THE MASSES OF NEARBY DWARFS AND BROWN DWARFS WITH THE HST

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ABSTRACT

The known nearby stars, moving in front of the background of distant stars and galaxies, create 'weak gravitational lensing' variations in their positions. These variations may be measurable with the HST, and they may allow a direct mass determination for the nearby stars. The cross section for the HST measurable astrometric effect is much larger than the cross section for the photometric effect which is measurable from the ground. The mass determination will be easier for the fainter nearby lenses which will be discovered in future searches of faint high proper motion stars.

Subject headings: gravitational lensing - stars: low mass, brown dwarfs

Any massive object moving in front of distant sources makes their positions vary by deflecting their light rays, the phenomenon known as gravitational lensing (cf. Paczyński 1996 for references and the derivation of all formulae). The effect is presented in Figure 1, where the trajectories of double images of distant sources with respect to the lens are shown with curved solid lines, and the Einstein ring is shown with the dashed line. One image is always formed on the outside of the ring, while the second image is always formed inside the ring.

The possibility of measuring gravitational lensing effects astrometrically was analyzed by Hog et al. (1995), Miyamoto and Yoshi (1995), and Gould (1996). They discussed lensing events caused by objects located at a distance of many kiloparsecs, so the two images were expected to be separated by about a milli arcsecond, and the lens itself was assumed to be too dark to be visible. Such lensing events may be resolved, or the light centroid displacement may be measured with future space based optical interferometers.

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Fig. 1.— The geometry of gravitational lensing of many sources by a single lens is presented in projection onto the sky. The trajectories of images of distant sources with respect to the gravitational lens are shown with solid lines, the Einstein ring is shown with the dashed line, and the lens is shown with a large dot at the center. One image is always formed on the outside of the Einstein ring, while the second image is always formed inside the ring.

Miralda-Escudé (1996) proposed to look at relatively bright nearby stars with ground based near-infrared interferometers, and to measure the astrometric displacement of the background stars caused by gravitational lensing by the nearby stars. Both approaches require new technology to be developed, and it is not clear how long we have to wait for that. The purpose of this paper is to point out that a similar astrometric effect may be measurable now with the HST for a few dozen of the nearest dwarf stars, leading to the determination of their masses. This is an astrometric analogue of the photometric project described before (Paczyński 1995).
The angular radius of the Einstein ring is given as

\[ \varphi_E = \left[ \frac{4GM}{c^2} \frac{D_s - D_d}{D_d D_s} \right]^{1/2} = 0.023 \times \left( \frac{M}{0.2 \, M_\odot} \frac{3 \, \text{pc}}{d_\pi} \right)^{1/2}, \tag{1} \]

where \( M \) is the lens mass and \( d_\pi \) is the parallax distance to the lens as measured with respect to the source:

\[ \frac{1}{d_\pi} = \frac{1}{D_d} - \frac{1}{D_s}. \tag{2} \]

We consider here a few dozen dwarf stars located at a distance of \( D_d \sim 3 \, \text{pc} \), with the sources being typically at \( D_s > 1 \, \text{kpc} \).

If a source is located at an angular distance \( \varphi_s \) from a lens, then the two images are located at angular distances \( \varphi_\pm \) from the lens, according to

\[ \varphi_\pm = 0.5 \, \varphi_E \left[ u \pm 0.5 \left( u^2 + 4 \right)^{1/2} \right], \quad u \equiv \frac{\varphi_s}{\varphi_E}. \tag{3} \]

For a large value of the dimensionless impact parameter \( u \) we have approximate relations

\[ \varphi_+ \approx \varphi_s + \frac{\varphi_E^2}{\varphi_s}, \quad \varphi_- \approx -\frac{\varphi_E^2}{\varphi_s}. \tag{4} \]

The magnification of the brightness of the two images is given as

\[ A_\pm = \frac{u^2 + 2}{2u (u^2 + 4)^{1/2}} \pm 0.5, \quad A = A_+ + A_- = \frac{u^2 + 2}{u (u^2 + 4)^{1/2}}, \tag{5} \]

i.e.

\[ A_+ = 1 + A_- \approx 1 + \left( \frac{\varphi_E}{\varphi_s} \right)^4, \quad \text{for} \quad u = \frac{\varphi_s}{\varphi_E} \gg 1. \tag{6} \]

Note, that for large values of the impact parameter \( u \) the increase in apparent brightness is proportional to \( u^{-4} \), while the displacement in the image locations is proportional to \( u^{-1} \). In other words, the effect of gravitational lensing on the image positions falls off with the impact parameter much less rapidly than the effect on the apparent brightness, as pointed out by Miralda-Escudé (1996), and references therein. Therefore, if the brighter of the two images and the lens are resolved then the cross section for an astrometric effect is much larger than the cross section for a photometric effect.

Let us consider as an example the Barnard’s star (cf. van de Kamp 1971). It has a parallax of 0.′′552 and a proper motion of 10.′′31 per year. If we adopt the mass of \( \sim 0.2 \, M_\odot \) we obtain for its Einstein radius

\[ \varphi_{E, \text{Barnard}} = 0.030 \times \left( \frac{M_{\text{Barnard}}}{0.2 \, M_\odot} \right)^{1/2}, \tag{7} \]
The HST can resolve two point like images at a separation of $\varphi_{HST} \approx 0.1$". The displacement due to gravitational lensing by the Barnard's star at the impact parameter of $\varphi_+$ is expected to be

$$\Delta \varphi_+ = \varphi_+ - \varphi_s = \frac{\varphi_{E,Barnard}^2}{\varphi_+} \approx 0.009 \left( \frac{M_{Barnard}}{0.2 \, M_\odot} \right) \left( \frac{0.1}{\varphi_+} \right),$$  \hspace{1cm} (8)

A displacement as small as $0.002$" should be accurately measurable with the HST, which gives $\sim 1$" as the total geometrical cross section for astrometrically measurable gravitational lensing. Combined with the high proper motion this gives the total area of $\sim 10 \times (1")^2$ covered by the Barnard's star in one year, which is much more than $\sim 0.02 \times (1")^2$ covered by a typical high proper motion object considered by Paczyński (1995). The good news is that the cross section for an astrometric effect is much larger than a cross section for a photometric effect. The bad news is that it may be very difficult for the HST to measure the positions of the faint background stars or galaxies located within $0.1$" of the bright Barnard's star, which has $V \approx 9.5$. However, with the spectral type of M5 the Barnard's star may be sufficiently faint in the ultraviolet to make the relative astrometry feasible. Note, that the mass of the lensing star as given with the equation (8) is proportional to the astrometric displacement $\Delta \varphi_+$, i.e. the measurement of the displacement is equivalent to a direct measurement of the lens mass.

There are a few dozen nearby stars in the van den Kamp's (1971) list, some of them considerably fainter than Barnard's star, like Wolf 359, with a parallax of $0.431$, a proper motion of $4.71$" per year, an apparent magnitude $V \approx 13.5$, and a spectral type M8e. The estimate of the Einstein radius for Wolf 359 is

$$\varphi_{E,Wolf\,359} = 0.023 \times \left( \frac{M_{Wolf\,359}}{0.15 \, M_\odot} \right)^{1/2},$$  \hspace{1cm} (9)

It is possible, and even likely, that large proper motion stars even fainter than Wolf 359 will be discovered (Alard 1996), and will become prime brown dwarf candidates. Accurate measurement of their masses with the HST will be crucial for the determination of their masses, and therefore their nature. Excellent field brown dwarf candidates have recently reported by Hawkins and Jones (1996) in their relatively low proper motion study, which revealed four objects at $\sim 30$ pc.

Let us consider now a case when with a proper choice of filters the lensing star can be extinguished, and all that is measurable is an unresolved pair of the two lensed images, with the locations and magnifications given with the eqs. (3) and (5). The displacement of light centroid can be calculated as

$$\Delta \varphi \equiv \varphi - \varphi_s = \frac{(\varphi_+ - \varphi_s)A_+ + (\varphi_0 - \varphi_s)A_0}{A} = \frac{u}{u^2 + 2} \varphi_E,$$  \hspace{1cm} (10)
The variations of the photometric ($\Delta m_{\text{eff}}$) and astrometric ($\Delta \varphi_{\text{eff}}/\varphi_E$) effects of gravitational lensing are shown as a function of dimensionless impact parameter ($u = \varphi_s/\varphi_E$) with solid and dashed lines, respectively. The five lines correspond to five values of the ratio: $A_d \equiv F_{\text{lens}}/F_{\text{source}} = 0, 0.3, 1, 3, 10$, respectively.

which reaches the maximum value of $2^{-3/2}\varphi_E \approx 0.354 \varphi_E$ for $u = \varphi_s/\varphi_E = 2^{1/2} \approx 1.414$. These displacements are large and easily measurable even from the ground with the existing instruments, provided the light from the nearby lens can somehow be extinguished, or the lens is simply very faint.

In some cases, perhaps even in most cases, the lens and the unresolved double image of the lensed star will combine to form a single composite image. Therefore, it is useful to compare the amplitude of the photometric and astrometric effects expected for such blended microlensing events. Let the observed flux from the lens be $A_d$ times the flux from the source. The amplitude of the astrometric effect, i.e. the difference in the position between the light centroid of the lens plus double image system and the centroid of the lens
plus source system in the absence of microlensing is given as

$$\Delta \varphi_{\text{eff}} = \Delta \varphi \frac{A(u)}{A(u) + A_d} = \frac{u}{u^2 + 2} A(u) \varphi_E, \quad A_d \equiv \frac{F_{\text{lens}}}{F_{\text{source}}}. \quad (11)$$

where $A(u)$ is given with the eq. (5). The photometric effect, i.e. the brightness of the composite image in units of the combined brightness of the lens and the unlensed source is given as

$$A_{\text{eff}} = \frac{A(u) + A_d}{1 + A_d}, \quad \Delta m_{\text{eff}} \equiv 2.5 \log A_{\text{eff}}. \quad (12)$$

The dependence of the astrometric effect, $\Delta \varphi_{\text{eff}}/\varphi_E$, and the photometric effect, $\Delta m_{\text{eff}}$, on the impact parameter $u$ is shown in Figure 2 for five values of the relative lens brightness: $A_d = 0, 0.3, 1, 3, 10$. $A_d = 0$ corresponds to the lens which is invisible, and $A_d = 10$ corresponds to the lens which is 10 times brighter than the source. Naturally, the fainter the lens the stronger the astrometric and photometric effects. Note, that the photometric effect becomes rapidly negligible for the impact parameter $u > 1$, i.e. $\varphi_s > \varphi_E$, while the astrometric effect may be measurable even for much larger values of the impact parameter. For example, for the lens as bright as the source ($A_d = 1$) and $\varphi_s = 3\varphi_E$ the effective displacement of the light centroid is $\Delta \varphi_{\text{eff}} = 0.14 \varphi_E$. This corresponds to $\sim 0.004''$ for the Barnard’s star. This is a small displacement, but it should be easily measurable with the HST, and perhaps even from the ground, from a site with an excellent seeing.

The cross section for astrometric effect of gravitational microlensing is larger than the photometric effect, but both require the lens to be relatively dim with respect to the source. As the two stars are likely to have different spectral types their brightness ratio can be optimized with a proper selection of filters. This task may be very difficult with the currently known nearby stars, as these are relatively bright. Future searches of faint high proper motion stars and/or brown dwarfs (Alard 1996, Hawkins and Jones 1996) will lead to a discovery of objects much more suitable for microlensing based mass determination. Note, that the faint objects will also be the most interesting, being candidates for brown dwarfs and the faintest (and hence the oldest) degenerate dwarfs. Cool objects like M dwarfs and brown dwarfs have strong molecular bands which should make it relatively easy to select filters in which these objects will appear very faint, i.e. with a very small value of $A_d$ (cf. eq. 11).

Let us compare the relative merits of the project proposed here with those proposed by Miralda-Escudé (1996) and Paczyński (1995). The project proposed here has an advantage over that proposed by Paczyński (1995) in that it can be carried out over the whole sky, not only in the Milky Way, thanks to the much larger cross section for the astrometric effects of gravitational lensing as compared to the cross section for the photometric effects.
The weakness is the requirement of the HST resolving power as opposed to a 1-meter class ground based photometric telescope. Still, if some objects are very rare, like hypothetical brown dwarfs, the ability to conduct the search over the whole sky may turn out to be essential. The current project may work best for the very faint high proper motion objects, while the project proposed by Miralda-Escudé (1996) requires relatively bright objects as lenses, so the two projects are complementary.

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