Deriving optimal transmission architecture of planetary gear assembly by mathematical modelling of design parameters

S Mandol¹, P K Dan¹ and M K Mondal¹

¹Rajendra Mishra School of Engineering Entrepreneurship, Indian Institute of Technology Kharagpur, West Bengal, India, PIN: 721 302

E-mail: sourabh.mandol@iitkgp.ac.in

Abstract. This paper aims to develop optimal transmission architecture, constrained by selection of parameters, to be used as a reference for design of planetary gear assembly comprising of gearsets. These parameters are CAD (Computer Aided Design) geometric attributes of gear members. Planetary gearset is used in fully automatic transmission, in automotive, since this is compact, accommodates gear shifting under load and possesses high torque density, which the parallel shaft variants of gearbox do not. Fully automatic transmission involves high maintenance cost and low fuel efficiency than the manual or the automated manual types. Modelling of transmission architecture has been attempted for hybrid electric vehicle (HEV) as per the available literature, but no mathematical modelling of design parameters has been reported. This research emphasises the significance of mathematical modelling of design parameters for a planetary gear assembly and gear members for deriving optimised architecture. Mathematical models, presented in this work, are gleaned from DoE (design of experiment) based response data and are built utilising General Linear Model (GLM) for obtaining good fit. Material failure based information is expounded in this work for designing vehicle transmission architecture. Effect of design parameters on fuel consumption is also explicated in this article.

1. Introduction

The power split Hybrid Electric Vehicle (HEV) is the most promising short-term solution to achieve high fuel efficiency and low heat loss [1]. A combination of electric power and mechanical power is used in the drivetrain to attain high fuel efficiency with low vehicle emission. In some systems, self-replenishment facilities are available where the battery is recharged through regenerative braking and/or the engine is coupled to the generator to aid similar situation. Other systems, such as a plug-in HEV, has a large capacity battery that requires charging from external source. The different architectures of HEV viz. parallel, series, complex, plug-in, etc. present distinct advantages and dis-advantages. In this case, an attempt has been made to convert architectural attributes into mathematical model. The mathematical model is developed using design parameters and inculcate deformation based information. Optimizing the mathematical model will not only yield best design, but also deduction of failure criteria for will be made possible for an architecture in this experiment. Numerous research in planning and designing robust transmission architecture suggest improvement in fuel efficiency and it plays a crucial role for the same for HEV [2-4]. Therefore, architecture of the transmission elements within the HEV, will contribute significantly towards the robust improvement in fuel efficiency. Analysis of
multiple architectural combinations of HEV transmission in real-world can be both very costly and time consuming. Virtual simulation of multiple product architectures in desired simulation condition is a proven method to deduce product behavior [5]. In our case, multiple architectural possibilities were tested using Design of Experiments (DoE) where, multiple types of architecture is incorporated as a level for a control parameter in the experiment. The power-split element in HEV is a planetary gearset. The propulsion sources are connected to a specific gear members in the planetary gearset. The power transmission modes in the HEV can be changed by locking/unlocking gear members. A good example is the Toyota Prius which uses a planetary gearset to strategically control product flow within the vehicle transmission where the ring gear is connected to the motor and differential, the sun gear connects the generator and the carrier connects the engine [6]. The 3D assembly of planetary of power split element (planetary gearset) was composed in solidworks. The assembly was analyzed in ANSYS Workbench to calculate the deformation with load constraints representing power transmission modes. Design of Experiments (DoE) is used to develop experimentation strategies for determining effective number of test runs with a finite number of samples [7]. The Taguchi method is a tool for identifying the significant parameters and deriving the best setting of control parameters to attain desired outcome [8 and 9]. With the criteria of least number of treatment condition, it is the best choice to perform DoE for assemblies with large number of components [9]. In this case, the signal to noise ratio approach was used to derive the best architecture. General linear model (GLM) is a combination of statistical models such as: linear regression, logistic regression and Poisson’s regression. It is a method for deriving maximum likelihood estimation of the model parameters [10]. For analyzing the possible fuel consumption, Simscape driveline in Matlab was used to model the feasible characteristics utilizing available HEV model in Simscape [11]. In the available HEV model, only the gearbox components were altered according to the design attributes of 3D assembly composed in solidworks (two planetary gearset taken into consideration). To attain high transmission efficiency, the deformation within the geared mechanism during operation is to be minimized [12]. Therefore, the deformation within the assembly is considered (recorded in the response column of DoE table) for analysis. Emphasis of design parameters is disseminated in this paper and the consequent deformation was analysed for multiple architecture of planetary gearset. Therefore, minimization of this mathematical model will result in improvement of the power transmission within the gearset. The fuel consumption is calculated by changing the gearset with the same simulation condition. Models deduced from this experimentation will ideate best suitable architectural configurations for a planetary gearset that takes into account deformation based information. The finding of the simulation experiment presents a useful relationship i.e. least fuel consumption is associated with the planetary gearset with least deformation value. The goal of this paper is to disseminate a novel design methodology that takes into consideration parametric constraints of CAD gearset assembly (in this case planetary gearset) for achieving the desired architectural configuration with high transmission and fuel efficiency.

![Figure 1. Experiment Workflow.](image)
2. Methods
The assembly of planetary gearsets was composed in Solidworks using the ISO standard. Four design parameters namely, Gearset, Power Input, Fixed Member and Moment (Nm) were analysed through Taguchi Orthogonal Array Design L18(2^1 3^3) with one factor at two level and other three at three levels with a total of 18 runs. Design considerations for planetary gearsets is stated in Table 1. The planetary gears designed in Solidworks with specifications as in Table 1.

Table 1. Gear design parameters.

| Gearset | Module | Face Width (mm) | Ring to Sun Gear teeth ratio |
|---------|--------|-----------------|-----------------------------|
| A       | 3      | 30              | 3.86                        |
| B       | 3      | 30              | 2.30                        |

Table 2. Control Parameters.

| Level | Gearset | Input | Fixed | Moment (Nm) |
|-------|---------|-------|-------|-------------|
| 1     | Type A  | Planet| Planet| 100         |
| 2     | Type B  | Carrier| Carrier| 200        |
| 3     | Ring    | Ring  |       | 300         |

Figure 2. Planetary gearset designed in SolidWorks.

Table 2 contains the select control parameters in the experiment. The selected parameters emphasize the operation of the power split device (PSD). The power flow in the gearset is framed according to the gear shifting configuration. The gearset assembly was analysed in ANSYS Workbench for calculating deformation (in mm) within the system. In this case, the total deformation was calculated using the assigned values of moment (in Nm) as present in the orthogonal array table. Structural steel material was assigned for the analysis under ideal/default simulation condition. Analysis settings were made in a manner so as to interpret the operation modes by locking and unlocking gear members. A combination of fixed and cylindrical support was used to relate similar conditions that are to found in operation. In this simulation, the power flow in HEV was analysed through DoE.

The control parameters in Table 3 illustrate the HEV transmission configurations. Locking/unlocking of gear members and power input was taken into considerations based on the topology schematics, as present in a typical HEV with planetary power split mechanism. For analysing the deformation within the system, a moment was induced in the assembly and the response was recorded as in Table 3.

Figure 3. Process Workflow.
Using the design configurations, the loading condition was set and the deformation of each treatment condition was calculated. Mathematical modelling of the data in OA table was analysed and a mathematical model was derived using Minitab. Poisson’s regression was used to derive the mathematical model and further the $R^2$ value was improved using GLM.

### 3. Results and Discussion

The DoE results using S/N ratio yield best possible configuration based on the select criterion to achieve design optimisation. Nominal-the-best criterion was used in Taguchi analysis for this experiment. From figure 2, it is observed that stage B with sun gear as input, planet gear fixed and 100 Nm of moment fetched the least deformation value. Using predict Taguchi design in Minitab, it was seen that at 100Nm with sun gear as input, planet gear fixed, for type B gearset, the S/N ratio calculated is 58.02 which is the highest amongst all responses in the OA table. The confirmatory test was performed using the following equation:

$$\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 - (Number\ of\ factors - 1) \times (Average\ of\ SN\ Ratio) = 76.55$$

| SI No. | Gearset | Input | Fixed | Moment (Nm) | Deformation (mm) |
|-------|---------|-------|-------|-------------|------------------|
| 01    | A       | P     | P     | 100         | 0.001            |
| 02    | A       | P     | S     | 200         | 0.240            |
| 03    | A       | P     | R     | 300         | 0.030            |
| 04    | A       | S     | P     | 100         | 0.031            |
| 05    | A       | S     | S     | 200         | 0.004            |
| 06    | A       | S     | R     | 300         | 0.150            |
| 07    | A       | R     | P     | 200         | 0.070            |
| 08    | A       | R     | S     | 300         | 0.530            |
| 09    | A       | R     | R     | 100         | 0.000            |
| 10    | B       | P     | P     | 300         | 0.003            |
| 11    | B       | P     | S     | 100         | 0.017            |
| 12    | B       | P     | R     | 200         | 0.034            |
| 13    | B       | S     | P     | 200         | 0.001            |
| 14    | B       | S     | S     | 300         | 0.002            |
| 15    | B       | S     | R     | 100         | 0.000            |
| 16    | B       | R     | P     | 300         | 0.110            |
| 17    | B       | R     | S     | 100         | 0.037            |
| 18    | B       | R     | R     | 200         | 0.000            |

**Figure 4.** S/N ratio results.
3.1 Poisson’s regression analysis

Poisson’s regression analysis was performed in Minitab. The deviance table is summarized in Table 4.

| Source    | DF | Adj Dev | Adj Mean | Chi-square | P-Value |
|-----------|----|---------|----------|------------|---------|
| Regression| 6  | 2.02    | 0.34     | 2.02       | 0.92    |
| Moment    | 1  | 0.44    | 0.44     | 0.44       | 0.51    |
| Stage     | 1  | 0.33    | 0.33     | 0.33       | 0.56    |
| Input     | 2  | 0.18    | 0.09     | 0.18       | 0.91    |
| Fixed     | 2  | 0.30    | 0.15     | 0.29       | 0.86    |
| Error     | 11 | 0.73    | 0.07     |            |         |
| Total     | 17 |         | 2.75     |            |         |

Minitab was used for regression analysis. Natural log link function was used for 18 data points. From Table 2 it can be seen that the P-value of the regression model is more than 0.05 and is insignificant. However, we see that the $R^2$ value is 73.42% which indicates that the model explains 73.42% of the variance of the independent variable w.r.t. the dependent variable. The adjusted $R^2$ is 0% which indicates that the sample size of the independent variables is inadequate for prediction and interaction in-between predictor variables is to be analysed. Multi-collinearity in between the predictor variables is observed in Table 5, with Variance Inflation Factor greater than 1.

| Terms     | Coefficient | SE Coefficient | VIF |
|-----------|-------------|----------------|-----|
| Constant  | -4.74       | 4.17           |     |
| Moment    | 0.0092      | 0.02           | 1.19|
| Stage     | -1.39       | 2.64           | 1.19|
| B         | -0.68       | 2.93           | 1.38|
| Input     | 0.44        | 2.36           | 1.7 |
| Fixed     | 0.91        | 2.68           | 2.03|
| S         | -0.28       | 3.44           | 2.10|

For better model fit, General Linear Model (GLM) was used to analyse the data, with the interaction of predictor variables where high VIF was observed in deviance model.

| Source     | DF | SS   | MS   | F-value | P-value |
|------------|----|------|------|---------|---------|
| Stage      | 1  | 0.040328 | 0.040328 | 6.74     | 0.122   |
| Input      | 2  | 0.028296 | 0.014148 | 2.36     | 0.297   |
| Fixed      | 2  | 0.042025 | 0.021013 | 3.51     | 0.222   |
| Moment     | 2  | 0.050196 | 0.025098 | 4.20     | 0.192   |
| Stage*Input| 2  | 0.007242 | 0.003621 | 0.61     | 0.623   |
| Stage*Fixed| 2  | 0.033334 | 0.016667 | 2.79     | 0.264   |
| Input*Fixed| 4  | 0.065272 | 0.016318 | 2.73     | 0.286   |
| Error      | 2  | 0.011965 | 0.005983 |          |         |
| Total      | 17 | 0.294506 |      |         |         |
The $R^2$ value got increased to 95.94% and the adjusted $R^2$ is 65.47%. Thus, model inadequacy was removed using the GLM method in ANOVA. From the above results, the regression equation is as follows:

$$\text{Deformation} = 0.0700 + 0.0473 \text{Stage}_A - 0.0473 \text{Stage}_B - 0.0158 \text{Input}_P - 0.0387 \text{Input}_S$$

$$- 0.0703 \text{Moment}_{200} + 0.1147 \text{Moment}_{300}$$

$$- 0.0112 \text{Stage}*\text{Input}_A P - 0.0112 \text{Stage}*\text{Input}_B P - 0.0170 \text{Stage}*\text{Input}_A R$$

$$+ 0.0170 \text{Stage}*\text{Input}_B R + 0.0123 \text{Stage}*\text{Fixed}_A P + 0.0123 \text{Stage}*\text{Fixed}_A R$$

$$+ 0.0282 \text{Stage}*\text{Input}_A S - 0.0282 \text{Stage}*\text{Input}_B S - 0.0761 \text{Stage}*\text{Fixed}_A S$$

$$- 0.0761 \text{Stage}*\text{Fixed}_B S - 0.0404 \text{Input}*\text{Fixed}_P P + 0.0404 \text{Input}*\text{Fixed}_S P$$

$$- 0.1318 \text{Input}*\text{Fixed}_S S + 0.0684 \text{Input}*\text{Fixed}_R S - 0.0356 \text{Input}*\text{Fixed}_R R$$

The above equation represents a general linear model or multivariate regression model. The variables represent power flow and loading condition. This regression model will predict the deformation value based on the setting of the predictor variables. In this model, the variables can take values of either 0 or 1. For example, if the power input is through planet carrier, then the variable Input_P will be taken as one throughout the regression model and other input constraint will be zero. This model relates the predictor variables with the response with good fit.

In this method, the equation suggests a compounded planetary gearset as having similar gear members with multiple design constraint can only be achieved when the two stages are compounded together.

### 3.2 Matlab Analysis

HEV model in Simscape Driveline was used for analysing the fuel consumption [11]. Tables 7 - 10 portrays the assumed values for the simulation.

**Table 7.** Parameters for gearset A.

| Parameter                          | Value          |
|------------------------------------|----------------|
| Ring to sun gear ratio             | 3.86           |
| Sun-planet and ring-planet efficiencies | 0.96, 0.98   |
| Sun-carrier and planet-carrier power threshold | 0.001, 0.001   |
| Planet inertia                     | 0.0012547 kg m$^2$ |
| Fuel consumption                   | 6.269 gm       |

**Table 8.** Parameters for gearset B.

| Parameter                          | Value          |
|------------------------------------|----------------|
| Ring to sun gear ratio             | 2.3            |
| Sun-planet and ring-planet efficiencies | 0.96, 0.98   |
| Sun-carrier and planet-carrier power threshold | 0.001, 0.001   |
| Planet inertia                     | 0.00021425 kg m$^2$ |
| Fuel consumption                   | 4.931 gm       |

**Table 9.** Vehicle parameters.

| Parameter            | Value          |
|----------------------|----------------|
| Vehicle mass         | 1200 kg        |
| Frontal area         | 3 m$^2$        |
| Drag coefficient     | 0.4            |
| Air density          | 1.18           |
| Tire radius          | 0.3 m          |
Table 10. Engine parameters.

| Parameter          | Value   |
|--------------------|---------|
| Maximum power      | 50 kW   |
| Speed at max power | 5500 rpm|
| Max speed          | 7000 rpm|
| Type               | SI Engine|

The winding ratio for DC to DC converter is 2.48, battery nominal voltage is set to 201.6 volts, motor power is 30 kW and generator power is 15 kW. Comparing the fuel consumption values for the two gearsets, the gearset B yields the lowest fuel consumption which confirms the result obtained from Taguchi method.

3.3. Limitations
a. Due to article length limitations, the control parameters analysed is set to four to demonstrate the possible outcome of this experiment. By increasing the number of parameters and corresponding levels, better optimization results can be achieved.

b. The parameters and their corresponding values were selected for elaborating the methodology for experimentation. For illustrating a real-world scenario, selection of significant control parameters and a large number of levels is recommended for achieving the accurate outcome.

c. From Table 6, it was observed that the parameters selected in this experiment have no significance in the mathematical model however, it addresses the cause as it explains about 95.94% variance of the independent variables selected for experimentation ($R^2 = 95.94\%$). Thus, larger sample size with interaction should be suitable for this case. [13].

d. The fuel consumption was calculated by changing the gearset. All other parameters were unchanged and ideal simulation condition was taken into consideration.

4. Conclusion
Mathematical modelling of design parameters reflecting architectural design range for a planetary gear assembly is illustrated in this work to derive optimised architecture from the simulation generated data arranged in Taguchi orthogonal array. The gearset B with ring to sun gear ratio 2.3 is found to be the best in terms of aforementioned design criteria which is also validated and confirmed with the MATLAB result. The control parameters selected in this experiment considers the operational characteristics of the power split mechanism in an HEV. The $R^2$ value explicates high predictive capacity of the model. Since the mathematical model explains 95.94% variations of the control parameters, the proposed model is suited for the design of planetary gear architecture for a PSD. This method is suitable for researchers and engineers to plan and design suitable HEV transmission architecture with high fuel efficiency and reliability.

Acknowledgement
Virtual simulation and analysis of result were conducted at the Product Analytics and Modelling (PAM) Lab facility which houses state-of-the-art Computer Aided Design (CAD) and Computer Aided Engineering (CAE) based computer software and hardware at Indian Institute of Technology Kharagpur, India.

References
[1] Liu J and Peng H 2008 Modeling and Control of a Power-Split Hybrid Vehicle, *IEEE Transactions on Control Systems Technology* **16**(6) 1242-1251
[2] Chan C C 2007 The State of the Art of Electric, Hybrid, and Fuel Cell Vehicles, *Proceedings of the IEEE* **95**(4) 704-718
[3] Chan C C, Bouscayrol A and Chen K 2010 Electric, Hybrid, and Fuel-Cell Vehicles:
Architectures and Modeling, *IEEE Transactions on Vehicular Technology* **59**(2) 589-598

[4] Sabri M F M, Danapalasingam K A and Rahmat M F 2016 A Review on Hybrid Electric Vehicles Architecture and Energy Management Strategies, *Renewable and Sustainable Energy Reviews* **53** 1433-1442

[5] Haug E J 1989 *Computer Aided Kinematics and Dynamics of Mechanical Systems*, Allyn & Bacon

[6] Burress T A, Campbell S L, Coomer C, Ayers C W, Wereszczak A A, Cunningham J P and Lin H T 2011 Evaluation of the 2010 Toyota Prius Hybrid Synergy Drive System (No. ORNL/TM-2010/253), Oak Ridge National Laboratory

[7] Condra L 2001 *Reliability Improvement with Design of Experiment*, CRC Press

[8] Kackar R N 1985 Off-Line Quality Control, Parameter Design, and the Taguchi Method, *Journal of Quality Technology* **17**(4) 176-188

[9] Mandol S, Bhattacharjee D and Dan P K 2016 Robust Optimization in Determining Failure Criteria of a Planetary Gear Assembly Considering Fatigue Condition, *Structural and Multidisciplinary Optimization* **53**(2) 291-302

[10] Minitab, *What is a general linear model?* https://support.minitab.com/en-us/minitab/18/help-and-how-to/modeling-statistics/anova/supporting-topics/basics/what-is-a-general-linear-model/

[11] MATLAB, *Hybrid-Electric Vehicle Model in Simulink*, https://in.mathworks.com/matlabcentral/fileexchange/28441-hybrid-electric-vehicle-model-in-simulink (accessed on February 2018)

[12] Pimsarn M and Kazerounian K 2002 Efficient Evaluation of Spur Gear Tooth Mesh Load using Pseudo-Interference Stiffness Estimation Method, *Mechanism and Machine Theory* **37**(8) 769-786

[13] Minitab, *http://blog.minitab.com/blog/adventures-in-statistics-2/how-to-interpret-a-regression-model-with-low-r-squared-and-low-p-values* (accessed on March 2018)