An Implementation of Nested Pattern Matching in Interaction Nets

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Reduction rules in interaction nets are constrained to pattern match exactly one argument at a time. Consequently, a programmer has to introduce auxiliary rules to perform more sophisticated matches. In this paper, we describe the design and implementation of a system for interaction nets which allows nested pattern matching on interaction rules. We achieve a system that provides convenient ways to express interaction net programs without defining auxiliary rules.

1 Introduction

Interaction nets [5] were introduced over 10 years ago as a new programming paradigm based on graph rewriting. Programs are expressed as graphs and computation is expressed as graph transformation. They enjoy nice properties such as locality of reduction, strong confluence and Turing completeness. The definition of interaction nets allows them to share computation: reducible expressions (active pairs) cannot be duplicated. For these reasons, optimal and efficient λ-calculus evaluators [2] [6] [8] based on interaction nets have evolved. Indeed, interaction nets have proved to be very useful for studying the dynamics of computation. However, they remain fruitful only for theoretical investigations.

Despite that we can already program in interaction nets, they still remain far from being used as a practical programming language. Drawing an analogy with functional programming, we only have the λ-calculus without any high level language constructs which provide programming comfort. Interaction nets have a very primitive notion of pattern matching since only two agents can interact at a time. Consequently, many auxiliary agents and rules are needed to implement more sophisticated matches. These auxiliaries are implementation details and should be generated automatically other than by the programmer.

In this paper we take a step towards developing a richer language for interaction nets which facilitates nested pattern matching. To illustrate what we are doing, consider the following definition of a function that computes the last element of a list:

\[
\text{lastElt} \ (x:[]) = x \\
\text{lastElt} \ (x:xs) = \text{lastElt} \ (xs)
\]

In this function [], is a nested pattern in (x:[]). We cannot represent functions with nested patterns in interaction nets. Hence, a programmer has to introduce auxiliary functions to pattern match the extra arguments.

\[
\text{lastElt} \ (x:xs) = \text{aux} \ x \ xs \\
\text{aux} \ x \ [] = x \\
\text{aux} \ x \ (y:ys) = \text{lastElt} \ (y:ys)
\]
In our previous work [4] we defined a conservative extension of interaction rules that allows nested pattern matching. The purpose of this paper is to bring these ideas into practise:

- we define a programming language that captures the extended form of interaction rules;
- we describe the implementation of these extended rules.

In [9] we defined a textual language for interaction nets (PIN) and an abstract machine that executes PIN programs. We take PIN as our starting point and extend the PIN language to allow the representation of rules with nested patterns.

There has been several works that extend interaction nets in some way. Sinot and Mackie’s Macros for interaction nets [11] are quite close to what we present in this paper. They allow pattern matching on more than one argument by relaxing the restriction of one principal port per agent. The main difference with our work is that their system does not allow nested pattern matching. Our system facilitates nested/deep pattern matching of agents.

The rest of this paper is organised as follows: In the Section 2 we give a brief introduction of interaction nets. In Section 3 we define a programming language that allows the definition of interaction rules with nested patterns. In Section 4 we give an overview of the implementation of nested pattern matching. A more detailed explanation of the algorithm is found in Section 5 (verification of well-formedness) and Section 6 (rule translation). Finally, we conclude the paper in Section 7.

2 Interaction Nets

We review the basic notions of interaction nets. See [5] for a more detailed presentation. Interaction nets are specified by the following data:

- A set $\Sigma$ of symbols. Elements of $\Sigma$ serve as agent (node) labels. Each symbol has an associated arity $ar$ that determines the number of its auxiliary ports. If $ar(\alpha) = n$ for $\alpha \in \Sigma$, then $\alpha$ has $n + 1$ ports: $n$ auxiliary ports and a distinguished one called the principal port. We represent an agent graphically as:

```
\begin{center}
\begin{tikzpicture}
  \node {$\alpha$} at (0,0) [circle,draw];
  \draw (\alpha) -- (1,0); \node at (1,0) {$x_1$};
  \draw (\alpha) -- (2,0); \node at (2,0) {$\cdots$};
  \draw (\alpha) -- (3,0); \node at (3,0) {$x_n$};
\end{tikzpicture}
\end{center}
```

and textually using the syntax: $x_0 \sim \alpha(x_1, \ldots, x_n)$ where $x_0$ is the principal port.

- A net built on $\Sigma$ is an undirected graph with agents at the vertices. The edges of the net connect agents together at the ports such that there is only one edge at every port. A port which is not connected is called a free port. A set of free ports is called an interface. A symbol denoting a free port is called a free variable.

- Two agents $(\alpha, \beta) \in \Sigma \times \Sigma$ connected via their principal ports form an active pair (analogous to a redex). An interaction rule $((\alpha, \beta) \Rightarrow N) \in \mathcal{R}$ replaces the pair $(\alpha, \beta)$ by the net $N$. All the free ports are preserved during reduction, and there is at most one rule for each pair of agents. The following diagram illustrates the idea, where $N$ is any net built from $\Sigma$.

```
\begin{center}
\begin{tikzpicture}
  \node {$\alpha$} at (0,0) [circle,draw];
  \node {$\beta$} at (1,0) [circle,draw];
  \draw (\alpha) -- (1,0); \node at (1,0) {$x_1$};
  \draw (\alpha) -- (2,0); \node at (2,0) {$\cdots$};
  \draw (\alpha) -- (3,0); \node at (3,0) {$x_n$};
  \draw (\beta) -- (1,0); \node at (1,0) {$y_1$};
  \draw (\beta) -- (2,0); \node at (2,0) {$\cdots$};
  \draw (\beta) -- (3,0); \node at (3,0) {$y_m$};
  \node {$N$} at (4,0) [rectangle,draw];
  \draw (N) -- (5,0); \node at (5,0) {$x_1$};
  \draw (N) -- (6,0); \node at (6,0) {$\cdots$};
  \draw (N) -- (7,0); \node at (7,0) {$y_1$};
  \draw (N) -- (8,0); \node at (8,0) {$\cdots$};
\end{tikzpicture}
\end{center}
```
For present purposes, we represent this rule textually using \( \langle \alpha(x_1, \ldots, x_n) \bowtie \beta(y_1, \ldots, y_n) \rangle \Rightarrow N \).

We use the notation \( N_1 \rightarrow N_2 \) for the one step reduction and \( \rightarrow^* \) for its transitive and reflexive closure. Interaction Nets have the following property [5]:

- **Strong Confluence**: Let \( N \) be a net. If \( N \rightarrow N_1 \) and \( N \rightarrow N_2 \) with \( N_1 \neq N_2 \), then there is a net \( N_3 \) such that \( N_1 \rightarrow N_3 \) and \( N_2 \rightarrow N_3 \).

In Figure 1 we give a simple example of an interaction net system that computes the last element of a list. We can represent lists using the agents Cons (:\:) of arity 2 and Nil of arity 0. The first port of Cons connects to an element of the list and the second port of Cons connects to the rest of the list. The agent Nil marks the end of the list. An active pair between Lst and Cons rewrites to an auxiliary agent Aux with its principal port oriented towards the second auxiliary port of Cons. This means that during computation, Aux will interact with either a Cons agent or a Nil agent\(^1\). To avoid blocking the computation, we define rules for active pairs (Aux, Nil) and (Aux, Cons). An active pair between Aux and Nil rewrites to a single wire, which connects the agents at the auxiliary ports of Aux. When paired with Cons, Aux is replaced by Lst, analogous to the recursive call of the LastElt function. The list element which is connected to the first port of Aux is deleted, as it is not the last element of the list. This is modeled by the agent \( \varepsilon \), which erases all other agents.

![Figure 1: Rules to compute last element of a list](image)

Figure 2 gives an example reduction sequence that computes the last element of a list that contains just one element: \([1]\). The second port of Lst is free and thus acts as the interface of the net. First, the active pair of Lst and Cons is rewritten, introducing an Aux agent. Now the second rule is applied to the pair (Aux, Nil), removing both agents and connecting 1 to the interface of the net. As expected, this final net contains only the agent 1, which is equivalent to the result of LastElt \( (1: []) \).

**Why Interaction Nets?** Interaction nets are a generalisation of proof nets for linear logic [11], in a similar way that term rewriting systems are a generalisation of the \( \lambda \)-calculus. Interaction nets are an important model of computation for several reasons:

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\(^1\)The second auxiliary port of a Cons agent will be connected to either a Cons agent or a Nil agent.
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1. All aspects of a computation are captured by the rewriting rules—no external machinery such as copying a chunk of memory, or a garbage collector, are needed. Interaction nets are amongst the few formalisms which model computation where this is the case, and consequently they can serve as both a low level operational semantics and an object language for compilation, in addition to being well suited as a basis for a high-level programming language.

2. Interaction nets naturally capture sharing—active pairs can never be duplicated. Thus only normal forms can be duplicated, and this must be done incrementally. Using interaction nets as an object language for compilers has offered strong evidence that this sharing will be passed on to the programming language being implemented. One of the most spectacular instances of this is the work by Gonthier, Abadi and Lévy, who gave a system of interaction nets to capture both optimal $\beta$-reduction [7] in the $\lambda$-calculus [2] (Lamping’s algorithm [6]), and optimal reduction for cut-elimination in linear logic [3].

3. There is growing evidence that interaction nets can provide a platform for the development of parallel implementations, specifically parallel implementations of sequential programming languages. Using interaction nets as an object language for a compiler offers strong evidence that the programming language being implemented may be executed in parallel (even if there was no parallelism in the original program). Evidence of this can be found in [10].

4. Finally, it is a formalism where both programs and data structures are represented in the same framework. In the light of the first point above, we have a very powerful mechanism to study the dynamics of computation.

The above points are clear indicators that interaction nets have a role to play in computer science every bit as important as the roles the $\lambda$-calculus or term rewriting systems have played over the last few decades. The aim of the present work is to realise some of the potential of interaction nets through the development of practical programming constructs which simplify the task of programming.

2.1 Interaction rules with nested patterns - INP

The above definition of interaction nets constraints pattern matching to exactly one argument at a time. Consequently, we have to introduce auxiliary agents and rules to perform deep pattern matching (as exemplified in Figure 1). Following [4], an interaction rule may contain a nested active pair with more than two agents on its left-hand side (lhs). A nested active pair is defined inductively as follows:

- Every active pair in ordinary interaction rules (ORN) is a nested active pair e.g. $P = \langle \alpha(x_1, \ldots, x_n) \gg \beta(y_1, \ldots, y_m) \rangle$
- A net obtained as a result of connecting the principal port of some agent $\gamma$ to a free port $y_j$ in a nested active pair $P$ is also a nested active pair e.g. $\langle P, y_j \sim \gamma(z_1, \ldots, z_l) \rangle$
As an example, Figure 3 gives a set of INP rules that will compute the last element of a list. In this Figure both rules contain a nested active pair on the lhs. The (non interacting) agents Nil and Cons on the lhs of the rules are nested agents. These rules are compiled into the set of ORN rules given in Figure 1 (See [4] for details of the compilation).

![Figure 3: example INP rules to compute the last element of a list](image)

To ensure that INP preserves the Strong Confluence property of interaction nets, rules in INP must satisfy the following constraints [4]:

**Definition 2.1 (Sequentiality).** Let P be a nested active pair. The set of nested active pairs \( \mathcal{P} \) is sequential iff when \( \langle P, y \sim \beta(x_1, \ldots, x_n) \rangle \in \mathcal{P} \) then

1. for the nested pair \( P \), \( P \in \mathcal{P} \) and,
2. for all the free ports \( y \) in \( P \) except the \( y_j \) and for all agents \( \alpha \), \( \langle P, y \sim \alpha(w_1, \ldots, w_n) \rangle \notin \mathcal{P} \)

**Definition 2.2 (Well-formedness).** A set of INP rules \( \mathcal{R} \) is well-formed iff

1. there is a sequential set of nested active pairs which contains every lhs of rules in \( \mathcal{R} \),
2. for every rule \( P \Rightarrow N \) in \( \mathcal{R} \), there is no interaction rule \( P' \Rightarrow N' \) in \( \mathcal{R} \) such that \( P' \) is a subnet of \( P \).

Intuitively, the Sequentiality property avoids overlaps between rules: a set of INP rules containing nested patterns that violate condition 2 of definition 2.1 can give rise to critical pairs, which potentially destroys the Strong Confluence property of interaction nets. Note that the definition of Sequentiality allows a nested active pair to be a subnet of another nested active pair (in the same sequential set) which may also give rise to critical pairs. Definition 2.2 ensures that there is at most one nested active pair in any given set of rules.

### 3 The Language

We represent nets in the usual way as a comma separated list of agents. This just corresponds to a flattening of the net, and there are many different (equivalent) ways to do this depending on the order in which the agents are enumerated. Using the net in Figure 2 as an example we write:

\[
p \sim \text{Lst}(r), p \sim \text{Cons}(x, x_8), x \sim 1, x_8 \sim \text{Nil}
\]

The symbol ‘\( \sim \)’ denotes the principal port of the agent. The variables \( p, x \) and \( x_8 \) are used to model the connection between two ports. All variable names occur at most twice: this limitation corresponds to the requirement that it is not possible to have two edges connected to the same port in the graphical setting. If a name occurs once, then it corresponds to the free ports of the net (\( r \) is free in the above). If a name occurs twice, then it represents an edge between two ports. In this latter case, we say that a variable is **bound**.

The syntax above can be simplified by replacing equals for equals:
In this notation the general form of an active pair is \( \alpha(\ldots) \sim \beta(\ldots) \).

We represent rules by writing \( l \Rightarrow r \), where \( l \) is the net on the left of the rule, and \( r \) is the resulting net. In particular, we note that \( l \) will always consist of two agents connected at their principal ports. As an example, the rules in Figure 1 are written as:

\[
\text{Lst}(r) \bowtie \text{Cons}(x,\text{x}s) \Rightarrow \text{x}s \sim \text{Aux}(x,r)
\]

\[
\text{Aux}(x,r) \bowtie \text{Nil} \Rightarrow x \sim r
\]

\[
\text{Aux}(x,r) \bowtie \text{Cons}(y,\text{ys}) \Rightarrow \text{Lst}(r) \sim \text{Cons}(y,\text{ys}), \varepsilon \sim x
\]

The names of the bound variables in the two nets must be disjoint, and the free variables must coincide, which corresponds to the condition that the free variables must be preserved under reduction. Note that we use the symbol ‘\( \bowtie \)’ for the active pair of the rule so that we can distinguish between an active pair and a rule.

We can simplify only optimised ORN rules i.e. rules that do not contain active pairs on their right-hand side (rhs). For example we can write the set of rules above in a simplified form:

\[
\text{Lst}(r) \bowtie \text{Cons}(x,\text{Aux}(x,r))
\]

\[
\text{Aux}(x,x) \bowtie \text{Nil} \Rightarrow x \sim r
\]

\[
\text{Aux}(x,r) \bowtie \text{Cons}(y,\text{ys}) \Rightarrow \text{Lst}(r) \sim \text{Cons}(y,\text{ys}), \varepsilon \sim x
\]

Note that once an optimised ORN rule is simplified, it’s rhs becomes empty and therefore we omit the ‘\( \Rightarrow \).’

We represent INP rules using a similar mechanism and allow the lhs of a rule to contain more than two agents. We restrict the simplification of INP rules such that the lhs and rhs are simplified independently of one another. As an example, the set of INP rules in Figure 3 can be written as:

\[
\text{Lst}(r) \bowtie \text{Cons}(x,\text{x}s), \text{x}s \sim \text{Nil} \Rightarrow r \sim x
\]

\[
\text{Lst}(r) \bowtie \text{Cons}(x,\text{x}s), \text{x}s \sim \text{Cons}(y,\text{ys}) \Rightarrow x \sim \varepsilon, p \sim \text{Lst}(r), p \sim \text{Cons}(y,\text{ys})
\]

or in a simplified form:

\[
\text{Lst}(r) \bowtie \text{Cons}(x,\text{Nil}) \Rightarrow r \sim x
\]

\[
\text{Lst}(r) \bowtie \text{Cons}(x,\text{Cons}(y,\text{ys})) \Rightarrow x \sim \varepsilon, \text{Lst}(r) \sim \text{Cons}(y,\text{ys})
\]

Other language constructs The PIN system provides other language constructs – modules, built-in operations on agent values, input/output e.t.c. These constructs remain unaffected by our extension and are out of scope in this paper. See [9] for a detailed description of the additional language features.

4 Translation

In this Section we give an overview of our translation. In general, the PIN compiler reads programs in our source language and builds the corresponding abstract syntax tree (AST). On the basis of the AST, the PIN compiler generates some byte codes which can be executed by an abstract machine or be further compiled into C source code. See [9] for a more detailed presentation of the PIN system.

Our translation function rewrites ASTs that represent INP rules into ASTs that represent ordinary interaction rules. Therefore, the back end of the PIN system remains unaffected by the translation. Overall, our translation function is similar to the compilation schemes defined in the original paper [4]. We summarise the translation algorithm in the following steps:
1. A rule is found in the AST. This rule can be either ORN or INP. All other nodes of the AST that are not rules (imports, variable declarations, ...) are ignored.

2. Check if the lhs is not a subnet of a previously translated lhs (and vice versa if the rule is INP). This is the first part of verifying the well-formedness property (Definition 2.2). We discuss this verification in Section 5.1.

3. If the rule is not INP, return.

4. If the rule is INP: check if the current and all previous nested active pairs can be added to a sequential set. This is the second part of verifying the well-formedness property (Section 5.2).

5. If both checks are passed, translate the rule (else, exit with an error message):
   
   (a) Resolve the first nested agent of the rule’s active pair.
   
   (b) Add an auxiliary rule to the AST.
   
   (c) The remaining nested agents are not (yet) translated. They are resolved by translating the auxiliary rule.

   We describe the translation algorithm in Section 6.

6. traverse the AST until the next (unprocessed) rule is found.

This algorithm allows for an arbitrary number of nested patterns (i.e., the number of nested agents in the lhs of an INP rule) and an arbitrary pattern depth.

5 Verifying the Well-Formedness Property

Our verification algorithm (see below) consists of two parts which correspond to the two constraints of the well-formedness property. We verify that the set of nested active pairs in a given PIN program are both disjoint and sequential.

We use the notation \(\emptyset\) for the empty list, \([1, \ldots, n]\) for a list of \(n\) elements and \(ps\@ ps2\) to append two lists.

**Definition 5.1** (Position Set). Let \(\alpha(l) \rightarrow r\) be a rule in PIN. We define the function \(\text{PosSym}(l)\) that given a nested active pair will return a set of pairs \((ps, u)\) where \(ps\) is a list that represents the position of a symbol \(u\) in \(l\).

\[
\text{PosSym}(\alpha(t_1, \ldots, t_n) \rightarrow \beta(s_1, \ldots, s_m)) = \text{PosSym}_1([1,1], t_1) \cup \ldots \cup \text{PosSym}_1([1,n], t_n) \cup \\
\text{PosSym}_1([2,1], s_1) \cup \ldots \cup \text{PosSym}_1([2,m], s_m)
\]

\[
\text{PosSym}_1(ps, x) = \emptyset
\]

\[
\text{PosSym}_1(ps, \alpha(x)) = \{(ps, \alpha)\}
\]

\[
\text{PosSym}_1(ps, \alpha(t_1, \ldots, t_n)) = \text{PosSym}_1(ps \@ [1], t_1) \cup \ldots \cup \text{PosSym}_1(ps \@ [n], t_n)
\]

where the sequence of terms \(t_1, \ldots, t_n\) in \(\text{PosSym}_1(ps, \alpha(t_1, \ldots, t_n))\) contain at least one term which is not a variable; and \(x\) is a sequence of zero or more variables.

The function \(\text{Pos}(l)\) returns a set of lists that represent the position of each nested agent in a nested active pair:

\[
\text{Pos}(l) = \pi_1(\text{PosSym}(l))
\]

\[
\pi_1(\emptyset) = \emptyset
\]

\[
\pi_1(\{(ps, s)\} \cup A) = \{ps\} \cup \pi_1(A - \{(ps, s)\})
\]
We extend these operations into the sequence \( l_1, \ldots, l_k \) of lhs of rules as follows:

\[
\text{PosSym}(l_1, \ldots, l_k) = \text{PosSym}(l_1) \cup \cdots \cup \text{PosSym}(l_k),
\]

\[
\text{Pos}(l_1, \ldots, l_k) = \text{Pos}(l_1) \cup \cdots \cup \text{Pos}(l_k).
\]

**Example 5.2.** For each rule in Figure 3 we can get sets of positions of nested agent pairs as follows:

- \( \text{PosSym}(\text{Lst}(r) >\times \text{Cons}(x,\text{Nil})) = \text{PosSym}(\text{Lst}_1([1,1],r)) \cup \text{PosSym}(\text{Lst}_2([2,1],x)) \cup \text{PosSym}(\text{Lst}_3([2,2],\text{Nil})) = \{(2,2),\text{Nil}\} \)
- \( \text{Pos}(\text{Lst}(r) >\times \text{Cons}(x,\text{Nil})) = \{(2,2)\} \)
- \( \text{PosSym}(\text{Lst}(r) >\times \text{Cons}(x,\text{Cons}(y,\text{ys}))) = \text{PosSym}(\text{Lst}_1([1,1],r)) \cup \text{PosSym}(\text{Lst}_2([2,1],y)) \cup \text{PosSym}(\text{Lst}_3([2,2],\text{Cons}(y,\text{ys}))) = \{(2,2),\text{Cons}\} \)
- \( \text{Pos}(\text{Lst}(r) >\times \text{Cons}(x,\text{Cons}(y,\text{ys}))) = \{(2,2)\} \)

### 5.1 Subnet property

Verifying the subnet property is straightforward. Since rules are represented as trees (subtrees of the AST), it is easy to verify if one rule’s lhs is a subtree of another. We compute the lhs subtree relation of the current rule \( P \) (to be translated) against all the rules \( Q \) which have already been translated. If \( P \) is in ORN, we verify the subtree relation in only one direction: the lhs of an INP rule cannot be a subnet of the lhs of an ORN rule. Otherwise we verify the subtree relation in both directions: \( P \) against \( Q \) and \( Q \) against \( P \). The case of two ORN rules with the same active agents is handled by the compiler at an earlier stage. If the current rule’s lhs is not a subnet of any previous rules’ lhs’s, we add it to the set of previous rules.

Note that we consider a rule to be a subtree of another rule up to alpha conversion, i.e., variable names are not considered.

### 5.2 Sequential set property

The check for the sequential set property is a bit more complicated than the subnet one. According to the definition of the well-formedness property, there must exist a sequential set that contains all nested active pairs in a given set of INP rules. Rather than attempting to construct such a sequential set, the algorithm tries to falsify this condition: it searches (exhaustively) for two nested patterns that cannot be in the same sequential set. This is done as follows:

For the current nested pattern \( P \) and all previously verified patterns \( Q \) with the same active agents:

1. Compare the sets \( \text{Pos}(P) \) and \( \text{Pos}(Q) \). We only consider the positions of agents at this point, not the agents themselves.
2. If one set is a subset of another, \( P \) and \( Q \) can be added to a sequential set \[^3\]. \( P \) is added to the set of previous nested patterns.
3. Else, we compare the actual nested agents at the common positions \( CP = \text{Pos}(P) \cap \text{Pos}(Q) \).
4. If for each element \( p \in CP, \alpha \) and \( \beta \) are the same where \( (p, \alpha) \in \text{PosSym}(P) \) and \( (p, \beta) \in \text{PosSym}(Q) \), no sequential set can contain \( P \) and \( Q \), as \( P \equiv \langle M, x \sim \alpha(...), a \rangle, Q \equiv \langle M, y \sim \beta(...), b \rangle \) with \( x \neq y \)

\[^3\]Note that \( P \) and \( Q \) have already passed the subnet check at this point. This means that (some of) the nested agents at the common positions are different. Hence, \( P \) and \( Q \) cannot give rise to a critical pair.
5. Else, $P$ and $Q$ can be added to a sequential set. $P$ is added to the set of previously verified nested patterns.

It is straightforward to see that after the full traversal of the AST, all possible pairs of nested patterns are considered. Hence, the search for a pair that violates the sequential set property is exhaustive.

**Proposition 5.3.** Let $R$ be a set of INP rules. $R$ is well-formed $\iff R$ is correctly verified to be well-formed using our verification algorithm.

**Proof.** \(\Leftarrow\):

Assume $R$ is not well-formed and passes the well-formedness checks. We proceed by a complete case distinction (according to the definition of well-formedness):

**Case 1.** There exist two rules $P \Rightarrow N, Q \Rightarrow M \in R$ where $P$ is a subnet of $Q$. But then, the pair $(P, Q)$ is tested for the subnet relation (Section 5.1). Hence, $R$ does not pass the well-formedness check.

**Case 2.** There exist two rules $A \Rightarrow N, B \Rightarrow M \in R$ where $A \equiv \langle P, x \sim \alpha(\ldots) \rangle, B \equiv \langle P, y \sim \beta(\ldots) \rangle$ for $x \neq y$. But since all pairs of nested patterns are checked for the sequential set property (Section 5.2), $(A, B)$ will be detected. Hence, $R$ does not pass the check.

In both cases, we reach a contradiction to the assumption above, hence it cannot be true.

\(\Rightarrow\):

Assume $R$ does not pass the well-formedness check, but is well-formed. Again, there are only two cases:

**Case 1.** $R$ does not pass the check because $\exists P \Rightarrow N, Q \Rightarrow N' \in R$ where $P$ is a subnet of $Q$. But then, $R$ is not well-formed (by the definition of well-formedness).

**Case 2.** $R$ does not pass because are two rules $A \Rightarrow N, B \Rightarrow M \in R$ where $A \equiv \langle P, x \sim \alpha(\ldots) \rangle, B \equiv \langle P, y \sim \beta(\ldots) \rangle$ for $x \neq y$. Then, there is no sequential set that contains both $A$ and $B$. Hence, $R$ is not well-formed.

Again, we reach a contradiction to the assumption above in either case.

\(\square\)

## 6 Rule Translation

We now describe the translation algorithm in more detail. As mentioned earlier, we translate INP rules to ORN rules by rewriting the AST. We perform a pre-order traversal of the AST and identify nodes that represent INP rules. Once we find an INP rule, we replace its nested agents with a fresh (variable) node $n$ and replace the subtree that represents the rhs of the rule with a new tree $N_t$. The nested agents and the rhs of the rule are stored for later processing. The tree $N_t$ represents an active pair between $n$ and a newly created auxiliary agent $Aux$. This auxiliary agent holds all the agents and attributes of the original active pair, with the exception of the former variable agent.

Now, we create an auxiliary rule with an active pair between $Aux$ and the current nested agent (initially connected to the interacting agent). We set the rhs of the auxiliary rule to be the rhs of the original INP rule. Finally, we add this auxiliary rule to the system.

Note that the auxiliary agent in the new rule may still contain additional nested agents, i.e., the auxiliary rule may be INP. Hence, the translation algorithm recursively translates the generated rules...
An Implementation of Nested Pattern Matching in Interaction Nets

until the lhs of each of the generated rules contains exactly two agents. The idea behind this is to resolve one nested agent per translation pass. Further nested agents are processed when the translation function reaches the respective auxiliary rule(s).

We can formalise the translation algorithm as a function \( \text{translate}(R, U, S) \), where \( R \) denotes the input set of interaction rules and \( U \) and \( S \) are stores. Intuitively, the components \( U \) and \( S \) are used to store previously processed rule patterns in order to verify the subnet and sequential set properties respectively. The function \( \text{translate} \) is defined as follows (in pseudo-code notation), where \( \text{FAIL} \) denotes termination of the program due to non well-formedness of \( R \):

\[
\text{translate}([], U, S) = []
\]

\[
\text{translate}(\langle P = \Rightarrow N \rangle : R, U, S) =
\]

if (\( P \) is a subnet of any \( Q \in U \) or vice versa)
    \( \text{FAIL} \)
else if (\( P \) is ORN)
    \( (P = \Rightarrow N): \text{translate}(R, P : U, S) \)
else
    if (\( P \) cannot be added to a sequential set with
        any \( Q \in S \))
        \( \text{FAIL} \)
    else
        \( (P' = \Rightarrow (P \sim p)): \text{translate}((P < A \Rightarrow N) : R, P : U, P : S) \)
where
- \( p \) = position of the first nested agent of \( P \)
- \( A \) = the nested agents at position \( p \)
- \( P' \) = \( P \) with all nested agents replaced by variables
- \( PX \) = auxiliary agent that contains all ports of \( P \)
except \( p \)

**Example 6.1.** Consider the interaction rules from Figure 3. \( R \) consists of two rules:

1. \( \text{Lst}(r) \gg \text{Cons}(x, \text{Nil}) \Rightarrow r \sim x \)
2. \( \text{Lst}(r) \gg \text{Cons}(x, \text{Cons}(y, ys)) \Rightarrow x \sim \varepsilon, \text{Lst}(r) \sim \text{Cons}(y, ys) \)

The translation works as follows:

- Rule 1 is INP (due to the \( \text{Nil} \) agent).
  Its lhs is checked for the subnet property. As there are no previous rules in \( U \), the check is passed.
- As the rule is INP, it is checked for the sequential set property. Again, there are no previous rules in \( S \), hence it passes the check.
- The rule is transformed, introducing the auxiliary agent \( \text{Lst}_\text{Cons} \)
  \[ 1. \text{Lst}(r) \gg \text{Cons}(x, \text{var}0) \Rightarrow \text{Lst}_\text{Cons}(r, x) \sim \text{var}0 \]

- A new auxiliary rule is added to \( R \):
  \[ 3. \text{Lst}_\text{Cons}(r, x) \gg \text{Nil} \Rightarrow r \sim x \]

The lhs pattern of rule 1 is added to \( U \) and \( S \).
• Rule 2 is INP (due to the nested Cons agents).
  Its lhs is checked for the subnet property. The only lhs pattern in $U$ is $\text{Lst}(r) \triangleright\!\!< \text{Cons}(x,\text{Nil})$. Due to different agents at the second auxiliary port of Cons, the lrhs cannot be subnets of one another. The check is passed.
• Rule 2 is checked for the sequential set property. First, the positions of nested agents of Rule 1 and 2 are compared. Since they are the same (both have their nested agent at the second auxiliary port of Cons), they can be added to a sequential set. There are no further rules in $S$, hence the check is passed.
• The rule is transformed and another auxiliary rule is added to $R$:
  2. $\text{Lst}(r) \triangleright\!\!< \text{cons}(x,\text{var1}) \implies \text{Lst}_{\text{Cons}}(r,x)\sim\text{var1}$
  4. $\text{Lst}_{\text{Cons}}(r,x) \triangleright\!\!< \text{cons}(y,\text{ys}) \implies x\sim\epsilon, \text{Lst}(r)\sim\text{Cons}(y,\text{ys})$
• Rule 3 is ORN.
  Its lhs is checked for the subnet property. As there are no with the same active agents in $U$, the check is passed. The lhs pattern of Rule 3 is added to $U$.
• Rule 4 is ORN.
  Its lhs is checked for the subnet property. Again, there are no rules with the same active agents in $S$. The check is passed and the lhs pattern is added to $U$.

This yields the translated set of rules

1. $\text{Lst}(r) \triangleright\!\!< \text{cons}(x,\text{var0}) \implies \text{Lst}_{\text{Cons}}(r,x)\sim\text{var0}$
2. $\text{Lst}(r) \triangleright\!\!< \text{cons}(x,\text{var1}) \implies \text{Lst}_{\text{Cons}}(r,x)\sim\text{var1}$
3. $\text{Lst}_{\text{Cons}}(r,x) \triangleright\!\!< \text{Nil} \implies r\sim x$
4. $\text{Lst}_{\text{Cons}}(r,x) \triangleright\!\!< \text{cons}(y,\text{ys}) \implies x\sim\epsilon, \text{Lst}(r)\sim\text{Cons}(y,\text{ys})$

Note that the rules 1 and 2 are identical (save variable names). Since only one of them is needed, rule 2 is discarded by PIN. As expected, we get the set of ORN rules given in Figure 1.

**Proposition 6.2. (Termination)** For a finite $R$, translate terminates.

**Proof.** Let $n$ be the number of rules in $R$ and $p$ be the sum of all nested agents of these rules. By a complete case distinction, we show that with each recursive call, $(n + p)$ decreases:

**Case 1** The current rule is ORN. At the recursive call, it is removed from $R$, hence $n$ decreases by 1.

**Case 2** The current rule has $i$ nested agents ($i > 0$). The rule is removed from $R$ and an auxiliary rule with exactly $i - 1$ nested agents is added to $R$. Hence, with the recursive call $p$ decreases by 1.

translate terminates if $n = 0$. $n$ only decreases if we encounter an ORN rule. Yet, since the number of nested agents for each rule is finite, all rules in $R$ will be ORN after finitely many decreases of $p$. ☐

### 6.1 Time and space complexity of translate

**Time complexity** To determine the time complexity of translate, we analyze the three main parts of the algorithm: the check for the subnet property, the check for the sequential set property and the actual rule translation. As a measurement of the input size, we consider the sum $n = r + a$, where $r$ is the number of rules and $a$ is the number of nested agents in the input set $R$. The idea behind this notion of size is that translate is invoked once for every interaction rule in the input. Additionally, each elimination of a nested agent yields a new auxiliary rule to be translated. Hence, translate is called $n = r + a$ times.
The subnet property check compares all pairs of rule patterns. For an input set of size \( n \), \( \frac{n(n-1)}{2} \) checks are performed. Therefore, the verification of the subnet property is quadratic to the input size, or \( O(n^2) \).

At first glance, the check for the sequential set property is similar to the one for the subnet property: all pairs of rule patterns need to be compared, resulting in a (worst-case) complexity of \( O(n^2) \). However, the verification algorithm only considers pairs of patterns with the same active agents (see \( \ref{5.2} \)), which is usually only a fraction of all pairs of rule patterns. On average, this results in a less than quadratic complexity for the verification of the sequential set property.

The actual translation is executed once for each nested agent in the input set. The elimination of a single nested agent is not influenced by the total number of nested agents or rules. The rule translation is thus linear to the number of nested agents \( r \).

This results in an overall time complexity of \( O(n^2) \) for \textit{translate}, which is mostly determined by the verification of the subnet property. Extended profiling on the implementation of the algorithm (including input sets with up to several thousand nested agents and rules) shows that also on practical cases the performance is bounded by a quadratic curve (with small constants involved).

**Space complexity**  
To analyze the space complexity of \textit{translate}, we consider the population of the stores \( U \) and \( S \). Every rule pattern of the input set is stored in \( U \), whereas only nested patterns are stored in \( S \). At the worst case, \( U \) and \( S \) will contain \( 2n \) rule patterns. This implies that space complexity is linear to the number of rules in the input. Again, this result is reflected by several profiling tests using practical examples.

### 6.2 Additional language features

The \texttt{PIN} language offers some features that are not considered in the original definition of the nested pattern translation function. Some important examples are data values of agents (integers, floats, strings, . . .), side effects (declaration and manipulation of variables, I/O) and conditions. These features are not involved in the process of nested pattern matching. Therefore, they do not need to be processed or changed by the translation function:

- with regard to nested pattern matching, data values can be considered as variable ports (they do not contain nested agents). Hence, they are unaffected by the translation.
- conditionals and side effects only occur in the rhs of a rule. Since the original rhs of an INP rule is propagated to the final auxiliary rule without a change, these features are not affected either.
- all auxiliary rules but the last one are responsible for pattern matching only, they do not do the “actual work” of the original rule. All special language features are simply passed to the next auxiliary rule.

### 6.3 The implementation

We have developed a prototype implementation which can be obtained from the project’s web page \texttt{http://www.interaction-nets.org/}. We have thoroughly tested the prototype implementation and developed several example modules. These examples include rule systems with a large number and depth of nested patterns as well as heavy use of state, conditionals and I/O. Additionally, we have designed several non well-formed systems in order to improve error handling. Some of these examples can be found in the current \texttt{PIN} distribution at the project’s web page.
7 Conclusion

We have presented an implementation for nested pattern matching of interaction rules. The implementation closely follows the definition of nested patterns and their translation to ordinary patterns introduced in [4]. We have shown nice properties of the algorithm such as its correctness and termination.

The resulting system allows programs to be expressed in a more convenient way rather than introducing auxiliary agents and rules to pattern match nested agents. We see this as a positive step for further extensions to interaction nets: future implementations of high-level language constructs can be built upon these more expressive rules.

References

[1] Jean-Yves Girard. Linear Logic. *Theoretical Computer Science*, 50(1):1–102, 1987.
[2] Georges Gonthier, Martín Abadi, and Jean-Jacques Lévy. The geometry of optimal lambda reduction. In *Proceedings of the 19th ACM Symposium on Principles of Programming Languages (POPL’92)*, pages 15–26. ACM Press, January 1992.
[3] Georges Gonthier, Martín Abadi, and Jean-Jacques Lévy. Linear logic without boxes. In *Proceedings of the 7th IEEE Symposium on Logic in Computer Science (LICS’92)*, pages 223–234. IEEE Press, 1992.
[4] Abubakar Hassan and Shinya Sato. Interaction nets with nested pattern matching. *Electronic Notes in Theoretical Computer Science*, 203, 2008.
[5] Yves Lafont. Interaction nets. In *Proceedings of the 17th ACM Symposium on Principles of Programming Languages (POPL’90)*, pages 95–108. ACM Press, January 1990.
[6] John Lamping. An algorithm for optimal lambda calculus reduction. In *Proceedings of the 17th ACM Symposium on Principles of Programming Languages (POPL’90)*, pages 16–30. ACM Press, January 1990.
[7] Jean-Jacques Lévy. Réductions Correctes et Optimales dans le Lambda-Calcul. Thèse d’état, Université Paris VII, January 1978.
[8] Ian Mackie. YALE: Yet another lambda evaluator based on interaction nets. In *Proceedings of the 3rd International Conference on Functional Programming (ICFP’98)*, pages 117–128. ACM Press, 1998.
[9] Ian Mackie, Abubakar Hassan, and Shinya Sato. Interaction nets: programming language design and implementation. *Proceedings of the Seventh International Workshop on Graph Transformation and Visual Modeling Techniques*, 2008.
[10] Jorge Sousa Pinto. *Parallel Implementation with Linear Logic*. PhD thesis, École Polytechnique, February 2001.
[11] François-Régis Sinot and Ian Mackie. Macros for interaction nets: A conservative extension of interaction nets. *Electronic Notes Theoretical Computer Science*, 127(5):153–169, 2005.