Direct search constraints on very heavy dark skyrmions

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Abstract In the standard dark matter creation scenario, dark matter arises from freeze-out due to decoupling from the thermal heat bath in the early universe. On the other hand, topological solitons can also emerge during phase transitions through the Kibble–Zurek mechanism or through bubble nucleation. In particular, Murayama and Shu found that the Kibble–Zurek mechanism can produce topological defects up to about 10 PeV, and Bramante et al. had recently pointed out that direct search constraints can be extrapolated to very large masses. Motivated by these observations, we examine direct search constraints for PeV scale dark skyrmions with a Higgs portal coupling to baryons. We find abundance constraints on the combination $g_{V}^2 M_S$ of Skyrme coupling $g_{V}$ and skyrmion mass $M_S$. We also find that extrapolation of the direct search constraints from XENON1T to very high masses constrains the combination $g_{wh}/g_{V}^4$ as a function of $M_S$, where $g_{wh}$ is the Higgs portal coupling of the dark skyrmions.

1 Introduction

The dark matter puzzle motivated numerous investigations of the question how dark matter may have been produced in the early universe. Dark matter up to a mass\textsuperscript{1} of about 100 TeV [1] can be produced in freeze-out from a thermal heat bath if the dark matter interaction rate with baryons becomes suppressed by the cosmic expansion [4]. Dark matter can also arise in the form of topological defects in phase transitions [5], or due to freeze-in if the interactions were too weak to ever establish thermal equilibrium in the dark sector [6]. Generation during or near the end of inflation is another possibility [7,8].

Numerous other proposals have been discussed in the literature, but here we would like to focus on direct search constraints on very heavy dark matter from the Kibble–Zurek mechanism [5,9–11]. This is motivated by the observation of Murayama and Shu that this mechanism can produce non-thermal PeV scale dark matter [12], and by the recent observation of Bramante et al. that direct search constraints can be extrapolated to very high mass values [13]. We assume that the dark matter can interact with the baryonic sector through a Higgs portal coupling for the calculation of the pertinent nucleon recoil cross section. We use a skyrmion scenario where the very heavy skyrmions are coherent states of heavy “mesons”, and it is the heavy mesons which couple to baryons through the Higgs portal. The ensuing separation of mass scales ensures that the very heavy dark matter can be non-thermally created as topological defects while the mesons have strong enough coupling to baryons to prevent the mesons’ freeze-out\textsuperscript{2} into a sizable dark matter component. This sets the present model apart from standard thermal Higgs portal models and implies different constraints from direct search experiments.

Higgs portal dark matter from thermal freeze-out has been extensively discussed and is well understood, see e.g. [14, 15] and references there. However, the mechanism of dark matter generation determines what (if any) relation exists between dark matter mass and coupling strength to baryons, thus also determining the relation between dark matter mass and nucleon recoil cross section from the theoretical side of matching to direct search constraints. We are therefore interested in the constraints on topological Higgs portal dark matter which arises through a phase transition in the dark

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\textsuperscript{2} The designation “freeze-out” is also extensively used in the literature to describe the formation of topological defects during quenching during a phase transition, as described by the Kibble–Zurek mechanism. Here we will use “freeze-out” only to refer to the standard cosmological freeze-out from the primordial heat bath, but not to the inhomogeneous formation of local ground states during a phase transition.
sector without subsequent thermalization and thermal freeze-out.

In particular, a chiral $G = SU_L(2) \times SU_R(2) \rightarrow H = SU_V(2)$ phase transition in the dark sector will create skyrmions from the non-trivial homotopy group $\pi_3(G/H) = \mathbb{Z}$ which arises from the mappings of compactified $\mathbb{R}^3 \rightarrow S^3$ into $S^3$ [16–18,20,21]. The compactification and the mapping arise from the boundary condition $\lim_{|x| \to \infty} U(x) = 1$ on the fields $U(x)$, which map $\mathbb{R}^3 \rightarrow S^3$ for fixed time $t$,

$$U(x) = \exp[i \mathbf{w}(x) \cdot \mathbf{\sigma} / f_w] = \cos(|\mathbf{w}(x)| / f_w) + i \hat{\mathbf{w}}(x) \cdot \mathbf{\sigma} \sin(|\mathbf{w}(x)| / f_w).$$

(1)

Here the $\sigma_i$ are the Pauli matrices of a dark $SU(2)$ isospin symmetry and the vector $\mathbf{\hat{w}} = \mathbf{w} / |\mathbf{w}|$ is the unit vector for the triplet $w_i(x)$ of dark mesons.

The winding number $W = \int d^3x W^0$ of the mapping $S^3 \rightarrow S^3$ is related to the conserved current (with the convention $e^{i123} = -1$)

$$W^\mu = \frac{1}{24\pi^2} e^{\mu\nu\rho\sigma} \text{Tr}(\partial_\nu U \cdot U^+ \cdot \partial_\rho U \cdot U^+ \cdot \partial_\sigma U \cdot U^+).$$

(2)

We will denote $W$ as skyrmion number, since the standard designation as “baryon number” is obviously inappropriate: The skyrmions in the dark sector are not baryons, neither in the hadronic sense nor in the cosmologist’s definition of baryonic matter.

The spherically symmetric skyrmions

$$w_S(x) = w_S(r)\hat{r}$$

(3)

are stable with skyrmion number $W = \pm 1$ if $w_S(0) = \pm \pi f_w$ [19,21,22]. It is therefore natural that skyrmions are discussed as dark matter candidates. Electroweak scale dark skyrmions from a non-standard Higgs sector are constrained to masses up to about 10 TeV [23–28]. If the coupling of the skyrmions to baryons is strong enough to thermalize the skyrmions after the phase transition, then their relic abundance is determined by thermal freeze-out [23,26–28]. However, if the skyrmions do not thermalize after the phase transition, then it is the phase transition itself which determines any relic skyrmion abundance. Such a scenario was discussed by Campbell et al. for electroweak skyrmions [24]. Furthermore, Murayama and Shu had pointed out that topological dark matter with masses up to 10 PeV could be generated through the Kibble–Zurek mechanism [12], and indirect signals from very heavy dark skyrmions through Higgs or neutrino portals were discussed in [29].

In the present paper we are interested in direct search constraints on very heavy dark skyrmions, since Bramante et al. had pointed out that limits from existing direct search experiments can principally be extrapolated to very high masses [13]. The non-thermalized dark skyrmions are not constrained by the standard indirect [30–40] and direct [14,15,41–56] search analyses for Higgs portal dark matter since the different creation mechanism changes the relation between mass and couplings. However, we can adopt estimates on the ratio $x_c = M_S / T_c$ from hadronic physics. This allows us to constrain the skyrmion parameter $g_V^2 M_S$ from the requirement $\Omega_S \leq \Omega_{\text{CDM}}$. Direct search constraints from XENON1T then allow us to constrain a combination of the skyrmion coupling and the skyrmion’s Higgs portal coupling as a function of skyrmion mass.

We will review some pertinent aspects of dark skyrmions in Sect. 2 and confirm that skyrmions are good candidates for PeV scale dark matter through a new analysis of their current abundance which takes into account that the Boltzmann equation erases initial conditions for $\Gamma \gtrsim H$ if the reverse process to $S^3$ annihilation is kinematically suppressed. Section 3 discusses constraints on the heavy $w$ mesons which arise from the requirement that the relic very heavy dark skyrmions, but not the heavy $w$ mesons, are the dominant dark matter component. The direct search constraints on PeV scale dark skyrmions are then discussed in Sect. 4. Section 5 summarizes our conclusions.

2 Dark skyrmions from chiral phase transitions

We parametrize chiral symmetry breaking in a dark sector with a Lagrangian

$$\mathcal{L} = \mathcal{L}_S + \mathcal{L}_{\text{DSM}}.$$  

(4)

The fourth order term in the Skyrme Lagrangian

$$\mathcal{L}_S = \frac{1}{16\lambda^2} \text{Tr}[\partial^\mu U \cdot \partial^\nu U^+ \cdot (\partial_\mu U \cdot \partial_\nu U^+ - \partial_\nu U \cdot \partial_\mu U^+)] - \frac{f_w^2}{4} \text{Tr}(\partial_\mu U \cdot \partial^\mu U^+)$$

(5)

stabilizes skyrmions as topological solitons against Derrick’s theorem [16–18,20,21]. The coupling constant $g_V$ is usually written as $e$, but we avoid this notation. The designation $g_V$ is motivated by the observation that the skyrmion coupling can arise from vector dominance due to a hidden local $SU_V(2)$ invariance [18,57].

The skyrmions (3) can be thought of as coherent excitations of the dark meson fields $\mathbf{w}(x)$, and the Skyrme Hamiltonian

$$\mathcal{H}_S = \frac{1}{16\lambda^2} \text{Tr}[\partial_\mu U \cdot \partial_\nu U^+ \cdot (\partial_\nu U \cdot \partial_\mu U^+ - \partial_\mu U \cdot \partial_\nu U^+)] + \frac{f_w^2}{4} \text{Tr}(\partial_\mu U \cdot \partial_\mu U^+)$$

(6)

defines the mass for the static skyrmion solutions through the energy functional

$$E_S = \int d^3x \mathcal{H}_S.$$  

(7)
with the result [18,21]

\[ M_S \simeq 73 f_w / g_V. \]  

(8)

Their effective extension can be estimated as \( L_S \simeq 1.2/ (g_V f_w) \) [58]. The representation (6) for the Skyrme Hamiltonian uses the convention that summation over lower pairs of 4-indices is defined as Euclidean, e.g. \( \partial_\mu U \cdot \partial_\mu U^+ \equiv \partial_0 u \cdot \partial_0 u^+ + \nabla U \cdot \nabla U^+ \).

The Skyrme Lagrangian (5) is symmetric under the chiral symmetry \( U(x) \rightarrow L \cdot U(x) \cdot R^+ \), but the boundary condition \( \lim_{|x| \rightarrow \infty} U(x) = 1 \) breaks this symmetry to the diagonal subgroup \( SU_v(2)_L \otimes SU_v(2)_R \rightarrow SU_v(2) \) in the form of a local axial \( SU_v(2) \) transformation (1), i.e. under parity \( P \), \( P \circ U(x) \circ P = U^{-1}(x) \). Just like for ordinary pions in chiral perturbation theory, the residual \( SU_v(2) \) transformation

\[
\begin{align*}
\tilde{w}^I(x) \cdot \sigma &= V \cdot w(x) \cdot \sigma \cdot V^+ \\
&= \exp(i \phi \cdot \sigma / 2) \cdot w(x) \cdot \sigma \cdot \exp(-i \phi \cdot \sigma / 2)
\end{align*}
\]

implies a standard spin-1 rotation representation on the dark mesons,

\[
\begin{align*}
\tilde{w}^I(x) &= R(\phi) \cdot w(x) = \exp(\phi \cdot \mathbf{L}) \cdot w(x) \\
&= \hat{\phi}(\hat{\phi} \cdot w(x)) + (1 - \hat{\phi} \otimes \hat{\phi}) \cdot w(x) \cos \phi \\
&= -\phi \times w(x) \sin \phi.
\end{align*}
\]

Here \( (\mathbf{L})_{jk} = \epsilon_{ijk} \) are the standard \( SO(3) \) generators. Adding a mass term for the Goldstone bosons,

\[ L_m = m_w^2 \hat{f}_w^2 \frac{\text{Tr}(U - 1)}{2} \]

(10)

respects the unbroken symmetry and is compatible with the effective Goldstone boson interpretation of the \( w \) triplet if \( m_w \ll M_S \).

Following Ref. [29] we also include a Higgs coupling for the dark mesons,

\[
\begin{align*}
L_{DSM}^{(b)} &= \frac{g_{wh} f_w^2}{2} (2H^+(x)H(x) - v_h^2) \text{Tr}(U(x) - 1) \\
&= g_{wh} f_w^2 v_h \left( h(x) + \frac{h^2(x)}{2v_h} \right) \text{Tr}(U(x) - 1).
\end{align*}
\]

The exact nature of chiral phase transitions for different chemical potentials and different numbers of underlying fermion flavors is still a matter of investigation. The non-thermal abundance of topological defects from a second order phase transition has been estimated by Muramaya and Shu [12], while Campbell et al. provided an estimate for a first order transition [24].

The finite correlation length \( \xi \) from quenching during a second order phase transition implies that anti-skyrmions and skyrmions with different centers will form with a density of order \( \xi^{-3} \) during the phase transition. The Kibble–Zurek based estimate of Muramaya and Shu for the abundance of particle-like topological defects from cosmological phase transitions therefore applies,

\[
\frac{n_S}{S} \bigg|_{T = T_c} \simeq 0.006 \left( \frac{T_c}{M_{Pl}} \right)^{1.2}.
\]

(12)

Here \( n_S \) denotes the sum of skyrmion and anti-skyrmion densities, and a critical exponent \( v \simeq 2/3 \) was assumed [12]. On the other hand, Campbell et al. assume a first order phase transition in an approximately scale-invariant model with chiral symmetry breaking and a pseudo-dilaton \( \chi \). They find an estimate for the skyrmion abundance

\[
\frac{n_S}{S} \bigg|_{T = T_c} \simeq 3.2 \left( \frac{m_\chi}{v_\chi} \right)^{3/2},
\]

(13)

depending on the ratio of dilaton mass to vacuum expectation value \( m_\chi/v_\chi \) [24]. We will find that the relic dark skyrmion abundance decouples from the order of the phase transition and the initial skyrmion abundance if \( \Gamma(T_c) \gtrsim H(T_c) \) while thermal skyrmion creation is kinematically suppressed.

Numerical simulations indicate that skyrmions annihilate with their classical cross section \( \sigma_{SS} = \pi L_5^2 \) into a number 2 \( < (N_w) < M_S/m_w \) of \( w \) particles [59,60]. The simulations for hadronic skyrmions actually yield 2 \( < (N_w) \lesssim 7 \), which is in remarkable agreement with observations of low-energy \( p\overline{p} \) annihilations. These annihilations produce 5 \pm 1 pions, with pion multiplicities 3 \( \leq n_\pi \leq 7 \) in 99% of measurements [61]. The \( w \)-bosons then annihilate into baryons with annihilation cross sections for collision energies which are much larger than the top mass, \( \sqrt{s} \gtrsim 2m_w \gg m_t \) [29],

\[
\nu_{\sigma_{ww} \rightarrow hh} = \nu_{\sigma_{ww} \rightarrow ZZ} = \frac{1}{2} \nu_{\sigma_{ww} \rightarrow W^+W^-} = g_{wh}^2 \frac{4\pi \xi_s}{\alpha_s}.
\]

(14)

and

\[
\nu_{\sigma_{ww} \rightarrow f\overline{f}} = N_c \frac{g_{wh}^2 m_f^2}{\alpha_s \xi_s}.
\]

(15)

Their leading order annihilation cross section into Standard Model states is therefore

\[
\nu_{\sigma_{ww}} = \frac{g_{wh}^2}{4\pi \xi_s}.
\]

(16)

Both quenching and bubble nucleation delay completion of a phase transition, such that topological defects would be born into a heat bath with lower temperature \( T_1 \) than their mass, \( T_1 \lesssim T_c \lesssim M_S \). Furthermore, we would generically need collision of \( (N_w) \sim M_S/m_w \gg 2 \) \( w \) particles to produce an \( SS \) pair, such that these processes are phase-space suppressed with a factor \( \exp (- (N_w)E_w / T) \). Both of these effects suppress the thermal creation of the dark skyrmions from baryonic matter. Therefore, similar to
magnetic monopoles [62], we need to solve the balance equation
\[
\frac{d}{dt}(n_S a^3) = -\frac{1}{2} n_S^2 a^3 \langle \sigma v_{SS} \rangle
\]  
(17)

with initial conditions given by Eqs. (12) or (13), respectively, to determine the relic skyrmion density today. We encounter a factor 1/2 in the skyrmion annihilation rate in equation (17) because \( n_S = \tilde{n}_S + \tilde{n}_S \simeq 2 \tilde{n}_S \) is the sum of skyrmion and anti-skyrmion densities, and the skyrmion annihilation rate is
\[
\frac{d}{dt}(\tilde{n}_S a^3) = -\tilde{n}_S \tilde{n}_S a^3 \langle \sigma v_{SS} \rangle \simeq -\frac{1}{4} n_S^2 a^3 \langle \sigma v_{SS} \rangle.
\]  
(18)

The temperature-dependence of \( \langle \sigma v_{SS} \rangle \) depends on how efficiently the dark mesons can couple the skyrmions to the primordial heat bath. The possible range of temperature dependences can be parametrized by
\[
\langle \sigma v_{SS} \rangle = \langle \sigma v_{SS} \rangle_c (T/T_c)^\beta \equiv \langle \sigma v_{SS} \rangle_c (t_c/t)^\beta/2,
\]  
(19)

with \( 0 \leq \beta \leq 1 \) [4].

Evolving the skyrmion density from the time \( t_c \) of creation to the epoch of radiation-matter equality yields for \( t_{eq} \gg t_c \)
\[
n_S(t_{eq}) \simeq \frac{n_S(t_c)}{1 + n_S(t_c)(1 + \beta)^{-1} \langle \sigma v_{SS} \rangle_c t_c} \left( \frac{t_c}{t_{eq}} \right)^{3/2}.
\]  
(20)

Annihilation is negligible at later stages, and we find a relic skyrmion density
\[
n_S(t_0) \simeq \frac{n_S(t_c)}{1 + n_S(t_c)(1 + \beta)^{-1} \langle \sigma v_{SS} \rangle_c t_c} \tilde{z}_c^{-3},
\]  
(21)

where \( \tilde{z}_c \) is the redshift at the epoch of the chiral phase transition in the dark sector. This can be expressed in terms of entropy ratios
\[
\tilde{z}_c^{-3} \simeq \frac{a_c}{a_{eq}} \simeq \frac{s_0}{s_c} = \left( \frac{2\pi^2/45 g_{*S}(T_c) T_c}{2\pi^2/45 g_{*S}(T_e) T_e} \right)^3.
\]  
(22)

The annihilation term in the denominator in Eq. (21) is related to the ratio of the skyrmion reaction rate and the expansion rate at the phase transition,
\[
n_S(t_c)(1 + \beta)^{-1} \langle \sigma v_{SS} \rangle_c t_c = \Gamma_S(t_c)/(1 + \beta) H(t_c).
\]  
(23)

This will be larger than 1 at \( t_c \), and we find
\[
n_S(t_0) \simeq \frac{(1 + \beta)}{1 + n_S(t_c)(1 + \beta)^{-1} \langle \sigma v_{SS} \rangle_c t_c} \frac{s_0}{(2\pi^2/45 g_{*S}(T_c) T_c)^3}.
\]  
(24)

Time and temperature at the phase transition are related by radiation domination,
\[
t_c T_c^2 = \frac{3}{4\pi} M_{PL} \sqrt{\frac{5}{\pi g_{*S}(T_c)}}.
\]  
(25)

3 Constraints on the heavy dark mesons

We are particularly interested in the very heavy dark skyrmions as the dominant dark matter component, and therefore presume that the heavy \( w \) mesons do not contribute to \( \Omega_{CDM} \) at a significant level. I.e. besides \( 1 \) TeV \( \leq m_w \ll M_S \), we also assume that \( g_{wh} \) is sufficiently large to prevent thermal freeze-out of the dark mesons at a level where they would contribute a significant fraction to the dark matter.
according to Figs. 1 and 2, \( m_w/g_y^2 M_S \sim 10^{-3} \) while \( g_{wh} \) is of order 1. Furthermore, we certainly have \( v_{S \Sigma} \ll 1 \) for the very heavy relic skyrmions, i.e. we can safely assume that the very heavy dark skyrmions are the dominant dark matter component in the model for the parameter ranges identified in Figs. 1 and 2. The “dark mediator bottleneck” that was considered as a generic possibility in Ref. [29] therefore does not materialize in this model. 4

### 4 Direct signals from the dark skyrmions

Bramante et al. have pointed out that existing exclusion limits from direct searches can be extrapolated to very high masses [13], and we are particularly interested in direct search constraints on PeV scale dark skyrmions.

To calculate the expected nucleon recoil cross section from virtual Higgs exchange, we consider the skyrmion as a coherent superposition of the \( w \) mesons, i.e. we use the coherent state

\[
|S(t)\rangle = \exp\left(-\frac{1}{2}\int d^3k \, |\xi(k, t)|^2\right) \times \exp\left(\int d^3k \, \xi(k, t) \cdot a^+(k)\right) |0\rangle
\]

(31)

with

\[
\xi(k, t) = \sqrt{\frac{\omega(k)}{2}} w_S(k) \exp[i\omega(k)t].
\]

(32)

Here \( \omega^2(k) = k^2 + m_w^2 \) and \( w_S(k) \) is the skyrmion in \( k \) space,

\[
w_S(k) = \int \frac{d^3x}{\sqrt{2\pi}} w_S(r) \hat{x} \exp(-ik \cdot x)
\]

\[
= \sqrt{\frac{2}{\pi}} \hat{k} \int_0^\infty dr \frac{r}{k} w_S(r) \sin(kr) = k w_S(k).
\]

(33)

The state (31,32) has the property to yield the skyrmion as the expectation value of the \( w \) meson operator

\[
w(x) = \int \frac{d^3k}{4\sqrt{\pi^2 \omega(k)}} \left[ a(k) \exp(ik \cdot x) + a^+(k) \exp(-ik \cdot x)\right],
\]

(34)

i.e.

\[
\langle S(t) | w(x) | S(t) \rangle = w_S(r) \hat{k}.
\]

(35)

The state (31) and operator (34) are the interaction picture state and operator, respectively.

\[
|S(t)\rangle = \exp(iH_0 t / \hbar) |S\rangle,
\]

(36)

---

3 Otherwise, if thermally created heavy \( w \) mesons would be the dominant dark matter component, the standard constraints on thermally created scalar Higgs portal dark matter would apply to them, see [14,15] and references there.

4 What has been missed in Ref. [29] was the energy conservation estimate for \( \langle N_w \rangle / (s_w^{-1}) \) and the fact that contrary to \( n_w / n_S \), the relevant ratio \( \Omega_w / \Omega_S \) comes with an extra power of \( m_w / M_S \).
Fig. 3 The skyrmion in $k$ space

$$w(x) = \exp(iH_0t/\hbar)w(x)\exp(-iH_0t/\hbar),$$  (37)

with $H_0 = \int d^3ka_\omega(k)a^+(k) \cdot a(k)$. The Schrödinger picture skyrmion state $|S\rangle \equiv |S(0)\rangle$ is of course time-independent for the static skyrmion solution. The time-dependent factor $\exp[i\omega(k)t]$ in $\zeta(k, t)$ arises from the shift of the factor from the interaction picture operator $a^+(k, t) = a^+(k)\exp[i\omega(k)t]$ into the amplitude $\zeta(k)$.

The expectation value of the $w$ meson number in the skyrmion state is

$$\langle n_w \rangle = \int d^3k |\zeta(k, t)|^2 = \frac{1}{2} \int d^3k \omega(k)w^2_S(k)$$  

$$= 2\pi \int_0^\infty dk k^2 \omega(k)w^2_S(k).$$  (38)

This yields for the dark skyrmion-nucleon recoil cross section the estimate

$$\sigma_{SN} = \langle n_w \rangle^2 \sigma_{wN},$$  (39)

with the nucleon recoil cross section of the $w$ particles (for $m_w \gg m_N$)

$$\sigma_{wN} = \frac{1}{\pi} \left( \frac{g_{wH_N} v_h m_N}{m_h^2 m_w} \right)^2.$$  (40)

We use the abundance weighted average nucleon mass in stable or long lived Xenon isotopes $m_N = 930.6$ MeV in evaluations of Eqs. (39, 40). The coupling constant $g_{hN}$ in Eq. (40) is the effective Higgs-nucleon coupling [64] with a modern value $g_{hN} v_h = 289$ MeV [65].

We use the semi-analytic approximation

$$\frac{w_S(r)}{f_w} = 2 \arccos \left[ 1 - \exp\left( -\frac{\sqrt{2}}{a} g_V f_w r \right) \right],$$  (41)

$a = 0.9451$ [19], for the skyrmion profile in numerical evaluations. This yields Fig. 3 for the skyrmion profile in $k$ space. The $w$ meson number is shown in Fig. 4.

On the other hand, for comparisons with direct dark matter searches it is more useful to express this in terms of the actual dark matter mass $M_S$,

$$\sigma_{SN} = \frac{1}{\pi} \left( \frac{g_{wH_N} v_h m_N}{m_h^2 m_w} \right)^2 \left( \frac{g_{wh}}{g_V^2 m_w} \right)^2$$  

$$= 2.071 \times 10^{-7} \times \left( \frac{g_{wh}}{g_V^4 M_S} \right)^2.$$  (42)

This yields the skyrmion-nucleon recoil cross sections in Fig. 5.
Fig. 6 The skyrmion-nucleon recoil cross section for $100\text{TeV} \leq \mu_S \equiv g_V^4 M_S / g_{wh} \leq 100\text{PeV}$

Fig. 7 Direct search constraints on very heavy dark skyrmion parameters from the XENON1T 1-year results

\[
\sigma_{SN} = 4.575 \times 10^{-4} \times \left( \frac{g_{wh}}{g_V^4 M_S} \right)^2.
\]

(43)

This yields the skyrmion-nucleon recoil cross sections in Fig. 6.

Extrapolation of the XENON1T 1-year constraints to high masses implies $\sigma_{SN} \lesssim 8.3 \times 10^{-46} \text{cm}^2 \text{MeV}^{-1} \times F(M_S)$ [66], where $F(M_S) = |S(1\text{TeV})/S(M_S)|^2$ is the corresponding ratio of Helm structure factors squared. Our result (43) then constrains the coupling constant $\eta \equiv g_{wh} / g_V^4$ as a function of skyrmion mass. This is displayed in Fig. 7.

The constraints yield $g_V \gtrsim 1.8 \times g_{wh}^{1/4}$ for $M_S = 100\text{TeV}$ and $g_V \gtrsim 0.32 \times g_{wh}^{1/4}$ for $M_S = 10\text{PeV}$, i.e. an underlying gauge theory to induce the very heavy dark skyrmion model through the BKUYY mechanism [57] could require non-perturbative coupling strength $g_V$ to satisfy direct search constraints at the lower end of the non-thermal dark matter mass range, but very weak gauge coupling $g_V$ would be still possible for higher dark skyrmion masses.

The improved sensitivities of next-generation and next-to-next generation direct search experiments to $g_{wh} / g_V^4$ are displayed in Fig. 8.

For actual parameters for the next-generation Xenon based experiments, we used XENONnT [67], but the anticipated sensitivities and time scales for LZ [68] and PandaX-4T [69] are comparable. For the DARWIN experiment see [70]. The neutrino floor is too close to the DARWIN limit in the high mass region to resolve on the logarithmic scales used in Fig. 8.

5 Conclusions

In the future, multi-tonne scale direct search experiments with long exposures will provide stronger constraints on very heavy dark matter, and this will eventually also probe the non-thermal dark matter mass range beyond the Griest-Kamionkowski bound around 100 TeV. Indeed, extrapolation from current direct search results already implies constraints for nucleon recoil cross sections at very high masses. It is therefore of interest to develop theoretical models and techniques for nucleon recoils from very heavy dark matter. We find that very heavy dark skyrmions from an early chiral phase transition provide an interesting avenue to theoretical descriptions of dark matter in the PeV mass range. Their non-thermal creation implies that the parameter $x_c = M_S / T_c$ is not determined from a freeze-out condition, but we can use estimates for $x_c$ from hadronic skyrmions. The requirement $\Omega_S \lesssim \Omega_{CDM}$ then constrains the parameter $g_V^2 M_S$ as a function of $x_c$, as shown in Fig. 1. We could also derive an estimate for the nucleon recoil cross section of the dark skyrmions through Higgs exchange (43). Comparison with direct search constraints then limits the parameter $\eta = g_{wh} / g_V^4$ as a function of skyrmion mass $M_S$, as shown in Fig. 7.

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