de Sitter Vacua, Renormalization and Locality

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Abstract: We analyze the renormalization properties of quantum field theories in de Sitter space and show that only two of the maximally invariant vacuum states of free fields lead to consistent perturbation expansions. One is the Euclidean vacuum and the other can be viewed as an analytic continuation of Euclidean functional integrals on $RP^d$. The corresponding Lorentzian manifold is the future half of global de Sitter space with boundary conditions on fields at the origin of time. We argue that the perturbation series in this case has divergences at the origin which render the future evolution of the system indeterminate, without a better understanding of high energy physics.
1. Introduction

In the recent outbreak of interest in de Sitter spacetimes, attention has been drawn again to the existence of a one (complex) parameter family of vacuum states (called the $\alpha$-vacua) for free quantum fields in de Sitter spacetime[6]. Experts in the field have long harbored a vague suspicion that only the standard Euclidean vacuum was sensible, but until now there has been no conclusive argument to this effect. The purpose of this note is to present one.

The argument is, in essence, very simple. Propagators in quantum field theory are singular on the light cone. The propagators in the $\alpha$-vacua are linear superpositions of a Euclidean\footnote{We use the short phrase Euclidean propagator to denote the propagator of a field in dS space, which is obtained by analytic continuation of the Euclidean functional integral on a sphere.} propagator evaluated between two points $x, y$, and the same propagator evaluated between $x$ and the antipodal point to $y$, $y^A$. The Feynman diagrams of interacting quantum field theory contain products of propagators between the same two points. These are not distributions, and a subtraction procedure must be supplied to
define them. The key point of standard renormalization theory is that the subtractions
all take the form of local contributions to the effective action, and can thus be viewed
as renormalizations of couplings in the theory. We will show by simple examples that
in the $\alpha$-vacua this is no longer true. The subtractions include non-local contributions
to the effective action of the form $\textit{e.g.}$

$$\delta S = \delta \lambda \int \phi(x) \phi(x^A)$$

(1.1)

where $\delta \lambda$ is a divergent constant. Thus, renormalized interacting field theory in a
generic $\alpha$ vacuum is intrinsically non-local, and presumably has no sensible physical
interpretation.

There are only two values of $\alpha$ for which this catastrophe is avoided. The first is
$\Re(\alpha) = a = -\infty$ which gives the standard Euclidean vacuum and has no antipodal
singularity. The second is $\alpha = 0$, which is the unique vacuum state invariant under
the antipodal map. The Green’s function in this vacuum appears to be the analytic
continuation of a Euclidean functional integral on $RP^{d2}$. In this vacuum state, which
we call the $\textit{antipodal vacuum}$ we must view the Lorentzian spacetime manifold as the
orbifold of de Sitter space by the antipodal map. Every point is identified with its
antipode, and the interaction 1.1 is local. From a physical point of view we have a
manifold with a past spacelike singularity and an asymptotic de Sitter future. We call
this spacetime the $\textit{antipodal universe}$.

In discussions of inflationary cosmology, one often invokes a Quantum No-Hair
Theorem for de Sitter space. According to this theorem, generic initial states of
quantum fields in dS space, evolve into a state indistinguishable from the Euclidean
vacuum after enough e-foldings. A crucial assumption in this theorem, is that the initial
state approaches the Euclidean vacuum for very high angular momentum modes (in
global coordinates - in planar coordinates we would say ordinary momentum modes).
Modes of any finite comoving wave number are redshifted to a size larger than the
horizon volume after a sufficient number of e-foldings, and are no longer observable by
a local measurement. If the initial state is the Euclidean vacuum for sufficiently high
momentum modes, then the local observer will eventually see a state indistinguishable
from the Euclidean vacuum.

The state implied by the orbifold boundary conditions does not satisfy the condi-
tions of this theorem. In global coordinates the Euclidean vacuum for a boson field is
a Gaussian with time dependent covariance, for each angular momentum mode. The
orbifold boundary conditions imply instead that the initial wave function of the even

\footnote{To our knowledge, E. Witten\cite{5} was the first to point out the significance of this special value of alpha and its Euclidean interpretation.}
angular momentum states is a field eigenstate, while that of the odd modes is an eigenstate of the canonical momentum. These are non-normalizable states, for each angular momentum mode, and differ from the Euclidean vacuum for arbitrarily large angular momentum. They do not obey the de Sitter no hair theorem. Thus, the future evolution of the antipodal universe depends on the initial conditions.

We argue further that the initial conditions may be subject to infinite ultraviolet corrections in higher orders of perturbation theory. These are the standard UV divergences of fixed time Schrödinger picture states in quantum field theory. If this were true, we would have to claim that, without a nonperturbative understanding of the state near the orbifold singularity, we could not make reliable predictions in the antipodal universe.

These considerations cast doubt on the identification of the Lorentzian antipodal vacuum with the analytic continuation of a Euclidean functional integral on $\mathbb{R}P^d$. The latter is renormalized by the standard counterterms for quantum field theory on smooth manifolds without boundary. It may be that the boundary conditions defined by the $\mathbb{R}P^d$ functional integral are a fixed point of the boundary renormalization group of the Lorentzian orbifold field theory, but we have not done enough computations to verify this conjecture.

All of these arguments are made in the context of quantum field theory in a fixed spacetime background. In quantum gravity, we have the additional problem that the antipodal initial state has infinite energy density, which leads us to expect a large back reaction. A much more extensive discussion of the back reaction problem in $\alpha$-vacua will be presented in [4].

Our conclusion is that only the Euclidean vacuum state has a chance of describing sensible physical processes in de Sitter space. The rest of this note is devoted to calculations which explicate the argument made above.

We note that after we submitted this paper to arXiv.org, two related papers appeared which have some overlap with our work. The first, by Einhorn and Larsen[1], discusses aspects of higher loop graphs in $\alpha$ vacua, and also concludes that these are generally ill-defined. The second[2] discusses the $\mathbb{Z}_2$ orbifold of dS space (and points out that it was first introduced long ago by Schrödinger). It is not clear to us that their definition of the quantum theory is the same as ours. They do not discuss divergences near the origin of time in this system.

2. Interacting Scalar Field Theory in an $\alpha$ Vacuum

In this section we will present a calculation of the two point function in a simple scalar field theory. We hope the reader will realize that our conclusions are quite
general. In particular, we began this project by computing the two point function of the renormalized stress tensor in an $\alpha$ vacuum. This computation would enter into any perturbative theory of quantum gravity in de Sitter space. This calculation is more divergent than any we will actually present, but exhibits the same non-locality that we find in our simple example. We decided that the extra indices and the subtleties of covariance would only distract the reader from the main point.

### 2.1 Notation

In the following we will consider 4-dimensional de Sitter space $dS^4$. It may be realized as the manifold

$$-X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 = l^2$$

(2.1)

embedded in the 5-dimensional Minkowski space $M^{4,1}$. We will use lower case $x$ to indicate 4-dimensional coordinates on $dS^4$ and upper case $X$ to denote embedding coordinates. We will denote the antipodal points by $X^A \equiv -X$. Henceforth we will set $l = 1$.

We are considering an interacting scalar field theory in $dS^4$ with action

$$S = \frac{1}{2} \int d^4x (-g(x))^\frac{1}{2} \left[ (\nabla \phi)^2 - m^2 \phi^2 - \frac{\lambda}{3!} \phi^3 \right]$$

(2.2)

In $dS^4$ there is a one complex parameter, $\alpha$, family of dS invariant vacua [6] that we will denote $|\alpha\rangle$. The associated de Sitter invariant family of two point Wightman functions is

$$\langle \alpha | \phi(x) \phi(y) | \alpha \rangle = W_\alpha(x, y) =$$

$$n^2 \left( W_e(x, y) + e^{\alpha + \alpha^*} W_e(y, x) + e^{\alpha} W_e(x, x^A) + e^{\alpha^*} W_e(x^A, y) \right)$$

(2.3)

with

$$\alpha \in \mathcal{C}$$

$$\Re(\alpha) = a < 0$$

(2.4)

$$n = n(\alpha) = \frac{1}{\sqrt{1 - e^{\alpha + \alpha^*}}}$$

(2.5)

Here we use the Euclidean two point Wightman function $W_e(x, y)$ defined in [7]. The Euclidean Wightman function and vacuum correspond to $a = -\infty$. 

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2.2 Computation

In this section we will compute a term in the 1-loop effective action, in a general $\alpha$-vacuum. The computation will lead to divergent non-local counterterms. Only the Euclidean vacuum produces a completely local counterterm action.

The 1-loop, two point contribution to the effective action in our simple field theory is

$$\Gamma(\phi) \sim \int d^4x d^4y \, (-g(x))^{\frac{1}{2}}(-g(y))^{\frac{1}{2}} \phi_d(x) F_\alpha(x, y) F_\alpha(x, y) \phi_d(y)$$

(2.6)

The Feynman propagator $F_\alpha(x, y)$ can be expressed in terms of the Wightman functions and the parameter $\alpha$ as

$$F_\alpha(x, y) = \Theta(x_0 - y_0) W_\alpha(x, y) + \Theta(y_0 - x_0) W_\alpha(y, x)$$

(2.7)

$$W_\alpha(x, y) = n^2 (W_e(x, y) + e^{\alpha + \alpha^*} W_e(y, x) + e^\alpha W_e(x, y^A) + e^{\alpha^*} W_e(x^A, y))$$

(2.8)

with

$$n = n(\alpha) = \frac{1}{\sqrt{1 - e^\alpha + \alpha^*}}$$

(2.9)

The behavior of the two point Euclidean Wightman function near the light cone is for $dS^4$

$$W_e(x, y) \sim \frac{C}{(x_0 - y_0 - i \epsilon)^2 - (x_s - y_s)^2}$$

(2.10)

where $x = (x_0, x_s)$, $y = (y_0, y_s)$ and $C$ is a constant whose value is not relevant for the following considerations.

We will show now that in $F_\alpha^2(x, y)$ only the terms $W_e^2$, having a singular behavior near the light cone of the form

$$W_e^2 \sim \frac{C^2}{(x - y)^4}$$

(2.11)

$$W_e^2 \sim \frac{C^2}{(x^A - y)^4}$$

(2.12)

$$W_e^2 \sim \frac{C^2}{(x - y^A)^4}$$

(2.13)
\[ W_e^2 \sim \frac{C^2}{(x - y^A)^2(x^A - y)^2} \]  

(2.14)

contribute to the divergent part of the effective action. In these equations, we suppress the \( i \epsilon \) prescription because it is not relevant at this point. Considering \( W^2(x, y) \) as a distribution on the space of test function \( \phi(x) \) we have

\[
T_{W^2}[\phi] = \int d^4x \, W_e^2(x, y) \phi(x)
\]

\[
= \int d^4x \left( W_e^2(x, y) - \frac{C^2}{(x - y)^4} \right) \phi(x) + \int d^4x \frac{C^2 \phi(x)}{(x - y)^4}
\]

\[
= \int d^4x \left( W_e^2(x, y) - \frac{C^2}{(x - y)^4} \right) \phi(x)
\]

\[
+ \int d^4x \frac{C^2 (\phi(x) - \phi(y))}{(x - y)^4} + \phi(y) \int d^4x \frac{C^2}{(x - y)^4}
\]

\[
= Regular + \int d^4z \, \delta(z - y) \phi(z) \int d^4x \frac{C^2}{(x - y)^4}
\]

(2.15)

where the regular part does not contribute to the divergent part of the effective action. Similarly, in the terms which contain squares of Wightman functions evaluated between points and their antipodes, we have

\[
T_{W^2}[\phi] = \int d^4x \, W_e^2(x^A, y) \phi(x)
\]

\[
= Regular + \int d^4z \, \delta(z - y^A) \phi(z) \int d^4x \frac{C^2}{(x^A - y)^4}
\]

(2.16)

and

\[
T_{W^2}[\phi] = \int d^4x \, W_e^2(x, y^A) \phi(x)
\]

\[
= Regular + \int d^4z \, \delta(z - y^A) \phi(z) \int d^4x \frac{C^2}{(x - y^A)^4}
\]

(2.17)

Similarly
\[ T_{W^2}[\phi] = \int d^4x W_e(x^A, y) W_e(x, y^A) \phi(x) \]

\[ = \text{Regular} + \int d^4z \delta(z - y^A) \phi(z) \int d^4x \frac{C^2}{(x^A - y)^2 (x - y^A)^2} \]  \hspace{1cm} (2.18)

All the other terms in \( W^2_e \) are regular and do not contribute to the divergent part of the effective action.

After eliminating the regular terms in \( F^2_\alpha(x, y) \) and doing the replacements \( \Theta(x_0 - y_0) (y_0 - x_0) \to 0 \), \( \Theta(x_0 - y_0)^2 \to \Theta(x_0 - y_0) \), and \( \Theta(y_0 - x_0)^2 \to \Theta(y_0 - x_0) \), we get

\[ F_\alpha(x, y)^2 = n^4 \Theta(x_0 - y_0) W_e(x, y)^2 + e^{2\alpha + 2\alpha^*} n^4 \Theta(y_0 - x_0) W_e(x, y)^2 \]

\[ + e^{2\alpha^*} n^4 \Theta(x_0 - y_0) W_e(x, y^A)^2 + 2 e^{\alpha + \alpha^*} n^4 \Theta(x_0 - y_0) W_e(x^A, y) W_e(x, y^A) \]

\[ + e^{2\alpha} n^4 \Theta(x_0 - y_0) W_e(x^A, y)^2 + 2 e^{\alpha + \alpha^*} n^4 \Theta(x_0 - y_0) W_e(x, y) W_e(y, x) \]

\[ + 2 e^{\alpha + \alpha^*} n^4 \Theta(y_0 - x_0) W_e(x, y) W_e(y, x) + e^{2\alpha + 2\alpha^*} n^4 \Theta(x_0 - y_0) W_e(y, x)^2 \]

\[ + n^4 \Theta(y_0 - x_0) W_e(y, x)^2 + e^{2\alpha^*} n^4 \Theta(y_0 - x_0) W_e(y, x^A)^2 \]

\[ + 2 e^{\alpha + \alpha^*} n^4 \Theta(y_0 - x_0) W_e(y, x^A) W_e(y^A, x) + e^{2\alpha} n^4 \Theta(y_0 - x_0) W_e(y^A, x)^2 \]  \hspace{1cm} (2.19)

Replacing the \( W_e \) terms with their singular behavior near the light cone, we find

\[ F_\alpha(x, y)^2 \sim \]

\[ \delta(x - y) \left( \frac{C^2 n^4 \Theta(x_0 - y_0)}{((x_0 - y_0 - i \epsilon)^2 - (x_s - y_s)^2)^2} + \frac{C^2 e^{2\alpha + 2\alpha^*} n^4 \Theta(x_0 - y_0)}{((y_0 - x_0 - i \epsilon)^2 - (x_s - y_s)^2)^2} \right) \]

\[ + \frac{2 C^2 e^{\alpha + \alpha^*} n^4 \Theta(x_0 - y_0)}{((x_0 - y_0 - i \epsilon)^2 - (x_s - y_s)^2) \left( (y_0 - x_0 - i \epsilon)^2 - (x_s - y_s)^2 \right)} \]
\[
\begin{aligned}
&+ \frac{C^2 e^{2\alpha+2\alpha^*} n^4 \Theta(y_0 - x_0)}{((x_0 - y_0 - i\epsilon)^2 - (x_s^4 - y_s^4)^2)} + \frac{C^2 n^4 \Theta(y_0 - x_0)}{((y_0 - x_0 - i\epsilon)^2 - (x_s^4 - y_s^4)^2)} \\
&+ \frac{2 C^2 e^{\alpha+\alpha^*} n^4 \Theta(y_0 - x_0)}{((x_0^4 - y_0 - i\epsilon)^2 - (x_s^4 - y_s^4)^2)}
\end{aligned}
\]

\[
\begin{aligned}
+\delta(x - y^A) \left( \frac{C^2 e^{2\alpha} n^4 \Theta(x_0 - y_0)}{((x_0^4 - y_0 - i\epsilon)^2 - (x_s^4 - y_s^4)^2)} + \frac{C^2 e^{2\alpha^*} n^4 \Theta(x_0 - y_0)}{((x_0^4 - y_0 - i\epsilon)^2 - (x_s^4 - y_s^4)^2)} \right) \\
+ \frac{2 C^2 e^{\alpha+\alpha^*} n^4 \Theta(y_0 - x_0)}{((y_0 - x_0^4 - i\epsilon)^2 - (x_s^4 - y_s^4)^2)} \left( (y_0^4 - x_0 - i\epsilon)^2 - (x_s^4 - y_s^4)^2 \right) + \frac{C^2 e^{2\alpha} n^4 \Theta(y_0 - x_0)}{((y_0^4 - x_0 - i\epsilon)^2 - (x_s^4 - y_s^4)^2)}
\end{aligned}
\]

(2.20)

The \(\delta(x - y^A)\) term gives rise to a non local, divergent, contribution to the effective action. The coefficient of \(\delta(x - y^A)\) is

\[
\begin{aligned}
&+ \frac{C^2 e^{2\alpha} n^4 \Theta(x_0 - y_0)}{((x_0^4 - y_0 - i\epsilon)^2 - (x_s^4 - y_s^4)^2)} + \frac{C^2 e^{2\alpha^*} n^4 \Theta(x_0 - y_0)}{((x_0^4 - y_0 - i\epsilon)^2 - (x_s^4 - y_s^4)^2)} \\
&+ \frac{2 C^2 e^{\alpha+\alpha^*} n^4 \Theta(y_0 - x_0)}{((x_0^4 - y_0 - i\epsilon)^2 - (x_s^4 - y_s^4)^2)} \left( (y_0^4 - x_0 - i\epsilon)^2 - (x_s^4 - y_s^4)^2 \right) + \frac{C^2 e^{2\alpha} n^4 \Theta(y_0 - x_0)}{((y_0^4 - x_0 - i\epsilon)^2 - (x_s^4 - y_s^4)^2)}
\end{aligned}
\]
\[
2 C^2 e^{\alpha + \alpha^*} n^4 \Theta(y_0 - x_0) \\
\left(\frac{(y_0 - x_0 - i \epsilon)^2 - (x_s - y_s)^2}{(y_0^A - x_0 - i \epsilon)^2 - (x_s - y_s^A)^2}\right)
\] (2.21)

After the substitutions \((x_0^A, x_s^A) \rightarrow (-x_0, -x_s), (y_0^A, y_s^A) \rightarrow (-y_0, -y_s), \Theta(x_0 - y_0) + \Theta(y_0 - x_0) \rightarrow 1\),

we find that the non-local part of the divergent counterterm is

\[
\left(\frac{C^2 e^{2\alpha} n^4}{((x_0 + y_0 + i \epsilon)^2 - (x_s + y_s)^2)^2} + \frac{C^2 e^{2\alpha^*} n^4}{((x_0 + y_0 - i \epsilon)^2 - (x_s + y_s)^2)^2}\right)
\]

\[
+ \frac{2 C^2 e^{\alpha + \alpha^*} n^4}{((x_0 + y_0 + i \epsilon)^2 - (x_s + y_s)^2)((x_0 + y_0 - i \epsilon)^2 - (x_s + y_s)^2)}
\] (2.22)

The three terms in this expression are different because they have distinct \(i\epsilon\) prescriptions and therefore diverse poles in the complex plane.

As a consequence to eliminate all the divergent, non local terms in the effective action we must set

\[
e^{\alpha} = e^{a+ib} = 0
\] (2.23)

\[
e^{\alpha^*} = e^{a-ib} = 0
\] (2.24)

\[\Rightarrow e^a = 0
\] (2.25)

\[\Rightarrow a = -\infty
\] (2.26)

This corresponds to the choice of the Euclidean vacuum as previously stated. We should remark that the constant \(n = \frac{1}{\sqrt{1-e^{a+\alpha^*}}} = \frac{1}{\sqrt{1-e^{2\alpha}}}\) can never be zero because the family of de Sitter invariant vacua is defined by \(\alpha \in \mathbb{C}, \Re(\alpha) = a < 0\). There is however another way to obtain a system with local effective action. The nonlocalities are all products of fields at points and their antipodes. For \(\alpha = 0\) we can interpret the Green’s functions as living on an orbifold of dS space, the antipodal universe, in which a point is identified with its antipode. On this spacetime, all of our counterterms can be viewed as local operators.
Witten[5] has suggested that for this value of $\alpha$ the Green’s functions can be viewed as analytic continuations of the Euclidean functional integral on the real projective space $RP^4$. Since $RP^4$ is a smooth manifold without boundary, this Euclidean functional integral should be renormalized by the same local counterterms that define the field theory on the sphere. We will discuss this interpretation in the next section.

3. The Wave Functional in the Antipodal Vacuum

We have seen that, with the exception of the Euclidean and Antipodal vacua, field theory in an $\alpha$ vacuum cannot be renormalized by local counterterms. We now want to investigate whether the Antipodal vacuum forms the basis for a sensible quantum field theory. Certainly, the Euclidean functional integral on $RP^d$ is well defined. However, it is not immediately apparent that the Green’s functions defined by this functional integral have a Hamiltonian interpretation. The conventional reflection positivity argument requires the reflected Euclidean points to be distinct from the points themselves.

Indeed, it would appear that the Lorentzian version of the Antipodal universe requires more renormalization than the corresponding Euclidean functional integral. $RP^d$ is a smooth manifold without boundary and the Euclidean functional integral on this manifold will be renormalized by the same local subtractions that are required for the Euclidean functional integral on the sphere. However, the Lorentzian version of the theory describes the evolution of a quantum field theory starting from a fixed state at a sharp time. It has been known since the work of Symanzik[3] that in renormalizable quantum field theories, the wave functional at a sharp time requires additional renormalizations, above and beyond those which render the Green’s functions finite. In modern parlance, the sharp time state introduces a boundary into the system and one must introduce counterterms for all relevant boundary operators at the fixed point of the bulk renormalization group.

Thus, it would seem that, if field theory is to be defined in the antipodal vacuum it requires additional definitions to determine the initial state. These remarks also seem to indicate that the connection between the Lorentzian theory and the Euclidean theory on $RP^d$ must somehow be valid only in the absence of boundary renormalizations. We have remarked that the Euclidean antipodal Green’s functions do not seem to require additional subtractions. It is possible that this means that the Lorentzian boundary conditions implied by continuation from $RP^d$ are automatically fixed points of the boundary renormalization group.

Indeed, the above discussion of boundary renormalization is valid for boundary conditions of the form $\phi(t = 0, x) = \phi_0(x)$, which would define the Schrodinger wave functional. We then think of the orbifold boundary condition as a restriction on the
allowed Schrödinger functionals. Perhaps, since the Lorentzian orbifold does not have a geometric boundary, all boundary counterterms will vanish in such a state\textsuperscript{3}. We have not been able to determine the validity of such a conjecture. In particular, in general field theories there would seem to be marginal and relevant boundary operators which are not projected out by the orbifold condition. We do not understand why additional counterterms proportional to these relevant operators are not generated by the Lorentzian Feynman rules.

Even apart from these additional renormalization effects, the state defined by the antipodal boundary conditions is somewhat singular. Classically, the field is required to be invariant under simultaneous reflection in the global coordinate time and spatial sphere. If we expand the field into spherical harmonics, then (for free field theory in dS space) each mode $\phi_L$ is a time dependent harmonic oscillator, with a frequency that is even under reflection about the point of minimal size. Under reflection in the sphere, $\phi_L \to (-1)^L \phi_L$. Thus, invariance under the antipodal map is equivalent to the quantum mechanical statement that the initial state is annihilated by $\phi_L$ for odd $L$ and by the conjugate momentum $\Pi_L$ for even $L$. The quantum system is then studied as a collection of time dependent oscillators, with these initial conditions, on the interval $t \in [0, \infty]$. Note that the quantum state defined by this boundary condition differs from the Euclidean state of the same system even for $L \to \infty$. The dS No-Hair Theorem is not applicable, and this state does not approach the Euclidean vacuum at large times.

Finally, we note that the deviation from the Euclidean vacuum for large $L$ also implies that the matrix elements of the renormalized stress tensor between states of the form

$$a_{L_1}^\dagger \ldots a_{L_n}^\dagger |A> \quad (3.1)$$

where $|A>$ is the antipodal vacuum and the operators are its associated creation operators, will blow up as $t \to 0$.

These additional divergences have little to do with renormalization. They are more analogous to the singularities at particular places in Lorentzian momentum space that one finds in the analytic continuation of renormalized Euclidean Green’s functions in any field theory. That is, they represent real physical processes, rather than virtual contributions to the effective action.

The consequence of these remarks is that, although the antipodal vacuum does not suffer from the renormalization problems of the generic $\alpha$ vacuum, its physics is not under control at $t = 0$.

\textsuperscript{3}TB thanks M. Douglas for a discussion of this point.
4. Conclusion

We have investigated the perturbative renormalizability of quantum field theories in rigid dS space, when the vacuum state of the free fields is chosen to be one of the non-Euclidean, dS invariant vacua. In general the renormalization program fails. Non-local counterterms, involving products of fields at both points and their antipodes, are necessary to render the interacting Green’s functions finite. Even if it were possible to prove that this nonlocal renormalization program could be carried out to all orders (which is by no means obvious), the resulting theory would probably not have a Hamiltonian interpretation. We consider this as evidence that quantum field theory in generic $\alpha$-vacua does not make sense.

Apart from the Euclidean vacuum, the antipodal vacuum is the only one where the non-local renormalization problem can be avoided. This vacuum state can be interpreted in terms of field theory on an orbifold of dS space, in which the non-local operators are local. It is possible that the resulting theory is just the analytic continuation of the Euclidean functional integral on $RP^d$, though one would have to do a more thorough study of boundary renormalizations in the Lorentzian orbifold in order to prove this.

Independently of this renormalization problem, there are clearly divergent matrix elements of local operators like the stress tensor on the fixed plane $t = 0$ of the Lorentzian orbifold. If we tried to couple gravity to the system this would lead to large back reaction effects. At the very least, a straightforward perturbative approach to the system would fail. Back reaction effects and the failure of the semiclassical approximation in general $\alpha$-vacua are discussed in more detail in [4].

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