Simulation and histogram modeling of conveyor systems

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Abstract. Using the example of transport conveyor systems, an approach that reduces the cost of resources and time for the development and application of a computer simulation model during the design of the system, which provides a combination of simulation with the method of histogram calculations, which is one of the numerical methods for working with random variables, is proposed. The approach involves replacing a unified simulation model of a system with a complex of submodels of its components, accumulating intermediate results on the basis of submodels and subsequent histogram processing of these results in order to determine the probabilistic characteristics and performance indicators of the system. An example of a histogram simulation of an airport baggage handling system is presented.

1. Introduction
An automated transport system designed to provide a given technological process, including a number of series-parallel connected conveyor lines is considered. Conveyor lines consist of processing devices and continuous transport devices, such as various types of conveyors: belt, plate, roller, including those built into processing devices, as well as lines of deviating pallets, autonomous trolleys, etc. The system receives material objects for processing - operands that generally have different properties, which determine the set of technological operations performed, the trajectory of movement within the system, and the duration of processing for each operation. In order to exclude system failures caused by delays in processing and transportation of material flows, it is possible to temporarily stop conveyor lines and its individual components. The functions of the components of the conveyor system in relation to operands are reduced to ensuring technological operations of processing, transportation, storage during stops in anticipation of further progress. The systems, all components of which are capable of providing the functions of transportation and storage, and some – even processing, are considered below. The conveyor system control object solves the problems of controlling the movement of operands within the system, changing the route of movement in the event of failure of individual elements, issuing information about the position in the system of each operand, etc.

As indicators of the effectiveness of conveyor systems at the stages of their design and operation, a complex of probabilistic numerical and functional characteristics is used that take into account the stochastic nature of the systems under consideration. In the most critical cases, the complexity and stochasticity of the system makes simulation computer modeling the only method for determining these characteristics that has acceptable accuracy, but requires significant resources and time to develop a computer model and simulation [1]. These costs are especially significant in optimal design problems, involving an iterative approach to solving with the need to repeat a simulation experiment.
with updated source data at each iteration. An approach that partially solves this problem is the combination of simulation with the method of histogram calculations, which is one of the numerical methods for working with stochastic variables (SV) [2].

2. The formation of a simulation-histogram model

The proposed histogram-simulation approach involves decomposing the initial “general” simulation model of the conveyor system into separate components (submodels), conducting the necessary studies on submodels with accumulation of intermediate statistical results, and then determining the desired characteristics of the system by “stitching” the obtained results using histogram calculations. The selection of submodels from the “general” model is logical and convenient (but not necessary) in accordance with the accepted division of the system into structural components (subsystems). Scattered fragments of the general model obtained by its formal fragmentation cannot be used directly for modeling without preliminary modification. The necessary refinement of the selected fragment consists in equipping it with blocks that perform, to one degree or another, the functions of model fragments adjacent to the fragment under consideration, and thereby allow keeping account of both the mutual influence of subsystems within a single system, and the impact on the system and subsystems of the environment. The purpose of the decomposition procedure of the general model will be achieved if additional blocks are much simpler than the submodels that they replace, and at the same time such replacement does not lead to an unacceptable decrease in the accuracy of the model. If the submodel selected as a result of decomposition is so simple that it allows an analytical solution, for example, on the basis of the queuing theory, then from the point of view of saving time and resource costs, it is advisable to replace it with an appropriate calculation algorithm that does not require simulation. The results of individual submodels that do not require the use of histogram calculations can be of independent value.

Let us dwell on the histogram approach to constructing the probabilistic distribution of the operand's residence time in the system, which is used to determine such significant performance indicators of the conveyor system as probabilistic-temporal indicators. In the framework of the histogram approach, the distribution function of the operand's residence time in the system can be obtained numerically as a distribution function of the composition of random operand residence times in various subsystems, the distributions of which must be determined in advance on the corresponding simulation submodels.

The method of histogram calculations involves performing numerical operations on the densities and distribution functions of CV and their functions. The first work on the computer implementation of histogram calculations appeared in the early 1990s [3, 4]. Modern directions of the use of the considered methods, the list of which is given in the monograph [2], are relatively diverse and numerous.

Histogram operations are carried out on the systems of independent SV \(X_1, X_2, \ldots, X_M\) in order to determine the functional relationship of the species \(Y = g(X_1, X_2, \ldots, X_M)\). The concept of a histogram SV is used, which is called SV X with a distribution given by a histogram — a piecewise constant function \(f_x(x)\) defined by a set \(\{x_i : x_i \in R, i = 0, \ldots, N\}\) which elements are subject to the condition \(x_{i-1} < x_i, i = 1, \ldots, N\) [4]. In the interval \([x_{i-1}, x_i]\) the histogram takes a constant value \(f_{x_i}\), defined as

\[
f_{x_i} = \int_{x_{i-1}}^{x_i} f(x) dx = \frac{F_x(x_i) - F_x(x_{i-1})}{x_i - x_{i-1}},
\]

where \(f_x(x)\), \(F_x(x)\) are respectively the density and probability distribution function of SV X.

The task is to determine in histogram form SV Y on the set \(\{y_j : y_j \in R, j = 0, \ldots, K\}\), \(y_{j-1} \leq y_j, j = 1, \ldots, K\).

Averaged values of probability density over the interval \([y_{j-1}, y_j]\) is determined as
\[ f'_{y_j} = \frac{F_j(y_j) - F_j(y_{j-1})}{y_j - y_{j-1}} = \frac{P(y_{j-1} < Y < y_j)}{y_j - y_{j-1}}, \]

where \( F_j(y) \) is a probability-distribution function of \( SV \), \( P(y_{j-1} < Y < y_j) \) is the probability of getting \( SV \) on a segment \([y_{j-1}, y_j] \).

In order to be able to define the probability-time characteristics of the system with the help of the histogram, it is necessary to have design formulas for the quantities \( F_j(y) \) and \( P(y_{j-1} < Y < y_j) \) for two design cases that correspond to two options for connecting the subsystems - serial and parallel.

Let the operand enter the conveyor system, consisting of \( M \) subsystems connected in series. The duration of the operand's stay in the subsystems \( X_1, X_2, ..., X_M \) can rightfully be considered independent \( SV \) given with the help of the histogram. Then, the total residence time of the operand in the \( M \) subsystems is a random amount \( Y = X_1 + X_2 + ... + X_M \), the probability \( P(y_{j-1} < Y < y_j) \) to fall into the segment \([y_{j-1}, y_j] \) is approximately calculated by the formula [3]:

\[
P(y_{j-1} < Y < y_j) = \int_{\{y_{j-1} < x_1 + x_2 + ... + x_M < y_j\}} \prod_{i=1}^{M} f_i(x_i) \, dx_1 \cdots dx_M, \quad (1)\]

using instead of \( f_i(x_i), f_2(x_2), ..., f_M(x_M) \) histogram-defined averaged densities \( f'_1(x_1), f'_2(x_2), ..., SV X_1, X_2, ..., X_M \), respectively.

Now let the operand enter the conveyor system, consisting of \( M \) subsystems connected in parallel. The probabilities of entry into each of the subsystems \( \pi_1, \pi_2, ..., \pi_M \) are known and subject to the normalization condition: \( \pi_1 + \pi_2 + ... + \pi_M = 1 \). The duration of the operand's stay in the subsystem (if it enters the given subsystem) is set for each system. As in the case of a serial connection, these durations \( X_1, X_2, ..., X_M \) are accepted by independent histogram-defined SV. In this case, the residence time of the operand in the system is \( SV \), formed as

\[
Y = \begin{cases} 
X_1 & \text{probability } \pi_1, \\
... & \\
X_M & \text{probability } \pi_M.
\end{cases}
\]

The distribution function \( F_j(y) \), according to the theorems of adding the probabilities of incompatible events and multiplying the probabilities of independent events, is defined as

\[
F_j(y) = \pi_1 F_{X_1}(y) + \pi_2 F_{X_2}(y) + ... + \pi_M F_{X_M}(y), \quad (2)
\]

where \( F_{X_1}(x_1), F_{X_2}(x_2), ..., F_{X_M}(x_M) \) are the distribution functions of \( SV X_1, X_2, ..., X_M \). These functions must be predefined or determined by simulation.

The use of expressions (1) and (2) in the required sequence ensures the construction of a histogram of the distribution of the operand's residence time in the system for any combination of options for connecting subsystems.

3. An example of a histogram-simulation

A simulation-histogram model of the airport conveyor system for baggage handling (BHS) of departing passengers is considered, a structural diagram of which is shown in Figure 1, where arrows indicate the direction of movement of the baggage, the lowercase letters indicate the most important devices of the system. BHS is designed to provide pre-flight check-in of passengers and baggage, conducting a three-level inspection of baggage and sorting by flights. At check-in, passengers place their baggage (B) on the weighing conveyor (c) with which each check-in place is equipped, thereby
entering baggage into BHS. Through the collector conveyor k, the baggage is sent to a high-performance screening device \( d_I \) that implements automatic screening “in the stream”. A baggage item in respect of which the need for a second inspection has been identified is sent by a reversing conveyor \( r_I \) to the inspection apparatus \( d_g \) for conducting an inspection with the participation of a human operator. Baggage, which aroused suspicion at this stage, is removed by the reverse conveyor \( r_g \) from BHS for conducting a “manual” inspection. Successfully inspected Bs are transferred to the sorting conveyor \( s \), from which they are distributed by executing mechanisms \( b \) (such as, for example, vertical deflecting conveyors) to drives \( g \) assigned to one or several flights.

BHS is equipped with one or more conveyor lines providing baggage registration and inspection, and a single sorting line. The number \( N^C \) of conveyor lines for registration and inspection, which coincides with the adopted design scheme with the number of inspection devices at each stage, at the design stage of BHS, should be determined taking into account the intensity of the baggage flow to be processed. The number \( N^C \) of check-in desks should be determined based on the intensity of the flow of passengers. As part of BHS, it is logical to distinguish the following three subsystems corresponding to its main functions - the registration subsystem \( S^C \), the inspection subsystem \( S^D \), and the sorting subsystem \( S^S \). In turn, the first two subsystems are divided by the number of conveyor lines for registration and inspection into \( N^D \) registration subsystems \( S^n_C \) and the same number of inspection subsystems \( S^n_D \), \( n = 1, ..., N^D \). The described structuring is illustrated in Figure 1 (where, in order to avoid clutter, the framework of the subsystems \( S^n_C \), \( S^n_D \), \( n = 2, ..., N^D \) is not marked).

The effectiveness of BHS will be the higher if it provides less time spending on passenger service and baggage handling. Two probabilistic characteristics defined by a similar scheme were selected as indicators of the effectiveness of BHS. The first is the probability \( P(T_{pas} \leq t_{pas}) \) of an event consisting in the fact that the actual time the passenger spent at check-in did not exceed a predetermined threshold level accepted as the maximum acceptable for the passenger, and the second, the probability \( P(T_{bag} \leq t_{bag}) \) of an event consisting in the fact that the actual time spent by the B in the BHS did not exceed the specified threshold the level \( t_{bag} \) accepted as the maximum ensuring compliance with the regularity of transportation.

In the presence of distribution functions \( F_{pas}(t) \) and \( F_{bag}(t) \) of random variables, respectively, \( T_{pas} \) and \( T_{bag} \), the probabilities \( P(T_{pas} \leq t_{pas}) \) and \( P(T_{bag} \leq t_{bag}) \) for any \( t_{pas} \geq 0 \) and \( t_{bag} \geq 0 \) are determined as

\[
P(T_{pas} \leq t_{pas}) = F_{pas}(t_{pas}), \quad P(T_{bag} \leq t_{bag}) = F_{bag}(t_{bag}).\]

![Figure 1. Diagram of baggage-handling system](image)
For the formation of functions \( F_{\text{pas}}(t) \) and \( F_{\text{bag}}(t) \) simulation-histogram modeling is used. A complex of simulation models of subsystems \( S^s \), \( S^c \), \( S^d \) and software for histogram processing of their results has been developed. The structure and parameters of all registration subsystems and inspection subsystems are assumed to be identical, which allowed restricting to a single model for each subsystem. Distribution functions are formed with the help of a histogram for random variables

\[
T_{\text{pas}} = \begin{cases} 
T_{\text{pas}}^{eq} & \text{probability } P_1, \\
... & \\
T_{\text{pas}}^{N_D} & \text{probability } P_{N_D},
\end{cases} \quad 
T_{\text{bag}} = \begin{cases} 
T_{1}^{C} + T_{1}^{D} + T_{1}^{S} & \text{probability } P_1, \\
... & \\
T_{N_D}^{C} + T_{N_D}^{D} + T_{N_D}^{S} & \text{probability } P_{N_D},
\end{cases}
\]

where \( T_{\text{pas}} \) is the time spent at the registration of the passenger who arrived at one of the places of registration of the \( n \) conveyor line; \( T_{1}^{C}, T_{1}^{D} \) is a B residence time in subsystems \( S^c, S^d \), respectively; \( T_{1}^{S} \) is time spent in the subsystem \( S^s \) of B entered in BHS in the subsystem \( S^c \); \( P_n \) is the probability of a passenger with baggage arriving at one of the places of registration of the \( n \) conveyor line, \( n = 1,...,N_D \).

BHS, equipped with three conveyor lines for registration and inspection (\( N_D = 3 \)) serving 12 locations of registration (\( N^c = 36 \)), was a model example. The average intensity of baggage flow in BHS, which is about 1100 MB/h, is determined by the characteristics of the flows of departing aircraft of various groups, their passenger capacity and level of occupancy, probabilistic distributions of the time spent by departing passengers at the airport terminal, the number of passengers traveling together, the number of B passengers group, and a number of other parameters taken into account in the simulation. Sources of data on the technical and technological characteristics of BHS were field observations at airports and materials on official websites of manufacturers of luggage systems [5, 6].

![Figure 2. Simulation predictions.](image)

The accuracy of the complex of simulation-histogram models was assessed using the specially developed complete simulation model of the considered BHS, specially developed for this purpose. The results of both models for comparison are shown in Figure 2, where the dash indicates the distribution functions obtained using histogram calculations. From the figure it follows that the distribution of the passenger’s stay at check-in, obtained on the two models, practically coincides. The distribution of baggage handling time in BHS is very close, while the average values of this parameter, determined by the two models, differ by less than 1%.

**4. Conclusion**

The presented results indicate the possibility of replacing the general simulation model with a much smaller and simpler simulation-histogram complex of models. In fairness, it should be noted that such a replacement in the described problem did not reduce the cost of computer time, since the solution in
the considered formulation assumed the accumulation of statistics for each subsystem, and therefore required the repetition of the simulation experiment with the submodel as many times as the number of subsystems this submodel corresponds to. However, if the task was to create a statistical portrait of not the entire system, but only a fragment of it, for example, the baggage handling process received in one of the registration and inspection lines, then, as the verification showed, with the structure of the system considered, the modeling time would be reduced by at least 2 - 2.5 times. Thus, under certain conditions, the use of simulation-histogram models is quite justified.

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