Recent observations show that planets can reside in close binary systems with stellar separation of only ~20 AU. However, planet formation in such close binary systems is a challenge to current theory. One of the major theoretical problems occurs in the intermediate stage—planetesimals accretion into planetary embryos—during which the companion’s perturbations can stir up the relative velocities ($\Delta V$) of planetesimals and thus slow down or even cease their growth. Recent studies have shown that conditions could be even worse for accretion if the gas-disk evolution was included. However, all previous studies assumed a two-dimensional disk and a coplanar binary orbit. Extending previous studies by including a three-dimensional gas disk and an inclined binary orbit with small relative inclination of $i_B = 0.1–5^\circ$, we numerically investigate the conditions for planetesimal accretion at 1–2 AU, an extension of the habitable zone (~1–1.3 AU), around α Centauri A in this paper. Inclusion of the binary inclination leads to the following: (1) differential orbital phasing is realized in the three-dimensional space, and thus different-sized bodies are separated from each other, (2) total impact rate is lower, and impacts mainly occur between similar-sized bodies, (3) accretion is more favored, but the balance between accretion and erosion remains uncertain, and the “possible accretion region” extends up to 2 AU when assuming an optimistic $Q^\star$ (critical specific energy that leads to catastrophic fragmentation), and (4) impact velocities ($\Delta V$) are significantly reduced but still much larger than their escape velocities, which infers that planetesimals grow by means of type II runaway mode. As a conclusion, the inclusion of a small binary inclination is a promising mechanism that favors accretion, opening a possibility that planet formation in close binary systems can go through the difficult stage of planetesimals accretion into planetary embryos.

Key words: methods: numerical – planetary systems: formation

Online-only material: color figures
whether accretion or erosion dominates. If $\Delta V$ is greater than the planetesimal escape velocity ($V_{esc} \approx 100R_p/(100 \text{ km}) \text{ m s}^{-1}$), runaway accretion is prevented and growth is slowed down. Furthermore, if $\Delta V$ is excited beyond the threshold velocity $V_{ero}$ (a few times of $V_{esc}$, depending on the prescription for collision, see the Appendix for details) at which erosion dominates accretion, planetesimal growth ceases. Fortunately, in the early stages, planetary disk was rich in gas, which could partly damp companion’s perturbations toward an equilibrium. Furthermore, Marzari & Scholl (2000) found that, in this equilibrium state, all the orbits of planetesimals with the same size and similar semimajor axes had the configuration of periastron aligned (so-called orbital alignment), hence reducing $\Delta V$ to very low values. Applying this mechanism to the $\alpha$ Cen AB system, accretion zone of km-sized bodies could extend to 2 AU to the primary star ($\alpha$ Cen A). Nevertheless, their work only took into account the $\Delta V$ of the equal-sized bodies. Detailed study by Thébault et al. (2006) found that the orbital alignment simultaneously increased $\Delta V$ of different-sized bodies, and it was so efficient that it led to significant $\Delta V$ increase for any departure from the exact equal-sized condition ($R_1 = R_2$, where $R_1$ and $R_2$ are the radii of the two colliding bodies). Extending their studies by including the effect of a fast gas-disk depletion procedure under UVO photoevaporation from the host star or nearby stars (Matsuyama et al. 2003; Alexander et al. 2006); Xie & Zhou (2008) found that the effect of differential orbital alignment could be reduced and all planetesimals eventually converged toward the same forced orbits, regardless of size. This mechanism indeed reduces all $\Delta V$ to low values, however it is not efficient enough for small bodies (radii less than 5 km) and it takes a long time ($\sim 10^5$–$10^6$ yr) to reach the “low-$\Delta V$” state, before which conditions are still accretion hostile. Recently, taking into account the differential phasing effect, Thébault et al. (2008, 2009) revisited the planetesimal accretion conditions in the $\alpha$ Cen AB system, and found that the “accretion-safe” area never extended beyond 0.75 AU around Cen A or 0.5 AU around Cen B. Their results also confirmed that the planetesimals-to-embryos phase was more affected by the binary environment than the last stages, and it might be one of the biggest difficulties of planet formation in close binary systems.

All previous studies belong to coplanar cases, which have not taken into account the inclination of the companion’s orbit relative to the planetary disk. According to Hale (1994), coplanarity is just an approximation with a deviation of the relative orbital inclination of $i_B \leq 10^\circ$ for solar-type binary systems with separations less than 30–40 AU, and systems with separations greater than 40 AU appear to have their orbital planes randomly distributed. Therefore, a small inclination between the binary orbit and the planetary disk (such as $i_B = 1^\circ$–$5^\circ$) is more realistic than exact coplanarity ($i_B = 0$) for the binary systems with separations of $\sim 20$ AU. In this paper, assuming a small inclination for the binary orbit, we reinvestigate the conditions for planetesimal accretion around $\alpha$ Cen A. Results are compared with the previous study of Thébault et al. (2008). As presented next, by considering the inclination of the companion star, planetesimal accretion is much more favored than in the coplanar case, thus providing a possible solution for the problem of planet formation in close binaries.

In Section 2, we describe our numerical model and initial conditions. In Section 3, we first compare two nominal cases: $i_B = 0$ and $i_B = 1^\circ$, then present the results of other cases. Some further discussions are in Section 4. Finally, we summarize in Section 5.

2. NUMERICAL MODEL AND INITIAL CONDITIONS

2.1. Gas-Disk Model

Gas drag is modeled by assuming a nonevolving axisymmetric gas disk with constant circular streamlines. Following Weidenschilling & Davis (1985), the drag force is expressed as

$$\mathbf{F} = -K \mathbf{v} \mathbf{v},$$

where $\mathbf{F}$ is the force per unit mass, $\mathbf{v}$ is the relative velocity between the planetesimal and gas, $V$ is the velocity modulus, and $K$ is the drag parameter, defined as

$$K = \frac{3 \rho_g C_d}{8 \rho_p R_p^2}.$$  \hspace{1cm} (2)

Here $\rho_g$ is the local gas density, and $\rho_p$ and $R_p$ are the planetesimal density and radius, respectively. $C_d$ is a dimensionless coefficient related to the shape of the body ($\approx 0.4$ for spherical bodies).

As we consider the case of $i_B \neq 0$, both the distribution and motion of gas should be modeled in the three-dimensional space. According to Takeuchi & Lin (2002), in cylindrical coordinates $(r, z)$, we adopt the gas-density distribution as

$$\rho_g(r, z) = \rho_{g0} \left( \frac{r}{\text{AU}} \right)^p \exp \left( -\frac{z^2}{2h_g^2} \right),$$

and the gas rotation as

$$\Omega_g(r, z) = \Omega_{g, mid} \left[ 1 + \frac{1}{2} \left( \frac{h_g}{r} \right)^2 \left( p + q + \frac{q z^2}{2 h_g^2} \right) \right],$$

where $h_g(r) = h_0 (\frac{r}{\text{AU}})^{q+3}/2$ is the scale height of gas disk, $\Omega_{g, mid}$ is the Keplerian rotation on the midplane, and the subscript “0” means the value at 1 AU. In this paper, we take the Minimum Mass of Solar Nebula (hereafter MMSN; Hayashi 1981) as the nominal gas disk, where $p = -2.75$, $q = -0.5$, $\rho_{g0} = 1.4 \times 10^{-6}$ g cm$^{-3}$, and $h_0 = 4.7 \times 10^{-2}$ AU. As stated in many previous studies (Scholl et al. 2007; Thébault et al. 2006), this axisymmetric gas-disk model is a crude simplification, and in reality, perturbations on the gas disk by the companion should be included (Artymowicz & Lubow 1994; Kley & Nelson 2007). A full description of the dynamical behavior of planetesimals in such a “real” disk requires hydrocode modeling of the gas in addition to N-body type models for planetesimals. However, there still exist some uncertainties even in a “real” disk model: how to choose a numerical wave-damping procedure (Paardekooper et al. 2008). On the other hand, such an all-encompassing gas-plus-planetesimals modeling costs much in calculation and goes beyond the scope of our study in this paper. Fortunately, the simplified axisymmetric gas-disk model, first of all, is valid on the average; second, it costs very little CPU in calculation and goes beyond the scope of our study in this paper.

2.2. Initial Setup

Table 1 lists most parameters for $\alpha$ Cen AB system, planetesimals and the gas disk. $10^4$ planetesimals with random eccentricities of $0$–$10^{-4}$, random inclinations of $0$–$5 \times 10^{-5}$ rad, random
semimajor axes of 1–2 AU, and random radii of 1–10 km, orbit around the primary star, \( \alpha \) Cen A. For the nominal gas disk (1 MMSN), we explore four cases with different binary inclinations from 0° to 5°; for the nominal binary inclination (\( i_B = 1° \)), we explore three cases with different gas densities (0.1, 1, and 10 MMSN). The inflated radius for collision search is set as \( 5 \times 10^{-5} (r_{col}/AU)^{1/2} \) AU, where \( r_{col} \) is the distance from a collision to the primary. This choice gives a precision of \( \sim 1.5 \text{ m s}^{-1} \) in every impact velocity estimate. All the cases evolve \( 10^4 \) yr by using the fourth-order Hermite integrator (Kokubo et al. 1998), including the gas drag force and companion’s perturbations.

### 3. RESULTS

#### 3.1. Coplanar Case Versus Inclined Case

In order to clearly see and fully understand the role of the binary inclination, we present a detailed comparison between the nominal coplanar and inclined cases: \( i_B = 0 \) and \( i_B = 1° \). Both cases have a nominal gas disk of 1 MMSN.

#### 3.1.1. Orbital Distribution

In Figure 1, we plot the evolutions of planetesimals’ distribution in the \( X-Z \) coordinate plane for both \( i_B = 0 \) and \( i_B = 1° \) cases. As can be seen in the \( i_B = 0 \) case, at beginning \( t = 0 \), planetesimals with random radii of 1–10 km are randomly mixed together. As planetesimals move in the gas disk, they are subject to a gas drag force, which progressively damps their orbital inclinations. This inclination damping process is more efficient for small bodies because gas drag force is inversely proportional to planetesimal radius. At the end of the run \( t = 10^4 \) yr, the average \( Z \) value of 1 km bodies is almost one order of magnitude lower than those of 10 km ones. Small bodies are crowded in the midplane of the disks while big ones are diffused on the surface. For the \( i_B = 1° \) case, although with the same initial condition as in the \( i_B = 0 \) case, the evolution of planetesimals’ orbits is entirely different. Planetesimals’ inclinations are not damped by the gas drag force but excited by the companion star (see the second panels on the right side). Under the coupled effect of the companion’s secular perturbation and gas drag force, different-sized bodies are eventually forced to orbits with different orbital inclinations. Because of the size-dependence of gas drag force, smaller planetesimals are damped more by gas drag and thus have smaller forced inclinations than bigger bodies. As can be seen in the two bottom panels on the right-hand side of Figure 1, different-sized bodies are grouped and separated by their radii because of the different forced inclinations. In the next two subsections, we will show that this orbital grouping significantly changes the distribution of impacts among a swarm of planetesimals, reduces their impact velocities \( \Delta V \) and finally favors planetesimal accretion.

#### 3.1.2. Impact Rate Distribution

As found by Thébault et al. (2006), \( \Delta V \) is very sensitive to the difference of radii sizes (\( R_1 \) and \( R_2 \)) of the two colliding bodies because of the size dependence of the orbital alignment. In order to understand the evolution and distribution of \( \Delta V \) among a swarm of planetesimals, we, therefore, should first investigate the impact distribution on the \( R_1-R_2 \) plane. In Figure 2, we track the evolution of the impact distributions for both \( i_B = 0 \) and \( i_B = 1° \) cases. In the first 5 \( 10^2 \) yr, the distributions are random and similar in both cases because they start with the same random initial condition and orbital alignments are not fully completed (see Figure 1). Afterwards, the distributions become entirely different: for \( i_B = 0 \) case, impacts occur much more often between bodies of different sizes; for \( i_B = 1° \) case, impacts mainly occur between similar-sized bodies. These two totally different impact distributions can be understood as follows:

1. In the \( i_B = 0 \) case, planetesimal disk is very flattened and can be treated as a two-dimensional disk. Because of orbital alignment in the two-dimensional disk, a planetesimal can collide with another similar-sized one only if their semimajor axes are very close since they have the similar forced orbits (means the similar orbital shapes and configurations) and radial drifts; in contrast, given the different forced orbits and radial drifts, a planetesimal can cross more orbits of other different-sized bodies over a much larger region.

2. In the \( i_B = 1° \) case, orbital alignment is in three-dimensional space and thus bodies of different sizes are separated from each other (see Figure 1), leading to much lower probability of impact between two planetesimals of different sizes.

| Parameters | Values | Units | Descriptions |
|------------|--------|-------|--------------|
| \( a_B \)  | 23.4   | AU    | Semimajor axis of companion |
| \( e_B \)  | 0.52   |       | Eccentricity of binary |
| FP*        |        |       |              |
| \( i_B \)  | 0, 0.1, 1, 5.0 | deg | Inclination of binary |
| \( M_A \)  | 1.1    | \( M_\odot \) | Mass of primary |
| \( M_B \)  | 0.93   | \( M_\odot \) | Mass of companion |
| UD*        | R\(_p\) | 1–10 km | Planetesimal radii |
| UD*        | \( a_p \) | 1.0–2.0 AU | Semimajor axes of planetesimals |
| UD*        | \( e_p \) | 0–10\(^{-4} \) | Eccentricities of planetesimals |
| UD*        | \( i_p \) | 0–5 \times 10\(^{-5} \) | Inclinations of planetesimals |
| UD*        | \( \rho_p \) | 3.0 g cm\(^{-3} \) | Planetary density |
| FP*        | \( \rho_{\text{g}O} \) | 1.4 \times 10\(^{-10} \), 10\(^{-9} \), 10\(^{-8} \) | Initial gas density at 1 AU |
| FP*        | \( \rho_\pi \) | \( r^{-11/4} \) | Initial gas density radial profile |
| FP*        | \( N_p \) | \( 10^6 \) | Number of test particles |
| FP*        | \( R_{\text{infl}} \) | 5 \times 10\(^{-5} (r_{col}/AU)^{1/2} \) AU | Inflated radius for collision search |
| FP*        | \( t_{\text{final}} \) | \( 10^6 \) yr | Final timescale for each run |

Notes. \( M_\odot \) stands for the solar mass. The fields marked by FP* are explored as free parameters in the simulations, while the ones marked by UD* have an uniform distribution.
In addition, a big enhancement of impact rate is observed in the $i_B = 0$ case since gas drag damps planetesimals' inclinations and thus disk becomes more dense. While in the $i_B = 1^\circ$ case, impact rate is much lower because planetesimals' inclinations are excited and thus the volume density of disk is reduced.

3.1.3. Accretion Versus Erosion

We plot Figure 3 to finally analyze the accretion or erosion conditions for the cases of $i_B = 0$ and $i_B = 1^\circ$. In the two top panels, as expected from the analysis of the last subsection, the impacts in $i_B = 0$ case are much more than those in $i_B = 1^\circ$ case. Since impacts between different-sized bodies are favored in the $i_B = 0$ case, the impact velocities are increased to relatively high values with average of $\sim 200 \text{ m s}^{-1}$; while in the $i_B = 1^\circ$ case, impacts mainly occur between similar-sized bodies with low impact velocities of 10–100 m s$^{-1}$ in average. In the two bottom panels, by comparing each $\Delta V$ with two threshold velocities: $V_{\text{low}}$ and $V_{\text{high}}$ (see the Appendix for detail), we plot the relative
weights of the possible types of collision outcomes as a function of distance to the primary star ($\alpha$ Cen A). For the case of $i_B = 0$, most impacts lead to erosion (red zone). The weight of accretion impact (green + blue zone) is only less than 5% or about 10%, if the uncertain ones (yellow zone) are included. These results are roughly comparable with the Figure 3 in Thébault et al. (2008), which also adopted the assumption of $i_B = 0$ but a Maxwellian distribution for planetesimal size. In contrast, for the case of $i_B = 1^\circ$, accretion is significantly favored: at 1.15 AU, center of the habitable zone ($\sim$1–1.3 AU, e.g. Barbieri et al. 2002), about 40% (80%, if including uncertain ones) collisions lead to accretion. If we define the “possible accretion zone” as green+blue+yellow zones $> 50\%$, it extends to as far as $\sim$2 AU, beyond which companion’s perturbations is too strong to reach orbital alignment for planetesimal of any size. Note that our “possible accretion zone” here is an optimistic definition, where all uncertain ones are taken into account as accretion collision. Actually, the balance between accretion and erosion has a large uncertainty (see the yellow region in Figure 3) because of the poor constraint on $Q^*$ (critical specific energy that leads to catastrophic fragmentation, see the Appendix for detail).

3.2. Other Cases

Beside the nominal case with $i_B = 1^\circ$ and gas-disk density of 1 MMSN, we explore some other cases with different $i_B$ and gas densities.

3.2.1. Different $i_B$

Figure 4 shows the results of two cases with the same gas density of 1 MMSN, but different binary inclinations: $i_B = 0.1$ and $5^\circ$, respectively. $i_B = 0.1$ is marginal case, where every planetesimal has a forced inclination in a very small range 0–0.1 and thus very close to each other. In such case, bodies of different sizes are not well separated from each other and impacts between different-sized bodies are still an important part (see the left up panel of Figure 4). Therefore, the average impact velocity is intermediate ($\Delta V \sim 100$ m s$^{-1}$), leading to, although more accretion-friendly than the case of $i_B = 0$, still an accretion-hostile condition (red zone exceeds 50% at least). For the larger inclination case of $i_B = 5^\circ$, although the planetesimal vertical excursions might actually exceed the typical thickness of the gas disk, the results are similar to the nominal case of $i_B = 1^\circ$, except for less impact events because of the larger inclination (see the three bottom panels in Figure 4).

3.2.2. Different Gas Disks

We also explore two cases with gas densities of 0.1 MMSN and 10 MMSN in Figure 5. By comparing these two cases with the nominal inclined case in Figure 3, we find: as gas density increases from 0.1 to 10 MMSN, the weight of accretion increases in most outer region but decreases in the inner region (say the habitable zone $\sim$1.0–1.3 AU). The probable explanations are:

1. In the inner region, the companion’s perturbations are weak and gas is dense; while in our region, perturbations are stronger and gas density is lower. As a result of these conditions, orbital alignment is realized much more easily in the inner than in the outer region.

2. For 0.1 MMSN gas density, orbital alignment is already realized in the inner region but not in outer region. A larger gas density still favors accretion in the outer region, but it can no longer improve the inner accretion condition. Furthermore, gas drag is a double-edged sword. If orbital alignment is already realized, increasing gas density can result in larger different radial drifts between different-sized bodies, which increase $\Delta V$ and finally prevents accretion.
4. DISCUSSION

4.1. Behaviors in the Full Gas Model

As mentioned in Section 2.1, our gas drag model is a simplified one where gas disk is assumed to be fully axisymmetric and non-evolving. A full gas model requires hydro-code for modeling the gas evolution and N-body type models for planetesimals. Recently, taking the full gas model, Paardekooper et al. (2008) have studied the dynamics and encounter velocities for a swarm of planetesimals in close binaries. They found that the inclusion of the full gas dynamics increased the encounter velocity and made planetesimal accretion even more difficult than in a static, axisymmetric gas disk. Note that their gas disk is two-dimensional, and the binary inclination is also not included. Nevertheless, they have shown some important results that we might infer the $\Delta V$ behavior in the real three-dimensional full gas model, where both the binary inclination and gas evolution are included. As shown in the Figures 9–11 of Paardekooper et al. (2008), orbital alignments, both of eccentricity and longitude of periastron, are still maintained for bodies of the same size, leading very low $\Delta V$ between equal-sized bodies as in a static, circular gas disk. This means that gas evolution only increases $\Delta V$ between different-sized bodies. In a real three-dimensional full gas model, orbital alignments should probably be maintained, but it is in the three-dimensional space, and thus different-sized bodies are separated from each other, leading to most collisions occurring between similar-sized bodies with low $\Delta V$. Therefore, even in the full gas model, the inclusion of a small binary inclination might also be a promising mechanism favoring planetesimal accretion. The only uncertainty is to what extent can it favor accretion, or whether it is enough to render accretion-friendly impacts dominant? This problem needs to be addressed by a future work which can model a three-dimensional evolving gas disk in an inclined binary system.

4.2. Planetesimal Growth: Type II Runaway Mode

The major result of this paper is: the inclusion of a small binary inclination favors planetesimal accretion significantly,
although it might not be enough (if only the green and blue region are counted as accretion) for planetesimal growth to proceed. Even if planetesimal growth can proceed, it cannot be classical single-star-like accretion, or normal accretion. Actually, normal accretion collisions never exceed 20% in all the cases we investigated. Growth of planetesimals in close binary systems is mainly through “high-$\Delta V$ accretion” or type II runaway mode (Kortenkamp et al. 2001). Type II growth mode is characterized by an initial orderly growth phase followed by a runaway stage. At the beginning, $\Delta V$ is increased and held on values exceeding the biggest objects’ escape velocities, but still leading to slow orderly growth. After this orderly phase, growth then switches to runaway when the biggest bodies reach a size large enough for significantly enhancing their gravitational focusing factors: $F_g = (V_{esc}/\Delta V)^2$. This turn-off size directly depends on $\Delta V$. For example, if the runaway growth is turned on at a gravitational focusing factor of 1–100, the biggest objects’ escape velocities should be 1–10 times as large as $\Delta V$. For the coplanar case considered in this paper, the average $\Delta V$ is about 200 m s$^{-1}$ at 1 AU, thus requiring a turn-off size of 200–2000 km. While for the inclined case with $i_B = 1$ (see Figure 3), it requires a turn-off size of only 50–500 km, since the average $\Delta V$ is reduced to 50 m s$^{-1}$. Therefore, with the inclusion of the binary inclination, the runaway growth is shifted to an earlier date.

5. SUMMARY

In this paper, assuming a small inclination of the binary orbit, we numerically investigate the conditions for planetesimal accretion around $\alpha$ Cen A. Different-sized (1–10 km, randomly) planetesimals are embedded in the gas disk and moving around the primary from 1 to 2 AU, an extension of its habitable zone ($\sim$1–1.3 AU). The gas drag is modeled with a simplified three-dimensional axisymmetric nonevolving gas disk. First, we study the effects of the binary inclination on planetesimal accretion by comparing two nominal cases with and without inclination ($i_B = 0$ and $i_B = 1^\circ$). Then we explore more cases with different binary inclinations and gas densities. The main results are summarized as follows:

1. In the nominal coplanar case ($i_B = 0$), planetesimal disk is almost damped to a two-dimensional plane, and thus impact rate is highly increased. While in the nominal inclined case ($i_B = 1^\circ$), bodies of different sizes are forced to orbits of different inclinations (three-dimensional differential orbital phasing) and well separated from each other, leading to
much lower total impact rate compared with the coplanar case.

2. In the nominal coplanar case, as differential orbital phasing is almost in a two-dimensional plane, impacts between different-sized bodies are favored, leading to high $\Delta V$ and planetesimal erosion in all the region studied in this paper (1–2 AU to the $\alpha$ Cen A). While in the inclined case ($i_B = 1^\circ$), because of the three-dimensional differential orbital phasing, impacts mainly occur between similar-sized bodies, thus leading to much lower $\Delta V$ and a possible accretion zone, which can extend to as far as about 2 AU to the $\alpha$ Cen A when assuming an optimistic $Q^*$ value.

3. In the small inclined case ($i_B = 0.1^\circ$), the results are marginal. Different-sized bodies are not fully separated from each other, and thus a large part of impacts still occur between them, leading to conditions, although much better than that in the coplanar case, still accretion hostile beyond 1 AU. While for the larger inclined case ($i_B = 5^\circ$), results are comparable with those in the nominal case ($i_B = 1^\circ$).

4. For the nominal binary inclination ($i = 1^\circ$), as gas density increases from 0.1 to 10 MMSN, condition becomes less (more) accretion hostile in most outer (inner) region.

5. The inclusion of a small binary inclination (say 1–5$^\circ$) significantly favors accretion by reducing $\Delta V$. However, the balance between accretion and erosion still has a large uncertainty, and the reduced $\Delta V$ is not low enough for normal accretion. Planetesimal growth in close binary systems, such as $\alpha$ Cen A, is probably through "high-$\Delta V$" collisions or type II runaway mode.

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**APPENDIX**

**THRESHOLD VELOCITY**

The threshold velocity of the erosion/accretion limit mainly depends on the critical specific energy $Q^*$. However, different $Q^*$ available in literature often significantly diverge from one another. As in Figure 8 of Benz & Asphaug (1999), for example, the difference in $Q^*$ between Benz & Asphaug (1999) and Holsapple (1994) can be as much as $\sim 2$ orders of magnitude in our studied size range (1–10 km). Such a broad $Q^*$ is less usable to interpret our results. Therefore, first, in order to have a clear erosion/limit, we adopt our nominal $Q^*_{nom} = Q^*_{H91}$ as in Housen et al. (1991), second, given the consideration of the uncertainty of $Q^*$, we assume an optimistic $Q^*_{high} = 5Q^*_{H91}$. The $Q^*$ in the range of $Q^*_{H91}$–$5Q^*_{H91}$ is a reasonable intermediate value between those in the Holsapple (1994) and Benz & Asphaug (1999).

Following the detail procedure as in the appendix B of Thébault & Augereau (2007), we then deduce an "optimistic threshold velocity" $V_{high}$, by adopting a critical specific energy of $Q^*_{high}$ and a hard excavation coefficient $\alpha = 10^{-5}$, and a "pessimistic threshold velocity" $V_{low}$, by adopting a critical specific energy of $Q^*_{nom}$ and a weak excavation coefficient $\alpha = 4 \times 10^{-8}$. Figure 6 shows $V_{high}$ and $V_{low}$ versus the radii of target for a projectile of radii of 1 km.

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