Evolution of clustered supernovae

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\textbf{ABSTRACT}

We study the merging and evolution of isolated supernovae (SNe) remnants in a stellar cluster into a collective superbubble, with the help of 3-D hydrodynamic simulations. We particularly focus on the transition stage when the isolated SNe remnants gradually combine to form a superbubble. We find that when the SN rate is high ($\nu_{\text{sn}} \sim 10^{-9}$ pc$^{-3}$ yr$^{-1}$), the merging phase lasts for $\sim 10^4$ yr, for $n = 1\text{--}10$ cm$^{-3}$, and the merging phase lasts for a longer time ($\sim 0.1$ Myr or more) for lower SN rates ($\nu_{\text{sn}} \lesssim 10^{-10}$ pc$^{-3}$ yr$^{-1}$). During this transition phase, the growing superbubble is filled with dense and cool fragments of shells and most of the energy is radiated away during this merging process. After passing through the intermediate phase, the superbubble eventually settles on to a new power-law wind asymptote that is smaller than estimated in a continuous wind model. This results in a significant (more than several times) underestimation of the mechanical luminosity needed to feed the bubble. We determine the X-ray and H$\alpha$ surface brightnesses as functions of time for such merging SNe in a stellar cluster and find that clusters with high SN rate shine predominantly in soft X-rays and H$\alpha$. In particular, a low value of the volume averaged H$\alpha$ to H$\beta$ ratio and its large spread can be a good indicator of the transition phase of merging SNe.

\textbf{Key words:} galaxies: ISM -- ISM: bubbles -- shock waves -- supernova remnants

\section{1 INTRODUCTION}

SNe (Supernovae) explosions and their remnants are believed to play a crucial role in shaping the evolution of galaxies. It became clear from the seminal paper by McKee & Ostriker (1977) that SN remnants may stay isolated during only a limited period in the beginning, and afterwards they merge with neighbouring remnants to form a percolating network. Depending on the environment, such merged SNe remnants can build giant supershells (as seen, e.g., in Holmberg galaxies Walter & Brinks 1999, Stewart & Walter 2000, Egorov et al. 2014, 2017), and large scale galactic outflows, called galactic winds. Galactic winds in turn act to enrich the Universe with heavy elements.

Observations of edge-on galaxies have demonstrated that most spiral galaxies have 5 to 10 kpc scale gaseous haloes, indicating activity of local outflows in the underlying disks. More recently, observational evidences of existence of huge – up to 150 to 200 kpc – metal bearing circumgalactic gaseous coronae have also appeared. Such scales suggest much more powerful outflows – galactic winds – to feed the coronae. Observations of absorptions in metal lines in the intergalactic medium at high redshifts ($z = 2\text{--}6$) show the existence of highly enriched – up to $\sim Z_{\odot}$ – gas. Typical intergalactic scales imply that galaxies develop a powerful stellar feedback process in the form of galactic winds that are able to carry metals through over the intergalactic space.

When the dynamical effects from collective SNe are studied in detail, a commonly practice is to introduce the concept of a mechanical luminosity $L = \tau_{\text{sn}}^{-1}E_{\text{sn}}$, where $\tau_{\text{sn}}$ is the time between successive SNe, and $E_{\text{sn}}$ is the typical mechanical energy released in a supernova. This assumption implicitly requires that explosion energy from different supernovae add up to produce a collective effect. This in turn means that the remnants from the subsequent supernovae overlap before becoming radiative (Nath & Shchekinov 2013). Recent calculations have found that when this condition is not fulfilled, the efficiency of SNe energy to convert into the collective action drops by roughly a factor of 10 (Vasiliev et al. 2014, Yadav et al. 2017).

In general, the dynamics of a sequence of concerted SNe can be understood in terms of the characteristic time scales relevant for an isolated SN remnant and their inter-
play with the time between sequential explosions. These are basically: 1) the time when the Sedov-Taylor (adiabatic) stage gives way to the radiative stage, 2) when radiative losses significantly decrease the remnant energy in one dynamical time, and 3) the time when the remnant velocity falls below the sound speed in ambient medium. The first time scale is marked by the shock velocity dropping roughly below $c_b \sim 100 \text{ km s}^{-1}$ and the remnant expanding further with decreasing energy as $E \propto R^{-2}$. The porosity of the remnants whose energy has decreased to half of the explosion energy $E_{\text{sn}}$ is estimated to be $P_{1/2} \sim 2 \times 10^{10} \nu_{\text{sn}}(E_{\text{sn}}/10^{51} \text{ erg}) \eta_w$ where $\nu_{\text{sn}}$ [pc$^{-3}$ yr$^{-1}$] is the supernova rate density (Nath & Shechkinov 2013). For a typical galactic environment, one has $P_{1/2} \approx 1$. The subsequent snowplough expansion of a radiative remnant continues until $u_{\text{shell}} \sim c_s$, and the corresponding porosity $P_{\text{shell}} \sim P_{1/2}(c_b/c_s) \sim 10 - 100$ for warm ($T \sim 10^4 \text{ K}$) and cold ($T \sim 10^2 \text{ K}$) gas. This shows that the probability for supernova to occur within a remnant formed by a previously exploded SN is rather large in typical galactic conditions, particularly during radiative stages. Thus an immediate consequence of this estimate is that the dynamical state and morphology of the interstellar medium in galaxies are mostly governed by supernova explosions as first recognized by McKee & Ostriker (1977) (see also more recent discussion by Vasiliev et al. 2015; Li et al. 2015).

It is therefore clear that simplified models of giant expanding supershells or galactic winds based on the assumption a single spherical bubble either from a central wind source with mechanical energy $L \sim n_{\text{sn}}^{-1} E_{\text{sn}}$ (Weizs et al. 2011; Egorov et al. 2014), or an instant explosion with energy $E \sim N_{\text{OB}} E_{\text{sn}}$ (Walter & Brinks 1999; Ott et al. 2001; Simpson et al. 2003; Weizs et al. 2009) may not be realistic. They may produce inconsistencies between the underlying stellar population and estimated energetics of an expanding flow. In order to correctly evaluate the limits of such inconsistencies, here we perform a 3D hydrodynamical study of the transition from single isolated SNe explosions to merging remnants and the formation of a collective large scale outflow, focusing mainly on observables accompanying the intermediate transitional stage.

The paper is organized as follows. In Section 2 we describe the details of the model. In Section 3 we consider two simple models: an ensemble of isolated SNe and multiple SNe exploded from the same point. In Section 4 we present the dynamics of clustered SNe and in Section 5 we describe possible observational consequences. Section 6 summarizes the results.

2 MODEL DESCRIPTION AND NUMERICAL SETUP

We carry out 3-D hydrodynamic simulations (Cartesian geometry) of multiple SNe explosions clustered in space. SNe are distributed uniformly and randomly inside a region with a fixed cluster radius $r_c$. Simulations are performed for a set of $r_c$, ranging from 30 to 90 pc for ambient gas density $n = 1 \text{ cm}^{-3}$, and up to 120 pc for $n = 10 \text{ cm}^{-3}$; as a fiducial model we assumed the density rate of SNe $\nu_{\text{sn}} = 10^{-9} (30 \text{ pc}/r_c)^3 \text{ pc}^{-3} \text{ yr}^{-1}$, which corresponds to one SN per $10^4 \text{ yr}$ in the cluster. We also ran models with the rate of one SN in $10^5 \text{ yr}$. We inject the mass and energy of each SN in a region of radius $r_0 = 2 \text{ pc}$. We assume that typical mass and energy of the SN are $10M_\odot$ and $10^{51} \text{ erg}$. The energy is injected in thermal form. The ambient gas density ranges in $1 - 10^3 \text{ cm}^{-3}$, while temperature is $10^4 \text{ K}$; gas metallicity is kept constant and equal to the solar value within the whole computational domain.

Recently Yadav et al. (2017) considered a similar model for a dynamical structure growing under the action of clustered multiple supernovae. They mostly focused on the study of dynamical features of the growing structure and related thermodynamic variables, such as the overpressure in the remnant, evolution of the fraction of SNe injected energy retained in thermal and kinetic energies, the fraction of gas removed from the cluster, etc. A major difference between their work and ours is connected to the regime of energy injection into the computational domain. In our case we inject thermal energy into a region of radius $r_0 = 2 \text{ pc}$ homogeneously, and the time lag between the subsequent instantaneous SNe was chosen to lie between $10^3$ to $10^5 \text{ yr}$. In contrast, Yadav et al. (2017) injected the energy smoothly over a sphere of radius $r_{\text{SN}} = 5 \text{ pc}$. The smoothing procedure is described by the core $\propto \exp(-[t - t_0]/\tau_{\text{inj}}^2) \times \exp(-[x - x_0]^2/r_{\text{SN}}^2)$ with $\tau_{\text{inj}} = \delta t_{\text{SN}}/10$, where

$$\delta t_{\text{SN}} = \frac{\tau_{\text{inj}}}{N_{\text{OB}}}$$

$\tau_{\text{inj}} = 30 \text{ Myr}$ is the lifetime of an OB-association, and $N_{\text{OB}}$ is the number of massive stars (supernovae progenitors) in the association. For their fiducial model with $N_{\text{OB}} = 100$, this means that the energy injection period is $\Delta t \sim \delta t_{\text{inj}} \sim 3 \times 10^4 \text{ yr}$, factor of 3 larger than the time lag between the subsequent SNe in our fiducial model. Moreover, the number of SNe exploded within $\Delta t \sim \delta t_{\text{inj}}$ in a volume occupied by one remnant is larger than unity. From this point of view the fiducial model of Yadav et al. (2017) corresponds to a continuous energy input, while in our fiducial model the injection is discrete. Asymptotic dynamics and kinematics on times $t > 1 \text{ Myr}$ in our simulations is similar.

Standard simulations are performed with a physical cell size of 1 pc in the computational domain of $300^3 \text{ pc}^3$. Several simulations are run with a box size of $800^3 \text{ pc}^3$, with a cell size of 0.37 pc to resolve the free expansion phase, as shown in Figure A1. The ejecta avoids artificial overcooling much longer than the free expansion phase (as discussed in detail in Sharma et al. 2014 and Yadav et al. 2017), as the cooling time $t_c \approx 10^{14} \text{ s}$ is much longer than the free-expansion time scale $t_f \geq 3 \times 10^{10} \text{ s}$.

The code is based on the unsplit total variation diminishing (TVD) approach that provides high-resolution capturing of shocks and prevents unphysical oscillations. We have implemented the Monotonic Upstream-Centered Scheme for Conservation Laws (MUSCL)-Hancock scheme and the Haarten-Lax-van Leer-Contact (HLLC) method (see e.g. Tord (1999) as approximate Riemann solver. This code has successfully passed the whole set of tests proposed in Klingenberg et al. 2007). In order to check convergence, we performed runs with half-size cells.

Simulations are run with radiative cooling processes with a tabulated non-equilibrium cooling curve fitting the calculated one (Vasiliev 2011, 2013). The is obtained for a
gas cooling isobarically from $10^8$ down to 10 K. Our choice of isobaric cooling rate is reasonable because of the fact that the typical sound crossing time (given the typical temperatures) of a resolution element in our simulation is of order $\sim 1000$ yr, much shorter than the relevant time scales in the problem.

The non-equilibrium calculation (Vasiliev 2011, 2013) includes the kinetics of all ionization states of H, He, C, N, O, Ne, Mg, Si, Fe, as well as kinetics of molecular hydrogen at $T < 10^4$ K. We perform non-equilibrium calculations of the hydrogen ionization fraction and emissivities in Balmer lines and implement them into the hydrodynamic step. The emissivities in Balmer lines have been calculated with making use the CLOUDY package as a subroutine (Ferland et al. 1998). The ionization fraction is utilized for calculating emission measure and then X-ray spectral intensity (e.g., Kaplan & Pikel’ner 1979). For this purpose the approximation for the Gaunt factor is taken from (Draine 2011). The heating rate is assumed to be constant, with a value chosen such as to stabilize the radiative cooling of the ambient gas at $T = 10^4$ K. This stabilization vanishes when the gas density and temperature deviate from the equilibrium state by less than 1%.

In order to localize the boundaries of isolated SNe remnants or the collective superbubble, we applied two techniques: the first (used as the fiducial) is based on the fact that any motion in the computational domain is being driven by a SN, i.e. a gas parcel is identified as a part of a bubble when its velocity squared is greater than a given small value. Even though generally this value should be zero in an unperturbed ambient medium, numerical errors introduce perturbations and we set this threshold 1 km s$^{-1}$. The second is based on the deviation of gas temperature from the temperature of unperturbed ambient gas, i.e. a gas parcel is located inside a bubble if its temperature is not equal to $T = 10^4$ K. In practice, the temperature of the ambient gas is kept within a narrow range around $10^4$ K. The last method can result in a loss of a fraction of cooling gas by a SN, i.e. a gas parcel is identified as a part of a bubble if any motion in the computational domain is being driven by a SN, i.e. a gas parcel is identified as a part of a bubble when its velocity squared is greater than a given small value. Even though generally this value should be zero in an unperturbed ambient medium, numerical errors introduce perturbations and we set this threshold 1 km s$^{-1}$. The second is based on the deviation of gas temperature from the temperature of unperturbed ambient gas, i.e. a gas parcel is located inside a bubble if its temperature is not equal to $T = 10^4$ K. In practice, the temperature of the ambient gas is kept within a narrow range around $10^4$ K. The last method can result in a loss of a fraction of cooling gas.

3 TOY MODELS

Before describing multiple SN explosions in a cluster we consider several simple models.

3.1 Isolated SN remnants

For an isolated SN at the very initial stages $t < t_f$, the exponent (that describes the evolution of the bubble radius in the form $R \propto t^n$) turns from the case of free expansion $\alpha \simeq 1$, through an intermediate ‘quasi-diffusive’ law with $\alpha \simeq 0.45$, (see, Appendix A), to Sedov-Taylor (ST) law $\alpha = 2/5$. Figure 1 presents the bubble radius from a single SN explosion with $E = 10^{53}$ erg (red symbols). In this Figure the expansion has already entered ST phase, and after roughly one radiative time $t_c \sim 10^5$ yr (estimated for a SN with $E = 10^{53}$ erg exploded in the medium with $n = 1$ cm$^{-3}$) the expansion converges to the momentum driven Oort stage with $\alpha \simeq 1/4$. The intermediate pressure-dominated stage with $\alpha = 2/7$ (McKee & Ostriker 1977; Blinnikov et al. 1982) continues for a short timescale $-t_{IP} \sim 0.05t_c$, close to the epoch $t \sim t_c$ (see Appendix B), and remains unresolved on Figure 1.

Let us consider an ensemble of isolated SNe of different ages. Supernovae explode with a constant rate of 1 SN per $\Delta t$, each passing through the standard sequence of phases: free expansion, adiabatic (Sedov-Taylor) expansion, and finally the radiative (snowplough) phase. After $k$ explosions the total volume occupied by all remnants is $V(t) = \sum_{i=1}^{k} t_i - (t - (i-1)\Delta t)$. For large $k$ the majority of SNe in the ensemble are at the radiative phase and $V(t) \propto t^{6/5-1}$, where the shell radius of a single SN at the radiative stage $r \sim t^{\alpha}$ with $\alpha = 1/4$ is assumed. It is seen that the expansion law asymptotically reaches the wind regime with the effective radius $R(t) = V^{1/3} \sim t^{0.6}$, nearly coincident with the radius of a wind driven bubble (Avedisova 1972; Castor et al. 1973). Throughout the paper, we will denote by effective radius the size defined as $R = V^{1/3}$.

Figure 1 depicts the effective radius of the “collective remnant” of 100 isolated SNe exploded with the rate of 1 SN per $10^4$ yr (green symbols) and the remnant radius from a single enhanced (with the energy $E = 10^{53}$ erg) SN explosion (red symbols) for comparison. Differences between the two cases are clearly seen: while multiple SNe exploding sequentially with a constant rate settle on to the wind mode, a single SN with an enhanced energy passes through the Sedov-Taylor to Oort stage and eventually occupies a lower volume.

3.2 Centred multiple SN explosions

Let us now consider another simple case – centred SNe explosions, i.e., multiple SNe exploding sequentially at the same
point. This commonly used model is considered reasonable since the typical size of a progenitor stellar cluster is much smaller than the size of the remnant produced by the explosion. A detailed description of such a remnant has been performed within a 1D approach (e.g., [Sharma et al. 2014]), which allowed for a high spatial resolution, homogeneously spread from the central region to the shell. However, hydrodynamic (Rayleigh-Taylor) and thermal instabilities can influence the dynamics of the shell evolution by enhancing radiation losses. Such an enhancement is mainly due to an increase of the remnant surface by growing "tongues" driven by Rayleigh-Taylor instability (Korolev et al. 2013). This motivates us to describe the centred SNe explosions in 3D.

Note though that the resolution we reach in our simulations (0.5 pc) is not quite sufficient for discriminate the clumps driven by thermal instability (the corresponding sizes are of $\sim 0.1$ pc), and may, in principle, suffer from numerical instabilities. This problem is well known for multi-dimensional simulations (e.g., Badjin et al. 2016).

It is worth noting that the time delay between subsequent explosions is an additional characteristic time scale in the problem and its ratio to the cooling time may be an important factor. Indeed, when the time gap between two subsequent explosions $\Delta t$ is shorter than the cooling time $t_c$, the energy supply into the shell can partly replenish the radiative losses and make the expansion more rapid, as seen in the left panel of Figure 2. However, as time elapses, radiation losses grow as $\propto t^2$. Ultimately the shell settles on to the wind regime, with radii larger in those cases when $\Delta t/t_c < 1$, as seen in Figure 2 where the effective bubble radii driven by collective SN explosions located in the centre are shown. The corresponding cooling time $t_c \approx kT/\Lambda n \approx 10^4 n^{-1} \text{yr}$ for the solar metallicity ($\Lambda(T \sim 10^8 \text{K}) \approx 10^{-22} \text{erg s}^{-1} \text{cm}^2$).

This figure shows the results for models with the rates 1 SN per $10^3$, $10^4$ and $10^5$ yr in the ISM of ambient density $n = 1 \text{ cm}^{-3}$ (left panel), and $n = 10 \text{ cm}^{-3}$ (right panel); radiative cooling times are $t_c \approx 10^4 \text{ yr}$ in the left panel and $t_c \sim 10^3 \text{ yr}$ in the right panel, depicted by red, green and blue symbols. It is readily seen that for a delay of $\Delta t = 10^3 \text{ yr}$, the shell radius initially grows as $\sim t^{0.57}$, close to the wind mode (Avedisova 1972, Caster et al. 1975). Initial shell dynamics in models with a longer delay shows a pressure-dominated mode $\sim t^{0.45}$ (see Appendix B), which asymptotically turns to the wind solution.

Ejecta from successive SN exploding into the already existing hot low-density bubble from earlier SNe expand without losing energy. Its initial dynamics is close to that of the free expansion solution, which evolves to the adiabatic mode (see Appendix A and Figure AX). Eventually, the ejecta merges with the shell produced by preceding explosions and transfer to it all their energy.

Figure 3 shows the time dependence of the total energy (left-side panels) and the remaining fraction of energy $f_E = E/E_{in}$ (right-side panels) in a remnant from a collective action of sequential SN centred explosions in two different environments: in a low ($n = 1 \text{ cm}^{-3}$, left panel) and high ($n = 10 \text{ cm}^{-3}$, right panel) density ambient gas. Several generic features are observed in Fig. 3 and from a comparison of Figs 2 and 4: i) a nearly order of magnitude decrease of the characteristic cooling time with the increase in ambient density, ii) during radiative stages, the remaining energy fraction decreases approximately as $\propto R^{-2}$ — much slower than that inferred from 1D simulations of a single SN remnant (Cox 1972), iii) remnants with less frequent SNe explosions enter the radiative stage, i.e. show decreasing energy fraction $f_E$, earlier than that with a higher SN rate, iv) in spite of the fact that at the beginning of the radiative regime the energy drop is larger for the remnants in a denser environment, the asymptotic behaviour of the remaining energy fraction $f_E$ flattens roughly as $R^{-1.8}$, and depends fairly weakly on the ambient density (nearly as $n^{0.2}$).

4 CLUSTERED SNE

Next we consider the effect of having the SNe explode at different (random) locations within a cluster of radius $r_c$. Figure 4 presents the evolution of density (left group of panels)
and temperature (right group of panels) in a bubble growing under the action of multiple SN explosions assembled in a cluster. The two models with a rate $\nu_{\text{sn}} = 10^{-9}$ pc$^{-3}$ yr$^{-1}$ (left column in the group) and $4 \times 10^{-11}$ pc$^{-3}$ yr$^{-1}$ (right column in the group) as seen in the plane $z = 150$ pc are shown. In the first case the SNe are spread in a volume of radius $r_c = 30$ pc, and in the second, over a volume of $r_c = 90$ pc. Throughout the rest of the paper we fix the time lag between subsequent SN explosions to be $\Delta t = 10^4$ yr, and connect the variations of the explosion rate $\nu_{\text{sn}}$ with variations of the cluster radius $r_c$. The accepted values of $\Delta t$ and $r_c$ reasonably fit real OB-associations.

In the case of higher SN rate, the collective bubble is already formed by the time $t = 5 \times 10^4$ yr (top panel), whereas for lower rate, individual remnants still continue to merge until $t = 5 \times 10^5$ yr (third panel). Therefore bubbles growing under collective explosions with a lower rate remain more irregular on longer time scales with dense walls and broken shells inside the bubble. These fragmentary structures represent either parts of the merged shells or shells from older explosions broken by younger ones. These features are transients with characteristic time scale $\Delta t \lesssim (\nu_{\text{sn}} r_c^2)^{-1} \sim 10^4$ yr. At the same time, signatures of separate SNe explosions within the bubble can be clearly found in the form of nearly spherical rings with higher density and temperature, particularly for the higher rate of SNe. A fraction of the gas inside these remnants from individual explosions becomes cool due to adiabatic expansion.

It is readily seen that the overall dynamics can be described qualitatively in terms of the total number of SNe occurred in the cluster volume with $r_c$ within the cooling time $t_c$, $N_{\text{sn,c}} = 4 \pi \nu_{\text{sn}} r_c^2 t_c / 3$; the lower the number $N_{\text{sn,c}}$, the more non-homogeneous and fragmentary is the density and temperature distribution within the superbubble and the larger fraction of energy is lost radiatively (see discussion in the next section).

Figure 4 presents the evolution of the effective radius $R(t) = V^{1/3}$ of the (super)bubble from multiple clustered SN for a set of explosion rates. In order to extract the volume occupied by remnants we fiducially applied the following technique: a single cell from the whole computational zone was counted as belonging to the remnant provided its velocity $v_i = \sqrt{v_{i,x}^2 + v_{i,y}^2 + v_{i,z}^2}$ exceeds 1 km s$^{-1}$ – the corresponding results are shown by thick lines on Figure 4.
As an alternative, we also applied a technique based on the temperature differences: a cell was assigned to the superbubble if gas temperature within the cell deviates from the unperturbed ambient temperature $T = 10^4$ K by $\Delta T \lesssim 100$ K — the corresponding effective radii are shown by thin lines in the right panel on Figure 4.

For the highest rate $\nu_\text{sn} = 10^{-9}$ pc$^{-3}$ yr$^{-1}$ (red lines in left panel of Figure 5), SN remnants merge at very early stages (see upper left panel in Figure 4), such that in the low-density environment, $n = 1$ cm$^{-3}$, several SNe explode in a single remnant before it enters the radiative cooling stage. As a result, the collective bubble transforms into a wind-like regime as manifested on Figure 5 at $t \sim 3 \times 10^4$ yr as $r(t) \sim a_1 t^{0.56}$ (with the exponent 0.56 reasonably close to the standard exponent 0.6 for a wind regime). However, quite soon radiative losses come into play and the expansion slows down to a slower scaling of $t^{0.45}$ and eventually sets again on to the same power-law $\sim a_2 t^{0.56}$ with $a_2 < a_1$. The factor $a_2$ is immediately connected to the supernova explosion energy and their rate as $a_2 \simeq (L/\rho)^{1/2}$ with the effective mechanical luminosity $L = 0.3 E/\Delta t$ (see Appendix C).

For lower rates $\nu_\text{sn} \leq 1.2 \times 10^{-10}$ pc$^{-3}$ yr$^{-1}$ (green and blue lines in the left panel of Figure 5), several initial SNe expand in isolation with the effective radius $r \sim t^{0.56}$ as described in Section 2.1. After $t \sim 3 \times 10^4$ yr, the remnant turns to the radiative stage and evolves further as in the previous case of $\nu_\text{sn} = 10^{-9}$ pc$^{-3}$ yr$^{-1}$. It is worth pointing out a seemingly unusual increase in the effective radius with decreasing explosion rate as seen in Figure 5. It can be understood in terms of the superposition of adiabatic remnants in limiting cases: in the low rate end, the total volume is assembled from several nearly isolated remnants $\langle R(t) \rangle \sim \left[ \sum_i R(t - t_i)^{1/3} \right]^{1/3} \propto t^{2/5+1/3}$, while in the high rate end, different explosions add energy approximately into the same remnant $\langle R(t) \rangle \sim \left[ \sum_i E_i(t - t_i)^{2/3} \right]^{1/3} \propto t^{1/2}$. However, at later stages when the remnants fully merge, the differences between the time dependences of effective radii vanish.

The evolution of a growing superbubble in gas with higher density is mostly similar to that described above with only minor deviations: they enter the radiative regime very early at around $3 \times 10^3$ yr, such that at initial stages the
5 OBSERVATIONAL CONSEQUENCES

5.1 Kinematics

Radius of a wind blown bubble evolves as \( r \simeq 0.76(L/\rho)^{1/3}t^{2/3} \), where \( L = M v^2/2 \) is the mechanical luminosity (Castor et al. 1975). In simulations of star-forming galaxies and the interpretation of observational data, the mechanical luminosity is commonly assumed to be the ratio of the total energy of all SNe to the duration of a starburst: \( L = NE_{SN}/t \) (e.g., Mac Low & McCray 1988). In case of a constant SN rate, \( L = E_{SN}/\Delta t \), where \( E_{SN} \) is the energy of a single SN, \( \Delta t \) is the time delay between successive SNe. Accordingly, the radius of a superbubble from this SNe ensemble can be expressed as

\[
r \simeq 160 \left( \frac{10^4 \text{yr} \ 1 \text{cm}^{-3} \ n^{-1/5}}{\Delta t^{1/3} \ n^{-3/5}} \right) \left( \frac{t}{10^6 \text{yr}} \right)^{3/5} \text{pc}.
\]

(2)

It coincides with the previously described numerical models with \( \Delta t = 10^3 \text{ yr} \) during the early evolution, before the radiative stage begins at \( t \sim 1.5 \times 10^5 \text{ yr} \) for \( n = 1 \text{ cm}^{-3} \) and \( t \sim 3 \times 10^4 \text{ yr} \) for \( n = 10 \text{ cm}^{-3} \) (Figure 2).

After passing through the intermediate phase, the superbubble eventually settles on to a new power-law wind asymptote but with the radius being a factor of 1.4 (\( n = 1 \text{ cm}^{-3} \)) to 2 (\( n = 10 \text{ cm}^{-3} \)) smaller than given by (2). This leads to an underestimate of the mechanical luminosity \( L \) by about \( 1.4^3 \simeq 5.4 \) to \( 2^3 = 32 \) when \( L \) is estimated from the directly observable quantities: radius \( R \), expansion velocity \( v \) and the shell mass \( M \).

5.2 Emission in Balmer lines

A useful diagnostic of the superbubble evolution is commonly thought to come from line emission of different elements that probe the physical conditions in gas. Giant holes in the gas distribution in galactic disks of several nearby dwarf galaxies have been studied with the help of H\( \alpha \) emission from the gas ionized by Ly-continuum photons produced by the underlying stellar population (e.g., Martinez-Delgado et al. 2007; Moiseev & Lozinskaya 2012; Egorov et al. 2014, 2017). In this environment the spatial H\( \alpha \) distribution traces the regions with a higher emission measure \( EM = \int n_e^2 dl \), and thus reflects variations of gas density and radiation field under an implicit assumption that photoionized gas is kept at \( T \simeq 10^4 \text{ K} \). Under such conditions the ratio of H\( \alpha \) to H\( \beta \) intensities is practically fixed at 2.86 (e.g., Draine 2011) provided the dust extinction is weak.
Figure 6. The total energy (left column) and ratio of the total energy to the injected one (right column) of the collective bubble formed by multiple SN explosions in a cluster with volume rate \(10^{-9}, 1.2 \times 10^{-10}, 4 \times 10^{-11}\) pc\(^{-3}\) yr\(^{-1}\) depicted by red, green and blue lines in both panels and \(1.5 \times 10^{-11}\) pc\(^{-3}\) yr\(^{-1}\) at bottom panel by purple lines. The ambient density equals \(1\) cm\(^{-3}\) (upper panels) and \(10\) cm\(^{-3}\) (lower panels). For illustration thermal (dashed line) and kinetic (dash-dotted line) energies (left panels) and their fractions (right panels) are shown for the rate \(4 \times 10^{-11}\) pc\(^{-3}\) yr\(^{-1}\).

The situation changes in the case when ionization is dominated by shock waves randomly impinging on a given gas element (see e.g., Raga et al. [2013]). Figure 8 presents the predicted H\(\alpha\) brightness maps for superbubbles with SN rate \(10^{-9}\) pc\(^{-3}\) yr\(^{-1}\) (left column) and \(4 \times 10^{-11}\) pc\(^{-3}\) yr\(^{-1}\) (right column) at epochs \(2.5 \times 10^5, 5 \times 10^5, 10^6\) yr (top to bottom); the maps correspond to the three bottom panels in Figure 4. It is worth noting here that in contrast with Raga et al. [2013] we calculated the emission in H\(\alpha\) and H\(\beta\) lines under the conditions of a non-steady radiatively cooling environment as described above in Section 2. At earlier times, H\(\alpha\) maps are smoother for the higher SN rate, while those for the lower SN rate reveal copious bright knots and walls inside. It is clear that more frequent SNe sweep the gas from the central part of the bubble outward during the initial explosion episodes, so that every subsequent SN explodes in a hot low-density environment. Less frequent SNe approach a similar behaviour at later stages when many isolated remnants merge. Thus, in this case the variations of H\(\alpha\) emission are due to variations in density and to some extent in temperature.

This circumstance is clearly seen in the distribution function of the H\(\alpha\) brightness on Figure 9; the distribution is normalized to the total number of cells inside the remnant. The distribution for higher SNe rate is slightly narrower, and over time, shifts towards a less bright state than that for the lower rate: if at \(t = 0.25\) Myr the distribution for the bubble with faster SNe peaks at higher brightness \(\approx 10^{-14.7}\) erg cm\(^{-2}\) s\(^{-1}\) arcsec\(^{-2}\) than for the slower SNe explosions at \(\approx 10^{-15.3}\) erg cm\(^{-2}\) s\(^{-1}\) arcsec\(^{-2}\), while at \(t = 1\) Myr they both peak at \(\approx 10^{-15.4}\) erg cm\(^{-2}\) s\(^{-1}\) arcsec\(^{-2}\). Thus, the H\(\alpha\) maps in bubble from more frequent SNe explosions “cool” faster than those with a lower SN rate. It stems from the fact that frequent SNe explosions empties the interior sweeping gas into the shell where the electrons recombine faster. Note that the distribution functions for H\(\beta\) brightness are practically identical to those for H\(\alpha\) though slightly shifted towards lower brightnesses. In general, the supershells produced by merging remnants from isolated
SNe are the brightest regions of the superbubbles. However, for low SN rates, a growing superbubble passes through the intermediate stages when the most prominent features of the merging process – namely, the fragments of dense overlapping shells and dense clumps – fill the interior and manifest themselves as the brightest spots, as seen in the maps on Figure 8 and in the histogram on Figure 9 as a wider distribution.

However, in practice, the spatial variations of the Hα brightness and their pattern similar to those shown on Figure 8 can hardly be distinguished observationally because of insufficient resolution: the pixel size in our maps is 1 pc, whereas even nearby galaxies are resolved at the best with 5 pc (e.g., Martinez-Delgado et al, 2007; Moiseev & Lozinskaya 2012). In order to compare the simulated Hα maps to those obtained observationally one would...
have to degrade the resolution \cite{Vasiliev_2015}, which would diminish the differences in the patterns of Hα brightness between the models shown in the left and in the right panels of Figure 8.

The ratio of Hα to Hβ intensities would be an additional indicator of the evolutionary status of the superbubble. In the conditions prevailing in the cooling gas in individual remnants and their shells, the fractional ionization stays nearly frozen because recombination is slower than cooling. Under such conditions, even rather cold (e.g., \( T \lesssim 10^2 \) K) regions can be luminous in Hα and Hβ. The ratio Hα/Hβ depends on temperature and under conditions of high fractional ionization (\( x > 0.5 \)), it can vary by factor of \( \sim 1.5 - 2.5 \) in the temperature range between \( 10^3 \) K to \( 10^{5.5} \) K \cite{Raga_2015}, and thus can be measured observationally. It is worth noting that in all the models considered here with \( \nu_{\text{sn}} = (4 \times 10^{-11} - 10^{-9}) \) pc\(^{-3}\) yr\(^{-1}\), dust extinction along the remnant radius is \( A_v \sim (0.1 - 0.2) \times n^{0.7} (r_{100} t_{\text{Myr}})^{0.3/5} \), where \( r_{100} = r_c/(100 \text{ pc}) \), \( t_{\text{Myr}} \) is time elapsed since the first SN in the cluster. This means that for superbubbles grown under a “normal” galactic environment with ambient density \( n \sim 1 \) cm\(^{-3}\), dust extinction cannot affect variations of Hα/Hβ due to evolutionary effects.

Radiatively cooling gas in the growing bubble stays highly ionized (\( x_c \gtrsim 1 \)), except in dense parts of the external shell and fragments of shells from overlapping remnants in the bubble. At the same time, temperature can considerably vary from cell to cell resulting in the variation of intensities of Hα and Hβ lines throughout the whole bubble. It differs from the case of a predominantly photoionized gas where the ratio is practically fixed at 2.86.

Figure 10 shows the distribution function of Hα to Hβ ratio for the two superbubbles driven by SNe with the higher (left panel) and the lower (right panel) rates over the time range from \( 10^4 \) to \( 10^6 \) yr. The colour bar in the right shows the logarithm of the fraction of volume occupied, such that the distribution function varies from \( f \sim 3 \times 10^{-3} \) to \( \sim 0.3 \), peaking at Hα/Hβ = 3.6 practically independent of the SN rate. It is readily seen, that the ratio Hα to Hβ, for the interior of superbubbles differs from the “standard” value of photoionized gas Hα/Hβ = 2.86 at \( T \sim 10^4 \) K. This difference, however, is small to be distinguished observationally. However, the spread of the Hα/Hβ ratio can reach upto factor 1.5, and this result may be important in distinguishing photoionized regions of a superbubble from those where ionization is predominantly collisional due to multiple randomly impinging shock waves. As mentioned above, the dust extinction in this context of superbubble dynamichas small effects on the variations of the Hα/Hβ ratio. For instance, in Holmberg II the total extinction in Hα emitting gas is only \( A_v \sim 0.1 \), assuming solar metallicity (e.g., \cite{Egorov_2017}).

5.3 X-ray maps

A simple inspection of Figure 11 confirms that a considerable fraction of the remnant from collective SNe is occupied by hot gas. Thus one can expect not only optical emission in hydrogen Balmer recombination lines from the cold remnant shell, but also X-ray photons from its hot interior. This issue is important because the gas density inside the remnants is low. It is clearly seen from Fig. 7 where for both models with the lower and higher SN rates, the gas density inside the growing bubble at late stages (\( t > 1 \) Myr) is mostly lower than \( n \lesssim 0.01 \) cm\(^{-3}\), such that pressure within it is a few \( \times 10^4 \) K cm\(^{-3}\).

Figures 11 and 12 demonstrate the spatial distribution of surface X-ray brightness for the remnants for SN rate \( 10^{-9} \) pc\(^{-3}\) yr\(^{-1}\) from a cluster of \( r_c = 30 \) pc, and \( 4 \times 10^{-11} \) pc\(^{-3}\) yr\(^{-1}\) in a cluster with \( r_c = 90 \) pc, respectively. The columns from left to right depict surface brightness for different energy bands: 0.1 – 0.3, 0.7 – 1.2, 1.2 – 2.1 and 1.6 – 8.3 keV (as in \cite{Suchkov_1994}) – the first three bands approximately correspond to the ROSAT bands R1= 0.11 – 0.284, R6= 0.73 – 1.56, R7= 1.05 – 2.04 keV. The rows from top to bottom are for the elapsed times 0.25, 0.5 and 1 Myr. Characteristic features of the X-ray surface brightness distribution can be understood in terms of the density and temperature maps shown in Fig. 6. The brightness mostly follows the product of \( n^2 T^\beta \), where \( \beta \) varies...
from $\beta \sim 0$ at $T \sim 10^5$ K, to $\beta \sim -0.5$ at $T \sim 4 \times 10^5 - 10^6$ and to $\beta \sim 0.5$ at $T \gtrsim 10^7$ K. At $t \approx 0.25$ Myr, the remnant is already radiative and a dense and cold ($T \sim 10^4$ K) shell has formed. However, there is a thin region just interior to the shell, where the density is less than the shell but the temperature grows up to $T \sim 10^7$ K. This thin region provides an efficient cooling and emission in the lower energy bands. Thus, the upper left map represents emission from this thin layer with a very weak concentration towards the shell. Higher energy bands are less efficiently produced within the bubble, as the internal temperature is either too high ($T \gtrsim 10^7$ K) or too low ($T \lesssim 10^5$ K in the central domain), except the $1.6 - 8.3$ keV band photons being emitted from the quasi-spherical high-temperature interior (see, Fig. 4 for 0.25 Myr). Subsequent evolution clearly shows diminished X-ray emission not only in the higher energy bands, but in the soft band as well, which is the result of evacuation of gas from the interior towards the shell and its cooling.

Explosions with a lower SN rate surprisingly produce more photons in higher energy bands as seen from Fig. 12. This is basically connected with the fact that less frequent shock waves inside the bubble evacuate gas outward less efficiently, and leave a larger amount of compressed and relatively hot gas in the form of clumps. Eventually, even at $t = 1$ Myr some fraction of the bubble volume emits at high energies $E > 1.8$ keV.

A generic and most obvious feature of the X-ray brightness distribution functions seen in Figure 10 is that the superbubbles produced by more frequent SNe explosions have narrower distributions in all energy bands and the gas cools faster, while those from slower SNe explosions are wider in photon energy spread and cool slower. This conclusion looks consistent with a narrow density and temperature distributions of a highly compressed gas swept up mostly into a thin shell by frequent supernovae explosions, and with a more spread and less concentrated gas distribution (generally with lower densities and temperatures) shocked by less frequent supernovae explosions. In addition, in the first case (higher SN rate) the brightness distributions in the high energy bands are narrower than those at lower energies. Moreover, they appear to be squeezed in time. On the contrary, the distributions in the second case (with a lower SN rate) are nearly equally wide. Such a behaviour is consistent with the overall dynamics discussed above.

6 CONCLUSIONS

We have studied how isolated supernova remnants merge into a collective superbubble depending on the SN rate ambient gas density. We have found the following:

- The number of SNe exploded in one cooling time ($N_{sn,c}$) is an important factor governing the superbubble growth: the lower is the number, the larger is the fraction of energy lost radiatively and the smaller is the asymptotic radius of the superbubble.
- Superbubbles from clustered SNe explosions pass through a merging phase when individual SN remnants in-
teract and gradually combine into a collective superbubble. For a high SN rate $N_{\text{sn},c} > 1$ and a low density environment ($n = 1 - 10 \text{ cm}^{-3}$) the merging phase lasts for a short period ($\sim 10^4 \text{ yr}$), and turns to an intermediate expansion law with the effective radius of the shell $r \sim t^{0.45}$. After about 1 Myr the expansion turns to a steady quasi-wind regime with $r \sim t^{0.56}$.

- In OB-associations with a lower SN rate ($N_{\text{sn},c} < 1$) individual SN remnants merge after formation of dense radiative shells. In these conditions a growing superbubble is filled with dense and cool fragments of shells broken in the process of merging. Most of energy is radiated during the merging. Eventually the effective radius of the superbubble sets on to a wind-like asymptotic $r(t) \sim t^{0.56}$ on a time scale about 1 Myr.

- After passing through the intermediate phase, the superbubble eventually settles on to a new power-law wind asymptote with the radius being smaller than that estimated by using the wind driven bubble law. It results in a significant (factor of a few to one or two orders of magnitude, depending on the ambient density) underestimation of the mechanical luminosity needed to sustain the bubble.

- Superbubbles with frequent SNe are dominated by the dynamical effects of sweeping the gas outwards and thereby enhancing its cooling in dense external supershells. Bubbles produced by clusters with a lower SN rate, shine brighter in X-ray predominantly, and show a wider brightness distribution function.

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Figure 12. Same as in Fig. 11 for SN rate $4 \times 10^{-11} \text{ pc}^{-3} \text{ yr}^{-1}$ clustered within 90 pc radius.
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Figure 13. Distribution functions of the surface brightness in X-ray emission for the models shown in Figs. [and [ for the lower $4 \times 10^{-11}$ pc$^{-3}$ yr$^{-1}$ (blue thin lines) and the higher $10^{-9}$ pc$^{-3}$ yr$^{-1}$ (red thick lines) SN rates clustered within 90 pc and 30 pc, respectively, for 0.25, 0.5 and 1 Myr (right to left). Panels from the uppermost to the lowermost correspond to X-ray energy bands: 0.1 – 0.3, 0.7 – 1.2, 1.2 – 2.1 and 1.6 – 8.3 keV as shown by legends.

APPENDIX A: A SINGLE SN REMNANT IN A LOW-DENSITY ENVIRONMENT

In our simulations energy and mass of a SN are injected into a small volume and the gas density inside turns to be several orders of magnitude higher than the ambient density. Before the shell forms, the dense hot ball expands freely into a tenuous medium. This process can be described as gas expansion out of the central part of the cluster. All subsequent SNe thus expand into a very dilute and hot medium. We consider the mass of swept out gas becomes comparable to the shock gradually collects ambient gas, decelerates and when the shell forms, the dense hot ball expands freely into a tenuous medium. This process can be described as gas expansion into vacuum. The ball radius evolves as $r(r_0 + \frac{c_s t}{2})$, where $r_0$ is the initial radius, $c_s$ is sound velocity. We assume flat initial radial profiles of density and pressure in the injection region. In general, there is no self-similar solution for this problem (e.g., Zel’dovich & Raizer [1967]), besides only some particular initial radial profiles (Stanyukovich [1960]) which are not applicable in our case.

Linear growth of radius is a part of ejecta-dominated phase (e.g., Gull [1973], Truelove & McKee [1999]). The shock gradually collects ambient gas, decelerates and when the mass of swept out gas becomes comparable to the ejecta the expansion makes a transition to Sedov-Taylor phase (Truelove & McKee [1999]). The duration of the free-expansion phase depends on the ambient density.

Multiple SN explosions efficiently sweep ambient gas out of the central part of the cluster. All subsequent SNe thus expand into a very dilute and hot medium. We consider a single SN explosion into a hot ($T = 2 \times 10^5$ K) low-density 1 Strictly speaking, fluid dynamics approach violates in the interface separating vacuum and the bulk of gas. However, when gas dynamics far from the interfacial region is concerned the numerical recipes for solving the Riemann problem in presence of vacuum can be found (Ford [1999]).

[^1]: Strictly speaking, fluid dynamics approach violates in the interface separating vacuum and the bulk of gas. However, when gas dynamics far from the interfacial region is concerned the numerical recipes for solving the Riemann problem in presence of vacuum can be found (Ford [1999]).
Figure A1. The density profiles (left column) and the evolution of SN shell radius (right column). The number density of the medium is 0.02, 0.1 and 1 cm$^{-3}$ (from top to bottom).

\( (n_{\text{amb}} = 0.02, 0.1 \text{ and } 1 \text{ cm}^{-3}) \) medium. In order to understand the details of the initial stages, the resolution is taken to be 0.15 pc, almost 7 times better than for the majority of runs on later stages. The injected mass and energy of the SN are \( 10M_\odot \) and \( 10^{51} \text{ erg} \).

Left panels of Figure A1 present the density profiles at several time moments. It is clearly seen that the shell in front of the expanding gas forms practically immediately – on times \( \approx 600 - 800 \) yr, which is a factor of 3 shorter than that follows from the formal estimate \( t_{\text{free}} = \left( \frac{3M_{\text{ej}}}{4\pi \rho} \right)^{1/3} v^{-1} \). \( M_{\text{ej}} \) is the ejecta mass, \( v \) is the initial velocity \( v \approx 3c_s \). It is seen also from the right panel on Fig A1 where the bubble radius \( r(t) \) versus time is shown. Before \( t < 600 \) yr the ejecta expands freely as \( r = 0.0049t + 2.5 \) with the constant term approximately equal to the initial ejecta radius and the expansion velocity \( v = 0.0049 \) pc yr$^{-1}$. This value is only slightly higher than \( v_f = 2(\gamma - 1)^{-1}c_s \approx 0.0034 \) pc yr$^{-1}$ – the expansion velocity into vacuum, calculated for \( T_0 = 3 \times 10^8 \) K, which corresponds to the mass \( M_{\text{ej}} = M_\odot \) and thermal energy \( E = 10^{51} \) erg injected into ambient gas with density \( n_{\text{amb}} = 0.02 \text{ cm}^{-3} \) within \( r_0 = 2 \) pc. After \( t \sim (2 - 3)t_f \), the expansion turns to a quasi-diffusive law \( R(t) \propto t^{0.45} \), apparently caused by a series of reversed shock waves; the contribution from numerical diffusion at these time scales is negligible: \( (\langle \Delta x \rangle^2)^{1/2} \sim 2Dt \sim 0.1 \) pc at \( t \sim 1000 \) yr with \( D \) determined by the product of the spatial resolution 1 pc and typical sound velocity 40 km s$^{-1}$. Afterwards, the expansion decelerates due to the rarefac-
tion wave propagating inward from the ambient dilute gas. Eventually after \( \simeq 7 t \sim t_{\text{free}} \) it turns to the Sedov-Taylor solution \( r \propto t^{2/5} \) (dash-dotted lines). In our models the very initial free-expansion velocity \( v_f \) weakly depends on the ambient density as \( v_f \propto (1 + 0.07 n_{\text{amb}}/M_{\text{ej}})^{-1/2} \), here \( M_{\text{ej}} \) is in units of \( 10^4 M_\odot \). Details of the dynamics are seen clearly on Figure A2 where the radial profiles of temperature, velocity and density a SN expansion in the environments with \( n = 0.02 \text{ cm}^{-3} \) are given. The structure (density, velocity and temperature profiles) of the bubbles presented here are different from the analytical results of Chevalier & Clegg (1985, hereafter CC85). However, it has been previously shown the CC85 results are valid for a comparatively large rate of energy deposition. Sharma et al. (2014) showed that CC85 profiles are valid when \( \nu_{\text{SN}} \geq 3.5 \times 10^{-6} \text{ pc}^{-3} \text{ yr}^{-1} \). The SN rate densities considered here are much smaller than this and therefore one does not expect the structure to follow the predictions of CC85, as borne out by our simulations.

APPENDIX B: PRESSURE-DRIVEN PHASE

Pressure-driven phase begins at stages when radiation losses come into play, dense shell forms around the hot remnant cavity and the rate of change of momentum is governed by the cavity pressure (McKee & Ostriker [1977; Blinnikov et al. [1982])

\[
\frac{d(MR)}{dt} = 4\pi R^2 P,
\]

where \( M \) is the shell mass, \( R \), its radius, \( P \), is pressure in the cavity. It can be readily shown that the characteristic time of momentum growth \( t_P \sim |M\dot{R}|/4\pi R^2 P \simeq 20\Delta t \), where \( \Delta t \) is time elapsed since the expansion escaped ST phase at \( t = t_c \). The shell is being driven by pressure until \( t_P \) becomes longer than dynamical time \( t_P > t \simeq t_c \), meaning \( t_P \simeq 0.05t_c \).

APPENDIX C: A POWER LAW WIND-LIKE EXPANSION FOR A CONSTANT SNE RATE

It can be readily shown that when the SN rate is constant the resulting bubble settles asymptotically on to standard wind regime \( R \simeq (L^3/\rho)^{1/5} \) with \( L \simeq 0.3\nu_{\text{SN}}E \) being slightly reduced compare to a commonly assumed \( L = \nu_{\text{SN}}E \) mechanical luminosity from cumulative SNe explosion. Indeed, assuming the energy injected continuously into the bubble to be written as

\[
L = L_0 - \frac{3}{2}k_B T n V,
\]

where \( L_0 = \nu_{\text{SN}}E \), \( V \) is the volume occupied by the shell, we explicitly assume here that thermal energy of shocked gas \( 3k_B T/2 = 9m_H R^2/32 \) is completely lost radiatively when it passes through the shell. Accounting that the postshock flow inside the bubble is subsonic we can seek the solution for the shell radius

\[
\frac{4}{3\pi pR^3} \dot{R} = 2 \frac{Ldt}{R} - 4\pi pR^2 R^2,
\]

as a power-law \( R(t) \propto t^\alpha \) to find \( R \simeq (0.3L dt^3/\rho)^{1/5} \).