The seesaw path to leptonic CP violation

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Abstract
Future experiments such as SHiP and high-intensity $e^+e^-$ colliders will have a superb sensitivity to heavy Majorana neutrinos with masses below $M_Z$. We show that the measurement of the mixing to electrons and muons of one such state could establish the existence of CP violating phases in the neutrino mixing matrix, in the context of low-scale seesaw models. We quantify in the minimal model the CP reach of these future experiments, and demonstrate that CP violating phases in the mixing matrix could be established at $5\sigma$ CL in a very significant fraction of parameter space.

1 Introduction

The simplest extension of the standard model that can accommodate naturally light neutrino masses is the well-known seesaw model [1–4], where at least two singlet Majorana fermions, $N^R_i$, are added and can couple to the lepton doublets via a Higgs–Yukawa coupling.

The Lagrangian of the model is given by

$$\mathcal{L} = \mathcal{L}_{\text{SM}} - \sum_{\alpha,i} \bar{L}^\alpha Y^{\alpha i} \Phi N^R_i - \frac{1}{2} \sum_{i,j=1}^{2} \tilde{N}^R_i M^{ij} N^R_j + \text{h.c.},$$

where $Y$ is a $3 \times 2$ complex matrix and $M$ is a two-dimensional complex symmetric matrix.

While the Majorana mass scale, $M$, has been traditionally assumed to be very large so that the light neutrino masses can be explained with Yukawa couplings of $O(1)$, the absence of any indication of new physics that solves the hierarchy problem suggests that a more natural option might be to assume a Majorana scale of the order of the electroweak scale. The lepton flavour puzzle would not be qualitatively very different from the quark one, in particular there would not be a large gap between neutrino Yukawas and those of the charged leptons, since the neutrino Yukawas would need to be only slightly smaller than the electron one:

$$\frac{y_\nu}{y_e} \simeq 0.2 \sqrt{\frac{m_\nu}{0.05\text{eV}}} \frac{M}{v}.$$  

(1)

From a phenomenological point of view this is of course a very interesting possibility since the heavy Majorana neutrino states might be searched for in fixed-target experiments and at colliders.

In this letter we explore the opportunities that this opens for the discovery of leptonic CP violation. In particular we concentrate on the observables that could be provided by direct searches of these heavy neutrino states, more concretely their mixings to electrons and muons. It is well known that in seesaw models there is a strong correlation between the mixings of the heavy states and the light and heavy neutrino spectrum. In particular for just one neutrino species/flavour, the mixing of the heavy state is fixed to be

$$|U_{\alpha h}|^2 = \frac{m_1}{M_h},$$

(2)

where $m_1$ is the light neutrino mass and $M_h$ is the heavy one. This is the naive seesaw scaling which implies that the mixing of heavy states is highly suppressed when $M_h \gg m_1$.

In the case of more families the correlation still exists but it is not so strong. In particular, for the minimal model with two heavy neutrinos, using the Casas–Ibarra parametrization of the model [5], the mixing of the heavy neutrinos is given in all generality by

$$U_{\alpha h} = i U_{\text{PMNS}} \sqrt{m_1} P_{\text{NO}} R^+ (\phi) M^{-1/2},$$

(3)

where $m_1$ is the diagonal matrix of the light neutrino masses (note that the lightest neutrino is massless) and $U_{\text{PMNS}}$ is the PMNS matrix in the standard parametrization.
depends only on two CP violating phases [6–8]. \( \rho_{\text{NO}} \) is a 3 \( \times \) 2 matrix that depends on the neutrino ordering (NH, IH)

\[
P_{\text{NH}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad P_{\text{IH}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix},
\]

where \( I \) is the 2 \( \times \) 2 identity matrix and \( \theta = (0, 0) \) [9]. The unknown parameters are \( M = \text{Diag}(M_1, M_2) \), the diagonal matrix of the heavy neutrino masses, and \( R \), a two-dimensional complex orthogonal matrix, that depends generically on one complex angle, \( z = \theta + i\gamma \). The entries in this matrix can be very large and therefore the naive seesaw scaling of Eq. (2) is not generically satisfied in the case of more than one family. In this work we will fix the known oscillation parameters to their best fit values as taken from [10].

In [11], we pointed out that in the region of large mixings the ratio of mixings to electron and muons of the heavy states are essentially fixed by the PMNS matrix, and is therefore very sensitive to its unknown CP phases. In this paper, our aim is to quantify the CP discovery potential of such measurement. Interestingly this measurement provides an example of a CP conserving observable that is sensitive to leptonic CP violation.

It should be noted that the results below will also apply to the model with three neutrinos, where one of them is sufficiently decoupled, as in the vMSM [12], and more generally to more complex seesaw models where the dominant contribution to neutrino masses comes from the two singlet states considered.

### 2 Heavy neutrino mixings versus leptonic CP phases

Given the upper bound on the light neutrino masses of \( \mathcal{O}(1) \) eV, the naive seesaw scaling formula of Eq. (2) implies that the mixings of the heavy states are of \( |U_{ab}|^2 \simeq \mathcal{O} \left( \frac{10^{-9} \text{GeV}}{M} \right) \), a value at the limit of sensitivity of SHiP [13] in the GeV range or FCC-ee in the \( \mathcal{O}(10) \) GeV range [14]. This implies that in most of the sensitivity range of SHiP and other collider experiments the entries of \( R \) need to be largish (i.e. \( \gamma \gtrsim 1 \)), and in this case the dependence of the mixings on the unknown parameters simplifies greatly. Indeed, a perturbative expansion to \( \mathcal{O}(\epsilon) \) in the small parameters

\[
\epsilon \sim r \equiv \sqrt{\frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2}} \sim |\theta_{13}| \sim e^{-\frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2}},
\]

shows that the ratio of electron/muon mixings does not depend on the complex angle \( \gamma, \theta \), nor on the masses of the heavy states, and only depends on the the mixing angles and CP phases of the PMNS matrix [11]. Defining \( A \equiv e^{i \sqrt{\Delta m_{\text{sol}}^2}} \), the result for the inverted ordering (IH) is

\[
|U_{\text{ee}}|^2 M_1 \simeq A \left[ (1 + \sin \phi_1 \sin 2\theta_{12})(1 - \theta_{13}^2) + \frac{1}{2} r^2 s_{12}(c_{12}\sin \phi_1 + s_{12}) + \mathcal{O}(\epsilon^3) \right],
\]

\[
|U_{\mu e}|^2 M_1 \simeq A \left[ (1 - \sin \phi_1 \sin 2\theta_{12} \left( \frac{1}{4} + r^2 + \frac{1}{4} r^2 c_{12}^2 \right) c_{23}^2 + \theta_{13} (\cos \phi_1 \sin \delta - \sin \phi_1 \cos 2\theta_{12} \cos \delta) \sin 2\theta_{23} + \theta_{13}^2 (1 + \sin \phi_1 \sin 2\theta_{12}) s_{23}^2 + \mathcal{O}(\epsilon^2) \right],
\]

while for the normal one (NH) we have

\[
|U_{\text{ee}}|^2 M_1 \simeq A \left[ r s_{12}^2 - 2 \sqrt{r} \theta_{13} \sin(\delta + \phi_1) s_{12} + \theta_{13}^2 + \mathcal{O}(\epsilon^{5/2}) \right],
\]

\[
|U_{\mu e}|^2 M_1 \simeq A \left[ s_{23}^2 - \sqrt{r} c_{12} \sin \phi_1 \sin 2\theta_{23} + r c_{23}^2 s_{23}^2 + 2 \sqrt{r} \theta_{13} \sin(\phi_1 + \delta) s_{12}^2 s_{23}^2 - \theta_{13}^2 s_{23}^2 + \mathcal{O}(\epsilon^{5/2}) \right].
\]

On the other hand, the parameter \( \gamma \) and the Majorana masses determine the global size of the mixings, and therefore the statistical uncertainty in the measurement of the ratio of mixings.

However, values of the heavy mixings much larger that the naive seesaw scaling in Eq. (2) imply generically large one loop corrections to neutrino masses [15] except in the region where an approximate lepton number symmetry holds [16–19], which implies highly degenerate neutrinos, besides large \( \gamma \).

We want to quantify the range of the PMNS CP phase parameter space, that is, the rectangle \( \delta, \phi_1 \in [0, 2\pi] \), in which leptonic CP violation can be discovered via the measurement of the electron and muon mixings of one of the heavy neutrinos. In other words, we wonder for which values of the true parameters \( (\delta, \phi_1) \) these mixings be distinguished from those at the CP conserving points \((0, 0), (0, \pi), (\pi, 0), (\pi, \pi)\).

### 3 Discovery of Majorana neutrinos

We consider two future experiments that have the potential to discover sterile neutrinos in complementary regions of masses: SHiP [13] and FCC-ee [14].

#### 3.1 The SHiP experiment

The SHiP experiment [13] will search for the heavy neutrinos of the seesaw model in charmed and b meson decays. Neutrinos with masses below charmed and b meson masses could...
be discovered, and the best sensitivity is expected for masses around 1 GeV. In order to carry out our study we would ideally need an estimate of the uncertainty in the determination of the mixing angles of the sterile neutrinos to electrons and muons as a function of those mixings and the heavy neutrino masses. Such information has not been published yet by the collaboration. The only publicly available result is the sensitivity reach (corresponding to 2 expected events in a 5 year run with 0.1 estimated background events) on the plane $U^2 \equiv \sum_{\alpha} |U_{\alpha i}|^2$ vs. $M_1$ for five scenarios, where the contribution of electron and muon channels is combined, and only the channels with two charged particles in the final state are considered. These scenarios also correspond to two highly degenerate neutrinos with equal mixings that contribute equally to the signal. Since we are interested in a more general case, where neutrinos might not be degenerate, we consider the contribution of just one of the states, so we will assume that at the sensitivity limit the number of events is one instead of two.

The number of events in the charged-current two-body decays $N \rightarrow l^\pm_a H \mp$, for different hadrons $H = \pi, \rho, \ldots, \pi$, is expected to scale as

$$N_\alpha(M_1) \propto |U_{\alpha i}|^2 U^2,$$

because $U^2$ controls the number of charm decays into sterile neutrinos, while $|U_{\alpha i}|^2$ controls the fraction of those that decay to the lepton flavour $\alpha$ via a charged current within the detector length (which is assumed to be much smaller than the decay length). Note that in the charged-current three-body decays $N \rightarrow l^+_a l^\mp_i H \mp$ with $\alpha \neq \beta$, the contributions from the charged-current muon and electron mixings cannot be distinguished, because the $N$ are Majorana. Also those decays with $\alpha = \beta$ cannot be distinguished from the corresponding neutral-current process. These three-body decays are, however, subleading in the sensitivity regime of SHiP so we will consider only the information from the two-body decays.

Among the five scenarios studied in [13,20], there are, however, two of them where either the mixing to electrons or the mixing to muons dominates by a large factor. In Fig. 1 we reproduce the SHiP sensitivity curves for these electron-dominated (IV) and muon-dominated (II) scenarios. As an illustration, we also indicate in Fig. 1 the mixings corresponding to different values of the Casas–Ibarra parameter $\gamma$. Stringent bounds exist from direct searches. We do not include them in the plot because they are flavour dependent and cannot easily be interpreted in terms of $U^2$. They restrict severely the region of masses below 0.5 GeV and cut slightly the range of largest mixings. We will therefore not consider this region of parameter space. In the figure we also indicate the line that corresponds to one-loop corrections to neutrino mass differences being of the same order as the atmospheric mass splitting, for a level of degeneracy $\Delta M/M_1 = 1$. No degeneracy is therefore required in this range of masses to reach mixings up to $U^2 \sim 10^{-6}$ without fine-tuning.

In order to estimate the number of electron and muon events as a function of the Casas–Ibarra parameters, we assume that the total number of events (with two charged particles in the final state) is one at the sensitivity limit curve in the two scenarios, and we compute the contribution to this total of the electron and muon two-body decay channels. Note that for $M \sim 1$ GeV the three-body decays are subleading. At each mass, the number of electron and muon events at different value of the mixings is obtained by scaling this number according to Eq. (8).

Besides the number of electron and muon events we assume that the mass of the heavy neutrino is measured, which should be easy in these two-body channels. Again the uncertainty in the determination of the mass as a function of the mass and mixing has not yet been presented by the SHiP collaboration. A realistic error is much harder to estimate in this case, since it involves reconstructing a peak in the invariant mass. We will therefore assume a fixed relative error of 1%. We note that this uncertainty is not expected to be very relevant, since most information on the phases comes from the ratio of mixings where the mass dependence drops.

### 3.2 FCC-ee collider

For masses above 2 GeV, the heavy neutrinos can be searched for in high luminosity colliders such as the future $e^+e^-$ circular collider (FCC-ee) [14,21–23] that can provide $10^{12} - 10^{13}$ $Z$ boson decays at rest per year. Sterile neutrinos with masses $m_N \leq M_Z$ can be produced in the decay $Z \rightarrow N\nu$, and they leave a very characteristic signal of a displaced vertex produced by the decay of the long-lived heavy neutrino. A simplified estimate of the number of events to visible lepton channels within the detector can be obtained as follows:
\[ N_{\text{total}} = N_Z \frac{\text{BR}(Z \to N\nu) \text{BR}(N \to \text{leptonic})}{(e^{-\frac{l_{\min}}{2\gamma_L}} - e^{-\frac{l_{\max}}{2\gamma_L}})^N}, \]

where \( N_Z \) is the number of \( Z \) decays, \( \gamma_L = \frac{1}{2} \left( \frac{M_{\ell}^2 + M_Z^2}{M_{\ell}^2} \right) \) is the heavy neutrino Lorentz boost gamma factor and \( \tau_{\ell} \) is its lifetime. \( l_{\min} \) and \( l_{\max} \) are the minimum and maximum displacement of the secondary vertex that can be measured. The \( Z \to N\nu \) branching ratio has been computed in [24] to be

\[ \text{BR}(Z \to N\nu) = 2U^2 \frac{\text{BR}(Z \to \nu\bar{\nu})}{(1 - \frac{M_{\ell}^2}{2M_Z^2})^2} \times \left(1 + \frac{1}{2} \frac{M_{\ell}^2}{M_Z^2} \right), \]

where \( \text{BR}(Z \to \nu\bar{\nu}) \) corresponds to one family.

The partial width to the leptonic channels can be found in many references, see for example [25,26], while the total width requires an estimate of the hadronic width. While in the SHiP case, exclusive hadronic channels where considered, for the heavier masses relevant at FCC-ee, the inclusive hadronic decay width is approximated by the parton model [27]. This is sufficient for our purposes. Note, however, that a precise determination of this width would be possible, applying similar methods to those used in \( \tau \) decays [28].

In the recent analysis of [20], a FCC-ee configuration with \( 10^{13} \) \( Z \) decays per year and an inner tracker that can resolve a displaced vertex at a distance between \( l_{\min} = 0.1 \) mm and \( l_{\max} = 5 \) m has been assumed in the context of the same two scenarios above. The sensitivity reach using only leptonic channels and assuming 100% efficiency and zero background is reproduced in Fig. 2, which agrees reasonably well with the results of [20]. The regions of mixings above the red lines would imply fine-tuning between the tree level and one-loop contributions to the light neutrino masses for a level of degeneracy of \( \Delta M/M_1 = 1 \) (solid) and \( \Delta M/M_1 = 0.01 \) (dashed).

Since we are interested in measuring separately the mixings to electrons and muons, we will separate the distinguishable leptonic channels \( l_\alpha l_\beta = ee, e\mu, \mu\mu \). Note that each of these channels depends on several mixings, either because they get a contribution from neutral currents in the case of \( \beta = \alpha \) or because the channels where the leading lepton (i.e. that coupled to the \( N_i \)) is \( \alpha \) or \( \beta \) \( \neq \alpha \) are indistinguishable. We also consider the inclusive hadronic ones, \( N \to l_\alpha + q + \bar{q}' \), that depend only on the mixing \( |U_{l\alpha}| \).

Finally we assume an angular acceptance of ~80% (i.e. we require the \( N \) direction is at least 15° off the beam axis) and include just statistical uncertainties.

### 4 CP reach

In order to quantify the discovery CP potential we consider that SHiP or FCC-ee will measure the number of electron and muon events in the decay of one of the heavy neutrino states (without loss of generality we assume to be that with mass \( M_1 \)), estimated as explained in the previous section. We will only consider statistical errors.

The test statistics (TS) for leptonic CP violation is then defined as follows:

\[ \Delta \chi^2 = -2 \sum_{\alpha=\text{channel}} N_{\alpha}^{\text{true}} - N_{\alpha}^{\text{CP}} + N_{\alpha}^{\text{true}} \log \left( \frac{N_{\alpha}^{\text{CP}}}{N_{\alpha}^{\text{true}}} \right) + \left( \frac{M_{\alpha} - M_{\alpha}^{\text{min}}}{\Delta M_{1,\alpha}} \right)^2, \]

where \( N_{\alpha}^{\text{true}} = N_{\alpha}(\delta, \phi_1, M_1, \gamma, \theta) \) is the number of events for the true model parameters, and \( N_{\alpha}^{\text{CP}} = N_{\alpha}(\text{CP}, \gamma^{\text{min}}, \theta^{\text{min}}, M_{1,\alpha}^{\text{min}}) \) is the number of events for the CP conserving test hypothesis that minimizes \( \Delta \chi^2 \) among the four CP conserving phase choices \( \text{CP} = (0/\pi, 0/\pi) \) and over the unknown test parameters. \( \Delta M_{1,\alpha} \) is the uncertainty in the mass, which is assumed to be 1%.

Surface plots for \( \Delta \chi^2 \) are shown in Fig. 3 for SHiP and the true parameters \( \{\gamma, \theta, M_1\} = \{3.5, 0, 1 \text{ GeV}\} \). The basic features of these contours can be understood analytically, as shown by the superimposed lines corresponding to a constant electron–muon mixing ratio, as obtained from the analytical formulae in Eqs. (6) and (7).

We have evaluated via Monte Carlo the statistical distribution of this test statistics in order to define confidence intervals for the exclusion of the CP conserving hypothesis, following the approach of [29]. In Fig. 4 we show the result of \( \mathcal{O}(10^7) \) experiments where the true value for the phases

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**Fig. 2** Lines corresponding to two expected events for scenarios II (solid) and IV (dashed) in [20]. The indicated points correspond to the CP conserving case and \( \gamma = 2, 3, 4, 5, 6 \) in ascending order for IH (circles) and NH (squares). The red lines in the top right corner correspond to one-loop corrections to neutrino masses being of the same order as the atmospheric splitting for \( \Delta M/M_1 = 1 \) (solid) and \( \Delta M/M_1 = 0.01 \) (dashed).
is any of the CP conserving hypotheses and the number of events is distributed according to Poisson statistics. The true distribution is very similar for NH/II and is very well approximated by the $\chi^2(1\text{dof})$. This is probably due to the strong correlation between the two CP phases. We conclude from this exercise that is a good approximation to define the $5\sigma$ regions in the following as those corresponding to $\Delta\chi^2 = 25$.

As indicated by the analytical results, we expect almost no dependence on the parameter $\theta$, while the sensitivity to CP violation is expected to depend significantly on $\gamma$ and $M_1$, which control the size of the mixing. In Fig. 5 we show the $5\sigma$ contours with $M_1 = 1$ GeV and for different values of true $\gamma$ on the plane $(\delta, \phi_1)$. The coloured regions limited by

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**Fig. 3** Surface plots of $\Delta\chi^2$ for $\gamma, \theta, M_1 = (3.5, 0, 1 \text{ GeV})$ as a function of the true $(\delta, \phi_1)$ of II (up) and NH (down) for SHiP. The lines correspond to the constant value indicated of the ratio of electron–muon mixing as obtained from the analytical formulae of Eqs. (6) and (7).

**Fig. 4** Distribution of the test statistics for $O(10^7)$ number of experimental measurements of the number of events for true values of the phases $(\delta, \phi_1) = (0, 0)$ for II and $(\gamma, \theta, M_1) = (3.5, 0, 1 \text{ GeV})$, compared to the $\chi^2$ distribution for 1 or 2 degrees of freedom.

**Fig. 5** 5$\sigma$ CP violation SHiP discovery regions on the plane $(\delta, \phi_1)$ for various true values of $\gamma$ for II (up) and NH (down) and for the true parameters $M_1 = 1 \text{ GeV}$ and $\theta = 0$. 
the contours indicate the range of true CP phases for which leptonic CP conservation can be excluded at $5 \sigma$ CL.

A nice way of quantifying the potential of SHiP and FCC-ee is provided by the CP fraction, $R_{CP}$, defined as the fraction of the area of the CP phase square $\delta, \phi_1 \in [0, 2\pi]$ where leptonic CP violation can be discovered (i.e. the CP conserving hypothesis excluded) at $5 \sigma$ CL, in complete analogy to the CP fraction for the $\delta$ phase, often used in evaluating the sensitivity to CP violation of future long-baseline neutrino oscillation experiments. In Fig. 6 we show the $R_{CP}$ as a function of $\gamma$ and $M_1$.

Some observations are in order. CP fractions are significantly larger for IH than for NH, for the same $\gamma$. This can be understood because the dependence of the mixings on the phases is more suppressed in the latter. There is a very significant sensitivity to leptonic CP violation in these observables (mass and mixing to electron and muon). In SHiP with $M_1 = 1$ GeV, we obtain fractions above 70% for $\gamma \geq 3$ for IH (mixings of $\geq \theta(10^{-8})$) and for $\gamma \geq 3.8$ for NH (mixings of $\geq \theta(3 \times 10^{-9})$). For FCC-ee, with a mass 30 GeV, we get fractions above 70% for $\gamma \geq 3.1$ (mixings above $\theta(4 \times 10^{-10})$) for IH and for $\gamma \geq 4.5$ (mixings above $\theta(4 \times 10^{-9})$) for NH. The complementarity of SHiP and FCC-ee is clear, since they have sensitivity in different mass ranges. For mixings in the middle of their sensitivity region, they can reach fractions above 50% (90%) for NH(IH) in the mass range 0.4–2 GeV in SHiP, and above 50% (80%) for NH (IH) in the range 7–30 GeV in FCC-ee.

For simplicity, we have not included errors on the known oscillation parameters. In order to get a rough idea of the effect, we have considered an error on the atmospheric angle of 0.02 radians. The effect is only visible in the high sensitivity region, where the CP fraction saturates at $\sim 90\%$. We have also studied the impact of increasing the error on the determination of $M_1$ to 5%, and we observe a small effect in the high sensitivity region, for IH the CP fraction decreases from $98 \rightarrow 96\%$, while for the NH the effect is a bit bigger changing from $96 \rightarrow 88\%$. These effects are not expected to change significantly the results, but an analysis including all errors in the oscillation parameters should be done in future work.

Our analysis relies on rather simple assumptions, and should be further improved by including a realistic detector response and systematic errors. However, the potential of this measurement is excellent and we do not expect a big difference in the results when a more realistic simulation is implemented. It will also be interesting to include the extra information, provided by the measurement of the mixing to $\tau$ leptons, that should break the $\delta, \phi_1$ correlation to a discrete degeneracy [11]. Note also that we have only considered the contribution of one sterile state, but if there is another state in the same mass range, the statistics would improve by a factor of 2.

5 Conclusions

In seesaw models, the size of the mixing of the heavy Majorana neutrinos is strongly correlated with their masses, while their flavour structure is strongly dependent on the structure of the PMNS matrix. In the minimal seesaw model with just two right-handed neutrinos giving the largest contribution to neutrino masses, this correlation is strong enough for the ratio of the mixings to electron and muon flavours to be essentially fixed by the PMNS CP phases. In this letter we have quantified for the first time the sensitivity of these observables to leptonic CP violation in the mixing matrix. We have considered two proposed experiments SHiP and FCC-ee that have a superb sensitivity to heavy neutrinos in the mass range

![Fig. 6](image-url)
between $\mathcal{O}(1-100)$ GeV. Within their range of sensitivity, we have demonstrated that the discovery of one of these massive neutrinos and the measurement of its mass and its mixings to electrons and muons can establish the existence of CP violating phases in the mixing matrix at $5\sigma$ CL, in a very significant fraction of the CP phase parameter space ($>70\%$ for mixings above $\mathcal{O}(1/3 \times 10^{-8})$ for IH/NH in SHiP and above $\mathcal{O}(4 \times 10^{-10}/4 \times 10^{-9})$ in FCC-ee).

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