LEPTON ACCELERATION IN THE VICINITY OF THE EVENT HORIZON: HIGH-ENERGY AND VERY-HIGH-ENERGY EMISSIONS FROM ROTATING BLACK HOLES WITH VARIOUS MASSES

Kouichi Hirotani1, Hung-Yi Pu1, Lupin Chun-Che Lin1, Hsiang-Kuang Chang2, Makoto Inoue1, Albert K. H. Kong2, Satoki Matsushita1, and Pak-Hin T. Tam3

1 Academia Sinica, Institute of Astronomy and Astrophysics (ASIAA), P.O. Box 23-141, Taipei, Taiwan 10617, R.O.C.; hirotani@tiara.sinica.edu.tw
2 Department of Physics, National Tsing Hua University, No. 101, Section 2, Kuang-Fu Road, Hsinchu, Taiwan 30013, R.O.C.
3 School of Physics and Astronomy, Sun Yat-Sen University, Zhuhai 519082, China

Received 2016 June 21; revised 2016 October 7; accepted 2016 October 21; published 2016 December 13

ABSTRACT

We investigate the electrostatic acceleration of electrons and positrons in the vicinity of the event horizon, applying the pulsar outer-gap model to black hole (BH) magnetospheres. During a low accretion phase, the radiatively inefficient accretion flow (RIAF) cannot emit enough MeV photons that are needed to sustain the force-free magnetosphere via two-photon collisions. In such a charge-starved region (or a gap), an electric field arises along the magnetic field lines to accelerate electrons and positrons into ultra-relativistic energies. These relativistic leptons emit copious gamma rays via curvature and inverse-Compton (IC) processes. Some of such gamma rays collide with the submillimeter-IR photons emitted from the RIAF to materialize as pairs, which polarize to partially screen the original acceleration electric field. It is found that the gap gamma-ray luminosity increases with decreasing accretion rate. However, if the accretion rate decreases too much, the diminished RIAF soft photon field can no longer sustain a stationary pair production within the gap. As long as a stationary gap is formed, the magnetosphere becomes force-free outside the gap by the cascaded pairs, irrespective of the BH mass. If a nearby stellar-mass BH is in quiescence, or if a galactic intermediate-mass BH is in a very low accretion state, its curvature and IC emissions are found to be detectable with Fermi/LAT and imaging atmospheric Cherenkov telescopes (IACT). If a low-luminosity active galactic nucleus is located within about 30 Mpc, the IC emission from its supermassive BH is marginally detectable with IACT.

Key words: acceleration of particles – gamma rays: stars – magnetic fields – methods: analytical – methods: numerical – stars: black holes

1. INTRODUCTION

It is widely accepted that an accreting black hole (BH) of an arbitrary size can produce a pair of relativistic plasma jets, which are often observed in various wavelengths from radio to very-high-energy (VHE) γ-rays. The most plausible mechanism for powering such jets is the extraction of the BH rotational energy through the Blandford–Znajec (BZ) process (Blandford & Znajek 1976). In this electromagnetic process, the magnetic field lines threading the event horizon exert a counter torque on it to spin down the BH, launching Poynting-flux-dominated outflows (Koide et al. 2002). Indeed, general relativistic (GR) magnetohydrodynamic (MHD) models show the existence of nearly steady, collimated, magnetically dominated jets in the polar regions (Hirose et al. 2004; McKinney & Gammie 2004; Tchekhovskoy et al. 2010), whose structures are similar to those in the force-free models (Hawley & Krolik 2006; McKinney & Narayan 2007a, 2007b). Since the centrifugal-force barrier prevents plasma accretion toward the rotation axis, the magnetic energy density dominates the plasmas’ rest-mass energy density in these polar funnels.

Even if the axial funnels are almost force-free in this sense, it is the electric current that sustains the electromagnetic power, and it is the charged particles that carry the electric current. That is, even under the assumption of masslessness, it must be the real charged particles that constitute the electric currents. In the direct vicinity of the horizon, causality requires that positive (or negative) charges must flow into the horizon when an electric current flows into (or out from) the horizon. Since accreting plasmas cannot easily penetrate into the funnels, and since they cannot emit sufficient MeV photons that are capable of materializing as electron–positron pairs when the accretion rate is very low, we need a process of plasma supply deep within the magnetosphere somewhere above the horizon.

To contrive a plasma source in the vicinity of the horizon, Beskin et al. (1992) extended the pulsar outer-magnetospheric lepton accelerator model (Cheng et al. 1986a, 1986b) to BH magnetospheres, and proposed the BH gap model. Extending this pioneering work, Hirotani & Okamoto (1998) demonstrated that a sufficient electric current can be supplied via copious pair production around supermassive BHs (SMBHs). However, the predicted γ-ray luminosity was too small to be detected by any instruments, because they assumed a substantial accretion rate (as in quasars), which leads to a very thin gap width along the magnetic field lines.

On these grounds, Neronov & Aharonian (2007) and Levinson & Rieger (2011) revisited the BH gap model and applied it to the central BH of radio galaxy M87 (i.e., M87*) and Sgr A*, adopting a much thicker gap width as large as the horizon radius. Their assumption of a thick gap was, indeed, reasonable, because such a low-luminosity active galactic nucleus generally possesses a less dense radiation field in the center and hence a geometrically extended gap slightly above the horizon. Then Broderick & Tchekhovskoy (2015) demonstrated that the two-stream instability does not grow in the ultra-relativistic, counter-streaming pairs in BH gaps. Subsequently, Hirotani & Pu (2016, hereafter HP16) showed that a gap arises around the so-called “null-charge surface,” on which the GR Goldreich–Julian (GJ) charge density vanishes, by solving the set of an inhomogeneous part of the Maxwell
equations, lepton equations of motion, and the radiative transfer equation. Then they applied their BH gap model to the radio galaxy IC310, whose central BH (i.e., IC310*) is accreting plasmas at much smaller rate than the Eddington rate. They demonstrated that the gap width becomes as large as the horizon radius when the accretion rate is very low, and that the observed VHE flux during the flare cannot be reproduced by their BH gap model, unless the magnetic field becomes much stronger than the equipartition value with the plasma accretions. Noting that GR effects most importantly appear in the formation of the null-charge surface through frame dragging, they evaluated the gap width, lepton equations of motion, and the radiative transfer calculations, extending the method of HP16. In the next section, we examine the detectability of BH gap emissions for various BH masses. Then in Section 3, we describe the background spacetime and derive the GR GJ charge density general-relativistically. However, they neglected the GR effects in any other terms of the basic equations as the first step, because the electromagnetic interaction dominates the gravitational one except for in the direct vicinity of the horizon, and because the photons emitted from the horizon vicinity will not strongly affect the emission spectra due to redshift.

In the present paper, to further quantify the gap model, we include the GR effects in all the basic equations and calculations, extending the method of HP16. In the next section, we examine the detectability of BH gap emissions for various BH masses. Then in Section 3, we describe the background spacetime and derive the GR GJ charge density in a rotating BH magnetosphere. We then formulate the basic equations of gap electrodynamics in Section 4, and investigate their emission properties in Section 5. In the final section, we discuss how to discriminate the gap emission from the jet emission.

2. DETECTABILITY OF GAP EMISSIONS

Since the gap liberates only a portion of the electromagnetic power extracted from a rotating BH, the upper limit of its luminosity can be given by the Blandford–Znajek power (Blandford & Znajek 1976)

$$L_{\text{BZ}} \approx 10^{27} a_*^2 M^2 \dot{M}^2 \text{ erg s}^{-1},$$  

where $a_* \equiv a/r_g$ denotes the dimensionless BH’s spin parameter, $a \equiv J/(Mc)$ the spin parameter, $J$ the BH’s angular momentum, $M$ the BH mass, $c$ the speed of light, $r_g \equiv GM/c^2$ the gravitational radius, $G$ the gravitational constant, $B$ the magnetic field strength in Gauss, and $M_\odot \equiv M/(10^5 M_\odot)$.

In the present paper, unless explicitly mentioned, we assume that the magnetic buoyancy balances disk gravity and evaluate $B$ with the equipartition value. Near the horizon, we obtain (Bisnovatyi-Kogan & Ruzmaikin 1974; Ghosh et al. 1977; Narayan et al. 2003; Levinson & Rieger 2011)

$$B_{\text{eq}} \approx 4 \times 10^6 \eta^{-1/2} M_\odot^{1/2} G,$$

where $\dot{m}$ refers to the dimensionless accretion rate near the horizon and is defined as $\dot{m} \equiv \dot{M}/M_{\text{Edd}}$. $\dot{M}$ denotes the mass accretion rate, $M_{\text{Edd}} \equiv L_{\text{Edd}}/\eta c^2$, $L_{\text{Edd}} = 1.25 \times 10^{39} M_\odot \text{ erg s}^{-1}$, and $\eta \approx 0.1$ (McKinney et al. 2012). Substituting $B = B_{\text{eq}}$ into Equation (1), we obtain

$$L_{\text{BZ}} \approx 1.7 \times 10^{38} a_*^2 \dot{m} M_\odot \text{ erg s}^{-1}.  \tag{3}$$

The magnetosphere becomes force-free, that is, an electric field does not arise along the magnetic field lines, if the pair density exceeds the GJ value, $N_{\text{GJ}} \sim \Omega_B B/(2\pi e)$, where $\Omega_B$ denotes the angular frequency of the magnetic field and $e$ the magnitude of the charge on the electron. In the vicinity of an accreting BH, a plasma accretion becomes radiatively inefficient when the accretion rate typically reduces to $\dot{m} < 10^{-2.5}$ (Ichimaru 1979; Narayan & Yi 1994, 1995). From such a radiatively inefficient accretion flow (RIAF), MeV photons are emitted via a free–free process, colliding with each other to materialize as electron–positron pairs in the magnetosphere. If the number density, $N_{\Lambda}$, of such created pairs becomes less than $N_{\text{GJ}}$, charges cannot completely screen an electric field, $E_\parallel$, along the magnetic field line; that is, a gap (i.e., a charge-starved region) appears. The RIAF theory gives $N_{\Lambda}$ as a function of $M$ and $\dot{m}$ (Levinson & Rieger 2011). Thus, putting $N_{\Lambda} < N_{\text{GJ}}$, we obtain a condition for a gap to appear,

$$\dot{m} < \dot{m}_{\text{up}} \equiv 3.1 \times 10^{-3} M_\odot^{-1/7}.  \tag{4}$$

In Figure 1, we plot this upper limit, $\dot{m}_{\text{up}} = \dot{m}_{\text{up}}(M)$ as the thick solid line. The dotted lines denote $L_{\text{BZ}}$ (Equation (3)) as labeled. The crossing of the solid line and a dotted line gives the Blandford–Znajek power at each BH mass. For example, the thick solid line and the dotted line labeled $L_{\text{BZ}} = 10^{39} \text{ erg s}^{-1}$ cross at $M \approx 10^5 M_\odot$; thus, we find that a SMBH with $M \approx 10^8 M_\odot$ has a gap whose luminosity can attain up to $10^{40} \text{ erg s}^{-1}$.

To consider the case of an efficient energy extraction from a BH, we assume $a_* = 0.9$ unless explicitly mentioned. Substituting $\dot{m} = \dot{m}_{\text{up}}$ into Equation (3), we then obtain the maximum gap luminosity

$$L_{\text{BZ}} \approx 5.2 \times 10^{35} a_*^2 M_\odot^{6/7} \text{ erg s}^{-1}.  \tag{5}$$

Assuming that 100% of this power is converted into radiation, we obtain the upper limit of its flux at Earth, $F_{\text{BZ}} = L_{\text{BZ}}/4\pi d^2$, where $d$ is the distance to the BH.

For stellar-mass BHs, we obtain the flux upper limit,

$$F_{\text{BZ}} = L_{\text{BZ}}/(4\pi d^2) = 4.1 \times 10^{-9} a_*^2 M_\odot^{6/7} \left( \frac{d}{\text{kpc}} \right)^{-2} \text{ erg s}^{-1} \text{ cm}^{-2}.  \tag{6}$$

![Figure 1. Dimensionless accretion rate, $\dot{m}$, vs. black hole mass, $M$. The thick solid line shows the upper limit of $\dot{m}$ above which the copious pair production by the RIAF emission prevents the formation of a gap (and ensures the force-free magnetosphere). The filled circles, open circles, and open squares denote the solved lower limits of $\dot{m}$ (Section 5.4) for $(a_*, L_{\text{BZ}}/\eta c, R_{\text{min}}/\eta c) = (0.9, 0.5, 6), (0.5, 0.5, 6),$ and $(0.9, 0.5, 12)$ respectively. The thin dashed line shows a linear fit of the filled circles, $\lg \dot{m}_{\text{up}} = -3.64 + 0.261 \lg (M/M_\odot)$. The dotted lines show the extracted power from the black hole (Equation (1)) for an extremely rotating case, $a = M$. For the explanation of the red-graded region, see the end of Section 5.3.](image-url)
As will be seen in Section 5.1, the gap emission spectrum peaks between GeV and 10 GeV for \( M \sim 10 \, M_\odot \). Thus, if a small portion of this BZ power (e.g., 0.1\%) is dissipated in the gap, we can expect a large time-averaged high-energy (HE) flux (e.g., >10^{-11} \text{erg s}^{-1}) that is detectable with Fermi/LAT.

There is another component, formed by the inverse-Compton (IC) scatterings, which appears in VHE. This component may be detectable with ground-based imaging atmospheric Cherenkov telescopes (IACTs). Note that Equation (6) merely gives the upper limit, and that the actual photon flux can be obtained when we solve the gap electrodynamics from the basic equations. Will we examine this issue in Section 5.1.

For intermediate-mass BHs, we obtain the maximum flux of

\[
F_{\text{BZ}} = 2.1 \times 10^{-9} \alpha_M^2 M_6^{1/2} \left( \frac{d}{10 \, \text{kpc}} \right)^{-2} \text{erg s}^{-1} \text{cm}^{-2},
\]

where \( M_6 = M/(10^6 M_\odot) \). The spectrum has two peaks: the curvature photons peak in 1–10 GeV and the IC ones above TeV. Both spectral components are potentially detectable in HE and VHE if they are located within our galaxy. We will examine this possibility in Section 5.2.

For supermassive BHs, we obtain the maximum flux of

\[
F_{\text{BZ}} = 3.0 \times 10^{-10} \alpha_M^2 M_9^{1/2} \left( \frac{d}{10 \, \text{Mpc}} \right)^{-2} \text{erg s}^{-1} \text{cm}^{-2},
\]

where \( M_9 = M/(10^9 M_\odot) \). The IC component, which appears in VHE, may be detectable with ground-based IACTs. We will examine this possibility in Section 5.3.

3. BACKGROUND GEOMETRY AND THE NULL CHARGE SURFACE

The self-gravity of the plasma particles and the electromagnetic field little affects the spacetime geometry. Thus, around a rotating BH, the background geometry is described by the Kerr metric (Kerr 1963). In the Boyer–Lindquist coordinates, it becomes (Boyer & Lindquist 1967)

\[
ds^2 = g_{tt}(dt)^2 + 2g_{t\varphi}dtd\varphi + g_{\varphi\varphi}(d\varphi)^2 + g_{rr}(dr)^2 + g_{\theta\theta}(d\theta)^2,
\]

where

\[
g_{\varphi\varphi} = -\frac{\Delta - a^2 \sin^2 \theta}{\Sigma}, \quad g_{t\varphi} = -\frac{2\Sigma}{\Delta}, \quad g_{tt} = -\frac{\Sigma}{\Delta}, \quad g_{\theta\theta} = \frac{\Sigma}{\Delta};
\]

\[
\Delta \equiv r^2 - 2GM/c^2r + a^2, \quad \Sigma \equiv r^2 + a^2 \cos^2 \theta, \quad A \equiv (r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta.
\]

At the event horizon, \( \Delta \) vanishes, giving \( r_+ \equiv r_h + \sqrt{r_h^2 - a^2} \) as the horizon radius. If the BH is extremely rotating (i.e., if \( a \to r_h \)) for instance, we obtain \( r_+ \to r_h \).

In a stationary and axisymmetric spacetime (as described by Equation (9)), Gauss’s law becomes

\[
\nabla_t F_{t\mu} = \frac{1}{\sqrt{-g}} \partial_\mu \left[ \frac{\sqrt{-g}}{\rho_w} \varepsilon_{\mu\nu\xi\varphi} ( -g_{\xi\varphi} F_{t\nu} + g_{t\nu} F_{\xi\varphi} ) \right] = 4\pi \rho,
\]

where \( \nabla \) denotes the covariant derivative, the Greek indices run over \( t, r, \varphi, \theta \), \( \varepsilon = \sqrt{g_{t\nu}g_{\xi\varphi}} = c^2 \sin \theta \) and \( \rho_w = \rho_{\text{BZ}} - \rho_{\text{IC}} = c^2 \Delta \sin^2 \theta, \rho \) is the real charge density. The electromagnetic fields are observed by an observer that is static with respect to asymptotic infinity, and are given by (Camenzind 1986a, 1986b)

\[
E_\varphi = E_{\varphi}^{\text{HE}}, \quad B_{\varphi} = B_{\varphi}^{\text{HE}} = E_{\varphi}/\sqrt{-g}, \quad B^\theta = (g_{\varphi\varphi} + g_{\varphi\theta}) \mathcal{F}_{\varphi\theta}/\sqrt{-g}, \quad B_r = -\rho_w^2 F_{\varphi r}/\sqrt{-g}, \quad \mathcal{F}_{\mu\nu} \equiv \partial_\mu \varepsilon_{\mu\nu\xi\varphi} A_\xi - A_\mu.\]

In this paper, we assume that the electromagnetic fields (i.e., all components of the Faraday tensor, \( F_{\mu\nu} \)) depend on \( t \) and \( \varphi \) through \( \varphi - \Omega t \). In this case, we can introduce the non-corotational potential \( \Phi \) such that

\[
F_{\mu\nu} + \Omega F_{\mu\varphi} = -\partial_\mu \Phi (r, \varphi, \varphi - \Omega t).
\]

If \( F_{t\theta} + \Omega F_{t\varphi} = 0 \) holds for \( A = r \) and \( \Omega \), \( \Omega \equiv F_{t\theta}/F_{\varphi} = F_{tt}/F_{\varphi} \) is conserved along the field line. However, in a particle acceleration region, \( F_{t\theta} + \Omega F_{t\varphi} \) deviates from zero and the magnetic field does not rigidly rotate. The deviation from rigid rotation is expressed in terms of \( \Phi \), which gives the strength of the acceleration electric field measured by an observer that is static to asymptotic infinity as

\[
E_i = \frac{B_i}{B} \cdot E = \frac{B^i}{B} (F_{t\theta} + \Omega F_{t\varphi}) = \frac{B_i}{B} \cdot (-\nabla \Phi),
\]

where the Latin index \( i \) runs over spatial coordinates \( r, \theta, \varphi \). Note that \( B^i F_{i\varphi} = 0 \).

Substituting Equation (13) into (12), we obtain the Poisson equation for the non-corotational potential (Hirotani 2006),

\[
- \frac{c^2}{\sqrt{-g}} \partial_\mu \left[ \frac{\sqrt{-g}}{\rho_w^2} \varepsilon_{\mu\nu\xi\varphi} \partial_\nu \Phi \right] = 4\pi (\rho - \rho_{\text{GI}}),
\]

where the GR Goldreich–Julian charge density is defined as

\[
\rho_{\text{GI}} = \frac{c^2}{4\pi \sqrt{-g}} \partial_\mu \left[ \frac{\sqrt{-g}}{\rho_w^2} \varepsilon_{\mu\nu\xi\varphi} (\Omega_{\xi\varphi} - \omega F_{\xi\varphi}) \right].
\]

In the limit \( r \gg r_h \), Equation (16) reduces to the ordinary, special-relativistic expression (Goldreich & Julian 1969; Mestel 1971),

\[
\rho_{\text{GI}} = -\frac{\Omega \cdot B}{2\pi c} + \frac{(\Omega \times r) \cdot (\nabla \times B)}{4\pi c}.
\]
have different signs at the two boundaries so that the gap may be closed. In a nearly vacuum gap, \( \rho \approx 0 \) shows that \( \rho_{GJ} \) should change sign within the gap. Therefore, a gap should appear around the null-charge surface, where \( \rho_{GJ} \) vanishes. The null surface is, therefore, a natural place for a particle accelerator (i.e., a gap) to arise, in the same way as pulsar vacuum gap models (Holloway 1973; Chiang & Romani 1992; Romani 1996; Cheng et al. 2000).

It should be noted that the null surface appears near the place where \( \Omega_{GJ} \) coincides with the spacetime dragging angular frequency, \( \omega \) (Beskin et al. 1992). The deviation of the null surface from this \( \omega(r, \theta) = \Omega_{GJ} \) is, indeed, small, as Figure 1 of Hirotani & Okamoto (1998) indicates. Since \( \omega \) matches \( \Omega_{GJ} \) only near the horizon, the null surface, and hence the gap, generally appears within one or two gravitational radii above the horizon, irrespective of the BH mass.

4. MAGNETOSPHERIC LEPTON ACCELERATOR NEAR THE HORIZON

In this section, we formulate the BH gap electrodynamics, extending the method described in Hirotani (2013) and HP16. Throughout this paper, we assume an aligned rotator in the sense that the magnetic axis coincides with the rotational axis of the BH, and seek an axisymmetric solution.

4.1. Magnetic Field Structure

As described in HP16, a stationary BH gap is formed around the null surface, as long as the injected current density across the inner or outer boundaries is much smaller than the GJ value. Since the null surface is formed by the frame-dragging effect, the gap electrodynamics is essentially governed by the frame-dragging effect rather than the magnetic field configurations. This forms a striking contrast to the pulsar outer-magnetospheric gap model, in which the null surface is formed by the convex geometry of the poloidal magnetic field lines. Thus, in a BH magnetosphere, the gap position and its spatial extent, as well as the exerted \( E_l = E_l(s) \) in the gap, depend little on the magnetic flux function, \( \Psi = A_{\varphi} \). We thus assume a radial magnetic field on the poloidal plane, \( \Psi = \Psi(\theta) \).

Because of axial symmetry, the gap electrodynamics structure can be described in the poloidal plane. For simplicity, we assume that the photons propagate radially in this 2D plane, which is justified if the photons have negligible angular momenta. Provided that the drift motion (e.g., due to toroidal radiation drag in a radial magnetic field, or due to radial gravity in a toroidal magnetic field) is small in the meridional direction, charged particles roughly move along the magnetic field lines in the poloidal plane. In this case, due to relativistic beaming, ultra-relativistic particles emit photons along the instantaneous magnetic field lines; thus, as long as the poloidal magnetic field is radial, photons propagate on the same magnetic flux surface, \( A_{\varphi} = A_{\varphi}(\theta) = \text{constant} \). As a result, we can solve the particle equations of motion and the radiative transfer equation along individual radial poloidal magnetic field lines separately.

Even when the magnetic field lines are radial in the poloidal plane, there exists a toroidal magnetic component, \( B_{\varphi} \), due to retardation, magnetospheric currents, and a frame dragging. It is, however, out of the scope of this paper to restrict the functional form of \( B_{\varphi}(r, \theta) \), taking account of such effects. In addition, \( B_{\varphi} \) does not affect \( \rho_{GJ} \) or the propagation direction of particles and photons in the poloidal plane. Thus, we do not specify \( B_{\varphi} \). Accordingly, instead of computing the curvature radius, \( R_c \), of the leptons from their 3D motion in the rotating magnetosphere, we parameterize \( R_c \) when we calculate the curvature emission. Although \( R_c \) affects the spectral shape of curvature emission, it affects the total luminosity little, because the latter is essentially determined by the potential drop within the gap, and because the potential drop is determined by the gap width, which is in turn predominantly determined by the pair production rate of the IC-emitted, VHE photons (not the curvature-emitted, lower energy photons). On these grounds, we adopt \( R_c = r_g \) in the present paper, leaving \( B_{\varphi} \) unconstrained.

4.2. Gap Electrodynamics

In the same way as HP16, we solve the stationary gap solution from the set of the Poisson equation for \( \Phi \), the equations of motion for electrons and positrons, and the radiative transfer equation for the emitted photons.

4.2.1. Poisson Equation

To solve the radial dependence of \( \Phi \) in the Poisson Equation (15), we introduce the following dimensionless tortoise coordinate, \( \eta_{\varphi} \),

\[
\frac{d\eta_{\varphi}}{dr} = \frac{r^2 + a^2}{\Delta} \frac{1}{r_g}.
\]

In this coordinate, the horizon corresponds to the “inward infinity,” \( \eta_{\varphi} = -\infty \). In this paper, we set \( \eta_{\varphi} = r/r_g \) at \( r = r_{\text{max}} = 25 r_g \), where the value of \( r_{\text{max}} \) can be chosen arbitrarily and does not affect the results in any way. The distribution of \( \eta_{\varphi} \) is depicted as a function of \( r/r_g \) in Figure 2. Note that the relationship between \( \eta_{\varphi} \) and \( r \) does not depend on the colatitude, \( \theta \).

Since the gap is located near the horizon, we take the limit \( \Delta \ll r_g^2 \). Assuming that \( \Phi \) does not depend on \( \varphi - \Omega_{GJ} t \), that
is, $\Phi = \Phi(r, \theta)$, we can recast the Poisson Equation (15) into the two-dimensional form,
\[
\left(\frac{r^2 + a^2}{\Delta}\right)^2 \frac{\partial^2 \Phi}{\partial \eta_x^2} + \frac{2(r - r_g)(r^2 + a^2)}{\Delta^2 r_g} \frac{\partial \Phi}{\partial \eta_x} = \frac{r_g^2}{\Delta \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \Phi}{\partial \theta} \right)
\]
\[
= \left( \frac{r^2 + a^2}{\Delta} \right)^2 (n_+ - n_- - n_{GJ}),
\]
where
\[
\Phi(\eta_x) = \frac{c}{2 \Omega_B B^2} \Phi(r)
\]
denotes the dimensionless non-coriolis potential. Dimensionless lepton densities per magnetic flux tube are defined by
\[
n_{\pm} = \frac{2 \pi c \nu}{\Omega_B} N_{\pm},
\]
where the number densities of positrons and electrons, $N_+$ and $N_-$, are computed from the pair production rate at each position (Hirotani & Okamoto 1998; Hirotani & Shibata 1999a, 1999b). Dimensionless GJ charge density per magnetic flux tube is defined by
\[
n_{GJ} \equiv \frac{2 \pi c}{\Omega_B} \rho_{GJ}.
\]

For a radial poloidal magnetic field, $\Psi = \Psi(\theta)$, we can compute the acceleration electric field by
\[
E_r \equiv -\frac{\partial \Phi}{\partial r} = -\frac{\Omega_B B}{c} \frac{r^2 + a^2}{\Delta} \frac{\partial \Phi}{\partial \eta_x}.
\]

Without loss of any generality, we can assume $F_{0r} > 0$ in the northern hemisphere. In this case, a negative $E_r$ arises in the gap, which is consistent with the direction of the global current flow pattern.

Equation (16) shows that $\rho_{GJ}$ is essentially determined by $B^r$, rather than $B^\theta$, near the horizon. In a stationary and axisymmetric magnetosphere, Equation (16) becomes
\[
\rho_{GJ} \propto \partial_{\theta}(G F_{r \theta}) + \frac{1}{\Delta} \partial_{\theta}(G F_{\theta \theta}),
\]
where $G \equiv (A \sin \theta / \Sigma)(\Omega_\Sigma - \omega)$. Since $G$, $F_{r \theta}$, and $F_{\theta \theta}$ are well-behaved at the horizon, we find that the second term dominates the first one at $\Delta \rightarrow 0$. Therefore, although the null surface itself is formed by the frame-dragging effect, the radial component of the magnetic field essentially determines $\rho_{GJ}$ near the horizon owing to the redshift effect.

4.2.2. Leptonic Densities

We next consider $n_-$ and $n_+$ in Equation (19). Because of $E_r < 0$, electrons are accelerated outwards, and positrons inwards. As a result, as long as there is no current injection across either outer or inner boundaries, charge density, $n_+ - n_-$, becomes negative (or positive) at the outer (or inner) boundary. In a stationary gap, $E_r$ should not change sign in it. In a vacuum gap, a positive (or a negative) $-n_{GJ}$ near the outer (or inner) boundary makes $\partial_r E_r > 0$ (or $\partial_r E_r < 0$), thereby closing the gap. In a non-vacuum gap, the right-hand side of Equation (19) should become positive (or negative) near the outer (or inner) boundary so that the gap may be closed. Therefore, $|n_+ - n_-|$ should not exceed $|n_{GJ}|$ at either boundary.

At the outer boundary, for instance, we can put
\[
(n_+ - n_-)_{r = r_2} = -n_-(r = r_2) = j_{n_{GJ}}(r = r_2),
\]
where the dimensionless parameter $j$ should be in the range, $0 < j < 1$, so that the gap solution may be stationary. Since $F_{0r} > 0$ is assumed, it is enough to consider a positive $j$. If $j = 1$, there is no surface charge at the outer boundary. However, if $j < 1$, the surface charge results in a jump of $\partial_r E_r$ at the outer boundary. That is, the parameter $j$ specifies the strength of $\partial_r E_r$ at the outer boundary. Thus, the inner boundary position, $r = r_1$, is determined as a free boundary problem by this additional constraint, $j$. The outer boundary position, $r = r_2$, or equivalently the gap width $w = r_2 - r_1$, is constrained by the gap closure condition (Section 4.2.6).

It is noteworthy that the charge conservation ensures that the dimensionless current density (per magnetic flux tube), $J_r \equiv -n_+ - n_-$ conserves along the flowline. At the outer boundary, we obtain
\[
J_r = -n_-(r = r_2) = j_{n_{GJ}}(r = r_2).
\]

Thus, $j$ specifies not only $\partial_r E_r$ at the outer boundary, but also the conserved current density, $J_r$.

In general, under a given electro-motive force exerted in the ergosphere, $J_r$ should be constrained by the global current flow pattern, which includes an electric load at the large distances where the force-free approximation breaks down and the trans-magnetic-field current gives rise to the outward acceleration of charged particles by Lorentz forces (thereby converting the Poynting flux into particle kinetic energies). However, we will not go deep into the determination of $J_r$ in this paper, because we are concerned with the acceleration processes near the horizon, not the global current closure issue. Note that $w$ (or $r_2$) is essentially determined by $\dot{m}$; thus, $j$ and $\dot{m}$ give the actual current density $(\Omega_B B / 2 \pi) J_r$, where $B$ should be evaluated at each position. On these grounds, instead of determining $J_r$ by a global requirement, we treat $j$ as a free parameter in the present paper.

It may be worth mentioning, in passing, that we may not have to consider a time-dependent solution, which may be obtained when $j > 1$ as in pulsar polar cap models (Harding et al. 1978; Daugherty & Harding 1982; Dermer & Sturmer 1994). In the polar cap model, the absence of the null surface results in a non-stationary gap solution (Timokhin 2010; Timokhin & Arons 2013; Timokhin & Harding 2015). However, in the pulsar outer-gap models (Shang & Cheng 1997; Takata et al. 2006; Romani & Watters 2010; Wang et al. 2011; Hirotani 2015) or in the present BH gap model, the existence of the null surface leads to the formation of a stationary gap around this surface. We thus adopt $0 \leq j \leq 1$ and consider a stationary gap solution.

4.2.3. Particle Motion

Let us describe the motion of electrons and positrons. For simplicity, we assume that the distribution functions of electrons and positrons are mono-energetic. We evaluate the Lorentz factors of electrons (or positrons) by the motion of a test particle injected across the inner (or the outer) boundary of the gap. For example, an injected test electron is accelerated by
a negative $E_\parallel$ outwards and loses the kinetic energy via curvature and IC processes. The former, curvature radiation rate (per particle), is computed by the standard synchrotron emission formula with the gyration radius replaced with $R_c$ (Rybicki & Lightman 1979). The latter, IC radiation rate, is computed by multiplying the scattering probability (per unit time) and the scattered photon energy. Thus, the particle Lorentz factor saturates at the curvature-or IC-limited value, whichever is smaller.

### 4.2.4. Radiative Transfer Equation

Throughout this paper, we assume that all photons are emitted with vanishing angular momenta. In this case, photons propagate on a constant-$\theta$ surface; thus, the radiative transfer equation is solved one-dimensionally along the radial magnetic field lines on the poloidal plane. These primary leptons emit photons via curvature and IC processes both inside and outside the gap. For the details of how to compute the emissivities of curvature and IC processes, see Sections 4.2 and 4.3 of HP16.

Some portions of the photons are emitted above 10 TeV via IC process. A significant fraction of such hard $\gamma$-rays are absorbed, colliding with the RIAF soft photons. If such collisions take place within the gap, the created electrons and positrons polarize to be accelerated in opposite directions, becoming the primary leptons. If the collisions take place outside the gap, the created, secondary pairs migrate along the magnetic field lines to emit photons via IC and synchrotron processes. Some of such secondary IC photons are absorbed again to materialize as tertiary pairs, which emit tertiary process. Some of such secondary IC photons are absorbed once more to materialize as quaternary pairs, which emit quaternary IC photons. This assumption simplifies the calculations of photon–photon pair production and IC scatterings, because photons are assumed to be emitted by leptons with vanishing angular momenta. To calculate the flux of ADAF photons, we assume that their number density is homogeneous and becomes $L_{\text{ADAF}}/(4\pi R_{\text{min}}^2 c)$ within $r < R_{\text{min}}$, where $L_{\text{ADAF}}$ denotes the ADAF luminosity given by Mahadevan (1997). We assume that the ADAF luminosity becomes $L_{\text{ADAF}}$ at $r = R_{\text{min}}$. Outside this radius, $r > R_{\text{min}}$, we assume that their number density decreases by $r^{-2}$ law. This treatment may be justified, because the submillimeter-IR photons, which most effectively work both for pair production and IC scatterings, are emitted from the innermost region of the ADAF.

#### 4.2.5. Boundary Conditions

We solve the gap in the 2D poloidal plane. We assume a reflection symmetry with respect to the magnetic axis. Thus, we put $\partial_\phi \Phi = 0$ at $\theta = 0$. We assume that the polar funnel is bounded at a fixed colatitude, $\theta = \theta_{\text{max}}$ and impose that this lower-latitude boundary is equi-potential and put $\Phi = 0$ at $\theta = \theta_{\text{max}}$.

Both the outer and inner boundaries are treated as free boundaries. Their positions are determined by two conditions: the value of $j$ along each magnetic field line (specified by $\Phi$), and the gap closure condition (to be described in Section 4.2.6). For simplicity, we assume that $j$ is constant for $\Phi$. At the outer boundary, $\partial_r E_\parallel = -\partial_\phi \Phi$ is specified by $j$. At the inner boundary, we impose $E_\parallel = -\partial_\phi \Phi = 0$. We assume that electrons, positrons or photons are not injected across either the outer or the inner boundaries.

#### 4.2.6. Gap Closure Condition

The set of Poisson and radiative-transfer equations are solved together with the terminal Lorentz factor $\gamma$ and the $n_\perp$, obtained by the local pair production rate. Unlike HP16, we discard the reflection symmetry (along radial magnetic field lines) with respect to the null-charge surface, and explicitly consider the asymmetric distribution of $E_\parallel$, $\gamma$ and $n_\perp$, and the photon specific intensity at each point $r$. Accordingly, the gap closure condition should be modified as $M_{\text{in}}M_{\text{out}} = 1$, where $M_{\text{in}}$ and $M_{\text{out}}$ denote the multiplicity (Equation (41) of HP16) associated with the in-going and out-going leptons, respectively. See Hirotani (2013) for a detailed treatment of the asymmetric multiplicities, $M_{\text{in}} = M_{\text{out}}$.

### 5. GAP SOLUTIONS

In Section 2, we examined the upper limits of the BH gap luminosity, imposing the charge-starvation condition (Equation (4)) that requires the ADAF to be less luminous so that their MeV photons may not produce pairs above the GJ density. In this section, solving the gap hydrodynamics by the method described in Section 4 for various BH masses, we demonstrate that the gap becomes most luminous when the gap longitudinal width becomes much greater than the horizon radius, and that there is a lower-limit accretion rate below which a stationary pair-production cascade cannot be maintained within the gap.

Throughout this paper, we assume a radial poloidal magnetic field, $\Psi = \Psi(\theta) \propto -\cos \theta$. The magnitude of $\Psi$ is adjusted so that $B = B_{\text{eq}}$ (Equation (2)) may be satisfied at $r = 2 r_g$. Assuming that the poloidal magnetic components dominate the toroidal one, we put $B(r) = B_{\text{eq}} (r/2 r_g)^{-2}$ and compute the synchrotron emission outside the gap. Unless explicitly mentioned, we adopt $a = 0.90 r_g$, $\Omega_p = 0.50 \Omega_{\text{Kerr}}$, $R_{\text{min}} = 6 r_g$, $j = 0.7$, and $R_c = r_g$. To solve the Poisson Equation (19), we set the meridional boundary at $\theta = \theta_{\text{max}} = 60^\circ$. We assume that the entire system is axisymmetric with respect to the rotation axis, which coincides with the magnetic axis.

#### 5.1. The Case of Stellar-mass BHs

Let us first examine the case of $M = 10 M_{\odot}$. We begin with describing the 2D distribution of the $E_\parallel$ in Section 5.1.1, and demonstrate that the gap emission becomes strongest within the colatitude, $\theta < 38^\circ$ along the magnetic axis in Section 5.1.2. Then in Sections 5.1.3–5.1.4, we examine the lepton densities and acceleration electric field. Adopting a distance of 1 kpc, we present the expected spectra of gap emissions in Section 5.1.5, and examine their dependence on $j$, $a_*$, and $\Omega_p$ in Sections 5.1.7–5.1.9. Finally, in Section 5.1.10, we demonstrate that the magnetosphere becomes entirely force-free (except for the gap region) by the cascaded pairs outside the gap.
5.1.1. Electric Field along the Magnetic Field Lines

We first present the distribution of the magnetic-field-aligned electric field on the poloidal plane. In Figure 3 we plot $E_\parallel$ (in statvolt cm$^{-1}$) as a function of the dimensionless tortoise coordinate, $\eta$, and the magnetic colatitude, $\theta$ (in degrees), for $\dot{m} = 1.00 \times 10^{-4}$. We also plot $E_\parallel$ at six discrete colatitudes in Figure 4. It follows that the $E_\parallel(\eta, \theta)$ distribution changes little in the polar region within $\theta < 38^\circ$.

5.1.2. Gap Emission versus Colatitudes

We next compare the $\gamma$-ray spectra of a BH gap emission as a function of the colatitude, $\theta$. In Figure 5, we compare the SEDs at the same six discrete $\theta$s as in Figure 4. It follows that the gap emission becomes most luminous if we observe the gap with a viewing angle $\theta < 38^\circ$. This conclusion is unchanged if we adopt different BH masses or spins. In what follows, we therefore adopt $\theta = 0^\circ$ as the representative colatitude to estimate the maximum $\gamma$-ray flux of BH gaps.

5.1.3. Created Lepton Densities

We plot the solved lepton densities at five discrete $\dot{m}$ in Figure 6. Since $E_\parallel$ is negative, electrons are accelerated outwards while positrons inwards. Thus, the dimensionless electronic density (solid curve), $n_e$, per magnetic flux tube, increases outwards, while the positronic one (dashed curve), $n_p$, decreases outwards. Note that the abscissa, $r - r_0$, is converted from the tortoise coordinate to the Boyer–Lindquist radial coordinate for presentation purpose. Thus, $r - r_0 = 0$ corresponds to the null-charge surface. These solved $n_e(r, \theta)$ are used to compute the real charge density $\rho = (\Omega B/2\pi c)(n_e - n_p)$ at each position, which is necessary to solve $E_\parallel$ on the poloidal plane. We continue iterations until $n_\pm(\eta, \theta), E_\parallel(\eta, \theta)$, and the photon specific intensity saturate.

5.1.4. Acceleration Electric Field

As $\dot{m}$ decreases, the reduced ADAF near-IR photon field leads to less efficient pair production, thereby resulting in an
extended gap to sustain the externally required current density, \( j \) per magnetic flux tube. We plot \( E(r, \theta = 0) \) for five discrete \( \dot{m} \)s in Figure 7. The cyan, blue, green, black, and red curves correspond to the cases of \( \dot{m} = 10^{-3.0}, 10^{-3.5}, 10^{-4.0}, 10^{-4.125} \) and \( 10^{-4.25} \), respectively; that is, the same as Figure 6. Integrating \( E \) over the gap width, we obtain the potential drop at each \( \dot{m} \). It becomes \(-6.3 \times 10^{11} \), \(-3.9 \times 10^{12} \), \(-2.5 \times 10^{13} \), \(-4.2 \times 10^{13} \), and \(-6.3 \times 10^{13} \) V, for \( \dot{m} = 10^{-3.0}, 10^{-3.5}, 10^{-4.0}, 10^{-4.125} \), and \( 10^{-4.25} \), respectively. Thus, the potential drop increases with decreasing \( \dot{m} \) because of the increased gap width, \( w \equiv r_2 - r_1 \). More specifically, as the accretion rate reduces, the decreased ADAF near-IR photon field results in a less effective pair production for the gap-emitted IC photons, thereby increasing the mean-free path for two-photon collisions. Since \( w \) essentially becomes the pair-production mean-free path divided by the number of photons emitted by a single electron above the pair production threshold energy (Hirotani & Okamoto 1998), the reduced pair production leads to an extended gap along the magnetic field lines. As a result, the smaller \( \dot{m} \) is, the greater the potential drop becomes.

As \( w \) increases, the trans-field derivative begins to contribute in the Poisson Equation (19). As a result, the \( E_\parallel \) distribution shifts outwards, in the same way as in pulsar outer-magnetospheric gaps (Figure 12 of Hirotani & Shibata 1999a). That is, a pulsar outer gap extends from the null surface to (or beyond) the light cylinder because the transverse thickness is limited by the efficient screening of \( E_\parallel \) due to the trans-field propagation in concave poloidal magnetic field lines, whereas a BH gap shifts from the null surface toward the outer light surface because the longitudinal width becomes comparable to the transverse thickness when the accretion rate is small.

Let us briefly examine how the gap width, \( w \), is affected when the ADAF soft photon field changes. In Figure 8, we plot the gap inner and outer boundary positions as a function of \( \dot{m} \), where the ordinate is converted into the Boyer–Lindquist radial coordinate. It follows that the gap inner boundary (solid curve, \( r = r_2 \)) infinitesimally approaches the horizon (dashed-dotted horizontal line, \( r = r_H \)), while the outer boundary (dashed curve, \( r = r_2 \)) moves outwards, with decreasing \( \dot{m} \). Below the accretion rate \( \dot{m} \sim \dot{m}_{\text{low}} = 2 \times 10^{-4} \), the outer boundary moves rapidly away from the horizon with decreasing \( \dot{m} \), so that the required current density, \( j = 0.7 \), may be produced within the gap under a diminished ADAF photon field. At

\[ \dot{m} = 5.62 \times 10^{-5} \text{ and } 4.21 \times 10^{-5} \], the outer boundary is located at \( r_2 = 9.35 r_g \) and 28.64\( r_g \). We consider that the solution with \( r_2 > 10 r_g \) may not have significant physical meaning, because the funnel boundary with the equatorial disk will deviate from a conical shape beyond this radius (Hirose et al. 2004; McKinney & Gammie 2004; Krolik et al. 2005; McKinney 2006; McKinney et al. 2012; O’Riordan et al. 2016). We thus define the lower-limit accretion rate, \( \dot{m}_{\text{low}} \), when \( r_2 \) exceeds 10\( r_g \) in this paper. Indeed, at further lower accretion rate, \( \dot{m} \ll 3.16 \times 10^{-5} \), we fail to find a 2D gap solution, because the weak ADAF photon field can no longer sustain the current density, \( j = 0.7 \), per magnetic flux tube. Thus, we obtain \( \dot{m}_{\text{low}} = 5.56 \times 10^{-5} \) for \( M = 10 M_\odot \), \( a = 0.90 r_g \), \( \Omega_f = 0.50 \omega_H \), and \( j = 0.7 \), interpolating \( r_2 = r_2(\dot{m}) \) and putting \( r_2 = 10 r_g \). Because \( r_2 \) rapidly increases near \( \dot{m} \sim \dot{m}_{\text{low}} \), the value of \( \dot{m}_{\text{low}} \) depends little on whether we define it by e.g., \( r_2 = 10 r_g \) or \( r_2 = 20 r_g \).

5.1.5. Spectrum of Gap Emission

The predicted photon spectra are depicted in Figure 9 for the same set of \( \dot{m} \) as in Figures 6 and 7. The thin curves on the left denote the input ADAF spectra, while the thick lines on the right show the output spectra from the gap. We find that the emitted flux increases with decreasing \( \dot{m} \), because the potential drop in the gap increases with decreasing \( \dot{m} \). The spectral peak around GeV is due to the curvature emission, while that around TeV is due to the IC scatterings. Provided that the distance is within several kpc, these HE and VHE TeV is due to the IC scatterings. Provided that the distance is located near GeV is due to the curvature emission, while that around
photons will be detectable if the duty cycle of the flaring activities is not too small (e.g., >0.1).

We plot the individual emission components in Figure 10, picking up the case of $m = 7.49 \times 10^{-5}$ (i.e., the case of the black solid line in Figure 9). The red dashed line shows the primary curvature component, and the red dashed–dotted line the primary IC component. The former component is not absorbed and appears as the spectral peak at several GeV when $m \sim m_{\text{thr}}$ (i.e., when the gap outer boundary is located at $r_2 \gg r_g$). The latter component is heavily absorbed by the ADAF near-IR photons to be reprocessed as the secondary component (blue dashed–dotted–dotted–dotted line). In this secondary component, IC emission dominates above GeV and the synchrotron component dominates only below this energy. The secondary component above 500 GeV is absorbed again to be reprocessed as the tertiary component (purple dotted).

Figure 9. SED of the gap emission for a stellar-mass black hole with $M = 10 M_\odot$ and $a = 0.9$, for five discrete dimensionless accretion rates, $m$ at 1 kpc. The thin curves denote the input ADAF spectra, while the thick lines show the output gap spectra. Each color corresponds to the same case of $m$ as in Figures 6–7. The thin solid curves (with horizontal bars) denote the Fermi/LAT detection limits after 10 years of observation, while the thin dashed and dotted curves (with horizontal bars) denote the CTA detection limits after a 50 hr observation. Magnetic field strength is assumed to be the equipartition value with the plasma accretion.

Figure 10. Similar to Figure 9, but only the case of $m = 7.49 \times 10^{-5}$ is depicted. The thick and thin black curves correspond to the same ones in Figure 9. The red dashed and dashed–dotted lines denote the primary curvature and inverse-Compton components, respectively. The blue dashed–dotted–dotted–dotted and purple dotted ones show the secondary and tertiary emission via synchrotron and inverse-Compton processes outside the gap.

To grasp the strength of the absorption taking place inside and outside the gap, we examine the optical depth for photon–photon collisions. For presentation purposes, we compute a representative optical depth for test photons emitted outwards at the gap inner boundary, $r = r_g$. With vanishing angular momenta, photons propagate in the $(r, \theta, \phi)$ surface (with constant $\theta$). Thus, the invariant distance, $ds$, of a photon path after propagating $dr$ and $d\phi$ in $r$ and $\phi$ coordinates becomes

$$ds^2 = g_{rr}dr^2 + g_{\theta\theta}d\theta^2 = \frac{\Sigma}{\Delta}dr^2 \left[ 1 + O\left( \frac{\Delta \sin^2 \theta}{r_g^2} \right) \right].$$

Thus, near the pole, $|\theta| \ll 1$, and near the horizon, $\Delta \ll r_g^2$, we can put $ds \approx \sqrt{\Sigma/\Delta}dr$. Integrating the local absorption probability over the photon ray, we can compute the optical depth by

$$\tau(\nu) = \int_{\nu_1}^{\nu_2} \frac{d\nu'}{d\nu} = \int_{r_1}^{r_2} \sqrt{\frac{\Sigma}{\Delta}} \frac{d\nu}{ds} dr,$$

where $ds$ coincides with the distance of radial interval, $dr$, measured by ZAMO. The radial gradient of the optical depth becomes

$$\frac{d\tau}{ds} = \frac{1}{c} \int_{0}^{1} \frac{d\mu'}{d\nu_{\text{th}}^\prime} \frac{d\sigma_{\gamma\gamma}(\nu', \nu', \mu')}{d\nu_{\text{th}}^\prime} \frac{dE_{\gamma}}{d\nu_{\text{th}}^\prime} dE_{\gamma},$$

where the primes denote quantities evaluated by ZAMO; $\mu'$ is the cosine of the collision angle of two photons, $\nu'$ the soft photon energy, and $F_{\gamma}$ the soft photon number flux. We employ ZAMO here, because the photons are assumed to be emitted with vanishing angular momenta in this paper. The threshold energy is defined by $\nu_{\text{th}}^\prime = [2/(1 - \mu')](m_e c)^2/h^2 \nu'$, where $h$ is the Planck constant. The photon frequency $\nu$ (at infinity) is redshifted to the ZAMO value, $\nu'$, at each altitude, $r$. Specifically, the local photon energy, $h\nu'$, is related to $h\nu$, by $h\nu' = i(h\nu + m \cdot d\nu'/d\tau)$, where $m$ denotes the photon angular momentum and $d\nu'/d\tau$ the local observer’s angular frequency in $dr$-basis at $r$. However, we here have $m = 0$, which gives the ZAMO angular frequency, $d\nu'/d\tau = g_{\nu\nu}/g_{\nu\nu}$. The quantity $i$ is given by the definition of the proper time, $i = \frac{g_{\theta\theta}}{\rho_w}$ for a ZAMO. Thus, we obtain $\nu' = \nu \frac{\rho_w}{\sqrt{g_{\nu\nu}}} = \nu \frac{\rho_w}{\sqrt{g_{\nu\nu}}}$ as the redshift relation between us and ZAMO. Note that the integral $\int_{r_1}^{r_2} \Delta^{-1/2} dr$ in Equation (28) is finite for a finite $r$, and that $dE_{\gamma}/d\nu_{\text{th}}^\prime$, and hence $dr/d\tau$, vanishes at large distances.

In Figure 11, we present $\tau(\nu)$ for five discrete accretion rates: the five lines correspond to the same cases of $m$ as in Figures 6, 7, and 9. It follows that the photon–photon absorption optical depth exceeds unity above 16 GeV, 90 GeV, 0.3 TeV, 0.5 TeV, and 0.9 TeV for $m = 1.00 \times 10^{-3}$, $3.16 \times 10^{-4}$, $1.00 \times 10^{-4}$, $7.49 \times 10^{-5}$, and $5.62 \times 10^{-5}$, respectively. The optical depth peaks at several TeV, because the ADAF photon spectrum peaks in near-IR wavelengths for stellar-mass BHs.

It should be stressed that the actual photons are absorbed by smaller optical depths than this figure, because the individual photons are emitted at different positions whose altitudes are always higher than the inner boundary.
5.1.6. Curvature versus Inverse-Compton Processes

It is worth examining the relative importance of the curvature and IC processes. In Figure 12, we plot the luminosity of the outward curvature-emitted photons as the solid curve, and that of the IC-emitted photons as the dashed-dotted one, as a function of $\dot{m}$. The outward curvature photons are mostly emitted inside the gap, while the IC photons are emitted both inside and outside the gap. Figure 12 shows that the curvature luminosity exceeds the IC one when $\dot{m} < 2 \times 10^{-4}$, or equivalently when the gap extends enough. This is because the curvature power is proportional to $\gamma^3$, and the IC power approximately to $\gamma^0 \sim \gamma^2$, depending on whether the collisions take place mainly in the extreme Klein–Nishina or the Thomson regime. The IC power also depends on the specific intensity of the soft photon field; thus, its dependence on $\gamma$ is more complicated than the curvature process.

5.1.7. Luminosity versus Created Current Density

Let us briefly examine the dependence of the spectrum on the created current density, $j$. In Figure 13, we plot the gap spectra at six discrete $j$s. It follows that the HE emission becomes most luminous when $0.5 \leq j \leq 0.7$ and the VHE one does when $j \sim 0.9$. In what follows, we thus adopt $j = 0.7$ as a compromise to optimize the HE and VHE fluxes. Note that we restrict our argument for $|j| \leq 1$, because $|j| > 1$ would incur a sign reversal of the $\rho - \rho_{\text{IC}}$ in Equation (19), thereby resulting in a sign reversal of $E_{\text{HE}}$ at the outer boundary, $r = r_2$, which would violate the present assumption of stationarity.

5.1.8. Luminosity versus BH Spin

We next briefly investigate how the spectrum depends on the BH spin parameter, $a_*=a/r_g$. In Figures 14 and 15, we present the gap spectra for $a_*=0.5$ and 0.998 when $\dot{m} = 1.00 \times 10^{-4}$. It follows that the gap flux increases with increasing BH spin. Thus, we adopt $a_*=0.9$ as the representative value in this paper.

5.1.9. Luminosity versus Magnetic Field Rotation

It is numerically suggested that the angular frequency, $\Omega_p$, of an accreting BH magnetosphere decreases from $0.3\omega_{\text{K}}$ in the middle latitudes to $-0.17\omega_{\text{K}}$ in the higher latitudes (i.e., near the pole; McKinney et al. 2012). Similar tendency, from $0.4\omega_{\text{K}}$ to $-0.2\omega_{\text{K}}$ is, indeed, analytically suggested (Beskin & Zheltoukhov 2013). Thus, in this subsection, we examine a
Figure 15. Similar to Figures 9 and 14, but the BH is maximally rotating. Other parameters are the same as Figures 9 and 14, including the accretion rate. W = 0.2FH. Other parameters are the same, including the accretion rate, ṁ.

Figure 16. Similar to Figure 9, but the magnetosphere is more slowly rotating, Ωe = 0.2ωH. Other parameters are the same, including the accretion rate, ṁ.

smaller Ωe case. In Figure 16, we plot the SED for Ωe = 0.2ωH, keeping other parameters unchanged from Figure 9. It follows that the HE flux decreases to about 17% of the Ωe = 0.5ωH case, while the VHE flux is roughly unchanged. The VHE flux changes mildly, because the IC process depends weakly on the lepton Lorentz factor compared to the curvature process. Note that the BZ power, which is proportional to Ωe(ωH − Ωe), reduces to only 64% of the Ωe = 0.5ωH case. This means that the gap becomes less efficient at smaller (in fact, also at greater) Ωe/ωH than 0.5. Thus, we adopt Ωe = 0.5ωH as the representative value.

5.1.10. Cascaded Pairs Outside the Gap

Let us investigate if a force-free magnetosphere is realized outside the gap. We compute the densities of the cascaded pairs within the cutting radius rcut = 60rg, which is well above the gap outer boundary, r = r2. In Figure 17, we plot the secondary, tertiary, and quaternary pair densities as the red dashed, green dashed–dotted, and blue dashed–dotted–dotted–dotted curves, respectively. Here, the secondary pairs denote those cascaded from the primary γ-rays, which are defined as being emitted by the primary electrons or positrons that are accelerated in the gap. The tertiary pairs denote those cascaded from the secondary γ-rays, which are defined as being emitted by these secondary pairs.

It follows that the density of the cascaded pairs exceeds the GJ one (black dotted line), as long as a stationary gap is formed. Thus, the magnetosphere becomes force-free outside the gap, as long as ṁ > ṁgap. This conclusion does not depend on the choice of rcut.

In short, gap solutions exist if the dimensionless accretion rate is in the range ṁgap < ṁ < ṁ<sub>low</sub>. For stellar-mass BHs, the gap emission peaks at several GeV and its maximum flux >10<sup>−12</sup> erg s<sup>−1</sup> is detectable with Fermi/LAT, if the duty cycle of the flaring activity is not too small. The gap luminosity increases with decreasing ṁ, because the gap is dissipating a portion of the BH’s spin-down luminosity. This forms a striking contrast to accretion-powered systems, whose luminosity will decrease with decreasing ṁ. The cascaded pairs outside the gap have a greater density than the Goldreich–Jullian value; thus, the magnetosphere becomes force-free in the downstream of the gap-generated flow (i.e., outside the gap outer boundary).

5.2. The Case of Intermediate-Mass BHs

Let us examine the gap emission from intermediate-mass BHs. We put M = 10<sup>3</sup> M<sub>☉</sub> and calculate the gap emission for a<sub>b</sub> = 0.9 and Ω<sub>fl</sub> = 0.5ω<sub>H</sub>. The predicted spectra become as in the top panel in Figure 18 for a distance of 10 kpc. It shows that the γ-ray fluxes are detectable in both HE and VHE if an IMBH is located within our galaxy, as long as the accretion rate is in the range, 2 × 10<sup>−5</sup> < ṁ < 4 × 10<sup>−5</sup>. Since the absolute luminosity of the gap increases with M, we also compute the SEDs for M = 10<sup>5</sup> M<sub>☉</sub> (bottom panel of Figure 18), assuming d = 100 kpc. It shows that such a heavy BH is detectable in HE or VHE only when the distance is comparable to or less than 100 kpc.

The curvature and IC luminosities are plotted as a function of ṁ in Figure 19. It shows that the curvature process is stronger than the IC when ṁ < 6 × 10<sup>−5</sup> for M = 10<sup>3</sup> M<sub>☉</sub>, and when ṁ < 1.5 × 10<sup>−5</sup> for M = 10<sup>5</sup> M<sub>☉</sub>. We examine if a force-free magnetosphere is sustained. Figure 20 shows the densities of the pairs cascaded between r<sub>2</sub> and 60rg. It follows that the density of the cascaded pairs

Figure 17. Density of the cascaded pairs outside the gap, within radius r<sub>2</sub> < r < 60rg, plotted as a function of ṁ for M = 10<sup>5</sup> M<sub>☉</sub> and a<sub>b</sub> = 0.9. The red dashed, green dashed–dotted, and blue dashed–dotted–dotted–dotted curves denote the the densities of the secondary, tertiary, and quaternary, pairs, respectively. The black dotted curve shows the Goldreich–Jullian charge density, which is volume-averaged within the sphere of radius r = 60rg.
exceeds the GJ density for both $M = 10^3 M_\odot$ and $M = 10^5 M_\odot$.

5.3. The Case of SMBHs

Next, let us examine SMBHs. We present the acceleration electric field in Section 5.3.1, gap spectrum in Section 5.3.2, and cascaded pair densities in Section 5.3.3. We briefly examine the dependence on the soft photon density in Section 5.3.4.

5.3.1. Acceleration Electric Field

Choosing a typical mass of $M = 10^9 M_\odot$, we plot an $E_\parallel(\eta_\chi, \theta)$ distribution in Figure 21, whose ordinate is the dimensionless tortoise coordinate, $\eta_\chi$. Comparing this with Figure 3, we find that the essential behavior of $E_\parallel$ is unchanged from the case of $M = 10 M_\odot$. In the Boyer–Lindquist radial coordinate, the gap outer and inner boundaries are distributed as a function of $\dot{m}$ as depicted in Figure 22. The gap outer boundary is located at $r_2 = 5.60 r_g$ and $r_2 = 10.32 r_g$ at $\dot{m} = 7.49 \times 10^{-7}$ and $5.62 \times 10^{-7}$, respectively; thus, we

Figure 19. Top panel: gap luminosity as a function of the dimensionless accretion rate for a BH with $M = 10^3 M_\odot$, $a_*= 0.9$, and $\Omega_F = 0.5 \omega_H$. The solid and dashed-dotted curves denote the luminosity of the curvature and inverse-Compton processes, respectively. Bottom panel: Similar figure to the top panel but for $M = 10^5 M_\odot$.

Figure 20. Similar to Figures 17 and 26, but for a BH with $M = 10^3 M_\odot$ (top panel) and $M = 10^5 M_\odot$ (bottom panel); $a_*= 0.9$ and $\Omega_F = 0.5 \omega_H$ are unchanged. The dashed and dashed–dotted curves denote the densities, $N_{\pm}$, of the secondary and tertiary pairs, while the dotted one is the GJ value, $N_{GJ}$. 
obtain $\dot{m}_{\text{low}} = 5.75 \times 10^{-7}$ for $M = 10^9 M_\odot$, $a = 0.9 r_g$, $\Omega_F = 0.50 \omega_{11}$, and $j = 0.7$. Figure 22 shows that the gap longitudinal width becomes comparable or greater than $r_g$ when the accretion rate reduces to $\dot{m} < 2 \times 10^{-6}$, whereas it is realized when $\dot{m} < 2 \times 10^{-4}$ for a stellar-mass case (Figure 8). It also follows that the gap outer boundary shifts outwards with decreasing $\dot{m}$, in the same manner as the $M = 10 M_\odot$ case.

5.3.2. Spectrum of Gap Emission

Figure 23 shows the gap spectra as the thick lines for five discrete $\dot{m}$, assuming a luminosity distance of 10 Mpc. When the accretion rate is in the range $5.6 \times 10^{-7} < \dot{m} < 10^{-6}$, we find that the gap emission will be marginally detectable with CTA, if the source is located in the southern sky. The emission components are depicted in Figure 24 for $\dot{m} = 7.49 \times 10^{-7}$. Since $\dot{m}$ is much smaller than the stellar-mass cases, the absorption optical depth decreases accordingly; as a result, most of the primary IC component (red dashed–dotted line) cascades only to the secondary generation pairs, whose emission is represented by the blue dashed–dotted–dotted–dotted line.

The curvature and IC luminosities are plotted as a function of $\dot{m}$ in Figure 25. It is clear that the IC process dominates the curvature one in the entire range of $\dot{m}$ for SMBHs.
5.3.3. Cascaded Pairs Outside the Gap

The created pair densities between \( r_2 \) and \( 60r_g \) are depicted as a function of \( \dot{m} \) in Figure 26. In the same way as for stellar-mass BHs (Section 5.1) and intermediate-mass BHs (Section 5.2), the magnetosphere becomes force-free outside the gap, as long as the gap solution exists.

Figures 17, 20 and 26 show that the BH magnetosphere becomes force-free for \( M = 10^3 M_\odot \), \( 10^5 M_\odot \), \( 10^6 M_\odot \), and \( 10^9 M_\odot \). Indeed, a BH magnetosphere becomes force-free irrespective of the BH mass, as long as a stationary gap is sustained in it by \( \dot{m} > \dot{m}_{\text{low}}(M) \).

5.3.4. Dependence on RIAF Soft Photon Field Density

Let us quickly examine how the gap luminosity depends on the ADAF photon field density, fixing \( \dot{m} \) so that the magnetic field near the horizon may not be changed. As a test model, we artificially reduce the photon density near the horizon to one fourth of its original value by doubling \( R_{\text{min}} \). In Figure 27, we plot the gap spectra for \( R_{\text{min}} = 12r_g \). We find that the gap emission increases because of the diminished soft photon density, as expected. To predict the gap spectrum of SMBHs more precisely, we must constrain the specific intensity of the RIAF photon field near the horizon for example by numerical simulations.

5.4. Gap Luminosity for Various BH Masses

Finally, we apply the method to various BH masses from \( 10 M_\odot \) to \( 6.4 \times 10^5 M_\odot \). The filled circles in Figure 1 denote the solved \( \dot{m}_{\text{low}}(M) \) for \((a_*, \Omega_*/\omega_*) \), \( R_{\text{min}}/r_g = (0.9, 0.5, 6) \). It follows that \( \dot{m}_{\text{low}}(M) \) always lies below the upper limit, \( \dot{m}_{\text{up}} \) (upper straight line). Therefore, BH gaps exist for arbitrary BH masses.

We fit the filled circles in Figure 1 with a straight line by the least square method to obtain \( \log \dot{m}_{\text{low}} = -3.6 - 0.26 \log(M/M_\odot) \). The luminosity, \( L_{\text{gap}} \), of a BH gap, could be estimated by substituting this \( \dot{m}_{\text{low}} \) into \( L_{\text{gap}} \approx 0.3 L_{\text{edd}} \approx 1.5 \times 10^{38} \dot{m}_{\text{low}} M_\odot \) erg s\(^{-1} \), where Equation (3) is used. The factor 0.3 comes from the fact that the gap luminosity can attain at most \( \sim 30\% \) of \( L_{\text{edd}} \), because \( E_{\text{ij}} \) is partly screened by the created pairs within the gap, and because a stationary gap solution can possess only a sub-GJ current density. It is also noteworthy that the BH gap solution at such a low \( \dot{m} \) corresponds to the case of the middle-aged pulsars whose outer-gap luminosity attains at most 30% of the spin-down luminosity for the same reasons (Hirotani 2013).

If we artificially change \( R_{\text{min}} \), we can consider the impact of the ADAF photon field density, without changing \( B \) at the same \( \dot{m} \). In Figure 1, we plot the \( \dot{m}_{\text{up}}(M) \) for \((a_*, \Omega_*/\omega_*) \), \( R_{\text{min}}/r_g = (0.9, 0.5, 12) \) as the open squares. It follows that the reduction of the soft photon field (by doubling \( R_{\text{min}} \)) changes the gap solution to some extent, because it affects the pair production process in the gap.

In Figure 1 we also plot a slower BH spin, \((a_*, \Omega_*/\omega_*) \), \( R_{\text{min}}/r_g = (0.5, 0.5, 6) \) as the open circles. It follows that \( \dot{m}_{\text{low}}(M) \) has a weak dependence on \( a_* \) due to the negative feedback effect (Hirotani 2013). To further constrain \( \dot{m}_{\text{low}}(M) \), we need to specify the specific intensity of the RIAF (e.g., ADAF) photon field near the horizon by numerical computation. Such details are, however, out of scope of the present paper. Therefore, the region \( \dot{m} < \dot{m}(M) \) (in which stationary gaps do not exist) is depicted in red gradient in Figure 1.

6. DISCUSSION

To sum up, we have solved stationary lepton accelerators (or gaps) in the magnetospheres of rotating BHs with arbitrary masses. By solving the set of an inhomogeneous part of the Maxwell equations, lepton equations of motion, and the radiative transfer equation, we demonstrate that an electric field arises along the magnetic field line around the null charge surface on which the frame-dragging angular frequency coincides with the magnetic-field angular frequency. In the gap, electrons and positrons are created via two-photon collisions and accelerated in opposite directions by the acceleration electric field into ultra-relativistic energies. Such leptons emit copious \( \gamma \)-rays via curvature and inverse-Compton (IC) processes, leading to a pair-production cascade inside and outside the gap. The gap longitudinal width is self-regulated so that a single electron eventually cascades into a single pair within the gap, and approximately coincides with the mean-free path (for an IC photon to materialize via two-photon collision) divided by the number of IC photons emitted by a single electron. As the accretion rate decreases, the increased mean-free path results in an extended gap, and hence an increased luminosity. The gap luminosity maximizes when the gap width becomes much greater than the horizon radius. For stellar-mass...
BHs, we can expect that their curvature emission is detectable with Fermi/LAT, and the IC emission with CTA, when the BH binary is in a quiescent state, provided that the distance is within several kpc. For SMBHs, their IC emission may be marginally detectable with CTA for low-luminosity AGN if their distances are within a few tens of Mpc.

6.1. Improvement form HP16

In the present work, there are mainly two improvements from HP16, who formulated a BH gap model and applied it to the radio galaxy IC310. First, in the present work, the Poisson Equation (15) is solved general-relativistically on the poloidal plane. However, in HP16, the left-hand side of Equation (15) was approximated one-dimensionally in the Newtonian limit. Second, in the present work, \( \rho (r, \theta) \) (in the Poisson equation) is solved from the local pair production rate in a manner consistent with the radiative transfer equation. However, HP16 assumed that \( \rho (r) \) changes linearly with \( r \) (see their Equation (10)).

Both above-mentioned simplifications suppress \( E_\parallel \) to some extent, when comparing the GR treatment. To overcome the difference, we apply the present 2D, GR method to IC310, assuming \( B = B_{\text{eq}} \). Figure 28 shows the resultant SED, where the parameter set is basically the same as HP16; namely, we adopt \( a_{\ast} = 0.998 \) and \( \Omega_F = 0.3 \omega_H \). Compared with Figure 13 of HP16, which was obtained for \( B = B_{\text{eq}} \), we find that the 1D Newtonian approximation in HP16 underestimated the gap luminosity about 1.5 times from the present 2D GR treatment.

6.2. Rigidity of Rotating Magnetic Field Lines

We have assumed that \( \Omega_{\parallel} = F_{\parallel}/F_{\phi \parallel} = F_{\phi \parallel}/F_{\phi \parallel} \) is constant along each magnetic field line. However, \( \Omega_F \) is not conserved when the electric potential drop along the magnetic field line becomes a non-negligible fraction of the electro-motive force exerted across the horizon. Nevertheless, for all the cases in the present paper, the gap luminosity is less than two percent of the Blandford–Znajek power. Noting that the particle energy is predominantly converted into radiation within the gap, we can conclude that the deviation of \( \Omega_F \) from a constant value (e.g., \( 0.5 \omega_H \)) is at most a few percent. Thus, the assumption of a constant \( \Omega_F \) along each magnetic field line is mostly justified.

6.3. Gap Position versus the Separation Surface

It is worth comparing the gap position with the separation surface where both inflows and outflows start without \( E_\parallel \). Since the plasma particles will not be accelerated either outward or inward at this surface, the sum of the gravitational, Lorentz and centrifugal forces will vanish along the magnetic field lines there. One of the convenient ways to find such a surface is to put both \( u_p = 0 \) and \( u_\parallel = 0 \) in the poloidal wind equation of MHD (Camenzind 1986a, 1986b), where \( u_\perp \) refers to the poloidal velocity of the fluid. Note that this MHD argument is valid in the magnetically dominated limit, which is close to (but not equal to) the force-free limit.

In a stationary and axisymmetric BH magnetosphere, the separation surface is determined by the condition \( k_0 = 0 \) in MHD (see end of Section 3; Takahashi et al. 1990), where the prime denotes the derivative along the poloidal magnetic field line. For radial poloidal field lines, \( k_0 \) attains its maximum value 0.651, 0.478, 0.372, and 0.310 at \( r = 8.34 r_g, 5.21 r_g, 4.02 r_g, \) and 3.45 \( r_g \), along the field lines at \( \theta = 15^\circ, 30^\circ, 45^\circ, \) and \( 60^\circ \), respectively. Thus, the separation surface is approximately located at \( 5 r_g < r < 8 r_g \) at \( \theta < 30^\circ \).

As for the gap position, Figures 3, 4, and 21 show that \( E_\parallel (r, \theta) \) distribution, and hence the gap longitudinal extent depends on little \( \theta \), except near the meridional boundary, \( \theta \sim \theta_{\text{max}} = 60^\circ \). Thus, near the magnetic pole (e.g., \( |\theta| < 30^\circ \)), the separation surface is generally located outside the gap (Figures 8 and 22). This conclusion depends little on the poloidal magnetic field structure, because the gap solution is essentially determined by the \( \rho_{\text{GJ}} \) distribution, which is governed by the radial component of the magnetic field near the horizon (Equation (24)) and because the separation surface distribution has a weak dependence on the poloidal field structure as well (Figure 2 of HP16).

As a plasma fluid begins to flow with \( u_\parallel \approx 0 \) from the separation surface, it is accelerated along the flow line by the MHD interactions; however, their Lorentz factors can attain only a few (e.g., Equation (5.2) of Hirotani et al. 1992). On the other hand, the electron–positron pairs that cascaded from outward-propagating PeV \( \gamma \) rays have typical Lorentz factors \( \sim 10^8 \) (for stellar-mass BHs). Thus, such pairs continue their outward motion across the separation surface, climbing up the “hill” of the MHD effective potential \( k_0 \) (Figure 2 of HP16) very easily. In other words, the large charge-to-mass ratio of the electron makes the electrostatic acceleration dominate the MHD acceleration, which is comparable to the centrifugal or gravitational one. Only in the direct vicinity of the horizon, do the plasma mass and the causality at the horizon make the gravitational interaction overcome the electromagnetic one. Put differently, we can forget about the separation surface when we consider the formation of a gap in a BH magnetosphere.

It is worth noting that the above-mentioned MHD separation surface is distinct from what is argued in recent MHD simulations. In numerical simulations, the flow density decreases to zero at a specific surface, which requires an
imposition of “density floors.” Without the density floors, a vacuum would develop in the funnel. However, such a limitation of numerical MHD has nothing to do with the gap formation discussed in the present paper.

6.4. The Case of Very Small Accretion Rate

Let us discuss what is expected when \( \dot{m} \) becomes even less than \( m_{\text{low}}(M) \). The ADAF photon field peaks around eV for stellar-mass BHs and around meV for SMBHs. These photons are emitted from the innermost region, \( r \sim R_{\text{min}} \sim 6g, \) and decreases outward approximately by \( r^{-2} \) law. Thus, when the gap outer boundary is located at \( r \gg R_{\text{min}} \), as in the case of \( \dot{m} \sim m_{\text{low}}(R) \), stationary pair production can be no longer sustained and a vacuum region develops in the entire polar funnel. In this vacuum region, migratory leptons are accelerated by the vacuum \( E_{\gamma} \) and cascade into copious primary electrons and positrons accelerated in opposite directions. Emissions from such cascading primaries will be time-dependent and may consist of many “shots,” whose power spectrum density (PSD) may show some characteristic power law. In this case, the lower cutoff frequency of the power-law PSD will be given by the reciprocal of the light crossing time of the extended gap, which is much longer than the horizon-light-crossing timescale.

6.5. Distinction Between Gap and Jet Emissions

We finally discuss how to discriminate the gap and jet emissions. It follows from Figure 9 that the gap HE and VHE fluxes increase with decreasing \( \dot{m} \). That is, we can predict an anti-correlation between the IR/optical and HE/VHE fluxes. It forms a contrast to the standard shock-in-jet scenario, in which the IR/optical and the HE/VHE fluxes will correlate. Therefore, we propose to simultaneously observe nearby BH transients during quiescence both in near-IR/optical and VHE. If their time-varying multi-wavelength spectra show anti-correlation, it strongly suggests that the photons are emitted from the BH gap. For nearby low-luminosity AGNs, the anti-correlation will appear between submillimeter wavelength and VHE.

In X-rays, the gap emission is very weak. Thus, if X-ray photons are detected, they are probably emitted from the jet or from the accretion flow.

Stellar-mass BHs exhibit the strongest gap emission in HE. Figure 9 shows that nearby BH transients are capable of emitting an order of magnitude greater fluxes than the LAT detection limit (with 10 years observation). It means that an HE “flare” could be detected by LAT if the flare lasts for a month or so, particularly for the sources located away from the galactic plane. We will examine the plausible sources located in a subsequent paper.

One of the authors (K. H.) is indebted to Dr. T. Y. Saito for valuable discussion on the CTA sensitivity, and to Drs. K. Kashyama, K. Asada, M. Nakamura, A. K. Harding, D. Kazanas, S. Shibata for fruitful discussion. This work is supported by the Theoretical Institute for Advanced Research in Astrophysics (TIARA) operated under Academia Sinica.

REFERENCES

Aleksić, J., Antonelli, L. A., Antonanz, P., et al. 2010, ApJL, 723, L207
Aleksić, J., Antonelli, L. A., Antonanz, P., et al. 2014a, A&A, 563, A91
Aleksić, J., Ansoldi, S., Antonelli, L. A., et al. 2014b, Sci, 346, 1080
Beskin, V. S., Istomin, Y. N., & Par’ev, V. I. 1992, SvA, 36, 642
Beskin, V. S., & Zheltoukhov, A. A., 2013, AstL, 39, 215
Bisnovatyi-Kogan, G. S., & Ruzmaikin, A. A., 1974, ApSSS, 28, 45
Blandford, R. D., & Znajek, R. L., 1976, MNRAS, 179, 433
Bogovalov, S. V. 1999, A&A, 349, 1017
Boyer, R. H., & Lindquist, R. W., 1967, JMP, 265, 281
Broderick, A. E., & Tchekhovskoy, A. 2015, ApJ, 809, 97
Camenzind, M. A. 1986a, A&A, 156, 137
Camenzind, M. A. 1986b, A&A, 162, 32
Cheng, K. S., Ho, C., & Ruderman, M. 1986a, ApJ, 300, 500
Cheng, K. S., Ho, C., & Ruderman, M. 1986b, ApJ, 300, 522
Cheng, K. S., Ruderman, M., & Zhang, L. 2000, ApJ, 537, 964
Chiang, J., & Romani, R. W. 1992, ApJ, 400, 629
Daugherty, J. K., & Harding, A. K. 1982, ApJ, 252, 337
Dermer, C. D., & Sturner, S. J. 1994, ApJL, 420, L75
Dolch, P., Pethick, C. J., & Lamb, F. K. 1977, ApJ, 217, 578
Goldreich, P., & Julian, W. H., 1969, ApJ, 157, 869
Harding, A. K., Tademaru, E., & Esposito, L. S. 1978, ApJ, 225, 226
Hawley, J. F., & Krolik, J. H. 2006, ApJ, 641, 103
Hirose, S., Krolik, J., de Villiers, J.-P., & Hawley, J. F., 2004, ApJ, 606, 1083
Hirota, K. 2006, MPLA, 21, 1319
Hiroto, K. 2013, ApJ, 766, 98
Hiroto, K. 2015, ApJ, 798, L40
Hiroto, K., & Okamoto, I. 1998, ApJ, 497, 563
Hiroto, K., & Pu, H.-Y. 2016, ApJL, 818, 50
Hiroto, K., & Shibata, S. 1999a, MNRAS, 308, 54
Hiroto, K., & Shibata, S. 1999b, MNRAS, 308, 67
Hiroto, K., Takahashi, M., Nitta, S., & Tomimatsu, A. 1992, ApJ, 386, 455
Holloway, N. J. 1973, NPS, 246, 6
Ichimaru, S. 1979, ApJ, 214, 840
Kerr, R. P. 1963, PhRvL, 11, 237
Koide, S., Shibata, K., Kudoh, T., & Meier, D. L., 2002, Scii, 295, 1688
Krolik, J. H., Hawley, J. F., & Hirose, S. 2005, ApJ, 622, 1008
Levinson, A., & Rieger, F. 2011, ApJ, 730, 123
Levinson, A., & Rieger, F. 2011, ApJ, 730, 123
Mahadevan, R. 1997, ApJ, 477, 585
McKinney, J. C. 2006, MNRAS, 368, 1561
McKinney, J. C., & Gammie, C. F. 2004, ApJ, 611, 977
McKinney, J. C., & Narayan, R. 2007a, MNRAS, 375, 513
McKinney, J. C., & Narayan, R. 2007b, MNRAS, 375, 531
McKinney, J. C., Tchekhovskoy, A., & Blandford, R. D., 2012, MNRAS, 423, 3083
Mestel, L. 1971, Natur, 233, 149
Miller-Jones, J. C. A., Jonker, P. G., MacCarone, T. J., Nelemans, G., & Calvelo, D. E. 2011, ApJL, 739, L18
Narayan, R., Igumenshchev, I. V., & Abramowicz, M. A., 2003, PASJ, 55, L69
Narayan, R., & Yi, I. 1994, ApJL, 428, L13
Narayan, R., & Yi, I. 1995, ApJ, 452, 710
Neronov, A., & Aharonian, F. A. 2007, ApJ, 671, 85
O’Riordan, M., Pe’re, A., & McKinney, J. C., 2016, ApJ, 819, 95
Petri, J. 2013, MNRAS, 434, 2636
Petri, J., & Kirk, J. G. 2005, ApJL, 627, L37
Plotkin, R. M., Gallo, E., & Jonker, P. G. 2013, ApJ, 773, 79
Romani, R. 1996, ApJ, 470, 469
Romani, R. & Watters, K. 2010, ApJ, 714, 810
Rybicki, G. B., & Lightman, A. P., 1979, Radiative Processes in Astrophysics (New York: Wiley)
Takahashi, M., Nitta, S., Tatematsu, Y., & Tomimatsu, A. 1990, ApJ, 363, 206
Timokhin, A. N., & Arons, J. 2013, MNRAS, 434, 2636
Timokhin, A. N., & Arons, J. 2013, MNRAS, 429, 20
Timokhin, A. N., & Harding, A. K. 2015, ApJ, 809, 144
Wang, Y., Takata, J., & Cheng, K. S. 2011, MNRAS, 414, 2664
Zhang, J. L., & Cheng, K. S. 1997, ApJ, 487, 370

HIROTANI ET AL.