DISSIPATIVE NONLINEAR SCHRÖDINGER EQUATIONS FOR LARGE DATA IN ONE SPACE DIMENSION

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Abstract. In this study, we consider the global Cauchy problem for the nonlinear Schrödinger equations with a dissipative nonlinearity in one space dimension. In particular, we show the global existence, smoothing effect and asymptotic behavior for solutions to the nonlinear Schrödinger equations with data which belong to $F^{H^\gamma}$, $1/2 < \gamma \leq 1$. In the proof of main theorem, we introduce a priori estimate for $H^\gamma$-type norm and the condition $F^{H^\gamma}$ for data relaxed into $F^{H^\gamma}$, $1/2 < \gamma \leq 1$.

1. Introduction. We consider the Cauchy problem for the nonlinear Schrödinger equations

$$
\begin{cases}
  i\partial_t u + (\partial_x^2/2)u = F(u), \\
  u(0) = \phi
\end{cases}
$$

in one space dimension $n = 1$, where $\mathbb{R}_+ = [0, \infty)$, $\partial_t = \partial/\partial t$, $\partial_x^2 = \partial^2/\partial x^2$, $u : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{C}$ is an unknown function and the long-range nonlinearity satisfies the following gauge condition:

$$
e^{i\theta} F(u) = F(e^{i\theta} u), \quad \theta \in \mathbb{R}.
$$

To be specific, we assume that the nonlinearity $F(u)$ has the following form:

$$
F(u) = \lambda |u|^{p-1}u
$$

where $1 < p \leq 3$, $\lambda \in \mathbb{C}$.

There is a large literature on the Cauchy problem for the nonlinear Schrödinger equations (see [3, 5, 13, 14, 19, 26, 27] and references therein).

The critical exponent between long-range and short-range in the scattering theory is

$$
p_* = 1 + \frac{2}{n}.
$$

If $\lambda > 0$, $1 < p \leq p_*$ and $n \geq 1$, the nonlinear Schrödinger equations do not have nontrivial asymptotically free solution in $L^2$ ([2, 25], see also Chapter 7 of [3]). The nonexistence of nontrivial asymptotically free solution in $L^1$ has been shown in [17].
On the other hand the $L^2$-critical exponent is

$$p_c = 1 + \frac{4}{n}$$

and (1)–(2) for $1 < p < p_c$, $\lambda \in \mathbb{C}$ with $\text{Im}(\lambda) \leq 0$ has global solutions for data $\phi \in L^2$, by the standard contraction argument with $L^2$-conservation law (see [3, 5, 19, 26, 27] for instance). If $p = p_c$, (1)–(2) with $\lambda \in \mathbb{C}$ has global solution for sufficiently small data $\phi \in L^2$.

In [21], the author has shown the existence of modified scattering state for the nonlinear Schrödinger equations with long-range potential in one space dimension. After this work the study of long-range scattering and asymptotic behavior for solutions to nonlinear Schrödinger equations are developed by many authors (see [4, 6, 7, 9, 10, 12, 15, 16, 18, 23, 24, 17] and references therein).

In [6, 7, 9, 10, 15, 16, 24, 17] (see also [18] and references therein), the authors have studied the asymptotic behavior for solutions to the nonlinear Schrödinger equations. More precisely, in [9] the authors assume $\text{Im}(\lambda) = 0$ and in [6, 7, 10, 15, 16, 24, 17] the authors assume $\text{Im}(\lambda) < 0$ (see also [18]). In particular in [10, 16], the authors do not need any smallness condition to the $H^1 \cap \mathcal{F}H^1$-norm of the data in one space dimension under the condition

$$\frac{5 + \sqrt{33}}{4} < p \leq 3$$

by using a priori estimate with the following condition for the coefficient $\lambda \in \mathbb{C}$ in front of the nonlinear term:

$$\text{Im}(\lambda) < 0 \quad \text{and} \quad \frac{p - 1}{2\sqrt{p}} |\text{Re}(\lambda)| \leq |\text{Im}(\lambda)|.$$ 

In [7], the authors have studied asymptotic behavior for solutions to (1)–(2) for large data

$$\phi \in \mathcal{F}L^\infty \cap \mathcal{F}\dot{H}^{\frac{1}{2} + \varepsilon} \cap \mathcal{F}\dot{H}^1$$

with some $\varepsilon > 0$ under the condition

$$\frac{5 + \sqrt{33}}{4} < p < 3.$$ 

Our aim of this study is to consider the global existence and asymptotic behavior for solutions to (1)–(2) for large data

$$\phi \in \mathcal{F}\dot{H}^\gamma$$

with $1/2 < \gamma \leq 1$, under the condition

$$\frac{(7 - 2\gamma) + \sqrt{(7 - 2\gamma)^2 + 8(2\gamma - 1)}}{4} < p \leq 3$$

and

$$\text{Im}(\lambda) < 0 \quad \text{and} \quad \frac{p - 1}{2\sqrt{p}} |\text{Re}(\lambda)| \leq |\text{Im}(\lambda)|$$

where the condition (3) appears in the argument to obtain the time decay estimate (see (9) in Sect.4, below) and we need the condition (4) to prove the a priori estimate.
To prove the global existence and the time decay estimate of solutions, we show the following a priori estimate for solutions with fractional order 0 < \gamma < 1, which has an important role in this study (see Lemma 3.6 in Sect.3, below):

$$\left\| e^{it\partial_x^2/2} |x|^{\gamma} e^{-it\partial_x^2/2} u(t) \right\|_{L^2} \leq \|x\| \phi_{L^2}.$$ 

The proof of this estimate is similar to the proof of $L^2$-conservation law like as the following argument. Let $G_y$ be the Galilei transform (see [19] and proof of Lemma 3.6 in Sect.3, below), then by the Galilei invariant property, we have

$$i\partial_t (G_y u - u) + (\partial_x^2/2)(G_y u - u) = \lambda \left( |G_y u|^{p-1} G_y u - |u|^{p-1} u \right).$$

Multiplying $G_y u - u$ by the above equation and integrating over $\mathbb{R}$ with taking imaginary part, we have

$$\lambda \lambda \frac{d}{dt} \int_\mathbb{R} |G_y u(t, x) - u(t, x)|^2 dx = 2\text{Im} \int_\mathbb{R} \left( |G_y u(t, x)|^{p-1} G_y u(t, x) - |u(t, x)|^{p-1} u(t, x) \right) \left( G_y u(t, x) - u(t, x) \right) dx.$$

If $\lambda \in \mathbb{C}$ satisfies (4), then (see [16, 20] and Lemma 4 in Sect.3 below)

$$\frac{d}{dt} \int_\mathbb{R} |G_y u(t, x) - u(t, x)|^2 dx \leq 0$$

and we obtain the above a priori estimate with the identity

$$\left\| e^{it\partial_x^2/2} |x|^{\gamma} e^{-it\partial_x^2/2} u(t) \right\|_{L^2} = C_{\gamma} \left( \int_\mathbb{R} \|G_y u(t) - u(t)\|_{L^2}^2 \right)^{1/2}$$

(see Proposition 1.37 in [1] for example).

In this study, we do not need any smallness condition and regularity assumption for the data $\phi \in \mathcal{F} H^\gamma$. The asymptotic behavior of solutions to the Cauchy problem for the dissipative nonlinear Schrödinger equations has been studied by [6, 7, 10, 16, 18, 23, 24, 17] (see also references therein).

$L^q = L^q(\mathbb{R})$ is the usual Lebesgue space with $1 \leq q \leq \infty$. $(A \cap B, \|\cdot\|_{A \cap B})$ is a Banach space with the norm $\|\cdot\|_{A \cap B} = \max (\|\cdot\|_A, \|\cdot\|_B)$, for two Banach spaces $(A, \|\cdot\|_A)$ and $(B, \|\cdot\|_B)$. The map $\mathcal{F} : \varphi \mapsto \hat{\varphi}$ is the Fourier transform defined by

$$\hat{\varphi}(\xi) = \frac{1}{(2\pi)^{1/2}} \int_\mathbb{R} e^{-i\xi x} \varphi(x) dx, \quad \xi \in \mathbb{R}$$

and the map $\mathcal{F}^{-1} : \varphi \mapsto \varphi^\vee$ is the inverse Fourier transform defined by

$$\varphi^\vee(x) = \frac{1}{(2\pi)^{1/2}} \int_\mathbb{R} e^{ix\xi} \varphi(\xi) d\xi, \quad x \in \mathbb{R}.$$
for any $\varphi \in \dot{H}^\gamma$ (see Proposition 1.37 in [1]). The free Schrödinger propagator is defined by

$$U(t)\varphi = e^{it\partial_x^2/2} \varphi = F^{-1} \left[ e^{-it\xi^2/2} \hat{\varphi} \right], \; t \in \mathbb{R}.$$ 

We often use the notation $U^{-1}(t)\varphi = U(-t)\varphi$, $t \in \mathbb{R}$. For $t \neq 0$, $U(t)$ has another representation

$$U(t)\varphi = M(t)D(t)F M(t)\varphi,$$

where the phase modulation operator $M(t) : \varphi \mapsto e^{ix^2/2t} \varphi$ and the dilations $D(t) : \varphi \mapsto (it)^{-1/2} \varphi\left(\frac{\cdot}{t}\right)$. The generator of Galilei transform is defined by

$$J(t)\varphi = (x + it\partial_x)\varphi = U(t)xU(-t)\varphi, \; t \in \mathbb{R}.$$ 

For $t \neq 0$, $J(t)$ is also represented as (see [8])

$$J(t)\varphi = M(t)it\partial_x (M(-t)\varphi).$$

Let $\gamma > 0$. We define the following Sobolev type space

$$A_{\gamma}^q(t) = \left\{ \varphi \in S' : \|\varphi\|_{A_{\gamma}^q(t)} < \infty \right\},$$

$$\|\varphi\|_{A_{\gamma}^q(t)} \equiv \max \left( \|\varphi\|_{L^q}, \|J^\gamma(t)\varphi\|_{L^q} \right), \; t \in \mathbb{R}$$

where

$$|J|^\gamma(t)\varphi = U(t)|x|^\gamma U(-t)\varphi, \; t \in \mathbb{R}.$$ 

and which has another representation

$$|J|^\gamma(t)\varphi = M(t)\left(-t^2\partial_x^2\right)^{\gamma/2} (M(-t)\varphi), \; t \neq 0.$$ 

Note that

$$A_{\gamma}^q(0) \equiv \mathcal{F}H^\gamma.$$ 

We introduce the following function spaces

$$X(I) = C \cap L^\infty (I; A_{\gamma}^q) \cap L^4 (I; L^\infty), \; I \subset \mathbb{R}^+,$$

$$X_{\text{loc}}(\mathbb{R}^+) = C \cap L^\infty (\mathbb{R}^+; A_{\gamma}^q) \cap L^4_{\text{loc}} (\mathbb{R}^+; L^\infty).$$

2. **Main result.** We introduce the exponent $p(\gamma)$ defined by

$$p(\gamma) = \frac{(7 - 2\gamma) + \sqrt{(7 - 2\gamma)^2 + 8(2\gamma - 1)}}{4}, \; 1/2 \leq \gamma \leq 1.$$ 

We state our main result:

**Theorem 2.1.** Assume that $1/2 < \gamma \leq 1$, $1 < p \leq 3$ and

$$\text{Im}(\lambda) < 0, \; \frac{p-1}{2\sqrt{p}} |\text{Re}(\lambda)| \leq |\text{Im}(\lambda)|.$$ 

If

$$\phi \in \mathcal{F}H^\gamma.$$ 

Then (1)–(2) has a unique global solution

$$u \in X_{\text{loc}}(\mathbb{R}^+) \cap L^4_{\text{loc}} (\mathbb{R}^+; A_{\infty}^\gamma)$$

satisfying the following properties:
1. For any $t > 0$
\[
(M^{-1}u)(t) \in H^\gamma \subset C(\mathbb{R}).
\]
2. There exist $C > 0$ such that
\[
\|u(t)\|_{L^\infty} \leq C \|\phi\|_{\mathcal{F}H^\gamma} t^{-1/2}, \quad t > 0.
\]
Furthermore,
\[
\|u(t)\|_{L^\infty} \leq Ct^{-1/2}(\log t)^{-1/2}, \quad \text{if } p = 3,
\]
\[
\|u(t)\|_{L^\infty} \leq Ct^{-\frac{p}{p-1}}, \quad \text{if } p(\gamma) < p < 3
\]
for $t \geq 2$.
3. There exists a unique $\psi_+ \in L^2 \cap L^\infty$ such that
\[
\left\| \exp \left( i \lambda \int_2^t s^{-\frac{p-1}{2}}|\bar{u}(s)|^{p-1} ds \right) \mathcal{F}U(-t)u(t) - \psi_+ \right\|_{L^2 \cap L^\infty} = O \left( t^{-\alpha} \right)
\]
as $t \to +\infty$ for $p(\gamma) < p \leq 3$ and some $\alpha > 0$.

**Remark 1.** We do not need the smallness condition on the weighted $L^2$-norm
\[
\|\phi\|_{\mathcal{F}H^\gamma} = \|(\langle x \rangle^\gamma \phi)\|_{L^2}
\]
to the data $\phi \in \mathcal{F}H^\gamma$ with $1/2 < \gamma \leq 1$. Also we do not need any regularity assumption on the data $\phi \in \mathcal{F}H^\gamma$, however the solutions belong to $C(\mathbb{R})$ (smoothing effect). For example the function $\phi_R = R\chi_{\{|x| \leq 1\}}$, $R > 0$ which belongs to $\mathcal{F}H^\gamma$, with large $\mathcal{F}H^\gamma$-norm and which is not continuous on $\mathbb{R}$, where the characteristic function
\[
\chi_{\{|x| \leq 1\}}(x) = \begin{cases} 
1, & |x| \leq 1, \\
0, & |x| > 1.
\end{cases}
\]

**Remark 2.** The solutions $u \in X_{loc}(\mathbb{R}_+)$ satisfy $u \in L^r(\mathbb{R}_+; L^\infty)$, with appropriate $1 \leq r < \infty$ because the $L^\infty$-norm of solutions: $\|u(t)\|_{L^\infty}$ have time decay estimate above for large $t \geq 1$ and $\|u\|_{L^\infty} \in L^4_{loc}(\mathbb{R}_+; L^\infty)$.

**Remark 3.** We see that
\[
p(1/2) = 3
\]
which says that, if $\gamma = 1/2$, then it is difficult to obtain the time decay estimate above, because we need the assumption $p(\gamma) < p < 3$ to show the time decay estimate. Also we see that
\[
p(1) = \frac{5 + \sqrt{33}}{4}
\]
and which is the same to the condition appears in the previous studies [10, 16]. Therefore $p(\gamma)$ is regarded as an extension of $p(1)$ and which has monotone decreasing property
\[
p(\gamma_1) < p(\gamma_0)
\]
for $1/2 \leq \gamma_0 < \gamma_1 \leq 1$. 

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3. Preliminaries.

Lemma 3.1. Let \( I \subset \mathbb{R}_+ \). The metric space \( (X_r(I), d) \) defined by
\[
X_r(I) = \left\{ u \in X(I); \|u\|_{X(I)} \leq r \right\},
\]
d\((u, v) = \|u - v\|_{L^\infty(I; L^2) \cap L^4(I; L^\infty)}
\]
is a complete metric space.

Proof. Let \( \{u_k\}_{k \geq 1} \subset X_r(I) \) be Cauchy sequence. Then there exists a
\[
u \in L^\infty(I; L^2) \cap L^4(I; L^\infty)
\]
such that
\[
\lim_{k \to \infty} \|u_k - u\|_{L^\infty(I; L^2) \cap L^4(I; L^\infty)} = 0
\]
and
\[
\|u\|_{L^\infty(I; L^2) \cap L^4(I; L^\infty)} \leq r.
\]
Because \( \{|J|^\gamma(t)u_k(t)\}_{k \geq 1} \) is bounded in \( L^2 \) and \( L^2 \) is a reflexive Banach space,
there exist sub sequence \( \{|J|^\gamma(t)u_{k_j}(t)\}_{j \geq 1} \), which converges weakly in \( L^2 \) and
\[
\left( |J|^\gamma(t)u_{k_j}(t), \varphi \right) = \left( u(t), |J|^\gamma(t)\varphi \right)
\]
\[
\to \left( u(t), |J|^\gamma(t)\varphi \right) = \left( |J|^\gamma(t)u(t), \varphi \right), \quad \text{as} \quad j \to \infty
\]
for all \( \varphi \in C_0^\infty(\mathbb{R}) \), where
\[
(f, g) = \int_\mathbb{R} f(x)\overline{g(x)}dx
\]
is the usual \( L^2 \)-scalar product. Therefore
\[
\| |J|^\gamma(t)u(t)\|_{L^2} \leq \liminf_{j \to \infty} \| |J|^\gamma(t)u_{k_j}(t)\|_{L^2} \leq r
\]
for \( t \in I \). This completes the proof. \( \square \)

Lemma 3.2. Let \( \gamma \geq 0 \) and \( I = [\tau, T] \) with \( \tau, T \in \mathbb{R}_+ \). Then we have the following estimates
\[
\|U(\cdot)\phi\|_{X(I)} \leq C \|\phi\|_{A_2^\infty(0)} , \quad \left\| \int_\tau U(\cdot - s)F(s)ds \right\|_{X(I)} \leq C \|F\|_{L^1(I; A_2^\infty)}
\]
with some constant \( C > 0 \).

Proof. By using the identities
\[
|J|^{\gamma(t)}U(t)\phi = U(t)|x|^{\gamma(t)}\phi, \quad t \in I,
\]
\[
|J|^{\gamma(t)} \int_0^t U(t - s)F(s)ds = \int_0^t U(t - s)|J|^{\gamma(s)}F(s)ds, \quad t \in I
\]
and the Strichartz estimate (\([3, 19, 26, 28]\)), we immediately have the desired estimates. \( \square \)

Lemma 3.3 (\([9]\)). Let \( 0 < \gamma \leq 1, \quad p > 1 \). Then there exist \( C > 0 \) such that
\[
\| |u|^{p-1}u \|_{A_2^{\infty}(t)} \leq C \|u\|_{L^\infty}^{-1} \|u\|_{A_2^{\infty}(t)}
\]
for \( t \in \mathbb{R} \).
Lemma 3.5. Let $t \neq 0$. By
\[ \| |J|^{\gamma}(t)u|^{p-1}u\|_{L^2} = |t|^{\gamma}\| |M^{-1}(t)u|^{p-1}M^{-1}(t)u\|_{H^2} \]
and the Leibniz estimate in the Appendix of [14], we have:
\[ |t|^{\gamma}\| |M^{-1}(t)u|^{p-1}M^{-1}(t)u\|_{H^2} \leq C|t|^{\gamma}\| |M^{-1}(t)u|^{p-1}\|_{L^\infty}\| M^{-1}(t)u\|_{H^2} \]
\[ = C\|u\|_{L^\infty}\| |t|^{\gamma}M^{-1}(t)u\|_{H^2} \]
\[ \leq C\|u\|_{L^\infty}\| |J|^{\gamma}(t)u\|_{L^2}. \]
This completes the proof.

We define the following two quantities:
\[ R_1 = \mathcal{F}(M^{-1} - 1)\mathcal{F}^{-1}|\mathcal{F}MU^{-1}u|^{p-1}\mathcal{F}MU^{-1}u \]
and
\[ R_2 = |\mathcal{F}MU^{-1}u|^{p-1}\mathcal{F}MU^{-1}u - |\mathcal{F}U^{-1}u|^{p-1}\mathcal{F}U^{-1}u. \]

Lemma 3.4 ([9]). Let $1/2 < \gamma \leq 1$, $1 < p \leq 3$ and $0 < 2k + 1/2 < \gamma$. Then we have
\[ \| R_j(t) \|_{L^\infty} \leq Ct^{-k}\| u(t) \|_{A_j^\gamma(t)}, \quad j = 1, 2 \]
for all $t \geq 1$, where $C > 0$.

Lemma 3.5 ([16, 20]). We assume that
\[ \text{Im}(\lambda) < 0, \quad \frac{p-1}{2\sqrt{p}}|\text{Re}(\lambda)| \leq |\text{Im}(\lambda)|. \]
Then for $z_1, z_2 \in \mathbb{C}$ and $p > 1$, we have
\[ \text{Im} \left( \lambda \left( |z_1|^{p-1}z_1 - |z_2|^{p-1}z_2 \right) \frac{z_1 - z_2}{\overline{z_1 - z_2}} \right) \leq 0. \]
The following lemma has an important role in this study.

Lemma 3.6. Let $1/2 < \gamma \leq 1$. We put $I = [0, T]$, $T > 0$ and we assume that
\[ \text{Im}(\lambda) < 0, \quad \frac{p-1}{2\sqrt{p}}|\text{Re}(\lambda)| \leq |\text{Im}(\lambda)|. \]
Let $u \in X(I)$ be solutions to (1)–(2). Then
\[ \| u(t) \|_{A^\gamma(t)} \leq \| \phi \|_{A^\gamma(0)}, \quad t \in I. \]

Proof. The desired result for the case of $\gamma = 1$ has been obtained in [16]. We assume that $1/2 < \gamma < 1$. Let $u \in X(I)$ be solutions to (1). We define the translation and the Galilei transform by
\[ \tau_y u(t) = u(t, \cdot + y) \]
and
\[ G_y u(t) = U(t)e^{-iyx}U(-t)u(t) \]
for $y \in \mathbb{R}$. By the Galilei invariant property of (1), we have
\[ G_y u(t) = U(t)e^{-iyx}\phi - i\lambda \int_0^t U(t - s) (|G_y u|^{p-1}G_y u) (s)ds \]
where we have used the another representation
\[ G_y u(t) = e^{-iyx - i2y^2} u(t, \cdot + ty) = M(t)\tau_{ty}(M(-t)u(t)). \]

Therefore, as in the similar way of [22], we have
\[
\|FU(-t)(G_y u(t) - u(t))\|^2_{L^2} = \left( U(-t)(G_y(t) - 1)u(t), U(-t)(G_y(t) - 1)u(t) \right)
\]
\[
= \|e^{-iyx} \phi - \phi\|^2_{L^2} + 2\text{Im} \left[ \int_0^t \left( \lambda((G_y - 1)|u|^{p-1}u)(s), ((G_y - 1)u)(s) \right) ds \right]
\]
\[
+ 2\text{Im} \left[ \int_0^t \left( \lambda((G_y - 1)|u|^{p-1}u)(s), i\lambda \int_0^s U(s - s') ((G_y - 1)|u|^{p-1}u)(s') ds' \right) ds \right]
\]
\[
+ \left\| \int_0^t \lambda U(-s)((G_y - 1)|u|^{p-1}u)(s)ds \right\|^2_{L^2}
\]
\[
= \|\tau_y \hat{\phi} - \hat{\phi}\|^2_{L^2} + 2\text{Im} \left[ \int_0^t \left( \lambda|G_y u|^{p-1}G_y u - |u|^{p-1}u)(s), (G_y u - u)(s) \right) ds \right]
\]
\[
\leq \|\tau_y \hat{\phi} - \hat{\phi}\|^2_{L^2}
\]
where we have used the Lemma 3.5 and the following identity

\[
\left\| \int_0^t \lambda U(-s)f(u)(s)ds \right\|^2_{L^2} = \int_0^t \int_0^s \left( \lambda f(u)(s), \lambda U(s - s') f(u)(s') \right) ds' ds
\]
\[
\quad + \int_0^t \int_0^s \left( \lambda f(u)(s), \lambda U(s - s') f(u)(s') \right) ds ds'
\]
\[
= -2\text{Im} \left[ \int_0^t \left( \lambda f(u)(s), i\lambda \int_0^s U(s - s') f(u)(s') ds' \right) ds \right]
\].

Therefore by the above estimate and the identity (5), we have

\[
\|J^\gamma(t)u(t)\|_{L^2} = \|\mathcal{F}U(-t)u(t)\|_{H^\gamma}
\]
\[
= C_\gamma \left( \int_R \frac{\|\mathcal{F}U(-t)(G_y u(t) - u(t))\|^2_{L^2}}{|y|^{1+2\gamma}} dy \right)^{1/2}
\]
\[
\leq C_\gamma \left( \int_R \frac{\|\tau_y \hat{\phi} - \hat{\phi}\|^2_{L^2}}{|y|^{1+2\gamma}} dy \right)^{1/2}
\]
\[
= \|\hat{\phi}\|_{H^\gamma}, \ t \in I.
\]

Also we see that

\[
\|u(t)\|_{L^2} \leq \|\phi\|_{L^2}, \ t \in I.
\]

This completes the proof. □
4. Proof of Theorem 2.1. We consider the map \( \Phi : u \mapsto \Phi u \) defined by
\[
\Phi u(t) = U(t)\phi - i\lambda \int_0^t U(t-s)(|u|^{p-1}u)(s)\, ds, \quad t \in I
\]
in the complete metric space \((X_r(I), d)\) with
\[
X_r(I) = \left\{ u \in X(I); \|u\|_{X(I)} \leq r \right\},
\]
\[
d(u, v) = \|u - v\|_{L^\infty(I; L^2) \cap L^4(I; L^\infty)}
\]
for interval \( I = [0, T] \). We have the following estimates
\[
\|\Phi u\|_{X(I)} \leq C \|\phi\|_{F^\gamma} + C \|u|^{p-1}u\|_{L^1(I; A^2)}
\]
\[
\leq C \|\phi\|_{F^\gamma} + C \left\| \|u(t)\|^\frac{p-1}{2} \|u(t)\|_{A^2(t)} \right\|_{L^4(I)}
\]
\[
\leq C \|\phi\|_{F^\gamma} + CT^{1-\frac{n-1}{p}} \|u\|^{p-1}_{L^4(I; L^\infty)} \|u\|_{L^\infty(I; A^2)}
\]
\[
\leq C \|\phi\|_{F^\gamma} + CT^{1-\frac{n-1}{p}} r^p
\]
and
\[
d(\Phi u, \Phi v) = \|\Phi u - \Phi v\|_{L^\infty(I; L^2) \cap L^4(I; L^\infty)}
\]
\[
\leq C \|(u|^{p-1} + |v|^{p-1})(u - v)\|_{L^1(I; L^2)}
\]
\[
\leq CT^{1-\frac{n-1}{p}} \left( \|u\|^{p-1}_{L^4(I; L^\infty)} + \|v\|^{p-1}_{L^4(I; L^\infty)} \right) \|u - v\|_{L^\infty(I; L^2)}
\]
\[
\leq CT^{1-\frac{n-1}{p}} r^p \|u - v\|_{L^\infty(I; L^2) \cap L^4(I; L^\infty)}
\]
\[
= CT^{1-\frac{n-1}{p}} r^p d(u, v).
\]
By the Banach fixed point theorem with
\[
r = 2C \|\phi\|_{F^\gamma}, \quad CT^{1-\frac{n-1}{p}} r^p = \frac{1}{2}
\]
we have the desired unique local solution
\[
u \in X(I).
\]
By the Strichartz estimate, we have
\[
\|u\|_{L^4(I; A^\infty)} \leq C \|\phi\|_{F^\gamma} + CT^{1-\frac{n-1}{p}} r^p \leq r.
\]
By a standard continuation argument with a priori estimate
\[
\|u(t)\|_{A^2(t)} \leq \|\phi\|_{A^2(0)}, \quad t \in I
\]
we obtain the desired unique global solution
\[
u \in X_{loc}(\mathbb{R}^+) \cap L^4_{loc}(\mathbb{R}^+; A^\infty).
\]
In the last part of the proof of Theorem 2.1, we follow the argument studied in [9, 10, 11]. By the relation
\[
i\partial_t + \partial_x^2/2 = U i\partial_t U^{-1}
\]
we have
\[
i\partial_t U^{-1} u = \lambda U^{-1} |u|^{p-1} u
\]
and by using the formulas
\[ U(t) = M(t)D(t)F M(t), \quad t \neq 0 \]
and
\[ U(-t) = M(-t)F^{-1}iD \left( \frac{1}{t} \right) M(-t), \quad t \neq 0 \]
we have
\[
U(-t) \left( |u|^{p-1}u \right) (t) = t^{-\frac{p-1}{2}} M(-t)F^{-1} \left[ |\mathcal{F}[M(t)U(-t)u(t)]|^{p-1} \mathcal{F}[M(t)U(-t)u(t)] \right]
\]
\[
= t^{-\frac{p-1}{2}} \left\{ (M(-t) - 1)F^{-1}[|\mathcal{F}[M(t)U(-t)u(t)]|^{p-1} \mathcal{F}[M(t)U(-t)u(t)]
\]
\[ + \mathcal{F}^{-1}[|\mathcal{F}[M(t)U(-t)u(t)]|^{p-1} \mathcal{F}[M(t)U(-t)u(t)]
\]
\[ - |\mathcal{F}[U(-t)u(t)]|^{p-1} \mathcal{F}[U(-t)u(t)] \} \right\}
\]
\[ + t^{-\frac{p-1}{2}} \mathcal{F}^{-1} \left[ |\mathcal{F}[U(-t)u(t)]|^{p-1} \mathcal{F}[U(-t)u(t)] \right].
\]
Therefore we obtain
\[
i\partial_t \mathcal{F}U^{-1}u - \lambda t^{-\frac{p-1}{2}} |\mathcal{F}U^{-1}u|^{p-1} \mathcal{F}U^{-1}u = \lambda t^{-\frac{p-1}{2}} (R_1 + R_2)
\]
where
\[ R_1 = \mathcal{F}(M^{-1} - 1)\mathcal{F}^{-1}|\mathcal{F}MU^{-1}u|^{p-1} \mathcal{F}MU^{-1}u \]
and
\[ R_2 = |\mathcal{F}MU^{-1}u|^{p-1} \mathcal{F}MU^{-1}u - |\mathcal{F}U^{-1}u|^{p-1} \mathcal{F}U^{-1}u. \]
Setting \( v = \mathcal{F}U^{-1}u \) and multiplying both sides of (6) by \( \overline{v} \) we have
\[
\overline{v} \left( i\partial_t v \right) - \lambda t^{-\frac{p-1}{2}} |v|^{p+1} = \lambda t^{-\frac{p-1}{2}} (R_1 + R_2) \overline{v}
\]
and
\[
\frac{1}{2} \partial_t |v|^2 - \text{Im}(\lambda)t^{-\frac{p-1}{2}} |v|^{p+1} = \text{Im} \left( \lambda t^{-\frac{p-1}{2}} (R_1 + R_2) \overline{v} \right)
\]
\[
\leq \left| \lambda t^{-\frac{p-1}{2}} (R_1 + R_2) \overline{v} \right| \leq Ct^{-\frac{p-1}{2}-k} \|\phi\|_{F^{H, \gamma}}^3 |v|
\]
for \( 1/2 < \gamma \leq 1 \) and \( 0 < 2k + 1/2 < \gamma \) by Lemma 4 and Lemma 6. Then by \( \partial_t |v|^2 = 2|v|\partial_t |v| \), we have
\[
\partial_t |v| - \text{Im}(\lambda)t^{-\frac{p-1}{2}} |v|^p \leq Ct^{-\frac{p-1}{2}-k} \|\phi\|_{F^{H, \gamma}}^3.
\]
In the Case: \( p = 3 \). If \( p = 3 \), then
\[
\partial_t |v| - \text{Im}(\lambda)t^{-1} |v|^3 \leq Ct^{-1-k} \|\phi\|_{F^{H, \gamma}}^3.
\]
Multiplying both sides of (8) by \((\log t)^{3/2}\) and by the identity
\[
\partial_t ((\log t)^{3/2} |v|) = \frac{3}{2}(\log t)^{1/2}t^{-1} |v| + (\log t)^{3/2} \partial_t |v|,
\]
we have
\[
\partial_t ((\log t)^{3/2} |v|) - \frac{3}{2}(\log t)^{1/2}t^{-1} |v| - (\log t)^{3/2} \text{Im}(\lambda)t^{-1} |v|^3 \leq C(\log t)^{3/2}t^{-1-k} \|\phi\|_{F^{H, \gamma}}^3.
\]
By the Young inequality: \( ab \leq \frac{2}{5}a^{3/2} + \frac{1}{3}b^3 \), we have

\[
\frac{3}{2}(\log t)^{1/2} |v| = \left( \frac{3}{2} \left| \text{Im}(\lambda) \right|^{-1/3} t^{-2/3} \right) \left( \frac{3}{2} |\text{Im}(\lambda)|^{1/3} t^{-1/3} (\log t)^{1/2} |v| \right)
\leq \frac{2}{3} \left( \frac{3}{2} \left| \text{Im}(\lambda) \right|^{-1/2} t^{-1} \right) + \frac{1}{3} \left( |\text{Im}(\lambda)| t^{-1} (\log t)^{3/2} |v|^3 \right)
= \frac{t^{-1}}{\sqrt{2|\text{Im}(\lambda)|}} + |\text{Im}(\lambda)| t^{-1} (\log t)^{3/2} |v|^3.
\]

Therefore

\[
\partial_t ((\log t)^{3/2} |v|) \leq \frac{t^{-1}}{\sqrt{2|\text{Im}(\lambda)|}} + C (\log t)^{3/2} t^{-1-k} \|\phi\|_{\mathcal{F}H^s},
\]

\[
\int_2^t \partial_s ((\log s)^{3/2} |v(s)|) ds \leq \int_2^t \left( \frac{s^{-1}}{\sqrt{2|\text{Im}(\lambda)|}} + C (\log s)^{3/2} s^{-1-k} \|\phi\|_{\mathcal{F}H^s} \right) ds
\]
and

\[
(\log t)^{3/2} |v(t)| \leq (\log 2)^{3/2} |v(2)| + \frac{1}{\sqrt{2|\text{Im}(\lambda)|}} (\log t - \log 2) + C \|\phi\|_{\mathcal{F}H^s}.
\]

Hence

\[
|v(t)| \leq \left( (\log 2)^{3/2} |v(2)| + C \|\phi\|_{\mathcal{F}H^s} \right) (\log t)^{-3/2} + \frac{1}{\sqrt{2|\text{Im}(\lambda)|}} (\log t)^{-1/2}
\leq (\log t)^{-1/2} \left( \frac{1}{\sqrt{2|\text{Im}(\lambda)|}} + (\log 2)^{-1} \left( (\log 2)^{3/2} |v(2)| + C \|\phi\|_{\mathcal{F}H^s} \right) \right)
\leq C (\log t)^{-1/2}
\]
for \( t \geq 2 \). We devide \( u \) into two terms

\[
u(t) = U(t)U(-t)u(t)
= M(t)D(t)FM(t)U(-t)u(t)
= M(t)D(t)v(t) + M(t)D(t)F(M(t) - 1)U(-t)u(t)
\]
and we estimate

\[
\|u(t)\|_{L^\infty} \leq Ct^{-1/2} (\log t)^{-1/2} + Ct^{-1/2-k}
\leq Ct^{-1/2} (\log t)^{-1/2},
\]
for \( t \geq 2 \). We put the phase factor \( \Gamma \) by

\[
\Gamma(t) = \lambda \int_2^t s^{-1} |\tilde{u}(s)|^2 ds, \ t \geq 2.
\]
and we put the new function \( f \) by

\[
f(t) = e^{i\Gamma(t)} FU(-t)u(t), \ t \geq 2.
\]
Then we have

\[
i\partial_tf = e^{i\Gamma} \lambda t^{-1} (R_1 + R_2)
\]
and

\[
f(t) - f(t') = -i \int_t^{t'} e^{i\Gamma(s)} \lambda s^{-1} (R_1(s) + R_2(s)) ds, \ t > t' \geq 2.
\]

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Therefore, we have
\[ \|f(t) - f(t')\|_{L^2 \cap L^\infty} \leq C \int_{t'}^t \|\phi\|_{F^H \gamma}^3 \exp \left( C \int_2^s \tau^{-1}(\log \tau)^{-1} d\tau \right) s^{-k-1} ds \]
\[ \leq C \int_{t'}^t \|\phi\|_{F^H \gamma}^3 \exp (C \log (\log s)) s^{-k-1} ds \]
\[ = C \int_{t'}^t \|\phi\|_{F^H \gamma}^3 (\log s)^C s^{-k-1} ds \]
\[ \leq C \|\phi\|_{F^H \gamma} (\log t')^{C (t')^{-k}} \to 0, \quad \text{as} \quad t' \to +\infty \]
and there exists a unique function \( \psi_+ \in L^2 \cap L^\infty \) such that
\[ \|f(t) - \psi_+\|_{L^2 \cap L^\infty} \leq C \|\phi\|_{F^H \gamma}^3 t^{-\alpha}, \quad t \geq 2 \]
where \( 0 < \alpha < k \).

In the Case: \( 1 < p < 3 \). If \( 1 < p < 3 \). Multiplying both sides of (7) by \( t^{-\frac{1}{2} + \frac{1}{p-1}} \) and by the identity
\[ \partial_t (t^{\frac{1}{2} + \frac{1}{p-1}} |v|) = \frac{3 - p}{2(p - 1)} t^{-1} t^{-\frac{1}{2} + \frac{1}{p-1}} |v| + t^{-\frac{1}{2} + \frac{1}{p-1}} \partial_t |v| \]
we have
\[ \partial_t (t^{\frac{1}{2} + \frac{1}{p-1}} |v|) - \frac{3 - p}{2(p - 1)} t^{-1} t^{-\frac{1}{2} + \frac{1}{p-1}} |v| \leq \text{Im}(\lambda) t^{\frac{1}{p-1} - \frac{1}{2}} |v|^p + Ct^{\frac{1}{p-1} - \frac{1}{2} - k} \|\phi\|_{F^H \gamma}^p. \]

Multiplying both sides of the above by \( t^{\frac{3p}{2p-1}} \) we obtain
\[ \partial_t \left( t^{\frac{3p}{2p-1}} t^{-\frac{1}{2} + \frac{1}{p-1}} |v| \right) \leq \left( \frac{3 - p}{2} \frac{p}{p - 1} \right) t^{\frac{3p}{2p-1}} t^{-\frac{1}{2} + \frac{1}{p-1}} |v| + \text{Im}(\lambda) t^{-1 - \frac{3p}{2p-1} t^{-\frac{1}{2} + \frac{1}{p-1}} |v|^p} \]
\[ + Ct^{\frac{3p}{2p-1} + \frac{1}{p-1} - \frac{1}{2} - k} \|\phi\|_{F^H \gamma}^p. \]

By the Young inequality: \( ab \leq \frac{p-1}{p} a^{\frac{p}{p-1}} + \frac{1}{p} b^p \), we have
\[ \left( \frac{3 - p}{2} \frac{p}{p - 1} \right) t^{\frac{3p}{2p-1}} t^{-\frac{1}{2} + \frac{1}{p-1}} |v| \]
\[ = \left( \frac{3 - p}{2} \frac{p}{p - 1} \right) p^{-\frac{1}{p}} |\text{Im}(\lambda)|^{-\frac{1}{p}} t^{-\left(\frac{3p}{2p-1} - \frac{1}{2}\right)} \left( p^{\frac{1}{p}} |\text{Im}(\lambda)|^{-\frac{1}{p}} t^{-\frac{1}{2} + \frac{2p}{2p-1} t^{-\frac{1}{2} + \frac{1}{p-1}} |v|} \right) \]
\[ \leq \frac{p-1}{p} \left( \left( \frac{3 - p}{2} \frac{p}{p - 1} \right) p^{-\frac{1}{p}} |\text{Im}(\lambda)|^{-\frac{1}{p}} \right)^{\frac{p}{p-1}} t^{-\frac{2p}{p-1}} - \text{Im}(\lambda) t^{-1 + \frac{3p}{2p-1} t^{-\frac{1}{2} + \frac{1}{p-1}} |v|^p}. \]

Therefore
\[ \partial_t \left( t^{\frac{3p}{2p-1}} t^{-\frac{1}{2} + \frac{1}{p-1}} |v| \right) \]
\[ \leq \frac{p-1}{p} \left( \left( \frac{3 - p}{2} \frac{p}{p - 1} \right) p^{-\frac{1}{p}} |\text{Im}(\lambda)|^{-\frac{1}{p}} \right)^{\frac{p}{p-1}} t^{-\frac{2p}{p-1}} + Ct^{\frac{3p}{2p-1} + \frac{1}{p-1} - \frac{1}{2} - k} \|\phi\|_{F^H \gamma}^p. \]
and
\[ \int_2^t \partial_s \left( s^{\frac{3-p}{2}} s^{-\frac{1}{p-1}} |v(s)| \right) ds \]
\[ \leq \int_2^t \left( \frac{p-1}{p} \left( \frac{3-p}{2} \frac{p}{p-1} \right) p^{-\frac{1}{p}} |\text{Im}(\lambda)|^{-\frac{1}{p}} \right)^{\frac{p}{p-1}} s^{\frac{3-p}{2}} \]
\[ + Cs^{\frac{3-p}{2} + \frac{1}{p-1} - \frac{5}{2} - k} \|\phi\|_{\mathcal{F}H^\gamma} ds \]
\[ = \frac{2}{3} \frac{p-1}{p} \left( \frac{3}{2} \frac{p}{p-1} \right) p^{-\frac{1}{p}} |\text{Im}(\lambda)|^{-\frac{1}{p}} \left( t - t^{\frac{3-p}{2}} \right) \]
\[ + C \left( t^{1+\frac{3-p}{2} + \frac{1}{p-1} - \frac{5}{2} - k} - 2^{1+\frac{3-p}{2} + \frac{1}{p-1} - \frac{5}{2} - k} \right) \|\phi\|_{\mathcal{F}H^\gamma}. \]

Hence
\[ t^{\frac{3-p}{2}} + \frac{1}{p-1} |v(t)| \]
\[ \leq C|v(2)| + \frac{2}{3} \frac{p-1}{p} \left( \frac{3}{2} \frac{p}{p-1} \right) p^{-\frac{1}{p}} |\text{Im}(\lambda)|^{-\frac{1}{p}} \left( t - t^{\frac{3-p}{2}} \right) \]
\[ - \frac{2}{3} \frac{p-1}{p} \left( \frac{3}{2} \frac{p}{p-1} \right) p^{-\frac{1}{p}} |\text{Im}(\lambda)|^{-\frac{1}{p}} \left( \frac{p}{2} - 2^{\frac{3-p}{2}} \right) \]
\[ + C \|\phi\|_{\mathcal{F}H^\gamma} t^{1+\frac{3-p}{2} + \frac{1}{p-1} - \frac{5}{2} - k} \]
and
\[ t^{\frac{1}{2} + \frac{1}{p-1}} |v(t)| \]
\[ \leq Ct^{\frac{3-p}{2}} + \frac{2}{3} \frac{p-1}{p} \left( \frac{3}{2} \frac{p}{p-1} \right) p^{-\frac{1}{p}} |\text{Im}(\lambda)|^{-\frac{1}{p}} \left( \frac{p}{2} - k \right) \]
\[ + C \|\phi\|_{\mathcal{F}H^\gamma} t^{1+\frac{1}{p-1} - \frac{5}{2} - k} \]
\[ = Ct^{\frac{3-p}{2}} + C \|\phi\|_{\mathcal{F}H^\gamma} t^{1+\frac{1}{p-1} - \frac{5}{2} - k} + \frac{3-p}{2(2p-1)|\text{Im}(\lambda)|} \left( \frac{p}{2} - k \right). \quad (9) \]

If
\[ p > p(\gamma) = \frac{(7 - 2\gamma) + \sqrt{(7 - 2\gamma)^2 + 8(2\gamma - 1)}}{4} \]
then
\[ 1 + \frac{1}{p-1} - \frac{p}{2} - k < 0 \]
for sufficiently large
\[ k \in (0, \gamma/2 - 1/4). \]

Therefore by the decomposition
\[ u(t) = M(t)D(t)v(t) + M(t)D(t)\mathcal{F}(M(t) - 1)U(-t)u(t) \]
we have
\[ \|u(t)\|_{L^\infty} \leq Ct^{-\frac{3-p}{2}}, \quad t \geq 2 \]
for $p(\gamma) < p < 3$. Let $f$ be the function defined as

$$\Gamma(t) = \lambda \int_2^t s^{-\frac{p-1}{2}} |\tilde{u}(s)|^{p-1} ds$$

and

$$f(t) = e^{i\Gamma(t)} FU(-t)u(t)$$

for $t \geq 2$. By (9) above, for any $\epsilon > 0$, there exists $T_\epsilon = T_* (\epsilon) > 0$ such that, if $t \geq T_\epsilon$, then

$$t^{\frac{1}{2}+\frac{1}{p-1}} |v(t)| \leq \left( \frac{\epsilon}{|\text{Im}(\lambda)|} + \frac{3-p}{2(p-1)|\text{Im}(\lambda)|} \right)^{\frac{1}{p-1}}.$$  \hfill (10)

Therefore for $t > t' \geq \max(T_\epsilon, 2)$, we have

$$f(t) - f(t') = -i \int_{t'}^t e^{i\Gamma(s)} \lambda s^{-\frac{p-1}{2}} (R_1(s) + R_2(s)) ds$$

and

$$\|f(t) - f(t')\|_{L^2 \cap L^\infty} \leq C \int_{t'}^t \|\phi\|_{F^p_{H^s}}^p \exp \left( |\text{Im}(\lambda)| c(\epsilon) \int_2^s \tau^{-1} d\tau \right) s^{-k-\frac{p-1}{2}} ds$$

$$\leq C \int_{t'}^t \|\phi\|_{F^p_{H^s}}^p \exp \left( |\text{Im}(\lambda)| c(\epsilon) \int_2^s \tau^{-1} d\tau \right) s^{-k-\frac{p-1}{2}} ds$$

$$\leq C \|\phi\|_{F^p_{H^s}}^p \int_{t'}^t s^{\text{Im}(\lambda)|c(\epsilon)|} s^{-k-\frac{p-1}{2}} ds \rightarrow 0, \quad \text{as} \ t > t' \rightarrow +\infty$$

for $p(\gamma) < p < 3$, where we have used the following fact

$$|\text{Im}(\lambda)| c(\epsilon) - k - \frac{p-1}{2} + 1 < 0$$

with

$$c(\epsilon) = \frac{\epsilon}{|\text{Im}(\lambda)|} + \frac{3-p}{2(p-1)|\text{Im}(\lambda)|}.$$  

by (10), above.

Note that the constant $c(\epsilon)$ is similar to the constant $K^{p-1}$ appears in Proposition 3.2. of [16].

Hence there exists a unique function $\psi_+ \in L^2 \cap L^\infty$ such that

$$\|f(t) - \psi_+\|_{L^2 \cap L^\infty} \leq C \|\phi\|_{F^p_{H^s}} \tau^{-\alpha}, \quad t \geq \max(T_\epsilon, 2),$$

where $0 < \alpha < k$. This completes the proof of Theorem 2.1.

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