Holographic Bosonic Technicolor

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Abstract

We consider a technicolor model in which the expectation value of an additional, possibly composite, scalar field is responsible for the generation of fermion masses. We define the dynamics of the strongly coupled sector by constructing its holographic dual. Using the AdS/CFT correspondence, we study the S parameter and the phenomenology of the light technihadrons. We find that the S parameter is small over a significant region of the model’s parameter space. The particle spectrum is distinctive and includes a nonstandard Higgs boson as well as heavier hadronic resonances. Technihadron masses and decay rates are calculated holographically, as a function of the model’s parameters.

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I. INTRODUCTION

The physics responsible for the breaking of electroweak symmetry will be studied at the LHC over the next few years. In anticipation of potentially definitive experimental results, a number of novel models of electroweak symmetry breaking (EWSB) have been proposed recently [1]. These models aim, in part, to address the hierarchy problem that is inherent to the Higgs sector of the minimal Standard Model, while satisfying the constraints posed by LEP data. Long ago, technicolor models were proposed as an alternative to the minimal Higgs sector [2]. Fermions coupling both to the electroweak and technicolor gauge sectors condense when the technicolor interactions become strong. The fermion condensate takes the place of the vacuum expectation value (vev) of the Higgs field in the breaking of electroweak symmetry. The hierarchy problem associated with radiative corrections to the Higgs mass is eliminated since no fundamental scalar fields are present in the theory. The large separation between the Planck and electroweak scales is understood as a natural consequence of the logarithmic running of the technicolor gauge coupling. Unfortunately, it was realized in the last millennium that technicolor models predict large corrections to precisely measured electroweak observables if the new strong sector is similar to QCD [3]. In particular, the Peskin-Takeuchi oblique parameter $S$ is predicted to be $\gtrsim 0.2$, while experiments constrain $S$ to be less than about 0.1 [4].

More recently, technicolor models with small or negative values of $S$ have been constructed using the AdS/CFT correspondence [5, 6, 7, 8]. In this holographic approach, the dynamics of the strongly coupled sector is not necessarily specified \textit{a priori}. Instead, a five-dimensional (5D) gauge theory in a warped background is postulated to define a strongly coupled four-dimensional (4D) technicolor sector via the rules of the AdS/CFT correspondence [9]. Beginning with a holographic theory that has properties similar to QCD, new couplings are introduced in the 5D model that alter the holographic prediction of current-current correlation functions in the strongly-coupled theory. It can then be shown that the parameter space of the 5D theory contains regions where the value of the $S$ parameter is in accord with experimental constraints [5]. In the present work, we also use the freedom to deviate from a QCD-like holographic theory in constructing a viable model of dynamical EWSB. In particular, we will allow for a separation of the technicolor confinement and chiral
symmetry breaking scales as another means for reducing the $S$-parameter.

A complete model of holographic technicolor must also provide a mechanism for generating Standard Model fermion masses. Four-dimensional extended technicolor (ETC) models provide the desired coupling between Standard Model fermions and the technicolor condensate through four-fermion operators that are generated when heavy ETC gauge bosons are integrated out of the theory \cite{10}. Unfortunately, ETC gauge boson exchange generically produces flavor-changing four-fermion operators that are excluded by experiment, if the ETC scale is low enough to account for a heavy top quark. An alternative means of generating fermion masses is possible in technicolor theories that have an additional, possibly composite, Higgs doublet field in the low-energy theory \cite{11, 12, 13, 14, 15, 16, 17, 18}. The coupling of the technifermion condensate to this field forces it to develop a vev, even if the Higgs mass squared is positive. Yukawa couplings of the Higgs field then provide the origin of Standard Model fermion masses. If the Higgs field is composite, we assume that the compositeness scale is higher than the technicolor scale, so that the Higgs may be treated as a new fundamental scalar in the low-energy effective theory. While it may seem unusual to consider electroweak symmetry breaking models with fundamental scalars, viable strongly coupled extended technicolor sectors that suppress flavor changing operators often have scalars in their effective description below the ETC scale \cite{19}. The basic features of this “bosonic technicolor” scenario are reviewed in Section II.

In this paper, we present a bosonic technicolor model in which the dynamics of the strongly coupled sector is defined through its holographic dual. The model is compatible with electroweak constraints and provides for the origin of fermion masses. Coefficients in the electroweak chiral Lagrangian of the theory, that would otherwise be unknown, are determined by the AdS/CFT correspondence, as we discuss in Section III. We then study the phenomenology of the model in Section IV. In particular, we compute the usually problematic contribution to the $S$ parameter, as a function of the technirho mass and the vev of the Higgs field, and find that a significant region of parameter space is allowed. We also study the decays of the technirho which, if observed at the LHC, could exclude regions of the model’s parameter space and potentially discriminate between different holographic technicolor scenarios. Neither the $S$ parameter nor the partial decay widths of the technicolor resonances were calculable in earlier versions of bosonic technicolor. We conclude in
II. VINTAGE TECHNICOLOR WITH A SCALAR

The gauge group of the model is \( G_{TC} \times \text{SU}(3)_C \times \text{SU}(2)_W \times \text{U}(1)_Y \), where \( G_{TC} \) represents the technicolor group. We will assume that \( G_{TC} \) is asymptotically free and confining, but make no other assumptions about the group. We assume two flavors of technifermions, \( p \) and \( m \), which transform in a nontrivial representation of \( G_{TC} \). In addition, these fields form a left-handed \( \text{SU}(2)_W \) doublet and two right-handed singlets

\[
\Upsilon_L \equiv \begin{pmatrix} p \\ m \end{pmatrix}_L , \quad p_R , \quad m_R , \tag{2.1}
\]

with hypercharges \( Y(\Upsilon_L) = 0 \), \( Y(p_R) = 1/2 \), and \( Y(m_R) = -1/2 \). With these assignments, the technicolor sector is free of gauge anomalies.

The technicolor sector has a global \( \text{SU}(2)_L \times \text{SU}(2)_R \) symmetry, which corresponds to independent special unitary rotations on the left- and right-handed technifermion fields. It is assumed that strong dynamics results in a technifermion condensate

\[
\langle \bar{p}p + \bar{m}m \rangle \approx 4 \pi f^3 \tag{2.2}
\]

that spontaneously breaks this symmetry. Here, \( 4 \pi f \) is traditionally identified as the chiral symmetry breaking scale \[20\]. The resulting Goldstone bosons may be described in an effective chiral lagrangian, where

\[
\Sigma = \exp(2i \Pi/f) \quad \text{and} \quad \Pi = \begin{pmatrix} \pi^0/2 & \pi^+ / \sqrt{2} \\ \pi^- / \sqrt{2} & -\pi^0/2 \end{pmatrix} , \tag{2.3}
\]

and where the \( \Sigma \) field transforms simply under the \( \text{SU}(2)_L \times \text{SU}(2)_R \) symmetry:

\[
\Sigma \rightarrow L \Sigma R^\dagger . \tag{2.4}
\]

A kinetic term for \( \Sigma \) may be constructed that is invariant under Eq. (2.4), and also under the Standard Model gauge symmetries,

\[
\mathcal{L}_{KE} = \frac{f^2}{4} \text{Tr} \left( D_\mu \Sigma^\dagger D^\mu \Sigma \right) , \tag{2.5}
\]
where the covariant derivative is given by

$$D^\mu \Sigma = \partial^\mu \Sigma - igW^\mu_a T^a \Sigma + ig' B^\mu \Sigma T^3.$$  \hspace{1cm} (2.6)

Here, the $T^a$ are the generators of SU(2), while $g$ and $g'$ are the SU(2) and U(1) gauge couplings, respectively. The quadratic terms in Eq. (2.5) include mixing between gauge fields and the pion fields, indicating that the latter are unphysical and can be gauged away. After doing so, the remaining quadratic terms in Eq. (2.5) give the gauge boson masses,

$$m_W = \frac{1}{2} gf \quad m_Z = \frac{1}{2} (g^2 + g'^2)^{1/2} f,$$  \hspace{1cm} (2.7)

which reproduce the correct experimental values for $f \approx 246$ GeV.

For $G_{TC} = SU(N)$, the theory described thus far corresponds to conventional technicolor, with all its well-known problems. Large contributions to the $S$ parameter will be avoided in our case by deforming the model away from one that could be naively interpreted as a scaled-up version of QCD. This will be discussed in the next section. Here, we extend the model to provide for an origin of fermion masses. We assume that the low-energy theory includes a scalar SU(2)$_W$ doublet $\phi \equiv (\phi^+, \phi^0)$ that can couple to the technifermions and to Standard Model fermions via ordinary Yukawa couplings:

$$\mathcal{L}_{\phi_T} = \overline{Y_L} \tilde{\phi} h_+ p_R + \overline{Y_L} \phi h_- m_R + \text{h.c.}, \hspace{1cm} (2.8)$$

$$\mathcal{L}_{\phi_f} = \overline{Y_L} \tilde{\phi} h_U E_R + \overline{Q_L} \tilde{\phi} h_U U_R + \overline{Q_L} \phi h_D D_R + \text{h.c.} \hspace{1cm} (2.9)$$

We will not assume that $\phi$ has a negative squared mass. The Yukawa coupling of $\phi$ to the technifermions produces a $\phi$ tadpole term when the chiral symmetry is dynamically broken and the technifermions condense. This guarantees that there is a non-zero vacuum expectation value for $\phi$, and hence, that masses for the Standard Model fermions are generated.

The origin of the $\phi$ doublet is worthy of some comment. The $\phi$ field either represents a fundamental particle in the ultraviolet (UV), or a composite one in the infrared (IR). Each possibility presents its own advantages and disadvantages. If $\phi$ is fundamental, then an ultraviolet completion that separately solves the hierarchy problem, such as supersymmetry, must be assumed. Some may object to such a hybrid proposal on philosophical grounds, but such arguments have little bearing on whether or not such a theory is realized in nature. On the other hand, if $\phi$ is composite, one avoids the problems of quadratic divergences, which
are regulated by the Higgs compositeness scale. In this case, however, other difficulties may occur. The couplings of the composite field to the fundamental fermions in the theory arise via higher-dimension operators, leading to suppression factors. One might worry that the top quark Yukawa coupling could be too small in a generic model of this sort, though such an outcome could be avoided, for example, if the third generation fermions are also composite. A Higgs compositeness sector may also provide a new source for dangerous flavor-changing neutral current effects, via higher-dimension operators suppressed by the Higgs compositeness scale. Whether such problems actually do arise, however, depends on the details of the theory in the UV, which are unknown. Since we work exclusively with the low-energy theory, it is only necessary that we assume that some adequate UV completion of our model exists. The same assumption is made in other popular models of EWSB.

We should also comment on naturalness of bosonic technicolor models. We assume that the scalar field in the effective low-energy Lagrangian has a positive squared mass. The scalar mass cannot be arbitrarily large, or else the scalar vacuum expectation value would not be large enough to account for fermion masses with perturbative Yukawa couplings. Therefore, the scalar sector of bosonic technicolor is comparable in naturalness to the Higgs sector of the Standard Model. We always assume that if bosonic technicolor is realized in Nature, then it is the low-energy effective description of a theory in which the scalar mass is stabilized by some additional mechanism. We stress that our purpose is not to solve the hierarchy problem, but to study the phenomenology of this class of electroweak symmetry breaking models.

We may incorporate $\phi$ into the chiral Lagrangian by defining the matrix field

$$\Phi = \begin{pmatrix} \phi^0 & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix},$$

which transforms precisely in the same way as $\Sigma$ under the chiral symmetry. For the case in which the technifermions have a common Yukawa coupling $h_+ = h_- \equiv h$, which we assume henceforth, the $\phi$ tadpole described above appears through the following term in the effective chiral lagrangian

$$\mathcal{L}_H = c_1 \cdot 4\pi hf^3 \text{Tr} (\Phi \Sigma^\dagger) + \text{h.c.}.$$  

The coefficient has been chosen such that $c_1$ is of order unity by naive dimensional analysis (at least in QCD-like models). It is now convenient to employ a nonlinear representation
of the $\Phi$ field,

$$\Phi = \left( \sigma + f' \right) \sqrt{2} \exp \left( 2 i \Pi'/f' \right) ,$$

(2.12)

where $f'$ is a vev and $\Pi'$ is defined in analogy to Eq. (2.13). The fields $\{\sigma, \Pi'\}$ are equivalent to the four real degrees of freedom in the original field $\phi$. Expanding Eq. (2.11), one obtains the mass matrix for the $\Pi$ and $\Pi'$ multiplets. One linear combination, which we call $\pi_a$, is exactly massless and becomes the longitudinal components of the weak gauge bosons in unitary gauge; the orthogonal combination, $\pi_p$, are physical and remain in the low-energy spectrum:

$$\pi_a = \frac{f \Pi + f' \Pi'}{\sqrt{f^2 + f'^2}} , \quad \pi_p = \frac{-f' \Pi + f \Pi'}{\sqrt{f^2 + f'^2}} .$$

(2.13)

Note that our phase conventions have been chosen to agree with those found in the previous literature [13]. The $f$ and $f'$ vevs, as well as the mass of the $\sigma$ field, can be determined in terms of the tadpole parameter $c_1$ in Eq. (2.11), the Yukawa coupling $h$, and the parameters that appear in the $\phi$ potential (for a detailed treatment, see Ref. [13]). For our present purposes, we will find it more convenient to express quantities of phenomenological interest in terms of $f$ and $f'$ directly. The mass of the physical pion multiplet also follows from $L_H$ in Eq. (2.11). One finds

$$m^2_\pi = 8 \sqrt{2} \pi c_1 h \frac{f v^2}{f'}$$

(2.14)

where

$$v \equiv \sqrt{f^2 + f'^2} = 246 \text{ GeV} .$$

(2.15)

In the holographic treatment of this model, the parameter $c_1$ can be calculated, as well as the pion couplings to the hadronic technicolor resonances in the theory.

III. HOLOGRAPHIC TECHNICOLOR WITH A SCALAR

We use the AdS/CFT correspondence [9] to model the strong dynamics of the technicolor sector. The holographic description allows us to calculate the masses and couplings of the technicolor resonances and to estimate the $S$ parameter. We take the geometry of the 5D spacetime to be a slice of anti-de Sitter (AdS) space, given by the metric,

$$ds^2 = \frac{1}{z^2} \left( -dz^2 + dx^\mu dx_\mu \right) , \quad 0 < z \leq z_m$$

(3.1)
where \( z = z_m \) is an infrared cutoff. We include in the 5D bulk a complex scalar field \( X \), whose boundary value is proportional to the source for the technifermion operator \( q_L \bar{q}_R \) in the 4D theory, where \( q = (p, m) \). The field \( X \) is a two-by-two matrix in flavor space and transforms as a bifundamental under the \( SU(2)_L \times SU(2)_R \) chiral symmetry, which becomes a gauge symmetry in the corresponding 5D model. Normalizable modes of the \( X \) field and the bulk gauge fields correspond to hadronic resonances, with \( 1/z_m \) setting the scale of confinement.

An ultraviolet cutoff may be introduced by moving the AdS boundary away from \( z = 0 \), to \( z = \epsilon \). One can think of \( 1/\epsilon \) as the scale at which the holographic model breaks down.

Although we work with a finite and small choice for \( \epsilon \) in our numerical calculations, we find that all our physical results remain convergent in the limit \( \epsilon \to 0 \). For definiteness, we also set the AdS scale to the electroweak scale, \( v = 246 \) GeV.

If we assign to the 4D operator \( q_L \bar{q}_R \) its UV conformal dimension 3, then according to the AdS/CFT correspondence, the corresponding 5D field \( X \) has a mass squared \( m_X^2 = -3 \) in units of the AdS curvature scale \[9\]. In principle, we can consider this mass, or equivalently the dimension of the techniquark bilinear, as another parameter in the model, and the running of the dimension can be included by a modification of the geometry. For simplicity we do not consider such modifications here. To summarize the model and our conventions, the 5D action is,

\[
S_{5D} = \int d^5x \sqrt{g} \cdot \text{Tr} \left\{ -\frac{1}{2g_5^2} (F_R^2 + F_L^2) + |Dx|^2 + 3|X|^2 \right\},
\]

where \( D_\mu X = \partial_\mu X - iA_{L,R} X + iX A_{R,L} \), \( A_{L,R} = A_{L,R}^a T^a \), and \( F_{L,R} = \partial_\mu A_{L,R} - \partial_\nu A_{L,R} - i[A_{L,R}, A_{L,R}] \). The profile of the \( X \) field is determined by solving the classical equations of motion with \( A_{L,R}^\mu = 0 \) and \( X(x, z) = X(z) \). There are two independent solutions, whose coefficients have the interpretation of the common techniquark mass, \( m_q = hf'/\sqrt{2} \), and condensate, \( \sigma \), so that \[22, 23\],

\[
X(z) = \frac{1}{2} \left( \frac{hf'}{\sqrt{2}} z + \sigma z^3 \right).
\]

In an SU(\( N \)) technicolor theory with two flavors, one may match the holographic prediction for the vector current-current correlator in the UV \[24\] to the result of a one-loop calculation, which implies \[22, 23\],

\[
g_5^2 = \frac{24\pi^2}{N} .
\]
Since we do not assume that $G_{TC} = SU(N)$ in the present model, however, $g_5$ is a free parameter. We will set $g_5$ to the value given by Eq. (3.4) with $N = 4$ or 8 for the purpose of numerical estimates. Our qualitative conclusions do not depend strongly on this choice. Finally, for fixed small Yukawa coupling $h$ the condensate $\sigma$ can be expressed as a function of the decay constant $f$ by a holographic calculation of the small $q^2$ behavior of the axial vector current correlator \cite{22, 23}, $\Pi_A(q^2) \to -f^2$. Taking into account the constraint that $f^2 + f'^2 = v^2$, the free parameters in the model are therefore $f$, $h$ and $z_m$.\footnote{The AdS/CFT correspondence allows us to calculate the pion decay constant in terms of the techniquark condensate in our model. This provides a test of the naive dimensional analysis (NDA) prediction, Eq. (2.2). We find that $f(\sigma)$ agrees with NDA to within a factor of $O(1)$. However, the discrepancy leads to qualitatively different estimates of the $S$ parameter and decay widths. In particular, the $S$ parameter is generally smaller than our quoted results if we take $\sigma = 4\pi f^3$.}

The type of holographic construction that we have just described is known to give a reliable description of the light pseudoscalar and vector mesons in QCD \cite{22, 23, 25}, so we anticipate that it will be equally successful in describing the technicolor sector of our 4D model. We will use the correlation functions, masses and couplings computed in this theory to determine unknown coefficients in the effective 4D chiral Lagrangian, described in Sec. III, which properly takes into account the gauging of electroweak symmetry and the mixing between the $\Phi$ and technipion fields. As we make explicit in the next section, our approach does not require that we include the weakly coupled degrees of freedom in the 5D theory to extract the desired results.

The 5D model that we have described is a simple holographic construction of the strongly coupled sector, but is by no means the only one. Additional interactions may be included in the 5D action, the metric may be allowed to deviate from the AdS metric away from $z = 0$, and the boundary conditions for the fields at $z_m$ may be altered. Such modifications make it possible to include power corrections to the vector and axial-vector current-current correlation functions, so that one may obtain negative values of the $S$ parameter \footnote{The AdS/CFT correspondence allows us to calculate the pion decay constant in terms of the techniquark condensate in our model. This provides a test of the naive dimensional analysis (NDA) prediction, Eq. (2.2). We find that $f(\sigma)$ agrees with NDA to within a factor of $O(1)$. However, the discrepancy leads to qualitatively different estimates of the $S$ parameter and decay widths. In particular, the $S$ parameter is generally smaller than our quoted results if we take $\sigma = 4\pi f^3$.}. Alternatively, $S$ may be reduced if the dimension of the operator $\overline{q}q$ is smaller than its UV dimension \footnote{The AdS/CFT correspondence allows us to calculate the pion decay constant in terms of the techniquark condensate in our model. This provides a test of the naive dimensional analysis (NDA) prediction, Eq. (2.2). We find that $f(\sigma)$ agrees with NDA to within a factor of $O(1)$. However, the discrepancy leads to qualitatively different estimates of the $S$ parameter and decay widths. In particular, the $S$ parameter is generally smaller than our quoted results if we take $\sigma = 4\pi f^3$.}. Although our model may be modified in these ways, we take a different approach. We work with the minimal theory, Eqs. (3.1) and (3.2), but allow the scale $z_m$ and the chiral symmetry breaking scale $4\pi f$ to be independent. This freedom provides another means of
obtaining a significant reduction in $S$ (see also Ref. [8]). With the confinement scale held fixed, we will also see that $S$ decreases as the vev of the field $\Phi$ approaches the electroweak scale $v$, the limit in which the technicolor sector no longer participates in EWSB.

IV. PHENOMENOLOGY

In this section, we compute what is usually the most dangerous contribution to the $S$ parameter, and show that an acceptably small value can be obtained without adding new parameters to the minimal 5D theory defined by Eqs. (3.1) and (3.2). We also consider some aspects of the phenomenology of our holographic bosonic technicolor model that are relevant to future collider searches.

A. The $S$-Parameter

One of the most serious problems with QCD-like technicolor models is the generically large value of the $S$-parameter. The oblique parameter $S$ may be defined in terms of correlation functions of the vector and axial-vector currents $J_V^{a\mu}$ and $J_A^{b\mu}$ at small momentum transfer [3],

$$S = 4\pi \frac{d}{dq^2} \left( \left| \Pi_V(-q^2) - \Pi_A(-q^2) \right|_{q^2 \to 0} \right). \quad \text{(4.1)}$$

with,

$$\int d^4xe^{iq\cdot x}\langle J_V^{a\mu}(x)J_V^{b\nu}(0)\rangle \equiv \delta^{ab}\left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right)\Pi_V(-q^2),$$

$$\int d^4xe^{iq\cdot x}\langle J_A^{a\mu}(x)J_A^{b\nu}(0)\rangle \equiv \delta^{ab}\left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right)\Pi_A(-q^2). \quad \text{(4.2)}$$

In the holographic model, the contribution of the strong technicolor sector to $\Pi_V(-q^2)$ and $\Pi_A(-q^2)$ are calculated by evaluating the part of the 5D action, Eq. (3.2), that is quadratic in the SU(2)$_V$ and SU(2)$_A$ gauge fields. According to the rules of the AdS/CFT correspondence, the variation of the action (twice) with respect to the 4D vector or axial vector gauge fields $V_\mu(q)$ or $A_\mu(q)$, which act as sources for $J_V^{a\mu}$ and $J_A^{a\mu}$, yields the correlators in Eq. (4.2). We define the vector bulk-to-boundary propagator $V(q,z)$ as the solution to the equations of motion for the SU(2)$_V$ gauge field $V_\mu(q,z) \equiv V(q,z)V_\mu(q)$, where $V(q,\epsilon) = 1$ if $z = \epsilon$ is the location of the spacetime boundary; similarly, the axial vector bulk-to-boundary propagator
$A(q, z)$ is defined by $A_{\mu}(q, z) \equiv A(q, z)A_{\mu}(q)$. The desired correlators are then determined to be

$$\Pi_V(-q^2) = \frac{2}{g_5^2} \left. \frac{1}{z} \partial V(q, z) \right|_{z \to \epsilon},$$

and similarly for $\Pi_A(-q^2)$ with the replacement $V(q, z) \to A(q, z)$. The equations of motion for the bulk-to-boundary propagators follow from the action, Eq. (3.2):

$$\partial_z \left( \frac{1}{z} \partial_z V(q, z) \right) + \frac{q^2}{z} V(q, z) = 0,$$

$$\partial_z \left( \frac{1}{z} \partial_z A(q, z) \right) + \frac{q^2}{z} A(q, z) - \frac{g_5^2}{2z^3} X_0(z)^2 A(q, z) = 0.$$

In our model, we allow for the possibility that confinement and chiral symmetry breaking occur at different scales. The confinement scale associated with the masses of the vector mesons is determined by the shape of the extra dimension away from the boundary at $z = \epsilon$. In our model, the location of the IR wall, at $z = z_m$, determines this scale. By increasing the confinement scale, physics around the $Z$ pole becomes increasingly like the Standard Model, and corrections to $S$ become negligible. Similarly, if the Higgs vev approaches the electroweak scale, $v = 246$ GeV, with the technirho mass held fixed, then the physics around the $Z$ pole again becomes increasingly like the Standard Model. (The technirho mass is calculated holographically, as will be discussed in Sec. IVB.) This behavior is reflected in Fig. 1. The different curves correspond to different masses of the lightest technivector resonance, and hence to different $z_m$. As $f/v \to 0$ the technicolor sector plays no role in EWSB, and as $f/v \to 1$ the Higgs sector plays no role in EWSB. Note that we take a reasonably small (though not atypical) value for the Yukawa coupling $h = 0.01$ in this example. We have studied the dependence of the $S$ parameter on $h$, for a technivector mass of 3 TeV, and have found that our results remain unchanged at the few percent level for any $h \lesssim 0.3$. We do not discuss larger $h$ since there are then regions of the parameter space for which the approximation of chiral symmetry breaks down. The 5D gauge coupling $g_5$ was taken to be as in Eq. (3.4) with $N = 4$. As we mentioned earlier, this identification is made for definiteness but is somewhat arbitrary, as the UV description of

\[2\] Perturbative unitarity places an upper bound on the technirho mass. However, for heavier technirho, the 5D theory becomes strongly coupled and 5D loop effects that we have ignored become important. We thank Csaba Csáki and Kaustubh Agashe for discussions of this issue.
FIG. 1: Technicolor contribution to $S$ parameter vs. technipion decay constant $f$ and lightest technivector meson mass. Curves for technivector masses of 1, 3, and 5 TeV are shown. In this example the technifermion Yukawa coupling was taken to be $h = 0.01$, and $g_5$ was chosen to match the UV behavior of the technicolor group SU(4) as described in the text. Note that the existence of the Higgs field allows $S < 0.05$ over a significant region of the parameter space.

the holographic technicolor theory is unconstrained for our purposes. It is clear from Fig. 1 that without a large hierarchy between the confinement and chiral symmetry breaking scales the $S$ parameter can be acceptably small. This is unlike traditional technicolor models.

B. Physical Spectrum and EWSB

Thus far, the SU(2)$_L \times$SU(2)$_R$ chiral symmetry of the technicolor theory on the 4D boundary has been assumed to be a global symmetry. Strictly speaking, the model described in Sec. III allows us to calculate hadronic properties in the limit that the electroweak gauge interactions and couplings to the $\Phi$ scalars are turned off. In order to become a model of EWSB, we must now consider the effect of gauging an SU(2)$\times$U(1) subgroup of the chiral symmetry. As far as the strong technicolor interactions are concerned, the only difference is that three Goldstone bosons are eaten through the usual Higgs mechanism and are replaced by the longitudinal components of the $W$ and $Z$ bosons. The 4D theory initially contains six pseudoscalar fields, the $\Pi_a$ and $\Pi'_a$ defined in Sec. III for $a = 1 \ldots 3$. One linear combination,
π_a, is eaten during EWSB, and the other, π_p, remains in the physical spectrum, as given in Eq. (2.13).

The Π^a components of π_a and π_p correspond to normal mode solutions of the bulk equation of motion in the holographic theory, as we shall now review. Ignoring its radial σ component, we may express the bulk scalar field as

\[ X(x, z) = \frac{X_0(z)}{2} \exp[2i\Pi_X(x, z)] \tag{4.6} \]

where \( X_0(z) = 2X(z) \), with \( X(z) \) given in Eq. (3.3). The quadratic part of the action, Eq. (3.2), includes mixing between Π_X and the longitudinal component of the axial vector field \( A_\mu(x, z) = \partial_\mu \varphi(x, z) \). The relevant terms in the action are:

\[ S_{5D} \supset \int d^5x \left[ -\frac{1}{4g_5^2 z} F_{AMN} F_M^{ANa} + \frac{1}{z^3} |D_M X|^2 \right] \tag{4.7} \]

\[ \supset \int d^5x \left[ -\frac{1}{4g_5^2 z} F_{AMN} F_M^{ANa} + \frac{X_0(z)^2}{2z^3} \left( \partial_M \Pi_X^a - \frac{A_M^a}{\sqrt{2}} \right)^2 \right]. \tag{4.8} \]

In the subsequent numerical analysis, we choose \( g_5 \) to match the UV behavior of the technicolor group SU(8). For the normalizable modes, we write \( \Pi_X(q, z) = \pi_X(z)\Pi(q) \) and \( \varphi(q, z) = \varphi(z)\Pi(q) \), where \( q^2 = m^2_\Pi \) is an eigenvalue of the equations of motion with appropriate boundary conditions. We use the gauge \( A_z = 0 \), in which case the equations of motion for \( \pi_X \) and \( \varphi \) are:

\[ \partial_z \left( \frac{1}{z} \partial_z \varphi^a \right) + \frac{g_5^2 X_0(z)^2}{z^3 \sqrt{2}} \left( \pi_X^a - \varphi^a \sqrt{2} \right) = 0, \tag{4.9} \]

\[ -\frac{\sqrt{2}q^2}{g_5^2} \partial_z \left( \frac{1}{z} \partial_z \varphi^a \right) + \partial_z \left( \frac{X_0(z)^2}{z^3} \partial_z \pi_X^a \right) = 0. \tag{4.10} \]

Note that the linearized equations of motion are invariant under the gauge transformation \( \varphi^a/\sqrt{2} \rightarrow \varphi^a/\sqrt{2} + \lambda^a(q) \), \( \pi_X^a \rightarrow \pi_X^a + \lambda^a(q) \). The boundary conditions for the normalizable modes are, \( \pi_X^a(\epsilon) = \varphi^a(\epsilon) = 0 \), and \( \partial_z \varphi^a(z)|_{z=z_m} = 0 \). The UV boundary conditions are determined by the normalizability of the modes (up to a gauge transformation as described above), and the gauge invariant form of the IR boundary condition is \( F_{z\mu} = 0 \) (although other choices are possible).

The eigenvalues of \( q^2 \) for the coupled equations of motion determine the mass squared term for the normalized \( \Pi(x) \) field. In the holographic approach, the \( \Pi^2 \) mass term in the 4D
FIG. 2: The physical pion mass as a function of the technipion decay constant $f$, for $h = 0.01$ and $m_\rho = 3$ TeV. Given a generic potential for the scalar doublet $\phi$ from Eq. (2.10), one finds that the limit $f/v \to 1$ is not physically accessible.

effective Lagrangian, arising from the expansion of Eq. (2.11), is roughly proportional to the techniquark mass and condensate (for small techniquark mass) by the Gell-Mann–Oakes–Renner relation [26], $m_\pi^2 f^2 \simeq 2m_q\sigma$. The techniquark mass and condensate appear in the profile of the bulk scalar field $X(z)$ as in Eq. (3.3), and indeed the Gell-Mann–Oakes–Renner relation can be derived from the holographic model [22]. Taking into account the mixing in Eq. (2.13), it follows that

$$m_\pi^2 \pi^a_\mu \pi^a_\mu = \frac{m_\pi^2}{v^2} \left[ f'^2 \Pi^a \Pi^a - 2f'f\Pi^a \Pi'^a + f^2 \Pi'^a \Pi'^a \right]. \quad (4.11)$$

The holographic calculation provides information on the $\Pi^2$ squared mass term alone, $m_\Pi^2 = m_\pi^2 f'^2/v^2$, with $v = 246$ GeV. This allows us to infer the physical pion mass in the full theory. In Fig. 2 we plot the physical technipion mass, $m_\pi$, as a function of $f/v$.

The analysis of the vector (technirho) and transverse axial vector sectors are more straightforward in this model. The lightest axial vector resonance will be heavier than the technirho, and of somewhat less interest in collider searches, so we will not discuss it further here. Considering only quadratic terms in the 5D action, Eq. (3.2), the vector part of the $\text{SU}(2)_L \times \text{SU}(2)_R$ gauge fields does not mix with either the axial part or with the bulk scalar $X$. The equation of motion for the transverse part of the vector field in the gauge
$V_z^a = 0$ is,
\[
\partial_z \left( \frac{1}{z} \partial_z V^a_\mu (q, z) \right) + \frac{q^2}{z} V^a_\mu (q, z) = 0,
\]
with boundary conditions $V_\mu (q, \epsilon) = 0$ and $F_{\mu z} (q, z_m) = 0$. The solutions are Bessel functions and the spectrum is given by zeroes of $J_0(q z_m)$. The mass of the lightest technirho in the model is therefore,
\[
m_\rho = \frac{2.405}{z_m},
\]
(4.13)

C. Decays of the Technirho

As long as phase space allows, the technirho will decay strongly about 100\% of the time. In our model the dominant decays of the neutral technirho $\rho^0$ will be to the longitudinal $W$ boson and to physical pions $\pi_p$. We will calculate the couplings $g_{\rho\pi_p\pi_p}$, $g_{\rho W_L W_L}$ and $g_{\rho W_L \pi_p}$ that appear in the effective 4D Lagrangian,
\[
L_{\rho XY} = ig_{\rho XY} \rho^\mu_0 \left[ (\partial_\mu X^+) Y^- - Y^+ (\partial_\mu X^-) \right],
\]
(4.14)
where $X$ and $Y$ represent either $W_L$ or $\pi_p$. By the Goldstone boson equivalence theorem we treat the longitudinal $W$ as a Goldstone scalar (an unphysical pion $\pi_a$) with mass $m_W$ coupled as in Eq. (4.14). The equivalence theorem is valid if the $W$ boson carries energy much larger than its mass, which is valid in our examples for all of the partial decays of the technirho except to $W_L \pi_p$ in the small region of parameter space for which $m_\pi \sim m_\rho$. In that regime, however, the branching fraction for decays to $W_L \pi_p$ is small anyway due to the reduced phase space, so our qualitative results remain unchanged.

A standard calculation of the partial decay widths gives,
\[
\Gamma_{\pi_p\pi_p} = \frac{1}{48\pi} m_\rho g_{\rho\pi_p\pi_p}^2 \left( 1 - \frac{m_\rho^2}{m_\rho^2} \right)^{3/2},
\]
(4.15)
\[
\Gamma_{W_L W_L} = \frac{1}{48\pi} m_\rho g_{\rho W_L W_L}^2 \left( 1 - \frac{m_W^2}{m_\rho^2} \right)^{3/2},
\]
(4.16)
\[
\Gamma_{W_L \pi_p} = \Gamma_{W_L \pi_p} = \frac{1}{48\pi} m_\rho g_{\rho W_L \pi_p}^2 \left( 1 + \frac{m_\pi^4}{m_\rho^4} + \frac{m_W^4}{m_\rho^4} - 2 \frac{m_W^2}{m_\rho^2} - 2 \frac{m_\pi^2}{m_\rho^2} - 2 \frac{m_\pi^2 m_W^2}{m_\rho^2} \right)^{3/2}.
\]
(4.17)
In terms of the mixing angles $\cos \theta \equiv f/v$, $\sin \theta = f'/v$, the couplings of the technirho are
related by,

\[ g_{\rho\pi\pi} = g_{\rho W_L W_L} \tan^2 \theta = g_{\rho W_L \pi} \tan \theta. \quad (4.18) \]

Defining \( \Gamma_{\text{Tot}} = \Gamma_{\pi^+ \pi^-} + 2 \Gamma_{W_L^+ \pi^-} + \Gamma_{W_L^+ W_L^-} \), the branching fractions \( \Gamma_{XY}/\Gamma_{\text{Tot}} \) depend only on the mixing angles, resonance masses and \( m_W \). To calculate the total width we need to know \( g_{\rho\pi\pi} \), which is obtained in the holographic model by integrating the 5D action over the extra dimension \( z \) for the lightest modes of \( V_\mu \) and \( \Pi_X \), and multiplying by the appropriate mixing angles to convert \( \Pi \), defined after Eq. (4.8), to the physical pion \( \pi_p \). The couplings arise from both the gauge field and scalar kinetic terms. In terms of the modes \( V_\mu(k,z) = \psi_\rho(z) V_\mu(k) \), \( \varphi(q,z) = \varphi(z) \Pi(q) \) and \( \Pi_X(q,z) = \pi_X(z) \Pi(q) \), where \( k^2 = m_\rho^2 \) and \( q^2 = m_\Pi^2 \) are the lowest eigenvalues for the bulk equations of motion, we obtain the \( \rho \Pi \Pi \) coupling:

\[ g_{\rho \Pi \Pi} = \frac{g_5}{\sqrt{2}} \int dz \psi_\rho(z) \left( \frac{\varphi'(z)^2}{g_5^2 z} + \frac{X_0(z)^2 (\pi_X(z) - \varphi(z)/\sqrt{2})^2}{z^3} \right). \quad (4.19) \]

The technirho wavefunction is normalized such that \( \int (dz/z) \psi_\rho(z)^2 = 1 \); the technipion wavefunctions \( \pi_X(z) \) and \( \varphi(z) \) are normalized such that the integral in Eq. (4.19) (without the prefactor) would equal 1 if \( \psi_\rho(z) \) were replaced by 1. These normalizations are chosen so that the modes are canonically normalized in the effective 4D theory \([22, 23]\). The remaining pion fields \( \Pi' \) do not couple strongly to the technirho in this model, so the contribution from the bulk 5D action completely determines the coupling of the technirho to physical technipions. Taking into account the mixing between the two sets of pions, the physical pion coupling is then given by,

\[ g_{\rho\pi\pi} = \sin^2 \theta g_{\rho \Pi \Pi}. \quad (4.20) \]

The branching fractions and total decay widths are plotted in Fig. 3 as a function of the mixing angle \( \cos \theta \) for a fixed \( z_m \) corresponding to a technirho mass \( m_\rho = 3 \) TeV. Note that the physical pions become heavy as \( \cos \theta \to 1 \), so the branching fractions to final states containing pions vanish in that limit for any \( z_m \).

V. CONCLUSIONS

The condensation of a fermion bilinear operator could provide a simple mechanism for the breaking of electroweak symmetry. Nature provides an example of this mechanism in
FIG. 3: Branching fractions and total decay width of the $\rho^0$, for $m_{\rho} = 3$ TeV, and $h=0.01$.

QCD: a quark condensate spontaneously breaks the global chiral symmetries of the theory, leading to the observed spectrum of pseudogoldstone bosons. Although the analogy to QCD is a source of inspiration for theories of dynamical electroweak symmetry breaking, it has also presented a challenge. A technicolor sector that is simply a scaled-up version of QCD produces a significant positive contribution to the electroweak $S$ parameter, leading to results that are in conflict with current experimental bounds.

There is no reason to believe a priori that nature should choose a technicolor sector that can be compared so easily to QCD. Since the technicolor gauge coupling is nonperturbative at the electroweak scale, however, alternative models have proved difficult to study. In the absence of simple scaling arguments that start with known results from hadronic physics, one in the past could only make the polite observation that the $S$ parameter might not be a problem in all models. The low-energy spectrum and dynamics of any specific proposal could not be determined with any degree of certainty.

Holography presents a way around this impasse by allowing one to work instead with an equivalent higher-dimensional theory that is weakly coupled. We have studied in this work a technicolor sector that is defined entirely in terms of its five-dimensional holographic dual, allowing us to deviate in a calculable way from the QCD-like limit. We have shown that appropriate choices of the parameters in the 5D theory exist where the otherwise leading
contribution to the $S$ parameter is small. Separation of the confining and chiral symmetry breaking scales in the holographic approach is possible (although there is not necessarily a simple 4D gauge theory description of such a theory), and provides a mechanism for reducing the $S$ parameter. We take this approach for simplicity, not out of necessity. The main issue that we address is the generation of fermion masses. We have added to the theory a (possibly composite) weak doublet field, whose vev is shifted from zero via its coupling to the technifermion condensate. Like a conventional Higgs doublet, the new field has Yukawa couplings to the Standard Model fermions, allowing their masses to be generated. Notably, the squared mass of the scalar is taken to be positive in our model, so that it is not the origin of electroweak symmetry breaking by itself.

The inclusion of this additional scalar doublet in our theory is not a radical proposition. It has been known for some time that models with strongly-coupled extended technicolor sectors can produce exactly this low-energy particle spectrum [19]. We have presented a holographic representation of a strongly-coupled ETC theory in which low energy properties of the theory are calculable. Our model presents one possible mechanism for the generation of masses in holographic technicolor models, an issue that has not yet been addressed in this context.

Using the AdS/CFT correspondence, we have calculated some basic quantities of phenomenological interest in our model, namely the $S$ parameter, the lightest technirho and technipion masses, and the dominant neutral technirho branching fractions. This provides a basis for future phenomenological studies. In particular, (i) a global electroweak analysis may provide useful constraints on the Higgs boson and technirho masses, (ii) scalar-mediated flavor-changing-neutral-currents, which were studied in similar effective theories [13, 14, 16], may also be re-examined in the present context, and (iii) the production of technipions, technivector and axial-vector resonances may be determined using holographic estimates of the relevant couplings. With the start of the LHC on the near horizon, a detailed collider simulation that incorporates these results would be well motivated.
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