Modeling of coupled-resonator optical waveguide (CROW) based refractive index sensors using pixelized spatial detection at a single wavelength

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Abstract: We model and analyze coupled-resonator optical waveguide (CROW) based refractive index (RI) sensors using pixelized spatial detection. Our modeled cascaded Fabry-Perot (FP) CROWs reveal that the intra-band states mode-field distributions vary upon effective RI change at a single wavelength. The spatial Fourier transform of the CROW mode-field distributions, with each cavity field intensity integrated as a pixel, shows spatial frequency peak shift, which constitutes the basis of such a spatial domain sensor. The spatial domain sensing performance depends on the cavity number, the cavity length and the inter-cavity coupling. Our modeled 21-element CROW sensor attains a detection limit of $10^{-4}$ refractive index unit (RIU) with a sensing dynamic range of $10^{-3}$ RIU. Detailed analysis of the spatial frequency harmonic peak amplitude variation further suggests an improved detection limit. Finite-difference time-domain (FDTD) simulations of an 11-element microring CROW device shows sensitivity consistent with the FP modeling.

OCIS codes: (130.6010) Sensor; (230.5750) Resonators; (230.4555) Coupled resonators.

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1. Introduction

Silicon microresonator-based optical sensors [1–3] have been attracting significant attention due to key merits of demonstrated sensing performance (~100nm resonance shift per refractive index unit (RIU); ~10^{-3} – ~10^{-5} RIU detection limit), compact size (tens to hundreds of μm) and the potential for large-scale integration in optofluidics applications [4–7]. The microresonator confines light at a discrete set of wavelengths, which are determined by the cavity resonance condition. The evanescent field of the light propagating in the microresonator extends outside and interacts with the cladding material. Biomaterials binding to the surface of a functionalized microresonator change the effective refractive index (RI) and shift the resonance wavelengths. Thus, the conventional microresonator sensor is based on measuring sharp resonance wavelength shifts or lineshape modulations [8–10]. However, such methods require microresonators with high quality (Q) factors and measurements with high-resolution wavelength-tunable lasers. Besides, the data accumulated from the spectral measurements are usually of single value in the form of either resonance wavelength shift or change of transmission intensity, which limit the accuracy and repeatability of the sensing results [10]. In practice, wavelength-tunable lasers are bulky, costly and the tuning step of the laser eventually limits the detection limit [5]. Therefore, sensors that enable a multiple-value measurement and only require a single-wavelength laser could potentially serve as a reliable, high-performance, relatively compact and low-cost sensor.

Compared with single-element microresonators, cascaded microresonators exhibit broadband transmission spectra rather than sharp resonances [11]. Two structures of cascaded microresonators, namely, side-coupled integrated sequence of spaced optical resonators (SCISSORs) and coupled-resonator optical waveguides (CROWs) have been widely studied...
as high-order optical filters [12,13], optical delay lines [14–16] and wavelength converters [17] for telecommunications and on-chip interconnects applications. Disorders- and localization-induced statistical phenomena of many-element cascaded microresonators have also been recently studied [18,19].

For characterizing single and cascaded microresonators, infrared imaging of the microresonator out-of-plane scattering light has been used [20,21]. Especially for cascaded microresonators, the spatial detection of the scattering light provides comprehensive information from individual cavities [21].

Previously, motivated by the above, we proposed a RI sensing scheme based on CROWs using multi-channel pixelized spatial detection [22]. The method relies on measuring the out-of-plane light scattering from each coupled microresonator acting as a pixel. Our preliminary experiment demonstrated an 11-element microdisk CROW-based sensor on a silicon nitride (SiN) on silica substrate.

In this paper, we use cascaded Fabry-Perot (FP) resonators model [23] to analyze the mode intensity distributions along the CROW device. Spatial Fourier transforms are used to analyze the pixelized intensity distributions. Finite-difference time-domain (FDTD) simulations of a microring CROW are used to compare with the cascaded FP resonators modeling.

2. Principle

Figure 1 illustrates the sensing principle of a generic CROW-based sensor. Figure 1(a) shows the structure of a microring CROW device. In the spectral domain, CROW comprising identical microresonators exhibit periodic broadband transmission due to the inter-cavity coupling-induced mode splitting [11]. In the case that the microresonators are single-mode, the number of modes (states) within each transmission band is equal to the number of coupled microresonators. In the spatial domain, the cascaded microresonators exhibit distinctive spatial mode-field pattern at each state.
Figure 1(b) depicts the transmission band spectrally shifts upon effective RI changes from \( n_0 \) to \( n_0 + \Delta n \). At a fixed wavelength \( \lambda_p \), the mode-field pattern varies corresponding to the state transition (Fig. 1(c)) while the transmission band spectrally shifts within the bandwidth (Fig. 1(b)). The field intensity scattered out of plane from each cavity is integrated as a pixel, thus the intensity distribution is described as a pixel array. Because the individual cavity is single-mode, integrating the mode-field intensity within each cavity maintains all relevant spatial information and enables far-field detection of the pixelized cavity mode intensity without concerning the near-field spatial information details. Through the spatial Fourier transform, we obtain the spatial frequency of a pixelized mode pattern. The spatial frequency peak shows discrete frequency shifts among states. Therefore, it is conceivable to extract discrete effective RI change at a single wavelength by imaging in the far-field pixelized spatial mode-field pattern change.

Based on the spatial frequency components of the pixel array, we develop two sensing schemes. For an effective RI change resulting in inter-state transitions, we can extract the RI change by directly identifying the state number from the dominant spatial frequency component (see sections 3.2–3.5). For an effective RI change resulting in intra-state transitions, we can extract the RI change from the detailed spatial frequency components peak amplitude change (see section 3.6).

3. Fabry-Perot modeling

3.1 Cascaded FP cavities model

We use a cascaded FP cavities model to investigate the pixelized mode-field patterns and sensing performance of the proposed refractive index sensor. The one-dimensional FP cavity model [23] is generic for single-mode microresonators and can be modified to account for specific structures such as microring CROWs [11,24].

![Schematic of the cascaded FP cavities model.](image)

Figure 2 shows the schematic of the cascaded FP cavities model. The cavity number of the cascaded FP cavities is \( N \) and the \( i^{th} \) cavity length is \( L_i \). \( a_0 \) and \( b_0 \) are the forward- and backward-propagating electric field complex amplitudes at the input. In the \( i^{th} \) cavity, \( a_i(x) \) and \( b_i(x) \) are the forward- and backward-propagating electric field complex amplitudes at position \( x \) within the \( i^{th} \) cavity (\( \sum_{j=1}^{i} L_j - L_i \leq x \leq \sum_{j=1}^{i} L_j \)). We assume there is no gap spacing between adjacent cavities. \( a_{N+1} \) and \( b_{N+1} \) are the forward- and backward-propagating electric field complex amplitudes at the output. The modeling assumes only scalar fields and steady-state analysis. The transfer matrix of the inter-cavity light transmission is described as [25]

\[
\begin{bmatrix}
    a_{i+1}(x = \sum_{j=1}^{i} L_j - L_{i+1}) \\
    b_{i+1}(x = \sum_{j=1}^{i} L_j - L_{i+1})
\end{bmatrix} = \begin{bmatrix}
    \frac{1}{\kappa_i} \left[ \kappa_i \kappa'_i - r_i r'_i \right] & r'_i \\
    -r_i & 1
\end{bmatrix} \begin{bmatrix}
    a_i(x = \sum_{j=1}^{i} L_j) \\
    b_i(x = \sum_{j=1}^{i} L_j)
\end{bmatrix} = Q_i \begin{bmatrix}
    a_i(x = \sum_{j=1}^{i} L_j) \\
    b_i(x = \sum_{j=1}^{i} L_j)
\end{bmatrix}
\]

(1)
where $Q_i$ is the amplitude transfer matrix between the $i^{th}$ and $(i+1)^{th}$ cavity. $\kappa_i$ ($\kappa'_i$) and $r_i$ ($r'_i$) are the complex coupling coefficient and reflection coefficient for the forward- (backward-) propagating waves. Time reversibility requires $r_i = -r'_i$ and $\kappa \kappa' - r r' = 1$; while reciprocity and energy conservation give the relations $\kappa'_i = \kappa_i^*$ and $r'_i = -r_i^* = -r_i$ [25]. Thus, the coupling and reflection coefficients satisfy the lossless coupling condition $|\kappa|^2 + |r|^2 = 1$. The transfer matrix $Q_i$ can be simplified as

$$Q_i = \frac{1}{\kappa'_i} \begin{bmatrix} 1 & -r_i \\ -r_i & 1 \end{bmatrix}$$

(2)

The internal fields at the two boundaries of each cavity are related as

$$\begin{bmatrix} a_i(x = \sum_{j=1}^i L_j) \\ b_i(x = \sum_{j=1}^i L_j) \end{bmatrix} = \begin{bmatrix} \exp(j\beta_i L - \frac{\alpha_i}{2} L_i) & 0 \\ 0 & \exp(-j\beta_i L + \frac{\alpha_i}{2} L_i) \end{bmatrix} \begin{bmatrix} a_i(x = \sum_{j=1}^i L_j - L_i) \\ b_i(x = \sum_{j=1}^i L_j - L_i) \end{bmatrix} = P \begin{bmatrix} a_i(x = \sum_{j=1}^i L_j) \\ b_i(x = \sum_{j=1}^i L_j) \end{bmatrix}$$

(3)

where $\beta_i = n_{i,\text{eff}} 2\pi/\lambda$ is the propagation constant in the $i^{th}$ cavity, $n_{i,\text{eff}}$ is the effective RI of the $i^{th}$ cavity at the propagation wavelength $\lambda$ in free space and $\alpha_i$ is the propagation power loss coefficient of the $i^{th}$ cavity. $P_i$ is the phase transfer matrix in the $i^{th}$ cavity.

Likewise, the input- and output-coupling transfer matrices $Q_{in}$ and $Q_{out}$ are defined as

$$\begin{bmatrix} a_i(x = 0) \\ b_i(x = 0) \end{bmatrix} = \begin{bmatrix} \kappa_{in} & -r_{in} \\ -r_{in} & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ b_0 \end{bmatrix} = \begin{bmatrix} 1 \\ \kappa_{in} \end{bmatrix} \begin{bmatrix} -r_{in} \\ 1 \end{bmatrix} \begin{bmatrix} a_0 \\ b_0 \end{bmatrix} = Q_{in} \begin{bmatrix} a_0 \\ b_0 \end{bmatrix}$$

(4)

$$\begin{bmatrix} a_N(x = \sum_{j=1}^N L_j) \\ b_N(x = \sum_{j=1}^N L_j) \end{bmatrix} = \begin{bmatrix} \kappa_{out} & -r_{out} \\ -r_{out} & 1 \end{bmatrix} \begin{bmatrix} a_N(x = \sum_{j=1}^N L_j) \\ b_N(x = \sum_{j=1}^N L_j) \end{bmatrix} = \begin{bmatrix} 1 \\ \kappa_{out} \end{bmatrix} \begin{bmatrix} -r_{out} \\ 1 \end{bmatrix} \begin{bmatrix} a_N(x = \sum_{j=1}^N L_j) \\ b_N(x = \sum_{j=1}^N L_j) \end{bmatrix}$$

(5)

where $\kappa_{in}$ ($\kappa'_{in}$) and $r_{in}$ ($r'_{in}$) are the complex coupling coefficient and reflection coefficient for the forward- (backward-) propagating waves at the input port, $\kappa_{out}$ ($\kappa'_{out}$) and $r_{out}$ ($r'_{out}$) are the complex coupling coefficient and the reflection coefficient for the forward- (backward-) propagating wave at the output port. The input- and output-coupling and reflection coefficients satisfy the lossless coupling condition $|\kappa_{in}|^2 + |r_{in}|^2 = 1$ and $|\kappa_{out}|^2 + |r_{out}|^2 = 1$. By cascading the transfer matrices $Q_i$, $P_i$, $Q_{in}$ and $Q_{out}$, we obtain the expression for the field components at the output after $N$ elements of FP cavities as

$$\begin{bmatrix} a_{N+1} \\ b_{N+1} \end{bmatrix} = Q_{out} P \left( \prod_{i=1}^N Q_i P \right) Q_{in} \begin{bmatrix} a_N \\ b_N \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} a_N \\ b_N \end{bmatrix}$$

(6)

The overall transfer matrix from the input-coupled electric field to the output-coupled electric field is expressed as a matrix comprising complex matrix elements $A$, $B$, $C$ and $D$. For a single input, we set the boundary condition as $b_{N+1} = 0$. Therefore, the field transfer
functions for the reflection field amplitude $b_0$ and the transmission field amplitude $a_{N+1}$ can be expressed as

$$b_0 = -\frac{C}{D} a_0 \quad (7)$$

$$a_{N+1} = (A - \frac{BC}{D}) a_0 \quad (8)$$

In order to study the internal electric field in each cavity, we define the reduced coordinate $x_i$ within the $i^{th}$ cavity ($0 \leq x_i \leq L_i$), which is related to the global coordinate $x$ as $x_i = x - \sum_{j=1}^{i-1} L_j$. The internal field at position $x_i$ within the $i^{th}$ cavity is calculated as

$$\left[ \begin{array}{c} a_i(x_i) \\ b_i(x_i) \end{array} \right] = \left[ \begin{array}{cc} \exp(j\beta_i x_i - \frac{\alpha}{2} x_i) & 0 \\ 0 & \exp(-j\beta_i x_i + \frac{\alpha}{2} x_i) \end{array} \right] \left[ \begin{array}{c} a(x = \sum_{j=1}^{i} L_j - L_i) \\ b(x = \sum_{j=1}^{i} L_j - L_i) \end{array} \right] \quad (9)$$

In the $i^{th}$ cavity at position $x$, internal field intensity $I_i(x)$ normalized to the input intensity $I_0$ is extracted by superposition of the two counter-propagating fields as

$$I_i(x) / I_0 = [a_i(x) + b_i(x)]^2 / [a_0]^2 \quad (10)$$

Without losing any relevant spatial information within each of the individual single-mode cavities, we integrate the internal field intensity within each cavity as a single pixel and describe the CROW mode-field intensity pattern as an $N$-element array.

In this paper, we focus on ideal CROWs with perfect uniformity and coherence. We assume ideal FP CROW structures with identical cascaded cavities ($L_i = L$, $n_{eff} = n_{eff}$, $\beta_i = \beta$, $\alpha_i = \alpha$), uniform inter-cavity coupling ($\kappa_i = \kappa$, $r_i = r$) and symmetric input/output-coupling ($\kappa_{in} = \kappa_{out}$, $r_{in} = r_{out}$). We note that the way we lay down our model allows setting non-uniform parameters among individual cavities. We will comment on effects of fabrication imperfection in section 5.

### 3.2 Pixelized mode-field intensity patterns and spatial Fourier analysis

In order to illustrate the basic modeling results, we choose $N = 11$, $L = 50 \mu m$, $\kappa_{in} = \kappa = 0.4$, $\alpha = 1.85 \ cm^{-1}$ and $n_{eff} = 1.6$. The cavity number is chosen to study a CROW with a fair number of coupled microresonators. We assume an effective RI and a loss coefficient for water-clad SiN microresonators sitting on a silica substrate as motivated by our previous work [16,22] and other SiN microresonator-based sensors [3]. In the modeling, we vary the effective RI and study the pixelized spatial mode patterns and the spatial frequency spectra at a fixed wavelength as described in the sensing principle.

Figure 3(a) shows the modeled periodic broadband transmission of the cascaded FP cavities in the 1550nm wavelength range. There are 11 resonance peaks within each passband corresponding to the states of the cascaded FP cavities. We label the states from 1 (long-wavelength edge state) to 11 (short-wavelength edge state). The bandwidth (defined as the spacing between the edge states 1 and 11) is 3.8 nm and the free-spectral range (FSR) is 15.1 nm, which is given by the FP cavity length $L$. We align the fixed probe wavelength $\lambda_p$ to state 1 ($\lambda_p = 1547.8 \ nm$). By increasing the effective RI uniformly among all the cascaded FP cavities in discrete steps, we model the transmission band redshifts and the state transitions from state 1 to 11.

Figure 3(b) shows the calculated pixelized mode-field intensity patterns in the form of 11-element arrays for the 11 states at $\lambda_p$ upon corresponding effective RI increments. States 7 to...
11 exhibit almost identical pixelized spatial intensity distributions as states 5 to 1. We term state 6 as the band-center state.

We apply spatial Fourier transform to the pixelized intensity arrays. The spatial frequency is defined as the number of the periodic pattern cycles within the length of the cascaded cavities. As the mode-pattern distribution is pixelized as an $N$-element array, all the spatial frequency used in the following text is normalized to the unit spatial frequency ($1/\text{NL} \mu\text{m}^{-1}$).

Figure 3(c) shows the spatial Fourier frequency spectrum of each state exhibiting a distinct peak component. For example, state 1 at the long-wavelength edge of the broadband transmission has an intensity distribution with one cycle, which corresponds to spatial frequency of one unit (fundamental). When state 2 shifts to $\lambda_p$ upon the effective RI increment, the modeled pixelized pattern shows two cycles and the spatial frequency peak component shifts to two units (second harmonic). Therefore, the spatial frequency analysis of the pixelized patterns provides a straight-forward approach to identify the inter-state transition and the corresponding discrete effective RI change.

Fig. 3. Modeling of the cascaded FP cavities with $N = 11$, $L = 50 \mu\text{m}$ and $\kappa = 0.4$. (a) Periodic broadband transmission with states from 1 to 11. (b) Pixelized mode-field intensity distributions of states 1 to 11 at a fixed probe wavelength $\lambda_p$ initially aligned to state 1 upon corresponding effective RI increases. (c) Spatial frequency spectra of the pixelized mode-field intensity patterns by spatial Fourier transform. Dashed arrows illustrate the shift of the dominant frequency component as the effective RI increases.
The spatial Fourier frequency spectra reflect the pixelized spatial intensity distribution symmetry around the band-center state. As the state shifts beyond the band-center state 6 (corresponding to the 6th harmonic in the frequency spectrum), further increase in the effective RI down-shifts the spatial frequency to lower harmonics till it reaches the fundamental (corresponding to the short-wavelength edge state 11).

3.3 Inter-state sensing scheme sensitivity and the detection limit

We define the inter-state sensing scheme sensitivity ($S_{\text{intertest}}$) of the CROW sensor as the ratio of the normalized inter-state discrete spatial frequency peak shift ($\Delta F$) to the effective RI change ($\Delta n_{\text{eff}}$) as follows:

$$S_{\text{intertest}} = \frac{\Delta F}{\Delta n_{\text{eff}}} = \frac{\Delta \lambda}{\delta \lambda \Delta n_{\text{eff}}}$$  \hspace{1cm} (11)

$\Delta F$ can be described as the number of state transitions at a fixed wavelength upon transmission band shift $\Delta \lambda$ induced by $\Delta n_{\text{eff}}$, and $\delta \lambda$ is the quasi-uniform state spacing measured in wavelength unit.

The inter-state sensing has a resolution given by unit normalized spatial frequency peak shift ($\Delta F = 1$). The detection limit ($DL_{\text{intertest}}$) or minimum effective RI change detectable is given as

$$DL_{\text{intertest}} = \frac{1}{S_{\text{intertest}}} = \frac{\delta \lambda \Delta n_{\text{eff}}}{\Delta \lambda}$$  \hspace{1cm} (12)

The inter-state sensitivity $S_{\text{intertest}}$ and the detection limit $DL_{\text{intertest}}$ relate to the state spacing $\delta \lambda$. A smaller state spacing gives a larger sensitivity and a smaller detection limit.

3.4 Inter-state sensing windows

![Diagram of inter-state sensing windows](image)

Fig. 4. Schematic of the inter-state sensing windows. (a) In the red sensing window, spatial frequency peak harmonic rises (drops) upon the effective RI increases (decreases). (b) In the blue sensing window, spatial frequency peak harmonic drops (rises) upon the effective RI increases (decreases). (c) At the band-center state, the spatial frequency drops independent of the sign of the effective RI change.

As shown in Fig. 3, because the pixelized arrays and their corresponding spatial Fourier spectra are symmetric around the band-center state, the useful dynamic range for the inter-
state sensing scheme becomes limited to only within half of the total number of states, namely, either between states 1 and 6 (denoted as the red sensing window) or between states 11 and 6 (denoted as the blue sensing window).

It should be noted that within each sensing window the inter-state sensing scheme allows distinguishing the sign of the effective RI change. Figures 4(a) and 4(b) illustrate that the probe wavelength initially aligned at a state in the red sensing window (blue sensing window), the spatial Fourier frequency increases (decreases) with the effective RI increases. In the case that the probe wavelength is aligned at the band-center state, however, the spatial Fourier frequency drops independent of the sign of the effective RI change (Fig. 4(c)).

Figure 5(a) shows the modeled spatial frequency peak shifts with the effective RI increases, assuming the fixed probe wavelength is initially aligned to the long-wavelength edge state. All the parameters except the \( N \) values used here follow those adopted in the 11-element FP CROW example in section 3.2. The slopes of the spatial frequency peak to the effective RI change curves are, however, only linear around the band-center state and reach the steepest slopes near the band-edge states. The dashed lines show the linear fits of the spatial frequency shift curves near the band-center states. The inter-state sensitivity of 11-, 21- and 51-element cascaded FP cavities are \( 2 \times 10^3 \) RIU\(^{-1} \), \( 4 \times 10^3 \) RIU\(^{-1} \) and \( 9 \times 10^3 \) RIU\(^{-1} \), respectively. The shaded regions depict the linear sensing windows, which span a sensing dynamic range of \( 10^3 \) RIU. In the following discussion, we only focus on these linear sensing windows.

Figure 5(b) shows the pixelized intensity distributions of 11-, 21- and 51-element cascaded FP cavities at the band-center states. The intensity values of 21- and 51-element FP cavities are artificially amplified by 2 times and 10 times in order to clearly show the patterns. With a large cavity number \( N \), the mode intensity distributions exhibit an obvious loss-induced gradient along the cascaded FP cavities. The gradient could cause distortion to the intensity distributions. Moreover, the mode intensity of each cavity decreases and may affect the signal level of the spatial domain detection. Therefore, although the large cavity number in principle gives a high sensitivity, the pixel intensity drop and the intensity gradient along the CROW eventually limit the cavity number.

3.5 Inter-state sensing scheme sensitivity analysis

In the inter-state sensing scheme, a high sensitivity requires a narrow state spacing \( \delta \lambda \) (see Eq. (11)). The state spacing \( \delta \lambda \) is determined by the transmission bandwidth \( BW \) and the number of states \( N \) within the band. In the weak coupling regime \( (\kappa^2 \ll 1) \), we can approximately express \( BW \) as [24]
\[ BW \approx \frac{\kappa \lambda_0^2}{\pi Ln_{eff}} \]  

where \( \lambda_0 \) is the band-center wavelength in free space. Assuming the single-mode condition, we express the state spacing \( \delta \lambda \) in the sensing windows as

\[ \delta \lambda \approx \frac{BW}{N-1} = \frac{\kappa \lambda_0^2}{(N-1)\pi Ln_{eff}} \]  

Therefore, the inter-state sensing scheme sensitivity as defined in Eq. (11) is given as

\[ S_{\text{interstate}} = \frac{(N-1)\pi Ln_{eff} \Delta \lambda}{\kappa \Delta n_{eff} \lambda_0^2} \]  

According to Eq. (15), in order to attain a high sensitivity, we need to design a CROW with a large \( N \) number of coherently coupled cavities, a large cavity size \( L \), a small coupling coefficient \( \kappa \), a short band-center wavelength \( \lambda_0 \) and a large effective RI \( n_{eff} \).

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Figure 6(a) shows the modeled sensitivity \( S_{\text{interstate}} \) as a function of cavity number \( N \). Given \( L = 50 \) μm, \( \alpha = 1.85 \) cm\(^{-1}\) and \( \kappa = 0.4 \), the sensitivity linearly increases from 1969 RIU\(^{-1}\) to 8445 RIU\(^{-1}\) with \( N \) increases from 11 to 51.

Figure 6(b) shows the modeled sensitivity \( S_{\text{interstate}} \) as a function of cavity length \( L \). The sensitivity increases linearly as \( L \) increases. For example, given \( \alpha = 1.85 \) cm\(^{-1}\) and \( \kappa = 0.4 \), the sensitivity of the 11-element CROW increases from 977 RIU\(^{-1}\) to 3972 RIU\(^{-1}\) as \( L \) increases from 25 μm to 100 μm.

Figure 6(c) shows the modeled sensitivity \( S_{\text{interstate}} \) as a function of coupling coefficient \( \kappa \). Given \( L = 50 \) μm and \( \alpha = 1.85 \) cm\(^{-1}\), the 51-element CROW exhibits a sensitivity of 31898 RIU\(^{-1}\) with \( \kappa = 0.1 \).

Figure 6(d) shows the modeled sensitivity \( S_{\text{interstate}} \) as a function of loss coefficient \( \alpha \) spanning from 1 cm\(^{-1}\) to 10 cm\(^{-1}\). The sensitivity is independent of the loss coefficient. This is because the inter-state sensing scheme only relies on identifying the spatial frequency peak harmonics without concerning the absolute Fourier frequency peak amplitudes that are determined by loss. However, as modeled and discussed in Fig. 5(b), the increment of loss...
gradually distorts and weakens the spatial patterns, thus eventually compromises the spatial domain sensing.

By increasing $N$ and $L$, we can in principle reduce $\delta \lambda$ and thus enhance $S_{\text{interstate}}$. However, in practice, disorders and inhomogeneity due to fabrication imperfections are inevitable. According to some best numbers in the literature [18,19], the number of coherently coupled microcavities is practically limited to the order of 100 using relatively large-sized microcavities (e.g. $L$ is in the order of tens to a hundred of microns).

3.6 Intra-state sensing scheme

The CROW sensor is not limited to the inter-state sensing scheme. Upon a transition within two adjacent states, the corresponding Fourier amplitudes vary. By measuring the Fourier amplitude change ($\Delta I$) relative to the input intensity ($I_0$), one can identify the relative RI change using this intra-state sensing scheme. We define the intra-state sensing scheme sensitivity as:

$$S_{\text{intra state}} = \frac{\Delta I}{I_0 \Delta n_{\text{eff}}}$$  \hspace{1cm} (16)

Figure 7(a) shows the spectra of the broadband transmission of the 11-element CROW example modeled in section 3.2. The zoom-in picture shows the modeled intra-state positions (labeled as A to E) between state 4 (A) and state 5 (E).

Figure 7(b) shows the pixelized field intensity distributions extracted from equally spaced positions between two adjacent states 4 and 5 as labeled in Fig. 7(a). The fixed probe wavelength is initially aligned with state 4. By increasing the effective RI the probe state transits from state 4 to state 5.

Figure 7(c) shows the Fourier transform of the pixelized field intensity distributions. According to the intra-state transition from state 4 to state 5, the Fourier amplitude of the $4^\text{th}$ harmonic drops while that of the $5^\text{th}$ harmonic rises.

Figure 8(a) shows the Fourier amplitudes variation with the effective RI increases, assuming the fixed probe wavelength is initially aligned to state 4. The slope of the red (blue) curve corresponding to the $4^\text{th}$-harmonic ($5^\text{th}$-harmonic) Fourier amplitude is linear and steepest near state 4 (state 5). The shaded regions depict the linear sensing windows, which span a sensing dynamic range of around $6 \times 10^{-5}$ RIU. The dashed lines show the linear fits of the Fourier amplitude variation curves in the linear regions. The sensitivities of the red and blue curves are $1507.4 \text{ RIU}^{-1}$ and $1373.7 \text{ RIU}^{-1}$.
Fig. 8. Intra-state sensing scheme sensitivity analysis. (a) Modeled normalized Fourier frequency amplitude as a function of effective RI increment. The red and blue curves illustrate the 4th and 5th harmonics of spatial Fourier amplitude normalized to the input $I_0$. (b) Intra-state sensing scheme sensitivity of CROW devices with various cavity numbers and coupling coefficients.

Figure 8(b) shows the modeled intra-state sensing scheme sensitivity $S_{\text{intrastate}}$ as a function of cavity number with various coupling coefficients. In contrast to the inter-state sensing scheme, there is an optimal cavity number for maximum sensitivity with a given coupling coefficient. For instance, with the coupling coefficient of 0.2, the 21-element CROW shows the maximal sensitivity of 4348.7 RIU$^{-1}$. With the coupling coefficient of 0.3, the 41-element CROW shows the maximal sensitivity of 4561.7 RIU$^{-1}$. With the coupling coefficient of 0.4, the sensitivity is maximized at around 4100 RIU$^{-1}$ with 41-element and 51-element CROWs. With the coupling coefficient of 0.6, the maximal sensitivity drops to about 1800 RIU$^{-1}$ with a 51-element CROW.

4. FDTD simulations

We investigate the spatial intensity patterns of a microring CROW with TE-polarized light (electric field in plane) using two-dimensional FDTD simulations. All the simulation parameters are chosen for SiN ($n = 1.9$) based devices with water ($n = 1.33$) upper-cladding and silica ($n = 1.44$) lower-cladding. Limited by the computation intensive simulations, here the mode-field intensity distributions are extracted at different resonance wavelengths at a fixed effective RI.

Figure 9(a) illustrates the structure of a cascaded microring CROW device. The cascaded microring array has 11 identical cavities with 10 μm diameters, 0.5μm waveguide width, 0.3μm gap widths for the waveguide input- and output-coupling and 0.2μm gap widths for the inter-cavity coupling. The waveguide-to-microring coupling coefficient is 0.57 and the microring-to-microring coupling coefficient is 0.6, as extracted from a single-microring-resonator FDTD simulation.

Figure 9(b) shows the simulated drop-port transmission spectrum exhibiting periodic broadband transmission with eleven intra-band resonance peaks, which is consistent with the cavity number. The state spacing is 2.5 nm.

Figure 9(c) shows the cavity electric field distributions at the intra-band states. The states that are symmetric about the band-center state (e.g. states 4 and 8) have almost identical amplitudes and intensity patterns but with $\pi$ phase difference. For example, the zoom-in pictures show the electric field distributions at the same coupling region at states 4 and 8. Two adjacent microrings display symmetric (S) in-phase mode-field patterns for state 4. While for state 8, two adjacent microrings display anti-symmetric (AS) $\pi$-out-of-phase mode-field patterns.

Figure 10(a) shows the spatial mode intensity patterns of the 11-element microring CROW at the 11 intra-band resonance wavelengths. We integrate the intensity of each
microresonator as a pixel. Figure 10(b) shows the pixelized intensity distributions. Figure 10(c) shows the harmonic shifts as a function of inter-state transition. The spatial frequency unit is given as $1/2NR$, where $2R = 10 \, \mu m$. The inter-state sensing scheme sensitivity extracted from the simulation is $407.5 \, \text{RIU}^{-1}$. This is consistent with the sensitivity $421.8 \, \text{RIU}^{-1}$ in a modeled 11-element FP CROW with the same cavity size ($L = 15.6 \, \mu m$) and coupling coefficients ($\kappa_{in} = 0.57$ and $\kappa = 0.6$).

Figure 11(a) shows the simulated spectra of the broadband transmission of the 11-element CROW. The zoom-in picture shows the intra-state positions (labeled as A to E) between state 4 (A) and state 5 (E).

Figure 11(b) shows the pixelized intensity distributions. The intensity values at positions B, C and D are artificially amplified by 5 times in order to clearly show the patterns. Figure 11(c) shows that spatial Fourier amplitude of the 4th harmonic drops gradually while the Fourier amplitude of the 5th harmonic rises from states A to E. The intra-state sensing scheme sensitivity extracted from the FDTD simulation is $29.7 \, \text{RIU}^{-1}$.

5. Discussion

Here, we compare the sensing performances of CROW sensors using spatial domain detection with single microresonator sensors using spectral domain detection. The sensitivity ($S_{\text{spectral}}$)
and detection limit ($D_{\text{spectral}}$) of single-element microresonator-based RI sensors in the spectral domain are described as [5]

\[ S_{\text{spectral}} = \frac{\Delta \lambda}{\Delta n_{\text{eff}}} \]  

(17)

\[ D_{\text{spectral}} = \frac{LW}{S_{\text{spectral}}} = \frac{LW \Delta n_{\text{eff}}}{\Delta \lambda} \]  

(18)

where $LW$ is the 3-dB linewidth of the resonance.

According to Eqs. (12) and (18), CROW-based sensors upon the inter-state sensing scheme have the same detection limit as single microresonator-based sensors under the condition that the state spacing $\delta \lambda$ equals to the resonance linewidth $LW$. For example, we target sensors with an effective RI detection limit of $10^{-4}$ RIU. For single microresonator-based sensors, the $LW$ should be as narrow as 0.1 nm, which requires a Q-factor of $1.5 \times 10^4$ in the 1550nm wavelength range. For CROW-based sensors, the inter-state also should be as
narrow as 0.1 nm, which can be obtained by choosing design parameters such as \( N = 21, L = 50 \ \mu m \) and \( \kappa = 0.2 \).

In practical applications, fabrication imperfections which alter the ring resonator parameters will limit the coherence of many-element large-sized CROWs [16,26], and thus will compromise the sensitivity and detection limit of the device. We are now working on the statistical analysis of fabrication errors induced effects using the transfer matrix model, and the results will be reported elsewhere. Although thermal tuning by means of a metal heater has been adopted as a technology to mitigate the fabrication inaccuracies, we do not regard such tunable configuration as a viable approach here. This is partially because the metal contacts could affect the ability to measure the out-of-plane light scattering. Another reason is that the tunable device tends to make the CROW structure much more complicated than a passive one as each of the coupled microresonators would need to be individually tunable. For practical sensor applications, we prefer a passive CROW design to an active one.

It should be mentioned that our approach inherently requires out-of-plane light coupling which does not allow totally in-plane integration. However, we believe that by collecting the out-of-plane light scattering using integrated microlenses and imaging the patterns onto a charge-coupled device (CCD) camera the approach can enable a portable sensor.

Table 1 summarizes the comparison between the single-element microresonator-based spectral domain sensing and the CROW-based spatial domain sensing.

|                      | Spectral domain sensing | Spatial domain sensing |
|----------------------|-------------------------|------------------------|
| **Device**           | Single microresonators  | CROWs                  |
| **Sensing method**   | Spectral resonance shift/ | Spatial pattern variation/ |
| **Light source**     | Wavelength-tunable lasers | Single-wavelength lasers |
| **Key parameter**    | Linewidth or Q-factor    | Intra-band state spacing |
| **Physical parameter dependence** | \( n, \kappa \) | \( N, L, \kappa \) |

6. Conclusion

We modeled a RI sensing scheme based on CROW structures using spatial domain detection. In our FP model, the spatial mode-field pattern varies upon effective RI change-induced state transition at a fixed probe wavelength. We adopted the spatial Fourier transform to identify the patterns in the spatial frequency domain and sense the effective RI variations.

For the inter-state sensing scheme, the sensitivity and the detection limit are determined by the state spacing within the transmission band. The CROW-based sensor with an inter-state detection limit of \( 10^{-4} \) RIU can be obtained by choosing design parameters such as \( N = 21, L = 50 \ \mu m \) and \( \kappa = 0.2 \). For intra-state sensing scheme, the spatial Fourier amplitude is the superposition of the harmonics of the adjacent two states. From the analysis of the spatial Fourier amplitude we can extract the effective RI change under the inter-state detection limit. Our FDTD simulations of an 11-element microring CROW demonstrated both the inter- and intra-state sensing schemes. Given the sensing dynamic range of a CROW-based sensor is relative wide (several nm), therefore the spatial domain sensing could be adopted using a single-wavelength light source.

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