A Neural Network Assisted Greedy Algorithm For Sparse Electromagnetic Imaging

A. I. Sandhu, S. A. Shaukat, A. Desmal, and H. Bağcı

1Division of Computer, Electrical, and Mathematical Science and Engineering (CEMSE), King Abdullah University of Science and Technology (KAUST), Thuwal, Saudi Arabia, 23955-6900.
email: aлимran.sandhu@kaust.edu.sa; hakan.bagci@kaust.edu.sa
2Department of Electrical Engineering, Higher Colleges of Technology (HCT), Ras Al-Khaimah, UAE.
email: adesmal@hct.ac.ae

Abstract—Greedy pursuit algorithms (GPAs), are well appreciated candidates for accurate and efficient reconstruction of sparse signal and image processing applications. Even though many electromagnetic (EM) imaging applications are naturally sparse, GPAs have rarely been explored for this purpose. This is because, for accurate reconstruction, GPAs require (i) the exact number of non-zeros, $k$, in the unknown to be reconstructed. This information is not available a-priori for EM imaging applications, and (ii) the measurement matrix to satisfy the restricted isometric property (RIP), whereas the EM scattering matrix is obtained by sampling the Green’s function between measurement locations and the unknowns does not satisfy the RIP. To address the aforementioned limitations, two solutions are proposed. First, an artificial neural network (ANN) is trained on synthetic measurements, such that given a set of measurements, the ANN produces an estimate of $k$. Second, Tikhonov second norm regularization term is added to the diagonal elements of the scattering matrix, which scales the eigenvalues of the scattering matrix such that it satisfies the RIP. The CoSaMP algorithm, which is at the heart of GPAs, is then applied, to accurately and efficiently reconstruct the unknown. The proposed scheme implicitly imposes the sparsity constraint, as the regularization parameter is specified by the ANN, hence no additional tuning is required from the user. Numerical results demonstrate the efficiency and superiority of the proposed scheme.

I. INTRODUCTION

Compressed sensing (CS) [1] has introduced several new approaches for sparse reconstruction in areas of signal and image processing [1], [2]. Given a reconstruction problem, a CS algorithm seek for the sparsest approximation to the solution, while requiring the cardinality of the solution (i.e. measured by its $L_0$-norm to be the minimum). It is well known that a direct solution of an $L_0$ - constraint minimization problem is not feasible [3], however greedy pursuit based algorithms (GPAs), under certain conditions provide a well-approximated solution to the $L_0$ - constraint linear inverse problems [3], [4]. These algorithms works by successively identifying, single or multiple, locally optimal candidate(s) that could best represent the signal at a given stage, with the hope to approximate a global optimal solution in a reasonable time. The algorithms that are most widely in use in the image and signal processing community, includes but not limited to orthogonal matching pursuits (OMP) [5], regularized OMP [4] and compressive sampling matching pursuit (CoSaMP) [3].

Even though many electromagnetic (EM) imaging applications are naturally sparse, such as nondestructive testing, crack detection and hydrocarbon reservoir exploration, the application of GPAs in this area is very limited. This is simply because, for reliable and efficient reconstruction, GPAs require (i) a-priori information about the exact number of non-zero elements, $k$, to be reconstructed. This information is not available for EM imaging applications. (ii) the measurement matrix to satisfy the restricted isometric property (RIP) [3]–[5]. The EM scattering matrix, which depends upon the physics of the problem and is obtained by sampling the background medium’s Green function between measurement locations and the unknowns, does not satisfy the RIP [6]–[8].

CS algorithms have been adopted, within the last decade, for EM imaging applications. In [8], a simultaneous OMP algorithm is used to resolve targets as a function of average number of transmitters and receivers used. Where on one hand, the contrast levels and reconstruction accuracy were not reported, the algorithm was provided with $k$, which eliminates its usefulness in EM imaging framework. In [6], a phaseless reconstruction scheme (i.e. using only the intensity of scattered fields) is used to image low contrast point like dielectric scatterers. The nonlinearity is alleviated using the Born approximation and the reconstruction is carried out using convex (i.e. $L_1$) programing. In a recent work [7], a flexible tree search based OMP algorithm is incorporated for the reconstruction of closely spaced, point like scatterers. $k$ is estimated by comparing the data misfit at each stage of the tree search-based approach. Within this framework, there exists a tradeoff between the reconstruction accuracy and computational complexity, which depends upon the search tree size. Numerical results presented accurate and sharp image, however the algorithm’s ability to reconstruct multiple connected or large objects is not demonstrated. This could be due to the associated computational complexity or a consequence of the underlying orthogonalization procedure associated with the OMP algorithm, which refrains it from recovering electrical large objects, hence limits its applicability in EM imaging problems. Another limitation associated with OMP is that if the very first estimate of the solution component is incorrect, the algorithm would converge to a local minimum or an entirely incorrect solution.
To address the aforementioned limitations, two solutions are proposed. First, an artificial neural network (ANN) is trained beforehand using the synthetic training set (i.e. synthetic measurements) generated using different scatterers, their orientations, contrast levels and discretization mesh sizes to achieve efficient and reasonable estimate \( \hat{k} \) of \( k \). Second, Tikhonov second norm regularization term is added to the diagonal elements of the EM scattering matrix, which scales the eigenvalues of the scattering matrix. This reduces the effect of noise on the reconstruction process and enables the scattering matrix to satisfy the RIP. The CoSaMP algorithm, which is at the heart of GPAs, is then applied to solve two-dimensional (2D) EM imaging problems. CoSaMP is more efficient and accurate in comparison to its OMP based counterparts [3]. It works by estimating multiple basis elements instead of one at a time and refines the support set iteratively. Consequently, it offers a faster rate of convergence and renders the reconstruction of large connected objects, which is a limitation usually associated with the OMP based algorithms.

The advantages of this Tikhonov and ANN-enhanced CoSaMP algorithm are threefold: (i) the CoSaMP estimates the unknowns by solving a least squares problem on the refined support set (i.e. it only considers \( \hat{k} \) out of \( N \), where \( N \) is the total number of unknowns, columns of the scattering matrix whose indices are identified in the support estimation step), consequently it offers significant computational savings in contrast to solving the least squares problem involving a full scattering matrix, (ii) it does not require tuning of a thresholding parameter [9, 10] since the sparsity parameter \( \hat{k} \) is implicitly determined by the ANN. It is important to note here that many different ANN architectures based on convolutional neural networks (CNNs), have been studied for EM imaging [11], [12]. Their work is based on training the network (mainly U-Net architectures) with a set of approximated contrast profiles either originated from first order approximation, e.g. first order Born approximation and back propagation, or smooth images that do not contribute higher frequency components [11], [12]. In such scenarios, the neural network will work as a regularizer that restores finer image details or higher frequency components. In this work, the ANN does not handle at its input, a set of first order profiles, nor it produce the image, instead it directly handles at its input, the measurements and generates the sparsity estimate \( \hat{k} \), and (iii) the reconstructed images are more accurate and sharper than those produced by smoothness promoting inverse algorithms.

II. FORMULATION

A. Electromagnetic Formulation and Discretization

Let \( S \) represent the support of a 2D inhomogeneous investigation domain residing in an unbounded background medium. The permittivity and permeability in \( S \) and in the background medium are \( \{\varepsilon(\mathbf{r}), \mu_0\} \) and \( \{\varepsilon_0, \mu_0\} \), respectively. It is assumed that \( S \) is illuminated by \( N^T \) line source transmitters which generate TM incident fields \( E_{inc}^i(\mathbf{r}) \), where the subscript \( i \) traces the transmitters, \( i = 1, \ldots, N^T \). Upon excitation by \( E_{inc}^i(\mathbf{r}) \), secondary electric current density induces on \( S \) which in turn generates the scattered electric field \( E_{sca}^i(\mathbf{r}) \), which satisfies [22]:

\[
E_{sca}^i(\mathbf{r}) = k_0^2 \int_S \tau(\mathbf{r'}) E_{tot}^i(\mathbf{r'}) G(\mathbf{r}, \mathbf{r'}) ds'.
\]

The scattered field is measured away from \( S \) at \( N^R \) receivers located at \( \mathbf{r}^m_r, m = 1, \ldots, N^R \). Here, \( G(\mathbf{r}, \mathbf{r'}) = H_0^2(k_0 |\mathbf{r} - \mathbf{r'}|)/(4\pi) \) is the 2D scalar Green’s function, \( k_0 = \omega\sqrt{\varepsilon_0\mu_0} \) is the wavenumber, \( \tau(\mathbf{r'}) = \varepsilon(\mathbf{r})/\varepsilon_0 - 1 \) is the contrast relative to the background medium and \( E_{tot}^i(\mathbf{r}) \) is the total electric field inside \( S \). To solve (1) numerically, \( S \) is discretized using \( N \) square cells, s.t. \( S = \bigcup_{n=1}^N S_n \), and inside each cell, the contrast and total electric field is assumed constant and simply expanded using pulse basis functions. The resulting equations are evaluated at the receiver locations, which yielded the following discretized systems

\[
\tilde{E}_{sca}^i = \tilde{G} \tilde{D}(\tilde{E}_{tot}^i) \tilde{\tau}
\]

here, \( \tilde{D}(\tilde{E}_{tot}^i) \) represents a diagonal matrix with the samples of total electric field on its diagonal and the entries of matrix \( \tilde{G} = \{G\}_{m,n} = k_0^2 \int_{S_n} G(\mathbf{r}^m_r, \mathbf{r'}) ds' \). Prior to analyzing the reconstruction efficiency of the unknown contrast by applying the CoSaMP algorithm to (2), the scheme to estimate the sparsity level is discussed below.

B. Sparsity Estimation

Useful prior information, about the object of interest under test, helps adopting efficient regularization and imaging techniques, thereby reduces the computational complexity and increases the reconstruction accuracy. Albeit greedy algorithms are super-efficient in recovering sparse signals, their reconstruction accuracy severely degrades if \( k \) is not known a-priori. This is a typical case with the EM imaging frameworks; consequently, greedy algorithms have rarely been explored in solving EM imaging problems. To this end, in this work, an ANN is trained and utilized to provide a good estimate \( \hat{k} \).

More precisely, in this work, a simple two-layer feedforward perceptron network is incorporated Fig. 2(a) The proposed network handles at its input layer, directly the measured electric field values. The training set is synthetically generated using the 2D volume integral equation solver formulated in the previous section. The scattered electric field corresponds
to uniformly distributed scatterers (i) of different shapes, in particular, circular rings with random radii, single and double cylinders with random radii, and with varied separation distance, (ii) with varying contrast levels i.e. 0.2, 0.4, 0.6 and 0.8, and (iii) with twofold and fourfold discretization elements \( N \) i.e \( N \in \{784, 3136, 12544\} \). A total of 14450 training examples are synthetically generated, out of which 70\% of the set is used for training and remaining 30\% is used for testing the ANN. The training step minimizes the cost, which is the mean squared difference between the normalized (i.e. normalization is done with respect to the discretization size \( N \), such that the training is discretization independent), \( k \) and contrast level in the investigation domain. It is important to note here that, the estimation of the contrast level at the ANN output, is merely to train the ANN better (i.e. it is observed that adding more information at the ANN output, while training, produces accurate results over a wider range of examples), and this additional information is not used with the integrated CoSaMP algorithm to reconstruct the profile.

Fig. 2(b) plots the convergence of the cost function while training and validating the ANN, in a mean squared error sense, with respect to the number of epochs. Once trained, the performance of the ANN on the testing set, can be analyzed considering following five factors, (i) the contrast in the investigation domain, which ranges from 0.2 to 0.8 (ii) number of discretization elements \( N \) where \( N \in \{784, 3136, 12544\} \) (iii) number of pixels in error i.e. the absolute difference between true and predicted number of non-zeros in the investigation domain, i.e. \( |k - \hat{k}| \) (iv) minimum size of the scatterer in pixels, i.e. \( k \), when the ANN produced a certain error in the estimate \( \hat{k} \), and (v) from the entire testing set, for how many examples the ANN produced a certain error. A compact demonstration with respect to all these parameters is not feasible, the histograms which could demonstrate factor (v) for each contrast as well as discretization size, are omitted here. This is simply to emphasize that for the given experiment it is more appropriate to observe factor (iv), irrespective of how many examples were in error.

In Figures 2(c)-(h), the \( y \)-axis represents factor (iv), i.e. the minimum value of \( k \) for a given discretization size or contrast in the investigation domain (observable along the \( x \)-axis), for which the ANN produced a certain error in the estimate \( \hat{k} \). The values are also labeled on top of the respective bars. The blue, green and yellow bars represents if \( k \) is incorrectly estimated by \( \pm\{1, 2 \text{ or } 3\} \) pixels, where the maximum error is \( \pm3 \) pixels over the range of testing examples. For instance, consider Fig. 2(e), the yellow bar shows that there are a minimum of \( k = 80 \) non-zeros with a contrast of 0.6 in the investigation domain having \( N = 784 \) elements, when the ANN produced an error of \( \pm3 \) pixels. Similarly a 0 anywhere in the bar plots represents that for the given experiment, \( \hat{k} = k \) i.e. not a single example resulted in an incorrect estimate. Fig. 2(g) shows similar error performance for fixed \( N = 784 \), but for contrast levels 0.3 and 0.7, which were not used while training. Fig. 2(h) demonstrates the prediction accuracy of the ANN for the Austria like profiles. This particular testing set consist of 400 examples, including (i) circular ring with a constant radius and the two outer cylinders rotated 360° in steps of 1° to generate 360 examples, and (ii) circular rings

Fig. 2. (a) Sparsity estimation framework. (b) Convergence of training and validation error in the ANN, and (c) testing analysis of the ANN.
with varied radii and outer cylinders rotated in steps of 90°, which contributed another 40 examples. It is important to note here that, the ANN is trained merely on independent set of rings, and cylinders, and the Austria like testing set is never used in the training, even then the proposed ANN architecture resulted in a maximum of 3. For this experiment, \( N \in \{3136, 12544\} \) for a fixed contrast level of 0.2. Several numerical examples are presented in the results section, which demonstrates the superiority of the proposed scheme over other first order reconstruction techniques.

C. CoSaMP Applied to EM Imaging

In this work, CoSaMP [3] is applied to (2) which constitutes a sparse linear inverse problem in the Born approximated regime [13]. The optimization problem can be formulated as

\[
\hat{\tau} = \min_{\tau} \| \hat{\tau} \|_0 \quad \text{s.t.} \quad \| \hat{E}^{\text{meas}} - \hat{H} \hat{\tau} \|_2^2 \leq \epsilon. \tag{3}\]

In the above, \( \hat{E}^{\text{meas}} \approx \hat{E}^\text{ca} + \hat{\eta} \) where \( \hat{\eta} \) contains the samples of additive white Gaussian noise. The subscript \( i \) is omitted such that \( \hat{E}^{\text{meas}} \) represents a cascaded vector corresponding to all illuminations. The scattering matrix \( \hat{H} \) has to satisfy the RIP for the CoSaMP algorithm to converge [3], which states such that

\[
\| \hat{\eta} \| \leq \epsilon \cdot \hat{H} \hat{\tau} \|_2^2 \leq \epsilon. \tag{4}\]

The lower bound on Eq. (4) is not satisfied due to the fact that \( \hat{H} \) is ill-conditioned and \( \text{eig}(\hat{H})_{\text{min}} = 0 \), which enforces \( \delta = 1 \), hence breaks the RIP. To address this problem, the data misfit \( ||\hat{E}^{\text{meas}} - \hat{H} \hat{\tau}||_2 \) in Eq. (3) is replaced with the second norm Tikhonov kind of system which yields

\[
\hat{\tau} = \min_{\tau} \| \hat{\tau} \|_0 \quad \text{s.t.} \quad \| \hat{E}^{\text{meas}} - \hat{H}^{\lambda} \hat{\tau} \|_2^2 \leq \epsilon. \tag{5}\]

In Eq. (5), \( \hat{H}^{\lambda} = \hat{H}^\dagger \hat{H} + \lambda \hat{I} \) where \( \hat{H}^\dagger \) is the complex conjugate of \( \hat{H} \) and \( \hat{E}^{\text{meas}} = \hat{H}^\dagger \hat{E}^{\text{meas}} \). Eq. (5) can also be written as

\[
\hat{\tau} = \min_{\tau} \| \hat{\tau} \|_0 \quad \text{s.t.} \quad \| \hat{E}^{\text{meas}} - \hat{H}^\dagger \hat{H} \hat{\tau} \|_2^2 + \lambda^2 \| \hat{\tau} \|_2^2 \leq \epsilon. \tag{6}\]

Eq. (6) represents an optimization problem, not only with the sparsity, but also a second norm regularization term with \( \lambda^2 \) as the regularization parameter. It is known that \( \hat{H} \) is ill-conditioned, so is \( \hat{H}^\dagger \hat{H} \), however by adding a selected parameter \( \lambda \) onto the diagonal elements of \( \hat{H}^\dagger \hat{H} \), the eigenvalues of \( \hat{H}^{\lambda} \) will be modified to \( \text{eig}(\hat{H}^\dagger \hat{H} + \lambda \hat{I}) \). While \( \lambda \) is a strictly positive number, \( \text{eig}(\hat{H}^{\lambda})_{\text{min}} = \lambda \), hence the lower bound of the RIP can be satisfied, as \( \delta \) can be strictly positive within (0,1). It should be noted here that \( \lambda \) is not introduced to promote smoothness in the solution, but only to satisfy the RIP. The CoSaMP algorithm is applied to Eq. (5) and an approximate sparse solution to Eq. (5) can be sought using the following proposed algorithm:

\[
\text{Step 1:} \quad \text{Initialize} \quad t^0 \leftarrow \hat{E}^{\text{meas}}, \quad n \leftarrow 0, k, \lambda
\]

\[
\text{Step 2:} \quad \text{repeat}
\]

\[
\text{Step 2.1:} \quad n \leftarrow n + 1
\]

\[
\text{Step 2.2:} \quad \text{evaluate} \quad \hat{y}^{(n)} \leftarrow ||(\hat{H},r^{(n-1)})||_2^2
\]

\[
\text{Step 2.3:} \quad \text{evaluate} \quad \Omega^{(n)} \leftarrow \text{sort}(\hat{y}^{(n)}_1)
\]

\[
\text{Step 2.4:} \quad \hat{r}^{(n)} \leftarrow \Omega^{(n)} \cup \text{supp}(\hat{r}^{(n-1)})
\]

\[
\text{Step 2.4:} \quad \text{estimate} \quad \hat{\tau}^{(n)} \leftarrow (\hat{H}^\dagger \hat{E}^{(n)})^{-1} \hat{E}^{\text{meas}}
\]

\[
\text{Step 2.4:} \quad \text{update} \quad r^{(n)} \leftarrow \hat{E}^{\text{meas}} - \hat{H} \hat{\tau}^{(n)}
\]

Several comments about the proposed scheme are in order: At step 1, several parameters are initialized. The sparsity level \( k \) is estimated by feeding the measurements to the already trained ANN. The parameter \( \lambda \) can be estimated from the noise level in the measurements. It makes \( \hat{H}^{\lambda} \) a full rank matrix that helps in satisfying the RIP criteria. At step 2.2 the residual from the last iteration is projected onto the model subspace to determine which components of the unknown model are still not determined. At the third step, set \( \Omega^{(n)} \) stores \( k \) column-indices from \( \hat{H} \) which contributed maximally towards the correlation in step 2.2. At step 2.4, the newly identified support set \( \Omega^{(n)} \) is unified with the final support set \( \text{supp}(\hat{r}^{(n-1)}) \) from the last iteration, in order to eliminate any repetitions in the support elements. At step 2.5 the solution coefficients are estimated by solving a least squares problem over the merged support set \( \hat{E}^{(n)} \). It should be noted here that, \( \hat{H}^{\lambda} \) contains only those columns of \( \hat{H} \) whose indices are in the merged support set \( \hat{E}^{(n)} \). This significantly reduces the computational cost in contrast to solving the least squares problem, which would involve the whole matrix \( \hat{H} \). Finally, the residual is updated so that it reflects only the part of unknowns that has yet not been estimated. This process continues until some specified halting criteria is met. The algorithm is set to terminate if the data misfit \( ||\hat{E}^{\text{meas}} - \hat{H} \hat{\tau}||_2 \leq 5\% \) or the residual between successive iterations does not change significantly i.e. \( ||r^{(n)} - r^{(n-1)}||_2 \leq 10^{-6} \).

III. NUMERICAL RESULTS

This section demonstrates the accuracy and efficiency of the proposed scheme via numerical experiments. First, \( \hat{E}^{\text{meas}} \) is added to the result to yield \( E^{\text{meas}} \). Here, \( \text{eig}(\tau^{\text{ref}})_p = \tau^{\text{ref}}(r_p), \) \( p = 1, \ldots, N \), are the samples of the actual contrast \( \tau^{\text{ref}}(r) \) being reconstructed.

Three different EM inversion schemes are compared: (i) the FTB-OMP algorithm [7] (ii) first order Born-approximation [14] with soft thresholding [10], and (iii) the algorithm proposed in this work. For all simulations, the quality of reconstruction is measured using

\[
e_{\text{err}}^n = \frac{||\tau^n - \tau^{\text{ref}}||_2}{||\tau^{\text{ref}}||_2} \tag{7}\]

where \( \tau^n \) stores the samples of the contrast reconstructed at convergence \( t \).
A. Closely Spaced Point Like Targets

The first example is reproduced from a recent article [7], which demonstrates the reconstruction of closely spaced point like targets for several SNR values ranging from 5dB to 50dB. The investigation domain is extremely sparse and discretized using 1369 square cells, surrounded by 30 transmitters operating at 300MHz and 50 receivers. It is clear from Fig. 3 that the proposed CoSaMP algorithm has yielded much higher reconstruction accuracy in comparison to the FTB-OMP algorithm over the range of SNR values and moreover the reconstructed image is sharper and accurate. In this example, \( \hat{k} \) is not estimated using the ANN, instead provided to the CoSaMP algorithm. This is to demonstrate that indeed the CoSaMP algorithm outperforms in comparison to other GPAs, and which is why it is considered in this work.

B. Closely Spaced Dielectric Cylinders

The second example demonstrates the reconstruction of closely located dielectric cylinders (i.e. multiply connected objects) using the proposed scheme. The electrical dimension of the investigation domain in Fig. 4(a), is about \( 2\lambda \times 2\lambda \) and is discretized using 3136 square cells, surrounded by 32 transmitter and receivers pairs operating at 130MHz. In the reference profile \( \hat{k} = 72 \), and the ANN estimated it exactly. The SNR is maintained at 30dB. The reconstructed images using CoSaMP and the first order Born approximation are shown in Fig. 4(c) and Fig. 4(e) respectively. Clearly CoSaMP produced a sharper and accurate image with a relative error of 28%. As we have already discussed that a maximum error of ±3 pixels is observed over all the testing set and even though \( \hat{k} = k \) for this example, the black curve in Fig. 4(g) represents the accuracy of the proposed scheme over the range of error in the estimate.
C. Austria

The third example is the well-known Austria profile, Fig. 5(b). All the simulation parameters, but the discretization size, are identical to the second example. The investigation domain is discretized using 784 square cells. In the reference profile $k = 66$, and the ANN estimated it exactly. The reconstructed images using CoSaMP and the first order Born approximation are shown in Fig. 5(d) and Fig. 5(f) respectively. Clearly CoSaMP produced a sparse and sharper image with a relative error of 43%. As we have already discussed that a maximum error of $\pm 4$ pixels is observed over all the testing data that is synthesized using variations of Austria, the red curve in Fig. 5(g) represents the accuracy of the proposed scheme over the range of error in the estimate.

D. Electrically Large Objects

To demonstrate that the reconstruction efficiency of the proposed algorithm is not limited to electrically very small targets, such as those presented in couple of earlier examples, Fig. 5(a)-(b) presents reconstructed images of a multi-layered cylinder whose diameter is on the order of a wavelength and of an L-shaped object respectively. The electrical dimensions of the investigation domains in Fig. 5(a)-(b), are $2.7\lambda \times 2.7\lambda$ and are discretized using 961 square cells, surrounded by 32 transmitter and receiver pairs operating at 160MHz. The reconstructed images using the CoSaMP, in Fig. 5(c)-(d), are sharper and accurate with a relative error of 24% and 31.6% for the multi-layered cylinder and the L-shaped object respectively.

References

[1] E. J. Candes and M. B. Wakin, “An introduction to compressive sampling,” *IEEE Signal Process. Mag.*, vol. 25, no. 2, pp. 21–30, 2008.
[2] R. G. Baraniuk, “More is less: signal processing and the data deluge,” *Science*, vol. 331, no. 6018, pp. 717–719, 2011.
[3] D. Needell and J. A. Tropp, “Cosamp: Iterative signal recovery from incomplete and inaccurate samples,” *Appl. Comput. Harmon. A.*, vol. 26, no. 3, pp. 301–321, 2009.
[4] D. Needell and R. Vershynin, “Signal recovery from incomplete and inaccurate measurements via regularized orthogonal matching pursuit,” *IEEE J. Sel. Top. Signals*, vol. 4, no. 2, pp. 310–316, 2010.
[5] J. A. Tropp and A. C. Gilbert, “Signal recovery from random measurements via orthogonal matching pursuit.” *IEEE Trans. Info. Theory*, vol. 53, no. 12, pp. 4655–4666, 2007.
[6] X. C. L. Pan and S. P. Yeo, “A compressive-sensing-based phase-less imaging method for point-like dielectric objects,” *IEEE Trans. Antennas Propag.*, vol. 60, no. 11, pp. 5472–5475, 2012.
[7] G. K. K. R. V. Senyuva, O. Ozdemir and E. Anarim, “Electromagnetic imaging of closely spaced objects using matching pursuit based approaches,” *IEEE Antennas Wirel. Propag. Lett.*, vol. 15, pp. 1179–1182, 2015.
[8] T. C. Ye and S. Y. Lee, “Non-iterative exact inverse scattering using simultaneous orthogonal matching pursuit (s-comp),” in 2008 *IEEE International Conference on Acoustics, Speech and Signal Processing*, IEEE, 2008, pp. 2457–2460.
[9] I. Daubechies, M. Defrise, and C. De Mol, “An iterative thresholding algorithm for linear inverse problems with a sparsity constraint,” *Commun. Pure Appl. Math.*, vol. 57, no. 11, pp. 1413–1457, 2004.
[10] A. Desmal and H. Bagci, “Shrinkage-thresholding enhanced Born iterative method for solving 2D inverse electromagnetic scattering problem,” *IEEE Trans. Antennas Propag.*, vol. 62, no. 7, pp. 3878–3884, 2014.
[11] e. a. L. Li, “Deepinis: Deep neural network for nonlinear electromagnetic inverse scattering,” *IEEE Trans. Antennas Propag.*, vol. 67, no. 3, pp. 1819–1825, 2018.
[12] Z. X. Y. Sun and U. Kamilov, “Efficient and accurate inversion of multiple scattering with deep learning,” *Optics express*, vol. 26, no. 11, pp. 14678–14688, 2018.
[13] M. Pastorino, *Microwave Imaging*. Wiley, 2010.
[14] M. Born and W. Wolf, *Principles of Optics*. Pergamon Press, 1965.