Robust Quantum Computation with Quantum Dots

C. Stephen Hellberg
Center for Computational Materials Science, Naval Research Laboratory, Washington, DC 20375
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Quantum computation in solid state quantum dots faces two significant challenges: Decoherence from interactions with the environment and the difficulty of generating local magnetic fields for the single qubit rotations. This paper presents a design of composite qubits to overcome both challenges. Each qubit is encoded in the degenerate ground-state of four (or six) electrons in a system of five quantum dots arranged in a two-dimensional pattern. This decoherence-free subspace is immune to both collective and local decoherence, and resists other forms of decoherence, which must raise the energy. The gate operations for universal computation are simple and physically intuitive, and are controlled by modifying the tunneling barriers between the dots—Control of local magnetic fields is not required. A controlled-phase gate can be implemented in a single pulse.

A quantum computer with a sufficient number of quantum bits “qubits” (on the order of 1000) would be able to solve certain problems that are intractable on classical computers. Building such a device is a formidable task, and several radically differing designs have been proposed [1]. One promising approach is to encode quantum information using the spin of single electrons confined in semiconductor quantum dots [2, 3, 4, 5]. Universal quantum computation [6, 7] in this approach uses a tunable kinetic exchange interaction between the dots (resulting in a Heisenberg interaction) and one-qubit rotations, which can be obtained by applying local magnetic fields in at least two directions.

The one-qubit rotations are much more difficult to control experimentally than the kinetic exchange interaction. This spurred a number of proposals of quantum computation schemes using the exchange interaction alone [8, 9, 10, 11, 12, 13]. To use this single interaction, the quantum information must be encoded in multiple (two or more) spins.

Decoherence due to interactions with the environment pose a much larger problem for qubits than for classical bits, and there has been a tremendous effort on developing ways of protecting quantum information from decoherence [12, 13, 14, 15, 16, 17, 18, 20, 22, 23, 24]. To shield quantum information from the environment, Zanardi and Rasetti [25] first proposed encoding quantum information in the “noiseless” singlet subspace of 4 (or more) 2-level systems. This subspace, often called a Decoherence Free Subspace (DFS), is immune to collective decoherence, that is, environment-induced dephasing that acts equally on each constituent element of the composite qubit [12, 15, 20, 22, 23, 24].

We propose adding an extra dot (and not an extra electron) in the middle of a square arrangement of four dots, as shown in Fig. 1. We separate the outer four dots so the direct tunneling between them is negligible. Since the ground state wave function of the middle dot will have s-wave character, the effective interaction between each pair of the outer four dots will be equal if the four tunnelings between the outer and middle dots are made equal (e.g. by tuning gates located above or below each tunneling region).
This design is inspired by the superexchange process, which uses empty (or filled) auxiliary quantum dots to mediate interactions between dots that are too widely separated to interact directly. Electrons (or holes) can reach distant quantum dots by hopping through the auxiliary dots.

We need to verify that four electrons can be placed in the five-dot system. We model the system with a Hubbard Hamiltonian using one orbital per quantum dot:

\[
H = \sum_{i,j,\sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_i (U_i n_{i\uparrow} n_{i\downarrow} - \mu_i n_i)
\]

where \(t_{ij}\) is the hopping amplitude between dots \(i\) and \(j\), \(U_i\) is the Coulomb repulsion between two electrons on dot \(i\), \(n_i = n_{i\uparrow} + n_{i\downarrow}\) is the total number of electrons on dot \(i\), and \(\mu_i\) is the onsite potential of dot \(i\). The calculations used \(t_{ij} = -1\), \(U_i = 8\), and an equal overall potential on each dot of \(\mu_i = \mu\). The total occupancy of the ground state of the system calculated by exact diagonalization in the grand canonical ensemble is shown in Fig. 2. The largest region of stability contains five electrons, which is to be expected in the large \(U\) limit, but there are significant ranges of the chemical potential \(\mu\) for which the ground state has four and six electrons. In these regions, the ground state is a doubly degenerate singlet, and quantum computation in this supercoherent subspace is possible.

The Hamiltonian used to generate Fig. 2 has equal onsite potentials at every dot, and thus has electron-hole symmetry. Raising the potential of just the middle dot breaks this symmetry and increases the range of chemical potentials yielding the \(N = 4\) ground state while lowering this potential increases the range of the \(N = 6\) ground state. Quantum computation is possible using either the \(N = 4\) or \(N = 6\) ground states, and the gate operations for the two cases are identical.

A simple way of describing eigenstates of the 5-dot composite qubit is shown in Fig. 3 using valence-bond representation. The ground states contain two separate singlet bonds. There are three ways to construct these bonds: Dot number 1 is bonded to any of the other three outer dots, and then the other bond is formed between the two outer dots not bonded to dot 1. Valence-bond states are not orthogonal in general, and two orthogonal states may be formed, for example, as \(|0\rangle = |a\rangle\) and \(|1\rangle = (|b\rangle + |c\rangle)/\sqrt{3}\).

The information encoded in the degenerate total-spin singlet subspace of the 5-dot composite qubit is immune to collective decoherence, that is, decoherence affecting all spins equally. It is also immune to local decoherence affecting only a single dot. To see this, consider a magnetic field or an extra electron coupling only to dot 1 in Fig. 3. The singlet bond connecting to spin 1 will be mixed with a triplet bond. However, this occurs equally to all three eigenstates in Fig. 3, so the degeneracy between these states is not broken. Multiple spins can cause decoherence, but these mechanisms must overcome the energy gap to the first excited state.

We now demonstrate the physically intuitive gate operations on the 5-dot qubit that allow universal quantum computation. Notice that \(|0\rangle = |a\rangle\) in Fig. 3 is odd under an exchange of sites 1 and 2, denoted by 1 ↔ 2, and this state is also odd under 3 ↔ 4. The other ground state \(|1\rangle = (|b\rangle + |c\rangle)/\sqrt{3}\) is even under both operations.

Increasing the tunneling between the middle dot and dots 1 and 2, denoted by \(H_{12}\) in Fig. 4, respects the \(1 \leftrightarrow 2\) and \(3 \leftrightarrow 4\) symmetries and does not mix the ground states, but \(H_{12}\) does break the degeneracy of the ground state. Thus it acts in the pseudospin space of \(|0\rangle\)
FIG. 4: Single-qubit rotations are performed by varying at least two of the tunneling parameters, shown by thicker lines. In $H_{12}$, for example, the tunnelings between the the central dot and dots 1 and 2 are increased relative to the tunnelings between the central dot and dots 3 and 4. $H_{12}$ splits but does not mix states $|0\rangle$ and $|1\rangle$, and thus functions as a field in the $\hat{z}$ direction in pseudospin space. $H_{14}$ splits and mixes the states as described in the text. Combinations of $H_{12}$ and $H_{14}$ allow arbitrary $SU(2)$ rotations of the single composite qubit.

and $|1\rangle$ as a magnetic field in the $\hat{z}$ direction:

$$H_{12} \propto \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$  

The tunnelings from the middle dot to dots 3 and 4 could have been increased to implement the same gate, so effectively $H_{34} = H_{12}$. Adiabatic variation of the tunneling rates is required to avoid mixing with excited states. Adiabaticity is also required in the conventional single-quantum-dot qubit implementation to avoid mixing with states containing dots occupied by two electrons [24,32].

To perform arbitrary $SU(2)$ rotations of the single composite qubit, we need to be able to perform rotations about two directions in pseudospin space. Therefore a gate in addition to $H_{12}$ is required. Varying three tunnelings in the 5-dot composite qubit can produce a rotation about the $\hat{x}$ axis in pseudospin space, but a simpler gate can be formed by increasing the tunneling between the middle dot and dots 1 and 4, denoted by $H_{14}$ in Fig. 4. This operation breaks the 1 ↔ 2 and 3 ↔ 4 symmetries, and can be shown to be

$$H_{14} \propto \begin{pmatrix} -1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix},$$  

which represents a rotation at an angle of 120 degrees from the $\hat{z}$ axis [8,12]. Again, increasing the tunnelings to dots 2 and 3 would implement the same gate, so $H_{23} = H_{14}$. With $H_{12}$ and $H_{14}$, any $SU(2)$ rotation can be performed on the composite qubit. The one-qubit operations are similar to operations described for the 4-spin DFS, but in that case the interaction takes place directly between the spins and not through an auxiliary fifth dot [12,13,29].

To form a two-qubit gate, two tunnelings must be turned on between adjacent qubits—A single tunneling interaction performs no operation due to the immunity of the composite qubit to local decoherence. An example of a two-composite-qubit gate is shown in Fig. 5 in which tunneling between pairs of outer dots on adjacent qubits has been turned on. This operation preserves the symmetries 1 ↔ 2 and 7 ↔ 8. Thus its action in the basis of $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ has the general form

$$H_{2\text{qubit}} = \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & B & 0 \\ 0 & 0 & 0 & C \end{pmatrix}$$  

which has been verified by exact diagonalization of the 10-dot 8-electron Hubbard model for the two-dot system. During both the single- and two-composite qubit gate operations, the system stays in the total singlet subspace and remains immune to collective decoherence [12].

FIG. 5: A two-qubit operation can be performed by turning on tunneling between two pairs of dots in neighboring composite qubits. Combined with the single-qubit rotations in Fig. 4, this operation allows any arbitrary unitary transformation to be performed.

Combining $H_{2\text{qubit}}$ with single qubit rotations on the individual dots allows the controlled-phase gate $C_P$ to be implemented with a single pulse in which six tunneling
rates are varied:

\[
\mathcal{C}_P = \exp\left(\frac{i}{\hbar} \int (H_{12}(t) + H_{2\text{qubit}}(t) + H_{78}(t)) \right) dt
\]

\[
= \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}
\]  \hspace{1cm} (5)

where \(H_{12}, H_{2\text{qubit}}, \text{and } H_{78}\) all commute \[1, 12, 29\].

Finally, it is interesting to note that the 5-dot configuration is actually more stable than a 4-dot setup to variations in the hopping parameters. Varying a single tunneling rate in Fig. 4 does not break the degeneracy of the ground state. Such a change modifies the effective interactions of a single outer dot with each of the other outer dots, and simply shifts the ground-state energy preserving the degeneracy. In contrast, varying one of the six tunneling rates in the 4-dot setup splits the ground states.

In summary, a five-dot composite qubit design was presented that operates in a decoherence-free subspace. Universal quantum computation is easily implemented by varying tunneling rates in a simple, physically intuitive manner—Generation of local magnetic fields is not required to perform the gate operations. Each qubit is encoded in the degenerate singlet ground-state of four (or six) electrons in a system of five quantum dots arranged in a two-dimensional pattern. This supercoherent subspace is immune to both collective and local decoherence, and resists other forms of decoherence, which must raise the energy.

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* Email: mylastname@dave.nrl.navy.mil

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