Slow sound laser in lined flow ducts

Antonin Coutant, Yves Aurégan, Vincent Pagneux

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**Analogy** [Unruh 1981]

**Waves in a spacetime**

\[ \partial_\mu \sqrt{|g|} g^{\mu\nu} \partial_\nu \phi = 0 \]

Picture of a Black Hole
(From: Event Horizon Telescope ’19)

**Acoustic** waves with mean flow

\[ (\partial_t + \nabla \cdot \mathbf{v}) \frac{\rho}{c^2} (\partial_t + \mathbf{v} \cdot \nabla) \phi - \nabla \cdot \rho \nabla \phi = 0 \]
What happens when flow becomes faster than waves?

- Wave trapping
  → acoustic analogue of a black hole: *Dumb hole*
- Exotic wave effects around black holes could be reproduced
What happens when flow becomes faster than waves?

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“Slow sound”: an opportunity to realize dumb holes?
Speed of sound:

\[ c_0 = \sqrt{\frac{K}{\rho_0}} \]

*\(K\): isentropic bulk modulus

*\(\rho_0\): density

**Slow sound:**

- Tubes mounted flush
  \(\rightarrow\) **Metamaterial like**
- **Lower the effective stiffness**
- Long wavelengths:
  \[ c_{\text{eff}} < c_0 \]

Waveguide with impedance treatment
Wave propagation in lined ducts

Units where duct height $H = 1$ and $c_0 = 1$

Potential perturbations $\mathbf{v} = \nabla \phi$

Inside the duct

$$\partial_t^2 \phi - \Delta \phi = 0.$$
Wave propagation in lined ducts

Boundaries:

- $y = 0$ hard wall: $\partial_y \phi = 0$
- $y = 1$ impedance wall: $p = Z \partial_y \phi$
- Complex impedance $Z$. Tube model:

$$Z(\omega) = \frac{i}{\sigma \tan(b\omega)} + \Gamma$$

- Dissipation $\Gamma$ neglected
- Low frequencies $\omega \ll \pi/2b$ (subresonance)
Wave propagation in lined ducts

- Fixed frequency $\omega = 2\pi f$
- **Two** acoustic modes
- $\omega \ll \pi/2b$:
  $$\omega^2 = \frac{k^2}{1 + b}$$

Hence $c_{\text{eff}} = 1/\sqrt{1 + b}$

- Increase $\omega \rightarrow$ **dispersion**
Lined ducts with mean flow

- Mean Mach number $M_0 = U_0/c_0$
- **Two** types of flow:
  - **Subsonic:** $M_0\sqrt{1+b} < 1$
  - **Supersonic:** $M_0\sqrt{1+b} > 1$
- Possible for $M_0 < 1$
- What is the dispersion relation?
Lined ducts with mean flow

- Inside the duct

\[(\partial_t + M \partial_x)^2 \phi - \Delta \phi = 0.\]

- **Hard problem:** vorticity modes, inhomogeneities, etc.
- \( M \approx M_0 \) constant except in **boundary layer** \( 1 - \delta \lesssim y < 1 \)
Lined ducts with **mean flow**

Asymptotic approach: **effective boundary condition**

[Brambley '13, AC, Auréган, Pagneux '19]

Our boundary condition

- pressure \( p \) is continuous
- displacement \( \eta \) is continuous
- **But** effective compliance

\[
C_{\text{eff}}(\omega) = \frac{\eta}{p} = C_0(\omega) + \delta C_1(\omega)
\]

- \( \delta = 0 \) is Ingard-Myers boundary conditions
Lined ducts with mean flow

Subsonic flows $M_0 < c_{\text{eff}}$

- Two acoustic modes (Doppler shifted)
- Two hydrodynamic modes (disappear for $M_0 \to 0$)
  - One with positive energy $k_S$
  - One with negative energy $k_N$
- One boundary layer mode $k_B$ (disappear for $\delta \to 0$)
Lined ducts with mean flow

Supersonic flows $M_0 > c_{\text{eff}}$

- Hydrodynamic modes disappear
- Two acoustic modes \textbf{co-moving} with flow
Impedance change $\rightarrow$ transsonic flow

- $M_0 > c_{\text{eff}} \rightarrow \text{wave trapping}$
- Acoustic modes cannot propagate against the flow
- **Acoustic analogue of a black hole**
  
  [Unruh ’81 [...], Auregan, Fromholz, Michel, Pagneux, Parentani 15’]
Impedance change $\rightarrow$ transsonic flow

Important properties of transsonic flows:

Couple very different wavelengths  
(due to Doppler effect)
Effective transsonic flows

- Supersonic flow
  \[ \gamma \rightarrow k_B \]
  \( \text{evanescent} \rightarrow \\rightarrow \rightarrow \rightarrow \rightarrow \)
  \[ 0 \rightarrow k_N \]
  \[ 0 \rightarrow k_{A+} \]

- Subsonic flow
  \[ k_N \rightarrow \beta \]
  \[ k_{A+} \rightarrow R \]
  \[ k_S \rightarrow \alpha \]
  \[ k_B \rightarrow 0 \]
  \[ k_{A-} \rightarrow 1 \]

- Impedance change \( \Rightarrow 4 \times 4 \) scattering matrix
- Energy conservation
  \[ |\alpha|^2 - |\beta|^2 + |\gamma|^2 + |R|^2 = 1 \]
- Possibility of amplification
  \( \Rightarrow \) analogue of the Hawking radiation of black holes
Double transsonic flows

Supersonic flow

- $k_B$  
- $k_{Lev}$  
- $k_N$  
- $k_{A+}$

Subsonic flow

- $k_B$  
- $k_{A-}$  
- $k_N$  
- $k_{A+}$  
- $k_S$

Supersonic flow

- $k_B$  
- $k_{Rev}$  
- $k_N$  
- $k_{A+}$
Double transsonic flows

Supersonic flow

- $k_B$
- $k_{Lev}$
- $k_N$
- $k_{A+}$

Subsonic flow

- $k_B$
- $k_{A-}$
- $k_N$
- $k_{A+}$
- $k_S$

Supersonic flow

- $k_B$
- $k_{Rev}$
- $k_N$
- $k_{A+}$

$k_N$

$E_N < 0$

Energy exchange

Resonant cavity

- $k_{A-}$
- $k_S$

$E_{cav.} > 0$

Energy exchange

- $k_{A+}$

$E_{A+,B} > 0$

$E_{N} < 0$

$E_{cav.} > 0$

$E_{A+,B} > 0$
Described by **complex eigen-frequencies** $\omega \in \mathbb{C}$

**Eigen-value problem**
Double transsonic flows

Eigen-value problem

- Boundary conditions
  - \( \text{Im}(\omega) > 0 \): decaying for \( x \to \pm\infty \)
    \(\rightarrow\) unstable solutions
  - \( \text{Im}(\omega) < 0 \): analytic continuation
    \(\rightarrow\) resonances

  \(\rightarrow\) equivalent to outgoing boundary conditions

- Spectrum symmetry \( \text{Re}(\omega) \to -\text{Re}(\omega) \)
Double transsonic flows

**Eigen-value problem**

- **Boundary conditions**
  - $\text{Im}(\omega) > 0$: decaying for $x \rightarrow \pm \infty$
    - $\rightarrow$ **unstable solutions**
  - $\text{Im}(\omega) < 0$: analytic continuation
    - $\rightarrow$ **resonances**
  
  $\rightarrow$ equivalent to outgoing boundary conditions

- **Spectrum symmetry** $\text{Re}(\omega) \rightarrow -\text{Re}(\omega)$

- **How does the spectrum change with external parameters?**
  - $\rightarrow$ **Two main properties**
Parameters $b_O = 15$, $b_C = 6$, $M = 0.3$, $\delta = 0.002$

- $L$ varies from 0.2 to 3.6

**Two types of unstable modes:**

- Static $\text{Re}(\omega) = 0$
- Dynamic $\text{Re}(\omega) \neq 0$
**Slow sound laser**

Mode profiles - Subsonic region in grey

- **Subwavelength** instability
- Governed by hydrodynamic wavelengths:
  Unstable mode if \( k_S L \approx \pi/2 \) with \( k_S \) hydrodynamic mode
Conclusion

Slow sound laser:

- Double transsonic flows
- Can be static or dynamic

[AC, Aurégan, Pagneux, JASA 2019, arXiv:1904.03079]

Analogous to “black hole laser”

→ leads to rich nonlinear phenomenology

- Undular bore? (static)
- Dispersive shock waves? (dynamic)
- Emission of solitons? (dynamic)
Conclusion

**Slow sound laser:**

- Double transsonic flows
- Can be *static* or *dynamic*

[AC, Aurégan, Pagneux, JASA 2019, arXiv:1904.03079]

Analogous to “black hole laser” → **leads to rich nonlinear phenomenology**

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Thank you.