η GEMINORUM: AN ECLIPSING SEMIREGULAR VARIABLE STAR ORBITED BY A COMPANION SURROUNDED BY AN EXTENDED DISC

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ABSTRACT

We report 11 yr of spectroscopic monitoring of the M-type asymptotic giant branch star η Gem, a semiregular variable and a known spectroscopic binary with a period of 8.2 yr. We combine our radial velocities with others from the literature to provide an improved spectroscopic orbital solution giving a period of 2979 days, which we then use to predict past times of eclipse. We examine archival photometry from amateur variable star observers, and other sources, and find many instances of dimmings that occurred at the right time. This confirms previous indications that the system is eclipsing, and it now ranks among those with the longest known periods. No secondary eclipses are seen. The \~0.4 mag eclipses lasting about 5 months are much too deep to be produced by a stellar companion. We propose instead that the companion is surrounded by a large disk that is at least 1.5 au in diameter, but is likely larger. We predict the center of the next eclipse will occur on New Year’s day, 2029.

Keywords: binaries: eclipsing – binaries: spectroscopic – stars, individual: η Geminorum – stars, variables: general – techniques: radial velocities.

1. INTRODUCTION

η Geminorum (HD 42995, HR 2216) is a late-type giant (M3.5 Ib-II; Abt 1985) and a well known semiregular variable of type SRA, with a fairly consistent periodicity but with modest changes in the amplitude and shape of the light curve. The photometric period is about 230 days (see, e.g., Hofmeister 1913, Vogelzang 1928, van Schewick 1950, Percy et al. 1996, 2001). η Gem has a visual companion approximately 3 mag fainter discovered in 1881 by Burnham (1887), which is currently at a separation of 1.8 arcsec. The binary star designation is WDS J06149+2230. The orbital period has been estimated to be about 470 yr (Baize 1980). It is also a spectroscopic binary, which makes the system a hierarchical triple. The velocity changes were first noticed by astronomers at the Lick Observatory in 1901 after the third observation (Campbell 1902), and a provisional orbital orbit was published by Christie (1936), although the reported period of 1875 days was erroneous. The correct period was established a few years later in a study by McLaughlin & van Dijke (1944), who derived an eccentric orbit \((e = 0.53)\) with a period \(P \approx 2983\) days, or 8.2 years.

In the same study those authors reported contemporaneous photometric observations over about 13 years, and pointed out that the two deepest minima, in 1931 and 1939, happened to be close to the times at which primary eclipses would be expected from their spectroscopic orbit, should the system be eclipsing. However, given the very wide separation of the components, they did not consider the coincidence to be conclusive. If confirmed, η Gem would rank among the longest period eclipsing binaries known, even today. van Schewick (1950) identified two other relatively deep minima in earlier brightness measurements that seemed to fit the pattern suggested by McLaughlin & van Dijke (1944), and proceeded to infer a photometric period of 2984 days, essentially identical to the one from the spectroscopic orbit. Neither period was given with an associated uncertainty.

After the 1950s, it seems that most studies dealing with other aspects of η Gem have generally assumed that the system is eclipsing, although others (e.g., Woolf 1973, Hassforther 2007) remained skeptical. Amateur variable star observers have been more confident, organizing several observing campaigns over the years hoping to record dips in brightness at the expected times, with partial success.

To our knowledge there has been no definitive study of the persistence of eclipses, although by now there are decades more of observations by both amateur and professional variable star observers that are potentially useful to that end. This is therefore one of the main goals of this paper. Additionally, there has been no modern redetermination of the spectroscopic orbit of η Gem after the one published nearly 80 years ago by McLaughlin & van Dijke (1944). That analysis relied on radial velocity measurements of considerably poorer precision than today’s instruments can deliver. Consequently, as a second goal of this paper we monitored the object spectroscopically over a period of 11.5 years (1.4 cycles of the spectroscopic orbit), aiming to confirm and improve upon the previous orbit. Finally, it is of interest to discuss in some detail the nature of the companion, in the context of other eclipsing binaries of very long period such as the well known systems ε Aur \((P = 27.1\) yr) and EE Cep \((P = 5.6\) yr), or the more recently discovered TYC-2505-672-1 system \((P = 69.1\) yr; Rodríguez et al. 2016; Lipunov et al. 2016). In all three of them, and in a

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See, e.g., Bohme (1980, 1989), and the following notes from the British Astronomical Association, Variable Star Section: http://www.britastro.org/vss/The2004EclipseofEtaGeminorum.pdf and http://www.britastro.org/vss/EtaGem_VSOtY2012.pdf
few others, the eclipsing object is surrounded by a large opaque disc. We will show that the evidence in \( \eta \) Gem points to a similar conclusion.

Our spectroscopic observations are presented in Section 2. In Section 3 we combine our radial velocities with other measurements from the literature to derive an improved spectroscopic orbit for \( \eta \) Gem. The extensive photometric observations available for the object are discussed in Section 4, from which we compile the most complete list of dimming events occurring near the times of primary eclipse, as predicted from our updated spectroscopic orbit. These instances leave no doubt about the eclipsing nature of \( \eta \) Gem, and allow a substantial improvement in the ephemeris of the system. The nature of the secondary is discussed in Section 5. Final remarks are given in Section 6.

2. SPECTROSCOPIC OBSERVATIONS

\( \eta \) Gem was placed on the observing program at the Center for Astrophysics (CfA) as part of an effort to monitor about a dozen semiregular variables. Observations began in September of 1993, and continued until April of 2005 with two nearly identical instruments on the 1.5m Wyeth reflector at the (now closed) Oak Ridge Observatory (Massachusetts, USA) and the 1.5m Tillinghast reflector at the Fred L. Whipple Observatory (Arizona, USA). These echelle instruments (Digital Speedometers; \( \text{Latham} [1992] \) used intensified photon-counting Reticon detectors that recorded a single order 45 Å wide centered at a wavelength of 5187 Å, featuring the Mg I b triplet. The resolving power was \( R \approx 35,000 \). We gathered 137 spectra with the Wyeth reflector, and 6 with the Tillinghast reflector. The signal-to-noise ratios (SNRs) range from 10 to 80 per resolution element of 8.5 km s\(^{-1}\). Wavelength solutions relied on exposures of a thorium-argon lamp bracketing each science exposure. The zero point of the velocity system was monitored with observations of the sky early in the evening and in the morning, and small run-to-run corrections based on them were applied as described by \( \text{Latham} [1992] \), and were usually smaller than 2 km s\(^{-1}\).

Our typical procedures for deriving radial velocities from similar material involve cross-correlation against a synthetic template, using the XCSAO task running under \text{IRAF} \(^{4}\) The late spectral type of \( \eta \) Gem is such that the spectrum is nearly saturated with molecular features, which theoretical models still struggle to reproduce. Although we had initially adopted a template from \( \text{Husser et al.} [2013] \) based on PHOENIX models, we found better results using instead a high-SNR spectrum of the same star as the template. The scatter from our orbital solution described later was smaller, and the correlation coefficients higher. To transfer these relative velocities to an absolute frame of reference, we added an offset derived by cross-correlating the observed template against the best-matching synthetic PHOENIX spectrum with a temperature of \( T_{\text{eff}} = 3700 \text{ K} \), \( \log g = 0.0 \), solar metallicity, and no rotational broadening. While the transfer to the absolute frame may not be perfect because the lines in the synthetic template do not exactly match those in \( \eta \) Gem, we are only concerned here with changes in the radial velocities, so a small systematic offset is of no consequence for this work. The velocities derived in this manner are presented in Table 1 along with their formal uncertainties as returned by XCSAO.

3. SPECTROSCOPIC ORBIT

Our main reason for attempting to improve upon the spectroscopic orbital solution of \( \text{McLaughlin \& van Dijke} [1944] \) is to use predictions for the times of eclipse to identify dips in brightness in the photometric observations going back a century or more, which might reasonably be interpreted as eclipse events. An initial orbital solution using our velocities gave elements quite similar to those of \( \text{McLaughlin \& van Dijke} [1944] \), with an uncertainty in the period of 9 days and an uncertainty in the time of periastron passage of 23 days. Projected backward to the earliest brightness measurements from the 1870s, the eclipse predictions are only good to about 140 days, a consequence of the fact that our observations cover just slightly over 1.4 binary cycles. This can be improved significantly by considering other sources of radial velocities together with ours.

Four small sets of historical measurements for \( \eta \) Gem are available between the years of 1900 and 1922: 11 velocities from the Lick Observatory (\( \text{Campbell} [1928] \), 3 from the Cape Observatory (\( \text{Lunt} [1919] \) of which the first is clearly erroneous and was rejected, 4 from the Mount Wilson Observatory (\( \text{Abt} [1970] \) and 2 from the Dominion Astrophysical Observatory (\( \text{Harper} [1934] \). To the extent possible we transformed them all to the system of the Lick measurements by applying small offsets published by \( \text{Moore} [1932] \), and we assigned them all uncertainties of 1 km s\(^{-1}\). These four groups of observations, and one additional measurement described below, were considered as a single data set. We also used the 142 velocities of \( \text{McLaughlin \& van Dijke} [1944] \) made between 1930 and 1941, with larger adopted uncertainties of 2 km s\(^{-1}\). For the next 50 years \( \eta \) Gem appears to have been almost completely neglected by spectroscopists: we have located only a single radial velocity measurement reported by \( \text{Beavers \& Eitter} [1986] \), made at the Fick Observatory in 1979, which the authors describe as being on the same system as the Lick velocities. We assigned it an uncertainty of 1 km s\(^{-1}\), and added it to the first data set for simplicity. A more recent and extensive set of 129 high-quality measurements was reported by \( \text{Eaton} [2020] \) covering the years 2003–2008. This partly overlaps with our own observations, and extends the total

| HJD (2,400,000+) | \( RV \) (km s\(^{-1}\)) | \( \sigma_{RV} \) (km s\(^{-1}\)) | Orbital Phase |
|------------------|-----------------|----------------|----------------|
| 49260.9110       | 21.72           | 0.21           | 0.602          |
| 49284.8984       | 22.55           | 0.32           | 0.610          |
| 49289.8034       | 22.17           | 0.21           | 0.612          |
| 49295.9109       | 21.81           | 0.21           | 0.614          |
| 49313.8304       | 22.29           | 0.21           | 0.620          |

Note. — Orbital phases were computed from the ephemeris given in Section 3. (The full version of this table is provided as supplementary material.)

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\(^{4}\) \text{IRAF} is distributed by the National Optical Astronomy Observatories, which is operated by the Association of Universities for Research in Astronomy, Inc., under contract with the National Science Foundation.
spectroscopic coverage by three years. Uncertainties for these measurements were taken to be 0.2 km s\(^{-1}\).

A joint orbital fit using all radial velocities was carried out with a Markov chain Monte Carlo (MCMC) procedure, using the emcee\(^5\) package of Foreman-Mackey et al.\(^6\). We used uniform priors over suitable ranges for all jump variables. In addition to the standard orbital elements \(P\), \(T_{\text{peri}}\), \(K_1\), and \(\gamma\), we elected to use eccentricity parameters \(\sqrt{e}\cos\omega_1\) and \(\sqrt{e}\cos\omega_2\) instead of \(e\) and \(\omega_1\), where \(\omega_1\) is the argument of periastron for the primary. Because there are four separate data sets that may be offset from one another, we solved for a different center-of-mass velocity for each one: \(\gamma_{\text{old}}\), \(\gamma_{\text{McL}}\), \(\gamma_{\text{CIA}}\), and \(\gamma_{\text{Eat}}\). Additionally, we solved for four radial velocity “jitter” terms \(\sigma_{j,\text{old}}\), \(\sigma_{j,\text{McL}}\), \(\sigma_{j,\text{CIA}}\), and \(\sigma_{j,\text{Eat}}\), to be added quadratically to the internal errors in order to account not only for the possibility that errors may be underestimated, but also for the extra scatter that may be caused by the \(\sim\)230-day pulsations in \(\eta\) Gem.

The results are given in the second column of Table 2 (Solution 1), together with derived properties listed at the bottom. The benefit of adding the other data sets to ours is seen in the significant reduction in the uncertainties for the period (a factor of 9) and time of periastron passage (a factor of 4). Predictions for the times of eclipse are about a factor of 7 better than before. All radial velocity observations are shown with our best-fitted model in Figure 1.

4. ECLIPSES

\(\eta\) Gem has been a target of variable star observers for a century and a half. Several organizations over the world have systematically collected those observations, made mostly by amateur astronomers, and in some cases they have made efforts to homogenize them. Here we consider four large sets of brightness measurements from the American Association of Variable Star Observers (AAVSO, \(\sim\)28,200 measurements)\(^7\), the Association Française des Observateurs d’Étoiles Variables (AFOEV, \(\sim\)10,800 measurements)\(^8\), the British Astronomical Association–Variable Star Section (BAA-VSS, \(\sim\)6,400 measurements)\(^9\), and the Variable Star Observers League in Japan (VSOLJ, \(\sim\)6,100 measurements)\(^10\), all with data as of June of 2022. We have used only observations in or near the visual band, which is the most common. In addition we have also examined shorter sets of data by McLaughlin & van Dijke\(^1\) (1944), Percy et al.\(^1\) (2001), and from the Czech Astronomical Society–Variable Star and Exoplanet Section (CAS-VSES)\(^2\). When possible, for the largest data sets we have separated the measurements provided with one decimal place (mostly made visually) from the more precise ones given to two or more decimal places. This is to guard against the possibility of slight differences in the magnitude scales. The format provided for the BAA-VSS observations did not allow this distinction. In most of these data sets, but especially among the more precise data, brightness fluctuations due to the \(\sim\)230-day semiregular pulsations are obvious and have total amplitudes of up to 0.5 mag in the visual (see, e.g., Cristian et al.\(^3\) 1995, Percy et al.\(^4\) 1996, Hassforther\(^5\) 2007).

Armed with the improved spectroscopic orbital elements from the previous section, we examined these sources of photometric observations and identified many instances where light minima were close to the predicted times of eclipse (see Figure 2). This leaves no doubt as to the eclipsing nature of \(\eta\) Gem. In all cases the events are primary eclipses; no compelling case of a secondary eclipse was found (secondary minima are expected about 504 days later than primary eclipse, though with somewhat larger uncertainties). We note that because of the intrinsic variability of the star, the primary minima vary in depth depending on whether or not the drop in brightness from the eclipse is superimposed with a change due to the semiregular variations. The eclipse centers may also be affected for the same reason. The gray shaded areas in the top third of each panel in Figure 2 mark the predicted times of eclipse and their formal uncertainties.

Eclipse timings provide a much stronger constraint on the ephemeris of a binary than radial velocities\(^11\). To take advantage of this information, we measured the center of each event in Figure 2 by eye, and assigned realistic uncertainties. These determinations are shown with red shaded areas in the lower two thirds of each panel, and are listed in Table 3. Two additional measurements were made graphically from series of observations in 1980 and 1988 shown in figures by Bohme\(^6\) (1980, 1989), and were added to the table. We have not included four additional timings reported by van Schewick\(^7\) (1950) because they are given without uncertainties, and we do not have access to the original observations. The dates of those events are JD 2,411,452, 2,423,387, 2,426,360, and 2,426,375.

We then performed a final orbital solution in which we combined the radial velocity measurements with the eclipse timings in Table 3. A scale factor \(f_{\text{lim}}\) for the formal timing errors was included as an additional adjustable parameter, with a log-uniform prior. The results are shown in the last column of Table 2 (Solution 2). The precision of the period is improved by more than a factor of 2 compared to the previous fit. Residuals for the timings are listed in Table 3.

5. DISCUSSION

Knowledge of the spectroscopic orbit of \(\eta\) Gem provides information about the mass of the companion. If we assume an edge-on orientation, which must be very close to the truth given the wide separation, then for

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\(^5\) https://emcee.readthedocs.io/en/stable/index.html
\(^6\) https://www.aavso.org/
\(^7\) https://cdsarc.u-strasbg.fr/ftp/pub/afoev/
\(^8\) https://britastro.org/photdb/data.php
\(^9\) http://vsoii.jetmis-net.org/database.html
\(^10\) http://var2.astro.cz/ER/
\(^11\) We note in passing that the AAVSO observations contain a valuable subset of measurements from the 1870s originally made by Edward Schoenfeld (1828–1891), and published by Valentinier (1908). These are the oldest publicly available brightness measurements for \(\eta\) Gem. They were later reprocessed by Burnashev & Burnasheva (2009) using modern magnitudes for the comparison stars, and as a result the precision is now considerably improved. These observations happen to be the first to have recorded an eclipse of \(\eta\) Gem, in January of 1874, which therefore carries the highest weight for improving the ephemeris. This important event seems to have been overlooked by previous investigators, even though the observations are contained in the public AAVSO archive.
a value of \( \theta \) to measure the angular diameter of the star, as follows.

Parallax determinations for \( \eta \) Gem have been problematic: the isolated 1979 measurement of Beavers & Eitter (1986) that we have included as part of the “historical” data set is not shown, for clarity. The solid curve is our best-fitted model described in Section 3 that incorporates the eclipse timings. They also performed a fit to the spectral energy distribution of the star to estimate the bolometric flux, \( F_{\text{bol}} \). From the angular diameter and flux they derived an effective temperature, changes in brightness across its surface have the potential to alter the center of light, affecting the parallax determinations (e.g., Mozurkewich et al. 2003; Richichi & Calamai 2003; Mondal & Chandrasekhar 2005).

Table 2

| Parameter | Solution 1 | Solution 2 |
|-----------|------------|------------|
| \( P \) (day) | 2978.4 ± 1.2 | 2978.90 ± 0.49 |
| \( T_{\text{peri}} \) (HJD – 2,400,000) | 44488.4 ± 5.5 | 44489.1 ± 4.5 |
| \( \sqrt{g} \cos \omega_1 \) | -0.753 ± 0.0044 | -0.7382 ± 0.0030 |
| \( K_1 \) (km s\(^{-1}\)) | 9.417 ± 0.086 | 9.438 ± 0.084 |
| \( \gamma_{\text{CIA}} \) (km s\(^{-1}\)) | +17.597 ± 0.062 | +17.601 ± 0.063 |
| \( \gamma_{\text{McL}} \) (km s\(^{-1}\)) | +18.96 ± 0.30 | +18.93 ± 0.28 |
| \( \gamma_{\text{ext}} \) (km s\(^{-1}\)) | +17.43 ± 0.25 | +17.42 ± 0.25 |
| \( \sigma_{\gamma_{\text{CIA}}} \) (km s\(^{-1}\)) | 0.683 ± 0.052 | 0.679 ± 0.051 |
| \( \sigma_{\gamma_{\text{McL}}} \) (km s\(^{-1}\)) | 0.74 ± 0.39 | 0.71 ± 0.38 |
| \( \sigma_{\gamma_{\text{ext}}} \) (km s\(^{-1}\)) | 2.05 ± 0.25 | 2.05 ± 0.25 |
| \( f_{\text{lim}} \) | 0.924 ± 0.065 | 0.929 ± 0.066 |

Derived quantities

| Parameter | Solution 1 | Solution 2 |
|-----------|------------|------------|
| \( \epsilon \) | 0.5463 ± 0.0061 | 0.5507 ± 0.0041 |
| \( \omega_1 \) (deg) | 174.13 ± 0.85 | 174.20 ± 0.82 |
| \( f(M) \) (\( M_\odot \)) | 0.1515 ± 0.0044 | 0.1509 ± 0.0042 |
| \( M_2 \sin i/(M_1 + M_2)^{2/3} \) (\( M_\odot \)) | 0.5330 ± 0.0052 | 0.5324 ± 0.0049 |
| \( a_1 \sin i \) (10\(^8\) km) | 323.0 ± 3.2 | 322.7 ± 3.0 |

Note. — Solution 1 uses only radial velocities. Solution 2 adds the eclipse timings from Table 3. The \( \sigma \) symbols are jitter terms added quadratically to the internal velocity errors, and \( f_{\text{lim}} \) represents a scale factor for the eclipse timing measurements. Posterior distributions for the derived quantities are constructed from those of the quantities involved.

Figure 1. Radial-velocity measurements for \( \eta \) Gem from various sources, as labelled. The isolated 1979 measurement of Beavers & Eitter (1986) that we have included as part of the “historical” data set is not shown, for clarity. The solid curve is our best-fitted model described in Section 3 that incorporates the eclipse timings.
Figure 2. Primary eclipses of η Gem recorded in archival photometric measurements since 1874, as labelled. The gray shaded areas in the top third of each panel mark the predicted eclipse times from our Solution 1 in Table 2, with their uncertainty. The red shaded areas in the bottom two thirds represent the times of minimum estimated by eye, with their uncertainties. These are the timing measurements incorporated into Solution 2 (along with two others; see the text). Two of the panels with no gridding and no lower shaded area show dimmings that happened approximately at the expected times, but were considered to be too poorly defined to measure and use.

Table 3
Primary Eclipse Timing Measurements for η Gem

| Cycle | Year | Time of Eclipse (JD−2,400,000) | σ (day) | O−C (day) | Source |
|-------|------|--------------------------------|--------|----------|--------|
| −13   | 1874 | 5537                           | 8      | −2       | AAVSO (two decimals) |
| −5    | 1939 | 29376                          | 10     | +5       | McLaughlin & van Dijke (1944) |
| −5    | 1939 | 29360                          | 20     | −11      | VSOJL (one decimal) |
| −1    | 1971 | 41281                          | 7      | −5       | BAA-VSS |
| 0     | 1980 | 44274                          | 7      | +9       | AAVSO (one decimal) |
| 0     | 1980 | 44271                          | 10     | +6       | BAA-VSS |
| 0     | 1980 | 44272                          | 6      | +7       | AFOEV (one decimal) |
| 1     | 1988 | 47253                          | 20     | +15      | Böhm (1980) |
| 1     | 1988 | 47254                          | 20     | +20      | AAVSO (one decimal) |
| 1     | 1988 | 47253                          | 7      | +9       | Böhm (1989) |
| 4     | 2012 | 56185                          | 10     | +4       | BAA-VSS |
| 5     | 2020 | 59160                          | 10     | 0        | AAVSO (one decimal) |
| 5     | 2020 | 59146                          | 7      | −14      | AAVSO (two decimals) |
| 5     | 2020 | 59174                          | 6      | +14      | BAA-VSS |
| 5     | 2020 | 59156                          | 8      | −4       | VSOJL (one decimal) |
| 5     | 2020 | 59147                          | 7      | −13      | VSOJL (two decimals) |

Note. — Cycle numbers in the first column are counted from the primary eclipse nearest to the reference time of periastron passage in Table 2. That eclipse is at JD 2,444,265.2. “One decimal” or “two decimals” in the last column refer to the formal precision with which the magnitudes are given in each data set (see the text). Timings from Böhm (1980, 1989) were determined by eye from figures presented in those reports.

of $T_{\text{eff}} = 3502 \pm 30$ K. Their inferred luminosity for the star is then $L_1 = 10.276 \pm 4.445 \ L_\odot$.

In Figure 3 we show the above radius and temperature of the star against a representative set of model isochrones from the MIST series of Choi et al. (2016). The location of η Gem is near the tip of the asymptotic giant branch. Based on these models we infer a primary mass of $M_1 \approx 5.1 \ M_\odot$ at an age of about 110 Myr. This then leads to a secondary mass of $M_2 \approx 2.0 \ M_\odot$ corresponding to an early A-type star ($T_{\text{eff}} \approx 9000$ K), provided it is a single star. The luminosity would be $L_2 \approx 16 \ L_\odot$, nominally some 600 times less luminous than the primary. Such a star would have a radius of $R_2 \approx 1.65 \ R_\odot$ according to the models, which is far too small to produce an eclipse of the depth shown in Figure 2. The inescapable conclusion is that the secondary must be surrounded by a much larger and at least partially opaque structure, which we speculate may be a
The disc around the companion of \( \eta \) Gem then appears to be another example of a class of wide binaries in which the primary is eclipsed by a disc around the companion. The prototype of this small and diverse class of objects is the much studied 27.1-yr binary \( \epsilon \) Aur, an F-type supergiant. The large tilted disk enshrouding the B-type companion of \( \epsilon \) Aur has been estimated to have a radius of 4–6 au (see, e.g., Hoard et al. 2010; Mourard et al. 2012), and causes eclipses that last about 2 years. A disc of similar size (roughly 2–7 au) has been proposed to be surrounding the hot (stripped red giant) pre-He white dwarf companion of the M-type giant primary in TYC-2505-672-1, currently the eclipsing binary with the longest orbital period (\( P = 67.1 \) yr; Lipunov et al. 2016; Rodriguez et al. 2016). In this case the eclipses last for an astonishing \( \sim 3.5 \) years. The short-period eclipsing binary EE Cep (\( P = 5.6 \) yr) has an invisible companion to the B5 giant primary embedded in a disc with a roughly 2 au diameter (Pieńkowski et al. 2020). The eclipses are highly variable in depth, and can last up to 60 days.

A disc such as the one we propose in \( \eta \) Gem can potentially contribute some amount of infrared excess to the system, which could provide information on its nature and composition. The spectral energy distribution of the star was examined by McDonald et al. (2012, 2017) based on brightness measurements between 0.42 and 25 \( \mu \)m, as part of much larger samples of Hipparcos and Tycho-2 sources. They reported finding no significant excess between 4.3 and 25 \( \mu \)m within the photometric uncertainties. Studies of this kind are made more challenging in \( \eta \) Gem because of the intrinsic variability, and to some extent also because of the presence of the visual companion that may not always be resolved in the photometric measurements.

To gain some understanding of the dimensions of the disc in \( \eta \) Gem, we use the shape of the eclipse as determined from the observations. Figure 4 shows a phase-folded plot of the photometric data near the primary eclipse, based on the best ephemeris from Section 3. The eclipse is roughly 0.4 mag deep, although there is significant variation in the depth of individual events caused by the pulsations, as seen earlier in Figure 2. This depth implies a flux decrement of about 30%. To first order this would be the ratio of the surface area of the occulting object relative to that of the primary. The total duration of the eclipse is about 5 months, although the precise contact points are difficult to establish given the scatter in the photometry. A binary model fitted to these data is indicated with the solid curve in Figure 4. The fit was performed using the JKTEBOP code of Southworth et al. (2004), with linear limb-darkening and gravity darkening coefficients appropriate for the primary star, and assuming that the secondary contributes no significant light given the lack of secondary eclipses. The ephemeris and eccentricity parameters were held fixed at the values in Table 2, and the mass ratio was set from the mass estimates given earlier, although the results are insensitive to this quantity.

By design JKTEBOP assumes the companion is a star, which will have an essentially circular profile, whereas a disc might be expected to be a flattened structure. Nevertheless, we may view the results from our fit as a limiting case of a companion that is a completely opaque and circular disc seen face-on, regardless of whether or not this is a realistic orientation. Light curve fits such as this have no absolute scale, and only yield relative quantities. The relative radius of the secondary returned by the model is \( R_2/a \approx 0.10 \), where \( a \) is the semimajor axis of the binary. Experiments show that this value appears quite robust against changes in limb darkening or gravity darkening. With the adopted masses and the orbital period, we derive a semimajor axis of \( a \approx 1670 \, R_\odot \), which leads to a diameter for the face-
on disc of $\sim 330 R_\odot$, or about 1.5 au. This is strictly a lower limit to its size, as any other orientation would lead to a smaller projected area, and the requirement to still block 30% of the surface of the primary means the disc diameter would have to be larger. Furthermore, the disc is not expected to be completely opaque, so meeting the same requirement with any radially-decreasing density distribution would imply an even larger size. We do not attempt here to perform a more sophisticated modelling of the disc properties or a full exploration of possible geometries (orientations, aspect ratio, etc.), as this likely requires better constraints on the eclipse shape than the current observations can provide.

The formal impact parameter from our light curve fit is $\sim 0.60$ (inclination angle $\sim 81^\circ$), although we do not attach a high significance to this result given the tentative nature of the solution. On the other hand, the more firmly determined relative radius of the primary star is $R_1/a \approx 0.17$. Interestingly, converting back to an absolute scale results in $R_1 \approx 280 R_\odot$, which happens to be very close to the primary radius derived from its measured angular diameter and the parallax ($R_1 = 275 R_\odot$: Baines et al. 2021).

Finally, the linear semimajor axis of the spectroscopic orbit corresponds to approximately 7.8 au, which translates to an angular semimajor axis of about 37 mas at the adopted distance. Projected angular separations can reach 57 mas (and will do so in mid 2025), which should be easily accessible by various astrometric techniques. Lunar occultation observations of $\eta$ Gem that have been used in the past to measure its angular diameter (Richichi & Calamai 2003; Mondal & Chandrasekhar 2005) have not revealed any sign of the spectroscopic companion. The 110 Myr MIST isochrone employed earlier predicts a magnitude contrast of 5 mag in $V$ (assuming the secondary is a single star), but much less favorable values in the near infrared ($\Delta H \approx 9.3$ mag, $\Delta K \approx 9.9$ mag). This assumes no obscuration of the secondary by its surrounding disc.

6. CONCLUSIONS

We have used our new radial velocity measurements combined with others from the literature to update the 2979-day (8.2 yr) spectroscopic orbit of the M3.5 Ib-II semiregular variable $\eta$ Gem. Inspection of nearly 150 years of archival photometry collected by several associations of variable star observers around the world, as well as by professional astronomers, have allowed us to identify a number of instances in which clear dimmings occur very near the times of primary eclipse predicted from our spectroscopic orbit. Eighteen independent detections from different data sets for 8 separate eclipses have been compiled, and remove any doubt that primary eclipses recur faithfully every 8.2 yr. No secondary eclipses are seen.

Combining the measured times of eclipse with the radial velocities yields a much improved ephemeris, with which we predict the center of the next eclipse will occur on January 1st 2029 (JD 2,462,138.6 $\pm$ 3.3).

$\eta$ Gem now ranks among the longest-period eclipsing binaries that have been confirmed to date. Only ten examples were previously known with periods longer than 5 yr, not all of which have known orbits (spectroscopic or astrometric). $\eta$ Gem is only the sixth case with an orbit. A listing of all such long-period eclipsing systems of which we are aware is presented in Table 1 with an indication of the variability type, spectral type, and which have reliable orbits determined. References are provided for the orbital determinations. Several other candidate long-period systems have been proposed, but require confirmation; they are not included in the table.

The primary eclipses of $\eta$ Gem are about 0.4 mag in depth, on average, and last for approximately 5 months. Given the large size of the primary star ($275 R_\odot$: Baines et al. 2021), the eclipses are far too deep to be produced by the spectroscopic companion. We propose they are instead caused by an extended disc surrounding the secondary, similar to the discs found in several other long-period eclipsing binaries with evolved primary components, such as $\epsilon$ Aur. The properties of the disc around $\eta$ Gem are not well established at present, although simple arguments suggest it is at least 1.5 au in diameter, and very likely larger. The difficulties are partly due to confusion stemming from the semiregular variations that add scatter to the light curve, and that have amplitudes similar to the depth of the eclipse. Further improvements in our knowledge may come from the next opportunity for study that will occur near the beginning of 2029. That eclipse will be well placed for observation, unlike others that have occurred with $\eta$ Gem too close to the Sun.

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7. DATA AVAILABILITY

The data underlying this article are available in the article and in its online supplementary material.

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Table 4
Confirmed Eclipsing Binaries with Orbital Periods Longer than 5 yr

| Name           | Period (yr) | Orbit | Variability Type | Spectral Type | Orbit reference       |
|----------------|-------------|-------|------------------|---------------|-----------------------|
| TYC-2505-672-1 | 69.07       | No    | ...             | M1III+sdB     | ...                   |
| ε Aur          | 27.09       | Yes   | EA/GS           | A9I           | Stefani et al. (2010) |
| VV Cep         | 20.34       | Yes   | EA/GS+SRC       | M2epIab-B8eV  | Wright (1977)          |
| V381 Sco       | 17.92       | No    | EA/GS+SRC       | A8I           | ...                   |
| γ Per          | 14.64       | Yes   | EA/GS+SRC       | G9III+A2-III: | Pourbaix (2000)        |
| V383 Sco       | 13.35       | No    | EA/GS+SRC+SRC   | F6Iab         | ...                   |
| 31 Cyg         | 10.36       | Yes   | EA/GS/D         | K5Ib+B2IV-V   | Videla et al. (2022)   |
| γ Per          | 14.64       | Yes   | EA/GS+SRC       | M3.5Ib-II     | Ashbrook (1939)        |
| V383 Sco       | 13.35       | No    | EA/GS+SRC       | F6Iab         | ...                   |
| AZ Cas         | 9.32        | Yes   | EA/GS+SRC       | K5Iab-Ib      | ...                   |
| γ Per          | 14.64       | Yes   | EA/GS+SRC       | M3.5Ib-II     | Ashbrook (1939)        |
| KIC 5273762    | 7.33        | No    | EA/GS+SR        | gM3.0         | ...                   |
| EE Cep         | 5.61        | No    | EA/GS           | B5:nev        | ...                   |

Note. — The variability type has been extracted from the General Catalog of Variable Stars (GCVS; Samus’ et al. 2017), or assembled from other sources in the literature. The GS notation refers to a system in which one or both components are giants or supergiants. SRA and SRC are two subtypes of semiregular variables defined in the GCVS. Spectral types are from SIMBAD or other sources in the literature. The γ Per binary has both a spectroscopic and an astrometric orbit. For KIC 5273762 we use this shorter designation in the table; the designation in the discovery paper is ASASSN-V J192543.72+402619.0 (Jayasinghe et al. 2018).

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