Probing superfluidity in a quasi two-dimensional Bose gas through its local dynamics

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Supplementary material for this article is available online

Abstract

We report direct evidence of superfluidity in a quasi two-dimensional Bose gas by observing its dynamical response to a collective excitation, the scissors mode. Relying on a novel local average analysis, we are able to probe inhomogeneous clouds and reveal their local dynamics. We identify in this way the superfluid and thermal phases inside the gas and locate the boundary at which the Berezinskii–Kosterlitz–Thouless crossover occurs. This new analysis also allows to evidence the coupling of the two fluids which induces at finite temperatures damping rates larger than the usual Landau damping.

Superfluidity is one of the most fascinating expressions of quantum mechanics at the macroscopic level. It manifests itself through specific dynamical properties, giving rise to collective effects as diverse as quantized vortices or second sound [1]. First observed in liquid helium [2], superfluidity has also been shown to occur for polaritons in semi-conductor micro-cavities or weakly interacting quantum gases [3]. Thanks to mature imaging techniques and a high degree of control over their parameters, the latter have proven to be particularly well suited for evidencing key features of superfluids: their linear excitation spectrum at low momentum [3] as well as the existence of a finite critical velocity have been observed [5–7]. Moreover when set into rotation quantum gases display vortices [8, 9] and present specific collective modes [10, 11]. Yet, extracting relevant information from the dynamics of these systems still represents an experimental challenge.

Properties of quantum gases are fully determined by the knowledge of their density and temperature. Hence, in order to explore different physical regimes, these quantities may have to be varied over a large range of values. While for homogeneous systems each repetition of an experiment will address a single point of this phase space, inhomogeneous quantum gases readily experience different regimes locally. This is the framework of the local density approximation (LDA), which has been extensively exploited while exploring physics of quantum gases at equilibrium. It has allowed for instance the determination of the equation of state of a Fermi gas at unitarity [12] and of a two-dimensional (2D) Bose gas both in the weakly and strongly interacting regimes [13–15]. In the same spirit, density fluctuations of a quasi-one-dimensional Bose gas have been described from the quasi-condensate to the nearly ideal gas regime [16].

In this work, we extend this approach to the in situ study of the dynamical properties of an inhomogeneous quantum gas, which is crucial to directly evidence superfluidity. We apply a novel local average analysis to probe the superfluidity of a quasi-2D Bose gas, observing its response to the so-called scissors excitation [11, 17]. It allows us to identify and locate the superfluid and normal phases within the system. In addition, at finite temperature we evidence a collisional coupling between them. In a sense, our scheme relying on the weak global excitation of a gas close to equilibrium probed locally is symmetric to the recent measurement of the critical velocity in an inhomogeneous quasi-2D Bose gas, where the effect of a strong local perturbation is probed through a global observable, the total heat deposited over the whole cloud [7].
On the other hand, these systems undergo a Berezinskii–Kosterlitz–Thouless (BKT) phase transition to a superfluid state [22–24]. This still holds for quantum gases confined in harmonic traps [25], and the normal to BKT crossover was first evidenced experimentally for quasi-2D Bose gases through the study of their phase coherence [26]. The equilibrium properties of 2D quantum gases have been explored in detail in the last few years: emergence of quasi long range order [26–28], role of the third, transverse, dimension in the emergence of phase coherence [29], phase fluctuations at finite temperature [30, 31], scale invariance [32], determination of the equation of state within LDA, both in the weakly and strongly interacting regimes [13–15], and crossover to BEC for vanishing interactions [33]. By contrast, here we study a 2D quantum gas out of equilibrium, and directly locate the boundary where the superfluid fraction vanishes across the BKT crossover through local average analysis.

The starting point for our experiments is a partially degenerate Bose gas of$^{87}$Rb atoms produced in a radio-frequency (rf) dressed quadrupole trap [34], loaded from an hybrid trap [35]. For our experimental parameters the measured vertical trapping frequency $\omega_z = 2\pi \times 1.83(1)$kHz is much larger than the in-plane trapping frequencies $\omega_x = 2\pi \times 33.8(2)$Hz and $\omega_y = 2\pi \times 48.0(2)$Hz, resulting in a pancake shaped atomic cloud. The trap in-plane geometry, defined by the direction of the eigenaxes (x,y) and the anisotropy $\epsilon = (\omega_x^2 - \omega_y^2)/(\omega_x^2 + \omega_y^2) = 0.34(3)$, is controlled by the rf polarization.

We prepare different samples by varying the total atom number and temperature. As both the temperature $k_B T$ and the chemical potential $\mu$ are comparable to the vertical oscillator energy level spacing $\hbar \omega_z$, the system is in the quasi-2D regime, and the effective dimensionless 2D interaction constant is $g = \sqrt{8\pi a/\ell_s} = 0.1056(3)$ [36], where $a$ is the scattering length, $\ell_s = \sqrt{\hbar/(m\omega_z)}$ the vertical harmonic oscillator length and $m$ the atomic mass. For most of our datasets the fraction of atoms out of the vertical oscillator groundstate is smaller than 20%, despite the relatively high temperatures and chemical potentials $^3$ (with respect to $\hbar \omega_z = h \times 1830$ Hz = $k_B \times 88$ nK). Therefore the system is well described by coupled 2D manifolds and exhibits 2D physics.

Specifically, the BKT transition is expected to occur in our trap at a phase space density $\phi = 8.188(3)$ for a reduced chemical potential $\alpha = \mu/(k_B T)$ equal to $\alpha_c = 0.162(1)$ [37]. These critical values increase slightly when the effect of the population in the transverse excited states at $k_B T \gtrsim \hbar \omega_z$ is taken into account [38]. The samples are prepared with a reduced chemical potential $\alpha$ in the range $[0.09–0.88]$, on both sides of the expected threshold (see supplementary material).

We excite the scissors oscillations by suddenly changing the orientation of the horizontal confinement axes by $\theta \approx 10^\circ$, while keeping the trap frequencies constant [39], see figure 1(a). The superfluid and thermal fractions of the cloud are subsequently expected to oscillate at different frequencies: $\omega_{\pm} = [\omega_x \pm \omega_y]$ for a normal gas close to the collisionless regime and $\omega_{bd} = \sqrt{\omega_x^2 + \omega_y^2}$ for the superfluid [11]. We typically record

See supplementary material for details on the imaging techniques, data processing and models.

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Figure 1. Principle of the experiment studied in this work. (a) A quasi-2D sample (blue shaded ellipse) is prepared in an anisotropic harmonic trap (dashed axes), and at $t = 0$ the trap axes are rotated by $\theta \approx 10^\circ$. (b) For $t > 0$, the cloud shape oscillates in the rotated trap (solid axes): the scissors mode is excited. (c) Examples of the evolution with holding time of the average $\langle \psi \rangle$ over the in situ atomic density distribution, for an almost purely degenerate sample ($\alpha = 0.78(18)$, open blue triangles) and a non-degenerate sample ($\alpha = 0.09(2)$, filled red squares), together with the corresponding fits (black solid lines). Each point is an average of two measurements. A constant offset has been subtracted from the first dataset for the sake of clarity.
one hundred \textit{in situ} pictures of the atoms after different holding times, spanning about 200 ms. In order to study the scissors mode at finite temperature, it is essential to be able to discriminate between the superfluid and thermal phases \cite{40}. However a direct fit of the 2D \textit{in situ} density profiles by a bi-modal function is quite imprecise because the typical width over which they extend are comparable.

Inspired by the theoretical works which use the average $\langle xy \rangle$ as an observable of the scissors mode excitation \cite{11,41,42}, we use an efficient algorithm to extract $\langle xy \rangle$ from our data (see supplementary material). For all of our datasets $\langle xy \rangle$ presents damped oscillations as illustrated in figure 1(c). In order to quantitatively analyze this signal, we take advantage of the classical description of the scissors oscillations \cite{11}. In this framework the complex oscillation frequencies $\omega$ of $\langle xy \rangle$ are parametrized by a characteristic relaxation time $\tau$:

$$
\frac{i\omega}{\tau}(\omega^2 - \omega_{hd}^2) + (\omega^2 - \omega^2_{\text{uid}})\omega = 0.
$$

This equation describes the scissors mode in a classical gas, from the collisionless to the hydrodynamic regime. In the hydrodynamic limit where $\tau$ vanishes, this model also describes the scissors mode in a superfluid \cite{11}. Applying this prediction to our data, we are able to fit (see supplementary material) the value of $\tau$ and deduce the frequencies $\omega_{\text{uid}}$ and damping rates $\Gamma_{\text{uid}}$ of the oscillations relying on the constraints of equation (1). In this way, we perform a double frequency fit with only three parameters (amplitude, offset and $\tau$). The model always predicts two frequencies, corresponding to an upper branch and a lower branch. The former spans frequencies in the range $[\omega_{\text{hd}}, \omega_{\text{uid}}]$ while the latter spans $[0, \omega_{\text{uid}}]$. We found this method to be more accurate than a fit to a sum of two damped cosines which requires at least nine free parameters, in particular for the determination of the lowest, highly damped, frequency. We checked that the fit constrained by equation (1) introduces no bias and that both models give the same results (see supplementary material).

Figure 2 reports the frequencies found as a function of the reduced chemical potential $\alpha$. Below the BKT transition ($\alpha < \alpha_c$, leftmost point of the graph), the two frequencies are very close to, but slightly below, the classical prediction $\omega_{\text{uid}}$. This, together with the large observed damping rates (see figure 1(c)), indicates that the gas is not fully in the collisionless regime, which would require $\omega_{\text{uid}} \tau \gg 1$ \cite{11}. For samples above the BKT transition ($\alpha > \alpha_c$) we find either a single frequency close to $\omega_{\text{hd}}$ or two frequencies, shifted below the collisionless frequencies $\omega_{\text{uid}}$. Our data are consistent with an increase of the upper frequency from $\omega_{\text{hd}}$ to $\omega_{\text{uid}}$ when $\alpha$ decreases, in agreement with 2D dynamical classical field simulations \cite{42}, and in stark contrast with what was observed in three-dimensional experiments \cite{40}. We note however that the simple criterion based on the critical value $\alpha_c$ is not sufficient to describe our datasets: in particular the thermal frequencies $\omega_{\text{uid}}$ are still observable beyond $\alpha = \alpha_c$. The global $\langle xy \rangle$ observable therefore fails to evidence the normal to superfluid crossover. Indeed, in these conditions the superfluid and normal phases coexist in the trap and the simple...
picture of equation (1) does not capture all the dynamics. Moreover, as the observable \( \langle xy \rangle \) is computed over the whole sample, both phases are combined in the total signal and it is difficult to isolate their respective signature.

In order to overcome this issue we introduce the concept of local average analysis. In the spirit of LDA, we define the rescaled radius \( \bar{r} = \sqrt{(\omega_x/\omega_y)^2 + (\omega_y/\omega_x)^2} \), following a 2D isodensity path around the trap center. We compute a partial average \( \langle xy \rangle_{\bar{r}} \), evaluated by including only the pixels at a rescaled distance \( \bar{r} < r_c \). In this way we expect to isolate the superfluid phase which is located at smaller \( \bar{r} \).

Figure 3 presents the frequencies found as a function of the cut-off radius \( r_c \), for three of the datasets. Below a cutoff radius of \( r_c \approx 10 \mu m \) there is no clear oscillation (indicated by the large error bars). This reflects the fact that the scissors excitation is a surface mode and does not affect the core of the cloud. Beyond this cutoff radius, we observe in most of the datasets a central region oscillating at a single frequency, in good agreement with the hydrodynamic prediction \( \omega_{hd} \) which was not apparent in the global analysis. For larger radii, we recover the results of the global analysis, showing that the contribution of the central region to the total signal is small, even if the density is higher at the center. The radius at which the crossover between these two regimes occurs depends on \( \alpha \). As discussed below, this result is a direct evidence of the coexistence of normal and superfluid scissors excitations at finite temperature in our system (see supplementary material).

To get more accurate results, we reduce further the region of interest to a thin annulus centered on \( \bar{r} = r_{eq} \), with a width \( \delta r = 4 \) pixels, thus probing an isodensity region of the cloud. Figure 4 displays the frequencies measured for this partial average \( \langle xy \rangle_{\bar{r}} \) as a function of \( r_{eq} \) for \( \alpha = 0.73(3) \), corresponding to one of the datasets of figure 3. The larger error bars result from a smaller number of pixels involved in the average. This local analysis is even more sensitive to frequency shifts and allows a more accurate determination of the position of the boundary between the two phases at \( \bar{r} \approx 22 \mu m \).

We attribute the hydrodynamic part of the gas present at the center to the superfluid phase of the degenerate quasi-2D Bose gas. One may wonder if a hydrodynamic classical gas at high density could explain our observation (see supplementary material). From the independent measurement of the equilibrium values of \( \mu \) and \( T \), we evaluate the scaled distance from the trap center at which the reduced local chemical potential \( \alpha(\bar{r}) = \mu_{loc}(\bar{r})/(k_B T) \), where \( \mu_{loc}(\bar{r}) = \mu - m\omega_y^2 r^2/2 \), reaches the critical value for the BKT transition [38]. This is indicated by the vertical blue solid line in figure 4, in good agreement with our observed crossover. The small discrepancy could be explained in two ways. First, close to the critical radius, the finite extension \( \delta r \) of our local probe averages the signal over both the superfluid and normal regions. Second, finite size effects for a trapped cloud, away from the thermodynamic limit, modify the position of the expected boundary anyway.

Indeed, a quantum Monte Carlo simulation of a quasi 2D trapped Bose gas with our parameters [4] has shown that the normal to superfluid crossover is broadened by about \( 2 \mu m \) in the sample, resulting in a non-zero superfluid

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4 Markus Holzmann, private communication.
fraction beyond the critical radius predicted by the LDA. We are thus confident that we observe the superfluid to classical transition signature in the dynamical response of the gas.

We now come to the interpretation of the local relaxation time \( \tau (r_i) \) deduced from the local analysis. We find that its value is in good agreement with the inverse of the local collision rate \( \Gamma_i (r_i) \simeq n_{2D} (r_i) \bar{v}^2 k_B / m \) evaluated in the confinement dominated three-dimensional regime [43], where \( n_{2D} (r_i) \) is the measured average 2D density on the annulus. This supports the use of equation (1) to describe the local dynamics. We point out that our analysis could give access to a local relaxation time of the excitation, a key ingredient in finite temperature two-fluid models [44, 45].

Conversely we use the measured \( \tau (r_i) \) to estimate the local collision rate and hence the local phase space density, knowing the sample temperature. The vertical dashed magenta line in figure 4 indicates the radius at which this estimated local phase space density becomes higher than the BKT critical phase space density \( \bar{D}_c = 11.2 \) computed for this quasi-2D gas [38]. The analysis of the dynamics thus confirms the location of the boundary between the superfluid and normal phases.

We now turn to a quantitative analysis of the damping rates \( \Gamma_{sc} \) of the local averages. We will only consider the upper frequency branch, sketched on figure 5(a), as the lower frequency is highly damped in our experiments. In order to get a reliable estimate of the frequency and damping rates of the scissors oscillation for both the central and outer regions, we exclude the crossover area and compute the \( \langle xy \rangle \) averages over a disc for the central part and over a large annulus for the outer part, as illustrated on figure 5(c). Figure 5(b) compares the measured reduced damping rate of the scissors oscillations \( \Gamma_{sc} / \omega_{sc} \) to the reduced Landau damping \( \Gamma_{Ls} / \omega_{Ls} \) [46, 47]:

\[
\frac{\Gamma_1}{\omega_{sc}} = \frac{3 \pi k_B T a}{8 \bar{c}} ,
\]

where \( \bar{c} = \sqrt{2 \mu / (3 m)} \) is the sound velocity for a pancake-shaped Bose gas [48]. For our coldest samples the measured damping rate for the superfluid phase is close to the damping predicted by equation (2). However for the datasets with a large thermal fraction, the damping in the superfluid phase deviates from the predicted Landau damping and is of the same order of magnitude than the damping in the normal phase, suggesting that this additional damping originates from collisional coupling to the normal gas [44]. The reduced damping rate in the normal phase is approximately constant for all our datasets. We note that in our 2D geometry the abnormal damping cannot be explained by a resonant Beliaev coupling to other modes as observed in three-dimensional experiments [49].

In conclusion we have studied the dynamics of a quasi-2D Bose gas near the BKT transition. We have shown the coexistence of a superfluid fraction and a thermal part by a direct, in situ, local analysis of its dynamical behavior. We use the frequency of the scissors mode as a local probe of superfluidity. Whereas a global analysis of the data fails to reveal the coexistence of the superfluid and normal oscillations, we demonstrate a new local...
average analysis, reminiscent of the LDA, which allows to locate the normal to superfluid transition, using a purely dynamical criterion. The position of the boundary is in good agreement with the crossover expected from the equilibrium properties of the gas. In principle it should be possible with this local probe to observe the superfluid density jump at the BKT transition \cite{24, 50}. This motivates future numerical studies of finite temperature dynamics to establish quantitatively the relation between local scissors frequency and local superfluid fraction.

Local average analysis could also be applied to numerical simulations. In particular, the local measurement of relaxation times would be useful to compare experiments to numerical calculations at finite temperature. We expect this new kind of local diagnosis to shed new light on the study of out-of-equilibrium systems.

Acknowledgments

We thank M Holzmann for enlightening discussions and for providing us with the QMC simulations of our experiment, S Stringari for helpful suggestions and acknowledge stimulating discussions with B Laburthe-Tolra. We thank D Guéry-Odelin for a critical reading of the manuscript. We acknowledge financial support from ANR project SuperRing (ANR-15-CE30-0012-01). LPL is a member of Institut Francilien de Recherche sur les Atomes Froids (IFRAF).

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Figure 5. (a) Measured damping rate and frequencies for the central superfluid phase (open blue circles) and for the normal phase (filled red squares), for all the datasets. As expected (see supplementary material), the data all gather on the black solid curve, which is the prediction of equation (1). The error bars are the standard deviations estimated using the fit residuals. (b) Measured damping rate, in units of the scissors oscillation frequency, for the central superfluid phase (open blue circles) and for the normal phase (filled red squares), as a function of the reduced damping Landau rate of equation (2), estimated for each dataset from the values of \( \mu \) and \( T \). The black solid line is a guide to the eye (with slope 1). The error bars are the standard deviations estimated using the fit residuals. (c) Contributions from the superfluid (central blue shaded area) and the normal phase (outer red shaded area) to the computation of \( \langle xy \rangle \) are isolated. The thin excluded area (white annulus) corresponds to the crossover evidenced using the method of figure 4, for each of the datasets.

\( \Gamma_{\text{sc}} \)
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