Amplitude-Fluctuation Separation Based on BPM Waveform Reconstruction in MIMO Systems

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Abstract. In Multi-input-multi-output (MIMO) systems, the orthogonality among sub-waves could be realized by binary phase modulation (BPM) encoding. Especially in the case of insufficient matched filters, Hadamard-matrix-based BPM encoding could be employed to separate the transmitted signal from echo effectively, which is verified firstly in this paper by utilizing target location, when introducing fluctuations in amplitude as well as in phase. Hence the amplitude-fluctuation separation method based on waveform reconstruction is put forward. The waveform is reconstructed on basis of the known array structure, and then the results are optimized obeying the minimum mean square error (MSE) criterion to obtain more accurate estimation. Compared with the traditional single-channel blind source separation approach, this method shows significant computation reduction by taking full use of the phase and structure information. Monte-Carlo experiments validate the method application in angle estimation improvement.

1. Introduction

Multi-input-multi-output (MIMO) systems, which are composed of multiple transmitters and receivers, use the orthogonal waveform to separate different transmit-receive channels, achieving the improvement of detection and estimation [1]-[3]. In contrast to traditional phased array radar (PAR), the orthogonal signals transmitted by MIMO radar could achieve waveform diversity through transmitting orthogonal sub-waves. Thus the design and blind source separation for MIMO radar has attracted attentions from researchers in recent years [4] [5].

Appropriate waveform design is a fastidious in practical operation to satisfy complete orthogonality among sub-waves in MIMO systems. To solve this problem, variety of optimization methods for MIMO radars are proposed. [6] has given a brief introduction of MIMO radar waveforms, including four types of orthogonalization schemes and a particular MIMO waveform. The characteristic of orthogonal waveforms is analysed in detail in [7], and also has provided an improved OFDM chirp waveform method. The performance analysis including ambiguity function, correlations, as well as ideal peak side lobe ratio (PLSR) are provided thoroughly by [8]-[11].

Advanced Driver Assistance Systems (ADAS), benefiting from increased angular resolution in radar system, especially in MIMO radar, require massive calculation [12]-[14], causing a conflict between performance and resource. So most systems adopt time division multiple access (TDMA) to realize the orthogonal MIMO waveform so as to reduce filters overhead [15]. However, according to
the reference [16], TDMA will increase coupling degrees on account of switching antennas as well as Doppler ambiguity.

This paper has proposed a Hadamard-matrix-based BPM encoding approach aiming at antenna coupling problem caused by TDMA. It is organized as follows. In Section II, the signal model is introduced in ideal condition. Hence, the influence of the fluctuation in amplitude as well as in phase are analyzed. In order to reduce the impaction on result, separation based on waveform reconstruction is put forward in Section III. This method shows computation reduction by utilizing the phase and significant structure information. Monte-Carlo experiments validate its application in angle estimation improvement. At last, conclusions are drawn and the future work is prospected.

2. Signal model and error analysis

2.1 Signal model

A co-located MIMO radar system is shown in Figure 1, including $M$ transmitting and $N$ receiving elements, operated in corresponding far-field uniform arrays. The distance between elements is set as $d = \lambda/2$. The chirp signal in each transmitting antenna element is described as

$$s(t) = \exp[j2\pi(f_c t + 0.5kt^2)]$$

where $f_c$ denotes the carrier frequency, and $k$ denotes the chirp slope. Thus the echo from a target at $(R_m, \theta')$, transmitted by $Tx_1$, and received by $Rx_1$ can be expressed as:

$$sr(t) = \exp[j2\pi(f_c(t - \tau_0) + 0.5k(t - \tau_0)^2)]$$

(1)

where $\tau_0 = c/2R$ represents the time delay. And the d-chirp signal is:

$$s_{m_1}(t) = sr(t) \cdot s(t) = \exp[-j2\pi(\tau_0 + \tau_m)] \exp[-j2\pi f_c \tau_0] \exp[jk\pi \tau_0^2]$$

(2)

Generally, we regard $Tx_1$ and $Rx_1$ as the reference antennas. Hence the d-chirp signal from $Tx_m$ to $Rx_n$ is expressed as:

$$s_{mn}(t) = \exp[-j2\pi(\tau_0 + \tau_m)] \exp[-j2\pi f_c \tau_0] \exp[jk\pi \tau_0^2]$$

(3)

Where $\tau_{mn}$ is the time delay, brought by array structure, and equals to $\tau_{mn} = (d + d_s) \sin \theta / c$. Because of a far-field target, which means $\tau_{mn} \ll \tau_0$, Equation (3) can be rewritten as:

$$s_{mn}(t) \approx \exp[-j2\pi f_c \tau_0] \exp[jk\pi \tau_0^2] \exp[-j2\pi f_c \tau_{mn}]$$

$$= s_{11} \exp[-j2\pi f_c \tau_{mn}] = s_{11} \exp[-j\pi((m-1)N + (n-1))\sin \theta]$$

(4)

Here a brief introduction about Hadamard-matrix-based BPM encoding is provided. For convenience, we divide the system into two parts. On the transmit side, we assume 3 transmit antennas, and each antenna transmits a chirp with same parameters. Each block of frame signals transmitted by 3 antennas are modulated by a 3-by-4 Hadamard matrix. On the receiving side, by multiplying the 4-by-4 Hadamard matrix, the echo can be converted to the signals from 3 transmit antennas. Figure 2 and Figure 3 give explanations about this process.
Figure 2. Illustration of transmitting of Hadamard coding MIMO waveform.

$$ H = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix} $$

Figure 3. Illustration of receiving of Hadamard coding MIMO waveform.

Therefore the echo signal can be rewritten as:

$$ X(t) = AS(t) + N(t) $$

Where $A$ is an array manifold of the virtual array:

$$ A = [a(\theta_1), a(\theta_2), \ldots, a(\theta_K)] $$

$$ a(\theta) = [e^{-j\pi 0 \sin \theta}, e^{-j\pi \sin \theta}, \ldots, e^{-j\pi (MN-1) \sin \theta}]^T $$

Considering a 77GHz chirp with 192MHz bandwidth and the maximum unambiguous speed is 120km/h, Figure 4 shows the receive signal amplitude simulation result from Rx1. Figure 5 (a)-(c) illustrates the signals from 3 transmit antennas and (d) shows the redundancy signal. In Figure 6 (a) and (b), the one-dimensional range profile and the DOA estimation of the target are displayed respectively.

Figure 4. The receive signal amplitude simulation result from Rx1.
Figure 5. The signals from 3 transmit antennas. (a) The signal from Tx1; (b) The signal from Tx2; (c) The signal from Tx3; (d) The redundancy signal.

Figure 6. One-dimensional range profile and the DOA estimation. (a) One-dimensional range profile; (b) The DOA estimation.

2.2. Error analysis

The estimation results are not right if the sick do not be a better deal which is due to the fluctuations are inevitable in the BPM encoding process. The mathematical models of fluctuations in amplitude as well as in phase are established. On the basis, the effects from the fluctuations on the spatial spectrum estimation are analyzed. The fluctuations can be introduced by error matrix $\Gamma$. With the assumption that only amplitude fluctuation exists, $\Gamma \in \mathbb{C}^{12 \times 12}$ can be expressed as:

$$\Gamma = \begin{bmatrix}
\rho_1 & 0 & 0 & 0 & \rho_2 & 0 & 0 & 0 & \rho_3 & 0 & 0 & 0 \\
0 & \rho_1 & 0 & 0 & 0 & \rho_2 & 0 & 0 & 0 & \rho_3 & 0 & 0 \\
0 & 0 & \rho_1 & 0 & 0 & 0 & \rho_2 & 0 & 0 & 0 & \rho_3 & 0 \\
0 & 0 & 0 & \rho_1 & 0 & 0 & 0 & \rho_2 & 0 & 0 & 0 & \rho_3 \\
\end{bmatrix} \quad (8)$$

where $\rho_1, \rho_2, \rho_3$ are:

$$\begin{bmatrix} \rho_1 & \rho_2 & \rho_3 \end{bmatrix} = \begin{bmatrix}
(\rho_{11} + \rho_{31} + \rho_{41}) & (\rho_{12} - \rho_{32} - \rho_{42}) & (\rho_{13} + \rho_{33} + \rho_{43}) \\
(\rho_{21} + \rho_{31} + \rho_{41}) & (\rho_{22} + \rho_{32} + \rho_{42}) & (\rho_{23} + \rho_{33} + \rho_{43}) \\
(\rho_{31} + \rho_{41}) & (\rho_{32} + \rho_{42}) & (\rho_{33} + \rho_{43}) \\
\end{bmatrix} \quad (9)$$

$\rho_j$ means the amplitude fluctuation of the $j$th sequence from the $j$th transmitting antenna. According to the reference [6], the covariance matrix can be deduced as:

$$R_{xx}^\prime = \Gamma A R_s A^H \Gamma^H + \sigma_s^2 I \quad (10)$$

and the spatial spectrum calculated by multiple signal classification (MUSIC) algorithm is:
\[
P_{\text{MUSIC}}(\theta) = \frac{1}{\bar{a}^H(\theta) \tilde{U} \tilde{U}^H a^H(\theta)}
\]

\[
\tilde{U}_N \tilde{U}^H_N = \sum_{i=M+1}^{N} \tilde{e}_i \tilde{e}_i^H = I - \sum_{i=1}^{M} \tilde{e}_i \tilde{e}_i^H
\]

Note that there is only one target which is located at \((R_m, \theta')\), the largest eigenvalue \(\lambda_i\) and corresponding eigenvector \(\tilde{e}_i = \Gamma \delta(\theta_i)\) can be achieved by eigenvalue decomposition. So Equation (12) turns into:

\[
\tilde{U}_N \tilde{U}^H_N = I - \sum_{i=1}^{M} \tilde{e}_i \tilde{e}_i^H = I - \frac{\Gamma a(\theta) a^H(\theta) \Gamma^H}{\Gamma^2}
\]

Therefore, the denominator of \(P_{\text{MUSIC}}(\theta)\) equals to:

\[
a^H(\theta) \tilde{U}_N \tilde{U}^H_N a^H(\theta) = N - \frac{\left| a^H(\theta) \Gamma a(\theta) \right|^2}{\Gamma^2}
\]

For convenience, \(\gamma\) and \(\eta\) are employed to replace \((\rho_1 + \rho_2 + \rho_3 + \rho_4)\) and \((\rho_1 + \rho_2 - \rho_3 - \rho_4)\) so that \(a^H(\theta) \Gamma a(\theta)\) are:

\[
a^H(\theta) \Gamma a(\theta) = e^{j0\pi \sin \theta} \left( e^{j0\pi \sin \theta} + \eta_1 e^{-j4\pi \sin \theta} + \eta_2 e^{-j8\pi \sin \theta} \right) + \cdots + e^{j1\pi \sin \theta} \left( \eta_2 e^{-j3\pi \sin \theta} + \eta_3 e^{-j7\pi \sin \theta} \right) + \cdots + e^{j0\pi \sin \theta} \beta_1 e^{j\phi_1} + e^{j3\pi \sin \theta} \beta_2 e^{j\phi_2} + \cdots + e^{j1\pi \sin \theta} \beta_{11} e^{j\phi_{11}}
\]

On account of fluctuations in amplitude, Equation (14) will not reach the minium when \(\theta = \theta_1\), indicating the evaluated errors. Similarly, in the case that only fluctuations existing in phase, \(a^H(\theta) \Gamma a(\theta)\) can be shown by:

\[
a^H(\theta) \Gamma a(\theta) = e^{j0\pi \sin \theta} K_1 e^{j\phi_1} + e^{j3\pi \sin \theta} K_2 e^{j\phi_2} + \cdots + e^{j1\pi \sin \theta} K_{11} e^{j\phi_{11}}
\]

Figure 7 (a) and (b) illustrates the simulations of amplitude and phase fluctuation, and Table 1 displays a part of results on the basis of 100 Monte-Carlo runs.
Table 1. The partial results of DOA estimation within 100 Monte-Carlo runs.

| No. | 15 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 70 | 75 | 80 | 85 |
|-----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| fluctuation in amplitude | 9.5 | 8 | 9 | 11 | 12 | 10 | 10.5 | 9.5 | 10 | 11.5 | 9.5 | 9.5 | 10.5 |
| fluctuation in phase | 9.5 | 10 | 10 | 9.5 | 10 | 9.5 | 10 | 9.5 | 10 | 9.5 | 10 | 9.5 |

Generally speaking, for the error of phase is fluctuating around ±5° as well as the cosine is nonlinear, the consequence resulted by phase fluctuation can be ignored reasonably.

3. Signal separation method based on waveform reconstruction

Supported by Equation (15), it can be deduced that incomplete signal separation mainly results in estimating errors. Considering the random amplitude fluctuation and the known array structure in MIMO radar, a method of waveform reconstruction fitting based on least mean-square error rule is put forward. Thus we provide the processing for fitting in signal separation:

- Initialize the arriving direction of the target echo by MUSIC algorithm, where \( \theta = \theta_0 \).
- Normalize the signal amplitude of each unit in ULA to get the \( A' = \begin{bmatrix} e^{j\theta_0}, e^{j\theta_1}, \ldots, e^{j\theta_N} \end{bmatrix}^T \).

For this part, each receiving antennas can separate different signals corresponding to different transmitting antennas. For a better estimation, normalization is necessary.

Set the size and direction of scanning step. The step parameter setting need to consider computation and accuracy.

- Waveform reconstruction supported by the information of the array structure. Reconstruct the manifold \( A' = \begin{bmatrix} e^{j\theta_0}, e^{-j\theta_0}, \ldots, e^{-j\theta_N} \end{bmatrix}^T \).
- Obeying least mean square error criterion.

Figure 8 shows the initialization of \( A' \) and normalized \( A' \). Figure 9 demonstrates the processing of iteration and get the optimal estimation. For obvious contrast from Table 2, this method can reduce the errors caused by amplitude fluctuation effectively, and the results of Monte Carlo experiments shows the estimation MSEs have been reduced by 0.4 when the amplitude fluctuation obeys Gaussian distribution indicated by \( N(1,0.5) \).

Figure 8. The phase of \( A' \) and \( A' \).

Figure 9. The processing of iteration

Table 2. The partial compare between the original and the proposed method

| No. | 15 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 70 | 75 | 80 | 85 |
|-----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| fluctuation in amplitude | 9.5 | 8 | 9 | 11 | 12 | 10 | 10.5 | 9.5 | 10 | 11.5 | 9.5 | 9.5 | 10.5 |
| proposed method | 10.0 | 9.8 | 9.8 | 10.1 | 10.2 | 9.9 | 9.8 | 9.9 | 10.1 | 10.0 | 10 | 9.9 |
4. Conclusions
In this paper, a Hadamard-matrix-based BPM encoding method was confirmed is suitable for MIMO radar systems, especially in the case of insufficient matched filters. This paper also had presented results on influence caused by fluctuations in amplitude as well as in phase, which pointed out that amplitude fluctuations have a bigger effect than phase. Within the known MIMO radar architecture, the paper proposed a waveform reconstruction fitting method based on minimum mean-square error rule. Monte-Carlo experiments validate the method application in angle estimation improvement.

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