Dynamics at a smeared phase transition

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Abstract. We investigate the effects of rare regions on the dynamics of Ising magnets with planar defects, i.e., disorder perfectly correlated in two dimensions. In these systems, the magnetic phase transition is smeared because static long-range order can develop on isolated rare regions. We first study an infinite-range model by numerically solving local dynamic mean-field equations. Then we use extremal statistics and scaling arguments to discuss the dynamics beyond mean-field theory. In the tail region of the smeared transition the dynamics is even slower than in a conventional Griffiths phase: the spin autocorrelation function decays like a stretched exponential at intermediate times before approaching the exponentially small equilibrium value following a power law at late times.

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1. Introduction

In recent years, the influence of quenched disorder on phase transitions and critical points has reattracted considerable attention. A particularly interesting source of new phenomena are the effects of rare spatial regions which can be locally in the wrong phase. These rare regions or "Griffiths islands" fluctuate very slowly because flipping them requires changing the order parameter in a large volume. Griffiths [1] showed that this leads to a non-analytic free energy everywhere in an entire temperature region close to the transition, which is now known as the Griffiths phase [2] or the Griffiths region. In generic classical systems with uncorrelated disorder, the contribution of the Griffiths singularities to thermodynamic (equilibrium) observables is very weak since the singularity in the free energy is only an essential one [1,3]. In contrast, the consequences for the dynamics are much more severe with the rare regions dominating the behavior for long times. In the Griffiths region, the spin autocorrelation function $C(t)$ decays very slowly with time $t$, like $\ln C(t) \sim -(\ln t)^{d/(d-1)}$ for Ising systems [2,4–7], and like $\ln C(t) \sim -t^{1/2}$ for Heisenberg systems [7,8]. Many of these results have been confirmed by rigorous calculations of disordered Ising models [9–11].

The effects of impurities are greatly enhanced by long-range spatial disorder correlations. In an Ising model with linear defects, the thermodynamic Griffiths singularities are given by power-laws with the average susceptibility actually diverging in a finite temperature region. The critical point itself is of exotic infinite-randomness
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type and displays activated rather than power-law scaling. This was first found in
the McCoy-Wu model [12, 13], a disordered 2D Ising model in which the disorder is
perfectly correlated in one dimension and uncorrelated in the other. Later it was
studied in great detail in the context of the quantum phase transition of the random
transverse-field Ising model where the imaginary time dimensions plays the role of the
"correlated" direction [14].

Recently, it has been shown that even stronger effects than these power-law
(quantum) Griffiths singularities can occur in Ising models with planar defects
[15, 16]. Because the disorder is perfectly correlated in two directions, the effective
dimensionality of the rare regions is two. Therefore, an isolated rare region can undergo
the phase transition independently from the bulk system. This leads to a destruction
of the global sharp phase transition by smearing. Similar smearing effects have also
been found in itinerant quantum magnets [17] and at a non-equilibrium transition in
the presence of linear defects [18]. In all of these cases, the effective dimensionality
of a rare region is above the lower critical dimension of the problem which allows the
rare region to order independently.

In this paper, we investigate the dynamics of an Ising model with planar defects
in the vicinity of this smeared phase transition. We show that the dynamics in the
"tail" of the smeared transition is even slower than in a conventional Griffiths phase.
The autocorrelation function decays like a stretched exponential at intermediate times
followed by power-law behavior at late times. The paper is organized as follows. In
section 2 we introduce the model and briefly summarize the results for the
thermodynamics of the smeared transition to the extent necessary for the discussion
of the dynamics. In section 3 we numerically solve dynamical mean-field equations
and compare the results to predictions based on a variational approach. In section 4,
we use general scaling arguments in connection with extremal statistics to go beyond
mean-field theory. Conclusions are presented in section 5.

2. The model and its thermodynamics

For definiteness we consider a d-dimensional Ising model with bond disorder
completely correlated in the $d_C = 2$ directions $x_1$ and $x_2$ but uncorrelated in the
remaining $d_\perp = d - d_C$ directions $x_3, \ldots, x_d$. Classical Ising spins $S_x = \pm 1$ reside on
the sites $x$ of a hypercubic lattice. They interact via nearest-neighbor interactions. In
the clean system all interactions are independent of the lattice site; their values are $J_0$
for bonds in the uncorrelated directions and $J_0/4$ for bonds in the $x_1$ and $x_2$ directions.
We have chosen this scaling of the interactions because it leads to mean-field equations
compatible with the corresponding thermodynamic mean-field theory [15]. The planar
defects are modelled by weakening the bonds in $(x_1, x_2)$-planes at random positions
in the uncorrelated directions. The system effectively consists of blocks separated by
parallel planes of weak bonds. The Hamiltonian of the system is given by:

$$H = -\frac{1}{4} \sum_x J(x_\perp) \left[ S_x S_{x+e_1} + S_x S_{x+e_2} \right] - \sum_{x, j=3}^d J_0 S_x S_{x+e_j}, \quad (1)$$

where $e_j$ is a unit vector in direction $x_j$, $x_\perp$ is the projection of $x$ on the uncorrelated
directions $x_3, \ldots, x_d$, and $J(x_\perp)$ is the random coupling constant in the $(x_1, x_2)$-planes.
The values of $J(x_\perp)$ are drawn from a binary distribution:

$$J(x_\perp) = \begin{cases} \ cJ_0 & \text{with probability } p \\ J_0 & \text{with probability } (1 - p) \end{cases} \quad (2)$$
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characterized by the concentration $p$ and the relative strength $c$ of the weak bonds ($0 \leq p \leq 1, 0 < c \leq 1$).

We now briefly summarize the results on the thermodynamics of this system [15, 16] to the extent necessary for the discussion of the dynamics. The clean system ($p = 0$) undergoes a magnetic phase transition at some temperature $T^0_c$, with the order parameter being the total magnetization

$$m = \frac{1}{V} \sum_x \langle S_x \rangle,$$

(3)

where $V$ is the system volume, and $\langle \cdots \rangle$ is the thermodynamic average. In the disordered system ($p > 0$), a crucial role is played by rare strong disorder fluctuations: There is a small but non-zero probability for finding large spatial regions in $x_\perp$ direction devoid of any impurities. These regions can be locally in the ordered phase even if the bulk system is in the disordered phase. Because of the disorder correlations, these rare regions are infinite in the $x_1$ and $x_2$ directions but finite in the remaining directions. Thus, each rare region is equivalent to a two-dimensional Ising system and can undergo a real phase transition independently of the rest of the system. The resulting effect is much stronger than the conventional Griffiths effects [1,2]: The global phase transition is destroyed by smearing [15]; and the order parameter develops very inhomogeneously in space with different parts of the system (i.e., different $x_\perp$ regions) ordering independently at different temperatures. Correspondingly, the correlation length in $x_\perp$ direction remains finite for all temperatures.

Using extremal statistics, the leading thermodynamic behavior can be easily worked out in the ‘tail’ of the smeared transition, i.e., in the parameter region where a few islands have developed static order but their density is so small that they can be treated as independent. The probability $w$ of finding a large region of linear size $L_R$ (in $x_\perp$-space) devoid of any impurities is, up to pre-exponential factors, given by

$$w \sim (1 - p) L_R^{d_\perp} \exp[L_R^{d_\perp} \ln(1 - p)].$$

(4)

As discussed above, such a rare region develops static long-range (ferromagnetic) order at some temperature $T_{c}(L_R)$ below the clean bulk critical temperature $T^0_c$. The value of $T_{c}(L_R)$ varies with the size of the rare region; the largest islands will develop long-range order closest to the clean critical point. According to finite size scaling we obtain

$$T_{c}(L) - T^0_c = r_c(L_R) = -AL_R^{-\phi},$$

(5)

where $\phi$ is the finite-size scaling shift exponent of the clean system and $A$ is the amplitude for the crossover from $d$ dimensions to a slab geometry infinite in two dimensions but finite in $d_\perp = d - 2$ dimensions. The reduced temperature $r = T - T^0_c$ measures the distance from the clean critical point. If hyperscaling is valid, the finite-size shift exponent fulfills $\phi = 1/\nu$. Combining (4) and (5) we obtain the probability for finding a rare region that becomes critical at $r_c$ as

$$w(r_c) \sim \exp(-B |r_c|^{-d_\perp/\phi}) \quad (\text{for } r_c \to 0-)$$

(6)

where the constant $B$ is given by $B = -\log(1 - p) A^{d_\perp/\phi}$. The total (or average) order parameter $m$ is obtained by integrating over all rare regions which are ordered at $r$, i.e., all rare regions having $r_c > r$. Since the functional dependence on $r$ of the order parameter on a given island is of power-law type it does not enter the leading exponentials but only the pre-exponential factors. To exponential accuracy, we therefore obtain

$$m \sim \exp(-B |r|^{-d_\perp/\phi}) \quad (\text{for } r \to 0-)$$

(7)
Thus, the total magnetization develops an exponential tail towards the disordered phase which reaches all the way to clean critical point. Analogous estimates show that the homogeneous magnetic susceptibility does not diverge anywhere in the tail region of the smeared transition because the exponentially decreasing island density overcomes the power-law divergence of the susceptibility of an individual island. However, there is an essential singularity at the clean critical point produced by the vanishing density of ordered islands.

3. Local dynamic mean-field approach

We now turn to the main topic of this paper, viz., the dynamic behavior in the vicinity of the smeared phase transition. We consider a purely relaxational local dynamics without any conservation laws, i.e., model A in the Hohenberg-Halperin classification [19]. Microscopically, it can be realized, e.g., by the Glauber algorithm [20].

In this section, we study the dynamics of the Ising model (1) by numerically solving local dynamic mean-field equations. For definiteness, we consider $d = 3$ spatial dimensions, i.e., $d_\perp = 1$ and $d_C = 2$. A local dynamic mean-field theory of the Hamiltonian (1) can be derived, e.g., by following Ref. [21]. The plane magnetizations

$$m_{x_3} = \frac{1}{L_C} \sum_{x_{1,2}} S_x$$

where $L_C$ is the system size in correlated direction, then fulfill the local mean-field equations of motion

$$\frac{1}{\alpha} \frac{d}{dt} m_x = -m_x + \tanh[(J_0 m_{x-1} + J(x)m_x + J_0 m_{x+1})/T].$$

Here, $\alpha$ is a microscopic relaxation rate which determines the overall time scale. Alternatively, the same equations can be obtained without further approximation from the infinite-range Hamiltonian used in the thermodynamic mean-field theory [15].

Note that the local mean-field equations contain the full spatial dependence of the magnetization on the uncorrelated dimension. In contrast to a global mean-field theory these local equations are therefore capable of describing rare region physics. The stationary solution of (9) corresponds to the thermodynamic local mean-field theory studied in Ref. [15]. The clean model ($p = 0$) undergoes a magnetic phase transition at $T_c^0 = 3 J_0$, displaying mean-field critical behavior. Below $T_c^0$, the diluted system ($p > 0$) develops an exponential magnetization tail of the form $m \sim \exp(-B(T_c^0 - T)^{-1/2})$ following the prediction (7) with $d_\perp = 1$ and $\phi = 2$.

To study the dynamics, we have numerically integrated the equations (9) for $\alpha = 1, J_0 = 1, c = 0.2$, dilutions of $p = 0.2, 0.3, 0.4, 0.5, 0.6, 0.7$ and 0.8 and system sizes (uncorrelated direction) up to $L_\perp = 10^6$ averaging over up to 25 disorder realizations. Let us first analyze the behavior at the clean critical temperature $T_c^0 = 3.0$ (at the threshold of rare region ordering). The left panel of figure [1] shows the time evolution of the total magnetization $m = (1/L_\perp) \sum_x m_x$ at $T_c^0 = 3$ for several dilutions. The figure shows that the long-time decay of the magnetization is governed by a stretched exponential of the form $\ln m(t) = -Et^{-1/3}$. The decay constant $E$ increases with increasing dilution. The right panel of figure [1] shows $E$ as a function of the dilution $p$. Clearly, the decay constant depends linearly on $[-\ln(1 - p)]^{2/3}$.

This behavior can be understood using variational arguments similar to that of Bray [6,7] for conventional Griffiths effects. At the clean critical point, the correlation
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Figure 1. Left: Time evolution of the magnetization at the clean critical temperature $T_c^0 = 3$ for several impurity concentrations $p$ starting from a fully polarized state at $t_0 = 0$ to $t_{max} = 2000$. Parameters are $L_\perp = 10^5$, $J_0 = 1$, $c = 0.2$. The data represent averages of 25 disorder configurations. The long-time behavior of $m$ follows $\ln m(t) \sim -E t^{-1/3}$. Error bars representing the standard deviation of $m$ are shown for $p = 0.3$. Right: Decay constant $E$ of the stretched exponential decay of the magnetization as a function of the impurity concentration $p$. The solid line is a linear fit to $[-\ln(1-p)]^{2/3}$.

The rare region contribution to the time-dependent magnetization $m(t)$ can be obtained by summing over the exponential time dependencies of the individual islands with $L_R$-dependent correlation time. Using (4) we obtain to leading exponential accuracy

$$m(t) \sim \int dL_R \exp \left[ L_R \ln(1-p) - t/\xi_t(L_R) \right]. \quad (10)$$

This integral can easily be evaluated variationally, i.e., within the saddle-point method. The main contribution to $m$ at time $t$ comes from islands of size $L_R^{SP}(t) \sim [t/(\ln(1-p))]^{1/3}$. The leading long-time decay of the magnetization at the clean critical point is then given by a stretched exponential

$$\ln m(t) \sim -t^{1/3} \left[ -\ln(1-p) \right]^{2/3} \quad (11)$$

in complete agreement with the simulation results presented in figure 1.

Before we continue, let us briefly comment on error bars and finite-size effects. A detailed analysis of finite-size effects in our model was performed for the thermodynamics in Ref. [15]. A finite-size sample contains only a finite number of islands of a certain size $L_R$. As long as the number of relevant islands of size $L_R^{SP}$ is large, finite-size effects on the time evolution, Fig. 1, are small and governed by the central-limit theorem. With increasing time, $L_R^{SP}$ increases; and the number of islands of this size decreases. When it becomes of order one, strong sample-to-sample fluctuations are expected. Finally, when $L_R^{SP}$ becomes larger than the largest island on the sample, the time evolution will cross over from stretched exponential to simple exponential decay. The data in Fig. 1 show that finite-size effects do not play a significant role in the parameter range shown. Sample-to-sample fluctuations (as represented by the standard deviation of the magnetization) remain small down to $m = 10^{-9}$. Also, no deviations from stretched exponential decay are observed even at the smallest magnetizations shown (they do start to occur below $m \approx 10^{-8}$, though).
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Figure 2. Time evolution of the magnetization at several temperatures from \( T = 2.8 \) to 3.25 (top to bottom) starting from a fully polarized state at \( t_0 = 0 \) to \( t_{\text{max}} = 2000 \). Parameters are as in figure 1. The data represent averages of 25 disorder configurations. The impurity concentrations are \( p = 0.2 \) (left panel) and \( p = 0.5 \) (right panel).

We now turn to the dynamic behavior for temperatures close to but not at the clean critical temperature \( T^0_c \). Figure 2 shows the time evolution of the magnetization for several temperatures from \( T = 2.8 \) to 3.25. At early times, all curves follow the stretched exponential found at \( T^0_c \). After reaching a (temperature-dependent) crossover time \( t_x \), the behavior qualitatively changes. For temperatures above the clean critical temperature, the magnetization decays faster than the stretched exponential. In fact, the asymptotic decay is exponential \( \ln m \sim t/\tau(T) \), as can be seen from a log-linear replot of the \( p = 0.2 \) data in the left panel of figure 3. The dependence of relaxation time \( \tau(T) \) on the temperature \( T \) is shown in the right panel of figure 3. Clearly, \( 1/\tau \) varies linearly with \( T - T^0_c \) in good approximation. This behavior can be understood as follows. For \( T > T^0_c \) the correlation time of the largest islands does not diverge, but it is cut off by the distance from the clean critical point, \( \xi_t \sim (T - T^0_c)^{-z\nu} \). For our infinite-range model, \( z\nu = 1 \). The large islands with this correlation time dominate the variational integral for \( m(t) \). This leads to a simple

Figure 3. Left: Replot of the the \( p = 0.2 \) data from figure 2. Right: Inverse decay time \( 1/\tau \) as a function of \( T - T^0_c \) for the curves above \( T^0_c \).
exponential decay with an inverse decay time that depends linearly on $T - T_c^0$.

The more interesting case are temperatures below the clean critical temperature $T_c^0$, i.e., the tail region of the smeared transition. Here, the total magnetization approaches an exponentially small but non-zero value in the long-time limit, in agreement with Ref. [15]. The approach to this stationary value is very slow. Fig. 4 shows $m(t) - m(\infty)$ vs. $t$ for $p = 0.5$ and three different temperatures. The data show that the asymptotic decay is governed by a power law with a non-universal, temperature-dependent exponent. To understand this, we note that below $T_c^0$, the island correlation time has a divergence of the form

\[ \xi_t(T, L_R) \sim |T_c - T - AL_R^{-2}|^{-1}. \]

If this is inserted into the variational integral (10), the main contribution comes from (finite-size) islands with $L_R^{-2} \approx (T_c - T)/A$ because they have diverging correlation time. Carrying out the integral leads to a power-law for $m(t) - m(\infty)$.

The distribution of island sizes leads to a corresponding distribution of local correlation times $\xi_t(x)$. Within our infinite-range model, they can be determined by analyzing the time evolution of the local magnetization via $\ln m_x \sim -t/\xi_t(x)$. Figure 5 shows the resulting probability distribution $P(\xi_t)$ of the local correlation times at the clean critical point $T_c^0$ for $p = 0.3$ as determined from 10 samples of $L_{\perp} = 10^6$ sites. The behavior of the large-$\xi_t$ tail of the probability distribution follows from combining eq. (10) with $\xi_t \sim L^2$ leading to $\ln P \sim \xi_t^{-1/2}$.

4. Dynamics from general scaling arguments

In this section, we use general scaling arguments to determine the dynamical behavior at the smeared transition of our disordered magnet beyond mean-field theory. The interesting physics in the tail of the smeared transition is local because the islands are effectively decoupled from each other, and the spatial correlation length remains finite. An appropriate quantity to study the rare region dynamics is therefore the spin autocorrelation function

\[ C(t) = \frac{1}{V} \sum_x \langle S_x(t) S_x(0) \rangle. \]
To determine the leading behavior of $C(t)$ in the tail of the smeared transition we generalize the variational extremal statistics calculation from section 3 to the non-mean-field case.

According to finite-size scaling [22], the behavior of the correlation time $\xi_t$ of a single rare region of size $L_R$ in the vicinity of the clean bulk critical point can be modelled by (for $T < T_c^0$, i.e., $r < 0$)

$$\xi_t(r, L_R) \sim L_R^{z\nu - \tilde{z}\nu} \left[ r + A L_R^{-1/\nu} \right]^{-\tilde{z}\nu}. \quad (13)$$

Here, $\nu$ and $z$ are the correlation length and dynamical exponents of a $d$-dimensional system, and $\tilde{\nu}$ and $\tilde{z}$ are the corresponding exponents of a $d_C$-dimensional system.

Let us first consider the time evolution of the autocorrelation function $C(t)$ at the clean critical point $T_c^0$. For $r = 0$, the correlation time (13) simplifies to $\xi_t \sim L_R^z$. Similar to the total magnetization in the mean-field treatment above, the rare region contribution to $C(t)$ is obtained by simply summing over the exponential time dependencies of the individual islands with $L_R$-dependent correlation time. Using (4) we obtain to exponential accuracy

$$C(t) \sim \int dL_R \exp \left( L_R^{d_A/2} \ln(1-p) - D t / L_R^z \right)\quad (14)$$

where $D$ is a constant. This integral can easily be evaluated within the saddle-point method. The leading long-time decay of the autocorrelation function $C(t)$ at the clean critical point is given by a stretched exponential,

$$\ln C(t) \sim -\left[ -\ln(1-p) \right]^{z/(d_A + z)} t^{d_A/(d_A + z)} . \quad (15)$$

For $T > T_c^0$, i.e. $r > 0$, the correlation time does not diverge for any $L_R$. Instead, the correlation time of the large islands saturates at $\xi_t(r, L_R) \sim r^{-z\nu}$ for $L_R > (r/A')^{-\nu}$. The autocorrelation function $C(t)$ can again be evaluated as an integral over all island contributions. For intermediate times $t < t_x \sim |r|^{-(d_A + z)\nu}$, the autocorrelation function follows the stretched exponential (15). For times larger than the crossover time $t_x$, we obtain a simple exponential decay $\ln C(t) \sim -t/\tau$ with the decay time $\tau \sim r^{-z\nu}$. Our results for $T \geq T_c^0$ agree with those for conventional Griffiths effects [7]. This is not surprising, because above $T_c^0$, there is no difference.
between the Griffiths and the smearing scenarios: In both cases, all rare regions are locally still in the disordered phase.

This changes below the clean critical point $T^c_0$. For $r < 0$, we repeat the saddle point analysis with the full expression (13) for the correlation length. Again, for intermediate times $t < t_x$, the decay of the average density is given by the stretched exponential (15). For times larger than the crossover time $t_x$ the system realizes that some of the rare regions have developed static order and contribute to a non-zero equilibrium value of the autocorrelation function $C(t)$. The approach of $C(t)$ to this equilibrium value is dominated by finite-size islands with $L_R \sim (-r/A)^{-\nu}$ because they have diverging correlation time. As a result, we obtain a power-law.

$$C(t) - C(\infty) \sim t^{-\psi}. \quad (16)$$

The value of $\psi$ cannot be found by our methods since it depends on the neglected pre-exponential factors.

The behavior of the local autocorrelation function $C(x,t) = \langle S_x(t)S_x(0) \rangle$ is spatially inhomogeneous because the local correlation time depends on the size of the underlying rare region. The probability distribution $P(\xi_t)$ of these of local (island) relaxation times can be obtained from (1) and (13). At the clean critical point $T^c_0$ we find

$$P(\xi_t) \sim \exp(-D [-\ln(1-p)] \xi_t^{d/z}) \quad (17)$$

where $D$ is a constant.

5. Conclusions

To summarize, we have studied the dynamic behavior of Ising magnets with planar defects. In these systems the magnetic phase transition is smeared because rare strongly coupled spatial regions independently undergo the phase transition. In the rare-region dominated tail of the smeared transition, the dynamics is very slow. Using general scaling arguments, we have shown that the spin autocorrelation function decays in a two stages, a stretched exponential decay at intermediate times followed by a power-law approach to the exponentially small stationary (equilibrium) value. We have illustrated these general scaling results by computer simulations of an infinite-range model.

Let us briefly compare our results to the dynamics in a conventional Griffiths phase (as produced by uncorrelated disorder) [2, 4–7]. In both cases, rare regions dominate the long-time dynamics. In a conventional Griffiths phase, the finite-size rare regions remain fluctuating for all temperatures above the dirty critical point, i.e., their island correlation times remain finite. As a result, the autocorrelation function decays like $\ln C(t) \sim -(\ln t)^{d/(d-1)}$ for Ising systems [2, 4–7], and $\ln C(t) \sim -t^1/2$ for Heisenberg systems [7, 8]. In contrast, in the tail of a smeared transition, the effects of the rare regions are even stronger because individual islands can undergo the phase transition independently, connected with a divergent island correlation time. This leads to an even slower power-law decay of the spin autocorrelation function as shown in section 1.

We emphasize that we have considered a purely relaxational (local) dynamics corresponding to model A in the Hohenberg-Halperin classification [19]. Other dynamic algorithms require separate investigations. For instance, model B dynamics globally conserves the order parameter. In this context, an interesting question is:
How does the interplay of the local thermodynamics of the rare regions and the global conservation law modify the dynamic behavior in the tail of the smeared transition.

Finally, we note that while the results here have been derived for an Ising model with planar defects, we expect analogous results for other disorder-smeared phase transitions. Indeed, in the tail of the smeared non-equilibrium phase transition of a contact process with extended impurities, a power-law decay of the density was recently found [18].

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