Determination of the large scale volume weighted halo velocity bias in simulations

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A profound assumption in peculiar velocity cosmology is $b_v = 1$ at sufficiently large scale, where $b_v$ is the halo(galaxy) velocity bias. However, this assumption is largely unverified in N-body simulations, due to the severe sampling artifact in measuring the volume weighted velocity power spectrum for sparse populations. With recently improved understanding of the sampling artifact 1,2, we are now able to measure the volume weighted halo velocity bias, after appropriate correction of the sampling artifact. (1) We verify $b_v = 1$ at $k \lesssim 0.1h/$Mpc and $z = 0-2$ for halos of mass $\sim 10^{12}$-$10^{13}h^{-1}M_\odot$, and therefore consolidates a foundation of peculiar velocity cosmology. (2) We also find statistically significant signs of $b_v \neq 1$ at $k > 0.1h/$Mpc. Whether this is real or caused by residual sampling artifact requires further investigation. Nevertheless, cosmology based on $k > 0.1h/$Mpc velocity data shall keep caution on this potential velocity bias.

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Introduction.—Large scale peculiar velocity is maturing as a powerful probe of cosmology. Peculiar velocity directly responds to gravitational pull of all clustered matter and energy, making it a precious tool to study dark matter (DM), dark energy and the nature of gravity (e.g. 3,4). Measuring peculiar velocity at cosmological distances with conventional method of distance indicators is challenging, albeit improving (e.g. 5,6). Alternatively, redshift space distortion (RSD) provides a way of measuring peculiar velocity at cosmological distances, free of the otherwise overwhelming contamination of Hubble flow. It enables $\sim 1\%$ accuracy in velocity power spectrum measurement at $z \sim 1$ 10, through stage IV dark energy surveys such as DESI and Euclid.

A profound assumption in cosmology based on peculiar velocity is that the velocity bias $b_v$ of galaxies vanishes at large scales ($b_v = 1$). The strong equivalence principle guarantees that galaxies sense the same acceleration as ambient DM particles. Hence one would naturally expect statistically identical velocity for galaxies and DM particles, at $\gtrsim 10$Mpc/$h$ scales where the only force operating is gravity. However, a loophole in this argument is that galaxies only reside in special regions (local density peaks). The same environmental difference is known to cause $b_v < 1$ in proto-halos 11,14. However, due to stochastic relation between proto-halos and real halos 13, it is non-trivial to extrapolate this prediction to real halos where galaxies reside and share the same large scale velocity 25. Since $v \propto fD b_v$ at large scale, uncertainties in $b_v$ lead to systematic error in all existing $fD$ measurements 16.
at \( k = 0.1h/\text{Mpc} \) for populations with number density \( \sim 10^{-3}(\text{Mpc}/h)^{-3} \), typical of \( \sim 10^{12}-10^{13}M_\odot \) halos \([2]\).

**Simulation specifications.—** We analyze the same J1200 N-body simulation \([2]\), run with a particle-particle-particle-mesh (P³M) code \([2]\). It adopts a \( \Lambda \)CDM cosmology with \( \Omega_m = 0.268, \Omega_\Lambda = 0.732, \Omega_b = 0.045, \sigma_8 = 0.85, n_s = 1 \) and \( h = 0.71 \). It has box size 1200Mpc/\( h \), 1024³ particles and mass resolution 1.2 \( \times 10^{11}M_\odot/h \). The halo catalogue is constructed by Friends-of-Friends (FOF) method with a linking length \( b = 0.2 \). Gravitationally unbound “halos” have been excluded from the catalogue. In total we have \( N_h = 6.18 \times 10^6 \) halos with at least 10 simulation particles, at \( z = 0 \). We choose the halo center as the mass weighted center and the halo velocity as the velocity averaged over all member particles. We try three mass bins detailed in table \( \textbf{I} \).

| Set ID | mass range | \( (M) \) | \( N_h/10^4 \) | \( n_h \) | \( b_n(\text{density}) \) |
|--------|------------|----------|----------------|--------|------------------|
| \( A1 \) \( (z = 0.0) \) | 10-3700 | 39 | 7.5 | 4.4 | 1.3 |
| \( z = 0.5 \) | 10-2000 | 30 | 5.9 | 3.5 | 1.8 |
| \( z = 1.0 \) | 10-950 | 24 | 3.7 | 2.1 | 2.6 |
| \( z = 2.0 \) | 10-400 | 19 | 1.3 | 0.76 | 4.3 |
| \( A2 \) \( (z = 0.0) \) | 1.2-10 | 2.8 | 54 | 32 | 0.8 |
| \( z = 0.5 \) | 1.2-10 | 2.8 | 52 | 31 | 1.1 |
| \( z = 1.0 \) | 1.2-10 | 2.7 | 46 | 27 | 1.5 |
| \( z = 2.0 \) | 1.2-10 | 2.5 | 31 | 18 | 2.4 |
| \( B(z = 0.0) \) | 2.3-3700 | 13 | 31 | 18 | 1.0 |

**TABLE I:** Three sets of halo mass bins. The mass unit is \( 10^{12}M_\odot/h \) and the number density \( n_h \) has unit of \( 10^{-4}(\text{Mpc}/h)^{-3} \). The density bias \( b_n(\text{density}) \) is averaged around \( k = 0.01h/\text{Mpc} \). The mass bin \( B \) at \( z = 0 \) has density bias of unity, designed for better control of the sampling artifact.

**Correcting the sampling artifact.—** We aim to measure the halo velocity bias defined in Fourier space,

\[
b_v(k) \equiv \sqrt{\frac{P_{h,E}(k)}{P_{DM,E}(k)}}.
\] (2)

The subscript “E” denotes the gradient (irrotational) part of velocity, which is most relevant for peculiar velocity cosmology. The subscript “h” and “DM” refer to halos and DM simulation particles respectively. Throughout this paper, we restrict to the volume weighted power spectrum. We adopt the NP (Nearest Particle) method \([19]\) to sample the velocity on 256³ uniform grids. Before correcting the sampling artifact, the measured velocity power spectrum \( P_v(k) \) differs from its true value by a factor \( C(k) \equiv \frac{P_v(k)}{P_v^{\text{true}}(k)} \). We found that \([2]\)

\[
C(k) \approx \langle e^{ik \cdot D} \rangle^2 e^{i k \cdot \xi_D/(r = \alpha/k)^3} \equiv C_T(k).
\] (3)

\( D \) is the deflection field pointing from a particle used for velocity assignment to the corresponding grid point that the velocity is assigned. \([1]\) showed that \( D \) fully captures the sampling artifact. \( \xi_D \) is the spatial correlation in \( D \). For \( \alpha = 1/2 \) and \( \bar{n}_P \sim 10^{-3}(\text{Mpc}/h)^{-3} \), \( C_T(k) \) agrees with the actual \( C(k) \) to \( \sim 1\% \) at \( k \leq 0.1h/\text{Mpc} \) \([2]\).

We take two steps to correct for the sampling artifact. **Step one.** We use Eq. \([3]\) to correct for the bulk of the sampling artifact. **Step two.** There are residual sampling artifact since our theory is not perfect \((C_T \neq C)\). We further correct this residual sampling artifact with the aid of DM control samples (DMCs). They are constructed by randomly selecting simulation DM particles from the full simulation sample, with the requirement \( \sigma_D(\text{DMC}) = \sigma_D(\text{halo}) \). \([2]\). \( \sigma_D \equiv \langle D^2 \rangle^{1/2} \) is the dominant factor determining the sampling artifact \([1,2]\). The halo sample and DMCs have identical sampling artifact at the \( k \to 0 \) limit and similar sampling artifact elsewhere. Hence to greater accuracy than Eq. \([3]\) we expect \( C_{h,T}/C_h \approx C_{DMC,T}/C_{DMC} \). We then obtain

\[
b_v(k) \approx \sqrt{\frac{P_{h,E}(k)}{P_{DM,E}(k)}} \sqrt{\frac{1}{C_{h,T}(k)}} \sqrt{\frac{C_{DMC,T}(k)}{C_{DMC}(k)}}.
\] (4)

The terms on the r.h.s. are the raw velocity bias measurement without correcting the sampling artifact, the step one correction, and the two step two correction, respectively. \( P_{DM,E} \) is measured from the full J1200 simulation sample, which is essentially free of sampling artifact due to its high \( \bar{n}_P \). All the correction terms \((C_{h,T}, C_{DMC,T} \text{ and } C_{DMC})\) are directly calculated from the J1200 simulation.

The inaccuracy of Eq. \([4]\) increases with \( k \). For peculiar velocity cosmology to be competitive, at least we shall utilize measurement at \( k \leq 0.1h/\text{Mpc} \). So we choose \( k = 0.1h/\text{Mpc} \) as the pivot scale for quoting the accuracy. Overall we expect \( \sim 1\% \) accuracy \([28]\), extrapolating from the DM cases. We caution the readers on this \( \sim 1\% \) uncertainty in the measured \( b_v(k) \) (Fig. \([1]\)).

**No velocity bias at \( k \lesssim 0.1h/\text{Mpc}.—** Fig. \([1]\) shows \( b_v \) for all mass bins listed in Table \( \textbf{I} \). The raw measurements (dashed curves) suggest “anti-bias” in all mass bins at all redshifts. This is most significant for \( A1 \). However, we have solid evidences that it is essentially an elusion caused by the sampling artifact. It causes systematic suppression of \( P_v \) \([1,2,20]\), mimicking an anti-bias. In another word, the apparent “anti-bias” is unreal, irrelevant for cosmology.

Theoretically, we expect the sampling artifact to exist for any populations of sparsely and inhomogeneously distributed objects \([1]\). It has been robustly detected for the case of DM simulation particles \([2]\). Therefore it must also exist for DM halos \([1]\). Fig. \([2]\) further consolidates this theoretical prediction. It shows that the DM control samples containing a fraction of DM simulation particles have smaller \( P_v \) than the full DM sample at \( k \lesssim 0.1h/\text{Mpc} \). Furthermore, \( P_v \) decreases with decreasing number density. If the number density of
the best that we can achieve. Again we find

tical large scale LSS fluctuation as DMCs, so we can

b0 \sim 1\% at \kappa > 0\text{h/Mpc}. Therefore we conclude that \b0 = 1 at \kappa < 0.1\text{h/Mpc} and \b0 > 1 at \kappa > 0.1\text{h/Mpc}.

The vanishing velocity bias (\bv = 1) at \kappa < 0.1\text{h/Mpc} verifies a fundamental assumption in peculiar velocity cosmology. However, from the theoretical viewpoint, this result is quite surprising, as linear theory predicts \bv(k = 0.1\text{h/Mpc}) \sim 0.9 for \sim 10^{13}M_\odot/h proto-halos (peaks in initial/linearly evolved density field) [13].

The predicted \bv < 1 arises from correlation between density gradient and velocity at initial density peaks. A number of processes may weaken/destroy this correlation and hence make the velocity bias disappear. First is the stochasticity in proto-halo-halo relation. A fraction of halos today do not correspond to initial density peaks and halos move from their initial positions. They tend to move towards each other and hence modify their velocity correlation. Third, the density and velocity evolution has non-negligible nonlinearity (e.g. [15]), and hence non-Gaussianity.

Velocity bias \bv \neq 1 at \kappa \gtrsim 0.1\text{h/Mpc}.—On the other hand, at \kappa \gtrsim 0.1\text{h/Mpc} and \sim 0.2). This excess is statistically significant. (2) In contrast, bin A1 (> 10^{13}M_\odot/h) has \bv(k = 0.1\text{h/Mpc}) < 1 at 1% to 10% level, also statistically significant. (3) Even more significant is that smaller halos seem to move faster at \kappa > 0.1\text{h/Mpc} and the difference reaches 10% at \kappa \sim 0.15\text{h/Mpc}. If this difference is indeed intrinsic, instead of residual sampling artifact, it could be caused by different environments that differ-
tent halos reside. Small halos tend to live in filaments and have extra infall velocity with respect to large halos. The infall velocity has a correlation length of typical filament length. Hence it can show up at \kappa \gtrsim 0.1\text{h/Mpc}. Unfortunately, our understanding of the sampling artifact at \kappa > 0.1\text{h/Mpc} is considerably poorer [2]. Therefore we are not able to draw decisive conclusions, other than that cosmology based on peculiar velocity at \kappa > 0.1\text{h/Mpc} must keep caution on this potential velocity bias.

Signs of \bv \neq 1 at \kappa < 0.1\text{h/Mpc}.—This paper presents determination of volume weighted halo velocity bias through N-body simulations. The raw measurements suffer from a severe sampling artifact which could be misinterpreted as a significant “velocity bias”. We are able to appropriately correct the sampling artifact following our previous works [1, 2] and obtain two major findings:

n_{v} = 1 at \kappa < 0.1\text{h/Mpc}. It consolidates the peculiar velocity cosmology;

• Signs of \bv \neq 1 at \kappa > 0.1\text{h/Mpc} and signs that
The sampling artifact in the velocity power spectrum measured in N-body simulations, which causes systematic underestimation at \( k \lesssim 0.1h/{\text{Mpc}} \). (1) The halo velocity power spectra (dash lines) are lower than measured from all DM simulations particles (dash-dot line). (2) The velocity power spectra of DM control samples containing a fraction of all simulation particles are also lower. Member particles in the control samples are randomly selected from the full simulation particles and hence must have statistically identical velocity power spectra. Therefore the observed deficit in the DM velocity power spectrum is caused by the sampling artifact. (3) When the number density of DM control samples and halo samples are identical, they have similar (but not identical) velocity power spectra and similar deficit with respect to the full DM sample. These are solid evidences of significant sampling artifact in the measured halo velocity power spectrum. The most crucial step in measuring the halo velocity bias is to understand and correct this sampling artifact. This is the sole purpose of our two preceding works [1, 2], which show that the sampling artifact depends on not only the number density, but the intrinsic LSS fluctuation and its correlation with the velocity field.

\[ b_v - 1 \text{ depends on redshifts, scales and halo mass.} \]

Although we are not able to robustly ruled out the possibility of residual sampling artifact, it raises the alarm of using \( k > 0.1h/{\text{Mpc}} \) velocity data to constrain cosmology.

Accurate measurement of the velocity bias in simulations heavily relies on robust correction of the sampling artifact. The sampling artifact depends on not only the halo number density, but also the intrinsic LSS fluctuation in the halo distribution and its correlation with the halo velocity field [2]. It is for this reason that our understanding of the sampling artifact at \( k \sim 0.1h/{\text{Mpc}} \) so far is no better than 1%. Therefore we caution the readers that the measured velocity bias has \( \sim 1\% \) model uncertainty (systematic error). For similar reason, we can not fully quantify the accuracy of Eq. 4 and the accuracy in the sampling artifact corrected \( b_v \) [29]. We know it is more accurate than Eq. 3 and have estimated its accuracy by extrapolating the DM cases [2]. Nevertheless, this ambiguity forbids us to draw unambiguous conclusion on whether the found \( b_v \neq 1 \) at \( k > 0.1h/{\text{Mpc}} \) is real. Therefor a major future work desired is to improve understanding of the sampling artifact (e.g. discussions in the appendix of [2]). We will also extend to galaxy velocity bias with mock galaxy catalogues, where the sampling artifact should also be corrected. Finally we address that RSD determines velocity indirectly through the galaxy number density distribution and therefore can be free of the sampling artifact. This is another advantage of RSD over conventional velocity measurement methods. However when comparing the RSD determined velocity with simulation, we must worry about the sampling artifact in simulation part.

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Galaxies in a halo have extra velocity relative to the host halo. The correlation length of this velocity is \( \lesssim 1 \text{Mpc}/h \), so it does not contribute to velocity at \( k \sim 0.1h/\text{Mpc} \) of interest. Therefore the large scale halo velocity and galaxy velocity are identical, statistically speaking.

For example, the velocity power spectrum determined from RSD is volume weighted in the framework of [24].

For this requirement, DMCs are more sparse than the halo samples.

For all the investigated halo bins, except A1(\( z = 2 \)), \( \bar{n}_h > 2 \times 10^{-4}(\text{Mpc}/h)^{-3} \). Based on [2], we expect \( \sim 1\% \) accuracy in the step one correction (Eq. 3). For A1(\( z = 2 \)), the number density is lower (0.76 \times 10^{-5}). However Eq. 3 is more accurate at \( z = 2 \) than \( z = 0 \) [2]. Hence we also expect Eq. 4 to be accurate at \( \sim 1\% \) level. The step two correction can further improve the accuracy.

Another reason is that we do not have corresponding halo samples of identical mass and intrinsic LSS clustering, but with \( \bar{n}_P \to \infty \) and hence free of sampling artifact. This differs from the case of DM, for which in principle we can increase the simulation mass resolution to reach this limit.