NEUTRINO AND CPT THEOREM

V.P. Efrosinin

Institute for Nuclear Research, Russian Academy of Sciences, pr. Shestidesyatiletiya Oktyabrya 7a, Moscow, 117312 Russia

Abstract

The situation connected with a possibility of CPT violation in neutrino sector is considered.

CPT theorem [1, 2, 3] states: the local theory of a quantum field, invariant concerning Lorentz-rotations and including a usual causal commutativity or an anticommutativity of operators of a field, is always invariant relative products of CPT transformations.

From CPT theorem, the equality of matrix elements of processes P and CPTP implies. And CPTP process turns out from P replacement of all particles by antiparticles, all spins on inverse and permutation of initial and final conditions. In particular, from CPT theorem equalities of masses and life times, and also difference only in a sign of the magnetic moments of particles and antiparticles follow.

Let us notice that the theory of mixing of a neutrino [4, 5, 6] is essentially non-stationary. The born beam of a neutrino is described not by a stationary state, as in traditional theories of strong, electromagnetic and weak interactions, and superposition of stationary conditions:

\[ | \nu_e(0) > = c_1 | \nu_1 > + s_1 c_3 | \nu_2 > + s_1 s_3 | \nu_3 >, \]

where \( c_i = \cos \theta_i, \) \( s_i = \sin \theta_i, \) \( \theta_i \) - angles of mixing matrix.

Condition evolution in time is defined by corresponding eigenvalues of an operator of energy at the fixed impulse \( p \):

\[ E_i^2 = p^2 + m_i^2, \]

or

\[ | \nu_e(t) > = c_1 e^{-iE_1t} | \nu_1 > + s_1 c_3 e^{-iE_2t} | \nu_2 > + s_1 s_3 e^{-iE_3t} | \nu_3 >. \]

The born beam has probability of a survival as oscillates owing to a difference of masses \( m_1, m_2, m_3 \). In model of two-neutrino mixing the probability of a survival...
can be noted in terms of parameter of mixing $\sin^2 \theta$ and a difference of quadrats of masses $\delta m^2 = |m_2^2 - m_1^2|$, properly:

$$P_{\nu_e \rightarrow \nu_e} = 1 - \sin^2 2\theta \sin^2 \left( \frac{\delta m^2 L}{4E} \right),$$

where $L$ - distance from a source to the detector.

If CPT theorem is fulfilled in neutrino sector, similar (4) relation can be fair and for $\bar{\nu}$ [7]:

$$P_{\bar{\nu}_e \rightarrow \bar{\nu}_e} = 1 - \sin^2 2\theta \bar{\nu} \sin^2 \left( \frac{\delta m^2 L}{4E} \right).$$

Formulas (4) and (5) should be fulfilled at the same distances from a source $L$ and at identical energies of neutrino and antineutrino $E$.

In [7] check of correspondence of probabilities of the disappearance of electron neutrino $P_{\nu_e \rightarrow \nu_e}$ (4) and antineutrino $P_{\bar{\nu}_e \rightarrow \bar{\nu}_e}$ (5) was spent. Data of the radioactive source neutrino experiments $^{51}$Cr of collaboration Gallex and $^{51}$Cr, $^{37}$Ar of collaboration Sage presented in [8] was thus used. And also were used reactor antineutrino disappearance experiments of collaborations Bugey [9] and Chooz [10].

For calculation of parameters $\delta m^2$, $\sin^2 2\theta$ and $\delta m^2$, $\sin^2 2\theta$ the maximum likelihood method was used. Also asymmetries for masses and mixing angles are calculated:

$$A_{\delta m^2}^{CPT} = \delta m^2_{\nu} - \delta m^2_{\bar{\nu}},$$
$$A_{\sin^2 2\theta}^{CPT} = \sin^2 2\theta_{\nu} - \sin^2 2\theta_{\bar{\nu}}.$$  

The best-fit values of the asymmetries with $\chi^2_{min}$ are [7]:

$$A_{\delta m^2}^{CPT} = 0.42, \quad A_{\sin^2 2\theta}^{CPT} = 0.37 eV^2.$$  

Authors [7] consider there are indications on CPT violation in disappearance experiments of electron neutrinos and antineutrinos by confronting the neutrino data and the antineutrino data.

However there are doubts in it considering essential unhomogeneity of experiments compared in [7] of Gallex-Sage and reactor antineutrino disappearance experiments on statistics and errors. For Gallex and Sage are available on pair experiments which essentially differ from each other:

\begin{align*}
(Galleyx) & Cr 1 \quad P_{\nu_e \rightarrow \nu_e} = 1.0 \pm 0.10; \quad Cr 2 \quad P_{\nu_e \rightarrow \nu_e} = 0.81 \pm 0.10, \\
(Sage) & \quad ^{51}Cr \quad P_{\nu_e \rightarrow \nu_e} = 0.95 \pm 0.12; \quad ^{37}Ar \quad P_{\nu_e \rightarrow \nu_e} = 0.79 \pm 0.10.
\end{align*}

Such scatter of results of experiments testifies either to game to the statistican or about systematic shift.

In the analysis [7] results of reactor experiments were used:

\begin{align*}
(Chooz) & \quad P_{\bar{\nu}_e \rightarrow \bar{\nu}_e} = 1.01 \pm 0.04, \\
(Bugey) & \quad P_{\bar{\nu}_e \rightarrow \bar{\nu}_e} = 1.0 \pm 0.035,
\end{align*}
that is results not displaced from unit. Let us notice, that if experiment Bugey was with short flying bases as well as the experiments Gallex-Sage that experiment Chooz was with intermediate base $\sim 1$ km.

Averages on a harmonics of $\delta m^2$ and $\sin^2 2\theta$ in [7] have essential uncertainty. Therefore is the most advisable check of CPT invariance at definition on experiments of the ratio of $P_{\nu_e \rightarrow \nu_e}/P_{\bar{\nu}_e \rightarrow \bar{\nu}_e}$ with an adequate accuracy. And experiments should be fulfilled at the same baseline $L$ both identical energies of a neutrino and antineutrino $E$ and to be homogeneous for statistics and errors.

In [11] the weighted average for results of definition of $P_{\nu_e \rightarrow \nu_e}$ in four experiments [8] is used. There is problem on correctness of association of results of pair experiments of $Cr_1$ and $Cr_2$ and also pair of $^{51}Cr$ and $^{37}Ar$.

We spend simple check on a homogeneity of experiments $Cr_1$ and $Cr_2$. Assuming normal distribution with experimental values of an average and dispersion for $P_{\nu_e \rightarrow \nu_e}$ (Fig.1), it is received for product of probability value 34%. That already guards. If the difference of averages in experiments $Cr_1$ and $Cr_2$ was up to level $1\sigma$ that product of probability would be 61%. That is association of results is more justified to pair experiments $Cr_1$ and $^{51}Cr$.

In the same way it is possible to check up on a homogeneity results of experiment of $Cr_1$ and experiment of $Bugey$ for an antineutrino that already reflects check on CPT invariance (Fig.2). Also we receive for product of probability value 53%. Experiment of $Chooz$ does not correspond to a principle of identical baseline for check of CPT invariance.

Probably to unite pair of experiments $Cr_1$ and $^{51}Cr$ and to receive a weighted average for this pair:

$$P_{\nu_e \rightarrow \nu_e} = 0.975 \pm 0.078. \tag{10}$$

Product of probability for this pair [10] and result of $Bugey$ [9] equals 61%.

For homogeneous experiments of $Cr_2$ and $^{37}Ar$ the weighted average equals:

$$P_{\nu_e \rightarrow \nu_e} = 0.80 \pm 0.071. \tag{11}$$

Product of probability for this pair of experiments [11] and experiment of $Bugey$ [9] equals 5%.

So we see that experimental data available now on disappearance of electron neutrino with short-baseline are unsatisfactory for a solution of the problem on CPT invariance in neutrino sector. The possible solution of this problem is connected with the future experiments at accelerators.

At last we will notice that in [7, 12] approach it is impossible to define errors in average $\delta m^2$ and $\sin^2 2\theta$. Therefore in [7] asymmetries [7] for masses and angles of mixing without their errors are presented. The confidence levels specified in [7] concern to $P_{\nu_e \rightarrow \nu_e}$ and $P_{\bar{\nu}_e \rightarrow \bar{\nu}_e}$. Also do not give us the information on uncertainty of asymmetries.

In summary it is necessary to tell about importance of check of CPT invariance independent on model in neutrino sector, including from the model of an oscillation of a neutrino accepted now. Definition of the ratio $P_{\nu_e \rightarrow \nu_e}/P_{\bar{\nu}_e \rightarrow \bar{\nu}_e}$ with an adequate accuracy can be such check.
In this direction experiment of Collaboration MINOS (Main Injector Neutrino oscillation Search) with long-baseline 734 km is perspective [13, 14]. MINOS employs two detectors to significantly reduce the effect that systematic uncertainties associated with the neutrino flux have upon the $\nu_\mu$ and $\bar{\nu}_\mu$ disappearance measurement. From recent results of MINOS follows that there is a certain difference between the oscillation parameters for $\nu_\mu$ and $\bar{\nu}_\mu$ [15]. At 90% confidence level, it reports that:

\begin{equation}
|\delta m^2_{32}| = 2.35^{+0.11}_{-0.08} \times 10^{-3} \text{eV}^2, \tag{12}
\end{equation}

\begin{equation}
|\delta \bar{m}^2_{32}| = 3.36^{+0.45}_{-0.40} \times 10^{-3} \text{eV}^2. \tag{13}
\end{equation}

Together with $\sin^2(2\theta_{23}) > 0.91$ и $\sin^2(2\bar{\theta}_{23}) = 0.86 \pm 0.11$.

Let us notice that on the long-baseline the effect of localisation of admissible space of the oscillation parameters takes place. That has allowed a goodness-of-fit method similar in [12] to receive errors for $\delta m^2_{32}$ [12,13] and intervals for $\sin^2(2\theta_{23})$. And from short-baseline experiments calculated in [7] asymmetries [6] have no corresponding errors. Therefore results of evaluation of asymmetries [7] do not allow to draw any conclusion on CPT violation. To be convinced of it, it is enough to look at Figs.1,3 from [11] and on Fig.37 from [16].

So search of CPT violation in neutrino sector with definition of oscillation parameters with short-baseline is represented unpromising. That confirms our conclusion that definition on the ratio $P_{\nu_e \rightarrow \nu_e}/P_{\bar{\nu}_e \rightarrow \bar{\nu}_e}$ with a sufficient statistic can be check of CPT invariance.

By the way in [15] along with research of oscillation parameters of atmospheric neutrino and antineutrino the ratio of final flaxes of a neutrino and an antineutrino normalised on Monte Carlo is presented also:

\begin{equation}
P_{\bar{\nu}/\nu}^{\text{data}} / P_{\bar{\nu}/\nu}^{\text{MC}} = 1.04^{+0.11}_{-0.10} \pm 0.10. \tag{14}
\end{equation}

Whence difference of thys ratio from unit within an error is visible that and is impossible to speak about CPT violation. There are foundation for research prologation in this direction.
References

[1] G. Luders, R. Oehme, W.E. Thirring, Z. Naturforsch 7a, 213 (1952).
[2] A. Pais, R. Jost, Phys. Rev. 87, 871 (1952).
[3] В. Паули, в книге "Niels Bohr and the Development of Physics", London, 1957.
[4] S.M. Bilenky, B. Pontecorvo, Phys. Rep. 41C, 225 (1978).
[5] V.N. Gribov, B. Pontecorvo, Phys. Lett. 28B, 495 (1969).
[6] B. M. Pontecorvo, JETF 20, 430 (1974).
[7] C. Giunty, M. Laveder, arXiv: 1008.4750 [hep-ph].
[8] C. Giunty, M. Laveder, arXiv: 1006.3244 [hep-ph].
[9] B. Achkar et al. (Bugey), Nucl. Phys. 434B, 503 (1995).
[10] M. Apollonio et al. (CHOOZ), Eur. Phys. J., 27C, 331 (2003), arXiv: hep-ph/0301017.
[11] M.A. Acero. C. Giunti. M. Laveder. arXiv: 0711.4222 [hep-ph].
[12] M. Maltoni, T. Schwetz, arXiv: 0304176 [hep-ph].
[13] D.G. Michael et al. [MINOS Collaboration], Phys. Rev. Lett. 97, 191801 (2006).
[14] P. Adamson et al. [MINOS Collaboration], Phys. Rev. Lett. 101, 131802 (2008).
[15] P. Vahle (MINOS) (2010). Presentation at the XXIV International Conference on Neutrino Physics and Astrophysics (Neutrino 2010) in Athens, Greece; slides available at http: //www.neutrino2010.gr/.
[16] P. Adamson et al. [MINOS Collaboration], Phys. Rev. D 77, 072002 (2008).
Figure captions

Fig. 1. Check on a homogeneity of results of experiments of $Cr_1$ and $Cr_2$.

Fig. 2. Check on a homogeneity of results of experiment $Cr_1$ and experiment of $Bugey$. 