Characterization of seismic energy during fault-slip induced by fluid injection using coupled and dynamic X-FEM analysis

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Abstract. Fluid injection into a rock mass from industrial processes can lead to reactivation of pre-existing faults at great depths. When the shear behaviour involves dynamic rupture, perceivable seismic events could be induced that may raise public concern. This seismicity can be caused by injection-induced fluid pressure in the rock mass causing slip on faults. This study provides a method to distinguish between aseismic and seismic fault movement induced by anthropogenic fluid injection with the aim of gaining an insight into the difference in seismic source parameters between the two types of fault behaviour, i.e., seismic and aseismic. This was achieved by using a two-dimensional fully coupled fluid and mechanical loading extended finite element model (X-FEM) with a dynamic analysis module. This code considers fluid flow along the fault as well as into the rock mass and uses a directly proportional equivalent injected flow rate into the fault as the input. This model was validated by comparing the resultant pressure and normal and shear displacements calculated at the centre of the fault against observations from a decametre-scale in-situ experiment. The main results were that not only the mechanics of the fault could be simulated using this approach, but the simulation correctly predicted the onset of seismicity and transition to dynamic analysis and at similar seismic magnitudes to observations. In terms of the difference in seismic source parameters between seismic and aseismic fault movements, it was shown that while the seismic shear movement to total shear movement is approximately 1%, the dynamic analysis produces approximately 75% of the total near field energy released in the simulation. This indicates the necessity of distinguishing dynamic and quasi-static shear movements in the numerical simulation of fluid injection-induced seismicity in order to quantify magnitude and seismic energy released accurately and to assess the risk of seismicity properly, since aseismic fault movements do not cause any damage to facilities on the surface. These results are important, since they demonstrate the applicability of this X-FEM approach in accurately predicting the mechanics of fault reactivation and the resultant seismicity, aiding in the design and scheduling of fluid injection and in the optimization of operational parameters.

1. Introduction
Fluid injection into or near a fault may induce slip and result in seismicity of sufficient magnitude to cause damage to surface and underground structures [1]. Fluid injection is synonymous with wastewater disposal and hydraulic fracturing used to develop sites for geothermal energy,
unconventional hydrocarbon production and in destressing in deep hard rock mines. Therefore, to obtain the anticipated seismicity in geological formations, understanding the mechanics of fault reactivation is important. Understanding the resultant behavior to fluid-injection is typically challenging due to the characteristically coupled nature of the fluid-transmission and mechanical processes. In addition, predicting the onset of seismicity, i.e., distinguishing aseismic fault movement from seismic slip, and then modelling the process under dynamic conditions remains a difficult task.

Only a few numerical methods have been utilized to model dynamic fault-slip behavior [2–5], still fewer have introduced numerical methods to model coupled (fluid pressure and mechanical behavior) fault-slip behavior [6–9] with very limited analyses representing fault-slip using coupled and dynamic analyses together [10]. Thus, the problem addressed in this study is the reproduction of the quasi-static and dynamic mechanics of a natural in-situ fault using a coupled extended finite element method (XFEM) approach. This approach uses dynamic analysis when unstable fault-slip conditions are met (considering rate and state friction behaviour), otherwise using quasi-static analysis. Results are benchmarked against observations from a decameter-scale in situ fault remobilization experiment [11]. The validated approach contains all the necessary processes contributing to the reactivation, i.e., fluid injection, leak-off, inertial terms, and stiffness differences in the system. The method presented could be applied to other faults that are perturbed by fluid injection to forecast the expected fault movement. Inclusion of the dynamic fault-slip process in this approach provides insights into induced seismicity and may contribute to the mitigation of its risks.

2. X-FEM formulation for a mixed aseismic/seismic fault-slip simulation based on a coupled hydro-mechanical model

An X-FEM approach was applied since it is computationally efficient and accurate when accommodating a discontinuity, compared to conventional continuum approaches. The X-FEM modelling implicitly represents individual cracks without requiring complex meshing and remeshing of the crack, resulting in decreased computation time. Specifically, the X-FEM approach enriches the FEM model by providing additional degrees of freedom (DOF) to the nodes of the element(s) that are crossed by the discontinuity. Therefore, a single mesh can be used for discontinuities of any length and orientation [12].

2.1. Spatial discretisation of the strong formulation for X-FEM
The weak forms of the governing equations are obtained by applying the well-known divergence theorem, which are then discretized spatially, based on the Galerkin discretization technique. The dynamic part of the analysis uses the following resulting system of linear equations, where the static analysis follows the same procedure as Schwartzkopff et al. [13]:

\[
\begin{align*}
&M \ddot{U} + C \dot{U} + K U - Q P + f^\text{int}_U - f^\text{ext}_U = 0 \\
&I n \ddot{P} + Q^T \dot{U} + H P + S P - q^\text{int}_P - q^\text{ext}_P = 0
\end{align*}
\]

where \( \bar{U} = \langle \bar{u}, \bar{a} \rangle \) and \( \bar{P} = \langle \bar{p}, \bar{c} \rangle \) are the standard and enriched degrees of freedom (DOF) of displacement and pressure, respectively. \( M \) is the mass matrix, \( C \) is the Rayleigh damping matrix, \( K \) is the stiffness matrix, \( Q \) is the coupling matrix, \( H \) is the permeability matrix, \( S \) is the compressibility matrix, \( I n \) is the fluid inertial matrix, and \( f^\text{int}_U \) and \( q^\text{ext}_P \) are the external force vectors. The matrices added (or redefined) compared with Schwartzkopff et al. [13] are:
\[ M_{\alpha\beta} = \int_{\Omega} \left( N^p_n \right)^T \rho N^p_n \, d\Omega \]

\[ C = \alpha_{\text{Rayleigh}} \mathbf{M} + \beta_{\text{Rayleigh}} \mathbf{K} \]

\[ \mathbf{In}_{\delta\gamma} = \int_{\Omega} \left( \mathbf{V} \mathbf{N}^\delta_n \right)^T k_f \rho \mathbf{N}^\delta_n \, d\Omega \]

where \((\alpha, \beta) \in \{\text{std}, \text{Hev}\}\) represent the standard and Heaviside functions of the displacement field and \((\delta, \gamma) \in \{\text{std}, \text{abs}\}\) are the standard and modified level set functions of the pressure field. In these definitions, A Rayleigh damping matrix has been introduced to prevent undesirable oscillations in the dynamic system. The Rayleigh parameters \((\alpha_{\text{Rayleigh}} \text{ and } \beta_{\text{Rayleigh}})\), are estimated using the procedure in Kontoe et al. [14].

2.2. Discretisation of the time domain and solution technique

The Newmark-Beta implicit time integration scheme is utilized for the temporal discretization of the unknown variables (of displacement and pressure), where \( t_{n+1} = t_n + \Delta t \). Note that when the model either switches to dynamic analysis, or back to static analysis, the velocity, acceleration, and pore gradient vectors are zeroed, since the model is transiting-from or -into static analysis where the material must be at rest.

Substituting the Newmark-Beta temporal discretization and then rearranging these equations into the spatially discretized system of linear equations results in the following system of linear equations (where the iteration number is defined as \( i \)):

\[
\begin{bmatrix}
-\frac{\gamma}{\beta \Delta t} M + \frac{\gamma}{\beta \Delta t} C + K + \frac{\partial f^\text{int}_U}{\partial U} \\
\frac{1}{\beta \Delta t^2} M + \frac{\gamma}{\beta \Delta t} C + K - \frac{\partial f^\text{int}_U}{\partial U} \\
\frac{1}{\beta \Delta t^2} \mathbf{In} + \frac{\gamma}{\beta \Delta t} \left( \mathbf{Q}^T - \frac{\partial f^\text{int}_U}{\partial P} \right) + H + \frac{1}{\theta \Delta t} S - \frac{\partial q^\text{int}_P}{\partial P}
\end{bmatrix}
= \frac{dU}{\Delta t}^n_{n+1}
\]

where the Jacobian (first matrix of equation 8) was made semi-symmetrical (except for the fluid inertial matrix \( \mathbf{In} \), which can be omitted, if required) by multiplying the first row by \(-\gamma/\beta \Delta t\) to reduce the computational cost. This defines the full suite of terms contributing to the Jacobian. Note that the Newton-Raphson iterative method is used to reduce the error to within a predefined tolerance - in this case <1% in each time step.
2.3. Frictional model

To capture changes in the friction coefficient from the evolution of slip velocity and healing rate, a modified version of the Coulomb friction law using rate and state parameters is implemented in the X-FEM model [15]. For cohesionless frictional faults, the original Coulomb friction law can be expressed simply as:

\[ |\tau| = \mu \sigma_n' \tag{4} \]

where \(\tau\) is the shear stress that is present at a location along the fault, \(\sigma_n'\) is the effective normal compressive stress, and \(\mu\) is the friction coefficient. The frictional coefficient can vary over the length of the fault. The effective normal compressive stress is simply the normal compressive stress minus the fluid pressure along the fault – assuming a Biot coefficient \(\alpha_{\text{Biot}}\) of unity. Rate state friction relates the friction coefficient to the rate of tangential shear displacement and the duration at that state. The Dieterich (aging) constitutive law, used here to simulate dynamic frictional response, can be expressed as:

\[ \mu = \mu_0 + \alpha \ln \left( \frac{V}{V_0} \right) + b \ln \left( \frac{V_0 \theta}{D_c} \right) \text{ and } \frac{d\theta}{dt} = 1 - \frac{V \theta}{D_c} \tag{5} \]

where \(\mu_0\) is the residual friction coefficient, \(\alpha\) is an empirical dimensionless coefficient which controls the velocity response, \(V\) is the tangential velocity, and \(V_0\) is a reference velocity. The parameter \(b\) is an empirical dimensionless coefficient that controls the state response, \(D_c\) has been interpreted as the slip required to renew surface contacts and \(\theta\) is the contact time parameter. This constitutive law provides a relationship that captures the time and velocity dependence of friction.

To account for the static frictional response, i.e. the degradation of the friction coefficient with the change in shear displacement from initial in-situ conditions (\(\Delta u_0\)), the residual friction coefficient \(\mu_0\) is reduced linearly over a critical slip weakening distance (\(\text{Slip}_c\)) from an initial value \(\mu_i\) to a final value \(\mu_f\), i.e.:

\[ \mu_0 = \begin{cases} \mu_i, & \text{if } |\Delta u_s| < \text{Slip}_c \\ \mu_f, & \text{if } |\Delta u_s| \geq \text{Slip}_c \end{cases} \tag{6} \]

2.4. Conditions for dynamic analysis

The analysis is transferred into dynamic mode when the following instability condition is met [16]:

\[ \frac{GD_c}{(b-a)} < -\sigma_n' L \tag{7} \]

where the shear modulus \(G\) is of the rock surrounding the fault, \(L\) is the continuous slipping area of the velocity weakening region and the other parameters are as defined earlier. Noting that the effective normal stress is the weighted average along the velocity weakening slipping area (and negative values are in compression, in this study). Note that the geometric constant is assumed to be unity, and therefore not explicitly shown in the above expression. This criterion is checked for every continuous velocity weakening area.

3. Verification of the X-FEM code

The aforementioned model is used to represent fault reactivation during a well-constrained field experiment [11]. See Schwartzkopff et al. [13] for a short description of the in-situ experiment, the numerical model setup, and boundary and analysis conditions used. Note that the time increment \(\Delta t\) for the dynamic analysis was determined to be 1\times10^{-4} \text{ seconds}. 
3.1. Verification using PEST software

History matching was used to verify the dynamic X-FEM code. History matching is a type of inverse problem in which the observations in the reservoir (pressures and displacements in the present study) are used to estimate model variables that caused that response. The process implies that the input parameters have some physical interpretation and optimizes these variables to reproduce the observed measurements (that is, in the present study, from the in-situ experiment). These problems are usually ill-posed with many parameter combinations that result in equally good matches to the past observations [17]. See Schwartzkopff et al. [13] for a discussion on previous uses of the PEST software suite in the literature.

4. Verification result

The Levenberg-Marquardt algorithm [18] is used in PEST to reduce the objective function, which is the summation of the squared weighted residuals. The smaller the objective function the closer the overall fit to the measurements (see Table 1 for the final calibrated parameters). The PEST calibration process reduced this initial objective function to 14.33 (approximately 29.2% of the initial weighted objective function), with individual contributions from the pressure, shear displacement and normal displacement of 3.13, 8.79, and 2.41, respectively (using the same weightings in Schwartzkopff et al. [13]). Using the calibrated parameters, the $R^2$ value was 0.8502 with a ratio of 1.0042 between the normalized measured data and the normalized model values (corresponding to a slight underestimation) and representing a good match.

| Parameter | Calibrated dynamic case |
|-----------|-------------------------|
| Damage zone elastic modulus $E$ (GPa) | 16.2 |
| Damage zone Poisson’s ratio $\nu$ | 0.34 |
| Intact zone elastic modulus $E$ (GPa) | 29.2 |
| Intact zone Poisson’s ratio $\nu$ | 0.33 |
| Density $\rho_s$ (kg/m$^3$) | 2364 |
| Porosity $n$ (%) | 14.25 |
| Fault damage zone permeability $k_f$ (m$^2$) | $1.40 \times 10^{-13}$ |
| Biot poroelastic constant $\alpha_{f,\text{biot}}$ | 0.78 |
| Initial hydraulic aperture $2h_0$ (m) | $4.71 \times 10^{-5}$ |
| Kappa factor $\kappa$ | 1.2 |
| Apparent normal stiffness $\overline{k}_n$ (GPa/m) | 28.0 |
| Apparent tangential stiffness $\overline{k}_t$ (GPa/m) | 12.5 |
| Dilation angle $\phi_d$ ($^\circ$) | 19 |
| Initial frictional coefficient $\mu_i$ | 0.95 |
| Final frictional coefficient $\mu_f$ | 0.80 |
| Critical slip weakening distance $Slip_c$ (m) | $1.0 \times 10^{-3}$ |
| $a$ parameter | $2.70 \times 10^{-2}$ |
| $b$ parameter | $3.70 \times 10^{-2}$ |
| $D_c$ (m) | $2.14 \times 10^{-5}$ |
| Reference velocity $V_0$ (m/s) | $8.4 \times 10^{-8}$ |
| Number of fractures | 83 |
| Standard deviation of non-logarithmized fracture asperity heights $\sigma_h$ (m) | $2.0 \times 10^{-4}$ |
4.1. Simulation result with calibrated parameters

Figure 1 illustrates the results using the calibrated values, for both the mixed dynamic/static simulation and the static only simulation, compared to the measured in-situ data.

![Figure 1](image-url)

**Figure 1.** (a) Pressure, (b) shear displacement, and (c) normal displacement at the injection point over time for the calibrated dynamic and static simulations.

4.2. Model verification in terms of aseismic to seismic slip ratio

The ratio of dynamic shear movement to total shear movement was 1.10%. The dynamic shear movement to total shear movement was calculated by the absolute cumulative weighted average shear movement along the fault during the dynamic analysis and compared to the total absolute cumulative weighted average shear movement along the fault. This is a reasonable value since other numerical results using rate and state friction state reported this ratio was approximately 1% for low amounts of seismic activity, with seismic magnitude ~0.3 [19].

The seismic moment was calculated at every time step:

\[ M_0 = G |\Delta u_s|A \]  

where \( G \) is the shear modulus, \( |\Delta u_s| \) is the absolute weighted shear displacement change from in-situ conditions for the slipping area \( A \). The seismic magnitude can be approximated from the seismic moment using the following expression [20]:

\[ M_w = \frac{2}{3} \log_{10} (M_0) - 6.0 \]

The seismic magnitude for both total and dynamic only can be seen in Figure 2. Interestingly, the final value for the dynamic only seismic magnitude was -1.97, which agrees with the calculated value of approximately below -2 for the in-situ experiment. Demonstrating that, as with the in-situ experiment, the slip was mostly aseismic in the simulation. In addition, the second dynamic event is at approximately 1,110 seconds, where the in-situ experiment recorded initial seismicity at about 1,100 seconds.
4.3. Simulated near field released energy

The near field released energy is estimated from [21]:

\[ E_{nf} = \frac{A}{2} \rho_s V_s \int \dot{d}(t)^2 \, dt \]

where \( A \) is the fault slip area, \( \rho_s \) is the density, \( V_s \) is the shear wave velocity, \( \dot{d} \) is the velocity, and \( t \) is the time.

The results are shown in Figure 3, where the majority (~75%) of the near field released energy is produced from the dynamic analysis. This illustrates why it is necessary to distinguish and model dynamic and quasi-static shear movements of fluid injection-induced seismicity. This assists the quantification of the magnitude and seismic energy released accurately. Via this quantification, the risk of seismicity can be assessed properly, since aseismic fault slip do not cause damage to structures on the surface.

5. Conclusions

In this study a static and dynamic coupled X-FEM approach is presented to predict the pressure and movement of an in-situ experiment. By using history matching the simulated values produced became close to the measured data. The dynamic analysis was used when instability conditions were met, using a direct implication of rate and state friction. This calibrated model predicted the main seismic event and corresponding magnitude that was recorded in the in-situ experiment. This result emphasizes that under the in-situ experimental conditions that fault-slip was mainly aseismic, however <2% of the fault-slip was seismic. In addition, ~75% of the near field released energy in the simulation is from the dynamic analysis. This illustrates the importance of considering and modelling the main mechanisms that contribute to fault-slip, including the fluid exchange between the rock mass and the fault core (approximated, in this case, by a through-going discontinuity). This demonstrates the importance of estimating or measuring important rock mass and fault properties before fluid injection takes place. By predicting the onset of seismicity and seismic magnitudes caused by fluid injection this would assist in determining fluid injection protocols for industry. Extension of this code to three-dimensional analysis could be considered. This may lead to a better understanding of seismicity from fluid injection, which could assist with mitigating the risks of injecting fluid underground.

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