Possibility of Measuring the Width of Narrow Fe II Astrophysical Laser Lines in the Vicinity of η Carinae by means of Brown-Twiss-Townes Heterodyne Correlation Interferometry

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Abstract

We consider the possibility of measuring the true width of the narrow Fe II optical lines observed in spectra of the Weigelt blobs in the vicinity of η Carinae. The lines originate as a result of stimulated amplification of spontaneous emission of radiation in quantum transitions between energy levels showing inverted population (Johansson & Letokhov, 2002, 2003, 2004). The lines should have a subDoppler spectral width of 30-100 MHz, depending on the geometry of the lasing volume. To make measurements with a spectral resolution of $R > 10^7$ and an angular resolution better than 0.1 arcsec, we suggest the use of the Brown-Twiss-Townes optical heterodyne intensity correlation interferometry. The estimates made of the S/N ratio for the optical heterodyne astrophysical laser experiment imply that it is feasible.

Key words: Atomic processes - Line: profiles - Instrumentation: high angular resolution - Techniques: interferometric - Stars:individual: η Carinae

1 Introduction

The conclusion about the existence of an Fe II astrophysical laser (APL) active in spectral lines in the range 0.9-2 μm in the Weigelt blobs in the vicinity of η Carinae (Johansson & Letokhov, 2002, 2003, 2004) was drawn on the basis of data obtained with the HST/STIS facility (Gull et al., 2001). The high spatial resolution of HST/STIS allows observation of the emission spectrum of the Weigelt blobs separated from the photospheric spectrum of the central source. The inference about the laser origin of some of the Fe II spectral lines was based on a detailed analysis of the photoselective excitation of the Fe\textsuperscript{+}
ions and the radiative decay pathways from some specific energy levels. These levels are resonantly excited by the intense H Lyα radiation coming from the HII region around the Weigelt blobs. The analysis was initiated by our finding of unexpectedly strong lines in the HST/STIS spectra that indicated a transformation of a series of weak transitions into intense (allowed-like) lines due to an inverted population. This can occur as a result of stimulated emission of radiation in transitions between energy levels having an inverted population.

However, the maximum spectral resolution of the HST/STIS instrument is $R \approx 10^5$, which is an order of magnitude less than the Doppler width of the spontaneous Fe II lines from the cold HI region of the Weigelt blobs. As discussed in Sec. 2 laser spectral lines can be much narrower. Hence, it seems very desirable to measure the true width of these laser lines at least in the range 0.9–1.0 $\mu$m with the aid of ground-based telescopes. This would be a direct proof of the laser effect in the Fe II lines, as was the case with the spectral measurements in the microwave region in radio astronomy that led to the discovery of astrophysical masers (Elitzur, 1992).

The Brown-Twiss correlation interferometry (Hanbury Brown & Twiss, 1956; Hanbury Brown, 1974), modified by heterodyne detection using a CO$_2$ laser as a local oscillator (Johnson et al., 1974) for the 10 $\mu$m region, is well suited to achieve very high angular and spectral resolution simultaneously. We will call this method the Brown-Twiss-Townes (BTT) technique. In the range 0.9-1.0 $\mu$m this can be done today by means of two spatially separated telescopes equipped with two heterodyne photoreceivers, e.g. avalanche diodes, and a tunable semiconductor laser diode transporting its radiation via an optical fiber (Sect. 3). A general approach of this method has already been considered and discussed in a non-astronomical journal (Lavrinovich & Letokhov, 1976; Letokhov, 1996). In the present paper, we focus on the use of the BBT correlation heterodyne interferometry to study the Fe II laser lines in the range 0.9-1.0 $\mu$m and the appropriate signal-to-noise (S/N) ratio estimates (Sect. 4).

2 Expected Spectral Width of the APL

The astrophysical laser is a laser amplifier that intensifies weak spectral lines of spontaneous emission of radiation in quantum transitions between pairs of energy levels having an inverted populations. The linear amplification factor $K = \exp(\alpha_0^0 D)$, where $\alpha_0^0$ (cm$^{-1}$) is the linear gain per unit length of the laser operating in the $3 \rightarrow 2$ transition (see Fig. 1) between inverted energy levels, and $D$ is the size of the amplifying region, for example, the diameter of the Weigelt blobs $D \approx 10^{15}$ cm (Johansson & Letokhov, 2002, 2003, 2004). Under linear amplification conditions, the intensity of spontaneous radiation
Fig. 1. (a) Change in the amplification coefficient $\alpha_{32}$ during the transition from the linear amplification regime to the saturated amplification regime; (b) Growth in the intensity of the amplified spontaneous radiation in the linear and saturated regimes ($L_{\text{lin}}$ is the length, over which the linear (non-saturated) amplification and exponential growth of intensity occur. The insert shows a schematic of the four-level radiative cycle involving the laser transition.

increases exponentially with the amplification length until the amplification is saturated (Fig. 1). Under saturated amplification conditions, the stimulated transition rate $W_{32}$ exceeds the pumping rate $W_{\text{exc}}$ of the excited level $3$ and the gain drops as (Siegman, 1966)

$$\alpha_{32} = \frac{\alpha_{32}^0}{1 + (I/I_{\text{sat}})},$$

(1)

where $I$ is the intensity of the radiation being amplified (in photons/cm$^2$·s). $I_{\text{sat}}$ is the amplification saturation intensity given by the expression

$$I_{\text{sat}} = \frac{\hbar \omega_{32}}{\sigma_{32} \tau_3},$$

(2)

where $\tau_3$ is the lifetime of upper level $3$ and $\sigma_{32}$ is the stimulated emission cross-section (in cm$^2$),

$$\sigma_{32} = \frac{\lambda_{32}^2}{2\pi} \frac{A_{32}}{2\pi \Delta \nu_D}.$$

(3)

The Einstein coefficient for the $3 \rightarrow 2$ transition is $A_{32}$, and $\Delta \nu_D$ is the Doppler width (in Hz) of the $\lambda_{32}$ line. When $I >> I_{\text{sat}}$, amplification varies linearly, not
Fig. 2. Illustration of the effect of the divergence of the monochromatic light field on the resonant interaction width from a non-homogeneously (Doppler) broadened spectral line: (a) Isotropic monochromatic field; (b) plane light wave; (c) light field with the angular divergence $\Theta$.

exponentially, with the amplification length (Siegman, 1986; Elitzur, 1992).

Under linear conditions, amplification takes place predominantly at the center of the bell-shaped Doppler profile, and the spectrum of the radiation being amplified narrows as (Casperson & Yariv, 1972)

$$\Delta \nu = \frac{\Delta \nu_D}{\sqrt{1 + \alpha_0^3 L_{\text{lin}}}},$$

(4)

where $L_{\text{lin}}$ is the linear amplification length, i.e., the length of the amplifying region where no amplification saturation occurs. The further spectral evolution of the amplified radiation depends on its angular divergence, i.e. essentially on the geometry of the amplifying region.

Before we consider how various geometries of the amplification volume influence the spectral width of an APL it is useful to discuss how the divergence of the monochromatic light field affects the resonant interaction width $\Delta \nu_{\text{int}}$ of the spectral line, non-homogeneously (Doppler) broadened by the moving atoms (ions), as illustrated in Fig. 2 (Letokhov & Chebotayev, 1977).
When spectral-line broadening is non-homogeneous, the light wave interacts only with particles with which it is in resonance. The portion of the particles interacting with the field depends both on the homogeneous spectral width $\nu_{\text{hom}}$, and on the divergence of the light field. If a monochromatic field is isotropic, all of the particles can interact with the field, whatever their velocity (Fig. 2a). On the other hand, a plane travelling wave $E \cos(2\pi \nu t - kr)$ interacts only with particles located within the spectral range of the homogeneous width $\Delta \nu_{\text{hom}}$ at the resonance frequency $\nu = \nu_0 + (kv/2\pi)$ (Fig. 2b). In other words, the field interacts only with particles having a definite velocity projection on the travelling-light-wave direction: $| \nu - \nu_0 + kv/2\pi | \leq \Delta \nu_{\text{hom}}/2$. In case of finite angular divergence $\Theta$ the interaction spectral width is proportional to $\Theta$ (Fig. 2c).

Let us for simplicity consider the following three limiting geometrical cases: (a) elongated volume, (b) spherical volume and (c) disk-like volume.

(a) *Elongated Amplification Volume* (Fig. 3a). In this case, the angular divergence $\Theta$ of the amplified radiation is governed by the angular spread of the amplifying region:

$$\Theta \approx \frac{a}{L},$$

(5)
where $a$ and $L$ are the lateral and longitudinal sizes of the volume, respectively. In rarefied gas condensations, the collisional broadening is very small. A more or less directed light beam interacts with the narrow spectral range $\Delta \nu_{\text{int}}$ of the non-homogeneously broadened Doppler profile of the spectral line (Letokhov & Chebotayev, 1977):

$$\Delta \nu_{\text{int}} = \left(\frac{\Theta}{\pi}\right) \cdot \Delta \nu_{\text{D}},$$

If $\Theta << \pi$, amplification gets saturated first at the center of the Doppler profile, and then the amplification saturation gradually spreads over the entire profile. As a result, the spectral profile of saturated amplification becomes flat (Fig. 3b), which gives rise to the reverse effect - a spectral rebroadening of the radiation being amplified under saturated conditions (the spectral rebroadening effect, Fig. 3c). This effect was analyzed by Litvak (1970) in the case of microwave space masers.

(b) **Spherical Amplification Volume** (Fig. 4a). With this geometry of the amplifying region, the amplified radiation is isotropic. The saturation of the Doppler profile under the effect of isotropic radiation occurs equally for the entire profile, i.e., without any change in the shape of the amplified line, as shown in Fig. 4b (Letokhov & Chebotayev, 1977). In this case, the effect of rebroadening of the amplified radiation is absent (Fig. 4c), and the spectral width of
the radiation is defined by

\[ \Delta \nu = \frac{\Delta \nu_D}{\sqrt{1 + \alpha_0^{32} D}}, \]

where \( D \) is the full size of the amplifying region, for instance, the diameter of a Weigelt blob, which can be much greater than the linear amplification length \( L_{\text{lin}} \). Since the factor \( \alpha_0^{32} D \) can be as great as 100, the width of the amplified line can be an order of magnitude smaller than \( \Delta \nu_D \).

(c) *Disk-Like Amplification Volume.* This is an intermediate case between (a) and (b). In the direction along the disk plane, amplification saturation takes place under the effect of radiation with a geometrical angular divergence of \( \Theta \sim a/b \), where \( a \) is the thickness of the amplifying disk and \( b \) is its diameter. When \( \Theta \ll \pi \) amplification should be saturated first in the narrow central region of the Doppler profile after which the top of the profile should become flat, as in the case with an elongated volume (a). Thus, in the present case (c) the width of the laser line should be comparable with the Doppler width \( \Delta \nu_D \). However, if saturation is attained in the direction perpendicular to the disk plane as well, the quasi-isotropic radiation will provide for saturation of the entire Doppler profile, as in the case with a spherical amplifying volume (b). One could then expect that the laser line rebroadening effect will be absent, so that the line will narrow in accordance with Eq. (7):

\[ \Delta \nu \simeq \frac{\Delta \nu_D}{\sqrt{1 + \alpha_0^{32} a}}. \]

For astrophysical lasers active in the wavelength range 0.9–1.0 \( \mu \)m in the Weigelt blobs of \( \eta \) Carinae, any of the different types of geometry of the amplifying region mentioned above can be realized, either (a), or (b), or else (c). The various types of geometry that can exist for different sets of parameters characterizing the Weigelt blob will be considered in a future publication. What is important at the moment is that the laser lines observable in the range 0.9 – 1.0\( \mu \)m can have a width, \( \Delta \nu \), somewhere between \( \Delta \nu_D \) and down to at least 0.1 \( \Delta \nu_D \). The magnitude of \( \Delta \nu_D \) for the Fe II lines from the HI region of the blobs depends on the temperature \( T \) and can amount to \( \Delta \nu_D \sim 300–1000 \) MHz, since \( T \) is in the range 100 to 1000 K. Accordingly, the laser line width \( \Delta \nu \) can lie between 30 and 1000 MHz. To measure such lines adequately requires a spectral resolution of \( R \sim 10^7 \), which is very difficult to achieve with the standard spectroscopic techniques. For this reason, it is expedient to use the Brown- Twiss-Townes correlation method, modified by utilizing the up-to-date capabilities of electronics, optics, and lasers.
In the 1950-1960’s, R. Hanbury Brown and Q. Twiss proposed and developed a new method under the name intensity interferometry (Hanbury Brown, 1974), which is characterized by its very high spatial (angular) resolution. It produced accurate diameter measurements of 36 bright stars, using a pair of 65-meter collectors and a 188-meter baseline. The method was perhaps the first among a series of new techniques based on advanced technology that led to a substantial progress in attaining levels of high and ultrahigh resolution in astronomy (see review by Saha, 2002). For example, the speckle interferometry technique helped to discover gas condensations (blobs) in the vicinity of Eta Carinae (Weigelt & Ebersberger, 1986). To master new wavelength regions and achieve high spectral resolution levels, the intensity interferometry method was modified to become heterodyne interferometry (Johnson et al., 1974). This technique uses a local monochromatic laser oscillator to produce beats between the light wave of the star of interest and the coherent laser wave of the local oscillator. The method can be considered intermediate between intensity interferometry and direct interferometry. Townes and co-workers made successful observations in the 10-µm infrared window of the atmosphere using a CO₂ laser as a local oscillator (Johnsson et al., 1974; Townes, 1977). The baseline used consisted of a pair of auxiliary telescopes spaced a few meters apart at Kitt Peaks solar telescope.

In Fig. 5 we present a schematic diagram of a Brown-Twiss-Townes (BTT) optical heterodyne interferometer that can be used to measure both the angular size and emission spectrum of Fe II laser line sources in the Weigelt blobs of η Carinae. A specific feature of such a correlation interferometer is the use of a 0.9-1 µm tunable monomode diode laser as a local oscillator and an optical fiber to transport this monomode laser radiation. This is much easier to
accomplish than to transport the space laser radiation received by the telescopes. The distance \(d\) between the telescopes should meet the requirement in angular resolution that the the radiation emitted by the Weigelt blob studied is separated from the photospheric radiation of \(\eta\) Carinae, i.e.

\[
d \simeq \lambda_0 \frac{L}{fD},
\]

where \(L = 10^{22}\) cm is the distance to \(\eta\) Car, \(D \sim 10^{15}\) cm is the diameter of the Weigelt blob, and \(f\) is the fraction of the blob region, in which the laser effect takes place to produce the radiation received by the ground-based telescopes. For estimation purposes, one can set \(f = 1\), and the necessary distance between the telescopes will then be \(d \sim 10^3\) cm. By choosing \(d \sim 10^4\) cm, one can, in principle, analyze laser radiation coming from blob regions as small as one-tenth of the size of the blob.

At the high-speed photomixer, the wavefront of the radiation being received should be matched with diffraction-limited accuracy to that of the local laser oscillator radiation over the entire area \(S\) of the photodetector in accordance with the antenna theorem for photomixing (Letokhov, 1965; Siegman, 1966). In that case, \(S\Omega \sim \lambda_0^3\), where \(\Omega\) is the field of view or aperture. The heterodyne field of view will then be \(\Theta \sim 1.2\lambda_0/a_0\), where \(a_0\) is the diameter of the primary telescope mirror. Modern avalanche photodiodes (Prochazka et al, 2004) with quantum efficiency \(\eta \sim 1\) are most suitable to use as a high-speed photomixer. The photomixer signals of intermediate frequency \(f = \nu_{\text{het}} - \nu\), where \(\nu_{\text{het}}\) is the laser heterodyne frequency and \(\nu\) is the frequency of the desired spectral line frequency of the astrophysical laser radiation being received, should be processed with a correlator. The measured correlation functions of the signal intensity fluctuations as a function of frequency (Fig. 6a) should provide information about the Fourier transformation of the spectral profile of the radiation being received. The spectral resolution \(R\) will be determined by the reception bandwidth \(B\) (Hz) of the photomixer radio signals:

\[
R = \frac{c}{\lambda_0 B}.
\]

At \(B \sim 10^6\) Hz the spectral resolution \(R \sim 3 \cdot 10^8\), which is sufficient to measure the emission spectrum of an astrophysical laser with a spectral width tens of times narrower than the Doppler width. The dependence of the correlation signal on the distance \(d\) between the telescopes should give information on the angular size \(\varphi\) of the blob region wherein the astrophysical laser of interest is active at the wavelength \(\lambda_0\) under study (Fig. 6b). However, the key problem for the experiment proposed is the signal-to-noise ratio \(S/N\) that poses the requirement for the primary mirror diameter \(a_0\) of the telescopes, the reception bandwidth \(B\), and the observation time \(\tau\).
Fig. 6. Expected dependence of the correlation signal intensity as a function of a) the heterodyne frequency detuning and b) the spacing of telescopes $d$.

4 S/N Ratio for Optical Heterodyne APL Experiment

With heterodyne detection, one can control the intensity of the local laser oscillator and thus raise the signal level above the noise level of the electronic circuits used. As a result, one can reach the quantum noise limit, at which the signal-to-noise level is given by (Abbas et al., 1976; Rothemal et al., 1983)

$$\frac{S}{N} = \frac{\eta P}{\hbar \nu B},$$

(11)

where $P$ is the power of the source, $B = \Delta \nu$ is the bandwidth of a single frequency resolution element of the mixing signal, and $\eta$ is the quantum efficiency of the photodetector-mixer. If the signal is integrated over time $\tau$, the $S/N$ ratio is enhanced by a factor of $(B\tau)^{1/2}$, and becomes

$$\frac{S}{N} = \frac{\eta P}{\hbar \nu B} \left( \frac{\tau}{B} \right)^{1/2}.$$  

(12)

The $S/N$ ratio drops with the increasing bandwidth $B = \Delta \nu$. To measure the profile of a spectral line having a width of tens and hundreds of MHz, i.e. much wider than $B$, it is not necessary to scan the entire profile with a spectral resolution of $B$. It is natural to suppose that the spectral profile of the APL radiation is bell-shaped. Therefore, it is sufficient to measure the signals within very narrow bands $B$, a few Hz wide, at several points of the profile and integrate over a sufficiently long time at each sampling point. In that case, one can achieve a sufficient increase in the $S/N$ ratio without any loss of essential information.

To estimate the $S/N$ ratio in the case of astrophysical laser lines from the Weigelt blobs in, one can use the HST/STIS data for the integrated intensity of one of these lines (Gull et al., 2001). For example, the intensity of the 9997
A line from the Weigelt blobs BD amounts to $2 \cdot 10^{12} \text{ erg/cm}^2 \cdot \text{s} \cdot \text{Å}$. With the primary telescope mirror area $A \approx a_0^2$ of the order of 1 m$^2$, one can expect a power of $2 \cdot 10^8 \text{ erg/s}$ within the limits of the Doppler profile $\Delta \nu_D \ll 1 \text{ Å}$. From this we get a lower limit estimate of the $S/N$ ratio:

$$\frac{S}{N} \geq 1.2 \cdot 10^4 \eta \left(\frac{\tau}{B}\right)^{1/2}. \quad (13)$$

For $\eta \sim 1$, $\tau \sim 10^2 \text{ s}$, and $B = 1 \text{ kHz}$, we get $S/N \sim 4 \cdot 10^3$.

The estimate based on Eq. 13 should be considered very approximate, as it can be either greater or smaller by at least an order of magnitude for the following reasons. On one hand, the size of the laser volume in the blob can be smaller than that of the whole blob whose emission is received by the HST/STIS. Therefore, the integrated intensity of the 9997 Å line can be lower than the value used for the estimation. On the other hand, the spectral width of the APL radiation can be an order of magnitude smaller than the Doppler width (Sect. 2). Furthermore, the divergence of the APL radiation in the case of an elongated volume for the amplifying medium can be much smaller than $4\pi$. In any case, the estimates above point to the feasibility of the proposed experiment with the power of the radiation received varying over wide limits because of the existing variable parameter $(\tau/B)^{1/2}$. Let us underline that the proposed experiment can already be performed using existing pairs of ground-based telescopes in the Southern hemisphere.

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