On the dynamo cycle variations with rotational period

V. Pipin

Institute of Solar-Terrestrial Physics, Russian Academy of Sciences, Irkutsk, 664033, Russia

13 August 2020

ABSTRACT
The paper reports results of calculations of the magnetic cycle parameters, like the dynamo cycle period, the amplitude of the total magnetic energy, and the Poynting flux luminosity from the surface for the dynamo models of solar analogs with rotation periods of range from 1 to 30 days. The computations were done using the non-linear mean-field dynamo models. We do simulations both for the kinematic and non-kinematic dynamo models. The kinematic dynamo models, which take into account the non-linear $\alpha$-effect and the loss of the magnetic flux due to magnetic buoyancy, show a decrease of the magnetic cycle with the decrease of the stellar rotation period. The stars with the rotational period less than 10 days show the non-stationary long-term variations of the magnetic activity. The non-kinematic dynamo models take into account the magnetic field feedback on the large-scale flow and heat transport inside the convection zone. They show the non-monotonic variation of the dynamo period with the rotation rate. For the range of periods from 15 to 30 days, the stars with sub-equipartition dynamo regimes show a similarity to the kinematic dynamo models. A decrease of the rotation period from 15 to 10 days results in doubling of the dynamo frequency relative to the frequency of the kinematic dynamo waves. The models for the rotational periods less than 10 days show the non-stationary evolution with a slight increase in the primary dynamo period with the increase of the rotation rate. The non-kinematic models show that the growth of the dynamo generated magnetic flux with the increase of the rotation rate saturates for the star rotating with period two days and less. The saturation of the magnetic activity parameters is accompanied by depression of the differential rotation.

Key words: Sun: magnetic fields; Sun: activity cycles; Stars: magnetic activity

1 INTRODUCTION
The partially convective stars, like the Sun, often show the cyclic magnetic activity Baliunas et al. (1995); Oláh et al. (2009); Olsert et al. (2018). The magnetic activity of the Sun and solar-type stars demonstrates the large-scale organization of the active phenomena both in time (cycles) and in space Donati & Landstreet (2009); See et al. (2016). It is widely accepted that the nature of the global magnetic activity in solar-type stars stems from the turbulent hydromagnetic dynamo operating in their convection zones. Parker (1955) suggested the basic dynamo scenario for the Sun. This scenario suggests the cyclic transformation of the large-scale poloidal magnetic field into the toroidal magnetic field and vice versa by means of the differential rotation and the cyclonic convection motions. Hence, the energy supply for the dynamo process comes from the energy of the global rotation and the turbulent energy of the convective motions. The partially-convective stars with higher rotation rates show a higher level of the magnetic activity Noyes et al. (1984); Baliunas et al. (1995); See et al. (2015). Noyes et al. (1984) estimated the magnetic cycle parameters in solar-type stars following the properties of the eigenmodes of the mean-field dynamo equations. They found that the linear theory predicts the growing level of the magnetic activity and the simultaneous decrease of the magnetic cycle period with the increase of the rotation rate of a star. Observations show that the magnitude of the surface latitudinal shear decreases inversely to the increase of the rotation rate in the solar-type stars Saar (2011). This was found in the mean-field models as well Kitatinov & Rüdiger (1999). Therefore, from the point of view of the linear theory, the dynamo efficiency of the differential rotation does not increase with the increase of the Sun’s rotation rate Kitatinov & Olemskoy (2011). On the other hand, the growing level of the magnetic activity with the increase of the rotation rate can be explained by the growing effect of the Coriolis force acting on the convective flows. In the stratified convection zone, this force results in the so-called $\alpha$-effect Krause & Rädler (1980). This may explain why the Rossby number

* email: pip@iszf.irk.ru

© 0000 The Authors
Ro = $P_{\text{rot}}/\tau_c$ is often a good parameter to trace the level of the magnetic activity and cycle’s parameters in the stars with the partial convection zone Noyes et al. (1984); Baliunas et al. (1995); Olsert et al. (2018).

Our main goal is to extend our previous study (see, Pipin (2015); Pipin & Kosovichev (2016)) for the higher rotation rates employing the non-kinematic dynamo model, which was developed recently by Pipin & Kosovichev (2019). The model takes into account the magnetic activity effect on the large-scale flow and heat transport in the convection zone. The latter effect seems to results in the so-called “extended cycle” of the solar torsional oscillations Ulrich (2001). The rotational and magnetic anisotropy of the convective heat transport is rather important for the large-scale flow organization in the stellar convection zone (see, e.g., Busse (1983); Kitchatinov & Rüdiger (1999); Simitev & Busse (2009); Käpylä et al. (2011); Gastine et al. (2012)). The recent global convection simulations of Strugarek et al. (2017); Warnecke (2018); Guerrero et al. (2019) show the variations of the large-scale flow organization with the rotation rate. Interestingly, that models of Warnecke (2018) show that variations of the large-scale flow organization are accompanied by the non-monotonic variations of the dynamo period with an increase of the rotation rate. This indeed was found by Lehtinen et al. (2016) in observations of the young solar-type stars. These works encourage us to try the non-kinematic mean-field dynamo models for the range of the rotational periods from 1 to 30 days. For the sake of simplicity, we use the reference state of the model which reproduces the one cell per hemisphere case. This type of meridional circulation structure is popular in solar dynamo studies Charbonneau (2011). Also, we do not take into account the evolutionary changes of the stellar structure accepting the solar interior model of the modern Sun. The next chapter describes some details of our dynamo model.

2 MODEL

We use the non-kinematic dynamo model developed by (Pipin & Kosovichev 2019, hereafter PK19). The model is based on the mean-field induction equation Krause & Rädler (1980):

$$\dot{B} = \nabla \times (\mathbf{E} + \mathbf{U} \times \mathbf{B}),$$

where the induction vector of the LSMF, $\mathbf{B}$, is represented as the sum of the toroidal and poloidal components:

$$\mathbf{B} = \hat{\phi} \Phi + \nabla \times \frac{A \hat{\phi}}{r \sin \theta},$$

where $r$ is the radial distance, $\theta$ is the polar angle, $\hat{\phi}$ is the unit vector in the azimuthal direction. The mean electromotive force $\mathbf{E}$ describes the turbulent generation effects, pumping, and diffusion:

$$\mathbf{E}_i = (\alpha_{ij} + \gamma_{ij}) \mathbf{B}_j - \eta_{ijk} \nabla \mathbf{B}_k,$$

where the symmetric tensor $\alpha_{ij}$ stands for the turbulent generation of the LSMF by kinetic and magnetic helicities; the antisymmetric tensor $\gamma_{ij}$ describes the turbulent pumping effect including the mean-field magnetic buoyancy Kitchatinov & Pipin (1993); the anisotropic (in the general case) tensor $\eta_{ijk}$ is the eddy diffusivity of the LSMF Pipin (2018).

We employ the $\alpha$-effect tensor, in the following form:

$$\alpha_{ij} = C_\alpha \psi_a(\beta) \alpha^{(H)}_{ij} + \alpha^{(M)}_{ij} \psi_a(\beta) \frac{\nabla \tau_c}{14 \pi^2},$$

where $\alpha^{(H)}_{ij}$ - hydrodynamic part of the $\alpha$-effect tensor; $\tau_c = \mathbf{a} \cdot \mathbf{b}$ is the magnetic helicity density (a and b are the turbulent parts of the magnetic vector potential and magnetic field vector), and tensor $\alpha^{(M)}_{ij}$ takes into account the effect of the Coriolis force. Function $\psi_a(\beta)$ stands for the “algebraic” saturation of the $\alpha$-effect caused by the small-scale Lorentz force which opposes convective motions across the field lines of the LSMF, where, $\beta = |\mathbf{B}|/\sqrt{4 \pi \rho \ell e_0}$. For strong LSMF, $\beta \gg 1$, $\psi_a(\beta) \sim \beta^{-3}$. A detailed description of $\alpha^{(H)}_{ij}$, $\alpha^{(M)}_{ij}$ and $\psi_a(\beta)$ is given by Pipin (2018). The magnetic helicity evolution follows the conservation law:

$$\frac{\partial \chi^{(tot)}}{\partial t} = - \nabla \cdot \left( \frac{\nabla \tau_c}{R_m \tau_c} - 2 \nu \epsilon_0 \mathbf{J} - \nabla \mathbf{F} - (\mathbf{U} \cdot \nabla) \chi^{(tot)} \right),$$

where

$$\chi^{(tot)} = \chi_{\text{M}} - \mathbf{B} \cdot \mathbf{A} + \mathbf{a} \cdot \mathbf{b},$$

and by equation for the azimuthal component of large-scale vorticity, $\varpi = (\nabla \times \mathbf{U}^m) \cdot \hat{\phi}$:

$$\frac{\partial \varpi}{\partial t} = r \sin \theta \mathbf{\nabla} \cdot \left( \frac{\dot{\phi} \nabla \tau_c \mathbf{T} - \mathbf{U}^m \varpi}{r \sin \theta} \right) + \mathbf{\nabla} \cdot \left( r \sin \theta \mathbf{B} \mathbf{\tau_c} \right),$$

and by the equation for the advection of $\mathbf{B}$:

$$\mathbf{B}_i = \frac{1}{4 \pi} \left( \mathbf{B} \cdot \mathbf{\nabla} \right) \varpi - \frac{1}{4 \pi} \left( \mathbf{B} \cdot \mathbf{\nabla} \right) \varpi,$$

(see detailed description in PK19). Also, $\varpi$ is the mean density, $\mathbf{B}$ is the mean magnetic field. $\eta$ is the mean entropy; $\partial / \partial \mathbf{x} = \cos \theta / \partial r - \sin \theta / r \partial / \partial \theta$ is the gradient along the axis of rotation. The mean heat transport equation determines the mean entropy variations from the reference state due to the generation and dissipation of LSMF and large-scale flows Pipin & Kitchatinov (2000):

$$\varpi \mathbf{T} \left( \frac{\partial \varpi}{\partial t} + (\mathbf{U} \cdot \nabla) \varpi \right) = - \mathbf{\nabla} \cdot \left( \mathbf{F}^e + \mathbf{F}^r \right) - \mathbf{T} \mathbf{\nabla} \cdot \left( \frac{\partial \varpi}{\partial \mathbf{x}} + (\mathbf{U} \cdot \nabla) \varpi \right),$$

where $\mathbf{T}$ is the mean temperature, $\mathbf{F}^e$ is the radiative heat
flux, $\mathbf{F}^c$ is the anisotropic convective flux. An analytical mean-field expression for $\mathbf{F}^c$ takes into account the effect of the Coriolis force, and the influence of the LSMF on the turbulent convection (see, PK19). The last two terms in Eq (9) take into account the convective energy gain and sink caused by the generation and dissipation of LSMF and large-scale flows. The kinetic coefficients in the mean-field analytical expressions of the mean electromotive force, $\mathcal{E}$, and the turbulent stress tensor $\mathbf{T}$, depend on profiles of the turbulent parameters of the convection zone, such as the typical convective turnover time, $\tau_c$, and the convective RMS velocity, $u_c$. The reference profiles of mean thermodynamic parameters, such as entropy, density, and temperature are determined from the stellar interior model MESA Paxton et al. (2011, 2013). The convective RMS velocity is determined from the mixing-length approximation,

$$u_c = \frac{\ell_c}{2} \left( \frac{g}{2\zeta_p} \right)^{1/2},$$

where $\ell_c = \alpha_{MLT} H_p$ is the mixing length, $\alpha_{MLT} = 1.9$ is the mixing length parameter, and $H_p$ is the pressure height scale. We determine the convective turnover time $\tau_c = \ell_c/u_c$ from the parameters of the MESA code output. We assume that $\tau_c$ does not depend on the magnetic field and global flows. Eq. (10) determines the reference profiles for the eddy heat conductivity, $\chi_T$, eddy viscosity, $\nu_T$, and eddy diffusivity, $\eta_T$, as follows,

$$\chi_T = \frac{i^2}{6} \left( \frac{g}{2\zeta_p} \right)^{1/2},$$
$$\nu_T = \frac{\ell_T}{2\nu_T},$$
$$\eta_T = \frac{\ell_T}{2\eta_T}.$$  

The model gives the best agreement of the angular velocity profile with helioseismology results for $\Omega_T = 3/4$ (PK19). Also, the dynamo model reproduces the solar magnetic cycle period, $\sim 22$ years, if $P_{\text{rot}} = 10$. For the solar case we use the period of rotation of solar tachocline determined from helioseismology, $\Omega_0/2\pi = 430$ nHz Kosovichev et al. (1997), which corresponds to the sideral period of about $P_{\text{rot}} = 25$ days.

### 2.1 Boundary conditions and tachocline

The position of the top boundary is $r_{\text{top}} = 0.99 R_\odot$, the bottom of the convection zone is fixed to $r_b = 0.728 R_\odot$, and the bottom of the tachocline is $r_{\text{ta}} = 0.68 R_\odot$. At the $r = r_{\text{ta}}$ we put the solid body rotation and the perfect conductor boundary conditions. We do not solve the heat transport equation for the tachocline region. Instead, we assume that within tachocline all turbulent coefficients (except the eddy viscosity and eddy diffusivity) decrease factor of $\exp(-100z/R)$, where $z$ is the distance from the bottom of the convection zone. We restrict the decrease of the eddy viscosity and eddy diffusivity by one order of magnitude for the numerical stability. At the top, $r = r_{\text{top}}$ we employ the stress-free boundary condition for the angular momentum problem. For the heat transport at the bottom of the convection zone, $r_b = 0.728 R_\odot$, we put the total flux $F_{\text{conv}} + F_{\text{rad}} = \frac{L_s (r_B)}{4\pi R_B^2}$, and for the external boundary, in following to Kitchatinov & Olemskoy (2011) we use

$$F_r = \frac{L_s}{4\pi r_{\text{top}}^2 \left( 1 + \left( \frac{r}{r_B} \right)^4 \right).$$

The relative variations of the radiation flux are

$$\frac{\delta F_r}{F_r} = \frac{1}{\left( 1 + \left( \frac{r}{r_B} \right)^4 \right).$$

Following ideas of Moss & Brandenburg (1992) and Pipin & Kosovichev (2011), we formulate the top boundary condition in the form that allows penetration of the toroidal magnetic field to the surface:

$$\frac{\delta B_{\phi}}{B_{\text{top}}} = \left( 1 + \left( \frac{r}{r_B} \right)^4 \right),$$

where $r_{\text{top}} = 0.99 R_\odot$, and parameter $\delta = 0.99$ and $B_{\text{top}} = 50 G$. The magnetic field potential outside the domain is

$$\frac{A(r, \mu)}{\ell_0} (r, \mu) = \sum_{\mu} a_n \left( \frac{r}{r_B} \right)^n \sqrt{1 - \mu^2} P_n^1 (\mu),$$

where $\mu = \cos \theta$. The boundary conditions Eq(16) provide the Poynting flux luminosity of the magnetic energy out of the convection zone:

$$L_P = -\frac{1}{2} \int_{-1}^1 \frac{\delta G_{\phi} \phi}{B_{\text{top}}} d\mu$$

$$= \frac{1}{2} \left( \frac{\delta}{1 - \delta} \right) \int_{-1}^1 B^2 \left( 1 + \left( \frac{r}{r_B} \right)^4 \right) d\mu.$$  

Also, we will consider the following integral parameters of the models, the total toroidal magnetic flux in the convection zone:

$$F_T = 2\pi \int_{-1}^1 \int_{-1}^{r_{\text{ta}}} \left| B_{\phi} \right| \sin \theta r^2 dr d\mu,$$

the total toroidal magnetic flux in subsurface layer

$$F_S = 2\pi \int_{-1}^1 \int_{r_{\text{ta}}}^{r_{\text{top}}} \left| B_{\phi} \right| \sin \theta r^2 dr d\mu,$$

where $r_s = 0.89 R_\odot$.

We define the parameters characterizing the energy of the symmetric and antisymmetric parts of the subsurface toroidal magnetic field:

$$\mathcal{E}_B^S = \frac{1}{2} \int_{-1}^1 \left[ B_{\phi} (\mu, t) + B_{\phi} (-\mu, t) \right]^2 d\mu,$$

$$\mathcal{E}_B^N = \frac{1}{2} \int_{-1}^1 \left[ B_{\phi} (\mu, t) - B_{\phi} (-\mu, t) \right]^2 d\mu.$$  

Then, the parity index, or the reflection symmetry index for this component of the magnetic activity is

$$P_B = \frac{\mathcal{E}_B^S - \mathcal{E}_B^N}{\mathcal{E}_B^S + \mathcal{E}_B^N}.$$  

### 2.2 Turbulence parameters and reference models of the large-scale flow

In this paper, we use the same reference convection zone model for all solar analogs rotating with periods from 1 to 30 days. Figure 1a) shows the radial profiles of the Rossby
number for each model. Following Castro et al. (2014) we define the stellar Rossby number, $R_\text{O}^\ast = \frac{P_{\text{rot}}}{\tau_\alpha}$, using value of $\tau_\alpha$ at the distance of one pressure height scale above the bottom of the convection zone. We find that for the Sun rotating with period of 25 days $R_\text{O}^\ast \approx 1.4$ which agrees roughly with the above-cited paper. Figures 1 b) and c) show the radial profiles of the eddy viscosity, eddy diffusivity, and the kinetic $\alpha$ -effect tensor for the models of the star rotating with the period of 25 and 2 days. Note, that we use the same $\alpha$-effect parameter, $C_\alpha = 0.04$ as in the paper PK19. We see that effect of rotation results in quenching of the eddy viscosity and the eddy diffusivity coefficients in the main part of the convection zone Kitchatinov et al. (1994). This quenching is, to some extend, compensated by the effect of rotation on the mean entropy profiles. Solution of the heat transport equation for the rotating star shows that the higher rotation rate, the stepper radial profile of the mean entropy. In following the mixing length theory this results to increase of the convective RMS velocity (see, the Eq10). Therefore we find that the strength of the equipartition magnetic field in the model M2 is about factor 2 higher than in the model M25. Note, the stepper radial profile of the mean entropy in case of the star rotating with period 2d results into the higher amplitude of the $\alpha$ -effect in comparison with the solar case.

The angular velocity profiles for the kinematic versions of the model are illustrated in Figure 2. The model of the differential rotation for the star rotating with a period of 25 days is the same as reported in PK19. The angular velocity profile in this model agrees well with helioseismology data. A more detailed discussion about mechanisms generating the large-scale flow can be found in Pipin & Kosovichev (2018) (also see, Kitchatinov & Rüdiger (1999, 2005)). For the star rotating with a period of 1 day, the angular velocity profile gets close to the cylinder. This is in agreement with the results of Kitchatinov & Olemskoy (2011). Note, that the differential rotation is concentrated on the equator. In the model M2, the meridional circulation in the depth of the convection zone is suppressed. The poleward circulation is concentrated on the surface. For the star rotating with the period more than 5 days the rotational profile is close to the modern Sun, except the magnitude of the differential rotation is different. Later we will discuss relationships between the magnitudes of the large-scale flows and the rotational and magnetic parameters of the star.

3 RESULTS

3.1 Kinematic models

As the first step, we discuss the results for the kinematic models with the nonlinear $\alpha$ and magnetic buoyancy effects. For the given parameters of the $\alpha$ -effect and the angular velocity profile the star with the rotation period of 30 days is slightly above the large-scale dynamo instability threshold. The Table1 lists the integral parameters for the kinematic dynamo models. Our results are in general agreement with the results of the previous paper Pipin (2015). The models show a decrease in the dynamo period with an increase in the rotation rate. Figure 3 shows the time-latitude diagrams for the large-scale toroidal magnetic field evolution at $r=0.9R$ in the kinematic models. The models with the period of rotation longer than 10 days show the solar-like butterfly diagrams. The star rotating with a period of 10 days shows rather weak migrating dynamo waves of the quadrupole parity. In this model, we see the first signs of different directions of the dynamo waves propagation at the high and low latitudes. The high latitudes show the equatorward propagation of the dynamo waves and the poleward propagation is at the solar equator. This property becomes very clear in the case of the M8, M5, M2, and M1 models. All those models show a mix of the magnetic parity modes and different systems of the dynamo waves at the high and low latitudes. The complicated dynamo wave patterns in these models result in identification of the multiple dynamo periods, see Table1. We deduce the dynamo periods using the time-latitude diagrams of the toroidal magnetic field in subsurface layer $r=0.9R$, the variations of the toroidal magnetic field flux, $F_S$, and the Poynting flux luminosity, $L_P$. To identify the dynamo periods we employ the standard wavelet package of the SCIPY distribution (www.scipy.org). All those parameters indicate the unique dynamo period for the models in the range of the rotational periods from 15 to 30 days. The models for the rotational period of less than 15 start to show the long-term variations. The different dynamo parameters show the different sets of periodic variations. For the long period, we choose the minimal value which is found both in the $F_S$ and $L_P$ and in the time-latitude diagrams. Also, the models for the fast rotating star, M1, M2 and M5, show the short-term periodicity of the $L_P$ parameter. Its period is about twice less of the main dynamo period.

In general, the found dynamo waves patterns are in agreement with the prediction of the Parker-Yoshimura rule Yoshimura (1975). For the fast rotating stars, the range of
Figure 2. The angular velocity (top) and meridional circulation at 45° (bottom) profiles for the star rotating with period: a) 2 days; b) 5 days; c) 25 days.

Table 1. The integral parameters of the kinematic dynamo models. \( P_{\text{rot}} \) is period of stellar rotation; \( R^* \) - estimation of the Rossby number, \( B_T \) is the magnitude of the total magnetic flux in the convection zone; \( F_S \) is the magnitude of the magnetic flux in subsurface layer, \( r = 0.89 - 0.99 R \); \( B_T/B_P \) is the ratio between the strength of the toroidal and poloidal magnetic field in the model; \( \beta_{\text{max}} \) is maximum \( \beta = |B|/\sqrt{4\pi \rho u^2 c} \) and its cycle variations; \( P_{\text{cyc}} \) stands for the dynamo periods found for in the model, the bold face font mark the primary period of the near surface toroidal magnetic field dynamo waves.

| Model | \( P_{\text{rot}} \), day | \( R^* \) | \( B_T [\text{MX}]_{10^{24}} \) | \( F_S [\text{MX}]_{10^{24}} \) | \( B_T/B_P \) | \( \beta_{\text{max}} \) | \( P_{\text{cyc}} \), year |
|-------|-----------------|------|-----------------|-----------------|-----------------|-----------------|-----------------|
| M1    | 1.38            | 0.08 | 20\pm1          | 7\pm0.1         | 1000-1650       | 2.3             | 1.3/2.7/11.9/25.4 |
| M2    | 2.08            | 0.12 | 12\pm1          | 7\pm0.1         | 1000-1650       | 1.7             | 1.36/2.43/24.4   |
| M5    | 5               | 0.29 | 7\pm0.5         | 4\pm0.15        | 450-1100        | 1.05            | 6.21/10.8/22.6   |
| M8    | 7.9             | 0.29 | 3.9\pm0.3       | 1.7\pm0.2       | 270-800         | 0.63            | 8.95            |
| M10   | 10              | 0.59 | 3.3\pm0.5       | 1.4\pm0.4       | 200-500         | 0.35            | 10.3            |
| M15   | 15              | 0.88 | 2.3\pm0.2       | 0.8\pm0.2       | 240-400         | 0.36            | 11.3            |
| M20   | 20              | 1.17 | 1.6\pm0.1       | 0.6\pm0.2       | 210-400         | 0.24            | 12.9            |
| M25   | 25              | 1.46 | 1\pm0.1         | 0.35\pm0.15     | 185-350         | 0.16            | 11.3            |
| M30   | 30              | 1.98 | 0.5\pm0.02      | 0.1\pm0.02      | 100-300         | 0.03            | 12.9            |

Figure 4 shows variations of the integral parameters of the magnetic activity in the kinematic dynamo models. The decrease of the rotation period from 30 to 1-day results into an increase of the dynamo generated toroidal magnetic field flux by two orders of magnitude. We compute the mean helicity density of the large-scale magnetic field in the model and the magnetic energy radiated off the star. The results are shown in Figures 4 c) and d). We find that our estimations of the mean helicity density are in agreement with the results of observations Lund et al. (2020). We postpone their analysis for the next subsections. The magnitude of the radiated magnetic energy increase from \( 10^{-7} \) of the solar luminosity to \( 10^{-4} \). The Poynting flux provides the energy input to the stellar corona. Our values can be used as an estimation of energy source for the magnetic cycle variation of the stellar X-ray luminosity. The solar observations show that variations in the X-ray background flux are the order of \( 10^{-6} \) Winter & Balasubramaniam (2014). Therefore, the estimation of the magnetic luminosity of the model M25 is enough to explain
the solar soft X-ray luminosity variations. We find that the models M1 and M2 show the saturation for this parameter.

3.2 Non-kinematic dynamo models

Table 2 lists the integral parameters of the non-kinematic dynamo models. The non-kinematic models M25dn, M20dn, and M15dn hold the qualitative properties of the magnetic field evolution the same as their kinematic versions. The patterns of the torsional oscillations and meridional circulation variations are qualitatively similar to the results of Pipin & Kosovichev (2019). The maximum magnetic field strength in these models is below half of the equipartition value. Figure 5 shows an example of the solar-type time-latitude diagram of the near-surface toroidal magnetic field for the model M15n. The dynamo period in this model is slightly longer than for the kinematic case. The same is found for the models M20n and M25n. Variations of the magnetic activity and the large-scale flow result in variations of the surface radiation flux. The models M20n and M25n show which results are qualitatively similar to Pipin (2004) and Pipin (2018) and we do not illustrate it here. It is found that the radiation flux is suppressed during the maximum of the magnetic cycle. This is due to the magnetic quenching of the convective heat flux. The period after a maximum of the magnetic cycle shows the relative increase of the radiation flux. The model M15n shows a small magnitude of radiation flux variations. It is an order of $10^{-5}$ and it is much less than that found in the solar observations. In the model M15n, we find that duration of the relatively high radiation flux is longer than the duration of the suppressed period.

The results for the model M10n are drastically different from the kinematic case. The non-kinematic model shows only the equatorward propagating dynamo waves. Their frequency is as twice as high in comparison to the model M10. The magnitude of the radiation flux variations in the time-latitude diagrams of the model M10n is a little higher than in the model M15n. We investigated the origin of the change of the magnetic butterfly diagrams in the model M10n in some details. We made additional runs where we alternately switched off the magnetic effects on the turbulent stresses tensor, $\hat{T}$, the large-scale Lorentz force, and the magnetic effects on the mean heat transport. Surprisingly, we find that despite the difference in the large-scale flow dynamics, these runs show the evolution of the large-scale magnetic fields which is similar to that in the fully nonlinear case M10n, see Figure 5. Further, we made another run where we employ
On the dynamo cycle variations with rotational period

Figure 4. a) The total toroidal magnetic field flux generated in the star for the different rotational periods; b) the same as a) for the total magnetic energy in the subsurface layer \( r = 0.89 - 0.99R_\odot \); c) the mean helicity density of the large-scale magnetic field at the surface; d) the total magnetic energy luminosity at the surface.

Table 2. The integral parameters of the non-kinematic dynamo models. Here, \( \Delta \Omega / \Omega \) is the relative variation of the latitudinal shear; \( \pm \delta U_\phi \) - magnitude of the torsional oscillations, \( \pm \delta U_\theta \) - magnitude of the meridional flow variations; the other parameters are the same as in the Table 1.

| Model | \( \Delta \Omega / \Omega \), \( 10^{-2} \) | \( \pm \delta U_\phi \), m/s | \( \pm \delta U_\theta \), m/s | \( F_T \), [MX] \( 10^{24} \) | \( F_S \), [MX] \( 10^{24} \) | \( B_T / B_P \) | \( \beta_{\text{max}} \) | \( P_{\text{cyc}}, \text{year} \) |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| M1n   | 0.1±1.1         | 184.0           | 3.25            | 35.4±4          | 20.4±4          | 1184±219        | 1.91            | 3.6/8.4/15.1    |
| M2n   | -0.1±0.8        | 148.3           | 3.86            | 29.7            | 16±4            | 1225±371        | 1.75            | 4.3/8.5/19.1    |
| M5n   | 3.6±1.1         | 54.2            | 5.92            | 7.8±1.1         | 3.9±0.8         | 245±294         | 1.01            | 2.7/4.6/19.1    |
| M8n   | 7.6±1.3         | 26.9            | 4.7             | 4.2±0.5         | 1.7±0.4         | 234±82          | 0.63            | 4.7/8.9         |
| M10n  | 9.±0.8          | 12.1            | 4.1             | 3±0.01          | 0.9±0.001       | 806±11          | 0.41            | 2.72            |
| M15n  | 15.5±0.52       | 7.5             | 2.3             | 1.95±0.2        | 0.8±0.15        | 328±42          | 0.35            | 9.5             |
| M20n  | 21.2±0.22       | 5.4             | 1.2             | 1.5±0.1         | 0.6±0.1         | 321±67          | 0.28            | 10.5            |
| M25n  | 25.7±0.20       | 3.3             | 0.6             | 1.4±0.1         | 0.37±0.15       | 301±75          | 0.21            | 11.7            |
| M30n  | 31.±0.05        | 0.3             | 0.1             | 0.5±0.01        | 0.1±0.01        | 238±90          | 0.09            | 12.8            |

The angular and meridional velocity profiles from the non-kinematic runs and found that the corresponding kinematic model shows the short periodic dynamo wave patterns similar to that shown in Figure 5. It seems that our model for the star rotating with a period of 10 days shows a marginal state of the dynamo solution, which can be easily perturbed by a variation of the large-scale flow distribution.

Model M7n shows similar changes as the model M10n. This model shows the non-stationary variations of the dynamo cycle. There are long and high cycles of a period of about 9 years and short cycles of a period of about 4.5 years. The model shows an asymmetric development of the magnetic activity in the North and South hemisphere. Another interesting phenomenon is the visible dominance of the long-term increase in radiation flux in the time-latitude diagrams. In the integral balance, this increase is compensated by the
deep suppresses of the radiation flux from time to time. The periods of the radiation flux increase corresponds to the decrease of the magnetic activity level.

The equatorward propagating dynamo waves are found in models M1n, M2n, and M5n, as well. However, the dynamo period in these models is longer than in the kinematic case. Figure 6 shows results for these models. The model M5n shows the qualitatively similar evolution as the model M8n. The models M1n and M2n show the longer dynamo periods than the models M5n and M8n. These models show the long-term suppression of the radiation flux because periods of high magnetic activity dominate.

For the range of the rotational periods from 1 to 10 days, we find that the major effect of the dynamo on the large-scale flow is the multi-cell meridional circulation. The model M10n is at the boundary of nonlinear bifurcation of one meridional cell into a multi meridional cell convection zone. Figure 7 shows the typical snapshots of the magnetic field distributions, the mean large-scale flows, and the α-effect profiles in the set of the non-kinematic dynamo models. In our previous paper, we found that the magnetic effects on heat transport can produce variations of the azimuthal flow and meridional circulation. When the magnetic field strength is much less than the equipartition value, the magnetic effect on convection is not strong. In this case, variations of the large-scale flow about the reference state are relatively small. For the case of weak magnetic activity, the reference profiles of the large-scale flow are determined by the effect of the Coriolis force on convection. Effect of rotation on the convective heat transport results in the difference between the temperature at the stellar equator and pole. For qualitative analysis, it is useful to see the magnitude of the convective velocity RMS variations due to the large-scale dynamo. We compute the relative deviations of the convective velocity RMS for each model. These deviations are calculated from the mixing-length expression Eq(10). For the reference state, we use the averaged over latitude profile of the mean entropy. The average is done for each snapshot. The results are illustrated in the third row of Figure 7.

In the model M25n, the magnetic perturbations of convection flow are small. In this case, the relative deviations of the convective velocity RMS are determined by the effect of the Coriolis force. Convection in the polar regions is suppressed relative to the equator. The effect is strong near the bottom of the convection zone and it is quenched toward the surface. So the temperature inhomogeneity is rather small at the solar surface Beckers (1960); Miesch et al. (2008); Teplitzskaya et al. (2015). The models for the rotation periods from 1 to 10 days show the stronger latitudinal contrast for the convective velocity RMS variations than the model M25n. These stars have a higher rotation rate and the stronger effect of the Coriolis force than the solar case. We find that the increase of the magnetic activity results in an increase of the inhomogeneity of the convective velocity RMS variations, especially, in the near-equatorial region. In the model M10n, there is a weak meridional cell of opposite sign at the bottom of the convection zone. Besides, the main anticyclone (in the North) cell is divided for two cell. It has two center-type stationary points and one saddle-type stationary point. The same is found for the models M8n and M5n. In these models, the location of the saddle-type stationary point corresponds to the local maximum of the convective velocity RMS variations. In the models M2n and M1n, the bottom clockwise cell (in the North) become dominant. These runs show a strong depression of the differential rotation in the main part of the convection zone.

The non-kinematic models M1n, M2n, and M5n show the high magnetic activity with the mean strength of the large-scale magnetic field exceeding the equipartition value, $\beta \geq 1$. There are about 4 activity nests of the toroidal magnetic field in each hemisphere. The strong magnetic field results in a high deviation of the mean entropy distribution from the pure hydrodynamic state. From Figure 7 we find that the maxims of the convective velocity RMS variations are located in the upper part of the convection zone. The differential rotation in these models is weak. The averaged angular velocity profile shows an accelerated rotation regions in the polar caps. This seems for the first time to be found in the mean-field dynamo models (cf. Kitchatinov & Rüdiger (1999)). In the model M2n, the mean circulation structure in the main part of the convection zone is opposite to the solar case, i.e., the main circulation cell is clockwise. Near the surface, there is a weak anti-clockwise circulation cell.

The mean α-effect is, in general, positive for all models. The models M1n and M2n show inversion of the α-effect in the near-equatorial regions. This inversion is due to the magnetic helicity conservation Pipin et al. (2013). The distributions of the α-effect, the angular velocity profiles together with the meridional circulation provide the equatorward propagation of the dynamo waves in the upper part of the convection zone. Figure 8 illustrates the radial profiles of the angular velocity in the non-linear dynamo models. It is found that at high and mid-latitudes the levels of the equal angular velocity decline toward the equator.

Figure 9 shows variations of the integral parameters for the non-kinematic dynamo models. It is found that the magnitude of the magnetic activity in the models for the star’s rotation period in the range of 10 to 25 days is decreased in comparison with the kinematic case. This result is in agreement with our previous calculations Pipin (2015); Pipin & Kosovichev (2016). Surprisingly, the non-kinematic models for the stars rotating with periods of 1, 2, and 5 days show the higher activity level than their kinematic analogs. This is due to a considerable reorganization of the large-scale flow in these models. Moreover the models M1dn and M2dn have a week anti-solar differential rotation in the depth part of the convection zone. The kinematic theory predicts the anti-solar differential rotation for the slow rotating stars, which have a high Rossby number Kitchatinov & Rüdiger (1995); Käpylä et al. (2011); Guerrero et al. (2013); Gastine et al. (2014); Brandenburg (2018); Rüdiger et al. (2019). On the other hand, Kitchatinov & Rüdiger (2004) showed that anti-solar differential rotation can be generated by means of the magnetically induced anisotropy of the heat-transport inside the convection zone. Variations of the magnetic activity parameters in the models for the rotational periods 8 and fewer days are non-stationary. We find that in these models the magnetic cycle is well seen in the butterfly diagrams of the toroidal magnetic field and in the integral parameters of the magnetic activity including the Poynting flux luminosity and the radiation flux variations. In variations of the total magnetic flux, the short cycles can disappear from time to time. Besides, this situation happens in the time series of the unsigned magnetic flux in the subsurface
On the dynamo cycle variations with rotational period

The dynamo cycle variations with rotational period can be observed through the time-latitude diagrams. Figure 5 shows the time-latitude diagrams for the near surface ($r=0.9R$) toroidal magnetic fields for the non-kinematic dynamo models M15n, M10n, and M8n. The bottom row shows the same for the relative variation of the radiation flux $\delta F/F_\odot$, see Eq(15).

Figure 6 shows the same as Figure 5 for models M5n, M2n, and M1n.

Such periods are characterized by the change of the magnetic parity. The strong parity variations can cause the variability of the primary cycle length as well. Figure 10 shows the evolution of the parity symmetry about the solar equator in the non-kinematic dynamo models. The models for the rotational periods from 15 to 30 days show $P_\delta \approx -1$, which means that the antisymmetric about the equator toroidal magnetic field dominates. The models for the range of periods from 1 to 8 days show the long-term variations of the $P_\delta$ from dipole to quadrupole type symmetry and back. The model M10n shows $P_\delta \approx 0$. We see that despite time-latitude variations of the toroidal magnetic field, this model shows no variations of the integral parameters $F_T$, $F_S$ and it shows a small magnitude variation of the
Figure 7. The first row shows snapshots of the toroidal magnetic field distribution (color image) and streamlines of the poloidal magnetic field for the non-kinematic dynamo models; the second row shows the mean over one magnetic cycle cycle the angular velocity distributions and streamlines of the meridional circulation; the third row shows the same for the convective RMS perturbation, where the reference RMS is calculated by averaging over latitudes; the bottom row show the same for the mean α-effect profiles at the latitudes 30° and 60°.
mean large-scale magnetic helicity density and the Poynting flux luminosity. The mean values of the integral parameters are slightly less than in the kinematic model M10.

### 3.3 Rotation-activity relations

The stellar rotation - magnetic activity relations are often considered as a major argument in favor of the turbulent dynamo action in convection zones of the late type stars Noyes et al. (1984); Baliunas et al. (1995). Figure 11a) shows the dependence of the magnitude of the total magnetic flux generated in the convection zone on the Rossby number. We find that the studied interval of rotation rates includes both saturation regimes. The star with the rotation period of 30 days shows a considerable drop in the generated magnetic flux in comparison with the solar case. At the opposite end, for the case of the small Rossby number we see a sign of plateau, which is characterized by an increase in the magnetic energy variability. This result is in agreement with observations of Noyes et al. (1984); Vidotto et al. (2014); See et al. (2015) From point of view of observations our simulations touch the low boundary of the dynamo saturation regime at the small Rossby number. Similarly, such a plateau is found for the magnitudes of the Poynting flux and the variations of the heat flux. The Poynting flux describes the magnetic energy input in the outer atmosphere of the star. Following Kleinerin et al. (1995); Blackman & Thomas (2015), the magnitude of the flux can serve as estimation of the X-ray radiation of the stellar corona. Wright & Drake (2016) found the saturation of the X-ray luminosity occurs at Ro<0.1. This roughly agrees with the results shown in Figure 11b). Still, our conclusions are preliminary because we have only two models that are in the saturation regime. These models show that the magnitude of the relative latitudinal shear varies with STD level σ ≈ 1 (Figure 11c)). The branch of models below the saturation regime shows relation ΔΩ/Ω ∝ Ro−1.16, which is in agreement with observations Barnes et al. (2005); Saar (2011) and expectations of the linear mean-field theory Kitchatinov & Rüdiger (1999).

Figure 11d shows relations between the magnitude of the large-scale magnetic helicity density and the Rossby number. Our results agree with the recent survey of Lund et al. (2020). Note, that a considerable set of stars from their survey are stars with mass low than the Sun and they can operate another type of the large-scale dynamo. Similarly to the above-cited paper, we looked at relations of the mean squares of the large-scale poloidal and toroidal magnetic fields on the surface with the magnitude of the mean large-scale helicity density, ⟨A·B⟩ ∝ ⟨B²⟩.16. They are shown in Figure 12. We find that our set of models is in satisfactory agreement with observations. It is found that restricting the ⟨A·B⟩ by the low ℓ ≤ 4, where ℓ is the order of the spherical harmonic decomposition, do not affect much the results for the coefficient α. The dependence of ⟨A·B⟩ on the magnitude of the toroidal magnetic field seems to show two distinct branches. The stars with the rotational periods form 10 to 30 days show the power law in agreement with the results of Lund et al. (2020). In comparison with that paper, our definition of the large-scale helicity density involve only the mean magnetic field components and vector potential. In the general case, in particular, when there is a considerable non-axisymmetric magnetic field, we can expect that ⟨A·B⟩ ≠ ⟨A·B⟩. This study should be extended further including the effects of the nonaxisymmetric magnetic fields.

We use the wavelet analysis to identify the dynamo periods. For the time series, we use both the time-latitude diagrams of the toroidal magnetic field and the radiation flux variations. We also studied the time-series of the integral parameters including the total magnetic flux, the Poynting flux luminosity, and the total irradiance variation. For the rotation periods longer than 10 days all parameters give a unique value for the dynamo period that corresponds to the period of the near-surface dynamo waves of the large-scale toroidal magnetic field. For the sample of the fast rotating stars, the time-latitude diagrams of the toroidal magnetic field give the most accurate estimation of the main dynamo period. The series of the Poynting flux luminosity and irradiance variations can give twice shorter dynamo periods, (cf., e.g. Fig5 and 6). Also, the series of the models for rotational periods 8 and fewer days are non-stationary. There we find the long periods in their dynamo patterns. We check the long periods on the all integral parameters we have.

Figure 13 compares results of a sample of F- and G-stars of Brandenburg et al. (2017), the young solar-type stars of Lehtinen et al. (2016), and different dynamo models for a relation of the dynamo cycle period on stellar rotation period and the reverse Rossby number. The latter is also called the Coriolis number. The earlier results of Saar & Brandenburg (1999) and Böhm-Vitense (2007) suggested that the decrease of the dynamo period with the increase of the rotation rate can have different branches which is attributed to the “active” and “quiet” states of the stellar magnetic activity. Their results were further refined by Brandenburg et al. (2017) and other studies Oláh et al. (2009) and Olspert et al. (2018). The latter two and the survey of Lehtinen et al. (2016) did
Figure 9. The same as Figure 4 for the non-kinematic dynamo models.

Figure 10. The magnetic parity parameter in the non-kinematic dynamo models. The line notation is the same as in Figure 9.

not show the clear division on the quiet and active branches. The set of the dynamo models which we use for comparison with observations includes the quasi-non-kinematic dynamo models of Pipin (2015), one kinematic dynamo model from results of Pipin & Kosovichev (2016), the results of global convection simulations of solar-like stars of Warnecke (2018) (we remove one extreme case from his set) and sets of the kinematic and non-kinematic models of this paper. For the rotation period interval of 10 to 30 days, our models (both versions) show the linear increase of the dynamo period with an increase of the rotational period. The slope of the linear expression is about 0.36 ± 0.03. Expressing the dynamo period in days we find the linear slope is 136 ± 13 which is about factor 4 higher than the results of Oláh et al. (2009). We find that for the linear branch of models in the interval from 5 to 30 days the dynamo period depends on the amplitude of the magnetic buoyancy effect. In our models, we include the default expression of the mean-field magnetic buoyancy which follows from the results of Kitchatinov & Pipin (1993). Pipin & Kosovichev (2016) studied the different types of kinematic dynamo models and found that neglecting the magnetic buoyancy results in inversion of the slope sign (see the set of the white crosses in Figure 13).
On the dynamo cycle variations with rotational period

ΔΩ/Ω ~ Ro^{1.16} 

Figure 11. a) The magnitude of the magnetic energy in upper part of convection zone and the Rossby number; b) the Poynting luminosity variations (blue triangles) and the irradiance variations (red triangles) vs the Rossby number; c) the magnitude of the latitudinal shear at the surface vs the Rossby number; d) the same as c) for the magnitude of the mean large-scale magnetic helicity density at the surface.

Figure 12. a) Relation of the large-scale helicity density with the mean energy of the surface large-scale toroidal magnetic field; b) shows the same for the poloidal magnetic field.
Such a slope is typical for the results of the flux-transport models Jouve et al. (2010). The results of Warnecke (2018) show about factor 2 larger slope than ours. His results fit well into the “active” branch of stars from a sample of Brandenburg et al. (2017). Our models for the period of rotation from 10 to 30 days fit approximately into the “quiet” branch of the sample from Brandenburg et al. (2017).

For the stars rotating with a period less than 10 days, we find a weak dependence of the main dynamo period on the stellar rotation rate. Similarly to Warnecke (2018), it is found that the decrease of the rotation period from 5 to 1 day results in a variation in the dynamo period from about 3 to 4 years. The models M1dn and M2dn show the non-stationary variations of the magnetic cycle parameters with strong cycle-to-cycle parity variations. In these models, we find the cyclic patterns with periods that are much longer than the main period determined by the near-surface dynamo waves. We find the long dynamo periods for about 12 and 16 years for models M1dn and M2dn respectively. In Figure 13a, these points are in the cloud of the young solar-type stars sample of Lehtinen et al. (2016).

Normalization of the dynamo cycle frequency to the stellar rotation rate was found to be a good parameter to distinguish between the “active” and “quiet” branches of the magnetic activity Saar & Brandenburg (1999). Figure 13b show this parameter against the reverse Rossby number. Indeed, the sample of F- and G-stars from Brandenburg et al. (2017) shows the distinct branches of stellar activity, where the “active” branch has longer periods. Similarly to Warnecke (2018), the set of our models fits well the sample of Lehtinen et al. (2016). Viviani et al. (2018) found similar dependence in global convection simulations non-axisymmetric dynamo models. We see, that the branch of models, where the dynamo period decreases with the increase of the rotation rate (rotation periods between 30-10 days), is slightly inclined from the direction of the abscissa axis. The slope is opposite to that in the samples of Brandenburg et al. (2017) and Warnecke (2018). This branch can be identified in the kinematic dynamo models or in the full nonlinear models when the effect of the magnetic field on the large-scale flow is not strong. The models M8dn M5dn, M2dn, and M1dn follows to results Lehtinen et al. (2016) both for the “short” and long dynamo periods. Note, that the slope of this branch agrees approximately with all models that show the increase of the dynamo period with the increase of the rotation rate, e.g., the sample of models from Pipin & Kosovichev (2016) or results of Strugarek et al. (2017) (see, Warnecke (2018)). In whole, the results presented in Figure 13 are compatible with the surveys of Olspert et al. (2018) and Boro Saikia, S. et al. (2018).

4 DISCUSSION

In the paper, we explore the magnetic cycle parameters for the solar-type star with a rotation period from 1 to 30 days. For this study, we employ the non-kinematic mean-field dynamo models which take into account the effect of the magnetic activity on the angular momentum and heat transport inside of the convection zone. The given model is fully compatible with the solar dynamo model suggested recently by Pipin & Kosovichev (2019) as the model of the solar torsional oscillations. In our previous study Pipin (2015); Pipin & Kosovichev (2016) we used the smaller interval of the rotational periods. Also, that studies did not take into account the effects of the meridional circulation neither in the advection of the large-scale magnetic field nor in the angular momentum and heat transport in the bulk of the convection zone. Despite the difference, both the previous and the current studies consider the dynamo process distributed in the whole convection zone. In all the models the
near-surface dynamo wave propagation patterns satisfy the Parker-Yoshimura rule (Yoshimura 1975). We find that the efficiency of the large-scale dynamo grows with the increase of the rotation rate. The equipartition strength of the large-scale magnetic field for the star rotating with a period of 2 days is twice as high as the solar analog rotating with a period of 25 days.

In the kinematic models, we find that the decrease of the rotation below 10 days results in the increasing complexity of the near-surface dynamo wave pattern. Near the pole we find the wave propagating toward the equator. Following the Parker-Yoshimura rule, this is because of the nearly radial angular velocity profiles and the positive (at the North) $\alpha$-effect in the main part of the convection zone. The cylinder-like angular velocity profiles near the equator cause the poleward propagation of the dynamo waves. The dynamo period of the kinematic models increases with the decrease of the rotation rate. This is compatible with the results of studies exploring the chromospheric and photometric variations of the solar-type stars Oláh et al. (2009); Olsert et al. (2018); Boro Saikia, S. et al. (2018). Similar results are suggested by the global simulations of Guerrero et al. (2019). In our previous study, we find that this relation can depend on details of the dynamo models. For example, the distributed dynamo model with the standard angular velocity and $\alpha$-effect profiles (see, Pipin & Kosovichev (2016)) can show the inverse “period-period” relation in case if the magnetic buoyancy effect is disregarded. Similar dependence was found in flux-transport models of Jouve et al. (2010) and global simulations of Strugarek et al. (2017). The kinematic dynamo models do not show a clear sign of the magnetic activity saturation with an increase in rotation rate.

The non-kinematic models show that saturation of the magnetic activity is likely happens for the rotation period less than 5 days. In general, this conclusion is in agreement with observations, e.g., Wright & Drake (2016). Note, that the partially- and fully-convective stars can show a difference in parameters of the magnetic activity saturation Nizamov et al. (2017). Therefore the validity of our conclusion can be questioned for the general case of the non-axisymmetric dynamo, which becomes dominant for the fast-rotating solar-type stars Viviani et al. (2018). The non-kinematic models in the range of rotation periods longer than 10 days agree qualitatively with their kinematic analogs. In the model with a rotation period of 10 days, we find the doubling of the magnetic cycle frequency. This model is likely to show the solution for the marginal state. The marginal state is caused by the re-organization of the large-scale flow which shows the multiple meridional circulation cell in the bulk of the convection zone. Interestingly, the model M10n shows the mixed parity solution with the approximate energy equipartition of the symmetric and antisymmetric about the solar equator toroidal magnetic field. Such type of dynamo solution is often considered to be typical for the solar case. The global convection simulations of Warnecke et al. (2017) do not show clear trend in this case as well. The primary and secondary periods in the models for the range of the rotational periods from 1 to 8 days agrees with the findings from observations of the fast rotating solar analogs Lehtinen et al. (2016).

We find that a study of the dynamo periods solely on the base of the integral proxies of stellar magnetic activity can bias conclusions about the magnetic periodicity of the solar-type stars. Using our results we can identify several traps of such studies. Firstly, the mix of the magnetic parity modes can result in variations of the integral parameters showing either a state of “Maunder minimum” or the double dynamo frequency oscillations. Secondly, the strong nonlinearity of the dynamo solution can cause variations of the global activity parameters with the double frequency Sokoloff et al. (2020), as well. In the solar case, the effect is...
not strong. However, it can bias the conclusion for the fast-
rotating solar analogs, which are expected to show a highly
nonlinear dynamo regime (cf., $\beta$-parameter in Tables 1 and
2). Finally, in the nonlinear dynamo regimes, the integral pa-
rameters show the non-stationary evolution where the main
period of the time-latitude dynamo waves is hard to identify
as the primary dynamo period. The models M1n and M2n show good examples of this type.

In our study, we investigated a few aspects of the observ-
utional trends of the magnetic variability of the solar-type
stars. Our discussion has been concentrated on the prop-
erties of the near-surface dynamo wave patterns and their
relation with the dynamics of the large-scale flow and vari-
ations of the integral proxies of the magnetic activity. We
did not touch numerous theoretical aspects of the magnetic
activity of the solar-type stars including changes in the mag-
netic field topology and the type of axial symmetry of the
large-scale magnetic field with the rotation rate of a star.
These and other interesting questions can be studied fur-
ther using the numerical simulations and the growing base
of stellar magnetic activity observations.

REFERENCES

Baliunas S. L., et al., 1995, ApJ, 438, 269
Barnes J. R., Collier Cameron A., Donati J.-F., James D. J.,
Brandenburg A., 2018, Journal of Plasma Physics, 84, 735840404
Barnes J. R., Collier Cameron A., Donati J.-F., James D. J.,
Paxton B., Bildsten L., Dotter A., Herwig F., Lesaffre P., Timmes F.,
Baliunas S. L., et al., 1995, ApJ, 438, 269
Böning V. G. A., Roth M., Jackiewicz J., Kholikov S., 2017, ApJ,
Baliunas S. L., et al., 1995, ApJ, 438, 269
Böning V. G. A., Roth M., Jackiewicz J., Kholikov S., 2017, ApJ,
Baliunas S. L., et al., 1995, ApJ, 438, 269

Kitchatinov L. L., Pipin V. V., Rüdiger G., 1994, Astronomische
Nachrichten, 315, 157
Kitchatinov L. L., Pipin V. V., Rüdiger G., 1994, Astronomische
Nachrichten, 315, 157
Kleiner N., Rogachevskii I., Ruzmaikin A., 1995, A&A, 297, 159
Kosovichev A. G., et al., 1997, Sol. Phys., 170, 43
Krause F., Rädler K.-H., 1980, Mean-Field Magnetohydrodynam-
ics and Dynamo Theory. Berlin: Akademie-Verlag
Lehtinen J., Jetsu L., Hackman T., Kajakari P., Henry G. W.,
Lund K., et al., 2020, MNRAS, 493, 1003
Miesch M. S., Brun A. S., Do Rosa M. L., Toomre J., 2008, ApJ,
Mitra D., Candelaresi S., Chatzopoulos M., Tavakol R., Brandenburg
A., 2010, Astronomische Nachrichten, 331, 130
Moss D., Brandenburg A., 1992, A&A, 236, 371
Nizamov B. A., Katsova M. M., Livshuits M. A., 2017, Astronomy
Letters, 43, 202
Noyes R. W., Weiss N. O., Vaughan A. H., 1984, ApJ, 287, 769
Oláh K., et al., 2009, A&A, 501, 703
Olsert N., Pelt J., Köppel M. J., Lehtinen J., 2018, A&A, 615, A111
Parker E., 1955, Astrophys. J., 122, 293
Paxton B., Bildsten L., Dotter A., Herwig F., Lesaffre P., Timmes F.,
Paxton B., 2013, ApJS, 208, 4
Pipin V. V., 2004, Astronomy Reports, 48, 418
Pipin V. V., 2015, MNRAS, 451, 1528
Pipin V. V., 2018, Journal of Atmospheric and Solar-Terrestrial
Physics, 179, 185
Pipin V. V., Kitchatinov L. L., 2000, Astronomy Reports, 44, 771
Pipin V. V., Kosovichev A. G., 2011, ApJL, 727, L45
Pipin V. V., Kosovichev A. G., 2016, ApJ, 823, 133
Pipin V. V., Kosovichev A. G., 2018, ApJ, 854, 67
Pipin V. V., Kosovichev A. G., 2019, ApJ, 887, 215
Pipin V. V., Sokoloff D. D., Zhang H., Kuzanyan K. M., 2013,
ApJ, 768, 46
Rajaguru S. P., Antia H. M., 2015, ApJ, 813, 114
Rüdiger G., Küker M., Köppel M. J., Strassmeier K. G., 2019,
A&A, 630, A109
Saar S. H., 2011, in Prasad Choudhary D., Strassmeier K. G.,
eds, IAU Symposium Vol. 302, IAU Symposium. pp 61–67,
doi:10.1017/S1743921311015018
Saar S. H., Brandenburg A., 1999, ApJ, 524, 295
Schrijver C., Harvey K., 1984, Sol.Phys., 150, 1
See V., et al., 2015, MNRAS, 453, 4301
See V., et al., 2016, MNRAS, 462, 4442
Simitev R. D., Busse F. H., 2009, EPL (Europhysics Letters), 85,
19001
Sokoloff D., Nesme-Ribes E., 1994, A&A, 288, 293
Sokoloff D. D., Shibulava A. S., Obriko V. N., Pipin V. V., 2020,
arXiv e-prints, p. arXiv:2007.14779
Strugarek A., Beaudoin P., Charbonneau P., Brun A. S., do Nasci-
mento J.-D., 2017, Science, 357, 185
Teplitskaya R. B., Ozhogina O. A., Pipin V. V., 2015, Astronomy
Letters, 41, 848
Ulrich R. K., 2001, ApJ, 560, 466
Vidotto A. A., et al., 2014, MNRAS, 441, 2361
Viviani M., Warnecke J., Köppel M. J., Köppel P. J., Olsert N.,
Cole-Kodikara E. M., Lehtinen J. J., Brandenburg A., 2018,
A&A, 616, A160
Warnecke J., 2018, A&A, 616, A72
Weiss N. O., Tobias S. M., 2016, MNRAS, 456, 2654
Winter L. M., Balasubramaniam K. S., 2014, ApJ, 793, L45
Wright N. J., Drake J. J., 2016, Nature, 535, 526
Wright N. J., Drake J. J., 2016, Nature, 535, 526
Yoshimura H., 1975, ApJ, 201, 740
Zhao J., Bogart R. S., Kosovichev A. G., Duvall Jr. T. L., Hartlep
T., 2013, ApJ, 774, L29

MNRAS 000, 000–000 (0000)