Assessment of Multiaxial Fatigue Damage Criteria Based on the Critical Plane Concept in Micro-EHL Line Contact.

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Abstract. Micropitting is a fatigue failure phenomenon that concentrates at the surface roughness level between interacting surfaces. This type of surface fatigue is commonly recognized to exist in gears and bearings, where specific film thicknesses are sufficiently low that the rough surfaces run in the EHL condition, where the direct asperity contacts are prominent. This paper is an experimental and theoretical study to investigate a number of fatigue failure theories concerning the multi-axial fatigue models which are depended on a critical plane analysis in the mixed lubrication regime. These failure theories are namely the Findley, the Matake, the Dang Van, McDiarmid and, Fatemi and Socie model, where they are used to perform fatigue investigation for Micro-EHL contacts problem. Numerical analysis to investigate the cumulative damage and fatigue parameter in a Micro-EHL contact is established in this paper. The results of applying failure theories have indicated that the different multiaxial fatigue criteria adopted provide significant results for contact analysis in lubricated conditions, and they are more relevant to the applications of the rough surface Micro-EHL mode.

1. Introduction

This study is an experimental and theoretical study to investigate a number of fatigue failure theories concerning the multi-axial fatigue models which are depending on a critical plane approach. The major objective of the current work is to calculate the fatigue parameter \( FP \) and accumulated damage \( D \) by using the following fatigue criteria: the shear strain model employed by Fatemi and Socie, Findley, Matake, McDiarmid and, Dang Van.

These theories are considered important and play a key role in the fatigue damage calculation for rough surfaces which experience elastohydrodynamic lubrication (EHL) condition. In such a situation, the oil film thickness which is estimated between two smooth contacting bodies is relatively small comparing with the height of surfaces asperity. The percentage of the computed smooth surface film thickness to the standard deviation of the composite surface roughness from its mean line is called \( \Lambda \) ratio. In general, this ratio in rough gear applications is less than \( 0.1 \). If \( \Lambda > 3 \) surface roughness is generally regarded as not having any significant influence on the pressure distribution, and the probability of direct contact between asperity features on the surfaces is remote. Furthermore, as \( \Lambda \) is reduced progressively there will be an increasing probability of surface asperity interactions either in the form of micro EHL lubricated contacts with very thin films or in the form of direct asperity contact.
via a boundary film. Under such operating conditions, the pressure distribution due to the direct contact of rough surfaces will be much higher than the pressure generated in the case of smooth surfaces.

There is a lot of research on the issue of fatigue failure and the effect of the rough surface on fatigue damage. In addition, numerous EHL numerical models of the rough surface which having the transient effect have been studied [1, 2 and 3]. Zhao solved numerically the point contact case by breaking the total contacting zone into zones of either solid interaction or separation by using lubricant [4]. A study about asperity contact problems has been done by [5], where a simple iterative algorithm was used to examine the rough surface, as well as contact pressure, was investigated. This pressure was used to predict a surface deformation based upon the hypothesis that the two surfaces behave as elastic half-spaces. It was found that this pressure differs significantly from the smooth case situation.

Many researchers have been using the transient roughness behaviour to investigate the transient EHL problem to monitor and evaluate the change of pressure and lubricant film within the EHL contact analysis. Holmes et al. [6 and 7] simulate the interaction of two rough profiles in point contact with rough surfaces. Holmes used a novel technique, which is called the differential deflection formulation to examine the EHL analysis. This approach has been used by many researchers because it gives a realistic and effective rough contact model result. Preliminary work was carried out by Tao et al. [8] to calculate the failure due to micropitting phenomena by using a rheological model based on Eyring non-Newtonian shear thinning. This paper showed factors obtained from the mixed EHL analyses that specify the degree of distress developed by the surfaces in Elastohydrodynamic contact conditions.

Qiao [9] and Al-Mayali et al. [10] investigated a failure of a rough surface in an Elastohydrodynamic lubrication regime. By adopting a line contact problem, the fatigue damage was examined by using a critical plane-based multi-axial fatigue approach. In addition, the relationship between fatigue damage and changes in working parameters such as lubricant viscosity and roughness effects were also investigated. The authors found that the fatigue life highly affected by surfaces with significant roughness features.

To study the roughness analyses, transient roughness mode has to be explored in order to simulate the effect of pressure and film thickness on the subsurface stresses and fatigue failure. Holmes et al. [11] was simulating the interaction of two rough profiles in point contact with asperity features. Holmes used a new method, which is called the differential deflection formulation to examine the EHL analysis.

Further analysis concerning contact fatigue has been done by Li and Kahraman [12], this research provided an approximation for a pitting crack, which is proposed for non-conformal point contact analysis. Elastic stress fields, as well as the surface roughness effects, were discussed. The crack initiation was predicted by a multi-axial fatigue criterion, while the location of the crack nucleation site was investigated by the Lagrangian–Eulerian approach. The results provided a very good agreement between the experiment and theory work. Evans et al. [13] made a further investigation to the problem of mixed EHL contact by adopting a novel numerical solution approach which is appropriate for issues of rough-surface thin-film conditions. Experiment and numerical study on micropitting was performed by AL-Mayali et al. [14]. They reached the conclusion that the running-in process leads to a rapid occurrence of permanent deformation during the first stage of running-in. They used a commercial finite element software (ABAQUS) to determine the residual plastic deformation which occurs at the asperity peaks. A numerical study has been conducted by Elcoate et al [15] to explore the micro-EHL line contact problem. This paper provided an important opportunity to advance the understanding of the micro-EHL problem. In this research, the researcher used a new numerical method to solve the problem of elastohydrodynamic lubrication by using the differential deflection formulation with fully coupling. This kind of mixed lubrication analysis for asperity contact has been improved by Holmes et al. [11]. In this method coupling using differential deflection equation incorporated without having a full matrix so that the solution of the matrix can be happen very rapidly.
The present study is reporting the application of this novel analysis technique to the simulation of real rough surfaces. Furthermore, the aims of this study are to determine the fatigue parameter (FP) and fatigue damage by using the following fatigue criteria: the shear strain model employed by Fatemi and Socie, Findley, Matake, McDiarmid, and Dang Van.

2. Hypothesis and method

2.1. Micro-Elastohydrodynamic Analyses

Assessment of the various fatigue criteria depends on the solution of a micro-EHL for a pair of roughness in sliding-rolling mode.

This paper is based on numerical analysis of micro-EHL that adopt a novel method developed by Evans et al. [16]. This method is depending on fully coupling the Reynolds and elastic equation. These equations are shown below:

\[ \frac{\partial}{\partial x} \left( \rho h^3 \frac{\partial p}{\partial x} \right) - \frac{\partial (\rho \bar{u}h)}{\partial x} - \frac{\partial (\rho h)}{\partial t} = 0 \]  

(1)

Where S, in the above equation is the non-Newtonian flow factor. So, the elastic deflection equation written in differential form,

\[ \frac{\partial^2 h}{\partial x^2} = \frac{\partial^2 \phi}{\partial x^2} + \frac{1}{R} + \frac{2}{\pi \alpha'} \sum_{i} \sum_{k} f_{ik} f_{ik-1} p_{ik} \]  

(2)

Where \( \phi (x,t) \) in equation (2) is the surface roughness composite. The elastic equation has been written in this such form so as to achieve the advantage of rapidity. This will achieve when the influence coefficients \( f_{ik} \) set to zero as the indice \( i \) rises from zero.

The viscosity equation is rearranged to the form,

\[ \eta = \eta_0 \{ \exp \{ \ln(\eta_0/\kappa) \ (1 + \chi p)^2 - 1) \} \} \]  

(3)

The density which is given by the Dowson and Higginson formula [17], shown below

\[ \rho = \rho_0 \left( 1 + \gamma p \right) \left( 1 + \lambda p \right) \]  

(4)

The objective of this article study is to execute micro-EHL numerical simulations for different fatigue models by taking into account the operational conditions of the twin disk rig tests by simulating the rolling/sliding contact of real roughness profiles.

The surface roughness profiles used throughout the study are acquired from profilometer traces obtained from unrun and run-in ground disk surfaces at Cardiff University. The profiles were measured using Talysurf profilometer which is shown in figures 1 and 2 and were taken across the face of the slow and fast disks. Measurements were taken from the un-run and run face of the ground disks in order to model the development of the surface as running in takes place. The original profiles were then filtered to remove the form of the disk from the profile, leaving only the surface roughness, before being smoothed to remove the high-frequency noise. The smoothed version of the 1-mm representative sections shown in figures 3 and 4 were imported into the EHL program simulation, to be prepared for contact loading and then for fatigue failure analysis. The EHL profile length is adjusted so that it begins and ends at a valley feature. The multi-profile formed from the representative profile by repeating the 2a length of roughness profile for each disk as shown in the figure 5, where a can be defined as the nominal Hertzian contact semi-dimension. This process is important to experience all possible asperity interactions. The join for the multi-profile is made at a valley feature in order to ensure that no artificial asperity is formed and to experience all possible asperity interactions. Micro-EHL analyses carried out for interacting profiles i.e. fast profile against the slow profile to experience all possible asperity interactions.
Figure 1. Talysurf setup on stage for measuring un-run and run face of the fast and slow disks. Cardiff university Lab - UK.

Figure 2. Test head of disk machine.

Figure 3. Real roughness profile section used in the micro-EHL simulation for the slow disk.
Figure 4. Real roughness profile section used in the micro-EHL simulation for the fast disk.

Figure 5. Representative profile for the slow disk as in example.

Figures 6 and 7 show two typical results obtained from the analysis of transient EHL – mixed lubrication of two real profiles. They show results for two different values of the lambda ratio ($\Lambda$) ratio which defined as,

$$\Lambda = \frac{h_{\text{min}}}{\sqrt{(R_{q_1}^2 + R_{q_2}^2)}}$$

Where $h_{\text{min}}$ is the theoretical film thickness which is determined by using the Dowson and Higginson equation with assumption for smooth surfaces. In addition, ($R_{q1}$) and ($R_{q2}$) are the standard deviation of the real rough profiles from its mean line.

Figures 6 and 7 correspond to the same timestep for each Micro-EHL simulation and illustration the contact pressure (red curve) and thickness of the lubricant film (blue curve) at that time step. In addition, these figures show the two-rough contacting profile (green lines) and shown offset below in their relative positions at particular timestep. The single most prominent observation to emerge from this simulation comparison is as $\Lambda$ is reduced progressively there will be an increasing probability of surface asperity interactions either in the form of micro EHL lubricated contacts with very thin films, or in the form of direct asperity contact via a boundary film. For example, in figure 6 there is only an asperity interaction occurring at $x/a = -0.875$. In the figure 7 the asperity interactions are effect by reducing the contact Lamda value to 0.1 and it is very clear that the two rough profiles in this case are seen to be in much closer asperity interaction. Several asperity interactions existing, for example at $x/a = -0.88$, -0.375, -0.125, 0.187 and 0.625.
Figure 6. Lubricant Film (blue line) and contact pressure (red line) for a single timestep, $\lambda = 0.5$ with slow and fast rough profiles (green lines) shown offset down.

Figure 7. Film thickness (blue line) and contact pressure (red line) for a single timestep, $\lambda = 0.1$ with slow and fast rough profiles (green lines) shown offset down.
2.2. Modelling of Elastic stress and high-cycle fatigue damage.

The evaluation of stress cycling in terms of subsurface elastic stress and strain is considered to be highly important for the simulation of micro-EHL. Thus, a precise and effective assessment of the stress history which including in the Micro-EHL analysis is significant in order to carry out a reasonable fatigue damage estimation. The results of Micro-EHL numerical simulation for each timestep which they are known as contact pressure, lubricant film and shearing stress are then used to find surface stress and strain components at every point in the representative block of material.

The following integrations are adopted to evaluate the instantaneous distribution of surface elastic stress and strain components, Johnson, [18], by using equations below:

\[\tau_{xz} = (-2z^2/\pi) \int_{p>0} \left( (p(s) \times (x-s) ds) \right) \times \left( \left( \frac{2z^2}{\pi} \right) \int_{r>0} \left( \tau(z) \times (s-s) ds \right) \right) \times \left( \left( \frac{2z^2}{\pi} \right) \right) \]

\[\sigma_z = (-2z^2/\pi) \int_{p>0} \left( \left( \frac{p(s) ds}{((x-s)^2+z^2)^2} \right) \right) \times \left( \left( \frac{2z^2}{\pi} \right) \int_{r>0} \left( \tau(z) \times (s-s) ds \right) \right) \times \left( \left( \frac{2z^2}{\pi} \right) \right) \]

Surface roughness topography leads to increase the level of stresses on surfaces and areas near the surfaces, and these stresses are at values higher than the maximum Hertzian contact pressure. These stresses decrease as we move further down from the rough surface and this is more likely to occur due to the lower pressure zones at that level. This may be visibly in the findings of stress simulations which are presented in figures 8 through 12. These figures display the effect of different velocity of the slow rough surface on the components of transient sub-surface stress. The focus will be given when analysing stresses and fatigue damage on the slow surface because it is exposed to a greater number of loading cycles compared to the faster surface. Significant stress concentrations can be seen in figures (8-12) in plot b and c within the sub-surface layer, which create directly by the asperities of the rough surface. In figures 8 to 12, b and c, severe stress concentrations corresponding to \(\tau_{xz}\) and \(\sigma_{zz}\) are noticed at the sub-surface over areas with \(z/a < 0.05\). It is interesting to note that all values of normal stress components \(\sigma_{zz}\) in all figures are negative everywhere and that because it’s under a compression loading. This result has further strengthened our confidence in the hypothesis that high stress concentration near the surfaces is causing by the surface irregularities. These stress components are employed to explore for the critical plane and evaluate the subsurface micropitting and accumulated damage which they are causing by the intense cyclic loading, as we will see in the next section.
Figure 8. Model predictions of the Micro-EHL transient simulation of the slow rough surface at a velocity of 200 rpm, (a) contact pressure, (b) stress fields ($\tau_{xz}$) and (c) stress fields ($\sigma_{zz}$).

Figure 9. Model predictions of the Micro-EHL transient simulation of the slow rough surface at a velocity of 500 rpm, (a) contact pressure, (b) stress fields ($\tau_{xz}$) and (c) stress fields ($\sigma_{zz}$).
Figure 10. Model predictions of the Micro-EHL transient simulation of the slow rough surface at a velocity of 1000 rpm, (a) contact pressure, (b) stress fields ($\tau_{xz}$) and (c) stress fields ($\sigma_{zz}$).

Figure 11. Model predictions of the Micro-EHL transient simulation of the slow rough surface at a velocity of 1500 rpm, (a) contact pressure, (b) stress fields ($\tau_{xz}$) and (c) stress fields ($\sigma_{zz}$).
Figure 12. Model predictions of the Micro-EHL transient simulation of the slow rough surface at a velocity of 2000 rpm, (a) contact pressure, (b) stress fields ($\tau_{xz}$) and (c) stress fields ($\sigma_{zz}$).

The transient subsurface stress and strain fields for the model block of material are used for post-processing to examine the accumulation damage of the rough profile at the surface and subsurface level.

Five fatigue criteria based on the critical plane approach, namely, Fatemi and Socie, Findley, Matak, McDiarmid and DangVan, are employed to the transient Micro-EHL contact fatigue solutions. The model proposes by Fatemi and Socie, [19] is given in equation below which estimate the number of effective loading cycles necessary for fatigue failure to occur ($N_f$).

$$\frac{\Delta\sigma_{\text{max}}}{2} \left(1 + k \frac{\sigma_{\text{max}}}{\sigma_y}\right) = \frac{\tau'_{f}}{G} \left(2N_f\right)^{\psi} + \gamma'_{f} \left(2N_f\right)^{\psi}$$

Therefore, the fatigue damage may be written for each effective loading cycle as:

$$\text{damage}_{\text{cycle}} = \frac{1}{N_f}$$

According to the Palmgren-Miner [20] equation, the fatigue failure will occur when the total damage ($D$) value equal to unity.
\[ D = \frac{1}{\sum_{\text{all effective loading cycles}} N_f} \]

where \( \gamma_{\text{max}} \) is amplitude of shear strain, \( \sigma_{\text{max}} \) the tensile stress, variable \( k \) is a material constant, and \( G \) is considered as a shear modulus. The operating variables used in accumulation damage modelling are shown in table 1. The material used in this simulation is SAE4340 steel, as considering by Zahavi and Torbilo [21].

| Table 1. operating variables of SAE4340 steel material. |
|---------------------------------------------------------|
| (exponent of fatigue strength) - b.                      | (-0.091) |
| (Fatigue ductility exponent) - a.                        | (-0.6)   |
| (Material constant)-k.                                   | (1.00)   |
| (Yield strength) - \( \sigma_0 \).                       | (827)    |
| fatigue ductility coefficient                            | (0.48)   |
| shear fatigue strength coefficient                       | (2 ) GPa |

Four other criteria based critical plane theory are employed in this paper to solve the Micro-EHL fatigue contact problem of the real rough surface. These criteria are the Matake, the Findley, the Mc Diarmid and DangVan. These fatigue criteria can be expressed in a general equation of the standard form as,

\[ A + kB \leq \bar{\lambda} \]

These criteria state that Fatigue failure will happen if the inequality is not satisfying. Therefore, the fatigue parameter may be written as,

\[ FP = \frac{A + kB}{\bar{\lambda}} \]

The variables A and B can be defined as the stress field on the critical plane. In addition, \( \bar{K} \) and \( \bar{\Lambda} \) are considered as material coefficients. These constants take various values for each criterion. The endurance limits tests is used to estimate the material coefficients considering fully-reversed torsion and bending. These criteria are based on the assumption that the failure mode depends on two important variables, namely shear stress and maximum normal stress. However, the shear strain model (Fatemi and Socie’s ) assume that the failure is driven by shear strain which is located on the critical plane and the normal stress.

The fatigue simulation results for the slow surface are presented in figures 13 to 16 for criteria considered. The slower disk is studied because it has been exposed to a greater number of loading cycles compared to the faster surface during asperities contact.

Figures 13 to 16 show contours plot of calculated \( FP \) for the slow profile, as well as the corresponding slow surface profile using for criteria considered. The fatigue damage is assumed to occur for \( 10^7 \) stress cycling when the FP is equals to value of one. What is interesting in this data is that the failure areas are located near the surface and subsurface of aggressive asperity topography for the four criteria considered. In figure 13 there is a clear trend of increasing the zones of high \( FP \) values calculated at a depth of \( z/a < 0.025 \). These results are significant at \( x/a = -0.9, -0.8, -0.6, -0.1, 0.1, 0.3 \) and 0.7 levels. Damage maps value can be seen to be growing and localized. The variation of damage value can be demonstrated as a contour plot. Fatigue failure zones (in dark blue colour) are those with \( FP \geq \)
1. Deeper investigation of figure 13 getting that the FP data have their highest difference at and close the asperities tips. This finding is in good agreement with Evans et al. [13] findings which showed that the high FP areas are localized near the subsurface of particular asperity features, which have sizes commensurate with the scale of asperities dimensions.

Figure 14 displays the contours of FP values based on the Matake fatigue model. The Matake criterion estimates a severely more aggressive damage. This occurs because it is driven by the shear stress, therefore, it can be considered as more responsive to the roughness effect. It can be clearly seen that there are fatigue zones between (0.2-0.4) shown in red colour in figure 14 located near the asperity feature. The red areas of fatigue damage are surrounding by a fatigue which is not subjected to the same value of FP. This indicated that the micro-pitting mode could be occurred at this level of asperities depth.

Figure 15 displays the contours of FP values based on the Dang Van fatigue model. It can be noticed that the Dang Van model gives almost results somewhat relatively lower than the Matake and Findley models. Figure 16 displays the contours of FP values based on the McDiarmid fatigue model. This criterion shows that the shear stress has a significant effect on the FP values. This FP is remarkable at the asperities surface level z/a < 0.025. Evaluation and analysis of results show that there are differences of an order of magnitude in the estimated FP regarding four criteria used.

![Figure 13](image)

**Figure 13.** Slow surface: Findley contours of FP for $10^7$ loading cycles, the top plot is the slow profile.
Figure 14. Slow surface: Matake contours of FP for $10^7$ loading cycles, the top plot is the slow profile.

Figure 15. Slow surface: Dang Van contours of FP for $10^7$ loading cycles, the top plot is the slow profile.
Figure 16. Slow surface: McDermid contours of FP for $10^7$ loading cycles, the top plot is the slow profile.

Figure 17 displays contour of accumulated damage based on Fatemi-Socie shear strain model for the slower profile during moving inside the EHL contact region. It can be seen that the slow profile subject to a damage (D) value of $10^{-5}$ which indicates fatigue in $10^5$ cycles. Fatemi-Socie shear strain model is recognized as being the most relevance criterion in the mixed lubrication situation. Thus, it is suitable to employ the variable amplitude multiaxial life prediction criteria to investigate the influences of surface roughness on surface damage analysis.

Figure 17. Slow surface: Contour of damage accumulation by using Fatemi-Socie shear strain model.
Figure 18 shows a comparison of the roughness profiles which were acquired from experiment of micropitting. These profiles data taken before and after each loading case. It can be noted that a plastic deformation has taken place in the initial loading cycles and some asperities replaced by new valley features which is shown in fatigue profile. This new valley feature can be considered as a result of micropitting initiation. This can be seen at \( x = 12193 \ \mu m \) where the new valley feature has been created at a depth of \( z = -0.5\mu m \). In addition, the depth of micropitting (new valley) depends on how much material has been removed.

![Figure 18](image)

**Figure 18.** A comparison of the roughness profiles showing formation of a micropit: unrun profile, Load stage 1, Load stage 2, Load stage 3 and Fatigue profile for slow disk.

**Conclusions.**

- This paper provides estimations of fatigue failure using the critical plane approach. The micro-EHL analysis is based on using real rough surfaces.
- Corresponding experimental work which shown in the figure 18 showed that a new valley feature has been created which is considered as a result of micropitting initiation.
- The aggressive asperities spikes are subjected to significant damage risks at asperities scale.
- The results have shown that velocity has a greater influence on fatigue failure calculation.
It has been believed that the variable amplitude multiaxial fatigue criterion is more suitable for fatigue modelling of lubricated contact problem considering real rough surfaces.

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