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Damage Model and Experimental Study of a Sand Grouting-Reinforced Body under Seepage

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Abstract: Grouting is a reinforcing method commonly used in underground engineering. The grouting-reinforced body is in saturated groundwater, which is due to long-term seepage, resulting in reinforcing performance attenuation. This is a guarantee for the safe operation of the underground structure’s long life cycle that obtains the performance attenuation law of the grouting-reinforced body under the action of seepage. This paper uses primitive to describe the mechanical properties of grouting-reinforced body under seepage erosion, establishing the damage mechanics model of grouting-reinforced body under seepage, designing deterioration experimental of grouting-reinforced body under seepage, studying the mechanism of the permeation water pressure on the permeability of the grouting-reinforced body, and obtaining the variable of the grouting-reinforced body damage variable with the permeation water pressure. Studies have shown that seepage effects have a “sudden jump” phenomenon in the process of grouting-reinforced body erosion, and the limit damage of grouting-reinforced body is 0.15~0.19. The relationship between the permeability and damage variables of grouting-reinforced body under seepage are verified, and the quantitative relationship of grouting-reinforced body between permeation pressure, time and damage variables are obtained. This research is of great importance for improving the deterioration theory under seepage and ensuring the long-term safety of tunnel operations.

Keywords: sand layer; grouting-reinforced body; seepage; damage model

1. Introduction

The disposal of poor geology of tunnels uses grouting reinforcing methods to improve their physical mechanics, thereby meeting engineering security and stability requirements. However, there is currently a reinforcement of tunnel adverse geology, less consideration of long-term seepage effect on long-term deterioration of sand layer reinforcing body, ignorance of the influence of seepage effect on the reinforcing body during design, resulting in poor grouting-reinforced body effect, occurrence of secondary disasters in construction and operation period, and difficulty in meeting the safety needs of engineering long life cycle. Therefore, the reinforced body damage model under seepage is studied, and it is important to ensure the safety of the tunnel long life cycle.

At present, domestic and foreign scholars have made some progress on the study of seepage effects on the fractured rock mass. Louis [1] a empirical formula for the negative index relationship between the permeability coefficient of rock mass and the normal stress is given according to some of the drilling water test results; Snow [2] proposed a multi-group parallel fracture penetration coefficient as the calculation formula of the normal stress distribution; Jones [3] proposed empirical formula of the granite fracture permeability coefficient and stress; Gale [4] that the negative index relationship equation of the transmissibility coefficient and stress is obtained through the indoor experiment of the
fracture of granite, dali rock and basalt—three kinds of rock mass. Liu Quansheng, Cui Guangze [5,6] explored the influence of the seepage velocity and the seepage direction on the secondary detachment of porous media particles, and the porosity attenuation model caused by particle deposition in the porous medium. Zhang Pengyuan [7] believes that the influence of seepage velocity and hydraulic effect on particle outflow concentrations are positively correlated, and there is a seepage speed threshold. When the threshold is above the threshold, the particle migration speed is greater than the tracer outflow speed, below the threshold, less than Tracer outflow speed. Bai Bing, Song Xiaoming, Chen Xingxin [8,9] believes that under determining the seepage speed, the influence of suspended particles on the particle concentration in the penetration curve has an intermediate value, under this value, they are positively correlated, above this value, they are negatively correlated. There are also many scholars who study seepage effects on the fractured rock mass [10–15].

At present, domestic and foreign scholars have launched a number of research on the damage of seepage effect on the fracture rock mass, but the grouting-reinforced body and the fracture rock mass have a large difference—under the infiltration, the grouting-reinforced body has undergone changes in permeability coefficient mutations, and there is no description of the process. At the same time, the fracture rock mass damage simulation experiment under seepage is less, and long-term permeability indicators of grouting-reinforced body under seepage were not accurately predicted through indoor trials. To this end, this paper uses primitive to describe the mechanical properties of grouting-reinforced body under seepage erosion, and the damage mechanical model of grouting-reinforced body under seepage is established. In designing the deterioration experimental of grouting-reinforced body under seepage and obtaining the variable of the grouting-reinforced body damage variable with the permeation water pressure, the relationship between the permeability and damage variables of grouting-reinforced body under seepage are verified, further obtaining the quantitative relationship of grouting-reinforced body. Research on damage theory of grouting-reinforced body damage under seepage ensures safety and longevity in tunnel operations under important permeation pressure, time and damage variables.

2. Establishment of Damage Model under Seepage

2.1. Non-Uniformity Description of Sand Layer Grouting-Reinforced Body

The sand layer if the grouting-reinforced body contains granular particles, cement slurry, and its various wrapping forms. Due to its special formation, it is difficult to ensure absolute uniformity. The composition of the grouting-reinforced body is extremely complex, which is difficult to accurately mathematically describe. However, if we discrete the grouting-reinforced body, we can use the mathematical description of the statistics. Take any section of grouting-reinforced body, and divide it into several primitive, the mechanical parameters of these primitive can be described by statistical methods. The primitive taken is characterized by the following characteristics: (1) Comparison with the macroscopic scale of the grouting-reinforced body, small enough to be considered to be a macroscopic nature of the entire grouting-reinforced body, and its physical mechanical properties can be ignored; (2) In terms of microscopic properties of grouting-reinforced body, it is necessary to be large enough to include slurry, granular particles, cement interface, and microporous micropores, and all contained media in the primer. In the primitive, all the contained media can be seen as uniform, the physical mechanical properties of the primitive unit are the average of the physical mechanical properties of the components. The physical mechanical properties of these primitive unit can be determined by a certain test manner, and their stress and strain may be described based on a continuous uniform theory of homogeneous media. The slurry, granular particles, the cement interface and the micropores contained in each of the bases are different, so the physical mechanical properties of each base element are not complete, that is, each of the bases is plunging. The solid space is discontinuous, uneven.

When damage from seepage effects on the grouting-reinforced body occurs by both permeability and pore water pressure, the grouting-reinforced body is formed by the
diffusion of the cement slurry in the sand grain, and the microcrack and the pore structure are relatively stable, so the permeability damage to the grouting-reinforced body is smaller. The external water pressure increases the external load of the grouting-reinforced body, and the internal pore water pressure causes damage to the structure of the agglomerate of the grouting-reinforced body, and the type of damage does not consume the original components of the grouting-reinforced body, so the damage of grouting-reinforced body under seepage was described with the primitive.

In response to this non-uniform medium, it is difficult to accurately measure its destruction, but at a given stress level, the probability of destruction can be defined. Non-uniformity of the material is described using a method of mathematical statistics proposed by Weibull, which is:

$$\varphi(\alpha) = \frac{m}{\alpha_0} \cdot \left(\frac{\alpha}{\alpha_0}\right)^{m-1} \cdot e^{-\left(\frac{\alpha}{\alpha_0}\right)^m}$$  \hspace{1cm} (1)

where: $\alpha$ is the grouting-reinforced body primitive mechanical properties parameter (single shaft compressive strength, elastic modulus, permeation coefficient, etc.); $\alpha_0$ is the average value of the corresponding primitive unit mechanical properties; $m$ is uniformity the coefficients, the uniformity of the reaction grout reinforcement; $\varphi(\alpha)$ is a statistical distribution density of the grouting-reinforced body primitive mechanical properties $\alpha$.

The Formula (1) reflects the distribution of non-uniformity of molecular strength in grouting-reinforced body. The homogeneous coefficient $m$ reflects the uniformity of the grouting-reinforced body. As the $m$ value increases, the mechanical properties of the grouting-reinforced body will be concentrated in a narrow range, indicating that the grouting-reinforced body is more uniform; in turn, with the reduction of the $m$ value, the mechanical properties distribution curve of the grouting-reinforced body is relatively gentle, indicating that the uniformity of the grouting-reinforced body is poor. Figure 1 shows the profile of grouting-reinforced body mechanical properties of different uniform coefficients.

**Figure 1.** The primitive property distribution of different uniformity coefficients.

### 2.2. Sand Grouting-Reinforced Body Mesoscopic Damage Equation

The effect of seepage on grouting-reinforced body is mainly reflected in the damage of permeability and pore water pressure. The damage variable $D$ is a measure of the degree of damage of grouting-reinforced body, and grouting-reinforced body is composed of several primitive bodies. It can be seen that the damage of the primitive body directly affects the mechanical properties of the primitive body. Therefore, the damage variable $D$ of
the grouting-reinforced body is the following relationship with the statistical distribution density of the damage of its constituent primitive body:

$$\frac{dD}{d\varepsilon} = \varphi(\varepsilon)$$

(2)

In the formula, \(\varphi(\varepsilon)\) is a measure of the damage rate of the primitive body during the process of grouting-reinforced body seepage, and it reflects the damage degree of grouting-reinforced body macroscopically. For a single primitive body, there are only two states, namely destruction and non-destruction. It is the state change from the non-destructive to the failure of each primitive body that constitutes the continuous change of the damage degree of grouting-reinforced body from small to large. The increase in the number of primitive body damage causes the continuous macroscopic damage of grouting-reinforced body. Then the grouting-reinforced body damage variable \(D\) can be expressed as follows:

$$D = \int_{0}^{\varepsilon} \varphi(x)$$

(3)

Assuming that the initial damage of grouting-reinforced body is \(D_0 = 0\), the grouting-reinforced body damage variable expressed by the statistical distribution of the strength of the grouting-reinforced body primitive body is:

$$D = \int_{0}^{\varepsilon} \varphi(x) = \int_{0}^{\varepsilon} \left( \frac{m}{\varepsilon_0} \left( \frac{x}{\varepsilon_0} \right)^{m-1} e^{-\left( \frac{x}{\varepsilon_0} \right)^m} \right) dx = 1 - e^{-\left( \frac{x}{\varepsilon_0} \right)^m}$$

(4)

Derivation of the above formula, the function of the destruction rate of the primitive body can be obtained. Figure 2 is a schematic diagram of the function.

![Figure 2. The damage rate variation function of primitives.](image)

Analysis of the above figure shows that at the initial stage of grouting-reinforced body deformation, a small number of low-strength primitives are destroyed, and the overall damage of grouting-reinforced body is small; as the deformation continues to increase, the number of destroyed primitives increases sharply, the reason is that, from the probability of the strength distribution of the primitive body, the strength of the primitive body is concentrated around a certain value, and the number is the largest. When the deformation reaches the number value, the primitive body that constitutes the grouting-reinforced body is largely destroyed, resulting in the rapid decrease of the overall strength of grouting-reinforced body; in the later stage of deformation, until the overall failure of grouting-reinforced body, there are still a small amount of primitives that remain intact, and the strength of these primitives is greater.

3. Seepage Simulation Test

3.1. Experimental Design

The effect of seepage on the mechanical properties of grouting-reinforced body is mainly to increase the porosity of grouting-reinforced body. According to the principle
of the effect of seepage on grouting-reinforced body, the factors related to the mechanical properties of grouting-reinforced body under the action of seepage are: the degree of cement hydration, the strength of hydrated cement, the pressure of external water, and the action time of seepage. Based on the study of the long-term stability of grouting-reinforced body under seawater erosion, cement slurries with high water-cement ratio are not suitable for grouting-reinforced body in seawater environments. Therefore, in order to study the effect of seepage on the strength and permeability of grouting-reinforced body, this test mainly studies the effect of grouting-reinforced body on grouting-reinforced body under different external water pressures and different seepage time conditions [16].

(1) Slurry water–cement ratio

The commonly used grout for sand grouting is cement grout. This test uses ordinary cement grout with a water-cement ratio of 0.8:1.

(2) Grouting pressure

According to the research on the conditions of penetration grouting, as well as the sand penetration grouting indoor test and a lot of engineering practice experience, when the penetration grouting pressure is 0.5 MPa, the penetration and diffusion of the grout in the sand layer are relatively sufficient, so the grouting pressure selected in this test is 0.5 MPa.

(3) Gradation of injected sand

According to the analysis of sand penetration test and reinforcement test, the sand gradation is in the range of 1~2 mm, and the grout diffuses more fully in the injected sand layer. Therefore, the sand size range selected in this test is 1~2 mm, the porosity and permeability coefficient of the injected medium was determined by standard for soil test method and constant head permeability test [17], porosity and permeability coefficient are shown in Table 1.

Table 1. Parameters of grouted sand.

| Particle Size Range/mm | Porosity $\varphi$ | Permeability Coefficient $k$/cm/s |
|------------------------|---------------------|-------------------------------|
| 1~2                    | 37.4%               | $5.857 \times 10^{-2}$        |

(4) External water pressure

According to the current buried depth of subways and tunnels in seawater environment, three variable conditions of 40 m water head, 80 m water head and 120 m water head are taken.

3.2. Experimental Scheme of Grouting Stage

(1) The permeable plexiglass tube was fixed with a fastening device through a screw connection. A hard disk filter screen was placed at the bottom of the plexiglass tube to support the weight of the sand-injected layer and prevent the loss of sand particles. The injected sand particles were put into the plexiglass tube in layers, and the sensor embedded. After the filling in the plexiglass tube is completed, the top of the tube was closed and fixed on the test bench.

(2) The grouting pipeline was connected, the pressure sensor connected, the flow sensor and the manual grouting pump in series in sequence, and the cement slurry prepared according to the set water-cement ratio, and placed in the storage tank for standby, the test device is shown in Figure 3.

(3) The slurry was pumped by a manual grouting pump, and the slurry feeding rate controlled to be uniform according to the designed grouting rate. The cement slurry in the slurry tank was constantly stirred to prevent the slurry from settling.

(4) When the slurry emerged from the top of the plexiglass tube, the grouting was stopped and the grouting pipeline cleaned.
3.3. Seepage Test Plan

(1) Test operation process

1) According to the design grout water-cement ratio, the design grouting pressure, and the design of the injected medium, carry out the grouting-reinforced body test, and take the core in time for maintenance.

2) Put the curing box into the standard curing box, select the constant temperature function of the standard curing box, and carry out clean water curing, the temperature is 20 °C, and the curing time is 360 d.

3) Put the sample cured for 360 days into the seepage device, and seal and fix it with silica gel. After the silicone gel is solidified, start the air compressor to supply water to the seepage device, seepage grouting and adding solids, seepage erosion test device is shown in Figure 4.

4) Measure the permeability coefficients of 3 d, 7 d, 14 d, 28 d, 56 d, 90 d, 180 d, 225 d, 270 d, 360 d samples under seepage conditions.

The main part of the seepage device is a detachable iron bucket. A small hole with a diameter of 10 mm is set under the iron barrel, and a water collection device is placed under each small hole to help collect the amount of seepage water passing through the test sample and test the permeability coefficient of the sample. Silica gel is used to fill between the samples, the specific details is shown in Figure 5.
Figure 5. The fabrication process of seepage device.

(2) Measurement method of permeability coefficient

The seepage erosion device is also a set of permeability coefficient testing devices. The hydraulic system uses compressed air from an air compressor to provide pressure to act on the pressurized water tank, convert it into water pressure, and act on one end of the tested sample. The other end of the sample is the free outflow end. By measuring the water seepage over a period of time, the grouting-reinforced body permeability coefficient can be obtained.

Grouting-reinforced body permeation speed can be expressed by the following formula:

$$v = \frac{Q}{AT}$$  \hspace{1cm} (5)

In the formula: $A$ is the cross-sectional area of the sample, $A = \pi (D/2)^2$, $D$ is the diameter of the grouting sample, $D = 5$ cm; $v$ is the water flow velocity; $T$ is the measurement time; $Q$ is the water volume at the outflow end during the measurement time.

According to Darcy’s law, the relationship between water seepage in grouting-reinforced body and its permeability coefficient is as follows:

$$v = \frac{K \Delta p}{\rho g \Delta l}$$  \hspace{1cm} (6)

where: $K$ is the permeability coefficient of grouting-reinforced body; $\rho$ is the density of water, $\rho = 1000$ kg/m$^3$; $g$ is the acceleration of gravity, $g = 9.8$ m/s$^2$; $\Delta p$ is the pressure difference between the grouting-reinforced body; $\Delta l$ is the seepage path length of the grouting sample.

In summary, the grouting sample coefficient can be expressed as:

$$K = \frac{\rho g Q \Delta l}{AT \Delta p}$$  \hspace{1cm} (7)

4. Variation Law of Permeability Coefficient of Grouting-Reinforced Body under Seepage

The deterioration of the infiltration of the grouting-reinforced body is mainly expanded by the original inner structure of the grouting-reinforced body. According to the composition and structure analysis, the grouting-reinforced body is formed by diffusion of the cement slurry in the sand, and its microcracks and pore structures are relatively stable. The seepage deterioration and the seepaining effect on the grouting-reinforced body is mainly due to the internal pore water pressure that generates damage to the original
microcrack and the pores of the grouting-reinforced body. Once the seepage act generates a certain degree of damage to the grouting-reinforced body, the internal microcracks and pore structures extend, forming stable seepage. Then, under the seepage water pressure, the seepage flow of the grouting-reinforced body will reach the balance, the structure of the reinforced body is dense, it is difficult for the internal pore water pressure to cause new damage, and the infiltration of the seepage is basically disappeared.

Under the action of different water pressures, seepage effect on the grouting-reinforced body deteriorated permeability coefficient of solids changes with time as shown in Figure 6.

![Figure 6. The seepage coefficient of grouting-reinforced body changes with time under seepage action.](image)

It can be seen that the permeability coefficient of grouting-reinforced body increases with the increase of seepage time. The greater the seepage water pressure, the faster the increase of the permeability coefficient, and the larger the permeability coefficient when it reaches stability. In the seepage degradation test with a water pressure of 40 m, the permeability coefficient changes suddenly when the seepage erosion time is 300–330 d, then the permeability coefficient is basically stable, and the subsequent erosion changes little. The stable permeability coefficient is about 10–15 times the initial permeability coefficient. In the seepage degradation test with a water pressure of 80 m, the permeability coefficient changes suddenly when the seepage erosion time is 240–270 d, then the permeability coefficient is basically stable, and the subsequent erosion changes little. The stable permeability coefficient is about 15–20 times the initial permeability coefficient. In the seepage degradation test with a water pressure of 120 m, the permeability coefficient changes suddenly when the seepage erosion time is 210–240 d, then the permeability coefficient is basically stable, and the subsequent erosion changes little. The stable permeability coefficient is about 25–30 times the initial permeability coefficient.

5. Evolution Law of Grouting-Reinforced Body Damage Variables

Picandet [18] conducted a related study on the relationship between concrete permeability and damage under seepage action, and obtained the relationship between concrete permeability and concrete damage under low damage, according to the analysis, the mechanism of seepage effect on the deterioration of sand grouting-reinforced body is similar to that of low-damage concrete, and the damage of sand grouting-reinforced body under seepage will reach a certain value. The magnitude of the value satisfies the concrete damage value range studied by Picandet. In addition, the composition of concrete is cement slurry, sand, and gravel, and the composition of sand grouting-reinforced body is cement slurry and sand. The two components are similar, and the cement hydration reaction and cementation structure are similar. Therefore, it is reasonable to use the relationship between
the permeability of low-damage concrete under seepage and the damage value to calculate
the damage value of sand grouting-reinforced body under seepage.

Picandet proposed the relationship equation between the permeability of the medium
and the damage variable:

\[ k_D = k_0 \cdot e^{(\alpha D)^\beta} \] (8)

where \( k_D \) is the permeability of grouting-reinforced body under seepage conditions, \( k_0 \) is
the initial permeability of grouting-reinforced body, \( \alpha \) and \( \beta \) are constants related to the
properties of the material itself, and the value of \( \alpha \) ranges from 9.4 to 12.3, usually 11.3; The
value of \( \beta \) ranges from 1.6 to 1.8, usually 1.64.

Definition of damage variable \( D \):

\[ D = \frac{E_0 - E_d}{E_0} = 1 - \frac{E_d}{E_0} \] (9)

In the formula, \( E_0 \) is the initial elastic modulus of grouting-reinforced body, and \( E_d \) is
the elastic modulus after grouting-reinforced body damage.

There is the following relationship between grouting-reinforced body permeability
and permeability coefficient:

\[ K = k \cdot \left( \rho g / \mu \right) \] (10)

In the formula, \( \rho \) is the density of water, \( g \) is the acceleration of gravity, and \( \mu \) is the
viscosity of water.

Substituting Equation (10) into Equation (8), the relationship between the grouting-
reinforced body permeability coefficient and the damage variable is obtained:

\[ K_D = K_0 \cdot e^{(\alpha D)^\beta} \] (11)

In order to verify the applicability of Formula (11) in the effect of seepage effect on
the permeability of grouting-reinforced body, a verification test was designed. Take the
grouting-reinforced body with a water-cement ratio of 0.8:1, perform 60 d, 90 d, and 180 d
seepage damage on it, and test the grouting-reinforced body permeability coefficient before
and after the damage, and calculate the damage variable with Equation (11). The test
results are shown in Table 2.

| Seepage Time/d | Initial Permeability Coefficient/$\times 10^{-8}$ cm/s | Permeability Coefficient after Damage/$\times 10^{-8}$ cm/s | Damage Variable |
|----------------|---------------------------------|---------------------------------|----------------|
| 60             | 0.754                           | 1.413                           | 0.066          |
| 90             | 0.754                           | 1.622                           | 0.075          |
| 180            | 0.754                           | 3.11                            | 0.109          |

The grouting-reinforced body elastic modulus before and after the damage is tested,
and Formula (9) is used to calculate the damage variable. The test results are shown in
Table 3.

| Seepage Time/d | Initial Modulus of Elasticity/MPa | Modulus of Elasticity after Damage/MPa | Damage Variable |
|----------------|---------------------------------|---------------------------------|----------------|
| 60             | 2238                            | 2092                            | 0.065          |
| 90             | 2238                            | 2073                            | 0.076          |
| 180            | 2238                            | 1994                            | 0.109          |

It can be seen from the verification test of the seepage effect on grouting-reinforced
body in Formula (11)—calculating the influence of seepage effect on grouting-reinforced
body—that it is reasonable and feasible to calculate the damage variable of the rein-
forced bodies from the permeability coefficient of the reinforced bodies before and after
the damage.
According to the change law data of grouting-reinforced body permeability coefficient under the action of seepage, the relationship between grouting-reinforced body damage variables and seepage erosion time under different water pressures and seepage conditions can be obtained by combining Equation (11). The damage variable changes of grouting-reinforced body with time is shown in Figure 7:

![Limit damage](image)

**Figure 7.** The damage variable changes of grouting-reinforced body with time under seepage action.

It can be seen from the figure that the damage variable caused by the deterioration of the grouting-reinforced body caused by the seepage effect increases with time. The greater the seepage water pressure, the larger the damage variable, but the stable value of the damage variable is within the satisfying range of Equation (8). Within this, the correctness of Formula (8) is verified. In the seepage degradation test with a water pressure of 40 m, the damage variable of grouting-reinforced body when the damage is stable is about 0.15–0.16. In the seepage degradation test with a water pressure of 80 m, the damage variable of grouting-reinforced body when the damage is stable is about 0.16–0.17. In the seepage degradation test with a water pressure of 120 m, the damage of the grouting-reinforced body is stable when the damage is stable. The damage variable is about 0.18–0.19. It can be seen that under the action of high head water pressure, the damage variable of grouting-reinforced body is gradually increasing, which is related to the structure of the grouting-reinforced body. The main components of grouting-reinforced body are sand particles, cement hydration, and the cementation of the two. The main effect of seepage damage is that under the action of the grouting-reinforced body under water pressure, the pore water pressure in the reinforcement body has an impact on the original interior of the reinforcement. The expansion of micro-crack pores and other structures, result in an increase in grouting-reinforced body porosity and an increase in permeability coefficient. Therefore, under the premise of a certain grouting-reinforced body structure, the higher the water head height that the reinforcement body can withstand, that is, the greater the water pressure, the greater the pore water pressure in the reinforcement body, and the greater the influence of pore water pressure on the grouting-reinforced body. When the stability is reached, the damage of grouting-reinforced body will be greater.

6. Determination of the Model of Grouting-Reinforced Body under the Action of Seepage

According to Figure 8 shows the curve form of grouting-reinforced body damage variable with time under the action of seepage, the function form \(D = a \cdot \ln(t) + b\) is proposed to describe the evolution of grouting-reinforced body damage variables with time under the action of seepage. Among them, \(D\) is the grouting-reinforced body damage variable, \(t\) is the seepage erosion time, and \(a\) and \(b\) are constants related to the seepage water pressure.
By fitting the above data, the relationship between grouting-reinforced body damage variables and seepage erosion time under different seepage water pressures is obtained:

\[
\begin{align*}
D_{0.4} &= 0.033 \cdot \ln(t) - 0.052 \rightarrow R^2 = 0.985 \\
D_{0.8} &= 0.036 \cdot \ln(t) - 0.053 \rightarrow R^2 = 0.994 \\
D_{1.2} &= 0.041 \cdot \ln(t) - 0.052 \rightarrow R^2 = 0.991
\end{align*}
\]  

(12)

It can be seen from the above fitting relationship that the constant \( a \) related to the osmotic water pressure has a linear relationship with the osmotic water pressure, and the constant \( b \) is a fixed value with a value of \(-0.052\). Fit the constant \( a \) and the osmotic pressure to get the relationship between the osmotic pressure \( p \) and the constant \( a \):

\[
a = 0.01 \cdot p + 0.0287 \rightarrow R^2 = 0.989
\]  

(13)

Substituting Equation (13) into Equation (12), the relationship between the variable \( D \) of grouting-reinforced body damage under seepage action and the seepage pressure \( p \) and seepage erosion time \( t \) is:

\[
D = (0.01p + 0.0287) \cdot \ln(t) - 0.052
\]  

(14)

7. Conclusions

(1) Based on the effective stress and damage mechanics, using the description of the mechanical properties of grouting-reinforced body under the action of seepage erosion by the primitive body, a mechanical model of grouting-reinforced body damage under the action of seepage is established.

(2) A simulation test of grouting-reinforced body degradation under the action of seepage was designed, and the evolution law of grouting-reinforced body permeability coefficient under different seepage water pressures studied. The results show that the permeability coefficient of the grouting-reinforced body increases with the growth of the seepage time, and with the increase of the external water pressure, pore water pressure has a greater influence on the original micro-cracks and pores of grouting-reinforced body, resulting in a shorter jump time for the permeability coefficient of grouting-reinforced body.
body under the action of seepage. In the process of grouting-reinforced body erosion, the permeability coefficient has a "sudden jump" phenomenon, and the limit damage of grouting-reinforced body is 0.15–0.19.

(3) Based on the relationship between the permeability of the medium and the damage variable, a design test is designed to verify the relationship between the grouting-reinforced body permeability and the damage variable under the action of seepage. The quantitative relationship between grouting-reinforced body osmotic pressure and time and damage variables is further obtained, determining the mechanical model of grouting-reinforced body damage under the action of seepage. The research results have improved the damage mechanism of grouting-reinforced body under seepage, and promoted further development of water-rich soft ground grouting reinforcement design theory to ensure the safety requirements and longevity.

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