On the hysteresis effect in transitions between accretion and propeller regimes

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ABSTRACT

Some observations and numerical simulations of disc-magnetosphere interaction show that accretion can proceed in the propeller regime. When the Alfvén radius is beyond the corotation radius, matter climbs up to the high latitudes where the Alfvén surface is inside the equilibrium surface and can accrete. We calculate the fraction of the mass flux in the disc that can accrete onto the neutron star depending on the fastness parameter and the inclination angle between rotation and magnetic axis. We find that, for a narrow range of the fastness parameter, the Alfvén and the equilibrium surfaces intersect at two different critical latitudes. While the system is transiting from the propeller to the accretion regime (the initial rise of an outburst), the disc is already thick and the part of the disc between these two critical latitudes cannot accrete. In transitions from the accretion to the propeller regime (decay of an outburst), the disc is thin, hence, full accretion of matter proceeds until the Alfvén radius moves beyond the equilibrium radius at the disc-midplane. Therefore, the accretion regime commences at a smaller fastness parameter than it ceases. As a result, the transition from the propeller to the accretion regime occurs at a luminosity higher than the transition from the accretion to the propeller regime. We discuss the implications of our results for spectral transitions exhibited by low-mass X-ray binaries.

Key words: accretion, accretion disks — stars: neutron — X-rays: binaries

1 INTRODUCTION

Neutron stars in low-mass X-ray binaries accrete matter from a disc (Shakura & Sunyaev 1973; Frank et al. 2002) fed by a low-mass companion (Pringle & Rees 1972). The interaction of the magnetosphere with the disc modulates the flow of matter onto the magnetic poles allowing for coherent X-ray pulsations revealing the spin frequency of the neutron star to be detected (Wijnands & van der Klis 1998). These systems are often transients due to thermal-viscous instability within the disc (see e.g. Dubus et al. 2018) and the X-ray luminosity of the system, determined by the accretion rate onto the neutron star, changes by 4 orders of magnitude during an outburst (see Patruno & Watts 2021; Di Salvo & Sanna 2022, for reviews). As the accretion rate declines, the system is expected to make a transition from the accretion to the propeller stage (Illarionov & Sunyaev 1975; Lovelace et al. 1999) during which the centrifugal barrier does not allow the matter to fall onto the neutron star.

Axisymmetric (2.5 dimensional) numerical simulations of the propeller regime (Romanova et al. 2004; Ustyugova et al. 2006; Zanni & Ferreira 2013; Romanova et al. 2018) suggest the presence of partial accretion together with the propelling of matter in outflows. This “partial accretion regime” is possible since the inner region of the disc becomes thicker and accretion can proceed from the regions away from the midplane. Menou et al. (1999) considered reduced accretion due to the propeller effect to address luminosity of neutron star systems at the quiescent stage. Ekşi & Kutlu (2011) employed this model to address the rapid decline stage in the outburst of SAX J1808.4–3658. Güngör et al. (2017) introduced a “reverse engineering” method to determine the fraction of mass flux that can accrete onto the star from the lightcurves of Aql X–1. Most recently, Lipunova et al. (2022) presented a detailed discussion of the lightcurves including the effects of irradiation of the disc.

The purpose of the paper is to investigate what fraction of the mass flux in the disc can reach the surface of the neutron star in the fast rotating regime, depending on the rotation rate of the neutron star and to uncover a hysteresis effect in transitions between accretion and propeller regimes. In the next section, the geometric arguments for partial accretion from a spherical flow and a disc is reviewed and improved. In § 3 we discuss the implications of our results for transient accreting systems with neutron stars.

2 ACCRETION IN THE PROPELLER REGIME

In this section we derive, from geometrical arguments, the fraction of accretion rate that can reach the surface of the neutron star depending on the fastness parameter, \( \omega_a / \Omega_K \) (here, \( \Omega_K \) is the Keplerian angular velocity at the inner radius of the accretion flow at the disc-midplane) and the inclination angle between rotation and magnetic axis, \( \alpha \). We start with the aligned case and depart to the inclined rotator configuration afterwards. This allows us to have a benchmark to check the calculations of the inclined rotator case at the \( \alpha = 0 \) limit.

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2.1 Aligned rotator

We first assume that the magnetic moment of the dipole is aligned with the rotation axis of the star. We further assume, for simplicity, that the presence of the disc does not change the field configuration from dipole which is obviously an oversimplification and results in a toy model. The only justification is that these also are the assumptions inherent in the derivation of the Alfvén radius.

The magnetic field $\mathbf{B}$ for an aligned dipole can be written as

$$ \mathbf{B} = \frac{\mu_s}{r} (2 \cos \theta \hat{\epsilon}_r + \sin \theta \hat{\epsilon}_\theta) $$

In spherical coordinates $d\mathbf{r} = d\mathbf{r} = d\theta \hat{\epsilon}_\theta + r \sin \theta d\phi \hat{\epsilon}_\phi$, the field lines are described by

$$ \frac{dr}{B_r} = \frac{r d\theta}{B_\theta} = \frac{r \sin \theta d\phi}{B_\phi}. $$

This can be integrated to give

$$ r = C \sin^2 \theta $$

where $C$ labels different field lines. The magnitude of the poloidal magnetic field is

$$ |B_p| = B_r^2 + B_\theta^2 = \frac{\mu_s^2}{\mu_\phi} \left(1 + 3 \cos^2 \theta\right) $$

where we used equation (1). The inner radius of a thin disc can be determined by the condition of the material and magnetic stresses at the disc-midplane (see equation 42 in Ghosh & Lamb 1979),

$$ \frac{1}{4\pi r^2} M r^2 \Omega_K = \frac{1}{4\pi} B_\phi B_\theta \Delta r, $$

Here, $\Delta r$ is the width of the transition region where the disc flow deviates from the Keplerian motion. For a non-thin disc, we generalise this condition as

$$ \frac{1}{4\pi r^2} M r^2 \Omega_K = \frac{1}{4\pi} \gamma |B_p|^2 \Delta r, $$

where

$$ \gamma = \frac{B_\phi}{\sqrt{B_r^2 + B_\theta^2}} $$

is the ratio of the toroidal magnetic field to the strength of the poloidal magnetic field. This can be used to define an Alfvén surface

$$ r_A = R_A \left(1 + 3 \cos^2 \theta\right)^{2/7} $$

where

$$ R_A = \xi \frac{\mu_\phi^2}{\sqrt{GM_s M}} \left(\frac{\mu_s^2}{\mu_\phi}\right)^{2/7} $$

is the Alfvén radius, the radius of the Alfvén surface at the disc-midplane and $\xi = \gamma \Delta r / r$ is a numerical factor at the order of unity. We will consider $\xi$ as unity throughout the paper.

For a point mass rotating with the stellar angular velocity, $\Omega_s$, the acceleration towards the rotation axis is $\mathbf{a}_r = \Omega_s^2 r \sin \theta \hat{\epsilon}_r + \cos \theta \hat{\epsilon}_\phi$. The balance of this acceleration with the gravitational acceleration, $\mathbf{a}_g = -\left(GM_s / r^2\right) \hat{\epsilon}_r$, along the magnetic field defines the equilibrium surface,

$$ r_{eq} = \left(\frac{2}{3}\right)^{1/3} R_{co} \sin^{-2/3} \theta, $$

(Lyutikov 2022) where the Keplerian angular velocity of the disc matches the stellar angular velocity, the so-called corotation radius, is given by

$$ R_{co} = \left(\frac{GM_s}{\Omega_s^2}\right)^{1/3}. $$

Note that the above condition for the equilibrium surface is not valid at the disc-midplane where the magnetic field is perpendicular to the gravitational and centrifugal accelerations. However, equation (10)
can be used at the disc-midplane by the continuity. Thus, the equilibrium radius at the disc-midplane is \( R_{\text{eq}} = (2/3)^{1/3} R_{\text{co}} \). Accordingly, the propeller regime starts when \( R_{\text{A}} \) is larger than \((2/3)^{1/3} R_{\text{co}}\) rather than \( R_{\text{co}} \).

The intersection of the equilibrium surface with the Alfvén surface defines a critical angle \( \theta_c \) below which the disc material does not meet with an equilibrium surface and so can accrete onto the star (see Fig. 1). This critical angle depends on the fastness parameter given implicitly by

\[
\omega_s^{-1} = \frac{3}{2} \sin \theta_c \left( 1 + 3 \cos^2 \theta_c \right)^{3/7},
\]  

(12) which is found by \( r_{\text{eq}} = r_{\text{A}} \) given in equations (8) and (10), and referring the definition of the fastness parameter\(^1\). The numerical solution of \( \theta_c \) from this equation is shown in Fig. 2. Note that, for \( 0.74 < \omega_s < 0.82 \), this equation has two solutions for \( \theta_c \) since the Alfvén surface intersects the equilibrium surface at two distinct altitudes. The matter within these two critical altitudes is beyond the equilibrium surface and hence is expelled. Accordingly, the matter can accrete from two distinct regions; between the disc-midplane and the higher altitude, and between the lower altitude and the spin axis (see the right panel of Fig. 1). We assume the disc flow might be channelled onto the star from these two regions simultaneously. On the other hand, when \( \omega_s > 0.82 \), the Alfvén radius is beyond the equilibrium radius at the disc-midplane and the Alfvén surface intersects with the equilibrium surface at one altitude, therefore, only the matter between \( 0 - \theta_c \) can accrete onto the star (see the left panel of Fig. 1).

The fraction of mass inflow that can accrete onto the star is then given by

\[
f = \frac{\dot{M}_*}{\dot{M}} = \frac{2 \int_0^{\sqrt{3}} 2 \pi r^2 \sin^2 \rho(r, \theta) v(r, \theta) d\theta}{2 \int_0^{\sqrt{3}} 2 \pi r^2 \sin^2 \rho(r, \theta) v(r, \theta) d\theta} + \frac{2 \int_0^{\sqrt{3}} 2 \pi r^2 \sin \theta \rho(r, \theta) v(r, \theta) d\theta}{2 \int_0^{\sqrt{3}} 2 \pi r^2 \sin \theta \rho(r, \theta) v(r, \theta) d\theta}
\]  

(13)

\(^\dagger\) Note that \( \text{Menou et al. (1999)} \) assumes \( r_{\text{A}} = R_{\text{A}} \) (spherical magnetosphere) and obtains \( \omega_s^{-1} = \sin \theta_c \) analytically which is accurate only for \( \theta_c = \pi/2 \).

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The transition from the propeller regime to the accretion regime and during the propeller, we assume the disc has a finite constant thickness otherwise it is thin. As a result of the different thickness parameter of the disc in transitions, the fraction is doubly-valued for 0.74 < \omega_s < 0.82 and takes different values depending on whether \omega_s is ascending or descending. Moreover, the partial accretion requires the inner part of the disc to be thick such as \zeta \geq 0.4.

In Fig. 4, we report the evolution of the lightcurves to demonstrate the effect of the partial accretion. The luminosity produced by the accretion is given as

\[ L = \frac{GM_s M}{R_s} \varphi \]  

where G is the gravitational constant, M_s and R_s are, respectively, the mass and the radius of the star. We assume the mass accretion rate evolves as

\[ M = \begin{cases} \frac{M_0}{6} \left(1 + \frac{5t}{t_0}\right), & t < t_0, \\
\frac{M_0}{6} \left(1 - \frac{t}{t_0}\right), & t > t_0, \end{cases} \]  

where M_0 and t_0 are some arbitrary constants. This toy model ensures that the mass accretion rate linearly increases until t = t_0, then, it decreases rapidly (see the bottom panel of Fig. 4). So, we can observe the transition from the propeller regime to the accretion regime and then to the propeller regime again. Additionally, we set the inner radius of the accretion flow to 0.7R_co at t = t_0 in all cases.

As mentioned above, the fraction of the mass inflow that can accrete onto the star takes different values in transitions (0.74 < \omega_s < 0.82). Therefore, the luminosity produced by the accretion would be different between the transition from the accretion to the propeller regime and the vice-versa transition as the full accretion commences or ceases at different fastness parameters (see the middle panel of Fig. 4). The ratio of the luminosities of these two transitions is 6 for \zeta \to \infty limit, and it increases as the thickness parameter of the disc reduces. Other than the thickness parameter of the disc, our results for the luminosity ratios depend on the choice of the inner radius of the accreting flow at t = t_0 while they are not affected by the toy model of the mass-accretion rate we employed. The difference between the transition luminosities exposes a hysteresis effect in the lightcurve as can be seen from Fig. 4.

### 2.2 Inclined rotator

The dipole magnetic field of the star can be written in a general form as

\[ \mathbf{B} = \frac{\mu}{r^3} \left(3 (\hat{\mu} \cdot \hat{e}_r) \hat{e}_r - \hat{\mu}\right), \]  

where \hat{\mu} is the unit magnetic dipole moment vector. If we choose the coordinate system such that z-axis is the rotation axis of the spherical star, the unit magnetic dipole moment vector can be written as

\[ \hat{\mu} = \sin \alpha \cos \chi \hat{e}_x + \sin \alpha \sin \chi \hat{e}_y + \cos \alpha \hat{e}_z, \]  

where \alpha is the inclination angle between the rotation axis and the magnetic dipole moment, and \chi = \Omega t is the angle between the x-z plane and the magnetic dipole moment.

Since the timescale of observations of lightcurves are much longer than the spin period of the star, we calculate the time-averaged Alfvén
For instance, the ratio of the transition luminosities is \( \frac{L_{\text{in}}}{L_{\text{out}}} \). When the transition luminosities decreases as the inclination angle increases. For instance, the ratio of the transition luminosities is 4 for \( \zeta \to \infty \), and it is 118 for \( \zeta = 0.4 \) when \( \alpha = 15^\circ \). When \( \alpha > 30^\circ \), the Alfvén surface does not intersect the equilibrium surface at two latitudes for any values of the fastness parameter as mentioned above. Therefore, the fraction is no longer doubly-valued and the hysteresis effect is lost.

3 DISCUSSION

We have given geometrical arguments to calculate the fraction \( f \) of mass-flux in the disc that can reach the surface of the neutron star depending on the fastness parameter, \( \omega_c \) and inclination angle \( \alpha \). We have seen that, for a range of the fastness parameter near the transition
Figure 6. The dependence of the critical angle $\theta_c$ on the fastness parameter, $\omega_s$, for the inclined rotator. Dashed lines represent the larger root of $\theta_c$ for a given $\omega_s$ for each alignment angle. Vertical dashed lines bound the interval of $0.74 < \omega_s < 0.82$.

region ($\omega_s \approx 0.82$ in the aligned case), the fraction $f$ is two-valued since the thickness of the disc is different in transitions and the Alfvén surface intersects the equilibrium surface at two different critical angles, $\theta_c$. If the system transits from the propeller to the accretion regime, such as at the commence of an outburst, the disc is already thick and some part of the disc might be outside the equilibrium surface. If, on the other hand, the system is transiting from the accretion to the propeller stage, such as at the decay of an outburst, the disc is thin and fully inside the equilibrium surface. This leads to a hysteresis effect in the lightcurves of transient accreting systems in the sense that the transition from the propeller to the accretion regime occurs at a luminosity higher than the luminosity at which the transition from the accretion to the propeller regime occurs.

Our results show that the thicker is the inner disc, the smaller is the luminosity difference in the transitions (see Fig. 4). The luminosity difference also decreases with the inclination angle (see Fig. 7) vanishing at $\alpha \approx 30^\circ$ where the hysteresis effect is lost since the critical angle at which the averaged Alfvén surface intersects with the averaged equilibrium surface is not doubly-valued for such large values of $\alpha$ (see Fig. 6).

We would like to note that our results depend on several assumptions, e.g. the assumption about the shape of the magnetosphere as an ideal dipole is expected to be modified in the presence of the disc material (see Lyutikov 2022). Moreover, by crudely generalising the condition of balance of stresses given in equation (42) of Ghosh & Lamb (1979), we obtain expression (8) and use it to determine the Alfvén surface for an accretion disc which is not necessarily thin although the Alfvén surface of a non-thin disc would require more detailed investigations.

Also, we use arbitrary constant thickness parameter values to determine the fraction of the mass-accretion rate. However, the accumulation of the matter in the propeller regime causes the disc to get thicker. Therefore, the fraction of the mass-accretion rate, as a boundary condition of the mass-loss determines the dynamics of the disc. Beside the thickness parameter, the simplistic functions we employed for the density and the velocity of the flow may in turn different. Moreover, the mass-accretion rate might be effected by the change of the thickness of the disc. Hence our solution for $f$ and the resulting lightcurves may be different in a self-consistent solution. As a follow up to our general relativistic magnetohydrodynamics (GRMHD) simulations (Çıkıntoğlu et al. 2022) with Black Hole Accretion Code (BHAC) (Porth et al. 2017), we will extend our analysis to rotating neutron stars and study the partial accretion regime.

We stress however that the presence of the hysteresis effect in transitions between the accretion and the propeller regimes does not depend on the details of the disc flow and on our specific assumptions about the $\theta$ dependence of $v$ or $\rho$, but on the thickness parameter being different in transitions, and on the shape of the magnetosphere, as given in equation (8), being non-spherical allowing for the intersection of the equilibrium surface at two different altitudes (consider the peanut shape the magnetosphere would have if Fig. 1 is drawn in 3-dimensions). If a spherical magnetosphere ($r_A \approx R_A$) is employed for the sake of simplicity, the hysteresis effect is lost. The most trivial prediction of the model presented here is that, for systems where $\alpha \leq 30^\circ$, the transition to the propeller regime would be exhibited in a lightcurve with an abrupt drop to a lower luminosity followed by a slower decay.

It is tempting to associate the hysteresis effect we discuss in this work with the hysteresis effect observed by Maccarone & Coppi (2003) in the spectral transitions of Aql X–1. Since then many systems exhibited the hysteresis effect the most recent being 4U 1730–22 Chen et al. (2022). These systems exhibit transitions from the low-hot state, associated with the propeller stage, to the high-cold state associated with the accretion stage. These works report that the former transitions occur at a few times higher luminosity compared to the latter transitions, a hysteresis effect that is akin to what we present here.

Although compelling, we must be cautious in making the above association since such hysteresis effect in spectral transitions is observed also from systems where the accreting objects are black holes (see e.g. Muñoz-Darias et al. 2014). Although black holes can not have magnetic fields themselves (no-hair theorem), they can have magnetospheres (Blandford & Znajek 1977; Komissarov 2004; Crinquand et al. 2020; Bransgrove et al. 2021; Crinquand et al. 2022) coupled to the inner disc, but it is unlikely that these magnetospheres would allow black holes to experience a propeller stage similar to the neutron stars. Thus the hysteresis effect for transitions between accretion and propeller regimes that we propose here can not be a favourable explanation of the hysteresis effect in spectral transitions observed in LMXBs if one insists on a common mechanism working both for black hole and neutron star accretors.

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DATA AVAILABILITY

This is a theoretical paper that does not involve any new data. The model data presented in this article are all reproducible.
On the hysteresis effect in transitions

Figure 7. Top panel: The evolution of the normalized lightcurve of inclined rotators. The black solid line represents full accretion stage, i.e., $f = 1$, for all cases. Bottom panel: The evolution of the fraction for the same cases.

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