Matching of Resummed NLLA with Fixed NNLO for Event Shapes

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We report work on the matching of the next-to-leading logarithmic approximation (NLLA) onto the fixed next-to-next-to-leading order (NNLO) calculation for event shape variables in electron-positron annihilation. The correction of the combined NLLA+NNLO computation in the three-jet region, relevant for precision phenomenology, is small compared with pure NNLO or NLLA+NLO.

1. INTRODUCTION

Event shape distributions in $e^+e^-$ annihilation processes are very popular hadronic observables, mainly due to the fact that they are well suited both for experimental measurement and for theoretical description because many of them are infrared and collinear safe. The main idea behind event shapes variables is to parameterize the energy-momentum flow of an event, such that one can smoothly describe its shape passing from pencil-like two-jet configurations, which are a limiting case in event shapes, up to multijet final states. Since the deviation from two-jet configurations is proportional to the strong coupling constant $\alpha_s$, the comparison of experimental measurements and theoretical prediction permits to determine $\alpha_s$.

At LEP a set of six different event shape observables were measured in great detail: thrust $T$ (which is substituted here by $\tau = 1 - T$), heavy jet mass $\rho$, wide and total jet broadening $B_W$ and $B_T$, C-parameter and two-to-three-jet transition parameter in the Durham algorithm $y_3$.

Until very recently, the theoretical state-of-the-art description of event shape distributions was based on the matching of the NLLA [2] onto the NLO [3, 4] calculation. Using the newly available results of the NNLO corrections for the standard set of event shapes [5] introduced above, we computed the matching of the resummed NLLA onto the fixed order NNLO.

2. FIXED ORDER AND RESUMMED CALCULATIONS

At NNLO the integrated cross section

$$ R(y, Q, \mu) \equiv \frac{1}{\sigma_{\text{had}}} \int_0^y d\sigma (x, Q, \mu) dx, $$

has the following fixed-order expansion:

$$ R(y, Q, \mu) = 1 + \bar{\alpha}_s (\mu) A (y) + \bar{\alpha}_s^2 (\mu) B (y, x_\mu) + \bar{\alpha}_s^3 (\mu) C (y, x_\mu) . $$

where $\bar{\alpha}_s = \alpha_s/(2\pi)$ and $x_\mu = \mu/Q$. Approaching the two-jet region the infrared logarithms in the coefficient functions becomes large, spoiling the convergence of the perturbation expansion. The main contribution in this case comes from the highest power of the logarithms which have to be resummed to all orders. For suitable observables resummation leads to exponentiation. At NLLA the resummed expression is

$$ R(y, Q, \mu) = (1 + C_1 \bar{\alpha}_s) e^{(12 \lambda^2(\alpha_s L) + F_2(\alpha_s L))} , $$

1 Recently an inconsistency in the treatment of large-angle soft radiation was discovered [6]. It is about to be corrected and it should result in numerically minor changes to the NNLO coefficients in the kinematical region of phenomenological studies here. The corrections turn out to be significant only in the deep two-jet region, e.g. $(1 - T) \ll 0.05$. 

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can be expanded as power series in $\bar{\alpha}$ where $\Sigma (\bar{\alpha}^3)$. MATCHING OF FIXED ORDER AND RESUMMED CALCULATIONS

Effective field theory methods [13]. Y remainder functions which tends to zero as $\bar{\alpha}$ the LL are term of the form $\alpha^3$. The terms in green are contained in the LL and NLL contributions and exponentiate trivially with them.

where the function $g_1 (\alpha_s L)$ contains all leading-logarithms (LL), $g_2 (\alpha_s L)$ all next-to-leading-logarithms (NLL) and $\mu = Q$ is used. Terms beyond NLL have been consistently omitted. The resummation functions $g_1 (\alpha_s L)$ and $g_2 (\alpha_s L)$ can be expanded as power series in $\bar{\alpha}_s L$

$$L g_1 (\alpha_s L) = G_{12} L^2 \bar{\alpha}_s + G_{23} L^3 \bar{\alpha}_s^2 + G_{34} L^4 \bar{\alpha}_s^3 + \ldots \text{ (LL)},$$

$$g_2 (\alpha_s L) = G_{11} L \bar{\alpha}_s + G_{22} L^2 \bar{\alpha}_s + G_{33} L^3 \bar{\alpha}_s + \ldots \text{ (NLL)}. \quad (1)$$

Table I shows the logarithmic terms present up to the third order in perturbation theory. At the fixed order level the LL are term of the form $\alpha^n_{\mu} L^{n+1}$, the NLL those which goes like $\alpha^n_{\mu} L^n$, and so on. Notice that this can be read off the expansion $[11]$ of the exponentiated resummation functions.

Closed analytic forms for functions $g_1 (\alpha_s L), g_2 (\alpha_s L)$ are available for $\tau$ and $\rho$ [2], $B_{W}$ and $B_{\tau}$ [8, 9], $C$ [10] and $Y_3$ [11]; and are collected in the appendix of [14]. Recently also $g_3 (\alpha_s L)$ and $g_4 (\alpha_s L)$ were computed for $\tau$ using effective field theory methods [13].

**3. MATCHING OF FIXED ORDER AND RESUMMED CALCULATIONS**

To obtain a reliable description of the event shape distributions over a wide range in $y$, it is mandatory to combine fixed order and resummed predictions. The two predictions have to be matched in a way that avoids the double counting of terms present in both. At NLLA the the expression which has to be matched with fixed NNLO is given by

$$R (y) = \left(1 + C_1 \alpha_s + C_2 \alpha_s^2 + C_3 \alpha_s^3\right) e^{L g_1 (\alpha_s L) + g_2 (\alpha_s L) + \bar{\alpha}_s G_{21} L + \bar{\alpha}_s^2 G_{32} L^2 + \bar{\alpha}_s^3 G_{33} L} + D (y),$$

$$= C (\alpha_s) \Sigma (y) + D (y), \quad (2)$$

where $\Sigma (\alpha_s)$ is the exponentiated part containing the resummed logarithms, $C (\alpha_s)$ is a constant and $D (\alpha_s)$ is a remainder functions which tends to zero as $y \to 0$.

A number of different matching procedures have been proposed in the literature, see for example [1] for a review. In the so-called $R$-matching scheme, the two expression for $R (y)$ are matched. In this case all the coefficients ($C_2$, $C_3$, $G_{21}$, $G_{32}$ and $G_{33}$) appearing in $[2]$ have to be extracted numerically from the distributions at fixed order. The increasing number of logarithms present in the fixed order coefficient functions of the NNLO distributions causes large errors on these coefficients. For this reason we computed the matching in the so-called ln $R$-matching [2] since in this particular scheme, all matching coefficients can be extracted analytically from the resummed calculation. The ln $R$-matching at NLO is described in detail in [2]. In the ln $R$-matching scheme, the NLLA+NNLO expression is

$$\ln (R (y, \alpha_s)) = L g_1 (\alpha_s L) + g_2 (\alpha_s L) + \bar{\alpha}_s \left(A (y) - G_{11} L - G_{12} L^2\right) +$$

$$+ \bar{\alpha}_s^2 \left(B (y) - \frac{1}{2} A^2 (y) - G_{22} L^2 - G_{23} L^3\right) +$$

$$+ \bar{\alpha}_s^3 \left(C (y) - A (y) B (y) + \frac{1}{3} A^3 (y) - G_{33} L^3 - G_{34} L^4\right). \quad (3)$$

Table I: Powers of the logarithms present at different orders in perturbation theory. The color highlights the different orders in resummation: LL (red) and NLL (blue). The terms in green are contained in the LL and NLL contributions and exponentiate trivially with them.
The matching coefficients appearing in this expression can be obtained from (1) and are listed in [14]. To ensure the vanishing of the matched expression at the kinematical boundary $y_{\text{max}}$ a further shift of the logarithm is made [1].

The renormalisation scale dependence of (3) is given by making the following replacements:

$$\alpha_s \rightarrow \alpha_s(\mu),$$
$$B(y) \rightarrow B(y,\mu) = 2\beta_0 \ln x_{\mu} A(y) + B(y),$$
$$C(y) \rightarrow C(y,\mu) = \{2\beta_0 \ln x_{\mu}\}^2 A(y) + 2 \ln x_{\mu} \{2\beta_0 B(y) + 2\beta_1 A(y)\} + C(y),$$
$$g_2 (\alpha_s L) \rightarrow g_2 (\alpha_s L, \mu^2) = g_2 (\alpha_s L) + \frac{\beta_0}{\pi} (\alpha_s L)^2 g_1' (\alpha_s L) \ln x_{\mu},$$
$$G_{22} \rightarrow G_{22}(\mu) = G_{22} + 2\beta_0 G_{12} \ln x_{\mu},$$
$$G_{33} \rightarrow G_{33}(\mu) = G_{33} + 4\beta_0 G_{23} \ln x_{\mu}.$$  

In the above, $g_1'$ denotes the derivative of $g_1$ with respect to its argument. The LO coefficient $A$ and the LL resummation function $g_1$, as well as the matching coefficients $G_{i,i+1}$ remain independent on $\mu$.

4. MATCHED DISTRIBUTIONS AND DISCUSSION

For the resulting plots of the matched distributions we refer to [14]. The most striking observation is that the difference between NLLA+NNLO and NNLO is largely restricted to the two-jet region, while NLLA+NLO and NLO differ in normalisation throughout the full kinematical range. This behavior may serve as a first indication for the numerical smallness of corrections beyond NNLO in the three-jet region. In the approach to the two-jet region, the NLLA+NLO and NLLA+NNLO predictions agree by construction, since the matching suppresses any fixed order terms. Although not so visible on these plots, the difference between NLLA+NNLO and NLLA+NLO is only moderate in the three-jet region. The renormalisation scale uncertainty in the three-jet region is reduced by 20-40% between NLLA+NLO and NLLA+NNLO. This effect is due to the smaller renormalization scale dependence of the NNLO contributions. It is also important to observe that the scale dependence remains the same and is larger in the two-jet region, because the resummed calculations at NLLA take into account only the one-loop running of the coupling constant. This has important consequences in the determination of $\alpha_s$ and we will comment more on this in the next section.

The description of the hadron-level data improves between parton-level NLLA+NLO and parton-level NLLA+NNLO, especially in the three-jet region. The behavior in the two-jet region is described better by the resummed predictions than by the fixed order NNLO, although the agreement is far from perfect. This discrepancy can in part be attributed to missing higher order logarithmic corrections and in part to non-perturbative corrections, which become large in the approach to the two-jet limit.

5. CONCLUSIONS AND OUTLOOK

After the extraction of $\alpha_s$ using only the NNLO distributions and the experimental data of ALEPH [12], a new extraction of $\alpha_s$ using the new matched results was performed [16] using JADE data. The improvement in the error coming from the inclusion of resummed calculation is not as dramatic as passing from NLO to NLLA+NLO calculations. As already anticipated, this is due to the fact that the NNLO coefficients compensate the two-loop renormalization scale variation, whereas the NLLA part only compensates the one-loop variation. A further improvement is possible by including the NNLL corrections into the calculations. These corrections are known only for $\tau$, where higher order logarithmic corrections have been computed [13] using soft-collinear effective theory (SCET). From these calculations one can extract the functions $g_3 (\alpha_s L)$ and $g_4 (\alpha_s L)$. The next step towards the further improvement in the extraction of $\alpha_s$ from event-shape distributions could be to compute them for all six observables mentioned here. As shown in [13], the subleading logarithmic corrections can also account for about half of the discrepancy between parton-level theoretical predictions and hadron-level experimental data.
Improvements can also come from non-perturbative corrections. A very recent non-perturbative study for $\tau$ using a low-scale effective coupling \cite{15} shows that non-perturbative $1/Q$ power corrections cause a shift in the distributions, which can account for an important part of the difference between parton-level distributions and hadron-level experimental data discussed in the previous section.

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