Autonomization of monoidal categories

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SYCO 3
Outline

1. Pregroup grammars and compositional semantics

2. Free yourselves from the strings of tensors!

3. Examples of applications
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Context

Pregroup grammars (Lambek, 1993, Lambek, 1999)

the film that Emily directed

$np \cdot n^l$ $n$ $n^r \cdot n \cdot np^{ll} \cdot s^l$ $np$ $np^r \cdot s \cdot np^l$
Pregroup grammars (Lambek, 1993, Lambek, 1999)

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the · film · that · Emily · directed
```

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np · n^l
n
n^r · n · np^ll · s^l

np

np^r · s · np^l
```
Pregroup grammars (Lambek, 1993, Lambek, 1999)

```
the film that Emily directed
np · n^l  n  n^r · n · np^ll · s^l  np  np^r · s · np^l
```

Context
Pregroup grammars (Lambek, 1993, Lambek, 1999)

the film that Emily directed

\[ np \cdot n^l \quad n \quad n^r \cdot n \cdot np^{ll} \cdot s^l \quad np \quad np^r \cdot s \cdot np^l \]
Autonomous (or rigid) categories

- **Objects (= types)**
  - are closed under \( \_ \otimes \_ \) (product of types), \( \_^l \) and \( \_^r \) (adjoints).
  - contain basic types, and \( I \), neutral for \( \otimes \).
- **Arrows (= type reductions) between two objects**
  - can be composed with \( \circ \) (sequential composition) and \( \otimes \) (parallel composition);
  - contain \( 1_A : A \rightarrow A \) (identity of \( A \)) and

\[
\begin{align*}
\epsilon^l &: A^l \otimes A \rightarrow I \\
\eta^l &: I \rightarrow A \otimes A^l \\
\epsilon^r &: A \otimes A^r \rightarrow I \\
\eta^r &: I \rightarrow A^r \otimes A
\end{align*}
\]

and such that some equations hold.
Representation

\[ f \circ g = \begin{array}{c} A \\ \downarrow f \\ B \\ \downarrow \end{array} = \begin{array}{c} A \\ \downarrow g \\ B \\ \downarrow f \\ C \\ \downarrow \end{array} \]

\[ f \otimes g = \begin{array}{c} A \otimes C \\ \downarrow f \otimes g \\ B \otimes D \\ \downarrow \end{array} = \begin{array}{c} A \\ \downarrow \end{array} \otimes \begin{array}{c} C \\ \downarrow \end{array} = \begin{array}{c} f \\ \downarrow \end{array} \otimes \begin{array}{c} g \\ \downarrow \end{array} \]

\[ f \circ (\mathbf{I}) = \begin{array}{c} A \\ \downarrow f \\ B \\ \downarrow \end{array} = \begin{array}{c} I \\ \downarrow f \\ I \\ \downarrow \end{array} \]
\( \epsilon \) and \( \eta \)

\[
\begin{align*}
\epsilon^r &= \begin{array}{c}
A \\
\otimes \\
I
\end{array} \begin{array}{c}
A^r
\end{array} \\
\epsilon^l &= \begin{array}{c}
A^l \\
\otimes \\
I
\end{array} \begin{array}{c}
A
\end{array} \\
\eta^r &= \begin{array}{c}
I
\end{array} \begin{array}{c}
A^r \\
\otimes
\end{array} \begin{array}{c}
A
\end{array} \\
\eta^l &= \begin{array}{c}
I
\end{array} \begin{array}{c}
A \\
\otimes
\end{array} \begin{array}{c}
A^l
\end{array} \\
I_A &= \begin{array}{c}
A
\end{array} \begin{array}{c}
A
\end{array}
\end{align*}
\]
Some equalities
Pregroup reductions as arrows

Clouzot directed an Italian movie

\[ n \quad n^r \quad s \quad n^l \quad d \quad d^r \quad d \quad d^r \quad n \]
Clouzot directed an Italian movie

\[ n \quad n^r \quad s \quad n^l \quad d \quad d^r \quad d \quad d^r \quad n \]
Pregroup reductions as arrows

Clouzot directed an Italian movie

\[
n \quad n^r \quad s \quad n^l \quad d \quad d^r \quad d \quad d^r \quad n
\]
Compositional semantics

Word semantics

Type reduction

Motto: Type reduction ∘ Word meanings = Sentence meaning
DisCoCat (Coecke, Sadrzadeh, and Clark, 2011): use \((\text{Vect}, \otimes, I)\), finite dimensional vector spaces over \(\mathbb{R}\) and linear maps between them.

\[
\begin{array}{c}
I_n = \begin{pmatrix}
0.73 \\
-2.3 \\
0.1 \\
1.4
\end{pmatrix} \\
I_{n n^r} = \begin{pmatrix}
-0.3 & 3.9 & -2.1 & 0.4 \\
-2.3 & 2.2 & 1.5 & -1.6 \\
0.1 & 0.3 & -3.8 & 1.2 \\
1.4 & 3.4 & 0.1 & 3.2
\end{pmatrix}
\end{array}
\]
DisCoCat (Coecke, Sadrzadeh, and Clark, 2011) : use \((\text{Vect}, \otimes, I)\), finite dimensional vector spaces over \(\mathbb{R}\) and linear maps between them.

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1.4 & 3.4 & 0.1 & 3.2 \\
\end{pmatrix}
\]

The dimension of a word representation is \textbf{exponential} in the length of the grammatical type.
Why should we use the tensor product?

The direct sum $\oplus$ is cartesian, so it cannot have cups and caps:
Why should we use the tensor product?

The direct sum $\oplus$ is cartesian, so it cannot have cups and caps:

$$
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\text{=}
\end{array}
\end{array}
\end{array}
\end{array}
$$

General belief in the community: “sticking with the categorical framework [...] forces us to stay within the world of linear maps” (Wijnholds and Sadrzadeh, 2018).
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Just cheat and be free!

Our semantic category does not need to have caps and cups: we can freely add them.

Trick: caps and cups can be eliminated in any sentence representation.
Constructing free autonomous categories

- Preller and Lambek (2007) construct the free autonomous category generated by a category.
- We need to start from a monoidal category instead. We factorize their construction:
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Examples of applications

Additive models

Observation by Mikolov et al. (2013):

\[ \text{queen} + \text{king} + \text{man} \rightarrow \text{man} \rightarrow \text{queen} \rightarrow \text{woman} \]

So, it tempting to define royal\((x) = x + \text{queen} - \text{woman} \).
Examples of applications

Additive models

Observation by Mikolov et al. (2013):

So, it tempting to define royal($x$) = $x + \text{queen} - \text{woman}$.
That is forbidden in ($\text{Vect}$, $\otimes$, $I$)!
Convolutional neural networks

Socher et al. (2013) combine vectors following a Chomskyian tree:

Lewis (2019) translates this approach to the categorical model, in \((\text{Vect}, \otimes, I)\).
Examples of applications

A man who ate a cake

\[ a \] \[ b \] \[ c \] \[ d \]

\[ d \] \[ d^r n_s \] \[ n^r s n^l \] \[ n^r s n^l \] \[ d \] \[ d^r n_s \]

= \[ a \] \[ d \] \[ c \] \[ b \]