An Improvement in Estimation of Population Mean using Two Auxiliary Variables and Two-Phase Sampling Scheme under Non-Response

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Abstract

In the present paper, we have proposed some improved ratio and regression-type estimators of the finite population mean utilizing the information on two auxiliary variables in the presence of non-response. The two-phase sampling scheme has been used to accomplish the job of estimating the desired parameter. The expressions for the basic properties such as bias and mean square error (MSE) of the proposed estimators have been derived up to the first order of approximation. A comparative study of the proposed estimators with some existing estimators has also been carried out through a real data set.

Keywords: Ratio and regression-type estimators, population mean, two-phase sampling scheme, auxiliary information, non-response.

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1 Introduction

There are several practical situations in which the survey (study) variable is highly correlated with more than one auxiliary variable. In such situations, the information on the auxiliary variables can be utilized to provide the improved estimates of the population parameters. Olkin (1958) proposed a multivariate ratio estimator utilizing the information on a number of auxiliary variables. Further, it has been extended by Srivastava (1966). Chand (1975) and Kiregyera (1980) have suggested some ratio-type estimators to resolve the problem of estimating the population mean with the help of two or more auxiliary variables.

The problem of non-response is very common in most of the sample surveys. The non-response arises from the fact that the surveyor fails to obtain the information from the respondents. It is of the great importance to deal with the problem of non-response through some innovative ideas or techniques. Hansen and Hurwitz (1946) resolved the problem of non-response by pioneering out a technique of sub-sampling of non-respondents. Besides it, there are several authors who have contributed a lot to deal with the problem of non-response. The authors such as Khare and Srivastava (1995), Singh and Kumar (2009), Singh et al. (2010), Shabbir and Nasir (2013), Verma et al. (2014) and Chaudhary and Kumar (2016) have tackled the problem of non-response in the case of two-phase sampling scheme.

In the subsequent sections, we have suggested some improved ratio and regression-type estimators of the finite population mean for the survey variable under non-response. We have utilized the information on two auxiliary variables with unknown means under the situation in which the first auxiliary variable is suffering from non-response, whereas the second (additional) auxiliary variable is free from the non-response. The basic properties of the proposed estimators have been discussed in detail. An empirical study has also been carried out to compare the efficiency of the suggested estimators with that of some existing estimators.

2 Sampling Strategy

Let \( U = \{1, 2, \ldots, N\} \) be a population of \( N \) units. Let \( Y \) be the study variable with population mean \( \bar{Y} \). Let \( X \) and \( Z \) be the auxiliary variables with respective population means \( \bar{X} \) and \( \bar{Z} \). Let \( y_i, x_i \) and \( z_i \) be the observations on the \( i \)th unit for the variables \( Y \), \( X \) and \( Z \) respectively. The aim of the present work is to estimate the population mean \( \bar{Y} \) using the information on
two auxiliary variables $X$ and $Z$ with unknown means $\bar{X}$ and $\bar{Z}$ under the assumption that the non-response occurs on study variable $Y$ and auxiliary variable $X$ whereas the auxiliary variable $Z$ is free from the non-response. Adapting the concrete idea of two-phase (or double) sampling scheme, we select a first-phase sample of $n'$ units using simple random sampling without replacement (SRSWOR) scheme and collect the information on the auxiliary variables $X$ and $Z$. Now, a second-phase sample of $n$ units is selected from the first-phase sample of $n'$ units by the method of SRSWOR and the information is collected on $Y$. Out of $n'$ units at the first-phase, it is noted that there are $n'_1$ responding units and $n'_2$ non-responding units for the auxiliary variable $X$. Further, we select a sub-sample of $h'_2 (h'_2 = n'_2 / L'; L' > 1)$ units from $n'_2$ non-responding units and collect the information on all $h'_2$ units. Following Hansen and Hurwitz (1946), the unbiased estimator of $\bar{X}$ at the first-phase is given by

$$\bar{x}' = \frac{n'_1 \bar{x}'_{n_1} + n'_2 \bar{x}'_{h_2}}{n'}$$

The unbiased estimator of $\bar{Z}$ at the first-phase is given as

$$\bar{z}' = \frac{1}{n'} \sum_{i}^{n'} z_i$$

where $\bar{x}'_{n_1}$ and $\bar{x}'_{h_2}$ are the means based on $n'_1$ responding units and $h'_2$ non-responding units respectively for the auxiliary variable $X$.

The expressions for the variance of the estimators $\bar{x}'$ and $\bar{z}'$ are respectively given by

$$V(\bar{x}') = \left( \frac{1}{n'} - \frac{1}{N} \right) S^2_X + \frac{(L' - 1)}{n'} W_2 S^2_{X2}$$

$$V(\bar{z}') = \left( \frac{1}{n'} - \frac{1}{N} \right) S^2_Z$$

where $S^2_X$ and $S^2_Z$ are the population mean squares of the entire group for the auxiliary variables $X$ and $Z$ respectively. $S^2_{X2}$ is the population mean square of the non-response group for the auxiliary variable $X$. $W_2$ is the non-response rate in the population.

At the second-phase, $n_1$ units do respond and $n_2$ units do not respond on the study variable $Y$ and auxiliary variable $X$ such that $n_1 + n_2 = n$. A sub-sample of $h_2 (h_2 = n_2 / L; L > 1)$ units is now selected from the $n_2$
non-responding units and the information is collected from all $h_2$ units on the variables $Y$ and $X$. Thus, the unbiased estimators of $Y$, $X$ and $Z$ at the second-phase are respectively given as [See Hansen and Hurwitz (1946)]

$$
\bar{y}^* = \frac{n_1 \bar{y}_{n1} + n_2 \bar{y}_{h2}}{n} \quad (5)
$$

$$
\bar{x}^* = \frac{n_1 \bar{x}_{n1} + n_2 \bar{x}_{h2}}{n} \quad (6)
$$

$$
\bar{z}^* = \frac{n_1 \bar{z}_{n1} + n_2 \bar{z}_{h2}}{n} \quad (7)
$$

where $\bar{y}_{n1}$, $\bar{x}_{n1}$ and $\bar{z}_{n1}$ are the means based on $n_1$ responding units for the variables $Y$, $X$ and $Z$ respectively. $\bar{y}_{h2}$, $\bar{x}_{h2}$ and $\bar{z}_{h2}$ are respectively the means based on the sub-sample of $h_2$ non-responding units for the variables $Y$, $X$ and $Z$.

**Note:** The functional form of the estimator $\bar{z}^*$ is similar to that of the estimators $\bar{y}^*$ and $\bar{x}^*$. The concept behind it is only to get the similar estimator, however the non-response does not occur on the auxiliary variable $Z$.

The expressions for the variance of the estimators $\bar{y}^*$, $\bar{x}^*$ and $\bar{z}^*$ are respectively given by

$$
V(\bar{y}^*) = \left( \frac{1}{n} - \frac{1}{N} \right) S_Y^2 + \frac{(L - 1)}{n} W_2 S_Y^2 \quad (8)
$$

$$
V(\bar{x}^*) = \left( \frac{1}{n} - \frac{1}{N} \right) S_X^2 + \frac{(L - 1)}{n} W_2 S_X^2 \quad (9)
$$

$$
V(\bar{z}^*) = \left( \frac{1}{n} - \frac{1}{N} \right) S_Z^2 + \frac{(L - 1)}{n} W_2 S_Z^2 \quad (10)
$$

where $S_Y^2$ and $S_{Y,2}^2$ are respectively the population mean squares of the entire group and non-response group for the study variable $Y$. $S_{Z,2}^2$ is the population mean square of the non-response group for the auxiliary variable $Z$.

Chaudhary and Kumar (2016) have suggested some ratio and regression-type estimators of the finite population mean $Y$ utilizing the information on the auxiliary variable $X$ with unknown mean under the situation in which both the variables $Y$ and $X$ are suffering from non-response as follows:

$$
T_1^* = \frac{\bar{y}^*}{\bar{x}^*} \bar{x}^* \quad (11)
$$

$$
T_2^* = \bar{y}^* + b^* (\bar{x}^* - \bar{x}^*) \quad (12)
$$
where \( b^* = s_{xy}^*/s_x^2 \cdot s_{xy}^* \) and \( s_x^2 \) are the unbiased estimators based on \((n_1 + h_2)\) units for \( S_{XY} \) and \( S_X^2 \) respectively. \( S_{XY} = \rho_{XY} S_X S_Y. \) \( \rho_{XY} \) is the population correlation coefficient between \( Y \) and \( X. \)

The expressions for the mean square error (MSE) of the estimators \( T_1^* \) and \( T_2^* \) up to the first order of approximation are respectively given as

\[
MSE(T_1^*) = \left( \frac{1}{n'} - \frac{1}{N} \right) S_Y^2 + \left( \frac{1}{n} - \frac{1}{n'} \right) \left( S_Y^2 + R^2 S_X^2 - 2 \rho_{XY} R S_X S_Y \right) + \frac{(L - 1)}{n} W_2 S_Y^2 + W_2(R^2 S_X^2 - 2 \rho_{XY} R S_X S_Y) \left[ \left( \frac{L - 1}{n} - \frac{(L' - 1)}{n'} \right) \right]
\]

\[
MSE(T_2^*) = \left[ \left( \frac{1}{n'} - \frac{1}{N} \right) + \left( \frac{1}{n} - \frac{1}{n'} \right) (1 - \rho_{XY}^2) \right] S_Y^2 + \frac{(L - 1)}{n} W_2 S_Y^2 + W_2(\beta_1^2 S_X^2 + 2 \beta_1 \rho_{XY} S_X S_Y) \left[ \left( \frac{L - 1}{n} - \frac{(L' - 1)}{n'} \right) \right]
\]

where \( \beta_1 \) is the population regression coefficient of \( Y \) on \( X \) and \( \rho_{XY} \) is the population correlation coefficient between \( Y \) and \( X \) of the non-response group. \( R = \overline{Y}/\overline{X}. \)

### 3 Proposed Estimators

We now propose some improved ratio and regression-type estimators of the finite population mean \( \overline{Y} \) utilizing the information on two auxiliary variables \( X \) and \( Z \) with unknown means under the given situation of non-response as

\[
T_1^{*R} = \frac{\overline{y}^*}{\overline{x}^*} \overline{x}^{*R}
\]

\[
T_2^{*R} = \overline{y}^* + b^*(\overline{x}^{*R} - \overline{x}^*)
\]

where \( \overline{x}^{*R} = \frac{\overline{x}^{*R}}{\overline{x}^*} \cdot \overline{z}^* \), \( \overline{x}^{*R} = \overline{x}^{*R} + b^*(\overline{z}^* - \overline{z}^*) \) and \( b^* = \frac{s_X^*}{s_Z^*}, s_{xz}^* \) and \( s_{z}^2 \) are respectively the unbiased estimators based on \((n_1' + h_2')\) units for \( S_{XZ} \).
and \( S_Z^2, S_{XZ} = \rho_{XZ} S_X S_Z \). \( \rho_{XZ} \) is the population correlation coefficient between \( X \) and \( Z \).

To obtain the biases and mean square errors of the proposed estimators \( T_1^* \) and \( T_2^* \), we use the concept of large sample approximations. Let

\[
\begin{align*}
\bar{y}^* &= \bar{Y}(1 + e_0), \quad \bar{x}^* = \bar{X}(1 + e_1), \quad \bar{z}^* = \bar{Z}(1 + e_2), \quad \bar{x}'^* = \bar{X}(1 + e'_1), \\
\bar{z}' &= \bar{Z}(1 + e'_2), \quad s_{xY}^* = S_{XY}(1 + e_3), \\
\bar{s}_{x'}^2 &= S_X^2(1 + e_4), \quad s_{xZ}^2 = S_{XY}(1 + e_5), \quad s_{z'}^2 = S_Z^2(1 + e_6)
\end{align*}
\]
such that \( E(e_0) = E(e_1) = E(e_2) = E(e_1') = E(e_2') = E(e_3) = E(e_4) = E(e_5) = E(e_6) = 0 \),

\[
\begin{align*}
E(e_0^2) &= \left( \frac{1}{n} - \frac{1}{N} \right) C_Y^2 + \frac{(L - 1)}{n} W_2 \frac{S_Y^2}{Y^2}, E(e_1^2) \\
&= \left( \frac{1}{n} - \frac{1}{N} \right) C_X^2 + \frac{(L - 1)}{n} W_2 \frac{S_X^2}{X^2}, \\
E(e_2^2) &= \left( \frac{1}{n} - \frac{1}{N} \right) C_Z^2 + \frac{(L - 1)}{n} W_2 \frac{S_Z^2}{Z^2}, E(e_1'^2) = E(e_1 e_1') \\
&= \left( \frac{1}{n'} - \frac{1}{N} \right) C_X^2 + \frac{(L' - 1)}{n'} W_2 \frac{S_X^2}{X^2}, \\
E(e_2'^2) &= E(e_2 e_2') = \left( \frac{1}{n'} - \frac{1}{N} \right) C_Z^2, E(e_0 e_1) \\
&= \left( \frac{1}{n} - \frac{1}{N} \right) \rho_{XY} C_X C_Y + \frac{(L - 1)}{n} W_2 \rho_{XY} \frac{S_Y}{Y}, \\
E(e_0 e_2) &= \left( \frac{1}{n} - \frac{1}{N} \right) \rho_{YZ} C_Y C_Z + \frac{(L - 1)}{n} W_2 \rho_{YZ} \frac{S_Z}{Z}, E(e_0 e_1') \\
&= \left( \frac{1}{n'} - \frac{1}{N} \right) \rho_{XY} C_X C_Y + \frac{(L' - 1)}{n'} W_2 \rho_{XY} \frac{S_Y}{Y}, \\
E(e_0 e_2') &= \left( \frac{1}{n'} - \frac{1}{N} \right) \rho_{YZ} C_Y C_Z, E(e_1 e_2) \\
&= \left( \frac{1}{n} - \frac{1}{N} \right) \rho_{XZ} C_X C_Z + \frac{(L - 1)}{n} W_2 \rho_{XZ} \frac{S_Z}{Z},
\end{align*}
\]
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\[ E(e_1e_2') = E(e_1'e_2') = \left( \frac{1}{n'} - \frac{1}{N} \right) \rho_{XZ}C_XC_Z, E(e_1e_3) \]

\[ = \frac{N(N-n)}{(N-1)(N-2)} \frac{\mu_{21}}{nXS_{XY}} + W_2 \left( \frac{L-1}{n} \right) \frac{\mu_{21(2)}}{X^2S_X}, \]

\[ E(e_1e_4) = \frac{N(N-n)}{(N-1)(N-2)} \frac{\mu_{30}}{nXS_X^2} + W_2 \left( \frac{L-1}{n} \right) \frac{\mu_{30(2)}}{X^2S_X}, \]

\[ E(e_2e_1') = \left( \frac{1}{n'} - \frac{1}{N} \right) \rho_{XZ}C_XC_Z + \frac{(L-1)}{n'} W_2 \rho_{XZ} \frac{S_X^2 S_Z^2}{X Z}, \]

\[ E(e_2e_5) = E(e_2e_4) = E(e_2e_3) = E(e_1e_3) \]

\[ = \frac{N(N-n)}{(N-1)(N-2)} \frac{\mu_{21}'}{nZS_{XZ}} + W_2 \left( \frac{L-1}{n} \right) \frac{\mu_{21(2)}}{ZS_{XZ}}, \]

\[ E(e_2e_6) = \frac{N(N-n)}{(N-1)(N-2)} \frac{\mu_{30}'}{nZS_{Z}^2} + W_2 \left( \frac{L-1}{n} \right) \frac{\mu_{30(2)}}{ZS_{Z}^2}, \]

\[ E(e_1'e_3) = \frac{N(N-n')}{(N-1)(N-2)} \frac{\mu_{21}}{nXS_{XY}} + W_2 \left( \frac{L-1}{n'} \right) \frac{\mu_{21(2)}}{X^2S_X}, \]

\[ E(e_1'e_4) = \frac{N(N-n')}{(N-1)(N-2)} \frac{\mu_{30}}{nXS_X^2} + W_2 \left( \frac{L-1}{n'} \right) \frac{\mu_{30(2)}}{X^2S_X}, \]

\[ E(e_2'e_3) = E(e_2'e_4) = E(e_2'e_5) = \frac{N(N-n')}{(N-1)(N-2)} \frac{\mu_{30}'}{n'ZS_{Z}^2}, \]

\[ \mu_{rs} = \frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{X})^r (y_i - \bar{Y})^s, \]

\[ \mu_{rs(2)} = \frac{1}{N_2} \sum_{i=1}^{N_2} (x_i - \bar{X}_2)^r (y_i - \bar{Y}_2)^s, \]

\[ \mu_{rs}' = \frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{X})^r (z_i - \bar{Z})^s, \]
\[ \mu_{rs}^{(2)} = \frac{1}{N_2} \sum_{i=1}^{N_2} (x_i - \bar{X}^{(2)})^r (z_i - \bar{Z}^{(2)})^s, \]

\[ \bar{Y}^{(2)} = \frac{1}{N_2} \sum_{i} y_i, \bar{X}^{(2)} = \frac{1}{N_2} \sum_{i} x_i, \bar{Z}^{(2)} = \frac{1}{N_2} \sum_{i} z_i, \]

\[ C_Y = \frac{S_Y}{\bar{Y}}, C_X = \frac{S_X}{\bar{X}} \quad \text{and} \quad C_Z = \frac{S_Z}{\bar{Z}} \]

where \( \rho_{YZ} \) is the population correlation coefficient between \( Y \) and \( Z \). \( \rho_{YX} \) and \( \rho_{XZ} \) are the population correlation coefficients between \( Y, X \) and \( X, Z \) respectively for the non-response group. \( N_1 \) and \( N_2 \) are respectively the numbers of responding and non-responding units in the population such that \( N = N_1 + N_2 \).

Expressing the equation (15) in terms of \( e_0, e_1, e'_1, e_2, e'_2 \) and then removing the terms having powers of \( e_0, e_1, e'_1, e_2, e'_2 \) greater than two, we get

\[ T_1^{(2)} - \bar{Y} = \bar{Y}(e_0 - e_1 - e_2 + e'_1 + e_2 + e_0^2 + e_1^2 + e_2^2 + e'_2 - e_0 e_1 \]
\[ - e_0 e_2 + e_0 e'_1 + e_0 e'_2 + e_1 e_2 - e'_1 e_2 - e'_1 e'_1 + e'_1 e'_2 - e_1 e'_2 \]
\[ - e'_2 e_2) \]

(17)

Now, taking expectation on both sides of the equation (17), we find the expression for the bias of \( T_1^{(2)} \) up to the first order of approximation as

\[ \text{Bias}(T_1^{(2)}) = \bar{Y} \left[ (f + f') C_X^2 + f C_Z^2 + (f - f') \right] \]
\[ (- \rho_{XY} C_X \bar{C}_Y - \rho_{YZ} C_Y \bar{C}_Z + \rho_{XZ} C_X \bar{C}_Z) \]
\[ - f' \rho_{XZ} C_C Z + \frac{(L - 1)}{n} W_2 S_{Z^2} \]
\[ - \frac{(L - 1)}{n} W_2 \rho_{YZ} \frac{S_{Y^2} S_{Z^2}}{YZ} - \frac{(L' - 1)}{n'} W_2 \rho_{XZ} \frac{S_{X^2} S_{Z^2}}{XZ} \]
\[ + \left( \frac{S_{X^2}}{X^2} - \rho_{XY} \frac{S_{X^2} S_{Y^2}}{XY} + \rho_{XZ} \frac{S_{X^2} S_{Z^2}}{XZ} \right) \]
\[ W_2 \left[ \frac{(L - 1)}{n} - \frac{(L' - 1)}{n'} \right] \]

(18)

where \( f = (\frac{1}{n} - \frac{1}{N}), f' = (\frac{1}{n'} - \frac{1}{N}). \)
Squaring both sides of the equation (17) and then taking expectation on avoiding the terms having powers of \(e_0, e_1, e'_1, e_2, e'_2\) higher than two, we get

\[
E \left[ T_{1*}^2 - \bar{Y} \right]^2 = \bar{Y}^2 \left[ E(e_0^2) + E(e_1^2) + E(e'_1^2) + E(e_2^2) + E(e'_2^2) \right. \\
- 2E(e_0e_1) - 2E(e_0e'_2) + 2E(e_0e'_1) + 2E(e_0e'_2) \\
+ 2E(e_1e_2) - 2E(e'_1e_2) - 2E(e'_1e_1) - 2E(e'_1e'_2) \\
- 2E(e'_2e_2) + 2E(e'_1e'_2) \right]
\]

Thus, the expression for the MSE of \(T_{1*}\) up to the first order of approximation is given as

\[
MSE(T_{1*}) = \bar{Y}^2 \left[ \left( \frac{1}{n} - \frac{1}{N} \right) C_y^2 + \left( \frac{1}{n} - \frac{1}{n'} \right) \right. \\
\left( C_x^2 + C_Z^2 - 2\rho_{XY}C_xC_y - 2\rho_{YZ}C_yC_Z \right. \\
+ 2\rho_{XZ}C_xC_Z \right) + W_2 \left( \frac{L - 1}{n} \right) \\
\left( \frac{S_{Y_2}^2}{Y^2} + \frac{S_{Z_2}^2}{Z^2} - 2\rho_{YZ} \frac{S_{Y_2}S_{Z_2}}{YZ} \right) \right. \\
+ W_2 \left( \frac{S_{X_2}^2}{X^2} - 2\rho_{XY} \frac{S_{X_2}S_{Y_2}}{XY} + 2\rho_{XZ} \frac{S_{X_2}S_{Z_2}}{XZ} \right) \right] \\
\left[ \left( \frac{L - 1}{n} - \frac{(L - 1)}{n'} \right) \right]
\]

(19)

Now, expressing the Equation (16) in terms of \(e_0, e_1, e'_1, e_2, e'_2, e_3, e_4, e_5, e_6\) and then neglecting the terms involving powers of \(e_0, e_1, e'_1, e_2, e'_2, e_3, e_4, e_5, e_6\) greater than two, we get

\[
T_{2*} = \bar{Y} + \beta_1 \bar{X} (e'_1 - e'_1e_4 + e'_1e_3 - e_1e_4 - e_1e_3) \\
+ \beta_1 \beta_2 \bar{Z} (e'_2 - e_2 - e'_2e_6 + e_2e_6 - e_4e'_2 - e_4e_2) \\
+ e_3e'_2 - e_3e_2 + e'_2e_5 - e_2e_5
\]

(20)

where \(\beta_2\) is the population regression coefficient of \(X\) on \(Z\).

The expression for the bias of \(T_{2*}\) up to the first order of approximation can be obtained on taking expectation of both sides of the equation (20). Thus,
we have

\[ \text{Bias}(T_2'^* ) \]

\[ = \beta_1 \left\{ \left( \frac{N(N - n')}{n'(N - 1)(N - 2)} - \frac{N(N - n)}{n(N - 1)(N - 2)} \right) \left( \frac{\mu_{21}}{S_{XY}} - \frac{\mu_{30}}{S_X^2} \right) \right. \]

\[ - W_2 \left( \frac{\mu_{21(2)}}{S_{XY}} - \frac{\mu_{30(2)}}{S_X^2} \right) \left( \frac{(L - 1)}{n} - \frac{(L' - 1)}{n'} \right) \}

\[ + \beta_1 \beta_2 \left\{ \left( \frac{N(N - n')}{n'(N - 1)(N - 2)} \right) \left( \frac{\mu_{21}'}{S_{XZ}} - \frac{\mu_{30}'}{S_Z^2} \right) \right. \]

\[ - \left\{ \left( \frac{N(N - n)}{n(N - 1)(N - 2)} \right) \left( \frac{3\mu_{21}'}{S_{XZ}} - \frac{\mu_{30}'}{S_Z^2} \right) \right. \]

\[ \left. - W_2 \left( \frac{L - 1}{n} \right) \left( \frac{3\mu_{21(2)}}{S_{XZ}} - \frac{\mu_{30(2)}}{S_Z^2} \right) \right\} \]}

(21)

Squaring both sides of the equation (20) and then captivating expectation on neglecting the terms having powers of \( e_0, e_1, e_2, e_2', e_3, e_4, e_5, e_6 \) larger than two, we get

\[ E(T_2'^* - \bar{Y})^2 = \bar{Y}^2 E(e_0^2) + \beta_1^2 \bar{X}^2 \left( E(e_1'^2) + E(e_1^2) - 2E(e_1'e_1') \right) \]

\[ + \beta_1^2 \beta_2^2 \bar{Z}^2 (E(e_2'^2) + E(e_2^2) - 2E(e_2'e_2')) + 2\bar{Y} \bar{X} \beta_1 \]

\[ \times (E(e_0'e_1') - E(e_0'e_1)) + 2\beta_1 \beta_2 \bar{Y} \bar{Z} (E(e_0'e_2') - E(e_0'e_2)) \]

\[ + 2\beta_1^2 \beta_2 \bar{X} \bar{Z} (E(e_1'e_2') - E(e_1'e_2) - E(e_1'e_2) + E(e_1'e_2)) \]

Therefore, the expression for the MSE of \( T_2'^* \) up to the first order of approximation is given by

\[ \text{MSE} (T_2'^* ) = \left( \frac{1}{n} - \frac{1}{N} \right) S_Y^2 + \left( \frac{1}{n} - \frac{1}{n'} \right) \]

\[ \times \left( \beta_1^2 S_X^2 + \beta_1^2 \beta_2^2 S_Z^2 - 2\beta_1 \rho_{XY} S_X S_Y \right) \]

\[ - 2\beta_1 \beta_2 \rho_{XZ} S_Y S_Z - \beta_1^2 \beta_2^2 S_X S_Z \]

\[ + \frac{(L - 1)}{n} W_2 \left( S_{Y^2} + \beta_1^2 \beta_2^2 S_{Z^2} - 2\beta_1 \beta_2 \rho_{YZ} S_Y S_Z \right) \]
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\[ + W_2 \left( \beta_1^2 S_{X2}^2 - 2\beta_1 \rho_{XY2} S_{X2} S_{Y2} + 2\beta_1^2 \beta_2 \rho_{XZ2} S_{X2} S_{Z2} \right) \]

\[ \times \left[ \frac{(L - 1)}{n} - \frac{(L' - 1)}{n'} \right] \]  

(22)

4 Empirical Study

Here, the data considered by Khare and Sinha (2007) have been used to demonstrate the theoretical results. The data relate to the physical growth of upper socioeconomic group of 95 school children of Varanasi district under an ICMR study, Department of Pediatrics, B.H.U., during 1983–84. In this data set, we have:

- \( Y \) = Weight (in kg.) of the children,
- \( X \) = Skull circumference (in cm.) of the children, and
- \( Z \) = Chest circumference (in cm.) of the children.

The first 25% units have been considered as the non-responding units for all the variables \( Y, X \) and \( Z \). The details are given below:

- \( N = 95 \), \( n' = 70 \), \( n = 35 \), \( \bar{Y} = 19.4968 \), \( \bar{X} = 51.1726 \)
- \( Z = 55.8611 \), \( C_Y = 0.15613 \), \( C_X = 0.03006 \),
- \( C_Z = 0.05860 \), \( S_{Y2} = 2.3542 \), \( S_{X2} = 1.2681 \), \( S_{Z2} = 3.0176 \),
- \( \rho_{YX} = 0.328 \), \( \rho_{YX2} = 0.477 \),
- \( \rho_{YZ} = 0.846 \), \( \rho_{YZ2} = 0.729 \), \( \rho_{XZ} = 0.297 \),
- \( \rho_{XZ2} = 0.570 \), \( W_2 = 0.25 \).

Table 1 shows the variance/MSE of the estimators \( \tilde{Y}^*, T_1^*, T_2^*, T_1'^*, T_2'^* \) along with their percentage relative efficiency (PRE). The PRE is computed with respect to \( \tilde{y}^* \).

| Variance/MSE | \( \tilde{y}^* \) | \( T_1^* \) | \( T_2^* \) | \( \tilde{y}^* \) | \( T_1'^* \) | \( T_2'^* \) |
|--------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( L = L' \) | \( \tilde{y}^* \) | \( T_1^* \) | \( T_2^* \) | \( \tilde{y}^* \) | \( T_1'^* \) | \( T_2'^* \) |
| 1.5 | 0.187 | 0.171 | 0.150 | 0.146 | 0.106 | 100.00 | 109.57 | 124.67 | 127.65 | 176.36 |
| 2.0 | 0.207 | 0.188 | 0.165 | 0.165 | 0.116 | 100.00 | 109.78 | 125.27 | 125.51 | 177.66 |
| 2.5 | 0.226 | 0.206 | 0.180 | 0.183 | 0.127 | 100.00 | 109.96 | 125.77 | 123.79 | 178.74 |
| 3.0 | 0.246 | 0.224 | 0.195 | 0.201 | 0.137 | 100.00 | 110.10 | 126.19 | 122.39 | 179.65 |
From the above table, it is revealed that the MSE of the proposed ratio-type estimator $T_1^{*1}$ is much smaller than that of the existing ratio-type estimator $T_1^{*}$ and hence the PRE of $T_1^{*1}$ is higher than that of $T_1^{*}$. The same pattern of results is seen in the case of the proposed regression-type estimator $T_2^{*2}$ and the existing regression-type estimator $T_2^{*}$. It is also revealed that the proposed estimators $T_1^{*1}$ and $T_2^{*2}$ perform very well than the usual mean estimator $\bar{y}^{*}$.

5 Concluding Remarks

We have proposed some improved ratio and regression-type estimators for estimating the finite population mean utilizing the information on two auxiliary variables in the presence of non-response. We have discussed the situation in which the knowledge about the population means of both auxiliary variables is not available and hence we have adopted the concrete idea of two-phase sampling scheme to get the desired information. A theoretical study along with the basic properties of the suggested estimators has been presented. To strengthen the theoretical results, we have carried out an empirical study by considering a natural data set. In Table 1, we have presented a comparative study of the proposed estimators with the usual mean estimator and some other existing estimators. The Table 1 shows that the proposed ratio and regression-type estimators $T_1^{*1}$ and $T_2^{*2}$ provide better estimates as compared to the usual mean estimator $\bar{y}^{*}$ and existing ratio and regression-type estimators $T_1^{*}$ and $T_2^{*}$. Thus, the proposed estimators may be preferred over the usual mean estimator and existing estimators.

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