Differential scattering measurements from a collider for ultracold atoms

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**Abstract.** We present a method for measuring differential cross sections from scattering halos produced by cold collisions between ultracold atomic clouds. Our scheme incorporates the dynamics of scattered particles within a harmonic potential that is used as a collider for the clouds. We demonstrate how to transform an experimentally acquired absorption image in order to obtain the angular scattering probability density.

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1. Introduction

The field of cold and ultracold collisions of atoms has attracted substantial attention in recent years [1] with an interdisciplinary impact on physics, chemistry, and quantum information science [2]. Adopting the convention of [3], cold collisions are associated with collision energies in the range from 1 \( \mu \)K to 1 mK (as measured in units of Boltzmann’s constant \( k_B \)) and ultracold collisions with energies less than 1 \( \mu \)K. In the latter regime, all the essential interaction properties are described by a single parameter, the s-wave scattering length. All collisions occur via radial motion since the cross section for angular motion vanishes asymptotically in the zero-energy limit. Experiments with atomic gases cooled to the ultracold regime have given rise to a wealth of important achievements, with the realization of Bose–Einstein condensation as a prominent example [4, 5].

Above the ultracold regime, atoms may collide through angular motion states. In a recent letter [6], we reported direct imaging of s and d partial-wave interference in cold collisions of atoms using a novel experimental tool for colliding two ultracold clouds [7]. Our method is reminiscent to that of particle colliders used extensively in high energy physics. Using electromagnetic fields, two ensembles of particles are guided and accelerated towards each other to collide at an interaction region where the scattered fragments are detected. In contrast to the high energy counterpart, our implementation does not rely on the particles being charged for the purpose of confinement and acceleration. Rather, the particles are neutral atoms with...
a magnetic moment. The two ensembles of particles are initially confined in a double-well magnetic trap [9, 10], where they are cooled to ultracold temperatures. By transforming the trap to a single-well configuration the atoms are accelerated to achieve essentially monoenergetic cold collisions along a well-defined axis in an anisotropic harmonic potential. The scattered particles are directly imaged using a laser absorption technique. The analysis of the data is complicated by the particle dynamics in the anisotropic harmonic potential subsequent to the collision and the two-dimensional projection operation of the imaging scheme. In the present paper, we discuss in detail the process through which the angular scattering distribution can be extracted.

This paper is organized as follows. In section 2, we derive the dynamics of particles in an anisotropic harmonic trap and the resulting transformation of the angular scattering distribution with respect to the free-space case. We use Abel inversion (a result from Fourier analysis) to reconstruct the 3D information contained in the projection image of a cylindrically symmetric particle distribution and introduce the expansion of the scattering amplitude into partial waves as is the custom in quantum scattering. In section 3, we describe our cold-collision experiments of ultracold doubly spin-polarized $^{87}\text{Rb}$ atoms which we analyse in section 4 in terms of the methods presented in section 2. Finally, section 5 provides a discussion of the results and outlines possible future directions for atomic colliders.

2. Theory

2.1. Kinematics in an anisotropic harmonic potential

In the following we shall consider binary elastic scattering of particles colliding at the centre of a cylindrically symmetric harmonic potential with axial and radial angular oscillation frequencies $\omega_z$ and $\omega_\rho$, respectively. Particles of mass $m$ enter the collision along the symmetry axis $z$ from opposite directions with identical speeds $v$. From the conservation of energy and momentum, it follows that a particle scattered at time $t = 0$ in a direction with angle $\theta$ to the $z$-axis will follow a trajectory $C_\theta$ given in cylindrical polar coordinates $(\rho, \phi, z)$ by

$$z(t) = v \frac{\cos \theta \sin(\omega_z t)}{\omega_z},$$  

(1)

and

$$\rho(t) = v \frac{\sin \theta \sin(\omega_\rho t)}{\omega_\rho}.$$  

(2)

Hence, at a given time $t$, a particle emitted at an angle $\theta$ will be at a position with angle $\vartheta$ related to the initial angle as

$$\tan[\vartheta(t)] = \frac{\omega_z \sin(\omega_\rho t)}{\omega_\rho \sin(\omega_z t)} \tan \theta \equiv \kappa(t) \tan \theta.$$  

(3)

To accommodate the experiment to be described in section 3, we shall consider a cigar-shaped potential for which $\omega_\rho/\omega_z > 1$ (i.e., the harmonic potential has a stronger confinement in the radial direction). At any given time $t$, (1) and (2) describe a spheroid $E_t$ with semiaxes...
The trajectories $C_\theta$ for particles emitted at the same speed $v$ at angles $0, \Delta \theta, 2\Delta \theta, \ldots, 2\pi$ with $\Delta \theta = \pi/20$ in an anisotropic harmonic potential characterized by oscillation frequencies $\omega_\rho/\omega_z = 12$. Superimposed is an isochronic map (red lines), which shows possible progress of travel from the centre in specified time intervals of $\pi/10\omega_\rho$. The isochrones constitute ellipses with semiaxes $v \sin(\omega_z t)/\omega_z$ and $v \sin(\omega_\rho t)/\omega_\rho$, where $t$ is the time travelled after emission at the origin. The outermost ellipse corresponds to a quarter of a radial trap period ($t = \pi/2\omega_\rho$). For times longer than this, trajectories will begin to intersect.

$\begin{align*}
\frac{\sin(\omega_z t)}{\omega_z} \text{ and } \frac{\sin(\omega_\rho t)}{\omega_\rho} \text{ in the axial and radial directions, respectively. The factor } \kappa(t) \text{ in (3) can be approximated as } \\
\kappa(t) \approx \begin{cases} 
\frac{\sin(\omega_\rho t)}{\omega_\rho t} & \text{if } t \ll \frac{\pi}{2\omega_z}, \\
1 & \text{if } t \ll \frac{\pi}{2\omega_\rho} < \frac{\pi}{2\omega_z}.
\end{cases}
\end{align*}

For short times $t \ll \pi/2\omega_\rho$, $\mathcal{E}_1$ approaches a sphere of radius $vt$, which is the result for particles scattered in free space (the so-called Newton sphere). In figure 1 we show trajectories for particles emitted in a plane at uniformly distributed polar angles $\theta$ in steps of $\pi/20$ radians when $\omega_\rho/\omega_z = 12$.

2.2. Transformation of the angular scattering distribution

Particles colliding at the centre of the harmonic potential will be scattered out in a given direction with a probability dictated by the differential cross section $d\sigma/d\Omega$. If the collisional interaction is assumed to be spherically symmetric, $d\sigma/d\Omega$ will have cylindrical symmetry about the $z$-axis (the collision axis). Hence, the angular probability density distribution of a scattered particle $p_{sc}(\theta) \propto d\sigma/d\Omega$ is independent of the azimuthal angle $\phi$. Assuming several particles to be scattered at the rate $R(t)$ from the origin of the coordinate system within a finite time interval $t_{sc} \ll \pi/2\omega_\rho$ beginning at time $t = 0$, a three-dimensional scattering halo with a cylindrically symmetric density distribution $n(\rho, z)$ will be formed. The distribution of particles in the halo is determined by $R(t)$, $p_{sc}(\theta)$, and the dynamics in the harmonic well. For observation times shorter than $\pi/2\omega_\rho$ the mapping to spatial coordinates is one-to-one. Assume for example that
we had measured \(n(\rho, z)\) in an experiment and wanted to extract the angular distribution \(p_{sc}(\theta)\). It follows from (1) and (2) that the region contained within \(E_t = \pi/2\omega\rho\) in any \(\rho, z\)-plane is covered by a family of curves \(\varphi(\rho, z) = \theta\) in such a way that through each point of that plane there passes one, and only one, curve of the family (see figure 1). Formally, the angular probability density is then given by the path integral along \(C_{\theta}\) of arc length \(s\)

\[
p_{sc}(\theta) = \frac{1}{N \sin \theta} \int_{C_{\theta}} n(\rho, z) \rho \frac{1}{\sqrt{\varphi^2 + \varphi_z^2}} \, ds,
\]

(5)

where \(N = \int_0^t R(t) \, dt\) is the total number of scattered particles. However, in practice one would typically consider the number of particles \(\Delta N\) emitted in angular bins \(\theta_i\) of some finite width \(\Delta \theta\)

\[
p_{sc}(\theta_i) \approx \frac{\Delta N(\theta_i)}{2\pi \sin \theta_i \Delta \theta} = \frac{1}{N \sin \theta_i \Delta \theta} \int_{\theta_i - \Delta \theta/2}^{\theta_i + \Delta \theta/2} \int_{C_{\theta}} n(\rho, z) \rho \frac{1}{\sqrt{\varphi^2 + \varphi_z^2}} \, ds \\
= \frac{1}{N \sin \theta_i \Delta \theta} \int_{S_i} n(\rho, z) \rho \, d\rho \, dz \\
\approx \frac{1}{N \sin \theta_i \Delta \theta} \sum_{(\rho_j, z_k) \in S_i} n(\rho_j, z_k) \rho_j \Delta \rho \Delta z,
\]

(6)

where \(S_i\) is the area bounded by the trajectories \(C_{\theta_i - \Delta \theta/2}\) and \(C_{\theta_i + \Delta \theta/2}\), and \((\rho_i, z_k)\) \(\rho_m = m \Delta \rho\) and \(z_m = m \Delta z\) define a rectangular grid for \(m = \ldots -1, 0, 1, \ldots\) with spacings \(\Delta \rho\) and \(\Delta z\), respectively, in the radial and axial directions (see, e.g., [11]). In (6), \(2\pi \sum_{(\rho_j, z_k) \in S_i} n(\rho_j, z_k) \rho_j \Delta \rho \Delta z\) is the total number of particles within the rotational volume of \(S_i\) as illustrated in figure 2.

2.3. Absorption imaging and Abel inversion

Absorption imaging is a standard method of probing the density of atomic clouds [12]. The atoms are illuminated with a beam of laser light resonant with an electronic transition and the resulting shadow is imaged by a CCD camera. As a result, a projection of the atom density onto a plane—the so-called column density—is obtained:

\[
P(x, z) = \int_{-\infty}^{\infty} n(x, y, z) \, dy.
\]

(7)

If there is rotational symmetry about the \(z\)-axis, this may be expressed as

\[
P(x, z) = 2 \int_{0}^{\infty} \mathcal{H}(\rho - |x|) \frac{n(\rho, z)}{\sqrt{\rho^2 - x^2}} \, d\rho,
\]

(8)

where \(\mathcal{H}\) is the Heaviside step function and we have used the substitution \(\rho^2 = x^2 + y^2\). Equation (8) can be recognized as the canonical form of the Abel transform \(\mathcal{A}\) of \(n(\rho, z)\), i.e., \(P(x, z) = \mathcal{A}[n(\rho, z)]\) [13]. Hence, given the column density in the \(x, z\)-plane, we may obtain \(n(\rho, z)\) from the inverse Abel transform \(n(\rho, z) = A^{-1}[P(x, z)]\). Formally, an inverse Abel transform can be obtained by first taking the Fourier transform of \(P\) and subsequently the Hankel transform.

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Figure 2. Visualization of the rotational volume of an angular bin at θi of area \( S_i \). The particles contained within this volume all passed through a solid angle area \( 2\pi \sin \theta_i \Delta \theta \). The normalized number of particles per unit solid angle is given by (6).

2.4. Quantum scattering

Using the above formalism we can extract the differential cross section for a collision process from scattering halos projected onto a plane and evolving in an anisotropic harmonic potential. In this procedure, which is the focus of this paper, only classical concepts are involved. Quantum mechanics, however, is essential for understanding the scattering patterns arising when two atoms collide. As in classical mechanics, the two-body problem is conveniently described in the centre-of-mass system of the particles where it is reduced to the equivalent problem of a single particle moving in a potential. The wave function solving the Schrödinger equation is at large distances (beyond the range of the interaction potential) represented by an incoming plane wave along the \( z \)-axis and an outgoing spherical wave (see, e.g., [14])

\[
\psi \sim e^{ikz} + f(\theta) e^{ikr/r},
\]

where \( k \) is the magnitude of the relative wave-vector of the colliding particles. \( f(\theta) \) is the energy-dependent complex scattering amplitude which depends on the details of the interaction potential. In partial wave analysis the scattering amplitude is expanded as

\[
f(\theta) = (1/2i k) \sum_{l=0}^{\infty} (2l + 1)(e^{2i\eta_l} - 1) P_l(\cos \theta),
\]

where \( P_l \) is the Legendre polynomial of order \( l \) and \( \eta_l \) are the partial wave phase shifts (see the appendix). The differential cross section is the squared modulus of the scattering amplitude

\[
\frac{d\sigma}{d\Omega} = |f(\theta)|^2,
\]
and has an angular pattern which depends crucially on the quantum mechanical interference between the partial wave states as dictated by the phase shifts. The total elastic cross section can be expressed as

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = 2\pi \int_0^\pi |f(\theta)|^2 \sin \theta \, d\theta = \sum_l \sigma_l, \quad (12)$$

where

$$\sigma_l = 4\pi(2l + 1) \sin^2 \eta_l / k^2 \quad (13)$$

are the partial cross sections. By Levinson’s theorem the partial cross sections at zero energy can be written as

$$\sigma_l \rightarrow k \rightarrow 0 \frac{4\pi(2l + 1) \tan^2 \eta_l / k^2}{k^2}. \quad (14)$$

For potentials falling off more rapidly than $1/r^4$, it can be shown [14] that $\tan \eta_l$ vanishes at least as rapidly as $k^2$ when $k \rightarrow 0$ and $l \geq 1$. Hence, in the zero-energy limit $\sigma_0$ is the only nonvanishing partial cross section. This may written as

$$\sigma_0 \rightarrow k \rightarrow 0 \frac{4\pi a^2}{k^2}, \quad (15)$$

where $a$ is the scattering length defined by

$$\tan \eta_0 \rightarrow k \rightarrow 0 -ak. \quad (16)$$

The scattering length is the essential and sole parameter needed to describe ultracold gases, notably Bose–Einstein condensates, entering in the many-body description (see, e.g., [15] for an account on scattering theory in relation to Bose–Einstein condensates). We note that in the case of indistinguishable bosons, equation (15) gains a factor of 2 (i.e., $\sigma_0(k \rightarrow 0) = 8\pi a^2$) and that scattering of indistinguishable fermions via even angular momentum states is forbidden so that $\sigma_0$ is identically zero for all $k$.

3. Experiment

3.1. System

In the present paper we consider collisions between gases of $^{87}\text{Rb}$ atoms in the $|F = 2, m_F = 2\rangle$ hyperfine substate. Effectively, two colliding atoms are both identical composite bosons and the extended version of the Pauli principle then requires the total wave function to be symmetric. As a result, only terms of even $l$ contribute to the partial wave expansion (10) and the sum acquires a degeneracy factor of 2. $^{87}\text{Rb}$ is among the most popular species for Bose–Einstein condensation experiments. This quantum degenerate regime is associated with such low collision energies that only the $l = 0$ term of (10) remains nonzero—all elastic scattering has an isotropic (s-wave) nature. For increasing collision energies differential scattering via higher-order partial waves sets in. Notably, the so-called
d-wave \((l = 2)\) shape resonance is known to occur for \(^{87}\text{Rb}_{|F=2,m_F=2}\) \[16\] and in a recent paper \[6\] we investigated the interplay and interference between s- and d-wave differential scattering using the experimental method of the present paper. The differential cross section in the special case of bosonic scattering restricted to s- and d-wave states is given by

\[
\frac{d\sigma}{d\Omega} = k^{-2} \left[ (e^{2i\eta_0} - 1) + 5(e^{2i\eta_2} - 1)(3\cos^2\theta - 1)/2 \right]^2
\]

\[
= k^{-2} \left[ 4\sin^2\eta_0 + 25\sin^2\eta_2(3\cos^2\theta - 1)^2 + 20\sin\eta_0\sin\eta_2\cos(\eta_0 - \eta_2)(3\cos^2\theta - 1) \right] + \text{s+d interference term}
\]

\[17\]

3.2. Setup

3.2.1. Preparation of two ultracold atomic clouds. In our experiment, \(^{87}\text{Rb}_{|F=2,m_F=2}\) atoms are loaded into an Ioffe–Pritchard magnetic trap using a standard double-MOT scheme \[17\]. The magnetic trap is in the three-coil quadrupole-Ioffe-configuration (QUIC) \[18\] with the axis of the two quadrupole coils in the vertical direction \((x\text{-axis})\) and the axis of the Ioffe-coil oriented horizontally \((z\text{-axis})\). The QUIC trap has the advantage that all coils can be driven in series using a single current supply reducing magnetic field noise. While in the QUIC trap the atomic gas is rf evaporatively cooled to a temperature of \(\sim\)12 \(\mu\)K. By increasing the currents in the quadrupole coils (engaging an additional supply in shunt) we raise a potential barrier in the \(z\) direction, splitting the cloud into two. In order to avoid zero field at the minima of the resulting double well (where Majorana spin-flip transitions would then occur), an additional rotating magnetic bias field of 5 G is applied just before the double-well split \[9\]. This field rotates at a frequency of 2.78 kHz about the \(z\)-axis and creates a time-averaged harmonic trapping potential near each well minimum. While in the double well the two clouds are further evaporatively cooled to temperatures of typically a few hundred nano-Kelvin.

3.2.2. A collider for ultracold atoms. Collision between the two clouds will occur if the double well is transformed back into the original single-well configuration. The clouds accelerate from two opposite sides of the potential to collide at the centre with an energy given by their initial positions. After preparation of two ultracold ensembles of atoms, their separation is adiabatically adjusted via the quadrupole gradient to select the desired collision energy and the rotating bias field is reduced to 2 G to increase the cloud densities. The transformation to a single well is achieved by linearly ramping down the quadrupole current to its QUIC value in 20 ms. The atoms are probed by pulsing on a resonant laser beam at a given delay \(t_{\text{delay}}\) (controlled by a high-precision pulse-delay generator) after this ramp is initiated. In figure 3 we show the measured axial positions of the two clouds as a function of \(t_{\text{delay}}\) (the figure is accompanied by a movie showing the collision and particle dynamics in the trap). For delay times larger than 20 ms (i.e., after the ramp to a single well has finished), the two data sets are fitted simultaneously to sinusoidal functions with a common oscillation frequency, which is found to be \(\omega_z = 2\pi \times 14.6\) Hz.
Figure 3. Measured positions of the two clouds (open and closed circles) as functions of the time after the ramp to a single-well trap is initiated. The lines are a simultaneous sinusoidal fit to the data for $t_{\text{delay}} > 20$ ms yielding an axial oscillation frequency of $\omega_z = 2\pi \times 14.6$ Hz. The images from which the positions have been extracted have been combined in movie1 (from $t_{\text{delay}} = 3$ ms to $t_{\text{delay}} = 69$ ms in steps of 1 ms), which shows the dynamics of particles scattered when the two clouds collide.

4. Analysis

We obtain the collision energy by determining the velocity of each cloud from a linear fit to the position of each cloud versus time, over a period of 1.5 ms either side of collision. The collision energy is calculated in the centre-of-mass frame and is quoted in units of $k_B$. For the examples presented in this paper we determine a value of $192(8) \mu K$ (in the present setup we can achieve collision energies of up to more than 1 mK).

Figure 4(a) shows an absorption image acquired at a time of 1.8 ms after the collision. This gives the column density, along a radial direction, of the 3D particle distribution. Using a software implementation provided by Dribinski et al [19], we apply the inverse Abel transform to the absorption image column density. By definition, the inverse Abel transform returns a distribution with no azimuthal dependence, and as a consequence the resulting image (figure 4(b)) has reflection symmetry about the axial direction. Hence it is sufficient to consider only one half of the image in determining the angular scattering probability. To find the number of collision products per scattering angle, the Abel-inverted image is divided into a number of curved angular bins, defined by equations (1) and (2) over some angle $\Delta \theta$ (see figure 1). In the experimental examples presented in this paper, we have used $\Delta \theta = \pi/32$. Because we wish to superimpose curved bins onto a discretized image it is preferable to reduce the size of the discrete steps, and we do this by increasing the number of pixels by a factor of 100. We count only the number of scattered particles in each bin that are bounded by two ellipses (see figure 4(b)), corresponding to times between the earliest and latest scattering events. Furthermore, we do not include bins in which the bounded section contains unscattered particles. We extract the angular probability...
Figure 4. (a) Absorption image acquired shortly before $t = \pi/2\omega_r$ for a collision energy of 190 µK. (b) Inverse Abel-transformed image obtained from figure 4(a) (colour map has been scaled relative to (a) to emphasize the scattered particles). The position of particles at times of 1.0 ms (inner) and 1.8 ms (outer) after the collision are shown by the semi-ellipses. On the lower half, the solid lines show the division of the bins. (c) Corresponding polar plot of the normalized angular scattering probability density (dots). The solid line is a fit to the data.

density distribution $p(\theta)$ as given in (6). For binary collisions, $p(\theta) = p(\pi - \theta)$. However, it is apparent from figure 4 that this symmetry is not perfect near the collision axis. The broken symmetry might be caused by multiple scattering events within the differently sized clouds as discussed in section 5. Using (17), a fit to the data from bins between angles of $6\pi/32$ and $26\pi/32$ (shown by the solid line in figure 4(c)) yields partial-wave phase shifts of $\eta_0 = -1.18(3)$ and $\eta_2 = 0.43(2)$, corresponding to s- and d-wave contributions respectively. Since (17) is invariant under a simultaneous sign reversal of $\eta_0$ and $\eta_2$, we can in fact only determine the magnitudes $|\eta_0|$, $|\eta_2|$, and $|\eta_0 - \eta_2|$ from our data (we note that this is sufficient to compute the partial cross sections). The signs as stated above were chosen according to the model of [6] where
Figure 5. Absorption images for a collision energy of 192 µK acquired at times approximately (a) 1.2 ms, (b) 1.4 ms, (c) 1.6 ms and (d) 1.8 ms since the start of the collision. (e) Polar plots of the normalized angular scattering probability density shown by ♦, ◯, □ and △ corresponding to (a), (b), (c) and (d) respectively. (See movie2 for a sequence showing the evolution.)

$\eta_0 < 0 < \eta_2$ for collision energies up to at least 600 µK. In general, complementary information about the interaction potential is required to determine the correct signs for the phases (see [8] for a method comparing accumulated phases in the tail of a van der Waals potential with known $C_6$ coefficient).

Due to evolution of the scattered particles in the harmonic trap, we only consider images taken at times prior to $t = \pi/2\omega_0$ so that an angular bin corresponds uniquely to one scattering angle on collision. Using the curved bins described above, the measured angular scattering probability is expected to be independent of the evolution time since collision (for $t < \pi/2\omega_0$). Figure 5 shows a series of images following collision and a comparison between the obtained data. The dynamics is also illustrated in the sequence movie2. The amplitudes of the data series
in figure 5(e) have been scaled by a factor of up to 1.11, and these differences in amplitude can be accounted for in terms of shot-to-shot number variation.

5. Discussion and concluding remarks

5.1. Discussion of the experiment

In practice, the colliding clouds do not have the same speed or contain the same number of particles. For the example shown in figures 4 and 5, the larger (smaller) cloud has a speed of 0.125(3) ms\(^{-1}\) (0.146(3) ms\(^{-1}\)) and contains 2.3 \times 10^5 (1.3 \times 10^5) atoms. The latter is a result of different trap frequencies in each well of the double-well potential, and different rf evaporation rates for each, whereas the differing speeds are due to ramping to a slightly anharmonic single-well potential. These factors are due to experimental difficulty in creating a perfectly symmetric potential. If the colliding clouds have different speeds, the trajectories of scattered particles are not identical for angles \(\theta_i\) and \(\pi - \theta_i\). However, in calculating the boundaries for the bins we have assumed that the clouds are moving at equal speeds. The centre-of-mass velocity of \(\sim 0.02\) ms\(^{-1}\) in the presented examples has been calculated to have negligible effect due to the weak trapping potential in the axial direction. We note that the absorption imaging technique has the advantage over fixed particle detectors that no transformation between laboratory frame to centre-of-mass frame [20] is necessary. The origin of angular bins can simply be chosen with respect to the centre of the collision halo.

The spatial extent of the clouds along the axial direction implies that the binary collisions will occur over some finite time, which is accounted for in our summation of particles between two ellipses [as in figure 4(b)]. For simplicity, we have made the assumption that all scattering events occur at a single point in space. This is not strictly true because the overlap of the clouds actually extends over \(\sim 100\) \(\mu\)m. However, if this was a major effect under the present conditions, it would modify the observed angular scattering probability between different observation times after collision. This is clearly not the case in figure 5. Of more concern are multiple scattering effects where scattered particles undergo subsequent collisions. Due to the prolate shapes of the colliding clouds, primarily particles scattered at angles close to the collision axis will be affected. If such a scattered particle moving to the right collides a second time with a particle in the left-going cloud, the emission pattern for the secondary collision will be similar to that of the primary collision. The net effect of several events is to deplete particles scattered at these small angles and redistribute them over the entire angular range. If, however, the scattered particle moving right collides with a particle of the right-going cloud their relative momentum will be small. This type of secondary collision will have a completely different centre-of-mass frame (close to a frame following the unscattered cloud), and lower energy than for the primary collision. This may give rise to small s-wave-like halos surrounding the outgoing clouds. The effect could possibly be enhanced by collisional avalanches [21]. We speculate that this might explain the left–right asymmetry for the measured angular scattering probability as is apparent in figure 4.

Despite these complications, the phase shifts \(\eta_0 = -1.18(3)\) and \(\eta_2 = 0.43(2)\), as given by the fit of (17) to the measured angular distribution, do not seem unreasonable. From the theoretical model presented in [6] we expect an s-wave phase shift between \(-1.01\) and \(-1.04\) and a d-wave phase shift between 0.29 and 0.38 for the bounds of our collision energy. We note
that due to the interference term in (17) the phase shifts can be extracted without knowledge about absolute particle numbers. Independent of our work, a similar interferometric method to determine the phase shifts was reported very recently [8].

5.2. Future directions

We point out that our collider method could readily be extendable to other atomic systems with magnetic moments. Notably, several future candidates can be chosen from the list of species routinely cooled to temperatures in the ultracold range [22]. For example, $^{85}\text{Rb}$ has a $g$-wave ($l = 4$) shape resonance at $\sim 700 \mu \text{K}$ [23]. Perhaps even more interesting is the possibility of colliding ultracold mixtures of atoms. We are presently pursuing experiments on scattering between spin mixtures $^{87}\text{Rb}$ in the $F = 1$ and 2 ground states. Besides opening up the possibility for elastic scattering via odd partial waves, inelastic processes are expected to play a pronounced role in the case of $|F = 2, m_F = 1\rangle + |F = 1, m_F = -1\rangle$ collisions [24]. A further step in the direction of mixtures would be the study of heteronuclear collisions. With recent progress in cooling and confining two different atomic species simultaneously [25] using two-species-MOTs and sympathetic cooling in magnetic traps, this indeed seems to be feasible. Although the techniques described in the present paper are essentially applicable to the case of mixtures, the situation will generally be more complicated. If the mass or the magnetic moment differs between two species, so will the trapping frequencies for each. When a mixture of species A and B with trapping frequencies $\omega_A > \omega_B$ is accelerated from the double well configuration one would expect to first see A + A collisions at the single-well trap centre, subsequently A+B collisions at two locations axially offset to the trap centre, and finally B+B collisions at the trap centre. If the spatial separation between these events is larger than the axial size of the clouds, this effect might be advantageous (especially if the laser probing employs a resonant transition of B such that the initial A halo is undetected).

5.3. Conclusion

In conclusion, we have described a method for measuring differential scattering in cold atomic collisions. Our experimental setup constitutes a linear collider for ultracold atoms. Using Abel inversion we infer the 3D distribution of scattered particles from absorption images of the scattering halos. By accounting for the dynamics in the harmonic potential of the collider we are able to deduce the angular scattering probabilities. In the presented example of collisions between $^{87}\text{Rb}$ atoms at $192 \mu \text{K}$, the s and d partial wave phase shifts were extracted from the measured interference pattern. While we have shown our method to be robust with respect to the evolution dynamics in the harmonic trap (up to one-quarter of a radial trap period), we note the observable effect of multiple scattering on the differential cross section, which can occur for small scattering angles. These effects remain uncharacterized in terms of a theoretical model, which is beyond the scope of this paper.

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Appendix. Partial waves and scattering phase shifts

In quantum scattering of a particle, with mass $\mu$ and wave vector $k$, by a central potential $V(r)$ (see, e.g., [14]) we seek a solution for the time-independent Schrödinger equation

$$\left[ \nabla^2 - \frac{2\mu}{\hbar^2} V(r) + k^2 \right] \psi(r) = 0, \quad (A.1)$$

by considering the ansatz

$$\psi(r) = \sum_{l=0}^{\infty} i^l (2l + 1) F_l(r) P_l(\cos \theta). \quad (A.2)$$

Equation (A.1) is separable (upon substitution of (A.2) into angular equations

$$\left[ \frac{1}{\sin \theta} \frac{d}{d\theta} \left( \sin \theta \frac{d}{d\theta} \right) + l(l + 1) \right] P_l(\cos \theta) = 0, \quad (A.3)$$

fulfilled by construction of Legendre polynomials $P_l$, and radial equations

$$\left[ \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d}{dr} \right) - \frac{l(l + 1)}{r^2} - \frac{2\mu}{\hbar^2} V(r) + k^2 \right] F_l(r) = 0. \quad (A.4)$$

If $r^2 V(r) \rightarrow 0$, the radial function $F_l$ has the asymptotic form

$$F_l(r) \rightarrow r \rightarrow \infty \psi_{\eta_l} \sin(kr - \frac{l}{2} \pi + \eta_l) \frac{1}{kr}, \quad (A.5)$$

where $\eta_l$ is the scattering phase shift for the $l$th partial wave.

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