Sounding Rockets Scattering Factors and their Influence on Motion Accuracy

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Abstract. The increase of sounding rocket active part of trajectory compared to the "normal" can be due to the launches of ballistic flying vehicle for the maximum specified range. In this case, rockets should use propulsion systems with a relatively low fuel specific impulse. This effect also occurs in case of implementation of minimum scattering programs during launches at intermediate ranges (between the minimum and maximum mission ranges). In proposed research was developed methods for analysing the influencing of ballistic and navigation support the uncertain factors on the accuracy of sounding rocket motion with an extended active part of trajectory and was given a relevant assessment to the contribution of each component in relation to specific cases of their occurrence.

1. Introduction
The current level of rocket technology development is characterized by extremely high requirements for guaranteed mission accuracy. These requirements practically apply for all flying vehicles classes, but they are especially hard for ballistic type vehicles, or Sounding Rockets (SR). By sounding rockets we mean a wide class of ballistic vehicles, which, along with the active, controlled part, also has a passive, uncontrolled part of the trajectory [1].

Generally, active and passive parts of trajectory can be several and they can alternate with each other. The controlled movement of the centre of mass of this type of flying vehicle is usually considered as a guidance task, the solution of which should ensure that the vehicle could reach to a given point in inertial space, or to achieve a predetermined mission range [2, 3].

High precision Sounding Rocket mission could be possible only if available control algorithms, reliable mission’s ballistic and navigation support (BNS). BNS based on the available primary and source navigation information, which are contains information about the physical environment, and directly during the motion control, in the structure of the a priori model of the SR dynamics motion, etc.

Development of methods and means of improving the reliability of the BNS has always been among the priorities of the SR dynamic design. Currently, a lot of publications have been published related to solving problems of increasing the reliability of measurement information [4-9].

In a much lesser extent, these issues were raised in relation to the assessment of the impact of ballistic motion errors. It should, however, bear in mind that whatever approaches are not used for the reliability and accuracy of the initial and primary navigation information always remain uncontrollable factors that affect the accuracy of controlled motion.
There is a number of problems in which their importance is growing significantly. Such tasks include flight control problems with Sounding Rockets extended active part of trajectory (APT). Longer APT may be due to ballistic SR for a predetermined maximum mission range. In this case, launchers should use propulsion systems with a relatively low specific impulse.

This effect also occurs in case of implementation the minimum scattering programs [1, 10-13] during launches at intermediate ranges. The SR mission trajectory under the minimum scattering program is steeper than the maximum range mission trajectory. The implementation of this type of program requires a significantly higher thrust-to-weight ratio, and, consequently, a longer APT. If we using minimum range launch it is also interlinked with the discussed effect. This is explained by the fact that this removal trajectory leads to a reduction of the SR transit time through high-dense atmosphere, as well as with the possibility the increasing afterburning of fuel, compared to the nominal version.

There are quite few publications devoted to this kind of research. It should be noted that the BNS, as a whole, are often not objects of decomposition and require a comprehensive approach in assessing the influence in both the measurement information and the initial (ballistic) data. First of all, it has to do with the level of reliability of the used model of the Earth's gravitational field.

The above gives reason to believe relevant the topic of the study, which analyses the influence of BNS uncertain factors on the accuracy of SR motion with extended APTs.

The purpose of the study is to develop methods for analysing the influence of BNS uncertain factors on the accuracy of SR motion with an extended APT and to give a relevant assessment to the contribution of each component to specific cases of their occurrence.

In this research, we used methods of differential equations, probability and statistics methods, linear algebra, matrices, ballistics, rocket dynamics and SR navigation.

2. Scattering Components of Sounding Rockets with Inertial Control System
Scattering of the trajectory is caused by deviations from the nominal (calculated) values of the design characteristics of the complex (Sounding Rocket and its systems), as well as flight parameters.

Possible unforeseen factors connected with the launch of SR and its provision are the reasons for these deviations [2, 3, 11-17]. Scattering is a measure criteria of the rocket accuracy and its main characteristic functioning as an integral evidence of the accuracy of different types of SR systems. Due to the functional diversity, there are reasons to consider the scattering components as a statistical sum of some physical independent components. This should correspond to the stratified level of the target accuracy structure, which correspond to the SR at certain structurally aggregated levels [11, 18].

![Command Instrumentation Complex (CIC) Classification](image)

**Figure 1.** Command instrumentation complex (CIC) classification.
Command instrumentation complex (CIC) for Sounding Rockets classification is represents on Figure 1. The resulting scattering value shall be presented as an additive sum of three main components [12, 17, 18]:

\[
\delta = \delta_{\text{grav}} + \delta_{\text{tech}} + \delta_{\text{init}},
\]

where \(\delta_{\text{grav}}\) – gravimetric and geodetic errors; \(\delta_{\text{tech}}\) – technical errors; \(\delta_{\text{init}}\) – errors in data preparation.

Gravimetric and geodetic are including next inaccuracies:
1. In true parameters of Earth shape.
2. In passive part of the trajectory (PPT).
3. In geodetic reference.

The above types of inaccuracies are characterized by the fact that they announced in total deviation of the SR from the target. That are because of errors in the initial conditions definition of motion and calculating some capture trajectory inaccuracies, which is a reference to determine are limit sizes of the scattering ellipse [4, 12, 19].

Generally, considered appropriate to subdivide technical preparation inaccuracies into the following components [20]:
1. Methodical control errors from the deviation point.
2. Instrumental errors in control system.
3. GPS orientation errors for platform INS.
4. GINS orientation errors.
5. Errors in the PPT.

The above errors systematization cause by technical scattering. They are explained by the peculiarities of their expression in the total deviation from the target of the point of incidence of the SR last stage.

For example, with the perfect operation of the SR angular stabilization system, the orientation errors in the axes of sensitive measuring systems are announced only through isochronous variations of the motion parameters. And they do not depend on DPI control action, and, consequently, on the methodical control errors. DPI caused by instrumental errors in CS. In the first approximation shall be determined only by the instrumental component of the switch-off time variation for the propulsion system (PS) arising due to errors in the all system elements [2, 21]. As for the methodological error, it practically does not depend on from any instrumental errors in DPI CS. And it does not depend on errors inside devices sensitivity axes orientation. For most control methods, this error is announced in DPI through the motion parameters isochronous deviation, as well as the methodological component of the variation of PS switch-off time.

Finally, the errors (perturbations) in PPT mainly at the stage of afterburning effect, separation phase and descend in the atmosphere, announced in DPI, can be considered independent from the groups of errors, which have been analysed. Whereby:

- the pulse scatter after thrust cut-off;
- separation perturbations;
- deviation of the velocity vector, as well as angle-of-attack and angle-of-slip deviations from nominal values, when entering the dense layers of the atmosphere, are usually classified as perturbations of the initial conditions of motion.

External perturbations:
1) perturbations of atmospheric thermodynamic parameters (deviations from normal conditions);
2) wind (speed and direction);
3) deviation of the mass distribution parameters in the last SR stage (centre of mass displacement, angular misalignment of the main inertia axes);
4) form deviation, and, consequently, deviation of aerodynamic force coefficients and moments of SR last stage.
Errors in data preparation for SR launch are directly related to errors during solving the inverse boundary problem, which is determine the ballistic control parameters. They are generating the following errors:

1. The capture trajectory calculation errors.
2. Errors in CS settings calculation and on-board entering data errors during its preparation for launch.

These errors occurrence may be depending from:

1. Methodical errors in calculation and formulæ.
2. Calculation methods errors (interpolation errors, numerical integration, etc.).
3. Computational errors (finite presentation of numbers in the bit grid of the on-board computer, the use of rounding in the calculation process, etc.).
4. Input and storage data errors.

It is assumed that data preparation errors can be reduced to negligible values. This gives us a reason to confine to the first two components in the formula (1) while studying the scattering characteristics, i.e., to consider only the components due to gravimetric and geodetic errors, as well as technical errors.

3. The Capture Trajectory Calculation

Currently, there are many approaches and methods, which are used for calculation of capture trajectory, i.e. a nominal trajectory, which goes through the target point \([12, 13, 15, 17, 18, 20, 22, 23]\). This circumstance makes it impossible to describe in sufficient detail the possible ways to solve the problem within the framework of this study. In this regard, we should limit ourselves to the problem general mathematical formulation, and a brief description of its solution aspects.

We assume that the coordinates of the point of incidence are given in the form of nonlinear spherical coordinates. And they uniquely determined by the SR motion parameters at the time \(t_k\) command is sent to the last stage or head section (SP). These coordinates can be represented by the following functional dependencies \([18]\):

\[
D_c = D_c[q_{a1}(t_k), q_{a2}(t_k), \ldots, q_{a6}(t_k), t_k] \\
Z_c = Z_c[q_{a1}(t_k), q_{a2}(t_k), \ldots, q_{a6}(t_k), t_k],
\]

(2)

where \(q_{a}(t_k)\) — the absolute motion parameter values at the time moment \((i = 1, \ldots, 6)\) \(t_k\).

The motion parameters \(q_{a1}(t_k), q_{a2}(t_k), \ldots, q_{a6}(t_k), t_k\) for the given programs uniquely depend on the of the take-off point \(H_0, B_0, L_0\) coordinates, targeting azimuth \(A_0\) and the time \(t_k\). In this case, the dependences (2) can be presented as follows:

\[
D_c = D_c[H_0, B_0, L_0, A_0, t_k]; Z_c = Z_c[H_0, B_0, L_0, A_0, t_k].
\]

(3)

For a known take-off point, the dependencies (3) may be written as follows:

\[
D_c = D_c[A_0, t_k]; Z_c = Z_c[A_0, t_k].
\]

(4)

The coordinates determination under given mission parameters and time is a direct ballistic problem, which is solved by numerical integration of a system of differential equations of nominal motion \([12, 18, 20]\). Determination of the initial data for the FV launch is based on the inverse ballistic problem solution \([12, 18]\).

Nevertheless, the calculation of the capture trajectory can be made by successive approximations during solving a direct ballistic problem. This method presented here as follows. Using simple, approximate dependencies, the first approximation of the \(A_0\) and \(t_k\) parameters shall be determined. Then the direct problem, based on the obtained coordinates of the point of incidence \(D_c, Z_c\) shall be determined by the corrections \(\delta A_0\) and \(\delta t_k\), which shall be introduced into the values \(A_0\) and \(t_k\). The calculation will be repeated until the next condition is fulfilled a required accuracy as in (4).

The main input data for the calculation of the capture trajectory are:

1) geodetic coordinates of the take-off point \(H_0, B_0, L_0\);
2) geodetic coordinates of the target \(H_T, B_T, L_T\);
3) model of Earth shape parameters;
4) model of the Earth's gravitational field parameters;
5) the Earth's standard atmosphere parameters;
6) FV control system characteristics;
7) errors $\Delta D_{\text{allowable}}, \Delta Z_{\text{allowable}}$ in the calculation of the range and direction of the capture trajectory.

4. Discussion

The calculation of the capture trajectory can be made according to the following scheme.

1. A linear spherical range to the target $D_T$ and a spherical targeting azimuth $A_{\text{spheric}}$ shall be determined from the known geodetic coordinates of the take-off point $B_0, L_0$ and the target $B_T, L_T$ by the formulas:

$$D_T = R_N \Phi_N = R_N \left( \frac{\pi}{2} - \arcsin \left[ \sin \phi_T \sin \phi_0 + \cos \phi_T \cos \phi_0 \cos (L_T - L_0) \right] \right),$$

$$A = \begin{cases} \frac{\pi}{2} - A^*_T; & \text{when } \sin(L_T - L_0) > 0; \\ \frac{3\pi}{2} + A^*_T; & \text{when } \sin(L_T - L_0) < 0; \end{cases}$$

where $A^*_T = \arcsin \left( \frac{\sin \phi_T - \sin \phi_0 \cos \phi_T}{\cos \phi_0 \cos \phi_T} \right); \sin \phi_T = \frac{(1 - e^2) \sin \beta_t}{(1 - e^2 \sin^2 \beta_t)}$, $(i = 0,T a r g e t)$.

2. The spherical azimuth $A_{\text{spheric}}$ accepted as the first approximate of the targeting azimuth $A_0 = A_{\text{spheric}}$ and approximate time for sending for a separation command of the last stage or a nose cone of FV. Knowing the time limits change $t k_{\text{max}}$, and corresponding minimum and maximum range changes $D_{\text{max}}$ and $D_{\text{min}}$ it is possible to determine $t_k$, the appropriate ranges $D_T$ by linear interpolation from the formula:

$$t_k = t^*_k + \frac{t_{k_{\text{max}}} - t_{k_{\text{min}}}}{D_{k_{\text{max}}} - D_{k_{\text{min}}}} (D_T - D^{'})$$

where $D'$ is the range corresponding to the time $t^*_k$;

$$t^*_k = \left[ tT \left(D_{\text{min}}/2; \left| t \right(D_{\text{max}}/2 \leq D_T \leq D_{\text{max}})k_{\text{max}} \right) \right] \text{min}(k_{\text{min}}).$$

3. Then we can calculate coordinates $x^r_r, y^r_r, z^r_r$ using the equations of nominal motion for example, the equations with the specified parameters $A_0, t_k$ and the coordinates of the incidence point $D_c, Z_c$ are determined from the next formulas:

$$D_c = R_N \Phi_c = R_N \left( \frac{\pi}{2} - \arcsin \left[ x^r_r + \frac{(y^r_r)^2 + (z^r_r)^2}{r_{r_c}} \right] \right);$$

$$Z_c = -R_N E_c \sin \Phi_c = -R_N (A_0 - A_c) \sin \Phi_c,$$

where $A_c - 1$ the azimuth of incidence point;

$$A_c = \begin{cases} \frac{\pi}{2} - A^*_c; & \text{when } z^r_r < 0; \\ \frac{3\pi}{2} + A^*_c; & \text{when } z^r_r > 0; \end{cases}$$

$$A_c = \arcsin \left( \frac{x^r_r y^r_r - x^r_r y^r_r}{r_{r_c}} \right),$$

$$r_{r} = \sqrt{\left( x^r_r \right)^2 + \left( y^r_r \right)^2 + \left( z^r_r \right)^2}; \quad r_0 = \sqrt{\left( x^r_0 \right)^2 + \left( y^r_0 \right)^2};$$

$$x^r_0 = (N e_0 + H_0) \cos B_0; \quad y^r_0 = \left[ (N e_0 + H_0) - e_c N e_0 \right] \sin B_0; \quad N e_0 = \frac{1}{\sqrt{1 - \frac{e_c^2 N e_0}}},$$

4. Now we may verify the conditions:

$$|D_c - D_T| \leq \Delta D_{\text{extra}}; \quad |Z_c - Z_T| \leq \Delta Z_{\text{extra}}.$$
If these conditions are not satisfied, we shall improve \( A_0 \) and \( t_k \), correcting by formulas:

\[
\begin{align*}
t_k^{(i+1)} &= t_k^{(i)} + \delta t_k^{(i)}; \\
A_0^{(i+1)} &= A_0^{(i)} + \delta A_0^{(i)}.
\end{align*}
\]  

(9)

Corrections \( \delta A_0 \) and \( \delta t_k \) shall be defined in different ways. Particularly, these corrections can be determined by solving a system of two linear equations obtained from decomposing the functions (4) up to terms of the 2nd order of smallness:

\[
\begin{align*}
\delta D &= \dot{D}_k \delta t_k + \frac{\partial D}{\partial A_0} \delta A_0; \\
\delta Z &= \dot{Z}_k \delta t_k + \frac{\partial Z}{\partial A_0} \delta A_0,
\end{align*}
\]  

(10)

where \( \dot{D}_k \) and \( \dot{Z}_k \) are the full derivatives from the point of incidence coordinates in APT flight; \( \frac{\partial D}{\partial A_0}, \frac{\partial Z}{\partial A_0} \) - are partial derivatives from the point of incidence coordinates in targeting azimuth \( A_0 \).

Derivatives \( \dot{D}_k \) and \( \dot{Z}_k \) can be calculated, for example, using the following formulas:

\[
\begin{align*}
\dot{D}_k &= \sum_{j=1}^{n} \frac{\partial D}{\partial q_j^{(k)}} q_j^{(k)}; \\
\dot{Z}_k &= \sum_{j=1}^{n} \frac{\partial Z}{\partial q_j^{(k)}} q_j^{(k)},
\end{align*}
\]  

(11)

where \( \frac{\partial D}{\partial q_j^{(k)}}, \frac{\partial Z}{\partial q_j^{(k)}} \) - are partial derivatives of the coordinates \( D_c, Z_c \) in terms of the motion parameters \( q_j^{(k)} \) in APT end (so-called ballistic derivatives); \( q_j^{(k)} \) - are full derivatives of the motion parameters \( q_j^{(k)} \) in APT flight time.

The process of improving successive approximations in determining the input data \( t_k \) and \( A_0 \) would be continue until the conditions (8) are met.

The ballistic derivatives in the expression (11) and purposed to construct the capture trajectory are necessary for the scattering characteristics analysis as well. Within the solution of the problem under consideration, which are often called sensitivity functions, and have a physical meaning of weight functions. The weight functions determine how single deviations of different perturbing motion factors influence to the total range. The weigh function is expressed in terms of the point of incidence coordinates.

Among the sensitivity function methods determining, the following are the most common [2]:

- numerical methods (finite difference methods and variation methods);
- analytical method based on Kepler's model of FV motion along PPT.

In this publication, used the finite difference method in its simplified version in order to determine desired values of the sensitivity functions.

The method of variations based on a single integration of the system of differential equations of motion (SDEM), describing the SR nominal motion, and multiple integration of linearized SDEM under single initial conditions.

In contrast to numerical methods, the analytical method allows [8, 11, 18] to get a solution based on finite analytical relations. But there is possible to find the corresponding relations only for simple models, among which, is the Kepler model. This model is obtained under the following assumptions: 1) the Earth is taken as a fixed sphere of radius \( R_E \); 2) the gravitational field of the Earth is determined by the Newtonian potential \( U = \frac{\pi_0}{r} \) where \( \pi_0 = \mu = f M = 3.986 \times 10^{14} \frac{m^3}{sec^2} \) - is the coefficient; 3) the atmosphere is absent; 4) SR is considered as a material point with a mass equal to the mass of FV and concentrated in its center of mass.

Under these assumptions the trajectory of SR motion on PPT, will be a curve lying in a plane, which passes through the center of the attracting body. In this case, it is completely determined by the motion parameters \( r_k, V_k, \theta_{FV} \) and time \( t_k \) (see Figure 2) [20, 21].
Figure 2. Main points of SR trajectory.

The main advantage of analytical method compared to the numerical one is in reduction of calculation time required to implement the algorithm for calculating ballistic derivatives. However, the analytical method has its drawbacks. And the essential drawback of this method are arising errors from the model of Kepler motion, which is not quite accurate [22, 23].

5. Conclusion

Proposed general estimation methodology for influence scattering factors of ballistic and navigation support uncertain factors on the accuracy of sounding rocket motion with an extended active part of trajectory and was given a relevant assessment to the contribution of each component in relation to specific cases of their occurrence.

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