In this paper, a class of impulsive neutral stochastic functional partial differential equations driven by Brownian motion and fractional Brownian motion is investigated. Under some suitable assumptions, the $p$th moment exponential stability is discussed by means of the fixed-point theorem. Our results also improve and generalize some previous studies. Moreover, one example is given to illustrate our main results.

1. Introduction

In recent years, stochastic differential equations (SDEs) have come to play an important role in many areas such as physics, population dynamics, electrical engineering, medicine biology, ecology, economics and other areas of science, and engineering. Because of their great applications, stochastic differential equations have been developed very fast; see, for example, [1–27].

In observing the process of stock price fluctuations, it is found that the fluctuations of stock prices are not self-similar; on a larger time scale (month or year), these processes are more stable and more stable than on a small time scale (hour or day). One reason is that random noise in the market is a sum of irregular “trading” noise. Therefore, it can be assumed that the stock price is affected by two random phenomena; one is that the incremental process is independent, and the other is that the incremental process is related. Generally speaking, the random perturbation of stock prices consists of two parts: one is the basic part, that is, the overall economic situation of the society, comes from the actual financial background of the stock market and has a long correlation, so it can be expressed by fractional Brownian motion; the other is trading part, that is, the random trading conditions of stockholders in the stock market, is derived from the stochastic inherent factors of stockholders, so it can be expressed by Brownian motion. In addition, similar phenomena have appeared in the research of fluid mechanics, electrical communication, economics, and finance. Therefore, the mixed model has been considered by many authors; see, for example, [7, 10, 12, 16, 20, 23, 24, 27].

On the other hand, impulsive effects are caused by instantaneous perturbations at a certain moment which can be used to model many practical problems that arise in the areas of mechanics, electrical engineering, medicine biology, ecology, and so on. Therefore, there has been increasing interest in the theory of impulsive differential equations (for example, [2, 6, 10, 21, 22]). Stochastic partial differential equation is one of the most important, active, and rapidly developing key research fields in probability due to its wide and great applications in physics, chemistry, biology, economic, finance, and so on. On the other hand, many dynamical systems not only depend on present and past states but also involve derivatives with delays. Neutral stochastic functional differential partial differential equations are often used to describe such systems. It is well known that the time delay and stochastic perturbations may cause oscillation and instability in systems. It is important to consider the influence of delay and stochastic perturbations in the investigation of these
systems. Therefore, the stability of neutral stochastic functional partial differential equations has been studied by many researchers (see, for instance, [2, 4, 6, 10, 12, 13, 15, 27] and the reference therein).

\[ \begin{aligned}
&d[x(t) + G(t, x_t)] = [Ax(t) + f(t, x_t)]dt + g(t, x_t)dB(t) + \sigma(t)dW(t), \\
&\Delta x(t_k^n) = x(t_k^n) - x(t_k^n) = I_k(x(t_k^n)), \\
x(t) = \varphi(t),
\end{aligned} \]

under suitable conditions on the operator \( A \); the coefficient functions \( G, f, g, \sigma, I_k \); and the initial value \( \varphi \). Here, \( W(t) \) denotes a Brownian motion and \( B^H(t) \) denotes an fBm with the Hurst parameter \( H \in (1/2, 1) \).

The contents of this paper are as follows. In Section 2, some necessary notions, definitions, and lemmas are introduced. In Section 3, the \( p \)th moment exponential stability of a class of impulsive neutral stochastic functional partial differential equations driven by Brownian motion and fractional Brownian motion is investigated by means of the fixed point theorem. In Section 4, one example is given to illustrate our main results. At last, in Section 5, our conclusion is presented.

2. Preliminaries

In this section, we collect some notions, definitions, and lemmas which will be used throughout the whole of this paper.

**Definition 1** (see [5]). Given \( H \in (0, 1) \), a continuous centered Gaussian process \( \{ B^H(t), t \in \mathbb{R} \} \) with the covariance function

\[ R_H(t, s) = \mathbb{E}[B^H(t)B^H(s)] = \frac{1}{2}(|t|^{2H} + |s|^{2H} - |t-s|^{2H}), \quad t, s \in \mathbb{R}, \]

is called a two-sided one-dimensional fractional Brownian motion (fBm) and \( H \) is the Hurst parameter.

Let \((\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})\) stand for a complete probability space equipped with some filtration \( \{ \mathcal{F}_t \}_{t \geq 0} \) satisfying normal assumptions. \( Y_1, Y_2, \) and \( X \) stand for three real Hilbert spaces, respectively. \( \mathcal{L}(Y_i; X) \) represents the space of all bounded linear operators from \( Y_i \) to \( X, i = 1, 2 \). Let \( \{ \phi_n \}_{n \in \mathbb{N}} \) be a complete orthonormal basis in \( Y_i \).

Motivated by the above discussion, this paper is concerned with the exponential stability results for a class of neutral stochastic functional partial differential equations driven by standard Brownian motion and fractional Brownian motion with impulses:

\[ Q(\cdot) \in \mathcal{L}(Y_i; X) \] be a operator defined by \( Q(\cdot) = \sum_{n=1}^{\infty} \lambda_n e_n(\cdot) \)

with finite trace \( \text{tr} Q(\cdot) = \sum_{n=1}^{\infty} \lambda_n(\cdot) < \infty \), where \( \{ \lambda_n(\cdot) \}_{n \geq 1} \) are nonnegative real numbers. Then there exists a \( \mathbb{R} \)-valued sequence \( \{ \omega_n(t) \}_{n \geq 1} \) of one-dimensional Brownian motions mutually independent over \((\Omega, \mathcal{F}, \{ \mathcal{F}_t \}_{t \geq 0}, \mathbb{P})\) such that

\[ W(t) = \sum_{n=1}^{\infty} \lambda_n e_n(\cdot) \omega_n(t), \quad t \geq 0, \]

and the infinite-dimensional cylindrical \( Y_2 \)-valued fBm \( B^H(t) \) is defined by the formal sum (see [5]).

\[ B^H(t) = \sum_{n=1}^{\infty} \lambda_n e_n(\cdot) \xi_n(t), \quad t \geq 0, \]

where the sequence \( \{ \xi_n(\cdot) \}_{n \geq 1} \) are stochastically independent scalar fBms with Hurst parameter \( H \in (1/2, 1) \).

Let \( \mathcal{L}^0(Y_i; X) \) be the space of all \( Q(\cdot) \) Hilbert–Schmidt operators from \( Y_i \) to \( X, i = 1, 2 \). Now we show the following definition.

**Definition 2** (see [20]). Let \( \xi_i \in \mathcal{L}(Y_i; X) \) and define

\[ \| \xi_i \|^2_{\mathcal{S}^2(\mathcal{Q})} = \text{tr} \left( \xi_i Q(\cdot) \xi_i^* \right) = \sum_{n=1}^{\infty} \left\| \lambda_n(\cdot) \xi_i e_n(\cdot) \right\|^2_X. \]

If \( \| \xi_i \|^2_{\mathcal{S}^2(\mathcal{Q})} < \infty \), then \( \xi_i \) is called a \( Q(\cdot) \) Hilbert–Schmidt operator and the space \( \mathcal{L}^0(\mathcal{Q}) = \mathcal{L}^0(\mathcal{Q}; Y_i; X) \) is a real separable Hilbert space equipped with the inner product

\[ \langle \varphi, \psi \rangle_{\mathcal{S}^2(\mathcal{Q})} = \sum_{n=1}^{\infty} \langle \lambda_n(\cdot) \varphi_n(\cdot), \psi_n(\cdot) \rangle, \quad i = 1, 2. \]

In order to set our problem, we need the following lemmas.

**Lemma 1** (see [19]). For any \( r \geq 1 \) and for arbitrary \( \mathcal{L}^0(\mathcal{Q}) \) predictable process \( \phi(\cdot) \),

\[\sup_{s \in [0, T]} \int_0^s \| \phi(u) \|^2_X dW(u) \leq (r(2r-1))^r \int_0^T \left( \mathbb{E} \| \phi(s) \|^2_{\mathcal{S}^2(\mathcal{Q})} \right)^{1/r} ds, \quad t \in [0, T]. \]

**Lemma 2** (see [4]). If \( \psi: [0, T] \rightarrow \mathcal{L}^0(\mathcal{Q}) \) satisfies

\[ \int_0^T \psi(s) \| \phi(s) \|^2_{\mathcal{S}^2(\mathcal{Q})} ds < \infty \]

then, we have

\[ \mathbb{E} \left| \int_0^T \psi(s) dB^H(s) \right|^2_X \leq 2H^{2H-1} \int_0^T \| \psi(s) \|^2_{\mathcal{S}^2(\mathcal{Q})} ds, \quad t \in [0, T]. \]
where $H \in (1/2, 1)$.

**Lemma 3** (see [10]). For any $p \geq 2$ and for arbitrary $\mathcal{D}^p_{\mathbb{Q}, 0}(Y; X)$ predictable process $\psi(\cdot)$,

$$
E\left[ \int_0^t |\psi(s)dB^H(s)|^p \right] \leq C_p \left( \int_0^t \|\psi(s)dB^H(s)\|^2 \right)^{p/2}, \quad t \in [0, T].
$$

(8)

Now, we turn to state some notations and basic facts about the theory of semigroups and fractional power operators. Let $A: \mathcal{D}(A) \rightarrow X$ be the infinitesimal generator of an analytic semigroup $\{S(t), t \geq 0\}$ of bounded linear operators on $X$. It is well known that there exists a pair of constants $\lambda \in \mathbb{R}$ and $M \geq 1$ such that $\|S\| \leq Me^{\lambda t}$ for every $t \geq 0$. If $|S(t)| \geq 0$ is a uniformly bounded and analytic semigroup such that $0 \in \rho(A)$, where $\rho(A)$ is the resolvent set of $A$, then it is possible to define the fractional power $(-A)^{\alpha}$ for any $\alpha \in [0, 1]$ which is a closed linear operator with its domain $\mathcal{D}((-A)^{\alpha})$. Furthermore, the subspace $\mathcal{D}((-A)^{\alpha})$ is dense in $X$, and the expression $\|\cdot\|_p = \|\cdot\|_X \cdot \eta \in \mathcal{D}((-A)^{\alpha})$ defines a norm in $\mathcal{D}((-A)^{\alpha})$. If $X_\alpha$ represents the space $\mathcal{D}((-A)^{\alpha})$ equipped with the norm $\|\cdot\|_p$, then the following properties are well known (see [28]).

**Lemma 4.** Suppose that the preceding conditions are satisfied.

(1) Let $0 \leq \alpha \leq 1$, then $X_\alpha$ is a Banach space.

(2) If $0 \leq \beta \leq \alpha$, then the injection $X_\alpha \rightarrow X_\beta$ is continuous.

(3) For every $0 \leq \alpha \leq 1$, there exists a constant $M_\alpha > 0$ such that

$$
(-A)^{\alpha}S(t) \leq M_\alpha t^{-\alpha}e^{-\lambda t}, \quad t > 0, \lambda > 0.
$$

(9)

### 3. The $p$th Moment Exponential Stability

Consider $(\Omega, \mathcal{F}, \mathbb{P})$ the complete probability space which was introduced in Section 2. Denote $\mathcal{F}_t = \mathcal{F}_0$, for all $t \leq 0$. We denote by $\mathcal{C} = \mathcal{C}([-\tau, 0]; X)$ the Banach space of all continuous $X$-value functions $\phi$ defined on $[-\tau, 0]$ equipped with the norm $\|\phi\|_{\mathcal{C}} = \sup_{-\tau \leq \theta \leq 0} \|\phi(\theta)\|_X$, where $\|\cdot\|_X$ denotes the Euclidean norm. Let $\mathcal{P}(J, \mathbb{R}^n) = \{\phi: J \rightarrow \mathbb{R}^n \text{ is continuous for all but at most a finite number of points } t \in J \text{ and at these points } t \in J, \phi(t+) \text{ and } \phi(t-) \text{ exist, } \phi(t+) = \phi(t), \text{ where } J \subset \mathbb{R} \text{ is a bounded interval. } \phi(t+) \text{ and } \phi(t-) \text{ stand for the right-hand and left-hand limits of the function } \phi(t), \text{ respectively.}

Define $\mathbb{P}_\alpha = \mathbb{P}_\alpha([-\tau, 0]; X).$ Let $\mathbb{P}_{\mathcal{C}_\alpha}([-\tau, 0]; X)$ be the family of all bounded $\mathcal{F}_0(\mathcal{C}_\alpha)$-measurable, $\mathbb{P}([-\tau, 0]; X)$-value random variables $\phi$, satisfying $\|\phi\|_{\mathbb{P}_\alpha} \leq \sup_{-\tau \leq \theta \leq 0} \|\phi(\theta)\|^p < +\infty$ for $p > 0$.

In this section, we consider the $p$th moment exponential stability of mild solutions of the following impulsive neutral stochastic functional partial differential equations driven by Brownian motion and fractional Brownian motion:

$$
\begin{align*}
&\left\{ \begin{array}{ll}
d\left[ x(t) + G(t, x_t) \right] = [Ax(t) + f(t, x_t)]dt + g(t, x_t) dW(t) + \sigma(t) dB^H(t), & t \geq 0, t \neq t_k, \\
x(t) = \phi \in \mathbb{P}_{\mathcal{C}_\alpha}([-\tau, 0]; X),
\end{array} \right.
\end{align*}
(10)

and

$$
\begin{align*}
&x(t) = \begin{cases} S(t)[\varphi(0) + G(0, \varphi)] - G(t, x_t) & t = t_k, k = 1, 2, \ldots, \\
- \int_0^t AS(t-s)G(s, x_s)ds & -\tau \leq t \leq 0,
\end{cases}
\end{align*}
(11)

Definition 4. Equation (10) is said to be exponentially stable in $p$th ($p \geq 2$) moment, if for any initial value $\varphi$, there exists a pair of positive constants $\gamma$ and $C$ such that

$$
E\|x(t)\|_p^p \leq C\|\varphi\|_{\mathbb{P}_\alpha}^p e^{-\gamma t}, \quad t \geq 0.
$$

(12)

In order to set the stability problem, we assume that the following conditions hold.

**Condition 1.** A is the infinitesimal generator of an analytic semigroup $\{S(t), t \geq 0\}$ of bounded linear operators on $X$ satisfies
\(\|S(t)\|_X \leq M e^{-\lambda t}, \quad \forall t \geq 0, \text{ where } M \geq 1 \text{ and } \lambda > 0.\) \hfill (13)

**Condition 2.** There exist constants \(L_f \geq 0, a_f \geq 0 \text{ and } y_f > 0\) such that for any \(x, y \in \mathcal{P}C\) and for all \(t \in [0, +\infty),\)
\[
\|f(t, x_t) - f(t, y_t)\|_X \leq L_f \|x_t - y_t\|_X,
\]
\[
\|f(t, 0)\|_X \leq a_f e^{-y_f t}, \quad \text{as.}
\] \hfill (14)

**Condition 3.** There exist constants \(L_g \geq 0, a_g \geq 0 \text{ and } y_g > 0\) such that for any \(x, y \in \mathcal{P}C\) and for all \(t \in [0, +\infty),\)
\[
\|g(t, x_t) - g(t, y_t)\|_X \leq L_g \|x_t - y_t\|_X,
\]
\[
\|g(t, 0)\|_X \leq a_g e^{-y_g t}, \quad \text{as.}
\] \hfill (15)

**Condition 4.** There exist constants \(\alpha \in (0, 1] \text{ and } L_G \geq 0, a_G \geq 0 \text{ and } y_G > 0\) such that \(G\) is \(X_\alpha\)-valued, for any \(x, y \in \mathcal{P}C\) and for all \(t \in [0, +\infty),\)
\[
\|A(t)x\|_X \leq M_1(t) + a_G \|x\|_X, \quad \text{as.}
\]
\[
\|A(t)\|_X \leq M_2(t) + a_G \|x\|_X, \quad \text{as.}
\]
\[
\|\phi(t, w)\|_X \leq M_3(t) + a_G \|w\|_X, \quad \text{as.}
\]
\[
\|\phi(t, 0)\|_X \leq a_G e^{-y_G t}, \quad \text{as.}
\]
\[
\|\phi(t, w)\|_X \leq a_G e^{-y_G t}, \quad \text{as.}
\]
\[
\|\phi(t, 0)\|_X \leq a_G e^{-y_G t}, \quad \text{as.}
\]

**Proof.** Denote by \(\mathcal{D}\) the Banach space of all \(\mathcal{F}\)-adapted processes \(\phi(t, w) \in [-\tau, +\infty) \times \Omega \rightarrow \mathbb{R}, \) which satisfies \(\phi(s, w) = \phi(s)\) for \(s \in [-\tau, 0)\) and \(e^{\mu t}\|\phi(t, w)\|_X^p \rightarrow 0\) as \(t \rightarrow +\infty,\) where \(\mu\) is a positive constant such that \(0 < \mu \leq \min\{y_G, y_f, y_p, y_G, \lambda\}.\)

Define an operator \(\pi : \mathcal{D} \rightarrow \mathcal{D}\) by \(\pi(x)(t) = \phi(t)\) for \(t \in [-\tau, 0)\) and for \(t \in [0, +\infty),\)
\[
\pi(x)(t) = S(t)[\phi(0) + G(0, \varphi)] - G(t, x_t)
\]
\[
- \int_0^t AS(t - s)G(s, x_s)ds
\]
\[
+ \int_0^t S(t - s)f(s, x_s)ds + \int_0^t S(t - s)g(s, x_s)dw(s)
\]
\[
+ \int_0^t S(t - s)\sigma(s)dB^H(s) + \sum_{0 < t_k < t} S(t - s)I_k(x(t_k))
\] \hfill (20)

Next, we show that \(\pi(\mathcal{D}) \subseteq \mathcal{D}\). It follows from (20) that

\[
\pi(x)(t) \leq 7^{p-1} e^{\mu t}E\|S(t)[\varphi(0) + G(0, \varphi)]\|_X^p
\]
\[
+ 7^{p-1} e^{\mu t}E\|S(t)[\varphi(0) + G(0, \varphi)]\|_X^p
\]
\[
+ 7^{p-1} e^{\mu t}E\|G(t, x_t)\|_X^p
\]
\[
+ 7^{p-1} e^{\mu t}E\|\phi(t, w)\|_X^p
\]
\[
+ 7^{p-1} e^{\mu t}E\int_0^t S(t - s)f(s, x_s)ds_x^p
\]
\[
+ 7^{p-1} e^{\mu t}E\int_0^t S(t - s)g(s, x_s)dw(s)_x^p
\]
\[
+ 7^{p-1} e^{\mu t}E\int_0^t S(t - s)\sigma(s)dB^H(s)_x^p
\]
\[
+ 7^{p-1} e^{\mu t}E\sum_{0 < t_k < t} S(t - s)I_k(x(t_k))_x^p
\] \hfill (21)
Now we estimate the terms on the right-hand side of (21). Firstly, by condition 1, we can obtain

\[ \mathbb{E}\| k_1(t) \|^p_X \leq M^p e^{-(p-k)t} \| \varphi(0) \|^p_X + G(0, \varphi) \| X \longrightarrow 0, \quad \text{as } t \longrightarrow +\infty. \]  \hfill (22)

Secondly, Hölder’s inequality and condition 4 yield

\[ \mathbb{E}\| k_2(t) \|^p_X \leq e^{\alpha t} \mathbb{E}\| G(t, x_t) - G(t, 0) + G(t, 0) \|^p_X \]
\[ \leq \| (-A)^{-\alpha} \|^p_X e^{\alpha t} \| (-A)^\alpha [G(t, x_t) - G(t, 0) + G(t, 0)] \|^p_X \]
\[ \leq 2^{p-1} L^p_{11} \| (-A)^{-\alpha} \|^p_X e^{\alpha t} \sup_{\theta \in [-\tau, 0]} \mathbb{E}\| x(t + \theta) \|^p_X + 2^{p-1} \| (-A)^{-\alpha} \|^p_X e^{\alpha t} \| (-A)^\alpha G(0, 0) \|^p_X \]
\[ = 2^{p-1} L^p_{11} \| (-A)^{-\alpha} \|^p_X k_21(t) + 2^{p-1} \| (-A)^{-\alpha} \|^p_X k_22(t). \]  \hfill (23)

For any \( x(t) \in \delta \) and any \( \varepsilon_1 > 0 \), there exists a \( T_1 > 0 \), such that \( e^{\alpha t} \| x(t) \|^p_X < \varepsilon_1 \) for \( t \geq T_1 - \tau \); thus, we can get

\[ k_21(t) = e^{\alpha t} \sup_{\theta \in [-\tau, 0]} \mathbb{E}\| x(t + \theta) \|^p_X \leq e^{\alpha t} \sup_{\theta \in [-\tau, 0]} \mathbb{E}\| x(t + \theta) \|^p_X \leq e^{\alpha t} \varepsilon_1, \quad t \in [T_1, +\infty). \]  \hfill (24)

So \( k_21(t) \longrightarrow 0 \) as \( t \longrightarrow +\infty \). From condition 4, we get \( k_22(t) \longrightarrow 0 \) as \( t \longrightarrow +\infty \). That is to say

\[ \mathbb{E}\| k_2(t) \|^p_X \longrightarrow 0, \quad \text{as } t \longrightarrow +\infty. \]  \hfill (25)

Further, Hölder’s inequality, Lemma 4, and condition 4 yield

\[ \mathbb{E}\| k_3(t) \|^p_X \leq e^{\alpha t} \int_0^t (-A)^{-\alpha} S(t-s)(-A)^\alpha G(s, x_s) ds \| X \]
\[ \leq M^p_{1-a} e^{\alpha t} \int_0^t \frac{e^{-\lambda(t-s)}}{(t-s)^{1-a}} (-A)^\alpha G(s, x_s) ds \| X \]
\[ \leq M^p_{1-a} e^{\alpha t} \left( \int_0^t \frac{e^{-\lambda(t-s)}}{(t-s)^{1-a}} ds \right)^{p-1} \int_0^t \frac{e^{-\lambda(t-s)}}{(t-s)^{1-a}} \mathbb{E}\| (-A)^\alpha [G(s, x_s) - G(s, 0) + G(s, 0)] \|^p_X ds \]
\[ \leq M^p_{1-a} \Gamma^{p-1} (a) (-A)^{-\alpha} e^{\alpha t} \int_0^t \frac{e^{-\lambda(t-s)}}{(t-s)^{1-a}} \sup_{\theta \in [-\tau, 0]} \mathbb{E}\| x(s + \theta) \|^p_X ds \]
\[ + 2^{p-1} M^p_{1-a} \Gamma^{p-1} (a) (-A)^{-\alpha} e^{\alpha t} \int_0^t \frac{e^{-\lambda(t-s)}}{(t-s)^{1-a}} \mathbb{E}\| (-A)^\alpha G(s, 0) \|^p_X ds \]
\[ = 2^{p-1} L^p_{11} M^p_{1-a} \Gamma^{p-1} (a) (-A)^{-\alpha} k_31(t) + 2^{p-1} M^p_{1-a} \Gamma^{p-1} (a) (-A)^{-\alpha} k_32(t). \]  \hfill (26)
For any \( x(t) \in S \) and any \( \varepsilon_2 > 0 \), there exists a \( T_2 > 0 \), such that \( e^{t\varepsilon_2} \mathbb{E} \| x(t) \|_{\mathcal{X}}^{p} < e^{\mu t} \varepsilon_2 \) for \( t \geq T_2 - \tau \). Thus, we can get
\[
e^{\mu t} \sup_{\theta \in [\tau, t]} \mathbb{E} \| x(s + \theta) \|_{\mathcal{X}}^{p} < e^{\mu t} \varepsilon_2, \quad s \in [T_2, +\infty),
\]
then
\[
e^{\mu t} \int_{T_2}^{t} \frac{e^{-\lambda (t-s)}}{(t-s)^{1-a}} e^{-\mu s} \sup_{\theta \in [\tau, t]} \mathbb{E} \| x(s + \theta) \|_{\mathcal{X}}^{p} ds < e^{\mu t} \varepsilon_2,
\]
so that for any \( e \in \tau \), we have
\[
e^{(\lambda-p) t} \longrightarrow 0 \quad \text{as} \quad t \longrightarrow +\infty,
\]
then
\[
As \quad e^{(\lambda-p) t} \longrightarrow 0 \quad \text{as} \quad t \longrightarrow +\infty,
\]
there exists \( T_3 \geq T_2 \) such that for any \( t \geq T_3 \), we have
\[
\mathbb{E} \| k_3(t) \|_{\mathcal{X}}^{p} < e^{\mu t} \varepsilon_2.
\]
So from the above, we obtain for any \( t \geq T_3 \)
\[
k_{31}(t) < \left[ \Gamma(\alpha)(\lambda - \mu)^{\alpha} + 1 \right] \varepsilon_2.
\]
As for the fifth term on the right-hand side of (21), we have
\[
e^{(\mu - \lambda) t} \int_{0}^{T_4} e^{\lambda s} \sup_{\theta \in [\tau, t]} \mathbb{E} \| x(s + \theta) \|_{\mathcal{X}}^{p} ds < e^{\mu t} \varepsilon_3,
\]
then
\[
e^{(\lambda-p) t} \longrightarrow 0 \quad \text{as} \quad t \longrightarrow +\infty,
\]
there exists \( T_5 \geq T_4 \) such that for any \( t \geq T_5 \), we have
\[
\mathbb{E} \| k_4(t) \|_{\mathcal{X}}^{p} \longrightarrow 0, \quad \text{as} \quad t \longrightarrow +\infty.
\]
As for the last term on the right-hand side of (21), we get

\[ \mathbb{E}\left\| k_7(t) \right\|_X^p \leq M^p \mathbb{E}\left( \sum_{0 \leq t_k < t} \mathbb{E}\left( S(t - t_k) \mathcal{I}_k \left( x(t_k) \right) \right) \right)^{p/2} \]

\[ \leq M^p \mathbb{E}\left( \sum_{0 \leq t_k < t} d_k e^{-\lambda(t-t_k)} \mathbb{E}\left( \left\| x(t_k) \right\|_X^p \right) \right)^{p/2} \]

\[ \leq M^p \mathbb{E}\left( \sum_{0 \leq t_k < t} d_k e^{-\lambda(t-t_k)} \mathbb{E}\left( \left\| x(t_k) \right\|_X^p \right) \right)^{p/2} \]

As for the sixth term on the right-hand side of (21), by Lemmas 2 and 3, we have

\[ \mathbb{E}\left\| k_6(t) \right\|_X^p \leq e^{\alpha t} C_p \left( \mathbb{E}\left( \left\| \int_0^t S(t-s) \sigma(s) dB^H(s) \right\|_X^2 \right) \right)^{p/2} \]

\[ \leq e^{\alpha t} C_p \left( 2M'^2 H^{2H-1} \mathbb{E}\left( \left\| S(t-s) \right\|_X^2 \right) \right)^{p/2} \]

\[ \leq C_p 2^{pH} M'^p t^{(2H-1)p/2} e^{\alpha t} \left( e^{-2\lambda t} \int_0^t a e^{(2\lambda - \gamma_4)y} ds \right)^{p/2} \]

\[ \leq C_p 2^{pH} M'^p a^{p/2} t^{(2H-1)p/2} e^{\alpha t} \left( e^{\gamma_4 t} - e^{-2\lambda t} \right)^{p/2} \]

\[ \leq C_p 2^{pH} M'^p a^{p/2} t^{(2H-1)p/2} e^{\alpha t} \left( \frac{\min\{\gamma_4, 2\lambda\} t}{2\lambda - \gamma_4} \right)^{p/2} \]

As for the sixth term on the right-hand side of (21), we have

\[ \mathbb{E}\left\| k_5(t) \right\|_X^p \leq \frac{p(p-1)}{2} M^p e^{\alpha t} \left( \mathbb{E}\left( \int_0^t e^{-\lambda(t-s)} \mathbb{E}\left( \left\| g(s,x) \right\|_{\mathcal{F}_{p(t+1)}}^p \right) ds \right) \right)^{p/2} \]

\[ \leq \frac{p(p-1)}{2} M^p e^{\alpha t} \left( \mathbb{E}\left( \int_0^t e^{-2\lambda(t-s)} \mathbb{E}\left( \left\| g(s,x) \right\|_{\mathcal{F}_{p(t+1)}}^p \right) ds \right) \right)^{p/2} \]

\[ \leq \frac{p(p-1)}{2} M^p e^{\alpha t} \left( \mathbb{E}\left( \int_0^t e^{-2\lambda(t-s)} \mathbb{E}\left( \left\| g(s,x) \right\|_{\mathcal{F}_{p(t+1)}}^p \right) ds \right) \right)^{p/2} \]

\[ \leq \frac{p(p-1)}{2} M^p e^{\alpha t} \left( \mathbb{E}\left( \int_0^t e^{-2\lambda(t-s)} \mathbb{E}\left( \left\| g(s,x) \right\|_{\mathcal{F}_{p(t+1)}}^p \right) ds \right) \right)^{p/2} \]

\[ \leq \frac{p(p-1)}{2} M^p e^{\alpha t} \left( \mathbb{E}\left( \int_0^t e^{-2\lambda(t-s)} \mathbb{E}\left( \left\| g(s,x) \right\|_{\mathcal{F}_{p(t+1)}}^p \right) ds \right) \right)^{p/2} \]
For any $x(t) \in \mathcal{S}$ and any $\varepsilon_4 > 0$, there exists a $T_6 > 0$, such that $e^{\mu t}\|x(t)\|_X^p < \varepsilon_4$, for $t \geq T_6$. Thus, we can get
\[ e^{\mu x}\|x(t_k)\|_X^p < \varepsilon_4, \quad t \in [T_6, +\infty), \tag{42} \]

\[
M^p \left( \sum_{k=1}^{\infty} d_k \right)^{p-1} \left( \sum_{T_k < t_k < t} d_k e^{-p(t-t_k)} e^{\mu t} \|x(t_k)\|_X^p \right) \leq M^p \left( \sum_{k=1}^{\infty} d_k \right)^{p-1} \left( \sum_{T_k < t_k < t} d_k e^{-(p-\mu)(t-t_k)} \right) \varepsilon_4 \leq M^p \left( \sum_{k=1}^{\infty} d_k \right)^{p-1} \varepsilon_4. \tag{43} \]

As $e^{-(\lambda - \mu)t} \to 0$ as $t \to +\infty$, then there exists $T_7 \geq T_6$ such that for any $t \geq T_7$, we have
\[
M^p \left( \sum_{k=1}^{\infty} d_k \right)^{p-1} \left( \sum_{0 < t_k \leq t} d_k e^{-p(t-t_k)} e^{\mu t} \|x(t_k)\|_X^p \right) \leq e^{-(p-\mu)t} M^p \left( \sum_{k=1}^{\infty} d_k \right)^{p-1} \left( \sum_{0 < t_k \leq t} d_k e^{p(t-t_k)} \|x(t_k)\|_X^p \right) < \varepsilon_4. \tag{44} \]

So from the above, we obtain for any $t \geq T_7$
\[
k_7(t) \leq \left( M^p \left( \sum_{k=1}^{\infty} d_k \right)^p + 1 \right) \varepsilon_4. \tag{45} \]

So we have
\[
\mathbb{E}\|k_7(t)\|_X^p \to 0, \quad \text{as} \quad t \to +\infty. \tag{46} \]

Thus, from (20)–(32), (37), and (39)–(46), we know that $e^{\mu t}\|\pi(x)(t)\|_X^p \to 0$ as $t \to +\infty$. So, we conclude that $\pi(\mathcal{S}) \subseteq \mathcal{S}$.

Thirdly, we will show that $\pi$ is contractive. Let $x, y \in \mathcal{S}$ by using the inequality
\[
\mathbb{E}\|\pi(x)(t) - \pi(y)(t)\|_X^p \leq \frac{1}{k^{p-1}} \mathbb{E}\|G(t, x_t) - G(t, y_t)\|_X^p
\]
\[
+ \frac{4^{p-1}}{(1-k)^{p-1}} \mathbb{E}\int_0^t (-A)^{-q} S(t-s) (-A)^q [G(s, x_s) - G(s, y_s)] ds_{X}^p
\]
\[
+ \frac{4^{p-1}}{(1-k)^{p-1}} \mathbb{E}\int_0^t S(t-s) [f(s, x_s) - f(s, y_s)] ds_{X}^p
\]
\[
+ \frac{4^{p-1}}{(1-k)^{p-1}} \mathbb{E}\int_0^t S(t-s) [g(s, x_s) - g(s, y_s)] dW(s)_{X}^p
\]
\[
+ \frac{4^{p-1}}{(1-k)^{p-1}} \mathbb{E}\int_0^t S(t-s) (I_k(x(t_k)) - I_k(y(t_k))) ds_{X}^p. \tag{48} \]

where $d_1, d_2, d_3, d_4, d_5$ are nonnegative constants and $k = L_0 \|(-A)^{-\eta}\|_X < 1$. For any fixed $t \in [0, T]$, we have
Hence,
\[
\sup_{s \in [-T,T]} \mathbb{E} \| \pi(x)(s) - \pi(y)(s) \|_X^p \leq \sup_{s \in [-T,T]} \mathbb{E} \| x(s) - y(s) \|_X^p \times \left( k + 4p^{-1}M^p_1 \Gamma^p (a) L^p G \right)
\]
\[
+ 4p^{-1} M^p L^p_f \left( \frac{1}{1 - k} \right)^{p-1} \lambda^\alpha \left( 1 - \frac{1}{p} \right)^{p-1} \lambda^\alpha
\]
\[
+ 4p^{-1} \left( \sum_{k=1}^{\infty} k \right)^{p-1} \left( \frac{1}{1 - k} \right)^{p-1} \lambda^\alpha
\]
\[
\left( \frac{1}{1 - k} \right)^{p-1} \lambda^\alpha
\]
\[
(49)
\]

Thus, by (19), we know that \( \pi \) is a contraction mapping. Hence, by the Contraction Mapping Theorem, \( \pi \) has a unique fixed point \( x(t) \) in \( \mathcal{D} \), which is a solution of equation (10) with \( x(s) = \phi(s) \) on \([ -\tau, 0 ] \) and \( e^{\omega t} \mathbb{E} \| x(t) \|_X \rightarrow 0 \) as \( t \rightarrow +\infty \). This completes the proof. \( \square \)

Remark 1. Zhang and Ruan in [27] considered (10) without impulsive effects and required the following conditions to ensure the \( p \)th \( (p \geq 2) \) moment exponential stability: \( (1/p) < \alpha \leq 1 \) and
\[
4p^{-1} L^p \left( \frac{a}{1} \right)^{p}\left( \frac{a-1}{1} \right)^{L^p G} \lambda^\alpha
\]
\[
+ 4p^{-1} M^p L^p_f \lambda^\alpha
\]
\[
+ 4p^{-1} M^p L^p_a \left( \frac{1}{1 - k} \right)^{p-1} \lambda^\alpha \left( \frac{1}{1 - k} \right)^{p-1} \lambda^\alpha
\]
\[
\leq 1,
\]
where \( 0^p = 1 \). Our condition is \( 0 < \alpha \leq 1 \) and
\[
k + \frac{3p^{-1} M^p_1 \Gamma^p (a) L^p G}{(1 - k)^{p-1} \lambda^\alpha} + \frac{3p^{-1} M^p L^p_f}{(1 - k)^{p-1} \lambda^\alpha}
\]
\[
+ 3p^{-1} \left( \frac{1}{1 - k} \right)^{p-1} \left( \frac{1}{2} \right)^{p-1} \lambda^\alpha
\]
\[
\leq 1,
\]
where \( 0^p = 1 \). Our condition is \( 0 < \alpha \leq 1 \) and
\[
k + \frac{3p^{-1} M^p_1 \Gamma^p (a) L^p G}{(1 - k)^{p-1} \lambda^\alpha} + \frac{3p^{-1} M^p L^p_f}{(1 - k)^{p-1} \lambda^\alpha}
\]
\[
+ 3p^{-1} \left( \frac{1}{1 - k} \right)^{p-1} \left( \frac{1}{2} \right)^{p-1} \lambda^\alpha
\]
\[
\leq 1.
\]

Now we show that why (51) is weaker than (50). Obviously,
\[
\Gamma^p (a) = \left( \int_0^{\infty} e^{-x \alpha^{-1}} \, ds \right)^p = \left( \int_0^{\infty} e^{-(x/p) e^{-((p-1)/p) x \alpha^{-1}}} \, ds \right)^p
\]
\[
+ \left( \int_0^{\infty} e^{-x \alpha^{-1}} \int_0^{\infty} e^{-((p-1)/p) x \alpha^{-1}} \, ds \right) \Gamma^p (a)
\]
\[
\leq \Gamma^p (a) \left( \frac{p - 1}{p} \right)^{p} \left( \frac{2}{2p - 1} \right)^{p-1}, \quad p > 2.
\]
\[
(52)
\]

Then we can transfer to prove (51) is weaker than following condition:
\[
4p^{-1} L^p \left( \frac{a}{1} \right)^{p}\left( \frac{a-1}{1} \right)^{L^p G} \lambda^\alpha
\]
\[
+ 4p^{-1} M^p_1 \Gamma^p (a) L^p G \lambda^\alpha
\]
\[
+ \frac{4p^{-1} \left( \frac{1}{1 - k} \right)^{p-1} \lambda^\alpha}{(2 \lambda)^{p-1}} < 1.
\]
\[
(53)
\]

Obviously, (53) is not hold for \( k = L^p \left( \frac{a}{1} \right)^{p}\left( \frac{a-1}{1} \right)^{L^p G} \lambda^\alpha \) in \( [1 - (1/p) \lambda^\alpha, 1] \) and (51) may hold for \( k \in [1 - (1/p) \lambda^\alpha, 1] \). On the other hand, when \( k = L^p \left( \frac{a}{1} \right)^{p}\left( \frac{a-1}{1} \right)^{L^p G} \lambda^\alpha \), conditions (51) and (53) can be transferred into the following conditions, respectively.
\[
\frac{3p^{-1} M^p_1 \Gamma^p (a) L^p G}{(1 - k)^p \lambda^\alpha} + \frac{3p^{-1} M^p L^p_f}{(1 - k)^p \lambda^\alpha}
\]
\[
+ 3p^{-1} \left( \frac{1}{1 - k} \right)^{p-1} \lambda^\alpha \lambda^\alpha
\]
\[
\leq 1,
\]
\[
(54)
\]

Obviously,
\[
\frac{3p^{-1} M^p_1 \Gamma^p (a) L^p G}{(1 - k)^p \lambda^\alpha} + \frac{3p^{-1} M^p L^p_f}{(1 - k)^p \lambda^\alpha}
\]
\[
+ 3p^{-1} \left( \frac{1}{1 - k} \right)^{p-1} \lambda^\alpha \lambda^\alpha
\]
\[
\leq 1,
\]
\[
(54)
\]

Then we conclude that (51) is weaker than (50); in this sense, this paper improves and generalizes the results in [27].

Remark 2. Theorem 1 does not ask for \( G(t, 0) \equiv 0, \ f(t, 0) \equiv 0, \) and \( g(t, 0) \equiv 0, \) which are imposed in [10]. Even in this special case, our results also improve the result in [10]. Our conditions are \( 0 < \alpha \leq 1 \) and
\[
k + \frac{3p^{-1} M^p_1 \Gamma^p (a) L^p G}{(1 - k)^{p-1} \lambda^\alpha} + \frac{3p^{-1} M^p L^p_f}{(1 - k)^{p-1} \lambda^\alpha}
\]
\[
+ 3p^{-1} \left( \frac{1}{1 - k} \right)^{p-1} \left( \frac{1}{2} \right)^{p-1} \lambda^\alpha
\]
\[
\leq 1.
\]
\[
(56)
\]

However, the corresponding conditions in [15] are \( (1/p) < \alpha \leq 1 \) and
\[
\rho = 7^{-P} \| \Pi^{P} (\lambda - a) \Pi^{P} \|^{P} \left( \frac{7^{-P} M^{\alpha} \Pi^{P} ((p - 1)/(p - 1)) M^{\alpha} \Pi^{P} (1 - k) M^{\beta} \Pi^{P}}{1 - k} \right)^{P/2} + \frac{14^{-P} M^{\alpha} \Pi^{P}}{\lambda^{P}} \left( \frac{14^{-P} M^{\alpha} \Pi^{P} (p(p - 1))/2 \Pi^{P} ((p - 2)/(2(p - 1)))^{P/2 - 1}}{\lambda^{P/2}} \right) + 7^{-P} M^{\alpha} \left( \sum_{k=1}^{\infty} d_{k} \right)^{P} < 1.
\]

(57)

In this sense, this paper improves the results in [10].

Remark 3. A similar discussion can be used to show that this paper improves and generalizes the results in [6, 15].

4. Example

Consider the following neutral stochastic functional partial differential equation driven by Brownian motion and fractional Brownian motion with impulses:

\[
\begin{align*}
\frac{dx(t, \xi)}{dt} + \frac{L_{1} x(t - \tau_{G}(t, \xi)) + a_{1} e^{-\gamma_{t}}}{M_{1 - \alpha}} = & \left[ \frac{\partial^{2} x(t, \xi)}{\partial \xi^{2}} + L_{2} x(t - \tau_{f}(t, \xi)) + a_{4} e^{-\gamma_{t}} \right] dt + \int_{a}^{b} \left[ L_{3} x(t - \tau_{g}(t, \xi)) + a_{3} e^{-\gamma_{t}} \right] dW(t) + a_{4} e^{-\gamma_{t}} dB^{H}(t), \\
\Delta x(t_{k}, \xi) = & I_{k} x(t_{k}(\xi)) = \frac{C_{k} x(t_{k}(\xi))}{k(k + 1)}, \\
x(t, 0) = & x(t, \pi) = 0, \\
x(t, \xi) = & \varphi(t, \xi), \\
\end{align*}
\]

where \( W(t) \) is the Brownian motion and \( B^{H}(t) \) is the fractional Brownian motion with Hurst parameter \( H \in ((1/2), 1) \), \( \gamma_{1}, \gamma_{2}, \gamma_{3}, \gamma_{4} > 0 \), and \( 0 \leq \tau_{G}(t), \tau_{f}(t), \tau_{g}(t) \leq \tau \). Let \( X = \mathbb{R}^{2} \) and \( Y = \mathbb{R} \), and define the operator \( A : X \rightarrow X \) given by \( A = (\partial^{2}/\partial \xi^{2}) \) with domain

\[
\mathcal{D}(A) = \{ v(\cdot) \in X, v, v' \text{ are absolutely continuous, } v'' \in X, v(0) = v(\pi) = 0 \}.
\]

(59)

It is well known that an analytic semigroup \( \{ S(t), t \geq 0 \} \) generated by the operator \( A \) on \( X \) satisfies \( \| S(t) \|_{X} \leq e^{-\sigma t} \). Furthermore, \( -A \) has a discrete spectrum, the eigenvalues are \( n^{2} \), \( n \in \mathbb{N} \), with the corresponding normalized eigenvectors \( e_{n}(x) = \sqrt{2/\pi} \sin(nx) \). Then, for \( y \in \mathcal{D}(A) \), we have

\[
\begin{align*}
Ay = & \sum_{n=1}^{\infty} n^{2} \langle y, e_{n} \rangle e_{n}, \\
S(t)y = & \sum_{n=1}^{\infty} e^{-\pi^{2}t} \langle y, e_{n} \rangle e_{n}.
\end{align*}
\]

(60)

Let

\[
\begin{align*}
G(t, x(t - \tau_{G}(t, \xi))) = & \frac{L_{1} x(t - \tau_{u}(t, \xi)) + a_{1} e^{-\gamma_{t}}}{M_{1 - \alpha}}, \\
f(t, x(t - \tau_{f}(t, \xi))) = & L_{2} x(t - \tau_{f}(t, \xi)) + a_{4} e^{-\gamma_{t}}, \\
g(t, x(t - \tau_{g}(t, \xi))) = & L_{3} x(t - \tau_{g}(t, \xi)) + a_{3} e^{-\gamma_{t}}, \\
\sigma(t) = & a_{4} e^{-\gamma_{t}}.
\end{align*}
\]

(61)
Obviously,

\[ \| (-A)^{\alpha} G(t, x(t - \tau_G(t), \xi)) - (-A)^{\alpha} G(t, y(t - \tau_G(t), \xi)) \|_X \]
\[ \leq \left| \frac{L_1}{M_{1-\alpha}} \right| \| x(t - \tau_G(t), \xi) - y(t - \tau_G(t), \xi) \|_X, \]
\[ \| f(t, x(t - \tau_f(t), \xi)) - f(t, y(t - \tau_f(t), \xi)) \|_X \]
\[ \leq |L_2| \| x(t - \tau_f(t), \xi) - y(t - \tau_f(t), \xi) \|_X, \]
\[ \| g(t, x(t - \tau_g(t), \xi)) - g(t, y(t - \tau_g(t), \xi)) \|_{\mathcal{X}^{\alpha}_0} \]
\[ \leq |L_3| \| x(t - \tau_g(t), \xi) - y(t - \tau_g(t), \xi) \|_{\mathcal{X}^{\alpha}_0}, \]
\[ \| (-A)^{\alpha} G(t, 0, \xi) \|_X \leq |a_1| e^{-\gamma t}, \]
\[ \| f(t, 0) \|_X \leq |a_2| e^{-\gamma t}, \]
\[ \| g(t, 0) \|_{\mathcal{X}^{\alpha}_0} \leq |a_3| e^{-\gamma t}, \]
\[ \| \sigma(t) \|_{\mathcal{X}^{\alpha}_0} = |a_4| e^{-\gamma t}. \]

From the definition of \((-A)^{-\alpha}\), we get

\[ \| (-A)^{-\alpha} \|_X \leq \frac{1}{\Gamma(\alpha)} \int_0^\infty t^{\alpha - 1} \| S(t) \|_X dt \]
\[ \leq \frac{1}{\Gamma(\alpha)} \int_0^\infty t^{\alpha - 1} e^{-\pi^2 t} dt \leq \frac{1}{\pi^{2\alpha}}. \]  

(63)

It is obvious that all the assumptions are satisfied with

\[ \lambda = \pi^2, \]
\[ L_G = \frac{|L_1|}{M_{1-\alpha} \pi^{2\alpha}}, \]
\[ L_f = |L_2|, \]
\[ L_g = |L_3|, \]
\[ \sum_{k=1}^\infty d_k = 1. \]

Then, the mild solution of (58) is exponentially stable in \(p\)th moment provided that

\[ k + \frac{4^{p-1} \Gamma(p)(|L_1|^p + |L_2|^p)}{(1 - k)^{p\alpha} \pi^{4\alpha}} + \frac{4^{p-1} |L_3|^p}{(1 - k)^{p\alpha}} \]
\[ + \frac{4^{p-1} (p(p - 1)/2)^{p/2} |L_4|^p}{(1 - k)^{p\alpha} (2\pi^2)^{p/2}} + \frac{5^{p-1} |C_1|^p}{(1 - k)^{p\alpha}} < 1, \]

where \( k = |L_1|/M_{1-\alpha} \pi^{4\alpha} \) and \( \alpha \in (0, 1] \).

5. Conclusion

In this paper, a class of neutral stochastic functional partial differential equations driven by Brownian motion and fractional Brownian motion with impulses is investigated by means of the fixed point theorem. Some sufficient conditions to ensure that the mild solution is exponentially stable in \(p\)th moment are established. The obtained results improve and generalize the results in [6, 10, 15, 27].

In our next paper, we will investigate the global attracting set and quasi-invariant set of a class of neutral stochastic functional partial differential equations driven by Brownian motion and fractional Brownian motion with impulses by means of the fixed point theorem.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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