Morphological characterization of shocked porous material

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Abstract
Morphological measures are introduced to probe the complex procedure of shock wave reaction on porous material. They characterize the geometry and the topology of the pixelized map of a state variable like temperature. The relevance to thermodynamical properties of a material is revealed and various experimental conditions are simulated. Numerical results indicate that the shock wave reaction results in a complicated sequence of compressions and rarefactions in porous material. The increasing rate of the total fractional white area \( A \) roughly gives the velocity \( D \) of a compressive-wave series. When a velocity \( D \) is mentioned, the corresponding threshold contour level of the state variable, such as temperature, should also be stated. When the threshold contour level increases, \( D \) becomes smaller.

The area \( A \) increases parabolically with time \( t \) during the initial period. The \( A(t) \) curve goes back to being linear in the following three cases: (i) when the porosity \( \delta \) approaches 1, (ii) when the initial shock becomes stronger and (iii) when the contour level approaches the minimum value of the state variable. The area with high temperature may continue to increase even after the early compressive waves have arrived at the downstream free surface and some rarefactive waves have come back into the target body. In the case of energetic material needing a higher temperature for initiation, a higher porosity is preferred and the material may be initiated after the precursory compressive waves have scanned the entire target body. In some cases we need scattered hot spots, but in others we need connected ones. One may desire the fabrication of a porous body and choose the appropriate shock strength according to what is needed. With the Minkowski measures, the dependence on experimental conditions is reflected simply by a few coefficients. They may be used as order parameters to classify the maps of physical variables in a similar way to thermodynamic phase transitions.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

A porous material contains voids or tunnels of different shapes and sizes. Such materials are commonly found in nature and as industrial materials such as wood, carbon, foams, ceramics, bricks, metals and explosives. They have also been used in surgical implant design to fabricate devices to replace or augment soft and hard tissues, etc. In order to use them effectively, their mechanical and thermodynamical properties must be understood in relation to their mesoscopic structures [1, 2].

In this work we focus on porous materials under shock wave reaction. When a porous material is shocked, the cavities inside the sample may result in jets and influence its back velocity [3]. Cavity nucleation due to tension waves controls the spallation behaviour of the material [4]. Cavity collapse plays a prominent role in the initiation of energetic reactions in explosives [5]. In this regard, most of the previous studies concerned the Hugoniots [6–13] and the equation of state [14–16]. It is known that, under strong shocks, the porous material is globally in a nonequilibrium state and shows complex dissipative structures. How to describe and pick up information from such a system is still an open problem. In this work we introduce the Minkowski functionals to measure the morphological behaviour of the map of a state variable and use them to probe the procedure of shock wave reaction on the porous material.

This study also needs a powerful simulation tool. The molecular dynamics can discover some atomistic mechanisms of shock-induced void collapse [17, 18], but the spatial and...
temporal scales it may cover are far from those comparable to experiments. To overcome this scale limitation, we resort to a newly developed mesoscopic particle method, the material-point method (MPM) [19–24]. The MPM was originally introduced in fluid dynamics by Harlow et al [19], extended to solid mechanics by Burgess et al [20] and then developed by various researchers, including us [25–27]. The other reason for using the MPM is related to the severe difficulties of the traditional Eulerian and Lagrangian methods in treating shocked porous materials. The material under investigation is generally highly distorted during the collapsing of cavities. The Eulerian description is not convenient for tracking interfaces. When the Lagrangian formulation is used, the original element mesh becomes distorted so significantly that the mesh has to be re-zoned to restore the proper shapes of elements. The state fields of mass density, velocities and stresses must be mapped from the distorted mesh to the newly generated one. This mapping procedure is not a straightforward task, and introduces errors. The MPM not only takes advantage of both the Lagrangian and the Eulerian algorithms but makes it possible to avoid their drawbacks as well. At each time step, calculations consist of two parts: a Lagrangian part and a convective one. Firstly, the computational mesh deforms with the body and is used to determine the strain increment and the stresses in the sequel. Then, the new position of the computational mesh is chosen (particularly, it may be the previous one), and the velocity field is mapped from the particles to the mesh nodes. Nodal velocities are determined using the equivalence of momentum calculated for the particles and for the computational grid.

The following part of the paper is planned as follows. Section 2 briefly reviews the Minkowski descriptions. Section 3 presents the theoretical model of the material under consideration. Simulation results are shown and analysed in section 4. Section 5 presents the conclusions.

2. Brief review of morphological characterization

A variety of techniques can be used to describe the complex spatial distribution and the time evolution of state variables in the shocked porous material. In this study we concentrate on the set of statistics known as Minkowski functionals [28]. A general theorem of integral geometry states that all properties of a \( d \)-dimensional convex set (or more generally, a finite union of convex sets) which satisfy translational invariance and additivity (called morphological properties) are contained in \( d + 1 \) numerical values [29]. For a pixelized map \( \psi(x) \), we consider the excursion sets of the map, defined as the set of all map pixels with value of \( \psi \) greater than some threshold \( \psi_{\text{th}} \) (see, e.g., [30, 31]), where \( x \) is the position, \( \psi \) can be a state variable like temperature \( T \), density \( \rho \) or pressure \( P \); \( \psi \) can also be the velocity \( v \) or its components, some specific stress, etc. Then the \( d + 1 \) functionals of these excursion sets completely describe the morphological properties of the underlying map \( \psi(x) \). In the case of two or three dimensions, the Minkowski functionals have intuitive geometric interpretations.

For a two-dimensional map, the three Minkowski functionals correspond geometrically to the total fractional area \( A \) of the excursion set, the boundary length \( L \) of the excursion set per unit area and the Euler characteristic \( \chi \) per unit area (equivalent to the topological genus). Such a description has been successfully used to describe patterns in reaction–diffusion systems [32], the cosmic microwave background temperature fluctuations [33] and patterns in phase separation of complex fluids [34–37], etc.

In this work we probe the shocked porous material via checking the temperature map \( T(x, t) \), where the time \( t \) is explicitly denoted. The maps of other physical variables can be analysed in a similar way. When the temperature \( T(x) \) is beyond the threshold value \( T_{\text{th}} \), the grid node at position \( x \) is regarded as a white (or hot) vertex, otherwise it is regarded as a black (or cold) one. For the square lattice, a pixel possesses four vertices. A region with connected white (hot) or black (cold) pixels is defined as a white (hot) or black (cold) domain. Two neighbouring white and black domains present a clear interface or boundary. When we increase the threshold contour level \( T_{\text{th}} \) from the lowest temperature to the highest one in the system, the white area \( A \) will decrease from 1 to 0; the boundary length \( L \) first increases from 0, then arrives at a maximum value and finally decreases to 0 again. There are several ways to define the Euler characteristic \( \chi \). Two of the simplest ones are

\[
\chi = N_W - N_B,
\]

or

\[
\chi = \frac{N_W - N_B}{N},
\]

where \( N_W \) (\( N_B \)) is the number of connected white (black) domains and \( N \) is the total number of pixels. The only difference between the two definitions is that the first keeps \( \chi \) an integer. In contrast to the white area \( A \) and boundary length \( L \), the Euler characteristic \( \chi \) describes the connectivity of the domains in the lattice. It describes the pattern in a purely topological way, i.e. without referring to any kind of metric. It is negative (positive) if many disconnected black (white) regions dominate the image. A vanishing Euler characteristic indicates a highly connected structure with equal numbers of black and white domains. Specifically, for definition (1), the integer \( \chi \) equals \(-1\) when one has a black drop in a large white lattice, and vice versa for the +1 case, since the surrounding white (black) region conventionally does not count. In this paper, we use the second definition without creating any ambiguity. The ratio

\[
\kappa = \frac{N_W - N_B}{NL}
\]

describes the mean curvature of the boundary line separating the black and the white domains. Despite having global meaning, the Euler characteristic \( \chi \) can be calculated in a local way using the additivity relation [32].

3. Theoretical model of the material

In this study the material is assumed to follow an associative von Mises plasticity model with linear kinematic and isotropic hardening [38]. Introducing a linear isotropic elastic relation,
the volumetric plastic strain is zero, leading to a deviatoric–volumetric decoupling. Hence, it is convenient to split the stress and strain tensors, \( \sigma \) and \( \varepsilon \), as

\[
\sigma = s - P I, \quad P = \frac{1}{3} \text{Tr}(\sigma), \quad \varepsilon = e + \frac{1}{2} \theta I, \quad \theta = \frac{1}{3} \text{Tr}(\varepsilon),
\]

where \( P \) is the pressure scalar, \( s \) the deviatoric stress tensor and \( e \) the deviatoric strain. The strain \( e \) is generally decomposed as \( e = e^e + e^p \), where \( e^e \) and \( e^p \) are the traceless elastic and plastic components, respectively. The material shows a linear elastic response until the von Mises yield criterion, the deviatoric strain. The strain

\[
\sqrt{\frac{2}{3} ||s||} = \sigma_Y,
\]

is reached, where \( \sigma_Y \) is the plastic yield stress. The yield \( \sigma_Y \) increases linearly with the second invariant of the plastic strain tensor \( e^p \), i.e.

\[
\sigma_Y = \sigma_{Y0} + E_{\text{un}} \| e^p \|,
\]

where \( \sigma_{Y0} \) is the initial yield stress and \( E_{\text{un}} \) the tangential module. The deviatoric stress \( s \) is calculated by

\[
s = \frac{E}{1 + \nu} e^e,
\]

where \( E \) is Young’s module and \( \nu \) Poisson’s ratio. Denote the initial material density and the sound speed by \( \rho_0 \) and \( c_0 \), respectively. The shock speed \( U_s \) and the particle speed \( U_p \) after the shock follows a linear relation, \( U_s = c_0 + \lambda U_p \), where \( \lambda \) is a characteristic coefficient of the material. The pressure \( P \) is calculated by using the Mie–Grüneisen state of equation, which can be written as

\[
P - P_H = \frac{\gamma(V)}{V} [E - E_H(V_H)].
\]

In equation (9), \( P_H \) and \( V_H \) are pressure, specific volume and energy on the Rankine–Hugoniot curve, respectively. The relation between \( P_H \) and \( V_H \) can be estimated by experiment and can be written as

\[
P_H = \begin{cases} 
\rho_0 c_0^2 \left( \frac{1 - V_H}{V_H} \right), & V_H \leq V_0 \\
\rho_0 c_0^2 \left( \frac{V_H}{V_0} - 1 \right), & V_H > V_0
\end{cases}
\]

In this paper, the transformation of specific internal energy \( E - E_H(V_H) \) is taken as the plastic energy. Both the shock compression and the plastic work cause an increase in temperature. The increase in temperature from shock compression can be calculated as:

\[
dT_H = \frac{c_v^2 \cdot \lambda (V_0 - V_H)^2}{c_v ((\lambda - 1)V_0 - \lambda V_H)^3} - \gamma(V) T_H, \quad (11)
\]

where \( c_v \) is the specific heat. Equation (11) can be derived from thermal equation and the Mie–Grüneisen state of equation [39]. The increase in temperature from plastic work can be calculated as

\[
dT_p = \frac{dW_p}{c_v}.
\]

Both equations (11) and (12) can be written in incremental form.

In this paper we choose aluminium as the sample material. The corresponding parameters are \( \rho_0 = 2700 \text{ kg m}^{-3}, \)

\( E = 69 \text{ MPa}, \nu = 0.33, \sigma_{Y0} = 120 \text{ MPa}, \)

\( E_{\text{un}} = 384 \text{ MPa}, \)

\( c_0 = 5.35 \text{ km s}^{-1}, \lambda = 1.34, \)

\( c_v = 880 \text{ J (kg K)}^{-1}, \)

\( k = 237 \text{ W (m K)}^{-1} \) and \( \gamma_0 = 1.96 \) when the pressure is below 270 GPa. The initial temperature of the material is 300 K.

4. Simulation results and physical interpretation

In our numerical experiments the porous material is fabricated by a solid material body with a number of voids randomly embedded. We denote the mean density of the porous body as \( \rho \) and the density of the solid portion as \( \rho_0 \). The porosity is defined as \( \delta = \rho_0/\rho \). This work concentrates on the two-dimensional case and the porosity \( \delta \) is controlled by the total number \( N_{\text{void}} \) and the mean size \( r_{\text{void}} \) of voids embedded. The shock wave reacting on the target porous body is loaded via a collision by a rigid wall with the same material. We choose the coordinate system where the rigid wall is horizontal and keeps static at the position \( y = 0 \), the target porous body is on the upper side of the rigid wall and moves towards the rigid wall at a velocity \( -v_{\text{init}} \). The porous body begins to touch the rigid wall at the time \( t = 0 \). The simulated porous body is initially 1 mm in width and 5 mm in height, as shown in figure 1. Periodic boundary conditions are set in the horizontal directions, which means the real system under consideration is composed of many of the simulated ones aligned periodically in the horizontal direction.

4.1. Case with \( \delta = 0.2 \) and \( v_{\text{init}} = 1000 \text{ m s}^{-1} \)

Figure 1 shows a set of snapshots for a procedure that a shock wave is reacting on a porous body, where the colour contours denote temperature (colour online only). The porosity \( \delta = 0.2 \) and \( v_{\text{init}} = 1000 \text{ m s}^{-1} \). It is clear that, in contrast to the case with uniform material, the original shock wave is scattered and dispersive in the porous body. The first two snapshots show the loading procedure. When \( t = 500 \text{ ns} \), the early compressive waves arrive at about \( y = 1 \text{ mm} \); when \( t = 1500 \text{ ns} \), they arrive at about \( y = 3.1 \text{ mm} \). The last two snapshots show the procedure of downloading. When compressive waves arrive at the upper free surface, rarefactive waves are reflected back into the target porous body. Under the tension wave, the height of the porous body increases with time. In fact, before the compressive waves arrive at the upper free surface, a large number of local downloading phenomena have occurred within the porous body. When the initial shock wave or a compressive wave encounters a void, rarefactive waves are reflected back and propagate within the compressed portion, which destroys the original possible equilibrium state there. Since the details of the wave series are very complex, when we mention the
value of a state variable, for example the density, we refer to its local mean value.

To perform the Minkowski functional analysis for the temperature map, we can choose a threshold temperature $T_{th}$ and pixelize the map into white regions (with $T \geq T_{th}$) and black regions (with $T < T_{th}$). Figure 2 shows the Minkowski measures for the same procedure as in figure 1. ‘DT’ in the legend means $T_{th} - 300$. The unit of temperature is K. The time unit is ns. When DT is very small, the wave front is nearly a plane, which is similar to the case with shock reacting on a uniform solid material. When DT = 10 K, the total fractional white area $A$ increases to be nearly 1 at the time $t = 1600$ ns and keeps this value until the time $t = 2600$ ns, before showing a slight decrease. This means the early compressive wave arrives at the upper free surface at about, in fact before, the time $t = 1600$ ns, nearly all material particles in the target body have a temperature beyond 310 K during the following 1000 ns. In the downloading procedure the rarefactive waves make a very small fraction of material particles decrease their temperature to below 310 K. When DT increases from 200 to 300 K, the white area arrives at a steady value of 0.96, which means 4% of the material particles could not get a temperature higher than 310 K up to this time. When DT = 500 K, the white area keeps nearly zero during the whole procedure shown here, which means no local temperature is higher than 300 K in the system up to the time $t = 3000$ ns. For cases with DT = 300, 330, 360 and 400 K, after the initial slow increasing period, the white (hot) area has a quick increasing period. The latter indicates that a large number of ‘hot-spots’ in the previously compressed region coalesced during that period. After that the increasing of $A$ with $t$ shows a slowing down. The slope of the $A(t)$ curve corresponds approximately to the mean propagation speed of some components of the compressive waves. Therefore, the first Minkowski measure indicates that, in porous material, when a velocity $D$ of the compressive-wave series is mentioned, the corresponding contour level of a state variable like temperature should also be stated. From this figure, it is clear that $D(T_{th})$ decreases with increasing $T_{th}$; the total fractional white (hot) area $A(t)$ shows parabolic behaviour during the initial period; when DT approaches 0, the $A(t)$ behaviour goes back to being linear.

Now we go to the second Minkowski measure, the boundary length $L$. To understand this measure, we can
The increase results from collision by the target body. The compressive wave is propagating towards the upper free boundary, where the mountain has more than one peak, the situation will decrease, while the total fractional white area decreases quickly. These results show that the increase in the white area $A$ is mainly due to the coalescence of previous scattered 'hot-spots'. The curves for $DT = 330$ K and $DT = 360$ K can be understood in a similar way. For the present shock strength, only very few material particles can get a temperature beyond 700 K before the time $t = 2000$ ns. Therefore, the boundary length $L$ for the case with $DT = 400$ K has a meaningful increase only after $t = 2000$ ns.

When $DT$ is small, $T > T_{\text{th}}$ in (nearly) the entire compressed portion and $T < T_{\text{th}}$ in the uncompressed part of the material body. The temperature map shows a highly connected structure with (nearly) equal and very small numbers of black and white domains. Hence, the Euler characteristic $\chi$ keeps close to zero in the whole shock-loading procedure and the mean curvature $\kappa$ is nearly zero. The value of $\chi$ decreases to be evidently less than zero in the downloading procedure, which indicates that the number of domains with $T < T_{\text{th}}$ increases. (See the $\chi(t)$ curves for cases with $DT = 10$, $DT = 100$ and $DT = 200$ in figure 2.) With the increase in the contour level $T_{\text{th}}$, more regions change their colour from white ($T > T_{\text{th}}$) to black ($T < T_{\text{th}}$). The pattern evolution in the shock-loading procedure can be regarded as that scattered white domains appear gradually with time in the black background. So the Euler characteristic $\chi$ is positive and increasing with time. (See the $\chi(t)$ curves for cases with $DT = 300$, $DT = 330$ and $DT = 360$ in figure 2.) When the contour level $T_{\text{th}}$ is further increased up to 700 K, a meaningful fraction of the material particles could not get a temperature higher than the contour level $T_{\text{th}}$. The saturation phenomenon in the $\chi$ curve during the period 550 ns < $t$ < 2100 ns indicates that the numbers of connected 'hot' and 'cold' domains vary with time in a similar way. The increase in $\chi$ in the period 2100 ns < $t$ < 2500 ns is because the rarefactive waves make the mean temperature decrease, and correspondingly, some connected 'hot-domains' are disconnected as scattered 'hot-spots' again. For the case of $DT = 500$ K, the pixelized temperature map is nearly all black. Hence, the Euler characteristic $\chi$ is nearly zero.

4.2. Effects of porosity

Figure 3 shows a set of snapshots for the case with a lower porosity, $\delta = 1.4$. The other conditions are the same as in figure 1. From left to right, the four configurations correspond to the times, $t = 500$, 1100, 1400 and 1700 ns. Compared with the snapshots in figure 1, it is clear that the propagation velocity of compressive wave increases with decreasing porosity. At time $t = 500$ ns, in the system with $\delta = 1.4$, the compressive wave arrives at about $y = 1750 \mu$m; while in the system...
Figure 3. Configurations with temperature contours. $\delta = 1.4$ and $v_{\text{init}} = 1000 \text{ m s}^{-1}$. From left to right, $t = 500 \text{ ns}, 1100 \text{ ns}, 1400 \text{ ns}$ and $1700 \text{ ns}$, respectively. The length unit here is $10 \mu\text{m}$. (Colour online.)

Figure 4. Minkowski measures for cases with various porosities. $T_0 = 400 \text{ K}$. The values of porosity are shown in the legend. (Colour online.)

Figure 5. Minkowski measures for cases with various porosities. $T_0 = 500 \text{ K}$. The values of porosity are shown in the legend. (Colour online.)
with $\delta = 2$, the compressive wave arrives only at about $y = 1000 \mu m$. In the case of $\delta = 1.4$, the compressive wave has arrived at the top free surface and the rarefactive wave has been reflected back to the target body before the time $t = 1400$ ns; while in the case of $\delta = 2$, the shock-loading procedure has not finished up to $t = 1500$ ns.

Figure 4 shows the Minkowski measures for cases with various porosities, where $T_{th} = 400$ K and the values of porosity, $\delta = 2.45, 2, 1.7, 1.4, 1.22, 1.15, 1.1$, are shown in the legend. In the subfigure for white area $A$, the initial shock-loading part presents meaningful information: the velocity $D$ of the compressive-wave series is smaller for a higher porosity $\delta$. The most significant property in the subfigure for boundary length $L$ is that the largest boundary length $L_{max}$ increases as $\delta$ decreases. When $\delta = 1.1$, the total boundary length $L$ gets the maximum value at about $t = 1250$ ns. This result indicates that the highest temperature in shocked porous material decreases when the porosity approaches 1. The Euler characteristic $\chi$ becomes more negative when the porosity $\delta$ decreases from 2.45 to 1.1, which means the disconnected ‘cold’ domains with $T < 400$ K dominate the image.

Figures 5 and 6 show the Minkowski measures for the same porosities but higher temperature thresholds, $T_{th} = 500$ K and $T_{th} = 600$ K. They present supplementary information to figure 4. For cases with $\delta = 1.4, 1.22, 1.15$ and 1.1, only 88%, 55%, 36% and 15% of the material particles get temperature higher than 500 K. For cases with $\delta = 1.4$ and 1.22, only 16 and 6% get temperature higher than 600 K in the shock-loading procedure. When $T_{th} = 500$ K, the case with $\delta = 1.15$ has the maximum boundary length and the case with $\delta = 1.1$ has the maximum Euler characteristic. When $T_{th} = 600$ K, the case with $\delta = 1.4$ has the maximum boundary length and the maximum Euler characteristic, which means
the ‘hot-spots’ with \( T > 600 \text{K} \) are scattered in the ‘cold’ background with \( T < 600 \text{K} \).

### 4.3. Effects of initial shock-wave-strength

We now study the effects of different initial impacting speeds. Figure 7 shows a set of snapshots for the case with \( \delta = 1.4 \) and \( v_{\text{init}} = 500 \text{m s}^{-1} \). From left to right, the four configurations are for the times \( t = 500, 1500, 2000 \) and \( 2500 \text{ns} \). From the first two, we observe the upward propagation of compressive wave in the target body. From the last two, we observe the downward rarefactive effects. Compared with figure 3, it is clear that the velocity \( D \) of the compressive-wave series and the highest temperature \( T_{\text{max}} \) decreased. The Minkowski measures for this procedure are shown in figure 8. Such a shock procedure could not produce ‘hot-spot’ with \( T = 500 \text{K} \). The high-temperature ‘hot-area’ continues to increase even after some precursory compressive waves have scanned the whole target body and some rarefactive waves have come into the target body from the upper free surface. Up to the time \( t = 3000 \text{ns} \), the fractional area of ‘hot-spots’ with \( T > 400 \text{K} \) reaches 40%, the fractional area for \( T > 380 \text{K} \) reaches 74% and that for \( T > 360 \text{K} \) reaches 91%. The contour level with \( T = 380 \text{K} \) has the largest boundary length at about \( t = 1500 \text{ns} \) when the ‘hot-spots’ are mainly scattered in the ‘cold’ background. Figures 9 and 10 show the Minkowski measures for cases with the same porosity but lower initial impacting speeds. \( v_{\text{init}} = 400 \text{m s}^{-1} \) in figure 9 and \( v_{\text{init}} = 300 \text{m s}^{-1} \) in figure 10. With the decrease in the initial impact speed, the highest temperature \( T_{\text{max}} \) in the system further decreases; the total fractional white area \( A \) for the low contour level, for example \( DT = 10 \text{K} \), increases with time in a more linear way.

We compare Minkowski measures for different initial impacting speeds in figure 11, where \( \delta = 1.4, DT = 50 \text{K} \), \( v_{\text{init}} = 1000, 500, 400, 300 \) and \( 200 \text{m s}^{-1} \). It is clear that the higher the initial impacting speed, the closer the \( A(t) \) curve is to being linear. The case of \( v_{\text{init}} = 400 \text{m s}^{-1} \) has the longest total boundary separating the ‘hot’ and ‘cold’ domains. For this case, disconnected ‘hot’ regions dominate the image from the topology side in the shock-loading procedure; disconnected ‘cold’ regions dominate in the downloading procedure.
5. Conclusions

Under shock wave reaction, the porous material is globally in a nonequilibrium state and shows complex dissipative structures. We pixelize the map of temperature into Turing patterns and introduce morphological measures for it. The relevance of the total fractional white area $A$, boundary length $L$ and the Euler characteristic $\chi$ to the thermodynamical properties of material is revealed. Various experimental conditions are simulated via the MPM. Numerical results indicate that the shock wave reaction results in a complicated sequence of compressions and rarefactions in porous material. The increasing rate of $A$ roughly gives the velocity $D$ of a compressive-wave series. When a velocity $D$ is mentioned, the corresponding threshold contour level of the temperature should also be stated. When the threshold contour level increases, $D$ becomes smaller. The area $A$ increases parabolically with time $t$ during the initial period. The $A(t)$ curve goes back to being linear in the following three cases: (i) when the porosity $\delta$ approaches 1, (ii) when the initial shock becomes stronger and (iii) when the contour level approaches the minimum value of the temperature. The area with high temperature may continue to increase even after the early compressive waves have arrived at the downstream free surface and some rarefactive waves have come back into the target body. In the case of energetic material needing a higher temperature for initiation, a higher porosity is preferred and the material may be initiated after the precursory compressive-waves have scanned the entire target body. In some cases we need scattered hot-spots, but in others we need connected ones. One may desire the fabrication of a porous body and choose the appropriate shock strength according to what is needed. The same measures can also be used to analyse the maps of other physical variables, such as the density, velocity or various stresses. With the Minkowski measures, the dependence on experimental conditions is reflected simply by a few coefficients. They may be used as order parameters to classify the maps of a state variable in a similar way to thermodynamic phase transitions.

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