Analysis of high-resolution foreign exchange data of USD-JPY for 13 years

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Abstract

We analyze high-resolution foreign exchange data consisting of 20 million data points of USD-JPY for 13 years to report firm statistical laws in distributions and correlations of exchange rate fluctuations. A conditional probability density analysis clearly shows the existence of trend-following movements at time scale of 8-ticks, about 1 minute.

Key words: Econophysics, Foreign exchange, Fat-tail, Correlation
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1 Introduction

In the case of transactions of stocks, the places and times for trading are limited, but in the case of transactions of foreign exchanges dealers in the world are trading 24 hours except weekends. About 200 trillion yen is traded by the exchange market in the world in one day. This amount of money is equivalent to about 2.5 times the national budget of Japan for one year. Due to these continuous and gigantic properties the characteristics of the open market appear most notably in the foreign exchange.
The fat-tail distributions of price fluctuations [1], the diffusions of price [2,3],
the correlations of price fluctuations [4,5,6] are well-known characteristics of
the open market. However, some of them are not conclusive enough because
the terms of data are not long enough. We analyze all the Bid record (about
20 million ticks) of the exchange rates for "Yen/Dollar" that were traded by
the term from 1989 to 2002 to report the firm statistical laws. We use so-called
CQG database in the present study. Fluctuations as large as 1.5 yen/dollar or
more are observed in the high-frequency (= tick) data of CQG, but we have
deleted them by the filter in the analysis of this paper because it is reasonably
thought that these fluctuations are caused by manual errors.

2 The statistical laws of Exchange rate fluctuations

It is important to observe the distribution of price difference in a study of the
dynamics of price fluctuations. In Fig. 1 we show the cumulative probability
distributions of the absolute value of (a) positive \( (dP(t) > 0 \) means weaker-yen
& stronger-dollar, because we observe the Yen/Dollar rate.) and (b) negative
(=stronger-yen & weaker-dollar) price fluctuations. Time scale unit \( (dt) \) is
one minute, and each rate change distribution is normalized by the standard
deviation. First, we focus the distribution in the time scale of 1 minute \( (dt = 1) \).
The dashed lines in Fig.1 (a) and (b) are the normal distributions. It is
found that the price change distribution for 1 minute has tails much fatter
than those of the normal distribution. We can use the power distribution
to approximate the fat tails of this distribution. In both cases of positive and
negative differences, the power exponent of distribution for 1 minute time scale
is about \(-2.5\). These figures are quite symmetrical. Next we pay attention to
the larger time scales. In the positive case the rate change distribution tends
to converge to the normal distribution. When time scale \( (dt) \) is 10000 minutes
(about 1 week), the rate change distribution is nicely approximated by the
normal distribution. However, in the negative case the distributions keep the
power law even in the case of \( (dt = 10000) \) nearly 1 week in contrast to
the positive case. Namely, the assumption of normal distribution which is
commonly applied in the financial technology is not valid for the negative
changes even in the case of large time scales. Therefore, many theories in
financial technology have the inconsistency for all time scales. It should be
noticed from these figures that the distribution of large time scales is more
asymmetrical than the distribution of the short time scales. To clarify the
asymmetry, we introduce the skewness, which is defined by third moments as
described by the following equation,

\[
\text{Skewness} = \frac{\langle (dP(t, dt) - \langle dP(t, dt) \rangle)^3 \rangle}{\sigma^3}
\] (1)
Fig.1a,b The distributions of fluctuation with (a) the weaker-yen, (b) the stronger-yen. Cumulative probability distributions of fluctuation of Yen/Dollar rate. The time scale \( dt \) is 1 minute scale. \( dt = 10000 \) is about 1 week. The dashed lines represent the straight line of the power-law with exponent \(-2.5\) and standard normal distribution.

Fig.2 Skewness of the rate change fluctuation.

where, \( \sigma \) is the standard deviation and \( dP(t, dt) \) is the price difference over time scale \( dt \). The value of skewness is zero for symmetric case. In Fig.2 we show the scale dependence of the skewness. The skewness of the rate change distribution of short time scale is almost zero. However, the skewness shows the negative value for large time scale because in the case of large time scales the change of very strong yen often occurs. The largest asymmetry of the rate change distribution is observed with the time scale of \( dt = 3 \) days.

In October, 1998, the dollar rate changed from 140 yen to 119 yen, about 20\% in 4 days and violent fluctuations followed for a while after this event. This large rate change is special. In Fig.3, the rate change distribution in the 4 days is shown. The distribution observed with \( dt = 500 \) ticks (about 1 hour) is quite asymmetric. However, when observed with \( dt = 1 \) tick, the distribution is almost symmetric. From this extreme example, we believe short time statistics of exchange rate is always symmetric, and the asymmetric properties appear for larger time scales.

We discuss about the power exponent of the price change distribution of the short time scale. The power exponent of distribution shows the market characteristics, but the power exponent of each market is different [4]. As shown in
Fig. 4 Time series of the distribution exponent. The transition of distribution exponent ($\beta$). The starting point ($t = 1$) is the exponent for July 1989.

Fig. 1, the power exponent of Yen/Dollar rate for 13 years is about $-2.5$. Because data covers a long term, the characteristic of the market is not uniform we think. We investigate time change of a market by estimating a locally defined power exponent $\beta$ in each month observing the date with $dt = 1$ minute. In Fig. 4, we show the time series of the distribution exponent $\beta$. As known from this figure we find the exponent $\beta$ is slowly changing in the range from $-1.9$ to $-3.8$. Hence, it is impossible to assume that the value of the exponent is a universal constant; the market’s conditions have been changing slowly.

3 Correlation

We focus on the predictability of the rate changes. The autocorrelation of the rate changes is defined by the following equation,

$$C(dt) = \frac{\langle dP(t) \cdot dP(t + dt) \rangle - \langle dP(t) \rangle \langle dP(t + dt) \rangle}{\sigma^2},$$

where, $dP(t) = P(t + 1\text{tick}) - P(t)$. This correlation vanishes after $dt = 2$ ticks (about 10 seconds) [5]. Therefore, it is difficult to predict the future by this correlation function. However, higher order correlations are expected to exist in the rate changes because dealers are estimating the future by some ways. Although the volatility ($= |P(t + dt) - P(t)|$) cannot predict whether price goes up or down, long correlation is observed [4]. We investigate how long this correlation continues. For the volatility we use the absolute value of the rate change for 1 minute and calculate the correlation function of the volatility. As shown in Fig. 5, we find that the correlation of volatility continues for about 3 months following a power law with exponent about $-0.2$. This result may imply that the market conditions are changing slowly with time scale of 3 months, a season.
Next, we investigate higher order correlation by considering the dealer’s strategies. We assume there are basically two strategies in dealers. The first strategy is so-called against-the-trend; the Fundamentalists’ believe in the equilibrium exchange rate where market rates should move around, so that they trend to respond against local trends. The other strategy is the trend followers’ who try to ride on the trends. We examine which strategy dominates the market from the data. To this aim we introduce the up-down analysis [6]. The up-down signal of ”+” is shown when the exchange rate goes up and the up-down signal of ”−” is shown when the exchange rate goes down. The even cases are neglected. We observe the existence of ”trends” with the help of the conditional probability after n-successive signs. For example, the conditional probability after 3-successive signs is \( P(\text{+} | \text{+++} \text{−}) \) or \( P(\text{−} | \text{−−−} \text{+}) \). The conditional probability distributions of \(+\) and \(−\) are almost symmetric. In Fig.6 we show conditional probability after n-successive signs. When the time unit is about 30 minutes, the conditional probability is always about 0.5, namely, the fluctuations can be regarded roughly as random. Now in the case of 1 tick, if the number of succession \( n \) is 7 ticks or less, the joint probability shows the moves against trends are more frequent than random cases. This shows that the strategy of against-the-trend is stronger in very short time scale. When \( n \) is 8 ticks or more, we can find the tendency that the same sign will appear with probability larger than 0.5, namely, the trend-followers become dominant. From this analysis we find that the existence of trend can be judged by 8 ticks (or about a minute), namely, when the exchange rates move monotonically for more than a minute, the dealers may feel the existence of the trend and a large rate change will be resulted.

4 Discussion

Many statistical laws of the foreign exchange rates have been confirmed in the analysis with high precision. In the next step, we need to establish the dynamics which satisfies these statistical laws of microscopic time scales from
1 tick up to several hundred minutes scales. A set of market price equations has already been proposed [7] and by extending this set of equations we have derived the basic price equation that is valid also for much longer time scales including the inflations [8]. Deriving a macroscopic price dynamics from the statistics of microscopic fluctuations should be a challenging topic in the near future. We hope that the statistical laws in the microscopic time scales will be helpful to the stabilization of the world economy.

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