Dynamic Ensemble Size Adjustment for Memory Constrained Mondrian Forest

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Abstract—Supervised learning algorithms generally assume the availability of enough memory to store data models during the training and test phases. However, this assumption is unrealistic when data comes in the form of infinite data streams, or when learning algorithms are deployed on devices with reduced amounts of memory. Such memory constraints impact the model behavior and assumptions. In this paper, we show that under memory constraints, increasing the size of a tree-based ensemble classifier can worsen its performance. In particular, we experimentally show the existence of an optimal ensemble size for a memory-bounded Mondrian forest on data streams and we design an algorithm to guide the forest toward that optimal number by using an estimation of overfitting. We tested different variations for this algorithm on a variety of real and simulated datasets, and we conclude that our method can achieve up to 95% of the performance of an optimally-sized Mondrian forest for stable datasets, and can even outperform it for datasets with concept drifts. All our methods are implemented in the OrpailleCC open-source library and are ready to be used on embedded systems and connected objects.

Index Terms—Mondrian Forest, Trimming, Concept Drift, Data Stream, Memory Constraints.

I. INTRODUCTION

Supervised classification algorithms mostly assume the availability of abundant memory to store data and models. This is an issue when processing data streams — which are infinite sequences by definition — or when using memory-limited devices as is commonly the case in the Internet of Things. We focus on the Mondrian forest, a popular online classification method. Our ultimate goal is to optimize it for data streams under memory constraints to make it compatible with connected objects.

The Mondrian forest is a tree-based, ensemble, online learning method with comparable performance to offline Random Forest [1]. Previous experiments highlighted the Mondrian forest sensitivity to the ensemble size in a memory-constrained environment [2]. Indeed, introducing a memory limit to the ensemble also introduces a trade-off between underfitting and overfitting. On the one hand, low tree numbers make room for deeper trees and increase the risk of overfitting. On the other hand, large tree numbers constrain tree depth due to the memory limitation and increase the risk of underfitting. Depending on the memory limit and the data distribution, a given ensemble size may either overfit or underfit the dataset. The goal of this paper is to address this trade-off by adapting the ensemble size dynamically.

In summary, this paper makes the following contributions:

1) Highlight the existence of an optimal tree count in memory-constrained Mondrian forest;
2) Propose a dynamic method to optimize the tree count;
3) Compare this method to the Mondrian forest with a optimally-sized tree count.

II. MATERIALS AND METHODS

All the methods presented in this section are implemented in the OrpailleCC framework [3]. The scripts to reproduce our experiments are available on GitHub at https://github.com/big-data-lab-team/benchmark-har-data-stream.

In this section, we start by presenting background on Mondrian Forests (II-A and II-B), then presents the main contribution of the paper, namely dynamically adjusting the ensemble size of a memory-constrained Mondrian forest (II-C, II-D, II-E, II-F), then describes the experimental evaluation framework (II-G, II-H).

A. Mondrian Forest

The Mondrian forest [1] is an ensemble method that aggregates Mondrian trees. Each tree recursively splits the feature space, similar to a regular decision tree. However, the feature used in the split and the value of the split are picked randomly. The probability to select a feature is proportional to its range, and the value for the split is uniformly selected in the range of the feature. In contrast with other decision trees, the Mondrian tree does not split leaves to introduce new nodes. Instead, it introduces a new parent and a sibling to the node where the split occurs. The original node and its descendant are not modified and no data point is moved to that new sibling besides the data points that initialized the split. This approach allows the Mondrian tree to introduce new branches to internal nodes. This training algorithm does not rely on labels to build the tree, however, each node maintains counters for each label seen. Therefore, labels can be delayed, but are needed before the prediction. In addition to the counters, each node keeps track of the range of its feature which represents a box containing all data points. A data point can create a new branch only if it is sorted to a node and falls outside of the node’s box.

B. Mondrian Forest for Data Stream Classification

The implementation of Mondrian forest presented in [1, 4] is online because trees rely on potentially all the previously seen data points to grow new branches. To support data

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streams, the Mondrian forest has to access data points only once as the dataset is assumed to be infinite in size.

The work in [2] describes a Data Stream Mondrian forest with a memory bound. The ensemble grows trees from a shared pool of nodes and the trees are paused when there is no node left. This work also proposed out-memory strategies to keep updating the statistics for the trees without creating new branches. In particular, this work recommend using the Extend Node strategy when the memory is full, a strategy where statistics of the node boxes are automatically extended to fit all data points, and the counters automatically increased.

The attributes of a node are an array for counting labels that fall inside a leaf, and two arrays lower_bound and upper_bound that define a box of the node. The Extend Node strategy automatically increases the counter of the label in the leaf and automatically adjusts lower_bound and upper_bound so the new data point fits inside the box.

Having a shared pool of nodes for the ensemble has a direct impact on the number of trees. As mentioned before, having more trees limits the tree depth and may lead to underfitting, whereas having less trees increases the risk of overfitting.

C. Dynamic Tree Count Optimization

Algorithm 1 describes the function that trains the forest with a new data point and dynamically adjusts the ensemble size. The main idea is to compare pre- and post-quential errors to decide whether or not to adjust the forest size.

Algorithm 1: Training function for a data stream Mondrian forest with a dynamic ensemble size.

Data: f = a Mondrian forest
Data: x = a data point
Data: l = the label of data point x

Function train_forest(f, x, l) is
1. predicted_label = f.test(x);
2. prequential.update(predicted_label, l);
3. for i do
   a. train_tree(i, x, l);
   b. predicted_label = f.test(x);
   c. postquential.update(predicted_label, l);
   d. post_metric = postquential.metric();
   e. pre_metric = prequential.metric();
   f. if post_metric > pre_metric then
      a. trim_trees(f);
      b. add_tree(f);
4. end

D. Comparison Test

In Algorithm 1, the update process determines if it needs to add a tree based on a comparison between the prequential and the postquential accuracies. If both accuracies are significantly different, the forest is deemed to overfit and thus, the algorithm adds a tree to compensate.

There are different methods to test the statistical difference between two accuracies and we experiment with four in this paper: the sum of variances, the t-test, the z-test, and the sum of standard deviations.

Notations for the following equations include: µ_pre and µ_post the mean of respectively the prequential and postquential accuracies, σ²_pre and σ²_post the variance of respectively the prequential and postquential accuracies, n the size of the sample, µ and σ² respectively the mean and variance of µ_post − µ_pre.

1) Sum of Variances: Equation 1 shows the sum of variances as a comparison test. The two accuracies are different when the distance it is higher than the square root of the prequential and postquential variances.

\[
\mu_{post} - \mu_{pre} > \sqrt{\sigma^2_{post} + \sigma^2_{pre}}
\] (1)

2) T-test: Equation 2 describes the t-test used to compare the prequential and postquential accuracies. We apply a one-sample t-test where we check if µ is different from 0 with a 99% confidence [6].

\[
\frac{\mu}{\sigma} > 2.326
\] (2)

3) Z-test: Equation 3 shows how we computed the two proportion z-test pooled for µ_pre equal µ_post. We first compute the z-score, then we compare it with the Z-value that ensures confidence of 99% [7], [8]. The z_score is the observed difference (a) divided by the standard error of the difference (b) pooled from the two samples (p).
is sufficient to compute the mean accuracy and its variance:

\[ A = \text{faded accuracy of the prediction} \]

\[ P = \text{points} \in \mathbb{N}, \text{the faded count of data points} \]

\[ n = 1 \] 

then only the last point is taken into account. 

\[ f = 0.995 \] 

In this experiment, we use a fading factor of 0.995.

\[ \mu = \mu_{\text{pre}} - \mu_{\text{post}} \]
\[ p = \frac{\mu_{\text{pre}} + \mu_{\text{post}}}{2} \]
\[ b = \sqrt{\frac{2p(1-p)}{n}} \]
\[ z_{\text{score}} = \frac{a}{b} \]
\[ z_{\text{score}} > 2.576 \]

\[ \mu_{\text{post}} - \mu_{\text{pre}} > \sigma_{\text{post}} + \sigma_{\text{pre}} \tag{4} \]

\[ 4) \text{Sum of Standard Deviations: Equation 4 shows the use of the sum of standard deviations as comparison test. The difference between posterior and prequential is significant when that difference is higher than the sum of standard deviations.} \]

\[ a = \mu_{\text{pre}} - \mu_{\text{post}} \]
\[ p = \frac{\mu_{\text{pre}} + \mu_{\text{post}}}{2} \]
\[ b = \sqrt{\frac{2p(1-p)}{n}} \]
\[ z_{\text{score}} = \frac{a}{b} \]
\[ z_{\text{score}} > 2.576 \]

\[ E. \text{ Pre- and Postquential Statistics Computation} \]

In Algorithm 1 we mention that the accuracy and its variance are evaluated both prequentially and posteriorly. However, only the most recent data points are relevant for these statistics. We compared two ways of computing the mean (\( \mu \)) and variance (\( \sigma^2 \)): sliding and fading.

The sliding version uses a sliding window to store the statistics. Let \( P_i \in \{0, 1\} \) be the correctness of the prediction for data point \( i \). The values of \( P_i \) are stored in a binary sliding window \( W \) of size \( W_{\text{size}} \). \( W \) is updated with the most recent \( P_i \) and the mean and variance of the accuracy are computed as follows:

\[ \mu = \frac{\sum_{i\in W} P_i}{W_{\text{size}}} \]
\[ \sigma^2 = \mu(1 - \mu) \tag{5} \]

This expression of \( \sigma^2 \) comes from the fact that \( P_i \) is a binary variable.

The sliding version increases memory consumption because it needs to keep \( W \) in memory. The fading version addresses this downside of the sliding version. To reduce memory usage, the fading version relies on a fading factor [5]. The sum maintained to compute the accuracy and the variance are faded. Which mean these sums are multiplied by a fading factor \( f \in [0, 1] \) before being updated. It gives a weight to all elements with older elements having a smaller weight. If \( f = 1 \) then the sum is computed for the entire stream. If \( f = 0 \) then only the last point is taken into account.

To compute the fading statistics, we need the count of data points \( n \), the faded count of data points \( N = \sum_{i=1}^{n} f^{n-i} \), and the faded accuracy of the prediction \( A_n = \sum_{i=1}^{n} f^{n-i} P_i \). This is sufficient to compute the mean accuracy and its variance:

\[ \mu = \frac{A_n}{N} \]
\[ \sigma^2 = \mu(1 - \mu) \tag{6} \]

In this experiment, we use a fading factor of 0.995.

\[ F. \text{ Tree Addition Method} \]

In the Data Stream Mondrian forest, adding a tree simply implants a root in the node pool. The main issue in adding trees revolves around the number of nodes available for that tree to grow. Indeed, if the number of nodes available is too small, the tree won’t grow much and it will underfit the data.

Therefore, to make space for new trees, we need to trim the leaves of existing trees. The trimming phase ensures that every tree has a similar size while accommodating enough memory space for the new tree to grow. We test three approaches for trimming trees: Add random, Add depth, and Add count. The Add random approach randomly selects the leaves to remove. The Add depth approach removes the deepest leaves first. The Add count approach focuses on the leaves that contain the least amount of data points.

\[ G. \text{ Datasets} \]

We used six datasets to evaluate our proposed methods: three synthetic datasets to mimic real-world situations and to make comparisons with and without concept drifts, and two real Human Activity Recognition datasets.

1) Banos et al: The Banos et al dataset [9] is a human activity dataset with 17 participants and 9 sensors per participant. Each sensor samples a 3D acceleration, gyroscope, and magnetic field, as well as the orientation in a quaternion format, producing a total of 13 values. Sensors are sampled at 50 Hz, and each sample is associated with one of 33 activities. In addition to the 33 activities, an extra activity labeled 0 indicates no specific activity.

We pre-processed the Banos et al dataset as in [9], using non-overlapping windows of one second (50 samples), and using only the 6 axes (acceleration and gyroscope) of the right forearm sensor. We computed the average and standard deviation over the window as features for each axis. We assigned the most frequent label to the window. The resulting data points were shuffled uniformly.

In addition, we constructed another dataset from Banos et al, in which we simulated a concept drift by shifting the activity labels in the second half of the data stream.

2) Recofit: The Recofit dataset [10], [11] is a human activity dataset containing 94 participants. Similar to the Banos et al dataset, the activity labeled 0 indicates no specific activity. Since many of these activities were similar, we merged some of them together based on the table in [12].

We pre-processed the dataset similarly to the Banos et al dataset, using non-overlapping windows of one second, and only using 6 axes (acceleration and gyroscope) from one sensor. From these 6 axes, we used the average and the standard deviation over the window as features. We assigned the most frequent label to the window.

3) MOA Datasets: We generated two synthetic datasets using Massive Online Analysis [13] (MOA) is a Java framework to compare data stream classifiers. In addition to classification algorithms, MOA provides many tools to read and generate datasets. We generate two synthetic datasets (MOA commands are available here).
available [here](#) using the RandomRBF algorithm, a stable dataset, and a dataset with a drift. Both datasets have 12 features and 33 labels, similar to the Banos *et al* dataset. We generated 20,000 data points for each of these synthetic datasets.

4) **Covtype**: The Covtype dataset\(^2\) is a tree dataset. Each data point is a tree described by 54 features including ten quantitative variables and 44 binary variables. The 581,012 data points are labeled with one of the seven forest cover types and these labels are highly imbalanced. In particular, two labels represent 85% of the dataset.

### H. Evaluation Metric

The models start without prior knowledge of the datasets. We evaluated our methods using a prequential fading macro F1-score. We focused on the F1 score because most datasets are imbalanced. We used the prequential evaluation because we process a data stream [5]. The prequential evaluation or *interleaved-test-then-train* evaluation is the most popular method to evaluate data stream models. It first tests the model with the data points, then trains the model with it. We used a fading factor to minimize the impact of old data points, especially data points at the beginning or data points seen before a drift occur. To obtain this fading F1 score, we multiplied the confusion matrix with the fading factor before incrementing the cell in the confusion matrix. The model is continuously evaluated throughout the data stream and we report only the final evaluation metric. The F1 scores are averaged across 20 repetitions.

### III. Results

In this section, we first highlight the existence of an optimal number of trees dependent on the data and the memory. Then we evaluate the performance of the tree-adding methods independently from the dynamic update process. Finally, we assess the complete dynamic update method and all its parameter.

#### A. Optimal Forest Size

Figure 1 shows the relation between the number of trees in the Mondrian forest and the F1 score. We notice that in most configurations, there is an optimal number of trees located between 1 and 15, except for 10MB and the datasets RBF stable, in which case the F1 score keeps increasing without reaching a maximum.

A particular situation occurs on the Covtype and the RecoFit datasets with 0.2MB and 0.6MB: the optimal ensemble size is one, which suggests that the memory limit is not high enough for a tree to overfit the datasets. Therefore, adding trees will always underfit.

We observe significant performance differences between the best-performing and least-performing number of trees, in particular for low memory amounts. Therefore, optimizing the number of trees is necessary to achieve the best performances.

\(^2\)available [here](#)
D. Comparison to Fixed Ensemble Size

We tested all possible combinations of:

- Tree addition methods (Section II-F)
- Pre- and Postquential statistics (Section II-E)
- Comparison Tests (Section II-D)

The tree addition includes Add random, Add depth, and Add count. The pre- and postquential statistics include fading and sliding. Finally, the comparison test contains the sum of standard deviation (sum-std), the sum of variance (sum-var), the t-test (t-test), and the z-test (z-test).

Figure 3 shows how the top combinations compare to Fixed for each dataset and memory limit. The score is relative to the F1 score of Fixed with the optimal number of trees.

We observe that for most datasets, at least one dynamic forest reaches the performance of the Fixed method. The only exception is the RBF drift dataset with 0.2MB where all the dynamic approaches are substantially under the performance obtained by the Fixed method.

For the drift datasets, some dynamic forests surpass the performance of the Fixed method. This is due to the introduction of new trees that are not influenced by older concepts, and thus are more accurate to the new data points.

Nevertheless, no single dynamic method consistently reaches the performance of Fixed. Indeed, the count fading t-test method (purple on Figure 3) reaches or surpasses the performance of the Fixed method for RBF stable and drift (except 0.2MB), Banos et al and Banos et al drift, however, it underperforms on the Recofit and Covtype datasets where the count fading sum-std (blue on Figure 3) and the random fading sum-std (orange on Figure 3) perform significantly better. Conversely, the depth fading sum-std method (brown on Figure 3) reaches the Fixed performance for Covtype, Recofit, and Banos et al datasets, but significantly underperforms on RBF stable and RBF drift.

We computed the average rank of the dynamic forests and reported the top 10 methods in Table I. The best average rank of 6.50 out of 24 indicates the lack of a clear winner among all combinations.

Moreover, we observed that varying the fading factor made some methods consistently approach the Fixed performance.
In particular, when reducing the fading factor, the methods using the z-test and the t-test approach fixed on all datasets. This is explained by the fact that the experimental conditions are closer to the assumptions made by the t-test and the z-test. Nevertheless, these fading factor explorations were not done with a proper cross-validation, and cannot be reported in this paper because it would require an analysis on independent datasets.

IV. RELATED WORK

The work in [14], [15] proposes methods to adjust ensemble size based on accuracy contribution of the sub-classifier. A sub-classifier that significantly decreases the accuracy of the ensemble should be removed, whereas the new sub-classifier built on the last chunk of data should be added if it significantly increases the ensemble accuracy. However, their hypotheses are not valid in our study, since growing a new tree influences the performance of existing trees. Indeed, the new tree requires nodes to grow and these nodes will be taken from existing trees.

The work in [16] explores an algorithm to adjust hyperparameters on data streams with concept drift. When a concept drift is detected, the algorithm makes a list of hyperparameters configurations to evaluate on the most recent data points. The best configuration provides the new hyper-parameters until the next concept drift. This method is not suited for our hypothesis since it assumes we have enough memory to store at least two models.

V. CONCLUSION

In this paper, we showed experimentally that a memory-bounded Mondrian forest has an optimal ensemble size that depends on the dataset and the memory limit. To find this optimal, we proposed a dynamic ensemble-sized Mondrian forest that estimates overfitting to drive the ensemble toward the optimal number of trees. The overfitting measure relies on the postquential accuracy, an innovative concept to estimate the training accuracy of a data stream classifier.

We introduced the use of fading factor to keep track of the mean and variance, needed for the comparison test. We tested our algorithm on six datasets with different combinations of comparison tests, different methods to add trees, and different ways of computing the mean and variance.

From this experiment, we observed that some of the proposed methods were able to reach the performance of a Mondrian forest with an optimal number of trees. However, none of the methods consistently achieve that optimal. In future work, we suggest investigating the role of the fading factor.

In addition, further investigations are required to design a functional tree removal method that resists two issues: detecting underfitting in the forest and continuing the growth of paused Mondrian trees.

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REFERENCES

[1] B. Lakshminarayanan, D. M. Roy, and Y. W. Teh, “Mondrian Forests: Efficient Online Random Forests,” in Advances in Neural Information Processing Systems 27, Z. Ghahramani, M. Welling, C. Cortes, N. D. Lawrence, and K. Q. Weinberger, Eds. Curran Associates, Inc., 2014.
[2] M. Khannouz and T. Glarat, “Mondrian Forest for Data Stream Classification Under Memory Constraints,” 2022.
[3] M. Khannouz, B. Li, and T. Glarat, “OrpailleCC: A Library for Data Stream Analysis on Embedded Systems,” The Journal of Open Source Software, vol. 4, p. 1485, 07 2019.
[4] Balaji. Lakshminarayanan. (2014) Python implementation of the mondrian forest. [Online]. Available: https://github.com/balajiln/mondrianforest
[5] J. a. Gama, R. Sebastião, and P. P. Rodrigues, “Issues in Evaluating Stream Learning Algorithms,” Proceedings of the 15th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, p. 329–338, 2009.
[6] Student, “The probable error of a mean,” Biometrika, pp. 1–25, 1908.
[7] R. C. Sprinthall, Basic Statistical Analysis 9th ed., Pearson Education, 2011.
[8] Wikipedia contributors, “Z-test — Wikipedia, the free encyclopedia,” 2004, [Online; accessed 30-September-2022]. [Online]. Available: https://en.wikipedia.org/wiki/Z-test
[9] O. Banos, J.-M. Galvez, M. Damas, H. Pomares, and I. Rojas, “Window Size Impact in Human Activity Recognition,” Sensors, vol. 14, no. 4, pp. 6474–6499, apr 2014.
[10] D. Morris, T. S. Saponas, A. Guillory, and I. Kelner, “RecoFit: Using a Wearable Sensor to Find, Recognize, and Count Repetitive Exercises,” in Proceedings of the SIGCHI Conference on Human Factors in Computing Systems, ser. CH ’14 – New York, NY, USA: Association for Computing Machinery, 2014, p. 3225–3234.
[11] ——. (2014) RecoFit: Using a Wearable Sensor to Find, Recognize, and Count Repetitive Exercises. [Online]. Available: https://msropendata.com/datasets/799c1167-28f4-44c4-929c-227fb04e2b9a
[12] A. Dehghani, O. Sarbishie, T. Glarat, and E. Shihab, “A Quantitative Comparison of Overlapping and Non-Overlapping Sliding Windows for Human Activity Recognition Using Inertial Sensors,” Sensors, vol. 19, no. 22, 2019.
[13] A. Bifet, G. Holmes, R. Kirkby, and B. Pfahringer, “MOA: Massive Online Analysis,” Journal of Machine Learning Research, vol. 11, no. May, pp. 1601–1604, 2010.
[14] L. Pietruczuk, L. Rutkowski, M. Jaworski, and P. Duda, “A Method for Automatic Adjustment of Ensemble Size in Stream Data Mining,” in 2016 International Joint Conference on Neural Networks (IJCNN), 2016, pp. 9–15.
[15] P. Duda, M. Jaworski, and L. Rutkowski, “On ensemble components selection in data streams scenario with recoccurring concept-drift,” in 2017 IEEE Symposium Series on Computational Intelligence (SSCI), 2017, pp. 1–7.
[16] J. L. Lobo, J. Del Ser, and E. Osaba, “Lightweight Alternatives for Hyper-parameter Tuning in Drifting Data Streams,” in 2021 International Conference on Data Mining Workshops (ICDMW), 2021, pp. 304–311.

| Tree Addition | Pre & Postquential | Comp. Test | Avg. Rank |
|---------------|-------------------|------------|-----------|
| count         | fading            | sum-std    | 6.50      |
| random        | fading            | sum-std    | 7.50      |
| random        | sliding           | z-test     | 7.61      |
| count         | sliding           | z-test     | 7.67      |
| count         | fading            | t-test     | 8.39      |
| random        | fading            | z-test     | 8.67      |
| count         | fading            | sum-std    | 9.50      |
| count         | sliding           | t-test     | 10.44     |

TABLE I: The best-ranked component combinations out of the 24 combinations of Algorithm 1 across datasets and memory limits. The top lines rank better on average than the bottom lines.