We report on a new mechanism for heat conduction in the neutron star crust. We find that collective modes of superfluid neutron matter, called superfluid phonons (sPhs), can influence heat conduction in magnetized neutron stars. They can dominate the heat conduction transverse to magnetic field when the magnetic field $B \gtrsim 10^{13}$ G. At density $\rho \approx 10^{12} - 10^{14}$ g/cm$^3$, the conductivity due to sPhs is significantly larger than that due to lattice phonons and is comparable to electron conductivity when temperature $\approx 10^8$ K. This new mode of heat conduction can limit the surface anisotropy in highly magnetized neutron stars. Cooling curves of magnetized neutron stars with and without superfluid heat conduction could show observationally discernible differences.

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Multiwavelength observations of thermal emission from the neutron star (NS) surface and explosive events such as superbursts in accreting NS and giant-flare in magnetars provide a real opportunity to probe the NS interior (see 1 and 2 for recent reviews). Theoretical models of these phenomena clearly underscore the importance of heat transport in the neutron star (NS) crust and have shown that it directly impacts observations. For example, nearby isolated compact X-ray sources that have also been detected in the optical band have a significant optical excess relative to the extrapolated X-ray blackbody emission (a factor 5 to 14) 3. This optical excess can arise naturally if heat conduction in the NS crust is anisotropic due to the presence of a large magnetic field leading to an anisotropic surface temperature distribution 4,5,6. In accreting NSs, the thermal conductivity directly affects the observed thermal relaxation time of the crust 7, and the superburst ignition depth and recurrence time-scale 8.

Typically, heat conduction is thought to arise due to the flow of relativistic electrons. Electrons are numerous, degenerate, and provide an efficient mode to transport heat. Conduction due to lattice phonons has been considered, but shown to be unimportant when the crust temperature $T \gtrsim 10^9$ K 9,10 and in NSs with low magnetic fields. In this letter we demonstrate that a new mechanism for heat transport arising due to the superfluid nature of the inner crust is important at high temperature and/or large magnetic fields. Here the heat is carried by the collective excitations of the neutron superfluid called superfluid phonons (sPhs).

This mode of conduction is especially relevant in NSs with moderate to high magnetic fields ($B \gtrsim 10^{13}$ G). In these cases, electron heat transport is very anisotropic because electrons can only move freely along magnetic field lines. Motion perpendicular to the field is restricted because the electron-cyclotron frequency is large compared to the inverse collision time. In contrast, sPhs, being electrically neutral excitations, are not directly affected by magnetic fields. We find that sPh conduction is typically much larger than the electron conduction transverse to the field and this limits the degree of surface temperature anisotropy in magnetized NSs. In NSs with low magnetic fields, sPh conduction can still be relevant when the temperature is in the range $10^8 - 10^9$ K.

Free neutrons in the inner crust are known to form Cooper pairs and become superfluid at temperatures below the critical temperature $T_c \approx 10^{10}$ K. The superfluid ground state spontaneously breaks baryon-number symmetry and gives rise to a new massless Goldstone mode - the superfluid phonon (sPh). At long wavelengths, these phonons have a linear dispersion relation $\omega = v_s q$, where $v_s \approx k_F n / \sqrt{3} M$, $M$ is the mass of the neutron and $k_F n$ is the neutron Fermi momentum. The thermal conductivity of a weakly interacting gas of sPhs can be computed from kinetic theory and is given by

$$\kappa_{sPh} = \frac{1}{3} C_V v_s \lambda_{sPh}$$

where $C_V = 2 \pi^2 T^3 / (15 v_s^3)$ is the specific heat of the phonon gas and $\lambda_{sPh}$ is the typical mean free path of a thermal sPh. We can rewrite Eq. 1 using fiducial values of the temperature and the phonon velocity as

$$\kappa_{sPh} = 1.5 \times 10^{22} \left( \frac{T}{10^8 \text{ K}} \right)^3 \left( \frac{v_s}{0.1 \text{ cm s}^{-1}} \right)^2 \left( \frac{\lambda_{sPh}}{\text{ cm}} \right) \text{ erg cm}^{-1} \text{ s}^{-1} \text{ K}^{-1}$$

This is to be compared with the thermal conductivity of electrons in the inner crust which is approximately $10^{18}$ erg cm$^{-1}$ s$^{-1}$ K$^{-1}$ when $T = 10^8$ K 11. In the presence of large magnetic fields electron conduction in the direction perpendicular to the field can be several orders of magnitude smaller. Although sPhs are less numerous than the electrons they have large mean free paths because they interact weakly at long-wavelengths. In what follows we will estimate $\lambda_{sPh}$ and its temperature dependence.

The low energy degrees of freedom in the inner crust are lattice phonons (lPhs), electrons, and sPhs. At
long-wavelengths, the excitations of the ion-lattice with mean number density \( n_I \), mass number \( A \) and charge \( Z \), are the longitudinal and transverse IPHs with velocities \( c_s = \omega p / q T F e \) and \( c_t \approx 0.7 \omega p / q B z \), respectively \([11]\). Here \( \omega p = \sqrt{4 \pi e^2 Z^2 n_I / (AM)} \) is the ion-plasma frequency, \( q T F e = \sqrt{4 e^2 / \pi} p_{Fe} \) is the inverse Thomas-Fermi screening length of the electrons, \( p_{Fe} \) is the electron Fermi momentum, \( q B z = (4.5\pi)^{1/3} a_i^{-1} \) and \( a_i \) is the inter-ion distance. The electrons are characterized by their Fermi momentum \( p_{Fe} = (3\pi^2 Z n_I)^{1/3} \gg m_e \gg T \) and form a highly degenerate, relativistic Fermi gas. As mentioned earlier, the superfluid phonons (sPHs) are collective excitations of the neutron superfluid and their interactions are weak at low temperature. Their scattering cross section is parametrically suppressed by the factor \( T^8 / \mu_n^8 \) where \( \mu_n = k_F^2 / 2M \) is the neutron chemical potential \([12]\). We note that when \( T \approx T_c \), sPHs are strongly damped as they can easily decay into neutron particle-hole excitations. However, since the crust cools to \( T \ll T_c \), within several hours of the birth of the NS, this process is exponentially suppressed by the factor \( \exp (-2T_c / T) \) and the mean free path of sPHs is limited only by their interaction with IPHs, electrons and impurities.

The relevant processes are illustrated in Fig. 1: (i) Rayleigh scattering of phonons due to interactions with (compositional) impurities in the solid lattice \([13]\); (ii) absorption of sPHs due to their mixing with the longitudinal lattice phonons which are absorbed efficiently by electrons; and (iii) the decay of a sPH into two lattice phonons (IPHs).

At long-wavelength, the cross section for Rayleigh scattering of phonons by impurities is

\[
\sigma_R = \pi r_0^2 \frac{(q r_0)^4}{1 + (q r_0)^4}, \tag{3}
\]

where \( q \) is the phonon momentum and \( r_0 \) is an intrinsic strong-interaction length-scale related to the scattering length of the neutron-impurity system. This depends on the impurity composition and is expected to be about 10 fm. At low temperature, when \( q r_0 \ll 1 \), the mean free path of sPHs with thermal energy \( \omega_{th} = 3T \) is

\[
\lambda_{Ray} = (n_{Im} \sigma_R)^{-1} = \frac{v_s^4}{81 n_{Im} \pi r_0^6 T^4}, \tag{4}
\]

where \( n_{Im} = 3/(4\pi d^3) \) is the impurity number density and \( d \) is the inter-impurity distance. Using fiducial values we can rewrite Eq. 4 as

\[
\lambda_{Ray} = 450 \left( \frac{v_s}{0.1} \right)^4 \left( \frac{x}{10} \right)^3 \left( \frac{10 \text{ fm}}{r_0} \right)^3 T_7^{-4} \text{ cm} \tag{5}
\]

where \( x = d/r_0 \) is the diluteness parameter for the impurities and \( T_7 \) is the temperature in \( 10^7 \) K.

We now show that inelastic processes such as sPH absorption by electrons and decay to IPHs shown in Fig. 1 (B) and (C) are more relevant. To study these processes we need a low-energy effective theory which couples the sPHs and IPHs. On general grounds, this effective theory can only contain derivative terms and at low temperature it is sufficient to retain the leading terms. The leading order Lagrangian that describes these interactions is of the form

\[
\mathcal{L}_{\text{eff}} = g_{mix} \partial_0 \phi \partial_i \xi^i + \frac{g_{mix}}{\Lambda^2} \partial_0 \phi \partial_i \xi_i \partial_\xi_i + \frac{g_{mix}}{\Lambda^2} \partial_0 \phi \partial_i \xi_j \partial_\xi_j + \cdots \tag{6}
\]

where \( \phi \) and \( \xi^i \) are the sPH and IPH (canonically normalized) fields, respectively. The first term describes the mixing between longitudinal IPHs and sPHs, and the second and third terms describe the process for the decay of a sPH into two longitudinal or transverse IPHs, respectively.

The coefficients of this effective theory are determined by computing the coupling between neutrons and lattice phonons in a manner similar to the calculation of the electron-phonon coupling in condensed matter physics \([14]\). We begin with the fundamental short-range interaction between neutrons and ions given by

\[
\mathcal{H}_{nl} = \int d^3x \ d^3x' \ \Psi_I^\dagger(x) \mathcal{V}(x - x') \ \Psi_I(x) \ \psi_n^\dagger(x) \ \psi_n(x)
\]

where \( \mathcal{V}(x - x') = 2\pi a_{nl} \delta^3(x - x')/M \), is the neutron-ion low-energy potential and \( a_{nl} \) is the neutron-ion scattering length. By expanding the ion and neutron density fluctuations in terms of their collective modes we obtain

\[
g_{\text{mix}} = 2 a_{nl} \sqrt{n_I k_F^2 / AM^2}, \ \ \ \ \Lambda^2 = \sqrt{n_I AM}, \tag{7}
\]

respectively. For typical conditions in the NS crust \( g_{\text{mix}} \approx 10^{-3} \) and \( \Lambda \approx 50 \text{ MeV} \). The specific details of this calculation and possible quantum corrections to the sPH-IPH interaction will be reported elsewhere \([15]\). Here we employ these results to estimate \( \lambda_{IPh} \).

Mixing between sPHs and IPHs will damp sPH propagation because IPHs are strongly damped due to their interaction with electrons. To leading order in the small mixing parameter \( g_{\text{mix}} \), the mean free path of sPHs with energy \( \omega \) is given by

\[
\lambda_{abs}(\omega) = \frac{v_s^2}{g_{\text{mix}}^2} \frac{1 + (1 - \alpha^2)^2 (\omega / \tau_{ph})^2}{\alpha (\omega / \tau_{ph})^2} \lambda_{IPh}(\omega), \tag{8}
\]
where \( \alpha = c_s/v_s \), and \( \lambda_{\text{lPh}} \) and \( \tau_{\text{lPh}} = \lambda_{\text{lPh}}/c_s \) are the mean free path and lifetime of the lPh, respectively. At low temperature, where the Umklapp processes are frozen, we can estimate \( \lambda_{\text{lPh}} \) by assuming that the lPhs decay primarily by producing particle-hole excitations (normal processes) in the degenerate electron gas. We find that

\[
\lambda_{\text{lPh}}^{-1}(\omega) = f_{\text{ep}}^2 \frac{2\pi c_s^2}{p_{\text{F}}^2} \frac{\omega}{\omega_x} = \frac{2}{3\pi} \frac{Z}{\lambda_{\text{lPh}}} \frac{p_{\text{e}}}{\omega} = \frac{2}{\pi} \frac{\omega}{\omega_x} \tag{9}
\]

where \( f_{\text{ep}} = Z \sqrt{n_1/AM/c^2} \) is related to the electron-phonon coupling constant [10].

When \( T \gtrsim T_{\text{Um}} = Z^{1/3} e^2 \omega_p/3 \), the Umklapp processes begin to dominate [17] and in this regime \( \lambda_{\text{lPh}}^{-1} \approx 100 \alpha \simeq 2000 \text{ fm} \) [11]. The corresponding sPh mean free path is in the range \( 10^{-3} - 10^{-6} \text{ cm} \). At intermediate temperature we employ a simple interpolation formula given by

\[
\lambda_{\text{sPh}}^{-1}(\omega) = f_u \lambda_{\text{lPh}}^{-1} + (1 - f_u) \lambda_{\text{lPh}}^{-1}, \tag{10}
\]

where \( f_u = \exp(-T_{\text{Um}}/T) \) [17].

Finally we estimate the mean free path due to the decay process shown in the Fig. 1 (C). When the sPh velocity is greater than the lPh velocity this decay is kinematically allowed. The amplitude for the sPh of momentum \( q \) to split into two longitudinal lPh of momentum \( q_1 \) and \( q_2 \) is given by

\[
A \simeq \frac{g_{\text{mix}}}{A^2} \omega q_1 q_2. \tag{11}
\]

The corresponding mean free path for a thermal phonon with energy \( \omega = 3T \) can be calculated and we find that

\[
\lambda_{\text{decay}} \simeq \frac{863}{f(\alpha)} \left( \frac{10^{-6}}{g_{\text{mix}}} \right) \left( \frac{c_s}{0.01} \right)^7 \left( \frac{v_s}{0.1} \right) \lambda_0^3 T^{-5} \text{ cm} \tag{12}
\]

where \( \Lambda_0 = \Lambda/50 \text{ MeV} \), \( f(\alpha) = 1 - 2\alpha^2/3 + \alpha^4/5 \), and \( \alpha = c_s/v_s \). For the temperatures of relevance we find that \( \lambda_{\text{abs}} \ll \lambda_{\text{decay}} \) and \( \lambda_{\text{abs}} \ll \lambda_{\text{Ray}} \). Consequently, only the sPh absorption process with the mean free paths given by Eq. 12 is relevant.

In Fig. 2 we employ the above result (Eq. 12) for the sPh mean free path and compare the thermal conductivity due to electron, lPhs, and sPhs transport in a typical NS crust, at temperatures of \( 10^8 \) K and \( 10^7 \) K. We show the electron contribution parallel (\( e^\parallel \)) and perpendicular (\( e^\perp \)) to the magnetic field lines for \( B = 10^{13} \text{ G} \) and \( B = 10^{14} \text{ G} \), according to [18]. Heat conduction due to lattice vibrations is primarily carried by the transverse modes because of their larger number density and we use the results from [13]. We can see that, above the neutron drip point, where a significant number of superfluid neutrons coexist with the electrons and the lattice, heat conduction due to sPhs is relevant, and is always significantly more efficient than conduction due to lPhs. The sPh conductivity decreases with depth and reaches a minimum when the velocity of the sPh becomes equal the longitudinal speed of sound due to the resonant mixing. Near neutron drip, where \( v_s \ll c_s \), sPhs can even dominate over electron conductivity along the field. In the direction transverse to magnetic field the sPh conduction is shown to be relevant over much of the inner crust for fields \( B \geq 10^{14} \text{ G} \).

![FIG. 2: Thermal conductivity in a NS crust for \( T = 10^8 \) K (left) and \( T = 10^7 \) K (right). The dot–dashed (red) curve shows the contribution due to sPhs. The thick (thin) solid lines show the electron conductivity in the direction longitudinal (perpendicular) to the magnetic field lines for \( B = 10^{13} \text{ G} \) (i) and \( B = 10^{14} \text{ G} \) (ii). The dashed lines (yellow) is the lPh contribution.](image)

![FIG. 3: Cooling curves of magnetized NS with and without considering sPhs. Left panel: \( T_0 \) at the neutron drip point vs. age. Right panel: \( T \) vs. \( \rho \) for fixed times. The polar temperature (solid lines) is not affected, but the temperatures at the equator with sPhs (dash-dotted lines) and without sPhs (dashes) differ significantly.](image)
To assess how this new mode of heat conduction might affect observable aspects of NS thermal evolution, we have performed magnetar cooling simulations with and without including sPh conduction. Details regarding the simulation and the input physics are described in earlier work [15]. The new ingredients here are the sPh contribution to the thermal conductivity and the use of an updated $^1S_0$ neutron superfluid gap obtained from Quantum Monte Carlo simulations [20]. The cooling curves and temperature profiles for a NS model (Model A from Ref. [19]) with a poloidal, crustal magnetic field of strength $B = 5 \times 10^{13}$ G at the magnetic pole, are shown in Fig. 4. Temperatures at the magnetic equator are shown as dashed lines for the model without sPhs and as dashed-dotted lines for the model with sPhs. The temperature at the pole is shown by solid lines and there is no significant difference between the two cases. The left panel shows $T_{\nu}$, the temperatures at the neutron drip point ($\rho \approx 3 \times 10^{43}$ g cm$^{-3}$), as a function of the NS age, $t$. In the right panel we show the temperature profiles in the NS crust as a function of density, for three fixed times $t = 2 \times 10^3, 10^4, 10^5$ yr. The effect of sPhs is important, specially in the inner crust when $T \approx 10^8$ K. In the right panel one can see that sPhs partially reduce the otherwise strong temperature gradient generated along the inner crust. As a consequence, the temperature anisotropy in the inner crust is limited by sPh conduction. We note, however, that an anisotropy generated in the outer layers at density below neutron drip cannot be suppressed by sPhs.

Finally, we remark that even the phonon conductivity can be anisotropic. Large magnetic fields will induce an anisotropy in the electron response because of the gap in the electron excitation spectrum due to Landau quantization. When this gap is much larger than the temperature, phonons propagating transverse to the magnetic field cannot excite single electron-hole states and this is likely to greatly enhance both the lPh and sPh conductivity transverse to the field. Rotation will create superfluid vortices that will preferentially scatter sPhs propagating perpendicular to the rotation axis. Both these effects warrant further investigation.

Our estimate for the sPh conductivity, based on Eq. 1, can be improved by replacing the typical mean free path by an appropriate thermal average, and by accounting for composition and nuclear structure dependence of the neutron-nucleus scattering potential for neutron-rich nuclei in the crust. Nonetheless, our study clearly demonstrates that sPhs are important for heat conduction and are primarily damped through their mixing with the longitudinal lattice phonons. The sPh contribution is significantly larger than lPhs and the magnitude of the temperature anisotropy generated at deep crustal layers is likely to be limited by superfluid heat conduction. If future observations allow us to establish a clear correlation between surface temperature distribution, magnetic field orientation and age, we may be able to probe the superfluid nature of the NS inner crust.

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