Theoretical Analysis of Residual Head Velocity after the Jet Penetrating the Multi-layer Metal-Liquid Composite Structure

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Abstract. The residual head velocity after the jet penetrating a target plate is an indication of the residual penetration capability of the jet, which changes obviously with the penetration time. In this study, the residual head velocity after the jet penetrating through the $n$-layer metal and the $n$-layer liquid was deduced by using the quasi-steady penetration model theory. The influence of shock wave to the residual head velocity of jet was analyzed. The calculation results show that the residual head velocity of the jet decreases with the increase of the number of layers of the composite structure, and the shock wave obviously affects the velocity of the residual head of the jet but not very strong. According to the difference between the experimental value and the theoretical value of the jet penetrating the single-layer composite structure, a method of modifying the theoretical mathematical model by using constant $C$ as the calibration value is proposed. The mathematical model can be extended to calculate the residual head velocity of the jet penetrating the multi-layer metal-liquid composite structure with different materials and thickness.

1. Introduction

Penetration and protection have always been the relationship between spears and shields, but also mutual promotion. It is because of this relationship that there has been continuous scientific and technological progress and innovation on penetration and protection. Brikhoff et al put forward the theory of jet forming for the first time, and established a one-dimensional hydrodynamic penetration model [1]. But this theory is only suitable for continuous jet penetration. For this reason, combined with Bernoulli equation, Pack and Evans give the penetration equation with jet fracture [2]. Considering the effect of target plate strength on jet penetration, Eichelberger introduced the plastic deformation impedance term [3], which more accurately describes the penetration process of ideal incompressible viscous steady state jet. In practice, however, the velocity of the jet's head, tail and diameter is constantly changing. Based on this, Allison and Vitali put forward a quasi-steady penetration model with virtual origin as the origin point [4]. Furthermore, on the basis of virtual distance, Dipersio and Simon give the penetration equation of jet in three different cases [5], which is closer to the actual description of jet penetration. Recent years, Wang Jing et al get the virtual origin by using the method of the least square fitting [6]. Chen Chuang et al established a double virtual
origin calculation model by referring to the improved PER theory, which more accurately described the penetration process of the jet [7].

At present, there are many researches on jet penetration into multi-layer spacer targets and composite targets, such as aluminum foam, ceramics, rubber, explosives and other composite armor [8-12]. White, Andersson, Shi Jinwei, Zhang Xian, Gao Zhenyu, Shan Feng and others have carried out studies on the anti-jet penetration performance of mono-layer metal-liquid composite structures [13-22], and obtained some results that can be used for reference. However, the multi-layer metal-liquid composite structure has not been analyzed, which is worth studying about the unknown effects. In order to solve the above problems, the equation of jet penetrating multi-layer metal-liquid composite structure of is established, and the residual head velocity after jet penetrating into metal-liquid composite structure under the influence of shock wave is analyzed, and the mathematical model of residual head velocity modified by calibration method is proposed.

2. Theoretical Analysis

2.1 Quasi-steady Penetration Model

According to the virtual origin theory redefined by Dipersio and Simon, the jet is formed through the virtual origin and penetrates the 1st layer of metal plate after passing through the air. The penetration model is shown in Fig. 1. After the shaped charge is initiated, the detonation wave axial collapses the charge cover, and the residual head velocity of the jet after forming is very high. The quasi-steady penetration model can be used to describe both jet and target as fluid before the residual head velocity of the jet is not reduced to the $v_{\text{min}}$, also known as hydrodynamic model. The quasi-steady penetration model is the most widely used penetration model and the most basic penetration model. Therefore, we can reference this theory to analysis our study.

Fig. 1 Geometric model of jet penetration into metal-liquid composite structure

Assuming the jet flow is an ideal steady and incompressible non-viscous fluid and the interaction process with metal is a steady process, and the diameter and velocity of the micro-element of the jet do not change. We assume the velocity ($v_j$) of the head after jet forming as the residual head velocity of the jet before penetration, that is, the influence of air and other external factors is not considered. The residual head velocity ($v_{j1}$) after jet penetration into the 1st metal plate is
\[ v_j = \frac{v_{j0} \cdot Y_0}{(Y_0 + d_1) \sqrt{\frac{\rho_{m1}}{\rho_j}}} \]  

(1)

Where, \( Y_0 \) is the distance between the virtual origin and the fist layer of metal, \( \rho_{m1} \) is density of the fist layer of metal, \( \rho_j \) is density of the jet, \( d_1 \) is thickness of the fist layer of metal. So, \( v_{j1} \) is the initial head velocity of the jet penetrating the 1st layer of fluid.

According to the equation (1), the residual head velocity (\( v'_{j1} \)) after the jet penetrating the 1st layer of liquid is

\[ v'_{j1} = v_{j1} \cdot \sqrt{\frac{\rho_{m1}}{\rho_j}} \sqrt{\frac{\rho_j}{(Y_0 + d_1) \sqrt{\frac{\rho_{m1}}{\rho_j}}}} \]  

(2)

Where, \( v'_{j1} \) is the initial head velocity of jet penetration into the second layer of metal plate, \( \rho_{m1} \) is density of the fist layer of liquid, \( h_1 \) is thickness of the fist layer of liquid.

Similarly, the residual head velocity (\( v_{j2} \)) after the jet penetrates the second layer of metal plate and the second layer of liquid are respectively as follows

\[ v_{j2} = v'_{j1} \cdot \sqrt{\frac{\rho_{m2}}{\rho_j}} \sqrt{\frac{\rho_j}{(Y_0 + d_1 + h_1 + d_2) \sqrt{\frac{\rho_{m1}}{\rho_j}}}} \]  

(3)

\[ v_{j2} = v_{j1} \cdot \sqrt{\frac{\rho_{m2}}{\rho_j}} \sqrt{\frac{\rho_j}{(Y_0 + d_1 + h_1 + d_2) \sqrt{\frac{\rho_{m1}}{\rho_j}}}} \]  

(4)

Where, \( d_2 \) is thickness of the second layer of metal, \( \rho_{m2} \) is density of the second layer of metal, \( \rho_{f2} \) is density of the second layer of liquid, \( h_2 \) is thickness of the second layer of liquid.

And so on, according to the above iterative law, the residual head velocity (\( v_{jn} \)) after the jet penetration into the n-layer metal plate and the n-layer liquid are obtained

\[ v_{jn} = v_{jn-1} \cdot \sqrt{\frac{\rho_{mn}}{\rho_j}} \sqrt{\frac{\rho_j}{(Y_0 + \sum_{i=1}^{n} d_i + \sum_{i=1}^{n} h_{n-1}) \sqrt{\frac{\rho_{m(n-1)}}{\rho_j}}}} \]  

(5)

\[ v_{jn} = v_{jn-1} \cdot \sqrt{\frac{\rho_{mn}}{\rho_j}} \sqrt{\frac{\rho_j}{(Y_0 + \sum_{i=1}^{n} d_i + \sum_{i=1}^{n} h_{n-1}) \sqrt{\frac{\rho_{m(n-1)}}{\rho_j}}}} \]  

(6)

Where, \( d_n \) is thickness of the n-layer of metal, \( \rho_{mn} \) is density of the n-layer of metal, \( \rho_{ln} \) is density of the n-layer of liquid, \( h_n \) is thickness of the n-layer of liquid.
2.2 The Influence of Shock Wave
When the jet formed by explosion runs through the 1st layer of metal plate, the acoustic velocity \( c_0 \) of the undisturbed liquid is far lower than \( v_j \), so the motion of the jet in the liquid is supersonic. According to Gao Zhen-yu's study [23], the cone-like shock wave produced by the jet will move the liquid through the side face of the closed metal structure and cause the liquid to converge in the radial direction while the jet penetrates the liquid radial reaming hole. This will interfere with the jet, and lead the residual head velocity of the jet to drop so that reduces the ability to continue to penetrate, so it is necessary to modify the above equation.

The micro-elements at the point where the jet head interacts with the liquid in the hole at a certain time are taken for analysis. According to the momentum conservation principle, the head velocity is positive, the momentum conservation differential equation is

\[
\dot{v}_{jn} dm_{jn} = u_n dm_n \cos \theta + v_{jn} \frac{dm'}{jn-shock \ wave} \tag{7}
\]

Where, \( dm_{jn} \) is the mass differential element of jet, \( dm_n \) is the mass differential element of liquid, \( dm'_{jn} \) is the mass differential element of jet after the jet penetrating liquid, \( \theta \) is the angle between \( u_n \) and \( v_{jn} \), \( v_{jn-shock \ wave} \) is the residual head velocity of the jet under the influence of shock wave. According to the relationship between mass and volume, equation (7) can be rewritten to

\[
v_{jn} \rho_j dV_{jn} = u_n \rho_n dV_n \cos \theta + v_{jn-shock \ wave} \rho dV'_{jn} \tag{8}
\]

Where, \( dV_{jn} \) is the volume differential element of jet, \( dV_n \) is the volume differential element of liquid, \( dV'_{jn} \) is the volume differential element of jet after the jet penetrating liquid.

Due to the liquid is in-compressible, the shape of the micro-element changes and the volume remains unchanged, taking the unit volume micro-element, there is

\[
\frac{u_n \rho_n \cos \theta + v_{jn-shock \ wave} \rho_j}{\rho_j} = \frac{v_{jn-shock \ wave} \rho}{\rho_j} - u_n + v \tag{9}
\]

Thus, the residual head velocity of the jet under the influence of shock wave is obtained

\[
v_{jn-shock \ wave} = \frac{u_n \rho_n \cos \theta + v_{jn-shock \ wave} \rho}{\rho_j} - u_n \tag{10}
\]

It can be seen that the residual head velocity will decrease after the jet is disturbed by liquid, and the decrease is related to \( \rho_n, \rho_j \) and \( u_n \).

For obtaining the penetration velocity \( u \), we can only consider the effect of the first shock wave because the time of interaction between jet and liquid is very short. And the velocity of the shock wave is assumed to be constant and the attenuation of the wave is not taken into account. According to Xia-Qing's study [24], the penetration velocity \( u_n \) is

\[
u_n = \frac{1}{2(\lambda_2 \rho_j - (1 + \lambda_2) \rho_n)} \left\{ 2 \lambda_2 \rho_j \left[ -\rho_j \cos \epsilon - \epsilon \rho_j - \epsilon \rho_n \right] + \frac{8 \lambda_2 \rho_j \left[ (1 + \lambda_2) \rho_j - \epsilon \rho_j - \epsilon \rho_n \right]}{\rho_n} + \frac{2 \lambda_2 \rho_j \left[ (1 + \lambda_2) \rho_j - \epsilon \rho_j - \epsilon \rho_n \right]}{\rho_j} \right\} \tag{11}
\]

Where, \( \lambda_n \) is the Hugoniot parameter of the liquid and \( R_n \) is the dynamic yield strength of the liquid, \( c_n \) is the acoustic velocity of the n-layer undisturbed liquid.

When the penetration condition is \( v_{jn} > c_n \), the effect of shock wave should be taken into account. The initial velocity of jet penetration into the head of the n-layer metal plate should be calculated by using the \( v_{jn-shock \ wave} \) which associated with \( u_n \). The effect of \( v_{jn} < c_n \) shock wave on penetration condition is negligible, and the initial head velocity of jet penetration into the n-layer metal plate should by using \( v_{jn} \).

Note that the condition of the above analysis is that \( v_{jn} > v_{jmin} \) and \( v_{jn} > c_n \). When \( v_{jn} < v_{jmin} \), the strength of the metal plate should be considered. And when \( v_{jn} < c_n \), the motion is no longer supersonic, and the penetration will be basically stopped because the velocity of the head is too low.
3. Calculation Example and Model Correction

In order to simplify the study, we take one to five layers metal-liquid composite structure to study. Define that the thickness of each layer of metal and liquid is the same, and all metals are LC4 aluminum alloy, all liquids are diesel oil.

Where, \( d=5 \text{ mm} \) and \( h=12.5 \text{ mm} \), \( Y_0=28.85 \text{ mm} \) and \( v_{j0}=6510 \text{ m/s} \) are known. According to the material parameters in Table 1, \( \rho_{mn}=2770 \text{ kg/m}^3 \), \( \rho_f=837 \text{ kg/m}^3 \), \( c_f=1775 \text{ m/s} \), \( R_t=150 \text{ Pa} \), \( \rho_j=8960 \text{ kg/m}^3 \), \( \lambda_n=1.725 \). In combination with the formula (1)-(11), the theoretical results about one to five layers metal-liquid composite structure are calculated by programming.

According to the experimental data in reference [11], the residual head velocity is 6038 m/s after the jet penetrating the mono-layer metal-liquid composite structure, and the theoretical result of this paper is 5110.77 m/s, the relative error is 15.4%. In order to improve the accuracy of the mathematical model in the theoretical analysis, the experimental data can be used as the calibration value to modify the results obtained from the equations (5) and (6). Define the calibration value as

\[
C = v_e - v_{j2}
\]  

(12)

Where, \( v_e \) is the experimental value of residual head velocity of the jet penetrating the mono-layer metal-liquid composite structure.

The function of the calibration value \( C \) is to make the original theoretical calculation result (the relation curve between \( n \) and \( v \)) to translate \( C \) units along the positive direction of the velocity axis. That is, \( C \) is an intercept correction value. Through this method, the theoretical results modified by the calibrated value \( C=927.25 \) are shown in Fig. 2.

![Fig. 2 Jet residual head velocity change trend under quasi-steady penetration model and under the influence of shock wave](image)

From the theoretical calculation results, \( v \) decreases with the increase of \( n \) under the influence of quasi-steady penetration model and shock wave. However, the \( v \) under the influence of shock wave is lower than the quasi-steady value before a certain critical point, and then it higher than the quasi-steady value. The critical position is the 4th layer composite structure.

4. Summary

The liquid-filled unit cell was studied by Gao Zhen-yu and Zhang Xian [11,16-23], which was characterized by only one layer or many mono-layer with transverse arrangement. The disadvantage is that only some of the composite structures can play a protective role when the jet penetrates vertically, so the protective ability is limited. In contrast, the innovation of this paper lies in the multi-layer axial arrangement of the composite structure, which can effectively and continuously interfere with the jet.
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