ANOMALY CANCELLATIONS AND OPEN-STRING THEORIES

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ABSTRACT

In models of oriented closed strings, anomaly cancellations are deeply linked to the modular invariance of the torus amplitude. If open and/or unoriented strings are allowed, there are no non-trivial modular transformations in the additional genus-one amplitudes (Klein bottle, annulus and Möbius strip). As originally recognized by Green and Schwarz, in the ten-dimensional type-I superstring the anomaly cancellation results from a delicate interplay between the contributions of these additional surfaces. In lower-dimensional models, the possible presence of a number of antisymmetric tensors yields a generalization of the Green-Schwarz mechanism. I illustrate these results by referring to some six-dimensional chiral models, and I conclude by addressing the additional difficulties that one meets when trying to extend the construction to chiral four-dimensional models.

1. Introduction

In ten dimensions, the unique Lagrangian of $N = 1$ supergravity coupled to $N = 1$ supersymmetric Yang-Mills theory [1] describes the low-energy interactions of the massless modes of two vastly different string models, the heterotic string [2] and the type-I superstring. As a result, the anomaly cancellations at work in the two models take a unique form, the Green-Schwarz mechanism [3]. There are two crucial ingredients in this mechanism. The first, a proper choice of gauge group, does not suffice to eliminate the whole anomaly polynomial. Rather, it disposes only of the terms containing irreducible traces of six gauge-field strengths and of six Riemann tensors. The second, key ingredient, is the presence of an antisymmetric two-tensor, that is to acquire proper transformations under gauge and Lorentz symmetries. One may then exhibit a local counterterm that would suffice to cancel the residual anomaly polynomial, in this case the product of an eight form and a four form.

Though quite suggestive, the low-energy analysis should be supplemented by a proper study of string amplitudes. The crucial differences between the two string models are then evident. In the heterotic string, a model of oriented closed strings only, the modular invariance of the torus amplitude may be held responsible for the cancellation of the residual anomaly. A neat discussion of this result may be found in ref. [4]. On the other hand, in the type-I superstring, containing both unoriented closed strings and open strings, three additional surfaces (the Klein bottle, the
annulus and the Möbius strip) contribute to the anomaly. The cancellation results in this case from a delicate interplay between the contributions of these three surfaces that, by a suitable choice of “time” coordinate on the world sheet, may all be associated to the propagation of closed strings [3,5,6]. A related, crucial feature of the type-I superstring has to do with the Chan-Paton construction of the gauge symmetry [7,8]. This is the origin of long-known restrictions on the gauge group [8]: all exceptional groups are excluded, even \( E_8 \otimes E_8 \), despite its being allowed by the low-energy analysis. The lesson that should be drawn from all this is that lower-dimensional models are likely to exhibit further crucial differences between the two classes of string theories.

The study of open-string models has long been hampered by the lack of a procedure to construct new interesting solutions. In ref. [9] I proposed that consistent open-string models should be defined in terms of an orbifold-like construction in parameter space, and I argued that they should be somehow associated to arbitrary closed-string models with a symmetry under the interchange of left and right modes. In a series of subsequent papers [10,11,12] these observations were turned into an algorithm to construct rational open-string theories. Cardy’s analysis of the annulus amplitude in rational conformal field theory [13] proved to be the key to achieve Chan-Paton symmetry breaking.

A procedure that uses closed-string models as the starting point has the potential to lead to some surprising results. Thus, the chiral six-dimensional models of ref. [11] contain in their spectra a number of (anti)self-dual two-tensors that draw their origin from Ramond-Ramond sectors of the “parent” type-IIb string. I have no good argument to justify the presence of these fields starting from the type-I theory, since the key feature of the construction is using the closed-string theory as the starting point. Still, the antisymmetric tensors play a crucial role in the anomaly cancellation procedure [14], and give it some distinctive features that I would like to discuss in the remainder of this talk. Other, related, developments are reviewed in ref. [15].

2. Six - Dimensional Chiral Models and the Generalized Mechanism

I will begin by discussing two classes of six-dimensional models. They are obtained starting from different rational models describing the compactification of the type-IIb superstring on \( K_3 \). In the first class of models [11], the torus amplitude is

\[
T = \sum_{i=1}^{8} |\chi_i|^2 + \sum_{i=1}^{8} |\tilde{\chi}_i|^2 ,
\]

where the sixteen generalized characters are suitable combinations of the four characters of \( SO(4) \) level one. Defining \( Q_o = VO - CC \), \( Q_v = OV - SS \), \( Q_s = OC - SO \) and \( Q_c = VS - CV \), the sixteen generalized characters are

\[
\chi_1 = Q_o OO + Q_v VV , \quad \tilde{\chi}_1 = Q_s SO + Q_c CV ,
\]
The model has \( N = 2 \) supersymmetry in six dimensions (thus, it would have \( N = 4 \) supersymmetry if trivially reduced to four dimensions), and all massless modes are associated to the terms (or supersymmetry if trivially reduced to four dimensions), and all massless modes are associated to the terms \( |\chi_1|^2, |\chi_5|^2, |\tilde{\chi}_6|^2, |\tilde{\chi}_7|^2 \) and \( |\tilde{\chi}_8|^2 \). The massless spectrum contains a supergravity multiplet (in the notation of ref. [16] the \( N = 4b \) multiplet) and a total of 21 tensor multiplets. This field content is fixed completely by the anomaly analysis, as pointed out in refs. [17,18]. Starting from this model and proceeding as in ref. [11], one may derive a class of open-string “descendants” whose anomaly polynomials do not contain any irreducible traces, provided the Chan-Paton multiplicities satisfy the tadpole conditions

\[
\sum_{i=1}^{8} n_i = \sum_{i=1}^{8} \tilde{n}_i = 16 ;
\]

\[
\begin{align*}
n_5 - n_1 + \tilde{n}_1 + \tilde{n}_6 + \tilde{n}_7 + \tilde{n}_8 &= 8 ; \\
n_6 - n_2 + \tilde{n}_2 + \tilde{n}_5 + \tilde{n}_7 + \tilde{n}_8 &= 8 ; \\
n_7 - n_3 + \tilde{n}_3 + \tilde{n}_5 + \tilde{n}_6 + \tilde{n}_8 &= 8 ; \\
n_8 - n_4 + \tilde{n}_4 + \tilde{n}_5 + \tilde{n}_6 + \tilde{n}_7 &= 8 .
\end{align*}
\]  

These allow, for instance, a \( USp(8)^4 \) gauge group. The resulting massless spectrum contains chiral fermions in the representations \((8,1,1,8), (1,8,8,1), (8,1,8,1)\) and \((1,8,1,8)\). In addition to the scalar multiplets containing these fermions, the massless spectrum contains the \( N = 2b \) supergravity multiplet, five tensor multiplets and sixteen scalar multiplets from the closed sector, as well as the gauge multiplet from the open sector. I would like to stress that the model contains a number of antisymmetric two-tensors, to wit five self-dual tensors from the tensor multiplets and one antiself-dual tensor from the \( N = 2b \) supergravity multiplet. This is rather fortunate since, even after imposing the tadpole conditions of eq. (3), the anomaly polynomial \textit{does not} factorize. Therefore, in these models the Green-Schwarz mechanism may not work in the standard fashion. For instance, for the \( USp(8)^4 \) model one finds the residual polynomial

\[
A = \frac{1}{8} \left\{ (\text{tr}F_1^2)^2 + (\text{tr}F_2^2)^2 + (\text{tr}F_7^2)^2 + (\text{tr}F_8^2)^2 \right\} \\
+ \frac{1}{16} \left\{ (\text{tr}F_1^2 + \text{tr}F_2^2 + \text{tr}F_7^2 + \text{tr}F_8^2) \right\} \text{tr}R^2 \\
- \frac{1}{4} \left\{ (\text{tr}F_1^2 \text{tr}F_7^2 + \text{tr}F_1^2 \text{tr}F_8^2 + \text{tr}F_2^2 \text{tr}F_7^2 + \text{tr}F_2^2 \text{tr}F_8^2) \right\} \\
- \frac{1}{32} (\text{tr}R^2)^2 ,
\]  

\[ (2.4) \]
where the two-forms are defined as follows: $R^{ab} = \frac{1}{2} R_{\mu\nu}^{ab} dx^\mu dx^\nu$ and $F^a = \frac{1}{2} F_{\mu\nu}^a dx^\mu dx^\nu$.

If the polynomial is diagonalized, the end result is rather pleasing, since there are precisely six non-zero eigenvalues, as many as the antisymmetric tensors, and

$$A = -\frac{1}{32} \left\{ \text{tr} F_1^2 + \text{tr} F_2^2 + \text{tr} F_7^2 + \text{tr} F_8^2 - \text{tr} R^2 \right\}^2 + \frac{3}{32} \left\{ \text{tr} F_1^2 + \text{tr} F_2^2 - \text{tr} F_7^2 - \text{tr} F_8^2 \right\}^2 + \frac{1}{32} \left\{ \text{tr} F_1^2 - \text{tr} F_2^2 + \text{tr} F_7^2 - \text{tr} F_8^2 \right\}^2 + \frac{1}{32} \left\{ \text{tr} F_1^2 - \text{tr} F_2^2 - \text{tr} F_7^2 + \text{tr} F_8^2 \right\}^2 .$$

(2.5)

It should be appreciated that only one combination contains the gravitational two-form. Strictly speaking, only the different normalization of the second term suggests that the quadratic form has six non-zero eigenvalues. This, however, may be seen quite clearly if all sixteen charge sectors are allowed, subject only to the tadpole conditions of eq. (3). The various combinations of field traces correspond precisely to the rows of the $S$ matrix acting on the sixteen characters of eq. (2) that identify the sectors containing the antisymmetric tensors, and

$$A = - \frac{1}{2} \left\{ \sum_m S_{1m} \text{tr} F_m^2 - 4 \text{tr} R^2 \right\}^2 + \frac{1}{2} \sum_k \left\{ \sum_m S_{km} \text{tr} F_m^2 \right\}^2 ,$$

(2.6)

where $k = 5, \tilde{1}, \tilde{6}, \tilde{7}, \tilde{8}$.

The first line contains the sum of all gauge-field strengths and is the only one containing the Riemann curvature. In addition, the various contributions enter the anomaly polynomial in a way that corresponds to a Minkowski metric with signature $(1 - n)$. If, following standard practice, the eight-form in eq. (6) is converted into a Green-Schwarz counterterm

$$\Delta L = + \frac{1}{2} \sum_{ij} \eta_{ij} F^{(i)} B^{(j)} ,$$

(2.7)

where $F^{(i)}$ denote combinations of Yang-Mills (and gravitational) curvatures, the modified field strengths for the antisymmetric tensors are

$$H^{(i)} = dB^{(i)} + \omega^{(i)} ,$$

(2.8)

with $\omega^{(i)}$ proper combinations of Yang-Mills and gravitational Chern-Simons forms. The couplings between combinations of Yang-Mills Chern-Simons forms and antisymmetric tensors may be explored by constructing the field equations of the low-energy theory (there are long-standing problems with action principles for (anti)self-dual bosons [18]). On the other hand, as in ten dimensions, the coupling to the gravitational Chern-Simons form is not in the low-energy field theory. The counterterm of eq. (7) has precisely the $SO(1, n)$ symmetry that one would expect to be present in this class of supergravity models [16].
Before displaying the structure of the generalized Chern-Simons couplings, I would like to repeat the exercise for another class of models discussed in ref. [11]. In this case the torus amplitude is

\[
T = |\chi_1|^2 + |\chi_2|^2 + |\chi_5|^2 + |\chi_6|^2 + "\text{tilde}"
\]

\[
\chi_3\bar{\chi}_4 + \chi_4\bar{\chi}_3 + \chi_7\bar{\chi}_8 + \chi_8\bar{\chi}_7 + "\text{tilde}"
\]

(2.9)

where “tilde” stands for the corresponding characters from the “twisted” sector. Following the procedure described in ref. [11] one may construct a class of open-string “descendants” whose annulus amplitudes are built out eight composite characters, \(\chi_1 + \chi_2, \chi_3 + \chi_4, \chi_5 + \chi_6, \chi_7 + \chi_8\), and the corresponding ones from the “twisted” sector. The resulting anomaly polynomials do not contain irreducible traces, provided the eight independent Chan-Paton multiplicities satisfy the tadpole conditions

\[
\sum_i n_i = \sum_i \tilde{n}_i = 8
\]

\[
n_1 + n_2 - n_3 - n_4 = \tilde{n}_3 + \tilde{n}_4 - \tilde{n}_1 - \tilde{n}_2
\]

\[
n_1 - n_2 - n_3 + n_4 = \tilde{n}_3 - \tilde{n}_4 - \tilde{n}_1 + \tilde{n}_2
\]

(2.10)

For instance, one may choose a gauge group \(USp(4)^4\). Then, apart from the gaugini, the resulting model contains chiral fermions in the representations \((4,1,4,1)\) and \((1,4,1,4)\), as well as two families in each of the representations \((4,1,1,4)\) and \((1,4,4,1)\). In addition to the scalar multiplets containing these fermions, the massless spectrum contains the \(N = 2b\) supergravity multiplet, seven tensor multiplets and fourteen scalar multiplets from the closed sector, as well as the gauge multiplet from the open sector. In this case there are only eight types of quantum numbers, and the residual anomaly polynomial may be written

\[
A = -\frac{1}{16} \left\{ \sum_i \text{tr} F_i^2 + \sum_i \text{tr} \tilde{F}_i^2 - \frac{1}{2} \text{tr} R^2 \right\}^2
\]

\[
+ \frac{1}{16} \left\{ \sum_i \text{tr} F_i^2 - \sum_i \text{tr} \tilde{F}_i^2 \right\}^2
\]

\[
+ \frac{1}{16} \left\{ \text{tr} F_1^2 + \text{tr} F_2^2 - \text{tr} F_3^2 - \text{tr} F_4^2 + "\text{tilde}" \right\}^2
\]

\[
+ \frac{1}{16} \left\{ \text{tr} F_1^2 - \text{tr} F_2^2 - \text{tr} F_3^2 + \text{tr} F_4^2 - "\text{tilde}" \right\}^2
\]

(2.11)

Why is the Green-Schwarz mechanism using only four of the eight available two-tensors? This may be understood quite naturally in terms of the construction of ref. [11]. Indeed, only eight of the sixteen characters are allowed in the transverse annulus amplitude. They are \(\chi_1, \chi_2, \chi_5, \chi_6\), and the corresponding characters from the twisted sector. Out of these, only \(\chi_1, \chi_5, \tilde{\chi}_1\) and \(\tilde{\chi}_6\) yield antisymmetric two-tensors. Thus, the model uses precisely the four antisymmetric tensors that are allowed in the vacuum channel of the annulus. These are the only ones that may take part in the Green-Schwarz mechanism.
It is amusing to construct the field equations for $N = 2b$ supergravity coupled to $n$ tensor multiplets and to gauge multiplets via the generalized Chern-Simons couplings required by the anomaly analysis. These equations extend the previous work of ref. [16], where the authors considered the two cases of $n$ tensor multiplets with no gauge multiplets and of a single tensor multiplet.* The low-energy supergravities corresponding to the two classes of open-string models I have described contain matter multiplets as well, but these coupled equations suffice to display the structure of the generalized couplings.

In the notation of ref. [14], the $n$ scalar fields parametrize the coset space $SO(1,n)/SO(n)$, and are conveniently described using the $SO(1,n)$ matrix

\[ V = \begin{pmatrix} v_0 & v_M \\ x^m_0 & x^m_M \end{pmatrix} \]  

(2.12)

Out of the elements of $V$ one may construct the composite $SO(n)$ connection

\[ S^{[mn]}_{\mu} = (\partial_\mu x^m_r) \tilde{x}_n^r \]  

(2.13)

antisymmetric in $(m,n)$ because of the (pseudo)orthogonal nature of $V$. The scalar kinetic term is then built out of

\[ P^m_{\mu} = \sqrt{\frac{1}{2}} (\partial_\mu v_r) c^{rz} ) \]  

(2.14)

where $P$ satisfies $D_{\mu} P^m_{\nu} = 0$. The model contains $(n+1)$ tensor fields $A_{\mu \nu}$ that transform in the fundamental representation of $SO(1,n)$. Combining these fields and the Chern-Simons forms one may construct the field strengths

\[ F^r = dA^r - c^{rz} \omega_z \]  

(2.15)

and, from these,

\[ H_{\mu \nu \rho} = v_r F^r_{\mu \nu \rho} \]  

\[ K^{m \mu \nu} = x^m_r F^r_{\mu \nu \rho} \]  

(2.16)

The spinor fields are a left-handed gravitino $\psi_\mu$, $n$ right-handed spinors $\chi^m$ from the tensor multiplets and the gaugini $\lambda$. All spinors are $Sp(2)$ Majorana-Weyl.

The field equations of the spinor fields are

\[ \gamma^{\mu \nu \rho} D_\nu \psi_\rho + H^{\mu \nu \rho} \gamma_{\nu \psi_\rho} - \frac{i}{2} K^{mn \mu \nu \rho} \gamma_{\nu \psi_\rho} \chi^m - \frac{i}{\sqrt{2}} P^m_{\mu \nu} \gamma^\nu \gamma^\mu \chi^m \\
- \frac{1}{2\sqrt{2}} \gamma^{\sigma \tau} \gamma_{\mu} v_r c^{\tau z} \text{tr}_z (F^{\sigma \tau} \lambda) = 0 \]  

(2.17)

\[ \gamma^\mu D_\mu \chi^m - \frac{1}{12} H_{\mu \nu \rho} \gamma^{\mu \nu \rho} \chi^m - \frac{i}{2} K^{mn \mu \nu \rho} \gamma_{\nu \psi_\rho} + \frac{i}{\sqrt{2}} P^m_{\mu \nu} \gamma^\mu \gamma^\nu \psi_\mu \\
- \frac{i}{2\sqrt{2}} x^m_r c^{\tau z} \text{tr}_z (\gamma^{\mu \nu} F_{\mu \nu}) = 0 \]

\[ (v_r c^{rz} ) \gamma^\mu D_\mu \lambda + \frac{1}{2\sqrt{2}} P^m_{\mu \nu} (x^m_r c^{rz} ) \gamma^\mu \lambda + \frac{1}{2\sqrt{2}} (v_r c^{rz} ) F_{\lambda \tau} \gamma^\mu \gamma^{\lambda \tau} \psi_\mu \\
+ \frac{i}{2\sqrt{2}} (x^m_r c^{rz} ) \gamma^{\mu \nu} \chi^m F_{\mu \nu} = 0 \]

* In this case the self-dual two-tensor joins the antiself-dual one in the $N = 2b$ supergravity multiplet to yield a single two-tensor with no self-duality, and one may write a Lagrangian in standard form.
while the field equations of the Bose fields are

\[
D_\mu P^{\mu\nu} - \frac{\sqrt{2}}{3} H^{\mu\nu\rho} K^m_{\mu\nu\rho} + \frac{1}{2\sqrt{2}} x^m r c^{rz} \text{tr}_z (F_{\alpha\beta} F^{\alpha\beta}) = 0
\]

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - H_{\mu\rho\sigma} H^{\rho\sigma} - K^m_{\mu\rho\sigma} K^m_{\nu\rho\sigma} - 2 P^m_{\mu} P^m_{\nu} + 2 g_{\mu\nu} P^m_{\rho} P^m_{\sigma} + 2 v_r c^{rz} \text{tr}_z (F_{\lambda\mu} F^{\lambda\nu} - \frac{1}{4} g_{\mu\nu} F^2) = 0
\]

\[
(v_r c^{rz}) D_\mu F^{\mu\nu} + \sqrt{2} (x^m r c^{rz}) P^m_{\mu} F^{\mu\nu} - (v_r c^{rz}) F^{\rho\sigma} H^\nu_{\rho\sigma} = 0
\]

(2.18)

together with the (anti)self-duality conditions for the antisymmetric two-tensors, that read

\[
H_{\mu\nu\rho} = \bar{H}_{\mu\nu\rho}
\]
\[
K^m_{\mu\nu\rho} = - \bar{K}^m_{\mu\nu\rho}
\]

(2.19)

A good consistency check comes from the supersymmetry transformations,

\[
\delta e^{\mu}_m = - i \bar{\epsilon} \gamma^m \psi^\mu
\]
\[
\delta \psi^\mu = D_\mu \epsilon + \frac{1}{4} H_{\mu\nu\rho} \gamma^{\nu\rho} \epsilon
\]
\[
\delta A^\nu_{\mu\nu} = i \bar{\psi}^\nu \gamma_{\nu\nu}^\mu \epsilon - \frac{1}{2} \bar{x}^m r \chi^{\mu\nu\rho} \epsilon - c^{rz} \text{tr}_z (A_{\mu} \delta A_{\nu})
\]
\[
\delta \chi^{\mu} = - \frac{i}{\sqrt{2}} \gamma^\nu P^m_{\mu} \epsilon + \frac{i}{12} K^m_{\mu\nu\rho} \gamma^{\mu\nu\rho} \epsilon
\]
\[
\delta v_r = \bar{x}^m r \bar{\epsilon} \chi^m
\]
\[
\delta \lambda = - \frac{1}{2\sqrt{2}} F^\mu_{\nu\mu} \gamma^{\mu\nu} \epsilon
\]
\[
\delta A_{\mu} = - \frac{i}{\sqrt{2}} (\bar{\epsilon} \gamma_{\mu} \lambda)
\]

(2.20)

that close on the Bose fields in terms of all local symmetries in the model. Under the transformations of eq. (20) the fermionic field equations turn into the bosonic ones, as discussed in ref. [14]. The constants $c^{rz}$ determine the generalized Chern-Simons couplings and the effective gauge charges of the vectors, thus limiting the effective range of the scalar fields. As in ref. [20], all these equations have been constructed to lowest order in the spinor fields. The methods of ref. [21] should provide a convenient way of completing the construction.

3. Four-dimensional Models

I would like to conclude by pointing out the type of difficulties one meets when trying to construct chiral four-dimensional models. A convenient class of four-dimensional models may be used to illustrate the nature of the problem. The “parent” closed strings are obtained in this case as $Z_2 \otimes Z_2$ orbifolds of the type-IIb
superstring compactified to four dimensions on the $SO(12)$ torus. I will confine my attention to two choices for the torus partition function,

$$T_1 = \sum_i |\chi_i|^2,$$

the diagonal modular invariant, and

$$T_2 = \sum_{i,j} C_{ij} \chi_i \chi_j,$$

the “charge-conjugation” modular invariant. The $\chi_i$ are a set of 64 generalized characters, and the label $i$ is a shorthand for a triple of indices. The first index, taking the values $o,v,s,c$, relates the characters to projections and/or twistings of the four $SO(12)$ characters $O_{12}, V_{12}, S_{12}, C_{12}$. The last two indices, taking the values $o,g,h,f$, are the usual labels for the four sectors and for the four projections. Thus, for instance,

$$\chi_{o,oo} = \tau_{oo} OOO + \tau_{og} VVO + \tau_{oh} OVV + \tau_{of} VOV,$$

$$\chi_{v,oo} = \tau_{oo} OVO + \tau_{og} VOO + \tau_{oh} OOV + \tau_{of} OVV,$$

while

$$\chi_{o,go} = \tau_{go} OOS + \tau_{gg} VVS + \tau_{gh} OVC + \tau_{gf} VOC,$$

$$\chi_{v,go} = \tau_{go} OVS + \tau_{gg} VOS + \tau_{gh} OOVC + \tau_{gf} VVC.$$

In eqs. (3.3) and (3.4) the space-time characters are

$$\tau_{oo} = oo00 + o0vv - scsc - cssc \quad \tau_{og} = oo00 + o0vv - scsc - cssc,$$
$$\tau_{oh} = o000 + v000 - cssc - sccc \quad \tau_{of} = oo00 + o0vv - sscc - cssc,$$
$$\tau_{go} = v0ss + ovcc - scvo - svco \quad \tau_{gg} = o0ss + v0cc - svoc - csvo,$$
$$\tau_{gh} = o0sc + v0cs - ccvs - ssoo \quad \tau_{gf} = o0cs + v0sc - ssuv - csov,$$
$$\tau_{ho} = v0so + occv - sovc - cvos \quad \tau_{hg} = ocs0 + vosc - sosv - cvvc,$$
$$\tau_{hh} = os0v + vccv - svos - ovsc \quad \tau_{hf} = os0v + vccv - svos - ovsc,$$
$$\tau_{fo} = vsus + acoc - soco - cuvc \quad \tau_{fg} = ocus + usvc - sosv - cvco,$$
$$\tau_{fh} = osvc + vcos - cocv - suvc \quad \tau_{ff} = osos + vcvc - svsv - coso.$$

I will confine my attention to one choice of “open-string Wilson lines” on the Möbius strip [11] such that the $P$ matrix acts as follows:

$$\chi_{o,oo} \rightarrow \frac{1}{2} \left( - \chi_{o,oo} + \chi_{o,og} + \chi_{o,oh} + \chi_{o,of} \right),$$
$$\chi_{o,og} \rightarrow \frac{1}{2} \left( \chi_{o,oo} - \chi_{o,og} + \chi_{o,oh} + \chi_{o,of} \right),$$
$$\chi_{o,oh} \rightarrow \frac{1}{2} \left( \chi_{o,oo} + \chi_{o,og} - \chi_{o,oh} + \chi_{o,of} \right),$$
$$\chi_{o,of} \rightarrow \frac{1}{2} \left( \chi_{o,oo} + \chi_{o,og} + \chi_{o,oh} - \chi_{o,of} \right).$$
Other consistent choices of the $P$ matrix lead to similar conclusions.

If the torus partition function is the one given in eq. (1) the vacuum channel of the annulus may only accommodate the 16 “untwisted” characters. As a result, the identity of the annulus is associated to the sum of the four characters $\chi_{o,oo}$, $\chi_{o,og}$, $\chi_{o,oh}$ and $\chi_{o,of}$. Given the first of eqs. (6), this is quite consistent with the structure of the $P$ matrix, since in this case the transverse Möbius channel may only accommodate the identity. There is no problem with the tadpole conditions, and the resulting open-string models have $N = 1$ supersymmetry in four dimensions but are not chiral, on account of the structure of the identity. On the other hand, if the torus invariant is the one given in eq. (2), all 64 characters may flow in the annulus amplitude. The models are chiral, have a large Chan-Paton charge space, and one may show that the tadpole conditions from the “twisted” sectors, easy to solve, imply the cancellation of all gauge anomalies.* The trouble comes from the four tadpole conditions from the “untwisted” sector. In this case all four characters in eq. (6) are allowed in the vacuum channels of the Klein bottle and Möbius amplitudes, and one finds the conditions

$$
\begin{align*}
    n_o + n_g + n_h + n_f &= 32 \\
    n_o + n_g - n_h - n_f &= -32 \\
    n_o - n_g + n_h - n_f &= -32 \\
    n_o - n_g - n_h + n_f &= -32
\end{align*}
$$

where $n_i$ denote the total Chan-Paton multiplicities from the four sectors of the spectrum. The different numbers of “minus” signs in the two sides of eq. (3), necessary in order to obtain a consistent symmetrization of the annulus amplitude, force some of the multiplicities to be negative, and therefore one may not impose these tadpole conditions in a consistent fashion. Though not directly related to four-dimensional anomalies, from a two-dimensional viewpoint these conditions have a dignity comparable to the others coming from the “twisted” sectors, since they are also related to the decoupling of spurious states [6] from vacuum channels. For the time being, one is therefore forced to live with other, less handy cases, where the available gauge groups of chiral models with $N = 1$ supersymmetry are small, typically products of $U(2)$ factors. I will stop here, leaving a proper discussion of these models to a future work.

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* These tadpole conditions are homogeneous, since all chiral fermions are in fundamental representations of unitary groups.
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