Modelling of a high-current magnetron discharge in a plasma electron emitter

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Abstract. An analytical model of a high-current form of a low-pressure glow discharge in an inverted cylindrical magnetron, which performs the function of plasma electron emitter, is shown. Were found conditions of the discharge self-sustaining, allowing to estimate the voltage of the discharge and determine the critical value of the magnetic field and residual gas pressure below which the existence of this type of discharge is impossible. A comparison of the calculated discharge characteristics with experimental data obtained on the setup for studying the emission properties of the magnetron discharge was carried out.

1. Introduction

For generation of high-current electron beams with pulse duration of microsecond range, it is reasonable to use the plasma emitters with cold cathodes. An efficient source of high-current electron beam was created on the basis of a plasma discharge in crossed electric and magnetic fields excited in the electrode system of an inverted cylindrical magnetron type [1–3]. Due to the electron oscillations the discharge in crossed fields is easily ignited at low pressures, providing a high degree of plasma gas ionization and stability of the plasma parameters. In [1] is shown a possibility of using discharge system of an inverted magnetron type for receiving a tubular beam of charged particles. In [3] the results of experimental research of a high-current magnetron discharge in conditions of generating tubular electron and ion beams in pulsed and in continuous modes are shown.

Created plasma emitter on the basis of the magnetron discharge system is a part of an electron accelerator designed for the relativistic microwave electronics. At an accelerating voltage of 200 kV and pulse duration of 15 µs electron beam current is 500 A.

For further upgrade of such a source of electrons, optimization of its geometry and parameters of plasma discharge, and also for definition of the discharge self-sustaining conditions in the present work modelling of the discharge cell is carried out.

2. Experimental setup

Figure 1 shows a diagram of the experimental setup with electrode system of an inverted magnetron type [2, 3]. The discharge is ignited between the cathode 1 and the anode 2. The diameter of the
The cathode is changed from 80 to 140 mm, the interelectrode gap varies in the range of 10–20 mm, and the length of the electrodes is 60–100 mm. The end electrode 5 is electrically connected with the cathode. The electron emission from the discharge is carried out from the annular surface of plasma through a slit in the electrode 5 with a width of 2 mm. Particle acceleration is carried out when voltage is applied between the electrodes 5 and 8. The magnetic field in the discharge and accelerating gaps is created using solenoids 3 and 4, respectively. The magnitude of the magnetic field varies from 100 to 500 G, pressure of the plasma gas (xenon, nitrogen, argon) in the discharge chamber is $10^{-3}–6\cdot10^{-3}$ Torr.

![Figure 1. The experimental setup for studying the emission properties of the magnetron discharge: 1 – cathode; 2 – anode; 3, 4 – solenoids; 5 – end electrode; 6 – Langmuir probe; 7 – emission probe; 8 – accelerating electrode; 9 – collector.](image)

When the gas pressure is below $10^{-3}$ Torr, discharge exists in a low-current form with current less than 1 A and discharge voltage of several kV. With growing pressure, increases the intensity of ionization and occurs an abrupt transition to a high-current form of the discharge, which is accompanied by a reduction of discharge voltage to several hundred volts, raise of the current by hundreds of times and a significant increase in the brightness of the discharge.

3. Model of a discharge cell and comparison of the theoretical and experimental results

A discharge cell of an inverted magnetron is a hollow cylinder with radius $R$, on the axis of which is situated the anode in a form of a rod with radius $r_0$. The cylindrical surface acts as the cathode. Uniform magnetic field $B$ is directed along the cylinder axis. We investigate only stationary high-current mode when the voltage drop $U$ is concentrated in a thin near-cathode layer, and all the interelectrode space is occupied by the positively charged plasma. Cathode layer is formed by a stream of fast electrons with the energy corresponding to the voltage of the discharge $U$. These electrons with velocity $V_f(R) = V_{fr} = (2eU/m)^{1/2}$ come into the weakly ionized quasi-neutral plasma and maintain the necessary level of the gas ionization. Slow plasma electrons are not able to produce the ionization of atoms and provide a discharge current to the anode.

Let us consider the one-dimensional problem in the cylindrical coordinate system and analyze the conditions of discharge maintenance, considering that the transfer of fast electrons along the radius of the cylinder is in the diffusive regime, and the influence on the movement of fast particles of weak electric field in quasi-neutral plasma can be neglected. With the help of the stationary equation of continuity and the equation for the flow of fast electrons [4, 5] we can get the following equation for radial distribution of particle concentration $n_f$:

$$
\frac{1}{\rho} \frac{d}{dp} \left( \rho \frac{dn_f}{dp} \right) - n_f = -\frac{\gamma n_e V_{fr}(R-r_0)}{2(R-r_0)} = -n_{f0},
$$

(1)
where $\rho = r/\lambda_f$; $\lambda_f = (D_f \tau_f)^{1/2}$; $D_f = eU\gamma_f/3m(\omega_{Be}^2 + v_{f0}^2)$ – diffusion coefficient; $v_{f0}$ – frequency of elastic collisions with gas atoms; $\omega_{Be} = eBlm$ – Larmor frequency; $\gamma$ – effective coefficient of ion-electron emission; $\tau_f = eU/\gamma_fW$ – characteristic time of relaxation of fast electrons; $v_i$ – ionization frequency; $W$ – energy spent by fast electrons on ionization of the gas atoms.

The term on the right side of equation (1) describes the appearance of fast electrons in the volume due to the $\gamma$-processes on the cathode under the action of incident plasma ions. Here $n_i$, and $V_s = (T_e/M)^{1/2}$ – concentration and the rate at which ions leave the region of quasi-neutral plasma to the cathode; $T_e$ – temperature of the plasma electrons; $M$ – mass of the ions. The solution of the equation (1) is

$$n_f(p) = n_{f0}(1 - AI_0(p) - BK_0(p)),$$

where $A, B$ – constants; $I_0$, $K_0$ – modified Bessel functions of zero order of the first and second kind respectively. This solution satisfies the boundary conditions at the anode and cathode:

$$n_f(r = r_0) = 0; \quad (n_fV_f)_r = D_f \frac{dn_f}{dr}.$$  \hspace{1cm} (2)

In the right part of the second condition of (2) there is no minus sign, because as the positive direction for the flow of fast and slow electrons we consider the direction from the cathode to the anode, i.e. against the positive direction of the cylindrical coordinate $r$. Taking into account the boundary conditions we can get the following expression for the concentration of fast electrons:

$$\frac{n_f(p)}{n_{f0}} = 1 - \frac{I_0(p)K_i(p_r) + I_i(p_r)K_0(p)}{I_0(p_r)K_0(p_r)} + 2(p_r - p_{r0})(I_0(p_r)K_0(p_r) - I_0(p)K_0(p_{r0})),$$

where $p_r = R/\lambda_f$; $p_{r0} = r_0/\lambda_f$.

The condition of the discharge self-sustaining in a cylindrical magnetron can be found in the following way. As follows from the work [4], the multiplication factor of the plasma electrons $a = eU/W$ is connected with the ratio of the fast electrons current at the anode to the complete discharge current $\psi = I_{fa}/I$:

$$\alpha \approx \frac{1}{\gamma - \psi}. \hspace{1cm} (4)$$

Current of electrons at the anode is transferred mainly by plasma electrons $I = I_{fa} + I_{ea} \approx I_{ea}$, that is formula (4) is valid when $\psi \ll 1$. Determining the flow of fast electrons from the equation that was used for the second boundary condition (2), and the flow of plasma electrons to the anode

$$j_{ea} = \frac{1}{r_0} \int e_r n_f r dr,$$

using (4) we can find the condition of the discharge self-sustaining in the magnetron:

$$u = 1 + \frac{1}{\frac{\rho_{r0}N}{2}(5R^2/r_0^2 - 4R/r_0 - 1)} - 1,$$

where

$$N = \frac{I_0(p_r)K_1(p_r) + I_1(p_r)K_0(p_r)}{I_0(p_r)K_0(p_{r0}) - I_1(p_r)K_1(p_r) + 2(p_r - p_{r0})(I_0(p_r)K_1(p_r) + I_1(p_r)K_0(p_{r0})).}$$

In the formula (6) are used for convenience the dimensionless variables $u = U/\U_0 = \gamma v_f \tau_f$ and $b = BlB_0$, where $\U_0 = Wl\gamma_f$; $B_0 = 1.5(mWl/\gamma_f)^{1/2} \nu_f(R - r_0)$, besides $\nu_f \approx 2v_i$; $p_r = (2.61a/\U_0)R/(R - r_0)$ and $p_{r0} = p_r r_0/R$ in the case of magnetized electrons, when $\omega_{Be}^2 >> v_{f0}^2$.
Figure 2 presents the dependence of the discharge sustainment voltage from an induction of the magnetic field obtained using equation (6).

\[ V_s \propto B^\alpha \]

It can be seen that in the inverted magnetron the minimum value of the magnetic field for which the existence of the discharge is still possible is more than in planar magnetron [6]. The magnitude of the magnetic field \( B_0 \), shown in the formula (6), and curve 2 in figure 2 are in a good agreement with experimental results [3], shown in figure 3.

From these results it also follows that an increase in gas pressure leads to a decrease in the discharge voltage and shifts the voltage curve to smaller values of the magnetic field.

This model does not describe the dependence of the voltage of a high-current discharge from the residual gas pressure. Influence of the gas pressure on the conditions of discharge self-sustaining can be considered if in the equation (4) for the current density of plasma electrons to the anode across the magnetic field instead of (5) use the following expression [4]:

\[ j_{ea} = \frac{en_e V_{Te}}{4(1 + \beta_e^2)} \]

where \( n_0 \) – plasma density in the interelectrode gap; \( V_{Te} = (T_e/m_e)^{1/2} \), \( \beta_e = 1/r_{Te} n_e \sigma_{e0}, r_{Te} = mV_{Te}/eB \) – thermal velocity, Hall parameter and cyclotron radius of the plasma electrons, respectively; \( n_e \) – density of the gas atoms; \( \sigma_{e0} \) – transport cross section of electron-atom collisions. Then in the assumption of the constant concentration of plasma in the interelectrode gap, i.e. \( n_0 = n_{ic} \) (see formula (1)) instead of (6) we have:
\[ u \left(1 - \frac{\psi}{\gamma}\right) = 1; \]  

where \[ \frac{\psi}{\gamma} = \frac{1}{(\rho - \rho_0) p} \]; \( p \) - gas pressure; \[ T = \frac{I_1(\rho_\infty)K_1(\rho_0) - I_1(\rho_0)K_1(\rho_\infty) + (\rho_\infty - \rho_0)I_1(\rho_0)K_0(\rho_\infty) + I_0(\rho_0)K_1(\rho_\infty)}{I_0(\rho_\infty)K_1(\rho_0) + I_1(\rho_0)K_0(\rho_\infty)} \].

Figure 4 shows the dependence of the discharge voltage from the xenon pressure at different values of the magnetic field, plotted according to the formula (7). Here \( b = 1 \) corresponds to the magnetic field \( B = B_0 = 140 \) G; \( \gamma = 0.1 \); \( W = 30 \) eV; \( \sigma_{\infty} = 1.5 \times 10^{-15} \text{ cm}^2 \). The calculated dependence of the discharge voltage from the pressure confirms the results of the experiment – when the gas pressure is lower than 1 mTorr magnetron discharge exists in a high voltage form and has a voltage more than 1 kV.

![Figure 4. Dependence of the discharge voltage from the xenon pressure: \( l - b = 1.2; 2 - b = 1; 3 - b = 0.9 \).](image)

4. Conclusions
In this work was built a hydrodynamic model of a high-current plasma discharge in the crossed electric and magnetic fields excited in the electrode system of an inverted cylindrical magnetron type.

Were found the self-maintaining conditions for a high-current discharge in a cylindrical magnetron cell. The minimum value of the magnetic field induction for which in such geometry the existence of a discharge is still possible is somewhat larger than in the planar magnetron. As well as in direct magnetron, with a decrease in the magnitude of the magnetic field increases the minimum gas pressure that is necessary to maintain a high-current discharge form in an inverted magnetron.

Minimum gas pressure for which may exist a high-current magnetron discharge is on the order of magnitude smaller than the corresponding pressure of \( 2 \times 10^{-2} \) Torr in the combined reflective discharge with a hollow cathode [4].

Built in this model dependences of the discharge voltage of a high-current plasma discharge from the pressure and magnitude of the magnetic field are in a good agreement with the corresponding experimental curves.

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