HARD EXCLUSIVE SCATTERING AT JLAB

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The various factorization schemes for hard exclusive processes and the status of their applications is briefly reviewed. Invited talk presented at the workshop on Exclusive Processes at High Momentum Transfer, Newport News, Virginia USA, May 2007

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1. Introduction

In large momentum transfer exclusive processes the probe, say, a virtual photon has a wave length that is much shorter than the spacial extension of the hadronic target. This allows to look inside the hadrons and to study the interactions of their constituents, quarks and gluons. There is overwhelming evidence, mainly from inclusive reactions, that QCD is the correct theory for the interactions between quarks and gluons. QCD is a complicated theory. Quarks and gluons are confined, only their bound states - the hadrons - can be observed experimentally. The formation of hadrons from quarks and gluons occurs at soft scales where QCD perturbation theory is inapplicable. But, with the exception of lattice QCD, there is no analytical or numerical method known to solve QCD in the soft region. In any scattering process as hard as the external scale, for instance the virtuality, $Q^2$, of the probing photon, may be, soft hadronization is unavoidably involved too. Thus, one may wonder whether it is possible to calculate observables for hard processes. This is indeed possible in a number of cases thanks to the factorization properties of QCD: hard processes factorize into parton-level subprocesses amenable to perturbative QCD (and/or QED), and in soft hadronic matrix elements which embody the non-perturbative physics. For a number of processes there are rigorous proofs of factorization available, e.g. the pion electromagnetic form factor, deeply virtual lepton-
nucleon scattering (DIS), deeply virtual Compton scattering (DVCS). For others factorization is a hypothesis with often good arguments for its validity. However, we have to be cautious in these cases. Collins and Qiu\(^1\) found a counterexample, namely \(h_1 h_2 \rightarrow h_3 h_4 X\) where \(h_i\) denotes a hadron, for which \((k_L)\) factorization breaks down. Given the theoretical complications involved in exclusive scattering and with regards to the large number of successful tests of QCD properties accumulated over the last 30 years, the experimental and theoretical investigation of hard exclusive processes will not contribute towards the verification of QCD, rather we will learn about methods how to apply QCD.

In the following I will briefly review the factorization schemes used in exclusive scattering (Sects. 2 and 3). In Sect. 4 I will summarize our present knowledge on the generalized parton distributions (GPDs), the soft hadronic matrix elements occurring in the handbag factorization scheme. Next I will turn to applications of the handbag factorization to deeply virtual exclusive scattering (Sect. 5), discuss alternative theoretical approaches such as the Regge model (Sect. 6), and turn finally to wide-angle exclusive reaction (Sect. 7). Special emphasis is laid on the role of JLab in this physics - what has been achieved by JLab till now, what will be done in the future. In Sect. 8 I will present the summary.

2. The ERBL factorization scheme

A first factorization scheme for hard exclusive processes has been invented around 1980. Efremov and Radyushkin\(^2\) as well as Brodsky and Lepage\(^3\) showed that factorization holds for the pion form factor at large \(Q^2\). This factorization scheme has been generalized later on to many other exclusive processes, lacking however proof in most cases \(^4\). Since the evolution equation for the associated soft matrix element, the so-called distribution amplitude (DA), is named after these authors, I take the liberty to give the full factorization scheme also this name - ERBL factorization. Other frequently used names for it are either misleading or lead to a clash of notation.

In order to sketch the ERBL scheme let me consider Compton scattering off protons at large Mandelstam variables \(s, -t, -u\) as a typical and important example and let me consider only the at large scales dominant valence Fock state of the proton. The amplitudes of this process factorize into partonic subprocess \(\gamma qqq \rightarrow \gamma qqq\) (see Fig. 1) and in proton DAs

\(^{4}\)Many authors have also contributed to the development of that field, e.g. Refs. 4,5.
\[ \Phi_p(x_1, x_2, x_3) \] where the \( x_i \) are the usual momentum fractions. All partons of the valence Fock state participate in the subprocess, they are emitted or absorbed collinearly from their parent hadron and are quasi on-shell. This necessitates the exchange of at least two hard gluons. The Compton amplitudes are given by convolutions of subprocess amplitudes and DAs

\[ M \sim \Phi_p \otimes H \otimes \Phi_p. \tag{1} \]

One may also consider higher Fock states of the involved hadrons but these contributions are suppressed by inverse powers of the hard scale as compared to the valence Fock state contribution.

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**Fig. 1.** Left: A typical graph for Compton scattering within the ERBL factorization scheme. Right: The Compton cross section, scaled by \( s^6 \), at a scattering angle of 90°. Data taken from Ref. 8.

The ERBL factorization scheme implies dimensional counting\(^6,7\) which means that at large momentum scales (or short distances) exclusive observables exhibit scaling, i.e. the fall off as a certain power of the hard scale asymptotically. The power laws are modified by perturbative logs generated by the running of \( \alpha_s \) and the evolution of the DAs. Scaling often holds approximately in experiment although there seems to be no evidence for the perturbative logs. Recent precision data are often in conflict with dimensional counting. As an example the JLab Hall A data\(^8\) on Compton scattering are shown in Fig. 1. Clearly the cross section does not drop as \( s^{-6} \) as predicted by dimensional counting. Violations of dimensional counting are also seen in the Pauli form factor\(^9\) or in the precise BELLE data\(^10\) on \( \gamma \gamma \to p\bar{p} \). These counterexamples do not disprove dimensional counting. They merely indicate that the experimentally available scales for these data are not sufficiently large for applying dimensional counting and, hence, the ERBL factorization scheme.
The ERBL factorization scheme has been frequently applied to various exclusive processes, e.g. electromagnetic form factors, Compton scattering, photoproduction of mesons and various time-like processes. It turned out however that with very few exceptions the size of the ERBL contribution is too small, often by order of magnitude, in comparison with experiment. What does this mean? Are the scales available in present-day experiments, typically about 10 GeV$^2$, too low for applying ERBL factorization or is it possible to improve the results within that scheme? For instance, one may follow the suggestion by Chernyak and Zhitnitky and use DAs which are concentrated in the end-point regions where one of the momentum fractions tends to zero. Such DAs provide much larger ERBL contributions, in some cases even agreement with experiment is achieved, e.g. for the pion form factor. It has been argued that the use of these CZ-type DAs lead to theoretical inconsistencies since the bulk of the perturbative contribution is accumulated in the end-point regions where perturbation theory breaks down. One may also suspect that higher order pQCD corrections lead to a large K-factor but this has not yet been elaborated. However, the known NLO corrections for the pion form factor do not suffice for solving the difficulties with the size of the ERBL contribution if evaluated from DAs close to the asymptotic one, $\Phi_{AS} = 6x(1-x)$.

In order to cure some of the deficiencies of the ERBL factorization scheme Sterman and Li invented to so-called modified perturbative approach in which the quark transverse momenta are retained and Sudakov suppressions are taken into account. Configurations with large transverse separations of the quarks which occur in the end-point regions are suppressed and theoretically consistent results are obtained. For consistency the DAs are to be replaced by transverse momentum dependent light-cone wavefunctions. In general the contributions obtained with the modified perturbative approach are also too small even if CZ-type wavefunctions are used, see for instance Ref. 15.

3. Handbag factorization in exclusive reactions

A new factorization scheme became popular in 1996. In constrast to the ERBL scheme there is only one active parton that participates in the partonic subprocess, e.g. for Compton scattering $\gamma q \rightarrow \gamma q$, see Fig. 2. Similar to the ERBL scheme the active parton is emitted and reabsorbed by the hadron collinearly and is quasi on-shell. The soft hadronic matrix elements are now GPDs. The handbag factorization applies to two different kinematical regions of exclusive reactions. The deeply virtual region is characterized
by large $Q^2$ but small Mandelstam $-t$. In the wide-angle region, on the other hand, $Q^2$ is assumed to be small while $-t$ and $-u$ are considered as large. Reactions studied in both the regions are Compton scattering and photo- and electroproduction of mesons in the space-like region as well as the crossed processes (e.g. $\gamma^{(*)}\gamma \leftrightarrow p\bar{p}$, $p\bar{p} \rightarrow \gamma^{(*)}M$) in the time-like region (see Fig. 2). Related to these processes is the photon-pseudoscalar-meson (P) transition form factor (see Fig. 2). The partonic subprocess is identical to that of Compton scattering but the hadronic matrix element for the $q\bar{q} \rightarrow P$ transition is just the DA appearing in the ERBL factorization scheme. The transition form factor is an exceptional case since the handbag and the ERBL factorization schemes fall together for it. The theoretical result for it, say, to NLO and evaluated from the asymptotic $\pi$ DA is very close to experiment.\(^{19}\) Only the origin of the remaining about 10% is still under debate. Suggested have been NNLO corrections, deviations from the asymptotic DA and/or power corrections.

![Fig. 2. Handbag factorization in the space- and time-like regions and the form factor for photon-pseudoscalar-meson transitions.](image)

For parton helicity non-flip there are four GPDs for the proton in the space-like region denoted by $H$, $\tilde{H}$, $E$ and $\tilde{E}$. They exist for each quark flavor and for the gluon and are functions of three variables, a momentum fraction $x$, the skewness $\xi$ and $t$. For the GPDs a number of properties are known. Thus, $H$ and $\tilde{H}$ reduce to the ordinary unpolarized and polarized parton distribution functions (PDFs) in the forward limit $\xi, t \rightarrow 0$

$$
H^q(x, 0, 0) = q(x), \quad \tilde{H}^q(x, 0, 0) = \Delta q(x), \\
H^g(x, 0, 0) = xg(x), \quad \tilde{H}^g(x, 0, 0) = x\Delta g(x).
$$

The forward limits of $E$ and $\tilde{E}$ are not accessible in DIS. The GPDs are related to the proton form factors by sum rules, e.g. for the Dirac form
factor
\[ F_1^a(t) = \int_{-1}^{1} dx H^a(x, \xi, t), \quad F_1(t) = \sum_a F_1^a(t). \] (3)

Analogous sum rules for \( E \) being related to the Pauli form factor, \( \bar{H} \) (related to the axial form factor) and \( \bar{E} \) (related to the pseudoscalar form factor) hold. Other known properties of the GPDs are polynomiality, universality, evolution, Ji’s sum rule and a couple of positivity constraints.

One may also consider parton helicity flip. These configurations define four more GPDs, termed \( H_T, \bar{H}_T, E_T \) and \( \bar{E}_T \) for each quark flavor and for the gluon. These functions are practically unknown. They are very hard to access since parton helicity flip is frequently suppressed in partonic subprocesses. One may proceed and consider two (or more) active partons. It is straightforward to show that in order to match the requirement of collinear emission and absorption of quasi on-shell partons by the hadrons, at least one hard gluon is to be exchanged between the active partons. These contributions which have not yet been investigated, are therefore expected to be suppressed. It is interesting to note that, say, for Compton scattering off protons the case of three active partons is just the ERBL contribution if dominance of the valence Fock state is assumed.

4. What do we know about the GPDs?

A popular model which allows to construct the GPDs from the PDFs is the double distribution ansatz
\[ f_i(\beta, \alpha, t) = g_i(\beta, t) h_i(\beta) \frac{\Gamma(2n_i + 2)}{2^{2n_i + 1} \Gamma^2(n_i + 1)} \frac{(1 - |\beta|)^2 - \alpha^2}{{(1 - |\beta|)^{2n_i + 1}}}. \] (4)

The functions \( h_i \) represent the PDFs. In the case of \( H \) for instance
\[ h_g = |\beta| g(|\beta|), \quad h_{sea}^a = q_{sea}(|\beta|) \text{ sign}(\beta), \quad h_{val}^q = q_{val}(\beta) \Theta(\beta), \] and \( g_i(\beta, t = 0) = 1, n_i \) either 1 or 2. The GPD is obtained from the double distribution by the following integral representation
\[ H_i(\bar{x}, \xi, t) = \int_{-1}^{1} d\beta \int_{-1}^{1-|\beta|} d\alpha \delta(\beta + \xi \alpha - \bar{x}) f_i(\beta, \alpha, t) + D - \text{term}. \] (6)

For the \( t \) dependence of the GPDs, embodied in the function \( g_i \), several ansaetze are to be found in the literature. The simplest idea is to assume that it represents a \( \beta \) independent kind of form factor but the implied \( \beta - t \) factorization seems to be unrealistic. Another idea is to generalize the
Regge behaviour of the PDFs, \( q(\beta) \rightarrow \beta^{-\alpha(0)} \) for \( \beta \rightarrow 0 \), to non-zero values of \( t \):

\[
g_i(\beta, t) = e^{b_i t} |\beta|^{-\alpha_i' t},
\]

(7)

Here a linear Regge trajectory, \( \alpha_i = \alpha_{i}(0) + \alpha_{i}' t \), is assumed and an exponential \( t \) dependence of the corresponding residue (with a parameter \( b_i \)). There are many applications of the double distribution model, reggeized or not, for instance Refs. 24–29. The advantage of the double distribution model is that the reduction formulas (2) and polynomiality are automatically satisfied. The \( D \)-term which is not related to the PDFs and hence a free function, provides the largest power of \( \xi \) in the moments. It only contributes to the real parts of the gluon and the flavor-singlet quark GPDs. Its quantitative role is not clear.

Alternatively, one may try to extract the GPDs from experimental data in analogy to the determination of the PDFs. First attempts to determine at least the zero-skewness GPDs this way have been published.\(^{31,32}\) The idea is to exploit the sum rules (3) at zero skewness, e.g.

\[
F_1^w = \int_0^1 dx H_v^w = 2F_1^p + F_1^n, \quad F_1^d = \int_0^1 dx H_v^d = 2F_1^n + F_1^p,
\]

(8)

where the valence quark GPDs are defined by \( H_q^v = H_q - H^\bar{q} \). A possible contribution from \( H^s - H^\bar{s} \) has been neglected in (8). The measurements of the strangeness form factors seem to indicate that this contribution is small although non-zero.\(^{36}\) A weak evidence for \( s(x) \neq \bar{s}(x) \) has been found by the CTEQ group.\(^{37}\)

To determine the integrand from the integral is an ill-posed problem in a strict mathematical sense. But using an ansatz for the GPDs with a few free parameters adjusted to experiment, it is possible to extract the GPDs \( H, \bar{H} \) and \( E \) for valence quarks. Admittedly the results on the GPDs depend on the ansatz which one may take as

\[
H_v^q = q_v(x) \exp [f_q(x)t],
\]

(9)

in which

\[
f_q = [\alpha' \log(1/x) + B_q] (1 - x)^{n+1} + A_q x(1 - x)^n,
\]

(10)

and analogously for the other two GPDs. In Ref. 32 a standard slope for the Regge trajectory is assumed (\( \alpha' = 0.9 \text{ GeV}^2 \)), \( n = 2 \) taken and the CTEQ6 PDFs are used as input. The parameters \( A_q \) and \( B_q \) are fitted to the form factor data. In Ref. 31, on the other hand, \( A_q = B_q = 0 \) is assumed as well as \( n = 0 \) while \( \alpha' \) is fitted to the data. The ansatz (9), (10)
is motivated by overlap of Gaussian light-cone wavefunctions at large $-t$ and large $x$ and by Regge behavior at low $-t$ and small $x$ (cf. the double distribution ansatz (4) and (6) in the limit $\xi \to 0$). It should be noted that there is a third somewhat different attempt to extract the zero-skewness GPDs from the form factors.\textsuperscript{39}

The GPDs $H$ and $\tilde{H}$ extracted from the form factor data look similar to the corresponding PDFs at low $-t$ while, at larger $-t$ (beyond the zero of the Regge trajectory), all GPDs exhibit a pronounced peak which moves towards $x = 1$ with increasing $-t$. The GPDs $H_u^v$ and $H_d^v$ are both positive while $\tilde{H}_u^v$ and $\tilde{H}_d^v$ as well as $E_u^v$ and $E_d^v$ have opposite signs. The double distribution model (4), (6) possess also this property. The signs and sizes of the valence quark GPDs are fixed by the known lowest moments of the GPDs at $\xi = t = 0$ $(e_u^v(x) = E_v^u(x, \xi = 0, t = 0))$

\[
\int_0^1 dx u(x) = 2, \quad \int_0^1 dx \Delta u(x) = 0.926, \quad \int_0^1 dx e_u^v(x) = 1.67,
\]

\[
\int_0^1 dx d(x) = 1, \quad \int_0^1 dx \Delta d(x) = -0.341, \quad \int_0^1 dx e_d^v(x) = -2.03. \tag{11}
\]

The moments of $\tilde{H}$ are known from $\beta$ decays, those of $E$ from the anomalous magnetic moments of proton and neutron. Given that the GPDs are smooth functions without zeros they should reflect the properties of the moments at least at low $\xi$ and low $-t$.

With the valence quark GPDs at hand one may evaluate Ji’s sum rule\textsuperscript{18} and determine the total and orbital angular momentum the valence quarks carry. Thus, for instance, from the GPDs derived in Ref. 32 one obtains

\[
L_u^v = -(0.24 \div 0.27), \quad J_u^v = 0.21 \div 0.24,
\]

\[
L_d^v = 0.15 \div 0.19, \quad J_d^v = -0.02 \div 0.02. \tag{12}
\]

The opposite signs of $L_u^v$ and $L_d^v$ but nearly the same magnitude are related to the corresponding property of $E_v$. Fourier transforming the zero-skewness GPDs with respect to the momentum transfer $\Delta$ ($\Delta^2 = -t$)\textsuperscript{40} one learns about the transverse localization of partons, i.e. about their densities in the hybrid representation of longitudinal momentum fraction and transverse configuration space. One may also evaluate various moments of the GPDs and, with regard to their universality property, they provide the soft physics input for the calculation of hard wide-angle exclusive processes as for instance real Compton scattering, see Sect. 7.

Lattice QCD provides a method to calculate moments of the GPDs. In fact the lowest three moments of the GPDs have been worked out as yet\textsuperscript{41}
in scenarios with pion masses between 350 and 800 MeV. The extrapolation to the chiral limit cannot be performed with a sufficient degree of accuracy as yet. In so far the comparison of the lattice results with experiment or other theoretical or phenomenological results is to be done with reservation. Nevertheless, the $t$ dependencies of ratios of moments either obtained from lattice QCD\textsuperscript{41} or from phenomenology\textsuperscript{32} are surprisingly close each other for $-t \leq 1.2$ GeV$^2$ and for a large range of pion masses in the lattice calculation. Also the lattice results\textsuperscript{41} on the orbital angular momentum are in fair agreement with (12) given the uncertainties in both the approaches.

![Graph of $F_d^1/F_u^1$ vs $-t$ and $Q^2$](image)

Fig. 3. The ratio of moments $F_d^1/F_u^1$ from Refs. 32 (left) and 42 (right) versus $-t = Q^2$.

A particularly interesting feature of the GPD $H$ is that the ratio of the lowest moments for $d$ and $u$ valence quarks, $F_d^1/F_u^1$, drops rapidly with increasing $-t$, see Fig. 3. This feature is seen in both the phenomenological\textsuperscript{32} and the lattice\textsuperscript{42} analysis. It seems also to be demanded by experiment although presently an extrapolation of the neutron’s electric form factor is needed for the extraction of these moments from data. The JLab Hall A collaboration (E02-013) will provide data on $G_E^n$ up to about 3.5 GeV$^2$ in the near future which will render an extrapolation unnecessary. Thus we have an indication that $u$ quarks may dominate over $d$ quarks in the proton form factor at large $-t$, a behavior that corresponds to that of the PDFs at large $x$:\textsuperscript{38} $d_{\text{val}}/u_{\text{val}} \propto (1 - x)^{1.6}$. This $t - x$ correlation of form factors and PDFs is a property of the ansatz (9), (10). Indeed one can show that the moments of the form factors drop as

$$F_q^n \propto |t|^{-(1+\beta_q)/2},$$

where $\beta_q$ is the power of $1 - x$ with which the PDFs fall towards $x = 1$ (CTEQ6M:\textsuperscript{38} $\beta_u = 3.4$, $\beta_d = 5$). These results shed doubts on the asser-
tion that the behavior of the Dirac form factor at intermediate values of momentum transfer is a consequence of dimensional counting.

5. Deeply virtual exclusive scattering

Hard electroproduction of photons, vector mesons (V) and pseudoscalar mesons constitute an important class of processes to which the handbag factorization scheme can be applied to. In fact for these processes rigorous proofs of factorization exist in the limit $Q^2 \to \infty$.\textsuperscript{17,43,44} In Fig. 4 typical Feynman graphs are shown which contribute to these processes to leading-twist and LO pQCD accuracy. The dominant helicity amplitudes $(\nu, \nu' (\lambda, \lambda')$ label the helicities of the incoming and outgoing proton (parton), explicit helicities refer to those of photons and mesons) read

$$M_{\nu', \nu}^{\gamma} \sim \sum_a e_a^2 \int_{x_1}^{x_2} d\bar{x} \left[ \frac{F_{a\nu'}^a + \tilde{F}_{a\nu'}}{\bar{x} - \xi + i\epsilon} + \frac{F_{a\nu'}^a - \tilde{F}_{a\nu'}}{\bar{x} + \xi - i\epsilon} \right],$$

$$M_{0\nu', 0\nu}^{M(q)} \sim \sum_a C_V \int_{x_1}^{x_2} d\bar{x} \left[ \sum_{\lambda} \mathcal{H}_{0\lambda, 0\lambda}^M(q) F_{\nu'}^{a\nu} + \sum_{\lambda} 2\lambda \mathcal{H}_{0\lambda, 0\lambda}^{M(q)} F_{\nu'}^{a\nu} \right],$$

(14)

where

$$F_{\nu'}^{a\nu} = H^a - \frac{\xi^2}{1 - \xi^2} E^a, \quad F_{-\nu'}^{a\nu} = 2\nu \frac{\sqrt{t_0 - 1}}{2m(1 - \xi^2)} E^a,$$

(15)

and analogously for $\tilde{F}$. For vector-meson production there is an analogous contribution from the gluonic subprocess (see Fig. 4) to be added. Skewness is fixed in electroproduction by Bjorken-$x$: $\xi \approx x_{\text{Bj}}/(2 - x_{\text{Bj}})$ at small $x_{\text{Bj}}$. Since the interest lies in small $-t$, the $\gamma^* \to \gamma, V, P$ helicity non-flip transitions dominate. I.e. for the Compton process the transverse-transverse transition is leading while the longitudinal-longitudinal transition obviously dominates for the production of pseudoscalar mesons but also for vector mesons. This is so since the subprocesses shown in Fig. 4, suppress transversely polarized vector mesons. Parity conservation tells us furthermore that $\sum_{\lambda} \lambda \mathcal{H}_{0\lambda, 0\lambda} = 0$ for vector mesons while, for pseudoscalar mesons, $\sum_{\lambda} \mathcal{H}_{0\lambda, 0\lambda} = 0$. In other words, electroproduction of pseudoscalar mesons probes the GPDs $\tilde{H}$ and $\tilde{E}$, vector mesons the GPDs $H$ and $E$ to leading-twist order. To DVCS, on the other hand, all four GPDs contribute. The $t$ dependence of the subprocess amplitudes is usually neglected in contrast to that of the GPDs since it provides corrections of order $t/Q^2$. With regard to flavor it is evident that DVCS probes the valence and sea quark GPDs to LO pQCD, $\rho$ and $\omega$ production the gluon GPD in addition. The production of $\phi$ mesons is sensitive to the gluon and sea GPDs, $J/\Psi$ production...
only to the gluon GPD since the charm content of the proton is tiny. The production of $\pi^0$, on the other hand, is only fed by the valence quark GPDs.

Different experiments probe different regions of $\xi$: $\simeq 10^{-3}$ by HERA, $\simeq 10^{-2}$ COMPASS, $\simeq 10^{-1}$ HERMES and $\simeq 0.2 - 0.6$ JLab. Guided by the double distribution model (4), (6), one expects that the role of gluons and sea quarks is diminishing with increasing skewness while that of the valence quarks is increasing. Thus, provided LO handbag physics is dominant at a given hard scale, the study of the mentioned processes over a wide range of $x_{Bj}$ may allow to disentangle the various GPDs.

Vector-meson electroproduction is dominated by the GPD $H$, the others play a minor role. They are noticeable only in spin asymmetries like $A_{LL}$ or $A_{UT}$ measured with longitudinally polarized beam and target or a transversally polarized target, respectively. This is particularly the case for $\rho$ production, for $\omega$ production these effects are larger. Model estimates indicate that for $\tilde{H}$ and $E$ the valence quarks dominate for $\xi \gtrsim 0.01$, sea quarks and gluons contributions seem to be small and cancel each other to some extent. Now, for valence quarks the following combinations occur

$$ F_v^\rho = e_u F_v^{u} - e_d F_v^{d}, \quad F_v^\omega = e_u F_v^{u} + e_d F_v^{d}, \quad (16) $$

where $F_v = H_v, \tilde{H}_v, E_v$. Given the signs of the GPDs discussed in Sec. 4, we see that $H_v^\rho$ is large but $H_v^\omega$ is small while we have the opposite situation in the case of $\tilde{H}_v$ and $E_v$. Thus, $\omega$ production is probably a very good case for studying $\tilde{H}_v$ and $E_v$. This seems to be a rewarding task for JLab.

5.1. **Deeply virtual Compton scattering**

This process is considered to be the theoretical cleanest one and therefore a lot of theoretical and experimental work is devoted to its investigation. Still it is not a simple process. At NLO there are enhanced corrections from the gluonic GPDs which are particularly large at low $\xi$ and overcompensate the suppression by $\alpha_s$. Another interesting feature of DVCS is the
interference with the Bethe-Heitler process for which the final state photon is emitted from the lepton. Since the Bethe-Heitler amplitude is known for given nucleon form factors, the interference region of both the contributions allows to study DVCS at amplitude level. Measurements of $ep \rightarrow e\gamma p$ with a polarized beam or target allows to filter out the interference term.\textsuperscript{46–48} In Fig. 5 a recent result from the Jlab Hall A collaboration\textsuperscript{47} is shown. Clearly seen are the interference regions and a region where the DVCS contribution dominates. Whether the DVCS contribution seen in this experiment can be understood within the handbag framework is still a pending issue, detailed phenomenological analyses of DVCS data have not yet been performed. Only the HERA DVCS cross section data\textsuperscript{49,50} have already been analyzed\textsuperscript{25} to NLO with GPDs obtained from the double distribution model (4), (6). Recently methods have been developed that provide fitting schemes to the data by using a kind of partial wave expansions of the DVCS amplitudes.\textsuperscript{51} These methods are not yet probed in detail.

![Graph of $d^0 \gamma (\text{nb/GeV})$ vs $\theta_\gamma \text{ (deg)}$](image.png)

Fig. 5. Left: The cross section for $ep \rightarrow e\gamma p$. The dash-dot-dotted line represents the Bethe-Heitler contribution. Data taken from Ref. 47.

### 5.2. Electroproduction of mesons

The disadvantage of meson electroproduction as compared to DVCS is that a second soft hadronic matrix element is required, namely the meson wavefunction or DA. This is to be traded for the advantage of separating the GPDs $H$ and $E$ from $\tilde{H}$ and $\tilde{E}$ at leading-twist accuracy. While there is a large set of accurate data available for vector meson electroproduction, only a few data exist as yet for $\pi$ production.\textsuperscript{52} Here I will restrict myself to a few comments on vector-meson electroproduction.

There are several leading-twist, LO pQCD handbag calculations of vector-meson electroproduction\textsuperscript{24,28,29} for which the basic graphs are shown
in Fig. 4. It turned out that the handbag contribution overestimates the cross section for longitudinal photons (\(\gamma^*_L p \rightarrow V_L p\)) although with the tendency of approaching experiment with increasing \(Q^2\), see Fig. 6. This an example of power corrections that persist up to very large scales.

![Fig. 6. Left: The longitudinal cross section for \(\rho\) production versus \(Q^2\) at \(W = 75\) GeV. The solid line represents the handbag predictions, the dashed line the leading-twist contribution. The bands indicate the theoretical uncertainties. Data taken from Ref. 53,54. Right: The \(\rho\) Regge trajectory. Cross section data are taken from Ref. 70.]

It has recently been shown that NLO corrections\(^{55,56}\) are very large due to BFKL-type logarithms \(\sim \ln 1/\xi\) and cancel to a large extent the LO term at low \(Q^2\) and low \(x_{\text{Bj}}\). A recent attempt\(^{57}\) to resum higher orders with methods known from DIS seems to indicate that the sum of all higher order corrections to the LO term is not large. Thus the issue of the size of higher order corrections is still unsettled.

A LO calculation that includes power corrections (modeled by quark transverse momenta) in order to suppress the leading-twist contribution to the \(\gamma^*_L p \rightarrow V_L p\) amplitude and which also allows to calculate the \(\gamma^*_T p \rightarrow V_T p\) amplitude is advocated for in Refs. 28,58. Only the subprocesses are calculated within the modified perturbative approach while the partons are still emitted and reabsorbed by the proton collinearly. The results for \(\sigma_L(\rho)\) obtained in Ref. 28 are shown in Fig. 6. With this approach good agreement with experiment is also achieved for the ratio \(R = \sigma_L/\sigma_T\), some spin density matrix elements, \(A_{LL}\) and the target asymmetry \(A_{UT}\).

Extension of this approach to other transitions is in principle possible. Interestingly, while to the longitudinal amplitude only \(H\) and \(E\) contribute, the other amplitudes are also fed by \(\tilde{H}\) and \(\tilde{E}\). As shown in Ref. 59 the two types of GPDs \(H, E\) and \(\tilde{H}, \tilde{E}\) lead to special symmetry relations among the
helicity amplitudes which are known from the exchange of particles with natural parity \((N)\), and unnatural parity \((U)\), respectively,
\[
\mathcal{M}_\mu'\nu',\mu\nu = \frac{(-1)^{\mu'\nu'\mu}}{\mu\nu} \mathcal{M}_{\mu'\nu',\mu\nu}^{N(U)}. \tag{17}
\]
These symmetry relations prevent interferences between \(N\) and \(U\) type contributions in unpolarized vector-meson electroproduction. Such terms however appear for instance in double spin asymmetries like \(A_{LL}\).

A final remark concerning vector-meson electroproduction is in order. The cross section data\(^{53,54,60,61}\) reveal an asymmetric minimum at \(W \simeq 3-4\) GeV and fixed \(Q^2\). The mild increase of the cross section towards larger energies is well described by the handbag physics but not the sharp increase in the opposite direction. Whether a new dynamical mechanism sets in at low \(W\) or whether it is still handbag physics but with more complicated GPDs remains to be seen. The upcoming data on \(\rho\) electroproduction from CLAS may help in unravelling the physics in that kinematical region.

6. Alternative approaches to deeply virtual processes

Vector-meson electroproduction has a long history. Its main feature is that it behaves similar to elastic two-body reactions. At the beginning this diffractive nature was understood with the help of vector-meson dominance which views the photon as a superposition of vector mesons and, hence, the process as elastic vector-meson proton scattering. Pomeron exchange, supplemented by subleading Regge poles, lead to a fair description of vector-meson electroproduction at least at low photon virtualities. More complicated versions of the Pomeron (soft and hard ones, BFKL Pomeron) allowed for an extension of the Regge model to larger values of \(Q^2\). Later on the Pomeron was viewed as two gluons\(^{62}\) which couple perturbatively to the \(q\bar{q}\) pair created by the virtual photon. Brodsky \textit{et al}\(^{63}\) discussed the limit of large \(Q^2\) but small \(x_{\text{Bj}}\) and showed that the Pomeron-proton vertex is approximately given by the gluon PDF \(g(x_{\text{Bj}})\). This so-called leading-log(1/\(x_{\text{Bj}}\)) approximation which has frequently been applied\(^{64-66}\) similar to that approach is the color-dipole model\(^{67}\). For the HERA setting of the kinematics, i.e. for \(x_{\text{Bj}}\) of the order of \(10^{-3}\), the leading-log approximation is close to the handbag approach, the latter is only enhanced by the skewness effect of about 20\%. For larger values of \(x_{\text{Bj}}\) the leading-log approximation breaks down. It is also not clear how to generalize it to quarks.

There is a renewed interest in Regge ideas, not only for vector-meson electroproduction and the small \(x\) behaviour of the PDFs and GPDs but also for \(\pi\) production and even for DVCS. Complete Regge fits to data exist,
e.g. Ref. 68,69. The spectrum of hadrons forms linear Regge trajectories (see Sect. 4) which means that \( J_\jmath = \alpha_i (t = m^2_\jmath) \) for a hadronic resonance with mass \( m_\jmath \) and spin \( J_\jmath \). The remarkable observation is that these Regge trajectories, continued to negative \( t \), describe the energy dependence of the cross sections of soft two-body reactions at small \(-t\). For instance, for \( \pi^- p \rightarrow \pi^0 n \) to which only the \( \rho \) trajectory contributes, one finds

\[
d\sigma/dt(\pi^- p \rightarrow \pi^0 n) \propto s^{2(\alpha_\rho(t)-1)},
\]

(18) see Fig. 6. In other cases the cross section is subject to a superposition of several Regge trajectories. To each Regge trajectory a residuum is associated which is a free function of \( t \). In spite of this interesting connection between the particle spectrum and the energy dependence of cross sections, the predictiveness of the Regge model is low. It often fails with polarization observables but this can easily be cured by adding other Regge poles and/or cuts. The Regge model lacks an important property any theory and model should have - it cannot be disproved.

7. Wide-angle scattering

It has been argued\(^2\) that at large \( s, -t, -u \) the amplitude for real and virtual \( (Q^2 < -t) \) Compton scattering factorizes in analogy to DVCS (see Fig. 2). The cross section for real Compton scattering reads in this case

\[
\frac{d\sigma}{dt} = \frac{d\hat{\sigma}}{dt} \left\{ \frac{1}{2} \left[ R_V^2(t) + \frac{t}{4m^2} R_T^2(t) + R_A^2(t) \right] \right. \\
- \frac{us}{s^2 + u^2} \left[ R_V^2(t) + \frac{t}{4m^2} R_T^2(t) - R_A^2(t) \right] \left\},
\]

(19)

\( (d\hat{\sigma}/dt \) is the Klein-Nishina cross section). Instead of a convolution as in (14) \( 1/x \) moments of zero-skewness GPDs occur now \( (F_{vi} = H_v, \tilde{H}_v, E_v) \)

\[
R_i(t) \simeq \sum_{a=u,d} e^2_a \int_0^1 \frac{dx}{x} F_{vi}^a(x, 0, t),
\]

(20)

The tensor form factor \( R_T \) describes proton helicity flip.\(^7\) With the zero-skewness GPDs,\(^3\) discussed in Sect. 4, at hand these Compton form factors can be evaluated and the Compton cross section predicted; there is no free parameter. A very good agreement with the recent JLab Hall A data\(^8\) is achieved for sufficiently large Mandelstam variables. The handbag approach also predicts interesting spin effects. For instance, the helicity transfer from the initial photon to the outgoing proton reads

\[
K_{LL} \simeq \frac{s^2 - u^2}{s^2 + u^2} \frac{R_A(t)}{R_V(t)}.
\]

(21)
Also this result is in agreement with a JLab measurement. The large positive value of $K_{LL}$ found in Ref. 73, is very difficult to achieve in the ERBL factorization scheme.

For the corresponding time-like process $\gamma\gamma \rightarrow p\bar{p}$ a cross section, similar to (19), can be derived but the form factors are unknown and have to be extracted from experiment. It turns out that these form factors are larger than the corresponding space-like form factors, a feature that is known from the electromagnetic form factors. The handbag approach accounts for all the features of the BELLE data. It can be extended to other two-photon channels like pairs of hyperons or mesons. It also applies to photoproduction of mesons and to $p\bar{p} \rightarrow \gamma M$. As for deeply virtual meson electroproduction (see Sect. 5.2) there are difficulties with the normalization of the cross sections which have not yet been settled. Other features of these processes, as for instance the ratio of the $\gamma n \rightarrow \pi^- p$ and $\gamma p \rightarrow \pi^+ n$ measured at Jlab, are quite well understood in the handbag approach.

8. Summary

In this talk I have sketched the factorization schemes in use for hard exclusive scattering processes and discussed their applications in some detail. The main interest has been focussed on the handbag approach since its prospects of becoming the standard description of both the deeply virtual and the wide-angle exclusive processes are best although a detailed comparison between theory and experiment is still pending. With the exception of vector-meson electroproduction for which already a vast amount of data exist, data for hard exclusive processes which cover a wide range of kinematics are still lacking but are expected to become available in the near future from all pertinent experiments. The upgraded JLab will provide even more data on these processes in a few years. A definite judgement of the handbag approach cannot be given at present. In case that the handbag approach survives the detailed future tests we will learn much about the GPDs and the structure of the proton.

A special case are the valence quark GPDs at zero skewness which, with a few assumptions, can be accessed through the data on the nucleon form factors. JLab is in the position of providing more form factor data in the near future ($G_{E}^{n}$ from CLAS, $G_{E}^{p}$ from PR01-109, $G_{E}^{p}$ from E02-013) which will lead to improved GPDs. From the upgraded JLab more form factor data can be expected that will allow for an extension of the $t$ range in which the zero-skewness GPDs can be extracted. Lattice QCD results on moments of GPDs, provided these are reliably extrapolated to the limit of
the physical mass of the pion, may diminish the dependence of these GPDs on the chosen parameterization.

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