A relation among the effective nucleon mass, the incompressibility and the effective \(\sigma\)-meson mass in nuclear matter

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Abstract

A relation among the effective nucleon mass \(M^*\), the incompressibility \(K\) and the effective \(\sigma\)-meson mass \(m^*_\sigma\) in nuclear matter is studied by using the relativistic nuclear model. We found that there is a strong correlation between \(M^*\) and \(m^*_\sigma\), while there is only a weak correlation between \(K\) and \(m^*_\sigma\). At the normal density, \(m^*_\sigma\) is smaller than the one at zero density, if \(M^*\) is smaller than 0.8 times of the nucleon mass at zero density. It is also found that the off-shell effective mass \(\mu^*_\sigma\) is related directly to \(K\) and \(M^*\) at the normal density.

1 Introduction

Recently, the \(\pi-\pi\) scattering phase shift is reanalyzed and the existence of the light iso-singlet scalar \(\sigma\)-meson is strongly suggested. [1] The similar results are also obtained by reanalyzing the \(\pi^0\pi^0\) mass spectra and angular distributions around \(K\bar{K}\)-threshold and at 1.5GeV in \(p\bar{p}(\text{at rest}) \to 3\pi^0\). [2]

Although the existence of the \(\sigma\)-meson is not still established, this meson play an important role for the nuclear matter properties in the quantum hadrodynamics(QHD). For example, the nuclear saturation properties are realized by a balance of attractive effects of the \(\sigma\)-meson and repulsive effects of the \(\omega\)-meson. [3]

The effective self-interactions (or potentials) of \(\sigma\)-meson play an important role in determining the effective nucleon mass and the incompressibility of the nuclear matter. [4] Inversely, in QHD, the properties of the effective potentials in the symmetric nuclear matter are almost determined if the values of the effective nucleon mass \(M^*_N\) at the normal density and the incompressibility \(K\) are given as input parameters. The effective \(\sigma\)-meson mass is also determined if the values of these two quantities are given, since it can be defined as a second derivative of the effective potential with respect to the \(\sigma\)-meson field. In this paper, we study the relation among the effective nucleon mass, the incompressibility and the effective \(\sigma\)-meson mass within the framework of QHD.

2 Formalism

We use the relativistic Hartree approximation (RHA) [5] based on the \(\sigma-\omega\) model. [6] The Lagrangian density is composed of three fields, the nucleon \(\psi\), the scalar \(\sigma\)-meson \(\phi\) and the vector \(\omega\)-meson \(V^\mu\), and is given by

\[
L = \bar{\psi}(i\gamma^\mu \partial^\mu - M + g_s \phi - g_v \gamma^\mu V^\mu) \psi + \frac{1}{2} \phi \partial^\mu \phi \partial_\mu \phi - \frac{1}{2} \mu^2 \phi^2
- \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \mu^2 \phi^2 \psi^4 + \frac{1}{4} \sum_{n=0}^4 C_n \phi^4; \quad F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu \quad (1)
\]
where \( M, \mu_s, \mu_v, g_s, g_v \) and \( C_n \) are constant parameters. The last term in (1) is a counter term and \( C_n \) is determined by the phenomenological renormalization conditions at zero baryon density, namely,

\[
C_n = \frac{1}{n! \partial^n} U_{\text{1-loop}}^V(<\phi>)\quad (n = 0, 1, 2, 3, 4),
\]

where \( U_{\text{1-loop}}^V \) is the unrenormalized 1-loop effective potential induced by the vacuum fluctuations and \( <\phi> \) is the ground state expectation value of the \( \sigma \)-meson field.

Replacing the meson fields by their ground state expectation values \( <\phi> \) and \( <V^0> \), we obtain the equation of motions for \( <\phi> \) and for \( <V^0> \), namely,

\[
\rho_s = \frac{1}{C_s^2} g_s <\phi> = \frac{1}{C_s^2} (M - M^*)
\]

and

\[
\rho = \frac{1}{C_v^2} g_v <V^0>.
\]

where \( M^* (\equiv M - g_s <\phi>) \), \( \rho_s \) and \( \rho \) are the effective nucleon mass, the scalar density and the baryon density, respectively, and \( C_s = g_s/\mu_s \) and \( C_v = g_v/\mu_v \).

The scalar density \( \rho_s = <\bar{\psi}\psi> \) and the baryon density \( \rho = <\bar{\psi}\gamma_0\psi> \) are given by

\[
\rho_s(k_F, <\phi>) = \frac{\lambda}{2\pi^2} k_F^3 \left[ k_F E_F^* - M^* k_F E_F^* \ln \left( \frac{k_F}{E_F^*} \right) \right]
\]

\[
- \frac{1}{g_s} \frac{d}{d <\phi>} U_{\text{1-loop}}^{V,R} \equiv \rho_s^D(k_F, <\phi>) + \rho_s^V(<\phi>)
\]

and

\[
\rho(k_F) = \frac{\lambda}{3\pi^2} k_F^3,
\]

where \( \lambda \) is the number of degree of freedom of nucleons, \( k_F \) is the Fermi momentum, \( E_F^* = \sqrt{k_F^2 + M^*} \), respectively. The \( \rho_s^D \) and \( \rho_s^V \) are the density part and vacuum fluctuation part of the scalar density, respectively. The \( U_{\text{1-loop}}^{V,R} \) is the renormalized effective potential induced by the 1-loop vacuum fluctuation effects and is given by

\[
U_{\text{1-loop}}^{V,R}(<\phi>) = - \frac{\lambda}{8\pi^2} [M^* \ln(M^*/M) + M^3(M - M^*) - \frac{7}{2} M^2(M - M^*)^2 + \frac{13}{3} M(M - M^*)^3 - \frac{25}{12} (M - M^*)^4].
\]

The energy density of the nuclear matter is also given by

\[
\varepsilon(k_F, <\phi>, <V^0>) = \varepsilon_N(k_F, M^*) + \varepsilon_v(<V^0>)
\]

\[
+ \frac{1}{2} \mu_s^2 <\phi>^2 + U_{\text{1-loop}}^{V,R}(<\phi>)
\]

where

\[
\varepsilon_N(k_F, <\phi>) = \frac{\lambda}{12\pi^2} \left[ 3k_F^2 E_F^* + \frac{3}{2} M^2 k_F E_F^* - \frac{3}{2} \ln \left( \frac{k_F + E_F^*}{M^*} \right) \right]
\]

\[
\varepsilon_v(<V^0>) = \frac{\lambda}{8\pi^2} \left[ M^* \ln(M^*/M) + M^3(M - M^*) - \frac{7}{2} M^2(M - M^*)^2 + \frac{13}{3} M(M - M^*)^3 - \frac{25}{12} (M - M^*)^4 \right].
\]
\[ \varepsilon_v(k_F, V_0) = g_v < V^0 > \rho - \frac{\mu_v^2}{2} < V^0 >^2 = \frac{\mu_v^2}{2} < V^0 >^2. \] (10)

It is easy to show that
\[ \rho^D_s = - \frac{1}{g_s} \frac{\partial \varepsilon_N}{\partial \langle \phi \rangle} \] (11)

Using the thermodynamical identity, we obtain
\[ \varepsilon + P = \mu = E_F^* + C_v^2 \rho, \] (12)

where \( P \) and \( \mu \) are the pressure and the baryonic chemical potential of the nuclear matter, respectively. At the normal density \( \rho_0 \), the pressure \( P \) vanishes. Then, Eq. (12) yields
\[ C_v^2 = \left( -a_1 + M - \sqrt{k_F^2 + M_0^{*2}} \right)/\rho_0, \] (13)

where the quantity with zero subscript shows the one at the normal density and \( a_1 \) is the value of the binding energy. Equation (13) and the condition \( C_v^2 > 0 \) gives a condition \( M_0^*/M < 0.944 \). The incompressibility \( K \) of the nuclear matter is given by
\[ K = 9 \rho_0 \left( \frac{\partial^2 (\varepsilon/\rho)}{\partial \rho^2} \right) \bigg|_{\rho = \rho_0} = 9 \left. \frac{\partial P}{\partial \rho} \right|_{\rho = \rho_0} = 9 \left. \frac{\partial \mu}{\partial \rho} \right|_{\rho = \rho_0} \] (14)

Putting \( \mu = E_F^* + C_v^2 \rho \) into (14), we obtain
\[ K = 9 \rho_0 \left( \frac{k_F^3}{3 \rho E_F^*} + \frac{g_s^2}{\mu_v^2} + \frac{M^*}{E_F^*} \frac{dM^*}{d \rho} \right) \bigg|_{\rho = \rho_0} \] (15)

Next we calculate the self-energy \( \Pi_s \) of the \( \sigma \)-meson by using the random phase approximation (RPA). [6] Using the same renormalization conditions as (2) for the effective potential and the usual renormalization conditions for the \( \sigma \)-meson wave function, we obtain
\[ \Pi_s(q; < \phi >, k_F) = \Pi_s^V(q^2; < \phi >) + \Pi_s^D(q; < \phi >, k_F). \] (16)

The particle-antiparticle excitation part \( \Pi_s^V \) does not depend explicitly on \( k_F \) and is given by
\[ \Pi_s^V(q^2; < \phi >) = \frac{3g_s^2}{4\pi^2} \int_0^1 dx \left[ 3M^{*2} + M^2 - 4MM^* - q^2x(1 - x) - A^{*2} \ln \frac{A^{*2}}{M^2} \right], \] (17)

where \( A^{*2} = M^{*2} - q^2x(1 - x) \). The \( \Pi_s^D \) which includes the particle-hole excitations and the Pauli-blocking effects depends explicitly on \( k_F \) and is given by
\[ \Pi_s^D(q; < \phi >, k_F) = - i\frac{g_s^2}{2(2\pi)^4} \text{Tr}[G_D(k)G_F(k + q) \right. \]
\[ + G_F(k)G_D(k + q) + G_D(k)G_D(k + q)] \] (18)
where \( G_D \) and \( G_F \) are the density part and the Feynman part of the nucleon propagator in RHA.

Now we define two kinds of effective \( \sigma \)-meson mass. First, we define an "off-shell" effective mass \( \mu_s^* \) by

\[
\mu_{s}^{*2} \equiv \mu_{s}^{2} + \lim_{q_0 \to 0} \lim_{|q| \to 0} \Pi_{s}(q).
\]  

(19)

The \( \mu_s^* \) can be regarded as a range of the nuclear force which is mediated by the \( \sigma \)-meson in the nuclear matter. The \( \mu_s^* \) can be related to the effective potential of the nuclear matter. In fact, it is easy to show that

\[
\mu_{s}^{*2} = \frac{\partial^2 \varepsilon}{\partial \langle \phi \rangle^2}.
\]  

(20)

Differentiating the equation of motion \((3)\) for the \( \sigma \)-meson field with respect to the baryon density and using Eqs. \((11)\) and \((20)\), we obtain

\[
\frac{dM^{*}}{d\rho} = -\frac{g_{s}^{2} M^{*}}{E_{F}^{*} \mu_{s}^{*2} E_{F}^{*}} + \frac{g_{s}^{2} M^{*}}{\mu_{s}^{2} E_{F}^{*}}
\]  

(21)

Putting \((21)\) into \((15)\), we obtain

\[
K = 9\rho_{0}\left(\frac{k_{F}^{3}}{3\rho E_{F}^{*}} + \frac{g_{s}^{2} M^{*}}{\mu_{s}^{2} E_{F}^{*}}\right)_{\rho = \rho_{0}}.
\]  

(22)

The \( \mu_s^* \) is not an "on-shell" mass which is defined by the pole of the propagator

\[
\Delta(q) = \frac{1}{q^2 - (\mu_{s}^{2} + \Pi_{s}(q))}.
\]  

(23)

We define the "on shell" effective mass \( m_{s}^{*} \) by the equation

\[
m_{s}^{*2} \equiv \mu_{s}^{2} + \Pi_{s}(q)_{\delta_{0} = m_{s}^{*}}.
\]  

(24)

In particular, at \( \rho = 0 \), we define

\[
m_{s}^{2} \equiv \mu_{s}^{2} + \Pi_{s}(q)_{\delta_{0} = m_{s}^{2}}.
\]  

(25)

where \( m_s \) is the physical mass of \( \sigma \)-meson.

In the ordinary RHA \( K = 473 \text{MeV} \), which is much larger than the empirical value \( 150 \sim 350 \text{MeV} \). \cite{7, 8} Therefore, we add an additional potential of the \( \sigma \)-meson self-interaction

\[
U_{H}(\phi) = \sum_{n=5}^{\infty} D_{n}(g_{s} \phi)^{n} = \sum_{n=5}^{\infty} D_{n}(M - M^{*})^{n}
\]  

(26)

to the Lagrangian \((\Phi)\). We regard \( U_{H}(\phi) \) as the effective potential induced by the higher-order quantum corrections beyond 1-loop approximation. We remark that, in Eq. \((26)\), the terms of \( \phi > 0 \sim \phi > 4 \) have been canceled by
the counter term to the higher-order quantum corrections just as Eq. (2). By this modification, Eqs (8) and (16) are modified as

$$
\varepsilon(k_F, <\phi>, <V^0_0>) = \varepsilon_N(k_F, M^*) + \varepsilon_v(<V^0_0>) + \frac{1}{2} \mu^2_s <\phi>^2 + U_1^{V,R}(<\phi>) + U^H(<\phi>)
$$

and

$$
\Pi_s(q; <\phi>, k_F) = \Pi^V_s(q^2; <\phi>) + \Pi^D_s(q; <\phi>, k_F) + g_s^2 \frac{d^2}{dM^*} U^H(<\phi>).
$$

We remark that Eqs. (20) and (22) are still valid, after this expansion was carried out.

3 Numerical calculation

Equation (22) gives the relation among the effective nucleon mass, the incompressibility and the effective $\sigma$-meson mass $\mu^*_s$ at the normal density. If the values of $M^*_0$ and $K$ is given, $C_v = g_v/\mu_v$ is determined by Eq. (13) and we can calculate the ratio $\mu^*_s/g_s$ at the normal density. In Fig. 1, we display the ratio $\mu^*_s/g_s$ as a function of $M^*_0$ with several values of $K$. In the numerical calculations, we set $a_1 = 15.75 \text{MeV}$, $\rho_0 = 0.15 \text{fm}^{-3}$ and $M = 939 \text{MeV}$. The ratio $\mu^*_s/g_s$ increases as $M^*_0$ increases, while the ratio depends on $K$ only slightly.

Since $U^H$ does not appear explicitly in (22), the result in Fig. 1 is established regardless of the details of the potential form. However, to calculate $\mu^*_s$ itself, we must determine $g_s$. For this purpose, we assume that

$$
U^H(<\phi>) = D_5(M - M^*)^5 + D_6(M - M^*)^6.
$$

In this approximation, we have four parameters for RHA calculation, namely, $C_s$, $C_v$, $D_5$ and $D_6$. As is seen in Eq. (13), $C_v$ is determined if the value of $M^*_0$ is given. We have two conditions for the saturations at $\rho = \rho_0$:

$$
\varepsilon(\rho_0) = (M - a_1) \rho_0 \quad \text{and} \quad P(\rho_0) = 0
$$

Therefore, if the value of $K$ is given, the remaining three parameters $C_s$, $D_5$ and $D_6$ are determined.

Using the same potential as (29), we obtain

$$
\Pi_s(q; <\phi>, k_F) = \Pi^V_s(q^2; <\phi>) + \Pi^D_s(q; <\phi>, k_F) + 20 g_s^2 D_5 (M - M^*)^3 + 30 D_6 g_s^2 (M - M^*)^4,
$$

From Eqs. (29) and (32), at zero density, we obtain

$$
1 + C_s g_s^2 \frac{\Pi^V_s(q)}{g_s^2} \bigg|_{q^2=0, q_0=m_s^*} = \frac{m_s^2}{\mu_s^2}.
$$

Since $\Pi^V_s/g_s^2$ does not depend on $g_s$, we can calculate $\mu_s$ by putting the value of $C_s$ and $m_s = 550 \text{MeV}$ into Eq. (22). After determining $\mu_s$, we can also determine the value of $g_s$. In Figs. 2 and 3, we display $\mu_s$ and $g_s$ as a function of $M^*_0$. As
$M_0^*$ increases, $g_s$ is suppressed more strongly than $\mu_s$. Since, as is seen in $C_2^s$ approaches zero as $M_0^* \to 0.944M$, $g_s$ becomes small more quickly than $\mu_s$ to keep the saturation conditions. Both of $\mu_s$ and $g_s$ depend on $K$ only slightly.

After determining $\mu_s$ and $g_s$, we can calculate $\mu_s^*$ and $m_s^*$. In Figs. 4 and 5, we display $\mu_s^*/\mu_s$ and $m_s^*/m_s$ at the normal density as a function of $M_0^*$. As $M_0^*$ increase, the ratios $\mu_s^*/\mu_s$ and $m_s^*/m_s$ increase. The ratio $\mu_s^*/\mu_s$ is smaller than 1 for $M_0^* < 0.560(0.542, 0.526)M$, when $K = 200(300, 400)$MeV. Similarly, the ratio $m_s^*/m_s$ is smaller than 1 for $M_0^* < 0.815(0.802, 0.793)M$, when $K = 200(300, 400)$MeV. Both of two ratios depend on $K$ only slightly.

In Figs. 6 and 7, we display $\mu_s^*/\mu_s$ and $m_s^*/m_s$ as a function of baryon density with several values of $M_0^*$. Since two ratios depend on $K$ only slightly, we have fixed the value of $K$ at 300MeV. The density dependence of the two ratios also changes, when $M_0^*$ changes. The ratio $\mu_s^*/\mu_s \gtrsim 1$ in the region of $\rho \lesssim 1.3\rho_0$, while the ratios $m_s^*/m_s < 1$ except for the case with $M_0^* = 0.85M$.

4 Summary

In summary, we have studied the relation among the effective nucleon mass $M^*$, the incompressibility $K$ and the effective $\sigma$-meson mass $m_s^*$ in nuclear matter by using the relativistic nuclear model. We found that there is a strong correlation between $M^*$ and $m_s^*$, while there is only a weak correlation between $K$ and $m_s^*$. At the normal density, $m_s^*$ is smaller than the one at zero density for $M_0^* \leq 0.8M$, while $\mu_s^*$ hardly decreases. We remark that it is interesting that the off-shell mass $\mu_s^*$ at the normal density is related directly to $M_0^*$ and $K$ which can be determined phenomenologically.

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Figures

Figure 1: The ratio $\mu^*_0/g_*$ as a function of $M^*_0$. The solid, the dashed and dashed-dotted curves show results for $K = 200\text{MeV}, 300\text{MeV}$ and $400\text{MeV}$, respectively. The cross show the result for the original RHA.
Figure 2: The $\mu_s$ as a function of $M_0^*$. The various curves and the cross have the same notation as in Fig. 1.

Figure 3: The $g_s$ as a function of $M_0^*$. The various curves and the cross have the same notation as in Fig. 1.
Figure 4: The ratio $\mu_{s0}^*/\mu_s$ as a function of $M_0^*$. The various curves and the cross have the same notation as in Fig. 1.

Figure 5: The ratio $m_{s0}^*/m_s$ as a function of $M_0^*$. The various curves and the cross have the same notation as in Fig. 1.
Figure 6: The ratio $\mu_s^*/\mu_s$ as a function of the baryon density. The solid, the dashed and dashed-dotted curves show results for $M_0^* = 0.65M$, $0.75M$ and $0.85M$, respectively. The dotted curve shows the result for the original RHA.

Figure 7: The ratio $m_s^*/m_s$ as a function of the baryon density. The various curves have the same notation as in Fig. 6.