Faithful qubit transmission against collective noise without ancillary qubits *

Xi-Han Li, Fu-Guo Deng,† and Hong-Yu Zhou

The Key Laboratory of Beam Technology and Material Modification of Ministry of Education, and Institute of Low Energy Nuclear Physics, Beijing Normal University, Beijing 100875, China (Dated: February 1, 2008)

We present a faithful qubit transmission scheme with linear optics against collective noise, not resorting to ancillary qubits. Its set-up is composed of three unbalanced polarization interferometers, based on a polarizing beam splitter, a beam splitter and a half-wave plate, which makes this scheme more feasible than others with present technology. The fidelity of successful transmission is 1, independent of the parameters of the collective noise, and the success probability for obtaining an uncorrupted state can be improved to 100% with some time delayers. Moreover, this scheme has some good applications in one-way quantum communication for rejecting the errors caused by the collective noise in quantum channel.

The main task of quantum communication is transmitting and exchanging quantum information between two remote parties [1]. In the last two decades, quantum communication had a drastic progress both in theory and experiment. In laboratory, the polarization state of photons is usually chosen as the qubit for quantum communication due to its maneuverability. For example, the famous BB84 [2] quantum key distribution (QKD) scheme uses two conjugate polarization bases of photons to create a secret key, and the first quantum teleportation [3] scheme selects the polarization states of photons to teleport an unknown quantum state. However, the polarization freedom of photons is incident to be influenced by the thermal fluctuation, fluctuation and the imperfection of the fiber, i.e., the noise in quantum channel. This is a serious obstacle to the application of quantum communication in practice. Then, various error correction and error rejection methods are proposed. For instance, Walton et al. [4] proposed a scheme for rejecting the errors introduced by noise with decoherence-free subspaces [5]. Quantum redundancy code [6] is introduced by entangling the signal with some ancillary qubits before the transmission. After the transmission, the ancillary qubits will be measured, and some unitary operations will be performed on the signal to correct the errors according to the measurement results. Theoretically, the number and the kind of errors that can be corrected depend on the number of the ancillary qubits and the kind of code. In a long-distance quantum communication, entanglement purification [7] is introduced to decrease the influence arisen from the noise, with which a higher fidelity state is obtained by sacrificing several qubits. On the other hand, the steps to get a genuine pure entangled state are always infinity and the cost of resource grows with the distance between the two parties.

In 2005, Kalamidas [8] proposed two linear-optical single-photon schemes to reject and correct arbitrary qubit errors without additional particles. The first one obtains an uncorrupted qubit at a definite time of arrival. Its success probability is 0.5 when varying the noise parameters over their entire ranges; otherwise, the probability depends on the parameters of noise. The second scheme is based on self-correcting and its success probability is 1. That is, the receiver can always get an uncorrupted state. Later Han et al. [9] proved that these two schemes are also suitable for mixed state. However, in these two protocols, at least two fast polarization modulators (Pockels cell), whose synchronization makes it difficult to be implemented with current technology [10], are employed. Subsequently, de Brito and Ramos [10] presented a different setup to realize error correction without Pockels cells, in which coherent states were used instead of single-photon pulses. They also use Faraday mirror and common mirror substituting the half wave plate. This new setup is passive and suitable for bright coherent states, but due to the repetitious reflection, a majority of energy is lost in the useless pulses, which would result in a low efficiency.

In this letter, we present a new setup for a single-photon qubit against collective noise without ancillary qubits. It is made up of three unbalanced polarization interferometers, based on a polarizing beam splitter (PBS), a beam splitter (BS: 50/50), and a half wave plate (HWP). The sender encodes a qubit with time code, and the receiver can obtain an uncorrupted state in a definite time of arrival. The fidelity of transmission through the noisy channel is hold by sacrificing some success percent. The success probability is independent of the parameters of noise and can be improved to 1 with some time delays. Moreover, this scheme has some good applications in quantum communication. We discuss its application in the famous BB84 QKD for rejecting the errors caused by the collective noise in quantum channel.

Let us assume that the initial state to be transmitted is \( |\psi\rangle = \alpha |H\rangle + \beta |V\rangle \) (\(|\alpha|^2 + |\beta|^2 = 1\)). Here, \(|H\rangle\) and \(|V\rangle\) denote the horizontal and the vertical polarization modes of photons, respectively. Our setup is shown in Fig. 1. The sender Alice and the receiver Bob have the similar set-ups, an encoder and a decoder, including

*Published in Applied Physics Letters 91, 144101 (2007)
†Author to whom correspondence should be addressed. Also at: Department of Physics, Beijing Normal University, Beijing 100875, China; Electronic mail: fgdeng@bnu.edu.cn
an unbalanced polarization interferometer composed of a PBS, a BS, and a HWP. The first PBS in Alice’s side transmits the horizontal polarization mode $|H\rangle$ and reflects the vertical polarization mode $|V\rangle$. In this way, the state $|H\rangle$ propagates through the short path (S) and the state $|V\rangle$ goes through the long path (L). The time of flight difference of the unbalanced interferometer is set to be on the order of a few nanoseconds, which is much lesser than the time of fluctuation in the fiber. The two proximate parts will have the same infection by the noisy channel $^{[11]}$. The first HWP accomplishes the transformation $|V\rangle \rightarrow |H\rangle$.

The state launched into the noisy channel can be described as follows

$$|\psi\rangle \rightarrow \alpha |H\rangle + \beta |V\rangle$$

$$\overset{PBS}{\rightarrow} \alpha |H\rangle + \beta |V\rangle$$

$$\overset{HWP}{\rightarrow} \alpha |H\rangle + \beta |L\rangle$$

$$\overset{BS}{\rightarrow} \frac{1}{\sqrt{2}} \left\{ (\alpha |H\rangle_S + i \beta |H\rangle_L)_1 + i (\alpha |H\rangle_S - i \beta |H\rangle_L)_2 \right\} \quad (1)$$

The subscripts 1 and 2 represent the two output ports of Alice’s BS, shown in Fig.1, called them as channels 1 and 2, respectively. The coefficient $i$ comes from the phase shift aroused by the BS reflection. Owing to the fact that the two states in two output ports can be transformed into each other with a unitary operation, we just consider the channel 1 in detail below, and the same way can be used to the state coming from the channel 2 with a little modification.

The state in the input of the channel 1 is $\frac{1}{\sqrt{2}} (\alpha |H\rangle_S + i \beta |H\rangle_L)_1$. Notice that the polarization states of the wave packets in the two time bins are both $|H\rangle$, the influences of the noise in the quantum channel on these two pulses are the same one. The noise of channel can be expressed with a unitary transformation

$$|H\rangle_j = \delta |H\rangle_j + \eta |V\rangle_j, \quad (2)$$

$$|\delta|^2 + |\eta|^2 = 1, \quad (3)$$

$j = S, L$ denote the two time-bins. Different kinds of noises in the quantum channel have the similar form shown in the equation (4), and just are different in their parameters. The evolution of the state from Alice to Bob through the channel 1 can be written as

$$\frac{1}{\sqrt{2}} (\alpha |H\rangle_S + i \beta |H\rangle_L) \overset{\text{channel}}{\rightarrow} |\psi\rangle_u$$

$$= \frac{1}{\sqrt{2}} (\alpha \delta |H\rangle_S + \alpha \eta |V\rangle_S + i \beta \delta |H\rangle_L + i \beta \eta |V\rangle_L) \quad (4)$$

The subscript 1 was omitted for simpless. When Bob receives the state with the noise parameters $\delta$ and $\eta$, the action of his unbalanced interferometer (i.e., his decoder) is given by

$$|\psi\rangle_u \overset{BS}{\rightarrow} \frac{1}{2} (\alpha \delta |H\rangle_S + \alpha \eta |V\rangle_S + i \beta \delta |H\rangle_L + i \beta \eta |V\rangle_L - \beta \delta |H\rangle_{LL} - \beta \eta |V\rangle_{LL})$$

$$\overset{HWP}{\rightarrow} \frac{1}{2} (\alpha \delta |H\rangle_S + \alpha \eta |V\rangle_S + i \beta \delta |H\rangle_L + i \beta \eta |V\rangle_L + i \alpha \delta |H\rangle_{SL} + i \alpha \eta |V\rangle_{SL} - \beta \delta |V\rangle_{LL} - \beta \eta |H\rangle_{LL})$$

$$\overset{PBS}{\rightarrow} \frac{1}{2} (\alpha \delta |H\rangle_S - \beta |V\rangle_{LL} + i \beta \delta |H\rangle_{LS} + \alpha |V\rangle_{SL})_a + \frac{\eta}{2} (\alpha |V\rangle_{SS} - \beta |H\rangle_{LL} + i \beta |V\rangle_{LS} + \alpha |H\rangle_{SL})_b. \quad (5)$$

The subscripts $a$ and $b$ represent the two outputs of Bob’s PBS (see Fig. 1). From the last line of expression (5), one can see that the terms with underlines indicate the states which will arrive at a definite time in the two outputs $a$ and $b$. LS or SL means being delayed once, transmitting once through S and once through L. Other states may arrive too late (LL) or early (SS). Therefore, Bob can get the uncorrupted states in the determinate time corresponding to SL and LS. If the photon arrives at the output $a$, a bit flip operation is needed to get the original
state. In contrast, there is nothing needed to perform on the photon arriving at the output b. The evolvement of the states transmitting through the channel 2 is similar to that through the channel 1. From Eqs. (5), one can see that the success probability is $\frac{1}{4}$ in path 1, independent of the noise parameters $\delta$ and $\eta$. Consider the contribution of the channel 2, the total probability to obtain an uncorrupted state is $\frac{1}{4}$.

With some time delayers Bob can obtain an uncorrupted state with the success probability of 1 as he can also select the state at the time bins SS and LL. In this time, he can first manipulate the relative phases of the wave packets SS, LL and LS (SL), and then delay the signal arriving at the time bin SS twice of the interval of the short path (S) and the long path (L) and those at the time bins LS (SL) once.

This quantum error rejection scheme has some important characters. First, an arbitrary qubit error caused by the noisy channel can be rejected with the success probability of 1. The significant hypothesis is that the two time bins are so near that they suffer from the same noise infection [11]. Secondly, neither additional qubits nor entangled states are employed, so the resource needed will not increase with the channel length. Moreover, the setup is absolutely passive linear optics without the use of high polarization modulator. Thirdly, the success probability is independent of the noise parameters. In other schemes proposed [6, 8], the probability always relates to the noise. They can get a certain number of success probability only by varying the noise parameters over their ranges. Moreover, this scheme is efficient for the transmission of a subsystem of a larger quantum system. That is, it is also suitable for a mixed state as one needs only to replace the state $\alpha|H\rangle + \beta|V\rangle$ with $\alpha'|H\rangle_h|H\rangle_t + \beta'|V\rangle_h|V\rangle_t \equiv \alpha''|H\rangle_t + \beta''|V\rangle_t$ in the equations (1)-(5) and obtain the same result. Here $h$ and $t$ represent the home particle and the traveling particle, respectively.

The present scheme has some good applications in almost all one-way quantum communication protocols [1] for rejecting the errors caused by the collective noise. For example, the BB84 QKD protocol [2] can be carried out through a noisy channel using this time-bin encoding method. In this time, the two parties Alice and Bob can choose the two nonorthogonal bases, X basis $|\pm x\rangle = \frac{1}{\sqrt{2}}(|H\rangle \pm |V\rangle)$ and Y basis $|\pm y\rangle = \frac{1}{\sqrt{2}}(|H\rangle \pm i|V\rangle)$, to prepare and measure the quantum states. Alice products single photons randomly in one of the four states and sends them to Bob using this error rejecting device (her encoder). Bob first recovers the original state with his decoder and some time delayers, and then selects one of the two bases randomly to measure the photon. Owing to the fact that half of particles are measured with uncorrelated bases, only 50% of the photons are used to create the raw key, the same as the original BB84 QKD protocol. Apparently, the efficiency of this BB84 realization is same as that in the original one [2] and it rejects the errors caused by the channel noise. Without time delayers, Bob can decode the quantum state with success probability of $\frac{1}{4}$ and accomplish the BB84 QKD securely, although the two parties have a low generating-key bit rate.

In summary, we have proposed a single-photon error rejection scheme against a collective noise with linear optics. In this scheme, additional qubits and fast polarization modulators are not required. Qubits are encoded in time bins and the uncorrupted state will arrive at definite time slots. The success probability for obtaining an uncorrupted state can be improved to 100% with some time delayers, which is independent of the parameters of the collective noise. This scheme is passive and is also suitable for a mixed state. It has some good applications in one-way quantum communication.

This work is supported by the National Natural Science Foundation of China under Grant Nos. 10604008 and 10435020, A Foundation for the Author of National Excellent Doctoral Dissertation of China, and Beijing Education Committee under Grant No. XK100270454.

[1] N. Gisin, G. Ribordy, W. Tittel and H. Zbinden, Rev. Mod. Phys. 74, 145 (2002).
[2] C. H. Bennett and G. Brassard, in: Proceedings of IEEE International Conference on Computers, Systems and Signal Processing, Bangalore, India, IEEE, New York, 1984 p. 175.
[3] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. 70, 1895 (1993).
[4] Z. D. Walton, A. F. Abouraddy, A. V. Sergienko, B. E. A. Saleh, and M. C. Teich, Phys. Rev. Lett. 91, 087901 (2003).
[5] D. A. Lidar and K. B. Whaley, Eprint: arXiv: quant-ph/0301032
[6] M.A. Nielsen and I.L. Chuang, Quantum Computation and Quantum Information, Cambridge Univ. Press, Cambridge, 2000.
[7] C. H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. A. Smolin, and W. K. Wootters, Phys. Rev. Lett. 76, 722 (1996).
[8] D. Kalamidas, Phys. Lett. A 343, 331 (2005).
[9] C. Han, Z. W. Zhuo, and G. C. Guo, J. Phys. B 39, 1677 (2006).
[10] D. B. de Brito and R. V. Ramos, Phys. Lett. A 352, 206 (2006).
[11] T. Yamamoto, J. Shimamura, S. K. Özdemir, M. Koashi, and N. Imoto, Phys. Rev. Lett. 95, 040503 (2005).