Electroweak Standard Model and Precision Tests

Jens Erler

Instituto de Física, Universidad Nacional Autónoma de México, 01000 México D.F., México

Abstract. I give an introduction and overview of recent developments in high precision tests of the Standard Model. This includes a summary of Z-pole measurements, a brief account of the NuTeV result on neutrino-nucleon scattering, the anomalous magnetic moment of the muon, and implications for the Higgs boson mass.

INTRODUCTION

The most fundamental observable related to the weak interaction is the muon lifetime, $\tau_\mu$. With the electromagnetic two-loop contribution obtained in Ref. [1], $\tau_\mu$ can be used unambiguously to extract the Fermi constant, $G_F = 1.16637(1) \times 10^{-5}$ GeV$^{-2}$, where the uncertainty is completely dominated by experiment. With the electromagnetic fine structure constant, $\alpha$, as an additional input, we can then obtain the quantity,

$$A^2 = \frac{\pi \alpha}{\sqrt{2} G_F} = (37.2805 \pm 0.0003 \text{ GeV})^2,$$

and use it to write down relations between the intermediate gauge boson masses, $M_{W,Z}$, and the weak mixing angle; for example, in the $\overline{\text{MS}}$ scheme one has [2],

$$\sin^2 \hat{\theta}_W \equiv \hat{s}^2 = \frac{A^2}{M_W^2 (1 - \Delta \hat{r}_W)}, \quad \hat{s}^2 (1 - \hat{s}^2) = \frac{A^2}{M_Z^2 (1 - \Delta \hat{r}_Z)},$$

where $\Delta \hat{r}_W$ and $\Delta \hat{r}_Z$ are electroweak radiative correction parameters. Most of the Z-pole asymmetries discussed in the next section are basically measurements of $\sin^2 \theta_{\text{eff}} = \hat{\kappa}_e \hat{s}^2$, where $\hat{\kappa}_f$ denotes a flavor dependent form factor which for $f = e$ is numerically very close to unity with little sensitivity to the Standard Model (SM) input parameters. Since furthermore $M_Z$ is known to great accuracy, the second Eq. (2) implies that the Z-pole asymmetries effectively determine,

$$\Delta \hat{r}_Z = \frac{\alpha}{\pi} \hat{\Delta}_\gamma + F_1 (m_t^2, M_H, \ldots).$$

Asymptotically for large top quark masses, $m_t$, the function, $F_1$, grows like $m_t^2$. This is because the large mass hierarchy, $m_t \gg m_b$, breaks isospin symmetry in W boson self-energy diagrams with a bottom and a top quark in the loop. This effect has been absorbed

---

1 Talk given at the X Mexican School of Particles and Fields, Playa del Carmen, México, 2002.
into $G_F$, but now reappears in $\Delta r$ when $M_Z$ is computed in terms of it. For the same reason, there is an $m_t^2$ effect in the low-energy $\rho$-parameter \([3]\), which is defined as the ratio of neutral-to-charged weak current interaction strengths. The first Eq. \([2]\) shows that a determination of the $W$ boson mass can then be used to measure

$$\Delta r_W = \frac{\alpha}{\pi} \hat{\Delta} \gamma + F_2 (\ln m_t, M_{H}, \ldots), \quad (4)$$

where indeed $F_2$ has a milder $m_t$ dependence. $F_1$ and $F_2$ are complicated functions of the Higgs boson mass, $M_H$, which are asymptotically logarithmic. Eqs. \([3]\) and \([4]\) also show that $M_H$ can be extracted from the precision data only when the quantity,

$$\hat{\Delta} \gamma = 4 \pi^2 \hat{\Pi}_{\gamma\gamma}^{(f)} + \text{bosonic terms}, \quad (5)$$

which characterizes the renormalization group (RG) evolution (“running”) of $\alpha$,

$$\hat{\alpha}(M_Z) = \frac{\alpha}{1 - \frac{\alpha}{\pi} \hat{\Delta} \gamma}, \quad (6)$$

is known accurately. While it can be computed rigorously for leptons, there is a problem for quarks (hadrons). This is best seen by noting that the one-loop fermion contribution to the photon self-energy, $\hat{\Pi}_{\gamma\gamma}^{(f)}$, is proportional to $\ln M_Z^2 / m_f^2$, and it is not clear what mass definition is to be used here for quarks. This is a question of loop corrections proportional to powers of the QCD coupling, $\alpha_s$, and can indeed be dealt with perturbatively for charm and bottom quarks \([3]\). For the light quarks, however, perturbation theory breaks down and one needs a different strategy: one uses analyticity and the optical theorem which in essence delivers $\hat{\Pi}_{\gamma\gamma}^{(f)}$ from its imaginary part and thus from (a weighted integral over) the cross-section $\sigma(e^+ e^- \rightarrow \text{hadrons})$ for which experimental data are available. Incidentally, a similar strategy is used to estimate the hadronic two- and three-loop contributions to the muon anomalous magnetic moment, $g_{\mu} - 2$, which amounts to an integral over the same data, but with a different weight. Notice that the uncertainty in the cross-section data induces correlations between $\Delta \gamma$, $M_H$, $g_{\mu} - 2$, and (currently of less importance) the running of the weak mixing angle relevant for weak neutral current precision observables at low energies (such as in atomic parity violation) which is also subject to this kind of treatment and the same data.

By assuming isospin symmetry and correcting for kinematics, isospin violating effects \([5]\), electroweak radiative corrections \([6, 7, 8]\), etc., one can use the invariant mass spectrum in hadronic $\tau$ decays to obtain additional information \([9]\). Kinematic suppression limits this method mainly to two pion (and to a lesser extent four pion) final states. Hadronic $\tau$ decay data are of particular relevance to $g_{\mu} - 2$ (see below).

**Z-POLE OBSERVABLES**

The first part of Table \([1]\) shows the $Z$ line shape and leptonic forward-backward (FB) cross section asymmetry, $A_{FB}(\ell)$, measurements from LEP 1 \([10]\). They include the
Table 1. Results from Z-pole precision measurements compared to the SM predictions obtained from a global analysis of high and low energy experiments. The deviations from the predictions (in terms of the pulls) are also shown.

| Quantity | Group(s) | Value     | Standard Model | pull  |
|----------|----------|-----------|----------------|-------|
| $M_Z$    | LEP      | $91.1876 \pm 0.0021$ | $91.1874 \pm 0.0021$ | $0.1$ |
| $\Gamma_Z$ | LEP      | $2.4952 \pm 0.0023$ | $2.4972 \pm 0.0011$ | $-0.9$ |
| $\Gamma^{(\text{inv})}$ | LEP      | $499.0 \pm 1.5$ | $501.74 \pm 0.15$ | —     |
| $\sigma_{\text{had}}$ | LEP      | $41.541 \pm 0.037$ | $41.470 \pm 0.010$ | $1.9$ |
| $R^+_c$  | LEP      | $20.804 \pm 0.050$ | $20.753 \pm 0.012$ | $1.0$ |
| $R^+_b$  | LEP      | $20.785 \pm 0.033$ | $20.753 \pm 0.012$ | $1.0$ |
| $R^-_t$  | LEP      | $20.764 \pm 0.045$ | $20.799 \pm 0.012$ | $-0.8$ |
| $A_{FB}(\ell)$ | LEP      | $0.0145 \pm 0.0025$ | $0.01639 \pm 0.00026$ | $-0.8$ |
| $A_{FB}(\mu)$ | LEP      | $0.0169 \pm 0.0013$ | $0.4$ |
| $A_{FB}(\tau)$ | LEP      | $0.0188 \pm 0.0017$ | $1.4$ |
| $R_b$    | LEP + SLD | $0.21644 \pm 0.00065$ | $0.21572 \pm 0.00015$ | $1.1$ |
| $R_c$    | LEP + SLD | $0.1718 \pm 0.0031$ | $0.17231 \pm 0.00006$ | $-0.2$ |
| $A_{FB}(b)$ | LEP      | $0.0995 \pm 0.0017$ | $0.1036 \pm 0.0008$ | $-2.4$ |
| $A_{FB}(c)$ | LEP      | $0.0713 \pm 0.0036$ | $0.0741 \pm 0.0007$ | $-0.8$ |
| $A_{b}$  | SLD      | $0.922 \pm 0.020$ | $0.93477 \pm 0.00012$ | $-0.6$ |
| $A_c$    | SLD      | $0.670 \pm 0.026$ | $0.6681 \pm 0.0005$ | $0.1$ |
| $A_{LR}$ (hadrons) | SLD      | $0.15138 \pm 0.000216$ | $0.1478 \pm 0.0012$ | $1.6$ |
| $A_{LR}$ (leptons) | SLD      | $0.1544 \pm 0.00060$ | $1.1$ |
| $\bar{\theta}_q$ | SLD      | $0.142 \pm 0.015$ | $-0.4$ |
| $\bar{\theta}_t$ | SLD      | $0.136 \pm 0.015$ | $-0.8$ |
| $\theta_{\tau}^e$ | LEP      | $0.1439 \pm 0.0043$ | $-0.9$ |
| $\theta_{\tau}^{LR}$ | LEP      | $0.1498 \pm 0.0049$ | $0.4$ |
| $Q_{FB}$ | LEP      | $0.0403 \pm 0.0026$ | $0.0424 \pm 0.0003$ | $-0.8$ |

total Z decay width, $\Gamma_Z$, the hadronic peak cross section, $\sigma_{\text{had}}$, and for each lepton flavor the ratio of hadronic to leptonic partial Z widths, $R_\ell$. The invisible Z partial width, $\Gamma^{(\text{inv})}$, is derived from $\Gamma_Z$, $\sigma_{\text{had}}$, and the $R_\ell$, and is not independent. It is smaller than the SM prediction by almost 2 $\sigma$, which can be traced to $\sigma_{\text{had}}$ which deviates by a similar amount. Conversely, one can use the data to determine the number of standard neutrinos, $N_v = 2.986 \pm 0.007$, again showing a $2 \sigma$ deviation from the expectation, $N_v = 3$.

The second part of Table 1 shows the results from Z decays into heavy flavors. For bottom and charm quarks the partial Z width normalized to the hadronic partial width, $R_b$, is shown, as well as the FB-asymmetry, $A_{FB}(q)$, and $A_q$ which is proportional to the combined left-right (LR) forward-backward asymmetry, $A_{LR}^{LR}(q)$. The latter is equivalent to $\sin^2 \theta_q^{eff}$. $A_{FB}(q)$ is proportional to $A_eA_q$ and primarily sensitive to $\sin^2 \theta_e^{eff}$ with $A_{FB}(b)$ providing one of its best determinations. It shows a $2.4 \sigma$ deviation, and favors larger values of $M_H$. It is tempting to suggest new physics effects in the factor $A_b$ to reconcile this deviation and the disagreement with $A_{LR}$ discussed below. However, one would need a $(19 \pm 7)\%$ radiative correction to $\kappa_b$, while typical electroweak radiative corrections are of $\mathcal{O}(1\%)$ or smaller. New physics entering at tree level is generally not resonating and/or strongly constrained by other processes. At any rate, $R_b$ is in reasonable agreement with the SM and some tuning of parameters would be required.

The last part of Table 1 shows further measurements with sensitivity to $\sin^2 \theta_e^{eff}$. The
as well as experiments from CERN and FNAL. The ratio of neutral-to-charged current higher than the SM expectation. Just as polarization, LR cross section asymmetry, these mass measurements with all other (indirect) data, and the SM prediction for various searches at LEP 2 [13], the SM predictions of numerous observables (for example through $A_{FB}(\tau)$) [12], as well as the hadronic charge asymmetry [10], which are excluded by the direct searches at LEP 2 [13].

\[ M_H \geq 114.4 \text{ GeV} \ (95\% \text{ CL}) \]  

(7)

The other determinations are from $A_{FB}(\mu)$ and $A_{FB}(\tau)$ [12], from the final state $\tau$ polarization, $\mathcal{P}_\tau$, and its angular dependence [10], as well as the hadronic charge asymmetry [10], which is a weighted sum over $A_{FB}(q)$.

**OTHER OBSERVABLES**

The first part of Table 2 shows the (direct) $m_t$ measurement from the Tevatron [14, 15], as well as $M_W$ from LEP 2 [10] and $p\bar{p}$ collisions [10, 17]. The combined $M_W$ is 1.8 $\sigma$ higher than the SM expectation. Just as $A_{LR}$ it favors smaller values of $M_H$. We compare these mass measurements with all other (indirect) data, and the SM prediction for various values of $M_H$ in Figure 1. The bottom and charm quark masses, $m_b$ and $m_c$, which enter the SM predictions of numerous observables (for example through $\delta_{\gamma}$) are constrained using a set of inclusive QCD sum rules [18] and are recalcualted in each call within the fits as functions of $\alpha_s$ and other global fit parameters.

The second part of Table 2 shows results of neutrino-nucleon deep inelastic scattering experiments from CERN and FNAL. The ratio of neutral-to-charged current $\nu_\mu$-cross

**TABLE 2.** Precision observables away from the Z-pole. The first error for the measurement values is experimental and (where applicable) the second refers to theory or model uncertainties.

| Quantity | Group(s)       | Value          | Standard Model       | pull    |
|----------|----------------|----------------|----------------------|---------|
| $m_t$ [GeV] | Tevatron       | 174.3±5.1     | 174.4±4.4            | 0.0     |
| $M_W$ [GeV] | LEP           | 80.447±0.042  | 80.391±0.019         | 1.3     |
| $M_W$ [GeV] | Tevatron + UA2 | 80.454±0.059  |                      | 1.1     |
| $g_2^V$ | NuTeV         | 0.30005±0.00137 | 0.30396±0.00023     | -2.9    |
| $g_3^V$ | NuTeV         | 0.03076±0.00110 | 0.03005±0.00004     | 0.6     |
| $R^\ell$ | CCFR          | 0.5820±0.0027 | 0.5833±0.0004       | -0.3    |
| $R^q$  | CDHS          | 0.3096±0.0033 | 0.3092±0.0002       | 0.1     |
| $R^\nu$ | CHARM         | 0.3021±0.0031 | 0.3026±0.0002       | -1.7    |
| $R^\nu$ | CDHS          | 0.384±0.016   | 0.3862±0.0002       | -0.1    |
| $R^\nu$ | CHARM         | 0.403±0.014   |                      | 1.0     |
| $R^\nu$ | CDHS 1979     | 0.365±0.015   | 0.3817±0.0002       | -1.0    |
| $g_2^{\nu_e}$ | CHARM II  | -0.035±0.017  | -0.0398±0.0003      | —       |
| $g_2^{\nu_e}$ | all          | -0.041±0.015  |                      | -0.1    |
| $g_2^{\nu_e}$ | CHARM II     | -0.503±0.017  | -0.5065±0.0001      | —       |
| $g_2^{\nu_e}$ | all          | -0.507±0.014  |                      | 0.0     |
| $Q_{W}(\text{Cs})$ | Boulder    | -72.69±0.44   | -73.10±0.04         | 0.8     |
| $Q_{W}(\text{TI})$ | Oxford + Seattle | -116.6±3.7   | -116.7±0.1          | 0.0     |
| $10^9 \left( a_{\mu} - \frac{\alpha}{2\pi} \right)$ | BNL + CERN | 4510.64±0.79 | 4508.28±0.33        | 2.5     |
sections, $R_v = \frac{\sigma_{VN}^{NC}}{\sigma_{VN}^{CC}}$, is more sensitive to the weak mixing angle than the analogous $\bar{\nu}_\mu$-ratio, $R_{\bar{\nu}_\mu}$. Both are sensitive to charm threshold effects which introduce the dominant theoretical uncertainty. This uncertainty largely cancels in the ratio \cite{19},

$$R^- = \frac{\sigma_{V\mu}^{NC} - \sigma_{V\mu}^{NC}}{\sigma_{V\mu}^{CC} - \sigma_{V\mu}^{CC}} = g_L^2 - g_R^2,$$

which was used by the NuTeV Collaboration \cite{20} (who had a clean \(\bar{\nu}_\mu\)-beam at their disposal) to measure the weak mixing angle precisely off the $Z$-pole. In the presence of new physics, however, which will in general affect $\nu_\mu$ and $\bar{\nu}_\mu$ cross sections differently, one should rather monitor $R_v$ and $R_{\bar{\nu}_\mu}$ independently, or equivalently, the effective four-Fermi $\nu_\mu$-quark couplings,

$$g_L^2 = \frac{1}{2} - \sin^2 \theta_W + \frac{5}{9} \sin^4 \theta_W, \quad g_R^2 = \frac{5}{9} \sin^4 \theta_W,$$

which are shown in the Table. One sees that the quoted \cite{20} 3 $\sigma$ deviation in the weak mixing angle can be traced to a 2.9 $\sigma$ deviation in $g_L^2$. The weak charge of Cs \cite{21}.
$Q_W(Cs)$, is currently the only other precise determination of $\sin^2 \theta_W$ off the Z-pole, while the results of $\nu e$ scattering in the third part of Table 2 and of $Q_W(Tl)$ are less precise. The extraction of weak charges from atomic parity violation is complicated by atomic theory uncertainties and $Q_W(Cs)$ was deviating from the SM prediction in the past. Thus, it is important to have complementary determinations of $\sin^2 \theta_W$ which (as indicated in Figure 2) will come from polarized Møller scattering by the E158 Collaboration at SLAC [23] and elastic $\vec{e}p$ scattering by the QWEAK Collaboration at JLAB [24] which will determine the weak charges of the electron and the proton, respectively. A new deep inelastic $eD$ scattering experiment has also been suggested [25].

The last entry in Table 2 refers to $g_\mu - 2$ [26] which shows a deviation of about 2.5 $\sigma$ from the SM prediction. This could be interpreted as a hint at supersymmetry or many other types of physics beyond the SM [27]. However, as discussed in the introduction, there are complications from the two-loop hadronic vacuum polarization. In fact, data based on $\sigma(e^+e^- \rightarrow \text{hadrons})$ suggest even a 3 $\sigma$ deviation [9, 28], while data based on $\tau$ decays suggest agreement with the SM within about one standard deviation [8]. Moreover, the three-loop hadronic light-by-light contribution can be modeled [29], but methods based on first principles such as chiral perturbation theory are not precise enough to confirm the model estimates [30].

**FIGURE 2.** Scale dependence of the weak mixing angle in the SM and the minimal supersymmetric SM. The error bars for E158, QWEAK, and eD DIS are projections (the vertical location is arbitrary).
CONCLUSIONS

Despite of the various deviations described above, it must be stressed that the overall agreement between the data and the SM is reasonable. The $\chi^2$ per degree of freedom of the global fit is 49.1/40 where the probability for a larger $\chi^2$ is 15%.

The global fit to all precision data currently favors values for the Higgs boson mass

$$M_H = 86^{+49}_{-32} \text{ GeV}, \quad (M_H = 93^{+52}_{-35} \text{ GeV}),$$

where the central value is slightly below the lower LEP 2 limit in Eq. (7). The result in parentheses refers to the case where the $g_\mu - 2$ result is excluded from the fit. The two results differ not because the SM prediction for $g_\mu - 2$ has a strong $M_H$ dependence, but rather due to the correlation described in the Introduction. If one includes the Higgs search information from LEP 2, one obtains the probability density shown in Figure 3.

---

2 See Ref. [31] and the talk of W. Hollik at this meeting for a discussion of Higgs boson masses in the minimal supersymmetric SM.
The global fit result,

\[ m_t = 174.2 \pm 4.4 \text{ GeV}, \quad (m_t = 174.0^{+9.9}_{-7.4} \text{ GeV}), \quad (11) \]

is dominated by the Tevatron measurement, \( m_t = 174.3 \pm 5.1 \text{ GeV} \). One can exclude this from the fit and determine \( m_t \) from the (indirect) precision data alone. This is the result in parentheses, which is seen in spectacular agreement with the Tevatron value.

**ACKNOWLEDGMENTS**

It is a pleasure to congratulate Augusto García and Arnulfo Zepeda to their 60th birthday, and to thank the organizers for inviting me to a very enjoyable meeting.

**REFERENCES**

1. van Ritbergen, T., and Stuart R. G., Phys. Rev. Lett. **82**, 488 (1999).
2. Degrassi, G., Fanchiotti, S., and Sirlin, A., Nucl. Phys. B **351**, 49 (1991).
3. Veltman, M. J., Nucl. Phys. B **123**, 89 (1977).
4. Erler, J., Phys. Rev. D **59**, 054008 (1999).
5. Cirigliano, V., Ecker, G., and Neufeld, H., Phys. Lett. B **513**, 361 (2001).
6. Marciano, W. J., and Sirlin, A., Phys. Rev. Lett. **56**, 22 (1986), and *ibid.* **61**, 1815 (1988).
7. Braaten, E., and Li, C. S., Phys. Rev. D **42**, 3888 (1990).
8. Erler, J., preprint [hep-ph/0211345](http://arxiv.org/abs/hep-ph/0211345).
9. Davier, M., Eidelman, S., Höcker, A., and Zhang, Z., preprint [hep-ph/0208177](http://arxiv.org/abs/hep-ph/0208177).
10. ALEPH, DELPHI, L3, and OPAL Collaborations, LEP Electroweak Working Group and SLD Heavy Flavor and Electroweak Groups: Abbaneo, D., *et al.*, preprint [hep-ex/0112021](http://arxiv.org/abs/hep-ex/0112021).
11. SLD Collaboration: Abe, K., *et al.*, Phys. Rev. Lett. **84**, 5945 (2000).
12. SLD Collaboration: Abe, K., *et al.*, Phys. Rev. Lett. **86**, 1162 (2001).
13. Holzner, A. G., preprint [hep-ex/0208045](http://arxiv.org/abs/hep-ex/0208045), talk presented at BEACH 2002.
14. DØ Collaboration: Abbott, B., *et al.*, Phys. Rev. D **58**, 052001 (1998).
15. CDF Collaboration: Abe, F., *et al.*, Phys. Rev. Lett. **82**, 271 (1999).
16. DØ Collaboration: Abbott, B., *et al.*, Phys. Rev. Lett. **84**, 222 (2000).
17. CDF Collaboration: Affolder, T., *et al.*, Phys. Rev. D **64**, 052001 (2001).
18. Erler, J., and Luo, M., preprint [hep-ph/0207114](http://arxiv.org/abs/hep-ph/0207114).
19. Paschos, E. A., and Wolfenstein, L., Phys. Rev. D **7**, 91 (1973).
20. NuTeV Collaboration: Zeller, G. P., *et al.*, Phys. Rev. Lett. **88**, 091802 (2002).
21. Bennett, S. C., and Wieman, C. E., Phys. Rev. Lett. **82**, 2484 (1999).
22. Kuchiev, M. Y., and Flambaum, V. V., preprint [hep-ph/0206126](http://arxiv.org/abs/hep-ph/0206126).
23. Carr, R., *et al.*, SLAC-PROPOSAL-E-158, [http://www.slac.stanford.edu/exp/e158/documents/proposal.ps.gz](http://www.slac.stanford.edu/exp/e158/documents/proposal.ps.gz).
24. Armstrong, D., *et al.*, JLAB-PROPOSAL-E-02-020, [http://www.jlab.org/exp_prog/proposals/02/PR02-020.pdf](http://www.jlab.org/exp_prog/proposals/02/PR02-020.pdf).
25. Reimer, P., [http://mocha.phys.washington.edu/~int_talk/WorkShops/int_02_3/People/Reimer_P/DIS-Parity_Seattle.pdf](http://mocha.phys.washington.edu/~int_talk/WorkShops/int_02_3/People/Reimer_P/DIS-Parity_Seattle.pdf), talk presented at INT-02-3.
26. Muon g-2 Collaboration: Bennett, G. W., *et al.*, Phys. Rev. Lett. **89**, 101804 (2002).
27. Czarnecki, A., and Marciano, W. J., Phys. Rev. D **64**, 013014 (2001).
28. Hagiwara, K., Martin, A. D., Nomura, D., and Teubner, T., preprint [hep-ph/0209187](http://arxiv.org/abs/hep-ph/0209187).
29. Knecht, M., Nyffeler, A., Perrotta, M., and De Rafael, E., Phys. Rev. Lett. **88**, 071802 (2002).
30. Ramsey-Musolf, M., and Wise, M. B., Phys. Rev. Lett. **89**, 041601 (2002).
31. Frank, M., Heinemeyer, S., Hollik, W., and Weiglein, G., preprint [hep-ph/0212037](http://arxiv.org/abs/hep-ph/0212037).
32. Erler, J., Phys. Rev. D **63**, 071301 (2001).