Preventive Equipment Repair Planning Model

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Abstract

Any large functioning system consists of equipment that needs to be repaired during its lifetime. The methods of mathematical programming are used to formalize the optimization problem of preventive equipment repair planning in this paper. The method is used to find the annual optimal plan of preventive equipment repair. The goal of such plan is the uniform distribution of repair works by the months. It is supposed, that the deviation from the standard plan of repair works can not be more than one month. An algorithm to find the optimal plan of repair works is provided. A numerical example is given.

Keywords: optimization problem, preventive equipment repair
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1 Introduction

Different problems of preventive equipment repair using various mathematical methods are considered in many papers. For example, in [1] the methods of mathematical programming are applied to study the problems of preventive maintenance scheduling. In [1] authors illustrate how linear programming can be effectively used to solve problems of such class and propose...

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different optimality criteria. Several articles discuss the problem of preventive maintenance efficacy testing. The hypothesis that repairs have no effect on the sequence of times to failure or on the costs of the failures is tested in [2]. Other mathematical models that can be used for formalization and analysis of the problem of preventive repair planning are illustrated in [3–51].

Equipment which fails during its lifetime and needs to be repaired is considered in this paper. For such cases, it is usually necessary to ensure a uniform plan of repair works. This plan should contain information about the equipment repair periods for a year. Such plan can be considered as a normative annual plan of equipment repair. It is formed in accordance with standard monthly volumes of the repair works. But usually, in real life, the volumes of repair works do not correspond to the actual plan of equipment repairs.

Therefore there is a need of regulatory adjustments to the annual plan of equipment repairs through changes in the time periods of repair works. However, such changes in the time periods of repair works should not be great. It is supposed that repair works can be moved for no more than one month (forward or backward) within a year. Under this constrain, it is possible not to obtain a uniform distribution of monthly volume of repair works. Then the difference between the normative annual plan of equipment repairs and a new distribution of monthly volume of repair works can be analyzed. If this difference is more than one month, then the correction of the normative annual plan can be carried out. And in this case the revised plan is taken as the new normative annual plan.

2 The formalization of the optimization problem of planning preventive equipment repair

Let us suppose $A_1, \ldots, A_{12}$ are the monthly values of the repair works expressed in repair units(hours). Each $j$-th monthly repair volume consists of the elements $\begin{pmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{kj} \end{pmatrix}$, $j = 1,12$, where $A_{ij} = \sum_{i=1}^{k} a_{ij}$, $j = 1,12$. Thus we obtain the matrix $A = \begin{pmatrix} a_{1,1} & \cdots & a_{1,12} \\ a_{2,1} & \cdots & a_{2,12} \\ \vdots & \cdots & \vdots \\ a_{k,1} & \cdots & a_{k,12} \end{pmatrix}$, where $a_{ij} \geq 0$, $A$ is a standard matrix of normative annual equipment repair plan. $A_j = \sum_{i=1}^{k} a_{ij}$ is the total monthly work hours in a given month $j$. Indices $ij$ give the information about equipment, which needs to be fixed this month: a) if $a_{ij} > 0$, then $ij$-
th equipment needs to be fixed. The repair will last \( a_{ij} \) hours; b) if \( a_{ij} = 0 \), then \( ij \)-th equipment does not need to be repaired. The average monthly repair works last \( A_{cp} = \frac{\sum_{i=1}^{12} A_j}{12} = \frac{\sum_{i=1}^{k} \sum_{j=1}^{2a_{ij}}}{12} \). The transfer of repair works can be carried out only one month backward or forward. This means, that the element \( a_{ij} \) can be transferred either to the column \((j+1)\), or to the \((j-1)\)-th column. It is necessary to find transfers, that have a minimal deviation the monthly volumes of repair works from the average volume of repair works.

If \( A'_j \) is the new monthly volume of repair works in hours, then it is necessary to minimize the expression \( \sum_{j=1}^{12} |A'_j - A_{avg}| \). Let us denote by \( x_k > 0 \) the volume of repair works transferred from \( k \)-th month to \((k+1)\)-th. Let us denote by \( x_k < 0 \) the volume of repair works transferred from \((k+1)\)-th month to \( k \)-th. If \( x_k = 0 \) there are no transfers. Here \( x_k \) is integer number. Let \( X = (x_{ij})_{k \times 11} \), \( i=1, k, j=1,11 \) be a matrix, where

\[
\begin{cases}
0, & \text{if nothing is transferred} \\
I, & \text{if some volume of repair works is transferred from the } j \text{-th month to } (j+1) \text{-th} \\
-I, & \text{if some volume of repair works is transferred from the } (j+1) \text{-th month to } j \text{-th}
\end{cases}
\]

Then the problem is subdivided in two problems.

**Problem 1.** It’s necessary to find the monthly volumes of transferable repair works in hours, that is \( x_k \), \( k=1,11 \).

**Problem 2.** The problem of integer choice, that is the calculation of the matrix \( X \).

The solution for the problem 1.

**Method I.** The problem can be solved by methods of quadratic programming. We use the quadratic function as the measure for deviation of the repair works volume from the average volume of repair works

\[
V = \sum_{i=1}^{n} (A_i - A_{avg} - x_i + x_{i-1})^2 + (A_n - A_{avg} + x_{n-1})^2,
\]

that should be minimized subject to

\[
-A_{i+1} \leq x_i \leq A_i, \ i = n-1
\]

and additional condition of integrality \( x_i \). Consider the following transformations: \( \hat{A}_i = A_i - A_{avg} \). Then the function \( V \) takes the form

\[
V = 2 \sum_{i=1}^{n} \left( \hat{A}_{i+1} - \hat{A}_i \right) x_i + \sum_{i=1}^{n-1} x_i^2 - 2 \sum_{i=2}^{n} x_{i-1} x_i + \sum_{i=1}^{n} \hat{A}_i^2, \ \hat{A}_i = A_i - A_{avg}. \]

Let us suppose \( z = -(V - \sum_{i=1}^{n} \hat{A}_i^2) \). Then

\[
z = -\sum_{i=1}^{n-1} \left( \hat{A}_{i+1} - \hat{A}_i \right) x_i + 2 \sum_{i=2}^{n-1} x_{i-1} x_i - 2 \sum_{i=1}^{n} x_i^2.
\]
Let us introduce the variables $x'_i$ and $x''_i$ to transform the inequality (2) into equations. Then restrictions will take the form of

$$\begin{cases}
  x_i + x'_i = A_i \\
  -x_i + x''_i = A_{i+1} \\
  x'_i \geq 0 \\
  x''_i \leq 0
\end{cases}$$

Let us introduce a new variable $x_0 \geq 0$ and express each variable $x_i$ as a difference: $x_i = x_i - x_0$, where $x_i \geq 0$; $x_0 \geq 0$. Then the target function $V$ would be

$$V = n - \sum_{i=1}^{n-1} \left( \hat{A}_i - (\overline{A}_i - x_0) + (\overline{A}_i - x_0) \right)^2 + (A_n + (\overline{A}_{n-1} - x_0) - \overline{A}_{avg})^2.$$ 

Let us rewrite the equation (3) in the form $z = -2 \sum_{i=1}^{n-1} \left( \hat{A}_{i+1} - \hat{A}_i \right) \overline{A}_i - x_0 \left( 2\hat{A}_i - 2\hat{A}_n \right) + \sum \overline{A}_i^2 - 2x_0^2$. Since $\min z = \max(-z)$. The problem can be represented in the form:

$$\max z = -2 \sum_{i=1}^{n-1} \left( \hat{A}_{i+1} - \hat{A}_i \right) \overline{A}_i - x_0 \left( 2\hat{A}_i - 2\hat{A}_n \right) + 2\overline{A}_i x_0 + 2\overline{A}_{n-1} x_0 -$$

$$- 2 \sum_{i=1}^{n-1} \overline{A}_i^2 - 2x_0^2 (4)$$

under constraints:

$$(\overline{A}_i - x_0) + x'_i = A_i, -(\overline{A}_i - x_0) + x''_i = A_{i+1}, \overline{A}_i \geq 0, x_0 \geq 0, x'_i \geq 0, x''_i \geq 0 (5)$$

and $\overline{A}_i, x_0$ are integers. The problem (4), (5) is the standard problem of mathematical programming in the form: $\max z = cx + x' D x, A x = B, x \geq 0$.

**Method 2.** The algorithm is as follows: first, the value of $x_6$ is determined. This value minimizes the deviations of the volume repair works of the first half of the year from the volume of repair works of the second half the year. Then similarly $x_3$ and $x_9$ are determined. After determination of $x_3, x_6, x_9$ the optimal distribution of the repair works volume is produced for each quarter.

**Method 3.** Let us suppose $A_1, A_2, \ldots, A_{12}$. Let us start from the first month. If $A_1 = A_{avg}$ in the first month, then we go to the second month. If $A_1 < A_{avg}$, then we subtract the missing hours from the second month. If we have an excess of repair works in the first month, then we transfer them to the second month. As a result of using this algorithm we have the following: the transfer from the $i$-th to the $i + 1$-th month is denoted by $x > 0$, the lack of transfers is denoted by $x = 0$, transfer to the $i$-th month from $i + 1$ is denoted by $x < 0$.

Let us describe the algorithm by steps.
Step 1. Let us suppose $I = 1$.

Step 2. Let us calculate $X = A_i - A_{\text{avg}}$. Then let us consider the $i$-th month: a) if $A_i > A_{\text{avg}}$, then the value of repair works equals $A_{\text{avg}}$ in the $i$-th month. Then we transfer to the $i + 1$-month the rest of the repair works: $A_i = A_i - X_i$. Then the step 4 is executed. b) if $A_i = A_{\text{avg}}$, then nothing is transferred from the $i$-th month. Then the step 4 is executed. c) if $A_i < A_{\text{avg}}$, then we execute to the step 3.

Step 3. Let us compare $A_{i + 1}$ and ($-X_i$): a) if $A_{i + 1} \geq -X_i$, then $A_i = A_i - X_i$; $A_{i + 1} = A_{i + 1} + X_i$. Then we execute the step 4. b) if $A_{i + 1} < X_i$, then $X_i = A_{i + 1}$; $A_i = A_i - X_i$, $A_{i + 1} = A_{i + 1} + X_i$. Then we execute the step 4.

Step 4. Let us suppose $I = I + 1$.

Step 5. If $I \geq 12$, then we execute the step 2, otherwise we execute the step 6.

Step 6. As a result we have $X_i, A_j, i = 1, \ldots , 12, j = 1, 12$.

Step 7. Algorithm stops.

**The solution of the problem 2.**

Let us suppose, that the monthly transfers $X = X_1, \ldots , X_{11}$ are found using one of the three methods. With the help of simple transformation we get the following sets consisting of the elements of original set. The first set $z = \{z_1, \ldots , z_{11}\}$, where $z_i > 0$ if $z_{ij} \in X$. Otherwise $z_i = 0$. If $z_i > 0$, then we transfer from $i$-th month to $i + 1$-th month. The second set $y = \{y_1, \ldots , y_{12}\}$, where $y_i > 0$ if $y_i \in X$ (transfer from $j$-th month to $j + 1$-th month). Otherwise $y_i = 0$ (nothing is transferred from $j$-th month to $j + 1$-th month). The goal of introducing the sets $Y$ and $Z$ is the transformation of the problem to any convenient type suitable for application of the methods of dynamic programming. Let us suppose $D$ is the volume of repair works that is transferred. The vector of monthly repair works is $\delta_n = \{0, 1\}$. This vector consists of the numbers $a_{ij}$, that we now denote by $a_i$. Let us consider the problem $(D - \sum_{i=1}^{N} a_i \delta_i) = \max \sum_{\{\delta_i\}}^{N} (a_i \delta_i - D/N) = \max \sum_{\{\delta_i\}}^{N} q_i(\delta_i),$ where $q_i(\delta_i) = a_i \delta_i - D/N$ under the constraints \[ \sum_{i=1}^{N} \delta_i a_i \leq D, \delta_i = \{0, 1\} \]. Let us write a recurrent relation of dynamic programming in the form of Bellman function: $f_1(D) = \max \sum_{\{\delta_i\}}^{N} q_i(\delta_i),$ $f_D(D) = \max \{\delta_N a_N + f_{N-1}(D - \delta_N a_N)\}$, $\delta_N = 0.1$, $\delta_N a_N \leq D$, $N = 1, \ldots , N$. The above algorithm was implemented for the matrix

\[
\begin{pmatrix}
10 & 20 & 30 & 40 \\
5 & 8 & 6 & 6 \\
21 & 11 & 3 & 2 \\
14 & 1 & 5 & 3 \\
\end{pmatrix}
\]

The solution has a form:

\[
\begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & -1 & 0 & -1 \\
1 & -1 & -1 & 0 \\
0 & 1 & 0 & 0 \\
\end{pmatrix}
\]
3 Conclusion

The problem of optimal preventive equipment repair planning is formalized and investigated in this paper. The optimal annual plan provides a uniform distribution of repair works by the months. An algorithm for finding the optimal plan of repair works is described. A numerical example is calculated.

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