Spin-pairing instabilities at the coincidence of two Landau levels

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The effect of interactions near the coincidence of two Landau levels with opposite spins at filling factor $1/2$ is investigated. By mapping to Composite Fermions it is shown that the fluctuations of the gauge field induces an effective attractive Fermion interaction. This can lead to a spin-singlet ground state that is separated from the excited states by a gap. The magnitude of the gap is evaluated. The results are consistent with the recently observed half-polarized states in the FQHE at a fixed filling factor. It is suggested that similar anomalies exist for other spin configurations in degenerate spin-up and spin-down Landau levels. An experiment for testing the spin-singlet state is proposed.

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It is well established that in two dimensions (2D) the interacting electrons in a half-filled Landau level (LL) can be mapped via the Chern-Simons transformation to a weakly interacting Fermi liquid of Composite Fermions (CF) \cite{1, 2, 3}. This singular gauge transformation is equivalent to attaching two flux quanta to each electron. The CF feel an effective magnetic field that is smaller than the external one. It vanishes, on the average, at half filling. By introducing the CF, one can understand the principal features of the incompressible states in the fractional quantum Hall effect (FQHE) as an integer quantum Hall effect of CFs. The CF also help clarifying the nature and the properties of the compressible phases at other even-denominator filling factors in the first LL \cite{3}. The theoretical expectations concerning the transport properties of the latter states have been confirmed in surface-acoustic waves experiments. In transport measurements on anti-dot lattices near half filling, commensurability oscillations of CFs were observed. This strongly suggests that CFs can be viewed as real objects which, in certain situations, behave almost as classical point-like particles \cite{3}. This view is strongly supported by the recent report of CF cyclotron resonance \cite{3}.

Via radiative recombination of electrons in the inversion layer of high-electron mobility AlGaAs-heterostructures with holes bound to acceptors in the delta-doping region, the spin polarizations of several FQHE-ground states at fixed filling factors have been measured \cite{4} as a function of the ratio between Zeeman and the Coulomb energies, $\xi \equiv E_Z/E_C$. Crossovers between FQHE-ground states with different spin polarizations have been detected. Upon varying $\xi$, the polarization of the ground state remains constant within large intervals until a certain critical value $\xi_c$ is approached. Then, the system is transferred into a stable, differently polarized ground state. The experimental data have been found to be consistent with the model of non-interacting CFs with spin, and with an effective mass that scales with Coulomb interaction ($\propto \sqrt{B}$). The broad plateaus in the spin-polarization are then due to the occupation of a fixed number of spin split LL of the CF (CFLL). The crossovers occur when intersections of CFLLs coincide with the chemical potential.

Most strikingly, the experimentally determined spin polarizations, when extrapolated to zero temperature, show additional plateaus for magnetic flux densities near the centers of the crossovers. The corresponding polarizations are within the experimental uncertainty almost exactly intermediate between those in the neighboring broad plateaus. This indicates additional features beyond the non-interacting CF model and could be signature of a new collective state since one can expect that if two CFLLs are degenerate, residual interactions become very important and cannot be treated perturbatively. In these experiments, the CFLLs with different spins have been tuned to degeneracy via the dependence of the effective mass of the CFs on the magnetic field which was aligned perpendicular to the 2D system.

By measuring via NMR the spin polarization at filling factor $\nu = 2/3$, a remarkably abrupt transition from a fully polarized state to a state with polarization $3/4$ has been found when decreasing $\xi$ \cite{5}. This has been associated with a first order quantum phase transition. For $\nu > 2/3$, strong depolarization has been observed that is associated with two spin flips per additional flux quantum. In these measurements, $\xi$ has been tuned via tilting the magnetic field such that the electron system is also subject to an in-plane component of the magnetic field.

The nature of collective states under such conditions has been addressed in several works. For instance, several spin polarization instabilities has been found assuming the tilted field geometry for LLs \cite{3, 6}. Directly related to the above optical measurements, a non-translationally invariant charge-density-wave-state of CFs has been proposed on the basis of restricted Hartree-Fock calcula-
tions \[10\]. From exact diagonalizations of few interacting Fermions, a liquid of non-symmetric excitons has been suggested \[11\].

In this paper, it is proposed that a condensate of spin-singlet pairs of CF, could account for some of the anomalies observed near degeneracy. We investigate the coincidence of two LLs with opposite spins using the Chern-Simons gauge transformation and considering the effective interactions induced by the gauge field. The two intersecting LL can then be imagined as two degenerate CF-Fermi seas with opposite spins coupled by interactions. Our main result is that the fluctuations of the gauge field generate an effective interaction between CFs with opposite spins and momenta which is attractive in the limit of small frequency and long wavelengths. This indicates an instability towards a condensate of spin-singlet pairs. The gap between the ground state and the energetically lowest excited state is evaluated by solving the Eliashberg equation, similar to earlier work \[12, 13\]. We suggest that the half-polarized states of CFs observed in the experiments mentioned above can be understood by generalizing our model to pairing of second-generation CFs. We provide a quantitative estimate of the gap which is found to be roughly consistent with the experimental data. Similar to the previously discussed pairing instability in well-separated double layer QHE-systems \[14\], the condensate of CFs should show macroscopic quantum effects that could be experimentally addressed.

We consider two half-filled, degenerate LL with opposite spin. Using the Chern-Simons transformation we obtain CFs with spin that form 2D Fermi seas with a Fermi wave number \(k_F = \sqrt{2\pi p} (\rho \text{ total average electron number density})\) \[15\]. This system is formally analogous to a double-layer quantum Hall system with \(\nu = 1/2\) in each layer. The layer index is here the spin orientation \(s = \uparrow, \downarrow\) and the layer separation is zero.

In order to formally extract the effective CF-interaction we consider the Lagrangian density of the two coupled Chern-Simons liquids of charge \(e\)

\[
\mathcal{L}(r, t) = \mathcal{L}_F(r, t) + \mathcal{L}_{CS}(r, t) + \mathcal{L}_1(r, t)
\]

with the kinetic term of the Fermions

\[
\mathcal{L}_F(r, t) = \sum_{s=\uparrow, \downarrow} \psi_s(r, t) \left\{ \frac{1}{2m} \left( i \hbar \nabla + \frac{e}{c} \left( \mathbf{A}(r) - \mathbf{a}^s(r, t) \right) \right)^2 \right\} \psi_s(r, t),
\]

the gauge term \(\mathbf{a}\) perpendicular to the 2D plane

\[
\mathcal{L}_{CS}(r, t) = -\frac{e}{\phi_0} \sum_{s=\uparrow, \downarrow} a^s_0(r, t) \mathbf{z} \cdot \nabla \times \mathbf{a}^s(r, t),
\]

and the contribution of the Coulomb interaction

\[
\mathcal{L}_1(r, t) = -\frac{1}{2} \sum_{s, s'} \int d^2r' \rho_s(r, t) V(r-r') \rho_{s'}(r', t).
\]

Here, \(\rho_{s}(r, t) \equiv \psi^\dagger_{s}(r, t) \psi_s(r, t)\) is the density of the Fermions with spin orientation \(s\), \(\mathbf{A}\) the vector potential of the external magnetic field, \(\langle a^\mu_0, a_0^\alpha \rangle\) the gauge field for spin orientation \(s\), and \(V(r) = e^2/kr\) the Coulomb interaction.

The Chern-Simons Lagrangian \(L_{CS}\) is responsible for the attachment of \(\vec{\varphi}\) flux quanta \(\phi_0 \equiv \hbar c/e\) to each Fermion, as can be seen by minimizing the action with respect to the \(a^\mu_0\)-gauge field. This gives the constraint \(\mathbf{z} \cdot \nabla \times \mathbf{a}^s(r, t)/\vec{\varphi} \phi_0 = \rho_s(r, t)\). Here, the flux attachment for the two species of Fermions has been performed independently. We assume \(\vec{\varphi} = 2\), such that the mean fictitious magnetic field cancels the external one at half filling, \(\nu = \rho \phi_0/2B = 1/2\).

We use the transverse gauge, \(\nabla \cdot \mathbf{a}^s = 0\). Then, the Bosonic variables associated with the gauge field fluctuations are the *transverse* components of their Fourier transforms, \(a^\mu_0(q, \omega) \equiv \mathbf{z} \cdot \mathbf{q} \times \mathbf{a}^s(q, \omega) - (\mathbf{a}^s(q, \omega))\). By introducing the mean gauge field into \(L_T\) the external field \(\mathbf{A}\) is canceled. From the terms linear in the charge \(e\) and the momentum \(-i\hbar \nabla\), one easily can extract the form of the vertices connecting two Fermions with one gauge field fluctuation operator \(a^\mu_0(q, \omega) (\mu = 0, 1)\)

\[
u^\mu_0(k, k + q) = \left( \frac{\hbar c}{me} \mathbf{z} \cdot \mathbf{k} \times \mathbf{q} \right).
\]

In addition, there is a Fermion-gauge field coupling term quadratic in the fluctuation operators.

In order to derive the effective interaction between the CFs we generalize the diagrammatic Chern-Simons Fermi liquid description of FQHE states to include the spin. First, one inserts the above relation between the charge density and the gauge field into \(L_T\) and introduces the gauge field fluctuations such that \(L_{CS} + L_1\) describes the free gauge field. The effective CF interaction can then be obtained from the coupling terms in \(L_T(r, t)\) by tracing out the gauge field fluctuations. At imaginary time, one gets for the effective interaction in the Euclidean action

\[
\nu^\mu_0(k, k + q; \Omega_n) = \nu^\mu_0(k, k + q) \nu^\nu_0(k', k' - q)
\times [D^\nu_\mu(q, \Omega_n) + (2\delta_{ss'} - 1) D^\nu_\mu(q, \Omega_n)].
\]

This describes scattering of CFs from states with spin \(s, s'\) and momenta \(\hbar \mathbf{k}\) and \(\hbar \mathbf{k'}\) into states with \(\hbar (\mathbf{k} + \mathbf{q})\), \(\hbar (\mathbf{k}' - \mathbf{q})\) by exchanging a gauge field quantum with momentum \(\hbar \mathbf{q}\) and frequency \(\Omega_n = 2\pi nk_B T/\hbar (n \text{ integer})\).

The effective interaction contains the RPA gauge field propagators \(D^\alpha_\mu(q, \tau) \equiv (1/\hbar)(\langle T_\tau \sigma^\mu_\alpha(q, \tau) \sigma^\nu_\alpha(-q, 0) \rangle\) \((T_\tau\text{ time ordering operator and }\alpha = \pm)\) evaluated in terms of the symmetric and antisymmetric combinations of the gauge field fluctuations \(a^\mu_\alpha = (a^\mu_0 + a^\nu_0)/2\). In terms of the current-current correlation functions for free Fermions at zero magnetic field, \(\Pi^0_\mu = \Pi^0_{\psi\psi}(q, \Omega_n)\) one has

\[
(D^{-1})^\alpha_\mu(q, \Omega_n) = \left( -\frac{\Pi^0_0(q, \Omega_n)}{\Omega_n} \delta^\alpha_\mu - \Pi^0_1 \right).
\]
It can be shown that the dominant small-momentum small-energy contributions of the above symmetric and antisymmetric propagators correspond to \( \mu = \nu = 1 \). For \( \Omega_n \ll v_F q \ll v_F k_F \), \( \Pi^0_{ij} \approx e^2 m / \pi^2 h^2 \), \( \Pi^1_{ij} \approx -(e^2 q^2 / 12 + 2 \Omega_n)^2 / mc^2 \), such that

\[
D^{+\pm}(q, \Omega_n) \approx q \alpha_{\pm} q^{(5 + 3 \Omega)/2} + \eta \Omega_n
\]

with the constants \( \eta = 2 e^2 \rho / mc^2 v_F \), \( \alpha_+ = 2 e^2 / e \phi_0^2 \), \( \alpha_- = 4 \pi h^2 / 3 m v_F^2 \). For small wave numbers, and frequency \( \Omega_n \to 0 \), the antisymmetric propagator \( D^{+\pm}(q, \Omega_n) \) dominates. This does not contain the Coulomb interaction, as pointed out earlier \[14\]. In the Cooper channel, the effective interaction \( V^{+\pm}(q, \Omega_n) \) is attractive for \( \Omega_n \to 0 \) due to \( D^{+\pm}(q, 0) \propto q^{-2} (q \to 0) \) and can lead to an instability towards the formation of spin-singlet pairs of CFs in a single QHE layer.

We calculate the quasi-particle energy gap for the Cooper channel in mean field approximation \[12, 14\] using the Nambu field \( \Phi_{\mu}(\tau) = [c^\dagger_{\sigma}(\mathbf{k}, \tau), c_{\sigma}(\mathbf{k}, \tau)] \) where \( c_{\sigma}(\mathbf{k}, \tau), c^\dagger_{\sigma}(\mathbf{k}, \tau) \) are the Fermion annihilation and creation operators, respectively, for spin \( s \) and momentum \( h\mathbf{k} \) at imaginary time \( \tau \). The Green function \( \mathcal{G}(\mathbf{k}, \tau) = -(1 / h) \langle T \Phi_{\mu}(\tau) \Phi_{\mu}^\dagger(0) \rangle \) is a 2 x 2 matrix, \( \mathcal{G}_{ij} \), that is obtained by inverting the Dyson equation

\[
\mathcal{G}^{-1}(\mathbf{k}, \omega_n) = \mathcal{G}^{-1}_0(\mathbf{k}, \omega_n) - \Sigma(k, \omega_n).
\]

Here, \( \mathcal{G}_0(\mathbf{k}, \omega_n) = [i \sigma_0 \gamma_{\omega_n} - \sigma_3 (h^2 k^2 / 2 m - \mu)]^{-1} \) is the (diagonal) Green function for a free Fermion pair. Here, \( \sigma_0 \) is the unit matrix, \( \sigma_3 \) the Pauli matrix, \( \mu \) the chemical potential, and \( \omega_n = (2 n + 1) \pi k_B T / h \) a Fermionic frequency. The self-energy \( \Sigma(k, \omega_n) \) is a 2 x 2 matrix \( \Sigma_{ij} \) with the diagonal elements describing the renormalization of the Fermion mass, while the off-diagonal matrix element defines the pair-breaking gap \( \Delta \)

\[
\Delta(k, \omega_n) \equiv \frac{i \hbar \omega_n \Sigma_{12}(k, \omega_n)}{i \hbar \omega_n - \Sigma_{11}(k, \omega_n)}.
\]

For the self-energy we use only the Fock term \[12, 13\],

\[
\Sigma_{ij}(k, \omega_n) = \frac{k_B T}{(2 \pi)^2} \sum_{n'} \int dq \mathcal{G}_{ij}(k - q, \omega_n - \omega_n') \times [\delta_{ij} V^{+\pm}_{11}(q, k, q; \omega_n') + (\delta_{ij} - 1) V^{+\pm}_{11}(q, -k, q; \omega_n')]
\]

In order to evaluate this explicitly, we assume \( k \approx k_F \). Carrying out the necessary integrations, by analytical continuation to real frequencies, \( \omega_n \rightarrow -i \omega + \delta \), using the spectral representation of the Green function, and combining the self-energy equation with the above \[10\] one obtains an implicit equation for \( \Delta(\omega) \equiv \Delta(k_F, \omega) \) in the zero-temperature limit,

\[
\Delta(\omega) = \frac{C}{\omega} \int_{-\infty}^{\infty} dz \Theta(h | z | - \Delta(z)) \times \frac{[z K_1(z) \Delta(\omega) - \omega \Delta(z) K_2(z)]}{[h^2 z^2 - \Delta^2(z)]^{1/2}}
\]

with the constant \( C = e^2 h k_F / 8 \pi^3 m c^2 \) (\( E_F \) Fermi energy),

\[
K_j(z) \equiv \text{sgn}(z) [F^j_+(z) - (-1)^j F^j_-(z)],
\]

\((j = 1, 2)\) and

\[
F^+_j(z) = \int_{-\infty}^{\infty} \frac{d\zeta}{\zeta + z - \omega - i\delta} \text{Im} D^\pm(\zeta)
\]

where \( D^\pm = - \int_0^{\infty} dq D^{+\pm}(q, -i\zeta) \). By evaluating the integrals one obtains for \( \omega \to 0 \) to leading order the self-consistency condition

\[
1 = C^{-1/3} - C^+ \left[ \ln \left( \omega_0 / C^+ \right) \right]^2.
\]
the number of "effective" flux quanta crossing the sample. In analogy to the above, one can perform a gauge transformation leading to "second generation" CFs. The corresponding gauge fluctuations mediate an effective attractive interaction. This leads to the formation of a condensate. From the experiments, one estimates $k_F e_C \approx 1$ and $E_F / 2\pi e_C \approx 0.01$, such that $\Delta \approx 0.3 E_F$. This value is more or less consistent with the experimental observations concerning the spin flip gap which give values of the order 0.2 K and the experimental Fermi energy of 1 K [3], though this numerical estimate is already somewhat outside the range of validity of the asymptotic condition $\gamma_2$. The existence of the gap at the crossing point implies that in a region of magnetic fields around this point, where the energy difference between the CFLLs is less then $\Delta$, the condensate remains stable. The formation of such a state of singlet CF-pairs was then responsible for the formation of a plateau exactly at half the distance between the neighboring plateaus in an interval of magnetic fields near the crossover point.

Second, one can suspect that mechanisms similar to the above can also lead to instabilities at other filling factors. Near degeneracy, non-equal but commensurate fillings of spin-up and spin-down states might yield collective instabilities with intermediate spin polarizations. For instance, consider $\nu = 2/3$. This corresponds to two (Zeeman-split) CFLLs with $p = -2$. When changing the magnetic field, the uppermost (spin-down) Zeeman level of the lowest CFLL and the lower (spin-up) Zeeman level of the first CFLL intersect. When the electron density is adjusted to the degeneracy point, the lowest spin-up CFLL is fully occupied, while only half of the states in the degenerated levels are filled. A priori, it is not clear how the Fermions are distributed among these states. For the ground state all possible configurations have to be taken into account. Above we have considered the special case of both levels being half-filled. However, also other configurations can yield stable states. Assume, for instance, that $3/4$ of the spin-up level of the first CFLL and $1/4$ of the lowest spin-down CFLL are occupied at degeneracy and that this, together with the totally occupied lowest spin-up Zeeman level, would correspond to a stable state via the effective interaction. This would give an average spin polarization of $\gamma = 3/4$. If such a state was stable in the presence of an in-plane component of the magnetic field, it could account for the recently reported 3/4-state observed in NMR at $\nu = 2/3$ [3].

Third, the possibility of generating long-range spin-pairing correlations in a single 2D Hall sample, similar to those discussed previously for QHE double layers [13, 15], by tuning the density and the magnetic field to induce the above crossing between spin-up and spin-down LL leads to interesting speculations. For instance, consider two QHE systems in the same plane, say at $\nu = 2/5$, separated by a tunnel junction. By tuning the two densities to the value of the point of degeneracy a "Josephson current" should flow. Such a current should vanish as soon as one of the two densities was detuned.

In conclusion, we have considered two Landau levels with opposite spins tuned to intersect at filling factor 1/2 at the Fermi level. By applying the Chern-Simons gauge transformation, we have derived an effective attractive CF interaction. This yields an instability towards a spin-singlet condensate. We have discussed several experimental consequences. In order to observe the predicted spin-singlet state, a close-to-zero in-plane component of the magnetic field should be necessary as has been achieved in the spin-polarization experiments done in the region of the FQHE. Our results suggest that different occupations of spin-up and spin-down LLs could account for instabilities at other fractional polarizations and that an in-plane component of the magnetic field could account for an anisotropic spin-singlet condensate.

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[1] J. K. Jain, Phys. Rev. Lett. 63, 199 (1989).
[2] B. I. Halperin, P. A. Lee, and N. Read, Phys. Rev. B 47, 7312 (1993).
[3] Composite Fermions , edited by O. Heinonen (World Scientific 1998).
[4] R. L. Willett, Adv. Phys. 46, 447 (1997) and references therein.
[5] I. V. Kukushkin, J. H. Smet, K. v. Klitzing, and W. Wegscheider, Nature 415, 409 (2002).
[6] I. V. Kukushkin, K. v. Klitzing, and K. Eberl, Phys. Rev. Lett. 82, 3665 (1999); I. V. Kukushkin, K. v. Klitzing, K. G. Levchenko, and Yu. E. Lozovik, Pis’ma Zh. Éksp. Teor. Fiz. 70, 722 (1999) [JETP Letters 70, 730 (1999)]; I. V. Kukushkin, J. H. Smet, K. v. Klitzing, and K. Eberl, Phys. Rev. Lett. 85, 3688 (2000).
[7] N. Freytag, Y. Tokunaga, M. Horvatić, C. Berthier, M. Shayegan, and L. P. Levy, Phys. Rev. Lett. 87, 136801 (2001).
[8] G. F. Giuliani, and J. J. Quinn, Phys. Rev. B 31, 6228 (1985).
[9] S. Yarlagadda, Phys. Rev. B 44, 13101 (1991).
[10] G. Murthy, Phys. Rev. Lett. 84, 350 (2000).
[11] V. M. Apalkov, T. Chakraborty, P. Pietiläinen, and K. Niemela, Phys. Rev. Lett. 86, 1311 (2001).
[12] D. V. Khveshchenko, Phys. Rev. B 47, 3446 (1993).
[13] N. E. Bonesteel, Phys. Rev. B 48, 11484 (1993).
[14] N. E. Bonesteel, I. A. McDonald, and C. Nayak, Phys. Rev. Lett. 77, 3009 (1996).
[15] N. Nagaosa, and P. A. Lee, Phys. Rev. Lett. 64, 2450 (1990).
[16] N. E. Bonesteel, Phys. Rev. Lett. 82, 984 (1999).
[17] M. M. Fogler and F. Wilczek, Phys. Rev. Lett. 86, 1833 (2001).
[18] I. B. Spielman, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. 84, 5808 (2000).