Quarkyonic Matter and the Revised Phase Diagram of QCD

Larry McLerran

Presented at the 21st International Conference on Ultrarelativistic Nucleus-Nucleus Collisions
Knoxville, Tennessee
March 30 to April 4, 2009

Physics Department
Nuclear Theory Group
Brookhaven National Laboratory
P.O. Box 5000
Upton, NY 11973-5000
www.bnl.gov

Notice: This manuscript has been authored by employees of Brookhaven Science Associates, LLC under Contract No. DE-AC02-98CH10886 with the U.S. Department of Energy. The publisher by accepting the manuscript for publication acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this manuscript, or allow others to do so, for United States Government purposes.

This preprint is intended for publication in a journal or proceedings. Since changes may be made before publication, it may not be cited or reproduced without the author's permission.
DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or any third party’s use or the results of such use of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.
Quarkyonic Matter and the Revised Phase Diagram of QCD

Larry McLerran

RIKEN Brookhaven Center and Brookhaven National Lab., Physics Dept., Upton, NY 11973-5000 USA

Abstract

At high baryon number density, it has been proposed that a new phase of QCD matter controls the physics. This matter is confining but can have densities much larger than $\Lambda_{QCD}^3$. Its existence is argued from large $N_c$ approximations, and model computations. It is approximately chirally symmetric.

1. Introduction

Rob Pisarski and I recently proposed the existence of Quarkyonic Matter.[1] These arguments were originally in the context of the large number of colors approximation, $N_c \rightarrow \infty$ with the number of quark flavors held fixed, $N_f$ finite. The arguments were then generalized to the large $N_c$ limit, with $N_f/N_c$ fixed.[2] Computations based on the PNJL model,[3],[4],[5] were subsequently performed that argued that such a phase of matter may occur in systems with $N_c = 3$, and that the quarkyonic transition might be associated with chiral symmetry restoration.[6]-[7] In the paper with Redlich and Sasaki, a model was proposed that could continuously vary $N_c$, and the behaviour of the phase transition boundaries could be studied as a function of $N_c$.[7] A picture of the revised phase diagram of QCD is shown in Fig. 1 The purpose of this talk is to provide a brief explanation of these developments.

![Figure 1: The revised phase diagram of QCD](image-url)
2. The Large $N_c$ Approximation

The large $N_c$ approximation is the limit where the number of colors of QCD is taken to be very large, but the coupling strength $\lambda = g^2 N_c / 4\pi$ is fixed. This theory is asymptotically free, and confining in vacuum. There is a Hagedorn spectrum of mesons, and mesons made of quarks interact with strength $1/N_c$ and glueballs with strength $1/N_c$. Baryons are made of $N_c$ quarks, have a mass of order $N_c\Lambda_{QCD}$ and interact strongly with strength $N_c$.

At finite temperature and zero baryon number density, the low temperature phase is composed of light mass mesons. The number of degrees of freedom are of order one in the number of colors. At very low temperature, the Goldstone degrees of freedom are important, and there are $N_c^2 - 1$ degrees of freedom. Baryons degrees of freedom vanish in the confined phase since $e^{-MB/T} \sim e^{-N_c}$. At very high temperatures, the degrees of freedom are $2(N_c^2 - 1) + 4N_fN_f$ arising from the gluons and quarks. Baryons degrees of freedom are unsuppressed in the Quark Gluon Plasma.

Since the number of degrees of freedom of is of order one in the hadron phase and is of order $N_c^2$ in the Quark Gluon Plasma, there must be a jump in the energy density at a de-confinement temperature. Viewed from the perspective of the hadron phase, there is a limiting temperature where the energy density diverges. For finite but large $N_c$, the Hagedorn limiting temperature, the density of mesons becomes enormous, and at some point they must interact with one another. As this occurs, one makes a transition to a Quark Gluon Plasma, that has a large but finite number of degrees of freedom. The rapid cross over for QCD at $N_c = 3$ is a remnant of this Hagedorn transition of large $N_c$ QCD.[8]

At finite baryon density, we see that $e^{(\mu_B - M_B)/T}$ will be of order $e^{-N_c}$ for $\mu_B$ much less than $M_B$. In fact unless $(\mu_B - M_N)/M_N \sim 1/N_c$, the baryon number density is exponentially small. Therefore when $\mu_B = M_N$, there is a baryon number generating transition. Of course, due to the very strong interactions of baryons in the large $N_c$ limit, the threshold for baryon number generation is not the free nucleon mass, but is the mass of a nucleon in bound nuclear matter.

High density baryonic matter however remains confined. The contribution of fermion loops is suppressed by $\lambda^2 N_c$, where $\mu_Q = \mu_B/N_c$ is a quark chemical potential. Such loops cannot Debye screen the confining potential until $\mu_Q \sim \sqrt{N_c}\Lambda_{QCD}$ This means that the de-confinement phase transition is independent of baryon density In spite of the fact that the baryonic matter is confined, at high density the typical bulk quantities such as pressure and energy density should be described by a weakly coupled gas of quarks. This is because for particles inside a Fermi sea, interactions are not infrared sensitive, and bulk quantities should be dominated by quark kinetic energies.

This matter that is simultaneously confined and yet has its energy density and pressure well approximated by a quasi-free gas of quarks, Rob Pisarski and I have named as Quarkyonic.

For $N_c = 3$, the phase transitions of the large $N_c$ world can become cross overs. The Quarkyonic transition remains at $\mu_B = M_N$ for $T = 0$, since this is the physical threshold for producing baryon number. As $T$ is increased, presumably the transition is at small $\mu_B$. At high density, the confinement transition should become weaker since the effect of quarks will begin to reduce the effect of the confinement temperature at higher density. It also should move to lower values of $T$ as $\mu_B$ increases. The width of the Quarkyonic transition is estimated from the baryon chemical potential which for Fermi momenta $k_F << M_N, \mu_B = M_N + k_F^2/2M_N = M_N(1 + O(1/N_c^2))$, since the transition should be completed when $k_F \sim \Lambda_{QCD}$. 


3. Physical Picture of Quarkyonic Matter

The Hadron Phase is composed of mesons and glueballs. In the large \(N_c\) limit, there are no baryons. The number of Goldstone degrees of freedom are \(N^2_c - 1\). The Quark Gluon Plasma is composed of quarks and gluons with \(2(N^2_c - 1) + 4N_f N_c\) degrees of freedom.

The Quarkyonic phase is composed of quarks, mesons and glueballs. The number of quark degrees of freedom are \(2N_c N_f\) and in addition there are \(N^2_f - 1\) Goldstone boson degrees of freedom. The transition between these various phases should be thought of as a change in the number of degrees of freedom in a narrow range of temperature and baryon chemical potential. A phenomenological parameterization that embodies these constraints should be straightforward.

The Quarkyonic phase can be thought of as a Fermi gas composed of quasi-free quarks. Near the Fermi surface the degrees of freedom are confined baryons. The thermal excitations are mesons and glueballs. Color superconductivity in the quarkyonic phase would not be allowed in the large \(N_c\) limit, but surely for finite \(N_c\) such phenomena are possible.

One can understand the Quarkyonic transition in large \(N_c\) with fixed \(N_c/N_f\) in a simple way: For large \(N_f\) there are exponentially large numbers of degenerate lowest mass baryon states, \(N \sim e^{N_c F(N_c/N_f)}\). Near the Quarkyonic transitions, these states become important, since the probability that they contribution can be of order \(e^{N_c F(N_c/N_f)/\mu B/T-M_N/T}\). The accumulating density is however rapidly cutoff by the strong interactions of the baryons.

The nature of chiral symmetry breaking is not so simple to understand. In the high density phase, one expects that the effects of high density matter may lower or perhaps reduce to zero the chiral quark condensate.[9],[10],[11],[12],[13] Intuitively, this is because a high baryon density will exclude quarks unless they have energy near the Fermi surface. This means that if pairs form, they arise from particle hole pairs near the Fermi surface. It is indeed possible that pairing phenomena near the Fermi surface might generate chiral symmetry breaking through chiral density waves.[14]-[15] These pairs would break the translational invariance and a crystal would result.[16] (In previous considerations of chiral density waves, it was assumed that the potential between quarks was Coulombic. In Quarkyonic matter, the potential should be linear, and the possibility of condensation may be easier to realize.)

One can question whether or not the large \(N_c\) limit is applicable for \(N_c = 3\). On the plus side of the equation is the fact that quarks seem to little influence the confining potential in vacuum out to a distance of about 2 \(Fm\), as seen from lattice Monte-Carlo computations. The linear potential begins at a distance of about 0.2 \(Fm\). One may expect that there is some finite range of density before media quarks may short out the potential, so there is room for a Quarkyonic phase. There have also been model computations that argue for a Quarkyonic phase.[6],[7] These computations also argue that the chiral transition is approximately coincident with the Quarkyonic transition.

The negative side follows from some generic features of the large \(N_c\) limit that are not observed in baryons. The large \(N_c\) limit predicts that pion exchange has an intrinsic strength of order \(N_c\) at large distances. This requires that the ground state of large \(N_c\) baryonic matter is a Skyrme crystal.[16] On the other hand nuclear matter for \(N_c = 3\) is a liquid, not a solid, and the biding energy is of order 15 \(MeV\), not of order the nucleon mass. Moreover, lattice gauge theory computations argue that at long and intermediate distance, the nuclear force is not of strength of order \(N_c\). How these facts become consistent with the large \(N_c\) approximation is a mystery. Perhaps the small binding energy of nuclear matter is an accident peculiar to \(N_c = 3\). If not, then we do not understand something fundamental about the nature of QCD.
Acknowledgments

I thank my colleague Rob Pisarski, Yoshimasa Hidaka, Krzysztof Redlich, Chhiro Sasaki and Toru Kojo with whom I have collaborated in the development of these ideas. The research of L. McLerran is supported under DOE Contract No. DE-AC02-98CH10886.

References

[1] L. McLerran and R. D. Pisarski, Nucl. Phys. A 796, 83 (2007) [arXiv:0706.2191 [hep-ph]].
[2] Y. Hidaka, L. D. McLerran and R. D. Pisarski, Nucl. Phys. A 808, 117 (2008) [arXiv:0803.0279 [hep-ph]].
[3] K. Fukushima, Phys. Lett. B 591, 277 (2004) [arXiv:hep-ph/0310121].
[4] C. Ratti, M. A. Thaler and W. Weise, Phys. Rev. D 73, 014019 (2006) [arXiv:hep-ph/0506234].
[5] R. D. Pisarski, Phys. Rev. D 62, 111501 (2000) [arXiv:hep-ph/0006205].
[6] K. Fukushima, Phys. Rev. D 77, 114028 (2008) [Erratum-ibid. D 78, 039902 (2008)] [arXiv:0803.3318 [hep-ph]].
[7] L. McLerran, K. Redlich and C. Sasaki, Nucl. Phys. A 824, 86 (2009) [arXiv:0812.3585 [hep-ph]].
[8] C. B. Thorn, Phys. Lett. B 99, 458 (1981).
[9] L. Y. Glozman and R. F. Wagenbrunn, Mod. Phys. Lett. A 23, 2385 (2008) [arXiv:0802.0276 [hep-ph]].
[10] L. Y. Glozman, arXiv:0803.1636 [hep-ph].
[11] L. Y. Glozman and R. F. Wagenbrunn, arXiv:0805.4799 [hep-ph].
[12] L. Y. Glozman, arXiv:0812.1101 [hep-ph].
[13] L. Y. Glozman, arXiv:0907.1473 [hep-ph].
[14] E. Shuster and D. T. Son, Nucl. Phys. B 573, 434 (2000) [arXiv:hep-ph/9905448].
[15] B. Y. Park, M. Rho, A. Wirzba and I. Zahed, Phys. Rev. D 62, 034015 (2000) [arXiv:hep-ph/9910347].
[16] I. R. Klebanov, Nucl. Phys. B 262, 133 (1985).