The Research of Computational Method by Using Beam Element for Rope Structure

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Abstract. On the basis of in-depth study of mechanical behavior of rope, according to the catenary theory the equation of guy rope is built. Based on this equation, the guy rope is established in FEM which is analogized by using beam elements with little bending stiffness, and non-linear finite element calculations are applied for the guy rope. Take the theoretical shape of guy rope as study point, according to example calculations, computational accuracy about using beam element to simulate is quantificationally analyzed, and discipline curve of flexural rigidity to computational result is obtained and the results is also analyzed.

1. Introduction
Rope structure are widely used in engineering, and their mechanical characteristics are highly flexible and extremely geometric nonlinear. Theoretical research on rope structure is relatively mature, and there are generally analytical methods and finite element methods for its calculation methods [1, 4-5]. The analytical method is generally used for the case of simple structure and force, and the finite element method is the mainstream method for rope structure calculation. At present, the finite element calculation of rope structure is mainly divided into two types: one is to establish a rope structure model directly in the finite element software, and the calculation model of the rope structure is established by the method of coupling the degrees of freedom between the cable elements [7-10]. The assumption of this kind of calculation method is that the rope is completely flexible, and only the degree of freedom of movement is constrained between the cable elements. The unconstrained degree of freedom in the overall model is too much, which will bring difficulties to the convergence of the calculation, and the modeling is complicated and the calculation amount is large. Another method is to derive the stiffness matrix of the computational model based on the catenary theory for the specific problem, and finally calculate it by self-programming or combined with the secondary development of finite element software. This type of method is difficult and has poor versatility for different problems. There are few studies on the finite element calculation method which is fast in calculation speed, good in convergence, and can satisfy the rope structure of engineering application. No detailed research results have been published.

Based on the catenary theory [1, 6], based on the linear equation of the rope structure, a mathematical model of the rope structure fixed at both ends is established [11-12], and the beam unit with extremely weak bending stiffness is used to simulate the long span. The rope structure forms its finite element analysis process [2, 3, 8, 13]. The study uses the catenary theory as the basis for the investigation. The
accuracy of the rope structure is simulated by numerical examples. The influence curves of different bending stiffness coefficients on the linear shape of the rope are given quantitatively.

2. Basic Assumptions for Rope Structure Calculation

In the calculation and analysis of the rope structure, it is inevitable that the model will be simplified. Therefore, the following basic assumptions are made on the rope structure before establishing the rope structure calculation model:

1) The rope works in the elastic phase;
2) The rope satisfies the catenary equation;
3) The rope belongs to a large displacement and small strain.

3. Rope Structure Calculation Model

3.1. Rope Structure Linear Equation

The rope structure is fixed at both ends and is in a natural overhanging state under the action of gravity. The force diagram is shown in Fig. 1 [6].

The rope is a flexible, uniform suspension cable [2], point A is a fixed point and point B is a free end. Under the action of gravity, take the rope micro-element as the research object, according to the force balance:

\[ \sum X = 0, F_x + dF_x - F_x = 0 \]  
\[ \sum Y = 0, F_x \frac{dy}{dx} + dF_x \frac{dy}{dx} + qds - F_x \frac{dy}{dx} = 0 \]

It is obtained from (1), \( dF_x = 0 \), which means that the horizontal component of the rope tension is equal everywhere when only gravity is applied, that is, the horizontal tension of the rope is constant, which is obtained by (2):

\[ dF_x \frac{dy}{dx} + qds = 0 \]

![Figure 1. Schematic diagram of the tension of the wire rope](image-url)
Equation (3) is the differential equation of the rope. According to the curve integral formula:

\[ ds = \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx \, , \]

Then (3) can be transformed into:

\[ \frac{d^2 y}{dx^2} = -\frac{q}{Fx} \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \]  \hspace{1cm} (4)

Using the hyperbolic sine and cosine function and the variable substitution method, the final result is obtained:

\[ y = -\frac{Fx}{q} \cosh\left( -\frac{q}{Fx} x + c_1 \right) + c_2 \]  \hspace{1cm} (5)

Where \( c_1 \) and \( c_2 \) are integral constants, and the Cartesian coordinate system is established with the point A as the origin, then there are boundary conditions: \( f(0) = 0; f(l) = h \), which yields:

\[ c_1 = \sinh^{-1}\left( \frac{-qh}{2 \sinh\left( \frac{-q}{2Fx} \right)} \right) + \frac{ql}{2Fx} \quad c_2 = \frac{Fx}{q} \cosh\left( \sinh^{-1}\left( \frac{-qh}{2 \sinh\left( \frac{-q}{2Fx} \right)} \right) + \frac{ql}{2Fx} \right) \]

Equation (5) is the linear equation of the rope. When the ends of the rope are fixed, the linear equation of the rope is a nonlinear function of the horizontal tension \( Fx \). In order to uniquely determine the shape of the rope, it is necessary to give the horizontal tension \( Fx \) of the end of the total tension of the rope. Here, let \( Fx \) be a known amount and study the wire shape in a specific state.

3.2. Finite Element Model

The finite element method replaces the complex problem with a simpler problem. It considers the solution domain as a small number of finite element small subdomains that are interconnected by nodes. Therefore, the finite element method can use a finite number of unknowns to go to an infinitely unknown real system to get a real solution. The finite element method not only has high calculation accuracy, but also has high computational efficiency and can cope with complex structures, so it is called an effective engineering analysis method.

According to the structural characteristics and mechanical model analysis of the rope, the three-dimensional elastic beam element beam4 with small bending rigidity is used to simulate. The beam4 unit is an elastic beam unit with 6 degrees of freedom that can withstand stretching and bending, as shown in Fig. 2.
Let A and B be the support points. Establish a finite element model of the rope between the two points, and constrain the three degrees of freedom of A, B, X, Y, and Z, as shown in Fig. 3.

4. Calculation of Examples and Analysis of Results

4.1. Rope finite element model verification
In order to verify the accuracy of simulating the rope structure in the finite element with the beam element with very weak bending stiffness, a two-point support, initial fixed length rope beam element model is established in the finite element, as shown in the initial model curve in Fig. 4. Detailed parameters are shown in Table 1.
### Table 1. Rope structure parameters

| Material       | Modulus of elasticity / Gpa | Poisson's ratio | Density / Kg.m\(^{-3}\) | Cross-sectional area / mm\(^2\) |
|----------------|-----------------------------|-----------------|---------------------------|---------------------------------|
| Wire rope      | 150                         | 0.3             | 7850                      | 1962                            |

![Rope finite element model verification results](image)

**Figure 4.** Rope finite element model verification results

Analysis of Fig.4, only consider the weight of the rope, after ANSYS nonlinear iteration, can converge to the o curve in the figure. The horizontal tension of the end of the finite element calculation is extracted. The calculated linear shape of the rope based on the rope equation obtained after calculation is in good agreement with the finite element analysis. According to the analysis result, the finite element calculation result is larger than the maximum displacement \(\delta_{\text{max}} = 16\text{mm}\) of the theoretical linear vertical direction. It is feasible to simulate the super-lifting cable with beam element in finite element with high precision.

### 4.2. Influence of the bending stiffness coefficient of the beam on the calculation results

The bending stiffness refers to the ability of an object to resist bending deformation. For a beam unit, when the bending rigidity is large, the drape is poor. Therefore, in the calculation of the rope structure with the beam unit, if the bending stiffness coefficient is not properly set, the calculation accuracy is directly affected. In order to analyze the influence of bending stiffness on the calculation results, the calculation example of IV-A is used as the calculation model, and the method of optimization iteration is used to calculate the bending force stiffness based on the maximum displacement of the linear theoretical direction of the rope. The coefficient-displacement curve is shown in Fig.5.
Figure 5. Influence curve of bending stiffness coefficient on calculation results

The analysis results show that when the beam structure is used to simulate the rope structure calculation, setting the bending stiffness coefficient of the beam too large or too small will reduce the calculation accuracy. Before the calculation, the bending stiffness coefficient should be optimized and the optimal value is obtained. For the calculation model in III-A, the bending stiffness is set to 0.46 to obtain the optimal calculation result.

5. Conclusion

Based on the catenary theory, a two-point rope structure calculation model is established by using the space beam element. The accuracy of the model is verified by an example. The influence of the bending stiffness on the calculation results of the rope structure is analyzed quantitatively. The following conclusions are drawn:

1) It is feasible to use the beam element simulation rope structure with extremely weak bending stiffness to calculate the finite element. For the calculation example of the span of 60m in the example 3.1, the calculation error is 16mm compared with the theoretical linear shape;

2) When using the beam unit to simulate the rope mechanism, it is necessary to set the bending stiffness coefficient, and there is an optimal value. For example, in the example of Section 2.2, the bending stiffness coefficient is set to 0.46 to obtain the most accurate calculation result of the rope structure.

3) The calculation method proposed in this paper can be applied to the design and analysis of any span flexible rope structure.

This study provides a new idea for the design and calculation of the rope structure. The purpose is to develop the research and exploration of the rope structure calculation method, and to explore a more accurate, simple and efficient rope structure calculation method through research.

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