How much security does Y-00 protocol provide us?

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New quantum cryptography, often called Y-00 protocol, has much higher performance than the conventional quantum cryptographies. It seems that the conventional quantum cryptographic attacks are inefficient at Y-00 protocol as its security is based on the different grounds from that of the conventional ones. We have, then, tried to cryptoanalyze Y-00 protocol in the view of cryptographic communication system. As a result, it turns out that the security of Y-00 protocol is equivalent to that of classical stream cipher.

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Quantum cryptography has appeared as a promising way to achieve security without depending on any computational complexity assumption. However, most of the proposed schemes up to date are based on single photon states (QCSPS) thus, presenting a well-known negative characteristic, namely, its bit rate is much slower than that of normal optical communication systems. Recently a quantum cryptography scheme which uses mesoscopic coherent states has been reported [1]. This scheme has much higher performance than conventional ones which are based on single photon states[2, 3, 4]. Because this scheme based on mesoscopic coherent states, often called “Y-00 protocol[5]” has an average photon number of 100-1,000 photons per pulse, its bit rate is expected to be 100-1,000 times faster than that of QCSPS. In addition, the required technical level to realize the protocol is supposed to be quite the same as in conventional optical systems. Y-00 protocol would be, then, a sufficiently fast and easily realizable quantum cryptography scheme, if it had actually perfect security.

In this paper, we show that the Y-00 protocol does not provide perfect security, even against the simplest of cryptographic attacks, ciphertext-only ones. A usual cryptographic system consists of two channels (Fig.1): an open channel for exchanging encrypted messages and a secure channel for key distribution. Quantum cryptography based protocols, including Y-00 protocol, provide a realization of the secure channel for key distribution. Let us underline that our attack targets are not only the secure channel, but also the open channel for messages.

We also show that the security of Y-00 protocol is just equivalent to that of a classical stream cipher. In other words, we can safely say that Y-00 protocol has no perfect security and that its security depends on just a common computational complexity assumption, being then no better than currently used schemes.
where \( k \in \{0, \ldots, M-1\} \) and the pairs are orthogonal to each other. A specific pair, then, determines a specific polarization base.

At first Alice and Bob generate a pseudo-random number stream \( k_i \in \{0, \ldots, M-1\} \) from the secret key \( K_s \) in synchronized manner:

\[
PRNG : K_s \rightarrow k_i, \quad i \in N,
\]

where \( PRNG \) is a pseudo-random number generator and the number \( k_i \) determines the polarization base. Alice generates a number \( r_i \in \{0,1\} \) with a physical random number generator (PhRNG) and modulates the \( r_i \) into a qumode in the \( k_i \)-base. Bob observes the qumode sent from Alice with the \( k_i \)-base and de-modulates \( r_i \) from the qumode. The mechanism is called “M-ary level ciphering.”

Y-00 protocol has another important mechanism called “Ciphering Wheel.” Ciphering Wheel is a rule for assigning opposite bit values. Thus one must distinguish closest qumodes in the neighboring bases are generally assigned opposite bit values. Thus one must distinguish a correct qumode to get a correct bit. However, because of the fundamental quantum fluctuation in any measurement by eavesdroppers, the discrimination of the qumode is impossible for eavesdroppers when \( M \) is sufficiently large, unless they know the information of \( k_i \).

These circumstances can be easily understood with the so-called Poincaré representation. The qumode is represented by a point on the Poincaré sphere and all qumodes are located on the equator including the \( x \)-axis and \( y \)-axis defined by the Stokes operators in Fig. 2:

\[
S_z = \frac{1}{2}(a^\dagger a - b^\dagger b), \quad S_y = \frac{1}{2}(a^\dagger b + b^\dagger a),
\]

where \( a \) is an annihilation operator for the horizontal mode and \( b \) is an annihilation operator for the vertical mode. The qumode is, then, represented by the point

\[
(S_z, S_y) = \frac{1}{2} |\alpha|^2 (\cos \theta_k, \sin \theta_k),
\]

and it has the following isotropic quantum fluctuation

\[
\Delta S_z = \Delta S_y = \frac{1}{2} |\alpha|.
\]

\( N_s \)-other qumodes are included in the fluctuation where \( N_s = M/(\pi |\alpha|) \) and no one can distinguish the correct qumode from the others. Therefore nobody except Alice and Bob draw out the correct bit whereas Bob’s decision has to be made only between two nearly orthogonal state in the same basis defined by a given \( k_i \).

From the above argument, one can see that Alice and Bob can surely safely share the new random bits \( \{r_i\} \) whose length is longer than the original shared random common key \( K_s \). Alice and Bob seem to be able to realize QKE by using the Y-00 protocol. However, a careful consideration tells us that this protocol doesn’t help Alice and Bob to expand their key in a perfect secure way.

Let us imagine that Eve classifies the bases into two classes: The one class \( C_+ \) consists of bases which has the same bit-assignment as the \( k_i \) = 0 base and the other class \( C_- \) consists of the rest of the bases. She can, then, define a mono-bit mapping \( CW(\cdot) \) from the base \( k_i \):

\[
CW : k_i \rightarrow \tilde{k}_i = \begin{cases} 0 & \text{if } k_i \in C_+ \\ 1 & \text{if } k_i \in C_- \end{cases}.
\]

Since Eve seizes the qumode and its base vaguely, she, then, selects an appropriate base which belongs to the class \( C_+ \) from the candidates and gets a bit \( l_i \) from observation under the selected base. Since the candidates are not always the same as the true bases, the bit \( l_i \) cannot always be equal to \( r_i \). However one will find that the following important relation holds:

\[
l_i = r_i \oplus \tilde{k}_i.
\]

Notice that the quantum error due to the miss choice of base is absorbed in the second term of the right side. Moreover, since the inner product of the true qumode \( |\Psi(\theta_k)\rangle \) and the orthogonal qumode on the incorrect base \( k + \Delta k \) is given by

\[
|\langle \Psi(\theta_k) | \Psi(\theta_{M+k+\Delta k}) \rangle |^2 = e^{-2|\alpha|^4 (1+\sin(\pi \Delta k /2M))},
\]

(the product is extremely small if the qumode is a mesoscopic state where the order of \( \Delta k \) is at most \( N_s \), the error derived from M-ary ciphering mechanism is negligible.

In addition, Eve can make her measurement undetectable by Alice and Bob, by resending a similar quantum state to Bob who is interested in only the discrimination of the possible two states. In principle, Bob can

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**FIG. 2: qumode in Poincaré sphere at \( M = 16 \)**

\[ S_z, S_y \]
check if the state is really one of two possible state, if the quantum channel can be assumed free from noise. However, it is much harder to check it than only to distinguish from each other. Moreover, it requires much higher technology than what is assumed in the original Y-00 protocol. Also, the assumption that the quantum channel is free from noise is a quite unrealistic one.

Again, we would like to stress that the detection of Eve’s measurement is impossible for Alice and Bob under the relevant assumptions assumed in the Y-00 protocol, paying serious attention to the philosophy of the original proposal, that is, an easily realizable protocol with conventional optical technologies. Also, note that signal amplification, which is considered as one of the mains advantages of the protocol, is impossible under the assumption that Eve’s activities can be detected.

There is a small technical problem that the classification is not globally well-defined because the base space is topologically homomorphic to the Möbius ring. The vertical polarization mode on the $k$-base space has cyclic structure of a module $M$ vertical polarization mode on the horizontal one on the $k$-base, though the base space has cyclic structure of a module $M$. However one can solve this problem, by introducing the following “local” classification in the neighborhood of a given $k$-base in Fig. 3.

1. Fix the most far base $k_{cut} \equiv k + \lfloor M/2 \rfloor \mod M$ as the cut base if a certain $k$ is given.

2. Classify each bases from $k = 0$ to $k_{cut}$ in both additive and subtractive directions.

The local classification is well-defined in the neighborhood of $k$ and unique on any $k$. It is no problem that the classification is ill-defined because the qumodes on the $k$-base and $k_{cut}$ are orthogonal to each other, i.e.

$$|\langle \Psi(k) | \Psi(k_{cut}) \rangle|^2 \simeq e^{-2(2-\sqrt{2})|\alpha|^2} \to 0. \quad (12)$$

Let us recall the important fact that, in the Y-00 protocol, Eve can get $|\psi\rangle$ without being detected by Alice and Bob. First of all, one should notice that this situation can be simulated in a classical way.

Let us investigate the Y-00 protocol as a cryptographic communication system where one-time pad is used as encryption algorithm for messages in Fig. 4.

Eve gets two clues to cryptanalysis: One clue is a ciphertext $c_i$ of the message:

$$c_i = p_i \oplus r_i, \quad (13)$$

where $p_i$ is a plaintext of the message which is legible to anybody and has often language characteristics. The other clue is an inaccurate qumode that Eve observes in the quantum channel.

As Eve knows the two bit streams $|\psi_1\rangle$ and $|\psi_0\rangle$ in this way, she gets another bit stream

$$c_i \oplus l_i = p_i \oplus \tilde{k}_i. \quad (14)$$

The bit stream is nothing but a classical stream cipher where $c_i \oplus l_i$ is a ciphertext, $\tilde{k}_i$ is a key stream, and its generator algorithm is $CW \circ PRNG(\cdot)$. Therefore Y-00 protocol has no perfect security and its security is based on computational complexity.

The above discussion is independent of the encryption algorithm. In the case of a block cipher algorithm instead of one-time pad, we may observe $|\psi_0\rangle$ in a block whose size is $N$

$$L_J = R_J \oplus \tilde{K}_J, \quad (15)$$

where $\oplus$ is bitwise XOR and $L_J$, $R_J$, and $\tilde{K}_J$ are concatenations of $l_i$s, $r_i$s, and $\tilde{k}_i$s, for example,

$$L_J = l_{(J-1)N+1} || l_{(J-1)N+2} || \cdots || l_{JN}. \quad (16)$$

The ciphertext is given by

$$C_J = E_{R_J}(P_J), \quad \text{and} \quad P_J = D_{R_J}(C_J), \quad (17)$$

where $P_J$ and $C_J$ are block sequences instead of each bit streams and $E(\cdot)$, $D(\cdot)$ are encryption and decryption algorithm. We, then, get the following relation

$$P_J = D_{L_J \oplus \tilde{K}_J}(C_J), \quad (18)$$

FIG. 3: Local Classification on bases at $M = 16$

FIG. 4: Y-00 communication system
instead of the simple relation \[14\]. Brute-force attack on the freedom of \(K_s\) is applicable to \([12]\) in the worst case that we have no clue to cryptoanalyze the encryption algorithm. Therefore its security is never beyond that of a scheme based on computational complexity.

This attack is not applicable to QCSPS like BB84 because it has the feature of eavesdropping detection. QCSPS with non-separable carrier abandons the communication itself as soon as eavesdropping is detected.

It is interesting to analyze our results in comparison to the ones related to secret key agreement over classical noisy channels. It is known that certain noisy correlated data can provide finite secrecy capacity and perfect secure key agreement between Alice and Bob even when the correlation between Alice and Eve is less noisy than the correlation between Alice and Bob (if an authenticated noiseless public channel is provided) \[8\], \[9\], \[10\]. Y-00 protocol would be, then, expected as a practical and efficient implementation of this noisy correlation using quantum noise. The result \[10\], however, tells us that the correlation implemented by Y-00 protocol is not a stochastic one but one which comes from a deterministic bit-flip channel owing to the local classification and the appropriate base-selection. Therefore, Y-00 protocol does not provide “genuine” noisy correlations between Alice, Bob and Eve and, hence, does not satisfy the conditions specified in \[3\], \[4\], \[5\], \[6\].

We assume that qumodes are mesoscopic, \(|\alpha|^2 \gg 1\) in the above discussion. If \(|\alpha|^2\) is sufficient small, quantum effects are expected to be no longer negligible. In our attack, if the two qumodes on the most far base \(k_{cut}\) are not efficiently orthogonal to the true qumode: \(|\alpha|^2 < 1 + 1/\sqrt{2}\) from \[12\], the clue \[10\] is not correct and the eavesdropping channel becomes a stochastic noisy channel. Y-00 protocol would, then, recover perfect security if it used microscopic coherent states instead of mesoscopic ones, but its extremely high performance would be lost.

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