CONFINEMENT, QUARK MATTER EQUATION OF STATE
AND HYBRID STARS

S.B. Khadkikar, A. Mishra, H. Mishra

Theory Group, Physical Research Laboratory, Navrangpura, Ahmedabad 380 009, India

Abstract

We consider here quark matter equation of state in a relativistic harmonic confinement model at zero temperature. The same is considered to study phase transition of neutron matter to quark matter at high densities. This, along with a phenomenological equation of state in the neutron matter sector is used to study hybrid stars. Using Tolman Oppenheimer Volkoff equations, stable solutions for hybrid stars are obtained with a maximum mass of 1.98 $M_\odot$ and radii around 10 kms with a quark core of about 1 to 2 kilometers.

(To appear in Mod. Phys. Lett. A)
I. INTRODUCTION

It is believed that hadronic matter undergoes a phase transition to quark matter at high temperatures and/or high densities. We may have these transitions possible in heavy ion collision experiments. However, signals here may get masked since such a phase lasts for a very short time followed by hadronisation. On the other hand we could also have compact stellar objects formed in a gravitational collapse where quark gluon matter could exist at sufficiently high densities. Since quarks and gluons are confined with absence of totally asymptotic states the overall confinement can affect the equation of state for quark matter. A phase transition of neutron matter to quark matter at zero temperature or temperature small compared to degeneracy temperature allows the existence of hybrid stars i.e., stars having a quark core and crust of neutron matter with appropriate pressure balancing at the interface [1-6]. In fact, the large magnetic field of the pulsars could be held by quark matter with nonzero electrically charged constituents rather than neutron matter.

Although there is a naive wish that quantum chromodynamics (QCD) at high temperature and/or densities should approach free field limits, there have been many rather convincing results that QCD remains as nonperturbative as the confined phase at high temperatures [7]. Expecting a similar behaviour at finite baryon densities also, we are motivated to study thermodynamic properties, e.g., the equation of state at finite densities including the effect of confinement explicitly.

In the present paper we first analyse the quark matter equation of state that explicitly takes into account the overall confinement of the quarks using a relativistic harmonic oscillator potential. The harmonic confinement model as used here has been applied earlier for study of hadronic properties as well as to consider equation of state at finite temperature and zero baryon density [8]. We consider this model at finite baryon density as may be relevant for hybrid stars. In the neutron matter sector we shall use a phenomenological equation of state. This is then used to consider phase transition to quark matter and to study hybrid stars.
It is generally felt that it is difficult to take correct thermodynamic limit in the presence of confinement. However the oscillator states of the relativistic harmonic confinement model are exactly analogous to the Landau orbits of electrons in a magnetic field [8]. The Landau orbits are characterised by an arbitrary centre of confinement. The collection of such orbits has no difficulty in forming a large system with appropriate thermodynamic limit. However, if few such orbits form a colourless bound state like a hadron the centre of mass of the system is free to move and the centre of mass correction of the bound state has to be applied. In the context of neutron stars the gravitational field crunches the neutrons and a large macroscopic quark matter consisting of the Landau like states is formed. In any confinement model the energy per particle grows rapidly with the number of particles. This would mean that one requires infinite energy in the thermodynamic limit. However, one has to keep in mind that the gravitational potential which grows as square of the number of particles is primarily responsible in producing the macroscopic state. The thermodynamic properties of this system are calculated by using the partition function which does not involve gravitational field but takes account of modified phase space occupation because of long range confinement. Then the full system is considered inclusive of gravitational field in TOV equations. The analogue of an electron in a magnetic field is used for obtaining the statistical weights in converting momentum space integrals to sums over possible states in the confining field.

We organise the paper as follows. In section 2, we derive the equation of state for quark matter in relativistic harmonic confinement model and use Gibbs criterion to consider the possibility of a phase transition of neutron matter to quark matter. In section 3 we shall use these equations of state to consider hybrid stars using Tolman Oppenheimer Volkoff equations. In section 4, we discuss and summarise our results.
II. RELATIVISTIC HARMONIC CONFINEMENT MODEL AND QUARK MATTER EQUATION OF STATE

In the harmonic confinement model [9] the relativistic equation for the quarks is given as

\[ [i\gamma^\mu \partial_\mu - M_i - V(r)]\psi_i(\vec{r}) = 0 \]  \hspace{1cm} (1)

where \( V(r) \) is the 'potential' with a Lorentz scalar plus vector structure given as

\[ V(r) = \frac{1}{2}(1 + \gamma_0)\alpha^2 r^2. \]  \hspace{1cm} (2)

In the above, the subscript ‘i’ in \( \psi(r) \) represents the colour and flavour index of the quark. This model had been applied earlier to study hadronic properties [9] and nucleon nucleon angular distributions [10]. In terms of the two component single particle wave functions, equation (1) may be written in the simple form

\[ [p^2 + \Omega^2 r^2]\phi_i(\vec{r}) = (E^2 - M_i^2)\phi_i(\vec{r}), \]  \hspace{1cm} (3)

where \( \Omega = \alpha \sqrt{E + M_i} \) are "frequencies" (assumed to be constant) of the quark fields. We may note that the confining term \( \Omega^2 r^2 \) is given by \( Tr(A_\mu^b A^{\mu b}) \), where \( A^b_\mu \) is the background field [8] similar to quantum motion of an electron in external magnetic field giving rise to Landau orbits. The 'potential' \( V(r) \) in equation (2) which has both Lorentz scalar and vector components gives rise to the energy levels as [8][9]

\[ E_n^2 = M_i^2 + (2n + 1)\Omega \quad (n = 1, 2, \cdots). \]  \hspace{1cm} (4)

For the study of hadronic properties the parameters chosen were [9]

\[ M_{u,d} = 160.66 \text{MeV} \]

\[ M_s = 460.66 \text{MeV} \]

\[ \sqrt{\Omega} = 1.162 \text{fm}^{-1}, \]
which we shall be using to derive the quark matter equation of state. The thermodynamic pressure $P_i$ (i=u,d,s) is then given by

$$\frac{P_i V}{k_B T} = g_V g_I \sum_n g_n \left[ \log \left( 1 + e^{-\beta (E_n^i - \mu^i)} \right) + \log \left( 1 + e^{-\beta (E_n^i + \mu^i)} \right) \right], \tag{5}$$

where $g_V$, $g_I$ and $g_n$ are respectively the statistical weights corresponding to volume, internal degrees of freedom (spin and colour) and the degeneracy factor for the nth energy level. These are given as $g_V = 4\pi/3\Omega^3/2 \times V/(2\pi)^3$, $g_I = 2 \times 3$ and $g_n = n(n + 1)/2$. Further, $\mu^i$ is the chemical potential associated with the $i$-th quark. We may note that no correct thermodynamic limit can be considered when there is harmonic confinement as the system does not have translational invariance. One has to include center of mass motion corrections to be able to obtain correct thermodynamic limit. Such corrections, though important for hadron spectroscopy and nucleon - nucleon scattering involving few particles, are expected to be small ($\simeq 1/N$, N is the number of particles) and are not included.

As we shall be considering hybrid stars, we shall consider the equation of state at zero temperature and for quark matter, along with u and d quarks, we include strange quarks and electrons which could be produced at high densities through weak interactions and be in equilibrium. The chemical potentials for each species of fermion may be written down in terms of the two independent chemical potentials $\mu_B$ and $\mu_E$, the baryon and electric charge chemical potentials respectively as

$$\mu_u = \frac{1}{3} \mu_B + \frac{2}{3} \mu_E \tag{6a}$$

$$\mu_d = \frac{1}{3} \mu_B - \frac{1}{3} \mu_E \tag{6b}$$

$$\mu_s = \frac{1}{3} \mu_B - \frac{1}{3} \mu_E \tag{6c}$$

$$\mu_e = -\mu_E \tag{6d}$$

Thermodynamic pressure for quark matter is then given as

$$P_{qm} = \sum_{i=u,d,s,e} P_i \tag{7}$$
where the electron pressure is given as

\[ P_e = \frac{\mu_e^4}{12\pi^2} \]  \hspace{1cm} (8)

These chemical potentials are fixed from the electrical charge neutrality condition for a given baryon density. The baryon number density and the electric charge densities are given as

\[ \rho_B = \frac{1}{3} \sum_{i=u,d,s} \rho_i \]  \hspace{1cm} (9)

and

\[ \rho_E = \frac{2}{3}\rho_u - \frac{1}{3}\rho_d - \frac{1}{3}\rho_s - \rho_e, \]  \hspace{1cm} (10)

where the quark densities at zero temperature are

\[ \rho_i = \sum_n \theta(\mu_i - E_n^i) g_V g_{1g} g_{1n} / V \]  \hspace{1cm} (11)

and the electron density is

\[ \rho_e = \frac{\mu_e^3}{3\pi^2}. \]  \hspace{1cm} (12)

It is then straightforward to evaluate the sums in equation (5) for each quark flavour to obtain the equation of state.

In the neutron matter sector, the equation of state is still a matter of debate particularly at higher densities. Different parametrisations for the same are there for the description of the structure of neutron stars [13]. In contrast to sophisticated parametrisations designed for neutron star matter, we shall use throughout in this paper a simple parametrisation for the neutron matter equation of state by Sierk and Nix [14]

\[ \epsilon_N(\rho) = \rho \left[ \frac{2}{9} K \left( \sqrt{\frac{\rho}{\rho_0}} - 1 \right)^2 + W_0 + W_{sym} + M_N \right]. \]  \hspace{1cm} (13)

In the above, \( W_0 = -16 \text{ MeV} \) is the binding energy per nucleon at normal nuclear matter density, \( \rho_0 = 0.145 \text{ fm}^{-3} \), \( M_N = 939 \text{ MeV} \), the rest mass of neutron and \( W_{sym} = 32 \text{ MeV} \) is the symmetry energy of neutron matter as estimated from liquid drop model calculations. While
deriving the equation of state for the neutron matter, we take the value of compressibility, 
\( K = 550 \text{ MeV} \) \(^2\). Below the nuclear matter densities the star matter is no longer determined 
by nucleon nucleon interactions only and we shall use for subnuclear density regions the 
equation of state given by Baym, Bethe and Pethick \(^17\) in the region \( 0.00267 \text{ fm}^{-3} < \rho < 1.6 \text{ fm}^{-3} \) and the equation of state given by Baym, Pethick and Sutherland \(^18\) for 
densities less than the above region. Different density regions of each equation of state were 
so taken that the pressure and energy density varied smoothly from one equation of state 
to the other \(^2\)\(^19\) in the neutron matter sector.

Once the energy density is known the pressure at zero temperature can be obtained from 
the thermodynamic relation \(^15\)

\[
P_{nm}(\rho) = \rho \frac{\partial \epsilon_N}{\partial \rho} - \epsilon_N,
\]

and the chemical potential \( \mu_B \) is given as

\[
\mu_B = \frac{\partial \epsilon_N}{\partial \rho}.
\]

We shall now consider the scenario of phase transition from cold neutron matter to 
quark matter. As usual, the phase boundary of the coexistence region will be given by 
Gibbs criteria. The critical pressure and the critical chemical potentials are given by the 
condition

\[
P_{nm}(\mu_B) = P_{qm}(\mu_B).
\]

The left hand side of the above equation is given through equations (13) and (14) and the 
right hand side is the zero temperature limit of equations (5) and (7).

We may note that in the calculations of pressure and chemical potentials in equations 
(5), (9) and (10) in the quark matter sector the zero point energy and in equations (13), 
(14) and (15) for the neutron matter sector the nucleon mass have been subtracted. This is 
because we wish to compare the pressure and chemical potential to consider phase transition 
for which the ground state energy is not relevant. However these will be included in the 
energy densities when we consider hybrid star as they contribute to the gravitational effects.
In Fig. 1, we plot \((P, \mu_B)\) curves for quark matter and neutron matter which yield the critical parameters \(P_{cr} = 300\text{MeV/fm}^3\) and \((\mu_B)_{cr} = 610\text{ MeV}\). In Fig. 2, we have plotted the baryon number density in the quark matter sector as a function of the baryon chemical potential. The shape of the curve is indicative of the discrete energy levels characteristic of an oscillator potential. The baryon number density in the quark matter sector corresponding to the critical \(\mu_B\) is calculated to be \(\rho^{qm}_{B} = 1.341\text{fm}^{-3}\). The same in the neutron matter sector at the critical \(\mu_B\) is \(\rho^{nm}_{B} = 0.784\text{fm}^{-3}\), about 5.4 times the nuclear matter density, which seems reasonable. At the critical pressure, the energy densities for the quark matter and neutron matter sectors, \(\epsilon^{qm}_{cr}\) and \(\epsilon^{nm}_{cr}\) are found to be 2542 \(\text{MeV/fm}^3\) and 918 \(\text{MeV/fm}^3\) respectively. The discontinuity in the number density as well as energy density indicates a first order phase transition. We may note here that the calculation for the two phases are done in two different models—a phenomenological parametrisation in the neutron matter sector and the oscillator confinement model in the quark matter sector. The different treatments of the two phases essentially leads to a first order phase transition as above.

### III. HYBRID STARS

For the description of neutron star, which is highly concentrated matter so that the metric of space-time geometry is curved, one has to apply Einstein’s general theory of relativity. The space-time geometry of a spherical neutron star described by a metric which in Schwarzschild coordinates has the form [2][13]

\[
ds^2 = -e^{\nu(r)}dt^2 + [1 - 2M(r)/r]^{-1}dr^2 + r^2[d\Theta^2 + \sin^2\Theta d\phi^2]
\]  

(17)

The equations which determine the star structure and the geometry are, in dimensionless forms,

\[
\frac{d\hat{P}(\hat{r}r_0)}{d\hat{r}} = -\hat{G}\frac{\hat{\epsilon}(\hat{r}r_0) + \hat{\hat{P}}(\hat{r}r_0)[\hat{\hat{M}}(\hat{r}r_0) + 4\pi\hat{a}\hat{r}^2\hat{\hat{P}}(\hat{r}r_0)]}{\hat{r}^2[1 - 2\hat{G}\hat{M}(\hat{r}r_0)/\hat{r}]},
\]

(18)

\[
\hat{\hat{M}}(\hat{r}r_0) = 4\pi\hat{a} \int_{0}^{\hat{r}r_0} d\hat{r}'\hat{r}'^2\hat{\epsilon}(\hat{r}'r_0),
\]

(19)
and the metric function, \( \nu(r) \), relating the element of time at \( r = \infty \) is given by

\[
\frac{d\nu(\hat{r}r_0)}{d\hat{r}} = 2\hat{G}[\hat{M}(\hat{r}r_0) + 4\pi a\hat{r}^3\hat{P}(\hat{r}r_0)]
\]

\[
\hat{r}^2[1 - 2GM(\hat{r}r_0)/\hat{r}]
\]  \hspace{1cm} (20)

In equations (18) to (20) the following substitutions have been made.

\[
\hat{\epsilon} \equiv \epsilon/\epsilon_c, \quad \hat{P} \equiv P/\epsilon_c, \quad \hat{r} \equiv r/r_0, \quad \hat{M} \equiv M/M_\odot,
\]  \hspace{1cm} (21)

where, with \( f_1 = 197.329 \text{ MeV fm} \) and \( r_0 = 3 \times 10^{19} \text{ fm} \), we have

\[
a \equiv \epsilon_c r_0^3/M_\odot, \quad \hat{G} \equiv (G/f_1)/(r_0/M_\odot)
\]  \hspace{1cm} (22)

In the above, quantities with hats are dimensionless. \( G \) in equation (22) denotes the gravitational constant \( (G = 6.707934 \times 10^{-45} \text{ MeV}^{-2}) \).

Further the surface condition for the metric function is given by

\[
\nu(R) = \ln(1 - 2GM/R)
\]

In order to construct a stellar model, one has to integrate equations (18) to (20) from the star’s center at \( r=0 \) with a given central energy density \( \epsilon_c \) as input until the pressure \( P(r) \) at the surface vanishes. As stated, with central density greater than the critical energy density as dictated by the last section, we expect that we shall have quark matter, and not neutron matter at the center of the star. Hence we shall be using here the equation of state for quark matter using the zero temperature limit of equation (7) with \( \hat{P}(0) = P(\epsilon_c) \). We then integrate the TOV equations until the pressure and density decrease to their critical values, so that there is a first order phase transition from quark matter to neutron matter at radius \( r = r_c \) for a given central density. For \( r > r_c \), we shall have equation of state for neutron matter where pressure will change continuously (but the energy density will have a discontinuity at \( r = r_c \)). The TOV equations with neutron matter equation of state as given in equations (13) and (14) are continued. As described earlier we use the equation of state by Baym, Bethe and Pethick [17] for subnuclear density region of neutron rich nuclei and of Baym, Pethick and Sutherland [18] for the lower density crystalline lattice bathed
in relativistic electrons. The TOV equations are continued with the above equations of state till the pressure vanishes which defines the surface of the star. This will complete the calculations for stellar model for a hybrid “neutron” star, whose mass and radius can be calculated for different central densities.

The energy density profile obtained from (18) to (20) are plotted in Fig. 3 for central densities $\epsilon_c = 3250 \text{ MeV/fm}^3$ and $\epsilon_c = 4000 \text{ MeV/fm}^3$. For core energy densities greater than the critical energy density ($\simeq 2542 \text{ MeV/fm}^3$) the core consists of quark matter. As we go away from the core towards the surface through TOV equations, when the critical pressure is reached, the density drops discontinuously indicating a first order phase transition. Thus e.g. for central density of 3250 MeV/fm$^3$ such a star has a quark matter core of radius 1 km with a crust of neutron matter of about 10 kms, whereas for $\epsilon_c=4000 \text{ MeV/fm}^3$, the quark matter core radius is 1.8kms with neutron matter crust of 9 kms.

Fig. 4 depicts mass of a hybrid star as a function of its central energy density. Here we shall have two branches of solutions. Pure neutron matter stars at lower central densities $\epsilon_c < \epsilon_{nm}^c$ and hybrid stars at central energy densities $\epsilon_c > \epsilon_{qm}^c$. Taking into account the stability of such stars under density fluctuations require $dM/d\epsilon_c > 0$ [20]. We find stable hybrid stars with central energy densities $\epsilon_c \approx 3 - 4.6 GeV/fm^3$ and radii around 10 to 11 kilometers, the critical mass being around $1.98 M_\odot$, beyond which they may collapse to black holes. The instability region is shown as dashed curves in the above figure.

Fig. 5 describes the mass radius relationship for hybrid star. This is important to estimate how rapidly the stars with a given equation of state can rotate without mass shedding at the equator.

We may take the relativistic Keplerian angular velocity $\Omega_K$ given as [21]

$$\frac{\Omega_K}{10^4 \text{sec}^{-1}} = 0.72 \sqrt{\frac{M/M_\odot}{(R/10km)^3}}.$$  \hspace{1cm} (23)

as for neutron stars and estimate the same. Figure 6 shows the variation of $\Omega_k$ as a function of its mass for such a star.

We wish to estimate the moment of inertia of these stars. The expression for the moment
of inertia of pulsars as well as the equilibrium for the rotating hybrid stars is given as

\[ I = \frac{8\pi}{3} \epsilon_c r_0^5 \int_0^{Rr_0} d\hat{r} \hat{r}^4 \left[ \hat{\epsilon}(\hat{r}r_0) + \hat{P}(\hat{r}r_0) \right] \exp\left(-\nu(\hat{r}r_0)/2\right) \sqrt{1 - 2GM(\hat{r}r_0)/\hat{r}}. \]  

(24)

We further estimate the surface gravitational red shift \( Z_s \) of photons as

\[ Z_s = \frac{1}{\sqrt{1 - 2G M/R}} - 1. \]  

(25)

This is plotted in Figure 7 as a function of mass. In this context it may be worth while to mention here that the surface redshifts as determined from gamma ray bursters seem to lie in the range 0.2 to 0.5 where as the masses seem to lie in the range 1 to 1.85 \( M_\odot \) with an error of \( \pm 0.3 \) at both ends.

IV. CONCLUSIONS

Let us summarise the findings of the present paper.

Using a relativistic harmonic confinement model for quark matter and with a phenomenological parametrisation for neutron matter we saw that a first order phase transition exists between the neutron phase and quark phase at about five to six times the nuclear matter density.

We have used here the parameters of the harmonic confinement model fixed from hadronic properties. In this context or otherwise, we may note that the harmonic confinement model has its origin through Prasad – Sommerfeld dyon like configurations in QCD. When we include temperature and/or density effects we may expect that they will affect such configurations in a dynamical manner and the parameters of the confinement model may change. However to get an insight into the equation of state including the effect of harmonic confinement we have approximated \( \Omega \) to be independent of temperature and/or density.

To study the possibility of a hybrid star, namely a star consisting of both quark matter and neutron matter, we applied the TOV equation to the appropriate equation of state with
a given central energy density $\epsilon_c$. It turns out that a stable hybrid star with a quark core and a neutron matter crust can exist upto $\epsilon_c \simeq 4.6 \text{ GeV/fm}^3$ beyond which instability may result. For $\epsilon_c$ from 3000 to 4600 MeV/fm$^3$, the mass of the star varies between 1.795 to 1.98 $M_\odot$ and the radius of the star between 10 to 11 kms. Consistently, the bulk of the hybrid star is provided by the neutron matter, the quark matter providing core of about 1 to 2 kms. The detailed properties of neutron matter e.g. whether it is soft or stiff do not seem to be reflected in the gross properties of hybrid stars.

ACKNOWLEDGMENTS

The authors are thankful to A.R. Prasanna, J.C. Parikh for discussions. AM and HM acknowledge many discussions with S.P. Misra, N. Barik and P.K. Panda.
REFERENCES

[1] N. K. Glendenning and F. Weber, preprint LBL-31613, 1992; N.K. Glendenning, Phys. Rev. D46 (1992) 1274.

[2] A. Rosenhauer et al Nucl. Phys. A540 (1992) 630.

[3] H. Mishra, S.P. Misra, P.K. Panda and B.K. Parida, Int. J. Mod. Phys. E2 (1993) 547.

[4] T. Øvergård and E. Øvergård, Class. Quantum Grav. 8 1191 L49.

[5] T. Øvergård and E. Øvergård, Astro. Astrophys. 243, 412 (1991); ibid, Class. Quantum Grav. 8, L49, 1991; B.Datta, P. K. Sahu, J. D. Anand and A. Goyal, Phys. Lett. B283, 313 (1992).

[6] A. Mishra, H. Mishra, S.P. Misra and P.K. Panda preprint- IP/BBSR/94-1, PRL-TH-94/2, To appear in Z. Phys. C.

[7] J. W. Negle, Nucl. Phys. A55 (1993) 47c; E. Manousakis and J. Polonyi, Phys. Rev. Lett. 58 (1987) 847.

[8] S.B. Khadkikar, J.C. Parikh and P.C. Vinodkumar, Mod. Phys. Lett. A8 (1993) 749; P.C. Vinodkumar and S. B. Khadkikar, Phys. Lett. B329 (1994) 81.

[9] S.B. Khadkikar and S.K. Gupta, Phys. Lett. B124 (1983) 523; S.B. Khadkikar, Pramana - J Phys. 24 (1985) 63; S.K. Gupta and S.B. Khadkikar, Phys. Rev. D36 (1987) 307.

[10] S.B. Khadkikar and K.B. Vijaykumar Phys. Lett. B254 (1991) 320.

[11] H. Huang, Statistical Mechnics (Wiley, 1963).

[12] There is a difference of factor $4\pi/3$ as compared to Ref. 7 in the expression of $g_{V}$. When this factor is taken into account, the critical temperature of Ref. 7 reduces from 170 MeV to 110 MeV.

[13] W.D. Arnett and R. L. Bowers, Astrophys. J. Suppl. Series 33 (1977) 415.
[14] A.J. Sierk and J.R. Nix, Phys. Rev. C22 (1980) 1920.

[15] See for example in “Quantum Theory of Many Particle System” A.L. Fetter and J.D. Walecka, *McGraw Hill Book Company*, 1971.

[16] F. Weber and M.K. Weigel, Nucl. Phys. A493 (1989) 549.

[17] G. Baym, H.A. Bethe and C. J. Pethick, Nucl. Phys. A175 (1971) 225.

[18] G. Baym, C.J. Pethick and P. Sutherland, Ap. J. 170 (1971) 299.

[19] S. L. Shapiro and S. A. Teukolsky, Black holes, white dwarfs and neutron stars (Wiley, New York, 1983).

[20] S. Weinberg, Gravitation and cosmology (Wiley, New York, 1972).

[21] J. L. Friedman, J.R. Ipser and L. Parker, Phys. Rev. Lett. 62 (1989) 3015.

[22] K. Brecher, Astro. Phys. J. 215 (1977) L17.

[23] J.B. Hartle, Phys. Reports 47 (1978) 201.

[24] N.K. Glendenning, F. Weber and S.A. Moszkowski, Phys. Rev. C45 (1992) 844.

[25] E.P. Liang Astrophys. J. 304 (1986) 682.

[26] P.C. Joss and S.A. Rappaport, Annu. Rev. Astron. Astrophys. 22 (1984) 537.

[27] A. Mishra, H. Mishra, S.P. Misra and S.N. Nayak, Z. Phys. C57 (1993) 233; A. Mishra, H. Mishra and S.P. Misra, Z. Phys. C59 (1993) 159.
Figure captions

**Fig.1:** We plot pressure, $P$ in MeV/fm$^3$ as a function of the baryon chemical potential, $\mu_B$ in MeV for the quark matter (the solid line) and neutron matter (dashed line).

**Fig.2:** We plot here baryon number density, $\rho_B$ in fm$^{-3}$ as a function of baryon chemical potential $\mu_B$ in MeV for quark matter.

**Fig.3:** We plot here the energy profile curves inside the hybrid star for central densities 3250 MeV/fm$^3$ (the solid line) and 4000 MeV/fm$^3$ (dot dashed line). Discontinuity at the critical energy density is shown by the dashed line.

**Fig.4:** Mass of the hybrid star as a function of central density, $\epsilon_c$ in MeV/fm$^3$ is plotted here. The dashed portion indicates instability.

**Fig.5:** We plot here mass of the star in units of solar mass as a function of radius of the star in kilometers.

**Fig.6:** We plot here the Keplerian angular velocity, $\Omega_K$ in $10^4 sec^{-1}$ as a function of $M/M_\odot$.

**Fig.7:** We plot here the surface gravitational red shift, $Z_s$ as a function of $M/M_\odot$. 
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9510391v1
This figure "fig2-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9510391v1
This figure "fig3-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9510391v1
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9510391v1
This figure "fig2-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9510391v1
This figure "fig3-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9510391v1
This figure "fig1-3.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9510391v1