The Use of Logistic Model in RUL Assessment

R Gumiński 1, S Radkowski
Institute of Vehicles, Warsaw University of Technology
Narbutta 84, 02-524 Warsaw, Poland
E-mail: rgumin@simr.pw.edu.pl

Abstract. The paper takes on the issue of assessment of remaining useful life (RUL). The goal of the paper was to develop a method, which would enable use of diagnostic information in the task of reducing the uncertainty related to technical risk. Prediction of the remaining useful life (RUL) of the system is a very important task for maintenance strategy. In the literature RUL of an engineering system is defined as the first future time instant in which thresholds of conditions (safety, operational quality, maintenance cost, etc) are violated. Knowledge of RUL offers the possibility of planning the testing and repair activities. Building models of damage development is important in this task. In the presented work, logistic function will be used to model fatigue crack development. It should be remembered that modeling of every phase of damage development is very difficult, yet modeling of every phase of damage separately, especially including on-line diagnostic information is more effective. Particular attention was paid to the possibility of forecasting the occurrence of damage due to fatigue while relying on the analysis of the structure of a vibroacoustic signal.

1. Introduction
Defining (estimating) the remaining useful life is a key task in the maintenance system of an object or a group of objects. Determination of RUL is connected with several issues: limited time of use as a result of degradation processes, often of random character, taking place. Knowledge of the damage phase, its progressing development, allows for estimation of the useful life. Information about the damage can be generated by means of the appropriately prepared model of damage development. In real life, it proves to be the case that prediction on the basis of prepared models renders results with a wide margin of uncertainty. It is caused by varying conditions of object’s operation, which correlates with initiation time and then, with the pace of the damage development. Therefore, in order to diminish the range of uncertainty of the conducted analyses, parameter updating of the assumed model or of the form of the model itself is necessary. Another issue worth emphasising is relating the information on the damage development to the remaining useful life. This task requires determining the limit value of the damage size or the related probability of damage and the scale of the damage, i.e. the level of acceptable risk.
The alternative to the damage development model and the need to its updating is the use of diagnostic information; the advantage of this approach being the information about the actual object and not the model object. Such an approach requires developing diagnostic parameters but eliminates uncertainty connected with the model.

1 To whom any correspondence should be addressed.
A different approach uses statistical models [1, 2] describing the object’s behaviour. Such an approach requires an access to statistical data, which is often a substantial restraint, especially if a small group of objects is taken into consideration. Additionally, inference with the help of statistical methods renders good results in the case of the maintenance system of a large group of objects. However, in the case of maintenance of individual units or even a single object, it can lead to great uncertainty. Inference about population on the basis of not particularly large sample size, and inversely, expressing opinion about a single element on the basis of statistical data can lead to major errors. Therefore, in such cases an individualised approach is necessary, i.e. prediction of the reliability indicators on the basis of the general model of the “personalised” damage development for the specific group of objects or even a specific technical object, for example working for one end-user, under defined conditions, technical culture, etc. Such an approach requires determining the model parameters for the group of objects or a specific object but as a consequence offers a more precise prediction. Particularly good results can be obtained when prognostics is enhanced with diagnostic data conveying information about a potential damage. In this context, several issues should be raised: possibility of detection of early phases of damage development/initiation, monitoring of the development, determination of the acceptable level of damage.

A review of the methods for determination of RUL should be conducted in the context of uncertainty [3] and particularly of its sources and methods of their elimination. This is particularly vital because of the fact that, as often proved by the maintenance practice, inaccuracies of RUL estimates range from a few tens to several hundred percent which leads to the disastrous aftermath of the damage or an overly early putting an object out of operation. Both cases create the cost increase, nonetheless of different character – random occurrence of high costs (in form of the failure effects) or the continuous increase in the object’s maintenance costs (in form of too frequently performed maintenance routines and preventive repairs). In the case of the occurrence of the estimate uncertainty of reliability characteristics, and RUL as a consequence, taking maintenance-related decision can be seen as an optimization task between the occurrence of the increased maintenance costs and the increased risk level.

It should be kept in mind that detection of damage initiation does not necessarily mean putting the object out of operation, which depends on the adopted/required level of acceptable risk. For a better demonstration of the subject of the influence reliability characteristics have on the maintenance decisions, the curve of the exemplary reliability function with the assumed uncertainty range has been shown. The uncertainty range of the characteristics translates into the uncertainty range of the estimated time (with the assumed probability of damage occurrence). It should be emphasised that the width of the uncertainty range of the maintenance time depends on the steepness of the reliability function.
Another aspect that needs to be discussed is multimodal damaging, i.e. every possible type of damage and occurring of successive phases of its development is related to a specific curve of the reliability function. A fact which complicates the inference under the conditions of multimodal damaging is the possibility of initiation of subsequent types of damage at different times and their development with varying intensity, which as a result can lead to a situation, in which for example the damage initiated as the first one can develop with little intensity, however, after a longer period of use, another or a different type of damage can be initiated, which will develop with greater intensity and in spite of its occurring relatively late can be decisive, i.e. it can reach the threshold value earlier. The abovementioned issues regarding the process of determining the RUL result in various methods of its estimation.

2. Review of RUL estimation methods
Generally, the RUL prediction methods should be divided into: based on the data and based on models [4].

Physical model of an object usually describes the dynamic behaviour and kinematic relations. Having been built, it requires selection of parameters and validation of the whole model, which is connected with conducting the appropriate experiments [5]. Moreover, as mentioned before, such a model can require updating so as to diminish the discrepancy in the results obtained from the model and on the basis of the actual object’s behaviour. In order to reduce the uncertainty of the simulation results it is worthwhile to model additional phenomena taking place on a real object, which often leads to building multidomain models. Modelling and verification of such models is both time- and cost consuming but leads to obtaining results burdened with lesser uncertainty. Additionally, such an approach causes better understanding of phenomena and mutual relations taking place in a real object, which is valued highly in dynamic states of work.

Methods based on knowledge, most frequently use historical data recorded in data bases. Customarily, using the data focuses on describing relations between observation of incidents (undesirable) and the data recorded in the data base [6]. Relations built on the basis of historical data can be used to create prognostics of the future damage occurrences with the simultaneous taking into
account the current measurements or historical and current measurements (for example using the trend analysis). Description of the abovementioned relation by means of function relationships can lead to designing of models for changes in diagnostic parameters as a function of time, numerous cycles, etc. Considering the fact that data can be burdened with high risk of error and indeterminacy, a substantial dynamics of phenomena occurs so as a consequence, also of the data; methods based on knowledge often use expert systems (based on artificial intelligence). It is caused also by the fact that building mathematical model is expensive and often impossible to carry out at the required level of detailedness. Referring to the definition, systems based on artificial intelligence have the feature of self-teaching that causes flexible adjusting to changing conditions. A good proposition in inference with the foundation of a knowledge base is employing the theory of fuzzy sets [7], offering the possibility of taking decisions on the basis of the uncertain input data. In order to make the building process of the fuzzy logic system more efficient, they are often supported with neural networks (neuro-fuzzy systems). In explorations of knowledge and decision-taking process regarding operation, the Bayesian probability is also used, especially Bayes networks.

In recent times, methods bordering on physical models and models based on knowledge, the so-called hybrid methods have been gaining popularity. Such an approach aims at eliminating the disadvantages of one method (e.g. expensive building of a very detailed physical model), by means of providing support with expert-system based methods. The simplest case of RUL determination takes place when there is a one-dimension correlation of the diagnostic parameter with the remaining useful life, such a case however, occurs rarely, i.e. the correlation is often multi-dimensional, and the character of the relationship changes with occurrence of the subsequent phases of damage development. The most recent tendencies in modelling damage development take into consideration a multi-phase development of the damage [8], i.e. because of the changing character of the phenomena taking place at the stage of initiation and damage development, each of the stages is described by a separate model, which leads to a greater precision of results obtained from the simulation. An important issue in application of such approach is the decision about changing the model being used.

In the following section of this work, the analysis will be shown of the results of an experiment consisting in fatigue breaking of the gear tooth. An attempt was made to describe changes in the parameter of a vibration signal using the logistic function determined by the relationship (1):

$$PD = \frac{a}{1 + be^{-\gamma}}$$  \hfill (1)

Estimation of parameters of the logistic function was conducted by means of linearisation of the relationship (1) and by juxtaposing it with the equation of the straight line. Line coefficients were determined on the basis of measuring data using the method of the least squares.

3. The test stand

The experiment was conducted at the FZG back to back test-bed. The test-bed consists of two toothed gears operating in a revolving power setup and it enables examination of both toothed wheels as well as gear lubricants. The diagram showing the test-bed is presented in Figure 2.

The shaft connecting the pinions is divided, which enables rotating one of its sections versus the other and thus introducing relevant meshing forces. Strain gauges are affixed to the shaft and they enable measuring the torque. Wheels with straight teeth are installed in the examined gear, while wheels with helical teeth are installed in the closing gear. Thanks to such a set up it was the examined toothed gear that was subject to defect-development during the experiment. Toothed wheels made of 20H2N4A carburized steel, hardened to 60 HRC hardness were used for the research (Figure 3). They were subjected to accelerated fatigue test. It should be emphasised that the examined toothed wheel did not have any defects that were previously made, such as tooth cuts. During the experiment, the vibration accelerations of the tested gear housing were registered, measured on the cover, directly over the pinion shaft bearing, in the direction X – radial – horizontal, and Z – radial – vertical. A detailed description of the test stand was given in [9].
Using the measurements of vibration accelerations conducted in such a way, the Fourier transform was designated (spectrum of vibration accelerations), on the basis of which changes in vibration energy were determined in the course of the experiment, i.e. starting from installation of the new toothed wheel up until the fatigue tooth breaking. Figure 4 shows the curve of the vibration energy and the logistic function describing energy.
Unfortunately, an attempt to describe changes in energy by means of one model throughout the whole experiment time, i.e. during occurrence of the successive phases of the fatigue breaking, leads to big errors. Because of the fact that the successive phases of damage are related to the occurrence of phenomena of different character, an attempt was made to describe the curve of vibration energy, dividing the operation time into two sub-periods (Figure 5).

Parameter estimation of the logistic curve in two phases causes a substantial decrease in the model error in relation to the values of energy obtained from measurement. This relation is of the repetitive

**Figure 4.** Energy of vibrations in the course of experiment.

**Figure 5.** Energy of vibrations in the course of experiment (with division into two phases).
character towards other examined gearwheels. Table 1 shows estimated parameters of the logistic curve for the exemplary gearwheels.

| Table 1. Logistic function parameters. | Phase I | Phase II |
|----------------------------------------|---------|---------|
| **Wheel no.1**                         |         |         |
| $a$                                    | 2,0E+07 | 7,6E+07 |
| $b$                                    | -9,1E-03| 4,6E+10 |
| $c$                                    | -2,2E-03| 1,3E-02 |
| **Wheel no.2**                         |         |         |
| $a$                                    | 5,5E+07 | 2,8E+08 |
| $b$                                    | -4,0E-02| 5,4E+01 |
| $c$                                    | -3,0E-03| 4,1E-03 |
| **Wheel no.3**                         |         |         |
| $a$                                    | 8,5E+07 | 2,8E+08 |
| $b$                                    | -4,0E-02| 7,8E+01 |
| $c$                                    | -3,0E-03| 5,6E-03 |

Comparing values of the obtained parameters, the following regularities can be observed: all three parameters in Phase II assume much greater values than in Phase I, parameters $b$ and $c$ additionally have the sign changed from negative to positive.

Analysing the obtained curves of energy and the logistic model curves, it should be noticed that the first part (slow increase) is relatively short and as a consequence the second part (rapid value increase) takes place early. What ensues is that monitoring energy allows for a relatively early detection of the changes taking place, which is an undisputed advantage. This causes a significant utility of a proposed parameter in the systems with the required high level of safety. The possibility of description of changes in vibration energy with a function relationship offers a possibility of prediction. On the other hand, long period of saturation of the parameter value and of the function relationship describing it, causes much smaller usefulness in the case of application in managing the system operation, where the acceptable level of safety is lower and the maintenance and repair activities undertaken too early are avoided so as to reduce the operation costs. In such a situation, in order to avoid prediction errors, other diagnostic methods should be referred to.

4. Conclusions

The article presents a suggestion of using the energy of vibrations in a toothed gear as a diagnostic parameter correlated with the fatigue crack propagation at the base of a tooth in the toothed gear wheel.

An attempt was made to describe changes in vibration energy as a function of time using the logistic curve. The conducted analyses indicated the need of multi-phase modelling of the occurring changes, which results from the physics of the fatigue breaking phenomenon. The problem of decision-taking regarding the change within the model, i.e. moving from Phase I to Phase II remains unresolved. However, the obtained results are promising and are of great utility especially in the operation of systems with demanding requirements defined by the acceptable level of safety.

5. References

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