The \( \omega \) meson (\( \omega \)) is one of the most important particles for describing the nuclear force between two nucleons (\( Ns \)), giving a short-ranged repulsive central force together with a strong spin-orbit force. Nevertheless, scattering between \( \omega \) and \( N \) has not been well established. The structure of hadrons and dynamical hadron-mass generation are the most important subjects to be studied in the non-perturbative domain of quantum chromodynamics (QCD). Detailed information on \( \omega N \) scattering would not only reveal highly excited nucleon resonances (\( N^* \)) but also provide modification of the \( \omega \) properties in the nuclear medium.

The low-energy \( \omega N \) scattering is characterized by the scattering length \( a_{\omega N} \) and effective range \( r_{\omega N} \) through an effective-range expansion of the \( S \)-wave phase shift \( \delta(p) \):

\[
p \cot \delta(p) = \frac{1}{a_{\omega N}} + \frac{1}{2} r_{\omega N} p^2 + O(p^4),
\]

where \( p \) denotes the momentum of \( \omega \) in the \( \omega N \) center-of-mass (CM) frame. In this definition, a positive (negative) \( a_{\omega N} \) gives attraction (repulsion). Recently, the A2 collaboration at the Mainz MAMI accelerator facility has reported \( |a_{\omega N}| = 0.82 \pm 0.03 \text{ fm} \), which is extracted from \( \omega \) photoproduction on the proton (\( \gamma p \to \omega p \)) near the threshold assuming a vector meson dominance (VMD) model. The obtained value is a combination of two independent \( S \)-wave scattering lengths with total spins of 1/2 and 3/2. The unknown sign of \( a_{\omega N} \) leaves the naive question of whether low-energy \( \omega N \) scattering is repulsive or attractive.

Theoretically estimated values of \( a_{\omega N} \) are scattered in a wide range from attractive to repulsive ones. The effective Lagrangian approach based on chiral symmetry gives an attractive value of \( a_{\omega N} = +1.6 + i0.30 \text{ fm} \). A QCD sum-rule analysis provides a weakly attractive value of \( a_{\omega N} = +0.41 \pm 0.05 \text{ fm} \). The coupled-channel unitary approach gives repulsive values of \( a_{\omega N}^{(1/2)} = -0.45 + i0.31 \text{ fm} \) and \( a_{\omega N}^{(3/2)} = -0.43 + i0.15 \text{ fm} \) for the two total spins, giving a spin-averaged value of \( a_{\omega N} = -0.44 + i0.20 \text{ fm} \). The coupled-channel analysis of \( \omega \) production in pion and photo-induced reactions gives a very weakly repulsive value of \( a_{\omega N} = -0.026 + i0.28 \text{ fm} \). The dynamical coupled-channel analysis resulted in \( a_{\omega N}^{(1/2)} = 0.0454 + i0.0695 \text{ fm} \) and \( a_{\omega N}^{(3/2)} = -0.180 + i0.0597 \text{ fm} \), giving a repulsive spin-averaged value of \( a_{\omega N} = -0.135 + i0.0630 \text{ fm} \). Neither the coupled-channel analyses nor the VMD analysis by the A2 collaboration incorporates the finite width of \( \omega \) in the final state.

To determine the low-energy \( \omega N \) scattering parameters \( a_{\omega N} \) and \( r_{\omega N} \) experimentally, we investigate the

\[1\] We adopt \( a_{\omega N} = (\frac{1}{3})a_{\omega N}^{(1/2)} + (\frac{2}{3})a_{\omega N}^{(3/2)} \) for the spin average using the convention of Lutz et al. [11].
\( \gamma p \rightarrow \omega p \) reaction very close to the reaction threshold. Several collaborations have already measured the total cross sections near the threshold using the \( \omega \rightarrow \pi^+\pi^-\pi^0 \) decay mode (SAPHIR \[3\] and CLAS \[3\] collaboration), and the \( \omega \rightarrow \pi^0\gamma \) decay mode (CBELSA/TAPS \[10\] and A2 \[3\] collaborations). Currently, the data points for the total cross section near the threshold (\( E_\gamma \lesssim 1.2 \) GeV), where the S-wave \( \omega N \) contribution is dominant, are not enough for determining \( a_{\omega N} \) and \( r_{\omega N} \) from the shape of the total cross section as a function of the incident energy (excitation function) through \( \omega N \) rescattering in the final-state interaction. We have measured ten data points of the total cross section at incident photon energies ranging from 1.09 to 1.15 GeV. The \( \omega \) meson mainly decays in the \( \omega \rightarrow \pi^0\pi^+\pi^- \) mode with a branching ratio of 89.2\% \[11\]. It is, however, difficult to reproduce the background shapes in the \( \pi^0\pi^+\pi^- \) invariant mass distributions measured with poor identification for charged particles \[12\]. Thus, we determined the cross sections using the \( \omega \rightarrow \pi^0\gamma \) decay mode with a branching ratio of 8.40\%. In this letter, we present \( a_{\omega N} \) and \( r_{\omega N} \) extracted from the shape of the excitation function for the \( \gamma p \rightarrow \omega p \) reaction.

A series of meson photoproduction experiments were conducted \[13\] using the FOREST detector \[14\], which was installed on the second photon beamline \[15\] at the Research Center for Electron Photon Science (ELPH), Tohoku University, Japan. In the present experiments, bremsstrahlung photons were produced from 1.2-GeV circulating electrons in a synchrotron \[16\] by inserting a carbon thread (radiator) \[17\]. The photons collimated with two lead apertures of 10 and 25 mm in diameter located 4.2- and 12.9-m downstream from the radiator, respectively, were incident on a 45-mm thick liquid-hydrogen target located at the center of FOREST. The energies of the incident photons were analyzed up to 1.15 GeV by detecting the post-bremsstrahlung electrons with a photon-tagging counter, STB-Tagger II \[17\]. FOREST consists of three different electromagnetic calorimeters (EMCs): 192 undoped CsI crystals, 252 lead scintillating-fiber modules, and 62 lead glasses. A plastic-scintillator hodoscope (PSH) is placed in front of each EMC to identify charged particles. FOREST covers a solid angle of \( \sim 88\% \) in total. The typical photon-tagging rate was 20 MHz, and the photon transmittance (the so-called tagging efficiency) was \( \sim 53\% \) \[13\]. The trigger condition of the data acquisition (DAQ), which required for an event to have more than one final-state particles in coincidence with a photon-tagging signal \[14\], was the same as that in Ref. \[17\]. The total number of collected events was \( 1.79 \times 10^6 \). The average trigger rate was 1.6 kHz, and the average DAQ efficiency was 80\%.

Event selection is made for the \( \gamma p \rightarrow \pi^0\gamma p \rightarrow \gamma\gamma\gamma p \) reaction. At first, events containing three neutral particles and a charged particle are selected. The time difference between every 2 neutral EMC clusters out of 3 is required to be less than thrice that of the time resolution for the difference. The two neutral EMC clusters giving the \( \gamma\gamma \) invariant mass ranging from 50 to 220 MeV are selected, and the other EMC cluster is required to have an energy higher than 200 MeV. The charged particles are detected with the forward PSH. Further selection is made by applying a kinematic fit with five constraints: energy and three-momentum conservation, and \( \gamma\gamma \) invariant mass being the \( \pi^0 \) mass. The momentum of the charged particle is obtained from the time delay assuming that the charged particle has proton mass. Events for which the \( \chi^2 \) probability is higher than 0.1 are selected. When the number of combinations is more than 1 in an event, the combination with the minimum \( \chi^2 \) is adopted. Sideband-background subtraction is performed for accidental-coincidence events detected in STB-Tagger II and FOREST.

All the data for incident energies above 1.09 GeV (\( E_\gamma = 1.09-1.15 \) GeV) are divided into ten bins (every bin includes 4 photon-tagging channels), and 10 angular bins of the \( \pi^0\gamma \) emission angle \( \cos\theta \) in the \( \gamma p \)-CM frame. The typical \( \pi^0\gamma \) invariant mass (\( M_{\pi^0\gamma} \)) distributions are shown in Fig. 1. Each \( M_{\pi^0\gamma} \) distribution shows a prominent peak with a centroid of \( \sim 0.78 \) GeV, and has a broad background contribution in the lower side. This background contribution is well reproduced by a Monte-Carlo (MC) simulation based on Geant4 \[18\] for the \( \gamma p \rightarrow \pi^0\omega^0 p \rightarrow \gamma\gamma\gamma p \) reaction, where 1 out of 4 is not detected with FOREST. In the simulation, the five-fold differential cross sections are assumed to be the same as those provided by the 2-PION-MAID calculation \[19\]. The \( M_{\pi^0\gamma} \) distributions for the \( \gamma p \rightarrow \pi^0\pi^0 p \) reaction are also plotted in Fig. 1 where the same analysis is applied as for the \( \gamma p \rightarrow \pi^0\gamma p \) reaction.

![FIG. 1. Typical \( M_{\pi^0\gamma} \) distributions for the highest incident energy group (\( E_\gamma = 1.144-1.149 \) GeV). In each panel, the histogram (blue) shows the experimentally obtained \( M_{\pi^0\gamma} \) distribution, and the solid curve (red) shows the sum of the \( M_{\pi^0\gamma} \) distributions obtained in the simulation for the \( \gamma p \rightarrow \pi^0\gamma p \) and \( \gamma p \rightarrow \pi^0\pi^0 p \) reactions. The dashed (magenta) and dotted (cyan) curves show these contributions. The angular region of \( \omega \) emission in the \( \gamma p \)-CM frame is described in each panel. The vertical lines show the lower limit \( M_{\pi^0\gamma} = 0.76 \) GeV for selecting the \( \omega \) produced events.](image-url)
ulation are fitted to the measured $M_{\pi\gamma}$ distribution for each emission-angle incident-energy bin only by changing the normalization coefficients. Here, the events are generated according to the pure phase space for the $\gamma p \rightarrow \omega p$ reaction. The number of the $\omega$ produced events $N_\omega$ is estimated for $M_{\pi\gamma} \geq 0.76$ GeV after subtracting the background $\gamma p \rightarrow \pi^0\pi^0p$ contribution for each bin. The angular differential cross section is obtained from $N_\omega(\cos \theta)$:

$$\frac{d\sigma}{d\Omega} = \frac{N_\omega(\cos \theta)}{2\pi \Delta \cos \theta N_\gamma N_{\eta \text{acc}}(\cos \theta) \text{BR} (\omega \rightarrow \gamma \gamma \gamma)}$$

(2)

with the incident photon flux including the DAQ efficiency correction $N_\gamma$, the number of target protons $N_\gamma$, the multiplication of branching ratios for the $\omega \rightarrow \pi^0\gamma$ and $\pi^0 \rightarrow \gamma \gamma$ decays BR($\omega \rightarrow \gamma \gamma \gamma$), and the detector acceptance calculated in the simulation $\eta_{\text{acc}}(\cos \theta)$. Fig. 2 shows the typical $d\sigma/d\Omega$ distributions. The systematic uncertainty of $d\sigma/d\Omega$ is also given in Fig. 2. It includes the uncertainty of event selection in the kinematic fit, that of counting $N_\omega$ due to the $M_{\pi\gamma}$ threshold, that of acceptance owing to the uncertainties of the $d\sigma/d\Omega$ distributions for event generation in the simulation, that of detection efficiency of protons, and that of normalization resulting from $N_\gamma$ and $N_\gamma$. Every $d\sigma/d\Omega$ distribution shows a slight increase with increase of $\cos \theta$. A finite $P$-wave amplitude must produce asymmetric behavior of the angular distribution through the interference with the $S$-wave amplitude although the $S$-wave contribution is expected to be dominant near the threshold.

The total cross section $\sigma$ is obtained by integrating $d\sigma/d\Omega$s all over the emission-angle bins:

$$\sigma = \sum 2\pi \Delta \cos \theta \frac{d\sigma}{d\Omega}.$$  

(3)

Fig. 3 shows $\sigma$ as a function of the incident photon energy. The excitation function shows a monotonic increase, and finite yields are observed below the threshold $E_\gamma^{\text{thr}}$ for production of $\omega$ having the centroid mass.

FIG. 2. Typical angular differential cross sections $d\sigma/d\Omega$ as a function of the $\omega$ emission angle $\cos \theta$ in the $\gamma p$-CM frame. The range of the incident photon energies is described in each panel. The filled circles (blue) represent the measured $d\sigma/d\Omega$ in this work. The shaded areas represent the systematic uncertainties of $d\sigma/d\Omega$s. The solid curves show the fitted distribution with a $P$-wave contribution of $\sigma^p_{\text{max}}/5$ (see text). The $d\sigma/d\Omega$ results from SAPHIR [8], CLAS [9], and A2 [2] collaborations are depicted by open boxes (green), open triangles (black), and open circles (magenta), respectively.

We determine $a_{\omega p}$ and $r_{\omega p}$ from the shape of the excitation function. We evaluate the excitation function for the $\gamma p \rightarrow \omega p$ reaction using a model with final-state $\omega p$ interaction (FSI) based on the Lippmann-Schwinger equation. We assume that the $S$-wave contribution is dominant at $E_\gamma = 1.09-1.15$ GeV. The total cross section for a fixed $\omega$ mass $M$ and $\gamma p$-CM energy $W$ can be calculated using a transition amplitude $T_{\gamma p \rightarrow \omega p}(W, M)$:

$$\sigma_0(W, M) = \frac{1}{16\pi W^2} \frac{p(W, M)^2}{k} |T_{\gamma p \rightarrow \omega p}(W, M)|^2,$$

(4)

where $k$ and $p$ denote the momenta of an initial- and a final-state particles, respectively, in the $\gamma p$-CM frame. The total cross section $\sigma$ as a function of $E_\gamma$ is obtained by averaging $\sigma_0(W(E_\gamma), M)$ over available $\omega$ masses:

$$\sigma(E_\gamma) = \int_{m_\omega}^{W(E_\gamma)-m_p} \sigma_0(W(E_\gamma), M) L_\omega(M) dM,$$

(5)

where the probability $L_\omega(M)$ stands for a Breit-Wigner function with a centroid of $M_\omega = 782.65$ MeV and a width of $\Gamma_\omega = 8.49$ MeV [11].
The $T_{\gamma p \rightarrow \omega p}$ is expressed by

$$T_{\gamma p \rightarrow \omega p} = V_{\gamma p \rightarrow \omega p} + T_{\omega p \rightarrow \omega p} G_{\omega p \rightarrow \omega p} V_{\gamma p \rightarrow \omega p},$$  \hspace{1cm} (6)$$

where $T_{\omega p \rightarrow \omega p}$ stands for the $\omega p$ scattering amplitude, $G_{\omega p \rightarrow \omega p}$ denotes the $\omega p$ propagator, and $V_{\gamma p \rightarrow \omega p}$ is the production amplitude without FSI. We evaluate the matrix element for $T_{\gamma p \rightarrow \omega p}$ with on-shell approximations for $T_{\omega p \rightarrow \omega p}$ and $V_{\gamma p \rightarrow \omega p}$, and introduce a Gaussian form factor in the integration of $G_{\omega p \rightarrow \omega p}$. This leads the matrix element of $T_{\gamma p \rightarrow \omega p}$ to the equation:

$$\langle \omega(p) \left| T_{\gamma p \rightarrow \omega p} \right| \gamma(p) \rangle = \left( \omega(p) \left| V_{\gamma p \rightarrow \omega p} \right| \gamma(p) \right) + \int \langle \omega(p) \left| T_{\omega p \rightarrow \omega p} \right| \omega(q) \rangle \frac{\delta^3 (\vec{q} - \vec{q}')}{W - H_\gamma + i\epsilon} \langle \omega(q) \left| V_{\gamma p \rightarrow \omega p} \right| \gamma(p) \rangle d\vec{q} d\vec{q'},$$ \hspace{1cm} (7)

where $H_\gamma$ stands for the free Hamiltonian for the final-state $\omega p$, and $\mu$ denotes a reduced mass between $\omega$ (with a mass of $M$) and the proton. Here, we use a cut-off parameter $\Lambda = 0.8$ GeV/c. The $\langle \omega(p) \left| T_{\omega p \rightarrow \omega p} \right| \omega(p) \rangle$ is given by $a_{\omega p}$ and $r_{\omega p}$:

$$\langle \omega(p) \left| T_{\omega p \rightarrow \omega p} \right| \omega(p) \rangle = \frac{1}{(2\pi)^2} \mu \left( \frac{1}{a_{\omega p}} + \frac{1}{2} r_{\omega p} p^2 - ip \right)^{-1}.$$  \hspace{1cm} (8)$$

The parameters may be somewhat affected by the adopted $\Lambda$. We also determine $a_{\omega p}$ and $r_{\omega p}$ for $\Lambda = 0.6$ and 1.0 GeV/c. The obtained values are $a_{\omega p} = \{ -1.11^{+0.14+0.03}_{-0.16-0.02} \} + i \{ 0.12^{+0.17+0.01}_{-0.17-0.01} \}$ fm and $r_{\omega p} = \{ 2.78^{+0.81+0.04}_{-0.57-0.11} \} + i \{ 0.00^{+0.44+0.31}_{-0.57-0.11} \}$ fm for $\Lambda = 0.6$ GeV/c. Those are $a_{\omega p} = \{ -0.89^{+0.16+0.01}_{-0.18-0.00} \} + i \{ 0.04^{+0.14+0.13}_{-0.12-0.08} \}$ fm and $r_{\omega p} = \{ -2.78^{+0.62+0.13}_{-0.50-0.05} \} + i \{ 0.00^{+0.47+0.03}_{-0.50-0.05} \}$ fm for $\Lambda = 1.0$ GeV/c. Although $\text{Re} \ a_{\omega p}$ and $\text{Im} \ a_{\omega p}$ become larger with decrease of $\Lambda$, changes of $a_{\omega p}$ and $r_{\omega p}$ are not significant among the realistic $\Lambda$ values.

The asymmetric behavior of the angular distribution mainly comes from interference between $S$- and $P$-wave contributions. The dotted curve (magenta) in Fig. 3 shows the shape of the $P$-wave excitation function where $\sigma_0^p(W, M) \propto p^3/k$ is assumed. The finite width of $\omega$ makes the $P$-wave excitation function rather flat, and the $P$-wave contribution does not explain the gap at higher incident energies between the data and calculation without FSI. We also fit the excitation function adding a $P$-wave contribution to the experimental data by fixing $\text{Im} \ r_{\omega p} = 0$ fm. The deduced values are $a_{\omega p} = \{ -0.96^{+0.16+0.03}_{-0.14-0.01} \} + i \{ 0.10^{+0.14+0.11}_{-0.14-0.01} \}$ fm and $r_{\omega p} = \{ 2.85^{+0.77+0.10}_{-0.53-0.15} \} + i0.00$ fm. The optimal coefficient to the $P$-wave contribution is 0, and the $P$-wave total cross section is 0 with an error of $\sigma_{\text{max}}^p$. The dotted curve in Fig. 3 corresponds to $5\sigma_{\text{max}}^p$. The asymmetric behavior in the angular distribution shown in Fig. 2 re-

The [$\omega(p) \left| T_{\omega p \rightarrow \omega p} \right| \omega(p) \rangle$ is assumed to be a constant value of 1 in the incident-energy region of interest.

The dashed curve (gray) in Fig. 3 shows the excitation function $\sigma_{\text{nonFSI}}$ with $a_{\omega p} = 0$ fm and $r_{\omega p} = 0$ fm corresponding to non FSI condition, which does not reproduce the experimental data. FSI is necessary and the optimal set of $a_{\omega p}$ and $r_{\omega p}$ are determined to reproduce the experimentally obtained excitation function. The $\chi^2$ corresponding to the reproducibility is defined as

$$\chi^2 = \sum_{i=1}^{10} \frac{(\sigma_i - \alpha Y_i)^2}{(\sigma_{i, \text{stat}}^ \text{measured})^2 + (\sigma_{i, \text{sys}}^ \text{stat})^2},$$ \hspace{1cm} (9)$$

where $\sigma_i$, $\sigma_{i, \text{stat}}$, $\sigma_{i, \text{sys}}$, and $Y_i$ denote the measured total cross section, its statistical error, its systematic error, and the yield estimated in Eq. 3 by taking the coverage of incident energies into account, respectively, for the $i$-th incident-energy bin. The coefficient $\alpha$ for the overall normalization is determined to minimize $\chi^2$ for each parameter set. The deduced values are $a_{\omega p} = \{ -0.97^{+0.16+0.03}_{-0.16-0.00} \} + i \{ 0.07^{+0.15+0.17}_{-0.15-0.00} \}$ fm and $r_{\omega p} = \{ 2.78^{+0.68+0.11}_{-0.54-0.09} \} + i \{ -0.01^{+0.46+0.07}_{-0.50-0.06} \}$ fm. The first and second errors for each parameter refer to the statistical and systematic uncertainties, respectively. The systematic uncertainty is estimated from that of the mean incident energy ($\pm 3\%$) for each photon-tagging bin. The solid (red) curve in Fig. 3 shows the excitation function with the optimal parameters.
quires a finite $P$-wave contribution. The solid curve in Fig. 2 corresponds to a solution under the condition that the $P$-wave contribution in $\sigma$ is $\sigma_{\text{max}}^{P}/5$. We can conclude that the $P$-wave contribution in $\sigma$ is negligibly small in determination of $a_{\omega p}$ and $r_{\omega p}$.

We have assumed that $V_{\gamma p \rightarrow \omega p}$ is constant since the coverage of incident energies is narrow ($E_{\gamma} = 1.09$–$1.15$ GeV) for several overlapping $N^*$s with a very wide width. We deduce the scattering parameters with $\text{Im } r_{\omega p} = 0$ fm by assuming a single $N^*$ contribution $D_{13}(1700)$ as an extreme condition:

$$V_{\gamma p \rightarrow \omega p} \propto (W^2 - M_{N^*}^2 + iM_{N^*}\Gamma_{N^*})^{-1}$$  \hspace{1cm} (10)

where $M_{N^*} = 1.7$ GeV and $\Gamma_{N^*} = 0.2$ GeV $[11]$. The obtained scattering parameters are $a_{\omega p} = (-0.87 \pm 0.12 \pm 0.05) + i (0.22 \pm 0.14 \pm 0.11)$ fm and $r_{\omega p} = (-2.69 \pm 0.62 \pm 0.16) + i (-0.04 \pm 0.04 \pm 0.00)$ fm. The change of each parameter from the constant-$V_{\gamma p \rightarrow \omega p}$ result is not significant. In addition, our calculation with a constant $V_{\gamma p \rightarrow \omega p}$ and $\omega$ of zero-width correctly reproduces the theoretically predicted $\sigma$ for a single partial wave ($S$-wave $\omega N$: $S_{11}$, and $D_{33}$; $P$-wave $\omega N$: $P_{11}$, $P_{13}$, and $P_{15}$) $[7]$ below the incident energy of 1.15 GeV.

Fig. 4 shows the real and imaginary parts of 1/2 and 3/2 spin-averaged $a_{\omega N}$ obtained by assuming a constant $V_{\gamma p \rightarrow \omega p}$ in this work together with the previously obtained values. It is consistent with $|a_{\omega N}| = 0.82 \pm 0.03$ fm given by the A2 collaboration $[2]$. The positive $\text{Re } a_{\omega N}$ value, giving an attraction, is rejected at a confidence level higher than 99.9%. The repulsion is found to be much stronger than the $\pi N$ ones, and no bound or virtual state is expected for $\omega N$. Slightly attractive $\omega$-nucleus interactions are reported with potential depths at normal nuclear density of $-42 \pm 17 \pm 20$ MeV $[20]$ and $-15 \pm 35 \pm 20$ MeV $[21]$ from $\omega$ photoproduction from nuclei. These potential depths seem to contradict the obtained $a_{\omega p}$. Spin-dependent terms including higher partial waves should be considered for understanding the slight attraction in nuclei. Measurement of the $\omega$ line shape shows a decrease of the $\omega$ mass by $9.2\% \pm 0.2\%$ without any in-medium broadening $[22, 23]$. It also seems inconsistent with the obtained $a_{\omega p}$. More detailed analyses of $\omega N$ scattering are desired to explain the discrepancy.

In summary, the total cross sections have been measured for the $\gamma p \rightarrow \pi^0 \gamma$ reaction near the threshold. The $\omega$ is identified through the $\omega \rightarrow \pi^0 \gamma$ decay. The spin-averaged scattering length $a_{\omega p}$ and effective range $r_{\omega p}$ between the $\omega$ and proton are estimated from the excitation function at incident photon energies ranging from 1.09 to 1.15 GeV: $a_{\omega p} = (-0.97 \pm 0.36 \pm 0.03) + i (0.07 \pm 0.15 \pm 0.17)$ fm and $r_{\omega p} = (+2.78 \pm 0.68 \pm 0.11) + i (-0.01 \pm 0.46 \pm 0.07)$ fm. The real and imaginary parts for $a_{\omega p}$ and $r_{\omega p}$ are determined separately for the first time. A small $P$-wave contribution does not affect the obtained values. The positive $\text{Re } a_{\omega p}$ value indicates repulsion.

The authors express gratitude to the ELPH accelerator staff for stable operation of the accelerators in the FOR-EST experiments. They acknowledge Mr. Kazue Matsuda, Mr. Ken’ichi Nanbu, and Mr. Ikuro Nagasawa for their technical assistance in the FOREST experiments. They received help at the early stage of this work from Dr. Hiroyuki Kamano. They also thank Prof. Igor I. Strakovsky for providing all the available numerical values of cross sections for the $\gamma p \rightarrow \omega p$ reaction. They are grateful to Prof. Mark W. Paris for giving us the numerical values on the total cross sections of a single partial wave. One of the authors (TI) expresses heartfelt gratitude to Dr. Shuntaro Sakai for several useful conversations. This work was supported in part by the Ministry of Education, Culture, Sports, Science and Technology, Japan (MEXT) and Japan Society for the Promotion of Science (JSPS) through Grants-in-Aid for Scientific Research (B) No. 17340063, for Specially Promoted Research No. 19002003, for Scientific Research (A) No. 20244022, for Scientific Research (C) No. 26402087, for Scientific Research (A) No. 16H02188, and for Scientific Research on Innovative Areas No. 18H05407.

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