LOW $R_v$ FROM CIRCUMSTELLAR DUST AROUND SUPERNOVAE

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ABSTRACT

The effective extinction law for supernovae surrounded by circumstellar dust is examined with Monte Carlo simulations. Grains with light scattering properties as for interstellar dust in the Milky Way (MW) or the Large Magellanic Clouds (LMC), but surrounding the explosion site, would cause a semidiffusive propagation of light up to the edge of the dust shell. Multiple scattering of photons predominantly attenuates photons with shorter wavelengths, thus steepening the effective extinction law as compared to the case of single scattering in the interstellar medium. Our simulations yield typical values for the total-to-selective extinction ratio $R_v\sim1.5–2.5$, as seen in recent studies of Type Ia supernova colors, with a steepening differential extinction toward shorter wavelengths.

Subject headings: dust, extinction — supernovae: general

1. INTRODUCTION

The uncertainties in the brightness corrections of Type Ia supernovae (SNe Ia) for color excess is among the largest systematic uncertainties in the use of SNe Ia to measure cosmological distances (Nordin et al. 2008). The standard interpretation of color excess being due to extinction by interstellar dust in the supernova host galaxy has recently been challenged by the empirically deduced color-brightness relation for SNe Ia. The standard interstellar extinction law for supernovae (SNe Ia) for color excess is among the largest systematic uncertainties in the use of SNe Ia to measure cosmology benefits significantly from the understanding the total-to-selective extinction ratio $R_v$.

1 It should be noted that low $R_v$ values to individual QSO systems have been found in, e.g., Ostman et al. (2008) and Wang et al. (2004).

For $R_v \gg R_{CS}$, the single scattering approximation is valid and the light beam reaching the observer is attenuated as $e^{-\tau}$, with $\tau = R_{CS}/\lambda_{eff}$. This is the case applicable for extinction by dust in the interstellar medium.

Next, we examine the possibility that low values of $R_v$ stem from the semi-diffusive propagation of photons in the neighborhood of the site of the supernova explosion.

2. QUASI-DIFFUSIVE LIGHT PROPAGATION AROUND THE SUPERNOVA

Photon propagation around a medium of scatterers can be described by a quasi-random-walk picture. The reader is referred to Chandrasekhar (1943) for a beautiful introduction to this subject. Let us consider a localized distribution of dust particles within a distance $R_{CS}$ from the explosion site, negligibly small compared to the distance to the observer, $d$, i.e., $R_{CS} \ll d$, where $R_{CS}$ corresponds to the radius from where the SN radiation emerges. The trajectory of a photon will be straight until it hits a dust particle at which point the photon can be either scattered or absorbed. If the photon is scattered in a new direction, it follows a straight path until the next encounter and so on until $r > R_{CS}$. The mean free path between interactions, $\lambda_{eff}$, is thus determined by the number density of scatterers, $n$, and their effective cross section for scattering and absorption of light, i.e., $\sigma_{eff} = \sigma_s + \sigma_a$:

$$\lambda_{eff} = \frac{1}{n \sigma_{eff}}. \quad (1)$$

For $\lambda_{eff} \gg R_{CS}$, the single scattering approximation is valid and the light beam reaching the observer is attenuated as $e^{-\tau}$, with $\tau = R_{CS}/\lambda_{eff}$. This is the case applicable for extinction by dust in the interstellar medium.

For $\lambda_{eff} \ll R_{CS}$, corresponding to a (local) high number density of scatterers, the situation is different. If the absorption probability is much lower than for scattering, photon propagation is diffusive and the average properties can described analytically using the formulas for random-walk (Askebjer et al. 1997). The case we are considering here is for $\lambda_{eff} \sim R_{CS}$, where the scattering cross section exceeds the absorption cross...
section, \(a_l > a_s\), in a wavelength-dependent manner. In particular, we examine the cases where the light scattering properties of dust particles in the CS matter are similar to what has been modeled for interstellar dust grains in the MW (Draine 2003) or the LMC (Weingartner & Draine 2001).

Table 1 shows the wavelength-dependent albedo factor \([= a_s/(a_s + a_l)]\) and the average of the cosine of the scattering angle for interactions between light and dust particles. Also tabulated is the absorption cross section divided by dust mass. Note that the MW parameters correspond to a dust size distribution matching \(R_C = 3.1\) for dimming of stars in the Galaxy. Wang (2005) considered the impact of circumstellar dust on the measured value of \(R_C\), but only the extreme case where all scattered photons may reach the observer. That assumption overlooks an important aspect of the problem: while the bluer photons scatter more, they are also more likely to be absorbed. This leads to a steeper wavelength dependence of the effective extinction law, possibly explaining the unusual total-to-selective extinction ratios found in studies of SNe Ia.

3. MONTE CARLO SIMULATION OF LIGHT PROPAGATION AROUND THE SUPERNova

In order to estimate the net effect of scattering and absorption on the light reaching the outer edge of a shell of circumstellar dust around the SN site, \(R_{CS}\), a Monte Carlo simulation was performed. Photons with energies corresponding to the central wavelengths of the \(UBVRIJHK\) photometric system were generated and subsequently followed as they propagate in the dusty medium. A uniform distribution of scatterers within a sphere of radius \(R_{CS}\) is used in the calculations. Our treatment is rather insensitive to the physical size of \(R_{CS}\) since what governs the differences in path lengths of photons at different wavelengths are the optical depths, \(\tau_R = R_{CS}/\lambda_0\) and \(\tau_s = R_{CS}/\lambda_s\). Thus, for a fixed color excess, a larger \(R_{CS}\) can be compensated by a lower number density, \(n\), thus keeping \(\tau_R\) and \(\tau_s\) unchanged. Figure 1 shows the wavelength dependence of \(\tau_s\) and \(\tau_R\) in our calculation for MW and LMC dust types for a reddening at \(r = R_{CS}\) of \(E(B-V) = 0.1\). The key observation is that at short wavelengths, both \(\tau_s\) and \(\tau_R\) increase, while \(\tau_s\) dominates. Since they scatter more, photons with short wavelength leaving the CS region to eventually reach the observer must propagate a larger path length than photons at longer wavelengths. However, since \(\tau_s\) also increases at shorter wavelengths, the relative attenuation of bluer photons is enhanced, thus generating an effective extinction law with steeper wavelength dependence, i.e., lower \(R_C\).

SMC-type dust, as parameterized in Weingartner & Draine (2001), has a smaller albedo in the optical to near-IR region (see Fig. 23 in Weingartner & Draine 2001), leading to a larger value of \(\tau_s\) compared to LMC or MW dust. Thus, SMC dust is not suitable to explain the anomalous SN colors with the scenario presented in this work.

For each photon starting at \(R_{SN} \leq R_{CS}\), straight path lengths between interactions \((L_x, L_y)\) were generated from an exponential distributions \(\lambda_x^{-1} \exp(-L_x/\lambda_x)\) and \(\lambda_y^{-1} \exp(-L_y/\lambda_y)\), where the mean free paths for scattering and absorption, \(\lambda_x = (n a_s)^{-1}\) and \(\lambda_y = (n a_l)^{-1}\), were calculated from the wavelength-dependent parameters in Table 1. The scattering angle for each interaction was generated in the simulation following the Henyey-Greenstein approximation (Henyey & Greenstein 1941):

\[
\frac{d\sigma}{d[\cos(\theta)]} = \frac{1 - g^2}{[1 + g^2 - 2g \cos(\theta)]^{1/2}},
\]

where \(g = \langle \cos(\theta) \rangle\) (also listed in Table 1). The density of dust in the shell was varied to cover a wide range of reddening at \(r = R_{CS}\), \(0 \leq E(B-V) \leq 0.4\). The probability of photons 

\[\text{Notes.—Scattering parameters for optical and near-IR photons (Weingartner & Draine 2001; Draine 2003) corresponding to interstellar extinction in the Large Magellanic Clouds (LMC) and Milky Way (MW) with } R_C = 3.1.\]
reaching \( R_{cs} \) without absorption is calculated by repeating the ray-tracing Monte Carlo simulation 5 \times 10^5 times for each wavelength.

4. \( R_v \) FROM SIMULATIONS

Figure 2 shows that the attenuation of light after the circumstellar shell yields low values of \( R_v = A_v/E(B-V) \), as anticipated. In particular, the simulations using LMC dust result in an effective \( R_v = 1.65 \), compatible with the best-fit result of Nobili & Goobar (2008). We note, however, that the CCM extinction law (Cardelli et al. 1989) does not accurately reproduce the wavelength dependence of photon attenuation over the entire optical range, as shown in Figure 3.\(^3\) The deviations are significant for the \( U - B \) color, in agreement with the observations reported in Guy et al. (2005).

Instead, a power-law relation gives a good fit to the simulations (see Fig. 4):

\[
\frac{A_{\lambda}}{A_v} = 1 - a + a \left( \frac{\lambda}{\lambda_v} \right)^\nu, \tag{3}
\]

where \( \lambda_v = 0.55 \, \mu m \) is the central wavelength of the V-band filter. Since the attenuation at B band (central wavelength \( \lambda_B \)) is given by \((R_v + 1)E(B-V)\), the parameters in equation (3) are related to \( R_v \) as

\[
R_v = a^{-1} \left[ \left( \frac{\lambda_B}{\lambda_v} \right)^\nu - 1 \right]^{-1} = \frac{1}{a(0.8^\nu - 1)}. \tag{4}
\]

It is encouraging for the use of Type Ia SNe for precision cosmology that the reddening corrections may take a simple and general analytical form, if the model is confirmed. Further studies of SN colors, also including near-IR data, should be used to test this relation.

\(^3\) The validity of the simulation code was tested by accurately reproducing the standard extinction laws when the diffusive process was switched off.

5. SUMMARY AND CONCLUSIONS

Simple simulations show that circumstellar material, detected in at least one Type Ia supernova, SN 2006X (Patat et al. 2007; Wang et al. 2008b), could potentially explain the empirically determined extinction law for low-redshift SNe Ia, especially if the circumstellar material resembles LMC dust grains. Adopting the CS shell size of Patat et al. (2007), \( R_{cs} \sim 10^{16} \, cm \), we find that for \( \tau \sim 1 \), the required mass in dust around the supernova is \( M_{dust} \sim 4\pi R_{cs}^2\sigma_{dust} \sim 10^{-4} \, M_\odot \) when inserting typical values of the absorption cross section from Table 1.

A simple power-law expression fits is found to fit very well the effective extinction law for dust in the CS environment of the supernova produced by Monte Carlo simulations.

Depending on the thickness of the CS shell, shifts in the time of light curve maximum may be expected for different bands since the amount of quasi–random-walk will differ. In particular, photons in redder bands will suffer less scattering and thereby less time delay. This effect should correlate with the measured reddening, \( E(B-V) \). The assumption of a uniform density is not expected to be critical for the results at first
order. However, a second-order effect may be expected since a large scale of $R_{\text{cs}}$ would result in a longer time for photons being “trapped” in the scattering sphere. As the intrinsic colors of Type Ia change on a timescale of days (Nobili & Goobar 2008), time delays of photons of that timescale would affect the measured colors as a function of time.

In a forthcoming paper, potential direct observables from interaction between photons and dust will be investigated. Also, the sensitivity of the effective extinction law to the dust grain sizes and density profile in CS medium and the combination of both scattering in the circumstellar material and the interstellar medium needs to be further investigated. If the presence of CS material is indeed the source of the color-brightness relation found in SNe Ia, the case for rest-frame near-IR observations is further strengthened; the peak magnitude corrections, and their model dependence, are smaller than at optical wavelengths.

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