Holography, inflation, and quantum fluctuations in the early universe

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Abstract

In this paper the relevance of holographic entropy bounds in the context of inflation is investigated. We distinguish between entropy on large and small scales and confront the entropy of quantum fluctuations in an inflating cosmology with the appropriate entropy bounds. In conclusion we do not find any constraints on inflation from holography, but some suggestions for future studies are given.
1 Introduction

One of the most intriguing challenges in modern physics is to find observable consequences of quantum gravity. Recently the attention has focused towards cosmology, where in particular inflation might serve as a useful testing ground for new ideas. One proposal is that the magnifying effect of inflation might allow physics beyond the Planck scale to show up in the CMBR-spectrum. For reviews see [1][2]. Another approach, based on the successful introduction of holography in string theory, aims at finding meaningful constraints on inflation through holography. Some years ago there were several attempts in this direction, but the general conclusion, [3][4][5], was that a correct implementation of holography did not give anything more than what follows from the generalized second law of thermodynamics [6].

More recently there has been several interesting works discussing various aspects of holography and cosmology, [7][8][9][10][11][12][13][14][15][16][17], where many conceptual problems of physics in de Sitter space are addressed. Furthermore, in [16] and [17], it has been argued that there are, in fact, situations where holography actually do imply non trivial constraints on inflation and that the effects might even be detectable. Other groups, like [15], do not find any such constraints on inflationary physics from holography and are therefore more in line with the earlier investigations.

In the present paper we will find evidence in favor of the conservative point of view. We will discuss the various entropy bounds during inflation and explain why they do not seem to yield any new constraints. The outline is as follows. In section two we review the Hubble and D-bounds for the entropy in an expanding universe. In section three we investigate more carefully where the entropy is from the point of view of different observers. In particular we pay attention to the scale on which the entropy can be found. We also discuss the entropy of the quantum fluctuations responsible for the fluctuations in the CMBR. In section four we investigate in more detail the entropy bounds in a slowly rolling universe without finding any constraints from holography. We end, in section five, with some speculations on how to find other ways of applying holography to inflation.

2 The Hubble and D-bound

The entropy bound that will serve as a starting point for our discussion is the Bekenstein bound in asymptotically flat space, [19], which states that

\[ S \leq S_B = 2\pi E R, \]  

where \( E \) is the energy contained in a volume with radius \( R \). There are several arguments in support of the bound when gravity is weak [20][21], and it is widely believed to hold true for all reasonable physical systems. Furthermore, in the case of a black
hole where $R = 2E l_p^2$, we have an entropy given by

$$S_{BH} = \frac{A}{4 l_p^2} = \frac{\pi R^2}{l_p^2},$$

which exactly saturates the Bekenstein bound. We will consequently put $\hbar = c = 1$, but explicitly write the Planck length, $l_p = \sqrt{\frac{G \hbar}{c^3}}$, to keep track of effects due to gravity.

Beginning with [22], there have been many attempts to apply similar entropy bounds to cosmology and in particular to inflation. (See for instance [23].) The idea has been to choose an appropriate volume and argue that the entropy contained within the volume must be limited by the area. An obvious problem in a cosmological setting is, however, that for a constant energy density a bound of this type always will be violated if the radius $R$ of the volume is chosen to be big enough. However, as was explained in [3][4][24], it is not reasonable to discuss radii which are larger than the Hubble radius in the expanding universe. See also [25][26][27][28]. This, then, suggests that the maximum entropy in a volume of radius $R > r$, where $r$ is the Hubble radius, is obtained by filling the volume with as many Hubble volumes as one can fit – all with a maximum entropy of $\frac{\pi r^2}{l_p^2}$. This gives rise to the Hubble bound, which states that

$$S < S_H \sim \frac{R^3}{r^3 l_p^2} = \frac{R^3}{r l_p^2}. \tag{3}$$

As an illustrative example of how the Hubble bound works, we might consider a radiation field with a temperature. We begin by considering a case where the Hubble radius is not changing – not even through slow roll. The maximum entropy of the gas (attained in thermal equilibrium) is given by

$$S_g \sim (ER)^{3/4}.$$  

This is clearly much smaller than the naive Bekenstein bound given by $S = 2\pi ER$. But the important point in a cosmological setting, is that there is nothing that prevents the energy $E$ to exceed the critical value $\frac{R}{2 l_p^2}$, which in flat space would have corresponded to the formation of a black hole. In other words, the entropy can easily exceed the value given by $S_{BH}$, even if it can not exceed the Hubble bound $S_H$. In fact, if the matter is in the form of a gas it can not even saturate the bound. This follows since the entropy can not reach the Hubble bound value $S_H$ until the energy density is a factor $\left(\frac{r}{l_p}\right)^{2/3}$ larger than the energy density $\frac{1}{r l_p^2}$ in the cosmological constant. For saturation of the Hubble bound one would, instead, need a collection of black holes.

The Hubble bound is a bound on the entropy that can be contained in a volume much larger than the Hubble radius. It is, therefore, a bound that is meaningful only from the point of view of a global observer, typically using the standard, time dependent, FRW-coordinates. In order to be able to make actual measurements of
associated quantities one needs, therefore, inflation to eventually stop and a non-accelerating phase to take over. The notion of a cosmological horizon and the corresponding area does not play an important role from this point of view since all entropy is present in matter, possibly in the form of black holes.

If we, on the other hand, want to discuss things from the point of view of what a local observer, that do not have time to wait for inflation to end, can measure, we must be more careful. In this case one has a cosmological horizon with an area that it is natural to give an entropic interpretation \cite{29}. Since the area of the horizon grows when matter is passing out towards the horizon, from the point of view of the local observer, it is natural to expect the horizon to encode information about matter that, in its own reference frame, has passed to the *outside* of the cosmological horizon of the local observer. From the point of view of the observer, the matter will never be seen to leave but rather become more and more redshifted. The outside of the cosmological horizon should, therefore, be compared with the inside of a black hole. It follows that the horizon only indirectly provides bounds on entropy within the horizon as is nicely exemplified through the *D-bound* introduced in \cite{30}. The cosmological horizon area in a de Sitter space with some extra matter is smaller than the horizon area in empty space. If the matter passes out through the horizon, the increase in area can be used to limit the entropy content in matter. This is the content of the D-bound which turns out to coincide with the Bekenstein bound. The D-bound, therefore has not, necessarily, that much to with de Sitter space or cosmology. It is more a way to use de Sitter space to derive a constraint on matter itself.

3 Entropy in an expanding universe

3.1 Where is the entropy?

In this section we will try to understand the nature and relations between the various entropy bounds a little bit better. In particular we must find out on what scales the entropy is stored. If we assume that all entropy is stored on short scales smaller than the horizon scale \( r \), we can consider each of the horizon bubbles separately and use the Bekenstein bound (or D-bound) on each and everyone of these volumes. We conclude from this that the entropy, under the condition that it is present only on small scales, is limited by

\[ S < S_{LB} = 2\pi Er, \]

which we will refer to as the *local Bekenstein bound*. It is interesting to compare this with the entropy of a gas in thermal equilibrium. One then finds \( S_g \lesssim Er \) for high temperatures where \( T \gtrsim 1/r \), and \( S_g \gtrsim Er \) for low temperatures where \( T \lesssim 1/r \). This is quite natural and a consequence of the fact that most of the entropy in the gas is stored in wavelengths of the order of \( 1/T \). This means that the entropy for low
temperatures is stored mostly in modes larger than the Hubble scale and are therefore unaffected by the local Bekenstein bound $S_{LB}$.

One can also imagine a situation starting out at high temperature, with entropy stored on small scales. As the universe expands, and the gas is diluted, the entropy is transferred to larger scales. From the point of view of the local observer the entropy is no longer stored in local matter, instead it has receded towards the horizon. The size of the horizon therefore limits the amount of information on scales larger than the Hubble scale, or, more precisely, the large scale information that once was accessible to the observer on small scales. If the horizon is smaller than its maximal value this is a sign that there is matter on small scales and the difference limits the entropy (or information) stored in the matter. This is the role of the D-bound. We conclude, then, that a system with an entropy in excess of $S_{LB}$ (but necessarily below $S_H$) must include entropy on scales larger than the horizon scale.

3.2 How does the entropy change?

While the entropy bounds above are rather well understood, the way entropy can flow and change involve some rather subtle issues. This is in particular true when the size of the horizon is slowly changing as in a slowly rolling inflationary scenario. As a start let us consider a slow roll universe and compare the horizon size at two different times. We have $r_1 = r(t_1)$ and $r_2 = r(t_2)$ with $r_2 \gg r_1$ for $t_2 \gg t_1$. From the previous section it is quite clear that it is possible to have a much larger entropy within the radius $r_2$ at time $t_1$ than the area of a sphere with radius $r_2$ without violating the Hubble bound. At time $t_2$, however, the entropy within $r_2$, which now has become the horizon, is limited by the horizon area. As a consequence, the entropy that has passed out through the fixed sphere with radius $r_2$ between time $t_1$ and $t_2$ will exceed the limit set by the area of the horizon. It is important to realize that there is no contradiction in this. The crucial issue is how much entropy has passed out through the apparent horizon which grows from $r_1$ to $r_2$ during the time interval, and this will, indeed, be limited by the horizon size at $t_2$. This is a direct consequence of the D-bound, and is explicit in the calculation of [18].

So far we have discussed entropy in the form of a diluting gas. As the universe expands this implies a flow of entropy out through the horizon, but as the gas eventually is completely diluted the flow of entropy taps off. Whether or not the horizon radius is changing, one will never be able to violate the Hubble bound or get an entropy flow through an apparent horizon violating the bound set by the area. But we also need to be able to go back in time where we will note that the entropy flow increases. The crux of the matter is, however, that if we go far enough back in time, the energy density will be comparable with the one of the cosmological constant, and the space time geometry will be dramatically different. As a consequence, we can not argue that the total entropy flow will be larger than is allowed for by the area of the horizon.
A potentially more disturbing situation is obtained if we consider an empty universe (apart from a possibly changing cosmological constant), which can be traced arbitrarily far back in time, with entropy generated through the quantum fluctuations that are of importance for the CMBR. As discussed in several works, there is an entropy production that can be associated with these fluctuations and one can worry that this will imply an entropy flow out through the horizon that eventually will exceed the bound set by the horizon. This is the essence of the argument put forward in [17].

To understand this better one must have a more detailed understanding of the cause of the entropy. Entropy is always due to some kind of coarse graining where information is neglected. In the case of the inflationary quantum fluctuations we typically imagine that the field starts out in a pure state\(^1\), with a subsequent unitary evolution that keeps the state pure for all times. This is true whether we take the point of view of a local observer or use the global FRW-coordinates. To find an entropy we obviously must introduce a notion of coarse graining. Various ways of coarse graining have been proposed, but they all imply an entropy that grows with the squeezing parameter \(r_k\). The squeezing formalism was first applied to cosmology in [31] [32]. For large squeezing the entropy in a mode with comoving momentum \(k\) is typically given by, \[ S_k = 2r_k. \] (4)

The squeezing parameter for a massless scalar in an inflating cosmology obeys

\[ \sinh^2 (r_k) = \frac{1}{4k^2 \eta^2}, \] (5)

which for late times (small negative conformal time \(\eta\)) can be approximated as

\[ r_k \sim \frac{1}{2} \ln \frac{1}{k^2 \eta^2} = \ln \frac{H}{p}. \] (6)

Most of entropy is produced at large scales (when the modes are larger than the horizon), and, as we will show in a moment, well below the Hubble bound.

This is all in terms of the FRW-coordinates, but let us now take the point of view of the local observer. In this case the freedom to coarse grain is more limited. In order to generate entropy we must divide the system into two subsystems and trace out over one of the subsystems in order to generate entropy in the other. As an example consider a system with \(N\) degrees of freedom divided into two subsystems with \(N_1\) and \(N_2\) degrees of freedom, respectively, with \(N = N_1 + N_2\) and \(N_2 > N_1\). If the total system is in a pure state it is easy to show that the entropy in the larger subsystem is limited by the number of degrees of freedom in the smaller one,

\(^1\)In standard treatments this is done in the infinite past at energy scales infinitely higher than the Planck scale. In practice the initial conditions should be set at a finite scale supplied by quantum gravity. This is, however, not important for the present discussion.
i.e. $S_2 < \ln N_1$. Applied to our case, this means that the entropy flow towards the horizon must be balanced by other matter with a corresponding ability to carry entropy within the horizon. Since the amount of such matter is limited by the D-bound, the corresponding entropy flow is also limited. As a consequence, there can not be an accumulated flow of entropy out towards the horizon that is larger than the area of the horizon. This does not mean that inflation can not go on for ever, rather it implies that the local observer will not be able to do an arbitrary amount of coarse graining. The analysis of [18] shows, indeed, how inflationary quantum fluctuations leave the horizon without actually affecting the horizon area. It is just the flow of energy, present in the case of slow roll, that makes the horizon grow. The size of the horizon should be viewed as the upper limit on the amount of entropy or information. Whether there is entropy in the fluctuations is not addressed in [18] – this is a matter of a subjective coarse graining. As an example of how this can happen one can consider the effect discussed in [29], where it was noted that if the fluctuations are actually measured this leads to an increase in energy of the detector and the horizon area shrinks. This implies that entropy is generated from the fluctuation and stored, not at large scales, but at small.

It is instructive to actually calculate the total amount of entropy in the fluctuations. We will focus on the entropy on large scales. The total entropy in a volume with radius $R$ on scales larger than the Hubble scales is of the order

$$S \sim R^3 \int_{1/R}^{H} d^3p \sim R^3 \int_{1/R}^{H} dp f \sim \frac{R^3}{p^3} \ll \frac{R^3}{r_p^2} \sim S_H.$$  

That is, there is only of the order of one degree of freedom per horizon volume and we are therefore far from any holographic bound. On the other hand, if we trace the volume $R^3$ back in time through the expanding universe, it will eventually become small enough to be contained within just one horizon volume. Hence all the entropy in (7) are carried by field modes that were once inside of the horizon of a single local observer. As explained above, this observer will not, however, be able to assign such a huge entropy to the fluctuations that have left, simply because the necessary coarse graining requires many Hubble volumes to be achieved. To summarize: from a local point of view the production of entropy in quantum fluctuations is limited by the ability to coarse grain; from a global point of view entropy is created on scales larger than the Hubble scale.

One can also note that the concept of thermalization will be quite different from the two points of view. The early universe will contain, from the point of view of the FRW-observer, thermal degrees of freedom as well as non thermal. It is the non thermal ones (well correlated and information carrying) that eventually leads to the CMBR-fluctuations. In fact, as explained in [37], one should not view the entropy in (7) as the actual entropy of the fluctuations. It is more appropriate to view (7) as the upper limit on the amount of information that can be present in the fluctuations. The actual entropy that we should assign to the fluctuations are much smaller than

\footnote{A simple proof can be found in [41] in the context of the black hole information paradox.}
the value given in (7) – the difference is the information contained in the acoustic
peaks. From the point of view of the local observer, both thermal and non-thermal
fluctuations start out on small scales but expand and is eventually contained in the
degrees of freedom of the horizon. From the local observer point of view this means
that they all have thermalized.

4 The slow roll

Let us investigate how the above ideas work in more detail in the presence of a slowly
changing Hubble constant in a slow roll universe. The Friedman equations in the
presence of a scalar inflaton is given by

\[ H^2 = \frac{8\pi l_p^2}{3} \left( \frac{1}{2} \dot{\phi}^2 + V(\phi) \right), \quad (8) \]

while the equation of motion for the scalar field is given by

\[ \ddot{\phi} + 3H \dot{\phi} = -V'(\phi). \quad (9) \]

It is useful to define a slowroll parameter according to

\[ \varepsilon = \frac{1}{16\pi l_p^2} \left( \frac{V'(\phi)}{V(\phi)} \right)^2, \quad (10) \]

and assume \( \varepsilon \) to be small. To lowest, zeroth, order in \( \varepsilon \) the Hubble constant will be
fixed at

\[ H^2 \sim \frac{8\pi l_p^2}{3} V(\phi), \quad (11) \]

(that is, the potential energy will dominate over the kinetic energy in contributing to
the vacuum energy) while \( \phi \) is slowly varying according to

\[ 3H \dot{\phi} \sim -V'(\phi). \quad (12) \]

To first order in \( \varepsilon \) there will, however, be a variation in \( H \) determined by

\[ \dot{H} = -\varepsilon H^2, \quad (13) \]

which is easily integrated to give

\[ H(t) = \frac{H_0}{1 + \varepsilon H_0 t}, \quad (14) \]

and

\[ a(t) = (1 + \varepsilon H_0 t)^{1/\varepsilon}. \quad (15) \]
We have chosen the constants of integration such that $H(0) = H_0$ and $a(0) = 1$.

A useful model for a slow roll is to assume a cosmological constant together with another matter component with an equation of state given by $p = w\rho$, with $w$ constant. In this setup the first Friedman equation becomes

$$H^2 = \frac{8\pi l_p^2}{3} (\rho + \rho_\Lambda),$$

where

$$\rho = \frac{\rho_0}{a^q},$$

with $q = 3(1 + w)$. We assume $q > 0$; $q = 0$ would just give an additional contribution to the true cosmological constant. The Friedman equation at $t = 0$ now tells us that

$$H_0^2 = \frac{8\pi l_p^2}{3} (\rho_0 + \rho_\Lambda),$$

while first order in $t$ implies

$$q\Omega = 2\epsilon,$$

where we have defined

$$\rho_0 = \Omega \rho_c = \frac{3H_0^2\Omega}{8\pi l_p^2}.$$

As expected, the energy density in the extra matter is suppressed compared to the cosmological constant. Let us now compare the area of the final horizon given by

$$A_f = 4\pi \left(\frac{8\pi l_p^2}{3} \rho_\Lambda\right)^{-2},$$

and the (smaller) one at $t = 0$ given by

$$A_0 = 4\pi \left(\frac{8\pi l_p^2}{3} (\rho_0 + \rho_\Lambda)\right)^{-2}.$$

A short calculation (assuming $\Omega$ small) reveals that

$$\Delta A = A_f - A_0 = \frac{2\epsilon}{q} A_f.$$  

The matter energy contained within the volume is given by

$$E = \frac{4\pi}{3} \rho_0 H_0^{-3} = \frac{\epsilon r}{q l_p^2},$$

and we therefore conclude that the entropy contained in the matter is limited by

$$S \leq \frac{1}{4} \frac{\Delta A}{L_p^2} = 2\pi EH_0^{-1} = 2\pi Er.$$
This is simply an example of the D-bound at work.

It is instructive to compare with the calculation in [17] where the horizon was obtained from (8) either by keeping the kinetic term (the exact apparent horizon), or dropping the kinetic term (the slow roll approximation). This lead to a difference in area given by

$$\Delta A = A_f - A_0 = \frac{\varepsilon}{3} A_f,$$

and a corresponding entropy bound. It is easy to see that this is a special case of the discussion above. The parameter $w$ in the equation of state can be written

$$w = \frac{p}{\rho} = \frac{1}{2} \frac{\dot{\phi}^2 - V}{\frac{1}{2} \dot{\phi}^2 + V}.$$  \hspace{1cm} (27)

The choice to remove just a kinetic term, as in [17], from the Friedman equation corresponds to having matter contributing to $\rho$ with $w = 1$, which leads to $q = 6$ and therefore agreement between equations (23) and (26). This, then, indicates that an entropy bound of the form (23) (or (26)) should be viewed as a bound on the entropy contained in the matter within the horizon.

5 Conclusions

It seems difficult to find clear cut implications of holography in the context of inflation. As advocated in, e.g., [3][4][5], holography does not seem to say anything more than an appropriate use of the generalized second law of thermodynamics. Furthermore, the latter, in its physical consequences, is always a consequence of using standard laws of general relativity and field theory in a proper way.

To get truly new physics one seems to need more than just a counting of the degrees of freedom. One such possibility would be to study the concept of horizon complementarity in a cosmological setting. In the black hole case, complementarity has been used to resolve the black hole information paradox [42][43][44][45]. According to complementarity the same physics will look very different from the point of view of a local observer and from the point of view of FRW-coordinates. (Actually, one would need to wait until inflation is over to make real measurements relevant to these coordinates.) We have already seen how coarse graining and entropy necessarily will be treated in different ways. While objects passing out through the horizon do not experience anything dramatic from the point of view of FRW-coordinates, the local observer will see how the object experiences effectively higher and higher temperatures as the horizon is approached, and eventually observe how the object is boiled to pieces and thermalized. Could these phenomena, and the way complementarity makes the different pictures compatible, lead to detectable effects? In [46] it is argued that Poincare recurrences play an important role in resolving possible paradoxes in a universe where inflation ends. Unfortunately the analysis also shows that the time
scales of inflation make it unlikely that anything of this will be relevant for observable physics. It would, however, be interesting to push these ideas further.

Another approach to finding effects of quantum gravity through inflation, is to use its ability to magnify small scale physics to cosmological scales. In particular one could hope that subtleties in how the effective initial conditions for the inflaton are chosen by high energy physics could leave a detectable imprint on the CMBR. These ideas have been investigated in several recent works, see [1][2] for reviews. In order to have any chance of finding something detectable, the scale of new physics can not be higher than the GUT-scale at $10^{16}$ GeV, see [47] for a detailed analysis, which happen to coincide with the string scale in many realistic heterotic string theories. One might also speculate that holography would provide a new scale. These kind of ideas were pursued in [48] in an attempt to understand the cosmological constant in the present universe, but it is easy to apply their ideas to an inflationary scenario. An obvious way to find a holographic scale is simply to say that the highest energy scale where field theory make sense, is limited through $r^3 \Lambda^3 \sim \frac{r^2}{l_p}$, which leads to an energy scale a bit below Planck scale, that is, $\Lambda \sim \left( \frac{r}{r_p} \right)^{1/3} \frac{1}{l_p}$. In [48] it was argued that the limit can be improved by observing that $\Lambda$ should be limited by the highest entropy that you can have with out forming a black hole. That is, $r^3 \Lambda^3 \sim \left( \frac{r^2}{l_p} \right)^{3/4}$. This leads to a characteristic scale given by $\Lambda \sim \left( \frac{r}{r_p} \right)^{1/2} \frac{1}{l_p}$. (This is actually the temperature of a gas with an energy density equal to the cosmological constant.) Using the largest possible value of $H$, around $10^{14}$ GeV, one finds $\Lambda \sim 10^{16}$ GeV, which is, indeed, a potentially interesting energy scale.\(^3\) Unfortunately it is difficult to give a firm argument for why this is a relevant limit. It remains, therefore, a challenge to find non-trivial limits on inflationary physics imposed by holography.

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**References**

[1] R. H. Brandenberger, “Trans-Planckian physics and inflationary cosmology,” arXiv:hep-th/0210186

\(^3\)This is the same scale as was argued for in [17] using a different interpretation than in the present paper of the results in section 4.
[2] W. H. Kinney, “Cosmology, inflation, and the physics of nothing,” Lectures given at the NATO Advanced Study Institute on Techniques and Concepts of High Energy Physics, St. Croix, USVI (2002) [arXiv:astro-ph/0301448].

[3] R. Easther and D. A. Lowe, “Holography, cosmology and the second law of thermodynamics,” Phys. Rev. Lett. 82, 4967 (1999) [arXiv:hep-th/9902088].

[4] G. Veneziano, “Pre-bangian origin of our entropy and time arrow,” Phys. Lett. B 454, 22 (1999) [arXiv:hep-th/9902126].

[5] N. Kaloper and A. D. Linde, “Cosmology vs. holography,” Phys. Rev. D 60, 103509 (1999) [arXiv:hep-th/9904120].

[6] J. D. Bekenstein, “Generalized Second Law Of Thermodynamics In Black Hole Physics,” Phys. Rev. D 9, 3292 (1974).

[7] T. Banks, “Cosmological breaking of supersymmetry or little Lambda goes back to the future. II,” [arXiv:hep-th/0007146]

[8] T. Banks and W. Fischler, “M-theory observables for cosmological space-times,” [arXiv:hep-th/0102077]

[9] S. Hellerman, N. Kaloper and L. Susskind, “String theory and quintessence,” JHEP 0106, 003 (2001) [arXiv:hep-th/0104180].

[10] W. Fischler, A. Kashani-Poor, R. McNees and S. Paban, “The acceleration of the universe, a challenge for string theory,” JHEP 0107, 003 (2001) [arXiv:hep-th/0104181].

[11] T. Banks and W. Fischler, “An holographic cosmology,” [arXiv:hep-th/0111142]

[12] E. Witten, “Quantum gravity in de Sitter space,” [arXiv:hep-th/0106109]

[13] L. Dyson, J. Lindesay and L. Susskind, “Is there really a de Sitter/CFT duality,” JHEP 0208, 045 (2002) [arXiv:hep-th/0202163].

[14] L. Dyson, M. Kleban and L. Susskind, “Disturbing implications of a cosmological constant,” JHEP 0210, 011 (2002) [arXiv:hep-th/0208013].

[15] Y. S. Myung, “Holographic entropy bounds in the inflationary universe,” [arXiv:hep-th/0301073].

[16] C. J. Hogan, “Holographic discreteness of inflationary perturbations,” Phys. Rev. D 66, 023521 (2002) [arXiv:astro-ph/0201020].

[17] A. Albrecht, N. Kaloper and Y. S. Song, “Holographic limitations of the effective field theory of inflation,” [arXiv:hep-th/0211221].
[18] A. Frolov and L. Kofman, “Inflation and de Sitter thermodynamics,” arXiv:hep-th/0212327.

[19] J. D. Bekenstein, “A Universal Upper Bound On The Entropy To Energy Ratio For Bounded Systems,” Phys. Rev. D 23, 287 (1981).

[20] J. D. Bekenstein, “Entropy Content And Information Flow In Systems With Limited Energy,” Phys. Rev. D 30, 1669 (1984).

[21] M. Schiffer and J. D. Bekenstein, “Proof Of The Quantum Bound On Specific Entropy For Free Fields,” Phys. Rev. D 39, 1109 (1989).

[22] W. Fischler and L. Susskind, “Holography and cosmology,” arXiv:hep-th/9806039.

[23] S. Kalyana Rama and T. Sarkar, “Holographic principle during inflation and a lower bound on density fluctuations,” Phys. Lett. B 450 (1999) 55 arXiv:hep-th/9812043.

[24] D. Bak and S. J. Rey, “Cosmic holography,” Class. Quant. Grav. 17, L83 (2000) arXiv:hep-th/9902173.

[25] R. Brustein, “The generalized second law of thermodynamics in cosmology,” Phys. Rev. Lett. 84 (2000) 2072 arXiv:gr-qc/9904061.

[26] R. Brustein, S. Foffa and R. Sturani, “Generalized second law in string cosmology,” Phys. Lett. B 471 (2000) 352 arXiv:hep-th/9907032.

[27] R. Brustein and G. Veneziano, “A Causal Entropy Bound,” Phys. Rev. Lett. 84 (2000) 5695 arXiv:hep-th/9912055.

[28] R. Brustein, “Causal boundary entropy from horizon conformal field theory,” Phys. Rev. Lett. 86 (2001) 576 arXiv:hep-th/0005266.

[29] G. W. Gibbons, S. W. Hawking, “Cosmological event horizons, thermodynamics, and particle creation,” Phys. Rev. D 15 (1977) 2738.

[30] R. Bousso, “Bekenstein bounds in de Sitter and flat space,” JHEP 0104 (2001) 035 arXiv:hep-th/0012052.

[31] L. P. Grishchuk and Y. V. Sidorov, “On The Quantum State Of Relic Gravitons,” Class. Quant. Grav. 6 (1989) L161.

[32] L. P. Grishchuk and Y. V. Sidorov, “Squeezed Quantum States Of Relic Gravitons And Primordial Density Fluctuations,” Phys. Rev. D 42 (1990) 3413.

[33] R. H. Brandenberger, V. Mukhanov and T. Prokopec, “Entropy of a classical stochastic field and cosmological perturbations,” Phys. Rev. Lett. 69, 3606 (1992) arXiv:astro-ph/9206005.
[34] T. Prokopec, “Entropy of the squeezed vacuum,” Class. Quant. Grav. 10 (1993) 2295.

[35] M. Kruczenski, L. E. Oxman and M. Zaldarriaga, “Large squeezing behavior of cosmological entropy generation,” Class. Quant. Grav. 11 (1994) 2317 arXiv:gr-qc/9403024.

[36] D. Polarski and A. A. Starobinsky, “Semiclassicality and decoherence of cosmological perturbations,” Class. Quant. Grav. 13 (1996) 377 arXiv:gr-qc/9504030.

[37] C. Kiefer, D. Polarski and A. A. Starobinsky, “Entropy of gravitons produced in the early universe,” Phys. Rev. D 62, 043518 (2000) arXiv:gr-qc/9910065.

[38] M. Gasperini and M. Giovannini, “Entropy production in the cosmological amplification of the vacuum fluctuations,” Phys. Lett. B 301 (1993) 334 arXiv:gr-qc/9301010.

[39] M. Gasperini, M. Giovannini and G. Veneziano, “Squeezed thermal vacuum and the maximum scale for inflation,” Phys. Rev. D 48 (1993) 439 arXiv:gr-qc/9306015.

[40] M. Gasperini and M. Giovannini, “Quantum squeezing and cosmological entropy production,” Class. Quant. Grav. 10 (1993) L133 arXiv:gr-qc/9307024.

[41] U. H. Danielsson and M. Schiffer, “Quantum Mechanics, Common Sense And The Black Hole Information Paradox,” Phys. Rev. D 48 (1993) 4779 arXiv:gr-qc/9305012. Reprinted in Information theory in physics, 2000, AAPT, editor W.T. Grandy.

[42] L. Susskind, L. Thorlacius and J. Uglum, “The Stretched horizon and black hole complementarity,” Phys. Rev. D 48 (1993) 3743 arXiv:hep-th/9306069.

[43] L. Susskind, “String theory and the principles of black hole complementarity,” Phys. Rev. Lett. 71 (1993) 2367 arXiv:hep-th/9307168.

[44] L. Susskind and L. Thorlacius, “Gedanken experiments involving black holes,” Phys. Rev. D 49 (1994) 966 arXiv:hep-th/9308100.

[45] L. Susskind and J. Uglum, “String Physics and Black Holes,” Nucl. Phys. Proc. Suppl. 45BC (1996) 115 arXiv:hep-th/9511227.

[46] U. H. Danielsson, D. Domert and M. Olsson, “Miracles and complementarity in de Sitter space,” arXiv:hep-th/0210198

[47] L. Bergström and U. H. Danielsson, “Can MAP and Planck map Planck physics?,” JHEP 0212 (2002) 038 arXiv:hep-th/0211006.
[48] A. G. Cohen, D. B. Kaplan and A. E. Nelson, “Effective field theory, black holes, and the cosmological constant,” Phys. Rev. Lett. 82 (1999) 4971 arXiv:hep-th/9803132.