First principles cosmology of the Standard Model epoch

Daniel Friedan

New High Energy Theory Center and Department of Physics and Astronomy, Rutgers, The State University of New Jersey, Piscataway, New Jersey 08854-8019 U.S.A. and Science Institute, The University of Iceland, Reykjavik, Iceland
dfriedan@gmail.com physics.rutgers.edu/~friedan

April 14, 2022

Abstract

This is a summary of a project to construct a first principles cosmology of the Standard Model epoch, the period starting shortly before the electro-weak transition. The cosmology is derived from a simple initial state entirely within the SM and General Relativity. The initial state is semi-classical — concentrated near a classical solution of the SM equations of motion — and is precisely specified by a few simple conditions. The dark matter is a classical effect, a coherent state of the SU(2)-weak gauge field and the Higgs field. The leading order, classical universe contains only the dark matter. Ordinary matter is a correction due to the fluctuations of the SM fields around the classical trajectory. The initial state produces a homogeneous, isotropic, flat universe. There are no adjustable parameters. No physics beyond the SM is invoked. Only the classical calculations have been done so far. The time evolution of the fluctuations remains to be calculated.

1 Introduction

The Standard Model cosmological epoch is the period starting at energy scale roughly 1 TeV somewhat before the electro-weak transition The project is to formulate a simple initial quantum state whose time evolution accurately explains the SM epoch entirely within the SM and classical GR. It is a top-down approach to cosmology. The initial state is completely specified by a few simple conditions:

1. The universe is governed by the SM and classical GR (with \( \Lambda \)).
2. The universe is a 3-sphere.
3. The initial state of the SM epoch is semi-classical.
4. The initial state has a certain Spin(4) symmetry.
5. The initial energy is > \( 10^{107} \) in natural units.

The cosmology is a systematic expansion around a classical solution of the SM, a coherent state of the Higgs field and the SU(2) gauge field (the cosmological gauge field or CGF). The CGF acts as a perfect fluid with \( w_{\text{CGF}} \approx 0 \). It is dark matter. The classical CGF universe contains only the dark matter. Ordinary matter appears as a sub-leading correction due to fluctuations of the SM fields around the classical trajectory.
The classical solution has the basic structure of the SM epoch: the electro-weak transition followed by an expanding universe that contains only dark matter and is homogeneous, isotropic, and flat. This classical CGF universe is the dark matter skeleton of the SM epoch, to be fleshed out by the fluctuations. The time evolution of the fluctuations is a perturbative calculation within the SM and GR that remains to be done.

Laboratory high energy physics looks for new physics in small discrepancies from the SM. The idea is to put cosmology in the same situation. If this project works out, if the initial condition accounts accurately for the SM epoch, then any discrepancies will be signs of new physics. Of course the project may be too ambitious. It could be that the SM epoch depends on undiscovered particles and fields. Still, until those particles or fields are actually discovered, the top-down approach seems worth trying.

Dark matter being a SM effect explains why no dark matter particles have been found. It predicts that no such particles will be found. Dark matter being a classical effect and ordinary matter a sub-leading correction explains why most of the matter in the universe is dark matter.

It seems nontrivial that any simple, natural initial state should give the basic structure of the SM epoch as first approximation, with a systematic method to calculate corrections. It might be worthwhile to calculate the corrections to see if the details come out right.

The initial condition was proposed in [1]. The classical time evolution was calculated in [2] and the dark matter identified. In [3] a first step is taken towards identifying detectable signals of the dark matter. In [4] a first step is taken towards constructing the initial state of the fluctuations.

2 The initial state

The leading order approximation is a Spin(4)-symmetric classical solution of the SM equations of motion. SO(4) is the symmetry group of the 3-sphere in euclidean 4-space. Spin(4) = SU(2) × SU(2) is the simply connected covering group. Spin(4) acts on spinors on the 3-sphere. The SU(2) gauge bundle of the SM is identified with the spinor bundle. The U(1) and SU(3) gauge bundles are trivial product bundles over the 3-sphere. This defines the action of Spin(4) on the space-time metric and on the SM fields.

The metric (in \( c = 1 \) units) is
\[
ds^2 = R(\hat{t})^2 (-d\hat{t}^2 + \hat{g}_{ij}(\hat{x})d\hat{x}^id\hat{x}^j)\]
where \( \hat{t} \) is conformal cosmological time, \( \hat{g}_{ij}(\hat{x}) \) is the metric of the unit 3-sphere in 4-space, and \( R(\hat{t}) \) is the radius of the universe. Co-moving time \( t \) is given by \( dt = R(\hat{t})d\hat{t} \).

The SM fields with nontrivial classical values are the Higgs field \( \phi \) and the SU(2) gauge field. The Higgs field is fixed at \( \phi = 0 \) by the symmetry. The SU(2) covariant derivative is \( \hat{D}_i = \hat{\nabla}_i + \hat{b}(\hat{t})\hat{\gamma}_i \) where \( \hat{\gamma}_i(\hat{x}) \) are the Dirac matrices on the unit 3-sphere and \( \hat{\nabla}_i \) is the metric covariant derivative on spinors. The degree of freedom \( \hat{b}(\hat{t}) \) is the cosmological gauge field (CGF). The Yang-Mills action is
\[
\frac{1}{\hbar} S_{\text{gauge}} = \frac{6\pi^2}{g^2} \int \left[ -\frac{1}{2} \left( \frac{d\hat{b}}{d\hat{t}} \right)^2 + \frac{1}{2} (\hat{b}^2 - 1)^2 \right] d\hat{t} \quad g^2 = 0.426 \tag{2.1}
\]
\( g \) is the SU(2) gauge coupling constant of the SM [5]. The conserved quantity

\[
\hat{E}_{\text{CGF}} = \frac{1}{2} \left( \frac{d\hat{b}}{d\hat{t}} \right)^2 + \frac{1}{2} (\hat{b}^2 - 1)^2 \tag{2.2}
\]
is the dimensionless energy (in energy units $\hbar/R$). The CGF is an anharmonic oscillator. The classical equation of motion is solved by a Jacobi elliptic function

$$
\hat{b}(t) = \frac{k \text{cn}(z, k)}{\epsilon} \quad z = \frac{\hat{t}}{\epsilon} \quad \hat{E}_{\text{CGF}} = \frac{1}{8\epsilon^4} \quad k^2 = \frac{1}{2} + \epsilon^2
$$

(2.3)

The energy condition $\hat{E}_{\text{CGF}} > 10^{107}$, $\epsilon < 10^{-27}$ will come later as a physical condition needed to produce the observed flatness of the present universe.

Scale the coordinates and degrees of freedom by $\epsilon$,

$$
x^0 = z = \frac{\hat{t}}{\epsilon} \quad x^i = \frac{x^i}{\epsilon} \quad a(z) = \epsilon R(\hat{t}) \quad b(z) = \epsilon b(\hat{t}) = k \text{cn}(z, k)
$$

(2.4)

The metric and Dirac matrices become

$$
ds^2 = a(z)^2 \left( -dz^2 + g_{ij}(x)dx^i dx^j \right) \quad \gamma_i \gamma_j + \gamma_j \gamma_i = -\frac{1}{2} g_{ij}
$$

(2.5)

$g_{ij}(x)$ is the metric on the 3-sphere of radius $1/\epsilon$. The SU(2) covariant derivative is

$$
D_i = \nabla_i + b(z) \gamma_i(x) \quad \nabla_i = \partial_i + \epsilon \gamma_i
$$

(2.6)

The action becomes

$$\frac{1}{\hbar} S_{\text{gauge}} = \frac{6\pi^2}{g^2 \epsilon^3} \int \left[ -\frac{1}{2} \left( \frac{db}{dz} \right)^2 + \frac{1}{2} \left( b^2 - \epsilon^2 \right)^2 \right] dz
$$

(2.7)

Local physics in the scaled coordinate system is independent of $\epsilon$ when $\epsilon$ is very small. So the value of the initial energy $\hat{E}_{\text{CGF}}$ does not matter as long as it is very large.

The Jacobi elliptic function $\text{cn}(z, k)$ is doubly periodic in $z$. The real period is $z \sim z + 4\pi K$. The imaginary period is $z \sim z + 4\pi K'$. The numbers $K$ and $K'$ are the complete elliptic integrals of the first kind. The CGF $b(\hat{t})$ oscillates with period $t \sim t + 4K/a$ in real co-moving time. The CGF is periodic in imaginary co-moving time with period $t \sim t + 4K'a_i$. The period in imaginary time defines a temperature $k_B T_{\text{CGF}} = \hbar/(4K'a)$. The initial state of the SM fluctuations is specified by requiring correlation functions to be periodic in imaginary time with the period $4K'a_i$. The CGF acts as thermal bath for the fluctuations. The initial state is specified by the five conditions above and the condition that the fluctuations are in the natural thermal state defined by the CGF.

### 3 The electro-weak transition

The Higgs action is

$$\frac{1}{\hbar} S_{\text{Higgs}} = \int \left[ a^{-2} D_\mu \phi^\dagger D^\mu \phi + \frac{1}{2} \lambda^2 \left( \phi^\dagger \phi - \frac{v^2}{2} \right)^2 \right] a^4 \sqrt{-g} d^4 x
$$

(3.1)

$$
D_\mu \phi^\dagger = \partial_\mu \phi^\dagger + \frac{1}{2} b(z)(\partial^\mu \phi \gamma_i - \phi \gamma_i \partial^\mu \phi) + \frac{3}{4} b(z)^2 \phi^\dagger \phi
$$

$\lambda$ is the Higgs coupling constant, $\lambda^2 = 0.258$, and $m_{\text{Higgs}} = \hbar \lambda v = 125 \text{ GeV}$ is the Higgs mass [5]. We have replaced $\nabla_i = \partial_i + \epsilon \gamma_i$ with $\partial_i$ assuming $\epsilon$ to be very small.
The energy in the CGF and the Higgs field at $\phi = 0$ drives an expanding universe. The CGF oscillates much faster than the expansion so the adiabatic approximation is accurate. Averaging $b$ and $b^2$ over the oscillation gives the effective potential for $\phi$.

$$V_{\text{eff}}(\phi) = \frac{\lambda^2 v^4}{8} + \left( \frac{3}{4} \frac{\langle b^2 \rangle}{a^2} - \frac{\lambda^2 v^2}{2} \right) \phi^4 + \frac{\lambda^2}{2} (\phi^4)$$

\[ (3.2) \]

At early times when $a(z)$ is small the coefficient of $\phi^4$ is positive so $\phi = 0$ is stable. The coefficient of $\phi^4$ turns negative when $a(z)$ reaches $a_{\text{EW}}$.

$$a_{\text{EW}} = \left( \frac{3}{2} \frac{\langle b^2 \rangle}{\lambda^2 v^2} \right)^{\frac{1}{2}} = \frac{(6\pi)^{\frac{1}{2}}}{4K\lambda v} = 0.5854 \frac{\hbar}{m_{\text{Higgs}}} = 3.08 \times 10^{-27} \text{ s}$$

\[ (3.3) \]

### 4 Classical solution after $a_{\text{EW}}$

After $a(z) = a_{\text{EW}}$ the Higgs field moves away from $\phi = 0$ towards its vacuum expectation value, tracking the minimum of the effective potential (3.2),

$$\langle \phi^4 \rangle_0 = \frac{v^2}{2} - \frac{3}{4\lambda^2} \frac{\langle b^2 \rangle}{a^2}$$

\[ (4.1) \]

$\phi \neq 0$ breaks the Spin(4) symmetry but the gauge field action remains Spin(4)-symmetric because, for $B_i$ the SU(2) gauge field, $\phi^4 B_i B_i = \phi^4 \phi \text{ tr}(B^i B_i)/2$. The Spin(4)-symmetric CGF continues to oscillate but now with action and dimensionless energy

$$\frac{1}{\hbar} S_{\text{gauge}} = \frac{6\pi^2}{g^2 e^3} \int \left[ -\frac{1}{2} \left( \frac{db}{dz} \right)^2 + \frac{1}{2} \mu^2 b^2 + \frac{1}{2} b^4 \right] dz$$

\[ (4.2) \]

$$E_{\text{CGF}} = \frac{1}{2} \left( \frac{db}{dz} \right)^2 + \frac{1}{2} \mu^2 b^2 + \frac{1}{2} b^4 \quad \mu^2 = \frac{1}{2} g^2 a^2 (\phi^4)$$

The adiabatic approximation remains valid. The oscillation changes slowly with $\mu^2$. The classical solution is again a Jacobi elliptic function,

$$b(z) = \frac{k \text{cn}(u, k)}{\alpha} \quad dz = \alpha du \quad \alpha^2 \mu^2 = 1 - 2k^2 \quad \alpha^4 E_{\text{CGF}} = \frac{k^2(1 - k^2)}{2}$$

\[ (4.3) \]

now parametrized by $k^2$ and $\alpha$ instead of $\mu^2$ and $E_{\text{CGF}}$. The solution $b(z)$ determines the value of $\phi^4$ by (4.1) and the identity

$$\alpha^2 \langle b^2 \rangle = \frac{1}{4K} \int_0^{4K} \alpha^2 b^2 du = \frac{1}{4K} \int_0^{4K} k^2 \text{cn}^2(u, k) du = k^2 - 1 + \frac{E}{K}$$

\[ (4.4) \]

$E$ is the complete elliptic integral of the second kind. Equations (4.1) and (4.2) combine to parametrize the scale $a$ by $k^2$.

$$\hat{a} = \frac{m_{\text{Higgs}}}{\hbar} \quad \alpha^2 a^2 = \frac{3}{2} \alpha^2 \langle b^2 \rangle + \frac{4\lambda^2}{g^2} \alpha^2 \mu^2$$

\[ (4.5) \]
Finally, the adiabatic invariant $\oint p \, dq$ is a constant of the motion for an adiabatically evolving oscillator $q$. The adiabatic equation for the CGF is

$$\frac{(1-k^2)K + (2k^2 - 1)E}{\alpha^3} = \text{constant, with } \alpha = 1 \text{ at } k^2 = 1/2. \quad (4.6)$$

The classical time evolution is now completely parametrized by $k^2$. As $k^2$ evolves from $1/2$ to 0, the scale $a$ goes from $a_{\text{EW}}$ to $\infty$ and $\phi^\dagger \phi$ goes from 0 to $v^2/2$.

## 5 CGF equation of state

The energy-momentum tensors of both the gauge field and $\phi$ are SO(4)-symmetric so the CGF is a perfect fluid. The total density $\rho_{\text{CGF}}$ and total pressure $p_{\text{CGF}}$ are obtained by substituting the classical solutions in the energy-momentum tensor. Define the dimensionless density and pressure

$$\hat{\rho}_{\text{CGF}} = \frac{\rho_{\text{CGF}}}{\rho_b} \quad \hat{p}_{\text{CGF}} = \frac{p_{\text{CGF}}}{\rho_b} \quad \rho_b = \frac{m_{\text{Higgs}}^4}{\hbar^3} = 5.68 \times 10^{28} \text{ kg/m}^3 \quad (5.1)$$

$\hat{a} = m_{\text{Higgs}} a / \hbar$ is the dimensionless scale. Before $a_{\text{EW}}$ the gauge field is pure radiation, and $\phi$ contributes vacuum energy.

$$\hat{\rho}_{\text{CGF}} = \frac{3}{8g^2} \frac{1}{\hat{a}^4} + \frac{1}{8\lambda^2} \quad \hat{p}_{\text{CGF}} = \frac{1}{8g^2} \frac{1}{\hat{a}^4} - \frac{1}{8\lambda^2} \quad (5.2)$$

After $a_{\text{EW}},$

$$\hat{\rho}_{\text{CGF}}(k^2) = \frac{1}{\hat{a}^4} \left( \frac{3E_{\text{CGF}}}{g^2} + \frac{9\langle b^2 \rangle^2}{32\lambda^2} \right) \quad \hat{p}_{\text{CGF}}(k^2) = \frac{1}{\hat{a}^4} \left( \frac{E_{\text{CGF}} - \mu^2 \langle b^2 \rangle}{g^2} - \frac{9\langle b^2 \rangle^2}{32\lambda^2} \right) \quad (5.3)$$

In both regimes the equation of state relating $p$ to $\rho$ is given implicitly by the two functions $\hat{\rho}, \hat{p}$. The boundary between the two regimes is at $\hat{a}_{\text{EW}} = 0.585$, $\hat{p}_{\text{EW}} = 7.97$.

The equation of state parameter $w = p/\rho$ is plotted in Figure 1. The CGF behaves as a nonrelativistic perfect fluid, $w_{\text{CGF}} \approx 0$, from about $10 a_{\text{EW}}$ or $10^2 a_{\text{EW}}$ onward, which is to say that the CGF is cold dark matter.
6 Present flatness

The Friedmann equation normalized by the Hubble constant is

$$\frac{H^2}{H_0^2} = \Omega_m + \Omega_\Lambda + \Omega_{\text{curvature}}$$

$$H = \frac{1}{a} \frac{da}{dt} = \frac{1}{a^2} \frac{da}{dz}$$

$$\rho_c = \frac{3H_0^2}{\kappa} \quad \Omega_m = \frac{\rho_m}{\rho_c} \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} \quad -\Omega_{\text{curvature}} = \frac{1}{H_0^2} \frac{1}{R^2} = \frac{\epsilon^2}{H_0^2 a^2}$$

The dark energy density is $\Omega_\Lambda = 0.685$ [5]. It is assumed due to the cosmological constant so is constant in time. The present curvature is small, $|\Omega_{\text{curvature}}| < 0.001$ [5]. The only matter in the classical CGF universe is the CGF.

$$\rho_m = \rho_{\text{CGF}} \quad \Omega_m = \Omega_{\text{CGF}} = \frac{\rho_{\text{CGF}}}{\rho_c}$$

The present time is identified by the condition $H = H_0$ which is $\Omega_m = 0.315$. Solving (6.2) gives the present value $k_0^2 = 7.89 \times 10^{-56}$ and

$$a_0 = 4.54 \times 10^{18} a_{\text{EW}} \quad -\Omega_{\text{curvature}} = \frac{\epsilon^2}{H_0^2 a_0^2} = 1.07 \times 10^{51} \epsilon^2$$

$|\Omega_{\text{curvature}}| < 0.001$ is $\epsilon < 10^{-27}$ which is the initial energy condition $\dot{E}_{\text{CGF}} > 10^{107}$.

The dimensionless energy $\dot{E}_{\text{CGF}}$ is the only adjustable parameter in the initial state. If $\dot{E}_{\text{CGF}} > 10^{107}$ then $\epsilon$ is so small that no local physics depends on the value of $\dot{E}_{\text{CGF}}$. In effect there are no adjustable parameters.

7 Dark matter stars

The classical CGF cosmology has no ordinary matter, only dark matter. The dark matter is the CGF. The actual universe is a perturbation of this dark matter universe by the fluctuations of the SM fields. Constructing the initial fluctuations and calculating their time evolution is a well-defined calculation within the SM and GR. Detailed quantitative checks of the theory depend on that calculation, which remains to be done.

Meanwhile, having in hand the equation of state of the classical CGF allows solving the TOV stellar structure equations to find the possible stars made of the CGF. Density fluctuations in the CGF presumably collapsed gravitationally to form self-gravitating bodies of which the simplest are the spherically symmetric non-rotating stars governed by the TOV equations. Figure 2 shows the results of solving the TOV equations numerically. The gravitational scales are set by the CGF density scale

$$r_b = (4\pi G \rho_b)^{-1/2} = 4.34 \text{ cm} \quad m_b = G^{-1} r_b = 2.94 \times 10^{-5} M_\odot = 5.26 \times 10^{42} \text{ J}$$

$$\hat{R} = \frac{R}{r_b} \quad \hat{M} = \frac{M}{m_b} \quad \hat{\dot{E}} = \frac{BE}{m_b}$$

The left plot is the mass-radius curve. The right plot is the mass-binding energy curve. The curves spiral inwards, parametrized by increasing central density.

The dark matter universe is presumably populated with such stars. The abundance distribution of their masses is a fluctuation calculation still to be done. Microlensing puts
an upper limit $10^{-11} \text{M}_\odot$ on such compact dark matter objects as the halos [6]. So the halos must consist mostly of dark matter stars of radius $R = 13.6 \text{ cm}$ at the low mass end of the curve. It seems challenging to detect dark matter in such a form. The rapidly oscillating CGF will have no significant non-gravitational interactions.

The binding energy curve shows the possibility of metastable dark matter stars that could undergo explosive collapse to smaller radius, emitting a burst of gravitation energy on the order of $10^{41} \text{ J}$ in $10^{-10} \text{ s}$. Such bursts might be observable, perhaps taking place in the centers of ordinary stars or out in the open.

Finally, a dark matter star near the asymptotic fixed point of the spiral has high central density. Such objects could probe physics beyond the SM.

I thank C. Keeton for advice on microlensing and for suggesting reference [6]. This work was supported by the Rutgers New High Energy Theory Center and by the generosity of B. Weeks. I am grateful to the Mathematics Division of the Science Institute of the University of Iceland for its hospitality.

References

[1] D. Friedan, “Origin of cosmological temperature,” arXiv:2005.05349 [astro-ph.CO]. May, 2020.

[2] D. Friedan, “A theory of the dark matter,” arXiv:2203.12405 [astro-ph.CO]. March, 2022.

[3] D. Friedan, “Dark matter stars,” arXiv:2203.12181 [astro-ph.CO]. March, 2022.

[4] D. Friedan, “Thermodynamic stability of a cosmological SU(2)-weak gauge field,” arXiv:2203.12052 [hep-th]. March, 2022.

[5] Particle Data Group Collaboration, P. Zyla et al., “Review of Particle Physics,” PTEP 2020 no. 8, (2020) 083C01. and 2021 update, http://pdg.lbl.gov/.

[6] H. Niikura et al., “Microlensing constraints on primordial black holes with Subaru/HSC Andromeda observations,” Nature Astron. 3 no. 6, (2019) 524–534, arXiv:1701.02151 [astro-ph.CO].