Quantum key distribution using bright polarized coherent states

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Abstract

We propose a novel protocol for quantum key distribution based on the quantum polarization properties of the coherent states. The qubits are encoded either in the component amplitude difference or in the phase difference of strong coherent fields. The quantum counterpart (Stokes operators) of the classical parameters which represent the preponderance of $x$-linear polarization over $y$-linear polarization and phase difference information of the electrical field components are used as the non-commuting operators which are essential, together with the no-cloning theorem, for the security of our protocol.

1 Introduction

The stimulating question whether quantum continuous variables may provide a valid alternative to the usual “single photon” quantum key distribution schemes [1] has been answered by Grangier et al using Gaussian-modulated coherent states [2, 3, 4]. In particular, Grangier have shown that there is actually no need for squeezed and entangled beams because an equivalent level of security is obtained by simply generating and transmitting random distributions of coherent states. The security of these protocols is related to the continuous variable version of the no-cloning theorem [5, 6], which limits possible eavesdropping.

In contrast to Grangier’s work, which used homodyne detection to measure the amplitude and phase of coherent light pulses, we propose a novel protocol based on the quantum polarization properties of the coherent states. The qubits are encoded either in component amplitude difference or in the optical phase difference of strong coherent fields. The quantum counterpart (Stokes operators) [7, 8] of the classical parameters [9], which represent preponderance of $x$-linear polarization over $y$-linear polarization and phase difference information of the electrical field components are used as the non-commuting operators which are essential, together with the no-cloning theorem [5, 6], for the security of our protocol.

The main advantages of our scheme are: readily available sources of coherent light; robustness of coherent states upon dissipation during the transmission; the measurement of the Stokes operators is easily accomplished with the presently available technology [10], and requires just linear optical elements such as beam splitters, wave plates and direct photocounting, making unnecessary more complicated set ups such as homodyne detection, which requires a separate (local oscillator) beam.

2 Stokes operators

The Hermitian Stokes operators are defined analogously with their classical counterparts [9], and may be written as [7]

\[ \hat{S}_0 = \hat{a}_x^\dagger \hat{a}_x + \hat{a}_y^\dagger \hat{a}_y = \hat{n}_x + \hat{n}_y = \hat{n} \] (1)

\[ \hat{S}_1 = \hat{a}_x^\dagger \hat{a}_x - \hat{a}_y^\dagger \hat{a}_y = \hat{n}_x - \hat{n}_y \] (2)

\[ \hat{S}_2 = \hat{a}_x^\dagger \hat{a}_y + \hat{a}_y^\dagger \hat{a}_x \] (3)

\[ \hat{S}_3 = i (\hat{a}_y^\dagger \hat{a}_x - \hat{a}_x^\dagger \hat{a}_y) \] (4)

where $\hat{a}_x^\dagger$ ($\hat{a}_x$) and $\hat{a}_y^\dagger$ ($\hat{a}_y$) denote the creation (annihilation) operators associated with $x$ and $y$ photon polarization modes, and $\hat{n}$ is the total photon-number operator. The creation and annihilation operators satisfy the usual commutation relations,
\[
\{\hat{a}_j, \hat{a}_k^\dagger\} = \delta_{jk}, \quad j, k = x, y.
\] (5)

Therefore the commutation relations for the Stokes operators are
\[
[\hat{S}_0, \hat{S}_i] = 0, \quad i = 1, 2, 3,
\] (6)

and
\[
[\hat{S}_1, \hat{S}_2] = 2i\hat{S}_3.
\] (7)

This means that simultaneous exact measurements of the Stokes operators \(\hat{S}_1\) and \(\hat{S}_2\) for instance, are not possible. As a consequence, the product of the Stokes parameters variances must obey the following relations
\[
V_2 V_3 \geq \left| \left\langle \hat{S}_1 \right\rangle \right|^2, \quad V_3 V_1 \geq \left| \left\langle \hat{S}_2 \right\rangle \right|^2, \quad V_1 V_2 \geq \left| \left\langle \hat{S}_3 \right\rangle \right|^2.
\] (8)

Here \(V_j\) is the variance \(\left\langle \hat{S}_j^2 \right\rangle - \left\langle \hat{S}_j \right\rangle^2\) of the quantum Stokes parameter \(\hat{S}_j\). We note that we could use any pair (or even three) of the Stokes operators for our protocol, except \(\hat{S}_0\), which commutes with all the others.

3 The protocol

3.1 All-continuous scheme

We propose a all-continuous quantum cryptographic scheme based on the Gaussian modulation of coherent states [2,3,4], but using the quantum polarization variables instead of the quadrature variables. Gaussian-modulated continuous variables states are proven to be secure against optimum eavesdropping techniques [11]. Moreover, cryptographic protocols based on displaced Gaussian states are equivalent to entanglement-based protocols, although in former there is no entanglement. Such a “virtual” entanglement provides an upper bound on the mutual information between Alice and Bob as if they had used entanglement. This is an indication that coherent state quantum cryptography may be unconditionally secure [12]. Besides, it has been recently proved that continuous variable quantum cryptography is secured against non-Gaussian attacks [13]. Our proposition requires a simpler measurement set-up, without the need of synchronized local oscillators at Alice’s and Bob’s stations necessary for a common phase for homodyne detection [14]. Alice prepares a Gaussian continuously modulated polarized beam\(^1\) (represented by \(|\psi_{xy}\rangle\)), which is basically a two-mode state where both modes (\(x\) and \(y\)) are excited to independent single-mode coherent states
\[
|\psi_{xy}\rangle = |\alpha_x\rangle_x |\alpha_y\rangle_y = \hat{D}_x (\alpha_x) \hat{D}_y (\alpha_y) |0\rangle_x |0\rangle_y,
\] (9)

where \(\hat{D}_j (\alpha_j), j = x, y\), are Glauber’s displacement operators.

Alice then sends the beam to Bob through a Gaussian noisy channel. Bob randomly measures, for instance, either \(\langle \hat{S}_1 \rangle\) or \(\langle \hat{S}_2 \rangle\) of the incoming beam. After that Bob informs Alice, via a public authenticated channel, which Stokes parameter \((\langle \hat{S}_1 \rangle\) or \(\langle \hat{S}_2 \rangle\)) he has measured. At this stage Bob and Alice share two sets of quantum correlated continuous variables, which may be transformed into errorless bit strings via a reconciliation algorithm [16,17]. Finally, they should use a standard protocol for privacy amplification [18,19] in order to distill the private key.

3.2 Measurement

As seen above, the security of the all-continuous Gaussian variables protocols depends on the precise measurements of intensities and intensity variances. In reference [10] a simple scheme for the measurement of the quantum Stokes parameters and their fluctuations and correlations is proposed. For Gaussian states

\(^1\)This allows an optimal information rate [15].
all the fluctuations and correlations can be related to second order expectations values, which could be measured in an experimental set-up using a polarizing beam splitter and a combination of quarter wave and half wave plates.

Note:
We would like to inform that our paper (with some modifications) was submitted to the XXVII National Conference on Condensed Matter Physics (Optics session) in 16/02/2003. A related paper, using polarization encoding has been recently published, S. Lorentz, N. Korolkova and G. Leuchs, [quant-ph/0403064](quant-ph/0403064)

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