Effective One-Body Approach to Impurities in One-Dimensional Trapped Bose Gases

S. I. Mistakidis,1 A. G. Volosniev,2 N. T. Zinner,3,4 and P. Schmelcher1,5

1Zentrum für Optische Quantentechnologien, Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany
2Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany
3Department of Physics and Astronomy, Aarhus University, DK-8000 Aarhus C, Denmark
4Aarhus Institute of Advanced Studies, Aarhus University, DK-8000 Aarhus C, Denmark
5The Hamburg Centre for Ultrafast Imaging, Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany

(Dated: September 7, 2018)

We investigate the quench dynamics of impurity atoms in a one-dimensional trapped Bose gas. Our focus is to explore the effects of inhomogeneity due to the harmonic confinement generally present in current cold-atom experiments. We show that inhomogeneity can be taken into account by an effective one-body model where both the mass and the string constant are renormalized. This is in contrast to the classic renormalization which addresses only the mass. We propose an effective single particle Hamiltonian and use the multi-layer multi-configuration time-dependent Hartree method for bosons to explore its validity. The numerical simulations reveal that there exist two parameter regimes, which we call the ‘miscible phase’ and ‘immiscible phase’. Our one-body description is valid for the former but must be modified for the latter regime. Importantly, it turns out that the mass of the ‘polaron’ is smaller than the impurity mass, which means that it cannot straightforwardly be extracted from translationally-invariant models.

Introduction.— Often, low-energy quantum states of an impurity in a homogeneous infinite environment can be well characterized by only the momentum $P$ of the impurity, such that the energies are

$$E(P) \simeq \epsilon + \frac{P^2}{2m_{\text{eff}}},$$

(1)

where $\epsilon$ and $m_{\text{eff}}$ are parameters. $E(P)$ is the energy of a free particle with the mass $m_{\text{eff}}$, which enables the notion of a polaron: A quasiparticle that describes the response of an impurity in a medium to certain low-energy perturbations [1,2]. The idea of a polaron was put forward in solid state physics to understand the motion of an electron in a polarizable solid [3]. However, the universality of the polaron mechanism makes it useful to describe many other impurities such as a $^3$He particle in superfluid $^4$He [3], a proton or a Λ-baryon in nuclear matter [5,6], or a proton in proton conductors [7]. Concerning applications, the polaron concept is vital to computing properties of various strongly correlated electronic materials [8,9], and organic semiconductors of technological significance [10].

To employ the picture of the polaron, the values of $m_{\text{eff}}$ and $\epsilon$, and the limits of applicability of Eq. 1 must be specified. To this end, one must use experimental data or (in the absence of such data) ab initio calculations. The latter are arguably the simplest that give insight into the interplay between one- and many-body physics. Still, only approximate solutions are available as various assumptions have to be made along the way. Mixtures of cold atoms that put these solutions to the test [11–17] inspire the discussion between the polaron theories and experiments, which sheds new light onto the underlying physics not only in cold atoms, but also in more practically relevant materials.

This Letter contributes to this dialogue by studying effective descriptions of an atom that moves in a harmonic trap, which also confines a weakly-interacting one-dimensional Bose gas. Such a study is necessary because a harmonic confinement is often present in cold-atom experiments, and, therefore, it must be taken into account if one wants to analyze measured data using the predictions of classic homogeneous models. In one-dimensional systems the properties of the system depend very strongly on the confinement [18], which motivates theoretical studies of trap effects on the ground state [19,20] and quench dynamics [21–25]. The latter studies, however, rely on various approximations, e.g., on the local density or mean-field approximations, whose validity is hard to test for mesoscopic experimental set-ups. Here we perform a variational beyond-mean-field study of the quench dynamics using the multi-layer multi-configuration time-dependent Hartree method for bosons (ML-MCTDHB) [26–32], which focuses on weak and intermediate interaction strengths. The ML-MCTDHB results allow us to identify parameters for which an effective one-body description is appropriate. They demonstrate the necessity to renormalize both the mass and the string constant in the corresponding effective Hamiltonian. Furthermore, they reveal that the mass of the ‘polaron’ can be smaller than the bare mass as it incorporates certain effects of the trap. These findings are important for interpreting current cold-atom set-ups, and for simulating unconventionally small effective masses.

Hamiltonian.— We consider a single impurity atom that moves in a system of $N$ bosons. The one-dimensional Hamiltonian of interest is motivated by the
current cold-atom experiments \cite{18,35,37}

\[ H = \sum_i h(x_i) + h(y) + g \sum_{i>j} \delta(x_i - x_j) + c \sum_i \delta(x_i - y), \tag{2} \]

where \( y \) is the position of the impurity, and \( x_i \) is the coordinate of the \( i \)th boson. The zero-range interactions are parametrized by \( g > 0 \) and \( c > 0 \). The one-body Hamiltonian reads \( h(x) = -\frac{g}{\rho} \frac{\delta^2}{\delta x^2} + \frac{m\Omega^2 x^2}{2} \), where \( m \) is the mass and \( \Omega \) is the trap frequency. Note that, for the sake of argument, the impurity and a boson obey the same one-body Hamiltonian \( h \). For later convenience, let us introduce the string constant \( k \equiv m\Omega^2 \).

To find the low-energy eigenstates of \( H \), one has to address an \((N+1)\)-body problem. In the homogeneous case qualitative features of these states are well understood \cite{24,35,39}. The trapped case requires, however, further exploration. In particular, the quench dynamics is of immediate interest because it potentially can be used to study polarons \cite{18,24}. The analysis of the time evolution upon the change: \( c = 0 \) at \( t < 0 \) to \( c > 0 \) at \( t \geq 0 \), which can be realized in experiments (e.g., by tuning an external magnetic field \cite{41,48}) is the main objective of our work. However, to set the stage we first consider the homogeneous case, i.e., \( \Omega = 0 \).

**Homogeneous Case.**—In the homogeneous case (in the limit \( N \to \infty \) and finite density \( \rho \)) the low-lying excited eigenstates of \( H \) related to the motion of the impurity are approximately parametrized by the momentum of the impurity (cf. Ref. \cite{32}). The corresponding energies are given by Eq. (1). This knowledge motivates the use of the effective Hamiltonian for the description of the low-energy dynamics of the impurity

\[ H_{\text{eff}} = \epsilon - \frac{\hbar^2}{2m_{\text{eff}}(y)} \frac{\partial^2}{\partial y^2} + \frac{ky^2}{2}, \tag{3} \]

where \( \epsilon(g/\rho,c/\rho) \) and \( m_{\text{eff}}(g/\rho,c/\rho) \) are referred to as the self-energy and the effective mass of the polaron, respectively. These quantities can be measured in a laboratory \cite{15,16,18}; in particular, \( \epsilon \) can be related to the frequency shift of the spectroscopic signal; see, e.g., Ref. \cite{19}, whereas \( m_{\text{eff}} \) can be extracted from the quench dynamics; see, e.g., Ref. \cite{18}. The values of \( \epsilon \) and \( m_{\text{eff}} \) can also be theoretically computed; see, e.g., Refs. \cite{24,32,38,39}. We summarize the theoretical expectations on the effective mass and self-energy in Fig. 1. To obtain the data shown in this figure, we have extended the theory developed in \cite{39} (see Ref. \cite{32}). It allowed us to find the function \( m_{\text{eff}}(g/\rho,c/\rho) \), which will be needed below. Both the self-energy and the effective mass in Fig. 1 are increasing functions of the boson-impurity interaction \( \eta = cm/(\hbar^2 \rho) \), so that \( m_{\text{eff}} \) is always larger than \( m \): The impurity gains ‘weight’ when in media. This gain can be large even in the weakly-interacting regime \((\eta \to 0, \gamma \to 0)\) given that \( \eta \gg \gamma \); the energy correction in this regime is not important – it scales as \( \sqrt{\gamma} \) (see, e.g., Refs. \cite{39}). Finally, we note that in the weakly-interacting regime the effective mass (using perturbation approach based on the Bogoliubov approximation) is \cite{38}

\[ \frac{m_{\text{eff}}}{m} = 1 + \frac{2c^2}{3g^{3/2}\pi} \sqrt{\frac{m}{\hbar^2 \rho}}. \tag{4} \]

For the considered cases (see Fig. 1) this form is accurate at \( c \lesssim g \).

**Harmonic Trap Impact.**—Cold-atom experiments have a finite number of particles and are often performed in harmonic traps, hence, the Hamiltonian \( H \) must be modified to take this into account. A natural extension of Eq. (1) (assuming that the density of the Bose gas does not vary appreciably on the length scale given by the healing length) to the trapped case is

\[ H_{\text{trap}}^{\text{eff}} = \epsilon(y) - \frac{\hbar^2}{2m_{\text{eff}}(y)} \frac{\partial^2}{\partial y^2} + \frac{ky^2}{2}, \tag{5} \]

where \( \epsilon(y) \equiv \epsilon(g(\rho(y),c(\rho(y))) \) and \( m_{\text{eff}}(\rho(y)) \equiv m_{\text{eff}}(g(\rho(y),c(\rho(y))) \) with \( \epsilon \) and \( m_{\text{eff}} \) from Eq. (3). Naturally, this extension makes sense only if the impurity is inside the cloud, i.e., it never probes the region with \( \rho(y) = 0 \) where \( \epsilon(y) \) and \( m_{\text{eff}}(y) \) are not defined. To receive some insight into the properties of the Hamiltonian \( H \), let us consider \( c \to 0 \). In this limit the leading order correction to the energy can be assessed using first order perturbation theory: \( \epsilon \approx c\rho \); see also Ref. \cite{32} and Fig. 1. The correction to \( m_{\text{eff}} \) is given by Eq. (4), thus, \( m_{\text{eff}}/m \sim c^2 \) and can be neglected in the leading order. The density profile \( \rho(x) \) can be estimated from
the Thomas-Fermi approximation (see, e.g., Ref. [19]):
\[ \rho(|x| < R) = \frac{k_R^2}{2g} \left(1 - \frac{x^2}{R^2}\right), \text{ and } \rho(|x| > R) = 0, \]
where \( R = (3gN/(2k))^1/3 \). Therefore, the leading order effective Hamiltonian for \( c \to 0 \) reads
\[ H_{\text{eff}}^{\text{trap}} \simeq \frac{\hbar^2}{2m} \frac{\partial^2}{\partial y^2} + \left(1 - \frac{c}{g}\right) \frac{m\Omega^2 y^2}{2}. \] (6)

Three comments are in order here: i) the string constant has to be always renormalized; ii) for \( c < g \) and small \( g \) the dynamics are determined mainly by the renormalization of the string constant; iii) the effective Hamiltonian \( (6) \) is valid only for \( c < g \), otherwise the system is unstable – the harmonic oscillator in Eq. \( (6) \) acquires a negative string constant. Due to iii) below we consider the case with \( c < g \) (referred to as the ‘miscible’ case, cf. \([51, 51]\) and \( c > g \) (‘immiscible’ case) separately. Note that Eq. \( (6) \) is expected to describe qualitatively the \( c < g \) regime for weak and moderate interactions, where the approximations used to derive Eq. \( (6) \) are accurate; see Fig. \( (4) \).

To renormalize the mass and string constant beyond Eq. \( (6) \), maintaining a simple form, we propose the following Hamiltonian for \( c < g \)
\[ H_{\text{eff}}^{\text{trap}} \simeq \frac{\hbar^2}{2m} \frac{\partial^2}{\partial y^2} + \left(1 - \frac{c}{g}\right) \frac{m\Omega^2 y^2}{2}, \] (7)
where \( m_{\text{eff}} \) and \( k_{\text{eff}} \) are independent from one another. The parameters \( \tau, m_{\text{eff}} \) and \( k_{\text{eff}} \) must describe the ‘average’ action of \( \epsilon(y) \) and \( m_{\text{eff}}(y) \). As such they simplify the description: They do not depend on the position, and do not require the knowledge of the density \( \rho(y) \). Note also that even though the Hamiltonian \( (7) \) is more complicated than the model in Eq. \( (3) \), its propagator is well-known (see, e.g., Ref. \([52]\)), which grants an easy access to various observables \([52]\). However, there is a tradeoff: The parameters \( \tau, m_{\text{eff}} \) and \( k_{\text{eff}} \) must be calculated directly for an inhomogeneous problem of interest.

‘Miscible’ (\( c < g \)) Case.— To test the proposed effective Hamiltonian \( (7) \), we investigate the quench dynamics that obeys \( H \) from Eq. \( (2) \). We assume that at \( t < 0 \) the impurity is non-interacting, i.e., \( c = 0 \), and the system is in the ground state of \( H \). At \( t = 0 \) there is a change to \( c > 0 \). We study the time evolution at \( t > 0 \). For the sake of argument, we set \( N = 100, \Omega = 2\pi \times 20Hz, g = 10^{-37}Jm \) and \( m = m(87\text{Rb}) \) – these values resemble the experiments with quasi-one-dimensional Rb atoms \([53, 54]\). Note that for \( N = 100 \) the Thomas-Fermi radius is \( R \simeq 19\mu m, \rho(0) \simeq 4/\mu m \), hence, the dimensionless interaction parameter \( \gamma(x) \equiv gm/\rho(x)\hbar^2 \) in the center of the trap is \( \gamma(0) \simeq 0.33 \). Finally, we note that for these parameters and \( g = \text{the effective mass in the center of the trap} \) is \( m_{\text{eff}}/m \simeq 1.1 \): The renormalization of the mass is significant; it is larger on the edges of the trap where the density is smaller (see Eq. \( (4) \)).

![Fig. 2. The size of the impurity cloud \( \langle y^2(t) \rangle /\langle y^2(0) \rangle \) as a function of time (in ms). The (black) solid curves show the ML-MCTDHB results. The yellow dots (on top) show the fits to the effective Hamiltonian \( (7) \). The purple dots are the results of the Hamiltonian \( (6) \) (no fit parameters). The (black) dashed curves show the mean-field (in \( (a) \)) and the species mean-field (in \( (b) \)) results, the corresponding (green) dots are the fits to \( (7) \). Panel \( (a) \) is for \( c = 0.1g \), the parameters fitted to the ML-MCTDHB results are \( m_{\text{eff}}/m = 0.983, \ k_{\text{eff}}/k = 0.87 \); to the mean-field results they are \( m_{\text{eff}}/m = 1.006, \ k_{\text{eff}}/k = 0.9054 \). Panel \( (b) \) is for \( c = 0.9g \), the effective parameters are \( m_{\text{eff}}/m = 0.937, \ k_{\text{eff}}/k = 0.104 \) (ML-MCTDHB); and \( m_{\text{eff}}/m = 1.5, \ k_{\text{eff}}/k = 0.205 \) (species mean-field). The other parameters of the system are \( N = 100, \Omega = 2\pi \times 20Hz, g = 10^{-37}Jm \) and \( m = m(87\text{Rb}) \).]

For the natural time scales given by the trap and experimental limitations, we calculate the size (variance) of the impurity cloud \( \langle y^2 \rangle \) first using ML-MCTDHB; see Ref. \([52]\) for the description and estimates of the accuracy. Then we calculate \( \langle y^2 \rangle \) within our polaron model \([7]\). The latter model has the parameters \( m_{\text{eff}} \) and \( k_{\text{eff}} \), which are found by fitting to the numerical results; see Fig. \( (2) \). The parameter \( \tau \) defines an overall energy shift and therefore cannot be obtained from the quench dynamics. We compare \( \langle y^2 \rangle \) for \( c = 0.1g \) (weakly interacting impurity) and \( c = 0.9g \) (all interactions are of the same order); see Fig. \( (2) \). The Hamiltonian \( (7) \) describes both cases well. For \( c = 0.1g \) the agreement is excellent; for \( c = 0.9g \) we see that the polaron model is less accurate: There is a damping which is beyond Eq. \( (7) \). This damping sig-
nals the presence of higher-order correlations. Note that even for very small interaction strengths the renormalization of the mass and string constant has to be performed beyond Eq. (5) to accurately describe the ML-MCTDHB data (see also Fig. 3). The fitting to the data in Fig. 2 dictates that $m_{\text{eff}}/m < 1$, which is at odds with our intuition from the homogeneous model (see Fig. 1); $m_{\text{eff}}/m < 1$ even for $c \to 0$ where ML-MCTDHB captures all important particle correlations (see Ref. [22]). Apparently, this unusual mass renormalization is due to the trap, and cannot be straightforwardly related to the physics in the homogeneous case. Note that the entanglement between the species, which is a beyond-mean-field effect, is crucial for this conclusion; see Fig. 2 where in (a) we present the calculations using the two coupled Gross-Pitaevski equations [22] [49] [50] [55] and in (b) the results of the species mean-field calculations [55]. The mean-field results disagree with the full calculations and can fit well with $m_{\text{eff}}/m > 1$. Finally, we note that it is necessary to renormalize both the effective mass and the string constant (cf. Ref. [18]) because they are related to different effects. Indeed, one can first calculate $k_{\text{eff}}/m_{\text{eff}}$ independently from the period of oscillation of $\langle y^2 \rangle$, and then compute $m_{\text{eff}}$ by fitting to the amplitude.

To calculate the ‘best’ values of $m_{\text{eff}}/m$ and $k_{\text{eff}}/k$, we minimize the sum

$$
\chi^2 = \sum_{i=1}^{M} \left( \langle y^2 \rangle^H(t_i) - \langle y^2 \rangle^{\text{trap}}(t_i) \right)^2 / M \langle y^2 \rangle^0(0),
$$

(8)

where $M = 1500$ is the number of the data points we use; the time interval $[t_1, t_M]$ is shown in Fig. 2. The values of $m_{\text{eff}}/m$ and $k_{\text{eff}}/k$ that minimize $\chi^2$ for $c/g \in [0.1, 1]$ are presented in Fig. 3. The one-body description is valid for $c/g \lesssim 0.8$, which is quantified by small values of $\chi^2$. For larger values of $c/g$ the impurity probes the edges of the cloud which is not captured by $\mathcal{H}_{\text{trap}}$, see the next subsection. It is worthwhile noting that the values of $m_{\text{eff}}$ and $k_{\text{eff}}$ that minimize Eq. (8) describe accurately other quantities, e.g., the density (results not shown here for the sake of brevity). Therefore, the effective Hamiltonian $\mathcal{H}_{\text{trap}}$ is universal, in the sense that it does not depend on the low-energy observable of interest.

‘Immiscible’ ($c > g$) Case. — Even though there is only one particle of a different kind, the time evolution of the impurity is sensitive to the phase separation point for two harmonically trapped gases [24] [51]; see the increase of $\chi^2$ in Fig. 3(ii). For $c > g$ the impurity is pushed to the edge of the Bose gas (see also Refs. [19] [20]), and, hence, the part of the space that is not occupied by the Bose gas must be included into the effective description [57]. One can extend Eq. (5) to $y > |R|$ by using a free oscillator or some other effective Hamiltonian, e.g., from Ref. [19]. However, it is not clear that the ‘polaron’ description is valid for these large interactions and momenta. Indeed, even in the homogeneous case it might well be that the involved momenta are larger than the critical momentum for the stability of a polaron. This investigation is beyond the scope of this Letter.

Discussion. — The main ideas and results of this work are summarized in Eq. (7) and Fig. 3. Arguably, the most unexpected finding of our study is that the effective mass in Fig. 3 is smaller than the bare mass. We will now relate this observation to the homogeneous model, which predicts $m_{\text{eff}} > m$; see Fig. 1 also Refs. [24] [58]. First, we argue that Eq. (5) is valid for $c \to 0$. The ‘size’ of the polaron approximately given by $\sqrt{2/(\gamma(0) \rho(0)^2 \approx 0.6 \mu m}$ is ‘much’ smaller than the relevant lengths of the problem, e.g., the harmonic oscillator length $\sqrt{\hbar/(m \Omega)} \approx 2.5 \mu m$. Therefore, we may treat the external trap as a low-energy perturbation for which the polaron description is expected to be accurate. However, the validity of Eq. (5) does not immediately imply that $m_{\text{eff}} > m$. Indeed, $m_{\text{eff}}$ might be related not only to $m_{\text{eff}}(y)$ from Eq. (5) but also to $\epsilon(y)$. Therefore, for a fair comparison one must estimate the $\epsilon^2$-order correction to $\epsilon$, and the corrections to the Thomas-Fermi density. We focus on the latter as it contributes directly to the leading order in $c$. Using the ideas of Ref. [58], one can show that the corrections to the density within the mean-field approach yield terms $O(1/R^4)$ in the renormalized parameters. This is a marginal effect that can be neglected. However, effects beyond the Gross-Pitaevski equation (cf. [22] [50]) captured by our numerical method might explain Fig. 3. One can show that the contribution $Q \sqrt{1 - z^2/r^2}$ to the density [51], where $Q > 0$ and $r$ are fitted parameters can lead to $k_{\text{eff}}/k < 1 - c/g$ and $m_{\text{eff}}/m < 1$. Note that the competition between this beyond mean-field contribution and the mean-field re-
results in the order $c^2$ might explain the non-monotonic behavior in Fig. 3. The effective mass reaches its minimum ($\frac{\pi_{eff}}{m} \approx 0.91$) at $c/g \approx 0.8$. The string constant crosses the line $k(1 - c/g)$ at around this point.

We thank Oleksandr Marchukov for comments on the manuscript, and Martin Zwierlein for bringing Ref. [33] to our attention. A. G. V. and N. T. Z. thank Hans-Werner Hammer and Amin Dehkharghani for many useful discussions concerning the Bose polaron problem. This work was supported by the Deutsche Forschungsgemeinschaft (DFG) in the framework of the SFB 925 “Light induced dynamics and control of correlated quantum systems” (S. I. M. and P. S.); the Humboldt Foundation and the Ingenuity organization of TU Darmstadt (A. G. V.); the Danish Council for Independent Research and the DFF Sapere Aude program (N. T. Z.).

S. I. Mistakidis and A. G. Volosniev contributed equally to this work.

[1] S. Pekar, *Issledovaniya po Elektronnoj Teorii Kristallov* (Gostekhizdat, Moskva (1951). English translation: *Research in Electron Theory of Crystals*, AEC-tr-555, US Atomic Energy Commission (1963).

[2] N. N. Bogolubov and N. N. Bogolubov (Jr.), *Aspects Of Polaron Theory: Equilibrium and Nonequilibrium Problems*, World Scientific Publishing Company (2008).

[3] L. D. Landau and S. I. Pekar, JETP 18, 419 (1948).

[4] G. Baym and C. Pethick, *Landau Fermi-Liquid Theory: Concepts and Applications*, John Wiley and Sons (2008).

[5] R. F. Bishop, Annals of Physics 78, 391 (1973).

[6] M. Kutscher and W. Wójcik, Phys. Rev. C 47, 1077 (1993).

[7] A. Braun and Q. Chen, Nature Commun. 8, 15830 (2017).

[8] M. B. Salamon and M. Jaime, Rev. Mod. Phys. 73, 583 (2001).

[9] P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. 78, 17 (2006).

[10] M. E. Gershenson, V. Podzorov, and A. F. Morpurgo, Rev. Mod. Phys. 78, 973 (2006).

[11] A. Schirotzek, C.-H. Wu, A. Sommer, and M. W. Zwierlein, Phys. Rev. Lett. 102, 230402 (2009).

[12] F. Chevy and C. Mora, Rep. Prog. Phys. 73, 112401 (2010).

[13] N. Spethmann, F. Kindermann, S. John, C. Weber, D. Meschede, and A. Widera, Phys. Rev. Lett. 109, 235301 (2012).

[14] P. Massignan, M. Zaccanti, G. M. Bruun, Rep. Prog. Phys. 77, 034401 (2014).

[15] N. B. Jørgensen, L. Wacker, K. T. Skalmstang, M. M. Parish, J. Levinse, R. S. Christensen, G. M. Bruun, J. J. Arit, Phys. Rev. Lett. 117, 055302 (2016).

[16] M.-G. Hu, M. J. Van de Graaff, D. Kedar, J. P. Corson, E. A. Cornell, and D. S. Jin, Phys. Rev. Lett. 117, 055301 (2016).

[17] R. Schmidt, M. Knap, D. A. Ivanov, J.-S. You, M. Cetina, and E. Demler, Rep. Prog. Phys. 81, 024401 (2018).

[18] J. Catani, G. Lamporesi, D. Naik, M. Gring, M. Inguscio, F. Minardi, A. Kantian, and T. Giamarchi, Phys. Rev. A 85, 023623 (2012).

[19] A. S. Dehkharghani, A. G. Volosniev, and N. T. Zinner, Phys. Rev. A 92, 031601(R) (2015).

[20] A. S. Dehkharghani, A. G. Volosniev, and N. T. Zinner, Phys. Rev. Lett. 121, 080405 (2018).

[21] A. G. Volosniev, H. W. Hammer, and N. T. Zinner, Phys. Rev. A 92, 023623 (2015).

[22] J. Akram and A. Pelster, Phys. Rev. A 93, 033610 (2016).

[23] M. Schelter, D. M. Gangardt, and A. Kamenev, New J. Phys. 18, 065002 (2016).

[24] F. Grusdt, G. E. Astrakharchik, and E. A. Demler, New J. Phys. 19, 103035 (2017).

[25] A. Lampo, S. Hoe Lim, M. Á. García-March, and M. Lewenstein, Quantum 1, 30 (2017).

[26] L. Cao, S. Krönke, O. Vendrell, and P. Schmelcher, J. Chem. Phys. 139, 134103 (2013).

[27] L. Cao, V. Bolsinger, S. I. Mistakidis, G. M. Koutentakis, S. Krönke, J. M. Schurer, and P. Schmelcher, J. Chem. Phys. 147, 044106 (2017).

[28] S.I. Mistakidis, G.C. Katsimiga, P.G. Kevrekidis, and P. Schmelcher, New J. Phys. 20, 043052 (2018).

[29] G.C. Katsimiga, G.M. Koutentakis, S.I. Mistakidis, P.G. Kevrekidis, and P. Schmelcher, New J. Phys. 19, 073004 (2017).

[30] G.M. Koutentakis, S.I. Mistakidis, and P. Schmelcher, arXiv:1804.07199 (2018).

[31] P. Siegl, S.I. Mistakidis, and P. Schmelcher, Phys. Rev. A 97, 053626 (2018).

[32] See the Supplementary Material for the description of 1) the ML-MCTDHB; 2) the polaron using a Gross-Pitaevskii-type equation; 3) the evolution governed by the effective Hamiltonian 7. The Supplementary Material includes Refs. [33, 34].

[33] M. Ishikawa and H. Takayama, J. of Phys. Soc. of Jap. 49, 1242 (1980).

[34] V. Hakim, Phys. Rev. E 55, 2835 (1997).

[35] S. Palzer, C. Zipkes, C. Sias, and M. Köhl, Phys. Rev. Lett. 103, 150601 (2009).

[36] T. Fukuhara, A. Kantian, M. Endres, M. Cheneau, P. Schauß, S. Hild, D. Bellem, U. Schollwöck, T. Giamarchi, C. Gross, I. Bloch, and S. Kuhr, Nature Physics 9, 235 (2013).

[37] F. Meinert, M. Knap, E. Kirilov, K. Jag-Lauber, M. B. Zvonarev, E. Demler, H.-C. Nagerl, Science 356, 945 (2017).

[38] L. Parisi and S. Giorgini, Phys. Rev. A 95, 023619 (2017).

[39] A. G. Volosniev and H.-W. Hammer, Phys. Rev. A 96, 031601(R) (2017).

[40] V. Pastukhov, Phys. Rev. A 96, 043625 (2017).

[41] N. J. Robinson, J.-S. Caux, and R. M. Konik, Phys. Rev. Lett. 116, 145302 (2016).

[42] A. S. Campbell and D. M. Gangardt, SciPost Phys. 3, 015 (2017).

[43] G. E. Astrakharchik and L. P. Pitaevskii, Phys. Rev. A 70, 013608 (2004).

[44] K. Sacha, and E. Timmermans, Phys. Rev. A 73, 063604 (2006).

[45] M. Bruderer, W. Bao, and D. Jaksch, arXiv:1804.07199 (2018).
[57] The Hamiltonian must be corrected when the harmonic oscillator length associated with the trap in Eq. (6) is of the order of $R$, i.e., $\sqrt{\frac{\hbar}{m\Omega\sqrt{1-c/g}}} \approx R$. For our system it leads to $c \approx 0.99g$.

[58] A. Fetter and D. Feder, Phys. Rev. A 58, 3185 (1998).

[59] K. Sakmann and M. Kasevich, Nature Phys. 12, 451 (2016).

[60] O. V. Marchukov and U. R. Fischer, arXiv:1701.06821 (2017).

[61] This correction to the density is needed to describe strong correlations present in one spatial dimension [62]. In particular, this contribution allows to account for low-energy scattering for which the bosons fermionize [63].

[62] E. B. Kolomeisky, T. J. Newman, J. P. Straley, and X. Qi, Phys. Rev. Lett. 85, 1146 (2000).

[63] M. D. Girardeau, J. Math. Phys. 1 516 (1960).

[64] Note that for $g \to 0$ the $c^2$-corrections can be computed in the Bogoliubov theory [58]: $-\sqrt{\frac{\mp(x)}{\pi\hbar\Gamma_g}} c^2$, or using a suitable Gross-Pitaevskii equation [39]: $-\frac{W(x)}{8\pi\hbar^2a^2} c^2$. Both results lead to the attraction to the center of the trap, which cannot explain Fig. 3. For example, these corrections imply that $k_{\text{eff}}/k > (1-c/g)$.