Thermal conductivity of Toyoura sand at various moisture and stress conditions

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Abstract. Thermal conductivity of soils is a crucial characteristic in various geotechnical applications, such as geothermal pumps, energy piles and buried pipelines. Previous researchers have done extensive works on the factors that may affect the soil thermal conductivity, including soil porosity, degree of saturation, mineralogy, testing temperatures, particle size and gradation. A modified oedometer frame that can incorporate the transient heat probe method is adopted to investigate the influence of stress state on thermal conductivity of Toyoura sand. Preliminary test under 1-D compression shows that the thermal conductivity of sand increases with the rise of vertical stress, and the variation exhibits hysteresis during a loading and unloading cycle. In addition, the effects of void ratio and water content were also studied and test results agreed well with previous values reported in the literature.

1 Background

Due to the increasing cost of energy and the environmental impacts of burning fossil fuels for energy, geothermal structures and energy technologies are gaining more and more attentions. One of the most important factors affect the performance of geo-energy sources or sinks is the thermal properties of soils [1], which governs the heat transferring or preservation in related projects. There are mainly three ways of heat exchange existing, which are conduction, fluid convection and radiation, respectively [2]. The dominant heat transferring happened in soils is through conduction, except for cases where there is significant fluid flow occurring [3].

There has been extensive studies about the thermal conductivity of soils with a consideration of various factors, such as void ratio, degree of saturation, particle size, mineral component, salt concentration, microstructures, etc. [4-13]. However, the studies about the stress effect on the soil thermal conductivity are limited [14; 15]. Although there are efforts made regarding the pressure/stress effect on the thermal properties of rocks [16;17], the mechanism of stress-heat coupling of soil has not been fully understood.

This paper presents some preliminary test results regarding the stress effect on thermal conductivity, through a series of measurements carried out on a standard material, Toyoura sand. A traditional oedometer is modified to allow transient method at different one-dimensional stress level. Besides, the influences of void ratio and water content were also tested and compared with existing data.

2 Test method

There are two widely used categories for measuring the thermal conductivity of soils, which are steady state method and transient method, respectively [18].

In the steady state measurement, usually there is a heat resource and a relatively low-temperature heat sink set at the edges of specimen to generate heat flow, with environmental conditions controlled. When the temperature recorded at the boundaries remains constant, it is indicated that the heat flow comes to a steady state. Then the thermal conductivity is determined by measuring the temperature difference at a distance, together with the steady state heat flow through the specimen. The steady state methods is easy to carry out analysis, but it is often time consuming to reach a thermal equilibrium, moreover, the errors from heat losses and contact resistance of sensors are also difficult to quantify [19].

On the other hand, various transient techniques have been developed to overcome the shortage of steady state methods, which requires measurement of temperature change of heat source and the corresponding amount of heat generated. The material properties are determined while the sample temperature still changes. Only a short measurement time is required and the sample dimensions do not necessarily enter the equation.

The current study uses the transient method. A small size non-steady-state probe, denoted as TP08 (Hukseflux Thermal Sensors), was adopted, which is also suggested by ASTM D5334 [20].The principle of this transient probe method relies on a unique property of a line source: after a short transient period the temperature rise, ΔT, only

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depends on heater power, $Q$, and medium thermal conductivity, $\lambda$, as shown below:

$$\Delta T = \frac{Q(\ln t + B)}{4\pi \lambda}$$  \hspace{0.5cm} (1)

with $\Delta T$ in K, $Q$ in W·m$^{-1}$·s$^{-1}$, $\lambda$ in W·m$^{-1}$·K$^{-1}$, $t$ the time in seconds and $B$ is a constant.

The thermal conductivity can be calculated from two measurements at $t_1$ and $t_2$, as follow:

$$\lambda = \frac{Q[\ln(t_2 \div t_1)]}{4\pi \Delta T}$$  \hspace{0.5cm} (2)

where $t_1$ and $t_2$ is the chosen start and end point for calculation in heating stage.

For sensor TP08, it is suitable for measuring the thermal conductivity of granular materials such as soils, gravel (grain size smaller than 0.5mm) and soft rock, within the range of 0.1 to 6 W·m$^{-1}$·K$^{-1}$. The applied stress level cannot exceed the durability of probe. The normal and shear stress between particles and probe may change the interface contact, thus lead to variation of measurements. The expected accuracy for carefully made measurements of suitable media at 20 °C is +/-0.02% W·m$^{-1}$·K$^{-1}$. At temperatures different from 20 °C, the temperature dependence of the sensor must be taken into account, which adds extra uncertainty about +/- 0.02%/K.

3 Thermal conductivity of Toyoura sand

3.1 Test set-up

3.1.1 Test soil

Toyoura sand, from Yamaguchi, Japan, is a well-sorted fine sand, which is commonly used as standard sands in various civil engineering testing and applications. It mainly contains light brown, rounded to sub-angular grains, with particle size from 0.1 mm to 0.2 mm. The mineral component includes 75% of quartz, 22% of feldspar, and 3% of magnetite [21]. After dry density tests, the minimum and maximum dry density are about 1.33 and 1.65 g/cm$^3$, respectively [22; 23]. Some basic properties of Toyoura sand are given in Table 1.

| Specific gravity $G_s$ | 2.65 |
|-----------------------|------|
| Mean diameter $D_{50}$ | 0.17 mm |
| Uniformity coefficient $U_c$ | 1.7 |
| Maximum void ratio $e_{max}$ | 0.98 |
| Minimum void ratio $e_{min}$ | 0.62 |

3.1.2 Equipment

TP08 thermal probe together with oedometer and modified container were adopted in measurement of thermal conductivity. A sketch of test setup is shown in Figure 1 below.

![Fig. 1. Measurement of thermal conductivity.](image)

The needle in sensor TP08 is equipped with a heating wire. Providing that the resistance is known, the energy generated is controlled by the heating time. There is also a thermal-couple at the tip of the needle, and a PT1000 RTD sensor outside the heating area, serving the purpose of detecting the needle and environmental temperature, respectively.

Additionally, existing oedometer is modified with a new frame to accommodate a large sample with diameter of 100 mm and height of 100 mm. The purpose of adopting a large container is to reduce the boundary effect as much as possible, with combination of choosing a low heating power. The manufacturer suggests a minimum sample radius of 30 mm and a minimum needle-covering length of 35 mm. The one-dimensional pressure can still be applied through oedometer. The vertical stress achieved using this new frame at the top surface of sample can be up to 1.2 MPa.

3.1.3 Test procedure

The dry soil samples were prepared using vibration method; the unsaturated samples were prepared using compaction method with desired water content; the fully saturation samples were vacuum saturated from dry samples and submerged in water during the whole test process.

In each measurement, first 30 min to 1 hour is spent to ensure there is no change of room temperature. After thermal equilibrium is reached, the switch of heating wire is turned on, while the current is adjusted to provide different power according to the thermal properties of samples. The heating normally lasted for 3 min, after trying different time periods. Then the heating circuit was immediately turned off, and the temperature recording was continued for the same time as heating. Although it was stated that the probe method provides reliable measurements [21], more than 3 measurements were repeated for each sample.
3.2 Results and discussion

3.2.1 Calculation of \( \lambda \)

Thermal conductivity is calculated according to the linear section of temperature-logarithmic time curve. To choose this linear section, two parameters need to be determined, which are the start time and duration. Additionally, the cooling data is suggested to be adopted in calculation to avoid the possible temperature drift during test [20], and the corresponding log-time coordinate needs to be recalculated as:

\[
t' = t/(t-t_h)
\]

(3)

where \( t_h \) is the duration of heating.

The selection of linear section could be subjective. It is recommended by ASTM that the selection should be fixed for all measurements to avoid any biasing results. There is also study [24] of sensitivity of thermal conductivity to start point and duration, where the part of curve corresponding to low sensitivity is suggested to be adopted. This kind of selection does not explain why the convergence point can represent the actual properties, it is in fact, still depends on subjective judgement to some extent.

In tests reported herein, the processing of data is conducted as follows: the first 20 to 80 seconds of heating and cooling data is fixed, and the average value is taken as the final value of each measurement; the resulted R-squared for linear regression is required to be higher than 0.98, otherwise the test will be repeated.

3.2.2 Effect of void ratio

Dry samples were prepared with different pre-determined void ratios, and then the thermal conductivity of each sample is measured, the results are shown in Figure 2 below.

![Graph of thermal conductivity vs. void ratio](image)

**Fig. 2.** \( \lambda \) of dry Toyoura sand at different void ratio.

There were some measurements regarding the thermal conductivity of Toyoura sand reported before [21]. It is found that the thermal conductivity of dry Toyoura sand is around 0.2 and 0.3, and it increases linearly with the decrease of void ratio in the range from 0.6 to 1. The results from current tests generally agree with previous one, and the maximum deviation is around 10% (the point corresponding to void ratio of 0.81).

However, based on tests conducted herein, it appears that the thermal conductivity comes to a stable stage or even decreases slightly when the sand approaches the densest state. The difference is within the error of this transient method, so this observation may need verification from other kinds of thermal conductivity tests.

Results of this series of tests also serves as a verification of equipment and test parameters, such as the linear section choice, which will be followed in other tests. It provided confidence for the setup of the entire system.

3.2.3 Effect of saturation degree

The effect of degree of saturation on the thermal conductivity of Toyoura sand was also investigated.

Soil specimens with different water content were prepared with void ratio of 0.72 ± 0.01, and the corresponding saturation degree is calculated. The water content of soil mass was weighted before and after thermal conductivity measurements, and the results is summarized in Table 2. Note that the water content before tests was calculated based on weight measurement during specimen preparation, while the water content after tests was based on three samples taken at different depth from the specimen, and the average value is presented herein.

| Test No. | Before test | After test |
|----------|-------------|------------|
|          | Sr(%) | \( \omega \)(%) | Sr(%) | \( \omega \)(%) |
| 1        | 0     | 0          | 0     | 0          |
| 2        | 18.5  | 5.4        | 18.1  | 4.9        |
| 3        | 43.0  | 11.6       | 41.1  | 11.1       |
| 4        | 63.0  | 17.0       | 61.1  | 16.5       |
| 5        | 81.5  | 22.0       | 77.4  | 20.9       |
| 6        | 88.9  | 24.1       | 80    | 21.6       |
| 7        | 100   | 27.0       | 100   | 27.0 (submerged) |

The previous measurements of the same material was also collected, and the comparison with current results is shown in Figure 3.

Similarly as the dry sand, some data from the literature [25] was extracted and presented for comparison. Generally, these two series of tests agree with each other, though the current tests exhibit a large deviation, especially for the one with high water content, which can be up to 15%.
3.2.4 Preliminary test of stress effect

Utilizing the modified loading frame, the thermal conductivity of Toyoura sand under one dimensional loading and unloading was also measured.

A dry soil specimen was prepared with void ratio of 0.714. The vertical stress was increased to 1,200 kPa by several stages with the settlement at the top surface of specimen recorded. The recorded magnitude of compression was less than 0.1mm, which corresponds to vertical strain of 0.1%, and the resulted change of void ratio is less than 0.002. This is confirmed with previous 1-D silica sand compression test [26], which shows that the change of void ratio is marginal until the reach of yield stress (usually larger than 2 MPa). The varying thermal conductivity with vertical stress is presented in Figure 4.

Preliminary test results show that the thermal conductivity of dry sand increases with the rise of vertical stress, and the increase is about 0.1 W·m⁻¹K⁻¹ for a load of 1.2 MPa, which is around 40% compared to that of soil under no stress. Meanwhile, the change of thermal conductivity shows slight hysteresis during a loading and unloading cycle. The value corresponding to the unload condition was observed to be slightly higher than that on the loading path. However, this phenomenon needs to be verified by more tests.

4 Summary

Thermal conductivity of Toyoura sand was experimentally investigated using transient methods, where a thermal probe was inserted into soil specimens and temperature changes during heating and cooling process were recorded. The effects of void ratio and water content (degree of saturation) was tested. It was found that the thermal conductivity increases with the decrease of void ratio and with the increase of water content, which agrees well with previous reported measurements.

The stress effect was also examined through a modified oedometer frame that is able to incorporate thermal probe. Preliminary tests showed that thermal conductivity increases with vertical stress level, and that hysteresis may exist during loading and unloading.

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