Multi-objective Optimization Design of Helical Gear in Centrifugal Compressor Based on Response Surface Method

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Abstract: Based on ANSYS Workbench, this paper adopts response surface method, in order to multi-objectively optimize the helical gear in centrifugal compressor. The local sensitivity and global sensitivity are analyzed using the constructed response surface. The linear relationship between the three dimension parameters and the mass of the helical gear, as well as the non-linear relationship between the three dimension parameters and the maximum stress on the helical gear are discussed. And it is found that changing the dimensions of the minor radius and tensile depth of the ring-groove weight loss makes little influence to the maximum stress on the helical gear. Then the helical gear is multi-objectively optimized using the constructed response surface. Premised on the same application requirements, the mass of the helical gear is reduced 27.4% and the maximum stress on the helical gear is within the allowable range.

1. Introduction

A manufacturer of centrifugal compressor helical gear, though reliable, but due to the unreasonable design, quality, and the moment of inertia is big, not only wastes materials, also can make the motor starting current fluctuations, reduce the transmission efficiency, so the structure size needs to be designed. The optimization goal is to reduce the quality by over 20%, and the maximum stress on the helical gear should not exceed the material yield limit of 200MPa.

The response surface method was first proposed by Box and Wilson[1] in 1951 and initially applied to the fitting of physical experiments. Compared with the traditional optimization design method, the response surface method is faster, and the result is generally more accurate and has been applied in many applications[2-5] The response surface method is also used to solve the problem in this paper.

2. Establishing the three-dimensional model of helical gear

The parameters of the helical gear of the gear pair used in centrifugal compressor are \( m_r = 2.5 \), the normal pressure Angle \( \alpha_n = 20^\circ \), and the helix Angle \( \beta = 15.74055^\circ \). The export parameters of the helical gear and the pinion axis are shown in Tab1
Tab1. Helical gear and small gear shaft export parameters

|                      | Helical gear | Pinion shaft |
|----------------------|--------------|--------------|
| Number of teeth      | $Z_1 = 123$  | $Z_2 = 31$   |
| Tooth width          | $b_1 = 102$  | $b_2 = 100$  |
| Pitch diameter       | $d_1 = 319.4085$ | $d_2 = 80.5195$ |
| Center distance      | $a = (d_1 + d_2) / 2 = 200$ |

The two ends of the helical gear have two symmetrical circular weight reduction grooves, and the small diameter of optimizing the front groove is $r_1 = 107.5 \text{ mm}$. Large diameter is $r_2 = 104 \text{ mm}$. The stretching depth is $h = -20 \text{ mm}$.

UG8.0 is adopted to set up the gear sub-assembly model, and the built 3d model is imported into ANSYS 14.5 through the special interface.

3. Establishing the finite element model and performing static stress analysis

The material properties and mesh division to the 3d model is added. The static stress analysis includes applying load and constraint boundary conditions and solving the stress of the helical gear.

The material of pinion shaft and helical gear is 45 steel, and the young's modulus of elasticity is $52.09 \times 10^3 \text{ MPa}$ and Poisson's ratio is 0.269. Mesh subdivision of meshing area, unit size of 2mm.

Firstly, the constraint conditions are added: the two supporting surfaces of the pinion shaft are fixed and restrained, and the cylinder constraints are applied to the inner circle of the helical gear, wherein the axial and radial fixation are arranged, and the cutting direction is free.

Then the loading condition is added: considering the pressure on the side of the two key ways of the helical gear, the pressure is applied in both sides of the two sides. The key way width of the helical gear is 16mm, and the width of the large gear tooth is 100mm, so $b \times h \times l = 16 \text{ mm} \times 10 \text{ mm} \times 80 \text{ mm}$ is selected as the type A common flat key connection.

The results of constraints and loads are shown in figure 1. Then, the stress of the helical gear is solved, and the maximum stress appears on the gear tooth, which is 155.1MPa as shown in figure 2.

![Fig. 1 Gear pair load and constraint diagram](image-url)
4. Response surface and sensitivity analysis
In the polynomial response surface model, suppose that the relationship between $y$ and the random parameter $x = [x_1, x_2, \cdots, x_k]$ is described in equation (1). By random variable method, $N$ sample values of the random parameters $(y_1, y_2, \cdots, y_r)$ were obtained, and a set of sample values of the system response were calculated for the $N$ sample values, and the system function was obtained by using the least square method.

$$
y = a_0 + \sum_{i=1}^{R} a_i x_i + \sum_{i=1}^{R} \sum_{j=i+1}^{R} a_{ij} x_i x_j$$

(1)

Where $a_0, a_i, a_{ij} (i = 1, \cdots, R; j = i, \cdots, R)$ is the undetermined coefficient, $R$ is the number of design variables.

The response function is described in equation (2). Where, $f(x)$ is a polynomial function of $x$, $y$ is mathematical expectation; $z(x)$ is a stochastic process.

$$
y = f(x) + z(x)$$

(2)

The response surface method includes four basic processes: experimental design, model selection, response surface fitting and fitting evaluation.

The input variables for the P1 (stretching depth $h$), P2 (the small diameter $r_1$ of the circular weight reduction groove), P3 (large diameter $r_2$), a total of three variables, output variables for P4 (helical gear quality $m$), P5 (maximum stress on helical gears). The available formula (3) is used to describe the multi-objective optimization model of helical gears.

The 15 design points for input variables, P1, P2 and P3, as shown in fig.3. The samples of fitting response surface is shown in fig. 4. The samples of fitting response surface is shown in fig. 4.

Design variables: P1, P2, P3, P4, P5.
Objective function: min $P_4$, min $P_5$.
Constraint function:

\[
\begin{align*}
-50 \text{mm} & \leq P_1 \leq -15 \text{mm} \\
75 \text{mm} & \leq P_2 \leq 110 \text{mm} \\
120 \text{mm} & \leq P_3 \leq 145 \text{mm} \\
\text{s.t.} & \\
P_4 & \leq 36.08 \text{kg (45.1 kg} \times 80\%) \\
P_5 & \leq 200 \text{MPa}
\end{align*}
\]

(3)
There are many evaluation indexes of model precision, and two commonly used methods are used in this paper. One is the decision coefficient $R^2$. The closer $R^2$ is to 1, the smaller is the error. The second is the RMS error $RMSE$. The closer the $RMSE$ is to 0, the error is smaller, and regression equation is more accurate. The evaluation results are shown in Tab2.

| P4 | P5 |
|----|----|
| **Coefficient of decision $R^2$** | 1   | 1   |
| **Root mean square error $RMSE$**  | 0.0014 | 0.0005 |

It can be seen from Tab2 that the determination coefficient $R^2$ meets the requirements. According to the analysis of the reference[6], it can be seen that the accuracy of the model is satisfied when the $RMSE$ of the root mean square error of the response surface is less than 0.04, which indicate the reliability of the response surface obtained by fitting.

The response surface contour and local sensitivity of P4 are shown in fig.5. (a), (b) and (c) respectively represent the response of the other two input parameters to P4 when P1, P2 and P3 are unchanged.
The above sensitivity analysis is only for one output. A series of response surface contours are flat, response value with the change of the input is almost linear increase or decrease. The fig (d) for the local sensitivity curve of P4, it is known that the three input parameters and P4 are both linear, and the result is consistent with the high cloud image of the response surface.

The response surface of the maximum stress P5 on the helical gear is shown in fig6. (a), (b) and (c) respectively denote the response of the other two input parameters to the P5 when P1, P2 and P3 are unchanged.

A series of response surface cloud graphs on the maximum stress P5 on the helical gear are irregular surfaces, and the response values change as the input changes. (d) for the local sensitivity curve of the P5, it is known that the three input parameters and the P5 are non-linear, and the sensitivity curves of P2 and P3 and P5 are non-monotonic and have extreme values.
Fig. 7 Global sensitivity of P4 and P5.

The global sensitivity of P4 and P5 is shown in figure 7. It can be seen from figure 7 that the influence of three input parameters on the quality P4 of the helical gear is much greater than that of the maximum stress P5 on the helical gear. The maximum stress on the helical gear is very small, and the results are consistent with the literature [6].

5. Optimization result analysis

The optimal solution is selected in these sampling points, and the decision function needs to be used for evaluation. These variables are of different magnitude and cannot be directly compared, the variables need to be regularized. The regularized function is described in equation (4) and (5).

\[ N_i = \left\{ \frac{x_i - x_i}{x_u - x_l} \right\} \quad (4) \]

\[ M_j = \left\{ \frac{y_j - y_{min}}{y_{max} - y_{min}} \right\} \quad (5) \]

Type: \( N_i \) input variable target regularized value of \( M_j \) for the output variable target regularization values, \( x_i \) and \( y_j \) as input or output variables of target, \( x, y \) for the current value of the input or output variables, \( x_i, x_l \) for the input variables of the upper and lower bounds.

The decision function can be described in (6).

\[ \Phi = \sum_{i=1}^{n} \omega_i N_i + \sum_{j=1}^{m} \omega_j M_j \quad (6) \]

Where: \( n \) is the number of input variables, \( \omega_i \) is the penalty value of input variable \( N_i \), \( m \) is the number of output variables, and \( \omega_j \) is the penalty value of output variable \( M_j \).

The optimal solution is shown in Tab 3. The corresponding finite element simulation values and relative errors of the response values are also given in the Table. Can see the response surface method is used to get the three alternate optimal solutions meet the constraints of optimization, and the maximum stress on the helical gear (P5 response surface optimal solution compared with the finite element simulation error is very small.

In order to compare with the bevel gear before optimization, an optimal solution is selected for comparison. Select the alternative solution 1 in Tab 3. The small change in size has little effect on the quality of the helical gear and the maximum stress on the helical gear, as shown in Tab 4. The quality of the helical gear reduced 27.49%, the maximum stress on the helical gear is increased, but still less than the yield limit of material, satisfy the constraint conditions.
Tab.3 Optimal alternatives.

| Optimal alternative | Alternative solution 1 | Alternative solution 2 | Alternative solution 3 |
|---------------------|------------------------|------------------------|------------------------|
| P1 (mm)             | -35.1                  | -36.9                  | -41.2                  |
| P2 (mm)             | 93.8                   | 90.2                   | 103.7                  |
| P3 (mm)             | 142.6                  | 135.4                  | 143.7                  |
| P4 (kg)             | 33.06                  | 34.46                  | 32.92                  |
| P5 (MPa) Response values | 172.4                  | 165.1                  | 182.6                  |
| P5 (MPa) Simulation value | 169.7                  | 160.5                  | 181.8                  |
| P5 (MPa) Relative error | 1.59%                  | 2.87%                  | 0.44%                  |

Tab.4 Comparison of skew gear before and after optimization.

| h     | r1   | r2   | m    | σ   |
|-------|------|------|------|-----|
| Before| -20  | 107.5| 140  | 45.10| 155.1|
| After | -35  | 94   | 143  | 33.06| 170.6|
| change| The weight was reduced by 27.49% and the maximum stress was less than 200MPa. |

6. Conclusion
In this paper, the three-dimensional solid model and finite element model of the helical gear of centrifugal compressor are established, and the static stress analysis is carried out according to the working conditions. Then established multi-objective optimization model of helical gear, based on the input parameters and output parameters of experimental data of the response surface is constructed, and the response surface and the sensitivity of each parameter was analyzed, found that weight loss annular groove path and tensile depth within the scope of a certain size changes have little impact on the maximum stress on the helical gear. The optimal solution 1 was compared with the pre-optimization phase, and the quality of the helical gear was reduced by 27.49%, the maximum stress was less than the material yield stress of 200MPa, and the optimized target was achieved.

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