Constraining Bianchi Type I Universe With Type Ia Supernova and H(z) Data

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Abstract. We use recent 36 observational Hubble data (OHD) in the redshift range $0.07 \leq z \leq 2.36$, the "joint light curves" (JLA) sample, comprised of 740 type Ia supernovae (SNIa) in the redshift range $0.01 \leq z \leq 1.30$, and their joint combination datasets to constrain anisotropic Bianchi type I (BI) dark energy model (DE). To estimate model parameters, we used Metropolis-Hasting algorithm to perform Monte Carlo Markov Chain analysis. We also compute the covariant matrix for BI dark energy model considering different datasets to compare the correlation between parameters of the model. To check the acceptability of our fittings, all results are compared with those obtained from 9 year WMAP as well as Planck (2015) collaboration. Our estimations show that at 68\% confidence level, the dark energy equation of state (EOS) parameter for JLA data varies in quintessence-phantom region while for OHD and the joint combination of datasets only varies in phantom region. It is found that the current cosmic anisotropy is of order $10^{-3}$ which imply that OHD and JLA datasets do not put tight constrain on this parameter. The deceleration parameter is obtained as $q = -0.46^{+0.09}_{-0.36}$ from OHD, $q = -0.619^{+0.12}_{-0.41} - 0.37$, $q = -0.619^{+0.12}_{-0.41} - 0.37$, and $q = -0.52^{+0.06}_{-0.04}$ for H(z), SNIa, and H(z)+SNIa data respectively.

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1 Introduction

The cornerstone of recent day cosmology is the belief that the place we live in universe has no privileged position in the universe. This simple an powerful idea is called cosmological principle (CP). Mathematically this means that there are necessarily translational symmetries from any point of space to any other which implies that space should be homogeneous (universe looks the same at any point). Moreover, at enough large scales since universe looks the same a any direction, there should be rotational symmetries which imposes the isotropic property to the geometry of space. A maximally symmetric space-time satisfying the cosmological principle is given by Friedmann-Robertson-Walker (FRW) metric. From observational point of view, it is widely believed that our universe could be accurately described by FRW model as the cosmic microwave background (CMB) temperature is highly isotropic about our position. Nevertheless, the high symmetry of FRW models represents a very high degree of fine tuning of initial conditions which implies that this models are infinitely improbable in the space of all possible cosmologies. Although, the observed universe could be describes by FRW models at current epoch, there are some important questions (1) does our universe necessarily posses the same symmetries outside the particle event horizon? (2) Are there possible models that will fit the observations rather than FRW models? furthermore, recent observations indicate small variations between the intensities of cosmic microwave background (CMB) coming from different directions which may be related to the origin of structure in the universe. Of course, a more general and realistic metric posses both inhomogeneity and anisotropy properties, but in this case the exact solution of Einstein’s field equations are almost impossible. Therefore, we usually simplify this general metric in following two sub classes: (1) isotropic and inhomogeneous models given by Lemaitre-Tolman-Bondi (LTB) metric \cite{11234} (2) anisotropic and homogeneous models given by Bianchi metrics \cite{9}. In fact, at least, these models provide an arena for testing the accuracy of FRW models in describing our universe at the present epoch. It is worth to mention that some Bianchi models isotropize due to inflation \cite{8}.

According to recent observations the expansion rate of universe is accelerating \cite{7890}. In the context of General Relativity (GR) this means that there must be an extra component in the cosmic fluid which acts against gravity. Because of the lack of our knowledge, this extraordinary element is called dark energy (DE). Since we still could not detect any interaction between DE and ordinary matter, in spite of many efforts, our informations about this component is pretty less. Fortunately, the nature of DE could be investigated through it’s equation of state parameter (EOS) which defined as the ratio of pres-
sure to energy density $\omega = p/\rho$. Recent 9 year WMAP [10] and Planck (2015) [11] collaboration results, at 68 confidence level, show that $-1.16 < \omega_X < -0.983$ and $-1.099 < \omega_X < -0.944$ respectively ($\omega_X$ refers to the dark energy EOS parameter). The dark energy EOS parameter could be considered as a constant parameter (i.e $\omega_X = -1$) described by cosmological constant or a dynamical time varying function of time (or equivalently redshift) which could be described by scalar fields. As interval $-1/3 > \omega_X > -1$ is called quintessence region, phantom region indicates by $\omega_X < -1$. While cosmological constant scenario faces two serious theoretical problems namely the fine-tuning and the coincidence problems [12,13], phantom scenario suffers from ultraviolet quantum instabilities [14] and quintessence does not match with recent observations [15] which indicate the possibility of crossing phantom divide line (PDL) at 68 confidence level (CL). The thing which we almost know precisely is that at matter dominated era the cosmic expansion was decelerating then, at a certain redshift called transition redshift, dark energy dominated over universe and hence the expansion phase changed to accelerating one. We can investigate this phase transition by tracing the sign change of the universal deceleration parameter $q(z)$ in the history of cosmic evolution. In general, basic characteristics of the cosmological evolution could be expressed in terms of the Hubble parameter $H_0$ and the deceleration parameters $q_0$ [15].

On the bases of inhomogeneous LTB spacetime, Zibin [16,17], Valkenburg et al [18], Zumalacarregui et al [19] and Tokutake et al [20] have recently investigated dark energy in different context of use. Very recently Authors of Ref [21] have studied some DE models in the scope of LTB spacetime. There are increasing interest in the study of dark energy models in the scope of anisotropic Bianchi spacetimes (for example see [22,23,24,25,26,27,28] also see [29,30] for recent review). Recently, we have studied viscous dark energy in the scope of Bianchi type V spacetime [31]. It is worth noting that for more than five decades there have been considerable studies of CMB temperature in spatially homogeneous universes have used the observed temperature anisotropy to place constrain on the overall anisotropy of the cosmic expansion [32,33]. Motivated by the situation discuses above, we investigate dark energy in the scope of Bianchi type I (ωBI) universe through the recent 36 observational Hubble data (OHD) in the intermediate $0.07 \leq z \leq 2.36$ compiled by Yu et al [34], “latest joint light curves” (JLA) dataset [1] comprised of 740 type Ia supernovae in the redshift range $z \in [0.01, 1.30]$, and their combination. Note that JLA dataset provides model-independent apparent magnitudes instead of model-dependent distance moduli whereas several SN datasets such as Union provide cosmological distance moduli that are derived assuming a flat $\Lambda$CDM model and hence can not be applied to other models such as Bianchi spacetimes. We estimate all parameters of BI dark energy model (some other parameters are also derived from fit parameters) by the aide of Markov Chain Monte Carlo (MCMC) technique and compare our results to the 9years WMAP & Planck(2015) to evaluate the robustness of our fits. The plane of this paper is as follows. In section 2 we briefly discuss the theoretical DE models. Section 3 deals with the summary of computational technique we have used to fit parameters to data. In section 4 we study $\omega BI$ dark energy model and fit its parameters to OHD, JLA, and their joint combination datasets. Finally, we summarize our findings and conclusions in section 5.

## 2 Theoretical Models

In synchronous coordinate system we construct the following general $(N + 1)$-dimensional inhomogeneous and anisotropic Lorentzian spacetime with the metric

$$ds^2 = -dt^2 + \delta_{ij} g_{ij} dx^i dx^j, \quad i, j = 1, 2, \ldots N,$$

where $g_{ij}$ are functions of $(t, x^1, x^2, x^3)$ and $t$ refers to the cosmological (or cosmic) time. In 4-dimensions, we could generate FRW and Bianchi type I models from above equation as

$$\begin{cases} 
\text{if } \delta_{ij} g_{ij} = a^2(t, x) & \text{FRW model} \\
\text{if } \delta_{ij} g_{ij} = a_0^2(t, x) & \text{BI model}
\end{cases}$$

Above relations show that for FRW universe all three metric potentials are equal (i.e $g_{11} = g_{22} = g_{33} = a^2(t, x)$) which demonstrates an isotropic but inhomogeneous spacetime whereas for BI model, metric components are different functions ($g_{11} = A^2(t, x), g_{22} = B^2(t, x), g_{33} = C^2(t, x)$) which indicates an anisotropic and inhomogeneous spacetime. It is worth to noting that in BI case, the average scale factor is defined as $a = \langle ABC \rangle^{1/3}$. In an inhomogeneous universe, metric components are function of time and spatial coordinates. But, for simplicity, we assume that in both FRW and BI models, metric components are functions of time only. Hence, FRW describes a homogeneous and isotropic universe which obeys the cosmological principle (CP) whereas BI is homogeneous but anisotropic which violates CP. We consider the possible constituents of the universe to be in the perfect fluids form, meaning that we neglect the effect of viscosity or heat flow. Under this condition, the perfect fluid energy-momentum tensor could be written as

$$T_{ij} = diag(-\rho c^2, p, p, p),$$

where $\rho$ is the total energy density, $p$ is pressure and $c$ is the speed of light. The Einstein’s field equations (in gravitational units $8\pi G = c = 1$) read as

$$R_{ij} - \frac{1}{2} R g_{ij} = T_{ij}. $$

Given the general metric eq. (1), the $0 - 0$ and the $i - i$ components of Einstein’s equation lead to the following equations [35]

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{1}{3} (\rho_m + \rho_X + p_r) + \frac{k}{a^2},$$

---

1. All data used are available on [http://supernovae.in2p3.fr/sdss-snls-jla/ReadMe.html](http://supernovae.in2p3.fr/sdss-snls-jla/ReadMe.html)
and
\[
\frac{\dddot{a}}{a} = -\frac{1}{6}(\rho_m + \rho_X + \rho_r + 3p), \tag{6}
\]
respectively. Here \((\rho_m, \rho_r, \rho_X)\) are the DE, DM, and radiation energy densities and \(k = (k, A_0)\) for FRW and BI models respectively, where \(k\) stands for curvature and \(A_0\) indicates the anisotropy amount of BI model\(^2\). The density fractions \(\Omega_m, \Omega_r, \Omega_X, \Omega_k,\) and \(\Omega_A\) are defined by
\[
\begin{align*}
\Omega_m &= \frac{\rho_m}{3H^2}, \quad \Omega_r = \frac{\rho_r}{3H^2}, \quad \Omega_X = \frac{\rho_X}{3H^2}, \\
\Omega_k &= \frac{k}{H^2}, \quad \Omega_A = \frac{A_0}{H^2}, \tag{7}
\end{align*}
\]
Therefore, from \(\text{(5)}\) the Hubble parameter \(H(z)\) is
\[
H(z)^2 = H(z)^2 = H_0^2 \left[ \Omega_m(1 + z)^3 + \Omega_r(1 + z)^4 + \Omega_X(1 + z)^{3(1 + \omega_X)} + \Omega_k(1 + z)^{\omega_k} \right], \tag{8}
\]
where \(\omega_X = p_X/\rho_X\) is the equation of state parameter of DE fluid (note that, as usual, \(\rho_m = 0\) which imply \(\omega_m = 0\)) and \(a = (1 + z)^{-1}\). Requiring the consistency of \(\text{(8)}\) at \(z = 0\), gives
\[
\begin{align*}
&\left\{ \begin{array}{ll}
\Omega_m + \Omega_r + \Omega_X + \Omega_k = 1 & \text{FRW model} \\
\Omega_m + \Omega_r + \Omega_X + \Omega_A = 1 & \text{BI model}
\end{array} \right. \tag{9}
\end{align*}
\]
From \(\text{(8)}\) the possible cosmologies that could be considered in our study are shown in Table 1.

Table 1: Three possible cosmological models which could be derived from eq (8)

| Parameter | Flat \(\omega\)CDM | Non-Flat \(\omega\)CDM | \(\omega\)BI |
|-----------|------------------|-------------------|------------|
| \(n\)     | =2               | =2                | =6         |
| \(H_0\)   | \checkmark       | \checkmark        | \checkmark |
| \(\Omega_m\) | \checkmark     | \checkmark        | \checkmark |
| \(\Omega_r\) | \checkmark   | \checkmark        | \checkmark |
| \(\Omega_X\) | \checkmark | \checkmark        | \checkmark |
| \(\Omega_k\) | \times          | = \Omega_k       | = \Omega_A |
| \(\omega_X\) | \checkmark | \checkmark        | \checkmark |

In the history of cosmic evolution, first, our universe was undergoing a decelerating expansion, then at a certain redshift (time) \(z_t\) dark energy dominates over the Universe in terms of the redshift range \(0 < z < 2.36\). This transition redshift could be obtained by condition \(q(z_t) = \dddot{a}/a = 0\). The deceleration parameter is defined as
\[
q(z) = \frac{1}{H^2} \frac{\dddot{a}}{a} = \frac{1}{H(z)} \frac{dH(z)}{dz} - 1, \tag{10}
\]
which in turn gives
\[
q(z) = \frac{1}{2}(1 + 3\omega_X \Omega_X - \Omega_k(1 + z)^\omega_k), \tag{11}
\]
From eqs \((9)\), \((7)\) and \((10)\) we get
\[
\begin{align*}
\Omega_m &= \frac{\Omega_m}{(\Omega_m + \Omega_r + \Omega_k - 1)(1 + 3\omega_X)} \frac{1}{x^2} - 1. \tag{12}
\end{align*}
\]
For all three models, one also could obtain the age of the Universe in terms of the redshift \(z\) as
\[
dt = \frac{da}{H} \Rightarrow t_0 = \int_0^{\infty} \frac{dz}{(1 + z)^H}, \tag{13}
\]
which in turn gives (by the aid of eq \((8)\))
\[
\begin{align*}
t_0 &= \int_0^{\infty} \frac{H_0^4 dz}{\sqrt{\Omega_m(1 + z)^3 + \Omega_X(1 + z)^{3(1 + \omega_X)} + 2 + \Omega_k(1 + z)^{\omega_k + 2}}}.
\end{align*}
\tag{14}
\]

It is worth to mention that since almost all observations put a very tight constraint on \(\Omega_m\) which is in order of \(\sim 10^{-5}\), we have neglected this parameter from our estimations. In next section we fit \(\omega\)BI dark energy model to the OHD, JLA, and their combination to compare the values of estimated parameters to the parameters of Table 2.

Table 2: Results from 9years WMAP and Planck 2015 collaboration for \(\Lambda\)CDM model at 1 \(\sigma\) confidence level.

| Parameter | WMAP+eCMB+BAO+H0 | TT+TE+EE+lensing+BAO+JLA+H0 |
|-----------|------------------|-------------------------------|
| \(H_0\)   | 68.92\(\pm\)0.04 | 67.74 \(\pm\) 0.46          |
| \(\Omega_m\) | 0.285\(\pm\)0.004 | 0.3099 \(\pm\) 0.0062       |
| \(\Omega_k\) | 0.717 \(\pm\) 0.011 | 0.6911 \(\pm\) 0.0062       |
| \(\Gamma_h\) | -0.0027\(\pm\)0.0039 | 0.0085\(\pm\)0.0040       |
| \(\omega_X\) | -1.074\(\pm\)0.000 | -1.109\(\pm\)0.007        |
| \(\omega_k\) | -0.000\(\pm\)0.000 | -0.000 \(\pm\) 0.000       |
| \(t_0\)   | 13.88 \(\pm\) 0.16 | 13.799 \(\pm\) 0.021       |

3 Data Sets And Method

In what follows we use Metropolis-Hasting algorithm to perform to generate MCMC chains and place constraints on cosmological parameters of \(\omega\)BI dark energy model. To do so, we use independent observables that are (1) Observational Hubble data (OHD) including 36\(H(z)\) data-points (see Table 3 in the redshift range \(0.07 \leq z \leq 2.36\) (note that because of the partial overlap of the WiggleZ and BOSS spatial regions (see Beutler et al [39]), we drop the three Blake et al [37] WiggleZ radial BAO points from Table 1 of Farooq et al [38] but include the recent redshift \(z = 0.47\) cosmic chronometric measurement [39]), (2) JLA dataset comprised of 740 type Ia supernovae in the redshift range \(0.01 \leq z \leq 1.30\) [40], and their joint combination which could increase the sensitivity of our

\(^2\) In Ref [35] it has been shown that the anisotropy parameter in BI model decays as \(A = A_0a^{-6}\).
estimates. In case of OHD dataset, we minimize the following likelihood log marginal likelihood function
\[
\ln L = -\frac{1}{2} \sum_{i,j}^N \left[ H_{th}(z) - H_{obs}(z_i) \right] (c^{-1})_{ij} \left[ H_{th}(z) - H_{obs}(z_j) \right],
\]
(15)
where \(c^{-1}_{ij}\) is the inverse of covariance matrix of the observed data. In this case, since three galaxy distribution radial BAO \(H(z)\) measurements \([48]\) are correlated, it is straightforward to find correlation matrix from Table 3 which is given by
\[
c = \begin{pmatrix}
3.65 & 1.78 & 0.93 \\
1.78 & 3.65 & 2.20 \\
0.93 & 2.20 & 4.45
\end{pmatrix}.
\]
(16)
In case of SNIa dataset, we minimize
\[
\ln L = -\frac{1}{2} \sum_{i,j}^N \left[ \mu_{th}(z) - \mu_{obs}(z_i) \right] (c^{-1})_{ij} \left[ \mu_{th}(z) - \mu_{obs}(z_j) \right],
\]
(17)
where the predicted distance modulus, \(\mu(z)\), for a flat space-time may be given by
\[
\mu(z) = 5 \log_{10} \left[ 3000(1+z) \int_0^z \frac{dz}{E(z)} \right] + 25 - 5 \log_{10}(h)
\]
(18)
Here \(E(z) = H(z)/H_0\) is the reduced Hubble parameter given by eq \([8]\) and \(H_0 = 100h\) km s\(^{-1}\)Mpc\(^{-1}\). It is clear that \(h\) is an additive constant and hence marginalizing over it does not affect the SNe results. It is worth to mention that for uncorrelated data (including all JLA data) we have \(c_{ij} = \text{diag}(\sigma_i^2)\). Since two datasets are assumed to be independent, the total likelihood could be defined as the product of the likelihoods of the single datasets. The total likelihood is given by
\[
L_{tot} = L_{OHD} \times L_{SNIa}.
\]
(19)
In following section, we estimate parameters of flat \(\omega BI\) dark energy model. We also derive transition redshift \(z_t\), deceleration parameter \(q\), and age of universe \(t_0\) for this model. The prior for all parameters of model are assumed to be Uniform.

4 Results and discussion

Dark energy \(\omega BI\) model has five unknown parameters to be estimated from 36\(H(z)\), JLA, and their joint combination. The base parameters set for this model is
\[
\theta = \{ \Omega_m, \Omega_X, \Omega_A, \omega_X, H_0 \}.
\]
(20)
Table 3 demonstrates the results of statistical analysis for \(\omega BI\) DE model using OHD, JLA, and their combination. The contour plots, at 1\(\sigma\) and 2\(\sigma\) confidence levels, of the model parameters for OHD, JLA, and OHD+JLA

| \(H(z)\) | \(\sigma_H\) | \(\sigma_\mu\) | \(z\) | Reference |
|-------|--------|--------|-----|---------|
| 69    | 19.6   | 0.070  |  |        |
| 69    | 12.2   | 0.090  |  |        |
| 68.6  | 26.2   | 0.120  |  |        |
| 83    | 8      | 0.170  |  |        |
| 75    | 4      | 0.179  |  |        |
| 75    | 5      | 0.199  |  |        |
| 72.9  | 29.6   | 0.200  |  |        |
| 77    | 14     | 0.270  |  |        |
| 88.8  | 36.6   | 0.280  |  |        |
| 83    | 14     | 0.352  |  |        |
| 81.9  | 1.9    | 0.350  |  |        |
| 83    | 13.5   | 0.3802 |  |        |
| 95    | 17     | 0.400  |  |        |
| 77    | 10.2   | 0.4004 |  |        |
| 87.1  | 11.2   | 0.4247 |  |        |
| 92.8  | 12.9   | 0.4407 |  |        |
| 89    | 50     | 0.47   |  |        |
| 80.9  | 9      | 0.4783 |  |        |
| 97    | 62     | 0.480  |  |        |
| 90.8  | 1.9    | 0.520  |  |        |
| 104   | 13     | 0.593  |  |        |
| 97.8  | 2.1    | 0.610  |  |        |
| 92    | 8      | 0.660  |  |        |
| 105   | 12     | 0.741  |  |        |
| 125   | 17     | 0.875  |  |        |
| 90    | 40     | 0.890  |  |        |
| 117   | 23     | 0.890  |  |        |
| 114   | 30     | 1.037  |  |        |
| 168   | 17     | 1.300  |  |        |
| 160   | 33.6   | 1.363  |  |        |
| 177   | 18     | 1.420  |  |        |
| 140   | 14     | 1.530  |  |        |
| 202   | 40     | 1.750  |  |        |
| 186.5 | 80.4   | 1.965  |  |        |
| 222   | 7      | 2.310  |  |        |
| 227   | 8      | 2.360  |  |        |

are also depicted in Figure 1. In this case, at 1\(\sigma\) confidence levels, for JLA data the dark energy EOS parameter varies between quintessence and phantom regions \((-1.36 \leq \omega_X \leq -0.84)\) whereas for OHD and OHD+JLA data the EOS parameter only varies in phantom region (for OHD \(-1.54 \leq \omega_X \leq -1.404\) and for JLA \(-1.46 \leq \omega_X \leq -1.08\)). It is obvious that our results support phantom dark energy scenario in BI dark energy model. This result is in agreement with WMAP & Planck (2015) collaboration. When we use joint OHD+JLA dataset, the values of estimated parameters are obtained in close agreement with the concordance model. Our joint analysis constrains anisotropy parameter as \(-38 \times 10^{-4} \leq \Omega_A \leq -16 \times 10^{-4}\) at 1\(\sigma\) error which is 10 times larger than \(\sim 10^{-5}\) level anisotropies in the CMB. This result shows that using these two datasets are not enough to constrain anisotropy parameter, \(\Omega_A\), in BI universe. It is worth mentioning that \(H(z)\) data are not sensitive to the behavior of cosmological spatial inhomogeneities \([50]\). Hence, more precise measurements of \(H(z)\) at higher redshift are needed for tighter constraints on \(\Omega_A\). Also we observe that the for joint combination of two datasets, the estimated value of the current expansion rate of universe \(H_0\) is in good agreement to the Adel et al, Planck, \((67.74 \pm 0.46)\) \([11]\), Hinshaw et al, WMAP, \((68.92 \pm 0.84)\) \([10]\), but deviates from Riess et al \((73.4 \pm 1.74)\) \([51]\). It is worth noting that our estimated \(H_0\) is in excellent agreement with Chen & Ratra \((68 \pm 2.8)\) \([52]\). Figures 2 and 3 depict the robustness of our fits. From Figure 2 we observe that the joint dataset gives raise to quite better fit. It is worth noting that al-

Table 3: Hubble parameter versus redshift data.
though JLA dataset by itself is not sensitive to the expansion rate $H_0$, but when we combine it to OHD dataset, in the joint analysis, JLA constrains other parameters of the Model which in turn affect the estimate of $H_0$. This is why in Table 4 we observe a change in the value of $H_0$ when fitting model to the joint OHD+JLA dataset. From the $H(z)$ data we find evidence for the cosmological deceleration-acceleration transition to have taken place at a redshift $z_t = 0.72 \pm 0.14$ which is in good agreement with the Farooq et al. [38] determination of $z_t = 0.72 \pm 0.05$ as well as Busca et al. [53] determination of $z_t = 0.82 \pm 0.08$ at 1σ error. The deceleration-acceleration transition takes place at $z_t = 0.72 \pm 0.14$ at 68% CL which is in good agreement with the results in Tables 1 and 2 of Ref [54]. From The join combination of $H(z)$ and SNIa datasets we find that the change of the BI expansion phase from deceleration to acceleration takes place at a redshift $z_t = 0.57_{-0.15}^{+0.037}$ which is in good agreement with the results obtained by Vargas dos Santos et al. [55].

In Figures 4, we have plotted the dependence of deceleration parameter, $q(z)$ as a function of redshift $z$ for OHD (fig. 4a), JLA (fig. 4b), and OHD+JLA (fig. 4c) at 1σ & 2σ error. The solid lines show the mean value of $q(z)$ and filled circles indicate the best fit value of deceleration parameter at $z_t$ in each figure. We obtained $q = -0.46{}_{-0.41}^{+0.36}$, $q = -0.619{}_{-0.095}^{+0.12}{}_{-0.24}$, and $q = -0.55{}_{-0.046}^{+0.080}{}_{-0.14}$ for $H(z)$, SNIa, and $H(z)+SNIa$ data respectively. These results are in good agreement with those reported in Refs [35, 53, and 54].

A useful tool to study the degeneracy direction between estimated parameters is covariance matrix. The covariance matrix $C$ of the parameter space $\{\theta\}$ could be
Table 4: Results from the fits of the flat $\omega BI$ model to the data at 1$\sigma$ confidence level.

| Parameter       | OHD   | JLA   | OHD+JLA |
|-----------------|-------|-------|---------|
| Fit parameters  |       |       |         |
| $H_0$           | 69.49±0.70 | -     | 69.2±1.2 |
| $\Omega_m$      | 0.4300±0.0074 | 0.315±0.14 | 0.364±0.054 |
| $\Omega_X$      | -0.00423±0.00013 | -0.0037±0.0006 | -0.00246±0.00006 |
| Derived parameters |     |       |         |
| $H_X$           | 0.547±0.0073  | 0.488±0.017  | 0.638±0.050 |
| $z_t$           | 0.4097±0.0089  | 0.80±0.61    | 0.57±0.15 |
| $t_0$           | 11.5±0.72     | 12.23±0.24   | 12.36±0.12 |
| $q$             | -0.359±0.011  | -0.619±0.012 | -0.52±0.08 |

Fig. 2: The Hubble rate of the flat $\omega BI$ model versus the redshift $z$ at 1$\sigma$ and 2$\sigma$ confidence level. The points with bars indicate the experimental data summarized in Table 3. The solid line shows the mean value of $H(z)$.

Fig. 3: Schematic representation of $H_0$ (at 1$\sigma$) for flat $\omega BI$ model (OH$D$(purple color) and OHD + JLA(cyan color)). Constraints from the direct measurement by Riess et al. (2016) (red color) WMAP (green color), and the Planck Collaboration (2015) (blue color) are also shown.

Fig. 4: Plots of the deceleration parameter of $\omega BI$ model using: (a) Hubble (OHD), (b) SNIa (JLA), and (c) OHD + JLA combination data. The solid lines show the mean value of $q(z)$ and filled circles indicate the best fit value of deceleration parameter at $z_t$ in each figure.

defined as

$$C_{ij} = \rho_{ij}\sigma(\theta_i)\sigma(\theta_j),$$

where $\rho_{ij}$ is called as the correlation coefficient between parameters $\theta_i$ and $\theta_j$. $\sigma(\theta_i)$ and $\sigma(\theta_j)$ are the 1$\sigma$ uncertainties in parameters $\theta_i$ and $\theta_j$. Note that $\rho$ varies from 0 (independent) to 1 (completely correlated). We can estimate the covariance matrix $C$ from the MCMCs. Figure 5 depicts the correlation matrix for OHD (fig. 5a), JLA (fig. 5b), and OHD+JLA (fig. 5c). It is clear that when we apply joint combination, the correlations between estimated parameters decreases.
According to the recent observations there is a tiny difference between intensities of microwaves coming from different directions of the sky. This fact motivated us to study universe in the scope of anisotropic Bianchi type I in such a way to describe our universe in more realistic situation with respect to FRW universe. We considered two independent observational datasets namely OHD and JLA as well as their joint combination to constrain $\omega_B I$ dark energy model which is inherently a flat spacetime. We compared our results to the recent results of WMAP and Planck (2015) collaboration. We found that for JLA data, at $1\sigma_{CL}$, the estimated value of the dark energy EOS parameter varies between quintessence and phantom regions whereas for OHD and joint OHD+JLA dataset the estimated value of the EOS parameter only varies in phantom region. By looking at Table 2 one could see that both WMAP & Planck (2015) predict the possibility of DE to vary in phantom region. The joint analysis constraints the anisotropy parameter of $\omega_B I$ DE model to vary in the range $-38 \times 10^{-4} \leq \Omega_A \leq -16 \times 10^{-4}$ at $1\sigma$ confident level which is not in good agreement with the recent CMB observations which indicate that this parameter is of order $\sim 10^{-5}$. In fact, this parameter is important in the study of early universe i.e at high redshifts, when the anisotropy plays more effective role in the structure formation of our universe. It is worth to mention that the measure of the Hubble parameter at high redshifts will be possible by detecting the Sandage-Loeb signal (SL signal)\(^3\). For example, the undergoing project CODEX (COSmic Dynamics and EXo-earth experiment)\(^4\) aims to detect the SL signal with the European Extremely Large Telescope (E-ELT)\(^2\). It is also found that the estimated value of the Hubble rate $H_0$ for joint OHD+JLA dataset is in excellent agreement with recent observations from WMAP and Planck 2015 collaboration but it has meaningful deviation from Riess et al\(^1\)\(^1\)\(^\text{(2011)}\) result. The age ($t_0$), transition redshift ($z_t$), and deceleration parameter ($q$) are seen to be estimated much better to the joint HOD+JLA with respect to any individual dataset. These parameters are in good agreement with those obtained in Refs.\(^3\)\(^4\)\(^5\). It is worth noting that other data, such as BAO, growth factor, or CMB anisotropy data can tighten the constraints on the parameters of $\omega_B I$ model, and it is of interest to study how the other data constrains parameters when used in conjunction with the H(z) and SNIa data we have used in this work.

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