Fibre-optics implementation of asymmetric phase-covariant quantum cloner

Lucie Bartůšková,1 Miloslav Dušek,1 Antonín Černoch,1,2 Jan Soubusta,2 and Jaromír Fiurášek1

1Department of Optics, Palacký University, 17. listopadu 50, 772 00 Olomouc, Czech Republic
2Joint Laboratory of Optics of Palacký University and Institute of Physics of Academy of Sciences of the Czech Republic, 17. listopadu 50A, 772 00 Olomouc, Czech Republic

(Dated: September 14, 2018)

We present the experimental realization of optimal symmetric and asymmetric phase-covariant 1 → 2 cloning of qubit states using fiber optics. State of each qubit is encoded into a single photon which can propagate through two optical fibers. The operation of our device is based on one- and two-photon interference. We have demonstrated creation of two copies of any state of a qubit from the equator of the Bloch sphere. The measured fidelities of both copies are close to the theoretical values and they surpass the theoretical maximum obtainable with the universal cloner.

The quantum no-cloning theorem[1] lies at the heart of quantum information theory. The apparently simple observation that perfect copying of unknown quantum states is impossible has profound consequences. On the fundamental side, it prevents superluminal communication with entangled states, thereby guaranteeing the peaceful coexistence of quantum mechanics and theory of relativity. On the practical side, this theorem is behind the security of the quantum key distribution schemes which rely on the fact that any attempt to measure or copy an unknown quantum state results in the disturbance of this state. Going beyond the no-cloning theorem, Bužek and Hillery in a seminal paper introduced the concept of the universal approximate quantum cloning machine that optimally approximates the forbidden transformation $|ψ⟩ → |ψ⟩|ψ⟩$[2]. Today, optimal quantum cloners are known for many different cases and scenarios[3, 4]. During recent years, growing attention has been paid to the experimental implementation of quantum cloning machines and, in particular, optimal cloning of polarization states of single photons via stimulated parametric downconversion or via photon bunching on a beam splitter has been successfully demonstrated[3, 6, 7, 8, 9, 10].

Besides giving an insight into the fundamental limits on distribution of quantum information, the quantum cloning machines turned out to be very efficient eavesdropping attacks on the quantum key distribution protocols[11, 12, 13, 14]. In this context one is particularly interested in the asymmetric quantum cloners that produce two copies with different fidelities. In this way, the eavesdropper can control the trade-off between the information gained on a secret cryptographic key and the amount of noise added to the copy which is sent down the channel to the authorized receiver. While the theory of optimal asymmetric quantum copying machines is well established (see, e.g. the recent reviews[3, 4]), the experimental optical realization of such machines has received considerably less attention. This might be attributed to the fact that the asymmetric cloning operations exhibit much less symmetry than the corresponding symmetric ones. To the best of our knowledge, asymmetric quantum cloning of single-photon states has been so far achieved only in a single experiment, where universal asymmetric copying of polarization states was performed by means of partial quantum teleportation[15].

In this Letter, we report on the experimental implementation of the optimal 1 → 2 phase-covariant asymmetric cloning of photonic qubits represented by a single photon propagating in two single-mode optical fibers. The phase-covariant copying machine optimally clones all states on the equator of the Bloch sphere, $|ψ⟩ = 1/√2(|0⟩ + e^{iϕ}|1⟩)$. Our experiment is based on the interplay of single- and two-photon interference of two photons in an optical network built from optical fibers. This approach has several important technological advantages. First, the single-mode fibers guarantee very high interference visibility. Second, variable ratio couplers enable to easily change in a controlled way the cloning transformation and we are thus able to demonstrate the whole class of the optimal asymmetric cloners.

In contrast to our previous experiment on the optimal symmetric phase-covariant cloning of polarization states of single photons[16], with the present fiber-based scheme[17] we are able to achieve fidelities exceeding the limit of optimal universal cloning machine. This is rather challenging because the fidelities of the optimal universal and phase-covariant cloners are very close. For instance, for a symmetric cloner we have $F_{\text{univ}} = 5/6 ≈ 0.833$ and $F_{\text{pc}} = 7/8 ≈ 0.875$, so the fidelities differ only by 2.1%.

The optimal asymmetric phase-covariant cloning transformation requires only a single blank copy in addition to the input qubit to be cloned and reads[18],

$$|0⟩ \mapsto |00⟩,$$

$$|1⟩ \mapsto √q|10⟩ + √{1-q}|01⟩,$$  \hspace{1cm} (1)

where $q ∈ [0, 1]$ characterizes the asymmetry of the clones and for the symmetric cloner $q = 1/2$. The fidelities of the
two clones are given by

\[ F_A = \frac{1}{2} (1 + \sqrt{q}), \quad F_B = \frac{1}{2} (1 + \sqrt{1 - q}). \]  

(2)

In our scheme (see Fig. 1) each qubit is represented by a single photon which may propagate in two optical fibers and the basis states \(|0\rangle\) and \(|1\rangle\) correspond to the presence of the photon in the first or second fiber, respectively. The state of ancilla photon is initially \(|0\rangle\) while the signal photon can be prepared in an arbitrary state from the equator of the Bloch sphere. The two photons impinge on two unbalanced beam splitters (variable ratio couplers VRC0 and VRC1) with different splitting ratios. Let us suppose that real amplitude transmittances and reflectances of VRC0 and VRC1 are \(t_0, r_0\) and \(t_1, r_1\), respectively. We use the notation \(R_j = t_j^2\) and \(T_j = t_j^2\) for the intensity reflectances and transmittances and \(R_j + T_j = 1\) for a lossless beam splitter. In the experiment, we accept only the events when there is a single photon detected in each output pair of fibers corresponding to the clone A and B, respectively. The cloning transformation is thus implemented conditionally, similarly to other optical cloning experiments. The resulting conditional transformation reads \[19\]

\[ |0\rangle_{\text{Sig}} |0\rangle_{\text{Anc}} \rightarrow (r_0^2 - t_0^2) |00\rangle, \]

\[ |1\rangle_{\text{Sig}} |0\rangle_{\text{Anc}} \rightarrow r_0 r_1 |10\rangle - t_0 t_1 |01\rangle. \]  

(3)

This becomes equivalent to the optimal cloning operation \[11\] up to a constant prefactor representing the probability amplitude of successful cloning, if the following equations hold,

\[ r_0 r_1 = \sqrt{q(r_0^2 - t_0^2)}, \quad t_0 t_1 = -\sqrt{1 - q(r_0^2 - t_0^2)}. \]

Taking the square of the ratio of these two equations, we arrive at

\[ R_1 = \frac{q(1 - R_0)}{q(1 - R_0) + (1 - q)R_0}. \]  

(4)

and from the normalization \(T_1 + R_1 = 1\) we find after some algebra that \(R_0\) can be determined as a root of a cubic polynomial,

\[ R_0 (1 - R_0) + [R_0(2q - 1) - q] (2R_0 - 1)^2 = 0. \]  

(5)

The resulting reflectances are given in Tab. 1 for several values of the asymmetry parameter \(q\). The equations have always two physically significant solutions that also require different signs of amplitude reflectances and transmittances. We have always selected the “less unbalanced” splitting ratios as they are more convenient from the experimental point of view.

Our experimental setup is shown in Fig. 1. A pair of signal and ancilla photons is prepared by means of frequency-degenerate type-I spontaneous parametric down-conversion in a 10-mm-long LiIO3 nonlinear crystal pumped by a krypton-ion cw laser (413.1 nm), similarly as in our previous experiments \[16, 17\]. The signal photon is split by a fiber coupler FC into two fibers. The basis states of the signal qubit, \(|0\rangle\) and \(|1\rangle\), correspond to the presence of a photon either in fiber \(f2\) or \(f1\), respectively. The intensity ratio and phase difference between these two modes determine the input state of the signal qubit. Preparation of the state is affected by unequal losses in the two optical paths \(f1\) and \(f2\) which alter the effective splitting ratio of FC. This effective splitting ratio is measured with the help of a semiconductor laser and a PIN photodiode and the attenuator in mode \(f2\) is adjusted in such a way that the setup is balanced and at the end of the state preparation block the signal photon is evenly split between \(f1\) and \(f2\). Various equatorial qubit states \(\frac{1}{\sqrt{2}}(|0\rangle + e^{i\phi}|1\rangle)\) can be then prepared by changing only the voltage applied to the phase modulator PM which sets the relative phase \(\phi\). The ancilla is in a fixed state \(|0\rangle\) which corresponds to a single photon propagating through the fiber \(f3\).

The cloning operation is realized by two variable-ratio couplers VRC0 and VRC1. VRC0 forms the core of Hong-Ou-Mandel (HOM) interferometer \[20\]. For optical cloning it is necessary to achieve precise time overlap of the two photons at VRC0 and match their polarizations. To accomplish these tasks the splitting ratio of VRC0 is set to 50:50. We typically reach visibilities of HOM dip around 98%. Then the VRC0 splitting ratio is changed to the required value depending on the asymmetry parameter \(q\), c.f. Table I.

The two fiber-based Mach-Zehnder (MZ) interferometers are adjusted using only the signal beam from the nonlinear crystal, the ancilla beam is blocked. Detection rates at each detector are measured and used for the alignment of the setup. First the intensity transmittances of the whole arms of each MZ interferometer are balanced with the help of the attenuators in the “state verification” part of the setup, which compensates for the unequal losses caused by the splitting ratios of variable ratio couplers, the phase modulators, air-gaps and...
TABLE I: Asymmetric phase covariant cloner. Table shows calculated reflectances of variable ratio couplers and theoretical and measured fidelities for different parameters of asymmetry $q$. For $q < 0.5$ the clones are just interchanged. Error intervals represent statistical errors.
where \( p \) is the HOM dip lower than 100% and the inaccuracy of position part of ancilla went to clone B. Therefore the visibility of the measured coincidences. As expected, fidelities are simultaneously by all four detectors. The unequal deformation cannot be perfect at both output ports.

FIG. 3: Symmetric phase-covariant cloner. Fidelities \( F_A \) and \( F_B \) are plotted as functions of input-state phase \( \phi \). Symbols denote experimental data, solid line represents theoretical prediction for the phase-covariant cloner, and the dashed line shows theoretical prediction for the universal cloner. Error bars represent statistical errors.

Displayed fidelities are calculated from data measured over 40 three-second measurements were performed. The unequal detector efficiencies were compensated by proper rescaling of all four coincidence rates sequentially at only one pair of the coupler [21]. Therefore we have also measured the visibilities were maximized. Using only these two detectors we have obtained fidelities (averaged over all phases): \( F_A = 0.840 \pm 0.009 \), \( F_B = 0.850 \pm 0.009 \). In this kind of measurement no compensation for different detector efficiencies was needed.

In summary, we have demonstrated optimal symmetric and asymmetric phase-covariant cloning of single-photon states. Using fiber optics allowed us to reach very high visibilities and achieve fidelities exceeding the maximum obtainable by any universal cloning machine. Our implementation is compatible with fiber-based communication systems and represents a promising platform for realization of various protocols for quantum information processing.

This research was supported by the projects LC06007, 1M06002 and MSM6198959213 of the Ministry of Education of the Czech Republic and by the SECOQC project of the EC (IST-2002-506813).

[1] W.K. Wootters and W.H. Zurek, Nature (London) 299, 802 (1982); D. Dieks, Phys. Lett. 92A, 271 (1982).
[2] V. Bužek and M. Hillery, Phys. Rev. A 54, 1844 (1996).
[3] V. Scarani, S. Iblisdir, N. Gisin, and A. Acín, Rev. Mod. Phys. 77, 1225 (2005).
[4] N.J. Cerf and J. Fiurášek, In: Progress in Optics, vol. 49, Ed. E. Wolf (Elsevier, 2006), p. 455.
[5] A. Lamas-Linares, C. Simon, J.C. Howell, and D. Bouwmeester, Science 296, 712 (2002).
[6] F. De Martini, D. Pelliccia, and F. Sciarrino, Phys. Rev. Lett. 92, 067901 (2004).
[7] M. Ricci, F. Sciarrino, C. Sias, and F. De Martini Phys. Rev. Lett. 92, 047901 (2004).
[8] W.T. M. Irvine, A. Lamas-Linares, M. J. A. de Dood, and D. Bouwmeester, Phys. Rev. Lett. 92, 047902 (2004).
[9] I.A. Khan and J.C. Howell, Phys. Rev. A 70, 010303 (2004).
[10] F. Sciarrino and F. De Martini, Phys. Rev. A 72, 062313 (2005).
[11] C.A. Fuchs, N. Gisin, R. B. Griffiths, C.-S. Niu, and A. Peres, Phys. Rev. A 56, 1163 (1997).
[12] N.J. Cerf, M. Bourennane, A. Karlsson, and N. Gisin, Phys. Rev. Lett. 88, 127902 (2002).
[13] M. Dušek, N. Lütkenhaus, M. Hendrych, In: Progress in Optics, vol. 49, Ed. E. Wolf (Elsevier, 2006), p. 381.
[14] D. Bruss and C. Macchiavello, Phys. Rev. Lett. 88, 127901 (2002).
[15] Z. Zhao, A.-N. Zhang, X.-Q. Zhou, Y.-A. Chen, C.-Y. Lu, A. Karlsson, and J.-W. Pan, Phys. Rev. Lett. 95, 030502 (2005).
[16] A. Černoch, L. Bartušková, J. Soubusta, M. Ježek, J. Fiurášek, M. Dušek, Phys. Rev. A 74, 042327 (2006).
[17] L. Bartušková, Á. Černoch, R. Filip, J. Fiurášek, J. Soubusta, M. Dušek, Phys. Rev. A 74, 022325 (2006).
[18] C.-S. Niu and R.B. Griffiths, Phys. Rev. A 60, 2764 (1999).
[19] J. Fiurášek, Phys. Rev. A 67, 052314 (2003).
[20] C.K. Hong, Z.Y. Ou, and L. Mandel, Phys. Rev. Lett. 59, 2044 (1987).
[21] M. Hendrych, M. Dušek, O. Haderka, Acta Physica Slovaca 46, 393 (1996).