Stochastic and hybrid methods for the identification in the Bouc-Wen model for magneto – rheological dampers

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Abstract. Magneto-rheological (MR) dampers are semi-active control devices that require low power input and are considered fail-safe, i.e. in case of control hardware failure they become passive dampers. In the present work we consider a sensitivity analysis and use the deterministic gradient based Levenberg-Marquardt method, the stochastic methods Simulated Annealing and Genetic Algorithms, as well as hybridizations of these methods, for the estimation of parameters in the Bouc-Wen model, which is used to describe the dynamic behavior of MR dampers. The parameters associated with the hysteretic displacement evolution equation are the most difficult to be estimated, and therefore we use a strategy for estimating such parameters separately from other parameters of the model.

1. Introduction

Magneto-rheological (MR) dampers are semi-active control devices that in recent years have attracted the interest of several researchers. Such interest is justified by the large number of relevant applications for such devices in mechanical, civil and biomedical engineering.

Olabi and Grunwald [1] provided some examples of applications involving magneto-rheological fluids: dampers for knee prosthesis; vibration dampers; and seismic dampers for civil industry, among others. Thanth and Ahn [2] investigated the use of magneto-rheological brakes for the control of pneumatic artificial muscle manipulators.

Gu and Oyadiji [3] studied the application of MR dampers in structural control. Wei and Pinqi [4] presented an adaptive inverse control method in which the damper force of a MR damper is fit to a desired damping force.

Several models have been developed in order to describe the dynamic behavior of MR dampers. Stanway et al. [5] describes the Bingham model, Wereley et al. [6] the bi-viscous hysteresis model, Spencer et al. [7] the Bouc-Wen model, and Sims et al. [8] a unified model for ER (Electro-Rheological) and MR vibrations dampers. In fact it is observed a growing interest towards the development of models for magneto-rheological dampers behavior description [9-11].

Among these models, the one that describes well not only the bi-viscous behavior, but also the hysteretic behavior of MR dampers is the Bouc-Wen model.
Besides the interest in developing models for MR dampers, a lot of effort has also been devoted to parameter identification in such models. For that purpose, Kwok et al. [12] employed a particle swarm optimization (PSO) algorithm to identify parameters associated with a new MR damper model based on algebraic expressions, and the same authors used a Genetic Algorithm (GA) for the identification of parameters in the Bouc-Wen model [13]. Soeiro et al. [14] used a hybrid identification method – based on the Levenberg-Marquardt method (LM), the Simulated Annealing method (SA), and a combination of both methods (SA-LM) – in order to estimate the parameters of the Bouc-Wen model for a magneto-rheological damper.

In this work we first show the sensitivity analysis related to the parameters in the Bouc-Wen model [15]. We then apply the methods LM, GA, SA, and the hybridizations SA-LM and GA-LM, for the solution of the inverse problem of parameter estimation. We observe from the sensitivity analysis that the parameters associated with the hysteretic behavior of the MR damper are the most difficult to estimate. Therefore we apply a strategy in which such parameters are estimated while the other parameters of the Bouc-Wen model are considered fixed, being extracted directly from an observation of the experimental data.

2. Direct problem formulation: The Bouc-Wen model

The Bouc-Wen model is intended to describe the nonlinear dynamic behavior of the MR damper, and, as shown in Fig. 1, consists on a combination of an element of hysteresis, a viscous damper, and a spring.

![Figure 1. Schematic representation of the Bouc-Wen model for a MR damper.](image)

In the Bouc-Wen model the force \( u(t) \) generated by the MR damper, in response to a displacement \( q(t) \), is given by

\[
u(t) = c_0 \dot{q}(t) + k_0 (q(t) - x_0) + \alpha h(t), \tag{1}\]

where \( t \) stands for time, the overdot represents the time derivative, \( c_0 \) is the plastic damping coefficient, \( k_0 \) and \( x_0 \) account for the effects of the accumulator in the MR damper, which in a phenomenological perspective acts as a spring, and \( \alpha \) is a redundant coefficient that will be removed from the formulation. The internal variable \( h(t) \) in Eq. (1) represents the hysteretic displacement and its evolution equation is given by

\[
\dot{h}(t) = -\gamma \|\dot{q}(t)\| h(t) \|h(t)\|^{\alpha-1} - \beta \dot{q}(t) \|h(t)\|^{\alpha} + A \dot{q}(t), \tag{2}\]

where the parameters \( \gamma, \beta, A, n \) control the shape of the hysteresis curve.

In the model described by Eqs. (1) and (2) the dynamics of the MR damper is governed by eight parameters: \( c_0, k_0, x_0, \alpha, \gamma, \beta, A, n \). In the model described by Du et al. [9,10], which is based on the Bouc-Wen model, 14 parameters are considered.

Aiming to reduce the number of parameters to be estimated, the following hysteretic force is defined
and Eqs. (1) and (2) are rewritten as:

\[ u(t) = c_0 \dot{q}(t) + k_0 (q(t) - x_0) + Z(t); \]  

\[ \ddot{Z}(t) = -\tilde{\gamma} \dot{q}(t) \ |Z(t)|^{\alpha-1} - \tilde{\beta} \dot{q}(t) \ |\dot{Z}(t)|^{n} + \tilde{A} \dot{q}(t), \]  

where the parameters \( \tilde{\gamma}, \tilde{\beta}, \text{ and } \tilde{A} \) are given by:

\[ \tilde{\gamma} = \frac{\gamma}{\alpha^{n-1}}, \quad \tilde{\beta} = \frac{\beta}{\alpha^{n-1}}, \quad \tilde{A} = \alpha A. \]  

Therefore, the number of parameters to be estimated in the MR Bouc-Wen model is reduced from eight to seven and the parameter vector becomes

\[ \tilde{P} = \{c_0, k_0, x_0, \tilde{\gamma}, \tilde{\beta}, \tilde{A}, n\}^T. \]  

In the direct problem the vector \( \tilde{P} \) and the displacement \( q(t) \) are known, and then from Eqs. (4a) and (4b) the response force \( u(t) \) is calculated.

### 3. Inverse problem formulation

In the inverse problem we are interested in the vector \( \tilde{P} \), which represents the vector of unknowns. A displacement \( q(t) \) is imposed to the MR damper, and experimental data on the damper force is measured at different time instants.

The inverse problem of parameter estimation is stated as an optimization problem, in which we seek to minimize the squared residues functional

\[ \min_{\tilde{P}} Q(\tilde{P}) = \min_{\tilde{P}} \frac{1}{2} \tilde{F}(\tilde{P})^T \tilde{F}(\tilde{P}), \]  

where \( \tilde{F} \) is the vector of residues, whose components are defined as

\[ F_i(\tilde{P}) = u_i(\tilde{P}) - u_{\text{exp}}, \quad i = 1, 2, ..., N_t, \]  

where \( u_i \) and \( u_{\text{exp}} \) represent the calculated and experimental values for the damper force, at the time instant \( t_i \), and \( N_t \) is the total number of experimental data.

### 4. Inverse problem solution

For the solution of the minimization problem described by Eq. (7) we have considered a deterministic gradient based method, the Levenberg-Marquardt method (LM), and two stochastic methods: the Simulated Annealing method (SA); and the Genetic Algorithm method (GA). The hybridizations SA-LM and GA-LM were also considered.

#### 4.1 The Levenberg-Marquardt method (LM)

In the Levenberg-Marquardt method [16] we start the iterative procedure for the solution of the minimization problem, given by Eq. (7), with an initial guess for the vector of unknowns \( \tilde{P}^0 \), and new estimates are obtained with

\[ \tilde{P}^{n+1} = \tilde{P}^n + \Delta \tilde{P}^n, \]  

where \( n \) is the iteration index. The corrections \( \Delta \tilde{P}^n \) are obtained from the solution of the following system of linear equations..
\[
\left( J^T J + \lambda^n I \right) \Delta \tilde{P}^n = -J^T \tilde{F}^n ,
\]

where \( I \) is the identity matrix, \( \lambda \) is a damping parameter used to improve the convergence of the method, and \( J \) is the Jacobian matrix whose elements are given by

\[
J_{ij} = \frac{\partial u_i (\tilde{P})}{\partial P_j} , \quad i = 1, 2, \ldots, N_i ; \quad j = 1, 2, \ldots, 7.\]

The parameter \( \lambda^n \) is reduced from one iteration to the next according to the procedure proposed by Marquardt [16].

The LM iterative procedure is interrupted when a stopping criterion such as

\[
| \Delta P_j / P_j | < \varepsilon , \quad j = 1, 2, \ldots, 7
\]

is satisfied, where \( \varepsilon \) is a tolerance, say \( 10^{-5} \).

### 4.2 The Simulated Annealing method (SA)

Based on statistical mechanics reasoning applied to a solidification problem Metropolis et al. [17] introduced a simple algorithm that can be used to accomplish an efficient simulation of a system of atoms in equilibrium at a given temperature \( T \).

The main control parameters of the algorithm implemented (“cooling procedure”) are the initial “temperature”, \( T_0 \), the cooling rate \( r_t \), number of steps performed through all elements of vector \( \tilde{Z} \), \( N_s \), number of times the procedure is repeated before the “temperature” is reduced, \( N_r \), and number of points of minimum (one for each temperature) that are compared and used as the stopping criterion if they all agree within a tolerance \( \varepsilon \), after a number \( N_\varepsilon \) of “temperature” reductions. In the present work the following parameters were used: \( T_0 = 5.0 \), \( r_t = 0.75 \), \( N_s = 20 \), \( N_r = 5 \), \( \varepsilon = 10^{-6} \) and \( N_\varepsilon = 4 \).

### 4.3 Genetic Algorithms (GA)

Genetic Algorithms belong to the general category of stochastic global optimization methods [18]. They have their philosophical basis in a process found in nature related to the evolution of the different species. Darwin’s theory of survival of the fittest gives the main idea of the method. In the present work the following parameters were used in the GA algorithm: population size = 100, \( p_c = 0.8 \) (crossover probability), \( p_m = 0.01 \) (mutation probability) and uniform type of crossover.

### 4.4 Hybridizations SA-LM and GA-LM

The deterministic LM may not converge if a poor choice is made for the initial guess \( \tilde{P}^0 \), and if it converges it may in fact lead to a local minimum.

Silva Neto and Soeiro [19] have used previously the hybridizations SA-LM and GA-LM for the solution of inverse problems in heat conduction and radiative transfer.

In such hybridizations the stochastic method is used to generate an initial guess for the deterministic method i.e. \( \tilde{P}^0_{LM} = \tilde{P}_{SA} \) or \( \tilde{P}^0_{LM} = \tilde{P}_{GA} \) where \( \tilde{P}_{SA} \) and \( \tilde{P}_{GA} \) represent the estimates obtained with the Simulated Annealing method and the Genetic Algorithm, respectively. In order to save computational time the SA and GA are not run fully, i.e. in the former a not very strict convergence criterion tolerance is considered while in the latter a small number of generations is used.
5. Sensitivity analysis

The sensitivity analysis plays a major role in several aspects related to the formulation and solution of inverse problems, being a fundamental tool in the design of experiments [20, 21].

In the present work we investigate the behavior of the scaled sensitivity coefficients

\[ X_{P_i} = P_j \frac{\partial u_i}{\partial P_j}, \quad i = 1, 2, ..., N_i; \quad j = 1, 2, ..., N_u. \]  

(14)

The sensitivity of the damper force with respect to the unknowns we want to estimate must be high enough to allow that the estimates obtained with the solution of the inverse problem are within reasonable confidence bounds. Also, when two or more parameters are simultaneously estimated, their effects on the damper force must be independent (uncorrelated). Therefore, when represented graphically the sensitivity coefficients should not have the same shape, otherwise it means that two or more different parameters affect the damper force in the same way, being therefore difficult to distinguish their influences, which yields poor quality estimates.

In Fig. 2 are presented the scaled sensitivity coefficients with respect to the parameters \( c_0 \), \( k_0 \) and \( x_0 \), considering a sinusoidal excitation of the MR damper with an amplitude of 15 mm, for two different values of the excitation frequency: 0.5 Hz and 2.5 Hz, evaluated by finite differences.

![Figure 2. Scaled sensitivity coefficients with respect to \( c_0 \), \( k_0 \) and \( x_0 \) for a sinusoidal excitation.](image)

(a) \( X_{c_0} \)  
(b) \( X_{k_0} \)  
(c) \( X_{x_0} \)

The sensitivity with respect to \( c_0 \) is given by the speed at the tip of the damper rod, and therefore its amplitude varies with the excitation frequency, since a sinusoidal displacement was imposed at one of the damper ends. The sensitivity with respect to \( k_0 \) is given by the displacement imposed to the damper, and therefore its amplitude does not depend on the excitation frequency. The sensitivity with respect to \( x_0 \) is constant and independent of the frequency.

In Fig. 3 are shown the scaled sensitivity coefficients with respect to the parameters \( \gamma, \beta, A \) and \( n \), for the same amplitude and excitation frequencies considered before.

As observed in Fig. 3 the amplitude of the sensitivity coefficients with respect to \( \gamma, \beta, A \) and \( n \) do not depend on the excitation frequency. It is also observed a linear dependence of the curves, most of the time including the sensitivity to parameter \( n \), what is not desirable if the parameters are to be estimated simultaneously. Nonetheless, there are some periods of time in which the linear dependence is not observed, and they are longer for the lower excitation frequency. Therefore, it may be possible to obtain better estimates for the unknowns if only the data acquired at such periods of time are taken into account.
In the results section we considered two sets of unknowns. In the first case the seven unknowns shown in Eq. (6) are estimated simultaneously. In the second case we consider that \( c_0, k_0 \) and \( x_0 \) are fixed, and known previously, and then we estimate the following vector with only four unknowns which are related to the evolution equation for the hysteretic displacement.

\[
\bar{P} = \{\bar{\gamma}, \bar{\beta}, \bar{A}, n\}.
\]  

In fact, in real applications the unknowns \( c_0, k_0 \) and \( x_0 \) may be obtained directly from the observation of the experimental data, what makes the second case a very interesting one.

### 6. Results

The exact values of the seven parameters are: \( c_0 = 50 \text{ Nscm}^{-1}, \ k_0 = 25 \text{ Ncm}^{-1}, \ x_0 = 3.8 \text{ cm}, \ \gamma = 0.1136 \text{ (Ncm)}^{-1}, \ \beta = 0.1136 \text{ (Ncm)}^{-1}, \ A = 105.6 \text{ kNcm}^{-1}, \ n = 2.0. \) As mentioned in the previous section two cases were considered. In the first case the seven parameters were identified. Frequencies of 0.5 Hz and 2.5 Hz were analyzed. The SA and GA were employed, considering \( N_t = 1000 \) points in the range \( 0.0 \text{ s} \) to \( 1.0 \text{ s} \), and the results are presented in Table 1 and Table 2. The numbers shown in the tables are non-dimensional. The results were divided by the exact values above. The experimental data used in the inverse problem are synthetic, obtained from the presented Bouc-Wen model and corrupted with random numbers simulating errors in the acquisition process. The initial guess for parameter in the SA was considered 30% below and above the exact values. For the GA these were the upper and lower etc.
bounds for the design variables. For these values the LM was not able to converge. According to the results obtained both SA and GA performed well. SA was slightly more precise. The hybrid solution (SA/LM and GA/LM) presented the best performance of all. It decreased the computational time with very good precision. In general, the stochastic methods used about 50k function evaluations with 12 min of CPU time. Using only 3 cycles of SA (2.1k function evaluations) or 10 generations of GA (1k function evaluations) to obtain the initial guess for the LM the time is reduced to less than 20 sec with better precision.

Table 1. Seven design variables using SA , GA , SA/LM and GA/LM

| Variable | SA | GA | SA/LM | GA/LM |
|----------|----|----|-------|-------|
|          | Initial design: | (after 400 generations) | LM after 3 cycles | LM after 10 generations |
| c₀       | 0.996 | 0.987 | 0.999 | 1.008 |
| k₀       | 1.002 | 0.943 | 0.992 | 1.011 |
| γ        | 0.993 | 0.991 | 0.992 | 0.993 |
| n        | 0.981 | 0.989 | 0.990 | 0.990 |
| A        | 1.001 | 0.999 | 1.001 | 0.999 |
| β        | 0.985 | 0.985 | 0.998 | 1.001 |
| x₀       | 0.995 | 1.050 | 1.005 | 0.986 |

Table 2. Seven design variables using SA , GA , SA/LM and GA/LM

| Variable | SA | GA | SA/LM | GA/LM |
|----------|----|----|-------|-------|
|          | Initial design: | (after 400 generations) | LM after 3 cycles | LM after 10 generations |
| c₀       | 1.002 | 1.002 | 1.002 | 1.005 |
| k₀       | 1.001 | 1.019 | 0.999 | 1.010 |
| γ        | 1.068 | 1.014 | 1.005 | 1.015 |
| n        | 1.083 | 1.064 | 1.004 | 1.065 |
| A        | 0.980 | 1.004 | 0.987 | 1.003 |
| β        | 0.982 | 1.080 | 0.992 | 1.015 |
| x₀       | 1.001 | 0.994 | 1.002 | 1.002 |

In the second case (Tables 3 and 4) only four variables were identified (γ, n, A, and β) for frequency 0.5 Hz. Based on the sensitivity analysis shown in Fig. 3 the experimental results were concentrated in the range 0.5 s to 0.6 s with the same number of experimental points. The right solution was obtained with less computational effort (one half of the first case) and better precision even with high level of noise in the experimental data.

Table 3. Four design variables using GA and GA/LM

| Variables | GA (400 gen) | GA/LM (after 10 gen.) | GA (200 gen) | GA/LM (after 10 gen.) |
|-----------|--------------|-----------------------|--------------|-----------------------|
| γ         | 1.017        | 0.975                 | 1.004        | 1.003                 |
| n         | 0.989        | 0.975                 | 1.005        | 1.006                 |
| A         | 0.988        | 1.008                 | 0.999        | 1.003                 |
Table 4. Four design variables using GA and GA/LM

| Variables | GA (400 gen) | GA/LM | GA (200 gen) | GA/LM |
|-----------|-------------|-------|-------------|-------|
| $\gamma$  | 1.200       | 1.025 | 1.099       | 1.007 |
| $n$       | 1.177       | 1.035 | 0.993       | 1.004 |
| $A$       | 0.958       | 0.994 | 0.995       | 1.000 |
| $\beta$   | 0.888       | 0.987 | 0.972       | 1.000 |

7. Conclusions

An implicit formulation for the characterization of MR dampers using the Bouc-Wen model was presented. The consequent optimization problem was solved using stochastic methods such as SA and GA and a combination of these methods with the LM which brings some advantages over the pure stochastic ones from the standpoint of both computational effort and accuracy. Also the sensitivity analysis has been proved to be an important tool to verify which parameters could be more easily identified and also to help in the development of the experimental setup, indicating the points where the experimental data should be obtained.

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References

[1] A.G. Olabi and A. Grunwald, Design and Application of Magneto-Rheological Fluid, Materials and Design, Vol. 28, pp. 2658-2664, 2007.
[2] T.D.C. Thanh and K.K. Ahn, Intelligent Phase Plane Switching Control of Pneumatic Artificial Muscle Manipulators with Magneto-Rheological Brake, Mechatronics, Vol. 16, pp. 85-95, 2006.
[3] Z.Q. Gu and S.O. Oyadiji, Application of MR Damper in Structural Control Using ANFIS Method, Computers and Structures, 2007, in press.
[4] W. Wei and X. Pinqi, Adaptive Control of Helicopter Ground Resonance with Magnetorheological Damper, Chinese Journal of Aeronautics, Vol. 20, pp. 501-510, 2007.
[5] R. Stanway, J.L. Spronton and N.G. Stevens, Non-Linear Modeling of Electrorheological Vibration Damper, Journal of Electrostatics, Vol. 20, No. 2, pp. 167-184, 1987.
[6] N.M. Wereley, L. Pang and G.M. Kamath, Idealized Hysteresis Modeling of Electrorheological and Magnetorheological Dampers, Journal of Intelligent Material System and Structures, Vol. 9, pp. 642-649, 1988.
[7] B.F. Spencer Jr., S.J. Dyke, M.K. Sain and J.D. Carlson, Phenomenological Model of a Magnetorheological Damper, Journal of Engineering Mechanics, ASCE, Vol. 123, No. 3, pp. 230-238, 1997.
[8] N.D. Sims, N.J. Holmes and R. Stanway, A Unified Modeling and Model Updating Procedure for Electrorheological and Magnetorheological Vibration Dampers, Smart Materials and Structures, Vol. 13, No. 1, pp. 100-121, 2004.
[9] H. Du, J. Lam and N. Zhang, Modelling of a Magneto-Rheological Damper by Evolving Radial Basis Function Networks, Engineering Applications of Artificial Intelligence, Vol. 19, pp. 869-881, 2006.
[10] H. Du and N. Zhang, Application of Evolving Takagi-Sugeno Fuzzy Model to Nonlinear System
[11] A.E. Charalampakis and V.K. Kounousis, *On the Response and Dissipated Energy of Bouc-Wen Hysteretic Model*, Journal of Sound and Vibration, Vol. 309, pp. 887-895, 2008.

[12] N.M. Kwok, Q.P. Ha, T.H. Nguyen, J. Li and B. Samali, *A Novel Hysteretic Model for Magnetorheological Fluid Dampers and Parameter Identification Using Particle Swarm Optimization, Sensors and Actuators A*, Vol. 132, pp. 441-451, 2006.

[13] N.M. Kwok, Q.P. Ha, T.H. Nguyen, J. Li and B. Samali, *Bouc-Wen Model Parameter Identification for a MR Fluid Damper Using Computationally Efficient GA*, ISA Transactions, Vol. 46, pp. 167-179, 2007.

[14] F.J.C.P. Soeiro, L.T. Stutz, R.A. Tenenbaum and A.J. Silva Neto, *Estimation of parameters in the Bouc-Wen model for magnetorheological dampers using a hybrid method*, 19th International Congress of Mechanical Engineering, Brasilia, Brazil, 5-9 November, 2007.

[15] L.T. Stutz, R.A. Tenenbaum and A.J. Silva Neto, *The Bouc-Wen Model for Magnetorheological Dampers: A Sensitivity Analysis*, Inverse Problems Symposium, East Lansing, USA, 11-12 June, 2007.

[16] D.W. Marquardt, *An Algorithm for Least-Squares Estimation of Nonlinear Parameters*, Journal of the Society for Industrial and Applied mathematics, Vol. 11, pp. 431-441, 1963.

[17] N. Metropolis, A.W. Rosenbluth, M.N. Rosenbluth, A.H. Teller and E. Teller, *Equation of State Calculations by Fast Computing Machines*, Journal of Chemical Physics, Vol. 21, pp. 1087-1092, 1953.

[18] D.E. Goldberg, *Genetic Algorithms in Search, Optimization and Machine Learning*, Addison-Wesley, Reading, MA, 1989.

[19] A.J. Silva Neto and F.J.C.P. Soeiro, *Solution of implicitly formulated inverse heat transfer problems with hybrid methods*, 2nd MIT Conference on Computational Fluid and Solid Mechanics, Cambridge, USA, 2003.

[20] K.J. Dowding, B.F. Blackwell and R.J. Cochran, *Applications of Sensitivity Coefficients for Heat Conduction Problems*, Numerical Heat Transfer, Part B. Vol. 36, pp. 33-55, 1999.

[21] J.V. Beck, *Combined Parameter and Function Estimation in Heat Transfer with Application to Contact Conductance*, Journal of Heat Transfer, Vol.110, pp. 1046-1058, 1988.