Optical wavefront synthesis using digital holography

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Abstract. Here we investigate the method for optical wavefront spatial formation, based on the adaptive optimization of the wavefront algorithm (AOWF). We consider the regularities of this method for the basic problems. Among the results, it is worth highlighting the possibility of creating two-dimensional optical structures that can be implemented in holographic imaging technologies.

1. Introduction

The optical beam conversion for formation of the desired spatial distribution is necessary for many applications, for example, for optical tweezers [1] and non-invasive surgical interventions using lasers [2]. Digital holography based approach to solve this problem opens the possibility to create virtual optical devices that are impossible for physical realization due to a lack of technology.

To specify the necessary amplitude distribution in a certain space region, one needs the boundary conditions that describe the wavefront in the input plane. These boundary conditions can be formed by means of holographic optical elements (HOEs) [3], as well as dynamic optical elements based on spatial light modulators (SLMs) [4], etc.

As for today, the most promising types of boundary conditions definition are the digital optical methods. These methods do not require the manufacturing of optical elements, for example, the Gershberg-Saxton iterative algorithm [5]. The development of digital optical methods for wavefront manipulation is an important stage in the development of technologies using optical radiation.

2. Statement of the problem

Numerical simulation of propagation and transformation of optical wavefronts is a known problem for optical physics. Usually, for these purposes, the paraxial approximation and scalar theory of diffraction are used.

In this paper, we investigate the method for optical wavefront formation, based on the algorithm of adaptive wavefront optimization (AOWF). Initially, this algorithm was developed to focus laser radiation through strongly scattering media [6]. We propose the application of this technique to the task of digital optical elements (DOEs). Herewith we will consider the main features of this method for the basic problems of optical physics.
3. Used methods of optical wavefront formation

In this section, we consider the implemented approaches for formation of the optical wavefronts.

3.1. Scalar diffraction theory treatment

The description of wavefront propagation can be realized through a variety of methods of the diffraction theory. We shall elaborate the method of the scalar diffraction theory, namely, the angular spectrum (AS) of plane waves. This method has several advantages over other methods. For example, it describes the optical beam propagation without introducing additional approximations. In addition, calculations through this method can be markedly accelerated by implementation of the fast Fourier transform (FFT) \[7\]. The AS of plane waves defines the wavefront through the Fourier transformation:

\[
U(x, y) = \hat{F}^{-1}\{H(f_x, f_y, l) \cdot \hat{F}[U(x', y')]\},
\]

where \(H(f_x, f_y, l)\) is the transfer function, given by:

\[
H(f_x, f_y, l) = \begin{cases} 
\exp\left[i \frac{2\pi l}{\lambda} \sqrt{1 - \lambda^2 \cdot (f_x^2 + f_y^2)} \right], & f_x^2 + f_y^2 < \frac{1}{\lambda^2}, \\
0, & f_x^2 + f_y^2 \geq \frac{1}{\lambda^2}, 
\end{cases}
\]

where \(\lambda\) is the wavelength of the propagated optical field, and \(f_x, f_y\) are the spatial frequencies along \(x, y\) axes, respectively.

3.2. Adaptive optimization of the wavefront

Initially, the AOWF algorithm was developed to solve the problem of the beam focusing through the randomly scattering media \[6\]. Flat wavefront passed through a scattering sample. The resulting pattern of randomly distributed intensity maxima and minima appeared in the registration plane (see Fig. 1 (a)). After this, the AOWF algorithm was applied, which transformed the wavefront in such a way that the radiation from the source was collected at a single point on the registration plane (see Fig. 1 (b)).

An elementary case was considered earlier when there is no scattering medium in the object plane \[8\]. In this case, the AOWF algorithm generates an array of optimal phase delay values on the SLM, which is a discrete representation of the thin lens function (see Fig. 1 (c)). The rays of light from the source, after passing the SLM, converge at a single point on the registration plane. However, the method of target correction \[6\] used for the AOWF algorithm to focus into one or several focal spots is not the only possible one. If one uses more than one pixel of the matrix photodetector as a target, but a group of pixels, one can obtain a phase DOE that will focus the wavefront into a given number of points.

As a result of radiation scattering by the passage through the scattering medium, the interrelation between adjacent wavefront segments is lost in the output plane. It follows from the Huygens-Fresnel principle, which states that each point of the wavefront is a separate secondary source of spherical waves. Thus, the field in the input plane can be considered as a set of elementary radiators. Phase shift variation of radiators by means of SLM opens the possibility to configure the total wavefront in the registration plane. This fact allows to develop a simple and effective algorithm for finding the optimal configuration of elementary segments in the input plane in order to focus through the scattering medium \[9\].
Figure 1. (a) Flat wavefront passes through a scattering sample without conversion in SLM; (b) After setting of the phase delay for each segment of the SLM, the optical beam is focused into the target image on the output plane; (c) Radiation propagation through SLM and its focusing without a scattering sample; (d) An example of the algorithm for the phase delay setting is a line-by-line search of single SLM segments.

The principle of the algorithm can be briefly described as follows (see figure 2):

1. Flat wavefront from the laser source falls onto the SLM plane;
2. Separate segments of the wavefront acquire the phase delay specified on the SLM pixels. The transformed wavefront after the passage of SLM then propagates to the registration plane;
3. The intensity distribution obtained in the registration plane is further compared by PC with the feedback condition, that is, by the distribution of the energy (target), which must be obtained in the registration plane;
4. PC selects a phase delay in the range (-π, π] in the current segment (or segments) of SLM.

Then process 1 - 4 is repeated for the next segment (or segments) of SLM. There are several algorithms for selecting segments to find the optimal phase delay, for example, a line-by-line search of single SLM segments (see figure 1 (d)), binary search, or random phase masks. More details about these algorithms are given in [9].

Figure 2. Algorithm of AOWF operation without a scattering medium.
4. Results
In this section, we analyze the basic properties of the presented method.

4.1. Formation of the point source
Let us consider the simplest case of wavefront transformation, namely, focusing in a particular point on the registration plane. The result of applying the AOWF method to obtain a point source is shown in Fig. 3.

![Figure 3](image)

**Figure 3.** The target (a) and the result of the flat wavefront with homogenous intensity conversion by the AOWF method: phase (b) in the plane of SLM, amplitude (c) and phase (d) in the registration plane. Parameters: image size is 320 × 320 μm, the wavelength is $\lambda = 532$ nm, the gradation of the AOWF phase is $\varphi = 4$, the distance to the screen is $l = 2$ mm.

Uniformly distributed intensity in the plane of SLM is not shown. The wavefront from the source was redistributed to the point on the registration plane (Fig. 3 (c)). In this case, as a result of the calculation, we have the necessary phase distribution (Fig. 3 (b)) for uniform intensity (thin lens), which must be sent to SLM to obtain the required amplitude distribution.

4.2. Formation of the two dimensional image
Consider more complex intensity distribution in the plane away from the SLM. In contrast to the first problem, where the AOWF algorithm found an unambiguous distribution of phase delays for SLM, namely the function of a thin lens, with the presence of several intensity points, the solution of the problem can be ambiguous for analytical methods. The AOWF algorithm will automatically find the desired distribution, with the most uniform distribution of energy between the points of intensity. The result is shown in Fig. 4.

A beam of light with the uniform intensity distribution was redistributed into three points on the registration plane (Fig. 4 (c)). In this case, as a result of the calculation, we have the necessary phase distribution (Fig. 4 (b)), which is a sort of ‘cross-linking’ of phase distributions for three points separately. As seen from the comparison of Fig. 3 (c) and Fig. 4 (c), with the increase of the intensity points number undesirable additional diffraction orders appear. Methods for avoiding such effects are presented in [10].
Figure 4. The target (a) and the result of the flat wavefront with homogenous intensity conversion by the AOWF method: phase (b) in the plane of SLM, amplitude (c) and phase (d) in the registration plane. Parameters: image size is $320 \times 320 \, \mu m$, the wavelength is $\lambda = 532 \, nm$, the gradation of the AOWF phase is $\varphi = 4$, the distance to the screen is $l = 2 \, mm$. 

Let us now consider the case of the nontrivial intensity distribution in the registration plane, which is most interesting for imaging applications. In this problem, the AOWF algorithm also finds the integral phase distribution for SLM. The result of the algorithm application is shown in Fig. 5.

Figure 5. The target (a) and the result of the flat wavefront with homogenous intensity conversion by the AOWF method: phase (b) in the plane of SLM, amplitude (c) and phase (d) in the registration plane. Parameters: image size is $320 \times 320 \, \mu m$, the wavelength is $\lambda = 532 \, nm$, the gradation of the AOWF phase is $\varphi = 4$, the distance to the screen is $l = 2 \, mm$.

The light beam with the uniform intensity distribution was transformed into complex intensity configuration on the registration plane (Fig. 5 (c)). A large number of intensity points led to an
inefficient redistribution of energy, but the threshold intensity at the points of interest is reached, and
one can see the necessary object in the registration plane. Therefore, if the resulting two-dimensional
projection exposes a photographic plate, it is always possible to select the necessary threshold value of
the intensity and the exposure time at which the plate will be exposed only in the required region. This
refinement opens the possibility of the fast and universal assignment of two-dimensional intensity
distributions for holographic imaging problems.

5. Conclusion
In this paper, the method of optical wavefronts formation using adaptive wavefront optimization
(AOWF) was presented and studied. The method was applied to a number of basic problems, most
frequently encountered in optical physics. The obtained results showed the possibility of applying the
considered method to the assignment of two-dimensional distributions of optical energy in the needed
spatial area.

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