A Freeform Lens System Design and Simulation in the Polar Coordinate

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Abstract. In the field of illumination engineering, freeform technology is playing a more and more important role. Several methods have been reported in recent papers. The paper introduce a design method to generate an ellipse spot in the polar coordinate using the polar grids based on flux transportation mapping. In this method, the source is a point and the point is divided along the azimuth angle and zenith angle in the polar coordinate system. Then we divided the flux target in the polar coordinate, the grids of the target are achieved by the mapping between the source and the target. Based on the mapping and energy conservation, we can derive a set of the first-order PDE according to Snell’s law. We can acquire a points cloud after solving the PDE numerically, then we will construct a smooth freeform using Rhino. Finally, we will test the freeform using Light Tools. With this method, we can design a lens with the light utilization efficiency over 0.8.

Keywords: LED; Freeform; Mapping; Polar coordinate.

1. Introduction

Nowadays LED has become the most important light source in the 21st century, so the design for the uniform of a lens is becoming a hot research topic. There are many methods to get uniform illumination1234. Y Ding proposed a method using ODE to design a freeform lens with high light utilization efficiency5. Rengmao Wu presented a method using the M-A equation in the Cartesian coordinate system6. Qi-hui Zhang has shown a method to design a freeform reflector with the same method using MATLAB7. For far filed target L. B. Romijn shown a method using Monge-Ampere equation from point source8. All the methods we have mentioned have common characteristics: the lens is designed in the Cartesian coordinate system.

In this paper, we will introduce a method based on the polar coordinate, the data of points cloud is acquired by solving the first-order PDE using finite difference method in polar coordinate. We can get a point cloud by solving the equation using MATLAB. Then using softwares we can get the freeform lens. In this paper, a freeform lens to get an ellipse spot will be designed using the method mentioned above.

2. The Design of the Lens and Simulation

2.1. The Design of Freeform Lens

The design method can be summarized into three steps: 1. The derivation of the first-order PDE, 2. The establishment of mapping between source and target, 3. The construction of a freeform lens.
2.1.1. The derivation of first-order PDE. The first step to design free-form lens is the derivation of first-order PDE. Assume the LED is located at the origin of the optical system. \( I \) is the vector of the incident light, and \( O \) is the vector of emergent light, \( N \) is the normal vector of \( P \), the coordinates of \( P \) is \( P(\theta, \varphi, \rho(\theta, \varphi)) \), the relationship between these vectors is shown in figure 1.

Figure 1. The relation between the vectors of the incident ray \( I \), emergent ray \( O \), and the normal vector \( N \).

The vector of \( N, I, O \) can be expressed as follow:

\[
N = (N_x i, N_y j, N_z k), \\
I = (I_x i, I_y j, I_z k), \\
O = (t - p) / |t - p|.
\]

In equation (1) \( N \) is derived by the derivative of \( P \), in the \( \theta \) and \( \varphi \) directions.

\[
N = \left( P_\theta \times P_\varphi \right) / \left| P_\theta \times P_\varphi \right|.
\]

Using Snell’s laws the connection of these three vectors can be expressed as followed.

\[
t_x = N_y \left[ n_o (z - p_z) - n_i I_z |t - p| \right] / \left( n_o N_z + p_y + n_i I_y |t - p| \right), \\
t_y = N_y \left[ n_o (z - p_z) - n_i I_z |t - p| \right] / \left( n_o N_z + p_y + n_i I_y |t - p| \right).
\]

Where in the equation (3) \( t_x, t_y \) is the function of \( \rho, \theta \), \( n_i, n_o \) is the refractive index of the incident and emergence medium.

\[
t_x = f(\theta, \varphi, \rho(\theta, \varphi)), \\
t_y = f(\theta, \varphi, \rho(\theta, \varphi)).
\]

2.1.2. The mapping between the point source and target in the polar coordinate. The second step in our method is to establish the mapping between source and target, the difference between the traditional method and our method is that our method is in polar coordinate rather than Cartesian coordinate. The point source is divided in the polar coordinate \( (\theta, \varphi) \), we assume the luminous intensity of point light is \( I(\theta) \). Assume there is no other energy losses, the law of energy conservation can be described as

\[
\iint_{\Omega_t} I(\theta) \sin \theta d\theta d\varphi = \iiint_{\Omega_s} EdA.
\]
In equation (5) the solid angle is represented by $\Omega$, $\Omega$ is the illuminated target and the $A$ represents the area of the target. $I(\theta)$ is the emitting intensity of the LED, and $E$ is the luminance on the target surface. Suppose our target pattern is an ellipse. The target is divided as shown in figure 2, where the illuminated picture can be described by $\rho$ and $\theta$.

![Figure 2](image)

**Figure 2.** The illumination target is divided into $m \times n$ parts by $\rho$ and $\theta$, and $i \in m, j \in n$.

Using Snell’s law and Energy conservation we concluded first-order PDE in the polar coordinate

$$\frac{\partial \rho(\theta, \varphi)}{\partial \theta} + \frac{\partial \rho(\theta, \varphi)}{\partial \varphi} = f,$$

(6)

And from the flux mapping of point and target in polar coordinate, we can infer the relationship between the diametral distance of the ellipse and $\theta, \varphi$.

$$\rho = \rho_0 \sin \varphi \frac{P}{1 + e \cos \theta},$$

$$P = \frac{b^2}{a},$$

$$e = \frac{c}{a},$$

$$c = \sqrt{a^2 + b^2}.$$

(7)

In equation (7) $a, b$ is the length of the major and minor axes of the ellipse. We divide the $\varphi$ into $m$ parts and the $\theta$ into $n$ parts, the $\varphi, \theta$ can be expressed as the following equation.

$$\varphi_i = i^* d\varphi, i \in m, d\varphi = \frac{\varphi}{m},$$

$$\theta_j = j^* d\theta, j \in n, d\theta = \frac{\theta}{n}.$$

(8)

Then we will solve the first-order PDE to get the coordinate of the freeform lens. The Finite difference method (FEM) is the most common numerical method to solve PDE. The main idea of FEM is that the continuous definite solution area is replaced by a grid composed of finite discrete points, which are called the junction (node) points of the grid. The continuous variable function defined on the continuous definite solution region is approximated by the discrete variable function defined on the grid. The micro commercial difference quotient of the original equation and the definite solution condition is approximated, and the integral is approximated by the integral sum, then the original equation and the definite solution condition can be approximately replaced by algebraic equations, and the approximate
solution of the original problem can be obtained by solving the algebraic equations. This method is simple and universal, and easy to be realized on an electronic computer. We solve equation (7) using FEM in MATLAB. Then the coordinate of the points of the freeform can be acquired.

2.1.3. The construction of the optical lens. After getting the coordinate of the freeform lens, we need to construct the lens from these points, Rhino is a software which is widely used in furniture, model design, and other industries. It’s based on Non-Uniform Rational B-Spline (NURBS), which is a very excellent model approach that is supported in advanced 3D software. NURBS can control the curvilinear degree of object surface better than the traditional grid model method, thus can create a more realistic and vivid model. In our method, we use Rhino to construct the lens from the points cloud data acquired from the PDE. We import the calculated coordinates into Rhino, we will get the points could of the freeform lens. Then we can structure the freeform lens using NURBS. The general method of model software is that the lines are connected by points and the plane is connected by lines. For a freeform lens, the spatial structure of it is very complex. Up to now, the best way to construct complex freeform is Non-Uniform Rational B-Spline (NURBS). It was first used by French scientist Pierre Bezier and Paul de Casteljau for the design of freeform. The parameter equation of NURBS is follow

\[ \mathbf{r}(u) = \frac{\sum_{i=0}^{n} w_i N_{k,i}(u) \mathbf{C}_i}{\sum_{i=0}^{n} w_i N_{k,i}(u)} = \sum_{i=0}^{n} N_{k,i}^*(u) \mathbf{C}_i. \]  

In the equation (9)

\[ N_{k,i}^*(u) = \frac{w_i N_{k,i}(u)}{\sum_{i=0}^{n} w_i N_{k,i}(u)}. \]

\( \mathbf{C}_i \) is control points, \( W_i \) is the weighing function of the control point. By changing the weighing function, we can change the shape of the lines. In the equation (10) \( N_{k,i}(u) \) is control function, the mathematical expression of \( N_{k,i}(u) \) is

\[ N_{k,i}(u) = \frac{(u-u_j) N_{k-1,i}(u)}{(u_{i+k-1}-u_j)} + \frac{(u_{i+k}-u) N_{k-1,i+1}(u)}{(u_{i+k}-u_{i+1})}. \]

For the construction of surfaces, there are two directions: \( u \) and \( v \), so the equation (9) becomes

\[ \mathbf{r}(u,v) = \frac{\sum_{i=0}^{n} \sum_{j=0}^{m} w_{i,j} N_{p,i}(u) N_{q,j}(v) \mathbf{C}_{i,j}}{\sum_{i=0}^{n} \sum_{j=0}^{m} w_{i,j} N_{p,i}(u) N_{q,j}(v)}, \]

\[ 0 \leq u, v \leq 1. \]

In equation (12) \( p, q \) is the power of number of freeform and \( \mathbf{C}_{i,j} \) is the control points, \( W_{i,j} \) is weighing function of the control point.

For the freeform design, when we import the points cloud Rhino will construct the freeform lens using NURBS. After the construction of the mesh of freeform lens, we use Rhino to construct freeform.
2.2. Ray Tracing and Simulation Result
After completing the construction of the freeform lens using Rhino for LED. The next step is to test the performance of the lens using simulation software. Compared with other software Light Tools is more suitable for LED design and ray-tracing simulation. It can simulate luminescence characteristics of LED, what’s more, it can simulate the shape and illumination uniformity of the spot formed by the light passing through the lens, so we choose Light Tools to analyse the lens.
We import the lens model into Light Tools, setting point light source at (0,0,0) and the number of tracing light is 25000. The result of the simulation is shown in figure 4. In the picture, we can see that the ellipse is clear, and the luminous intensity distribution shows that the energy utilization rate is satisfactory.

3. Conclusion
The design method we mentioned above can design many targets such as ellipse, rectangle, and rhombus. Compared the method in the cartesian coordinate system, our method is easier when the target is described in the polar coordinate such as ellipse. Using this method, we can not only design a rotationally symmetric pattern but also no-rotationally symmetric, such as the Cartesian heart-shaped line. Many targets such as ellipse and Cartesian heart-shaped line when they are described in the polar coordinate, they are easier to build model and calculate, according to the results of simulation, when the number of points is the same, the luminous intensity distribution is much better than that described in the cartesian coordinate. But when the target is rectangle, it is better described in Cartesian coordinates, when they are described in polar coordinate the luminous intensity distribution is not very good, to solve this question, we can divide the target into three parts according the light intensity of the point source, then we use the method mentioned to calculate the points and construct the lens. Using this method the luminous intensity distribution can be improved.
The solution is practical, saves designer time to calculate, model, improve industry efficiency, particularly when the target can be described in polar coordinate. Using this method we can accomplish many lighting design. This method is complementary to cartesian coordinates. Using the two methods more diagram can be designed. With the development of the manufacturing industry, the freeform lens will be used more and more widely in our life and production.
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