Abstract

It is shown that all simple ecological, i.e. population dynamical equations (unlimited exponential population growth (or decrease) dynamics, logistic or Verhulst equation, usual and generalized Lotka-Volterra equations) hold a symmetry, called $C_EPT$ symmetry. Namely, all simple ecological dynamical equations are invariant (symmetric) in respect to successive application of the time reversal transformation - $T$, space coordinates reversal or parity transformation - $P$, and predator-prey reversal transformation - $C_E$ that changes preys in the predators or pure (healthy) in the impure (fatal) environment, and vice versa. It is deeply conceptually analogous to remarkable $CPT$ symmetry of the fundamental physical dynamical equations. Further, it is shown that by more accurate, "microscopic" analysis, given $C_EPT$ symmetry becomes explicitly broken.

1 Introduction. $CPT$ theorem in the fundamental physical dynamical equations

As it is well-known [1], [2] that $CPT$ symmetry theorem represents one of the most important theorem in the fundamental physics, more precisely relativistic quantum mechanic or local quantum field theory. Within less fundamental, classical physics or nonrelativistic quantum mechanics, $CPT$ theorem cannot be proved but some its consequences exist even in these less fundamental physical theories.

$CPT$ theorem is based and can be proved by some of the most fundamental concepts of the local quantum field theory. First one represents the dynamical, continuous Lorentz invariance (symmetry). It, simply speaking, admits only local (realizable luminal, i.e. by speed of light) and forbids any nonlocal (superluminal, i.e. faster than light) dynamical interaction. Or the final (in the future) physical dynamical state represents the result of the non-instantaneous dynamical
evolution of the initial (in the past) physical dynamical state. Second one represents the quantum field theory postulate that an elementary particle can have either integer spin (spin that equals even number of \( \frac{\hbar}{2} \) where \( \hbar \) represents reduced Planck's constant) or half-integer spin (spin that equals odd number of \( \frac{\hbar}{2} \)).

\( CPT \) theorem refers on the dynamical discrete symmetries and it states the following. All fundamental physical dynamical equations and theirs solutions are invariant (symmetric) in respect to successive application of the following three discrete transformations: time reversal transformation - \( T \), space coordinate reversal or parity transformation - \( P \), and charge conjugation transformation - \( C \). Simply speaking, \( T \) changes time moment \( t \) in \(-t\) and initial in the final conditions, \( P \) changes space coordinates \( x, y, z \) in \(-x, -y, -z\), while \( C \) changes any charge parameter \( c \) in \(-c\), or more generally, all particles in antiparticles, and vice versa.

On the basis of \( CPT \) theorem significant spin-statistic theorem can be proved. Also, on the basis of \( CPT \) theorem it follows that particle and antiparticle masses and lifetimes must be equivalent, while theirs electrical charges and magnetic moments must be opposite, i.e. must have opposite signature.

Neither \( T \) nor \( P \) nor \( C \) represents any real dynamical motion. In this sense all three given transformations are abstract. However, all of them, or at least \( P \) and \( T \), can be relatively simply formally demonstrated. \( P \) can be simply particularly demonstrated by a mirror, i.e. mirror transformation of the space. Also, \( T \) can be simply demonstrated by a video projector when it does a play back backward (corresponding to negative, i.e. reversal time direction). (Of course, when a video projector does play back forward it corresponds to usual, i.e. positive time direction. So if a physical process recorded by video camera and presented by video projector is time reversible or \( T \)-symmetric (as it is motion of a linear harmonic oscillator, e.g. undamped pendulum), then both, forward and backward, play backs are empirically ”natural”. But if a physical process recorded by video camera and presented by video projector is time irreversible or \( T \)-asymmetric (as it is diffusion of a drop of the ink in a glass of the water) then only one, forward play back is empirically ”natural”, while other, backward play back projection, is empirically ”strange” or ”impossible”.

In this work it will be shown that all simple ecological, i.e. population dynamical equations (unlimited exponential population growth (or decrease) dynamics, logistic or Verhulst equation, usual and generalized Lotka-Volterra equations) \([3]-[6]\) hold a symmetry, called \( C_{EP}T \). Namely all simple ecological dynamical equations are invariant (symmetric) in respect to simultaneous application of \( T, P \), and predator-prey reversal transformation - \( C_{E} \), that changes preys in the predators or pure (healthy) in the impure (fatal) environment, and vice versa. It is deeply conceptually analogous to \( CPT \) symmetry of the fundamental physical dynamical equations. Further, it will be shown that by more accurate, ”microscopic” analysis, \( C_{EP}T \) symmetry become explicitly broken.

2 \( C_{EP}T \) theorem in the simple ecological dynamical equations

Consider usual Lotka-Volterra, i.e. predator-prey differential equations system with significant applications in the ecology, biology, medicine (epidemiology, neurology) etc. \([3]-[6]\). Namely, as it is well-known, given system (including its different generalizations) represents one of the most simplest and most typical models of the ecological, i.e. population dynamics. This system is given
by the following two nonlinear differential equations of the first order

\[ \frac{dp}{dt} = ap - bpq \]  
\[ \frac{dq}{dt} = -cq + dpq \]  

Here, mathematically \( p, q \) represent the real, positive variables that depends of the time, \( t \), while \( a, b, c, d \), represent mathematically real, positive time independent constants, i.e. parameters.

But, ecologically, \( p \) represents the population or number of the individuals of a biological species of the preys (without physical dimension). Correspondingly \( q \) represents ecologically the population or number of the individuals of a biological species of the predators (without physical dimension). In this way, as it is well-known, (1), (2) refers on two competitive species living together. Also, ecologically, \( a \) represents the birth rate of the preys, \( b \) - the death rate of the preys, \( c \) - the death rate of the predators, and \( d \) - the birth rate of the predators, (all with \([s-1]\) physical dimension). Simply speaking \( a, b, c, d \) parameters, more precisely ecological dynamical parameters, express simplified or phenomenologically the basic dynamical interactions between predators, preys and environment (food resources, territory, etc.).

It is necessarily that the following be pointed out. System (1), (2) describes satisfactorily given population dynamics only then if \( p \) and \( q \) are sufficiently large or "macroscopic", i.e. for

\[ p \gg 1 \]  
\[ q \gg 1 \]

so that change of \( p \) or \( q \) for a small (eg. 1, 2 etc.) number of the corresponding individuals can be considered effectively as an infinitesimal change. It represents very important approximation condition. In other words presented simplified form of the exactly very complex interaction between given two animal species and environment, corresponding to described population dynamics (1), (2), is consistently applicable and can be correctly applied only then if given "macroscopic" approximation conditions (4), (5) are satisfied. Or, according to given "macroscopic" approximation conditions, more detailed analysis referring on the small number of the individuals, eg. one individual, based on (1), (2), is or can be (mostly) incorrect. Small values of \( p \) and \( q \) can be considered consistently only in the sense of an analytical extension of the solution of (1), (2) but not in the sense of a more accurate empirical analysis of the small number of the individuals of the species. For this reason more detailed (accurate), "microscopic", analysis of the ecological, i.e. population dynamics of a small number of the individuals can need significant corrections and changes of (1), (2).

Thus, ecological, i.e. population dynamics (1), (2), according to "macroscopic" approximation conditions (3), (4), represents implicitly a typical statistical method where populations \( p \) and \( q \) are proportional to statistical distributions that evolve during time. Explicit normalization of \( p \) and \( q \) to statistical distributions by some normalization parameters needs, since (1) and (2) are nonlinear, corresponding normalization of \( b \) and \( d \). In this way in different equivalent forms of the ecological dynamical system (1), (2) as well as in other ecological dynamical systems, statistical normalization parameters without real dynamical sense can appear.

It can be observed that \( p \) and \( q \) in the usual Lotka-Volterra equations system (1), (2) as well as similar simple ecological dynamical systems (that will be considered later) do not depend of the
space coordinates \(x, y, z\) that characterize predator and prey species territory (area). (Coordinate dependence of \(p\) and \(q\) can characterize some more complex ecological dynamical systems.) For this reason it can be considered that \(p\) and \(q\) are effectively space homogeneous and that they are invariant (symmetric) in respect to parity transformation \(P\) (that changes \(x, y, z\) in \(-x, -y, -z\) respectively). It means that both equations of the usual Lotka-Volterra equations system (1), (2) is invariant (symmetric) in respect to \(P\) too.

Further, it can be observed that (1) and (2) have not completely equivalent form. Namely, the coefficients before \(p\) and \(pq\) in (1), \(a\) and \(b\), are always positive and negative, while constant before \(q\) and \(pq\) in (2), \(-c\) and \(d\), are oppositely, always negative and positive, respectively. This in-equivalence or asymmetry causes that \(p\) can correspond to the preys species and that \(q\) corresponds to the predators species exclusively. Or this non-equivalence forbids that \(p\) corresponds to the predators species and that \(q\) corresponds to the preys species.

Define, however, such transformation, called predator-prey reversal transformation - \(C_E\), that changes \(a, b, c, d\) in \(-a, -b, -c, -d\) respectively. So ecologically, \(C_E\) represents a discrete transformation of the ecological dynamical parameters only that does not transform pure normalization parameters. Or generally, \(C_E\) denotes a change of the direction of the dynamical interactions between all constituents in the ecological system.

Obviously \(C_E\) holds idempotent characteristic \(C_E^2 = I\), where \(I\) represents the identity transformation.

Application of \(C_E\) at (1), (2) yields

\[
\frac{dp}{dt} = -ap + bpq
\]

\[
\frac{dq}{dt} = cq - dpq
\]

or after transposition of the equations,

\[
\frac{dq}{dt} = cq - dpq
\]

\[
\frac{dp}{dt} = -ap + bpq
\]

It is not hard to see that (7), (8) represents a typical usual Lotka-Volterra system but where, formally ecologically, \(q\) represents a new preys species population, \(p\) - a new predators species population, \(c\) - the birth rate of the new preys species, \(d\) - the death rate of the new preys species, \(a\) - the death rate of the new predators species, and \(b\) - the birth rate of the new predators species. Simply speaking \(C_E\) changes the previous preys species in the new predators species and the previous predators species in the new preys species.

In this way \(C_E\) represents a symmetry transformation of the set of all usual Lotka-Volterra equations systems (for all values of the parameters). Or in the domain of the applicability of all usual Lotka-Volterra equations systems, it is impossible differ represents some usual Lotka-Volterra system a real ecological system or the result of an abstract application of \(C_E\) at corresponding real Lotka-Volterra system. But \(C_E\) does not represent a symmetry transformation of any concrete usual Lotka-Volterra equations systems (for concrete values of the parameters).

Consider now an especial case

\[
b = d = 0
\]
when the usual Lotka-Volterra system (1), (2) turns it the following system

\[
\frac{dp}{dt} = ap \tag{10}
\]

\[
\frac{dq}{dt} = -cq \tag{11}
\]

Obviously, (10), (11) represent a system of two independent equations (and variables). They describe unlimited exponential population growth (increase) and decrease of the population, respectively, by interaction of the corresponding species with its environment. For this reason \( p \) species does not represents preys for \( q \) species. But, (10) and (11) have not completely analogous form since coefficient before \( p \) in (10), \( a \), is positive, while the coefficient before \( q \) in (11), \(-c\), is negative. Ecologically it means that environment positively stimulates increase of \( p \), i.e. first species population. Simply speaking, environment is pure (healthy) for the first species. Simultaneously, environment negatively stimulates \( q \) i.e. second species population. Or environment positively stimulates decrease of \( q \). Simply speaking, environment is impure (fatal) for the second species.

Application of \( C_E \) at (10),(11) yields

\[
\frac{dp}{dt} = -ap \tag{12}
\]

\[
\frac{dq}{dt} = cq \tag{13}
\]

or, after transposition of the equations,

\[
\frac{dq}{dt} = cq \tag{14}
\]

\[
\frac{dp}{dt} = -ap \tag{15}
\]

Obviously, (14)(15) represents again a typical system (10)(11) but now environment is pure (healthy) for \( q \) species, while environment is impure (fatal) for \( p \) species. Simply speaking here \( C_E \) changes pure (healthy) in the impure (fatal) environment and vice versa. It is again in full agreement with previous general definition that \( C_E \) denotes a change of the direction of the dynamical interactions between all constituents in the ecological system.

Also, it is obvious that \( C_E \) represents a symmetry transformation of the set of all (10), (11) equations systems (for all values of the parameters). Or, in the domain of the applicability of all (10), (11) equations systems it is impossible differ represents some (10), (11) equations system a real ecological system or the result of an abstract application of \( C_E \) at corresponding real (10), (11) equations system . But, of course, \( C_E \) does not represent a symmetry transformation of any concrete (10), (11) equations systems (for any concrete values of the parameters). It is in full agreement with previous conclusion on \( C_E \) as a symmetry transformation of the set of all usual Lotka-Volterra systems and an asymmetry transformation of any concrete usual Lotka-Volterra system.

It is not hard to see (which will not be considered explicitly here) that same conclusions on the ecological meaning and characteristics of \( C_E \) can be done even in case when unlimited exponential population growth (or decrease) dynamics is changed by limited exponential population growth.
(or decrease) dynamics, i.e. by logistic or Verhulst population dynamical equation [4]-[6]. Namely, in this case, instead of (10), (11) there are following two equations

\[
\frac{dp}{dt} = ap(p_0 - p) \frac{p_0}{p_0} \tag{16}
\]

\[
\frac{dq}{dt} = -cq(q_0 - q) \frac{q_0}{q_0} \tag{17}
\]

where \(p_0\) and \(q_0\) represent limits of the first and second species population, while \(a\) and \(it - c\) represent corresponding Malthusian parameters. It is very important that the following be pointed out. In distinction from \(a\) and \(it - c\) that represents real dynamical parameters, \(p_0\) and \(q_0\) do not represent real dynamical parameters but only normalization parameters. They do not represent any real ecologic dynamical interaction but express only proportion between variables and real numbers of the individuals in corresponding species. For this reason \(C_E\) does not any influence at given normalization parameters.

Now, apply time reversal \(T\) at usual Lotka-Volterra equations system (1),(2) that yields

\[
\frac{dp}{(-dt)} = ap - bpq \tag{18}
\]

\[
\frac{dq}{(-dt)} = -cq + dpq \tag{19}
\]

which, after simple transformations including transposition of the equations, becomes equivalent to system (7), (8) representing, as it has been shown, result of \(C_E\) application at the same equations system (1), (2).

Then, on the basis of the previous discussion of the characteristics of \(C_E\) transformation of usual Lotka-Volterra equations system (1), (2) it follows simply that \(T\) represents a symmetry transformation of the set of all usual Lotka-Volterra equations system (1), (2) (for all values of the parameters) and that, \(T\) does not represent a symmetry transformation of any concrete usual Lotka-Volterra equations system (1), (2) (for any concrete values of the parameters).

Also, on the same basis, it follows that successive application of \(C_E\) and \(T\) at any concrete usual Lotka-Volterra equations system (1), (2) (for any concrete values of the parameters) represents a symmetry, moreover the identity transformation of this system. Or, symbolically \(C_E T = I\).

In this way it is shown that usual Lotka-Volterra system (1), (2) is invariant (symmetric) in respect to \(C_E T\), moreover, \(C_E PT\) which means that it is invariant (symmetric) in respect to the successive application of \(T\), \(P\) and \(C_E\).

It is not hard to see that the equations system (10), (11), i.e. unlimited exponential population increase and decrease laws, as well as system (16), (17), i.e. limited exponential population increase and decrease laws or logistic, i.e. Verhulst equations, are, also, \(C_E T\), moreover, \(C_E PT\) invariant (symmetric).

Finally, consider a well-known [4]-[6] generalization of the Lotka-Volterra equations system

\[
\frac{dp_i}{dt} = a_i p_i + \sum_{ij} b_{ij} p_i p_j \quad \text{for} \quad i, j = 1, 2, \tag{20}
\]

where \(p_i\) represents corresponding real, time dependent variable and \(a_i\) and \(b_{ij}\) corresponding real constants for \(i, j = 1, 2\). Such system can describe a generalized ecological dynamics. It is not hard to see that given system is invariant (symmetric) in respect to \(C_E T\), moreover, \(C_E PT\) too.
So, it is shown that all simple ecological dynamical equations, definable at the "macroscopic" level of the analysis accuracy, are invariant (symmetric) in respect to $C_EPT$, or in respect to successive application of time reversal, parity and predator-prey reversal. This statement will be called ecological $C_EPT$ theorem.

3 Explicit breaking of $C_EPT$ symmetry by "microscopic" analysis

Now, it will be shown that $C_EPT$ symmetry of the simple ecological dynamical equations (that satisfy three important conditions) becomes explicitly broken at the level of a more detailed (accurate), "microscopic", analysis. For reason of the simplicity, it will be demonstrated explicitly for unlimited population growth and decrease dynamics only. But, as it is not hard to see, the same $C_EPT$ symmetry breaking by more detailed (accurate), "microscopic", analysis, is satisfied for all other simple ecological dynamical equations.

Consider a simple monosexual biological species without enemies, i.e. predators that uses unlimitedly environmental resources (food, territory, etc.). Suppose, firstly, that population, i.e. number of the individuals of given species, $p$, is sufficiently large so that approximation condition (3) is satisfied. Then "macroscopically" definable population dynamics is given by the expression (10) for pure (healthy) environment or (12) for impure (fatal) environment. As it has been discussed, from aspect of noted "macroscopic" accuracy of the analysis, set of all (10), (12) equations systems is $C_E$, $P$, $T$ and $C_EPT$ symmetric.

Suppose, however, that video camera records population dynamics with large, "microscopic", resolution (analysis accuracy). Precisely, suppose that given video camera does not observe characteristics of the environment but that it observes some characteristics of the individuals neglected by previous "macroscopic" resolution (analysis accuracy). It means that here approximation condition (3) is unsatisfied which causes that $p$ cannot represent a continuous time dependent variable so that dynamical equations (10) and (12) cannot be consistently applied.

For this reason, for "microscopic" description of the population dynamics, instead of the differential equations (10), (12) corresponding difference equations

\[
\frac{\Delta p_{n+1}}{\Delta t} = a p_n \quad \text{for} \quad n = 1,2, \tag{21}
\]

\[
\frac{\Delta p_{n+1}}{t} = -a p_n \quad \text{for} \quad n = 1,2, \tag{22}
\]

will be suggested which will be called correspondence condition (principle) (in sense that (10), (12) and (21), (22) have corresponding analogous forms). Here $\Delta t$ represents a finite time interval of the duration of one dynamical cycle, $p_n$ - natural number of the individuals at the beginning of $n$-th dynamical cycle, $p_{n+1}$ - natural number of the individuals at the beginning of $n + 1$-th dynamical cycle or at the end of $n$-th dynamical cycle, while a natural number

\[
\Delta p_{n+1} = p_{n+1} - p_n \tag{23}
\]

represents finite difference between population at the end and beginning of the $n$-th dynamical cycle, for $n = 1,2$. Also, according to the previous discussions, equation (21) refers on the positive influence of the environment on the species (pure environment), while equation (22) refers on the negative influence of the environment on the species (impure (fatal) environment).
Demand that population value, by ”microscopic” analysis, must be a natural number 1, 2, etc.
or eventually 0, represents a quite natural biological condition, called condition of the biological
discretization (discontinualization) of the population.

In equations (21), (22), theirs left hand-sides, representing change of the population during
one dynamical cycle, are proportional to the (initial) value of the population at the beginning
of this dynamical cycle. In some sense this ”retardation” represents a ”local” characteristic of
given population dynamics or locality condition. On the contrary, under a formal supposition that
change of the population during one dynamical cycle is proportional to the (final) value of the
population at the end of this dynamical cycle, this formal ”advance” would represent a ”nonlocal”
characteristic of such population dynamics or nonlocality condition.

It is very important that the following be pointed out. In distinction from ”macroscopic”
analysis, here, in ”microscopic” analysis, difference equations (21), (22) do not represent approxi-
mate form of the exact differential equations (10), (12). Here (21), (22) represent exact difference
equations that describe ecological dynamics since differential equations (10), (12) are definitely
inapplicable for ecological dynamics description.

Also, it can be supposed

$$a = \frac{k}{\Delta t}$$

(24)

where k represents some number whose meaning will be discussed later.

According to (24), (21) turns in

$$\frac{\Delta p_{n+1}}{\Delta t} = k \frac{p_n}{t}$$

(25)

and according to (23) in

$$p_{n+1} - p_n = kp_n$$

(26)

for n = 1,2, . Finally, it yields

$$p_{n+1} = (k + 1)p_n \quad for \quad n = 1,2,$$

(27)

Since both, $p_{n+1}$ and $p_n$, represent natural numbers for $n = 1,2,$, then $k + 1$ must represent a
natural number too. It means that $k$ can be 0 or any natural number.

For $k = 0$, or according to (24), $a = 0$, (27) yields

$$p_n = \text{const} \quad for \quad n = 1,2,$$

(28)

which means that here environment does not any influence at any individual of the species. In
this case we can speak on the neutral environment. But it represents ecologically trivial situation.

For $k > 0$, or, according to (24), $a > 0$, (27) defines an increasing geometric progression with
coefficient $k + 1$ greater than 1. It can be observed that here $k$ can be any natural number, i.e.
that there is no limit for $k$ value.

In this way, for $a > 0$ and $k > 0$, $\Delta t$ can consistently represent a finite time interval of
the individuals reproduction, or, time interval of the one reproduction cycle. Also, here $k$
consistently represent a natural number that denotes the additional number of the individuals in
respect to one individual that appear in one reproduction cycle. Or, here, it can be consistently
supposed (within an admitable idealization) that during one reproduction cycle there is splitting
of any individual (parent) in $k + 1$ individuals (descendants) where constant $k$ represents a natural
number.
According to (24), (22) turns in
\[
\frac{\Delta p_{n+1}}{\Delta t} = -\frac{k}{\Delta t} p_n
\]
and, according to (23), in
\[
p_{n+1} - p_n = -kp_n
\]
for \(n = 1, 2, \ldots\). Finally, it yields
\[
p_{n+1} = (1 - k)p_n \quad \text{for} \quad n = 1, 2, \ldots
\]
Since both, \(p_{n+1}\) and \(p_n\), represent natural numbers for \(n = 1, 2, \ldots\), then \(1 - k\) must represent a natural number too. It means that \(k\) can be 0 or 1. Namely, for \(k \neq 0, 1\) population becomes negative which has none ecological meaning, i.e. which contradicts to condition of the biological discretization of the population.

For \(k = 0\), or, according to (23), \(a = 0\), it follows
\[
p_n = \text{const} \quad \text{for} \quad n = 1, 2,
\]
which means that here we have again an ecologically trivial neutral environment.

For \(k = 1\), or, according to (23), \(|-a| = \frac{1}{\Delta t}\), it follows
\[
p_{n+1} = 0 \quad \text{for} \quad n = 1, 2, \quad \text{and for any} \quad p_1.
\]
It means that whole species, for any value of \(p_1\), die already during the first dynamical cycle and that \(\Delta t\) represents the life time of any individual. Or, ”microscopic” population dynamics for negative environmental influence needs that population (33) represents necessarily a step function of time whose initial value at the beginning of the first dynamical cycle \(p_1\) can be arbitrary natural number, but whose final value at the end of the first (and all other) dynamical cycle must be zero exclusively. Metaphorically speaking, environmental influence at given species is here absolutely fatal.

In this way it is shown that, in distinction from set of all ”macroscopic” population dynamics (10), (12), set of all ”microscopic” population dynamics (21)-(24) (based on the correspondence, biological discretization and locality condition) is \(C_E\) asymmetric. Or here change of \(a\) by \(-a\) in (21) does not yield (22), or, change of \(a\) by \(-a\) in (22) does not yield (21). Namely, here positive environmental influence admits that \(k\) can be any natural number, while negative environmental influence needs that \(k\) represents 1 exclusively.

Now, apply \(T\) at (21) or (25) for \(k > 0\). Practically it means that here \(n\) turns in some \(m\) while \(n + 1\) turns in some \(m - 1\) for \(n = 1, 2,\) and \(m = 2, 3, \ldots\). It yields
\[
-\frac{\Delta p_m}{\Delta t} = \frac{k}{\Delta t} p_m \quad \text{for} \quad m = 2,
\]
or
\[
\frac{\Delta p_m}{\Delta t} = -\frac{k}{\Delta t} p_m \quad \text{for} \quad n = 2,
\]
which yields
\[
p_{m-1} - p - m = kp_m \quad \text{for} \quad n = 1, 2,
\]
or, finally,
\[
p_m = \frac{p_{m-1}}{k + 1} \quad \text{for} \quad n = 1, 2,
\]
Obviously, (37) defines a decreasing geometric progression with coefficient $\frac{1}{k+1}$ smaller than 1.

Obtained geometric progression (37) represents numerically the inversion of the geometric progression (27) and vice versa.

Meanwhile, it is obvious that equation (35) does not satisfy locality condition but that it satisfies nonlocality condition. Concretely, change of the population during a cycle depends of the population at the end of this cycle. For this reason equation (35) represents neither the equation (29) nor equation (25) in which $k$ is changed by $-k$.

Moreover, a visual demonstration that $T$ applied on (35) yields neither (29) nor (25) in which $k$ is changed by $-k$ can be done. Namely, by time reversal, i.e. by video projector that does a play back backward, in this case, an observer can see the following. During $\Delta t$ it seems that $k+1$ initial individuals interact mutually which yields only one final individual. In other words it can seem that during $\Delta t$ one individual eats all $k$ other individuals, or, that within given biological species cannibalism appears.

Analogously, apply $T$ at (22) or (29) for $k=1$. It yields

$$\frac{-\Delta p_m}{\Delta t} = \frac{k}{\Delta t} p_m \quad for \quad m = 2,$$

or

$$\frac{\Delta p_m}{\Delta t} = \frac{k}{\Delta t} p_m \quad for \quad m = 2,$$

which yields

$$p_m - p_{m-1} = kp_m \quad for \quad m = 2,$$

or, finally,

$$(1-k)p_m = p_{m-1} \quad for \quad m = 2,$$

i.e., since $k=1$,

$$p_{m-1} = 0 \quad for \quad m = 2, \quad and \quad for \quad any \quad p_M$$

where $M$ represents maximal value of $m$. Obviously, according to (42), population at the beginning and end of any cycle (except at the end of the final cycle) must be 0, while population at the end of the final dynamical cycle can be any natural number. Mathematically, obtained time dependent population represents a discrete step function in some degree inverse to step function (33). But, ecologically, statement that population at the end of any cycle except the final, must be 0, while population at the end of the final dynamical cycle can be any natural number is meaningless. Namely, such statement means in fact that individuals of given species can be obtained by environment influence only without theirs parents.

Also, it is obvious that equation (39) does not satisfy locality condition but that it satisfies nonlocality condition. Concretely, change of the population during a cycle depends of the population at the end of this cycle. For this reason equation (39) represents neither the equation (25) nor equation (29) in which $k$ is changed by $-k$.

In this way it is shown that, in distinction from set of all "macroscopic" population dynamics (10), (12), set of all "microscopic" population dynamics (21)-(24) (based on the correspondence, biological discretization and locality condition) is $T$ asymmetric. Or here $T$ applied on (21) does not yield (22) and $T$ applied at (22) does not yield (21).

Now, on the basis of the previous discussions, it is not hard to see that by "microscopic" analysis $C_E T$ symmetry is broken, i.e. that neither (21) nor (22) is $C_E T$ invariant (symmetric). Simply
speaking here $T$ generates an ecologically inplausible nonlocality that cannot be trenaformed in ecologically plausible locality by $C_E$. Of course, even by ”microscopic” analysis (21) and (22) are $P$ invariant (symmetric), but neither (21) nor (22) is $C_EPT$ invariant (symmetric).

In this way it is shown that $C_EPT$ symmetry of the unlimited exponential population growth or decrease is explicitly broken by ”microscopic” analysis. Moreover, it is not hard to see that the same conclusion on the explicit breaking of the $C_EPT$ symmetry by ”microscopic” analysis can be done for all other simple population dynamical equations.

4 Conclusion

In conclusion the following can be repeated and pointed out. In ecology there is a $C_EPT$ symmetry theorem. It states that all simple ecological, i.e. population dynamical equations (unlimited exponential population growth (or decrease) dynamics, logistic or Verhulst equation, usual and generalized Lotka-Volterra equations ) are ”macroscopically” invariant (symmetric) in respect to successive application of the time reversal transformation, space coordinates reversal or parity transformation and predator-prey reversal transformation. However, such $C_EPT$ symmetry becomes explicitly broken by more accurate, ”microscopic” analysis.

5 References

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