Matrix Elements of Four-quark Operators Relevant to Lifetime Difference $\Delta \Gamma_{B_s}$ from QCD Sum Rules

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Abstract

We extract the matrix elements of four quark operators $O_{L,S}$ relevant to the $B_s$ and $\bar{B}_s$ lifetime difference from QCD sum rules. We find the vacuum saturation approximation works reasonably well, i.e., within 10%. We discuss the implications of our results and compare them with the recent lattice QCD determination.

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1 Introduction

The recent results on CP violation in $B_d - \bar{B}_d$ mixing have been reported by the BaBar and Belle Collaborations \[1\] in the ICHEP2000 Conference. More experiments on B physics have been planned in the present and future B factories \[2\]. Theoretical efforts to improve predictions and reduce uncertainties are expected and needed. It is well-known that mixing in neutral B meson systems provides a good place to examine CP violation as well as flavor physics in the standard model and beyond. For example, the mass difference between the mass eigenstates of neutral $B_d$ meson, $\Delta M_{B_d}$, gives an important constraint on CKM matrix element $V_{td}$ and the first indication of large mass of top quark. Similarly, the mass difference between the mass eigenstates of neutral $B_s$ meson, $\Delta M_{B_s}$, which will be precisely measured in the near future would give an valuable constraint on CKM matrix element $V_{ts}$. The another important observable for mixing in neutral B meson systems is the lifetime difference between the mass eigenstates of neutral B mesons, $\Delta \Gamma_{B_d}$ or $\Delta \Gamma_{B_s}$. The ratio $|V_{ts}/V_{td}|^2$ can be extracted from the measurement of $\Delta \Gamma_{B_s}$ \[3\]. The width difference of $B_d$ mesons is CKM suppressed and consequently not easy to be observed. In contrast, for $B_s$ mesons the width difference is large enough to be measured \[4\] and has been recently measured \[5\] with low statistics. Hopefully, it will be measured with high statistics in the near future.

As usual, The light $B^L_s$ and heavy $B^H_s$ mass eigenstates are defined by

$$|B_s^{L,H}\rangle = p|B_s^0\rangle \pm q|\bar{B}_s^0\rangle,$$

where $|B_s^0\rangle$ and $|\bar{B}_s^0\rangle$ are the flavor eigenstates. The mass difference and the width difference between the physical states are given by

$$\Delta m \equiv M_H - M_L, \Delta \Gamma \equiv \Gamma_H - \Gamma_L.$$

Because $|\Gamma_{12}| \ll |M_{12}|$ for $B_s$ mesons \[5\], to the leading order in $|\Gamma_{12}/M_{12}|$, $\Delta m_{B_s} = 2|M_{12}|$, $\Delta \Gamma_{B_s} = 2\Re(M_{12}\Gamma_{12}^* / |M_{12}|)$. Neglecting very small CP violating corrections, the width difference for $B_s$ mesons in SM has been given \[6, 7\]

$$\left( \frac{\Delta \Gamma}{\Gamma} \right)_{B_s} = \left( \frac{f_{B_s}}{210 \text{ MeV}} \right)^2 [0.006 B(m_b) + 0.150 B_S(m_b) - 0.063],$$

where $f_{B_s}$ is the decay constant of $B_s$, B and $B_S$ are the bag parameters related to the four quark operators $O_L$ and $O_S$ (see below). These hadronic quantities need to be calculated by non perturbative methods such as lattice, QCD sum rules, Bethe-Salpeter approach, etc.

The similar quantities related to $B^0_d - \bar{B}^0_d$ mixing have been estimated by Narison et al within the traditional QCD sum rules approach \[8\] at $\alpha_s$ order. Their conclusion is that the vacuum saturation values $B_B \simeq B_{B^*} \simeq 1$ are satisfied within 15%. Their sum rules are constructed through two-point correlation functions and depend on some phenomenological assumptions. In this letter we shall calculate the matrix elements of four-quark operators relevant to the $B_s$ meson lifetime difference through QCD sum rules in HQET. The sum rules are constructed with three-point correlation functions. Our calculation is carried out at the leading order in $1/m_b$ expansion in HQET for simplicity. In ref.\[8\] the effects of condensates are absorbed like other factorizable corrections into the contribution to $f_B$ and the available result of \[8\] (though not
explicitly said) has been used as the effects are small compared to the perturbative corrections. In our sum rules the nonperturbative contributions of condensates are explicitly included and the numerical results confirm the smallness of these corrections (see below).

2 Theoretical formalism

We employ the following three-point Green’s function,

$$\Gamma^O(\omega, \omega') = i^2 \int dx dy e^{ik' \cdot x + ik \cdot y} \langle 0 | \mathcal{T} [\bar{s}(x) \gamma_5 h^{(b)}_v(x)] O_{L,S}(0)[\bar{s}(y) \gamma_5 h^{(b)}_v(y)] | 0 \rangle,$$  

where $\omega = v \cdot k$, $\omega' = v \cdot k'$; $h^{(b)}_v$ is the b-quark field in the HQET with velocity $v$. And $O_{L,S}$ denotes the color-singlet four quark operators. They are

$$O_L = \bar{b} \gamma^\mu (1 - \gamma_5) s \bar{s} \bar{b} \gamma^\mu (1 - \gamma_5) s,$$

$$O_S = \bar{b} (1 - \gamma_5) s \bar{s} (1 - \gamma_5) s,$$

In terms of the hadronic expression, the correlator in Eq. (2) reads

$$\Gamma^O(\omega, \omega') = \frac{F^2_{B_s}}{4} \frac{\langle B_s | Q_{L,S} | B_s \rangle}{(\Lambda - \omega)(\Lambda - \omega')} + \text{resonances},$$

where $\Lambda = m_B - m_b$ and $F_{B_s}$ is the $B_s$ decay constant in the leading order of heavy quark expansion defined as

$$\langle 0 | \bar{s}(0) \gamma_5 h^{(b)}_v(0) | B_s \rangle = -i \sqrt{m_Q} F_{B_s}.$$  

In order to eliminate the contribution from the non-diagonal single pole terms and suppress the continuum contribution in Eq. (5), we make double Borel transformation to the correlator. The transformation is defined as

$$\hat{B} = \lim_{-\omega \to \infty} \lim_{\omega' \to \infty} \frac{(-\omega)^{n+1}}{n!} \left( \frac{d}{d\omega} \right)^n (\omega')^{m+1} \left( \frac{d}{d\omega'} \right)^m.$$  

There are two Borel parameters $\hat{\tau}$ and $\hat{\tau}'$, which appear symmetrically, so $\hat{\tau} = \hat{\tau}' = 2T$ are taken in the following analysis.

On the other hand the correlator can be calculated at the quark gluon level. For example for $O_L$ we may rewrite the right hand side of Eq. (2) as

*Note that $f_{B_s}$ in Eq. (1) is defined by

$$\langle 0 | \bar{s} \gamma^\mu \gamma_5 b | B_s^0 \rangle = -i f_{B_s} p^\mu.$$
\[-2 \int \! dx dy e^{i k' \cdot x + i k \cdot y} \{- \text{Tr}[\gamma_5 \cdot i S^{mi}_b(x) \cdot \gamma^\mu (1 - \gamma_5) \cdot i S^{in}_s(-y) \cdot \gamma_5 i S^{nj}_b(y) \cdot \gamma_\mu (1 - \gamma_5) \cdot i S^{jm}_s(-x)] + \text{Tr}[i S^{im}_s(-x)] \cdot \gamma_5 \cdot i S^{mi}_b(x) \cdot \gamma^\mu (1 - \gamma_5) \text{Tr}[i S^{jn}_s(-y)] \cdot \gamma_5 \cdot i S^{nj}_b(y) \cdot \gamma_\mu (1 - \gamma_5) \} \}

(9)

where \(i S^{jn}_s(x)\) is the full strange quark propagator with both perturbative term and condensates, \(i, j\) etc is the color index. \(i S^{nj}_b(x)\) is the leading order heavy quark propagator which has very simple form in coordinate space:

\[i S^{ij}_b(x) = \delta^{ij} \int_0^\infty dt \delta(x - vt) \]

(10)

Note the structure of color flow is quite different for the two terms in Eq. (9). For the perturbative part the first and second term is proportional to \(N_c^2\) and \(N_c^2\), respectively, where \(N_c = 3\) is the QCD color number. In the limit of \(N_c \to \infty\), the second term dominates! As shown below, the non-factorizable contribution in Fig. 1 d, f and g has different color structure from the factorizable terms in Fig. 1a, b, c and e. The condensates up to dimension six are kept in our calculation. We also expand the strange quark propagator and keep perturbative term of order \(O(m_s)\). The calculation is standard and we simply present final results after making the double Borel transformation.

3 Duality Assumption

We may write the dispersion relation for the three-point correlator \(\Gamma(\omega, \omega')\) as

\[\Pi(\omega, \omega') = \frac{1}{\pi^2} \int_0^\infty d\nu \int_0^\infty d\nu' \frac{\text{Im}\Pi(\nu, \nu')}{(\nu - \omega)(\nu' - \omega')}.\]

(11)

In order to subtract the continuum contribution, we have to invoke quark hadron duality assumption and approximate the continuum by the integral over the perturbative spectral density above a certain energy threshold \(\omega_c\).

With the redefinition of the integral variables

\[\nu_+ = \frac{\nu + \nu'}{2},\]

\[\nu_- = \frac{\nu - \nu'}{2},\]

(12)

the integration becomes

\[\int_0^\infty d\nu \int_0^\infty d\nu' \cdots = 2 \int_0^\infty d\nu_+ \int_{-\nu_+}^{\nu_+} d\nu_- \cdots .\]

(13)

It is in \(\nu_+\) that the quark-hadron duality is assumed \[10, 11, 12\].

\[\text{higher states} = \frac{2}{\pi^2} \int_{\omega_c}^\infty d\nu_+ \int_{-\nu_+}^{\nu_+} d\nu_- \frac{\text{Im}\Pi(\nu, \nu')}{(\nu - \omega)(\nu' - \omega')}.\]

(14)
This kind of assumption was suggested in calculating the Isgur-Wise function in Ref. [11] and was argued for in Ref. [12]. As pointed out in [10, 12], in calculating three-point functions the duality is valid after integrating the spectral density over the "off-diagonal" variable \( \nu_\perp = \frac{1}{2}(\nu - \nu') \). Such a duality assumption is favored over the naive one:

\[
\text{higher states} = \frac{1}{\pi^2} \int_0^\infty d\nu \int_0^\infty d\nu' \frac{\text{Im}\Pi(\nu, \nu')}{(\nu - \omega)(\nu' - \omega')}.
\] (15)

4 QCD sum rules

The spectral density \( \rho_{L,S}(s_1, s_2) \) of the perturbative term reads

\[
\rho_L(s_1, s_2) = \frac{N_c(N_c + 1)}{2\pi^4} s_1 s_2 [s_1 s_2 + m_s (s_1 + s_2)]
\] (16)

\[
\rho_S(s_1, s_2) = \frac{N_c(2N_c - 1)}{4\pi^4} s_1 s_2 [s_1 s_2 + m_s (s_1 + s_2)]
\] (17)

The sum rule for \( \langle \bar{B}_s | O_{L,S} | B_s \rangle \) after the inclusion of the condensates and the integration with the variable \( \nu_\perp \) is

\[
\frac{F_{B_s}^2}{4} \langle \bar{B}_s | O_L | B_s \rangle \exp \left( -\frac{\Lambda}{T} \right) = \frac{N_c(N_c + 1)}{\pi^4} \left\{ \int_0^\omega d\nu \exp \left( -\frac{\nu}{T} \right) \left[ \frac{16}{15} \nu^5 + \frac{8}{3} m_s \nu^4 \right] + \frac{4}{3} a_s T^3 (1 - \frac{m_0^2}{64T^2}) + \frac{1}{6} \frac{\alpha_s^2}{288} \right\}
\] (18)

where \( a_s = -2(2\pi)^2 \langle \bar{s}s \rangle \) and we have used the factorization assumption for the four-quark condensates. Similarly we have

\[
\frac{F_{B_s}^2}{4} \langle \bar{B}_s | O_S | B_s \rangle \exp \left( -\frac{\Lambda}{T} \right) = \frac{N_c(2N_c - 1)}{2\pi^4} \left\{ \int_0^\omega d\nu \exp \left( -\frac{\nu}{T} \right) \left[ \frac{16}{15} \nu^5 + \frac{8}{3} m_s \nu^4 \right] + \frac{4}{3} a_s T^3 (1 - \frac{m_0^2}{64T^2}) + \frac{1}{6} \frac{\alpha_s^2}{288} \right\}
\] (19)

We want to emphasize that in Eqs. (18), (19) the terms with color factor \( N_c(N_c + 1) \) and \( N_c(2N_c - 1) \) come from the factorizable diagrams in Fig. 1 a, b, c and e. The non-factorizable contribution has a color factor \( \frac{N_c^2 - 1}{2} \) which comes from the summation over color factors, \( \text{Tr} \left[ \frac{\chi_s}{2} \frac{\chi_s}{2} \right] = \frac{N_c^2 - 1}{2} \), in Fig. 1 d, f and g. A second observation is that the factorizable terms are all positive while nonfactorizable pieces are negative.

Now we turn to the numerical analysis. The decay constant and binding energy of the \( B_s \) meson at the leading order of heavy quark expansion can be obtained from the mass sum rule [13].

\[
F_{B_s}^2 \exp \left( -\frac{2\Lambda}{M} \right) = \frac{3}{8\pi^2} \int_0^{s_0} ds s (2m_s) e^{-s/M} - <\bar{s}s> \left( 1 - \frac{m_0^2}{4M^2} \right)
\] (20)
Note $M = 2T$, $s_0 = 2\omega_c$. We have not included $\alpha_s$ corrections in Eq. (20), because they are also neglected in the sum rule for $\langle \tilde{B}_s|O_{L,S}|B_s \rangle$ (18)-(19). The values of the parameters are calculated to be $F_{B_s} = (0.49 \pm 0.1) \text{ GeV}^{3/2}$, $\Lambda = (0.68 \pm 0.1) \text{ GeV}$ with the threshold $s_0$ to be $(2.2 \pm 0.3) \text{ GeV}$ and the Borel parameter $M$ in the window $(0.65 - 1.05) \text{ GeV}$ [13]. Numerically we use the following values of the condensates,

$$
\langle \bar{q}q \rangle \simeq -0.8 \times (0.23 \text{ GeV})^3,
$$

$$
\langle g_s^2 G^2 \rangle \simeq 0.48 \text{ GeV}^4,
$$

$$
\langle g_s \sigma_{\mu\nu} G^{\mu\nu} \rangle \equiv m_0^2 \langle \bar{s}s \rangle, \quad m_0^2 \simeq 0.8 \text{ GeV}^2.
$$

(21)

For the strange quark mass we use $m_s = 0.15 \text{ GeV}$.

In order to minimize the dependence of the parameters we divide Eqs. (18, 19) by Eq. (20) to extract the matrix elements, the variation of which with $\omega_c$ and $T$ are given in Fig. 2 and 3. The sum rule window is $T = (0.2 - 0.5) \text{ GeV}$, which is almost the same as that in the two-point correlator sum rule. We obtain

$$
\langle \tilde{B}_s|O_L|B_s \rangle = (0.85 \pm 0.20) \text{ GeV}^4,
$$

(22)

$$
|\langle \tilde{B}_s|O_S|B_s \rangle| = (0.55 \pm 0.15) \text{ GeV}^4
$$

(23)

where the central value corresponds to $T = 0.3 \text{ GeV}$ and $\omega_c = 1.1 \text{ GeV}$. The uncertainty includes the variation with $T$ and $\omega_c$. The bag parameters $B$ and $B_S$ are defined by

$$
\langle \tilde{B}_s|O_L|B_s \rangle = \frac{8}{3} f_{B_s}^2 M_{B_s}^2 B, \quad \langle \tilde{B}_s|O_S|B_s \rangle = -\frac{5}{3} f_{B_s}^2 M_{B_s}^2 \frac{M_{B_s}^2}{(m_b + m_s)^2} B_S.
$$

(24)

and they can be directly obtained from Eqs. (22), (23) and (24).

The ratio of these two matrix elements is very interesting. We divide Eq. (19) by Eq. (18) to extract the numerical value of the ratio. In such a way the dependence on the the Borel parameter and the continuum threshold is minimized as can be clearly seen in Fig. 4. Within the accuracy of QSR the curve in Fig. 4 is flat. The ratio is practically the same in the working region of $T$ and $\omega_c$. It reads

$$
R = \frac{|\langle \tilde{B}_s|O_S|B_s \rangle|}{\langle \tilde{B}_s|O_L|B_s \rangle} = (0.63 \pm 0.13)
$$

(25)

In our numerical calculation, the contribution of the perturbative term is about $45 - 65\%$ of the total contributions in preferred Borel variable region. We have used factorization approximation for the four quark condensates in numerical calculations. This may introduce some uncertainty. We may introduce a scale factor $\kappa$ to indicate the deviation from the factorization approximation as in [14]. In our calculations of sum rules the $1/M_b$ corrections in HQET have not been included in , which may bring a deviation from the numerical results of the matrix elements. However, for the ratio of the two matrix elements, we expect little change to the above analysis. Our numerical results are not sensitive to the mass of the strange quark. Actually, the effects due to the strange quark are very small so that the results for $B_s$ are almost the same as those for $B_d$. 
We now give a remark on the usual factorization assumption. In our Feynman diagram (Fig. 1) calculations, the contributions of nonfactorizable diagrams are around $-6\%$, $-7\%$ for $\langle \bar{B}_s|O_S|B_s \rangle$ and $\langle \bar{B}_s|O_L|B_s \rangle$ respectively, which means that the factorization approach works well even though our calculations are limited to the leading order in the $1/m_b$ expansion in HQET. That is, the conclusion in Ref.\[8\] remains unchanged when the nonperturbative condensate contributions are taken into account. If one considers $\alpha_s$ corrections, there is only one nonfactorizable perturbative diagram, in which the gluon line in Fig. 1 f is connected, in the fixed-point gauge in the leading of $1/M_b$ expansion. However, radiative corrections are generally of high order $\alpha_s \pi$ compared to the leading order, and the fact is that the perturbative term is about $45-65\%$ to the whole contribution, so the contribution from the diagram can be neglected compared to those in Fig. 1 (d), (f) and (g). The case here is different with that in the calculation of matrix elements of four-quark operators, relevant to the lifetime difference between heavy mesons, where the flavor changes $\Delta F = 0$. In that case, the perturbative contribution vanishes\[15\], and we can’t predict naively how large the radiative correction is compared to the nonperturbative terms.

5 The $B_s$ and $\bar{B}_s$ decay width difference

The complete expression for $\Delta \Gamma_{B_s}$ with short-distance coefficients at NLO in QCD is given by\[6\]

$$\left( \frac{\Delta \Gamma}{\Gamma} \right)_{B_s} = \frac{16 \pi^2 B(B_s \rightarrow X e \nu) f_{B_s}^2 m_{B_s} |V_{cs}|^2}{g(z)\eta_{QCD} m_b^3} \cdot \left( G(z) \frac{8}{3} B + G_S(z) \frac{M_{B_s}^2}{(m_b + m_s)^2} \frac{5}{3} B_S + \sqrt{1 - 4z} \frac{\delta_{1/m}}{m_b} \right),$$

where

$$G(z) = F(z) + P(z) \quad \text{and} \quad G_S(z) = -(F_S(z) + P_S(z)).$$

and $F$, $P$, $F_S$, $P_S$ can be found in Ref. \[7\]. We eliminated the total decay rate $\Gamma_{B_s}$ in favor of the semileptonic branching ratio $B(B_s \rightarrow X e \nu)$, as done in \[4\]. This cancels the dependence of $(\Delta \Gamma/\Gamma)$ on $V_{cb}$ and introduces the phase space function

$$g(z) = 1 - 8z + 8z^3 - z^4 - 12z^2 \ln z,$$

as well as the QCD correction factor \[16\]

$$\tilde{\eta}_{QCD} = 1 - \frac{2\alpha_s(m_b)}{3\pi} \left[ \left( \frac{\pi^2 - 31}{4} \right) (1 - \sqrt{z})^2 + \frac{3}{2} \right].$$

One can also express the width difference as

$$\frac{\Delta \Gamma_{B_s}}{\Gamma_{B_s}} = \left( \tau_{B_s} \Delta m_{B_d} \frac{m_{B_s}}{m_{B_d}} \right)^{(exp.)} \left( \frac{V_{ts}}{V_{td}} \right)^2 K \cdot \left( G(z) - G_S(z) \mathcal{R}(m_b) + \delta_{1/m} \right) \xi^2,$$
where
\[ \xi = \frac{f_{B_s} \sqrt{B_{B_s}}}{f_{B_d} \sqrt{B_{B_d}}}, \] (31)

\( K \) is the Eq. (7) in ref. [17],
\[ \tilde{\delta}_{1/m} = f_{B_s}^2 M_{B_s}^2 \delta_{1/m}, \] (32)
and the \( \tilde{\delta}_{1/m} \) represents the \( 1/m_b \) corrections and can be found in Ref. [3].

It is clear from the above equation that besides the ratio \( R \) of the matrix elements of four quark operators, which are those we have calculated in the paper, we only use the experimental \( B_d \)-meson mass difference, which is known with a tiny error \[ (\Delta m_{B_d})^{(\text{exp.})} = 0.484(15) \text{ ps}^{-1}, \] (33)
and another ratio of hadronic matrix elements, \( \xi \), which is rather accurately determined in lattice simulations \[ K \text{ and } \tilde{K}. \]

As it is well known, the quantities in Eq. (4) are calculated at the scale \( \text{O}(m_b) \), while our result Eq. (25) is calculated at the hadron scale \( \mu_{\text{had}} \). Therefore, We have to consider the renormalization scale dependence of those four-quark operators. The anomalous dimension matrix of these operators has been given in Ref. [7]. Using the anomalous dimension matrix and following the standard way, we obtain the scale dependence of \( R \)
\[ R(m_b) = 1.69 R(\mu_{\text{had}}) + 0.03. \] (34)

where \( R(\mu_{\text{had}}) \) is defined by Eq. (25). To obtain the numerical result, \( m_b = 4.8 \text{ GeV} \) and \( \mu_{\text{had}} = 1.0 \text{ GeV} \) have been used. It is obvious from Eq. (34) that the result depends on the renormalization scale heavily.

Numerically we have
\[ \frac{\Delta \Gamma_{B_s}}{\Gamma_{B_s}} = [(0.5 \pm 0.1) + (13.8 \pm 2.8)R(m_b) + (15.7 \pm 2.8)(-0.55 \pm 0.17)] \times 10^{-2} \]
\[ = (7.0 \pm 0.8) \times 10^{-2} \] (35)

Clearly such a life time difference is compatible with existed literatures. It’s interesting to compare our result to the two recent lattice QCD calculation: \[ \frac{\Delta \Gamma_{B_s}}{\Gamma_{B_s}} = (10.7 \pm 2.6 \pm 1.4 \pm 1.7) \times 10^{-2} \] in Ref. [21] and \[ \frac{\Delta \Gamma_{B_s}}{\Gamma_{B_s}} = (4.7 \pm 1.5 \pm 1.6) \times 10^{-2} \] in Ref. [17]. In Eq. (35) the numerical value of \( \tilde{\delta}_{1/m} \), which corresponds to the \( 1/m_b \) correction in the short distance expansion of the operator product \( H_{\text{eff}}(x)H_{\text{eff}}(0) \) [4], has been taken as \(-0.55[17]\). If it is taken as \(-0.30\), one has \( \Delta \Gamma/\Gamma = 10.9 \times 10^{-2} \), larger than \( 7.0 \times 10^{-2} \), while in the case of Ref. [17], \( \Delta \Gamma/\Gamma \) would remain in the 10\% range with the change from \(-0.55 \) to \(-0.30\). That is, the sensitivity to the final term in Eq. (35), i.e., the \( 1/m_b \) correction, increases in our result. Without a good control of the correction, a precise determination of the lifetime difference is impossible.
6 Conclusion and discussion

In summary we have calculated the matrix elements of the four-quark operators relevant to the $B_s$ meson lifetime difference in QCD sum rules in HQET. The sum rules are constructed with three-point correlators and both the perturbative and nonperturbative contribution are taken into account. Our result shows that the usual factorization assumption is indeed a good approximation. The numerical results show that the sum rules of those operators have a good platform. The perturbative contribution to sum rules are about $45 - 65\%$ of the total contribution. Our results are not sensitive to $m_s$. The life difference $\frac{\Delta \Gamma_{B_s}}{\Gamma_{B_s}}$ is found to be around $(7.0 \pm 0.8) \times 10^{-2}$. This result is compatible with those predicted by lattice calculations. The $\alpha_s$ corrections have not been taken into account in the sum rules and they will definitely have effects on the resulting numerical values. To get more accurate prediction, the $\alpha_s$ corrections should be taken into account, which is beyond the content of the letter.

Acknowledgments

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Figure 1: Dominant non-vanishing Feynman diagrams for $\Gamma^O(\omega, \omega')$

Figure 2: The dependence of $\langle \bar{B}_s | O_L | B_s \rangle$ on $T, \omega_c$
Figure 3: The variation of $|\langle B_s | O_S | B_s \rangle|$ with $T, \omega_c$.

Figure 4: The variation of $\mathcal{R}$ with $T, \omega_c$. 