Quark-Photon-Quark Correlation and Transverse Target Single Spin 
Asymmetry in Inclusive DIS

Matthias Burkardt and Tareq Alhalholy
Department of Physics, New Mexico State University, Las Cruces, NM 88003-0001, U.S.A.

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Abstract

We calculate the $q\gamma q$ correlation function associated to transverse target lepton-nucleon inclusive 
deep-inelastic scattering by direct evaluation of the corresponding matrix element. We use the electro-
magnetic impact parameter potential for a transversely polarized nucleon to define the $q\gamma q$ correlation 
function for each flavor in terms of the field components of that flavor. The results are compared with the 
two existing models for the $q\gamma q$ correlator. Using the calculated $q\gamma q$ correlation function, we estimated 
the transverse target SSA in IDIS process.
I. Introduction

Single spin asymmetry in inclusive and semi inclusive deep inelastic scattering is of a prime importance in studying and revealing the spin structure of the nucleon. In particular, asymmetries appear in inclusive DIS are important to study quark correlation $q\gamma q$ in the nucleon, while in semi-inclusive DIS the asymmetries give information about the quark-gluon correlation $qqq$ in the nucleon. Due to its importance in transverse target single spin asymmetry, the $qqq$ correlator is extensively studied [1-4], and extraction from data is done either directly from $pp^\uparrow$ collision [5] or by extracting the Sivers function from SIDIS data where the $qqq$ correlator is proportional to the first moment of Sivers function [6].

On the other hand, the $q\gamma q$ correlator (associated to a transversely polarized target in inclusive DIS) is modeled by relating it with the Qiu-Sterman function (or the Efremov-Teryaev-Qiu-Sterman (ETQS) twist-3 matrix element), by rescaling the $qqq$ correlator [7,8] utilizing the symmetry between the corresponding diagrams representing the two correlators. In the two existing models for the $q\gamma q$ correlator, there are disagreements in sign and magnitude in the correlators for the majority flavors (up quarks in the proton and down quarks in the neutron), while they agree in sign and magnitude for the proton and neutron minority flavors. In Ref. [9], the electromagnetic potential corresponding to a transversely polarized nucleon was calculated, this potential can be used to explicitly calculate the $q\gamma q$ correlator. In this paper, we present a direct calculation of the $q\gamma q$ correlator by evaluating the corresponding matrix element using the transverse (impact parameter) electromagnetic potential for a transversely polarized nucleon. The results are then compared with the existing models for the $q\gamma q$ correlator.

II. Transverse charge Density and Impact Parameter Potential for Transversely Polarized Nucleon

For a proton transversely polarized (with respect to the lepton plane), the transverse charge density (or the impact parameter dependent parton distribution function) is given by [10]

$$\rho_T(x, b_\perp)_{\text{proton}} = \int \frac{d^2q_\perp}{(2\pi)^2} e^{-i q_\perp \cdot b_\perp} \left( \sum_f e_f H^f_v(x, -q_\perp^2) + \frac{q_y}{2M} \sum_f e_f E^f_v(x, -q_\perp^2) \right),$$

(1)

here $f = u, d$ are used for up and down quarks, $H^f_v(x, -q_\perp^2)$, $E^f_v(x, -q_\perp^2)$ denote the proton valence GPDs for unpolarized quark of flavor $f$, the transverse position vector is $b_\perp = b \perp \cos(\phi_{b_\perp}) \hat{e}_x + b \perp \sin(\phi_{b_\perp}) \hat{e}_y$ and the transverse momentum transfer is $q_\perp = q \perp \cos(\phi_{q_\perp}) \hat{e}_x + q \perp \sin(\phi_{q_\perp}) \hat{e}_y$ with a nucleon target of spin vector $S = \cos(\phi_s) \hat{e}_x + \sin(\phi_s) \hat{e}_y$ such that $q_y = q \perp \sin(\phi_{b_\perp} - \phi_s)$. The neutron transverse charge density is obtained utilizing isospin symmetry. Decomposing the GPDs in terms of their flavor components and evaluating the angular integrals for a nucleon polarized in the $+x$ direction (i.e. $\phi_s = 0$), the proton transverse charge density becomes

$$\rho_T(x, b_\perp)_{\text{proton}} = \int_0^\infty \frac{dq_\perp}{2\pi} J_0(b \perp q_\perp) \left[ e_u H^u_u(x, -q_\perp^2) + e_d H^d_u(x, -q_\perp^2) \right]$$

$$- \sin(\phi_{b_\perp}) \int_0^\infty \frac{dq_\perp}{2\pi} \frac{q_\perp^2}{2M} J_1(b \perp q_\perp) \left[ e_u E^u_u(x, -q_\perp^2) + e_d E^d_u(x, -q_\perp^2) \right].$$

(2)

From the above equation, the up and down quark densities read

$$\rho_u(x, b_\perp) = \int_0^\infty \frac{dq_\perp}{2\pi} J_0(b \perp q_\perp) H^u_u(x, -q_\perp^2) - \sin(\phi_{b_\perp}) \int_0^\infty \frac{dq_\perp}{2\pi} \frac{q_\perp^2}{2M} J_1(b \perp q_\perp) E^u_u(x, -q_\perp^2)$$

(3)

$$\rho_d(x, b_\perp) = \int_0^\infty \frac{dq_\perp}{2\pi} J_0(b \perp q_\perp) H^d_u(x, -q_\perp^2) - \sin(\phi_{b_\perp}) \int_0^\infty \frac{dq_\perp}{2\pi} \frac{q_\perp^2}{2M} J_1(b \perp q_\perp) E^d_u(x, -q_\perp^2)$$

(4)
On the other hand, the electromagnetic potential corresponding to a transversely polarized nucleon is given by [9]

\[ A^{0+}(x, b_\perp) = -\frac{1}{2\pi} \int_0^\infty dq_{\perp} \left[ -J_0 (b_\perp q_{\perp}) + J_0 (b_0 q_{\perp}) \right] \sum_f e_f H^f (x, -q_{\perp}^2) + \frac{\sin(\phi_{b_\perp} - \phi_0)}{4\pi M} \int_0^\infty dq_{\perp} J_1 (b_\perp q_{\perp}) \sum_f e_f E^f (x, -q_{\perp}^2), \] (5)

where \( b_0 \) is a reference point for the potential; in our case it is equal to the size of the transverse charge density of the nucleon. The field strengths corresponding to this potential read

\[ F^{+i} (x, b_\perp) = -\nabla V (b_\perp) = \left[ \frac{1}{2\pi} \int_0^\infty dq_{\perp} J_1 (b_\perp q_{\perp}) \sum_f e_f H^f (x, -q_{\perp}^2) - \frac{\sin(\phi_{b_\perp} - \phi_0)}{8\pi M} \int_0^\infty dq_{\perp} q_{\perp} \left[ J_0 (b_\perp q_{\perp}) - J_2 (b_\perp q_{\perp}) \right] \sum_f e_f E^f (x, -q_{\perp}^2) \right] \hat{b}_{\perp} - \frac{\cos(\phi_{b_\perp} - \phi_0)}{4\pi M b_{\perp}} \int_0^\infty dq_{\perp} J_1 (b_\perp q_{\perp}) \sum_f e_f E^f (x, -q_{\perp}^2) \hat{\phi}_{b_\perp}, \] (6)

the \( x \) and \( y \) components of \( F^{+i} \) for proton become after writing the polar unit vectors in terms of the Cartesian ones

\[ F^{+x} (x, b_\perp)_{\text{proton}} = \frac{\cos(\phi_{b_\perp})}{2\pi} \int_0^\infty dq_{\perp} J_1 (b_\perp q_{\perp}) \sum_f e_f H^f (x, -q_{\perp}^2) - \frac{\cos(\phi_{b_\perp}) \sin(\phi_{b_\perp})}{8\pi M} \int_0^\infty dq_{\perp} q_{\perp} \left[ J_0 (b_\perp q_{\perp}) - J_2 (b_\perp q_{\perp}) \right] \sum_f e_f E^f (x, -q_{\perp}^2) + \frac{\sin(\phi_{b_\perp}) \cos(\phi_{b_\perp})}{4\pi M b_{\perp}} \int_0^\infty dq_{\perp} J_1 (b_\perp q_{\perp}) \sum_f e_f E^f (x, -q_{\perp}^2) \] (7)

\[ F^{+y} (x, b_\perp)_{\text{proton}} = \frac{\sin(\phi_{b_\perp})}{2\pi} \int_0^\infty dq_{\perp} J_1 (b_\perp q_{\perp}) \sum_f e_f H^f (x, -q_{\perp}^2) - \frac{\sin^2(\phi_{b_\perp})}{8\pi M} \int_0^\infty dq_{\perp} q_{\perp} \left[ J_0 (b_\perp q_{\perp}) - J_2 (b_\perp q_{\perp}) \right] \sum_f e_f E^f (x, -q_{\perp}^2) - \frac{\cos^2(\phi_{b_\perp})}{4\pi M b_{\perp}} \int_0^\infty dq_{\perp} J_1 (b_\perp q_{\perp}) \sum_f e_f E^f (x, -q_{\perp}^2), \] (8)

the above forms allow to perform a flavor decomposition for the fields, for example the \( y \)-component of the field due to up quarks is

\[ F^{+y}_{u} (x, b_\perp)_{\text{proton}} = \frac{e_u \sin(\phi_{b_\perp})}{2\pi} \int_0^\infty dq_{\perp} J_1 (b_\perp q_{\perp}) H^u (x, -q_{\perp}^2) - \frac{e_u \sin^2(\phi_{b_\perp})}{8\pi M} \int_0^\infty dq_{\perp} q_{\perp} \left[ J_0 (b_\perp q_{\perp}) - J_2 (b_\perp q_{\perp}) \right] E^u (x, -q_{\perp}^2) - \frac{e_u \cos^2(\phi_{b_\perp})}{4\pi M b_{\perp}} \int_0^\infty dq_{\perp} J_1 (b_\perp q_{\perp}) E^u (x, -q_{\perp}^2), \] (9)
and due to down quarks is

\[ F_{d\gamma q}(x, b_\perp)_{\text{proton}} = \frac{e_d}{2\pi} \sin(\phi_{b_\perp}) \int_0^\infty dq_\perp J_1(b_\perp q_\perp) H_\perp^d(x, -q_\perp^2) - \]

\[ \frac{e_d \sin^2(\phi_{b_\perp})}{8\pi M} \int_0^\infty dq_\perp q_\perp [J_0(b_\perp q_\perp) - J_2(b_\perp q_\perp)] E_\perp^d(x, -q_\perp^2) - \]

\[ \frac{e_d \cos^2(\phi_{b_\perp})}{4\pi M b_\perp} \int_0^\infty dq_\perp J_1(b_\perp q_\perp) E_\perp^d(x, -q_\perp^2). \] (10)

As we shall see, the above decomposition of the nucleon’s fields motivates to define the \(q\gamma q\) correlator for each flavor in terms of the field due to the corresponding flavor.

### III. \(q\gamma q\) Correlator in Inclusive DIS

The \(q\gamma q\) correlator (denoted by \(F_{FT}^q(x, x')\)) as defined in Eq. (10) in Ref. [7] is given by

\[ \int \frac{d\xi^- d\zeta^-}{2(2\pi)^2} e^{ix P^+ \xi^-} \langle P, S|\bar{\psi}^q(0)\gamma^+ e F_{QED}^{+i}(\zeta)\psi^q(\xi)|P, S\rangle = -Me^{ij}S_T^f F_{FT}^q(x, x) \] (11)

here \(F_{QCD}^{+i}(\zeta)\) is the electromagnetic field strength tensor. The above definition follows from an analogy with the so-called QCD soft gluon pole matrix element (Qiu-Sterman function) \(T_F^q(x, x')\) which is defined through [12]

\[ \int \frac{d\xi^- d\zeta^-}{4\pi} e^{ix P^+ \xi^-} \langle P, S|\bar{\psi}^q(0)\gamma^+ F_{QCD}^{+i}(\zeta)\psi^q(\xi)|P, S\rangle = -e^{ij}S_T^f T_F^q(x, x) \] (12)

where \(F_{QCD}^{+i}\) is the QCD field strength tensor and the corresponding matrix element represents the average force of the ejected quark in a SIDIS process [11], i.e. in the final state interaction. In the same way the matrix element in Eq.(11) is interpreted as the average transverse force on the electron in DIS process due to initial and final state interactions [3]. Based on the above analogy, and using the results of the previous section for the flavor decomposition of the fields of a transversely polarized nucleon, we define the \(q\gamma q\) correlator for each flavor in terms of the corresponding field component for that flavor such that

\[ \int \frac{d\xi^- d\zeta^-}{4\pi} e^{ix P^+ \xi^-} \langle P, S|\bar{\psi}^q(0)\gamma^+ e F_{q/N}^{+i}(x, \zeta)\psi^q(\xi)|P, S\rangle = -Me^{ij}S_T^f N_{q/N}(x) F_{FT}^{q/N}(x, x) \] (13)

where

\[ N_{q/N}(x) = (2\pi)^2 e_q^2 \rho_{q/N}(x) \]

and \(e = \sqrt{4\pi \alpha_{em}}, \alpha_{em} = 1/137, M = 0.938 \text{ GeV}, e_T^2 = 1, S_T^f = 1\) and the quark densities \(\rho_{q/N}(x, b_\perp)\) are give in Eqs. [3, 4]. The function \(N_{q/N}(x)\) is consistent with the results obtained for the \(qqq\) correlator using Sivers function extracted from SIDIS data [12] and also with the calculation of the T-odd \(qqq\) correlation functions in the diquark model [3] and with the parametrization of the Qiu-Sterman function \(T_F^q(x, x')\) from \(pp^+\) data [5], where in all of the above, similar normalization function is usually used. Notice the main difference between Eqs. (11,12) and Eq. (13); in Eq. (12), the matrix element of \(F_{QCD}^{+i}\) represents the average force experienced by the active quark from the nucleon remnants, i.e. in the FSI [11]. Similarly, in Eq. (11), the matrix element of \(F_{QED}^{+i}\) is the average force experienced by the electron when passing through the nucleon in the FSI and ISI [8]. On the other hand, in Eq. (13), \(F_{q/N}^{+i}\) is the field experienced by the electron in DIS process from quarks of flavor \(q\). Therefore, the \(q\gamma q\) correlator corresponding to flavor \(q\) can be expressed as

\[ F_{q/N}^{+i}(x, x) = \frac{-1}{M S_T^f e_T^2 N_{q/N}(x)} \int d^2b_\perp \rho_{q/N}(x, b_\perp) e F_{q/N}^{+i}(x, b_\perp), \] (14)
note that the only fields contribute to the above formula are $F_{q/N}^{y}(x,b_{\perp})$; since the average transverse momentum in the $x$-direction vanishes by symmetry. The figures below are plots of $xF_{q/N}^{y}(x,x)$ ($N = \text{proton or neutron}$), using three approaches, the impact parameter transverse potential, and two models for the correlator $F_{FT}^{q/N}(x,x)$, Burkardt’s model from Ref. [8] and Metz’s et al. model from Ref. [7]. In the calculations of $F_{FT}^{q/N}(x,x)$ using Burkardt and Metz models, the parametrization of the Qui-Sterman functions $T_{q}^{x}(x,x)$ used in these models was taken from Ref. [13], this parametrization is based on Sivers functions parametrization which is taken from Ref. [14]. In the calculation of $F_{FT}^{q/N}(x,x)$ using the transverse nucleon potential, the generalized parton distribution (GPD) parametrization was taken from Ref. [15] and the transverse radii of the charge densities for proton and neutron were taken from Ref. [16].

**FIG. 1.** $x$ times the correlator $F_{FT}^{q/p}(x,x)$ as a function of $x$ for proton using the impact parameter transverse potential from Ref. [9], Burkardt’s model from Ref. [8] and Metz’s model from Ref. [7].

**FIG. 2.** $x$ times the correlator $F_{FT}^{q/n}(x,x)$ as a function of $x$ for neutron using the impact parameter transverse potential from Ref. [9], Burkardt’s model from Ref. [8] and Metz’s model from Ref. [7].

**IV. Calculation of the Transverse Target Single Spin Asymmetry in IDIS**

In this section we will use the results obtained in the previous section for the correlator $F_{FT}^{q/N}(x,x)$ to calculate the SSA in inclusive DIS for transversely polarized proton and neutron. In Ref. [7] the transverse
target SSA is given by

\[ A_{UT}^N = \frac{2\pi M}{Q} \frac{2 - y}{\sqrt{1 - y}} \sum_q e_q^2 x \tilde{F}_{FT}^{q/N}(x,x), \]  

(15)

where

\[ \tilde{F}_{FT} = F_{FT}(x,x) - x \frac{d}{dx} F_{FT}(x,x), \]  

(16)

and \( f_1^q \) is the unpolarized quark distribution function for a quark of flavor \( q \) which are taken from Ref. [15].

The relation between the variables in (15) is given by \( y = Q^2/(xs) \), with \( s \) denotes the square of the center of mass energy and \( y \) is the fraction of the electron energy carried by the virtual photon. Figure 3 is a plot for proton and neutron transverse target asymmetry for typical Jefferson Lab kinematics at \( y = 0.6 \) and \( Q^2 = 1 \text{ GeV}^2 \).

![Graph showing proton and neutron transverse target asymmetry at \( y = 0.6 \) and \( Q^2 = 1 \text{ GeV}^2 \)](image)

FIG. 3. Proton and neutron transverse target asymmetry at \( y = 0.6 \) and \( Q^2 = 1 \text{ GeV}^2 \), corresponding to typical Jefferson Lab kinematics.

V. Conclusion

We calculated the \( q\gamma q \) correlator \( F_{FT}^{q/n}(x,x) \) associated with a transversely polarized nucleon in IDIS process by direct evaluation of the corresponding matrix element using the electromagnetic potential for a transversely polarized nucleon. The results are compared with the two existing models for the \( q\gamma q \) correlator. Clearly from the above figures, Burkardt’s and Metz’s models agree in the \( q\gamma q \) correlator for proton and neutron minority flavors but disagree in sign and magnitude for proton up quarks while for neutron down quarks, the two models significantly disagree in magnitude but agree in sign. On the other hand, the results obtained using the transverse nucleon potential are in agreement in sign and magnitude with Burkardt’s model for proton and neutron majority flavors. For the minority flavors, there is agreement in sign, but one notice a slight difference in magnitude for both proton and neutron. However, the results obtained for \( F_{FT}^{q/n}(x,x) \) using the transverse potential are in better agreement with the sum rule connecting the correlators of the two flavors [17].

Using the calculated \( q\gamma q \) correlator, we estimated the transverse target SSA for inclusive DIS, the neutron results are consistent with recent calculations based on Sivers function parametrization [17]. For the proton, the SSA results are almost half those for neutron, which is a main difference from the above reference and other calculations [18], however the sign of our calculations is consistent with the above references.
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