Phase readout of a charge qubit capacitively coupled to an open double quantum dot

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(Dated: March 9, 2010)

PACS numbers: 42.50.Dv, 73.23.-b, 03.67.Lx, 05.60.Gg

I. INTRODUCTION

Quantum algorithms usually terminate with qubit readout, i.e., a measurement of the quantum register’s state. Generally, the laws of quantum mechanics inhibit one to directly and fully determine the wave function of the qubit from a single measurement. With repeated projective measurements in the same basis, it is only possible to sample the probability that the qubit is in the one or the other of two orthogonal states. Such destructive measurements can nevertheless be used to demonstrate coherent oscillations by repeating the experiment many times. Such experiments have been performed for superconducting qubits5–7 as well as for charge qubits implemented with double quantum dots.8–10 Recently, the coupling of two charge qubits of the latter type has been demonstrated,11 which represents a first step toward performing gate operations.

In order to distinguish in an experiment between different charge states of a single quantum dot, one may couple the dot capacitively to a quantum point contact which acts as charge meter. Then the current through the meter depends on the number of electrons in the quantum dot. This also allows one to monitor the transport of individual electrons and to eventually determine the associated full counting statistics. It has also been proposed to couple a charge qubit to a point contact, such that the current through the latter depends on the location of the electron in the double quantum dot, i.e., on the state of a charge qubit.12,13 It is also possible to employ a voltage-biased open quantum dot as charge meter if a nearby additional charge shifts one energy level of the quantum dot across the Fermi surface of an attached lead, the current depends as well on the presence of the charge.12,14 When measuring the state of a charge qubit in that way, the measurement acts back on the coherence of the qubit which, thus, experiences decoherence and dissipation. This means that the qubit evolves into an incoherent mixture. The associated transient current allows one to infer on the initial charge state of the qubit.

A double quantum dot can be used as charge meter as well.15,16 There the main idea is that the monitored charge acts as gate voltage on one quantum dot, such that the energy levels of the double dot are tuned into resonance. The consequence is that the conductance of the double dot and, thus, the current increases, see Fig. 1(b). For the parameters of this figure, the current changes by roughly a factor 10 upon qubit tunneling from dot $D_2$ to dot $D_1$. Thus the achievable signal-to-noise ratio for this charge measurement is higher than the one for the single-dot meter.14,15 Moreover, different qubit states lead to significantly different currents, such that the measurement basis is not fluctuating.11,16 The central aim of this work is to demonstrate that the double-dot charge meter can even be used to distinguish between qubit states with identical population but different phase. This means that

![FIG. 1: (Color online) (a) Sketch of the qubit-meter setup consisting of an open and a closed double quantum dot. The closed double dot ($D_1$ and $D_2$) is occupied with one electron and forms a charge qubit. An electron in dot $D_1$ effectively shifts the onsite energy of dot $D_2$, such that dots $D_3$ and $D_4$ are tuned into resonance. Thus the current through the open double dot ($D_3$ and $D_4$) is sensitive to the state of the qubit. (b) Current as a function of the meter energy bias $\varepsilon = \varepsilon_4 - \varepsilon_3$ without coupling to the qubit, $U = 0$, for inter-dot tunneling $\Delta_m = 0.2$ meV and wire-lead coupling $\Gamma = 0.2$ meV. The marked values correspond to the energy shift induced by a qubit electron localized in dot $D_1$ and $D_2$, respectively, for the interaction strength $\Delta = 1$ meV.](arXiv:0904.2754v2 [cond-mat.mes-hall] 8 Mar 2010)
where the operator $c^\dagger$ creates an electron in state $k$ of lead $\ell = L, R$ with energy $\hbar \omega_{\ell k}$. Henceforth, we consider the limit of large bias voltage such that initially all relevant states of the left lead are occupied, while those of the right lead are empty. Then the electron transport becomes unidirectional. Electron tunnelling between the leads and the open double dot is described by the Hamiltonian
\begin{equation}
H_{m-1} = \sum_k (V_{Lk} c^\dagger_{Lk} c_3 + V_{Rk} c^\dagger_{Rk} c_4) + \text{H.c.}
\end{equation}
The coupling matrix elements $V_{\ell k}$ can be subsumed in the effective tunnel rates $\Gamma_\ell = (2\pi/\hbar) \sum_{\ell,k} |V_{\ell k}|^2 \delta(\epsilon - \epsilon_{\ell k})$, which within a wide-band limit are assumed to be energy independent.

In order to compute the time evolution of the system-meter setup, we derive a master equation for the reduced density operator $\rho$ of both double quantum dots by eliminating the leads within second-order perturbation theory. Starting from the Liouville-von Neumann equation $\dot{R}(t) = -(i/\hbar)[H, R(t)]$ for the total density operator $R$, we follow Ref. [20] and obtain by tracing out the leads the master equation,
\begin{equation}
\dot{\rho}(t) = -i/\hbar [H_s, \rho(t)] - \frac{1}{\hbar^2} \int_0^\infty d\tau \text{Tr}_{\text{leads}} \left\{ \left[ H_{\text{m-1}}, \dot{H}_{\text{m-1}}(-\tau) \right], \rho(t) \otimes \rho_{\text{leads}} \right\},
\end{equation}
with $\rho_{\text{leads}}$ being the density operator of both leads, each in a canonical state but with different Fermi energy. The factorization assumption for the density operator allows evaluating the trace over the lead states such that a closed equation for the reduced density operator of the quantum dots remains. The first term on the right-hand side refers to the coherent dynamics of the electrons in the two coupled double dots, while the second term describes incoherent tunnelling between the leads and the open double quantum dot. The tilde denotes the interaction picture with respect to the system Hamiltonian $H_s = H_q + H_m + H_{q-m}$. Defining the current operator in a symmetric manner, $J = (e/2)(\hat{N}_L - \hat{N}_R)$, we obtain the ensemble-averaged expectation value $J(t) = (e/2) \text{Tr} \{ (J^\text{in}_L + J^\text{out}_R) \rho(t) \}$. In the large-bias limit, the superoperators $J^\text{in}_L$, $J^\text{out}_R$ act on the reduced density operator according to
\begin{align}
J^\text{in}_L \rho &= \Gamma c_3 \rho c_3^\dagger, \\
J^\text{out}_R \rho &= \Gamma c_4 \rho c_4^\dagger.
\end{align}
They describe incoherent tunnelling of an electron from the left lead to dot $D_3$ and from dot $D_4$ to the right lead, respectively. Notice that for large bias voltage, the meter properties no longer depend on the absolute values of the onsite energies $\epsilon_3$ and $\epsilon_4$, but only on the energy bias $\epsilon = \epsilon_4 - \epsilon_3$. All numerical results presented below have been obtained by integrating master equation [7].

III. MEASUREMENT CONCEPT AND VISIBILITY

The central idea of our readout scheme is that the current through the open double dot depends on the position of the electron in the closed quantum dot, i.e., on
the state of the qubit. The qubit in turn is affected by
the coupling to the leads via the meter which thus ef-
fectively represents a macroscopic environment. There-
fore, the qubit will experience decoherence, such that the
qubit-meter setup will evolve into a generally unique sta-
tionary mixed state in which the current assumes a value
independent of the initial condition. This transition cor-
responds to a “collapse” of the wave function during a
finite time. Moreover, it implies that the readout is de-
structive, i.e., some information about the qubit state is
transferred to the leads. The full quantum state of the
lead, however, is not accessible such that the measure-
ment is quantum limited. Nevertheless, measurement of
macroscopic lead observables such as the current is pos-
able. In the present case, the current exhibits transients
that allow one to draw conclusions on the qubit’s initial
state which we propose to read out. In the following, we
reveal the underlying relation between the initial qubit
state and the transient current.

As a natural and experimentally relevant initial condi-
tion, we assume that in the beginning, the meter is not
coupled to the qubit ($U = 0$) and stays in the corre-
sponding stationary state which is a mixed non-equilibrium
state in which a current flows. At this stage, the qubit
is in a pure state which we parameterize on the Bloch
sphere as

$$|\psi(t = 0)|_q = \cos(\theta/2) |D_1\rangle + e^{i\phi} \sin(\theta/2) |D_2\rangle,$$

(10)

where the state $|D_{1,2}\rangle$ refers to an accordingly localized
electron. The angle $\theta = 0 \ldots \pi$ determines the posi-
tion $\langle \sigma_z \rangle = \langle n_1 \rangle - \langle n_2 \rangle = \cos \theta$ of the electron, while
$\phi = 0 \ldots 2\pi$ denotes the relative phase. State preparation
with such qubits has been demonstrated experimentally in
Refs. 14,15. At time $t = 0$, the qubit-meter coupling $U$
is switched on, such that the qubit influences the current
and the readout process starts.

Before addressing phase readout, we elucidate the underly-
ing mechanism for the more intuitive charge readout.10,14,15
Figures 2(a,b) show a typical time evolution of the qubit
population and the corresponding cur-
rent. If the qubit is initially in state $|D_2\rangle$, i.e. for $\theta = \pi$, it
will essentially stay there and, thus, the open double dot
remains off-resonant and the current small. This stems
from the fact that for a given $U$, the energy of the two
double dots is smaller when the qubit electron resides in
one particular dot, which for the present parameters is
dot $D_2$. This implies that the initial state $|D_2\rangle$ is already
close to the stationary state and not much dynamics is
going on. When starting in state $|D_1\rangle$ ($\theta = 0$), by con-
trast, the capacitive coupling tunes the levels of dots $D_3$
and $D_4$ into resonance and the current starts to increase
until the systems evolves into a stationary state with an
again smaller current. Thus, we observe a current peak.
The solid line in Fig. 2(c) shows that the peak current
$I$, i.e. the maximal current, is related to the population
parameter $\theta$. This clear dependence is quite important
for the readout scheme, because it implies that the mea-
surement of $I$ corresponds to the determination of the

FIG. 2: (Color online) Charge readout for $\Gamma_L = \Gamma_R = 0.2 \text{ meV}$, $\Delta_q = \Delta_m = 0.2 \text{ meV}$, and $U = \varepsilon = 1 \text{ meV}$. (a) Transient dynamics of the qubit population $\langle \sigma_z \rangle = \langle n_1 \rangle - \langle n_2 \rangle$ and (b) corresponding current for various initial localizations. (c) Height of the current peak as a function of the initial oc-
pupation $\cos(\theta/2)$ for different relative phases $\phi$. (d) Charge-
readout visibility $\nu_\theta$. The dashed-dotted line marks the “opti-
nal” visibility, for which backaction is ignored; see Fig. 1(b).

IV. PHASE-READOUT

We have already seen in the last section that phase
readout is possible in principle. This raises two intrigu-
ing questions. First, we would like to qualitatively understand why phase readout works, despite the fact that the capacitive coupling is sensitive to the location of the qubit electron only. The second question is of quantitative nature: can one achieve a visibility comparable to the one obtained for charge readout?

The relative phase is only meaningful when both qubit states are populated, and a noticeable influence on the transient current requires even significant population of both states. Therefore, we restrict ourselves to equal initial population, i.e., to $\theta = \pi/2$. In order to gain a physical picture of the phase readout, let us focus on the two phases $\phi =\pi$ and $\phi = 0$, i.e., on the states $(|D_1\rangle - |D_2\rangle)/\sqrt{2}$ and $(|D_1\rangle + |D_2\rangle)/\sqrt{2}$, which are the eigenstates of the qubit Hamiltonian $\{q\}$ in the one-electron subspace. Since the qubit couples via the meter to a macroscopic environment, it will evolve into an asymptotic state. Thus, for the ground state ($\phi = \pi$), the qubit will absorb energy from the meter, while for the excited state ($\phi = 0$), the qubit emits energy. Both processes leave their fingerprints in the transients of the current and, thus, allow one to discern different initial phases.

A typical transient current is shown together with the corresponding qubit dynamics in Figs. 3(a,b). The current peak resembles the one analyzed in the context of charge readout. Here however, the peak never vanishes completely, as it was the case above for the initial state $|D_2\rangle$. Thus, at first glance, phase readout seems to possess a much lower resolution than charge readout. For a quantitative analysis we consider the phase-readout visibility $\nu_\phi$ which we define according to Eq. 11 but with the population parameter $\theta$ replaced by the relative phase $\phi$.

Figure 3(c) reveals the clear $\phi$-dependence of the current peak height $I$, which forms the basis of our phase-readout scheme. The maximal value and the minimal value of $I$ are assumed for phases very close to $\phi = 0$ and $\phi = \pi$, respectively. Therefore the experimental distinction between these two phases can be achieved with the full visibility $\nu_\phi$ shown in Fig. 3(d). As compared to charge readout, the visibility exhibits a less regular structure. For the interaction strength $U \approx \epsilon/2$, it reaches a value $\nu_\phi \approx 0.25$ and remains of that order when $U$ is increased. In contrast to charge readout, we find a tendency toward higher visibility for larger qubit splitting. Nevertheless, for these parameters, $\nu_\phi$ still stays clearly below the charge readout visibility $\nu_\theta$. Therefore it is essential to optimize the setup.

Three routes toward an optimized phase readout come to mind. First, as already noticed above, the qubit splitting $\Delta_q$ should be larger than the tunnel matrix element $\Delta_m$ of the meter. Irrespective of any experimental constraints, increasing $\Delta_q$ is only of limited use, because beyond a certain limit, the qubit oscillations then become so fast that the meter is no longer able to follow. Consequently, the meter no longer contains information on the qubit and, thus, the readout quality will decrease. In our case, we find that $\Delta_q = 3\Delta_m$ is a good choice, while for larger qubit splittings, the visibility indeed decreases; see Fig. 3(d).

A second way for improving the visibility is to tune the meter into a regime of higher sensitivity which is mainly determined by the resonance curve shown in Fig. 1(b). When the dot-lead tunnel rates $\Gamma_{L,R}$ become smaller, the current maximum $I_1$ increases, while the current for an energy bias $\epsilon = U$, which is $I_2$, decreases. Consequently, the achievable visibility $\nu_{\text{opt}}$ becomes larger. The reduced dot-lead tunnelling, however, leads to a smaller current, such that the current measurement eventually will be difficult.

Alternatively, one can reduce the current $I_2$ by using a setup with a larger bias $\epsilon$ and an accordingly larger interaction energy $U$. The Coulomb interaction $U$, however, is determined by the distance between the dots $D_1$ and $D_3$ and, thus, is limited by the size of the top gates that define the quantum dots. Nevertheless, it is possible to enhance the qubit-meter coupling by choosing a setup in which the electrons in $D_2$ and $D_1$ repel each other. Then the qubit-meter Hamiltonian $\{q\}$ has to be extended by the term $U' n_2 n_4$. If now an electron tunnels from $D_2$ to $D_1$, the left meter level is raised by $U$, while the right meter level is no longer raised by $U'$, i.e., it is effectively lowered by $U'$. This implies that the relevant effective interaction strength is $U_{\text{eff}} = U + U'$. Notice that the resonance condition for the meter bias nevertheless reads $\epsilon = U$. This additional qubit-meter coupling represents our third way of optimization. We explore its ben-

![FIG. 3: (Color online) Phase readout of a qubit with energy splitting $\Delta_q = 0.2$ meV coupled to a meter with the same parameters as in Fig. 2. The qubit states are at initial time equally populated, i.e., $\theta = \pi/2$. (a) Transient qubit dynamics and (b) current for various initial phases $\phi$ and qubit-meter interaction $U = \epsilon = 1$ meV and $\Delta_q = 0.2$ meV. (c) Height of the current peak as a function of the initial phase. (d) Readout visibility as a function of the qubit-meter interaction strength $U$.](image-url)
and coupling only between dots

FIG. 4: (Color online) Phase-readout visibility for a meter with \( \Gamma_L = \Gamma_R = 0.2 \) meV and tunnel matrix element \( \Delta_m = 0.2 \) meV. Comparison of symmetric coupling (\( U = \varepsilon \)) and coupling between the left dots (\( U' = 0 \)) for qubit energy splitting (a) \( \Delta_q = 3\Delta_m \) and (b) \( \Delta_q = 2\Delta_m \). (c) Visibility as a function of the interaction strength \( U \) for symmetric coupling \( U' = U \), bias \( \varepsilon = 1.5 \) meV and various qubit splittings.

efits by comparing two situations with the same effective coupling strength: symmetric coupling \( U = U' = U_{\text{eff}}/2 \) and coupling only between dots \( D_1 \) and \( D_3 \), which means \( U = U_{\text{eff}} \) while \( U' = 0 \).

Figures 4(a,b) show the resulting visibilities as a function of the meter bias. We find that in the relevant regime with large \( U_{\text{eff}} \), the symmetric coupling is superior to the asymmetric coupling. The difference is up to roughly 30%. Finally, in order to explore the limits of the symmetrically coupled setup, we plot in Fig. 4(c) the visibility as a function of the interaction strength \( U \) for symmetric coupling \( U' = U \), bias \( \varepsilon = 1.5 \) meV and various qubit splittings.

V. CONCLUSIONS

We have investigated the transient current through an open double quantum dot with a capacitive coupling to a charge qubit. In particular, we focused on the impact of the initial qubit state on the current peak that emerges after the qubit is coupled to the open double quantum dot. Such qubit-meter setups have recently been proposed for monitoring the location of an electron in a closed double dot, i.e., for charge readout with single or double dot meters. Our results demonstrate that such a charge meter is useful for phase readout as well, despite the fact that the qubit phase possesses only indirect influence on the measured current. For an unbiased qubit, the relative phase determines whether the qubit is initially in its ground state or in its excited state. Thus, when coupled to a macroscopic device such as the meter, the qubit will absorb or emit energy, depending on its initial phase. This difference is visible in the height of a transient current peak, whose measurement thus corresponds to phase readout.

After having realized that phase readout is possible, in principle, we have investigated whether the measured signals are sufficiently pronounced, such that they allow one to reliably discern between different initial phases. Our results for the phase readout visibility, defined as the scaled difference of the current peaks, reveal that phase readout is only slightly more demanding than the previously proposed charge readout with single or double quantum dot meters. In conclusion, we believe that the experimental implementation of our phase-readout scheme opens a promising way for the observation of coherent tunnelling dynamics in double quantum dots.

Acknowledgments

This work has been supported by the DFG through SFB 631 and by the German Excellence Initiative via “Nanosystems Initiative Munich (NIM)”. S.K. acknowledges support by the Spanish Ministerio de Ciencia e Innovación through the Ramón y Cajal program.
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