In the machine industry, 34% of the elements are axially-symmetric with 12% being characterized by low-rigidity. Despite the fact that the dimensions of such parts are often characterized by the lack of proportion as well as low rigidity in certain sections and directions, they ought to fulfill certain requirements regarding the parameters of the geometrical shape, mutual location of the surfaces, linear dimensions and surface quality [9, 10, 23]. It is difficult to obtain such parameters of the accuracy of shape, dimensions and surface quality in the shaft treatment process [6, 7, 18]. The relatively low rigidity of the elements compared to the rigid systems of the machine tool causes vibrations to occur under certain conditions [12, 13, 15]. There are numerous factors disturbing and destabilizing the treatment process (significant own strain of the parts, tools, chips, dust etc.) [1, 10, 22, 27]. For this reason, it is necessary to seek for new methods of controlling the treatment of low-rigidity shafts.

The quality of the treatment of elements on machine tools directly depends on the automatic control system (ACS) of the machine tool and the parameters of the dynamic system [9, 23, 21]. The dynamic system (DS) of the treatment process consists of a machine tool-handle-device-tool (MHD) that is mass-dissipative-elastic (MDE) system of the machine tool with the machining processes (turning, grinding, drilling, milling) [4, 21].

ABSTRACT
The article presents the models of technological systems and the parameters of the control object allowing one to seek rational control algorithms, choose the structure of the control system and synthesize the correction devices. The generalized and detailed structural schemes of controlling the elastic-deformable state during working on low-rigidity shafts while applying the tensile force as well as with additional feedback and transfer functions of technological systems, considering the assumptions made and the results of theoretical and experimental testing. The structure of the control object and the dependencies describing the system and considering the specificity of forming the section of the machined surface of the elastic-deformable low-rigidity shafts at longitudinal feeds. As a result of the research a method of increasing the accuracy of controlling the parameters of dynamic systems at the change of the longitudinal feed and regulating the parameters of the elastic-deformable state of low-rigidity elements by introducing correction devices in form of negative feedback in accordance with the cutting force. Moreover, the possibility of creating adaptive units which are not very sensitive to the change of parameters in turning and grinding.

Keywords: low-rigidity shafts, elastic-deformable shafts, low-rigidity, machining, accuracy, control, correction, elastic-deformable state
The analysis and synthesis of the modern automatic control systems is conducted with a significant simplification of the physical and mathematical dependencies describing the processes in the system [2, 5, 8]. This results, to a great extent, from the imperfection of the applied testing machine as well as the a priori obtained information on the static and dynamic characteristics of the control object as well as external disturbing interactions [3, 20]. The available methods of synthesis of the automatic control systems allow for a certain indeterminacy of the character and magnitude of disturbances [14, 23]. To some extent, the aspects of dispersion of the parameters of the control object are considered, although currently this problem is extensively researched [5, 14, 16, 22, 27].

Designing such control systems that can operate in uncontrolled changeability of the parameters of the control object led to applying adaptive control (AC) in machine construction [1, 6, 14]. Despite its versatility as well as the possibility of obtaining the required quality of transient processes in a wide variety of objects, it is difficult to use them on machine tools due to the need to constantly measure the characteristics of the technological systems and disturbances [17, 27].

In the construction of rational structures of control systems, it is necessary to consider the possibility of increasing the quality of the sample. Due to the fact that elastic deformation control system (relative displacement of the cutting tool and the workpiece) is a static one, both in terms of controlling and disturbing interactions, the change causes errors to the relative location of the part and the cutting edge [9, 17, 25, 26].

### The parameters of the elastic-deformable state of low-rigidity shafts

The control system for the parameters of the elastic-deformable state is essentially a closed system. The cooperation of its basic elements is geared by the interactions between the working processes occurring in the elastic system.

The dependencies describing the system at turning and grinding elastic-deformable low rigidity parts were shown in [11, 16, 23]. The dynamic properties of the linear models are approximated by the transfer functions of typical dynamic elements. The obtained dynamical models of the technological system and the parameters of control system allow one to seek optimal control algorithms, choose the structure of the control system as well as synthesize the corrective devices [3, 27]. A generalized structural scheme of controlling the elastic-deformable state of a low-rigidity part at the application of tensile force $F_{x_1}$ is shown in Figure 1.

The transfer function of the technological system, considering the assumptions and the results of theoretical and experimental testing can be expressed as follows:

$$g_{t}(s) = \frac{1 + K_{t}K_{m_{r}}[1 - e^{-\tau_s}]}{1 + + K_{t}K_{m_{r}} + K_{t}K_{m_{r}} + \cdots + K_{t}K_{m_{r}}}[1 - e^{-\tau_s}]K_{t}K_{m_{r}}[1 - e^{-\tau_s}]K_{t}K_{m_{r}}$$

(1)

The structural scheme of the adaptive control system of elastic deformation for elastic-deformable parts in the technological system with additional feedback $g_{x}(s)$ according to cutting force $F_{p}$ is presented in Figure 2.

Thereby the condition (2) of full unchangeability of the control system in relation to the control interaction is obtained.

$$g_{xz}(s) = \frac{1 + g_{x}(s)G_{x}^{*}(s)}{G_{x}(s)G_{x}^{*}(s)}$$

(2)

Therefore, in this case there is no static error according to the controlling interaction $y_{x}(s)$ with an additional feedback [19, 21]. As a consequence, stability, operational speed and insensitivity to the change of allowance at a negative feedback increase. The expression determining $G_{x}(s)$ can be presented as:

$$g_{x}(s) = \frac{1 - e^{-\tau_s}}{1 + K_{x}K_{y}m_{y}m_{x}^{-\tau_s}}$$

(3)

Considering the dependencies (1) and (3) the transfer function of the corrected control system can be written in the following form:

$$g_{x}(s) = \frac{1 + K_{x}K_{y}m_{y}m_{x}^{-\tau_s}}{1 + K_{x}K_{y}m_{y}m_{x}^{-\tau_s} + K_{x}K_{y}m_{y}m_{x}^{-\tau_s} + \cdots + K_{x}K_{y}m_{y}m_{x}^{-\tau_s}}$$

(4)

and the error of the system according to the controlling interaction is determined form the following dependency:

$$\varepsilon_{x}(s) = \frac{1 + K_{x}K_{y}m_{y}m_{x}^{-\tau_s} + K_{x}K_{y}m_{y}m_{x}^{-\tau_s} + \cdots + K_{x}K_{y}m_{y}m_{x}^{-\tau_s} + K_{x}K_{y}m_{y}m_{x}^{-\tau_s}G_{x}(s)G_{x}^{*}(s)}{1 + K_{x}K_{y}m_{y}m_{x}^{-\tau_s} + K_{x}K_{y}m_{y}m_{x}^{-\tau_s} + \cdots + K_{x}K_{y}m_{y}m_{x}^{-\tau_s} + K_{x}K_{y}m_{y}m_{x}^{-\tau_s}}$$

(5)

The dependency (5) indicates that the error $\varepsilon_{x}(s)$ introduced to the control system of the parameters of the elastic-deformable state of the elements in the technological system, control
interactions can be eliminated if the structure and parameters of the transfer function of the positive feedback $G_{sz}(s)$ is selected in accordance with the dependency:

$$G_{sz}(s) = \frac{1 + K_{sz} K_x n_z + K_{xy} n_y + K_{yy} n_y}{n_y K_F a G_y(s)}$$  \hspace{1cm} (6)

When the influence of the component $F_f$ of the cutting force on the increase in plastic strain with the coordinate $y$ the structural scheme of the control object is not considered, the technological system of turning elastic-deformable low-rigidity shafts, may be transformed into the form presented in Figure 3, whereas the transfer function is expressed with the following dependency:

$$G_{z1}(s) = \frac{1 + K_{zz} K_x + K_{zy} n_y + K_{yy} n_y}{n_y K_F a G_y(s)}$$  \hspace{1cm} (7)

The structural scheme of the control system with a feedback concerning the cutting force is shown in Figure 3, where the transfer function $G_y(s)$ is described with the dependency (3).

The error of the control system introduced by control interaction is described as:

$$\epsilon_{y1}(s) = y_y(s) - \frac{1 + K_{zz} K_x n_z + K_{zy} n_y + K_{yy} n_y}{1 + K_{zz} K_x n_z + K_{zy} n_y + K_{yy} n_y} G_y(s)$$  \hspace{1cm} (8)

In order to eliminate the static error $\epsilon_{y1}(s)$, introduced to the system by control interactions, the structure and parameters of the transfer function of the positive feedback $G_{z1}(s)$ ought to be determined in the following manner:

$$G_{z1}(s) = \frac{1 + K_{zz} K_x n_z + K_{zy} n_y}{n_y K_F a G_y(s)}$$  \hspace{1cm} (9)

When the influence of the elastic deformation after the coordinates $z$ and $x$ on the change
to the cutting depth (according to coordinate $y$) is not considered, the structural scheme of a certain model of the technological system of turning an elastic-deformable shaft may be presented as in Fig. 4a, where the transfer function $G_i(s)$ is expressed by the notion (3). The structural scheme of the corrected adaptive control system of the parameters of the elastic-deformable state of a low-rigidity shaft allowing for removing the static error by introducing additional positive feedback is shown in Fig. 4b. The transfer function of the corrected special system of the model is described by the following dependency:

$$G_{c1}(s) = \frac{1 + K_{w}n_y}{n_y K_{F_p} G_{w}(s)}$$

(12)

The structural scheme of the adaptive control system of the parameters of the elastic-deformable state was shown in Figure 5, where:

$$G_{c2}(s) = \frac{1 + K_{w}n_y}{n_y K_{F_p} G_{w}(s)}$$

(13)

The transfer function of the corrected control system considering (13) and (14) takes the form of:
The structure and parameters of the transfer function of the positive feedback $G_{sz3}(s)$ are expressed by the notion:

$$G_{sz3}(s) = \frac{1 + K_{x}n_{x} + K_{y}n_{y} + n_{y}K_{p}G_{w}(s) + \left[1 - e^{-\tau}\right]\frac{1}{m_{x}K_{p} + \left[G_{w}(s)G_{p}(s)\right]K_{p}[n_{y} - K_{p}n_{x}n_{y}]}}{m_{x}}$$

(15)

The dependency indicates that by determining the structure and parameters in accordance with (16) the static error introduced to the adaptive control system at oscillatory grinding by control interaction $y_{0}(s)$ can be eliminated.

**Adaptive control systems realizing the integral quality criteria**

Increasing the operating speed of the control systems as well as their quality can be achieved by applying the quality criterion of the corrected integral square estimation in controlling elastic deformation as well as the elastic-deformable state.

Considering the fact that elastic system can be perceived as a vibrating linear segment of the second order that is a one-mass system concentrated at the edge [4], the vibrations forced during the cutting process are expressed by the following dependency:

$$y_{0}(s) = \frac{G_{u}(s)G_{w}(s)\frac{1}{1 + G_{u}(s)G_{w}(s)}}{a_{0} + a_{1}s + a_{2}s^{2} + a_{3}s^{3}}f(s)$$

(17)

where: $b_{0} = K_{y}K_{p}$, $a_{0} = T_{p}^{2}$, $a_{1} = T_{p}^{2} + T_{p}$, $a_{2} = T_{p}^{2} + T_{p}$, $a_{3} = 1 + K_{p}$, $K_{p} -$ the displacement proportionality coefficient and cutting coefficient, respectively, $T_{p}$ – time constant of cutting (occurrence of the chip).

The dependency (17) can be the basis for research on the accuracy of the technological
systems and the choice of rational cutting parameters. The criterion suggested for characterizing the speed of damping the forced vibrations of elastic deformation $y(t)$ and their deviations from the steady state is the integral square estimation:

$$I = \int_0^\infty y^2(t)dt$$

Integral estimation (18) can be assumed as an accuracy criterion at controlling the elastic deformation of the technological system. The lesser the value of $I$, the lesser the vibration $y(t)$ of the disturbing interaction.

At the jump change $f(t)$ the $I$ value may be determined from the dependency:

$$I = \frac{b_0}{2a_5} \left( \frac{a_2}{a_3} + \frac{a_1^2}{a_1 a_2 - a_0 a_3} \right)$$

The $I$ criterion is a function of the time constant of the occurrence of chip $T_p$ and the cutting rigidity $K_p$. The dependencies $I = I(T_p)$ determined for the 16K20 machine tool with the parameters of the elastic system $m_y = 0.1$ [Ns$^2$/mm], $n_y = 7.75$ [Ns/mm], $C_y = 6.3 \times 10^4$ [N/mm] are presented in Figure 6a.

The analysis of the obtained dependencies indicates that:

- A visible minimum as well as possibility of minimizing the $I$ criterion occurs;

- It is possible to create an extreme adaptive control system with elastic deformation of the technological system;

- Migration of the extremum point at the change of allowance of the semi-finished product occurs.

The choice of the optimal cutting parameters is limited to determining the speed of cutting and longitudinal feed, minimizing the value $I$ depending on the allowance.

Determining the optimum value of the longitudinal feed depending on the change of the allowance is shown in the nomograph (Fig. 6b) obtained, in the case of treating C45 grade steel, using the S20S carbide cutting edge (P20 according to ISO 513) ($\alpha = 5$, $\zeta = 6$, $K = 4000$ [N/mm$^2$]). When selecting the rational cutting parameters as well as solving the optimization problem, the conditions to be fulfilled by the control interaction – in this case longitudinal feed – must be considered.

The algorithm of controlling the longitudinal feed can be realized using the adaptive control system shown in Figure 7.

Since the extreme characteristic is ambiguous, measuring only one variable does not enable to determine the location points of the cutting parameter regarding the extremum. Due to this fact, the automatic regulator ought to perform a

Fig. 6. Dependencies for the change: a) $I = I(T_p)$; b) nomographs for the expression $f(x)$.
trial displacement and after taking an additional measure of the extremum indicator, the location of this point in relation to the extremum should be determined [23]. At the certain direction of the change to the longitudinal feed, considering the symbol of the increase of the value $I$ and at the same time the state of the system regarding the extremum is considered. The location of the system obtained in such a manner is a logical problem, since the answer is attained upon comparing the signs $\Delta I$ and $\Delta f$. Instead of a logical comparison, the sign of the derivative can be assessed. This operation allows one to unequivocally determine the place of location of the point of work. If the derivative is positive, the point is located left from the extremum and right if negative. Determining the symbol $dI/df$ eliminates the ambiguity of the characteristic, since the functional link is linear and reaching the extremum point equals the derivative being zero.

The adaptive control system of the machine tool (Fig. 7) contains: dynamic system of the machine tool comprised of the cutting process (1) and elastic system (2) with a natural in the case of a dynamic system of the machine tool constraint $y$, as a control object; a controller of the longitudinal feed value (3), power transducer (4); longitudinal feed drive (5); reducer (6); feedback sensor regarding the magnitude of elastic deformation (7); square nonlinear converter (8); electric tachometric generator of the control of the longitudinal feed (9); motor (10); logical device (11) determining the symbol of the derivative “+” or “−” and controls the reverse device (12), which conveys the signal through the comparing device to the power transducer (4). A change to the frequency of the rotation of the shaft of the motor of the longitudinal feed (5) realised by the reducer (6) causes a change to the value of longitudinal feed $f$ in the direction of the minimum $dI/df$. The controller (3) limits the maximum and minimum value of the feed.

The transient characteristics of the processes were presented in Fig. 8a, where curve 1 characterizes the forced vibrations of elastic deformation in the case of the parameters of the cutting process: $v = 60$ [m/min], $a_p = 1.5$ [mm], $f = 0.7$ [mm/rotation]. The analysis of the transient process indicates the sufficiently fast damping of the forced vibrations of the elastic deformation. The significant vibrations of the transient process may be undesirable in some cases, e.g. in finishing, since the porosity of the surface worsens.

This defect can be eliminated by implementing the following corrected integral estimation as a criterion of the quality of the control system:

$$I_0 = \int_0^\infty \left[ y^2(t) + r_1^2 y(t)^2 \right] dt$$

(20)

Considering the speed of the change of elastic deformation in the technological system $y(t) = dy(t)/dt$ with the weight $r_1$ gives new properties to the adaptive control system.

The integral (20) can be written as:

$$I_0 = \int_0^\infty \left[ y(t) + r_1 \dot{y}(t) \right]^2 dt - 2r_1 \int_0^\infty y(t) \dot{y}(t) dt$$

(21)

Because $y(\infty) = 0$, then:
\[ I_0 = \int_0^\infty y(t) + \tau_1 y(t) \cdot \tau_1^2 dt + \tau_1 y^2(0) \]  

(22)

The dependency (22) indicates that the minimum of the integral square estimation is determined by the dependency:

\[ I_{0\text{min}} = \tau_1 y^2(0) \]  

(23)

Considering that \( y(0) = K_p y / (1 + K_p K_y) \), it can be rendered as

\[ I_{0\text{min}} = \tau_1 \left( \frac{K a_p}{C_y + K a_p} \right)^2 \]  

(24)

The dependency (24) enables to determine such cutting depth \( a_p \) at which the required accuracy of the element is obtained, provided the values of the elastic element of the machine tool \( C_y \) and the cutting force are known. The numerical value of the weight coefficient \( \tau_1 \) determines the speed of the system operating as well as the fluency of the progression of the transient processes. The integral estimation (21) can be written as:

\[ I_0 = \int_0^\infty y^2(t) dt + \tau_1^2 \int_0^\infty y^2(t) dt = \int I_1 \]  

(25)

The \( I_1 \) value is determined from the following dependency:

\[ I_1 = \frac{(K_y K_p)^2 a_1 a_3^2}{2a_3^3(a_1 a_2 - a_0 a_3)} \]  

(26)

The corrected integral estimation takes form of

\[ I_0 = \frac{(K_y K_p)^2}{2a_3^3(a_1 a_2 - a_0 a_3)} \left( a_1 a_3^2 - a_0 a_2 a_3 - a_1^2 a_3 - \tau_1^2 a_1 a_3^2 \right) \]  

(27)

The dependency of the criterion \( I_0 \) on the time constant of the occurrence of chip \( T_s \) at the constant rigidity of cutting \( K_y = 6000 \) [N/mm] and various values of the weight coefficient \( \tau_1 \) are shown in Figure 8b. An analysis of the obtained dependencies indicates that decreasing the vibrations of the transient processes in the technological system reduces the accuracy of the element as a result of the increase in the weight coefficient \( \tau_1 \).

The choice of the optimum cutting parameters ought to consider the requirements in terms of the accuracy and quality of the transient processes. Eliminating the vibrations of the elastic deformation is only possible at great values of the time constant of the occurrence of the chip. Due to this fact, the cutting process ought to be conducted at sufficiently high values of longitudinal feed and low cutting speeds.

The curve (2) of the transient process of the forced vibrations of elastic deformation in the technological system in the case of \( \tau_1 = 0.5 \cdot 10^{-3} \) [s] and the values of the parameters of the cutting process \( v = 20 \) [m/min], \( a_p = 1.5 \) [mm], \( f = 0.9 \) [mm/rotation] is shown in the Figure 8a.

The block scheme of the adaptive control system, forming the functional – integral estimation

**Fig. 8.** Transient characteristics in the adaptive control system: a) curve 1 with implementing the integral square estimation, curve 2 of the corrected integral square estimation; b) dependencies of the criterion \( I_0 = f(T_s) \) in the case of various values of the weight coefficient \( \tau_1 \)
of the likelihood of vibration in the transient process is shown in Figure 9. Compared to the scheme in Figure 7, this system also comprises: derivative controller (9), square nonlinear converter (10), multiplier (11), adder (12), electric tachometric generator (13), divisor (14), logical device (15), adjustment element (16) and adder (17).

The elastic deformation $y$ occurring in the sensor (7) during the treatment of the elements are converted to the signal $U_y$ transferred through the square nonlinear converter (8) to the adder (12), on the second input of which the output signal from the chain containing the blocks 9–11 is transferred. As a result, a proportional signal is formed in the output of the adder

$$\frac{dI}{dt} = y^2(t) + \tau^2 y^2(t)$$

The value of the longitudinal feed $f$ is transformed in the electric tachometric generator (13), then the signals $df/dt$ and adder $dI/df$ are transmitted to the divisor (14) determining the value $dI/df$. The logical device (15) controls the element (16) in accordance to the symbol of the derivative $dI/df$. The output signal is sent to the output of the adder (17), and then to the transducer (3), motor (5), which changes the feed of the edge in the direction of the minimum of the dependency $dI/df$ using the reducer (6).

**CONCLUSIONS**

Within the research conducted, the dynamic models of the technological systems and the parameters of the control object were designed, which allows one to determine the optimum control algorithms, select the proper structure of the control system and synthesize the correction devices.

The analysis of the systems of automatic control for elastic deformation of the technological system indicates that these systems are static as a result of both the controlling and disturbing interaction, leading the relative location error for the part and the cutting edge. The accuracy and quality of the treatment can be increased by creating optimum structures of the control system and designing adaptive control systems.

The method of correcting the technological system as a result of introducing positive feedback according to the cutting force eliminates the static errors of controlling and disturbing interactions in controlling the parameters of accuracy and quality (the regulating interactions are changes of the longitudinal feed and the parameters of the elastic-deformable state of a low-rigidity part in the technological system).

The accuracy of controlling the parameters of the technological systems, change to the longitudinal feed and regulation of of the elastic-deformable state parameters of a low-rigidity element can be increased by implementing the negative feedback in accordance to the cutting force. The conditions for determining the structure and parameters of the additional feedback in accordance to the cutting force, ensuring elimination of the static errors in the system and giving it the properties of an adaptive system.

Introducing the correction elements in the form of negative feedback in accordance to the cutting force enables to increase the accuracy of controlling the parameters of technological systems as a result of a change to the longitudinal feed and regulation of the elastic-deformable state parameters of low-rigidity elements. This allows one to build adaptive systems with low sensitivity to the change of its parameters in the turning and grinding processes. The developed
methods of placement correction and the adaptive control systems can be implemented in other machining processes.

A better working speed and quality of the adaptive control systems can be obtained by implementing integral square estimation and a corrected integral square estimation as an accuracy criterion in controlling the elastic deformation in the elastic-deformable state. Numerical modeling of the curves of the transient processes in the control system indicates a sufficiently fast damping of the forced vibrations of elastic deformation in the technological system as well as the possibility to obtain elements of high quality.

Abbreviations

- \( F_p \) Cutting force
- \( F_{x1} \) Tensile force
- \( G_{sz}(s) \) Full unchangeability of the control system
- \( y_0(s) \) Controlling interaction
- \( \varepsilon_y(s) \) The error of the control system
- \( \varepsilon_{yz}(s), \varepsilon_{xp}(s) \) The static errors of the control system
- \( T_p \) The time constant of the occurrence
- \( K_p \) The cutting rigidity

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