Probing Pseudo-Dirac Neutrino through Detection of Neutrino
Induced Muons from GRB Neutrinos

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Abstract

The possibility to verify the pseudo-Dirac nature of neutrinos is investigated here via the detection of ultra high energy neutrinos from distant cosmological objects like GRBs. The very long baseline and the energy range from $\sim$ TeV to $\sim$ EeV for such neutrinos invokes the likelihood to probe very small pseudo-Dirac splittings. The expected secondary muons from such neutrinos that can be detected by a kilometer scale detector such as ICERCUBE is calculated. The pseudo-Dirac nature, if exists, will show a considerable departure from flavour oscillation scenario in the total yield of the secondary muons induced by such neutrinos.
1 Introduction

Evidence has been obtained from the satellite-borne observations, the existence of the Gamma Ray Bursts (GRB) from extra galactic (or galactic) sources characterised by sudden intense flashes of $\gamma$-rays. The GRBs can produce very high energy ($\gtrsim 10^{16}$ eV) cosmic rays. Although the mechanism of such GRBs are not fully understood but various model calculations suggest that neutrinos of that energy range are also produced in GRBs and should be detected by very large detectors like a kilometer cube water cherenkov detector (e.g. ICECUBE at south pole). Because of their astronomically long baseline ($\gtrsim$ Mega Parsec) these ultra high energy (UHE) neutrinos may open up a new window in very small mass square difference ($\Delta m^2$) regime and may throw more insight to neutrino physics. A viable way to detect such UHE neutrinos is to look for upward going secondary muons produced by the charged current (CC) interaction of such UHE $\nu_\mu$ with the terrestrial rock. Already installed AMANDA in the south pole and the future ICECUBE [1] detector, are able to detect such secondary muons. Both are water Cerenkov detectors and use south pole ice as the detector material.

The possibility that the UHE neutrinos from a distant GRB can probe very low $\Delta m^2$ region much below the solar or atmospheric neutrino regime, can be utilised in investigating the proposed pseudo-Dirac nature of the neutrinos [2, 3] through its detection in large ($\sim$ Km$^2$) detectors [4] like ICECUBE. In the pseudo-Dirac scenario, each active neutrino is split into two almost degenerate components of active and sterile part. This can be theoretically realised from the generic mass matrix in $[\nu_L, (\nu_R)^C]$ basis which can be written as (for one generation)

$$\begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix},$$

where the tiny Majorana mass terms $m_L$ and $m_R$ ($m_L, m_R << m_D$) are introduced to slightly lift the degeneracy of Dirac mass $m_D$. Thus the Dirac
neutrino is split into a pair of almost degenerate Majorana neutrinos with nearly maximum mixing angle given by \( \tan 2\theta = \frac{2m_D}{m_R - m_L} \). In this scenario, therefore, each of the three neutrino species is split up into a pair of neutrinos with very tiny mass square difference \( \Delta m^2 = 2m_D(m_L + m_R) \). For three generation scenario, therefore each of the three types of neutrinos has this pseudo-Dirac splitting of very small mass square difference.

The purpose of this work is to demonstrate the possibility of probing the pseudo-Dirac nature by studying the neutrino oscillation effects through the detection yield of neutrino induced muons in a large (Km^2) neutrino detector such as ICECUBE. For this purpose, the expected muon signal induced by neutrinos from a GRB at Mpc distance is calculated by separately folding (i) the flavour oscillation effects and (ii) the oscillation effects in pseudo-Dirac scenario to the GRB neutrino flux.

2 GRB flux, neutrino oscillations and number of secondary muons

The GRB neutrino flux is estimated considering the the relativistic fireball model [5]. In the relativistic expanding fireball model of GRB, protons (also electrons, positrons and photons) produced in the magnetic field of the rotating accretion disc around a possible black hole are accelerated perpendicular to the accretion disc at almost the speed of light forming a jet which is referred to as fireball. The burst is supposed to be the dissipation of kinetic energy of this relativistic expanding fireball. These very highly energetic protons in the jet then interact with the photons and produce pions (cosmological beam dump) through the process of \( \Delta \) resonance. (The pions are also produced through the \( pp \) process). These pions then decay to yield \( \nu_\mu \) and \( \nu_e \) in the approximate proportion of 2:1.

The neutrino flux from a GRB depends on several GRB parameters. Firstly, the Lorentz boost factor \( \Gamma \) is required for the transformation from
the fireball blob to observer’s frame of reference. As the shocked protons in
the blob photoproduce pions, the photon break energy (as the photon spec-
trum is considered broken) and the photon luminosity $L_\gamma$ (generally $\sim 10^{53}$
ergs/sec) are important parameters for determining the neutrino spectrum.

With all these, the neutrino spectrum from a GRB can be parametrised
as [6, 7]

$$\frac{dN_\nu}{dE_\nu} = A \times \min(1, E_\nu/E_\nu^b) \times \frac{1}{E_\nu^b}. \quad (1)$$

In the above, $E_\nu$ is the neutrino energy, $N_\nu$ is the number of neutrinos and

$$E_\nu^b \simeq 10^6 \frac{\Gamma_2^{2.5}}{E_\gamma^{0.3} \text{MeV}} \text{GeV}$$

$$\Gamma_2 = \Gamma/10^{2.5}$$

$$A = \frac{E_{\text{GRB}}}{1 + \ln(E_{\nu_{\text{max}}}/E_\nu^b)} \quad (2)$$

where $E_{\nu_{\text{max}}}$ is the cut-off energy for the GRB neutrinos and $E_{\text{GRB}}$ is the total
energy that a GRB emits. Now, the observed energy $E_\nu^{\text{obs}}$ of a neutrino with
the actual energy $E_\nu$ coming from a GRB at a redshift distance $z$ is given
by the relation $E_\nu^{\text{obs}} = E_\nu/(1 + z)$ and similarly, the maximum observable
neutrino energy $E_{\nu_{\text{max}}}^{\text{obs}}$ is $E_{\nu_{\text{max}}}^{\text{obs}} = E_{\nu_{\text{max}}}/(1 + z)$. The comoving distance $d$
of a GRB at redshift $z$ is given by

$$d(z) = \frac{c}{H} \int_0^z \frac{dz'}{\sqrt{\Omega_\Lambda + \Omega_M((1 + z')^3}} \quad (3)$$

where $\Omega_M$ is the matter density (both luminous and dark), $\Omega_\Lambda$ is the dark
energy density respectively in units of critical density of the universe and $c, H$
are the velocity of light in vacuum and Hubble constant respectively. In the
present calculation $c = 3 \times 10^5$ Km/sec and $H = 72$ Km/sec/Mpc (1 Mpc
= 3.086 $\times 10^{19}$ Km). Therefore the neutrinos from a single GRB that can be
observed on earth per unit energy per unit area of the earth is given by,

$$\frac{dN_\nu^{\text{obs}}}{dE_\nu^{\text{obs}}} = \frac{dN_\nu}{dE_\nu} \frac{1}{4\pi d^2(z)}(1 + z) \quad (4)$$
Here, as is evident from the above, the total flux per unit area per unit energy from a single GRB is being considered rather than the flux per unit time.

The production process of UHE neutrinos suggests that the neutrino flavours are produced in the ratio $\nu_e : \nu_\mu : \nu_\tau = 1 : 2 : 0$. Considering the maximal mixing between $\nu_\mu$ and $\nu_\tau$ (as indicated by the atmospheric neutrino data) and the element $U_{e3}$ of the mass-flavour mixing matrix to be zero, the flavour ratio on reaching the earth, for neutrino mass-flavour oscillation, becomes $\nu_e : \nu_\mu : \nu_\tau = 1 : 1 : 1$ irrespective of the solar mixing angle. Needless to say, because of the astronomical baseline ($L \sim \text{Mpc}$), the acquired relative phases of the propagating neutrino mass eigenstates is averaged out ($\Delta m^2 L/E >> 1$) and the UHE neutrinos from a GRB reaching the earth are incoherent mixture of mass eigenstates. Therefore, the probability for measuring a particular flavour $\beta$ by a terrestrial neutrino telescope, if only flavour oscillation is considered, is $P_\beta = 1/3$.

On the other hand, in pseudo-Dirac scenario, we have each of the three mass eigenstates $\nu_1$, $\nu_2$ and $\nu_3$ to be nearly degenerate pairs and thus one obtains a total of six mass eigenstates. Kobayashi and Lim [3] worked out the mixing in such scenario and calculated the oscillation probability. Following [3] and Beacom [4] the probability $P_\beta$ to detect a flavour $\beta$ by a neutrino telescope for pseudo-Dirac neutrinos is

$$P_\beta = \sum_\alpha w_\alpha \sum_{j=1}^3 |U_{\alpha j}|^2 |U_{\beta j}|^2 \left[ 1 - \sin^2 \left( \frac{\Delta m^2_{jj} L}{4E} \right) \right],$$

(5)

where $m_j(j = 1, 3)$ denotes the mass eigenstates for the three types of neutrinos, $\Delta m^2_j$ is the mass square difference due to pseudo-Dirac splitting of the mass eigenstate $\nu_j$. $\alpha, \beta, \ldots$ denote the flavour index and $U_{\alpha j}$ is the CKM matrix for three generation mass to flavour mixing. $w_\alpha$ is the relative flux of each of the neutrino flavours ($\alpha$) at the production point ($\sum_\alpha w_\alpha = 1$).

The total number of secondary muons induced by GRB neutrinos at a
detector of unit area is given by (following [8, 9, 6])

\[ S = \int_{E_{\text{thr}}}^{E_{\text{obs max}}} dE_{\nu} \frac{dN_{\nu}^{\text{obs}}}{dE_{\nu}^{\text{obs}}} P_{\text{surv}}(E_{\nu}^{\text{obs}}, \theta_z) P_{\mu}(E_{\nu}^{\text{obs}}, E_{\text{thr}}) \]  

(6)

In the above, \( P_{\text{surv}} \) is the probability that a neutrino reaches the detector without being absorbed by the earth. This is a function of the neutrino-nucleon interaction length in the earth and the effective path length \( X(\theta_z) \) (gm cm\(^{-2} \)) for incident neutrino zenith angle \( \theta_z \) (\( \theta_z = 0 \) for vertically downward entry with respect to the detector). The interaction length \( L_{\text{int}} \) is given by

\[ L_{\text{int}} = \frac{1}{\sigma_{\text{tot}}(E_{\nu}^{\text{obs}}) N_A} \]  

(7)

and

\[ P_{\text{surv}}(E_{\nu}^{\text{obs}}, \theta_z) = \exp[-X(\theta_z)/L_{\text{int}}] = \exp[-X(\theta_z)\sigma_{\text{tot}} N_A]. \]  

(8)

where \( N_A(= 6.022 \times 10^{23} \text{gm}^{-1}) \) is the Avogadro number and \( \sigma_{\text{tot}}(= \sigma_{\text{CC}} + \sigma_{\text{NC}}) \) is the total cross section. The effective path length \( X(\theta_z) \) is calculated as

\[ X(\theta_z) = \int \rho(r(\theta_z, \ell)) d\ell. \]  

(9)

In Eq. (9), \( \rho(r(\theta_z, \ell)) \) is the matter density inside the earth at a distance \( r \) from the centre of the earth for neutrino path length \( \ell \) entering into the earth with a zenith angle \( \theta_z \). The quantity \( P_{\mu}(E_{\nu}^{\text{obs}}, E_{\text{thr}}) \) in Eq. (6) is the probability that a secondary muon is produced by CC interaction of \( \nu_{\mu} \) and reach the detector above the threshold energy \( E_{\text{thr}} \). This is then a function of \( \nu_{\mu} N \) (\( N \) represents nucleon) - CC interaction cross section \( \sigma_{\text{CC}} \) and the range of the muon inside the rock.

\[ P_{\mu}(E_{\nu}^{\text{obs}}, E_{\text{thr}}) = N_A \sigma_{\text{CC}} \langle R(E_{\nu}^{\text{obs}}, E_{\text{thr}}) \rangle \]  

(10)

In the above \( \langle R(E_{\nu}^{\text{obs}}, E_{\text{thr}}) \rangle \) is the average muon range given by

\[ \langle R(E_{\nu}^{\text{obs}}, E_{\text{thr}}) \rangle = \frac{1}{\sigma_{\text{CC}}} \int_0^{1-E_{\text{thr}}/E_{\nu}} dy R(E_{\nu}^{\text{obs}}(1-y), E_{\text{thr}}) \frac{d\sigma_{\text{CC}}(E_{\nu}^{\text{obs}}, y)}{dy} \]  

(11)
where \( y = (E_{\nu}^{\text{obs}} - E_{\mu}) / E_{\nu}^{\text{obs}} \) is the fraction of energy loss by a neutrino of energy \( E_{\nu}^{\text{obs}} \) in the charged current production of a secondary muon of energy \( E_{\mu} \). Needless to say that a muon thus produced from a neutrino with energy \( E_{\nu} \) can have the detectable energy range between \( E_{\text{thr}} \) and \( E_{\nu} \). The range \( R(E_{\mu}, E_{\text{thr}}) \) for a muon of energy \( E_{\mu} \) is given as

\[
R(E_{\mu}, E_{\text{thr}}) = \int_{E_{\text{thr}}}^{E_{\mu}} \frac{dE_{\mu}}{\langle dE_{\mu} / dX \rangle} \approx \frac{1}{\beta} \ln \left( \frac{\alpha + \beta E_{\mu}}{\alpha} \right) \tag{12}
\]

The average lepton energy loss with energy \( E_{\mu} \) per unit distance travelled is given by [8]

\[
\langle \frac{dE_{\mu}}{dX} \rangle = -\alpha - \beta E_{\mu} \tag{13}
\]

The values of \( \alpha \) and \( \beta \) used in the present calculations are

\[
\begin{align*}
\alpha &= \{2.033 + 0.077 \ln[E_{\mu}(\text{GeV})]\} \times 10^{-3} \text{GeVcm}^2\text{gm}^{-1} \\
\beta &= \{2.033 + 0.077 \ln[E_{\mu}(\text{GeV})]\} \times 10^{-6} \text{cm}^2\text{gm}^{-1} \tag{14}
\end{align*}
\]

for \( E_{\mu} \lesssim 10^6 \text{ GeV} \) [10] and

\[
\begin{align*}
\alpha &= 2.033 \times 10^{-3} \text{GeVcm}^2\text{gm}^{-1} \\
\beta &= 3.9 \times 10^{-6} \text{cm}^2\text{gm}^{-1} \tag{15}
\end{align*}
\]

otherwise [11].

## 3 Calculations and results

The GRB neutrino flux is calculated for a GRB with energy \( E_{\text{GRB}} = 10^{53} \text{ ergs} \). The neutrino break energy \( E_{\nu}^{b} \) is calculated following Eq. (2) with the Lorentz factor \( \Gamma = 50.12 \) and corresponding photon break energy \( E_{\gamma,\text{MeV}}^{b} = 0.794 \). These values are obtained from Guetta et al [12] from their fireball model framework calculations. These values are tabulated in Ref. [6]. The calculation is performed for several values of redshift \( (z) \). In this calculation all length units are taken in Km.
The probability $P_\beta$ in Eq. (5) is computed with solar mixing angle $\theta_\odot = 32.31^\circ$, the atmospheric mixing angle $\theta_{\text{atm}} = 45.0^\circ$ and 1-3 mixing angle $\theta_{13} = 0$. Also, the representative pseudo-Dirac splittings are considered as $\Delta m_j^2 = 10^{-12} \text{eV}^2$ for each of the three species. Note that, in this case, $P_\beta$ for mass-flavour oscillation is $1/3$. GRB neutrino flux with redshift $z$, per square kilometer on earth per unit energy (GeV) for a zenith angle $\theta_z$, is obtained using Eq. (1 -4) and then multiplied by the probability $P_\beta$.

The secondary muon yield at a kilometre scale detector such as ICECUBE is calculated using Eqs. (6 - 15). The earth matter density in Eq. (9) is taken from [9] following the Preliminary Earth Reference Model (PREM). The interaction cross-sections - both charged current and total - used in these equations are taken from the tabulated values (and the analytical form) given in Ref. [13]. In the present calculations $E_\nu^{\text{obs}}_{\text{max}} = 10^{11} \text{GeV}$ and threshold energy $E_{\text{thr}} = 1 \text{TeV}$ are considered.

Fig. 1 shows the total yield of secondary muons induced by UHE neutrinos from GRBs of different red shift values ranging from 0.02 to 3.8, for three cases namely (i) no neutrino oscillation (ii) mass-flavour oscillation and (iii) the pseudo-Dirac oscillation. The zenith angle is taken to be $\theta_z = 180^\circ$ (vertically upwards neutrino, i.e. muons coming from vertically below the detector). Fig. 2 is same as Fig. 1 but for $\theta_z = 100.9^\circ$. The pseudo-Dirac case distinctly differs from flavour oscillation and no oscillation scenario. In the present case considered here, the secondary muon events for pseudo-Dirac oscillation is almost half the yield from expected events if the UHE neutrinos suffers only mass-flavour oscillation and same is around six times less in case there is no oscillation. In Fig. 3, the secondary muon events in each bin of $E_\nu^{\text{obs}}$ are shown. The events below $10^{-15}$ are not shown in Fig. 3 for clarity.
4 conclusion

In this report, the possibility of probing pseudo-Dirac neutrino through ultra high energy neutrinos from distant GRBs is considered. It is demonstrated, that such UHE neutrinos from GRB are indeed capable of producing oscillation effects clearly distinct from flavour oscillation or no oscillation of such neutrinos. This can be realised by looking at the yield of secondary muons, produced by these neutrinos, at a kilometer scale detector such as ICECUBE at south pole. Detecting UHE neutrinos is also a viable means to probe as also verify the pseudo-Dirac splitting if it really exists.

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Figure Captions

Fig. 1 The comparison of total secondary muon yields in cases of (i) pseudo-Dirac oscillation (ii) Mass-Flavour oscillation and (iii) No oscillation for ultra high energy neutrinos from GRBs at different zshift \(z\) distance. The zenith angle \(\theta_z\) is 180°. See text for other GRB parameters considered.

Fig. 2 Same as Fig. 1 but for \(\theta_z = 100.9°\).

Fig. 3 The secondary muon events in each bin of \(E_\nu^{\text{obs}}\).
Fig. 1

- pseudo-Dirac Osc.
- Mass-Flavor Osc.
- No Osc.

Total muon yield vs. Redshift (z)
Fig. 2

Total muon yield vs. Redshift (z)

- Pseudo-Dirac Osc.
- Mass-Flavor Osc.
- No Osc.
Fig. 3

Events vs. Neutrino Energy (GeV)