Quantum Gravity: physics from supergeometries

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Abstract

We show that the metric (line element) is the first geometrical object to be associated to a discrete (quantum) structure of the spacetime without necessity of black hole-entropy-area arguments, in sharp contrast with other attempts in the literature. To this end, an emergent metric solution obtained previously in [Physics Letters B 661, 186-191 (2008)] from a particular non-degenerate Riemmanian superspace is introduced. This emergent metric is described by a physical coherent state belonging to the metaplectic group $Mp(n)$ with a Poissonian distribution at lower $n$ (number basis) restoring the classical thermal continuum behaviour at large $n$ ($n \to \infty$), or leading to non-classical radiation states, as is conjectured in a quite general basis by mean the Bekenstein-Mukhanov effect. Group-dependent conditions that control the behavior of the macroscopic regime spectrum (thermal or not), as the relationship with the problem of area / entropy of the black hole are presented and discussed.
I. INTRODUCTION

The unification of gravity and quantum theory is one of the great challenges of physics. The last years were dominated by attempts to reach this goal by rather radical new concepts, as is exemplified by the string theory and loop quantum gravity. Then, one of the main points treated in the current literature is the close relation between the quantum structure of the spacetime and its discretization at particular scale. In the pioneering works of the last century, the concept of fundamental scale was associated to the minimal length or, geometrically speaking, to the metric (e.g. through the line element describing the spacetime). Actually, contrarily to these prior investigations, arguments favoring the use of the area as a fundamental entity were appearing in recent years. This fact was motivated strongly by the relation area / entropy of the black hole on one side, and the theoretical structure of theories such as loop quantum gravity (spin networks), dynamical triangulations and however, the string theory on the other side. Two questions motivated by the gravity-quantum unification immediately arise. One of them without answer until today: is the exact behaviour at macroscopic regime of the quantum gravity thermal or not? The another question is: there
exists a consistent formulation where the length is the minimum fundamental entity and that the spectrum of such theory meets the correct limits? (e.g. correct spacing between levels for n small and for large n). We will demonstrate through this letter that such consistent description certainly exists, and it can contemplate the classical (thermal) or not, behaviour of the spectrum. Our claim is based in some previous research of one of us trying to give an unambiguous quantum mechanical description of a particle in a general spacetime. Because the introduction of supersymmetry provide new approach to several physical problems of interest, in [1] a particular interesting Riemannian $N = 1$ superspace was introduced. The main feature of this superspace that makes it especially important, is that the corresponding supermetric, which is the basic ingredient of a Volkov-Pashnev particle action [8], is invertible and non-degenerate, that is, of $G4$ type in the Casalbuoni’s classification [10] As shown in [2,3,5], the non-degeneracy of the supermetrics (and therefore of the corresponding superspaces) leads to important consequences in the description of physical systems [5]. In particular, notorious geometrical and topological effects on the quantum states, namely, consistent mechanisms of localization and confinement, due purely to the supergeometrical character of the Lagrangian. Also an alternative to the Randall-Sundrum (RS) model without extra bosonic coordinates, can be consistently formulated in terms of such non degenerated superspace approach, eliminating the problems that the RS-like models present at the quantum level [1,2,3]. And from the probabilistic point of view was recently demonstrated in [4] that, using the probability current as the probability density, the quantum counterpart of the Fisher’s metric can be exactly implemented being all the relevant quantum operators exactly constructed in a manner that was already inferred in 1988 on a quite general basis by Caianiello. In this letter (strongly motivated by the above results) we will show, after brief description of the superspace and the emergent spacetime of ref.[1,2,3], that as a result of quantization of this supersystem[2,3], a discrete metric associated with coherent states (in obedience to a Poisson distribution), is immediately obtained without prescription of discretization. This discrete solution, that represents an emergent metric, is described by a physical coherent state belonging to the metaplectic group $Mp(n)$ with a Poissonian distribution at lower n (number basis) restoring the classical thermal continuum behaviour at large $n(n \rightarrow \infty$, number basis), or leading to non-classical radiation states, as is conjectured in a quite general form by the dynamic Bekenstein-Mukhanov effect. The results that we present here are absolutely without black/hole entropy arguments given a
Finally a discussion linking our results with the black hole entropy and spectrum are giving and some perspectives and future directions of research suggested.

II. SUPERMETRIC AND EMERGENT SPACETIME

The model introduced in [1,8], represents a free particle in a superspace with coordinates $z_A \equiv (x_\mu, \theta_\alpha, \bar{\theta}_\dot{\alpha})$. It is described by the Lagrangian density

$$L = -m \sqrt{\omega^A \omega_A} = -m \sqrt{\dot{\omega}_\mu \dot{\omega}^\mu} + a \dot{\theta}^\alpha \dot{\bar{\theta}}_{\dot{\alpha}} - a^* \dot{\bar{\theta}}^\dot{\alpha} \dot{\theta}_{\alpha}.$$  

(1)

where $\omega_\mu = \dot{x}_\mu - i(\dot{\theta} \sigma_\mu \bar{\theta} - \theta \sigma_\mu \dot{\bar{\theta}})$, and the dot indicates derivative with respect to the parameter $\tau$, as usual. In coordinates, the line element of the superspace reads,

$$ds^2 = \dot{z}^A \dot{z}_A = \dot{x}^\mu \dot{x}_\mu - 2i \dot{\theta}^\mu (\dot{\theta} \sigma_\mu \bar{\theta} - \theta \sigma_\mu \dot{\bar{\theta}}) + (a - \bar{\theta}^\dot{\alpha} \dot{\theta}_{\dot{\alpha}}) \dot{\theta}^\alpha \dot{\bar{\theta}}_{\dot{\alpha}} - (a^* + \theta^\alpha \bar{\theta}_{\dot{\alpha}}) \dot{\bar{\theta}}^\dot{\alpha} \dot{\theta}_{\alpha}.$$

Is important to note that the quantization was exactly performed by a new method introduced by one of us in [2,3] given the correct physical and mathematical interpretation to the square root Hamiltonian. The method is based on two fundamental points: first, introducing a modification of Lanczos technique [2] that permits to pick the correct phase space of the problem without modify the form of the relevant quantum geometrical operators (i.e.the particular form of the Hamiltonian or Lagrangian remains as square root ). And second, using the underlying covering group of $SL(2\mathbb{C})$ ( that is the Metaplectic group) to give a quantum meaning to the radical operator (Lagrangian or Hamiltonian). With these ingredients the problem is schematically reduced to $\mathcal{H} |\Psi\rangle \equiv \sqrt{m^2 - \mathcal{P}_\mu \mathcal{P}^\mu} - (\mathcal{P}_i \mathcal{P}^i + \frac{1}{a} \Pi^\alpha \Pi_\alpha - \frac{1}{a^2} \Pi^\alpha \Pi_\alpha) |\Psi\rangle = 0$ were $\mathcal{P}^\mu$, $\Pi^\alpha$ are the momenta corresponding to the supercoordinates.

Without lose generality and for simplicity, the ‘squared’ solution with three compactified dimensions ($\lambda = 2$ spin fixed) is [1,3,5]

$$g_{AB}(t) = e^{A(t)+\xi(t)} g_{AB}(0),$$  

(2)

where the initial values of the metric components are given by

$$g_{ab}(0) = \langle \psi(0)| \begin{pmatrix} a & \\ a^* & \end{pmatrix}_{ab} |\psi(0)\rangle,$$  

(3)
or, explicitly,
\[ g_{\mu\nu}(0) = \eta_{\mu\nu}, \quad g_{\mu\alpha}(0) = -i\sigma_{\mu\alpha\dot{\alpha}}, \quad g_{\mu\dot{\alpha}}(0) = -i\theta^\alpha\sigma_{\mu\dot{\alpha}}, \quad g_{\alpha\beta}(0) = (a - \bar{\theta}^\alpha\bar{\theta}_\alpha)\epsilon_{\alpha\beta}, \quad g_{\dot{\alpha}\dot{\beta}}(0) = -(a^* + \theta^\alpha\theta_\alpha)\epsilon_{\dot{\alpha}\dot{\beta}}. \] (4)

It worth mention here that these components were obtained in the simplest case in [9].

The bosonic and spinorial parts of the exponent in the superfield solution (2) are, respectively,
\[ A(t) = -\left(\frac{m}{m} \right)^2 t^2 + c_1 t + c_2, \]
\[ \xi(t) = \xi_0(t + \bar{\chi}(t)) = \theta^\alpha \left( \bar{\phi}_\alpha \cos(\omega t/2) + \frac{2Z_\alpha}{\omega} \right) - \bar{\theta}^\dot{\alpha} \left( -\bar{\phi}_\dot{\alpha} \sin(\omega t/2) - \frac{2Z_{\dot{\alpha}}}{\omega} \right) \] (6)
\[ = \theta^\alpha \bar{\phi}_\alpha \cos(\omega t/2) + \bar{\theta}^\dot{\alpha} \bar{\phi}_\dot{\alpha} \sin(\omega t/2) + 4|a|Re(\theta Z), \]
where \( \bar{\phi}_\alpha, Z_\alpha, \bar{Z}_{\dot{\alpha}} \) are constant spinors, \( \omega = 1/|a| \) and the constant \( c_1 \in \mathbb{C} \), due to the obvious physical reasons and the chiral restoration limit of the superfield solution [1,3,5].

## III. SUPERSPACE AND DISCRETE SPACETIME STRUCTURE

Now we will see how the discrete spacetime structure naturally arise from the model.
Expanding on a number basis, as usual
\[ \sum_m |m\rangle \langle m| = 1, \]
we have
\[ g_{ab}(0) = \sum_{n,m} \langle \psi(0)|m\rangle \langle m|L_{ab}|n\rangle \langle n|\psi(0)\rangle \]
then
\[ g_{ab}(t) = e^{A(t)\xi(t)} \sum_{n,m} \langle \psi(0)|m\rangle \langle m|L_{ab}|n\rangle \langle n|\psi(0)\rangle \langle m|L_{ab}|n\rangle \]
\[ \langle m|L_{ab}|n\rangle = \langle m| \begin{pmatrix} a \\ a^\dagger \end{pmatrix}_{ab} |n\rangle = \begin{pmatrix} \langle m|n + 1\rangle \sqrt{m} \\ \langle m|n - 1\rangle \sqrt{m} \end{pmatrix}_{ab} = \begin{pmatrix} \delta_{m,m+1}\sqrt{m} \\ \delta_{m,m-1}\sqrt{m} \end{pmatrix}_{ab} \] (7)
It follows
\[ g_{ab}(0) = \sum_{n,m} \langle \psi(0)|m\rangle \begin{pmatrix} \delta_{m,m-1}\sqrt{m} \\ \delta_{m,m+1}\sqrt{m} \end{pmatrix}_{ab} \langle m|\psi(0)\rangle \]
\[ g_{ab}(0) = \sum_n \sqrt{n} \langle \psi(0) | n - 1 \rangle \langle n | \psi(0) \rangle \begin{pmatrix} 1 \\ 0 \end{pmatrix}_{ab} + \sum_m \sqrt{n + 1} \langle \psi(0) | n + 1 \rangle \langle n | \psi(0) \rangle \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{ab} \]

From the equation above we see that the only clear sense for it is due the decomposition of \( \psi \) into the basic states of the metaplectic representation

\[ | \psi(0) \rangle = A | \alpha_+ \rangle + B | \alpha_- \rangle \]  

(8)

where the constants \( A \) and \( B \) are arbitrary and they control the classical behavior of the spectrum at the macroscopic level. We, without lose generality in this part of the discussion, take \( A = B \) such that \( | \psi(0) \rangle = | \alpha_+ \rangle + | \alpha_- \rangle \), but we will return to this important point later.

This, in fact, is the effect of the decomposition of the SO(2,1) group in two irreducible representations of the metaplectic group \( \text{Mp}(2) \): spanning even and odd \( n \) respectively. The important feature of the state \( | \psi(0) \rangle = | \alpha_+ \rangle + | \alpha_- \rangle \) is that is invariant (if \( A = B \)) to the action of operators \( a \) and \( a^\dagger \). This fact is because in the metaplectic representation the general behaviour of these states are:

\[ a | \alpha_+ \rangle = a^\dagger | \alpha_+ \rangle = | \alpha_- \rangle \quad \text{and} \quad a | \alpha_- \rangle = a^\dagger | \alpha_- \rangle = | \alpha_+ \rangle \]

(we will not enter in more details here).

Is easily checked from the Poissonian distribution for the coherent states: \( P_\alpha(n) = | \langle n | \alpha \rangle |^2 = \frac{a^n e^{-a}}{n!} \) obeying \( \sum_{n=0}^{\infty} P_\alpha(n) = 1 \), \( \sum_{n=0}^{\infty} n P_\alpha(n) = \alpha \) that it differs with the individual distributions coming from each one of the two irreducible representations of the metaplectic group \( \text{Mp}(2) \) (spanning even and odd \( n \) respectively):

\[ \sum_{n=0}^{\infty} P_{\alpha_+}(2n) = e^{-\alpha} \cosh(\alpha), \quad \sum_{n=0}^{\infty} P_{\alpha_-}(2n + 1) = e^{-\alpha} \sinh(\alpha) \to \sum_{n=0}^{\infty} \left( P_{\alpha_+}(n) + P_{\alpha_-}(n) \right) = 1 \]  

(9)

Although the different form between above equations, the limit \( n \to \infty \) is the same for the sum of the two distributions coming from the \( \text{Mp}(2) \) irreducible representations (Irreps). and for the SO(2,1) representation as it should be.

Having this in mind, the specific form of \( | \alpha_+ \rangle, | \alpha_- \rangle \) was given in [1,2,3] and are

\[ | \alpha_+ \rangle \equiv | \Psi_{1/4}(0, \xi, q) \rangle = \sum_{k=0}^{+\infty} f_{2k}(0, \xi) | 2k \rangle = \sum_{k=0}^{+\infty} f_{2k}(0, \xi) \frac{(a^\dagger)^{2k}}{\sqrt{(2k)!}} | 0 \rangle \]

\[ | \alpha_- \rangle \equiv | \Psi_{3/4}(0, \xi, q) \rangle = \sum_{k=0}^{+\infty} f_{2k+1}(0, \xi) | 2k + 1 \rangle = \sum_{k=0}^{+\infty} f_{2k+1}(0, \xi) \frac{(a^\dagger)^{2k+1}}{\sqrt{(2k + 1)!}} | 0 \rangle \]  

(10)

where in the parameter \( \xi \) all the possible remaining \( B_1 \) (odd) dependence is stored. Then,
we arrive to following result

\[
g_{ab}(t) = \frac{f(t)}{2} \sum_{m} \left\{ [P_{\alpha+}(2m) \cdot 2m + P_{\alpha-}(2m + 1) \cdot (2m + 1)] \left( \begin{array}{c} 1 \\ 0 \end{array} \right)_{ab} + \\
+ [P_{\alpha+}^*(2m) \cdot 2m + P_{\alpha-}^*(2m + 1) \cdot (2m + 1)] \left( \begin{array}{c} 0 \\ 1 \end{array} \right)_{ab} \right\} \quad (11)
\]

this expression is the core of our discussion: it shows explicitly the discrete structure of the spacetime as the fundamental basis for a consistent quantum field theory of gravity. By the other hand, when we reach the limit \( n \to \infty \) the metric solution goes to the continuum due:

\[
\sum_{n=0}^{\infty} [P_{\alpha+}(2m) \cdot 2m + P_{\alpha-}(2m + 1) \cdot (2m + 1)] = \alpha e^{-|\alpha|} (\cosh(\alpha) + \sinh(\alpha)) = \alpha
\]

and similarly for the lower part (spinor down) of above equation

\[
\sum_{n=0}^{\infty} [P_{\alpha+}(2m) \cdot 2m + P_{\alpha-}(2m + 1) \cdot (2m + 1)] = \alpha^*.
\]

Consequently, when the number of levels increase the metric solution goes to the continuum "manifold" general relativistic behaviour:

\[
g_{ab}(t)_{n \to \infty} \to \frac{f(t)}{2} \left\{ \alpha \left( \begin{array}{c} 1 \\ 0 \end{array} \right)_{ab} + \alpha^* \left( \begin{array}{c} 0 \\ 1 \end{array} \right)_{ab} \right\} = f(t) \langle \psi(0) | \left( \begin{array}{c} a \\ a^\dagger \end{array} \right)_{ab} | \psi(0) \rangle \quad (12)
\]

as expected.

IV. THE MINIMAL LENGTH

Is not difficult to see that for \( m = 0 \) the metric solution takes its minimal value

\[
g_{ab}(t) = \frac{f(t)}{2} \left[ P_{\alpha-}(1) \left( \begin{array}{c} 1 \\ 0 \end{array} \right)_{ab} + P_{\alpha-}^*(1) \left( \begin{array}{c} 0 \\ 1 \end{array} \right)_{ab} \right] \\
= \frac{f(t)}{2} e^{-|\alpha|} \left[ \alpha \left( \begin{array}{c} 1 \\ 0 \end{array} \right)_{ab} + \alpha^* \left( \begin{array}{c} 0 \\ 1 \end{array} \right)_{ab} \right] \quad (13)
\]

this evidently defines the minimal length due the metric axioms in a Riemannian manifold. This point will be not analyzed fully here, but in principle (due the existence of discrete Poincare subgroups of this supermetric) fundamental symmetries as the Lorentz one, can be preserved at this level of discretization. Some of interested issues related to this problem in general were given in ref. [11].
V. BLACK HOLE ENTROPY AND SUPERSPACE SOLUTION

As is well known, the black hole entropy \( S = k_B A_{bh}/4l_P^2 \) where is the horizon area and \( l_P \equiv \sqrt{\hbar G/c^3} \) is the Planck length was first found by Bekenstein and Hawking [18] using thermodynamic arguments of preservation of the first and second laws of thermodynamics. An information theory proof was also found by Bekenstein in which can be treated as the measure of “inaccessibility” of the information of an external observer on an actual internal configuration of the black hole realized in a given state (described by values of mass, charge, and angular momentum). The first controversial thing that immediately appears from the point of view of statistical mechanics (in which the entropy is the mean logarithm of the density matrix) is that the entropy of a black hole is proportional to its surface area. About this issue, Bekenstein propose a model of quantization of the horizon area in the section with the suggestive title ”Demystifying black hole’s entropy proportionality to area” in ref.[19]. Resuming the proposal, the horizon is formed by patches of equal area \( \delta l_P^2 \) (however, which are added one after another at a time). Their standard size is important and makes them all equivalent. The horizon can be regarded as having many degrees of freedom, one per each patch, due it is made from equivalent patches all with the same number \( \chi \) of quantum states. Consequently, the total number of quantum states of the horizon is \( \Omega_H = \chi A_{bh}/\delta l_P^2 \) and the statistical (Boltzmann) entropy associated with the horizon is \( S = k_B \ln \Omega_H = k_B (A_{bh}/\delta l_P^2) \ln \chi \): putting \( \delta = 4 \ln \chi \) Bekenstein arrived to the expected thermodynamical black hole formula. As was pointed out in [20], Bekenstein don’t gives account that putting \( \delta \) into the original black hole entropy formula we obtain the Poisson expression for the total number of states,

\[
\Omega_H = e^{A_{bh}/4l_P^2} \tag{14}
\]

being this precisely the link with the structure of the emergent coherent state metric. Considering the similar Poissonian expression for the number of states from \( g_{ab} \), namely \( e^{\mid \alpha \mid} \), the relation between the coherent state eigenvalue \( \alpha \) corresponding to the solution (11) and the above equation is clear:

\[
A_{bh}/4l_P^2 = \mid \alpha \mid \tag{15}
\]

this expression relates the space of phase of the coherent state solution \( g_{ab} \) and the \( A_{bh} \) through the \( l_P^2 \) and \( \mid \alpha \mid \). The linear behaviour of area and entropy with respect to \( \alpha \) is given
VI. IS THE BLACK HOLE RADIATION BLACK?

Recently was discussed the fact if the black hole Hawking’s radiation is finally thermal or it can be quanitically affected, as suggested again by Bekenstein and Mukhanov [13]. Due the interplay between the area of the black hole surface and the black hole mass, it is likely to be quantized as well. Then, the mass of the black hole decreases when radiation is emitted due the quantum jump from one quantized value of the mass (energy) to a lower quantized value. In consequence (because radiation is emitted at quantized frequencies, corresponding to the differences between energy levels) quantum gravity implies a discretized emission spectrum for the black hole radiation.

As is well known from the comments of the (LQG) loop quantum gravity community [12,14,16,17], the spectral lines can be very dense in macroscopic regimes leading physically no contradiction with Hawking’s prediction of a continuous thermal spectrum. Was also verified that the behavior of the spectrum is ansatz-dependent at macroscopic regimes: if we pick (as Bekenstein and Mukhanov does) the simplest ansatz for the quantization of the area -that the area is quantized in multiple integers of an elementary area $A_0$, then the
emitted spectrum turns out to be macroscopically discrete, this effect as the kinematical Bekenstein-Mukhanov effect. By the other hand, into the loop quantum gravity context, the celebrated thermal spectrum is reached because the density of levels increase in parallel with the number of levels. Then, is possible to circumvented the theoretical dilemma?: from the point of view of LQG the spectrum is always continuous at macroscopic regime, and from the viewpoint of Mukhanov and Bekenstein the spectrum may be macroscopically discrete (but ansatz-dependent finally). From the point of view of our model we can give an affirmative answer to this question: if now we suppose simply that $A \neq B$ in the state solution (8,11), that is one possibility in our proposal due the arbitrariness of the constants $A$ and $B$, we have $|\psi(0)\rangle = A|\alpha_+\rangle + B|\alpha_-\rangle$ then we cannot reach the thermal (Hawking) spectrum at the macroscopic level. This fact is clear because we need exact balance between the superposition of the two irreducible representations of the Metaplectic group as clearly given by expressions (11,9). This will leads, as a result, non classical states of radiation in the sense of [21] as can be easily seen putting, for example, $B$ (or $A$) zero:

$$g_{ab}(t) = A \frac{f(t)}{2} \sum_m \left[ P_{\alpha+}(2m) \cdot (2m) + P_{\alpha-}(2m+1) \cdot (2m+1) \right] \begin{pmatrix} 1 \\ 0 \end{pmatrix}_{ab}$$

notice that only the up spinor part survives and the classical (thermal) limit is not reached, even in the continuous limit were the number of levels increases accordingly to

$$g_{ab}(t)_{n\to\infty} \to \frac{f(t)}{2} A \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix}_{ab} = Af(t) \langle \psi(0) \rangle \begin{pmatrix} a \\ 0 \end{pmatrix}_{ab} |\psi(0)\rangle$$

In such a case $A = 0$ ($B = 0$), the spectrum will takes only even(odd) levels becoming evidently non thermal. Then, if $A = B$ the kinematical Bekenstein-Mukhanov effect disappears and the thermal Hawking spectrum is reached at the continuum classical gravity level (the Poissonian behaviour of the distribution is complete). Otherwise, with $A \neq B$, the spectrum belongs to a non classical one and the quantum properties of the gravity are macroscopically manifest.

VII. CONCLUDING REMARKS

Trough this paper we have been shown that a N=1 superspace equipped with a non-degenerate and invertible supermetric where the unconstrained quantization was exactly
performed by new methods based on coherent states and respecting the form of the Hamiltonian (modified Lanczos technique), a discrete structure of spacetime naturally emerges without any prescription of discretization (in sharp contrast of the other attempts in the literature). Due the Metaplectic representation (double covering of the $SL(2C)$) of the coherent state solution representing the emergent spacetime, the crossover from the quantum to the macroscopical regime (classical or not) is natural and consistent. This important fact permits us to conciliate the two apparently different pictures of the macroscopical quantum gravity regime given by the LQG claims supporting the Hawking (Thermal) spectrum) and the dynamical Bekenstein-Mukhanov effect that point out that quantum (non thermal) imprints can survive at macroscopical regime. Despite the simplicity of the model introduced here, we have been obtained physically and geometrically, an amount of important answers with respect to a consistent quantum gravity formulation. The main properties that any consistent formulation of quantum gravity must have, in the light of the results presented, are:

1) Emergent nature of the spacetime.
2) Independence of the discretization method.
3) Consistent suitable transition to the macroscopic (classical, semiclassical, etc.) regime.
4) Total and absolute independence of particular solutions or other arguments involving particular geometries (e.g. black-hole/area and the entropy).
5) Solutions, arguments involving particular geometries, etc. of the previous point, must be reached by the quantum gravity theory but not depending them at the fundamental level.

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