Multi-objective reactive power optimization of hybrid AC/DC power system considering power system uncertainty

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Abstract. With the High Voltage Direct Current (HVDC) transmission is connected to the AC system, the uncertainty of converter reactive power consumption has brought certain influence to the reactive power optimization of the power grid. For the uncertainty of converter reactive power consumption and load, the DC reactive power and load are expressed by trapezoidal fuzzy parameters, and the multi-objective fuzzy reactive power optimization model of AC / DC interconnected system is established which optimizes the total network loss and node voltage offset of the AC / DC transmission system. The multi-objective optimization is converted into single-objective optimization by using the objective membership function and the fuzzy algorithm of multi-objective nonlinear objective programming, and the biogeography-based optimization algorithm is used to solve the model. The results of extended 39-node system example show that the proposed model and algorithm are suitable for voltage and reactive power control optimization under the condition that the converter reactive power consumption and load are fuzzy parameters.

1. Introduction
Reactive power optimization of power system is to make the active power network loss minimum, the voltage level best or the reactive power reserve capacity optimal by adjusting the generator terminal voltage, adjustable transformer ratio and the switching of compensation equipment under the premise of meeting the system constraints such as power flow equation, node voltage and generator reactive power output[1]. At present, reactive power optimization is mainly focused on the AC system[2-3]. Tabu search method is applied to reactive power optimization of AC power grid and compared with genetic algorithm[2]. Reference [3] proposes the gradient optimization algorithm to solve the reactive power optimization of AC power system.

HVDC transmission is widely connected to the power system because of its advantages such as long distance transmission and large capacity. Due to the power electronic characteristics of converter stations, it is necessary to absorb a large amount of reactive power from the AC system, which brings adverse effects to the reactive power flow of power system. Therefore, the system network loss and voltage quality can be effectively improved by optimizing and coordinating reactive power and voltage control measures in AC/DC transmission system, and the adverse effects of HVDC transmission can be reduced to a certain extent. Therefore, reactive power optimization of AC / DC transmission systems has been attracted widespread attention[4-7]. Reference [4] proposes the right singular vector index of singular value decomposition method, and the weak node of voltage stability in AC/DC system is identified and used as reactive power compensation point. Reference [5] presents a hybrid algorithm, which uses genetic algorithm to deal with discrete variables and interior point
method to deal with continuous variables, effectively simplifying the traditional AC-DC hybrid power flow calculation method. An improved normalized planar constraint method is proposed to transform multi-objective optimization into a single objective optimization problem for reactive power optimization of AC/DC power systems[6]. Reference [7] presents a multi-objective reactive power optimization control method for AC/DC systems considering the detailed loss characteristics of converter stations.

The above studies mainly focus on the AC/DC reactive power optimization model under deterministic conditions, but there are a lot of uncertainties in power system [8]. Therefore, reactive power optimization of AC / DC system considering system uncertainties should be the direction of future research. Based on the normal distribution characteristics of load errors, a multi-objective reactive power optimization model for AC/DC systems considering load uncertainty is proposed by using the method of bilevel programming theory[9]. However, the load is often fuzzy, and the expression of fuzzy parameters is more suitable [10-12]. Moreover, the uncertainty in the operation of the converter station is not considered in the above paper[13].

Based on the above research, for the uncertainties of load and reactive power consumption of converter stations in AC/DC system, a multi-objective fuzzy optimization model of AC/DC reactive power is constructed with the total network loss and voltage offset as the multi-objective. The biogeography-based optimization algorithm is used to optimize the model. The results of the example show the feasibility of the proposed model and algorithm.

2. Fuzzy characteristics of AC/DC power system

2.1 Fuzzy characteristics of load
Real-time data are needed in reactive power optimization of power system. Because of the large number of nodes or investment, the real-time data of each node load can not be obtained in power grid operation. Some data can only be obtained from historical data or forecast data, so load data can only be regarded as fuzzy information of current operation data. Trapezoidal membership function is widely used in power system because of its rich theoretical basis and accord with people subjective judgment of thing in reality [10-12]. The trapezoid membership function of load is expressed by the predicted value (or historical value) and the four proportional parameters.

\[ P = (P_1, P_2, P_3, P_4) = P_0(w_1, w_2, w_3, w_4) \]  

where, \( P_0 \) represents the reference value (predicted or historical value) of load, and \( w_k \) is proportional parameter, \( k = 1, 2, 3, 4 \).

2.2 Fuzzy characteristics of DC
The AC system and DC system are mainly connected by converter. According to the principle of converter, the formula of reactive power consumption by converter as follows:

\[ Q_{dc} = P_{dc} \tan \phi \]  

\[ \tan \phi = \left( \frac{\pi}{180} \right) \mu - \sin \mu \cos \frac{2\alpha + \mu}{\sin \mu} \]  

\[ \mu = \cos^{-1}\left[ \frac{U_{d_0}}{U_{d_0} - \left( X_c / \sqrt{2} \right) I_d / E_{11}} \right] - \alpha \]  

\[ U_d / U_{d_0} = \cos \alpha - \left( X_c / \sqrt{2} \right) I_d / E_{11} \]  

where, \( U_{d_0} \), \( P_{dc} \), \( Q_{dc} \), \( \phi \), \( \mu \), \( X_c \), \( I_d \), \( \alpha \), \( E_{11} \), \( U_d \) are ideal no-load voltage, DC side power, reactive power consumption, power factor angle, commutation angle, per-phase commutation reactance, DC operating current, rectifier trigger angle, valves side winding no-load voltage, DC voltage.

It can be seen from the above formula that it needs to obtain reactive power from the AC system no matter whether the converter station is in rectifier or inverter operation state, that is, the converter station is a reactive load for the AC system. Moreover, the reactive power consumed by the converter
station is not only affected by the active power, but also related to many operating parameters. The reactive power consumption calculated by the above formulas is all calculated under certain equipment, operation and control parameters. In actual operation, the converter voltage is uncertain because of some parameters, such as bus voltage uncertainty and converter voltage, in addition to the given operational control and other deterministic parameters, when calculating the reactive power consumption of the converter station. Impedance tolerance of converter, inaccuracy of control parameters caused by some measurement data and control errors will lead to uncertainty of reactive power consumption of converter. Ref. [13] analyzes the reactive power consumption of converter under different working conditions and considerations in detail, as shown in Table 1.

Table 1. Reactive power consumption by rectifier side under normal operation.

| Operation mode | Consideration of factors | Reactive power(Mvar) |
|----------------|--------------------------|----------------------|
| Rated operating mode bipolar, ±500 kV 3000 MW | without consideration | 1492 |
| | considering control range | 1635 |
| | and considering measurement error | 1651 |
| | considering all factors | 1705 |

It can be seen that rectifier have different reactive power consumption under different operating conditions. Therefore, the trapezoidal membership function can be used to represent the uncertainty of reactive power dissipated by four different parameters, and the reference value can be expressed by the mean of the middle two values.

3. Multi-objective reactive power fuzzy optimization model

The multi-objective fuzzy chance-constrained reactive power optimization model makes use of the reliability theory measure parallel to the probability measure in probability theory, which can make up for the shortage of the possibility measure to express the risk and is widely used[12]. Based on the above model, a simpler expectation model is proposed to solve the complex problem of objective function [14].

Under the condition of credible constraints, the mathematical model of minimizing the uncertain objective function is minimized. The expected model is as follows

\[
\min E[f(\chi, \xi)] \\
\text{s.t.} \\
g_r(\chi, \xi) \leq 0, i = 1, 2, ..., p
\]

where, \(\chi\) is decision vector, \(\xi\) is fuzzy variable, \(f(\chi, \xi)\) is uncertain objective function, \(g_r(\chi, \xi)\) are constraint function.

The objective function \(F = \min E[f_{\text{loss}}, f_{\text{VD}}]\)

\[
f_{\text{loss}} = \sum_{i,j \in N_L} g_i(U_i^2 + U_j^2 - 2U_i U_j \cos \theta_{ij})
\]

\[
f_{\text{VD}} = \sum_{i \in N_{PQ}} (U_i^{\text{max}} - U_i^{\text{min}})^2
\]

where, \(f_{\text{loss}}\) is loss of total network, \(f_{\text{VD}}\) is voltage offset. \(U_i\), \(U_j\), \(U_{\text{spec}}\), \(U_{\text{max}}\), \(U_{\text{min}}\) are the voltage amplitude, voltage reference value, upper limit and lower limit of the voltage of node \(i\). \(N_L\), \(N_{PQ}\) are the set of transmission lines and the set of PQ nodes.
Equality constraint, the AC power flow equation:

\[
\begin{align*}
P_i - U_i \sum_{j=1}^{N_{PV}} (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0 & \quad i \in N_{PV} \\
Q_i - U_i \sum_{j=1}^{N_{PV}} (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) = 0 & \quad i \in N_{PV}, N_{PV}
\end{align*}
\]  

(8)

where, \(P_i, Q_i\) are the active power and reactive power, \(G_{ij}, B_{ij}\) are the conductance and the admittance between the nodes \(i, j\), \(N_{PV}\) are the PV node set.

The power balance equation of the AC node connected to the converter station is

\[
\Delta P_z = P_z^e - P_z^{(dc)} - P_z(U, \theta) = 0
\]

\[
\Delta Q_z = Q_z^e - Q_z^{(dc)} - Q_z(U, \theta) = 0
\]

(9)

where, the formula \(z\) represents the AC node connected with the converter station, \(P_z, Q_z\) are the active and reactive power of the AC side respectively, and \(P_z^e, Q_z^e\) are the active and reactive power given by the node respectively. \(P_z^{(dc)}, Q_z^{(dc)}\) are the active and reactive power of DC system is injected into the AC system.

Control variable constraint:

\[
\begin{align*}
U_{\min}^i \leq U_i \leq U_{\max}^i & \quad i = 1, 2, \ldots, N_G \\
Q_{\min}^i \leq Q_i \leq Q_{\max}^i & \quad i = 1, 2, \ldots, N_C \\
T_{\min}^i \leq T_i \leq T_{\max}^i & \quad i = 1, 2, \ldots, N_T
\end{align*}
\]  

(10)

where, \(N_G, N_C, N_T\) are the number of generator nodes, the number of compensation capacitors and adjustable transformers, \(U_{\min}^i, U_{\max}^i, Q_{\min}^i, Q_{\max}^i, T_{\min}^i, T_{\max}^i\) are respectively the upper and lower bounds of corresponding control variables.

State variable constraint:

\[
\begin{align*}
\text{Cr}(U_{\min}^i \leq U_i \leq U_{\max}^i) & \geq \alpha, i = 1, 2, \ldots, N_{PV} \\
\text{Cr}(Q_{\min}^i \leq Q_i \leq Q_{\max}^i) & \geq \alpha, i = 1, 2, \ldots, N_G
\end{align*}
\]  

(11)

where, \(Q_{\min}^i, Q_{\max}^i\) the upper and lower limits of the reactive power output of the corresponding generators respectively.

4. Solution of multi-objective fuzzy optimization model

4.1. Fuzzy power flow

There are fuzzy variables in the objective function and constraints of the fuzzy value model. How to get the expected value and credibility of the fuzzy power flow is the key to solving the model. The fuzzy membership function is used to solve the credible distribution of state variables in this paper. The calculation process is as follows:

1. Formulas (2), (3), (4), (5), (8) and (9) are used to form the power flow equations of AC/DC systems.
2. Without considering the fuzzy variables of the system, the power flow of AC/DC system with reference value is solved by Newton method\[15\]. and \(V_d, \theta_d, P_{\text{load}}, V_{\text{Ref}}\) are the reference value of node voltage, voltage phase angle, power loss, voltage offset. The subscript \(d\) is the reference value of the corresponding variable.
3. Solution of fuzzy increment of active and reactive power injected into nodes \(\Delta P, \Delta Q\).
4. Using the Jacobi matrix of Newton power flow algorithm to solve the fuzzy increment of node voltage and phase angle \(\Delta P, \Delta Q\).

\[
\begin{align*}
\Delta \theta &= H \Delta P \\
\Delta V &= L \Delta Q
\end{align*}
\]  

(12)
(5) Solving fuzzy variables $\theta$. 

(6) Solving fuzzy variables $Q$, $V_{FD}$ and $P_{loss}$.

$$\Delta y = \sum \frac{\partial y}{\partial x_j} \Delta x_j$$

(13)

where, $y$ are reactive power output, voltage deviation and network loss of power system, and $x_j$ and $y$ fuzzy variables for corresponding systems. After solving the fuzzy increment corresponding to the state variable, the membership function of the objective function is solved by expression 14.

$$y = y_d + \Delta y$$

(14)

(7) The reliability distribution of state fuzzy variable and the expected value of objective function fuzzy variable are solved by the relationship between membership function, reliability measure and expected value[14].

4.2 Multi-objective processing strategy

Pareto optimal frontier is the optimal solution set in the feasible domain, because of its large amount of calculation, and in practical problems, decision makers often make decisions according to demand which increases the amount of calculation. Therefore, this paper uses fuzzy programming to transform multi-objective optimization into single objective optimization [16].

Fuzzy programming needs to solve two key problems: 1) Fuzzy processing is achieved by establishing membership function of objective function; 2) Fuzzy operator is used to synthesize different objectives, and then the overall satisfaction is formed to obtain Pareto optimal solution. In this paper, linear functions are used to construct membership functions. If the decision maker is not satisfied with the local effective solution, the effective solution can be updated by changing the objective membership value until the satisfactory solution is obtained.

4.3 Biogeography-based optimization algorithm

The biogeography-based optimization algorithm (BBO) is an optimization algorithm based on swarm intelligence which is just put forward in 2008. It has the advantages of fewer parameters, simple calculation and fast convergence speed[17].

BBO algorithm simulates the mechanism of species migration between habitats. For solving the optimization problem, BBO constructs multiple habitats randomly as the initial solution of the optimization problem, and interacts information through species migration between habitats to improve the habitat. The species diversity of the land is improved, and the HSI of the habitat is improved, thus obtaining the optimal solution of the problem.

5. Example analysis

5.1 Description of the test systems

To verify the effectiveness of the proposed method, an improved IEEE 39-bus system is used to verify the proposed method. The system wiring diagram is shown in Figure 3. HVDC transmission system are introduced. The rectifier side is node 18, the inverter side is node 6. The IEEE 39 extended example is shown in Figure 1. The DC transmission system is controlled by constant current and constant power. The initial DC current is 1.0 p.u, the arc extinguishing angle is 8.0 degrees, and the upper and lower limits of node voltage are 1.10 and 0.92 respectively.
5.2 Analysis of optimization results

As shown in Table 2, the proposed uncertainty model is superior to the deterministic model in terms of network loss and voltage offset expectations. Fig. 2 compares the membership functions of the objective function of the uncertain model and the deterministic model. It can be seen that this model is more applicable in uncertain environment.

| Model                     | The network loss (pu.) | The voltage offset (pu.) |
|---------------------------|------------------------|--------------------------|
| The proposed uncertain model | 0.9231                 | 1.2273                   |
| The deterministic model    | 0.9285                 | 1.2652                   |

Figure 2 indicates the comparison of node voltages before and after optimization. The voltage of nodes before optimization is lower, especially the converter nodes (8 node and 16 node) connected with DC which near the lower limit of voltage. After optimization, the voltage quality of nodes is obviously improved.
Figure 4 shows the convergence curve of algorithms that particle swarm optimization (PSO) algorithm is easy to fall into local optimum and get local minimum. Although BBO converges slowly, it improves the global search ability and is superior to the PSO in search accuracy. Therefore, the algorithm used in this paper has better convergence and robustness.

6. Conclusions
Considering the influence of uncertainties on power system, this paper establishes the expected value model based on the credibility theory, and transforms the multi-objective function into a single objective function by using the fuzzy goal method of fuzzy programming theory, and solves the problem by using the BBO algorithm. A feasible reactive power optimization control scheme is obtained. The example analysis shows that the expected value model and algorithm proposed in this paper can effectively reduce the active power loss of the system and improve the voltage quality of the nodes.

References
[1] Grudinn N. Reactive power optimization using successive quadratic programming method, IEEE Trans. on Power Systems 4, 13(1998)
[2] Yutian L. Li M. Reactive power optimization based on Tabu search approach, Automation of Electric Power Systems 2, 24(2000)
[3] Hong Y. Sun D. Lin S. et al. Multi-year multi-case optimal VAR planning, IEEE Trans on PS 4, 5(1990)
[4] Tao D. Huaqiang Li. Pei F. Reactive Power Optimization of AC/DC System Based on Singular Value Decomposition and Interior Point Method, Transactions of China Electrotechnical Society 2, 24(2007)
[5] Ping Jiang . Le L. Combining Internal Point Method and Genetic Algorithm for AC/DC System Reactive Power Optimization, High Voltage Technology 3, 41(2015)
[6] Qing L. Mingbo L. Liuqing Y. INNC Method for Multi-Objective Reactive Power Optimization of AC/DC Interconnected Power Grid, Proceedings of the CSEE 7, 34(2014)
[7] Xin L . Zhibin Y. et al. Multi-objective reactive power optimal control of AC-DC systems including power loss characteristics of converter stations, Power System Protection and Control 9, 45(2017)
[8] Hongxin L. Study on voltage stability and reactive power optimization of power system considering uncertainties, Wu Han: Huazhong University of Science and Technology, 2013.
[9] Hong F. Xinbin J. et al. Multi-objective reactive power optimization of hybrid AC/DC power system considering load uncertainty, Electrical Measurement & Instrumentation 10, 55(2018)
[10] Junying S.Dichen L. Runping C. A fuzzy model and approach for power system reactive power optimization. *Power System Technology* 3,25(2001)
[11] Kun Y. Yijia C. Xingying C. et al. Reactive power and voltage optimization of the district grid with distributed generators, *Automation of Electric Power Systems* 8, 35(2011)
[12] Wenxue L. Jun L.Zhihao Y. et al. Multiobjective fuzzy chance constrainer optimal reactive power flow based on credibility theory, *Transactions of China Electrotechnical Society* 21, 30(2015)
[13] Wanjun Z. HVDC transmission engineering technology, Beijing: China power press, 2004
[14] Baoding L. Uncertain planning and application, Beijing:Tsinghua university press, 2003
[15] Zheng X. Dynamic behavior analysis of AC / DC power system,Beijing:Machinery industry press, 2004
[16] Jiuping X. Theory and method of multiobjective decision making, Beijing: Tsinghua university press press, 2005
[17] SIMON D. Biogeography-based optimization, *IEEETransaction on Evolutionary Computation* 6, 1(2008)