Phase-field model for erosion processes

Carla Henning\textsuperscript{1,*}, Maximilian Brodbeck\textsuperscript{1}, Christian Koch\textsuperscript{2}, Stephan Staudacher\textsuperscript{2}, and Tim Ricken\textsuperscript{1}

\textsuperscript{1} Institute of Mechanics, Structural Analysis and Dynamics, Faculty of Aerospace Engineering and Geodesy, University of Stuttgart, 70569 Stuttgart, Germany
\textsuperscript{2} Institute of Aircraft Propulsion Systems, Faculty of Aerospace Engineering and Geodesy, University of Stuttgart, 70569 Stuttgart, Germany

Solid particle erosion is one of the main damage mechanisms in high-pressure compressors of jet engines. The significant change in the compressor blades leads to performance degradation over lifetime. To enhance predictive capabilities of erosive wear measurements under conditions, related to high pressure compressors, were performed. Even if there is no general model to describe erosive wear, it is understood that the transfer of kinetic energy from the impacting particle into the material is a central aspect in modelling erosion.

In order to predict erosive wear of general geometries, Schrade et al. \cite{9} developed a numerical model, based mainly on experimentally determined erosion rates. The shortage of such modelling approaches is the limited applicability in situations referring to the measurement conditions. In the following, we will describe a physically based damage model, closing the gap between measurement and calculating shape changes of general specimens. Based on the phase-field approach for predicting crack evolution presented by Miehe et al. \cite{6}, an overall energy balance of the considered specimen is constructed. A dissipative portion describes the influence of propagating damage, while special boundary conditions provides information about the energy transfer between impinging particles and specimen.

1 Introduction

1.1 Motivation

The abrasive wear process of surfaces caused by the repeated impact of small but fast-moving solid particles entrained in a surrounding fluid is called solid particle erosion \cite{2}. This process is one of the most significant damage mechanisms in high-pressure compressors of jet engines leading to performance degradation with time \cite{11}, cf. Figure 1a. The complex process of solid particle erosion involving particle transport, impact and material abrasion can be characterised using a jet-blast test rig. Eroding flat plates at engine-representative test conditions delivers the erosion rates \( \varepsilon = \Delta m / m_P \) as an integral parameter. This parameter is used to derive the amount of material abrasion \( \Delta m \) as a function of the amount of particles \( m_P \) impinged on a given surface. Schrade et al. \cite{9} developed a numerical approach to compute the erosive change of shape of a component by discretising its geometry with flat surface elements. A cross section of such a discretised component is depicted in Figure 1b. The erosion rate of a particular surface element is determined as a function of the particle impingement conditions. The numerical simulation is carried out in small time steps \( \Delta t \) to reflect the change of particle impingement conditions due to the continuous change of shape.

Fig. 1: Eroded blade from high-pressure compressor \cite{3} (a) and a schematic representation of one calculation step within the numerical model from \cite{9} (b)
In the following, we will describe a model to predict the influence of erosive wear in a more physical-based way. Recent works of MIEHE ET AL. [6, 7] on phase-field models show good results for predicting crack propagation in two- and three-dimensional domains. In this approach, the state of the material between being intact and broken is described through a scalar valued diffuse quantity between zero and one. The overall energy balance of the solid contains a dissipative part, which is linked to the propagation of the damage process. Different damage mechanisms can be considered through different model approaches for the dissipation, especially whether the damage is caused by brittle or ductile mechanisms. In the present case, special boundary conditions have to be applied, which provide information about the energetic interaction of impinging particles and the growth of the variables describing the damaging process in the considered domain.

### 1.2 Relevant influencing factors for erosion

The erosion rate derived from jet blast test rigs and flat plate specimen are heavily influenced by the particular test setup. As a consequence, a direct application in numerical methods such as SCHRADE ET AL. [9] is only justified within the given boundary conditions. This limits the application of the vast amount of test data taken at different combinations of blade and particle materials, particle velocities, sizes and shapes as well as impact angles and surface roughness [1, 8, 9, 12]. This issue can be overcome by the application of non-dimensional parameters describing particle properties and impact conditions directly at the surface [4, 5]. The transfer of the tribological conditions from the flat plate experiment to the local tribology at a surface element of the numerical simulation is still left as a challenge. Nevertheless, it is understood that the kinetic energy transferred into the material by the impacting particles is a main driver of the local loss of material. Hence, thermodynamically consistent phase-field models offer an opportunity to provide a consistent model ranging from flat plate experiment to numerical computation of the change of shape.

### 2 Underlying physical model

#### 2.1 Governing equations

In the following, the governing equations for fracture modeling in linear, elastic, brittle solids are presented. In the approach developed by MIEHE ET AL. [6], the argument of virtual power is used to derive the macroscopic mass and momentum balance equation coupled with a phase-field, which captures the evolution of micro-structural damage. A quasi-static process as well as small deformations are assumed. The sum of the temporal change of the Helmholtz free energy \(\psi(\nabla_{\text{sym}}(\mathbf{u}))\), denoted by \(\dot{E}\) and the dissipated energy \(D\), leads to the power due to external forces \(\dot{P}\) with

\[
\dot{E} + D = \dot{P}.
\]

The dissipation \(D\) – the volume integral of a density-functional \(\phi\) – describes the contribution of the propagating crack as shown in section 2.2. Evaluating the equation (1) yields

\[
\nabla \cdot \mathbf{P} + \rho_0 \mathbf{g} = 0
\]

\[
\nabla \cdot [\partial_{\mathbf{C}} \phi] - [\partial_d \phi + \partial_d \phi] = 0,
\]

wherein \(\mathbf{P}\) denotes the first Piola-Kirchhoff stress tensor, \(\rho_0\) is the density in reference configuration, \(\mathbf{g}\) denotes the volume force, and the index \(d\) denotes the phase field parameter, cf. Section 2.2. The GALERKIN-type weak forms of the above-mentioned balance equations are solved numerically, using the finite element method (FEM).

#### 2.2 A phase-field approach for fracture modeling

Crack evolution is described using a phase-field \(d \in [0, 1]\). This scalar variable states whether the material is intact \((d = 0)\) or fully cracked \((d = 1)\). MIEHE ET AL. [6] motivated their approach in a fully geometric frame. For an infinite bar, cracked at \(x = 0\), the phase-field holds

\[
d(x) = \begin{cases}
1 & x = 0 \\
0 & \text{otherwise}.
\end{cases}
\]

To avoid sharp crack topologies, the non-smooth phase-field (3) is regularized

\[
d(x) = \exp(-|x|/\ell),
\]

using a length-scale parameter \(\ell \in \mathbb{R}^+\). Obviously, the regularized phase-field is a solution of the differential equation

\[
d(x) - \ell^2 d'' = 0,
\]
the EULER equation, related to the minimization of the crack surface. Generalizing this result onto multiple dimensions, the overall crack-surface $\Gamma_{\ell}(d)$ of a domain $\Omega$ with its boundary $\partial\Omega$ can be calculated using a volumetric crack density $\gamma(d)$

$$\Gamma_{\ell}(d) = \int_{\Omega} \gamma(d) \, dV \bigg| \gamma(d) = \frac{1}{2\ell} (d^2 + \ell^2 |\nabla d|) .$$

(6)

With these results at hand, it is possible to calculate the dissipation, stated in section 2.1. Defining $g_c$, a material constant related to Griffiths fracture energy, dissipation can be modelled based on the temporal change of the crack-surface $\Gamma_{\ell}$

$$D = \int_{\Omega} \phi \, dV \bigg| \phi = g_c \dot{\gamma} + I_+(d) .$$

(7)

Due to the irreversibility of crack propagation $D, \dot{d} > 0$ must apply. Therefore, a penalization term $I_+$ is embedded into the model. In addition to the dissipation function, the phase-field affects the energy storage in the bulk. If the solid is damaged, energy degradation takes place, compared to the unbroken state. Assuming that fracture occurs in tension as well as in compression, the Helmholtz free energy $\psi(\nabla_{\text{sym}} u)$ can be calculated from the energy stored in an undamaged solid $\psi_{ud}(\nabla_{\text{sym}} u)$ through

$$\psi(\nabla_{\text{sym}} u) = [(1 - d)^2 + k] \psi_{ud}(\nabla_{\text{sym}} u) .$$

(8)

The scalar value $k$ defines a small positive parameter to keep the numerical solution well-posed. Due to the relations between the free energy in damaged and undamaged solids (cf. equation (8)), the first Piola-Kirchhoff stress tensor $P$ can be obtained, using the undamaged stress tensor $P_{ud}$ with

$$P = [(1 - d)^2 + k] P_{ud} .$$

(9)

### 3 Model enhancements for erosion modelling

Using the equations presented above, the evolution of cracks due to external forces or prescribed displacements can be modelled. During this process, bulk free energy is dissipated over the crack-surfaces. As mentioned, erosion is affected by the stress state and external loads as well as surface penetration by particles. Assuming a case setup under test-rig conditions while measuring erosion-rates (cf. Hufnagel et al. [5]), the specimen is not affected by external forces. To describe the influence of the impacting particles, an additional boundary condition is required. Thereby, the impact of particles will be modelled as a diffusive energy-flux $f_d$ over the surface, using a gradient-diffusion hypothesis with a tensorial diffusion coefficient $D_{\text{Particle}}$

$$f_d = D_{\text{Particle}} \nabla d .$$

(10)

Using the boundary normal vector $n$, the energy transport through the surface can be calculated from the energy-flux $f_d$. According to Miehe et al. [6], the gradient of the state variable $\nabla d$ is assumed to be zero on the boundary. Thus, the definition of the crack-surface (cf. equation (6)) has to be modified

$$\dot{\Gamma}_{\ell} = \int_{\Omega} \frac{1}{\ell} [d - \ell^2 \Delta d] \cdot \dot{d} \, dV + \int_{\partial\Omega} \ell(\nabla d \cdot n) \cdot \dot{d} \, dA .$$

(11)

In addition, the new boundary condition has to be taken into account, defining the external power $\dot{P}$ in equation (1). Considering the presented changes, one of the major issues applying the phase-field approach to describe erosion will be the evaluation of the diffusion coefficient $D_{\text{Particle}}$. Due to its tensorial form, influences of local impacting angle and erosion mechanism, among others determined by the material type (ductile or brittle), can be considered.

### 4 Conclusion and further work

The tribology of erosive wear still lacks a description, which coherently builds upon the parameters delivered by erosion experiments or numerical particle impact simulations. This lack is overcome by superimposing generalized models of impact wear and cutting wear for form a particular erosion model. Consequently, such erosion models only are valid for a particular combination of particle and substrate material (cf. fig. 2). Whilst being successful, this semi-empirical approach does not necessarily reflect the underlying physics of material wear. As a consequence, the transfer of specific results to applications in other parameter domains is questionable. To overcome this, we outlined a physical based phase-field approach closing the gap actual models leave between measurement data and predicting erosive wear in general. Based on phase-field models for fracture, crack propagation can be considered, without treatment of discontinuities, due to the regularized crack topology. Applying this approach modelling erosive wear, a surface effect, can be modelled in a continuum-mechanical sense.
Implementing the outlined model approach, the following aspects will need special attention. With regard to numerics, it must be ensured that quadratic convergence within the Newton-Raphson scheme is guaranteed. For this purpose it is necessary to implement an algorithmically consistent incremental tangent module. This means for the current time increment the partial derivative of the incremental constitutive stress function with respect to the unknown strain under consideration of fixed density of the previous incremental time step, cf. Souza Neto et al. [10]. As already mentioned, one key aspect is the kinetic energy transfer from impinging particles into the domain. Main influence parameters such as impacting angle, material properties (brittle or ductile behavior) as well as material combinations or surface roughness have to be considered, determine the proposed diffusion coefficient. Due to the high sensitivity of erosion whether materials are brittle or ductile, it seems appropriate to consider the typical damaging mechanisms of those materials for the propagating phase-field. The equations presented so far were developed for brittle materials, but in their later work Miehe et al. [7] suggest modifications of the dissipation function for ductile fracture processes. In addition to those more physically motivated aspects, one detail of the numerical implementation should be discussed. With propagation of erosive wear, the real surface as well as the boundary conditions change. Taking this into account, fully damaged cells could be taken out of the domain, and boundary conditions for the actual surface can be calculated. The continuous redefinition of the problem setting leads, in the context of parallel computation, to performance penalties and should be done as rarely as possible.

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