Abstract—Nonlocal image representation or group sparsity has attracted considerable interest in various low-level vision tasks and has led to several state-of-the-art image denoising techniques, such as BM3D, LSSC. In the past, convex optimization with sparsity-promoting convex regularization was usually regarded as a standard scheme for estimating sparse signals in noise. However, using convex regularization cannot still obtain the correct sparsity solution under some practical problems including image inverse problems. In this paper we propose a non-convex weighted \( \ell_p \) minimization based group sparse representation (GSR) framework for image denoising. To make the proposed scheme tractable and robust, the generalized soft-thresholding (GST) algorithm is adopted to solve the non-convex \( \ell_p \) minimization problem. In addition, to improve the accuracy of the nonlocal similar patch selection, an adaptive patch search (APS) scheme is proposed. Experimental results demonstrate that the proposed approach not only outperforms many state-of-the-art denoising methods such as BM3D and WNNM, but also results in a competitive speed.

Index Terms—Image denoising, group sparsity, weighted \( \ell_p \) minimization, generalized soft-thresholding algorithm, adaptive patch search.

I. INTRODUCTION

The goal of image denoising is to restore the clean image \( X \) from its noisy observation \( Y \) as accurately as possible, while preserving significant detail features such as edges and textures. The degradation model for the denoising problem can be represented as: \( Y = X + V \), where \( V \) is usually assumed to be additive white Gaussian noise. Image denoising problem is mathematically ill-posed and image priors are exploited to adjust it such that meaningful solutions exist. Over the past few decades, numerous image denoising methods have been developed, including total variation based [1, 2], sparse representation based [3, 4], nonlocal self-similarity based [5–12] and deep learning based ones [9, 10, 38], etc.

Early models mainly consider the priors on level of pixel, such as total variation (TV) regularization methods [1, 2]. These methods actually assume that natural image gradients exhibit heavy-tailed distributions, which can be fitted by Laplacian or hyper-Laplacian models [11]. Since the TV model favors the piecewise constant image structures, it often damages the image details and tends to over-smooth the images.

As an alternative, another significant property of natural images is to model the prior on patches. The most representative work is sparse representation based scheme [3, 4], which encodes an image patch as a sparse linear combination of the atoms in an over-complete redundant dictionary. The dictionary is usually learned from natural images [12]. The seminal K-SVD dictionary [4] has not only confirmed promising denoising performance, but also extended and successfully exploited it in various image processing and computer vision tasks [13, 14]. However, patch-based sparse representation model usually suffers from some limits, such as dictionary learning with great computational complexity and neglecting the relationships among similar patches [7, 15, 16].

Motivated by the observation that nonlocal similar patches in a natural image are linearly correlated with each other, this so-called nonlocal self-similarity (NSS) prior was initially employed in the work of nonlocal means denoising [5], which has become the most effective priors for the task of image restoration [17, 18]. Due to its favorable reconstruction performance, a large amount of further developments have been proposed [6–8, 15, 16, 19, 20]. For instance, a very popular scheme is BM3D [6], which groups similar patches into 3D array and disposes these arrays by sparse collaborative filtering. Marial et al. [7] proposed the learned simultaneous sparse coding (LSSC) to improve the denoising performance of K-SVD [4] via group sparse coding. Gu et al. [19, 20] proposed the weighted nuclear norm minimization (WNNM) model, which turned the image denoising into the problem of low rank matrix approximation of noisy nonlocal similar patches. Lately, deep learning based techniques for image denoising have been attracting considerable attentions due to its impressive denoising performance [9, 10, 38].

Traditional sparse representation based image denoising methods exploit the \( \ell_1 \)-norm based sparsity of an image and the resulting convex optimization problems can be efficiently solved by the class of surrogate-function based methods [21, 22]. However, using convex regularization cannot still obtain the correct sparsity solution under some practical problems including image inverse problems [39].

Inspired by the success of \( \ell_p \) (\( 0 < p < 1 \)) sparse optimization [23–25, 40] and our previous work [39], this paper proposes a non-convex weighted \( \ell_p \) minimization based group sparse representation (GSR) framework for image de-
noising. To make the proposed scheme tractable and robust, the generalized soft-thresholding (GST) algorithm is adopted to solve the non-convex \( \ell_p \) minimization problem. Moreover, we propose an adaptive patch search (APS) scheme to improve the accuracy of the nonlocal similar patch selection. Experimental results show that the proposed approach not only outperforms many state-of-the-art denoising methods such as BM3D and WNNM, but also results in a competitive speed.

II. GROUP-BASED SPARSE REPRESENTATION

Recent advances have suggested that structured or group sparsity can offer powerful performance for image restoration [7, 8, 16]. Since the unit of our proposed sparse representation model is group, this section will give briefs to introduce how to construct the groups. More specifically, image \( X \) with size \( N \) is divided into \( n \) overlapped patches \( x_i \), of size \( \sqrt{d} \times \sqrt{d}, i = 1, 2, \ldots, n \). Then for each exemplar patch \( x_i \), its most similar \( m \) patches are selected from an \( L \times L \) sized searching window to form a set \( S_i \). Since then, all the patches in \( S_i \) are stacked into a matrix \( X_i \in \mathbb{R}^{d \times n} \), which contains every element of \( S_i \) as its column, i.e., \( X_i = \{x_{i,1},x_{i,2},...,x_{i,m}\} \). The matrix \( X_i \) consisting of all the patches with similar structures is called as a group, where \( x_{i,m} \) denotes the \( m \)-th similar patch (column form) of the \( i \)-th group. Finally, similar to patch-based sparse representation [3, 4], given a dictionary \( D_i \), we learn the principal component analysis (PCA) based dictionary [32], each group \( X_i \) can be sparsely represented as \( \alpha_i = D_i^{-1}X_i \) and solved by the following \( \ell_0 \)-norm minimization problem,

\[
\alpha_i = \arg\min_{\alpha_i} \sum_{i=1}^{n} \frac{1}{2}||X_i - D_i\alpha_i||_F^2 + \lambda_i||\alpha_i||_0 \tag{1}
\]

where \( || \cdot ||_F \) denotes the Frobenious norm and \( \lambda_i \) is the regularization parameter. \( || \cdot ||_0 \) is \( \ell_0 \)-norm, counting the nonzero entries of \( \alpha_i \).

In image denoising, each noise patch \( y_i \) is extracted from the noisy image \( Y \). We search for its similar patches to generate a group \( Y_i \), i.e., \( Y_i = \{y_{i,1},y_{i,2},...,y_{i,m}\} \). Thus, image denoising is translated into how to reconstruct \( X_i \) from \( Y_i \) by using group sparse representation,

\[
\alpha_i = \arg\min_{\alpha_i} \sum_{i=1}^{n} \frac{1}{2}||Y_i - D_i\alpha_i||_F^2 + \lambda_i||\alpha_i||_0 \tag{2}
\]

Once all group sparse codes \( \{\alpha_i\} \) are obtained, the latent clean image \( X \) can be reconstructed as \( X = D\alpha \), where the group sparse code \( \alpha \) includes the set of \( \{\alpha_i\} \).

However, since the \( \ell_0 \) minimization is discontinuous optimization and NP-hard, solving Eq. (2) is a difficult combinatorial optimization problem. For this reason, it has been suggested that \( \ell_0 \) minimization can be replaced by its convex \( \ell_1 \) counterpart,

\[
\alpha_i = \arg\min_{\alpha_i} \sum_{i=1}^{n} \frac{1}{2}||Y_i - D_i\alpha_i||_F^2 + \lambda_i||\alpha_i||_1 \tag{3}
\]

However, \( \ell_1 \) minimization is hard to achieve the desired sparsity solution in some practical problems, such as image denoising, image compressive sensing [26, 27], etc.

III. NON-CONVEX WEIGHTED \( \ell_p \) MINIMIZATION BASED GROUP SPARSE REPRESENTATION FRAMEWORK FOR IMAGE DENOISING

Conventional convex optimization with sparsity-promoting convex regularization is usually regarded as a standard scheme for estimating sparse signals in noise. However, using convex regularization cannot still obtain the correct sparsity solution under some practical problems including image inverse problems [39]. This section introduces a non-convex weighted \( \ell_p \) minimization based group sparse representation framework for image denoising. To make the optimization tractable, the generalized soft-thresholding (GST) algorithm [25] is adopted to solve the non-convex \( \ell_p \) minimization problem. To improve the accuracy of the nonlocal similar patch selection, an adaptive patch search scheme is proposed.

A. Modeling of Non-convex Weighted \( \ell_p \) Minimization

Inspired by the success of \( \ell_p \) \((0 < p < 1)\) sparse optimization [23–25, 40] and our previous work [39], to obtain sparsity solution more accurately, we extend the non-convex weighted \( \ell_p \) \((0 < p < 1)\) penalty function on group sparse coefficients of the data matrix to substitute the convex \( \ell_1 \) norm. Specifically, instead of Eq. (3), a non-convex weighted \( \ell_p \) minimization based group sparse representation framework for image denoising is proposed by solving the following minimization,

\[
\alpha_i = \arg\min_{\alpha_i} \sum_{i=1}^{n} \frac{1}{2}||Y_i - D_i\alpha_i||_F^2 + ||W_i\alpha_i||_p \tag{4}
\]

where \( W_i \) is a weight assigned to each group \( Y_i \). Each weight matrix \( W_i \) will enhance the representation capability of each group sparse coefficient \( \alpha_i \). In addition, one important issue of the proposed denoising approach is the selection of the dictionary. To adapt to the local image structures, instead of learning an over-complete dictionary for each group \( Y_i \) as in [7], we learn the principle component analysis (PCA) based dictionary [32] for each group \( Y_i \). Due to orthogonality of each dictionary \( D_i \), and thus, based on the orthogonal invariance, Eq. (4) can be rewritten as

\[
\alpha_i = \min_{\alpha_i} \sum_{i=1}^{n} \frac{1}{2}||\gamma_i - \alpha_i||_F^2 + ||W_i\alpha_i||_p \tag{5}
\]

\[
= \min_{\alpha_i} \sum_{i=1}^{n} \frac{1}{2}||\gamma_i - \tilde{\alpha}_i||_2^2 + \tilde{\gamma}_i||\tilde{\alpha}_i||_p \tag{5}
\]

where \( Y_i = D_i\gamma_i \), \( \tilde{\alpha}_i \) and \( \tilde{\gamma}_i \) denote the vectorization of the matrix \( \alpha_i \), \( \gamma_i \), and \( W_i \), respectively.

B. Solving the Non-convex Weighted \( \ell_p \) Minimization by the Generalized Soft-thresholding Algorithm

To achieve the solution of Eq. (5) effectively, in this subsection, the generalized soft-thresholding (GST) algorithm [25] is used to solve Eq. (5). Specifically, given \( p \), \( \gamma_i \) and \( \tilde{\gamma}_i \), there exists a specific threshold,

\[
\tau^{GST}_{\gamma_i} = (2\tilde{\gamma}_{i,j}(1-p))^{\frac{1}{p-1}} + \tilde{\gamma}_{i,j}p(2\tilde{\gamma}_{i,j}(1-p))^{\frac{p-1}{p}} \tag{6}
\]

where \( \tilde{\gamma}_{i,j} \), \( \tilde{\alpha}_{i,j} \) and \( \tilde{\gamma}_{i,j} \) are the \( j \)-th element of \( \gamma_i \), \( \tilde{\alpha}_i \) and \( \tilde{\gamma}_i \), respectively. Here if \( \tilde{\gamma}_{i,j} < \tau^{GST}_{\gamma_i} \), \( \tilde{\alpha}_{i,j} = \tilde{\gamma}_{i,j} \), etc.
Algorithm 1: Generalized Soft-Thresholding (GST) [25].

Input: $\tilde{\gamma}_{i,j}, \tilde{w}_{i,j}, p, J$.
1. $\gamma_{i,j}^{GST}(\tilde{w}_{i,j}) = (2\tilde{w}_{i,j}(1-p)\frac{1}{p} + \tilde{w}_{i,j}p(2\tilde{w}_{i,j}(1-p))\frac{1}{2p};$
2. If $|\gamma_{i,j}| \leq \gamma_{i,j}^{GST}(\tilde{w}_{i,j})$
3. $\theta_{i,j}^{GST}(\tilde{w}_{i,j}) = 0$;
4. Else
5. $k = 0, \hat{\alpha}_{1,j}^{(k)} = |\gamma_{i,j}|;
6. \text{Iterate on } k = 0, 1, \ldots, J
7. \hat{\alpha}_{1,j}^{(k+1)} = |\gamma_{i,j}| - \tilde{w}_{i,j}p(\hat{\alpha}_{1,j}^{(k)})^{p-1};
8. $k \leftarrow k + 1;
9. \theta_{i,j}^{GST}(\tilde{w}_{i,j}) = \text{sgn}(\hat{\alpha}_{1,j})\hat{\alpha}_{1,j}^{k};
10. \text{End if}
\text{Input: } \theta_{i,j}^{GST}(\tilde{w}_{i,j}).

C. Adaptive Patch Search

$k$ Nearest Neighbors (kNN) method [28] has been widely used to nonlocal similar patch selection. Given a noisy reference patch and a target dataset, the aim of $k$NN is to find the $k$ most similar patches. However, since the given reference patch is noisy, $k$NN has a drawback that some of the $k$ selected patches may not be truly similar to given reference patch. Therefore, to obtain an effective similar patches index via $k$NN, an adaptive patch search scheme is proposed. We define the following formula,

$$\varphi = \text{SSIM}(\theta, \hat{X}^{t+1}) - \text{SSIM}(\theta, \hat{X}^t)$$

(8)

where SSIM represents structural similarity [29], $\theta$ is pre-filtering denoised image and $\hat{X}^t$ represents the $t$-th iteration denoising result. We empirically define that if $\varphi < \rho$, $\hat{X}^{t+1}$ is regarded as target image to fetch the $k$ similar patch indexes of each group, otherwise $\theta$ is regarded as target image. $\rho$ is a small constant.

For the weight $W_i$ of each group sparse coefficient $\alpha_i$, large values of each $\alpha_i$ usually represent major edge and texture information. Therefore, we should shrink large values less, while shrinking smaller ones more [30]. Inspired by [31], the weight $W_i$ of each group $Y_i$ is set as $w_i = [\tilde{w}_{i,1}, \tilde{w}_{i,2}, \ldots, \tilde{w}_{i,J}]$, where $\tilde{w}_{i,j} = c*2\sqrt{\sigma^2/\sigma_i}$, $\sigma_i$ denotes the estimated variance of $\alpha_i$, and $c$ is a small constant.

In addition, we could execute the above denoising procedure for better results after several iterations. In the $t$-th iteration, the iterative regularization strategy [33] is used to update the estimation of noise variance. Then the standard deviation of noise in $t$-th iteration is adjusted as $(\sigma^t) = \delta * \sqrt{(\sigma^2 - ||Y - \hat{X}^t||_2^2)}$, where $\delta$ is a constant. The proposed denoising procedure is summarized in Algorithm 2.

This paper BM3D is chosen as a pre-filtering.

Algorithm 2: The Proposed Denoising Algorithm.

Input: Noisy image $Y$.
Initialization: $\hat{X} = Y, \theta, c, d, m, L, J, \sigma, \rho, \delta, \lambda$;
For $t = 1, 2, \ldots, K$ do
Iterative regularization $Y^{t+1} = \hat{X}^t + \lambda(Y - \hat{X}^t)$;
If $t = 1$
Similar patch selection based on $\theta$.
Else
If $\text{SSIM}(Y^{t+1}, \theta) - \text{SSIM}(Y^t, \theta) < \rho$
Similar patches index selection based on $Y^{t+1}$.
Else
Similar patches index selection based on $\theta$.
End if
End if
For each patch $y_i$ do
Find a group $\theta_i^{t+1}$ via $k$NN.
Constructing dictionary $D_i^{t+1}$ by $y_i$ by PCA operator.
Generating the group sparse coefficient $\gamma_i^{t+1}$ by $D_i^{-1}Y_i$.
Update $W_i^{t+1}$ computing by $\gamma_i^{t+1}$.
Update $\gamma_i^{t+1}$ computing by Algorithm 1.
Get the estimation $X_i^{t+1} = D_i^{t+1}\gamma_i^{t+1}$.
End for
Aggregate $X_i^{t+1}$ to form the recovered image $\hat{X}^{t+1}$.
End for
Output: $\hat{X}^{t+1}$.

Fig. 1. The six test images for denoising experiments.

Fig. 2. Denoising images of $plants$ by different methods ($\sigma = 50$). (a) Original image; (b) Noisy image; (c) BM3D [6] (PSNR=28.11dB); (d) LINC [34] (PSNR=27.96dB); (e) AST-NLS [35] (PSNR=28.04dB); (f) MSEPPLL [36] (PSNR=28.90dB); (g) WNMM [20] (PSNR=28.23dB); (h) Proposed (PSNR=28.60dB).

IV. EXPERIMENTAL RESULTS

To demonstrate the efficacy of the proposed denoising algorithm, in this section, we compare it with recently proposed state-of-the-art denoising methods, including BM3D [6], LINC [34], AST-NLS [35], MSEPPLL [36] and WNMM [20]. The experimental images are shown in Fig. 1. The Matlab code can be downloaded at: https://drive.google.com/open?id=0B0wKhwcnCjM0d0VFhlREIXWjg.

The parameter setting of proposed approach is as follows: the searching window $L \times L$ for similar patches is set to be $30 \times 30$. The searching matched patches $m$ is set to be 60. The size of each patch $\sqrt{d} \times \sqrt{d}$ is set to be $6 \times 6$ and $7 \times 7$. 0 is the global minimum. Otherwise, the optimum will be obtained at non-zero point. According to [25], for any $\tilde{\gamma}_{i,j} \in (\tau^{GST}_{p}(\tilde{w}_{i,j}), +\infty)$, Eq. (5) has one unique minimum $T_p^{GST}(\tilde{\gamma}_{i,j}; \tilde{w}_{i,j})$, which can be obtained by solving the following equation,

$$T_p^{GST}(\tilde{\gamma}_{i,j}; \tilde{w}_{i,j}) - \tilde{\gamma}_{i,j} + \tilde{w}_{i,j}p(T_p^{GST}(\tilde{\gamma}_{i,j}; \tilde{w}_{i,j}))^{b-1} = 0 \quad (7)$$

The complete description of the GST algorithm is exhibited in Algorithm 1. For more details about the GST algorithm, please refer to [25].
TABLE I
DENOISING PSNR (dB) RESULTS BY DIFFERENT DENOISING METHODS.

| Images | BM3D | LINC | AST-NLS | MSEPLL | WNNM | Proposed | BM3D | LINC | AST-NLS | MSEPLL | WNNM | Proposed |
|--------|------|------|--------|--------|------|----------|------|------|--------|--------|------|----------|
| House  | 33.77| 33.82| 33.87 | 33.27  | 34.04| 34.08    | 32.07| 32.26| 32.26  | 30.96  | 31.07| 31.14    |
| lin    | 32.83| 33.04| 33.84 | 32.80  | 33.00| 33.08    | 30.95| 31.03| 30.83  | 30.96  | 31.07| 31.11    |
| flower | 30.01| 30.50| 30.28 | 30.10  | 31.34| 30.48    | 27.97| 28.13| 28.20  | 28.05  | 28.26| 28.36    |
| foreman| 34.54| 34.76| 34.55 | 34.09  | 34.72| 34.86    | 32.75| 32.93| 32.79  | 32.34  | 33.00| 33.31    |
| plants | 32.68| 32.83| 32.75 | 32.58  | 32.04| 33.09    | 30.76| 30.67| 30.55  | 30.66  | 30.94| 31.05    |
| Miss   | 33.71| 33.64| 33.64 | 33.08  | 33.70| 33.80    | 31.89| 31.75| 31.72  | 31.92  | 31.93| 32.04    |
| Average| 32.92| 33.07| 32.99 | 32.80  | 33.14| 33.23    | 31.06| 31.13| 31.08  | 30.93  | 31.29| 31.42    |

TABLE II
AVERAGE PSNR (dB) RESULTS OF ADS AND NO-ADS ON 6 TEST IMAGES.

| σ   | BM3D | LINC | AST-NLS | MSEPLL | WNNM | Proposed |
|-----|------|------|--------|--------|------|----------|
| 20  | 30.65| 31.00| 30.91  | 30.47  | 31.31| 31.49    |
| 20  | 29.52| 29.94| 29.39  | 29.08  | 29.80| 29.89    |
| 30  | 26.48| 26.79| 26.75  | 26.64  | 26.85| 26.90    |
| 40  | 31.29| 31.31| 31.29  | 31.05  | 31.54| 32.08    |
| 50  | 29.14| 29.09| 29.05  | 29.25  | 29.28| 29.70    |
| 20  | 30.50| 30.29| 30.19  | 30.56  | 30.53| 30.78    |
| 30  | 29.59| 29.74| 29.60  | 29.61  | 29.88| 30.14    |
| 40  | 28.62| 28.59| 28.69  | 28.57  | 28.85| 29.10    |

We first evaluate the proposed approach and the competing algorithms on 6 test images. Table I shows the PSNR results. It can be seen that the proposed approach performs competitively compared to other methods. The proposed approach achieves 0.42dB, 0.34dB, 0.39dB, 0.51dB and 0.18dB improvement on average over the BM3D, LINC, AST-NLS, MSEPLL and WNNM, respectively. Fig. 2 shows the denoised image of plants by the competing methods. It can be seen that BM3D, LINC, AST-NLS, MSEPLL and WNNM still generate some undesirable artifacts and some details are lost. In contrast, the proposed approach not only preserves the sharp edges, but also suppresses undesirable artifacts more effectively than other competing methods.

TABLE III
AVERAGE RUN TIME (s) WITH DIFFERENT METHODS ON THE 6 TEST IMAGES (SIZE: 256 × 256).

| Methods | LINC | AST-NLS | MSEPLL | WNNM | Ours |
|---------|------|---------|--------|------|------|
| Average | 263  | 300     | 182    | 172  | 82   |

Second, to verify the proposed adaptive patch selection (APS) scheme effectively, we compare it with No-APS scheme. The average PSNR results of APS and No-APS schemes on 6 test images are shown in Table II. One can observe that the PSNR results of APS scheme are better than No-APS. Thus, under the task of image denoising, the proposed APS scheme can enhance the accuracy of nonlocal similar patch selection.

TABLE IV
AVERAGE PSNR (dB) RESULTS WITH DIFFERENT METHODS ON BSD200 DATASET [37].

| σ   | BM3D | LINC | AST-NLS | MSEPLL | WNNM | Ours |
|-----|------|------|--------|--------|------|------|
| 20  | 29.86| 29.92| 29.98  | 29.95  | 30.11| 30.14|
| 30  | 27.93| 27.94| 28.02  | 28.02  | 28.17| 28.15|
| 40  | 26.58| 26.61| 26.68  | 26.75  | 26.88| 26.89|
| 50  | 25.71| 25.64| 25.80  | 25.84  | 25.96| 25.97|

Third, to evaluate the computational cost of the competing algorithm, we compare the running time on 6 test images with different noise levels. All experiments are conducted under the Matlab 2012b environment on a machine with Intel (R) Core (TM) i3-4150 with 3.56Hz CPU and 4GB memory. The average run time (s) of the competing methods is shown in Table III. It can be seen that the proposed approach clearly requires less computation time than other methods. Note that the run time of the proposed approach includes the pre-filtering process.

Finally, we also comprehensively evaluate the proposed method on 200 test images from the BSD dataset [37]. Table IV shows qualitative comparisons of the competing denoising methods on four noise levels (σ = 20, 30, 40, 50). It can be seen that the proposed approach achieves very competitive denoising performance compared to WNNM.

V. CONCLUSION

Different from the conventional convex optimization, this paper proposed a non-convex weighted ℓ_p minimization based group sparse representation (GSR) framework for image denoising. To make the proposed scheme tractable and robust, we adopted the generalized soft-thresholding (GST) algorithm to solve the non-convex ℓ_p minimization problem. Moreover, we proposed an adaptive patch search (APS) scheme to boost the accuracy of the nonlocal similar patch selection. Experimental results have verified that the proposed approach outperforms many state-of-the-art denoising methods such as BM3D and WNNM, and results in a competitive speed.
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