Multi-Pass Q-Networks for Deep Reinforcement Learning with Parameterised Action Spaces

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Abstract

Parameterised actions in reinforcement learning are composed of discrete actions with continuous action-parameters. This provides a framework for solving complex domains that require combining high-level actions with flexible control. The recent P-DQN algorithm extends deep Q-networks to learn over such action spaces. However, it treats all action-parameters as a single joint input to the Q-network, invalidating its theoretical foundations. We analyse the issues with this approach and propose a novel method—multi-pass deep Q-networks, or MP-DQN—to address them. We empirically demonstrate that MP-DQN significantly outperforms P-DQN and other previous algorithms in terms of data efficiency and converged policy performance on the Platform, Robot Soccer Goal, and Half Field Offense domains.

1 Introduction

Reinforcement learning (RL) and deep RL in particular have demonstrated remarkable success in solving tasks that require either discrete actions, such as Atari [Mnih et al., 2015], or continuous actions, such as robot control [Schulman et al., 2015; Lillicrap et al., 2016]. Reinforcement learning with parameterised actions [Masson et al., 2016] that combine discrete actions with continuous action-parameters has recently emerged as an additional setting of interest, allowing agents to learn flexible behavior in tasks such as 2D robot soccer [Hausknecht and Stone, 2016a; Hussein et al., 2018], simulated human-robot interaction [Khamassi et al., 2017], and terrain-adaptive bipedal and quadrupedal locomotion [Peng et al., 2016].

There are two main approaches to learning with parameterised actions: alternate between optimising the discrete actions and continuous action-parameters separately [Masson et al., 2016; Khamassi et al., 2017], or collapse the parameterised action space into a continuous one [Hausknecht and Stone, 2016a]. Both of these approaches fail to fully exploit the structure present in parameterised action problems. The former does not share information between the action and action-parameter policies, while the latter does not take into account which action-parameter is associated with which action, or even which discrete action is executed by the agent. More recently, Xiong et al. [2018] introduced P-DQN, a method for learning behaviours directly in the parameterised action space. This leverages the distinct nature of the action space and is the current state-of-the-art algorithm on 2D robot soccer and King of Glory, a multiplayer online battle arena game. However, the formulation of the approach is flawed due to the dependence of the discrete action values on all action-parameters, not only those associated with each action. In this paper, we show how the above issue leads to suboptimal decision-making. We then introduce a novel multi-pass method to separate action-parameters, and demonstrate that the resulting algorithm—MP-DQN—outperforms existing methods on the Platform, Robot Soccer Goal, and Half Field Offense domains.

2 Background

Parameterised action spaces [Masson et al., 2016] consist of a set of discrete actions, \( \mathcal{A}_d = [K] = \{k_1, k_2, ..., k_K\} \), where each \( k \) has a corresponding continuous action-parameter \( x_k \in \mathcal{X}_k \subseteq \mathbb{R}^{m_k} \) with dimensionality \( m_k \). This can be written as

\[
\mathcal{A} = \bigcup_{k \in [K]} \{a_k = (k, x_k) | x_k \in \mathcal{X}_k\}. \tag{1}
\]

We consider environments modelled as a Parameterised Action Markov Decision Process (PAMDP) [Masson et al., 2016]. For a PAMDP \( M = (\mathcal{S}, \mathcal{A}, P, R, \gamma) \): \( \mathcal{S} \) is the set of all states, \( \mathcal{A} \) is the parameterised action space, \( P(s'|s, k, x_k) \) is the Markov state transition probability function, \( R(s, k, x_k, s') \) is the reward function, and \( \gamma \in [0, 1) \) is the future reward discount factor. An action policy \( \pi : \mathcal{S} \to \mathcal{A} \) maps states to actions, typically with the aim of maximising Q-values \( Q(s, a) \), which give the expected discounted return of executing action \( a \) in state \( s \) and following the current policy thereafter.

The Q-PAMDP algorithm [Masson et al., 2016] alternates between learning a discrete action policy with fixed action-parameters using Sarsa(\( \lambda \)) [Sutton and Barto, 1998] with the Fourier basis [Konidaris et al., 2011] and optimising the continuous action-parameters using episodic Natural Actor Critic (eNAC) [Peters and Schaal, 2008] while the discrete action policy is kept fixed. Hausknecht and Stone [2016a] apply artificial neural networks and the Deep Deterministic Policy...
2.1 Parameterised Deep Q-Networks

Unlike previous approaches, Xiong et al. [2018] introduce a method that operates in the parameterised action space directly by combining DQN and DDPG. Their P-DQN algorithm achieves state-of-the-art performance using a Q-network to approximate Q-values used for discrete action selection, in addition to providing critic gradients for an actor network that determines the continuous action-parameter values for all actions. By framing the problem as a PAMDP directly, rather than alternating between discrete and continuous action MDPs as with Q-PAMDP, or using a joint continuous action MDP as with PA-DDPG, P-DQN necessitates a change to the Bellman equation to incorporate continuous action-parameters:

\[
Q(s, k, x_k) = \mathbb{E}_{r, s'} \left[ r + \gamma \max_{k'} \sup_{x_{k'}' \in \mathcal{X}_{k'}} Q(s', k', x_{k'}') | s, k, x_k \right].
\]  

(3)

To avoid the computationally intractable calculation of the supremum over \(\mathcal{X}_k\), Xiong et al. [2018] state that when the Q function is fixed, one can view \(\text{argmax}_{x_k} Q(s, k, x_k)\) as a function \(x_k^Q : S \rightarrow \mathcal{X}_k\) for any state \(s \in S\) and \(k \in [K]\). This allows the Bellman equation to be rewritten as:

\[
Q(s, k, x_k) = \mathbb{E}_{r, s'} \left[ r + \gamma \max_{k'} Q(s', k', x_k^Q(s')) | s, k, x_k \right].
\]  

(4)

P-DQN uses a deep neural network with parameters \(\theta_Q\) to represent \(Q(s, k, x_k; \theta_Q)\), and a second deterministic actor network with parameters \(\theta_x\) to represent the action-parameter policy \(x_k(s; \theta_x) : S \rightarrow \mathcal{X}_k\), an approximation of \(x_k^Q(s)\). With this formulation it is easy to apply the standard DQN approach of minimising the mean-squared Bellman error to update the Q-network using minibatches sampled from replay memory \(D\) [Mnih et al., 2015], replacing \(a\) with \((k, x_k)\):

\[
L_Q(\theta_Q) = \mathbb{E}_{(s, k, x_k, r, s') \sim D} \left[ \frac{1}{2} (y - Q(s, k, x_k; \theta_Q))^2 \right],
\]  

(5)

where \(y = r + \gamma \max_{k' \in [K]} Q(s', k', x_k(s'; \theta_x); \theta_Q)\) is the update target derived from Equation (4). Then, the loss for the actor network in P-DQN is given by the negative sum of Q-values:

\[
L_x(\theta_x) = \mathbb{E}_{s \sim D} \left[ - \sum_{k=1}^K Q(s, k, x_k(s; \theta_x); \theta_Q) \right].
\]  

(6)

Although this choice of loss function was not motivated by Xiong et al. [2018], it resembles the deterministic policy gradient loss used by PA-DDPG where a scalar critic value is used over all action-parameters [Hausknecht and Stone, 2016a]. During updates, the estimated Q-values are back-propagated through the critic to the actor, producing gradients indicating how the action-parameters should be updated to increase the Q-values.

3 Problems with Joint Action-Parameters

The P-DQN architecture inputs the joint action-parameter vector over all actions to the Q-network, as illustrated in Figure 1. This was pointed out by Xiong et al. [2018] but they did not discuss it further. While this may seem like an in-consequential implementation detail, it changes the formulation of the Bellman equation used for parameterised actions (Equation 4) since each Q-value is a function of the joint action-parameter vector \(x = (x_1, \ldots, x_K)\), rather than only the action-parameter \(x_k\) corresponding to the associated action:

\[
Q(s, k, x) = \mathbb{E}_{r, s'} \left[ r + \gamma \max_{k'} Q(s', k', x^{Q}(s')) | s, k, x \right].
\]  

(7)

This in turn affects both the updates to the Q-values and the action-parameters. Firstly, we consider the effect on the action-parameter loss, specifically that each Q-value produces gradients for all action-parameters. Consider for demonstration purposes the action-parameter loss (Equation 6) over a single sample with state \(s\):

\[
L_{x}(\theta_x) = - \sum_{k=1}^K Q(s, k, x(s; \theta_x); \theta_Q).
\]  

(8)

The policy gradient is then given by:

\[
\nabla_{\theta_x} x(s; \theta_x) = - \sum_{k=1}^K \nabla_{x} Q(s, k, x(s; \theta_x); \theta_Q) \nabla_{\theta_x} x(s; \theta_x).
\]  

(9)
Expanding the gradients with respect to the action-parameters gives

$$
\nabla_x Q = \left( \frac{\partial Q_1}{\partial x_1}, \frac{\partial Q_2}{\partial x_1}, \ldots, \frac{\partial Q_K}{\partial x_1}, \frac{\partial Q_1}{\partial x_2}, \ldots, \frac{\partial Q_K}{\partial x_2}, \ldots, \frac{\partial Q_1}{\partial x_K}, \ldots, \frac{\partial Q_K}{\partial x_K} \right),
$$

where $Q_k = Q(s, k, x(s; \theta_k); \theta_Q)$. Theoretically, if each Q-value was a function of just $x_k$ as the P-DQN formulation intended, then $\partial Q_k/\partial x_j = 0 \forall k, j \notin [K], j \neq k$ and $\nabla_x Q$ simplifies to:

$$
\nabla_x Q = \left( \frac{\partial Q_1}{\partial x_1}, \frac{\partial Q_2}{\partial x_2}, \ldots, \frac{\partial Q_K}{\partial x_K} \right).
$$

However this is not the case in P-DQN, so the gradients with respect to other action-parameters $\partial Q_k/\partial x_j$ are not zero in general. This is a problem because each Q-value is updated only when its corresponding action is sampled, as per Equation 5, and thus has no information on what effect other action-parameters $x_j, j \neq k$ have on transitions or how they should be updated to maximise the expected return. They therefore produce what we term false gradients. This effect may be mitigated by the summation over all Q-values in the action-parameter loss, since the gradients from each Q-value are summed and averaged over a minibatch.

The dependence of Q-values on all action-parameters also negatively affects the discrete action policy. Specifically, updating the continuous action-parameter policy of any action perturbs the Q-values of all actions, not just the one associated with that action-parameter. This can lead to the relative ordering of Q-values changing, which in turn can result in suboptimal greedy action selection. We demonstrate a situation where this occurs on the Platform domain in Figure 2.

4 Multi-Pass Q-Networks

The naïve solution to the problem of joint action-parameter inputs in P-DQN would be to split the Q-network into separate networks for each discrete action. Then, one can input only the state and relevant action-parameter $x_k$ to the network corresponding to $Q_k$. However, this drastically increases the computational and space complexity of the algorithm due to the duplication of network parameters for each action. Furthermore, the loss of the shared feature representation between Q-values may be detrimental.

We therefore consider an alternative approach that does not involve architectural changes to the network structure of P-DQN. While separating the action-parameters in a single forward pass of a single Q-network with fully connected layers is impossible, we can do so with multiple passes. We perform a forward pass once per action $k$ with the state $s$ and action-parameter vector $xe_k$ as input, where $e_k$ is the standard basis vector for dimension $k$. Thus $xe_k = (0, \ldots, 0, x_k, 0, \ldots, 0)$ is the joint action-parameter vector where each $x_j, j \neq k$ is set to zero. This causes all false gradients to be zero, $\partial Q_k/\partial x_j = 0$, and completely negates the impact of the network weights for unassociated action-parameters $x_j$ from the input layer, making $Q_k$ only depend on $x_k$. That is,

$$
Q(s, k, xe_k) \cong Q(s, k, x_k).
$$

Both problems are therefore addressed without introducing any additional neural network parameters. We refer to this as the multi-pass Q-network method, or MP-DQN.

A total of $K$ forward passes are required to predict all Q-values instead of one. However, we can make use of the parallel minibatch processing capabilities of artificial neural networks, provided by libraries such as PyTorch and Tensorflow, to perform this in a single parallel pass, or multi-pass. A multi-pass with $K$ actions is processed in the same manner as a minibatch of size $K$:

$$
\begin{pmatrix}
Q(s_i, \cdot, xe_1; \theta_Q) \\
\vdots \\
Q(s_i, \cdot, xe_K; \theta_Q)
\end{pmatrix}
= \begin{pmatrix}
Q_{11} & Q_{12} & \cdots & Q_{1K} \\
\vdots & \vdots & \ddots & \vdots \\
Q_{K1} & Q_{K2} & \cdots & Q_{KK}
\end{pmatrix},
$$

where $Q_{ij}$ is the Q-value for action $j$ generated on the $i^{th}$ pass where $x_i$ is non-zero. Only the diagonal elements $Q_{ii}$ are valid and used in the final output $Q_i \leftarrow Q_{ii}$. This process is illustrated in Figure 3.
5 Experiments

We compare the original P-DQN algorithm with a single Q-network against our proposed multi-pass Q-network (MP-DQN), as well as against separate Q-networks (SP-DQN). We also compare against Q-PAMDP and PA-DDPG, the former state-of-the-art approaches on their respective domains. We are unable to use King of Glory as a PA-DDPG, the former state-of-the-art approaches on their re-

Experiments

Compared to separate Q-networks, our multi-pass technique introduces a relatively minor amount of overhead during forward passes. Although minibatches for updates are similarly duplicated $K$ times, backward passes to accumulate gradients are not duplicated since only the diagonal elements $Q_{ii}$ are used in the loss function. The computational complexity of this overhead scales linearly with the number of actions and minibatch size during updates. Unlike separate Q-networks (and even when a larger Q-network with more hidden layers and neurons is used) if the number of actions does not change, then the overhead of multi-passes would be the same as with a smaller Q-network, provided the minibatch is of a reasonable size and can be processed in parallel.

We introduce a passthrough layer to the actor networks of P-DQN and PA-DDPG to initialise their action-parameter policies to the same linear combination of state variables that Masson et al. [2016] use to initialise the Q-PAMDP policy. The weights of the passthrough layer are kept fixed to avoid instability; this does not reduce the range of action-parameters available as the output of the actor network compensates before inverting gradients are applied. We use an $\epsilon$-greedy discrete action policy with additive Ornstein-Uhlenbeck noise for action-parameter exploration, similar to Lillicrap et al. [2016], which we found gives slightly better performance than Gaussian noise.

5.1 Platform

The Platform domain [Masson et al. 2016] has three actions—run, hop, and leap—each with a continuous action-parameter to control horizontal displacement. The agent has to hop over enemies and leap across gaps between platforms to reach the goal state. The agent dies if it touches an enemy or falls into a gap. A 9-dimensional state space gives the position and velocity of the agent and local enemy along with the horizontal and vertical position of the agent. The agent has a collision mask to detect collisions and avoid enemies. The agent has a collision mask to detect collisions and avoid enemies. The agent has a collision mask to detect collisions and avoid enemies.

Complete source code is available online.1

1https://github.com/cycraig/MP-DQN
5.2 Robot Soccer Goal

The Robot Soccer Goal domain [Masson et al., 2016] is a simplification of RoboCup 2D [Kitano et al., 1997] in which an agent has to score a goal past a keeper that tries to intercept the ball. The three parameterised actions—kick-to, shoot-goal-left, and shoot-goal-right—are all related to kicking the ball, which the agent automatically approaches between actions until close enough to kick again. The state space consists of 14 continuous features describing the position, velocity, and orientation of the agent and keeper, and the ball’s position and distance to the keeper and goal.

Training consisted of 100 000 episodes, using the same hyperparameters for Q-PAMDP as Masson et al. [2016] except we set $\alpha_{\text{ENAC}} = 0.06$ and $\sigma = 0.0001$. P-DQN uses a single hidden layer (256), with $\alpha_Q = 10^{-3}$, $\alpha_x = 10^{-5}$, $\tau_Q = 0.1$, $\tau_x = 0.001$, and $B = 128$. Two hidden layers (128, 64) are used for PA-DDPG, with $\alpha_Q = 10^{-4}$, $\alpha_x = 10^{-5}$, $\tau_Q = 0.01$, $\tau_x = 0.01$, and $B = 64$. Both algorithms use a replay memory size of 20 000, $\gamma = 0.95$, gradients clipping at 1, and the same action-parameter policy initialisation as Q-PAMDP with additive Ornstein-Uhlenbeck noise.

5.3 Half Field Offense

The third and final domain, Half Field Offense (HFO) [Hausknecht and Stone, 2016a], is also the most complex. It has 58 state features and three parameterised actions available: dash, turn, and kick. Unlike Robot Soccer Goal, the agent must first learn to approach the ball and then kick it into the goals, although there is no keeper in this task.

We use 30 000 episodes for training on HFO. This is more than the 20 000 episodes (or roughly 3 million transitions) used by Hausknecht and Stone [2016a] and Xiong et al. [2018] so that ample opportunity is given for the algorithms to converge in order to fairly evaluate the final policy performance. We use the same network structure as previous works with hidden layers of (256, 128, 64) neurons for P-DQN and (1024, 512, 256, 128) neurons for PA-DDPG. The leaky ReLU activation function with negative slope 0.01 is used on HFO because of these deeper networks. Xiong et al. [2018] use 24 asynchronous parallel workers for n-step returns on HFO. For fair comparison and due to the lack of sufficient hardware, we instead use mixed n-step return targets [Hausknecht and Stone, 2016b] with a mixing ratio of $\beta = 0.25$ for both Q-PAMDP and PA-DDPG, as this technique does not require multiple workers. The $\beta$ value was selected after a search over $\beta \in \{0, 0.25, 0.5, 0.75, 1\}$. We otherwise use the same hyperparameters as Hausknecht and Stone [2016b] apart from the network learning rates: $\alpha_Q = 10^{-3}$, $\alpha_x = 10^{-5}$ for P-DQN and $\alpha_Q = 10^{-3}$, $\alpha_x = 10^{-3}$ for PA-DDPG. In the absence of an initial action-parameter policy, we use the same $\epsilon$-greedy with uniform random action-parameter exploration strategy as the original authors. In general we kept as many factors consistent between the two algorithms as possible for a fair comparison.

We select 10 of the most relevant state features for Q-PAMDP to avoid intractable Fourier basis calculations. These features include: player orientation, stamina, proximity to ball, ball angle, ball-kickable, goal centre position, and goal centre proximity. Even with this reduced selection, we could not achieve a satisfying performance.
found at most a Fourier basis of order 2 could be used. We use an adaptive step-size [Dabney and Barto, 2012] for Sarsa(λ) with an eNAC learning rate of 0.2. The Q-PAMDMP agent initially learns with Sarsa(λ) for a period of 1000 episodes before alternating between $\kappa = 50$ eNAC updates of 25 rollouts each, and 1000 episodes of discrete action re-exploration.

### 6 Results

The resulting learning curves of MP-DQN, SP-DQN, P-DQN, PA-DDPG, and Q-PAMDMP on the three parameterised action benchmark domains are shown in Figure 4, with mean evaluation scores detailed in Table 1.

| Algorithm            | Platform Return | Robot Soccer Goal P(\text{Goal}) | Half Field Offense P(\text{Goal}) | Avg. Steps to Goal |
|----------------------|-----------------|-----------------------------------|------------------------------------|--------------------|
| Q-PAMDMP             | 0.789 ± 0.188   | 0.452 ± 0.093                     | 0 ± 0                              | n/a                |
| PA-DDPG              | 0.284 ± 0.061   | 0.006 ± 0.020                     | 0.875 ± 0.182                      | 95 ± 7             |
| P-DQN                | 0.964 ± 0.068   | 0.701 ± 0.078                     | 0.883 ± 0.085                      | 111 ± 11           |
| SP-DQN               | 0.941 ± 0.164   | 0.752 ± 0.131                     | 0.718 ± 0.131                      | 99 ± 7             |
| MP-DQN               | 0.987 ± 0.039   | 0.789 ± 0.070                     | 0.913 ± 0.070                      | 99 ± 12            |
| PA-DDPG$^2$          | -               | 0.923 ± 0.073                     | 112 ± 5                           |                    |
| Async. P-DQN$^3$     | -               | 0.989 ± 0.006                     | 81 ± 3                            |                    |

Table 1: Mean evaluation scores over 30 random runs for each algorithm, averaged over 1000 episodes after training with no random exploration. We include previously published results from Hausknecht and Stone [2016a] and Xiong et al. [2018] on HFO, although they are not directly comparable with ours as we use a longer training period and have a much larger sample size of agents—30 versus 7 and 9 respectively—and asynchronous P-DQN uses 24 parallel workers to implement n-step returns rather than the mixing strategy we use.

### 7 Related Work

Many recent deep RL approaches follow the strategy of collapsing the parameterised action space into a continuous one. Hussein et al. [2018] present a deep imitation learning approach for scoring goals on HFO using long-short-term-memory networks with a joint action and action-parameter policy. Agarwal [2018] introduces skills for multi-goal parameterised action space environments to achieve multiple related goals; they demonstrate success on robotic manipulation tasks by combining PA-DDPG with hindsight experience replay and their skill library.

One can alternatively view parameterised actions as a 2-level hierarchy: Klimek et al. [2017] use this approach to learn a reach-and-grasp task using a single network to represent a distribution over macro (discrete) actions and their lower-level action-parameters. The work most relevant to this paper is by Wei et al. [2018], who introduce a parameterised action version of TRPO (PATRPO). They also take a hierarchical approach but instead condition the action-parameter policy on the discrete action chosen to avoid predicting all action-parameters at once. While their preliminary results show the method achieves good performance on Platform, we omit comparison with PATRPO as it fails to learn to score goals on HFO.

### 8 Conclusion

We identified a significant problem with the P-DQN algorithm for parameterised action spaces: the dependence of its Q-values on all action-parameters causes false gradients and can lead to suboptimal action selection. We introduced a new algorithm, MP-DQN, with separate action-parameter inputs which demonstrated superior performance over P-DQN and former state-of-the-art techniques Q-PAMDMP and PA-DDPG. We also found that PA-DDPG was unstable and converged to suboptimal policies on some domains. Our results suggest that future approaches should leverage the disjoint nature of parameterised action spaces and avoid simultaneous optimisation of the policies for discrete actions and continuous action-parameters.

$^2$Average over 7 runs [Hausknecht and Stone, 2016a].

$^3$Average over 9 runs with 24 workers [Xiong et al., 2018].
Acknowledgments

This work is based on the research supported in part by the National Research Foundation of South Africa (Grant Number: 113737).

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