Phenomenological analysis of $K^+$-meson production in proton-nucleus collisions

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(Dated: March 30, 2022)

We investigate the experimental data, total and differential cross sections, on the production of $K^+$-mesons in $pA$ interactions at projectile energies between $T_p=0.8$ and 2.9 GeV, covering the transition across the free nucleon-nucleon threshold at 1.58 GeV. No clear evidence for the expected change of the dominant reaction mechanism from two-step to direct kaon production is found. It is suggested that further data, in particular at forward angles, are taken in order to clarify the situation. It also is shown that, independent of the beam energy and emission angle, the invariant $K^+$-production cross sections show an overall exponential scaling behaviour with the squared four-momentum transfer between the beam proton and the produced $K^+$-meson for $t<0.05$ GeV$^2$. The most recent data from COSY-Jülich, differential cross sections measured for $t>0$ GeV$^2$, show a strongly different $t$ dependence. Further data at forward angles and different beam energies are needed in order to exploit this region of kinematically extreme conditions.

PACS numbers: 29.40.-n; 25.40.-h
Keywords: $K^+$-mesons; Meson production; Medium modifications

I. INTRODUCTION

The study of sub- and near-threshold proton-induced production of $K^+$-mesons in nuclei has received considerable interest during the last two decades. Due to the rather high $K^+$-production threshold in free nucleon-nucleon collisions ($T_{NN}=1.58$ GeV) and to the large mean free path of $K^+$-mesons in nuclear matter, one hopes to extract information about the intrinsic properties of the target nuclei and in-medium properties of the kaons. Obviously, in order to extract this information, it is essential to determine the $K^+$-production mechanisms.

Total $K^+$-production cross sections have been measured at the PNPI synchrocyclotron for targets between Be and Pb and projectile energies $T_p$ between 0.8 and 1.0 GeV [1], i.e., at beam energies far below the free nucleon-nucleon threshold. Inclusive differential cross sections have been studied at BEVALAC [2], SATURNE [3] and CELSIUS [4]. Partial momentum spectra have been obtained at different laboratory emission angles in the range $10^\circ-90^\circ$, at projectile energies between 1.2 and 2.1 GeV. Recently, full momentum spectra at forward emission angles $<12^\circ$ have been measured with the ANKE spectrometer of COSY-Jülich [5] at $T_p=1.0$ GeV. $K^+$ production has also been studied at ITEP [6, 7], where kaons with fixed momenta and emission angle $10.5^\circ$ were identified at projectile proton energies of 1.75 to 2.9 GeV.

The results were discussed in terms of different models [1, 2, 4, 6, 10, 11]; in particular of single or two-step reactions, the latter with the creation of an intermediate pion. It has been argued that the dependence of the $K^+$-production cross section on the target mass $A$ is sensitive on the mechanism dominantly contributing to kaon production.

In Sect. [1] it is studied whether a systematic analysis of the $A$ dependence of the existing data allows, indeed, to draw conclusions on the involved reaction mechanisms. In Sect. [11] it is shown that data taken at the same beam energy $T_p$ but for different kaon momenta and emission angles can be easily compared if plotted as a function of the four-momentum transfer $t$ between the beam proton and the outgoing kaon.

II. TARGET-MASS DEPENDENCE

The first aim of our analysis is to evaluate the exponent $\alpha$ by fitting the cross sections measured for different target nuclei with an $A^\alpha$ dependence. We analyze available data on $K^+$-production in $pA$ collisions at beam energies $T_p \leq 2.9$ GeV. The data from Refs. [4, 7] are not taken into account here, since only a single target material was used for these measurements. A similar analysis of the $A$ dependence for meson production at high energies can be found in in Refs. [12, 13, 14].

The target-mass dependence for $K^+$-meson production in $pA$ collisions can be factorized in terms of the $A$ dependence of the production process and that of the kaon propagation and distortion in the nuclear medium. Due to strangeness conservation, kaons with relatively low momenta which are considered in this work, are not absorbed in nuclear matter after their creation and can carry out the information about the production mechanism. Also quasi-elastic scattering of $K^+$-mesons in nu-
The $A$ dependence for direct kaon production in collisions of the beam proton with a single target nucleon is given by the inelasticity of the $pA$ interaction. This also holds for the production of other mesons, since the summation over all possible mesonic inelastic $pA$ reaction channels is proportional to the total inelastic $pA$ cross section. The data [14, 16] on total inelastic proton-nucleus cross sections can be well fitted by an $A^α$ dependence with $α=0.69±0.03$ at proton beam energies from 0.84 to 2 GeV. Modifications of the $A$ dependence can be due to the intrinsic momenta of the participating nucleons, to Pauli blocking and other nuclear effects, which essentially should not depend on the target mass [17]. Therefore, one expects that the $K^+$-production cross section is proportional to $A^{0.7}$ if the kaons are dominantly produced via the direct production mechanism.

The $K^+$-mesons can also be produced in two-step mechanisms with intermediate pion production in $pN \rightarrow πX$ reactions followed by a $πN→K^+X$ process on a second target nucleon. Since two nucleons are needed for the kaon production, a stronger $A$ dependence for two-step production as compared with direct kaon production is expected. Depending on the beam energy, $K^+$-production may be due to both, the direct and two-step reaction mechanisms. At low beam energies the two-step processes are energetically favorable since the intrinsic nucleon motion can be utilized twice.

It has also been suggested that, in particular at deep subthreshold energies, $K^+$-production is due to many-body interactions (like e.g. the formation of clusters in the target nucleus) or is a reflection of a high degree of collectivity in the target nucleus. We expect that such effects are proportional to the number of target nucleons and, therefore, the cross section should scale as $A^4$.

Total $K^+$-production cross sections $σ_{tot}$ in proton-nucleus collisions at $T_p≤1.0$ GeV for $Be$, $C$, $Cu$, $Sn$ and $Pb$ as target nuclei have been measured by Koptev et al. at the Petersburg Nuclear Physics Institute (PNPI). We recall that the threshold for $K^+$-production in free $pN$ collisions is $1.58$ GeV and, thus, the PNPI measurements were devoted to study deep-subthreshold strangeness-production mechanisms. At each beam energy $σ_{tot}$ was fitted by a function $σ(A)=const \cdot A^α$. Figure 1 shows the resulting parameters $α$ as a function of $T_p$. In the full energy range $0.842≤T_p≤0.990$ GeV, the data can well be described by a constant value $α=1.04±0.01$ as indicated by the solid line and listed in the first line of Table 1. The strong $A$ dependence of the total $K^+$-production cross section has been interpreted [14, 15] as an indication for the dominance of non-direct $K^+$-meson production in $pA$ collisions at energies far below the free nucleon-nucleon threshold.

To obtain more detailed information about the strangeness-production mechanisms, differential $K^+$-production cross sections were subsequently measured by several groups. Kaon production induced by 2.1 GeV protons (i.e. above the free $NN$ threshold) on $NaF$ and $Pb$ targets has been studied by Schnetzer et al. at the Lawrence Berkeley Laboratory (LBL). The kaons were measured at emission angles of $θ_K=15^\circ$, $35^\circ$, $60^\circ$ and $80^\circ$ and for momenta in the range $0.350$–$0.750$ GeV/$c$. We have fitted the mass dependence of the double differential cross sections by $d^2σ/dp\cdot dΩ(A)=const \cdot A^α$ and show in Fig. 2 the parameter $α$ as a function of the kaon momentum and for the different production angles. Since no dependence of $α$ on the kaon momenta is observed, we fit $α(p_K)$ by constant values. The results are shown in Fig. 2 by the solid lines and in Table 1.

The mass dependence is weak ($α=0.56±0.05$) for $θ_K=15^\circ$, and increases to larger angles ($α=0.88±0.08$ for $θ_K=80^\circ$). This might be explained by the fact that the direct reaction mechanism dominates at forward laboratory angles due to the Lorentz boost of this production process. It is also possible that the higher values of $α$, observed at larger angles, are related to a higher transparency of the nucleus in case of particle production with

![FIG. 1: Dependence of the parameter $α$ on the beam energy $T_p$ evaluated from the PNPI data [1] on total $K^+$-production cross sections. The solid line indicates the fitted average value $α=1.04±0.01$.](image)
high perpendicular momenta. This phenomenon is well established in many processes and is caused by the small size of the produced particles. In this case, however, it starts at rather low momenta. The dependence of $\alpha$ on the kaon momentum does not indicate a transition from the direct production process to two- or multi-step $K^+$-production. In the full momentum range, the LBL results are consistent with or close to the expectation for direct $K^+$-meson production and the $A$ dependence is weaker than the one of the PNPI data.

Double differential cross sections in $pC$ collisions at $T_p=1.2, 1.5$ and 2.5 GeV and in $pPb$ collisions at $T_p=1.2$ and 1.5 GeV at a production angle $\theta_K=40^\circ$ have been measured by Debowski et al. at SATURNE. The parameter $\alpha$ evaluated from the data at $T_p=1.2$ and 1.5 GeV is shown in Fig. 2 as a function of $p_K$. For both beam energies, the results can be well fitted by a constant value $\alpha=0.73\pm0.04$. In Table 1 the values for $\alpha$ are shown which were obtained from individual fits for 1.2 and 1.5 GeV. It can be seen that at 1.2 GeV the large error of $\alpha$ does not allow any conclusions about the reaction mechanisms. At this energy a dominance of two-step kaon production is expected. At 1.5 GeV the relatively small value of $\alpha=0.73\pm0.04$ is in line with the expected dominance of one-step kaon production.

Double differential cross sections for $K^+$-production with $Be$, $Al$, $Cu$ and $Ta$ targets were measured by Akindinov et al. at the Institute for Theoretical and Experimental Physics, Moscow (ITEP).

For $1.65\leq T_p \leq 2.91$ GeV, kaons with an emission angle $\theta_K=10.5^\circ$ and fixed momentum of $p_K=1.28$ GeV/c were detected. Below $T_p=2.2$ GeV, such high-momentum kaons cannot be produced in a free NN collision. Thus it is justified to assign these kaons to subthreshold particle production. We have fitted the $A^\alpha$ dependence and show $\alpha$ as a function $T_p$ in Fig. 4. The increase of $\alpha$ at low beam energies is not understood so far. It maybe either due to statistical fluctuations (indicated by the dashed line) or reflect that below $T_p\approx 2.2$ GeV there is a transition from direct to two-step $K^+$-production. Within the experimental uncertainties, the $A$ dependence of the ITEP data is compatible with the LBL and SATURNE results.

The results on the $A$ dependence of kaon production are collected in Table 1. The PNPI data on total $K^+$-production cross sections at beam energies $0.842\leq T_p \leq 0.990$ GeV indicate a strong, $\propto A^1$, dependence. The data on differential cross sections available for proton beam energies from 1.2 up to 2.9 GeV yield an $A^\alpha$ dependence with $\alpha$ varying between $0.54\pm0.02$ and $0.88\pm0.08$. Thus, for beam energies $1.2\leq T_p \leq 2.9$ GeV the average value of $\alpha=0.71\pm0.08$ is in reasonable agreement with the $A$ dependence expected from the direct production mechanism.

The inconsistency between the PNPI data and the other experiments might be related to the different beam energies. In fact, since the PNPI measurements were performed at $T_p<1.0$ GeV, thus substantially below the kaon production threshold in free space, two-step mechanisms or many body effects should be more pronounced. To draw final conclusions one would like to have data on differential cross sections at beam energies close to 1.0 GeV. The available data in-
obtained from the total cross sections \cite{1} (αKd).\footnote{The solid line indicates a fit to the data with a constant value α=0.54±0.02. The dashed line only serves to guide the eye.}

The PNPI and the ANKE measurements is that at ANKE we weaker data shown in Fig. 2. However, these data were obtained at a significantly higher beam energy where kaon production in single step reactions should dominate. It would be interesting to check with new data from ANKE whether at 2.1 GeV α is also significantly smaller at angles around 0°.

We also find that the momentum-integrated cross section from ANKE for angles θK≤12° corresponds to roughly 10% of the total cross section, σ_{tot} = 39 nb for pC collisions at \(T_p=0.990\) GeV \cite{18}, whereas the covered solid angle corresponds to less than 1% of 4\π \cite{18}. This indicates a strong forward peaking of the produced kaons in the laboratory system.

FIG. 4: \(\alpha\) as a function of the beam energy \(T_p\) evaluated from the ITEP data \cite{5} on differential \(K^+\)-meson production cross sections for \(p_K=1.28\) GeV/c and \(θ_K=10.5°\). The solid line shows a fit by a constant value α=0.54±0.02. The dashed line only serves to guide the eye.

FIG. 5: Dependence of α on \(p_K\) evaluated from the ANKE data \cite{5} at \(T_p=1.0\) GeV and \(0°≤θ_K≤12°\). The solid line shows the fit by a constant value α=0.74±0.05.

In contrast to previous studies at higher energies, the ANKE experiment covers the full momentum range of the produced kaons at 1.0 GeV \cite{5}. Figure 5 illustrates that the data do not show any dependence of α on \(p_K\) for the complete \(K^+\)-momentum spectrum. Thus there is no indication for a change of the \(A\) dependence due to a possible transition from the direct to the two-step production mechanism. We cannot provide any conclusive explanation for this observation and for the low values of α, rather than that for the forward subthreshold \(K^+\)-production the direct mechanism appears to be dominant. This is in strong contrast to results of model calculations \cite{8, 9, 10, 11} where most of the kaons are produced in the two-step mechanism. It can only be speculated here that two-step production combined with rescattering effects of the produced kaons may result in a weaker \(A\) dependence.

The results collected in Table \ref{tab:1} do not allow unambiguous conclusions about the reaction mechanisms. In particular, there is a strong disagreement between the 1.0 GeV data from ANKE \cite{5} measured under forward angles and the total cross sections from PNPI \cite{1} obtained at the same beam energy. In order to clarify the situation and to allow a comparison with the data on differential cross sections obtained at higher energies \cite{2, 3, 6, 19} it is necessary to systematically measure kaon production at forward angles and at higher energies. These data would.
also help to reconstruct the angular dependences of kaon production in pA collisions at the higher energies.

III. SYSTEMATICS OF KAON SPECTRA

At high energies data on hadron production in pA collisions are generally analyzed in terms of the Feynman variable $x_F$ and the transverse momentum $p_t$ of the produced particle. The Feynman-scaling variable is defined as $x_F = p_L / p_{\text{max}}$, where $p_L$ is the longitudinal momentum of the produced particle in the center-of-mass system (CMS) of the incident proton and the target nucleon, while $p_{\text{max}}$ is the maximum CMS hadron momentum at a given beam energy. However, one cannot apply the commonly adopted $x_F$ analysis of the $K^+$-data at energies below the free $NN$ threshold, since a $pN$ CMS with the nucleon at rest is kinematically not allowed. One may select an overall $pA$ CMS $\mathcal{J}$, however then the joint analysis of subthreshold kaon production and the data available above the $NN$ threshold becomes rather questionable.

In order to compare kaon spectra measured at different kinematical conditions, i.e. proton beam energies $T_p$ as well as the kaon momenta $p_K$ and emission angles $\theta_K$, we propose a more natural kinematical variable given by the squared four-momentum transfer $t$ between the produced kaon and the incident proton

$$t = m_p^2 + m_K^2 - 2\sqrt{(p_p^2 + m_p^2)(p_K^2 + m_K^2)} + 2p_p p_K \cos \theta_K$$

where $m_p$, $m_K$, $p_p$ and $p_K$ are the proton and kaon masses and laboratory momenta, respectively, while $\theta_K$ denotes the $K^+$-meson emission angle measured in the laboratory system relative to the direction of the proton beam. Since $t$ is Lorentz invariant the analysis becomes independent of the choice of the reference CMS, $pN$ or $pA$.

A large four-momentum transfered from the incident proton to the target (corresponding to large negative values of $t$) induces excitation and disintegration of the target nucleus. Reactions with small $|t|$ are those where the produced $K^+$-mesons carry away a substantial part of the momentum and energy of incident proton. For a $pA \rightarrow K^+X$ reaction the minimum and maximum values of $t$ follow from Eq. (1) as:

$$t_{\pm} = m_p^2 + m_K^2 - 2\sqrt{(q_p^2 + m_p^2)(q_K^2 + m_K^2)} \pm 2q_p q_K,$$ (2)

where the sign of the last term corresponds to kaon production in forward and backward direction relative to the proton-beam momentum in the overall $pA$ CMS. $q_p$ and $q_K$ are the proton and $K^+$-meson three momenta in the $pA$ CMS:

$$q_p^2 = \frac{(s - m_p^2 - m_A^2)^2 - 4m_p^2 m_A}{4s},$$

$$q_K^2 = \frac{(s - m_K^2 - m_A^2)^2 - 4m_K^2 m_A}{4s},$$

where $m_A$ denotes the target mass and $s$ the total invariant energy of the beam proton with $T_p$, and the target

$$s = m_p^2 + m_A^2 + 2m_A(m_p + T_p).$$ (4)

In Eq. (4) $m_X$ represents the invariant energy of the residual final system with respect to the detected kaon. The minimum and maximum values of $m_X$ are given as

$$m_X^{\text{min}} = m_A + m_A,$$ (5)

$$m_X^{\text{max}} = \sqrt{s} - m_K,$$

where $m_A$ is the $\Lambda$-hyperon mass. Note that by the definition of $m_X^{\text{min}}$ we neglect the $\Lambda$ binding energy, i.e. formation of $\Lambda$-hypermni. $m_X^{\text{max}}$ corresponds to $K^+$-meson production with transfer of all available collision energy to the invariant mass of the residual system $X$ leading to $q_K = 0$. The maximum and minimum values of $t$ are given by Eq. (2) with minimum mass of the residual system $m_X^{\text{min}}$.

The maximum positive squared four-momentum transfer $t_+$ depends both on the beam energy $T_p$ as well as the target mass $m_A$. Figure 6 shows the $T_p$ dependence of $t_+$ calculated for a carbon target and $m_X^{\text{min}}$. It is seen that $t_+$ can be positive and has its maximum at small beam energies around $T_p \approx 1.5$ GeV. It is of specific interest to check whether processes with such positive values of $t_+$ are experimentally accessible.

**FIG. 6:** Values for the maximum positive $t_+$ for $pA \rightarrow K^+X$ reactions as a function of the beam energy $T_p$. $t_+$ has been calculated for a minimum mass of the residual system $m_X = m_A + m_A$, neglecting binding energy of the $\Lambda$-hyperon in the target nucleus.

The $pA \rightarrow K^+X$ reaction amplitude $\mathcal{A}$ is a function of the invariant collision energy $s$, the squared four momentum transfer $t$ and the squared invariant mass $m_X^2$ of the residual system. Obviously, these Mandelstam variables can be expressed in terms of the laboratory observables as the $K^+$-emission angle $\theta_K$ and momentum $p_K$. Following Regge theory we factorize the $s$ dependence of the
reaction amplitude as
\[ \mathcal{A}(s, t, m_X^2) \propto f(t, m_X^2) \exp[\gamma(t) \ln(s/s_0)], \]
where functions f and \( \gamma \) are given by the Regge pole with quantum numbers of a strange baryon like a \( \Lambda \) or \( \Sigma \) hyperon and \( s_0 \) characterizes the domain of validity of the Regge-pole theory. Assuming a linear t dependence of the Regge trajectory one can expand \( \gamma(t) = \gamma_0 + \gamma_1 t \), with \( \gamma_0 \) and \( \gamma_1 \) being constant.

Within the beam-energy range \( 1.0 \leq T_p \leq 2.9 \) GeV, \( \ln(s/s_0) \) at \( s_0 = 1 \) GeV\(^2\) varies between 5.14 and 5.37 and, therefore, the reaction amplitude can be considered as a function of \( t \) and \( m_X^2 \) only. In the following we investigate whether the \( t \) dependence can be factorized out of both, the \( s \) and \( m_X^2 \) dependences, for the \( pA \rightarrow K^+X \) data. According to Eq. 6 we describe the invariant \( K^+ \)-production cross section as
\[ E \frac{d^3\sigma}{d^3p} = c_0 \exp[b_0 t], \]
with parameters \( c_0 \) and \( b_0 \) to be fitted to the data. Finally, we intend to investigate the dependence of the slope \( b_0 \) on the beam energy \( T_p \) and the kaon emission angle \( \theta_K \), which both are related to \( s \) and \( m_X \).

Figure 7 shows the invariant cross section \( E \frac{d^3\sigma}{d^3p} \) for \( K^+ \)-production in \( p(NaF) \) collisions at \( T_p = 2.1 \) GeV. The measurements cover a \( t \) range from \(-0.15 \) to \(-3.6 \) GeV\(^2\). For all measured angles and \( t \leq -0.7 \) GeV\(^2\) the invariant cross section can be well fitted by Eq. 6 with parameters \( c_0 \) and \( b_0 \) listed in Table II. At \(-0.7 \leq t \leq -0.15 \) the invariant cross section can be reasonably fitted by a constant value \( c_1 \), see Fig. 7 and Table II. The exponential dependence of the invariant production cross section is not surprising and quite typical for hadronic reactions. However, the constant \( t \) dependence at low squared four-momentum transfer (small kaon emission angles) seems to be unusual and needs further clarification.

The \( t \) dependence of the invariant cross section for \( K^+ \)-production in \( pPb \) collisions at \( T_p = 2.1 \) GeV is shown in the lower spectrum of Fig. 7. Again, the data show an overall exponential scaling behaviour in \( t \) and can be reasonably described by Eq. 6 at \( t \leq -0.7 \) GeV\(^2\), while they are almost constant at small \( t \). However, the \( pPb \) data indicate some systematic deviation from the exponential dependence at large angles which can be attributed to function \( f(t, m_X^2) \) in Eq. 6. The slope \( b_0 \) obtained from the \( pPb \) data is close to that fitted to the \( p(NaF) \) data, see Table II.

The \( t \) dependence of the invariant cross section for \( K^- \)-meson production in \( pC \) collisions at \( T_p = 1.2, 1.5 \) and 2.5 GeV and in \( pPb \) collisions at beam energies of 1.2 and 1.5 GeV is shown in Fig. 8. It is found that the data clearly follow an exponential dependence. The fitted slopes \( b_0 \), see Table II, substantially depend on the proton-beam energy. At small \( T_p \) the \( t \) dependence is steeper. However, when comparing the data with those from 2, the different slopes cannot completely be attributed to the variation of \( T_p \), see first and fifth line of Table II. The most probable explanation could be essentially different excitations of the residual nuclei in these cases. This problem clearly needs further investigation.

At low \( T_p \) the large values might be caused by the sub-threshold production mechanisms. With the presently available data it is not possible to extract a systematic dependence of the parameters \( c_0 \) and \( b_0 \) on \( T_p \).

Figure 9 shows the \( t \) dependence of the invariant \( K^+ \)-production cross section from \( pC \) collisions at \( T_p = 1.2 \) GeV and \( \theta_K = 90^\circ \). Again, the data can be well fitted by an exponential function with parameters given in Table II.

We also analyzed the data from Ref. 2, where the \( T_p \) dependence of \( K^+ \)-production in \( pA \) collisions was studied for fixed kaon momentum \( p_K = 1.28 \) GeV/c and production angle \( \theta_K = 15^\circ \). The invariant cross section for \( pBe \) collisions is shown in Fig. 10 as a function of \( t \). An important feature of the data is that they were essentially collected at positive values of \( t \). In principle, the measurements probe the region around the maximum positive squared momentum transfer \( t_+ \).
TABLE II: $t$ dependence evaluated from the data on $K^+$-production in $pA$ collisions for different proton-beam energies $T_p$. $t_{\min}$ and $t_{\max}$ indicate the range of the squared four-momentum transfer measured in the individual experiments. Parameters $c_0$ and $b_0$ were fitted with Eq. 7 to the data at large $|t|$, while $c_1$ was evaluated by fitting with a constant value at small $|t|$, see text. $c_0$ and $c_1$ are given in units of $(\text{mb} \cdot \text{GeV}^{-2} \cdot \text{c}^3 \cdot \text{sr}^{-1})$.

| $T_p$ (GeV) | $A$ | $t_{\max}$ | $t_{\min}$ | $c_0$ | $b_0$ | $c_1$ | Ref. |
|-------------|-----|------------|------------|-------|-------|-------|------|
| 2.1 NaF     | -3.58 ± 0.15 | 3.7±0.9 | 1.5±0.1 | 2.6±0.1 |
| 2.1 Pb      | -3.58 ± 0.15 | 32±8    | 1.6±0.1 | 9.5±0.8 |
| 1.2 C       | -0.41 ± 0.48 | 15.1±1.9 | 28±3     | —     |
| 1.5 C       | -0.59 ± 0.64 | 9±3     | 25.5±0.8 | —     |
| 2.5 C       | -1.22 ± 1.65 | 24±3    | 7.4±0.1  | —     |
| 1.2 Pb      | -0.41 ± 0.45 | 12±9    | 35±2     | —     |
| 1.5 Pb      | -0.58 ± 0.7  | 55±5    | 17.3±0.2 | —     |
| 1.2 C       | -1.10 ± 1.25 | 53±21   | 6.4±0.3  | —     |

As we found before, the $t$ dependence can not be trivially factorized from the $T_p$ dependence. For the data from Fig. 6 each experimental point corresponds to a different proton-beam energy $T_p$, for which, for fixed $p_K$ and $\theta_K$, can be calculated with Eq. 7. Also $t_+$ is a function of $T_p$, and different for each of the data points, which makes the analysis of the data quite ambiguous. Thus, with the data from Ref. 6 we cannot verify the results for the data from Ref. 7 indicating a constant $t$ dependence of the invariant kaon production cross section at $t\geq 0$. We do not show the results of 7 for the $Al$, $Cu$ and $Ta$ targets, which have similar $t$ dependence as the data for $pBe$ collisions.

Finally, the most recent results 8 on $K^+$-production in $pC$ collisions at $T_p=1.0$ GeV and $0\leq \theta_K \leq 12^\circ$ are shown in Fig. 10. It can be seen that the measurements probe the region of $t\geq 0$, as well as close to $t_+$ from Fig. 8. The data confirm that at small $|t|$ the invariant $K^+$-production cross section is almost independent of $t$. Within the range $-0.32 \leq t \leq -0.07$ GeV$^2$ we fit the data by a constant value $c_1=1.6\pm0.1$ mb GeV$^{-2}$ c$^3$ sr$^{-1}$.

For positive values of $t$ the dependence of the invariant $K^+$-production cross section obviously changes and becomes very steep in the range $-0.031 \leq t \leq 0.09$ GeV$^2$ where it can be fitted by Eq. 7 with

$$c_0 = 1.07 \pm 0.08 \ (\text{mb} \cdot \text{GeV}^{-2} \cdot \text{c}^3 \cdot \text{sr}^{-1})$$

$$b_0 = -11.1 \pm 1.7 \ (\text{GeV}^2)$$

The strong decrease of the cross section towards $t_+$ is not surprising since this region corresponds to the formation of hypernuclei which characteristically is accompanied by very small cross sections, see e.g. Ref. 18.

![FIG. 8: $t$-dependence of the invariant cross section for $pC\rightarrow K^+X$ reactions at $T_p=1.2, 1.5$ and 2.5 GeV (upper) and $pPb\rightarrow K^+X$ reactions at $T_p=1.2$ and 1.5 GeV (lower). The data were measured at $\theta_K=40^\circ$ and are taken from Ref. 11. The lines show a fit to the data using Eq. 7 with parameters listed in Table II.](image)

Table 7 shows that all available data taken at subthreshold energies cover only very limited ranges of $t$, making systematical analyses, like e.g. the extraction of the remaining $T_p$ and $\theta_K$ dependences, impossible. Obviously, more data are needed at different proton-beam energies $T_p$, as well as $K$ angles different from the previously measured ones. According to the reaction kinematics shown by Eq. 8, the extreme values $t_\pm$, corresponding to large $t$ intervals, are accessible at forward laboratory angles $\theta_K \approx 0^\circ$. In this respect the ANKE spectrometer 19 is particularly useful since it allows to measure kaon production at $0^\circ \leq \theta_K \leq 12^\circ$, and offers a wide kaon-momentum coverage for beam energies in the range $1.0 \leq T_p \leq 2.3$ GeV.

### IV. SUMMARY

Our analysis of the target-mass dependence of $K^+$-meson production in $pA$ collisions at $T_p \leq 2.9$ GeV show that, based on the existing data 4 6 7 8 9 10 11, it is
not possible to draw unambiguous conclusions about the underlying reaction mechanisms. In particular, at $T_p=1.0$ GeV there is a discrepancy between the PNPI data on total cross sections and those from ANKE obtained under forward angles. The data on differential cross sections indicate that the parameter $\alpha$ only weakly depends on the beam energy $T_p$ and kaon emission angle $\theta_K$ and is independent of the kaon momentum. Surprisingly, for all these data taken in the energy range $1.0 \leq T_p \leq 2.9$ GeV, the target-mass dependence is close to the expectation for direct kaon production. We conclude that further $K^+$-momentum spectra should be measured at forward angles and higher beam energies. The $A$ dependence of such data should be most sensitive to a transition from one- to two-step reactions. Furthermore, microscopical model calculations should be performed which include rescattering effects of the produced kaons in the nuclear medium.

The invariant $K^+$-production cross sections obtained under different kinematical conditions show an overall exponential scaling behaviour with the four-momentum transfer between the beam proton and the produced kaon for $t<-0.05$ GeV$^2$. This indicates that the Regge model is applicable at large negative values of $t$ even for subthreshold kaon production down to $T_p=1.2$ GeV. The dependence on $t$ becomes steeper for small beam energies and kaon emission angles $\theta_K$. It would be interesting to check by future measurements whether for $t<-0.05$ GeV$^2$ the exponential $t$ scaling is violated at $\theta_K \simeq 0^\circ$. It can be speculated that here the $t$ dependence becomes very strong.

At small negative values, $t>-0.05$, the invariant cross sections are almost independent of $t$, if measured at the same beam energy $T_p$. This indicates a transition from the Regge regime to boson-exchange models. For $t>0$ the data show a strong falloff towards the kinematical limit corresponding to the formation of hypernuclei.

**ACKNOWLEDGMENTS**

This work profited significantly from discussions with members of the ANKE collaboration, in particular W. Cassing, M. Hartmann and V. Hejny. Financial support by BMBF (WTZ grant RUS-685-99) and RMS (grant FNP-125.03) is gratefully acknowledged by one of the authors (V.K.). B.I. would like to thank J. Speth and
A. Sibirtsev for the hospitality at the IKP of the FZJ and the A.v. Humbold foundation for financial support of his visit. His work is supported in part by US CRDF (grant RP2-2247), INTAS (grant 2000-587) and RFBR grant (00-02 17808).

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