LETTER TO THE EDITOR

On the detectability of gravitational waves background produced by gamma ray bursts

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Abstract. In this paper we discuss a new strategy for the detection of gravitational radiation likely emitted by cosmological gamma ray burst. Robust and conservative estimates lead to the conclusion that the uncorrelated superimposition of bursts of gravitational waves can be detected by interferometric detectors like VIRGO or LIGO. The expected signal is predicted to carry two very distinctive signatures: the cosmological dipole anisotropy and a characteristic time scale in the auto correlation spectrum, which might be exploited, perhaps with *ad hoc* modifications and/or upgrading of the planned experiments, to confirm the non-instrumental origin of the signal.

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1. Introduction

Gamma Ray Bursts (GRB) are short and intense flashes of e.m. radiation in the keV-MeV range, accidentally discovered more then 30 years ago [1]. The observation with high spatial resolution by the Italian-Dutch satellite Beppo-SAX [2] of the X-ray afterglow, has allowed the optical identification of some of the burst, which has given the evidence of the cosmological origin of the GRB’s. It is now evident that for few seconds, at least twice a day, unknown sources become so bright to over shine all the visible Universe [3]. From the observed γ-ray fluence and the red shift of the host galaxy, it is estimated [4] that the average buster emits isotropically \( \langle E_{\gamma \text{ iso}} \rangle = (1.3^{+1.2}_{-1.1}) \times 10^{53} \) erg (We will assume [4] here and in the rest of the paper a flat Universe with \( H_0 = 65 \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1} \), \( \Omega_M = 0.3 \) and \( \Omega_V = 0.7 \)). The dispersion around this average is consistent with a log-normal distribution having logarithmic width \( \sigma_{\gamma \text{ iso}} = 1.7^{+0.4}_{-0.3} \) (about a factor 50). This energy corresponds in the average to \( \approx 0.1 \, M_\odot \, c^2 \), but arrives in the extreme case of GRB 990123 [5] to be \( \approx 1.4 \, M_\odot \, c^2 \), which poses serious problems for understanding what could be the “central engine” powering these events. In the currently accepted “fireball” model [6] the γ-rays are emitted by electron/positron pairs, accelerated by internal relativistic shocks, which radiates via synchrotron and/or synchro-Compton process. In order to be optically thin, the fireball should expand with ultra–relativistic velocity (bulk Lorentz factor \( \Gamma \approx 100 \)). Even if the conversion of the kinetic energy into γ radiation is estimated [8] to be very efficient
(\eta \gamma \approx 0.2), the formation of the relativistic wind with a kinetic energy \( \approx E_{\gamma iso}/\eta \gamma \) could be problematic.

The energy output of the central engine of GRB can be largely reduced if the \( \gamma \)-ray emission is beamed with a small opening angle. This possibility is supported by the observation of an achromatic break in the time evolution of the afterglow luminosity, occurring when the bulk Lorentz factor becomes \( \Gamma < \theta_{\text{jet}}^{-1} \). In a recent paper this hypothesis is applied to estimate the opening angle of the 10 GRB’s with observed afterglows, with the interesting result that the average opening angle in the sample is \( \langle \theta_{\text{jet}} \rangle \approx 4^\circ \). Furthermore, it appears from these data that the fluence of the GRB in the sample is correlated with the opening angle, in a way that justifies the large spread of the apparent fluence as entirely due to the spread of the opening angles. The intrinsic energy emitted by GRB in \( \gamma \)-rays, if estimated as

\[
E_{\gamma jet} = f_{\text{jet}} F_{\gamma} d_L^2 \left(1 + z\right)^{-1}
\]

where \( f_{\text{jet}} = \frac{1}{2}(1 - \cos \theta_{\text{jet}}) \) is the beaming factor, clusters around a value of \( \langle E_{\gamma jet} \rangle = 5 \times 10^{50} \) erg with a logarithmic width \( \sigma_{\gamma jet} \approx 0.3 \) that is about a factor 25 smaller than the one of the \( E_{\gamma iso} \). If this is true the kinetic energy would be of the order of \( E_{\text{GRB}} \approx 10^{-3} M_\odot c^2 \), about constant for all the bursts. The scenario emerging from this study is particularly appealing because one could assume that the central engine of the GRB produces always a similar amount of energy, while the complexity of the energy transfer from the central engine to the relativistic jet causes the wide range of opening angle, and consequently the wide range of apparent fluence that is observed.

This scenario implies also that the true rate of bursts \( R_{\text{jet}} \approx \langle f_{\text{jet}}^{-1} \rangle R_{\text{iso}} \), is enhanced by a factor \( \langle f_{\text{jet}}^{-1} \rangle \approx 500 \) respect to the observed rate. However clearly the total energy output per unit volume and unit time injected in the universe by GRB explosion \( \dot{\epsilon}_{\gamma} = R_{\text{jet}} E_{\gamma jet} = R_{\text{iso}} E_{\gamma iso} \) will be the same of the isotropic case.

In the next Sect. of this paper we discuss the implications of the possible beamed emission on the detectability of gravitational waves (GW) likely emitted by the collapse event that originates the GRB. It is to be expected that if the e.m. energy of the \( \gamma \)-ray burst is not extraordinary (about the same order of magnitude of the total output of a Type Ib SN), the energy output in gravitational waves should also be modest. But, as we observed above, in the latter case the number of bursts should be increased correspondingly. This suggests a new strategy for looking to gravitational radiation from GRB. As we will show in Sect. even if the GW’s produced by each GRB are below the detection threshold, integrating over one year one should find an excess of noise, due to the uncorrelated superimposition of many GW pulse trains. In Sect. we discuss two very distinctive signature that a genuine cosmological stochastic signal should carry. Those intrinsic signatures, perhaps detectable with modifications and/or upgrading of the planned experiments, could be of great help to check the non-instrumental origin of the stochastic signal. Finally in Sect. we summarize the results of this investigation.
2. Gravitational wave signal from GRB

The energy required to power the γ-ray burst, estimated by Eq. (1), is of the order of \( \approx 10^{-3} M_\odot c^2 \), not overwhelming but always much larger than the total nuclear binding energy of few-\( M_\odot \)'s stars. Therefore it requires in any case an energy release that is compatible only with the collapse of a several solar masses object. A natural assumption would be that the GRB are similar to ordinary supernovas, but perhaps the difference is in the final result of the collapse. Like for the “failed supernova” model [16] the collapse of a massive star to a black hole surrounded by a dense rotating torus of material that might result in a relativistic jet. The energy irradiated in this case as gravitational waves can be estimated [17] as \( E_{GW} \approx \epsilon_{GW} M_{BH} c^2 \) where \( M_{BH} \) is the mass of the black hole and \( \epsilon_{GW} \lesssim 10^{-4} \) is an efficiency parameter. As we observed above the bolometric energy released in the gamma ray burst is already of the order of \( E_{\gamma jet} \gtrsim 10^{-4} M_\odot c^2 \) and \( \eta_\gamma \approx 0.2 \), then we could expect that the energy provided by the central engine could be \( E_{GRB} \approx \epsilon_{GRB} M_{BH} c^2 \) where \( \epsilon_{GRB} \) is the fraction of the gravitational energy converted into kinetic energy of the ejecta subsequently radiated as γ-rays.

It could be possible under certain conditions [18] that \( \epsilon_{GRB} \approx \epsilon_{GW} \), but perhaps we should adopt the more conservative view that \( \epsilon_{GW} \lesssim \epsilon_{GRB} \) even if it is unlikely that \( \epsilon_{GW} \ll \epsilon_{GRB} \). Our ignorance of the actual mechanism that could transfer the energy from the accretion torus to the jet does not allow a solid prediction of the energetic of the central engine of the GRB. Nevertheless we can set a lower limit to this energy in the form \( E_{GRB} \gtrsim E_{\gamma jet}/\eta_\gamma \). We can parameterize the production of GW introducing a phenomenological \( \eta_{GW} \lesssim 1 \) putting \( E_{GW} = \eta_{GW} E_{\gamma jet}/\eta_\gamma \) and verifying the sensitivity of the GW detectors to the actual value of \( \eta_{GW} \).

The energy flux (viz. energy per unit surface and unit time) of GW produced by a burst of intrinsic luminosity (in gravitational waves) \( L_{GW} \) at a red shift \( z \) is given by [19]

\[
F^b_{GW} = \frac{(1 + z) L_{GW}}{4\pi d_L^2} \tag{2}
\]

where \( d_L \) is the luminosity distance. Assuming a typical time scale for the emission \( \Delta t \) we can estimate this flux at Earth for a typical red shift \( z = 1 \)

\[
F^b_{GW}(z = 1) \approx 4 \times 10^{-7} \left( \frac{f_{jet} E_{\gamma iso}/\eta_\gamma}{2.5 \times 10^{51} \text{ erg}} \right) \left( \frac{\eta_{GW}}{0.01} \right) \left( \frac{10 \text{ ms}}{\Delta t} \right) \text{ erg cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}
\]

In order to convert the flux of Eq. (2) into an adimensional amplitude we can use the classical formula [20]:

\[
F_b = \frac{c^3}{16\pi G} \left\langle \hat{h}_+^2 + \hat{h}_z^2 \right\rangle \tag{3}
\]

where the average is taken over several wavelengths. The amplitude of the signal produced depends from the direction and the beam pattern of the detector. In the best case we have integrating over time and applying the Parseval’s theorem

\[
\int_{-\infty}^{+\infty} \omega^2 \hat{h}_z^2(\omega) d\omega = \frac{16\pi G}{c^3} \frac{(1 + z) E_{GW}}{4\pi d_L^2} \tag{4}
\]
where $\tilde{h}_b^2(\omega)$ will be the Rayleigh power of the signal as a function of the frequency $\omega$. In order to estimate the order of magnitude of the amplitude of the GW signal we do not need a detailed shape of the spectral power density of the signal, but only the knowledge of the firsts two moments of the distribution $\bar{\omega}$ and $\Delta\omega$. In fact we can recast Eq. (4) in the form

$$\left(\bar{\omega}^2 + \Delta\omega^2\right) \int_{-\infty}^{+\infty} \tilde{h}_b^2(\omega) \, d\omega = \frac{16\pi G (1 + z) E_{GW}}{c^3 4\pi d_L^2}$$

It is remarkable that rather natural physical assumption on the first and second moment of the unknown Rayleigh power distribution $\tilde{h}_b^2(\omega)$ can be made. In fact we can assume that the first moment will be $\bar{\omega} \approx 2\pi c / r_S$ where $r_S$ will be the Schwarzschild radius of the collapsed object. The second moment will be the r.m.s. bandwidth of a wave packet that can be estimated $\Delta\omega \approx 2\pi / \Delta t$ where $\Delta t$ is the duration of the emission in the comoving frame. In this case we have, if $c \Delta t \gg r_S$, for the peak amplitude

$$\tilde{h}_b^{\text{peak}} \approx \frac{r_S}{d_L} \sqrt{\frac{G}{2\pi^3 c^5}} E_{GW} \Delta t$$

(5)

The expected amplitude for typical values is

$$d_L \tilde{h}_b^{\text{peak}} \approx 10^{-27} \left( \frac{r_S}{5 \text{ km}} \right) \left( \frac{E_{GRB}}{2.5 \times 10^{51} \text{ erg}} \right)^{1/2} \left( \frac{\eta_{GW}}{0.01} \right)^{1/2} \left( \frac{\Delta t}{10 \text{ ms}} \right)^{1/2} \text{Gpc}/\sqrt{\text{Hz}}$$

It is clear from this estimate that the probability of detecting a single burst is very low, unless $\eta_{GW} \gg 1$. However even if the individual burst could not be detected, it is to be remarked that in case of beaming the rate of explosion would be very large ($\approx 1500$ per day), therefore the stochastic accumulation of signal integrated over a long time could emerge from the noise.

3. Cosmological background

The energy flux of GW produced at Earth from a cosmological distribution of sources is given by

$$F_{GW}^{\text{diff}} = \int_0^{z_{\text{max}}} R(z) E_{GW} \frac{dz}{(1 + z) H(z)}$$

(6)

where $R(z)$ is the comoving rate of bursts per unit volume exploding at red shift $z$, $E_{GW}$ the average energy emitted in gravitational waves by each source that we have estimated in §2 and $H(z) = \dot{R}/R$ is the Hubble expansion rate $[19]$. As obvious, the lower apparent brightness of distant burst is compensated by the increase of volume up to a red shift 1-2, like in the classical Olbert’s paradox. It is also worth noticing that the flux will be the same for beamed or isotropic sources. The rate $R(z)$ can be obtained from the Log N-Log P distribution $[10, 11]$, even if its value depends from the assumption made on the evolution of the burst rate in the recent past. In practice the energy flux of the present stochastic background of GW is dominated by the rate of explosions at $z \gtrsim 1$. If the GRB are produced by a final catastrophic collapse event of an evolved massive star,
one expects that the large scale distribution of GRB should reflect the star formation rate. The latter is known \[12, 13\] to increase by at least a factor ten from \(z = 0\) to \(z \approx 2\) to flatten in the range \(2 \leq z \leq 3\) and to decrease exponentially for \(z > 3\), reaching a value similar to the present for \(z \approx 6\). The fit obtained with this evolution \[11\] is \(R_{iso}(z = 0) = 0.14 \pm 0.02 \, \text{Gpc}^{-3} \, \text{y}^{-1}\). On the other side if no evolution is assumed the constant rate is about one order of magnitude larger. But it is worth noticing that due to the constraint of having about 1000 bursts per year the only difference between the two cases is that in the first case the average distance of the bursts is slightly larger, due to the fast rise of the rate of explosion with red shift. In fact from Eq. (6) we estimate

\[
F_{\text{diff}}^{\text{GW}} \approx 10^{-10} \left( \frac{E_{iso}/\eta_{\gamma}}{6.5 \times 10^{53} \, \text{erg}} \right) \left( \frac{\eta_{GW}}{0.01} \right) \left( \frac{R_{iso}}{1 \, \text{Gpc}^{-3} \, \text{y}^{-1}} \right) \, \text{erg cm}^{-2} \, \text{s}^{-1} \, \text{sr}^{-1}
\]

while assuming evolution of the rate in the recent past (see below) the predicted flux is practically the same. Comparing with Eq. (2) we observe that the diffuse flux is consistent with the order of magnitude estimate

\[
F_{\text{diff}}^{\text{GW}} / F_{\text{b}}^{\text{GW}} \approx \left( \langle f_{\text{jet}}^{-1} \rangle \times 1,000 \, \text{bursts/1 year} \times \Delta t \right)
\]

Applying again Eq. (3) we have:

\[
\frac{c^3}{16\pi G} \left\langle \hat{h}_+^2 + \hat{h}_x^2 \right\rangle = \int_0^{z_{\text{max}}} R(z) E_{GW} \frac{dz}{(1 + z) H(z)}
\]

On the L.H.S. of this equation we have the amplitude of the wave that invests at a certain instant the detector. We have seen in the previous section that each of this burst will not have a detectable intensity, but if we average over an observation time \(T\) long compared to the GW burst duration but short compared to Hubble time scale (typically one year) we have a signal

\[
\frac{1}{T} \int_{-\infty}^{+\infty} \omega^2 \hat{h}_x^2(\omega) d\omega = \int_0^{z_{\text{max}}} R(z) E_{GW} \frac{dz}{(1 + z) H(z)}
\]

that will be detectable if the power spectral density is greater then the power spectral of the noise, averaged over the same observation time. The uncorrelated superimposition of bursts of gravitational waves will be well approximated, for the central limit theorem, by the superimposition of red shifted gaussian distributions. Therefore the power spectral density of the signal can be estimated by the integral

\[
\left\langle \hat{h}_x^2(\omega) \right\rangle_T \approx \frac{G}{\pi^2 c^6} \int_0^{z_{\text{max}}} R(z) E_{GW} r_S^2 \frac{d\omega}{\sqrt{2\pi} \Delta \omega} e^{-\frac{[(1+z)\omega-\omega_0]^2}{2\Delta \omega^2}} \frac{dz}{(1 + z) H(z)}
\]

where the normalization has been obtained from Eq. (8). We have reported in Figure 1, which is the central result of this paper, the Rayleigh power of the stochastic signal from the cosmological background for \(\eta_{GW} = 0.01\). The dashed curve in Figure 1 represents the power expected allowing for an evolution of the rate \(R(z)\) with the red shift similar to the luminosity density evolution of QSO’s \[21\]. This evolution is assumed to be \[24\]

\[R(z) = (1 + z)^{\alpha}\]

with \(\alpha \simeq 3\) at low red shift, \(z < 1.9\), \(\alpha = 0\) for \(1.9 < z < 2.7\) \[23\], and
Figure 1. Stochastic signal integrated over one year from cosmological GRB ($\eta_{GW} = 0.01$). In this plot the solid line corresponds to a flat rate, while the dashed one to a rate of bursts evolving like the QSO rate (see text). The dot-dashed line is the noise estimated in the VIRGO proposal \cite{25}, averaged over one year.

an exponential decay at $z > 2.7$ \cite{24}, similar to that describing the evolution of star formation rate that we have discussed above. The solid curve represents, on the contrary, the power expected if the rate of the GRB is assumed to be constant. For comparison we have also reported the noise expected in the VIRGO experiment \cite{25} averaged over one year of integration. This comparison shows that the cosmological signal should be detectable at the same level in both cases, because the overall normalization is constrained by the observed rate by BATSE of $\approx 1000$ bursts per year.

4. Signatures of the genuine signal

We expect for the stochastic signal produced by cosmological sources some very clear signatures showing its origin. We will not discuss in the following whether or not those signatures will be detectable by the two planned detectors VIRGO and LIGO, but we expect that an \textit{ad hoc} modification of those experiments could be designed in order to extract all the possible informations from the signal. The more evident of those signatures should be the dipole anisotropy, as observed for the cosmic microwave background (CMB). The dipole anisotropy of the CMB is interpreted as the result of Doppler shift caused by the solar system motion relative to the isotropic radiation field. This motion is confirmed by measurements of the apparent velocity of local galaxies \cite{26}. The motion of the observer (receiver) with velocity $\beta = v/c$ relative to a source
of gravitational waves with frequency $\omega_0$ produces a shift in the observed frequency $\omega'_0$ given by the formula [27]:

$$\omega'_0 = \omega_0 \left( \frac{1 - \beta^2}{1 - \beta \cos \theta} \right)^{\frac{1}{2}} \approx \omega_0 \left[ 1 - \beta \cos \theta + \mathcal{O}(\beta^2) \right]$$

(10)

where the velocity for the solar-system barycentre is $\beta = 0.001237 \pm 0.000002$ or $v = 371 \pm 0.5$ km s$^{-1}$ and $\theta$ is the angle formed with the direction of equatorial coordinates $(\alpha, \delta) = (11.20^h \pm 0.01^h, -7.22^\circ \pm 0.08^\circ)$. This frequency shift produces at a given frequency a diurnal modulation of the Rayleigh power in the true signal only, which depends from the slope of the spectrum. From the Figure 1 we can infer that this modulation will be particularly enhanced at the extremes of the range of cosmological red shifted distribution of the characteristic spectrum. In Figure 2 we have reported the theoretical maximum amplitude of the intrinsic modulation, induced by the dipole anisotropy as a function of the frequency, calculated substituting $\omega'_0$ given by Eq. (10) to $\omega_0$ into Eq. (9).

A second signature of the cosmological signal comes from the fact that the detectable power is obtained accumulating many short duration GW pulse trains.
signal auto correlation spectrum defined as

\[ A(\tau) = \frac{1}{T} \int_{0}^{T} h(t) h(t+\tau) \, dt \] (11)

should show the evidence of a correlation over the characteristic scale of the stellar collapse (of the order of 10 ms.) If the instrumental noise is white and the sampling frequency of the detector very high, the distinction could be very clear. In real life the detection of this feature could be much harder because the noise of the detector will be the superimposition of white noise due to microscopic processes and colored noise coming from other physical sources (like for example the mirror resonance or the 1/f noise) and the sampling frequency could be inadequate.

5. Conclusions

In §2 we have shown that the GW emission from single bursts at cosmological distances, if the e.m. emission is beamed with a small angle as suggested by afterglow observations, is expected to be, for conservative values of the source emissivity, \( d_L \tilde{h} \lesssim 10^{-27} \text{ Gpc}/\sqrt{\text{Hz}} \) which is well below the detection threshold of presently planned experiments. However, as we have discussed in §3, if the e.m. emission is beamed only a small fraction (one over 500) of the GRB are observed by \( \gamma \)-ray satellites. This implies that small amplitude pulse trains of GW impinge over the detector at a frequency that could be as high as one per minute, which is practically a continuous signal. Even if the individual pulse is small, integrating over a reasonable observation time (order of one year) an excess Rayleigh power should emerge from the instrumental noise. The frequency spectrum of this eventual excess should give a direct information on the characteristic time scale of the collapse and on the cosmological evolution of the GRB rate in the recent past \((z \approx 1 - 2)\). The predicted amplitude is conservatively estimated to be in the range of \( 5 \times 10^{-26} 1/\sqrt{\text{Hz}} \) averaging the Rayleigh power over one year. This estimate is rather robust because does not depends on the beaming factor and depends only slightly from the large scale distribution of the GRB sources. The cosmological origin of the excess noise can be proved by detecting a dipole anisotropy, that in the relevant frequency band could reach the level of a fraction of percent. In addition the auto correlation spectrum of the noise shall carry an imprint of the characteristic duration of the GW pulse trains. The detection of those two signatures, even if perhaps not possible with presently planned experiments, can give the important additional evidence of the cosmological origin of the stochastic signal and informations on the physics of GRB’s.

References

[1] Mészáros P 1998 Plenary talk given at the "19th Texas Symp. on Relativistic Astrophysics & Cosmology", Paris and references therein.
[2] Costa E et al 1997 Nature 387 783.
[3] Piran T 1998 Physics Reports 314 575; ibid. 1999 333 529; ibid. 2000 333 529.
[4] Jimenez R, Band D and Piran T 2001 Preprint astro-ph/0103258.
[5] Fukugita M and Hogan C J 2000 in Groom DE et al Eur. J. Phys. C15 1.
[6] Kulkarni S R et al 1999 Nature 398 389.
[7] Mészáros P and Rees M Astroph. J. (Lett.).......
[8] Guetta D, Spada M and Waxman E 2001 Accepted for publication in Astroph. J. (Preprint astro-ph/0011170).
[9] Frail D A et al 2001 Paper submitted to Nature (Preprint astro-ph/0102282).
[10] Komes et al 2000 Astrophys. J. 533 696.
[11] Wijers R, Bloom JS, Bagla JS and Natarajan P 1998 MNRAS 294 L17.
[12] Lilly SJ, Le Fervre O, Hammer F and D. Crampton D 1996 Astrophys. J. 460 L1.
[13] Madau P et al 1996 Mon. Not. Roy. Astron. Soc. 283 1388.
[14] Phinney ES 1991 Astrophys. J. 380, L17.
[15] Portegies Zwart SF and Spreeuw HN 1996 Astron. & Astroph. 312 670.
[16] Weisley SE 1993 Astroph. J. 405 273.
[17] Stark RF and Piran T 1985 Phys. Rev. Lett. 55 891.
[18] van Putten M 2001 Paper presented at the "2001 Aspen Winter Conference on Gravitational Waves", Feb. 4-10, 2001, Aspen (Preprint gr-qc/0102043).
[19] Kolb E W and Turner M S 2000 in Groom DE et al Eur. J. Phys. C15 1.
[20] Shapiro S L and Teukolsky S A, 1983, Black Holes, White Dwarfs and Neutron Stars (New York:Wiley Interscience Pub.) p 469.
[21] Waxmann and Bahcall 1998 Preprint astro-ph/9807282.
[22] Boyle B J and Terlevich R J 1998 Monthly Not. Roy. Astron. Soc. 293 L49.
[23] Hewett P C, Foltz C B and Chaffee F 1993 Astrophys. J. 406 43.
[24] Schmidt M, Schneider D P and Gunn J E 1995 Astronomical J. 110 68.
[25] Bradaschia C et al 1989 "The VIRGO project", INFN Sez. di Pisa, p. 62; see also Giazotto A 1989 Physics Reports 182 365.
[26] Dekel A 1999 in "Cosmic Flows: Towards an Understanding of Large-Scale Structure", ed. S. Courteau, M.A. Strauss, and J.A. Willick (ASP Conf. Ser.), in press, Preprint astro-ph/9911501.
[27] Smoot GF and Scott D 2000 in Groom DE et al Eur. J. Phys. C15 1.
[28] Kogut A et al 1993 Astrophys. J. 419 1.
[29] Lineweaver C et al 1996 Astrophys. J. 470 L28.
[30] Schwartz M and Shaw L 1975 "Signal Processing: Discrete spectral analysys, detection and estimation", Mc Graw Hill, New York, p. 159.