Polarized deep inelastic scattering at high energies
and parity violating structure functions

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ABSTRACT

A comprehensive analysis of deep inelastic scattering of polarized charged leptons on polarized nucleons is presented; weak interaction contributions, both in neutral and charged current processes, are taken into account and the parity violating polarized nucleon structure functions are studied. Possible ways of their measurements and their interpretations in the parton model are discussed.

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1 - Introduction

The nucleon internal structure, as probed in deep inelastic scatterings (DIS) at high \( Q^2 \) values, keeps revealing unexpected features and appears to be much less simple than naively expected according to the quark-parton model ideas. The recent experimental data on polarized structure functions [1] and the Gottfried sum rule [2] have indeed been surprising ones. In particular, the spin content of protons in terms of its constituents needs a better and deeper understanding before a clear pattern emerges and non perturbative effects could be under control; after many theoretical efforts some more experimental information would certainly help in casting some light on the subject.

We consider here the deep inelastic scattering of charged polarized leptons on polarized nucleons at very high energies, taking into account both electromagnetic and weak interactions, with both neutral and charged current contributions. We discuss in detail what can be learnt from such experiments, with a particular interest in the parity violating polarized structure functions which have not yet been measured, but might be experimentally accessible in a near future. We are thinking of deep inelastic scattering processes at HERA or SLAC facilities, with polarized electron, positron or muon beams and polarized nucleon targets. Also the possibility of polarized proton beams appears promising [3]. The extension of our results to neutrino initiated processes is trivial. However, we do not consider these processes explicitly here because of the technical difficulties in polarizing the large nucleon targets needed for neutrino scatterings, which make these experiments unrealistic at the moment.

There exist in the literature several works on the weak interaction contribution to polarized deep inelastic scattering, with some discrepancies among them [4-19]; our goal in this paper is that of presenting a complete and detailed analysis of all contributions to charged lepton–nucleon interactions, with explicit expressions of cross-sections and spin asymmetries, with arbitrary spin directions, which might be useful to experimentalists. Our emphasis is on the possible experimental study of the nucleon structure in very high energy deep inelastic scatterings within the Standard Model: we suggest some measurements which, although difficult, might soon be feasible and could allow to extract information on new polarized structure functions. The partonic interpretation of all structure functions is discussed and interesting combinations are indicated which single out particular quark distributions; most of them are independent of the axial anomaly, and allow an unambiguous interpretation. A full study of the gluonic contributions has been performed in Ref. [14] and a much more formal approach to deep inelastic scattering with electroweak currents can be found in Ref. [18].

The paper is organized as follows. In Section 2 we introduce the formalism and define the independent structure functions which appear in the most general hadronic tensor describing the electro–weak coupling of polarized nucleons to gauge bosons, both in case of neutral and charged currents; we also give explicit formulas for cross-sections with particular spin configurations. In Section 3 we give the parton model expressions of the scaling structure functions and in Section 4 we discuss how to perform useful measurements and to combine particular cross-sections, in order to extract meaningful combinations of quark and antiquark distribution functions.
2 - General formalism

Let us consider first the neutral current (nc) deep inelastic process $\ell N \rightarrow \ell' X$ with polarized charged leptons ($\ell$) and nucleons ($N$). In lowest order perturbation theory the amplitude for such a process is given by the sum of two contributions: the one photon exchange ($M_\gamma$) and the $Z^0$ exchange ($M_Z$). The differential cross–section then reads:

$$\frac{d^2\sigma^{\ell N}_{nc}}{d\Omega dE'} = \frac{1}{2m_N(4\pi)^2} \frac{E'}{E} \times |M_\gamma + M_Z|^2,$$

where $E$ and $E' = E - \nu$ are, respectively, the initial and final lepton energy in the laboratory frame where the nucleon of mass $m_N$ is at rest, and the scattered lepton, of mass $m_\ell$, emerges into solid angle $d\Omega$. Terms proportional to $m_\ell/E$ or $m_\ell/E'$ are neglected throughout the paper.

Eq. (1) receives contributions from a purely electromagnetic term ($|M_\gamma|^2$), a purely weak one ($|M_Z|^2$) and an interference one ($M_\gamma M_Z^* + M_Z M_\gamma^*$), so that it can be cast in the form

$$\frac{d^3\sigma^{\ell N}_{nc}}{dxdy\,d\phi} = \frac{y\alpha^2}{2Q^4} \sum_{i=\gamma,\gamma Z,z} L_{\mu\nu}^i W_{\mu\nu} \eta^i,$$

where $Q^2 = -q^2$ is the squared four-momentum transfer, $x = Q^2/(2m_N\nu)$, $y = \nu/E$ and $\phi$ is the azimuthal angle of the final lepton. $L_{\mu\nu}^i$ is the usual leptonic tensor deriving from the couplings of the $\gamma$ and the $Z^0$ to the lepton. For negatively charged leptons ($e^−, \mu^−$) one has

$$L_{\mu\nu}^\gamma = [\bar{u}(l')\gamma_\mu u(l)] [\bar{u}(l')\gamma_\nu u(l)] = 2[(l'_\mu l'_\nu + l_\mu l_\nu) - i\lambda\epsilon_{\mu\nu\alpha\beta} l^\alpha l^\beta]$$

$$L_{\mu\nu}^{\gamma Z} = [\bar{u}(l')\gamma_\mu (g_\nu - g_A \gamma_5) u(l)] [\bar{u}(l')\gamma_\nu u(l)] = (g_\nu - \lambda g_A) L_{\mu\nu}^\gamma$$

$$L_{\mu\nu}^Z = [\bar{u}(l')\gamma_\mu (g_\nu - g_A \gamma_5) u(l)] [\bar{u}(l')\gamma_\nu (g_\nu - g_A \gamma_5) u(l)] = (g_\nu - \lambda g_A)^2 L_{\mu\nu}^\gamma$$

where $l$ and $\lambda = \pm 1$ are, respectively, the four-momentum and the helicity of the initial lepton and $l' = l - q$ is the four-momentum of the final lepton. For positively charged leptons ($e^+, \mu^+$) one should simply replace, in the above formula, $g_A$ with $-g_A$ [20]. In our notations

$$g_\nu = -\frac{1}{2} + 2\sin^2 \theta_W \quad g_A = -\frac{1}{2}.$$  

The factors $\eta^i$ in Eq. (3) collect some kinematical quantities, coupling constants and the relative weights of different propagators, namely:

$$\eta^\gamma = 1$$

$$\eta^{\gamma Z} = \left(\frac{GM_Z^2}{2\sqrt{2}\pi\alpha}\right) \left(\frac{Q^2}{Q^2 + M_Z^2}\right)$$

$$\eta^z = (\eta^{\gamma Z})^2.$$  

(5)
where $G$ is the Fermi coupling constant and $M_Z$ is the $Z^0$ mass. Notice that 
$GM_Z^2/(2\sqrt{2}\pi\alpha) = (4\sin^2 \theta_W \cos^2 \theta_W)^{-1} \simeq 4/3$.

Finally, the hadronic tensor $W^\mu_\nu$ defines the coupling of the electromagnetic ($J_\gamma$) and
the weak ($J_z$) current to the nucleon with its transition to all possible final states $X$.

\[ W^\mu_\nu = \sum_X < X|J_\gamma^\mu|N >^* < X|J_\gamma^\nu|N > (2\pi)^3 \delta(p_X - p - q) \]

\[ W^\mu_\nu = \sum_X [< X|J_z^\mu|N >^* < X|J_z^\nu|N > + < X|J_\gamma^\mu|N >^* < X|J_z^\nu|N >] (2\pi)^3 \delta(p_X - p - q) \]

\[ W^\nu_\mu = \sum_X < X|J_z^\mu|N >^* < X|J_z^\nu|N > (2\pi)^3 \delta(p_X - p - q) \]

where $p_X$ is the total four-momentum of the state $X$. Exploiting Lorentz and CP invariance
the hadronic tensor can be expressed in terms of $q^\mu$, the nucleon four-momentum $p^\mu$, its
spin $S^\mu$ and 8 independent structure functions, as

\[ \frac{1}{2m_N} W^i_\mu_\nu = - \frac{g^\mu_\nu}{m_N} F_1^i + \frac{p_\mu p_\nu}{m_N (p \cdot q)} F_2^i \]

\[ + i \frac{\epsilon^{\mu\nu\alpha\beta}}{2(p \cdot q)} \left[ \frac{p^\alpha q^\beta}{m_N} F_3^i + 2 q^\alpha S^\beta g_1^i - 4 x p^\alpha S^\beta g_2^i \right] \]

\[ - \frac{p_\mu S_\nu + S_\mu p_\nu}{2(p \cdot q)} g_3^i + \frac{S \cdot q}{(p \cdot q)^2} p_\mu p_\nu g_4^i + \frac{S \cdot q}{p \cdot q} g_\mu g_\nu g_5^i. \]

Notice that terms proportional to $q^\mu$ or $q^\nu$ can be dropped in the definition of $W^i_\mu_\nu$ because
they give no contribution (in the $m_\ell/E \to 0$ limit) when contracted with $L^\mu_\nu$. Different
definitions of the hadronic tensor appearing in the literature differ from ours due to these
terms. In particular the coefficient of $g_2$ could be written in a more familiar way using the
identity

\[ \epsilon^{\mu\nu\alpha\beta} p^\alpha S^\beta = \frac{\epsilon^{\mu\nu\alpha\beta}}{2x(p \cdot q)} [(q \cdot S) q^\alpha p^\beta - (p \cdot q) q^\alpha S^\beta] \]

\[ - (q_\mu \epsilon_{\nu\alpha\beta\gamma} - q_\nu \epsilon_{\mu\alpha\beta\gamma}) p^\alpha q^\beta S^\gamma \]

and dropping the last two terms.

The structure functions $F_j(Q^2, p \cdot q)$ and $g_j(Q^2, p \cdot q)$ are expected to scale in the large
$Q^2$ limit and to depend only on the Bjorken scaling variable $x = Q^2 / (2p \cdot q)$. The $F_j$ are
the unpolarized structure functions and the $g_j$ the polarized ones: when averaging over the
nucleon spin one computes $W^\mu_\nu(p, q, S) + W^\mu_\nu(p, q, -S)$ and all terms proportional to $g_j$
cancel out. In Eq.(7) we have allowed for parity violations and indeed $W^i_\mu_\nu$ is a mixture of
second rank tensors and pseudotensors; in case of pure electromagnetic interactions ($i = \gamma$)
parity is conserved and one has

\[ F^\gamma_3 = g^\gamma_3 = g^\gamma_4 = g^\gamma_5 = 0. \]
\( F_3, g_3, g_4 \) and \( g_5 \) only contribute to parity violating interactions and are often referred to as the parity violating structure functions.

The structure functions defined according to Eq.(7) are such that one recovers the usual ones \([20]\) when considering unpolarized DIS, and electromagnetic polarized DIS.

We shall now consider some particular cases of Eq.(2), corresponding to specific nucleon spin configurations. Let us start from a longitudinally polarized nucleon. If we choose the \( z \)-axis as the direction of motion of the incoming lepton we then have

\[
\begin{align*}
  l^\mu &= E(1, 0, 0, 1) & l'^\mu &= E'(1, \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \\
  p^\mu &= (m_N, 0, 0, 0) & S^\mu &= S_L^\mu = (0, 0, 0, 1),
\end{align*}
\]  

and we obtain (after integration over the azimuthal angle \( \phi \)):

\[
\begin{align*}
  \frac{d^2 \sigma_{\ell N}^{\text{unp}}}{dx \, dy} (\lambda, S = S_L) &= 4\pi m_N E y \frac{\alpha^2}{Q^4} \sum_i \eta^i C^i \\
  &\quad \times \left\{ 2xy F_1^i + \frac{2}{y} \left( 1 - y - \frac{xym_N}{2E} \right) (F_2^i + g_3^i) \\
  &\quad - 2\lambda x \left( 1 - \frac{y}{2} \right) F_3^i - 2\lambda x \left( 2 - y - \frac{xym_N}{E} \right) g_1^i + 4\lambda \frac{x^2 m_N}{E} g_2^i \\
  &\quad - \frac{2}{y} \left( 1 + \frac{xm_N}{E} \right) \left( 1 - y - \frac{xym_N}{2E} \right) g_4^i + 2xy \left( 1 + \frac{xm_N}{E} \right) g_5^i \right\},
\end{align*}
\]  

where, for negatively charged leptons,

\[
  C^\gamma = 1 \quad C^{\gamma z} = (g_V - \lambda g_A) \quad C^z = (g_V - \lambda g_A)^2,
\]

and for positively charged leptons one simply replaces \( g_A \) with \(-g_A\); \( g_V \) and \( g_A \) are as in Eq.(4). Notice that when the lepton flips its helicity \( \lambda \) changes sign and when the nucleon flips its spin all terms containing a polarized structure function \( g_j (j = 1, 2, .., 5) \) also change sign. Upon averaging over \( \lambda \) and \( S \) one obtains the unpolarized cross-section

\[
\begin{align*}
  \frac{d^2 \sigma_{\ell N}^{\text{unp}}}{dx \, dy} (\text{unp}) &= \frac{1}{4} \sum_{\lambda, S} \frac{d^2 \sigma_{\ell N}^{\text{unp}}}{dx \, dy} (\lambda, S) \\
  &= 4\pi m_N E y \frac{\alpha^2}{Q^4} \sum_i \eta^i C^i \left\{ 2xy F_1^i + \frac{2}{y} \left( 1 - y - \frac{xym_N}{2E} \right) F_2^i \right\}.
\end{align*}
\]  

In the case of nucleons with transverse polarization, \textit{i.e.} with a spin orthogonal to the lepton direction (\( z \)-axis) at an angle \( \alpha \) to the \( x \)-axis, we have

\[
  S^\mu = S_T^\mu = (0, \cos \alpha, \sin \alpha, 0)
\]  

\( (14) \)
which, using Eqs. (2), (3) and (7), yields

\[
\frac{d^3\sigma^{\ell N}_{nc}}{dx\,dy\,d\phi}(\lambda, S = S_T) = 2m_N E y \frac{\alpha^2}{Q^2} \sum_i \eta^i C^i \times \left\{ 2xy F^i_1 + \frac{2}{y} \left( 1 - y - \frac{ym_N}{2E} \right) F^i_2 - 2\lambda x \left( 1 - \frac{y}{2} \right) F^i_3 \right. \\
+ \sqrt{ym_N [2(1-y)E - ym_N]} \frac{E}{E} \cos(\alpha - \phi) \\
\left. \times \left[ -2\lambda x g^i_1 - 4\lambda \frac{x}{y} g^i_2 + \frac{1}{y} g^i_3 + \frac{2}{y^2} \left( 1 - y - \frac{ym_N}{2E} \right) g^i_4 - 2xg^i_5 \right] \right\}.
\]

(15)

Again, a flip in the lepton helicity results in a change of the sign of \( \lambda \) (including in the expressions for \( C^i \), Eq. (12)) and a flip of the nucleon spin induces a change of sign in all terms containing a polarized structure function \( g_j \).

Before turning to a discussion of the parton model interpretation of the structure functions we consider also the case of charged current (cc) deep inelastic processes \( \ell^+ N \rightarrow \nu(\bar{\nu}) X \), which proceed, at lowest perturbative order, through the exchange of a charged vector boson \( W^\mp \). This case resembles the \( Z^0 \) contribution to the neutral current process, with the assignment \( g_V = g_A = 1 \). In fact Eq. (1) now reads

\[
\frac{d^2\sigma^{\ell N}_{cc}}{d\Omega\,dE'} = \frac{1}{2m_N (4\pi)^2} \frac{E'}{E} \times |M_W|^2,
\]

(16)

and Eq. (2) changes into

\[
\frac{d^3\sigma^{\ell N}_{cc}}{dx\,dy\,d\phi} = \frac{y\alpha^2}{2Q^4} L^W_{\mu\nu} W^\mu_{\nu} W^\nu_{\sigma} \eta^\sigma W,
\]

(17)

where, for a negatively charged lepton \( \ell^- \) (which couples to a \( W^- \)),

\[
L^W_{\mu\nu} = \left[ \bar{u}(l') \gamma_{\mu} (1 - \gamma_5) u(l) \right]^* \left[ \bar{u}(l') \gamma_{\nu} (1 - \gamma_5) u(l) \right] = (1 - \lambda)^2 L^\gamma_{\mu\nu}.
\]

(18)

Eq. (18) shows that fast \( \ell^- \) leptons only couple to a \( W \) if they have a negative helicity (\( \lambda = -1 \)). Eq. (17) is completed by

\[
\eta^w = \frac{1}{2} \left( \frac{G M_W^2}{4\pi \alpha} \frac{Q^2}{Q^2 + M_W^2} \right)^2,
\]

(19)

where \( M_W \) is the vector boson mass, and \( W^W_{\mu\nu} = W^i_{\mu\nu} \), Eq. (7). For a positively charged lepton \( (\ell^+) \) one simply changes, as usual, the sign of the axial coupling \( \gamma_{\mu} \gamma_5 \), that is one replaces \( \lambda \) with \( -\lambda \) in Eq. (18) to get \( L^W_{\mu\nu} = (1 + \lambda)^2 L^\gamma_{\mu\nu} \).

Eqs. (11) and (13) then hold also for the charged current interaction, with no \( \sum_i \), the factor \( \eta \) as in Eq. (13) and \( C = (1 - \lambda)^2 \) for \( \ell^- \), \( C = (1 + \lambda)^2 \) for \( \ell^+ \); they will be used in Section 4 when discussing meaningful possible measurements and we will drop the index \( nc \). All of the previous formulae can also be easily modified to describe neutrino initiated processes, which we do not consider here.
3 - Structure functions in the naive quark-parton model

According to the naive quark-parton model the lepton interaction with the nucleon is just the incoherent sum of all elementary interactions of the gauge bosons $\gamma, W^\pm, Z^0$ with the quarks, supposed to be free constituents, each of which carries a fraction $x$ of the proton four-momentum, $k^\mu = x P^\mu$. This leads to (20):

$$W^{i,\mu\nu} = \frac{1}{2x m_{N\nu}} \left[ \omega^{i,\mu\nu}_{q,S} q^\nu + \omega^{i,\mu\nu}_{q,-S} q^A + \omega^{i,\mu\nu}_{q,S} \bar{q}^\nu + \omega^{i,\mu\nu}_{q,-S} \bar{q}^A \right]$$ (20)

where $q^{P(A)}(x)$ is the number density of quarks with flavour $q$ and spin parallel (antiparallel) to the nucleon spin; similarly for $\bar{q}^{P,A}(x)$, which refer to antiquarks. $\omega^{i,\mu\nu}_{q,s}$ is the quark tensor, analogous to the leptonic tensor $L^{\mu\nu}$, and the index $i$ refers, as usual, to the different interaction contributions, $i = \gamma, \gamma Z, Z, W$:

$$\omega^{\gamma,\mu\nu}_{q,s} = \sum_{s'} e_q^2 \left[ \bar{u}(k', s') \gamma^\mu u(k, s) \right]^* \left[ \bar{u}(k', s') \gamma^\nu u(k, s) \right]$$

$$\omega^{\gamma Z,\mu\nu}_{q,s} = \sum_{s'} e_q \left[ \bar{u}(k', s') \gamma^\mu (g_\nu - g_A \gamma_5) q u(k, s) \right]^* \left[ \bar{u}(k', s') \gamma^\nu (g_\nu - g_A \gamma_5) q u(k, s) \right]$$

$$\omega^{\gamma Z,\mu\nu}_{q,s} = \sum_{s'} \left[ \bar{u}(k', s') \gamma^\mu (g_\nu - g_A \gamma_5) q u(k, s) \right]^* \left[ \bar{u}(k', s') \gamma^\nu (g_\nu - g_A \gamma_5) q u(k, s) \right]$$

$$\omega^{W,\mu\nu}_{q,s} = \sum_{s', q'} \left[ \bar{u}(k', s') \gamma^\mu (1 - \gamma_5) u(k, s) \right]^* \left[ \bar{u}(k', s') \gamma^\nu (1 - \gamma_5) u(k, s) \right] |(K.M.)_{qq'}|^2$$

where $k, k' = k + q, s$ and $s'$ are respectively the momentum and spin four-vectors of the initial and final quarks; the initial quark is not necessarily supposed to be in a helicity state but its spin is taken to be either parallel to the nucleon spin ($s = S$) or antiparallel ($s = -S$). $(g_\nu)_q$ and $(g_A)_q$ are the vector and axial couplings of the quark of flavour $q$ to the $Z^0$ and $e_q$ is the quark charge in units of the proton charge. In case of charged current, negative charge leptons only couple to $u$-type quarks (or $d$-type antiquarks) and positive charge leptons couple to $d$-type quarks (or $u$-type antiquarks); one has also to take into account the proper Cabibbo-Kobayashi-Maskawa matrix elements occurring in the transition coupling from a flavour $q$ to a flavour $q'$. However, if we consider the contribution of four flavours ($u, d, s$ and $c$), one always has $\sum_{q'} |(K.M.)_{qq'}|^2 = \cos^2 \theta_c + \sin^2 \theta_c = 1$, where $\theta_c$ is the Cabibbo angle.

Explicit expressions of Eqs. (21) can be obtained from the general form

$$\omega^{\mu\nu}_{q,s} = \sum_{s'} \left[ \bar{u}(k', s') \gamma^\mu (v_1 - a_1 \gamma_5) u(k, s) \right]^* \left[ \bar{u}(k', s') \gamma^\nu (v_2 - a_2 \gamma_5) u(k, s) \right]$$

$$= 2 (a_1 a_2 + v_1 v_2) [2k^\mu k^\nu - k \cdot q g^{\mu\nu}] - 4a_1 a_2 m_q^2 g^{\mu\nu}$$

$$- 2v_1 a_2 m_q [2k^\mu s^\nu - s \cdot q g^{\mu\nu}] - 2a_1 v_2 m_q [2s^\mu k^\nu - s \cdot q g^{\mu\nu}]$$

$$+ 2ie^{\mu\nu\alpha\beta} [(v_1 a_2 + a_1 v_2) k_\alpha q_\beta + a_1 a_2 m_q k_\alpha s_\beta + (a_1 a_2 + v_1 v_2) m_q q_\alpha s_\beta]$$

(22)
by properly fixing the values of \( v_{1,2} \) and \( a_{1,2} \); \( m_q \) is the quark mass and we have dropped terms proportional to \( q^\mu \) or \( q^\nu \) which, when contracted with the leptonic tensor \( L_{\mu\nu} \), give contributions proportional to \( m_\ell / E \), consistently neglected in the whole paper. If the quark has opposite spin it suffices to change the sign of \( s \).

The antiquark tensors \( \omega_{\bar{q},s}^{i,\mu\nu} \) are defined exactly as in Eq. (21) with the only replacement \( \gamma_5 \rightarrow -\gamma_5 \); Eq. (22) can then be exploited also for antiquarks and gives \( \omega_{\bar{q},s}^{i,\mu\nu} \) as a result of the usual replacement in the axial couplings, \( a_{1,2} \rightarrow -a_{1,2} \).

Eqs. (20)-(22) give the quark-parton model predictions for the hadronic tensor \( W_{i,\mu\nu} \); by comparing them with the general expression, Eq. (7), one obtains the quark-parton model results for the nucleon structure functions. For completeness we list all of them here, starting from the electromagnetic case \((i = \gamma)\):

\[
\begin{align*}
F_1^\gamma &= \frac{1}{2} \sum_q e_q^2 (q + \bar{q}) \\
F_2^\gamma &= 2x F_1^\gamma \\
g_1^\gamma &= \frac{1}{2} \sum_q e_q^2 (\Delta q + \Delta \bar{q}) \\
g_2^\gamma &= 0
\end{align*}
\]

where \( q = q^P + q^A \) is the number density of quarks of flavour \( q \) and \( \Delta q = q^P - q^A \); analogously for antiquarks.

The interference contribution \((i = \gamma Z)\) is:

\[
\begin{align*}
F_1^{\gamma Z} &= \sum_q e_q (g_V)_q (q + \bar{q}) \\
F_2^{\gamma Z} &= 2x F_1^{\gamma Z} \\
F_3^{\gamma Z} &= 2 \sum_q e_q (g_A)_q (q - \bar{q}) \\
g_1^{\gamma Z} &= \sum_q e_q (g_V)_q (\Delta q + \Delta \bar{q}) \\
g_2^{\gamma Z} &= g_4^{\gamma Z} = 0 \\
g_3^{\gamma Z} &= 2x \sum_q e_q (g_A)_q (\Delta q - \Delta \bar{q})
\end{align*}
\]

and the purely weak interaction \((i = Z)\) leads to:

\[
\begin{align*}
F_1^Z &= \frac{1}{2} \sum_q (g_V^2 + g_A^2)_q (q + \bar{q}) \\
F_2^Z &= 2x F_1^Z \\
F_3^Z &= 2 \sum_q (g_V g_A)_q (q - \bar{q}) \\
g_1^Z &= \frac{1}{2} \sum_q (g_V^2 + g_A^2)_q (\Delta q + \Delta \bar{q}) \\
g_2^Z &= -\frac{1}{2} \sum_q (g_A^2)_q (\Delta q + \Delta \bar{q}) \\
g_3^Z &= 2x \sum_q (g_V g_A)_q (\Delta q - \Delta \bar{q}) \\
g_4^Z &= 0 \\
g_5^Z &= 2x g_5^Z = g_3^Z
\end{align*}
\]
In case of charged current \((i = W)\), on performing explicitly the \(\sum q\), one obtains, for \(\ell^- N \rightarrow \nu X\) processes:

\[
\begin{align*}
F_{1W}^- &= u + c + \bar{d} + \bar{s} \\
F_{2W}^- &= 2x F_{1W}^- \\
F_{3W}^- &= 2(u + c - \bar{d} - \bar{s}) \\
g_{1W}^- &= (\Delta u + \Delta c + \Delta \bar{d} + \Delta \bar{s}) \\
g_{2W}^- &= 2 \Delta u + \Delta c - \Delta \bar{d} - \Delta \bar{s} \\
g_{3W}^- &= 2x(\Delta u + \Delta c - \Delta \bar{d} - \Delta \bar{s}) \\
g_{4W}^- &= 0 \\
g_{5W}^- &= 2xg^W_5 = g^W_3
\end{align*}
\]

where \(u\) stays for the number density of quarks \(u\) and so on. \(\ell^+ N \rightarrow \bar{\nu} X\) processes probe different quark flavours and one obtains the corresponding expressions of the structure functions \(F^W_j\) and \(g^W_j\) by the flavour interchanges \(d \leftrightarrow u\) and \(s \leftrightarrow c\) in the above Eq. (26).

Notice that in the quark-parton model the structure functions \(g_i^4\) are always zero and one finds \(g_3^1 = 2xg_5^1\) for any \(i = \gamma, \gamma Z, Z, W\). Indeed in the Bjorken limit a Callan–Gross–like relation holds, independently of the actual model of the nucleon employed in the calculation and of the kind of local interaction, provided that the elementary point–like constituents are fermions, as follows from an analysis of the helicity amplitudes \([3]\) or from the OPE of the hadronic tensor \([16]\): \(g_3 - g_4 = 2xg_5\). The quark–parton model only sets \(g_4 = 0\). The function \(g_5^1\) are nonzero only for pure weak interactions (both \(nc\) and \(cc\) ones). It is interesting to note that, in case of neutral current, the integral of \(g_2^Z\) appears to be directly proportional to the the total spin carried by the quarks and antiquarks, as can be seen from Eqs. (25) and the fact that \(g_2^Z = 1/4\) for any quark flavour. Its experimental determination, however, appears to be very difficult, as can be seen from Eqs. (11) and (15) and will be discussed in the next Section.

This completes our summary of the naive quark-parton model predictions for the different structure functions; we should now tackle the problem of experimental measurements which could single out the various contributions and possibly give new information on the polarized quark distributions inside the nucleons.

4 - Experimental measurements

We would like to suggest some ideal experimental measurements which could provide additional precious information on the quark and spin content of nucleons. We assume that high energy longitudinally polarized electron, positron or muon beams are available, as well as polarized proton and neutron targets or proton beams, and consider some suitable combinations of cross–sections. Such measurements might become affordable in the near future at HERA, CERN or SLAC facilities.

Let us define the nucleon spin asymmetries:

\[
\Delta^L \sigma^\ell N(\lambda) \equiv \frac{d^2 \sigma^\ell N}{dx\, dy}(\lambda, S = S_L) - \frac{d^2 \sigma^\ell N}{dx\, dy}(\lambda, S = -S_L)
\]

and

\[
\Delta^T \sigma^\ell N(\lambda) \equiv \frac{d^3 \sigma^\ell N}{dx\, dy\, d\phi}(\lambda, S = S_T) - \frac{d^3 \sigma^\ell N}{dx\, dy\, d\phi}(\lambda, S = -S_T)
\]

(27) and (28)
which single out the polarized structure functions. Eqs. (11) and (14), together with the parton model results \( g_4^i = 0 \) and \( g_3^i = 2xg_5^i \), yield

\[
\Delta^L \sigma_{\ell N} (\lambda) = 16\pi m_N E \frac{\alpha^2}{Q^4} \sum_i \eta^i C^i \\
\times \left\{ -\lambda xy \left( 2 - y - \frac{ym_N}{E} \right) g_1^i + 2\lambda \frac{x^2ym_N}{E} g_2^i \\
+ x \left[ y^2 + (1 - y) \left( 2 - \frac{ym_N}{E} \right) \right] g_5^i \right\}
\]

(29)

and

\[
\Delta^T \sigma_{\ell N} (\lambda) = 8m_N \frac{\alpha^2}{Q^4} \cos(\alpha - \phi) \sqrt{ym_N [2(1 - y)E - ym_N]} \\
\times \sum_i \eta^i C^i \left\{ -\lambda xy g_1^i - 2\lambda x g_2^i + x(1 - y) g_5^i \right\}
\]

(30)

which hold, depending on the values of the factors \( \eta^i C^i \), both for neutral and charged current processes. For the \( nc \) case the \( \eta^i \) are given in Eq.(5) and the \( C^i \) in Eq.(12); for the \( cc \) case there is only one term in the sum, with \( \eta \) given in Eq.(19) and \( C = (1 \mp \lambda)^2 \) respectively for \( \ell^\mp \). Notice that the transverse asymmetry (30) is suppressed by a factor \( \sqrt{m_N/E} \) with respect to the longitudinal one, Eq.(29), and the unpolarized cross-section, Eq.(13); this might make its measurement problematic.

Let us now discuss the charged current case. From Eqs.(29), (30) and the parton model results (26) one has, for negatively charged leptons (\( \ell^- \)) with helicity \( \lambda = -1 \) or for positively charged ones (\( \ell^+ \)) with helicity \( \lambda = 1 \):

\[
\Delta^L \sigma_{cc}^{\ell^\mp N} = 64\pi m_N E \frac{\alpha^2}{Q^4} \eta^w \times \left\{ \pm xy \left[ 2 - y + \frac{ym_N}{E} (1 - y) \right] g_1^{w^\mp} \\
+ x \left[ y^2 + (1 - y) \left( 2 - \frac{ym_N}{E} \right) \right] g_5^{w^\mp} \right\}.
\]

\[
\Delta^T \sigma_{cc}^{\ell^\mp N} = 32m_N \frac{\alpha^2}{Q^4} \eta^w \sqrt{ym_N [2(1 - y)E - ym_N]} \cos(\alpha - \phi) \\
\times x(1 - y) \left( \mp g_1^{w^\mp} + g_5^{w^\mp} \right).
\]

(31)

Eqs.(31) show how separate measurements of \( \Delta^L \sigma_{cc}^{\ell^\mp} \) and \( \Delta^T \sigma_{cc}^{\ell^\mp} \) could allow a determination of \( g_1^{w^\mp} \) and \( g_5^{w^\mp} \). By suitable combinations of these structure functions one could extract meaningful information on the spin content of the nucleon. For example:

\[
g_1^{w^-} + g_1^{w^+} = (\Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s} + \Delta c + \Delta \bar{c}) \\
= \sum_q (\Delta q + \Delta \bar{q})
\]

(32)

and

\[
g_5^{w^-} - g_5^{w^+} = (\Delta u + \Delta \bar{u} + \Delta c + \Delta \bar{c}) - (\Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s})
\]

(33)
Notice that the combination (32) supplies another way of getting information on the total spin carried by quarks and antiquarks; moreover, by scattering on an isoscalar target one could obtain information on the strange quark polarization,

\[(g_5^{w^{-}} - g_5^{w^{+}})_p + (g_5^{w^{-}} - g_5^{w^{+}})_n = 2(\Delta c + \Delta \bar{c} - \Delta s - \Delta \bar{s})_p \tag{34}\]

where the indexes \(p\) and \(n\) stay respectively for proton and neutron.

Let us now consider neutral current interactions. To simplify our results we notice that in the high energy region one can keep only leading terms in \(m_N/E\); moreover, from Eq.(33) and for \(Q^2\) values up to \(\sim 10^3\) GeV\(^2\) one has

\[\eta^Z \ll \eta^\gamma Z \ll \eta^\gamma. \tag{35}\]

Eqs.(29) and (30) and the parton model results (23)-(25) then give, with a very good approximation:

\[
\begin{align*}
\Delta^L \sigma^\ell N_{nc} (\lambda = 1) &= -16\pi m_N E \frac{\alpha^2}{Q^4} xy(2-y) g_1^\gamma \\
\Delta^T \sigma^\ell N_{nc} (\lambda = 1) &= -8m_N \frac{\alpha^2}{Q^4} \cos(\alpha - \phi) \sqrt{2xy m_N E(1-y)} xy g_1^\gamma
\end{align*}
\tag{36}
\]

which allow a direct measurement of \(g_1^\gamma\) and show how, in our kinematical region, such asymmetries are still dominated by electromagnetic interactions.

In order to extract information on the other polarized structure functions, or simple combinations of few of them, in neutral current processes, it is convenient to introduce further quantities:

\[
\begin{align*}
\Sigma_L (\lambda) &\equiv \Delta^L \sigma^\ell N_{nc} (\lambda) + \Delta^L \sigma^{\bar{\ell} N}_{nc} (\lambda) \\
\Sigma_T (\lambda) &\equiv \Delta^T \sigma^\ell N_{nc} (\lambda) + \Delta^T \sigma^{\bar{\ell} N}_{nc} (\lambda) \\
D_L (\lambda) &\equiv \Delta^L \sigma^\ell N_{nc} (\lambda) - \Delta^L \sigma^{\bar{\ell} N}_{nc} (\lambda) \\
D_T (\lambda) &\equiv \Delta^T \sigma^\ell N_{nc} (\lambda) - \Delta^T \sigma^{\bar{\ell} N}_{nc} (\lambda)
\end{align*}
\tag{37}
\]

whose full explicit expressions can be found from Eqs.(29) and (30). At large energy and in the \(Q^2\) range of validity of Eq.(35) they can be combined, using the parton model results,
and noticing, Eq. (4), that for the lepton $g_\nu \simeq -0.04$ whereas $g_\alpha = -0.5$, to give:

$$
\Sigma_L(\lambda = 1) + \Sigma_L(\lambda = -1) = 64\pi m_N e^{\alpha^2 Q^4} \frac{Q^4}{x(2 - 2y + y^2)} \left\{ g_\nu \eta^\gamma \gamma^\gamma g_5^\gamma + g_\alpha^2 \eta^\gamma \gamma_5^\gamma \right\}
$$

$$
\Sigma_T(\lambda = 1) + \Sigma_T(\lambda = -1) = 32m_N \alpha^2 \cos(\alpha - \phi) \sqrt{2xym_N E(1-y)} x(1- y) \times \left\{ g_\nu \eta^\gamma \gamma^\gamma g_5^\gamma + g_\alpha^2 \eta^\gamma \gamma_5^\gamma \right\}
$$

$$
D_L(\lambda = 1) - D_L(\lambda = -1) = -64\pi m_N e^{\alpha^2 Q^4} \frac{Q^4}{x(2 - 2y + y^2)} g_\alpha \eta^\gamma \gamma_1^\gamma
$$

$$
D_T(\lambda = 1) - D_T(\lambda = -1) = -32m_N \alpha^2 \cos(\alpha - \phi) \sqrt{2xym_N E(1-y)} x(1- y) \times x(1- y) g_\alpha \eta^\gamma \gamma_1^\gamma
$$

$$
D_T(\lambda = 1) + D_T(\lambda = -1) = 32m_N \alpha^2 \cos(\alpha - \phi) \sqrt{2xym_N E(1-y)} xy g_\alpha \eta^\gamma \gamma_1^\gamma
$$

(38)

From a measurement of these quantities and the spin asymmetries (39) one can obtain information on the structure functions $g_1^\gamma$, $g_1^\gamma^\gamma$, $g_5^\gamma$ and $g_5^\gamma$. Their parton model expressions, using Eqs.(23)-(25) and the quark weak coupling constants,

$$
(g_\nu)_{u,c} = \frac{1}{2} - \frac{4}{3} \sin^2 \theta_w \quad (g_\alpha)_{u,c} = \frac{1}{2}
$$

$$
(g_\nu)_{d,s} = -\frac{1}{2} + \frac{2}{3} \sin^2 \theta_w \quad (g_\alpha)_{d,s} = -\frac{1}{2},
$$

are given by

$$
g_1^\gamma = \frac{2}{9} (\Delta u + \Delta c + \Delta \bar{u} + \Delta \bar{c}) + \frac{1}{18} (\Delta d + \Delta s + \Delta \bar{d} + \Delta \bar{s})
$$

$$
g_1^\gamma^\gamma = \left( \frac{1}{3} - \frac{8}{9} \sin^2 \theta_w \right) (\Delta u + \Delta c + \Delta \bar{u} + \Delta \bar{c})
$$

$$
+ \left( \frac{1}{6} - \frac{2}{9} \sin^2 \theta_w \right) (\Delta d + \Delta s + \Delta \bar{d} + \Delta \bar{s}) \simeq \frac{1}{9} \sum_q (\Delta_q + \Delta \bar{q})
$$

(40)

$$
g_5^\gamma^\gamma = \frac{1}{6} \left[ 2 (\Delta u + \Delta c - \Delta \bar{u} - \Delta \bar{c}) + (\Delta d + \Delta s - \Delta \bar{d} - \Delta \bar{s}) \right]
$$

$$
g_5^\gamma = \frac{1}{2} \left( \frac{1}{2} - \frac{4}{3} \sin^2 \theta_w \right) (\Delta u + \Delta c - \Delta \bar{u} - \Delta \bar{c})
$$

$$
+ \frac{1}{2} \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) (\Delta d + \Delta s - \Delta \bar{d} - \Delta \bar{s})
$$

where the approximate equality holds when assuming $\sin^2 \theta_w = 1/4$. 

In principle, a measurement of the above four structure functions allows to extract information on the four combinations of quark distribution functions $\Delta u + \Delta c$, $\Delta d + \Delta s$, $\Delta \bar{u} + \Delta \bar{c}$ and $\Delta \bar{d} + \Delta \bar{s}$.

Throughout the paper we have assumed the initial leptons to be in pure helicity states (either $\lambda = 1$ or $\lambda = -1$); however, our formulae can easily be modified to take into account leptons with an arbitrary average helicity $\langle \lambda \rangle = P(1) - P(-1)$, 

$$\langle \lambda \rangle = P(1) - P(-1), \quad (41)$$

where $P(\lambda)$ is the probability of helicity $\lambda$. By suitably tuning the value of $\langle \lambda \rangle$ one could extract single structure functions. For example, from $\Delta L_{\sigma N}(\langle \lambda \rangle) = P(1) \Delta L_{\sigma N}(\lambda = 1) + P(-1) \Delta L_{\sigma N}(\lambda = -1)$ 

$$\Delta L_{\sigma N}(\langle \lambda \rangle) = P(1) \Delta L_{\sigma N}(\lambda = 1) + P(-1) \Delta L_{\sigma N}(\lambda = -1) \quad (42)$$

one has

$$\Delta L_{\sigma_{nc}^{\ell N}}(\langle \lambda \rangle) = \frac{g_\gamma}{g_A} + \Delta L_{\sigma_{nc}^{\ell N}}(\langle \lambda \rangle) = -\frac{g_\nu}{g_A} = 32\pi m_N \frac{\alpha^2}{Q^4} x (2 - 2y + y^2)(g_A^2 - g_\nu^2) \eta^z g_5^z. \quad (43)$$

Information on $g_{1}^\gamma^z$ and $g_{5}^\gamma^z$ could also be obtained, as pointed out in [22], by scattering unpolarized leptons off longitudinally polarized nucleons; in this case indeed there is no spin dependent e.m. contribution, and from Eqs.(29) and (35) one has:

$$\Delta L_{\sigma_{nc}^{\ell N}}(\langle \lambda \rangle = 0) = 16\pi m_N \frac{\alpha^2}{Q^4} x \left\{ y(2-y)g_A g_{1}^\gamma^z + (2-2y+y^2)g_\nu g_{5}^\gamma^z \right\}. \quad (44)$$

Eqs.(21), (36), (38), (43) and (44) are only few out of the many particular combinations of cross-sections which one might think of measuring in order to extract new information on the spin structure of nucleons; many others could be derived from our formulae and have not been explicitly presented here. Also, we have written our results always assuming the validity of the parton model predictions for the structure functions, like $g_{i}^4 = 0$, $g_{j}^3 = 2xg_{i}^5$, $g_{j}^3 = 0$, etc.. Of course, from Eqs.(11) and (15), one could obtain the most general expressions for our combination of cross-sections, independently of the parton model results. Such general expressions are somewhat more complicated than the ones shown here in that they contain more structure functions.

All of the measurements suggested here are extremely difficult and require careful and detailed analysis of very high energy deep inelastic scattering experiments; some of them might even be a prohibitive task. However, we think that the possible outcome – a better knowledge of the intimate nucleon structure – would certainly justify a serious consideration of such an analysis.

There is another consideration which increases the phenomenological interest of such measurements: some of the quantities we have dealt with are not affected by the axial anomaly. A general treatment of the perturbative gluonic contributions can be found in Ref.[14] and, for charged current DIS, in Ref.[17]. In the case of neutral current, only the parity violating spin structure functions, the ones which measure the “valence” C-odd spin contribution $\Delta q - \Delta \bar{q}$, like $g_{3}$ and $g_{5}$, appear to be independent of the axial anomaly. From a rigorous point of view, beyond the scope of this paper, only these structure functions are unambiguously defined, scale-independent, and describe well-defined quark spin degrees of freedom. The comparison between these C-odd spin structure functions and the C-even ones could be the only experimental test of whether the anomaly is a large or small $x$ effect [23].
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