1. Introduction

The excitation of global instabilities by super-thermal particles in hot plasmas and the related transport processes are of great interest for the fusion community, due to their importance for burning fusion plasmas. These energetic particles (EPs) are present in magnetic fusion devices due to external plasma heating and eventually due to fusion born $\alpha$ particles. It is necessary that the super-thermal particles are well confined while they transfer their energy to the background plasma. EPs are typically super-Alfvénic and can destabilise shear Alfvén, Alfvén-acoustic waves or other global plasma modes by resonant wave-particle interaction (inverse Landau-damping). The resulting nonlinear EP transport processes from the core to the edge and the consequential particle losses reduce the plasma heating and the fusion reaction rate. In addition, the EP losses may cause severe damage to the first wall of the device.

Within the last few years, significant advances on both the experimental and the theoretical sides have been made leading to a more detailed understanding of EP-driven instabilities. On the theoretical side, models have advanced from fluid models for the plasma background to a fully kinetic model for all the plasma species, i.e. background ions and electrons, as well as EPs [1]. This more accurate treatment of the background leads to changes in the linear mode properties such as frequency, damping/growth and mode structure, and can also influence the nonlinear dynamics. Since the new physics that is accessible due to a more comprehensive model can be directly validated with experimental data from present-day machines, predictions for several ITER scenarios can be attempted.

In the ITER 15 MA scenario [2], a ‘sea’ of small-amplitude perturbations is likely [3, 4]. This work cannot yet provide a realistic prediction for nonlinear multi-mode EP transport in this ITER scenario. It should rather be understood as a
necessary first step towards this demanding goal, since it demonstrates a physical effect that cannot be neglected for ITER: the role of linearly stable modes within the TAE spectrum. The mechanism described in the following is investigated with the hybrid HAGIS-LIGKA model. The nonlinear HAGIS code [5] treats the EPs’ drift kinetically but obtains the non-perturbative mode structures, frequencies and damping from the gyrokinetic eigenvalue solver LIGKA [1]. Recent realistic modelling of double mode scenarios with the HAGIS-LIGKA code [6] not only reproduced experimentally measured EP losses in ASDEX Upgrade [7], but also revealed the importance of linearly sub-dominant modes as well as the detailed mode structures. Hence, the crucial question for the ITER scenario arises: if the interaction between the ‘sea’ of perturbations with the EPs will drive linearly subdominant modes unstable such that EP transport occurs in a domino effect caused by the overlap of resonances [6, 8]. In this case, even modes localised outside the region of the strong EP drive could be excited nonlinearly by EP transport from more core-localised modes. As a consequence, gradient depletion and EP redistribution can exceed the quasilinear estimates. If so, the preconditions (q profile, background density profile) as well as the consequences (e.g. the losses) must be investigated to know if and how these conditions can be avoided.

2. The HAGIS-LIGKA model

To date, the nonlinear multi-scale problem of wave-particle interaction cannot be solved in a fully consistent way. Although recently, linear as well as the first nonlinear results for single and multi-mode cases were obtained with global codes originally designed for turbulence studies [9, 10], the large computational effort for these approaches limits the possibilities of extended, realistic simulation studies. The joint Culham-IPP code project HAGIS [5] therefore follows a hybrid approach (figure 1): the EP distribution is evolved in a drift kinetic model and the wave contribution enters the problem via a set of pre-calculated modes. This way, the EP nonlinearities are kept, but the magnetohydrodynamic (MHD)-nonlinearities are dropped—only the energy transfer between the waves and the particles is accounted for. This leads to a redistribution of the EP population in the phase space and to the self-consistent evolution of the amplitudes of (multiple) modes and their real frequencies. The wave structures and also their damping is kept fixed during a simulation. Saturation is reached in the nonlinear stage due to the local flattening of the driving gradient in the radial EP distribution. The stochastisation of the EP orbits caused by overlapping resonances with different modes influences the saturation level.

In [11] it was found that wave-wave nonlinearities, which excite zonal structures, can lead to steeper EP profiles for high amplitude cases. This stabilising effect is not captured in the present model. Further, it is assumed that the absence of sources, sinks and collisions in the model is not crucial for the effect that will be shown in this work, but it makes it easier to clearly demonstrate the underlying physics. The timescale of the observed nonlinear dynamics is a few milliseconds (< simulation time of ≤ 10 ms), which is well below plasma heating and slowing-down times (~100 ms). Therefore, it is expected that the timescales are separated well enough to neglect the slower processes within this physics-oriented study, and that the additional effects of sources, sinks and collisions may alter the qualitative results only mildly.

Mode frequencies, damping and structures that enter HAGIS are pre-calculated by the LIGKA code. The ability to predict the stability of EP-driven Alfvén eigenmodes requires a detailed understanding of the dissipative mechanisms that damp these modes. To cover also the important dissipation mechanism of large-scale MHD modes coupling to gyroradius scale-length kinetic Alfvén waves, a gyrokinetic description is necessary. With the linear gyrokinetic, electromagnetic and non-perturbative code LIGKA [1] not only the growth rate and damping are calculated, but also the global mode structure of the magnetic perturbation in realistic geometry due to the EPs and background kinetic effects.

3. ITER simulation conditions

The ITER 15 MA baseline scenario has been analysed in detail, as presented in [4, 12]. The findings which are relevant for this work are summarised in this section: figure 2 shows the kinetic shear Alfvén-wave continuum for the different toroidal mode numbers n and the q profile as shown in figure 3 (q0 = 0.986). The toroidal Alfvén eigenmode (TAE) gaps are closed around s ≈ 0.85. Due to the flatness of the q profile and the fact that q is very close to 1, the radial TAE positions qTAE = (m + 1/2)/n decrease monotonously with the mode number n from s ≈ 0.7 to s ≈ 0.35 (magenta line in figure 4) and cluster relatively densely in the radial direction. The resonance overlap leads to additional nonlinear effects, which will be investigated in the subsequent sections. Figure 5 shows the radial structure of the electrostatic field amplitude for three representative waves as calculated by LIGKA and applied in the HAGIS study of this work.

In this work, if the wave evolution is not fixed, or prescribed, it will be referred to as the ‘self-consistent’ mode evolution.

\[^{3}\text{In this work, if the wave evolution is not fixed, or prescribed, it will be referred to as the ‘self-consistent’ mode evolution.}\]
The modelled EP population consists of fusion-born $\alpha$ particles, i.e. distributed isotropic in pitch ($\lambda$). Figure 6 shows the EP radial distribution function, which was taken from the ITER database. For the energy ($E$) distribution, a slowing-down function \[ f(E) = \frac{1}{E^{3/2} + E_c^{3/2}} \text{erf} \left( \frac{E - E_0}{\Delta E} \right) \] has been assumed, with $\Delta E = 491 \text{ keV}$, $E_c = 816 \text{ keV}$, $E_0 = 3.5 \text{ MeV}$:

At this stage, the neutral beam injection (NBI) EP population is neglected, as discussed later. Effects due to the NBI ions are the subject of future investigations. Further, it should be noted, that the applied distribution function is assumed to be separable in $s$, $E$ and $\lambda$, i.e. $f(s) \cdot f(E) \cdot f(\lambda)$ instead of $f(s,E,\lambda)$. Due to the high number of modes with many poloidal harmonics, the computational effort for a nonlinear study is challenging, even with a relatively low-cost hybrid model. Together with the long timescales which have to be observed due to the generally low drive in the marginally stable regime, this leads to computationally (CPU)-intensive simulations. In addition, the combination of a large machine size with high toroidal mode numbers $n$ broadens the scales of the resolution needed in the simulation. As a particle-in-cell (PIC) code, Hagis resolution is determined by the number of markers. Convergence tests reveal that the necessary number of markers depends strongly on the scenario, e.g. on the radial distance and $n$ range of the relevant modes and can exceed 10 million for unfavourable constellations, up to 20 million for the 27-modes scenarios presented in section 5.

4. Linear stability analysis of the Alfvén eigenmodes

A detailed local and global stability analysis has been performed for the ITER 15 MA baseline scenario \[4\]. This
scenario is not only one of the *ITER* baseline scenarios, but it has also been chosen by the *ITER* Topical Group of the International Tokamak Physics Activity as a benchmark reference case for the study of EP behaviour in *ITER*. In the following, the results for the $q_0 = 0.986$ case which are relevant for the nonlinear study presented in section 5 are summarised. This specific $q$ profile case was chosen, since it represents a ‘worst-case’ scenario in the sense that it leads to the most unfavourable constellation of gaps for a weakly-damped TAE and $\alpha$ particle drive [12]. However, the results which will be shown here cannot be easily compared with earlier stability analysis (such as is presented in [14, 15], using Nova-K or in [16], using GEM) because the radial profiles and background magnetic equilibria are not the same. In [16] an analytic $q$ profile is studied, which is much less box-like. This leads to smaller radial mode structures and as a consequence, the strongest mode drive (due to the mode width being comparable to the orbit width) is found between $n = 10$ and $n = 20$. A recent (nonlinear) study of this *ITER* scenario is presented in [17], using the *Miracle* code. There, relevant modes are reported only for $n \lesssim 10$, which disagrees with the findings of this work. The reasons for this remain still to be investigated. The linear stability analysis as reported in the present section agrees well with the findings of [18], using a *MISHKA*-CASTOR-K hybrid model, as well as with the qualitative analytical estimates [12].

The *Ligka* eigenvalue solver finds relevant eigenmodes up to $n \approx 40$. For intermediate and low $n$, several weakly damped branches of TAE appear in the gap\(^5\): $\gamma_0 \lesssim 1.0\%$ for the low-$n$ (blue line in figure 7) and $\gamma_0 \lesssim 1.4\%$ for the intermediate-$n$ branch (green line in figure 7). The main (red line in figure 7) branch is characterised by the two main poloidal harmonics $m$ of the mode being $m = n, n + 1$, whereas for the intermediate-$n$ branch, it is $m = n + 1, n + 2$ and for the low-$n$ branch, it is $m = n + 2, n + 3$. With increasing $n$, the TAE modes become more localised, whereas especially the outer, low-$n$ modes have a large number of radially extended poloidal harmonics. This effect is due to the shape of the TAE gap towards the edge in the SAW. For the calculation of the mode structures, the $\alpha$ particle drive has been taken into account. However, the effect becomes slightly relevant only in the outer core region, modifying the coupling of the harmonics through the TAE gap. It should be noted here again, that the mode structures will be kept fixed during the investigation presented in the next section. The possibility of evolving the mode structures with time is about to be implemented currently, however, for the present investigation of marginally unstable modes, major changes in the mode structures are not expected.

Although the TAE position moves inwards, i.e. towards a higher ion background temperature with an increasing $n$, the damping decreases (figure 7). Two effects are responsible for this damping behaviour: the frequency increases (figure 6) due to $\omega\text{}_{\text{TAE}}/\omega_A = n/(2m + 1)$ (which decreases the ion Landau damping) and the diamagnetic effects become more important with the increasing mode numbers. Further, the modes move into the low-shear region, which decreases the radiative damping. This effect can compensate for the increase of radiative damping (via $k_i, \rho_i$) for the more localised mode structures. The least damped modes (of the higher-$n$ main branch) are found around $n = 27$.

Adding the $\alpha$ particle population, a slight destabilisation is found linearly ($\gamma < 1\%$) for the TAEs between $n = 20$ and 35, as can be combined from figures 7 and 8. This is in agreement with linear *HAGIS* results, except for the missing finite Larmor radius effects in *HAGIS*, leading to higher growth rates of about $(^{+1}\%\,\gamma)$. Wave-EP resonance occurs, if the following condition is met: $\omega = - n(\zeta \dot{\theta}) + (p \pm 1)(\dot{\theta})$, with $\theta$ and $\zeta$ the poloidal and toroidal angle coordinate, $m, n$ the poloidal and toroidal mode number and $p$ the harmonic of the resonance (i.e. $p = 0, \pm 1, \pm 2, \ldots$). Due to the flat $q$ profile, the resonance areas in the phase space are broad. Where the mode structures cluster radially, the resonances in the $(s, E)$ space are

\(^{5}\)The damping is named $\gamma_0$ while $\gamma_0$ denotes the linear mode growth without any damping. The effective growth rate is $\gamma = \gamma_L - \gamma_s$. Note that a convention to calculate the mode growth in ‘%’ is to normalise $\gamma$ in units $1\,$s$^{-1}$ by the mode frequency $\omega$ in rad s$^{-1}$. This definition is valid for all $\gamma$ throughout this article.

\(^{6}\)The Alfvén frequency at the magnetic axis: $\omega_A := v_\text{A}/R$, where $v_\text{A}$ is the Alfvén velocity and $R$ the major plasma radius (both at magnetic axis $s = 0$). In the presented scenario, $\omega_A = 178\,$kHz.
neighbouring, and thus easily overlap for different modes, especially at higher amplitudes. The resonance overlap in the velocity space is visualised in figure 9 for the three representative modes. Although these modes form different parts of the spectrum, one can clearly see the overlap, especially the lower energetic co- and counter-passing, as well as the high energetic counter-passing particles.

5. Modelling nonlinear multi-mode EP transport

To understand complex nonlinear multi-mode behaviour, realistic ITER conditions are established step-by-step, which also helps to gain confidence about the numerical conditions. To start with, a simplified ITER 15 MA scenario is modelled with HAGIS, e.g. only selected modes are used instead of all the TAEs given by LIGKA, and the (small) parallel electric field \( E_p \) is still neglected. Convergence was checked for all numerical parameters at every level of the investigation. In the following, only the most realistic compromise between computational effort and relevant physical features will be discussed. The respective scenario was set up with the least damped 27 TAE modes that were found in the linear analysis in the main branch (red in figure 7), \( n \in [12, 30] \) and in the low-\( n \) branch (blue in figure 7), \( n \in [5, 12] \). Since the importance of a detailed poloidal harmonics spectrum is known from earlier ASDEX Upgrade studies [6], all harmonics with a peak greater than 25% of the mode’s maximum peak were taken into account. This affects mostly the low-\( n \) branch being simulated with up to 12 poloidal harmonics, whereas the high-\( n \) branch is characterised sufficiently by two harmonics only. The major motivation to focus on the main and the low-\( n \) branch is not only the low damping: together with the effect of multi-mode coupling, this can lead—as will be shown later—to a nonlinear enhancement of the low-\( n \) branch, despite its radial location outside the maximum \( \alpha \) particle drive at \( s = 0.4 \). As a consequence, modes with rather high amplitudes can possibly cover a large radial range, and subsequently, EP redistribution, especially in the outer core region can exceed quasilinear estimates.

In order to test the validity of quasilinear models (which require a reduced computational effort) this section is dedicated to working out a comparison between a quasilinear (e.g. [19, 20]) and a nonlinear approach.

In a quasilinear model, one can estimate the mode growth rates \( \gamma \) from the linear single mode simulations, since \( \gamma \propto B_{EP} \) (see e.g. the \( \beta \) scaling of high-\( n \) single mode linear growth rates in figure 10). Mode saturation amplitudes \( A \) can be estimated from quasi-linear scaling \( A \propto (\gamma/\omega)^2 \) w.r.t. the single mode linear growth rates over the mode frequency \( \omega \). For the ITER 15 MA baseline scenario \( q = 0.986 \), the amplitudes in the single mode simulations follow quite well such quadratic scaling with the mode linear growth rate, as shown in figure 11. In general, a critical plasma \( \beta_{cr} \) can be determined, at which the linear growth rate \( \gamma_L \) equals the damping \( \gamma_d \) and the mode will be marginally stable \( \gamma \approx 0 \). In the quasilinear theory it is assumed that the overlap of resonances causes diffusive EP transport as soon as the linear instability threshold is exceeded, i.e. \( \beta > \beta_{cr} \). Subsequently, the diffusive transport relaxes the EP distribution function such that the local \( \beta \) takes values around \( \beta_{cr} \).

In the following, a general comparison between the nonlinear and the quasilinear model is carried out. The main purpose of sections 5.1 and 5.2 is to point out limits beyond which enhanced EP transport can happen, triggered by a domino effect. These limits concern damping as well as EP density—however, a thorough study scanning both parameters goes beyond the scope of this work. Instead, section 5.1 shows crossing the limit to trigger domino-like EP transport via reducing the damping, whereas section 5.2 explores the limit with respect to the EP density.

5.1. Nonlinear versus quasilinear transport under reduced damping

A general comparison between the nonlinear and the quasilinear model is carried out with reduced mode damping. The damping is reduced by a factor of 6, which is in a way, that domino-like EP behaviour can be observed nonlinearly, but critical gradients are still determinable. Further advantages are the reduced computational effort and the fact that a possible difference between the quasilinear and the nonlinear model can be shown very clearly.

Figure 12 shows the final EP density gradient depletion caused by all 27 relevant modes together, simulated with amplitudes fixed at their single mode (peak) saturation level (in the following, this model will be referred to as a ‘quasilinear’ HAGIS simulation). The derivative of it is comparable but slightly above the local gradient of \( \beta_{cr} \) at each mode location (thicker lines). Thus, \( \beta_{cr} \) provides a lower estimate for profile relaxation here.

In the following, it will be investigated, whether this quasilinear estimate is a valid assumption by modelling the mode evolution of all relevant 27 modes self-consistently. For a radial EP density profile scaled to 30% of the default

\[ \Lambda = \mu B_{mag}/E \] is a constant of motion related to the pitch, with \( \mu = E/B \), the magnetic moment, \( B_{mag} \) the equilibrium magnetic field at the axis \( s = 0 \).

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value, the quasilinear estimate predicts all modes as stable, except for \( n = 29 \). The self-consistent nonlinear multi-mode simulation reveals no mode growth for the low-\( n \) branch and very moderate growth (\( \gamma < 0.8\% \)) for the high-\( n \) branch. For a radial EP density profile scaled to 50% of the default value, the quasilinear estimate predicts mode growth for \( n \in [23, 30] \). In this case, the quasilinear approach is significantly underestimating the nonlinear multi-mode amplitude evolution: the modes \( n \in [23, 30] \) exceed the estimate (by a factor of up to 28). Also, the modes \( n \in [12, 22] \) grow moderately. Although the quasilinear prediction underestimates the nonlinear amplitude evolution, none of the modes exceed \( \delta B/B_{mag} \approx 6 \cdot 10^{-4} \), and the low-\( n \) branch remains marginally unstable. Thus, no significant EP redistribution is found, in agreement with the quasilinear approach.

If the radial EP density profile is scaled up to its default value as predicted for this ITER scenario, the self-consistent wave evolution in the multi-mode scenario differs significantly from the single mode simulation, i.e. from the quasilinear expectation: at around 3 ms, the waves of the low-\( n \) branch (blue in figure 13) get strongly excited and their amplification grows to values of roughly more than one order of magnitude (a factor of 5–65) higher than in the single mode saturation (light blue in figure 13). While nonlinearly, some of the low-\( n \) modes (especially \( n = 12, 11 \)) become one of the dominant modes, the ratio of the linear growths \( \gamma/\omega \) in the multi-mode over the single mode cases is similar (1 ± 0.1) for the modes \( n > 12 \) (for the \( n \leq 12 \) modes it is comparable (1 ± 0.6) in the early\(^9\) linear phase). This emphasises the strong nonlinear effect of the multi-mode behaviour, that cannot be foreseen linearly.

The amplitudes of the high-\( n \) branch modes (red in figure 13) in the multi-mode case reach values that are enhanced slightly, by a factor of 5 at most for \( n > 12 \). For both the low- and high-\( n \) branch the nonlinear multi-mode dynamics is non-trivial—modes reach high amplitude regimes at very different times \( t \). While the low-\( n \) branch grows in three phases \( t < 0.4 \) ms: like single mode, 0.4 ms \( < t < 3.0 \) ms: first enhancement until a slight ‘saturation’, \( t > 3.0 \) ms: second enhancement), the behaviour of the high-\( n \) branch starts to differ from single mode evolution only at the onset of its saturation (\( t \approx 0.6 \) ms). This saturation is higher than in the single mode case and followed by a second enhancement of high-\( n \) modes at around \( t \approx 4.5 \) ms, when the low-\( n \) modes have reached relevant amplitudes of \( \delta B/B_{mag} \approx 10^{-3} \).

To understand the nonlinear multi-mode behaviour, it is helpful to look at the temporal evolution of EP redistribution: until \( t \approx 4 \) ms, the radial redistribution does not exceed the relaxed profile observed in the fixed amplitude multi-mode simulation (green

\( \text{Figure 9.} \) Resonances \(( p = 0, \pm 1 \)) of representative modes (blue for the low-\( n \) branch \( n = 12, m = 14, 15, 16 \) mode, yellow for \( n = 21, m = 21, 22 \) mode and red for \( n = 30, m = 30, 31 \) mode) in the velocity space \(( E, \Lambda \)) at the radius \( s = 0.5 \), for co- (left) and counter-going (right) particles. For values \( \Lambda \gtrsim 0.8 \), the particles are trapped.

\( \text{Figure 10.} \) Scaling of the linear growth rate \( \gamma \) subtracted by the Légier damping \( \gamma_d \) over the EP density in the single mode cases for selected \( n \). Each solid line represents the linear fit for the higher \(( \beta, \gamma \) values of the respective mode \( n \) (annotated number), represented by the circles of the respective colour.

\( \text{Figure 11.} \) Saturation amplitude depending on the growth rate for single mode simulations at the default radial EP density profile. The annotated labels give the toroidal mode number \( n \).

\(^9\) in the later linear phase of the multi-mode simulations, the linear growth rate is not constant anymore for the low-\( n \) modes.
lines in figure 14). However, this redistribution not only leads to a steeper EP density gradient at the radial position of the low-\textit{n} modes, but also provides more EPs at the radial location of these modes. There is a critical redistribution that triggers the excitation of the low-\textit{n} modes, which can be reached only due to the large amount of more core-localised modes in the high-\textit{n} branch: a reduced scenario simulation of only the highest amplitudes and in the radially, most extended modes (\textit{n} = 8, 11, 12, 18, 21, 24, 30) did not lead to sufficient redistribution to excite the low-\textit{n} modes, although the high-\textit{n} modes reached similar saturation amplitude levels compared to the full scenario simulation. Only with the low-\textit{n} modes reaching levels around $\delta B/B_{mag} \approx 10^{-3}$, massive EP gradient depletion sets in, and EPs are redistributed radially outwards to $s \approx 0.8$ (see blue line in figure 14). A similar depletion occurs even if the already quasilinearly relaxed EP density profile was chosen as an initial condition (see red line in figure 14). This is remarkable, since it may indicate a possible overshoot effect, leading to domino-like behaviour from an initially quasilinear state.

In the evolution of the density gradient depletion, big differences can be observed between the self-consistent
nonlinear multi-mode scenario (with 27 modes) and the quasilinear Hagis simulation with the same modes (amplitudes fixed at the respective single mode saturation level): while the quasilinear case converges in time towards a depletion slightly above the local values of $\nabla / \beta_{\text{crit}}$ (figure 12), the nonlinear scenario also reaches this state (figure 15, left), but with the low-$n$ modes exceeding amplitude $\delta B / B_{\text{mag}}$ levels of $10^{-3}$, broad redistribution sets in. The density gradient in the outer core region is depleted rapidly (figure 15, right), triggered by the radial overlap of the growing low-$n$ modes with the higher-$n$ modes (this is visualised by the thick lines in figure 15, left, while the necessary overlap in the velocity space was shown in figure 9). This domino behaviour is clearly a non-local effect, which is avoided in the quasilinear scenario by a transport barrier between the outer, low-$n$ and the inner, high-$n$ branch: the locally very high EP density gradient (figure 15, left) around $s \approx 0.55$ cannot be depleted because there is no radial mode overlap at amplitudes above certain thresholds (around $\delta B / B_{\text{mag}} \approx 10^{-3}$). The situation changes rapidly, as soon as broad, low-$n$ modes grow, triggered by gradient steepening due to EP redistribution from the inner, high-$n$ modes.

5.2. Nonlinear versus quasilinear transport with damping given by LIGKA

In this section, the original damping is used as given by the LIGKA code. With the default EP density profile, the low-$n$ branch stays now at low saturation amplitudes, well separated from the high-$n$ branch (figure 16). As a consequence, there is no significant EP redistribution from the low-$n$ branch. Due to this fact and the stronger damping, the high-$n$ branch does not reach the level of massive redistribution either. Thus, no domino-like interaction similar to the situation described in the last section takes place.

On the way towards a more realistic ITER prediction, there are still missing effects to be included into the model. The most important one is the destabilisation caused by the NBI-generated EP distribution, as shown in figure 17: in [12], it is shown that the NBI drive can—depending on the radial position and the mode number—more than double the drive for the ITER scenario investigated here. To include this effect, a multi-species model is needed, which has been implemented into the Hagis code and is now in the testing phase. Results will be shown in a follow-up publication. In this article, the nonlinear EP transport investigation is restricted to the single-species model, using just an $\alpha$ particle drive. To investigate the possibility of a domino-effect under a stronger drive, the $\alpha$ particle density is scaled to 200%. The rest of this section will be dedicated to an investigation of the multi-mode behaviour under such scaling.

In this case, as shown in figure 18, a domino-like behaviour is visible, enabling the low-$n$ modes to reach higher amplitudes compared to the single mode case—some ($n = 11, 8, 10$) reach $\delta B / B_{\text{mag}} > 10^{-3}$ and become dominant in the nonlinear phase. As a consequence, a second enhancement of the high-$n$ mode saturation is triggered. However, with the higher drive and at the same time higher damping, the temporal behaviour of the mode amplitude evolution relative to each other is very different compared to the results of the last section: the high-$n$ modes grow very fast and have already started decaying by the time the highly damped low-$n$ modes reach the levels of significant redistribution. Therefore, the peak amplitudes of many of the high-$n$
modes are reached at the very beginning of the nonlinear phase, while the second peak is lower (up to a factor of $\lesssim 6$). As a consequence, radial redistribution is weaker (compare figure 19 with figure 14), leading to a higher final EP density gradient in the region around $s \approx 0.6$ (see figure 20, right). This is due to the high amplitudes of certain low-$n$ modes (mainly $n = 11$). However, the previously observed outer EP redistribution up to $s \approx 0.8$ is seen here as well (although weaker), and cannot be found in the quasilinear estimate (figure 20, left), although the quasilinear gradient depletion in the area $s \approx 0.45$ is stronger than reported in the previous section.

6. Conclusions and discussion

This work presented a nonlinear investigation into the interaction of energetic particles with multiple global modes in the ITER 15 MA baseline scenario using the hybrid HAGIS-LiGkA code package. The addressed question is, if the overall energetic particle transport remains within the quasilinear estimates or if possible nonlinear excitation of linearly stable TAEs via phase space coupling effects leads to enhanced, domino-like energetic particle transport. The challenge for an investigation arises not only from the high amount of modes and poloidal harmonics, but also from the high resolution range needed due to the large machine size combined with the high toroidal mode numbers $n$. For the ITER 15 MA scenario with $q_0 = 0.986$ the linear, gyrokinetic, non-perturbative code LiGkA predicts a radially dense cluster of TAEs up to toroidal mode number $n \approx 40$ that can be categorised into three different branches: low-$n$, intermediate-$n$ and high-$n$. As discussed in [4] a flat $q$ with $q_0$ close to 1 causes a dense cluster of modes that facilitates the excitation of edge TAEs. Therefore, the $q_0 = 0.986$ scenario investigated in this work is a ‘worst-case’ scenario, compared to other scenarios with different $q$ shaping (as provided in [12]).

Nonlinear HAGIS simulations were carried out, taking into account the least damped modes of the core-localised high-$n$ branch ($n \in [12, 30]$) and the weakly damped low-$n$ branch ($n \in [5, 12]$) with modes in the outer core region. It was shown that in our model, the EP redistribution stays well within quasilinear expectations—even if damping is neglected—if a certain EP density threshold is not overcome, the relaxed radial density gradient remains well above the critical gradient. In the scenario with the full, default EP density profile and neglected damping, it was found, that the phase space redistribution caused by the (sufficiently large) number of inner, high-$n$ modes triggers the excitation of the otherwise marginally unstable outer modes of the low-$n$ branch (enhancement of a factor between 5 and 60). This excitation is very sensitive to the amount of redistribution by the high-$n$ modes. It leads to the rapid depletion of the radial EP density gradient in the region of $s \approx 0.55$. This non-local effect is based on the radial overlap of the two branches that forms, at the time when the radially broad low-$n$ mode amplitudes grow towards $\delta B/B_{mag} \approx 10^{-3}$. Therefore, such depletion does

Figure 16. Default radial EP density profile case: self-consistent multi-mode evolution for all 27 modes. Blue represents modes of the low-$n$ branch, red of the high-$n$ branch. The integers denote the toroidal mode numbers $n$.

Figure 17. Radial profile of the $\alpha$ particle (green) and NBI (blue) drive, and both of them together (red). From [12].
not occur in the quasilinear scenario, with mode amplitudes fixed to the much lower single mode saturation levels, which effectively also restricts the modes locally and therefore creates a transport barrier. The same transport barrier is present for more favourable \( q \) profiles which leads to a different shear Alfvén gap structure with increased continuum damping, either via a steeper \( q \)-profile or a very flat background density profile. The resulting more localised modes would prevent radial resonance overlap and lead to radially well-separated redistribution. Comparable, but slightly weaker domino-like behaviour was found in the self-consistent nonlinear case with full damping and EP density scaled to 200%. This scaling can be motivated by the strong NBI drive which is expected to approximately double the drive w.r.t. to a pure \( \alpha \) particle population. However, since the NBI population differs significantly from the \( \alpha \) particle population, a multi-species analysis is needed to give a more realistic statement.

This work is intended to demonstrate the probability that a domino-like effect in EP transport can be relevant for ITER. However, due to the limitations of the model, no predictions can be made yet. The most important effects that should be investigated further are the NBI EP population and the sensitivity of different—including radius-dependent—velocity distribution functions. At this point of the study, the domino-like depletion in the nonlinear simulation shows, that for certain ‘worst-case’ scenarios, it might not be sufficient to investigate EP redistribution on a quasilinear basis only, without taking into account the non-local and multi-mode effects. Even if (on the transport timescale), the background density and current (thus, \( q \)) profiles are leading to a stable situation, small changes (towards a more box-like shape) in either the background density or current can lead to ‘overshooting’ domino-like EP redistribution and TAE drive. The final situation might even oscillate around the marginal stable profile.

7. Outlook

In the presented work, the EP population generated by NBI is not taken into account yet. Due to the strong anisotropy of the NBI ions in the velocity space, the phase space resonance regions for these particles are different. The NBI drive depends also on the mode number \( n \) in a different way w.r.t. to the \( \alpha \) particle drive. Further, the radial mode structures might slightly change if the NBI EP population is considered. Since a multi-species approach has already been implemented into HAGS and is in the phase of testing, a follow-up publication will present an investigation including information about the NBI population.

The presented work raises a question about whether a large amount of EP losses could be expected for this (or other) ITER scenarios. Since the redistribution in the worst-case scenario reaches out to \( s \approx 0.8 \), possible interaction with the 3D field ripple perturbation has to be taken into account. A collaboration with the ASCOT code [21] simulating the edge-localised field ripple with given LIGRA perturbations is already ongoing. It allows for a first, preliminary estimate, that the amount of EP losses would not be dangerously large, since the loss...
regions calculated by Ascot and the radial extension of the high amplitude areas in the Hagis simulation barely overlap. However also other Alfvén eigenmodes (here only TAEs) and global MHD modes such as islands, have to be included.

Beside the question of real losses, it is planned in the near future to investigate the physical mechanisms and relevant timescales of particle redistribution; special attention will be paid to examining whether EPs are moving from the center to the edge on the basis of resonant or diffusive processes in the different stages of multi-mode particle interaction. For that study, the newly implemented Hamiltonian Mapping Technique [22] will be used within Hagis in combination with prescribed amplitude evolution to lower the CPU costs of the simulation.

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