Proportion problems: analyzing common errors

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Abstract. Proportion problems are problems that are often found in real life. For that, we need to be able to solve problems related to proportion. But some of us make the mistake of solving problems about proportion. The purpose of this research is to identify the types of errors made by junior high school students working on proportion problems. The types of errors students make are identified based on the types of test-taking errors by Nolting. There are three types of errors analyzed in this research, namely careless errors, concept errors, and test-taking errors. This research is qualitative descriptive research. Subjects’ research included 51 junior high school students, 27 male and 24 female. The research data were obtained from a test instrument of proportion topic consisting of 3 essay questions. The results of the research analysis showed that in problem 1 as many as 3.91% of students made first type error, 11.76% for second type, and 33.33% for third type. In problem 2 as many as 7.84% of students made first type error, 41.18% for second type, and 7.84% for third type. Whereas in problem 3 as many as 0% of students made first type error, 21.57% for second type, and 62.75% for third type. Teachers need to design effective learning so that students' errors can be decreased.

1. Introduction
Proportion is one of the topics in mathematics learning. In middle school mathematics, proportion is an important topic [1]. Some problems in daily life can be solved through the concept of proportion. Scale on the map, calculating distances, etc. are examples of applying the proportion topic. The National Council of Teachers of Mathematics [2] revealed that proportion is one of the basic ideas in mathematics that helps students to understand other mathematical schemes and connect them. So, it is important for students to understand concept and improve their abilities on this topic.

One way to improve students' mathematical abilities, especially the proportion topic, is by analyzing errors from previous students' work [3]. Through error analysis, students can form learning experiences, so students have a deeper understanding of mathematics [4]. Furthermore, Sisman and Aksu [5] revealed that the analysis of student errors was one of the important contributions to overcome obstacles in learning mathematics. So, it is important for teachers and educators to analyze student errors. Teachers need to know the minds of students in order to create effective learning.

There are six types of test-taking errors by Nolting [3], three of which are careless errors, concept errors, and test-taking errors. Careless errors are errors made by students due to carelessness and lack of student accuracy. Concept errors are errors made by students when students do not understand concepts or principles that should be used to solve mathematical problems. And test-taking errors are errors that occur when students do not solve the problem as a whole or do not work at all.

Several factors that cause students to make errors, among others, students rush when reading the questions that are available, students cannot make mathematical models of the problems, lack of understanding the concepts of the topic, less thorough and skilled in solving problems, and students
have minimal experience when working on different problem variations [6]. Ansari et al. [7] also revealed that the lack of student attention during learning was also one of the factors causing students to make calculation errors and this also depends on individual differences and their mathematical competence. Students also need to improve their intuition in mathematics to reduce error rates [8], through routine practice.

Furthermore, teacher factors can contribute so students make errors. Teachers need to know what factors are causing students to make errors in translating problems [9]. In addition, teachers need to have the ability to provide feedback on student errors and be able to interpret errors made by students when learning mathematics [10, 11]. Through knowing various factors of student error, the teacher can design learning so as to reduce the number of errors made by students.

Evaluate based on student error to improve mathematics teaching is important [12]. So, it is necessary to analyze the errors in the students' answers. The purpose of this research is to identify errors made by 7th grade junior high school students on the topic of proportion. The error categories used in this research are based on the types of test-taking errors by Nolting [3]. There are 3 types of errors used in this research, namely careless errors, concept errors, and test-taking errors.

2. Method

This research is qualitative descriptive research. The purpose of this research is to determine and analyze errors made by students in solving proportion problems. Errors made by students were analyzed based on the type of test-taking errors by Paul D Nolting [3]. As for this research the type of test-taking errors used are:

1) Careless Errors
2) Concept Errors
3) Test-Taking Errors

The research subjects were 51 grade 7th junior high school students, consisting of 27 male students and 24 female students. Students involved in this research have been taught proportion topics in their classrooms. Then, the data collection in this research used the essay test instrument that has been validated by experts. The essay test consists of three questions related to the topic of proportion for 7th grade junior high school students. This test is used to obtain student answers that represent students' abilities in the topic of proportion. Data collection was carried out in January 2020. The steps of students in working on the problems are then analyzed and categorized based on types of error. The proportion questions given to students are as follows:

1) If \( N = \frac{2}{3} E \) dan \( E = \frac{2}{5} W \). What is the ratio N, E, and W?
2) If \( a : b = 3 : 5 \), then \( \frac{a}{a+b} + \frac{a^2}{b} = \ldots \)
3) On Monday the ratio of Alfi and Hasan's money is 2 : 1. The next day, Alfi gives Rp 100,000.00 to Hasan, so that the ratio of Alfi and Hasan's money becomes 1 : 3. How much is Alfi's money now?

Problem number 1 is a problem related to proportion, number 2 is a problem of proportion involving variables, and problem number 3 is related to the application of proportion in real life.

3. Result and Discussion

The results of the research below are the results of the analysis of student answers to the proportion problem. Some students make errors while solving a given problem. Table 1 shows the data of students who answered correct and incorrect in this research.

| Problem | Correct | Incorrect |
|---------|---------|-----------|
|         | Total   | Percentage| Total   | Percentage|
| Problem 1 | 28      | 54.90%    | 23      | 45.10%    |
| Problem 2 | 22      | 43.14%    | 29      | 56.86%    |
| Problem 3 | 8       | 15.69%    | 43      | 84.31%    |
Table 1 is a table that shows the number of students who answered correctly and made errors on the proportion questions. In addition to the overall data analysis, the data is also analyzed based on gender answers. Table 2 shows the data of students who answered correct and incorrect based on gender.

| Problem  | Correct Male | Incorrect Male | Correct Female | Incorrect Female |
|----------|--------------|----------------|----------------|------------------|
|          | Total | Percentage | Total | Percentage | Total | Percentage | Total | Percentage |
| Problem 1| 13   | 48.15%     | 14   | 51.85%     | 15   | 62.50%     | 9    | 37.50%     |
| Problem 2| 11   | 40.74%     | 16   | 59.26%     | 11   | 45.83%     | 13   | 54.17%     |
| Problem 3| 1    | 3.70%      | 26   | 96.30%     | 7    | 29.17%     | 17   | 70.83%     |

Based on Table 1 above, it can be seen that in problem 1 more than half the students can answer the question correctly. Problem 1 is the most problem that can be solved by students. Figure 1 is the work of students who answered correctly on problem 1. In problem 2 it appears that more than half of students make errors when solving this problem. In addition, it appears that students are the weakest in solving problems with the type of application of the concept of proportion in daily life. As many as 84.31% of students made errors on problem 3. Only 8 students answered correctly. Only 1 male student can answer this question correctly, while 7 female students.

The data above also shows that in sequence students have more difficulty in solving problem 3, problem 2, then problem 1. The difficulty level of students to solve problem 3 is greater than problem 2 and problem 2 is more difficult than problem 1. Besides, it can be seen that the percentage of error male students, more than female students. This is in accordance with research conducted by Agnesti and Amelia [13] that male students make more errors compared to female students on the problem of middle school proportion.

Figure 1 shows students' correct answers in problem 1. The figure shows that students do not write information that is known, asked, and resolved. Students immediately answer these questions and the answers given by students are correct. Students look for the least common multiple score of E to find the ratio of N and W. It can be said that the student understands how to solve problems with the type of problem 1. Most students who answer correctly use the method in figure 1. Students also need to understand the concept of proportion to be able to solve this problem. Students who master mathematical concepts will make fewer errors when solving problems [14].

Furthermore, students' incorrect answers are analyzed to determine the form of errors made by students. There are 3 errors analyzed in this research, namely careless errors, concept errors, and test-taking errors. The number and percentage of student error categories can be seen in table 3. Table 4 also shows the number and percentage of student errors by gender.

![Figure 1. Examples of student’s correctly answer in problem 1](Image)
Table 3. Total of student errors in each problem

| Problem | Careless Error | Concept Error | Test-Taking Error |
|---------|----------------|---------------|------------------|
|         | Total Percentage | Total Percentage | Total Percentage |
| Problem 1 | 2 3.92% | 6 11.76% | 17 33.33% |
| Problem 2 | 4 7.84% | 21 41.18% | 4 7.84% |
| Problem 3 | 0 0% | 11 21.57% | 32 62.75% |

Table 4. Total of student errors in each problem based on gender

| Error | Problem | Male | Female |
|-------|---------|------|--------|
|       | Total Percentage | Total Percentage |
| Careless Error | Problem 1 | 2 7.41% | 0 0% |
|       | Problem 2 | 1 3.70% | 3 12.50% |
|       | Problem 3 | 0 0% | 0 0% |
| Concept Error | Problem 1 | 5 18.52% | 1 4.17% |
|       | Problem 2 | 11 40.74% | 10 41.67% |
|       | Problem 3 | 8 29.63% | 3 12.50% |
| Test-Taking Error | Problem 1 | 8 29.63% | 9 37.50% |
|       | Problem 2 | 4 14.81% | 0 0% |
|       | Problem 3 | 18 66.67% | 14 58.33% |

Based on the above table, it can be seen that students make more test-taking error types in problem 1. Male and female students make the same error. If seen further, in problem 1, males are more likely to make careless errors and concept errors than females. Even female students don't make careless errors. This shows that the accuracy of female students is higher than that of male students for problem 1. Students must be careful when solving mathematical problems to avoid errors [6].

In problem 2, most students make concept errors when solving problems. Only a small proportion of students with careless errors and test-taking errors. The number of male students who make concept errors is greater than female students. Pala, Herman, and Prabawanto [15] in their research also showed that most students make concept errors when solving mathematical problems. Then, there were no female students who did test-taking errors in problem 2. But the number of female students in careless error was more in this problem.

Furthermore, problem 3 is a problem where students make the most errors. Nobody makes careless errors. Most students do test-taking errors and concept errors. The number of male students who make this error is not much different from female students.

It is necessary to further investigate the types of test-taking errors conducted by students. In this research, there are two types of test-taking errors in question, namely errors if students do not finish working on the problems and students do not answer the questions at all. The results of test-taking error data can be seen in table 5.

Table 5. Data students’ test-taking errors in each problem

| Problem | Not completed | Not Answered |
|---------|---------------|--------------|
|         | Total Percentage | Total Percentage |
| Problem 1 | 6 11.76% | 11 21.57% |
| Problem 2 | 0 0% | 4 7.84% |
| Problem 3 | 4 7.84% | 28 54.90% |

Table 5 shows that most students do not answer at all to all problems. Students leave their answer sheets blank, not trying to solve the problem. This can be because students do not understand the problem, students do not know how to solve it, students do not understand the concepts used to solve the problem. Students need to increase their initial knowledge and understanding of concepts to reduce the level of error in working on problems [16].
The research findings also showed that more than half of the students (54.90%) did not answer at all to problem 3. Problem 3 was a problem of daily life in the form of narration. This indicates that students have more difficulties in dealing with daily life problems. In the research conducted by Agnesti and Amelia [13] also found that students could not solve problems in the form of real problems. Students do not do it at all, students just write down information from the problem, or scribble on answer sheets.

Furthermore, the following figure is an example of an error made by students in problem 1 (figure 2), problem 2 (figure 3), and problem 3 (figure 4).

**Figure 2. Example of student's error in problem 1**

Figure 2 above shows the error made by students in problem 1. The student does not compare N, E, and W. However, the student compares separately N with E, E with W, and W with E. So the answers given by students are incorrect. Errors made by students in this proportion problem can be caused by students not understanding the concepts that must be used to solve the problem. Then the errors made by the students above are included in the concept error type. Concept errors can occur if students cannot express the ratio [17]. In addition, students have difficulty finding least common multiple value to solve proportion problems [16], so students make errors on this problem.

**Figure 3. Example of student's error in problem 2**

There is an error made by students in figure 3. The student immediately substitutes the value of proportion of a and b into the question variable. Students do not understand that the proportion value is not the actual value of variables a and b. This is because students do not understand the concept of proportion. Error students understand the concept can be experienced by students if the student does not understand the precondition before. Bohlmann, Prince, and Deacon [18] revealed that students who were not taught multiplication and division operations properly in elementary school could cause students to make errors later on. The prerequisites for the topic of proportion’s junior high school are basic operations, fractions, algebra, and one variable linear equation. Students' misconceptions in understanding questions also affect errors made by students when solving problems [19].
Figure 4 shows the concept errors made by students in problem 3. Students do not understand the meaning of the problem given. Students immediately calculate by using the ratio numbers in the problem, without understanding more deeply the meaning of the numbers. Many students have difficulty if given a real problem such as problem 3. Students usually have difficulty solving real problems because students make errors in calculations, students do not understand the meaning of the problem, and students do not know what they should find from the problem information [13].

Various errors made by students in solving mathematical problems can be a reflection of the teacher and students for the future. Seeing the various errors made by students on the topic of proportion, teachers need to arrange good learning. An and Zu [20] in their research revealed that it is important for teachers to analyze errors based on student work. That way the teacher can build student knowledge. Providing appropriate feedback to errors made by students can also reduce the number of student errors [10]. Not only teachers, all related parties also need to find solutions to overcome student errors [15].

4. Conclusion

Based on the results of the analysis, it can be seen that male students make more errors compared to female students. In addition, the majority of students have difficulty in solving real problems. Many students do not answer the real problem given. Taking-tests errors are the type of error most students make in this research. Some factors that can cause this error are the prerequisite material that is not mastered, students do not understand the problem, and students do not understand the concept of the topic. The teacher must ensure that the students' prerequisites are met only when meeting. In addition, teachers must create classroom learning that facilitates students' understanding of mathematical concepts. Further research is important to look into students' ways based on errors made. This can make it easier for teachers or students to improve learning.

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