Gauging of N=2 Supergravity Hypermultiplet and Novel Renormalization Group Flows

Klaus Behrndt\textsuperscript{a} and Mirjam Cveti\v{c}\textsuperscript{b}

\textsuperscript{a} Humboldt Universität zu Berlin, Institut für Physik, Invalidenstrasse 110, 10115 Berlin, Germany
\textsuperscript{b} Department of Physics and Astronomy, University of Pennsylvania, Philadelphia PA 19104-6396, USA
and
Institut Henri Poincaré, 11 rue Pierre et Marie Curie, F75231 Paris, Cedex 05, France

ABSTRACT

We provide the explicit gauging of all the $SU(2,1)$ isometries of one N=2 supergravity hypermultiplet, which spans $SU(2,1)/U(2)$ coset space parameterized in terms of two complex projective coordinate fields $z_1$ and $z_2$. We derive the full, explicit Killing prepotential that specifies the most general superpotential. As an application we consider the supersymmetric flow (renormalization group) equations for: (i) the flow from a null singularity to the flat, supersymmetric space-time and (ii) the flow that violates c-theorem with the superpotential crossing zero.

\textsuperscript{1}E-mail:behrndt@physik.hu-berlin.de
\textsuperscript{2}E-mail:cvetic@cvetic.hep.upenn.edu
1 Introduction

BPS domain wall configurations in five-dimensional $N = 2$ gauged supergravity provide a fertile ground to address (see, [1, 2, 3, 4] and references therein) in fundamental theory the candidate solutions for trapping of gravity [5] and within AdS/CFT correspondence [6] for the study of viable, non-singular, gravity duals of four-dimensional strongly coupled $N = 1$ supersymmetric field theories (see, [7, 8, 9] and references therein).

On the other hand, the gauging procedure of the five-dimensional supergravity has been rather poorly understood until recently. While the Abelian $U(1)_R$-gauging with the vector supermultiplets, only, was known for a while [10], the progress on non-Abelian gauging of vector and tensor multiplets was made only recently [11]. In addition, the most recent efforts are focused on the gauging of hypermultiplets [12, 13, 14, 15]. Nevertheless the full fledged gauging with even single hypermultiplet has not been done, yet.

The purpose of this paper is twofold. First, we provide an explicit gauging of one-hypermultiplet superfield of N=2 supergravity which spans the $SU(2,1)/U(2)$ coset space; we choose to parameterize in terms of projective coordinate fields $z_1$ and $z_2$. We find the explicit prepotential that specifies the most general super potential. The details are given in Section 2, and the explicit form of the eight prepotentials and the resulting superpotential is displayed in the Appendix.

Second, we employ this newly obtained theory to study novel domain wall solutions (which specify novel renormalization group (RG) flows of dual field theories). In particular, we focus on the potential, which involves the scalar fields of the hypermultiplets, only. In Section 3 we study examples of supersymmetric flows in the case of the Abelian gauging associated with the Killing directions in the subset of compact directions. In particular, we quantitatively analyze the flow from a null singularity to the flat space-time, reminiscent of the solution in [6], and another flow that violates the $c$-theorem and interpolates between “infra-red” anti-deSitter space time and null-singularity which is reminiscent of the solution in [16]. This latter solution was found as a geodesic extension the D3-brane configuration (behind the horizon) that is further compactified on a five-sphere; it was proposed as a possible candidate for a gravity trapping domain wall. Possible further applications and concluding remarks are given in Section 4.

While this work was in progress we were informed of a related work in progress [17], where hypermultiplet gauging has been pursued in a basis where the axionic $U(1)$ symmetry is manifest.
2 Gauging the Isometries of a Hypermultiplet

The focus of this paper will be on the scalar fields of the hyper-supermultiplet, only. Thus, as a concrete application we shall concentrate on the Abelian gauging of the most general hypermultiplet isometries. Of course, since we provide the explicit prepotential for the complete hypermultiplet isometry, generalizations to the non-Abelian gauging, that also involves vector supermultiplets is straightforward, and will be studied, along with applications to the RG flows, elsewhere [18].

We parameterize\(^{\text{3}}\) the coset space $SU(2,1)/U(2)$ of the universal hypermultiplet with two complex scalars $z_1$ and $z_2$ with the Kähler potential:

$$K = -\log(1 - |z_1|^2 - |z_2|^2),$$

with $|z_1|^2 + |z_2|^2 < 1$. The Kähler metric and the Kähler two-form take the form:

$$\partial_A \partial_B K dz^A dz^B = e^K \delta_{AB} dz^A dz^B + e^{2K} (\bar{z}_A dz^A) (z_B dz^B),$$

$$\partial_A \partial_B K dz^A \wedge dz^B = e^K \delta_{AB} dz^A \wedge dz^B + e^{2K} (\bar{z}_A dz^A) \wedge (z_B dz^B).$$

In the following Subsection we shall first focus on the quaternionic structure and the isometries. In the subsequent Subsections we provide the gauging of the isometries, determine the prepotentials associated with all the isometries (which are summarized in the Appendix) and provide the explicit form of the superpotential for the specific examples of Abelian gauging. These latter examples are then employed in Section 3 to illustrate novel RG flows.

2.1 The Quaternionic Structure and the Isometries

Let us start with a discussion of the quaternionic structure of this space. Following essentially the parameterization employed in [19], it turns out to be more convenient to introduce polar coordinates in the following way:

$$z_1 = r \left( \cos \theta/2 \right) e^{i(\psi+\varphi)/2}, \quad z_2 = r \left( \sin \theta/2 \right) e^{i(\psi-\varphi)/2}. \quad (3)$$

with $r \in [0, 1], \theta \in [0, \pi), \varphi \in [0, 2\pi)$ and $\psi \in [0, 4\pi)$. The Kähler metric then becomes:

$$\partial_A \partial_B K dz^A dz^B = \frac{dr^2}{(1-r^2)^2} + \frac{r^2}{4(1-r^2)} (\sigma_1^2 + \sigma_2^2) + \frac{r^2}{4(1-r^2)^2} \sigma_3^2, \quad (4)$$

Another possible parameterization involves the complex fields $S$ and $C$, where, e.g., the action of the axionic $U(1)$ symmetry is manifest, see [20]. There the Kähler potential is of the form $K = -\log(S + S - 2CC)$ and is related to that in Eq. (1) by the Kähler transformation combined with the reparameterization: $z_1 = (1 - S)/(1 + S)$ and $z_2 = 2C/(1 + S)$. The gauging procedure in either parameterization is expected to give equivalent results.
where the $SU(2)$ one-forms $(d\sigma_i + \frac{1}{2} \epsilon_{ijk} \sigma_j \wedge \sigma_k = 0)$ are given by:

$$\sigma_1 = \cos \psi \, d\theta + \sin \psi \, \sin \theta \, d\varphi ,$$

$$\sigma_2 = -\sin \psi \, d\theta + \cos \psi \, \sin \theta \, d\varphi ,$$

$$\sigma_3 = d\psi + \cos \theta \, d\varphi .$$

In terms of these one-forms we find the following expressions for the Vielbeine:

$$e^r = \frac{dr}{1 - r^2} , \quad e^3 = \frac{r}{2(1 - r^2)} \sigma_3 , \quad e^{1/2} = \frac{r}{2\sqrt{1 - r^2}} \sigma_{1/2} ,$$

which in the complex notation take the form:

$$v = \frac{1}{1 - r^2} (dr + \frac{i}{2} \sigma_3) , \quad u = -\frac{r}{2\sqrt{1 - r^2}} (\sigma_2 + i \sigma_1) .$$

The metric is then of the following form:

$$ds^2 = e^r e^r + e^1 e^1 + e^2 e^2 + e^3 e^3 = u\bar{u} + v\bar{v} .$$

Since this space is quaternionic, it allows for a triplet of the complex structure $J^i_{mn}$, giving rise to a triplet of Kähler two-forms: $\Omega^i = e^m J^i_{mn} \wedge e^n$, which can be written as:

$$\Omega^1 = \frac{r}{(1 - r^2)^{3/2}} [dr \wedge \sigma_1 + \frac{r}{2} \sigma_2 \wedge \sigma_3] ,$$

$$\Omega^2 = \frac{r}{(1 - r^2)^{3/2}} [-dr \wedge \sigma_2 + \frac{r}{2} \sigma_1 \wedge \sigma_3] ,$$

$$\Omega^3 = \frac{r}{(1 - r^2)^{2}} dr \wedge \sigma_3 + \frac{r^2}{2(1 - r^2)} \sigma_1 \wedge \sigma_2 .$$

The holonomy group of a quaternionic space is contained in $SU(2) \times SP(2m)$ and the Kähler two-forms have to be covariantly constant with respect to the $SU(2)$ connection $p^i$:

$$\nabla \Omega^i = d\Omega^i + \epsilon^{ijk} p^j \wedge \Omega^k = 0 , \text{ i.e. } \Omega^i \text{ preserves the quaternionic algebra.} \text{ For our specific case the } SU(2) \text{ connections are:}$$

$$p^1 = -\frac{\sigma_1}{\sqrt{1 - r^2}} , \quad p^2 = \frac{\sigma_2}{\sqrt{1 - r^2}} , \quad p^3 = -\frac{1}{2} (1 + \frac{1}{1 - r^2}) \sigma_3 ,$$

and fulfill the following relationship:

$$dp^i + \frac{1}{2} \epsilon^{ijk} p^j \wedge p^k = -\Omega^i .$$

The isometry group of this space is $SU(2, 1)$ whose the eight generators are specified by
the following eight Killing vectors, see also [19]:

\[
\begin{align*}
    k_1 &= \frac{1}{2i} \left[ z_2 \partial z_1 + z_1 \partial z_2 - \text{c.c.} \right], \\
    k_2 &= \frac{1}{2} \left[ -z_2 \partial z_1 + z_1 \partial z_2 + \text{c.c.} \right], \\
    k_3 &= \frac{1}{2i} \left[ -z_1 \partial z_1 + z_2 \partial z_2 - \text{c.c.} \right], \\
    k_4 &= \frac{1}{2i} \left[ z_1 \partial z_1 + z_2 \partial z_2 - \text{c.c.} \right], \\
    k_5 &= \frac{1}{2} \left[ (-1 + z_1^2) \partial z_1 + z_1 z_2 \partial z_2 + \text{c.c.} \right], \\
    k_6 &= \frac{1}{2} \left[ (1 + z_1^2) \partial z_1 + z_1 z_2 \partial z_2 - \text{c.c.} \right], \\
    k_7 &= \frac{1}{2} \left[ -z_1 z_2 \partial z_1 + (1 - z_2^2) \partial z_2 + \text{c.c.} \right], \\
    k_8 &= \frac{1}{2} \left[ z_1 z_2 \partial z_1 + (1 + z_2^2) \partial z_2 - \text{c.c.} \right].
\end{align*}
\]

The compact subgroup \( SU(2) \times U(1) \) is associated with the Killing vectors \((k_1, \cdots, k_4)\) and the non-compact isometries are parameterized by \((k_5, \cdots, k_8)\). The two Abelian isometries are the phase transformations of \(z_1\) and \(z_2\) and correspond to the Killing vectors \(k_3\) and \(k_4\), respectively. In addition, the action of the \(SU(2)\) subgroup corresponds to the “rotations” of the two complex coordinates \(z_{1,2}\) and the three generators, which are represented by \((k_1, k_2, k_3)\), fulfill the \(SU(2)\) algebra \([k_m, k_n] = i\epsilon_{mnp}k_p\).

### 2.2 Gauging of Abelian Isometries

We gauge only a single combination of the isometries \(k = a^n k_n\) associated with the graviphoton \(A\), i.e. the covariant derivative of the hyper scalar \(q^u\) becomes: \(dq^u \to dq^u + k^u A\), where \(u\) is an index of the quaternionic manifold. Supersymmetry with eight unbroken supercharges requires that the Killing vector has to be tri-holomorphic. (For details see [12, 14] and references therein.) This property is ensured if the Killing vector \(k_n\) can be expressed in term of a Killing prepotential \(P^i_n\):

\[
(\Omega^i \cdot k_n) = -dP^i_n - \epsilon^{ijk} p^j P^k_n,
\]

where “\(i\)” is the \(SU(2)\) index and the Kähler forms are defined in (9). We indeed derive the Killing prepotentials associated with all eight Killing vectors in (12). Their explicit form is given in the Appendix.

In addition, there is a further constraint coming from the fact, that the fermionic projector has to commute with the covariant derivative. This constraint, which was discussed as geodesic constraint in N=1,D=4 supergravity [22] and which we will discuss in more detail in [18], becomes for the case at hand: \(dq^u [P_n, \nabla_u P_m] a^u a^m = 0\) and reads in components

\[
\epsilon^{ijl} P^j_n \Omega^l_{uv} dq^u k^v a^n = 0.
\]

This constraint (14) puts severe restrictions on consistent superpotentials and especially seems to exclude a regular flow\(^4\).

\(^4\)In the previous version of this paper we derived a regular flow by considering Abelian gauging of compact
As a concrete example let us first start with a gauging of a linear combination of the two Abelian Killing vectors, only:

\[ k = a^3 k_3 + a^4 k_4 . \] (15)

where the special case \( a^3 + a^4 = 0 \) was already discussed in [15]. The (real-valued) superpotential \( W \), given by the determinant of the \( SU(2) \)-valued Killing prepotential [15], takes for the above Killing vector (15) the form:

\[
W^2 = \det(-\mathcal{P}) = -(P^1)^2 - (P^2)^2 - (P^3)^2 = \frac{1}{4(1-r^2)^2} \left[ 4(a^3)^2(1-r^2) \sin \theta^2 + (a^3(2-r^2) \cos \theta + a^4r^2)^2 \right].
\] (16)

Here \( \mathcal{P} \) is the matrix-valued Killing prepotential, i.e. \( \mathcal{P} \equiv P^i \tau_i \), where \( \tau_i \) are Pauli matrices \((i = 1,2,3)\). This potential is consistent with (14) at the critical values: \( \theta = 0, \pi \) and has one non-trivial fixed point at \( r = 0 \) (and for any value of \( \theta \)). At this fixed point only the \( r \)-direction is non-flat and we have

\[
(\partial^2 \log |W|)_0 = (1 + \frac{a^4}{a^3} \cos \theta) . \] (17)

If this expression is positive it is an UV attractive fixed point and an IR fixed point if it is negative. In both cases the flow goes towards a singularity, but in the latter case \( W \) passes a zero at \( \{\theta = 0, r^2 = \frac{4}{2-a^4/a^3}\} \). One can restrict oneself to the critical orbit with \( \theta = 0 \), where the superpotential becomes:

\[
W = \frac{a^3(2-r^2) + a^4r^2}{2(1-r^2)} = a^3 + \frac{(a^3 + a^4)r^2}{2(1-r^2)} . \] (18)

Next, we are going to discuss modifications if we gauge also the other compact Killing isometries, i.e. we consider the Killing vector:

\[ k = a^1 k_1 + a^2 k_2 + a^3 k_3 + a^4 k_4 . \] (19)

The superpotential \( W \) takes the form:

\[
W^2 = \left[ (P_1^1 a^n)^2 + (P_2^1 a^n)^2 + (P_3^1 a^n)^2 \right] , \] (20)

where the explicit form of the prepotentials \( P_n^i \) is given in the Appendix. In this case the constraint (14) implies non-trivial restrictions on the coefficients and consistent cases are:

(i) either \( a^1 = a^2 = 0 \) yielding the case discussed before or (ii) \( a^3 = a^4 = 0 \) combined with \( \theta = d\varphi = d\psi = 0 \), which yields the superpotential:

\[
W^2 = \frac{(a^1)^2 + (a^2)^2}{1-r^2} . \] (21)

However we did not take into account this constraint, which is not satisfied for the considered regular flow.
3 Application: Novel Renormalization Group Flows

The results of the previous Section and Appendix in principle allow for the full analysis of the supersymmetric extrema of the general potential as well as the study supersymmetric flows between such isolated extrema. However, due to the complexity of the potential we confine ourselves to special cases.

In a general case the linear combination of these isometries is weighed with the constant coefficients:

\[ a = (a^1, a^2, a^3, a^4, a^5, a^6, a^7, a^8). \]  

(22)

We analyze special cases that involve only isometries in the Cartan subalgebra, i.e. \( a^3, a^4 \neq 0 \). The corresponding superpotential was given in Section 2.2 [18]. A novel feature is that now the superpotential can pass a zero and in a special case this point can even be extremal, i.e. \( W = dW = 0 \). Namely, the zeros of the superpotential [18] take place at:

\[ r \equiv r_0 = \sqrt{-\frac{2}{1 - \delta}}, \]  

(23)

where \( \delta = \frac{a^4}{a^3} \) and thus the only value of \( \delta \), for which \( r_0 \leq 1 \), is \( \delta \leq -1 \). It is an extremum if \( \delta \to -\infty \), i.e. \( a^3 = 0, a^4 \neq 0 \) (which is obvious from the superpotential [18]). As pointed out at the end of Subsection 2.2, in this case we have only one extremum, which can be in the UV or IR regime (see eq. (17)). In the IR case, the superpotential necessarily passes zero along the flow.

In order to solve the flow equation, we introduce a coordinate system, where

\[ ds^2 = e^{2A} \left( -dt^2 + d\vec{x}^2 \right) + dy^2, \]  

(24)

and the flow equations become [11]:

\[ \partial_y A = -W, \quad \partial_y r = 3g_{rr} \partial_y W, \]  

(25)

and the solution for the superpotential [18] is

\[ r = e^{3(a^3 + a^4)(y-y_0)}, \quad e^{2A} = e^{-2a^3(y-y_0)} \sqrt{1 - e^{6(a^3 + a^4)(y-y_0)}}, \]  

(26)

Thus \( W = dW = 0 \) point is reached for the special case: \( a^3 = 0 \) and \( a^4 y \to -\infty \) (\( \delta = -\infty \)). This is a special example where the UV point is singular while the IR regime corresponds to the flat (supersymmetric) space-time. The solution resembles that of [11], where the UV region is formally singular (since it corresponds to a decompactification to a “dilatonic” D=7 space-time), while in the IR it becomes a flat D=5 space-time. That type of solutions
may provide useful supergravity duals for testing the IR behavior of N=1 supersymmetric field theories.

On the other hand, \( a^3 + a^4 < 0 \) and \( \delta < -1 \) corresponds to the flow that in the IR regime passes \( W = 0 \) and runs into the null-singularity with \( W \to -\infty \). This set of solutions is intriguing since it violates the c-theorem. It bears similarities with the solution in [14] describing the inside horizon region of D3-brane, that is subsequently compactified on a five-sphere. In the latter case the singularity is, however, naked; if one were able to identify a (stringy) mechanism to regulate this singularity such a domain wall solution could trap gravity.

Finally, let us mention that it is not enough to focus only on the superpotential. For example, consider the Killing vector

\[ k = a^3 k_3 + a^4 k_4 + a^5 k_5 \]  

where \( k_5 \) is a non-compact Killing vector. One finds that \( \partial_\theta W = 0 \) and the constraint (14) is satisfied if \( \theta = 0 \) yielding the superpotential:

\[ W = \frac{a^3 (r^2 - 2) + a^4 r^2 + 2a^5 r \sin \alpha}{2(1 - r^2)}, \]

with \( \alpha = \frac{1}{2} (\varphi + \psi) \). This superpotential has two extrema:

\[ r_+ = 0, \, \alpha = 0, \quad \text{and} \quad r_- = \frac{\gamma}{2} - \sqrt{\frac{\gamma^2}{4} - 1}, \, \alpha = \frac{\pi}{2}, \]

with \( \gamma = \frac{a^3 - a^4}{a^2} > 2 \). However, the first extremum is not a fixed point of the \( \alpha \)-flow, because the metric component \( g^{\alpha \alpha} \) has a pole at this point and one does not obtain an AdS vacuum.

How about the cases discussed before, is the extremum at \( r = 0, \, \theta = 0 \) perhaps also an artifact of the coordinate system? The explicit solution (26) already shows that there is good AdS vacuum, but one may also go back to the \( z_{1/2} \) coordinates with the metric given in (2). Using the relations \( r^2 = z_1 \bar{z}_1 + z_2 \bar{z}_2 \) and \( r^2 \cos \theta = z_1 \bar{z}_1 - z_2 \bar{z}_2 \) it is straightforward to transform the superpotential (14) in the \( z_{1/2} \) coordinates and the extremum at \( r = \theta = 0 \) translates into \( z_1 = z_2 = 0 \), which is a regular point for the corresponding metric (2).

### 4 Concluding Remarks and Open Avenues

In this paper we have provided the gauging of the full \( SU(2,1) \) isometry group of the universal hypermultiplet spanning the \( SU(2,1)/U(2) \) coset space. We have chosen the parameterization in terms of the complex projective space fields \( z_1 \) and \( z_2 \) and determined the full Killing prepotential.
We analyzed two sets of flows associated with the gauging of the Cartan subalgebra. It corresponds to the gauging of the single $U(1)$ and involves the flow from the UV singular point to the IR supersymmetric flat spacetime ($dW = W = 0$). Another flow, which corresponds to the gauging of both isometries in the Cartan subalgebra, provides a flow that violated the c-theorem.

Since we have derived the general prepotentials associated with the gauging of the full isometry group, one can now proceed with the general non-Abelian gauging which necessarily involves vector-multiplets as well. In this case the structure of the scalar potential is significantly more complicated and its analysis is deferred for further study [18].

The results here provide a stepping stone toward a general procedure to gauge an arbitrary number of hypermultiplets and the subsequent analysis of the vacuum structure for such general N=2 gauged supergravity theories that awaits further study.

**Acknowledgments**  We would like to thank Gianguido Dall’Agata for many enlightening discussions and suggestions and Anna Ceresole for pointing out typographical errors in the appendix of the original version. We would also like to thank him for informing us about the related work in progress [17]. M.C. would like to thank the Theoretical Particle Physics Group of the Humboldt University and the Center for Applied Mathematics and Theoretical Physics, Maribor, Slovenia for hospitality during the final stages of the project. The work is supported in part by a DFG Heisenberg Fellowship (K.B.), the U.S. Department of Energy Grant No. DOE-EY-76-02-3071 (M.C.), the NATO Linkage grant No. 97061 (M.C.) and the programme *Supergravity, Superstrings and M-theory* of the Centre Émile Borel of the Institut Henri Poincaré No. UMS-839-CNRS/UPMC (M.C.).

**References**

[1] K. Behrndt and M. Cvetič, *Supersymmetric domain-wall world from $D = 5$ simple gauged supergravity*, Phys. Lett. **B475**, 253 (2000) [hep-th/9909058].

[2] R. Kallosh, A. Linde and M. Shmakova, *Supersymmetric multiple basin attractors*, JHEP **9911**, 010 (1999) [hep-th/9910021].

[3] R. Kallosh and A. Linde, *Supersymmetry and the brane world*, JHEP **0002**, 005 (2000) [hep-th/0001071].

[4] K. Behrndt and M. Cvetič, *Anti-de Sitter vacua of gauged supergravities with 8 supercharges*, Phys. Rev. **D61**, 101901 (2000) [hep-th/0001159].
[5] L. Randall and R. Sundrum, *An alternative to compactification*, Phys. Rev. Lett. **83**, 4690 (1999) [hep-th/9906064].

[6] J. Maldacena, *The large N limit of superconformal field theories and supergravity*, Adv. Theor. Math. Phys. **2**, 231 (1998) [hep-th/9711200].

[7] D. Z. Freedman, S. S. Gubser, K. Pilch and N. P. Warner, *Renormalization group flows from holography supersymmetry and a c-theorem*, [hep-th/9904017].

[8] I. R. Klebanov and M. J. Strassler, *Supergravity and a confining gauge theory: Duality cascades and (chi)SB-resolution of naked singularities*, JHEP **0008**, 052 (2000) [hep-th/0007191].

[9] J. M. Maldacena and C. Nunez, *Towards the large n limit of pure N = 1 super Yang Mills*, [hep-th/0008001].

[10] M. Gunaydin, G. Sierra and P. K. Townsend, “Gauging The D = 5 Maxwell-Einstein Supergravity Theories: More On Jordan Algebras,” Nucl. Phys. **B253** 573 (1985).

[11] M. Gunaydin and M. Zagermann, *The vacua of 5d, N = 2 gauged Yang-Mills/Einstein/tensor supergravity: Abelian case*, Phys. Rev. **D62**, 044028 (2000) [hep-th/0002228].

[12] L. Andrianopoli, M. Bertolini, A. Ceresole, R. D’Auria, S. Ferrara, P. Fre and T. Magri, “N = 2 supergravity and N = 2 super Yang-Mills theory on general scalar manifolds: Symplectic covariance, gaugings and the momentum map,” J. Geom. Phys. **23** 111 (1997) [hep-th/9605032].

[13] A. Lukas, B. A. Ovrut, K. S. Stelle and D. Waldram, “Heterotic M-theory in five dimensions,” Nucl. Phys. **B552** 246 (1999) [hep-th/9806051].

[14] A. Ceresole and G. Dall’Agata, *General matter coupled N = 2, D = 5 gauged supergravity*, Nucl. Phys. **B585**, 143 (2000) [hep-th/0004111].

[15] K. Behrndt, C. Herrmann, J. Louis and S. Thomas, *Domain walls in five dimensional supergravity with non-trivial hypermultiplets*, [hep-th/0008112].

[16] M. Cvetič, H. Lü and C. N. Pope, *Localized gravity in the singular domain wall background?* [hep-th/0002054].

[17] A. Ceresole, G. Dall’Agata, R. Kallosh and A. Van Proeyen, work in progress.
[18] K. Behrndt and M. Cvetič, work in progress.

[19] R. Britto-Pacumio, A. Strominger and A. Volovich, *Holography for coset spaces*, JHEP **9911**, 013 (1999) [hep-th/9905211].

[20] S. Ferrara and S. Sabharwal, “Quaternionic Manifolds For Type II Superstring Vacua Of Calabi-Yau Spaces,” Nucl. Phys. **B332** 317 (1990).

[21] A. W. Peet and J. Polchinski, *UV/IR relations in AdS dynamics*, Phys. Rev. D **59**, 065011 (1999) [hep-th/9809022].

[22] M. Cvetič, S. Griffies and S. Rey, *Static domain walls in N=1 supergravity*, Nucl. Phys. **B381**, 301 (1992) [hep-th/9201007].
Appendix: Killing Prepotentials

The Killing prepotential associated with gauging of the isometries in the compact subgroup $SU(2) \times U(1)$, specified by the Killing vectors $(k_1, \cdots, k_4)$ defined in \[12\]:

$$P_1 = \frac{1}{\sqrt{1-r^2}} \begin{pmatrix} 
\cos \psi \sin \varphi + \cos \theta \sin \psi \cos \varphi \\
\sin \psi \sin \varphi - \cos \theta \cos \psi \cos \varphi \\
-\frac{2-r^2}{2\sqrt{1-r^2}} \sin \theta \cos \varphi 
\end{pmatrix},$$

$$P_2 = \frac{1}{\sqrt{1-r^2}} \begin{pmatrix} 
\cos \psi \cos \varphi - \cos \theta \sin \psi \sin \varphi \\
\sin \psi \cos \varphi + \cos \theta \cos \psi \sin \varphi \\
\frac{2-r^2}{2\sqrt{1-r^2}} \sin \theta \sin \varphi 
\end{pmatrix},$$

$$P_3 = \frac{1}{\sqrt{1-r^2}} \begin{pmatrix} 
\sin \psi \sin \theta \\
-\cos \psi \sin \theta \\
\frac{2-r^2}{2\sqrt{1-r^2}} \cos \theta 
\end{pmatrix}, \quad P_4 = -\frac{r^2}{2(1-r^2)} \begin{pmatrix} 
0 \\
0 \\
1 
\end{pmatrix}. \quad (30)$$

The prepotentials associated with the non-compact isometries, specified by the Killing vectors $(k_5, \cdots, k_8)$ defined in \[12\], are of the form:

$$P_5 = -\frac{r}{1-r^2} \begin{pmatrix} 
\sqrt{1-r^2} \sin \frac{\theta}{2} \cos \frac{\varphi - \psi}{2} \\
-\sqrt{1-r^2} \sin \frac{\theta}{2} \sin \frac{\varphi - \psi}{2} \\
\cos \frac{\theta}{2} \sin \frac{\varphi + \psi}{2} 
\end{pmatrix}, \quad P_6 = \frac{r}{1-r^2} \begin{pmatrix} 
\sqrt{1-r^2} \sin \frac{\theta}{2} \sin \frac{\varphi - \psi}{2} \\
\sqrt{1-r^2} \sin \frac{\theta}{2} \cos \frac{\varphi - \psi}{2} \\
-\cos \frac{\theta}{2} \cos \frac{\varphi + \psi}{2} 
\end{pmatrix},$$

$$P_7 = -\frac{r}{1-r^2} \begin{pmatrix} 
\sqrt{1-r^2} \cos \frac{\theta}{2} \cos \frac{\varphi + \psi}{2} \\
\sqrt{1-r^2} \cos \frac{\theta}{2} \sin \frac{\varphi + \psi}{2} \\
\sin \frac{\theta}{2} \sin \frac{\varphi - \psi}{2} 
\end{pmatrix}, \quad P_8 = \frac{r}{1-r^2} \begin{pmatrix} 
\sqrt{1-r^2} \cos \frac{\theta}{2} \sin \frac{\varphi + \psi}{2} \\
-\sqrt{1-r^2} \cos \frac{\theta}{2} \cos \frac{\varphi + \psi}{2} \\
-\sin \frac{\theta}{2} \cos \frac{\varphi - \psi}{2} 
\end{pmatrix}. \quad (31)$$