Correlations and the relativistic structure of the nucleon self-energy

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Abstract

A key point of Dirac Brueckner Hartree Fock calculations for nuclear matter is to decompose the self energy of the nucleons into Lorentz scalar and vector components. A new method is introduced for this decomposition. It is based on the dependence of the single-particle energy on the small component in the Dirac spinors used to calculate the matrix elements of the underlying NN interaction. The resulting Dirac components of the self-energy depend on the momentum of the nucleons. At densities around and below the nuclear matter saturation density this momentum dependence is dominated by the non-locality of the Brueckner G matrix. At higher densities these correlation effects are suppressed and the momentum dependence due to the Fock exchange terms is getting more important. Differences between symmetric nuclear matter and neutron matter are discussed. Various versions of the Bonn potential are considered.

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I. INTRODUCTION

The success of the phenomenological Walecka model has motivated various attempts to account for relativistic effects in microscopic nuclear structure calculations which are based on realistic nucleon-nucleon interactions. These attempts, which are often called relativistic Brueckner-Hartree-Fock or Dirac-Brueckner-Hartree-Fock (DBHF) calculations, lead to a significant progress in describing the saturation properties of symmetric nuclear matter. Conventional non-relativistic Brueckner-Hartree-Fock calculations or applications of other techniques to account for correlation effects typically yield a prediction for the saturation point which occurs at a too large density and/or with too little binding energy as compared to the empirical values. This disease of conventional many-body calculations can be cured by introducing three-nucleon forces. DBHF calculations, however, reproduce the empirical saturation properties of nuclear matter without the need to introduce a three-nucleon force.

This success of the DBHF approach does not originate only from the use of relativistic kinematics. The key-point is the decomposition of the single-particle potential or the nucleon self-energy into two large components, a scalar part \( \Sigma_s \) and a time-like component of the vector part \( \Sigma_0 \). Each of these components is quite large with absolute values of a few hundred MeV. This means they are of a size comparable to the nucleon rest mass. Employing a self-energy with this relativistic structure in a Dirac equation for the nucleons in nuclear matter yields nucleons, which are weakly bound, which is in agreement with the empirical fact that the binding energy of a nucleon in nuclei is very small as compared to the rest mass of the nucleon. This is because the two large components compensate each other to a large extent in calculating binding energies. The mean-field single-particle energies derived from relativistic and non-relativistic calculations are rather similar.

The large scalar component \( \Sigma_s \) in the relativistic self-energy, however, leads to Dirac spinors which are quite different from the Dirac spinors of free nucleons. In solving the Dirac equation \( \Sigma_s \) can be combined with the mass of the nucleon leading to an effective Dirac mass \( m_D^* \), which is significantly smaller than the bare nucleon mass. This means that the Dirac spinors for the nucleons in the medium show a small component which is enhanced as compared to the free Dirac spinor. The matrix elements of the nucleon-nucleon (NN) interaction calculated within the meson exchange model depend on the structure of the Dirac spinor. This means that the NN interaction between two nucleons within the medium of nuclear matter is different from the interaction of two corresponding nucleons in vacuum. This medium dependence of the NN interaction, which can be characterized by the effective Dirac mass \( m_D^* \), is the essential ingredient of DBHF calculations which has lead to the improvement as compared to non-relativistic calculations.

The analysis of the nucleon self-energy in terms of its relativistic components is a very important step in such relativistic calculations. Within the framework of DBHF this analysis is non-trivial. The DBHF approach accounts for the effects of correlations by solving a three-dimensional Bethe-Goldstone equation. This leads to matrix elements of the so-called G-matrix just between the Dirac states of positive energy. From this G-matrix one can evaluate the single-particle energy for a nucleon with momentum \( k \). The problem is how to derive from the G-matrix or the momentum dependence of the single-particle energy, which is just an information about the positive energy Dirac spinors, the relativistic structure of
the nucleon self-energy.

One possibility, which has been used e.g. by Brockmann and Machleidt \[5\] and Engvik et al. \[10\], is to extract the Dirac structure of the self-energy from the momentum dependence of the single-particle energy. One assumes that the structure of the self-energy is identical to the one obtained within the mean-field or Dirac-Hartree approach for nuclear matter. This means that the space-like vector component $\Sigma_v$ vanishes and the time-like vector component $\Sigma_0$ as well as the scalar component $\Sigma_s$ are constant, i.e. independent of the nucleon momentum $k$. These two constants are then adjusted to reproduce momentum dependence of the calculated DBHF single-particle energy spectrum $\epsilon(k)$. Attempts have been made to extend this scheme and to allow for momentum dependent components $\Sigma_s$ and $\Sigma_0$ \[11\]. It is evident, however, that it is impossible to derive two functions $\Sigma(k)$ from one observable in an unambiguous way.

There are various sources which define the momentum dependence of the single-particle energy or the non-locality of the single-particle potential. The relativistic effects contained in a Dirac-Hartree approximation represent one of the possible sources for this non-locality, which is often parametrized in terms of an effective mass. Other sources for the non-locality are the Fock exchange contributions to the mean field and the non-locality of the effective NN interaction, the G matrix. A careful discussion of these various contributions has been given by Jaminon and Mahaux \[12\]. It is the aim of the present work to disentangle these various contributions within the framework of DBHF.

Another scheme to extract the Dirac structure of the nucleon self-energy, which has been used by various groups \[13–15\], is based on the attempt to analyze the matrix elements of the G-matrix in terms of a relativistic operator for a two-body interaction. Five amplitudes multiplying the different invariant operators are adjusted to reproduce the G-matrix elements. The Dirac structure of the self-energy can then be derived from these amplitudes. Also this procedure, however, is model-dependent. The results depend on the chosen set of invariants. Furthermore the DBHF calculation only yields matrix elements between antisymmetrized two-nucleon states. The decomposition of these matrix elements into direct and exchange contributions, which is required for this analysis, cannot be obtained in a unique way.

To avoid such ambiguities one has to calculate the scattering amplitude or G matrix in the complete Dirac space and not only for the positive energy spinors. Such an approach has been used by Huber et al. \[8\] using the so-called $\Lambda$-approximation. In the investigation presented here, we would like to perform a step into this direction by evaluating the G matrix and the resulting single-particle energy for positive energy Dirac spinors with different decompositions into small and large components. The enhancement of the small component is characterized by the Dirac mass $m_D^*$. From the dependence of the two-body interaction and the single-particle energy on this parameter $m_D^*$, i.e. on the ratio of small to large component we can derive the Dirac structure of the underlying nucleon self-energy.

This technique will be applied to perform DBHF calculations for nuclear matter and neutron matter employing various versions of the Bonn potential \[4\]. These results are compared to DBHF calculations, in which the Dirac structure of the self-energy has been deduced from the momentum dependence of the single-particle potential \[5\]. We will pay special attention to the influence of correlation effects on the momentum dependence of the self-energy. The results of this analysis will also be used to derive an effective NN interaction,
II. DIRAC BRUECKNER HARTREE FOCK THEORY

In isotropic nuclear matter the self-energy of a nucleon with momentum \( k \) can be decomposed in the following components of the spinor representation

\[
\Sigma(k) = \tilde{\Sigma}_s(k) - \gamma^0 \tilde{\Sigma}_0(k) + \gamma \cdot k \Sigma_v(k).
\]  

All other components vanish because of isotropy of the system. Inserting this self-energy into the Dirac equation for a nucleon in the nuclear medium, we obtain

\[
\left[(1 + \Sigma_v(k)) \gamma \cdot k + \left(m + \tilde{\Sigma}_s(k)\right) - \tilde{\Sigma}_0(k) \gamma^0\right] u(k) = \tilde{\epsilon}(k) \gamma^0 u(k)
\]  

Now it is convenient to eliminate the space-like vector component \( \Sigma_v \) and rewrite this Dirac equation into a form which only contains a scalar and a time-like vector component

\[
\left[\gamma \cdot k + (m + \Sigma_s(k)) - \Sigma_0(k) \gamma^0\right] u(k) = \tilde{\epsilon}(k) \gamma^0 u(k)
\]  

where

\[
\Sigma_s = \frac{\tilde{\Sigma}_s - m \Sigma_v}{1 + \Sigma_v}
\]

\[
\Sigma_0 = \frac{\tilde{\Sigma}_0 - \epsilon \Sigma_v}{1 + \Sigma_v}
\]

Comparing the Dirac equation as displayed in (3) with the Dirac equation for a free nucleon, it is obvious that the solution of this Dirac equation for a nucleon in the medium is identical to the one for a free nucleon if we replace the bare mass \( m \) by the effective Dirac mass

\[
m_D^* = m + \Sigma_s(k)
\]  

This implies that the Dirac spinors which are obtained as solutions of this Dirac equation for states with positive energy take the form

\[
u_\alpha(k) = \sqrt{\frac{E_k^* + m_D^*}{2E_k^*}} \left(\frac{1}{\frac{\partial \tilde{\epsilon}_k}{\partial \tilde{\epsilon}_k}}\right) \chi_\alpha
\]  

where \( \alpha \) refers to the spin of the nucleon and \( \chi_\alpha \) represents the corresponding Pauli spinor. Note that this spinor is normalized in such a way that \( u^\dagger u = \bar{u} \gamma^0 u = 1 \). The constant \( E_k^* \) stands for

\[
E_k^* = \sqrt{k^2 + m_D^*}^2
\]
and can also be used to define the time-like component or single-particle energy of the four-momentum \( k \)

\[
k^0 = \epsilon(k) = E_k^* - \Sigma_0
\]

This single-particle energy can also be calculated as an “expectation value” of the Dirac operator

\[
\epsilon(k) = \bar{u} [\gamma \cdot k + m] u + \bar{u} \Sigma u = T(k) + U(k) \tag{7}
\]

with a contribution \( T(k) \) originating from the free part of the Dirac operator

\[
T(k) = \frac{mm^*_D + k^2}{E_k^*} \tag{8}
\]

and the expectation value for the self-energy, the single-particle potential

\[
U(k) = \frac{m^*_p}{E_k^*} \Sigma_s - \Sigma_0. \tag{9}
\]

Assuming that the NN interaction can be described in terms of a meson-exchange or One-Boson-Exchange (OBE) model, the self-energy can easily be calculated in the Dirac-Hartree-Fock approximation. As an example we consider the exchange of \( \sigma \) and \( \omega \) and show the expression for the scalar part of the self-energy, which can be written as

\[
\tilde{\Sigma}_s(k) = -\left( \frac{g_\sigma}{M_\sigma} \right)^2 \rho_s + \frac{1}{(4\pi)^2} \frac{1}{k^F} \int_0^{k^F} p \, dp \, \frac{m^*_D}{E_p^*} \times \left[ g_\sigma^2 \Theta_\sigma(k, p) - 4g_\omega^2 \Theta_\omega(k, p) \right] \tag{10}
\]

\( g_\sigma \) and \( g_\omega \) represent the meson-nucleon coupling constants for these mesons and \( M_\alpha \) represent the corresponding masses. The first term on the right hand side of (10) originates from the Hartree contribution and contains the scalar density

\[
\rho_s = \frac{2}{\pi^2} \int_0^{k^F} p^2 \, dp \, \frac{m^*_D}{E_p^*}
\]

with \( k_F \) the Fermi momentum. The functions \( \Theta_\alpha \) occurring in the Fock (exchange) contributions are defined by

\[
\Theta_\alpha(k, p) = \ln \left( \frac{k^2 + p^2 + M_\alpha^2 - (\epsilon(k) - \epsilon(p))^2 + 2pq}{k^2 + p^2 + M_\alpha^2 - (\epsilon(k) - \epsilon(p))^2 - 2pq} \right)
\]

Similar expression are obtained for the other components of the self-energy \( \tilde{\Sigma}_0 \) and \( \Sigma_v \). They are listed e.g. in reference [16].

Using the Hartree approximation, \( \tilde{\Sigma}_s \) is a constant, independent of the nucleon momentum \( k \) and originates completely from the \( \sigma \) exchange part of the NN interaction. Within this approximation \( \Sigma_v \) is identically zero and also the space-like vector component \( \tilde{\Sigma}_0 \) is constant. Within the Dirac-Hartree-Fock approximation, the momentum dependence of the
various components in the nucleon self-energy originates from the various Fock exchange terms like those in the second term of (10).

The expressions for the components of the self-energy depend on the effective Dirac mass \( m_D^* \). This dependence is due to the fact that the meson-exchange matrix elements, which are needed to determine the nucleon self-energy, are calculated using the Dirac spinors, which arise from the solution of the Dirac equation (3). The matrix elements of the NN interaction in the medium, calculated for the dressed nucleon spinors, are different from the corresponding ones in the vacuum. These differences are characterized by the effective Dirac mass \( m_D^* \). It is obvious that the solution of the Dirac equation and the calculation of the nucleon self-energy requires a self-consistent procedure.

The Dirac-Hartree-Fock approximation, however, is not a useful approximation to describe nuclear properties if one considers realistic meson-exchange potentials, which are adjusted to fit NN scattering data. It would yield unbound nuclei (see also below). The origin of this disease is the existence of the strong short-range and tensor components in such a realistic NN interaction, which must be accounted for by considering the effects of short-range NN correlations in the nuclear wave function. This can be done using the tools of the Brueckner Hartree Fock approximation.

As a first step of such a Dirac-Brueckner-Hartree-Fock approximation, one evaluates the matrix elements of the meson-exchange potential

\[
<k p | V(m_D^*) | k' p' > = \bar{u}(k)\bar{u}(p)\mathcal{V}u(k')u(p')
\]  

with \( \mathcal{V} \) representing the relativistic operator for the OBE approximation of the NN interaction. The medium dependence of these matrix elements originating from the medium dependence of the Dirac spinors \( u \) (see eq.(6)) is emphasized by the formal parameter \( m_D^* \) on the left hand side of (11). These matrix elements may now be used, like matrix elements of a conventional non-relativistic NN interaction, to solve the Bethe-Goldstone equation

\[
<k p | G(m_D^*, \Omega) | k p > = < k p | V(m_D^*) | k p > + \int d^3 p' \int d^3 k' < k p | V(m_D^*) | k' p' > \times \\
\frac{Q(k', p')}{\Omega - \epsilon_k' - \epsilon_{p'}} < k' p' | G(m_D^*, \Omega) | k p >
\]  

with the starting energy parameter \( \Omega \), the Pauli operator \( Q \) and the single-particle energies for the intermediate states \( \epsilon_k', \epsilon_{p'} \). If one ignores the medium effects expressed in terms of the Pauli operator, the dressed nucleon spinors \( (u, m_D^*) \) and energies and replaces these by the corresponding quantities of the free nucleon, this Bethe-Goldstone equation becomes the Thompson equation, a specific three-dimensional reduction of the Bethe-Salpeter equation for NN scattering. Since the three versions of the Bonn potential defined in table A.2 of \[4\] have been fitted to the NN scattering data using this Thompson equation, it is consistent to use the Bethe-Goldstone equation in the form just outlined in order to determine the effective NN interaction of two nucleons in the medium, which accounts for NN correlations. Note, however, that this procedure leads to matrix elements between nucleon states and does not keep track of any Dirac structure.

The effective interaction \( G \) can then be used to determine the single-particle potential of the nucleons in nuclear matter by
The problem is how to determine the Dirac structure of the self-energy from this function $U(k)$. One possibility is to assume that the self-energy components preserve the properties of the Dirac-Hartree approach, i.e. $\Sigma_v$ vanishes and $\Sigma_s$ and $\Sigma_0$ are constant, independent on the momentum of the nucleon. In this case one can identify the potential energy of (13) with (9) and determine the two constants $\Sigma_s$ and $\Sigma_0$ such that the Dirac-Hartree expression (9) fits the potential energy derived in the DBHF approach of (13). With this Dirac-Hartree assumption it is also consistent to parametrize the starting energy $\Omega$ and the single-particle energies $\epsilon_k$ occurring in the propagator of the Bethe-Goldstone eq.(12) in the same way

$$\epsilon_k = \sqrt{k^2 + m_{BG}^2} - \Sigma_0$$

(14)

using the same effective mass parameter

$$m_{BG} = m_D = m + \Sigma_s$$

(15)

An iterative procedure to solve these DBHF equations in a self-consistent way could proceed as follows: Assume a value for $m_D^*$ - calculate the matrix elements of the NN interaction for the corresponding spinors following (11) - solve the Bethe-Goldstone eq.(12) using the same effective mass to determine the single-particle spectrum - calculate $U(k)$ according (13) and determine a new value for $\Sigma_s$ which redefines $m_D^*$. Repeat these steps until convergence is achieved.

This procedure, which we will refer to in the following as the Dirac-Hartree (DH) self-consistency scheme has been used by various groups, see e.g. [5,10]. There seems to be a general consensus that the underlying assumption that $\Sigma_s$ and $\Sigma_0$ are essentially constant is fulfilled. Inspecting the momentum dependence of these self-energies, derived from the analysis of the G matrix elements, which we discussed already in the introduction [13–15,19], indicates that the variation of $\Sigma_s$ and $\Sigma_0$ with the momentum $k$ is weak on the scale of the absolute values of these quantities (typically below 10 percent) [21,22]. Such variations, however, are large on the scale of the single-particle potential $U(k)$ to which these two components add with opposite sign. Therefore it can be very dangerous to ignore even a small momentum dependence in analyzing $U(k)$. It has been shown that applying such an analysis to asymmetric nuclear systems, including neutron matter, yields isovector terms with even an opposite sign compared to those which were derived from a more detailed analysis.

Therefore we propose a method which improves this DH self-consistency scheme. It yields momentum dependent components in the nucleon self-energy and allows a separation of the exchange, correlation and relativistic effects leading to the momentum dependence of the single-particle potential $U(k)$. This is achieved by evaluating the single-particle potential of (13) for various kinds of Dirac spinors characterized by the effective Dirac mass $m_D^*$. This means that we calculate the single-particle potential, keeping the value $m_D^*$ fixed, fulfilling only the Brueckner self-consistency requirement. In the limit $m_D^* = m$ this procedure would correspond to the conventional BHF approach, which ignores any change of the Dirac spinors in the medium completely. In this limit of the conventional BHF approach we have to
satisfy just the BHF self-consistency condition, i.e. determine the effective mass $m_{BG}$ which parametrizes the single-particle spectrum $\epsilon_k$ to be used in the Bethe-Goldstone equation in a self-consistent way.

Employing this procedure for various values of $m^*_D$ we obtain a function $U(k, m^*_D)$. At each nucleon momentum $k$ one can analyze this potential $U$ as a function of the Dirac mass $m^*_D$ to probe the sensitivity of the calculated BHF potential on the structure of the underlying Dirac spinors. To do so, we define for each $k$ two effective coupling constants, $\Gamma_{\sigma}(k)$ and $\Gamma_{\omega}(k)$ for an effective scalar and vector meson, respectively. These coupling constants are adjusted to reproduce at each momentum $k$ the dependence of the single-particle potential on the Dirac mass $m^*_D$ by

$$U(k, m^*_D) = \int_0^{k_F} d^3p <kp|G(m^*_D, \Omega = \epsilon_k + \epsilon_p)|kp> = -\frac{\Gamma_{\sigma}(k)}{\pi^2} \int_0^{k_F} p^2 dp \frac{m^*_D}{\sqrt{p^2 + m^*_D^2}} + \frac{\Gamma_{\omega}(k)}{\pi^2} \rho$$

(16)

It turns out that the dependence of the DBHF single-particle potential (first line of eq.(16)) on the parameter $m^*_D$ is very well reproduced by the two parameter fit displayed in the second line of eq.(16) which corresponds to the Dirac Hartree expression with coupling constants adjusted at each momentum $k$. With these effective coupling constants $\Gamma(k)$ we can now determine the self-consistent value for the effective Dirac mass by

$$m^*_D(k) = m + \Sigma_s(k) = m - \frac{\Gamma_{\sigma}(k)}{\pi^2} \int_0^{k_F} p^2 dp \frac{m^*_D}{\sqrt{p^2 + m^*_D^2}}. \quad (17)$$

Note, that this procedure requires at each density $\rho$ the fulfillment of two self-consistency conditions: one is the self-consistent definition of the single-particle energies to be used in the Bethe-Goldstone equation. This single-particle spectrum can be characterized by the effective mass parameter $m^*_{BG}$. The other one is the self-consistent determination of the Dirac structure of nucleon spinors, which is represented by the effective Dirac mass $m^*_D$. These two effective masses need to be identical only, if the components of the self-energy were indeed independent of the nucleon momentum.

III. DISCUSSION OF RESULTS

In this section we will consider two examples of realistic OBE potentials, the potentials $A$ and $C$ which are defined in table A.2 of [4]. Both of them were adjusted to fit NN scattering phase shifts with good accuracy using the Thompson equation to solve the NN scattering problem in the vacuum. Therefore they are suited with the DBHF treatment outlined in the preceding section. The main difference between these two interactions is the strength of the tensor force. The d-state probability calculated for the Bonn $A$ potential is 4.47 percent while the Bonn $C$ potential yields 5.53 percent.

Results of DBHF calculations for symmetric nuclear matter at the empirical saturation density, which corresponds to a Fermi momentum $k_F$ of 1.35 fm$^{-1}$ using the potential $C$ are displayed in Fig. [4]. The left and the middle part of this figure show the scalar $\Sigma_s$ and
the vector part $\Sigma_0$ as a function of the nucleon momentum $k$. Note that we consider the quantities which are renormalized according to eq. (4), which includes effects of the space-like vector component $\Sigma_v$. These two components show values ranging between $-245$ MeV and $-270$ MeV for $\Sigma_s$ and $-165$ MeV and $-200$ MeV for $\Sigma_0$ considering nucleon momenta between $k = 0$ and the Fermi momentum. This means that the variation of these quantities with the nucleon momentum is as large as 20 percent, which seems to be non-negligible.

In order to explore this momentum dependence, we have calculated the Dirac-Hartree-Fock (DHF) contribution to these components separately. This means that we calculate and analyze in eq. (16) the contribution to the single-particle potential $U(k, m^*_D)$, which is obtained replacing the $G$ matrix by the bare potential, i.e. using the Born approximation. These DHF contributions are presented in Fig. 1 using dashed lines. The values of these DHF contributions are roughly twice as large as the total terms. This demonstrates the importance of the short-range correlations, which are taken into account using the $G$ matrix.

The momentum dependence of these DHF terms is also non-negligible (although the relative importance is decreasing as the absolute values are larger), the variation of these terms with momentum, however, is just opposite to the one obtained for the total BBHF. The decomposition of the DHF self-energy shown in Fig. 1 has been derived from the analysis of $U(k, m^*_D)$ as outlined in the previous section. For the case of the DHF approach, however, one can also determine the Dirac structure of the self-energy directly using expressions like the example of (10). For the Bonn potentials these expressions have to be extended to account for the formfactors and all mesons included in the Bonn potential. The direct calculation and the analysis of $U(k, m^*_D)$ yield identical results, a fact which has been used to test our procedure of the analysis. This means that one can understand the momentum dependence of the DHF contribution from analytic expressions like (10).

The increase of the DHF terms in the self-energy components as a function of the momentum is mainly due to the interplay between the strong exchange terms of the $\sigma$ and $\omega$ mesons. This can be seen from the dashed lines marked with little dots which are obtained if only the contributions of these two mesons are retained in evaluating $V$. Other mesons, in particular the exchange of the rho meson, enhance the momentum dependence of $\Sigma_s$ a bit and reduce the value of $\Sigma_0$.

As a next step we would like to discuss the effect of correlations within this simplified $\sigma - \omega$ version of the Bonn OBE potential. The self-energy components calculated with a $G(\sigma, \omega)$ which results from the solution of the Bethe-Goldstone eq. (12) using a NN interaction $V$ which just contains the $\sigma$ and $\omega$ exchange from the Bonn $C$ potential are presented by the solid lines marked with dots in Fig. 1. Note that these results are not results of a self-consistent DBHF calculation, the parametrization of the single-particle energies ($m^*_{BG}$) and the Dirac spinors is identical to the one derived from the DBHF for the complete NN interaction.

The correlations reduce the absolute values of the self-energy in a very substantial way also in this simplified $\sigma - \omega$ model. This feature is easily understood: Correlations, i.e. the reduction of the NN wave functions at short distances suppress the contributions of $\sigma$ and $\omega$ exchange to a large extent. In order to understand the influence of the correlation effects on the momentum dependence of the self-energy terms, one should realize that the contributions to $G$ which are of second order in $V$ are attractive because of the negative energy denominator in (12). The absolute values for these energy denominators tend to
decrease with increasing momentum of the nucleon for which the self-energy is calculated. This means that the correlation terms in $G$ tends to provide a more attractive contribution to the self-energy components for nucleon momenta close to the Fermi momentum than for those with momentum close to zero. This is just the effect which we see in Fig. 1 comparing the solid and the dashed lines.

This momentum or better energy dependence of contributions to the nucleon self-energy which are of second and higher order in the NN interaction has also been observed by Trasobares et al. [23], who studied the momentum and energy dependence of the nucleon self-energy in a simple $\sigma - \omega$ model. It is very difficult to deduce this feature from a Dirac analysis of the $G$ matrix elements as it has been done in [13–15,19] as such an analysis is based on an analysis of the $G$ matrix in terms of a local interaction. The correlation effects which we just discussed, however, yield non-local contributions to $G$.

In order to investigate the sensitivity of these results on the NN interaction used, we show in Fig. 2 the various components of the self-energy calculated at the same density as considered in Fig. 1 but using the version $A$ of the Bonn potential rather than $C$. The same features are obtained for the NN interaction Bonn $A$ as we have discussed before for Bonn $C$. The difference between these two potentials can be seen best by comparing the results for single-particle potential $U(k)$ (see eq. (9) or eq. (13)) which are given in the right wings of figures 1 and 2. The DBHF results for this $U(k)$ range between -80 MeV and -60 MeV for nucleon momenta below the Fermi momentum $k_F$ for both interactions considered. The contribution of the Born or HF term is quite different in these two cases. While the Bonn $A$ yields values between -20 MeV and +10 MeV, the HF results obtained for Bonn $C$ are between +10 MeV and +40 MeV. This difference can be traced back to the different strength of the tensor component in the NN interaction. The Bonn $C$ potential has a stronger tensor component than Bonn $A$ (see discussion of the $d$-state probabilities in the deuteron above). Therefore Bonn $C$ yields a larger attractive contribution originating from the iterated tensor force than Bonn $A$. As both interactions were adjusted to fit NN scattering phase shifts, the Born approximation to the scattering matrix, the bare potential $V$, must be less attractive for Bonn $C$ than for $A$.

The discussion above demonstrates that the momentum dependence of the nucleon self-energy calculated in the DBHF approximation is a result of the Fock contribution and effects of the correlation terms beyond the Born approximation for the effective NN interaction, which tend to compensate each other to some extent. This can also be seen from Fig. 3, which shows the momentum dependence of the scalar component $\Sigma_s$ of the self-energy calculated for nuclear matter of different densities, characterized by the corresponding Fermi momentum. The left part of this figure contains the self-energy contributions calculated in the Born approximation ($G = V$, denoted by DHF) while the part on the right hand side shows the DBHF result obtained for the complete $G$ matrix. For small densities the momentum dependence of $\Sigma_s$ is dominated by the correlation term. For nuclear matter at higher densities, however, the contributions to $G$ of second and higher order in $V$ are suppressed by the effects of the Pauli operator and the dispersive corrections in the two-particle propagator of the Bethe-Goldstone eq. (12). Therefore in the examples visualized in Fig. 3 for the Fermi momentum $k_F = 1.6 \text{ fm}^{-1}$, the momentum dependence of the Fock contribution overcompensates the momentum dependence originating from the correlation effects and $\Sigma_s$ increases with momentum also when calculated in the DBHF approach. Similar trends are
also observed for the momentum dependence of the vector component of the self-energy $\Sigma_0$.

In the next step we would like to study the effect of this improved analysis of the nucleon self-energy on the self-consistent DBHF calculations. For that purpose we present in Fig. 4 the value of the effective mass $m^*_D$ characterizing the structure of the Dirac spinors (see eq. (5)) and the value of $m^*_{BG}$ (see eq. (14)) representing the single-particle spectrum in the Bethe-Goldstone equation as a function of the Fermi momentum. One observes a non-negligible difference between these two effective masses in particular at small densities. This reflects the importance of the momentum dependence of the self-energy which we just discussed. These values are compared to the result for the effective mass, $m^*$, derived in the simplified DH self-consistency scheme (see eq. (15)). For densities around and below the empirical saturation density of nuclear matter ($k_F = 1.35 \text{ fm}^{-1}$) the effective mass $m^*$ underestimates the values of both the Dirac mass $m^*_D$ and the “Bethe-Goldstone mass” $m^*_{BG}$ by a considerable amount. These three values merge to essentially the same value only at densities above twice the empirical saturation density.

The situation is quite different for neutron matter, as can be seen from the right part of Fig. 4. Note that the range of Fermi momenta displayed in this right part covers the same densities as displayed for nuclear matter in the left part. All three effective masses coincide for neutron matter at small densities. In this region, the momentum dependence of the Dirac components of the self-energy is weaker than the momentum dependence in nuclear matter at the same density. This can be traced back to a slight reduction of the correlation effects in neutron matter as compared to nuclear matter. At large densities the effective Dirac mass $m^*_D$ is smaller than $m^*_{BG}$ which is close to $m^*$ derived in the DH approach to the DBHF self-consistency requirement. It should be noted that the effective Dirac mass determined for neutron matter is rather close to the value of $m^*_D$ derived for nuclear matter at the same density, while deviations can be found for the effective masses describing the energy spectra.

Although the effective masses derived within the DH self-consistency scheme deviate considerably from those obtained within the improved scheme, the calculated binding energies for nuclear matter and neutron matter are very close to each other (see Fig. 4). This is not very surprising. At small densities the binding energies calculated within the DBHF and BHF approach, i.e. with and without accounting for any change of the Dirac spinors at all, are close to each other (see e.g. [1,2]), therefore one should not expect any sensitivity of the calculated binding energies on the precise way of calculating the effective Dirac mass at these densities. At large densities of nuclear matter the DH scheme and its improvement yield results which are close to each other, therefore the two methods should not lead to any substantial differences in this area. Deviations are only visible at densities around the saturation point, where the new scheme predicts a binding energy which is slightly smaller than obtained within the old scheme.

We expect that these differences will show up in calculations of finite nuclei which are based on nuclear matter calculations at such small densities using a local density approximation [16,18,20]. For such calculations one defines an effective meson theory, typically considering the exchange of a scalar, $\sigma$ and a vector meson $\omega$ only, adjusting the coupling constants at each density in such a way that this effective meson exchange model used within the Dirac-Hartree approach reproduces some key quantities of the DBHF calculation of nuclear matter. For these key quantities to be fitted we will use here the value of $m^*_D$ and the
total binding energy.

It has been a problem of such investigations based on DBHF using the old DH self-consistency scheme that the results were sensitive to the extrapolation of the nuclear matter results to small densities, for which no stable nuclear matter calculation can be performed. The reason for this sensitivity is exhibited in Fig. 6. The effective coupling constants for $\sigma$ and $\omega$ derived within this scheme are getting larger for nuclear matter with decreasing density. If, however, the self-consistency scheme is employed, which separates the Dirac structure and the momentum dependence of the self-energy, one obtains effective coupling constants, which are weaker and depend much less on the nuclear density. Also the isospin dependence is weaker as can be seen from the comparison between nuclear matter and neutron matter. Therefore we expect more reliable results for finite nuclei using the effective coupling constants derived from these new DBHF calculations than were obtained within the old scheme. This should particularly be true for asymmetric systems [10,18].

IV. CONCLUSIONS

The single-particle potential of nucleons in nuclear matter, calculated in the framework of the Dirac Brueckner Hartree Fock (DBHF) approach, is non-local or momentum dependent. This momentum dependence is conveniently expressed in terms of effective masses of the nucleon. Possible sources for this non-locality are the Fock exchange terms, the nucleon-nucleon correlations accounted for by the Brueckner G matrix and the relativistic structure of the self-energy, i.e. its decomposition into Lorentz scalar and vector terms. It is the aim of the work presented here to explore the importance of these different sources and their mutual influence. In other words we want to explore the contribution of these sources to the effective mass of the nucleon.

For that purpose a new method has been presented to enable the decomposition of the self-energy into its various Dirac components. It is based on the dependence of the single-particle energy on the small component in the Dirac spinors used to calculate the matrix elements of the underlying NN interaction. This technique has successfully been tested for the case of the Dirac Hartree Fock (DHF) approach, for which the Dirac components can also be calculated directly. The comparison between DHF and DBHF results demonstrates the importance of the NN correlations taken into account in terms of the Brueckner G matrix. These correlations tend to reduce the absolute values of the self energies by almost a factor of two. The reduction is slightly larger for the vector component than for the scalar one, which leads to a bound system of nuclear matter, while the DHF approach using the bare Bonn potentials leads to unbound nuclear matter.

The correlation and non-locality effects contained in the Brueckner G matrix are also very important to understand the momentum dependence of scalar and vector components of the self-energy. At densities around and below the saturation density of nuclear matter the contributions to G beyond the Born approximation yield a momentum dependence of these components which is opposite to the one derived in the DHF approximation. At higher densities these correlation effects are suppressed and the momentum dependence due to the Fock exchange terms is getting more important. The importance of these nonlocal contributions to G cast some doubt on the validity of so-called projection scheme which
derives the Dirac structure of the self-energy from an expansion of the two-body interaction, assuming a local parametrization.

The effective mass of the nucleon in pure neutron matter is dominated by the relativistic effects, the effective mass describing the momentum dependence of the single-particle potential is essentially identical to the Dirac mass $m_D^*$. The same is true for nuclear matter at high densities. At smaller densities, however, the effective mass parametrizing the single-particle potential is significantly smaller than $m_D^*$. The Dirac masses for nuclear matter and neutron matter at the same density are rather close to each other. This reflects the fact that the contribution to the NN interaction obtained from the exchange of a scalar isovector meson, like the $\delta$ meson, is much weaker than the contribution of the scalar isoscalar ($\sigma$) meson.

The improved analysis of the Dirac structure of the nucleon self-energy has only little effect on the calculated binding energy of nuclear matter. The deviations between the simple Dirac Hartree analysis and the present scheme are largest at densities below the saturation density of nuclear matter. Therefore we expect larger differences in the calculation of finite nuclei.

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FIG. 1. Momentum dependence of the scalar $\Sigma_s$ (left part) and vector component $\Sigma_0$ (middle part) of the nucleon self-energy. Results of the DBHF calculation are represented by solid lines, while the contribution of the Born terms (denoted DHF) are shown by dashed lines. The Bonn $C$ potential has been used in a calculation of nuclear matter with $k_F = 1.35$ fm$^{-1}$ to derive these results. The curves marked with little dots (DBHF $\sigma\omega$ and DHF $\sigma\omega$) were obtained in calculations, in which only the $\sigma$ and $\omega$ exchange parts of Bonn $C$ were retained. The right part of the figure shows results for the single-particle potential $U(k)$ calculated in the various approximations.

FIG. 2. Self-energy and single-particle potential of nuclear matter with $k_F = 1.35$ fm$^{-1}$ calculated for the Bonn $A$ potential. Further explanations see Fig. [1]
FIG. 3. Momentum dependence of the scalar part, $\Sigma_s$ of the nucleon self-energy calculated for nuclear matter at various densities (as indicated by the value of the Fermi momentum $k_F$) using Bonn A interaction. The left part of this figure contains the Born contribution, the right part the complete DBHF result.

FIG. 4. Various effective masses, as discussed in the text, calculated for nuclear matter (left part) and neutron matter using Bonn A interaction. For the effective Dirac mass $m_D^*$, the value at $k = 0.8k_F$ is displayed. Results are presented as a function of the Fermi momenta $k_F$. 
FIG. 5. Energy of nuclear and neutron matter at various densities characterized by the Fermi momentum $k_F$. Results obtained within the conventional DH self-consistency scheme are represented by the dashed line, while those obtained using the scheme which separates momentum dependence and Dirac structure are shown using solid lines. Bonn $A$ potential has been used for the NN interaction.
FIG. 6. Effective Coupling constants for a scalar meson ($\sigma$, left part of the figure) and vector meson ($\omega$, right part of the figure). The coupling constants were adjusted at each density in such a way that a Dirac Hartree calculation using these coupling constants fits the Dirac mass $m^*_D$ and the calculated energy of the DBHF calculation. Solid lines refer to nuclear matter, while the dashed lines are obtained for neutron matter. The lines marked with triangles are derived from DBHF calculations using the old DH scheme. The other lines are obtained from the calculations with effective Dirac masses derived from eq.(17). The meson masses are 550 MeV and 783 MeV for the $\sigma$ and $\omega$, respectively.