Motion of particles (bodies) in presence of random effects can be considered stochastic process. However, application of widely known stochastic processes used for description of particle motion is reduced to relatively small class of particle transport phenomena. Stochastic state-transition-change (STC) process is suitable for description of many systems. In this paper it is shown under which assumptions formulae of time resolved velocity spectrometry can be derived with the help of STC process. It opens up new possibilities of unified description of particles moving in a force field in presence of random effects. It extends possibilities of applications of theory of stochastic processes in physics.

Keywords: stochastic state-transition-change process, stochastic particle motion, time resolved velocity spectrometry, particle transport

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I. INTRODUCTION

Motion of particles, or any other bodies, under various conditions has been studied in physics for a very long time. In some cases the motion can be described deterministically using, e.g., Newton’s second law of motion. In some other cases it is necessary to take into account several random effects. Brownian motion (random walk) is well known example of stochastic process. Stochastic cyclotron motion [1] is another example of stochastic process. It introduces randomness to motion of a charged particle in a magnetic field by adding dissipation and fluctuation terms to the corresponding deterministic equation of motion (the terms can be added to any ordinary differential equation having time as an independent variable).

However, widely known stochastic processes are not suitable for description of all particle transport phenomena in presence of random effects. Stochastic state-transition-change (STC) process introduced in sect. III B in [2] extends possibilities of descriptions of motion of particles which may have random initial properties (states), may or may not reach given position, and may or may not change their properties during transport. It will be shown in this paper under which conditions (assumptions) it is possible to derive main formulae of time resolved velocity spectrometry using STC process.

This paper is structured as follows. Derivation of formulae of widely known time resolved velocity spectrometry with the help of stochastic STC process is in sect. II. The spectrometry is one of well known time-of-flight (TOF) methods which allows determination of spectrum of emitted particles from a source as a function of emission time and velocity on the basis of experimental data, see sect. III. The possibilities of generalization of the time resolved velocity spectrometry with the help of STC stochastic process are discussed in sect. IV. Concluding remarks are in sect. V.

II. PROBABILITY MODEL - TIME RESOLVED VELOCITY SPECTROMETRY

A. Stochastic process

Stochastic STC process can describe particle motion of particles in a force field when initial conditions are characterized by probability (density) functions.

Consider a source emitting particles of different speeds in the same direction and at different times as an example of non-trivial particle motion. It may not be possible to detect the particles at the place where they are emitted but it may be possible to measure some quantities characterizing the particle transport at several distances $x_i$ from the source ($i \in \{0, \ldots, M\}$, $M > 0$, $x_i < x_{i+1}$ and the spatial $x$-axis has the same orientation as the direction of the velocities of the particles). One may ask how to determine the characteristics of the emitted particles at the place where they are emitted on the basis of quantities which can be measured.

Assumption II.1 (Time interval of emitted particles). The particles were emitted in a burst in time interval from $t_{\text{min}}$ to
Variables

The transport (considering only inertial motion).

Definition II.1.

If assumption II.4 holds then function \( \rho_{C,i}(X_i, X_{i+1}) \) defined by eq. (52) [2] can be written also as

\[
\rho_{C,i}(x_i, v_i, t_i, x_{i+1}, v_{i+1}, t_{i+1}) = \rho_{C,i}(X_i, X_{i+1})
\]

where non-random variables \( x_i \) and \( x_{i+1} \) have been written explicitly, and \( i \in (0, \ldots, M - 1) \). The function \( \rho_{C,i}(X_i, X_{i+1}) \) has meaning of probability function that a particle at position \( x_i \) of properties characterized by random variables \( X_i \) had properties characterized by random variables \( X_{i+1} \) at position \( x_{i+1} \).

Definition II.2. Let \( v_i \) be defined by eq. (45) in [2] can be written also as

\[
\rho_{C,i}(X_i, X_{i+1}) = \frac{\nu}{t_{i+1}^\min} \cdot \frac{\nu}{t_{i+1}^\max}
\]

\[
\rho_{C,i}(x_i, v_i, t_i, x_{i+1}, v_{i+1}, t_{i+1}) = \delta(t_i - t_{i+1} + \frac{x_{i+1} - x_i}{v_i})
\]

\[
\rho_{C,i}(x_i, v_i, t_i, x_{i+1}, v_{i+1}, t_{i+1}) = \delta(t_i - t_{i+1} + \frac{x_{i+1} - x_i}{v_i}).
\]
Proposition II.7. It holds (for all $i \in (0,\ldots,M-1)$)
\[
\text{dos}_{i+1}(x_{i+1},v_{i+1},t_{i+1}) = \begin{cases} 
\text{dos}_i(x_i,v_i) = v_{i+1} - \frac{x_{i+1} - x_i}{v_{i+1}} \\
0 
\end{cases}
\]
if $t_{i+1} \in \langle t_{i+1} \rangle$, and $v_{i+1} \in [v_{\min},v_{\max}]$
otherwise
\hspace{1cm} (15)
where $t_{i+1}^{\min}$ and $t_{i+1}^{\max}$ can be determined using proposition II.2. Equivalently, it holds (for all $i \in (0,\ldots,M)$)
\[
\text{dos}_i(x_i,v_i,t_i) = \begin{cases} 
\text{dos}_0(x_0,v_0,v_i) = v_i - \frac{x_i - x_0}{v_i} \\
0 
\end{cases}
\]
if $t_i \in \langle t_i \rangle$, and $v_i \in [v_{\min},v_{\max}]$
otherwise
\hspace{1cm} (16)
where $t_i^{\min}$ and $t_i^{\max}$ are given by eqs. (12) and (13).

Proof. According to assumptions II.2 and II.3 it must hold $v_i = v_{i+1}$. The number of particles in interval $(v_i, v_i + dv_i) \times (t_i, t_i + dt)$ at position $x_i$ and time $t_i$ divided by $dv_i dt_i$ is equal to $\text{dos}_i(x_i,v_i,t_i)$ (see definition of DOS given by eq. (38) in [2]). The particles reach $x_{i+1}$ at time $t_{i+1}$ given by eq. (8). It implies $\text{dos}_{i+1}(x_{i+1},v_{i+1},t_{i+1}) = 0$ in regions of $t_{i+1}$ and $v_{i+1}$ which are outside physical region. The equivalence of eqs. (15) and (16) can be proven using proposition II.1.

Remark II.2. Proposition II.5 can be derived in another way using theorem III.1 in [2] and eq. (3) and function $\rho^{C,j}(x_i,v_i,t_i,x_{i+1},v_{i+1},t_{i+1})$ expressed as a 2-dimensional delta function corresponding to assumptions II.2 to II.4. To work with n-dimensional delta functions is, however, in general more delicate than in 1-dimensional case.

Proposition II.6. It holds
\[
\text{dos}_i^y(x_i,t_i) = \begin{cases} 
\int_{v_{\min}}^{v_{\max}} \text{dos}_i(x_i,v,v_i)dv_i = 0 \\
\int_{v_{\min}}^{v_{\max}} \text{dos}_i(x_i,v,v_i)dv_i 
\end{cases}
\]
if $t_i \in \langle t_i \rangle$, and $v_i \in [v_{\min},v_{\max}]$
otherwise
\hspace{1cm} (17)
where $t_i^{\min}$ and $t_i^{\max}$ are given by eqs. (12) and (13).

Proof. Insertion of $\text{dos}_i(x_i,v_i,t_i)$ given by eq. (5) into eq. (5) implies eq. (17).

Proposition II.7. It holds (for all $i \in (0,\ldots,M-1)$)
\[
\text{dos}_{i+1}^y(x_{i+1},t_{i+1}) = \int_{v_{\min}}^{v_{\max}} \int_{t_i}^{t_{i+1}} \text{dos}_i(x_i,v_i,t_i)\delta(t_i - t_{i+1} + \frac{x_{i+1} - x_i}{v_i})dt_idv_i,
\]
if $t_{i+1} \in \langle t_{i+1} \rangle$ (see eqs. (10) and (11)), otherwise $\text{dos}_{i+1}^y(x_{i+1},t_{i+1}) = 0$. 

Proof. Let us consider $x_j = (x_j,v_j,t_j)$ for $j \in (0,\ldots,i)$ and $x_{i+1} = (x_{i+1},t_{i+1})$ for fixed index $i \in (0,\ldots,M-1)$. Theorem III.1 in [2] and eqs. (3) and (14) imply eq. (19).

Proposition II.8. It holds (for all $i \in (0,\ldots,M-1)$)
\[
\text{dos}_{i+1}^y(x_{i+1},t_{i+1}) = \int_{v_{\min}}^{v_{\max}} \text{dos}_i(x_i,v_i,v_{i+1})dv_{i+1},
\]
if $t_{i+1} \in \langle t_{i+1} \rangle$, otherwise $\text{dos}_{i+1}^y(x_{i+1},t_{i+1}) = 0$. Equivalently (for all $i \in (0,\ldots,M)$)
\[
\text{dos}_i^y(x_i,t_i) = \int_{v_{\min}}^{v_{\max}} \text{dos}_i(x_0,v_0,v_i)dv_i,
\]
if $t_i \in \langle t_i \rangle$, otherwise $\text{dos}_i^y(x_i,t_i) = 0$.

Proof 1. By integrating eq. (15) over $v_{i+1}$ and using eq. (17) one obtains eq. (21). By integrating eq. (16) over $v_i$ and using eq. (17) one obtains eq. (22). The equivalence of eqs. (21) and (22) can be proven using propositions II.1 and II.5.

Proof 2. Performing the integration over $t_i$ in eq. (19) implies eq. (21) ($v_i$ in eq. (19) and $v_{i+1}$ in eq. (21) are only integration variables, they can be renamed).

III. ANALYSIS OF EXPERIMENTAL DATA

Time dependent densities of states $\text{dos}_i(x_i,t_i)$ can be measured at several positions $x_i$. Unknown parameters $v_{\min},v_{\max}$, $t_i^{\min}$ and $t_i^{\max}$, and unknown function $\text{dos}_0(x_0,v_i,t_0)$ (time resolved velocity spectrum of emitted particles) can be determined on the basis of the measured data with the help of formulae derived in sect. II B (see mainly propositions II.2, II.5, II.6 and II.8), and (constrained) optimization techniques as discussed in sect. IV in [2].

If we put $M = 1$, $x_i = 0$, $x_{i+1} = x$, $t_i = 0$, $t_{i+1} = \tau$, $v_{\min} = v_1$, $v_{\max} = v_2$, $t_i^{\min} = 0$ and $t_i^{\max} = \Delta T$ then eqs. (19) and (21) are equivalent to eq. (2) in [3] where an extension of the time-of-flight (TOF) method for determination of the time resolved velocity spectrum of particles emitted in intense bursts has been presented for the first time. Many useful comments to the time resolved velocity spectrometry method are in [3] (including numerical solutions and tests).

In [4] this method is called extended TOF method to distinguish it from basic TOF method in which velocity spectrum of particles is determined independently on the emission time. It is shown in [4] under which conditions the former method is reduced to the later one (in the case of relatively short intense burst in comparison to the time of flight of particles from source to a detector). The basic TOF method provides less information, but it is significantly easier to use it from both the experimental and data analysis point of view (it is sufficient to use only one detector in sufficiently large distance from the source ensuring that $\Delta T$ is much smaller than the time needed...
by an emitted particle to travel from the source to the detector.

Both the types of TOF methods are widely known and have been adapted and successfully applied in various experiments. E.g., with the help of the extended TOF method time resolved neutron energy spectra from D(d,n)3He fusion reactions were determined in [4] using analog Monte Carlo reconstruction method (AMCRT). In sect. 2.2.1 in [4] several other existing reconstruction methods (algorithms) are mentioned. Efficiency of different methods depends on several factors including the dependence of the density of states $\text{dos}(x_0, v_0, t_0)$ (i.e., also measured densities of states $\text{dos}(x, t)$) which one is trying to determine.

The basic TOF method was applied, e.g., to experimental data of electrons and ions emitted by laser-produced plasmas in [5–8]; further details to TOF spectra for mapping of charge density of ions produced by laser are in [9].

IV. GENERALIZATION OF TIME-OF-FLIGHT METHODS

It has been shown in sect. II that with the help of stochastic STC process introduced in sect. III B in [2] leads to the well known time-of-flight (TOF) method for determination of time resolved velocity spectrum of emitted particles from a source (TOF-TV). There are other experimental techniques based on measurement of TOF, such as the time-of-flight mass spectrometry (TOF-MS). This method uses an electric field of known strength to determine mass-to-charge ratio of particles (ions). An electric and magnetic fields of known strength are commonly used (in various configurations) to determine mass and electric charge of charged particle by measuring and analysing trajectories of the particles with the help of the Lorenz force. All time-of-flight methods and many methods of mass spectrometry have something in common. They concern particle transport phenomena under various conditions and their aim is to determine properties of the particles.

Each of the methods is typically suitable for determination of partial spectra being functions of only some variables characterizing properties of particles emitted from a source such as mass, electric charge, velocity (energy) or time of their emission from the source. E.g., the TOF-TV method allows determination of time resolved velocity spectrum (function of “only” two random variables characterizing the emitted particles). With the help of stochastic STC process it is possible to generalize the formulae in sect. II B (TOF-TV method) by taking into account:

1. non-zero external force
2. general initial and final positions and velocities of particles
3. various properties of particles such as mass, electric charge, etc.

Application of this generalized TOF-TV method to data can be more complicated than application of the TOF-TV method to data (it may be necessary to consider more random variables). It requires more experimental information. One can take advantage of various experimental methods determining the “partial” spectra (integrated over some of the variables) to constrain the “full” spectrum, see general guidelines in sect. IV in [2]. One can then derive a more general equation than eq. (21). This allows to study forces acting on particles and their properties (some of them may be specified by random variables). Inertial mass increase in dependence on velocity may be also studied on the basis of experimental data under these conditions [10].

V. CONCLUSION

Stochastic STC process introduced in sect. III in [2] can significantly help to improve existing or develop new techniques of measurement of properties of particles moving in an external force field and being specified by random variables. The measurement is essential for characterization of various sources of emitting particles (see, e.g., laser-produced plasmas mentioned in sect. III, or development of deuterium z-pinch as a powerful source of multi-MeV ions and neutrons [11]). The sources of known properties can be used for various applications. The new types of sources of emitted particles place extra demands on particle detectors (see, e.g., design of a scintillator calorimeter for laser-plasma characterization, or magnetic electron spectrometer spectrometer [12]).

With the help of stochastic STC process it is possible to describe in a unified way motion of particles in an external force field and many other particle transport phenomena which looks very distinct at first glance, such as transmission of light through sequence of polarizers [13]. Several other applications of stochastic STC process for description of (physical) systems are discussed in [2].
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