Atmospheric neutrinos and $\nu$-mass hierarchy
Sergio Palomares-Ruiz$^{abc}$, José Bernabéu$^a$

$^a$Departamento de Física Teórica and IFIC, Universidad de Valencia-CSIC, 46100 Burjassot, Valencia, Spain

$^b$Department of Physics and Astronomy, UCLA, Los Angeles, CA 90095, USA

$^c$Department of Physics and Astronomy, Vanderbilt University, Nashville TN 37235, USA

We discuss the possibility for matter effects in the three-neutrino oscillations of the atmospheric \(\nu_e\) (\(\bar{\nu}_e\)) and \(\nu_\mu\) (\(\bar{\nu}_\mu\)), driven by one neutrino mass squared difference, \(|\Delta m^2_{31}| \gg |\Delta m^2_{21}|\), to be observable under appropriate conditions. We derive predictions for the Nadir angle (\(\theta_n\)) dependence of the ratio \(N_\mu/N_e\) of the rates of the \(\mu\)-like and \(e\)-like multi-GeV events which is particularly sensitive to the Earth matter effects in the atmospheric neutrino oscillations, and thus to the values of \(\sin^2 \theta_{13}\) and \(\sin^2 \theta_{23}\), and also to the type of neutrino mass spectrum.

1. Introduction

Present evidence for neutrino masses and mixings can be summarized as: 1) the atmospheric \(|\Delta m^2_{31}|\) is associated with a mixing \(\theta_{23}\) close to maximal [1]; 2) the solar \(\Delta m^2_{21}\) prefers the LMA-MSW solution [2,3]; 3) CHOOZ reactor data [4] give severe limits for \(|U_{e3}|\). Here, we discuss that contrary to a wide spread belief, Earth effects on the propagation of atmospheric neutrinos can become observable, in detectors with lepton charge-discrimination [5,6] and in water-Cherenkov ones [7], even if \(|U_{e3}|\) is small, yet non-vanishing. This fact could allow to determine the sign of \(\Delta m^2_{31}\) and obtain more stringent constraints on the values of \(\theta_{13}\) and \(\theta_{23}\). Getting more precise information about the value of these mixing angles and determining the type of the neutrino mass spectrum (with normal or inverted hierarchy) with a higher precision is of fundamental importance for the progress in the studies of neutrino mixing.

We study here the possibilities to obtain this type of information using the atmospheric neutrino data that can be provided by present and future water-Cherenkov detectors. For baselines \(L\) smaller than the Earth diameter, appropriate for atmospheric neutrinos, \(\frac{\Delta m^2_{31}}{4\pi^2}L \equiv \Delta_{21} \ll 1\), so that we will neglect the (1,2)-oscillating phase in vacuum against the (2,3)-one. This is a very good approximation for the presently best favored LMA-I solution to the solar neutrino problem [2].

The Earth matter effects, which can resonantly enhance the \(\nu_\mu \to \nu_e\) and \(\nu_e \to \nu_\mu(\tau)\) transitions, lead to the reduction of the rate of the multi-GeV \(\mu\)-like events and to the increase of the rate of the multi-GeV \(e\)-like events in a water-Cherenkov detector with respect to the case of absence of these transitions (see, e.g., [8]). Correspondingly, as observables which are sensitive to the Earth matter effects, we consider the Nadir-angle distribution of the ratio \(N_\mu/N_e\), where \(N_\mu\) and \(N_e\) are the multi-GeV \(\mu\)-like and \(e\)-like numbers of events, respectively.

2. The matter-induced neutrino spectrum

If \(V\) is the effective neutrino potential, in going from \(\nu\) to \(\bar{\nu}\), there are matter-induced CP- and CPT-odd effects associated with the change \(V \to -V\). The effects here discussed depend on the interference between the different flavors and on the relative sign between \(2EV\) and \(\Delta m^2_{31}\). For atmospheric neutrinos, an appreciable interference will be present if and only if there are appreciable matter effects, i.e., the “connecting” mixing \(U_{e3}\) between the \(\nu_e\)-flavor and the \(\nu_3\) mass eigenstate does not vanish.

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For \( s_{13} \equiv \sin \theta_{13} = 0 \), matter effects lead to a breaking of the (1,2)-degeneracy such that \( \bar{\nu}_2 \) coincides with \( \nu_e \). The net effect is that \( \nu_1^m \) and \( \nu_3^m \) lead to the atmospheric \( \nu_\mu \rightarrow \nu_\tau \) indicated by SK, likewise in vacuum the (2,3)-mixing does. The \( \nu_\tau \)-flavor decouples in matter, even if there was a large mixing in the (1,2)-system. No matter effects would then be expected when starting with \( \nu_\mu \), i.e., there would be no chances to distinguish the type of mass hierarchy by these means. However, for small \( s_{13} \), even if the effects on the spectrum are expected to be small, there could be a substantial mixing of \( \nu_e \) with \( \nu_3^m \). This would lead to a resonant MSW behaviour in the case of neutrinos crossing only the Earth mantle and to new resonant effects (NOLR) \[8\] in the case of neutrinos crossing also the Earth core. But still \((\nu_1^m|\nu_e) = 0\). This vanishing mixing in matter is responsible for the absence of fundamental CP-violating effects, even if there are three non-degenerate mass eigenstates in matter. In vacuum, the absence of genuine CP-odd probabilities was due to the degeneracy \( \Delta_{21} = 0 \).

3. Atmospheric neutrino oscillations in the Earth

The fluxes of atmospheric \( \nu_{\alpha, \mu} \) of energy \( E \), which reach the detector after crossing the Earth along a given trajectory specified by the value of \( \theta_n \), \( \Phi_{\nu_{\alpha, \mu}}(E, \theta_n) \), are given by the following expressions in the case of the three-neutrino oscillations under discussion \[5\]:

\[
\Phi_{\nu_e}(E, \theta_n) \cong \Phi_{\nu_e}^0 \left( 1 + \left| s_{23}^2 \right| r - 1 \right) P_{2\nu} \quad (1)
\]
\[
\Phi_{\nu_\mu}(E, \theta_n) \cong \Phi_{\nu_\mu}^0 \left( 1 + s_{23}^4 \left| (s_{23}^2 \, r)^{-1} - 1 \right| P_{2\nu} - 2s_{23}^2 s_{23}^2 \left[ 1 - \text{Re} \left( e^{-i\kappa A_{2\nu}} \right) \right] \right) \quad (2)
\]

where \( P_{2\nu} \equiv P_{2\nu}(\Delta m_{31}^2, \theta_{13}; E, \theta_n) \) is the probability of two-neutrino oscillations in the Earth, \( \kappa \) and \( A_{2\nu} \) are known phase and two-neutrino transition probability amplitude, \( \Phi_{\nu_{\alpha}}^0 \) is the \( \nu_{\alpha}(\mu) \) flux in the absence of neutrino oscillations and

\[
r \equiv r(E, \theta_n) \equiv \frac{\Phi_{\nu_\alpha}(E, \theta_n)}{\Phi_{\nu_\mu}(E, \theta_n)} . \quad (3)
\]

The predicted ratio \[7\] for atmospheric sub-GeV neutrinos is \( r \cong (2.0 - 2.5) \), whereas \( r \cong (2.6 - 4.5) \) for multi-GeV atmospheric neutrinos. If \( s_{23}^2 = 0.5 \), the possible effects of the \( \nu_\mu \rightarrow \nu_e \) and \( \nu_\tau \rightarrow \nu_\mu(\tau) \) transitions on the sub-GeV e--like events would be strongly suppressed even if these transitions were maximally enhanced by the Earth matter effects. On the other hand, \( r > 2 \) for the multi-GeV sample, and matter effects can show up. Thus, in the case under study, the effects of the \( \nu_\mu \rightarrow \nu_e \) and \( \nu_\tau \rightarrow \nu_\mu(\tau) \) oscillations, increase with the increase of \( s_{23}^2 \), are considerably larger in the multi-GeV samples of events than in the sub-GeV ones and in the multi-GeV case, they lead to an increase of the rate of e--like events and to a slight decrease of the \( \mu \)--like event rate. This discussion suggests that the quantity most sensitive to the effects of the oscillations of interest should be the ratio of the \( \mu \)--like and e--like multi-GeV events, \( N_\mu/N_e \).

If \( s_{13} \neq 0 \), the Earth matter effects can resonantly enhance \( P_{2\nu} \) for \( \Delta m_{31}^2 > 0 \) and \( P_{2\nu} \) if \( \Delta m_{31}^2 < 0 \) \[5\]. Due to the difference of cross sections for neutrinos and antineutrinos, approximately 2/3 of the total rate of the \( \mu \)--like and e--like multi-GeV atmospheric neutrino events in a water-Čerenkov detector, i.e., \( \sim 2N_\mu/3 \) and \( \sim 2N_e/3 \), are due to neutrinos \( \nu_\mu \) and \( \nu_e \), respectively, while the remaining \( \sim 1/3 \) of the multi-GeV event rates, i.e., \( \sim N_\mu/3 \) and \( \sim N_e/3 \), are produced by antineutrinos \( \bar{\nu}_\mu \) and \( \bar{\nu}_e \). This implies that the Earth matter effects in the multi-GeV samples of \( \mu \)--like and e--like events will be larger if \( \Delta m_{31}^2 > 0 \) (normal hierarchy), than if \( \Delta m_{31}^2 < 0 \) (inverted hierarchy). Thus, the ratio \( N_\mu/N_e \) of the multi-GeV \( \mu \)--like and e--like event rates measured in water-Čerenkov detectors is sensitive, in principle, to the type of the neutrino mass spectrum \[7\].

4. Results

In Fig. 1 we show the predicted dependences on \( \cos \theta_n \) of the ratios of the multi-GeV \( \mu \)-- and e--like events, integrated over the neutrino energy from the interval \( E = (2 - 10) \) GeV, for different cases (see figure).

For \( \cos \theta_n \lesssim 0.2 \) and \( |\Delta m_{31}^2| = 3 \times 10^{-3} \) eV$^2$, the oscillations of the atmospheric \( \nu_\mu \) and \( \nu_e \), with energies in the multi-GeV range \( E \sim (2 - 10) \)
1.5
2.5
1.5
2.5
0.5
1.5
2.5
0.5
1.5
2.5

\[ \Delta m^2_{31} \]

2.5
0.5
1.5
0.2
0.4
0.6
0.8
1

Figure 1. The dependence on \( \cos \theta_n \) of the ratios of the multi-GeV \( \mu^- \) and \( e^- \) like events, integrated over the neutrino energy in the interval \( E = (2 - 10) \) GeV, in the cases i) of two-neutrino \( \nu_{\mu} \rightarrow \nu_{e} \) oscillations in vacuum and no \( \nu_e \) oscillations, \( N_{\mu}^{3\nu}/N_{\mu}^{0} \) (solid lines), ii) three-neutrino oscillations in vacuum of \( \nu_{\mu} \) and \( \nu_e \), \( (N_{\mu}^{3\nu}/N_{e}^{3\nu})_{vac} \) (dash-dotted lines), iii) three-neutrino oscillations of \( \nu_{\mu} \) and \( \nu_e \) and in the Earth and neutrino mass spectrum with normal hierarchy \( (N_{\mu}^{3\nu}/N_{e}^{3\nu})_{\text{NH}} \) (dashed lines), or with inverted hierarchy, \( (N_{\mu}^{3\nu}/N_{e}^{3\nu})_{\text{IH}} \) (dotted lines). The results shown are for \( |\Delta m^2_{31}| = 3 \times 10^{-3} \) eV\(^2\), \( \sin^2 \theta_{23} = 0.36 \) (upper panels); 0.50 (middle panels); 0.64 (lower panels), and \( \sin^2 2\theta_{13} = 0.05 \) (left panels); 0.10 (right panels).

oscillation probability \( P_{2\nu} \), but can enhance the neutrino mixing in matter. However, since the neutrino path in the Earth mantle is relatively short, \( P_{2\nu} \ll 1 \).

At \( \cos \theta_n \gtrsim 0.4 \), the Earth matter effects in the oscillations of the atmospheric \( \nu_{\mu} \) and \( \nu_e \) can generate noticeable differences between \( N_{\mu}^{3\nu}/N_{\mu}^{0} \) (or \( (N_{\mu}^{3\nu}/N_{e}^{3\nu})_{vac} \)) and \( (N_{\mu}^{3\nu}/N_{e}^{3\nu})_{\text{NH(IH)}} \), as well as between \( (N_{\mu}^{3\nu}/N_{e}^{3\nu})_{\text{NH}} \) and \( (N_{\mu}^{3\nu}/N_{e}^{3\nu})_{\text{IH}} \).

5. Conclusions

We have studied the possibility to obtain evidences for Earth matter enhanced atmospheric neutrino oscillations involving, in particular, the \( \nu_e \), from the analysis of the \( \mu^- \) like and \( e^- \) like multi-GeV event data that can be provided by present and future water-\( \text{Cerenkov} \) detectors. We have seen that such evidences could give also important quantitative information on the values of \( \sin^2 \theta_{13} \) and \( \sin^2 \theta_{23} \) and on the sign of \( \Delta m^2_{31} \).

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