Landscape of pear-shaped even-even nuclei

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Background: The phenomenon of reflection-asymmetric nuclear shapes is relevant to nuclear stability, nuclear spectroscopy, nuclear decays and fission, and the search for new physics beyond the standard model. Global surveys of ground-state octupole deformation, performed with a limited number of models, suggest that the number of pear-shaped isotopes is fairly limited across the nuclear landscape.

Purpose: We carry out global analysis of ground-state octupole deformations for particle-bound even-even nuclei with $Z \leq 110$ and $N \leq 210$ using nuclear density functional theory (DFT) with several non-relativistic and covariant energy density functionals. In this way, we can identify the best candidates for reflection-asymmetric shapes.

Methods: The calculations are performed in the frameworks of axial reflection-asymmetric Hartree-Fock-Bogoliubov theory and relativistic Hartree-Bogoliubov theory using DFT solvers employing harmonic oscillator basis expansion. We consider five Skyrme and four covariant energy density functionals.

Results: We predict several regions of ground-state octupole deformation. In addition to the “traditional” regions of neutron-deficient actinide nuclei around $^{225}\text{Ra}$ and neutron-rich lanthanides around $^{146}\text{Ba}$, we identified vast regions of reflection-asymmetric shapes in very neutron-rich nuclei around $^{209}\text{Gd}$ and $^{288}\text{Pu}$, as well as in several nuclei around $^{112}\text{Ba}$. Our analysis suggests several promising candidates with stable ground-state octupole deformation, primarily in the neutron-deficient actinide region, that can be reached experimentally. Detailed comparison between Skyrme and covariant models is performed.

Conclusions: Octupole shapes predicted in this study are consistent with the current experimental information. This work can serve as a starting point of a systematic search for parity doublets in odd-mass and odd-odd nuclei, which can be of interest in the context of new physics searches.

I. INTRODUCTION

The majority of atomic nuclei have reflection-symmetric ground-states (g.s.), and exhibit either spherical or ellipsoidal (prolate or oblate) shapes. In rare cases, however, the nucleus can spontaneously break its intrinsic reflection symmetry as a result of a nuclear Jahn-Teller effect [1–4] and acquire non-zero octupole moments associated with pear-like shapes [5, 7] (see Refs. 8, 9 for comprehensive reviews).

Early systematic calculations of octupole shapes were carried out with a macroscopic-microscopic (MM) approach based on the shell correction method [10–12] (see also Ref. 13 for an update). Those were followed by self-consistent studies within nuclear DFT with Gogny [14–17], BCP [15, 18], Skyrme [19, 20], and covariant [21–24] energy density functionals (EDFs).

Except for the global surveys [13, 14, 20, 24, 25], the majority of the previous DFT studies were focused on three specific regions of octupole collectivity: neutron-deficient actinides, neutron-rich lanthanides, and neutron-rich heavy and superheavy nuclei that are important for the modeling of heavy-element nucleosynthesis. Consequently, to better understand systematic trends of octupole instability throughout the nuclear landscape, it is helpful to carry out additional global inter-model comparisons: that is the main objective of this work.

Additionally, the results of this study can provide robust candidates for atomic parity and time-reversal violation searches [26–28]. Of particular interest is the atomic electric dipole moment (EDM) [28]. The nuclear quantity behind the atomic EDM is the Schiff moment [29], which can be enhanced by the presence of nuclear octupole deformation [30] [34]. In particular, the recent study [35] has demonstrated a correlation between the nuclear octupole deformation and the Schiff moment. Thus it is believed that the best candidates for atomic EDM measurements, such as $^{225}\text{Ra}$, are nuclei with octupole shapes. A strong motivation of this work is to produce data for systematic Schiff moment calculations in odd-mass and odd-odd nuclei in the vicinity of the most robust octupole-deformed even-even candidates.

This paper is organized as follows. Section 1 describes the theoretical frameworks used. The results of our global calculations and an analysis of trends are presented in Sec. II. The discussion of local regions of octupole-deformed nuclei is done in Sec. IV. Finally, Sec. V contains a summary and conclusions.
II. THEORETICAL FRAMEWORK

Our calculations are performed in the framework of nuclear DFT [26]. The details pertaining to axial reflection-asymmetric Hartree-Fock-Bogoliubov (HFB) calculations (Sec. II A) and relativistic Hartree-Bogoliubov (RHB) calculations (Sec. II B) can be found in Refs. [37] and [23], respectively.

We studied particle-bound even-even nuclei with \( Z \leq 110 \) and \( N \leq 210 \). For nuclei with \( Z \geq 112 \), Coulomb frustration effects result in exotic topologies of nucleonic densities such as bubbles and tori [38–43], which give rise to instabilities of potential energy surfaces due to configuration changes.

The axial shape deformations \( \beta \lambda \) are defined through multipole moments \( Q_{\lambda} \):

\[
\beta_2 = Q_{20}/\left( \frac{16\pi}{5} \frac{3}{4\pi} AR_0^2 \right), \\
\beta_3 = Q_{30}/\left( \frac{3}{4\pi} AR_0^3 \right),
\]

where \( R_0 = 1.2A^{1/3} \) and

\[
Q_{20} = 2(z^2 - x^2 - y^2), \\
Q_{30} = \sqrt{\frac{2}{15\pi}} \langle z (2z^2 - 3x^2 - 3y^2) \rangle.
\]

Total/proton multipole moments are used in the HFB/RHB calculations, respectively. This difference is not critical since proton and neutron deformations are very similar in the range of nuclei considered.

The magnitude of octupole deformation alone is insufficient in determining whether robust octupole deformation is present since it does not provide any information on the softness of the potential energy surface in the octupole direction. To address this issue, we also look at the gain in binding energy \( \Delta E_{\text{oct}} \) due to octupole deformation:

\[
\Delta E_{\text{oct}} = E^a (\beta_2, \beta_3) - E^a (\beta'_2, \beta'_3 = 0),
\]

where \( E^a \) is the absolute binding energy obtained in reflection-asymmetric calculations, and \( E^a \) is the binding energy minimum from reflection-symmetric calculations. These two minima do not necessarily have the same quadrupole deformation.

A. Skyrme-Hartree-Fock-Bogoliubov calculations

In our study, we consider five Skyrme energy density functionals (SEDfs): UNEDF0 [44], UNEDF1 [45], UNEDF2 [46], SLy4 [47], and SV-min [48]. These SEDFs are described by means of 12–14 coupling constants. The root-mean-square (rms) error of binding energy of these SEDFs, compared to the experimental mass dataset AME2016 [49] ranges from 1.7 MeV (UNEDF0) to 5.3 MeV (SLy4). In the pairing channel, we took the mixed-type density-dependent delta interaction [50] with Lipkin-Nogami approximate particle-number projection as in Ref. [51]. The pairing strengths for SLy4 and SV-min were assumed to be \(-258.2\) MeV and \(-214.28\) MeV, respectively, assuming the same value for neutrons and protons.

The calculations were performed using the parallel DFT solver HFTHO (v3.00) [62] that solves the HFB equation in the cylindrical deformed harmonic oscillator basis. We utilized the “kick-off” mode [52], whereby the multipole moments are constrained in the initial “kick-off” stage and subsequently released when certain criteria are met. Dynamic MPI scheduling was implemented to further reduce computational cost in large-scale mass-table calculations.

The effectiveness and efficiency of the “kick-off” mode has been thoroughly tested and benchmarked with potential energy surface (PES) calculations of more than a hundred nuclei in various mass regions. A cylindrical harmonic oscillator basis of \( N = 20 \) major oscillator shells was used; this was tested to be equivalent (within a reasonable accuracy) in the prediction of g.s. masses compared to using larger shell numbers. Computational savings by using “kick-off” mode and dynamic MPI scheduling, compared with PES calculation under static MPI scheduling, were found to be very significant.

B. Relativistic Hartree-Bogoliubov calculations

In the covariant DFT, the nucleus is considered as a system of \( A \) nucleons which interact via the exchange of different mesons [53]. A global search for octupole deformed nuclei has been performed using four covariant energy density functionals (CEDFs): DD-ME2 [54], NL3* [55], PC-PK1 [56] and DD-PC1 [57]. These CEDFs represent three major classes of covariant DFT models: the non-linear meson-nucleon coupling model (represented by NL3*), the density-dependent meson exchange model (represented by the DD-ME2) and the point coupling model (represented by DD-PC1 and PC-PK1). These functionals typically contain 6 to 9 parameters which are fitted to experimental data on finite nuclei and nuclear matter properties [58]. We used separable pairing of finite range [59] with the strength defined as in Ref. [58]. As compared with the experimental AME2012 dataset, the rms error of binding energy of these CEDFs ranges from 2.15 MeV (DD-PC1) to 3.0 MeV (NL3*) [58].

The reflection-asymmetric RHB calculations are carried out using a parallel version of the computer code developed in Ref. [23], formulated in an axially deformed harmonic oscillator basis. In the present paper, additional calculations to those presented in Refs. [23, 24] have been performed to cover the same range of nuclei as in the Skyrme HFB calculations. The procedure, similar to the “kick-off” procedure employed in the HFTHO calculations, is also used in the RHB calculations. How-
ever, in this case the set of initial Woods-Saxon pear-like densities defined by the basis deformations are used at the initial step of the calculations to push the convergence to the octupole deformed minimum.

III. GLOBAL SURVEY

The g.s. octupole deformations $\beta_3$ obtained in our HFB calculations are displayed in Fig. 1. (For RHB results, see Refs. [23, 24].) There is a good inter-model consistency, with large octupole deformations predicted around $^{146}$Ba (neutron-rich lanthanides), $^{200}$Gd (very neutron-rich lanthanides), $^{224}$Ra (neutron-deficient actinides), and $^{288}$Pu (neutron-rich actinides), i.e., in the regions of strong octupole collectivity defined by the presence of close-lying proton and neutron shells with $\Delta f = \Delta j = 3$ [8]. This finding is consistent with previous global studies [13, 14, 20, 23, 25].

In each region of octupole-deformed nuclei, the magnitude of octupole deformation increases with the number of valence nucleons. All five SEDFs predict neutron-deficient and neutron-rich actinides to exhibit strong octupole deformations, while predictions in the lanthanide region are less uniform regarding which nuclei are deformed and how deformed they are. In general, UNEDF2 and SLy4 predict the largest number of octupole-deformed nuclei and also the larger values of $\beta_3$. In both models, proton-rich nuclei around $^{112}$Ba are expected to be reflection-asymmetric. The functional UNEDF0 predicts the least amount of octupole-deformed nuclei and smaller $\beta_3$ deformations overall.

The octupole deformation energies $\Delta E_{\text{oct}}$ predicted in our HFB calculations are shown in Fig. 2. (For RHB results, see Refs. [23, 24].) We can see that lanthanide nuclei have appreciably smaller $\Delta E_{\text{oct}}$ values as compared to the actinides in spite of similar octupole deformations. This indicates that most of the reflection-asymmetric lanthanide nuclei are predicted to have very soft PESs in the octupole direction, regardless of the equilibrium value of $\beta_3$.

Microscopically, octupole deformations can be traced back to close-lying pairs of single-particle (s.p.) shells coupled by the octupole interaction [8]. Each pair consists of the unusual-parity intruder shell $(\ell, j)$ and the normal-parity shell $(\ell - 3, j - 3)$. Consequently, the regions of nuclei with strong octupole correlations correspond to particle numbers near $34$ ($g_{9/2} \leftrightarrow p_{3/2}$ coupling), $56$ ($h_{11/2} \leftrightarrow d_{5/2}$), $88$ ($i_{13/2} \leftrightarrow f_{7/2}$), $134$ ($j_{15/2} \leftrightarrow g_{9/2}$), and $196$ ($k_{17/2} \leftrightarrow h_{11/2}$).

Figure 3 shows the energy splitting

$$\Delta e = e(\ell, j) - e(\ell - 3, j - 3),$$

between s.p. canonical shells obtained from spherical HFB/RHB calculations. In general, there is a systematic decrease of $\Delta e$ with mass, which – together with the increased degeneracy of s.p. orbits (and matrix elements of the octupole coupling) – results in enhanced octupole correlations in heavy nuclei. However, while this general trend is robust, the magnitude of $\Delta e$ is not a good indicator of octupole correlations when comparing different models. Indeed, when comparing different models one also needs to consider other factors related to each model’s structure. For instance, the isoscalar effective mass of SLy4 is close to 0.7, which effectively increases the s.p. splitting as compared to UNEDF models (which have effective mass close to one). As a result, although in most cases SLy4 has larger $\Delta e$ than UNEDF1, it predicts more octupole-deformed nuclei and larger $\Delta E_{\text{oct}}$ values. It is safer and more instructive to compare predictions of the UNEDF family of SEDFs, as their properties are not very different. Here, the UNEDF2 parametrization, constrained to the spin-orbit splittings in several nuclei, yields the lowest values of $\Delta e$ for neutrons and predicts the strongest octupole correlations, see Figs. 1 and 2.

In an effort to obtain a more robust picture of octupole deformations, we combined the octupole predictions from the five SEDFs and four CEDFs in Fig. 4. We define the model multiplicity $m(Z,N) = k$ if a nucleus $(Z,N)$ is predicted by $k$ models ($k = 1, \ldots 9$) to have a nonzero octupole deformation. Nuclei predicted by all nine EDFs as octupole-deformed (i.e., $m = 9$) are shown by stars. These are: $^{146}$Ba, $^{224,226}$Ra, $^{226,228}$Th, and $^{288}$Pu in the regions experimentally accessible, and in the very neutron-rich actinides: $^{298,300}$Pu, $^{288,290}$Cm, and $^{288,290}$Cf. The supplemental Table [63] contains $\Delta E_{\text{oct}}$ and $\beta_3$ values of nuclei with multiplicity $\geq 6$.

Apart from the overall agreement between SEDFs and CEDFs when it comes to the predicted regions of octupole-instability, we see systematic shifts (by 2-4 neutrons) between the regions of $\Delta E_{\text{oct}}$ and $\beta_3$ obtained by these two EDF families. This systematic effect is illustrated in Fig. 5, where dots mark the HFB predictions with $m \geq 3$, squares show the RHB predictions with $m \geq 2$, and diamonds mark the overlap of the two. This shift has been noticed in Ref. [24] pertaining to super-heavy nuclei.

IV. LOCAL TRENDS

The majority of octupole-deformed nuclei are found near the intersection between neutron numbers 88, 134, and 194 and proton numbers 56 and 88. This pattern is more pronounced in heavy nuclei, due to their lower values of $\Delta e$, see Fig. 3.

We note that all EDFs used in this study provide robust and consistent predictions for quadrupole moments, which generally agree well with available experimental data [64, 66]. This suggests that the quadrupole collectivity is well developed. On the other hand, in many nuclei, the octupole deformation energy has a modest value of less than 500 keV. Such small values of $\Delta E_{\text{oct}}$ indicate soft PESs resulting in the octupole collectivity of transitional character, i.e., between octupole rotational and vibrational motions [8]. While in this work we re-
FIG. 1. Total g.s. octupole deformations $\beta_3$ of even-even nuclei in the $(Z,N)$ plane predicted with the SEDFs UNEDF0, UNEDF1, UNEDF2, SLy4, and SV-min. The magic numbers are indicated by dashed lines.

FIG. 2. Similar to Fig. 1 but for the octupole deformation energy $\Delta E_{\text{oct}}$.

fer to a nucleus as octupole-deformed when its g.s. has $\beta_3 \neq 0$, this does not mean that this octupole deformation is static. For octupole-soft, transitional nuclei, beyond mean-field methods are needed to describe the system [15, 17, 18, 25, 67–70].

In the following, we discuss the local regions of octupole collectivity with a focus on the cases robustly predicted to be octupole-deformed in Skyrme EDF calculations. A detailed discussion of CEDF results can be found in Refs. [23, 24]. Note that this discussion is not intended to provide a comprehensive review. For a detailed experimental discussion and other recent calculations we refer the reader to Refs. [9] and [13, 14, 20, 25], respectively.

A. Neutron-deficient actinides

Because of large octupole correlation effects and experimental accessibility, neutron-deficient actinides have traditionally been in the spotlight of octupole deformation studies. As seen in Fig. 4, this region is abundant in octupole-deformed nuclei, with many systems predicted robustly by several models, i.e., having high octupole multiplicity. Figure 6 summarizes our SEDF results for
the isotopic chains of Rn, Ra, Th, and U.

The isotopes 218, 220 Rn and 224, 226 Rn have been found experimentally to be close to the octupole vibrational limit [73, 74, 75]. As seen in Fig. 6(a), ΔE_{oct} reaches its maximum for 226 Rn, with an average value around −0.5 MeV. These shallow octupole minima suggest that neutron-deficient Rn isotopes are transitional systems, consistent with experiment.

The search for octupole instability in neutron-deficient Ra isotopes has been of great interest [64, 74], also because of atomic EDM studies. According to numerous theoretical calculations, 224 Ra has the largest octupole deformation [9, 74], and is often predicted to have the largest ΔE_{oct} among the Ra isotopes. It is therefore highly surprising that 224 Ra, along with 226 Ra, is predicted to be octupole-deformed by all nine EDFs studied.

Within the SEDFs, the values of β_3, β_1, and ΔE_{oct} appear to be very consistent for 220, 222 Ra, cf. Fig. 6(b). The largest |ΔE_{oct}| is predicted for 222 Ra, followed by 220 Ra and 224 Ra. Recent experiments suggest 222 Ra has the largest octupole deformation among the Ra isotopes followed by 226 Ra, 228 Ra, and 224 Ra [74].

Experimentally, even-even 222–226 Th exhibit many signatures of stable octupole deformation [71, 75, 76], in agreement with the SEDFs’ predictions shown in Fig. 6(c). All SEDFs predict octupole deformations in 226, 224, 226, 228 Th.

The majority of SEDFs predict even-even 222–228 U to be octupole-deformed. As seen in Fig. 6(d), the largest octupole deformation energy exceeding 2 MeV is calculated for 224 U, followed by 222, 226 U. Experimentally, the nucleus 226 U has similar octupole characteristics as 222 Ra and 224 Th [77]. According to our study, the nucleus 224 U is a superb candidate for a pear-shaped system.

Neutron-deficient Pu isotopes have received little attention in octupole-instability studies as they are extremely difficult to access. The lightest-known Pu isotope, 228 Pu, has a half-life of 1.1 s [76] but spectroscopic information about this system is nonexistent. Likewise, virtually nothing is known about 230, 232, 234 Pu, except for their g.s. properties [62]. Interestingly the isotope 228 Pu is predicted by all our models to be octupole-deformed, followed by 226 Pu (m = 7) and 230 Pu (m = 8). The large values of |ΔE_{oct}| in 224, 226, 228 Pu (1.5–2 MeV) calculated by SEDFs are similar to those Ra, Th, and U isotopes that show evidence for stable octupole deformations.

The lightest Cm isotope known experimentally is 233 Cm, which is significantly heavier than our best Cm candidates for pear-like shapes: 228, 230 Cm. As seen in Fig. 6(e) in neutron-deficient actinides with Z ≥ 98, most of the best candidates for octupole deformation lie well beyond the current discovery range, and some appear to be close, or outside, the predicted two-proton drip line [60].

B. Neutron-rich lanthanides

The region of Ba, Ce, Nd, and Sm isotopes around 146 Ba constitutes the second largest concentration of octupole-unstable nuclei predicted theoretically that are within experimental range. Figure 7 summarizes our SEDF results for the isotopic chains of Ba, Ce, and Nd.

Intrinsic dipole moment measurements indicate appreciable octupole correlations in even-even 140–148 Ba [79, 81]. In particular, direct measurements of E3 transition strength made recently in 144, 146 Ba [83, 84] suggest similar octupole correlations in these nuclei (within large experimental uncertainties). As seen in Fig. 7(a), except for UNEDF1, the SEDF results are consistent with this discovery by predicting similar β_3 and ΔE_{oct} for these isotopes.

For the Ce isotopes, all SEDFs except UNEDF0 predict octupole deformations in 146, 148 Ce, with the largest |ΔE_{oct}| in 146 Ce, see Fig. 7(b). Experiment suggests enhanced octupole correlations in 146 Ce [85], 144 Ce [81, 85], and 148 Ce [86], and a weakened octupole collectivity in 150 Ce [57].

The stable isotopes 146, 148 Nd are predicted to be octupole-deformed. Experimental data suggests en-
hanced octupole collectivity in $^{146,148,150}$Nd [88,89]. Another stable isotope with high octupole multiplicity is $^{150}$Sm. Experimentally, there is some evidence for octupole collectivity in an excited band of $^{150}$Sm [93]. As seen in Fig. 4, the isotopes $^{146,148,150}$Nd and $^{150}$Sm are the only stable even-even candidates for octupole instability. The parity doublets in odd-mass nuclei from this region, such as $^{153}$Eu, can be excellent candidates for searches of T,P-violating effects with atoms, ions, and molecules [94].

C. Proton-rich nuclei around $^{112}$Ba

Strong octupole correlations, including octupole instability, were predicted theoretically in nuclei around $^{112}$Ba in the early 1990s [95,96]. As seen in Fig. 4, some of our models yield reflection-asymmetric shapes in a handful of nuclei from this region that lie close to, or beyond, the proton drip-line, with $^{112}$Ba being the best candidate. The experimental data in this region are scarce, with enhanced octupole correlations being suggested for $^{112}$Xe [97] and $^{114}$Xe [98]. The lightest observed Ba isotope is $^{114}$Ba [99], for which no spectroscopic information...
D. Proton-rich and neutron-rich zirconium regions

Shallow octupole minima are calculated in the Zr region around $N = 40$ and $N = 70$ by some CEDFs, see Fig. 5, and also by Gogny calculations of Ref. [14]. On the other hand, our SEDF models predict no octupole instability in this region.

E. Very neutron-rich nuclei around $^{200}$Gd

Many extremely neutron-rich nuclei around $^{200}$Gd are predicted to be octupole-deformed, see Fig. 4 and Supplemental Material [63]. While this region lies well outside experimental reach, the nucleosynthesis calculations suggest that it can be accessed in a very neutron-rich $r$ process [100]. The best candidates for octupole instability in this region are $^{196,198,200}$Sm, $^{196,198,200}$Gd, and $^{200}$Dy.

F. Very neutron-rich nuclei around $^{288}$Pu

Many extremely neutron-rich actinide and transactinide nuclei with $184 < N < 206$ are predicted to be pear-shaped, see Fig. 4 and Refs. [16, 19, 22, 24]. From a purely nuclear structure perspective, this broad region of octupole instability is of solely theoretical interest as it lies well outside experimental reach. While the production of nuclei heavier than $N = 184$ in the astrophysical $r$ process is expected to be strongly hindered by neutron-induced fission [42, 101], the magnitude of this suppression strongly depends on predicted fission barriers [102] and hence the question of their astrophysical relevance is still open.

V. SUMMARY

A systematic survey of reflection-asymmetric even-even nuclei has been carried out within the Skyrme-HFB approach. Among the five SEDFs employed, UNEDF2 and SLy4 predict the largest number of octupole-deformed nuclei, and also the largest octupole deformation energies $\Delta E_{\text{oct}}$. The functional UNEDF0, which was not optimized to experimental shell gaps, predicts the lowest number of octupole minima. This can be attributed to the larger energy splitting $\Delta e$ between

![Graph showing values of $\beta_2$, $\beta_3$, and $\Delta E_{\text{oct}}$ predicted by the SEDF models for the isotopic chains of Rn (a), Ra (b), Th (c), and U (d).]
octupole-driving ($\ell,j$) and ($\ell-3,j-3$) shells in this model.

These results are combined with those obtained with four CEDFs in Refs. [23, 24] and additional RHB calculations performed for the present manuscript. This makes it possible to produce the landscape of octupole deformations shown in Fig. 4 that displays reflection-asymmetric nuclei for non-relativistic and relativistic EDFs, thus limiting systematic model uncertainties. There are 12 even-even nuclei predicted by all nine models used and agree well with experiment.

In the neutron-deficient actinide region, in addition to the “usual suspects”, our study suggests stable g.s. octupole deformations in $^{224,226,228}$U, $^{226,228,230}$Pu, and $^{228,230}$Cm. The only stable pear-shaped even-even nuclei expected theoretically are $^{146,148,150}$Nd and $^{150}$Sm.

Our global survey predicts two exotic regions of octupole instability in extremely neutron-rich nuclei that are inaccessible experimentally. The first region, of lanthanide nuclei around $^{200}$Gd, is possibly populated in a very neutron-rich $r$ process. In the second region of actinide and transactinide nuclei with $184 < N < 206$, neutron-induced fission is likely to suppress the $r$-process production of nuclei with $N > 184$, but the magnitude of this hindrance strongly depends on predicted fission barriers.

It will be of great interest to expand this work by systematic DFT studies of octupole deformation and underlying single-particle structure in odd-mass and odd-odd nuclei. Progress has been made in exploring particle-odd systems by using projection techniques, primarily in the systematic computation of Schiff moments [35], but much work still remains to be done.

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[1] H. A. Jahn and E. Teller, “Stability of polyatomic molecules in degenerate electronic states. I. orbital degeneracy,” Proc. R. Soc. Lond. A 161, 220–235 (1937)
[2] P.-G. Reinhard and E. W. Otten, “Transition to deformed shapes as a nuclear Jahn-Teller effect,” Nucl. Phys. A 420, 173 (1984).

[3] W. Nazarewicz, “Microscopic origin of nuclear deformations,” Nucl. Phys. A 574, 27 – 49 (1994).

[4] S. Frauendorf, “Spontaneous symmetry breaking in rotating nuclei,” Rev. Mod. Phys. 73, 463–514 (2001).

[5] V. M. Strutinsky, “Remarks about pear-shaped nuclei,” Physica 22, 1166–1167 (1956).

[6] K. Lee and D. R. Inglis, “Stability of pear-shaped nuclear deformations,” Phys. Rev. 108, 774–778 (1957).

[7] S. A. Johansson, “Nuclear octupole deformation and the mechanism of fission,” Nucl. Phys. 22, 529 – 552 (1961).

[8] P. A. Butler and W. Nazarewicz, “Intrinsic reflection asymmetry in atomic nuclei,” Rev. Mod. Phys. 68, 349–421 (1996).

[9] P. A. Butler, “Octupole collectivity in nuclei,” J. Phys. G 43, 073002 (2016).

[10] A. Gyurkovich, A. Sobczewski, B. Nerlo-Pomorska, and K. Pomorski, “On the stable octupole deformation of nuclei,” Phys. Lett. B 105, 95 – 98 (1981).

[11] W. Nazarewicz, P. Olanders, I. Ragnarsson, J. Dudek, G. Leander, P. Möller, and E. Ruchowska, “Analysis of octupole instability in medium-mass and heavy nuclei,” Nucl. Phys. A 429, 269 – 295 (1984).

[12] P. Möller, J. Nix, W. Myers, and W. Swiatecki, “Nuclear ground-state masses and deformations,” At. Data Nucl. Data Tables 59, 185 – 381 (1996).

[13] P. Möller, R. Bengtsson, B. G. Carlsson, P. Olivius, T. Ichikawa, H. Sagawa, and A. Iwamoto, “Axial and reflection asymmetry of the nuclear ground state,” At. Data Nucl. Data Tables 94, 758–780 (2008).

[14] L. M. Robledo and G. F. Bertsch, “Global systematics of octupole excitations in even-even nuclei,” Phys. Rev. C 84, 054302 (2011).

[15] L. M. Robledo and R. R. Rodríguez-Guzmán, “Octupole deformation properties of actinide isotopes within a mean-field approach,” J. Phys. G 39, 105103 (2012).

[16] M. Warda and J. L. Egido, “Fission half-lives of superheavy nuclei in a microscopic approach,” Phys. Rev. C 86, 044322 (2012).

[17] L. M. Robledo, “Ground state octupole correlation energy with effective forces,” J. Phys. G 42, 055109 (2015).

[18] L. M. Robledo, M. Baldo, P. Schuck, and X. Vivas, “Octupole deformation properties of the Barcelona-Paris energy density functionals,” Phys. Rev. C 81, 034315 (2010).

[19] J. Erler, K. Langanke, H. P. Loens, G. Martinez-Pinedo, and P.-G. Reinhard, “Fission properties for r-process nuclei,” Phys. Rev. C 85, 025802 (2012).

[20] S. Ebata and T. Nakatsukasa, “Octupole deformation in the nuclear chart based on the 3D Skyrme Hartree-Fock plus BCS model,” Physica Scr. 92, 064005 (2017).

[21] K. Nomura, D. Vretenar, T. Nikšić, and B.-N. Lu, “Microscopic description of octupole shape-phase transitions in light actinide and rare-earth nuclei,” Phys. Rev. C 89, 024312 (2014).

[22] Z. Xu and Z.-P. Li, “Microscopic analysis of octupole shape transitions in neutron-rich actinides with relativistic energy density functional,” Chinese Phys. C 41, 124107 (2017).

[23] S. E. Agbemava, A. V. Afanasjev, and P. Ring, “Octupole deformation in the ground states of even-even nuclei: A global analysis within the covariant density functional theory,” Phys. Rev. C 93, 044304 (2016).

[24] S. E. Agbemava and A. V. Afanasjev, “Octupole deformation in the ground states of even-even Z ≈ 96, N ∼ 196 actinides and superheavy nuclei,” Phys. Rev. C 96, 024301 (2017).

[25] S. Y. Xia, H. Tao, Y. Lu, Z. P. Li, T. Nikšić, and D. Vretenar, “Spectroscopy of reflection-asymmetric nuclei with relativistic energy density functional,” Phys. Rev. C 96, 054303 (2017).

[26] W. C. Haxton and E. M. Henley, “Enhanced T-nonconsering nuclei,” Phys. Rev. Lett. 51, 1937–1940 (1983).

[27] W. C. Haxton, “Atomic parity violation and the nuclear anapole moment,” Science 275, 1753–1753 (1997).

[28] T. E. Chupp, P. Fierlinger, M. J. Ramsey-Musolf, and J. T. Singh, “Electric dipole moments of atoms, molecules, nuclei, and particles,” Rev. Mod. Phys. 91, 015001 (2019).

[29] L. I. Schiff, “Measurability of nuclear electric dipole moments,” Phys. Rev. 132, 2194–2200 (1963).

[30] N. Auerbach, V. V. Flambaum, and V. Spevak, “Collective T- and P-odd electromagnetic moments in nuclei with octupole deformations,” Phys. Rev. Lett. 76, 4316–4319 (1996).

[31] V. Spevak, N. Auerbach, and V. V. Flambaum, “Enhanced T-odd, P-odd electromagnetic moments in reflection asymmetric nuclei,” Phys. Rev. C 56, 1357–1369 (1997).

[32] J. Engel, J. L. Friar, and A. C. Hayes, “Nuclear octupole correlations and the enhancement of atomic time-reversal violation,” Phys. Rev. C 61, 035502 (2000).

[33] J. Engel, M. Bender, J. Dobaczewski, J. H. d. Jesus, and P. Olbratowski, “Time-reversal violating Schiff moment of 225Ra,” Phys. Rev. C 68, 025501 (2003).

[34] V. V. Flambaum, “Enhanced nuclear Schiff moment and time-reversal violation in 229-Th-containing molecules,” Phys. Rev. C 99, 035501 (2019).

[35] J. Dobaczewski, J. Engel, M. Kortelainen, and P. Becker, “Correlating Schiff moments in the light actinides with octupole moments,” Phys. Rev. Lett. 121, 232501 (2018).

[36] M. Bender, P.-H. Heenen, and P.-G. Reinhard, “Self-consistent mean-field models for nuclear structure,” Rev. Mod. Phys. 75, 121 (2003).

[37] J. Erler, N. Birge, M. Kortelainen, W. Nazarewicz, E. Olsen, A. M. Perhac, and M. Stoitsov, “The limits of the nuclear landscape,” Nature 486, 509–512 (2012).

[38] A. V. Afanasjev and S. Frauendorf, “Central depression in nuclear density and its consequences for the shell structure of superheavy nuclei,” Phys. Rev. C 71, 024308 (2005).

[39] B. Schuetrumpf, W. Nazarewicz, and P.-G. Reinhard, “Central depression in nucleonics densities: Trend analysis in the nuclear density functional theory approach,” Phys. Rev. C 96, 024306 (2017).

[40] W. Nazarewicz, “The limits of nuclear mass and charge,” Nat. Phys. 14, 537–541 (2018).

[41] A. V. Afanasjev, S. E. Agbemava, and A. Gyawali, “Hyperheavy nuclei: Existence and stability,” Phys. Lett. B 782, 533 (2018).

[42] S. A. Giuliani, Z. Matheson, W. Nazarewicz, E. Olsen, P.-G. Reinhard, J. Sadhukhan, B. Schuetrumpf, N. Schuck, and P. Schwerdtfeger, “Colloquium: Superheavy elements: Oganesson and beyond,” Rev. Mod.
[90] V. Iacob, W. Urban, J. Bacelar, J. Jongman, J. Nyberg, G. Sletten, and L. Trache, “Reflection asymmetric states in $^{146}$Nd,” Nucl. Phys. A 596, 155 – 170 (1996).

[91] R. Ibbotson, C. White, T. Czosnyka, P. Butler, N. Clarkson, D. Cline, R. A. Cunningham, M. Devlin, K. G. Helmer, T. H. Hoare, J. R. Hughes, G. D. Jones, A. E. Kavka, B. Kotlinski, R. J. Poynter, P. Regan, E. G. Vogt, R. Wadsworth, D. L. Watson, and C. Y. Wu, “Octupole collectivity in the ground band of $^{148}$Nd,” Phys. Rev. Lett. 71, 1990–1993 (1993).

[92] H. Friedrichs, B. Schlitt, J. Margraf, S. Lindenstruth, C. Wesselborg, R. D. Heil, H. H. Pitz, U. Kneissl, P. von Brentano, R. D. Herzberg, A. Zilges, D. Häger, G. Müller, and M. Schumacher, “Evidence for enhanced electric dipole excitations in deformed rare earth nuclei near 2.5 MeV,” Phys. Rev. C 45, R892–R895 (1992).

[93] S. P. Bvumbi, J. F. Sharpey-Schafer, P. M. Jones, S. M. Mullins, B. M. Nyakó, K. Juhász, R. A. Bark, L. Bianco, D. M. Cullen, D. Curien, P. E. Garrett, P. T. Greenlees, J. Hirvonen, U. Jakobsson, J. Kau, F. Komati, R. Julin, S. Juutinen, S. Ketelhut, A. Korichi, E. A. Lawrie, J. J. Lawrie, M. Leino, T. E. Madiba, S. N. T. Majola, P. Maine, A. Minkova, N. J. Ncapayi, P. Nieminen, P. Peura, P. Rahman, L. L. Riedinger, P. Ruotsalainen, J. Saren, C. Scholey, J. Sorri, S. Stolze, J. Timar, J. Uusitalo, and P. A. Vymers, “Octupole correlations in the structure of $^{0}_{+}$ bands in the $N = 88$ nuclei $^{150}$Sm and $^{152}$Gd,” Phys. Rev. C 87, 044333 (2013).

[94] V. V. Flambaum and H. Feldmeier, “Enhanced nuclear Schiff moment in stable and metastable nuclei,” Phys. Rev. C 101, 015502 (2020).

[95] J. Skalski, “Octupole deformation nuclei near $^{112}$Ba,” Phys. Lett. B 238, 6 – 10 (1990).

[96] P.-H. Heenen, J. Skalski, P. Bonche, and H. Flocard, “Octupole excitations in light xenon and barium nuclei,” Phys. Rev. C 50, 802–806 (1994).

[97] J. Smith, C. Chiara, D. Fossan, D. LaFosse, G. Lane, J. Sears, K. Starosta, M. Devlin, F. Lerman, D. Sarantites, S. Freeman, M. Leddy, J. Durell, A. Boston, E. Paul, A. Semple, I. Lee, A. Macchiavelli, and P. Heenen, “Excited states and deformation of $^{112}$Xe,” Phys. Lett. B 523, 13 – 21 (2001).

[98] S. L. Rugari, R. H. France, B. J. Lund, Z. Zhao, M. Gai, P. A. Butler, V. A. Holliday, A. N. James, G. D. Jones, R. J. Poynter, R. T. Tanner, K. L. Ying, and J. Simpson, “Broken reflection symmetry in $^{114}$Xe,” Phys. Rev. C 48, 2078–2081 (1993).

[99] L. Capponi, J. F. Smith, P. Ruotsalainen, C. Scholey, P. Rakilla, K. Auranen, L. Bianco, A. J. Boston, H. C. Boston, D. M. Cullen, X. Derkx, M. C. Drummond, T. Grahn, P. T. Greenlees, L. Grucott, B. Hadinia, U. Jakobsson, D. T. Joss, R. Julin, S. Juutinen, M. Labiche, M. Leino, K. G. Leach, C. McPeake, K. F. Mullholland, P. Nieminen, D. O’Donnell, E. S. Paul, P. Peura, M. Sandzelius, J. Sarén, B. Saygi, J. Sorri, S. Stolze, A. Thorwardaite, M. J. Taylor, and J. Uusitalo, “Direct observation of the $^{114}$Ba $\rightarrow$ $^{110}$Xe $\rightarrow$ $^{106}$Te $\rightarrow$ $^{102}$Sn triple $\alpha$-decay chain using position and time correlations,” Phys. Rev. C 94, 024314 (2016).

[100] J. Lippuner and L. F. Roberts, “$r$-process lanthanide production and heating rates in kilonovae,” APJ 815, 82 (2015).

[101] S. A. Giuliani, G. Martínez-Pinedo, and L. M. Robledo, “Fission properties of superheavy nuclei for $r$-process calculations,” Phys. Rev. C 97, 034323 (2018).

[102] S. A. Giuliani, G. Martínez-Pinedo, M. R. Wu, and L. M. Robledo, “Fission and the $r$-process nucleosynthesis of translead nuclei,” (2019) arXiv:1904.03733 [nucl-th].