Multi-Objective Optimization of the Vuilleumier Heat Pump under Adiabatic Condition

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Abstract. The Vuilleumier cycle adiabatic model has complex calculations and numerous structural parameters. In order to facilitate the analysis and optimization of the structural parameters of the adiabatic model, a dimensionless method is adopted, a system of dimensionless differential equations containing 7 dimensionless parameters is obtained. Among the 7 dimensionless parameters, 5 dimensionless structural parameters representing the volume structure characteristics of the system are selected as decision variables in the optimization. The non-dominated sorted genetic algorithm-II is used for multi-objective optimization with heating capacity and COP as the target parameters. Finally, the Pareto front was obtained, and the baseline case was optimized using the TOPSIS method. After optimization, the heating capacity and COP increased by 0.701 % and 0.848 %, respectively.

1. Introduction
In the field of industrial energy saving and waste heat utilization, heat-driven heat pumps (HDHP) have always been an important part of the energy system to improve overall energy efficiency [1,2]. Vuilleumier (VM) cycle heat pump is a new type of HDHP with the advantages of high efficiency, low noise, low vibration and long life. It has a good application prospect in the application of small capacity heat pumps. VM cycle heat pump can be seen as a combination of a Stirling engine and a Stirling refrigerator [3]. In the cycle, its total internal volume remains constant. The volume of different temperature zones can be changed by the movement of the dispacer, so that the internal working gas can be compressed and expanded, thereby realizing the heat pump cycle.

In order to study the thermal performance of the VM cycle, the researchers used the analysis method of the Stirling machine and established the VM cycle isothermal model and adiabatic model successively. Although the calculation and analysis process of the isothermal model is very convenient, when it is applied to the VM heat pump, the calculation results are quite different from the actual [4]. Therefore, most of the current researches on VM cycle heat pump uses adiabatic models, which have high accuracy, but the calculations are more complicated [5-8].

Homutescu et al. [5] compared the performance of the VM cycle isothermal model, semi-adiabatic model and adiabatic model under the same parameters, and the results showed that with an increasing degree of thermal insulation, the heating output increases, but the thermal efficiency decreases. Chen et al. [6] studied the performance of heat pumps under two different piston motion rules based on the adiabatic model, and the results showed that the dwell-based motion has greater heating output and greater adaptability to variable loads. Rogdakis et al. [7] carried out four scalings on the size of the cylinder diameter and stroke of the VM heat pump adiabatic model and studied the impact of the
cylinder diameter and stroke on the performance of the heat pump under different scaling conditions. Chen et al. [8] used a multi-objective genetic algorithm to optimize some structural parameters in the adiabatic model of the VM cycle heat pump.

Although there are many studies based on the VM cycle adiabatic model, there are few optimization studies on it. This is partly because the number of structural parameters of the VM cycle heat pump is twice that of the Stirling machine, and the thermal cycle is also more complicated. Secondly, the solution of the VM cycle adiabatic model usually adopts a complex and time-consuming numerical calculation method. Therefore, it is particularly important to simplify the VM cycle adiabatic model.

In this paper, a dimensionless VM cycle adiabatic model, which effectively reduces the number of parameters, is established. Through a multi-objective optimization of VM cycle heat pump, the influence of dimensionless structure parameters on system performance is studied.

2. The structure and temperature distribution of VM cycle adiabatic model

Figure 1(a) is a structural diagram of the VM cycle heat pump. It can be seen that VM cycle heat pump is composed of a high temperature chamber, an ambient temperature chamber, a low temperature chamber, two regenerators, three heat exchangers and a crankshaft driving mechanism. The three chambers are the main work area, and the work capacity of each chamber is equal to the heat transfer of adjacent heat exchangers [9]. The volume of three heat exchangers, two regenerators and the dead volume of three chambers are generally regarded as redundant volumes [10]. The existence of redundant volume usually reduces the heating output of the system.

![Figure 1](image)

**Figure 1.** The structure and temperature distribution of VM cycle adiabatic model

1 – low temperature chamber; 2 – cold temperature heat exchanger; 3 – cold temperature regenerator; 4 – ambient temperature heat exchanger; 5 – ambient temperature chamber; 6 – hot temperature regenerator; 7 – high temperature heat exchanger; 8 – high temperature chamber.

The assumptions are stated as follows [6]:

1) The three gas chambers (1, 5, 8 in Fig. 1) are all adiabatic, and the temperature of the gas in each chamber \(T_c, T_a, T_h\) will fluctuate during a cycle.

2) The regenerator is an ideal regenerator with no heat loss and flow resistance, and its internal temperature is linearly distributed.

3) The heat exchanger is an ideal heat exchanger, with no heat transfer temperature difference and flow resistance, and the internal temperature is a fixed value.

4) There is no leakage and heat conduction inside and between the components.

5) The working gas is an ideal gas.
3. **Dimensionless VM Cycle Adiabatic Model**

The VM cycle adiabatic model includes the differential equations of each component and the initial conditions of the variables. Low temperature $T_{hx,c}$, initial pressure $P_0$, cold cylinder stroke volume $V_{s,c}$, initial mass of cold cylinder stroke volume $m_{s,c}$ is used as a dimensionless characteristic scale of temperature, pressure, volume and quality. $m_{s,c}$ can be calculated by formula (1):

$$m_{s,c} = \frac{P_0 V_{s,c}}{RT_{hx,c}}$$

The system of differential equations of the dimensionless adiabatic model of VM cycle heat pump is:

$$dm^*_c = \left[ \frac{P^* dV^*_c + \frac{1}{\gamma} V^*_c dP^*}{T_{in,c}} \right]$$

$$dm^*_h = \left[ \frac{P^* dV^*_h + \frac{1}{\gamma} V^*_h dP^*}{T_{in,h}} \right]$$

$$dm^*_a = \left[ \frac{P^* dV^*_a + \frac{1}{\gamma} V^*_a dP^*}{T_{in,a}} \right]$$

$$dT^*_c = T^*_c \left( \frac{dV^*_c}{V^*_c} + \frac{dP^*}{P^*} \cdot \frac{dm^*_c}{m^*_c} \right)$$

$$dT^*_h = T^*_h \left( \frac{dV^*_h}{V^*_h} + \frac{dP^*}{P^*} \cdot \frac{dm^*_h}{m^*_h} \right)$$

$$dT^*_a = T^*_a \left( \frac{dV^*_a}{V^*_a} + \frac{dP^*}{P^*} \cdot \frac{dm^*_a}{m^*_a} \right)$$

$$dP^* = \frac{-\gamma P^* \left( \frac{dV^*_c}{T_{in,h} V^*_h} + \frac{dV^*_h}{T_{in,c} V^*_c} + \frac{dV^*_a}{T_{in,a} V^*_a} \right)}{V^*_h + V^*_c + V^*_a + \gamma \cdot \text{const}^*}$$

$$\text{const}^* = \frac{V_{hx,c}}{T_{hx,c}} + \frac{V_{rh,c} \ln \left( \frac{T_{hx,h}}{T_{hx,a}} \right)}{T_{hx,h} - T_{hx,a}} + \frac{V_{hx,a} \ln \left( \frac{T_{hx,a}}{T_{hx,c}} \right)}{T_{hx,a} - T_{hx,c}} + \frac{V_{hx,c}}{T_{hx,c}}$$

$$\begin{align*}
T^*_{in,c} &= T^*_{hx,c} \quad \text{if } dm^*_c > 0 \\
T^*_{in,c} &= T^*_c \quad \text{if } dm^*_c \leq 0 \\
T^*_{in,h} &= T^*_{hx,h} \quad \text{if } dm^*_h > 0 \\
T^*_{in,h} &= T^*_h \quad \text{if } dm^*_h \leq 0 \\
T^*_{in,a} &= T^*_{hx,a} \quad \text{if } dm^*_a > 0 \\
T^*_{in,a} &= T^*_a \quad \text{if } dm^*_a \leq 0
\end{align*}$$

The dimensionless volume changes of each working chamber are as follows:

$$V^*_c = 0.5(1 + \cos \theta) + V^*_{d,c}$$
This is an initial value problem, and the system could be solved using a Runge-Kutta method. The initial conditions of the dimensionless VM cycle adiabatic model are: \( R_0 = 1, T_{h,0} = T_{h,x,h}, T_{a,0} = T_{h,x,a}, T_{c,0} = 1, m_{d,c} = 0.5V_{h}\), \( m_{d,h} = 0.5V_{a}\), \( m_{d,a} = 0.5V_{a}\), \( V_{h,h} = 0.5V_{a}\), \( V_{h,a} = 0.5V_{a}\).

As the work capacity of each chamber is equal to the heat transfer of adjacent heat exchangers [9], the heating capacity, heat consumption, and COP of VM cycle heat pump can be calculated as follows:

\[
Q_{h,a} = nPV_{s,c} \int P' dV_{n}
\]

\[
Q_{h,h} = nPV_{s,h} \int P' dV_{n}
\]

\[
COP = \frac{Q_{h,a}}{Q_{h,h}}
\]

According to the differential equation, volume change equation and initial conditions, the input parameters needed to solve the dimensionless VM cycle adiabatic model include: \( T_{h,x,h}, T_{h,x,a}, V_{s,c}, V_{d,c}, V_{d,h}, V_{d,a}, const, \phi, \gamma \). Among these 7 non-dimensional parameters, \( T_{h,x,h} \) and \( T_{h,x,a} \) are the non-dimensional temperature of high temperature and ambient temperature, respectively, representing the influence of external working conditions on system performance. \( V_{s,c}, V_{d,c}, V_{d,h}, V_{d,a}, const \) represent the volume structure characteristics of the system in Figure 1, therefore these 5 dimensionless parameters are selected as decision variables in the following optimization.

4. Result and Discussion of Multi-Objective Optimization of the Vuilleumier Heat Pump

When using the dimensionless model for calculation, the VM cycle heat pump prototype structure parameters of Carlsen [11] are used as the baseline point. Stroke volume of hot and cold cylinder \( V_{s,c} = 4.42 \times 10^{-4} \) m\(^3\); dead volume of three chambers \( V_{d,c} = 1.41 \times 10^{-4} \) m\(^3\), \( V_{d,h} = 1.71 \times 10^{-4} \) m\(^3\), \( V_{d,a} = 2.40 \times 10^{-4} \) m\(^3\); regenerators volume \( V_{r,c} = 2.36 \times 10^{-4} \) m\(^3\), \( V_{r,h} = 3.67 \times 10^{-4} \) m\(^3\); heat exchanger volume \( V_{h,b,h} = 4.40 \times 10^{-4} \) m\(^3\), \( V_{h,b,c} = 2.12 \times 10^{-4} \) m\(^3\), \( V_{h,b,a} = 4.03 \times 10^{-4} \) m\(^3\); three temperature levels \( T_{h,x,h} = 873.15 \) K, \( T_{h,x,c} = 328.15 \) K, \( T_{h,x,a} = 278.15 \) K; phase separation angle \( \phi = 90^\circ \); initial pressure \( P_0 = 12 \) MPa, frequency \( n = 20 \) Hz; the working gas is helium, \( \gamma = 1.67, R = 2078 \) J/(kg·K). After calculation, the values of the 7 dimensionless parameters are:

\( T_{h,x,h} = 3.14, T_{h,x,a} = 1.18, V_{s,c} = 1, V_{d,c} = 0.32, V_{d,h} = 0.39, V_{d,a} = 0.54, const = 2.48 \)

Figure 2. Pareto front for COP and heating output of VM cycle heat pump
At present, there are many types of multi-objective optimization algorithms. Among them, the fast non-dominated sorting genetic algorithm (NSGA-II) is widely used in solving the problem of multi-objective conflicts [12]. Therefore, this paper adopts NSGA-II as multi-objective optimization algorithm, taking COP and heating capacity as the objective function to optimize the five non-dimensional structural parameters. The value ranges or constraints of the five decision variables are as follows:

\[
0.25 \leq V_{s,h}^* \leq 4, \quad 1 \leq const^* \leq 5, \quad 0.1 \leq V_{d,c}^* \leq 0.8, \quad 0.1 V_{s,h}^* \leq V_{d,h}^* \leq 0.8 V_{s,h}^*, \quad 0.1(V_{s,h}^*+1) \leq V_{d,a}^* \leq 0.8(V_{s,h}^*+1)
\]

The parameters of the NSGA-II are set as follows: the population size is 1000; the maximum number of iterations is 200; the crossover probability is 0.9; the mutation probability is 0.1; the competition method is selected, and the scale is 2.

Figure 2 shows the Pareto front obtained after 200 iterations. All points in the figure are the optimal value points of multi-objective optimization. Among them, case A is the optimal value point when only COP is the optimization target, and case B is the optimal value point when only the heating capacity \( Q_{hx,a} \) is the optimization target. The parameters of case A are: \( Q_{hx,a}=2.113 \text{ kW}, \ COP=4.260 \). The parameters of case B are: \( Q_{hx,a}=92.364 \text{ kW}, \ COP=2.238 \). Perform polynomial curve fitting on the best point in Figure 2, and get the following fitting relationship:

\[
COP = 4.386 - 6.029 \times 10^{-2} \times Q_{hx,a} + 8.075 \times 10^{-4} \times Q_{hx,a}^2 - 6.161 \times 10^{-6} \times Q_{hx,a}^3 + 1.907 \times 10^{-8} \times Q_{hx,a}^4 \quad (17)
\]

Table 1. Non-dominant solutions in the Pareto front better than the baseline case

| case | \( Q_{hx,a} \)/kW | COP | \( V_{s,h}^* \) | \( V_{d,c}^* \) | \( V_{d,h}^* \) | \( V_{d,a}^* \) |
|------|------------------|-----|---------------|---------------|---------------|---------------|
| 1    | 15.299           | 3.632 | 1.819        | 4.760         | 0.100         | 0.182         | 2.193         |
| 2    | 15.332           | 3.631 | 1.905        | 4.994         | 0.104         | 0.191         | 2.315         |
| 3    | 15.384           | 3.629 | 1.882        | 4.891         | 0.104         | 0.189         | 2.305         |
| 4    | 15.516           | 3.624 | 1.942        | 4.986         | 0.180         | 0.198         | 2.327         |
| 5    | 15.556           | 3.622 | 1.944        | 5.000         | 0.116         | 0.230         | 2.342         |
| 6    | 15.616           | 3.620 | 1.871        | 4.800         | 0.101         | 0.187         | 2.207         |
| 7    | 15.695           | 3.617 | 1.959        | 5.000         | 0.179         | 0.199         | 2.255         |
| 8    | 15.781           | 3.613 | 1.958        | 4.960         | 0.178         | 0.199         | 2.248         |
| 9    | 15.889           | 3.609 | 1.874        | 4.691         | 0.105         | 0.188         | 2.206         |
| 10   | 15.938           | 3.607 | 1.969        | 4.913         | 0.100         | 0.224         | 2.375         |
| 11   | 15.976           | 3.605 | 1.896        | 4.770         | 0.105         | 0.190         | 2.120         |
| 12   | 16.073           | 3.602 | 1.971        | 4.890         | 0.100         | 0.223         | 2.312         |

After substituting the baseline point into the dimensionless model, the calculated performance parameters are: \( Q_{hx,a}=15.225 \text{ kW}, \ COP=3.601 \). Table 1 shows all the non-dominant solutions in the Pareto front that are better than the baseline point. The TOPSIS method is used to evaluate the 12 solutions in Table 1. The results show that case 2 is the best. After optimization, \( Q_{hx,a} \) can be increased by 0.701%, and \( COP \) can be increased by 0.848%.

5. Conclusion

In order to solve the difficulty of optimization caused by excessive structural parameters in the VM cycle adiabatic model, this paper has carried out a dimensionless treatment on the VM cycle adiabatic model. In the dimensionless VM cycle adiabatic model, the input parameters that affect the thermal process include volume phase angle \( \varphi \), specific heat ratio \( \gamma \), 2 dimensionless temperature parameters \( T_{hx,b}, T_{hx,c} \), and 5 dimensionless structural parameters \( V_{s,h}, V_{d,c}, V_{d,h}, V_{d,a}, const \). Taking the 5 simplified dimensionless structural parameters as decision variables, the NSGA-II is used to carry out multi-objective optimization of the VM cycle heat pump. The results show that the COP and heating capacity of VM cycle heat pump have been improved, confirming the feasibility of using the dimensionless model for analysis and calculation.
6. Acknowledgments
The present study is supported financially by Natural Science Foundation of Hebei Province-China (E2019502151) and Fundamental Research Funds for the Central Universities (2018MS105)

Nomenclature

\[ \text{COP} \] coefficient of performance

\[ \text{const}^\ast \] dimensionless relative volume

\[ m \] mass, kg

\[ n \] frequency, Hz

\[ P \] pressure, Pa

\[ Q \] heat transfer rate, W

\[ R \] working gas constant, \( J/(kg \cdot K) \)

\[ T \] temperature, K

\[ V \] volume, \( m^3 \)

Superscripts/Subscripts

\[ \* \] dimensionless variables

\[ a \] ambient temperature chamber/ambient

\[ c \] low temperature chamber/cold

\[ d \] chamber dead volume

\[ hx \] heat exchanger

\[ h \] high temperature chamber/hot

\[ s \] stroke of dispacer

\[ in \] inflow gas parameters

\[ r \] regenerator

\[ 0 \] iteration initial parameters

Greek symbols

\[ \gamma \] specific heat ratio

\[ \theta \] crankshaft angle, rad

\[ \varphi \] phase separation angle, rad

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