A New Entropy Measurement for the Analysis of Uncertain Data in MCDA Problems Using Intuitionistic Fuzzy Sets and COPRAS Method

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Abstract: In this paper, we propose a new intuitionistic entropy measurement for multi-criteria decision-making (MCDM) problems. The entropy of an intuitionistic fuzzy set (IFS) measures uncertainty related to the data modelling as IFS. The entropy of fuzzy sets is widely used in decision support methods, where dealing with uncertain data grows in importance. The Complex Proportional Assessment (COPRAS) method identifies the preferences and ranking of decisional variants. It also allows for a more comprehensive analysis of complex decision-making problems, where many opposite criteria are observed. This approach allows us to minimize cost and maximize profit in the finally chosen decision (alternative). This paper presents a new entropy measurement for fuzzy intuitionistic sets and an application example using the IFS COPRAS method. The new entropy method was used in the decision-making process to calculate the objective weights. In addition, other entropy methods determining objective weights were also compared with the proposed approach. The presented results allow us to conclude that the new entropy measure can be applied to decision problems in uncertain data environments since the proposed entropy measure is stable and unambiguous.

Keywords: intuitionistic set; COPRAS method; MCDM

1. Introduction

Multi-criteria decision analysis (MCDA) is an approach to dealing more easily with complex decision problems, which applies to decision-making related to professional activity and everyday life problems. Many authors have used MCDA methods for a wide range of problems. One of the fields in which multi-criteria decision problems arise is economics. For example, Stević et al. presented research on sustainable production assessment for forestry companies in the Eastern Black Sea region [1]. This research is essential for the circular economy, which aims to reduce the waste of natural resources. The evaluation process used a Rough PIvot Pairwise RElative Criteria Importance Assessment (PIPRECIA) method belonging to the stream of multi-criteria decision analysis methods. Another example of an emerging problem in economics is the problem of installing dams on the Drina River [2]. This problem is essential for the economy of Yugoslavia because of the extracted benefits from the use of a renewable energy source.

The field in which the approach of multi-criteria decision analysis is used is sustainable transport. The problem that arises in the evaluation of urban transport is the selection.
of relevant criteria. Shekhovtsov et al. [3] proposed an approach based on a reference ranking that determines the relevance of criteria using four approaches. For the supplier selection problem, studies have also been conducted on selecting an appropriate weight selection method [4]. As a result, objective methods of selecting weights, such as entropy, equal, or standard deviation methods, were selected. Pamucar et al. [5] used an approach combining the Best Worst Method (BWM) and Compressed Proportional Assessment (COPRAS) techniques to evaluate off-road vehicles for the Serbian Armed Forces (SAF). Choosing the appropriate off-road vehicle to transport units has a massive impact on their safety. Therefore, this problem is also essential to fill the research gap on defining a proper vehicle selection methodology for the military.

One of the most important fields in which MCDA methods are used is the healthcare field. La Scalia et al. [6] proposed using Technique Ordered Preference by Similarity to the Ideal Solution (TOPSIS) to evaluate information related to pancreatic islet transplantation. It was decided to use a multi-criteria decision analysis method because of the many variables involved in this problem. The TOPSIS method was also modified in this study to fit the medical problem better. Multi-criteria decision analysis was also used in the problem of supplier selection in the healthcare industry in Bosnia and Herzegovina [7]. A new method, Measurement of Alternatives and Ranking according to COmpromise Solution (MARCOS), was used in this work. The sensitivity analysis of the proposed approach in the study showed that this method could be applied in a supplier selection problem in the healthcare industry.

MCDA methods are also applied to solid waste management problems. Muhammad et al. presented a study using the GREY-EDAS hybrid model to evaluate alternative waste treatment methods in Nigeria [8]. Due to Nigeria being a developing country, it is still struggling with the problem of municipal solid waste treatment. The application of the proposed approach, i.e., the GREY-EDAS method, yielded consistent results, i.e., the most suitable method for municipal solid waste treatment was achieved.

Another field in which multi-criteria decision analysis techniques are applied is renewable energy. Bączkiewicz et al. presented an approach in which they used the COMET-TOPSIS and SPOTIS methods to create a decision support system (DSS) for the solar panel selection problem [9]. Proper selection of photovoltaic systems for companies and households involves the possibility of discontinuing the use of energy obtained through conventional sources. Therefore, the creation of a decision support system for this problem proves its great practical usefulness. The problem of selecting the development of hydrogen buses is also an example of the application of MCDA methods in renewable energy. An approach to this problem was proposed by Pamucar et al. using a model composed of Best-Worst Method (BWM) and Measurement Alternatives and Ranking according to COmpromise Solution (MARCOS) [10]. The results obtained in the study show that the best solution is co-generated electricity from a municipality cogeneration power plant.

Multi-criteria decision analysis is also applied in E-commerce. For example, Bączkiewicz et al. [11] proposed a consumer decision support system based on five multi-criteria decision analysis methods for selecting the most suitable cell phone. The results obtained from the methods were the components of the trade-off ranking obtained using the Copeland method.

One of the most significant challenges of multi-criteria decision-making is to express the uncertainty in the data correctly. The source of such uncertainty can be unreliable information, information noise, or conflicting information. Greis et al. presented a study on the source of uncertain information from user inputs, considering that their input is not mainly controlled [12]. Decisions made with uncertainty in the data are more challenging to analyze and often require specialized tools. One such tool is interval fuzzy sets, which can be considered as an extension of the concept of real numbers [13]. Shekhovtsov et al. proposed a new approach in multi-criteria decision analysis, where they combined interval fuzzy sets with the Stable Preference Ordering Towards Ideal Solution (SPOTIS)
method. The proposed approach aims to deal with the rank reversal paradox with uncertain data [14].

Fuzzy sets are another popular tool used to deal with uncertainty in data. Zadeh proposed a fuzzy set as a class of objects with some continuum of degrees of membership [15]. An example application of fuzzy sets in MCDA methods is the problem of selecting the best solution for the business balance of a passenger rail carrier. Vesković et al. used a combination of the fuzzy Pivot Pairwise RElative Criteria Importance Assessment (F-PIPRECIA) weight selection method with the fuzzy Evaluation based on Distance from Average Solution (F-EDAS) method [16]. The results obtained were also compared with other methods based on fuzzy sets, i.e., Fuzzy Measurement Alternatives and Ranking according to the COMpromise Solution (F-MARCOS), Fuzzy Simple Additive Weighing (F-SAW) and the Fuzzy Technique for Order of Preference by Similarity to Ideal Solution (F-TOPSIS). The following example of using fuzzy sets in multi-criteria decision analysis is the housing selection problem. Kizielewicz and Bączkiewicz proposed using four MCDA methods extended with fuzzy sets to select the best housing alternative [17]. The results have been compared using the similarity coefficients of the rankings and indicate high similarity between the obtained rankings of the decision alternatives.

A tool that is also used in multi-criteria decision analysis to deal with uncertainty in data is Pythagorean fuzzy sets (PFSs). They were introduced to deal with ambiguity by Yager, where the degrees of membership are represented as pairs [18]. The proposed tool is close to intuitionistic fuzzy sets (IFS) [19]. One of the applications of Pythagorean fuzzy sets in MCDA is to extend the VIKOR method to it. This method was used to assess the occupational risk in constructing a natural gas pipeline through Mete et al. [20]. The built DSS system aims to improve the situation in the energy industry. Methods such as AHP and TOPSIS also have extensions in the form of Pythagorean fuzzy sets. Both methods were applied in the problem of evaluating the quality of hospital services [21]. In this approach, three hospitals were evaluated, of which one was private and two were public. Furthermore, Pythagorean fuzzy sets are also used in hybrid approaches, both in European school methods and in American school methods. An example of such an approach is the risk assessment problem in failure modes and effects analysis (FMEA) [22]. The Pythagorean fuzzy hybrid Order of Preference by Similarity to an Ideal Solution (PFH-TOPSIS) method from the American school and the Pythagorean fuzzy hybrid ELimination and Choice Translating REality I (PFH-ELECTRE I) method from the European school were used for the evaluation. The effectiveness of the proposed approach is demonstrated using the color super-twisted nematic (CSTN) as an example.

Hesitant fuzzy sets are another tool used to deal with uncertain data. Torr proposed it because of the difficulty of determining the membership of elements to a set in some cases [23]. It is used, for example, in decision-making problems concerning the selection of electric city buses. An approach in which the method of characteristic objects using hesitant fuzzy sets was presented for this problem by Salabun et al. [24]. Another example of HFS application in MCDA is the use of the hesitant fuzzy TOPSIS method in the problem of site selection for a hospital [25]. Selecting a location for a new hospital in Istanbul is difficult because many important factors, such as transportation, demographics and infrastructure, influence the location. Therefore, it was decided to use a multi-criteria decision analysis method.

Decision-making only in certain exceptional situations can be performed with crisp data. More and more frequently, MCDA methods support decision-makers in complex systems in an uncertain environment. In these cases, a decision-maker generally has uncertain or incomplete data at his disposal, and choosing the best alternative with often conflicting criteria is even more challenging to achieve. In the case of crisp data, sensitivity analysis is performed to determine how much impact a change in attribute values has on the final ranking positions of the alternatives. However, to properly deal with uncertainty in the data, an appropriate tool must be developed to model this kind of uncertainty well. This paper is focused on intuitionistic fuzzy sets (IFSs), which have repeatedly proven
their usefulness in the MCDA domain [26,27]. Intuitionistic fuzzy sets are an essential generalization of the fuzzy sets [15], where the main concept supposes that, for each value in a defined domain, we assign a membership value and a non-membership value. Using such tools in combination with multi-criteria decision analysis allows for better identification of data uncertainties and enables their proper modelling in the decision-making process. Therefore, many approaches in decision-making use fuzzy sets and their generalizations, including IFSs [28,29].

Intuitionistic fuzzy sets are still being developed, and there are discussions about their implementation. Szmidt and Kacprzyk proposed new definitions concerning the distance between intuitionistic fuzzy sets [30]. They aim to address the issue that the degree to which an element does not belong to an intuitionistic fuzzy set will not always be 1. Turanli et al. presented new types of fuzzy connectedness in intuitionistic fuzzy topological spaces [31]. Bustince presented several theorems that allow intuitionistic fuzzy relations to be formed on a set with specific properties [32]. Ciftcibasi and Altunay presented the concept of two-sided fuzzy reasoning and developed its mathematical structure [33].

One of the key features of IFS is to better account for uncertainty. Many MCDA methods have used this concept to solve problems with uncertain data, and we can mention here such methods as Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) [34,35], Preference Ranking Organization METHod for Enrichment of Evaluations (PROMETHEE) [36,37], VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) [38,39], Analytic Hierarchy Process (AHP) [40,41], Élimination et Choix Traduisant la Réalité (ELECTRE) [42,43], Multi-Attributive Border Approximation area Comparison (MABAC) [44], Combinative Distance-based Assessment (CODAS) [45,46], COMplex PRIportional ASsessment (COPRAS) [47], Multi-Objective Optimization Method by Ratio Analysis (MOORA) [48,49], Analytic Network Process (ANP) [50,51], Decision Making Trial and Evaluation Laboratory (DEMATEL) [52,53], Stepwise Weight Assessment Ratio Analysis (SWARA) [47,54] and so on. All of these mentioned approaches mainly use attributes stored as uncertain data. A separate issue is the use of entropy IFSs to determine significance weights for criteria [55,56], which we address in this paper. Classical Information Entropy defines the average amount of information per single message from an information source. Entropy can be interpreted as a measure of uncertainty, and when the uncertainty increases, the entropy value increases also [57,58].

Entropy describes the uncertainty information, and it is an important research topic in the decision-making domain [59]. In 1996, the notion of intuitionistic fuzzy entropy was proposed for measuring the degree of an intuitionistic fuzzy set. Xu [60] examined the intuitionistic fuzzy Multiple Attribute Decision Making (MADM) with a study concerning attribute weights. Many well-known MCDA methods have been extended by using the entropy concept. One example is the COPRAS method [61]. According to these advantages, the COPRAS method has been applied by numerous scholars from distinctive features of composition in most modern times [62]. Banaitiene et al. [63] considered the applicability of a methodology for the multivariate perspective and multiple criteria interpretation of the life cycle of a building based on COPRAS. Ghorabaee et al. [64] considered the COPRAS method in interval type-2 fuzzy sets for supplier selection. Mishra et al. [47] studied the COPRAS method for the sustainability evaluation of the bioenergy production process. Jurgis et al. [65] considered the COPRAS method to assess city compactness. Zavadskas and Kaklauskas [66] considered the COPRAS method for the selection of effective dwelling house walls.

This work proposes a novel entropy measure for intuitionistic fuzzy sets compared with other existing approaches. We prove that the proposed measure satisfies conditions of minimality, maximality, resolution and symmetry according to axioms of valid entropy. We also present an illustrative example of using the proposed entropy in decision-making analysis using the COPRAS method, where new entropy is used. The results are compared with existing approaches, and their analysis is conducted using WS and weighted Spearman
coefficient. This shows that novel entropy can be easily used instead of the old one. However, more extensive research to compare them is required.

The rest of the paper is organized as follows. In Section 2, basic definitions and notations of the IFSs are described. The novel proposed entropy measure is presented and verified in Section 3. Section 4 recalls the algorithm on the COPRAS method using the new entropy measurement. In Section 5, a simple example is presented, and the results are compared with various existing entropy measures defined on an intuitionistic set. Finally, the benefits and conclusions regarding the new entropy measure are discussed.

2. Preliminaries

An intuitionistic fuzzy set $M$ in the classical a non-empty and finite set $S$ is determined as the set of standardized triplets of the following Equation (1).

$$
M = \{(x, \mu_M(x), \theta_M(x)) | x \in S\}
$$

where $\mu_M(x) : S \to [0, 1]$ and $\theta_M(x) : S \to [0, 1]$ are the membership degree and non-membership degree, sequentially with the condition $0 \leq \mu_M(x) + \theta_M(x) \leq 1$.

The numbers $\mu_M(x) \in [0, 1]$ and $\theta_M(x) \in [0, 1]$ determine the degree of membership of the element $x \in S$ to the intuitionistic fuzzy set $M$ and the degree of non-membership of the element $x \in S$ to the intuitionistic fuzzy set $M$, sequentially. The compilation of all intuitionistic fuzzy sets in the classical set $S$ of the formal values is signified by IFS(S).

$\pi_M(x)$ is determined by the following Equation (2).

$$
\pi_M(x) = 1 - \mu_M(x) - \theta_M(x)
$$

is called the hesitancy degree (or the degree of indeterminacy of information) of the element $x \in S$ to the set $M$, and $\pi_M(x) \in [0, 1]$, $\forall x \in S$.

3. Entropy for Intuitionistic Fuzzy Set

The fuzzy set theory uses entropy to measure the degree of fuzziness in a fuzzy set, called fuzzy entropy. Fuzzy entropy is used to express the mathematical values of the fuzziness of fuzzy sets. Classical Shannon entropy concerns probabilistic uncertainties, whereas fuzzy entropy concerns randomness, vagueness, fuzziness and ambiguous uncertainties [67]. Axiomatic entropy of fuzzy sets is continued as the intuitionistic fuzzy sets.

**Definition 1.** A real function $E$: intuitionistic fuzzy set IFS($M$) $\to [0, 1]$ is characterized as entropy of intuitionistic fuzzy sets ($M$) if $E$ satisfies the latter axioms [68].

- **Minimality:** $E(A) = 0$, if $A$ is crisp set;
- **Maximality:** $E(A) = 1$, if $\mu_A(x) = \theta_A(x) = \frac{1}{2}$, $\forall x$;
- **Resolution:** $E(A) \leq E(B)$, if $A$ is less fuzzy than $B$,
  $i.e.$, $\mu_A(x) \leq \mu_B(x) \leq \frac{1}{2}$ and $\theta_B(x) \leq \theta_A(x) \leq \frac{1}{2}$ for $\mu_B(x) \leq \mu_A(x)$
- **Symmetry:** $E(A) = E(A^c)$, where $A^c$ is the complement of $A$.

In a sense, with intuitionistic fuzzy knowledge, we suggest the latter intuitionistic fuzzy entropy analogous to criteria such as:

Let $M = x_1, x_2, x_3, \ldots, x_n$ be the universal set. Let $S = \{(x_i, \mu_s(x_i), \theta_s(x_i)|x_i \in M\}$ be a intuitionistic fuzzy set on $M$.

$$
E_p(S) = \frac{1}{n} \sum_{i=1}^{n} E^p_i(S)
$$

where

$$
E^p_i(S) = \frac{\sec\left(\frac{1}{4}||3 - 2\mu_s(x_i) - 2\pi||\pi\right) + \sec\left(\frac{1}{4}||3 - 2\theta_s(x_i) - 2\pi||\pi\right) - \frac{334}{135}}{206}
$$

\[ \pi_s(x_i) = 1 - \mu_s(x_i) - \vartheta_s(x_i), \text{ for all } i = 1, 2, 3, \ldots, m. \]  

(5)

**Theorem 1.** The measure defined in Equation (3) is a valid entropy.

**Proof.** To prove that the proposed measure is valid, we must show that it satisfies the properties as provided in Definition 1.

1. **Minimality:** if \( S \) is a crisp set, i.e., \( \mu_s(x_i) = 1, \vartheta_s(x_i) = 0 \) or \( \mu_s(x_i) = 0, \vartheta_s(x_i) = 1 \) for all \( x_i \in M \), then,

\[
\frac{\sec \left( \frac{1}{2} \| \frac{1}{2} - \frac{3}{7} \| \pi \right) + \sec \left( \frac{1}{2} \| \frac{1}{2} - \frac{3}{7} \| \pi \right) - \frac{334}{135}}{206} = 0
\]

(6)

Therefore, \( E_p(S) = 0 \)

2. **Maximality:** for all \( x_i \in M \), if \( \mu_s(x_i) = \vartheta_s(x_i) = \frac{1}{2} \) for all \( x_i \in M \), then,

\[
\frac{\sec \left( \frac{1}{2} \| \frac{1}{2} - \frac{3}{7} \| \pi \right) + \sec \left( \frac{1}{2} \| \frac{1}{2} - \frac{3}{7} \| \pi \right) - \frac{334}{135}}{206} = 1
\]

(7)

3. **Resolution:** in order to prove the fourth property, consider the function \( f(\mu, \vartheta) \) such that

\[
f(\mu, \vartheta) = \frac{\sec \left( \frac{1}{2} \| \frac{1}{2} - \frac{3}{7} \| \pi \right) + \sec \left( \frac{1}{2} \| \frac{1}{2} - \frac{3}{7} \| \pi \right) - \frac{334}{135}}{206}
\]

(8)

where \( \mu, \vartheta \in [0, 1] \). The partial derivatives with respect to \( \mu \) and \( \vartheta \) are obtained as

\[
\begin{align*}
\frac{\partial f}{\partial \mu} & = - \frac{135\pi(\frac{3}{7} - 2\mu(x))(|\frac{3}{7} - 2\mu(x)| - \frac{2}{7}) \sec \left( \frac{1}{2} \| \frac{3}{7} - 2\mu(x) \| \pi \right) \sec \left( \frac{1}{2} \| \frac{3}{7} - 2\mu(x) \| \pi \right)}{721|\frac{3}{7} - 2\mu(x)|^2 - 2\mu(x) - \frac{2}{7}} \\
\frac{\partial f}{\partial \vartheta} & = - \frac{135\pi(\frac{3}{7} - 2\vartheta(x))(|\frac{3}{7} - 2\vartheta(x)| - \frac{2}{7}) \sec \left( \frac{1}{2} \| \frac{3}{7} - 2\vartheta(x) \| \pi \right) \sec \left( \frac{1}{2} \| \frac{3}{7} - 2\vartheta(x) \| \pi \right)}{721|\frac{3}{7} - 2\vartheta(x)|^2 - 2\vartheta(x) - \frac{2}{7}}
\end{align*}
\]

(9)

We obtain that \( \frac{\partial f(\mu, \vartheta)}{\partial \mu} \geq 0 \) when \( \mu \leq \vartheta \) and \( \frac{\partial f(\mu, \vartheta)}{\partial \mu} \leq 0 \) when \( \mu \geq \vartheta \), whereas \( \frac{\partial f(\mu, \vartheta)}{\partial \vartheta} \leq 0 \) when \( \mu \leq \vartheta \) and \( \frac{\partial f(\mu, \vartheta)}{\partial \vartheta} \geq 0 \) when \( \mu \geq \vartheta \). Thus, \( f \) is increasing with respect to \( \mu \) when \( \mu \leq \vartheta \) and decreasing when \( \mu \geq \vartheta \). Moreover, \( f \) is decreasing with respect to \( \vartheta \) when \( \mu \leq \vartheta \) and increasing when \( \mu \geq \vartheta \).

Now, by using this property of the function, we can conclude that \( E(S) \leq E(S') \), if \( A \) is less fuzzy than \( B \), i.e., \( \mu_A(x) \leq \mu_B(x) \leq \frac{1}{2} \) and \( \vartheta_B(x) \leq \vartheta_A(x) \leq \frac{1}{2} \) for \( \mu_B(x) \leq \mu_A(x) \) or \( \mu_A(x) \geq \mu_B(x) \geq \frac{1}{2} \) and \( \vartheta_B(x) \geq \vartheta_A(x) \geq \frac{1}{2} \) for \( \mu_B(x) \geq \mu_A(x) \).

4. **Symmetry:** for the property, we have \( S = (\mu_s, \vartheta_s) \) as \( S' = (\vartheta_s, \mu_s) \). Thus, we have

\[
E_p(S') = \frac{\sec \left( \frac{1}{2} \| \frac{1}{2} - \mu_s(x_i) - \frac{2}{7} \| \pi \right) + \sec \left( \frac{1}{2} \| \frac{1}{2} - \mu_s(x_i) - \frac{2}{7} \| \pi \right) - \frac{334}{135}}{206} = \frac{206}{135} E_p(S)
\]

(10)

\[ \square \]

**4. Intuitionistic Fuzzy Multi-Criteria Decision-Making Based on COPRAS Approach**

The “Complex Proportional Assessment”, commonly known as COPRAS, was first introduced in 1994 by Zavadskas and Kaklauskas [69]. This method is used to estimate the uniqueness of one alternative over another and presents it as reasonable to equate alternatives [70]. In addition, this method can be implemented to maximize and minimi-
mize criteria in an assessment where more than one criterion should be considered [66]. The COPRAS technique ranks and estimates alternatives step-by-step for their relevance and utility degree [71]. The development and extensions of this technique, e.g., in uncertainty problems [72,73], make it an advanced approach among the methods of multi-criteria decision-making. The method considers that the importance and advantage of the reviewed versions depend directly on and are comparable to a system of criteria appropriately specifying the alternatives and the values and weights of criteria [74]. The method determines a clarification and the ratio to the ideal solution and the ratio to the worst-ideal solution and consequently can be considered a compromising method. The method is applied for solving numerous difficulties by its exhibitors and their associates. Figure 1 can represent the stages of the COPRAS method.

Figure 1. Flowchart of COPRAS method.

Explanation of Figure 1:

Step 1: Establishment of intuitionistic fuzzy decision matrix.

The initial move is relevant to the establishment of the intuitionistic fuzzy decision matrix. We determine the decision matrix \( M = (H_i, J_j)_{r \times n} \), where \( H_i \) are the alternatives concerning the criteria \( J_j \), such that \( M_{i} = \{ M_{11}, M_{12}, \ldots, M_{1n} \} \), \( i = 1, 2, \ldots, m \) and \( J_j = \{ J_1, J_2, \ldots, J_n \} \), \( j = 1, 2, \ldots, n \).

\[
\begin{bmatrix}
 a & j_1 & j_2 & \cdots & j_n \\
 M_1 & (\mu_{11}, \vartheta_{11}) & (\mu_{12}, \vartheta_{12}) & \cdots & (\mu_{1n}, \vartheta_{1n}) \\
 M_2 & (\mu_{21}, \vartheta_{21}) & (\mu_{22}, \vartheta_{22}) & \cdots & (\mu_{2n}, \vartheta_{2n}) \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
 M_m & (\mu_{m1}, \vartheta_{m1}) & (\mu_{m2}, \vartheta_{m2}) & \cdots & (\mu_{mn}, \vartheta_{mn})
\end{bmatrix}
\]

Step 2: Judgment of the weights of criteria.

Criteria weights symbolize a significant part of the clarification of MCDM issues. The weights can be obtained in different ways [75]. In a decision-making system, the experience concerning criteria weights is seldom entirely unknown or imperfectly known and somewhat known at specific times.

- For unknown criteria weights:
  If weights of criteria are entirely unknown, then we determine the weights by utilizing the following equation:

\[
 w_j = \frac{1 - E_p(S)}{\sum_{t=1}^{m} (1 - E_p(S))}, \quad (t = 1, 2, \ldots, m) \tag{12}
\]

where \( E_p(S) = \frac{3}{4\sqrt{3} + 3} \sum_{l=1}^{m} E_p(S) \).

- For partially known criteria weights:
  Because of the increasing intricacy of decision-making issues, it may not frequently be reasonable for the decision-makers to describe their viewpoint as exact numbers. In case the criteria weights are imperfectly known for MCDM issues,
\[
\min A = \sum_{t=1}^{m} w_t E_p(S)
\]  

**Step 3:** Calculate the weighted decision matrix \( D = [d_{ij}]_{m \times n} \):

\[
d_{ij} = w_j n_{ij} = \left( \sqrt{1 - \left( 1 - \mu_{ij}^2 \right)^{w_u}}, \vartheta_{ij} \right)
\]

where \( (j = 1, 2, \ldots, m) \).

**Step 4:** Calculate the score function:

\[
g(d_{ij}) = \mu_{ij}^2 - \vartheta_{ij}^2
\]

where \( i = 1, 2, \ldots, m; j = 1, 2, \ldots, n \).

**Step 5:** Determine the maximizing and minimizing index:

\[
g(O_i) = \frac{1}{|B|} \sum_{j \in B} g(d_{ij})
\]

and

\[
g(R_i) = \frac{1}{|NB|} \sum_{j \in NB} g(d_{ij})
\]

where, \( B \) is the set of benefit criteria and \( NB \) is the set of non-benefit criteria, for all \( (i = 1, 2, \ldots, m) \).

**Step 6:** Determine the relative significance value of each alternative:

\[
T_i = g(O_i) + \frac{\sum_{i=1}^{m} g(R_i)}{\sum_{i=1}^{m} 1 / g(R_i)}
\]

where \( i = 1, 2, \ldots, m \).

**Step 7:** Determine the priority order:

\[
V_i = \frac{T_i}{\max T_i} \times 100
\]

where \( i = 1, 2, \ldots, m \).

**Step 8:** Ranking of the alternatives:

The ranking of the alternatives is regulated in declining order based on the values of priority order. Thus, the highest final value has the highest rank.

5. Numerical Example

Let us consider determining the best private hospital in a large city. First, we have to survey four hospitals named A, B, C and D, with different facilities. Next, we select the five essential criteria by which the hospitals are evaluated, which are management \( (J_1) \), waiting lines \( (J_2) \), services \( (J_3) \), charges \( (J_4) \) and developed area \( (J_5) \). Here, \( \{J_1, J_3, J_5\} \) are taken as benefit criteria, whereas \( \{J_2, J_4\} \) are taken as cost criteria. Finally, the intuitionistic fuzzy decision matrix is shown in Table 1. This study used the uncertain decisions library available at https://gitlab.com/kiziub/uncertain-decisions (accessed on 10 November 2021).

Using Equation (3), we calculate the entropies: \( e_1 = 0.7409, e_2 = 0.6374, e_3 = 0.7783, e_4 = 0.7979 \) and \( e_5 = 0.7525 \) (also shown in Table 1).
Table 1. Intuitionistic fuzzy decision matrix.

| Hospitals | J₁ | J₂ | J₃ | J₄ | J₅ |
|-----------|----|----|----|----|----|
| A         | 0.41, 0.38 | 0.48, 0.57 | 0.36, 0.43 | 0.33, 0.37 | 0.28, 0.34 |
| B         | 0.46, 0.37 | 0.48, 0.39 | 0.37, 0.41 | 0.35, 0.44 | 0.51, 0.39 |
| C         | 0.36, 0.39 | 0.21, 0.37 | 0.41, 0.38 | 0.28, 0.34 | 0.32, 0.46 |
| D         | 0.51, 0.39 | 0.37, 0.45 | 0.32, 0.46 | 0.37, 0.57 | 0.37, 0.45 |
| E \(p(S)\) | 0.7409 | 0.6374 | 0.7783 | 0.7979 | 0.7525 |

Corresponding to Equation (12), we compute the weights of each criterion across all the alternatives, utilizing the values of \(E_p(S)\):

\[
w_1 = 0.2004, \quad w_2 = 0.2804, \quad w_3 = 0.1714, \quad w_4 = 0.1563 \quad \text{and} \quad w_5 = 0.1914.\]

Using Equation (15), we calculate the weighted decision matrix, shown in Table 2.

Table 2. Weighted decision matrix.

| | J₁ | J₂ | J₃ | J₄ | J₅ |
|----|----|----|----|----|----|
| A  | 0.19028, 0.82374 | 0.26609, 0.85416 | 0.15334, 0.86528 | 0.13365, 0.85604 | 0.12452, 0.81342 |
| B  | 0.21569, 0.81935 | 0.26609, 0.76794 | 0.15787, 0.85825 | 0.14220, 0.87954 | 0.23671, 0.83506 |
| C  | 0.16562, 0.82804 | 0.11210, 0.75668 | 0.17623, 0.84714 | 0.11261, 0.84479 | 0.14306, 0.86187 |
| D  | 0.24202, 0.82804 | 0.20110, 0.79938 | 0.13546, 0.87535 | 0.15084, 0.91587 | 0.16669, 0.85825 |

After calculating the weighted decision matrix, we obtain the value of the score function \(g(d_{ij})\), maximizing \(g(O_i)\) and minimizing \(g(R_i)\) index, relative significance value \(T_i\) and priority order \(V_i\). In the final step, we rank the alternatives in declining order based on the values of priority order. The highest final value has the highest rank.

The order of the alternatives of the proposed entropy is given in Table 3. Here, we have taken the hesitant intuitionistic fuzzy set to describe the uncertainty in the actual world. This entropy consists of a membership function, non-membership function and hesitant function. Therefore, the obtained results are more precise.

Table 3. The score function and ranking of alternatives.

|     | J₁ | J₂ | J₃ | J₄ | J₅ | Oᵢ | Rᵢ | Tᵢ | Vᵢ | Rank |
|-----|----|----|----|----|----|-----|-----|-----|-----|------|
| A   | -0.64235 | -0.65879 | -0.72521 | -0.64614 | -0.67123 | -0.68686 | -0.14581 | 100 | IV   |
| B   | -0.62482 | -0.51892 | -0.71167 | -0.71494 | -0.72236 | -0.69459 | -0.70731 | 108.57 | III  |
| C   | -0.65823 | -0.56000 | -0.68659 | -0.70100 | -0.72521 | -0.70731 | -0.19243 | 131.97 | I    |
| D   | -0.62708 | -0.59856 | -0.74789 | -0.81606 | -0.72521 | -0.70731 | -0.14581 | 109.61 | II   |

6. Comparative Analysis

In this section, we compare the results of the entropy introduced in this paper with those entropies introduced in earlier works [76–79]. We have used the same example for the comparison. Those entropies that we have taken for this comparison are listed below.

1. Burillo and Bustince (1996) intuitionistic fuzzy set [76]:

\[
E(B) = \frac{1}{n} \sum_{i=1}^{n} [1 - (\mu_s(x_i) + \vartheta_s(x_i))] \tag{20}
\]

2. Szmidt and Kacprzyk (2001) intuitionistic fuzzy entropy [77]:

\[
E(K) = \frac{1}{n} \sum_{i=1}^{n} \min(\mu_s(x_i), \vartheta_s(x_i)) + \pi_s(x_i) \tag{21}
\]

3. Liu and Ren (2014) introduced intuitionistic fuzzy entropy [78]:

\[...\]
\[
E(L) = \frac{1}{n} \sum_{i=1}^{n} \cot \left( \frac{\pi}{4} + \frac{\left| \mu_s(x_i) - \mu_s(x_i) \right| \pi}{4} \right)
\]  

(22)

4. Ye (2010) intuitionistic fuzzy set [79]:

\[
E(Y) = \frac{1}{n} \sum_{i=1}^{n} \left[ \sin \left( \frac{\mu_s(x_i) + 1 - \mu_s(x_i)}{4} \pi + \sin \left( \frac{\mu_s(x_i) + 1 - \mu_s(x_i)}{4} \pi - 1 \right) \times \frac{1}{\sqrt{2} - 1} \right) \right]
\]  

(23)

We have analyzed the priority order and rank of the alternatives. The entropies (20)–(23) that we have taken for the resemblance contain only the membership function (\(\mu\)) and non-membership function (\(\theta\)), whereas the entropy that we have introduced involves a membership function (\(\mu\)), non-membership function (\(\theta\)) and the hesitant function (\(\pi\)). Therefore, due to the hesitancy measure, the alternative order that we have obtained is slightly different. The results obtained are presented using Table 4. However, the analysis shows that the most likely preferred alternative is C, unanimously selected by all the entropy measures. Therefore, hospital C is the best choice among the ranking of the top hospitals. Figure 2 shows a comparison of the obtained rankings using WS and weighted Spearman coefficients [80], which once again shows the high similarity of the obtained results.

| Proposed Entropy | Burillo and Bustince [76] | Szmidt and Kacprzyk [77] | Liu and Ren [78] | Ye [79] |
|------------------|---------------------------|--------------------------|------------------|---------|
| \(V_i\) | Rank | \(V_i\) | Rank | \(V_i\) | Rank | \(V_i\) | Rank | \(V_i\) | Rank |
| A | 100 | IV | 100 | IV | 100 | IV | 105.77 | III | 100 | IV |
| B | 109.61 | II | 103.74 | III | 103.152 | III | 100 | IV | 103.23 | III |
| C | 131.97 | I | 129.67 | I | 128.631 | I | 126.03 | I | 129.43 | I |
| D | 108.57 | III | 105.25 | II | 104.78 | II | 120.17 | II | 104.61 | II |

**Figure 2.** WS and \(r_w\) correlation heat maps for different entropy methods. (a) Comparison of rankings of different entropies using ranking similarity coefficient. (b) Comparison of rankings of different entropies using Spearman correlation coefficient.

The advantages are as follows: in a comparable study, it is apparent that the \(V_i\) for suggested entropy is more well-defined than existing intuitionistic fuzzy entropies.
Therefore, the proposed novel entropy measure is more reliable and feasible to solve complex multi-criteria decision-making problems.

7. Conclusions

In this paper, we have introduced a novel intuitionistic fuzzy entropy with hesitancy measure. The proposed entropy measure develops a new approach of the COPRAS method based on the MCDM problem. We have taken an example to compare our entropy to some current entropies of intuitionistic fuzzy sets. The results achieved from the example show that the submitted entropy is more reliable as it has a hesitancy measure. The results of the new entropy measure are examined and interpreted favorably with the selected entropy measure on an intuitionistic fuzzy set. The intuitionistic fuzzy set is very efficient and suitable for handling uncertainty in complex MCDA problems. The proposed measure can be used in future research on multi-criteria decision analysis as a component of new objective methods for determining attribute weights using intuitionistic fuzzy sets. In addition, taking part in the decision-making process can help rank and select problems in various domains as a standard method for uncertain decision analysis.

Author Contributions: Conceptualization, P.T., N.G., B.K., A.S., A.B.S. and W.S.; methodology, P.T., N.G., B.K., A.S., N.S., A.B.S. and W.S.; software, P.T., N.G., B.K., A.S., A.B.S. and W.S.; validation, P.T., N.G., B.K., A.S., A.B.S. and W.S.; formal analysis, P.T., N.G., B.K., A.S., N.S., A.B.S. and W.S.; investigation, P.T., N.G., B.K., A.S., A.B.S. and W.S.; resources, P.T., N.G., B.K., A.S., A.B.S. and W.S.; data curation, P.T., N.G., B.K., A.S., A.B.S. and W.S.; writing—original draft preparation, P.T., N.G., B.K., A.S., A.B.S. and W.S.; writing—review and editing, P.T., N.G., B.K., A.S., N.S., A.B.S. and W.S.; supervision, N.G., A.B.S. and W.S.; project administration, W.S.; funding acquisition, W.S. All authors have read and agreed to the published version of the manuscript.

Funding: The work was supported by the National Science Centre 2018/29/B/HS4/02725 (B.K., A.S. and W.S.).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: The authors would like to thank the editor and the anonymous reviewers, whose insightful comments and constructive suggestions helped us to significantly improve the quality of this paper.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

- AHP: Analytic Hierarchy Process
- ANP: Analytic Network Process
- BWM: Best Worst Method
- CODAS: COMbinative Distance-based ASsessment
- CSTN: Color Super-Twisted Nematic
- COMET: Characteristic Objects Method
- COPRAS: COMplex PRoportional ASsessment
- DEMATEL: Decision Making Trial and Evaluation Laboratory
- DSS: Decision Support System
- EDAS: Evaluation based on Distance from Average Solution
- ELECTRE: Élimination et Choix Traduisant la REalité
- FMEA: Failure Modes and Effects Analysis
- HFS: Hesitant Fuzzy Sets
- IFS: Intuitionistic Fuzzy Sets
References

1. Stević, Ž.; Karamaša, Č.; Demir, E.; Korucuk, S. Assessing sustainable production under circular economy context using a novel rough-fuzzy MCDM model: A case of the forestry industry in the Eastern Black Sea region. *J. Entrepr. Inf. Manag.* 2021. [CrossRef]

2. Kizielewicz, B.; Szyszewski, Z. Handling economic perspective in multicriteria model-renewable energy resources case study. *Procedia Comput. Sci.* 2020, 176, 3555–3562. [CrossRef]

3. Shekhovtsov, A.; Kozlov, V.; Nosov, V.; Salabun, W. Efficiency of Methods for Determining the Relevance of Criteria in Sustainable Transport Problems: A Comparative Case Study. *Sustainability* 2020, 12, 7915. [CrossRef]

4. Kizielewicz, B.; Więckowski, J.; Shekhovtsov, A.; Wątrobki, J.; Depczyński, R.; Salabun, W. Study towards the Time-Based MCDM Ranking Analysis—A Supplier Selection Case Study. *Facta Univ. Ser. Mech. Eng.* 2021, 19, 381–399. [CrossRef]

5. Pamucar, D.S.; Savin, L.M. Multiple-criteria model for optimal off-road vehicle selection for passenger transportation: BWM-COPRAS model. *Mil. Tech. Cour.* 2020, 68, 28–64.

6. La Scalina, G.; Aiello, G.; Rastellini, C.; Micale, R.; Cicalese, L. Multi-criteria decision making support system for pancreatic islet transplantation. *Expert Syst. Appl.* 2011, 38, 3091–3097. [CrossRef]

7. Stević, Ž.; Pauška, A.; Chatterjee, P. Sustainable supplier selection in healthcare industries using a new MCDM method: Measurement of alternatives and ranking according to Compromise solution (MARCOS). *Comput. Ind. Eng.* 2020, 140, 106231. [CrossRef]

8. Muhammad, L.; Badi, I.; Haruna, A.A.; Mohammed, I. Selecting the Best Municipal Solid Waste Management Techniques in Nigeria Using Multi Criteria Decision Making Techniques. *Rep. Mech. Eng.* 2021, 2, 180–189. [CrossRef]

9. Bączkiewicz, A.; Kizielewicz, B.; Shekhovtsov, A.; Yelmkheiev, M.; Kozlov, V.; Salabun, W. Comparative Analysis of Solar Panels with Determination of Local Significance Levels of Criteria Using the MCDM Methods Resistant to the Rank Reversal Phenomenon. *Energies* 2021, 14, 5727. [CrossRef]

10. Pamucar, D.; Iordache, M.; Deveci, M.; Schitea, D.; Iordache, I. A new hybrid fuzzy multi-criteria decision methodology model for prioritizing the alternatives of the hydrogen bus development: A case study from Romania. *Int. J. Hydrogen Energy* 2021, 46, 29616–29637. [CrossRef]

11. Bączkiewicz, A.; Kizielewicz, B.; Shekhovtsov, A.; Wątrobki, J.; Salabun, W. Methodical Aspects of MCDM Based E-Commerce Recommender System. *J. Theor. Appl. Electron. Commer. Res.* 2021, 16, 2192–2229. [CrossRef]

12. Greis, M.; Schuff, H.; Kleiner, M.; Henze, N.; Schmidt, A. Input controls for entering uncertain data: Probability distribution sliders. *Proc. ACM Hum. Comput. Interact.* 2017, 1, 1–17. [CrossRef]

13. Sengupta, A.; Pal, T.K. On comparing interval numbers. *Eur. J. Oper. Res.* 2000, 127, 28–43. [CrossRef]

14. Shekhovtsov, A.; Kizielewicz, B.; Salabun, W. New Rank-Reversal Free Approach to Handle Interval Data in MCDM Problems. In *Proceedings of the International Conference on Computational Science, Kraków, Poland, 16–18 June 2021*; pp. 458–472.

15. Zadeh, L. Fuzzy sets. *Inf. Control* 1965, 8, 338–353. [CrossRef]

16. Veskić, S.; Stević, Ž.; Karabašević, D.; Rajilić, S.; Stojić, G. A new integrated fuzzy approach to selecting the best solution for business balance of passenger rail operator: Fuzzy PIPRECIA-fuzzy EDAS model. *Symmetry* 2020, 12, 743. [CrossRef]

17. Kizielewicz, B.; Bączkiewicz, A. Comparison of Fuzzy TOPSIS, Fuzzy VIKOR, Fuzzy WASPAS and Fuzzy MMDORRA methods in the housing selection problem. *Procedia Comput. Sci.* 2021, 192, 4578–4591. [CrossRef]

18. Yager, R.R.; Abbasov, A.M. Pythagorean membership grades, complex numbers, and decision making. *Int. J. Intell. Syst.* 2013, 28, 436–452. [CrossRef]

19. Peng, X.; Yang, Y. Some results for Pythagorean fuzzy sets. *Int. J. Intell. Syst.* 2015, 30, 1133–1160. [CrossRef]

20. Mete, S.; Serin, F.; Oz, N.E.; Gul, M. A decision-support system based on Pythagorean fuzzy VIKOR for occupational risk assessment of a natural gas pipeline construction. *J. Nat. Gas Sci. Eng.* 2019, 71, 102979. [CrossRef]
21. Yucesan, M.; Gul, M. Hospital service quality evaluation: An integrated model based on Pythagorean fuzzy AHP and fuzzy TOPSIS. Soft Comput. 2020, 24, 3237–3255. [CrossRef]
22. Akram, M.; Luqman, A.; Alcantud, J.C.R. Risk evaluation in failure modes and effects analysis: Hybrid TOPSIS and ELECTRE I solutions with Pythagorean fuzzy information. Neural Comput. Appl. 2021, 33, 5675–5703. [CrossRef]
23. Torra, V. Hesitant fuzzy sets. Int. J. Intell. Syst. 2010, 25, 529–539. [CrossRef]
24. Salabun, W.; Karczmarczyk, A.; Wątrobki, J. Decision-making using the hesitant fuzzy sets COMET method: An empirical study of the electric city buses selection. In Proceedings of the 2018 IEEE Symposium Series on Computational Intelligence (SSCI), Bangalore, India, 18–21 November 2018; pp. 1485–1492.
25. Senvar, O.; Olay, I.; Bolturk, E. Hospital site selection via hesitant fuzzy TOPSIS. IFAC-PapersOnline 2016, 49, 1140–1145. [CrossRef]
26. Faiz, S.; Salabun, W.; Nawaz, S.; ur Rehman, A.; Wątrobki, J. Best-Worst method and Hamacher aggregation operations for intuitionistic 2-tuple linguistic sets. Expert Syst. Appl. 2021, 181, 115088. [CrossRef]
27. Afful-Dadzie, E.; Oplaktova, Z.K.; Prieto, L.A.B. Comparative state-of-the-art survey of classical fuzzy set and intuitionistic fuzzy sets in multi-criteria decision making. Int. J. Fuzzy Syst. 2017, 19, 726–738. [CrossRef]
28. Thakur, P.; Gondatra, N. Pythagorean fuzzy multi-criteria decision making and its application in fitting assembly. Mater. Today Proc. 2021, in press. [CrossRef]
29. Kumar, R.; Gondatra, N. A novel pythagorean fuzzy entropy measure using MCDM application in preference of the advertising company with TOPSIS approach. Mater. Today Proc. 2021, in press. [CrossRef]
30. Szmidt, E.; Kacprzyk, J. Distances between intuitionistic fuzzy sets. Fuzzy Sets Syst. 2000, 114, 505–518. [CrossRef]
31. Turanli, N.; Coker, D. Fuzzy connectedness in intuitionistic fuzzy topological spaces. Fuzzy Sets Syst. 2000, 116, 369–375. [CrossRef]
32. Bustince, H. Construction of intuitionistic fuzzy relations with predetermined properties. Fuzzy Sets Syst. 2000, 109, 379–403. [CrossRef]
33. Ciftcibasi, T.; Altunay, D. Two-sided (intuitionistic) fuzzy reasoning. IEEE Trans. Syst. Man Cybern. Part Syst. Hum. 1998, 28, 662–677. [CrossRef]
34. Kumar, K.; Garg, H. Connection number of set pair analysis based TOPSIS method on intuitionistic fuzzy sets and their application to decision making. Appl. Intell. 2018, 48, 2112–2119. [CrossRef]
35. Gerogiannis, V.C.; Fitsilis, P.; Kameas, A.D. Evaluation of project and portfolio Management Information Systems with the use of a hybrid IFS-TOPSIS method. Intell. Decis. Technol. 2013, 7, 91–105. [CrossRef]
36. Feng, F.; Xu, Z.; Fujita, H.; Liang, M. Enhancing PROMETHEE method with intuitionistic fuzzy soft sets. Int. J. Intell. Syst. 2020, 35, 1071–1104. [CrossRef]
37. Krishankumar, R.; Ravichandran, K.; Seaid, A.B. A new extension to PROMETHEE under intuitionistic fuzzy environment for solving supplier selection problem with linguistic preferences. Appl. Soft Comput. 2017, 60, 564–576.
38. Krishankumar, R.; Premaladha, J.; Ravichandran, K.; Sekar, K.; Manikanadan, R.; Gao, X. A novel extension to VIKOR method under intuitionistic fuzzy context for solving personnel selection problem. Soft Comput. 2020, 24, 1063–1081. [CrossRef]
39. Çali, S.; Balaman, Ş.Y. A novel outranking based multi criteria group decision making methodology integrating ELECTRE and VIKOR under intuitionistic fuzzy environment. Expert Syst. Appl. 2019, 119, 36–50. [CrossRef]
40. Sadiq, R.; Tesfamariam, S. Environmental decision-making under uncertainty using intuitionistic fuzzy analytic hierarchy process (IF-AHP). Stoch. Environ. Res. Risk Assess. 2009, 23, 75–91. [CrossRef]
41. Zhang, C.; Li, W.; Wang, L. AHP under the intuitionistic fuzzy environment. In Proceedings of the 2011 Eighth International Conference on Fuzzy Systems and Knowledge Discovery (FSKD), Shanghai, China, 26–28 July 2011; Volume 1, pp. 583–587.
42. Rouyendegh, B.D. The intuitionistic fuzzy ELECTRE model. Int. J. Manag. Sci. Eng. Manag. 2018, 13, 139–145. [CrossRef]
43. Wu, M.C.; Chen, T.Y. The ELECTRE multicriteria analysis approach based on Atanassov’s intuitionistic fuzzy sets. Expert Syst. Appl. 2011, 38, 12318–12327. [CrossRef]
44. Xue, Y.X.; You, J.X.; Lai, X.D.; Liu, H.C. An interval-valued intuitionistic fuzzy MABAC approach for material selection with incomplete weight information. Appl. Soft Comput. 2016, 38, 703–713. [CrossRef]
45. Karagoz, S.; Deveci, M.; Simic, V.; Aydin, N.; Bolukbas, U. A novel intuitionistic fuzzy MCDM-based CODAS approach for locating an authorized dismantling center: A case study of Istanbul. Waste Manag. Res. 2020, 38, 660–672. [CrossRef] [PubMed]
46. Angeline, L.A.; Mystica, A.R.; Mary, S.F.J.; Merlin, M.M.M. A new integrated approach of combined FCM and CODAS method in interval valued intuitionistic fuzzy cognitive map for multi criteria decision making to evaluate and prioritize the branded mobile phones. Malay. J. Mat. (MJM) 2020, 8, 230–234.
47. Mishra, A.R.; Rani, P.; Pandey, K.; Mardani, A.; Steimikis, J.; Steimikiene, D.; Alrasheedi, M. Novel multi-criteria intuitionistic fuzzy SWARA–COPRAS approach for sustainability evaluation of the bioenergy production process. Sustainability 2020, 12, 4155. [CrossRef]
48. Thao, N.X. MOORA models based on new score function of interval-valued intuitionistic sets and apply to select materials for mushroom cultivation. Neural Comput. Appl. 2021, 17, 1–11. [CrossRef]
49. Perez-Dominguez, L.; Alvarado-Iniesta, A.; Rodriguez-Borbón, I.; Vergara-Villegas, O. Intuitionistic fuzzy MOORA for supplier selection. Dyna 2015, 82, 34–41. [CrossRef]
50. Büyüköztkan, G.; Güleyüz, S.; Karpak, B. A new combined IF-DEMATEL and IF-ANP approach for CRM partner evaluation. Int. J. Prod. Econ. 2017, 191, 194–206. [CrossRef]
51. Shariati, S.; Abedi, M.; Saedi, A.; Yazdani-Chamzini, A.; Tamošaitienė, J.; Šaparauskas, J.; Stupak, S. Critical factors of the application of nanotechnology in construction industry by using ANP technique under fuzzy intuitionistic environment. J. Civ. Eng. Manag. 2017, 23, 914–925. [CrossRef]

52. Nikjoo, A.V.; Saeedpoor, M. An intuitionistic fuzzy DEMATEL methodology for prioritising the components of SWOT matrix in the Iranian insurance industry. Int. J. Oper. Res. 2014, 20, 439–452. [CrossRef]

53. Govindan, K.; Khodaverdi, R.; Vafadarnikjoo, A. Intuitionistic fuzzy based DEMATEL method for developing green practices and improvements in a green supply chain. Expert Syst. Appl. 2015, 42, 7207–7220. [CrossRef]

54. Amoozad Mahdiraji, H.; Zavadskas, E.; Arab, A.; Turskis, Z.; Sahebi, I. Formulation of Manufacturing Strategies Based on An Extended SWARA Method with Intuitionistic Fuzzy Numbers: An Automotive Industry Application. 2021. Available online: https://dora.dmu.ac.uk/bitstream/handle/2086/21082/SWARA%20IFS%2025-5-20%20Within.pdf?sequence=1&isAllowed=y (accessed on 4 April 2021).

55. Hung, C.C.; Chen, L.H. A fuzzy TOPSIS decision making model with entropy weight under intuitionistic fuzzy environment. In Proceedings of the International Multiconference of Engineers and Computer Scientists, Hong Kong, China, 18–20 March 2009; Volume 1, pp. 13–16.

56. Liu, X.; Qian, F.; Lin, L.; Zhang, K.; Zhu, L. Intuitionistic fuzzy entropy for group decision making of water engineering project delivery system selection. Entropy 2019, 21, 1101. [CrossRef]

57. Chen, T.Y.; Li, C.H. Determining objective weights with intuitionistic fuzzy entropy measures: A comparative analysis. Inf. Sci. 2010, 180, 4207–4222. [CrossRef]

58. Salabun, W.; Wątróbski, J.; Shekhtovtsov, A. Are mcda methods benchmarkable? a comparative study of topsis, vikor, copras, and promethee ii methods. Symmetry 2020, 12, 1549. [CrossRef]

59. Gandotra, N.; Bajaj, R.K.; Gupta, N. Sorting of decision making units in data envelopment analysis with intuitionistic fuzzy weighted entropy. In Advances in Computer Science, Engineering & Applications; Springer: Berlin/Heidelberg, Germany, 2012; pp. 567–576.

60. Xu, Z.S. Approaches to multiple attribute decision making with intuitionistic fuzzy preference information. Syst. Eng. Theory Pract. 2007, 27, 62–71. [CrossRef]

61. Kumari, R.; Mishra, A.R. Multi-criteria COPRAS method based on parametric measures for intuitionistic fuzzy sets: Application of green supplier selection. Iran. J. Sci. Technol. Trans. Electr. Eng. 2020, 44, 1645–1662. [CrossRef]

62. Kouckasaraei, R.H.; Zolfani, S.H.; Golabchi, M. Glasshouse locating based on SWARA-COPRAS approach. Int. J. Strateg. Prop. Manag. 2015, 19, 111–122. [CrossRef]

63. Banaitiene, N.; Banaitis, A.; Kaklakauskas, A.; Zavadskas, E.K. Evaluating the life cycle of a building: A multivariant and multiple criteria approach. Omega 2008, 36, 429–441. [CrossRef]

64. Ghorabae, M.K.; Amiri, M.; Sadaghiani, J.S.; Goodarzi, G.H. Multiple criteria group decision-making for supplier selection based on COPRAS method with interval type-2 fuzzy sets. Int. J. Adv. Manuf. Technol. 2014, 75, 1115–1130. [CrossRef]

65. Zagorskas, J.; Burinskienė, M.; Zavadskas, E.; Turskis, Z. Urbanistic assessment of city compactness on the basis of GIS applying the COPRAS method. Ekologija 2007, 53, 55–63.

66. Zavadskas, E.K.; Kaklakauskas, A.; Turskis, Z.; Tamošaitienė, J. Selection of the effective dwelling house walls by applying attributes values determined at intervals. J. Civ. Eng. Manag. 2008, 14, 85–93. [CrossRef]

67. De Luca, A.; Termini, S. A definition of a nonprobabilistic entropy in the setting of fuzzy sets theory. Inf. Control 1972, 20, 301–312. [CrossRef]

68. Hung, W.L.; Yang, M.S. Fuzzy entropy on intuitionistic fuzzy sets. Int. J. Intell. Syst. 2006, 21, 443–451. [CrossRef]

69. Zavadskas, E.K.; Kaklakauskas, A.; Sarka, V. The new method of multicriteria complex proportional assessment of projects. Technol. Econ. Dev. Econ. 1994, 1, 131–139.

70. Zavadskas, E.K.; Kaklakauskas, A.; Peldschus, F.; Turskis, Z. Multi-criteria attribute assessment of road design solutions by using the COPRAS method. Balt. J. Road Bridge Eng. 2007, 2, 195–203.

71. Kaklakauskas, A.; Zavadskas, E.K.; Naimavicienė, J.; Krutinis, M.; Plakys, V.; Venskus, D. Model for a complex analysis of intelligent built environment. Autom. Constr. 2010, 19, 326–340. [CrossRef]

72. Dhiman, H.S.; Deb, D. Fuzzy TOPSIS and fuzzy COPRAS based multi-criteria decision making for hybrid wind farms. Energy 2020, 202, 117755. [CrossRef]

73. Ansari, Z.N.; Kant, R.; Shankar, R. Evaluation and ranking of solutions to mitigate sustainable remanufacturing supply chain risks: A hybrid fuzzy SWARA-fuzzy COPRAS framework approach. Int. J. Sustain. Eng. 2020, 13, 473–494. [CrossRef]

74. Kizielewicz, B.; Więckowski, J.; Shekhovtsov, A.; Ziemba, E.; Wątróbski, J.; Salabun, W. Input data preprocessing for the MCDM COPRAS method case study. In Proceedings of the AMCIS 2021 Proceedings, Online, 9–13 August 2021; p. 11.

75. Kizielewicz, B.; Wątróbski, J.; Salabun, W. Identification of relevant criteria set in the MCDM process—Wind farm location case study. Energies 2020, 13, 6548. [CrossRef]

76. Burillo, P.; Bustince, H. Entropy on intuitionistic fuzzy sets and on interval-valued fuzzy sets. Fuzzy Sets Syst. 1996, 78, 305–316. [CrossRef]

77. Szmidt, E.; Kacprzyk, J. Entropy for intuitionistic fuzzy sets. Fuzzy Sets Syst. 2001, 118, 467–477. [CrossRef]

78. Liu, M.; Ren, H. A new intuitionistic fuzzy entropy and application in multi-attribute decision making. Information 2014, 5, 587–601. [CrossRef]
79. Ye, J. Two effective measures of intuitionistic fuzzy entropy. *Computing* **2010**, *87*, 55–62. [CrossRef]
80. Salabun, W.; Urbaniak, K. A new coefficient of rankings similarity in decision-making problems. In Proceedings of the International Conference on Computational Science, Amsterdam, The Netherlands, 3–5 June 2020; pp. 632–645.