The $\rho$ parameter and the CDF - II $W$-mass anomaly: observations on the role of scalar triplets

Rituparna Ghosh, Biswarup Mukhopadhyaya, Utpal Sarkar

Department of Physical Sciences
Indian Institute of Science Education and Research, Kolkata, Mohanpur - 741246, India

Abstract

The $\rho$-parameter, together with the W-and Z-masses, acts as Occam’s razor on extensions of the electroweak symmetry breaking sectors. We apply this to non-doublet Higgs scenarios, by examining the CDF-II claim on the $W$-boson mass. Suspending any judgement on the CDF claim, we show that in general, if one works at the tree level, theoretical models which predict $\rho = 1$ at the tree-level are inconsistent with the CDF claims at 4-6 standard deviations if one confines oneself to the existing Z-boson mass, and the earlier $M_W$ value from either the global fit or the ATLAS data. We take some well-motivated scenarios containing one or more scalar SU(2) triplets in addition to the usual doublet, and show that, both a scenario including a complex scalar triplet and one with a complex as well as a real triplet (the Georgi-Machacek model) can be made consistent with the new data, where a small splitting between the complex and the real triplet vacuum expectation values is required in the second scenario. We will explore the consequences of this splitting, either at tree level or via incalculable new physics contribution to $M_W$, and indicate, as illustrations its implications in $H^\pm W^\mp Z$ type interaction vertices.

1 Introduction

A pertinent question facing all of us today is: is electroweak symmetry breaking(EWSB) entirely driven by one or more scalar doublets or can scalars in other $SU(2)$ representations, too, have a role there? Since the symmetry breaking process is inevitably reflected in the weak gauge boson masses and their phenomenology, any new information of the W and Z boson masses gets linked with the above question. Of considerable relevance in this context is the recently announced estimate of the W-boson mass by the CDF-II collaboration setting the value at $M_W = 80.433.5 \pm 6.4_{\text{Stat}} \pm 6.9_{\text{Sys}}$ MeV [1], which is in apparent disagreement with the standard model (SM) expectation of $M_W = 80.357 \pm 4_{\text{inputs}} \pm 4_{\text{theory}}$ MeV [2] at the 7.2$\sigma$ level. While the claim is quite justifiably subject to closer scrutiny, it serves as a stimulant to theoretical introspection, keeping especially in mind the fact that it involves the EWSB sector. This sector associates a number of important issues, such as the role of additional fields in EWSB, the generation of neutrino Majorana masses, and hence the possibility of lepton number violating interactions. One thus feels inclined to forge options beyond the standard model (BSM) on the anvil of the recently published data, just to prepare ourselves for the eventuality that the announced disagreement persists. Several speculations in this direction have already appeared.[3]-[24]

A rather curious feature of the experimental results is that the Z-boson mass stands at the value measured rather precisely at the Large Electron-Positron (LEP) collider, namely, $M_Z = 91.1876 \pm 0.0021$GeV [25]. This generates an apparent tension with the value of the $\rho$-parameter, defined as $\rho = m^2_W/(m^2_Z\cos^2\theta_W)$ ($\theta_W$ being the weak mixing angle), whose tree-level value in the standard model (SM) is unity, and the experimentally identified range is $1.00038 \pm 0.0002$. The tension arises essentially because both of the gauge boson masses owe themselves to the same ‘effective’ Higgs vacuum expectation value (vev) $v$, namely, $246.22/\sqrt{2}$ GeV. On the other hand, any BSM option in EWSB, too, has a bearing, amongst other things, on the $\rho$-parameter. Many such options also affect different aspects of electroweak phenomenology.

While the jury is still out on the CDF claim, we consider it important as an interim step to connect it to already proposed extensions of the EWSB sector. An example is a class of models containing one or more scalar triplets in addition to the SM doublet. Over and above the neutrino mass generation, they have additional rich phenomenology in store, including the possibility of substantial non-doublet contributions to EWSB and the associated signal [26, 27, 28]. It is naturally of interest to see what kind of additional constraint comes on them.
from the recent CDF claim. Such constraints, in which the $\rho$-parameter plays an important role, are found by us, using earlier reference values of $M_W$ based on different sets of data. Thus, different sets of constraints appear on the triplet model parameter space, which we report below.

The ‘new’ conclusions we have arrived at are:

1. If one works at tree level and uses the presently accepted $Z$-mass, the $W$-mass as per CDF claims, and the ‘old’ $W$-mass following the global fit, then one has an inconsistency at the level of 6.78$\sigma$ with any theoretical scenario which predicts $\rho = 1$ at the tree-level.

2. As the simplest model deviating from the requirement of tree-level $\rho = 1$, a scenario including a single $Y = 2$ scalar triplet is consistent with the CDF-II claim once the oblique corrections are considered. The triplet vacuum expectation value (vev) remains as constrained as before.

3. The Georgi-Machacek scenario, comprising a complex ($\chi$) and a real ($\xi$) scalar triplet, the triplet vevs can make substantial contributions to the $W$-and $Z$-masses so long as the vevs are equal, namely, $v_\chi = v_\xi$. However, such equality is inconsistent unless one uses the $W$-mass based only on LEP and Tevatron data, and not one from a global fit. On the other hand, a region of the parameter space with $v_\chi \neq v_\xi$ is allowed, which we identify below.

In section 2, we present a general discussion on the (in)consistency of the recent data with general scenarios that lead to $\rho = 1$ at the tree-level. Section 3 contains analyses of two kinds of triplet models. The first of one, namely, the Georgi-Machacek model, has a complex and a real triplet, where $\rho = 1$ is restored via a custodial SU(2). The second of this is the model with a single complex triplet, which tends to shift the tree-level value of $\rho$ from unity. The constraints arising on each scenario from the recently reported value of $M_W$ are presented for each case. We conclude in section 4.

2 Gauge Bosons mass

The new result can be explained in two ways. One can enhance the mass of $W$ boson by including loop effects as suggested [11][20][29]. Another way of looking at is to accept that the scalar vev differs by a small amount [31] from what is indicated by earlier measurements, in such a way that it remains consistent with the mass of the $Z$ boson as measured at LEP [25]. We go by the latter approach, demanding consistency with independently measured quantities like the mass of the $Z$ boson and the $\rho$-parameter.

In a general model consisting of more than one Higgs-like multiplets, the masses of the $W$ and $Z$ bosons are given by [32],

$$M_W^2 = \frac{g^2}{4} \sum_i (4T_i(T_i + 1) - Y_i^2) c_i v_i^2 = \frac{g^2}{4} v_W^2$$

$$M_Z^2 = \frac{g^2 + g'^2}{4} \sum_i 2Y_i^2 c_i^2 = \frac{g^2 + g'^2}{4} v_2^2$$

$c_i = 1/2$ for $Y = 0$ multiplet and $c_i = 1$ for $Y \neq 0$ multiplet. $T_i$ and $Y_i$ is the isospin and hypercharge of the $i^{th}$ higgs multiplet. $v_W$ and $v_Z$ denotes the contribution of Higgs vev to the mass of the $W$ and $Z$ boson respectively. Note that the added contribution from a triplet serves to enhance both $M_W$ and $M_Z$. But the value of the $\rho$-parameter is reduced in the process, as can be seen below.

In general, the $\rho$ parameter is given by [32],

$$\rho = \frac{\sum_i (4T_i(T_i + 1) - Y_i^2)c_i v_i^2}{\sum_i 2Y_i^2 v_i^2}$$

$\rho = 1$ at tree-level requires $v_W^2 = v_Z^2 = v^2$ where $v \approx 246.22$ GeV. Now, if one goes by the recent CDF claim, namely, $M_W = 80.4335 \pm 0.0094$ GeV, let us concede that $v$ is changed by a small amount $\Delta v$ so, the $W$ mass is now given by

$$M_W^2 = \frac{g^2}{4} (v^2 + \Delta v^2)$$

$$M_W = \frac{g v}{2} (1 + \frac{\Delta v^2}{2v^2}) = M_{W,old} + \Delta M_W$$

We have attributed the change in $W$-mass to the driving vev, keeping the gauge coupling unchanged, which causes a lot less tension with weak universality etc.
\[
\Delta M_W = \frac{g}{2} \Delta v^2 = \frac{\Delta v^2}{2v^2} M_{W,old}
\]

Here \( M_W \) is the W mass recently announced by CDF, \( M_{W,old} \) is the old measured W mass and \( V \) is the ‘effective’ vacuum expectation value which is required to reproduce that mass. Hence \( \Delta v^2 \) is given by

\[
\frac{\Delta v^2}{2v^2} = \frac{\Delta M_W}{M_{W,old}}
\]

Now as there is no new measurement for the mass of the Z boson, this change in W mass should be consistent with the old measurement of the Z boson mass. For models which naturally keep \( \rho = 1 \) at tree level, the altered ‘effective’ vev should contribute to \( M_Z \) with similar functional dependence as what occurs in the expression for \( M_W \). The thus modified mass of the Z boson is given by

\[
M_Z^2 = \frac{g^2 + g'^2}{4} (v^2 + \Delta v^2)
\]

Hence,

\[
M_Z = M_{Z,mean} + \Delta M_Z
\]

where, \( M_{Z,mean} \) is the central value of the old measured Z mass, namely, 91.1876 GeV. The consequent shift in \( m_Z \) required to retain \( \rho = 1 \) at tree-level is denoted here by \( \Delta M_Z \), and is given by

\[
\Delta M_Z = \frac{\sqrt{g^2 + g'^2}}{2} \Delta v^2
\]

\[
= \frac{\Delta v^2}{2v^2} M_{Z,mean}
\]

\[
= \frac{M_{Z,mean}}{M_{W,old}} \Delta M_W
\]

The existing measurements \[25\] tell us that \( M_Z = 91.1876 \pm 0.0021 \) GeV. Table-1 then implies that for \( M_{W,old} \) standardised following the world average \[2\] and ATLAS measurements \[33\] respectively, the demand that \( \rho = 1 \) shifts \( M_Z \) to values disagreeing with \( M_{Z,mean} \) at the 4.23 \( \sigma \) and 6.78 \( \sigma \) level respectively. For \( M_{W,old} \) given by the LEP2 and Tevatron combined results, shifts \( M_Z \) by less than 1 \( \sigma \) from \( M_{Z,mean} \). Thus the first two choices of \( M_{W,old} \) make the data clearly incompatible with any theoretical scenario that predicts \( \rho = 1 \) at the tree level. We emphasize that such incompatibility occurs even when \( M_{W,old} \) is set at its maximum value at 2\( \sigma \) level (with \( M_Z \) appropriately adjusted within its 2\( \sigma \) allowed range), and \( M_{W,new} \), at its minimum 2\( \sigma \) value.

The possibilities are summarised in Table-1

| \( M_{W,old} \) | Shift from \( M_{Z,mean} \) |
|-----------------|--------------------------|
| ATLAS           | 4.23\( \sigma \)         |
| World Avg       | 6.78\( \sigma \)         |
| LEP2/Tevatron   | < 1\( \sigma \)          |

Table 1: The shift from \( M_{Z,mean} \) for the various cases mentioned in the text

3 Model with Higgs triplet(s)

We turn next to an illustrated class of BSM scenarios, where one or more \( SU(2) \) triplet scalars participate in EWSB. Such scenarios are particularly interesting in the context of the Type-II seesaw mechanism of neutrino mass generation. In some variants, which we examine below the triplet vevs may also contribute substantially to the weak gauge boson masses, and thus play important roles in weak-scale collider phenomenology.

3.1 A real and a complex triplet: the Georgi- Machacek (GM) model

We take up first a scenario \[34, 35\] where one has a \( Y = 2 \) complex triplet \( \chi = (\chi^+, \chi^0, \chi^-) \) and a \( Y = 0 \) real triplet \( \xi = (\xi^+, \xi^0, \xi^-) \). As discussed widely in the literature, \( \langle \chi^0 \rangle = \langle \xi^0 \rangle = v_\chi \), ensured via a custodial global
SU(2) retains $\rho = 1$ at the tree-level. The scalar potential in this case is,

$$V(\Phi, X) = \frac{\mu^2}{2} Tr(\Phi^\dagger \Phi) + \frac{\mu^2}{2} Tr(X^\dagger X) + \lambda_1 [Tr(\Phi^\dagger \Phi)]^2 + \lambda_2 Tr(\Phi^\dagger \Phi) Tr(X^\dagger X) + \lambda_3 Tr(X^\dagger XX^\dagger X) + \lambda_4 Tr(\Phi^\dagger \Phi t^a X^b)\tau^a t^b$$

with,

$$\Phi = \begin{pmatrix} \phi^0 & \phi^+ \\ \phi^- & \phi^0 \end{pmatrix}, \quad X = \begin{pmatrix} \chi^0 & \xi^+ & \chi^{++} \\ \chi^- & \xi^0 & \chi^+ \\ \bar{\chi}^- & \bar{\xi}^0 & \chi^- \end{pmatrix}, \quad U = \begin{pmatrix} -\frac{i}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{i}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{2}} \end{pmatrix}$$

$\tau^a s$ and $t^a s$ are the $2 \times 2$ and $3 \times 3$ SU(2) generators respectively.

If one wants to satisfy the CDF claim on $M_W$ without any new physics coming in, the resulting shift in $M_Z$ is shown in Fig. 1. Here $M_{W, old}$ was in accordance with the world average.

Equation (11) manifestly ensures $v_\chi = v_\xi$. However, as has been emphasised in the literature [37, 35], it corresponds to a rather fine-tuned situation. The custodial SU(2) is broken by the $U(1)_Y$ gauge interaction of the scalar triplets; quadratic divergences consequently crop up in higher order corrections to the weak gauge boson masses and the $\rho$-parameter. The cancellation of these divergences requires the introduction of counterterms which essentially depend on the UV-completion of the theory, with its accompanying new physics. The parameters in the GM scalar potential are thus subject to corrections that cannot be calculated within the model itself. One may at best use phenomenological values of these parameters. If the GM scenario has to be relevant in TeV-scale physics, then the loop-revised mass parameters and the UV limit of the theory need to be around that scale. This is an assumption underlying any model-building, where observable consequences are expected.

We, too, take such a phenomenological approach and express the corrected W and Z masses, following [37] as,

$$M_Z^2 = \frac{1}{4} (g^2 + g'^2) v^2$$

$$M_W^2 = \frac{1}{4} g^2 v^2 (1 + s_H^2 \delta + \frac{1}{2} s_H^2 \delta^2)$$

where $v^2 = v_\phi^2 + 8 v_\chi^2$, $s_H = \frac{2\sqrt{2} v_\chi}{v}$. It should be emphasised that, $\delta$ is not effecting $M_Z$ as it is a measure of the separation of the real triplet vev from that of the complex triplet, i.e, $v_\chi = v_\xi (1 + \delta)$. We take $v_\chi = v_\xi$ at the tree

![Figure 1: The 2σ region consistent with the CDF claim is shown in green and the region consistent at 1σ level is shown in red.](image)
Table 2: Values of various parameters used in our scan to get Fig. 2 and 3

| Parameter | Range                  |
|-----------|------------------------|
| $\lambda_2$ | [0.0, 0.8]         |
| $\lambda_3$ | [-1.5, 1.5]       |
| $\lambda_4$ | [-1.5, 1.5]       |
| $M_2$ | 10 GeV               |
| $M_M^{++}$ | 200-1000 GeV       |
| $M_M^+$ | $> M_M^{++}$        |
| $s_H$ | [0, 1]               |

level at this stage. Note that $M_Z$ is fixed at its measured value, and the scalar vevs are regulated accordingly, while the effect of higher order corrections is explicitly shown in the expression for $M_W$.

The quantity $\delta$ can be expressed as

$$\delta = \delta_0 + \delta_{div}$$

(15)

Where $\delta_{div}$ is the counterterm that cancels the quadratic divergence. Following the argument given above, the finite correction $\delta_0$ is not calculable without knowledge of the UV completion, and has to be fixed phenomenologically.

If now one tries to explain the CDF claim (say, at the 2$\sigma$ level), one obtains a range of $\delta_0$ depending on the value of $s_H$ which is a measure of the triplet contribution to the weak boson masses. The thus allowed region in the $s_H - \delta_0$ plane, consistent at the 2$\sigma$ level with the measured value of $\rho$ as well as the CDF claim, is shown in Fig. 2. Here one has to use the following expression for the $\rho$ parameter corrected by $\delta_0$

$$\rho = \frac{v_\phi^2 + 4v_\chi^2 + 4v_\chi^2(1 + \delta_0)^2}{v_\phi^2 + 8v_\chi^2}$$

(16)

The free parameters for our analysis were, $\lambda_2, \lambda_3, \lambda_4, M_2, M_5^{++}, M_3^+, s_H$ and the corresponding ranges are shown in Table 2. $\lambda_1$ is tuned such that one of the two custodial singlet CP-even Higgs masses is always fixed at 125 GeV [38]. $M_5^{++}$ and $M_3^+$ are the masses of the fiveplet state and three-plet state respectively and given by,

$$M_5^{\pm \pm} = \frac{M_1}{4v_\chi} v_\phi^2 + 12M_2v_\chi^2 + \frac{3}{2} \lambda_5 v_\phi^2 + 8\lambda_3 v_\chi^2$$

(17)

$$M_3^{+ \pm} = \left( \frac{M_1}{4v_\chi} + \frac{\lambda_5}{2} \right) v_\chi^2$$

(18)

The following features of the GM scenario, implied by the CDF claim emerge from Fig. 2:

1. The larger is $s_H$ the more is the value of $\delta_0$ restricted. It should be noted that limits from the LHC data allow $s_H$ to be up to 0.3-0.4 [38, 39] approximately, depending on details of the GM parameter space. Thus, one gets restricted to $\delta_0$ below 0.1 if the weak gauge bosons have to receive substantial contributions to their masses from triplet vevs.

2. On the other hand, one is constrained to relatively small values of $s_H$ ($\leq 0.1$) in order to allow $\delta_0 \geq 0.5$. A large finite correction to $M_W$ arises presumably from some strongly coupled UV completion, thus implies a small contribution of triplet vev to the W- and Z- boson masses.

3. Consistency with the observed values of oblique electroweak parameters is another question. In terms of oblique parameters $S, T$ and $U$ [40], the shift in $M_W$ is given as,

$$\Delta M_W = \frac{\alpha}{2c_W^2 s_W^2} \left( -\frac{1}{2} S + c_W^2 T + \frac{c_W^2 - s_W^2}{4s_W^2} U \right)$$

(19)

As seen from Fig. 2, the region in green is consistent with oblique corrections though these values of triplet vev is highly disfavoured by collider data [38, 39]. We have used reference [41] to obtain the oblique parameters for our case and our results are also consistent with reference [42] for its chosen value of T-parameter. If the range of T parameter is extended to 0.05 ± 0.06, the whole region becomes compatible with oblique correction.
Another remarkable feature of the GM model is the appearance of the $H^±W^±Z$ vertex with a relatively greater strength than, for example, a model with a single triplet. For $v_χ = v_ξ$ this is the $H^±_3 W^±Z$ vertex. Once the finite correction $δ_0$ is incorporated, a non-vanishing $H^±_3 W^±Z$ vertex also appears. To obtain these vertex strengths let us first construct the Goldstone mode\[43\] in this case and define its orthogonal states to be $ψ^+_1$ and $ψ^+_2$:

$$G^+ = \frac{v_φ}{v} χ^+ + \frac{2v_χ}{v}(1 + δ_0)ξ^+ = φ^+ cos α + χ^+ sin α cos β + ξ^+ sin α sin β$$

$$(20)$$

$$ψ^+_1 = -φ^+ sin α + χ^+ cos α cos β + ξ^+ cos α sin β$$

$$(21)$$

$$ψ^+_2 = -χ^+ sin β + ξ^+ cos β$$

$$(22)$$

where, $tan α = \frac{2v_χ\sqrt{1 + (1 + δ_0)^2}}{v_φ}$ and $tan β = 1 + δ_0$.

The singly charged physical states, $H^±_3$ and $H^±_5$ will be a linear combinations of $ψ^+_1$ and $ψ^+_2$. The mixing angle depends on the loop corrections to the parameters of the potential, which are in calculable within the ambit of the model itself as discussed above. This angle, viz. γ is a phenomenological input, yielding the charged scalar states as,

$$H^+_3 = ψ^+_1 cos γ + ψ^+_2 sin γ$$

$$(23)$$

$$H^+_5 = -ψ^+_1 sin γ + ψ^+_2 cos γ$$

$$(24)$$

With this we can write the $H^+_3 W^-Z$ and $H^+_5 W^-Z$ vertex strengths as\[43\],

$$V_{H^+_3 W^-Z} = \frac{g^2}{\sqrt{2} cos^2 θ_W}(C^φ_3(cos^2 θ_W - 1)\frac{v_φ}{\sqrt{2}} + C^α_3(cos^2 θ_W - 2)v_χ + C^ξ_3\sqrt{2} cos^2 θ_W v_ξ)$$

$$(25)$$

where, $C^φ_3 = -sin α cos γ$, $C^α_3 = (cos α cos β cos γ - sin β sin γ)$ and $C^ξ_3 = (cos α sin β cos γ + cos β sin γ)$

$$V_{H^+_5 W^-Z} = \frac{g^2}{\sqrt{2} cos^2 θ_W}(C^φ_5(cos^2 θ_W - 1)\frac{v_φ}{\sqrt{2}} + C^α_5(cos^2 θ_W - 2)v_χ + C^ξ_5\sqrt{2} cos^2 θ_W v_ξ)$$

$$(26)$$

where, $C^φ_5 = sin α sin γ$, $C^α_5 = -(cos α cos β sin γ + sin β cos γ)$ and $C^ξ_5 = (cos β cos γ - cos α sin β sin γ)$

Let us define,

$$κ_{H^+_3 W^-Z} = V_{H^+_3 W^-Z}/M_W$$

$$(27)$$

$$κ_{H^+_5 W^-Z} = V_{H^+_5 W^-Z}/M_W$$

$$(28)$$

Figure 2: The 2σ region consistent with both the CDF claim and ρ parameter shown in Red and the region consistent with the oblique corrections in 2σ level is shown in green.
Note that $\gamma = 0$ and $\beta = \frac{\pi}{4}$ in the limit $\delta_0 = 0$, in which case $V_{H_3^+W-Z}$ vanishes identically. We show below these two vertex strengths against $s_H$ and the colors indicate the values of $|\tan \gamma|$. It is seen in Fig. 3 that for

![Figure 3: The vertex strength modification factors $\kappa_{H_3^+W-Z}$ and $\kappa_{H_3^+W-Z}$ as function of $s_H$. The color axis represents the values of $|\tan \gamma|$.](image)

high $\gamma$, $V_{H_3^+W-Z}$ can be comparable to $H_3^+W-Z$, while for small $\gamma$, $V_{H_3^+W-Z}$, is always dominant. Obviously $\gamma$ can be large only when the UV completion is strongly coupled. As seen from Fig. 2 this can happen for large $\delta_0$ which in turn implies small values of $s_H$. Therefore, the $H_3^+W-Z$ interaction strength is likely to be of less phenomenological consequence if one has to believe in the CDF claims on $M_W$, some likely exception coming for moderate values of $\delta_0$, $s_H$.

This alerts one to the alternative possibility of allowing $v_\chi \neq v_\xi$ at the tree level itself, fitting the CDF claim in terms of this splitting of vevs. Note that the parameters in the potential become phenomenological inputs in this approach. While we are deliberately refraining from proposing theoretical mechanisms behind this, we remind the reader that an ‘approximate’ custodial symmetry is not at all an unfamiliar proposition. When the custodial SU(2) is broken, one needs to go beyond the form of scalar potential in equation (11). One of the ways to do so is to use the most general potential under $SU(2)_L \times U(1)_Y$, following reference [44].

$$V(\phi, \chi, \xi) = \tilde{\mu}_2^2 \phi^d + \tilde{\mu}_3^2 \chi^d \chi + \tilde{\mu}_4^2 \xi^d \xi + \tilde{\lambda}_1 (\phi^d \phi)^2 + \tilde{\lambda}_2 (\chi^d \chi)^2 + \tilde{\lambda}_3 (\phi^d \tau^a \phi)(\chi^d \ell^a \chi)$$

$$+ \left[ \tilde{\lambda}_4 (\phi^d \tau^a \phi)(\chi^d \ell^a \chi) + \text{h.c.} \right] + \tilde{\lambda}_5 (\phi^d \phi)(\chi^d \chi) + \tilde{\lambda}_6 (\phi^d \phi)(\xi^d \xi) + \tilde{\lambda}_7 (\chi^d \chi)^2$$

$$+ \tilde{\lambda}_8 (\xi^d \xi)^2 + \tilde{\lambda}_9 (\chi^d \chi)^2 + \tilde{\lambda}_{10} (\chi^d \chi)(\xi^d \xi) - \frac{1}{2} \left[ \tilde{M}_1^d \phi^d \Delta_2 \phi + \text{h.c.} \right]$$

$$+ \tilde{M}_1^d \phi^d \Delta_0 \phi - 6 \tilde{M}_2 \chi^d \tilde{\Sigma}_0 \chi \quad (29)$$

where,

$$\Delta_2 \equiv \sqrt{2} \tau^a U_{a1} \chi_i = \begin{pmatrix} \chi^+ / \sqrt{2} & \chi^0 & -\chi^+ / \sqrt{2} \end{pmatrix}$$

$$\Delta_0 \equiv \sqrt{2} \tau^a U_{a1} \xi_i = \begin{pmatrix} \xi^0 / \sqrt{2} & -\xi^+ & -\xi^0 / \sqrt{2} \end{pmatrix}$$

$$\tilde{\Sigma}_0 \equiv -\tau^a U_{a1} \xi_i = \begin{pmatrix} -\xi^0 & \xi^+ & 0 \\ -\xi^+ & 0 & \xi^0 \end{pmatrix} \quad (30)$$

If one minimises the above potential, neglecting for simplicity any CP-violating phase in $\tilde{\lambda}_4$ and $\tilde{M}_1$, one obtains the following conditions by setting the tadpoles to zero,
Table 3: Values of various parameters used in our scan to get Fig. 4 and 6. The quartic couplings (other than \( \lambda_1 \)) have been scanned in the range \([-4\pi, 4\pi]\) in order to reproduce the masses.

| Parameter | Value |
|-----------|-------|
| \( M_1 \) | 100 GeV |
| \( M_2 \) | 10 GeV |
| \( M_{5}^+ \) | 200-1000 GeV |
| \( M_{5}^+, M_{5}^- > M_{5}^{++} \) | |
| \( v_x \) | \([0, \frac{1}{2}\sqrt{1 - \left(\frac{v_x}{v}\right)^2}]\) |

\[
0 = \frac{\partial V}{\partial \phi} = v_\phi \tilde{\phi}^2 + \lambda_1 v_\phi^2 + \frac{\lambda_3}{2} v_\phi^2 + \sqrt{2}\lambda_4 v_\phi v_\xi
+ \lambda_5 v_\phi^2 + \lambda_6 v_\xi^2 - M_1 v_\phi - \frac{M_1}{2} v_\xi,
\]

(31)

\[
0 = \frac{\partial V}{\partial \chi} = 2\tilde{\mu}_1^2 v_\chi + \frac{\lambda_3}{2} v_\phi^2 v_\chi + \frac{\lambda_4}{\sqrt{2}} v_\phi^2 v_\xi + \lambda_6 v_\xi^2 v_\chi
+ 4\lambda_7 v_\chi^3 + 2\lambda_10 v_\chi v_\xi^2 - \frac{M_1}{2} v_\phi^2
- 12M_2 v_\phi v_\chi,
\]

(32)

\[
0 = \frac{\partial V}{\partial \xi} = \tilde{\rho}_1^2 v_\phi + \frac{\lambda_4}{\sqrt{2}} v_\phi^2 v_\chi + \lambda_6 v_\chi^3 v_\phi + 4\lambda_8 v_\xi^3
+ 2\lambda_10 v_\chi v_\xi^2 - \frac{M_1}{4} v_\phi^2 - 6M_2 v_\chi^2,
\]

(33)

which reveals that \( v_\chi \neq v_\xi \) in general. The ranges of various parameters as used in our analysis are indicated in Table 3. \( \lambda_1 \) is tuned such that one of the three CP-even Higgs masses is always fixed at 125 GeV\(^4\). \( M_{5}^{\pm} \) denotes the mass of the doubly charged state. \( M_{5}^{\pm} \) denotes the mass of the singly charged scalar that has the least superposition with \( \phi^+ \) and has the smallest mass splitting with \( M_{5}^{++} \). \( M_{5}^{\pm} \) is the mass of the other singly charged scalar. The masses are obtained by exactly diagonalizing the resulting mass matrix from the potential given in (29). \( M_{5}^{++} \) is given by,

\[
\tilde{M}_{5}^{++} = 4\lambda_2 v_\chi^2 - \frac{\lambda_3 v_\phi^2}{2} - \frac{\lambda_4 v_\phi^2 v_\xi}{2\sqrt{2}v_\chi} + \frac{M_1}{4} v_\phi^2 + 12M_2 v_\chi.
\]

(34)

As is evident from the entries in the table, a combination of physical mass and gauge basis parameters have been used as our inputs.

\( v_\chi \neq v_\xi \) at tree level frees one from the restriction imposed by equation (10). As a result, an allowed region of the parameter space opens up, where the incompatibility between the existing \( M_Z \) and the CDF measurement \( M_W \) goes away. In this case

\[
M_W^2 = \frac{g^2}{4}(v_\phi^2 + 4v_\chi^2 + 4v_\xi^2)
\]

(35)

\[
= \frac{g^2}{4}(v_\phi^2 + 8v_\chi^2) + g^2(v_\xi^2 - v_\chi^2)
\]

The custodial symmetry breaking effect should be small as reflected in the \( \rho \) parameter. We can equate the first term to \( M_{W, old} \) (Since it is consistent, with measured \( M_Z \) as per the discussion in section 2) and expect the second term to generate the shift required to match the CDF claim. Hence,

\[
\Delta M_W = \frac{4(v_\xi^2 - v_\chi^2)}{2v_\chi} M_{W, old}
\]

(36)

Fig. 4a and 4b exhibit the allowed regions corresponding to \( M_{W, old} \) fixed according to the ATLAS result and global average respectively. The regions are specified by the co-ordinates \( v_\chi \) and \( \Delta v_\chi \) where \( \Delta v_\chi = v_\xi - v_\chi \).
\[ \Delta v_\chi < 0 \] is not used in the scan, since such a hierarchy does not serve to raise the W-mass to match the CDF claim. The scan over parameters in each case is once more subject to the SM-like Higgs mass being in the right region, quartic couplings being perturbative and vacuum stability being satisfied. The results corresponding to the two aforementioned benchmarks of \( M_{W,\text{old}} \) are shown in Fig. 4. For illustration, we also show \( \Delta M_W \) against \( v_\chi \) in Fig. 5.

The points to note further are as follows:

- The generalized Georgi-Machacek scenario with custodial symmetry broken at tree level has a multidimensional parameter space, since its potential has 3 mass terms, 10 quartic interactions, and 3 trilinear terms involving the doublet and triplet scalars. It is not our purpose here to scan the entire parameter space. We instead demonstrate its consistency with the CDF result in terms of some illustrative situations with \( v_\chi \neq v_\xi \), with random scan over some parameters. The EWSB conditions are satisfied for each point in the parameter space displayed in Fig. 4 in \( \Delta v_\chi = v_\xi - v_\chi \) vs \( v_\chi \) plane.

- The lower limits on \( v_\chi \) arise when one has non-zero values of the trilinear term parameters \( \tilde{M}_1', \tilde{M}_2 \) and both the fields \( \phi \) and \( \xi \) acquired a nonzero vev. The parameters are further constrained such that the SM-like scalar mass has the right value, the doubly charged scalar mass, \( \tilde{M}_5^{++} \) lies between 200-1000 GeV and \( \tilde{M}_5^{++} < \tilde{M}_3^+, \tilde{M}_4^+ \).

- The maximum value of \( \Delta v_\chi \) in each case comes from the requirement of satisfying the 2\( \sigma \) limit on the \( \rho \)-parameter. The small minimum value for each \( v_\chi \) answers to the smallest split in vev required to reconcile \( M_W \) with \( M_Z \) and \( \rho \).

- The doubly charged scalar mass used for illustration is between 200-1000 GeV, and for such a combination of parameters that it decays only into the channel \( H_5^{++} \rightarrow W^+W^+ \). The limits on the parameter space from the data on the \( H^{++} \) search in vector-boson fusion channel \[38, 45\] are used for all points generated in the random scan, consistently with the above benchmarking. It may also be noted that, for the points in the parameter space used for our illustration, the constraints from VBF search mostly turns out to be the strongest. Flavour constraints such as those from \( b \rightarrow s\gamma \) and \( B_s \rightarrow \mu^+\mu^- \), and also that from the S-parameter are weaker\[39, 38\].

As can be seen from Fig. 4, one obtains rather non-negligible regions satisfying all constraints, for the illustrative benchmarks. It is thus possible to reconcile the GM scenario with the CDF claim, so long as a broken custodial SU(2) is conceded, without compromising on the limits from the \( \rho \)-parameter.

\[ \text{Figure 4: (a) and (b): Regions in the } v_\chi - \Delta v_\chi \text{ plane which satisfy theoretical as well as } \rho \text{ parameter constraints but are excluded by VBF data of } H_5^{++} \text{ search at 35.9 } fb^{-1} \text{ are shown in red, and those that satisfy the VBF constraints are in green. (a) uses the ATLAS measurement for } M_{W,\text{old}}, \text{ while (b) uses the global fit, both at 2}\sigma \text{ levels.} \]

---

\[ ^2M_{W,\text{old}} \text{ equated with } LEP + Tevatron \text{ measurement continues to be consistent with the CDF claim in anyway.} \]
The $H^\pm W^\mp Z$ vertex strengths in this case are shown in Fig. 6. Here we define $v_{\text{triplet}} = \sqrt{4v_\chi^2 + 4v_\xi^2}$. Note that the vertex like $H^+_2 W^- W^-$ also appears with higher strength in GM like scenario with respect to the models with a single triplet. But irrespective of the presence of custodial symmetry this vertex strength is always proportional to $v_\chi$ and since for the custodial symmetric case consistency with oblique correction requires triplet vev to be high, this vertex strength is always high i.e $0.6 \leq \frac{V_{H^+_2 W^- W^-}}{|\sqrt{2}g_{MW}|} \leq 0.7$. When the custodial symmetry is broken at tree level the vertex strength has a larger range to vary i.e $0.1 \leq \frac{V_{H^+_2 W^- W^-}}{|\sqrt{2}g_{MW}|} \leq 0.9$ (These limits are given without collider data). On the other hand, custodial symmetry has a more significant effect on the vertex strength of $H^\pm W^\mp Z$ and hence here we are interested about this vertex only.

3.2 A single scalar triplet

For the sake of completeness, we include below a scenario with a $Y = 2$, complex triplet $\Delta$ in addition to the SM doublet $\Phi = (\phi^+, \phi^0)$. It should be noted that the CDF-claim has also been analyzed with the addition of just one real triplet to the usual Higgs doublet [46]. We however, discuss the complex triplet case only, because
of its relevance in neutrino mass generation. The scalar potential has the following form,

\[ V(\Phi, \Delta) = -m_\Delta^2 \Phi^\dagger \Phi + M^2 Tr \Delta^\dagger \Delta + \mu \Phi^T i \sigma^2 \Delta^\dagger \Phi + h.c. + \frac{\lambda}{4} (\Phi^\dagger \Phi)^2 + \lambda_1 \Phi^\dagger \Phi Tr \Delta^\dagger \Delta + \lambda_2 (Tr \Delta^\dagger \Delta)^2 + \lambda_3 Tr (\Delta^\dagger \Delta)^2 + \lambda_4 \Phi^\dagger \Delta \Delta^\dagger \Phi \]  

(37)

with

\[ \Delta = \begin{pmatrix} \Delta^0 & \Delta^{++} \\ \Delta^+ & -\Delta^0 \end{pmatrix} \]  

(38)

The ranges of our scanned over parameters are given in Table 3. \( \lambda \) is tuned such that one of the two CP-even Higgs masses is always fixed at 125 GeV\(^47\). \( M_H^{++} \) is the mass of the doubly charged scalar. The mass of the doubly charged and the singly charged scalar are given by,

\[ M_{H^{++}}^2 = \frac{\mu v_t^2}{\sqrt{2} v_t} \frac{\lambda_4}{4} v_t^2 - \lambda_3 v_t^2 \]  

(39)

\[ M_{H^-}^2 = \frac{2 \sqrt{2} \mu - 4 \lambda_4 v_t}{4 v_t}(v_3^2 + 2 v_t^2) \]  

(40)

The neutral component of \( \Delta \) is instrumental in generating neutrino masses when one allows gauge-invariant \( H^{++} \) giving rise to \( \ell^+ \ell^+ \) or \( W^+W^+ \) pairs, where \( H^{++} = \Delta^{++} \). Simultaneous nonzero values of \( \langle \phi^0 \rangle = v_3/\sqrt{2} \) and \( \langle \Delta^0 \rangle = v_t/\sqrt{2} \), automatically shifts the value of \( \rho \) away from 1. Therefore, it is imperative for \( v_t \) to be small enough not to overshoot of the allowed interval of \( \rho \), as derived from the oblique electroweak parameter \( T \)\(^{40}\). Electroweak precision data, mainly the \( T \)-parameter, restrict the mass splitting between the doubly-and singly-charged scalars. In this study we have selected our benchmarks satisfying \( |M_{H^{++}}^2 - M_{H^-}^2| \leq 40 \text{ GeV} \), which is consistent with the \( T \)-parameter. We have calculated the oblique parameters following reference \(^{47}\)\(^{49}\). The triplet couplings to leptons are taken to be diagonal. Among collider constraints, the production of doubly charged Higgs via vector boson fusion (VBF) does not appear to be significant as the triplet vev is already small from the restriction imposed by the \( T \)-parameter. Instead, the Drell Yan(DY) pair production of doubly charged Higgs and the dilepton decay of the same puts a major constraint on the parameter space, although such constraints depend on the decay branching ratios of \( H^{\pm \pm} \). The ATLAS search for DY production of \( H^{++} \) pair and its subsequent decay to diboson channel restricts \( M_{H^{++}}^2 \geq 220 \text{ GeV} \). The constraint on a leptonically decaying doubly charged scalar becomes stringent in the low triplet vev limit. For \( v_t = 10^{-4} \text{ GeV} \), this constraint requires \( M_{H^{++}}^2 \geq 450 \text{ GeV} \)\(^{47}\). Once more, our results correspond to benchmarks that appropriately satisfy all these constraints.

In Fig. 7 we show the allowed region at 2\( \sigma \) level in the \( v_t - v_3 \) parameter space of a scenario that yields the SM-like scalar mass in its allowed interval around 125 GeV. The allowed region is further restricted by the new measurement of \( M_W \) and the existing value of \( M_Z \), consistently with the requirements of vacuum stability and perturbative unitarity\(^{47}\)\(^{45}\). Fig. 7 thus reflects the capability of a single-triplet scenario in simultaneously explaining \( M_W \) as announced by CDF, \( M_Z \), and also the \( \rho \)-parameter. Also, this scenario continues to be useful in neutrino mass generation, since the low-\( v_t \) ranges are unaffected.

### 4 Summary and conclusions

We have shown that, if both the value of \( M_W \) announced by CDF and the extant value of \( M_Z \) continue to remain within their respective 2\( \sigma \) intervals, then any EWSB sector which theoretically predicts \( \rho = 1 \) at the tree-level becomes inconsistent. This is true if one accepts (a) the earlier value of \( M_W \) obtained from global...
fits, or (b) the current value from the ATLAS collaboration. On the other hand, The new \( M_W \) and the extant \( M_Z \) are consistent within 1\( \sigma \) with \( \rho = 1 \) if the earlier \( M_W \) obtained by using just the LEP + Tevatron data.

Keeping this in view, we have considered the Georgi-Machacek scenario comprising a complex \( Y = 2 \) scalar triplet, a real \( Y = 0 \) triplet, and the SM Higgs doublet, which retains \( \rho = 1 \) at the tree-level. This \textit{prima facie} disqualifies this model at the tree level for two of the three reference points of \( M_W, old \). However we show, that the CDF claim together with the experimental value of \( \rho \) parameter can be satisfied if one includes the finite corrections to \( M_W \) which is not calculable from the first principles unless one knows the UV completion of the GM scenario. It is also shown that this finite correction should be small for higher triplet vev around 30 GeV or above. On the other hand, a low triplet vev may allow relatively large correction. We have also explored the possibility of an explicit small splitting between the two triplet vevs at the tree level itself and have identified the allowed region in \( \Delta v_\chi - v_\chi \) plane. In context of colliders, both of these scenarios are interesting due to the presence of nonzero \( H^+W^-Z \) vertices. It has been shown that as long as the tree level scalar potential respects custodial \( SU(2) \), only \( H^+_\chi W^-Z \) vertex can be significant. On the other hand, for either of the two schemes outlined to accomodate the the CDF claims the \( H^+_\chi W^-Z \) vertex strengths can be not only significant but also sometimes comparable to the \( H^+_\chi W^-Z \) vertex strength.

For the sake of completeness we take into account, the simplest extension of the EWSB sector with \( \rho \neq 1 \), namely a scenario with a scalar doublet and a singlet complex \( Y = 2 \) scalar triplet. It turns out to be compatible with the new \( M_W \), the existing \( M_Z \) and the experimental range of \( \rho \), for all the three ways of standardizing \( M_W, old \) mentioned above once the oblique parameters are considered.

Before we conclude, some comments are in order regarding the additional, \( \Delta L = 2 \) Yukawa couplings involving the \( Y = 2 \) triplet and the leptons. In principle, these couplings may be subject to contraints from flavour-changing neutral current (FCNC) process such as \( \mu \to e\gamma \) and \( \mu \to 3e \). Moreover such couplings may contribute to muon decay and thus tend to alter the Fermi coupling constant extracted therefrom. These and similar kinds of issues are addressed by the fact that the \( (\Delta L = 2) \) Yukawa coupling strengths relevant in the parameter region in Fig. 7 corresponds to \( v_\chi \leq 10^{-11} \). Such values are too small to effect either \( G_F \) or FCNC rates, even if there remain flavour-changing \( \Delta L = 2 \) Yukawa interactions. On the contrary, Yukawa couplings which can affect the above processes will require non-vanishing triplet vev \( \leq 10^{-6} \) GeV (from neutrino mass limits)\cite{47,50}. Such a vev has no perceptible consequence on the W-boson mass, and is thus not relevant for the present discussion.

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