Reverse flow and vortex breakdown in a shear-thinning fluid

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Abstract. The effect of polymer concentration on the development of reverse secondary flow and vortex breakdown was studied using a viscoelastic solution of polyacrylamide in water. The fluid was contained in cylindrical containers of two different radii, the top end wall of which rotated at a varying speed, thus, imparting a circulating motion to the fluid. Whereas using a newtonian fluid, streamlines will occupy the entire container, the flow of a shear-thinning fluid may divide into two cells of opposite circulating motion. The curve of critical Reynolds and elasticity numbers (Re, E) values corresponding to the development of reverse flow was obtained over a wide range of Re values. Vortex breakdown was found to occur at extremely low Re values.

1. Introduction
The instabilities and structural changes undergone by confined rotating flows have been a matter of interest from the scientific as well as the technological standpoint.

One largely studied system consists of a newtonian or viscoelastic fluid confined in a cylindrical container fitted with a rotating end wall. Using newtonian fluids, an increased angular velocity eventually results in a dominant centrifugal force that induces secondary flow in the positive radial direction in the proximity of the rotating end wall and in the opposite direction in the vicinity of the fixed end [1,2]. However, using viscoelastic fluids, the resulting flow may divide into two cells, in one of which the direction of secondary rotation is opposite to that found in newtonian fluids [3,4,5,6]. In addition, under certain conditions, viscoelastic fluids may flow in more varied patterns than those found in newtonian fluids.

Both newtonian and non-newtonian fluids show structural changes referred to as vortex breakdown (VB), characterized by the appearance of a point of stagnation and the onset of reverse flow development. Stokes et al [4,5] studied the development of VB in polymer solutions in a water-glycerine mixture, using a large proportion of the latter to ensure that the overall viscosity was only slightly dependent on the shear rate. The authors reported that VB occurred at an increasing value of the Reynolds number (Re) with increasing polymer concentration, and that no VB was observed above a critical concentration value, the two polymer types used by the authors (polyacrylamide and xantham gum) leading to qualitatively similar behaviour.
Confined viscoelastic fluids flowing at low \( Re \) values have been reported to develop elasticity-driven turbulence similar to that developed by Newtonian fluids at high \( Re \) values [7,8], in addition to certain instabilities similar to those found in laminar flow [4].

Despite the multitude of parameters that may be used to describe flow patterns, the overall behaviour of viscoelastic fluids is generally described by \( Re \) and the Deborah number (\( De \)). The former is defined as that used for Newtonian fluids, \( Re = U L / v \), where the viscosity \( v \) depends on the shear rate; and the latter is defined as \( De = \tau / \dot{\gamma} \), where \( \tau \) is the polymer relaxation time and \( \dot{\gamma} \) is the shear rate. Whereas other dimensionless parameters may also influence the behaviour of viscoelastic fluids, their influence has been assumed to be low and has generally been neglected (see above references).

In this work, an aqueous solution of polyacrylamide at a concentration varying from 0.03% to 1.0% was used to study the structural changes undergone by shear-thinning fluids. The conditions that define the development of reverse flow were studied in detail, and the capacity of \((Re, De)\) plots to adequately describe the critical conditions resulting in such flow revised. The relative relevance of additional parameters was considered. The elasticity number \( E = De / Re \), a measure of the relative importance of elastic and inertial effects on the flow dynamics) and the \( Re \) number were used for dimensionless characterization of reverse flow. The \( Re \) value required for the occurrence of VB was determined under the assayed conditions and compared with the critical value leading to VB in Newtonian fluids.

2. Experimental set-up

Two cylindrical containers of internal diameter \( R_1 = 40.0 \) mm and \( R_2 = 67.5 \) mm, respectively, were constructed of acrylic plastic. The top end wall of the containers was fixed to the shaft of a DC motor rotating at an electronically-controlled speed varying between \( \omega = 0.06 \) rad/s and \( \omega = 42.77 \) rad/s. The shaft-motor system was so designed that no shaft precession was observed. The height of the container end wall was adjustable to within an accuracy of 1.0%. An aspect ratio \( H/R = 2 \) was used. In order to optimize the optical system and minimize the effects of light refraction, the cylindrical container was placed inside an outer container, square in cross-section, of side \( L = 150.0 \) mm.

An aqueous polyacrylamide solution was used as study fluid. A water-soluble, non-ionic polyacrylamide powder (Sigma nº 92560) was dissolved to different concentrations (0.03%, 0.1%, 0.5% and 1.0% by weight). Sample rheology was analysed on a Physica MCR 300 rheometer\(^1\).

Digital particle image visualization (DPIV) was used to obtain velocity fields within the fluid. A 2-mm-thick vertical laser sheet (500mW green laser) crossing the container along its axis was used as illumination means. The fluid was seeded with 50-\( \mu \)m neutral polyacrylamide particles. Images were recorded on a Cmos camera (PL 741 Pixelink) at 90 fps. To obtain velocity profiles, standard cross-correlation algorithms were used between consecutive images with 64x64-pixel interrogation windows and 16-pixel overlapping.

The \( Re \) number, defined as \( Re = \rho \omega R^2/\eta \) (where \( \rho \) is the fluid density, \( \omega \) is the angular velocity, \( R \) the container radius and \( \eta \) is the dynamic viscosity), varied between 0.009 and 1800, as determined with an accuracy lower than 2%. The fluid temperature was kept at 20ºC at all times during all runs. The rotation speed was varied very slowly, allowing the necessary time (between 3 and 5 min, depending on the polymer concentration) between increments for flow patterns to become steady before the visualization of structures developing within the fluid. At a concentration of 0.5% or 1.0%, the fluid flowed in transient state for around 4 min. Moreover, at these concentrations, the system also

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1 Samples were characterized by Grupo de medios Porosos, Facultad de Ingeniería, Universidad de Buenos Aires, Argentina
showed hysteresis – i.e., critical $Re$ values at which structures developed differed according to the increasing or decreasing nature of speed variation.

**Figure 1:** Experimental set-up

### 3. Results

Figure 2 shows typical reverse flow patterns, as well as velocity profiles on the $z$ and radial axes. The reverse flow zone is situated between the fixed end wall of the cylinder and a plane crossing its axis perpendicularly at a distance $z^*$, defined as the point of stagnation – i.e., where the vertical velocity $v_z$ along the cylinder axis changes sign. As the rotation speed increases, the reverse flow zone decreases in size, eventually disappearing above a critical value.

**Figure 2:** a) Streamlines of an aqueous solution of 0.5% w/w PAA; container radius: R1; Re=9x10$^{-3}$, De= 0.45. b) Vertical velocity profile as a function of height along the cylinder axis, represented in a) by a vertical red line. c) Radial velocity as a function of radius on the plane represented in a) by a horizontal red line
Figure 3 shows the experimental flow pattern and $v_z$ values within an interval around the point of stagnation.

**Figure 3:** a) Streamlines of an aqueous solution of 0.5% w/w PAA; container radius: R2; Re = 0.22, De=3.8 b) Vertical velocity profile as a function of height along the cylinder axis within a zoomed-in plot area around its centre.

Table 1 shows critical $Re$ and $De$ values leading to the disappearance of reverse flow, using fluids of different polymer concentration in the two containers. As both the fluid viscosity and relaxation time were found to depend on the angular velocity of the rotating end wall, $Re$ and $De$ were calculated accordingly, using the $\eta$ and $\tau$ values determined at the angular frequency being used. Table 1 also shows those critical values leading to bubble formation, associated with VB. It did not occur at high polymer concentrations in the fluid, consistently with previously reported results [4,5]. However, these results show that VB developed at fairly low $Re$ values ($Re = 100$), as opposed to the reported data ($Re = 1000$).

| Concentration | Flow reversal | Flow reversal | Vortex breakdown | Vortex breakdown |
|---------------|---------------|---------------|------------------|------------------|
|               | Radius=4.00cm | Radius=6.75 cm | Radius=4.00cm    | Radius=6.75 cm   |
| 0.03%         | No flow reversal | No flow reversal | Re = 58.85 De = 8.35 | Re = 57.52 De = 3.78 |
| 0.1%          | Re = 17.85 De = 3.36 | No flow reversal | Re = 4.81 De = 11.10 | Re = 128.00 De = 101.10 |
| 0.5%          | Re = 1.51 De = 13.20 | Re = 0.71 De = 7.50 | No bubble formation | No bubble formation |
| 1%            | Re = 0.71 De = 17.4 | Re = 0.22 De = 7.50 | No bubble formation | No bubble formation |

**Table 1:** Critical values of the Reynolds and Deborah numbers according to polymer concentration. Values show the disappearance of reverse flow and the formation of recirculation bubbles (vortex breakdown).
The experimental data appear to follow a $Re$ vs $E$ curve. Figure 4 describes the development of reverse flow as a function of $Re$ and $E$ on logarithmic axes. As shown in Figure 4, the data appear to fit a smoothly varying curve, suggesting that $Re$ and $E$ may be used as suitable descriptors of the reverse flow zone, with reverse flow developing at higher $(Re, E)$ values than those shown by the curve.

![Figure 4](image)

**Figure 4:** Critical $E$ and $Re$ values leading to the disappearance of reverse flow. Full symbols correspond to the cylinder of radius $R_2$ (67.5 mm); open symbols correspond to the cylinder of radius $R_1$ (40.0 mm); triangles, squares and circles correspond to PAA concentrations of 1.0%, 0.5% and 0.1%, respectively.

As $E = \lambda \eta / \rho R^2$, and $\eta$ decreases with increasing rotation speed ($\omega$), $E$ increases with decreasing $\omega$, which should be expected to favour the stabilization of reverse flow. As shown in Table 1, reverse flow did not develop at low concentrations, even when very low $\omega$ values were used. There appears to be a finite critical concentration value below which reverse flow does not develop. The fact that $E$ is a measure of the relative importance of elastic and inertial forces is consistent with the existence of a minimal $E$ value at which the influence of elastic forces is sufficiently strong for reverse flow to develop.

4. Conclusions

Using different configurations of cylinder radius and rotation speed, these results showed that the critical $Re$ and $E$ values leading to flow reversal follow a smooth curve such that reverse flow occurred at $(Re, E)$ values above this curve, suggesting the descriptive capacity of $Re$ and $E$ for dimensionless characterization of systems similar to the one being described.

Under the studied conditions, VB occurred at extremely low $Re$ values. The effect of polymer concentration on VB was found to be similar to that on turbulence, suggesting a possible connection.
between the two phenomena. Unlike the case with the development of reverse flow, VB did not follow a simple \((Re,E)\) curve, suggesting the need of a third parameter to adequately describe the VB phenomenon.

A preliminary study of the dimensionless parameters resulting from the Oldroyd-B model [9] suggests that a dimensionless variable accounting for the characteristic polymer length may play an important role in the overall flow characterization. The predictive relevance of such a parameter should be the focus of further research.

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