The Ten-dimensional Effective Action of Strongly Coupled Heterotic String Theory

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Abstract

We derive the ten-dimensional effective action of the strongly coupled heterotic string as the low energy limit of M–theory on $S^1/Z_2$. In contrast to a conventional dimensional reduction, it is necessary to integrate out nontrivial heavy modes which arise from the sources located on the orbifold fixed hyperplanes. This procedure, characteristic of theories with dynamical boundaries, is illustrated by a simple example. Using this method, we determine a complete set of $R^4$, $F^2 R^2$, and $F^4$ terms and the corresponding Chern-Simons and Green-Schwarz terms in ten dimensions. As required by anomaly cancelation and supersymmetry, these terms are found to exactly coincide with their weakly coupled one-loop counterparts.

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1 Introduction

Almost two years ago, Hořava and Witten completed the cycle of string theory dualities by relating the strongly coupled limit of the $E_8 \times E_8$ heterotic string to M–theory compactified on a $S^1/Z_2$ orbifold [1, 2]. The low-energy effective action of M–theory is usually simply that of eleven-dimensional supergravity. However, the presence of the orbifold projection means that the gravitino fields are chiral ten-dimensional fields on the two fixed hyperplanes of the orbifold. In order to cancel the corresponding anomalies which appear in fermion loops, it is necessary to introduce two sets of ten-dimensional $E_8$ gauge fields, one on each fixed hyperplane. Supersymmetry then requires that the bulk and hyperplane theories are not independent. In particular, the gauge fields act as magnetic sources for the four-form field strength of the bulk supergravity, and as stress-energy sources for the graviton.

When further compactified on a Calabi-Yau space, at tree level, the strong limit provides a better match to the predicted four-dimensional gravitational and grand-unified couplings than does the weakly coupled heterotic string. Witten has shown that such compactifications exist [3], although the internal space becomes distorted, while Hořava showed that the theory has a topological version of gaugino condensation [4]. This has led to a number of papers reconsidering basic four-dimensional string phenomenology in this new limit [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21].

To discuss the low-energy physics, one derives a four-dimensional effective action by dimensional reduction. The full eleven-dimensional effective action is an expansion in powers of the eleven-dimensional gravitational coupling constant $\kappa$. The lowest dimension operators of the four-dimensional theory then have a double expansion in terms of the Calabi-Yau and orbifold sizes compared to the eleven-dimensional Planck length $\kappa^{2/9}$. To zeroth order, the dimensional reduction to four dimensions is simple because the source terms on the orbifold fixed planes can be ignored. To this order, the derivation is identical to reducing on a circle. However, to the next order the reduction is more complicated. The presence of sources localized on the fixed planes means that one cannot consistently take all fields to be independent of the orbifold direction, as one would in a conventional truncation on a circle. In order to match the boundary conditions implied by the sources, the bulk fields must vary across the interval. The inclusion of extra terms which arise in such a reduction was an important ingredient in the derivation of the full low-energy action, including some terms of order $\kappa^{4/3}$, given in two previous papers [17, 20].

The purpose of the present paper is to refine exactly how this reduction works and to use it to calculate the ten-dimensional effective action for the strongly coupled $E_8 \times E_8$ heterotic string. This is the limit where the orbifold interval remains large compared to the eleven-dimensional Planck length but all fields are assumed to have wavelengths much longer than the orbifold size. This action can then be compared with the corresponding weakly coupled theory. In particular, we will
concentrate on deriving the local higher-order terms in the Riemann curvature $R_{AB}$ and gauge curvature $F_{AB}$, up to quartic order.

In general curvature terms enter the ten-dimensional action has in an expansion of the form

$$S = \int \sum_{n=1}^{\infty} a_n(\phi) A^n$$

where $A$ represents either the Riemann or gauge curvature, and $\phi$ is the dilaton, which is the modulus of the compact eleventh dimension in the strongly coupled theory. In the perturbative limit, in the string frame, the coefficients can be expanded as $a_n = b_{n,0} e^{-2\phi} + b_{n,1} + b_{n,2} e^{2\phi} + \cdots$. For the lower-dimension terms, supersymmetry implies a number of strong constraints on the form of $a_n$. Up to quadratic order, it is well known that the only supersymmetric invariants are $e^{-2\phi} R$, $e^{-2\phi} R^2$ and $e^{-2\phi} F^2$, where the last two terms must be paired with Chern-Simons terms in the three-form field $H$. Furthermore there are no supersymmetric invariants of the form $A^3$. At quartic order there are two types of invariant. There is a parity-odd term $t_8 A^4 - \frac{1}{2\sqrt{2}} e^{(10)} \epsilon^{(10)} B A^4$, with no dilaton dependence and so with only a one-loop contribution in the weak expansion. (For notation and a definition of $t_8$ see section 3 below.) There is also a parity-even term including $f(\phi) t_8 t_8 R^4$ and which does not include a coupling to $B$, which appears to allow arbitrary dilaton dependence. It is terms of this form which appear as stringy tree-level corrections in the weakly coupled limit. The parity-odd invariants include the Green-Schwarz term required for anomaly cancelation. This, in fact, then fixes the coefficients of these terms with respect to the lower-dimension terms. Consequently, ignoring the parity-even quartic terms, the form of the effective action is completely fixed to this order by supersymmetry and anomaly cancelation. (This is the simpler heterotic analog of the non-renormalization of $R^4$ terms found in type II theories.) In this paper we will show how these terms appear when the strongly coupled theory is reduced to ten-dimensions. We will not explicitly include the parity even terms, but will make some comments about how they might arise.

A calculation of some such terms and a discussion of anomaly cancelation was first presented by Dudas and Mourad, who noted that to obtain the full result it would be necessary actually to integrate out the massive Kaluza-Klein modes. As we will see, this is exactly the procedure we will perform. By clarifying the form of the dimensional reduction, we will find that we can reproduce the full parity-odd $R^4$, $R^2 F^2$ and $F^4$ supersymmetric invariants. While we will not explicitly consider other terms of the same dimension but involving other fields in the supergravity and gauge multiplets, we will find that the dimensional reduction procedure provides a convenient way of deriving complete higher-order supersymmetric actions. The result that the $R^4$, $R^2 F^2$ and $F^4$ terms do not renormalize beyond one loop helps explain the result that the low-energy four-dimensional effective actions derived in have the same form as those derived in the weakly coupled limit including loop corrections on a large Calabi-Yau manifold.
Let us end this introduction by summarizing our conventions. We denote the coordinates in the eleven-dimensional spacetime $M_{11}$ by $x^0, \ldots, x^9, x^{11}$ and the corresponding indices by $I, J, K, \ldots = 0, \ldots, 9, 11$. The orbifold $S^1/Z_2$ is chosen in the $x^{11}$–direction, so we assume that $x^{11} \in [-\pi \rho, \pi \rho]$ with the endpoints identified as $x^{11} \sim x^{11} + 2\pi \rho$. The $Z_2$ symmetry acts as $x^{11} \to -x^{11}$. Then there exist two ten-dimensional hyperplanes, $M^{(i)}_{10}$ with $i = 1, 2$, locally specified by the conditions $x^{11} = 0$ and $x^{11} = \pi \rho$, which are fixed under the action of the $Z_2$ symmetry. We will sometimes use the “downstairs” picture where the orbifold is considered as an interval $x^{11} \in [0, \pi \rho]$ with the fixed hyperplanes forming boundaries to the eleven-dimensional space. In the “upstairs” picture the eleventh coordinate is considered as the full circle with singular points at the fixed hyperplanes. We will use indices $A, B, C, \ldots = 0, \ldots, 9$ to label the ten-dimensional coordinates.

2 Dimensional reduction with orbifold sources

As we have stressed in the introduction, new features arise when making a dimensional reduction on a $S^1/Z_2$ orbifold with sources on the orbifold fixed planes. In this section we will show how such a reduction can be performed consistently in the case when it is possible to make a perturbative expansion in the strength of the sources. Rather than consider the full eleven-dimensional description of the strongly coupled heterotic string, we will illustrate the issues involved in a simpler toy model with only a scalar field.

Consider a scalar field $\phi$ in the bulk of the eleven-dimensional spacetime, together with two sources $J^{(1)}$ and $J^{(2)}$ localized on the two fixed hyperplanes of the orbifold. In M–theory on $S^1/Z_2$, the rôle of $\phi$ will be played by the eleven-dimensional metric and the three-form, while the sources are provided by gauge fields and $R^2$ terms on the orbifold fixed planes. Therefore, in general, the sources will be functionals of fields living on the orbifold fixed planes.

Let us consider the following simple theory, in the upstairs picture, with a standard kinetic term for $\phi$ and a linear coupling of $\phi$ to the sources,

$$ S = -\frac{1}{2} \int_{M^{(1)}_{11}} \langle \partial \phi \rangle^2 - \int_{M^{(2)}_{11}} J^{(1)} \phi - \int_{M^{(2)}_{11}} J^{(2)} \phi, $$

(2.1)

which is sufficient to illustrate the main points. The scalar field must have definite charge under the orbifold symmetry. Since the sources involve the value of $\phi$ on the fixed planes, we must take $\phi$ to be even under $Z_2$, so that $\phi(-x^{11}) = \phi(x^{11})$.

The equation of motion for $\phi$ reads

$$ \partial^2 \phi = J^{(1)} \delta(x^{11}) + J^{(2)} \delta(x^{11} - \pi \rho) $$

(2.2)

By Gauss’s theorem, for a small volume intersecting the orbifold plane we can integrate this equation
to give, near \( x^{11} = 0 \) and \( x^{11} = \pi\rho \) respectively

\[
\partial_{11} \phi = \frac{1}{2} J^{(1)} \epsilon(x^{11}) + \ldots
\]

\[
= \frac{1}{2} J^{(2)} \epsilon(x^{11} - \pi\rho) + \ldots
\]

where \( \epsilon(x) \) is the step function, equal to 1 for \( x > 0 \) and \(-1\) for \( x < 0 \). To derive this expression, we have used that fact that \( \partial_{11} \phi \) must be odd under the \( Z_2 \) orbifold symmetry. The dots represent terms which vanish at \( x^{11} = 0 \) in the first line, or at \( x^{11} = \pi\rho \) in the second line.

In the downstairs picture, the orbifold is an interval bounded by the hyperplanes. Rather than having an equation of motion with delta function sources, we then have the free equation \( \partial^2 \phi = 0 \) together with the boundary conditions, corresponding to the limiting expressions for \( \partial_{11} \phi \) as one approaches the boundaries,

\[
n_I^{(1)} \frac{\partial I \phi}{M_{10}^{(1)}} = \frac{1}{2} J^{(1)} \quad n_I^{(2)} \frac{\partial I \phi}{M_{10}^{(2)}} = -\frac{1}{2} J^{(2)}.
\]

Here \( n^{(i)} \) are normal unit vectors to the two hyperplanes, pointing in the direction of increasing \( x^{11} \). They are introduced solely in order to write these expressions in a covariant way. Note that the second expression comes with a negative sign since it is evaluated just to the left of the \( x^{11} = \pi\rho \) orbifold plane, while the first expression comes with a positive sign since it is evaluated just to the right of the \( x^{11} = 0 \) plane.

In a conventional dimensional reduction, one makes a Fourier expansion in the compact direction. The massive Kaluza-Klein modes have no linear coupling to the massless modes and have masses set by the size of the compact dimension. Thus at low energies they decouple and the effective theory can be obtained by simply dropping them from the action. More formally, this is equivalent to integrating them out at tree-level. (Massive loops can however contribute, but since in this paper we will be reducing what is already an effective action, we will not be interested in this possibility.) It is clear from the \( \phi \) equation of motion \((2.2)\) that a similar truncation will not work here. We might imagine trying to assume that both \( \phi \) and the sources \( J^{(i)} \) are independent of the eleventh coordinate. However, the delta functions in the sources mean that even though \( J^{(i)} \) are independent of \( x^{11} \) we must have \( \phi \) depending on the orbifold coordinate if we are to solve the equation of motion. Equivalently, we clearly can not satisfy the boundary conditions \((2.4)\) without \( \phi \) depending on \( x^{11} \). There is no consistent solution where the massive modes are set to zero. Instead we must consider more carefully what happens when these modes are integrated out.

First, we can consider expanding both \( \phi \) and the total source in the scalar equation of mo-
tion (2.2) in Fourier modes. We write
\[ \phi = \phi^{(0)} + \sum_n \tilde{\phi}^{(n)} \cos(nx^{11}/\rho) = \phi^{(0)} + \phi^{(B)} \]
\[ J = J^{(1)} \delta(x^{11}) + J^{(2)} \delta(x^{11} - \pi \rho) = J^{(0)} + \sum_n \tilde{J}^{(n)} \cos(nx^{11}/\rho) = J^{(0)} + J^{(B)}. \] (2.5)

Since both \( \phi \) and the total source \( J \) must be even under the \( Z_2 \) symmetry, only the cosine terms in the Fourier expansion contribute. We can give an explicit form for the Fourier components \( \tilde{J}^{(n)} \), but all we will require in what follows is that
\[ J^{(0)} = \frac{1}{2\pi \rho} \left( J^{(1)} + J^{(2)} \right) \]
\[ J^{(B)} = J^{(1)} \delta(x^{11}) + J^{(2)} \delta(x^{11} - \pi \rho) - \frac{1}{2\pi \rho} \left( J^{(1)} + J^{(2)} \right). \] (2.6)

We note, confirming the discussion above, that even with \( J^{(i)} \) independent of \( x^{11} \), the massive mode source \( J^{(B)} \) is non-zero. By definition, with the eleven dimensional average given by
\[ \langle F \rangle_{11} = \frac{1}{\pi \rho} \int_0^{\pi \rho} dx^{11} F \] (2.7)
the massive modes average to zero, \( \langle \phi^{(B)} \rangle_{11} = \langle J^{(B)} \rangle_{11} = 0. \)

We can now substitute these expansions in the action (2.1) and integrate out the massive Kaluza-Klein modes. Rather than performing this integration separately for each mode, it is easier to integrate out \( \phi^{(B)} \) as an eleven-dimensional field. Substituting into the action we have
\[ S = -2\pi \rho \int_{M_{10}} \left\{ \frac{1}{2} \left( \partial \phi^{(0)} \right)^2 + J^{(0)} \phi^{(0)} \right\} - \int_{M_{11}} \left\{ \frac{1}{2} \left( \partial \phi^{(B)} \right)^2 + J^{(B)} \phi^{(B)} \right\} \] (2.8)
where \( M_{11} = M_{10} \times S^1/Z_2. \) Because the massless and massive modes are orthogonal, they separate in the action. The massive modes can now be integrated out. A simple Gaussian integration gives
\[ S = -2\pi \rho \int_{M_{10}} \left\{ \frac{1}{2} \left( \partial \phi^{(0)} \right)^2 + J^{(0)} \phi^{(0)} \right\} - \int_{M_{11}} \frac{1}{2} J^{(B)} \Phi^{(B)} \] (2.9)
where \( \Phi^{(B)} \) is the solution of the massive equation of motion
\[ \partial^2 \phi^{(B)} = J^{(B)} \quad \iff \quad \Phi^{(B)}(x) \equiv \phi^{(B)}(x) = \int_{x'} G(x - x') J^{(B)}(x') \] (2.10)
with \( G(x - x') \) the eleven-dimensional Green's function. Using the form of \( J^{(0)} \) and \( J^{(B)} \) given in (2.6) and the fact that \( \langle \Phi^{(B)} \rangle_{11} \) is zero, we find that the action can be written as the ten-dimensional action
\[ S_{10} = -2\pi \rho \int_{M_{10}} \left\{ \frac{1}{2} \left( \partial \phi^{(0)} \right)^2 + J^{(0)} \phi^{(0)} \right\} + \frac{1}{4\pi \rho} \left( J^{(1)} \Phi^{(B)} \bigg|_{M_{10}^{(1)}} + J^{(2)} \Phi^{(B)} \bigg|_{M_{10}^{(2)}} \right). \] (2.11)
Since the solution (2.10) for $\Phi(B)$ is linear in the sources $J^{(i)}$, we see that by integrating out $\phi(B)$ we have generated a new term quadratic in the sources in the ten-dimensional effective action. Furthermore, since the calculation is purely classical and $\phi(B)$ enters the action quadratically, the process of integration is identical to substituting the solution $\Phi(B)(x)$ directly into the action (2.8). As we have mentioned, the sources will, in general, be given in terms of other fields of the theory, including perhaps $\phi(0)$. The corresponding equations of motion now arise by varying the fields in the ten-dimensional effective action (2.11). By integrating out the massive modes we ensure that reducing the action is equivalent to reducing the equations of motion.

We would like to have an exact expression for the ten-dimensional effective action in terms of the sources $J^{(i)}$ only. However, we cannot give a closed form expression for the solution $\Phi(B)$. Nonetheless we can make an approximation. Since the sources are assumed to vary slowly on the scale of the orbifold size, we can write $\Phi(B)$ as a momentum expansion in the inverse wavelength of the sources. To zeroth order we can ignore the ten-dimensional derivatives in the massive field equation of motion (2.10). Recalling that $\langle \Phi(B) \rangle_{11} = 0$ and that $\Phi(B)$ is even under $Z_2$, we find the solution

$$\Phi(B) = -\frac{\pi \rho}{12} \left[ 3 \left( x^{11}/\pi \rho \right)^2 - 6 \left| x^{11}/\pi \rho \right| + 2 \right] J^{(1)} + \left( 3 \left( x^{11}/\pi \rho \right)^2 - 1 \right) J^{(2)} + \ldots \quad (2.12)$$

Here the dots represent higher-order terms in the momentum expansion. They are of the form $f(x^{11}/\rho)^{2n+1} \phi^{2n+1} J^{(i)}$ where $\partial^2_{(10)}$ is the ten-dimensional Laplacian. These correspond to higher-dimension terms usually dropped in a Kaluza-Klein reduction since they are suppressed by $(\rho/\lambda)^{2n}$ where $\lambda$ is the wavelength of the field in ten dimensions. They have been computed in ref. [30] for an explicit five-brane soliton solution of M–theory on $S^1/Z_2$.

Substituting into the action (2.11), keeping only the lowest dimension terms so that the correction terms in the solution (2.12) can be dropped, we find

$$S_{10} = -2\pi \rho \int_{M_{10}} \left\{ \frac{1}{2} \left( \partial \phi(0)^2 \right) + \frac{1}{2\pi \rho} \left( J^{(1)} + J^{(2)} \right) \phi(0) - \frac{1}{24} \left( J^{(1)} - J^{(2)} \right)^2 + \frac{1}{24} \left( J^{(1)} + J^{(2)} \right)^2 \right\} \quad (2.13)$$

When performing the reduction of the full eleven-dimensional description of the strongly coupled heterotic string we will always find this characteristic form $J^{(1)}^2 - J^{(1)}J^{(2)} + J^{(2)}^2$ appearing.

In the discussion so far, we have glossed over one subtlety. We have taken an example where the bulk field $\phi$ enters only quadratically and couples linearly to the boundary sources. In general, the situation will be more complicated. There may be non-linear sources for $\phi$ both in the bulk and in the boundary. Fortunately, in the strongly coupled string theory, we can in general treat these sources perturbatively. The boundary sources are suppressed by a power of the eleven dimensional gravitational coupling, namely $\kappa^{2/3}$, with respect to the bulk, while the bulk sources are further suppressed. Thus there is a perturbative expansion of the reduced action in $\rho \kappa^{-2/9}$. Taking a scalar field example with an analogous structure, we treat the massive mode as a small perturbation, and
expand the eleven-dimensional action as a power series in $\phi^{(B)}$. To first non-trivial order in $\kappa$, the boundary sources are independent of the massive field $\phi^{(B)}$. We then have the approximate solution discussed above which, by analogy with the strongly coupled string theory case, we will assume to be of order $\kappa^{2/3}$. To this order, there is no contribution from bulk sources for the massive mode. We then iterate, using this solution at linear order in the boundary and bulk sources to calculate a corrected solution $\Phi^{(B)}$, which will include pieces of higher order in $\kappa$.

Suppose we are interested only in an effective action to order $\kappa^{4/3}$. To what order must we keep the solution? At first sight it would appear we need to keep terms up to order $\kappa^{4/3}$. However, we have the familiar result that substituting a solution to the equations of motion into the action gives no contribution to linear order, precisely because the solution is a point where the first-order variation of the action vanishes. The leading behavior of the solution $\Phi^{(B)}$ is of order $\kappa^{2/3}$. Thus, to obtain all the terms of order $\kappa^{4/3}$, we need only substitute the leading order solution and keep terms quadratic in $\Phi^{(B)}$. This corresponds to substituting precisely the linearized solution (2.10) and (2.12) we derived above.

\section{$F^4$, $F^2R^2$ and $R^4$ terms in the ten-dimensional effective action}

We would now like to use the dimensional reduction procedure defined above to reduce the eleven-dimensional description of the strongly coupled $E_8 \times E_8$ heterotic string to a ten-dimensional theory. This can then be compared with the loop expansion of the ten-dimensional effective action for the weakly coupled string.

As we discussed in the introduction, we will not derive all possible terms in the ten-dimensional theory. We will concentrate on terms up to quartic order in the Riemann and gauge curvatures. Furthermore, we will only explicitly consider the parity-odd quartic terms. This will be equivalent to including only terms appearing up to order $\kappa^{4/3}$ in an expansion in the eleven-dimensional gravitational coupling. However, if required, the procedure could be used to calculate a much larger class of terms.

Witten and Hořava have argued that the low-energy effective action for the strongly coupled heterotic string is described by eleven-dimensional supergravity on a $S^1/Z_2$ orbifold with gauge fields on each of the orbifold planes $[1, 2]$. The resulting action has an expansion in the eleven-dimensional coupling constant $\kappa$, with terms appearing at increasing powers of $\kappa^{2/3}$. If we write for the bosonic fields

\begin{equation}
S = S_0 + S_{\kappa^{2/3}} + S_{\kappa^{4/3}} + \ldots
\end{equation}
then, in the upstairs picture, $S_0$ is the usual eleven-dimensional supergravity theory,

$$
S_0 = \frac{1}{2\kappa^2} \int_{M^{11}} \sqrt{-g} \left\{ -R - \frac{1}{24} G_{IJKL} G^{IJKL} - \frac{\sqrt{2}}{1728} \epsilon_{I_1 \ldots I_{11}} C_{I_1 I_2 I_3} G_{I_4 \ldots I_8} G_{I_9 \ldots I_{11}} \right\},
$$

(3.2)

where $G_{IJKL} = 24 \partial_{[I} C_{JKL]}$ is the field strength of the three-form $C_{IJK}$. Under the $Z_2$ orbifold symmetry, $g_{AB}$, $g_{1111}$ and $C_{11AB}$ are even, while $g_{11A}$ and $C_{ABC}$ are odd. To next order, the term $S_{c,2/3}$ is localized purely on the fixed planes, and is given by

$$
S_{c,2/3} = -\frac{c}{8\pi\kappa^2} \left( \frac{\kappa}{4\pi} \right)^{2/3} \int_{M^{10}_{10}} \sqrt{-g} \left\{ \text{tr}(F^{(1)})^2 - \frac{1}{2} \text{tr}R^2 \right\} - \frac{c}{8\pi\kappa^2} \left( \frac{\kappa}{4\pi} \right)^{2/3} \int_{M^{10}_{10}} \sqrt{-g} \left\{ \text{tr}(F^{(2)})^2 - \frac{1}{2} \text{tr}R^2 \right\}.
$$

(3.3)

Here we have included the $R^2$ terms which were argued for in [7]. As discussed there, these terms cannot be fixed up to the addition of terms quadratic in the Ricci tensor and scalar. Since we will only be interested in Riemann-tensor terms in the reduced ten-dimensional theory, here we keep only the Riemann-squared term, the coefficient of which is fixed.

In Hořava and Witten’s original formulation of the theory, it was argued that anomaly cancellation implied that the $c = 1$ in the action (3.1). However, subsequently Conrad has argued that the correct factor should be $c = 2^{-1/3}$ [31] (see also [32]). Since we will mostly be interested in the form of the final result rather than explicit coefficients, in what follows we will keep $c$ general to allow for either possibilities.

We see that the presence of $S_{c,2/3}$ introduces a source, localized on the orbifold planes, to the gravitational equations of motion. One finds

$$
R_{IJ} - \frac{1}{2} g_{IJ} R = -\frac{1}{24} \left( 4 G_{IKLM} G_{J}^{KLM} - \frac{1}{2} g_{IJ} G_{KLMN} G^{KLMN} \right)
$$

$$
- \frac{c}{2\pi} \left( \frac{\kappa}{4\pi} \right)^{2/3} \left( \delta(x^{11}) T_{1J}^{(1)} + \delta(x^{11} - \pi \rho) T_{1J}^{(2)} \right),
$$

(3.4)

where

$$
T_{AB}^{(i)} = (g_{11,11})^{-1/2} \left\{ \text{tr} F_{AC}^{(i)} F_{B}^{(i)C} - \frac{1}{4} g_{AC} \text{tr}(F^{(i)})^2 - \frac{1}{2} \left( \text{tr} R_{AC} R_{B}^{C} - \frac{1}{4} g_{AB} \text{tr} R^2 \right) \right\}.
$$

(3.5)

In order to keep the action $S_0 + S_{c,2/3}$ supersymmetric, a source must also be appended to the Bianchi identity for $G$. One finds

$$
(dG)_{1ABCD} = -\frac{c}{2\sqrt{2\pi}} \left( \frac{\kappa}{4\pi} \right)^{2/3} \left\{ J_{ABCD}^{(1)} \delta(x^{11}) + J_{ABCD}^{(2)} \delta(x^{11} - \pi \rho) \right\}
$$

(3.6)

with the sources $J^{(i)}$ defined as

$$
J_{ABCD}^{(i)} = 6 \left[ \text{tr} F_{[AB}^{(i)} F_{CD]}^{(i)} \right] - \frac{1}{2} \text{tr} R_{[AB} R_{CD]} = \left[ d\omega_3^{(i)} \right]_{ABCD}.
$$

(3.7)
The three-forms $\omega_3^{(i)}$ can be expressed in terms of the Yang-Mills and Lorentz Chern-Simons forms $\omega_{YM}^{(i)}$ and $\omega_3^L$ as

$$\omega_3^{(i)} = \omega_{YM}^{(i)} - \frac{1}{2} \omega_3^L.$$ (3.8)

As in the example in the previous section, the theory can also be formulated in the downstairs picture. First, the zeroth-order action is written with a factor $1/\kappa^2$ rather than $1/(2\kappa^2)$ since the integration range is now restricted to $x^{11} = [0, \pi\rho]$. If we assume, as is usual, that the variation of the bulk fields is taken to be zero at the boundaries when calculating equations of motion, then the $g$ and $C$ equation of motion have no contributions from the boundaries. However, we must impose the effects of the modified Bianchi identity. Recalling $G_{ABCD}$ is odd under the $Z_2$ symmetry, one can integrate the Bianchi identity over a small volume intersecting the orbifold plane to get the equivalent boundary conditions for $G$

$$G_{ABCD}|_{x^{11}=0} = -\frac{c}{4\sqrt{2}\pi} (\kappa/4\pi)^{2/3} J_{ABCD}^{(1)}$$

$$G_{ABCD}|_{x^{11}=\pi\rho} = \frac{c}{4\sqrt{2}\pi} (\kappa/4\pi)^{2/3} J_{ABCD}^{(2)}. $$ (3.9)

From the form of the equations of motion derived in the upstairs picture, we know that the Einstein equation also has a source localized on the boundaries. It is easy to show that this translates into a boundary condition on the intrinsic curvature of the orbifold planes. One finds

$$K_{IJ}^{(1)} - \frac{1}{2} h_{IJ}^{(1)} K^{(1)} = -\frac{c}{2\pi} (\kappa/4\pi)^{2/3} T_{IJ}^{(1)}$$

$$K_{IJ}^{(2)} - \frac{1}{2} h_{IJ}^{(2)} K^{(2)} = -\frac{c}{2\pi} (\kappa/4\pi)^{2/3} T_{IJ}^{(2)}.$$ (3.10)

where $T^{(i)}$ was defined in (3.3), the intrinsic curvature $K_{IJ}^{(i)}$ is given by [33]

$$K_{IJ}^{(i)} = h_{IJ}^{(i)K} \nabla_K n_I^{(i)}$$ (3.11)

and $h_{IJ}^{(i)} = g_{IJ} - n_I^{(i)} n_J^{(i)}$ is the induced metric on the boundary and $n_I^{(i)}$ are the normal vectors.

Finally, we should discuss the order $\kappa^{4/3}$ terms in the eleven-dimensional theory which are relevant to our calculation. There are two types of such terms, namely Green-Schwarz terms needed to cancel gravitational anomalies in M-theory on $S^1/Z_2$ and $R^4$ terms which are paired to the former by supersymmetry. Both types of terms are bulk terms and are not specific to M-theory on $S^1/Z_2$ but rather are always present. The Green-Schwarz terms can be obtained from five-brane anomaly cancelation [34, 35], by comparison to type IIA string theory [36] or from gravitational anomaly cancelation in M-theory on $S^1/Z_2$ [2, 32, 31]. All approaches lead to

$$S_{\kappa^{4/3}, GS} \propto \int_{M_{11}} C \wedge \left(-\frac{1}{8} \text{tr} R \wedge R \wedge R \wedge R + \frac{1}{32} \text{tr} R \wedge R \wedge \text{tr} R \wedge R \right).$$ (3.12)
The corresponding $R^4$ terms are given by

$$S_{\kappa^{4/3} R^4} \propto \int_{M_{11}} \sqrt{-g} t_8^{I_1 \ldots I_8} R_{I_1 J_1 J_2} \ldots R_{I_7 J_7 J_8} .$$

(3.13)

A general definition of the tensor $t_8$ can be found in [37]. Acting on antisymmetric tensors $Y_{I J}$, it takes the form

$$t_8^{I_1 \ldots I_8} Y_{I_1 J_1} Y_{I_2 J_2} Z_{I_3 J_3} = -2 Y_{I J} Y^{I J} Z^{KL} Z^{KL} - 4 Y_{I J} Y_{K L} Z^{I J} Z^{KL}$$

$$+ 16 Y_{I K} Y^{I K} Z^{I L} Z_{J L} + 8 Y_{I J} Y_{K L} Z^{J K} Z^{L I} .$$

(3.14)

Thus, for instance, we have in short hand notation

$$t_8 t_8 R^4 = 6 t_8 (4 tr R^4 - tr R^2 tr R^2) .$$

(3.15)

The latter result can be used to rewrite the $R^4$-terms (3.13) and combine them with the Green-Schwarz terms (3.12) into the expression, in the upstairs picture,

$$S_{\kappa^{4/3}} = \frac{c'}{2\kappa^2} \left( \frac{\kappa}{4\pi} \right)^{4/3} \frac{1}{3 \cdot 2^{11} \pi^2} \int_{M_{11}} \left( t_8 - \frac{1}{\sqrt{2}} \epsilon^{(11)} C \right) (4 tr R^4 - tr R^2 tr R^2)$$

(3.16)

where

$$\epsilon^{(11)} C = \epsilon^{I_1 \ldots I_8 J K L} C_{J K L} .$$

(3.17)

Again, since there is some debate over the coefficient in this term [32, 31], we have introduced a parameter $c'$. This form of the $\kappa^{4/3}$ terms is adapted to the supersymmetric invariant combinations of the reduced ten-dimensional theory, as we will see below. More specifically, the reduction of $t_8 - \frac{1}{\sqrt{2}} \epsilon^{(11)} C$ acting on a fourth power of the curvature leads to a $N = 1$ supersymmetric invariant in ten dimensions. This shows that a $D = 11$ supersymmetric invariant of fourth power in the curvature should at least contain either the terms proportional to $tr R^4$ or the terms proportional to $tr R^2 tr R^2$ in eq. (3.16). Presumably, both terms are required by eleven-dimensional supersymmetry so that eq. (3.16) precisely represents the $D = 11$ supersymmetric invariant.

We note that in general other explicit quartic terms are allowed. In particular, there can be terms on the orbifold fixed planes. However, these enter at a higher order in the $\kappa$ expansion. They have a relative coefficient of $\kappa^{14/9}$, and so do not contribute to order $\kappa^{4/3}$. Such terms are expected to contribute to the parity-even quartic invariants, since they include a non-trivial dilaton dependence.

We are now ready to apply the reduction procedure, which we have explained in section 2, in order to find the $F^4$, $F^2 R^2$ and $R^4$ terms in a ten-dimensional low-momentum limit of the above theory. We note at this point that, since the sources for $G$ appear in the Bianchi identity, they
cannot be directly incorporated into the action as stands. Nonetheless, the reduction procedure
described in the previous section can be simply generalized to include this case. The result is
that, as before, one simply substitutes the linear massive solution for $G$ into the action to obtain
the correct, dimensionally reduced, effective action. To do the reduction, we split the bulk fields
according to

$$
C_{IJK} = C_{IJK}^{(0)} + C_{IJK}^{(B)}
$$

$$
G_{IJKL} = G_{IJKL}^{(0)} + G_{IJKL}^{(B)}
$$

$$
g_{IJ} = g_{IJ}^{(0)} + g_{IJ}^{(B)}
$$

into zero mode and massive background pieces. The former represent the actual ten-dimensional
degrees of freedom which must be $Z_2$-even components of the eleven-dimensional fields. For these,
we write

$$
C_{AB}^{(0)} = \frac{1}{6} B_{AB}
$$

$$
G_{ABC\ell}^{(0)} = 3 \partial_{[A} B_{BC]}\]

$$
ds^{(0)} = g_{IJ}^{(0)} dx^I dx^J = e^{-2\phi/3} g_{AB} dx^A dx^B + e^{4\phi/3} (dx^{11})^2,
$$

where $B_{AB}, g_{AB}$ and $\phi$ are $x^{11}$ independent and represent the two-form field, the ten-dimensional
metric and the dilaton, respectively. The factor $e^{-2\phi/3}$ in front of the ten-dimensional part of the
metric $ds^{(0)}$ has been included for convenience to arrive at the ten-dimensional string frame.

As in section two, we also separate the sources for $G$ and $g$ into massive and massless parts. The
background fields $G_{IJKL}^{(B)}$ and $g_{IJ}^{(B)}$ then include explicitly $x^{11}$-dependent pieces needed to properly
account for the source terms on the orbifold fixed hyperplanes. Expanding to the order $\kappa^{2/3}$, they
are determined in the upstairs picture by the equations

$$
D_I G^{(B) IJKL} = 0
$$

$$
dG_{11ABCD}^{(B)} = -\frac{c}{2\sqrt{2\pi}} \left(\frac{\kappa}{4\pi}\right)^{2/3} \left\{ J^{(1)} \delta(x^{11}) + J^{(2)} \delta(x^{11} - \pi \rho) - \frac{1}{2\pi \rho} \left( J^{(1)} + J^{(2)} \right) \right\}_{ABC\ell}
$$

$$
R_{IJ}^{(lin)} = \frac{1}{2} \left( D^2 g_{IJ}^{(B)} + D_I D_J g^{(B)} - D^K D_I g_{JK}^{(B)} - D_J D_K g_{IK}^{(B)} \right)
$$

$$
= \frac{c}{2\pi} \left(\frac{\kappa}{4\pi}\right)^{2/3} \left\{ \delta(x^{11}) S_{IJ}^{(1)} + \delta(x^{11} - \pi \rho) S_{IJ}^{(2)} - \frac{1}{2\pi \rho} \left( S_{IJ}^{(1)} + S_{IJ}^{(2)} \right) \right\}
$$

where $S_{IJ}^{(i)}$ is given in terms of the energy momentum tensor $T_{IJ}^{(i)},$ eq. (3.5), as

$$
S_{IJ}^{(i)} = T_{IJ}^{(i)} - \frac{1}{9} g_{IJ} T^{(i)} , \quad T^{(i)} = g^{IJ} T_{IJ}^{(i)} .
$$
To lowest order in the momentum expansion explained in section 2, we find the solution [17]

\[ G_{ABCD}^{(B)} = -\frac{c}{4\sqrt{2\pi}} \left( \frac{\kappa}{4\pi} \right)^{2/3} \left\{ \epsilon(x^{11})J^{(1)} - (x^{11}/\pi\rho)(J^{(2)} + J^{(1)}) \right\}_{ABCD} \]

\[ G_{ABC11}^{(B)} = -\frac{c}{4\sqrt{2\pi^2\rho}} \left( \frac{\kappa}{4\pi} \right)^{2/3} \left( \omega_3^{(1)} + \omega_3^{(2)} \right)_{ABC} \]

\[ g_{ij}^{(B)} = \frac{c\rho}{12} \left( \frac{\kappa}{4\pi} \right)^{2/3} e^{\frac{2}{3} \phi} \left\{ \left( 3 \left( x^{11}/\pi\rho \right)^2 - 6 \left| x^{11}/\pi\rho \right| + 2 \right) S_{ij}^{(1)} + \left( 3 \left( x^{11}/\pi\rho \right)^2 - 1 \right) S_{ij}^{(2)} \right\} \]

The currents \( J^{(i)} \) are defined in eq. (3.7) and the explicit form of \( S_{ij}^{(i)} \) can be read off from eq. (3.21) and (3.5). Note that these expressions have the correct \( Z_2 \) symmetry properties; that is, \( G_{ABCD}^{(B)} \) is odd and \( G_{ABC11}^{(B)} \), \( g_{AB}^{(B)} \) and \( g_{11,11}^{(B)} \) are even while the off-diagonal entries \( g_{ij}^{(B)} \) of the metric which are odd vanish since \( S_{ij}^{(i)} = 0 \). The above expressions satisfy the upstairs equations of motion (3.20). The \( \delta \)–function sources in these equations arise from the step-function discontinuities of \( G_{ABCD}^{(B)} \) and \( \partial_{11}g_{ij}^{(B)} \) at \( x^{11} = 0, \pi\rho \). Eq. (3.22) can, however, also be interpreted as the solution in the boundary picture. In this case, \( x^{11} \) is restricted to \( x^{11} \in [0, \pi\rho] \) and the step function in the expression for \( G_{ABCD}^{(B)} \) and the modulus in the expression for \( g_{ij}^{(B)} \) become obsolete. Then the downstairs equations of motion, which can be obtained from eq. (3.20) by omitting the \( \delta \)–function source terms, are fulfilled and the boundary conditions (3.3) and (3.10) are properly matched.

Before we proceed to the computation of the higher-order terms, let us first derive the effective ten-dimensional action to order \( \kappa^{2/3} \) to settle our conventions. Inserting the fields specified by eqs. (3.18), (3.19) and (3.22) into the action (3.1), (3.2), (3.3) we obtain, to order \( \kappa^{2/3} \)

\[ S_{10} = \frac{1}{2\kappa_{10}^2} \int_{M_{10}} \sqrt{-g} e^{-2\phi} \left[ -R + 4(\partial\phi)^2 - \frac{1}{6} H^2 - \frac{\alpha'}{4} \left( \text{tr}(F^{(1)})^2 + \text{tr}(F^{(2)})^2 \right) + \frac{\alpha'}{4} \text{tr}R^2 \right], \]

\[ (3.23) \]

where the three-form field strength \( H_{ABC} \) is defined by

\[ H_{ABC} = 3\partial_i [A B_{BC}] - \frac{\alpha'}{2\sqrt{2}} \left( \omega_3^{YM,(1)} + \omega_3^{YM,(2)} - \omega_3^L \right)_{ABC}. \]

\[ (3.24) \]

Here, we have made the identifications

\[ \kappa_{10}^2 = \frac{\kappa^2}{2\pi\rho}, \quad \alpha' = \frac{c}{2\pi^2\rho} \left( \frac{\kappa}{4\pi} \right)^{2/3}. \]

\[ (3.25) \]

We recognize eq. (3.23) as the zero slope effective action of the weakly coupled heterotic string to the first order in \( \alpha' \). It is known [23] that the \( R^2 \) term in this action is required by supersymmetry once the Lorentz Chern-Simons form \( \omega_3^L \) is included in the definition (3.24) of \( H \). Such a term would not appear in the dimensional reduction procedure unless it is explicitly included in the boundary actions of the eleven-dimensional theory, as done in eq. (3.3). This constitutes one rationale for the presence of such terms in the eleven-dimensional theory, as pointed out in ref. [17].
We are now going to calculate some order $\kappa^{4/3}$ corrections to the action (3.23), namely terms of the form $R^4, R^2F^2, F^4$ which consist of four powers of curvatures and the corresponding Green-Schwarz terms of the form $BR^4, BR^2F^2, BF^4$. From the eleven-dimensional action and the field configuration we are going to use for the reduction, we can already identify various sources for those terms. First, and most obviously, such terms arise from the explicit eleven-dimensional $R^4$ terms given in eq. (3.16). Those terms, however, cannot account for the full spectrum of expected terms in ten dimensions and, in particular, they do not lead to any such terms involving gauge fields. At this point the $x^{11}$ dependent background fields $G^{(B)}_{ABCD}$ and $g^{(B)}_{IJ}$ come into play. As can be seen from eq. (3.22), they are of order $\kappa^{2/3}$ and are proportional to $\text{tr}R^2$ and $\text{tr}F^2$ so that quadratic expressions of those backgrounds lead to terms of the right structure. Green-Schwarz terms in ten dimensions can only arise from the eleven-dimensional “Chern-Simons” term $CGG$ where $C$ and $G$ are replaced by $B$ and $G^{(B)} \sim \text{tr}R^2, \text{tr}F^2$ respectively. Pure curvature terms of the form $R^4, R^2F^2, F^4$, on the other hand, result from three distinct sources, namely from the bulk curvature expanded up to second order in the metric background $g^{(B)}_{IJ}$, from the four-form kinetic term $\sqrt{-g}G^2$ with $G$ replaced by $G^{(B)}_{ABCD}$ and from the expansion of the boundary actions up to first order in the metric background $g^{(B)}_{IJ}$ taken at $x^{11} = 0, \pi \rho$. The explicit calculation is performed by inserting the background (3.22) into the action specified by (3.1), (3.2), (3.3) and (3.16). This leads to

$$S_{10}(R^4, R^2F^2, F^4) = \frac{\epsilon^2}{3 \cdot 16 \pi^5 \alpha'} \int_{M_{10}} \sqrt{g} \left( t_8 - \frac{1}{2\sqrt{2}} \epsilon^{(10)} B \right) W_8 ,$$  

(3.26)

where

$$W_8 = 8 \left( \text{tr}F^{(1)^2} \text{tr}F^{(1)^2} - \text{tr}F^{(1)^2} \text{tr}F^{(2)^2} + \text{tr}F^{(2)^2} \text{tr}F^{(2)^2} \right)$$

$$- 4 \text{tr}R^2 \left( \text{tr}F^{(1)^2} + \text{tr}F^{(2)^2} \right) + 2 \text{tr}R^2 \text{tr}R^2 + \frac{\epsilon'}{\epsilon^2} \left( 4 \text{tr}R^4 - \text{tr}R^2 \text{tr}R^2 \right)$$  

(3.27)

and

$$\epsilon^{(10)} B = \epsilon^{I_1...I_8JK} B_{JK} .$$  

(3.28)

Let us discuss some basic properties of the above result. If $Y$ and $Z$ are curvatures the combination

$$\left( t_8 - \frac{1}{2\sqrt{2}} \epsilon^{(10)} B \right) X , \quad X = \text{tr}Y^2 \text{tr}Z^2 \quad \text{or} \quad X = \text{tr}Y^4$$  

(3.29)

constitutes an invariant under $N = 1$ supersymmetry in ten dimensions [25, 22]. Our result, eq. (3.26), is expressed as a sum of such supersymmetric invariants and, hence, is supersymmetric. The appearance of these supersymmetric combinations, though expected, is by no means a trivial consequence of the reduction process. While this is true for the terms resulting from the explicit $R^4$ terms in the eleven-dimensional theory which correspond to the last term in parentheses in
the polynomial $W_8$, eq. (3.27), all other terms result from various sources and include bulk and boundary contributions as described in detail above.

Comparing with the known form of the Green-Schwarz anomaly term, we see that the $c$ and $c'$ coefficients must be related

$$c' = c^2$$  \hspace{1cm}

Furthermore, a careful consideration of the exact overall coefficient of the Green-Schwarz term will also fix $c$, just as it would in the full eleven-dimensional theory. We note that, comparing with the expression for one-loop quartic curvature terms [38, 39, 40] quoted by Abe et al. [40], we have $c = 1$. However, the main result here is that, by the somewhat complicated procedure of integrating out the massive modes in the eleven-dimensional theory, we have succeeded in reproducing the full supersymmetric invariants of the ten-dimensional theory, up to parity-odd quartic terms.

4 Conclusion

We have seen that, when making a consistent dimensional reduction on a $S^1/Z_2$ orbifold with sources on the orbifold fixed planes, it is not possible to simply truncate and drop the massive Kaluza-Klein modes. Fields on the fixed planes, even if they are assumed to be independent of the circle coordinate, always provide a source for the massive modes. Thus the massive modes do not decouple from the zero mode fields, and integrating them out leads to new terms in the effective action. When the sources on the fixed planes can be treated perturbatively, this provides a new procedure for making a consistent dimensional reduction.

One can use this procedure to calculate terms in the ten-dimensional effective action of the strongly coupled $E_8 \times E_8$ heterotic string. In this paper, we have isolated the terms which are quartic in the curvature and, in the string frame, are independent of the dilaton. Such terms first appear at one loop in the weakly coupled theory and include the Green-Schwarz terms. In the dimensional reduction they come from two sources, both from explicit quartic terms in eleven-dimensions and from integrating out the massive modes. We find the full $N = 1$ supersymmetric invariants in ten-dimensions. Furthermore, these enter in the correct combination to give the full anomaly-canceling Green-Schwarz terms.

Unlike the case of the type II string where there is strong evidence for a non-renormalization theorem for the corresponding parity-odd quartic terms [28, 29], what we have done here is simply to show how the weak-coupling one-loop terms arise from the reduction of the strongly coupled theory, expanding up to $\kappa^{4/3}$. It is possible that other terms in the eleven-dimensional theory lead to parity-odd terms with non-trivial dilaton dependence. A good example is an explicit quartic term in the orbifold fixed-plane action. However, we know such terms cannot be supersymmetric
and so we expect them not to be present. In a sense it is more natural to reverse the argument and use a non-renormalization constraint to exclude certain terms in the eleven-dimensional theory. (Something which is simpler than investigating the supersymmetry of the eleven-dimensional theory directly.)

One set of terms which we do expect to be present in the ten-dimensional action are the parity-even terms corresponding to tree-level $\alpha'$ corrections, which have the characteristic coefficient $\zeta(3) e^{-2\phi}$. It is natural to ask how such terms appear here. There are two obvious candidates. Explicit quartic terms on the orbifold fixed planes can have the correct curvature structure, but enter with the power $e^{-2\phi/3}$, and so are non-perturbative from a weakly coupled perspective, and cannot be the source. However, as has been discussed in the context of type II theories in \cite{27, 28}, we can expect the bulk $R^4$ term in eleven dimensions gets a correction when the theory is compactified on a finite interval. This is essentially a one-loop Casimir effect. The term as stands corresponds to a one-loop supergravity correction in an infinite space. On a finite interval, the momentum modes become quantized and this shifts the form of the one-loop $R^4$ term. The authors of \cite{27, 28} have shown how this leads to a term corresponding to the tree-level $R^4$ term in type II theories and we expect the same effect here.

Two further points are worth making. First, the dimensional reduction procedure introduced here is a useful method for deriving higher-order supersymmetric invariants. While we concentrated only on the quartic curvature terms, the same calculation could also produce the corresponding terms involving higher powers of $H$ and the dilaton, as well as higher order fermion terms, needed to make the full supersymmetric invariant. Calculation of such invariants directly in ten dimensions is an extremely laborious task \cite{25}.

Secondly, the non-renormalization of the one-loop term provides part of the explanation of why the strong and weakly coupled theories, reduced to four dimensions on a Calabi-Yau manifold, have the same form \cite{17}. As an expansion in $\kappa$, one finds that it is precisely the one-loop terms which give the corrections to the lowest order effective action in both limits. Thus, since the ten-dimensional actions agree, the form of the correction in four dimensions is the same in each case. The actual situation is a little more complicated since in the strongly coupled theory one is never really reducing a ten-dimensional action. The Calabi-Yau space is actually smaller than the orbifold size. However, as discussed in \cite{17}, for the leading corrections, the heavy modes of the compactification do not contribute. Thus the form of the effective action is, in fact, independent of the relative sizes of the Calabi-Yau space and the orbifold. Consequently, all terms in the four-dimensional effective action resulting from the one-loop operators are of the same form in the strong and weakly coupled limits. Nonetheless it is important to note that while, to this order, the form of the actions is the same, the parameters are not, and this can lead to quite different and interesting low-energy phenomenology.
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