On the justification of the value of the apparent mass of soil in rheological modeling of the process of soil compaction by a vibrating roller

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Abstract. To solve the problem of justifying the characteristics of vibrating rollers, choosing a roller model for working under specific conditions, setting up the technological modes of compaction in a vibrating roller, it is required to model the dynamic interaction of a vibratory drum with the compacted soil. The mathematical model should allow us to study both the continuous and alternating contact operation modes of the vibratory drum and be consistent with the experimental results. The paper presents the results of the computational experiment based on a triple-mass rheological model of interaction between a vibrating roller and soil. It shows that the proposed model adequately reproduces vertical displacement and acceleration, as well as spectra of the vertical accelerations of a vibratory drum in a DM-614 roller during soil compaction. The conducted computational experiment of justifying the value of the apparent mass of soil showed that the best results can be achieved with value of the apparent mass of soil equal to 20% of the mass of the vibratory drum, which corresponds to the results of other researchers.

1. Introduction

Despite many years of practice of producing vibrating rollers and using them for soil compaction in construction, the problem of justifying the selection of a model of vibrating drum for soil compaction in specific conditions and determining the operation modes (the number of passes for compaction of a specific kind of soil in a layer of a specific thickness to the required compaction factor) [1] has not yet been solved. Due to this, it is difficult to solve a number of following problems: forming a fleet of vibrating rollers in construction and leasing companies; justifying characteristics of vibrating rollers during their design and improvement; justifying the selection of a model of vibrating roller for work under specific technological and soil conditions; choosing operation modes (number of passes, movement speed, the moments of turning on and off, and controlling the driving force and vibration frequency); planning time and cost of soil compaction; increasing the risks of insufficient soil compaction in construction [2].

To solve the aforementioned problems, we need to develop a methodology for determining the results of soil compaction with a vibrating roller with specific technical specifications when operating on specific soil in given technological conditions. The development of this methodology is a complex and not yet solved problem. The overview [3] of possible approaches to solving this problem showed the promise of using the rheological modeling method. The researchers have proposed many rheological models to describe the dynamics of interaction between a vibrating roller and their compacted soil. One-mass rheological models [4-7] consider only the mass of the working body (vibratory drum). Two-mass rheological models [8-13] consider the mass of the working body and the
mass of the frame of the vibratory drum, or the mass of the vibratory drum and the mass of the apparent soil. Three-mass rheological models [14] consider the mass of the working body, the mass of the vibratory drum's frame, and the mass of the apparent soil. There are even more complex multi-mass rheological models [15].

Three-mass models reflect the main features of the vibratory drum's design and the components of resistance of soil to deformation and, at the same time, don't have excessive complexity, which makes them preferable for modeling the dynamic interaction of a vibratory drum with the compacted soil. However, the justification of the value of the apparent soil that significantly affects the results the models are still a matter of debate.

This paper is dedicated to studying the impact of the mass of apparent soil to the results of rheological modeling of the dynamic interaction of a roller's vibratory drum with the soil and comparing the obtained results with the experimental data.

2. Problem statement

The mass of apparent soil models the inertial components of the resistance of soil to deformation by a vibratory drum of a vibrating roller. It is difficult to determine experimentally the soil mass that vibrates together with the vibratory drum during soil compaction, therefore we assume that the mass of apparent soil is proportionate to the mass of the vibratory drum during rheological modeling. Currently there is no consensus regarding how to calculate the mass of apparent soil. It is hypothesized that the mass of apparent soil is 10% [16], 20% [14], 36% [17] и 62% [18] of the mass of the vibratory drum. Some researchers believe that the mass of apparent soil varies from one pass of the vibrating roller to another [19], [20].

In this work, the value of mass of apparent soil was justified by comparing the calculated result of the rheological model and the experimental results of determining the displacement and acceleration scale of the vibrating roller DM-614 during soil compaction. The vibrating roller DM-614 made by Zavod Dorozhnih mashin, Rybinsk, has the following specifications: vibratory drum mass $m_d = 4000$ kg; vibratory drum's frame mass $m_f = 4000$ kg; driving force $P = 215$ kN; vibration frequency $f = 30$ Hz. As for vibration dampers of the drum, the metal-rubber dampers U150.030 were used in the amount of 20 units (10 dampers on each side of the drum). Elastic resistance was $k_f = 0.362$ MN/m [21]. Considering [14], viscous resistance rate of the damper was found to be $b_f = 0.006$ MN·s/m. The equipment, methodology, and description of conducting experimental studies are presented in [22].

3. Theory

The computational experiment was conducted with a three-mass rheological model [22] (Figure 1), which includes the vibratory drum $m_d$, which, in turn, vibrates vertically under the harmonic driving force $P$. The frame of the drum with the mass $m_f$ connects to the drum via the dampers. The elastic and viscous properties of the dampers are modeled with the Hooke and Newton elements characterized by the elastic $k_f$ and viscous $b_f$ resistances. The inertial resistances to soil deformation are modeled by the mass of apparent soil $m_s$. The elastic and viscous components of the resistance to soil deformation are also modeled by the corresponding Hooke and Newtons elements characterized by $k_s$ and $b_s$.

Similar to [14], the model makes it possible to implement various operating modes of the vibrating roller: continuous contact, partial uplift, and double jump [23], [24]. To do this, we composed the differential equations of movement for the vibratory drum, the frame, and soil in relation to the state of contact of the drum with soil and the state of uplift of the drum from soil and developed the conditions for changing these states [22]. When the drum is in contact with soil, the soil reaction force $F_s$ is calculated based on the weight of soil, as well as inertial, elastic and viscous resistances to its deformation. During the uplift, the soil reaction force is zero (Figure 1). Differential equations were solved numerically using the simulation environment SIMULINK in MATLAB software.
4. Experiment results

The computational experiment consisted of calculating the scale of displacements and accelerations of the drum in the vibrating roller DM-614 under various values of mass of apparent soil and comparing the calculated results with the corresponding experimental values. We also compared the calculated and experimental spectra of vertical accelerations of the vibratory drum. The values of apparent mass of soil were 5%, 20%, 40%, and 60% of the drum's mass.

During the experimental research, we evaluated the changes in the soil's properties during compaction with the dynamic modulo of soil deformation $E_{vd}$. Calculation of the elastic soil resistance rate $k_s$ depending on $E_{vd}$ was carried out according to the method described in [22]. The viscous soil resistance rate $b_s$ was constant and equal $b_s = 0.212 \text{ MN} \cdot \text{s} / \text{m}$ [14].

The graphs of vertical displacements and accelerations of the vibratory drum, as well as spectra of vertical acceleration with the dynamic soil deformation modulo $E_{vd} = 15.1 \text{ MPa}$ are shown in the Figures (Figure 2, Figure 3, Figure 4). The solid line in the graphs of calculated vertical displacements of the drum (Figure 5, Figure 8, Figure 11) corresponds to the vertical displacement of the drum, and the dotted line corresponds to the vertical displacement of soil. The graphs of the calculated vertical accelerations of the drum, as well as the calculated spectra of vertical accelerations of the drum are (Figure 6, Figure 9, Figure 12) and (Figure 7, Figure 10, Figure 13) accordingly. The results of the processing of the graphs obtained are in the table (Table 1).

| Parameter                      | Measurement units | Section number | 1   | 2   | 3   | 4   | 5   |
|--------------------------------|-------------------|----------------|-----|-----|-----|-----|-----|
| $E_{vd}$                        | MPa               |                | 6.8 | 7.6 | 10.0| 12.4| 15.1|
| Compaction factor $K_y$         | -                 |                | 0.926| 0.935| 0.948| 0.961| 0.972|
| $k_s$                           | MN/m              |                | 34  | 41  | 53  | 66  | 81  |

**Table 1.** Vertical displacements and accelerations of a vibratory drum with various values of apparent mass of soil.

| Vertical displacement scale of the drum of the vibrating roller DM-614 |
|--------------------------------------------------|
| Experimental value                               |
| mm                                               |
| 3.10                                             |
| 3.40                                             |
| 3.50                                             |
| 3.00                                             |
| 3.30                                             |
| When $m_s = 0.05 \cdot m_d$                      |
| mm                                               |
| 3.35                                             |
| 3.55                                             |
| 3.80                                             |
| 3.50                                             |
| 12.3                                            |
| When $m_s = 0.20 \cdot m_d$                      |
| mm                                               |
| 2.50                                             |
| 2.65                                             |
| 2.85                                             |
| 3.15                                             |
| 3.30                                             |
When $m_s = 0.4 \cdot m_d$ mm 1.90 1.94 2.10 2.39 3.00
When $m_s = 0.6 \cdot m_d$ mm 1.55 1.55 1.60 1.70 1.40

Vertical acceleration scale of the drum of the vibrating roller DM-614

| Experimental value | m/s² | ±54 | ±55 | ±55 | ±54 | ±56 |
|--------------------|------|-----|-----|-----|-----|-----|
| When $m_s = 0.05 \cdot m_d$ m/s² | ±60 | ±64 | ±70 | ±74 | ±94 |
| When $m_s = 0.2 \cdot m_d$ m/s² | ±45 | ±47 | ±50 | ±55 | ±60 |
| When $m_s = 0.4 \cdot m_d$ m/s² | ±33 | ±35 | ±37 | ±39 | ±42 |
| When $m_s = 0.6 \cdot m_d$ m/s² | ±26 | ±28 | ±29 | ±30 | ±32 |

**Figure 2.** The displacement of the vibratory roller drum DM-614 (Section No. 5, $E_{vd} = 15.1$ MPa).

**Figure 3.** The acceleration of the vibratory roller drum DM-614 (Section No. 5, $E_{vd} = 15.1$ MPa).
Figure 4. The spectra of vertical accelerations of the vibratory roller drum DM-614 (Section No. 5, $E_{vd} = 15.1$ MPa).

Figure 5. Calculated vertical displacement of the drum and soil when $m_s = 0.05 \cdot m_d$.

Figure 6. Calculated vertical accelerations of the drum when $m_s = 0.05 \cdot m_d$. 
Figure 7. The spectrum of vertical accelerations of the drum when $m_s = 0.05 \cdot m_d$.

Figure 8. Calculated vertical displacement of the drum and soil when $m_s = 0.20 \cdot m_d$.

Figure 9. Calculated vertical accelerations of the drum when $m_s = 0.20 \cdot m_d$. 
Figure 10. The spectrum of vertical accelerations of the drum when $m_s = 0.20 \cdot m_d$

Figure 11. Calculated vertical displacement of the drum and soil when $m_s = 0.40 \cdot m_d$

Figure 12. Calculated vertical accelerations of the drum when $m_s = 0.40 \cdot m_d$. 
5. Results discussion
The analysis of the results of the computational experiment (Figures 5…13 and Table 1) shows the significant impact of the mass of apparent soil \( m_s \) on the calculated vertical displacements and accelerations of the drum, as well as the calculated spectrum of vertical accelerations.

When calculated with \( m_s = 0.05 \cdot m_d \), the pattern shown on the graph of displacements of the drum speaks about how the vibratory drum works in a complex vibration mode uplifting from soil with the calculated vibration scale of 3.5...3.7 times higher than the experimental values. The calculated vertical accelerations of the drum are 1.7 higher than the experimental values. We can clearly see on the calculated spectrum of vertical accelerations of the drum (Figure 7) that there are the sub-harmonics 0.5f and 1.5f (where f - is the basic vibration frequency of the drum, \( f = 30 \) Hz), which indicates the double jump mode (there is a sub-harmonic at a frequency 0.5f). At the same time, the spectrum of experimental vertical accelerations of the drum has no sub-harmonics.

The results of the computational experiment with \( m_s = 0.2 \cdot m_d \) correspond to the constant contact mode of drum vibrations. The calculated scale of vibrations of the drum correlates to the experimental values at various \( E_{vd} \) values in an acceptable manner. The calculated spectrum of vertical accelerations of the drum (Figure 10) has no sub-harmonics, but we see the origin of a harmonic at the 2f frequency, which corresponds to the constant contact vibration mode and the beginning of the partial uplift vibration mode (there is the origin of a harmonic at the 2f frequency) and falls in line with the spectrum of experimental vertical accelerations of the drum (Figure 4). The calculated vertical accelerations of the drum at various \( E_{vd} \) correlate well with the experimental values. (Table 1).

During the computational experiment with \( m_s = 0.4 \cdot m_d \), the pattern shown on the graph of calculated displacements of the drum speaks about how the vibratory drum operates in the constant contact mode. However, the calculated scale of vibrations of the drum is lower than the experimental values across the whole variation range of \( E_{vd} \) (Table 1). The calculated spectrum of vertical accelerations of the drum (Figure 13) has only one harmonic at the base frequency. The harmonic at the 2f frequency is missing, which indicates the constant contact operating mode and doesn't quite match the spectrum of experimental accelerations. The variation range of calculated accelerations of the drum is also smaller than the experimental values across the whole variation range of \( E_{vd} \).

When calculating with \( m_s = 0.6 \cdot m_d \), both the calculated vibration scale and the calculated vertical accelerations are significantly lower than the existing experimental values. (Table 1).
The obtained results are consistent with the results obtained by V Susante and M Mooney [14], who also concluded that the results of the modeling correlate better with the experimental values with 

\[ m_s = 0.2 \cdot m_d \]

In general, the computational experiment conducted with the developed mathematical model showed its ability to display both the alternating and the constant contact operating modes of the drum, which meet the model requirements. When the mass of apparent soil is 

\[ m_s = 0.2 \cdot m_d \]

the calculated results of vertical displacements and accelerations of the drum, as well as the calculated spectrum of accelerations are consistent with the experimental data, which allows us to conclude that the display of the developed mathematical model of the dynamics of interaction between a vibratory drum of the vibrating roller DM-612 and compacted soil is adequate. In the future, it might be rational to compare the calculated results of the vertical displacements and accelerations of the drum's frame with the experimental data.

6. Conclusions

The authors have developed a mathematical model for the analysis of dynamic processes of interaction of vibrating roller's vibratory drum with compacted soil. The computational experiment conducted for the vibrating roller DM-614 showed the adequacy of reproducing the vertical displacements and accelerations of the model, as well as the spectrum of vertical accelerations of the vibratory drum of the vibrating roller DM-614 during soil compaction. Using the developed mathematical model, the conducted research of justifying the value of the mass of apparent soil showed the result consistent with the experimental results and the results obtained by V Susante and M Mooney [14]. The future studies of the vibratory soil compaction processes are advised to determine the mass of apparent soil via the expression 

\[ m_s = 0.2 \cdot m_d \]

The developed mathematical model should become a part of the methodology to determine the results of soil compaction by a vibrating roller with specific technical specifications when operating on a specific kind of soil in given technological conditions.

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