Theoretical Analysis and Finite Element Verification on Natural Vibration of Special Water Tower Structure System

Jing Ji1, Wei Kang2, Ming Xu1, Zhihui Li1 and Liangqin Jiang2*

1Department of Architectural Engineering, Qiqihar Institute of Engineering, Qiqihar, Heilongjiang, 161005, China
2College of Civil and Architectural Engineering, Northeast Petroleum University, Daqing, Heilongjiang, 163318, China

*Corresponding author’s e-mail: jijing1977@163.com

Abstract. In order to master the dynamic characteristics of this special engineering structure, based on the advanced structural dynamics, the water tower structure is simplified as a single degree of freedom vibration model according to the mass concentration method, and the analytical solution of the free vibration fundamental frequency of the water tower structure is derived. The three-dimensional finite element model of the water tower is established by using the finite element software ABAQUS, and the modal analysis of the water tower structure is carried out. The simulation fundamental frequency and vibration mode of the water tower structure are obtained. Through comparison, it can be seen that the analytical solution of the free vibration fundamental frequency of the water tower structure is in good agreement with the simulation result, which verifies the correctness and application scope of the analytical solution of the free vibration fundamental frequency of the water tower structure. It can be seen that the mass concentration method proposed in this paper can accurately calculate the fundamental frequency of water tower structure. Finally, the seismic design suggestions of water tower structure are given.

1. Introduction

Water tower is a kind of outdoor water supply structure which has been built since 1960s in China. It mainly regulates and stabilizes water pressure, stores and distributes water[1]. Although the urban water supply network system is developing rapidly, water towers still play an important role in some small and medium-sized cities. As a special architectural form, water tower has been damaged in different degrees in recent years (Tangshan earthquake and Wenchuan earthquake), so it is necessary to study the dynamic characteristics of water tower structure. In The Code for Seismic Design of Buildings (GB50011-2010), the seismic requirements of water tower structures are not stated. In The Standard for Seismic Design of Hydraulic Structures(GB51247-2018), only the provisions and requirements for seismic calculation of intake tower are described. Now the design calculation and construction of water tower are based on the atlas of water tower. Due to the particularity of the water tower structure, most of the current research is considering the mechanical properties of the water tower system under the solid-liquid coupling and Soil-Solid coupling. P. Qian and S. Zhu proposed to simplify the water tower structure into a vibration system with mass concentration [2,3], but they all proposed the calculation model and did not give the specific treatment method of mass concentration.

Based on the principle of advanced structural dynamics [4], in this paper, the water tower is simplified as a single degree of freedom system, and the analytical solution of the free vibration
frequency of the single degree of freedom water tower structure is given. Based on the finite element software ABAQUS [5], the modal analysis of water tower system is carried out [6], and the basic frequency and modal shape of water tower structure with different heights are obtained, and the correctness of the theoretical simplified analysis method is verified. Finally, the seismic design suggestions of water tower structure are given based on specific engineering examples.

2. Theoretical analysis on free vibration of water tower structure

2.1. Simplification of calculation model of water tower structure

The water tower structure system is composed of a concentrated upper water tank structure and a slender lower supporting tube structure, as shown in figure 1. In order to study the mechanical characteristics of water tower structure, it is necessary to simplify the structure system and obtain a reasonable calculation model. The upper water tank of the water tower can be simplified as the central point with mass, while the mass of the support tube of water tower is continuous and evenly distributed, and the mass can not be ignored (figure 2). Therefore, the mass concentration method is adopted to concentrate one-half of the member mass to both ends of the member, and the structural system is simplified as a typical single degree of freedom system, as shown in figure 3. The generalized mass is expressed as formula (1).

\[ M^* = M_1 + \frac{1}{2} M_2 \]  

(1)

The generalized stiffness is \( k^* \). From the structural mechanics, the generalized stiffness of the branch tube can be obtained.

\[ k^* = \frac{3(E_1 I_1 + E_2 I_2)}{L^3} \]  

(2)

Where, \( E_1 \) and \( E_2 \) are the modulus of elasticity of concrete and reinforcement, \( I_1 \) and \( I_2 \) are the moment of inertia of concrete and reinforcement sections, \( L \) is the height of the support tube.

2.2 Vibration frequency of water tower structure

According to advanced structural dynamics, the simplified equation of motion of single degree of freedom water tower system is established.

\[ M^* \ddot{x} + C^* \dot{x} + K^* x = p(t) \]  

(3)

When the structure has no external load, the reinforced concrete water tower system vibrates freely, and the damping effect of the water tower structure is very small, which can be ignored, so the motion equation can be simplified as:
The calculation formulas of circular and natural frequency of water tower system are as follows:

\[ M^* \ddot{x} + K^* x = 0 \]  
(4)

\[ \omega = \sqrt{\frac{k}{M^*}} \]  
(5)

\[ f = \frac{\omega}{2\pi} \sqrt{\frac{k}{4\pi^2 M^*}} \text{(Hz)} \]  
(6)

3. Finite element analysis of water tower

3.1 Structural system design of water tower

Referring to the actual project, nine cantilever systems with different heights are designed in this paper. The upper part adopts the same water tank structure, and the lower tube structure is 20m, 25m, 30m, 35m, 40m, 50m, 60m, 70m and 80m, respectively. The specific parameters of the nine water towers are shown in Table 1. The inner diameter of the cross section of the support cylinder is 2m and the outer diameter is 4m. The structure height of the upper water tank is 5m, and the inner diameter of the water tank structure is 9m and the outer diameter is 10m. HRB335 grade steel bar and HPB300 grade are used as longitudinal reinforcement and stirrup, and C30 concrete is adopted.

| The specimens | L (m) | \( M_1 \) (Kg) | \( \overline{M}_2 \) (Kgm\(^{-1}\)) | \( M^* \) (Kg) | \( E_1 I_1 \) (Nm\(^2\)) | \( E_2 I_2 \) (Nm\(^2\)) | \( K^* \) (Nm\(^{-1}\)) |
|---------------|---|------------|-----------------|------------|-----------------|-----------------|-----------------|
| TS-1          | 20 | 596548     | 23317           | 829717.79  | 3.50×10\(^{11}\) | 2.02×10\(^{10}\) | 138959054.36    |
| TS-2          | 25 | 596548     | 23317           | 888010.24  | 3.50×10\(^{11}\) | 2.02×10\(^{10}\) | 71147035.83     |
| TS-3          | 30 | 596548     | 23317           | 947302.69  | 3.50×10\(^{11}\) | 2.02×10\(^{10}\) | 41173053.14     |
| TS-4          | 35 | 596548     | 23317           | 1004595.13 | 3.50×10\(^{11}\) | 2.02×10\(^{10}\) | 25928220.06     |
| TS-5          | 40 | 596548     | 23317           | 1062887.58 | 3.50×10\(^{11}\) | 2.02×10\(^{10}\) | 17369881.80     |
| TS-6          | 50 | 596548     | 23317           | 1179472.48 | 3.50×10\(^{11}\) | 2.02×10\(^{10}\) | 8893379.48      |
| TS-7          | 60 | 596548     | 23317           | 1296057.37 | 3.50×10\(^{11}\) | 2.02×10\(^{10}\) | 5146631.64      |
| TS-8          | 70 | 596548     | 23317           | 1412642.27 | 3.50×10\(^{11}\) | 2.02×10\(^{10}\) | 3241027.51      |
| TS-9          | 80 | 596548     | 23317           | 1529227.16 | 3.50×10\(^{11}\) | 2.02×10\(^{10}\) | 2171235.22      |

3.2 Finite element model of water tower

In this paper, the finite element model of water tower structure is established by using the finite element software ABAQUS. In order to simplify the calculation, the influence of water sloshing on the structure mode is not considered, and the water is treated as a solid state. The concrete and solid water adopt eight node three-dimensional solid element (C3D8R), and the reinforcement framework adopts two node linear three-dimensional truss element (T3D2). During the structural modal analysis, materials are in the stage of elastic deformation, so all kinds of materials are regarded as isotropic materials. The bottom of the finite element model of the water tower is completely fixed, that is, the linear displacement (U1, U2, U3) along the x, y and z directions and the angular displacement (UR1, UR2, UR3) along the x, y and z directions are fixed. The mapping method is used for mesh generation. The geometric model of water tower structure is shown in figure 4, and the finite element model after mesh generation is shown in figure 5.
3.3 Modal analysis

The ABAQUS software is used to carry out the modal analysis of the water tower. The frequency \( f_s \) and mode shape of the free vibration of the water tower system can be obtained through the extended mode. The specific basic frequency is shown in table 2. The first six modes of TS-5 are shown in figure 6. It can be seen from the vibration mode that the first mode of water tower structure is mainly bending deformation.

| Specimens | L (m) | \( f_s \) (Hz) | \( \omega \) | \( f_{\text{theory}} \) (Hz) | \( \frac{f_s - f_{\text{theory}}}{f_{\text{theory}}} \times 100\% \) |
|-----------|------|---------------|-------------|-----------------|----------------------------------|
| TS-1      | 20   | 1.81          | 12.94       | 2.06            | 12.14\%                          |
| TS-2      | 25   | 1.34          | 8.95        | 1.42            | 5.63\%                           |
| TS-3      | 30   | 1.03          | 6.59        | 1.05            | 1.90\%                           |
| TS-4      | 35   | 0.82          | 5.08        | 0.81            | 1.23\%                           |
| TS-5      | 40   | 0.67          | 4.04        | 0.64            | 4.69\%                           |
| TS-6      | 50   | 0.47          | 2.75        | 0.44            | 6.82\%                           |
| TS-7      | 60   | 0.35          | 1.99        | 0.32            | 9.37\%                           |
| TS-8      | 70   | 0.27          | 1.51        | 0.24            | 12.50\%                          |
| TS-9      | 80   | 0.22          | 1.19        | 0.19            | 15.79\%                          |
3.4 Comparison between theoretical solution and numerical solution

According to the theoretical formula, the fundamental frequency of nine water tower structures is obtained, as shown in Table 2. Through comparison in Figure 7, when the length of the tube structure is 20m and 70m, the fundamental frequency errors are 12.14% and 12.50% respectively, which are large and beyond the scope of the project requirements. Therefore, it is reasonable to use the single degree of freedom system to calculate the height of the branch cylinder of the fundamental frequency of the water tower structure within the range of 25m-60m.

![Figure 7. Comparison of theoretical solution and numerical solution](image)

Figure 7. Comparison of theoretical solution and numerical solution

4. Conclusions

Through the establishment of simplified single degree of freedom calculation model and the finite element model of water tower structure, the analytical and numerical solutions of the fundamental frequency of nine water tower structures are obtained, and the corresponding modes are obtained.

1. The analytical solution of the fundamental frequency of water tower structure in the range of 25m-60m can be obtained by simplifying the structure of water tower to a single degree of freedom vibration model.

2. According to the finite element simulation and analysis, when the lower abutment tube structure of the water tower is dumpy or too slender, the fundamental frequency error will gradually increase.

3. Good agreement
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