Inhomogeneous Fixed Point Ensembles Revisited

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Abstract

The density of states of disordered systems in the Wigner-Dyson classes approaches some finite non-zero value at the mobility edge, whereas the density of states in systems of the chiral and Bogoliubov-de Gennes classes shows a divergent or vanishing behavior in the band centre. Such types of behavior were classified as homogeneous and inhomogeneous fixed point ensembles within a real-space renormalization group approach. For the latter ensembles the scaling law \( \mu = d\nu - 1 \) was derived for the power laws of the density of states \( \rho \propto |E|^\mu \) and of the localization length \( \xi \propto |E|^{-\nu} \). This prediction from 1976 is checked against explicit results obtained meanwhile.

1 Introduction

Some time ago I used real-space renormalization group arguments in analogy to the cell model of Kadanoff\cite{1} in order to investigate the critical behavior\cite{2} close to the mobility edge of the Anderson model\cite{3}. Two types of ensembles were considered, a homogeneous and an inhomogeneous one.

**Homogeneous fixed point ensemble (HFPE).** This ensemble is homogeneous in energy \( \epsilon \). It is invariant under the transformation \( \epsilon \to \epsilon + \text{constans} \). Since the density of states \( \rho \) stays constant during the renormalization group procedure the scale change

\[
 r \to r/b \quad \text{implies} \quad \epsilon \to \epsilon b^d
\]

with dimension \( d \) of the system. We assume one relevant perturbation to this system which grows like

\[
 \tau \to \tau b^\nu.
\]

Depending on the sign of \( \tau \) the perturbation produces localized and extended states, resp. This perturbation is added to the HFPF in a strength increasing linearly in energy \( E \)

\[
 \tau = cE,
\]

where the mobility edge is taken at \( E = 0 \), and extended states at \( \tau > 0 \) and localized ones for \( \tau < 0 \). \( c \) transforms under RG.
Inhomogeneous fixed point ensemble (IHFPEnsemble). In this ensemble the scale factors for length and energies are independent from each other. The ensemble is inhomogeneous in the energy,
\[ r \rightarrow r/b, \quad \epsilon \rightarrow \epsilon b^\nu. \] (4)

It is assumed that there is no relevant perturbation to such an ensemble.

Both ensembles yield power and homogeneity laws. The density of states obeys
\[ \rho_{\text{hom}} = \text{const}, \quad \rho_{\text{inh}} \propto |E|^\mu, \quad \mu = d/y - 1. \] (5)

The localization length yields in both cases
\[ \xi \propto |E - E_c|^{-\nu}, \quad \nu = 1/y. \] (6)

The low-temperature a.c. conductivity obeys the homogeneity relation
\[
\sigma(\omega, \tau) = \begin{cases} 
 b^{2-d} \sigma(\omega b^d, \tau b^\nu) & \text{IHFPEnsemble} \\
 b^{2-d} \sigma(\omega b^\nu, \tau b^\nu) & \text{IHFPEnsemble} \end{cases}.
\] (7)

One deduces the d.c. conductivity in the region of extended states
\[ \sigma(0, \tau) \sim \tau^s, \quad s = (d - 2)/y = (d - 2)\nu. \] (8)

What comes out correctly on the basis of these ideas? Not only the scaling and homogeneity laws shown above can be deduced, but also such laws for averaged correlations, including the inverse participation ratio and long-range correlations between states energetically close to each other including those in the vicinity of the mobility edge. What has to be added are averages of matrix elements and of their powers for the transformation step by the linear scale factor \( b \) of the cell model.

A short historical digression may be allowed. The oldest paper on the mobility edge i.e. the separation of localized and extended states of a disordered system was given by Phil Anderson\[3\] (1958) (Well aware of possible complications by the Coulomb interaction he considered the transition from spin diffusion to localized spin excitations). It is a nice accident that its page number 1492 coincides with the year of another important discovery. Earlier papers on disordered systems, which became important for the development of this field was Wigner's\[4, 5, 6\] Gaussian matrix ensemble (1951) for nuclei. Probably the oldest paper on chiral systems is Dyson’s paper\[7\] on disordered chains (1953). Other early contributions on disordered chains were by Schmidt\[8\] and arguments that states in one dimension are localized\[9, 10\]. In 1962 Dyson gave the threefold classification of ensembles of orthogonal, unitary and symplectic symmetry depending on the behaviour under time-reversal invariance\[11, 12\].

Since these early developments a lot of progress has been made. There are numerous calculations for the behaviour around the mobility edge both analytic and numerical. I refer to the review by Evers and Mirlin\[13\]. 1979 marked important break-throughs: The scaling theories of localization by Abrahams et al.\[14\] and by Oppermann and Wegner\[15\] appeared. The mapping
onto a non-linear sigma-model was conjectured 16, brought into its bosonic-replica 17, its fermionic-replica 18 and finally in its supersymmetric 19 form. A self-consistent approximation for the Anderson transition was put forward by Götze 20, Vollhardt and Wölfle 21 22. A numerical renormalization scheme was devised by MacKinnon and Kramer 23.

Since then many more results and techniques were developed. Here I mention only a few: The complete classification of ten symmetry classes of random matrix theories, \(\sigma\)-models, and Cartan’s symmetric spaces was given by Zirnbauer and Altland 24 25 and by Schnyder, Ryu, Furusaki, and Ludwig 26 after several occurrences of chiral and Bogolubov-de Gennes classes 27 28 29 30 31 32 33 34. These classes are listed in table 1 since I will refer later to this nomenclature.

Table 1: Symmetry classes of single particle Hamiltonians defined in terms of presence or absence of time-reversal symmetry (TRS) and particle-hole symmetry (PHS). Absence is denoted by 0, presence by the symmetry square \(\pm 1\). SLS indicates absence (0) and presence (1) of sublattice or chiral symmetry. After ref. 26.

| System                  | Cartan nomenclature | symmetry | TRS | PHS | SLS |
|-------------------------|---------------------|----------|-----|-----|-----|
| standard                | A                   | unitary  | 0   | 0   | 0   |
| (Wigner-Dyson)          | AI                  | orthogonal | +1  | 0   | 0   |
|                         | AII                 | symplectic | -1  | 0   | 0   |
| chiral                  | AIII                | unitary  | 0   | 0   | 1   |
| (sublattice)            | BDI                 | orthogonal | +1  | +1  | 1   |
|                         | CII                 | symplectic | -1  | -1  | 1   |
| Bogolubov-de Gennes     | D                   |          | 0   | +1  | 0   |
|                         | C                   |          | 0   | -1  | 0   |
|                         | DIII                |          | -1  | +1  | 1   |
|                         | CI                  |          | +1  | -1  | 1   |

Transfer matrix approaches originally used for linear chains were developed for the non-linear \(\sigma\)-model 19 as well as for the for the distribution function of the transfer matrix of chains with many channels (DMPK-equation 35 36). These techniques allowed the determination of correlations, wave-function statistics and transport properties. Such chains can have broad distributions of conductivities and even cases of perfect transmissions were found 37 38.

In two dimensions some of these symmetry classes allow the inclusion of a topological \(\theta\)-term. As observed by Pruisken et al. 39 40 the Wigner-Dyson unitary class with this term describes the integer quantum Hall effect. Another term which may be added is a Wess-Zumino term. Such terms are of importance in the study of disordered Dirac fermions, which appear in dirty d-wave superconductors 41 42 43 and in disordered graphene 44 45 46 47 48 49.

Network models originally introduced by Shapiro 50 are very useful for the description of quantum Hall systems as first shown for the integer quantum Hall effects 51.
effect in the Chalker-Coddington-model\textsuperscript{51}.

Obviously the HFPE applies to the Wigner-Dyson classes, whereas the IHFPE applies to the chiral and the Bogoliubov-de Gennes classes.

The main object of this paper is the comparison of the scaling law for the IHFPE

\[ \mu = d \nu - 1. \]  

(9)

derived from \textsuperscript{5} \textsuperscript{9} with various results meanwhile obtained. I will shortly comment on the scaling law \textsuperscript{8} for the conductivity in subsection 4.1.

2 One dimensional chains

2.1 Thouless relation

Thouless\textsuperscript{52} following Herbert and Jones\textsuperscript{53} considered a one-dimensional chain governed by the Hamiltonian

\[ H = \sum_{i=1}^{N} c_i |i\rangle\langle i| - \sum_{i=1}^{N-1} (V_{i,i+1} |i\rangle\langle i+1| + V_{i+1,i} |i+1\rangle\langle i|) \]  

(10)

and found in the limit \( N \rightarrow \infty \) that the function

\[ K(z) = \int dx \rho(x) \ln(x - z) - \ln|V|, \quad -\pi < \arg \ln(x - z) < \pi \]  

(11)

connects both the integrated density of states \( I(E) \) and the exponential decrease of eigenfunctions \( \lambda(E) \) (inverse correlation length \( \xi \))

\[ K(E \pm i0) = \lambda(E) \mp i\pi I(E). \]  

(12)

The density of states is symmetric in chiral and Bogoliubov-de Gennes classes \( \rho(-E) = \rho(E) \). Then besides \( K(z^*) = K^*(z) \) also \( K(-z) = K(z) + i\pi s(z) \) with \( s(z) = \text{sign} \Im(z) \) holds. If \( K(z) + i\pi s/2 \propto z^\gamma \) for small \( z \), then

\[ K(z) + i\pi s/2 = cr^\gamma e^{i\gamma(\phi - s\pi/2)}, \quad z = re^{i\phi} \]  

(13)

with real \( c \). Then

\[ K(E + i0) = c|E|^{\gamma} \left( \cos\left(\frac{\pi}{2}\gamma\right) - i \text{sign}(E) \sin\left(\frac{\pi}{2}\gamma\right) \right) - \frac{\pi}{2}i, \]  

(14)

from which \( \lambda \propto |E|^\gamma \), \( \rho(E) \propto |E|^{\gamma-1} \) follows in agreement with \textsuperscript{9}. One observes that

\[ \frac{dK(z)}{dz} = \int dx \frac{\rho(x)}{z - x}. \]  

(15)

Thus the sign of the imaginary part of \textsuperscript{13} is opposite to the sign of \( \Im z \). This implies that \( \gamma \leq 1 \). If a contribution with \( \gamma > 1 \) appears, then there is also a contribution with \( \gamma = 1 \), which according to \textsuperscript{14} does not contribute to \( \lambda \),
but to a finite density of states in the center of the spectrum. Such a system is described by the homogeneous fixed point ensemble.

For \( \gamma \leq 0 \) the integrated density of states would diverge. Thus these arguments can only be applied for \( 0 < \gamma < 1 \).

In a number of cases the asymptotic behaviour is given by a power multiplied by some power of the logarithm. Then

\[
K(z) + \frac{i\pi s}{2} = c e^{-\gamma s^2 (\phi - s\pi/2)} (\ln r + i(\phi - s\pi/2))^g. \tag{16}
\]

This yields for \( \gamma = 0 \) and \( \gamma = 1 \)

\[
\begin{array}{c|c|c}
\gamma & K(E + i0) & \lambda \\
\hline
0 & c(\ln |E|)^g - i\gamma \pi g \frac{1}{2} \ln(|E|)^{g-1} & (\ln |E|)^g \\
1 & -icE(\ln |E|)^g - \gamma \pi g \frac{1}{2} |E|(\ln |E|)^{g-1} & |E|(\ln |E|)^{g-1}
\end{array} \tag{17}
\]

and thus

\[
\begin{array}{c|c|c}
\gamma & \lambda \sim & \rho \sim \\
\hline
0 & (\ln |E|)^g & \frac{(\ln |E|)^{g-2}}{|E|} \\
1 & |E|(\ln |E|)^{g-1} & (\ln |E|)^g
\end{array} \tag{18}
\]

Dyson\[7\] calculated the averaged density of states for the chain (10) with \( \epsilon_i = 0 \) and random independently distributed matrix elements \( V \), for which he assumed a certain distribution and obtained

\[
\rho(E) \sim \frac{1}{|E(\ln |E|)^{g}|}. \tag{19}
\]

which corresponds to the case (18) with \( \gamma = 0 \) and \( g = -1 \). Indeed Theodorou and Cohen\[54\] and Eggarter and Ridinger\[55\] found the averaged localization length diverging

\[
\xi \sim |\ln |E|| \tag{20}
\]

in agreement with (18).

### 2.2 Ziman’ s model

Ziman\[56\] (compare also Alexander et al.\[57\]) considered a one-dimensional tight-binding model (his case II) (10) requiring the diagonal matrix elements to vanish, \( \epsilon_i = 0 \), and the hopping matrix-elements to agree pairwise \( V_{2m,2m+1} = V_{2m+1,2m+2} \). Apart from this restriction he assumed the \( V \)s to be independently distributed with probability distribution

\[
\rho(V) = (1 - \alpha) V^{-\alpha}, \quad 0 < V < 1, \quad -\infty < \alpha < 1. \tag{21}
\]

He obtained for these distributions

\[
\begin{array}{c|c|c|c}
\alpha & \nu & \mu \\
\hline
-1 < \alpha < 1 & \frac{2(1-\alpha)}{1-\alpha} & \frac{-1-\alpha}{1-\alpha} \\
-3 < \alpha < -1 & \frac{1-\alpha}{1-\alpha} & 0 \\
\alpha < -3 & 0 & 0
\end{array} \tag{22}
\]

Obviously the first row describes models in accordance with the IHFPE, the second and third row with the HFPE.
2.3 Further one-dimensional results

Titov et al. [58] have summarized and completed results for the density of states of all classes of chains with $N$ channels as shown in table 2.

| Class           | ρ(E)                              | $x = E\tau$ |
|-----------------|-----------------------------------|-------------|
| Chiral all classes, odd $N$ | $x^{-1} \ln^{-3}(x)$ | $x^{-1} \ln^{-3}(x)$ |
| AIII, even $N$   | $x \ln x$                        | $x \ln x$  |
| CII, even $N$    | $x^3 \ln x$                      | $x^3 \ln x$ |
| BDI, even $N$    | $\ln x$                         | $\ln x$    |
| BdG CI           | $|x|$                            | $|x|$       |
| C                | $x^2$                            | $x^2$      |
| D, DIII, $N \neq 2$ | $x^{-1} \ln^{-3}(x)$ | $x^{-1} \ln^{-3}(x)$ |
| D, DIII, $N = 2$ | two mean free paths              | two mean free paths |

All chiral classes are equivalent by a gauge transformation for $N = 1$ and yield the Dyson result (19) and $\xi \propto |\ln |E||$ for this case in agreement with (18). Due to Gruzberg et al. [59] also the BdG classes BD and DIII fall into the same universality class. The localization length does not diverge for the chiral classes, if $N$ is even. The same holds for (general $N$) for the BdG classes C and CI.

3 Bosons From One To Two Dimensions

3.1 One-dimensional chain

Whereas the Hamiltonian (10) yields the equation for eigenstates $|\psi\rangle = \sum_i \psi_i |i\rangle$

$$E\psi_i = \epsilon_i \psi_i - V_{i,i-1} \psi_{i-1} - V_{i,i+1} \psi_{i+1}$$ (23)

one obtains a similar equation for harmonic phonons governed by the Hamiltonian

$$H = \sum_i \frac{p_i^2}{2m} + \sum_i \frac{W_i}{2} (x_{i+1} - x_i)^2,$$ (24)

which reads

$$m\omega^2 x_i = W_i (x_i - x_{i+1}) + W_{i-1} (x_i - x_{i-1}).$$ (25)

Thus Thouless’ arguments can be applied again with $x = \omega^2$. Since there are no states for $\omega^2 < 0$, one has

$$K(z) = cr^\gamma e^{i\gamma(\phi - s\pi)}, \quad z = re^{i\phi},$$ (26)

which yields

$$K(\omega^2 + i0) = c\omega^{2\gamma} e^{-i\pi\gamma}$$ (27)
and thus
\[ \lambda(\omega) = c\omega^{2\gamma}\cos(\pi\gamma), \quad I(\omega) = c\omega^{2\gamma}\sin(\pi\gamma), \quad 0 < \gamma < 1. \] (28)

Alexander et al. [57] (cases a, b, and c) and Ziman [56] (case II) determined the density of states \( \rho(\omega) \propto \omega^\mu \) for harmonically coupled phonons with independently distributed spring constants
\[ p(W) = (1 - \alpha)W^{-\alpha}. \] (29)

Ziman moreover determined the localization length \( \xi \propto \omega^{-\nu} \) and obtained
\[
\begin{array}{ccc}
0 < \alpha < 1 & \frac{\nu}{2-\alpha} & \frac{\mu}{2-\alpha} \\
-1 < \alpha < 0 & 1 - \alpha & 0 \\
\alpha < -1 & 2 & 0
\end{array}
\] (30)

Again the first line is in agreement with the IHFPE, whereas the two other cases correspond to the HFPE.

3.2 Bosonic excitations discussed by Gurarie and Chalker

Gurarie and Chalker [60] point out that bosonic systems with and without Goldstone modes show different localization behavior.

John et al. [61] investigated localization in an elastic medium with randomly varying masses. For \( d > 2 \) they found extended states for small frequencies \( \omega \). The phonon states are localized beyond some critical \( \omega_c \). This transition is described by the orthogonal ensemble. For \( d < 2 \) all states are localized and obey \( \nu = 2/(2-d) \). The density of states for phonons shows the same power law \( \rho(\omega) \propto \omega^{d-1} \) as in the ordered case. In this system with Goldstone modes the critical density below which the density of states would differ from that of the ordered system is \( d_c = 0 \).

In a disordered antiferromagnet one obtains below the critical dimension \( d_c = 2 \)
\[ \rho(\omega) \sim \omega^\mu, \quad \mu = \frac{3d - 4}{4 - d}, \quad \xi(\omega) \sim \omega^{-\nu}, \quad \nu = \frac{2}{4 - d}, \] (31)

where the result of [62] and the argument of [60] have been generalized from \( d = 1 \) to general \( d < d_c \). This is in agreement with the IHFPE. These results rest on the assumption that there is a single length scale \( \xi \propto 1/k \).

4 Electronic Systems In Two Dimensions

4.1 Conductivity in two dimensions

From the homogeneity law \( s = (d - 2)\nu \), which works well for \( d > 2 \), I concluded [2] \( s = 0 \) for dimensionality \( d = 2 \) and thus a jump to a minimum
metallic conductivity. At that time I did not expect that \( \nu \) may diverge as \( d \) approaches 2. This was found three years later by means of explicit renormalization group calculations\[14, 15\]. Thus the idea of a minimum metallic conductivity was in error for the orthogonal and unitary Wigner-Dyson classes, where all states are localized in \( d = 2 \). The critical conductivity in the symplectic class shows some distribution\[63\] and is of order \( e^2/h \).

In many two-dimensional models of chiral and Bogolubov-de Gennes classes including the classes applying to d-wave superconductors and graphene one obtains a finite non-zero conductivity of order \( e^2/h \) at criticality. This is to a large extend due to edge currents, as first observed by Pruisken et al.\[39, 40\] for the integer quantum Hall effect. Thus although the prediction\[2\] turns out to be correct, the true mechanism is more complex.

### 4.2 Chiral and Bogolubov-de Gennes models in \( d = 2 \) dimensions

The unitary case of chiral models (Gade and Wegner, Gade\[31, 32\]) yields at intermediate energies effective exponents

\[
\nu = \frac{1}{B}, \quad \mu = -1 + \frac{2}{B}
\]

in accordance with \( \text{(32)} \). At asymptotically low energies \( \rho \propto E^{-1}\xi^2(E) \) corresponds to the limit \( B \to \infty \). These results as well as many similar results for various disordered Dirac hamiltonians are obtained under the assumption that the localization length is given by the cross-over length from chiral to Wigner-Dyson behaviour without taking further renormalization into account\[32\]; see also the argument after eq. (6.60) of the review by Evers et Mirlin\[13\]. Alternatively the integrated density of states from the band center up to energy \( E \) is set to \( \xi^{-2} \) as in Motrunich et al.\[64\], which yields \( \text{(9)} \) per definition. It is important that only one coupling yields a relevant perturbation. The conductivity itself stays constant for the chiral models in \( d = 2 \). The exponent which drives the renormalization of the energy is usually called dynamical exponent \( z \), which is identical to the exponent \( y \) of \( \text{(33)} \). A more rigorous investigation of the localization of such systems taking into account any dependence of the initial couplings on the energy and of the cross-over would be of interest.

The spin quantum Hall effect yields at the percolation transition point\[65, 66, 67\]

\[
\nu = 4/7, \quad \mu = 1/7
\]

in agreement with \( \text{(33)} \). The same behavior is obtained for the Bogolubov-de Gennes class C if two of the four nodes of a dirty d-wave superconductor are coupled\[41, 43\].

### 4.3 Power law for density of states, finite localization length

The two fixed point ensembles describe the situation, in which the localization length diverges and the density of states either approaches some finite non-
zero value (HFPE) or diverges or goes to zero by a power law, which may be augmented by a logarithmic term. As mentioned above this holds for chains with an even number of channels in the chiral classes and for the Bogolubov-de Gennes classes C and CI. Certain single-channel models of class D and DIII show a divergence of the density of states $\rho \propto |E|^{-1+\delta}$ without divergence of the localization length\[68\].

Gurarie and Chalker\[69\] found that bosonic excitations in random media, which are not Goldstone modes, obey $\rho \propto \omega^4$ with finite localization length at low frequencies.

Apparently this type of behavior is not covered by HFPE and IHFPE.

5 Conclusion

The scaling prediction \[9\] of the IHFPE relating the exponent of the density of states and of the localization length yields correct results in the cases, in which I found both exponents. The author appreciates the wealth of systems, which has been found and investigated over the years.

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References

[1] L.P. Kadanoff, Physics 2 (1966) 263
[2] F.J. Wegner, Z. Physik B25 (1976) 327
[3] P.W. Anderson, Phys. Rev. 109 (1958) 1492
[4] E.P. Wigner, Ann. Math. 53 (1951) 36
[5] E.P. Wigner, Ann. Math. 62 (1955) 548
[6] E.P. Wigner, Ann. Math. 67 (1958) 325
[7] F.J. Dyson, Phys. Rev. 92 (1953) 1331
[8] H. Schmidt, Phys. Rev. 105 (1957) 425
[9] R.E. Borland, Proc. Roy. Soc. A274 (1963) 529
[10] H. Furstenberg, Trans. Amer. Math. Soc. 108 (1963) 377
[11] F.J. Dyson, J. Math. Phys. 3 (1962) 140
[12] F.J. Dyson, J. Math. Phys. 3 (1962) 1199
[13] F. Evers, A.D. Mirlin, Rev. Mod. Phys. 80 (2008) 1355
[14] E. Abrahams, P.W. Anderson, D.C. Licciardello, and T.V. Ramakrishnan, Phys. Rev. Lett. 42 (1979) 673
[15] R. Oppermann and F. Wegner, Z. Phys. B34 (1979) 327
[16] F. Wegner, Z. Phys. B35 (1979) 207
[17] L. Schäfer and F. Wegner, Z. Phys. B38 (1980) 113
[18] K.B. Efetov, A.I. Larkin, D.E. Khmel'nitskii, Zh. Eksp. Teor. Fiz. 79 (1980) 1120; Sov. Phys. JETP 52 (1980) 568
[19] K.B. Efetov, Adv. Phys. 32 (1983) 53
[20] W. Götze, J. Phys. C12 (1979) 1279
[21] D. Vollhardt and P. Wölfle, Phys. Rev. Lett. 45 (1980) 842
[22] D. Vollhardt and P. Wölfle, Phys. Rev. Lett. 48 (1982) 699
[23] A. MacKinnon and B. Kramer, Phys. Rev. lett. 47 (1981) 1546
[24] M.R. Zirnbauer, J. Math. Phys. 37 (1996) 4986
[25] A. Altland and M.R. Zirnbauer, Phys. Rev. B55 (1997) 1142
[26] A.P. Schnyder, S. Ryu, A. Furusaki, and A.W.W. Ludwig, Phys. Rev. B78 (2008) 195125
[27] S. Hikami, Nucl. Phys. B215 (1983) 555
[28] R. Oppermann, Nucl. Phys. B280 (1987) 753
[29] R. Oppermann, Physica A167 (1990) 301
[30] F. Wegner, Nucl. Phys. B316 (1989) 663
[31] R. Gade and F. Wegner, Nucl. Phys. B360 (1991) 213
[32] R. Gade, Nucl. Phys. B398 (1993) 499
[33] K. Slevin and T. Nagao, Phys. Rev. Lett. 70 (1993) 635
[34] J.J.M. Verbaarschot and I. Zahed, Phys. Rev. Lett. 70 (1993) 3852
[35] O.N. Dorokhov, JETP Lett. 36 (1982) 318
[36] P.A. Mello, P. Pereyra and N. Kumar, Ann. Phys. (N.Y.) 181 (1988) 290
[37] M.R. Zirnbauer, Phys. Rev. Lett. 69 (1992) 1584
[38] A.D. Mirlin and Y.V. Fyodorov, Phys. Rev. Lett. 72 (1994) 526
[39] H. Levine, S.B. Libby, A.M.M. Pruisken, Phys. Rev. Lett. 51 (1983) 1915
[40] A.M.M. Pruisken, Nucl. Phys. B235 (1984) 277
[41] A.A. Nersesyan, A.M. Tsvelik and F. Wenger, Nucl. Phys. B438 (1995) 561
[42] M. Bocquet, D. Serban, and M.R. Zirnbauer, Nucl. Phys. B578 (2000) 628
[43] A. Altland, Phys. Rev. B65 (2002) 104525
[44] I.L. Aleiner and K.B. Efetov, Phys. Rev. Lett. 97 (2006) 236801
[45] D.V. Khveshchenko, Phys. Rev. Lett. 97 (2006) 036802
[46] E. McCann, K. Kechedzhi, V.I. Fal’ko, H. Suzuura, T. Ando, and B.L. Altshuler, Phys. Rev. Lett. 97 (2006) 146805
[47] P.M. Ostrovsky, I.V. Gornyi, and A.D. Mirlin, Phys. Rev. B74 (2006) 235443
[48] P.M. Ostrovsky, I.V. Gornyi, and A.D. Mirlin, Phys. Rev. Lett. 98 (2007) 256801
[49] P.M. Ostrovsky, I.V. Gornyi, and A.D. Mirlin, Eur. Phys. J. Spec. Top. 148 (2007) 63
[50] B. Shapiro, Phys. Rev. Lett. 48 (1982) 823
[51] J.T. Chalker and P.D. Coddington, J. Phys. C21 (1988) 2665
[52] D.J. Thouless, J. Phys. C 5 (1972) 77
[53] D.C. Herbert, R. Jones, J. Phys. C 4 (1971) 1145
[54] G. Theodorou and M.H. Cohen, Phys. Rev. B13 (1976) 4597
[55] T.P. Eggarter and R. Riedinger, Phys. Rev. B18 (1978) 569
[56] T.L.A. Ziman, Phys. Rev. Lett. 49 (1982) 337
[57] S. Alexander, J. Bernasconi, W.R. Schneider, R. Orbach, Rev. Mod. Phys. 53 (1981) 175
[58] M. Titov, P.W. Brouwer, A. Furusaki and C. Mudry, Phys. Rev. B 63 (2003) 235318
[59] I.A. Gruzberg, N. Read, S. Vishveshwara, Phys. Rev. B71 (2005) 245124
[60] V. Gurarie and J.T. Chalker, Phys. B 68 (2003) 134207
[61] S. John, H. Sompolinsky, M.J. Stephen, Phys. Rev. B27 (1983) 5592
[62] R.B. Stinchcombe and I. R. Pimentel, Phys. Rev. B38 (1988) 4980
[63] B. Shapiro, Phil. Mag. B56 (1987) 1031
[64] O. Motrunich, K. Damle, D.A. Huse Phys. Rev. B65 (2002) 064206
[65] H. Saleur, B. Duplantier, Phys. Rev. Lett. 58 (1987) 2325
[66] I.A. Gruzberg, N. Read, A.W.W. Ludwig, Phys. Rev. Lett. 82 (1999) 4524
[67] E.J. Beamond, J. Cardy, J.T. Chalker, Phys. Rev. B65 (2002) 214301
[68] O. Motrunich, K. Damle, D.A. Huse Phys. Rev. B63 (2001) 224204
[69] V. Gurarie and J.T. Chalker, Phys. Rev. Lett. 89 (2002) 136801