Particle deposition in circular pipes with variable bed height

JFW Adams, M Fairweather and J Yao
Institute of Particle Science and Engineering, School of Process, Environmental and Materials Engineering, University of Leeds, Leeds LS2 9JT, UK

E-mail: m.fairweather@leeds.ac.uk

Abstract. Reynolds-averaged Navier-Stokes modelling of particle-laden turbulent flows is studied for circular pipes with simulated bed heights of 0, 0.25 and 0.5 of the pipe diameter. A Lagrangian particle tracking technique is used to predict the deposition of spherical particles with sizes ranging from 5-500 μm. Secondary flows are observed to be present in the circular pipe flows with bed heights of 0.25 and 0.5. For the larger particles the presence of a stationary flat bed is found not to influence the deposition rate. For particles <50 μm an increasing bed height, in general, is seen to lead to a decrease in the particle mean displacement from the pipe walls with time. For the smallest particles, the secondary flows in the pipe with a bed height of 0.5 are found to contribute to some re-circulation of the particles, with the mean displacement of particles from the pipe walls found to decrease with time before a further increase is observed.

1. Introduction
Particle-laden flows are of concern to a wide range of industries. Particle deposition in pipes of circular cross-section is also of significant industrial relevance, with flows containing particulates prevalent throughout the nuclear, pharmaceutical, chemical, mining and agricultural industries. In these cases, it is valuable to further understand the behaviour of particles with flows in order to aid the design process and ensure efficient operation. The motivation for this work is taken from the need for the safer and more efficient processing of nuclear waste sludge. With nuclear waste in the form of a solid-liquid sludge it is important to understand the overall characteristics of the flow, with particular interest in the settling characteristics of the particles. Knowledge of the propensity of such flows to form solid beds is therefore important in avoiding unwanted blockages in pipelines and pumping systems and, in cases where the formation of a solid bed is unavoidable, it is important to know how the modified flow characteristics in the reduced pipe cross-section will affect the particle deposition and re-suspension behaviour. Flows in pipes of non-circular cross-section are also of interest due to the presence of secondary flows, with such motions within duct flows playing an important role in the particle deposition and re-suspension processes. It is therefore necessary to understand how the presence of a bed in a pipe will affect the overall flow characteristics, and how any resultant secondary flows will, in turn, affect particle behaviour.

Over the years there have been numerous experimental works on particle-laden flows in straight pipes of circular cross-section, e.g. Tsuji & Morikawa (1982), Caraman et al. (2003) and Boree & Caraman (2005). Particle deposition in a circular pipe has also been the subject of a number of modelling and simulation studies, e.g. Uijttewaal & Oliemans (1996), Portela et al. (2002) and Marchioli et al. (2003). In terms of studies which address pipes of circular cross-section with the presence of a stationary bed there has been comparatively little work published. Investigations
including a stationary bed have been conducted in channel flows by Alvarez-Hernandez (1990) and Hoohlo (1994), and circular pipe flows with a solid stationary bed have been the subject of an experimental investigation by Matousek (2009). To the authors’ knowledge, however, there has been no published work that considers particle deposition in a circular pipe containing beds of variable height.

This work considers the deposition of spherical particles in circular pipes with simulated bed heights $(B_h)$ of 0, 0.25 and 0.5 of the pipe diameter using a Reynolds-averaged Navier-Stokes (RANS) modelling approach coupled with a Lagrangian particle tracking (LPT) technique. The particle deposition rate in these flow geometries is compared over particle sizes ranging from 5-500 μm, with the role the bed height plays in particle deposition behaviour the main focus of the investigation.

2. Mathematical model

The three-dimensional flows are described below using a Cartesian co-ordinate system in which the x, y and z-directions correspond to the u (floor normal), v (transverse) and w (streamwise) velocities respectively. In predicting these flows, the boundary conditions for the momentum equations were no-slip at the pipe walls. The instantaneous flow field was assumed periodic along the streamwise direction, with the pressure gradient that drives the flow adjusted dynamically to maintain a constant mass flux through the pipes. The RANS predictions were based on time-dependent solutions of the Navier-Stokes equations, closed using the second-moment turbulence model of Jones & Musonge (1988) with standard model constants. The computer program BOFFIN (Jones 1991) was used to perform the computations. This code utilises an implicit finite-volume incompressible flow solver using a co-located variable storage arrangement, with solutions second-order accurate in time and space.

From the obtained single-phase flow field solutions the particle trajectories were predicted using a LPT technique (Fan et al. 2002) whereby the particle motion was simulated through the unsteady, non-uniform flow field based on the force balance equation:

$$\frac{dV_p}{dt} = \frac{3}{4} \frac{\rho_p C_d}{d_p} (V - V_p) |V - V_p| + (1 - \frac{\rho}{\rho_p}) g + 1.615 d_p \mu \text{Re}_s c_0 (V - V_p) \times \omega |\omega|$$

Here, $V$ is the fluid velocity, $\rho$ the fluid density, $V_p$ the particle velocity, $\rho_p$ the particle density, $d_p$ the particle diameter, and $g$ is gravity. The term $\omega = \nabla \times V$ is the fluid rotation, $\text{Re}_s = \rho D^2_p |\omega| / \mu$ is the particle Reynolds number of the shear flow, $c_0$ is the ratio of the extended lift force to the Saffman lift force, and $C_d$ is the Stokes coefficient for drag. A fourth-order Runge-Kutta scheme was used to solve the equation of motion, given the initial particle location and velocity. The initial particle positions conformed to a uniform wall-normal particle number density profile. The particles interaction time was assigned to be the lesser of the eddy lifetime and the transition time. Instantaneous fluid velocities were derived from time-averaged RANS solutions using a random Fourier series method (Fan et al. 2002).

The density of the fluid phase was $\rho_f = 1000$ kg m$^{-3}$, with the Reynolds number based on the bulk velocity $\text{Re}_f = 10,000$. In cases where a flat bed was incorporated, the bulk velocity of the flow was maintained. The particle phase was considered to be dilute and therefore one-way coupled. The particles, with diameters $d_p = 5, 10, 50, 100$ and 500 μm, were considered to be spherical with $\rho_p = 2500$ kg m$^{-3}$, and particle-wall collisions were taken to be elastic. The total number of particles considered in the computational domain was sufficient to ensure statistical independence of the results.

3. Results and discussion

Figure 1 shows contour plots of the streamwise velocity for the three test cases considered. As expected, no secondary flows were found to occur in the full pipe, although both of the pipes...
containing a stationary bed where found to contain such flows, with figure 1(d) highlighting the secondary flow velocity vectors present in the pipe with $B_h = 0.5$.

It can be seen from the results given in figure 2(a) and (b) that the addition of a stationary bed to a circular pipe flow has a significant effect on the mean displacement from the wall, normalised by the initial displacement from the wall ($D_w/D_{wi}$), for the 5 and 10 µm particles. The particles in pipes with $B_h = 0.25$ and 0.5, on average, finish closer to the walls by the simulation end point when compared to the pipe with $B_h = 0$. This effect is likely a result of the effective increase in Reynolds number and thus the magnitude of the secondary flows brought about by the presence of a stationary bed. For the pipe with $B_h = 0.5$, the effect of the secondary flows is most noticeable and, in this case, with approximately 65% of the simulation time (normalised by the total simulation time, $t_f$) elapsed, both the 5 and 10 µm particles on average start to see an increased displacement from the walls. This is due to the secondary flows causing re-suspension of the 5 and 10 µm particles as they approach the walls of the pipe back into the main body of the flow. It should be noted that in all cases the net movement of the particles compared to their initial distribution is still very small. This is likely attributable to the fact that the Reynolds number considered, ≈10k, is relatively low. It is also apparent that there is little difference between the mean displacement from the walls for the 5 and 10 µm particles, with the 10 µm particles finishing only marginally closer to the walls of the pipe in all cases when compared to the smaller particles. This is not surprising as the relative difference in size between the two sets of particles is small, and in both cases their trajectories are dominated by the drag force.

Figure 1. Streamwise velocity contour plots for (a) $B_h = 0$, (b) $B_h = 0.25$, (c) $B_h = 0.5$ and (d) secondary flow velocity vector plot for $B_h = 0.5$.

Figure 2. Comparison of mean particle displacement from wall for (a) 5 µm and (b) 10 µm particles in pipes with $B_h = 0$, 0.25 and 0.5.
Figure 3(a) displays the average particle distance from the nearest wall for the 50 µm particles. In this case, it can be clearly seen that for all values of $B_h$ the average particle displacement from the walls decreases significantly with time. Also apparent is the fact that the rate at which the particles approach the walls increases with increasing $B_h$. One interesting observation can be made during the first quarter of the simulation, during which time the 50 µm particles in pipes with $B_h = 0$ and 0.25 approach the walls at a very similar rate. From this point onwards, however, the rate of approach to the walls increases in both cases, although it can be seen that the 50 µm particles in the pipe with $B_h = 0.25$ approach the walls at an increased rate when compared to those in the pipe with no bed. The reason for this is because at early times in the simulation the particles are evenly distributed throughout the pipe cross-sections. During these early stages, therefore, there are a large number of particles still present in the upper part of the pipe cross-section, with flow similar to that in the pipe with $B_h = 0$. As the simulation then proceeds, in the case of the pipe with $B_h = 0.25$, more particles enter the region of the flow where the secondary flows are prevalent, close to the solid bed, through gravitational settling. At this point more particles become entrained within the secondary flows and circulate closer to the walls, further decreasing the mean particle displacement from the walls.

**Figure 3.** Comparison of mean particle displacement from wall for (a) 50 µm and (b) 100 µm particles in pipes with $B_h = 0$, 0.25 and 0.5 (key as figure 2).

**Figure 4.** Comparison of (a) mean vertical displacement from pipe bed for 100 µm particles and (b) mean particle displacement from wall for 500 µm particles in pipes with $B_h = 0$, 0.25 and 0.5 (key as figure 2).
When the pipe with $B_h = 0.5$ is considered it is apparent that the rate at which the 50 µm particles approach the walls is significantly higher than for the other two test cases. It can also be seen that this rate is constant for just under half of the total simulation time. After this point the rate begins to decrease slightly, but settles to a value very similar to that of the pipe with $B_h = 0.25$. This points to the fact that after the mid-point of the simulation both sets of 50 µm particles are predominantly in the region of the pipe flow that is influenced by the secondary motions. The faster initial rate at which the particles approach the walls in the $B_h = 0.5$ case can be explained as before. From figure 1(d) it is apparent that secondary flows are present across the whole cross-section of the pipe with $B_h = 0.5$. These secondary flows have an immediate effect on the particle behaviour, and begin to circulate them towards the walls from the onset of the simulation.

Mean particle displacement from the walls for the 100 µm particles is displayed in figure 3(b). From this it can be seen that for these particles there is again a difference in the deposition rates across the pipes with variable bed heights. However, due to the fact that the 100 µm particles in the pipes with $B_h = 0.25$ and 0.5 take approximately 75% and 50%, respectively, of the simulation time of the $B_h = 0$ case to deposit on the walls, and that these figures correspond to the relative reductions in distance to the base of the pipe, it is clear that the deposition of these particles is strongly influenced by the force of gravity. The varying rates at which the 100 µm particles approach the base of the pipes for all test cases can be attributed to the differences in the geometric cross-sections of the pipes. Indeed, the peculiarity in the mean displacement from the walls in the case of the pipe with $B_h = 0.25$, at 50% of the total simulation time, is a result of the combination of the nature of this particular pipe geometry and the way that the mean displacement from wall is defined. This can be further explained by considering figure 4(a), which shows the mean vertical particle displacement from the bed, normalised by the mean initial displacement ($D_h/D_{in}$). In this figure it can be seen that the final deposition location for the 100 µm particles is different for the pipe with $B_h = 0$ when compared to the pipes with $B_h = 0.25$ and 0.5. This arises from the fact that depositing particles in the latter cases descend onto a horizontal plane, whereas when no flat bed is present, as in the $B_h = 0$ case, the deposition plane is not constant. When comparing figures 3(b) and 4(a) it can be seen that the time at which the mean particle displacement from the walls reaches zero and the time at which the particles have all reached the base of the pipe are coincident in each case. This reaffirms the suggestion that the mechanism of deposition is predominantly gravitational, and by considering the mean vertical displacement from the bed instead of the mean displacement from the walls, the peculiarity previously noted in the pipe with $B_h = 0.25$ is no longer observed.

Lastly, figure 4(b) presents results for the average displacement from the walls for 500 µm particles in pipes with $B_h = 0$, 0.25 and 0.5. For the 500 µm particles there is only a small difference in the deposition rate with varying $B_h$, with the addition of a flat bed leading to a slight increase in deposition rate. This is largely due to the fact that the motion of these particles is totally dependent on the force of gravity, and that the fluid flow has little effect on the particle movement in any direction other than the streamwise direction. The increased deposition rate is therefore a result of the smaller distance a particle is required to travel, on average, to come into contact with a wall for pipes with an increasing bed height.

4. Conclusions

Using a RANS-Lagrangian particle tracking methodology to predict turbulent flows in circular pipes with variable simulated bed heights, $B_h$, the presence of a stationary flat bed is predicted to have little effect on the deposition rate of large particles whose motion is dominated by the force of gravity. For smaller particles, dominated by drag forces, the impact of an increasing $B_h$ leads to a decreased mean particle displacement from the walls of the pipe with time. The cause of this effect is the increasing strength of the secondary cross-stream motions present in pipe flows with a stationary bed. Additionally, and for smaller particles, the secondary flows in the pipe with a bed height of 0.5 are found to contribute to some re-suspension of the particles, with the mean displacement of the particles from the pipe walls found to decrease with time before a further increase is observed. Overall, the
presence of a stationary bed with variable height is found to have a significant effect on the deposition behaviour of some sizes of particle, and the study highlights that the secondary motions present in pipe flows with beds are worthy of further investigation to fully quantify the role that they play in particle deposition and re-suspension behaviour.

**Acknowledgments**
Some of the work described was carried out as part of the TSEC programme KNOO and as such we are grateful to the EPSRC for funding under grant EP/C549465/1. The authors are also grateful for additional financial support from the Nuclear Decommissioning Authority. Finally, the authors would like to express their gratitude to Prof. W.P. Jones for providing the BOFFIN code and for helpful discussion on its use.

**References**
ALVAREZ-HERNANDEZ, E.M. 1990, The influence of cohesion on sediment movement in channels of circular cross-section, PhD Thesis, University of Newcastle.
BORREE, J. & CARAMAN, N. 2005, Dilute bidispersed tube flow: Role of interclass collisions at increased loadings, *Phys. Fluids* 17, 055108.
CARAMAN, N., BORREE, J. & SIMONIN, O. 2003, Effect of collisions on the dispersed phase fluctuation in a dilute tube flow: Experimental and theoretical analysis, *Phys. Fluids* 15, 3602-12.
FAN, J.R., YAO, J. & CEN, K.F. 2002, Antierosion in a 90° bend by particle impaction, *AIChE J.* 48, 1401-12.
HOOHLO, C. 1994, A numerical and experimental study of open-channel flow in a pipe of circular cross-section with a flat bed, PhD Thesis, University of Newcastle.
JONES, W.P. 1991, BOFFIN: A computer program for flow and combustion in complex geometries. Dept. Mech. Eng., Imperial College of Science, Technology and Medicine.
JONES, W.P. & MUSONGE, P. 1988, Closure of the Reynolds stress and scalar flux equations, *Phys. Fluids* 31, 3589-604.
MARCHIOLI, C., GIUSTI, A., VITTORIA SALVETTI, M. & SOLDATI, A. 2003, Direct numerical simulation of particle wall transfer and deposition in upward turbulent pipe flow, *Int. J. Multiphase Flow* 29, 1017-38.
MATOUSEK, V. 2009, Concentration profiles and solids transport above stationary deposit in enclosed conduit, *J. Hydraul. Eng.* 135, 1101-6.
PORTELA, L.M., COTA, P. & OLIEMANS, R.V.A. 2002, Numerical study of the near-wall behaviour of particles in turbulent pipe flows, *Powder Technol.* 125, 149-57.
TSUJI, Y. & MORIKAWA, Y. 1982, LDV measurements of an air-solid two-phase flow in a horizontal pipe, *J. Fluid Mech.* 120, 385-409.
UIJTTEWAAL, W.S.J. & OLIEMANS, R.V.A. 1996, Particle dispersion and deposition in direct numerical and large eddy simulations of vertical pipe flows, *Phys. Fluids* 8, 2590-604.