Conquering systematics in the timing of the pulsar triple system J0337+1715: Towards a unique and robust test of the strong equivalence principle

To cite this article: N V Gusinskaia et al 2017 J. Phys.: Conf. Ser. 932 012003

View the article online for updates and enhancements.
Conquering systematics in the timing of the pulsar triple system J0337+1715: Towards a unique and robust test of the strong equivalence principle

N V Gusinskaia¹, A M Archibald¹, J W T Hessels¹,², D R Lorimer³,⁴,⁵, S M Ransom⁶, I H Stairs⁷ and R S Lynch⁵,⁴

¹ Anton Pannekoek Institute for Astronomy, University of Amsterdam, Science Park 904, 1098 XH Amsterdam, The Netherlands.
² ASTRON, the Netherlands Institute for Radio Astronomy, Postbus 2, 7990 AA, Dwingeloo, The Netherlands
³ Department of Physics and Astronomy, West Virginia University, PO Box 6315, Morgantown, WV 26506, USA
⁴ Center for Gravitational Waves and Cosmology, West Virginia University, Chestnut Ridge Research Building, Morgantown, WV 26505, USA
⁵ Green Bank Observatory, Green Bank, WV 24944, USA
⁶ National Radio Astronomy Observatory, Charlottesville, VA 22903, USA
⁷ Department of Physics and Astronomy, University of British Columbia, Vancouver, BC V6T 1Z1, Canada

E-mail: N.Gusinskaia@uva.nl

Abstract. PSR J0337+1715 is a millisecond radio pulsar in a hierarchical stellar triple system containing two white dwarfs. The pulsar orbits the inner white dwarf every 1.6 days. In turn, this inner binary system orbits the outer white dwarf every 327 days. The gravitational influence of the outer white dwarf strongly accelerates the inner binary, making this system an excellent laboratory in which to test the strong equivalence principle (SEP) of general relativity – especially because the neutron star has significant gravitational self-binding energy. This system has been intensively monitored using three radio telescopes: Arecibo, Green Bank and Westerbork. Using the more than 25000 pulse times of arrival (TOAs) collected to date, we have modeled the system using direct 3-body numerical integration. Here we present our efforts to quantify the effects of systematics in the TOAs and timing residuals, which can limit the precision to which we can test the SEP in this system. In this work we describe Fourier-based techniques that we apply to the residuals in order to isolate the effects of systematics that could masquerade as an SEP violation. We also demonstrate that tidal effects are insignificant in the modeling.

1. The pulsar triple system PSR J0337+1715

PSR J0337+1715 is a millisecond radio pulsar that was discovered in 2012 using the Green Bank Telescope (GBT) during a 350-MHz drift-scan survey [1]. It was initially identified as a member of a 1.6-day binary system with an optically detected white dwarf (WD) companion (hereafter WD₁). Soon it became clear that this system has another gravitationally significant component, which orbits the pulsar-WD₁ binary every 327 days. This component (hereafter WD₂) has mass 0.4 M☉, and is an old, cold white dwarf that is invisible in optical observations. As such, PSR J0337+1715 is a member of a hierarchical triple system with two WD components. It is the only known pulsar in a stellar triple system.

Due to the gravitational interactions between the components, it is possible to completely characterize the orbits of this system using direct numerical integration of the three-body equations of motion with perturbations between companions. Fitting this orbital model using
pulsar times of arrival (TOAs) gives precise values for the orbital parameters and the masses of all stellar components. Owing to the hierarchical structure of this system, the gravitational influence of the outer white dwarf strongly accelerates the inner binary. This gives an opportunity to test the strong equivalence principle (SEP) of general relativity: i.e. whether the strongly self-gravitating neutron star (PSR J0337+1715) and weakly self-gravitating WD$_i$ fall in the same way in the external gravitational potential of WD$_o$. Precision timing of the millisecond pulsar PSR J0337+1715 (sub-$\mu$s TOAs), the hierarchical structure of the system and information about the precise masses and orbital parameters of the companions make this system an excellent laboratory in which to test the SEP.

In order to perform the best possible timing of the millisecond pulsar PSR J0337+1715, it was intensively monitored using three world-class radio telescopes: Arecibo, GBT and Westerbork. It was monitored almost daily with Westerbork during the first $\sim$1.5 years after discovery and it has continued to be monitored regularly with GBT and Arecibo for the past 6 years (figure 1). These observations make PSR J0337+1715 one of the most intensively monitored pulsars and provide a unique data set for testing gravitational theories.

![Figure 1. Residuals of the full numerical fit. Each color represents a different telescope: orange – Westerbork, green – GBT and blue – Arecibo. Dashed vertical lines represent $T_{asc}=0$ of the outer orbit.](image)

2. Einstein’s strong equivalence principle

The Einstein strong equivalence principle is a fundamental tenet of general relativity (GR). It postulates that gravitational mass equals inertial mass, even for objects with strong gravity. In other words, the gravitational interaction does not depend on the gravitational binding energy of the self-gravitating bodies. If the gravitational mass ($M_G$) and the inertial mass ($M_i$) are related according to equation:

$$M_G = (1 + \Delta)M_i,$$

where $\Delta$ is a dimensionless parameter, then the SEP states that $\Delta = 0$. This principle holds only in GR and all plausible alternatives to GR violate the SEP at some level ($\Delta \neq 0$). Thus, testing the SEP is an important way to test the validity of GR and to put constraints on its alternatives.

A straightforward method of testing the SEP is to compare how two bodies that have different gravitational binding energies fall in the presence of an external gravitational potential. Thus, the ideal test-beds for the SEP are hierarchical triple systems. If the SEP is violated ($\Delta \neq 0$), then the fractional difference in acceleration of the two bodies gives rise to excess eccentricity of their orbit in the direction of the external acceleration. In the case of the pulsar triple system, the pulsar inner orbit (pulsar - WD$_i$ binary) would acquire an excess eccentricity towards the outer WD$_o$. As such, the pulsar inner orbit will be polarized in the direction of the WD$_o$, keeping this orientation as it moves in its outer orbit. This effect would lead to periodic changes of the orbital parameters with an exact frequency of $(-2\varphi_i + 1\varphi_o)$, where $(\varphi_i)$ and $(\varphi_o)$ are the inner and outer orbital frequencies ($\varphi = 1/P_{orb}$).
3. SEP in the pulsar triple system
In order to model orbital motion of the pulsar triple system we performed direct numerical integration of the three-body equations of motion in the first post-Newtonian approximation. We used this numerical orbital model and standard pulsar timing techniques (see e.g. [2]) to model the observed TOAs of PSR J0337+1715 (see Section 1), fitting for the pulsar and orbital parameters, including a possible difference between gravitational and inertial masses ($\Delta \neq 0$).

As a product of the linear part of the fit, we additionally produced derivatives, which represent the change of residuals corresponding to a linear change of each parameter of the numerical linear fit. The derivative of $\Delta$ represents the response of the timing residuals to a non-zero value of $\Delta$, i.e. to the SEP violation effect. The signature of $\Delta \neq 0$ produces structure in the residuals, here folded at the inner orbital period (figure 2). Structure in the values of the residuals (diagonal stripes) illustrate the inner binary phase shift due to its eccentricity excess towards WD$_o$. The separation and orientation of the stripes confirms that this effect is periodic with frequency: $(-2\varphi_i + 1\varphi_o)$.

4. Quasi-Fourier search
Ideally, the fitted value of $\Delta$ and its uncertainty would determine how well we constrain an SEP violation and whether GR is violated. However, fit uncertainties are only correct once we account for systematics in the residuals. Most importantly, such systematics could potentially masquerade as an SEP violation, especially if they have similar frequency structure. To quantify this, we used a Fourier-based technique to search for systematics whose frequency is harmonically related to the inner and outer orbital frequencies, assuming that these systematics have the same origin and, thus, roughly the same amplitude as the one related to $\Delta$ with $(-2\varphi_i + 1\varphi_o)$ frequency.

We perform a linear least-squares fit of the periodic signal to the residuals of the pulsar timing fit, in the form:

$$f(t) = A \{ \cos(k\varphi_i(t - T_{asc i}) + j\varphi_o(t - T_{asc o})) + \sin(k\varphi_i(t - T_{asc i}) + j\varphi_o(t - T_{asc o})) \}$$

(2)

Here, $t$ represents the TOAs (in MJD), $T_{asc i}$ and $T_{asc o}$ are times when the ascending nodes of the inner and outer orbits, respectively, equal zero, $k$ and $j$ are integer number of inner and outer orbital frequencies, and $A$ is the amplitude of the harmonic systematic. As parameters of the fit, we use two vector components of the amplitude $A$: the amplitudes of sinusoid and cosinusoid with the same frequency. We fit these amplitudes to the residuals (and their uncertainties) provided by the pulsar timing fit. As a result, we measure the amplitude and phase of the signal at each frequency where $k \in [-3; 3]$ and $j \in [0; 3]$. Superposition of these components
is analogous to the discrete two-dimensional Fourier-transform. Note that the sinusoids with components \([k; j]\) are precisely the same as those with components \([-k; -j]\), thus we ignore the half-plane with negative \(j\) components.

We present the results of the quasi-Fourier search in vector plot form (figure 3): each arrow represents a frequency component. The position of the arrow represents the frequency of the systematics as a combination of \(k\) and \(j\) (horizontal and vertical axes values). The length of the arrow represents the amplitude \((A)\) of this systematic, and the direction of this arrow represents its phase offset: i.e. sine and cosine components of \(A\) as vertical and horizontal projections of the arrow. Circles represent uncertainties on the linear least-squares fit parameters (projections of the amplitude \(A\)). We derived these uncertainties using the covariance of \([A_{\cos}; A_{\sin}]\) parameters for each systematic in the fit.

We performed our quasi-Fourier fit to all PSR J0337+1715 timing fit residuals (see blue arrows in figure 3). Our results show that there are still some periodic structures in our pulsar timing fit residuals that are not yet accounted for. The largest systematic that we found in our data has amplitude \(\approx 50\) ns, which is a factor of a few larger than our formal uncertainties.

![Figure 3. “Arrow plot” of the residuals of the full fit (blue arrows) and the signal of non-zero \(\Delta\) (red arrows). The length of each arrow represents the amplitude \(A\) of the harmonic systematic in the data, whose phase is the combination of integer inner and outer orbital frequencies represented by values on the x and y axes, respectively (see eq 2). The direction of each arrow is the phase offset. Circles represent uncertainties on the linear least-squares fit.](image)

With the red arrows in figure 3 we show the result of the quasi-Fourier search performed on the ‘derivative’ of the parameter \(\Delta\). These arrows represent the response of the timing residuals to the linear change of \(\Delta\), and this illustrates that a significant part of the \(\Delta \neq 0\) signal is concentrated, indeed, in the \((-2\varphi_i + 1\varphi_o)\) component. The lengths of these arrows are arbitrary, and correspond to a value of \(\Delta\) chosen for a convenient illustration.

The blue arrow that corresponds to the \((-2\varphi_i + 1\varphi_o)\) frequency is smaller than the fit uncertainty, which means that the timing fit provided the correct amplitude of the harmonic signal with this frequency. However, this signal does not necessarily correspond to the effect of \(\Delta \neq 0\), since there are many other arrows that have amplitude significantly larger than the statistical uncertainty. Thus, the amplitude of the signal that the pulsar timing fit returns as a \(\Delta\) value can actually be the result of an unknown systematic, similar to what is present in other frequency components. That means that we can not use the fitted value of \(\Delta\) and its uncertainty provided by the fit to constrain an SEP violation, but instead our constraint should be based on a measure of the arrow length in the quasi-Fourier fit. In this way, we can use our quasi-Fourier search to put a reliable constraint on the parameters of the numerical fit and, consequently, on the SEP test results.

5. Tidal effects
One of the effects that could produce systematics in the data is secular changes of the pulsar orbit caused by tidal deformation of the inner WD companion. The neutron star raises tides on the
inner WD, and this quadrupole moment introduces a non $1/r^2$ dependence of the gravitational pull. We do not model tidal effects in our timing fit. Thus, if they are present, they might appear as systematics in the timing residuals. However, even the inner binary of the triple system has a relatively wide orbit (16 lt-s), which suggests that tidal effects should be small.

In this section we provide calculations of the changes of longitude of periastron ($\dot{\omega}$) and shrinking of the orbit ($\dot{P}_{\text{orb}}$) of the pulsar – WD inner binary due to tidal effects. In our calculations we followed formulas provided by [3]. Since all the tidal effects strongly depend on the distance between objects, we do not provide the values of the tidal effects of the outer WD, whose orbit is a factor of 200 larger.

**Secular periastron advance:** The change of the longitude of periastron of the pulsar orbit caused by tidal deformation of its inner WD companion can be calculated using eq 3.7 of [3]:

$$\Delta \chi_{\text{tidal}} = k_2 \left( \frac{m_p}{m_c} \right) \left( \frac{R_c}{a} \right)^5 F(e) = 0.0036 \text{ deg/year}$$

(3)

Here, $R_c = 0.091 R_\odot$ is the radius of WD$_i$, derived from optical observations [4], $m_p = 1.4478 M_\odot$ is the mass of the pulsar, $m_c = 0.19751 M_\odot$ is the mass of the WD$_i$, $a = 16 \text{ lt-s}$ is the semi-major axis of the pulsar – WD$_i$ binary and $F(e) = 1$ is a function of its eccentricity. The orbital parameters and masses are derived from the numerical fit of the three-body interactions [1]. We used eq 27 from [5] to calculate the Love number: $k_2 =0.102$, which is consistent with other results for WDs (see e.g. [6]).

In summary, the change of $\chi_{\text{tidal}}$ of the pulsar inner orbit is about three orders of magnitude smaller than relativistic periastron advance and about one order of magnitude smaller than what can be detected with the current precision of the 6 years of PSR J0337+1715 timing. However this effect may be significant after $\sim 10$ more years of observations.

**Dissipative effect:** The tidal deformation of WD$_i$ can cause a loss of energy from the system (tidal lag) and thus shrinking of the orbit. We used eq 3.19 from [3] to calculate the characteristic timescale of the orbital period ($\dot{P}_{\text{orb}}$) decrease:

$$\tau_p \equiv \frac{P_{\text{orb}}}{\dot{P}_{\text{orb}}} \approx \frac{3}{2}(1 - e^2)^6 \left( \frac{m_c^2}{m_p(m_p + m_c)} \right) \left( \frac{n}{\omega_c} \right) \left( \frac{a}{R_c} \right)^8 \left( \frac{m_c}{\mu > R_c} \right) = 1.16 \times 10^{14} \text{ yr}$$

(4)

Following [3], here we make a conservative estimate, assuming that the inner WD$_i$ has a high rotation rate ($\omega_c = 0.1 \text{ rad/s}$) and a standard viscosity $<\mu> = 0.02 \times 10^{13} \text{ g/cm/s}$. The characteristic timescale of the PSR J0337+1715 inner binary orbital decay due to tidal lag $\tau_p$ is $\sim 2$ orders of magnitude longer than the characteristic timescale of the orbital decay due to gravitational wave emission $\tau_{gw}$, which is $\sim 2 \times 10^{12} \text{ yr}$ (see e.g. [7] for formula for $\tau_{gw}$). The current precision with which we can measure $P_{\text{orb}}$ of the inner binary is $\approx 10^{-8} \text{ days}$. This means that we need about $10^4$ years of observations in order to be able to detect dissipative tidal effects in the inner binary of the pulsar triple system.

**References**

[1] Ransom S M 2014 Nature 505 520–4
[2] Lorimer D R and Kramer M (eds) 2012 *Handbook of Pulsar Astronomy* (Cambridge, UK: Cambridge University Press)
[3] Smarr L L and Blandford R 1976 *Astrophysical Journal* 207 574–88
[4] Kaplan D L 2014 *The Astrophysical Journal Letters* 783 L23
[5] Hinderer T 2008 *The Astrophysical Journal* 677 1216–20
[6] DallOsso S and Rossi E 2013 *MNRAS* 428 518–31
[7] Wagoner R V 1975 *The Astrophysical Journal Letters* 196 L63–5