Seismic Analysis of a Limestone Rock Slope Through Numerical Modelling: Pseudo-Static vs. Non-Linear Dynamic Approach

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Abstract. In the present work, a seismic analysis was performed in advance on a limestone rock slope (height = 150 m) outcropping along the Tagliamento River valley, in the Friuli Venezia Giulia Region, north-eastern Italy. The analysed slope is characterised by strong rock mass damage, thus resulting in a critical stability condition (unstable volume = 110,000–200,000 m³). The seismic analysis was performed adopting the 2D finite difference method (FDM) and employing both a pseudo-static approach and a non-linear dynamic approach. Model outcomes demonstrate that the seismic motion induces internal, localised ruptures within the rock mass. Some important differences in the mechanical behaviour of the rock slope were highlighted, depending on the specific modelling approach assumed. When adopting a pseudo-static approach, the slope failure occurs for PGA values ranging between 0.056 g and 0.124 g, depending on the different initial static stability condition assumed for the slope (Strength Reduction Factor SRF = 1.00–1.15). According to the non-linear dynamic approach, the slope failure is achieved for PGA values varying between 0.056 g and 0.213 g. Pre-collapse slope displacements calculated with the pseudo-static approach (12–15 cm) are much more greater than those obtained through the non-linear dynamic approach (0.5–3 mm). The modelling results obtained through the non-linear dynamic analysis also testify that the seismic topographic amplification is 1.5 times the target acceleration at the slope face and 2.5 times the target acceleration at the slope toe.

1. Introduction
Earthquake-induced landslides represent one of the main hazardous effects of strong seismicity in mountainous areas and pose a severe threat to human lives and settlements [1]. When considering rock slopes subjected to seismic waves, cycle loading associated with the ground motion can determine localised, overstressed areas within the rock mass where intact rock fracturing initiates and propagates, leading to the enucleation of a fully persistent failure surface and the eventual slope collapse. Jibson [2] classified the methods developed to assess the stability or performance of slopes during earthquakes into three general categories: (i) pseudo-static analysis, (ii) permanent-displacement analysis (e.g., Newmark’s rigid-block method), and (iii) stress-deformation analysis. In the pseudo-static approach, the seismic shaking is simply represented by a constant inertial force applied to a sliding mass. However, the pseudo-static approach is commonly implemented in a static limit equilibrium analysis in order to calculate the factor of safety of the slope, and though possible, it is rarely employed in numerical modelling to evaluate pre-collapse plastic deformations [3]. On the other
hand, the suitability of the stress-strain approach to investigate co-seismic slope deformations has largely been demonstrated, despite the fact that this method is much more computationally demanding [4–7]. Numerical modelling is frequently used in seismic analyses of rock slopes owing to its capability of taking into account a real earthquake record [4, 7–9], even if a simplified or idealised slope geometry is sometimes considered [5, 10, 11]. However, very little research compares findings from the numerical modelling of natural rock slopes that employs both the pseudo-static and the non-linear dynamic approach.

In the present work, a seismic analysis has been performed on a natural rock slope (height = 150 m) outcropping along the Tagliamento River valley, in the Friuli Venezia Giulia Region, north-eastern Italy (figure 1). The seismic analysis was performed through two-dimensional stress-strain modelling using the Finite Difference Method (FDM) and employing both the pseudo-static and the non-linear dynamic approach in order to compare different possible slope seismic responses depending on the different approaches used in the analysis.

![Figure 1. Shaded relief map of the Passo della Morte site along the Tagliamento River valley (Carnic Alps, Friuli Venezia Giulia Region). The trace of the reference geological section is also shown.](image)

2. The unstable slope
The investigated slope is located in the Tagliamento River valley, in the Friuli Venezia Giulia Region (north-eastern Italy), at the “Passo della Morte” site (figure 1). The Passo della Morte slope is crossed by a tunnel related to a former national road. The slope is 150 m high and is made up of high-tilted (73°, on average) and thinly stratified limestone (bed thickness: 10–30 cm), highlighting a particular pyramidal shape (limestone spur) that clearly contrasts with the rear scarps constituted by massive dolomitic limestone (bed thickness: 1–3 m). The limestone spur is completely bounded at its back by two large and transcurrent faults (Passo della Morte and Scluses Stream faults, figure 2a). The geological and geomechanical survey that was performed on the Passo della Morte slope [12] proved that the limestone spur is characterised by strong rock mass damage, emphasising the occurrence of a progressive failure mechanism involving the unstable slope. The limestone spur is crossed by a number of large intersecting faults that subdivide the whole slope into many secondary blocks with different structural conditions (BLOCKS 1–4, figure 2a). The fault slips identified on these major
Discontinuities testify to relative movements among the secondary blocks. The fracturing pattern of the unstable slope highlights remarkable rock mass jointing, including fractures of gravity-induced origin that were not detected in the surrounding slopes considered as stable [12]. The block localised at the toe of the slope (BLOCK2) is characterised by the highest degree of damage, with a poorly interlocked rock mass. Two failure scenarios involving the limestone spur were identified: a smaller failure related to the collapse of BLOCK1 (volume of 110,000 m³) and a larger and deeper failure involving BLOCKS 1–3 (volume of 200,000 m³). When considering the reference geological cross-section of the Passo della Morte slope (figure 2b), the potential basal failure surfaces of the unstable blocks are characterised by typical bi-linear patterns.

![Figure 2](image-url)

**Figure 2.** (a) Frontal view of the Passo della Morte unstable slope showing the faults delimiting the secondary internal blocks (modified from [13]). (b) Reference geological section B’–B” of the Passo della Morte slope. See figure 1 for the cross-section location (modified from [12]).
However, when considering the larger failure scenario (BLOCKS 1–3), the basal rupture surface is not fully formed, and the slope collapse requires the formation of a new rupture surface at the slope toe. The two-dimensional (2D) and three-dimensional (3D) static stress-strain analyses of the Passo della Morte slope [13] highlighted a critical stability condition that is close to the limit equilibrium, with a Strength Reduction Factor (SRF) in the range SRF = 1.03–1.13, depending on the mechanical properties of the local faults that delimit the adjacent blocks.

3. Seismic analysis of the Passo della Morte slope
The study area falls in an Alpine sector characterised by strong seismicity, in which several historical earthquakes have been registered. The 1976 Friuli earthquake that occurred on May 6, 1976 was the most powerful quake recorded in the last 600 years in the area (Mw = 6.4). The seismic analysis of the Passo della Morte slope was performed through 2D numerical modelling, employing the Finite Difference Method (FDM) and using the commercial code FLAC [14]. The numerical simulations were aimed at evaluating the effects of a potential earthquake on the stability condition and stress-strain behaviour of the unstable slope. For comparison purposes, the stress-strain analyses were performed adopting two distinct approaches: the pseudo-static approach and the non-linear dynamic approach. In both approaches, the stresses caused by the ground motion were evaluated in the model by considering reference values of the Peak Ground Acceleration (PGA) that were extrapolated from the seismic hazard map of Italy (“Mappa di Pericolosità Sismica Italiana MPS04” [15]). The assumed reference values of PGA have specific annual rates of exceedance and corresponding return periods and are related to an earthquake with characteristics expected to occur at the Passo della Morte site (table 1).

| PGA (g) | Annual rate of exceedance (λ) | Probability of exceedance in 50 years (Pv) | Return period (T, years) |
|---------|-------------------------------|------------------------------------------|------------------------|
| 0.415   | 0.0004                        | 2%                                       | 2475                   |
| 0.287   | 0.0010                        | 5%                                       | 975                    |
| 0.213   | 0.0021                        | 10%                                      | 475                    |
| 0.147   | 0.0050                        | 22%                                      | 201                    |
| 0.124   | 0.0071                        | 30%                                      | 140                    |
| 0.106   | 0.0099                        | 39%                                      | 101                    |
| 0.090   | 0.0139                        | 50%                                      | 72                     |
| 0.074   | 0.0200                        | 63%                                      | 50                     |
| 0.056   | 0.0333                        | 81%                                      | 30                     |

The seismic analyses performed allowed us to assess the threshold values of PGA related to a shake that can trigger the slope failure, as well as to evaluate any permanent deformations (internal ruptures and block displacements) that cumulate within the unstable rock mass before failure. In the 2D calculation section, the local faults identified on the field and bounding the adjacent blocks were considered (figure 3). The mechanical parameters of the local faults (interfaces) and rock blocks used in the numerical modelling (table 2) were assessed on the basis of the geomechanical field survey performed on the Passo della Morte slope [12]. In the seismic analysis, three different initial values of the strength reduction factor (SRF) of the slope were considered: (1) SRF = 1.00, (2) SRF = 1.08, and (3) SRF = 1.15. These three reference configurations are representative of a stability condition of the Passo della Morte slope that is close to the limit equilibrium and reflect the variability in the mechanical properties of the materials involved [13].
Figure 3. Finite different mesh, initial stresses and boundary conditions of the 2D numerical analyses of the Passo della Morte slope. The location of the control points (CP) is also shown (modified from [13]).

Table 2. Geomechanical parameters used in the numerical modelling (modified from [13]).

| Material     | Normal stiffness $k_n$ (Pa/m) | Shear stiffness $k_s$ (Pa/m) | Unit weight $\gamma$ (kN/m$^3$) | Young’s modulus $E$ (MPa) | Friction angle $\phi$ (°) | Cohesion $c$ (kPa) | Tensile strength $\sigma_t$ (kPa) |
|--------------|-------------------------------|-------------------------------|---------------------------------|--------------------------|--------------------------|-------------------|---------------------------------|
| Interfaces   | 1.0E10                        | 1.0E9                         | –                               | –                        | 25, 30, 35               | 0–300             | 0                               |
| BLOCK1       | –                             | –                             | 25.0                            | 3500                     | 40                       | 500               | 500                             |
| BLOCK2       | –                             | –                             | 25.0                            | 2500                     | 40                       | 400               | 400                             |
| BLOCK3       | –                             | –                             | 25.5                            | 4500                     | 40                       | 600               | 600                             |
| BLOCK4       | –                             | –                             | 25.5                            | 6500                     | 40                       | 800               | 800                             |
| DOLOMITIC BLOCK | –                             | –                             | 26.0                            | 7500                     | 40                       | 900               | 900                             |

3.1. Pseudo-static approach

In the pseudo-static analysis, the seismic stresses generated by a ground motion were simulated by considering equivalent inertial forces that are applied at the centre of gravity of each mesh element. These equivalent inertial forces were implemented in the model simply by varying the direction and magnitude of the gravity force, according to the desired PGA value. Only horizontal accelerations were considered, and vertical accelerations were neglected.

For an initial slope stability condition equal to the limit equilibrium (SRF = 1.00), the lowest seismic acceleration considered (PGA = 0.056 g) would cause slope failure (figure 4a). However, the collapse would only involve the upper BLOCK1. When considering an initial stability condition of SRF = 1.08, slope failure would occur for an earthquake characterised by a PGA value equal to 0.106 g. In this case, the collapse would involve a larger and deeper portion of the slope, including BLOCKS 1–3 (figure 4b). Numerical simulations show the occurrence of pre-collapse plastic deformations. Internal ruptures (both shear and tensile failures) diffusely formed within the unstable blocks. Permanent block displacements up to about 15 cm were accumulated within the rock mass. In
particular, relative movements occurred between BLOCKS 2–3 and the overlying BLOCK1, which moves along fault F4. For an initial stability condition of SRF = 1.15, the ground motion that would cause the slope failure is characterised by a PGA value of 0.124 g (figure 4c). The slope collapse involves the larger multiblock (BLOCKS 1–3), even displaying relative movements among the blocks. The cumulated displacements before failure are about 12 cm. This value is lower than the one calculated in the previous case (15 cm). Lower movements are probably caused by the higher strength properties of the local faults that were adopted in this last model, which is consequently characterised by a higher initial value of SRF.

![PSEUDO-STATIC ANALYSIS](image)

**Figure 4.** Calculated displacement vectors associated with the slope collapse for different reference values of PGA and initial slope stability condition: (a) SRF = 1.00, (b) SRF = 1.08, and (c) SRF = 1.15.

### 3.1. Non-linear dynamic approach

The non-linear dynamic analysis of the Passo della Morte slope allowed for the investigation of the slope mechanical response to an earthquake represented by a real ground motion. In fact, the non-linear dynamic analysis takes into account the cycle effects of the seismic loading, with stress states that vary within the rock mass over the shaking time history. Particular attention was paid to the choice and elaboration of a proper seismic signal. A real accelerogram of the 1976 Friuli earthquake that was registered by a seismic station located on a rock outcrop near the village of Barcis (Pordenone, about 25 km away from the Passo della Morte site) was chosen. Owing to its proximity to the investigated site and considering that the seismic signal was registered at the bedrock (target input motion), this accelerogram was considered as representative of an earthquake with characteristics expected to occur at the Passo della Morte. The real accelerogram has a duration of 16.59 seconds and a peak ground acceleration of 29.92 cm/s² registered at 1.68 s (figure 5a). The acceleration response spectrum highlights a predominant period of 0.16 s (6.25 Hz) and a significant frequency content starting from 0.5 Hz (figure 5b).

For each numerical simulation that was carried out with FLAC, the recorded seismic signal was firstly scaled to the assumed reference values of PGA and subsequently deconvolved [16], filtered, corrected and converted [14] in order to obtain the target input motion of the model. Some control points were defined in the calculation section in correspondence with mesh nodes at the toe, face and crest of the slope with the aim of evaluating the pattern of the slope displacements as well as the seismic topographic amplification over the shaking stress-time history. In the numerical analyses, the slope failure condition was established when the system is in a state of continuing plastic flow (i.e., continuing motion).
Figure 5. (a) Accelerogram and (b) acceleration response spectrum (E–W component) of the seismic signal of the Friuli earthquake (May 6, 1976) registered by the seismic station of Barcis.

For an initial slope stability condition corresponding to the critical one (SRF = 1.00), an earthquake characterised by a PGA value equal to 0.056 g (figure 6a) would be strong enough to cause the block collapse. The slope failure is reached in correspondence with the abrupt exponential increase of the calculated displacements (figure 6b) that occurs at 1.2–1.4 s of the earthquake acceleration-time history, that is before reaching the peak ground acceleration. When the initial slope stability condition is SRF = 1.08, the slope failure occurs for a shake characterised by a PGA value of 0.106 g (figure 6c). In this case, the exponential increase in the slope displacements takes place at 1.7 s (figure 6d), i.e., in correspondence with the occurrence of the peak acceleration. Before failure, localised ruptures occur within the unstable block (mainly shear failures), whereas the permanent pre-collapse displacements are very small (0.5–1.5 mm). For an initial stability condition higher than the critical one (SRF = 1.15), slope failure is achieved for an earthquake with a PGA value of 0.213 g (figure 6e), as a result of shear yielding that mainly occurs at the toe of the unstable slope. The permanent displacements cumulated within the unstable block are slightly higher (1–3 mm, figure 6f). When considering the seismic topographic amplification, outcomes of the numerical simulations have shown that the ground motion in correspondence with the face, the toe and the crest of the slope is 1.5 times, 2.5 times and 4 times the target acceleration, respectively.

4. Discussion and conclusions
The seismic analyses performed on the unstable Passo della Morte slope demonstrate that an earthquake with characteristics expected to occur at the study site represents a potential landslide trigger. Modelling outcomes obtained from both the pseudo-static and non-linear dynamic analyses are only partially consistent. For a hypothesised initial stability degree of the Passo della Morte slope coincident or slightly higher than the critical condition (SRF = 1.00 and SRF = 1.08), slope failure is achieved for the same reference values of PGA, for both analysis approaches (PGA = 0.056 g and PGA = 0.106 g, respectively). These reference values of PGA are quite low when considering the
earthquake characteristics that are expected to occur at the Passo della Morte site, since their corresponding return periods are $T_r = 30$ years and $T_r = 101$ years, respectively. Differently, should the initial slope stability degree be higher than the critical condition ($SRF = 1.15$), slope failure would be reached for a higher reference value of PGA if calculated with the non-linear dynamic analysis ($PGA = 0.213 \ g$) rather than a PGA value obtained through the pseudo-static analysis ($PGA = 0.124 \ g$).

**Figure 6.** Target accelerograms and corresponding displacements calculated at a control point at the toe of the unstable slope (CP1 in figure 3), for different initial slope stability conditions: (a), (b) $SRF = 1.00$; (c), (d) $SRF = 1.08$; and (e), (f) $SRF = 1.15$. 
This results in quite a different return period of the earthquake that would cause the rock slope failure \( (T_r = 475 \text{ years and } T_r = 140 \text{ years, respectively}) \). In addition, the magnitude of the simulated plastic deformations accumulated before slope failure (i.e., slope displacements and localised internal ruptures) are significantly different between the two modelling approaches. Slope displacements calculated through the non-linear dynamic analysis are, on average, up to two orders of magnitude lower than movements obtained by means of the pseudo-static analysis (0.5–3.0 mm vs. 10–150 mm). According to the pseudo-static analysis, internal ruptures are widely distributed within the unstable rock mass; whereas the non-linear dynamic analysis shows plasticity indicators that are mainly localised at the toe of the slope, where a newly formed rupture surface has to occur to enable the slope collapse. The latter failure mechanism is consistent with the pre-collapse behaviour and internal damage that were previously established in the static stress-strain simulations of the Passo della Morte slope \[13\].

The aforementioned discrepancies are the result of the two different approaches used in the seismic analysis of the Passo della Morte slope. In fact, the stress distribution within the unstable rock mass is significantly different between the two calculation methods. The main drawback in the adoption of the pseudo-static analysis is that of reducing the seismic motion to equivalent inertial forces that have constant magnitude and direction over the numerical simulation, neglecting the typical cycle effects of the seismic load and the modes of vibration of the slope structure. This simplification results in very heavy stress state conditions for the slope, which may not be representative of the actual stresses occurring within the rock mass. In fact, the mechanical response of the slope is strongly dependent on the cyclic loading associated with the ground motion, along with the frequency content of the seismic signal and the direction of the wave train. These factors can only be taken into account by adopting a non-linear dynamic analysis. The analyses carried out on the Passo della Morte slope demonstrate that the pseudo-static approach may not be suitable, and not particularly precautionary, when investigating the seismic response of a rock slope.

Finally, the non-linear dynamic analysis shows that the cumulated permanent displacements occurring within the unstable slope before failure are very low in magnitude (0.5–3.0 mm), owing to the prevailing brittle behaviour of the rock mass. These very small slope movements are practically unnoticeable in the field, thus preventing a clear assessment of the effects of an earthquake on the cumulated damage of the rock mass, and in turn, on the stability condition of the slope. For this reason, it is not easily predictable if a rock slope could achieve failure as a consequence of a single strong earthquake rather than repeated and weaker shaking that causes cumulative internal damage of the rock mass and a progressive decrease in the overall resistance of the rock slope.

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