$ | Closest ranges from the antenna to the reference target. |
| $R_0$ | Reference slant range |
| $\lambda, f_c$ | Carrier wavelength and carrier frequency of the transmitted signal. |
| $c$ | Speed of light. |
| $f_r, f_\eta$ | Range and azimuth frequency variables. |
| $\gamma$ | Chirp rate of the transmitted signal. |

Fig.1 geometry of the highly squinted SAR

The platform flies through the X coordinate. The zero of the azimuth-time is defined at the time when the platform is at the point A. At the same time, the beam center points to B. P is an arbitrary point target in the scene. Supposing the transverse range between P and B is $X_s$, the instantaneous slant range between radar and the target P at the slow time $t_m$ is

$$R(\eta; R_0) = \sqrt{(V\eta - X_s)^2 + R_0^2 - 2R_0(V\eta - X_s)\sin\theta_0}$$

(1)

Supposing the transmitted signal is chirp signal

$$s(t_r) = a_r(t_r)\exp\left(j2\pi f_c t_r\right)\exp\left(j\pi f_\eta^2\right)$$

(2)

where $a_r(t)$ is the envelope of the signal. The baseband signal of the point P is

$$s(t_r, \eta, R_0) = \exp\left[j\pi\frac{2R(\eta, R_0)}{c}\right] \exp\left[-j\frac{4\pi}{\lambda} R(\eta; R_0)\right]$$

(3)

Defining $f_{\eta \mu} = \frac{2V}{\lambda}$ and $\cos \theta = \sqrt{1 - \frac{\lambda^2 f_{\eta \mu}^2}{4V^2}}$, after range walk removal, and then through range domain Fourier Transform, the equation (3) can be expressed as

$$s(f_r, \eta, R_0) = \exp\left[-j\frac{f_r^2}{\gamma}\right] \exp\left[-j\frac{4\pi}{c} R(\eta; R_0)(f_r + f_c)\right]$$

(4)

where
\[ R_0(\eta; R_0) = \sqrt{R_0^2 + V^2 \cos^2 \theta_0 (\eta - X_n/V)^2 + X_n \sin \theta_0 + \frac{V^3 \sin \theta_0 \cos^2 \theta}{2R_0^2} (\eta - X_n/V)^3} \quad (5) \]

Transforming the equation (4) to the two-dimensional (2-D) frequency domain, after that, expanding the equation through Taylor series, we obtain

\[
S(f_x, f_y; R_0) = \exp[-\frac{4\pi}{c}(f_x + f_y)X_n \sin \theta_0] \exp[-j2\pi f_x X_n \sin \theta_0] \exp[j \Phi_0(f_x, R_0)]
\]

\[
\times \exp\left[j \left( \Phi_1(f_y; R_0) f_x + \Phi_2(f_x; R_0) f_y^2 + \Phi_3(f_x; R_0) f_y^3 \right) \right]
\]

where

\[
\Phi_0(f_y; R_0) = -\frac{2\pi R_0}{V \cos \theta_0} \sqrt{f_y^2 - f_y^2 + \frac{2\pi R_0 \tan \theta_0 \sin \theta}{\lambda \cos^3 \theta}}
\]

\[
\Phi_1(f_y; R_0) = -\frac{4\pi}{c} \left( \frac{1}{\cos \theta} - \frac{\tan \theta_0 \sin^2 \theta (\cos^2 \theta - 3)}{2 \cos^3 \theta} \right)
\]

\[
\Phi_2(f_x; R_0) = -\pi \left( \frac{1}{\gamma} - \frac{2\lambda \sin^2 \theta}{c^2 \cos^3 \theta} \left( 1 + \frac{3 \tan \theta_0 \sin \theta (2 + 3 \sin^2 \theta)}{2 \cos^3 \theta} \right) \right)
\]

\[
\Phi_3(f_x; R_0) = -\frac{2\pi R_0 \lambda^2 \sin^2 \theta}{c^4 \cos^3 \theta} \left( 1 - \frac{\tan \theta_0 \sin \theta (30 \cos^2 \theta - 3 \cos^4 \theta - 35)}{2 \cos^4 \theta} \right)
\]

In equation (6), \( \Phi_0(f_y; R_0) \) is the azimuth modulation term, which corresponds to the azimuth position and the azimuth compression signal of the point target, \( \Phi_1(f_y; R_0) \) is the range cell migration(RCM), while \( \Phi_2(f_x; R_0) \) is the range modulation term, \( \Phi_3(f_x; R_0) \) corresponds to the third phase term, respectively.

3. **AFNCS algorithm**

3.1. **Range dimension processing**

Before azimuth dimension processing, the range dimension processing of missile-borne SAR is first considered. Considering the space variance of RCM, chirp scaling (CS) algorithm is adopted here to eliminate the range space variance.

Compared to the traditional CS algorithm[8][9], Due to the range walk removal, the new scaling factor \( H_{CS} \) can be expressed as

\[
H_{CS} = \exp\left[ j \pi R_c(f_y; R_0) a(f_y) \left( t_r - \frac{2R_0(f_y; R_0)}{c} \right)^2 \right]
\]

(8)

After CS operation, the signal can be expressed as

\[
S(t_x, f_y; R_0) = \text{sinc} \left( B \left( t_x - \frac{2R_{\text{sw}}}{c} \right) \right) \exp[j \Phi_0(f_y; R_0)]
\]

(9)

3.2. **AFNCS processing**

For the targets in different range cells, the range walk removal operation could make these targets located in the same range cell. However, after the range walk removal operation, because of the fact that the space variance of the quadratic term, cubic term and high-order term of the azimuth frequency, the azimuth filter will be mismatched if the azimuth dimension processing is performed uniformly,


thus causing the problem of the focus depth of the scene. Therefore, an AFNCS algorithm is proposed
to improve the problem of the focus depth.

Neglecting the space variance of the fourth and higher order terms in phase, the compensation
function can be expressed as

\[ H_n(f_n; R_{puc}) = \exp\left[-j\sum_{n=4}^{\infty} \phi_{ab} f_n^n \right] \] (10)

Multiplying equation (10) and equation (9), ignoring the constant term, and then transforming the
signal to the 2-D time domain, we obtain

\[ s(t_r, \eta; R_{puc}) = \sin c\left[ B_s \left( t_r - 2 \frac{R_{puc}}{c} \right) \right] a_0(\eta) \times \exp\left[-j\pi \frac{4R_{puc}}{\lambda} + j\pi K_a(\eta-t_p)^2 + j\pi K_i(\eta-t_p)^3 \right] \] (11)

where

\[ K_a = -\frac{2V^2 \cos^2 \theta_0}{\lambda \left( R_{puc} - VT_p \sin(\theta_0) \right)} \] (12)
\[ K_i = \frac{2V^4 \sin^3 \theta_0 \cos^2 \theta_0}{\lambda \left( R_{puc} - VT_p \sin(\theta_0) \right)^3 t_p} \] (13)

Expanding \( K_a \) and \( K_i \) through Taylor series

\[ \begin{align*}
K_a &= b_0 + b_1 t_p + b_2 t_p^2 \\
K_i &= K_0 t_p
\end{align*} \] (14)

From equation (11), we notice that the filtering factor cannot be directly introduced into the time
domain to correct the space variance of azimuth frequency modulation and cubic term. In order to
increase the degree of freedom of the system, we first introduce the high-order filtering function in the
time domain, and then introduce the scaling factor in the frequency domain to correct the space
variance of azimuth frequency modulation and cubic term. Therefore, the focused signal is performed
by using a uniform focusing factor.

The high-order filtering function in time domain can be expressed as

\[ H_f = \exp\left( j\pi Y f_n^n \right) \] (15)

After the high-order phase filtering in time domain, transforming the signal to azimuth frequency
domain, and then the phase filtering function \( H_f \) is constructed in frequency domain

\[ H_f = \exp\left[ j\pi \left( q_3 f_n^3 + q_4 f_n^4 \right) \right] \] (16)

Multiplying the equation (15) with the signal after high-order filtering in time domain, and
transforming it to azimuth time domain, we obtain

\[ s(t_r, \eta, t_p) = \sin c\left[ B_s \left( t_r - 2 \frac{R_{puc}}{c} \right) \right] \exp\left( j\left( \Phi(t_r, \eta, t_p) \right) \right) \] (17)

where

\[ \Phi(t_r, \eta, t_p) = A(\eta, \eta^3, \eta^4, R_{puc}, X_m, Y_m) + B(R_{puc}, X_m, Y_m) \eta_p + C(R_{puc}, X_m, Y_m) \eta_p^2 \]
\[ + D(R_{puc}, X_m, Y_m) \eta_p t_p + E(R_{puc}, X_m, Y_m) \eta_p^2 t_p^2 + F(t_r, \hat{t}^2, t_p^3, R_{puc}, X_m, Y_m) \] (18)

In order to eliminate the space variance of azimuth frequency modulation, letting \( C=D=E = 0 \)
and solving the binary linear equation group, we obtain
\[
\begin{align*}
Y_i &= -\frac{h_i}{3} \\
q_i &= -\frac{Y_i}{b_i^3} \\
q_i &= \left(3K_i + 9q_i b_i^2 b_i - b_i - 9Y_i^2 / b_i^2\right) / (6b_i^4)
\end{align*}
\]

Multiplying the equation (18) with the constructed deramp function

\[
H_{de-\eta} = \exp\left(-j\pi b_i^3 \eta^2 - j\pi q_i b_i^3 \eta^4\right)
\]

and then transforming the signal to the rang-doppler domain, the focused signal can be expressed as

\[
S(t_c, f_q, R_{\text{inc}}) = \sin c \left( B \left( t_c - \frac{2R_{\text{inc}}}{c} \right) \right) \sin c \left( \frac{1}{B_c} \left( f_q - b_p - \frac{h}{2} t_p^2 \right) \right)
\]

4. Simulation Result

In this section, simulated results are provided to demonstrate the performance of the proposed algorithm. The missile-borne SAR system works in the highly squinted mode. Using the parameters of Table 1, we simulate nine point targets, in which we select the points which are located at A= (-150, 300) m, B= (0, 0) m, C= (150, -300) m for analysis.

To further validate the improved algorithm, a quantitative analysis has been carried out here. The contour plots of the impulse response function of the three targets are shown in Fig.2. From Fig.2, we notice that the contour plots validates the improved algorithm.

| Parameters                      | Value |
|---------------------------------|-------|
| Carrier frequency              | 35 GHz|
| Pulse duration                 | 2.5 μs|
| Bandwidth                      | 150 MHz|
| System PRF                     | 1143 Hz|
| Reference slant range          | 20 km |
| Altitude                       | 10 km |
| Velocity                       | 500 m/s|
| Squint angle                   | 50 deg|
| Sample rate                    | 200 MHz|

![Fig.2 AFNCS algorithm (a) target A. (b) target B. (c) target C](image-url)

5. Conclusion

The AFNCS algorithm proposed in this paper is used to process highly squinted missile-borne SAR data. The key of the proposed algorithm is that a higher filtering factor is introduced in azimuth time domain, and then a new filtering factor is introduced in azimuth frequency domain to correct the azimuth frequency modulation and cubic cavitation. Finally, the signal is focused in the Doppler
domain by SPECtral Analysis technique. Numerical examples show the effectiveness of this method in highly squinted missile-borne SAR imaging.

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