Detector Time Offset and Off-line Calibration in EAS Experiments

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Abstract

In Extensive Air Shower (EAS) experiments, the primary direction is reconstructed by the space-time pattern of secondary particles. Thus the equalization of the transit time of signals coming from different parts of the detector is crucial in order to get the best angular resolution and pointing accuracy allowed by the detector. In this paper an off-line calibration method is proposed and studied by means of proper simulations. It allows to calibrate the array repeatedly just using the collected data without disturbing the standard acquisition. The calibration method is based on the definition of a Characteristic Plane introduced to analyze the effects of the time systematic offsets, such as the quasi-sinusoidal modulation on azimuth angle distribution. This calibration procedure works also when a pre-modulation on the primary azimuthal distribution is present.

Key words: extensive air showers, timing calibration, Characteristic Plane, quasi-sinusoidal modulation, geomagnetic effect
PACS: 96.50.sd, 06.20.Fn, 06.30.Ft

1 Introduction

In EAS experiments, the space-time information of the secondary particles is used to reconstruct the primary direction [1,2,3]. The space information refers to the detector unit position while the time information is achieved usually
by TDC (Time to Digital Converter). The former is easy to measure and stable in a long period, while the latter depends on detector conditions, cables, electronics, etc, and usually varies with time and environment. The time offsets are the systematic time differences between detector units, which lead to worse angular resolution, and more seriously, to wrong reconstruction of the primary direction. As a consequence the azimuthal distribution is deformed according to a quasi-sinusoidal modulation [4]. Thus the correction of these systematic time offsets [5] is crucial for the primary direction reconstruction, much more when the EAS detector is devoted to gamma ray astronomy and the pointing accuracy is required in order to associate the signals with astrophysical sources. Usually manual absolute calibration by means of a moving probe detector is used in EAS arrays, but this method takes time and manpower. The difficulty increases taking into account that periodical checks are necessary to correct possible time-drift of the detector units due to change in the operation conditions. Furthermore the number of detector units in current EAS arrays is getting larger and larger. As a conclusion, effective off-line calibration procedures are greatly needed because they do not hamper the normal data taking and can be easily repeated to monitor the detector stability.

Here a new off-line calibration procedure is presented. It does not depend on simulation and is very simple in the case of a uniform azimuthal distribution. It works also when some small modulation of the azimuthal distribution is expected, for instance due to the geomagnetic field. The correctness of this calibration method has been checked by means of simple simulations both in the case of uniform and modulated azimuthal distribution.

### 2 Characteristic Plane

In EAS experiments, for an event $i$ the time $t_{ij}$ is measured on each fired detector unit $j$, whose position $(x_j, y_j)$ is well known. The primary direction cosines $l_i = \sin \theta_i \cos \phi_i$, $m_i = \sin \theta_i \sin \phi_i$ ($\theta_i$ and $\phi_i$ are zenith and azimuth angles) can be reconstructed by a least squares fit. Taking into account the time offset $\Delta_j$ typical of the detector unit and assuming that the shower front is plane and the time-spread due to its thickness is negligible, the plane-equation is

$$c(t_{ij} - \Delta_j - t_{0i}) = l_i x_j + m_i y_j$$

(1)

where $c$ is the light velocity, and $t_{0i}$ is another parameter of the fit. But the time offset $\Delta_j$ is unknown and the goal of the calibration is just to determine it. A traditional off-line calibration method is based on the study of the time-residuals but their removal does not guarantee the removal of the complete
offset. Therefore one can assume that the time offset $\Delta_j$ is the sum of two terms: the residual term and another unknown term. Being unaware of $\Delta_j$ the plane-equation goes like:

$$c (t_{ij} - t_{0i}') = t_{ij}'x_j + m_{ij}'y_j$$

(2)
giving the fake direction cosines $l_{ij}' = \sin \theta_i' \cos \phi_i'$, $m_{ij}' = \sin \theta_i' \sin \phi_i'$. From Eq.s 1 and 2 and neglecting the residuals, it results:

$$\Delta_j = a_i \frac{x_i}{c} + b_i \frac{y_j}{c} + \delta_{0i}$$

(3)

where $a_i = l_i' - l_i$, $b_i = m_i' - m_i$, and $\delta_{0i} = t_{0i}' - t_{0i}$ is an irrelevant time-shift equal for all the units. One can conclude that the offset $\Delta_j$ is correlated with the position of the detector unit. The quantities $a_i$, $b_i$ define a Characteristic Plane (CP) in the $(x, y, \Delta)$ space, depending only on the fired unit pattern, representing the difference between the reconstructed plane without considering the time offset (FP: Fake Plane) and the real one (RP: Real Plane). Events firing different sets of units have different CPs, while events firing the same set of units have the same CP, that is the difference between the FP and RP is the same. We define the CP of an EAS array like the average difference between FPs and RPs, i.e. the systematic deviation between FP and RP (the pointing accuracy). The CP is fully determined by the direction cosines

$$a = \langle l' \rangle - \langle l \rangle = \sin \theta_0 \cos \phi_0, \quad b = \langle m' \rangle - \langle m \rangle = \sin \theta_0 \sin \phi_0$$

(4)

associated to the angles $\theta_0$, $\phi_0$.

2.1 Quasi-Sinusoidal Modulation

If the probability density function (PDF) of the primary azimuth angle is $f(\phi|\theta)$, one can deduce that the presence of the CP introduces a quasi-sinusoidal modulation of the reconstructed azimuth angle distribution:

$$f'(\phi'|\theta) = f(\phi|\theta) \left[ 1 + \frac{r}{\sqrt{1 - r^2 \sin^2(\phi' - \phi_0)}} \cos(\phi' - \phi_0) \right]$$

(5)

where $r = \sin \theta_0 / \sin \theta$. The PDF of the reconstructed azimuth angle is a combination of multi-harmonics of odd orders with the the amplitude approximately proportional to $r^{2n+1}$ ($n = 0, 1, 2...$) when $r << 1$. The time offset
does not introduce even order modulations into the reconstructed azimuth angle distribution. When \( f(\phi|\theta) = 1/2\pi \), the first harmonic becomes dominant and the PDF of the reconstructed azimuth angle goes as

\[
f'(\phi'|\theta) = \frac{1}{2\pi} [1 + r \cos(\phi' - \phi_0)]
\]  

(6)

One can observe that the modulation parameters depend on the angles \( \theta_0 \) and \( \phi_0 \) connected to the CP (see Eq.s 4). The phase is just \( \phi_0 \), while the amplitude is proportional to \( r \). By integrating \( f'(\phi'|\theta) \) over \( \theta \) it results

\[
f'(\phi') = \frac{1}{2\pi} \left[ 1 + \sin \theta_0 \left\langle \frac{1}{\sin \theta} \right\rangle \cos(\phi' - \phi_0) \right]
\]  

(7)

A fast Monte Carlo simulation was done to check the above conclusion. The azimuth angle was sampled uniformly over \([0, 2\pi]\) and the zenith angle from a typical distribution modulated according to \( \cos^6 \theta \) (the mode value is \( \sim 22^\circ \) and \( \langle 1/\sin \theta \rangle = 3.44 \)). CPs with different \( \theta_0 \) and \( \phi_0 \) were assumed, subtracting \( \sin \theta_0 \cos \phi_0 \) and \( \sin \theta_0 \sin \phi_0 \) from the original direction cosines, respectively, in order to get the new direction cosines. Fig. 1 shows the reconstructed azimuth distributions for two different CPs with \( \theta_0 = 0.02 \) rad, \( \phi_0 = 0.4 \) rad and \( \theta_0 = 0.20 \) rad, \( \phi_0 = 1.2 \) rad, respectively. The first distribution is well reproduced by a best-fit function like that of the Eq. 7 as expected for small values of \( \theta_0 \). Also the fit parameters are in agreement with the simulation parameters. The higher order harmonics must be taken into account in order to well reproduce the second distribution (\( A2 \) and \( A3 \) are the amplitudes of 2nd and 3rd harmonics) because in this case \( \theta_0 \) and \( r \) are larger.

![Fig. 1. Azimuth angle distributions fitted with harmonic functions (see the text for comments).](image-url)
3 Characteristic Plane Method

According to Eq. 1, if \( l_i \) and \( m_i \) were exactly known, then any event can be used to relatively calibrate all the detector units hit by that shower, while \( l'_i - a \) and \( m'_i - b \) can be taken as unbiased estimate of \( l_i \) and \( m_i \). Therefore the time correction is determined by \( a \) and \( b \), i.e. the CP of the EAS array, according to Eq. 3.

Suppose that the primary azimuth angle is independent on the zenith angle and distributes uniformly, then \( \langle l \rangle = 0 \), \( \langle m \rangle = 0 \). Thus \( a = \langle l' \rangle - \langle l \rangle = \langle l' \rangle \), \( b = \langle m' \rangle - \langle m \rangle = \langle m' \rangle \), which means that the CP of an EAS array can be determined by the mean values of the reconstructed direction cosines. Then the time offsets can be calculated by means of the off-line analysis of the collected data.

3.1 A simple simulation as a check of the CP method

Another fast geometrical simulation was implemented in order to check the CP method. One million of showers were extracted from the same distributions of \( \theta \) and \( \phi \) used in Sec. 2.1. The arrival primary directions were reconstructed by an array of detector units (10 \( \times \) 10 units on a surface of 40 \( \times \) 40 \( m^2 \)). The times measured by each unit were shifted by systematic time offsets (first plot of Fig. 2). As a consequence the primary directions were reconstructed with respect to a CP with \( a = 3.97 \times 10^{-3} \) and \( b = 7.34 \times 10^{-3} \) (mean values of the reconstructed direction cosines). From the Eq.s 4 it is trivial to estimate \( \theta_0 = 8.3 \times 10^{-3} \) rad and \( \phi_0 = 1.07 \) rad.

The reconstructed azimuth distribution is fitted according to Eq. 7 (see the first plot of Fig. 3). As expected the modulation coefficient \( p_1 \) and the phase \( p_2 \) are compatible with \( \sin \theta_0 (1/\sin \theta) = 0.029 \) and \( \phi_0 \), respectively. The angles between reconstructed and ”true” direction are shown in the first plot of Fig. 4.

The calibration based on the CP method allows to correct the time measurements, removing the effect of the time offset on each detector unit. In the second plot of Fig. 2 the offset-calibration differences are almost null and the RMS is lower than \( 2.3 \times 10^{-2} \) ns. As an effect of the CP calibration the modulation disappears in the azimuth distribution (second plot of Fig. 3) and the reconstructed directions are very close to the ”true” ones (see the second plot of Fig. 4). Then the validity of the CP method is fully confirmed.
Fig. 2. First plot: systematic time offsets introduced in the simulation of the time measurement. Second plot: differences between systematic offset and calibration correction.

Fig. 3. Azimuth distribution (first plot: before CP calibration, second plot: after CP calibration). In the first plot the sinusoidal fit is superimposed.

Fig. 4. Angles between "true" and reconstructed directions (first plot: before CP calibration, second plot: after CP calibration).
3.2 Pre-modulation on the primary azimuth angle

The assumption for the CP method is that the mean values of the primary direction cosines are null. Generally this is not true for EAS experiments. The possible primary anisotropy, the detection efficiency depending on the azimuth angle, the geomagnetic effect, and so on, introduce pre-modulation into the azimuth angle distribution. Assuming that the $\phi$-distribution is independent on $\theta$, the pre-modulation can be described typically as:

$$ f(\phi) = \frac{1}{2\pi} \left[ 1 + \sum_{n=1}^{\infty} g_n \cos(n\phi + \phi_n) \right]. \quad (8) $$

Only $g_1 \cos(\phi + \phi_1)$ contributes to the mean values of the primary direction cosines. Therefore they result

$$ \langle l \rangle = +\frac{g_1}{2} \cos \phi_1 \langle \sin \theta \rangle, \quad \langle m \rangle = -\frac{g_1}{2} \sin \phi_1 \langle \sin \theta \rangle. \quad (9) $$

The CP method annulls $\langle l \rangle$ and $\langle m \rangle$ leaving a sinusoidal modulation on the distribution of the new $\phi''$ azimuth angle. When $g_1$ and $g_2$ are small enough and the higher order harmonics can be ignored (see Sec. 2.1) the distribution approximately is

$$ f''(\phi'') = \frac{1}{2\pi} \left[ 1 + g'_1 \cos(\phi'' + \phi_1) + g_2 \cos(2\phi'' + \phi_2) \right] \quad (10) $$

where

$$ g'_1 = g_1 \left[ 1 - \frac{1}{2} \langle \sin \theta \rangle \left( \frac{1}{\langle \sin \theta \rangle} \right) \right]. \quad (11) $$

On the basis of this result one can conclude that the calibration with the CP method does not remove completely the pre-modulation on the primary azimuthal distribution. The $g_1$, $g_2$ amplitudes and the $\phi_1$, $\phi_2$ phases can be determined from the reconstructed azimuth angle distribution according to Eq.s 10 and 11. Then the direction cosines of the real CP can be determined by subtracting the pre-modulation term (Eq.s 9).

Fast simulations have been used also to check the calibration method in the case of pre-modulation with one and two harmonics ($g_1 = 0.05$, $\phi_1 = 0.3 \text{ rad}$ and $g_2 = 0.02$, $\phi_2 = 1.2 \text{ rad}$). The results are very similar to those of Sec. 3.1 confirming that the method works also when a pre-modulation is present. In Fig. 5 the "true" azimuthal distribution and the distribution after the first step of the calibration are shown. As expected the second distribution is well reproduced by Eq.s 10 and 11.
Fig. 5. The pre-modulation "true" azimuth angle distribution is fitted by the function
\[ f(\phi) = k [1 + g_1 \cos(\phi + \phi_1) + g_2 \cos(2\phi + \phi_2)] \]. The after-calibration azimuthal distribution is fitted according to Eqs. 10 and 11. The fit-parameter values are in full agreement.

3.3 Geomagnetic Effect

The geomagnetic field inflects the charged primaries and leads to the well known East-West effect (with the modulation period of \( \pi \) which does not modify the mean values of the reconstructed direction cosines and does not invalidate the CP method), while the secondary charged particles of EAS are separated in the geomagnetic field with the lateral distribution getting wider and flatter, thus affecting the detection efficiency [6]. A non-vertical geomagnetic field destroys the uniformity of the detection efficiency along the azimuth angle which will further leads to quasi-sinusoidal modulation on the azimuth angle distribution [7]. The geomagnetic effect on the secondaries is typically the most significant pre-modulation (as described in Sec. 3.2) with amplitude of the order of few percent and very slight variations with the zenith angle. This is just the case discussed in the above section and the modulation can be determined according to Eq. 10, after which the time can be off-line calibrated using the CP method.

4 Conclusion

The definition of the CP makes it easier to understand the effects of the detector time offsets in EAS experiments, and makes the off-line calibration possible. One can successfully correct the time offsets and remove the quasi-sinusoidal azimuthal modulation (with \( \phi \) depending on \( \theta \)). The calibration
procedure has been analytically defined and checked by means of fast simulations. The CP calibration is very simple when the "true" azimuthal distribution is uniform (this feature can be also achieved by selecting special event sample). The CP method works also when a "true" pre-modulation (with $\phi$ independent on $\theta$) of the azimuth distribution is present. The improvement in the pointing accuracy is well shown in Fig. 4 for the simulation of Sec. 3.1. In real cases, the pointing accuracy will depend on the detector performances and on quality and statistics of the data used for the CP calibration.

This method has been successfully applied to calibrate EAS detectors [8]. It has been also checked [9] by Monte Carlo full simulation and by sampling manual calibration. The experimental results of the CP method application will be the topic of a future paper.

5 Acknowledgements

We are very grateful to the people of the ARGO-YBJ Collaboration, in particular G. Mancarella, for the helpful discussions and suggestions. This work is supported in China by NSFC(10120130794), the Chinese Ministry of Science and Technology, the Chinese Academy of Sciences, the Key Laboratory of Particle Astrophysics, CAS, and in Italy by the Istituto Nazionale di Fisica Nucleare (INFN).

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