Abstract—Building structures by low capability robots is a very recent research development [3]. A robot (or a mobile agent) is designed as a deterministic finite automaton. The objective is to make a structure from a given distribution of materials (bricks) in an infinite grid $Z \times Z$. The grid cells may contain a brick (full cells) or it may be empty (empty cells). The field, a sub-graph induced by the full cells, is initially connected. At a given point in time, a robot can carry at most one brick. The robot can move in four directions (north, east, south, and west) and starts from a full cell. The Manhattan distance between the farthest full cells is the span of the field. We consider the construction of a fort, a structure with the minimum span and maximum covered area. On a square grid, a fort is a hollow rectangle with bricks on the perimeter. We show that the construction of such a fort can be done in $O(z^2)$ time – with a matching lower bound $\Omega(z^2)$ – where $z$ is the number of bricks present in the environment.

Index Terms—Automaton, Programmable Matter, Fort Formation, Infinite Grid, Mobile Robots.

I. INTRODUCTION

A. The Problem

Czyzowicz et al. [3] have initiated a study on building structures by a robot (or mobile agent) designed as a deterministic finite automaton in an infinite grid with materials. The motivation behind building such a structure stems from many areas, such as the demarcation of land using a robot in inaccessible areas or building structures in space where the materials are limited. In these cases, it is important to utilize the available resources to their complete capacity. This paper explores how a connected structure can be formed enclosing the maximum possible area with a given number of bricks. We denote the resulting structure as a fort.

B. Environment Model

We are given an infinite oriented grid $Z \times Z$, which can be viewed as a grid of cells in a two-dimensional plane, with the cells having either horizontal or vertical common edges. Thus, each cell can have four adjacent cells to its North, East, South, and West. We denote a cell having a brick as a full cell and a cell without a brick as an empty cell. The sub-graph induced by the full cells is called a field. Initially, the field is connected, i.e., there exists a rectilinear path between any two full cells, which consists of full cells. However, after operations by the robot, the field may get disconnected. The maximally connected sub-graph of a field is called its component. The number of cells in a field is called its size and the maximum Manhattan distance between two cells in a field is called its span.

C. Robot Model

The robot starts at one of the full cells. It can move in one of the four directions (East, North, West, or South) from a cell. It is oriented towards one of the directions, and when we say the robot turns left, it reorients itself locally (A robot facing north turns left to face west). It can only observe the state of its current cell, i.e., the cell contains a brick or not. It can pick up a brick from a full cell and drop the brick in an empty cell. It can move with or without carrying a brick (makes the robots light or heavy). It can carry at most one brick at once. The robot can move through full cells while carrying a brick. It is formalized as a Mealy automaton $R = (X, Y, \delta, \lambda, S_0, S_f)$, where $X = \{e, f\} \times \{l, h\}$ is the input, $Y = \{N, E, S, W\} \times \{e, f\} \times \{l, h\}$ is the output, $\delta$ is the set of states, $\lambda$ is the transition function with $S_0$ as initial state and $S_f$ as the final state. The alphabet denote the following in order: $N$: North, $E$: East, $S$: South, $W$: West, $e$: empty cell, $f$: full cell, $l$: light robot (not carrying a brick), $h$: heavy robot (carries a brick). The state of the robot contains its current orientation, the current cell’s status, and if the robot carries a brick. We follow the same model for the robot introduced by Czyzowicz et al. [3]. The robot has a very limited amount of memory, which maintains the set of states. Even with limited memory, it can perform a bounded exploration up to a small distance (say 8) and keep the state of the cells encoded in its states. In other words, at any point in time, the robot can have information about all the cells at a distance at most 8 from its current cell.

D. Target Structure

The largest connected structure with the maximum enclosed area in a square grid is a hollow rectangle, i.e., the bricks are only on the sides of the rectangle. The target is to construct this hollow rectangle – denoted as fort – using the bricks available. A perfect fort is the fort having all its sides made up of an equal number of bricks. The Manhattan distance between the diagonally opposite bricks is the span of the fort. This span will always be an even number for a perfect fort. For a perfect fort, it is easy to deduce the relation $z = 2s'$ where $z$ is the total number of bricks in the fort, and $s'$ is the span of the

$1$Manhattan distance between two cells at $(x, y)$ and $(x', y')$ is $|x - x'| + |y - y'|$. 

Fort Formation by an Automaton

Kartikey Kant
Indian Institute of Technology Guwahati
Guwahati, India
kartikeykant@gmail.com

Debasish Pattanayak
Indian Statistical Institute
Kolkata, India
drdebmth@gmail.com

Partha Sarathi Mandal
Indian Institute of Technology Guwahati
Guwahati, India
psm@iitg.ac.in
fort. The fort, shown in Fig. [2] is an example of a perfect fort. A perfect fort contains \(4m\) bricks, where each wall of the fort contains \(m + 1\) bricks. Note that the bricks at the corner are shared by two walls. Any fort consisting of odd number of bricks is a rough fort (ref. Fig. [3] (b), (d), (f) and (h)). The rough fort is a rectangle where an additional brick may be attached to one of its corners since the number of bricks is odd.

E. Challenge

Designing algorithms for a low memory robot with minimal capability is challenging, given the absence of the information. If the robot has sufficient memory, it suffices to keep track of the entire structure of the field by doing a simple zig-zag exploration and build any structure. Due to the lack of memory, the robot can neither remember its past actions nor the positions of the bricks in the infinite grid. Since the grid does not have any markers, it is easy for a robot to get lost, and systematic exploration of an infinite grid is impossible with limited memory [1]. It is also important to keep track of the state of the structure and achieve termination after all bricks have been used by the robot.

F. Our Contribution

We consider the problem of the construction of a fort with a finite automaton in an infinite square grid. Specifically, we show that

- any algorithm that builds a fort from a given initial connected field of span \(s = O(\sqrt{z})\) and size \(z\) must use at least \(\Omega(z^2)\) time.
- Algorithm BUILD FORT builds a fort from a given initial connected field of span \(s\) and size \(z\) in \(O(z^2)\) time. Hence, our algorithm is optimal.

G. Related Works

Many problems with robots or mobile agents exploring an unknown environment have been studied over the years [4], [6]. The studies can broadly be categorized in two ways based on the environment in which the robot operates; it can be a perfect disc has all the bricks on the boundary at a distance \(r\) from the center.

**Theorem 2.1:** There exists an initial field of size \(z\) and span \(s = O(\sqrt{z})\) such that any algorithm that builds a fort starting from this field must use \(\Omega(z^2)\).

**Proof.** Consider a rough disc \(D_1\) with \(z\) bricks and radius \(r \geq 9\). Then \(z\) satisfies \(z_1 \leq z < z_2\), where \(z_1 = 2r^2 + 2r + 1\) and \(z_2 = 2r^2 + 4r + 2 = (2r^2 + 2r + 1 + 2r + 1)\). The span of this initial arrangement of bricks is \(s = 2r\) and \(z = O(r^2)\), so we have \(s = O(\sqrt{z})\). Consider the two opposite walls of the resulting fort. The distance between the two opposite walls is \(z/4\), and there are at least \(z/4\) bricks on each wall. Suppose the bricks in \(D_1\) are at a distance \(k\) from one wall, then they are at a distance at least \(z/4 - k - s\) from the opposite wall. Now, the bricks would occupy their target position on one of the walls. So at least \(z/4\) bricks would move a distance \(k' = \max(k, z/4 - k - s)\). Since \(s = O(\sqrt{z})\), we have \(k' \geq z/10\) for \(z \geq 100\). Any algorithm must move at least \(z/4\) bricks by a distance \(z/10\) and must use \(\Omega(z^2)\) time to build the fort. □

Note that the span of the resulting fort is \(O(z)\). Also, the maximum span of the initial configuration can be at most \(z\).

III. Fort Formation

The algorithm for fort formation works like the following. The robot first creates two special components, the marker and the first cell of the fort, to initiate the construction of the fort.
(ref. Fig. 2). Any other component is called a free component.
A marker is used to determine when the robot is in the vicinity of the fort. The marker is always positioned closer to the fort and farther from the free component. The construction of a fort constitutes of four main steps:

1) create enough area for the fort to extend
2) pick a brick from the field
3) bring the brick back to the fort
4) add the brick to the fort to extend it

To achieve this goal, we use three procedures developed by Czyzowicz et al. [3] for nest formation as subroutine for fort formation, that are SWEEP, FINDNEXTBRICK and RETURNTOMARKER. We present a summary of those subroutines and refer the reader to the paper by Czyzowicz et al. [3] for more details.

**SWEEP**: This procedure is called every time a brick is added to the structure starting from the first brick. As the name suggests, in this procedure, the robot sweeps the bricks nearby the structure to a distance such that there is enough space to build the structure. Specifically, the robot makes a counterclockwise traversal of the fort and checks for full cells at a distance at most seven from the fort, which is not part of the fort. It finds an empty cell at a distance at least seven from the fort, which is in a direction away from the fort and places the brick that it has picked from the full cell. This procedure may create multiple components. The components are created at a distance of seven while the robot has information about all the bricks up to distance eight; hence all the components remain accessible. At the end of the procedure, the marker is placed at a distance three from the fort and a distance four from a component.

**FINDNEXTBRICK**: Once the first cell of the fort is established, we need to add bricks to the fort to make it larger. This procedure finds the next brick that is to be added to the fort. It performs a switch-traversal of a search-walk in one of the components until it reaches a leaf cell and then picks the brick at the leaf cell. If it does not reach a leaf cell at the end of the search walk, it performs shifting to get a free brick.

**RETURNTOMARKER**: After the robot picks the free brick, it performs a reverse switch-traversal. It reaches near the marker at the end of the reverse switch-traversal, and then it returns to the marker.

### Procedure 1 SWEEP

1: $M = \text{Marker}$
2: if the robot is at $M$ then
3: go to the first cell of the fort $F$
4: end if
5: perform a full counterclockwise direction of the border of $F$, perform the following actions after every step:
6: for each full cell $c \notin F \cup \{M\}$ at distance at most 7 from the robot do
7: $c' = c \in F$ at distance at most 7 from the robot do
8: go to $c$ and pick the brick
9: move in direction away from $F$ and stop at the first empty cell at distance at least 7 from $F$
10: drop and brick return to $c'$
11: end for
12: if there exists a free component $C$ then
13: pick the brick from marker and place it at distance 3 from the first cell of $F$ and at distance 4 from $C$, creating a new marker
14: end if
15: go to the new marker

### Procedure 2 FINDNEXTBRICK

1: $W = \text{the search walk that starts at the cell where the robot is located}$
2: go to the nearest cell belonging to a free component
3: perform a switch-traversal of $W$
4: if the robot is at the leaf then
5: pick the brick
6: else
7: perform shifting
8: end if

In Example [4] we describe the switch traversal of a field and how the robot obtains a free brick to bring back to the marker. We consider a simple example. There can be much more complicated cases.

**Example 1**: Consider a field, as shown in Fig. 3(a). The robot starts at $w_1$ and facing north. The robot starts with a left-oriented traversal. It moves straight until it is possible to move left and then it turns left. After the turn, it switches its orientation to the right and moves straight until it is possible to move right. The search walk $W$ starts at $w_1$ with a left-free segment $S_1 = (w_1, w_2, w_3, w_4)$ and then the right-free segment $S_2 = (w_4, w_5, w_6, w_7)$ which ends at the leaf cell $w_7$. Observe

*A full cell with exactly one adjacent cell*
Procedure 3 RETURNToMarker
1: \( W = \) the search walk traversed in the last cell to FINDNextBrick
2: let \( S_1, S_2, \ldots, S_l \) be the segments in \( W \)
3: if the robot is at the first cell of \( S_l \) then
4: \( \text{turn towards the penultimate cell of } S_l \)
5: \( \text{move } \min\{2, |S_{l-1}|-1\} \text{ cells forward} \)
6: if \( |S_{l-1}| = 2 \) and \( l > 2 \), then make a turn towards the
penultimate cell of \( S_{l-2} \)
7: end if
8: starting at the current location, perform a switch-traversal of \( \psi(W) \)
9: go to the marker

![Diagram]

Fig. 3. (a) The search-walk \( W \) starts at \( w_1 \), follows a left-free segment to \( w_4 \), and a right free segment up to the leaf node at \( w_7 \). (b) Switch traversal moves the brick at \( f_6 \) to \( e_6 \). (c) The robot picks the brick at leaf cell \( w_7 \). (d) The robot performs switch traversal on the reverse search walk of \( W \) to return to \( w_1 \).

that, for the walk to proceed further, the robot has to turn right at \( w_7 \) or go straight, but there are no full cells after \( w_7 \). The robot performs a switch traversal, as shown in Fig. 3(b), and arrives at \( w_7 \). To keep track of the path it has traversed, it converts the left-free segments into right-free segments and vice versa. As shown in Fig. 3(b), it moves the brick from left to right from \( f_6 \) to \( e_6 \). Now, the reverse of the segment \( S_2 \) becomes a right-free segment starting from \( w_7 \). As per Fig. 3(c), it picks up the leaf cell at \( w_7 \) and turns around to reach \( w_6 \). Finally, while the robot performs the reverse search walk, it restores the brick from \( e_6 \) to \( f_6 \), and the connectivity of the component is reestablished. Thus we pick a brick from the component and arrive at the starting point \( w_1 \).

In the next call to FINDNextBrick, the robot will again perform a switch traversal and arrive at \( w_6 \) as shown in Fig. 4(a). Now, it cannot turn right at \( w_6 \). So, the robot has to pick a brick from there, since the position of a leaf node may not be close to \( w_6 \). To get a free brick, the performs shifting after picking the brick at \( w_6 \). In shifting, the robot moves \( w_5 \) to \( e_5 \) and reaches \( w_4 \) carrying a brick as shown in Fig. 4(b). It makes sure that the component remains connected while a free brick is obtained by the robot, and then it performs a reverse switch traversal to return to \( w_1 \). It obtains a brick at the end of the traversal.

A. Constructing the Fort

As described in Example 1, the Procedure 1 (SWEEP), Procedure 2 (FINDNextBRICK) and Procedure 3 (RETURNToMarker) work in tandem to provide a free brick. Next, we describe the construction of the fort as we add bricks one by one to extend the structure. The robot carrying a free brick arrives at a marker after RETURNToMarker. From the marker, the robot comes to the first cell of the fort and invokes Procedure 4 (EXTENDFORT). Until the size of the fort is 4, the robot follows a strict rule of adding bricks to the fort in the manner shown in Figure 5.

![Diagram]

Fig. 5. The first four stages of construction of a fort where the newly added brick in each step is marked with a red shade.

These configurations are handled when the value of the variable stage = 0. Once the fourth brick is added, the value of stage becomes 1. Now the bricks are added in a manner that the resulting configuration is similar after every four steps. This can be seen in Figure 6.

In EXTENDFORT, one should note that turning right means the robot orients itself in the direction which is to its right originally. The same goes for turning around and turning left.

A configuration of the fort is controlled by the variables stage and counter. Depending on this configuration, the robot, starting from the first cell, traverses the perimeter of the fort and extends the fort. Sometimes, it needs to shift the bricks to the left. It does this by dropping the brick it currently carries to the left, picking the brick from the current cell, and moving forward. After adding the brick and changing the variables accordingly and SWEEP is called.
Procedure \textsc{ExtendFort} invokes Procedure \textsc{ TraverseWall} and Procedure \textsc{ ShiftBricks}. In \textsc{TraverseWall}, the robot moves along the wall of the fort until it reaches the end of the wall. In \textsc{ShiftBricks}, the robot shifts the bricks of the wall outwards. Since we traverse the walls of the fort in a clockwise manner, \textsc{ShiftBricks} always moves the bricks to the left of it.

\textbf{B. Algorithm \textsc{BuildFort}}

This section describes Algorithm \textsc{BuildFort} for the robot to build a fort from a given initial connected field. First, the robot points out the marker and the first cell of the fort. Then \textsc{Sweep} is called to create space for fort formation, and the variables are declared. The robot brings bricks one by one and adds them to the fort to extend it. At last, the robot brings the brick to the marker and adds it to the fort.

\textbf{IV. Correctness and Complexity}

To prove the correctness of the procedures, we will define a few terms first. A fort has a \textit{gap of width} $k$ if each free component is at a distance $k + 1$ from the fort. A field is structured if it satisfies the following conditions:

- the marker is at a distance three from the first cell of the fort and a distance four from some free component.
- the fort has a gap of width seven.

If a component is at a distance larger than 7 from the fort, it is called a \textit{lost component}. A field is \textit{strongly structured} if it satisfies the following conditions:

- the field is structured
- there are no lost components
- the robot is at the marker.

\textbf{A. Correctness}

We refer the reader to the paper by Czyzowicz et al. \cite{3} for the correctness of the procedures \textsc{Sweep}, \textsc{FindNextBrick}, and \textsc{ReturnToMarker}. After execution of \textsc{Sweep}, the robot moves the bricks nearby the fort to a distance at least 7 to create space for expansion. The robot takes the bricks from full cells within distance seven from the fort and drops them in an empty cell next to a full cell so that the brick remains part of a free component which is at most distance seven from the fort. This may result in multiple components, but \textsc{Sweep} ensures that there are no lost components and properly repositions the marker such that a free component can always be found for the robot to execute \textsc{FindNextBrick} on the free component. We state the following lemma.

\textit{Lemma 4.1:} \textsc{Sweep} results in a strongly structured field. \cite{3}

Next, we show the correctness of procedures \textsc{ShiftBricks},

\begin{verbatim}
Procedure 4 ExtendFort
1: Arrive at the first cell of the fort oriented to east
2: if stage = 0 then
3:    if counter = 1 then
4:      Place the brick in the front
5:      Increase counter by 1
6:    else if counter is 2 then
7:      Move one step forward
8:      Place brick on the right
9:      Increase counter by 1
10:   else if counter is 3 then
11:      Place the brick on the right
12:      Increase counter by 1
13:      Increase stage by 1
14:   end if
15: else
16:   if counter is 0 then
17:     Call TraverseWall
18:     Drop Brick in Front
19:     Increase counter by 1
20:   else if counter is 1 then
21:     Call TraverseWall
22:     Move one step back
23:     Turn Right
24:     Call ShiftBricks
25:     Increase counter by 1
26:   else if counter is 2 then
27:     Call TraverseWall
28:     Turn Right
29:     Call TraverseWall
30:     Drop Brick in front
31:     Increase counter by 1
32:   else if counter is 3 then
33:     Call TraverseWall
34:     Turn Right
35:     Call TraverseWall
36:     Move one step back
37:     Turn Right
38:     Call ShiftBricks
39:     Increase counter by 1
40: end if
41: end if
42: counter ← counter mod 4
43: Call Sweep
\end{verbatim}
Procedure 5 TraverseWall
1: while the cell in front is full do
2: move one step forward
3: end while

Procedure 6 ShiftBricks
1: while the cell in front is full do
2: move one step forward
3: drop the brick on the left
4: if the cell in front is full then
5: pick the brick from the current cell
6: end if
7: end while

Algorithm 7 BuildFort
1: if the span of the field is at most 2 then
2: exit
3: end if
4: the cell occupied by the robot is at the marker.
5: a full cell at distance 2 from the marker becomes the first cell of the fort (i.e., the north-west corner cell of the fort)
6: Call Sweep
7: Set counter = 1
8: Set stage = 0
9: while there exists a free component C do
10: Call FindNextBrick
11: Call ReturnToMarker
12: Call ExtendFort
13: end while
14: pick the marker and add it using ExtendFort

Lemma 4.2: Procedure ShiftBricks correctly shifts the bricks on one wall of the fort outwards apart from the first and last brick of that wall.

Proof. The robot carrying a brick has three different situations when Procedure ShiftBricks is invoked. It shifts all the bricks to the left of its current orientation until it encounters an empty cell. In Fig. 7(a), the robot carries a brick. Since the cell in front of the robot is not an empty cell, it moves forward and drops the brick on its left, as shown in Fig. 7(b). Now it picks up the brick from its current cell, and the resulting configuration of bricks looks like Fig. 7(c). Finally, the procedure terminates when it encounters an empty cell as per Fig. 7(d) and places the brick to its left and does not pick up the brick in its current cell. Note that the configurations of bricks in Fig. 7(a) and Fig. 7(d) correspond to a rough fort with an odd number of bricks and a rectangular fort.

If the cell in front of the current cell of the robot is empty, then the condition on line 1 of Procedure ShiftBricks returns false, and the procedure terminates. If the first cell of the wall has an empty cell in front of it along the robot’s orientation, it is also the last. If the first cell has a full cell in front, then line 2 gets executed, and the cell moves forward without picking the brick from the first cell of the row. Hence the first cell of the row is not shifted. This is due to the fact that the cell to the left of the first cell is full, and ShiftBricks is invoked to align the wall with the cell to its left.

The robot moves one cell forward according to line 2, and this cell contains a brick as it satisfied the condition in line 1. Now the brick is dropped to the left as per line 3 and will be the new wall. Since ShiftBricks is invoked after Sweep, the left cell would be empty since there is a gap of width seven from any nearest component. Hence it can drop the brick it is carrying on its left in line 3 correctly. Now, since the robot does not carry any brick, it can pick the brick from the current cell if the condition in line 4 is satisfied, i.e., the robot is not at the last cell of the wall. Only the last cell of the wall will not have a full cell in the front, and thus all the cells of the wall will shift one cell to the left except the last one.

In the following lemma, we prove the correctness of ExtendFort.

Lemma 4.3: The robot correctly adds a brick to the fort in Procedure ExtendFort.

Proof. Before the start of Procedure ExtendFort, the robot is at the marker and carries a brick. This procedure works in two stages with stage = 0 and stage = 1.

When stage = 0, the robot follows a defined set of instructions to place the first 3 bricks to get a perfect fort of size 4. In line 1, it arrives at the first cell of the fort since it is at a bounded distance from the marker, and no other brick is closer to the marker. Initially, counter is 1, lines 4-5 get executed. Shifting done in FindNextBrick moves a brick by at most two cells, and from lemma 4.1 we know before FindNextBrick, the field was strongly structured. Thus the fort had a gap of width seven, and after its execution, the fort has a gap of width at least 5. Thus the cell on the east of the first cell should be empty, and hence the robot can place the brick in front as it is oriented to the east. The robot increases counter by 1 so that in the next call of ExtendFort lines 6-9 get executed. Since the fort now has two bricks, in line 8, the robot can move one cell forward and place the brick in the empty adjacent cell. The robot extends the fort and increases counter by 1. Thus, when the robot arrives to add the next brick, lines 10-14 get executed. The robot places the brick to get a perfect fort of span 2 correctly since the gap is at least five, and thus, the cell on the south of the first cell of the fort must be empty. The robot increases counter by 1 and stage by 1. Thus, in every call to ExtendFort hereafter,

Fig. 7. An execution of Procedure ShiftBricks
the robot goes on to pick another brick. Thus the procedure XTEND south by one brick. In the next call to E ensures that it reaches the south of the east wall. The robot 4 of bricks is the south-east corner of the fort. In the next call, the number of bricks in the fort is of the form 4t + 1, and counter is 1. When the next brick is added, lines 20-25 get executed. The call to TRAVERSEWALL ensures that the robot reaches the last cell of the north wall. Then it moves one step back and arrives at the penultimate cell, which is also the north of the east wall of the fort. It turns to the right and moves along the east wall. Lemma 4.2 ensures that the call to SHIFTBRICKS shifts all the bricks of this row to one cell to the left. As a result, the new brick is placed at the south-east corner of the fort. In the next call, the number of bricks is 4t + 2, and counter is 2 results in the execution of lines 26-31. The first call to TRAVERSEWALL ensures that the robot reaches the last cell of the north wall, and then it turns right to the east wall where another call to TRAVERSEWALL ensures that it reaches the south of the east wall. The robot places the brick in the front, extending the east wall to the south by one brick. In the next call to EXTENDFORT, lines 32-39 get executed. After two calls to TRAVERSEWALL and a turn between them, the robot arrives at the southernmost cell of the east wall. Then it moves one step and turns right onto the south wall. It invokes SHIFTBRICKS procedure and moves the bricks on the south wall one step to the south (ref. Lemma 4.2). After this, the robot places brick on the south-west corner of the fort, extending the size of the fort to 4t + 4 and counter to 4. After line 42 is executed, counter resets to 0, and the procedure runs repetitively. In line 43, Procedure SWEEP ensures that the gap of the fort is of width 7 before the robot goes on to pick another brick. Thus the procedure EXTENDFORT correctly adds a brick to the fort. □

Theorem 4.4: BUILDFORT correctly builds a fort.

Proof. To prove this theorem, we use induction on the number of iterations of the while loop in line 9 in BUILDFORT. SWEEP is called in line 6 before the while loop. From lemma 4.1 we know that the execution of SWEEP has resulted in a strongly structured field. Thus before the first iteration, i.e., i = 1, of the while loop, the field is strongly structured. We assume the induction hypothesis holds after the ith iteration that the field is strongly structured and prove this remains true after the (i + 1)th iteration. Using procedures FINDNEXTBRICK and RETURNTOMARKER, the robot carries a brick after it performs the search walk on one of the components. From Lemma 4.3 we know that the robot correctly adds a brick to the fort. Also, SWEEP is called again in EXTENDFORT, which results in a strongly structured field. Thus, we proved our induction argument. Thus, the field is strongly structured before each iteration of the while loop in BUILDFORT and correctly builds the fort. □

B. Complexity

From [3], we have the complexity of SWEEP, FINDNEXTBRICK and RETURNTOMARKER to be O(s) for each of the procedures. We show the time complexity of Procedure 4 in Lemma 4.5 and then we show the complexity of Algorithm 7 in Theorem 4.6.

Lemma 4.5: The execution of EXTENDFORT requires O(z) time.

Proof. Since we add bricks to the fort starting from the north-west corner, adding a brick in the south-east corner requires the robot to travel a distance s', where s' is the span of the resulting fort. In case of the fort, s' = O(z). Hence EXTENDFORT takes O(z) time. □

Theorem 4.6: A fort is created by BUILDFORT given an initial connected field in time O(z^2) where z is the number of bricks in the field.

Proof. There are a total of z bricks in the field. Initially, one brick was chosen as the fort and another as the marker. Thus the rest of z - 2 bricks will be picked in the while loop of BUILDFORT. Thus there will be z - 2 iterations of this loop. Also, we know that O(s) is the length of the search walk, and the robot traverses it twice, once while picking the brick and the other while returning from it. We know that a single execution of SWEEP takes time O(s). Also, we saw in lemma 4.5 that adding a brick to the fort requires time O(z), which is done in one execution of EXTENDFORT. The span of the initial configuration can be at most z. Thus, BUILDFORT requires O(z^2) time. □

V. Conclusion and Future Work

We proposed a Fort formation problem by a robot modeled as a finite automaton in an infinite grid and established a lower bound for the same. We developed an algorithm that builds a Fort from a given initial connected field of bricks in an infinite grid by a mobile robot in worst-case optimal time. The research area is quite nascent, and a lot can be done further. One can explore in the direction of building different structures given an initial field. A variation of the problem can be considered instead of a square bricks. Extension to 3 dimensions with six directions of movement can also be considered as a future work. Further research using multiple mobile agents to build the structures can be performed to determine if the time complexity can be improved.

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