Simulation and Analysis of ESB Algorithm Based on Frequency Diverse Array

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Abstract. The beam of phased array radar has angular resolution and no range resolution. The frequency diverse array adds a frequency increment which is much smaller than the carrier frequency between the array elements, thus forming a beam which is related to angle, range and time. On the basis of analyzing the characteristic of frequency diverse array beam, the mechanism of receiving only the transmitting array element frequency signal is introduced. The eigenspace-based beamforming algorithm is used to simulate the transmitting pattern, the receiving pattern and the dual path pattern, and the simulation results are analyzed. The performance of SINR under the ESB algorithm is simulated and analyzed. The results verify the effectiveness of the application of the ESB algorithm in the frequency diverse array.

1. Introduction
Phased array can efficiently change the beam space pointing, and is widely used in radar, electronic warfare, radio astronomy and airport security and other fields [1]. Phased array can form high gain in the specified direction, to detect/track the weak signal and suppress other aspects of interference signal [2]. However, the limitation of phased array is that the spatial distribution of beam is only related to angle, and has nothing to do with range. That is, the beam has only angle resolution and no range resolution. In addition, the transmitting range dependent beams become more and more urgent in some specific fields, such as range dependent interference suppression, directional communication and range ambiguity suppression, [3]-[5].

In 2006, P. Antonik proposed the concept of frequency diverse array (FDA) radar at the international radar conference, which attracted wide attention from scholars both at home and abroad [6], [7]. Different from phased array, FDA adds an increment which is much smaller than the carrier frequency across the array element, so that the antenna beam spatial distribution will change with time, range and angle [8]. There is a lot of literature on FDA. Zhuang et al. [9] points out that beam scanning characteristics of FDA is related to frequency offset between the array elements. The research status of FDA is summarized, and the structure and beam characteristic of FDA are analyzed, and the application is prospected in [10]-[12]. Basit et al. [13] studies the application of FDA in multiple target localization in the same direction and different ranges. A logarithmically increasing frequency offsets is introduced in [14], and decoupled the range-angle beampattern.

Most of the current research focuses on the structure characteristics of FDA and its application, while the beamforming algorithms of FDA are rarely involved. For phased array, the essence of beamforming is to adjust the weight coefficients of each array element, so that the high gain is aligned to the desired signal direction, and the nulls are aligned to the direction of the interference, so as to
maximize the reception of useful signals. For FDA, because the spatial distribution of beam is related to range and angle, we need to get the high gain of the beam to the specific location of the desired signal, and zero notch is aligned to the location of the interference, so as to receive useful signals as far as possible. There are many beamforming algorithms in phased array, such as linear constrained minimum variance (LCMV) beamforming algorithm [15], and Eigenspace-based beamforming (ESB) [16].

In this paper, the ESB algorithm in phased array is introduced into the FDA, and the performance of the algorithm is analyzed. Finally, the effectiveness of the algorithm in the frequency control array is verified by simulation.

2. FDA signal model
The uniform linear array FDA structure is shown in Figure 1.

Under the narrow band condition, the signal of each array element can be expressed as

$$s_n(t) = \exp(j2\pi f_n t), \quad n = 0, 1, ..., M-1$$

(1)

Where $f_n = f_0 + n\Delta f, n = 0, 1, ..., M-1$ is the radiation frequency, $f_0$ is the carrier frequency, $\Delta f$ is the frequency increment, and $M$ is the number of array elements.

The signal arriving at a far-field point $(R, \theta)$ can be expressed as

$$s_m(t) = \exp\left[j2\pi f_0\left(t - \frac{R}{c}\right)\right]$$

(2)

Where $R = R - nd\sin\theta$ is the target slant range for the $n$th element, $d$ is element spacing and $c$ is the speed of light. The array factor at position $(R, \theta)$ can be expressed as

$$AF(t; R, \theta) = \sum_{n=0}^{M-1} \frac{1}{R_n} s_m(t) = \sum_{n=0}^{M-1} \frac{1}{R_n} \exp\left[j2\pi f_0\left(t - \frac{R}{c} + \frac{nd\sin\theta}{c}\right)\right] \approx \exp[j\phi_n] \cdot \sin\left[N\pi \left(\Delta ft - \frac{\Delta fR}{c} + \frac{df_0 \sin\theta}{c}\right)\right]$$

(3)

With $\phi_n$ being

$$\phi_n = 2\pi f_0\left(t - \frac{R}{c}\right) + \pi(N-1)\left(\frac{f_0 d\sin\theta}{c} + \Delta ft - \frac{\Delta fR}{c}\right)$$

(4)

When $f_0 = 3$GHz, $\Delta f = 2$KHz, $M = 10$, $d = 0.5c / f_0$, $\theta = 20\degree$, $R = 100$km, the transmitting beampattern of the ULA-FDA is shown in Figure 2, and Figure 3 shows the transmitting beampattern of the ULA-PAR.

Figure 1. Uniform linear array frequency diverse array.
As is obvious from figures 2-3, the beampattern of FDA is range-angle-dependent, and the phased array beampattern has only angle resolution.

3. Receiving signal processing

A steering vector is added to the FDA transmitting antenna.

\[ w_i = \left[ \alpha_0(\hat{R}, \hat{\theta}), \alpha_1(\hat{R}, \hat{\theta}), \ldots, \alpha_{M-1}(\hat{R}, \hat{\theta}) \right]^T \] (5)

Where \( \alpha_n(\hat{R}, \hat{\theta}) = \exp(-j2\pi f_n \hat{R}/c) \) is the weight coefficient of the \( n \)th element. For a fixed target position \((\hat{r}, \hat{\theta})\) in the far field, the weighted arrival target position of the signal transmitted by the array element \( n \) can be expressed as

\[ s(t; \hat{R}, \hat{\theta}) = \sum_{n=0}^{M-1} \exp \left\{ j2\pi f_n \left( t - \frac{R - \hat{R}}{c} + \frac{nd\sin \theta - \sin \hat{\theta}}{c} \right) \right\} \] (6)

The echo signal received by the \( m \)th array element is

\[ r(t; \hat{R}, \hat{\theta}) = \sum_{n=0}^{M-1} \exp \left\{ j2\pi f_n \left( t - \frac{2R - \hat{R}}{c} + \frac{nd\sin \theta - \sin \hat{\theta}}{c} + \frac{md\sin \theta}{c} \right) \right\} \] (7)

At this time, the echo signal received by each element contains all the transmitting signals. By adding filters, the \( m \)th element only extracts the signal with \( f_m \), and by weighting, the output can be obtained.

\[ y(t; \hat{R}, \hat{\theta}) = \sum_{m=0}^{M-1} w_m^H(r(t; \hat{R}, \hat{\theta})) = \sum_{n=0}^{M-1} \exp \left\{ j2\pi f_n \left( t - \frac{2(R - \hat{R})}{c} + \frac{2md\sin \theta - \sin \hat{\theta}}{c} \right) \right\} \] (8)

4. The ESB algorithm

Consider an expected signal and \( J \) interference signals in a space. The eigendecomposition of the sample covariance matrix \( \hat{R} \) is decomposed.
\begin{equation}
\hat{R} = \sum_{i=1}^{J-1} \lambda_i u_i u_i^H + \sigma_n^2 \sum_{i=J+2}^{M} u_i u_i^H
\end{equation}

Where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{J-1} > \lambda_{J+2} = \cdots = \lambda_M = \sigma_n^2$ are the corresponding $M$ eigenvalues, and the corresponding eigenvector is $u_i, i = 1,2,\cdots M$.

\begin{equation}
D_x = \text{diag}(\lambda_1, \ldots \lambda_{J-1}), D_n = \text{diag}(\lambda_{J+2}, \ldots \lambda_M)
\end{equation}

\begin{equation}
U_s = [u_1, u_2, \cdots, u_{J-1}], U_n = [u_{J+2}, \cdots, u_M]
\end{equation}

The column vectors of $U_s$ constitute the subspace of the signal, and the column vectors of $U_n$ constitute the noise subspace. In the LCMV algorithm, the adaptive weight is

\begin{equation}
w_0 = \frac{\hat{R}^{-1} a(R, \theta)}{a^H(R, \theta) \hat{R}^{-1} a(R, \theta)}
\end{equation}

In the ideal case, the expected signal is in the signal subspace, then $U_s a(R, \theta) = 0$. So the weight vector $a(R, \theta)$ is only the component of the signal subspace, and does not contain the subspace component of the noise. ESB algorithm is based on this principle, the weight vector onto the signal subspace, so as to give in the noise subspace components retained in the signal subspace of the component, i.e.

\begin{equation}
w_p = U_s U_s^H w_0
\end{equation}

5. Simulation and analysis

Consider an 10 element uniform linear FDA at the carrier frequency $f_0 = 3\text{GHz}$, with the element spacing being fixed to be $d = 0.5c / f_0$. Simulation have been carried out for $J = 4, \Delta f = 4500\text{Hz}$, $\hat{R} = 50\text{km}$, $\hat{\theta} = 20^\circ$, SNR=10dB, INR=10dB. The azimuth of the interference is $-60^\circ, 15^\circ, 20^\circ, 25^\circ$, respectively. And the corresponding slant range of the interference is 30km, 90km, 65km, 45km in turn. Simulations of transmitting, receiving, and dual path pattern under the ESB algorithm of FDA are shown in Figure 4-6.

![Figure 4. The transmitting beam pattern with ESB algorithm.](image1)

![Figure 5. The receiving beam pattern with ESB algorithm.](image2)
The dual path beampattern with ESB algorithm.

The beampattern of the range dimension with $\hat{\theta} = 20^\circ$.

The beampattern of the range dimension with $\hat{R} = 50\text{km}$.

The change diagram of SINR with SNR.

The green circle represents the target location, and the blue square represents the interference location. From the graph, we can see that using the ESB algorithm can form high gain at the target position and form zero trapping in the interference location, so as to realize the function of the beamforming algorithm.

In order to observe the convenience, the gain of the angle dimension and range dimension of the target position are simulated respectively. It is clear from the Figure 7-8 that the ESB algorithm is used to maximize the gain value at the target location.

Finally, the performance of signal-to-interference-plus-noise ratio after using the algorithm is simulated, and compared with the basic FDA and PAR, as shown in Figure 9. As can be seen from Figure 9, after using the ESB algorithm, the SINR performance of the system is the best, with higher noise and robustness.

6. Conclusion

FDA radar can produce range-angle-time correlation beam. It is the innovation of the radar system, which has brought more potential for radar application. Based on the analysis of FDA beam characteristics, we introduce a signal receiving mechanism which accepts the frequency of the corresponding transmitting array element only. With the application of ESB algorithm, we simulate the transmitting beampattern, receiving beampattern, dual path beampattern, and the results show the effectiveness of the application of ESB algorithm in FDA. The next step will be further studied in the improved beamforming algorithm of FDA.
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