Abstract

We studied the statistical distribution of student’s performance, which is measured through their marks, in university entrance examination (Vestibular) of UNESP (Universidade Estadual Paulista) with respect to (i) period of study - day vs. night period (ii) teaching conditions - private vs. public school (iii) economical conditions - high vs. low family income. This examination reflect quality of high schools education. We observed long ubiquitous power law tails in Physical and Biological sciences in all cases. The mean value increases with better study conditions followed by better teaching and economical conditions. In humanities, the distribution is close to normal distribution with very small tail. This indicate that these power law tail in science subjects are due to the nature of the subject itself. Further better study, teaching and economical conditions are more important for physical and biological sciences in comparison to humanities. We explain these statistical distributions through Gradually Truncated Power law distributions. We discuss the possible reason for this peculiar behaviour and make suggestions to improve science education at high school level.

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I. Introduction

Power law distribution [1-5] has been first noted by Pareto in economics [6] in 1897, and afterward, by others in many physical [7-10], biological [11,12], economical [13-17] and more recently in educational [18] complex systems. Recently, we studied the statistical distribution of student’s performance, which is measured through their marks, in university entrance examination (Vestibular) of UNESP (Universidade Estadual Paulista) for years 1998, 1999, and 2000. We observed long ubiquitous power law tails in physical and biological sciences as have been observed in many physical and economical systems. We explain these statistical distributions through Gradually Truncated Power law distributions which we defined in line with Gradually Truncated Lévy Flight [19, 20] for stochastic processes. In humanities we have almost normal distribution.

We postulate that these long ubiquitous power law tails may be due to the nature of subject itself and/or due to better economical, teaching and studying conditions. In the present paper we study the statistical behaviour of candidates marks in physical, biological and humanities with respect to candidate’s economical, teaching and studying conditions for the University Entrance Examination (Vestibular) of UNESP (Universidade Estadual Paulista) - São Paulo- Brazil. We found that power law tails are present in physical and biological sciences in all cases. The mean value increases more rapidly with better studying conditions followed by better teaching conditions and economical conditions. In Humanities the statistical behaviour is approximately normal distribution with very small tails. The mean value increases in the same way as observed in physical and biological sciences but in small magnitude.

This study is interesting as treat educational system as complex system and bring out relative importance of different factors on education and peculiar nature of mathematics and science subjects. This in turn can help to improve science education at high school.

II. Data Analysis

Pareto [6] proposed a power law distribution based on positive feedback. He found that the distribution of income is well approximated by an inverse power law. This feedback effect decreases gradually after certain step size due to physical limitation of the components of the system. The whole process can be described through Gradually Truncated power law distribution given as follow [19,20]:
\[ P(x) = \frac{c_1}{c_2 + (|x - x_m|)^{1+\alpha}} f(x) \]  

(1)

where \( P(x) \) is the probability of taking a step of size \( x \), \( x_m \) is the value of \( x \) for maximum probability, \( c_1 \) and \( c_2 \) are constants and are related through:

\[ c_1 = c_2 P(x_m) \]  

(2)

c_2 can be obtained through normalization condition. Further

\[ f(x) = \begin{cases} 
1 & \text{if } |x| \leq x_c \\
\exp\left\{-\left(\frac{|x| - x_c}{k}\right)^\beta\right\} & \text{if } |x| > x_c
\end{cases} \]  

(3)

where \( x_c \) \((x_c > x_m)\) is the critical value of step size, where the probability distribution began to deviate from power law distribution due to physical limitation, \( k \) gives the sharpness of the cut-off. It is a general tendency in these systems to approach a normal distribution in the large scale and thus we choose:

\[ \beta = 2 - \alpha \]  

(4)

\((1 + \alpha)\) is the power of the power law distribution.

The normal distribution [21] is given by:

\[ P(x) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{(x - \bar{x})^2}{2\sigma^2}\right] \]  

(5)

where \( \bar{x} \) and \( \sigma \) are respectively mean value and standard deviation.

In the present paper, we analyze data of marks obtained by candidates in University Entrance examination of Universidade Estadual Paulista (UNESP) in state of São Paulo - Brazil, in years 1998, 1999 and 2000 [22]. About sixty thousand candidates appear each year in this examination. This examination is divided in three groups depending on the course
chosen by the candidate. These groups are i) Physical sciences, ii) Biological sciences and iii) Humanities. Each candidate takes three examinations: General Knowledge, Specific Knowledge of the area and Portuguese Language. General Knowledge and Portuguese Language examinations are the same for all candidates, while the examination of Specific Knowledge is different and is on the area chosen.

In our earlier paper [18], we observed peculiar behaviour in marks obtained by candidates in the examination of specific knowledge. We therefore, in the present paper are considering statistical distribution of candidates marks only in this examination. To see the effect of various factors on the evolution of student’s knowledge, we compare the statistical distribution in physical sciences, biological sciences and humanities for: i) Period of study - Day time vs. Night time students, ii) Private vs. Public schools and iii) High vs. low family income.

In Figure 1, we compare the marks obtained by students studying in day time and night times for the year 2000. Generally night time students work during the day and thus have less time to study. We compare physical sciences, biological sciences and humanities students in Figures 1a, 1b and 1c respectively. In physical and biological sciences, the distribution is given by gradually truncated power law, while in humanities it approaches to normal distribution with small tail.

In Figure 2 we compare the marks obtained by students studying in private and public schools in physical sciences (Figure 2a), biological sciences (Figure 2b) and humanities (Figure 2c). Normally teaching conditions are better in private schools because in Brazil, the existence of these schools depend on the performance of their students in these types of examinations. Here again the distribution is given by gradually truncated power law in physical and biological sciences while normal distribution with small tail in humanities.

In Figure 3 we compare the marks obtained by students coming from high income group, i.e. total family income more than ten minimum salary ($\approx$US 800.00 dollars/month), and low income group, i.e. total family income less than ten minimum salary. In Brazil, families with income above than ten minimum salary normally have appropriate conditions to live and study. Again in this case distributions are the same as observed earlier.

The parameters of the distribution for drawing theoretical curves for physical and biological sciences are given in Table 1 and 2 respectively. The value of $(1 + \alpha)$ is between 1 and 2 in all cases, thereby giving long distribution tail. The values of $x_C$ is around 80 in all cases, where positive feedback began to decrease gradually, i.e. probability decreases more rapidly
than given by power law. The agreement of theoretical curves is good with empirical curves in all cases.

Table 1 - Physical Sciences

|                | (1 + \(\alpha\)) | \(x_m\) | \(x_c\) | \(k\) | \(\bar{x}\) |
|----------------|-------------------|---------|---------|-------|------------|
| Day time       | 1.11              | 6       | 83      | 200   | 31.6       |
| Night time     | 1.47              | 4       | -       | -     | 13.2       |
| Private School | 1.19              | 7       | 75      | 200   | 37.9       |
| Public School  | 1.41              | 4       | -       | -     | 17.5       |
| High income    | 1.0               | 6       | 77      | 200   | 34.2       |
| Low income     | 1.33              | 4       | 87      | 227   | 18.9       |

Table 2 - Biological Sciences

|                | (1 + \(\alpha\)) | \(x_m\) | \(x_c\) | \(k\) | \(\bar{x}\) |
|----------------|-------------------|---------|---------|-------|------------|
| Day time       | 1.35              | 6       | 82      | 62    | 32.3       |
| Night time     | 1.80              | 2       | -       | -     | 12.6       |
| Private School | 1.51              | 14      | 85      | 23    | 37.6       |
| Public School  | 1.53              | 4       | 91      | 14    | 16.6       |
| High income    | 1.42              | 5       | 82      | 50    | 34.4       |
| Low income     | 1.63              | 2       | 83      | 27    | 20.6       |

Table 3 - Humanities

|                | \(\bar{x}\) | \(\sigma\) |
|----------------|-----------|-----------|
| Day time       | 43.4      | 15.5      |
| Night time     | 31.8      | 14.5      |
| Private School | 47.7      | 14.7      |
| Public School  | 35.2      | 14.9      |
| High income    | 45.6      | 15.4      |
| Low income     | 35.1      | 15.1      |
We further observe that all the curves of different years collapse in a single curve so far as general behaviour is concerned [23]. A small variation in the magnitude is due to the relative variation of the difficulties felt by the candidates in answering the questions in the examination, from one year to another year. The basic facts of the process of teaching and evaluation remain the same.

III. Discussion

In general, the educational system is very complex. As this examination involves students who studied in schools with different facilities, traditions and teaching levels and come from families of different social, racial and economical groups, which in turn provide different financial and emotional conditions, the problem becomes still more complex. Broadly speaking, we can say the following:

(i) The better study conditions, classroom teaching and economical facilities improve the performance of students in all the areas. However in physical and biological science group (physics, chemistry, mathematics and biology) it is more important than humanities. We found that the average marks obtained by day time students are better than night time students by 140% in physical sciences, 157% in biological sciences and 37% in humanities. Further the average marks obtained by private school students are better than public school students by 117% in physical sciences, 126% in biological sciences and 36% in humanities. Finally, the average marks obtained by high income group students compare to low income group students are better by 81% in physical sciences, 67% in biological sciences and 27% in humanities.

(ii) Better study conditions, are more important followed by teaching and economical conditions.

(iii) We observed long power law distribution tail in physical and biological sciences in all group. This shows that this peculiar behaviour is due to nature of the subject itself. In science and mathematics, all topics are inter-related. To understand a topic, a student need to know the earlier topics given. This creat a long term memory effect thereby giving power law effect. In case of humanities, topics are independent and in most cases one can understand a topic without knowing a topic given earlier particularly at this level of education. This gives normal distribution.

As far as we know, this kind of study is just beginning in the field of education. It is
therefore not possible for us to speculate some definite reasons for this kind of behaviour. However, we feel that similar kind of studies with different examinations, like school leaving certificate examination etc., in different countries with different social, economical and educational structures and social inhomogenity, can provide a better clarification of the relative importance of each factor in the preparation of a student in a particular area of study.

In view of our observations, we make the following suggestions to improve science and mathematics education at high school level:

A) Scholarship should be given to talented poor student in science and mathematics at high school level. This will give them sufficient time to study. The student can be selected through a appropriate state or national examination.

B) Generally students feel science and mathematics as some thing abstract not related to our day life. Good science laboratories are a necessary part of science education [24] and make science ducation more interesting.

C) In schools, facilities should be provided for students to resolve their problems in science subjects. It can help them to have good idea of basic concepts. Perhaps tutorial is a good idea for this purpose.
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Figure Captions

Figure 1: Number of candidates obtaining marks $x \ (N(x))$ vs. marks obtained $(x)$ for specific knowledge examination for year 2000 among day time and night students. Figure 1a is for Physical science subjects; Figure 1b is for Biological science subjects and Figure 1c is for Humanities subjects. The theoretical curves are through Gradually Truncated power law distribution in (a) and (b), while through normal distribution in (c).

Figure 2: Number of candidates obtaining marks $x \ (N(x))$ vs. marks obtained $(x)$ for specific knowledge examination for year 2000 among private and public schools. Figure 2a is for Physical science subjects; Figure 2b is for Biological science subjects and Figure 2c is for Humanities subjects. The theoretical curves are through Gradually Truncated power law distribution in (a) and (b), while through normal distribution in (c).

Figure 3: Number of candidates obtaining marks $x \ (N(x))$ vs. marks obtained $(x)$ for specific knowledge examination for year 2000 among high and low income family students. Figure 3a is for Physical science subjects; Figure 3b is for Biological science subjects and Figure 3c is for Humanities subjects. The theoretical curves are through Gradually Truncated power law distribution in (a) and (b), while through normal distribution in (c).
The diagram shows a plot of $N(x)$ versus $x$ for different income categories: high_income (squares), low_income (circles), theory_high (upward triangles), and theory_low (downward triangle dashes). The x-axis represents $x$, and the y-axis represents $N(x)$. The data points are spread across the range of $x$ from 0 to 100, with $N(x)$ ranging from 0 to 350.
(c)