Mathematics Teachers Technological Content Knowledge (TCK) in using Dynamic Geometry Software

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Abstract. Technological content knowledge (TCK) is a framework that describes the knowledge required by teachers to represent material using technology. This paper reveals the knowledge of mathematics teachers in representing geometry material in software called GeoGebra. We analyzed the process of constructing geometrical objects in geometry problems by teachers who participated in a workshop on the use of GeoGebra in teaching mathematics. The approach used is a qualitative case study. 47 in-service mathematics teachers participated on training on the use of Geogebra in mathematics teaching. Researchers distributed questionnaires to 47 participants, and determined 7 subjects representing high, medium, and low questionnaires of Technological Pedagogical Content Knowledge (TPACK). Subject were given a task, to find out Technological Content Knowledge (TCK) of teachers in using geogebra, and observed them while they were doing the task. Observation was made when the process of constructing geometric objects into geogebra was taking place. Interviews were conducted to obtain more in-depth information that was not found by researchers in the observation process. Teachers’ knowledge in representing the geometry material in GeoGebra was reviewed based on how the knowledge supports the problem solving process. The results show that the knowledge can be divided into two groups: Constructive TCK and Obstructive TCK.

1. Introduction
The use of technology has widely penetrated into various aspects of human life. Technology can help people in various activities, including education. Teachers must have knowledge on how to integrate technology into the learning process. Many studies have been conducted on the knowledge required by teachers when integrating technology into learning, especially in mathematics [1]–[5]. The technologies used include a wide variety of software and hardware, such as devices that are specifically used in learning mathematics (GeoGebra, Geometer's Sketchpad, Cabri Geometry, graphing calculators, etc.) or common devices that can be used in various fields (tablet PC, spreadsheets, virtual blackboards, PowerPoint, java applets, etc.).

Dynamic geometry software (DGS) is often used in learning and is becoming an important tool because of its potential to improve the quality of learning [6]. DGS is a cognitive technological tool in learning mathematics [7] because it provides the ability to manipulate relations and mathematical objects intuitively. DGS can be used by teachers to represent and explain mathematical concepts more easily than traditional equipment, such as pencils and paper [8], [9]. A previous study has shown that
the use of DGS has a positive impact on student achievement [10], including the activities and motivation of students.

DGS has many names and includes free and paid versions, such as Geometer's Sketchpad, Cabri Geometry, Cabri 3D, Cinderella, Archimedes Geo3D, Compass and Ruler, Geometria, Wingeom, and GeoGebra. GeoGebra in particular has many advantages as an innovative tool to integrate technology in learning mathematics [11]. The combination of algebra and geometry is the heart of GeoGebra [12]. Because GeoGebra represents points, lines, and planes, it is easier and more attractive to use.

Many studies conclude that GeoGebra is an effective tool in the teaching and learning process, especially in the field of geometry. GeoGebra can motivate students to think critically when exploring mathematical problems [11]. GeoGebra can also help students to understand coordinates [13]. In addition, two experimental studies show that a group that used GeoGebra had better learning outcomes than a control group [14], [15]. Another study concludes that GeoGebra can increase students' motivation [16].

The Technological Pedagogical Content Knowledge (TPCK) framework is a theoretical framework that reflects the knowledge that teachers need in integrating technology in learning [17]–[20]. The TPCK acronym was changed to TPACK by [21] for easier pronunciation. Currently developing technology can be used as cognitive tools in learning [22]. For example, with web-based technology, students can learn to gain knowledge in a constructive manner by creating web contents by writing entries in Wikipedia or adding notes on Youtube. Similarly, when teachers use technology in the learning process, it also stimulates the process of cognition within the teachers themselves. The cognition process occurs before, during, and after the use of technology in learning activities. The cognition process before learning occurs when teachers represent the material using technology, which is a form of technological content knowledge (TCK). TCK is knowledge about how to represent material using technology [23], such as how to make a circle within a triangle using GeoGebra.

TCK is one of the domains of TPACK and is one area that may require some clarifications [24]. Only a few researchers have examined this component of TPACK [25]. To our knowledge, very few studies have been conducted concerning TCK. However, TCK is very important for the actualization of the TPACK framework. TCK is a way for teachers to apply other knowledge domains. As an illustration, to explain the process of line division using GeoGebra (TPACK), the teacher must be able to make the midpoint of a line segment using GeoGebra (TCK). Thus, the study of TCK is important. This research looks at the characteristics of TCK for mathematics teachers in using GeoGebra as DGS. The results could be useful for mathematics teachers in using technology when preparing teaching materials and using software in accordance with the characteristics of learning materials.

2. Research method
The participants in this study were 47 in-service mathematics teachers who participated in training on the use of GeoGebra in mathematics teaching held in May 2017. The researchers distributed questionnaires to the participants and determined seven subjects representing high, medium, and low questionnaires of TPACK. The data collection procedures were as follows:

1. Teachers were given a task that was adapted from Ramatlapana [26] to find out their TCK in using GeoGebra. This instrument is adapted because it tests TCK that requires competence to use geogebra to mediate geometry proficiency. Teachers required to identify the geometrical relationships between the objects created on the computer and original constructions. To successfully do the identification, Teachers need to visualize the different configurations of the figures and use GeoGebra construction tools such as the ‘drag mode’ tool.

2. Teachers were observed while they were doing or task of constructing geometric objects in GeoGebra.

3. Interviews were conducted to obtain more in-depth information that was not found by the researchers in the observation process. The interviews were used to gather deeper information about the opinions, impressions, experiences, thoughts of the subjects. The interviews were semi-structured interviews. If there was an answer beyond the scope of the structured questions, it was recorded to gain deeper knowledge about the research subjects.
4. The data collected from teachers were examined for their consistency. If any data were inconsistent, another interview was done for clarification. If the data remained inconsistent, then it was not used.

5. When a teacher had been interviewed and it was still necessary to conduct another interview, the same interview was conducted with the other teachers until the data obtained were considered sufficient (snow ball technique).

The data analysis in this study used Miles and Huberman’s measures of data reduction, data display, conclusion drawing, and verification. To find out the characteristics of TCK among mathematics teachers, the researchers made a framework for the instrument used before the task was given to subjects. The possible task completion steps are shown in Figure 1.

Figure 1. Problem framework
3. Results and Discussion

After receiving the task of constructing geometry problems in GeoGebra, subject S1 completed the task through completion steps 1, 2, 3, 4, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, and 20, as shown in Figure 2. Subject S5 completed the task through steps 1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 13, 15, 17, 18, 19, and 20. These two subjects can resolve the problem appropriately in accordance with the steps that have been created within the framework of the problem. From the review of TCK, these subjects have Content Knowledge (CK) that is integrated well with Technological Knowledge (TK). This is evident from the statement of S5: "I know that in GeoGebra there is a direct menu to make a midpoint, but I chose to use the precise geometry procedure." This TCK supports the completion of geometry problems. We refer to this type of TCK as constructive TCK. In this TCK, TK and CK are well integrated.

![Figure 2. Completion structure of S5](image)

Subjects S2, S3, and S6 completed the task with steps 1, 2, 3, 4, 11, 12, 14, 15, 16, 17, 18, 19, and 20. At a glance, the representation of geometry generated by these subjects is appropriate for the instructions, as shown in the results of S3 in Figure 3. However, if point B in Figure 3 is dragged, then the triangle ABC will no longer be an isosceles triangle, as shown in Figure 4.
The incorrect action done by S3 occurs because the subject does not perform proper geometry procedures when creating isosceles triangle ABC. In this case, the determination of point A as the high point of triangle ABC is not determined by the proper geometry procedure, but with an estimate only. The interviews indicate that S3 knows that there are actual procedures to determine the high point on an isosceles triangle. S3 said that "the high point of this triangle should be in the middle of BC," which means that the altitude passing through point A must divide BC into two equal parts. S3 also knows the steps for determining the altitude of triangle ABC that intersects circle S at point A so that point A becomes the high point of isosceles triangle ABC. This shows that the CK of S3 is actually good enough to solve the problem. As for TK, S3 also knows how to make a bisector and create a line segment. Accordingly, it appears that the CK of S3 cannot be integrated well with his TK.

Meanwhile, S2 and S6 do not know the proper geometric procedures for how to make an isosceles triangle, although in terms of TK, S2 and S6 can make a bisector and perpendicular line passing through a bisector. This TCK does not help to solve the given mathematical problem and can obscure students' understanding because when asked, S2, S3, and S6 initially did not realize that the picture they made was wrong. We refer to this TCK as obstructive TCK.
Subjects S4 and S7 were unable to complete all the tasks given and did not make isosceles triangles on the circle. The subjects made a misstep that was not contained in the problem structure created by the researchers, as shown in Figure 6. More specifically, the subjects did not put a triangle vertex on the circle and were unable to solve this problem.

Figure 6. (a) The initial construction of S4  
(b) Point A is not on the circle
Figure 7. Completion structure of S4

In the cases of S2, S3, S4, S6, and S7, it appears that their TCK cannot support the process of problem solving. This could happen because their CK cannot be integrated well with their TK or indeed because of the poor CK of the subjects. This TCK is also called obstructive TCK. There were two different obstructive TCK characteristics among subjects S2, S3, S4, S6, and S7. Subjects S2, S3, and S6 were not aware that their TCK does not support the process of problem solving. S2, S3, and S6 felt that their completion of the task was appropriate. The subjects were not aware that their completion of the task does not use the proper geometry procedures. This TCK can obscure and create confusion in the understanding of subjects on the task completed. Meanwhile, subjects S4 and S7 were fully aware that their knowledge is not enough to complete the given task.

4. Conclusions
Based on the findings, the TCK characteristics of mathematics teachers in using DGS to solve problems were determined as follows:
1. Constructive TCK
   Constructive TCK can support the task completion or problem solving of mathematics teachers. The forming elements of this knowledge (CK and TK) can be well integrated to generate
constructive knowledge for problem solving.

2. Obstructive TCK
   a. Obstructive consciousness
      Obstructive consciousness is TCK that is unable to support the process of task completion or problem solving without the individual realizing it. This TCK can obscure and create confusion in the understanding of subjects about the completed task.
   b. Obstructive lack of process
      Obstructive TCK is unable to support the process of task completion or problem solving, and the individual is fully aware that their knowledge is not enough to complete the given task.

This study could give opportunities to other researchers to conduct other research dealing with TCK. For example, future studies could examine how mental processes occur. Moreover, the causes of obstructive TCK have not been disclosed in this paper as well. Studies could also be done on how the parts of TCK (CK and TK) interact, the occurrence of constructive TCK, and how TCK can play a role in the formation of TPACK.

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