A Fluctuation Dissipation Relation for Relativistic stars

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A fluctuation-dissipation relation for perturbed configuration of a relativistic star is obtained. The stochastic fluctuations of the classical stress tensor comprising the matter content (a perfect fluid) of the relativistic star, act as the source in a classical Einstein-Langevin equation describing the system. We discuss the linear response of these fluctuations of the stress tensor and develop a fluctuation-dissipation relation from the first principles. Thus a system-bath separation in terms of spacetime metric and matter content is proposed, to study equilibrium and non-equilibrium statistical properties for a relativistic star. This is not derived from or related to the scalar fields or quantum stress tensors and their fluctuations, as is usually the case for semiclassical stochastic gravity.

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I. INTRODUCTION

Perturbations in relativistic stars [1,2] and black holes [3,4,5,7,8] comprise a vast literature, addressing stability issues and oscillations as in the area of asterosiesmology.

Often the compact stars are modeled by perfect and imperfect fluids, via the classical stress tensor, which can be treated as a random quantity, with randomness showing up in physical variables. In this article we follow the theoretical framework, that of taking into account the fluctuations of the classical stress tensor of a perfect fluid and its effects in the interior of the star. Though at a first glance such fluctuations in the matter field of the star may seem to be of negligible astrophysical significance, a closer look at adopting this approach indicates that these could be important towards the end or critical phases of dynamical collapse. It is to realise that the near equilibrium and non-equilibrium configurations would be affected by these fluctuations and their characterization.

Thus including stochasticity is a basic step further for the statistical analysis, using linear response theory with a system-bath separation for the perturbations of a relativistic star. The spacetime geometry is treated as the system and the matter content (given by the stress tensor) as the bath for a statistical analysis. The perturbations in a relativistic star are known in general to arise due to external or internal effects. It is thus possible to view the externally sourced perturbations having relation with the fluctuations of the interior matter content using principles of the linear response theory [9]. We work out a fluctuation dissipation theorem for a simple case, using a first principles approach.

In order to study perturbations sourced by stochastic effects in the interior of a relativistic star, the idea of using an Einstein-Langevin Equation has been introduced in [10,11]. This is adopted from semiclassical stochastic gravity [11,12]. It is important to note that there is a fundamental difference in what we attempt to do here and the semiclassical case which is well established now. The semiclassical Einstein-Langevin equation takes account of the fluctuations of the quantum stress tensor and its backreaction, with interesting applications to early universe cosmology [13,14] and black hole physics [15,16,17]. We have borrowed the idea of setting up a Langevin approach in the classical domain from the semiclassical case, but this calls for a very different basic formalism regarding mathematical and physical issues to tackle with. It also has a very different regime of proposed applications in astrophysics compared to that of semiclassical stochastic gravity. While semiclassical Einstein Langevin equation is shown to be of importance to cosmology and quantum aspects of black hole physics, the classical counterpart can be seen to cover the areas of stellar dynamics, asteroseismology and collapse of relativistic (compact) stars. There has been a recent effort to explore the connections of quantum stress fluctuations with the classical perfect fluid picture in terms of fluctuations of an effective fluid in the decoherence limit [18]. However at this point of time such a connection needs to be probed further in details to be directly applicable here to see if the fluctuation dissipation relation of the semiclassical case and the classical case match or have any direct connections.

We begin with the picture of a perfect fluid model which is often the simplest case considered for a relativistic star. The microscopic particles in the perfect fluid collide frequently and their mean free path is short compared to...
Thus $\Delta$ and Lagrangian displacement vector field for noise are worked out, obtained by defining the two point noise kernel as basic formalism correctly.

In this article, we restrict ourselves to the stress tensor for a perfect fluid as the simplest example in order to set up the mechanism and thermal effects explicitly. Our formalism here can be extended very easily to all these cases. However in realistic relativistic stars. In these models it may be appropriate to associate the fluctuations with dissipative mechanisms and thermal effects explicitly. Our formalism here can be extended very easily to all these cases. However in this article, we restrict ourselves to the stress tensor for a perfect fluid as the simplest example in order to set up the basic formalism correctly.

II. THE MODEL USING EINSTEIN LANGEVIN EQUATION

Recently a classical Einstein-Langevin equation has been proposed [19, 20], where a toy model for relativistic stars with radial perturbations induced by stochastic effect of the fluid has been discussed. Here we enhance the same formalism, by taking into account Lagrangian displacement vector $\xi(x)$ for describing the perturbations of the fluid variables while keeping $h_{ab}(x)$ separate for the metric perturbation. Such separations for perturbative analysis of the system are well known and used thoroughly in literature [4]. Advanced studies in the area of rotating relativistic stars usually follow the same practice of keeping the two separate and is well established.

Thus to consistently take into account the fluctuations of the stress tensor for a general relativistic treatment a classical Einstein Langevin equation can be written as (geometric units with $c = 1$, and $8\pi G = 1$ are used throughout the manuscript.)

$$G^{ab}[g + h](x) = <T^{ab}[g + h; \xi](x)> + \tau^{ab}[g](x)$$

where $\tau^{ab}(x) \equiv T^{ab}(x) - <T^{ab}(x)>$ ($T^{ab}$ being the classical stress tensor and a stochastic variable itself in the above equation) is a stochastic term and defined on the background spacetime $g^{ab}(x)$ such that $<\tau^{ab}(x)> = 0$ which is covariantly conserved, $\nabla_a \tau^{ab}(x) = 0$. This follows from the covariance of $T^{ab}(x)$. In [20] the explicit expressions for noise are worked out, obtained by defining the two point noise kernel as

$$N^{abcd}(x, x') = <\tau^{ab}(x)\tau^{cd}(x')>$$

Perturbations $\xi$ are defined to be deformations in the fluid elements, by way of shift from the equilibrium configuration and are deterministic in nature. These perturbations in spacetime and fluid variables can be described by $h_{ab}$ and Lagrangian displacement vector field $\xi^a$ respectively.

We denote the Lagrangian change for a quantity by $\Delta z$ and Eulerian change by $\delta z$ such that,

$$\delta z = \Delta z - \mathcal{L}_\xi z = \Delta z - \xi^a \nabla_a z$$

Thus $\Delta g_{ab} = h_{ab} + \nabla_a \xi_b + \nabla_b \xi_a$. The perturbation in pressure and density are given by

$$\delta p = -\frac{1}{2} \Gamma p q_{cd} \Delta g^{cd} - \xi^a \nabla_a \epsilon$$

where $q_{ab} = u_a u_b + g_{ab}$ and $\Gamma$ is the usual adiabatic index defined by $\frac{\epsilon + p}{\rho}$. Equation (5) can be shown to be gauge invariant under the change of metric perturbations given by $h'_{ab} = h_{ab} + \nabla_a \zeta_b + \nabla_b \zeta_a$, where $\zeta^a$ is a stochastic vector field on the background manifold. It corresponds to the following perturbed form,

$$\delta G^{ab}[h](x) = \delta T^{ab}[h; \xi](x) + \tau^{ab}[g](x)$$

Here $p$ denotes pressure and $\epsilon$ the matter energy density.
A spherically symmetric (non-rotating) relativistic star is represented in Schwarzschild coordinates by
\[ ds^2 = -e^{2\nu(r)}dt^2 + e^{2\psi(r)}dr^2 + r^2d\Omega^2 \] (7)
The energy momentum tensor for a perfect fluid in these coordinates is of the form:
\[ T_{ab}(x) = \begin{pmatrix} -e^{2\psi(r)} & 0 & 0 & 0 \\ 0 & e^{2\nu(r)} & 0 & 0 \\ 0 & 0 & \frac{r^2}{e^{2\psi(r)}} & 0 \\ 0 & 0 & 0 & e^{2\psi(r)}u^a u^b \end{pmatrix} \] (8)
We assume \( p_r = p_\theta = p_\phi \) for a simple model, and the four-velocity vector is given by \( u^a = (1, 0, 0, 0) \) which satisfies the condition \( u^a u_a = -1 \). Considering only radial perturbations here, the \( \xi^r \) component for the Lagrangian displacement vector is non-zero.

We now move on to discuss a fluctuation dissipation theorem which is of central importance to address stochastic effects in a system and holds a significant place in linear response theory. This relates an external perturbation to fluctuations in the system in thermal equilibrium. Is is known that perturbations in a star can be introduced by external effects like accreting matter etc. We attempt to relate these to internal fluctuations in the stress tensor for a star in thermal equilibrium. This is the first step aiming further to investigate similar results for dynamical configurations and collapse stages.

In what follows, we work out a first principle approach to a fluctuation dissipation relation for a simple model.

### III. A FLUCTUATION DISSIPATION THEOREM FOR PERTURBATIONS OF A STATIC SPHERICALLY SYMMETRIC STAR

In this section, we obtain a fluctuation-dissipation relation for a perturbed static spherically symmetric relativistic star with stochastic fluctuations in the fluid variables. A toy model for perturbations of such a system has been discussed in [19], with the classical Einstein- Langevin equation giving
\[ \delta G^i_r = 8\pi\delta T^i_r(x) + \tau^i_r(x) : \]
\[ \delta\lambda\left(\frac{1}{r^2} - \frac{2}{r}\lambda'\right) + \frac{1}{r}\delta\lambda' = 4\pi e^{2\lambda}(\delta\epsilon + \tau^i_r) \] (9)
\[ \delta G^r_r(x) = 8\pi\delta T^r_r(x) + 8\pi\tau^r_r(x) : \]
\[ \frac{1}{r}\delta\nu' - \delta\lambda\left(\frac{1}{r^2} + \frac{2}{r}\nu'\right) = 4\pi e^{2\lambda}(\delta\rho + \tau^r_r) \] (10)
where Schwarzschild coordinates are used, such that
\[ ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2d\Omega^2 \] (12)
Here we enhance the model by including the Lagrangian displacement vector \( \xi^r \), keeping only the radial component non-zero for simplicity. Thus using TolmanOppenheimer-Volkoff equation \( p' = -\nu'(\epsilon + p) \) and equations (9),
\[ \delta\epsilon = \Gamma\epsilon[-\delta\lambda - \xi'' + \xi'(\lambda' - \frac{2}{r} + \nu'\frac{\Gamma}{w})] \] (13)
\[ \delta\rho = \Gamma p[-\delta\lambda - \xi'' + \xi'(\lambda' - \frac{2}{r} + \nu'\frac{\Gamma}{w})] \] (14)
while we use the perturbed equation of state \( \delta p = wp\delta\epsilon \). As has been mentioned earlier that the perturbed configuration of the relativistic star is decomposed into the metric as the system and the fluid variables as the bath, one needs equation of motion for the bath ( fluid displacement vector ). This can be obtained by considering the perturbed Euler equation \( \nabla_a \delta T^a_b = 0 \), for which of relevance in our case is the component \( \nabla_a \delta T^a_1 = 0 \), giving,
\[ e^{2(\lambda - \nu)}\frac{\delta\epsilon}{(\epsilon + p)} + w\frac{\delta\nu'}{(\epsilon + p)} = 0 \] (15)
We note here that $\delta T^0_0 \neq 0$. This can be shown since, for the perturbed Einstein Langevin Equation, we have the component

$$\delta G^0_1 = 8\pi \delta T^0_1$$

$$-\frac{2}{r} e^{-2\nu} (\delta \lambda) = 8\pi e^{2\lambda - 2\nu}(\epsilon + p) \dot{\xi}$$

(16)

From now on $\xi$ denotes only the radial component of $\xi^a$, where $\dot{\xi}$ follows from taking $\delta u = e^{-\nu} \dot{\xi}$ even when for the unperturbed configuration $u' = 0$. This can be shown to follow from taking the coordinate velocity $\dot{r} = dr/dt$ and $v = e^{\lambda - \nu} \dot{r}$, thus $\delta v = \delta (e^{\lambda - \nu} \dot{r}) = e^{\lambda - \nu} \dot{\xi}$, which holds even for a strictly static case of the background when $v$ can be taken to be zero, since $u' = 0$.

We also assume that as for all perturbed quantities for the static background, $\xi(r, t) = \xi(r)e^{i\omega t}$. It follows that the equation of motion for $\dot{\xi}$ (note that for radial perturbations $\delta \lambda(r, t) \equiv \delta \lambda(r)e^{i\omega t}$ as usual).

$$\xi''(r) + \xi'(r)\left(\frac{2}{r} - \lambda'(r) - \frac{\nu(r)}{w}\right) - \xi(r)(\lambda''(r) + \frac{2}{r^2} + \frac{\nu''(r)}{w} + \frac{\nu^2}{2w^2} e^{2(\lambda - \nu)}) = -\delta \lambda'(r)$$

(17)

The above equation is a second order differential equation with variable coefficients. This can be solved if a particular solution of the linear second order homogeneous equation that by taking the rhs =0 in the above is known. For this we have to assume a solution $y_1 \neq 0$ such that the other for the same can be obtained from it using the usual methods. We assume a first solution of $\xi(r) = O(r)$ (for the homogeneous case) as it is known to exist in literature [4] based on the smoothness and differentiability for $\xi$. This also implies that $\xi(0) = 0$. The simplest form for such an assumed solution which is physically viable can be taken to be $y_1 = r$ for the solution of the homogeneous part, from which the second one can be constructed which reads $y_2 = r \int e^{\lambda + \nu}/w r^4 dr'$.

We use the formal way to solve the two coupled differential equations, one obtained from the Einstein-Langevin equation and the other from the perturbed Euler equations. As we see that the noise in the system given by below and as described in detail in [13], shows dependency on the radial variable (along with the 't' variable with $e^{i\omega t}$ dependence which is ohmic), we give a treatment of the equations where the 't' dependence is taken into consideration for identification of motion and for identifying the dissipation term as well consistently as in the following.

The solution of equation (17) can be shown to take the form

$$\xi_\lambda = c_1 r + c_2 \int \frac{e^{(\lambda(r) + \nu(r))/w}}{(r)^4} dr$$

(18)

$$\xi_\nu = -r \int \left[\frac{e^{(\lambda(r) + \nu(r))/w}}{(r)^4}\right] (r')^3 e^{-(\lambda(r') + \nu(r'))/w} \delta \lambda'(r') dr'dr$$

(19)

where $\xi_\lambda$ is the homogenous solution and $\xi_\nu$ is the particular solution. It is also evident from physical considerations that at $r = 0$ the Lagrangian displacement $\xi, \dot{\xi} = 0$. It can easily be seen that $c_1$ vanishes (assuming $\delta \lambda'(r) = 0$ at $r = 0$, so that there are no cusps at $r = 0$). Thus we will keep $c_2$ as a constant throughout our calculations, and put $c_1 = 0$. One can thus see, that a little ambiguity in the exact form for the first (assumed) solution of the homogeneous part $O(r)$ can be ignored, in that there may be other parameters present there. Since $c_1$ is taken to be zero in all the calculations that follow, this term would not affect our result, but stands just as a mathematical artefact in obtaining our expressions, without affecting the final results. It should however be noted that the assumed solution for the homogeneous part does fulfill the essential minimal physically relevant conditions that of differentiability and smoothness at the boundary of the star with the form that we have taken, hence is fulfills the minimal criteria of mathematical physical validity.

Subsituting equation (13) in (9)

$$\frac{\delta \lambda'}{r} + \delta \lambda(\frac{1}{r^2} - \frac{2}{r} \lambda') + 4\pi e^{2\lambda} \Gamma \epsilon \{\delta \lambda + \xi' - \xi(\lambda - \frac{2}{r} + \frac{\nu'}{w})\} = 4\pi e^{2\lambda} r_i^4$$

(20)

using the the expression obtained for $\xi(r)$ it is simple to rearrange the terms and re-write the above equation as

$$\left\{\frac{\delta \lambda'}{r} + \delta \lambda(\frac{1}{r^2} - \frac{2}{r} \lambda') + 4\pi e^{2\lambda} \Gamma \epsilon \{\delta \lambda + r \int \frac{1}{(r)^2} \delta \lambda(r) dr \right.$$

$$- \int \int \{\lambda'(r) + \frac{\nu'(r)}{w}\} (r')^3 e^{-(\lambda(r') + \nu(r'))/w} \delta \lambda'(r') drdr'$$

$$- (\lambda(r) + \frac{\nu(r)}{w}) r \int \delta \lambda(r) \{(r)^3 e^{-(\lambda(r) + \nu(r))/w}\} \int \frac{e^{(\lambda(r') + \nu(r'))/w}}{(r')^4} dr'dr$$

$$+ (\lambda(r) + \int \frac{\nu'(r)}{w} + 3) \int \delta \lambda(r) \{\int (r)^3 e^{-(\lambda(r) + \nu(r))/w}\} \int \frac{e^{(\lambda(r') + \nu(r'))/w}}{(r')^4} dr'dr\} e^{i\omega t} = 4\pi e^{2\lambda} r_i^4 (r) e^{i\omega t}$$

(21)
where the fluctuations on the rhs of the above follow from the homogeneous part of the solution for $\xi$. As is usual
with obtaining the fluctuation and dissipation relation from equation of motion, we have used the particular solution
part of $\xi$ for $\delta\epsilon$ in equation (9). The noise in the system as usual can be related to the homogeneous part as
\begin{equation}
\tau^*_i(r) = -\Gamma^2\epsilon(r)\{\xi^*_h(r) - \xi^*_h(r') - \frac{2}{r} + \frac{\nu'}{w}\}
\end{equation}
which gives
\begin{equation}
\tau^*_i(r)e^{i\omega t} = -c_2\{\Gamma\epsilon(r)\int(\lambda'(r'') + \nu(r''))(r'' \lambda'(r'') + \nu(r'')) dr'\}e^{i\omega t}
\end{equation}
The two point correlation then reads
\begin{equation}
<\tau^*_i(r)\tau^*_i(r')> e^{i\omega(t-t')} = c_2^2 \{\epsilon(r)e^{i\omega t} \int \int(\lambda'(r_1)+\nu(r_1))(\lambda'(r_2)+\nu(r_2)) e^{i\lambda(r_1)+\nu(r_1)} e^{i\lambda(r_2)+\nu(r_2)} dr_1 dr_2\}e^{i\omega(t-t')}
\end{equation}
while the dissipation term in equation (21) above reads
\begin{equation}
\{\epsilon(r)\int \int(\lambda'(r)+\nu(r)) e^{i\lambda(r)+\nu(r)} e^{i\lambda(r)+\nu(r)} (r'' \lambda'(r'') + \nu(r'')) \delta\lambda'(r') dr'\}e^{i\omega t}
\end{equation}
The fluctuation and the dissipation terms then can be related to give a fluctuation-dissipation relation which can be
written as
\begin{equation}
<\tau^*_i(r)\tau^*_i(r')> e^{i\omega(t-t')} = \int K(r, r')D(r'')d(r''e^{i\omega(t-t')}
\end{equation}
where
\begin{equation}
K(r, r') = \Gamma \{\lambda'(r'') + \nu(r'')\}e^{2i\lambda(r'') + \nu(r'')/w} e^{i\lambda(r'') + \nu(r'')} \int (\lambda(r)+\nu(r)) e^{i\lambda(r)+\nu(r)} dr
\end{equation}
and
\begin{equation}
D(r'', r') = \Gamma e^{i(r'')^3} e^{i\lambda(r'')} c_2^2
\end{equation}
One can easily verify that the same relation is obtained from the rest of the non zero components of the Einstein
Langevin equation and the perturbed Euler equation, by using the equation for state $p = \epsilon^2$ and its perturbed form.
Thus we can write the general fluctuation dissipation theorem as
\begin{equation}
N^{a}_{b d}(r, r', t, t') = N^{a}_{b d}(r, r') e^{i\omega(t-t')} \int K(r, r'')D^{a}_{b d}(r''', r'')d(r''e^{i\omega(t-t')})
\end{equation}
where $N^{a}_{b d}(r, r', t, t')$ is the noise kernel and $D^{a}_{b d}(r'', r')e^{-i\omega t}$ is the dissipation kernel, while $K(r, r'')e^{i\omega t}$ being the
fluctuation-dissipation kernel. The non-zero components of the dissipation kernel ( radial/spatial part ) are given by
( we denote the indices $t, r, \theta, \phi$ by $0, 1, 2, 3$ in the expressions below ).
\begin{equation}
D^{0}_{0 0}(r, r') = D(r, r'), D^{1}_{1 1}(r, r') = D^{2}_{2 2}(r, r') = D^{3}_{3 3}(r, r') = D^{1}_{1 2}(r, r') = D^{2}_{2 1}(r, r') = D^{3}_{3 1}(r, r') = D^{1}_{1 3}(r, r') = D^{2}_{2 3}(r, r') = D^{3}_{3 2}(r, r') = \epsilon\epsilon(r')/w
\end{equation}
while the radial ( spatial ) part of the noise kernel as obtained from the definition (3) has non-zero components given by
\begin{equation}
\begin{align*}
N^{0}_{0 0}(r, r') &= \epsilon\epsilon(r')/w \\
N^{1}_{1 1}(r, r') &= N^{2}_{2 2}(r, r') = N^{3}_{3 3}(r, r') = \epsilon\epsilon(r')/w \\
N^{2}_{2 3}(r, r') &= N^{3}_{3 2}(r, r') = N^{3}_{3 1}(r, r') = \epsilon\epsilon(r')/w \\
N^{0}_{1 0}(r, r') &= N^{0}_{0 2}(r, r') = N^{0}_{0 3}(r, r') = \epsilon\epsilon(r')/w \\
N^{1}_{1 0}(r, r') &= N^{2}_{2 0}(r, r') = N^{3}_{3 0}(r, r') = \epsilon\epsilon(r')/w \\
N^{3}_{3 0}(r, r') &= \epsilon\epsilon(r')/w
\end{align*}
\end{equation}
It is important to note here that, we have modeled stochasticity in the stress tensor such that the fluctuations in the pressure and density in the star show up as the significant stochastic variables while the four-velocity does not. This is in contrast to taking deterministic perturbations for stress tensors as is usually the case. However taking into consideration stochastic effects in the interior of the star does not call upon the necessity to have stochastic fluctuations of the four velocity of the fluid as significant, though such a model could be considered as well. But we keep and introduce the above model for mathematical simplicity, which is simpler still physically consistent. The source of noise in the system is thus seen to arise due to the pressure and density fluctuations in the interior of the star. This does not necessarily call upon the four velocity for the static configuration of the star to contribute as the source of fluctuations, though the perturbed star may have induced effect on the perturbed four velocity, while the background four-velocity can be treated deterministically. As is usually the case with Langevin approach, one can model stochastic fluctuations of the source phenomenologically. As mentioned earlier, the above model of noise is same as in \cite{19} for which the classical Einstein Langevin equation has been worked out, we have carried on here with the same model of noise to work out the fluctuation-dissipation theorem as well.

All the above components of noise, reduce to the following expression

$$\text{Cov}[\epsilon(r)\epsilon(r')]e^{i\omega(t-t')} = \int K(r, r'')D(r'', r')e^{i\omega(t-t')}dr''$$

(32)

where $K(r, r'')$ and $D(r'', r')$ are given by \cite{27} and \cite{28}. Thus equation (32) is the fluctuation dissipation relation for stochastic perturbations in a static spherically symmetric relativistic star.

This kind of analysis may also be very useful to identify or establish unexplored modes of oscillations in relativistic starts correctly due to stochastic effects in the interiors of stars or gravitating bodies, induced either due to external sources or internal stochastic dynamics of the massive stars, specially more significant in a dynamical system than a static one. Here we have worked out results for a static case, which is the first basic step of theoretical development on the lines we plan to proceed in future.

IV. DISCUSSION

The fluctuation dissipation theorem for a relativistic star as obtained above is the first of its kind of results for stellar dynamics and the simplest model. It should be noted that the Einstein Langevin approach which we use here, is not the same as the semiclassical references as in \cite{11, 24}, and is designed as in \cite{14, 20} to suit a similar analysis for the relativistic stars made up of compact matter of which the interiors can be modeled by perfect fluids etc. There are similar approaches in the semiclassical case for cosmological spacetime and black hole physics giving fluctuation-dissipation relations \cite{25}, but they bear no resemblance to our case here. Firstly, in these references the fluctuation and dissipation arise out of the quantum stress tensors and secondly, these are discussed for cosmological spacetimes and black holes which are coupled to some quantum field. Hence the basic set up there is very different astrophysically as well than the case we deal with here. In the present article we obtain the FD relation for asterioseismology as with oscillations in stars and their perturbations, we have not addressed any black hole configurations here as the basic source of fluctuation here lies in the interior matter content of the star, that of the pressure and density of the fluid. So this is fundamentally different than the other references in the area. This is also clear if one compares our expression for the FD with that of \cite{28}, thus the difference in both mathematical as well as physical form is apparent, even though the basic approach of the analysis remains same. It would be perhaps an interesting exercise to check in future if, using the results of recently established correspondence between the noise kernels of semiclassical gravity and that for the effective fluid models \cite{18} we can also solve the semiclassical Einstein Langevin equation or find out an expression for a similar FD theorem for the relativistic star model (in semiclassical case) and compare our results. This would hold in the decoherence limit of the quantum fields only. It may then be useful to comment over any useful correspondence over sources of stochasticity in the star. However it is important to note here, that in this article we have just considered perfect fluid model of the stars in its very basic form, and have not taken any effective fluid limits or traced it back to any connections with quantum or classical fields.

V. CONCLUSIONS

In this article we have obtained a fluctuation dissipation relation for a static configuration of a relativistic star with perturbations. This is the simplest model and has been worked out as a first step to establish the basic result, more realistic cases will be explored later. This little step in the program for developing a stochastic analysis (that of Einstein Langevin equation and its implications) for relativistic stars and compact objects is of central importance
for studying non-equilibrium and equilibrium statistical physics in the context. It is important to note here that this is different from the semiclassical counterpart.

As concluding remarks, we highlight few additional issues that can be raised over the stochasticity introduced here in the stellar model. A collapsing star goes through different phases, where classical and quantum effects become important to study the interior regions of the star and overall dynamics. The behaviour of these fluctuations and their effect on parameters which govern the final stages of collapse and critical points in the dynamics can be an interesting direction of investigation.

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