Single-quantum interferometry: which-way versus which-phase information stored in an ancillary quantum system

Soroush Khademi\(^1\) and Ali Reza Bahrampour

Department of Physics, Sharif University of Technology, PO Box 11155-9161, Tehran, Iran

E-mail: khademi_soroush@physics.sharif.edu and bahrampour@sharif.edu

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Abstract
In interferometers, the more information about a quantum’s path that is available in an ancillary quantum system (AQS), the less visibility the interference has. By use of Shannon entropy, we try to compare the amount of which-phase information with the amount of which-way information stored in the AQS of two-path interferometers with symmetric beam merging. We show that the former is less than or equal to the latter if the bipartite system of the single quantum and the AQS is initially prepared in a pure state and the interaction between the two parts is unitary. The equality holds when symmetry exists. No which-way information is obtained by the measurement that we use for extracting the which-phase information and vice versa. In order to verify the results experimentally, we propose assembling a new single-photon interferometer.

Keywords: complementarity, wave–particle duality, quantum erasure, accessible information

1. Introduction
There are two classical pictures of motion: a classical particle travels along one path and a classical wave can propagate through several paths. But in the quantum realm, a single quantum showing particle-like behavior in one interferometry experiment may exhibit wavelike behavior in another. According to wave–particle complementarity, such a change in behavior may happen only if the experimental setups are mutually exclusive [1]; so the results of such experiments can be described by the use of classical pictures of motion in a complementary way and without any contradiction [2]. Many experiments support this principle, e.g. [3, 4].

\(^1\)Author to whom any correspondence should be addressed.
While experimental investigations about the reality of a single quantum’s propagation, if any reality exists, continue—see, for example, the discussions around [5] including the recent paper [6], wave–particle complementarity does not deliver any ontological message about how a single quantum propagates. It just claims that speaking about the quantum’s path, with our everyday language formed in the classical world, is still possible; e.g. a physicist is able to use phrases such as ‘as if the single photon has propagated through several paths in this experimental setup’ or ‘as if the single photon has propagated through just one path in that experimental setup’ in order to describe the outputs of every single-photon interferometry experiment.

There exist interferometry setups in which the quantum’s behavior is partially particle-like and partially wavelike. Sometimes, this partial behavior is observed when the quantum’s path information is partially available in the environment, e.g. in an ancillary quantum system (AQS). The information can be extracted by an appropriate measurement. Based on the probability of success of path attribution, a quantity called distinguishability ($D$) is defined [7] for measuring the particle-like behavior. The visibility ($V$) of the interference quantifies the wavelike behavior. Englert [7] proves that

$$D^2 + V^2 \leq 1,$$

which means, as expected by wave–particle complementarity, that a trade-off exists between the particle-like behavior and the wavelike behavior.

Different entropic measures are also employed for quantifying the partial behaviors or similar purposes. Such measures were used, first in 1979, by Wootters and Zurek [8] and then by others [9, 10]. Although [11] and [12] criticized such measures, work on entropic measures continued, e.g. [13–15], and in 2014, Coles et al [16] took an interesting step in this direction. They defined a which-way guessing game and a which-phase guessing game played with a two-path interferometer. The possibility of winning each game represents one partial behavior and is determined by a special entropy. Finally, a very general relation has been derived that is basically an entropic uncertainty relation and shows the trade-off between the partial behaviors. The approach of [16] is employed for multipath interferometers in [17].

Recently, some measures of quantum coherence have been widely used as the measure of wavelike behavior. In this new trend, different non-entropic relations [18–21] and entropic relations [19, 22, 23] have been derived that show the trade-off between the partial behaviors.

Measuring the AQS in an inappropriate basis erases the which-way information, instead of extracting it. But, interestingly, such a measurement allows us to recognize sub-ensembles of the interferometry data showing greater interference visibility. This phenomenon is called quantum erasure [24, 25]. It has been observed in several experiments, e.g. [26, 27]. The first attempt to quantify this phenomenon was made by [28]. After that, [29] derived an erasure relation by setting an upper bound on the visibility of the sub-ensembles. The results of [29] are re-derived by [16] with the entropic approach. Glick and Adami [30] use von Neumann entropy in order to quantify the phenomenon in a special scenario called Bell-state quantum erasure.

We consider two guessing games like [16]; we use Shannon entropy as a standard information theoretic tool and try to compare the amount of which-way information stored in the AQS with the amount of which-phase information obtainable by using the AQS. We are also curious to know whether extracting one of these two types of information is at the cost of losing the other—this can be expected from what is learnt from quantum erasure.

In section 2, we determine our framework and elaborate the problem that we are going to deal with. The problem is solved for a special case with some interesting results in section 3. We propose a new interferometry setup in section 4 for experimental verification of the results.
In section 5, the strategies used in the games and the probable general answer of the problem are discussed. The conclusions are in section 6.

2. Framework

Figure 1(a) shows a two-path interferometer. A qubit with Bloch vector \( \mathbf{q} \) can represent the quantum’s path; the upper path and the lower one are characterized by \( q_z = -1 \) and \( q_z = 1 \) respectively. The single quantum passes through a beam splitter (BS) represented by a single-qubit gate. It interacts with a phase shifter (PS). Gate \( |0\rangle\langle 0| + e^{i\phi}|1\rangle\langle 1| \) plays the role of the PS. Then, it passes through a beam merger (BM). As we consider a symmetric beam merging for a single quantum, gate \( \exp(i\frac{\pi}{2}\sigma_\chi) \) can represent the BM. Finally, the quantum is detected in one of the paths—the path qubit is measured in the computational basis.

In figure 1(b), we let an AQS interact with the quantum. Generally, there is no restriction on the AQS and the interaction; it can even change the path of the quantum. Correlations that may exist between the AQS and the quantum after the interaction make it possible to obtain some information about the quantum’s path by using the AQS.

In order to find a value for the which-way information that may be stored in the AQS, let us consider a which-way guessing game (WWGG) played by Alice and Bob. While the BM is removed, Alice asks Bob to guess which detector will click or which one has clicked. Random variable \( W \) represents the detector that clicks. Bob uses the AQS. He performs a measurement on the AQS such that the amount of mutual information between \( W \) and the result of his measurement is maximized. Random variable \( M \) represents the result of such an optimum measurement. We take \( I(W : M) \), which is the amount of mutual information, as the amount of which-way information stored in the AQS.

Let us come back to the closed interferometer. Alice and Bob play a which-phase guessing game (WPGG), like the one defined in [16]. One of the two values of random variable \( \Phi = \{ \phi_0, \phi_0 + \pi \} \) is randomly applied to the interferometer as the phase difference by Alice. Bob should guess which value has been chosen. He can see the detectors and use the AQS. First of all, Bob looks at the detectors to see which one clicks. Random variable \( D \) represents the result. Then, given the value of \( \phi_0 \), he performs a measurement on the AQS that minimizes the amount of conditional Shannon entropy \( H(\Phi|D, E) \), the measurement’s result. This measurement’s result is represented by random variable \( E \). Alice’s choice of \( \phi_0 \) has already maximized \( H(\Phi|D, E) \) over all possible values of \( \phi_0 \). We take \( H(\Phi|D) - H(\Phi|D, E) \) as the amount of which-phase information obtainable by Bob by using the AQS.

We want to know if there is a meaningful relationship between \( I(W : M) \) and \( H(\Phi|D) - H(\Phi|D, E) \). Furthermore, we should survey how much information Bob gains if he uses the measurement of the WPGG in the WWGG and vice versa.

In the WWGG, we are faced with the problem of accessible information about quantum states, which is still, to the best of our knowledge, an open problem for the general case. In the WPGG, there are two quantum states; one of them is randomly prepared by Alice’s choice between \( \phi_0 \) and \( \phi_0 + \pi \). Bob’s measurement is supposed to maximize the value of

\[
H(\Phi|D) - H(\Phi|D, E) = I(\Phi : D, E) - I(\Phi : D) = I(\Phi : E | D)
\]

(2)

where the last term equals the expected value of mutual information between \( \Phi \) and \( E \) given the value of \( D \). So, again, we are faced with the problem of accessible information. The problem has been solved [31] for two pure states. The accessible information about \( |\psi^{(1)}\rangle \) and \( |\psi^{(2)}\rangle \) with occurrence probabilities \( p \) and \( 1 - p \), respectively, is a function of \( r = |\langle \psi^{(1)} | \psi^{(2)} \rangle| \) and \( p \). We show this function with \( I_A(r, p) \)—see appendix A for the exact form. As expected,
I_A(r, p) \leq I_A(r, 0.5),
(3a)

I_A(r_1, p) \leq I_A(r_2, p) \quad \text{for } r_2 \leq r_1.
(3b)

In the next section, we will examine the situations in which two pure states should be discriminated in the guessing games.

### 3. Results

The spatial degree of freedom of the single quantum and the AQS form a bipartite system. We consider that the system’s initial state is pure and the interaction between the two parts is unitary. After the interaction, the state of the system is of the form

$$ |\Psi\rangle = \sqrt{p} |0\rangle \otimes |w_0\rangle + e^{i\gamma} \sqrt{1-p} |1\rangle \otimes |w_1\rangle $$
(4)

where $|w_0\rangle$ and $|w_1\rangle$ are two normalized states of the AQS; $0 \leq p \leq 1$ and $\gamma$ has been chosen such that $r_w = <w_0|w_1>$ is a real non-negative number. As a result of the PS, the system’s state evolves into

$$ |\Psi'\rangle = \sqrt{p} |0\rangle \otimes |w_0\rangle + e^{i(\gamma+\phi)} \sqrt{1-p} |1\rangle \otimes |w_1\rangle. $$
(5)

Obviously, in the WWGG, Bob should discriminate $|w_0\rangle$ from $|w_1\rangle$, while the former exists with probability $p$. So

$$ I(W: M) = I_A(r_w, p). $$
(6)
If the quantum passes through the BM, the system finds the state
\[ |\Psi'\rangle = C_0 |0\rangle \otimes |d_0^{\phi}\rangle - C_1 |1\rangle \otimes |d_1^{\phi}\rangle \] (7)
where
\[ C_j = (0.5 + (-1)^j r \sqrt{p(1-p)} \cos(\gamma + \phi))^0.5, \] (8a)
\[ |d_j^{\phi}\rangle = \frac{\sqrt{p} |w_0\rangle + (-1)^j e^{i(\gamma+\phi)} \sqrt{1-p} |w_1\rangle}{\sqrt{2} C_j}, \quad j = 0, 1. \] (8b)

In the WPGG, if the upper (lower) detector clicks, Bob should discriminate \(|d_0^{\phi}\rangle\) (\(|d_1^{\phi}\rangle\)) from \(|d_0^{\phi+\pi}\rangle\) (\(|d_1^{\phi+\pi}\rangle\)). Since \(|d_0^{\phi+\pi}\rangle = |d_1^{\phi}\rangle\) and \(|d_1^{\phi+\pi}\rangle = |d_0^{\phi}\rangle\), \(H(\Phi|D) - H(\Phi|D, E)\) equals the accessible information about two equiprobable states \(|d_0^{\phi}\rangle\) and \(|d_1^{\phi}\rangle\):
\[ H(\Phi|D) - H(\Phi|D, E) = I_A(r_d, 0.5) \] (9)
where
\[ r_d = \frac{|\langle d_0^{\phi}|d_0^{\phi}\rangle - \langle d_1^{\phi}|d_0^{\phi}\rangle|^2}{\left(\frac{2p - 1}{2} + 4r_w^2 p(1-p) \sin^2(\gamma + \phi_0)\right)^{0.5}}. \] (10)

Now, we should fix the value of \(\phi_0\). \(r_d\) reaches its maximum at
\[ \phi_0 = \frac{\pi}{2} - \gamma. \] (11)

So, based on (3b) and (9), the minimum of \(H(\Phi|D) - H(\Phi|D, E)\) is at this value of \(\phi_0\). Furthermore, by considering (7) and (8a), it is easy to notice that \(H(\Phi|D)\) reaches its maximum at the same value of \(\phi_0\). As a conclusion, \(H(\Phi|D, E)\) is maximized by this value of \(\phi_0\). Thus Alice plays the WPGG with this value of \(\phi_0\) and \(r_d\) equals \(\sqrt{(2p - 1)^2 + 4r_w^2 p(1-p)}\).

For \(p = 0.5\), \(r_d\) equals \(r_w\) and, based on (6) and (9), \(H(\Phi|D) - H(\Phi|D, E) = I(\mathcal{W} : \mathcal{M})\). This is an interesting result. It states that, in the case for which equality holds in (1), the amount of which-phase information obtainable by using the AQS equals the amount of which-way information obtainable by using it.

For an arbitrary \(p\), \(r_w \leq r_d\). So it is not easy to compare \(I_A(r_w, p)\) with \(I_A(r_d, 0.5)\)—see (3a) and (3b). By explicit calculation of the value of these two functions for \(0 \leq r_w \leq 1\) and \(0 \leq p \leq 1\), and considering (2), (6) and (9), we have
\[ I(\Phi : \xi | D) \leq I(\mathcal{W} : \mathcal{M}) \] (12)
where equality holds for \(p = 0, 0.5\) or 1, or \(r_w = 1\). Figure 2 shows how relation (12) holds for some example values of \(r_w\). We note that the asymmetry makes it hard to reveal the value of the phase difference, as it does in an ordinary two-path interferometer (figure 1(a)).

The measurement found by [31] for extracting the accessible information about \(|w_0\rangle\) and \(|w_1\rangle\) is a projective measurement in the basis \(\{|m_+\rangle, |m_-\rangle\}\) where
\[ |w_0\rangle = \cos \frac{\alpha}{2} |m_+\rangle + \sin \frac{\alpha}{2} |m_-\rangle, \] (13a)
\[ |w_1\rangle = \cos \frac{\alpha'}{2} |m_+\rangle + \sin \frac{\alpha'}{2} |m_-\rangle; \]
\(\alpha\) and \(\alpha'\) are functions of \(p\) and \(r_w\) and change from 0 to \(\pi\)—see appendix A. By considering (8b) with \(\phi = \pi/2 - \gamma\) and (13a), it is straightforward to show that \(|\langle m_\pm|d_0^{\phi}\rangle|^2 = |\langle m_\pm|d_1^{\phi}\rangle|^2\).
So if Bob, in the WPGG, uses the measurement basis \( \{|m_+\rangle, |m_-\rangle\} \), he gains no which-phase information.

Similarly, the measurement used for extracting the accessible information about \( |d_{00}\rangle \) and \( |d_{10}\rangle \) is a projective measurement in the basis \( \{|e_+\rangle, |e_-\rangle\} \) where

\[
e^{i\delta} |d_{00}\rangle = \cos \beta |e_+\rangle + \sin \beta |e_-\rangle,
\]

\[
|d_{10}\rangle = \sin \beta |e_+\rangle + \cos \beta |e_-\rangle;
\]

(14)

\( \delta \) is the argument of the complex number \( \langle d_{00}|d_{10}\rangle \) and \( \beta \) equals \( \arcsin(r_{w}) \)—see appendix A.

By considering (8b) with \( \phi = \pi/2 - \gamma \) and (14), it can be shown that \( ||e_+|w_0\rangle|^2 = ||e_+|w_1\rangle|^2 \).

So if Bob, in the WWGG, measures the AQS in the basis \( \{|e_+\rangle, |e_-\rangle\} \), he gains no which-way information. The measurement used in the WPGG is an erasing measurement.

4. Proposal for experimental verification

In order to check relation (12) experimentally, we propose assembling the interferometer depicted in figure 3. It is a modified Mach–Zehnder interferometer. A single photon with linear polarization \( V \) enters the interferometer through a BS. In the lower arm, a half-wave plate changes its polarization to \( H \) (perpendicular to \( V \)) and the polarization beam splitter (PBS) lets it pass. Its polarization is changed to circular by a quarter-wave plate. Then it interacts with an atom trapped in a cavity (as an AQS); after being reflected, it passes through the quarter-wave plate for the second time. Since the photon’s polarization is changed to \( V \), the PBS reflects it this time. An adjustable PS introduces a phase shift \( \phi \); we note that the optical path lengths of
In order to describe the AQS, let us consider an atom with three levels \( \{ |0\rangle, |1\rangle, |e\rangle \} \) trapped in a cavity. The transition frequency between \( |1\rangle \) and \( |e\rangle \) equals the resonance frequency of the cavity but the transition frequencies between \( |0\rangle \) and \( |1\rangle \) and between \( |0\rangle \) and \( |e\rangle \) are far from it. By making the cavity’s left reflector have a little leakage, the system can interact with a (circularly polarized) ultra-narrowband single photon coming from the left side. The photon is finally reflected. Due to this interaction, the atomic state

\[
|+\rangle = \frac{(1 + e^{0\eta})}{\sqrt{2}}
\]

is transformed to

\[
|+\rangle = \frac{(1 + e^{0\eta})}{\sqrt{2}}
\]

where \( \eta \) can be set by adjusting the central frequency of the incoming photon—for more details, see appendix B. This system has been implemented very well [32] and has been proposed for several applications, e.g. [32–34].

In the interferometer, we prepare the atom in \(+\rangle\) and send the single photon. Let us consider the situation in which the first BS is symmetric \((p = 0.5)\) and \( \eta = \pi/2 \). In comparison with section 3, \( |w_0\rangle = |+\rangle \) and \( |w_1\rangle = \exp(-i\pi/4)(|1\rangle + i|0\rangle)/\sqrt{2} \).

In order to play the WWGG with this setup, the second BS is discarded; the atom is prepared in \(+\rangle\) and the single photon is sent. Bob should guess which detector will click. He uses the AQS by performing a measurement on the atom in the optimum basis

\[
\{(e^{i\pi/8}|1\rangle + e^{-i\pi/8}|0\rangle)/\sqrt{2}, (e^{-i\pi/8}|1\rangle + e^{i\pi/8}|0\rangle)/\sqrt{2}\},
\]

Alice and Bob repeat this procedure many times and estimate \( I(W: M) \).
In the WPGG, Alice randomly adjusts the PS such that \( \phi = 3\pi/4 \) or \( \phi = 7\pi/4 \) and runs the prepared setup—she is playing with the cleverest choice of \( \phi_0 \). Bob should make a guess about Alice’s choice. He sees which detector has clicked and measures the energy level of the atom, i.e. in the basis \( \{|0\rangle, |1\rangle\} \). They repeat this procedure again and again and estimate 
\[
H(\Phi|D) - H(\Phi|D,E).
\]

The two amounts of information estimated on the basis on the results of these guessing games will be approximately equal.

5. Discussion

In the WPGG, Alice does her best to make the game hard. In both games, Bob does his best to win the games. We should keep in mind that their strategies are based on maximization or minimization of \( H(\Phi|D,E) \). Nevertheless, in the cases that we examined in section 3, if their strategies were based on Bob’s mean error probability, they would have the same choices for the value of \( \phi_0 \) and the bases of the measurements [35, 36].

The problem of accessible information has also been solved for some other special situations including that of discriminating between two mixed states of a qubit that have the same determinant [31, 37]. This allows us to investigate the validity of (12) in some special examples where Bob should discriminate between two mixed states, e.g. when the initial state of the single quantum or the AQS is not pure. Based on our results in section 3 and such examples, we conjecture that relation (12) holds for the general case of all binary interferometers—such interferometers have just two interfering paths [16]—with symmetric beam merging.

6. Conclusions

By using Shannon entropy, we set the measures of which-phase and which-way information that are achievable through the AQS of two-path interferometers with symmetric beam merging. We investigated their relationship for the situations in which the bipartite system of the single quantum and the AQS was prepared in a pure state and the interaction between the two parts was unitary. It was shown that the amount of which-phase information that can be obtained is less than or equal to the amount of which-way information. For the symmetric case, the equality holds—this is the case for which there exists a sharp trade-off between distinguishability and visibility. The measurement that we used for extracting the which-phase information erases all the which-way information and vice versa. We also proposed a setup for experimental verification of the results; it is feasible with today’s technology.

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Appendix A. The explicit form of \( I_A(r,p) \) and the measurement used for extracting the information

In this appendix, we review some results of [31]. The accessible information about two pure quantum states with density matrices \( \rho^{(1)} = |\psi^{(1)}\rangle\langle\psi^{(1)}| \) and \( \rho^{(2)} = |\psi^{(2)}\rangle\langle\psi^{(2)}| \), which occur
respectively with probabilities $p^{(1)} = p$ and $p^{(2)} = 1 - p$, is obtained by a projective measurement in the basis \{\ket{\varphi^{(1)}}, \ket{\varphi^{(2)}}\}; in this basis,
\[ p \rho^{(1)}_{nn'} = (1 - p) \rho^{(1)}_{nn'} \quad \text{for} \ n \neq n'. \] (A.1)
So, if we write
\[ \rho^{(j)} = I + s^{(j)} \cdot \sigma \quad j = 1, 2, \] (A.2)
we have
\[ p(s^{(1)}_z + i s^{(1)}_y) = (1 - p)(s^{(2)}_z + i s^{(2)}_y). \] (A.3)
We are allowed to consider \( r = \langle \psi^{(1)} | \psi^{(2)} \rangle \) as a real non-negative number and set \( s^{(1)}_z = s^{(2)}_x = 0 \). By considering \( \text{tr}(\rho^{(1)} \rho^{(2)}) = r^2 \) and the unity of Bloch vectors \( s^{(1)} \) and \( s^{(2)} \), it is straightforward to find \( s^{(j)}_z \) and \( s^{(j)}_x \) for \( j = 1, 2 \).

In order to find the measurement basis for \( p = 0.5 \), we note that for this value of \( p \)
\[ s^{(1)}_z = s^{(2)}_x = r, \quad s^{(1)}_x = -s^{(2)}_y = \sqrt{1 - r^2}. \] (A.4)
So we can write
\[ \ket{\psi^{(1)}} = \cos \frac{\chi}{2} \ket{\varphi^{(1)}} + \sin \frac{\chi}{2} \ket{\varphi^{(2)}}, \]
\[ \ket{\psi^{(2)}} = \sin \frac{\chi}{2} \ket{\varphi^{(1)}} + \cos \frac{\chi}{2} \ket{\varphi^{(2)}} \] (A.5)
where \( \chi \) equals \( \arcsin(r) \). For an arbitrary \( p \), we have
\[ \ket{\psi^{(j)}} = \cos \frac{\theta^{(j)}}{2} \ket{\varphi^{(1)}} + \sin \frac{\theta^{(j)}}{2} \ket{\varphi^{(2)}}, \]
\[ \theta^{(j)} = \arccos(s^{(j)}_z), \quad j = 1, 2. \] (A.6)

In order to find the value of accessible information, we note that
\[ I_A(r, p) = -\sum_{n=1}^{2} \rho_{nn} \log_2 \rho_{nn} + \sum_{j=1}^{2} \sum_{n=1}^{2} p^{(j)} \rho^{(j)}_{nn} \log_2 \rho^{(j)}_{nn} \] (A.7)
where \( \rho = p \rho^{(1)} + (1 - p) \rho^{(2)} \). By defining \( C = \sqrt{1 - 4p(1 - p)r^2} \) and using the values of \( s^{(1)}_z \) and \( s^{(2)}_x \), it can be shown that
\[ I_A(r, p) = (1/2C) \left\{ p \left[ (C + 1 - 2(1 - p)r^2) \log_2 (C + 1 - 2(1 - p)r^2) \right. \right. \]
\[ + (C - 1 + 2(1 - p)r^2) \log_2 (C - 1 + 2(1 - p)r^2) \left. \right] \]
\[ + (1 - p) \left[ (C - 1 + 2pr^2) \log_2 (C - 1 + 2pr^2) \right. \right. \]
\[ + (C + 1 - 2pr^2) \log_2 (C + 1 - 2pr^2) \left. \right] \]
\[ - (C + 1 - 2p) \log_2 (C + 1 - 2p) - (C - 1 + 2p) \log_2 (C - 1 + 2p) \right\}. \] (A.8)
Appendix B. The AQS of the proposed setup

In order to describe the AQS, let us consider an atom with three levels \{\ket{0}, \ket{1}, \ket{e}\} trapped in a cavity. The transition frequency between \ket{1} and \ket{e} equals the resonance frequency of the cavity \(f_0\) but the transition frequencies between \ket{0} and \ket{1} and between \ket{0} and \ket{e} are far from it. When the atom’s state is in the subspace spanned by \ket{1} and \ket{e}, strong coupling between the atom and the cavity leads to mode splitting; that is to say, the system finds entangled energy levels, and the transition frequency between these new levels is different from \(f_0\). Now, we make the cavity’s left reflector have a little leakage and let the system interact with a (circularly polarized) ultra-narrowband single photon coming from the left side. What happens depends on the atom’s initial state and the photon’s frequency.

To be accurate, we use the relation between input and output operators of this system derived by [33]:

\[
\hat{a}_{\text{out}} \approx \frac{i\Delta - \kappa/2}{i\Delta + \kappa/2} \hat{a}_{\text{in}}
\]

where \(\kappa\) is the cavity decay rate and

\[
\Delta = 2\pi(f_p - f_s);
\]

\(f_p\) is the frequency of the single photon and \(f_s\) is the resonance frequency of the system that the photon faces. If the atom is initially prepared in \ket{0}, it is not coupled with the cavity; the photon faces just the cavity and \(f_s = f_0\). If the atom is initially prepared in the subspace spanned by \ket{1} and \ket{e}, it is strongly coupled with the cavity; \(f_s\) equals the transition frequency between the entangled energy levels of the atom and the cavity.

We are curious about the situation in which \(f_p\) is around \(f_0\). When the atom is prepared in \ket{0}, \(\Delta\) is comparable with \(\kappa\) and the photon experiences a phase shift of \(\eta \approx \pi - 2 \arctan(2\Delta/\kappa)\) by being reflected (for \(\Delta \geq 0\)). But when the atom is prepared in the subspace spanned by \ket{1} and \ket{e}, \(\Delta \gg \kappa\) and the photon does not experience a phase shift after being reflected. The atomic state \((\ket{1} + \ket{0})/\sqrt{2}\) evolves into \((\ket{1} + e^{i\eta}\ket{0})/\sqrt{2}\) by the interaction.

ORCID iDs

Soroush Khademi \(\oplus\) https://orcid.org/0000-0002-0871-3342

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