Chiral dynamics and Zitterbewegung of Weyl quasiparticles in a magnetic field

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Abstract

The relativistic dynamical properties of chiral Weyl quasiparticles (WQPs) are investigated in a stationary magnetic field. The visualized evolution process of quasiparticles’ wavepackets has been calculated with different angles between the spinor and the magnetic field. The results reveal that the chiral WQPs feature anisotropic dynamics, which falls into two typical motion modes, i.e. directional drift (chiral magnetic effect) and periodic oscillation (chiral Zitterbewegung). We theoretically find that the mechanism behind this interesting dynamical phenomenon is the special chiral relativistic Landau level. Since the frequency, amplitude and drift velocity of chiral WQPs can be controlled in a measurable range, one can expect the observation of the phenomenon in the cold atomic system.

1. Introduction

In early 1980s, physicists Yuri I Manin and Richard P Feynman proposed the concept of quantum simulation [1, 2]. 40 years later, quantum simulation techniques have been successfully implemented in numerous systems [3], such as trapped ions, waveguide array, superconducting circuit and cold atomic lattice systems. Among them, the cold atoms loaded in an optical lattice have found wide application in simulating condensed matter physics, nonlinear dynamics and high energy physics phenomena [5–22]. The study of artificial gauge field [10], spin–orbit coupling [8, 11] and topological state of matter [18, 20] has been facilitated by the fine manipulation of ultra-cold atoms in optical lattices. Furthermore, manipulable interatomic interaction has also been realized by adjusting Feshbach resonance parameters [23, 24]. New research fields, such as quantum many-body problem, quantum information scrambling [25–29], nonlinear and relativistic dynamics [4], have also been successively unfolded.

On the other hand, the Weyl fermion denotes a massless relativistic particle with specific chirality in quantum field theory [30]. Due to its similar properties to the Dirac and Majorana fermion, Weyl fermion was initially considered as a candidate for neutrinos [31, 32]. Researchers have long been expecting to find this intriguing relativistic particle in high-energy physics experiments, but chances are slim that they will succeed. Currently, it was demonstrated that Weyl fermion can emerge as a quasiparticle in a condensate matter system—the Weyl semimetal [33–42]. Non-magnetic [37, 38], magnetic [41, 42] and optical crystal Weyl semimetal [39, 40] (such as TaAs, Co5S2Sn2S2, et al) have been realized in table-top experiments one by one. In Weyl semimetals, two linear dispersion bands in three-dimensional (3D) momentum space intersect as a single degenerate point—the Weyl point, which appears as a pair with opposite chirality. The low energy
excited quasiparticles satisfy the relativistic Weyl equation, thus exhibit Weyl fermions’ dynamical properties. Recent years have witnessed an upsurge in the research on Weyl semimetallic materials and Weyl quasiparticles (WQPs). Great research achievements have been scoblue in this field. Fermi arc surface states [33, 37, 38], chiral magnetic effect and anormal chiral magnetic effect [43–46], negative magnetoresistance effect [47, 48] and other novel phenomena have been found in Weyl semimetal one after another [49–52]. When Weyl semimetal meets cold atomic system, more fascinating physics can be expected.

In 2015, Prof. Ketterle’s group first proposed that the Weyl semimetal can be realized with ultracold atoms loaded in a tunable cubic lattice [53–56]. This provides a versatile platform to study Weyl semimetal and the dynamics of the WQPs [57]. The artificial magnetic field [58–64] can be introduced to the system by means of optical techniques [10, 14]. So far, while there is massive research on Weyl semimetals with the cold atom system, only a few focus on relativistic dynamics [15, 22, 57, 65]. This paper is mainly devoted to the relativistic dynamical properties of WQPs in stable magnetic fields. Through analytical derivation, we believe that there are two typical motion modes in the system: chiral magnetic effect and chiral Zitterbewegung. Then, by numerical simulation, the visualized evolution of WQPs’ density distribution versus time has been obtained. The results show that the polar angle between the magnetic field and the quasiparticles’ spinor plays a key role in the dynamical properties, while the azimuth angle affects the Zitterbewegung phenomenon on the plane perpendicular to the magnetic field.

The paper is organized as follows. In section 2, a scheme for the realization of WQPs in the magnetic field is proposed with ultracold atoms loaded in an optical lattice. In section 3, the visualized dynamics of atomic clouds as well as the motion of quasiparticles’ centroid are studied by both numerical and analytical methods. In section 4, the underlying physics of dynamical properties has been discussed by examining different chiral relativistic Landau levels. A summary is provided in section 5.

2. WQPs in a magnetic field

One can obtain pseudospin-1/2 ultracold atoms in a cubic lattice by general cold atomic preparation technique, and then impose an assisted Raman laser on atoms along the y direction to ensure controllable the phase of the hopping along the x-direction. The corresponding Hamiltonian of the system reads [46, 60–62]

\[
\hat{H} = \sum_j \left\{ -iJ_x (\hat{a}_{j+e_x}^\dagger \hat{a}_{j} + \hat{a}_{j+e_x}^\dagger \hat{a}_{j}) e^{-i\Phi} + J_z (\hat{a}_{j+e_z}^\dagger \hat{a}_{j} - \hat{a}_{j+e_z}^\dagger \hat{a}_{j}) \right\} - J_z (\hat{a}_{j+e_z}^\dagger \hat{a}_{j} - \hat{a}_{j+e_z}^\dagger \hat{a}_{j}) + H.c. + m_z (\hat{a}_{j+e_z}^\dagger \hat{a}_{j} - \hat{a}_{j+e_z}^\dagger \hat{a}_{j}) \right\},
\]

where \( J_{x,y,z} \) denotes the nearest-neighboring hopping amplitudes and \( m_z \) represents the on-site energy. The magnetic flux \( \Phi = \delta k \cdot a \), in which \( \delta k \) is the momentum transferring in the Raman process. Thus, as shown in figure 1, the effective magnetic field of the system is \( B = (0, B_z) \) and \( B_z \approx \Phi / a^2 \). \( \hat{a}_{j+e_z}^\dagger (\hat{a}_{j+e_z}) \) is the creation (annihilation) operator on the site \( j \). By introducing Fourier transform \( \hat{a}_k^\dagger = \frac{1}{\sqrt{N}} \sum_{j} e^{ik \cdot j} \hat{a}_j^\dagger \), where \( k = (k_x, k_y, k_z) \) is the Bloch wave vector, one can get the corresponding Bloch Hamiltonian

\[
\hat{H}(k) = 2J_x \sin(k_x - B_2 y) \sigma_x + 2J_y \sin(k_y \sigma_y + |m_z| - 2J_z \cos k_z) \sigma_z,
\]

where \( \sigma_{x,y,z} \) is the Pauli matrix, \( y = j \cdot a \).

Firstly, we consider a simple case with \( B_z = 0 \). The Hamiltonian is a Weyl semimetal one. Thus, in the Brillouin zone, the paiblue Weyl points with opposite chirality locate in \( k_w = (0, 0, \pm k_w) \) with \( k_w = \arccos(m_z/2J_z) \). In the vicinity of the Weyl points, the Hamiltonian (2) is approximately linear, i.e.

\[
\hat{H}_\pm = v_x k_x \sigma_x + v_y k_y \sigma_y \pm v_z k_z \sigma_z.
\]

One can see that the Weyl points exhibit the characteristic of massless Weyl fermions with left-(right-) handed chirality (marked as \( +(-) \)). Here, \( v_x = 2J_x, v_y = 2J_y, \) and \( v_z = \sqrt{(2J_x + m_z)(2J_z - m_z)} = 2J_z \) with \( m_z = 0 ( -2J_z < m_z < 2J_z ) \). Since the left-and-right chirality is mirror symmetrical, we just investigate the dynamics of the left-handed WQPs in the magnetic field. The corresponding effective Hamiltonian can be described as

\[
\hat{H}_{\text{eff}} = v_x (k_x - B_2 y) \sigma_x + v_y k_y \sigma_y + v_z k_z \sigma_z.
\]
Then, the evolution of WQPs in the magnetic field satisfies the equation of motion,

\[ i\hbar \frac{\partial}{\partial t} \Psi = \hat{H}_{\text{eff}} \Psi. \]  

Hereinafter we take \( \hbar = 1 \) for simplicity. The effective light velocity \( v_{\text{eff}} = v = 1 \), and \( m_z = 0, v_z = 2J_Z, k_w = (0,0,\frac{\pi}{a}) \). Then, by considering the previous assumption \( J_{x,z} = J = \frac{1}{\hbar} \) and \( a = 1 \), both [\text{momentum}] = \( \hbar/a = 1/a \) and [\text{time}] = \( \hbar/J = 1/J \) will therefore become dimensionless in the following numerical computation.

3. Chiral wavepacket dynamics and Zitterbewegung of WQPs

Here we consider Bose–Einstein condensates (BEC, or a cold atomic ensemble). The evolution process of density distribution of the cold atomic cloud can be obtained by directly solving the Weyl equation. Since the eigenfunction of the Hamiltonian (4) is a plane wave in \( x \) and \( z \) directions, while a harmonic oscillator solution in \( y \) direction, a pancake wavepacket is considered as the initial state, i.e.

\[ |\Psi\rangle = \sqrt{\frac{1}{\sqrt{\pi}l_x}} \sqrt{\frac{1}{\sqrt{\pi}l_y}} \sqrt{\frac{1}{\sqrt{\pi}l_z}} e^{-\frac{\phi}{l_y}} e^{\frac{\phi_z}{l_z}} S |S\rangle, \]  

where \( l_y \ll l_x = l_z \) denotes the width of the initial wavepacket in \( x \), \( y \) and \( z \) direction. Without loss of generality, we use \( l_y = 2 \) and \( l_x = l_z = 10 \) as an approximate in the following computation. In a general way, any parameter is allowed to be taken as the width of the initial wavepacket. The evolution results will only find quantitative differences, but no qualitative ones. The initial spinor is chosen as

\[ |S\rangle = \left( \cos \frac{\theta}{2}, \sin \frac{\theta}{2} e^{i\phi} \right)^T, \]  

where \( T \) stands for matrix transposition. \( \theta \) is the polar angle and \( \phi \) is the azimuth angle in the Bloch sphere. One can obtain the relations

\[ \langle S| (\sigma_x, \sigma_y, \sigma_z) |S\rangle = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta). \]  

3.1. The quantum simulation of WQPs in a magnetic field

The visualized density distribution and the corresponding centroid of cold atomic gases versus time is plotted in figure 2. As shown in figures 2(a) and (d), a periodic oscillation of WQPs occurs along \( z \) direction with \( \theta = 0 \), which is the evidence of Zitterbewegung [66–72]. Here, the condition \( \theta = 0 \) represents \( |S\rangle = (1, 0)^T \), which means the initial spinor of WQPs and the magnetic field are parallel to each other. For the case of \( \theta = \pi \), the atomic cloud drifts in a straight line along the opposite direction of the \( z \) axis without any oscillation, which exhibits the chiral magnetic effect [43–46] (see figures 2(c) and (e)). In this case, the corresponding initial spinor \( |S\rangle = (0, 1)^T \), which is antiparallel to the direction of the magnetic field. As shown in figure 2(b), for \( \theta = 0.5\pi \) and \( \phi = 0 \), the wavepacket evolution process is special. On the one hand, the atomic cloud shows both oscillating and drifting along \( z \) direction. On the other hand, an additional periodic oscillation occurs in the \( x \) direction. Under such circumstances, the spinor \( |S\rangle = \frac{1}{\sqrt{2}}(1, 1)^T \) is parallel to the \( x \) direction, which is perpendicular to the magnetic field. The trajectory of quasiparticles’...
centroid on the $x$–$z$ plane is provided for $\theta = 0.5\pi, \phi = 0$ (see figure 2(f)). For clarity, the relations between the magnetic field and the spinor are plotted in figure 2(g).

Notably, as shown in figure 2(b), the density distribution of atomic cloud evolves in an exotic way. The atomic cloud splits into two parts, one part oscillates around the initial position and the other part drifts in the negative direction of the $z$-axis. How comes such a bizarre evolving process? Actually, for $\theta = 0.5\pi$, it can be seen clearly that the initial spinor $|S\rangle = \sqrt{2} (1, 1)^T / \sqrt{\sqrt{2} - 1} = \frac{1}{\sqrt{2}} (1, 0)^T + \frac{1}{\sqrt{2}} (0, 1)^T$, which means that the atomic cloud evolves into a superposition state of $(1, 0)^T$ and $(0, 1)^T$. Half of the atomic cloud evolves with $(1, 0)^T$, while the rest with $(0, 1)^T$. More details will be discussed in section 4.

3.2. The theoretical analysis
To gain insight of the intriguing phenomena, we analytically calculate the expectation value of WQPs’ centroid, which is the crucial observable quantity in studying dynamical properties and Zitterbewegung. The eigenvalues $E_n$ and the eigenfunctions $\psi_n$ of the effective Hamiltonian (4) are given by

$$ E_n = \sqrt{\langle v k_z \rangle^2 + n \omega^2}, $$

$$ \psi_n = \frac{1}{\sqrt{n \omega^2 + \langle v k_z - E_n \rangle^2}} \left( \frac{\omega \sqrt{n} |n - 1\rangle}{\langle v k_z - E_n | n \rangle} \right), $$

$$ E_0 = -v k_z, $$

$$ \psi_0 = \left( \begin{array}{c} 0 \\ 0 \end{array} \right), $$

$$ E_{-n} = -\sqrt{\langle v k_z \rangle^2 + n \omega^2}, $$

$$ \psi_{-n} = \frac{1}{\sqrt{n \omega^2 + \langle v k_z - E_{-n} \rangle^2}} \left( \frac{\omega \sqrt{n} |n - 1\rangle}{\langle v k_z - E_{-n} | n \rangle} \right), $$

where $\omega = v \sqrt{2B_z}$ is one half of the Landau gap, and $|n\rangle$ is the normalized Hermite function defined by

$$ |n\rangle = \sqrt{\frac{1/\ell_B}{\sqrt{\pi} 2^n n!}} H_n(\xi) e^{-\xi^2}. $$

$H_n(\xi)$ denotes the Hermite polynomial of order $n$, where $\xi = (y - k_zl_B)/\ell_B$ with $l_B = 1/\sqrt{2B_z}$ being the magnetic length. The initial state equation (6) can be expanded by introducing the eigenfunctions equation (9) as

$$ |\Psi\rangle = \int d k_x d k_z \sqrt{\frac{l_B}{\sqrt{\pi}}} e^{-\frac{1}{2} k_z^2} \sqrt{\frac{l_z}{\sqrt{\pi}}} e^{-\frac{1}{2} k_x^2} e^{i(k_x x + k_z z)} \left[ C_0 \psi_0 \sin \frac{\theta}{2} e^{i\phi} + (C_1 \psi_{+1} + C_2 \psi_{-1}) \cos \frac{\theta}{2} \right], $$

Figure 2. (a)–(c) Numerically calculated probability distributions, $\int dY |\Psi(x, y, z, t)|^2$, at time $t = 0, 7, 14, 21$ with $\phi = 0$ and $\theta = 0, 0.5\pi, \pi$ (from panels (a) to (c)). The motion of the wavepacket’s centroid versus time along the $z$ direction with $\theta = 0$ (d) and $\theta = \pi$ (e). The motion trail of the wavepacket’s centroid on the $x$–$z$ plane with $\theta = 0.5\pi$ (f). (g) The relation of the magnetic direction and spin direction. Solid lines (symbols) represent the analytical (numerical) results. The value of the probability is rescaled from 0 to 1.
where \( C_0 = 1, C_1 = \sqrt{\omega^2 + (v_{k_0} - E_{1,0})^2} \) and \( C_2 = \sqrt{\omega^2 + (v_{k_0} - E_{1,0})^2} \) are the expansion coefficient with \( E_{1,0} = \pm E_1 = \pm \sqrt{(v_{k_0})^2 + \omega^2} \). The magnetic length \( l_B = l_y \) and \( l_y \ll l_x = l_z \) are chosen to approximate a plane wave in the \( x \) and \( z \) direction, while Gaussian in \( y \) direction. Then, we get the time-dependent wave function

\[
\Psi(t) = \int dk_x dk_y \sqrt{\frac{l_y}{2\pi}} e^{-i\phi_k} \sqrt{\frac{l_z}{2\pi}} e^{-i\phi_k} \exp[i(k_x x + k_y y)]
\]

\[
\times \left[ C_0 \psi_0 \sin \frac{\theta}{2} e^{i\phi_0} e^{-iE_0 t} + (C_1 \psi_{1+} e^{-iE_1 t} + C_2 \psi_{1-} e^{-iE_{1,0} t} \cos \theta) \right].
\]

Through the gymnastics of operators, the expressions of \( \bar{x}(t) \), \( \bar{y}(t) \) and \( \bar{z}(t) \) can be obtained as

\[
\bar{x}(t) = \int dk_x \frac{l_y}{\sqrt{2\pi}} e^{-i\phi_0} \omega \sin \theta \times \left[ \cos \psi \cos(E_0 t) + \sin \psi \sin(E_0 t) \right] \sin(E_1 t) \right],
\]

\[
\bar{y}(t) = \int dk_x \frac{l_y}{\sqrt{2\pi}} e^{-i\phi_0} \omega \sin \theta \times \left[ \cos \psi \cos(E_0 t) - \sin \psi \sin(E_0 t) \right] \sin(E_1 t) \right],
\]

\[
\bar{z}(t) = \int dk_x \frac{l_y}{\sqrt{2\pi}} e^{-i\phi_0} \omega \sin \theta \times \left\{ -vt \sin \frac{\theta}{2} + \cos^2 \frac{\theta}{2} \left[ \frac{v\omega^2}{2E_1^3} \sin(2E_1 t) + \frac{v^2 k^2}{E_1^3} \right] \right\}.
\]

Both analytical and numerical results of \( \bar{x}(t) \), \( \bar{y}(t) \) and \( \bar{z}(t) \) are plotted in figures 3(a)–(c), respectively, where the analytical results (solid lines) show good agreement with the numerical results (symbols). From the expression of \( \bar{x}(t) \), we find that Zitterbewegung only occurs when both \( \sin \theta \neq 0 \) and \( \cos \phi \neq 0 \), and the corresponding frequency \( \omega_z = \omega \). Moreover, there is a damping term \( e^{-t^2 \omega_z^2} \cos(E_0 t) \), which results in an exponential damping of amplitude (see figure 3(a)). For \( \bar{y}(t) \), Zitterbewegung occurs when both \( \sin \theta \neq 0 \) and \( \sin \phi \neq 0 \), and the corresponding frequency is also \( \omega_y = \omega \). Similarly, the damping term \( e^{-t^2 \omega_y^2} \cos(E_0 t) \) leads to a damping amplitude (see figure 3(b)). However, as for \( \bar{z}(t) \), a complete Zitterbewegung occurs with \( \theta = 0 \), while a complete unidirectional drift with \( \theta = \pi \) (see figure 3(c)). Moreover, the maximum oscillating amplitude of \( \bar{x}(t) \), \( \bar{y}(t) \) and \( \bar{z}(t) \) are plotted in figures 3(d), (e) and (f), respectively. The drift velocity with different polar angle is shown in the insert of figure 3(f).
4. Underlying mechanism of Weyl dynamics

Since a pancake is chosen as the initial state, which is the eigenfunction of the effective Hamiltonian (4), the time-dependent evolution wave function equation (12) can be rewritten as

$$|\psi(t)\rangle = \int dk_x dk_z \sqrt{\frac{I_0}{\sqrt{\pi}}} e^{-ik_0k_x} \sqrt{\frac{I_2}{\sqrt{\pi}}} e^{i\frac{k_0^2}{2}t} e^{i(k_0k_x+k_2z)} \times \left[ C_0|\psi_0\rangle e^{-iE_0t}|S_1\rangle + (C_1|\psi_{+1}\rangle e^{-iE_{+1}t} + C_2|\psi_{-1}\rangle e^{-iE_{-1}t})|S_2\rangle \right],$$

(16)

where the spinor is written as $|S\rangle = (S_z, S_x)^T$ with $S_z = \cos \frac{\theta}{2}$ and $S_x = \sin \frac{\theta}{2} e^{i\phi}$. The relativistic Landau levels of the system are plotted in figure 4.

As shown in figure 4(a), when $\theta = 0$, the system evolves into the superposition state of $\psi_{+1}$ and $\psi_{-1}$, which is similar to a band insulator phase with Landau gap $2\omega$ in $k_z$ direction. However, when $\theta = \pi$, the system only evolves into a single state $|\psi_0\rangle$, which is a linear dispersion with the velocity $v$ in $k_z$ direction (see figure 4(a)). Furthermore, in the case $0 < \theta < \pi$, a part of atomic cloud evolves into the superposition state of $\psi_{+1}$ and $\psi_{-1}$ with the coefficient $S_1 = \cos \frac{\theta}{2}$, while the rest evolves into the single state $|\psi_0\rangle$ with the coefficient $S_2 = \sin \frac{\theta}{2} e^{i\phi}$. As shown in equation (16), since the two parts evolve independently, the motion of quasiparticles’ centroid exhibits a bizarre feature of both oscillating and drifting in the $z$ direction (see figure 3(c)). Moreover, in the case $0 < \theta < \pi$, spin component in $x$ or $y$ direction is not zero, which results in wavepacket oscillation in $x$ or $y$ direction. Furthermore, a linear dispersion $(E_0 = -vk_z)$ and the finite width effect [65] result in the oscillation amplitude damping with time (see figures 3(a) and (b)). From figure 4(b), one can find that a $\omega$ gap in the energy spectrum corresponds to the frequency of the quantum oscillation in the $x$ and $y$ direction. Thus, the frequency of Zitterbewegung in the $z$ direction is twice as higher as that on the $x$–$y$ plane (see figure 3).

5. Conclusion and outlooks

In summary, the dynamic properties of Weyl quasiparticles in a magnetic field are investigated. By fixing the direction of the magnetic field as $z$-axis, we discussed the dynamic evolution of quasiparticles with different spin directions. The image of the visualized density distribution versus time has been obtained. Besides, we analyzed the relativistic Landau level of the system in detail, which proved to be consistent with the dynamic behavior predicted by the theory. On the one hand, the results reveal that the quasiparticles’ center of mass in the $z$ direction is related merely to the polar angle $\theta$. Specifically, when $\theta = \pi$, quasiparticles exhibit directional transport without oscillation, which is the evidence of chiral magnetic effects. When $\theta = 0$, one can observe an oscillation with frequency $2\omega$ and without any drift in the $z$ direction, which is the evidence of chiral Zitterbewegung. Other cases ($0 < \theta < \pi$) are the combination of the above two motion modes. Furthermore, when the spinor is not parallel to the magnetic field plane, the quasiparticles’ center of mass is affected by both polar angle $\theta$ and the azimuth angle $\phi$. Under such circumstance, the quasiparticles exhibit an additional damping Zitterbewegung on the $x$–$y$ plane.

Experimentally, one can prepare $^{87}$Rb BECs in a magneto-optical trap. After a certain duration of expansion, a pancake initial wavepacket can be obtained. Then, load the atomic cloud into a phase controllable cubic lattice. Next, shut down the Harmonic trap and release the atoms to evolve in the optical lattice. Finally, the quasiparticles’ density distribution can be observed by absorption imaging. In optical lattice cold-atom systems, there are many candidates to realize Weyl quasiparticles [53, 73–78]. For...
example, \( |F = 5/2, m_F\rangle (m_F = -5/2, -1/2) \) of \(^{173}\text{Yb} \), \( |F = 9/2, m_F\rangle (m_F = -9/2, -7/2) \) of \(^{40}\text{K} \), and \( |F = 2, m_F\rangle (m_F = -2, -1) \) of \(^{85}\text{Rb} \) can be used as two spin states. The way of hopping in the model can be realized by laser-assisted hopping between lattice points. The magnetic field is obtained by adjusting phase in the lattice through auxiliary lasers. Recently, a report (arXiv:2012.07605) has shown that the lifetime of the cold atom can be enhanced to the magnitude of second (even minutes for \(^{84}\text{Sr} \) [79]), which would be of great help to the realization, manipulation and detection of the aforementioned dynamical phenomenon. In addition to optical lattice cold-atom systems, chiral magnetic dynamics can also be realized in quantum walk experiments [80] and Floquet systems [81–84]. Since the amplitude and frequency of ZB effect are able to be controlled in a detectable range in the above-mentioned systems, chiral Zitterbewung and chiral dynamics will bring the promise of experimental realization in multiple platforms.

Furthermore, the mechanism of chiral dynamics has much potential in many application fields, including chiral interaction between light and matter [86], Chiral edge mode in quantum hall system and topological insulators [18, 85], as well as the manipulation of chiral quasiparticles in cold-atom systems [53, 54, 57]. Research vitality will be injected into these areas through the idea of coupling between chiral states of matter and magnetic fields, moreover, study on chiral dynamics in many-body systems will also benefit a lot from the research work of chiral quasiparticle dynamics.

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Appendix A. The time-operator method

In general, the time evolution of the wave function can be written as

\[
\Psi(x, y, z, t) = \mathcal{T} \exp \left[ -\frac{i}{\hbar} \int_0^t \mathcal{H}_{\text{eff}}(t') dt' \right] \Psi(x, y, z, 0), \tag{A1}
\]

where \( \mathcal{T} \) denotes the time-ordering operator, \( \mathcal{H}_{\text{eff}} \) is the effective Hamiltonian (equation (4) in the main text) and \( \Psi(x, y, z, 0) \) is the initial wave packet (equation (6) in the main text). The effective Hamiltonian can be expressed as the sum of operators corresponding to the kinetic and potential energies of the system in the form \( \mathcal{H}_{\text{eff}} = \mathcal{T} + V \). Because the effective Hamiltonian (equation (4) in the main text) is time-independent, by using the standard split-operator method, equation (A1) can be rewritten as

\[
\Psi(x, y, z, t + \delta t) = \left[ e^{-i(\hat{T}/(2\hbar))t} e^{-i(\hat{V}/\hbar)t} e^{-i(\hat{T}/(2\hbar))t} + O(\delta t^3) \right] \Psi(x, y, z, t), \tag{A2}
\]

where \( \hat{T} = v_x \hat{k}_x \sigma_x + v_y \hat{k}_y \sigma_y + v_z \hat{k}_z \sigma_z \), \( \hat{V} = -v_x B_y \sigma_y - v_y B_x \sigma_x \). In the sufficiently short time \( \delta t \), the high-order term \( O(\delta t^3) \) (due to noncommutation) can be safely neglected. One can connect the position and the momentum spaces through the Fourier transform. Therefore, we can finally get the numerical solution of \( \Psi(x, y, z, t) \) following the computation procedure step by step with time-step \( \delta t \) [87–89].

Appendix B. The right-handed chirality Weyl quasiparticles in the magnetic field

The left-handed chirality WQPs has been discussed in the main text. Interestingly, the right-handed one has the opposite chiral properties. The wavepacket dynamical properties have the same behavior with opposite motion direction in \( z \)-axis (see the video [http://stacks.iop.org/NJP/23/073031/mmedia] in the attachment). Furthermore, the motion of the wavepacket’s centroid with time has been plotted in figure 5.
Figure 5. The motion of the wavepacket centroid with time along the $z$ direction with $\theta = 0$ (a) and $\theta = \pi$ (b). The motion trail of the wavepacket centroid on the $x$–$z$ plane (c) with $\theta = 0.5\pi$, $\phi = 0$.

Appendix C. Visualized chiral dynamics and chiral Zitterbewegung

The visualized evolution of quasiparticles’ wavepackets has been provided in the file of wavepacket evolution.mp4 in the attachment.

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