Finite temperature effects on baryon transport scattering in the early Universe

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We have computed finite temperature corrections to the electron-hadron scattering cross sections. These are based upon the renormalized electron mass and the modified density of states due to the presence of a background thermal bath. It is found that the electron-hadron thermal transport scattering cross section can be much larger than the zero temperature one. In the case of electron-neutron transport scattering, we find \( \sigma_{ne}(T)/\sigma_{ne}(T = 0) \simeq 5 \) at \( T \approx 0.1 \text{ MeV} \).

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I. INTRODUCTION

Finite temperature effects on elementary processes are significant from the point of view of cosmology and astrophysics. The early universe is usually described as a hot gas of particles in nearly thermodynamical equilibrium. Temperature effects enter through the statistical distribution functions. These can renormalize the masses and the wave functions. These renormalized masses and wave functions can then affect scattering processes and decay rates. Several authors \cite{1} have generalized the electron-mass and wave-function renormalization to all temperatures and densities. Dicus et al. \cite{2} and independently, Cambier et al. \cite{3} included the finite temperature effects on weak reaction rates in calculations of standard big-bang nucleosynthesis (BBN). They obtained the corrected light-element abundance and found that the corrections are only of order of a few percent. After that, Saleem \cite{4} included the effects of the electron mass shift at finite temperature on BBN and Baier et al. \cite{5} examined the finite temperature radiative corrections to the weak neutron-proton decay rates. More recently, Fornengo et al. \cite{6} have considered the finite temperature effects on the neutrino decoupling temperature which is important in the evolution of the early universe. In the present work we consider finite temperature corrections to electron-hadron scattering which is important for baryon inhomogeneous cosmologies.

Baryon inhomogeneities might have been produced during the cosmological quark-hadron phase transition in the early universe \cite{7}. If such inhomogeneities were present, then the different diffusion lengths for neutrons and protons could lead to the formation of high-baryon density proton-rich regions and low-baryon density neutron-rich regions. The light element nucleosynthesis yields from such regions can differ significantly from those of standard homogeneous big-bang nucleosynthesis \cite{8}. In view of the importance of using light-element yields from the BBN to constrain the baryon-to-photon ratio as well as various cosmological and particle physics theories, such inhomogeneous models must be examined seriously. It is therefore important to quantify the effects of baryon diffusion as accurately as possible.

In this regard Applegate, Hogan and Scherrer (AHS) \cite{8} have calculated the diffusion rate of baryons through the electron-positron plasma in the early universe. Subsequently, several authors used their results in calculations of inhomogeneous BBN \cite{9,10}. In AHS it was suggested that the diffusion coefficients could be derived from the mobility of the heavy particles, and that the mobility is determined from the distribution functions of the background plasma and the transport cross section. It is important, therefore, to carefully quantify the values of the distribution functions and the transport cross sections.

However, in all previous baryon diffusion coefficient calculations, vacuum transport scattering cross sections have been used. Therefore, in order to estimate the baryon diffusion coefficients more precisely, in the present...
work we take into account the finite temperature effects in the calculation of baryon diffusion coefficients at temperatures \( \lesssim M \text{eV} \).

Specifically, we calculate the transport scattering cross section of elastic electron-hadron scattering at finite temperature. Here we shall treat hadrons as particles which have an internal structure and an anomalous magnetic moment (although we do not have a good field theory for the magnetic moments of protons or neutrons at finite temperature). Also, we assume that their internal structure is not affected by finite temperature since their mass is more than about 1000 times the temperatures of interest.

The plan of the paper is as follows. In Section II, we discuss how to include finite temperature effects in the calculation. In particular, we will briefly discuss the effective mass of an electron in the MeV temperature range. In Section III, we evaluate the electron-hadron transport scattering cross section at finite temperature. Finally, we summarize our results and discuss some astrophysical applications. We shall employ units in which the physical mass of a particle modifies its contribution to the energy density of the universe, and therefore the expansion rate. The modification of the electron mass also changes the relationship between the neutrino and photon temperature. At finite temperature, the photon propagator is modified even at the tree level. Since these break the Lorentz invariance, they are absorbed into the effective mass for the particle. This effect can be evaluated by calculating the electron self-energy in the heat bath. The temperature-dependent physical mass of the electron is given as

\[
m_T^2 \equiv m^2 = E^2 - \vec{p}^2 = m_0^2 + \frac{2}{3} \pi \alpha T^2 + \frac{4}{\pi} a T^2 B(x) + \frac{m_0^2}{2 \pi^2} a J(p),
\]

where \( m_0 = 0.511 \text{ MeV} \) is the electron rest mass in vacuum. The function \( B(x) \) with \( x = T/m_0 \), is defined as

\[
B(x) \equiv \frac{1}{x} \int_{1/x}^{\infty} ds \sqrt{s^2 x^2 - 1} \frac{e^s + 1}{e^s - 1},
\]

and

\[
J(p) = \int \frac{d^3k}{E_k} f_F(E_k) \left[ \frac{1}{E_k E_k + m_0^2 - \vec{p} \cdot \vec{k}} - \frac{1}{E_k E_k - m_0^2 + \vec{p} \cdot \vec{k}} \right],
\]

where \( E_k^2 = m_0^2 + k^2 \). In Ref. [1] it has been shown that the \( J(p) \) term is negligible for \( T \sim M \text{eV} \). We will therefore neglect this term in our analysis.

Eq. (2) is valid for all temperatures. It gives the correct result \( m_T = m_0 \) at \( T = 0 \). Around \( T \sim m_0 \), however, the third term becomes important and has to be taken into account [for example, \( B(x = 1) \approx 0.543 \)]. It also has been shown in [1] that the thermal corrections to the electron mass at \( T \sim \text{MeV} \) are sizeable. At \( T = 1 \text{ MeV} \) the electron mass increases by 4.1%, and at \( T = 2 \text{ MeV} \) the correction is as large as 16%. The change in effective mass of a particle modifies its contribution to the energy density of the universe, and therefore the expansion rate. The modification of the electron mass also changes the relationship between the neutrino and photon temperature.

The dynamics of electrons in a thermal bath is modified by the electromagnetic interactions with background photons and electrons themselves. Therefore, the effect of the thermal bath on the propagation of an electron is expressed by calculating the electron self-energy in the presence of the ambient \( e^+, e^- \) and \( \gamma \)’s [1]. The temperature corrected electron physical mass is then obtained by evaluating the renormalized propagator and finding the zero of its inverse. Thereby, the temperature-dependent physical mass of the electron is given as

\[
m_T^2 \equiv m^2 = E^2 - \vec{p}^2
\]

\[
= m_0^2 + \frac{2}{3} \pi \alpha T^2 + \frac{4}{\pi} a T^2 B(x) + \frac{m_0^2}{2 \pi^2} a J(p),
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In brief, the influence of finite temperature on the particle evolution in the early universe is due to the temperature-dependent shifts in the dynamical mass of the particles and the temperature-dependent modification of the interactions between particles. Both modifications will be included in the calculation of the scattering cross section described in Section III.
III. ELECTRON–HADRON TRANSPORT SCATTERING AT FINITE TEMPERATURE

In this section we calculate the transport cross section of electron–hadron scattering at finite temperature which is one of the fundamental processes in cosmology and astrophysics. In 1950, Rosenbluth [12] calculated the electron-hadron differential cross section (the so called Rosenbluth formula) under the assumption that the electron is ultrarelativistic ($m \ll E$) and in the rest frame of the incoming hadron. This treatment takes into account the internal structure and anomalous magnetic moment. Applegate, Hogan and Scherrer [9] used the Rosenbluth formula in the calculation of their neutron diffusion coefficient for electron-neutron scattering with the assumption that the electron energy is much less than neutron mass $M$. They obtained a constant transport cross section $\sigma_t$ for the vacuum interaction between an electron and a neutron [8],

$$\sigma_t^{AHS} = 3\pi \left( \frac{\alpha \kappa_n}{M} \right)^2 \approx 8 \times 10^{-31} \text{ cm}^2 \quad (5)$$

where $\alpha = e^2 / h c$ is the electron fine structure constant and $\kappa_n$ is the anomalous magnetic moment of the neutron.

However, at $T \sim M \text{ eV}$, the dynamical properties of electrons and photons in this thermal bath would be changed (though hadrons would not be affected by the background thermal bath since their masses are are more than about 1000 times the mass of the electron). We therefore only have to take into account the effect of finite temperature on the interaction between electrons and hadrons.

The transport cross section $\sigma_t(T)$ of the process

$$e^- (E, \vec{p}) + H(\epsilon, \vec{k}) \leftrightarrow e^- (E', \vec{p}') + H(\epsilon', \vec{k}') \quad (6)$$

where $H$ denotes a hadron, is defined by

$$\sigma_t(T) = \int d\theta(1 - \cos \theta') \quad (7)$$

where $\theta'$ is the scattering angle. The differential cross section (including the thermal phase space) is

$$d\sigma = \frac{m}{E} \frac{1}{|\vec{V}|} \frac{d^3p'}{E'} \frac{d^3k'}{2(2\pi)^3} \frac{d\epsilon'}{\epsilon'} S(E', \epsilon') \times (2\pi)^4 \delta(4)(p + k - p' - k') |\tilde{M}|^2 \quad (8)$$

where $m$ is the mass of electron, $M$ is the mass of hadron, $V$ is the normalization volume, and the statistical factor $S(E', \epsilon')$ is

$$S(E', \epsilon') = [1 - f(E')][1 - f(\epsilon')] \quad (9)$$

where $f(E)$ is the Fermi-Dirac distribution function. The flux is given by the number of particles passing through a unit area per unit time, $|\vec{J}_{inc}| = |\vec{v} - \vec{V}| / V$, $\vec{v} = \vec{p} / \epsilon$, $\vec{V} = \vec{k} / \epsilon$, where $\vec{v}$ and $\vec{V}$ denote the initial velocities of electrons and hadron, respectively. The square of the spin-averaged scattering matrix element is given by

$$|\tilde{M}|^2 = \frac{1}{4} \sum_{\text{spins}} |\tilde{u}(p', s')\gamma^\mu u(p, s)|^2 \frac{\alpha D_F(q^2)}{2M} (k' + k)_\mu \quad (10)$$

where $s$ and $S$ denote the electron and hadron spin respectively. The vertex function $\Gamma_\mu(k', k)$ is defined using the Gordon decomposition is [3]

$$\Gamma_\mu(k', k) = \gamma_\mu (F_1(q^2) + F_2(q^2)) - \frac{F_2(q^2)}{2M} (k' + k)_{\mu} \quad (11)$$

Here, $q = k' - k$, is the momentum transfer and $F_1(q^2)$ and $F_2(q^2)$ are hadron form factors. The $q^2 \rightarrow 0$ limit of the proton and neutron form factors are known from scattering experiments [13]:

$$F_1^p(0) = 1, \quad F_2^p(0) \equiv \kappa_p = 1.792, \quad F_1^n(0) = 0, \quad F_2^n(0) \equiv \kappa_n = -1.913. \quad (12)$$

The lepton tensor in Eq. (11) is

$$L_{\mu\nu} = \frac{1}{2m^2} \left[ p'_{\mu} p_{\nu} + p_{\mu} p'_{\nu} - g_{\mu\nu}(p' \cdot p - m^2) \right],$$

and the hadron tensor is

$$H_{\mu\nu} = \frac{1}{2M^2} \left\{ (F_1 + F_2)^2 [k_{\mu} k_{\nu} + k_{\mu}' k_{\nu} - g_{\mu\nu}(k' \cdot k - M^2)] - \left[ (F_1 + F_2) F_2 - \frac{1}{4} F_2^2 \left( 1 + \frac{k' \cdot k}{M^2} \right) \right] (k_{\mu}' + k_{\nu}) \times (k_{\nu}' + k_{\mu}) \right\} \quad (14)$$

The finite temperature parts of the photon propagator give additional radiative corrections to the Feynman diagrams involving virtual photons. The photon propagator in the Landau gauge is [1]

$$D_F(q^2) = -4\pi \left[ \frac{i}{q^2 + i\epsilon} + 2\pi f_B(\omega)\delta(q^2) \right] \quad (15)$$

where the phase-space density $f_B(\omega)$ is the Bose-Einstein distribution function for photons. [The factor of $-4\pi$ arises from our use of Gaussian units; in “rationalized” units, this factor in Eq. (15) is replaced by $-1$]

In the rest frame of the incoming hadron, $k = (k^0 = \epsilon = M, \vec{k} = 0)$. Integrating Eq. (7) over scattering angle, we obtain the temperature dependent transport cross section

$$\sigma_t(T) = \frac{1}{2\pi} \frac{m^2M}{|\vec{p}|^2} \int_{E_{\text{min}}'}^{E_{\text{max}}'} dE'(1 - \beta(E')) |\tilde{M}|^2 S(E', \epsilon') \quad (16)$$
\[ \beta(E') = \frac{E'(M + E) - m^2 - ME}{|\vec{p}|\bar{p}}. \]  

For a given initial electron energy \( E \), the kinematical limits for the final-state energy of the electron, \( E'_{\text{min}} \) and \( E'_{\text{max}} \), can be determined from the constraint \( |\cos \theta'| \leq 1 \). The squared spin-averaged scattering matrix element in Eq. (16) becomes

\[ |\mathcal{M}|^2 = 2 \left( \frac{\pi \alpha}{MM} \right)^2 F(E'), \]  

where

\[ F(E') = \left( \frac{2EE' + M(E' - E)}{(E' - E)^2} \right) \left[ F_1^2 - \frac{1}{2} F_2^2 \left( \frac{E' - E}{M} \right) \right] + (F_1 + F_2) \left[ \frac{m^2}{M(E' - E)} \right]. \]  

Since \( q^2 = 2M(E' - E) \) in the hadron rest frame, the second term in Eq. (15) does not contribute in the calculated transport scattering cross section (because \( \beta(E) = 1 \) for \( E' = E \)). From Eqs. (16) and (18), we finally find that the electron-hadron transport cross section at finite temperature is

\[ \sigma_t(T) = \pi \alpha^2 \frac{1}{M|\vec{p}|^2} \int_{E'_{\text{min}}}^{E'_{\text{max}}} dE' (1 - \beta(E')) F(E') S(E', \epsilon'), \]  

where \( |\vec{p}|^2 = E^2 - m^2 \).

For neutron scattering, \( F_1 = 0 \) and \( F_2 = \kappa_n \). Thus, Eq. (19) becomes

\[ F(E') = \kappa_n^2 \left( \frac{1}{2} - \frac{EE' + m^2}{M(E' - E)} \right). \]  

In the case of ultrarelativistic (UR) electrons in which the electron mass can be ignored (\( m \ll E \)), the electron-neutron transport cross section at finite temperature is given by

\[ \frac{\sigma_{\text{ne}}^{\text{UR}}(T)}{\sigma_t^{\text{AHS}}} = \frac{M^2}{6E^2} \int_{E'_{\text{min}}}^{E'_{\text{max}}} dE' \left( \frac{E - E'}{EE'} + \frac{2}{M} \right) S(E', \epsilon'). \]  

Using a Taylor series expansion, at zero temperature and for \( E \ll M \), analytically we can obtain

\[ \frac{\sigma_{\text{ne}}^{\text{UR}}(T)}{\sigma_t^{\text{AHS}}} \approx 1. \]  

We can see this result in Fig. 2 for the numerically evaluated \( \sigma_{\text{ne}}^{\text{UR}}(T) \).

For the scattering of an electron with a proton, Eqs. (13) and (18) yield

\[ F(E') = \left( \frac{2EE' + M(E' - E)}{(E' - E)^2} \right) \left[ 1 - \kappa_p^2 \left( \frac{E' - E}{2M} \right) \right] + (1 + \kappa_p)^2 \left[ \frac{m^2}{M(E' - E)} \right]. \]  

In the case of ultrarelativistic electrons, Eq. (24) becomes

\[ F(E') = \left( \frac{2EE' + M(E' - E)}{(E' - E)^2} \right) \left[ \frac{1 + \kappa_p^2}{2} + \frac{EE'}{M(E' - E)} \right]. \]  

For electron-proton collisions, the most important scattering mechanism is Coulomb scattering and the differential cross section is given by the Mott formula. In which case the Coulomb transport cross section is

\[ \sigma_{\text{pe}}^{\text{Coul}}(T) = 4\pi \alpha^2 \frac{E}{|\vec{p}|^2} \Lambda(T) [1 - f(E)], \]  

where \( |\vec{p}| \) is the electron momentum. Because Eq. (26) diverges at small angles, the usual approximation is to cut-off the angular integration at an angle given by the ratio of the Debye shielding length, \( \lambda_D = (T/e^2 n_e)^{1/2} \), to the thermal wave length, \( \lambda_{th} = (2\pi/mT)^{1/2} \). This defines to Coulomb logarithm \( \Lambda(T) = \ln(\lambda_D/\lambda_{th}) \). With this we can evaluate numerically the transport cross sections for a given initial electron energy.

In the early universe, the number density and energy density of electrons are given by

\[ n_e(T) = g_e \int \frac{d^3p}{(2\pi)^3} f(E), \]  

\[ \rho_e(T) = g_e \int \frac{d^3p}{(2\pi)^3} Ef(E), \]  

where \( g_e \) is the electron degeneracy. For an electron in thermal equilibrium, the phase space occupancy \( f(E) \) is given by the Fermi-Dirac distribution

\[ f(E) = \frac{1}{e^{(E-\mu)/T} + 1}. \]  

where \( \mu \) is the electron chemical potential. In the early universe, \( \mu/T < 10^{-9} \). Therefore, density effects which are parameterized by \( \mu \) are much less significant than temperature effects which are parameterized by \( T \). Hence, we will work in the approximation \( \mu = 0 \).

At high temperature, when the electron can be considered massless (i.e. ultrarelativistic), the average energy of the electron is, \( \langle E \rangle_{\text{UR}} \approx 3.15 T \). But at lower temperatures for which the temperature-dependent electron mass cannot be neglected, the average electron energy \( \langle E \rangle = \rho_e/n_e \) is given by

\[ \langle E \rangle = T \left( \int_{1/z}^{\infty} dy \frac{y^2\sqrt{y^2z^2 - 1}}{e^y + 1} / \int_{1/z}^{\infty} dy \frac{y\sqrt{y^2z^2 - 1}}{e^y + 1} \right), \]  

where \( z = m/T \).
where \( z = T/m \). But for \( T \ll m \), the electron can be considered non-relativistic (\( NR \)), then \( \langle E \rangle_{NR} \approx m + \frac{z}{2} \). Fig. 1 shows the average electron energies \( \langle E \rangle \) and temperature dependent electron mass [see, Eq. (2)]. Here we can see that Eq. (30) goes to \( \langle E \rangle_{UR} \) for \( m_\circ < T \) and becomes \( \langle E \rangle_{NR} \) for \( T < m_\circ \) approximately. Therefore, we can use Eq. (30) for the initial electron energy in numerical calculations of the scattering cross section. Fig. 2 shows the transport scattering cross section for electron-neutron scattering \( \sigma_{ne}(T) \) as a function of \( x = T/m_\circ \), for the initial electron energy \( E = \langle E \rangle \) [Eq. (30)]. Fig. 3 shows the transport scattering cross sections for electron-proton scattering \( \sigma_{pe}(T) \) and the Coulomb scattering \( \sigma_{Coul}(T) \) as a function of \( x = T/m_\circ \) for an initial electron energy \( E = \langle E \rangle \) [Eq. (30)].

### IV. CONCLUSIONS

We have calculated temperature-dependent electron-hadron transport cross sections. These are important, for example, in the calculation of baryon diffusion coefficients at finite temperature. The major motivation here has been to investigate whether finite temperature effects can significantly change the baryon transport cross section \( \sigma_t \). In this work, we have treated hadrons as particles which have an internal structure and an anomalous magnetic moment. Also, we have assumed that their internal structure is not affected by the finite temperature since their mass is more than about 1000 times the temperature of interest.

Two major features of the finite temperature effects on the light particles have been included in the calculation: (1) finite temperature Dirac spinors which are recast into the form of an effective electron mass ; (2) finite temperature modifications to the phase space distribution of the electrons. We find that, for \( m_\circ < T \), both \( \sigma_{ne}(T) \) and \( \sigma_{pe}(T) \) approach the ultrarelativistic limit (where the electron mass can be ignored). In the case of electron-proton scattering, we have compared it with the Coulomb scattering cross section at finite temperature. In particular, for the case of electron-neutron transport scattering, we find \( \sigma_{ne}(T)/\sigma_{ne}(T=0) \approx 5 \) at \( T \approx 0.1 \text{ MeV} \).

In conclusion, the baryon diffusion coefficients which affect baryon inhomogeneities during big-bang nucleosynthesis could be changed significantly by our temperature dependent electron-hadron transport cross sections. Up to the time of weak decoupling (\( T \approx 1 \text{ MeV} \)) there is little change in the cross sections. However, during the epoch of nucleosynthesis (\( T \lesssim 0.2 \text{ MeV} \)) when baryon diffusion is most important, the transport cross sections increase as the temperature decreases. On the other hand, baryon diffusion at low temperature is strongly affected by proton-neutron scattering for which these finite temperature effects are insignificant. Clearly, a study of the effects of these new cross sections on the baryon diffusion coefficients and inhomogeneous primordial nucleosynthesis is desired. These will be the subject of a subsequent paper.

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FIGURE CAPTIONS

FIG. 1 The average electron energy $\langle E \rangle$ and temperature-dependent effective electron mass (dot line). The solid line denotes $\langle E \rangle_{UR}$ and the long dashed-dot line is $\langle E \rangle_{NR}$. The dashed line denotes $E = \langle E \rangle$ [Eq. (30)].

FIG. 2 Transport scattering cross section for electron-neutron scattering $\sigma_{ne}(T)$ (dashed line) compared with the ultrarelativistic approximation $\sigma_{ne}^{UR}(T)$ (solid line) in units of $cm^2$ as a function of $x = T/m_0$. This calculation assumes an initial electron energy $E = \langle E \rangle$ [Eq. (30)].

FIG. 3 Transport scattering cross section for electron-proton scattering $\sigma_{pe}(T)$ (dashed line) and ultrarelativistic case $\sigma_{pe}^{UR}(T)$ (solid line) and Coulomb transport cross section $\sigma_{pe}^{Coul}(T)$ (dot line) in units of $cm^2$ as a function of $x = T/m_0$, assuming an initial electron energy $E = \langle E \rangle$ [Eq. (30)].
Fig. 1
Fig. 2
Fig. 3