Fiber optic gyro de-noising based on VMD algorithm

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Abstract. In order to utilize the fiber optic gyroscope (FOG) in the inertial navigation system, it is necessary to eliminate the noise of FOG correctly and precisely. A de-noising method for signal of FOG based on variational mode decomposition (VMD) algorithm is proposed in this paper. Experiment result indicate that the VMD filter can de-noise the signal of FOG effectively. The amplitude of various random errors in the output can be obtained through Allan variance method. Comparing the error coefficients value before decomposition-reconstruction procedure with that after the procedure, the effectiveness of VMD filter can be verified.

1. Introduction
The precise model of gyroscope is vital to inertial navigation systems, and the de-noising signal of the gyroscope could affect equipment readiness and safety. Many different de-noising methods for the signal of the gyroscope have been proposed. The analysis based on wavelet is widespread through the research in de-noising method of FOG[1]. However, the process of the wavelet analysis is too complex. There may be some fake harmonic waves that are introduced into the analysis due to the limits of the algorithm, and these fake harmonic waves could make the analysis have no physical meaning. Neural network algorithm[2] is also used widely in de-noising the signal of the gyroscope, and the algorithm could get compensation models containing all nonlinear factors. But the accuracy of the model would degrade when the signal contains large randomness, and it isn’t suitable for the analysis of the signal of gyroscopes in the vibration environment. Empirical mode decomposition (EMD) algorithm was proposed by Huang N. E in 2003. This algorithm[3] and its improved method[4] have been used in the de-noising the signal of gyroscopes effectively. The reaction of EMD to noise and sampling is sensitive. The minimum and maximum calculations at each stage linger the process and the cubic spline method of interpolation adds up error to the analysis. EMD fails to decompose close multi-tone signals. In view of above questions, a modeling method based on variational mode decomposition is proposed. It could solve above question and improve the accuracy of the fitting curves of the reconstructed signal.

2. Data Aanalyzing Methods

2.1. Brief Description of VMD filter
Variational mode decomposition decomposes the real value signal into discrete number of sub-signals (modes), and the definition of modes is the same as that defined in [5].

\[ \mu_k = A_k(t) \cos \phi_k(t) \]  

(1)

Where \( A_k \) is the instantaneous amplitude of \( \mu_k(t) \).
\[ \omega_k(t) = \phi_k(t) = \frac{d\phi_k(t)}{dt} \]  

\( A_k(t) \) and \( \omega_k(t) \) changes more slowly than \( \phi_k(t) \), and \( \mu_k(t) \) could be consider as harmonic signal with amplitude \( A_k(t) \) and frequency \( \omega_k(t) \) during the period \([t - \delta, t + \delta]) \) . This algorithm also has certain sparsity property while producing the decomposed signal.

The signal \( x(t) \) is decomposed into several modes \( u(k) \), and the central frequencies of the modes are presented as \( \omega_k \). The decomposition is given as follows:

\[
\min_{\{u_k\}_{k=1}^{n}} \left\{ \sum_k \left\| \delta \left( \delta(t) + \frac{j}{\pi t} \ast u_k(t) \right) \right\|_2^2 \right\}
\]

\[ s.t \sum_k u_k = x \]  

Where \( \{u_k\} = \{u_1, \ldots, u_k\} \) and \( \{\omega_k\} = \{\omega_1, \ldots, \omega_k\} \) are the set of the modes and their center frequencies. Quadratic penalty term and Lagrangian multipliers \( \lambda \) are used to reconstruction constraint of signal \( x(t) \). Therefore, the augmented Lagrangian \( \mathcal{L} \) is introduced

\[
\mathcal{L}(\{u_k\}, \{\omega_k\}) = a \sum_k \left\| \delta \left( \delta(t) + \frac{j}{\pi t} \ast u_k(t) \right) \right\|_2 \]

\[
+ \left\| f(t) - \sum_k u_k(t) \right\|_2 + \lambda \left(t \right) f(t) - \sum_k u_k(t) \right\|_2 \]

The solution of (1) is found as the saddle point of the augmented Lagrangian \( \mathcal{L} \) in alternate direction method of multipliers (ADMM) [6-8]optimization algorithm. The algorithm is summarized as follows:

Initialize \( \{u^1_k\}, \{\omega^1_k\}, \lambda ^1, n \leftarrow 0 \)

Repeat  
for \( k = 1:K \) do
update \( \hat{u}_k \) for all \( \omega \geq 0 \):

\[
\hat{u}_k^{*+1} \leftarrow \frac{\hat{x}(\omega) - \sum_{i=1}^{k} \hat{u}_i^{*+1} + \sum_{i=1}^{k} \hat{u}_i^{*} + \frac{\hat{\lambda}^{*}(\omega)}{2}}{1 + 2a(\omega - \omega_k^{*})^2} \]

update \( \omega_k^{*} \):

\[
\omega_k^{*+1} = \frac{\int_0^{\infty} \omega |\hat{u}_k(\omega)|^2 \omega d\omega}{\int_0^{\infty} |\hat{u}_k(\omega)|^2 \omega d\omega} \]

end for.
Dual ascent \( \omega \geq 0 \):

\[
\hat{\lambda}^{*+1}(\omega) = \hat{\lambda}^{*}(\omega) + \tau \left( \hat{x}(\omega) - \sum_k \hat{u}_k^{*+1}(\omega) \right) \]

Until convergence: \( \sum_k \left\| \hat{u}_k^{*+1} - \hat{u}_k^* \right\|_2 / \left\| \hat{u}_k^* \right\|_2 < \epsilon \).

2.2. Allan variance

Allan variance (AVAR) is the standard method for parameter analysis of optical gyroscope recognized by IEEE[9]. Allan variance defines five basic noise terms and these terms are expressed in a notation appropriate for optic gyroscopes data reduction. The five terms are quantization noise, angle random walk, bias instability, rate random walk, and rate ramp.
Assume that there are $N$ data points from FOG during the sampling time, and each point has a sample time of $t_0$. Form a group of $k$ consecutive data arrays (with $k < N/2$), and define $\Omega(t)$ as the angular velocity outputted by FOG. The random variances clusters of the average values differences between adjacent arrays can be defined as (13):

$$A(t_0, \tau) = \overline{\Omega}_{t_0+\tau} - \overline{\Omega}_{t_0}$$  \hspace{1cm} (8)

Where $\overline{\Omega}_{t_0+\tau}$ and $\overline{\Omega}_{t_0}$ is the average angular velocity at time $t_0 + \tau$ and $t_0$. Then Allan variance could be defined as follows

$$\sigma^2(\tau) = \frac{1}{2} < A(t_0, \tau)^2 > = \frac{1}{2} < \overline{\Omega}_{t_0+\tau} - \overline{\Omega}_{t_0} >$$  \hspace{1cm} (9)

where $< >$ presents overall average. Different kinds of random processes can be tested by different sampling time $\tau$. Therefore, Allan variance can provide a method for distinguishing and quantifying the different noise terms in the signal.

2.3. Continuous mean square error

Assume that deterministic signal $x(t)$ is interfered by an additive white noise $z(t)$:

$$y(t) = x(t) + z(t)$$  \hspace{1cm} (10)

For observed signal $y(t)$, the main purpose of filtering is to obtain the approximate signal $\tilde{x}(t)$ of the deterministic signal $x(t)$ and to guarantee that the mean square error (MSE) between the deterministic signal and the approximate signal is minimum.

$$MSE(y, \tilde{y}) = \frac{1}{N} \sum_{i=1}^{N} [y(t_i) - \tilde{y}(t_i)]^2$$  \hspace{1cm} (11)

$$y = [y(t_1), y(t_2), ..., y(t_N)]^T \hspace{1cm} (12)$$

$$\tilde{y} = [\tilde{y}(t_1), \tilde{y}(t_2), ..., \tilde{y}(t_N)]^T \hspace{1cm} (13)$$

Where $N$ is the length of the signal. Because it is hard to obtain $x(t)$, continuous mean square error (CMSE) is usually regarded as assessment criteria of the filtering result.

$$CMSE(\tilde{y}_k, \tilde{y}_{k+1}) = \frac{1}{N} \sum_{i=1}^{N} [\tilde{y}_k(t_i) - \tilde{y}_{k+1}(t_i)]^2$$  \hspace{1cm} (14)

$$k = 1, ..., C-1$$

Equation (14) can be simplified as follows:

$$CMSE(\tilde{y}_k, \tilde{y}_{k+1}) = \frac{1}{N} \sum_{i=1}^{N} [imf_k(t_i)]^2$$  \hspace{1cm} (15)

CMSE criteria will improve SNR of $x(t)$ as high as that of the kth IMF. Therefore, the mathematical expression of index $j$, could be presented

$$j_* = \text{argmin}_{1 \leq k \leq C-1} \left[ CMSE(\tilde{y}_k, \tilde{y}_{k+1}) \right]$$  \hspace{1cm} (16)

Where $\tilde{y}_k$ and $\tilde{y}_{k+1}$ are reconstructed signal of IMF at index $k$ and $k+1$, respectively.

3. Result and discussion

FOG data $y(n)$ is sampled from Flagship-15 compass, and the sampling rate is 100Hz. The length of the data is 900000, i.e. the sampling time is 9000 seconds. The FOG data, its double logarithm Allan variance curve and the result fitting curve are shown in Figure 1.
Figure 1. The original signal of the FOG and its Allan variance analysis

The parameters in VMD filter are set as follows: $K = 5$, $a = 200000$ and $\tau = 0$. The signal of FOG is de-noised with five levels of decomposition to the output noise. Figure 2 shows the five IMFs decomposed from the signal of FOG.

Figure 2. The five IMFs decomposed from the signal of FOG

Figure 3 shows the spectrum of the five IMFs. This figure shows clearly that the most energy of the signal is concentrated mainly in the low frequency region.

Figure 3. The spectrum of five IMFs decomposed from the signal of FOG

The reconstructed signal and its Allan variance analysis is shown in figure 4.

Figure 4. The reconstructed signal and its Allan variance analysis
by comparison between identified error coefficients in table 1, we can find that all error coefficients reduce clearly, and it means VMD filter could de-noise the signal of FOG and improve the accuracy of the FOG measurement effectively.

Table 1. Identified error coefficients for original signal and reconstructed signal

|                  | quantization noise (arcsec) | angle random walk (deg/√sec) | bias instability (deg/sec) | rate random walk (deg/sec²) | rate ramp (deg/h²) |
|------------------|-----------------------------|-----------------------------|---------------------------|----------------------------|-------------------|
| Original Signal  | 7.4860e-7                   | 4.8190e-5                   | 8.7337e-6                 | 7.8012e-7                  | 7.3450e-9         |
| Reconstructed    | 1.8058e-7                   | 5.1361e-7                   | 8.3765e-6                 | 3.5266e-7                  | 4.7113e-9         |

4. Conclusion
A de-noising method for signal of fiber optic gyroscope based on variational mode decomposition algorithm is proposed in this paper and this de-noising method could limit the noise level at the output of FOG. Experimental results demonstrate that VMD filter could reduce quantization noise, angle random walk. However, the effect for de-noising bias instability, rate random walk and rate ramp is insignificant. This method could be applied to not only FOG, but also other inertial sensors such as accelerometer. It suggests that the accuracy of inertial navigation system could be improved when the inertial navigation system utilizes a pre-de-noising stage based on VMD filter during its alignment and navigation procedure. The selections of mode number \( K \) and other parameters still lack the mathematical bases, and these selections are still empirical. These problems should be studied in the future work.

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