Mathematical model and analysis of PV Converter - Inverter System

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Abstract— Photo Voltaic (PV) system is commonly preferred renewable energy source for most of the residential, commercial as well as for utility applications. Photo Voltaic systems are extensively used for standalone and grid tied applications. To utilize the PV generated power to the best possible extent, control and conversion of power in PV systems plays an important role. In this paper, mathematical model of the complete PV system that comprises of solar panel, DC-DC Boost converter and a H bridge inverter is developed. The developed model is validated for constant load condition at varying irradiation levels and at fixed duty cycle. The performance analysis of the PV system in terms of efficiency is carried out for 200W resistive load.

Keywords: Photo Voltaic (PV), converter-Inverter, DC-DC converter, Inverter, modeling

1. Introduction

PV system used in most of grid tied and standalone application consists of a PV module, DC-DC converter and Inverter as shown in Figure 1. The variable DC from PV panel is initially boosted to a higher voltage using DC-DC converter and then converted to AC using inverter that can be fed into the grid or used by standalone load [1]. The performance of PV converter–Inverter system depends on irradiation, temperature and conversion efficiency. The current generated from PV panel varies with irradiation and output obtained is unregulated DC. To obtain steady and regulated power, DC-DC converters are used in PV system. DC-DC converter output power is controlled by varying duty ratio. To reduce the voltage ripple a suitable DC capacitor is connected across PV module [2].

PV converter-Inverter system design includes PV panel identification and power converter topology selection which is based on cost, efficiency and reliability. Various control strategies are used in grid tied and standalone applications. Inverters used in PV system can have bipolar or unipolar PWM switching. PWM technique is generally used in controlling inverter output. PWM technique compares high frequency triangular carrier signal with low frequency sinusoidal signal. Power injected to grid or supplied to a standalone load can be controlled using voltage or current controlled PWM technique. With proper filter design pure sine wave can be obtained from PWM Inverter [3][5][6][7].

The performance of PV converter-Inverter system can be analyzed using mathematical modeling approach. The mathematical modeling describes physical system by a set of equations that relates a set of variables between input and output of a system. These mathematical equations provide an insight of
theoretical and numerical data of PV converter-Inverter system variables which helps in analyzing the PV system precisely and apply various computation methods for better results for a particular application. The PV system is represented as a set of differential equations that describe the dynamic behavior of the PV converter-Inverter system. Solution of these differential equations are analyzed and redesigned as per the desired output [4][6][7]. In this paper, the mathematical modeling of PV system that includes modeling of PV module, DC-DC converter, and Inverter is carried out. The PV system performance and conversion efficiency is analyzed for a resistive load. The performance of the developed system is validated for variation in irradiation condition. The effect of irradiation on efficiency of the PV system is analyzed.

2. Design and Simulation

The simulation model for the PV converter-Inverter system represented in Figure 1 is shown in Figure 2. The selected PV module generates maximum power of 213W at 1000W/m² at constant temperature of 25°C at maximum load condition. PV module output voltage of 29V DC is boosted to 120V DC using boost converter. To obtain 120V DC output, the boost converter is operated with 75.8% duty ratio. With this duty ratio 120V DC output from converter is converted to AC of 120V peak at 50Hz frequency using single phase H Bridge Inverter with amplitude modulation 1 and frequency modulation ratio 20. Inverter output is supplied to a standalone resistive load.

The analysis of PV converter–Inverter system includes

- Modeling and analysis of effect of irradiation on 213 W PV panel.
- Modeling and control of 200W DC-DC Boost converter.
- Modeling and control of 200VA single phase H Bridge Inverter.
3. Modeling of PV module

PV module parameters used for modeling in Simulink is shown in Table 1. The solar irradiance $G$ is set to 1 representing 1000W/m$^2$ as a standard test condition.

| Parameter                        | Value             |
|----------------------------------|-------------------|
| Short Circuit Current ($I_{sc}$) | 7.84A             |
| Open Circuit Current ($V_{oc}$)  | 36.3V             |
| Current at maximum power ($I_{mp}$) | 7.35A         |
| Voltage at maximum power ($V_{mp}$) | 29V             |
| Number of cells per module (N)  | 60                |
| Diode saturation current ($I_d$) | 2.9259*10^{-10}  |
| Ideality factor (A)             | 0.98117           |
| Series Resistance ($R_s$)        | 0.39383Ω          |
| Shunt Resistance ($R_{sh}$)      | 313.3991Ω         |
| Electron charge (q)             | 1.6*10^{-19}C     |
| Boltzmann constant (k)          | 1.38*10^{-21}     |

The equation that represents the photovoltaic model is shown in equation (1). The current $I_{ph}$ generated from photovoltaic module is proportional to the solar irradiance $G$ and is shown in equation (2). The equation used for calculating thermal voltage $vt$ of diode is shown in equation (3).

Mathematical model of PV module implemented in Simulink using equations (1-3) is shown in Figure 3.

$$I = I_{ph} - I_s \left( e^{\frac{(V+I_R)R_s}{vt}} - 1 \right) - \frac{V+I_R R_s}{R_{sh}}$$  \hspace{1cm} (1)

$$I_{ph} = I_{sc} \cdot G$$  \hspace{1cm} (2)

$$vt = \frac{(A K T N)}{q}$$  \hspace{1cm} (3)

![Figure 3. Mathematical model of PV module](image)

As per the selected PV module parameters maximum power will be generated from PV module at 1000W/m$^2$ at maximum load condition. With maximum voltage from PV module as 29V and with
maximum current 7.35A, maximum power of 213W will be obtained when load resistance is nearly 3.945Ω. The simulated I-V and P-V curves of 213W panel are shown in Figure 4.

![I-V and P-V curves](image)

**Figure 4.** I-V and PV curve of PV module

4. **Modeling of Boost converter**

In Boost converter, the output voltage is always greater than the input voltage. When switch is ‘on’ input supplies energy to the inductor. When the switch is ‘off’ output stage receives energy from the inductor as well as from input [6] [7]. The physical model of Boost converter is shown in Figure 5 which uses MOSFET as the switch. Table 2 shows Boost converter specifications for 200W output power. Input to boost converter is supplied from PV module.

![Physical Model of Boost converter](image)

**Figure 5.** Physical Model of Boost converter

| Input voltage(V) | Output voltage(V) | Duty cycle(%) | Switching frequency(kHz) |
|------------------|-------------------|---------------|--------------------------|
| 29               | 120               | 75.8          | 30                       |

**Table 2.** Boost Converter specification

The relation between duty cycle, input and output voltage in Boost converter is shown in equation (4).

\[
\frac{V_0}{V_{IN}} = \frac{1}{1-D}
\]  

(4)

With input voltage \(V_{IN}=29V\) and duty ratio \(D=0.758\) output voltage of Boost converter \(V_0 = 120V\). The dynamic equations are obtained by applying KVL to the circuit during on and off state of the switch [6] [7].
When switch is ‘off’, by applying KVL to the resultant circuit, the voltage and current equations as shown in equations (5-7) are obtained for inductor.

\[ V_L = V_{IN} - V_0 \] (5)

\[ L \frac{di_L}{dt} = V_{IN} - V_0 \] (6)

\[ i_L = \frac{1}{L} \int (V_{IN} - V_0) dt \] (7)

When switch is ‘on’, by applying KVL to the resultant circuit, the voltage and current equations as shown in equations (8-10) are obtained for inductor.

\[ V_0 = -V_L \] (8)

\[ -V_0 = L \frac{di_L}{dt} \] (9)

\[ di_L = \frac{1}{L} \int -V_0 dt \] (10)

Voltage and current equations of capacitor are obtained as shown in equations (11-14)

\[ V_C = V_0 \] (11)

\[ i_C = C \frac{dv_C}{dt} \] (12)

\[ (i_L - i_0) dt = C dv_C \] (13)

\[ V_C = \frac{1}{C} \int (i_L - i_0) dt \] (14)

Using the dynamic equations of Boost converter mathematical model is developed as shown in Figure 6.

\[ \text{Figure 6. Mathematical model of Boost converter} \]

The required value of inductor and capacitor when operated at different frequency and with 10% ripple is calculated using equations(15-16)[6]. Table 3 shows values of L and C calculated for 10% ripple voltage and ripple current. For 200W boost converter with 120V output voltage, output current is 1.6667A.
\[ \Delta I = \frac{V_{IN} D}{f_s L} \quad (15) \]

\[ \Delta V_C = \frac{i_{0} D}{f_s C} \quad (16) \]

**Table 3.** Boost converter L and C design

| Ripple(%) | f_s(kHz) | \( \Delta I \)(A) | \( \Delta V_C \)(V) | L(mH) | C(μF) |
|-----------|-----------|-------------------|-------------------|-------|-------|
| 10        | 30        | 0.1667            | 12                | 4.349 | 3.472 |
| 10        | 15        | 0.1667            | 12                | 8.69  | 6.9   |

5. Modeling of single phase bridge inverter

A single phase H Bridge Voltage Source Inverter consists of four switches S1, S2, S3 and S4 as shown in Figure 7. When S1 and S2 are turned on simultaneously the input voltage appears across the load. When S3 and S4 are turned on simultaneously voltage across the load is reversed [6] [7]. Table 4 shows Single phase H Bridge Inverter specification for 200VA output power.

![Physical model of Single phase H Bridge Inverter](image)

**Figure 7.** Physical model of Single phase H Bridge Inverter

**Table 4.** Inverter specification

| Input voltage(V) | Output power(VA) | Output frequency(Hz) | Switching frequency(kHz) |
|------------------|-------------------|----------------------|--------------------------|
| 120              | 200               | 50                   | 1                        |

When switches S1 and S2 are ‘on’ the voltage and current equations (17-19) are obtained for inductor.

\[ V_L = V_{IN} - V_0 \quad (17) \]
\[ L \frac{di_L}{dt} = V_{IN} - V_0 \]  

(18)

\[ i_L = \frac{1}{L} \int (V_{IN} - V_0) \, dt \]  

(19)

When switches S3 and S4 are ‘on’ the voltage and current equations (20-22) are obtained for inductor.

\[ V_L = -V_{IN} - V_0 \]  

(20)

\[ L \frac{di_L}{dt} = -V_{IN} - V_0 \]  

(21)

\[ i_L = \frac{1}{L} \int -(V_{IN} + V_0) \, dt \]  

(22)

Voltage across capacitor is obtained as shown in equation (23).

\[ V_C = \frac{1}{C} \int (i_L - i_0) \, dt \]  

(23)

Using the dynamic equations of single phase H Bridge Inverter mathematical model is developed as shown in Figure 8.

![Mathematical model of Single phase H Bridge Inverter](image)

**Figure 8.** Mathematical model of Single phase H Bridge Inverter

The required values of filter components L and C of Inverter to obtain sine output is calculated using equations (24-25) [10][11] and is tabulated in Table 5.

\[ L = \frac{v_{dc}}{4f_s \Delta I_0} \]  

(24)

\[ C = \frac{\Delta I_0}{8f_s \Delta V_0} \]  

(25)

| Ripple(%) | f_s(kHz) | \Delta I_0(A) | \Delta V_0(V) | L(mH) | C(\mu F) |
|-----------|----------|--------------|--------------|-------|--------|
| 10        | 1        | 0.1667       | 12           | 179.9 | 1.736  |
| 10        | 5        | 0.1667       | 12           | 35.99 | 0.34   |
6. Modeling of PV converter inverter system

Implementation of the mathematical model of PV converter-Inverter system is shown in Figure 9. A 213 W capacity PV module is connected to 200 W boost converter which in turn is connected to 200VA inverter. Generated power from PV module, power output from Boost converter and Inverter is calculated for ideal conditions neglecting switching losses of the power semiconductor devices. The simulation results obtained are tabulated in Table 6 for different modulation index.

**Table 6. Voltage output at various stages in a PV converter-Inverter system**

| Modulation Index | Value  | Panel output (V) | Boost Output(V) | Inverter output (V_{Peak}) |
|------------------|--------|-----------------|-----------------|---------------------------|
| Ma=0.6 M=20      | Ideal  | 29              | 120             | 72                        |
|                  | Simulation | 29            | 117             | 61.37                     |
| Ma=0.9 M=20      | Ideal  | 29              | 120             | 108                       |
|                  | Simulation | 29            | 118.4           | 103                       |
| Ma=1 M=20        | Ideal  | 29              | 120             | 120                       |
|                  | Simulation | 29            | 118.4           | 115.54                    |

**Figure 9. Mathematical model of PV converter-Inverter system**

Implementation of the mathematical model of PV converter-Inverter system is shown in Figure 9. A 213 W capacity PV module is connected to 200 W boost converter which in turn is connected to 200VA inverter. The simulation results obtained are tabulated in Table 6 for different modulation index.
7. Simulation results and discussion

The analysis of PV converter-Inverter system is carried out for a resistive load of 200W. Simulations are carried out for the following two cases.

- Constant load at different modulation index.
- Varying irradiation with fixed duty ratio.

By varying modulation index inverter output is controlled. Power output and conversion efficiency at various stages in PV system with varying irradiation is shown in Table 7. For the designed values at constant load of 33ohm with constant duty ratio 75.8%, Boost converter, Inverter and overall PV system efficiency is around 94%, 82.6% and 77.8% at any irradiation level and at any modulation index. The variation in percentage of power output from PV module and power output at various stages of the PV converter–Inverter system for varying irradiation is tabulated in Table 8. The percentage panel output power is calculated with respect to the maximum power output capacity of the panel which is 213 W for an irradiance of 1000W/m².

| Irradiation (W/m²) | Ppanel (W) | Pdc (W) | Pac (W) | ηBoost(%) | ηInverter(%) | ηOverall(%) |
|-------------------|------------|---------|---------|-----------|-------------|-------------|
| 1                 | 206.7      | 194.6   | 160.9   | 94.14     | 82.68       | 77.84       |
| 0.9               | 167.5      | 157.8   | 130.4   | 94.2      | 82.63       | 77.85       |
| 0.8               | 132.3      | 124.6   | 103     | 94.17     | 82.66       | 77.85       |
| 0.7               | 101.1      | 95.17   | 78.67   | 94.13     | 82.66       | 77.81       |
| 0.6               | 73.93      | 69.61   | 57.54   | 94.15     | 82.66       | 77.83       |
| 0.5               | 50.95      | 47.97   | 39.65   | 94.15     | 82.65       | 77.82       |

| Irradiation (W/m²) | Ppanel (W) | Pdc (W) | Pac (W) | Ppanel(%) |
|-------------------|------------|---------|---------|-----------|
| 1                 | 206.7      | 194.6   | 160.9   | 97        |
| 0.9               | 167.5      | 157.8   | 130.4   | 78.63     |
| 0.8               | 132.3      | 124.6   | 103     | 62.11     |
| 0.7               | 101.1      | 95.17   | 78.67   | 47.46     |
| 0.6               | 73.93      | 69.61   | 57.54   | 34.7      |
| 0.5               | 50.95      | 47.97   | 39.65   | 23.92     |

7.1. PV output simulation results

PV panel output voltage, current and power generated at 1000W/m² is shown in Figure 10. 213W power is generated from PV panel with maximum voltage of 29V and with maximum current 7.35A with a load resistance of 3.945Ω.
7.2. DC-DC Converter Simulation results

Figure 11 shows DC-DC converter output voltage, current and power at constant irradiation of 1000W/m². With DC input of 29V to Boost converter with 75.8% duty ratio, an output of 120V DC is obtained. 200W output is obtained from Boost converter when connected to load resistance of 72Ω.

7.3. Single phase H Bridge Inverter Simulation results

The single phase H Bridge Inverter output with bipolar voltage switching is obtained as per the equation (26)

\[ V_{01} = M_a * V_{dc} \quad (M_a \leq 1.0) \]  

(26)

For amplitude modulation index 1 and frequency modulation index 20 output obtained is shown in Figure 12. It is observed that, the peak value of Inverter voltage increases from 61.37V to 115.54V when modulation index \( M_a \) is changed from 0.6 to 1.
7.4. PV converter –Inverter system Simulation results

For amplitude modulation index of 1 and frequency modulation index of 20 Figure 13 shows PV system output for fixed load resistance of 33ohms at 75.8% fixed duty cycle .Maximum output voltage from panel 29V is boosted to 120V DC and is converted to 120Vac peak which is shown in Figure.13. With lower value of modulation index it can be observed that the output voltage decreases as shown in Figure 14. With modulation index Ma as 1, maximum output is obtained from single phase inverter. As observed from the results obtained, irrespective of variation in modulation index and irradiation level, the power conversion efficiency of DC-DC converter, inverter and overall system remains constant.
8. Conclusion

In this paper, simulation and analysis of mathematical model of the PV converter – Inverter system is carried out. The simulation is carried out for constant resistive load at varying irradiation and also for constant load at varying modulation index. Power output at various stages of PV system and power conversion efficiency is discussed. It is observed that the power available from the panel is reduced with reduction in the irradiation. However, it is also observed that, the overall efficiency of the PV converter-inverter system remains to be a constant for different levels of irradiation as it is operated at fixed duty cycle at constant load condition.

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