1. Introduction

Bundles of parallel particle conveying channels, each of limited capacity, may be subject to blockage. If the blockage is irreversible, the failure of one channel results in an increased load on the remaining open channels that can trigger a cascade, ultimately leading to a complete breakdown of the system [1–4]. If the blockage is of finite duration, however, the system will eventually reach a steady state. An analogous phenomenon of multiple failures can be observed in the exertion of an external force on textile fibres. The fibre bundle model [5–7] consists of a number of parallel threads subjected to an applied load. If the load on a single thread exceeds its threshold, the thread breaks and the global load is then redistributed over the remaining intact threads [8], with the attendant possibility of the material tearing.

Self-healing materials [9] are examples of multilayer network structures which have local finite storage capacities. When an overload occurs in a node of the network, this leads to a local failure and the network is able to rebuild a new local node. To describe these phenomena, Manfredi et al [10] proposed a stochastic model showing that the optimal performance is reached via complex trade-offs between physical parameters. This feature can be viewed as analogous to the ‘slower-is faster’ phenomenon [11], which is also present in pedestrian traffic flows [12, 13].
Further examples include blackouts in power distribution networks, that are generally preceded by a cascade of failures resulting from local overloads [14], Earthquakes [15, 16], vehicular traffic jams [17], network traffic jams [18–21], matteral failures [22, 23] and internet attacks (DoS) [24] all exhibit similar features.

We focus on particulate flow in channels with a limited carrying capacity of $N$ particles. When this threshold is reached, a blockage occurs and no additional particles can enter until the channel is emptied. For purely ballistic transport, Gabrielli et al [25] introduced non-Markovian models in which particles randomly enter a channel and traverse it at a constant velocity, meaning that the time spent in the channel is constant. For irreversible blockages, exact solutions can be obtained [25, 26] for small thresholds ($N = 2, 3$), but only numerical results are available for larger values of $N$. Several extensions of these models have been considered, including a time-dependent incoming flux [27], temporary blockage [28] and multiple channel transport where cascades of failures were observed [29]. However, as the model complexity increases it becomes increasingly difficult to obtain exact solutions.

In this article we propose a class of Markovian models, which can be viewed as an extension of approaches developed in queuing theory [30, 31]. Particles enter a channel according to a Poisson process of intensity $\lambda$, whereas the exit from the channel is given by a Poisson process of intensity $\mu$, given that the total number of particles present is less than the threshold capacity, $N$. When blocked, the channel may reset, with all particles exiting the channel, after a time given by a Poisson process of intensity $\mu^\ast$. The proposed model can encompass situations characterised by temporary or permanent blockages by assuming that the transit time of a particle is a stochastic variable. We study single and multi-channel systems, for which steady-state and time-dependent solutions can be obtained. The previously studied ballistic models with constant residence and blocking times do not permit a description of the system’s time evolution using differential equations. In contrast, the time evolution of the stochastic model presented here is always given by a set of differential equations, for which exact solutions can in principle be obtained. These results provide an effective tool for optimizing particle transport in channels and for other applications, such as self-healing materials.

### 2. Model

We consider a system composed of $N_c$ channels. A single channel is open if the number of particles inside is less than $N$ and blocked at the threshold of $N$ particles. When blocked, no more particles can enter until the channel is flushed. Particles randomly enter an open channel according to a Poisson process with intensity (rate) $\Lambda/(N_c - k)$ where $k$ is the number of blocked channels at time $t$ (see figure 1).

The queuing discipline which dictates the particle ejection is ‘first come first serve’. This may be contrasted with the discipline employed in [32]. It is worth noting that the choice of discipline has no effect on a channel, or bundle of channels, when each has a threshold capacity of $N = 2$. Particle egress from the channel is therefore modeled as a Poisson process with rate $\mu$. As a result, the exit rate remains constant for any number of particles present below the blocking threshold. A blocked channel reopens, and releases $N$ blocked particles, after an exponentially distributed time with rate $\mu^\ast$.

The following section focuses on single channel models for a general capacity, $N$. Section 4 considers $N_c$ channels with a fixed threshold $N = 2$.

### 3. Single channel models

For the single channel models, the stochastic dynamics is described by using $N+1$ state probabilities, $\pi(0, t), \pi(1, t), \ldots, \pi(N-1, t)$ and $\pi(N, t)$ which corresponds to $0, 1, \ldots, N - 1$ and $N$, particles in the channel, respectively.

Let $P(t) = (\pi(0, t), \pi(1, t), \ldots, \pi(N-1, t), \pi(N, t))$ denote the state vector, the time evolution of the process is therefore described by

$$\frac{dP(t)}{dt} = P(t) \cdot Q_{N+1}.$$  \hfill (1)

$Q_{N+1}$ is the $(N + 1) \times (N + 1)$ matrix,

$$Q_{N+1} = \begin{pmatrix}
-\lambda & \lambda & 0 & \ldots & 0 & 0 \\
\mu & -(\lambda + \mu) & \lambda & \ldots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & \ldots & \mu & -(\lambda + \mu) & \lambda & 0 \\
0 & \ldots & 0 & \mu & -(\lambda + \mu) & \lambda \\
\mu^\ast & \ldots & 0 & 0 & 0 & -\mu^\ast
\end{pmatrix}. \hfill (2)$$

The change of state $i$, in all but the first and last columns, contains two gain terms and two loss terms. The former correspond to the entrance of a particle (with a rate $\mu$) in a channel with $i - 1$ particles, and to the exit of a particle (at rate $\mu$) from a channel with $i + 1$ particles. The two loss terms correspond to a channel with $i$ particles (with $0 < i < N$) where a particle enters at time $t$ (with a rate $\lambda$) or where a particle exits (at rate $\mu$). The description is completed by including the previously omitted boundary situations: the empty channel has a loss term corresponding to a particle entrance and two gain terms. The first term corresponds to a particle exit from an empty channel and the second to the release of a blocked channel (with $N$ particles). Conversely, a blocked channel has a single loss term corresponding to a release with rate $\mu^\ast$ and a gain term corresponding to a particle entering the channel while in the $N - 1$th state.

The time evolution of $P(t)$ is supplemented by the conservation of total probability, $\sum_{i=0}^{N} \pi(i, t) = 1$. Consequently, the sum of each row of the transition matrix is equal to 0, which leads to a zero eigenvalue.

The channel throughput is given by

$$j(t) = \mu \sum_{k=1}^{N-1} \pi(k, t) + N\mu^\ast\pi(N, t). \hfill (3)$$
The first term accounts for the exit of one particle at time $t$ while the channel is still open. The second term describes the re-opening of a blocked channel that instantaneously releases $N$ particles. In the stationary state, $j$ can be expressed as

$$j = \lambda (1 - \pi(N)),$$

i.e. the incoming flux multiplied by the probability that the channel is open.

Figure 2 shows the stationary flux with respect to $\lambda$ for $N = 2, 3, 4, 6$ (from bottom to top) with $\mu = 1$ and for (left) $\mu^* = 1/2$ and (right) $\mu^* = 0.1$. The dashed lines show the limiting value, $N\mu^*$. 

and by using equation (4), the stationary flux of exiting particles $j$ reads

$$j = \frac{\mu^* \sum_{i=1}^{N} (N + 1 - i) \left(\frac{\lambda}{\mu} \right)^{i-1}}{1 + \frac{\mu^*}{\lambda} \sum_{i=1}^{N} (N + 1 - i) \left(\frac{\lambda}{\mu} \right)^{i-1}}.$$  

(6)

The asymptotic regimes can be easily analyzed. When $\lambda \to \infty$, the flux $j$ reads

$$j \approx N\mu^* + \frac{\mu^*}{\lambda} ((N - 1)\mu - N^2\mu^*).$$  

(7)

The leading term corresponds to an alternation of open (empty) and closed (blocked) states. The former is of an infinitesimally short duration where no particles exit and the latter is followed by the release of $N$ blocked particles. This explains why the exiting flux $j$ is independent of $\lambda$ at large intensity. The second term of the asymptotic expansion, equation (7), shows that the limit is approached from below when $\mu^* < \frac{N-1}{N} \mu$ and from above when $\mu^* > \frac{N-1}{N} \mu$. This implies that, in the latter case, the flux $j$ displays a maximum at a finite value of $\lambda$, whereas $j$ is a monotonically increasing function of $\lambda$ in the other case (the same behavior is observed in a related model [32]).

To solve equation (1), one notes that the process belongs to the class of circular Markov chains [30, 33] whose stationary state permits a solution. After some calculations, the blocking probability is found as

$$\pi(N) = \frac{1}{1 + \frac{\mu^*}{\lambda} \sum_{i=1}^{N} (N + 1 - i) \left(\frac{\lambda}{\mu} \right)^{i-1}},$$  

(5)

and at small $\lambda$, $j$ is given by

$$j \approx \lambda \frac{N^{N+1}}{\mu^{N-1} \mu^*}.$$  

(8)
The leading term of this expansion represents all particles leaving the channel without blockage and the sub-leading term corresponds to a decrement that becomes very small as the threshold $N$ increases.

The stationary probabilities allow the computation of many other quantities. For example, the variance of the stationary flux $\sigma_j^2$ is given by

$$
\sigma_j^2 = \mu^2 \sum_{k=1}^{N-1} \pi(k) + N^2 \mu^2 \pi(N) - \left( \mu \sum_{k=1}^{N-1} \pi(k) + N \mu \pi(N) \right)^2.
$$

(9)

For $N = 2$, one has

$$
\sigma_j^2 = \mu^2 \lambda \left[ \frac{(\mu^2 - 4 \mu \mu^* + 8 \mu^2 \lambda^2) \lambda^2 + \mu \mu^*(\mu + 4 \mu^*) \lambda + \lambda^3 \mu^*}{(\lambda^2 + 2 \lambda \mu^* + \mu^2 \lambda)^2} \right].
$$

(10)

For $N > 2$, the expressions are lengthy and so are not displayed. For small $\lambda$, the variance vanishes as

$$
\sigma_j^2 \approx \mu \lambda
$$

(11)

and is independent of $\mu^*$. This can be interpreted by noting that blockages are rare in this regime and do not contribute to the leading term of variance. For large $\lambda$, the variance also vanishes as

$$
\sigma_j^2 \approx \frac{\mu^*}{\lambda} \left[ (N-1)(\mu - N \mu^*)^2 + N^2 \mu^* \lambda^2 \right].
$$

(12)

Figure 3 shows the variance of the stationary flux $\sigma_j^2$ as a function of $\lambda$ for $\mu = 1$ and $\mu^* = 0.1, 0.5$. As shown above, the variance increases linearly at small values of $\lambda$, and decays to 0 at large $\lambda$. These results confirm that for low intensity the system is most likely to be in state $1$, $\lambda \ll 1$, $\pi(N) \ll \pi(1)$, while for high intensity the system is most likely to be in the blocked state, $\lambda \gg 1$, $\pi(N) \gg \pi(1)$.

4. Multi-channel models

A channel can be in one of $N+1$ states, corresponding to an index, $i$, ranging from 0 when the channel is empty to $N$, when it is blocked. The time evolution of $N_c$ channels can be described by introducing $N_c$ indices, which gives a number of states $(N+1)^{N_c}$. The flux of exiting particles depends on the number of channels in state $i$. The number of states can be labelled using indices $i_j$ (whose values range from 0 to $N_c$) associated with the number of channels in a state $j$, with the global constraint $\sum_{j=0}^{N} i_j = N_c$.

For $N = 2$, the state probabilities are given by $\pi(i,j,t)$, where $i$ is an index counting the number of blocked channels and $j$ is an index for the total number of open channels (i.e. with one particle present). Because the total number of channels is given by $\sum_{j=0}^{N} i_j = N_c$, the index $i_0$ is not necessary to define the state probabilities. This labeling reduces the number of state probabilities to $(N_c + 1)(N_c + 2)/2$. Figure 4 displays state diagrams for $N_c = 2$ (left) and $N_c = 3$ (right).

The generalization of the graphical method can be performed as follows: for $N = 3$, the state probabilities are defined by three indices $i,j,k$ denoting the number of channels with 3, 2, 1 particles, respectively. For example, for $N_c = 2$ when one channel is blocked and the other has two particles present...
is denoted by $(1, 1, 0)$. The number of state probabilities is $(N_e + 1)(N_e + 2)(N_e + 3)/6$. By using a three dimensional cubic lattice (whose nodes are labelled with the indices corresponding to the axes), one needs to add the different arrows connecting nearest neighbors nodes with the rules described above. It is clear when one increases both $N_e$ and $N$, it is necessary to code these rules in order to build the transition matrix and finally to obtain the stationary probabilities by using a matrix inversion, which remains feasible.

4.1. $N = 2, N_e = 2$

The kinetic equations of the model are given by the matrix differential equation

$$\frac{dP(t)}{dt} = P(t) \cdot Q_6,$$

where $P(t)$ is the state vector with 6 components, $P(t) = (\pi(0, 0, t), \pi(0, 1, t), \pi(0, 2, t), \pi(1, 0, t), \pi(1, 1, t), \pi(2, 0, t))$ and the transition probability matrix $Q_6$ is given by

$$Q_6 = \begin{pmatrix}
-\Lambda & \Lambda & 0 & 0 & 0 & 0 \\
\mu & -(\Lambda + \mu) & \frac{\mu}{2} & \frac{\mu}{2} & 0 & 0 \\
\mu^* & 0 & 0 & -(\Lambda + \mu^*) & \Lambda & 0 \\
0 & 2\mu & -(\Lambda + 2\mu) & 0 & \Lambda & 0 \\
0 & \mu^* & 0 & \mu & -(\Lambda + \mu^* + \mu) & \Lambda \\
0 & 0 & 0 & 2\mu^* & 0 & -2\mu^*
\end{pmatrix}.\quad (14)$$

The non-zero coefficients of the matrix correspond to the different arrows shown in figure 4. For clarity, we shall examine two example terms in detail. The time evolution of $(0, 1, t)$ has three gain terms and two loss terms: the gain terms correspond to the entrance of one particle in an empty system, a particle exit from a state where each channel contains one particle (thus contributing a factor of 2), and the release of a blockage from a system with one blocked channel and a channel with one particle. The loss terms correspond to the entrance or exit of a particle for a system in the state $(0, 1)$. As a second example, the time evolution of $(0, 2, t)$ has one gain term and two loss terms. The gain term is associated with the entry of a particle in an empty channel, while the other channel already contains one particle (resulting in the factor of 1/2). The two loss terms are associated with the entrance of a new particle and the exit of one particle (the factor 2 comes from the fact that a particle can exit from either channel).

The throughput of the two channel system is given by

$$J = \mu[\pi(0, 1, t) + 2\pi(0, 2, t)] + 2\mu^*\pi(1, 0, t) + (\mu + 2\mu^*)\pi(1, 1, t) + 4\mu^*\pi(2, 0, t),$$

where the first term corresponds to the exit of a particle from a system with no blocked channels. The second term corresponds to a blockage release (the factor 2 is the number of particles in the blocked channel). The third term contains two contributions: a particle exit from the open channel and a blockage release. The last term corresponds to the release of two particles from one of the two blocked channels.

In the stationary state,

$$J = \Lambda(1 - \pi(2, 0)),$$

which corresponds to the entrance flux times the probability that at least one of the channels is open. We find

$$\pi(2, 0) = \frac{2\Lambda^5 + (2\mu + \mu^*)\Lambda^4}{\Delta},$$

with

$$\Delta = 2\Lambda^5 + (2\mu + 9\mu^*)\Lambda^4 + 4\mu^*(3\mu + 5\mu^*)\Lambda^3 + 2\mu^*(2\mu^2 + 21\mu\mu^* + 4\mu^2)\Lambda^2 + 16\mu\mu^2(2\mu + \mu^*)\Lambda + 8(\mu\mu^2)^2(\mu + \mu^*).$$

By using equation (15), one obtains the stationary throughput $J$.

At small $\Lambda$, $J$ behaves as

$$J \simeq \Lambda - \frac{2\mu + \mu^*}{8(\mu\mu^*)^2(\mu + \mu^*)}\Lambda^5.$$\quad (19)

Comparing to the single channel model (with $N = 2$) (equation (8)), where the first term has a $\Lambda^3$ dependence, the first term has now a $\Lambda^5$ dependence, which corresponds to a smaller probability of a full blockage of the system. At large $\Lambda$, one obtains

$$J \simeq 4\mu^* - \frac{2\mu^*(\mu - 4\mu^*)}{\Lambda}.\quad (20)$$

As expected, in this limit the only contribution to the throughput are deblocked particles. We note that the limit is approached from below for $\mu^* < \mu/4$ and from above for $\mu^* > \mu/4$. The latter case results in a maximum of the flux for a finite value of $\Lambda$. Both behaviors are shown in figure 5. The change of behavior for the two-channel model at $\mu^* = \mu/4$ is also present in the previous one channel model [32].

4.2. $N = 2, N_e > 2$

It is possible to derive the time evolution for a model with $N_e > 2$. The time evolution of the process with $N_e = 3$ is given by a system of differential equations similar to equation (13). The state vector $P(t)$ is now given as a 10-component vector, $P(t) = (\pi(0, 0, t), \pi(0, 1, t), \pi(0, 2, t), \pi(0, 3, t), \pi(1, 0, t), \pi(1, 1, t), \pi(1, 2, t), \pi(2, 0, t), \pi(2, 1, t), \pi(3, 0, t))$. The matrix $Q_{10}$ is given by:
The exact solution is too lengthy to be displayed, but we focus on some partial results. The stationary throughput is given by

\[ J = \Lambda (1 - \pi(3,0)). \]  

At low \( \Lambda \), one obtains

\[ J \simeq \Lambda - \frac{18\mu^3 + 39\mu^2\mu^* + 22\mu\mu^{*2} + 4\mu^{*3}}{324(\mu\mu^*)^3(\mu + 2\mu^*)(\mu + \mu^*)(2\mu + \mu^*)} \Lambda^7 \]  

We have also solved the model for \( N_c = 4 \) with 15 probabilities. Since the expressions are very lengthy, we present the results graphically in figure 5. For larger values of \( N \), a complete solution can be obtained, by using symbolic software, once the transition matrix is obtained by using the graphical method depicted in the above section. However, because the linear size of the matrix increases as \( (N_c + 1)(N_c + 2)/2 \), the calculation rapidly becomes cumbersome. For general \( N_c \) we conjecture that the small \( \Lambda \) expansion is

\[ J \simeq \Lambda - \Lambda^{2N_c+1} f(\mu, \mu^*), \]

where \( f(\mu, \mu^*) \) is a positive function of \( \mu \) and \( \mu^* \), and the \( \Lambda \to \infty \) limit is

\[ J = 2N_c\mu^* + \frac{N_c\mu^*(\mu - 4\mu^*)}{\Lambda}. \]

Figure 5 shows the stationary flux \( J \) as a function of \( \Lambda \) for \( N_c = 1, 2, 3, 4 \). As discussed above, \( J \) is a monotonically increasing function when \( \mu^* = 0.5 \), whereas \( J \) displays a maximum for \( \mu^* = 0.1 \). This result is independent of the number of channels \( N_c \).

5. Optimized transport

Here we examine two different scenarios for conveying a particulate flux of given intensity \( \Lambda \). The configurations studied are shown in figure 6. The first scenario compares a single, high capacity (HC), channel with a threshold equal to \( N_{HC} = 2N_c \) with a bundle of \( N_c \) identical channels each with a low capacity (LC) of \( N = 2 \). For a bundle of identical channels, the intensity is equally distributed. The second scenario compares a system of \( N_c \) coupled channels, i.e. the total intensity is equally distributed over all open channels, each with threshold \( N = 2 \) with \( N_c \) independent channels each with threshold \( N = 2 \). In both cases we seek to determine which of the two configurations optimizes the steady state throughput.

5.1. One HC channel compared with several LC channels

At low intensity, both configurations present few blockage events and so their fluxes are equal to \( \Lambda \). At large intensity, the throughput of the HC channel is equal to \( 2N_c\mu^* \). Since each LC channel has a throughput equal to \( 2\mu^* \), the total outgoing flux is also equal to \( 2N_c\mu^* \).
In order to determine the most efficient system for different values of the intensity $\Lambda$, one compares both systems by calculating the difference of their stationary fluxes.

$$\Delta J(\Lambda) = J_{NHC}(\Lambda) - N_c J_2 \left( \frac{\Lambda}{N_c} \right),$$

where $J_{NHC}(\Lambda)$ denotes the stationary flux of the HC channel with entering intensity $\Lambda$ and $J_2 \left( \frac{\Lambda}{N_c} \right)$ the stationary flux of a LC channel of threshold $N = 2$ with a shared intensity $\Lambda/N_c$. This quantity clearly goes to zero for small and large $\Lambda$, illustrating the fact the two systems have the same stationary flux in these limits.

Figure 7 displays the difference of stationary flux $\Delta J(\Lambda)$ between the HC channel $J_{NHC}$ and the total flux of the LC channels, $J_2$, as a function of $\Lambda$ for $\mu^* = 0.5, 0.1, (\mu = 1)$ and for $N_{HC} = 4, 6, 8, 10$. One observes that the HC channel is slightly more efficient at low intensity. At large intensity the HC channel is always less efficient than the bundle of LC channels. This behavior is more pronounced when $N_c$ increases.

In summary, the throughput difference reaches a positive maximum for a finite value of $\Lambda$ and then passes through zero before attaining a negative minimum, corresponding to the maximum efficiency of the bundle of LC channels. By expanding equation (26) to the first order it is clear that $\Delta J$ is not a flat function of $\Lambda$:
\[ \Delta J \approx \frac{\Lambda^3}{\mu^4 \mu^2} \left[ N_c - \left( \frac{\Lambda}{\mu} \right)^{2(N_c-1)} \right] , \] (27)

which is positive for \( N_c > 1 \). Conversely at high intensity

\[ \Delta J = -\frac{(N_c - 1)^2 \mu^* \mu}{\Lambda} , \] (28)

which is always negative.

5.2. \( N_c \) coupled channels compared with \( N_c \) independent channels, both with \( N = 2 \)

We consider the flux difference between the two systems

\[ \Delta J^c(\Lambda) = J^c_{2N_c}(\Lambda) - N_c J^c_2 \left( \frac{\Lambda}{N_c} \right) \] (29)

where \( J^c_{2N_c} \) denotes the total stationary flux of \( N_c \) coupled channels of threshold 2.

At low intensity, there are few blockages in either system and the throughput is \( \Lambda \) in both. At large intensity, the throughput of the independent channels and the coupled correlated channels are both equal to \( 2N_c \mu^* \).

By calculating the flux difference, one can determine the most efficient system for different values of intensity. Figure 8 shows the flux difference, \( \Delta J^c(\Lambda) \), between the \( N_c \) coupled channels and the total flux of the independent channels as a function of \( \Lambda \) for two values of \( \mu^* \), \( (\mu = 1) \) and for \( N_c = 2, 3, 4 \).

If the deblocking rate is sufficiently large, \( \mu^* > 0.25 \), the independent channels always convey the flux less effectively than the coupled channels. If \( \mu^* < 0.25 \) the behavior is similar to the first scenario: \( \Delta J \) reaches a maximum for a finite value of \( \Lambda \). At higher intensities, the coupled channels are less efficient and \( \Delta J \) shows a minimum. At very large intensity, both models converge to the same limit as expected.

This behavior can be understood by examining the limiting behavior of \( \Delta J \). For small \( \Lambda \) one has (for \( N_c > 1 \))

\[ \Delta J \approx \frac{\Lambda^3}{N_c \mu^* \mu^2} , \] (30)

which is always positive, while at high intensity

\[ \Delta J \approx -\frac{N_c(N_c - 1) \mu^*}{\Lambda} \mu - 4 \mu^* , \] (31)

which is negative if \( \mu^*/\mu < 0.25 \) and positive otherwise. Coupled channels are always more efficient at low intensity and also at high intensity if the deblocking rate is sufficiently high. If, however, \( \mu^* < \mu/4 \) the coupled channels convey the flux less efficiently due to an accelerating cascade of blockages that is reminiscent of the irreversible model [29].

6. Summary

We have presented a stochastic model of blockage in a channel bundle of \( N_c \) individual channels. A particulate flux enters the system according to a Poisson process of intensity \( \Lambda \). Particles exit the open channels at a rate \( \mu \). An individual channel is blocked if \( N \) particles are simultaneously present in it. In this case, the flux that would have entered is evenly distributed over the remaining open channels. A channel remains blocked for an exponentially distributed time with rate \( \mu^* \). If all channels are blocked, the input flux is rejected. We have provided a framework to obtain both the time-dependent and steady state properties and have presented explicit results for the steady state throughput for \( N_c = 2, 3, 4 \).

We used the analytical results to compare different configurations for transporting a particulate flux of given intensity. A single HC channel is more efficient than several LC channels at low intensity, but the reverse is true at higher values of \( \Lambda \). We also compared \( N_c \) coupled channels with capacity, \( N = 2 \) with its uncoupled version. The coupled channels always have a higher throughput if \( \mu^*/\mu > 0.25 \). For \( \mu^*/\mu < 0.25 \) the coupled channels are more efficient at low intensity, but at higher intensities the order reverses. It will be interesting to see if this effect is still present when the dynamics is non-Markovian.

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