Designing flexible graph kernels that can run well on various platforms is a crucial research problem due to the frequent usage of graphs for modeling data and recent architectural advances and variety. In this work, we propose a novel graph processing framework, PGAbB (Parallel Graph Algorithms by Blocks), for modern shared-memory heterogeneous platforms. Our framework implements a block-based programming model. This allows a user to express a graph algorithm using kernels that operate on subgraphs. PGAbB support graph computations that fit in host DRAM but not in GPU device memory, and provides simple but effective scheduling techniques to schedule computations to all available resources in a heterogeneous architecture. We have demonstrated that one can easily implement a diverse set of graph algorithms in our framework by developing five algorithms. Our experimental results show that PGAbB implementations achieve better or competitive performance compared to hand-optimized implementations. Based on our experiments on five graph algorithms and forty-four graphs, in the median, PGAbB achieves 1.6, 1.6, 5.7, 3.4, 4.5, and 2.4 times better performance than GAPBS, Galois, Ligra, LAGraph Galois-GPU, and Gunrock graph processing systems, respectively.

Index Terms
Parallel graph processing, Block-based, Heterogeneous

1 INTRODUCTION

"Mobilis in Mobili", — Jules Verne

Graphs are one of the dominant data structures to model irregular and complex data. Hence, graph analysis is crucial for many data analytics applications. High-performance parallel graph processing has been an active research area for decades, and has become even more active in recent years as computer architectures have evolved. Reducing the running times of graph algorithms lowers overall costs and eases pressure on the environment, while easy implementation of efficient algorithms increases productivity. However, developing high-performance parallel graph algorithms is a challenging task. Several well-known hardware and software-related challenges arise [1]. Graph algorithms have random data access patterns, limiting their locality. Furthermore, these computations are memory and network bound rather than compute bound since the ratio of data access to computation is high, and data re-use is low. Finally, graph algorithms are data-driven and graph datasets tend to be highly skewed by vertex degree. This causes a significant workload imbalance between computational loads. An efficient parallel graph
algorithm must address these challenges. Different architectures are ideal for addressing different kinds of problems. Therefore, heterogeneous systems can be very beneficial in addressing wide range of challenges instead of solely depending on an architecture.

**Limitation of state-of-art approaches:** For a long time, distributed systems were de facto high-performance solutions to graph problems. Then, since CPUs’ clock speed increases slowed down, multicore technology has become ubiquitous. Recently, hardware accelerators such as GPUs (graphics processing units) and FPGAs (field programmable gate arrays) have emerged to serve different parallelization needs, and more are coming. The current computation environment is heterogeneous: it consists of multicore servers with hardware accelerators. Such an environment increases the importance of designing flexible graph kernels that can run well on various platforms.

Systems researchers have proposed many graph processing frameworks to simplify parallel graph algorithm design. Those systems mostly adopts one of the vertex-centric [2]–[19], edge-centric [7], [13], [16], [18], [20], block-centric [21]–[24] or linear-algebra-based [25]–[29] programming abstractions. For a detailed discussion, we encourage the reader to see Section 2. Most of the existing frameworks are designed to run solely on one type of architecture, and they do not support heterogenous execution. There exist a few shared-memory heterogeneous systems [8], [30]. Those systems adopts vertex-centric APIs. Best of our knowledge PGAbB is the first block-based heterogeneous graph processing framework.

A graph application may require running a variety of graph algorithms in an analysis pipeline. For instance, we may first run a connected-components algorithm to find the largest component and extract it. We may next run a BFS (breadth-first search) algorithm to re-order the vertices, then a triangle counting algorithm to measure clustering coefficients or more complicated features. All of the implementations in the pipeline should perform well to be able to achieve good overall performance. Vertex-centric and edge-centric programming abstractions are very restrictive and restrictive programming models are not good for expressing a variety of graph algorithms efficiently, and in general require complex workarounds [31].

**Key insights and contributions:** In this work, we propose a novel graph processing framework for modern shared-memory heterogeneous machines. Our framework has three design goals: First, the framework can execute kernel operations of a graph computation on different architectures and combine the results. Second, the framework addresses major parallel graph algorithm design challenges in the background. Third, the programming model is expressive to implement a diverse set of graph algorithms that achieve competitive performance.

We claim that a block-based programming model is suitable to design architecture-agnostic and efficient parallel graph algorithms and we propose PGAbB (Parallel Graph Algorithms by Blocks) as our solution. The literature includes a few block-based systems: Giraph++ [21], Blogel [22], and GoFFish [23] are three popular distributed graph processing frameworks. Those systems use a connectivity-based partitioner to reduce communication. They extend vertex-centric APIs: vertices in the same block can access each other’s information without communication. MultiGraph [24], a graph processing framework for GPUs, generates dense, light, and sparse edge blocks. It uses a different data representation for each type of block. This method allows MultiGraph to implement block-specific kernels. PGAbB differs from those systems in the following ways: First, the PGAbB user expresses the algorithm using a functor that operates on an ordered list of blocks (block-list), not on a single block. Second, PGAbB targets heterogeneous execution environments and does not solely depend on a single architecture. Third, PGAbB uses spatial partitioning.

In a block-based execution model, one might need to access data from multiple blocks. Pattern-mining algorithms are good examples. For instance, the triangle counting problem seeks the count of mutually-connected sets of three vertices (and their associated three edges). Edges of triangles may appear in three different blocks. To process a block, we need two other blocks. There are two common approaches for solving this issue: adding a communication step where each worker requests data from other workers, or breaking the decentralized computation model and making the graph (i.e., blocks) visible to everyone. The former solution brings addition synchronization and message passing costs. Many shared-memory systems [6], [16] applied the latter and achieved good performance. However, this approach is not efficient for heterogeneous execution due to the limited memory of hardware accelerators: they cannot process graphs larger than the device memory size. To solve those problems, we introduce the block-list concept and build our framework around it. In brief, a block-list is an ordered list of references to blocks (i.e., sub-graphs) of a graph. The PGAbB user expresses an algorithm using a functor that operates on a block-list
Modeling the computation using block-lists brings several advantages. First, we can process graphs larger than the device memory. PGAbB only needs the blocks of a single block-list to process a task. Second, we can localize the computation. Third, we can categorize graph algorithms using three block-list generation styles that we specify and name: activation-based, single block bulk-synchronous, and multi-block pattern-based (see Section 3). Fourth, we can bound the required data movement to the maximal size of a block-list.

The PGAbB user can run any computation on the host (CPU only), on the device (GPU only), or both (collaborative). In the collaborative setting: PGAbB generates tasks and sorts them based on their workload estimations. The goal is to detect bottleneck tasks. PGAbB assigns bottleneck tasks to the device and the others to the host. PGAbB uses streams and asynchronous calls to overlap the data copy time with the computation.

PGAbB’s programming model does not depend on a partitioning scheme. However, we encourage the usage of conformal two-dimensional spatial partitioning, especially in heterogeneous settings. In data-driven computations, conformal partitioning bounds the number of blocks that we should fetch, makes reasoning about graph algorithms easier, and can localize graph computation.

The contributions of our paper are as follows:

- We propose a novel block-based graph processing model directly motivated by heterogeneous architectures.
- We show that PGAbB can process graphs that state-of-the-art GPU based systems cannot.
- We provide a classification method to categorize graph algorithms.
- We analyze PGAbB’s and prior arts’ performances on 44 real-world and synthetic graphs against six state-of-the-art libraries/frameworks.

Experimental methodology and artifact availability. We have implemented five different graph algorithms on PGAbB and compared with the implementations of these five algorithms on six state-of-the-art libraries/framework. GAPBS [32], a hand-optimized lightweight graph library; three CPU-based graph processing frameworks: Galois [33], Ligra [6], LAGraph [34]; and two GPU-based graph processing frameworks: Galois-GPU [35], and Gunrock [16]. All competitive frameworks (i.e., other than GAPBS) provides a Vertex/Edge centric API. For parallelization, Galois, Ligra and LAGraph use OpenMP, and Galois-GPU and Gunrock use CUDA. Totem [8] and MultiGraph [24] are designed for older GPU architectures therefore were not able to include those systems in our experiments. Best of our knowledge Blogel [22] outperforms Giraph++ [21], and GoFFish [23]. However, on a single node Blogel was a magnitude of time slower than PGAbB. Therefore we did not include it in our experiments for fairness.

As we will present in more detail in Section 5, we performed our experiments on a heterogenous system with with 2 Power 9 CPUs, and 2 Nvidia Volta GPUs. And carried on experiments on 44 widely used benchmark graphs that have edges from 100 millions to 2.1 billions.

After the double-blind review and our internal pre-release processes are complete, we plan to release PGAbB on GitHub as open source under the BSD-3 license.

Limitations of the proposed approach. Our current PGAbB implementation is only for shared-memory heterogeneous systems with CPUs and GPUs. PGAbB explicitly, and efficiently, manages the data transfers between host and GPU memory. Adding different type accelerators with their own local memory, and/or supporting distributed-memory execution will necessitate modifications to PGAbB.

2 Related Work

Programming abstractions of current frameworks can be broadly divided into five categories: vertex-centric, edge-centric, block-centric, linear-algebra based and domain-specific-language (DSL) based. Vertex-centric and edge-centric programming abstractions allow flexibility to the framework developers in system implementation by restricting the API [36]. This trade-off is acceptable for some simple graph algorithms such as PageRank [37], breadth-first search (BFS) [38], or label-propagation-based connected components. However, those API’s restrictive nature makes it hard to implement non-trivial graph algorithms efficiently. One needs to break the decentralized computation model to implement non-trivial graph algorithms [6], [33], [34], [39].
In recent years some linear-algebra-based graph processing frameworks are proposed. The primary goal of these frameworks is to use hand-optimized linear-algebra kernels at the back-end while users write their algorithms in the language of linear algebra. This is a good idea however, there are two drawbacks. First, one cannot use these frameworks without linear-algebra knowledge. Second, in general, pure linear algebraic formulations require more number of operations than graph-based solutions and perform worse. Careful implementation is required to achieve a competitive performance [40] which requires algorithm-specific changes on the linear-algebra kernels.

In the literature, there exists a few block-centric or block-based works. Giraph++ [21], Blogel [22] and GoFFish [23] are three popular block-centric distributed graph processing frameworks. The primary goal of those systems is to reduce communication costs by using a connectivity-based partitioner to partition a given graph among processors and extending vertex-centric API in a way that vertices in the same block can access each other’s information without any communication. This approach has two problems. First, good connectivity-based graph partitioners are computationally expensive. Second, those systems still tend to have vertex-centric API restrictions but with some relaxation for the vertices in the same block. MultiGraph [24] proposes a different block-based approach for GPU-based graph processing. MultiGraph generates dense, light and sparse edge blocks using vertex order by degree. Besides, Multigraph uses a different data representation for each type of block. This method allows them to implement different kernels for each type of block for maximal utilization. This approach has two problems. First, in this approach reasoning on graph algorithms is hard due to the complex nature of blocks and their representations. Second, MultiGraph uses a kind of jagged partitioning which leads to higher communication costs on distributed and CPU/GPU heterogeneous settings.

3 Programming Model

PGAbB is is a header-only library. The PGAbB user implements their code and functions in C++ using PGAbB’s API. The computation takes a set of disjoint blocks (i.e., sub-graphs) as input. The user can define vertex, edge, and global attributes. There is no pre-defined output format, the user can store output in an attribute. The PGAbB user can provide six functors: two for doing the main computation, one for block-list composition, two for controlling the iterative execution process, and one for helping the scheduler. Listing 1 illustrates a high-level view of user-defined functors.

The user writes at least one of two kernels: a host kernel \(K_H\), and a device kernel \(K_D\). Both take as input a block-list: an ordered list that contains references to blocks. The user decides the number of blocks and the order of blocks in block-lists. The host kernel \(K_H\) runs on a CPU and the device kernel \(K_D\) runs on a GPU. The user can access the data of all blocks in the given block-list. PGAbB supports both CSR and COO representations.

The PGAbB user can compose block-lists using two different approaches. First, the user can generate all block-lists and provide them to PGAbB with the \(P_C\) functor. Second, the user can provide the \(P_G\)
functor. That functor takes a block-list as input and returns \texttt{true} if the given block-list is a member of the computation. In the latter, PGAbB checks all possible block-list combinations of the given size and only keeps block-lists that \( P_G \) returns \texttt{true}.

To support iterative computations the PGAbB user can use two functors: \( I_B \) and \( I_A \). PGAbB executes the \( I_B \) functor before starting the computation \((K_H, K_D)\). After the computation, PGAbB executes the \( I_A \) functor. The \( I_A \) functor defines the termination condition. PGAbB iterates while \( I_A \) functor returns \texttt{true}.

To customize the scheduling, the PGAbB user can provide an optional estimation functor \( E \). \( E \) returns a weight for a given block-list.

In brief, designing an algorithm using PGAbB's programming model involves four stages: block-list composition \((P_G \text{ or } P_C)\), attribute assignment (vertex, edge and/or block), execution handling \((I_B, I_A, \text{ and } E)\), and implementation of the kernel \((K_H \text{ and } K_D)\).

### 3.1 PGAbB Preliminaries

A graph \( G = (V, E) \), consists of a set of vertices \( V \) and a set of edges \( E \). An edge \( e \) is referred to as \( e = (u, v) \in E \) where \( u, v \in V \). PGAbB assumes that the given graph, \( G \), is divided into sub-graphs that we call "blocks." Let \( B_i = (V_i = (S_i \cup D_i), E_i) \) denote a block. We specify a block as three subsets: a subset of source vertices \( S_i \subset V \), a subset of destination vertices \( D_i \subset V \), and a subset of edges \( E_i \subset E \).

The PGAbB user can define vertex attributes, edge attributes and a global attribute. Let \( A_V \) denote the vertex attributes where \( A_V(S_i) \) and \( A_V(D_i) \) represent source and destination vertex attributes of the block \( B_i \), respectively. Let \( A_E \) denote the edge attributes where \( A_E(E_i) \) represents the edge attributes of the block \( B_i \), and \( A_G \) represents the global attribute. Let \( A_G \) denote the global attributes.

A block-list (e.g., \( L_i = (B_j, B_k, B_l) \)), is an ordered list of blocks and the user decides the size and the order of blocks in the block-list. We denote \( L = \{L_0, \ldots, L_i\} \) as the set of all block-lists for a graph algorithm. PGAbB models the computation using block-lists.

The computation of a kernel operation of a graph algorithm takes a block-list \( L_i \) as input. The PGAbB user provides the \( K_H \) functor for CPUs and \( K_D \) for GPUs. Both functors take a block-list as input. A kernel and a block-list define a task, \( T_i \), such that; \( T_i = K_H \ (L_i) \) or \( T_i = K_D \ (L_i) \). \( T \) represents the set of all tasks.

### 3.2 Categorization of graph algorithms in PGAbB

Parallel algorithm design for irregular graphs is a challenging research problem. Therefore, in the literature there exist many algorithms to solve the same graph problem. In general, there is no clear winner because both the structure of the graph and the nature of the architecture impact performance. To accommodate these variables, API permissiveness is important. On the other hand, more restrictive APIs can help system developers [36]. To become more permissive PGAbB supports three execution modes: \textit{single block bulk synchronous execution}, \textit{activation-based execution}, and \textit{multi-block pattern-based execution}. Fig. 1 illustrates some of the popular graph algorithms and which PGAbB execution mode is a good fit for that algorithm.

![Image of graph algorithms in PGAbB](image_url)

Fig. 1. A classification of some graph algorithms in PGAbB. We will cover the ones written in italic font.
**Activation-based execution:** Many graph computations are iterative and data-driven. In this execution model, the next iteration’s computation (i.e., active vertices/edges) depends on the vertices visited in the current iteration. Therefore, only some portion of the graph contributes to the computation. Traversal (i.e., BFS, Dijkstra [41]) and peeling-based graph algorithms (i.e., kTruss [42]) are good examples for that execution mode.

**Single block bulk synchronous execution:** Another common execution pattern among different graph algorithms is visiting all of the edges in an iterative fashion. PageRank [37], HITS [43], Shiloach-Vishkin [44] algorithms are good examples for this execution mode. The computation visits all of the edges in each iteration. Therefore, in this execution mode composing block lists from single blocks is sufficient to obtain efficient parallelism.

**Multi-block pattern-based execution:** Non-trivial graph algorithms that have more complicated computation structure may require to access different parts of the graph. Therefore we need to compose block lists using multiple blocks depending on the pattern of the computation structure. Triangle counting [45], k-clique finding, and butterfly counting algorithms are good examples for this execution mode.

Note that those execution modes are not strict and a graph algorithm may use two execution modes.

### 3.3 Parallel dispatches and basic API

PGAbB provides parallel for loop, and parallel reduction primitives for host and device. The PGAbB user can define the parallel body of those primitives using functors or lambdas. Parallel for loop primitives; `for_host` and `for_dev`, take the size of loop, and functor (or lambda function) written by the user. Parallel reduction primitives; `reduce_host` and `reduce_dev`, take an additional variable to represent reduction variable.

The PGAbB user can access source and destination vertices of the block $B_i$ using `Vertices ($S_i$ — $D_i$)`. `Edges ($E_i$, $u$)` returns neighbors of the vertex $u \in S_i$ in the block $B_i$. `Attribute ($A_V(S_i$ — $D_i$)`) returns attributes of source or destination vertices of $B_i$. `Attribute ($A_E(E_i)$) returns edge attributes of $B_i$, and the global attributes can be accessed using `Attribute ($A_G$)`.

PGAbB can do all read and write operations atomically. In this case the user does not need to worry about race conditions. However, atomic operations are expensive. To avoid this overhead, PGAbB provides atomic functions to users who have basic parallel programming knowledge: `Add (a, b)`; atomically adds $b$ to $a$. `CAS (a, b, c)`, compare-and-swap, takes three arguments; a memory location ($a$), an old value ($b$) and a new value ($c$). If the value stored at $a$ is equal to $b$ it atomically swaps $c$ with $a$ and returns `true`, and otherwise it does not update $a$ and returns `false`.

### 3.4 An example: Shiloach-Vishkin’s algorithm.

In this section we are going to explain how the PGAbB user can implement Shiloach-Vishkin’s [44] algorithm to solve weakly connected components problem. This is an iterative algorithm consists of two steps. In the first step, the algorithm iterates over edges and combines vertices into trees: for each edge, if roots of source and destination vertices are different, then the algorithm tries to hook the greater root to the smaller root. In the second step, the algorithm links each vertex to the root of its tree using pointer jumping. Algorithm stops when there is not any hooking operation performed.

**Block-list composition:** The PGAbB user designs the computation using block-lists. Therefore, the first step is the decision of block-list composition: Number of blocks per block-list, and their order. Shiloach-Vishkin’s algorithm iterates all edges during the hooking step. Therefore, we can set block-list size to one, so each block-list has a block. Providing $P_G$ functor that returns `true` is sufficient.

**Attribute assignment:** Note that within a block the user can access source vertex, destination vertex, and edge attributes. However, pointer jumping may jump outside vertices. Therefore we are going to assign and use global attributes: An array of size $|V|$; $C$, to store parent ids of each vertex. A variable, $H$, to store number of hook updates.

**Execution handling:** Shiloach-Vishkin’s algorithm is iterative and each iteration consists of hooking and linking steps. In our design, during the even iterations we do the hooking and during the odd iterations we do the linking: Hook $\rightarrow$ Link $\rightarrow \cdots \rightarrow$ Hook $\rightarrow$ Link. In $T_B$ functor, before each hooking iteration we should reset $H$ to 0. In $T_A$ functor, we should check if a hooking operation is done during the hooking
auto K_D = [](BlockList L_i){
    auto B_i = L_i(0);
    // Global attribute
    auto [C, H] = Attribute(A_G);
    if (pgabb::Iteration % 2 == 0){
        reduce_dev(Vertices(S_i).size(), [] (auto u){
            int h = 0; // local number of hooks
            for (auto v : Edges(E_i, u)){
                auto r1 = max(C(u), C(v)); // max root
                auto r2 = min(C(u), C(v)); // min root
                if (r1 == r2) continue;
                if (r1 == C(r1)){
                    C(r1) = r2;
                    ++h;
                }
            }
            H = H + h;
        }, H);
    } else{
        auto [u, v] = GetInterval(i, |C|)
        for_dev(v-u, [] (auto k){
            while (C(u+k) != C(C(u+k))){
                C(u+k) = C(C(u+k));
            }
        });
    }
};

Listing 2. Shiloach-Vishkin device kernel on PGAbB.

iteration if so return true otherwise return false. We are going to use number of edges in a block-list for estimating the workload: E functor returns the number of edges in a block-list.

Kernel development: Listing 2 illustrates an implementation of K_D. In our implementation, first, we get the block reference, B_i, from the block-list, L_i. Note that, each block-list has one block. Then we fetch block attributes, C and H. After, we check whether we are in an odd or even iteration. If it is even, then we iterate all edges in the block and do hooking operations. Here we store local number of hook operations, h, and at the end add it atomically to H. If we are in an odd iteration then we do the linking operations. Linking operations depends on the C array. Therefore, first we should divide the array C equally among different block-lists for parallel processing. The PGAbB user can use GetInterval(id, size) function to get a unique interval. Such an approach becomes highly useful when we partition the graph in two-dimension. K_H differs only in outer loop execution; line 6 and line 21 in Listing 2. K_D uses parallel reduction and for loop dispatches to leverage from the massive parallelization capabilities of GPUs. In K_H we use regular for loops.

3.5 An example: direction-optimized breadth-first search (BFS) algorithm

The BFS problem is to create a tree starting from a source vertex such that each level represents the distance between the source vertex, and every other vertex. The BFS algorithm explores all nodes at the present depth prior to moving on to the nodes at the next depth level. In each level, we insert unvisited neighbors of vertices in the frontier queue, to the next queue. In undirected graphs, one can perform this operation in two ways: Top-down: for all vertices in the current queue adding unvisited neighbors to the next queue. Bottom-up: visiting all vertices and adding them to the next-level queue if one of their neighbors is in the current queue. Beamer et al. [38] proposed to use a direction optimization for traversing less number of edges when the frontier queue becomes large. We are going to implement a version of this algorithm.

Attribute assignment: We are going to use vertex attributes to store the parent of each vertex. We are going to use a level-based queue data structure, Q, as the global attribute. Depending on the partitioning scheme, a block might use its own queue only. Therefore, assume that, Q(S_i) and Q(D_i) represent queues
of source and destination vertices of the block, \( B_i \), respectively. In addition we are going to use a variable, \( n_q \), to count the number of vertices that we inserted in an iteration.

**Block-list composition:** Each block composes a block-list. We are going to use activation-based execution model. Therefore, in each iteration we are going to create block-lists from the blocks whose source or destination vertex queues are not empty.

**Execution handling:** We are not going to use the \( \mathcal{I}_B \) functor for this algorithm, and the \( \mathcal{I}_A \) functor is going to finalize the iterations when we did not push any vertices to \( Q \).

**Kernel development:** By their nature GPUs are more suitable to run bottom-up BFS algorithm while CPUs are better at top-down BFS algorithm. Therefore, in our design we are going to implement top-down BFS in \( K_H \) and bottom-up BFS in \( K_D \). In \( K_H \), after fetching the block information and corresponding queue data-structure we are going to visit neighbors of each vertex in the frontier and push them to our queue if they are not visited before. This process continues until there is not any push operation in an iteration. Listing 4 illustrates this algorithm. In \( K_D \), we implement bottom-up BFS algorithm. In this kernel, after fetching the block information and required attributes, we visit all of the vertices. If we have not visited a vertex and one of its neighbors appears in the frontier then we insert that vertex to our queue and stop visiting the other neighbors. Again we keep track the number of push operation to understand if we have finished execution or not. Listing 3 illustrates this kernel.

### 3.6 An example: triangle counting algorithm

The triangle counting problem is to find the number of mutually connected sets of three vertices in an undirected graph. Common approach to solve this problem is to compute common neighbors between
source and destination vertices of each edge. In this work we are going to implement a two-dimensional triangle counting algorithm [46].

**Block-list composition:** Each block-list is going to have three blocks. For each edge in the first block, the second block is going to contain source vertex partial-adjacency list and the third block is going to contain destination vertex partial-adjacency list: $L_i = (B_k, B_l, B_m)$ such that $S_l = D_k$ and $S_m = D_l$.

**Attribute assignment:** We are going to use a variable, $n_t$, as the global attribute to store the number of triangles found.

**Execution handling:** This algorithm does not require iterative execution. Therefore, we are not going to use $I_B$ and $I_A$ functors.

**Kernel development:** The goal is counting the number of triangles that appear in three blocks of a block-list, $L_i = (B_k, B_l, B_m)$. For each edge, $(u, v)$ in $E_k$, this operation can be done by counting the number of common neighbors between the partial-neighbor list of $u$ in $B_l$ and the partial neighbor list of $v$ in $B_m$. Similar to regular triangle counting algorithms this can be computed using list or hashmap-based intersection algorithms [45]. Listing 5 illustrates $K_D$. We use parallel reduction to achieve parallelization within a block-list on GPUs. $K_H$ uses regular for loop and differs only in line 6 of Listing 5.

![Fig. 2. High-level execution flow.](image-url)
4 DESIGN OF THE PGAbB FRAMEWORK

We target single node heterogeneous environments: multicore CPUs and a GPU. PGAbB consists of four components; I/O Handler, Layout Manager & Partitioner, Scheduler, and a user API. As backend, we use Kokkos [47]; OpenMP on the host machine and CUDA on the device.

4.1 Execution flow

First, PGAbB reads the graph from disk and partitions it into blocks. The user can use one of the existing partitioners or provide a custom partitioner. After the block composition, PGAbB assigns an id to each block and orders them.

Generation of block-lists is the third step. Using \( P_C \) or \( P_G \), PGAbB generates block-lists, assigns them a weight using \( E \) functor and sorts them in decreasing order. The goal is distinguishing bottleneck tasks to be able to assign them to GPU. PGAbB aims to assign lighter tasks to CPUs.

PGAbB parallelizes a computation using tasks and utilizes CPUs and GPU streams. A CPU thread executes a single task. A GPU stream uses multiple threads to execute a task. After the ordering, PGAbB initialized the scheduler and executes \( I_B \) functor. Then, it assigns heavy tasks to GPU streams, and light ones to CPUs. When all tasks are processed, PGAbB executes \( I_A \) functor. If \( I_A \) functor returns false PGAbB stops, otherwise starts a new iteration. Fig. 2 illustrates high-level view of this execution flow.

4.2 I/O Handler

Most of the graph kernels are computationally lightweight, and disk I/O can become the bottleneck. Therefore, almost all of the graph processing frameworks use custom binary format to store the graph. PGAbB also uses a custom binary format to make reading process faster. In addition, to be able to read ASCII formatted graph inputs in the most efficient way, PGAbB adopts and extends PIGO [48], a library for reading and writing ASCII formatted graph files in parallel.

4.3 Partitioner and Layout Manager

PGAbB does not dictate any partitioning scheme to form blocks. However, based on our experiences, we strongly encourage the use of a symmetric two-dimensional partitioner [49] in hybrid settings for three reasons: First, gathering information and scattering computation results become straightforward. Gathering information from the in-edges of the vertices, computing on those and then scattering the result to all the processors that has the outer-edges is a common communication pattern in graph analysis, such as BFS [50], and PageRank [51]. Second, this type of partitioning becomes highly useful to reason about graph algorithms: Each block is a sub-graphs where diagonal blocks are the owners of the vertex meta-data and any other block represents the edges between two sub-graphs. Third, we can define a partitioning as conformal when connecting row and column lengths of different tiles match. Conformal partitioning is very crucial for many graph applications that access neighbors of neighbors or tiled matrix-matrix multiplication based operations. Because, for those kind of applications if one does not use a conformal partitioning for inputs and outputs, converting outputs of the previous iteration to inputs of next iteration would require additional communication. By its nature, symmetric rectilinear partitioning is a conformal partitioning and it is suitable for those applications.

Besides, one dimensional or vertex partitioning is very useful for CPU only execution because it increases the locality of threads. Therefore, PGAbB also provides a one-dimensional optimal partitioning algorithm. The PGAbB user can also override PGAbB partitioner class.

4.3.1 Block layout:

PGAbB assigns an integer ID for each block and align them from the lowest to the largest. By default, PGAbB uses row-major order to align blocks (see Fig. 2). The PGAbB user can override the partitioner and assign IDs using any space-filling curve mapping or another method.
4.3.2 Block storage data-structures:
Each block represents a subgraph. By default PGAbB rearranges vertex ids within a block and uses Compressed Sparse Row (CSR) data structure to store sub-graphs. In addition, PGAbB also supports Coordinate List (COO), and Compact Coordinate List (CCOO) data structures. Depending on the algorithmic need the PGAbB user can initiate those data structures.

4.4 Scheduler
Scheduler is responsible for assigning tasks to the GPU and CPUs. PGAbB stores the graph in the host memory. Therefore, the scheduler copies blocks in the BlockList of a task to the GPU memory after assigning that task to the GPU. We use asynchronous memory copies from host to device (device to host) to perform those operations. When available GPU memory is under a threshold, scheduler waits GPU tasks’ termination, then clears the GPU memory before assigning new tasks. While designing PGAbB, we also experimented using UVM (Unified Virtual Memory) but the performance was \( \approx 5 \times \) slower. Therefore, we do not use UVM or present UVM-based results in this paper.

Our scheduler aims to assign computationally heavy tasks to the GPU and lighter tasks to CPUs. The goal is leveraging the massive parallelism available on GPUs. PGAbB orders tasks based on their estimations and starts assigning heavier tasks to GPUs, and lighter ones to CPUs. For sorting, PGAbB estimates the workload of a task, \( T_i \), using \( E \) functor if defined, otherwise PGAbB uses the total number of edges within a block-list as the weight of a task. To decrease the initial synchronization cost of the GPU and guarantee the assignment of heavier tasks to the GPUs, a cut-off can be predefined. CPUs do not go past the cut-off.

We use CUDA streams to execute several tasks on the GPU simultaneously. Increasing the number of CUDA streams, initially helps to improve the performance and then starts hurting due to scheduling overhead. We experimentally selected four as the number of CUDA streams. We create four CUDA streams, then one of the CPU threads is assigned to a stream. That CPU thread is responsible for waiting on that stream, synchronizing the stream, sending a task to a device through that stream, and gathering information from a device through that stream. All of these operations use asynchronous function calls. When we create streams and assign a thread for each of them, GPUs and CPUs compete for tasks and get a new one from the queue when they finish executing a task. Then a stream thread can overlap copying the blocks of the next assignment with the computation.

PGAbB’s ultimate goal is to simultaneously leverage both CPUs and GPU. Such an approach is crucial for achieving three goals: maximizing memory utilization, processing graphs that cannot fit into GPU memory, and overlapping the data-copy time with the computation. However, in some lightweight graph kernels such an approach might not be suitable. Therefore, the PGAbB user can choose to run computation using only CPUs or the GPU. The user can do this decision before starting an iteration using \( I_B \) functor.

5 Experimental Evaluation
We performed our experiments on a Power9 architecture with 2 CPUs, and 2 Volta GPUs. Each CPU has 16 cores and each core has 4 SMTs (Simultaneous Multi-Threading). The machine has 320GB memory and each Volta GPU has 32 GB memory. We measured data transfer rate between CPU and GPU as \( \approx 11 \text{GB/s} \) with pageable memory and \( \approx 60 \text{GB/s} \) with pinned memory. We compiled codes using GNU compiler (g++) version 7.4, CUDA runtime version 10.0 and OpenMP version 4.0.

In our experiments, we ran each implementation ten times and we report the median of them. We only report the execution time of the graph kernels. In all systems, we excluded disk I/O and other pre-processing overheads. Data transfer overheads between the host and the device are included in PGAbB’s execution time but for others it was considered as part of the pre-processing time.
TABLE 1
Speedups over the GAPBS reference implementation. Each cell represents a framework’s speedup on a graph and a graph algorithm. Heat map indicates where speedup is lower (RED < 1.0), equal (WHITE = 1.0) or higher (GREEN > 1.0).
PGAbB-GPU: GPU-only results. PGAbB: Results with CPU and GPU.

|         | Social   |        | Web     |        | Gene   |        | Road    |        | Synthetic |
|---------|----------|--------|---------|--------|--------|--------|---------|--------|-----------|
|         | twitter7 | Orkut  | sk-2005 | kmer_V1r | eu_osm | myciel19 | kron21 |
| Galois  | PR       | 0.83   | 1.01    | 1.01   | 0.89   | 1.03    | 6.96    | 0.78   |
|         | SV/LP    | 8.40   | 1.71    | 1.68   | 2.29   | 1.81    | 1.25    | 1.12   |
|         | CC       | 0.84   | 1.56    | 0.98   | 0.64   | 0.64    | 2.94    | 0.81   |
|         | BFS      | 0.26   | 0.59    | 0.46   | 0.34   | 2.14    | 0.39    | 0.18   |
|         | TC       | 0.69   | 1.06    | 0.63   | 0.90   | 1.21    | 0.44    | 0.40   |
| Ligra   | PR       | 0.39   | 0.60    | 0.99   | 0.43   | 0.53    | 2.59    | 0.72   |
|         | SV/LP    | 1.24   | 0.70    | 1.05   | 0.18   | 0.02    | 0.58    | 0.66   |
|         | CC       | 0.02   | 0.04    | 0.00   | 0.02   | 0.01    | 0.03    | 0.02   |
|         | BFS      | 0.61   | 0.67    | 0.93   | 0.68   | 0.16    | 1.37    | 0.82   |
|         | TC       | 0.31   | 0.35    | 0.12   | 0.30   | 0.17    | 0.43    | 0.69   |
| LAGraph | PR       | 0.75   | 0.98    | 0.60   | 0.75   | 0.65    | 3.21    | 0.71   |
|         | SV/LP    | 14.24  | 1.64    | 0.89   | 0.30   | 0.13    | 7.70    | 0.92   |
|         | CC       | 0.17   | 0.21    | 0.12   | 0.14   | 0.05    | 0.27    | 0.09   |
|         | BFS      | 0.79   | 0.33    | 0.77   | 0.27   | 0.33    | 0.75    | 0.30   |
|         | TC       | 0.38   | 0.87    | 0.66   | 0.29   | 0.16    | 0.52    | 0.37   |
| Galois-GPU | PR   | 0.00   | 2.72    | 0.00   | 1.01   | 1.49    | 12.12   | 1.62   |
|         | SV/LP    | 0.00   | 3.67    | 0.00   | 2.43   | 2.71    | 2.65    | 1.57   |
|         | CC       | 0.00   | 0.46    | 0.00   | 1.16   | 0.99    | 0.09    | 0.15   |
|         | BFS      | 0.00   | 0.00    | 0.00   | 0.00   | 0.00    | 0.00    | 0.00   |
|         | TC       | 1.03   | 0.85    | 0.90   | 0.00   | 0.00    | 0.38    | 0.65   |
| Gunrock | PR       | 0.00   | 1.28    | 0.00   | 1.44   | 1.34    | 5.42    | 0.97   |
|         | SV/LP    | 0.00   | 1.88    | 0.00   | 3.18   | 1.22    | 3.90    | 0.97   |
|         | CC       | 0.00   | 0.24    | 0.00   | 1.51   | 0.44    | 0.14    | 0.09   |
|         | BFS      | 4.61   | 1.48    | 0.00   | 3.59   | 0.80    | 3.45    | 5.73   |
|         | TC       | 0.00   | 0.74    | 0.00   | 0.04   | 0.02    | 0.29    | 0.23   |
| PGAbB-GPU | PR   | 4.20   | 4.72    | 0.74   | 0.53   | 0.64    | 13.60   | 2.30   |
|         | SV/LP    | 19.19  | 9.96    | 3.16   | 6.45   | 3.63    | 9.21    | 3.85   |
|         | CC       | 1.68   | 1.08    | 5.52   | 3.56   | 1.37    | 0.64    | 0.31   |
|         | BFS      | 0.18   | 0.85    | 0.97   | 0.28   | 0.32    | 1.06    | 0.27   |
|         | TC       | 3.09   | 3.39    | 2.34   | 0.52   | 0.32    | 2.87    | 2.33   |
| PGAbB   | PR       | 4.64   | 4.67    | 0.80   | 0.53   | 0.64    | 10.76   | 1.79   |
|         | SV/LP    | 18.02  | 5.95    | 1.90   | 5.73   | 2.95    | 7.70    | 1.98   |
|         | CC       | 1.25   | 1.53    | 2.14   | 1.91   | 0.96    | 2.40    | 0.87   |
|         | BFS      | 0.16   | 0.89    | 0.77   | 0.90   | 0.33    | 1.00    | 0.29   |
|         | TC       | 3.02   | 3.01    | 1.69   | 1.11   | 3.91    | 5.39    | 3.48   |

5.1 Dataset
To include different kind of medium size graphs; we downloaded a total of 44 graphs that have edges from 100 millions to 2.1 billion from SuiteSparse\textsuperscript{1}, Konect\textsuperscript{2}, and Snap\textsuperscript{3} dataset repositories. We transformed all graphs to undirected, and removed duplicate edges.
Due to space limitations, we selected seven different types of graphs from our dataset for detailed results; two social networks, two synthetic graphs, a road network, a gene graph, and a web graph. In

1. SuiteSparse: https://suitesparse-collection-website.herokuapp.com
2. Konect: http://konect.cc/networks/
3. Snap: http://snap.stanford.edu/data/index.html
PGAbB selected seven graphs can fit into GPU memory. Therefore we also included PGAbB’s GPU-only performance on those graphs. Note that, to process larger problem sizes we need heterogeneous execution model of PGAbB. We are going to use GAPBS as the reference implementation in our comparisons. Table 1 groups performances of frameworks on five algorithms. Each cell presents a framework’s speedup on a graph and a graph algorithm. In a particular case the value of a cell is: (1) 1.0 if the framework performs the same as GAPBS. (2) 2.0 if the framework performs two times faster than GAPBS. (3) 0.5 if the framework performs two times slower than GAPBS. We use a heat map to visually represent performances. In recent work, Azad et al. [52] provide a similar table to present the results of different frameworks. Our findings match that work.

5.2 Evaluation of bulk graph computations

5.2.1 PageRank algorithm

There exist several graph algorithms that follow SpMV (Sparse matrix-vector multiplication) type of computation. PageRank is one of the most popular ones. In this experiment, we used SpMV like PageRank implementations. We choose damping factor as 0.85, error tolerance as 0.0001, and iteration limit as 20. We report the average execution time of a PageRank iteration. Gunrock and Galois-GPU fail to process twitter7 and sk-2005 graphs due to GPU memory limitations.

PGAbB gives the best performance in twitter7, Orkut, and kron21 graphs, and the second performance on the myciel19 graph. Atomic updates becomes the bottleneck on sk-2005, kmer_V1r and eu_osm graphs. Because, in our PageRank algorithm implementation we use atomic operations to update ranks of the vertices even in the pull direction due to two-dimensional block layout. sk-2005 graphs high locality, kmer_V1r and eu_osm graphs very high low degree vertices (≈ 99% of the vertices have less than two neighbors) increases the cost of atomic operations drastically. We observe that when graphs fit into GPU memory PGAbB’s GPU-only execution performs slightly better than hybrid execution. Bandwidth between CPU and GPU is the primary factor of this. The gain comes from CPUs cannot compensate the synchronization cost of the arrays between GPU and CPU.

PageRank algorithm implementations of these systems are very competitive. On the complete dataset; in median, PGAbB performs 2.4×, 1.3×, 1.8×, 1.7×, 1.1×, and 1.6× better than GAPBS, Galois, Ligra, LAGraph Galois-GPU, and Gunrock, respectively.

5.2.2 Shiloach-Vishkin or Label Propagation algorithms

In this experiment, we are going to evaluate performances of SV (Shiloach-Vishkin) or LP (Label Propagation) algorithm implementations in different systems. Because, some systems provide an implementation of SV algorithm and some LP algorithm. Both algorithms have similar execution principles. The main difference between those two algorithms is the upper bound of the number of iterations: SV algorithm’s iteration bound is $O(\log(|V|))$ and LP algorithms iteration bound is $O(D)$ where $D$ is the diameter of the graph. SV algorithm’s compression step causes this difference. Gunrock and Ligra implement LP algorithm. The other systems implement SV algorithm. LAGraph implements an optimized SV algorithm [53] which can converge faster on some graphs.

To achieve better performance, PGAbB executes hooking step in the GPU and linking step in CPUs. Between those steps PGAbB synchronizes the component array (a global attribute). That synchronization cost is included in the reported execution times.

Galois-GPU and Gunrock fail to process the largest twitter7 and sk-2005 graphs, due to device memory limitations. PGAbB performs the best. Galois, and Gunrock have the second and the third best performances. Ligra’s poor performance has two reasons; more number of iterations (LP algorithm) and imbalance between computational loads. OpenMP might be the cause of the latter reason, because Ligra is primarily designed using Cilk. Similar to PageRank algorithm, we observe that when graphs fit into GPU memory PGAbB’s GPU-only execution performs better than hybrid execution. Synchronization between CPU and GPU becomes the bottleneck.

On the complete dataset; in median, PGAbB performs 3.9×, 2.0×, 20.0×, 3.9×, 1.4×, and 1.9× better than GAPBS, Galois, Ligra, LAGraph Galois-GPU, and Gunrock respectively.
5.2.3 Best connected-components algorithm

In this experiment, we are going to evaluate performances of best performing connected component algorithm implementations in different systems. GAPBS, Galois and PGAbB implement Afforest algorithm [54]. Ligra implements low-diameter graph decomposition based algorithm [55]. The other systems do not have a specific implementation and implement SV or LP algorithms.

To achieve better performance, PGAbB executes sampling step in the GPU and finalization step in CPUs. Again, synchronization cost is included in the reported execution times.

Galois-GPU and Gunrock fail to process the largest twitter7 and sk-2005 graphs. We observe that Afforest algorithm’s lower computational complexity is the main advantage of GAPBS, Galois and PGAbB. Even though Ligra’s implementation is work-efficient it requires more read/write operations and perform poor. Ligra suffers from bad workload imbalance again. Besides of eu_osm and kron21 graphs, PGAbB’s implementation outperforms Galois, and GAPBS. Similar to previous algorithms, on majority of the graphs PGAbB’s GPU-only execution performs better than hybrid execution due to synchronization between CPU and GPU. However, we also observe that on more irregular graphs with many components (mycielskian19 and kron21) GPU-only execution performs worse than hybrid execution. Because, CPUs handle linking operation more efficiently and the gain compensates the synchronization cost.

On the complete dataset; in median, PGAbB performs $1.5 \times$, $1.4 \times$, $105 \times$, $11.2 \times$, $3.7 \times$, and $4.7 \times$ better than GAPBS, Galois, Ligra, LAGraph Galois-GPU, and Gunrock respectively.

5.3 Evaluation of traversal computations

In this experiment, we are going to evaluate performances of different systems on a traversal graph algorithm; direction-optimized BFS algorithm [38]. Galois-GPU hanged and could not process those graphs in our machine. Gunrock failed to process the sk-2005 graph due to device memory limitations.

Gunrock performs the best. The others have similar performances, but with slightly better execution times Ligra and PGAbB gives the second and third best performances. Gunrock’s design is a perfect fit for the BFS problem and it leverages massive parallelization capabilities of GPUs. As the runtime is small, the overheads of graph processing systems are significant in the final performance even if they implement the same algorithm as GAPBS therefore we observe a significant slow down on the other systems. In addition, depending on the graph structure, two-dimensional layout also brings additional cost to PGAbB. In an adversarial case PGAbB might require to visit all edges and cannot benefit from the direction optimization. We observe that PGAbB’s GPU-only and hybrid execution are close.

On the complete dataset; in median, PGAbB performs $1.6 \times$, $1.2 \times$, and $3.1 \times$, worse than GAPBS, Ligra and Gunrock, respectively. PGAbB performs $1.7 \times$, and $1.1 \times$ better than Galois and LAGraph, respectively.

5.4 Evaluation of pattern-based computations

In this experiment, we are going to evaluate performances of different systems on a non-trivial graph algorithm; triangle counting. Degree-based vertex ordering is a commonly used heuristic in triangle counting algorithms. Therefore we enabled degree ordering in all systems. We do not report time spent for this process.

Gunrock fails to process twitter7 and sk-2005 graphs. Triangle counting algorithm only requires half of the edges, therefore Galois-GPU was able to process all graphs. Thanks to design flexibilities PGAbB performs the best. GAPBS, and Galois have the second and the third best performances. Workload imbalance becomes the bottleneck for Ligra. PGAbB’s GPU-only execution is more fragile to the graph structure. Because, sparse tasks are more bandwidth bounded than the denser tasks and takes more time on GPUs. We observe that on this problem hybrid execution handles the performance variety caused by the very sparse tasks more successful by assigning them to CPUs.

On the complete dataset; in median, PGAbB performs $1.7 \times$, $2.6 \times$, $4.9 \times$, $2.9 \times$, $5.9 \times$, and $10.5 \times$ better than GAPBS, Galois, Ligra, LAGraph Galois-GPU, and Gunrock respectively.
5.5 Overall results

Fig. 3 illustrates performance profiles [56] of seven systems on complete 220 tests; 5 algorithms on 44 graphs. In $\approx 46\%$ of the test instances PGAbB gives the best performance. Considering overall performances of those seven systems we can subjectively rank them as follows: 1$^{st}$ PGAbB, 2$^{nd}$ Galois, 3$^{rd}$ GAPBS, 4$^{th}$ Gunrock, 5$^{th}$ Galois-GPU, 6$^{th}$ LAGraph, and 7$^{th}$ Ligra. In the median, PGAbB performs $1.6 \times$, $1.6 \times$, $5.7 \times$, $3.4 \times$, $4.5 \times$, and $2.4 \times$ better than GAPBS, Galois, Ligra, LAGraph, Galois-GPU, and Gunrock respectively.

6 Conclusion

This paper introduces PGAbB: a block-based algorithmic framework for parallel graph processing on heterogeneous platforms. In a heterogeneous setting, PGAbB aims to maximally leverage from different architectures by implementing a task-based execution on top of a block-based programming model. Our experimental results show that in the median, PGAbB performs $1.6 \times$ to $5.7 \times$ better than state-of-the-art four CPU-based and two GPU-based graph processing frameworks over 44 graphs in the range of 100M to 2.1B edges.

As a future work, our goal is to make PGAbB distributed by integrating locality-aware schedulers and garbage collectors. In addition, we plan to investigate, developing automatic parameter tuning approaches that will take into account properties of the graph, the kernel, and compute architectures.

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