THEORY AND TESTS OF CPT AND LORENTZ VIOLATION

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A general Lorentz-violating extension of the standard model of particle physics, allowing for both CPT-even and CPT-odd effects, is described. Some of its theoretical aspects and experimental implications are summarized.

1 Introduction

Nature appears to be covariant both under the discrete transformation CPT, formed from the product of charge conjugation C, parity inversion P, and time reversal T, and under the continuous Lorentz transformations including rotations and boosts. The CPT theorem links these symmetries, stating that under mild technical assumptions CPT is an exact symmetry of local Lorentz-covariant field theories of point particles.

High-precision tests of both CPT and Lorentz invariance exist. According to the Particle Data Group, the best figure of merit for CPT tests involves the kaon particle-antiparticle mass difference, which has been bounded by experiments at Fermilab and CERN to

$$\frac{|m_K - m_{\bar{K}}|}{m_K} \lesssim 10^{-18}.$$  (1)

Indeed, at present CPT is the only combination of C, P, T observed as an exact symmetry of nature at the fundamental level.

The existence of high-precision experimental tests and of the general CPT theorem for Lorentz-covariant particle theories means that the observation of CPT or Lorentz violation would be a sensitive signal for unconventional physics beyond the standard model. It is therefore interesting to consider possible theoretical mechanisms through which CPT or Lorentz symmetry might be violated. Most suggestions along these lines in the literature either have physical features that seem unlikely to be realized in nature or involve radical revisions of conventional quantum field theory, or both.

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Nonetheless, there does exist at least one promising theoretical possibility, based on spontaneous breaking of CPT and Lorentz symmetry in an underlying theory that appears to be compatible both with experimental constraints and with established quantum field theory. It suggests that apparent breaking of CPT and Lorentz symmetry might be observable in existing or feasible experiments, and it leads to a general phenomenology for CPT and Lorentz violation at the level of the standard model and quantum electrodynamics (QED). The formulation and experimental implications of this theory are briefly described in this talk.

2 Framework

In principle, one can attempt to circumvent the difficult issue of developing a satisfactory microscopic theory allowing CPT and Lorentz breaking by adopting a purely phenomenological approach. This can be done by identifying and parametrizing observable quantities that allow for CPT or Lorentz violation. A well-known example is the phenomenology of CPT violation in oscillations of neutral kaons. In the neutral-kaon system, linear combinations of the strong-interaction eigenstates $K^0$ and $\bar{K}^0$ form the physical eigenstates $K_S$ and $K_L$. These combinations contain two complex parameters, $\epsilon_K$ and $\delta_K$, parametrizing CP violation. One, $\epsilon_K$, governs T violation with CPT symmetry while the other, $\delta_K$, governs CPT violation with T symmetry. The standard model of particle physics has a mechanism for T violation, and so $\epsilon_K$ is in this context nonzero and in principle calculable. However, CPT is a symmetry of the standard model and so $\delta_K$ is expected to vanish. The possibility of a nonzero value of $\delta_K$ is from this perspective only a phenomenological choice. It has no grounds in a microscopic theory and $\delta_K$ is therefore not calculable. Indeed, in the absence of a microscopic theory, it is even unclear whether this parametrization makes physical sense. Moreover, without a microscopic origin $\delta_K$ cannot be linked to other phenomenological parameters for CPT tests in different experiments.

Evidently, it is more attractive theoretically to develop an explicit microscopic theory for CPT and Lorentz violation. With a theory of sufficient scope, a general and quantitative phenomenology for CPT and Lorentz violation could then be extracted at the level of the standard model. This would allow calculation of phenomenological parameters, direct comparisons between experiments, and perhaps the prediction of signals.

The development of a microscopic theory of this type is feasible within the context of spontaneous CPT and Lorentz breaking. The idea is that the underlying theory of nature has a Lorentz- and CPT-covariant action, but
apparent violations of these symmetries could result from their spontaneous violation in solutions to the theory. It appears that this mechanism is viable from the theoretical viewpoint and is an attractive way to violate CPT and Lorentz invariance.

Since spontaneous breaking is a property of the solution rather than the dynamics of a theory, the broken symmetry plays an important role in establishing the physics. In the case of CPT and Lorentz violation, spontaneous breaking has the advantage that many of the desirable properties of a Lorentz-covariant theory can be expected. This is in sharp distinction to other types of CPT and Lorentz breaking, which often are inconsistent with theoretical notions such as causality or probability conservation.

The physics of a particle in a vacuum with spontaneous Lorentz violation is in some respects similar to that of a conventional particle moving inside a biaxial crystal. This system typically breaks Lorentz covariance both under rotations and under boosts. However, instead of leading to fundamental problems, the lack of Lorentz covariance is merely a result of the presence of the background crystal fields, which leaves unaffected features such as causality. Indeed, one can explicitly confirm microcausality in certain simple models arising from spontaneous CPT and Lorentz breaking.

In a Lorentz-covariant theory, certain types of interaction among Lorentz-tensor fields could trigger spontaneous breaking of Lorentz symmetry. The idea is that these interactions could destabilize the naive vacuum and generate nonzero Lorentz-tensor expectation values, which fill the true vacuum and cause spontaneous Lorentz breaking. This also induces spontaneous CPT violation whenever the expectation values involve tensor fields with an odd number of spacetime indices. Provided components of the expectation values lie along the four macroscopic spacetime dimensions, apparent violations of CPT and Lorentz symmetry could arise at the level of the standard model. This could lead to observable effects, some of which are described in the following sections.

Conventional four-dimensional renormalizable gauge theories such as the standard model lack the necessary destabilizing interactions to trigger spontaneous Lorentz violation. However, the mechanism may be realized in some string (M) theories because suitable Lorentz-tensor interactions occur. This can be investigated using string field theory in the special case of the open bosonic string, where the action and equations of motion can be analytically derived for particle fields below some fixed level number $N$. Obtaining and comparing solutions for different $N$ allows the identification of solutions that persist as $N$ increases. For some cases this procedure has been performed to a depth of over 20,000 terms in the static potential.
stable as $N$ increases include ones spontaneously breaking Lorentz symmetry.

In standard field theories, spontaneous breaking of a continuous global symmetry is accompanied by the appearance of massless modes, ensured by the Nambu-Goldstone theorem. Promoting a global spontaneously broken symmetry to a local gauge symmetry leads to the Higgs mechanism: the massless modes disappear and a mass is generated for the gauge boson. Similarly, spontaneous breaking of a continuous global Lorentz symmetry would also lead to massless modes. However, although the inclusion of gravity promotes Lorentz invariance to a local symmetry, no analogue to the Higgs effect occurs. The dependence of the connection on derivatives of the metric rather than the metric itself ensures that the graviton propagator is affected in such a way that no graviton mass is generated when local Lorentz symmetry is spontaneously broken.

3 Standard-Model and QED Extensions

Assuming spontaneous CPT and Lorentz violation does occur, then any apparent breaking at the level of the $SU(3) \times SU(2) \times U(1)$ standard model and QED must be highly suppressed to remain compatible with established experimental bounds. If the appropriate dimensionless suppression factor is determined by the ratio of a low-energy (standard-model) scale to the (Planck) scale of an underlying fundamental theory, then relatively few observable effects of Lorentz or CPT violation would arise. To study these, it is useful to develop an extension of the standard model obtained as the low-energy limit of the fundamental theory.

To gain insight about the construction of such an extension, consider as an example a possible coupling between one or more bosonic tensor fields and fermion bilinears in the low-energy limit of the underlying theory. When the tensors acquire expectation values $\langle T \rangle$, the low-energy theory gains additional terms of the form

$$L \sim \frac{\lambda}{M^k} \langle T \rangle \cdot \bar{\psi} \Gamma(k\partial) \chi + h.c. \quad .$$

(2)

Here, the gamma-matrix structure $\Gamma$ and the $k$ spacetime derivatives $i\partial$ determine the Lorentz properties of the bilinear in the fermion fields $\psi, \chi$ and hence fix the type of apparent CPT and Lorentz violation in the low-energy theory. The effective coupling involves an expectation $\langle T \rangle$ together with a dimensionless coupling $\lambda$ and a suitable power of a large (Planck or compactification) scale $M$ associated with the fundamental theory.

Proceeding along these lines, one can determine all possible terms arising at the level of the standard model from spontaneous CPT and Lorentz breaking in any underlying theory (not necessarily string theory). This leads
to a general Lorentz-violating extension of the standard model that includes both CPT-even and CPT-odd terms. It contains all possible allowed hermitian terms preserving both $SU(3) \times SU(2) \times U(1)$ gauge invariance and power-counting renormalizability. It appears at present to be the sole candidate for a consistent extension of the standard model based on a microscopic theory of Lorentz violation.

Despite the apparent CPT and Lorentz breaking, the standard-model extension exhibits several desirable properties of conventional Lorentz-covariant field theories by virtue of its origin in spontaneous symmetry breaking from a covariant underlying theory. Thus, the usual quantization methods are valid and features like microcausality and positivity of the energy are to be expected. Also, energy and momentum are conserved provided the tensor expectation values are independent of spacetime position (no soliton solutions). Even one type of Lorentz symmetry remains: the theory is covariant under rotations or boosts of the observer’s inertial frame (observer Lorentz transformations). The apparent Lorentz violations appear only when (localized) fields are rotated or boosted (particle Lorentz transformations) relative to the vacuum tensor expectation values.

In the case of the conventional standard model, one can obtain the usual versions of QED by taking suitable limits. For the standard-model extension, it can be shown that the usual gauge symmetry breaking to the electromagnetic $U(1)$ occurs, and taking appropriate limits yields generalizations of the usual versions of QED. It turns out that the apparent CPT and Lorentz breaking can arise in both the photon and fermion sectors. These extensions of QED are of particular interest because many high-precision QED tests of CPT and Lorentz symmetry exist.

An explicit and relatively simple example is the restriction of the standard-model extension to an extension of QED involving only photons, electrons, and positrons. The usual lagrangian is:

\begin{equation}
L_{\text{QED}} = \overline{\psi} \gamma^\mu \left( \frac{1}{2} i \hat{\sigma} \cdot \partial \mu - q A_\mu \right) \psi - m \overline{\psi} \psi - \frac{1}{4} F_{\mu \nu} F^{\mu \nu}.
\end{equation}

(3)

Apparent Lorentz violation can occur in both the fermion and photon sectors, and it can be CPT even or CPT odd. The CPT-violating terms are:

\begin{align}
L_{\gamma}^{\text{CPT}} &= -a_\mu \overline{\psi} \gamma^\mu \psi - b_\mu \overline{\psi} \gamma_5 \gamma^\mu \psi,
L_{\gamma}^{\text{CPT}} &= \frac{1}{2} (k_A F) \epsilon_{\kappa \lambda \mu \nu} A^\kappa F^{\mu \nu}.
\end{align}

(4)

The CPT-preserving terms are:

\begin{align}
L_{\gamma}^{\text{Lorentz}} &= c_{\mu \nu} \overline{\psi} \gamma^\mu \left( \frac{1}{2} i \hat{\sigma} \cdot \partial^\nu - q A^\nu \right) \psi + d_{\mu \nu} \overline{\psi} \gamma_5 \gamma^\mu \left( \frac{1}{2} i \hat{\sigma} \cdot \partial^\nu - q A^\nu \right) \psi - \frac{1}{2} H_{\mu \nu} \overline{\psi} \sigma^{\mu \nu} \psi.
\end{align}
\[ \mathcal{L}_\gamma^{\text{Lorentz}} = -\frac{1}{4}(k_F)_{\kappa\lambda\mu\nu} F^{\kappa\lambda} F^{\mu\nu}. \]

The reader is referred to the literature for details of the notation and conventions and for information about the properties of the extra terms. Note, however, that all these terms are invariant under observer Lorentz transformations, whereas the expressions in Eqs. (4) and (5) violate particle Lorentz invariance: the coefficients of the extra terms behave as (minuscule) Lorentz- and CPT-violating couplings. Note also that not all the components of the coefficients appearing are physically observable. For example, field redefinitions can be used to eliminate some coefficients of the type \( a_\mu \) in the standard-model extension. It turns out that these can be directly detected only in flavor-changing experiments, and so they are unobservable at leading order in experiments restricted to electrons, positrons, and photons.

### 4 Experimental Tests

The standard-model extension described above forms a quantitative framework within which various experimental tests of CPT and Lorentz symmetry can be studied and compared. Moreover, potentially observable signals can be deduced in some cases. Evidently, any tests seeking to establish nonzero CPT- and Lorentz-violating terms in the standard-model extension must contend with the expected heavy suppression of physical effects.

Although many tests of CPT and Lorentz symmetry lack the necessary sensitivity to possible signals, a few special ones can already place useful constraints on some of the new couplings in the standard-model extension. Several of these tests are discussed elsewhere in these proceedings. Among the ones investigated to date are experiments with neutral-meson oscillations, comparative tests of QED in Penning traps, spectroscopy of hydrogen and antihydrogen, measurements of cosmological birefringence, and observations of the baryon asymmetry. The remainder of this talk provides a brief outline of some of these studies. Other work is in progress, including an investigation of constraints from clock-comparison experiments.

#### 4.1 Neutral-Meson Oscillations

Flavor oscillations occur or are anticipated in a variety of neutral-meson systems, including \( K, D, B_d \), and \( B_s \). A neutral-meson state evolves in time according to a non-hermitian two-by-two effective hamiltonian in the meson-antimeson state space. The effective hamiltonian involves complex parameters \( \epsilon_P \) and \( \delta_P \) that govern (indirect) CP violation, where the neutral meson is denoted by \( P \). In the \( K \) system, \( \epsilon_K \) and \( \delta_K \) are the same phenomenological quantities mentioned in section 2. The parameter \( \epsilon_P \) governs \( T \) violation,
while $\delta_P$ governs CPT violation. Bounds on CPT violation can be obtained by constraining the magnitude of $\delta_P$ in experiments with meson oscillations.

In the context of the usual standard model, which preserves CPT, $\delta_P$ is necessarily zero. In contrast, in the context of the standard-model extension $\delta_P$ is a derivable quantity. It turns out that at leading order $\delta_P$ depends only on a single type of extra coupling in the standard-model extension. This type of coupling has the form $-a^q_{\mu} \bar{q} \gamma^\mu q$, where $q$ represents one of the valence quark fields in the $P$ meson and the quantity $a^q_{\mu}$ is spacetime constant but depends on the quark flavor $q$.

Since Lorentz symmetry is broken in the standard-model extension, the derived expression for $\delta_P$ varies with the boost and orientation of the $P$ meson. Denoting by $\beta^\mu \equiv \gamma(1, \vec{\beta})$ the four-velocity of the $P$-meson in the frame in which the quantities $a^q_{\mu}$ are specified, it can be shown that $\delta_P$ is given at leading order in all coupling coefficients in the standard-model extension by

$$
\delta_P \approx i \sin \hat{\phi} \exp(i \hat{\phi}) \gamma(\Delta a_0 - \vec{\beta} \cdot \Delta \vec{a})/\Delta m.
$$

(6)

For simplicity, subscripts $P$ have been omitted on the right-hand side. In Eq. (6), $\Delta a_\mu \equiv a^q_{\mu} - a^{q_1}_{\mu}$, where $q_1$ and $q_2$ are the valence-quark flavors for the $P$ meson. Also, $\hat{\phi} \equiv \tan^{-1}(2\Delta m/\Delta \gamma)$, where $\Delta m$ and $\Delta \gamma$ are the mass and decay-rate differences, respectively, between the $P$-meson eigenstates.

This result has several implications. One is that tests of CPT and Lorentz symmetry with neutral mesons are independent at leading order of all other types of tests mentioned here. This is because $\delta_P$ is sensitive only to $a^q_{\mu}$ and because this sensitivity arises from flavor-changing effects. None of the other experiments described here involve flavor changes, which can be shown to imply that none are sensitive to any $a^q_{\mu}$.

The result (6) also makes predictions about signals in experiments with neutral mesons. For example, the real and imaginary parts of $\delta_P$ are predicted to be proportional. Similarly, Eq. (6) suggests that the magnitude of $\delta_P$ may be different for different $P$ due to the flavor dependence of the coefficients $a^q_{\mu}$. For example, if the coefficients $a^q_{\mu}$ grow with mass as do the usual Yukawa couplings, then the heavier neutral mesons such as $D$ or $B_d$ may exhibit the largest CPT-violating effects.

The dependence of the result (6) on the meson boost magnitude and orientation implies several notable effects in the signals for CPT and Lorentz violation. For example, two different experiments may have inequivalent CPT and Lorentz reach despite having comparable statistical sensitivity. This could arise if the mesons for one experiment are well collimated while those for the other have a $4\pi$ distribution, or if the mesons involved in the two experiments
have very different momentum spectra. Another interesting effect is the possibility of diurnal variations in the data, arising from the rotation of the Earth relative to the orientation of the coupling coefficients. This issue may be of some importance because the data in neutral-meson experiments are typically taken over many days.

At present, the tightest clean experimental constraints on CPT violation come from observations of the $K$ system. Some experimental results are now also available for the heavier neutral-meson systems. Two collaborations at CERN have performed analyses to investigate whether existing data suffice to bound CPT violation. The OPAL collaboration has published the measurement $\text{Im} \delta_{B_d} = -0.020 \pm 0.016 \pm 0.006$, while the DELPHI collaboration has announced a preliminary result of $\text{Im} \delta_{B_d} = -0.011 \pm 0.017 \pm 0.005$. Further theoretical and experimental studies are underway.

4.2 QED Experiments

High-precision measurements of properties of particles and antiparticles can be obtained by trapping individual particles for extended time periods. Comparison of the results provides sensitive CPT tests. Such experiments can constrain the couplings in the fermion sector of the QED extension.

Penning traps can be used to obtain comparative measurements of particle and antiparticle anomaly and cyclotron frequencies. The QED extension predicts direct signals and also effects arising from diurnal variations in the Earth-comoving laboratory frame. Appropriate figures of merit for the various signals have been defined and the attainable experimental sensitivity estimated.

As one example, comparing the anomalous magnetic moments of electrons and positrons would generate an interesting bound on the spatial components of the coefficient $b'_e$ in the laboratory frame. Available technology could place a limit of order $10^{-20}$ on the associated figure of merit. A related test involves the search for diurnal variations of the electron anomaly frequency, for which a new experimental result with a figure of merit bounded at $6 \times 10^{-21}$ is presented in these proceedings by Mittleman, Ioannou, and Dehmelt. Analogous experiments with protons and antiprotons may be feasible.

Particle and antiparticle cyclotron frequencies can also be compared. In these proceedings, Gabrielse and coworkers present the results of an experiment comparing the cyclotron frequencies of $H^-$ ions and antiprotons in the same trap. The leading-order effects in this experiment provide a test of Lorentz violation in the context of the standard-model extension, with an associated figure of merit bounded at $4 \times 10^{-25}$. 
Tests of CPT and Lorentz symmetry are also possible via high-precision comparisons of spectroscopic data from trapped hydrogen and antihydrogen. An investigation of the possible experimental signals within the context of the standard-model and QED extensions has been performed. Direct sensitivity to CPT- and Lorentz-violating couplings, without suppression factors associated with the fine-structure constant, arises for certain specific 1S-2S and hyperfine transitions in magnetically trapped hydrogen and antihydrogen. In principle, theoretically clean signals might be observed for particular types of CPT and Lorentz violation.

The photon sector of the QED extension can also be tightly constrained from a combination of theoretical considerations and terrestrial, astrophysical, and cosmological experiments on electromagnetic radiation. It is known that the pure-photon CPT-violating term in Eq. (3) can generate negative contributions to the energy, which may limit its viability and suggests the coefficient \((k_{AF})^\mu\) should be zero. In contrast, the CPT-even term in the following equation maintains a positive conserved energy.

The solutions of the extended Maxwell equations with CPT- and Lorentz-breaking effects involve two independent propagating degrees of freedom as usual. Unlike the conventional propagation of electromagnetic waves in vacuum, however, in the extended Maxwell case the two modes have different dispersion relations. This means the vacuum is birefringent. Indeed, the effects of the CPT and Lorentz violation on an electromagnetic wave traveling in the vacuum are closely analogous to those exhibited by an electromagnetic wave in conventional electrodynamics that is passing through a transparent optically anisotropic and gyrotropic crystal with spatial dispersion of the axes.

The sharpest experimental limits on the extra coefficients in the extended Maxwell equations can be obtained by constraining the birefringence of radio waves on cosmological distance scales. Considering first the CPT-odd coefficient \((k_{AF})^\mu\), one finds a bound of the order of \(\lesssim 10^{-42}\) GeV on its components. A disputed claim exists for a nonzero effect at the level of \(|k_{AF}| \sim 10^{-41}\) GeV.

For the CPT-even dimensionless coefficient \((k_F)_{\kappa\lambda\mu\nu}\), the single rotation-invariant irreducible component is constrained to \(\lesssim 10^{-23}\) by the existence of cosmic rays and other tests. Rotation invariance is broken by all the other irreducible components of \((k_F)_{\kappa\lambda\mu\nu}\). Although in principle it might be feasible to constrain these coefficients with existing techniques for measuring cosmological birefringence, no limits presently exist. It is plausible that a bound at the level of about \(10^{-27}\) could be placed on components of \((k_F)_{\kappa\lambda\mu\nu}\).

The sharp experimental constraints obtained on \((k_{AF})_\mu\) are compatible with the zero value needed to avoid negative-energy contributions. However,
no symmetry protects a zero tree-level value of $(k_{AF})_{\mu}$. It might therefore seem reasonable to expect $(k_{AF})_{\mu}$ to acquire a nonzero value from radiative corrections involving CPT-violating couplings in the fermion sector. Nonetheless, this does not occur: an anomaly-cancellation mechanism can ensure that the net sum of all one-loop radiative corrections is finite. The situation is technically involved because the contribution from each individual radiative correction is ambiguous, but the anomaly-cancellation mechanism can hold even if one chooses to define the theory such that each individual radiative correction is nonzero. Thus, a tree-level CPT-odd term is unnecessary for one-loop renormalizability. Similar effects may occur at higher loops. This ability to impose the vanishing of an otherwise allowed CPT-odd term represents a significant check on the consistency of the standard-model extension.

For the CPT-even Lorentz-violating pure-photon term there is no similar mechanism, and in fact calculations have explicitly demonstrated the existence of divergent radiative corrections at the one-loop level. This therefore leaves open the interesting possibility of future detection of a nonzero effect via measurements of cosmological birefringence.

Various other possible observable CPT effects have been identified. For example, under suitable conditions the observed baryon asymmetry can be generated in thermal equilibrium through CPT- and Lorentz-violating bilinear terms. A relatively large baryon asymmetry produced at grand-unified scales would eventually become diluted to the observed value through sphaleron or other effects. This mechanism represents one possible alternative to the conventional scenarios for baryogenesis, in which nonequilibrium processes and C- and CP-breaking interactions are required.

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