The study of the motion of the truck vibration motors by the oscillating of a pendulum of variable length

Sergey V Strygin, Vladimir D Levin, Nikolai V Loschinin, Ilya N Kiryushin, Sergey A Parshukov and Vladimir V Metik
Ryazan Institute (branch) of Moscow Polytechnic University, Ryazan, Russia
strsw@mail.ru

Abstract. The article provides a solution to the problem, according to the condition of which there is a body – a cart, which must be brought into directional motion by means of a swinging elastic pendulum of variable length, pivotally mounted on it. A variant of creating the initial momentum of the trolley movement due to the movement of the pendulum, made in the form of a spring-loaded load with the possibility of movement along a straight guide when it rotates relative to the pendulum suspension point, is proposed and justified. The mode of movement of the cart-pendulum system is studied when the individual components of the system are gradually released from the connections to provide the initial impulse of the cart movement. The full-turn shape of the trajectory of a spring-loaded pendulum pivotally mounted on a cart is shown. We use numerical analysis methods (numerical integration of systems of nonlinear differential equations) with modeling by means of the Mathcad package.

The development of mechanical engineering, transport, and other branches of science and technology requires the use of new effective ways to move transport vehicles.

The goals of this work are:

- identification of the dependence of the speed of the pendulum vibration motor at the same initial angle of deviation of its pendulum from the value of the spring stiffness of the pendulum, made in the form of a spring-loaded load with the ability to move along a straight guide when it rotates relative to the point of suspension of the pendulum;
- identification of the dependence of the speed of the pendulum vibration motor on the value of the initial angle of deflection of its pendulum with the same stiffness of the pendulum spring;
- identification of the dependence of the speed of the pendulum vibration motor on the mass of the vibration motor trolley with different spring stiffness of the pendulum.

To achieve these goals, you need to solve the following tasks:

1. Generalization of theoretical foundations and practical experience in creating vibration motors.
2. Research and identification of the main characteristics of the pendulum vibration motor.

In [1], four geometric forms of periodic trajectories of movement of a load attached to a spring suspended from a movable carriage are considered. In this case, the swinging spring is used as a mover when moving the carriage along the horizontal support surface. Also shown is a method for determining the trajectory of a load attached to the end of the specified spring. In the formulation of the dynamic analysis described thus transport machines libredigital determine the initial length of the spring, which provides a periodic trajectory of movement of freight and translational movement of the
carriage in the horizontal direction. In this case, the mass of the carriage, the weight of the load, the spring stiffness, the initial position of the spring with the load (other than vertical), and the initial impulse applied to the carriage are taken as known parameters. However, there are other geometric forms of periodic trajectories of movement of the load attached to a spring suspended from a movable carriage, in particular, this is a full-turn rotation of the spring with the load. Some drive systems of pendulum vibration motors are reduced to such a dynamic model, for example, shown in computer modeling on the Internet site [2]. Because of the potential structural simplification of the pendulum vibration motor, its dynamic model with a special operating mode is also of interest, in which the initial impulse applied to the carriage is provided only by the initial position of the spring with the load and the next two stages of starting work [3].

1. The Load is released from the deflected position at an angle $\phi_0$ at a fixed length of the spring (released from the connection that restrains its rotation) no initial speed. The cart position is also fixed.

2. At the moment when the spring with the load passes the vertical position, the load and the cart are released (released from ties) and acquire the ability to move with the spring deformation along the pendulum and translational horizontal movement, respectively.

Consider the movement of the trolley suspended by a spring, the weight of the load $m_1$ (figure 1).

Figure 1 shows: $\phi_0$ – the angle by which the spring with the load is deflected at the initial time; $L$ – the length of the spring in the undeformed state; $r$ – its length in the deformed state at a given time $t$; $\phi$ – the angle corresponding to a given time; $a_r$, $a_\phi$ - components of the acceleration vector in the polar coordinate system.

Since the movement of the load occurs along a flat trajectory different from the circle, the movement of the load in relative motion is convenient to describe in the polar coordinate system $(r, \phi)$.

In the polar coordinate system, the velocities along the coordinate lines $r = \text{const}$, $\phi = \text{const}$, are determined by the formulas:

\[
\begin{align*}
  v_r &= \dot{r}, \\
  v_\phi &= r \dot{\phi},
\end{align*}
\]

acceleration –

\[
\begin{align*}
  a_r &= \ddot{r} - r \dot{\phi}^2, \\
  a_\phi &= \frac{1}{r} \frac{d}{dt} (r^2 \cdot \dot{\phi})
\end{align*}
\]

When the truck is moving, the load makes a plane-parallel movement: translational motion together with the truck and rotational with a variable radius $r$ around the point $A$ of suspension.

To derive the equations of motion, we apply the method of kinetostatics [4]. According to this method, the load will be in equilibrium if it is applied along with the specified forces ($m_1 g$, $m_2 g$ – gravity of the load and the cart, $F_{qn}$ – spring elastic force) apply inertia forces $F_r$, $F_\phi$, $F_{xt}$, $F_x$
Their directions are opposite to the corresponding acceleration projections. We make the equilibrium equation in the projections on the coordinate lines \((r, \varphi)\) for the forces acting on the load:

\[
\begin{align*}
\sum r &= -F_{\varphi n} - F_r + m_1 g \cos \varphi - F_x \cdot \sin \varphi = 0, \\
\sum \varphi &= -F_{\varphi} - m_1 g \sin \varphi - F_x \cdot \cos \varphi = 0,
\end{align*}
\]  

(3)

where according to (2)

\[
\begin{align*}
F_r &= m_1 \cdot (\ddot{r} - r \cdot \dot{\varphi}^2), \\
F_{\varphi} &= m_1 \cdot (2\dot{r} \dot{\varphi} + r \cdot \ddot{\varphi}), \\
F_{\varphi n} &= c \cdot (r - L),
\end{align*}
\]  

(4)

\(c\) – spring stiffness.

\[\text{Figure 2. Dynamic model of a trolley with a swinging pendulum of variable length.}\]

Since the cart makes a translational motion, the speed and acceleration of all its points are the same. Then the force of inertia of the portable motion of the load will be equal to:

\[
F_x = m_1 \cdot \ddot{x},
\]

(5)

Substituting the expression of forces (4), (5) in equation (3), we obtain the differential equations of motion of the load relative to the fixed coordinate system \(xoy\):

\[
\begin{align*}
\ddot{r} + \frac{c}{m_1} (r - L) - r \cdot \dot{\varphi}^2 + \ddot{x} \cdot \sin \varphi &- g \cdot \cos \varphi = 0, \\
\ddot{\varphi} + \frac{2r \dot{\varphi}}{r} + \frac{\ddot{x}}{r} \cdot \cos \varphi + \frac{g}{r} \cdot \sin \varphi &= 0.
\end{align*}
\]  

(6)

The third equation is obtained by considering the motion of the cart. The impact of the truck load is carried out by means of the elastic force of the spring. The force of inertia acting on the cart will be associated with the acceleration of the cart in its forward motion.

\[
F_{xt} = m \cdot \ddot{x}
\]

(7)

Projecting the forces acting on the cart on the \(ox\) axis, we obtain the equation:

\[
\ddot{x} = \frac{c}{m} (r - L) \cdot \sin \varphi
\]

(8)

Substitute equality (8) in equations (6) and solve them with respect to the higher derivatives.

\[
\ddot{r} = -\frac{c}{m_1} (1 + \frac{m_1}{m} \sin^2 \varphi) (r - L) + r \dot{\varphi}^2 + g \cos \varphi,
\]
Using Lagrange equations of the 2nd kind, the authors also obtained three differential equations composed with respect to $r$, $\phi$, $x$ (the corresponding calculations are not given in this article due to their bulkiness).

\begin{align}
    m_1 (\ddot{x} + x \sin \phi - r \dot{\phi}^2) &= -c (r - L) + m_1 g \cos \phi , \tag{10} \\
    2 \ddot{\phi} + r \ddot{x} + x \cos \phi &= -g \sin \phi , \tag{11} \\
    (m + m_1) \ddot{x} + m_1 (\ddot{r} \sin \phi + 2 \dot{r} \dot{\phi} \cos \phi + r \dot{\phi}^2 \cos \phi - r \dot{\phi}^2 \sin \phi) &= 0 . \tag{12}
\end{align}

Expressing $\ddot{r}$ and $\ddot{\phi}$ from equations (10), (11) and substituting them into equation (12), we obtain the equation

$$
\ddot{x} = \frac{c}{m} (r - L) \sin \phi .
$$

Thus, the result obtained by the method of Lagrange equations of the 2nd kind coincides with the result obtained by the method of kinetostatics.

The initial conditions must be added to the system of differential equations (9).

Consider the movement of cargo, consisting of two stages:

1. The Load is released from the deflected position at an angle $\phi_0$ clockwise with a fixed length of the spring without the initial speed. The cart position is also fixed.

2. At the time of passage by a spring with the load the vertical position, load and truck are released (freed from bonds). This gives the cart an impulse that drives it forward.

When the spring passes the vertical position ($\phi = 0$) after releasing the load from the bonds, the spring receives an instantaneous elongation

$$
\Delta r = \frac{m_1 \dot{\phi}_0^2 L}{c} = \frac{m_1 g (1 - \cos \phi_0)}{c} \tag{13}
$$

and has an angular velocity

$$
\dot{\phi}_K = \frac{(2 g L (1 - \cos \phi_0))^{0.5}}{L} = \left[ \frac{2 g L (1 - \cos \phi_0)}{L} \right]^{0.5} \tag{14}
$$

Thus, the initial conditions for the second stage of movement, on the basis of (13) and (14) are:

$$
r(0) = L + \Delta r , \quad \dot{r}(0) = 0 , \quad \phi(0) = 0 , \quad \dot{\phi}(0) = \dot{\phi}_K . \tag{15}
$$

The numerical solution of the system of equations (9) with initial conditions (12) can be obtained using the computational complex Mathcad.

An interesting case is when $\phi_0 = \pi$. In this case, the load makes a continuous rotational movement around point $A$, and the cart moves on average at a constant speed.

It should be noted that perfectly smooth connections were considered in the problem.

Figures 3-8 show graphs of the movement of the trolley at the values: $L = 0.1$ m, $\phi_0 = 0.5 \pi$ and $\phi_0 = \pi$, $m_1 = 1$ kg, $m = 2$ kg and $m = 4$ kg, $C = 490$ Nm$^{-1}$ and $C = 49000$ Nm$^{-1}$. 

\[d\]
Figure 3. Numerical solution of the system of equations (9) with initial conditions (15) at values:
$L = 0.1 \, \text{m}, \phi_0 = 0.5\pi, m_1 = 1 \, \text{kg}, \, m = 2 \, \text{kg}, \, C = 490 \, \text{Nm}^{-1}$.

Figure 4. Numerical solution of the system of equations (9) with initial conditions (15) at values:
$L = 0.1 \, \text{m}, \phi_0 = 0.5\pi, m_1 = 1 \, \text{kg}, \, m = 2 \, \text{kg}, \, C = 49000 \, \text{Nm}^{-1}$.
The considered problem is devoted to the study and identification of individual characteristics of a pendulum vibration motor. Statement of the problem of translational directional movement of the cart of a pendulum vibration motor on a horizontal support surface due to the movement of a load attached to a spring suspended from the cart. For a fixed value of the initial length of the spring, determine the effect of the initial angle of deflection of the pendulum, spring stiffness, trolley mass, pendulum load on providing the initial impulse of the trolley for its directional movement in the horizontal plane with the gradual release of individual components of the system from connections. There are two stages. 1. the Load is released from the deflected position at an angle $\phi_0$ at a fixed length of the spring (released from the connection that restrains its rotation) no initial speed. The cart position is also fixed. 2. At the moment when the spring with the load passes the vertical position, the load and the cart are released from the bonds and acquire the ability to move with the spring deformation along the pendulum and translational horizontal movement, respectively. The direction of movement of the cart coincides with the direction of the circumferential speed of the load in stage 2.

Figure 5. Numerical solution of the system of equations (9) with initial conditions (15) at values: $L = 0.1 \text{ m}$, $\phi_0 = \pi$, $m_1 = 1 \text{ kg}$, $m = 2 \text{ kg}$, $C = 490 \text{ Nm}^{-1}$. 
Figure 6. Numerical solution of the system of equations (9) with initial conditions (15) at values: \( L = 0.1 \text{ m}, \phi_0 = \pi, m_1 = 1 \text{ kg}, m = 2 \text{ kg}, C = 49000 \text{ Nm}^{-1} \).

Figure 7. Numerical solution of the system of equations (9) with initial conditions (15) at values (e): \( L = 0.1 \text{ m}, \phi_0 = \pi, m_1 = 1 \text{ kg}, m = 4 \text{ kg}, C = 490 \text{ Nm}^{-1} \).
Figure 8. Numerical solution of the system of equations (9) with initial conditions (15) at values: 
\( L = 0.1 \, m, \phi_0 = \pi, m_1 = 1 \, kg, m = 4 \, kg, C = 49000 \, Nm^{-1} \).

As follows from the graphs of the obtained solutions, the rate of directional movement of the pendulum libredigital with the same initial deflection angle of the pendulum increases it by increasing the spring rate of the pendulum and decreases with decrease in the initial deflection angle of the pendulum and/or the increase of the weight of the truck libredigital with the same rigidity of a spring of pendulum. However, the identified mode, wherein the increase in the rate of libredigital occurs with a decrease in spring rate, excluding full-circle rotation of the pendulum, when the initial deflection angle of the pendulum \( \phi_0 = \pi \). In this case, the load without a spring makes a continuous rotational movement around the suspension point of the pendulum of the vibration motor, and the cart moves on average at a constant speed. If the load is connected to the pendulum through a spring, the speed of the cart increases in comparison with the operation of the pendulum without a spring, all other things being equal, and the full-rotation movement of the pendulum is excluded.

The paper describes the theoretical regularities of mechanical movements in the design of mechanisms for reproducing movements, including vibration. It is shown that there is a full-turn shape of the trajectory of a spring-loaded pendulum pivotally mounted on a cart. The substantiation of the main characteristics of the pendulum vibration motor, which allows the design of transport and technological means for various applications, is carried out.

References

[1] Kutsenko L, Vanin V, Zapolskiy L, Yablonskyi P, Vasyliev S, Danylenko V, Sukharkova O, Rudenko S and Zhuravskij M 2019 Synthesis and classification of periodic motion trajectories of the swinging spring load Eastern-European J. of Ent. Technol vol 2 Issue 7 (98) pp 26–37

[2] Strygin S 2018 Pendulum motor with self-oscillations (https://grabcad.com/library/pendulum-motor-with-self-oscillations-1)
[3] Kalinkin D S and Strygin S V 2018 Pendulum three-movable vibration motor *A collection of High school. New technologies of science, technology, and pedagogy: Materials of the all-Russian scientific and practical conference* (Moscow: Moscow Polytechnic University) pp 487-90

[4] Loitsyansky L G and Lurie A I 1983 *Course of theoretical mechanics: in 2 volumes. Vol. II. Dynamics* (Moscow: Nauka, Main edition of physical and mathematical literature)