The Nonsplit Resolving Domination Polynomial of a Graph

Pushpa N1, Dhananjayamurthy B V2

1 Department of Mathematics, REVA University, Bangalore, India
2 Department of Mathematics, Nitte Meenakshi Institute of Technology, Yelahanka, Bengaluru, India
*Corresponding author. Email: pushparao3@gmail.com

ABSTRACT
Metric representation of a vertex v in a graph G with an ordered subset \( R = \{a_1, a_2, ..., a_k\} \) of vertices of G is the k-vector \( r(v|R) = (d(v, a_1), d(v, a_2), ..., d(v, a_k)) \), where \( d(v, a) \) is the distance between v and a in G. The set \( R \) is called a Resolving set of G, if any two distinct vertices of G have distinct representation with respect to \( R \). The cardinality of a minimum resolving in G is called a dimension of G, and is denoted by \( \text{dim}(G) \). In a graph \( G = (V, E) \), A subset \( D \subseteq V \) is a nonsplit resolving dominating set of G if it is a resolving, and nonsplit dominating set of G. The minimum cardinality of a nonsplit resolving dominating set of G is known as a nonsplit resolving domination number of G, and is represented by \( \gamma_{nsr}(G) \). In network reliability domination polynomial has found its application [20], a resolving set has diverse applications which includes verification of network and its discovery, mastermind game, robot navigation, problems of pattern recognition, image processing, optimization and combinatorial search [19]. Here, we are introducing nonsplit resolving domination polynomial of G. Some properties of the nonsplit Resolving domination polynomial of G are studied and nonsplit resolving domination polynomials of some well-known families of graphs are calculated.

Keywords: Dimension of a graph, Graph polynomial, Resolving domination polynomial, Resolving dominating set.

1. INTRODUCTION

To analyse the mathematical models, graphs are widely used. At present time the study related to connected domination set has become a very hot topic of research in the field of computer science [1-2]. Connected domination sets can be considered as the virtual backbone of wireless networks. The application of dominating sets and domination number is studied by many research in graph theory to name a few H.L. Abbott, T.V.Wimer, Sampathkumar, Arumugam, H.B.Walikar, B.Zelinka and many more [21].

An isolated vertex is a vertex with degree zero and a pendant vertex is a vertex with degree one. A pendant edge in a graph G is an edge incident with a pendant vertex. The complement of a graph G is a graph \( G^c \), with vertex set V(G) in which any two vertices are adjacent if and only if they are not adjacent in G. A graph \( K_n \) is the empty (totally disconnected), if no two vertices in it are adjacent. If a graph G consists of disconnected components \( H_1 \) and \( H_2 \), then we write \( G = H_1 \cup H_2 \). If \( G \) consists of \( p \geq 2 \) disjoint copies of a graph H, then we write \( G = pH \). The corona \( G_1 \circ G_2 \) of two graphs \( G_1 \) and \( G_2 \) is the graph obtained by taking one copy of \( G_1 \) (which has \( n_1 \) vertices) and \( n_2 \) copies of \( G_2 \) and then joining the \( i^{th} \) vertex of \( G_1 \) to every vertex in the \( i^{th} \) copy of \( G_2 \). A bipartite graph G is a graph whose vertex set can be partitioned into two subsets \( V_1 \) and \( V_2 \) such that every edge of G joins a vertex of \( V_1 \) with a vertex of \( V_2 \). If every vertex of \( V_1 \) is joined with every vertex of \( V_2 \), then G is said to be complete bipartite graph and denoted by \( K_{n,m} \) where \( |V_1| = m, |V_2| = n \). In particular, a complete bipartite graph \( K_{n,n-1} \) is called a star. The join \( G = G_1 \cup G_2 \) of graphs \( G_1 \) and \( G_2 \) with disjoint vertex sets \( V_1 \) and \( V_2 \) and edge sets \( E_1 \) and \( E_2 \) is the graph union \( G_1 \cup G_2 \) together with all the edges joining \( V_1 \) and \( V_2 \). Graph theory notations and terminologies are not described here [3-8].
Consider $n$th order connected graph $G$ and $R = \{a_1, a_2, ..., a_n\}$ be a subset of vertices of $G$. The $k$-vector $r(\{v/R\}) = (d(v, a_1), d(v, a_2), ..., d(v, a_k))$ is called the representation (or the code) of a vertex $v$ with respect to a subset $R$, where $d(v, u)$ is the distance between the vertices $v$ and $u$ in a graph $G$. The subset $R$ is called a resolving set of $G$ if $r(u/R) \neq r(v/R)$ for every distinct pair of vertices $u, v$ of $G$. A Resolving set of minimum cardinality is called a minimum resolving set (or a basis) of a graph $G$ and its dimension $\text{dim}(G)$ is the cardinality of a basis of $G$. The concepts of resolving set and minimum resolving set has appeared in [16], and later in [17]. Slater introduced these terminologies and he has used locating set for Resolving set. He has used the word resolving set in a graph $G$. Harary and Melter [9] also introduced the concept of nonsplit sets. There are three nonsplit resolving sets but it is not a resolving set, and also, the subsets are only two nonsplit resolving dominating sets of $G$. Harary and Melter [9] also studied these concepts. Metric dimension has appeared in various applications of graph theory as a parameter, as diverse as, pharmaceutical chemistry [4], robot navigation [11], combinatorial optimization [12], and sonar and coast guard Loran [16], to name a few. Considerable bibliography is in the paper by Hernando et al. [10], contains.

### 1.1 Related works

In 2003, the new concept resolving domination in graphs was introduced by Robert and et al. [15]. Resolving dominating set is the one which is both Resolving and dominating set of graph $G$. Naji and Soner in [13] studied Connected resolving domination of graphs and Subramanian and Arasappan studied the secure Resolving domination in [18]. Recently the new concept of non-split Resolving domination of graphs was introduced by Pushpa and Dhananjayamurthy [14]. A connected dominating set is a set of nodes of a network such as WSN (Wireless sensor network), which has small sensing nodes which are capable of computations and wireless communication to monitor geo-fencing of gas and oil pipelines, air pollution, also in health monitoring machines it is used extensively.

One of the branches of algebraic graph theory is graph polynomials and there are many graph polynomials that have been introduced and studied widely. Farrell [26-28] proposed the most general approach to graph polynomials. We refer the interested readers to [1,5,6,7], for more information on this topic. Alikani et al. [28], in (2009), introduced domination polynomial in graphs to count the number of dominating sets in a graph of different size. In a polynomial

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$$

where $a_0$ is called the leading coefficient of $P(x)$, the polynomial $P(x)$ is called monic for $a_n = 1$.

Motivated by domination polynomial of graphs, here we introduce a non-split resolving domination polynomial of a graph and study some properties of the non-split resolving domination polynomial. For graphs like paths $P_n$, cycles $C_n$, complete bipartite graph $K_{r,s}$, star graph $K_{1,n-1}$, bi-star graph $B_{r,s}$ and friendship graph $F_n$, for join and corona product of graphs, the non-split resolving domination polynomial will be found.

Some fundamental results which will be required for many of our arguments in this paper are as follows:

**Lemma 1.1.** [14] For a positive integer number $n \geq 2$,

- $\gamma_{n,s}(K_n) = n - 1$.
- $\gamma_{n,s}(K_{1,n}) = n - 1$.
- $\gamma_{n,s}(P_n) = \begin{cases} 1, & n = 2; \\ n - 2, & \text{otherwise}. \end{cases}$
- $\gamma_{n,s}(C_n) = n - 2, n \geq 3$.
- $\gamma_{n,s}(K_{r,s}) = r + s - 2, r \geq s \geq 2$.
- $\gamma_{n,s}(F_{1,n}) = \frac{n}{2}$
- $\gamma_{n,s}(B_{r,s}) = n - 4$.

where $B_{r,s}$, for $r \geq s \geq 1$ is a bistar graph formed from the two star $K_{1,s}$ and $K_{1,r}$ by joined the central vertices by an edge [22-25].

### 2. PROPERTIES OF NONSPLIT RESOLVING DOMINATION POLYNOMIAL OF GRAPHS

In this section, we investigate non-split resolving domination polynomial of a graph and we study some its properties.

**Definition 2.1.** Let $G$ be a graph of orddern. The non-split resolving domination polynomial of $G$ is the polynomial $P(G, x) = \sum_{i=1}^{n} r(G, i) x^i$

Where $r(G, i)$ is the number of non-split resolving dominating sets of $G$ of size $i$.

To illustrate the concept of non-split domination polynomial of a graph.

Consider the graph $G_1$ shown in figure 1.

**Figure 1** A graph $G_1$ with $\gamma_{n,s}(G_1) = 2$

It is clear, by easy chick, that $\gamma_{n,s}(G_1) = 2$ and there are only two non-split resolving dominating sets of $G_1$ of size two namely $\{v_1, v_2\}$ and $\{v_3, v_4\}$. For more clarity, the subsets $\{v_1, v_2\}$ and $\{v_3, v_4\}$ are non-split dominating sets but it is not a resolving set, and also, the subsets $\{v_1, v_4\}$, $\{v_2, v_3\}$ and $\{v_2, v_4\}$ are resolving sets of $G_1$ but are not non-split sets. There are three non-split resolving
dominating sets of size three are \( \{v_1, v_2, v_3\}, \{v_1, v_3, v_4\} \) and \( \{v_3, v_4, v_5\} \). There are five non-split resolving dominating sets of size four which they are: \( \{v_1, v_2, v_3, v_4\}, \{v_1, v_2, v_3, v_5\}, \{v_1, v_2, v_4, v_5\}, \{v_1, v_3, v_4, v_5\} \) and \( \{v_2, v_3, v_4, v_5\} \). Also there is only one non-split resolving dominating set of size five which is \( V(G_1) \).

Here \( r(G_1, 1) = 0, r(G_1, 2) = 2, r(G_1, 3) = 3, r(G_1, 4) = 5 \) and \( r(G_1, 5) = 1 \).

Therefore, \( P(G_1, x) = x^5 + 5x^4 + 3x^3 + 2x^2 \).

In the following result, we present some properties of the coefficients of the non-split resolving domination polynomial of a graph.

**Proposition 2.2.** Let \( G \) be a graph of order \( n \geq 1 \). Then

Since the only non-split resolving dominating set of \( G \) with cardinality \( n \) is only the set \( V(G) \), so \( r(G, n) = 1 \) and hence \( P(G, x) \) is a monic polynomial.

In a graph \( G \) without isolated vertices, there are \( n \) possible different ways to choose the non-split resolving dominating sets of \( G \) of size \( n - 1 \). Therefore,

\[ r(G, n - 1) = n. \]

\( P(G, x) \) has no constant term for every \( i < \gamma_{n_{sr}}(G) \). Hence the non-split resolving domination root of \( P(G, x) \) is zero with multiplicity \( \gamma_{n_{sr}}(G) \).

\( P(G, x) \) is strictly increasing function in \([0, \infty)\). For any subgraph \( H \) of a graph \( G \), \( \deg(P(G, x)) \geq \deg(P(H, x)) \).

**Proof.** The proof is immediate consequences the definition of the non-split resolving domination polynomial of a graph.

Now, we show that, from the non-split resolving domination polynomial [29] of a graph \( G \), we can determine the number of isolated vertices in \( G \).

**Theorem 2.3.** Let \( G \) be a graph of order \( n \geq 2 \) with \( s \) isolated vertices. If \( P(G, x) = \sum_{i=1}^{s} r(G, i)x^i \), is its non-split resolving domination polynomial, then \( s = n - r(G, n - 1) \).

**Proof.** Let \( G \) be a graph of order \( n \geq 2 \) and \( S \subseteq V(G) \) be the set of all isolated vertices in \( G \), with \( |S| = s \). Then for any vertex \( v \in V(G) - S \), the set \( V(G) - \{v\} \) is a non-split resolving dominating set of \( G \). Therefore, \( r(G, n - 1) = |V(G) - S| = n - s \), and hence \( s = n - r(G, n - 1) \).

From Theorem 5.2 in [14], and by Proposition 2.2, part (3), the following result immediate consequence.

**Theorem 2.4.** Let \( g \) be a graph with \( n \) vertices. Then \( r(g, 1) \neq 0 \), if and only if \( g \cong K_2 \) or \( K_{1,3} \). Math and equations

3. THE NONSPLIT RESOLVING DOMINATION POLYNOMIAL FOR SOME WELL-KNOWN GRAPHS

In this section, we present the explicit formulas of non-split resolving domination polynomial for some well-known classes of graphs.

**Theorem 3.1.** For the complete graph \( K_n \), for \( n \geq 2 \),

\[ P(K_n, x) = x^{n-1}(x + n). \]

**Proof.** Let \( K_n \) be the complete graph with at least two vertices. Then by Lemma 1.1, \( \gamma_{n_{sr}}(K_n) = n - 1 \) and hence by Proposition 2.2, parts (1) and (2), \( r(K_n, n) = 1 \) and \( r(K_n, n - 1) = n \). Therefore

\[ P(K_n, x) = \sum_{i=n-1}^{n} r(K_n, i)x^i \]

The non-split polynomial representation of a complete graph is given by (1).

**Theorem 3.2.** For the trivial graph \( K_1 \), \( P(K_1, x) = x \).

**Theorem 3.3.** Let \( P_n \) for \( n \geq 2 \), be the path. Then

\[ P(P_n, x) = \begin{cases} x(x + 2), & \text{if } n = 2; \\
(x^n - 2)(x^2 + nx + (n - 3)), & \text{otherwise.} \end{cases} \]

**Proof.** Let \( P_n \) be the path with at least two vertices. From Lemma 1.1, we have \( \gamma_{n_{sr}}(P_n) = 1 \), if \( n = 2 \) and \( \gamma_{n_{sr}}(P_n) = n - 2 \), if \( n \geq 3 \). Hence, we consider the following two cases:

**Case 1:** If \( n = 2 \), then \( P_2 = K_2 \), and hence by Theorem 3.6 \( P(P_2, x) = x(x + 2) \).

\[ P(P_n, x) = \sum_{i=n-2}^{n} r(P_n, i)x^i \]

\[ P(P_n, x) = r(P_n, n - 2)x^{n-2} + r(P_n, n - 1)x^{n-1} + r(P_n, n)x^n \]

\[ P(P_n, x) = (n - 3)x^{n-2} + nx^{n-1} + x^n \]

\[ P(P_n, x) = x^{n-2}(x^2 + nx + (n - 3)) \]
**Theorem 3.4.** Let $C_n$, for $n \geq 3$, be the cycle graph. Then

$$P(C_n, x) = \begin{cases} x^2(x + 3), & \text{if } n = 3; \\ x^{n-2}(x^2 + nx + n), & \text{otherwise.} \end{cases}$$

**Proof.** Let $C_n$ be the cycle graph of order $n \geq 3$. Then by Lemma 1.1, we have $\gamma_{nsr}(C_n) = 2$, if $n = 3$ and $\gamma_{nsr}(C_n) = n - 2$, for every $n \geq 3$. Hence we have the following two cases:

**Case 1:** If $n = 3$, then $C_3 \cong K_3$, and hence by Theorem 3.6 $P(C_3, x) = x^2(x + 3)$. 

**Case 2:** If $n \geq 4$, then by Proposition 2.2, $r(C_n, n) = 1$ and $r(C_n, n - 1) = n - 1$. To select a nonsplit resolving dominating set $D$ of cardinality $n - 2$ such that $V - D$ is connected and $|V - D| = 2$. Then $V - D$ is an edge in $C_n$, and since $C_n$ has size $m = n$ and $V(C_n) = e$, for any edge $e = uv$, $u, v \in V(C_n)$ is a nonsplit resolving dominating set of $C_n$. Then we can choose $D$ by $n$ ways and hence, $r(C_n, n - 2) = n$. Therefore,

$$P(C_n, x) = \sum_{i=1}^{n} r(C_n, i)x^i$$

$$P(C_n, x) = r(C_n, n - 2)x^{n-2} + r(C_n, n - 1)x^{n-1} + r(C_n, n)x^n$$

$$P(C_n, x) = nx^{n-2} + nx^{n-1} + x^n$$

**Theorem 3.5.** For the star graph, for

$$n \geq 2, P(K_{1,n-1}, x) = x^{n-1}(x + n).$$

**Proof.** Let $K_{1,n-1}$ be the star graph. Then by Lemma 1.1, $\gamma_{nsr}(K_3) = n - 1$ and hence by Proposition 2.2, parts (1) and (2), $r(K_1, n - 1, n) = 1$ and $r(K_1, n - 1, n - 1) = n$.

Therefore

$$P(K_{1,n-1}, x) = \sum_{i=1}^{n} r(K_{1,n-1}, i)x^i$$

$$P(K_{1,n-1}, x) = r(K_{1,n-1}, n - 1)x^{n-1} + r(K_{1,n-1}, n)x^n$$

$$P(K_{1,n-1}, x) = x^{n-1}(x + n)$$

**Theorem 3.6.** For a complete bipartite graph $K_{rs}$, for $r \geq s \geq 2$, $P(K_{rs}, x) = x^{r+s-2}(x^2 + (r + s)x + rs)$. 

**Proof.** Let $K_{rs}$ be a complete bipartite graph with $n = r + s$ vertices and let $V_1 = \{u_1, u_2, \ldots, u_r\}$ and $V_2 = \{v_1, v_2, \ldots, v_s\}$ be the vertex partition sets of $K_{rs}$. Then by Lemma 1.1, $\gamma_{nsr}(K_{rs}) = r + s - 2 = n - 2$ and hence by Proposition 2.2, parts (1) and (2), $r(K_{rs}, n) = 1$ and $r(K_{rs}, n - 1) = n = r + s$. To select a nonsplit resolving dominating set $D$ of $K_{rs}$ with cardinality $n - 2$ such that $V - D$ is connected and $|V - D| = 2$. Then $V - D$ is an edge in $K_{rs}$, and since $K_{rs}$ has size $m = rs$ and $V(K_{rs}) = e$, for any edge $e = u_i, v_j, i = 1, 2, \ldots, r$ and $j = 1, 2, \ldots, s$ is a nonsplit resolving dominating set of $K_{rs}$. Then we can choose $D$ by $rs$ different ways and hence, $r(K_{rs}, n - 2) = rs$. Therefore,

$$P(K_{rs}, x) = \sum_{i=1}^{n} r(K_{rs}, i)x^i$$

$$P(K_{rs}, x) = r(K_{rs}, r + s - 2)x^{r+s-2} + r(K_{rs}, r + s - 1)x^{r+s-1} + r(K_{rs}, r + s)x^{r+s}$$

**Corollary 3.7.** For a complete bipartite graph $K_{rs}$, for $n \geq 4$, $P(K_{rs}, x) = x^{n-2}(x^2 + nx + \frac{n^2}{4})$. 

Bistar graph, shown in figure 2, is a graph constructed from $P_2$ by attaching $r$ edges in one vertex and $s$ edges in the other vertex, and is denoted by $S(r, t)$. 

Figure 2 Bistar graph $S(4,3)$

**Theorem 3.8.** For a bistar graph $S(r, s)$, for $r \geq s \geq 2$, with $n = r + s + 2$ vertices,

$$P(S(r, s), x) = x^n - 2(x^2 + nx + 1).$$

**Proof.** Let $S(r, s)$ be the bistar graph with $n = r + s + 2$ vertices. Then by easy check one get that $\gamma_{nsr}(S(r, s)) = n - 2$. It is clear that there is only one nonsplit resolving dominating set of $S(r, s)$ with size $n - 2$, that is containing all the pendant vertices. Thus, $r(S(r, s), n - 2) = 1$, and by Proposition 2.2, $r(S(r, s), n - 1) = n$ and $r(S(r, s), n) = 1$. Therefore,

$$P(S(r, s), x) = x^n - 2(x^2 + nx + 1)$$

In the following result, we compute the nonsplit resolving domination polynomial of the friendship graphs $F_p$, for $p \geq 2$, where the friendship graph is a graph formed by joining copies of $K_3$ with a common vertex, as shown in figure 3. Thus, $F_p$ has $n = 2p + 1$ vertices, and $m = 3p$ edges [30-33].
Figure 3 Friendship graph F₃

**Theorem 3.9.** For a friendship graph $Fₚ$, for $p ≥ 2$, with $n = 2p + 1$ vertices,

$$P(Fₚ,x) = x^{2p+1} + 2px^{2p} + \sum_{i=0}^{p} (\binom{p}{i}) 2^{p-1}x^{p+1}.$$ 

**Proof.** Let $Fₚ$ for $p ≥ 2$ be a friendship graph with $n = 2p + 1$. Then by Lemma 1.1, $γ_{rn}(Fₚ) = p$. It is clear that the central vertex of $Fₚ$ does not belong to any nonsplit resolving dominating set $D$ with cardinality at least or equal to $n - 2$, because $V - D$ in this case is disconnected. Thus we can choose a nonsplit resolving dominating set of size $p$, by chose one vertex from two for every copy of $K₂$. Then we have $2²$ different ways and hence, $r(Fₚ,p) = 2p$ [40]. To choose a nonsplit resolving dominating set of size $p + 1$, firstly, we chose the vertices of any copy of $K₂$ from the popies of $Fₚ$, so we have $(\binom{p}{0})$ ways and by chose one vertex from two vertices of the reminding copies of $K₂$ of $Fₚ$. Thus we have $2^{p-1}$ different ways. Hence, $r(Fₚ,p + 1) = (\binom{p}{0}) 2^{p-1}$. By continuous in the same way we obtain that $r(Fₚ,p + i) = (\binom{p}{i}) 2^{p-i}$, for every $2 ≤ i ≤ n - 2 = 2p - 1$ and by Proposition 2.2 $r(Fₚ,n - 1) = n = 2p + 1$ and $r(Fₚ,n) = 1$. Therefore

$$P(Fₚ,x) = \sum_{i=0}^{n} (\binom{p}{i}) 2^{p+i}x^{p+i} = \sum_{i=0}^{p} (\binom{p}{i}) 2^{p-1}x^{p+1}.$$ 

4. NONSPLIT RESOLVING DOMINATION POLYNOMIAL OF GRAPHS UNION

This section will provide formula for the nonsplit resolving domination polynomial of the graphs union [34-39] and we will apply it on the union of some standard graphs. The following result required to prove our main result.

**Theorem 4.1.** [14] Let $G₁, G₂, ..., Gₖ$, for $k ≥ 2$, be pairwise vertex-disjoint graphs with $n_i$ vertices, $i = 1, 2, ..., k$. Then for $G = G₁ ∪ G₂ ∪ ... ∪ Gₖ$,

$$γ_{rn}(G) = \left(\sum_{i=1}^{k} n_i\right) + γ_{rn}(G_i)$$

such that $n_j - γ_{rn}(G_i) = max(n_i - γ_{rn}(G_i); 1 ≤ i ≤ k)$. Then $k = \sum_{i=1}^{p} n_i$. Then by Theorem 4.1,

$$γ_{rn}(\bigcup_{i=1}^{p} G_i, x) = x^p P(G_i, x).$$

**Proof.** Let $G₁, G₂, ..., Gₚ$ be pairwise vertex-disjoint connected graph with vertices $n₁, n₂, ..., nₚ$, respectively. If $G_i$, for some $j = 1, 2, ..., p$, is the graph with $n_j - γ_{rn}(G_i) = max(n_i - γ_{rn}(G_i); 1 ≤ i ≤ p)$, and $k = \sum_{i=1}^{p} n_i$. Then by Theorem 4.1,

$$γ_{rn}(\bigcup_{i=1}^{p} G_i) = k + γ_{rn}(G_i)$$

If $D$ is the non-split resolving dominating set of $\bigcup_{i=1}^{p} G_i$, and $D_j$ is the non-split resolving dominating set of a graph $G_i$ for $i = 1, 2, ..., p$, and $i ≠ j$, then

$$D = (\bigcup_{i=1}^{p} V(G_i)) ∪ D_j$$

Thus, the different ways to choose a nonsplitly resolving dominating set $\bigcup_{i=1}^{p} G_i$ of size $k + γ_{rn}(G_i)$ are the same ways to choose a nonsplit resolving dominating set of Size $γ_{rn}(G_i)$, hence

$$r(\bigcup_{i=1}^{p} G_i, k + γ_{rn}(G_i)) = r(G_i, γ_{rn}(G_i))$$

By continuing with similar discussion, we obtain that

$$r(\bigcup_{i=1}^{p} G_i, k + γ_{rn}(G_i) + t) = r(G_i, γ_{rn}(G_i) + t),$$

for $1 ≤ t ≤ n_i - γ_{rn}(G_i)$. Therefore,

$$P(\bigcup_{i=1}^{p} G_i, x) = \sum_{t=γ_{rn}(G_i)}^{n_i} (\binom{p}{i}) 2^{p+i}x^{p+i} + (2p + 1)x^{2p} + x^{2p+1}$$

$$P(\bigcup_{i=1}^{p} G_i, x) = x^{2p+1} + (2p + 1)x^{2p} + x^{2p+1}$$

$$+ \sum_{i=0}^{p-1} (\binom{p}{i}) 2^{p-1}x^{p+1}.$$ 

(9)
Since
\[ r\left(\bigcup_{i=1}^{p} G_i, k + \gamma_{nss}(G_j) + t\right) = r\left(G_j, \gamma_{nss}(G_i) + t\right), \text{ for } 0 \leq t \leq n_j - \gamma_{nss}(G_j), \]
\[ n = n_1 + n_2 + \ldots + n_p, \text{ and } n = k + n_j. \]

Then
\[ P(\bigcup_{i=1}^{p} G_i, t) = r\left(G_j, \gamma_{nss}(G_i)\right) x^{k + \gamma_{nss}(G_j)} + \]
\[ r\left(G_j, \gamma_{nss}(G_i) + 1\right) x^{k + \gamma_{nss}(G_j) + 1} + \]
\[ \ldots + r\left(G_j, n_j - 1\right) x^{k + n_j - 1} + r\left(G_j, n_j\right) x^{n_j}. \]
\[ P(\bigcup_{i=1}^{p} G_i, t) = x^k \frac{\gamma_{nss}(G_j)}{r(G_j, t)} x^{n_j}. \]
\[ P(\bigcup_{i=1}^{p} G_i, t) = x^k P(G_j, t) x^{t}. \]

We obtain the pH graph G by Theorem 4.2, empty and p-components of some standard graphs.

**Proposition 4.3** If H is a connected graph of order n and let G = pH, then \( P(G, x) = x^{(p-1)n} P(H, x). \)

**Proof.** Let H be a connected graph of order n and let G = pH. Since
\[ G = \bigcup_{i=1}^{p} H. \]

Then by Theorem 4.1, \( \gamma_{nss}(G) = (\sum_{i=1}^{p-1} n_i) + \gamma_{nss}(H) = (p-1)n + \gamma_{nss}(H) \) and henceas in Theorem 4.2, \( k = (p-1)n. \) Therefore, by using Theorem 4.2, we obtain,
\[ P(G, x) = x^{(p-1)n} P(H, x). \]

**Corollary 4.4.** For \( n \geq 2, \)
1. \( P(K_n, x) = x^n. \)
2. \( P(pK_n, x) = x^{pn-1}(x + n). \)
3. \( P(pP_n, x) = x^{pn-2}(x^2 + nx + n - 3). \)
4. \( P(pC_n, x) = x^{pn-2}(x^2 + nx + n). \)

5. CONCLUSION

Studies on the concept of domination and also dominating sets plays a predominant role in graph theory with enormous applications to the networks in real-world.

Connected dominating polynomials have applications in networks such as WSN (wireless sensor networks), WAN (wireless ad hoc networks) and also in linked with few broadcast problems, recently it has found the application in network reliability as well.

Here we have considered one such domination called nonsplit resolving domination. In particular, the nonsplit resolving domination polynomial representation of certain graphs namely paths, cycles, complete graphs, complete bipartite graph, star graph, bi-star graph and friendship graph. The same polynomial representation is considered for Union of graphs and corona product of graphs. This work has some applications in the field of biology, biomedicine and biochemistry.

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