Large and robust mechanical squeezing of optomechanical systems in a highly unresolved sideband regime via Duffing nonlinearity and intracavity squeezed light

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Abstract: We propose a scheme to generate strong and robust mechanical squeezing in an optomechanical system in the highly unresolved sideband (HURSB) regime with the help of the Duffing nonlinearity and intracavity squeezed light. The system is formed by a standard optomechanical system with the Duffing nonlinearity (mechanical nonlinearity) and a second-order nonlinear medium (optical nonlinearity). In the resolved sideband regime, the second-order nonlinear medium may play a destructive role in the generation of mechanical squeezing. However, it can significantly increase the mechanical squeezing (larger than 3dB) in the HURSB regime. Finally, we show the mechanical squeezing is robust against thermal fluctuations of the mechanical resonator. The generation of large and robust mechanical squeezing in the HURSB regime is a combined effect of the mechanical and optical nonlinearities.

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1. Introduction

Optomechanical systems has received a lot of attentions due to the wide range of applications such as highly sensitive measurement of tiny displacement and quantum information processing [1–7]. In the highly sensitive measurement of tiny displacement, quantum squeezing of mechanical mode is indispensable. In principle, quantum squeezing can be accomplished by the parametric interaction of a quantum system [8]. However, quantum squeezing in this scheme can not be larger than 3dB since a quantum system becomes unstable if the quantum squeezing is larger than 3dB as pointed out by Milburn and Walls [9].

In recent years, many schemes have been proposed to generate strong mechanical squeezing beyond the 3dB limit including continuous weak measurement and feedback [10–13], squeezed light [14, 15], quantum-reservoir engineering [16–24], strong intrinsic nonlinearity [25, 26], and frequency modulation [27]. For instance, large steady-state mechanical squeezing can be achieved by applying two driving lasers to a cavity in an optomechanical system [22]. In this scheme, the power of the red-detuned driving field should be larger than that of the blue-detuned driving field. This scheme was realized experimentally in 2015 [23]. Very recently, the authors of [24] have shown that larger mechanical squeezing can also be achieved with only one periodically amplitude-modulated external driving field. The Duffing nonlinearity of the mechanical mode can be used to generate strong mechanical squeezing beyond the 3dB limit [26]. In addition, the mechanical squeezing is robust against the thermal fluctuations of the mechanical resonator with the help of the Duffing nonlinearity.

It is worth noting that the above schemes [10–26] are not valid to realize larger mechanical squeezing beyond the 3dB limit in the HURSB regime with \( \kappa \gg \omega_m \). Here, \( \kappa \) is decay rate of the cavity and \( \omega_m \) is the frequency of the mechanical resonator. In order to generate large
mechanical squeezing beyond the resolved sideband regime, the authors of [27] suggested to use frequency modulation acting on both the cavity field and mechanical resonator. They have shown that mechanical squeezing beyond 3dB can be achieved in the presence of frequency modulation beyond the resolved sideband and weak-coupling limits. It was shown that the strong mechanical squeezing beyond 3dB in the unresolved sideband regime ($\kappa \approx 30\omega_m$) can also be achieved by adding two auxiliary cavities since the unwanted counter-rotating terms could be suppressed significantly with the help of quantum interference from the auxiliary cavities [28]. However, the decay rates of the auxiliary cavities must be much smaller than the frequency of the mechanical resonator in the above scheme. Later, we proposed a scheme to generate large mechanical squeezing beyond 3dB in the HURSB regime by adding two two-level atomic ensembles and two driving lasers with different amplitudes [29]. Very recently, it was shown that the quantum ground-state cooling of mechanical resonator in an optomechanical system can be accomplished using intracavity squeezed light produced by a second-order nonlinear medium in the optomechanical system [30–32].

In the present work, we propose a scheme to generate large and robust mechanical squeezing in the HURSB regime via the Duffing nonlinearity of the mechanical mode and a second-order nonlinear medium in the cavity. The mechanical squeezing of the mechanical resonator can be larger than 3dB and is robust against the thermal fluctuations of the mechanical resonator. This is a combined effect of nonlinearity-induced parametric amplification (Duffing nonlinearity) and quantum ground-state cooling of the optomechanical system (intracavity squeezed light). On the one hand, in the resolved sideband regime, the second-order nonlinear medium may decrease the quantum ground-state cooling of the optomechanical system (intracavity squeezed light). On the other hand, the second-order nonlinear medium can significantly increase the mechanical squeezing in the HURSB regime for realistic parameters.

2. Model and Hamiltonian

In the present work, we consider an optomechanical system formed by two mirrors. One mirror is fixed and partially transmitting. The other mirror is movable and perfectly reflecting. In addition, a second-order nonlinear medium $\chi^{(2)}$ is put into the Fabry-Perot cavity. The fundamental mode and second-order optical mode are represented by $a_1$ and $a_2$ with frequencies $\omega_c$ and $2\omega_c$. The decay rates of the two optical modes are $\kappa_1$ and $\kappa_2$. The movable mirror (mechanical oscillator) is denoted by $b$ with frequency $\omega_m$ and decay rate $\gamma_m$. In addition, two driving fields with amplitudes $\epsilon_1$ and $\epsilon_2$ are applied to fundamental and second-order modes. The Hamiltonian of the present model is (we set $\hbar = 1$)

\[
H = H_0 + H_{dr} + H_I + H_D + H_N, \quad (1)
\]

\[
H_0 = \omega_c a_1^{\dagger}a_1 + 2\omega_c a_2^{\dagger}a_2 + \omega_m b^{\dagger}b, \quad (2)
\]

\[
H_{dr} = i(\epsilon_1 e^{-i\omega_L t} a_1^{\dagger} + \epsilon_2 e^{-2i\omega_L t} a_2^{\dagger} - H.c.), \quad (3)
\]

\[
H_I = -g_1 a_1^{\dagger}a_1 (b^{\dagger} + b) - g_2 a_2^{\dagger}a_2 (b^{\dagger} + b), \quad (4)
\]

\[
H_D = \frac{\eta}{2}(b^{\dagger} + b)^4, \quad (5)
\]

\[
H_N = \frac{i\chi_0}{2}(a_1^{\dagger 2}a_2 - a_2^{\dagger 2}a_1^{\dagger}), \quad (6)
\]

where $H_0$ is the free Hamiltonian of the whole system. $H_{dr}$ is the Hamiltonian for driving fields applied to the fundamental and second-order modes with frequencies $\omega_L$ and $2\omega_L$. $H_I$ is the interaction between the optical and mechanical modes. The coupling strength between the mechanical mode and fundamental mode (second-order mode) is denoted by $g_1$ ($g_2$). The Duffing nonlinearity of the mechanical mode is represented by $H_D$. It was pointed out that a nonlinear amplitude of $\eta = 10^{-4}\omega_m$ can be achieved by coupling the mechanical mode to an
Fig. 1. Schematic representation of our model. The movable mirror is perfectly reflecting. However, the fixed mirror is partially transmitting. A second-order nonlinear medium denoted by $\chi^{(2)}$ is put into the cavity. The fundamental and second-order optical modes with frequencies $\omega_c$ and $2\omega_c$ are denoted by $a_1$ and $a_2$. Here, $\kappa_1$ and $\kappa_2$ are the decay rates of the fundamental and second-order optical modes, respectively. The mechanical resonator (movable mirror) with frequency $\omega_m$ and decay rate $\gamma_m$ is denoted by $b$.

ancilla system [26]. The Hamiltonian of a second-order nonlinear medium is denoted by $H_N$ with $\chi_0$ being the interaction between the fundamental and second-order optical modes [30–32].

In a rotating frame defined by the unitary transformation $U(t) = \exp \left\{-i \omega_L t(a_1^\dagger a_1 + 2a_2^\dagger a_2)\right\}$, we obtain the Hamiltonian as follows

$$H = U^\dagger H U - iU^\dagger \dot{U}$$

$$= \overline{\Delta_c} a_1^\dagger a_1 + 2\overline{\Delta_c} a_2^\dagger a_2 + \omega_m b^\dagger b$$

$$+ i(\epsilon_1 a_1^\dagger + \epsilon_2 a_2^\dagger - \epsilon_1 a_1 - \epsilon_2 a_2)$$

$$- (g_1 a_1^\dagger a_1 + g_2 a_2^\dagger a_2)(b^\dagger + b)$$

$$+ \frac{\eta}{2}(b^\dagger + b)^2 + \frac{i\chi_0}{2}(a_1^\dagger a_2 - a_2^\dagger a_1),$$

with $\overline{\Delta_c} = \omega_c - \omega_L$.

3. Quantum Langevin equations

First, we linearize the above Hamiltonian by employing the following displacement transformations $a_1 \rightarrow a_1 + \delta a_1$, $a_2 \rightarrow a_2 + \delta a_2$, and $b \rightarrow \beta + \delta b$. The quantum Langevin equations
\[ \dot{a}_1 = -(i\Delta_c + \frac{k_1}{2})a_1 + \chi_0 a_1^\dagger a_2 + \epsilon_1, \]
\[ \dot{a}_2 = -(i\Delta_c' + \frac{k_2}{2})a_2 - \frac{\chi_0}{2}a_1^\dagger + \epsilon_2, \]
\[ \dot{\beta} = -(i\omega_m + \frac{\gamma_m}{2})\beta - i\eta(16\beta^3 + 12\beta) + ig_1|a_1|^2 + ig_2|a_2|^2, \]
\[ \delta \dot{a}_1 = -(i\Delta_c + \frac{k_1}{2})\delta a_1 + iG_1(\delta b^\dagger + \delta b) + \chi_0 a_1^\dagger \delta a_2 + \sqrt{\kappa_1}a_{1,\text{in}}, \]
\[ \delta \dot{a}_2 = -(i\Delta_c' + \frac{k_2}{2})\delta a_1 + iG_2(\delta b^\dagger + \delta b) - \chi_0 a_1^\dagger \delta a_2 + \sqrt{\kappa_2}a_{2,\text{in}}, \]
\[ \delta \dot{b} = -(i\omega_m + \frac{\gamma_m}{2})\delta b - 2i\Lambda(\delta b^\dagger + \delta b) + i(G_1\delta a_1^\dagger + G_2^*\delta a_2) + \sqrt{\gamma_m}b_{\text{in}}, \]

where \( \Delta_c = \Delta_c - g_1(\beta^* + \beta), \Delta_c' = 2\Delta_c - g_2(\beta^* + \beta), \Lambda = 3\eta(4\beta^2 + 1), \) and \( G_{1,2} = g_{1,2}a_{1,2}. \)

In the limit of large \( \kappa_2, \) the fluctuations of mode \( a_2 \) can be neglected and the adiabatic approximation is valid [31]. Thus, the quantum Langevin equations can be reduced to

\[ \delta \dot{a}_1 = -(i\Delta_c + \frac{k_1}{2})\delta a_1 + iG_1(\delta b^\dagger + \delta b) + \chi_0 a_1^\dagger \delta a_2 + \sqrt{\kappa_1}a_{1,\text{in}}, \]
\[ \delta \dot{b} = -(i\omega_m + \frac{\gamma_m}{2})\delta b - 2i\Lambda(\delta b^\dagger + \delta b) + iG_1(\delta a_1^\dagger + \delta a_1) + \sqrt{\gamma_m}b_{\text{in}}, \]

where \( \chi = \chi_0a_2 = |\chi|^2\sqrt{\Delta_c}. \) Without loss of generality, \( G_1 \) has been assumed to be real.

Now, we define the following quadrature operators \( X_{O=0,b} = (\delta O^\dagger + \delta O)/\sqrt{2} \) and \( Y_{O=0,b} = i(\delta O^\dagger - \delta O)/\sqrt{2}. \) The noise quadrature operators are defined as \( X_{O=0,b}^\dagger = (O^\dagger + O)/\sqrt{2}, \) and \( Y_{O=0,b}^\dagger = i(O^\dagger - O)/\sqrt{2}. \) From the above quantum Langevin equations, we obtain

\[ \vec{f} = A\vec{f} + \vec{n}, \]

where \( \vec{f} = (X_{a_1}, Y_{a_1}, X_b, Y_b)^T \) and

\[ \vec{n} = (\sqrt{\kappa_1}X_{a_1}^\dagger, \sqrt{\kappa_1}Y_{a_1}^\dagger, \sqrt{\gamma_m}X_b^\dagger, \sqrt{\gamma_m}Y_b^\dagger)^T, \]

\[ A = \begin{pmatrix}
|\chi| \cos 2\phi - \kappa_2 & |\chi| \sin 2\phi + \Delta_c & 0 & 0 \\
|\chi| \sin 2\phi - \Delta_c & -|\chi| \cos 2\phi - \kappa_2 & 2G_1 & 0 \\
0 & 0 & -\frac{\gamma_m}{2} & \omega_m \\
2G_1 & 0 & -\omega_m - 4\Lambda & -\frac{\gamma_m}{2} \\
\end{pmatrix}. \]

Note that the dynamics of the present system described by Eq. (10) can be completely described by a \( 4 \times 4 \) covariance matrix \( V \) with \( V_{jk} = \langle f_j f_k + f_k f_j \rangle/2. \) Using the definitions of \( V, \vec{f}, \) and the above equations, we obtain the evolution of the covariance matrix \( V \) as follows

\[ \dot{V} = AV + VA^T + D, \]

where \( D \) is the noise correlation defined by \( D = \text{diag}\left[ \frac{\gamma_m}{2}, \frac{\gamma_m}{2}, \frac{\omega_m}{2}(2n_{th} + 1), \frac{\omega_m}{2}(2n_{th} + 1) \right]. \) Here, \( n_{th} \) is the mean phonon number of the mechanical resonator.
4. Large and robust mechanical squeezing beyond resolved sideband regime

4.1. Stability

We first investigate influence of the Duffing nonlinearity and optomechanical coupling strength on the stability of the present system. The Duffing nonlinearity and optomechanical coupling constant are related to parameters \( \Lambda = 3\eta(4\beta^2 + 1) \) and \( G_1 = g_1\alpha_1 \), respectively. It is well known that the system described by Eq. (13) is stable only if all the real parts of the eigenvalues of the matrix A are negative according to the Routh-Hurwitz criterion [33].

From Fig.2, we find the system is unstable for \( 0.53 < \phi < 0.97 \). Comparing three panels of this figure, one can see that the areas of the unstable regions could increase with the increase of the coupling constant \( G_1 \). For example, the system is stable with \( \phi = 0.2 \), \( \Lambda = 5\omega_m \) and \( G_1 = 0.1\omega_m \), as one can find in the upper panel of Fig.2. However, if we increase the effective optomechanical coupling strength \( G_1 \) from \( 0.1\omega_m \) to \( 1.6\omega_m \), the system is not stable with \( \Lambda = 5\omega_m \) and \( \phi = 0.2 \). Fortunately, the system can be stable if the Duffing nonlinearity \( \Lambda \) is increased to about \( 7.9\omega_m \) (see the lower panel of this figure). Thus, the present system could be stable even for the strong and deep-strong coupling cases in the presence of the Duffing nonlinearity.
Fig. 3. Mechanical squeezing of the mechanical resonator (in units of dB) versus \( \Lambda \) and \( \Delta_c \) for different values of \( \kappa_1 \) and \( |\chi| \). The white solid lines correspond to mechanical squeezing at 3dB. Other parameter values are \( \gamma_m/\omega_m = 10^{-6} \), \( n_{th} = 0 \), \( G_1/\omega_m = 0.1 \), \( \phi = 0.5\pi \), and \( |\chi| = \sqrt{(\kappa_1/2)^2 + (\Delta_c - \omega_m)^2} \) for Figs. 2(d)-2(f).

4.2. Mechanical squeezing in HURSB regime

In Fig. 3, we plot the mechanical squeezing of the mechanical resonator (in units of dB) versus \( \Lambda \) and \( \Delta_c \) for different values of \( \kappa_1 \) and \( |\chi| \). In Figs. 3(a)-3(c), the second-order nonlinear medium is not put into the cavity. From Fig. 3(a), one can see the mechanical squeezing can be larger than 3dB if the Duffing nonlinearity is strong enough in the resolved sideband regime in the absence of second-order nonlinear medium \( \chi^{(2)} \). This is consistent with the results of [26]. The mechanical squeezing decreases with the increase of the decay rate of cavity \( \kappa_1 \). For instance, if the decay rate of the cavity is much larger than the frequency of the mechanical resonator \( (\kappa_1 = 100\omega_m) \), then the mechanical squeezing of the mechanical resonator could not be larger than 3dB (see Fig. 3(c)). If the second-order nonlinear medium is put into the cavity, then the mechanical squeezing overcomes the 3dB limit even in the HURSB regime as one can see from Figs. 3(d)-3(f).

On the one hand, in the resolved sideband regime, the second-order nonlinear medium may play a destructive role in the generation of mechanical squeezing of the mechanical resonator (see Figs. 3(a) and 3(d)). On the other hand, the situation is very different for the sideband
Fig. 4. Mechanical squeezing of the mechanical resonator (in units of dB) versus $\Lambda$ and $|\chi|$. The white solid line corresponds to mechanical squeezing at 3dB. Other parameter values are $\gamma_m/\omega_m = 10^{-6}$, $n_{th} = 1000$, $G_1/\omega_m = 1.5$, $\Delta_c/\omega_m = 10$, $\kappa_1/\omega_m = 100$, and $\phi = 0.5\pi$.

unresolved regime. In the absence of the second-order nonlinear medium with $|\chi| = 0$, the mechanical squeezing depends heavily on the decay rate $\kappa_1$, i.e., it decreases with the increase of the decay rate $\kappa_1$ significantly (Figs.3(b)-3(c)). However, if we put the second-order nonlinear medium into the cavity, the mechanical squeezing is insensitive to the decay rate $\kappa_1$ as one can find in Figs.3(e)-3(f).

4.3. Robustness against thermal fluctuations of mechanical mode

In Fig.4, we plot the mechanical squeezing of the mechanical resonator versus $\Lambda$ and $|\chi|$ with $\gamma_m/\omega_m = 10^{-6}$, $n_{th} = 1000$, $G_1/\omega_m = 1.5\omega_m$, $\Delta_c = 10\omega_m$, $\kappa_1 = 100\omega_m$, and $\phi = 0.5\pi$. If there is no Duffing nonlinearity or second-order nonlinear medium, large mechanical squeezing cannot be achieved. However, if the second-order nonlinear medium and Duffing nonlinearity are chosen appropriately the mechanical squeezing can overcome the 3dB limit even in the HURSB regime and in the presence of thermal fluctuation of the mechanical mode with $n_{th} = 1000$. This shows that the large and robust mechanical squeezing is a combined effect of the Duffing nonlinearity of the mechanical mode and the second-order nonlinearity medium $\chi^{(2)}$ in the cavity.

In order to show the influence of the thermal fluctuations on the mechanical squeezing more
clearly, we plot the mechanical squeezing (in units of dB) as functions of $\Delta_c$ for different values of $G_1$ and $|\chi|$. The black solid lines correspond to mechanical squeezing at 3dB. The red, green, blue, and cyan lines correspond to $n_{th} = 0, n_{th} = 500, n_{th} = 1000$, and $n_{th} = 10000$, respectively. Other parameter values are $\gamma_m/\omega_m = 10^{-6}$, $\Lambda/\omega_m = 8$, $\phi = 0.5\pi$, and $|\chi| = \sqrt{(\kappa_1/2)^2 + (\Delta_c - \omega_m)^2}$ for Figs.3(d)-3(f).

5. Conclusion

In the present work, we have proposed an efficient scheme to generate large and robust mechanical squeezing beyond the 3dB limit in the HURSB regime for realistic parameters. The system was formed by a standard optomechanical system with a second-order nonlinear medium $\chi^{(2)}$ in a cavity and the Duffing nonlinearity of the mechanical mode. In fact, a strong Duffing nonlinearity...
could be achieved by coupling the mechanical mode to an ancilla system as point out in [26]. There are two modes in the cavity. One is the fundamental mode. The other is the second-order mode. We assumed the decay rate of the second-order mode is very large. In the adiabatic approximation, we derived effective quantum Langevin equations of the model. The influence of the second-order nonlinear medium $\chi^{(2)}$ and Duffing nonlinearity on the mechanical squeezing was discussed carefully.

In the absence of the second-order nonlinear medium $\chi^{(2)}$, the mechanical squeezing $S_{dB}$ decreases with the increase of the decay rate of the cavity significantly and it could be negative for HURSB regime. However, if we put the second-order nonlinear medium into the cavity, the mechanical squeezing is insensitive with the decay rate of the cavity and $S_{dB}$ can be larger than 3dB even when the decay rate of the cavity is much larger than the frequency of the mechanical resonator.

Then, we discussed the influence of the thermal fluctuations of the mechanical mode on the mechanical squeezing in the HURSB regime. On the one hand, the mechanical squeezing can not be larger than 3dB without the second-order nonlinear medium if the thermal fluctuations of the mechanical mode is considered. On the other hand, the mechanical squeezing could be larger than 3dB even for high temperature when the second-order nonlinear medium is put into the cavity. Thus, we have shown that large and robust mechanical squeezing beyond the 3dB limit can be generated in the HURSB regime.

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**Disclosures**

The authors declare no conflicts of interest.

**References**

1. M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, “Cavity optomechanics,” Rev. Mod. Phys. 86, 1391–1452 (2014).
2. W. P. Bowen and G. J. Milburn, *Quantum Optomechanics* (CRC Press, 2015).
3. X. Y. Lü, Y. Wu, J. R. Johansson, H. Jing, J. Zhang, and F. Nori, “Squeezed optomechanics with phase-matched amplification and dissipation,” Phys. Rev. Lett. 114, 093602 (2015).
4. T. S. Yin, X. Y. Lü, L. L. Zheng, M. Wang, S. Li, and Y. Wu, “Nonlinear effects in modulated quantum optomechanics,” Phys. Rev. A 95, 053861 (2017).
5. H. Xiong, J. Gan, and Y. Wu, “Kuznetsov-ma soliton dynamics based on the mechanical effect of light,” Phys. Rev. Lett. 119, 153901 (2017).
6. J. S. Zhang, W. Zeng, and A. X. Chen, “Effects of cross-kerr coupling and parametric nonlinearity on normal mode splitting, cooling, and entanglement in optomechanical systems,” Quantum Inf. Process. 16, 163 (2017).
7. J. S. Zhang, M. C. Li, and A. X. Chen, “Enhancing quadratic optomechanical coupling via a nonlinear medium and lasers,” Phys. Rev. A 99, 013843 (2019).
8. D. F. Walls and G. J. Milburn, *Quantum optics* (Springer, Berlin, 2008, 2008).
9. G. Milburn and D. Walls, “Production of squeezed states in a degenerate parametric amplifier,” Opt. Commun 39, 401–404 (1981).
10. R. Ruskov, K. Schwab, and A. N. Korotkov, “Squeezing of a nanomechanical resonator by quantum nondemolition measurement and feedback,” Phys. Rev. B 71, 235407 (2005).
11. A. A. Clerk, F. Marquardt, and K. Jacobs, “Mechanical squeezing via parametric amplification and weak measurement,” New J. Phys. 10, 095010 (2008).
12. A. Szorkovszky, A. C. Doherty, G. I. Harris, and W. P. Bowen, “Mechanical squeezing via parametric amplification and weak measurement,” Phys. Rev. Lett. 107, 213603 (2011).
13. A. Szorkovszky, G. A. Brawley, A. C. Doherty, and W. P. Bowen, “Strong thermomechanical squeezing via weak measurement,” Phys. Rev. Lett. 110, 184301 (2013).
14. K. Jähne, C. Genes, K. Hammerer, M. Wallquist, E. S. Polzik, and P. Zoller, “Cavity-assisted squeezing of a mechanical oscillator,” Phys. Rev. A 79, 063819 (2009).
15. S. Huang and G. S. Agarwal, “Reactive coupling can beat the motional quantum limit of nanowaveguides coupled to a microdisk resonator,” Phys. Rev. A 82, 033811 (2010).
16. P. Rabl, A. Shnirman, and P. Zoller, “Generation of squeezed states of nanomechanical resonators by reservoir engineering,” Phys. Rev. B 70, 205304 (2004).
17. A. Mari and J. Eisert, “Gently modulating optomechanical systems,” Phys. Rev. Lett. 103, 213603 (2009).
18. J. Zhang, Y. X. Liu, and F. Nori, “Cooling and squeezing the fluctuations of a nanomechanical beam by indirect quantum feedback control,” Phys. Rev. A 79, 052102 (2009).
19. W. J. Gu, G. X. Li, and Y. P. Yang, “Generation of squeezed states in a movable mirror via dissipative optomechanical coupling,” Phys. Rev. A 88, 013835 (2013).
20. D. Y. Wang, C. H. Bai, H. F. Wang, A. D. Zhu, and S. Zhang, “Steady-state mechanical squeezing in a hybrid atom-optomechanical system with a highly dissipative cavity,” Sci. Rep. 6, 24421 (2016).
21. D. Y. Wang, C. H. Bai, H. F. Wang, A. D. Zhu, and S. Zhang, “Steady-state mechanical squeezing in a double-cavity optomechanical system,” Sci. Rep. 6, 38559 (2016).
22. A. Kronwald, F. Marquardt, and A. A. Clerk, “Arbitrarily large steady-state bosonic squeezing via dissipation,” Phys. Rev. A 88, 063833 (2013).
23. E. E. Wollman, C. U. Lei, A. J. Weinstein, J. Suh, A. Kronwald, F. Marquardt, A. A. Clerk, and K. C. Schwab, “Quantum squeezing of motion in a mechanical resonator,” Science 349, 952–955 (2015).
24. C.-H. Bai, D.-Y. Wang, S. Zhang, S. Liu, and H.-F. Wang, “Strong mechanical squeezing in a standard optomechanical system by pump modulation,” Phys. Rev. A 101, 053836 (2020).
25. M. Asjad, G. S. Agarwal, M. S. Kim, P. Tombesi, G. D. Giuseppe, and D. Vitali, “Robust stationary mechanical squeezing in a kicked quadratic optomechanical system,” Phys. Rev. A 89, 023849 (2014).
26. X. Y. Lü, J. Q. Liao, L. Tian, and F. Nori, “Steady-state mechanical squeezing in an optomechanical system via duffing nonlinearity,” Phys. Rev. A 91, 013834 (2015).
27. X. Han, D. Y. Wang, C. H. Bai, W. X. Cui, S. Zhang, and H. F. Wang, “Mechanical squeezing beyond resolved sideband and weak-coupling limits with frequency modulation,” Phys. Rev. A 100, 033812 (2019).
28. R. Zhang, Y. N. Fang, Y. Y. Wang, S. Chesi, and Y. D. Wang, “Strong mechanical squeezing in an unresolved-sideband optomechanical system,” Phys. Rev. A 99, 043805 (2019).
29. J.-S. Zhang and A.-X. Chen, “Large mechanical squeezing beyond a 3db of hybrid atom-optomechanical systems in a highly unresolved sideband regime,” Opt. Express 28, 12827 (2020).
30. H.-K. Lau and A. A. Clerk, “Ground-state cooling and high-fidelity quantum transduction via parametrically driven bad-cavity optomechanics,” Phys. Rev. Lett. 124, 103602 (2020).
31. M. Asjad, N. E. Abari, S. Zippilli, and D. Vitali, “Optomechanical cooling with intracavity squeezed light,” Opt. Express 27, 32427 (2019).
32. J.-H. Gan, Y.-C. Liu, C.-C. Lu, X. Wang, M.-K. Tey, and L. You, “Intracavity-squeezed optomechanical cooling,” Laser Photonics Rev. 2019, 1900120 (2019).
33. E. X. DeJesus and C. Kaufman, “Routh-hurwitz criterion in the examination of eigenvalues of a system of nonlinear ordinary differential equations,” Phys. Rev. A 35, 5288–5290 (1987).