Design of robust systems by means of the numerical optimization with harmonic changing of the model parameters

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Abstract. The design of robust feedback systems by means of the numerical optimization method is mostly accomplished with modeling of the several systems simultaneously. In each such system, regulators are similar. But the object models are different. It includes all edge values from the possible variants of the object model parameters. With all this, not all possible sets of model parameters are taken into account. Hence, the regulator can be not robust, i.e. it can not provide system stability in some cases, which were not tested during the optimization procedure. The paper proposes an alternative method. It consists in sequent changing of all parameters according to harmonic low. The frequencies of changing of each parameter are aliquant. It provides full covering of the parameters space.

1. Introduction

Methods of the design of robust regulators are relevant. They are necessary in robotics, since the mathematical model of mechanical elements can change at least due to changing of the load. This is also relevant in industry, science and technology [1–14]. For the most complex objects, not every method can be used. Nevertheless, the method of numerical optimization can be used for the any complex object if the software allows the use of the demanded amount of blocks [1–2]. This method is useable even for the task, which cannot be resolved by any other method at all.

The task of the robust control for the object which parameters are given in some intervals is very seldom discussed in literature.

The paper discusses the task of the control of object which model is given with transfer function in the field of Laplace. Its parameters are given in the wide intervals. This task can be resolved with the robust regulator if it exits for it. If it is not so, then the adaptive regulator is necessary.

An adaptive regulator is much more complex and expensive. It should change its coefficient and maybe structure to provide the stability of the system. For this, the determination of the object model during its working is necessary, which is quite difficult. Robust regulators provide good quality of the...
One of the methods of the robust regulator design is the numerical optimization of the regulator during simultaneous modeling of several systems. In each of them regulators are the same, and object models differ. For example, if the only parameter of the object is changing, then for the regulator optimization two systems with the edge values of this parameter are used. If two parameters are changing, then the simultaneous modeling of the four systems with edge values is necessary. For an object with three changing parameters, eight models are necessary, and so on. If the number of the changing parameters is \( N \), then the number of the necessary models is \( N^3 \).

With all this, the method researches only action of the system in which every parameter has an edge value. It is possible that some set of parameters with the middle values can destroy the system stability. Hence, the system will become unstable.

This paper proposes an alternative method of the numerical optimization of the regulator, which consists in modeling of the only system, but the object parameters in this system are changing inside the prescribed intervals. In this case, the system should be in the state of the transient response during the modeling. For this aim, the prescribed value is periodically changing to avoid the static regime.

The difficulty of this approach is in the problem of the realization of the object model in which the parameters are changing during the time. The modeling has shown that the parameters' changing is difficult. The use of the unit of the “variable value” is not a proper decision in program VisSim. In this case, the software uses its initial value. The changing value can be introduced using the multiplication unit or by means of keys. But this approach is too complex.

The most available versions of VisSim are not proper for this approach. But the advantage of this approach is in the possibility to investigate the system working not only with fixed edge values of the parameters of the model, but also with them changing during the time.

### 2. Task Statement

Let the object model be a transfer function of the following kind:

\[
W(s) = \frac{X(s)}{U(s)} = \frac{b_0 + b_1 s}{1 + a_1 s + \ldots + a_n s^n}.
\]  

(1)

Let the model parameters belong to intervals \( a_i \in [a_{i_{\text{min}}}, a_{i_{\text{max}}}] \), \( i = 1, \ldots, n \) and \( a_n \in [a_{n_{\text{min}}}, a_{n_{\text{max}}}] \).

The robust regulator providing the stable feedback control with any value of the parameters inside the intervals is necessary.

### 3. The method of solving the problem

The paper proposes transforming of the object model into the following equation:

\[
x(p) = \frac{1}{p} \left\{ \frac{1}{p} \left[ -a_n x + \frac{1}{p} \left[ -a_{n-1} x + \ldots + \frac{1}{p} \left[ -a_1 x - b_1 u + b_0 u \right] \right] \right] \right\}.
\]  

(2)

This equation is the model of the system with many feedbacks. Such structure is proper for the modeling with changing values of \( a \).

Model (2) is equivalent to the \( n \)-order filter model (1), but it is more convenient because its parameters are working as gain in the loops. For the modeling of the changing parameters, it is possible to use multiplication blocks. It is sufficient to give the sinusoidal changes of each parameter. The frequencies should be aliquant. The set of rectangular pulses is applied to the system input. Each pulse initiates its own transient response. These processes are different, since the model parameters are different. In addition, a typical optimization procedure should be started. The result gives values of the coefficients of the regulator, which provides good working of the system with any values of the object model parameters, which were realized during the optimization procedure. It allows considering that the system is stable with any set of these parameters.
4. A numerical example and results

Let the object model be the following transfer function:

\[ W(s) = \frac{X(s)}{U(s)} = \frac{1-s}{1+a_1s+a_2s^2+a_3s^3+a_4s^4}. \]  

(3)

The structure for the modeling of object (3) is shown in Figure 1.

Let the model parameters belong to intervals: \( a_1 \in [2, 8] \), \( a_2 \in [3, 7] \) and \( a_3 \in [3, 7] \).

First, let us transform object model (3) into the following form:

\[ x(p) = \frac{1}{p} \left\{ a_1 x + \frac{1}{p} \left\{ a_2 x + \frac{1}{p} \left\{ a_3 x - u + \frac{u}{p} \right\} \right\} \right\}. \]  

(4)

The program modulates the parameters values according to the following equations:

\[ a_1 = 5 + 3 \sin(\omega_1 + \varphi_1), \quad a_2 = 5 + 2 \sin(\omega_2 + \varphi_2), \quad a_3 = 5 + 2 \sin(\omega_3 + \varphi_3). \]  

(5)

Here, the frequencies are aliquant. The phases can be chosen for the best covering of the parameters field, which can be controlled with the help of the according graphs. The structure of the block for modulation of parameter \( a_1(t) \) according to equation (5) is shown in Figure 2. Blocks for other parameters are similar. The structure of the total system is shown in Figure 3.

In this structure, the cost function for the optimization is calculated according to the following relations:

\[ \Psi[T, e(t)] = \int_0^T \left\{ \psi_1[e(t)] + 50\psi_2[e(t)] \right\} dt, \]  

(6)

\[ \psi_1[e(t)] = \max\{0, e(t) \frac{de(t)}{dt} \}, \]  

(7)

\[ \psi_2[e(t)] = \|e(t) - R(t)\|. \]  

(8)
Figure 3. The structure for optimization of the regulator.

Here, \( R(t) \) is a saw-shape signal. The system input signals are rectangular pulses, its frequency is equal to the half of the frequency of the saw-shape signal. The resulting values of the PID-regulator coefficients are shown in the indicators in Figure 3. Figure 4 shows the system output signals.

Figure 4. The transient response of the system with the obtained regulator coefficients in the model according to Figure 3

Figure 5 shows the changing of the model parameters. The abscissa axe is parameter \( a_1 \), and the ordinate axe is parameters \( a_2 \) (blue line) and \( a_3 \) (black line). It is seen that the parameters values cover
the entire field. Figure 6 shows the typical transient response system in detail. This response has a rather good quality.

![Figure 6](image)

**Figure 6.** Resulting processes in the system.

5. **Possible disadvantages and shortcomings of the proposed method**

Possible disadvantages or shortcomings of the proposed method are the following:

1. The main disadvantage is the common for any method of the robust regulator design. It is the possibility of the absence of the decision. In this case, only the adaptive regulator can resolve the task. Hence, this disadvantage is not the feature of the method, but the feature of the task statement for some cases.

2. Not all possible variants of the parameters are tested in the modeling. Figure 5 has its blank spaces. The choice of the longer time of modeling can overcome this disadvantage.
3. It can occur that although some parameters set is not good for the calculated regulator, the modeling does not reveal this situation because this set was realized during too short time. In addition, the transient process is stable because the system stability was affected for a very short interval of the time. The use of less values of $\omega$ in equation (5) allows overcoming this disadvantage.

6. Conclusion
Hence, the research has shown that the proposed method is effective. If the result is not found, then the adaptive regulator will be the only decision for this task.

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