Dynamic, fatigue and harmonic analysis of a beam to beam system with various cross-sections under impact load

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ABSTRACT

Contact of materials is inevitable in machine environment where a number of elements interact with each other causing impact force and consequent stress and strain at point of contact. The study of this behaviour is essential for machine design applications as it helps to predict life of a particular element as per the amount of stress it can sustain. This study is based on dynamic, fatigue and harmonic analysis of two-beams in contact with a combination of cross-section geometries of the beams. ANSYS, which is a Finite Element Analysis software is used for numerical modelling of stresses and deformations developed within the two-beam system. Three different combination of beam cross-sections were simulated: square-square, circular-circular and square-circular. Under dynamic and fatigue analysis, the results show that the least deformation coupled with the highest fatigue life and safety factor occurs when one beam is square and the other is circular (Square-circular), a slightly higher deformation occurs when two square beams (square-square) interact with each other while the highest deformation occurs when two circular beams interact with each other. This shows that Square cross-section beams have higher bending resistance while circular cross-section beams have low bending resistance and thus higher deformation. Under vibration analyses, due to higher natural frequencies, both the square-square and square-circular section beams show excellent durability and better margin of safety to the same extent during vibration. The results of this work will be useful in enlightening design engineers on the combination of beam cross sections that will give optimum performance in a beam-to-beam system for structural and machine design applications.

1. Introduction

Cantilever beams serve as a reference for various structural applications. This is verified by numerous studies in mathematical and numerical modelling of structures in terms of beams [1, 2, 3]. Analysing the phenomenon of beams has been widely studied in literature, however, contact of various beams still requires an in-depth research and analysis. Contact of two or more beams is considered an important phenomenon in various application relating to moving parts of machines, robotic arms, building structures etc [4, 5]. In addition, one of the common examples of beams in contact is stringing of a tennis racket. Figure 1 below displays the 3D geometrical model of two cantilever beams making contact during vibrations.

In Engineering applications, contact between materials is inevitable and there are plenty of examples and techniques in literature that analyze and cater for such conditions.

Some researchers have studied Contact of beams in terms of vibration and frequency analysis using both numerical and experimental approaches [6]. Complex eigenvalue and dynamic transient analyses were performed to understand the linear and non-linear aspects of a frictional contact between the two beams. The results show a good correlation between numerical and experimental results.

Specific formulations have been proposed for interaction of beam elements [7]. The authors presented the theory related to the point interaction among circular cross section beams without any distinction of target body of contact. Both beams are considered to be 3D curves and
the smallest distance between the 3D curves is measured, which is called the gap function. Later, this method was redefined for frictional contacts [8] and validation for square cross section of beams was presented [9].

The interaction between curves was evaluated by using a Frenet coordinate system that describes kinematics, and also a term that defines relative torsion among the curves [10]. Thereafter, the authors went on to present a concrete mathematical correlation for a wide range of beam contacts [11]. Further, a beam to flat surface interaction was studied for DOF in rotation of a beam [12]. The study is based on the moment induced due to friction of contact. This study was practically analysed for study of offshore riser structures interacting with the seabed [13].

Furthermore, mathematical formulation of pointwise contact between beams at the edges was presented [16]. Later, a mathematical model analysing the basic geometric parameters involved in beam-to-beam contact was also presented [17]. In addition to contact analysis, research has also been proposed by considering pin-on-disc contact as beam-on-surface for analysis of wear and its inter-related properties [18, 19, 20].

A transition procedure between point-to-point and line-to-line contact schemes in terms of contact force and contact energy was presented [24]. The authors observed that the presented algorithm produces a small number of potential contact pairs and allows for subdivision into potential point and line contact pairs, which is required to fully exploit the efficiency potential of the proposed contact formulation.

Some group of researchers have examined the contact dynamics of beams experiencing significant overall motion with large deformations when making self-contact [25]. The contact procedure presented consists of a contact search algorithm combined with two approaches for imposing contact constraints. The results show that the employed contact detection method was found to be sufficiently accurate when used in conjunction with the researched contact constraint imposition models in simulation of the contact dynamics. It was also demonstrated that in the case of studied 2D problems, the optimization-based complementarity problem method is computationally more economical than the classical penalty method.

The Numerical method for the solution of pointwise contact between surfaces has been presented [26]. The primary contribution of the study is in the discussion, characterization, and algorithmic solution proposal of the Local Contact Point in the context of the master-master contact between surfaces. Using mortar method, the formulations for the static frictionless beam to beam contact was presented [27]. The authors discovered the mortar method is suitable and appropriate for modeling beam-to-beam contact and self-contact. It was concluded that the employed mortar method is suitable for future research.

Some researchers have proposed a nonlinear model for the dynamic analysis of vehicle-cableway bridge systems [28]. The authors employed an updated Lagrangian expression and the virtual work principle to present the nonlinear equations of motion by considering cableway bridges and vehicles as a whole dynamic system. The authors observed and concluded that the proposed method can efficiently perform a nonlinear dynamic analysis of a vehicle-cableway bridge system.

Some Researchers have studied the harmonic analysis of a cantilever beam with and without cracks [29]. The impact of crack parameters (crack position and crack depth) on the vibration parameters of the cracked cantilever beam were inspected and compared with uncracked beam by unique systems using finite element analysis (FEA) in SAP2000 software. The authors found out that displacement was greater for cracked beams than uncracked beams due to the reduction in stiffness of cracked beam. Lastly, the eigen frequencies of cracked beam were found to be lower than that of uncracked beam with the difference depending on the location of the crack from the fixed support Fatigue failure of materials refers to their failure under the action of cyclic elastic stress. Generally, fatigue involves the formation and gradual growth of cracks which ultimately leads to fracture as a result of reduced load carrying capacity of the structure. Some researchers have investigated the fatigue analysis of an I section beam using ANSYS software [30]. The authors fixed the beam at the end area so as to behave like a cantilever for the first scenario and was thereafter fixed on both sides so as to behave like a fixed beam in the second scenario. The results show that the maximum deflection, equivalent stress, and factor of safety occurred at the end of the beam in the cantilever condition and at the top of the beam in the fixed condition.

The design and analysis of a U-notched fatigue sensor to predict fatigue life before catastrophic failure occurs was presented [31]. The authors employed finite element modelling in ANSYS workbench and 7075-T6 aluminium alloy. The results show that increasing the U-notch radius increased fatigue life while the maximum equivalent stress, on the other hand, decreased as the notch radius increased. Lastly, fatigue damage was observed around the U-notch radius when it has a value less than or equal to 6.4 mm.

Some researchers have statically and cyclically loaded Miniaturized cantilever beams made of cement paste with different water/cement ratios to carry out the preliminary study on the fatigue behaviour of the cement paste at the micrometre level [32]. The authors found out that for...
1000 loading cycles, little damage was observed in the cantilever beams under applied stress levels ranging from 50% to 70%. The authors concluded that the test method used in the study can reliably produce cement paste’s micromechanical properties.

A method for predicting the crack growth and fatigue life of an Acrylonitrile Butadiene Styrene (ABS) cantilever beam produced using fused deposition modelling was proposed [33]. Three beam configurations based on length (L = 110, 130, and 150 mm) are considered during the study. When the residual fatigue life of the ABS is compared to that of a similar configuration metallic structure (Aluminium 1050), the authors found out that the ABS material has a longer residual fatigue life than the metallic structure at the same frequency drop.

From previously published works, studies related to contact of materials and the associated forces and heat distribution have been extensively analysed but with little attention to the simulation of stresses, deformations, and fatigue life evaluation of the beam-to-beam system under different cross-sectional geometries. Even, some of the related available literature deal with either circular cross-section beams or one beam in contact with other bodies [14, 15]. Hence, there is a need to explore the dynamics of the beam-to-beam system with different cross-sectional geometries.

In this work, a beam-to-beam system has been analysed using different combinations of three different cross-section geometries of the beams. The variety of the cross-sectional geometries is to replicate industrial machinery environment and structural applications where various beam structures with different cross-sections come in contact with each other. The beams are designed in such a way that one beam impacts the other with a contact force, which in turn leads to the second beam applying a reaction force to the first beam. The simulation is done using ANSYS Workbench and the results of the dynamic, fatigue and vibration analyses (modal and harmonic) of the system of the two beams in contact are presented.

2. Methodology

2.1. The governing equations

2.1.1. Euler-Bernoulli beam theory

The Euler–Bernoulli beam theory is a popular beam theory that is commonly used in the natural vibration of a cantilever beam. Let’s take each of the two beams in this study as a Euler-Bernoulli uniform cantilever beam with width “a”, length “L”, thickness “b”, Young’s modulus E, density ρ undergoing transverse vibration under the influence of a time dependent load P as shown in Figure 2 below.

![Figure 2. A cantilever undergoing transverse vibration.](image)

The cantilever beam is oriented in such a way that the length is along the x-principal axis, y is the vertical axis, and z is the lateral axis. The governing equations of motion for the flexural vibrations of the cantilever beam, where w is the cantilever beam deflection in the y-direction as a function of x and t can be represented as shown in Eq. (1) below [21]:

\[
EI \frac{\partial^4 w(x,t)}{\partial x^4} + c \frac{\partial w(x,t)}{\partial t} + \rho A \frac{\partial^2 w(x,t)}{\partial t^2} = P(x,t)
\]  (1)

Further, Eqs. (2), (3) and (4) below present the natural frequency, elastic strain and resonant frequency of the vibrating cantilevers respectively.

\[
\omega = \sqrt{\frac{K}{M}}
\]  (2)

\[
\varepsilon = \frac{\sigma}{E}
\]  (3)

\[
\omega_n = \frac{\omega}{\sqrt{1 - \frac{\rho c^2}{24EI}}}
\]  (4)

where:

- \(w(x,t)\): Cantilever lateral deflection
- \(I\): moment of inertia about the y-axis
- \(A\): Cross sectional area of the cantilever
- \(c\): Damping coefficient
- \(\rho\): Density of the cantilever
- \(\omega\): Natural frequency
- \(\omega_n\): Resonant frequency
- \(P(x,t)\): Force applied on the cantilever

2.1.2. Fatigue failure analysis

Fatigue failure of materials refers to their failure under the action of cyclic elastic stress. Fatigue generally involves the formation and gradual growth of cracks and ultimately to fracture as a result of reduced load carrying capacity. For the purpose of this work, we estimate the fatigue safety factor of the vibrating cantilever beams using an appropriate failure curve on the modified Goodman diagram which is referred to as the DE-Goodman criteria in Eq. (5) as follows:

\[
\frac{1}{n} = \frac{\sigma}{S_e} + \frac{\sigma}{S_u}
\]  (5)

Additionally, Eqs. (6), (7) and (8) below represent the Von-mises mean stress, Von-mises amplitude stress and yielding factor of safety respectively.

\[
\sigma'_m = \frac{1}{2} (\sigma_{max} + \sigma_{min})
\]  (6)

\[
\sigma'_a = \frac{1}{2} (\sigma_{max} - \sigma_{min})
\]  (7)

\[
n_e = \frac{S_y}{\sigma_{max}}
\]  (8)

where:

- \(n\): Fatigue safety factor
- \(\sigma'_m\): Von-mises amplitude stress
- \(\sigma'_m\): Von-Mises mean stress
- \(S_e\): Endurance strength
- \(S_u\): Ultimate strength
- \(n_e\): Yielding factor of safety
- \(S_y\): Yielding strength
- \(\sigma_{max}\): Von-Mises maximum stress

Additionally, To predict the fatigue life, the fatigue life relation based on S–N curve of structural steel is employed as based in the Eq. (9) below:

\[
\log_{10} N = \log_{10} a - m \log_{10} S
\]  (9)

where \(N\) is the fatigue life in cycles, \(S\) is the stress amplitude for failure and \(a\) and \(m\) are material constants.

The S–N curve for structural steel used by ANSYS for the study is presented in Figure 3 below.
2.1.3. Beam contact analysis

Contact is regarded as the physical enforcement of the condition that no point in space can possibly be occupied by more than one body at the same time [22]. For the purpose of this study, let’s consider two elemental bodies from the two respective beams that can be labeled as contactor/contact and target as displayed in the Figure 4 below.

The fundamental contact condition along the interacting boundaries is such that no material overlap can occur.

The contact problem between the two cantilever beams in the current study is solved by ANSYS using the robust Lagrange Multiplier Method which is formulated below:

$$\pi_{LM} = \pi^b + \pi^c$$ (11)

Substituting Eq. (10) into (11) to give Eq. (12) for the total potential energy of both beams as presented below:

$$n_{LM}(u) = \pi^b(u) + \lambda^T g^c(u)$$ (12)

With the stated conditions in Eqs. (13), (14) and (15) as presented below:

$$g^c(u) > 0$$ (13)

$$\lambda \leq 0$$ (14)

$$\lambda^T g^c(u) = 0$$ (15)

where:

- $\lambda$ Lagrange multipliers
- $g^c(u)$ Contact constraints functions
- $\pi^b$ Potential energy of both beams
- $\pi^c$ Contact contribution of global Potential energy
- $n_{LM}$ Lagrangian

2.2. The numerical simulation

In this work, we have carried out the dynamic, fatigue and harmonic analysis of interaction of two beams in contact with different cross-sections. The 3D model of the 2 meshed beams in contact is displayed in Figure 5 below. Also, the geometrical and material properties which are common for both beams are presented in Table 1.

Figure 3. S-N curve of structural steel from ANSYS.

Figure 4. The general beam contact problem [23].

The fundamental contact condition along the interacting boundaries is such that no material overlap can occur.

The contact problem between the two cantilever beams in the current study is solved by ANSYS using the robust Lagrange Multiplier Method which is formulated below:

$$n_{LM}^i = \lambda g^c_i(u)$$ (10)

Figure 5. Mesh of two beams (Circular-Circular cross-section).
The numerical simulation is carried out using ANSYS Workbench toolbox. As shown in the Table above, materials for both beams are selected to be Structural steel, which is a default material in ANSYS Workbench. The modelling of both beams is developed with 0.2 m cross sections for each beam (for both circular and square). However, it is worth mentioning at this juncture that in many practical applications, the cross section dimensions of the beams could be 0.2 m, 0.3 m or more or even less depending on the design requirements and the kind of application. Thus, the 0.2 m dimension selected for the beam cross-section in this work is just for the purpose of this study, not to specifically replicate a particular application situation of the beam to beam system. Further, a connection between the two beams is created. The connection type selected is a frictionless contact for all the three combinations of the beam cross-sections. Meshing is done for both beams and all the necessary boundary conditions are specified. In the boundary conditions, far edges of both beams are selected as fixed supports in order for the beams to act like cantilevers. Further, time dependent (transient) forces presented in Table 3 below are applied to the top beam in the downward direction in order to fluctuate it and make contacts with the other beam. The meshing details for both of the cantilever beams are displayed in Table 2 below and Table 3 below shows the time history of the force. The location of the force is selected to be the top edge of the top beam’s cross-sectional face.

For each of the dynamic and modal analysis, the following set of procedures is followed on ANSYS Workbench.

2.3. Dynamic and fatigue analysis (Transient Structural)

1. Select ‘Transient Structural’ from the list of Analysis systems on the left-hand side.
2. Select structural steel as the material of the beams (it is a default material in ANSYS).
3. Create 3D models of both beams using the dimension (10 m by 0.2 m by 0.2m). Place the top beam over the other beam with a distance of 0.2 m in between and finally drag the lower beam away from the top one until only 1 m edge of the lower beam is just below the top beam. Same steps are followed for all the cross-sections.
4. Once a correct 3D model of the two-beam system is achieved, create a ‘Manual Connection’ which is a ‘bonded’ connection by default and change it to ‘frictionless.’
5. Select the Bottom face of the top beam to be the ‘Contact Region’ and the top face of the bottom beam to be the ‘Target Region’.
6. Go to Advance settings and select ‘Pinball region’ to ‘radius’ and ‘Pinball radius’ to 0.2 m.
7. Mesh the two beams 3D model and apply sizing to refine the mesh.
8. Go to ‘Analysis Settings’ and turn off ‘Auto Time Stepping,’ select 4 steps, and fill each step as per Table 2.
9. Insert fixed supports at the far ends of the two beams. Also, insert a downward force at the edge of the contact end of the top beam. Fill in the transient force as “tubular data” as per Table 2.
10. Solve the model for ‘Total Deformation,’ ‘equivalent stress’ and ‘equivalent elastic strain’ and review all the results.
11. Select the fatigue tool, select all necessary fatigue metrics and solve the model for the fatigue analysis.

2.3.1. Modal and harmonic analysis

Modal analysis was also performed on the interaction of circular beams using ANSYS. The following procedures are used.

1. Select ‘Modal’ from the list of Analysis systems on the left-hand side.
2. Create 3D Geometry of the beams using the same steps as in the dynamic analysis.
3. Insert a ‘Manual Connection’ and change the type as described in the dynamic analysis.
4. Define the ‘Contact Region’ and ‘Target Region.’
5. Use a pinball radius of 0.2m.
6. Mesh the two beams 3D model and apply sizing to refine the mesh.
7. In default analysis settings, number of modes to find are set to 5. You can change to more number modes to find the actual natural frequencies. Also, to have a better understanding of the actual natural frequencies with the modes, carryout harmonic analysis with a damping coefficient of 0.2.
8. Right click on Modal and insert ‘Fixed supports’ at the far ends of the two beams.
9. Finally, in the ‘Solution’ tab, insert ‘Total Deformation’ and solve for each mode and then review all the results.
3. Results and discussion

Having developed the 3D model in ANSYS design modeller, all the necessary boundary conditions are specified and the simulation is carried out for each of the dynamic and modal analysis of the beam-on-beam system. The simulated values for the resulting total deformation, maximum principal stress and natural frequencies for the first five modes of vibration are presented. The results are discussed in the following sections with respect to each of the dynamic and modal analysis carried out on the two beam system.

3.1. Dynamic and fatigue analysis (transient structural)

The simulated results recorded for a 2 s period of the total deformation and maximum principal stress developed in the beam-on-beam system with all the three combinations of cross sections are displayed in the Figures 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19 and 20 below.

1) Square-square section beams

The dynamic and fatigue analysis results of the square-square section beams are displayed in the Figures 6, 7, 8, 9, and 10 below.

Figure 6. Total deformation for square-square section beams.

Figure 7. Equivalent (von-mises) stress of square-square section beams.

Figure 8. Equivalent strain of square-square section beams.
For the square-square section beams, the maximum deformation experienced by the beam-on-beam system is 0.12,904 m while the equivalent (von-mises) stress and equivalent strain developed are 83.94 MPa and 0.00042 respectively. All these are shown in the Figures 6, 7 and 8 above. Also, as shown in Figures 9 and 10 above, the fatigue life and fatigue safety factor are $10^6$ cycles and 1.0269 respectively. As can be seen in the Figure 10 above, the lowest safety factor of the most stressed regions of the square-square section beams have a value that is greater than 1 and an infinite fatigue life. This means the square-square sections beams can sustain long term stress under the dynamic loading without failure.

2) Circular-circular section beams

The dynamic and fatigue analysis results of the circular-circular section beams are presented in the Figures 11, 12, 13, 14, and 15 below.

Figure 9. Fatigue life of square-square section beams.

Figure 10. Fatigue safety factor of square-square section beams.

Figure 11. Total Deformation for circular-circular section.
Figure 12. Equivalent (von-mises) stress of circular-circular section beam.

Figure 13. Equivalent strain of circular-circular section beams.

Figure 14. Fatigue life of circular-circular section beams.
Similarly, as shown in Figures 11, 12 and 13 for the circular-circular section beams, the maximum deformation suffered by the two-beam system is 0.28514 m while an equivalent stress of 200.84 MPa and an equivalent strain of 0.001009 is experienced by the beam system. Also, as shown in Figures 14 and 15, the fatigue life and fatigue safety factor of the circular-circular section beams are 25,242 cycles and 0.42919 respectively. As can be seen in the Figure 15 above, the fatigue safety factor at some regions of the circular-circular section beams is less than 1. This shows that circular section beams have low fatigue life and thus cannot sustain such long term cyclic stress under the dynamic loading without failure.

3) Square-circular section beams

The dynamic and fatigue analysis results of the circular-circular section beams are presented in the Figures 16, 17, 18, 19 and 20 below.

For the square-circular section which is illustrated in the Figure 16, the beam-on-beam system has a maximum deformation of 0.08371 m. Also, an equivalent stress of 62.47 MPa and equivalent elastic strain of 0.000315 exist within the beams system as shown in Figures 17 and 18 respectively. Further, as shown in Figures 19 and 20, the fatigue life and fatigue safety factor of the square-circular section beams are 10⁶ cycles and 1.3799 respectively. As can be seen in the Figure above, the lowest safety factor of the most stressed regions of the square-circular section beams has a value that is greater than 1 and infinite fatigue life. This means the square-circular sections beams can sustain long term stress under the loading without failure.

Figure 15. Fatigue safety factor of circular-circular section beams.

Figure 16. Total Deformation of square-circular section beams.

Figure 17. Equivalent (von-mises) stress of square-circular section beams.
3.2. Comparison of dynamic and fatigue analysis results

3.2.1. Deformation, stress and factor of safety

Comparing the dynamic analysis results of the three sections of the beam-on-beam systems shown in the Table 5 above, it can be clearly seen that the square-circular section beams show best performance due to its lowest deformation, equivalent stress and equivalent strain. This is due to the high moment of inertia offered by square cross-section beams which makes them more resistant to bending and deformation as compared to circular cross-section beams. Also, the reaction force provided by the target beam (lower beam) which depends on the cross sections of the beams meeting at the contact region plays an important role in increasing or decreasing the deformation values.

Comparing the fatigue analysis results for all the cross sectional combinations, it can be clearly seen that the square-circular section beam...
system is the safest as it has the highest fatigue safety factor and an infinite fatigue life followed by the square-square section beams and both can withstand long term stress under the dynamic loading without failure. This essentially can be attributed to the high moment of inertia possessed by square section beams which makes them experience lower stress concentrations during vibration. As a result, square section beams are significantly employed in many structural applications where resistance to bending and excessive vibrations are design requirements. Such areas of applications include but not limited to design and construction of building structures, beam connections in construction of bridges, industrial machinery complex, offshore oil rig, space structures and many more [4, 5, 7, 34].

As for the circular-circular section beams, the fatigue safety factor at some regions of the circular section beams is far less than 1 which means that circular-circular section beams have low fatigue life and thus cannot sustain long term stress under the dynamic loading without failure. This is due to the high stress concentrations and distributions on the circular section beams due to their low value of moment of inertia. Additionally, this can also be caused by the small size of the surface area of body curvature of the circular section beams meeting at the contact region which would significantly lead to high contact pressures and stress concentrations. This makes circular section beams unsuitable and less applicable in many structural applications as they would fail when subjected to high and long term stress. Consequently, Circular beams are most widely used in applications where high bending of beams is required, for example, electric switch contact and coupled electromechanical fields as they have low resistance to bending.

3.3. Modal analysis

1) Square-square section beams

The following presentations in Figures 21, 22, 23, 24 and 25 below display the simulated results representing the total deformations recorded for the square-square cross-section under modal analysis of the beam-on-beam contact system.

![Figure 21](image1.png)

**Figure 21.** Natural frequency and deformation at the first mode for square-square section.

![Figure 22](image2.png)

**Figure 22.** Natural frequency and deformation at the second mode for square-square section.

![Figure 23](image3.png)

**Figure 23.** Natural frequency and deformation at the third mode for square-square section.
As can be seen in the above set of Figures, the total deformation slightly decreases from a maximum value of 0.035699 m– 0.035675 m for the square-square section as the natural frequencies increase along the first five vibrating modes. This is also presented in Table 6 shown below. Figure 26 below shows the frequency response for the square-square section beams validating the natural frequencies.

2) Circular-circular section beams

Similarly, the following set of Figures (Figures27, 28, 29, 30 and 31) display the simulated results for the total deformations recorded for the circular-circular cross-section under modal analysis of the two beam system.

As shown in Figures27, 28, 29, 30 and 31, the deformation increases from 0.040297 m to 0.040379 m for the circular-circular section as the natural frequency increases along the first five modes of vibration. This is due to the low moment of inertia of circular beams which results in low resistance to bending and rotational acceleration of the beams. This is also presented in Table 7 below. Figure 32 shows the frequency response for the circular-circular section beams validating the natural frequencies.

3) Square-circular section beams

Further, the presentations in Figures33, 34, 35, 36 and 37 represent the simulated results for the total deformations generated for the circular-circular cross-section beams under modal analysis of the beam-on-beam system.

Similar to square-square section, the deformation values for the square-circular beams slightly decreases from 0.035699 m to 0.035676 m as the natural frequency increases for the first five modes of vibration. This can be explained using beam bending theory as it is largely due to the higher resistance offered to bending and rotational acceleration at higher natural frequencies by the square section beams. The results are also tabularized in Table 8 below. Also, Figure 38 below shows the frequency response for the square-circular section beams validating the natural frequencies.

3.3.1. Comparison of modal analysis results

Table 9 with Figure 39 and Table 10 with Figure 40 compare the deformations and the natural frequencies for the three cross-sectional combinations investigated in this study. To have a deeper understanding and accuracy of the first five frequencies, we have also presented the frequency response diagrams from the forced vibration analysis for all the three sectional combinations as shown in the next section. The essence of the frequency response diagram is to validate our values for the first five natural frequencies through resonance frequencies and also to compare the amplitudes of

Table 6. Modal analysis results for the square-square section during the first five modes.

| Modes | Frequency (Hz) | Deformation (m) |
|-------|---------------|-----------------|
| 1     | 1.6322        | 0.035699        |
| 2     | 10.21         | 0.035691        |
| 3     | 28.503        | 0.035684        |
| 4     | 55.617        | 0.035679        |
| 5     | 91.435        | 0.035675        |

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3.3.1. Comparison of modal analysis results

Table 9 with Figure 39 and Table 10 with Figure 40 compare the deformations and the natural frequencies for the three cross-sections as shown below. As shown in Table 9 and Figure 39, the circular-circular section experienced the highest deformation while the other two sections suffer the same level of deformation.

Also, Table 10 and Figure 40 above compares the first five natural frequencies of the three cross-sectional combinations investigated in this study. To have a deeper understanding and accuracy of the first five frequencies, we have also presented the frequency response diagrams from the forced vibration analysis for all the three sectional combinations as shown in the next section. The essence of the frequency response diagram is to validate our values for the first five natural frequencies through resonance frequencies and also to compare the amplitudes of
Figure 26. Frequency response for the square-square section of the beams.

Figure 27. Natural frequency and deformation at the first mode for circular-circular section.
Figure 28. Natural frequency and deformation at the second mode for circular-circular section.

Figure 29. Natural frequency and deformation at the third mode for circular-circular section.

Figure 30. Natural frequency and deformation at the fourth mode for circular-circular section.

Figure 31. Natural frequency and deformation at the fifth mode for circular-circular section.
vibration of each sectional combination of the beams. The frequency values at the first five crest points on the frequency response diagram validates the first five natural frequencies from modal analysis.

As presented in the Figure 39 and on Table 9, circular-circular section beams have larger deformations due to their low resistance to bending and vibrations. Also, as shown in the table Table 10 above, both the square-square section and the square-circular section beams have the higher set of natural frequencies than the circular-circular section. The reason for this is not far-fetched, it is due to the higher moment of inertial offered by square section beams which makes them more resistant to bending and thus vibrate at higher frequencies.

| Modes | Frequency (Hz) | Deformation (m) |
|-------|---------------|-----------------|
| 1     | 1.4129        | 0.040297        |
| 2     | 8.8428        | 0.040302        |
| 3     | 24.707        | 0.040317        |
| 4     | 48.265        | 0.040343        |
| 5     | 79.451        | 0.040379        |

Figure 32. Frequency response for the circular-circular section of the beams.
Figure 33. Natural frequency and deformation at the first mode for square-circular section.

Figure 34. Natural frequency and deformation at the second mode for square-circular section.

Figure 35. Natural frequency and deformation at the third mode for square-circular section.

Figure 36. Natural frequency and deformation at the fourth mode for square-circular section.
Although the square section beams have larger mass due to their larger cross section area and would be expected to have lower frequencies, the significant effect of the higher moment of inertia of the square beam cross sections plays important role in the high values of the natural frequencies of the square section beams. The practical implication of this is that square beam section beams would find significant applications in structures subjected to heavy loading and vibrations as they would offer higher durability, service life and optimum performance than circular section beams. This is because the higher natural frequencies of the square section beams would lead to a better margin of safety during vibration.

Finally, it can be easily observed that the natural frequencies of both the square-square section and the square-circular section beams have the same values. This is because natural frequency of a two beam system is based on the higher set of frequencies provided by the two beams in contact. Since the square beam (contact beam) in a square-circular cross section beam system will produce the higher set of frequencies when compared to the circular beam (target beam), hence its set of natural frequencies is representing the natural frequencies for the beam-to-beam system which will be similar to the frequencies of a square-square section beam system.

### 3.4. Harmonic analysis

The presentations in Figures 41, 42 and 43 below represent the simulated results plotted for the frequency response functions of the three cross-sectional combinations under harmonic analysis of the beam-to-beam system.

Figures 41, 42 and 43 above show the frequency-deformation, frequency-strain and frequency -stress response diagram of the square-square, circular-circular and square-circular section beams. It is worth noting that the results simulated and presented under the harmonic analysis is always built on the modal analysis results.

Observing the amplitudes of vibration as shown in the Figure 41, it is clear that both the square-square and square -circular section beams suffer the lower deformations (same) during forced vibration. Further, Considering the amplitude of vibration as shown in the Figures 42 and 43 above, it is also clear that both the square-square and square -circular section beams similarly suffer the same lower strains and stress during forced vibration. These can only be attributed to the resistance of the square section beams to forced vibration and this would result in lower stresses and strains experienced by the beam-to-beam system.

Finally, going by this analysis, both the square-square and square -circular section beams have shown excellent performance due to their lower deformations coupled with their higher set of natural frequencies especially at the important early modes of vibrations when compared to the circular-circular section beams. This practically means both the square-square and square-circular section beams are excellent for heavy loading and heaving vibrating conditions and can also sustain long term dynamic loading due to their strength to withstand deformation. Additionally, the higher natural frequencies of the square section beams would be beneficial by resulting in a better margin of safety for the square section beams during such vibration conditions.

On the other hand, due to their highest values of deformations, normal stress and normal strains, circular section beams can only be suitably applied in situations in which high bending and large deformations of beams is required. For instance, they are significantly used in electric switch contact, robotic arms in motion, electro-mechanical coupling [7, 22], vehicle cableway bridge, springs and ropes consisting of multiple strings and many more [27, 28, 54].

### 4. Conclusion

In the current study, the dynamic, fatigue and harmonic analysis of a beam-on-beam contact system and interaction have been carried out. The author analysed different cross-section of beams when in contact and discovered that the shape and cross-section of beams play a vital role in resisting bending, deformation and failure. Based on the results of the numerical simulations in this study, the following conclusions are made:

- Cross-section of beams is important when bending stresses are involved.
- Square cross-section beams have higher bending resistance and thus low deformation due to high moment of inertia.
- Circular cross-section beams have low bending resistance and thus high deformation due to high moment of inertia.
Figure 38. Frequency response for the square-circular section of the beams.

Figure 39. Comparison of total deformation (m) of the square-square, circular-circular and square-circular section beams.
- Combination of square and circular cross-section beams gives the best performance (lowest deformation, highest fatigue life and safety factor)
- Failure of interacting beams is far-fetched where square cross-section beams are used whereas failure is much more likely to occur with beams of circular cross-sections.

5. Limitation and recommendation

This study, however, is subject to some limitations. This research has only focused on investigating the dynamic, fatigue and harmonic analysis of a beam-to-beam system with different cross-sectional geometries all at the same cross-section length of 0.2m while the cross-sectional area is different. However, cases with equivalent cross-sectional areas but different cross section length can also be encountered in other structural applications but are not addressed in the current study.

Further, this research has only studied the overall dynamic, fatigue and harmonic analysis of the beam-to-beam system without paying attention to the detailed analysis of the contact forces and pressures at each combination of cross-sectional geometries of the beam-to-beam system. Also, only three cross section combinations have been studied in the current research while there are many more beam cross sections that could be potentially combined especially in structural applications.

| Modes | Square-Square | Circular-Circular | Square-Circular |
|-------|---------------|------------------|-----------------|
| 1     | 1.6322        | 1.4129           | 1.6322          |
| 2     | 10.21         | 8.8428           | 10.21           |
| 3     | 28.503        | 24.707           | 28.503          |
| 4     | 55.617        | 48.265           | 55.617          |
| 5     | 91.435        | 79.451           | 91.441          |

Table 10. Comparison of Frequency (Hz) for the three cross sections during the first five modes.

Figure 40. Comparison of Frequency (Hz) of the square-square, circular-circular and square-circular section beams.

Figure 41. Frequency-deformation response diagram of the three section beams.

Figure 42. Frequency-strain response diagram of the three section beams.

Figure 43. Frequency-stress response diagram of the three section beams.
Hence, for further study on this work, it is recommended that cases with equivalent cross-sectional areas but different cross section length should be fully investigated. Also, a detailed analysis of the contact forces and pressures should also be studied with many more cross-section combinations to replicate more structural and machinery arrangements.

Declarations

Author contribution statement

Saeed Asiri, Ph. D: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

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Data availability statement

Data will be made available on request.

Declaration of interests statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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