Tension and constraints on modified gravity parametrizations of $G_{\text{eff}}(z)$ from growth rate and Planck data

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We construct an updated and extended compilation of growth rate data based on recent Redshift Space Distortion (RSD) measurements. The dataset consists of 34 datapoints and includes corrections for model dependence. In order to minimize overlap and maximize the independence of the datapoints we also construct a subsample of this compilation (a ‘Gold’ growth dataset) which consists of 18 datapoints. We test the consistency of this dataset with the best fit Planck15/ΛCDM parameters in the context of General Relativity (GR) using the evolution equation for the growth factor $\delta(a)$ with a wCDM background. We find tension at the ~3σ level between the best fit parameters $w$ (the dark energy equation of state), $\Omega_{\text{m}}$ (the matter density parameter) and $\sigma_8$ (the matter power spectrum normalization). We show that the tension disappears if we allow for evolution of the effective Newton’s constant, parametrized as $G_{\text{eff}}(a)/G_N = 1 + g_a(1-a)^n - g_a(1-a)^2$ with $n \geq 2$ where $g_a, n$ are parameters of the model, $a$ is the scale factor and $z = 1/a - 1$ is the redshift. This parametrization satisfies three important criteria: a. Positive energy of graviton ($G_{\text{eff}} > 0$), b. Consistency with Big Bang Nucleosynthesis constraints ($G_{\text{eff}}(a < 1)/G_N = 1$), c. Consistency with solar system tests ($G_{\text{eff}}(a = 1)/G_N = 1$ and $G_{\text{eff}}'(a = 1)/G_N = 0$). We show that the best fit form of $G_{\text{eff}}(z)$ obtained from the growth data corresponds to weakening gravity at recent redshifts (decreasing function of $z$) and we demonstrate that this behaviour is not consistent with any scalar-tensor Lagrangian with a real scalar field. Finally, we use MGCAMB to find the best fit $G_{\text{eff}}(z)$ obtained from the Planck CMB power spectrum on large angular scales and show that it is a mildly increasing function of $z$, in 3σ tension with the corresponding decreasing best fit $G_{\text{eff}}(z)$ obtained from the growth data.

I. INTRODUCTION

Despite of the vast improvement in quality and quantity of the cosmological observations during the past 18 years, the simplest cosmological model predicting an accelerating expansion of the Universe, known as the ΛCDM [1], has remained viable and consistent with observations [2–4]. Crucial assumptions of this model are the validity of General Relativity (GR) on cosmological scales, flatness homogeneity, isotropy and the invariance of dark energy in both space and time (cosmological constant). The parameters of this model have been pinned down to extraordinary accuracy by the Planck [5] mission. These parameter values define the concordance Planck15/ΛCDM model and are shown in Table I. This model is consistent with a wide range of independent cosmological observations testing mainly the large scale cosmological background $H(z)$. Such observations include earlier analyses of cosmic microwave background (CMB) fluctuations [6], large scale velocity flows [7], baryon acoustic oscillations [8, 9], Type Ia supernovae [10], early growth rate of perturbations [11–14], gamma ray burst data [15–17], strong and weak lensing data [18], $H(z)$ (Hubble parameter) data [19], HII galaxy data [20], cluster gas mass fraction data [21, 22].

Despite of the consistency of Planck15/ΛCDM with large cosmological scales background data, it has become evident recently that a mild tension appears to exist between Planck15/ΛCDM and some independent observations in intermediate cosmological scales ($z \leq 0.6$) [23]. Such tensions include estimates of the Hubble parameter [24–28] in the context of ΛCDM, estimates of the amplitude of the power spectrum on the scale of $8h^{-1}$Mpc ($\sigma_8$ [1] and estimates of the matter density parameter $\Omega_{\text{m}}$ [29].

In addition, there are theoretical arguments based on naturalness that may hint towards physics beyond the concordance ΛCDM model [2–4].

The data that are in some tension with Planck15/ΛCDM appear to indicate consistently that there is a lack of gravitational power in structures on intermediate-small cosmological scales. This lack of power may be expressed through different cosmological parameters in a degenerate manner. For example it may be expressed as a lower value of $\Omega_{\text{m}}$ at redshifts less than about 0.6, or as smaller value of $\sigma_8$ or as a dark energy equation of state that becomes smaller than −1 at low redshifts.

The situation is reminiscent of the corresponding situation in the early 90's before the confirmation of ΛCDM by...
factor $b$ divided values of the growth rate in terms of the galaxy density surveys can provide measurements of the perturbations identified in galaxy redshift surveys. In general such peculiar velocities obtained from RSD measurements [48] and $b$ matter perturbations through the bias parameter $z$. This growth rate has been measured in several surveys in redshifts today. Three such possible mechanisms are the following

- A Hot Dark Matter component induced e.g. by a sterile neutrino[45]

- Dark matter clusters differently at small and large scales, a possibility explored in [46].

- Modifications of GR[47] which attenuate the growth rate of perturbations.

In the present study we focus on the third mechanism. If a modification of GR is responsible for the observed cosmological accelerating expansion it would also lead to a modified growth rate of cosmological density perturbations compared to the one predicted in GR. This growth rate has been measured in several surveys in redshifts ranging from $z = 0.02$ up to $z = 1.4$ and is defined as

$$f(a) = \frac{d\delta(a)}{d\ln a}$$

where $\delta(a) \equiv \frac{\delta_0}{a}$ denotes the cosmological overdensity and $a(t)$ is the scale factor.

Most growth rate measurements are obtained using peculiar velocities obtained from RSD measurements [48] identified in galaxy redshift surveys. In general such surveys can provide measurements of the perturbations in terms of the galaxy density $\delta_g$, which are related to matter perturbations through the bias parameter $b$ as $\delta_g = b \delta_m$. Thus early growth rate measurements provided values of the growth rate $f$ divided by the bias factor $b$ leading to the parameter $\beta f$.

This measured parameter is sensitive to the value of the bias $b$ which can vary in the range $b \in [1,3]$. This uncertainty factor makes it difficult to combine values of $\beta$ from different regions and different surveys leading to unreliable datasets of $\beta(z_i)$.

A more reliable combination is the product $f(z)\sigma_8(z) \equiv f\sigma_8(z)$, as it is independent of the bias, and may be obtained using either weak lensing or RSD. Thus in the present study we only consider surveys that have reported the growth rate in the robust form $f(z)\sigma_8(z)$. These surveys along with the corresponding datapoints are shown in Table II where the data are shown in chronological order, along with the assumed fiducial cosmology and other notes, e.g. their covariance matrix and so on.

Some of these points are in fact highly correlated with other points since they were produced by analyses of the same sample of galaxies. Also, it is clear from Table II that there has been a dramatic increase and improvement of the growth rate data during the past five years. This is mainly due to the SDSS, BOSS, WiggleZ and Vipers surveys that have dramatically increased the number of growth rate data and their constraining power. The quality and quantity of the growth rate data is expected to improve dramatically in the coming years with the Euclid [49] and LSST [50] surveys.

Despite of the dramatic improvement of the quality and quantity of the growth rate data their combination into a single uniform and self-consistent dataset remains a challenge. There are two basic reasons for this:

- Model Dependence: Since surveys do not measure distances to galaxies directly, they have to assume a specific cosmological model in order to infer distances. All growth rate datapoints shown in Table II assume a flat ΛCDM cosmological background albeit with different $\Omega_m$ and/or $\sigma_8$. The actual values of these parameters used for each datapoint are shown in Table II. This model dependence requires a correction before the data are included in a single uniform dataset.

- Double Counting: Some of the data points shown in Table II correspond to the same sample of galaxies analyzed by different groups/methods and the inclusion of all these points without proper corrections would lead to double-counting and artificial decrease of the error regions.

In the present analysis we address the above issues and construct a new large, uniform and reliable growth rate dataset which consists of independent datapoints that are corrected for model dependence by rescaling growth rate measurements by proper ratios of $H(z)DA(z)$ where $DA(z)$ is the angular diameter distance. We use this dataset to investigate the tension level with a Planck15/ΛCDM background model under the assumption of validity of GR.

The tension we find can be eliminated by either changing the background Hubble parameter $H(z)$ or by al-
TABLE I. Planck15/ΛCDM parameters with 68% limits. Based on TT,TE,EE+lowP and a flat ΛCDM model (middle column) or a wCDM model (right column), see Table 4 of [5] and the Planck chains archive a.

| Parameter | Value (ACDM)       | Value (wCDM)       |
|-----------|--------------------|--------------------|
| Ωb h^2   | 0.02225 ± 0.00016  | 0.02229 ± 0.00016  |
| Ωc h^2   | 0.1198 ± 0.0015    | 0.1196 ± 0.0015    |
| n_s      | 0.9645 ± 0.0049    | 0.9649 ± 0.0048    |
| H_0      | 67.27 ± 0.66       | 81.3               |
| Ω_m     | 0.3156 ± 0.0091    | 0.203±0.022        |
| w        | -1                 | -1.55±0.19         |
| σ_s      | 0.831 ± 0.013      | 0.983±0.003        |

a A pdf describing the data contained in the Planck archive can be found here [https://wiki.cosmos.esa.int/planckpla2015/index.php/Cosmological_Parameters](https://wiki.cosmos.esa.int/planckpla2015/index.php/Cosmological_Parameters)

Following modifications of GR through a scale independent effective Newton’s constant G_{eff}(z). We follow that later route and assuming that the Planck15/ΛCDM background is correct, we find the best fit form of G_{eff}(z) using the Planck15/ΛCDM H(z) and our growth rate dataset. The derivation of the best fit effective Newton’s constant along with the Planck15/ΛCDM H(z) allows the reconstruction of the underlying fundamental model Lagrangian density in the context of specific classes of models.

A general and generic such class of models is scalar-tensor theories where the action in the Jordan frame is determined by the scalar field potential U(φ) and the nonminimal coupling F(φ) in the form

\[ S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} F(φ) R - \frac{1}{2} Z(φ) g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi - U(φ) \right] + S_m, \]

where R is the Ricci scalar. We have set 8πG_N ≡ 1 for simplicity (and therefore F_0 = 1 at the present time) and S_m is the matter action of arbitrary matter fields, i.e. does not involve the scalar field φ. Even though the scalar field is fully described by the set of F(φ), Z(φ) and U(φ), a convenient reduction to two parameters can be applied (e.g. [51–53]) by a rescaling of the scalar field. For example we may have the Brans-Dicke reduction where F(φ) = φ, Z(φ) = ω(φ)/φ or alternatively we can obtain Z(φ) = 1 with arbitrary F(φ) as done in the present analysis. We note that all of the above are applied in the Jordan frame where the model is studied. In addition, F(φ) > 0 is required so that gravitons have positive energy and dF/dφ < 4 × 10^{-4} according to solar system tests, (see Ref. [52, 54]). As discussed in section IV, the effective Newton’s constant G_{eff}(z) is approximately inversely proportional to the non-minimal coupling F(φ(z)) and is observable through the growth of cosmological perturbations.

It is thus possible to use the best fit form of G_{eff}(z) along with the Planck15/ΛCDM H(z) to reconstruct the underlying scalar-tensor theory potential U(φ) that would produce the observed functional forms of G_{eff}(z) and H(z). This scalar-tensor theory is defined by the functional forms of the scalar field potential U(φ) and non-minimal coupling F(φ) that are reconstructed uniquely using the method of Refs [51, 52, 55]. However, as also noted in Ref. [52], this task is not always possible as the reconstructed kinetic term of the scalar field in many cases becomes negative at some redshift range z, i.e. φ′(z)^2 < 0 and as a result the field itself becomes imaginary. In what follows we derive the properties of functions F(z) that lead to positive kinetic terms for a real scalar field when used in a reconstruction. These properties come from the fact that F(z) satisfies a differential inequality and we can deduce them by using the Chaplygin theorem on differential inequalities.

The structure of this paper is the following: In the next section we introduce the new robust and extended growth dataset (Table III) and use it to investigate the tension level between growth data and Planck15/ΛCDM in the context of GR. In section III we allow for extensions of GR and introduce G_{eff}(z) parametrizations consistent with solar system tests and nucleosynthesis. We then find the best fit form of G_{eff}(z) for each parametrization and investigate the effect of the evolving Newton’s constant on the tension between growth data and Planck15/ΛCDM. In section IV we use the best fit forms of G_{eff}(z) to implement the reconstruction method for the derivation of the underlying scalar-tensor potential. We find that for the particular form of the best fit G_{eff}(z), no consistent reconstruction of a realistic scalar-tensor model can be implemented due to the fact that the kinetic term of the scalar field becomes negative, i.e. φ′(z)^2 < 0. Then by using the Chaplygin theorem on differential inequalities we derive the required properties of the observed G_{eff}(z) in the context of a ΛCDM background so that a well defined scalar-tensor theory can be reconstructed. In section V we determine the effects of the G_{eff}(z) parametrization on the low-ℓ multipoles of the CMB, while in section VI we conclude, summarize and discuss future extensions of the present work.

II. EXTENDED CALIBRATED GROWTH RSD DATASET: TENSION WITH PLANCK15/ΛCDM

II.1. Theoretical Background

In order to discriminate between GR and modified gravity theories we need an extra observational probe which can track the dynamical properties of gravity. One such probe is the growth function of the linear matter density contrast δ ≡ \frac{δρ_m}{ρ_m}, where ρ_m represents the background matter density and δρ_m its first order perturbation.

It can be shown that in many classes of modified gravity theories the growth factor δ(a) satisfies the following
\begin{equation}
\delta''(a) + \left( \frac{3}{a} + \frac{H'(a)}{H(a)} \right) \delta'(a) - \frac{3}{2} \Omega_m G_{\text{eff}}(a, k)/G_N \delta(a) = 0,
\end{equation}

where primes denote differentiation with respect to the scale factor and \(H(a) \equiv \frac{\dot{a}}{a}\) is the Hubble parameter and \(G_{\text{eff}}(a, k)\) is the effective Newton’s constant which is constant and equal to \(G_N\) in GR. In modified gravity theories \(G_{\text{eff}}\) depends on both the scale factor \(a\) (or equivalently the redshift \(z\)) and the scale \(k\). However, \(G_{\text{eff}}\) is independent of the scale \(k\) for scales smaller than the horizon \((k \gg aH)\) [80]. Thus on subhorizon scales we may ignore the dependence on the scale \(k\) for both \(\delta\) and \(G_{\text{eff}}\).

For the growing mode we assume the initial conditions \(\delta(a \ll 1) = a\) and \(\delta'(a \ll 1) = 1\), where in practice we will choose a small enough value of the scale factor so that we are well within the matter domination era, e.g. \(a_{\text{ini}} \sim 10^{-3}\). Note that this equation is only valid on sub-horizon scales, i.e. \(k^2 \gg a^2 H^2\) where \(k\) is the wave-number of the modes of the perturbations in Fourier space. The effects of modified gravity theories enter Eq. (2.1) via both \(H(a)\) and \(G_{\text{eff}}(a, k)\). This is due to the fact that the growth of large scale structure is a result of the motion of matter and therefore is sensitive to both the expansion of the Universe and the evolution of Newton’s “constant”.

In the case of GR, the exact solution of Eq. (2.1) for a flat model with a constant dark energy equation of state \(w\) is given for the growing mode by [81, 82]:

\begin{equation}
\delta(a) = a \cdot 2F_1 \left( -\frac{1}{3w}; \frac{1}{2} - \frac{1}{2w}; \frac{1}{1 - \frac{5}{6w}}; a^{-3w(1 - \Omega^{-1}_m)} \right),
\end{equation}

where \(2F_1(a; b; c; z)\) is a hypergeometric function defined by the series

\begin{equation}
2F_1(a; b; c; z) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)n!} z^n
\end{equation}
on the disk $|z| < 1$ and by analytic continuation elsewhere (see Ref. [83] for more details). In general, it is impossible to find analytical solutions to Eq. (2.1) for a generic modified gravity model, so numerical methods for solving it have to be used.

As discussed in the Introduction, a robust measurable quantity in redshift surveys is not the growth factor $\delta(a)$. Instead, it is the combination

$$f\sigma_8(a) = f(a) \cdot \sigma(a)$$

$$= \frac{\delta_8}{\delta(1)} \cdot a \cdot \delta'(a), \quad (2.4)$$

where $f(a) = \frac{d\ln a}{d\ln a}$ is the growth rate and $\sigma(a) = \sigma_8 \cdot \frac{\delta_8}{\delta(1)}$ is the redshift-dependent rms fluctuations of the linear density field within spheres of radius $R = 8h^{-1}$Mpc while the parameter $\sigma_8$ is its value today. This combination is used in what follows to derive constraints for theoretical model parameters.

### II.2. RSD Measurements

Redshift-space distortions are very important probes of large scale structure providing measurements of $f\sigma_8(a)$. This can be achieved by measuring the ratio of the monopole and the quadrupole multipoles of the redshift-space power spectrum which depends on $\beta = f/b$, where $f$ is the growth rate and $b$ is the bias, in a specific way defined by linear theory [56, 84, 85]. The combination of $f\sigma_8(a)$ then is bias-free as both $f(a)$ and $\sigma_8(a)$ have a dependence on bias which is the inverse of the other, thus cancels out, and it has been shown to be a good discriminator of DE models [56].

In Table II we present a collection of recent $f\sigma_8(z)$ measurements from different surveys, ordered chronologically. In the columns we show the name and year of the survey that made the measurement, the redshifts to distances, an important step in the derivations of the data. This can be corrected by either taking into account how the correlation function $\xi(r)$ transforms by changing the cosmology, an approach followed by Ref. [86], or by simply rescaling the growth-rate measurements by the ratios of $H(z)D_A(z)$ of the cosmology used to that of the fiducial one as in Ref. [87]. As noted in [87], the correction itself is quite small, so we follow the latter method for simplicity.

### TABLE III. A compilation of robust and independent $f\sigma_8(z)$ measurements from different surveys, based on Table II.

In the columns we show in ascending order with respect to redshift, the name and year of the survey that made the measurement, the redshift and value of $f\sigma_8(z)$ and the corresponding reference and fiducial cosmology. These datapoints are used in our analysis in the next sections.

| Index | Dataset           | $f\sigma_8(z)$ | Refs. | Year | Notes |
|-------|-------------------|----------------|-------|------|-------|
| 1     | 6dFGS+SnLS        | 0.02 ± 0.065   | [72]  | 2016 | $(\Omega_m, h, \sigma_8) = (0.3, 0.683, 0.8)$ |
| 2     | SnLS+IRAS         | 0.02 ± 0.065   | [59],[58] | 2011 | $(\Omega_m, \Omega_K) = (0.3, 0)$ |
| 3     | 2MASS             | 0.02 ± 0.048   | [57],[58] | 2010 | $(\Omega_m, \Omega_K) = (0.266, 0)$ |
| 4     | SDSS+veloc        | 0.10 ± 0.130   | [70]  | 2015 | $(\Omega_m, \Omega_K) = (0.3, 0)$ |
| 5     | SDSS+MGS          | 0.15 ± 0.145   | [69]  | 2014 | $(\Omega_m, h, \sigma_8) = (0.31, 0.67, 0.83)$ |
| 6     | 2dFGRS            | 0.17 ± 0.060   | [56]  | 2009 | $(\Omega_m, \Omega_K) = (0.3, 0)$ |
| 7     | GAMA              | 0.18 ± 0.090   | [65]  | 2013 | $(\Omega_m, \Omega_K) = (0.27, 0)$ |
| 8     | GAMA              | 0.38 ± 0.060   | [65]  | 2013 | $(\Omega_m, \Omega_K) = (0.25, 0)$ |
| 9     | SDSS-LRG-200      | 0.25 ± 0.0583  | [60]  | 2011 | $(\Omega_m, \Omega_K) = (0.24, 0)$ |
| 10    | SDSS-LRG-200      | 0.37 ± 0.0378  | [60]  | 2011 | $(\Omega_m, h, \sigma_8) = (0.37115, 0.6777, 0.8288)$ |
| 11    | BOSS-LOWZ         | 0.32 ± 0.095   | [60]  | 2013 | $(\Omega_m, h, \sigma_8) = (0.27, 0.71)$ |
| 12    | SDSS-CMASS        | 0.50 ± 0.060   | [67]  | 2013 | $(\Omega_m, h, \sigma_8) = (0.3, 0.045)$ |
| 13    | WiggleZ           | 0.44 ± 0.080   | [61]  | 2012 | $C_{ij} \rightarrow$ Eq. (2.8). |
| 14    | WiggleZ           | 0.60 ± 0.063   | [61]  | 2012 | $C_{ij} \rightarrow$ Eq. (2.8). |
| 15    | WiggleZ           | 0.73 ± 0.072   | [61]  | 2012 | $C_{ij} \rightarrow$ Eq. (2.8). |
| 16    | Vipers PDR-2      | 0.60 ± 0.120   | [73]  | 2016 | $(\Omega_m, h, \sigma_8) = (0.3, 0.045)$ |
| 17    | Vipers PDR-2      | 0.86 ± 0.110   | [73]  | 2016 | $(\Omega_m, h, \sigma_8) = (0.3, 0.045)$ |
| 18    | FastSound         | 1.40 ± 0.116   | [71]  | 2015 | $(\Omega_m, \Omega_K) = (0.270, 0)$ |
Specifically, we implement the correction as follows: First, we define the ratio of the product of the Hubble parameter \( H(z) \) and the angular diameter distance \( d_A(z) \) for the model at hand to that of the fiducial cosmology, \( i.e. \)

\[
\text{ratio}(z) = \frac{H(z)d_A(z)}{H_{fid}(z)d_A^{fid}(z)},
\]

(2.5)

where the values of the fiducial cosmology, namely \( \Omega_{0m} \), are given in Table III. Note that the combination \( H(z)d_A(z) \) does not depend on \( H_0 \), so it could be equivalently written in terms of the dimensionless Hubble parameter \( E(z) = H(z)/H_0 \) and angular diameter distance \( D_A(z) = \frac{H_0}{c}d_A(z) \).

Having done this, we can now define the \( \chi^2 \) as usual for correlated data. We can define a vector \( V^i(z_i,p^j) \), where \( z_i \) is the redshift of \( i \)th point and \( p^j \) is the \( j \)-th component of a vector containing the cosmological parameters \( (\Omega_{0m}, w, \sigma_8 ... ) \) that we want to determine from the data. This vector contains the differences of the data and the theoretical model, after we implement our correction. Specifically, it is given by:

\[
V^i(z_i, p^j) = f\sigma_{8,i} - \text{ratio}(z_i)f\sigma_8(z_i, p^j)
\]

(2.6)

where \( f\sigma_{8,i} \) is the value of the \( i \)th datapoint, with \( i = 1, \ldots, N \) where \( N \) is the total number of points, while \( f\sigma_8(z_i, p^j) \) is the theoretical prediction, both at redshift \( z_i \).

Then, the \( \chi^2 \) can be written as:

\[
\chi^2_{\text{growth}} = V^i C^{-1} \text{growth} V^j,
\]

(2.7)

where \( C^{-1} \text{growth} \) is the inverse covariance matrix of the data and for compactness we only used the superscripts \( i, j \) for the data vectors. As an approximation we will assume that most of the data are not correlated, with the exception of the ones from Wigglez, where the covariance matrix is given by [61]

\[
C_{ij}^{\text{WiggleZ}} = 10^{-3} \begin{pmatrix}
6.400 & 2.570 & 0.000 \\
2.570 & 3.969 & 2.540 \\
0.000 & 2.540 & 5.184
\end{pmatrix}.
\]

(2.8)

Therefore, the total covariance matrix will be the identity \( N \times N \) matrix, but with the addition of a \( 3 \times 3 \) matrix at the position of the WiggleZ data, \( i.e. \) schematically we could write it as

\[
C_{ij}^{\text{growth, total}} = \begin{pmatrix}
1 & 0 & 0 & \cdots \\
0 & C_{ij}^{\text{WiggleZ}} & 0 & \cdots \\
0 & 0 & 1 & \cdots
\end{pmatrix}.
\]

(2.9)

An alternative approach would be that of Ref. [86] where the authors approximated the total covariance matrix of all the measurements as the fraction of overlap volume between the surveys to the total volume of the two surveys combined. However, this approach obviously cannot take into account any possible negative correlations between the data as the effect of the correlations can be due to more than the overlapping survey volumes. Thus this approach can lead to a potentially biased covariance matrix. This issue will be resolved in the near future when upcoming surveys like Euclid and LSST will provide consistent growth-rate measurements in both the low and high redshift regime.

Using the corrected \( \chi^2 \) and our “Gold-2017” compilation given by Table III, we now proceed to extract the best-fit cosmological parameters and discuss the results. First, we assume GR with a constant \( w \) model and a flat Universe. Then, the Hubble parameter is given by

\[
E(a)^2 = \frac{H(a)^2}{H_0^2} = \Omega_{0m}a^{-3} + (1 - \Omega_{0m})a^{-3(1+w)},
\]

(2.10)

where we have ignored the radiation as at late times it has a negligible impact. This case is rather simple, so in order to speed up the code it is convenient to use the analytical expression for the growing mode of the growth factor given by Eq. (2.2) and the analytical expression for the luminosity distance, which follows after a quick calculation using the definition, given by

\[
\frac{H_0}{c} d_L(a) = \frac{2}{\sqrt{\Omega_{0m}}} \int_0^a \kappa \left( \frac{1}{2} \frac{1}{1 - \frac{1}{6}w} : 1 - \frac{1}{\Omega_{0m}} \right) - \frac{2}{\sqrt{\kappa \Omega_{0m}}} \int_0^a \kappa \left( \frac{1}{2} \frac{1}{1 - \frac{1}{6}w} : 1 - \frac{1}{\Omega_{0m}(a)} \right),
\]

(2.11)

where \( \Omega_{0m}(a) = \frac{\Omega_{0m}a^{-3}}{E(a)^2} \) and then the angular diameter distance is given by \( d_A(z) = \frac{d_L(z)}{\sqrt{1+z}} \) as usual. In the more complicated cases discussed in the next sections, \( e.g. \) modified gravity models, we will perform the corresponding calculations numerically.

After fitting the data we obtain the 68.3%, 95.4% and 99.7% confidence contours in the \( (w, \sigma_8, \Omega_{0m}) \) parameter space, shown in Fig. 1. As it can be seen, the current growth rate data are at a \( \sim 3\sigma \) tension with the Planck15/\( \Lambda \)CDM best-fit cosmology, indicated with the red dot. For completeness we also overlap the corresponding Planck15/\( w \)CDM contours even though our goal here is to identify the tension level with Planck15/\( \Lambda \)CDM. We will attempt to alleviate this tension in the next section, by considering modified gravity models, as the extra degrees of freedom provided by the theories may allow a Newton’s constant of the form \( G_{\text{eff}}(a,k) \) to account for the tension.

Remarkably, we find that compared to previous studies, \( e.g. \) [88] or even the Planck 2015 data release [5], all of which use outdated growth data, with our new ‘Gold-2017’ compilation we identify a 3\( \sigma \) tension. Given the Planck15/\( \Lambda \)CDM background and the fact that we have corrected for the diverse fiducial cosmologies used, this tension could potentially be explained either by assuming that the growth rates \( f\sigma_8 \) suffer from an unaccounted for as yet systematic or by new physics perhaps affecting
of this model from ΛCDM is more transparent and second, by performing a Taylor expansion around \( b = 0 \) we can obtain analytical approximations for \( H(z) \) which are accurate to better than 0.1% for \( b \lesssim 1 \) and better than \( 10^{-5}\% \) for \( b \lesssim 0.1 \). The Lagrangian for the Hu and Sawicki model, as written equivalently in Ref. [90], is:

\[
 f(R) = R - \frac{2\Lambda}{1 + (\frac{2\Lambda}{R})^n} \quad (3.4)
\]

where \( n \) is a constant of the model, usually chosen as \( n = 1 \) without loss of generality as it only adjusts the steepness of the deviation from the ΛCDM model.

As mentioned we can also obtain a very accurate Taylor expansion of the solution to the equations of motion around \( b = 0 \), i.e. the ΛCDM model, as

\[
 H^2(a) = H^2_\Lambda(a) + \sum_{i=1}^{M} b^i \delta H^2_i(a), \quad (3.5)
\]

where

\[
 \frac{H^2_\Lambda(a)}{H^2_0} = \Omega_{0m}a^{-3} + \Omega_{r0}a^{-4} + (1 - \Omega_{0m} - \Omega_{r0}) \quad (3.6)
\]

and \( M \) is the number of terms we keep before truncating the series. However, we have found that keeping only the two first non-zero terms are more than enough to have better than 0.1% accuracy with the numerical solution. The functions \( \delta H^2_i(a) \) are just algebraic expressions and can be easily determined from the equations of motion (see Ref. [90]). Finally, we also follow Ref. [90] and set \( k = 0.1\mathrm{Mpc}^{-1} \sim 300H_0 \), which is necessary as now the Newton’s constant depends on the scale \( k \) as well.

One can generalize the above model to an action that includes a scalar field with arbitrary kinetic term non-
minimally coupled to gravity\(^1\). Such a model has the following action [77]:

\[
S = \int d^4x \sqrt{-g} \left( \frac{1}{2} f(R, \phi, X) + \mathcal{L}_m \right),
\]

where \(X = -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi\) is the kinetic term of the scalar field. In this case, Newton’s constant is given by [77]:

\[
G_{\text{eff}}(a, k)/G_N = \frac{f_X}{F} f_X + 3 \left( \frac{f_X}{F^2} + \frac{F_X}{F} \right).
\]

Clearly, the gravitational slip and the anisotropic stress are related via \(\eta = \frac{1}{G_{\text{eff}}} - 1\) and in GR we have that \(\gamma_{\text{slip}} = 1\) and \(\eta = 0\). In Ref. [77] it was shown that in scalar-tensor theories the anisotropic stress is given by:

\[
\eta = \frac{F_{\phi}^2}{F_{\phi} + F_{\phi}^2}
\]

and Eq. (3.15) implies that the quantities \(G_L\) and \(G_M\) are related via

\[
G_L = \frac{1}{2} \eta + 1, G_M = \eta - 1.
\]

In order to have agreement with the solar system tests viable models must satisfy \(F_{\phi} \simeq 0\) at \(z \simeq 0\), which from Eqs. (3.15) and (3.16) implies that \(\eta \simeq 0\). Similarly, from (3.17) we infer that at \(z \simeq 0\) \(G_L \simeq G_M \simeq 1\) and \(\gamma_{\text{slip}} \simeq 1\).

Any of the above quantities \(G_M, G_L, \gamma_{\text{slip}}\) or \(\eta\) can be used to construct null test for GR. Alternative approaches like the growth index [92–99] can also be used for parametrizing deviations from GR. However, they are not as efficient in distinguishing the effects of the background \(H(z)\) from the effects of modified gravity since they do not enter explicitly in the dynamical growth equations.

In the present analysis we focus on \(G_M = G_{\text{eff}}/G_N\) to parametrize deviations from GR since this is the only quantity that enters directly in the dynamical equation that determines the growth of density perturbations (eq (2.1)). We thus use the parametrization:

\[
\frac{G_{\text{eff}}(a, n)}{G_N} = 1 + g_a (1 - a)^n - g_a (1 - a)^{2n}
\]

\[
= 1 + g_a \left( \frac{z}{1 + z} \right)^n - g_a \left( \frac{z}{1 + z} \right)^{2n}
\]

Clearly, this parametrization mimics the large \(k\) limit of the above models, \(i.e.,\) scales small compared to the horizon, which is a reasonable approximation even for large surveys. In addition, the parametrization (3.18) may be viewed as an extended Taylor expansion around \(a = 1\) for a fixed number of two parameters. The second term describes \(G_{\text{eff}}\) for low and intermediate values of \(z\), while the third term for larger values of \(z\). A similar parametrization concerning the dark energy equation of state was introduced in Ref. [100]. The parametrization (3.18) is only viable for \(n \geq 2\) due to the solar system tests that demand that the first time derivative of \(G_{\text{eff}}\) should be zero. Therefore, in what follows, we will focus on values of \(n\) with \(n \geq 2\).

Furthermore, this parametrization is motivated by considering that any viable modified gravity model must satisfy the following conditions:

- \(G_{\text{eff}} > 0\) in order for the gravitons to carry positive energy;
- \(G_{\text{eff}}/G_N = 1.09 \pm 0.2\) to be in agreement with the Big Bang Nucleosynthesis,
Note that for $g_a < 0$, the parametrization of Eq. (3.18) has a minimum at $a = a_{\text{eff}, \min} = 1 - 2^{1/n}$ with a value of $G_{\text{eff}}/G_N = 1 + \frac{1}{2^n}$, hence in order to have $G_{\text{eff}} > 0$ we need $g_a > -4$ and as we will see later on, the best-fit for various $n$ satisfies that.

For a similar reason GR-quintessence does not allow crossing of the phantom divide line $w = -1$ as this crossing would require a change of sign of the scalar field kinetic term.
TABLE IV. The best fit values of \( g_n \) with errors bars for \( n = 2, 3, \ldots, 6 \). As we describe in Appendix B, this parametrization has several distinct minima, but here we show only the global one when both \( g_{n} \) and \( n \) are free (first row) and then for integer values of \( n = 2, \ldots, 6 \), the minima corresponding to the lowest \( g_{n} \) which are also the global ones for low values of \( n \).

| \( n \) | \( g_{n} \) |
|-------|---------|
| 0.343 | \(-1.200 \pm 1.025\) |
| 2     | \(-1.156 \pm 0.341\) |
| 3     | \(-1.334 \pm 0.453\) |
| 4     | \(-2.006 \pm 0.538\) |
| 5     | \(-2.542 \pm 0.689\) |
| 6     | \(-3.110 \pm 0.771\) |

alleviating the tension found between the growth rate data and the Planck15/ΛCDM best fit, reducing it from \( \sim 3\sigma \) to less than \( 1\sigma \), thus offering potential hints for new physics.

As mentioned above, the consistency of any modified gravity model with the solar system tests is paramount as they place stringent constraints on the evolution of \( G_{\text{eff}} \). Hence viable models like the Hu and Sawicki model [89] that evade them are effectively small perturbations around the ΛCDM (see e.g. Eq. (3.18)). From a phenomenological point of view it is also interesting to consider direct parametrizations of \( G_{\text{eff}} \) like the one of Eq. (3.18). Such a consideration leads to the following question: Are the best forms of \( G_{\text{eff}} \) able to lead to a reconstruction of self consistent scalar-tensor quintessence with the Planck15/ΛCDM background? We will address this question in the next section.

IV. RECONSTRUCTION OF SCALAR-TENSOR QUINTESSENCE

The line element for the FLRW metric corresponding to a flat universe is given by

\[
ds^2 = -dt^2 + a^2(t) \left[ dr^2 + r^2(d\theta^2 + \sin^2 \theta \, d\phi^2) \right]. \quad (4.1)
\]

Using this metric in the action (1.2) and assuming a homogeneous scalar field and a perfect fluid background we find the dynamical equations of the system as

\[
3FH^2 = \rho + \frac{1}{2} \dot{\phi}^2 - 3H \dot{F} + U \quad (4.2)
\]

\[
-2FH' = (\rho + p) + \dot{\phi}^2 + \ddot{F} - H \dot{F} \quad (4.3)
\]

We eliminate the kinetic term \( \dot{\phi}^2 \) in equation (4.3) and we set the squared rescaled Hubble parameter as

\[
q(z) \equiv E^2(z) = \frac{H^2(z)}{H_0^2} \quad (4.4)
\]

while a new rescaling to potential is applied, i.e. \( U \to U \cdot H_0^2 \). We thus obtain the dynamical equations in terms of the redshift \( z \) as

\[
F''(z) + \left[ \frac{q'(z)}{2q(z)} - \frac{4}{1+z} \right] F'(z) + \left[ \frac{6}{(1+z)^2} - \frac{2}{1+z} \right] U(z) - \frac{6}{(1+z)^2 q(z)} = 0 \quad (4.5)
\]

where the differentiation with respect to the redshift \( z \) is denoted by the prime and we have assumed a matter perfect fluid with \( p = 0, \Omega_{\text{om}} = 3\rho_{\text{om}}/H_0^2 \). In addition, equations (4.5),(4.6) satisfy the initial conditions \( \phi(0) = 0, F(0) = 1 \) and \( F'(0) = 0 \) for consistency with solar system tests \( (dF/d\phi) \sim dF/dz \simeq 0 \) [54, 101, 102].

In scalar-tensor theories, the effective Newton’s constant with respect to \( z \) is of the form (see [51])

\[
G_{\text{eff}}(z) = \frac{1}{2} \left[ F + 4 \left( \frac{dF}{d\phi} \right)^2 \right] F \quad (4.6)
\]

where \( G_N \) is the well known Newton’s constant in GR.

Equations (4.5) and (4.6) form the system of equations for \( \{ U(z), \phi'(z) \} \) that can be used for the reconstruction of the theory (derivation of functions \( U(\phi), F(\phi) \)), assuming that the functions \( F(z) \) (or \( G_{\text{eff}}(z) \)) and \( H(z) \) are observationally obtained ([85, 103, 104]). The function \( H(z) \) is well approximated by the Planck15/ΛCDM fit with parameters shown in Table I. The function \( G_{\text{eff}}(z) \) may be obtained using the growth data of Table III in the context of the parametrization (3.18) that satisfies the three basic conditions discussed in the previous section (solar system tests, nucleosynthesis constraints and proper normalization at the present time).

Even after the observational determination of \( G_{\text{eff}}(z) \) and \( H(z) \) the self-consistent reconstruction of a modified theory is not always possible. For example, in the case of a scalar-tensor theory the sign of \( \phi'(z)^2 \) obtained from Eqs. (4.5) and (4.6) may turn out to be negative leading to a complex predicted value of the scalar field. This violates that assumption of a real scalar field on which the theory is based and leads to inconsistencies that may be difficult to overcome.

As shown in Fig. 4 and in Table IV it is clear that the growth data indicate that the gravitational strength
may be a decreasing function of the redshift in the redshift range $[0,0.4]$ compared to its present value. The question that we want to address is the following: Can this weakening effect of gravity be due to an underlying scalar-tensor theory? If the answer is positive then the sign of the reconstructed $\phi'(z)^2$ should be positive so that the scalar field of the theory is real. We will show that a $G_{\text{eff}}$ that is decreasing with redshift at low $z$ is not consistent with positive $\phi'(z)^2$ and therefore this behavior can not be due to an underlying scalar-tensor theory. This is shown numerically in Fig. 5 where we show the reconstructed form of $\phi'(z)^2$ under the assumption of the best fit forms of $G_{\text{eff}}$ ($n = 2, 3, 4, 5, 6$) shown in Fig. 4 and the Planck15/ΛCDM background $H(z)$ obtained with the parameters of Table I. Clearly for all values of $n$ considered $\phi'(z)^2$ is negative for low $z$ leading to an unacceptable scalar-tensor theory.

This result may be generalized analytically as follows: Using Eqs. (4.5) and (4.6) and demanding that $\phi'^2(z) \geq 0$ we obtain

$$F''(z) + \frac{F'(z)}{2q(z)} \frac{q'(z)}{q(z)} + \frac{2}{z+1} F(z) \frac{q'(z)}{(z+1)q(z)} + \frac{3\Omega_m(z+1)}{q(z)} \leq 0,$$

which is a second order differential inequality for $F(z)$.

A useful theorem for dealing with such inequalities is the Chaplygin theorem (see Ref. [105] and Appendix A for details). In order to bring the inequality (4.8) to the form required by the theorem, we first set $F(z) = 1 - \delta f(z)$ and deduce the corresponding inequality for $\delta f(z)$. We then find:

$$\delta f''(z) + \frac{\delta f'(z)}{1+z} \frac{2}{1+z} + \frac{q'(z)}{2q(z)} - \frac{\delta f(z)}{(1+z)q(z)} \frac{q'(z)}{(1+z)q(z)} + \frac{3\Omega_m(1+z)}{q(z)} \geq 0.$$  

By applying the theorem, as described in Appendix A, we find that the inequality (4.8) is satisfied for an ΛCDM background only when $\delta f(z) \geq 0$ or $F(z) \leq 1$ ($G_{\text{eff}}(z)/G_N \geq 1$) for a range that includes all $z \geq 0$, as we found by a numerical analysis.

To summarize, in order to satisfy the inequality (4.8) along with the viability constraints (positive energy for the graviton etc.) and be able to reconstruct the scalar-tensor Lagrangian in a ΛCDM background, we need to have $0 < F(z) \leq 1$ ($G_{\text{eff}}(z)/G_N \geq 1$). This result explains why the reconstruction as seen in Fig. 5 does not work. Every one of these cases has a negative value for $\phi'(z)^2$ at some $z$ and, as seen in Fig. 4, it also has $F(z) > 1$ ($G_{\text{eff}}(z)/G_N < 1$) in some region. Several numerical tests we performed with several models seem to corroborate the result of this theorem. This issue has also been discussed in Ref. [52] even though no general rule was derived for the viability of the reconstruction.

Therefore, we conclude that the only viable models of scalar-tensor theories that can be reconstructed in the context of a ΛCDM background $H(z)$ are the ones where the non-minimal coupling function satisfies $0 < F(z) \leq 1$ for all $z \geq 0$. In the context of the reconstruction analysis we have used the approximation that $G_{\text{eff}} \approx \frac{1}{\phi}$, which we find is valid everywhere except when $\phi'(z)^2$ changes sign.

V. EFFECTS OF $G_{\text{eff}}(z)$ ON THE CMB

In this section we investigate the effects of a redshift dependent $G_{\text{eff}}(z)$ on the CMB spectrum. We anticipate (and verify with MGCAMB below) that $G_{\text{eff}}(z)$ affects only the large angular CMB spectrum scales (low-ℓ) through the Integrated Sachs Wolfe (ISW) effect while smaller scales (the acoustic peaks) depend only on the background $H(z)$ through the angular diameter distance $d_A = \frac{1}{H(z)} \int_0^z \frac{1}{H(z')}dz'$. The ISW effect is significantly affected by the redshift dependence of $G_{\text{eff}}$ because it depends on the time evolution of the potential $\Phi(z)$ which in turn depends on $G_{\text{eff}}$ due to the Poisson equation $\delta k^2 \Phi(k,z) \propto (\delta \zeta \cdot G_{\text{eff}}(k,z))$, where $\delta = \frac{\delta \rho}{\rho}$ is the growth factor.

In Fig. 6 we show a comparison of the theoretically predicted low-ℓ multipoles of the TT part of the CMB spectrum including the ISW effect for the best fit $G_{\text{eff}}$ models (Table IV) (continuous lines left panel). The Planck15 low-ℓ binned $C_\ell^{TT}$ data are also shown. The theoretically predicted spectra were obtained with a modified version of MGCAMB [106] with $G_{\text{eff}}(z)/G_N$ given by (3.18), anisotropic stress $\eta(z) = 0$ and with the parameter values shown in Table IV for $n = 2, 3, 4$ and for $G_{\text{eff}}/G_N = 1$ for GR. The right panel of Fig. 6 shows the theoretically predicted CMB spectra for $n = 2$ and various values of $g_a$.

Clearly the higher the exponent $n$ of our parametrization for $G_{\text{eff}}$, the stronger the ISW effect and it’s devia-
tion from the ΛCDM model. Thus the cases for \( n = 5, 6 \) are not included in Fig. 6 as they are not consistent with the observed CMB power spectrum. As shown in Fig. 4, a higher \( n \) means that gravitational strength varies more rapidly at low \( z \) leading to the stronger ISW effect shown in Fig. 6.

Also, we performed a simple \( \chi^2 \) analysis with the low-\( \ell \) data, where we defined

\[
\chi^2_{\text{low}-\ell} = \sum_{i=1}^{N} \left( \frac{D_{\ell i}^{\text{Pl}} - D_{\ell i}^{\text{th}}}{\sigma_{D_{\ell i}^{\text{th}}}} \right)^2
\]

and \( D_{\ell} = \frac{\ell(\ell+1)}{2\pi} C_{\ell}^{TT} \). In this case, we kept all other parameters except \( g_{a} \) and \( n \) fixed to their Planck15/ΛCDM values. We found that the ΛCDM model (\( n = 0 \) or \( g_{a} = 0 \)) has \( \chi^2_{\text{GR}} = 22.394 \) and the rest of the models have \( \chi^2_{n=2} = 255.683, \chi^2_{n=3} = 723.922 \) and \( \chi^2_{n=4} = 2086.69 \). Thus, these models are strongly disfavoured with respect to ΛCDM due to their rapid variation of \( G_{\text{eff}} \) leading to strong effects on the ISW effect. In the case of fixed \( n \), we find that \( \chi^2_{g_{a}=0.5} = 66.346, \chi^2_{g_{a}=0} = 22.394, \chi^2_{g_{a}=-0.5} = 42.755 \) and \( \chi^2_{g_{a}=-1} = 186.969 \), or in the case of \( g_{a} = -0.5 \) a \( \delta \chi^2 = 29.361 \), corresponding to a 4.1σ deviation. Thus the ISW effect provides significantly stronger constraints on \( G_{\text{eff}}(z) \) than the growth data.

Fig. 7 also show the contours for the \( G_{\text{eff}} \) model in the \((g_{a}, n)\) parameter space based on the low-\( \ell \) TT CMB data (red lines) and the growth rate data (blue lines). The black dashed line at \( g_{a} = 0 \) and the axis at \( n = 0 \) correspond to GR and the ΛCDM model since the last two terms in (3.18) in both cases cancel out. The green, blue and red dots correspond to the best-fit for \( n = 2, i.e. (g_{a}, n) = (-1.156, 2) \), the global minimum for \((g_{a}, n) = (-1.200, 0.343)\) and the minimum for the low-\( \ell \) data, i.e. \((g_{a}, n) = (1.227, 0.091)\), respectively. Clearly there is a strong tension between the best fit growth data and the Planck low-\( \ell \) power spectrum (ISW effect).

Finally, it should be mentioned that the \( \chi^2 \) analysis with the low-\( \ell \) TT CMB data should be interpreted with extreme caution as it neglects the covariances of the data.

In addition all other parameters, such as \( \Omega_m, H_0 \) etc., are fixed to their Planck15/ΛCDM values. Doing a full MCMC and exploring the whole parameter space is left for the future. Thus our analysis indicates that even though the tension between the growth data and the Planck15/ΛCDM background in the context of GR is re-
moved by allowing a redshift evolution of $G_{\text{eff}}(z)$, the required $G_{\text{eff}}(z)$ is not consistent with either scalar-tensor theories or the low-$\ell$ CMB spectrum as determined by the ISW effect.

VI. CONCLUSIONS-DISCUSSION

We presented a collection of 34 growth rate data based on recent RSD measurements obtained from several surveys and studies over the last 10 years. In an effort to maximize robustness and independence of the data we selected 18 of the 34 growth rate data to construct a ‘Gold-2017’ growth rate dataset. Using this dataset we fit a $w$CDM cosmology and find that the best fit parameters $(w, \sigma_8, \Omega_{\text{om}})$ are in $3\sigma$ tension with the corresponding parameters obtained with the Planck15 CMB data in the context of GR. In order to resolve this tension we consider a simple parametrization for $G_{\text{eff}}$ given by Eq. (3.18), we show that the tension in the parameters of the data gets now reduced to the $1\sigma$ level.

Despite of this reduction of the tension between the growth data and the Planck indicated background, this best fit parametrization of $G_{\text{eff}}(z)$ was shown to have two important problems:

1. It is a decreasing function of the redshift and therefore according to general rule whose validity we demonstrated, it can not be supported by a self consistent scalar-tensor theory because it leads to a negative scalar field kinetic term.

2. It predicts a large ISW effect that is not consistent with observed large scale (low-$\ell$) CMB spectrum.

These problems could potentially be resolved by considering more general modified gravity models which can potentially support the derived best fit $G_{\text{eff}}(z)$ such as Hordenski models [107] or bimetric gravity[108]. The tension of the best fit $G_{\text{eff}}(z)$ with the low-$\ell$ CMB spectrum induced by the ISW effect is more difficult to resolve and may indicate either required modifications on the background Planck15/$\Lambda$CDM $H(z)$ or systematics in the growth data.

We have pointed out the need for the construction of optimized, large, self-consistent compilations of the emerging growth data and have made a first attempt in that direction. Our updated ‘Gold-2017’ dataset compilation comes from reliable sources, i.e. major surveys and international collaborations. However, the fact that it consists of only a small amount of points indicates that there is significant potential for improvement. This situation will definitely improve in the coming decade as the Euclid [49] and LSST [50] surveys will release a significant amount of new high quality data points and as a result, very soon we will be able to detect any possible deviations from GR with a high level of confidence.

Numerical Analysis Files: See Supplemental Material at here for the Mathematica files used for the production of the figures, as well as the figures.

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Appendix A: Chaplygin theorem

The Chaplygin theorem [105] states that if $y(x)$ satisfies the $n$th-order differential inequality

$$L[y] \equiv y^n(x) + a_1(x)y^{n-1}(x) + \cdots + a_n(x) > b(x), \quad (A1)$$

where the $a_n(x)$ can be integrated and the function $f(x)$ satisfies the differential equation

$$L(f) = b(x) \quad (A2)$$

with the same initial conditions as Eq. (A1), i.e. $f(x_0) = y(x_0), \ldots, f^{n-1}(x_0) = y^{n-1}(x_0)$, then there is a region $x \in (x_0, x_*)$ such that $y(x) > f(x)$ and $x_*$ is specified by the region for which for every $\xi \in [x_0, x]$ we have $G(s, \xi) \geq 0$, where $G$ satisfies the Green equation with initial conditions

$$L[G] = 0$$

$$G(x = \xi) = \cdots = G^{n-2}(x = \xi) = 0, \quad G^{n-1}(x = \xi) = 1.$$  

By specifying the region where $G \geq 0$ we can thus determine where $y(x) > f(x)$.

Appendix B: Multiple minima

A rather interesting feature that arises by minimizing the $\chi^2$ of the ‘Gold-2017’ dataset using the $G_{\text{eff}}$ parametrization (3.18) is the one of the multiple minima. Specifically, as the number of $n$ increases the more minima we observe. This effect is due to the fact that the solution of the growth rate ODE of Eq. (2.1) contains Bessel functions which have degeneracies in their arguments. In order to keep things simple we will now consider a toy model with $\Omega_m = 1$ and $G_{\text{eff}}/G_N = 1 + g_n(1 - a)^n$, even though this model does not satisfy the viability criteria described in the text. Then, for $n = 1$ and $\Omega_m = 1$ the solution to the differential equation (2.1) is

$$\delta_n=1(a) = c_1 a^{-1/4} J_m \left( \sqrt{6 \ a \ g_n} \right)$$

$$+ c_2 a^{-1/4} J_m \left( \sqrt{6 \ a \ g_n} \right), \quad (B1)$$

where
while for $n = 2$ we have

$$
\delta_{n=2}(a) = e^{-\frac{1}{2} \beta a} a^{\frac{m-1}{2}} \left( c_1 U \left( \frac{1}{2} (m - \beta + 1), m + 1, a\beta \right) + c_2 L_n^m(\beta - m - 1)(a\beta) \right),
$$

where $c_{1,2}$ are constants to be determined for the growing and decaying mode, $m = \frac{1}{2} \sqrt{24g_n + 25}$, $\beta = \sqrt{6g_n}$ and $J_{\pm m} (z)$, $U(\kappa_1, \kappa_2, z)$ and $L_n^m$ are the BesselJ, the confluent hypergeometric $U$ and the Laguerre-$L$ functions respectively. As can be seen in this case, the presence of these functions in the solution of the growth rate is the root cause of the multiple minima since the variable $g_n$ of the model appears both in the order and the argument of the functions. As a result, the growth will be degenerate with respect to $g_n$, i.e. for many different values of $g_n$ we will have the same growth factor. Similar arguments can be made for any value of $\Omega_m$, since the case studied here ($\Omega_m = 1$) is just a limit of the $\Lambda$CDM model, but also for $G_{\text{eff}}$ models, other than the one used in the main analysis.

Note that the first minimum is not always the global one, i.e. the minimum with the smallest value of $\chi^2$. In Fig. 8 we show the $\chi^2(g_n)$ plot corresponding to our parametrization (3.18) for $n = 2, 3, 4$, where for $n = 2$ the first minimum is the global one, e.g. for $n = 3$ the first minimum corresponds to $(\chi^2 = 14.3, g_n = -1.534)$, while the second one to $(\chi^2 = 14.6, g_n = -11.14)$. On the other hand for $n = 4$ the three first minima from right to left correspond to $(\chi^2 = 14.9, g_n = -2.006), (\chi^2 = 12.6, g_n = -10.56)$ and $(\chi^2 = 14.8, g_n = -21.87)$ respectively and therefore the global minimum is the second one (albeit with a small difference). However, since $g_n$ must be small we consider only the first minimum, which for small values of $n$ is the global one as well.

FIG. 8. Plots of the $\chi^2(g_n)$ for $n = 2, 3, 4$ that clearly show the degeneracy of the model and the many minima of the $\chi^2$ in terms of $g_n$.
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