Evidence for non-universal scaling in dimension four Ising spin glasses

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The critical behavior of the Binder cumulant for Ising spin glasses in dimension four are studied through simulation measurements. Data for the bimodal interaction model are compared with those for the Laplacian interaction model. Special attention is paid to scaling corrections. The limiting finite size value at criticality for this dimensionless variable is a parameter characteristic of a universality class. This critical limit is estimated to be equal to 0.523(3) in the bimodal model and to 0.473(3) in the Laplacian model.

In Ref. [1] Jörg and Katzgraber used an elegant scaling display of raw numerical data to test for universality in Ising Spin Glasses (ISGs). They plot the ratio $g(\beta, L)/g(\beta, 2L)$ against $x(\beta, L) = \frac{g(\beta, L)}{(3-(q^2)/[(q^2)^2]/2}$ is the Binder cumulant for inverse temperature $\beta$ and lattice size $L$, with $q$ the spin glass order parameter. They studied numerically two ISGs in dimension 4, one with a Gaussian interaction and one with a diluted bimodal distribution. Over the range of temperatures used for the measurements, which extended well into the ordered phase, the scaled data points were independent of $L$ and followed the same curve for the two systems to within the statistics. Jörg and Katzgraber concluded that these results were evidence of universality in ISGs. Other authors, using other arguments, have also claimed that universality holds in ISGs [2]. Indeed it has been stated recently “The issue was settled in 2008 by Ref. [3] who emphasized the role of corrections to scaling, thus convincing the community that universality holds [in ISGs]” [4] and “the model with $\pm J$ interactions ... should be in the same universality class [as the Gaussian model].” [2].

In the Fig. [1] we show the same scaling plot as that of Ref. [1] for data on two other ISG models in dimension 4, with temperatures spanning the critical temperatures. We compare the Laplacian (decreasing exponential) interaction ISG to the standard bimodal ISG with undiluted interactions. Data on all sizes from 3 to 12 (Laplacian) or to 14 (bimodal) were summed over $2^{14}$ samples for sizes up to $L = 7$ and over $2^{13}$ samples for larger sizes. For the Laplacian ISG, our data show scaling with no correction term to within the statistics; the scaling curves are almost indistinguishable from those for the models of Ref. [1]. The bimodal data on the other hand show a strong $L$ dependence due to large finite size scaling corrections; the scaling curve $g(x)$ moves continuously to the right with increasing $L$. With a natural extrapolation the thermodynamic (large $L$) limit scaling curve for the bimodal interaction ISG will lie well to the right of the $L$-independent Laplacian curve, so the two models appear not to be in the same universality class.

Standard finite size scaling expressions which include a single leading conformal correction term lead to a size dependence $[g_{\text{cross}}(\infty) - g_{\text{cross}}(L)] = AL^{-\omega+1/\nu}$ for the crossing point $g(\beta, 2L) = g(\beta, L)$ (represented by $y(x) = 1$ on the plot), where $\omega$ is the correction to scaling exponent. In the dimension 4 bimodal ISG, $\omega$ has been estimated by simulations to be 1.04(0.1) [3]. From high temperature series expansion (HTSE) measurements $\theta = \omega \nu \approx 1.5$ [3] so $\omega \approx 1.3$. A natural extrapolation of the present bimodal data to infinite $L$ assuming $\omega \approx 1.2$ gives a thermodynamic limit estimate which is certainly considerably larger than the Laplacian crossing point limit.

Data near criticality for the bimodal and Laplacian ISGs are shown in a different form in Fig. [2] and Fig. [3] respectively. Near criticality

$$g(\beta, L) = g_c + AL^{-\omega} + B(\beta - \beta_c)L^{1/\nu}$$

(1)

The bimodal data are consistent with $\beta_c = 0.505(1), \omega \approx 1.2$ and $g_c = 0.523(3)$. The Laplacian data are consistent with $\beta_c = 0.622(1), g_c = 0.473(3)$ and a negligible correction. The $g_c$ values estimated for the Gaussian and dilute bimodal models in Ref. [3] are 0.470(5) and 0.472(2) which are similar to the Laplacian value.

The bimodal $\beta_c$ value is confirmed independently by thermodynamic derivative data on dimensionless observables $U(\beta, L)$. With increasing $L$, each derivative $\partial U(\beta, L)/\partial \beta$ tends to a step function centered on $\beta_c$. The inverse derivative peak height $1/D_{\text{max}}(L) = \partial^2 U(\beta, L)/\partial \beta^2$ both scale as $L^{1/\nu}[1 + aL^{-\omega}]$ [12]. So at large $L$, $\beta(L)$ plotted against $1/D_{\text{max}}(L)$ extrapolates linearly to $\beta_c$. From these extrapolations $\beta_c = 0.505(1)$ for the bimodal model in 4d [12].

For the $g(\beta_c, L)$ bimodal values to extrapolate finally to a limiting $g_c$ value at infinite $L$ consistent with that of the Laplacian model would require putative bimodal ISG data for very large $L$ (data inaccessible with current numerical resources) to bend back to the left in Fig. [1] or to sharply bend down in Fig. [2] (in an unlikely looking
respectively. In addition to the conformal correction term in the 4d bimodal ISG near criticality, there can also be an analytic correction in principle there can also be an analytic correction. This term would have an exponent $\theta_a = 1$. As $\nu \approx 1.1$, $\omega_a = \theta_a/\nu \approx 0.9$, so such an analytic correction cannot play the role of the hypothetical very small exponent term. Turning back to the conformal corrections, the first term in the RGT $\epsilon$-expansion for the ISG leading irrelevant operator exponent $\theta(d) = (6 - d)$ [4, 10]. Leading $\epsilon$-expansions terms in ISGs give useful qualitative indications for other critical exponents, and it turns out that the $\epsilon$-expansion values for $\theta(d) : \theta(5) \sim 1$, $\theta(4) \sim 2$, $\theta(3) \sim 3$, are qualitatively consistent with published effective $\theta(d)$ and $\omega(d) = \theta(d)/\nu(d)$ values from simulations, and from quite independent HTSE results [7, 11]. Bimodal ISG finite size scaling (FSS) estimates in 3d are $\omega(3) = 1.12(10)$ and $\nu(3) = 2.56(4)$ [4], so $\theta(3) = \omega \nu \approx 3.0$. We have seen that in 4d FSS estimates are $\theta(4) \approx 1.15$ or $\theta(4) \approx 1.35$ [13]. From FSS data for different 5d ISG models $\omega(5) \approx 1$ and $\nu(5) \approx 0.75 [13]$ so $\theta(5) \approx 1.2$. HTSE estimates in 4d ISG models are $\theta(4) \approx 1.4$ [7] and in 5d $\theta(5) \approx 1.0$ [7, 11]. Consistency therefore definitively excludes a hypothetical leading conformal correction term in the 4d bimodal ISG having an exponent $\omega_b(4)$ much smaller than 1. The correction term with $\omega \approx 1.2$ for the bimodal ISG can be confidently identified with the leading conformal correction. By definition, no conformal correction term with a smaller exponent exists. It can be concluded that there is no backbending correction, and that the natural extrapolations of the bimodal model data to the large $L$ limit with $\omega \approx 1.2$ are valid.

Systems in the same universality class must have identical values for the infinite size critical limit of a dimensionless parameter such as the Binder cumulant $g_c$. The observation of a critical limit for the bimodal ISG which is very different from those of the other three models dis-
proves universality in these 4d ISGs. Claims of universality for ISGs in other dimensions should be re-examined critically.

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