Modified Friedmann Equations From Debye Entropic Gravity

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A remarkable new idea on the origin of gravity was recently proposed by Verlinde who claimed that the laws of gravitation are no longer fundamental, but rather emerge naturally as an entropic force. In Verlinde derivation, the equipartition law of energy on the holographic screen plays a crucial role. However, the equipartition law of energy fails at the very low temperature. Therefore, the formalism of the entropic force should be modified while the temperature of the holographic screen is very low. Considering the Debye entropic gravity and following the strategy of Verlinde, we derive the modified Newton’s law of gravitation and the corresponding Friedmann equations which are valid in all range of temperature. In the limit of strong gravitational field, i.e. high temperature compared to Debye temperature, \( T \gg T_D \), one recovers the standard Newton’s law and Friedmann equations. We also generalize our study to the entropy corrected area law and derive the dynamical cosmological equations for all range of temperature. Some limits of the obtained results are also studied.

I. INTRODUCTION

Although gravity is the most universal force of nature, however, the origin of it in quantum level is still unclear. This is due to the fact that it is remarkably hard to combine gravity with quantum mechanics compared with all the other forces, and hence the final theory of the quantum gravity has not been established yet. The universality of gravity suggests that its emergence should be understood from general principles that are independent of the specific details of the underlying microscopic theory.

According to Einstein’s theory of general relativity, the concept of gravity has strongly connected to the spacetime geometry. Indeed, Einstein field equations tell us that the presence of energy or stress causes the deformation of the spacetime geometry. In 1970’s Bekenstein and Hawking [1] discovered black holes thermodynamics. With combination of quantum mechanics and general relativity they predicted that a black hole behaves like a black body, emitting thermal radiations, with a temperature proportional to its surface gravity at the black hole horizon and with an entropy proportional to its horizon area. The Hawking temperature and the horizon entropy together with the black hole mass obey the first law of thermodynamics,
\[
dM = T dS.
\]
Since the discovery of black hole thermodynamics in 1970’s, physicists have been speculating that there should be a direct relation between thermodynamics and Einstein equations. This is expected because the geometrical quantities like horizon area and surface gravity are proportional to entropy and horizon temperature, respectively. After Bekenstein and Hawking a lot of works have been done to disclose the connection between thermodynamics and gravity [2, 3]. In 1995 Jacobson [4] put forward a great step and showed that the Einstein field equation is just an equation of state for the spacetime and in particular it can be derived from the proportionality of entropy and the horizon area together with the fundamental relation \( \delta Q = T dS \). Following Jacobson, however, an overwhelming flood of papers has appeared which attempt to show that there is indeed a deeper connection between gravitational field equations and horizon thermodynamics. It has been shown that, not only in Einstein gravity but also in a wide variety of theories, the gravitational field equations for the spacetime metric has a predisposition to thermodynamic behavior. This result, first pointed out in [5], has now been demonstrated in various theories including f(R) gravity [6] and cosmological setups [7–14]. For a recent review on the thermodynamical aspects of gravity and complete list of references see [15].

Recently, Verlinde [16] has invented a conceptual theory that gravity is no longer fundamental, but is emergent. According to Verlinde, one can start from the first principles, and gravity appears as an entropic force naturally and unavoidably in a theory in which space is emergent through a holographic scenario. Similar discoveries are also made by Padmanabhan [17] who observed that the equipartition law for horizon degrees of freedom combined with the Smarr formula leads to the Newton’s law of gravity. In addition, Verlinde’s arguments reveal a fact that the key to understanding gravity is information (or entropy). In Verlinde derivation the holographic principle and the equipartition law of energy play a crucial role. The holographic principle was originally proposed by ’t Hooft [18] and then developed in cosmology by Susskind and others [19]. According to this principle the combination of quantum mechanics and gravity requires the three dimensional world to be an image of data that can be stored on a two
dimensional projection much like a holographic image. The studies on the entropic gravity scenario have arisen a lot attention recently [20–31].

It is well-known that the equipartition law of energy for a system of particles only valid for the situation where the kinetic energy of the particles is much larger than the effective interacting potential between particles. This means that the equipartition law of energy break down at very low temperatures. It is found that Debye model, which modified the equipartition law of energy, is in good agreement with experimental results for most solid objects. According to Verlinde, we know that the gravity can be explained as an entropic force, it means that the gravity may have a statistical thermodynamics explanation. Therefore, the formalism of the entropic force should be modified while the temperature of the holographic screen is very low. This means that Newtonian gravity takes a different form in the background of an extreme weak gravitational field. In the present work, inspired by the Debye’s model in statistical thermodynamics, we generalize the formalism of the entropic gravity to the very low temperatures.

The outline of the present paper is as follows. The modified Newton’s law of gravitation and the corresponding Friedmann equations which are valid in all range of temperature are extracted in the next section. Sec. III is devoted to the derivation of the Entropy corrected Friedmann equations in Debye entropic force scenario. The paper ends with a conclusion, which appears in Sec. IV.

II. DEBYE ENTROPIC GRAVITY AND FRIEDMANN EQUATION

Consider a closed holographic screen and a free particle of mass \( m \) near it on the side that spacetime has already emerged. In Verlide’s picture when the particle has an entropic reason to be on one side of the screen and the screen carries a temperature, it will experience an effective macroscopic force due to the statistical tendency to increase its entropy. This is described by

\[
F = T \frac{\Delta S}{\Delta x}.
\]

(1)

where \( \Delta x \) is the displacement of the particle from the holographic screen, while \( T \) and \( \Delta S \) are the temperature and the entropy change on the screen, respectively. According to the Unruh formula, the temperature in Eq. (1) associated with the holographic screen is

\[
k_B T = \frac{\hbar g}{2 \pi c}.
\]

(2)

where \( g \) represents the proper gravitational acceleration on the screen which is produced by the matter distribution inside the screen. Suppose we have a mass distribution \( M \) which induces a holographic screen \( S \) at some distance \( R \) that has encoded on it gravitational information. Suppose we have also a test mass \( m \) which is assumed to be very close to the holographic screen as compared to its reduced Compton wavelength \( \lambda_m = \frac{\hbar}{mc} \). Assuming the holographic screen forms a closed surface. The key statement is that we need to have a temperature in order to have a force. One can think about the boundary as a storage device for information. Assuming that the holographic principle holds, the maximal storage space, or total number of bits, is proportional to the area \( A \). Let us denote the number of used bits by \( N \). It is natural to assume that this number will be proportional to the area \( A = 4\pi R^2 \). Thus we write

\[
A = NQ.
\]

(3)

where \( Q \) is a fundamental constant which should be specified later. Note that \( N \) is the number of bits and thus for one unit change we have \( \Delta N = 1 \), hence from (3) we find \( \Delta A = Q \). Motivated by Bekenstein’s area law of black hole entropy, we assume the entropy associated with the holographic screen obey the area law, namely

\[
S = \frac{A}{4\ell_p^2}.
\]

(4)

where \( \ell_p^2 = G\hbar/c^3 \) is the Planck length. Following [10] we also assume the energy on the holographic screen is proportional to the mass distribution \( M \) that would emerge in the part of space enclosed by the screen

\[
E = Mc^2.
\]

(5)

According to statistical thermodynamics the equipartition law of energy for free particle only valid for the situation in which the kinetic energy of particle is much larger than the effective interacting potential between them. Therefore, the equipartition law of energy fails in the very low temperatures. It is found that Debye model, which modified
the equipartition law of energy, is in good agreement with experimental results for most solid objects. Following Verlinde’s scenario, the laws of gravitation are no longer fundamental, but rather emerge naturally as an entropic force. It means that the gravity may have a statistical thermodynamics origin. Thus, any modification of statistical mechanics should modify the laws of gravity accordingly. Motivated by this point, we modify the equipartition law of energy as

\[ E = \frac{1}{2} N k_B T D(x), \] (6)

where the Debye function is defined by

\[ D(x) ≡ \frac{3}{x^3} \int_0^x \frac{y^3}{e^y - 1} dy. \] (7)

Here \( x \) is related to the temperature \( T \) as follows

\[ x = \frac{T_D}{T} = \frac{\hbar \omega_D}{k_B}, \] (8)

where \( T_D \) is the Debye critical temperature and \( \omega_D \) is the Debye frequency. Combining Eqs. (3), (5) and (6), we obtain the temperature of the holographic screen as

\[ T = \frac{2Mc^2Q}{4\pi R^2 k_B D(x)}. \] (9)

Substituting Eqs. (9) and (4) in (1), and using relation \( \Delta A = Q \), we get

\[ F = -Mm \frac{1}{R^2 D(x)} \left( \frac{Q^2 c^3}{8\pi \ell_p^4 k_B \hbar} \right), \] (10)

where we have taken \( \Delta x = -\frac{\hbar}{mc} \) for one fundamental unit change in the entropy and the entropy gradient points radially from the outside of the surface to inside. In order to derive the Newton’s law of gravitation we must define \( Q^2 = 8\pi k_B \ell_p^4 \). Finally we reach

\[ F = -G \frac{Mm}{R^2} \frac{1}{D(x)}. \] (11)

The corresponding gravitational acceleration is obtained as

\[ g = \frac{GM}{R^2} \frac{1}{D(x)}. \] (12)

Using relation (2) we can define the Debye acceleration \( g_D \) which is related to the Debye temperature as

\[ T_D = \frac{\hbar g_D}{2\pi k_B c}, \quad x = \frac{T_D}{T} = \frac{g_D}{g}. \] (13)

Eq. (11) is the Newton’s law of gravitation resulting from Debye entropic force. Let us study two different limits of Eq. (11). In the strong gravitational field limit, i.e. at high temperature, \( T \gg T_D \) \( (x \ll 1) \), the Debye function reduces to

\[ D(x) \approx 1. \] (14)

As a result in this limit, the usual Newtonian gravity is restored. In the weak gravitational field limit, i.e. at very low temperature, \( T \ll T_D \) \( (x \gg 1) \) we have

\[ D(x) = \frac{\pi^4}{5x^3} = \frac{\pi^4}{5} \left( \frac{g}{g_D} \right)^3 \] (15)

In this limit, the Newton’s law is modified as

\[ F = \frac{5GMm}{R^2} \frac{g_D^3}{\pi^4 g^3}, \] (16)
while the gravitational acceleration becomes

\[ g = \left( \frac{5GMg_0^3}{\pi^4} \right) \frac{1}{\sqrt{R}} \Rightarrow g \propto \frac{1}{\sqrt{R}} \quad (17) \]

Therefore in this limit the gravitational field differs from Newtonian gravity. Let us then consider the cosmological implications of the presented model. We assume the background spacetime is spatially homogeneous and isotropic which is described by the line element

\[ ds^2 = h_{\mu\nu}dx^\mu dx^\nu + R^2(d\theta^2 + \sin^2 \theta d\phi^2), \quad (18) \]

where \( R = a(t)r, \) \( x^0 = t, \) \( x^1 = r, \) the two dimensional metric \( h_{\mu\nu} = \text{diag} \left( -1, a^2/(1-kr^2) \right). \) Here \( k \) denotes the curvature of space with \( k = 0, 1, -1 \) corresponding to flat, closed, and open universes, respectively. The dynamical apparent horizon, a marginally trapped surface with vanishing expansion, is determined by the relation \( h^{\mu\nu} \partial_\mu R \partial_\nu R = 0. \) A simple calculation gives the apparent horizon radius for the Friedman-Robertson-Walker (FRW) universe

\[ R = ar = \frac{1}{\sqrt{H^2 + k/a^2}} \quad (19) \]

The matter source in the FRW universe is assumed as a perfect fluid with stress-energy tensor

\[ T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}. \quad (20) \]

Now we are in a position to derive the dynamical equation for Newtonian cosmology. Consider a compact spatial region \( V \) with a compact boundary \( \mathcal{S} \), which is a sphere with physical radius \( R = a(t)r. \) Note that here \( r \) is a dimensionless quantity which remains constant for any cosmological object partaking in free cosmic expansion. Combining the second law of Newton for the test particle \( m \) near the surface, with gravitational force (11) we get

\[ m\ddot{R} = m\ddot{ar} = -GMmR^2 \left( \frac{1}{D(x)} \right). \quad (21) \]

We also assume \( \rho = M/V \) is the energy density of the matter inside the the volume \( V = \frac{4}{3}\pi a^3 r^3. \) Thus, Eq. (21) can be rewritten as

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \rho \frac{1}{\sqrt{D(x)}} \quad (22) \]

This is the dynamical equation for Newtonian cosmology which is valid in all range of the temperature. For strong gravitational field \( (D(x) \approx 1) \) we reach the well-known formula

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \rho. \quad (23) \]

Next we want to derive the Friedmann equations of FRW universe. For this purpose we need to employ the concept of the active gravitational mass \( \mathcal{M} \) \cite{32}, since this quantity produces the acceleration in general relativity. From Eq. (22) with replacing \( M \) with \( \mathcal{M} \) we have

\[ \mathcal{M} = -\frac{\ddot{a}a^3r^3}{G} \quad (24) \]

On the other side, the active gravitational mass is defined as \cite{20}

\[ \mathcal{M} = 2 \int_V dV \left( T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu} \right) u^\mu u^\nu. \quad (25) \]

A simple calculation gives

\[ \mathcal{M} = (\rho + 3p) \frac{4\pi G}{3} a^3 r^3. \quad (26) \]

Equating Eqs. (24) and (26) we obtain

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p) \frac{1}{\sqrt{D(x)}}. \quad (27) \]
This is the modified acceleration equation for the dynamical evolution of the FRW universe. Multiplying $\ddot{a}a$ on both sides of Eq. (27), and using the continuity equation

$$\dot{\rho} + 3H(\rho + p) = 0,$$  \hspace{1cm} (28)

after integrating we find

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3a^2} \int \frac{d(\rho a^2)}{D(x)}.$$  \hspace{1cm} (29)

This is the first Friedmann equation resulting from Debye entropic force. Eqs (29) and (28) together with the equation of state $p = w\rho$ govern the evolution of the universe. It is important to note that since $x = x(T)$ and the temperature is also a function of scale factor namely, $T = T(a)$, thus in general we cannot integrate Eq. (29) and derive the simplified result. When $x \ll 1 \Rightarrow D(x) \approx 1$, the well-known Friedmann equation in standard cosmology is recovered. For $x \gg 1$, using Eq. (15) we find

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3a^2} \left( \frac{\rho}{\pi^4} \right)^2.$$  \hspace{1cm} (30)

In order to derive the second Friedmann equation (29), we have to combine the first Friedmann equation with continuity equation (28). Let us put $k = 0$ for simplicity, which has been confirmed by recent observations. Differentiating Eq. (29) we find

$$2H \dot{H} = -\frac{8\pi G}{3} \left[ -\frac{2}{a^2} \frac{d}{a} \int d(\rho a^2) \frac{1}{D(x)} + \frac{1}{D(x)} \right].$$  \hspace{1cm} (31)

Multiplying $\frac{a}{H}$ on both sides of Eq. (31) and dividing by $dt$, we find

$$3H \dot{H} = -\frac{8\pi G}{a^2} \int d(\rho a^2) \frac{1}{D(x)} + 4\pi G \rho \frac{1}{D(x)} + 8\pi G \rho \frac{\dot{a}}{a} \frac{1}{D(x)}.$$  \hspace{1cm} (32)

Using the continuity equation (28), the above equation can be written as

$$- \left[ \dot{H}D(x) + \frac{8\pi G}{3a^2} D(x) \int d(\rho a^2) \frac{1}{D(x)} \right] = 4\pi G (\rho + p).$$  \hspace{1cm} (33)

For $x \ll 1$ we have $D(x) \approx 1$, and the well-known second Friedmann equation of FRW universe in flat spacetime is recovered, namely

$$- \dot{H} = 4\pi G (\rho + p).$$  \hspace{1cm} (34)

It is worth noting that the Friedmann equation in Debye entropic force scenario was first studied in [33]. Let us stress the difference between our derivation in this section and that of [33]. The author of [33] has derived Friedmann equations, following the method of [34], by applying the equipartition law of energy, $E = NT/2$, to the apparent horizon of a FRW universe with the assumption that the apparent horizon has the temperature $T = h/(2\pi R)$, where $R$ is the apparent horizon radius. Thus the total energy change of the system is obtained as [34]

$$dE = \frac{1}{2} NdT + \frac{1}{2} T dN = \frac{dR}{G},$$  \hspace{1cm} (35)

during the infinitesimal time interval $dt$, where the apparent horizon radius evolves from $R$ to $R + dR$. Indeed, the above equation is just the first law of thermodynamics in the form $dE = TdS$ on the apparent horizon, where $T = h/(2\pi R)$ and $S = A/(4hG)$ is the entropy of the system which assumed to obey the area-law and $A = 4\pi R^2$ is the apparent horizon area. While in the present work we have not employed the first law of thermodynamics for deriving the modified Friedmann equations. Therefore, our result is independent of the definition of the temperature in a dynamical spacetime.
III. ENTROPIC CORRECTED FRIEDMANN EQUATION IN DEBYE ENTROPIC GRAVITY

In this section we would like to consider the effects of the quantum correction terms to the entropy expression, on the laws of gravity in Debye model of entropic gravity. The result we will obtain are valid in all range of temperature. The correction terms to the entropy expression originate from the loop quantum gravity (LQG). The quantum corrections provided to the entropy-area relationship leads to the curvature correction in the Einstein-Hilbert action and vice versa [35, 36]. In the presence of quantum corrections the entropy takes the following form [37]

\[ S = \frac{A}{4\ell_p^2} - \beta \ln \frac{A}{4\ell_p^2} + \frac{\ell_p^2}{\pi^2} A + \text{const}, \]  

(36)

where \( \beta \) and \( \gamma \) are dimensionless constants of order unity. These corrections arise in the black hole entropy in LQG due to thermal equilibrium fluctuations and quantum fluctuations [38]. We will show that these corrections modify the Newton’s law of gravitation as well as the Friedmann equations. First of all we rewrite Eq. (36) in the following form

\[ S = \frac{A}{4\ell_p^2} + s(A), \]  

(37)

where \( s(A) \) represents the correction terms in the entropy expression. In this case the entropy change is obtained as

\[ \Delta S = \frac{\partial S}{\partial A} \Delta A = \left( \frac{1}{4\ell_p^2} + \frac{\partial s(A)}{\partial A} \right) \Delta A. \]  

(38)

Substituting Eqs. (9) and (38) in Eq. (1) and using relations \( \Delta x = -\frac{\hbar}{m c} \) and \( \Delta A = Q \), one can easily find

\[ F = -GMm \frac{1}{R^2 D(x)} \left( \frac{Q^2c^3}{2\pi k_B R} \right) \left[ 1 + \frac{\partial s(A)}{\partial A} \right] A = 4\pi R^2. \]  

(39)

If we define \( Q^2 \equiv 8\pi k_B \ell_p^4 \), as before, we immediately derive the modified Newton’s law of gravity in Debye entropic gravity

\[ F = -GMm \frac{1}{R^2} \left[ 1 - \frac{\beta \ell_p^2}{\pi R^2} - \frac{\gamma \ell_p^4}{4\pi^2 R^4} \right]. \]  

(40)

In the absence of correction terms (\( \beta = \gamma = 0 \)), the above equation reduces to the result of the previous section. Let us study two different limit of the above equation. In the strong gravitational limit, i.e. at high temperature, \( T_D \ll T \) \( (D(x) \approx 1) \) we have

\[ F = -GMm \frac{1}{R^2} \left[ 1 - \frac{\beta \ell_p^2}{\pi R^2} - \frac{\gamma \ell_p^4}{4\pi^2 R^4} \right], \]  

(41)

which is exactly the result obtained in [26]. When \( \beta = \gamma = 0 \), one recovers the well-known Newton’s law. On the other hand, at very low temperature \( T_D \gg T \) we have \( D(x) = \frac{\hbar^2}{m c^2} \) and Eq. (10) reduces to

\[ F = -GMm \frac{5}{R^2 \pi^4} \frac{g_D^2}{g^2} \left[ 1 - \frac{\beta \ell_p^2}{\pi R^2} - \frac{\gamma \ell_p^4}{4\pi^2 R^4} \right]. \]  

(42)

To derive Friedmann equation we follow the method of the previous section. Combining the second law of Newton for the test particle \( m \) near the screen with gravitational force (42) we obtain

\[ F = m\ddot{R} = m\ddot{a} = -\frac{MmG}{a^2 r^2} \frac{1}{D(x)} \left[ 1 - \frac{\beta \ell_p^2}{\pi R^2} - \frac{\gamma \ell_p^4}{4\pi^2 R^4} \right]. \]  

(43)

which from it we can derive the acceleration equation

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \rho \frac{1}{D(x)} \left[ 1 - \frac{\beta \ell_p^2}{\pi R^2} - \frac{\gamma \ell_p^4}{4\pi^2 R^4} \right]. \]  

(44)
With the entropic corrections terms, the active gravitational mass $\mathcal{M}$ will be modified accordingly. The active gravitational mass $\mathcal{M}$ in this case is obtained as

$$\mathcal{M} = -\frac{\ddot{a}a^2}{G} r^3 D(x) \left[ 1 - \frac{\beta}{\pi} \frac{\ell_p^2}{R^2} - \frac{\gamma}{4\pi^2} \frac{\ell_p^4}{R^4} \right]^{-1}. \quad (45)$$

Equating the above equation with Eq. (26) yields

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p) \frac{1}{D(x)} \left[ 1 - \frac{\beta}{\pi} \frac{\ell_p^2}{R^2} - \frac{\gamma}{4\pi^2} \frac{\ell_p^4}{R^4} \right]. \quad (46)$$

Next we multiply the both sides of the above equation by $\ddot{a}a$, after using the continuity equation (28) and integrating we find

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3a^2} \left[ \int \frac{d(\rho a^3)}{D(x)} - \frac{\beta}{\pi} \frac{\ell_p^2}{a^2} \int \frac{d(\rho a^2)}{D(x)} - \frac{\gamma}{4\pi^2} \frac{\ell_p^4}{a^4} \int \frac{d(\rho a^3)}{D(x)} \right]. \quad (47)$$

Where $k$ is an integration constant. Unfortunately, the above equation cannot be integrated in general for an arbitrary $D(x)$. In the limiting case $D(x) \approx 1$, the integrations can be done following the method developed in [26]. We find (see [26] for details)

$$ \left( H^2 + \frac{k}{a^2} \right) + \frac{\beta \ell_p^2 (1 + 3\omega)}{3\pi (1 + \omega)} \left( H^2 + \frac{k}{a^2} \right)^2 + \frac{\gamma \ell_p^4 (1 + 3\omega)}{4\pi^2 (5 + 3\omega)} \left( H^2 + \frac{k}{a^2} \right)^3 = \frac{8\pi G}{3} \rho. \quad (48)$$

Again we see that in the absence of correction terms ($\beta = 0, \gamma = 0$) the well-known Friedmann equation is recovered. For $x \gg 1$ ($D(x) = \frac{a^3}{\pi x^3}$) Eq. (47) can be written

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \left[ \frac{1}{a^2} \int \frac{d(\rho a^3)}{a^3} \right] \left[ \frac{5x^3}{\pi^4} \right] - \frac{\beta}{\pi} \frac{\ell_p^2}{a^2} \frac{5x^3}{\pi^4} - \frac{\gamma}{4\pi^2} \frac{\ell_p^4}{a^4} \frac{5x^3}{\pi^4} \right]. \quad (49)$$

IV. CONCLUSION

Verlinde proposal on the entropic origin of the gravity is based strongly on the assumption that the equipartition law of energy holds on the holographic screen induced by the mass distribution of the system. However, from the theory of statistical mechanics we know that the equipartition law of energy does not hold in the limit of very low temperature. By low temperature, we mean that the temperature of the system is much smaller than Debye temperature, i.e. $T \ll T_D$. It was demonstrated that the Debye model is very successful in interpreting the physics at the very low temperature. Since the discovery of black holes thermodynamics, physicist have been thought that the gravitational systems such as black hole and our universe can also be regarded as a thermodynamical system. Hence, it is expected that the equipartition law of energy for the gravitational system should be modified in the limit of very low temperature (or very weak gravitational field).

In this paper inspired by the Verlinde proposal and following the Debye model of equipartition law of energy in statistical thermodynamics, we modified the entropic gravity. First, we studied the Debye entropic gravity and derived the modified Newton’s law of gravitation and the corresponding Friedmann equations which are valid in all range of temperature. We found that the modified entropic force returns to the Newton’s law of gravitation while the temperature of the holographic screen is much higher than the Debye temperature. Then we extended our study to the case where there are correction terms such as logarithmic correction in the entropy expression. In this case we again reproduced the gravitational equations for all range of temperature. Our study shows a deep connection between Debye entropic gravity and modified Friedmann equation. The microscopic statistical thermodynamical model of spacetime may shed light on the origin of the Debye entropic gravity and the microscopic origin for the Newton’s law of gravity and also Friedmann equations in cosmology.
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