VPPP Algorithms of Baseline Vector Estimation among Multiple Antennas

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Abstract
In this paper, we study the VPPP (Very Precise Point Positions) algorithm based on GR (GNSS Regression) models by using multiple antennas for estimating baseline vectors. The multiple antennas are disposed with solid geometrical distances which provide the constraints of the estimated parameters, but both antennas’ positions are unknown. By using the baseline-vector length constraints, we show the VPPP algorithms based on the double-difference (DD) observables by each pair of two antennas, and apply the algorithm for estimating baseline vectors and GNSS Gyro. Finally we show the experimental results of the estimation algorithms for the baseline vectors and Euler angles.

1 Introduction
In general, not only the positions, velocities, and accelerations but also the body attitude are important information for the navigation or the system control of the vehicle mobile applications. Basically the more reliable and higher performed attitude determination system (ADS) are developed by combining the GNSS receiver systems with the inertial navigation systems (INS). The ADS performance can be achieved by approximately 0.5 degrees as 1-sigma errors within one minute at three-dimensional performance, however, the several expensive multi-frequency receivers are utilized to solve the attitude accuracy, the reliability, and the output rate. Any expensive receiver cannot resolve the limitation of non-positioning state under the tunnels and the underground parking and higher output rate, therefore the INS could complement the GNSS receiver for the disadvantages of ADS [1].

The relative positioning by the low-cost single frequency GNSS receivers and their patch antennas are applied for the vehicle heading determination [2]. The integer ambiguity resolution of the double-difference carrier-phase observables are needed. In order to stably obtain the fixed integer ambiguities, the detection and correction of cycle slips are needed. The inertial measurement units (IMU) with gyro sensors are utilized for the detection and correction.

In this paper, we present a novel baseline-vector estimation algorithm based on precise point positioning (PPP) algorithms with the double difference (DD) for Global Navigation Satellite System (GNSS) observables among multiple antennas. The update algorithms by the baseline-vector length constraints are applied to the estimation algorithm.

We have developed the GR models of PPP/VPPP algorithms among multiple antennas [3]-[8]. PPP also utilizes the carrier-phase observables, and is an ultimately desirable technology in the GPS/GNSS positioning community [9]. PPP/VPPP also can correct or estimate the parameters which cause satellite or receiver-related biases, or signal delays. PPP/VPPP also utilizes the carrier-phase observables, and achieves the positioning accuracy in decimeter level by utilizing multi-frequency high-end GNSS receivers. We also have been developing the GR models of DD-PPP/VPPP algorithms based on DD observables. The DD-PPP/VPPP can cancel the biases or the delays by single (SD) or double-difference (DD) methods [10, 11], and achieves more precise positioning than PPP/VPPP methods when low cost single frequency GNSS receivers are utilized.

In order to estimate the body attitude by the array-aided PPP comprised of multiple antennas, the observation equations are defined to estimate the baseline vectors between two antennas [12]. The equations for the SD or DD-based observables are derived, and the gradient vectors from two antennas to a same satellite are assumed as the same. We present a novel GR model of the baseline-vector estimation derived from the DD-based PPP algorithm. The gradient vectors
for the baseline vectors are the average of two different vectors from two antennas to a same satellite. The baseline-vector lengths can be utilized as constraints by the similar method of VPPP/DD-VPPP [13]–[16].

We present an Euler-angle estimation method among multiple baseline vectors. The rotation matrix for the Euler-angle calculation is defined by Z-Y-X rotation sequence of the vectors. The parameters of Euler’s principal rotation theorem are applied to derive the matrix, and estimated by the least-squares method among multiple vectors. Although each antenna position by DD-PPP has offset errors, the error of each baseline vector between two antennas is decreasing as time goes by. The least-squares method also make the Euler angles more precise.

This paper is organized as follows. Section 2 shows the GR models for the baseline-vector estimation in a static situation. The model is based on the DD-based PPP equations. In Section 3, in order to improve the vector estimate accuracy, the update algorithm based on the baseline-vector length constraints between two antennas are shown. Section 4 shows the Euler-angle estimation by the least-squares method based on the estimated baseline vectors. The parameters of Euler’s principal rotation theorem are used for the method. Section 5 shows the experimental results according to the baseline-vector and Euler-angle estimation algorithms in a static situation. Finally the concluding remarks are given in Section 6.

2 GR Models for DD-based Baseline-Vector Estimation

The derivations of the PPP equations based on the DD-based observables are explained in detail in [11]. For the case of \( p = 1, q = 2, \ldots, n_s \) and \( u_1 = u_1, u_j = u_2 \) \( (n_s = 2) \), we have the following measurement equation for antennas of \( u_1 \) and \( u_2 \) and for \( n_s \) satellites [15, 12]:

\[
y_{u_2u_1}^s = C_{u_2u_1}^s \psi_{u_2u_1}^s + \tilde{v}_{u_2u_1}^s, \tag{1}
\]

where

\[
y_{u_2u_1}^s \equiv \begin{bmatrix} y_{CA,u_2u_1} \\ y_{L1,u_2u_1} \end{bmatrix}, \quad \psi_{u_2u_1}^s = \begin{bmatrix} \psi_{u_2u_1}^s \\ \psi_{u_2u_1}^s \end{bmatrix}, \quad C_{u_2u_1}^s \equiv \begin{bmatrix} -G_{u_2u_1}^{s,1} & G_{u_2u_1}^{s,2} \\ -G_{u_2u_1}^{s,1} & G_{u_2u_1}^{s,2} \end{bmatrix} \lambda_1 I, \quad G_{u_2u_1}^{s,1} = \begin{bmatrix} g_{u_2u_1}^{s,1} & \ldots & g_{u_2u_1}^{s,n_s} \end{bmatrix}^T : (n_s - 1) \times 3. \tag{2}
\]

\( y_{CA,u_2u_1}^s \) and \( y_{L1,u_2u_1}^s \) are \( (n_s - 1) \times 1 \) vectors which are DD observables from raw C/A code pseudoranges and L1 carrier phases, respectively. \( s' (p = 1) \) is the reference satellite. \( N_{L1,u_2u_1}^s \) and \( \tilde{v}_{u_2u_1}^s \) are DD-based integer ambiguity and observation noise vector, respectively. \( G_{u_2u_1}^{s,1} \) is the known \((n_s - 1) \times 3\) matrix which consists of gradient vectors \( \dot{g}_{u_1}^{s,v} \) from the estimated antennas position \( \hat{u}_1 \) to the estimated satellite position \( \hat{k}_i \) by the receiver \( u_i \).

Furthermore let us consider GNSS Gyro by estimating the baseline vectors \( u_1, u_j = u_j - u_i \) of multiple antennas disposed in the same plane. The following equations are the GR models for baseline-vector estimation of two antennas \( u_1, u_2 \):

\[
y_{u_2u_1}^s = C_{u_2u_1}^s \psi_{u_2u_1}^s + \tilde{v}_{u_2u_1}^s, \tag{3}
\]

where

\[
G_{u_2u_1}^{s,1} = \begin{bmatrix} y_{CA,u_2u_1}^s \\ y_{L1,u_2u_1}^s \end{bmatrix}, \quad \psi_{u_2u_1}^s = \begin{bmatrix} \psi_{u_2u_1}^s \\ \psi_{u_2u_1}^s \end{bmatrix}, \quad C_{u_2u_1}^s = \begin{bmatrix} G_{u_2u_1}^{s,1} & 0 \\ G_{u_2u_1}^{s,2} & \lambda_1 I \end{bmatrix},
\]

\[
G_{u_2u_1}^{s,1} = \begin{bmatrix} g_{u_2u_1}^{s,1} \\ g_{u_2u_1}^{s,2} \end{bmatrix}, \quad \lambda_1 = \begin{bmatrix} \lambda_1^{u_1} & \ldots & \lambda_1^{u_n} \end{bmatrix}^T, \quad \dot{g}_{u_2u_1} = \sum_{i=1}^{n_s} \left[ \frac{1}{2} \left( \dot{g}_{u_1i}^{s,v} + \dot{g}_{u_2i}^{s,v} \right) \right] = \dot{g}_{u_2u_1}^{s,v}. \tag{4}
\]

The adjacent antenna \( u_1 \) and \( u_2 \) are very close compared with the distance between the satellites and the antennas (receivers), namely approximately 20,000 kilometers, therefore the gradient vectors \( \dot{g}_{u_2u_1}^{s,v} \) of baseline vector \( u_2u_1 \) are assumed as the average values of \( \dot{g}_{u_1i}^{s,v} \) and \( \dot{g}_{u_2i}^{s,v} \). We can apply these equations to the configuration of multiple antennas.

2.1 State Equations for the Static Case

In the static case, we utilize the state vector \( y_{u_2u_1}^s \) in Eq. (1) and \( \psi_{u_2u_1}^s \) in Eq. (3) for antennas of \( u_1 \) and \( u_2 \) \( (n_s = 2) \) and for \( n_s \) satellites. In order to simplify the expression, superscripts \( s \) and \( s' \) of subscripts \( u_1, u_2 \) are omitted hereafter. Then in the case of \( \xi \), the state equation is described by

\[
\xi_{t+1} = \xi_t, \quad \xi_t : n' \times 1, \quad n' = 5 + n_s, \tag{5}
\]

and the measurement equation is

\[
y_t = C_t \xi_t + \nu_t, \tag{6}
\]
Therefore the positioning algorithms based on the Kalman filter for Eqs. (5) and (6) are given as follows

\[ \hat{\xi}_{t+1|t} = \hat{\xi}_{t|t} \]  
\[ \hat{\xi}_{t|t} = \hat{\xi}_{t|t-1} + K_t \nu_t \]  
\[ \nu_t \equiv y_t - C_t \hat{\xi}_{t|t-1} \]  
\[ K_t = \Sigma_{\xi,t|t-1} C_t^T \left[ C_t \Sigma_{\xi,t|t-1} C_t^T + R_t \right]^{-1} \]  
\[ \Sigma_{\xi,t+1|t} = \Sigma_{\xi,t|t} \]  
\[ \Sigma_{\xi,t|t} = \Sigma_{\xi,t|t-1} - K_t C_t \Sigma_{\xi,t|t-1} \]  
Init. Cond.: \[
\begin{align*}
\hat{\xi}_{t,0} \equiv \hat{\xi}_{t,0} = \hat{\xi}_{t,0} \\
\Sigma_{\xi,t|t} \equiv \Sigma_{\xi,0} 
\end{align*}
\]  

3 Update by Constraints

The constraint conditions are applied to update DD-based baseline-vector estimation as follows. In the case of \( n_t = 2 \), define the baseline-vector length constraints at time \( t \) as follows:

\[ l_{21,t} = ||u_{21,t}|| + c_{l21,t} \]  
\[ \text{where } c_{l21,t} \text{ is a Gaussian white noise caused by the phase center variations of antennas with} \]  
\[ c_{l21,t} \sim \mathcal{N}(0, r_{l21,t}) \]  

where

\[ ||u_{21,t}|| = \sqrt{u_{21,t}^Tu_{21,t}}. \]  

When we apply the constraint conditions to the filtering estimates \( \hat{\xi}_{t|t} \) and the error covariance matrix \( \Sigma_{\xi,t|t} \), we can consider the following relations of the conditional probability density function (CPDF):

\[ p(\xi_t|Y^t, l_{21,t}) = \frac{p(\xi_t, l_{21,t}|Y^t)p(Y^t)}{p(Y^t, l_{21,t})} = \frac{p(l_{21,t}|\xi_t, Y^t)p(\xi_t|Y^t)p(Y^t)}{p(Y^t, l_{21,t})} = K_\xi(Y^t, l_{21,t}) p(\xi_t|Y^t)p(l_{21,t}|\xi_t). \]  

Then we have relations:

\[ p(\xi_t|Y^t) = \frac{1}{(2\pi)^{n_t/2} |\Sigma_{\xi,t|t}|^{1/2}} \exp \left\{ -\frac{1}{2} (\xi_t - \hat{\xi}_{t|t})^T \Sigma_{\xi,t|t}^{-1} (\xi_t - \hat{\xi}_{t|t}) \right\} \]  

And we have relations from Eq. (13):

\[ p(l_{21,t}|\xi_t) = \frac{1}{\sqrt{2\pi r_{l21}}} \exp \left\{ -\frac{(l_{21,t} - ||u_{21,t}||)^2}{2r_{l21}} \right\} \]  

Therefore, Eq. (16) is expressed as follows:

\[ p(\xi_t(Y^t, l_{21,t})) = K_\xi(Y^t, l_{21,t}) \frac{1}{(2\pi)^{n_t/2} |\Sigma_{\xi,t|t}|^{1/2}} \exp \left\{ -\frac{1}{2} (\xi_t - \hat{\xi}_{t|t})^T \Sigma_{\xi,t|t}^{-1} (\xi_t - \hat{\xi}_{t|t}) \right\} \frac{1}{\sqrt{2\pi r_{l21}}} \exp \left\{ -\frac{(l_{21,t} - ||u_{21,t}||)^2}{2r_{l21}} \right\} \]  

In order to express the exponent part in Eq. (19) by the quadratic form of \( \xi_{21} \), we linearize \( u_{21} \) by Taylor series expansion.

\[ ||u_{21}|| = \sqrt{u_{21}^Tu_{21}} = \sqrt{\sum_{i=1}^{3} u_{21,i}^2} \rightarrow \]  

\[ \frac{\partial}{\partial u_{21,k}} \left( \sum_{i=1}^{3} u_{21,i}^2 \right) = \frac{2}{\sqrt{\sum_{i=1}^{3} u_{21,i}^2}}, k = 1, 2, 3 \]

Then we remark that the power term of \( p(l_{21,t}|\xi_t) \) in Eq. (18) is expressed by the quadratic form of \( \xi_{21} \) as follows:

\[ \frac{(l_{21,t} - ||u_{21,t}||)^2}{2r_{l21}} = \frac{1}{2r_{l21}} \left( l_{21,t}^2 + u_{21,t}^Tu_{21,t} - 2l_{21,t}||u_{21,t}|| \right) \approx \frac{1}{2} \left( u_{21,t}^T I - u_{21,t}^Tu_{21,t} - 2 \frac{l_{21,t}}{r_{l21}} \xi_{21} + \frac{l_{21,t}^2}{r_{l21}} \right) \]

\[ = \frac{1}{2} \left( \xi_{21}^T M_{\xi_{21}} \xi_{21} - 2 \xi_{21}^T + \frac{l_{21,t}^2}{r_{l21}} \right), \]  

where

\[ M_{\xi_{21}} = \begin{bmatrix} K_{l21} & O \\ O & 0_{n_{s-1}} \end{bmatrix}, \quad \xi_{21}^T = \begin{bmatrix} \xi_{21}^T \\ 0_{n_{s-1}} \end{bmatrix}, \quad K_{l21} = \begin{bmatrix} I \\ r_{l21} \end{bmatrix}, \quad c_{l21}^T = -\frac{2u_{21}^T K_{l21}}{r_{l21}}. \]

\( O_{n_{s-1}} \) is \((n_s - 1) \times (n_s - 1)\) zero matrix, and \( 0_{n_{s-1}} \) is \( 1 \times (n_s - 1) \) zero vector. Therefore, finally we have
Then the updated matrix is as follows:

\[ \Sigma_{\xi_21} = (\Sigma_{\xi_21}^{-1} + M_{\xi_21})^{-1} (\Sigma_{\xi_21}^{-1} \xi_21 - \frac{1}{2} c_{\xi_21}) \]

\[ \Sigma_{\xi_21} = (\Sigma_{\xi_21}^{-1} + M_{\xi_21})^{-1}. \]  

4 Euler-angle estimation by parameters of Euler’s principal rotation theorem

The rotation angles \( \alpha, \beta, \gamma \) around X,Y,Z-axes, respectively, are called Euler angles, and the corresponding rotation matrices are as follows:

\[ \Gamma(X, \alpha) \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix}, \Gamma(Y, \beta) \equiv \begin{bmatrix} \cos \beta & 0 & -\sin \beta \\ 0 & 1 & 0 \\ \sin \beta & 0 & \cos \beta \end{bmatrix}, \]

\[ \Gamma(Z, \gamma) \equiv \begin{bmatrix} \cos \gamma & \sin \gamma & 0 \\ -\sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}. \]  

In the case of \( n_r = 2 \), the rotation matrix of Z-Y-X sequence from the vector \( r_{f,N,21} \) on the reference frame to the estimated baseline vector \( \hat{u}_{N,21} \) on the body frame is as follows:

\[ \hat{u}_{N,21} = \Gamma(X, \alpha) \Gamma(Y, \beta) \Gamma(Z, \gamma) r_{f,N,21} = \Gamma(\alpha, \beta, \gamma) r_{f,N,21} \]  

where \( N \) denotes normalized,

\[ r_{f,N,21} \equiv \frac{r_{f,N,21}}{||r_{f,N,21}||} = [x_{N,21}^R, y_{N,21}^R, z_{N,21}^R]^T, \]

\[ \hat{u}_{N,21} \equiv \frac{\hat{u}_{21}}{||\hat{u}_{21}||} = \left[ \hat{x}_{N,21}, \hat{y}_{N,21}, \hat{z}_{N,21} \right]^T. \]  

The composition of rotation matrix \( \Gamma(\alpha, \beta, \gamma) \) in Eq. (24) is expressed by the parameters based on Euler's principal rotation theorem as follows [1, 17]:

\[ \Gamma(\alpha, \beta, \gamma) = \hat{\Gamma}(n, e) = \begin{bmatrix} e + h n_x^2, h n_x n_y - w n_z, h n_x n_z + w n_y \\ h n_x n_y + w n_z, e + h n_y^2, h n_y n_z - w n_x \\ h n_x n_z - w n_y, h n_y n_z + w n_x, e + h n_z^2 \end{bmatrix}, \]

\[ \mathbf{e} = r_{f,N,21} \cdot \hat{u}_{N,21} = \cos \phi \]

\[ h = 1 - e, \quad w = \sqrt{1 - e^2} = \sin \phi. \]  

In the case of multiple antennas, we derive the computational algorithms by estimating the multiple baseline vectors \( \hat{u}_{N,j} \) from the reference antenna \( i \) to other antennas \( j \) disposed in a plane [1, 18]. The appropriate rotation matrix \( \Gamma \) to minimize the rotation errors from the multiple reference baseline vectors \( r_{f,N,ji} \) to the multiple baseline vectors \( \hat{u}_{N,j} \) is estimated by the least-squares method, and that is the corresponding
equations to estimate the parameters $\theta$ of Euler’s principal rotation theorem as follows (See Appendix):

$$\min_{\theta} \sum_{i<j} ||\hat{u}_{N,ji} - H_{ji}\theta||^2 = \min_{\theta} \sum_{i<j} ||\hat{u}_{N,ji} - H_{ji}\theta||^2$$

(28)

where

$$\hat{u}_{N,ji} = \left[ \begin{array}{c} \hat{x}_{N,ji} \\ \hat{y}_{N,ji} \\ \hat{z}_{N,ji} \end{array} \right] , \quad \theta = \left[ \begin{array}{c} \cos \phi \\ n_x \sin \phi \\ n_y \sin \phi \\ n_z \sin \phi \end{array} \right] ,$$

$$H_{ji} = \left[ \begin{array}{ccc} x^{(R)}(R) & \varrho^{(R)}_{ji} & -y^{(R)}(R) \\ y^{(R)}(R) & 0 & x^{(R)}(R) \\ z^{(R)}(R) & -z^{(R)}(R) & 0 \end{array} \right].$$

(29)

The Euler-angle estimation by the parameters of Euler’s principal rotation theorem under Z-Y-X sequence is as follows. First of all, the multiple vectors $r_{f,ji}$ and $\hat{u}_{ji}$ are normalized as shown in Eq. (25), and mapped on the X-Y-X-Z, and Y-Z planes to estimate the rotation angles in the sequence.

Then the parameters $\hat{\theta}(Z, \gamma)$, $\hat{\theta}(Y, \beta)$, and $\hat{\theta}(X, \alpha)$ for the rotations around Z-axis, Y-axis, and X-axis, respectively are estimated by the least-squares method in the sequence. Eq. (28) is developed and expressed by the quadratic form of $\theta$ as follows:

$$\min_{\theta} \sum_{i<j} ||\hat{u}_{N,ji} - H_{ji}\theta||^2$$

$$= \sum_{i<j} (\hat{u}_{N,ji} - H_{ji}\theta)^T (\hat{u}_{N,ji} - H_{ji}\theta)$$

$$= \sum_{i<j} (\hat{u}_{N,ji} - H_{ji}\theta)^T (\hat{u}_{N,ji} - H_{ji}\theta) + \theta^T H_{ji}^T H_{ji}\theta$$

$$= \theta^T \sum_{i<j} (H_{ji}^T H_{ji}) \theta - \theta^T \sum_{i<j} (H_{ji}^T H_{ji}) \theta - \theta^T \sum_{i<j} (H_{ji}^T H_{ji}) \theta$$

(30)

Equation (30) is differentiated by $\theta$,

$$2 \sum_{i<j} (H_{ji}^T H_{ji}) \theta - 2 \sum_{i<j} (H_{ji}^T \hat{u}_{N,ji}) = 0$$

(31)

and finally the estimate $\hat{\theta}$ is obtained by the least-squares method as follows:

$$\hat{\theta} = \left( \sum_{i<j} (H_{ji}^T H_{ji}) \right)^{-1} \sum_{i<j} (H_{ji}^T \hat{u}_{N,ji}).$$

(32)

Then each rotation matrix $\Gamma(Z, \gamma)$, $\Gamma(Y, \beta)$, and $\Gamma(X, \alpha)$ in Eq. (26), respectively is derived through

$$\text{yaw} : \gamma = \tan^{-1} \frac{\Gamma_{21}}{\Gamma_{11}}, \quad \text{pitch} : \beta = \tan^{-1} \frac{-\Gamma_{31}}{\sqrt{\Gamma_{22}^2 + \Gamma_{32}^2}},$$

$$\text{roll} : \alpha = \tan^{-1} \frac{\Gamma_{32}}{\Gamma_{33}}.$$  

(33)

where $\Gamma_{ij}$ denotes the matrix element of the second row and first column.

5 Experimental results

We have carried out the experiments of using the GPS data (see Table 1) from the antennas ANT-1,2,3,4 located at the corners of a square board (see Fig. 2), and the coordinates of their reference positions in the WGS84 system are listed in Table 2. The positioning error applying the relative positioning method is less

| Date          | March 8, 2015 |
|---------------|---------------|
| GPS-Time      | 06:00:00~06:10:00 |
| Location      | BK Campus, Ritsumeikan Univ. |
| Antenna(ANT)  | u-blox patch antenna for automobile |
| Receiver      | u-blox GPS-module NEO-M8 |
| Epoch interval| 1[s] |
| Elevation angle mask | 15 [deg.] |
| Measurement Data | C/A code, L1 carrier-phase |

| Table 1: Experimental Conditions |
|----------------------------------|
| Date               | March 8, 2015 |
| GPS-Time            | 06:00:00~06:10:00 |
| Location            | BK Campus, Ritsumeikan Univ. |
| Antenna(ANT)        | u-blox patch antenna for automobile |
| Receiver            | u-blox GPS-module NEO-M8 |
| Epoch interval      | 1[s] |
| Elevation angle mask| 15 [deg.] |
| Measurement Data    | C/A code, L1 carrier-phase |

| Table 2: Antennas’ positions |
|-------------------------------|
| WGS-84 | X [m] | Y [m] | Z [m] |
| ANT-1 | -3761236.1924 | 3636879.2811 | 3635962.9360 |
| ANT-2 | -3761236.7109 | 3636878.7092 | 3635962.9204 |
| ANT-3 | -3761236.9118 | 3636878.3835 | 3635962.6700 |
| ANT-4 | -3761235.8330 | 3636878.9540 | 3635962.6893 |

Fig. 2: GNSS antenna array on a square board

| Table 2: Antennas’ positions |
|-------------------------------|
| WGS-84 | X [m] | Y [m] | Z [m] |
| ANT-1 | -3761236.1924 | 3636879.2811 | 3635962.9360 |
| ANT-2 | -3761236.7109 | 3636878.7092 | 3635962.9204 |
| ANT-3 | -3761236.9118 | 3636878.3835 | 3635962.6700 |
| ANT-4 | -3761235.8330 | 3636878.9540 | 3635962.6893 |
Fig. 3: Reference and baseline vectors on ENU coordinates

Table 3: Six vectors on the reference and the body frame

| E [m] | N [m] | U [m] | E [m] | N [m] | U [m] |
|-------|-------|-------|-------|-------|-------|
| $r_{f,21}$ | 1 | 0 | 0 | $u_{21}$ | 0.799 | -0.015 | -0.006 |
| $r_{f,31}$ | 1 | 1 | 0 | $u_{31}$ | 0.812 | 0.783 | -0.003 |
| $r_{f,41}$ | 0 | 1 | 0 | $u_{41}$ | 0.013 | 0.797 | 0.000 |
| $r_{f,32}$ | 0 | 1 | 0 | $u_{32}$ | 0.012 | 0.798 | 0.002 |
| $r_{f,42}$ | -1 | 1 | 0 | $u_{42}$ | -0.786 | 0.812 | 0.006 |
| $r_{f,43}$ | -1 | 0 | 0 | $u_{43}$ | -0.799 | 0.014 | 0.005 |

than a few cm. We apply the ENU (ENU: East, North, and Height(Up)) coordinates to the reference frame and the body frame. The ANT-1 position is the origin of the ENU coordinates. As shown in Fig.3, the red vectors $r_{f,21}, r_{f,31}, r_{f,41}, r_{f,32}, r_{f,42}, r_{f,43}$ show the reference vectors of the reference frame, and ideally located along the axes of the ENU coordinates. The green vectors $u_{21}, u_{31}, u_{41}, u_{32}, u_{42}, u_{43}$ show the baseline vectors of the body frame on the ENU coordinates, and are fixed to the body of the square board. Table 3 shows the six reference vectors $r_{f,ji}$ on the reference frame, and the six baseline vectors $u_{ji}$ calculated from the relative positioning in Table 2. The reference vectors are reference values on the local tangent plane, namely the EN plane. The vectors’ values are approximately the same, and the rotational errors of the corresponding vectors are less than a few degrees.

Fig. 4 shows the six baseline vectors $u_{21}, u_{31}, u_{41}, u_{32}, u_{42}, u_{43}$ estimated by Eq. (3) and the vector $u_{21}$ updated by the vector length constraint in Eq. (13) on the EN plane. The measurement time is 20 minutes (1200 epochs [seconds]). All estimates without constraints are gradually converged to the real positions which are the end points of the baseline vectors (green arrows). On the contrary, $u_{21}$ (C) is converged adjacent to the real position in several seconds after the estimation start.

Fig. 5 shows the estimation results of the Euler-angles from the reference frame to the body frame. (a) utilizes only $u_{21}$ baseline-vector estimates, (b) utilizes six baseline-vector estimates, and (c) utilizes only $u_{21}$ updated by constraints to estimate the Euler angles. Although the real Euler angles are less than 1 degree derived from the real position data in Table 3, (a) shows that the rotation angles $\gamma, \beta$ are approximately 10 de-
degrees even after 1200 seconds. On the contrary, (b) shows that all rotation angles are gradually converged as time goes by, and less than 5 degrees in approximately 600 seconds, and less than 2 degrees in approximately 1200 seconds. (c) shows that the rotation angle $\gamma$ in the X-Y plane is less than 2 degrees in several seconds, because the $u_{21}$ vector is precisely estimated by the constraints, and converge to the fixed point around the real position on the EN coordinates. However, the upward errors of the vector is not small, therefore the rotation angle $\beta$ is more than 20 degrees.

The baseline-vector estimates are gradually converged to the real positions, however, it takes approximately 20 minutes. The improvements of the vector positions are caused by improving the estimation of the integer ambiguities $N_{L_1, L_2}^{a_1, a_2}$ in Eq. 4 with the lapse of time. When the fixed integer ambiguities are used as the initial values for the vector estimations, the more precise accuracy is obtained than usual. The Euler-angle estimation by six baseline vectors are worse than one vector just after the estimation, however, the better rotation angle estimates are obtained with the lapse of time. It is high possibility for the Euler angles to be more precise for shorter time as the number of baseline vectors is increased.

6 Conclusions

We have presented a novel baseline-vector estimation algorithms based on the double-difference positioning DD-PPP, and applied a novel VPPP algorithm based on the vector length constraints. GPS data are applied to the coupled GR equations for multiple antennas in the case of unknown positions. The experiments for the baseline-vector estimations among four antennas in the static situation during 20 minutes have been carried out. The six baseline vectors are estimated at a time, and the vectors’ positions gradually converge to the real positions. When the vector length constraints are applied, the positions can be reached adjacent to the real position in several seconds.

We have estimated the Euler angles from the reference frame to the body frame by the least-squares method of six baseline vectors through the parameters of Euler’s principal rotation theorem. The experiments for Euler-angle estimation by utilizing the vectors have been carried out for 20 minutes. The Euler angles by six baseline vectors are more precisely estimated compared with one vector, and gradually converged to less than 2 degrees in approximately 1200 seconds.

Presently the DD-based algorithms for PPP and VPPP provide the positioning accuracy in sub-meter error level, and has the potential capability for estimating the baseline vectors, namely the position differences between two antennas. When the DD-PPP/VPPP positioning performance would be improved, the Euler-angle estimation by the baseline vectors would be more precise. In the future, we will apply the positioning and the estimation algorithms for the kinematic environment.

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obtained from the outer product of two vectors $\mathbf{s}, \mathbf{t}$.

\[
\begin{align*}
\cos \phi &= \mathbf{s} \cdot \mathbf{t} \\
&= x_0 x_1 + y_0 y_1 + z_0 z_1, \quad (35) \\
\mathbf{n} &= (n_x, n_y, n_z)^T = \frac{\mathbf{s} \times \mathbf{t}}{|\mathbf{s} \times \mathbf{t}|} \\
&= \frac{1}{\sin \phi} (y_0 z_1 - z_0 y_1, z_0 x_1 - x_0 z_1, x_0 y_1 - y_0 x_1). \\
&= \frac{1}{\sin \phi} (y_0 z_1 - z_0 y_1, z_0 x_1 - x_0 z_1, x_0 y_1 - y_0 x_1). \\
&= \frac{1}{\sin \phi} (n_x, n_y, n_z).
\end{align*}
\]

From Eq. (35)-(36), we have following equations:

\[
\begin{align*}
\cos \phi &= x_0 x_1 + y_0 y_1 + z_0 z_1, \quad (37) \\
n_x \sin \phi &= y_0 z_1 - z_0 y_1, \quad (38) \\
n_y \sin \phi &= z_0 x_1 - x_0 z_1, \quad (39) \\
n_z \sin \phi &= x_0 y_1 - y_0 x_1. \quad (40)
\end{align*}
\]

Transforming Eq. (37)-(40), $(x_1, y_1, z_1)$ are expressed by $(x_0, y_0, z_0), (n_x, n_y, n_z)$, and $\phi$:

\[
\begin{align*}
x_1 &= x_0 \cos \phi + z_0 n_y \sin \phi - y_0 n_z \sin \phi, \quad (41) \\
y_1 &= y_0 \cos \phi - z_0 n_x \sin \phi + x_0 n_z \sin \phi, \quad (42) \\
z_1 &= z_0 \cos \phi + y_0 n_x \sin \phi - x_0 n_y \sin \phi. \quad (43)
\end{align*}
\]

Eq. (41)-(43) are expressed by matrix form as follows:

\[
\begin{bmatrix}
x_1 \\
y_1 \\
z_1
\end{bmatrix} =
\begin{bmatrix}
x_0 & 0 & z_0 - y_0 \\
0 & y_0 - z_0 & x_0 \\
z_0 & y_0 & -x_0
\end{bmatrix}
\begin{bmatrix}
\cos \phi \\
n_x \sin \phi \\
n_y \sin \phi \\
n_z \sin \phi
\end{bmatrix}. \quad (44)
\]

When we obtain two baseline vectors $\mathbf{s}, \mathbf{t}$, we estimate the parameters $(n_x, n_y, n_z), \cos \phi$, and $\sin \phi$. Then we derive the rotation matrix in Eq. (26) by the estimates.