Quantum Vacuum in Hot Nuclear Matter – A Nonperturbative Treatment

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Abstract

We derive the equation of state for hot nuclear matter using Walecka model in a nonperturbative formalism. We include here the vacuum polarisation effects arising from the nucleon and scalar mesons through a realignment of the vacuum. A ground state structure with baryon-antibaryon condensates yields the results obtained through the relativistic Hartree approximation (RHA) of summing baryonic tadpole diagrams. Generalization of such a state to include the quantum effects for the scalar meson fields through the σ-meson condensates amounts to summing over a class of multiloop diagrams. The techniques of thermofield dynamics (TFD) method are used for the finite temperature and finite density calculations. The in-medium nucleon and sigma meson masses are also calculated in a self consistent manner. We examine the liquid-gas phase transition at low temperatures (∼ 20 MeV), as well as apply

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the formalism to high temperatures to examine for a possible chiral symmetry
restoration phase transition.
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I. INTRODUCTION

The understanding of hot and dense matter is an interesting and important problem in the context of heavy ion collision experiments as well as to study different astrophysical objects such as neutron stars. Quantum Hadrodynamics (QHD) is a general framework which has been extensively used to study the nuclear matter, both at zero \[1\] and finite temperatures \[2\], as well as to describe the properties of finite nuclei \[3\]. In the Walecka model (QHD-I), the nucleons interact via the scalar (\(\sigma\)) and vector (\(\omega\)) mesons with the scalar and vector couplings fitted from the saturation density and binding energy of nuclear matter \[1\]. The hot nuclear matter has been studied neglecting the Dirac sea \[2\], i.e., in the so called no–sea approximation.

To study the nuclear matter at zero temperature including the sea effects in the relativistic Hartree approximation (RHA), one does a self consistent sum of the tadpole diagrams for the baryon propagator \[4\]. There have also been calculations including corrections to the binding energy up to two-loops \[5\], which are seen to be rather large as compared to the one-loop results. However, it is seen that using phenomenological monopole form factors to account for the composite nature of the nucleons, such contribution is reduced substantially \[6\] so that it is smaller than the one-loop result. However, without inclusion of such form factors the mean-field theory is not stable against a perturbative loop expansion. This might be because the couplings involved here are too large (of order of 10) and the theory is not asymptotically free. Hence nonperturbative techniques need to be developed to consider nuclear many-body problems.

The approximation scheme adopted here is nonperturbative and, it uses a squeezed coherent type of construction for the ground state \[7,8\] which amounts to an explicit vacuum realignment. The input here is equal-time quantum algebra for the field operators with a variational ansatz for the vacuum structure and does not use any perturbative expansion or Feynman diagrams. We have earlier seen that this correctly yields the results of the Gross-Neveu model \[9\] as obtained by summing an infinite series of one-loop diagrams. It was
also seen to reproduce the gap equation in an effective QCD Hamiltonian [10] as obtained through the solution of the Schwinger-Dyson equations for the effective quark propagator. In an earlier work [11], such a nonperturbative method was applied to consider vacuum polarization effects in nuclear matter, where it was shown that a realignment of the ground state in nuclear matter with baryon–antibaryon condensates is equivalent to the relativistic Hartree approximation (RHA). We had then included the quantum corrections arising from the scalar meson in a similar way through sigma condensates, amounting to summing over a class of multiloop diagrams. Recently, the formalism has also been generalised to include strange baryons and its effects on the composition of neutron star matter as well as gross structural properties of neutron stars [12]. In the present paper, we study hot nuclear matter including the vacuum polarisation effects arising from the nucleon and scalar meson fields. The method of thermofield dynamics (TFD) is used here to study the “ground state” (the state with minimum thermodynamic potential) at finite temperature and density. The temperature and density dependent baryon and sigma masses are also calculated in a self-consistent manner in the present framework. We note that in the Walecka model, TFD has been applied to compute perturbatively quantum corrections to the temperature dependent Hartree mean field [13] and effect of such corrections on the equation of state. Here we shall however use a nonperturbative variational approach to study symmetric nuclear matter within Walecka model using TFD for small temperatures (associated with liquid-gas phase transition) as well as at high temperatures to discuss possible chiral restoration transition. The ansatz functions involved in such an approach shall be determined through minimisation of the thermodynamic potential.

We organize the paper as follows. In section II, we study the quantum correction effects from the nucleon and sigma fields in nuclear matter as simulated through baryon–antibaryon and scalar meson condensates. The quantum vacuum in nuclear matter is generalized to finite temperatures using the thermofield dynamics method. The corresponding condensate functions as well as the functions introduced in the definition of the thermal vacuum are determined through the minimisation of the thermodynamic potential. This enables us
to obtain the equation of state (EOS) and various thermodynamic quantities for the hot nuclear matter. In section III, we discuss the results obtained in the present nonperturbative framework of inclusion of quantum corrections. Unlike the mean field calculations [2], we observe that, at temperatures of around 200 MeV, the decrease in the nucleon mass is slower when one includes such quantum correction effects, and need not be indicative of a chiral symmetry restoration phase transition. We also study the effect of quantum corrections on the liquid gas phase transition. The value of critical temperature for such a phase transition is seen to be lowered due to quantum effects arising from the scalar mesons. In section IV, we give a brief summary of the present work and discuss possible outlook.

II. QUANTUM VACUUM IN NUCLEAR MATTER

We shall start with briefly recapitulating the formalism for studying the nuclear matter including the quantum correction effects as arising through a realignment of the vacuum with baryon–antibaryon and scalar meson condensates [11]. The Lagrangian density is given as

\[ \mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - M - g_\sigma \sigma - g_\omega \gamma^\mu \omega_\mu)\psi + \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m^2_\sigma \sigma^2 - \frac{1}{4} \lambda \sigma^4 + \frac{1}{2} m^2_\omega \omega_\mu - \frac{1}{4} (\partial_\mu \omega_\nu - \partial_\nu \omega_\mu)(\partial^\mu \omega^\nu - \partial^\nu \omega^\mu). \]  

(1)

In the above, \( \psi, \sigma, \) and \( \omega_\mu \) are the fields for the nucleon, \( \sigma, \) and \( \omega \) mesons with masses \( M, m_\sigma, \) and \( m_\omega \) respectively. The quartic coupling term in \( \sigma \) is necessary for the sigma condensates through a vacuum realignment, to exist [11]. We retain the quantum nature of both the nucleon and the scalar meson fields, where as, the vector \( \omega \)-meson is treated as a classical field, using the mean field approximation for \( \omega \)-meson, given as \( \langle \omega^\mu \rangle = \delta^\mu_0 \omega_0 \). The reason is that without any quartic or any other higher order term for the \( \omega \)-meson, the quantum effects generated due to \( \omega \)-meson through the present variational ansatz turns out to be zero.

The Hamiltonian density can then be written as

\[ \mathcal{H} = \mathcal{H}_N + \mathcal{H}_\sigma + \mathcal{H}_\omega, \]  

(2)
with

\[ H_N = \psi^\dagger (-i \alpha \cdot \nabla + \beta \mathbf{M}) \psi + g_\sigma \sigma \bar{\psi} \psi, \]  

(3a)

\[ H_\sigma = \frac{1}{2} \sigma^2 + \frac{1}{2} \sigma (-\nabla^2) \sigma + \frac{1}{2} m_\sigma^2 \sigma^2 + \lambda \sigma^4, \]  

(3b)

\[ H_\omega = g_\omega \omega_0 \psi^\dagger \psi - \frac{1}{2} m_\omega^2 \omega_0^2. \]  

(3c)

We may now write down the field expansion for the nucleon field \( \psi \) at time \( t = 0 \) as given by [11]

\[ \psi(x) = \frac{1}{(2\pi)^{3/2}} \int [U_r(k)c_{I r}(k) + V_s(-k)\tilde{c}_{I s}(-k)] e^{ik \cdot x} dk, \]  

(4)

with \( c_{I r} \) and \( \tilde{c}_{I s} \) as the baryon annihilation and antibaryon creation operators with spins \( r \) and \( s \) respectively, and \( U \) and \( V \) are the spinors associated with the particles and antiparticles respectively [11]. Similarly, we may expand the field operator of the scalar field \( \sigma \) in terms of the creation and annihilation operators, at time \( t = 0 \) as

\[ \sigma(x, 0) = \frac{1}{(2\pi)^{3/2}} \int \frac{dk}{\sqrt{2\omega(k)}} \left( a(k) + a^\dagger(-k) \right) e^{ik \cdot x}, \]  

(5)

In the above, \( \omega(k) = \sqrt{k^2 + m_\sigma^2} \). The perturbative vacuum is defined corresponding to this basis through \( a \mid \text{vac} \rangle = 0 = c_{I r} \mid \text{vac} \rangle = \tilde{c}_{I r}^\dagger \mid \text{vac} \rangle \).

To include the vacuum polarisation effects for hot nuclear matter, we shall now consider a trial state with baryon–antibaryon and scalar meson condensates and then generalize the same to the finite temperatures and densities [11]. We thus explicitly take the ansatz for the trial state as

\[ |F\rangle = U_\sigma U_F |\text{vac}\rangle, \]  

(6)

with

\[ U_F = \exp \left[ \int dk \ f(k) \ c_{I r}^\dagger(k) \ a_{rs} \tilde{c}_{I s}(-k) - h.c. \right] \]  

(7)
Here $a_{rs} = \sigma_I \cdot \hat{k}) v_I s$ and $f(k)$ is a trial function associated with baryon-antibaryon condensates. For the scalar meson sector, $U_\sigma = U_{II} U_I$ where $U_i = \exp(B_i^\dagger - B_i)$, ($i = I, II$).

Explicitly the $B_i$ are given as

$$B_i^\dagger = \int d\mathbf{k} \sqrt{\omega(k)/2} f_\sigma(k) a^\dagger(k), \quad B_{II}^\dagger = \frac{1}{2} \int d\mathbf{k} g(k) a'^\dagger(k) a'^\dagger(-k).$$

In the above, $a'(k) = U_I a(k) U_I^{-1} = a(k) - \sqrt{\omega(k)/2} f_\sigma(k)$ corresponds to a shifted field operator associated with the coherent state $|\Psi\rangle$ and satisfies the usual quantum algebra. Further, to preserve translational invariance $f_\sigma(k)$ has to be proportional to $\delta(k)$ and we take $f_\sigma(k) = \sigma_0 (2\pi)^{3/2} \delta(k)$. $\sigma_0$ corresponds to a classical field of the conventional approach [11]. Clearly, the ansatz state is not annihilated by the operators, $c$, $\tilde{c}^\dagger$ and $a$. However, one can define operators, $d$, $\tilde{d}^\dagger$ and $b$, related through a Bogoliubov transformation to these operators, which will annihilate the state $|F\rangle$.

We next use the method of thermofield dynamics [14] to construct the ground state for nuclear matter at finite temperature. Here the statistical average of an operator is written as an expectation value with respect to a ‘thermal vacuum’ constructed from operators defined on an extended Hilbert space. The ‘thermal vacuum’ is obtained from the zero temperature ground state through a thermal Bogoliubov transformation. We thus generalise the state, as given by (6) to finite temperature and density as [11]

$$|F, \beta\rangle = U_\sigma(\beta) U_F(\beta)|F\rangle.$$  

The temperature-dependent unitary operators $U_\sigma(\beta)$ and $U_F(\beta)$ are given as [14]

$$U_\sigma(\beta) = \exp \left( \frac{1}{2} \int d\mathbf{k} \theta_\sigma(k, \beta) b^\dagger(k) b(\beta) - h.c. \right).$$

and

$$U_F(\beta) = \exp \left( \int d\mathbf{k} \left[ \theta_-(k, \beta) \tilde{d}^\dagger_{Ir}(k) \tilde{d}_{Ir}(-k) + \theta_+(k, \beta) \tilde{d}_{Ir}(k) \tilde{d}^\dagger_{Ir}(-k) \right] - h.c. \right).$$

The underlined operators are the operators corresponding to the doubling of the Hilbert space that arise in thermofield dynamics method. We shall determine the condensate func-
tions $f(k)$ and $g(k)$, and the functions $\theta_\sigma(k, \beta)$, $\theta_-(k, \beta)$ and $\theta_+(k, \beta)$ of the thermal vacuum through minimisation of the thermodynamic potential. The thermodynamic potential is given as

$$\Omega \equiv -p = \epsilon - \frac{1}{\beta} S - \mu \rho_B,$$

where $\epsilon$ and $S$ are the energy- and entropy-densities of the thermal vacuum, and $\rho_B$ is the baryon density. On evaluation, the energy density of the thermal vacuum is given as,

$$\epsilon \equiv \langle H \rangle_\beta = \epsilon_N + \epsilon_\omega + \epsilon_\sigma,$$

with

$$\epsilon_N = -\frac{\gamma}{(2\pi)^3} \int dk \left[ \epsilon(k) \cos 2f(k) + \frac{g_\sigma \sigma_0}{\epsilon(k)} \left( M \cos 2f(k) + |k| \sin 2f(k) \right) \right] (\cos^2 \theta_+ - \sin^2 \theta_-),$$

$$\epsilon_\omega = g_\omega \omega_0 \frac{\gamma(2\pi)^3}{3} \int dk (\cos^2 \theta_+ + \sin^2 \theta_-) - \frac{1}{2} m_\omega^2 \omega_0^2,$$

and

$$\epsilon_\sigma = \frac{1}{2} \frac{1}{(2\pi)^3} \int \frac{dk}{2 \omega(k)} \left[ k^2 (\sinh 2g + \cosh 2g) + \omega^2(k) (\cosh 2g - \sinh 2g) \right] \cosh 2\theta_\sigma(k, \beta)$$

$$+ \frac{1}{2} m_\sigma^2 I(\beta) + 6 \lambda \sigma_0^2 I(\beta) + 3 \lambda I(\beta)^2 + \frac{1}{2} m_\sigma^2 \sigma_0^2 + \lambda \sigma_0^4,$$

with

$$I(\beta) = \frac{1}{(2\pi)^3} \int \frac{dk}{2 \omega(k)} (\cosh 2g + \sinh 2g) \cosh 2\theta_\sigma(k, \beta).$$

The entropy density

$$S = -\frac{\gamma}{(2\pi)^3} \int dk \left[ \sin^2 \theta_- \ln(\sin^2 \theta_-) + \cos^2 \theta_- \ln(\cos^2 \theta_-) \right]$$

$$+ \sin^2 \theta_+ \ln(\sin^2 \theta_+) + \cos^2 \theta_+ \ln(\cos^2 \theta_+)$$

$$+ (2\pi)^3 \int dk \left[ \cosh^2 \theta_\sigma \ln(\cosh^2 \theta_\sigma) - \sinh^2 \theta_\sigma \ln(\sinh^2 \theta_\sigma) \right] + S_\omega$$

and the baryon density
\[ \rho_B = \gamma (2\pi)^{-3} \int d\mathbf{k} (\cos^2 \theta_+ + \sin^2 \theta_-). \]  

(17)

In the above, \( \gamma \) is the spin isospin degeneracy factor and is equal to 4 for nuclear matter. Further, \( S_\omega \) is the contribution to the entropy density from \( \omega \)-meson. Extremising the thermodynamic potential, \( \Omega \) with respect to the condensate function \( f(\mathbf{k}) \) and the functions \( \theta_\mp \) corresponding to the nucleon sector yields

\[ \tan 2f(\mathbf{k}) = \frac{g_\sigma \sigma_0 |\mathbf{k}|}{\epsilon(\mathbf{k})^2 + Mg_\sigma \sigma_0} \]

(18)

and

\[ \sin^2 \theta_\mp = \frac{1}{\exp(\beta(\epsilon^*(\mathbf{k}) \mp \mu^*)) + 1} \]

(19)

with \( \epsilon^*(\mathbf{k}) = (k^2 + M^{*2})^{1/2} \) and \( \mu^* = \mu - g_\omega \omega_0 \) as the effective energy and effective chemical potential, where the effective nucleon mass \( M^* = M + g_\sigma \sigma_0 \).

For the sigma meson sector, on extremising the thermodynamic potential, the functions \( g(\mathbf{k}) \) and \( \theta_\sigma \) are obtained as

\[ \tanh 2g(\mathbf{k}) = -\frac{6\lambda I(\beta) + 6\lambda \sigma_0^2}{\omega(\mathbf{k})^2 + 6\lambda I(\beta) + 6\lambda \sigma_0^2} \]

(20)

and

\[ \sinh^2 \theta_\sigma = \frac{1}{e^{\beta \omega'(\mathbf{k}, \beta)} - 1}; \quad \omega'(\mathbf{k}, \beta) = (\mathbf{k}^2 + M_\sigma(\beta)^2)^{1/2}. \]

(21)

In the above, the effective scalar meson mass is given as

\[ M_\sigma(\beta)^2 = m_\sigma^2 + 12\lambda I(\beta) + 12\lambda \sigma_0^2 \]

(22)

with

\[ I(\beta) = \frac{1}{(2\pi)^3} \int d\mathbf{k} \frac{1}{2 \left( \mathbf{k}^2 + M_\sigma(\beta)^2 \right)^{1/2}} \]

(23)

It is clear from the equation (20) that in the absence of a quartic coupling no such condensates are favoured since the condensate function, \( g(\mathbf{k}) \) vanishes for \( \lambda = 0 \).
Then the expression for the energy density becomes

\[ \epsilon = \epsilon_N + \epsilon_\omega + \epsilon_\sigma, \]  

(24)

with

\[ \epsilon_N = \gamma (2\pi)^{-3} \int dk (k^2 + M^*^2)^{1/2} (\sin^2 \theta_- - \cos^2 \theta_+), \]  

(25a)

\[ \epsilon_\omega = g_\omega \omega_0 \gamma (2\pi)^{-3} \int dk (\sin^2 \theta_- + \cos^2 \theta_+) - \frac{1}{2} m_\omega^2 \omega_0^2, \]  

(25b)

\[ \epsilon_\sigma = \frac{1}{2} m_\sigma^2 \sigma_0^2 + \lambda \sigma_0^4 + \frac{1}{2} m_\omega^2 \omega_0^2 \int dk (k^2 + M_\sigma (\beta)^2)^{1/2} \cosh 2\theta_\sigma - 3\lambda I(\beta)^2. \]  

(25c)

After subtracting out the vacuum contributions ($\theta_\pm = 0, f=0$ part) for the nucleon sector, one obtains,

\[ \Delta \epsilon_N = \epsilon^N_{finite} + \Delta \epsilon, \]  

(26)

where,

\[ \epsilon^N_{finite} = \gamma (2\pi)^{-3} \int dk (k^2 + M^*^2)^{1/2} (\sin^2 \theta_- + \sin^2 \theta_+) \]  

(27)

and

\[ \Delta \epsilon = -\gamma (2\pi)^{-3} \int dk \left[ (k^2 + M^*)^{1/2} - (k^2 + M^2)^{1/2} \right], \]  

(28)

The contribution arising from the Dirac sea effect given by (28), is identical to that of summing over the baryonic tadpole diagrams of RHA, before renormalisation. This is renormalised by adding the counter terms, as for the zero temperature situation, given as

\[ \epsilon_{ct} = \sum_{n=1}^{4} C_n \sigma_0^n, \]  

(29)

since finite temperature does not introduce any fresh divergences.

For the $\omega$-sector, the pure vacuum contribution to $\rho_B$ is subtracted out, which amounts to a normal ordering of the number operator $\psi^\dagger \psi$. This yields the usual energy density for the $\omega$-sector,
\[ \epsilon_\omega = g_\omega \omega_0 \rho_B^{ren} - \frac{1}{2} m_\omega^2 \omega_0^2, \]  

(30)

with

\[ \rho_B^{ren} = \gamma (2\pi)^{-3} \int d\mathbf{k} (\sin^2 \theta_- - \sin^2 \theta_+). \]  

(31)

For the \( \sigma \) sector, since there are no additional divergences arising from finite temperatures, we adopt the same renormalization procedure as in Ref. [11]. This yields the gap equation for field dependent effective sigma mass, \( M_\sigma(\beta) \), in terms of the renormalised parameters as

\[ M_\sigma(\beta)^2 = m_R^2 + 12 \lambda_R \sigma_0^2 + 12 \lambda_R I_f(M_\sigma(\beta)), \]  

(32)

where

\[ I_f(M_\sigma(\beta)) = \frac{M_\sigma(\beta)^2}{16\pi^2} \ln \left( \frac{M_\sigma(\beta)^2}{m_R^2} \right) + \frac{1}{(2\pi)^3} \int d\mathbf{k} \frac{\sinh^2 \theta_\sigma(\mathbf{k}, \beta)}{(k^2 + M_\sigma(\beta)^2)^{1/2}}, \]  

(33)

Simplifying equation (25c) and subtracting the vacuum contribution, we obtain the energy density for the \( \sigma \),

\[ \Delta \epsilon_\sigma = \frac{1}{2} m_R^2 \sigma_0^2 + 3 \lambda_R \sigma_0^2 + \frac{M_\sigma^4}{64\pi^2} \left( \ln \left( \frac{M_\sigma^2}{m_R^2} \right) - \frac{1}{2} \right) - 3 \lambda_R I_f^2 \]

\[ - \frac{M_{\sigma,0}^4}{64\pi^2} \left( \ln \left( \frac{M_{\sigma,0}^2}{m_R^2} \right) - \frac{1}{2} \right) + 3 \lambda_R I_{f0}^2, \]  

(34)

where \( M_{\sigma,0} \) and \( I_{f0} \) are the expressions as given by eqs. (32) and (33) with \( \sigma_0 = 0 \). We might note here that the gap equation given by (32) is identical to that obtained through resumming the daisy and superdaisy graphs [15] and hence includes higher order corrections from the scalar meson field.

In the absence of the quartic interaction (\( \lambda_R = 0 \)), equation (34) reduces to

\[ \Delta \epsilon_\sigma = \frac{1}{2} m_R^2 \sigma_0^2, \]  

(35)

which refers to the RHA. Also, we note that the sign of \( \lambda_R \) must be chosen to be positive, because otherwise the energy density would become unbounded from below with vacuum fluctuations [16, 18].
One then obtains the renormalised energy density as

$$\epsilon_{\text{ren}} = \epsilon_{\text{finite}}^{(N)} + \Delta\epsilon_{\text{ren}} + \epsilon_\omega + \Delta\epsilon_\sigma,$$

(36)

with,

$$\Delta\epsilon_{\text{ren}} = -\frac{\gamma}{16\pi^2}(M^{*4}\ln\left(\frac{M^*}{M}\right) + M^3(M - M^*) - \frac{7}{2}M^2(M - M^*)^2$$

$$+ \frac{13}{3}M(M - M^*)^3 - \frac{25}{12}(M - M^*)^4)$$

(37)

as the contribution from the Dirac sea. The thermodynamic potential, $\Omega$, given by equation (12), after subtracting out the vacuum contributions, is now finite and is given in terms of the meson fields, $\sigma_0$ and $\omega_0$.

Extremisation of the thermodynamic potential with respect to the meson fields $\sigma_0$ and $\omega_0$ give the self–consistency conditions for $\sigma_0$ (and hence for the effective nucleon mass, $M^* = M + g_\sigma\sigma_0$), as

$$\frac{d(\Delta\epsilon_\sigma)}{d\sigma_0} + \frac{\gamma}{(2\pi)^3}g_\sigma\int\frac{dk}{(k^2 + M^{*2})^{1/2}}(\sin^2\theta_- + \sin^2\theta_+) + \frac{d(\Delta\epsilon_{\text{ren}})}{d\sigma_0} = 0$$

(38a)

and, for the vector meson field, $\omega_0$, as

$$\omega_0 = \frac{g_\omega}{m_\omega^2}\frac{\gamma}{(2\pi)^3}\int d\epsilon(\sin^2\theta_- - \sin^2\theta_+)$$

(38b)

where $\sin^2\theta_\pm$ as the thermal distribution functions for the baryons and antibaryons, given through equation (13).

### III. RESULTS AND DISCUSSIONS

We now proceed with the finite temperature calculations for the nuclear matter. The values for $C_\sigma^2 = g_\sigma^2M^2/m_\sigma^2$ and $C_\omega^2 = g_\omega^2M^2/m_\omega^2$, are given as $C_\sigma^2=183.3,167.5,137.9$ and $C_\omega^2=114.7,96.45,63.7$, for the RHA and with quantum corrections from sigma meson, for $\lambda_R=1.8$ and 5 respectively [11]. These values were fitted from the nuclear matter saturation properties as $\rho_0 = 0.193fm^{-3}$ and binding energy as $-15.75$ MeV. For given values of the
temperature and the baryon chemical potential, $\mu$, we calculate the different thermodynamic quantities, with the meson fields determined self consistently from equation (38).

We plot the pressure as a function of the baryon density, $\rho_B$ for low temperatures to examine the liquid gas phase transition, in figures 1 and 2, for the RHA and $\lambda_R=1.8$ respectively. At zero temperature, the pressure decreases with density, reaches a minimum, then increases and passes through $p = 0$ at $\rho = \rho_0$, where the binding energy per nucleon is a minimum. The negative pressure indicates a mechanical instability of the uniform nuclear matter of saturation density, $\rho_0$. This has however an interesting physical interpretation. For the densities below the saturation density, the uniform nuclear matter is unstable and will break up into regions of nuclear matter with density, $\rho = \rho_0$ and zero pressure, surrounded by regions of vacuum with $\rho = 0$ and $p = 0$ [19]. This pocket in the pressure versus density curve, disappears at the liquid gas phase transition point. For RHA, this transition appears to occur at around 23 MeV. Inclusion of sigma condensates reduces this critical temperature, $T_c$ to 22 MeV for $\lambda_R=1.8$ and to 21 MeV for $\lambda_R=5$.

For higher temperatures (beyond the liquid gas phase transition), we plot the EOS for RHA and for $\lambda_R=1.8$ in figures 3 and 4 respectively. These plots show a softening of the EOS due to quantum effects arising from the scalar meson sector. Such a softening of the EOS with inclusion of the quantum corrections was already observed for the zero temperature situation [11], which had given rise to a lower value for the incompressibility. For a given $\rho_B$, the pressure has the usual trend of increasing with temperature [2]. The pressure for $\rho_B = 0$ is seen to be already nonzero and appreciable at around a temperature of 150 MeV for RHA and 200 MeV for $\lambda_R=1.8$. This has contributions arising from the thermal distributions of baryons and antibaryons, as well as from a nonzero value for the sigma field.

The magnitude of the scalar meson field, $\sigma$, as obtained through the self-consistency condition (38a), is plotted as a function of the baryon density for various temperatures in figure 5 for $\lambda_R=1.8$. It may be noted that for $\rho_B=0$, $\sigma$ field becomes nonzero at a temperature of around 160 MeV due to thermal effects, and is observed to have attained an appreciable value at 200 MeV due to contributions from the thermal distribution functions.
The sigma field attaining a nonzero value was also observed for nuclear matter in the mean field approximation in Walecka model [2], which had led to a sharp fall in the effective nucleon mass between 150 MeV and 200 MeV. The rapid fall of $M^*_N$ with increasing temperature, was ascribed to resemble the situation, when the system becomes a dilute gas of baryons in a sea of baryon-antibaryon pairs.

The density and temperature dependent nucleon mass is plotted in figures 6 and 7, for RHA and $\lambda_R$=1.8 respectively. The quantum effects are seen to increase the effective nucleon mass. It may be noted that the in-medium baryon mass increases with larger value of the quartic coupling. Hence, with inclusion of quantum effects, the rapid fall in effective nucleon mass as observed in the mean field calculations [2] is not seen in the present calculation. The nucleon mass here becomes about half of its vacuum value at $T$=250 MeV, as compared to $M^*/M \approx 0.2$ in Ref. [2] at $T$=200 MeV.

The dependence of the entropy density on the temperature and density for $\lambda_R$=1.8 is shown in figure 8. This becomes nonzero for zero baryon density at around a temperature of 160 MeV, with contributions from the nonzero value for the sigma field. However, the increase with temperature, for $\rho_B$=0, is rather gradual and need not be associated with a phase transition. A similar behaviour of entropy density becoming nonzero at higher temperatures, for $\rho_B$=0, was also observed in the Walecka model mean field calculations [2] as well as in the QMC model [20].

Finally, the inclusion of quantum corrections through scalar meson condensates enables us to determine the in-medium sigma meson mass in a self–consistent manner, which we plot in figure 9 for $\lambda_R$=1.8. An increase in the quartic coupling, $\lambda_R$ increases the value for the effective sigma mass [11].

**IV. SUMMARY**

To summarize, in the present work, we have studied the hot nuclear matter taking into account the vacuum polarisation effects within a nonperturbative variational calculation.
The approximation here lies in the ansatz for the ground state. A realignment of the ground state with baryon-antibaryon condensates takes into account the summing over baryonic tadpole diagrams. Generalization of the vacuum with mesons gives rise to summing over multi-loop diagrams. We could have generalized the ansatz to include vector meson ($\omega$)-condensates in a similar way. However, such an ansatz with the present Lagrangian would lead to a trivial solution for the condensate function (thus giving us the mean field result). However, with a quartic term $[21]$ for the $\omega$ meson, the present ansatz for the ground state would give rise to a nontrivial solution for the corresponding gap equation. Including such a term, it would be interesting to study the quantum effects arising from the vector meson.

We would also like to mention that the thermal distribution and the condensate functions are obtained here through a minimisation of the thermodynamic potential which included interactions with the corresponding effective masses for the $\sigma$-meson and nucleon determined self-consistently through the equations (12) and (38a) at a given temperature.

We have looked into the liquid-gas phase transition, and observed that the critical temperature for the phase transition is lowered due to quantum correction effects from the scalar meson sector. At high temperatures, with quantum effects, the effective nucleon mass does not decrease as rapidly as in the mean field results $[2]$. With increase in temperature, the decrease of nucleon mass as well as the increase in the entropy density appear to be rather gradual, i.e., without any sudden change and is not indicative of any (chiral or otherwise) phase transition.

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FIG. 1. Pressure versus the baryon density for RHA. The disappearance of the pocket at temperature of around 23 MeV is indicative of a liquid-gas phase transition.

FIG. 2. Pressure versus the baryon density for $\lambda_R = 1.8$. Quantum corrections due to scalar meson sector gives rise to a smaller value of $T_c$ of 22 MeV as compared to that of RHA.
FIG. 3. The equation of state for hot nuclear matter for the relativistic Hartree approximation.

FIG. 4. The equation of state for $\lambda_R = 1.8$. The quantum correction from scalar mesons gives a softer equation of state.
FIG. 5. The meson field strengths as functions of baryon density.

FIG. 6. Effective baryon masses in the medium for RHA.
FIG. 7. Effective baryon masses in the medium for \( \lambda_R = 1.8 \). The in-medium baryon mass increases with quantum correction effects.

FIG. 8. Entropy density versus baryon density for \( \lambda_R = 1.8 \).
FIG. 9. In medium scalar meson mass versus baryon density for $\lambda_R = 1.8$. 