Limits on Black Hole Formation from Cosmic String Loops

Jane H. MacGibbon\textsuperscript{1*}, Robert H. Brandenberger\textsuperscript{2†} and U. F. Wichoski\textsuperscript{2‡}

\textsuperscript{1}Code SN3, NASA Johnson Space Center, Houston, TX 77058, USA
\textsuperscript{2}Physics Department, Brown University, Providence, RI 02912, USA.

In theories with cosmic strings, a small fraction of string loops may collapse to form black holes. In this Letter, various constraints on such models involving black holes are considered. Hawking radiation from black holes, gamma and cosmic ray flux limits and constraints from the possible formation of stable black hole remnants are reanalyzed. The constraints which emerge from these considerations are remarkably close to those derived from the normalization of the cosmic string model to the cosmic microwave background anisotropies.

\textsuperscript{*}email: macgibbon@snmail.jsc.nasa.gov
\textsuperscript{†}email: rhb@het.brown.edu
\textsuperscript{‡}email: wichoski@het.brown.edu
I. INTRODUCTION

Cosmic strings are linear topological defects which are predicted to form during phase transitions in the very early Universe in many particle physics models of matter (for recent reviews see e.g. Refs. [1–3]). If the scale of symmetry breaking $\eta$ in the theory is of the order $10^{16}\text{GeV}$ and therefore the mass per unit length $\mu$ of the string satisfies $G\mu/c^2 \sim 10^{-6}$, then the strings provide a possible mechanism of producing the structure observed today on cosmological scales.

The cosmic string theory of structure formation leads to many specific predictions for observations. However, many of these predictions are almost identical to those of inflation-based models. This is one reason for studying other astrophysical constraints on particle physics models with strings, in addition to the constraints derived from structure formation arguments (we include constraints from cosmic microwave background (CMB) anisotropy measurements in the latter category). The second and maybe even more important reason for studying other astrophysical constraints is that strings created in phase transitions at symmetry breaking scales much lower than $10^{16}\text{GeV}$ will contribute negligibly to structure formation and the CMB anisotropies and therefore other methods are needed to constrain such models. When studying astrophysical constraints, it is thus important to keep the scale of symmetry breaking as a free parameter.

In this Letter, we shall reconsider the astrophysical constraints which arise from the probability that a small subset of string loops collapses to form black holes. We will work within the “standard” cosmic string model [4,5], according to which the network of linear defects formed during a symmetry breaking phase transition in the very early Universe quickly reaches a “scaling” solution characterized by having the statistical properties of the string distribution independent of time if all lengths are scaled to the Hubble radius.

Strings are either infinite or closed loops. For practical purposes one counts as “long” strings all infinite strings or string loops with curvature radius larger than the Hubble radius, and as “small” loops all remaining loops. Small string loops are continuously produced during the expansion of the Universe by the interaction of long strings. Recent numerical simulations [6–8] show that there is a substantial amount of small-scale structure on the long strings. This small-scale structure leads to the production of loops with formation radii substantially smaller than the Hubble radius $t$. (In contrast, most early work on cosmic strings and structure formation assumed that loops were created with radii comparable to the Hubble radius). We take the length of a string loop at the time of formation $t'$ to be $l = \alpha ct'$ and hence the string loop mass to be
\[ m(t') = \alpha c \mu t', \]  

where \( \alpha \) is a constant which from recent numerical work \[6-8\] is expected to be much smaller than 1. Once formed, string loops slowly decay by emitting gravitational radiation \[9\]. Hence, the string scenario predicts a large stochastic background of gravity waves.

As was shown in the initial work on the string scenario of structure formation (see e.g. Refs. \[10-12\]), a value of \( G\mu/c^2 \sim 10^{-6} \) leads to a reasonable amplitude for gravitational clustering (as expressed, for example, by the galaxy correlation function). The exact amplitude depends on \( \alpha \), on the average number \( \nu \) of long strings crossing each Hubble volume, and on the amount of small-scale structure on the strings (see e.g. \[13\] and references quoted therein). Since cosmic strings also lead to CMB anisotropies, the string model can be normalized by the recent COBE observations \[14\]. This normalization is more certain because the calculations involve only linear gravitational perturbations. Initial computations of CMB anisotropies gave the constraints \[15\] \( G\mu/c^2 \leq 1.5(\pm 0.5) \times 10^{-6} \) and \[16\] \( G\mu/c^2 \leq 1.7(\pm 0.7) \times 10^{-6} \). A more detailed recent calculation \[17\] which includes not only the scalar but also the vector and tensor modes leads to a very similar result \( G\mu/c^2 \leq 1.7 \times 10^{-6} \). Note that in these constraints, the equality sign holds if cosmic strings are responsible for the CMB anisotropies.

The most stringent current limits on \( G\mu \), independent of and somewhat weaker than structure formation considerations, are the gravitational wave constraints based on millisecond pulsar timing. The pulsar limits, however, are dominated by their sensitivity to the value of \( \alpha \). For the values of \( \alpha \) indicated by the work of Refs. \[6-8\] the pulsar limits are consistent with the value of \( G\mu/c^2 \) needed if strings are to provide the seeds for structure (see e.g. Refs. \[19,20\] for recent work on this issue).

As previously stated, in this Letter, we will address the astrophysical constraints on black hole formation from strings. These bounds can work two ways. For a fixed value of \( G\mu \) we can constrain the fraction \( f \) of string loops which forms black holes. Alternatively, given \( f \), we can derive constraints on \( G\mu \). The physical origins of the bounds we analyze are threefold. Firstly, we demand that the \( \gamma \)-ray flux from black holes formed by collapsing loops does not exceed the observed \( \gamma \)-ray background. Secondly, we use the observed cosmic ray fluxes (in particular the antiproton flux) to limit the contribution of black holes to \( \Omega \). Finally, exploring the possibility that black holes do not completely

\[ ^1 \text{No error bars are given in [17]. However, the errors are presumably comparable to those of a calculation [18] using the same cosmic string simulation but only scalar modes, namely } \pm 0.35. \]
vanish after their evaporation lifetime but remain as stable Planck scale massive relics we can derive additional limits on the string and relic scenarios by demanding that black hole relics do not overclose the Universe.

Cosmic string theory constraints based on the possibility that string loops will collapse to black holes were first studied by Hawking \cite{21} and by Polnarev and Zembowicz \cite{22}. They pointed out that string loops which collapse under their own tension to a radius smaller than their Schwarzschild radius will form black holes. For example, a planar circular string loop after a quarter period will collapse to a point and hence form a black hole. On the other hand, a typical string loops emerging either immediately after a phase transition in the early Universe or subsequently at a much later time due to the intersection of long strings will mostly be asymmetric, and hence have a tiny, but still significant, probability of collapsing to a radius small enough to form a black hole. We will denote by $f$ the fraction of loops produced during the scaling epoch which collapse to form black holes. Initial estimates \cite{21–23} of the constant $f$ differed widely. The most recent estimate of $f$ is due to Caldwell and Casper \cite{24} who found, by numerically simulating loop fragmentation and evolution, that the fraction of loops which collapse to form a black hole within the first oscillation period of the loop is

$$f = 10^{1.9\pm0.2}(G\mu/c^2)^{4.1\pm0.1},$$

for $G\mu/c^2$ in the range $10^{-3} \lesssim G\mu/c^2 \lesssim 3 \times 10^{-2}$. Caldwell and Casper argue that this parameterization can be extrapolated down to $G\mu/c^2 \simeq 10^{-6}$.

While the amplitude of the mass spectrum of black holes formed by string loops collapse is unknown in the cosmic string model, the spectral shape is determined by the scaling argument for strings. The number density $dn_l/dt$ of string loops formed per unit time can be determined from the conservation of string energy

$$\dot{\rho}_\infty - 2H\rho_\infty = -\frac{dn_l}{dt} \alpha \mu t,$$

where $\rho_\infty \sim \nu \mu t^{-2}$ is the energy density in the long string network. Since a fixed fraction $f$ of these loops forms black holes, and the collapse has the greatest probability of occurring on the first oscillation of the loop \cite{21}, we obtain

$$\frac{dn_{BH}^p(t)}{dt} \propto t^{-4}$$

for the number density (in physical coordinates) of black holes forming per time interval. The initial mass $M$ of the black hole formed at time $t$ is given by \cite{30}.
If we neglect the mass loss of black holes with initial mass greater than the formation mass of a black hole just expiring today $M_*$ (we later justify this to be a good approximation), the present number density $dn_{BH}/dM$ of black holes with mass $M$ can be obtained by redshifting the distribution (3) from the time of formation $t(M) > t(M_*)$ to the present time $t_0$. For black holes formed during the radiation-dominated epoch, we thus have

\[
\frac{dn_{BH}(M)}{dM} \propto M^{-2.5}, \quad M_* \lesssim M. \tag{5}
\]

The constraints on black hole formation from cosmic strings set by the observed $\gamma$-ray background at $100\text{MeV}$ have been reanalyzed most recently by Caldwell and Gates [25] and by Caldwell and Casper [24]. In this Letter, we review and correct the analytic arguments in Ref. [25], and so derive a stronger limit on $G\mu/c^2$. We also make use of the newly published EGRET data [26] on the observed $30\text{MeV} - 120\text{GeV}$ extragalactic $\gamma$-ray flux. Further constraints on black hole formation from cosmic strings are then considered. We first reanalyze the limits from the observed $\gamma$-ray background. Second, we determine the limits from the cosmic ray fluxes. Finally, we explore the bounds which can be derived under the postulation that black hole relics form.

**II. GAMMA-RAY FLUX CONSTRAINTS**

It is well known [27–29] that the extragalactic $\gamma$-ray flux observed at $100\text{MeV}$ provides a strong constraint on the population of black holes evaporating today. Too many black holes would lead to an excess of such radiation above the observed value.

In particular, it was shown that if the present day number density distribution of black holes of mass $M$ has the form [27]

\[
\frac{dn}{dM} = (\beta - 2) \left( \frac{M}{M_*} \right)^{-\beta} M_*^{-2} \Omega_{PBH} \rho_{crit}, \quad M_* \lesssim M, \tag{6}
\]

where $\beta = 2.5$ for black holes formed in the radiation-dominated era, $M_*$ is the mass of a black hole whose lifetime is the present age of the Universe, $t_o$, and $\Omega_{PBH}$ is the present fraction of the critical density in primordial black holes, then comparing the Hawking emission from the black hole distribution with the $\gamma$-ray background observed by the SAS-2 satellite [28] requires that [23]

\[
\Omega_{PBH} \lesssim 8 \times 10^{-9} h^{-2}. \tag{7}
\]
where \( h \approx 0.5 - 1.0 \) is the Hubble parameter in units of 100 km s\(^{-1}\)Mpc\(^{-1}\). (Note that this limit was incorrectly interpreted in Eq. (4.1) of [27].) The newly published 30MeV–120GeV extragalactic γ-ray background measured [26] by the EGRET experiment implies an updated limit on the present black hole density of [82]

\[
\Omega_{PBH} \lesssim (5.1 \pm 1.3) \times 10^{-9} h^{-1.95 \pm 0.15}.
\] (8)

The γ-ray flux per unit energy from the black hole distribution [31] turns over from an \( E^{-1.3} \) slope below about 10MeV to a steeper slope around \( E \approx 100 \)MeV, the peak energy of the instantaneous emission from a black hole with mass \( M_* \). The emission from the black hole distribution falls off as \( E^{-3} \) above about 1GeV. The origin of the observed keV - GeV extragalactic γ-ray background is unknown. Since the observed γ-ray background falls off as \( E^{-2.10 \pm 0.03} \) between 30MeV and 120GeV, this raises the possibility that black hole emission may [32] explain, or contribute significantly, to the observed extragalactic background between about 50 – 200MeV.

The distribution of black holes given by (6) is precisely the distribution (5) predicted by the cosmic string model. (Such a distribution will also be produced if the black holes form in the early Universe from scale-invariant adiabatic density perturbations [27,31].)

The fraction of the critical density today in black holes created from collapsing cosmic string loops is [25]

\[
\Omega_{PBH}(t_o) = \frac{1}{\rho_{crit}(t_o)} \int_{t_i}^{t_o} dt' \frac{dn_{BH}}{dt'} m(t', t_o),
\] (9)

where the integral is over the time \( t' \) when the black holes formed, \( \frac{dn_{BH}}{dt'} \) is the comoving number density of black holes created from loops at time \( t' \) and \( m(t', t_o) \) is the mass at the present time \( t_o \) of a black hole formed at time \( t' \).

In (3), \( t_i \) is the formation time of a black hole whose initial Schwarzschild radius is equal to the string thickness (the minimum initial radius possible from loop collapse),

\[
t_i = \alpha^{-1} \left[ \frac{G\mu}{c^2} \right]^{\frac{2}{3}} t_{pl}, \quad t_{pl} = \left[ \frac{Gh}{c^3} \right]^{\frac{2}{3}}
\] (10)

and \( t_* \) is the formation time of a black hole with initial mass \( M_* = \alpha \mu c t_* \), i.e.

\[
t_* = \alpha^{-1} \mu^{-1} c^{-1} M_*
= Gc^{-3} \alpha^{-1} \left[ \frac{G\mu}{c^2} \right]^{-1} M_*
\] (11)

\[\gg t_i\]

For an \( \Omega = 1 \) Universe,
\[ M_* \approx 4.4 \times 10^{14} h^{-0.3} \text{ gm}, \] (12)

independent of the formation scenario (see \[33\]). Because black holes with initial masses less than or equal to \( M_* \) will have evaporated by today, the lower limit of the integral in (9) is \( \max(t_i, t_*) = t_* \).

We can approximate \( m(t', t_0) \) by the initial mass of the black hole as given in (1):

\[ m(t', t_0) \approx \alpha \mu c t'. \] (13)

Since black holes with initial masses greater than \( M_* \) will have evaporated little by today, the approximation (13) adds an uncertainty of less than 6\% to the value of \( \Omega_{PBH} \) in (9). This can be shown by taking the mass loss rate of an individual black hole \[33, 34\]

\[ \frac{dM}{dt} \approx 5.34 \times 10^{25} \phi(M) M^{-2} \text{ gm sec}^{-1}, \] (14)
solving for \( m(t', t_0) \), and comparing a numerical evaluation of the resulting integral (9) with that obtained from the analytical approximation using (13). In (14), \( \phi(M) \) is a slowly increasing function which depends on the number of particle species emitted by the black hole. \( \phi(M) \) is normalized to unity when only massless particles are emitted, and \( \phi(M_*) \approx 2 \). Note that although the mass approximation to be used in (9) is also implicit in (8), the limit on \( \Omega_{PBH}(t_0) \) in (9) was derived using the exact evolved present day spectrum of black holes masses.

If \( f \) is the fraction of cosmic string loops which collapse to form black holes in the first period of oscillation, then the number density of black holes formed from loops at time \( t \) is \( d\bar{n}_{BH}/dt = f d\bar{n}_l/dt \). The assumption that the black hole forms on the first loop oscillation or not all has been shown in \[21\] to be a good approximation. From (3) and (4) it then follows that the number of black holes created in a volume \( V(t) \) at time \( t \) is

\[ \frac{d\bar{n}_{BH}}{dt} = 4f \frac{A}{\alpha} c^{-3} t^{-4} \frac{a(t)^3}{a(t_0)^3}, \] (15)

where \( A \) is proportional to the number \( \nu \) of long strings per Hubble volume in the scaling solution, adopting the notation of Ref. \[25\], and is found from numerical simulations \[6–8\]. Because the cosmological scale-factor \( a(t) \) is proportional to \( t^{\frac{1}{2}} \) in the radiation-dominated era, and proportional to \( t^{\frac{1}{3}} \) in the matter-dominated era, the integral in (9) is dominated by the black holes created about the time \( t_* \) in the radiation-dominated era. Noting also that \( t_* \ll t_{eq} \) for \( G\mu/c^2 \gtrsim 10^{-18} \), we have from (9)
\[ \Omega_{PBH}(t_o) = \frac{1}{\rho_{crit}(t_o)} \int_{t_o}^{t_e} dt' 4f A c^{-3} t'^{-\frac{3}{2}} \mu a^3(t') \]
\[ \approx \frac{4f A \mu c^{-2}}{\rho_{crit}(t_o)} \left[ \frac{t_{eq}}{t_o} \right]^2 \left[ \frac{1}{t_{eq}} \right]^\frac{3}{2} \int_{t_e}^{t_{eq}} dt' t'^{-\frac{3}{2}} \]  
\[ \approx \frac{8f A}{G \rho_{crit}(t_o)} \left[ \frac{G \mu}{c^2} \right] \left[ \frac{t_{eq}}{t_o} \right]^2 \left[ \frac{1}{t_{eq}} \right]^\frac{3}{2} t_{eq}'^{-\frac{3}{2}} \]

(16)

where \[ t_{eq} \approx 3.2 \times 10^{10} h^{-4} \text{sec}. \]

Substituting for the critical density of the Universe today \( \rho_{crit}(t_o) \) and the age of the Universe \( t_o \)

\[ \rho_{crit}(t_o) = \frac{3H_0^2}{8\pi G} \]
\[ t_o = \frac{2H_o^{-1}}{3}, \]

for an \( \Omega = 1 \) Universe, (16) becomes

\[ \Omega_{PBH}(t_o) \approx 48\pi f A \left[ \frac{G \mu}{c^2} \right] \left[ \frac{t_{eq}}{t_o} \right]^2 \]
\[ \approx 5.7 \times 10^{11} \left[ \frac{A}{10} \right] f \alpha^{-\frac{1}{2}} \left[ \frac{M_*}{4.4 \times 10^{14} h^{-0.3} \text{ gm}} \right]^{-\frac{1}{2}} \left[ \frac{G \mu/c^2}{1.7 \times 10^{-6}} \right]^{\frac{1}{2}} h^{-1.85}. \]

Since this must be less than or equal to the observational constraint on \( \Omega_{PBH} \) given by (8) for an \( \Omega = 1 \) Universe, then

\[ f \lesssim 8.9(\pm 2.3) \times 10^{-21} \left[ \frac{A}{10} \right]^{-1} \alpha^{-\frac{1}{2}} \left[ \frac{M_*}{4.4 \times 10^{14} h^{-0.3} \text{ gm}} \right]^{\frac{1}{2}} \]
\[ \times \left[ \frac{G \mu/c^2}{1.7 \times 10^{-6}} \right]^{-\frac{1}{2}} h^{-0.1 \pm 0.15} \left[ \frac{t_{eq}}{3.2 \times 10^{10} h^{-4} \text{ sec}} \right]^{-1/2}. \]

(19)

Because string loops are predominantly formed from the small-scale structure on long strings, the initial loop radius and hence the value of \( \alpha \) are determined by the physics which sets the scale for the small-scale structure. Ref. 36 provides evidence that this scale may be determined by the strength of the gravitational radiation from string loops. If this is so, then we would have the relation \( \alpha = \gamma G \mu/c^2 \), where \( \gamma \) is a dimensionless coefficient describing the strength of gravitational radiation generated by string loops 31, and (14) becomes

\[ f \lesssim 6.8(\pm 1.7) \times 10^{-19} \left[ \frac{A}{10} \right]^{-1} \left[ \frac{\gamma}{100} \right]^{\frac{1}{2}} \left[ \frac{M_*}{4.4 \times 10^{14} h^{-0.3} \text{ gm}} \right]^{\frac{1}{2}} \]
\[ \times \left[ \frac{G \mu/c^2}{1.7 \times 10^{-6}} \right]^{-2} h^{-0.1 \pm 0.15} \left[ \frac{t_{eq}}{3.2 \times 10^{10} h^{-4} \text{ sec}} \right]^{-1/2}. \]

(20)
This is significantly stronger than the Caldwell and Gates [25] limit (4.1) of \( f \lesssim 10^{-17} \). The main reason for the difference is that in Ref. [25] the limit of Ref. [33] was applied incorrectly. In order to also compare our new limit to that of Ref. [24], we rewrite (19) in the same form as Eq. (5.1) of [24]:

\[
f \lesssim 2.0(\pm 0.5) \times 10^{-30} \left[ \frac{A}{10} \right]^{-1} \left[ \frac{G\mu}{c^2} \right]^{-2} \left[ \frac{\gamma G\mu/c^2}{\alpha} \right]^{\frac{1}{2}} \left[ \frac{\gamma}{100} \right]^{-\frac{1}{2}} \times \left[ \frac{M_\ast}{4.4 \times 10^{14} h^{-0.3} \text{ gm}} \right]^{\frac{1}{2}} h^{-0.1 \pm 0.15} \left[ \frac{t_{eq}}{3.2 \times 10^{10} h^{-4} \text{ sec}} \right]^{-1/2}.
\]

(21)

Thus, our bounds on \( f \) are an order of magnitude stronger than those of Caldwell and Casper.

So far, we have viewed the constraint on \( f \) as an upper bound on the admissible value of \( f \) given any value of \( G\mu \). Conversely, if we assume the validity of expression (3) for \( f \) which was derived from the numerical simulation of cosmic string evolution, we can deduce an upper bound on the value of \( G\mu \). Combining (21) and (3) yields

\[
\left( \frac{G\mu}{c^2} \right)^{6.1 \pm 0.1} < 10^{-34.6 \pm 0.3} \left( \frac{A}{10} \right)^{-1} \left( \frac{G\mu}{\alpha c^2} \right)^{1/2} \left( \frac{\gamma}{100} \right)^{-1/2} \left( \frac{M_\ast}{4.4 \times 10^{14} h^{-0.3} \text{ gm}} \right)^{1/2} \left( \frac{\Omega_{P BH}}{5.1 \pm 1.3 \times 10^{-9} h^{-1.95 \pm 0.15}} \right) \left( \frac{t_{eq}}{3.2 \times 10^{10} h^{-4} \text{ sec}} \right)^{-1/2}.
\]

(22)

For \( \Omega_{P BH} \) as given by the observed \( \gamma \)-ray background, this requires

\[
G\mu/c^2 \leq 2.1(\pm 0.7) \times 10^{-6},
\]

(23)

which is in remarkable agreement with the bounds [15, 33] on \( G\mu/c^2 \) obtained by normalizing the cosmic string model according to the CMB anisotropy data and quoted in the Introduction. We also note that our results are considerably improved compared to the original estimates in Refs. [21, 23], because we have been able to make use of the numerical simulations of Ref. [24] to better determine \( f \) as a function of \( G\mu/c^2 \) and make use of the updated observational limits on \( \Omega_{P BH} \).

III. CONSTRAINTS FROM THE COSMIC RAY FLUXES

The limit on \( \Omega_{P BH} \) stated in (1) was found by comparing the \( \gamma \)-ray emission from a cosmological distribution of black holes with the observed diffuse extragalactic \( \gamma \)-ray background around 100 MeV. Significant limits on \( \Omega_{P BH} \) can also be derived by considering the antiproton, electron and positron emission from a distribution of black holes [31, 37]. However, these limits are more uncertain because, unlike the \( \gamma \)-ray limit, the \( \bar{p}, e^- \) and \( e^+ \) limits depend on the degree to which black holes cluster in the Galactic halo and on the propagation of charged particles in the Galaxy.
Since the black hole distribution is dominated by holes created before the era of galaxy formation, one would expect the black holes to cluster in the Galactic halo along with other Cold Dark Matter. This leads to an enhancement in the predicted black hole contribution to the local cosmic ray flux. Assuming a halo model in which the spatial distribution of black holes is proportional to the isothermal density distribution of dark matter within the Galactic halo and simulating the diffusive propagation of antiprotons in the Galaxy, Maki et al. \cite{37} derive an upper limit on $\Omega_{PBH}$ of

$$\Omega_{PBH} < 1.8 \times 10^{-9} h^{-4/3}$$  \hspace{1cm} (24)

based on antiproton data from the BESS ‘93 balloon flight \cite{38}. This value would imply a limit on $f$ in (19), (20) and (21) that is stronger by a factor of about 3 and a corresponding limit on $G\mu/c^2$ in (23) of

$$G\mu/c^2 < 1.8(\pm0.5) \times 10^{-6},$$ \hspace{1cm} (25)

(23) contains a negligible dependence on $h$. We note that the antiproton limit on $\Omega_{PBH}$ \cite{24} was calculated from the BESS ‘93 data by employing the same solar demodulation parameter which is applicable for demodulating the proton flux at 1 AU. It is presently a matter of debate as to whether there is a charge asymmetry in the solar modulation of the proton and antiproton fluxes.

Future long-duration balloon flights \cite{37} or observations at solar minimum \cite{39} may allow an order of magnitude greater sensitivity to $\Omega_{PBH}$ than stated in (24) or, alternatively, offer the possibility of detection of black hole emission. If no black hole antiprotons are detected by the proposed experiments with an order of magnitude greater sensitivity to $\Omega_{PBH}$, the constraint in (25) would then be

$$G\mu/c^2 < 1.2(\pm0.4) \times 10^{-6},$$ \hspace{1cm} (26)

which would only be in marginal agreement with the requirements on $G\mu$ for the cosmic string model of structure formation. Note, however, that the method of constraining $G\mu$ using charged particle cosmic ray data is intrinsically less certain that the $\gamma$-ray constraints because of the uncertainties in the clustering of black holes in the Galaxy, and the propagation and demodulation of charged particles in the Galaxy and Solar System.

The limits on $\Omega_{PBH}$ derived \cite{31} by matching the emission from a distribution of black holes clustered in the Galaxy to the demodulated interstellar electron and positron fluxes at 300\textit{MeV} are also comparable with, and overlap, the $\gamma$-ray limit \cite{8}. The origins of the observed antiproton, electron and positron spectra between 1\textit{MeV} and 1\textit{GeV} have yet to be fully understood. This raises the possibility that black hole emission may consistently be contributing significantly to the extragalactic $\gamma$-ray background and the Galactic antiproton, electron and positron backgrounds around 100\textit{MeV}.

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We can derive further limits on the parameters for black hole production from cosmic string loops by considering the possibility that black holes evolve into stable massive relics after completing their lifetime of emission \([40]\). The final state of an expiring black hole is unknown. The Hawking derivation \([41, 42]\) for black hole emission and all semiclassical analyses break down when the black hole mass approaches \(m_{pl}\). At this scale, the size of the black hole is comparable to the Compton wavelength associated with its mass, and the timescale of the mass loss is comparable to the frequency of the quantum radiation. Semiclassical back-reaction studies as well as perturbative quantum gravitational effects indicate that in this regime higher derivative terms become important in the gravitational action. Such terms may \([43]\) stabilize black holes against collapse at \(m_{pl}\) or well before the mass reaches \(m_{pl}\). (See also \([44, 45]\) for earlier work, \([46]\) for a recent review, and \([47]\) for a study of nonsingular black holes in the context of string theory). In an alternative scenario, the black hole terminates in an explosion which leaves behind a space-time singularity which then vanishes \([48]\) or continues to exist as a massless naked singularity \([49]\).

Let us consider the consequences if the black holes created from string loops remain as relics with residual mass \(M_{\text{relic}} \geq m_{pl}\) at the end of their evaporation. The present fraction of the critical density in black hole relics is then

\[
\Omega_{\text{relic}} = \frac{M_{\text{relic}}}{m_{pl}} \frac{m_{pl}}{\rho_c(t_0)} \int_{t_{\text{evap}(t_i)}}^{t_0} dt_{\text{evap}} \frac{dn_{\text{evap}}}{dt_{\text{evap}}},
\]

where \(t_{\text{evap}}\) is the time when an individual black hole ceases evaporating (i.e. reaches mass \(M_{\text{relic}}\)), and \(dn_{\text{evap}}/dt_{\text{evap}}\) is the comoving number density of black holes expiring per unit time at time \(t_{\text{evap}}\). \(t_i\) is defined in \([10]\). By conservation of number density, the comoving number density of black holes expiring at \(t_{\text{evap}}(t)\) is equal to the corresponding comoving number density of black holes formed from string loop collapse at time \(t\). Hence, \((27)\) becomes

\[
\Omega_{\text{relic}} = \frac{M_{\text{relic}}}{m_{pl}} \frac{m_{pl}}{\rho_c(t_0)} \int_{t_i}^{t'(t_0)} dt \frac{dn_{BH}}{dt},
\]

where \(t'(t)\) is the time when a black hole expiring at time \(t\) formed. Since \(M_* \gg M_{\text{relic}}\), we have to good approximation \(t'(t_0) \simeq t_* \ll t_{eq}\), and the time interval in the integration lies entirely in the radiation dominated epoch. Making use of \((10)\), \((11)\) and \((15)\), it follows that

\[
\Omega_{\text{relic}} = \frac{4 f A}{\alpha c^4 m_{pl} \rho_c(t_0)} \frac{M_{\text{relic}}}{m_{pl}} \int_{t_i}^{t'(t_0)} dt' t'^{-5/2} \approx 16 \pi f A^{1/2} \frac{t_{pl}^{1/2}}{t_{pl}^{1/2}} \frac{M_{\text{relic}}}{m_{pl}} \left( \frac{G t_{pl}^{3/2}}{c^2} \right)^{9/4}.
\]

10
The limit $\Omega_{\text{relic}} \leq 1$ then requires

$$f \leq 2.5 \times 10^{-17} \left( \frac{A}{10} \right)^{-1} \alpha^{-1/2} \left( \frac{m_{\text{pl}}}{M_{\text{relic}}} \right) \left( \frac{G\mu/c^2}{1.7 \times 10^{-6}} \right)^{-9/4} h^2 \left[ \frac{t_{eq}}{3.2 \times 10^{10} h^{-4} \text{ sec}} \right]^{-1/2},$$

(30)

or, if $\alpha = \gamma G\mu/c^2$,

$$f \leq 1.9 \times 10^{-15} \left( \frac{A}{10} \right)^{-1} \left( \frac{\gamma}{100} \right)^{-1/2} \left( \frac{m_{\text{pl}}}{M_{\text{relic}}} \right) \left( \frac{G\mu/c^2}{1.7 \times 10^{-6}} \right)^{-11/4} h^2 \left[ \frac{t_{eq}}{3.2 \times 10^{10} h^{-4} \text{ sec}} \right]^{-1/2}. $$

(31)

This bound (31) is substantially weaker than the bound on $f$ derived from the emission constraints (see (20)). However, in the black hole relic scenario, the previous emission constraints on $f$ must also still hold. We can thus combine these two analyses and find the maximum fraction $\Omega_{\text{relic}}$ of the critical density which is permitted in black hole relics under the condition that the cosmic string model does not violate the $\gamma$-ray bounds. Inserting the constraint on $f$ from (20) in (29), we obtain

$$\Omega_{\text{relic}} \leq 3.6 (\pm 0.9) 10^{-4} \left( \frac{M_{\text{relic}}}{m_{\text{pl}}} \right) \left( \frac{M_*}{4.4 \times 10^{14} h^{-0.3} \text{ gm}} \right)^{1/2} \left( \frac{G\mu/c^2}{1.7 \times 10^{-6}} \right)^{3/4} h^{-2.1 \pm 0.15}. $$

(32)

Thus, the bound on the black hole formation efficiency factor $f$ given by (20) implies that black hole remnants from collapsing loops can only contribute significantly to the dark matter of the Universe in the cosmic string scenario of structure formation ($G\mu/c^2 \simeq 1.7 \times 10^{-6}$) if the black hole remnants have a relic mass larger than about $10^3 m_{\text{pl}}$. Such relic masses naturally arise for example in the $SU(N)/Z_N$ theories of Ref. [50] in which quantum hair leads to stability. If the black hole formation process is less efficient than the equality in (20), the remnant mass required for an interesting contribution to the cosmological critical density is linearly increased.

**V. CONCLUSIONS**

We have reconsidered the astrophysical constraints on black hole formation from collapsing cosmic string loops, correcting a misinterpretation in [2], and compared the results with the most recent normalizations of the string model for structure formation to the CMB anisotropy data. We investigated limits arising from the latest available $\gamma$-ray and cosmic ray flux observations, and constraints stemming from the possible formation of black hole remnants.

The $\gamma$-ray emission constraint on the black hole formation efficiency implies that, for the cosmic string parameters which follow from the most recent cosmic string COBE normalizations [12] and from numerical simulations of cosmic string evolution [5], the maximal fraction of string loops which can collapse to form black holes is about $f \simeq 2 \times 10^{-18}$. The $G\mu$ dependence of the formation efficiency is not known, but taking the best available analysis [5], we obtain a
bound on $G_\mu$ which is remarkably close to the value required in the cosmic string model of structure formation. The bounds derived from the cosmic ray antiproton, electron and positron fluxes are of similar magnitude, but presently afflicted with larger uncertainties.

Our results raise the scenario that if cosmic string are responsible for the CMB anisotropies on large angular scales, then the Hawking emission from black holes created by string loop collapse is contributing significantly to the observed diffuse extragalactic $\gamma$-ray background and to the observed antiproton, electron and positron cosmic ray fluxes around 100$\text{MeV}$. Conversely, future tighter observational limits on the cosmic ray backgrounds, in particular on the antiproton flux below 1$\text{GeV}$, will imply constraints on $G_\mu/c^2$ stronger than those presently given by the anisotropies detected by the COBE satellite.

We have also investigated the constraints which can be derived if black holes do not evaporate completely, but instead evolve into stable massive remnants at the end of their life. We have found that unless the mass of the black hole remnants is larger than $10^3m_{pl}$, these remnants will contribute negligibly to the dark matter of the Universe, even if the black hole formation rate has the maximal value allowed by the $\gamma$-ray flux constraints. A remnant mass of $10^3m_{pl}$, however, can arise naturally in some models of black hole evaporation. In this case, cosmic strings could consistently provide an explanation for the origin of cosmological structure, for the dark matter, and for the origin of the extragalactic $\gamma$-ray and Galactic cosmic ray backgrounds around 100$\text{MeV}$.

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