Linear response in topological materials

Jonathan Noky and Yan Sun

Max Planck Institute for Chemical Physics of Solids, 01187 Dresden, Germany

Abstract

The discovery of topological insulators and semimetals has opened up a new perspective to understand materials. Owing to the special band structure and enlarged Berry curvature, the linear responses are strongly enhanced in topological materials. The interplay of topological band structure and symmetries plays a crucial role for designing new materials with strong and exotic new electromagnetic responses and provides promising mechanisms and new materials for the next generation of technological applications. We review the fundamental concept of linear responses in topological materials from the symmetry point of view and discuss their potential applications.
I. BACKGROUND AND INTRODUCTION

After the discovery of topological insulators (TIs) [1–8], topological band theory was successfully generalized into condensed matter [9–12]. In the last decade, the combination of topological band theory and symmetry in solids has led to different types of quantum topological states from insulators to semimetals [13–38]. Depending on the topological charges and symmetries, different exotic topological surface states and bulk transport properties were theoretically predicted and experimentally observed. Owing to these attractive properties, a lot of effort has been devoted to the study of topological materials for the next generation of technological applications.

The interplay between electromagnetic response theory and symmetry breaking is another crucial part for the understanding of transport properties in quantum materials. Materials with strong or quantum electromagnetic response have an extensive impact on the development of data storage, information processing, energy conversion, etc. Topological band structures characterized by band inversion, linear band crossings, and band anti-crossings with different topological charges can strongly enhance the electromagnetic responses, and even lead to quantized response effects in some special situations. Since the perturbation from external fields can change the symmetry of a system, the electromagnetic response effects also offer an effective way to manipulate the topological states and, in turn, to control the information processing.

The electromagnetic response can be understood from the perturbation approximation with different orders of the perturbation series. In electrical transport, the higher order responses are normally much smaller than the linear response term. Therefore, most of the electromagnetic responses can be understood in the linear response region, especially the different types of the Hall effect in topological materials. However, in some special situations, the linear response term is forbidden by the symmetry of the system. Consequently, the higher order response effect will dominate in these cases, although the signal is normally very weak and not easy to detect experimentally. In this paper, we give a review of electromagnetic responses in topological materials in the linear response region.
II. LINEAR RESPONSE THEORY

In the linear response approximation, the change of an observable ˆA with respect to an external field ˆF = ˆBF can be evaluated via a linear response as [39, 40]

$$\delta A_i = \chi_{ij}^A F_j$$  \hspace{1cm} (1)

with the linear response tensor

$$\chi_{ij}^A = \lim_{\varepsilon \to 0} \sum_{m,n} \frac{f(E_m) - f(E_n)}{E_m - E_n - i\varepsilon} \langle n|\hat{A}_i|m\rangle\langle m|\hat{B}_j|n\rangle,$$  \hspace{1cm} (2)

where $n$ and $m$ are the band indices. In a periodic system, the eigenstates and eigenvalues are labeled by band index and momentum $k$ as $|n,k\rangle$ and $E_{n,k}$, respectively.

In electrical transport, we are interested in the linear response with respect to an external electric field, which means $\hat{F} = eE\hat{r}$. By using the commutation relation $\hat{v} = \frac{i}{\hbar}[\hat{H},\hat{r}]$, one can transform the position operator to the velocity operator and the position operator matrix can be replaced by

$$\langle n,k|\hat{r}|m,k\rangle = -i\hbar\frac{\langle n,k|\hat{v}|m,k\rangle}{E_{n,k} - E_{m,k}}.$$  \hspace{1cm} (3)

With this replacement, one arrives at the commonly used form of the linear response tensor

$$\chi_{ij}^A = i\hbar e \lim_{\varepsilon \to 0} \sum_k \sum_{m\neq n} \frac{f(E_{n,k}) - f(E_{m,k})}{(E_{n,k} - E_{m,k} - i\varepsilon)(E_{n,k} - E_{m,k})} \langle n,k|\hat{A}_i|m,k\rangle\langle m,k|\hat{v}_j|n,k\rangle.$$  \hspace{1cm} (4)

The linear response tensor can be further separated into inter-band and intra-band contributions. This separation is essential for the study of topological materials.

For the inter-band part, because $\varepsilon \to 0$, one can easily get

$$\chi_{ij}^{A-I} = i\hbar e \sum_k \sum_{m\neq n} \frac{f(E_{n,k}) - f(E_{m,k})}{(E_{n,k} - E_{m,k})^2} \langle n,k|\hat{A}_i|m,k\rangle\langle m,k|\hat{v}_j|n,k\rangle.$$  \hspace{1cm} (5)

For the intra-band contribution, it becomes the Boltzmann formula with the constant relaxation time approximation ($\varepsilon = \hbar/\tau$) [41]

$$\chi_{ij}^{A-II} = -e\tau \sum_k \sum_n \delta(E_F - E_{n,k}) \langle n,k|\hat{A}_i|n,k\rangle\langle n,k|\hat{v}_j|n,k\rangle.$$  \hspace{1cm} (6)

The relaxation time for the intra-band contribution is hard to estimate correctly, therefore it is not easy to give an accurate theoretical evaluation. Nevertheless, one can use it to
estimate the response qualitatively, which is especially important for the symmetry analysis. In contrast, the inter-band part is easier to deal with in theory and numerical calculations. It can be accurately predicted in real materials as long as the electronic band structure is accurate enough. In particular, when we are interested in electrical current, the intrinsic part is just the Berry phase of the electronic band structure. Hence, the inter-band part plays a crucial role in topological materials.

III. ANOMALOUS HALL EFFECT IN FERROMAGNETIC TOPOLOGICAL MATERIALS

Considering the observable $\hat{A}$ as electrical current density $\hat{A} = -e\hat{v}/V$, the inter-band part of linear response tensor for each separated $k$-point becomes the so-called Berry curvature \[ \Omega^k(n, k) = \sum_{\text{m} \neq n} \langle n, k|\hat{v}_i|m, k\rangle \langle m, k|\hat{v}_j|n, k\rangle (E_{n,k} - E_{m,k})^2 \] (7) and the anomalous Hall conductivity (AHC) is dependent on the integral of the Berry curvature over the whole Brillouin zone (BZ). Because the Berry curvature is odd with respect to time reversal symmetry, the anomalous Hall effect (AHE) can only exist in magnetic systems in the linear response approximation.

The AHE was first observed in 1879 by E. H. Hall in two-dimensional ferromagnetic systems [44]. According to the current understanding, there are mainly two contributions for the AHE: one is the extrinsic contribution from impurity scattering and the other is the intrinsic contribution from Berry phase effects of the electronic band structure of the ideal single crystal [42, 43, 45–47]. Because both symmetry and topology are tied to the ideal crystal, it is not possible to discuss the the extrinsic contribution from the topological point of view. In the following, we therefore focus on the intrinsic part.

In topological materials, owing to the special band structure, the local Berry curvature is strongly enhanced, and the AHE is dominated by the intrinsic contribution. The extreme example is the quantum anomalous Hall effect (QAHE), where only intrinsic contributions exist, with a quantized transverse anomalous Hall conductance in quanta of $e^2/h$ and zero longitudinal resistance [48]. The topological charge in the QAHE is indexed by a topological invariant, the Chern number, which is given by the integral of the Berry curvature over the
whole 2D BZ. In contrast to the quantum Hall effect, which is decided by the external magnetic field [49], the QAHE originates from the spin–orbit coupling (SOC) and intrinsic magnetism. The QAHE was theoretically proposed by F. D. M. Haldane in 1988 by a model defined on a honeycomb lattice [48]. Owing to the development of topological band theory and thin film growth techniques, the first QAHE was experimentally observed in chromium-doped (Bi,Sb)$_2$Te$_3$ [50,51].

An effective model for the QAHE can be described by a $2\times2$ nodal line band structure [52]

$$H_{QAHE} = A(k_x\sigma_x + k_y\sigma_y) + M(k)\sigma_z, \quad (8)$$

where $\sigma_i$ are the Pauli matrices and $M(k) = M_0 + M_1(k_x^2 + k_y^2)$.

Because this model includes a mirror symmetry with respect to $x - y$ plane, the band structure forms a nodal loop in the absence of SOC, as presented in Figure 1a. This nodal line is broken by SOC and a band gap opens, which generates non-zero Berry curvature in the band gap and forms a hot loop in 2D $k$ space (see Figure 1b). The integral of the Berry curvature in the whole 2D BZ leads to a quantized anomalous Hall conductance in the band gap, which is just corresponding to the chiral edge state, as presented in Figure 1c.

A stack of 2D QAHE systems was proposed to be an effective way to obtain magnetic Weyl semimetals (WSMs) with the QAHE in its 3D version [53–55]. The simplest model can be obtained by including the coupling effect along $z$ via adding a $k_z$ term $M_1 k_z^2 \sigma_z$ into Equation (8) [55], where one pair of Weyl points is located on the $k_z$-axis at $(0,0,\pm k_w)$ with $k_w = \sqrt{M_0/M_1}$, as presented in Figure 2a.

The peak value of the anomalous Hall conductivity (AHC) lies at the energy of the pair of Weyl points (Figure 2b). The AHC in magnetic Weyl semimetals can be understood from the QAHE. The Weyl points act as a sink and a source of the Berry curvature, just like monopoles in reciprocal space, therefore the $k_x - k_y$ planes host a non-zero quantized anomalous Hall conductance when $k_z$ is lying between the pair of Weyl points, and the contribution is zero for the planes with $k_z > |\sqrt{M_0/M_1}|$ [15,16]. As a result, the AHC for the magnetic WSMs with one pair of Weyl point is $\sigma^z = \frac{e^2}{\hbar} \frac{k_w}{\pi}$ and its magnitude is decided by the separation of the Weyl points in $k$ space. Because the time reversal symmetry is broken due to the magnetization along $z$, only the $z$-component of the Berry curvature shows a non-zero value after the integral over the whole $k$ space (see Figure 2d). In contrast, the other two components still follow time reversal symmetry, as shown in Figure 2e,f, and therefore
they do not contribute to the AHE.

FIG. 1. Schematic of a quantum anomalous Hall insulator (QAHI) with a nodal line band structure. (a) The band inversion forms a nodal ring in the situation without including SOC. The Berry curvature is zero in the whole $k$ space. (b) The nodal line is broken by SOC with opening a band gap, and non-zero Berry curvature forms a hot ring located in the band gap. The color bar for the Berry curvature is in arbitrary units. (c) A quantized anomalous Hall conductance ($e^2/h$) in the band gap.

Because the nodal lines and Weyl points can generate strong local Berry curvature, magnetic nodal line semimetals and WSMs are expected to have a strong intrinsic AHE. Recently, a large AHE was observed in the Heusler compound Co$_2$MnGa [56, 57] and the quasi-two-dimensional compounds Co$_3$Sn$_2$S$_2$ [58, 59] and Fe$_3$GeTe$_2$ [60], where the mirror symmetry protected nodal line band structure is the most important feature for this phenomenon. Particularly, owing to the topological band structures and low charge carrier density, Co$_3$Sn$_2$S$_2$ and Co$_2$MnGa are so far the only two examples of all known materials hosting both giant AHC and anomalous Hall angle (AHA) [57–59].

Taking Co$_3$Sn$_2$S$_2$ as an example, the Co atoms form Kagome lattices stacked along the $c$ direction (see Figure 3a). Here, the crucial symmetries are the three mirror planes parallel to the $c$ direction, which results in three pairs of nodal lines connected by a $C_3$ rotation symmetry, as presented in Figure 3c–e. Because the magnetization is aligned along the $z$
direction, the mirror symmetries are broken by SOC, and the nodal lines are not longer protected. Therefore, a band gap opens. Meanwhile, one pair of Weyl points with opposite chirality remains along each of the former nodal lines (see Figure 3d). The Berry curvature distribution in $k$ space is dominated by the SOC-gapped nodal line (see Figure 3e), leading to an AHC of $\sim 1100$ S/cm. Together with the low charge carrier density, the AHA can reach up to $\sim 20\%$, which is a new record in 3D materials [58, 59].
FIG. 3. AHE in the magnetic WSM Co$_3$Sn$_2$S$_2$. (a) Crystal and magnetic structure for Co$_3$Sn$_2$S$_2$. The three mirror planes are represented by the green lines. (b) Energy dependent AHC. (c,d) Energy dispersion along high symmetry lines for the cases without and with SOC, respectively. The local energy dispersion for the nodal line and the Weyl points is shown in the inset. The green line represents the nodal line. The blue and red points represent Weyl points. (e) Berry curvature distribution in the 3D BZ. The color bar is in arbitrary units.

IV. ANOMALOUS HALL EFFECT IN ANTIFERROMAGNETIC TOPOLOGICAL SEMIMETALS

From the symmetry point of view, the AHE in ferromagnets is easy to understand due to the net magnetization, and the important insight is that the special topological band structure can enhance the signal dramatically. However, for a long time, it was believed that the AHE cannot exist in antiferromagnets (AFMs), because the measured Hall resistivity normally follows $\rho_{Hall} = R_0\mu_0H + (\alpha\rho_{xx}^2 + \beta\rho_{xx})$, where the fist term is the ordinary Hall from the Lorentz force and the second term is the anomalous component from extrinsic scattering and intrinsic contributions.

Because the intrinsic contribution can be understood from the integral of the Berry curvature in $k$ space, and the Berry curvature is odd with respect to time reversal symmetry, the AHE can only exist in systems with broken time reversal symmetry. In most collinear
AFMs, although the time reversal symmetry $\hat{T}$ is broken, it is possible to find a combined symmetry of $\hat{T}$ and a space group operation $\hat{O}$, which is preserved in the system and still reverses the sign of Berry curvature. Therefore, the AHE is forced to zero in these systems. However, such joint symmetry $\hat{T}\hat{O}$ does not necessarily exist in all AFM systems. Two counter-examples are non-collinear AFMs and compensated ferrimagnets.

The existence of an AHE in AFMs was first proposed by R. Shindou and N. Nagaosa in 2001 by predicting a non-zero AHE in non-coplanar AFM face-centered-cubic lattices via Berry curvature analysis [61]. Further, it was predicted in 2014 via Berry curvature calculations in cubic Mn$_3$Ir [62] and hexagonal Mn$_3$X ($X$ = Ga, Sn or Ge) [63] that the intrinsic AHE can even exist in coplanar non-collinear AFMs. The latter was experimentally observed in Hall measurements soon after the theoretical prediction [64, 65].

Recently, it was found that the AHE in Mn$_3$Ge and Mn$_3$Sn is strongly dependent on the symmetry of the magnetic structure and the topological band structure [66]. As presented in Figure 4a, the triangular lattices formed by Mn are stacked along the $c$ direction, and the magnetic structure follows a glide mirror symmetry $\{M_y|(0, 0, 1/2)\}$, see Figure 4b. Because the Berry curvature is invariant with respect to glide operations, the shape of the AHC tensor is only decided by the mirror $M_y$. $M_y$ is similar to a time reversal operation for the $x$ and $z$ components of the spin and will change the sign of $\Omega^x$ and $\Omega^z$ while keeping $\Omega^y$ invariant. As a result, the AHC vector takes the form $\vec{\sigma}^{AHE} = \{ 0, \sigma^y, 0 \}$, with only one non-zero component (see Figure 4c). In addition, it was found that the size of $\sigma^y$ is related to the Weyl points in Mn$_3$Ge and Mn$_3$Sn. The strong AHE in Mn$_3$Ge and Mn$_3$Sn inspired the investigation of their band structures from a topological point of view. It was found that there are six pairs of Weyl points near the Fermi level in Mn$_3$Sn according to first principles calculations [67], which were experimentally observed in ARPES measurements soon after the theoretical prediction [68].

Apart from a non-collinear magnetic structure, the combined symmetry that reverses the sign of the Berry curvature can also be broken in collinear magnetic structure with zero net moment. As presented in Figure 4d, the sign of the magnetic moments is changed by the combined time reversal and translation symmetry. This translation symmetry can be broken by an atomic replacement while keeping local moments invariant (see Figure 4e). In this case, the system can obtain a non-zero net Berry curvature and therefore a finite intrinsic AHC. This kind of magnetic structure can exist in compensated ferrimagnets. Because the
charge carrier density is normally very small in compensated ferrimagnets, the signal of the AHE is very weak. However, if there are some special topological band structures, such as nodal lines and Weyl points, the AHE should be strongly enhanced. A possible candidate is the Heusler compound Ti$_2$MnAl [69], where the zero net magnetic moment is due to the opposite magnetization from Ti and Mn sites.

V. ANOMALOUS HALL EFFECT IN THIN FILMS

In the above sections, we only focus on the AHE in bulk crystals. However, in micro-electronic applications, thin films are used most of the time. When discussing thin films from a theory point of view, one has to distinguish between two cases: On the one hand, the film can be above a certain thickness threshold and behave as the bulk system. On the other hand, the film can be of very small thickness. In this case, the bulk picture is not
longer valid and extrinsic effects are more pronounced. The thickness threshold is a material dependent value, e.g., $\approx 80$ nm in Co$_2$MnGa [70].

When the film thickness is below the threshold value, surface effects get stronger compared to the bulk properties. This can lead to a larger extrinsic contribution to the AHE due to enhanced scattering at the surfaces. Depending on the sign of the extrinsic part with respect to the intrinsic one, this can either enhance or decrease the intrinsic AHE [27, 71, 72]. For very thin films, it can even be possible that the confinement leads to a changed band structure where the Weyl points are annihilated and an inverted band gap is created [73–75], leading to a 2D QAHE system.

VI. ANOMALOUS NERNST EFFECT IN TOPOLOGICAL SEMIMETALS

Apart from the AHE, there is also the corresponding thermoelectric effect, the anomalous Nernst effect (ANE), which can be utilized for measurements of Berry phase effects in band structures. In the ANE, a transverse electrical current in magnetic materials is generated by an applied longitudinal temperature gradient, instead of an electric field [76–78], as presented in Figure 5a. Because also the ANE is dependent on the Berry curvature, a strong ANE is expected in magnetic topological materials. Very recently, large ANEs were observed in the magnetic WSMs Co$_2$MnGa [79, 80] and Co$_3$Sn$_2$S$_2$ [81], which are believed to originate from the Weyl points and the nodal line band structures. Owing to the contribution of the topological band structure, the anomalous Nernst conductivity (ANC) in Co$_2$MnGa and Co$_3$Sn$_2$S$_2$ can reach values one order of magnitude larger than in traditional magnetic materials (see Figure 5d).

Although AHE and ANE share the same symmetry and both can be understood from the Berry curvature, their dependence on the Berry curvature is different [78]. The ANC is generated by the combination of Berry curvature ($\Omega$), distribution function $f(n)$, and temperature $T$ in the form of

$$\alpha^i = \frac{1}{T} \frac{e}{\hbar} \sum_k \sum_n \Omega^i (E_n - E_F) f_n + k_B T \ln \left( 1 + e^{\frac{E_n - E_F}{k_B T}} \right). \quad (9)$$

Similar to other thermoelectric signals, the ANE is more sensitive to the first derivative of the electrical characteristics with respect to energy, rather than to the value itself. As discussed above, the AHC shows a peak value at the energy of the Weyl points in high symmetric WSMs due to the big contribution from the Weyl points. In contrast to that,
FIG. 5. ANE in magnetic materials. (a) Schematic of the ANE. (b) Energy dispersion of an effective model for a magnetic WSM. The gray plane represents the energy with the peak value of the ANC. (c) Energy dependent ANC. (d) Absolute value of the ANC for different magnetic metals, with data collected from Refs. [82–89].

the ANC is zero at the energy of the Weyl points (see Figure 5b,c), and two peak values appear away from that energy. Owing to the different mechanism in the ANE, it can be used to detect topological band structure features away from the Fermi level by the means of thermoelectric transport measurements, even if the electronic transport signal is weak [90].

VII. SPIN HALL EFFECT IN TOPOLOGICAL MATERIALS

Because the Berry curvature is odd with respect to the time reversal operation, the AHE can only exist in magnetic systems. However, although the AHE is forbidden, the spin Hall effect (SHE) is allowed in time reversal symmetric systems. In the SHE, a transversal spin current is generated by an applied longitudinal spin current due to the SOC [91,94] (see
Figure 6a. Conversely, a transverse charge current can be generated via a longitudinal spin current. Hence, the SHE provides an effective method to generate and manipulate the spin current without a magnetic field.

The extrinsic SHE was proposed by M. I. Dyakonov and V. I. Perel in 1971 [91], and the SHE received extensive attention after the theoretical studies of its intrinsic mechanism [95–97] and its experimental observations [93, 98, 99]. The study of the quantum version of the SHE led to the 2D TI [1–3], and the spin current from spin–momentum locked topological surface states opens up new approaches to the SHE.

![FIG. 6. SHE in topological materials. (a) Schematic of the SHE. A transverse pure spin current is generated by an applied longitudinal electric field. (b) The table for some typical SHE materials, with data collected from Refs. [100–105]. The unit of SHC is $\frac{e^2}{\hbar} S \text{cm}$.](image)

Similar to the AHC, the spin Hall conductivity (SHC) can be formulated via the spin Berry curvature in the Kubo formula approach [95–97], where the observable is the spin current density with $\hat{A} = \frac{-e}{2\hbar} \left( \hat{\sigma}_s \hat{v} \right)$. The spin current $\hat{J}^k_{s-i}$ is generated by the applied electric field $\vec{E}_j$ following the linear response $\hat{J}^k_{s-i} = \sigma^k_{ij} \vec{E}_j$. Here, the SHC tensor $\sigma^k_{ij}$ describes a spin current along $i$ with spin polarization along $k$ and an applied field in the $j$ direction. Because the AHC and SHC share a similar formalism, the linear band crossings and anti-crossings can also generate strong local spin Berry curvature in both magnetic and nonmagnetic systems. Additionally, because the SHE originates from the SOC, a strong intrinsic SHE is expected in both TI and topological metals.

In SHE experiments, apart from the SHC, there is another important parameter, the spin Hall angle (SHA). It represents the efficiency of conversion from charge current to spin current. Thus far, there are mainly two approaches for SHE materials: one is based on heavy transition metals [94, 106–109], and the other is based on TIs or topological semimetals [103–105, 110]. The SHC and SHA for some reported SHE materials are given in Figure 6b. Very
recently, it was found that the nodal line band structures play a crucial role for a strong SHE even in transition metals [111].

VIII. SPIN NERNST EFFECT IN TOPOLOGICAL MATERIALS

The relationship between AHE and ANE can also be transferred to spin current generation. In addition to SHE where the driving force is an electric field, a spin current can also be generated by a temperature gradient, which is known as the spin Nernst effect (SNE) [112]–[114]. In the SNE, a transverse spin current $\mathbf{J}_{s-i}$ can be generated by an applied temperature gradient $\nabla T_j$ via the linear response $\mathbf{J}_{s-i} = \alpha_{ij} \nabla T_j$, where $\alpha_{ij}$ is the linear response tensor of the spin Nernst conductivity (SNC). Very recently, the SNE was observed in heavy transition metals in thermoelectric transport measurements [112]–[114].

Because the SNE shares similar mechanisms with the ANE, only with different symmetry requirements, a strong SNE is also expected in topological materials. Figure 7 gives a general understanding of the relation between SNE and topological band structures. For a symmetry model of a TI with a global band gap, a constant SHC exists in the inverted band gap. However, because the SNE is dependent on the first derivative of the SHE with respect to the chemical potential, the SNE vanishes at the charge neutral point, as presented in Figure 7a–c. To obtain a non-zero finite SNE, breaking the balance of the SHC distribution in energy space is necessary. An effective way to achieve this is to decrease the symmetry and tilt the inverted band gap with a finite Fermi surface, as the schematically shown in Figure 7d. With the tilted inverted band gap, the distribution of SHC is no longer symmetric in energy space and therefore a finite SNC appears at the Fermi energy (see Figure 7e,f). From the model analysis, one can find that a topological semimetal or a doped topological insulator is expected to host a strong SNE, which is similar to the ANE in magnetic topological semimetals.

IX. SPIN–ORBIT TORQUE IN MAGNETIC SEMIMETALS

Further breaking inversion symmetry in magnetic materials, one can have another linear response effect to control the magnetization orientation by electrical current [115], [116]. Utilizing the inverse spin–galvanic effect in a non-centrosymmetric magnetic system, the
non-equilibrium spin polarization will interact via exchange-coupling with the magnetization and acts as an effective field. This effective field originates from the spin–orbit coupling and can generate torques on the magnetic moments, which allows the control of the magnetic states by an electric field. Therefore, it is called spin–orbit torque (SOT).

Most SOT studies have focused on ferromagnetic systems where the inversion symmetry is broken by either the bulk crystal unit cell [117, 118] or due to heterostructures [100, 119–124]. As AFMs are much less sensitive to external magnetic fields and their dynamics of the magnetic moments are much faster than those in FMs, spintronics in AFMs have attracted intensive attention [125–127]. Because the manipulation of the magnetic order through the traditional approach of using an external field requires too large fields, the electrical manipulation of AFMs via SOT is much more desirable. Recently, a SOT phenomenon has been theoretically proposed [128] and experimentally observed in bulk collinear antiferromagnetic Dirac semimetal CuMnAs [129, 130]. In CuMnAs, the inversion symmetry is broken by mag-
netic order, but the band spin degeneracy is protected by joint inversion and time-reversal symmetry. In such an AFM, an electric current can generate a staggered torque for each sub-lattice, which is an effective way to manipulate the magnetic order [131].

CuMnAs can exist in both orthorhombic and tetragonal crystal structures in experiments [129–131], and both can host the antiferromagnetic Dirac semimetal phase. Taking the orthorhombic phase as an example, four Mn sites form two spin sub-lattices, which are connected by the combined $\hat{P}\hat{T}$ symmetry, as presented in Figure 8a,c. When the spin polarization is along [001] direction, the crystal has a screw rotation symmetry $S_z = \{C_2z|(0.5, 0, 0.5)\}$, in which the bands with opposite $S_z$ eigenvalues guarantee the linear band crossing in the inverted band structure (see Figure 8b). The spin orientation can be tuned by an applied electrical current via the SOT, as presented in Figure 8c. As the spin polarization moves away from [001]-axis, the screw rotation symmetry is broken, and the Dirac point obtains a non-zero mass (see Figure 8d). Therefore, the SOT provides an effective method to tune the topological band structures in magnetic topological states.

X. SUMMARY

We introduce the linear response effects in topological materials based on fundamental theories and recent developments of experiments. The interplay of symmetry and electromagnetic structures tells us how to obtain corresponding materials with non-zero linear response signals. The topological band theory offers a direction to achieve enhanced and even quantum linear response effects, which is essential to find the correct materials for the next generation of technological applications.

XI. AUTHOR CONTRIBUTIONS

Conceptualization, S.Y. and J.N.; writing, J.N.; and supervision, S.Y.
FIG. 8. SOT in AFM CuMnAs. (a) Magnetic structure for orthorhombic CuMnAs with spin polarization along the $z$-axis. (b) Energy dispersion of CuMnAs. It is a Dirac semimetal when spin polarization is along the $z$-axis. (c) The spin orientation is moved away from the $z$-axis by an applied electric field via the SOT. The torques for the two spin sub-lattice A and B are opposite. (d) The Dirac point gets a mass term and a band gap opens when the spin polarization moves away from the $z$ direction.

XII. CONFLICTS OF INTEREST

The authors declare no conflict of interest.

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