Determination of Gravitomagnetic Field through GRBs or X-ray Pulsars

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Abstract

In gauge theory of gravity, there is direct coupling between the spin of a particle and gravitomagnetic field. In the surface of a neutron star or near black hole, the coupling energy between spin and gravitomagnetic field can be large and detectable. Precise measurement of the position of spectrum lines of the corresponding emission or absorption can help us to determine the gravitomagnetic field and electromagnetic field simultaneously. The ratio $\Delta E_e/\Delta E_p$ can be served as a quantitative criteria of black hole. In GRBs or X-ray pulsar, absorption spectral lines of electron were observed. If the absorption spectral lines of electron and proton can be observed simultaneously, using the method given in this paper, we can determine the gravitomagnetic field in the surface of the star, and discriminate black hole from neutron star.

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1 Introduction

The classical effects of gravitomagnetism were studied for more than one hundred years. The close analogy between Newton's gravitation law and Coulomb's law of electricity led to the birth of the concept of gravitomagnetism in the nineteenth century\cite{1, 2, 3, 4}. Later, in the framework of General Relativity, gravitomagnetism was extensively explored\cite{5, 6, 7} and recently some experiments are designed to test gravitomagnetism effects. Some recently reviews on gravitomagnetism can be found in literatures \cite{8, 9, 10}.

In quantum gauge theory of gravity\cite{11, 12, 13, 14, 15, 16}, gravitoelectric field and gravitomagnetic field are naturally defined as component of field strength of gravitational gauge field. In gauge theory of gravity, the definition of gravitomagnetism is more general, that is, the gravitomagnetism defined in the literature \cite{5, 6, 7, 8, 9, 10} only corresponds to the time components of the gravitomagnetism defined in gauge theory of gravity. It is known that gravitational Lorentz force is a direct classical gravitomagnetic effect of a moving mass point in gravitomagnetic field\cite{17, 18}.

In classical level, gauge theory of gravity returns to general relativity. In quantum level, gauge theory of gravity is a perturbatively renormalizable quantum theory, so based on it, quantum effects of gravity can be explored. Based on a unified theory of gravity and electromagnetic interactions\cite{19, 20, 21}, some known quantum effects related to gravitational interactions are discussed in the literature \cite{22}. In this paper, emission and absorption of a spin particle in strong gravitomagnetic field is studied. A proposal to detect such kind of emission or absorption is given and the meaning of such kind of measurement of gravitomagnetism is discussed. Determining the gravitomagnetic field of a star can help us to known which star is black hole, for can only black hole have very strong gravitomagnetic field.

2 Coupling between spin and gravitomagnetic field

In gauge theory of gravity, gravitational field is represented by gravitational gauge field $C_\mu^\alpha$. Its field strength is $F^\alpha_{\mu\nu}$. The gravitational field and gravitoelectric field are defined by

\begin{align}
E_i &= F_{0i} = E_i^\alpha \hat{P}_\alpha, \\
B_i &= -\frac{1}{2} \varepsilon_{ijk} F_{jk} = B_i^\alpha \hat{P}_\alpha,
\end{align}

where $\hat{P}_\alpha$ is the generator of global gravitational gauge group.
The equation of motion of a Dirac particle in both electromagnetic field and gravitational field is\[22\]
\[
\gamma^\mu (\partial_\mu - ieA_\mu - gC_\mu^a \partial_a) + m \psi = 0. \tag{2.3}
\]
Making non-relativistic limit of the above equation, we can obtain the following Schrödinger equation for a non-relativistic Dirac particle in strong gravitomagnetic field and electromagnetic magnetic field is\[22\]
\[
i \frac{\partial \psi}{\partial t} = \left[ \frac{1}{2m} \left( -i \vec{D} - e \vec{A} + mg \vec{C}^0 \right)^2 + mgC^0_0 - eA_0 \right. \\
\left. \frac{g}{2} \vec{\sigma} \cdot \vec{B} + \frac{e}{2m} \vec{\sigma} \cdot \vec{B}_e - \frac{ig}{2} \vec{\sigma} \cdot \vec{E} - \frac{ie}{2m} \vec{\sigma} \cdot \vec{E}_e \right] \Psi, \tag{2.4}
\]
where
\[
\vec{C} = (C^0_i),
\tag{2.5}
\]
\(\vec{B}_e\) and \(\vec{E}_e\) are electromagnetic field, \(\vec{\sigma}\) is the spin matrix of the Dirac particle, \(g\) and \(e\) the coupling constant of gravitational interactions and electromagnetic interactions respectively, \(A_\mu\) is electromagnetic gauge field, and \(\vec{A}\) its vector potential. It could be seen that the classical Newtonian gravitational potential naturally enters into the Schrödinger equation. Besides, there is direct coupling between spin and gravitomagnetic field, no matter the Dirac particle carries electric charge or not.

When there are strong gravitomagnetic field and electromagnetic magnetic field, there is coupling between spin and gravitomagnetic field and electromagnetic magnetic field, which has the following coupling energy
\[
\Delta H = -\frac{g}{2} \vec{\sigma} \cdot \vec{B}^0 + \frac{e}{2m} \vec{\sigma} \cdot \vec{B}_e. \tag{2.6}
\]
When spin transition from down to up in gravity, it will radiate the following energy
\[
\Delta E = \left| g \vec{B}^0 + \frac{e}{m} \vec{B}_e \right|. \tag{2.7}
\]

The above results are only applicable to point-like particles. It is known that neutron and proton are not point-like particle, for they have inner structures. Now, we extend the above results to general cases. If we denote the magneton \(\vec{\mu}\) of a particle as
\[
\vec{\mu} = g_L \frac{e}{2m} \vec{J}, \tag{2.8}
\]
where $\vec{J}$ is the spin of the particle and $g_L$ is the Lande $g$-factor, the coupling energy $\Delta H$ should be
\[
\Delta H = -g_L \frac{g}{2} \sigma \cdot \vec{B}_0 - \mu \cdot \vec{B}_e .
\] (2.9)

Then, the particle will emit the following energy during spin transition
\[
\Delta E = \frac{g_L}{2} \left| g \vec{B}_e + \frac{e}{m} \vec{B}_e \right| .
\] (2.10)

The separations of different Landau levels is integer multiples of $\Delta E$. Detecting such kind of radiation can directly measure the gravitomagnetic field on the surface of the star.

### 3 Determination of Gravitomagnetic Field from GRBs or X-ray Pulsars

It is known that, in Gamma-Ray Bursts (GRG), some absorption spectral lines are detected\[23, 24, 25\]. Spectral features due to electron cyclotron resonance have been discovered from some X-ray pulsars\[26, 27, 28, 29\]. These spectral lines are traditionally interpreted as cyclotron harmonic in strong magnetic fields of about $10^{12}$ gauss. Now, according to equation (2.7), gravitomagnetic fields also contribute to such kind of excitation. It is known that ordinary neutron stars are spinning with high angular velocity, and their angular momenta are also very high, so the gravitomagnetic fields generated by neutron stars should also be quite large. For a spinning black hole, its gravitomagnetic field should be very strong near the event horizon. Therefore, in these cases, the positions of the spectral lines contains the information of gravitomagnetic fields in the surface of the star, and we can determine the gravitomagnetic field through precise measuring the positions of those spectral lines. But just from one serial of spectral lines, we can not simultaneous determine the gravitomagnetic field $|g \vec{B}_0|$ and magnetic field $|\vec{B}_e|$. So, besides detecting the spectral lines of electrons, we need also detect the corresponding spectral lines of protons or neutrons. The masses of proton, neutron and electron are different, and their energy levels of spin transitions are also different. For electron, the position of the lowest spectral line is
\[
\Delta E_e = |g_{Le}| \cdot \left| g \vec{B}_e + \frac{e}{m_e} \vec{B}_e \right| ,
\] (3.1)
and those for proton and neutron are
\[
\Delta E_p = |g_{Lp}| \cdot \left| g \vec{B}_e + \frac{e}{m_p} \vec{B}_e \right| .
\] (3.2)
and
\[ \Delta E_n = |g_{Ln}| \cdot \left| g \otimes B + \frac{e}{m_n} \vec{B}_e \right| \] (3.3)
respectively, where \( m_e \), \( m_p \) and \( m_n \) are masses of electron, proton and neutron respectively, and \( g_{Le} \), \( g_{lp} \) and \( g_{Ln} \) are Lande g-factors of electron, proton and neutron respectively. It is know that
\[ m_e : m_p : m_n \simeq 1 : 1836 : 1839, \] (3.4)
\[ g_{Le} : g_{lp} : g_{Ln} \simeq 1 : 2.79 : -1.91. \] (3.5)
If the excitation is pure electromagnetic or dominantly electromagnetic
\[ |g \otimes B| \ll \left| \frac{e}{m_e} \vec{B}_e \right|, \] (3.6)
then we should observe three serials of spectral lines, and the positions of the lowest spectral lines of electron, proton and neutron should obey the ratio
\[ \Delta E_e : \Delta E_p : \Delta E_n \simeq 961 : 1.46 : 1. \] (3.7)
Suppose that the detected spectral lines reported in literature [23, 24] are from excitation of electrons in strong electromagnetic field, that is \( \Delta E_e \) is about 20 KeV, then \( \Delta E_p \) should be about 30 eV. In this case, we should also observe some spectral lines with equally spaced at about 30 eV, 60 eV, \cdots. But if those spectral lines are from excitation of protons, that is \( \Delta E_p \) is about 20 KeV, then \( \Delta E_e \) should be about 13 MeV. If we precisely measure the position of spectral lines and find that they obeys the above ratio, we can conclude that the GRBs comes from a neutron star or that X-ray pulsar is not a black hole, and the gravitomagnetic field in that neutron star is relatively very weak.

It is known that, at the event horizon, the gravitational gauge field is divergent, and the field strength of gravitational gauge field is also divergent. It means that, near the event horizon, the gravitomagnetic field can be very strong, or can be much stronger than electromagnetic field. So near a black hole, the excitation can be pure gravitational or dominantly gravitational, then
\[ \Delta E_e : \Delta E_n : \Delta E_p \simeq 1 : 1.91 : 2.79, \] (3.8)
and we should not observe any absorption lines at lower energy region. In this case, the position of the lowest spectral line of neutron is close to that of the second harmonic of electron, and the position of the lowest spectral line of proton is close to the third harmonic of electron. If the position of the first, the second and the third spectral lines satisfies the above relation, not exact 1 : 2 : 3, and there are no
harmonic spectral lines at much lower or much higher positions, it is possible that the GRB comes from a black hole or the X-ray pulsar is a black hole.

If both gravitomagnetic field and electromagnetic field contribute to the excitation, then
\[
0.36 < \frac{\Delta E_e}{\Delta E_p} < 658. \tag{3.9}
\]
Therefor, directly observing the absorption spectral lines of proton and precise measuring the position of the spectral line will help us to understand the mechanism of the excitation and the nature of GRBs or X-ray pulsars.

For a spinning black hole, its gravitomagnetic field will be very strong and its electromagnetic field will be relatively weak. So, the spin transition or cyclotron harmonic is dominantly gravitational, and
\[
\frac{\Delta E_e}{\Delta E_p} \sim 0.36. \tag{3.10}
\]
So, the ratio \(\frac{\Delta E_e}{\Delta E_p}\) can be served as a quantitative criteria of black hole. In other words, using this ratio, we can discriminate black hole from neutron star.

4 Summary and Discussions

In gauge theory of gravity, there is direct coupling between spin and gravitomagnetic field. In GRBs or X-ray pulsars, absorption spectral lines of electron were observed. If these lines originate from strong coupling between magneton and electromagnetic fields, there must exists new serials of spectral lines at much lower or much higher positions. Simultaneously determination of the absorption lines of electron, proton and neutron can help us to determine both the gravitomagnetic field and electromagnetic field, and help us to understand the nature of GRBs or X-ray pulsars. The ratio of the position of the lowest spectral line can be served as a criteria of black hole.

In order to observe the absorption spectral lines of both electron and proton, experimental observation should be performed in a large scope of energy region, at least from 1 eV to 100 MeV, which covers the spectral range from visible light to \(\gamma\) rays. Therefore, a combine observation of astronomical telescope, X-ray telescope and \(\gamma\)-ray telescope is needed.
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