Waveguide and $\Gamma$-factor optimization for low-divergence ridge lasers

I K Boikov\textsuperscript{1}, A V Savelyev\textsuperscript{2}, A E Zhukov\textsuperscript{2}

\textsuperscript{1} Department of Applied Mathematics and Physics, St. Petersburg Academic University of the Russian Academy of Sciences, St. Petersburg 194021, Russia
\textsuperscript{2} Nanophotonics Lab, St. Petersburg Academic University of the Russian Academy of Sciences, St. Petersburg 194021, Russia

E-mail: ik.boikov@gmail.com

Abstract. The work is aimed for analytical approach to maximize ridge laser modal gain at a given beam divergence in a vertical direction. When the gain/loss profile of planar waveguide is known the equation for optimum near-field profile was obtained. Surprisingly, it is appeared to be Schrödinger type equation where normalized gain/loss profile plays role of potential energy. The developed approach was applied to a few examples widely met in practice. Also it was checked whether usage of leaky waveguides is justified or not.

1. Introduction

Semiconductor ridge emitting lasers have a broad range of applications from medical applications to material processing, therefore, lasers parameters optimization is a matter of particular interest. One of goals is maximizing Γ-factor, which means increased field profile confinement. But it’s shown that it results in rising far-field beam divergence, so a trade-off takes place. In this paper we look for analytical approach to this problem.

2. Analytical approach

Consider planar waveguide with TE-polarized wave propagating along $x$-direction and let $F(z)$ be its near-field profile of $y$-component of electric field, where $z$-axis points growth direction in planar waveguide. As it was shown in [1], measure of beam divergence can be introduced as squared average angle $\Theta$:

$$\Theta^2 = -k_0^2 \int_{-\infty}^{\infty} F(z) \frac{d^2 F}{dz^2} dz , \text{while} \int_{-\infty}^{\infty} |F(z)|^2 dz = 1 , \quad (1)$$

where $k_0$ is the light wave number in vacuum for the lasing wavelength $\lambda_0 = 2\pi/k_0$. Let $g(z)$ be gain profile. Modal gain $G$ can be found via:

$$G = \int_{-\infty}^{\infty} |F(z)|^2 g(z) dz . \quad (2)$$
In order to find best possible modal gain keeping the value of \( \Theta \) fixed we use variational approach with the Lagrange multipliers, which is described in [2]. To do so, we introduce functional \( S [F(z)] \) as well as Lagrange multipliers \( \rho \) and \( \lambda \):

\[
S [F(z)] = \int_{-\infty}^{\infty} |F(z)|^2 g(z) dz + \rho \left( \Theta^2 k_0^2 + \int_{-\infty}^{\infty} F(z) \frac{d^2 F}{dz^2}(z) dz \right) + \lambda \left( \int_{-\infty}^{\infty} |F(z)|^2 dz - 1 \right).
\]

Hence, we convert initial task of maximum value of \( G \) to the problem of finding extremum of \( S [F(z)] \). The conventional variational procedure lead us to the following equation on \( F(z) \):

\[
-\frac{d^2 F}{dz^2} - \frac{1}{\rho} F(z)g(z) = \frac{\lambda}{\rho} F(z).
\]

One can clearly see here, that this is Shr"{o}dinger eigenvalue problem where \(-\rho^{-1} g(z)\) plays role of potential energy. Hence we can use well developed methods to solve this problem for different \( g(z) \) and to interpret physical meaning of its solution.

We can introduce \( \alpha = \rho^{-1} \) as well as \( \xi = \lambda \rho^{-1} \) as only their quotient plays role. From this point onwards \( \alpha \) is considered a given constant.

\[
\frac{d^2 F}{dz^2} + F(z) (\alpha g(z) + \xi) = 0
\]

This is a Schr"{o}dinger’s equation for particle in potential \(-\alpha g(z)\) with energy \( \xi \). The variable \( \xi \) can be found during solution of this eigenproblem. Hence, only \( \alpha \) could be varied during optimization procedure. From point of view of Shr"{o}dinger equation varying \( \alpha \) would lead to changing of depth of “potential well” that corresponds to the gain region. Higher values of the \( \alpha \) corresponds to more tight localization of the field profile \( F(z) \) around gain region, higher \( \Gamma \)-factor and higher beam divergence. Still the solution at any fixed \( \alpha \) is optimal in the sense that \( \Gamma \) could not be better for a given \( g(z) \) profile and beam divergence. While \( g(z) \) is arbitrary, we cannot proceed any further analytically and bring here few examples.

3. Examples

3.1. Example 1: single gain layer

In that case gain be introduced as \( g(z) = A \delta(z) \). Solution of (5) is well known to be

\[
F(z) \sim \exp \left( -\sqrt{\alpha A} |z| \right),
\]

where \( \alpha \) could be any positive value. Properties of that solution have been studied in details in [1]. We note here that actual value of \( A \) is of no importance, since \( \alpha \) is arbitrary and should be “scanned” in order to find \( F(z) \) with required divergence and, therefore, we ommit it in the following.

3.2. Example 2: two gain layers

Two gain layers in an active media can represent a couple of quantum wells as an example. With the gain profile written as

\[
g(z) = g_0 (\delta(z-a) + \delta(z+a)),
\]

the solution of (5) is following:

\[
F(z) = \begin{cases}
N \exp (\sqrt{\xi}(z+a)), & z < -a \\
N \text{sech} (\sqrt{\xi}a) \cosh (\sqrt{\xi}z), & z \in [-a,a] \\
N \exp (-\sqrt{\xi}(z-a)), & z > a
\end{cases}
\]
where $N$ is normalization constant and $\xi$ can be found as the solution of following transcendent equation:

$$\tanh \left( \sqrt{\frac{a}{\xi}} \right) = 2\alpha\xi^{-1/2} - 1$$

(9)

The profiles of $F(z)$ are shown in Figure 1 for different values of $a$. At the Figure 2 best achievable values of $\Theta$ are shown as a dependence of modal gain ($g_0$ is unity for the choice $A = 1$, see above).

![Figure 1: Optimal near-field intensity profiles for different values of $a$ that are shown at the right side of graph.](image1)

![Figure 2: $\Theta(G)$ dependency on the gain for various $a$.](image2)

As we can see from the Figure 2, at low gain and $a < \lambda_0$ small increase in gain results in the proportional increase in divergence for all relevant $a$ due to field distribution being "blob" of width much more than $\lambda_0$ so field will not respond to small $a$ change as much as at high gain. Also it is worth noting that best divergence angle is reached at $a = 0$, thus gain profile confinement is preferred.

### 3.3. Example 3: symmetric slab

There are cases when gain of single layer is not sufficient for effective laser operation, namely, in lasers with quantum dot active media. This problem is typically solved by stacking of gain layers.

As an example, in InGaAs/GaAs quantum dot lasers up to 10 layers may be stacked. The total width of the gain area in that case is up to few hundreds nanometres and is compared with the wavelength in the material. In that case gain profile could be well approximated as a “gain slab” of thickness $2a$ (see Figure 3):

$$g(z) = g_0 \theta(a + z)\theta(a - z),$$

(10)

where $\theta(z)$ is Heaviside step function.

![Figure 3: Gain profile with $a = \lambda_0$.](image3)
The optimum profile is a symmetric one:

\[ F(z) = \begin{cases} 
N \exp (k_1(z + a)), & z < -a \\
N \sech (k_2a) \cosh (k_2z), & z \in [-a, a] \\
N \exp (-k_1(z - a)), & z > a 
\end{cases} \tag{11} \]

Here \( k_1 = \sqrt{-\xi} \) and \( k_2 = \sqrt{-\xi - \alpha} \). \( \xi \) can be found as solution of the following equation:

\[ \tanh (k_2a) = -\frac{k_1}{k_2} \tag{12} \]

Note that there is modal gain limit \( g_0 \), which means field is being contained in \( g(z) \) completely. Also, better result is reached for greater slab width.

3.4. Example 4: single gain layer with losses on the sides

Let us consider laser from Example 1. In emitters electrons and holes absorb light.

\[ g(z) = g_0 \lambda_0 \delta(z) - g_1 \theta(z - a) - g_2 \theta(-z - a) \tag{13} \]

However, electrons’ and holes’ absorption differs explicitly, hence (e.g. \( g_2 \)) is considered negligibly small and omitted. Substituting

\[ \begin{cases} 
k_1 = \sqrt{-\xi} \\
k_2 = \sqrt{-\xi - \alpha g_1} 
\end{cases} \]

one of possible solutions (symmetric one) of (5) can be written as:

\[ F(z) = \begin{cases} 
N \exp (k_1(z + a)), & z < -a \\
N \left(1 - \frac{\alpha}{k_1}\right) \sinh (k_1z) + A \cosh (k_1z), & z \in [-a, a] \\
N \left(\left(1 - \frac{\alpha}{k_1}\right) \sinh (k_1a) + \cosh (k_1a)\right) \exp (-k_2(z - a)), & z > a 
\end{cases} \tag{14} \]
Figure 6: Optimal near-field intensity distributions for fixed modal gain.

4. Optimal refractive index profile equation
Consider $n_0$ as a substrate’s refractive index and $\Delta n(z)$ as a waveguide’s refractive index profile.

$$n(z) = n_0 + \Delta n(z)$$  \hspace{1cm} (16)

Then, looking at optimal near-field and wave equations, certain symmetry can be seen

$$\begin{cases}
  \frac{d^2F}{dz^2} + k_0^2 \left( (\Delta n(z))^2 + 2n_0 \Delta n(z) \right) F(z) = k_0^2 \left( n_{\text{eff}}^2 - n_0^2 \right) F(z) \\
  \frac{d^2F}{dz^2} + \alpha g(z) F(z) = -\xi F(z) 
\end{cases}$$ \hspace{1cm} (17)

where $\xi$ is a solution of the following equation:

$$-\frac{k_2}{k_1} = \frac{\left( 1 - \frac{\alpha}{k_1} \right) + \tanh \left( k_1 a \right)}{\left( 1 - \frac{\alpha}{k_1} \right) \tanh \left( k_1 a \right) + 1}$$ \hspace{1cm} (15)

The field distribution (Figure 6) visibly shifts to the left as the absorption on the right increases.

5. Leaky waveguides
Leaky modes appear in lasers when the barrier between waveguide and substrate is thin enough and refractive indexes are very close, allowing electric field to tunnel through barrier. What it allows is to achieve less far-field divergence through wide near-field distribution. In this section we want to compare efficiencies of laser designs with guided and leaky modes in terms of modal gain vs beam divergence.
Unlike in guided modes, $|F(z)|$ grows exponentially in substrate towards $z = \infty$, so $F(z)$ can’t be normalized, rendering (2) useless, thus modal gain should be found by other means. The method we use is based on physical meaning of effective refractive index, $n_{\text{losses}}$.

With leaky modes without pumping $\text{Im}(n_{\text{losses}}) > 0$, which means energy decreases as wave travels through waveguide it flows into substrate. If active layer is being inserted with refractive index $n = \text{Re}(n) - i\gamma$, where $\gamma > 0$, one can write an equation:

$$\text{Im}(n_{\text{losses}}) = \Delta n_{\text{losses}} - A\gamma$$  \hspace{1cm} (19)

Here $\Delta n_{\text{losses}} > 0$ stands for energy leak and $A > 0$ is a dimensionless constant which can be found through calculating $\text{Im}(n_{\text{eff},1})$ for $\gamma = 0$ and $\text{Im}(n_{\text{eff},2})$ for fixed known $\gamma \neq 0$. $\gamma$ won’t change field profile and $\Delta n_{\text{losses}}$, so:

$$\begin{cases} 
\text{Im}(n_{\text{eff},1}) = \Delta n_{\text{losses}} \\
A = \text{Im}(n_{\text{eff},1} - n_{\text{eff},2})/\gamma
\end{cases}$$  \hspace{1cm} (20)

Consider $\Phi$ as energy flux and $\alpha_i$ as number representing energy losses in a waveguide. Then, for modal gain we can write:

$$ \begin{cases} 
\Phi = \Phi_0 \exp((G - \alpha_i)x) \\
\Phi \sim |E|^2 = \exp((-2\text{Im}(n_{\text{losses}})k_0x))|F(z)|^2
\end{cases}$$  \hspace{1cm} (21)

From here, $G$ can be derived from known variables:

$$G = 2k_0\text{Im}(n_{\text{eff},1} - n_{\text{eff},2})$$  \hspace{1cm} (22)

To demonstrate, we’ll look into simple laser, with varied distance between waveguide and substrate, which is infinity for guided mode and $\sim 0.1$ micron for leaky.
Consider placing a 0.02 micron quantum well in the middle of a waveguide with $\gamma = 0.0191$ (or 1200 cm$^{-1}$ for 1 $\mu$m wavelength). The result as follows:

| Mode    | Gap, $\mu$m | Gain, 1/cm | $\Theta$, deg |
|---------|-------------|------------|---------------|
| Guided  | $\infty$    | 51.97      | 24.28         |
| Leaky   | 0.1         | 51.44      | 23.08         |
| Leaky   | 0.04        | 49.28      | 14.23         |

As seen from the table, with small gain penalty beam divergence can be decreased dramatically. On the contrary, this laser will radiate sideways.

6. Conclusion
We have developed variation approach that in analytical terms describes the near-field profiles that corresponds to best possible modal gain for a given divergence angle or vice versa. Though it is obvious that this approach could not be always used in real material systems due to limited refractive index variations, the approach still is useful to determine how far from the “ideal” any given structure is.

Also it was shown that leaky waveguides are superior to guided ones in terms of modal gain and beam divergence.

7. Acknowledgements
This work is supported by the Russian Science Foundation (Project No. 14-42-00006).

References
[1] Korenev V V, Savelyev A V, Zhukov A E, Maximov M V and Omelchenko A V 2015 Waveguide and active region structure optimization for low-divergence InAs/InGaAs quantum dot comb lasers Proc. SPIE 9503 950305
[2] Dimitri P Bertsekas 1982 Constrained Optimization and Lagrange Multiplier Methods. (New-York: Academic Press) p 72
[3] Bruce van Brunt 2003 The Calculus of Variations. (Berlin: Springer Science & Business Media) p 28
[4] Casey H C Jr. 1978 Heterostructure lasers. (New-York: Academic Press) p 94
[5] Shveikin V I, Bogatov A P, Drakin A E and Kurnyavko Yu V 1999 Directional radiation pattern of quantum dimensional InGaAs/GaAs leaky-mode lasers