Theoretical uncertainty in sin 2\(\beta\): An update

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The source of theoretical uncertainty in the extraction of sin 2\(\beta\) from the measurement of the golden channel \(B_d \to J/\psi K^0\) is briefly reviewed. An updated estimate of this uncertainty based on \(SU(3)\) flavour symmetry and the measurement of the decay \(B_d \to J/\psi \pi^0\) is also presented.

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1 Introduction

The decay $B_d \to J/\psi K$ was recognized long ago as a golden mode for extracting $\sin 2\beta$, $\beta$ being one of the angles of the Unitarity Triangle (UT), by measuring the time-dependent CP asymmetry [1]. In fact, its decay amplitude is strongly dominated by a single term with the consequence that the hadronic uncertainties largely cancel out in the CP asymmetry, making this measurement the prototype of the “theoretically clean” measurements in $B$ physics.

Yet a subleading amplitude with a different weak phase, however small, is present and introduces a theoretical uncertainty in the extraction of $\sin 2\beta$. This uncertainty needs to be evaluated, in view of the remarkable accuracy on the measurement of the $B_d \to J/\psi K$ CP asymmetry reached at the $B$ factories and even more of the high precision expected at LHCb and at the next-generation super $B$ factories.

Unfortunately, no reliable purely theoretical estimate of the $B_d \to J/\psi K$ decay amplitude is available as this amplitude does not factorize and is not readily computable using non-perturbative techniques, such as lattice QCD or QCD sum rules. However, back in 1999, Robert Fleischer pointed out that the theoretical error in the extraction of $\sin 2\beta$ from the $B_d \to J/\psi K$ CP asymmetry could be estimated from data using the decay $B_s \to J/\psi K_{S,L}$ and the $SU(3)$ flavour symmetry with no additional assumptions [2]. At that time, however, no measurement of $B_s \to J/\psi K_{S,L}$ was available. Recently CDF measured the CP-averaged Branching Ratio ($BR$) [3], but the time-dependent analysis is still missing. For this reason the method cannot be used yet.

Later on, we proposed an alternative method still based on flavour symmetry, but requiring few additional assumptions on hadronic amplitudes [4]. This method makes use of time-dependent measurements of the channel $B_d \to J/\psi \pi^0$ available from the $B$ factories. We obtained an estimate of the theoretical uncertainty in the extraction of $\sin 2\beta$ that was non-negligible with respect to the experimental errors. A conservative evaluation of $SU(3)$-breaking effects was used in the absence of additional experimental information.

More recently, an updated analysis based on this method appeared. Using more precise data and an estimate of $SU(3)$ breaking mainly based on factorization, the authors of ref. [5] found that $\beta$ could be shifted by as much as $[-3.9, -0.8]^\circ$ at 1$\sigma$.

In this proceedings, we briefly review the issue of theoretical uncertainties in the extraction of $\sin 2\beta$ from $B_d \to J/\psi K^0$ and present an update of our estimate based on ref. [4].
2 CP violation in $B_d \to J/\psi K^0$

The decay amplitude for $B_d \to J/\psi K^0$ in the Standard Model (SM) can be written as

$$A(B_d \to J/\psi K^0) = \lambda_c^s A_c(B_d, K^0) - \lambda_u^s A_u(B_d, K^0)$$

$$A(\overline{B}_d \to J/\psi \overline{K}^0) = \lambda_c^s A_c(B_d, K^0) - \lambda_u^s A_u(B_d, K^0),$$  \hspace{1cm} (1)

where $\lambda_{ud} = V_{ud} V_{ub}^*$ and $V$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix \[6, 7\]. The hadronic amplitudes $A_{u,c}(B_d, K^0)$ can be written in terms of the Renormalization Group Invariant (RGI) amplitudes introduced in ref. [8] as

$$A_c(B_d, K^0) = E_2(c, c, s; B_d, J/\psi, K^0) + P_3(s, c; B_d, J/\psi, K^0),$$

$$A_u(B_d, K^0) = P_2^{GIM}(s, c; B_d, J/\psi, K^0) - P_2(s, c; B_d, J/\psi, K^0).$$  \hspace{1cm} (2)

The time-dependent CP asymmetry in $B_d \to J/\psi K_{S,L}$ is given by

$$a_{CP}^{B_d \to J/\psi K_{S,L}}(t) = \frac{\Gamma(B_d(t) \to J/\psi K_{S,L}) - \Gamma(\overline{B}_d(t) \to J/\psi \overline{K}_{S,L})}{\Gamma(B_d(t) \to J/\psi K_{S,L}) + \Gamma(\overline{B}_d(t) \to J/\psi \overline{K}_{S,L})} = C_{B_d \to J/\psi K_{S,L}} \cos(\Delta m_{Bd} t) - S_{B_d \to J/\psi K_{S,L}} \sin(\Delta m_{Bd} t),$$  \hspace{1cm} (3)

where

$$S_{B_d \to J/\psi K_{S,L}} = \frac{2 \text{Im}(\lambda_{B_d \to J/\psi K_{S,L}})}{1 + |\lambda_{B_d \to J/\psi K_{S,L}}|^2}, \hspace{1cm} C_{B_d \to J/\psi K_{S,L}} = \frac{1 - |\lambda_{B_d \to J/\psi K_{S,L}}|^2}{1 + |\lambda_{B_d \to J/\psi K_{S,L}}|^2},$$  \hspace{1cm} (4)

with

$$\lambda_{B_d \to J/\psi K_{S,L}} = \eta_{K_S,K_L}(q/p)_{B_d} \frac{A(\overline{B}_d \to J/\psi \overline{K}^0)}{A(B_d \to J/\psi K^0)} (q/p)_{K^0}$$  \hspace{1cm} (5)

and

$$(q/p)_{B_d} = -\frac{V_{tb} V_{td}^*}{V_{tb} V_{td}^*}, \hspace{1cm} (q/p)_{K^0} = -\frac{V_{cd} V_{cd}^*}{V_{cd} V_{cd}^*}.$$  \hspace{1cm} (6)

The factors $\eta_{K_S} = -1$ and $\eta_{K_L} = 1$ account for the CP eigenvalue of the final state (neglecting CP violation in kaon mixing).

In the limit of vanishing $A_u(B_d, K^0)$, one has

$$\lambda_{B_d \to J/\psi K_{S,L}} = \eta_{K_S,K_L} \left( \frac{V_{cb} V_{td}^*}{V_{cd} V_{cd}^*} \right) \left( \frac{V_{cb} V_{cd}^*}{V_{cd} V_{cd}^*} \right) = \eta_{K_S,K_L} e^{-2i\beta},$$  \hspace{1cm} (7)

where the UT angle $\beta = \arg(-(V_{cb}^* V_{cd})/(V_{tb} V_{td}^*))$, so that

$$S_{B_d \to J/\psi K_{S,L}} = -\eta_{K_S,K_L} \sin 2\beta, \hspace{1cm} C_{B_d \to J/\psi K_{S,L}} = 0.$$  \hspace{1cm} (8)
A nonvanishing $A_u(B_d, K^0)$ induces a theoretical uncertainty in the extraction of $\sin 2\beta$ and possibly a value of $C_{B_d \to J/\psi K_{S,L}}$ different from zero. Indeed, for $A_u(B_d, K^0) \neq 0$ one expects a nonvanishing

$$\Delta S_{B_d \to J/\psi K_{S,L}} = S_{B_d \to J/\psi K_{S,L}} + \eta_{K_{S,L}} \sin 2\beta$$  \hspace{1cm} (9)$$

Let us now discuss how to estimate the value of $A_u(B_d, K^0)$ and thus the value of $\Delta S_{B_d \to J/\psi K_{S,L}}$ using flavour symmetry.

### 3 Evaluation of the theoretical uncertainty of the golden mode $B_d \to J/\psi K^0$ in the Standard Model

The basic idea of the $SU(3)$-based methods is to use the flavour symmetry to extract $A_u(B_d, K^0)$ from a decay channel where $A_u$ is not Cabibbo suppressed, thereby obtaining an estimate of the departure from eq. (8). In particular, the method discussed in refs. [4] uses the two $SU(3)$-related channels $B_d \to J/\psi K^0$ and $B_d \to J/\psi \pi^0$. The amplitude of $B_d \to J/\psi \pi^0$ can be written as

$$A(B_d \to J/\psi \pi^0) = \frac{1}{\sqrt{2}} \left\{ \lambda_c^d \left( A_c(B_d, \pi^0) + \Delta_2 A_c(B_d, \pi^0) \right) - \lambda_u^d \left( A_u(B_d, \pi^0) + \Delta_2 A_u(B_d, \pi^0) \right) \right\},$$  \hspace{1cm} (10)$$

where, using the notation of ref. [8],

$$A_c(B_d, \pi^0) = E_2(c, c, d; B_d, J/\psi, \pi^0) + P_2(d, c; B_d, J/\psi, \pi^0),$$

$$\Delta_2 A_c(B_d, \pi^0) = EA_2(c, c, d; B_d, J/\psi, \pi^0) - EA_2(c, c, u; B_d, J/\psi, \pi^0) + P_4(d, c; B_d, \pi^0, J/\psi) - P_4(u, c; B_d, \pi^0, J/\psi),$$

$$A_u(B_d, \pi^0) = P_2^{GIM}(d, c; B_d, J/\psi, \pi^0) - P_2(d, c; B_d, J/\psi, \pi^0) - EA_2(u, u, c; B_d, \pi^0, J/\psi),$$

$$\Delta_2 A_u(B_d, \pi^0) = P_4^{GIM}(d, c; B_d, \pi^0, J/\psi) - P_4^{GIM}(u, c; B_d, \pi^0, J/\psi) - P_4(d, c; B_d \pi^0, J/\psi) + P_4(u, c; B_d \pi^0, J/\psi).$$  \hspace{1cm} (11)$$

In the $SU(3)$ limit, with the additional assumption of negligible electroweak penguins ($\Delta_2 A_c$ and $\Delta_2 A_u$) and $E A_2$, one has $A_c^{SU(3)} = A_c(B_d, K^0) = A_c(B_d, \pi^0)$ and $A_u^{SU(3)} = A_u(B_d, K^0) = A_u(B_d, \pi^0)$. Therefore there are three independent hadronic parameters ($|A_c^{SU(3)}|$, $|A_u^{SU(3)}|$ and the relative strong phase) and six measurements ($S$, $C$, and the CP-averaged BR in each channel). Using all these measurements but $S_{B_d \to J/\psi K_{S,L}}^{exp}$, it is possible to extract the hadronic parameters, thus making predictions for $\Delta S_{B_d \to J/\psi K_{S,L}}$. This is the theoretical correction to be used in the
also estimated non-factorizable $A$ channels the experimental situation has improved considerably. First, the errors in the two $A$ ref. \cite{5}. The authors of this paper used exact $A$ similar estimate of $\Delta A$ following we give results for the normalized amplitudes extraction of $\sin 2\beta$ from $S_{\exp}^{B_d\rightarrow J/\psi K_{S,L}}$. For the sake of simplicity, the correlation between $\Delta S_{B_d\rightarrow J/\psi K_{S,L}}$ and $S_{\exp}^{B_d\rightarrow J/\psi K_{S,L}}$ is discarded. Its inclusion is straightforward, but would require the simultaneous fit of the CKM phase and $\Delta S_{B_d\rightarrow J/\psi K_{S,L}}$ within the UT analysis.

Clearly, as $SU(3)$ is not an exact symmetry, the main issue of this method is to quantify the effect of the $SU(3)$ breaking. In ref. \cite{4} we tried to reduce the usage of $SU(3)$ to a minimum, extracting from $B_d \rightarrow J/\psi \pi^0$ only the $4\sigma$ range of $|A_u^{SU(3)}|$, and leaving the phase unconstrained. This was a conservative choice in the absence of independent tests of $SU(3)$ but $A_c(B_d, K^0) \sim A_c(B_d, \pi^0)$. More recently, a similar estimate of $\Delta S_{B_d\rightarrow J/\psi K_S}$ using these two decay modes has been presented in ref. \cite{5}. The authors of this paper used exact $SU(3)$ taking $A_u(B_d, K^0)/A_c(B_d, K^0) = A_u(B_d, \pi^0)/A_c(B_d, \pi^0)$, and included an estimate of $SU(3)$ breaking in the ratio $A_c(B_d, K^0)/A_c(B_d, \pi^0)$ based on factorization. In this way they obtained a negative $\Delta S_{B_d\rightarrow J/\psi K_S}$ corresponding to a shift of $2\beta$ by $[-3.9,-0.8]$° at 1σ. They also estimated non-factorizable $SU(3)$-breaking effects, keeping however the sign of $A_u(B_d, K^0)/A_c(B_d, K^0)$ fixed, obtaining the range $[-6.7,0]$°.

In the rest of this section we update the analysis of ref. \cite{4}. In the past five years, the experimental situation has improved considerably. First, the errors in the two channels $B_d \rightarrow J/\psi K_S$ and $B_d \rightarrow J/\psi \pi^0$ have shrunk by a factor of two. Second, the $BR(B_s \rightarrow J/\psi K^0)$ has been measured, providing an independent test of $SU(3)$.

The input values of the theoretical and CKM parameters used in the present analysis are given in Table 1. No error is attached to Wilson coefficients, form factors and decay constants, as factorized amplitudes only provide the normalization of the hadronic amplitudes which are fitted from the data. In particular, we define

$$A_{u,c}(B_d, \pi^0) = \frac{G_F}{\sqrt{2} m_{B_d}^2} F(B \rightarrow \pi) f_{J/\psi} \left( C_2 + \frac{1}{3} C_1 \right) \overline{A}_{u,c}(B_d, \pi^0),$$

$$A_{u,c}(B_d, K) = \frac{G_F}{\sqrt{2} m_{B_d}^2} F(B \rightarrow K) f_{J/\psi} \left( C_2 + \frac{1}{3} C_1 \right) \overline{A}_{u,c}(B_d, K),$$

(12)

where $G_F$ is the Fermi constant and the other parameters are listed in Table 1. In the following we give results for the normalized amplitudes $\overline{A}_{u,c}$. The CKM parameters are taken from the Summer 2010 UT fit analysis without the $\sin 2\beta$ constraint \cite{9}.

|  | $C_1$ | $C_2$ | $F^{B\rightarrow K}/F^{B\rightarrow \pi}$ |
|---|---|---|---|
| $F^{B\rightarrow \pi}(m_{J/\psi}^2)$ | 1.083 | -0.185 | 1.2 |
| $f_{J/\psi}$ | 0.4 | - | 5.2795 |
| $A$ | 0.80 ± 0.01 | - | 0.2255 ± 0.0005 |
| $\bar{\rho}$ | 0.164 ± 0.025 | - | 0.397 ± 0.023 |

Table 1: Input values used in the analysis. Dimensionful quantities are given in GeV.
Table 2: Results of the fit of $B_d \to J/\psi\pi^0$ (see the text for details).

Experimental inputs and results used in the fit of the $B_d \to J/\psi\pi^0$ amplitude are given in Table 2. As also shown in Figure 1, the value of $\overline{A}_c$ is compatible with one, meaning that, even in the absence of compelling theoretical arguments, naïve factorization provides a reasonable estimate of this amplitude within $\sim 20$–$30\%$. On the other hand, $A_u$ is not as well determined. We notice that, with the new data, the possibility of exchanging the role of $A_c$ and $A_u$ is more disfavoured than in our previous analysis. Therefore we no longer need to introduce a cut to retain the $SU(3)$ compatible result only, as we did in ref. [4]. In addition, the relative strong phase is now better determined, showing a preference for positive values albeit with a large uncertainty. In this proceedings, we stick to our original proposal and discard the phase information (more refined analyses will be presented in a forthcoming paper). Therefore the $4\sigma$ range $\overline{A}_u < 2.5$ extracted from this fit is the only $SU(3)$-based information we use to evaluate the theoretical error on $\sin 2\beta$.

Figure 1: Hadronic parameters extracted from $B_d \to J/\psi\pi^0$ data.

With this $a$-priori cut on $A_u$, we can perform a fit to the $B_d \to J/\psi K^0$ data. Experimental inputs and results can be found in Table 3. In this case, $A_c$ is determined much better than in the $B_d \to J/\psi\pi^0$ case. Again it lies within $\sim 30\%$ of its factorized value, namely it is compatible with factorization given the typical uncertainties attached to decay constants and form factors in Table 1. As expected, both $A_u$ and the relative phase are practically unconstrained, showing the importance of the information coming from $B_d \to J/\psi\pi^0$ for estimating $\Delta S$. The corresponfing
Table 3: Results of the fit of $B_d \rightarrow J/\psi K^0$. $S^{\exp}$ is not used in the fit.

|            | $B^{\text{th}}$ | $BR^{\text{th}}$ | $C^{\text{th}}$ | $C^{\exp}$ | $S^{\text{th}}$ | $S^{\exp}$ | $|A_c|$ | $|A_u|$ | arg $A_u - \arg A_c$ |
|------------|-----------------|-----------------|-----------------|------------|-----------------|------------|-------|-------|---------------------|
|            | $(8.63 \pm 0.35) \times 10^{-4}$ | 0.00 $\pm$ 0.01 | 0.77 $\pm$ 0.04 | 1.24 $\pm$ 0.03 | 0.56 $\pm$ 0.56 | $(160 \pm 20)^{\circ}$ |
| $BR^{\exp}$ | $(8.63 \pm 0.35) \times 10^{-4}$ | 0.00 $\pm$ 0.02 | 0.655 $\pm$ 0.024 |

Using these results, we find

$$\Delta S_{B_d \rightarrow J/\psi K^0} = 0.00 \pm 0.02.$$  \hspace{1cm} (13)

Since our method discards the phase information on $A_u$ from $B_d \rightarrow J/\psi \pi^0$, the correction we obtain does not shift the central value of $S_{B_d \rightarrow J/\psi K_S}$ but just introduces a theoretical uncertainty.

Figure 2 shows that the theoretical uncertainty on $\sin 2\beta$ is not entirely negligible with respect to the present experimental error. We do not find a correction as large as in ref. [5], although the agreement is reasonable considering the aforementioned differences in the two methods (notice in addition that the variable $\Delta \phi_d$ defined in ref. [5] to account for the deviation of $S_{B_d \rightarrow J/\psi K_S}$ from $\sin 2\beta$ is $\Delta \phi_d \sim \Delta S_{B_d \rightarrow J/\psi K_S} / \cos 2\beta$).

It is very important to stress that the evolution of the $B \rightarrow J/\psi \pi$ data is expected to match the $B \rightarrow J/\psi K$ one so that this method will be always able to keep the theoretical error on the $\sin 2\beta$ extraction under control, even reaching the high precision expected at the super-$B$ factories [5, 10]. LHCb, on the other hand, will be able to exploit the $B_s \rightarrow J/\psi K^0$ data to achieve the same goal with no need of neglecting any hadronic amplitude.
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References

[1] I. I. Y. Bigi and A. I. Sanda, Nucl. Phys. B 193 (1981) 85.

[2] R. Fleischer, Eur. Phys. J. C 10 (1999) 299 [arXiv:hep-ph/9903455].

[3] CDF Collaboration, http://www-cdf.fnal.gov/physics/new/bottom/100708.blessed-BsJpsiK/cdf10240_SuppresBsPublicNote.pdf

[4] M. Ciuchini, M. Pierini and L. Silvestrini, Phys. Rev. Lett. 95 (2005) 221804 [arXiv:hep-ph/0507290].

[5] S. Faller, M. Jung, R. Fleischer and T. Mannel, Phys. Rev. D 79 (2009) 014030 [arXiv:0809.0842 [hep-ph]].

[6] N. Cabibbo, Phys. Rev. Lett. 10 (1963) 531.

[7] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49 (1973) 652.

[8] A. J. Buras and L. Silvestrini, Nucl. Phys. B 569 (2000) 3 [arXiv:hep-ph/9812392].

[9] M. Bona et al. [ UTfit Collaboration ], PoS EPS-HEP2009 (2009) 160. [arXiv:0909.5065 [hep-ph]]; A. Bevan et al. [ UTfit Collaboration ], [arXiv:1010.5089 [hep-ph]]; UTfit Collaboration, http://www.utfit.org/UTfit/ResultsSummer2010PreICHEP

[10] B. O’Leary et al. [ SuperB Collaboration ], [arXiv:1008.1541 [hep-ex]].