Application of software complex turbo problem solver to rayleigh-taylor instability modeling

S V Fortova, P S Utkin, V V Shepelev
Institute for Computer Aided Design of the Russian Academy of Sciences, 19/18 2nd Brestskaya str., Moscow 123056, Russian Federation
E-mail: sfortova@mail.ru

Abstract. The dynamic processes which take place during high-speed impact of two metal plates with different densities are investigated using three-dimensional numerical simulations. It is shown that as a result of the impact the Rayleigh-Taylor instability forms which leads to the formation of three-dimensional ring-shaped structures on the surface of the metal with smaller density. The comparative analysis of the metals interface deformation process with the use of different equations of state is performed. The numerical study is carried out by means of special software complex Turbo Problem Solver developed by the authors. The software complex Turbo Problem Solver implements generalized approach to the construction of hydrodynamic code for various computational fluid dynamics problems. Turbo Problem Solver provides several numerical schemes and software blocks to set initial, boundary conditions and mass forces. The solution of test problem about Rayleigh-Taylor instability growth and development for the case of very rapid density growth is also presented.

1. Introduction
The problem of explosive hardening and explosive welding is the actual issue for production of high-strength bimetallic compounds and constructions with the desired characteristics [1]. Explosive welding is a process of metallic compound production as a result of high-speed impact due to the energy of condensed explosives detonation products. The process is followed by the complex dynamic effects which are the subject of a number of experimental and numerical studies, see [2] for example. In [2] on the example of high-speed impact of lead plate with the plates of different metals the features of the processes in the near interface regions of the colliding plates are investigated experimentally.

The scheme of the experiment [2] is the following. The lead plate is thrown by the products of condensed explosive detonation to the steel plate under some angle with the speed several hundred meters per second. The plates become deformed with the formation of solid compound in some cases. The partial melting of the colliding surfaces takes place and the transition of the metals to the elastoplastic state occurs due the high energy release. So during some time (about 10 $\mu$s) after an impact the metals are in the elastoplastic state and behave as pseudo-fluids before the backward transition starts. Circumstantial proof of that fact is the existence of crateriform splashes on the surface of steel plate in the direction of lead plate. The explanation of the splashes was given in [2] on the assumption of the Rayleigh-Taylor instability (RTI) development. Such type of instability can be realized in the rarefaction wave (RW) which moves through the plates interface from the lead plate free side if the initial disintegration of discontinuity occurred with
the formation of two shock waves (SW). Acceleration in the RW is directed to the side of the metal with smaller density that is the necessary condition of the RTI development (the velocity and density gradients have different signs).

It should be noted that there are a lot of difficulties in experimental observations of the process in consideration due to the extremely high temperatures and pressures and very short times. In such conditions numerical modeling provides the possibility to obtain very precise effects that could not be obtained in natural experiments. The paper is devoted to the numerical simulation of high-speed impact of two metal plates in the statement qualitatively similar to [2] using three-dimensional (3D) Euler equations and different equations of state.

2. Statement of the problem
Consider the interaction of the lead plate with the density \( \rho = 11300 \text{ kg/m}^3 \) and thickness \( h = 0.002 \text{ m} \) with the steel plate (\( \rho = 7900 \text{ kg/m}^3 \)) or the copper plate (\( \rho = 8900 \text{ kg/m}^3 \)) of the same thickness. The lead plate is thrown in the vertical direction with the speed \( w = 500 \text{ m/s} \). As the initial disturbance we set the point disturbance \( 500 \text{ m/s} \) of the velocity on the throwing plate surface. The initial pressure is equal to \( p = 10^{12} \text{ Pa} \), acceleration is equal to \( g = 10^7 \text{ m/s}^2 \) and directed to the metal with smaller density. The computational area is the cube with the edge length \( 0.004 \text{ m} \). We set slip conditions on the upper (upper surface of the throwing plate) and lower (lower surface of the target plate) boundaries and periodic conditions of the side faces.

Note that due to the large computational costs we do not consider the whole development of the process during plates impact which is described above. Instead our statement qualitatively corresponds to the moment when the RW from the free surface of the throwing plate reaches the interface. Calculations are carried out up to the moment when the disturbances from the initial impact reach the upper and lower boundaries of the computational area.

3. Mathematical model and numerical algorithm
Mathematical model is based on 3D non-stationary two-component Euler equations [3] supplemented by different equations of state (EOS) (Cartesian frame, the notations are standard):

\[
\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} + \frac{\partial H}{\partial z} = S,
\]

\[
U = \begin{bmatrix}
\rho_1 \\
\rho u \\
\rho v \\
\rho w \\
e
\end{bmatrix},
F = \begin{bmatrix}
\rho_1 u \\
\rho u^2 + p \\
\rho uv \\
\rho uw \\
(e + p) u
\end{bmatrix},
G = \begin{bmatrix}
\rho_1 v \\
\rho uv \\
\rho v^2 + p \\
\rho vw \\
(e + p) v
\end{bmatrix},
H = \begin{bmatrix}
\rho_1 w \\
\rho uw \\
\rho vw \\
\rho w^2 + p \\
(e + p) w
\end{bmatrix},
S = \begin{bmatrix}
0 \\
0 \\
0 \\
-\rho g \\
0
\end{bmatrix},
\]

\[e = \frac{\rho (u^2 + v^2 + w^2)}{2} + \rho \epsilon (p, \rho), \quad \rho = \rho_1 + \rho_2.\]

We use three different EOS, namely: (i) ideal gas, (ii) barotropic gas and (iii) wide-range EOS for real metals.

As the first approach to explain the experimental results the model of ideal gas with the specific heat ratio \( \gamma = 5/3 \) was considered. The second considered EOS was the barotropic EOS:

\[p = p_0 + E \frac{\rho - p_0}{\rho_0},\]

where \( p_0 \) corresponds to the curve of “cold” compression of the substance which coincides with the “cold” isentropic line, \( E \) — module of elasticity in the solid phase. As the third model
we chose the wide-range semi-empirical EOS [4]. The last EOS not only well describes the condensed phase of the substance under the low temperatures but also takes into account the phase transition from the solid to the liquid one. Wide-range semi-empirical EOS are based on the extensive experimental data concerning shock compression of the solid and porous substances under normal conditions. At the extremely high pressures and temperatures they have the asymptotics of Tomas-Fermi and Debye-Huckel relations.

Numerical algorithm is based on the physical processes and spatial directions splitting techniques. The set of equations is written in the characteristics form. We solve three one-dimensional systems along each coordinate directions independently using Roe numerical flux function. Time integration is performed using explicit Euler scheme. Spatial approximation order of the scheme is increased up to second with the use of hybrid schemes approach. In this approach the switching between the central and upwind differences is realized for each characteristics based on the sign of characteristics. So the the numerical algorithm is characterized by theoretical first approximation order in time and the second approximation order in space on smooth solutions. The detailed description of the numerical algorithm could be found elsewhere [5]. The solutions of test cases including well-known Riemann problems could be found in [3]. As an example Figure 1 demonstrates the solution of the so called Sod problem [6]. The initial disintegration of discontinuity leads to the formation of “right” SW, contact surface and “left” RW. The SW smearing on three-four computational cells which is seen in Figure 1 is typical for the second approximation order schemes.

![Figure 1. Predicted density distribution (dots) in comparison with the exact solution (solid line) in the Sod problem [6]](image)

The typical size of the computational grid is 100 × 100 × 100. For the numerical simulations of the problems which are described by the hyperbolic system of equations the authors developed computer code Turbo Problem Solver (TPS) [7]. TPS has modular structure and consists from the independent blocks responsible for different parts of the numerical method. TPS provides to the user the possibility to change numerical scheme, initial and boundary conditions and mass forces. The code is written in C++ and is parallelized using Message Passing Interface (MPI) package using domain decomposition approach.
4. RTI development and growth on the interface between fluids
As a test case we chose the problem very similar to the well-known experiments of K.I. Read [9]. Consider very thin squared-shape tank, approximately $15 \times 13$ mm size. The upper part of a tank is filled with dense fluid (ethanol) and the lower part is filled with air. The densities ratio is $\rho_1/\rho_2 = 600$. 2D approximation is used. Initial disturbance is applied to the between-fluid interface itself: $\xi = a_0 \sin(2\pi X/\lambda)$ is the disturbed boundary. For our simulation $a_0 = 0.5$ mm and $\lambda = 65$ mm are taken.

Large particles method [8] is used for the simulation up to 100 ms on the mesh of $150 \times 130$ cells. Figure 2 shows the development of the instability, trajectories of the mixing zone boundaries and their velocities evolution.

![Figure 2](image)

Figure 2. Growth and development of RTI on the interface between relatively dense (e.g. ethanol) and relatively sparse (e.g. air) fluids. $\rho_1/\rho_2 = 600$.

5. Results of numerical experiments
The presented results concern the ideal gas EOS and barotropic EOS for the case of steel target plate. For the ideal gas EOS with the point disturbance in the center of one of the plates we obtained the splash in the direction to the lead plate. The shape of the splash is similar to that observed in [2]. Figure 3 shows density isosurface at the time moment $5 \mu s$ after the impact. The domain $z > 0$ corresponds to the lead plate and $z < 0$ to the steel plate.

For the barotropic EOS we also get the splash in the direction to the lead plate. The splash has the crateriform shape. Figure 4 shows density isosurface at the time moment $5 \mu s$ after the impact. The results obtained using two different EOS are in qualitative agreement.

6. Conclusions
To conclude let us figure out the main features of the process obtained in the natural experiment and confirmed by our numerical study:

(i) The instability development on the interface of the metal plates is characterized by splashes from the metal plate with the smaller density to the plate with the greater density.

(ii) The increase of the throwing plate thickness with the fixed velocity leads to the disturbance wavelength growth on the interface.
The numerical simulations are carried out with the use of software complex Turbo Problem Solver developed by the authors and suitable for the three-dimensional numerical solution of the hyperbolic problems.

References

[1] Deribas A A 1972 Physics of hardening and explosive welding (Novosibirsk: Nauka)
[2] Yakovlev I V 1973 Fiz. Gorenia i Vzryva 9(3) 447
[3] Belotserkovskii O M and Oparin A M 2001 Numerical experiment in turbulence. From order to chaos. (Moscow: Nauka)
[4] Bushman A V, Lomonosov I V and Fortov V E 1989 The models of wide-range equations of state under the high energy densities (Preprint JIHT) 6-287
[5] Fortova S V 2014 Comp. Math. and Math. Phys. 54(3) 553
[6] Toro E F 2009 Riemann Solvers and Numerical Methods for Fluid Dynamics (3d Eds., Springer)
[7] Fortova S V, Kraginskii L M, Chikitkin A V and Oparina E I 2013 *Matem. Mod.* **25**(5) 123
[8] Belotserkovskii O M 1994 *Numerical modeling in the continuum media mechanics* (Moscow: Fizmatlit)
[9] Read K I 1984 *Physica D* **12** 45.