Motion control and its ground-based experiment of a tethered subsatellite with a controllable rigid arm

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Abstract. The paper presents an experimental research on motion control of a ground-based tethered subsatellite with a controllable rigid arm between the subsatellite and the tether. The forces acting on the tethered subsatellite are first obtained through the dynamic similarity between the subsatellite and a tethered satellite orbiting the Earth. And then, an experimental system for simulating the tethered subsatellite is designed and elaborated in detail, including the airflow embranchment, the measuring and control subsystems, the power and communication subsystems, et al. Furthermore, the operating principle of the ground-based experimental system is illustrated, and the applicability of which is validated by a test of an on-orbit nonlinear optimal control problem.

1. Introduction

The Tethered Satellite System (TSS), which is deployed from the other flying vehicle in space via a tether, is a new technology for artificial gravity creation, LEO to GEO tethers transportation, scientific and technological applications, electro-dynamic study, and so on. As an extension of the application, TSS with an extra offset mechanism on board/subsatellite has been widely used for its highly controllability and repeatability.

Modi and Misra had committed themselves to investigate the dynamics and control of the TSS with an offset from the end of 20th century. They first introduced a three-dimensional offset on a plate-type space station to study the deployment/retrieval dynamics of the TSS in 1987 (Lakshmanan et al., 1987). The controllability of the linearized equations was analyzed numerically, and the control was achieved through a simple velocity feedback using thrusters and momentum wheels. Later, they (Lakshmanan et al., 1989) applied the tension control strategies to the same TSS and found that the proposed scheme damped a given disturbance in a shortest time during station-keeping process and, that the offset control is the most efficient one in terms of energy consumption. Afterwards, Modi and Pidgeon (Modi and Pidgeon, 1994) used the linear quadratic regulator in conjunction with the platform, on which a momentum gyros mounted, to alleviate the longitudinal oscillations of tether during retrieval phase, and designed a ground-based experimental facility to demonstrate the validity of the mathematical model with offset control strategy (Modi et al., 1990; Pradhan et al., 1996).

Nohmi et al. (Nohmi et al., 1998; Nohmi, 2004; Takehara et al., 2008; Nohmi et al., 2007; Nohmi et al., 2010) employed a mechanism with an arm on subsatellite to create tether tension torque. Microgravity experiments showed that the rotation motion can be suppressed by the tether tension torque. Kumar (Kumar and Kumar, 2001) investigated offsets mounted on both the platforms instead
of on one end and developed an open-loop strategy and showed the feasibility of those offsets by numerical simulations. In specific missions, however, the highly precision attitude of a satellite is required, such as like NASA’s SPECS (Submillimeter Probe of the Evolution of Cosmic Structure) mission, stellar interferometry. While the classical control algorithms were available for controlling the motions of the end-body of tethered satellite system during attitude maneuver, yet it took long time to get the desired states, requiring a high level of energy efforts. For example, the maximum moment for controlling the attitude motion of a tethered subsatellite would arrive at about 0.24Nm, which largely exceeded the usual moment in use of momentum wheel (Wang et al., 2010).

In order to achieve the highly control accuracy with less energy consumption, a controllable rigid arm between the subsatellite and the tether is proposed as a control strategy in this paper. The Gauss pseudospectral method and nonlinear programming are employed in motion control. The dynamic equivalence relative to a tethered satellite orbiting the Earth are detailed. The numerical and experimental results show the effectiveness of the proposed method and control scheme, according to the use of less energy.

2. Dynamic equivalence

2.1. Derivation of nonlinear equations on orbit

Consider the in-plane dynamics of a tethered subsatellite deployed/retrieved from a mother satellite. The subsatellite is envisioned to be a disk of radius $r$ and connected to the tether by a massless controllable arm of length $\lambda$ at points $A$ and $B$ shown in Figure 1. The coordinate frames of reference are defined as follows: the earth-centered Cartesian inertial frame $OXY$ with unit vectors $e_1$ and $e_2$, and the orbit coordinate frame $ox\bar{y}$ with unit vectors $b_1$ and $b_2$ placed on the mass center of mother satellite. As shown in the Figure 1, the $x$-axis is aligned with the local vertical position, and the $y$-axis is tangential to the orbit. Suppose that the mother satellite moves in an elliptical orbit, which is characterized by the following parameters, i.e., orbital eccentricity $e$, apogee radius $r_a$, perigee radius $r_p$, longitude ascending node $\Omega$, orbital inclination angle $\delta$, argument of perigee $\Omega_p$ and true anomaly $\nu$. Suppose that the current deployed tether length is $l$, while the deviation angle of the tether is $\theta_1$, the incremental angle of the controllable arm is $\theta_2$, and the attitude of the subsatellite is $\phi$. Let the tension force of the tether and the torque exerted on the subsatellite is $T$ and $M$, respectively.

![Figure 1. In-plane tethered subsatellite system with a rigid arm](image)

The assumptions employed in the derivation of nonlinear equations are as follows: (1) Solar radiation, air drag force or RF interference from satellite thrusters are negligible. (2) The massless tether is assumed to be a straight, inextensible string. (3) The mass of the mother satellite is assumed to be much larger than the mass of subsatellite $m$, so that the orbit dynamics of which cannot be influenced by the subsatellite and the tether.
According to Figure 1, it is easy to write out the position vector of the subsatellite in the orbital frame

\[
\mathbf{r} = x\mathbf{b}_1 + y\mathbf{b}_2 = -[l \cos \theta_1 + \lambda \cos \theta + r \cos \phi] \mathbf{b}_1 - [l \sin \theta_1 + \lambda \sin \theta + r \sin \phi] \mathbf{b}_2
\]

where \( \theta = \theta_1 + \theta_2 \) and \( \phi = \theta + \phi \). The acceleration of the subsatellite is

\[
\ddot{\mathbf{r}} = \ddot{\mathbf{R}} + (\ddot{x} - \dot{v} \dot{y} - 2 \dot{v} \dot{y} - \dot{v}^2 x) \mathbf{e}_1 + (\ddot{y} + \dot{v} \dot{x} + 2 \dot{v} \dot{x} - \dot{v}^2 y) \mathbf{e}_2
\]

and

\[
\ddot{\mathbf{R}} = \ddot{\mathbf{R}}_1 + \ddot{\mathbf{R}}_2 = \tau_a (1 - e^2) \{ [(e \cos \nu - 2 e^2 \sin^2 \nu + 1) \nu^2 \kappa^{-1} - \nu e \sin \nu] \mathbf{e}_1 + 2 e^2 \kappa^{-1} \sin \nu + \nu \} \mathbf{e}_2
\]

where \( \dot{v} = [\mu r_a^{-3}(1 - e^2)^3]^{1/2} \kappa^{-2} \), \( \nu = -2 \mu e \kappa^3 r_a^{-3}(1 - e^2)^3 \sin \nu \), \( \kappa = 1 + e \cos \nu \), and \( \mu \) is the gravitational parameter of Earth. The overdot represents the derivative with respect to time. The gravity force acting on the subsatellite can be described as

\[
F_g = F_{gx} \mathbf{e}_1 + F_{gy} \mathbf{e}_2 \approx m \dot{v} \{ (2x - r_a) \mathbf{e}_1 - y \mathbf{e}_2 \}
\]

Applying Newton’s law to the subsatellite, one obtains

\[
m(\ddot{x} - \dot{v} \dot{y} - 2 \dot{v} \dot{y} - \dot{v}^2 x - \dot{v}^2 r_a) = F_{gs} + T \cos \theta_1
\]

\[
m(\ddot{y} + \dot{v} \dot{x} + 2 \dot{v} \dot{x} - \dot{v}^2 y) = F_{gy} + T \sin \theta_1
\]

\[
J(\ddot{\phi} + \dot{\phi}) = M - T[\dot{\lambda} \sin \theta_2 + r \sin (\theta_2 + \phi)]
\]

where \( J \) is the moment of inertia of subsatellite.

2.2. Ground-based system

To reveal the dynamics of a tethered satellite system by physical simulation, it needs to construct a ground experiment system that is able to modeling the space environment of the mechanics of the tethered satellite system, according to dynamic similarity. Figure 2 is a schematic diagram of the ground experiment system. The coordinate \( \text{oxy} \) is settled as the same as that of on-orbit tethered satellite system. A tether controller having the measuring capacity of the length and tension of tether is mounted on the origin of the coordinate \( \text{oxy} \) for reeling in/out the tether. The subsatellite, called simulator in what follows, can float on the marble flat by three air-bearings so as to simulate the micro-gravity environment of subsatellite in the direction of deployment or retrieval. The position and attitude of the simulator can be measured by the dynamic measuring system, and controlled by four thrusters and a reaction wheel. The controlling forces are resultant forces \( F_t \) and \( F_c \) and force moment \( M_0 \) as shown in Figure 2. The deflection angle \( \phi \) of the controllable arm can be controlled by a DC motor.

![Figure 2. The schematic diagram of ground experiment system](image-url)

The equations of motion of the simulator is
\[ m_0\ddot{x} = F_I \sin \phi - F_I \cos \phi + T_0 \cos \theta_I \]  
\[ m_0\ddot{y} = -F_I \cos \phi - F_I \sin \phi + T_0 \sin \theta_I \]  
\[ J_0\ddot{\phi} = M_0 - T_0[\lambda \sin \theta_2 + r_0 \sin(\theta_2 + \phi)] \]

where \( m_0 \) and \( J_0 \) represent the mass and moment of inertia of simulator, respectively. \( r_0 \) is the radius of simulator and \( \lambda_0 \) is the length of controllable arm.

2.3. Equivalence Principle
Comparing Equations (5) with (6), one can find that if let
\[ F_I = m_0(\dot{v}x + 2\dot{v}\dot{x} - \dot{v}^2 + F_g \cos \theta) \]  
\[ M_0 = \{M - T[\lambda \sin \theta_2 + r \sin(\theta_2 + \phi)]\} \]

there is no difference between the two dynamical systems. The dimensionless equations of motion (6) can be rewritten as
\[ l\dot{\theta}_1^\prime = m_1^\prime (Q_1 \sin \theta_1 - Q_2 \cos \theta_1 - 2\lambda_2 Q_2 \xi_2^2) + l\lambda_3 -(2l^\prime + l\dot{v}^\prime)\theta_1^\prime \]  
\[ l\dot{\theta}_2^\prime = m_1^\prime [-Q_1 \sin \theta_1 - Q_2 \sin \theta_1 + 2(\lambda_2 + 1)Q_2 \xi_2^2 - l\lambda_3 - l(\dot{v}^2 \phi^2 + \dot{v}\dot{\phi} - \dot{\phi}^2)^2 + 2l^\prime \theta_1^\prime] \]

\[ l^\prime = m_1 (-Q_1 \cos \theta_1 - Q_2 \cos \theta_1 + 2\lambda_2 Q_2 \xi_2^2) + \lambda_4 + l\dot{\theta}_2^2 - l\dot{v}^2 \]

where
\[ \frac{d}{dt} = \dot{v} \frac{d}{dt} + \dot{v} \frac{d}{dt} \]
\[ \xi_1^\prime = \frac{\lambda l}{l} \]
\[ \xi_2^\prime = \frac{r l}{l} \]
\[ m_1^\prime = \frac{1}{m} \]
\[ Q_1 = (F_I \sin \phi - F_I \cos \phi + T_0 \cos \theta) \dot{v}^2 \]
\[ Q_2 = (-F_I \cos \phi - F_I \sin \phi + T_0 \sin \theta) \dot{v}^2 \]
\[ Q_\phi = (\bar{M} \dot{\phi}^2 - \bar{T}[\xi_1 \sin \theta_2 + \xi_2 \sin(\theta_2 + \phi)]) \dot{\phi}^2 \]

3. Ground-based experimental system

3.1. Experimental setup
Figure 3 shows the experimental platform of the tethered satellite system in State Key Laboratory of Mechanics and Control of Mechanical Structures. The satellite simulator, of diameter 0.2m and mass 12.02kg, can float on the table with the help of three air bearings. It has three degrees of freedom, i.e., two translational motions and one rotation. The tether is made of ultra high molecular weight polyethylene fiber (DYNEEMA), which has extremely long chains with molecular weight numbering in the millions, usually between 2 and 6 million. The air gap between the air bearing and the marble table is less than 5μm and each bearing can sustain a load up to 335N.
The ground-based experimental system consists of five subsystems: the pneumatic subsystem, the measuring and control subsystems, the power and communication subsystems. The components of the pneumatic subsystem includes one carbon dioxide tank, one regulator, three air bearings, four pressure-type solenoid valves, four pressure sensors, and four Rafael benitez’s nozzles. These parts constitute two airflow embranchments on the simulator, one is for floating and other for control. The airtank is of working capacity of approximately 1 hour for floating and 20 minutes for controlling.

The measuring subsystem can acquire the pressure in front of the valves, the location and the rotation rate of the satellite simulator. A 3D-dynamic-measuring-system (DMS) captures the frame of the ground-based experimental system at a high speed to recognize the markers located on points $o$, $A$, $B$, and $C$, respectively.

There are four groups of actuators to control the position and attitude of the simulator. The first is the tether reel in/out mechanism, fixed on one side of the marble platform, to control the tether tension or the reel rate. Four solenoid valves, in two orthogonal directions, are mounted on the satellite simulator for position control. The attitude motion is controlled by the reaction wheel placed on the bottom of the simulator. A controllable rigid arm is selected as the offset mechanism for attitude tuning.

Two rechargeable lithium batteries with DC/DC converter are chosen as power supply. They can keep the satellite simulator working for no less than 13 hours. The output voltage and the capacity of one battery are 22.2V and 2600mAh, respectively. The DC/DC converter converts the output voltage to $+12V$, $±15V$, and $5V$.

The main function of the communication subsystem is to read out the location information at assigned port generated by the DMS first. Three Digi international XBee OEM RF modules are the embedded solutions for providing wireless end-point connectivity to DMS, computer onboard and the reel mechanism, respectively. The data stream is transmitted in an 8 byte float format according to IEEE 802.15.4 networking protocol for fast point-to-multipoint networking and operate at 57.6kbs each.

3.2. Experimental Principle

Figure 4 illustrates the working flow of the ground-based experimental system. The state of the satellite simulator is first measured by DMS and exchanged in the Intel format. Floating point values are transferred in an 8 byte double format according to IEEE 754. The control commands determined by Equations (7) are sent to the tether reel mechanism and the industrial computer on the satellite simulator synchronously by the wireless modules as well. The satellite simulator floating on the marble platform is simultaneously controlled by the reel motor, the solenoid valves, the reaction wheel, and the offset mechanism. Once the computer gets the signals from the receiver, it sends them to a
Digital Signal Processor (DSP) and two positioning control unit of the motor. The DSP converts the digital signals sent by the computer through serial communications interface (SCI) to analog signals as the inputs of the solenoid valves. The reaction wheel can provide a linear moment to control the attitude motion of the satellite simulator.

3.3. Numerical and experimental results

In this paper, the nonlinear optimal control problem is discretized to a large-scale optimization problem by using the Gauss Pseudospectral method (Benson et al., 2006), and then the method of nonlinear programming is used to obtain numerical solutions.

Some constraints are imposed on the terminal states, and the cost functional is selected to be

\[
J = (u_1^2 + u_2^2 + u_3^2)dt + \sum_{i=1}^{n} [x_i(t_f) - x_{i,0}]^2
\]

where \(x = (\theta_1, \theta_2, \xi, \phi, \theta_1', \theta_2', \xi', \phi')\) are the states, \(n=8\) is the number of system states, \(\xi = l/L_f\) is the non-dimensional tether length normalized with respect to the fully deployed tether length \(L_f\). The integral term in Equation (10) represents the energy cost during the optimal and the second specification requires that the final states meet some specified endpoint conditions. Solutions are obtained in Matlab 7.3 using GPOPS as the optimization software. The final time is selected as \(t_f = 5\) for 30 nodes. Different eccentricities of the orbit are chosen to verify the optimal control method. Some physical constraints are enforced during the optimization process, i.e., the tension forces of the tether are maintained to be positive and less than its tensile strength. Meanwhile, the tether’s length is positive but 1.5 times shorter than the fully deployed tether length. The range of the tether angle is from \(-\pi/2\) to \(\pi/2\) so that the tether cannot swing to the back of the mother satellite. The force moment supplied by reaction wheel is no more than \(\pm 0.05\)Nm.

For simplicity, take the orbital parameters as same as that in Reference (Williams, 2005), namely \((R, \Omega, \delta, \omega, v, e) = (6770.882km, 0.05, 0, 0, 0, 0)\). The initial and terminal states are set as \((\theta_1, \theta_1', \theta_2, \theta_2', \xi, \xi', \phi, \phi')|_{t=0} = (0, 0, 0, 0, 1, 0, \pi, 0)\) and \((\theta_1, \theta_1', \theta_2, \theta_2', \xi, \xi', \phi, \phi')|_{t=f} = (0.042, 1.4973, 0, 0, 1.0152, 0, \pi, 0)\).

The experimental parameters are listed in Table 1. The profiles of the states and the control inputs are illustrated in Figure 5 and Figure 6. As shown in Figure 6, the tether tension force is revealed to be the primary energy input in the control process, the maximum of which reaches 11.43N. Also note that the moment of momentum wheel changes gently in the whole procedure, and the maximum of the moment, presented at much less than saturation values, is 3.944mNm only. The experimental control inputs according to Equations (7) are showed in Figure 7. The maximum amplitudes of the tangential and normal forces are \(F_r = 0.1291\)N and \(F_n = 0.0588\)N, respectively. The maximum and minimum values of the tether tension are 0.22403N and 0.02663N. One can see from Figure 5 that the motion of the tether exhibits an sinusoid-like oscillation in the process of optimization and the maximum
amplitude of the oscillation is \(-0.413\) rad. As a result of the initial errors between the numerical simulation and the experiment, the satellite simulator gradually increases to the designation.

| Symbol               | Pendulum | Experimental |
|----------------------|----------|--------------|
| Disc mass \( m \) (kg) | 500      | 12.02        |
| Tether length \( L_f \) (m) | 100      | 1.0689       |
| Disc radius \( r \) (m)    | 1        | 0.101        |
| Controllable arm length \( \lambda \) (m) | 2        | 0.1          |
| Moment of inertia \( J \) (kg m\(^2\)) | 0.06     | 0.035725     |

Figure 5. Profiles of the states (—Numerical, ——Experimental)
4. Conclusions
The possibility of using a controllable rigid arm connected to satellite for motion control of tether satellite system is numerically and experimentally studied. The dynamic equivalent principle and the experimental setup are presented. To show the controllable rigid arm is quite efficient in decreasing the control moment of reaction wheel, the numerical simulations are first made. The controlled satellite simulator can be brought to the destination at a preassigned motion on the marble table.
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