Modeling the influence of regular wall roughnesses on the heat transport

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Abstract. We investigate the effect of regular wall roughness on the heat transport in thermal convection. The roughness is introduced by a set of rectangular heated/cooled obstacles located at the corresponding plates. An analytical model to estimate the Nusselt number deviations caused by the wall roughness is developed, which is based on the Prandtl–Blasius boundary layer equations and is valid for moderate Rayleigh numbers ($\lesssim 10^8$) and a regular wall roughness, for which the height of the obstacles and the distance between them are significantly larger than the thickness of the thermal boundary layers. The model is validated in two-dimensional simulations, for different aspect ratios of the obstacles. It is found that the model predicts the heat transport with errors $\leq 6\%$ for all investigated cases.

1. Introduction

Turbulent thermal convection between two horizontal plates with lower heated and upper cooled flat surfaces, has been the subject of numerous experimental and numerical studies; for reviews we refer to Ahlers, Grossmann & Lohse (2009); Lohse & Xia (2010). This problem is known as turbulent Rayleigh–Bénard convection (RBC). The scaling law $\dot{N}u \sim ARa^\beta$ of the heat transport in terms of the Nusselt number $\dot{N}u$ and Rayleigh number $Ra$, is the main aspect of many theoretical, experimental and numerical studies.

The influence of the wall roughness on the heat transport is important in experimental studies of classical RBC, since with growing $Ra$ the thermal boundary layer thickness decreases. Therefore any wall roughness becomes influential above a certain $Ra$. Considering that many industrial investigations of the heat transfer aim at the development of more effective heating or cooling devices, which can reduce energy consumption, underlines the importance of this problem. Also the objective of applied research is to increase the heat transfer, i.e. the constants $A$ or $\beta$ in the power law $\dot{N}u \sim ARa^\beta$, while keeping the temperature difference between the heated and cooled plates constant. One of the possible ways to increase $\dot{N}u$ is to use rough surfaces of the plates instead of smooth ones. Other important applications of turbulent natural convection over rough surfaces are found in geophysics and meteorology.

Some experimental studies Du & Tong (2000); Shen, Xia & Tong (1996) reported that rough plates cause an increase of the prefactor $A$ in the scaling law $\dot{N}u \sim ARa^\beta$ if the thermal boundary layer thickness becomes smaller than the roughness height. Other studies Qiu, Xia & Tong (2005); Roche et al. (2001); Strigano, Pascazio & Verzicco (2006) found that $\dot{N}u$ increases, but in their case the exponent $\beta$ grew up significantly. Ciliberto & Laroche (1999) observed in their
experiment that $\beta$ increases if the roughness is based on a power law distributed heights and remains unchanged in the case of a periodic roughness. Villermaux (1998) developed a model reflecting the influence of the wall roughness on the exponent in the $\mathcal{N}_u$-scaling, which is based on the assumption that $\mathcal{N}_u$ is determined by the total covering area of the thermal boundary layers. According to this model, the exponent $\beta$ is unchanged if the roughness is introduced uniformly by elements of one size. In contrast, in an experimental study Qiu, Xia & Tong (2005) found a growth of the exponent $\beta$ up to 0.35 for $Ra \geq 10^8$ even for regular roughness elements. The above mentioned studies deliver contradicting conclusions concerning the influence of the regular wall roughness on the heat transport. Therefore a comprehensive analysis and advancement of the theory of the heat transfer in convection cells with regular rough walls is required.

2. Numerical simulations and developing of the model

Assuming that the Nusselt number $\mathcal{N}_u$ is known for the case of smooth walls, we address the following question: how does the Nusselt number $\mathcal{N}_u$ deviate in the case of rough heated/cooled plates compared to the case of flat plates for moderate Rayleigh numbers $Ra \leq 10^8$?

Based on the two-dimensional (2D) Prandtl–Blasius boundary layer equations in Shishkina & Wagner (2011) a physical model to approximate the mean heat transfer between the lower heated and upper cooled rough plates, in terms of the geometrical characteristics of the roughness and $\Delta T_V \equiv T_h - T_c$, $\Delta T_H \equiv T_h - T_0$, was developed. Here $\Delta T_V$ denotes the difference between the temperatures $T_h$ and $T_c$, respectively, at the lower heated and and upper cooled plates and $\Delta T_H$ the difference between $T_h$ and the temperature $T_0$ outside the thermal boundary layer near the heated vertical surface. Introducing the ratio $\Delta \Upsilon/\Upsilon$ of the roughness volume $\Delta \Upsilon$ and the cell volume $\Upsilon$, the typical height of the roughness $h$, the typical width of the roughness elements $d$ and the height $H$ of the Rayleigh–Bénard cell, in Shishkina & Wagner (2011) the following formula was derived from the Prandtl–Blasius equations:

$$\mathcal{N}_u \approx \mathcal{N}_u_0 \left(1 + \frac{\Delta \Upsilon}{\Upsilon} \left(\frac{h^{3/4} H^{1/4}}{d} \left(\frac{\Delta T_H}{\Delta T_V} \right)^{1/4} - \frac{3}{4}\right)\right).$$  \hspace{1cm} (1)

The temperature drop $\Delta T_H$ is modelled as in Shishkina & Thess (2009).

To validate the developed model (1), 2D Navier-Stokes simulations were conducted in a square domain for $Ra = 10^8$, $Pr = 1$, $m = 4$, and for all possible combinations of $h = 0.025, 0.050, 0.075, 0.1$ and $0.125$ and $d = 0.025, 0.075$ and $0.125$. Additionally the simulations were conducted for $Ra = 10^7$, $5 \times 10^7$ and $10^8$ and 3 different roughness types (see Fig. 1 and 2). To perform 2D simulations is justified, since the developed model (1) is two-dimensional and since according to Ahlers, Grossmann & Lohse (2009) the classical integral boundary layer parameters are characterised by a laminarlike scaling in the considered range of $Ra$.

The numerical method described in Shishkina & Wagner (2005) was applied to rectangular domains. The Poisson solver for the pressure is similar to that used in Kaczorowski et al. (2008) but uses also the capacitance matrix technique as proposed by Shishkina, Shishkin & Wagner (2009). The computational mesh consisting of $600 \times 400$ nodes is fine enough to resolve the flows in the boundary layers and in the bulk, according to Shishkina et al. (2010).

The results of the simulations reveal that neither the relative increase of the total covering area, nor the relative covering area or volume of the roughness obstacles determine the Nusselt number deviations caused by the wall roughness, if considered alone. The mean heat flux is found to be generally larger for slender roughness elements (with relatively large heights and small widths) if the distance between them is large enough (larger than the obstacle height). In this case the flow field reflects secondary rolls develop between the obstacles of the same
Figure 1. Instantaneous temperature distributions \((T_c \leq T \leq T_h)\) with superimposed velocity vectors for \(Ra = 10^8\), \(Pr = 1\) and 3 different roughness types \((k)\).

Figure 2. (a) Nusselt numbers \(\dot{N}_u\) obtained in numerical simulations of thermal convection over rough heated/cooled horizontal plates (\(\diamond, \triangle, \odot\)) compared to \(\dot{N}_u\) predicted by the model (1) (\(+\)), for \(Ra = 10^8\) (\(\diamond\)), \(Ra = 5 \times 10^7\) (\(\triangle\)) and \(Ra = 10^7\) (\(\odot\)), \(Pr = 1\) and different roughness types \(k = 1, \ldots, 15\). The simulations for \(Ra = 5 \times 10^7\) and \(Ra = 10^7\) were conducted only for \(k = 05, 08, 13\) and 16. The case \(k = 16\) corresponds to the classical Rayleigh–Bénard convection with smooth walls. (b) A close-up view of (a) for \(Ra = 10^8\).

temperature, which ease the heat transport as shown in Fig. 1. For very small distances between the roughness elements, the fluid between the obstacles of the same temperature has the tendency to stagnate, which impedes the heat transfer. This can lead to only a tiny increase or even to a decrease of \(\dot{N}_u\), although the wall roughness is coherent. For a Nusselt number which is valid for a convection cell with smooth horizontal plates, the developed model predicts the Nusselt number for the case of rough plates with an error within 6% with respect to the results of the 2D numerical simulations (see Fig.2).

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References

Ahlers, G., Grossmann, S. & Lohse, D. 2009 Heat transfer and large-scale dynamics in turbulent Rayleigh–Bénard convection. Rev. Mod. Phys. 81, 503.

Ciliberto, S. & Laroche, C. 1999 Random roughness of boundary increases the turbulent scaling exponents. Phys. Rev. Lett. 82, 3998–4001.

Du, Y.B. & Tong, P. 2000 Turbulent thermal convection in a cell with ordered rough boundaries. J. Fluid Mech. 407, 57–84.

Kaczorowski, M., Shishkin, A., Shishkina, O. & Wagner, C. 2008 Development of a numerical procedure for direct simulations of turbulent convection in a closed rectangular cell, New Results in Numerical and Experimental Fluid Mechanics VI, Notes on Numerical Fluid Mechanics and Multidisciplinary Design, V. 96, E.H. Hirschel et al. (eds.), Springer Verlag, 381–388.

Lohse, D. & Xia, K. Q. 2010 Small-scale properties of turbulent Rayleigh–Bénard convection. Annu. Rev. Fluid Mech. 42, 335–364.

Qiu, X.-L., Xia K.-Q. & Tong, P. 2005 Experimental study of velocity boundary layer near a rough conducting surface in turbulent natural convection. J. Turbulence 30, 1–13.

Roche, P.-E., Castaing, B., Chabaud, B. & Hebral, B. 2001 Observation of the 1/2 power law in Rayleigh–Bénard convection. Phys. Rev. E 63, 045303.

Shen, Y., Xia K.-Q. & Tong, P. 1996 Turbulent convection over rough surfaces. Phys. Rev. Lett. 76, 908–911.

Shishkina, O., Shishkin, A. & Wagner, C. 2009 Simulation of turbulent thermal convection in complicated domains. J. Comput. & Appl. Maths 226, 336–344.

Shishkina, O., Stevens, R., Grossmann S. & Lohse, D. 2010 Boundary layer structure in turbulent thermal convection and its consequences for the required numerical resolution. New J. Physics 12, 075022.

Shishkina, O. & Theiss, A. 2009 Mean temperature profiles in turbulent Rayleigh–Bénard convection of water. J. Fluid Mech. 633, 449–460.

Shishkina, O. & Wagner, C. 2005 A fourth order accurate finite volume scheme for numerical simulations of turbulent Rayleigh-Benard convection in cylindrical containers. C. R. Mecanique 333, 17–28.

Shishkina, O. & Wagner, C. 2011 Modelling the wall roughness influence on the heat transfer in thermal convection. Submitted.

Strigano, G., Pascazio, G. & Verzicco, R. 2006 Turbulent thermal convection over grooved plates. J. Fluid Mech. 557, 307–336.

Villermaux, E. 1998 Transfer at rough sheared interfaces. Phys. Rev. Lett. 81, 4859–4862.