ρ meson decays of heavy hybrid mesons*

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Abstract: We calculate the ρ meson couplings between the heavy hybrid doublets $H^b/S^b/M^b/T^b$ and the ordinary $qQ$ doublets in the framework of the light-cone QCD sum rule. The sum rules obtained rely mildly on the Borel parameters in their working regions. The resulting coupling constants are rather small in most cases.

Keywords: heavy hybrid meson, QCD sum rule, heavy quark effective theory

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1 Introduction

Hadron states that do not fit into the constituent quark model have been studied widely in the past several decades. In recent years, the discovery of a number of unexpected exotic resonances such as the so-called XYZ states has revitalized the research of the existence of unconventional hadron states and their nature.

Theoretically, Quantum Chromodynamics (QCD), the fundamental theory of the strong interaction, may allow a far richer spectrum than the conventional quark model. For example, hybrids ($qar{q}g, \cdots$), glueballs ($gg, ggg, \cdots$), and multi-quark states ($qqar{q}q, qqgq, \cdots$) may not be prohibited by QCD. Those with the ordinary mesons. They have attracted much interest because they are not allowed by the constituent quark model and do not mix with the ordinary mesons.

Evidence of exotic mesons with $J^{PC} = 1^{-+}$, e.g. $\pi_1(1400)$ [1], $\pi_1(1600)$ [2], have emerged in the last few years. They are usually considered as candidates for hybrid mesons and have been studied extensively in various frameworks such as QCD sum rules, lattice QCD, AdS/QCD, the flux tube model, etc. The masses and decay properties of the $1^{-+}$ states have been studied in the framework of QCD sum rules [3, 4].

Based on the accumulated evidence of these light hybrid mesons, it is plausible to assume the existence of heavy quarkonium hybrids ($Q\bar{Q}g$) and heavy hybrid mesons containing one heavy quark ($q\bar{Q}g$) which may be not exotic. Govaerts et al. have studied these states in several works [5]. In Ref. [6], the masses of $Q\bar{Q}g$ were calculated at the leading order of heavy quark effective theory (HQET) [7]. In Ref. [8], the masses of $Q\bar{Q}g$ and their pionic couplings to ordinary heavy mesons were calculated.

In the heavy quark limit, the binding energy and the pionic couplings of $Q\bar{Q}g$ to $q\bar{Q}$ were worked out in Ref. [9] by the Shifman-Vainshtein-Zakharov (SVZ) sum rules [10]. HQET describes the large mass ($m_q$) asymptotics. At the leading order of this theory, the Lagrangian is endowed with the heavy quark flavor-spin symmetry, and the spectrum of $Q\bar{Q}$ consists of degenerate doublets. The components of a doublet share the same $j_i$, the angular momentum of the light degrees of freedom. For example, we denote the doublet $(0^-,1^-)$ as $H$, which consists of two $j_i = \frac{1}{2}$ S-wave $q\bar{Q}$. Similarly, the $P$-wave doublets $(0^+,1^+)/(1^+,2^+)$ are denoted as $S/T$ and the $D$-wave doublets $(1^-,2^-)/(2^-,3^-)$ as $M/N$. We denote the two $j_i = \frac{1}{2} q\bar{Q}g$ doublets with parity $P = +$ and $P = -$ as $S^b$ and $P^b$, respectively. Similarly, we use $T^b$ and $M^b$ to denote the two $j_i = \frac{3}{2}$ doublets with positive parity and negative parity, respectively.

In this work, we adopt the light-cone QCD sum rules (LCQSR) approach [11] to investigate the $\rho$ meson couplings between $q\bar{Q}g$ and $Q\bar{Q}$. We derive the sum rules for the $\rho$ meson couplings between doublets $D^b$ and $D$ ($D = H/S/T/M$) in Section 2. The numerical analysis is

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given in Section 3, followed by a brief conclusion in Section 4. The details of the partial amplitudes of these \( \rho \) decay channels are presented in Appendix A. The light-cone wave functions of the \( \rho \) meson involved in our calculation are listed in Appendix B.

## 2 \( \rho \) meson couplings

The interpolating currents for \( H^b \) and \( M^b \) adopted in our calculation can be written as

\[
J_{h_0}^1 = \sqrt{\frac{1}{2}} \bar{h}_v g_5 \gamma_{\alpha} \sigma_{1} \cdot G q , \\
J_{h_0}^{\alpha} = \sqrt{\frac{1}{2}} \bar{h}_v g_5 \gamma_{\alpha} \sigma_{1} \cdot G q , \\
J_{m_1}^{\alpha} = h_v g_5 \left[ 3G_{\alpha}^0 \gamma_{\beta} + i\gamma_{\alpha} \sigma_{1} \cdot G \right] q , \\
J_{m_0}^{\alpha \beta} = \frac{3}{2} \bar{h}_v g_5 \left[ G_{\alpha \beta}^0 \gamma_{\alpha} \gamma_{\beta} + G_{\alpha \beta}^1 \gamma_{\alpha} \gamma_{\beta} \right] q ,
\]

where \( G_{\alpha \beta} = G_{\alpha \beta}^0 \lambda^0 / 2 \) and \( h_v(x) = e^{im_{Q潇}x} + \frac{1}{2} Q(x) \).

The subscript \( t \) means that the corresponding Lorentz tensor is perpendicular to \( v \), the 4-velocity of the heavy quark. \( g_{\alpha \beta} = g_{\alpha \beta} - v^\alpha v^\beta \). For any asymmetric tensor \( A^{a_1 a_2 \ldots a_n} \), we can define

\[
A^{a_1 a_2 \ldots a_n} = A^{a_1 a_2 \ldots a_n} - \sum_{i=1}^{n} (A^{a_1 \ldots a_i \ldots a_{i+1} \ldots a_n} v_\alpha)^{a_i} .
\]

We define the overlapping amplitudes between these interpolating currents and the corresponding hybrids as

\[
\langle 0 | J_{h_0}^{0} (0) | H_0^0 (v) \rangle = f_{h_0} , \\
\langle 0 | J_{h_0}^{\alpha} (0) | H_0^0 (v, \lambda) \rangle = f_{h_0} \eta_{h_0}^{\alpha} (v, \lambda) , \\
\langle 0 | M_1^{\alpha} (0) | M_1^0 (v, \lambda) \rangle = f_{m_1} \eta_{m_1}^{\alpha} (v, \lambda) , \\
\langle 0 | M_0^{\alpha \beta} (0) | M_0^0 (v, \lambda) \rangle = f_{m_0} \eta_{m_0}^{\alpha \beta} (v, \lambda) ,
\]

where \( \eta(v, \lambda) \) denotes the polarization of the heavy hyperb. These symmetric traceless tensors are perpendicular to \( v \), namely \( \eta_{a_2 \ldots a_n} v^{a_\alpha} = 0 \).

We obtain the interpolating currents for the doublets \( S^b \) and \( T^b \) by simply inserting \( \gamma_5 \) into the currents in Eq. (1):

\[
J_{s_0}^1 = \sqrt{\frac{1}{2}} \bar{h}_v g_5 \gamma_{\alpha} \sigma_{1} \cdot G q , \\
J_{s_0}^{\alpha} = \sqrt{\frac{1}{2}} \bar{h}_v g_5 \gamma_{\alpha} \sigma_{1} \cdot G q , \\
J_{t_1}^{\alpha} = h_v g_5 \left[ 3G_{\alpha}^0 \gamma_{\beta} + i\gamma_{\alpha} \sigma_{1} \cdot G \right] q ,
\]

The corresponding overlapping amplitudes and projection operators can be defined similarly to Eq. (3).

The interpolating currents for \( qQ \) doublets \( H \) and \( S \) read:

\[
J_{h_0}^{\alpha} = \sqrt{\frac{1}{2}} \bar{h}_v \gamma_{\alpha} q , \\
J_{h_1}^{\alpha} = \sqrt{\frac{1}{2}} \bar{h}_v \gamma_{\alpha} q , \\
J_{s_0} = \sqrt{\frac{1}{2}} \bar{h}_v q , \\
J_{s_1}^{\alpha} = \sqrt{\frac{1}{2}} \bar{h}_v \gamma_{\alpha} q .
\]

The amplitudes between the ordinary heavy mesons and the states created by these currents acting on the vacuum state are

\[
\langle 0 | J_{h_0}^{0} (0) | H_0^0 (v) \rangle = f_{h_0} , \\
\langle 0 | J_{h_1}^{\alpha} (0) | H_1^0 (v, \lambda) \rangle = f_{h_1} \epsilon_{h_1}^\alpha (v, \lambda) , \\
\langle 0 | J_{s_0} (0) | S_1^0 (v) \rangle = f_{s_0} , \\
\langle 0 | J_{s_1}^{\alpha} (0) | S_1 (v, \lambda) \rangle = f_{s_1} \epsilon_{s_1}^\alpha (v, \lambda) .
\]

Here we outline the deduction of the sum rules for \( g_{H_0^0 H_1^0 p}^\rho \) and \( g_{H_1^1 H_1^1 p}^\rho \), where \( p \) is the orbital angular momentum of the \( \rho \) meson, the superscript ‘0’ and ‘1’ are the total angular momentum of the \( \rho \) meson. We define \( g_{H_0^0 H_1^1 p}^\rho \) and \( g_{H_1^1 H_1^1 p}^\rho \) in terms of the decay amplitude of the process \( H_1^1 \to H_1 + \rho \):

\[
\mathcal{M}(H_1^1 \to H_1 + \rho) = I \left[ e^\ast \cdot \eta_h (e^\ast \cdot q_h) - (e^\ast \cdot \epsilon_h^\ast) (\eta_h \cdot q_h) \right] g_{H_1^1 H_1^1 p}^\rho + I (e^\ast \cdot q_h) (e^\ast \cdot \eta_h) g_{H_0^0 H_1^1 p}^\rho ,
\]

where \( \eta_h \), \( e^\ast \) and \( e^\ast \) are the polarization of \( H_1^1 \), \( H_1 \) and \( \rho \), respectively, and \( q_h \) denotes the momentum of the \( \rho \). For the charged \( \rho \) meson, \( I = 1 \), and \( I = 1 / \sqrt{2} \) if the final \( \rho \) meson is neutral.

We consider the following correlation function:

\[
i \int dx e^{-i k \cdot x} \langle \rho(q) | J_{h_1}^{\alpha} (0) J_{h_1}^{\beta} (x) | 0 \rangle
\]

\[
= I \left[ \epsilon_{h_1}^\alpha q_h^\beta - q_h^\alpha \epsilon_{h_1}^\beta \right] g_{H_0^0 H_1^1 p}^\rho (\omega, \omega') + I q_h^\alpha (e^\ast \cdot \epsilon_h^\ast) g_{H_0^0 H_1^1 p}^\rho (\omega, \omega') ,
\]

where \( \omega = 2k \cdot v \) and \( \omega' = 2(k - q) \cdot v \), and we have the following dispersion relation

\[
g_{H_0^0 H_1^1 p}^\rho (\omega, \omega')
\]

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\begin{align}
&= \int_0^\infty ds_1 \int_0^\infty ds_2 \frac{\rho_{H_H\rho}^{\tilde{\mu}}(s_1, s_2)}{(s_1 - \omega - i\epsilon)(s_2 - \omega' - i\epsilon)} + \int_0^\infty ds_1 \frac{\rho_{H_H\rho}^{\tilde{\mu}}(s_1)}{s_1 - \omega - i\epsilon} + \int_0^\infty ds_2 \frac{\rho_{H_H\rho}^{\tilde{\mu}}(s_2)}{s_2 - \omega' - i\epsilon} = \ldots \ldots \ldots (9),
\end{align}

with

\begin{align}
G_{H_H\rho}^{\tilde{\mu}}(\omega, \omega') &= \frac{1}{4} \int dt \int \mathcal{D}\alpha e^{it(\tilde{\omega} + \tilde{\omega}')} \frac{m}{(q \cdot v)^3} \left[ -2f_\rho m^2 \bar{\psi}(\alpha) - 2f_\rho m^2 T(\alpha)(q \cdot v) + f_\rho m[6\bar{\psi}(\alpha)
+ 2\bar{\psi}(\alpha) + A(\alpha)](q \cdot v)^2 + 2f_\rho \bar{T}(\alpha) + 2T(\alpha) + 2T(\alpha)])(q \cdot v)^3 \right],
G_{H_H\rho}^{\tilde{\mu}}(\omega, \omega') &= \frac{1}{4} \int dt \int \mathcal{D}\alpha e^{it(\tilde{\omega} + \tilde{\omega}')} \frac{m}{q \cdot v} \left[ f_\rho m^2 [\mathcal{V}(\alpha) + A(\alpha)] - 2f_\rho m[T_1(\alpha) - T_3(\alpha) + S(\alpha)](q \cdot v) - 2f_\rho \mathcal{V}[\mathcal{V}(\alpha) + A(\alpha)](q \cdot v)^2 \right],
\end{align}

in which $u \equiv \alpha_2 + \alpha_3$ and $\tilde{u} \equiv 1 - u$.

The double Borel transformation $B_1^T B_2^T$ eliminates the terms on the right side of Eq. (9), except the first one which is a double dispersion relation. Now we arrive at

\begin{align}
&f_{HH} f_{H_H\rho} \rho_{H_H\rho}^{\tilde{\mu}} \rho_{H_H\rho}^{\tilde{\mu}} e^{-2\bar{\alpha}_0\bar{\alpha}_0 T_H - 2\bar{u}_0\bar{u}_0 T_H} = m_\rho \left\{ -\frac{1}{2} f_\rho m^2 \bar{\phi}(-3) - \frac{1}{2} f_\rho m^2 T(-2) - f_\rho m_0 [6\bar{\phi}^{[-1]}(u_0) + 2\bar{\phi}^{[-1]}(u_0) + A^{[-1]}(u_0)]
+ f_\rho \bar{T}^{[0]}(u_0) + 2\bar{T}^{[0]}(u_0) + 2T^{[0]}(u_0)Tf_0 \left( \frac{\omega}{T} \right) \right\},
&f_{HH} f_{H_H\rho} \rho_{H_H\rho}^{\tilde{\mu}} \rho_{H_H\rho}^{\tilde{\mu}} e^{-2\bar{\alpha}_0\bar{\alpha}_0 T_H - 2\bar{u}_0\bar{u}_0 T_H} = m_\rho \left\{ f_\rho m^2 [\mathcal{V}^{[-1]}(u_0) + A^{[-1]}(u_0)] + f_\rho m_0 [T_1^{[0]}(u_0) + T_2^{[0]}(u_0) + T_2^{[0]}(u_0) + S^{[0]}(u_0)]Tf_0 \left( \frac{\omega}{T} \right)
- \frac{1}{2} f_\rho \mathcal{V}^{[1]}(u_0) + A^{[1]}(u_0)T^2f_1 \left( \frac{\omega'}{T} \right) \right\},
\end{align}

with

\begin{align}
T = \frac{T_1T_2}{T_1 + T_2}, \quad u_0 = \frac{T_1}{T_1 + T_2}, \quad f_n(x) = 1 - e^{-x} \sum_{i=0}^{n} \frac{x^i}{i!},
\end{align}

Here we employ functions $f_n(x)$ to subtract the contribution of the continuum. $\mathcal{F}^{[\alpha_1]}$s are defined as

\begin{align}
&\mathcal{F}^{[0]}(u_0) \equiv \int_0^{u_0} \mathcal{F}(\tilde{u}_0, \alpha_2, u_0 - \alpha_2) d\alpha_2,
&\mathcal{F}^{[1]}(u_0) \equiv \int_0^{u_0} \mathcal{F}(\tilde{u}_0, u_0, 0) - \frac{\partial \mathcal{F}(1 - \alpha_2 - \alpha_3, \alpha_3)}{\partial \alpha_3} \bigg|_{\alpha_3 = u_0 - \alpha_2},
&\mathcal{F}^{[2]}(u_0) \equiv \left. \frac{\partial^2 \mathcal{F}(1 - \alpha_2 - \alpha_3, \alpha_3)}{\partial \alpha_2 \partial \alpha_3} \right|_{\alpha_2 = u_0, \alpha_3 = 0} - \int_0^{u_0} \left. \frac{\partial \mathcal{F}(1 - \alpha_2 - \alpha_3, \alpha_3)}{\partial \alpha_3} \right|_{\alpha_3 = 0} d\alpha_2\left. \frac{\partial^2 \mathcal{F}(1 - \alpha_2 - \alpha_3, \alpha_3)}{\partial \alpha_3^2} \right|_{\alpha_3 = u_0 - \alpha_2},
&\mathcal{F}^{[-1]}(u_0) \equiv \int_0^{1 - \alpha_2} \mathcal{F}(1 - \alpha_2 - \alpha_3, \alpha_2, \alpha_3) d\alpha_3 d\alpha_2 - \int_0^{u_0} \int_0^{u_0 - \alpha_2} \mathcal{F}(1 - \alpha_2 - \alpha_3, \alpha_2, \alpha_3) d\alpha_3 d\alpha_2,
&\mathcal{F}^{[-2]}(u_0) \equiv \int_0^{1 - \alpha_2} \int_0^{1 - \alpha_2} \mathcal{F}(1 - \alpha_2 - \alpha_3, \alpha_2, x) d\alpha_3 d\alpha_2 - \int_0^{u_0} \int_0^{u_0 - \alpha_2} \int_0^{u_0} \mathcal{F}(1 - \alpha_2 - \alpha_3, \alpha_2, x) d\alpha_3 d\alpha_2 d\alpha_2
- \bar{u}_0 \int_0^{1 - \alpha_2} \mathcal{F}(1 - \alpha_2 - \alpha_3, \alpha_2, \alpha_3) d\alpha_3 d\alpha_2.
\end{align}

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Using the above mentioned method, we obtain the sum rules of other $\rho$ meson coupling constants as follows. Their definitions are presented in Appendix A.

\[
\begin{align*}
&f_{4\rho} g^{\rho}_{H_{1}H_{1}p} e^{-2a_{1}[A_{H}/T - 2a_{2}A_{S}/T]}
=f_{\rho} m_{p}^{2} \left[ 4\Phi - 2\Phi - 2\tilde{\Phi} - A \right]^{-2} + 4f_{\rho} T m_{p}^{3}[T^{-1}(u_{0}) - 2T_{1}^{-1}(u_{0}) + 2T_{2}^{-1}(u_{0})]
\qquad + 4f_{\rho} T m_{p}^{2}[4\Phi(0) - 4\Phi(0) + 2\tilde{\Phi}(0) + 2\tilde{\Phi}(0) - 3A(0)] T f_{0} \left( \frac{\omega_{1}}{T} \right)
\qquad + 2T_{2}^{-1}(u_{0}) - 2T_{4}^{-1}(u_{0}) - 2\tilde{S}_{1}^{-1}(u_{0}) T^{2} f_{1} \left( \frac{\omega_{1}}{T} \right) - f_{\rho} [T^{2}(0) + A(0)] T^{3} f_{2} \left( \frac{\omega_{1}}{T} \right) \Bigg],
\end{align*}
\]

\[
\begin{align*}
&f_{H_{1}} g^{\rho}_{H_{1}H_{1}p} e^{-2a_{1}[A_{H}/T - 2a_{2}A_{S}/T]}
=f_{\rho} m_{p}^{2} \left[ 4\Phi - 2\Phi - 2\tilde{\Phi} - A \right]^{-2} + 4f_{\rho} T m_{p}^{3}[T^{-1}(u_{0}) - 2T_{1}^{-1}(u_{0}) + 2T_{2}^{-1}(u_{0})]
\qquad + 4f_{\rho} T m_{p}^{2}[4\Phi(0) - 4\Phi(0) + 2\tilde{\Phi}(0) + 2\tilde{\Phi}(0) - 3A(0)] T f_{0} \left( \frac{\omega_{1}}{T} \right)
\qquad + 2T_{2}^{-1}(u_{0}) - 2T_{4}^{-1}(u_{0}) - 2\tilde{S}_{1}^{-1}(u_{0}) T^{2} f_{1} \left( \frac{\omega_{1}}{T} \right) - f_{\rho} [T^{2}(0) + A(0)] T^{3} f_{2} \left( \frac{\omega_{1}}{T} \right) \Bigg],
\end{align*}
\]

\[
\begin{align*}
&f_{H_{1}} g^{\rho}_{H_{1}H_{1}p} e^{-2a_{1}[A_{H}/T - 2a_{2}A_{S}/T]}
=f_{\rho} m_{p}^{2} \left[ 4\Phi - 2\Phi - 2\tilde{\Phi} - A \right]^{-2} + 4f_{\rho} T m_{p}^{3}[T^{-1}(u_{0}) - 2T_{1}^{-1}(u_{0}) + 2T_{2}^{-1}(u_{0})]
\qquad + 4f_{\rho} T m_{p}^{2}[4\Phi(0) - 4\Phi(0) + 2\tilde{\Phi}(0) + 2\tilde{\Phi}(0) - 3A(0)] T f_{0} \left( \frac{\omega_{1}}{T} \right)
\qquad + 2T_{2}^{-1}(u_{0}) - 2T_{4}^{-1}(u_{0}) - 2\tilde{S}_{1}^{-1}(u_{0}) T^{2} f_{1} \left( \frac{\omega_{1}}{T} \right) - f_{\rho} [T^{2}(0) + A(0)] T^{3} f_{2} \left( \frac{\omega_{1}}{T} \right) \Bigg],
\end{align*}
\]
\[ + 2 T_2^{[1]}(u_0) - 2 T_4^{[1]}(u_0) - 2 S^{[1]}(u_0) \right) T^2 f_1 \left( \frac{\omega'}{T} \right) - f_\rho [V^{[2]}(u_0) + A^{[2]}(u_0)] T^3 f_2 \left( \frac{\omega'}{T} \right) \right) ,
\]
\[ f_{sh} f_{h0} g_{nH_{SH_1}} e^{-2u_0 A_{gh}/T - 2u_0 A_{hi}/T} \]
\[ = \frac{1}{4} m_\rho \left\{ f_\rho m_\rho^2 [4 \Phi - 2 \tilde{\Phi} - 2 \tilde{\Psi}] \right\} \]
\[ \left[ 1 \right. 
\left. - 8 f_\rho^2 m_\rho^2 [4 \Phi - 2 \tilde{\Phi} - 2 \tilde{\Psi} - A^{[-2]}] \right. 
\left. - 2 f_\rho^2 m_\rho^2 [4 \Phi - 2 \tilde{\Phi} - 2 \tilde{\Psi} + A^{[-2]}] \right\} \]
\[ f_{sh} f_{h0} g_{nH_{SH_1}} e^{-2u_0 A_{gh}/T - 2u_0 A_{hi}/T} \]
\[ = m_\rho^2 \left\{ \frac{1}{2} f_\rho m_\rho^2 \tilde{\Psi}^{(-3)} - \frac{1}{2} f_\rho m_\rho^2 \tilde{\Psi}^{(-2)} + f_\rho m_\rho [6 \tilde{\Phi}^{(-1)}(u_0) + 2 \tilde{\Psi}^{(-1)}(u_0) + A^{[-1]}(u_0)] \right\} \]
\[ f_{sh} f_{h0} g_{nH_{SH_1}} e^{-2u_0 A_{gh}/T - 2u_0 A_{hi}/T} \]
\[ = - m_\rho \left\{ f_\rho m_\rho^2 [4 \Phi - 2 \tilde{\Phi} - 2 \tilde{\Psi} + A^{[-2]}] \right\} \]
\[ \left[ 1 \right. 
\left. - 2 f_\rho m_\rho^2 [4 \Phi - 2 \tilde{\Phi} - 2 \tilde{\Psi} + A^{[-2]}] \right. 
\left. - 2 f_\rho m_\rho^2 [4 \Phi - 2 \tilde{\Phi} - 2 \tilde{\Psi} + A^{[1]}(u_0)] \right. 
\left. \right] \]
\[ f_{sh} f_{h0} g_{nH_{SH_1}} e^{-2u_0 A_{gh}/T - 2u_0 A_{hi}/T} \]
\[ = \frac{1}{4} \sqrt{2} m_\rho \left\{ 2 f_\rho m_\rho^2 [4 \Phi - 2 \tilde{\Phi} - 2 \tilde{\Psi} + A^{[-2]}] \right\} \]
\[ \left[ 1 \right. 
\left. - 4 f_\rho m_\rho^2 [4 \Phi - 2 \tilde{\Phi} - 2 \tilde{\Psi} + A^{[-2]}] \right. 
\left. - 2 f_\rho m_\rho^2 [4 \Phi - 2 \tilde{\Phi} - 2 \tilde{\Psi} + A^{[1]}(u_0)] \right. 
\left. \right] \]
\[ f_{sh} f_{h0} g_{nH_{SH_1}} e^{-2u_0 A_{gh}/T - 2u_0 A_{hi}/T} \]
\[ = \frac{3}{4} \sqrt{2} m_\rho \left\{ 4 f_\rho m_\rho^2 \tilde{\Psi}^{(-3)} + 2 f_\rho m_\rho^2 \tilde{\Psi}^{(-2)} - 4 f_\rho m_\rho^2 [4 \Phi - 2 \tilde{\Phi} - 2 \tilde{\Psi} + A^{[-2]}(u_0)] \right\} \]
\[ \left[ 1 \right. 
\left. - 4 f_\rho m_\rho^2 [4 \Phi - 2 \tilde{\Phi} - 2 \tilde{\Psi} + A^{[-2]}(u_0)] \right. 
\left. - 4 f_\rho m_\rho^2 [4 \Phi - 2 \tilde{\Phi} - 2 \tilde{\Psi} + A^{[1]}(u_0)] \right. 
\left. \right] \]
\[ f_{sh} f_{h0} g_{nH_{SH_1}} e^{-2u_0 A_{gh}/T - 2u_0 A_{hi}/T} \]
\[ = \frac{3}{4} \sqrt{2} m_\rho \left\{ 2 f_\rho m_\rho^2 \tilde{\Psi}^{(-3)} - 2 f_\rho m_\rho^2 \tilde{\Psi}^{(-2)} + 8 f_\rho m_\rho^2 [4 \Phi - 2 \tilde{\Phi} - 2 \tilde{\Psi} + A^{[-2]}(u_0)] \right\} . \]
3 Numerical analysis

The parameters in the distribution amplitudes of the $\rho$ meson take their values from Ref. [12]. In this work, we take the values with $\mu = 1$ GeV, realizing that the heavy quark behaves almost as a spectator of the decay processes in our discussion at the leading order of HQET:

$$f^p_{\rho} / \text{MeV} = 216(3), f^I_{\rho} / \text{MeV} = 105(9), a_0^\| = 0.15(7), a^\perp = 0.14(6), \omega^\parallel = 0.30(10), \omega^\perp = -0.09(3)$$

$$\omega^\parallel_{\pi^0} = 0.15(5), \omega^\perp_{\rho} = 0.07(3), \omega^\parallel_{\rho} = -0.03(1), \omega^\perp_{\rho} = -0.03(5), \omega^\perp_{\rho} = -0.08(5)$$

For the mass sum rules of H and S, the working region of the Borel parameter $T$ is about $0.8 < T < 1.1$ GeV [13], which is in the vicinity of that of the mass sum rules for $D^b$ (D=H/S/M/T) [9]. So we choose $u_0 = 1/2$ in our calculation. The continuum contribution can be subtracted cleanly with this choice. An asymmetric choice of $u_0$, on the other hand, would result in a fuzzy continuum subtraction [14].

The binding energy and the overlapping amplitudes of doublets $H/S$ [13] and $H^b/M^b$, $S^b/T^b$ [9] involved in our numerical analysis are as follows.

| $A / \text{GeV}$ | $0.50$ | $1.15$ | $2.0$ | $2.5$ |
|-----------------|-------|-------|------|------|
| $f$ GeV$^{-3/2}$ | $0.25$ GeV$^{-3/2}$ | $0.40$ GeV$^{-3/2}$ | $1.1$ GeV$^{7/2}$ | $1.6$ GeV$^{7/2}$ |

The working region of $T$ is determined by the insensitivity of the coupling constant to the variation of $T$ and by the requirement that the pole contribution should be not less than 40%. We display the sum rules for these $\rho$ couplings with $\omega^\| = 2.8, 3.0, 3.2$ GeV in Fig. 1.

The following relations arise naturally in our calculation

$$g^{\rho_0}_{H^1H^0} = g^{\rho_0}_{H^1H^1} = -g^{\rho_0}_{H^1H^0},$$
$$g^{\rho_1}_{H^1H^0} = g^{\rho_1}_{H^1H^1} = g^{\rho_1}_{H^1H^0},$$
$$g^{\rho_1}_{H^1S^0} = g^{\rho_1}_{H^1S_1} = g^{\rho_1}_{H^1S_0} = g^{\rho_1}_{H^1S_1},$$
$$g^{\rho_1}_{H^1S^0} = g^{\rho_1}_{H^1S_1} = g^{\rho_1}_{H^1S_1},$$
$$g^{\rho_1}_{M^1H^0} = g^{\rho_1}_{M^1H^1} = -\frac{1}{2} g^{\rho_1}_{M^1H_0} = -\frac{\sqrt{3}}{2} g^{\rho_1}_{M^1H^0},$$
$$g^{\rho_2}_{M^1H^0} = g^{\rho_2}_{M^1H^1} = \frac{\sqrt{3}}{2} g^{\rho_2}_{M^1H_0} = -\frac{3}{2} g^{\rho_2}_{M^1H^0},$$
$$g^{\rho_1}_{M^2H^0} = g^{\rho_1}_{M^2H^1} = -\frac{\sqrt{3}}{2} g^{\rho_1}_{M^2H_0} = -\frac{3}{2} g^{\rho_1}_{M^2H^0},$$
$$g^{\rho_2}_{M^2H^0} = g^{\rho_2}_{M^2H^1} = \frac{\sqrt{3}}{2} g^{\rho_2}_{M^2H_0} = \frac{3}{2} g^{\rho_2}_{M^2H^0},$$
$$g^{\rho_1}_{M^1S^0} = g^{\rho_1}_{M^1S_1} = -\frac{1}{2} g^{\rho_1}_{M^1S_0} = \frac{\sqrt{3}}{6} g^{\rho_1}_{M^1S_0},$$
$$g^{\rho_1}_{M^1S^0} = g^{\rho_1}_{M^1S_1} = \frac{\sqrt{3}}{6} g^{\rho_1}_{M^1S_0},$$
$$g^{\rho_1}_{M^2S^0} = g^{\rho_1}_{M^2S_1} = \frac{\sqrt{3}}{6} g^{\rho_1}_{M^2S_0},$$
$$g^{\rho_2}_{M^2S^0} = g^{\rho_2}_{M^2S_1} = \frac{\sqrt{3}}{6} g^{\rho_2}_{M^2S_0},$$
$$g^{\rho_1}_{M^1S^0} = g^{\rho_1}_{M^1S_1} = -\frac{1}{2} g^{\rho_1}_{M^1S_0} = \frac{\sqrt{3}}{6} g^{\rho_1}_{M^1S_0},$$
$$g^{\rho_2}_{M^2S^0} = g^{\rho_2}_{M^2S_1} = \frac{\sqrt{3}}{6} g^{\rho_2}_{M^2S_0},$$
$$g^{\rho_1}_{M^1T^0} = g^{\rho_1}_{M^1T^1} = -\frac{1}{2} g^{\rho_1}_{M^1T_0} = -\frac{\sqrt{3}}{3} g^{\rho_1}_{M^1T_0},$$
$$g^{\rho_2}_{M^2T^0} = g^{\rho_2}_{M^2T^1} = \frac{\sqrt{3}}{3} g^{\rho_2}_{M^2T_0},$$
$$g^{\rho_1}_{M^1T^0} = g^{\rho_1}_{M^1T^1} = -\frac{1}{2} g^{\rho_1}_{M^1T_0} = -\frac{\sqrt{3}}{3} g^{\rho_1}_{M^1T_0},$$
$$g^{\rho_2}_{M^2T^0} = g^{\rho_2}_{M^2T^1} = \frac{\sqrt{3}}{3} g^{\rho_2}_{M^2T_0},$$
$$g^{\rho_1}_{M^1T^0} = g^{\rho_1}_{M^1T^1} = -\frac{1}{2} g^{\rho_1}_{M^1T_0} = -\frac{\sqrt{3}}{3} g^{\rho_1}_{M^1T_0},$$
$$g^{\rho_2}_{M^2T^0} = g^{\rho_2}_{M^2T^1} = \frac{\sqrt{3}}{3} g^{\rho_2}_{M^2T_0}.$$ (16)

These simple proportional relations among the obtained couplings result from the heavy quark flavor-spin symmetry. They also justify our construction of the interpolating currents for heavy hybrid mesons. The spin of the interpolating currents can be deduced from the symmetry of their Lorentz indices. The $P$ parity can be obtained directly from the $P$-transformation property of these currents. The tensor structure of the correlation functions considered above verifies their $J^P$ quantum
numbers. For example, if the $J^P$ quantum number of $J^{10}_{T_1} = \bar{h}_s g_s \gamma_5 [3G^{\rho\gamma_\rho} - i\gamma_\rho \sigma]$, $G^{\rho}$ is not 1, the tensor structure of the correlation function

$$i \int dx e^{-ik \cdot x} \langle \rho(q) | J_{h_1}^\alpha (0) J_{h_{10}}^{10} (x) | 0 \rangle$$

(17)
cannot include (only) $s_1$, $d_1$ and $d_2$.

Furthermore, if $J_{h_1}^{10} = J_{h_1}^{10} + \lambda J_{h_0}^{10} (\lambda \neq 0)$, where $J_{h_1}^{10}$ and $J_{h_0}^{10}$ are pure interpolating currents with $j_1 = 3/2$ and $j_2 = 1/2$, respectively, we have

$$i \int dx e^{-ik \cdot x} \langle \rho(q) | J_{h_0}^\alpha (0) J_{h_{10}}^{10} (x) | 0 \rangle$$

$$= i e^{i\beta \phi} v G_{T_{h_1}^{10}} + i \left[ e^{i\beta \phi} (e^* \cdot q_1) - \frac{1}{3} e^{i\beta \phi} q_1^2 \right] G_{T_{h_1}^{10}}$$

$$+ i \left[ e^{i\beta \phi} (e^* \cdot q_1) + e^{i\beta \phi} (e^* \cdot q_1) \right] G_{T_{h_1}^{10}}$$

and

$$i \int dx e^{-ik \cdot x} \langle \rho(q) | J_{h_0}^\alpha (0) J_{h_{10}}^{10} (x) | 0 \rangle$$

$$= i e^{i\beta \phi} v G_{T_{h_0}^{10}} + i \left[ e^{i\beta \phi} (e^* \cdot q_1) - \frac{1}{3} e^{i\beta \phi} q_1^2 \right] G_{T_{h_0}^{10}}$$

When $G_{T_{h_1}^{10}}$ is proportional to $G_{T_{h_0}^{10}}$, we have $c_1 = c_2$, namely,

$$G_{T_{h_1}^{10}} / G_{T_{h_0}^{10}} = G_{T_{h_1}^{10}} / G_{T_{h_0}^{10}}$$

This is inconsistent with the results (see Eq. (16)) we just obtained: $G_{T_{h_1}^{10}} / G_{T_{h_0}^{10}} \neq G_{T_{h_1}^{10}} / G_{T_{h_0}^{10}}$. This implies $J_{h_1}^{10} = J_{h_0}^{10}$. In other words, the interpolating current $J_{h_1}^{10}$ carries $j_1 = 3/2$. The $J$, $P$ and $j_1$ quantum numbers of other interpolating currents can be verified in a similar way.

The final values of these couplings are listed in Table 1. In most channels they are rather small, which may be attributed to the fading of the gluon degree of freedom in the decay.

### 4 Conclusion

At the heavy quark limit, we have constructed interpolating currents respecting the flavor-spin symmetry for $qQg$ and $qQ$. With these currents, the $p$ meson couplings between $qQg$ and $qQ$ have been worked out by means of LCQSR. The derived sum rules rely mildly on the Borel parameters in their working regions. The resulting coupling constants are rather small in most cases.

The main error of our calculation originates from the inaccuracies of the LCQSR: truncation of the OPE near the light-cone, the uncertainty of the parameters in the light-cone wave functions, the dependence of the coupling constant on the continuum threshold $\omega_c$ and the Borel parameter in the working region, the uncertainty of the binding energy $\tilde{A}$'s and the overlapping amplitudes $f$'s. As far as the charm quark is concerned, the $1/m_Q$ correction may be significant, while the correction from the finite mass of the bottom quark should be negligible.
We hope that our calculation may be helpful to experimental searches for these heavy hybrid mesons and the understanding of their strong interaction with conventional heavy mesons. Moreover, the coupling constants calculated in our work might shed further light on the nature of the XYZ mesons.

Appendix A

The $\rho$ decay amplitudes of heavy hybrid mesons

The $\rho$ decay amplitudes considered in the text are as follows.

\[
\mathcal{M}(H_0^h \to H_0 + \rho) = \langle e^* \cdot q \rangle g_{H_0^h H_0 \rho}^\rho \\
\mathcal{M}(H_0^h \to H_1 + \rho) = \langle e^* \cdot q \rangle g_{H_0^h H_1 \rho}^\rho + \langle e^* \cdot q \rangle \langle e^* \cdot \eta_1 \rangle (\eta^* \cdot q_1) \\
\mathcal{M}(H_1^h \to S_0 + \rho) = \langle e^* \cdot q \rangle g_{H_1^h S_0 \rho}^\rho + \langle e^* \cdot q \rangle \langle e^* \cdot q \rangle (\eta^* \cdot q_1) \\
\mathcal{M}(M_1^h \to H_0 + \rho) = \langle e^* \cdot q \rangle g_{M_1^h H_0 \rho}^\rho \\
\mathcal{M}(M_2^h \to H_1 + \rho) = \langle e^* \cdot q \rangle g_{M_2^h H_0 \rho}^\rho + \langle e^* \cdot q \rangle \langle e^* \cdot q \rangle (\eta^* \cdot q_1) \\
\mathcal{M}(M_1^h \to S_0 + \rho) = \langle e^* \cdot q \rangle g_{M_1^h S_0 \rho}^\rho \\
\mathcal{M}(M_2^h \to S_1 + \rho) = \langle e^* \cdot q \rangle g_{M_2^h S_0 \rho}^\rho + \langle e^* \cdot q \rangle \langle e^* \cdot q \rangle (\eta^* \cdot q_1) \\
\mathcal{M}(S_0^h \to S_0 + \rho) = \langle e^* \cdot q \rangle g_{S_0^h S_0 \rho}^\rho \\
\mathcal{M}(S_1^h \to S_1 + \rho) = \langle e^* \cdot q \rangle g_{S_1^h S_0 \rho}^\rho \\
\mathcal{M}(S_1^h \to S_1 + \rho) = \langle e^* \cdot q \rangle g_{S_1^h S_0 \rho}^\rho + \langle e^* \cdot q \rangle \langle e^* \cdot q \rangle (\eta^* \cdot q_1)
\]
\[
\mathcal{M}(S^b_1 \rightarrow S_1 + \rho) = I (e^+ \cdot q_1) (e^- \cdot \eta_1) g^{p_0}_{SS_1} + I \left[ (e^+ \cdot q_1) (e^- \cdot \eta_1) - (e^+ \cdot \eta_1) (e^- \cdot q_1) \right] g^{p_1}_{SS_1},
\]

\[
\mathcal{M}(S^b_0 \rightarrow H_1 + \rho) = I (e^+ \cdot \eta_1) g^{p_1}_{ss_1} + I \left[ (e^+ \cdot q_1) (e^- \cdot \eta_1) - \frac{1}{3} (e^+ \cdot \eta_1) q_1 \right] g^{d_1}_{SH_1},
\]

\[
\mathcal{M}(S^b_1 \rightarrow H_0 + \rho) = I (e^+ \cdot \eta_1) g^{p_1}_{ss_1} + I \left[ (\eta \cdot q_1) (e^+ \cdot \eta_1) - \frac{1}{3} (e^+ \cdot \eta_1) \eta_1 \right] g^{d_1}_{SH_0},
\]

\[
\mathcal{M}(S^b_1 \rightarrow H_1 + \rho) = I e^{\mu \nu} \varepsilon_{\mu \nu} g^{p_1}_{ss_1} + I \left[ e^{\mu \nu} q_1 (e^+ \cdot \eta_1) - \frac{1}{3} e^{\mu \nu} \eta_1 q_1 \right] g^{d_1}_{SH_1},
\]

\[
\mathcal{M}(T^b_1 \rightarrow H_0 + \rho) = I (e^+ \cdot \eta_1) g^{p_1}_{ss_1} + I \left[ (\eta \cdot q_1) (e^+ \cdot \eta_1) - \frac{1}{3} (e^+ \cdot \eta_1) \eta_1 \right] g^{d_1}_{SH_0},
\]

\[
\mathcal{M}(T^b_1 \rightarrow H_1 + \rho) = I e^{\mu \nu} \varepsilon_{\mu \nu} g^{p_1}_{ss_1} + I \left[ e^{\mu \nu} q_1 (e^+ \cdot \eta_1) - \frac{1}{3} e^{\mu \nu} \eta_1 q_1 \right] g^{d_1}_{SH_1},
\]

\[
\mathcal{M}(T^b_1 \rightarrow S_0 + \rho) = I e^{\mu \nu} \varepsilon_{\mu \nu} g^{p_1}_{ss_1},
\]

\[
\mathcal{M}(T^b_1 \rightarrow S_1 + \rho) = I \left[ (e^+ \cdot \eta_1) (e^+ \cdot q_1) - (e^+ \cdot \eta_1) (q_1 \cdot \eta_1) \right] g^{p_1}_{SS_1} + I \left[ (e^+ \cdot \eta_1) (q_1 \cdot \eta_1) + (q_1 \cdot \eta_1) (e^+ \cdot \eta_1) - \frac{2}{3} (e^+ \cdot \eta_1) (e^+ \cdot q_1) \right] g^{p_2}_{SS_1}
\]

\[
\mathcal{M}(T^b_2 \rightarrow H_1 + \rho) = I \eta_{\alpha_1 \alpha_2} \left( e^{\alpha_1 \alpha_2} q_1 \eta_1 + e^{\alpha_2 \alpha_1} q_1 \eta_1 \right) g^{d_2}_{SH_1},
\]

\[
\mathcal{M}(T^b_2 \rightarrow H_0 + \rho) = I \eta_{\alpha_1 \alpha_2} \left[ e^{\alpha_1 \alpha_2} q_1 \eta_1 + e^{\alpha_2 \alpha_1} q_1 \eta_1 \right] g^{d_2}_{SH_0},
\]

\[
\mathcal{M}(T^b_2 \rightarrow S_1 + \rho) = I \eta_{\alpha_1 \alpha_2} \left[ e^{\alpha_1 \alpha_2} q_1 \eta_1 + e^{\alpha_2 \alpha_1} q_1 \eta_1 \right] g^{d_2}_{SS_0},
\]

\[
\mathcal{M}(T^b_2 \rightarrow S_0 + \rho) = I \eta_{\alpha_1 \alpha_2} \left[ e^{\alpha_1 \alpha_2} q_1 \eta_1 + e^{\alpha_2 \alpha_1} q_1 \eta_1 \right] g^{d_2}_{SS_0}.
\]

\[
\mathcal{M}(T^b_2 \rightarrow S_1 + \rho) = I \eta_{\alpha_1 \alpha_2} \left[ e^{\alpha_1 \alpha_2} q_1 \eta_1 + e^{\alpha_2 \alpha_1} q_1 \eta_1 \right] g^{d_2}_{SS_1},
\]

\[
\mathcal{M}(T^b_2 \rightarrow S_0 + \rho) = I \eta_{\alpha_1 \alpha_2} \left[ e^{\alpha_1 \alpha_2} q_1 \eta_1 + e^{\alpha_2 \alpha_1} q_1 \eta_1 \right] g^{d_2}_{SS_0}.
\]
Appendix B

The definitions of the $\rho$ meson light-cone distribution amplitudes

The definitions of the distribution amplitudes of the $\rho$ meson used in the text read as [12, 15]

$$
\langle 0|\bar{u}(z)\gamma_\mu d(-z)|\rho^-(P, \lambda)\rangle = f_\rho m_\rho \left[ p_\mu \frac{e^{(\lambda)\cdot z}}{p\cdot z} \int_0^1 du e^{i\xi_{\mu\nu} z} \varphi_{1}(u, \mu^2) + e^{(\lambda)\cdot \mu} \int_0^1 du e^{i\xi_{\mu\nu} z} g_{3}(u, \mu^2) \right],
$$

$$
\langle 0|\bar{u}(z)\gamma_\mu\gamma_5 d(-z)|\rho^-(P, \lambda)\rangle = \frac{1}{2} f_\rho m_\rho e^{(\lambda)\cdot \mu} e^{(\lambda)\cdot \mu} \int_0^1 du e^{i\xi_{\mu\nu} z} g_{3}(u, \mu^2),
$$

$$
\langle 0|\bar{u}(z)\sigma_{\mu\nu} d(-z)|\rho^-(P, \lambda)\rangle = i f_\rho \left[ \left( e^{(\lambda)\cdot \mu} p_\nu - e^{(\lambda)\cdot \nu} p_\mu \right) \int_0^1 du e^{i\xi_{\mu\nu} z} \varphi_{1}(u, \mu^2) + \left( p_\mu z_\nu - p_\nu z_\mu \right) \frac{e^{(\lambda)\cdot z}}{p\cdot z} m_\rho^2 \int_0^1 du e^{i\xi_{\mu\nu} z} h_{(t)}(u, \mu^2) \right],
$$

$$
\langle 0|\bar{u}(z)d(-z)|\rho^-(P, \lambda)\rangle = -i f_\rho \left[ e^{(\lambda)\cdot z} m_\rho^2 \int_0^1 du e^{i\xi_{\mu\nu} z} h_{(u)}(u, \mu^2) \right]. \quad (B1)
$$

The distribution amplitudes $\varphi_{1}\parallel$ and $\varphi_{1}\perp$ are of twist-2, $\varphi_{7}^{(v)}$, $g_{\perp}^{(a)}$, $h_{(s)}^{(a)}$ and $h_{(s)}^{(t)}$ are twist-3 and $g_3$, $h_3$ are twist-4. All functions $\phi = \{ \varphi_{1}\parallel, \varphi_{1}\perp, g_{\perp}^{(a)}, h_{(s)}^{(a)}, h_{(s)}^{(t)}, g_3, h_3 \}$ are normalized to satisfy $\int_0^1 du \phi(u) = 1$.

The 3-particle distribution amplitudes of the $\rho$ meson are defined as [12, 15]

$$
\langle 0|\bar{u}(z)g_{\mu\nu}\gamma_\alpha d(-z)|\rho^-(P, \lambda)\rangle = f_\rho m_\rho p_\alpha [p_\mu e^{(\lambda)\cdot z} - p_\nu e^{(\lambda)\cdot \mu}] A(v, p z)
$$

$$
+ f_\rho m_\rho^3 \frac{3}{2} \frac{e^{(\lambda)\cdot z}}{p\cdot z} [p_\mu g_{a\nu}^1 - p_\nu g_{a\mu}^1] \bar{\Psi}(v, p z) + f_\rho m_\rho^3 \frac{e^{(\lambda)\cdot z}}{p\cdot z} [p_\mu z_\nu - p_\nu z_\mu] \bar{\Psi}(v, p z),
$$

$$
\langle 0|\bar{u}(z)g_{\mu\nu}\gamma_\alpha d(-z)|\rho^-(P, \lambda)\rangle = f_\rho m_\rho p_\alpha [p_\mu e^{(\lambda)\cdot z} - p_\nu e^{(\lambda)\cdot \mu}] V(v, p z)
$$

$$
+ f_\rho m_\rho^3 \frac{3}{2} \frac{e^{(\lambda)\cdot z}}{p\cdot z} [p_\mu g_{a\nu}^1 - p_\nu g_{a\mu}^1] \Phi(v, p z),
$$

$$
\langle 0|\bar{u}(z)\sigma_{\alpha\beta}g_{\mu\nu}(v)g_{\alpha\beta}(v) d(-z)|\rho^-(P, \lambda)\rangle = f_\rho \frac{m_\rho^4}{2} \frac{e^{(\lambda)\cdot z}}{2(p\cdot z)} [p_\alpha p_\beta g_{a\nu}^1 - p_\beta p_\alpha g_{a\mu}^1] T(v, p z)
$$

$$
+ f_\rho m_\rho^6 \frac{3}{2} \frac{e^{(\lambda)\cdot z}}{p\cdot z} [p_\alpha e^{(\lambda)\cdot \nu} - p_\beta e^{(\lambda)\cdot \nu} - p_\alpha g_{a\nu}^1 + p_\beta g_{a\nu}^1] T_1(v, p z)
$$

$$
+ f_\rho m_\rho^6 \frac{3}{2} \frac{e^{(\lambda)\cdot z}}{p\cdot z} [p_\alpha e^{(\lambda)\cdot \mu} - p_\beta e^{(\lambda)\cdot \mu} - p_\alpha g_{a\mu}^1 + p_\beta g_{a\mu}^1] T_2(v, p z)
$$

$$
+ f_\rho m_\rho^6 \frac{3}{2} \frac{e^{(\lambda)\cdot z}}{p\cdot z} [p_\alpha p_\beta e^{(\lambda)\cdot \nu} - p_\beta p_\alpha e^{(\lambda)\cdot \nu} - p_\alpha p_\nu e^{(\lambda)\cdot \mu} + p_\beta p_\nu e^{(\lambda)\cdot \mu}] T_3(v, p z)
$$

$$
+ f_\rho m_\rho^6 \frac{3}{2} \frac{e^{(\lambda)\cdot z}}{p\cdot z} [p_\alpha p_\beta e^{(\lambda)\cdot \nu} - p_\beta p_\alpha e^{(\lambda)\cdot \nu} - p_\alpha p_\nu e^{(\lambda)\cdot \mu} + p_\beta p_\nu e^{(\lambda)\cdot \mu}] T_4(v, p z),
$$

$$
\langle 0|\bar{u}(z)g_{\mu\nu}(v)g_{\alpha\beta}(v) d(-z)|\rho^-(P, \lambda)\rangle = i f_\rho \frac{m_\rho^4}{2} \frac{e^{(\lambda)\cdot z}}{p\cdot z} [p_\alpha p_\beta e^{(\lambda)\cdot \nu}] S(v, p z),
$$

$$
\langle 0|\bar{u}(z)g_{\mu\nu}(v)\gamma_\alpha d(-z)|\rho^-(P, \lambda)\rangle = i f_\rho \frac{m_\rho^4}{2} \frac{e^{(\lambda)\cdot z}}{p\cdot z} [p_\alpha p_\beta e^{(\lambda)\cdot \nu}] \bar{S}(v, p z). \quad (B2)
$$

The distribution amplitudes $A$, $V$ and $T$ are of twist-3 and the other 3-particle distribution amplitudes are of twist-4.
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