Testing Nonstandard Neutrino Properties with a Mössbauer Oscillation Experiment

P.A.N. Machado,$^{a,b}$ H. Nunokawa,$^{c}$ F. A. Pereira dos Santos$^{c}$ and R. Zukanovich Funchal$^{c}$

$^a$Instituto de Física, Universidade de São Paulo
São Paulo, C.P. 66.318, 05315-970 São Paulo, Brazil

$^b$Institut de Physique Théorique, CEA-Saclay, 91191 Gif-sur-Yvette, France

$^c$Departamento de Física, Pontifícia Universidade Católica do Rio de Janeiro
C.P. 38071, 22452-970, Rio de Janeiro, Brazil

E-mail: accioly@fma.if.usp.br, nunokawa@puc-rio.br, fabio.alex@fis.puc-rio.br, zukanov@if.usp.br

ABSTRACT: If the neutrino analogue of the Mössbauer effect, namely, recoiless emission and resonant capture of neutrinos is realized, one can study neutrino oscillations with much shorter baselines and smaller source/detector size when compared to conventional experiments. In this work, we discuss the potential of such a Mössbauer neutrino oscillation experiment to probe nonstandard neutrino properties coming from some new physics beyond the standard model. We investigate four scenarios for such new physics that modify the standard oscillation pattern. We consider the existence of a light sterile neutrino that can mix with $\bar{\nu}_e$, the existence of a Kaluza-Klein tower of sterile neutrinos that can mix with the flavor neutrinos in a model with large flat extra dimensions, neutrino oscillations with nonstandard quantum decoherence and mass varying neutrinos, and discuss to which extent one can constrain these scenarios. We also discuss the impact of such new physics on the determination of the standard oscillation parameters.

KEYWORDS: Neutrino Physics, Beyond Standard Model
1 Introduction

Shortly after the discovery of the Mössbauer effect [1], the resonant and recoil-free emission and absorption of photons by atoms bound in a crystal, Visscher [2] suggested that neutrinos could also be emitted and absorbed in a similar fashion. In the early eighties, Kells and Schiffer [3] proposed that a bound-state beta decay [4, 5] could produce a recoil free emission of antineutrinos with ultramonochromatic energy, necessary to accomplish the neutrino Mössbauer effect. Such monochromatic antineutrino could be resonantly absorbed by an induced orbital electron capture [6].

More recently, in 2005, Raghavan [7, 8] rekindle this idea studying the possibility for the recoilless $\bar{\nu}_e$ emission by the bound-state beta decay [4, 5]

$$^3\text{H} \rightarrow ^3\text{He} + e^-(\text{bound}) + \bar{\nu}_e, \quad (1.1)$$
producing a $\bar{\nu}_e$ with energy $E = 18.6$ keV, and the subsequent resonant $\bar{\nu}_e$ capture by the inverse reaction [6],

$$^{3}\text{He} + e^{-} (\text{bound}) + \bar{\nu}_e \rightarrow ^{3}\text{H},$$

(1.2)

where the number of $\bar{\nu}_e$ captured can be inferred either by observing the subsequent decay of $^3\text{H}$ or by directly counting the number of $^3\text{H}$ atoms produced using some chemical technique. After this first study, a considerable amount of related works [9–30] appeared in the last several years.

Due to the resonance nature of the detection process, it was estimated in ref. [7, 8] that the $\bar{\nu}_e$ absorption cross section would be 12 orders of magnitude larger than the standard non-resonant weak interaction cross section for the same energy. This would allow for rather compact detectors, of mass of about a kg, or so, instead of a ton or larger.

It was demonstrated in [14] that a Mössbauer neutrino experiment based on the $^3\text{H}-^3\text{He}$ system, due to the very low energy of $\bar{\nu}_e$ emitted (18.6 keV), can be used to study neutrino oscillations driven by the mass squared difference relevant to atmospheric neutrinos, $\Delta m_{31}^2$, with a baseline of only $\sim 10$ m. This experiment could provide precise measurements of $\theta_{13}$ and $|\Delta m_{31}^2|$.

Moreover, if $\theta_{13}$ is not so small, which the recent T2K result [31] seems to indicate, by extending the baseline to a few hundred meters, where the oscillation effect due to the solar mass squared difference $\Delta m_{21}^2$ becomes relevant, this experiment has the potential to determine also the neutrino mass hierarchy [15], as first considered for reactor neutrinos [32, 33].

Currently, almost all the existing neutrino data are very well described by the standard three flavor massive and mixed neutrinos. However, there are some experimental data which favor more than three neutrino species. Sterile neutrinos, phenomenologically motivated by the results of LSND [34, 35] and supported by MiniBooNe data [36], seem to have gained a new élan. The reactor antineutrino anomaly [37], discovered recently after a new calculation of the reactor antineutrino fluxes [38, 39], as well as the cosmological data [40], also seem to indicate the presence of light sterile neutrino(s).

In this paper, assuming that $\theta_{13}$ is not so small ($\sin^2 2\theta_{13} \gtrsim 0.01$), we consider the possibility to probe nonstandard neutrino properties coming from some new physics beyond the standard model by Mössbauer neutrinos. We will consider four scenarios: the possible presence of a light sterile neutrino, mixing with a Kaluza-Klein tower of sterile neutrinos in a model with large extra dimensions (LED) [41–43], nonstandard quantum decoherence (NQD) [44], and a model with the so-called mass varying neutrinos (MaVN) [45, 46].

Since the standard three neutrino flavor framework provides an excellent fit of almost all the experimental data we assume that new physics produce, at the most, subdominant effects on top of the standard oscillation pattern. Under this assumption, one can try to detect small deviations from the standard oscillation and study how to constrain new physics using Mössbauer neutrinos, in a similar way as done in ref. [47] for a future accelerator neutrino oscillation experiment using conventional neutrino beam from pion decays.
2 Mössbauer $\bar{\nu}_e$: Current Status

In last several years, there have been various works on Mössbauer $\bar{\nu}_e$, both from a theoretical and an experimental point of view [9–30]. Let us make a brief summary of the current status of the prospect of a Mössbauer $\bar{\nu}_e$ oscillation experiment.

2.1 Theoretical considerations on Mössbauer $\bar{\nu}_e$ oscillation

Despite that neutrino oscillations are believed to have been observed and confirmed experimentally, a complete consensus on the formalism of neutrino oscillations seems to be still lacking (see e.g. [48]). Indeed, due to the very special nature of Mössbauer $\bar{\nu}_e$, there has been some controversy in the literature whether or not Mössbauer $\bar{\nu}_e$ would indeed oscillate [19–24].

It was argued in refs. [20, 21, 30] that the very small energy uncertainty on Mössbauer $\bar{\nu}_e$ (due to its ultramonochromatic nature) is in conflict with the energy uncertainty required to observe neutrino oscillations. In this case, Mössbauer neutrinos could be used to test different approaches on the formalism of neutrino oscillations.

In ref. [19] the oscillation probability of Mössbauer $\bar{\nu}_e$ was calculated based solely on quantum field theory without making any a priori assumption about the energy and momentum of the intermediate neutrino state. It was concluded that despite the nearly perfect monochromaticity of the beam, Mössbauer neutrinos do oscillate (see also [26]). The same conclusion was also drawn in ref. [23].

In this work we assume that Mössbauer $\bar{\nu}_e$ do oscillate and that the standard expression for three active neutrino flavors oscillation probability can be used. We modify this expression accordingly to the new physics models we consider.

2.2 Experimental feasibility of a Mössbauer $\bar{\nu}_e$ experiment

The natural line width of a $\bar{\nu}_e$ from a $^3$H (with life time $\tau = 17.8$ yr) decay is $\Gamma = \hbar/\tau \simeq 1.17 \times 10^{-24}$ eV and if there is no recoil, this implies the extremely small energy uncertainty, $\Delta E/E \sim 10^{-31}$, which, however, is impossible to reach experimentally. In order to prevent recoil, Raghavan [7, 8] considered that both $^3$H and $^3$He should be embedded in Nb metal lattices, and estimated that, due to several line broadening effects, the relative energy uncertainty would be $\Delta E/E \sim 5 \times 10^{-16}$, implying a resonant capture cross section of the order of $\sim 10^{-35}$ cm$^2$.

If such a large value of the cross section can be realized, in the absence of oscillation, about one million events per day would be expected for 1 MCi source and 100 g $^3$He target at a baseline of $\sim 10$ m. However, in [13], it was argued that this value could be significantly reduced by some other line broadening effects missed in the estimation done in [7, 8].

More recently, it was claimed in ref. [9–12] that, due to motional averaging by lattice vibrations, the decay of $^3$H in crystals can emit a hypersharper neutrino with $\Delta E/E \sim 5 \times 10^{-29}$, implying a capture cross section of $\sim 10^{-17}$ cm$^2$. This conclusion was criticized by Refs. [27–29] claiming it would be impossible to reach such a value and that it would be impractical to perform the experiment using the $^3$H-$^3$He system. For example, it was
stressed [28, 29] that \(^3\text{H}\) and \(^3\text{He}\) atoms occupy differently the lattice space, implying some energy difference (by lattice expansion or contraction) before and after the emission and absorption of \(\bar{\nu}_e\), which would broaden the natural line width by many orders of magnitude.

In ref. [29] it was proposed that another system, \(^{163}\text{Ho}-^{163}\text{Dy}\), would be more promising than the \(^3\text{H}-^3\text{He}\) one. We note that this new system would imply an even smaller baseline \(\lesssim 2\) m in order to study \(\Delta m_{31}^2\) driven-oscillations, due to a lower \(\bar{\nu}_e\) energy, \(E = 2.6\) keV.

While it is yet far from clear if a Mössbauer \(\bar{\nu}_e\) experiment can be really realized, we assume that it will become possible in the future and for definiteness, throughout this work, we consider the \(^3\text{H}-^3\text{He}\) system as \(\bar{\nu}_e\) emitter/absorber with \(E = 18.6\) keV. We note, however, that our analysis method can be applied to other systems by appropriately re-scaling neutrino energies and baselines.

3 On the Framework and Assumptions on New Physics

In this section we describe the framework as well as the assumptions for the new physics to be probed by a Mössbauer neutrino experiment. Since all of these new physics models are already described in detail in previous works, we will provide only a brief descriptions of each model and refer the readers to the appropriate references in each case.

As mentioned in the introduction, most of the experimental data are well described by the standard three flavor oscillation scheme, allowing us to assume that the effect coming from new physics is small (subdominant). Therefore, throughout this work, even in the presence of new physics, we consider, to a good approximation, the following true (input) values of the standard oscillation parameters determined by the three flavor analysis of experimental data: \(\Delta m_{21}^2 = 7.6 \times 10^{-5}\) eV\(^2\), \(\sin^2 \theta_{12} = 0.31\), \(|\Delta m_{31}^2| = 2.4 \times 10^{-3}\) eV\(^2\) where the mass squared differences are defined as \(\Delta m_{ij}^2 \equiv m_i^2 - m_j^2\) with \(m_i (i = 1, 2, 3)\) being the neutrino mass. For the most recent global analyses of the neutrino oscillation data which have taken into account the new T2K result [31] as well as new calculations of the reactor neutrino fluxes [38, 39], see Refs. [49, 50].

As long as the mixing among the standard active three neutrino flavors is concerned, we consider the parameterizations found in ref. [51]. We note that the values of the CP phase \(\delta\) and of the angle \(\theta_{23}\) are irrelevant for the \(\bar{\nu}_e \rightarrow \bar{\nu}_e\) channel, even in the presence of new physics. We define the lightest neutrino mass \(m_0\) as \(m_0 = m_1 (m_3)\) for normal (inverted) mass hierarchy. As we will see, unlike the standard oscillation case, for LED and MaVaN, the oscillation probabilities depend also on the absolute neutrino mass scale \(m_0\).

3.1 A Light Sterile Neutrino

The original motivation for considering a light sterile neutrino was the result of the LSND experiment [34, 35], now also supported by MiniBooNe [36], where the data can be interpreted as oscillation between active and sterile neutrinos with a mass squared difference of \(\sim 0.1 - 1\) eV\(^2\). Another hint in favor of a light sterile neutrino comes from the GALLEX [52, 53] and SAGE [54] \(^{51}\text{Cr}\) neutrino source experiments. Both measured a
deficit of $\nu_e$ events with respect to the prediction. This can be a signal of oscillation from active to sterile neutrinos \cite{55, 56}.

More recently, the so called reactor antineutrino anomaly \cite{37} supports also the possibility of oscillation to a sterile neutrino driven by a mass squared difference compatible with LSND and MiniBooNe. In addition, though the significance is not yet strong, cosmological data also favors the presence of sub-eV mass sterile neutrinos \cite{40}.

We note, however, that the significance of the LSND excess was diminished from 3.8 to 2.9 $\sigma$ according to the new result on pion production from the HARP-CDP collaboration \cite{57}, and more recent MiniBOONE result, based on the $8.58 \times 10^{20}$ POT, also reduced the significance of the $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ excess to 0.84 $\sigma$ \cite{58}.

Oscillation between active and sterile neutrinos can be tested using the $\bar{\nu}_e$ disappearance mode \cite{59} in reactor neutrino experiments. In ref. \cite{60} the impact of sterile neutrinos on the determination of the standard oscillation parameters $\theta_{13}$ and $\Delta m^2_{31}$ for reactor neutrinos was studied.

Here we consider the so called 3+1 model where one species of light sterile neutrino is added to the standard 3 flavor framework. See ref. \cite{61–67} for a partial list of works that studied this possibility. In this model, the mixing between 4 neutrinos (3 active and 1 sterile) is described by six mixing angles and 3 CP phases. For simplicity, we take only one of the mixings, the one which involves the 4th mass eigenstate (mainly the sterile neutrino state), $\theta_{14}$, different from zero.

For definiteness, we consider the mixing between active and sterile neutrinos as

$$\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau \\
\nu_s
\end{pmatrix} = U \begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3 \\
\nu_4
\end{pmatrix},$$

where

$$U = \begin{bmatrix}
c_{14} & 0 & 0 & s_{14} \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
-s_{14} & 0 & 0 & c_{14}
\end{bmatrix}, \quad \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & c_{23} & s_{23} & 0 \\
0 & -s_{23} & c_{23} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}, \quad \begin{bmatrix}
c_{13} & 0 & s_{13} & 0 \\
0 & 1 & 0 & 0 \\
-s_{13} & 0 & c_{13} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix},$$

$$\begin{bmatrix}
c_{12} & s_{12} & 0 & 0 \\
-s_{12} & c_{12} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}, \quad \begin{pmatrix}
c_{14} \\
0 \\
0 \\
0
\end{pmatrix}, \quad \begin{pmatrix}
0 \\
0 \\
0 \\
0
\end{pmatrix}, \quad \begin{pmatrix}
0 \\
0 \\
0 \\
0
\end{pmatrix},$$

with the notation $c_{ij} \equiv \cos \theta_{ij}$ and $s_{ij} \equiv \sin \theta_{ij}$. Since CP violation is not observable in the $\bar{\nu}_e \rightarrow \bar{\nu}_e$ channel, we ignore all CP phases.

Under this parameterization, vacuum oscillation probabilities can be easily calculated without any approximation. In figure 1 we show the $\bar{\nu}_e$ survival probability as a function of the baseline for the 3+1 model, for $\sin^2 2\theta_{14} = 0.01$ and $|\Delta m^2_{31}| = 1, 0.1, 0.01$ and 0.001 eV$^2$. Assuming that the mixing angle $\theta_{14}$ is small, we see that in this range of distances the presence of a sterile neutrino induces an extra smaller modulation on top of the standard oscillation pattern. We note that for larger value of $\Delta m^2_{31}$, the net effect is expected to be similar to that of LED to be discussed in section 3.2.
Figure 1. $\bar{\nu}_e$ survival probability as a function of the distance from the source for the 3+1 model, the standard 3 active flavors plus one light sterile neutrino, and $E = 18.6$ keV. We set the mixing angle between active and sterile as $\sin^2 2\theta_{14} = 0.01$ ($\theta_{1i} = 0$ for $i \neq 1$) and $|\Delta m^2_{41}| = 1, 0.1, 0.01$ and 0.001 eV$^2$. Here $\sin^2 2\theta_{13} = 0.1$. For the purpose of comparison, the probability for the standard oscillation scenario without a sterile neutrino is also shown by the solid blue curve.

3.2 Large Extra Dimensions

We consider the model of large extra dimensions discussed in [68–74] in connection with neutrino physics, based on the so called flat large extra dimension (LED) scenario [41–43]. In this model, it is assumed that right handed neutrinos (Standard Model singlet fields) can, as well as gravity, propagate in the $d$-dimensional bulk, while Standard Model (SM) particles can only propagate in a brane of 3+1 dimensions. While LED induced neutrino oscillations is not favored by most of the neutrino data [73], in ref. [74] it was demonstrated that gallium [52–56] and reactor antineutrino [37] anomalies can be explained by this scenario.

As in [73], we do not consider explicitly how many extra spatial dimensions do exist, but we assume that the largest one, compactified on a torus of radius $a$, is sufficiently larger than the others so effectively only 4+1 dimensions can be considered. In other words, only the largest LED in practice contribute to modify the oscillation probabilities. Since in any case for us the LED effect is a subdominant one in neutrino oscillations, this assumption looks reasonable.

For this effective model, the 4-dimensional Lagrangian which describes the charged current interaction of the brane neutrinos with the $W$ as well as the mass term resulting from these couplings with the bulk fermions in the brane, after electroweak symmetry
breaking and dimensional reduction, can be written as \[69\],

\[
\mathcal{L}_{\text{LED}}^\text{eff} = \mathcal{L}_{\text{mass}} + \mathcal{L}_{\text{CC}} = \\
\sum_{\alpha,\beta} m_{\alpha\beta}^{\text{D}} \left[ \nu_{\alpha L}^{(0)} \nu_{\beta R}^{(0)} + \sqrt{2} \sum_{N=1}^{\infty} \nu_{\alpha L}^{(N)} \nu_{\beta R}^{(N)} \right] + \sum_{\alpha} \sum_{N=1}^{\infty} \frac{N \nu_{\alpha L}^{(N)} \nu_{\alpha R}^{(N)}}{a} + \frac{g}{\sqrt{2}} \sum_{\alpha} \nu_{\alpha} \gamma^\mu \left( 1 - \gamma_5 \right) \nu_{\alpha}^{(0)} W^\mu + \text{h.c.},
\]

\[(3.3)\]

where the Greek indices \(\alpha, \beta = e, \mu, \tau\), the capital Roman index \(N = 1, 2, 3, \ldots, \infty\), \(m_{\alpha\beta}^{\text{D}}\) is a Dirac mass matrix, \(\nu_{\alpha R}^{(0)}\), \(\nu_{\alpha R}^{(N)}\) and \(\nu_{\alpha L}^{(0)}\) are the linear combinations of the bulk fermion fields that couple to the SM neutrinos \(\nu_{\alpha L}^{(0)}\) which is identified, from now on, as \(\nu_{\alpha} (\alpha = e, \mu, \tau)\) for simplicity.

After performing unitary transformations in order to diagonalize \(m_{\alpha\beta}^{\text{D}}\) we arrive at the neutrino evolution equation (see eq. (A7) of ref. \[73\]) that can be solved to obtain the eigenvalues \(\lambda^{(N)}_j\) and amplitudes \(W_{ij}^{(0)(N)}\) (see Appendix of ref. \[73\]).

Then the \(\bar{\nu}_e\) survival probability at a distance \(L\) from production

\[P(\bar{\nu}_e \to \bar{\nu}_e; L) = |A(\bar{\nu}_e \to \bar{\nu}_e; L)|^2, \]

\[(3.4)\]
can be given in terms of the transition amplitude

\[A(\bar{\nu}_e \to \bar{\nu}_e; L) = \sum_{i,j,k=1}^{3} \sum_{N=0}^{\infty} U_{ei} U_{ek}^{*} W_{ij}^{(0)(N)} W_{kj}^{(0)(N)} \times \exp \left( \frac{i \lambda^{(N)}_j L}{2Ea^2} \right), \]

\[(3.5)\]

where \(E\) is the neutrino energy, \(L\) is the baseline distance, \(a\) is the size of the largest extra dimension, \(U\) and \(W\) are the mixing matrices for active and KK (Kaluza-Klein) neutrino modes, respectively (see \[73\]), \(\lambda^{(N)}_j\) are the eigenvalues of the Hamiltonian matrix described in eq. (A11) in \[73\], and the index \(N\) refers to the KK modes.

In figure 2 we show an example of the \(\bar{\nu}_e\) survival probability with the effect of LED, for \(a = 0.4 \, \mu\text{m}\) and both mass hierarchies. As discussed in \[68–73\], the presence of LED induces conversion from active to sterile KK mode neutrinos with rapid oscillations (or smaller oscillation lengths) and reduce further the \(\bar{\nu}_e\) survival probability when compared to the standard oscillation without LED. In addition to the overall reduction of the probability, LED induces some shift (distortion) of the oscillation minimum though this effect is not so large.

In agreement with the behavior of the \(\bar{\nu}_e\) survival probability for reactor neutrinos shown in figure 1 of ref. \[73\], for a given value of \(a\), the impact of LED is significantly larger for the case of the inverted hierarchy (orange curve) than that of the normal one (purple curve). The reason why the LED effect is larger for the inverted hierarchy is that for the normal one, there is a suppression due to small \(\theta_{13}\) \[73\].

Due to the ultra monochromatic energy, in principle, a Mössbauer neutrino experiment could be highly sensitive to the LED effect. However, due to the uncertainties on the exact production and detection points, \(i.e.\) the finite size of the source and detector, the large
Figure 2. $\bar{\nu}_e$ survival probability as a function of the distance from the source for the LED model with $a = 0.4 \mu m$ and the lightest neutrino mass set to zero for the normal ($m_1 = m_0 = 0$) (solid magenta curve) and inverted ($m_3 = m_0 = 0$) hierarchy (solid orange curve). Here $\sin^2 2 \theta_{13} = 0.1$. We also show the case where the probabilities are averaged over the production and detection (interaction) points using the Gaussian smearing function described in eq. (A.1) of the Appendix with $\sigma_L = 10$ cm, by the dashed red (normal) and green (inverted) curves. For the purpose of comparison, the standard survival probability without LED is shown by the solid blue curve.

LED effect can be significantly washed out. This is exemplified by the dashed curves shown in figure 2.

3.3 Nonstandard Quantum Decoherence

Even in the absence of new physics, loss of coherence can occur in standard neutrino oscillations if neutrinos travel further than the coherence length (see e.g. [75]). This, in fact, can be important for neutrinos from astrophysical sources traveling very long distances. Decoherence can also happen in dense media when collisions become important (see e.g. [76]), in particular, when neutrino-neutrino interactions are significant (see e.g. [77]), or matter density perturbations/fluctuations are relevant [78]. Here we focus on a different kind of decoherence, what we will refer to as nonstandard quantum decoherence (NQD), a decoherence that could be induced by quantum gravity [79].

We assume that the survival probability of $\bar{\nu}_e$ in the presence of the nonstandard decoherence effect in the 1-3 sector, is given by [44, 80],

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \frac{1}{2} \sin^2 2 \theta_{13} \left[1 - e^{-\gamma(E)L} \cos(\Delta_{31})\right] + P_\odot,$$

(3.6)
where $\Delta_{31} \equiv \Delta m^2_{31} L / (2E)$ and $P_{\odot}$ is the part of the probability that depends on the solar oscillation parameters,

$$P_{\odot} \equiv \frac{1}{2} s_{12}^2 \sin^2 2\theta_{13} \sin(\Delta_{31}) \sin(\Delta_{21}) + \left[ c_{13}^4 \sin^2 2\theta_{12} + s_{12}^2 \sin^2 2\theta_{13} \cos(\Delta_{31}) \right] \sin^2 \left( \frac{\Delta_{31}}{2} \right),$$

where $\Delta_{21} \equiv \Delta m^2_{21} L / (2E)$ and $c_{ij} \equiv \cos \theta_{ij}$ and $s_{ij} \equiv \sin \theta_{ij}$. Strictly speaking, there should be some interference term which depends on both, the decoherence parameters and the solar parameters, but since we consider the case where decoherence is a subdominant effect, we assume that such term is negligible in our case.

As in previous works [44, 47, 80], we assume that the parameter $\gamma(E)$ can be phenomenologically parameterized as,

$$\gamma(E) = \gamma_0 \left( \frac{E}{\text{GeV}} \right)^{\beta},$$

where $\gamma_0$ and $\beta$ are constant. In this work, the parameter $\beta$ is restricted to be in the range $-2 \leq \beta \leq 2$.

![Figure 3](image_url)  

**Figure 3.** $\bar{\nu}_e \to \bar{\nu}_e$ survival probability as a function of the distance from the source for the decoherence model, for the cases $(\gamma_0, \beta) = (8 \times 10^{-23} \text{ GeV}, -1), (1.8 \times 10^{-18} \text{ GeV}, 0)$ and $(1.5 \times 10^{-13} \text{ GeV}, +1)$. Here $\sin^2 2\theta_{13} = 0.1$.

To illustrate the effect of decoherence in terms of probability, we show in figure 3 how the survival probability is modified by this effect for $\sin^2 2\theta_{13} = 0.1$ and three cases $(\gamma_0, \beta) = (8 \times 10^{-23}\text{GeV}, -1), (1.8 \times 10^{-18}\text{GeV}, 0)$ and $(1.5 \times 10^{-13}\text{GeV}, +1)$. As expected, the net effect of NQD is to reduce the oscillation amplitudes as compared to the standard oscillation case.
3.4 Mass Varying Neutrinos

Some years ago a connection between neutrino mass and dark energy was proposed in a scenario known as Mass Varying Neutrinos (MaVaN) [45]. The idea was that neutrino mass comes about from the interaction with a scalar field whose effective potential changes with the local neutrino density. So the neutrino mass would be a dynamical variable that depends on the local neutrino density (therefore vary as the Universe evolves). Due to the connection field the dark energy density could keep track of the matter densities (dark matter, baryons and neutrinos) throughout the evolution of the Universe. One could further consider that if the scalar field is in some way coupled to visible matter, the neutrino mass could depend on the local matter density as well [46, 81].

One can find in the literature phenomenological studies of MaVaN models involving solar [82–85] and atmospheric neutrinos [86]. (See ref. [87] where the cosmological impact of MaVaN was studied.) We adopt here essentially the same framework of these references but for the 1-3 sector (mixing between the first and third generation), as discussed in ref. [88] for future reactor neutrino experiments.

If the effect of MaVaN is present and significant, a Mössbauer neutrino oscillation experiment should be able to detect some deviation of the oscillation probability, which will depend on the matter present between the source and the detector, from the standard vacuum oscillation. One of the advantages of a Mössbauer experiment is that it is very easy to switch on and off the matter induced MaVaN effect by placing (and removing) matter between the source and the detector which are separated by only $\sim O(10)$ m.

The Lagrangian we consider has the same form as assumed in ref. [84], and given by

$$L_{\text{MaVaN}}^{\text{eff}} = \sum_i \bar{\nu}_i (i\partial - m_i) \nu_i + \sum_f \bar{f} (i\partial - m_f) f + \frac{1}{2} \phi (\partial^2 - m_S^2) \phi + \sum_{ij} \lambda^{\nu}_{ij} \bar{\nu}_i \nu_j \phi + \sum_{f} \lambda^{f} \bar{f} f \phi,$$

(3.9)

where $m_i$ ($i = 1, 2, 3$) are neutrino masses in the presence of the cosmic neutrino background, which are regarded as vacuum neutrino masses, $m_f$ is the mass of fermion of $f$-species, $m_S$ is the mass of the scalar particle (acceleron) responsible for the accelerated expansion of the universe (which behaves as the dark energy), and $\lambda^{\nu}_{ij}$ and $\lambda^{f}$ are, respectively, the effective neutrino-scalar and matter-scalar couplings, and $f$ refers to fermions $e$, $n$ and $p$.

The effective neutrino evolution equation in the MaVaN scenario considered in this work is given by

$$\begin{align*}
\frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_{\mu} \\ \nu_{\tau} \end{pmatrix} &= \frac{1}{2E} \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + U M^2 U^\dagger \begin{pmatrix} \nu_e \\ \nu_{\mu} \\ \nu_{\tau} \end{pmatrix},
\end{align*}$$

(3.10)

where $A \equiv 2\sqrt{2}G_{F}n_{e}E$, $G_{F}$ and $n_{e}$ are the Fermi constant and the electron number density, respectively, and the effective mass squared matrix is given by

$$M^2 \equiv \begin{pmatrix} m_1^2 & 0 & M_{13}^2(r) \\ 0 & m_2^2 & 0 \\ M_{31}^2(r) & 0 & \{m_3 - M_{33}(r)\}^2 \end{pmatrix}.$$

(3.11)
As in the framework considered in \[84\], \( M_{ij} \) is related to more fundamental parameters of the MaVaN model as

\[
M_{ij}(r) = \frac{\lambda^i}{m_S} \sum_f \lambda^f n_f(r),
\]

(3.12)

where \( n_f(r) \) is the number density of fermion of \( f \)-species. For simplicity, we only consider the case of vanishing lightest neutrino mass \( m_1 = 0 \) (\( m_3 = 0 \)) for normal (inverted) mass hierarchy.

Following ref. \[84\], we introduce the effective MaVaN parameters \( \alpha_{13} \) and \( \alpha_{33} \) for the 1-3 sector as

\[
M_{ij}(r) \equiv \alpha_{ij} \left[ \frac{\rho}{\text{g/cm}^3} \right], \quad (i, j) = (1, 3), (3, 3),
\]

(3.13)

where \( \rho \) is the density of the matter present along the neutrino trajectory. In vacuum our evolution equation coincides with the standard one.

**Figure 4.** \( \nu_e \rightarrow \nu_e \) survival probability as a function of the distance from the source for the MaVaN model and four cases: \((\alpha_{33}, \alpha_{31}) = (\pm 5.7 \times 10^{-4} \text{ eV}, 0), (0, 2.5 \times 10^{-4} \text{ eV}) \) and \((0, 2.5i \times 10^{-4} \text{ eV})\). Here \( \sin^2 2\theta_{13} = 0.1 \). To study the MaVaN effect, we consider the case where we place iron (with density \( \rho = 7.9 \text{ g/cm}^3 \)) between the source and the detector.

In figure 4 we show how the survival probability can be modified by this effect for the four cases of \((\alpha_{33}, \alpha_{31}) = (\pm 5.7 \times 10^{-4} \text{ eV}, 0), (0, 2.5 \times 10^{-4} \text{ eV}) \) and \((0, 2.5i \times 10^{-4} \text{ eV})\). As expected from the effective mass squared matrix in eq. (3.11), we can see in figure 4, the effect of a nonzero \( \alpha_{33} \) is to shift the position of the oscillation minimum, whereas that of \( \alpha_{31} \) is to modify the oscillation amplitude (or effective mixing).
4 Constraining New Physics Models

In this section we present the sensitivity of a Mössbauer experiment to constrain new physics based on the results of our $\chi^2$ analysis, described in the Appendix.

4.1 Light Sterile Neutrino

In figure 5 we show the region of the sterile neutrino mixing parameters that can be excluded if the data are consistent with the standard three flavor active neutrino framework.

![Figure 5](image)

**Figure 5.** Regions of the parameters $\Delta m^2_{41}$ and $\sin^2 \theta_{14}$ which can be excluded if the data are consistent with the standard oscillation with $\sin^2 \theta_{13} = 0.1$ by the red curves (see upper two lines of legend) or 0.01 by the blue curves (lower two lines of legend). The two dotted lines correspond to benchmarks of ref. [89], see text.

We observe that the exclusion curves show somewhat complicated and strange oscillatory behaviours, leading to significant reductions of the sensitivity for particular values of $|\Delta m^2_{41}|$. We note that most of these behaviours are not physical and is caused by the fact that we have a finite number of detector positions. Below let us try to explain qualitatively the cause of such behaviours.

First of all, being a disappearance oscillation experiment, it is clear that the standard oscillation driven by $(\Delta m^2_{31}, \sin^2 \theta_{13})$ without sterile neutrino, can always be mimicked by the same values of $(\Delta m^2_{41}, \sin^2 \theta_{14})$ with vanishing $\theta_{13}$. This explains the “dip” behavior of the exclusion contours around $\Delta m^2_{41} = 2.4 \times 10^{-3} \text{ eV}^2$ in figure 5. This loss of sensitivity
can not be avoided even if we consider larger number of detector positions (unless we use independent information from some other experiment).

Second, for larger values of $|\Delta m_{41}^2|$ when $\bar{\nu}_e$ survival probabilities exhibit many rapid oscillations, due to the finite number detector positions, there exist some special values of $|\Delta m_{41}^2|$ which reproduce quite well the original probabilities at all of the detector positions we considered. This is the cause of the loss of sensitivity at several particular values of $|\Delta m_{41}^2|$ larger than $\sim 5 \times 10^{-3}$ eV$^2$ see in figure 5. We note, however, that in principle, by increasing the number of detector positions, such a loss of sensitivity can be avoided.

We also show in figure 5 two dotted lines that correspond to benchmarks for the sterile neutrino contributions to the active neutrinos mass matrix. According to ref. [89], if the oscillation parameters lie above the lower (upper) dotted lines, the sterile neutrino can influence sub-leading structures in the degenerate (normal hierarchy) neutrino mass spectrum. We observe that the Mössbauer experiment can exclude a large region in the plane $\sin^2 2\theta_{14}$ versus $|\Delta m_{41}^2|$, in particular, it closes the lower mass window for any significant induced effect of the sterile neutrino in the active neutrinos mass matrix for the $\nu_e$-$\nu_s$ channel [89].

4.2 Large Extra Dimensions

In figure 6 we show the sensitivity region in the LED parameter plane $a$-$m_0$. Here $a$ is the size of the largest extra dimension and $m_0$ is the lightest neutrino mass, $m_0$ being $m_1(m_3)$ for the normal (inverted) mass hierarchy. The parameter region on the top-right side of the curves can be excluded by Mössbauer neutrinos if the data are consistent with the standard oscillation (including the case where $\theta_{13} = 0$).

In obtaining these regions, we have also varied freely the LED parameters $a$ and $m_0$ and as described in the Appendix we have taken into account the finite size of the source and detector by using the Gaussian smearing function given in eq. (A.1) with $\sigma_L = 10$ cm. As expected from figure 2, we obtained better sensitivity for the case of the inverted mass hierarchy. The value $\sin^2 2\theta_{13} = 0.1$ was used as input but we verified that in practice the results do not depend on the true value of $\theta_{13}$.

Compared to the current bound coming from CHOOZ, KamLAND and MINOS obtained in [73], the sensitivity we obtained here is somewhat better but very similar to the one which is expected from the Double CHOOZ experiment [73].

We observe that there are mainly two factors that reduce significantly the sensitivity of the Mössbauer experiment to LED. First, as mentioned in the previous section, despite the ultra monochromatic beam energy, due to the finite size of the source and detector, the LED effect which exhibit large oscillatory behavior, is averaged out and the net effect is significantly reduced. Second, the rather large correlated systematic uncertainty of 10% which we assumed following [14] also reduces the sensitivity significantly for this model.

4.3 Nonstandard quantum decoherence

We show in figure 7 the sensitivity regions in the plane of the NQD parameters $\beta - \gamma_0$ for the cases where the true value of the standard parameter $\sin^2 2\theta_{13} = 0.1$ (lower two lines) or 0.01 (upper two lines). The NQD parameters $\beta$ and $\gamma_0$, as well as the standard
\( \theta_{13} \) and \( |\Delta m_{31}^2| \) were varied freely in fitting the data according to what is described in the Appendix. We find that the values of \( \beta \) and \( \gamma_0 \) that lie above the diagonal lines of figure 7 are not compatible with the simulated data and can be excluded by the Mössbauer neutrino experiment, if the data are consistent with the standard oscillation. On the other hand, the Mössbauer experiment has the potential of observing NQD effects in this region. Unlike the case of LED, the sensitivity to NQD essentially does not depend on the mass hierarchy but strongly depends on the true value of \( \theta_{13} \). This can be easily understood from the expression of the probability shown in eq. (3.6).

We note that these results are worse, by \( \sim 2\text{-}3 \) orders of magnitudes, than the current bounds on NQD obtained in ref. [90] which used solar and KamLAND neutrino data (see figure 1 of this reference). However, the bounds obtained in [90] can not be compared directly to the results we obtained in this work because what was constrained by solar and KamLAND data was the decoherence effect relevant for oscillation between the first and second generation whereas we consider here the one between the first and the third generation. This has not yet been constrained for small \( \theta_{13} \) allowed by current data, see [80].
Figure 7. Regions of the nonstandard decoherence parameters $\gamma_0$ and $\beta$ which can be excluded if the data are consistent with the standard oscillation and $\sin^2 2\theta_{13} = 0.1$ (upper two lines) or 0.01 (lower two lines). The region above the diagonal lines can be excluded (or probed) by the Mössbauer neutrino experiment.

4.4 Mass Varying Neutrinos

In figure 8 we show the sensitivity regions of the MaVaN parameters of $\alpha_{13}$ and $\alpha_{33}$. Again we consider two possible input values for $\theta_{13}$, $\sin^2 2\theta_{13} = 0.1$ (upper panel) or $\sin^2 2\theta_{13} = 0.01$ (lower panel). For simplicity, as in ref. [84], we have considered the case where CP violation is absent, so $\sin(\text{arg}[\alpha_{13}]) = 0$ ($\alpha_{13}$ is real or pure imaginary). In order to determine these exclusion (sensitivity) regions, we have combined the results from two cases where the data are taken by inserting the matter between the source and the detectors (we assume the iron with $\rho = 7.9$ g/cm$^3$) and without matter which is considered practically as vacuum, which is crucial to constrain any MaVaN induced effect (comparison of these two cases is very important).

We can exclude the parameter region outside the closed contours of figure 8 for normal (blue lines) and inverted (red lines) mass hierarchy, if the data are consistent with standard oscillation. However if this is not the case, the Mössbauer experiment has the potential to discover MaVaN effects in this region.

For the case where the mass hierarchy is normal, if $\sin^2 2\theta_{13} = 0.1$ (0.01), a Mössbauer neutrino experiment can exclude $|\alpha_{31}| \gtrsim 6 \times 10^{-4}$ ($12 \times 10^{-4}$) eV and $|\alpha_{33}| \gtrsim 10^{-4}$ ($10 \times 10^{-4}$) eV at 3 $\sigma$. For the case where the mass hierarchy is inverted, if $\sin^2 2\theta_{13} = 0.1$...
(0.01), we can exclude the same range of $|\alpha_{31}|$ as in the case of normal hierarchy and $|\alpha_{33}| \gtrsim 10^{-3} \times 3 \times 10^{-3} \text{ eV} at 3 \sigma$.

Following [84], one can try to describe the bounds we obtained in terms of more fundamental MaVaN parameters using eq. (3.12). We conclude that, roughly speaking, for the normal mass hierarchy, if $\sin^2 2\theta_{13} = 0.1 (0.01)$ the ranges of

$$\left| \lambda^\nu \lambda^f \right| \left( \frac{10^{-7} \text{eV}}{m_S} \right)^2 \gtrsim 10^{-27} \left(10^{-26}\right),$$

can be excluded by the Mössbauer experiment. For the inverted mass hierarchy, the bounds would be about one order of magnitude weaker.

Comparing our results with the ones obtained in [84] which used solar neutrino data, the sensitivity we obtained is worse by a factor of $\sim 10$ or more. However, we should note that we are probing a different set of MaVaN parameters, relevant for the 1-3 sector which are not yet constrained by data.

**Figure 8.** Regions of MaVaN parameters $\alpha_{13}$ and $\alpha_{33}$ (outside the closed curves) which can be excluded if the data are consistent with the standard oscillation with $\sin^2 2\theta_{13} = 0.1$ (upper panel) or $\sin^2 2\theta_{13} = 0.01$ (lower panel).
5 Impact of New Physics on the determination of the standard parameters

When one investigates the presence of any nonstandard property of neutrinos, one also should worry about the impact these new effects may have on the determination of the less known standard mixing parameters. In this section we discuss how the new physics studied in this paper can aggravate the determination of $\theta_{13}$ and $|\Delta m^2_{31}|$ in a Mössbauer neutrino oscillation experiment (here we are mainly interested in the impact for $\theta_{13}$ since $|\Delta m^2_{31}|$ is already rather well determined.)

This can be easily achieved by projecting the four dimensional allowed parameter regions determined by our $\chi^2$ analysis described in the Appendix, into the plane of the standard mixing parameters $\sin^2 2\theta_{13}$ and $|\Delta m^2_{31}|$.

![Figure 9](image)

**Figure 9.** Impact of the presence of a light sterile neutrino on the determination of $\theta_{13}$ and $|\Delta m^2_{31}|$. We show the 2 and 3 $\sigma$ CL regions allowed with (color shaded areas) and without (black solid and dashed curves) sterile neutrino parameters included in the fit. Here the input values are $\sin^2 2\theta_{13} = 0.1 (0.01)$ and $|\Delta m^2_{31}| = 2.4 \times 10^{-3}$ eV$^2$. We also show, by the red solid and dashed curves, the case where the information on the determination of $|\Delta m^2_{31}|$ from MINOS is combined.

When the presence of a light sterile neutrino is allowed in the fit we observe a large
impact on the determination of the $|\Delta m^2_{31}|$ and $\theta_{13}$ oscillation parameters as can be seen in figure 9 for the case where the true value of $\sin^2 2\theta_{13} = 0.1$ (upper panel) and 0.01 (lower panel).

In the case where $|\Delta m^2_{41}|$ and $\theta_{14}$ are varied freely, the precise determination of $|\Delta m^2_{31}|$ and $\theta_{13}$ by the Mössbauer experiment alone is severely limited. Similar argument applies to the case of $\theta_{13}$ measurement by reactor $\bar{\nu}_e$ alone. This is because one can not identify if the reduction of the $\bar{\nu}_e$ survival probability is due to nonzero $\theta_{13}$ or nonzero $\theta_{14}$, with $|\Delta m^2_{41}|$ similar to $|\Delta m^2_{31}|$. In particular, for vanishing $\theta_{13}$, arbitrary value of $|\Delta m^2_{31}|$ is allowed (see figure 9) as the input can be easily mimicked by $\sin^2 2\theta_{14} \sim 0.1$ (0.01) and $|\Delta m^2_{41}| \sim 2.4 \times 10^{-3}$ eV$^2$. Even if we add information on the allowed values of $|\Delta m^2_{31}|$ from another experiment, like MINOS, one can not anymore determine the mixing angle, even if it is rather large, by the Mössbauer experiment.

In Figs. 10 and 11, for the case of $\sin^2 2\theta_{13} = 0.1$ and 0.01, respectively, we show how the presence of LED (upper panel), NQD (middle panel) and MaVaN (lower panel) can influence the determination of the standard mixing parameters.

For LED one loses sensitivity in the determination of both $|\Delta m^2_{31}|$ and $\sin^2 2\theta_{13}$ but only towards the larger values of these parameters. This is because in the limit of small LED effects, consistent with current experimental bounds, one can write \[69, 74\] the effect of LED in terms of the effective mass squared difference

$$\Delta m^2_{31}^{(\text{eff})} \approx \Delta m^2_{31} - \frac{\pi^2}{3} a^2 \Delta m^4_{31},$$

and the mixing angle

$$\sin^2 \theta_{13}^{(\text{eff})} \approx \sin^2 \theta_{13} (1 - \frac{\pi^2}{3} m_3 a^2),$$

so that bigger values of $|\Delta m^2_{31}|$ can fit the data as long as they can be compensated by a corresponding increase of size of the largest extra dimension $a$. At the same time when one increases $a$, one decreases $\sin^2 \theta_{13}^{(\text{eff})}$ so one needs to increase $\sin^2 \theta_{13}$ in order to fit the data. Also because of the above behavior the minimum values of $|\Delta m^2_{31}|$ and $\sin^2 \theta_{13}$ allowed by LED coincide with the ones allowed by the standard analysis.

For the case of NQD, the allowed parameter regions distorted only towards larger values of the mixing angle. This is because the net effect of NQD is to reduce the amplitude and therefore, NQD can be compensated (canceled) to some extent by a larger value of the mixing angle. We conclude that, NQD, if present, could induce a significant overestimation of the mixing angle $\theta_{13}$.

For the MaVaN model we considered in this work, in principle, we do not have any problem in determining the standard mixing parameters because one can remove the matter inserted between source and detector for this determination. The case where only the atmosphere is present can be regarded as vacuum as the density of the atmosphere is too small to induce any MaVaN effect. However, for the sake of discussion we present the results for the case where the experiment is performed with and without matter (iron) and combined.

From the bottom panels of figure 10 and 11, we can see that the allowed parameter region for $|\Delta m^2_{31}|$ and $\theta_{13}$ are slightly increased, which was expected, since the parameters
\[ \sin^2 \theta_{13} = 0.1 \text{ (input)} \]

| \( |\Delta m_{31}^2| \times 10^{-3} \text{ eV}^2 \) |
|------------------|------------------|------------------|
| \( \sigma \) (standard osc.) | \( \text{Input} \) | \( \sigma \) (w. new physics) |

Figure 10. Impact of LED (upper panel), NQD (middle panel) and MaVaN (bottom panel) on the determination of the \( \sin^2 2\theta_{13} \) and \( |\Delta m_{31}^2| \). We show the 2 and 3 \( \sigma \) CL allowed parameter regions determined when the true (input) value is \( \sin^2 2\theta_{13} = 0.1 \) (color shaded areas). For reference we also show the 2 and 3 \( \sigma \) CL allowed regions without considering the possibility of new physics in the fit (solid and dashed lines).

\( \alpha_{33} \) and \( \alpha_{31} \) can mimic the mass squared difference and the mixing angle, respectively, as we can see from eq. (3.11). We conclude that even in this case the impact of MaVaN in the determination of \( |\Delta m_{31}^2| \) and \( \theta_{13} \) is small.

6 Discussion and Conclusions

In this work, we discussed the potential of a short baseline (\( \sim O(10 \text{ m}) \)) Mössbauer neutrino oscillation experiment based on the \( ^3\text{H}-^3\text{He} \) system to probe new physics beyond the standard model. We investigate four different scenarios: the presence of a light sterile neutrino that can mix with \( \bar{\nu}_e \), a model where a tower of sterile neutrinos can change the \( \bar{\nu}_e \) osci-
Figure 11. Same as in figure 10 but when the true value is $\sin^2 \theta_{13} = 0.01$.

lation pattern due to large flat extra dimensions, neutrino oscillations with nonstandard quantum decoherence and mass varying neutrinos.

When a single light sterile neutrino is added to the standard three active flavor neutrinos, we conclude that a Mössbauer oscillation experiments can probe (exclude) parameter regions still not yet excluded by other experiments. In particular, it can close the lower mass window for any significant induced effect of the sterile in the active neutrino mass matrix for the $\nu_e - \nu_s$ channel [89].

For the LED model, Mössbauer neutrinos can exclude the size of the largest extra dimension $a \gtrsim 1$ (0.45) $\mu$m at 3 $\sigma$ CL for the normal (inverted) mass hierarchy for a vanishing lightest neutrino mass ($m_0 = 0$). If $m_0$ is larger, say 0.2 eV, for example, this experiment can exclude $a \gtrsim 0.15 \mu$m at 3 $\sigma$ CL. The sensitivity we obtained is somewhat better than the current bounds but not better than what can be achieved in the near future by reactor experiments such as Double CHOOZ. We note, however, the sensitivity of the
Mössbauer experiment can be improved if one can reduce the uncertainties on the neutrino production and detection positions as well as the correlated systematic uncertainty on the initial neutrino flux and/or capture cross section.

For NQD, due to the low energy, Mössbauer neutrinos are most sensitive to the case of \( \beta = -2 \), where \( \gamma_0 \gtrsim 10^{-27} \) (10^{-26}) GeV can be excluded if \( \sin^2 2\theta_{13} = 0.1 \) (0.01). If \( \beta \) is increased by a unity, the sensitivity would be reduced by about a factor of five orders of magnitude. The sensitivity we obtained for the NQD parameter are worse than the existing ones derived from solar and KamLAND neutrino data [90]. However, the existing bounds are for the mixing between first and second generation whereas Mössbauer neutrinos can put limit on the decoherence parameter for first and third generation, where no bounds currently exist (for small \( \theta_{13} \)).

Regarding the MaVaN model, for \( \sin^2 2\theta_{13} = 0.01 - 0.1 \), Mössbauer neutrinos can exclude the range of \( |\alpha_{31}| \) and \( |\alpha_{33}| \) parameters larger than \( \sim 10^{-4} - 10^{-3} \) eV, depending on the precise value of \( \theta_{13} \) and of the mass hierarchy. This is worse than the bounds obtained in [84] which used solar neutrino and KamLAND data but as in the case of the decoherence effect, the existing bounds apply only to the 1-2 sector whereas Mössbauer neutrinos can probe the 1-3 sector which is not bounded yet.

We still do not know if a Mössbauer neutrino oscillation experiment can be really feasible due to several technical (experimental) difficulties. Nevertheless, we hope that new technologies will come about permitting it to happen in the future. It would be fantastic to be able to build such a compact experiment capable to explore not only standard but also nonstandard neutrino oscillation physics.

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A Analysis Method

Here we give a very brief description of our analysis method which is basically the same as used in [14, 15] apart from the fact that we have more free parameters due to new physics.

We adopt the experimental setup referred to as Run IIB in [14] where the detectors occupy 10 different positions corresponding to the following baselines: \( L_i = L_{OM}/5, L_{i+1} = L_i + (2/5)L_{OM}, \) \( i = 1, \ldots, 9, \) where \( L_{OM} = 4\pi E/|\Delta m_{31}^2| \approx 9 \) m, is the distance which corresponds to the first oscillation minimum. We assume that each detector is exposed to \( 10^6 \bar{\nu}_e \) events.

We simulate the input data assuming only standard oscillation physics. In doing this, throughout this work, we assume that the true values (input values for the simulation) of
the standard mixing parameters are $\Delta m^{2}_{21} = 7.6 \times 10^{-5}$ eV$^2$, $\sin^2 \theta_{12} = 0.31$, $|\Delta m^{2}_{31}| = 2.4 \times 10^{-3}$ eV$^2$, and $\sin^2 2\theta_{13} = 0.1$ or 0.01. In our $\chi^2$ analysis (described below), when fitting the simulated data, we vary freely, in addition to the new physics parameters of each model, the standard mixing parameters $\theta_{13}$ and $|\Delta m^{2}_{31}|$. We do not vary the other oscillation parameters since the impact of their uncertainties is quite small. In order to illustrate the standalone potential of this experiment, we do not use any biases information on the values of $|\Delta m^{2}_{31}|$ and $\theta_{13}$ from existing or future experiments, except for results shown in figure 9 where the results from MINOS experiment is combined only for the purpose of illustration of loss of the sensitivity.

When we calculate the survival probability for the LED model we observed many rapid oscillations due to the conversion of $\bar{\nu}_e$ into KK modes, as can be seen in figure 2 in section 3.2. Due to the finite size of the source and detector (uncertainties on the exact production and detection positions), such rapid oscillations must be averaged out over the size of the source and detector, which tends to “wash out” the LED effect. In order to take this finite size effect into account, we average the probability over the baseline, allowing for the uncertainty on the production/detection points through a Gaussian smearing function defined as

$$f(L, L') = \frac{1}{\sqrt{2\pi} \sigma_L} \exp \left[ -\frac{(L - L')^2}{2\sigma_L^2} \right],$$

(A.1)

where we set $\sigma_L = 10 \text{ cm}$.

To evaluate the sensitivity to constrain new physics described by the parameters, say, $\alpha$ e $\beta$, we compute

$$\Delta \chi^2_{\text{min}}(\alpha, \beta) = \chi^2_{\text{min}}(\alpha, \beta) - \chi^2_{\text{min}}(\alpha = \beta = 0),$$

(A.2)

where $\chi^2_{\text{min}}(\alpha, \beta)$ is the minimum of $\chi^2(\alpha, \beta, \Delta m^{2}_{31}, \sin^2 2\theta_{13})$ given by

$$\chi^2(\alpha, \beta, \Delta m^{2}_{31}, \sin^2 2\theta_{13}) = \sum_{i,j=1}^{10} \left( \frac{N^{\text{obs}}_i - N^{\text{theo}}_i}{N^{\text{theo}}_i} \right) (V^{-1})_{ij} \left[ \frac{N^{\text{obs}}_j - N^{\text{theo}}_j}{N^{\text{theo}}_j} \right],$$

(A.3)

where $N^{\text{obs}}_i$ is the number of observed (simulated) events at baseline $L_i$ for a given fixed values of the standard oscillation parameters (no new physics), while $N^{\text{theo}}_i = N^{\text{theo}}_i(\alpha, \beta, \theta_{13}, \Delta m^{2}_{31})$ is the theoretically expected number of events at baseline $L_i$ for a given set of oscillation and new physics parameters. We use, as in ref. [14], the correlation matrix defined by the elements

$$(V^{-1})_{ij} = \frac{\delta_{ij}}{\sigma_{ui}^2} - \frac{1}{\sigma_{ui}^2 \sigma_{uj}^2} \left[ 1 + \left( \sum_{k} \frac{1}{\sigma_{uk}^2} \right) \sigma_c^2 \right].$$

(A.4)

where $\sigma_{ui}^2 = \sigma_{uys}^2 + 1/N^{\text{theo}}_i$. For the uncorrelated systematic uncertainty, we take the optimistic choice considered in[14] $\sigma_{uys} = 0.2 \%$. For the correlated systematic uncertainty we set, as in[14], $\sigma_c = 10 \%$.

We determine the new physics parameter regions that can be excluded by the Mössbauer experiment by imposing the condition $\Delta \chi^2_{\text{min}} > 6.18$ and 11.83, respectively, for 2 and 3 $\sigma$ significance level and 2 degrees of freedom.
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