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Study on the detecting ability of the adaptive astronomical telescopes

Tan BiTao$^{1,2,3,4}$, RM Myers$^4$ and Chen HongBin$^1$

$^1$Institute of Optics and Electronics, Chinese Academy of Sciences, Chengdu, China
$^2$University of Chinese Academy of Sciences, Beijing, China
$^3$Northwest Institute of Nuclear Technology, Xi’an, China
$^4$University of Durham, Durham, United Kingdom

Email: tanbitao@aliyun.com

Abstract. Adaptive imaging systems have been developed to compensate for distortion introduced by atmospheric turbulence. The performance of its imaging quality can be evaluated by the Strehl ratio, but this does not directly quantify the detecting ability of an adaptively correcting telescope. Combining the normally detecting SNR and the telescope Strehl ratio, a new method evaluating the detecting ability of an adaptive astronomical telescope is put forward, which can give a quantified value. The new method is simulated on a computer, the simulation results indicated that the detecting ability of an adaptively correcting a 2-m telescopes improves 2 visual magnitudes. The effects of different atmospheric coherence length on detecting ability also can be quantitatively studied through the new method, which can give a scientific basis for the optimization of the design of the system and the development of implementations.

1. Introduction

Nowadays, due to the development of the space technology, there are more and more satellite objects in our space. The observation, the tracking and the recognizing of these objects is an interesting and very important task. As we known, the object itself cannot radiate, and depends on the reflecting of the sun rays illuminating on its surface, so the main observation method is to use the optical method with a telescope. Although the larger-aperture telescopes are efficient to collect the light, they are not able to resolve objects any better than can a telescope of diameter $\sim r_0$, the characteristic size of wave-front phase variations, and the detecting ability is also reduced by the phase variations causing the Strehl ratio to be reduced. However, telescopes with adaptive optical systems, which partially correct for wavefront distortion introduced by the atmosphere, make it possible to operate large ground based telescopes at or near the diffraction limit$^{1-3}$. At present, the study of the detecting ability of such telescopes is only based on the energy passed through the system$^4$-$^10$, using the detected signal to noise ratio for analysis, but in the practical process of detecting, the detecting ability is also affected by the quality of the space resolution performance, so the detecting signal to noise ratio method can not be used well to analyze telescopes with adaptive optical systems, and it can not give the difference...
between telescopes with adaptive optical systems and the ones without it. The Strehl ratio is usually
used to evaluate the performance of the telescope with adaptive optical systems. This is the ratio of the
on-axis irradiance in the focal plane to the on-axis irradiance that would be produced with a diffraction
limited telescope. It is evident that the detecting ability is also related to the Strehl ratio: a low Strehl
ratio make the details of the image margin blur, which results in detecting difficulty, and reduces the
detecting ability lastly.

After analysis of the characteristic of the detecting process of the telescope with adaptive optical
systems, a new signal to noise model based on the Strehl ratio is put forward (SNRS), which
synthesizes the SNR and Strehl ratio. Based on the SNRS, a new evaluating method for the detecting
ability of the telescope is presented. The new evaluating method can quantify the effects of adaptive
optical system and different atmospheric coherence lengths on detecting ability. The simulation result
indicates that the detecting ability of an adaptively correcting 2-m telescopes improves 2 visual
magnitudes, and the detecting ability improves nearly 1 visual magnitude as r0 increased from 5cm to
20cm.

2. Telescopes with adaptive optical systems

2.1. Schematic of the adaptive optical system

An arrangement for an adaptive optical system is shown in Fig.1.

![Figure 1. Schematic of an adaptive optical system](image)

The telescope is imaging a distant point like object above the atmosphere. Although the light arriving at
the telescope should be nearly a plane wave, it is instead highly distorted by passage through many
localized air masses of varying temperatures (and varying indices of refraction). This aberrated wavefront
is divided using a beamsplitter and sent to the main imaging camera (or other detection instrument such as
an imaging spectrometer) and a wavefront sensor that controls some device to correct the wavefront phase.

2.2. Performance criteria Strehl ratio

The optical transfer function or its Fourier transform, the point-spread function, provides most of the
information that is needed to describe the performance of a telescope with adaptive optical system
quantitatively. For reasons of simplicity, designers of adaptive optical systems typically try to
maximize a single number, the normalized Strehl ratio, which is the ratio of the on-axis irradiance in
the focal plane to the on-axis irradiance that would be produced with a diffraction limited system. This
is under the assumption that a high Strehl ratio, combined with an unaberrated optical train, will most
likely result in a sharp point-spread function.

For telescopes without adaptive optical systems, and only affected by a turbulent atmosphere, the
Strehl ratio is
\[
\text{Strehl} = \frac{4}{\pi D^2} \int_0^D A(\alpha) \exp[-1/2D(\alpha)]d\alpha
\]  
\text{(1)}

where \(A(\alpha)\) is the convolution of the aperture function, which for a circular aperture is given by
\[
A(\alpha) = \frac{2}{\pi} \left\{ \cos^{-1} \left( \frac{|\alpha|}{D} \right) - \frac{|\alpha|}{D} \left[ 1 - \left( \frac{|\alpha|}{D} \right)^2 \right]^{1/2} \right\}
\]  
\text{(2)}

for \(|\alpha| < D\) and \(A(\alpha)=0\) otherwise. \(D(\alpha)\) is the structure function of the atmospherically induced phase fluctuations, defined by
\[
D(r_1 - r_2) = \langle [\Phi(r_1) - \Phi(r_2)]^2 \rangle
\]  
\text{(3)}

where \(\Phi(r)\) is the accumulated random phase error, which is due to the turbulent atmosphere on the aperture plane of the telescope at coordinate \(r\). The phase structure function that is due to atmospheric turbulence has been well studied and has been determined to be
\[
D(\alpha) = 6.88|\alpha/ r_0|^{5/3}
\]  
\text{(4)}

where \(r_0\) is Fried’s coherence diameter:
\[
r_0 = \left[ 0.422(2\pi/\lambda)^2 \sec(\psi) \int_0^{\infty} C_N^2(h)dh \right]^{1/5}
\]  
\text{(5)}

where \(\lambda\) is the wavelength of the light, \(\psi\) is the angle from zenith, and \(C_N^2(h)\) is the height dependent refractive-index structure parameter.

For a partially corrected telescope, the distorted PSF consists of a sharp central peak surrounded by a halo. The width of this central peak is approximately the same as for the unaberrated system, but its amplitude is decreased. For small wavefront distortions the Strehl ratio can be approximated as
\[
\text{Strehl} = \exp(-\sigma_{\text{tot}}^2)
\]  
\text{(6)}

where \(\sigma_{\text{tot}}^2\) is the total phase variance. which is
\[
\sigma_{\text{tot}}^2 = \sigma_{\text{servo}}^2 + \sigma_{\text{dm}}^2 + \sigma_{\text{max}}^2 + \sigma_{\text{sep}}^2 + \sigma_{\text{time}}^2 + \sigma_{\text{es}}^2
\]  
\text{(7)}

where \(\sigma_{\text{servo}}^2\) is the RMS residual phase error in the closed loop, \(\sigma_{\text{dm}}^2\) is the residual wavefront error due to deformable mirror, \(\sigma_{\text{max}}^2\) is the total wavefront measurement error of the sensor, \(\sigma_{\text{sep}}^2\) is the RMS error due to angular separation between the beacon source and the astronomical light of interest, \(\sigma_{\text{es}}^2\) is the variance due to the finite size of the guide star.

3. Theory of SNRS

3.1. Detecting SNR

The detecting SNR of the telescope is a RMS values, and can be calculated as follow \textsuperscript{12}:
\[
\text{SNR} = \frac{S(t)}{\sqrt{B n_{\text{psr}} + I_d n_{\text{psr}} + I_r^2 n_{\text{psr}}}}
\]  
\text{(8)}

where \(S\) is the signal photons produced in a unit time, \(B\) is the background photons produced in a unit time, \(t\) is the integration time, \(I_d\) is the photons caused by the dark current, \(I_r\) is the photons caused by the readout noise, \(n_{\text{psr}}\) is the numbers of pixels of the target image.

Here we discuss the detection in the daytime, so it is a detection limiting by the background noise, so
\[
\text{SNR} = \frac{S(t)}{\sqrt{B n_{\text{psr}}}}
\]  
\text{(9)}

The signal photons produced in a unit time can be calculated as follow:
\[
S = \frac{\pi D^2 q_s \lambda_s c^2}{4} \epsilon^2 r_e (1 - \epsilon^2) 3.9 \times 10^{-9} \times 2.512 \times 3.9 \times 10^{-9} \times 0.2512 = S_0 \times 2.512 \times 3.9 \times 10^{-9}
\]  
\text{(10)}

where \(D\) is the diameter of the system, \(q_s\) is the quanta efficiency, \(r_o\) is the optical permeation efficiency of the system, \(r_e\) is the atmosphere permeation efficiency, \(\lambda_s\) is the average wavelength, \(ch\) is Planck’s constant, \(c\) is the velocity of light, \(\epsilon\) is the obstruct ratio of the system.
The background photons produced in a unit time can be calculated as follow:

\[
B = \frac{\pi^2}{16} d^2 q_b \lambda_n h^{-1} c^{-1} B_0 (1 - e^{-\lambda}) D^2 f^{-2} \tau_o = B_0 \cdot \frac{\pi}{4} d^2 f^{-2}
\]

where \(D\) is the diameter of the system, \(d\) is the size of the pixel, \(f\) is the focus of the system, \(q_b\) is the quanta efficiency, \(\tau_o\) is the optical permeation efficiency of the system, \(\lambda_n\) is the average wavelength, \(h\) is Planck’s constant, \(c\) is the velocity of light, \(\varepsilon\) is the obstruct ratio of the system.

3.2. Model of SNRS

In the unaberrated case, most of the image flux is focused into the central peak. For the partially corrected system, some of the image flux is scattered from the central peak to the surrounding halo. We know that the energy cannot be lost, and the imaging area becomes bigger than the unaberrated case. Then the total signal in the main lobe of the image is the same as the unaberrated case, which can be defined as

\[
S_c = S = S_0 \cdot 2.512^{-m}
\]

And the total background photons produced in a unit time can be defined as

\[
B_c = B_0 \cdot \omega_c
\]

where \(\omega_c\) is the apparent solid angle subtended by the imaged object as observed by the corrected telescope, which can be regarded as the solid angle related to the imaging area. For objects of small spatial extent, \(\omega_c\) is approximately given by the corrected telescope resolution:

\[
\omega_c = \left(2 \times 1.22 \frac{\lambda}{D}\right)^2 Strehl^{\frac{1}{2}}
\]

Finally, the model SNRS can be defined as

\[
SNRS = S_c B_c^{-\frac{1}{2}} = Strehl^{\frac{1}{2}} \cdot D \cdot S_0 \cdot 2.512^{-m} (1.22 \lambda)^{-1} B_c^{-\frac{1}{2}}
\]

The limit on how faint an object can be detected is found by setting the expression for the SNRS Eq.(15) equal to a certain value, According to empirical evidence, it is usually defined as 6 when the telescope can detect the object stably. Finally, the limited detecting ability of the telescope can be used to express the result in terms of the visual magnitude as follows.

\[
m = -\log_{10}(3.66 \lambda B_0^{\frac{1}{12}} Strehl^{\frac{1}{2}} \cdot D^{-1} \cdot S^{-1} o) [\log_{10}(2.512)]^{-1}
\]

4. Simulation

The new method of evaluating on the detecting ability based on the SNRS is simulated through computer modelling. Before the simulation, the background brightness is 14 W/m\(^2\)-Sr, the system permeation ratio is 85%, the atmosphere permeation ratio is 75%, the quantum efficiency is 85%, the integration time is 0.02s, the diameter of the system is 0.6m, the focus is 3m, and the pixel size is 16um.

In figure 2, the detecting ability of different diameters at some given atmospheric coherence length is illustrated. The given atmospheric coherence length is 5cm, 10cm, 20cm and no atmospheric effect. From Fig.2, we can find that the detecting ability is affected seriously by the atmospheric conditions, it improves as the increase of the atmospheric coherence length at the same diameter, and it also improves as increasing diameter at the same atmospheric coherence length. The detecting ability improves nearly 1 visual magnitude as the \(r_0\) improved from 5cm to 20cm. The atmosphere affects the telescope seriously, the difference in the detecting ability between atmospheric effects and no atmosphere is nearly 3 visual magnitudes, and as the coherence length increased to a certain value, the detecting ability improves a little.
In figure 3, the detecting ability of different background brightness at some given diameter is illustrated: the given diameter is 0.6m, 1m and 2m. From Fig.3, we can find that the detecting ability is reducing by the increasing background brightness.

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Figure 3. The magnitude curve of different background brightness

In figure 4, the detecting ability of the adaptive astronomical telescopes of a given phase error is illustrated, and this is compared to the diffraction limited telescope and the uncorrected telescope. From Fig.4, it can be seen that the detecting ability of the adaptive astronomical telescope is improved significantly compared to the uncorrected telescope, such that it improves about 2 visual magnitudes at aperture diameter 2m, and that 3 visual magnitudes at aperture diameter 4m, and the detecting ability of an adaptive astronomical telescope is near that of a the diffraction limited telescope when the phase error is below $\lambda/10$.

Figure 4. The magnitude curve for different conditions.
5. Conclusion
A new evaluating method for the detecting ability of the telescope, the SNRS, is defined in this paper which can give a quantified result for an adaptive astronomical telescope. The new method is used to simulate the detecting ability of the diffraction limited telescope, the atmospheric telescope and the adaptive astronomical telescope. From the simulation results, we can find that the detecting ability of the system is affected seriously by the atmosphere and residual phase error. The detecting ability improves slowly along with the increasing of the diameter under bad atmosphere conditions.

The detecting ability of astronomical telescopes improves about 2 visual magnitudes at aperture diameter 2m, and 3 visual magnitude at aperture diameter 4m, and it will be near the diffraction limited telescope when the phase error is below $\lambda/10$. The effects on detecting ability can quantitatively studied through the new method, which can give a scientific basis for the optimization of the design of the system and the development of implementations.

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