Intrinsic photon loss at the interface of superconducting devices

Igor Diniz\textsuperscript{1,2, \ast} and Rogério de Sousa\textsuperscript{1}

\textsuperscript{1}Department of Physics and Astronomy, University of Victoria, Victoria, British Columbia, Canada V8W 2Y2
\textsuperscript{2}Instituto de Ciências Exatas, Universidade Federal Rural do Rio de Janeiro, Seropédica, Brazil

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We present a quantum theory of dielectric energy loss arising from the piezoelectric coupling between photons and phonons in superconducting devices. Photon loss is shown to occur predominantly at the interface, where the piezoelectric effect is non-zero even when the materials are perfectly crystalline and free of two-level system defects. We present explicit numerical calculations for the value of the intrinsic loss tangent at several interfaces to conclude that the energy relaxation time in superconducting qubits may be increased by a factor of 10 – 100 if the device is made with defect-free interfaces.

Qubits based on Josephson junctions have come a long way and became one of the most promising devices for quantum information processing. Although coherence times have improved by several orders of magnitude in the past two decades \cite{1–3}, relatively short coherence is still arguably the main obstacle in the implementation of large scale quantum computation.

Coherence times in state-of-the-art Josephson devices are limited by energy relaxation ($T_1$ decay) \cite{4} and there are many noise/relaxation sources that can play a role. Understanding the physical origin of these sources is key to make further progress on coherence times. While any excitation with electric dipole moment can contribute to electric (photon) energy loss, a large number of experiments with superconducting resonators provide evidence of loss dominated by extrinsic sources that can be modelled as a bath of two-level-systems (TLSs) \cite{5–12}. The evidence for TLSs is based on the observation that the loss tangent (proportional to the inverse quality factor of the circuit, $1/Q$) always decreases with increasing microwave power, and this can only be explained by TLS saturation. At high power, when TLSs are saturated, the origin of the residual loss is not understood \cite{9}.

An additional mechanism of loss is phonon radiation due to the piezoelectric effect. It is well known that Josephson junctions radiate phonons at the Josephson frequency, but it is still not clear whether this occurs due to presence of TLSs or due to the piezoelectric effect \cite{13}. Ioffe et al. \cite{14} proposed a mechanism of phonon radiation due to piezoelectricity in disordered junctions; assuming the qubit electrical energy was mostly concentrated at the Josephson junction led to the conclusion that this effect could be responsible for the typical $T_1$ observed in superconducting qubits \cite{14}. However, systematic studies of qubit relaxation for varying qubit geometries showed that $1/T_1$ was proportional to the electrical energy at the interfaces away from the Josephson junction. Thus it was concluded that interface dielectric dissipation is the major limiting factor for both coherence and energy relaxation, leading to loss that is much larger than the one happening at the junctions \cite{15, 16}.

In spite of its ubiquity, the contribution of piezoelectricity to the quality factor of superconducting qubits is not known.

In this Letter we describe a quantum theory of photons and phonons coupled by the piezoelectric effect. In the microwave range the resulting loss is ineffective in large dielectrics such as bulk materials. However, the loss is found to be greatly enhanced at the interface. We show that an interface between two materials with different elastic constants always has a non-zero piezoelectric coefficient, so that the interface acts as an efficient phonon emitter. As a result, dielectric loss due to phonon radiation is an \emph{intrinsic effect}, that is present even when the materials/interfaces are perfect crystals.

\textit{Quantum theory of photons and phonons coupled by the piezoelectric effect.}– When a photon travels inside an insulator it inevitably has a finite lifetime, in that the pure photon is no longer an eigenstate of the material’s Hamiltonian. This occurs because the material has excitations and defects with electric dipole moment.

The coupling is most effective when the frequency of the photon is resonant with the frequencies of the excitations contributing to the material’s polarization $P$ (electric dipole moment per volume). In the microwave range a large density of acoustic phonons always satisfies these conditions; the phonons acquire electric dipole moment whenever the material or device lacks inversion symmetry, e.g. due to the presence of an interface or disorder. The Hamiltonian for a single “cavity” photon plus phonons can be written as

$$
\mathcal{H}_0 = \hbar \Omega \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right) + \sum_k \hbar \omega_k \left( \hat{b}_k^\dagger \hat{b}_k + \frac{1}{2} \right),
$$

where the operator $\hat{a}^\dagger$ creates a photon with frequency $\Omega$, and the operator $\hat{b}_k^\dagger$ creates a longitudinal acoustic phonon with wavevector $k$ and frequency $\omega_k = v |\mathbf{k}|$, with $v$ the phonon velocity.

For simplicity we consider a single photon mode, with
electric field operator
\[ \hat{E} = \sqrt{\frac{\hbar \Omega}{2 \varepsilon V_a}} [\psi(r) \hat{a} + \psi^*(r) \hat{a}^\dagger], \]
where \( \psi(r) \) is the shape function for the cavity mode, pointing along the mode's electric field and normalized to the cavity volume, \( \int d^3r |\psi|^2 = V_a \). The constant \( \varepsilon \) is the high frequency dielectric constant, which arises from non-resonant mechanisms such as electronic and optical phonon excitations.

We also take a simple model for piezoelectricity, assuming that the material's polarization is proportional to the divergence of the phonon displacement operator, \( \hat{P} = \mathbf{p} \nabla \cdot \hat{u} \) [17]. The constant of proportionality \( \mathbf{p}(r) \) is denoted “piezoelectric vector” and is here assumed to depend on position because we describe inhomogeneous systems such as interfaces and junctions. Inserting the usual expression for phonon displacement \( \hat{u} \) we get
\[ \hat{P}(r) = \mathbf{p}(r) \frac{\hbar \omega}{2 \rho V^{3/2}} (b_k e^{i k \cdot r} - b_k^* e^{-i k \cdot r}), \]
where \( V \) is the volume of the insulator (e.g. the dielectric substrate, which is assumed to be different than \( V_a \), the volume of the photon cavity), and \( \rho \) is its mass density. Note that \( \mathbf{p} \) has the same dimensions as \( \mathbf{P} \), charge/area, and does not point along the phonon wavevector \( \mathbf{k} \) (for example, simple symmetry considerations show that \( \mathbf{p} \) points perpendicular to the interface).

The interaction between photons and phonons is given by
\[ \mathcal{H}_{\text{int}} = - \int d^3r \hat{P}(r) \cdot \hat{E}(r) \]
\[ = \sum_k (\xi_k \hat{a} \hat{b}_k^\dagger + \text{H.c.}), \]
with coupling amplitude
\[ \xi_k = i \sqrt{\frac{\hbar \Omega \omega_k}{4 \rho \varepsilon V_{a}^2}} \int d^3r \mathbf{p}(r) \cdot \mathbf{\psi}(r) e^{-i k \cdot r}. \]
In Eq. (4) we neglected terms such as \( \hat{a} \hat{b}_k \) and \( \hat{a}^\dagger \hat{b}_k^\dagger \), because they can't conserve energy so they don't contribute to energy loss. The terms that conserve total energy lead to energy dissipation for the photon system, with rate given by
\[ \mathcal{R}_{\text{diss}} = \hbar \Omega (\Gamma_a \to b - \Gamma_b \to a), \]
where \( \Gamma_a \to b \) is the rate for processes that convert a photon into a phonon (energy loss), with \( \Gamma_b \to a \) the opposite process of energy gain. The former and the latter are induced by the terms \( \hat{a} \hat{b}_k^\dagger \) and \( \hat{b}_k \hat{a}^\dagger \) in Eq. (4), respectively. Using Fermi's golden rule we get
\[ \Gamma_a \to b = \frac{2 \pi}{\hbar} \sum_k |\xi_k|^2 n_a (n_k + 1) \delta (\hbar \Omega - \hbar \omega_k), \]
where \( n_a \) and \( n_k \) are the number of photons in mode \( a \) and the number of phonons in mode \( k \), respectively. The expression for \( \Gamma_b \to a \) is obtained by replacing \( n_a (n_k + 1) \) for \( (n_a + 1) n_k \).

Plugging the amplitudes (5) into Eqs. (6) and (7) leads to a general expression for the inverse quality factor \( 1/Q \), which is the fractional energy lost per cycle:
\[ \frac{1}{Q} = \frac{1}{\Omega \hbar \Omega} \frac{\mathcal{R}_{\text{diss}}}{(n_a + \frac{1}{2}) \cdot \frac{n_B(\Omega)}{4 \pi \rho \varepsilon V_a (n_a + \frac{1}{2})}} \int d^3r \int d^3r' |\mathbf{p}(r) \cdot \mathbf{\psi}(r)| \]
\[ \times \sin((\Omega \varepsilon) |\mathbf{r} - \mathbf{r'}|) |\mathbf{p}(r') \cdot \mathbf{\psi}^*(r')|, \]
where we assumed that the phonons are at thermal equilibrium at some temperature \( T \), i.e. their occupation is equal to the Bose distribution \( n_B(\Omega) = 1/\exp(\hbar \Omega/k_B T) - 1 \). If in addition the photon system is also at thermal equilibrium, \( n_a \) will also be equal to \( n_B(\Omega) \) and Eq. (8) will become exactly equal to zero. This shows that Eq. (8) satisfies detailed balance.

It is straightforward to generalize Eq. (8) to an arbitrary number of photon modes. The final answer is to replace \( \mathbf{\psi}(r) \) by \( \mathbf{E}(r)/\sqrt{2/\int d^3r |\mathbf{E}(r)|^2/V_a} \), where \( \mathbf{E}(r) \) is the space-dependent electric field (a classical field).

**Key role of photon confinement.** Consider a bulk material so that \( V_a \to \infty \) and assume that \( \mathbf{p}(r) \) is a constant. In this case the photons can be regarded as plane waves, \( \psi(r) = e^{i q \cdot r} \mathbf{\hat{e}} \), and Eq. (5) is non-zero only for phonons with \( \mathbf{k} = \mathbf{q} \) (conservation of momentum). Since Eq. (7) requires conservation of energy (\( \Omega = \omega_k \) or \( c |\mathbf{q}| = v |\mathbf{k}| \)), it yields \( 1/Q = 0 \) for \( \Omega > 0 \). Therefore, the piezoelectric mechanism yields zero dissipation in bulk.

Now consider what happens at an interface. A generalization of the usual microscopic model of piezoelectricity yields (See e.g. Section 3.9 of [17])
\[ \mathbf{p}(r) = e \frac{c}{2 \ell_f} \left( C_1 - C_2 \right) \delta(z) \mathbf{\hat{n}}, \]
where \( e \) is the electron’s charge, \( t_f \approx 4 \) Å is the atomic interface thickness, \( \mathbf{\hat{n}} \) is the unit vector perpendicular to the interface (pointing from material 1 to 2), and \( C_i = \rho_i v_i^2 \) are the elastic constants for the materials forming the interface. We assume an interface with area \( A \to \infty \), and photon shape function \( \psi(r) = e^{i q \cdot r} \mathbf{\hat{r}} \), with photon wavevector \( \mathbf{q} \perp \mathbf{\hat{n}} \). Now the phonon-photon momentum conservation in Eq. (5) is reduced to \( k_{\perp} = q_{\perp} \), with \( \mathbf{k} \cdot \mathbf{\hat{n}} \) arbitrary. This freedom allows satisfaction of energy conservation with phonon momentum perpendicular to the interface equal to \( \mathbf{k} \cdot \mathbf{\hat{n}} = \pm \Omega \sqrt{1/v^2 - 1/c^2} \approx \pm \Omega/v \). These considerations allow exact evaluation of Eq. (8) for an interface leading to \( 1/Q = (t_f A/V_a) \tan(\delta_f) \). The prefactor \( f_t = (t_f A/V_a) \) is the fraction of total electrical energy.
at the interface, denoted participation ratio [15]. The intrinsic loss tangent for the interface is given by

$$\tan(\delta_I) = \sum_{i=1,2} \frac{\Omega e^2 [n_a - n_B(\Omega)]}{16\pi^2 \rho_i v_i^3 \varepsilon (n_a + \frac{1}{2})} \left( \frac{C_1 - C_2}{C_1 + C_2} \right)^2,$$

(10)

where the label $i = 1, 2$ corresponds to the two materials forming the interface. Table I shows explicit calculations of Eq. (10) for a variety of interfaces, for $\Omega/2\pi = 10$ GHz, $T = 10$ mK, $n_a = 1$, $t_I = 4$ Å, and $C_i = \rho_i v_i^2$. The table shows a factor of 10−100 decrease in loss can be obtained if the extrinsic mechanism due to TLSs is suppressed.

For a small Josephson junction with lateral size $\ll v/\Omega$ we may approximate $p(\mathbf{r}) = p_0 V_J \hat{n} \delta(\mathbf{r})$ and $\text{sinc}(\Omega(\mathbf{r} - \mathbf{r}'))/v \approx 1$ in Eq. (8) leading to $1/Q = (V_J/V_a) \tan(\delta_J)$, with

$$\tan(\delta_J) = \frac{\Omega^2 v_0^2 V_J [n_a - n_B(\Omega)]}{4\pi \rho e^5 \varepsilon (n_a + \frac{1}{2})},$$

(11)

where $V_J$ is the volume of the junction. This expression contains an additional prefactor of $[n_a - n_B(\Omega)]/[4\pi(n_a + 1/2)]$ when compared to the result obtained in [14].

For more complex devices such as qubits, one can use $\langle \hat{N}_j \rangle$ to denote the average number of photons inside a particular region $j$. The rate for energy relaxation of the qubit is then

$$\frac{1}{T_1} = \frac{R_{\text{diss}}}{\hbar \Omega \sum_j \left( \langle \hat{N}_j \rangle + \frac{1}{2} \right)} = \Omega \sum_i f_i \tan(\delta_i),$$

(12)

where $f_i = (\langle \hat{N}_i \rangle + 1/2)/\sum_j \langle \hat{N}_j \rangle$ is the participation ratio for the electric energy in region $i$, and $\tan(\delta_i)$ is the loss tangent calculated from Eq. (8) in region $i$. 

| Metal/Dielectric | $\tan(\delta_I)$ |
|------------------|------------------|
| Al/Al2O3        | $6 \times 10^{-6}$ |
| Al/amorphous-Al2O3 | $2 \times 10^{-6}$ |
| Nb/Nb2O5    | $7 \times 10^{-7}$ |

| Metal/Air       |                  |
|-----------------|------------------|
| Al              | $2 \times 10^{-4}$ |
| Nb              | $9 \times 10^{-5}$ |

| Dielectric/Air  |                  |
|-----------------|------------------|
| SiO2            | $6 \times 10^{-9}$ |
| amorphous-SiO2  | $1 \times 10^{-4}$ |
| Al2O3           | $1 \times 10^{-5}$ |
| amorphous-Al2O3 | $1 \times 10^{-5}$ |
| Nb2O5           | $9 \times 10^{-6}$ |

Phonon interference as the experimental signature of the intrinsic piezoelectric effect.—For devices with interfaces separated by a distance of the order of the phonon wavelength $\lambda_{\text{phonon}} = 2\pi v/\Omega (\sim 1$ µm for $\Omega \sim $ GHz), the phonons emitted by the interface piezoelectric effect will show signatures of interference. Consider the microstrip line shown in Fig. 1a; it can be modelled by the piezoelectric vector $p(\mathbf{r}) = p_0 \theta [\hat{n}(z-d/2) - \hat{n}(z+d/2)]$, with $p_0 = (e/2\pi v^2)(C_1 - C_2)/(C_1 + C_2)$ as in Eq. (9). Explicit calculations of Eq. (8) show that the loss tangent becomes oscillatory as a function of frequency for interface.

FIG. 1. (Color online) (a) Photon loss due to the piezoelectric effect at the interface. A photon travelling in the dielectric waveguide formed by a superconducting microstrip line and ground plane spontaneously decays into an acoustic phonon. The selection rules for photon to phonon conversion ensure the phonon propagates nearly perpendicular to the interface, with emission from the top and bottom interfaces interfering with each other. (b) Calculated loss tangent as a function of photon frequency $\Omega$ for the microstrip line shown in (a) with parameters for Al/Al2O3 and $d = 10$ µm. When stripline lateral width $W$ is comparable or larger than $d$, the loss tangent becomes oscillatory. The oscillations are the signature of phonon interference; they can be used to distinguish the piezoelectric effect from other sources of loss.
whose participation ratio is 100 – vices most electric energy is concentrated at the interface, emphasized that in the majority of superconducting de-

ton confinement in interfaces or junctions. It should be

bulk materials, it only occurs in the presence of pho-

to relaxations are free from TLSs. Therefore, superconducting qubits with optimal interfaces can reach $T_1 \sim 1000 \mu s$, above the threshold for quantum error correction [4].

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* igordiniz@ufrrj.br

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| $\rho$ (g/cm$^2$) | $v$ (km/s) | $\varepsilon/\varepsilon_0$ |
|-----------------|-----------|------------------|
| Al              | 2.7       | 6.51             |
| Nb              | 8.57      | 5.43             |
| SiO$_2$         | 2.66      | 6.40             |
| amorphous-SiO$_2$ | 2.20 | 5.80             |
| Al$_2$O$_3$     | 3.97      | 11.23            |
| amorphous-Al$_2$O$_3$ | 2.66 | 8.70             |
| Nb$_2$O$_5$     | 4.60      | 33.0             |

Conclusions.— We presented a theory of photon loss due to the piezoelectric effect. Our main result is Eq. (8), the explicit expression for the fraction of photon loss per cycle ($1/Q$) in a general inhomogeneous structure.

We showed that piezoelectric loss does not occur in bulk materials, it only occurs in the presence of photon confinement in interfaces or junctions. It should be emphasized that in the majority of superconducting devices most electric energy is concentrated at the interface, whose participation ratio is 100 – 1000 larger than the participation ratio of the junction [15]. This leads to the conclusion that the metal interfaces of a superconducting device radiate acoustic phonons more efficiently than their Josephson junctions [13, 14].

In current devices, photon loss is dominated by the presence of extrinsic TLS defects with localized dipole moment. Substantial effort is underway to make devices with crystalline interfaces, free of TLS defects. For these perfect devices, piezoelectricity provides the ultimate loss mechanism: Even perfect interfaces break inversion symmetry and are piezoelectric (unless their elastic constants are matched, see Eq. (9)). Table I shows explicit numerical predictions of the intrinsic loss tangent in several different crystalline and amorphous interfaces. These results show that loss reduction of the order of 10 – 100 is expected in devices with perfectly crystalline interfaces.