Ratio of kaon-to-pion production cross-sections in $BeBe$ collisions as a function of $\sqrt{s}$

G. I. Lykasov$^a$, A. I. Malakhov, A. A. Zaitsev
Joint Institute for Nuclear Research, Joliot-Curie 6, 141980 Dubna, Russian Federation

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Abstract The inclusive spectra of pions and kaons produced in $BeBe$ collisions as functions of their transverse momentum $p_T$ at mid-rapidity are calculated within the approach, which is based on the assumption of the similarity of inclusive spectra of hadrons produced in $AA$ collisions at their small transverse momenta in the mid-rapidity region. The essence of the modification of the self-similarity approach consists in the inclusion of a quark-gluon dynamics in hadron production in nucleon–nucleon interaction at mid-rapidity. We focus mainly on the ratio of cross-sections of $K^\pm$ to $\pi^\pm$ mesons produced in $BeBe$ collisions as a function of $\sqrt{s}$. A fast rise of this ratio, when the initial energy increases starting from the kaon production threshold up to $\sqrt{s} \simeq 20–30$ GeV, is revealed together with its very slow increase up to LHC energies. The energy dependence of this ratio is due to the conservation laws of four-momenta and quantum numbers of the initial and produced hadrons and the Regge asymptotic behavior of the total and inelastic cross-sections of nucleon–nucleon collisions at large initial energies. The satisfactory description of NA61/SHINE data on these ratios is demonstrated. Some predictions for RHIC and LHC on the $K^+/\pi^+$ and $K^-/\pi^-$ yield ratios as functions of $\sqrt{s}$ in $BeBe$ collisions as functions of $\sqrt{s}$ are presented. The similarity of these observables to the ones for $pp$ collisions at mid-rapidity and in the wide range of initial energies is illustrated.

1 Introduction

The investigation of strange hadron and pion production in heavy-ion collisions is a promising tool to search for new physical properties of such processes. The observation of a sharp peak in the production ratio of $K^\pm$ mesons to $\pi^\pm$ mesons in central $PbPb$ and $AuAu$ collisions at mid-rapidity [1,2] has attracted the attention of both theoreticians and experimentalists, see [3–25] and references therein. When the initial energy $\sqrt{s_{NN}}$ per nucleon becomes larger than 30 GeV this ratio falls down. According to the suggestion of [3,4] this peak (so-called “horn”) can be due to formation of a quark-gluon plasma (QGP) phase at the center-of-mass energy $\sqrt{s} \simeq 7$ GeV. There are statistical models [5–8], which provide a statistical description of the heavy-ion collision processes. However, the authors do not interpret the “horn” within dynamical approaches for heavy-ion collisions, like microscopic transport models. The fast rise and drop in the energy dependence of the $K^+/\pi^+$ and ($\Lambda^0 + \Sigma^0$)/$\pi^0$ yield ratios is explained in [10] by the chiral symmetry restoration (CSR) in the string decay for hadron particle production at low energies and the deconfined partonic medium at high energies.

It has been shown in the recent paper [11] that sharp “horn”-type signal in the $K^+/\pi^+$ ratio in heavy-ion collisions can be also obtained if the possible pion condensation is taken into account within the three-fluid hydro-dynamics model, see also [12]. Another possible explanation of the “horn” in the $K^+/\pi^+$ ratio is presented in [27] within the hadronic kinetic model. The calculations of excitation functions of multiplicity ratios of $K^+/\pi^+$, $K^-/\pi^-$ and $\Lambda/\pi$ for central $PbPb$ collisions result in the satisfactory description of data at $4 \text{ GeV} \leq \sqrt{s} \leq 18 \text{ GeV}$ [1]. There are also some attempts to describe this “horn” in the $K^+/\pi^+$ and $\Lambda^0/\pi^0$ ratios in heavy ion collisions within the Boltzmann transport equation assuming the creation of initial partonic phase at $\sqrt{s}$ beyond a certain threshold [28].

However, the ratio of $K^+\pi^-$ mesons produced in collisions of nuclei lighter than $Pb$ and $Au$, as a function of the initial energy $\sqrt{s_{NN}}$, in particular, $BeBe$ [23] and $ArSc$ [24] has no peak, as revealed by the NA61/SHINE Collaboration [23,24]. The fast increase of this ratio, when $\sqrt{s_{NN}}$ grows from the kaon threshold up to 20-30 GeV and the
slow increase at larger energies have been observed \[23,25\]. Moreover, the energy dependence of $K/\pi$ ratios observed in $BeBe$ collisions \[23\] is similar to the one in $pp$ collisions.

The $p_T$ spectra of pions and kaons produced within the mid-rapidity region of $BeBe$ collisions and their $K/\pi$ ratios as functions of $\sqrt{s}$ were calculated within different models: Epos 1.99 \[13\], URQMD 3.4 \[14,15\], AMPT 1.26 \[16–18\], PHSD 4.00 \[19,20\], SMASH 1.6 \[21,22\]. In \[23\] the comparison of results obtained within these models at the center rapidity region ($y \approx 0$) with the NA61/SHINE data was performed. It was illustrated in Figs. 34, 35 of this paper that all these models do not result in a complete description of the NA61/SHINE data at the initial momentum of about $150A$ GeV/c. The energy dependence of the $K/\pi$ cross-sections ratio is described more or less satisfactorily within the URQMD \[14,15\] and SMASH 1.6 \[21,22\] models, see Fig. 37 of \[23\]. However, these models do not describe the $p_T$ and $y$ spectra of pions and of kaons satisfactorily at the initial momentum of about $150A$ GeV/c, as it is seen from Figs.(34, 35) of the NA61/SHINE paper \[23\].

In this paper we analyze the production of kaons and pions in $BeBe$ collisions at mid-rapidity and focus on ratios between their cross-sections as functions of the initial energy within the same theoretical approach, which was presented in \[29\] for $pp$ collisions. This approach is based on the similarity of spectra of hadrons produced in $AA$ collisions at $y = 0$ and on the conservation laws of four-momenta and quantum numbers suggested in \[30–33\]. It was a continue of the approach suggested earlier in pioneering papers \[35–38\]. Further development of this approach was presented in \[29,34,39–41\].

### 2 The self-similarity approach for $AA$ collisions

The inclusive production of hadron 1 in the interaction of nucleus $A$ with nucleus $B$

$$A + B \rightarrow 1 + \cdots,$$  \hspace{1cm} (1)

is satisfied by the conservation law of four-momenta in the following form \[31,33\]

$$(N_A p_A + N_B p_B - p_1)^2 = (N_A m_0 + N_B m_0 + M)^2,$$  \hspace{1cm} (2)

where $N_A$ and $N_B$ are the fractions of the four-momentum transmitted by nucleus $A$ and nucleus $B$, the forms of $N_A$, $N_B$ are presented in \[33,40\]; $p_A$, $p_B$, $p_1$ are the four-momenta of nuclei $A$, $B$ and particle 1, respectively; $m_0$ is the mass of the nucleon; $M$ is the mass of the particle providing for conservation of the baryon number, strangeness and other quantum numbers. Equation 2 was introduced in \[31,33\] for the production of hadrons in $AB$ collisions in the kinematics forbidden for free nucleon–nucleon collisions. In fact, it is valid for initial energies of colliding nuclei close to the threshold of hadron production. It allows us to find the minimal value of $M$, which provides for the conservation of quantum numbers. For $\pi$-mesons $m_1 = m_\pi$ and $M = 0$. For antinuclei $M = m_1$ and for $K^-$-mesons $M = m_1 = m_K, m_K$ is the mass of the $K$-meson. For nuclear fragments $M = -m_1$. For $K^+$-mesons $m_1 = m_K$ and $M = m_A - m_0, m_A$ is the mass of the $\Lambda$-baryon. Let us note that the isospin effects of the produced hadrons and other nuclear effects are out of this approach. Therefore, it is assumed that within the self-similarity approach there is no big difference between the inclusive spectra of $\pi^+$ and $\pi^-$ mesons produced in $pp$ and $AA$ collisions. However, there is a difference between similar spectra of $K^+$ and $K^-$ mesons, because the values of $M$ are different. This is due to the conservation law of strangeness.

In \[31,33\] the parameter of self-similarity is introduced in the following form

$$\Pi = \min \frac{1}{2} \left[ (u_A N_A + u_B N_B)^2 \right]^{1/2},$$  \hspace{1cm} (3)

where $u_A$ and $u_B$ are the four-velocities of nuclei $A$ and $B$, respectively. The minimization over $N$ presented in Eq. (3) allows us to find the parameter $\Pi$. This parameter introduced in \[31\] was obtained as the analytical form in \[33\] for nucleus-nucleus collisions in the mid-rapidity region, however, it can also be applied successfully for the analysis of pion production in $pp$ collisions, as it was shown in \[39–41\].

The inclusive spectrum of particle 1 produced in the $AB$ collision can be parametrized as a general universal function dependent on the self-similarity parameter $\Pi$, as it was shown in \[34\].

$$Ed^3\sigma_{AB}/d^3p = A^{\alpha(N_A)}_A \cdot A^{\alpha(N_B)}_B \cdot F(\Pi)$$  \hspace{1cm} (4)

where $\alpha(N_A) = 1/3 + N_A/3, \alpha(N_B) = 1/3 + N_B/3$ and function $F(\Pi)$ is the inclusive spectrum of hadron production in the $AB$ collision \[40,41\].

The form of Eq. 4 allowed us to describe satisfactorily inclusive spectra of hadrons produced in $pA$ and $AB$ collisions in kinematics forbidden at hadron production in $pp$ interactions and of particles produced close to the threshold of nucleon–nucleon collisions. This form of the inclusive hadron spectrum also results in the satisfactory description of $p_T$ spectra of pions produced in $AB$ collisions at the mid-rapidity region and not large transverse momenta $p_T$ of produced pions in the wide range of initial energies \[39–42\]. Therefore, we apply it to describe $p_T$ spectra of kaons produced in $AB$ collisions at mid-rapidity and not large transverse momenta.

For symmetric colliding nuclei $N_A = N_B = N$ the function $\Pi$ is found from the minimization of Eq. 3 by solving the equation. This assumption has been suggested in \[31,33\]
The exact solution of Eq. 5 at \( y = 0 \), as
\[
 N = \frac{\Pi}{\cosh(Y)} = \frac{2m_0\Pi}{\sqrt{s}}, \tag{6}
\]
was obtained in [33], for details see, also [40]. In Eq. 6 \( Y \) is rapidity of colliding nuclei. Therefore, \( \alpha(N) = 1/3 + 2m_0\Pi/(3\sqrt{s}) \). For symmetric nuclei Eq. 4 is presented in the following form
\[
 E^3\sigma_{AA}/d^3p = \Lambda^{2\alpha(N)} \cdot F(\Pi) \tag{7}
\]
where
\[
 F(\Pi) = A_q\exp\left(-\frac{\Pi}{C_q}\right) + A_g\sqrt{\frac{\Pi}{\sqrt{s}}}\phi_1(s)\exp\left(-\frac{\Pi}{C_g}\right) \tag{8}
\]
\[
 \Pi(s, m_{1T}, y) = \left\{ \frac{m_{1T}}{2m_0\delta_h} + \frac{M}{\sqrt{s}\delta_h} \right\} \cosh(y)G, \tag{9}
\]
\[
 G = \left\{ 1 + \frac{M^2 - m_1^2}{(m_{1T} + 2Mm_0/\sqrt{s})^2\cosh^2(y)} \cdot \delta_h \right\}. \tag{10}
\]

Here \( \phi_1(s) = 1 - \sigma_{nd}(s)/\sigma_{tot}(s) \), see [40,41],
\[
 \delta_h = \left(1 - \frac{s_0^2}{s}\right); \quad s_0 = 4m_0^2; \quad s_{th} = (m_0 + m_K + m_L)^2; \\
 s_{th}^K = (2m_0 + 2m_K)^2; \quad M = m_L - m_0; \quad m_L = 1.115 \text{ GeV}; \\
 m_K = 0.494 \text{ GeV}; \quad s_0 = 1 \text{ GeV}; \quad m_0 = 0.938 \text{ GeV}; \quad p_{1T} \text{ and } m_{1T} \text{ are the transverse momentum and transverse mass of the produced hadron 1; } \\
 \sigma_{nd} = (\sigma_{tot} - \sigma_{el} - \sigma_{SD}) \text{ is the non-diffractive cross-section; } \sigma_{tot}, \sigma_{SD} \text{ and } \sigma_{el} \text{ are the total cross-section, the single diffractive cross-section and the elastic cross-section of } pp \text{ collisions, respectively. They were taken from } [46,47] \text{ and, together with parameters } A_q, C_q \text{ and } A_g, C_g, \text{ they are presented in the Appendix.} \\
\]

Note that the self-similarity parameter \( \Pi \) depends not only on the transverse mass \( m_{1T} \) and the rapidity \( y \) of the produced hadron but also on the initial energy \( \sqrt{s} \). It leads to the non-factorized form of the inclusive spectrum presented by Eqs. (4, 7–10), as a function of \( s, mT, y \). At large \( \sqrt{s} \) the energy dependence of \( \Pi \) vanishes. This is the main difference of this approach from other models. For the pion production \( M = 0 \), and at transverse momenta less than 1 GEV/c in the mid-rapidity region the parameter function \( \Pi \) can be presented in the simple form
\[
 \Pi \simeq \frac{m_{1T}}{m_0(1 - 4m_0^2/s)} \equiv \frac{m_{1T}}{T}, \tag{11}
\]
where \( T = m_0(1 - 4m_0^2/s) \). It leads to the rapid rise and slow increase of the inverse slope \( T \) in the \( m_{1T} \) distribution of produced pions in \( pp \) collisions, as a function of \( \sqrt{s} \). It coincides very well with experimental data at 2 GEV \( \leq \sqrt{s} \leq \) 20 GEV, see [40], Fig. 2, bottom. There is the similar energy dependence of the inverse slope of the \( m_{1T} \) distribution of pions produced in \( AA \) collision at not large of their transverse momenta at the mid-rapidity. As for the production of \( K \)-mesons, \( \Pi \) has the more complicated dependence on their transverse momenta and \( \sqrt{s} \) because \( M \) is not equal to zero in Eq. 9. Therefore, one cannot determine the universal inverse slope in the transverse mass distribution of \( K \)-mesons.

In fact, the function \( F(\Pi) \) in Eq. (8) is the inclusive spectrum of hadrons produced in \( pp \) collisions at the mid-rapidity region. It was calculated within the approach suggested in [43,44], where the data on inclusive \( p_T \) spectra of charged hadrons produced in \( pp \) collisions at LHC energies in the mid-rapidity region and \( p_T < 2 \text{ GEV/c} \) were described satisfactorily assuming the nonperturbative gluons in \( NN \) collisions at low square transfers \( Q^2 \) about 1-2 (GEV/c)^2. It has been shown that the \( p_T \)-spectrum \( F(x \approx 0, p_T) \) can be presented in the form of Eq. (8), the sum of the quark contribution \( F_q = A_q\exp(-p_{1T}/C_q)\sigma_{tot} \) and the gluon one \( F_g = A_g\sqrt{\Pi/\sqrt{s}}\phi_1(s)\exp(-p_{1T}/C_g)\sigma_{tot} \). In this paper and in [29,39–41], according to [30–33], the transverse momentum of the produced hadron \( p_{1T} \), in functions \( F_q \) and \( F_g \) is replaced by the similarity parameter \( \Pi \), which goes to \( p_{1T} \) asymptotically at LHC energies. From the best description of the ATLAS and CMS data on \( p_T \) distribution of charged hadrons produced at the mid-rapidity region of \( pp \) collisions the unintegrated distribution of gluons \( (g_0(x, k_T)) \) as a function of their fraction \( x \) and transverse momentum \( k_T \) was found. Using the evolution equation of the noncollinear QCD the unintegrated gluon density \( xg(x, k_T) \) was calculated at any \( Q^2 \) and it was applied to describe rather satisfactorily data on many observables of hard \( pp \) collisions at LHC energies and HERA and ZEUS data on deep inelastic scattering (DIS) [42,45]. The forms of \( F(\Pi) \) and \( E d^3\sigma_{AA}/d^3p \) given by Eqs. (8,7) allowed us to describe satisfactorily the data on inclusive spectra of hadrons produced in \( pp \) and \( AA \) collisions at \( y \approx 0 \) and in the wide range of the initial energies \( \sqrt{s} \) [29,39–41]. Therefore, we can quite reasonably apply \( F(\Pi) \) in the form of Eq. (8) entered into Eq. 7 to analyze the \( p_T \) spectra of hadrons produced in \( AA \) collisions at the mid-rapidity range at not large \( p_T \).

3 Results and discussion

In the case of the process \( AA \rightarrow h + X \) Eq. 4 looks as the following
\[
 \rho^h_{AA}(p_{hT}, y) = E_h d^3\hat{\sigma}^h_{AA}/d^3p_1 = \frac{1}{\pi} \frac{d\hat{\sigma}^h_{AA}}{dp_{1T}dy}, \tag{12}
\]
\[
 = \frac{1}{\pi} \frac{d\hat{\sigma}^h_{AA}}{dm_{1T}dy} = A^{2\alpha(N)}F(\Pi(s, m_{1T}, y)),
\]
Then, the production cross-section of hadron \( h \) in \( AA \) collisions integrated over its transverse momentum \( p_{1T} \) or trans-
verse mass \( m_{1T} \) at \( y = 0 \) and \( s \geq s_{1h}^2 \) can be presented in the following form:

\[
\frac{d\sigma_{h_1}}{dy}(s, y = 0) = 2\pi \int_{p_{1T}^{\text{min}}}^{p_{1T}^{\text{max}}} \frac{\rho_{1h}(s, p_{1T}, y = 0) p_{1T} dp_{1T}}{p_{1T}^{\text{max}} - p_{1T}^{\text{min}}}
\]

(13)

Then, the ratio of production cross-sections for hadrons \( h_1 \) and hadrons \( h_2 \) in AA collisions and at \( y = 0 \) as \( \sqrt{s} \) can be presented in the following form

\[
P_{AA}^{h_1/h_2} = \frac{d\sigma_{h_1}}{d\gamma}(s, y = 0) / \frac{d\sigma_{h_2}}{d\gamma}(s, y = 0)
\]

(14)

The \( p_T \) spectra of \( \pi^- \), \( K^+ \), and \( K^- \) mesons are the sums of quark and gluon contributions including uncertainties due to the fit of data that are presented in Figs. 1, 2, 3. By fitting NA61/SHINE data on \( p_T \) spectra at mid-rapidity the parameters \( C_q, A_g, C_g \) were found to be independent of the initial energy \( \sqrt{s} \), they depend on the kind of produced mesons: \( \pi, K^+, K^- \). However, the parameter \( A_q \) varies very weakly at initial momenta from 30A GeV/c up to 150A GeV/c. All the bottom lines in Fig. 1 with \( A_q = 11.7\pm0.1 \) correspond to the bottom curve at \( P_{in} = 30A \) GeV/c described data satisfactorily. All the upper lines in this figure correspond to the upper curve at \( P_{in} = 150A \) GeV/c calculated with \( A_q = 17.7\pm0.1 \). The uncertainties in \( p_T \) spectra and ratios of yields, \( K^+ / \pi^+ \) and \( K^- / \pi^- \), are due to the uncertainties in the parameter \( A_q \). The parameters \( A_q, C_q, A_g, C_g \) are presented in the Appendix. Similar spectra with quark and gluon contributions are also presented in the Appendix.

In Figs. 4, 5 the respective yield ratios, \( K^+ / \pi^+ \) and \( K^- / \pi^- \), are presented as functions of \( \sqrt{s} \). From these figures one can see their fast rise from the threshold energy of \( K^+ \) or \( K^- \) production up to \( \sqrt{s} = 20–30 \) GeV and their further slow increase with energy.

The upper line in Fig. 4 corresponds to the fit of data for \( \pi^+ \) and \( K^+ \) mesons at \( P_{in} = 30A \) GeV/c, and the lower line corresponds to the similar fit at \( P_{in} = 150A \) GeV/c. The upper curve in Fig. 5 corresponds also to the fit of data for \( \pi^- \) and \( K^- \) mesons at \( P_{in} = 30A \) GeV/c and the lower line corresponds to the similar fit at \( P_{in} = 150A \) GeV/c.

4 Conclusion

In this paper we have applied the self-similarity approach, which is based on the assumption of the similarity of inclu-
sive spectra of hadrons produced in AA collisions at their low transverse momenta and in the mid-rapidity region. The essence of our modification of this approach is as follows. The quark-gluon dynamics of hadron production in nucleon–nucleon interaction at mid-rapidity is suggested. It leads to a more complicated form of the $p_T$ spectrum of hadrons produced in $NN$ interaction compared to the simple exponential form used in the self-similarity approach [31, 32]. Moreover, the correct Regge asymptotic behavior of total and inelastic $NN$ cross-sections is taken into account. It results in the realistic energy dependence of inclusive $p_T$ spectra of hadrons produced in $pp$ and $AA$ collisions at not large values of $p_T$ in the mid-rapidity region. Within this approach we analyze the production of both kaons and pions in $BeBe$ collisions at mid-rapidity $y \approx 0.2$ within a wide range of initial energies. We have presented a self-consistent satisfactory description of the NA61/SHINE data on $p_T$-spectra of pions and kaons at $7.62$ GeV $\leq \sqrt{s} \leq 16.84$ GeV. The fast rise of the $K^+/\pi^+$ and $K^-/\pi^-$ yield ratios as functions of $\sqrt{s}$ from the threshold energy of $K^+$ or $K^-$ production up to $\sqrt{s} = 20–30$ GeV has been demonstrated as well as their further slow increase with growing energy. The ratio of kaon yields to those of pions, produced in $BeBe$ collisions, has been calculated within this approach as a function of $\sqrt{s}$.

The energy dependence of the ratio of kaon yields to those of pions in $BeBe$ collision is the same as in $pp$ collision considered earlier in [29]. The fast rise with energy of the kaon and pion production cross-sections, when $\sqrt{s}$ grows from the threshold energy, is due to the conservation law of the four-momenta of initial and produced particles and the factor $\delta_h = 1 - s_{th}^{1/2}/s$ entering into the self-similarity function $\Pi(s, m_{1T}, y)$ given by Eq. 9. The non-zero value of $M$ in the kaon production cross-section results in the fast rise of the $K^+ / \pi^+$ yield ratios because in the pion production cross-section $M = 0$. When $\sqrt{s} \gg \sqrt{s_{th}}$ and $\sqrt{s} \gg M$, the pion and kaon production cross-sections and their ratios become insensitive to factors $\delta_h$ and $M$, however, they are sensitive to the difference between the quark and gluon contributions to the pion and kaon spectra as functions of $p_T$ and $\sqrt{s}$. That is why the $K^+ / \pi^+$ yield ratios exhibit two kinds of energy dependence, a fast rise, when $\sqrt{s_{th}} < \sqrt{s} < 20–30$ GeV and slow increase, when $\sqrt{s} > 20–30$ GeV.

Let us note that no fast rise and no sharp peak in the ratio between the yields of $K^+$ and $\pi^+$ mesons produced in central $BeBe$ collisions are observed in the NA61/SHINE experiment, according to [23]. This ratio is very similar to the same $K^+ / \pi^+$ ratio measured in $pp$ collisions by the NA61/SHINE Collaboration.

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Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: We have no own data and use the published data referred in our paper.]

5 Appendix

The parameterizations of $\sigma_{tot}$, $\sigma_{SD}$ and $\sigma_{el}$ have the following forms [46, 47]

$$\begin{align*}
\sigma_{tot} &= (21.7 (s/s_0)^{0.0808} + 56.08 (s/s_0)^{-0.4525}) \text{mb}; \\
\sigma_{el} &= (12.7 - 1.75 \ln (s/s_0) + 0.14 \ln^2 (s/s_0)) \text{mb}; \\
\sigma_{SD} &= (4.2 + \ln (\sqrt{s/s_0})) \text{mb}.
\end{align*}$$

In Fig. 6 the $p_T$-spectra of pions and kaons, produced in the mid-rapidity of $BeBe$ collisions within the initial momentum range of (30–150) GeV/c, fitted by the NA61 data, are presented. The black dashed line corresponds to the quark contribution, the blue dash-dotted curve is the gluon contri-
Fig. 6 The $p_T$ spectra of $\pi^-$, $K^+$ and $K^-$ mesons produced at $y \approx 0$ in inelastic $BeBe$ interactions at SPS energies $\sqrt{s} = (7.68–16.84)A$ GeV or $P_{in} = (30–150)A$ GeV/c. The NA61/SHINE data were taken from [23,62].

Table 1 Table of parameters found from the fit of NA61/SHINE data

| Reaction | $\sqrt{s}$, A GeV | $P_{in}$, A GeV/c | $A_q$ | $C_q$ | $A_g$ | $C_g$ |
|---------|-----------------|-----------------|------|------|------|------|
| Be+Be→ $h + X$ | 16.8 | 150 | 17.71 | 0.147 | 6.85 | 0.22 |
| | 11.9 | 75 | 15.9 | 0.148 | 2.963 | |
| | 8.8 | 40 | 13.9 | | | |
| | 7.6 | 30 | 11.73 | | | |
| | 7.6 | 150 | 3.83 | | | |
| $\pi^-$ | 16.8 | | 3.32 | | | 0.2271 |
| $K^+$ | 11.9 | 150 | 2.75 | | | |
| $K^-$ | 8.8 | 75 | 3.19 | | | |
| | 7.6 | 40 | 9.142 | | | |
| | 7.6 | 150 | 7.87 | | | |
| | | | 7.062 | | | |
| | | | 7.547 | | | |

As it is shown in [39–41], the form of inclusive pion spectra versus $p_T$ at mid-rapidity given by Eqs. (6–8) describes satisfactorily data in a wide range of $\sqrt{s}$ at $p_T < 2-3$ GeV/c. Moreover, as it is shown in [42–45], the contribution of gluons to the pion spectrum is related to the gluon distribution at low $Q^2 = 1–2$ (GeV/c)$^2$, the use of which results in a satisfactory description of data on hard $pp$ processes at LHC energies and of proton structure functions at low $x$. Therefore, we use Eqs. (4–7) for the description of data on pion $p_T$-spectra in $BeBe$ collisions, only improving the fit of data.

As for $K^{\pm}$ production in $BeBe$ collisions at not large initial energies we take into account the additional contribution due to the one Reggeon exchange diagram, which has a $\sqrt{s_{th}/s}$ dependence. It leads to modification of parameter $A_q$ in the following form $A_q(1 + \sqrt{s_{th}/s})$, which can be approximated by $A_q \exp(\sqrt{s_{th}/s})$. This correction vanishes at RHIC and LHC energies, however, it allows us to describe data at $\sqrt{s} < 10$ GeV satisfactorily.

Parameters $A_q$ for $\pi$, $K^+$ and $K^-$ meson production were found from the fit of NA61 data [23,62] at initial energies $P_{in} = (30–150)A$ GeV/c. Parameters $A_g$ for $\pi$, $K^+$ and $K^-$ meson production were found from the fit of NA61 data at $P_{in} = 150A$ GeV/c. Other parameters $C_q$ and $C_g$ were taken from fits of NA61 data in $pp$ collisions.

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