Abstract

We propose the measurement of the decay angular distribution of leptons from $J/\psi$’s produced at high transverse momentum balanced by a photon [or gluon] in hadronic collisions. The polar and azimuthal angular distribution are calculated in the color singlet model (CSM). It is shown that the general structure of the decay lepton distribution is controlled by four invariant structure functions, which are functions of the transverse momentum and the rapidity of the $J/\psi$. We found that two of these structure functions [the longitudinal and transverse interference structure functions] are identical in the CSM. We present analytical and numerical results in the Collins-Soper and in the Gottfried-Jackson frame.
The measurement of the angular distribution of lepton’s from $J/\psi$’s provides a detailed test of the production and decay mechanism of the $c\bar{c}$ bound state. So far there has been intensive experimental studies of the $J/\psi$ production rate and transverse momentum distributions in hadronic collisions both at UA1 [1] and CDF [2]. However, the observed $J/\psi$ rate was found to be markedly higher than the predicted one’s in [3], where in addition to the direct charmonium production also contributions from the $\chi$ and the production resulting from $B$ decays were taken into account. It has recently been pointed out in [4] that at large transverse momentum of the $J/\psi$ an additional production mechanism comes from fragmentation contributions of a gluon or a charm quark into charmonium states.

In this letter we restrict ourselves to the study of $J/\psi$ produced in association with a photon [5]. This has several advantages. First of all the experimental signature of a $J/\psi$ [decaying into an $e^-e^+$ or $\mu^\pm\mu^\mp$ pair] and a $\gamma$ with balancing transverse momentum is a very clean final state. Second, the fragmentation contributions from radiative $\chi$ decays to $J/\psi+\gamma$ production are small and the dominant subprocess contributing to a $J/\psi+\gamma$ final state is the gluon gluon fusion process.

For $J/\psi$’s produced with transverse momentum $p_T$ [balanced by the additional photon] one can define an event plane spanned by the beam and the $J/\psi$ momentum direction which provides a reference plane for a detailed study of angular correlations. The decay lepton distribution in $J/\psi \rightarrow l^-l^+$ in the $J/\psi$ rest frame is determined by the polarization of the $J/\psi$. Therefore, the study of the angular distribution can be used as an analyzer of the $J/\psi$-polarization. It is thus possible to test the underlying $J/\psi$-production dynamics [in our case the color singlet model (CSM) [6]] in much more detail than is possible by rate measurements alone [5].

For definiteness we consider the angular distribution of the leptons coming from the leptonic decay of $J/\psi$’s produced with non-zero transverse momentum in association with a photon in high energy proton-antiproton collisions:

$$p(P_1) + \bar{p}(P_2) \rightarrow J/\psi(P) + \gamma(k) + X \rightarrow l^-(l) + l^+(l') + \gamma(k) + X,$$

where the quantities in the parentheses denote the four-momenta. In leading order of perturbative QCD [$O(\alpha_s^2)$], $J/\psi + \gamma$ can only be produced in $gg$ fusion:

$$g(p_1) + g(p_2) \rightarrow J/\psi(P) + \gamma(k) + X .$$

Denoting hadron level and parton level quantities by upper and lower case characters, respectively, the hadron and parton level Mandelstam variables are defined by

$$S = (P_1 + P_2)^2 \, , \quad T = (P_1 - P)^2 \, , \quad U = (P_2 - P)^2 ,$$

and

$$s = (p_1 + p_2)^2 = x_1x_2S \, ,$$
$$t = (p_1 - P)^2 = x_1(T - P^2) + P^2 ,$$
$$u = (p_2 - P)^2 = x_2(U - P^2) + P^2 ,$$

$\ast$ The radiative $\chi_J$ decays can produce $J/\psi$ at both low and high $p_T$, but the photon produced will be soft [$E \sim O(400 \text{ MeV})$].
The angular distribution of the leptons from the $J/\psi$ has the general form

$$\frac{d\sigma}{dp_T^2\,dy\,d\theta\,d\phi} = \frac{3}{16\pi}\,\frac{d\sigma^{U+L}}{dp_T^2\,dy} \left[ (1 + \cos^2 \theta) + \frac{1}{2} A_0 \,(1 - 3 \cos^2 \theta) \right.$$  
+ $A_1 \,\sin 2\theta \cos \phi$  
+ $\frac{1}{2} A_2 \,\sin^2 \theta \cos 2\phi \right].$  

(5)

The angles $\theta$ and $\phi$ in Eq. (5) are the polar and azimuthal decay angles of the leptons in the $J/\psi$ rest frame with respect to a coordinate system described below and $y$ ($p_T$) denotes the rapidity (transverse momentum) of the $J/\psi$ in the laboratory frame. Note that the unpolarized differential production cross section denoted by $\sigma^{U+L}$ is factored out from the r.h.s of Eq. (5). The angular coefficients $A_i$ characterize the polarization of the $J/\psi$ (see below). They are dependent on the choice of the $z$ axis in the $J/\psi$ rest frame and are defined by the following ratios of helicity cross sections

$$A_0 = \frac{2}{d\sigma^{U+L}}, \quad A_1 = \frac{2\sqrt{2}}{d\sigma^{U+L}}, \quad A_2 = \frac{4}{d\sigma^{U+L}},$$

(6)

The hadronic helicity cross sections $\frac{d\sigma^{U+L,T,I}}{dp_T^2\,dy}$ are obtained by convoluting the partonic helicity cross sections $s\frac{d\hat{\sigma}^{U+L,T,I}}{dt\,du}$ with the parton densities. One has

$$\frac{d\sigma^{U+L,T,I}}{dp_T^2\,dy} = \int dx_1 dx_2 g_{h_1}^1(x_1,\mu_E) g_{h_2}^2(x_2,\mu_E) s\frac{d\hat{\sigma}^{U+L,T,I}}{dt\,du}.$$  

(7)

Each of the partonic helicity cross sections is calculated in the CSM. The unpolarized differential production cross section is denoted by $\hat{\sigma}^{U+L}$ whereas $\hat{\sigma}^{L,T,I}$ characterize the polarization of the $J/\psi$, i.e. the cross section for the longitudinal polarized $J/\psi$'s is denoted by $\hat{\sigma}^L$, the transverse-longitudinal interference by $\hat{\sigma}^I$, and the transverse interference by $\hat{\sigma}^T$ (all with respect to the $z$-axis of the chosen $J/\psi$ rest frame). The results for the helicity cross sections $\hat{\sigma}^{U+L,T,I}$ are dependent on the choice of the $z$ axis in the rest frame of the $J/\psi$. We present explicit results for the Collins-Soper (CS) and the Gottfried-Jackson (GJ) frame:

In the CS frame (8) the $z$-axis bisects the angle between $\vec{P}_1$ and $-\vec{P}_2$

$$CS: \quad \vec{P}_1 = E_1 (\sin \gamma_{CS}, 0, \cos \gamma_{CS}), \quad \vec{P}_2 = E_2 (\sin \gamma_{CS}, 0, -\cos \gamma_{CS}),$$

(8)

with $\cos \gamma_{CS} = \sqrt{\frac{m_S^2}{(T-m_S^2)(U-m_S^2)}} = \sqrt{\frac{m_E^2}{m_S^2 + p_T^2}}, \quad \sin \gamma_{CS} = -\sqrt{1 - \cos^2 \gamma_{CS}}$.

In the GJ frame (also known as $t$-channel helicity frame) the $z$-axis is chosen parallel to the beam axis

$$GJ: \quad \vec{P}_1 = E_1 (0, 0, 1), \quad \vec{P}_2 = E_2 (\sin \gamma_{GJ}, 0, \cos \gamma_{GJ}),$$

(9)

with
\[
\cos \gamma_{GJ} = 1 - \frac{2m_{\psi}^{2}S}{(T-m_{\psi}^{2})(U-m_{\psi}^{2})}, \quad \sin \gamma_{GJ} = -\sqrt{1-\cos^{2}\gamma_{GJ}}.
\]
The beam and target energies in the rest frame are \(E_{1} = (m_{\psi}^{2} - T)/(2m_{\psi}), \quad E_{2} = (m_{\psi}^{2} - U)/(2m_{\psi})\).

The partonic helicity cross sections in Eq. (7) are calculated applying the technique described in [10] to the CSM [details of the calculation will be presented elsewhere]. They are given by [R(0) denotes the radial wave function of the bound state]:

\[
\frac{d\sigma_{CS,GJ}}{dtdu} = \frac{16\pi\alpha\alpha_{s}^{2}m_{\psi}}{27s} |R(0)|^{2} H_{CS,GJ}^{\alpha}(s + t + u - m_{\psi}^{2}), \quad (10)
\]
with

\[
H_{CS}^{U+L} = \frac{s^{2}}{(t - m_{\psi}^{2})^{2}(u - m_{\psi}^{2})^{2}} + \frac{t^{2}}{(u - m_{\psi}^{2})^{2}(s - m_{\psi}^{2})^{2}} + \frac{u^{2}}{(s - m_{\psi}^{2})^{2}(t - m_{\psi}^{2})^{2}} , \quad (11)
\]

\[
H_{CS}^{L} = \frac{ut}{2(t - m_{\psi}^{2})(u - m_{\psi}^{2})} \left( \frac{4m_{\psi}^{2}s^{3}}{(s - m_{\psi}^{2})^{2}(s - m_{\psi}^{2})^{2}} + H_{CS}^{U+L} \right) , \quad (12)
\]

\[
H_{CS}^{T} = \frac{1}{2} H_{CS}^{L} , \quad (13)
\]

\[
H_{CS}^{I} = \frac{m_{\psi}\sqrt{sut}s(s^{2} - ut)(t - u)}{\sqrt{2}(s - m_{\psi}^{2})^{2}(s - m_{\psi}^{2})^{3}(u - m_{\psi}^{2})^{3}} , \quad (14)
\]

and for the GJ frame

\[
H_{GJ}^{U+L} = H_{CS}^{U+L} , \quad (15)
\]

\[
H_{GJ}^{L} = \frac{2m_{\psi}^{2}stu(s^{2} + u^{2})}{(s - m_{\psi}^{2})^{2}(s - m_{\psi}^{2})^{2}} , \quad (16)
\]

\[
H_{GJ}^{T} = \frac{1}{2} H_{GJ}^{L} , \quad (17)
\]

\[
H_{GJ}^{I} = -\frac{m_{\psi}\sqrt{sut}(s - u)[s^{2}(u - t) + u^{2}(s - t)]}{\sqrt{2}(s - m_{\psi}^{2})^{2}(s - m_{\psi}^{2})^{2}} , \quad (18)
\]

As mentioned before, \(H_{GJ}^{U+L}\) denotes the matrix element contribution for the production rate and was first calculated in the first paper of [7]. All other matrix elements correspond to the production of polarized \(J/\psi\)'s and are given here for the first time. Replacing 16\(\alpha\) by 15\(\alpha_{s}\) in Eq. (10), the results in Eqs. (10-18) are also valid for \(J/\psi + g\) production within the CSM.

We will now present numerical results for \(J/\psi + \gamma\) production at the Tevatron collider center of mass energy \(\sqrt{S} = 1.8\) TeV including the decay \(J/\psi \rightarrow \mu^{-}\mu^{+}\). All results are obtained using the gluon density parametrization from GRV [11] with \(\Lambda_{(4)MS}^{(4)} = 200\) MeV and the one-loop formula for \(\alpha_{s}\) with 4 active flavours. If not stated otherwise, we identify the renormalization scale \(\mu_{R}^{2}\) and the factorization scale \(\mu_{F}^{2}\) and set them equal to \(\mu^{2} = \mu_{F}^{2} = \mu_{R}^{2} = (m_{\psi}^{2} + p_{T}^{2}(J/\psi))\). The value for the bound state wave function at the origin \(|R(0)|^{2}\) is determined from the leptonic decay width of \(J/\psi\): \(\Gamma(J/\psi \rightarrow e^{-}e^{+}) = 4.72\) keV, therefore \(|R(0)|^{2} = 0.48\) GeV. 

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In Figs. 1 and 2 we show numerical results for the coefficients $A_i$ in Eq. (5) as a function of $p_T(J/\psi)$ and $y(J/\psi)$ in the CS [Figs. 1,2(a)] and GJ [Figs. 1,2(b)] frame. One observes that the coefficients are strongly dependent on $p_T(J/\psi)$ both in the CS and GJ frame. As mentioned before, the coefficients $A_0$ and $A_2$ are exactly equal in lowest order in both lepton pair rest frames. The angular coefficient $A_1$ is zero in the CS frame for all values of $p_T(J/\psi)$. The reason is that the matrix element for $A_1$ is antisymmetric in $u$ and $t$ [see Eqs. (14,15)] and therefore in $x_1$ and $x_2$, whereas the product of the gluon distributions is symmetric under the interchange of $x_1$ and $x_2$. As a consequence, the rapidity distribution for $A_1$ in the CS frame [Fig. 2(a)] is also antisymmetric around $y = 0$. However, this is different for the GJ frame [see Eq. (18)]. All coefficients $A_i$ vanish in the limit $p_T(J/\psi) \to 0$, which can be directly seen from our analytical expressions in Eqs. (11-18).

A similar relation $A_0^{DY} = A_2^{DY}$ was found in LO $[O(\alpha_s)]$ in the Drell-Yan process $p + \bar{p} \to V + X \to l^+l^- + X$, where $V$ denotes a gauge boson produced at high $p_T$. In Ref. [10], the complete NLO corrections to the coefficients $A_i^{DY}$ are calculated and the corrections are found to be fairly small for the ratio’s $A_i$ of the corresponding helicity cross section. We expect also here, that the LO results for the ratio’s $A_i$ in $J/\psi$ production are almost not affected by higher order QCD corrections. Note also, that the coefficients $A_i$ are not dependent on the bound state wave function. The measurement of these coefficients would be a sensitive test of the production mechanism of $J/\psi$'s.

To summarize: hadronic $J/\psi + \gamma$ production has been evaluated in the nonrelativistic bound state model. In leading order this final state can only be produced by gluon fusion. Analytical formula for the decay lepton distributions in terms of four structure functions are presented. The angular distribution in the $J/\psi$ rest frame is determined by the polarization of the $J/\psi$.

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Figure captions

Fig. 1 Angular coefficients $A_0$, $A_1$ and $A_2$ for $J/\psi + \gamma$ production and decay in the CS frame (a) and GJ frame (b) as a function of the $J/\psi$ transverse momentum at $\sqrt{S} = 1.8$ TeV. No cuts have been applied.

Fig. 2 Same as Fig. 1 for the $y(J/\psi)$ distribution.

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Figure 2

\[(\Lambda \Theta^\Lambda) (\Phi / f)^\Lambda\]

\[\Lambda^2 \Theta^I I = s^\Lambda\]

- C-frame
- CS-frame

\[X + \lambda^+ \rightarrow X + \lambda + A / f \rightarrow \frac{df}{d}\]