Simulation Driven Approach For Deriving The PID Parameters For The Control Of An Electric Resistance Furnace (ERF)

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Abstract
Modelling plays a major role in predicting the behaviour of any system. It is also one of the crucial parts in the design of the control system and tuning of the controller. Tuning parameters are developed based on the transfer function obtained from the modelling process. The primary goal of this paper is to model and simulate a research grade Electric Resistance Furnace (ERF) under transient conditions and derive the transfer function using simulation data. Using this transfer function the gain parameters (PID, Proportional, Integral and Derivative) of the controller will be derived using conventional tuning approaches. The secondary goal is to provide an optimized model for future development of a Neural Network based controller for the furnace. The controller will be equipped with an auto tuning algorithm based on Model Predictive Control (MPC) which will have a Recurrent Neural Network (RNN) as a system model. The training data set for the RNN will be obtained by using the PID parameters derived in this paper. This would enable precise control of the furnace with lower rise and settling time, reduced or no overshoot, maximum stability and ramp hold ability.

Keywords: Transfer function, Transient thermal analysis, Control system, Zeigler-Nicholas method, Cohen-Coon method, PID tuning

1. Introduction
System modelling involves analysing every aspect of a particular system and developing a model that predicts its behaviour. The element of compromise is quite large in mathematical modelling because of the complexity of the real world systems involved [1]. Various assumptions are to be made to simplify the system. This paper deals with modelling and simulation of an Electric Resistance Furnace (ERF) under transient conditions and obtaining transfer function that will be used in the future for design of a PID controller for the furnace. A precise model is the key element in obtaining effective control over any system [2]. Transient problems can be tricky because of the unsteadiness involved. The temperature varies with respect to both time and spatial coordinates.

1.1 Problem definition
The Electric Resistance Furnace (ERF) used here consists of 4 heating elements each made of high grade silicon carbide and body made of Ceramic. The primary mode of heat transfer will be radiation. A transient analysis will be performed to find out the temperature distribution at various locations in the furnace. Material properties of different components of furnace as follows. From the transient simulation results a transfer function will be derived using system identification which will be used in derivation of the PID parameters.

1.2 PID Tuning
PID controllers are either tuned by the following methods: Analytical methods, Heuristics methods, frequency response methods, optimization methods or by adaptive tuning methods [5]. In this paper we will be using frequency response method and derive the parameters based on step response test by implementing Zeigler Nicholas method and Cohen Coon method and discuss various possibilities of
optimization. Most applications involving thermal systems make use of PID controllers for temperature control. But conventional controllers cannot tackle discontinuities in the system [4]. In [6] authors have used lag and unit reaction rates of the system as quantitative measures and this concept serves as a basis for developing the PID parameters based on the Zeigler-Nicholas method. The PID parameters are obtained by performing an open loop step response test on the plant, from which process gain and delay are obtained. For this purpose a mathematical model of the system is required which may be transfer function model, state space model or differential equations defining the system. In this paper, a transfer function model is used for analysing of the system behaviour and to obtain the tuning parameters. Future work of this paper involves precise modelling of the system using artificial neural network and later implementing Model Predictive Control (MPC) for precise temperature control. The PID parameters derived from this paper will serve as base parameters for obtaining input/output data for system identification.

2. Development of geometric model
The geometry was modelled using Ansys DesignModeler which was found suitable for this problem. The model was developed based on a bottom up approach (Figure 1).

Fig.1.1 Isometric view
Fig.1.2 Sectional view

After the geometric modelling process the model was imported to Transient Thermal module of Ansys workbench for Meshing and simulation (Figure 2).

Fig.2.1 External mesh
Fig.2.2 Internal Mesh

The mesh size was carefully chosen to obtain proper temperature distribution even in the thinnest regions of the model.

2.1 Boundary conditions and loading
The thermal loading was specified in terms of internal heat generation at the heating filaments. Calculation of internal heat generation
Equivalent resistance of the filaments, \( R = 20 \ \Omega \) (4 Filaments, 5 \( \Omega \) each connected in series)
\[ Voltage = 220 \, V \]
\[ Power = \frac{V^2}{R} \]
\[ P = 2420 \, W \]

Combined volume of heating elements
\[ Volume = 4 \times \pi \times (0.004)^2 \times 0.290 \, m^3 \]
\[ v = 5.827 \times 10^{-5} \, m^3 \]

Heat generation rate = Power (P)/Volume (v)
\[ HGR = 4.1503 \times 10^7 \frac{W}{m^3} \]

**Boundary conditions:** Surface to Surface radiation (S2S) between filaments, crucible and furnace walls, convection at Filament-Fluid, Fluid-body and Fluid-Crucible interfaces, convection between furnace outer body and outside environment. Setting used in analysis are summarized in table 1.

Table 1: Analysis settings were used for the simulation

| Total number of steps | 200 |
|-----------------------|-----|
| Step end time         | 1 sec |
| Step interval         | 0.5 sec |

2.2 Simulation results

**Fig.3.1** Temperature distribution in filaments
Temperature near the filaments was measured using probe feature to simulate a thermocouple and the plotted against time (Figure 3).

**Fig.3.2** Temperature distribution in fluid domain
2.3 Mathematical Modelling

Governing equations

For unsteady transient problems,
\[
\frac{\partial T}{\partial t} = \alpha \nabla^2 T \tag{1}
\]

Where T - Temperature, \( \alpha \) - Thermal diffusivity \( \alpha = \frac{k}{\rho c_p} \)

Suitable approach

Lumped parameter approach is an effective way for solving transient problems, where the system parameters are function of time. Whole body is assumed to be a single lump at a certain temperature. \( Bi < 0.1 \) is the necessary condition for proceeding with Lumped parameter approach. Bi- Biot number, which compares internal conduction and external convection resistance.

Lumped parameter equations for the heating filament

Input energy is given by
\[
I^2 R t \tag{2}
\]

where \( I \) is the current through the circuit and \( R \) is the resistance

Change in temperature of the filaments is given by
\[
m c_p \frac{dT}{dt} \tag{3}
\]

Heat transfer due to convection is given by
\[
h A (T_t - T_i) \tag{4}
\]

Where \( A \) is the combined surface area of the filaments and \( h \) is the convective heat transfer coefficient

Heat radiated from the surface of the filaments is given by
\[
\frac{\sigma (T_t^4 - T_i^4)}{1-\varepsilon} \tag{5}
\]

Where \( T_i \) is the initial temperature of the filament surface and \( T_t \) is the temperature after time ‘t’ and \( \varepsilon \) is the emissivity of the filament surface.

Applying energy balance we get,
\[
I^2 R t = m c_p \frac{dT}{dt} + h A (T_t - T_i) + \frac{\sigma (T_t^4 - T_i^4)}{1-\varepsilon} \tag{6}
\]

Where \( I \) is the current flowing through the filaments and \( R \) is equivalent resistance of the filaments (Series connection).
The above equation can be used to study the nature of the system by obtaining a transfer function from it. It can be done by taking Laplace Transforms of each term and combining into a single transfer function.

Laplace transform of a function \( f(t) \) is given by

\[
F(s) = \int_0^\infty f(t) e^{-st} dt
\]

(7)

2.4 Transfer function using system identification

Once the simulation results were obtained, from the Time vs. Temp data, a transfer function was extracted using the MATLAB system identification tool.

MATLAB command used: \textit{ident}

Input and outputs were specified in the workspace for later use in the system identification.

From the mathematical model of the thermal system it can be seen that it is a first order system and hence the number of poles and zeros were defined so as to obtain a first order transfer function. After defining the time domain data obtained from Ansys, the model was solved. The transfer function obtained was found to be 99.08% fit to the simulation data. The following transfer function was obtained from MATLAB:

\[
\frac{9.046s + 5.073}{s + 0.04371}
\]

2.5 Estimation of PID parameters

Control systems implemented using Zeigler-Nicholas and Cohen-Coon methods are tuned for quarter-wave damping in which overshoot of any given wave is \( \frac{1}{4} \) of the previous wave. It involves obtaining a compromise between excessive oscillations and settling time [7]. Controller tuning can be done either by closed loop method or process reaction method. In this paper process reaction method is used.

| PID parameters | Controller gain | Integral time | Derivative time |
|----------------|-----------------|---------------|----------------|
| Z-N method     | \( \frac{1.2 \tau}{T_d K} \) | \( 2T_d \)     | 0.5 \( T_d \)   |
| Cohen-Coon method | \( \frac{1 \tau}{K T_d} \left[ \frac{16 \tau + 30}{12 \tau} \right] \) | \( T_d \left[ 32 + 6(T_d / \tau) \right] + 13 + 8(T_d / \tau) \) | \( 4 T_d \) |

| Table 1. [7] |

From the obtained transfer function of the model, the system behaviour is observed by applying step input also known as step response test from which process gain, dead time and time lag values are obtained. Using these values we can implement Z-N and Cohen-Coon methods for deriving the PID parameters.

2.6 Analysing the transfer function

2.6.1 Step response test

A step response test is done to study the behaviour of the system. We can also obtain different system parameters like process gain and dead time of the system which will be used for obtaining the tuning parameters for the controller as discussed above.

Let the obtained transfer function be

\[
T(s) = \frac{9.046s + 5.073}{s + 0.04371}
\]

We formulate the function for step response such that

\[
C(s) = R(s). T(s)
\]

R(s) = 1/s for step response test
Now we apply Inverse Laplace transform to the above system to convert it from frequency domain to
time domain. \[ C(s) \text{ to } C(t) \]
\[
L^{-1}[C(s)] = \frac{1}{s}L^{-1}[T(s)]
\]
Now plotting the step response in time domain we get,

![Step Response](image)

**Fig.4** Step response of the obtained transfer function

From the step response test (Figure 4), we obtain the following process parameters

- Process gain, \( K = 116.03 \)
- Time lag, \( \tau = 22.836 \text{ s} \)
- Dead time, \( T_d = 0.133 \text{ s} \)

Substituting the above obtained values in the Table 1. We can find out the optimum PID parameters using both Zeigler-Nicholas and Cohen-Coon methods.

**Table 2. PID parameter obtained from Z-N and Cohen-Coon methods**

| Method                  | P (Controller gain) | I (Integral time) | D (Derivative time) |
|-------------------------|---------------------|-------------------|---------------------|
| Zeigler-Nicholas method | 1.775               | 0.266             | 0.0665              |
| Cohen-Coon method       | 2.135               | 0.3265            | 0.04831             |

3. Future scope

The development of microcontrollers integrated in the control system allows the user to implement
various tuning algorithms and to collect data points from the process and improve the process
parameters further based on the data points [8]. Machine learning plays a vital role in analysing time
series data. Using this feature we can train and develop a neural network which would be able to
represent the nonlinear dynamic system (The furnace). Thus the trained model would be able to
predict the future outcome of the system based on the past states the system has been through and the
present input to the system. This is the key element in designing the Model Predictive Controller
(MPC) for the furnace. MPC generates control actions by optimizing an objective function repeatedly
over a predefined horizon [9]. The PID parameters obtained from this paper will be used to generate
training data which will be used to train the Recurrent Neural Network (RNN). The training data
obtained from a PID controlled plant is more efficient in training a Neural Network than the data
obtained from open loop training [10].

4. Conclusion

The simulation driven way of obtaining transfer function can be helpful in designing control systems
for the complex process where obtaining a proper mathematical model is difficult. As discussed
above, the transfer function will be used for development of an intelligent PID controller based on
Machine Learning. Discontinuities can be a big challenge in the design of control systems. Machine learning is very effective at analysing discontinuities. This leads to precise control of furnace temperature. Critical manufacturing processes like investment casting, metal injection moulding and other thermal cycling processes like physical vapour deposition/chemical vapour deposition, e-waste processing, which require precise control of temperature are expected to be benefited by ML-integrated PID parameter auto-tuning and control.

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