Probing penguin coefficients with the lifetime ratio $\tau(B_s)/\tau(B_d)$

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Abstract

We calculate penguin contributions to the lifetime splitting between the $B_s$ and the $B_d$ meson. In the Standard Model the penguin effects are found to be opposite in sign, but of similar magnitude as the contributions of the current-current operators, despite of the smallness of the penguin coefficients. We predict

$$\frac{\tau(B_s)}{\tau(B_d)} - 1 = (-1.2 \pm 10.0) \cdot 10^{-3} \cdot \left( \frac{f_{B_s}}{190 \text{ MeV}} \right)^2,$$

where the error stems from hadronic uncertainties. Since penguin coefficients are sensitive to new physics and poorly tested experimentally, we analyze the possibility to extract them from a future precision measurement of $\tau(B_s)/\tau(B_d)$. Anticipating progress in the determination of the hadronic parameters $\varepsilon_1, \varepsilon_2$ and $f_{B_s}/f_{B_d}$ we find that the coefficient $C_4$ can be extracted with an uncertainty of order $|\Delta C_4| \simeq 0.1$ from the double ratio $(\tau(B_s) - \tau(B_d))/(\tau(B^+) - \tau(B_d))$, if $|\varepsilon_1 - \varepsilon_2|$ is not too small.
1. Introduction

The theoretical achievement of the Heavy Quark Expansion (HQE) [1] has helped a lot to understand the inclusive properties of B-mesons. The measurements of lifetime differences among the b-flavoured hadrons test the HQE at the order $(\Lambda_{QCD}/m_b)^3$. Today's experimental information on the B-meson lifetimes is in agreement with the expectations from the HQE, but the present theoretical predictions still depend on 4 poorly known hadronic parameters $B_1$, $B_2$, $\varepsilon_1$ and $\varepsilon_2$ [2,3]. Recently they have been obtained by QCD sum rules [4]. Lattice results are expected soon from the Rome group [5] and will allow for significantly improved theoretical predictions of the lifetime ratios.

Weak decays are triggered by a hamiltonian of the form

$$H = \frac{G_F}{\sqrt{2}} \left[ V_{\text{CKM}} \sum_{j=1}^{2} C_j Q_j - V'_{\text{CKM}} \left( \sum_{k=3}^{6} C_j Q_j + C_8 Q_8 \right) \right]. \quad (1)$$

Here $Q_1$ and $Q_2$ are the familiar current-current operators, $Q_3 \ldots Q_6$ are penguin operators and $Q_8$ is the chromomagnetic operator. Their precise definition is given below in (3). The factors $V_{\text{CKM}}$ and $V'_{\text{CKM}}$ represent the factors stemming from the Cabibbo-Kobayashi-Maskawa matrix and are specific to the flavour structure of the decay. Feynman diagrams in which the spectator quark participates in the weak decay amplitude induce differences among the various b-flavoured hadrons. Such non-spectator effects have been addressed first by Bigi et al. in [6] evaluating the matrix elements in the factorization approximation in which $Q$ here the deviation of $\varepsilon_1$ from unity has been estimated to be below 1% in [2,6] and the detailed analysis of Beneke, Buchalla and Dunietz [3]. Here $\tau(B_s)$ is the average lifetime of the two CP-eigenstates of $B_s$. Experimentally the ratio $\tau(B_s)/\tau(B_d)$ can also be addressed by the measurements of the corresponding semileptonic branching fractions. Since spectator effects in the semileptonic decay rate are negligible, one may use $\tau(B_s)/\tau(B_d) = B_{SL}(B_s)/B_{SL}(B_d)$.

So far only the effect of $Q_1$ and $Q_2$ has been considered in [2,3,6]. Taking into account the present experimental uncertainty and the fact that $C_1$ and $C_2$ are much larger than $C_{3-8}$ in the Standard Model this is justified. Yet once the lifetime ratio $\tau(B_s)/\tau(B_d)$ is measured to an accuracy of a few permille, the situation will change: The smallness of $|\tau(B_s)/\tau(B_d) - 1|$ is caused by the fact that the weak annihilation contribution of $Q_{1,2}$ depicted in Fig. 1 almost yields the same contribution to the decay rates of $B_s$ and $B_d$. The difference in the CKM-factors is negligible and the lifetime difference is induced by the small difference of the $(c, \tau)$ vs. $(c, \pi)$ phase space and by $SU(3)_F$ violations of the hadronic parameters. These effects suppress $|\tau(B_s)/\tau(B_d) - 1|$ by roughly an order of magnitude compared to $|\tau(B^+)/\tau(B_d) - 1|$. The contributions stemming from the penguin operators and the chromomagnetic operator, however, do not exhibit such a cancellation. Their contribution to the non-spectator rate of $B_s$ comes with the same power of the Wolfenstein parameter $\lambda = 0.22$ as the contribution of $Q_{1,2}$. In contrast the effects of $Q_{3-8}$ to the non-spectator rate of $B_d$ or $B^+$ are suppressed by two powers of $\lambda$ and are therefore negligible. Hence one expects the contributions of $Q_{3-6}$ and $Q_8$ to $|\tau(B_s)/\tau(B_d) - 1|$ to be of the
same order as those of $Q_1$ and $Q_2$, $\tau(B^+)/\tau(B_d)$ is not modified, so that the phenomenological conclusions drawn from this ratio in [2] are unchanged. Observables sensitive to $C_{3-8}$ like $\tau(B_s)/\tau(B_d)$ are phenomenologically highly welcome. The smallness of $C_{3-8}$ is a special feature of the helicity structure of the corresponding diagrams in the Standard Model. In many of its extensions the values of these coefficients can easily be much larger. Such an enhancement due to supersymmetric contributions has been discussed in [7]. Up to now the focus of the search for new physics has been on new contributions to $C_8$ [7]. Yet many interesting possible non-standard effects modify $C_{3-6}$ rather than $C_8$: New heavy particles mediating FCNC at tree-level or modifications of the $b$-$s$-$g$ chromoelectric formfactor affect $C_{3-6}$, but not $C_8$. Likewise new heavy coloured particles yield extra contributions to $C_{3-6}$, e.g. in supersymmetry box diagrams with gluinos modify $C_{3-6}$.

It is especially difficult to gain experimental information on the numerical values of the penguin coefficients $C_{3-6}$. Even penguin-induced decays to final states solely made of $d$ and $s$ quarks do not provide a clean environment to extract $C_{3-6}$: Any such decay also receives sizeable contributions from $Q_2$ via CKM-unsuppressed loop contributions [8, 9]. In exclusive decay rates these “charming penguins” preclude the clean extraction of the effects of penguin operators [8]. In semi-inclusive decay rates like $B \to X_s \Phi$ the situation is expected to be similar. In inclusive decay rates such as the total charmless $b$ decay rate the effect of “charming penguins” can be reliably calculated in perturbation theory. Yet these rates are much more sensitive to new physics contributions in $C_8$ rather than in $C_{3-6}$, because $Q_8$ triggers the two-body decay $b \to s g$, while the effects of $Q_{3-6}$ involve an integration over three-body phase space [9]. Notice from Fig. 2 and Fig. 3, however, that this phase space suppression of the terms involving $C_{3-6}$ is absent in the non-spectator diagrams inducing the lifetime differences.

This work is organized as follows: In the following section we calculate the contributions to $\tau(B_s)/\tau(B_d)$ involving $Q_{3-6}$ or $Q_8$. Here we also obtain the dominant part of the radiative corrections to order $\alpha_s$. In sect. 3 we discuss the phenomenological consequences within the Standard Model and with respect to a potential enhancement of $C_{3-8}$ by new physics.
2. Penguin Contributions

For the non-spectator contributions to the $B_s$ decay rate we need the $|\Delta B| = |\Delta S| = 1$-hamiltonian:

\[
H = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \left[ \sum_{j=1}^{6} C_j Q_j + C_8 Q_8 \right]
\]  

(2)

with

\[
Q_1 = (\bar{s}c)_{V-A} \cdot (\bar{c}b)_{V-A} \cdot \mathbb{I}, \quad Q_2 = (\bar{s}c)_{V-A} \cdot (\bar{c}b)_{V-A} \cdot \bar{\mathbb{I}}
\]

\[
Q_3 = \sum_{q=u,d,s,c,b} (\bar{s}b)_{V-A} \cdot (\bar{q}q)_{V-A} \cdot \mathbb{I}, \quad Q_4 = \sum_{q=u,d,s,c,b} (\bar{s}b)_{V-A} \cdot (\bar{q}q)_{V-A} \cdot \bar{\mathbb{I}}
\]

\[
Q_5 = \sum_{q=u,d,s,c,b} (\bar{s}b)_{V-A} \cdot (\bar{q}q)_{V+A} \cdot \mathbb{I}, \quad Q_6 = \sum_{q=u,d,s,c,b} (\bar{s}b)_{V-A} \cdot (\bar{q}q)_{V+A} \cdot \bar{\mathbb{I}}
\]

\[
Q_8 = -\frac{g}{8\pi^2} m_b \bar{s}\sigma^{\mu\nu} (1 + \gamma_5) T^a b \cdot G_{\mu\nu}^a.
\]  

(3)

The colour singlet and non-singlet structure are indicated by $\mathbb{I}$ and $\bar{\mathbb{I}}$ and $V \pm A$ is the Dirac structure. For more details see [9, 10]. In (2) we have set $V_{ub} V_{us}^* = O(\lambda^4)$ to zero. The diagram of Fig. 1 has been calculated in [2, 3] and yields contributions to the non-spectator part $\Gamma_{\text{non-spec}}$ of the $B_s$ decay rate proportional to $C_2^2, C_1 \cdot C_2$ and $C_1^2$. The result involves four hadronic matrix elements, which are parametrized by the B-factors $B_1, B_2, \varepsilon_1$ and $\varepsilon_2$ [2]:

\[
\begin{align*}
\langle B_s | \bar{s}\gamma_\mu (1 - \gamma_5) b T^\mu \gamma_\mu (1 - \gamma_5) s | B_s \rangle &= f_{B_s}^2 M_{B_s}^2 B_1, \\
\langle B_s | \bar{s} (1 + \gamma_5) b (1 - \gamma_5) s | B_s \rangle &= f_{B_s}^2 M_{B_s}^2 B_2, \\
\langle B_s | \bar{s}\gamma_\mu (1 - \gamma_5) T^a b \gamma_\mu (1 - \gamma_5) T^a s | B_s \rangle &= f_{B_s}^2 M_{B_s}^2 \varepsilon_1, \\
\langle B_s | \bar{s} (1 + \gamma_5) T^a b (1 - \gamma_5) T^a s | B_s \rangle &= f_{B_s}^2 M_{B_s}^2 \varepsilon_2.
\end{align*}
\] 

(4)

Here $T^a$ is the colour $SU(3)$ generator, $M_{B_s} = 5369 \pm 2$ MeV and $f_{B_s}$ are the mass and decay constant of the $B_s$ meson. $\tau(B_s)/\tau(B_d) - 1$ is proportional to $\Gamma_{\text{non-spec}}(B_d) - \Gamma_{\text{non-spec}}(B_s)$. The main differences between the result of Fig. 1 for these two rates are due to the different mass of $u$ and $c$ and the difference between $f_{B_d}$ and $f_{B_s}$. Hence the current-current parts of $\tau(B_s)/\tau(B_d) - 1$ proportional to $C_2^2, C_1 \cdot C_2$ or $C_1^2$ are suppressed by a factor of $z$ or $\Delta$ with

\[
z = \frac{m_c^2}{m_b^2} = 0.085 \pm 0.023, \quad \Delta = 1 - \frac{f_{B_s}^2 M_{B_s}}{f_{B_d}^2 M_{B_d}} = 0.23 \pm 0.11.
\] 

(5)

The result for $\Delta$ in (5) is the present world average of lattice calculations [11]. There are also $SU(3)_F$ violations in the B-factors, but they are expected to be small from the experience with those appearing in $B^0 - \bar{B}^0$-mixing. We want to achieve an accuracy of 2 permille in our prediction for $\tau(B_s)/\tau(B_d)$, which corresponds to an accuracy of 20-30% in $\tau(B_s)/\tau(B_d) - 1$. Therefore we use the same $B_1, B_2, \varepsilon_1$ and $\varepsilon_2$ in $\tau(B_s)$ and $\tau(B_d)$. Likewise there is $SU(3)_F$-breaking in the matrix elements of the b-quark kinetic energy operator and the chromomagnetic...
Figure 3: Contribution of $Q_8$ to $\Gamma_{\text{non-spec}}(B_s)$. In the Standard Model the diagram is of the same order of magnitude as radiative corrections to Fig. 2 and therefore negligible. Yet in models in which quark helicity flips occur in flavour-changing vertices $|C_8|$ can easily be ten times larger than in the Standard Model [7]. The contribution of $Q_1$ vanishes.

Figure 4: Penguin diagram contribution to $\Gamma_{\text{non-spec}}(B_s)$. The final state corresponds to a cut through either of the $(\bar{c}, c)$-loops. The contributions of $Q_1$ vanish by colour. This is the only NLO contribution to $\tau(B_s)/\tau(B_d) - 1$ involving $Q_{1,2}$ without suppression factors of $\Delta$ or $z$.

moment operator. These effects are suppressed by a factor of $m_b/(\Lambda_{QCD} \cdot 16\pi^2)$ with respect to those discussed above. In [3] they have been estimated from heavy meson spectroscopy to be an effect of order one permille in $\tau(B_s)/\tau(B_d)$.

We are now interested in the diagram of Fig. 2 involving one large coefficient $C_{1,2}$ and one small penguin coefficient $C_{3-6}$. Diagrams with two insertions of penguin operators yield smaller contributions proportional to $C_{2}^{2}C_{3-6}$ and are neglected here. To order $\lambda^2$ in $H$ we have $V'_{\text{CKM}} = 0$ in (1) for the $B_d$ system and penguin effects are only relevant in $\tau(B_s)$. Hence the penguin contributions to $\tau(B_s)/\tau(B_d) - 1$ do not suffer from the suppression factors $z$ and $\Delta$. Next we want to evaluate the diagram of Fig. 3 which encodes the interference of $Q_{1,2}$ with the chromagnetic operator $Q_8$. This part of $\Gamma_{\text{non-spec}}$ already belongs to the order $\alpha_s$ and is small in the Standard Model, but it can be sizeable in the new physics scenarios discussed in [7].

We also must discuss radiative corrections to the contributions involving the large coefficients $C_1$ and $C_2$. Dressing the diagram in Fig. 1 with gluons gives contributions to $\Gamma_{\text{non-spec}}$ for both $B_d$ and $B_s$ and therefore yield small corrections of order $C_2^2\Delta\alpha_s/\pi$ or less. The penguin diagram of Fig. 4, however, contributes only to $\Gamma_{\text{non-spec}}(B_s)$ in the order $\lambda^4$. Hence Fig. 4 yields an unsuppressed contribution of order $C_2^2\alpha_s/\pi$ to $\tau(B_s)/\tau(B_d) - 1$ and cannot be neglected. The result of these penguin loop diagrams can easily be absorbed into the penguin coefficients $C_{3-6}$:

In the result of the diagram of Fig. 2 one must simply replace $C_j$ by

$$C_j' = C_j^{\text{NLO}} + \frac{\alpha_s}{4\pi}C_2\text{Re} \left[ r_{2j} \left( 1, \sqrt{z}, \mu/m_b \right) \right], \quad j = 3, \ldots, 6. \quad (6)$$

Here $r_{2j}$ encodes the result of the penguin diagram and can be found in [9] in the NDR scheme. To cancel the scheme dependence of $r$ we must also include the next-to-leading order (NLO) corrections to $C_j$ as indicated in (6). More precisely: We must include the NLO mixing of $C_2$ into $C_j$ in $C_j^{\text{NLO}}, j = 3, \ldots, 6$, but the penguin-penguin mixing only to the LO. The difference between these partial NLO-coefficients, which are tabulated in [9], and the full $C_j^{\text{NLO}}$'s has a
negligible impact on our result. Here we bypass this technical aspect of scheme independence by tabulating the $C_j''$'s in Tab. 1.

Our result for the non-spectator part of the $B_s$ decay rate reads:

$$
\Gamma_{\text{non-spec}}(B_s) = -\frac{G_F^2 m_b^2}{12\pi} |V_{cb} V_{cs}|^2 \sqrt{1 - 4 z f_{B_s}^2 M_{B_s}} [a_1 \varepsilon_1 + a_2 \varepsilon_2 + b_1 B_1 + b_2 B_2] \quad (7)
$$

with

$$
a_1 = [2 C_2^2 + 4 C_2 C_4'] [1 - z] + 12 z C_2 C_6 + [1 + 2 z] \frac{\alpha_s}{\pi} C_2 C_8
$$

$$
a_2 = -[1 + 2 z] \left[ 2 C_2^2 + 4 C_2 C_4' + \frac{\alpha_s}{\pi} C_2 C_8 \right]
$$

$$
b_1 = [C_2 + N_c C_1] \left\{ (1 - z) \left[ \frac{C_2}{N_c} + C_1 + 2 C_3' + 2 \frac{C_4'}{N_c} \right] + 6 z \left[ C_5' + \frac{C_6}{N_c} \right] \right\}
$$

$$
b_2 = -[1 + 2 z] [C_2 + N_c C_1] \left\{ \frac{1}{N_c} [C_2 + N_c C_1] + 2 \left[ C_3' + \frac{C_4'}{N_c} \right] \right\} \quad (8)
$$

Here $N_c = 3$ is the number of colours. By setting $C_j', j = 3, \ldots, 6, \text{ and } C_8$ in (8) to zero one recovers the result of [2].\(^3\) The result for the non-spectator contributions to the $B_d$ decay rate reads [2]:

$$
\Gamma_{\text{non-spec}}(B_d) = \frac{G_F^2 m_b^2}{12\pi} |V_{ud}|^2 (1 - z)^2 f_{B_d}^2 M_{B_d} (\Delta - 1) \left[ a_1^d \varepsilon_1 + a_2^d \varepsilon_2 + b_1^d B_1 + b_2^d B_2 \right] \quad (9)
$$

with\(^4\)

$$
a_1^d = 2 C_2^2 \left( 1 + \frac{z}{2} \right), \quad a_2^d = -2 C_2^2 (1 + 2 z),
$$

$$
b_1^d = \frac{1}{N_c} (C_2 + N_c C_1)^2 \left( 1 + \frac{z}{2} \right), \quad b_2^d = -\frac{1}{N_c} (C_2 + N_c C_1)^2 (1 + 2 z) \quad (10)
$$

---

\(^3\) Notice that our notation of $C_1$ and $C_2$ is opposite to the one in [2].

\(^4\) In the large $N_c$ limit one finds $\Gamma_{\text{non-spec}}$ helicity suppressed in analogy to the leptonic decay rate. This shows that one cannot neglect the $O(1/N_c)$ terms.
When we combine (7-10) in order to predict $\tau(B_s)/\tau(B_d) - 1$:

$$\frac{\tau(B_s)}{\tau(B_d)} - 1 = \frac{\Gamma^{\text{non-spec}}(B_d) - \Gamma^{\text{non-spec}}(B_s)}{\Gamma^{\text{total}}} + O(10^{-3})$$

$$= K(z) \cdot \left\{ \Delta \left[ 2C_2^2 (\varepsilon_1 - \varepsilon_2) + \frac{(C_2 + N_c C_1)^2}{N_c} (B_1 - B_2) \right] - 3C_2^2 z\varepsilon_1 - \frac{3}{2} \frac{(C_2 + N_c C_1)^2}{N_c} zB_1 ight.$$  

$$+ \Delta z \left[ C_2^2 (\varepsilon_1 - 4\varepsilon_2) + \frac{(C_2 + N_c C_1)^2}{2N_c} (B_1 - 4B_2) \right]$$

$$+ \left[ 4C_2 C_4' + (1 + 2z) \frac{\alpha_s}{\pi} C_2 C_8 \right] (\varepsilon_1 - \varepsilon_2)$$

$$+ 2 \left( C_2 + N_c C_1 \right) \left( C_3' + \frac{C_1'}{N_c} \right) (B_1 - B_2)$$

$$- 4z C_2 C_4' (\varepsilon_1 + 2\varepsilon_2) + 12z C_2 C_6' \varepsilon_1$$

$$+ 2z \left( C_2 + N_c C_1 \right) \left[ -\left( C_3' + \frac{C_1'}{N_c} \right) (B_1 + 2B_2) + 3 \left( C_5' + \frac{C_6'}{N_c} \right) B_1 \right] \right\} + O(2 \cdot 10^{-3})$$

Here $K(z)$ reads

$$K(z) = \frac{16\pi^2 |V_{ud}|^2 B_{SL}}{m_b^3 f_1(z) \left[ 1 + \alpha_s(\mu)/(2\pi) h_{SL}(\sqrt{z}) \right]} f_{B_s}^2 M_{B_s} \left[ 1 - 2z \right]$$  

$$\simeq \frac{0.060}{1 - 4 \left( \sqrt{z} - 0.3 \right)} \frac{B_{SL}}{0.105 \frac{(4.8)}{m_b}^3 \left( \frac{f_{B_s}}{190 \text{ MeV}} \right)}.$$

In (12) we have used the common trick to evaluate the total width $\Gamma^{\text{total}}$ in terms of the semileptonic rate and the measured semileptonic branching ration $B_{SL}$ via $\Gamma^{\text{total}} = \Gamma_{SL}/B_{SL}$. $f_1$ and $h_{SL}$ are the phase space and QCD correction factor of $\Gamma_{SL}$ calculated in [12]. We use the notation of [9]. The approximation in (13) reproduces $K(z)$ to an accuracy of 3%. The numerical value of $h_{SL}$ entering (13) corresponds to the use of the one-loop pole mass ($\simeq 4.8$ GeV) for $m_b$. For simplicity we have expanded $K(z)$ and the terms in the curly braces in (11) up to the first order in $z$. The size of the error in (11) is estimated as $2 \cdot 10^{-3}$. Its main source is the $SU(3)_F$-breaking in the kinetic energy and chromomagnetic moment matrix elements appearing at order $\Lambda_{QCD}^2/m_b^2$ of the HQE, which has been calculated to equal $(0-1) \cdot 10^{-3}$ in [3]. Then terms of order $16\pi^2 \Lambda_{QCD}^4/m_b^4$ can maximally be of the same order of magnitude. Conversely the remaining NLO correction of order $C_2^2 \Delta \alpha_s/\pi$ and the CKM-suppressed contributions are much smaller. Likewise the $SU(3)_F$-breaking in $\varepsilon_1, \varepsilon_2, B_1$ and $B_2$ is expected to be at the level of a few percent and therefore smaller than the present uncertainty in $\Delta$.

The first three lines (11a-11c) contain the result of the current-current operators calculated in [2, 3]. The remaining lines comprise the penguin effects. Note that the terms in (11d-11e) are
neither suppressed by $\Delta$ nor by $z$. For $z = 0$ the hadronic parameters in (11) only appear in the combinations $\varepsilon_1 - \varepsilon_2$ and $B_1 - B_2$, both of which are of order $1/N_c$. The coefficients of $B_1 - B_2$ suffer from numerical cancellations, e.g. $0.09 \leq C_2 + 3C_1 \leq 0.57$ (cf. Tab. 1), so that for most values of the input parameters only the terms involving $\varepsilon_1$ and $\varepsilon_2$ in (11a), (11b) and (11d) are important.

Finally we discuss a potential systematic uncertainty: The derivation of (11) has assumed quark-hadron duality (QHD) for the sum over the final states. QHD means that inclusive observables are unaffected by the hadronization process of the quarks and gluons in the final state. The new results for inclusive observables in $B$ decays presented at the 1997 summer conferences are consistent with QHD [13]. There are two potential sources of QHD violation in our problem: First it may be possible that the spectator decay rate of the $b$-quark is affected by the hadronization process. Yet the ballpark of this effect is independent of the flavour of the spectator quark and cancels out in the ratio $\tau(B_s)/\tau(B_d)$. $SU(3)_F$-breaking can only appear in the hadronization of the final state antiquark which picks up the spectator quark and we do not expect the $SU(3)_F$-breaking in the spectator decay rate to be larger than the $SU(3)_F$-breaking in the $(\Lambda_{QCD}/m_b)^2$-terms of the HQE. This effect should further not depend on whether the hadron containing the spectator quark recoils against other hadrons or against a lepton pair. Hence one can control the $SU(3)_F$-breaking in the spectator decay rate by comparing the hadron energy in semileptonic $B_d$ and $B_s$ decays. More serious is a potential violation of QHD in the non-spectator contribution $\Gamma_{\text{non-spec}}$ itself. In a theoretical analysis for the similar case of the width difference $\Delta\Gamma_{B_s}$ of the two $B_s$ eigenstates the size of QHD violation has been estimated to be moderate, maximally of order 30%. We can incorporate this into (11) by assigning an additional error of $\pm 0.3$ to $\Delta$. In any case the issue of QHD violation in lifetime differences will be experimentally tested in the forthcoming years, when high precision measurements of $\tau(B^+)/\tau(B_d)$ and of $\Delta\Gamma_{B_s}$ are confronted with accurate lattice results for the hadronic parameters.

3. Phenomenology

In the following we want to investigate the numerical importance of the penguin contribution. Then we analyze which accuracy is necessary to detect or constrain new physics contributions to $C_{3-6}$ by a precision measurement of $\tau(B_s)/\tau(B_d)$.

The three main hadronic parameters entering (11) are $f_{B_s}$ and $\varepsilon_1 - \varepsilon_2$, while $B_1$ and $B_2$ come with small coefficients. The canonical sizes of the B-factors are $\varepsilon_i = O(1/N_c)$ and $B_i = 1 + O(1/N_c)$. An important constraint on the $\varepsilon_i$’s is given by the measured value of $\tau(B^+)/\tau(B_d)$ [2]. The result of [2] for $\Gamma_{\text{non-spec}}(B^+)$ is obtained from (9) by replacing the $a^d_i$, $b^d_i$’s with

$$a^u_1 = -6 \left( C_1^2 + C_2^2 \right), \quad b^u_1 = -\frac{3}{N_c} \left( C_2 + N_c C_1 \right)^2 + 3N_c C_1^2, \quad a^u_2 = b^u_2 = 0. \quad (14)$$

The experimental world average [14]

$$\frac{\tau(B^+)}{\tau(B_d)} = 1.07 \pm 0.04 \quad (15)$$
There is an overall error of \( \pm 2.0 \) (see 11) for all entries.

Table 2: Standard Model prediction for \( 10^3 \cdot \frac{\tau(B_s)}{\tau(B_d)} - 1 \) obtained from (11) for \( f_{B_s} = 190 \text{ MeV}, \mu = m_b = 4.8 \text{ GeV}, z = 0.085, \alpha_s(M_Z) = 0.118 \) and \( B_1 = B_2 = 1 \). The entries marked with * are in conflict with the experimental constraint (15), which also implies \( \varepsilon_1 \lesssim 0 \).

Table 3: The columns labeled with ‘peng’ list the penguin contribution to \( 10^3 \cdot \frac{\tau(B_s)}{\tau(B_d)} - 1 \) as a function of \( \varepsilon_1 - \varepsilon_2 \) and \( f_{B_s} \). The other input parameters have little impact on the size of the penguin contribution. The current-current part of \( 10^3 \cdot \frac{\tau(B_s)}{\tau(B_d)} - 1 \) is listed for \( \varepsilon_1 = -0.1 \) and \( \Delta = 0.23 \). For the remaining parameters see Tab. 2.

leads to the following constraint:

\[
\varepsilon_1 \simeq (-0.2 \pm 0.1) \left( \frac{0.17 \text{ GeV}}{f_B} \right)^2 \left( \frac{m_b}{4.8 \text{ GeV}} \right)^3 + 0.3 \varepsilon_2 + 0.05. \tag{16}
\]

In [4] the \( \varepsilon_i \)'s and \( B_i \)'s have been calculated with QCD sum rules within the heavy quark effective theory (HQET). The results are \( \varepsilon_1(\mu = m_b) = -0.08 \pm 0.02 \) and \( \varepsilon_2(\mu = m_b) = -0.01 \pm 0.03 \) and \( B_{1,2} = 1 + O(0.01) \). In view of the smallness of the \( \varepsilon_i \)'s, however, it is conceivable that other neglected effects are numerically relevant. For example a NLO calculation of the matching between HQET and full QCD amplitudes replaces \( \varepsilon_i \) in (7) and (9) by \( \varepsilon_i + d_i B_i \), where \( d_i \) is a coefficient of order \( \alpha_s(m_b)/\pi \). Here we will consider the range \( |\varepsilon_1|, |\varepsilon_2| \leq 0.3 \), and further obey (15).

In Tab. 2 we have tabulated \( \tau(B_s)/\tau(B_d) - 1 \) for various values of \( \Delta \) and \( \varepsilon_1, \varepsilon_2 \). We have further split \( \tau(B_s)/\tau(B_d) - 1 \) into its current-current part consisting of (11a-11c) and the new penguin part involving \( C_{3-6}^d, C_8 \). These results can be found in Tab. 3. From Tab. 3 we realize that the penguin contributions calculated in this work are comparable in size, but opposite in sign to the current-current part obtained in [3]. This makes the experimental detection of any deviation of \( \tau(B_s)/\tau(B_d) \) from 1 even more difficult, if the penguin coefficients are really dominated by
Standard Model physics. The results of Tab. 2 can be summarized as

\[
\frac{\tau(B_s)}{\tau(B_d)} - 1 = (-1.2 \pm 8.0 \pm 2.0) \cdot 10^{-3} \cdot \left( \frac{f_{B_s}}{190 \text{ MeV}} \right)^2 \left( \frac{4.8 \text{ GeV}}{m_b} \right)^3. \tag{17}
\]

Here the first error stems from the uncertainty in \(\varepsilon_1\) and \(\varepsilon_2\) and will be reduced once lattice results for the hadronic parameters are available. The second error summarizes the remaining uncertainties. If \(\Delta\) and \(\varepsilon_2\) simultaneously acquire extreme values, \(\tau(B_s)/\tau(B_d) - 1\) can be slightly outside the range in (17) (see Tab. 2).

Today we have little experimental information on the sizes of the penguin coefficients. Their smallness in the Standard Model allows for the possibility that they are dominated by new physics. The total charmless inclusive branching fraction \(\text{Br}(B \to \text{no charm})\) is a candidate to detect new physics contributions to \(C_8\) [7], but it is much less sensitive to \(C_{3-6}\) [9]. The decreasing experimental upper bounds on \(\text{Br}(B \to \text{no charm})\) [14] therefore constrain \(C_8\) but leave room for a sizeable enhancement of \(C_{3-6}\). Now (11) reveals that \(\tau(B_s)/\tau(B_d)\) is a complementary observable mainly sensitive to \(C_4\), while \(C_8\) is of minor importance. As mentioned in the introduction, many interesting new physics scenarios affect \(C_{3-6}\), but not necessarily \(C_8\).

We remark here that we constrain ourselves to new physics scenarios, in which the CKM factors of the new contributions are the same as the ones of the Standard Model. This is fulfilled to a good approximation in most interesting models [7]. Now any new physics effect modifies (11) at some high scale of the order of the new particle masses, while the Wilson coefficients entering (11) are evaluated at a low scale \(\mu \approx m_b\). The renormalization group evolution down to \(\mu \approx m_b\) mixes the new contributions to \(C_{3-6}\). New physics contributions \(\Delta C_{3-6}(\mu = 200\text{GeV})\) affect \(C_4(\mu = 4.8\text{GeV})\) by

\[
\Delta C_4(\mu = 4.8\text{GeV}) = -0.35 \Delta C_3(200\text{GeV}) + 0.99 \Delta C_4(200\text{GeV})
- 0.03 \Delta C_5(200\text{GeV}) - 0.22 \Delta C_6(200\text{GeV}).
\]

Observe that \(\Delta C_4(200\text{GeV}) = -0.05\) already increases \(C_4'(m_b)\) by more than a factor of two. Clearly the usefulness of \(\tau(B_s)/\tau(B_d)\) to probe \(C_{3-6}\) crucially depends on the size of \(|\varepsilon_1 - \varepsilon_2|\) and \(f_{B_s}\). We now investigate the sensitivity of \(\tau(B_s)/\tau(B_d)\) to \(\Delta C_4(\mu = m_b)\) in a possible future scenario for the hadronic parameters. We assume

\[
\varepsilon_1 = -0.10 \pm 0.05, \quad \varepsilon_2 = 0.20 \pm 0.05, \quad B_1, B_2 = 1.0 \pm 0.1, \\
f_{B_s} = (190 \pm 15)\text{ GeV}, \quad \Delta = 0.23 \pm 0.05, \quad m_b = (4.8 \pm 0.1)\text{ GeV}. \tag{18}
\]

The assumed accuracy for \(f_{B_s}\) will be achieved, once more experimental information on the \(B_s\) system is obtained, e.g. after the detection of \(B_s - \overline{B}_s\)-mixing. Also a more precise measurement of \(f_{D_s}\) is helpful, because lattice QCD predicts the ratio \(f_{B_s}/f_{D_s}\) much better than \(f_{B_s}\) [11]. The error bars of the other hadronic parameters likewise appear within reach, if one keeps in mind that information on \(\varepsilon_1\) and \(\varepsilon_2\) will not only be obtained from the lattice but also from other observables like \(\tau(B^+)/\tau(B_d)\). Experimental progress in (15) and a next-to-leading order calculation of the coefficients in (14) and (10) will significantly improve the constraint in (16). In Fig. 5 we show
the dependence of $\tau(B_s)/\tau(B_d) - 1$ on $\Delta C_4(\mu)$ for the scenario in (18). A cleaner observable is the double ratio

$$\frac{\tau(B_s) - \tau(B_d)}{\tau(B^+) - \tau(B_d)} = \frac{B_{SL}(B_s) - B_{SL}(B_d)}{B_{SL}(B^+) - B_{SL}(B_d)},$$

which depends on $\varepsilon_1, \varepsilon_2$ and $\Delta$, while the dependence on $f_B$ and $m_b$ cancels. The corresponding plot for the parameter set of (18) can be found in Fig. 6.

We find a smaller error band for $(\tau(B_s) - \tau(B_d))/(\tau(B^+) - \tau(B_d))$ than for $\tau(B_s)/\tau(B_d) - 1$. If $\Delta C_4 < -0.075$ or $\Delta C_4 > 0.140$, we find the allowed range for $(\tau(B_s) - \tau(B_d))/(\tau(B^+) - \tau(B_d))$ incompatible with the Standard Model. An experimental lower bound $\tau(B_s)/\tau(B_d) > 1.005$ would indicate a new physics contribution $\Delta C_4 < -0.063$ in our scenario. Likewise the experimental detection of a sizeable negative lifetime difference $\tau(B_s) - \tau(B_d)$ may reveal non-standard contributions to $C'_4$ of similar size as its Standard Model value. Fig. 6 shows that e.g.
4. Conclusions

We have calculated the contributions of the penguin operators $Q_{3-6}$, of the chromomagnetic operator $Q_8$ and of penguin diagrams with insertions of $Q_2$ to the lifetime splitting between the $B_s$ and $B_d$ meson. In the Standard Model the penguin effects are found to be roughly half as big as the contributions from the current-current operators $Q_1$ and $Q_2$, despite of the smallness of the penguin coefficients. Yet they are opposite in sign, so that any deviation of $\tau(B_s) - \tau(B_d)$ from zero is even harder to detect experimentally. Assuming a reasonable progress in the determination of the hadronic parameters a precision measurement of $\tau(B_s)/\tau(B_d)$ can be used to probe the

Figure 6: Dependence of $(\tau(B_s) - \tau(B_d))/((\tau(B^+) - \tau(B_d))$ on $\Delta C_4$ for the parameter set in (18). This double ratio depends on $f_{B_s}$ and $f_{B_d}$ only through $\Delta$, and the factor of $m_b^{-3}$ in (11) cancels.

the bound $\tau(B_s) - \tau(B_d) < -0.20(\tau(B^+) - \tau(B_d))$ would indicate $\Delta C_4 > 0.051$. We conclude that the detection of new physics contributions to $C_4$ of order 0.1 is possible with precision measurements of $\tau(B_s)/\tau(B_d)$. 
coefficient $C_4$ with an accuracy of $|\Delta C_4| = 0.1$. Hence new physics can only be detected, if $C_4$ is dominated by non-standard contributions. The sensitivity to $C_4$ depends crucially on the difference of the hadronic parameters $\varepsilon_1$ and $\varepsilon_2$. For the extraction of $C_4$ the double ratio \((\tau(B_s) - \tau(B_d))/(\tau(B^+) - \tau(B_d))\) turns out to be more useful than $\tau(B_s)/\tau(B_d)$.

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