DEBYE SCREENING IN THE QCD PLASMA

O. PHILIPSEN

CERN Theory Division, 1211 Geneva 23, Switzerland

Various definitions for the QCD Debye mass and its evaluation are reviewed in a non-perturbative framework for the study of screening of general static sources. While it is possible to perturbatively integrate over scales \( \sim T \) and thus construct a 3d effective theory, the softer scales \( \sim gT \) and \( \sim g^2T \) are strongly coupled for temperatures \( \lesssim 10^7 \) GeV and require lattice simulations. Within the effective theory, a lattice treatment of screening at finite quark densities \( \mu \lesssim 4/T \) is also possible.

1 Introduction

One goal of heavy ion collision experiments is to probe the deconfined phase of QCD, in which matter is expected to exist in a state of quark-gluon plasma. At present, we are still far from a satisfactory understanding of its properties, and hence of unambiguous signals for its detection. Qualitative expectations are based on asymptotic freedom, i.e. the coupling weakening with increasing temperature. However, for any reasonable temperatures below \( 10^{11} \) GeV, perturbative treatments are badly convergent due to infrared-sensitive contributions. On the other hand, the only non-perturbative first principles approach, four-dimensional (4d) lattice simulations, is severely restricted in the presence of light fermions, and as of yet fails completely at finite baryon density.

Considerable progress has been made over the last years in studying the screening of static sources in an equilibrated plasma by simulating dimensionally reduced QCD, as well as 4d pure gauge theory. In this contribution these developments are reviewed with special emphasis on Debye screening, which is believed to play a central role for deconfinement. While at present it is not clear whether screening of static charges can be directly probed experimentally, it is a well-defined physical plasma property, providing the dynamical length scales over which interactions are effective.

One difficulty with Debye screening in QCD is properly defining it. After briefly recalling the QED analogue and pointing out the differences in QCD, a non-perturbative framework to study screening of a general static source is given. Next, an effective theory description for hot QCD is reviewed, permitting a convenient non-perturbative study and interpretation of screening...
lenghts, before returning to the question of a gauge-invariant definition and non-perturbative evaluation of Debye screening. Finally, Debye screening in the presence of a finite baryon density is discussed.

1.1 Debye screening in QED

Consider first a QED plasma in equilibrium, into which an external static charge is introduced. The modified Hamiltonian is then

\[ H' = H + H_{\text{ext}} = \frac{1}{2}((E + E_{\text{ext}})^2 + B^2) \]  \hspace{1cm} (1)

In linear response theory the perturbation of the electric field is

\[ \delta \langle E_i(x) \rangle = \lim_{t \to \infty} i \int_{t_0}^{t} dt' \text{Tr}(\rho[H_{\text{ext}}(x', t'), E_i(x, t)]) = \lim_{t \to \infty} -i \int_{t_0}^{t} dt' \int d^3 x' E_{i}^{\text{ext}}(x') \langle [E_i(x, t), E_j(x', t')] \rangle \]  \hspace{1cm} (2)

With \( E_i(x) = -\nabla_i V(r) \), the potential of the static source is determined by the correlation of electric fields, or equivalently by the Fourier transform of the \( A_0 \) propagator,

\[ V(r) = Q \int \frac{d^3 k}{(2\pi)^3} \frac{e^{ikr}}{k^2 + \Pi_{00}(0, k)} = Q \frac{e^{-m_D r}}{4\pi r} \]  \hspace{1cm} (3)

Thus, the electric field of the source is screened by the charges in the plasma, and the Debye mass, or inverse screening length, is defined by the pole of the photon propagator,

\[ \Pi_{00}(k_0, 0) = -m_D^2 \]  \hspace{1cm} (4)

The leading order one-loop diagram gives \( m_D^0 = eT / \sqrt{3} + O(e^2 T) \). On the other hand, there is no magnetic screening to all orders of perturbation theory, i.e. \( \Pi_{ii}(k_0 = 0, k \to 0) = 0 \).

1.2 Non-abelian Debye screening

Following the same steps in an SU(N) theory with \( N_f \) fermions and evaluating the pole of the one-loop \( A_0 \) propagator, one finds the leading order result

\[ V(r) \sim \frac{e^{-m_D^0 r}}{4\pi r}, \quad m_D^0 = \left( \frac{N}{3} + \frac{N_f}{6} \right)^{1/2} gT \]  \hspace{1cm} (5)
However, at next-to-leading order the problem becomes non-perturbative. The general form of the series in $g$ can be shown to be

$$m_D = m_0^0 + \frac{N}{4\pi} g^2 T \ln \frac{m_D}{g^2 T} + c_N g^2 T + \mathcal{O}(g^3 T),$$

(6)

which is non-analytic in the coupling constant. While the coefficient of the logarithm is fixed perturbatively, $c_N$ is entirely non-perturbative. The reason is that, starting from this order in $g$, the non-abelian $A_0$ couples to the soft magnetic gluons $A_i \sim g^2 T$, and hence becomes sensitive to the non-abelian infrared divergencies in the magnetic sector for which there is no perturbative cure $\Pi_{ii}(k_0 = 0, k \to 0) \sim g^2 T \neq 0$, with contributions from all loop orders two and larger.

This raises the conceptual problem whether a perturbative definition of Debye screening in QCD is at all sensible. Moreover, neither $\Pi_{00}$ nor the colour-electric field $\mathbf{E}$ are gauge-invariant physical concepts in a non-abelian theory. It can be shown that the pole of the $A_0$ propagator is gauge-invariant order by order in a (resummed) perturbation series $\Delta$. However, this does not guarantee the existence of a pole in the full propagator. Even if it exists, its non-perturbative relation to physical quantities remains unclear in a situation where the potential of a single static charge is not defined.

There are then two ways to deal with this problem non-perturbatively. One is to assume a pole in the full propagator and to determine its value from the exponential fall-off of the gauge-fixed propagator in space. Results of this approach are presented in another contribution to this conference $\Pi$. The second option, which will be discussed here, is to seek a manifestly gauge-invariant definition of Debye screening.

### 1.3 Non-perturbative description of screening

Let us first recall the general definitions needed to non-perturbatively describe screening of any gauge-invariant source. Consider gauge-invariant, local operators $\mathbf{A}(x)$. Static equilibrium physics is described by spatial correlation functions of euclidean time averages

$$C(|\mathbf{x}|) = \langle \bar{\mathbf{A}}(\mathbf{x})\mathbf{A}(0) \rangle_\epsilon \sim e^{-M|\mathbf{x}|}, \quad \bar{A}(x) = T \int_0^{1/T} d\tau A(x, -i\tau).$$

(7)

These fall off exponentially with distance. The “screening masses” $M$ have a precise non-perturbative definition: they are the eigenvalues of the space-wise transfer matrix in the corresponding lattice field theory. Physically, they correspond to the inverse length scale over which the equilibrated plasma is
sensitive to the insertion of a static source carrying the quantum numbers of \( A \). Beyond \( 1/M \), the source is screened and the plasma appears undisturbed. With these definitions, all screening lengths corresponding to gauge-invariant sources can be computed on the lattice in principle.

2 Effective theory description

If we are interested in length scales larger than the inverse temperature, \( |x| \sim 1/gT \gg 1/T \), the situation simplifies considerably. In this case the integration range for euclidean time averaging in Eq. \( \mathcal{C}[5] \) becomes very small, and the problem effectively three-dimensional (3d). The calculation of the correlation function \( C(|x|) \) can be factorized: the time averaging may be performed perturbatively by expanding in powers of the scale ratio \( gT/T \sim g \), which amounts to integrating out all modes with momenta \( \sim T \) and larger (i.e. the non-zero Matsubara modes, in particular the fermions). This procedure is known as dimensional reduction \( \mathcal{C}[6] \). The correlator \( C(|x|) \) of 3d fields is then to be evaluated with a 3d purely bosonic effective action, describing the non-perturbative physics of the modes \( \sim gT \) and softer, which can be done on the lattice. The screening masses \( M \) now are the eigenvalues of the 3d transfer matrix. The 3d purely bosonic simulations are much easier and thus more accurate than simulating directly in 4d. However, the perturbative integration over hard modes and neglecting higher dimension operators in the effective action introduces a relative error, which at two-loop level is\( \mathcal{C}[7] \)

\[
\frac{\delta C}{C} \sim \mathcal{O}(g^3). \tag{8}
\]

In the treatment of the electroweak phase transition, this error is less than 5\( \% \), but for hot QCD the coupling and thus the error is larger. An estimate of its size will be given later, when results from the full and the effective theory are compared.

The effective theory emerging from hot QCD by dimensional reduction is the SU(3) adjoint Higgs model with the action

\[
S = \int d^3x \left\{ \frac{1}{2} \text{Tr}(F_{ij}F_{ij}) + \text{Tr}(D_iA_0)^2 + (m_D^0)^2 \text{Tr}(A_0^2) + \lambda_3 (\text{Tr}(A_0^2)^2) \right\}. \tag{9}
\]

Apart from the SU(3) gauge symmetry, the action respects SO(2) planar rotations, two-dimensional parity \( P \), charge conjugation \( C \) and \( A_0 \)-reflections \( R \). Its parameters are via dimensional reduction functions of the four-dimensional gauge coupling \( g^2 \), the number of colours \( N \) and flavours \( N_f \), the fermion masses and the temperature \( T \). In all of the following fermion masses are
assumed to be zero, but in principle any other values may be considered as well. At leading order in the reduction step one then has

\[ g_3^2 = g^2(\bar{\mu})T, \quad \lambda_3 = \frac{1}{24\pi^2}(6 + N - N_f)g^2(\bar{\mu})T. \]  

(10)

The reduction step has been performed to two-loop order at which parameters have relative accuracy \( O(g^4) \). Specifying \( T \) and a renormalization scale \( \bar{\mu} \) completely fixes the effective theory.

If we are interested in the longest scales of order \( \sim 1/g^2T \), one may be inclined to integrate out the fields \( A_0 \sim gT \) as well, leaving a 3d pure gauge theory. In order to decide which is the correct effective theory in practice, simulation results for screening lengths will now be compared between the reduced and the full theory.

Results from simulations of the gauge-invariant screening spectrum with the effective theory Eq. (9) are displayed in Fig. 1. Screening masses are classified according to the symmetries of the 3d Hamiltonian by the quantum numbers \( J^P C R \). The open symbols show states dominated by scalar operators such as \( \text{Tr}(A_0^2) \), \( \text{Tr}(A_0F_{12}^2) \) etc., whereas full symbols show states receiving only gluonic contributions \( \text{Tr}(F_{12}^2) \) etc. It is a remarkable finding of detailed mixing analyses in several models that the latter are quantitatively consistent with the 3d glueball states and completely insensitive to the presence of the \( A_0 \) scalar field. Because of this pronounced non-mixing, we can state a first important result: For any reasonable temperatures the largest correlation length of gauge-invariant, local operators belongs to the \( A_0 \) degrees of freedom and not to the \( A_i \) in contrast to the naive parametric picture. This demonstrates that the physics from the scale \( gT \) down to \( g^2T \) is completely
non-perturbative, and hence $A_0$ may not be integrated out perturbatively. Only for temperatures $T > 10^7 T_c$ becomes the coupling small enough that the parametrically expected ordering of screening masses is realized.

Next, we are interested in the accuracy of the reduced theory. Fig. 2 compares the results for hot SU(2) gauge theory as obtained in the full lattice theory \(^{13}\) with those from the effective theory \(^{14}\). Note that the effective theory is only valid up to its cut-off $M/T \sim 2\pi$ and above this level disagreement is to be expected. The apparent excellent agreement for the two lowest lying states should be taken with care given some systematic uncertainties. In 4d simulations, the projection properties of operators and the scaling behaviour are more difficult to control than in 3d. In the effective theory, there is an ambiguity in the choice of renormalization scale. The uncertainty in the comparison may thus be as large as 20%. For the case of SU(3), one finds again quantitative agreement in the largest correlation length but about 20% deviation in the next shorter one.

One advantage of the effective theory approach is its easy inclusion of fermions, which are treated analytically and just amount to changing the parameters Eq. (10) of the action to be simulated. However, the screening masses $M/T$ are found to increase with $N_f$ \(^{14}\), and at $N_f = 4$ the lowest of them is already of the order $\sim 1/2\pi T$ and hence not expected to be quantitatively accurate. Moreover, close to $T_c$ the fermionic modes in the 4d theory feel the chiral phase transition, such that for $T \lesssim 2 - 3 T_c$ the correlation length

Figure 2. Comparison of screening masses in hot SU(2) (left) and SU(3) (right) pure gauge theory as determined in 4d (empty symbols) \(^{13}\) and 3d (full symbols) \(^{14}\) effective theory simulations.
obtained from pion operators becomes the longest one. In this situation fermions may not be integrated out and the purely bosonic effective theory is invalid. We may thus conclude that dimensional reduction gives a reasonable description of the largest correlation lengths of hot QCD for temperatures \( \geq 2 - 3T_c \), with an error \( \lesssim 20\% \) that decreases with temperature.

3 The Polyakov line and the heavy quark potential

A gauge-invariant, non-local operator that has been suggested in the literature to describe Debye screening is the Polyakov line

\[
L(x) = \frac{1}{N} \text{Tr} P \exp \left( -ig \int_0^\beta d\tau A_0(x, \tau) \right).
\]

Its spatial correlator is related to the free energy of a quark anti-quark pair in the plasma, and thus to the static potential at finite temperature

\[
\frac{\langle L^1(r)L(0) \rangle}{\langle |L| \rangle^2} = \exp -\beta F_{q\bar{q}} = \exp -\frac{V(r)}{T}.
\]

The idea is that by considering a source together with an anti-source gives a manifestly gauge-invariant operator related to heavy quark systems. In leading order perturbation theory one has

\[
L = 1 - \frac{g^2}{2} \text{Tr} (\int d\tau A_0)^2 + \ldots,
\]

and hence the correlator at this order is given by the exchange of two electric gluons leading to

\[
\langle L^1(r)L(0) \rangle_c = \frac{(N^2 - 1)g^4 e^{-2m_{0D}r}}{8N^2T^2 (4\pi r)^2},
\]

with the leading order Debye mass \( m_{0D} \). However, this state of affairs is spoilt by higher loop diagrams, as several authors have pointed out. Fig. 3 left shows a diagram of order \( g^8 \), where now \( A_0 \) can couple to two magnetic gauge fields \( A_i \), and hence the exponential fall-off of this diagram is governed purely by magnetic gauge fields. Indeed, through a fermion loop a similar coupling is possible even in QED, cf. Fig. 3 right. There, magnetic gauge fields are strictly massless, and the correlator falls off algebraically rather than exponentially. It thus fails to produce the Debye mass for electric fields even in QED.

With our previous discussion of screening masses at hand, it is not difficult to understand how the Polyakov line correlator should fall off away from \( T_c \). \( L \) contains a sum over all \( J^{PC} \) sectors in the fundamental representation. Furthermore, because of gauge-invariance it couples only to gauge-invariant intermediate states. Its correlator should therefore asymptotically fall off with the lightest gauge-invariant screening mass of the spectrum. Indeed, a recent
Figure 3. Higher loop diagrams contributing to the Polyakov line correlator in QCD (left) and QED (right).

4d simulation of the SU(3) Polyakov line correlator \( \frac{M}{T} \approx 2.5(3) \) finds it exponential decay to be governed by \( \frac{M}{T} \approx 2.5(3) \), which is fully compatible with the lightest 0\(^{++} \) mass given in the table in Fig. 2. The same observation is made to rather high precision in a hot (2+1)-dimensional gauge theory [2].

4 A gauge-invariant definition for \( m_D \)

A few years ago it was suggested to use a symmetry to distinguish between the electric and the magnetic sectors of the theory [18], namely euclidean time reflection \( T : \tau \to -\tau \). This operation changes the sign of the electric gauge fields, \( A_0 \to -A_0 \), but not that of the magnetic ones. The non-perturbative Debye screening length \( 1/m_D \) may then be defined to be the largest correlation length measured from a correlator \( \langle A(x)B(0) \rangle \), where \( A, B \) are gauge-invariant operators which are local in 3-space and odd under \( T \). Examples of four-dimensional operators with these properties are \( \text{Tr}(\text{Im}L), \text{Tr}(\text{Im}LF_{12}), \text{Tr}(F_{03}F_{12}^0), \text{Tr}(F_{03}F_{12}) \) etc.

In the 3d effective theory, where "time" has been reduced away, the remnant of euclidean time reflection is the scalar reflection symmetry \( R : A_0 \to -A_0 \). The Debye mass then corresponds to the lightest eigenstate with quantum number \( R = - \). It is now easy to go back to the results for the spectrum in Fig. 1 and identify the lowest mass in the 0\(^{+-} \) channel as the Debye mass according to this definition\(^a\). The corresponding operator is \( \text{Tr}(A_0 F_{ij}) \), which may be interpreted as an electric gauge field dressed by magnetic gluons to make it gauge-invariant. Despite its coupling to \( A_i \), \( R \)-symmetry prohibits its correlation to be governed by the 3d pure gauge sector. This definition certainly refers to a physical quantity. However, as it couples to colour singlet sources, its relation to screening of colour flux and to deconfinement of a quarkonium system is not obvious.

\(^a\) Alternatively, it was proposed to measure matrix elements between \( R \)-odd operators and a \( Z_N \) domain wall, introduced by a twist in the effective action [2].
Table 1. The different contributions to the Debye mass, Eq. (6). (1+2) refers to the first two terms in units of $g^2T$.

| $T$       | $m_D/g^2T$ | (1+2) | $c_3$ | $\mathcal{O}(g^2T)/(g^2T)$ |
|-----------|------------|-------|-------|-----------------------------|
| $T = 2T_c$| 1.82 (3)   | 0.514 | 1.65(6)| -0.35(8)                    |
| $T = 10^{11}T_c$| 3.83 (9) | 2.165 | 1.65(6)| 0.015(90)                  |

4.1 Perturbative and non-perturbative contributions to $m_D$

We may now ask to what extent this Debye mass can be computed in perturbation theory. First, note that the perturbative expansion follows the same pattern as in Eq. (6). At tree level, the correlator is given by exchange of one electric and two magnetic gluons. Since the latter are massless at tree level, the leading order fall-off is yet again $m_D^0$. However, at higher orders the contributions are different from those to the $A_0$ propagator.

There is a recipe to compute the coefficient $c_N$ non-perturbatively: if temperature is sent to infinity, the $A_0$ becomes infinitely heavy and decouples, so it may be integrated out. In this case, $A_0$ turns into a static external source, and the only dynamical fields left are the $A_i$ of a 3d pure gauge theory. We thus have

$$(\text{Tr}(A_0 F_{ij})(\mathbf{x})\text{Tr}(A_0 F_{lm})(0)) \rightarrow e^{-m_D^0|x|}/4\pi|x| \langle F^a_{ij}(\mathbf{x})W_{ab}(\mathbf{x},0)F^b_{lm}(0) \rangle,$$  

(14)

where $W_{ab}(\mathbf{x},0)$ is an adjoint representation Wilson line.

As a result, our correlator factorizes into the free $A_0$ propagator and a correlator of the magnetic field strength, connected by an adjoint Wilson line representing the static $A_0$. The field strength correlator falls off exponentially $\sim \exp(-(|\Delta + c_N|)|x|)$, where $\Delta$ contains a logarithmic divergence which gets absorbed by the renormalisation of the $A_0$ self-energy.

Measuring $c_N$ in a 3d pure gauge theory and comparing with the measurements of the full $m_D$ we can now summarise the various contributions to the SU(3) Debye mass in Table 1. We observe that at reasonable temperatures the next-to-leading order coefficient $c_3$ is larger than the leading contribution, and thus the Debye mass is entirely non-perturbative. Again the reason is its coupling to the non-perturbative magnetic sector $\sim g^2T$. Note also that the $\mathcal{O}(g^3T)$ contributions are small compared to the $g^2T$ contributions. Together with the previous observations about scales involved, one may then conclude that the physics of screening for all temperatures of practical interest is dominated by the scale $\sim g^2T$. Only at asymptotically high temperatures $T \gtrsim 10^{7}T_c$ is the perturbative picture restored.
Debye screening at finite baryon density

In heavy ion collisions the initial state has non-vanishing baryon number, and so does any subsequent plasma state. We would therefore like to know if and how a chemical potential for quarks affects the physics of screening. Unfortunately, finite baryon density cannot be addressed by standard lattice QCD because of the so-called “sign-problem”: when a chemical potential term is added to the theory, the fermion determinant becomes complex prohibiting Monte Carlo importance sampling, which requires a strictly positive measure. To date, no working cure has been found for this problem.

Recently it was demonstrated that in dimensionally reduced finite density QCD the sign problem is in fact numerically tractable. Inclusion of a chemical potential term for quarks leads to one extra term in the action \( S \rightarrow S + \mu \frac{N_f}{3\pi^2} \int d^3x \text{Tr}A_0^3 \), \( m_D^0 \rightarrow m_D^0 \left[ 1 + \left( \frac{\mu}{\pi T} \right)^2 \frac{3N_f}{2N + N_f} \right] \). This action is complex and cannot be simulated with standard methods. Instead, a reweighting procedure has to be performed, where the complex term of the action is absorbed into the observable, so that

\[
\langle O \rangle = \frac{\langle O e^{-iS_\mu} \rangle_0}{\cos(S_\mu)_0},
\]

and the averaging is done with \( \mu = 0 \). The sign problem, i.e. cancelling contributions to the expectation value, occur whenever \( S_\mu \gg 1 \). Fortunately, the width of the distribution of this quantity, growing as \( \sim (\mu/T)^{V^{1/2}} \), is narrow enough to permit large enough volumes for simulated masses to reach their infinite volume levels, as long as \( \mu < \sim 4T \).

There is one important change of the symmetries of the theory, which is relevant for Debye screening: the chemical potential term violates \( R \) and \( C \), and thus only \( P \) is left as a good symmetry to classify the screening states. Since the definition of \( m_D \) was based on the use of \( R \), it seems no longer applicable in the finite density situation. In the 3d effective theory, the corresponding state can mix with any lighter \( R = + \) states and decay. However, for \( \mu = 0 \), we found the Debye mass to be the lowest state in the \( 0^-_+ \) channel. This is also the lowest state with \( P = - \). As Fig. 4 shows, this remains to be the case as \( \mu/T \) is switched on, and by continuity one may now identify the lowest \( 0^- \) mass to correspond to the Debye mass.

Finally, it has been demonstrated, that screening masses for \( \mu < T \) should be accessible to 4d simulations with imaginary \( \mu \) and analytic continuation.
6 Conclusions

The analogy of Debye screening in QED and QCD is rather limited, because in QCD it is not possible to define the electric field induced by an isolated charge: neither of these are gauge-invariant physical concepts. The problem then is to specify what we mean by Debye screening in QCD. Depending on the definition, one gets different answers and different interpretations of the results. Defining the Debye mass through the pole of the full $A_0$ propagator has the disadvantage that its existence is not guaranteed, and its non-perturbative connection to physical quantities is not clear. Employing the Polyakov line correlator describing a heavy quark anti-quark pair does not probe Debye screening: it mixes electric and magnetic sectors, failing to fall off with $m_T$ even in QED. A gauge-invariant definition is possible through correlations of operators odd under euclidean time reflection, which reflects $A_0$ but not $A_i$.

In general, the screening of static sources in equilibrium is a well-defined problem that has been studied non-perturbatively by lattice QCD and with effective field theory methods. A detailed picture of the static length scales for non-abelian plasma physics has emerged. An important lesson for constructing effective theories is that, for all temperatures $T < 10^7$ GeV, the scales $\sim gT$ and $\sim g^2T$ are not separable non-perturbatively. All static screening physics seems to be dominated by the dynamics on soft scales $\sim g^2T$.

Finally, dimensionally reduced QCD gives a realistic description of static phenomena down to temperatures of a few $T_c$, and is able to non-
perturbatively accommodate chemical potentials for quarks $\mu \lesssim 4T$. It can therefore address the physics of screening in the parameter range relevant for heavy ion collisions.

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