Living islands of driven two-dimensional magnetic colloids on the disordered substrate

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Abstract
We investigate, by Langevin simulations, the depinning of driven two-dimensional magnetic colloids on a substrate with randomly distributed pinning centers. The magnetic colloids are modeled as particles interacting with each other through repulsive magnetic dipole and attractive Lennard–Jones potentials. We find living islands above the depinning when the attraction between colloidal particles dominates. The critical pinning force increases and the living islands disperse gradually with an increasing strength of the pinning centers. But with an increasing strength of the repulsion between colloidal particles, the critical pinning force increases first and then decreases, and the living islands appearing at the low repulsion strength disappear when the repulsion strength is increased above a certain value at which the peak effect takes place. When the repulsion strength is further increased, moving smectic and moving crystal structures arise above the depinning where the repulsion between colloidal particles dominates. On increasing the attraction strength, we find that the critical pinning force, which remains unchanged basically at the low attraction strength, decreases and the living islands form gradually in the plastic flows. When the attraction strength is increased to be large enough, the critical pinning force becomes unchanged again and the living islands and phase separation between different islands occur manifestly. The thermal fluctuations at low temperatures compete with the pinning potential, conducive to the formation of living islands, but they smooth the pinning potential at high temperatures, leading to the rapid decrease in the critical pinning force and the dispersion of the living islands.

1. Introduction

Active colloids have become one of the hotspots in the fields of materials science, physical chemistry, condensed matter physics and even life sciences. Active systems can obtain initiatively energy from the outside and overcome resistance through their own storage [1–3]. This is different from that of the passive systems, which gain energy from the surrounding environments, and then restore the energy to the environments. It is known that there is a self-propelled interaction between active colloidal particles [4, 5]. Thanks to this property, they could present a series of novel behaviors that are not attainable in the passive systems, such as living clusters and living islands [6–8], which are closely related to the biological self-organization [5, 7, 9, 10]. In addition, the self-propelled property can also be used to design the micro/nanomachines that operate in microscopic environments, such as micro/nanomotors [1, 11–15], single particle pumps [14, 16], nanorotors [17], and so forth [18–21].

While most of studies of colloids have been devoted to the equilibrium properties [22–24], recent attention has been aroused to the non-equilibrium behaviors, particularly the responses to an external field [25–29]. Using an external field, we can control and track the motion of colloidal particles, and thereby probing the microscopic rheological properties and viscous elastic responses of the biological cells. But an external field will cause an equilibrium system into a non-equilibrium moving state, and thus there will exhibit a series of unusual
phenomena, such as self-assembly [5, 6, 10, 30], clustering [7–10, 31–35], swarming [36, 37], phase separation [25, 38, 39], odd rheological and phase behaviors [8, 40, 41].

A very active research area is the formation of self-organized clusters and the self-organization behaviors. We have examined the slip of fluid particles interacting with Lennard–Jones potential and found self-organized clusters above the onset of slip [42]. Chen et al have investigated the dynamical phases and phase transitions of charged colloids with competing attraction and repulsion, and they found a growth-melting process of the clusters as the temperature was increased [43]. Palacci et al demonstrated a form of self-organization from nonequilibrium driving forces in a suspension of synthetic photoactivated colloidal particles and found two-dimensional (2D) 'living crystals' which form, break, explode, and reform elsewhere [9]. This method of production of self-organization by the active colloids and non-equilibrium forces provides a new way to design the novel moving structures.

The living clusters and living crystals are shown to form in the low density active colloids [7], and there will exhibit non-equilibrium stripes either in the direction of the driving force or in the transverse direction, depending on the pinning strength [26]. It is pointed out that adjusting the short-range attraction between colloidal particles could control the structure and growth of kinetic clusters [44], and the long-range repulsion between the colloidal particles as well as the attraction between colloidal particles and pinning centers in the substrate will lead to rich phase behaviors [32]. This provides a way to manipulate the active particles by changing the boundaries [33] and the environments [15, 34, 45].

The mechanism of self-organization remains unclear to date. In this paper, we will investigate systematically the depinning of driven 2D magnetic colloids with competing repulsive and attractive interactions. In reality, a 2D system is usually subjected to the surface of a solid or liquid substrate. Hence the interactions between colloidal particles and point-like pinning centers randomly distributed in the substrate will also be considered. The aim is to reveal the mechanism of self-organization.

The paper is organized as follows. In section 2, we will describe the model, and the results and discussion will be presented in section 3. Section 4 will give the summary of our findings.

2. Model

We consider \( N_c = 400 \) magnetic colloidal particles, which are initially placed in an ideal triangular lattice subjected to periodic boundary conditions. A realistic quenched substrate is considered and we simulate the disorder with \( N_p = 400 \) point-like pinning centers which are randomly distributed in the substrate.

The motion of colloidal particles is described by the Langevin equation [26, 28, 29, 42, 43, 46–48]

\[
\eta \frac{d\mathbf{R}_i}{dt} = -\sum_{i\neq j} \nabla_i U_i(\mathbf{R}_i - \mathbf{R}_j) - \sum_j \nabla_j U_{ip}(\mathbf{R}_i - \mathbf{R}_j) + \mathbf{F}_i + \mathbf{F}_i^T,
\]

where \( \eta \) is the viscosity coefficient, \( \mathbf{R}_i \) and \( \mathbf{r}_j \) are the positions of the \( i \)th colloid and the \( j \)th pinning center, respectively, \( U_i \) is the interaction potential between colloidal particles, \( U_{ip} \) is the interaction potential between colloidal particles and pinning centers in the substrate, \( \mathbf{F}_i \) is the external driving force, and \( \mathbf{F}_i^T \) is the thermal stochastic force, which satisfies [26, 28, 29, 42, 43, 46–48]

\[
\langle \mathbf{F}_i^T(t) \rangle = 0
\]

and

\[
\langle \mathbf{F}_i^T(t) \mathbf{F}_j^T(t') \rangle = 2\eta T \delta_{ij} \delta_{\alpha\beta} \delta(t - t'),
\]

where \( T \) is the system temperature and the subscripts \( \alpha \) and \( \beta \) indicate the components of \( \mathbf{F}_i^T \) and \( \mathbf{F}_j^T \), respectively.

The total interaction potential between colloidal particles \( U_i \) is the sum of a repulsive potential \( U_{rp} \) [28, 29, 47–49] and an attractive potential \( U_{ap} \) [7, 8, 43], i.e.,

\[
\sum_{i\neq j} U_i(\mathbf{R}_i - \mathbf{R}_j) = \sum_{i\neq j} (U_{rp}(\mathbf{R}_i - \mathbf{R}_j) + U_{ap}(\mathbf{R}_i - \mathbf{R}_j)).
\]

Some colloidal particles are superparamagnetic, and when a magnetic field \( B \) is applied, a magnetic moment \( \mathbf{M} \) will be induced along the magnetic field \( \mathbf{B} \) in the superparamagnetic colloidal particles [49]. The magnitude \( M \) scales linearly with the magnetic field strength \( B \) as \( M = \chi B \) [48, 49], where \( \chi \) is the magnetic susceptibility. Thus the interaction between a colloidal pair confined to the plane perpendicular to the magnetic field \( \mathbf{B} \) is that of parallel dipoles [49]. In this paper, we choose the parallel dipole potential, forming as [28, 29, 47, 48]
for the repulsive interaction between colloidal particles. Here, $|\mathbf{r}_i - \mathbf{r}_j|$ is the distance between the $i$th and the $j$th colloidal particles, $A_v = \left( \frac{\mu_0}{\sqrt{4\pi}} \right) M = \left( \frac{\mu_0}{\sqrt{4\pi}} \right) \chi B$ represents the strength of the repulsive interaction [28, 29, 47, 48] and $\mu_0$ is the magnetic permeability of free space.

The attractive interaction between colloidal particles is described by the Lennard–Jones potential [7, 8, 10, 43]

$$U_{np}(\mathbf{r}_i - \mathbf{r}_j) = 4A_v \left[ \left( \frac{\sigma}{|\mathbf{r}_i - \mathbf{r}_j|} \right)^{12} - \left( \frac{\sigma}{|\mathbf{r}_i - \mathbf{r}_j|} \right)^{6} \right],$$

which arises from the instantaneous electric dipole interaction between colloidal particles. Here $\sigma$ acts as the diameter of colloidal particles and $A_v$ is the depth of the potential well, reflecting the strength of the attractive interaction between colloidal particles.

The interaction between colloidal particles and pinning centers in the substrate is short range, and chosen as the traditional Gaussian attractive potential [28, 43, 46–48, 50]

$$U_{np}(\mathbf{r}_i - \mathbf{r}_p) = -A_p e^{-|\mathbf{r}_i - \mathbf{r}_p|^2/\sigma_p^2},$$

where $|\mathbf{r}_i - \mathbf{r}_p|$ is the distance between the $i$th colloidal particle and the $j$th pinning center, $A_p$ and $\sigma_p$ are the strength and the size of the pinning centers, respectively.

The driving force $F_d$ is exerted along the direction of horizontal symmetry axis $x$, and the average velocity [28, 42, 43, 46–48]

$$V_d = \left( \frac{1}{N_c} \right) \sum_{i=1}^{N_c} v_i \cdot x$$

is measured. Here $N_c$ is the number of colloidal particles.

In our simulations, all the quantities are given in dimensionless units. All the lengths are scaled with respect to the lattice constant of ideal triangular lattice $a_0$, and $\sigma = r_p = 0.2a_0$. The scale for temperature is chosen as the 'bare' Kosterlitz–Thouless melting temperature $T_{m0}$ [51]. The time scale is $1/\mu_0^2$. A time integration step $\Delta t = 0.001$ is used, and the averages are calculated during $2 \times 10^5$ steps after $10^5$ steps for equilibrium.

### 3. Results and discussion

Figure 1 presents a series of velocity-force dependence (VFD) curves for different values of $A_p$, with given the strength of repulsive interaction between colloidal particles (i.e., $A_v = 0.1$) and the temperature (i.e., $T = 0.1T_{m0}$). One can find a critical pinning force, i.e., threshold of the driving force, $F_c$ in each VFD curve. For $F_d < F_c$, the thermally assisted advances of colloids are so small that they can be neglected and the average moving velocity approximately equals to zero, i.e., $V_d \approx 0$. 

![Image](image.png)
Above $F_c$, the VFD is linear basically for the weak pinning (see, e.g., the curve of $A_p = 0.1$ in the figure 1), and colloidal particles move in different living islands, as shown in the inset (a) of figure 2 where living islands are clearly visible. As an example, we give the time evolution of colloid coordinates above the depinning for $A_p = 0.01$ as an example. The inset (a) of figure 2 is a snapshot of the final colloid coordinates at $t = 40$.

Figure 2. The $A_p$ dependence of $F_c$. Insets (a)–(c) are the snapshots of colloid coordinates at a drive above the depinning ($F_d = 1.01F_c$) for $A_p = 0.01$, 0.25 and 0.6, respectively.

Figure 3. The time evolution of colloid coordinates at a drive above the depinning ($F_d = 1.01F_c$) for $A_p = 0.01$ as an example. The inset (a) of figure 2 is a snapshot of the final colloid coordinates at $t = 40$.

Above $F_c$, the VFD is linear basically for the weak pinning (see, e.g., the curve of $A_p = 0.1$ in the figure 1), and colloidal particles move in different living islands, as shown in the inset (a) of figure 2 where living islands are clearly visible. As an example, we give the time evolution of colloid coordinates above the depinning for $A_p = 0.01$ in figure 3. We can find that colloidal particles quickly assemble into different living islands, as shown in the inset (a) of figure 2, from the initial triangular lattice. In such a case, the attraction between colloidal particles overcomes the repulsion between them, forming different assembled states, i.e., living islands, and the phase separation between different islands is apparent.

With an increase in the pinning strength $A_p$, $F_c$ increases, as also illustrated in the figure 2 where we plot the $A_p$ dependence of $F_c$, the average velocity $V_d$ increases nonlinearly with $F_d$ above $F_c$ (see, e.g., the curve of $A_p = 0.3$ in the figure 1), and the living islands disperse and the phase separation disappears gradually, as seen in insets (b) and (c) of figure 2, when the attraction between colloidal particles and pinning centers could compete with the attraction between colloidal particles. When $A_p$ is increased to be large enough, homogeneous plastic flows take place eventually.

It should be noted that the island structures and phase separation have never been found in the passive colloids [28, 46–48].

Figure 4 gives a series of VFD curves for different values of $A_v$. The other parameters are fixed, such as $A_p = 0.2$, $A_v = 500$ and $T = 0.1T_{mrb}$. We find that $F_c$ increases with $A_v$ for the weak repulsion between
colloidal particles, when the attraction between colloidal particles dominates \((A_v < 0.5)\), as also shown in figure 5 where we present the \(A_v\) dependence of \(F_c\) and above the depinning the living islands which form at \(A_v = 0\) disperse gradually with the increasing \(A_v\) but the phase separation and the nonlinear VFD still maintain, as seen when we compare the insets (a) with (b) of figure 5.

When \(A_v\) is increased above certain value \((A_v \approx 0.5)\), the repulsion between colloidal particles dominates and the phase separation disappears, leading to the decrease in \(F_c\) and the occurrence of homogeneous plastic flows above the depinning, as shown in the inset (c) and the upper left of inset (c) in figure 5. Further increasing \(A_v\) will cause a rapid decrease in \(F_c\) and the restoration of linear VFD above the depinning, as seen when we compare the curves of \(A_v = 0.5\) and \(A_v = 1\) in the figure 4. The moving smectic structures appear, as presented in the inset (d) and the upper left of inset (d) in figure 5, when the attraction between colloidal particles becomes small, compared with the repulsion between colloidal particles.

As the repulsion between colloidal particles is increased to be strong enough, see, e.g., \(A_v = 1.5\), the attraction between colloidal particles could be neglected, moving crystal structures arise, as shown in the inset (e) and the upper left of inset (e) in figure 5. In addition, we find a significant phenomenon that the occurrence of the peak in \(F_c\) is accompanied with a crossing of VFD curves, as seen in the figure 4. The peak effect has been observed in vortex lattice \([10]\) and passive colloids \([47]\), and implies the disappearance of living islands and phase separation.

**Figure 4.** A series of VFD curves for different values of \(A_v\).

**Figure 5.** The \(A_v\) dependence of \(F_c\). Insets (a)–(e) are the snapshots of colloid coordinates at a drive above the depinning \((E_d = 1.01E_0)\) for \(A_v = 0, 0.4, 0.6, 1, 1.5\), respectively. The upper lefts of insets (c)–(e) show their structure factors defined as

\[
S(k) = \left\langle \frac{1}{N \sum_{m=1}^{N} \exp (i \mathbf{k} \cdot \mathbf{R}_m) \right\rangle.
\]
With given the values of $A_e = 0.1$, $v = 0.1p$ and $T = 0.1T_{mob}$, a series of VFD curves for different values of $A_e$ are presented in figure 6, from which we find that the VFD is nonlinear above the depinning for the weak attraction between colloidal particles when the attraction between colloidal particles and pinning centers dominates, see the curve of $A_e = 100$. $F_c$ in such a case does not change with $A_e$, as shown in figure 7 where the $A_e$ dependence of $F_c$ is plotted, and homogeneous plastic flows emerge, as presented in the inset (a) of figure 7.

When the attraction between colloidal particles becomes strong and competes with the repulsion between colloidal particles, $F_c$ decreases and the phase separation appears and is becoming increasingly clear with increasing the strength of the attraction between colloidal particles, as seen when we compare the insets (b) with (c) of figure 7, and living islands come into being and become more and more evident.

As the attraction between colloidal particles is increased to be strong enough and overcomes the repulsion between colloidal particles, small islands grow into large islands, as shown in the inset (d) of figure 7, and $F_c$ is invariable with $A_e$ again. The coarsening of living islands has been observed in experiments [9].

Figure 8 shows the temperature dependence of $F_c$ with the given $A_p = 0.2$, $A_v = 0.1$ and $A_e = 500$. $F_c$ is found to decrease with the temperature and a rapid decrease is observed when the temperature is increased above $10T_{mob}$. At low temperatures, thermal fluctuations are small and the attraction between colloidal particles compete with pinning centers, resulting in dispersed living islands, as shown in the inset (a) of figure 8. At an increased temperature, the thermal fluctuations conquer the pinning disorder, reducing the effectiveness of the intrinsic pinning, and clearly separated living islands appear, as shown in the inset (b) of figure 8. At a further increased temperature, the thermal fluctuations become large, causing the islands to melt, as shown in the inset
(c) of figure 8, and eventually the islands are melted into homogeneous plastic flows when the temperature is increased enough and the thermal fluctuations dominate, as presented in the inset (d) of figure 8.

4. Conclusion

Using Langevin simulations, we have numerically studied the depinning of driven 2D disordered magnetic colloids. We have found living islands above the depinning and that the attraction between colloidal particles is responsible for the formation of living islands. With an increase in the pinning strength of the substrate, the critical pinning force increases and the clearly separated islands disperse gradually. On increasing the strength of the repulsive interaction between colloidal particles, the critical pinning force increases first, then decreases, and the living islands disperse into homogeneous plastic flows and even crystal flows. Increasing the attraction between colloidal particles leads the dispersed islands into clearly separated living islands. Small thermal fluctuations at low temperatures are conducive to the formation of living islands, which are melted into uniform plastic flows when the thermal fluctuations become large at high temperatures.

Our results are valuable for the understanding of formation of biological self-organization and the control of self-organization by the external fields. It has been shown that self-organization is dominated by the interparticle attraction and formed in a plastic environment. In addition, we have found that the formation of self-organization and the self-organization behavior could be tuned by an external field and an environment, as recently pointed out in [15, 45].

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References

[1] Ramaswamy S 2010 Annu. Rev. Condens. Matter Phys. 1 323
[2] Vicsek T and Zafeiris A 2012 Phys. Rep. 517 71
[3] Marchetti M C, Joanny J F, Ramaswamy S, Liverpool T B, Prost J, Rao M and Simha R A 2012 Rev. Mod. Phys. 85 1143
[4] Baskaran A and Marchetti M C 2008 Phys. Rev. Lett. 101 268101
[5] Stenhammar J, Wittkowski R, Marenduzzo D and Cates M E 2015 Phys. Rev. Lett. 114 018301
[6] Theurkauff I, Cottin-Bizonne C, Palacci J, Ybert C and Bocquet L 2012 Phys. Rev. Lett. 108 268303
[7] Mognetti B M, Saric A, Angloitti-UBerti S, Cacciuto A, Valeriani C and Frenkel D 2013 Phys. Rev. Lett. 111 245702
[8] Zhang T H, Kuipers B W M, Tian W D, Groenewold J and Kegel W K 2015 Soft Matter 11 297
[9] Palacci J, Sacanna S, Steinberg A P, Pine D J and Chaikin P M 2013 Science 339 936
[10] Reichhardt C and Olson Reichhardt C J 2017 Rep. Prog. Phys. 80 026501
[11] Wang W, Duan W, Ahmed S, Sen A and Mallouk T E 2015 Acc. Chem. Res. 48 1938
[12] Lin X, Wu Z, Wu Y, Xuan M and He Q 2016 Adv. Mater. 28 1060
[13] Badic J D, Balzani V, Credi A, Silvi S and Stoddart J F 2004 Science 303 1845
[14] Wong F, Dey K K and Sen A 2016 Ann. Rev. Mater. Res. 46 407
[15] Chen J X, Chen Y G and Ma Y Q 2016 Soft Matter 12 1876
[16] Hong Y, Diaz U, Cordova Figueroa U M and Sen A 2010 Adv. Func. Mater. 20 1568
[17] Fournier-Bidoz S, Arsenaault I C, Manners I and Oizn G A 2005 Chem. Commun. 41 441
[18] Hanggi P and Marchesoni F 2009 Rev. Mod. Phys. 81 387
[19] Hauser M J B and Schimansky-Geier L 2015 Eur. Phys. J. Spec. Top. 224 1147
[20] Erdmann U, Ebeling W, Schimansky-Geier L and Schweiter F 2000 Eur. Phys. J. B 15 105
[21] Cates M E 2012 Rep. Prog. Phys. 75 046601
[22] Fernández I A, Martin-Mayor V and Verroccchio P 2007 Phys. Rev. Lett. 98 085702
[23] Nellissen K,Partoens B and Peeters F M 2005 Phys. Rev. E 71 066204
[24] Babic D, Schmitt C and Bechinger C 2005 Chaos 15 026114
[25] Zhao H J, Misko V R and Peeters F M 2013 Phys. Rev. E 88 022914
[26] Zhao H J, Misko V R and Peeters F M 2012 Physica C 479 130
[27] Persitsinidis A and Ling X S 2008 Phys. Rev. Lett. 100 028303
[28] Cao Y G, Zhang Z F, Zhao M H, Fu G F and Ouyang D X 2012 Physica A 391 2940
[29] Song K N, Wang H L, Ren J and Cao Y G 2015 Physica A 417 102
[30] Leunissen M E, Christova C G, Hyninnen A P, Royall C P, Campbell A I, Imhof A, Dijkstra M, Van Roij R and Van Blaaderen A 2005 Nature 437 235
[31] Chen X, Yang X, Yang M and Zhang H P 2015 Europhys. Lett. 111 54002
[32] Tierno P 2016 Phys. Rev. Lett. 116 038303
[33] Nikol N, Solon A P, Kafri Y, Kardar M, Tailleur J and Voituriez R 2016 Phys. Rev. Lett. 117 090601
[34] Bechinger C, Di Leonardo R, Löwen H, Reichhardt C, Volpe G and Volpe G 2016 Rev. Mod. Phys. 88 045006
[35] Takatori S C and Brady J F 2017 Phys. Rev. Lett. 118 018003
[36] Thutupalli S, Seemann R and Herminghaus S 2011 New J. Phys. 13 073021
[37] Zöttl A and Stark H 2012 Phys. Rev. Lett. 108 218104
[38] Schwarz-Linek J, Valeriani C, Cacciuto A, Cates M E, Marenduzzo D, Morozov A N and Poon W C K 2012 Proc. Natl Acad. Sci. USA 109 4052
[39] Aidan B and Wilson P 2014 Soft Matter 10 4016
[40] Herrera-Velarde S and Von Grünberg H H 2009 Soft Matter 5 391
[41] Shen T and Wolynes P G 2004 Proc. Natl Acad. Sci. USA 101 8547
[42] Cao Y G and Chow W K 2005 Physica A 348 74
[43] Chen J X, Mao J W, Thakur S, Xu J R and Liu F Y J 2011 J. Chem. Phys. 135 094504
[44] Zhang T H, Klok J, Hans Tromp R, Groenewold J and Kegel W K 2012 Soft Matter 8 667
[45] Chen J X, Zhu J X, Ma Y Q and Cao Y G 2014 Europhys. Lett. 106 18003
[46] Chen J X, Cao Y G and Jiao Z K 2004 Phys. Rev. E 69 041403
[47] Cao Y G, Li Q X, Fu G F and Ouyang D X 2012 Physica A 391 2940
[48] Cao Y G and Li Q X 2008 Physica A 387 4755
[49] Löwen H 2001 J. Phys.: Condens. Matter 13 R415
[50] Brandt E H 1983 J. Low Temp. Phys. 53 71
[51] Fisher D S 1980 Phys. Rev. B 22 1190