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Studies of Light Neutron-Excess Systems from Bounds to Continuum

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Abstract The generalized two-center cluster model, which can handle various single particle configurations in general two center systems, is applied to the light neutron-rich system, $^{12}\text{Be} = \alpha + \alpha + 4N$. We discuss the change of the neutrons’ configuration around two $\alpha$-cores as a variation of an excitation energy. We show that the excess neutrons form various chemical-bonding-like configurations around two $\alpha$ cores in the unbound region above the $\alpha$ decay threshold. The possibility of the $\alpha$ cluster formation in the heavier neutron-excess system, $^{28}\text{Ne}$, is also discussed.

In the last two decades, developments of experimental techniques with secondary RI beam have extensively advanced the studies on light neutron-rich nuclei. In particular, much efforts have been devoted to the investigation of molecular structure in Be isotopes. The Be isotopes can be considered as typical examples of two-center superdeformed systems which build on an $\alpha + \alpha$ rotor of $^8\text{Be}$. Theoretically, molecular orbital (MO), such as the $\pi^-$ and $\sigma^+$ orbitals associated with the covalent bonding in atomic molecules, have been successful in understanding the low-lying states of these isotopes [1].

The MO model can describe many kinds of characteristic properties of these isotopes, but they are mainly limited to the analysis on low-lying bound states, and theoretical studies on highly excited states above the particle-decay threshold is still open area. In contrast to the situation of theoretical studies, recent experiments on $^{12}\text{Be}$ revealed the existence of many resonant states [2–6], which strongly decay into $^6\text{He}_{g.s.} + ^6\text{He}_{g.s.}$ and $\alpha + ^8\text{He}_{g.s.}$. Similar resonances, decaying to He isotopes, have also been observed in other Be isotopes, such as $^{10}\text{Be} = \alpha + ^6\text{He}$ and $^{14}\text{Be} = ^6\text{He} + ^8\text{He}$.

In the present study, we investigate the structural changes appearing in an unbound region of $^{12}\text{Be}$. In order to investigate the continuum states above the particle decay threshold, the intrinsic structures and their coupling to the scattering states should be treated in a unified manner. For this purpose, we introduce the generalized two-center cluster model (GTCM) [7]. In this model, the covalent MO configuration can be smoothly connected to the atomic or ionic one, in which valence neutrons are localized around one of the $\alpha$ cores. Furthermore, it becomes possible to describe both the formation of the covalent MO structures and the reaction process induced by the collision of the ionic state. In this report, we investigate the chemical-bonding-like structure based on two $\alpha$ cores in $^{12}\text{Be}$, which appears in the continuum energy region. In addition, we also discuss
the formation of the \( \alpha \) cluster in the heavier neutron excess system, \(^{28}\text{Ne}\), based on a simple \( \alpha + ^{24}\text{O} \) cluster model.

The detailed explanation of GTCM has already been published in Ref. [7], and we briefly show the formulation of GTCM in the following. In GTCM, the total wave function of \(^{12}\text{Be}\) is given by the superposition of the basis \( \{ \Phi^J_K(S) \} \), where,

\[
\Phi^J_K(S) = \hat{P}^J_K \cdot A \left\{ \psi_L(\alpha) \psi_R(\alpha) \prod_{j=1}^{4} \varphi_j(m_j) \right\}_S.
\]

The \( \alpha \)-cluster \( \psi(p) (n = L, R) \) is expressed by the (0s)\(^4\) configuration of the harmonic oscillator (HO) centered at the left(\( L \))- or right(\( R \))-side with the relative distance parameter \( S \). The single-particle wave function for the four valence neutrons localized around one of the \( \alpha \) clusters is given by an atomic orbital (AO) \( \varphi(p) \), and \( p \)-orbitals \( p_k (k = x, y, z) \) around \( i \) (= \( L \) or \( R \)) with the spin \( \tau \) (= \( \uparrow \) or \( \downarrow \)). Here, \( \{ m_j \} \) are indices of AO \( \varphi(p) \) and \( m \) represents a set of AOs for the four neutrons, where \( m = (m_1, m_2, m_3, m_4) \). The intrinsic basis functions with full anti-symmetrization \( A \) are projected to the eigenstate of the total spin \( J \), its intrinsic angular projection \( K \), and the total parity \( \pi \) by the projection operator \( \hat{P}^J_K \). In the present study, we discuss the level structure in the \( J^\pi = 0^+ \) states. Thus, only the case of \( J = K = 0 \) is considered. In the present calculation, we include all possible AO bases, which have an axial symmetric \( K = 0 \) configuration.

The total wave function is finally given by taking the superposition over \( S, m \) as

\[
\Psi^\nu = \int dS \sum_m C^\nu_m(S) \Phi^0_m(S) \equiv \Psi^\nu.
\]

The coefficients for the \( 0^+_1 \) state (\( \nu \)-th \( 0^+ \)), \( C^\nu_m(S) \), are determined by solving an eigenvalue problem. As for the nucleon–nucleon (NN) interaction, we use the Volkov No.2 and the G3RS for the central and spin-orbit parts, respectively. The parameters of the NN interactions and a size for HO are shown in Ref. [7].

In the calculation for the \( J^\pi = 0^+ \) state, we identified the two-bound and four-resonant states below and above the \( \alpha + ^8\text{He}_{g.s} \) threshold, respectively. The ground state has the well developed MO configuration of \( (\pi_{3/2}^-)^2(\sigma_{1/2}^+)^2 \), and it contains two degree of freedoms such as the \( \alpha - \alpha \) relative motion and the single particle motions for the four valence neutrons. By analyzing the properties of the wave functions in the excited states, we find that all the excited states can be characterized in terms of the excitation degree of freedoms included in the ground states. We summarized the identified levels and their intrinsic characters in Fig. 1.

![Fig. 1](image-url)  
**Fig. 1** Energy spectra classified by the excitation mode (\( J^\pi = 0^+ \)). See text for details. The threshold energies of the \( \alpha \) emission is taken to be the origins.
Table 1 The decay width and branching ratio for the $\alpha + ^{24}\text{O}$ state. The energy and $\Gamma_{tot}$ are written in units of MeV. The branching ration of an $\alpha$ emission, $P(\alpha)$, is defined by $\Gamma_\alpha / \Gamma_{tot}$.

| Energy | $J^\pi$ | $\Gamma_{tot}$ | $P(\alpha)$ |
|--------|---------|----------------|-------------|
| 13.4   | $0^+$   | 1.45           | 0.69        |
| 15.9   | $2^+$   | 0.86           | 0.35        |

1) Molecular orbit excitation mode The $0_2^+$ state has the MO configuration, $(\pi_3^+/2)^2(\pi_1^-/2)^2$, while the $0_5^+$ state has a hybrid structure of the MO and AO configurations. Namely, two of the valence neutrons are localized around individual $\alpha$ as $^5\text{He}+^3\text{He}$, which is the AO structure, and the remaining two neutrons occupy the $\sigma^+$ orbital and rotate around both clusters, which represents the MO character. These states can be generated by the excitation of the neutrons’ molecular orbit around two $\alpha$ cores.

2) Cluster excitation mode The $0_3^+$ and $0_4^+$ states have a large component of the ionic structures of $\alpha + ^8\text{He}_{g.s.}$ and $^5\text{He}_{g.s.} + ^7\text{He}_{g.s.}$, respectively. These two states are the excitation modes of the $\alpha-\alpha$ relative motion from the $0_1^+$ and $0_2^+$ states, respectively.

3) Double excitation mode The $0_1^-$ state has the atomic configuration, $^6\text{He}_{g.s.} + ^6\text{He}_{g.s.}$. It corresponds to the simultaneous excitation mode of the relative motion of the two $\alpha$-cores and the single particle motions of the excess neutrons.

The result shown in Fig. 1 means that the cluster structures can change from level to level, and various cluster configurations coexist with a small energy interval of about 1–2 MeV in $^{12}\text{Be}$.

We also discuss the $\alpha$ cluster formation in the heavier system, $^{28}\text{Ne}$. This nucleus has the almost same $N/Z$ ratio as that of $^{12}\text{Be}$ and hence, the formation of the ionic $\alpha + ^{24}\text{O}$ state is a natural extension of the $\alpha + ^8\text{He}$ appearing in $^{12}\text{Be}$. The $\alpha$ cluster formation is expected at the excitation energy of $E_x \sim 10$ MeV, which is the dissociation energy into $\alpha + ^{24}\text{O}$, according to the prediction of the threshold rule [8]. In the heavier system, however, there are two effects which will disturb the $\alpha$ cluster formation: First, the level density of $^{28}\text{Ne}$ is larger by about 2–4 order of the magnitude than that of $^{12}\text{Be}$. Second, one neutron emission channel opens in the lower excitation energy than the $\alpha$ threshold energy ($E_x \sim 4$ MeV). These two factors strongly suggest that an $\alpha$ cluster state couples to the many compound nucleus (CN) states, which finally emit a neutron. Therefore, we should consider the competition of the $\alpha$ decays and decays into the CN states from the $\alpha$ cluster states.

In the present study, we discuss the decay width for the $\alpha$ emission, and its branching ratio is evaluated. The escape width of the $\alpha$ decay from the $\alpha$ cluster state, $\Gamma_\alpha$, can be estimated from the $\alpha + ^{24}\text{O}$ cluster wave function. Here we applied the Orthogonalized Condition Model (OCM) to $\alpha + ^{24}\text{O}$ and found that the $\alpha$ cluster state appears around the respective threshold. From the amplitude of the wave function at the surface region and the penetration factor, we calculate $\Gamma_\alpha$. As for the decay width into the compound state, $\Gamma_{CN}^\alpha$, we employ a simple perturbation formula of

$$\Gamma_{CN}^\alpha = 2\pi <CN|V|CL>_0^2 \rho(E_x),$$

where $\rho(E_x)$ represents the level density at the excitation energy of $E_x$. In this estimation we use the density shown in Ref. [9]. $<CN|V|CL>_0$ represents the averaged coupling matrix element of the compound state ($CN$) and the cluster state ($CL$). Since there is no theoretical analysis on the matrix element, we speculate the order of the magnitude for this matrix element from the experimental spectra in the $\alpha$ inelastic scattering of $^{24}\text{Mg}$ [10]. The estimated $<CN|V|CL>_0$ is about $10^{-3}$ MeV$^2$.

In Table 1, the estimated branching ratios are shown. Here the total decay width, $\Gamma_{tot}$, is defined by $\Gamma_\alpha + \Gamma_{CN}$. The $\alpha’s$ branching ratios, which are denoted by $P(\alpha)$, is calculated by the ratio of $\Gamma_\alpha / \Gamma_{tot}$. In the $J^\pi = 0^+$ state, the total decay width is about 1.5 MeV with the dominant branching ratio of the $\alpha$ decay, $\sim 0.7$. The ratio is suppressed in $J^\pi = 2^+$, but the ratio is still large, which is about 0.4. These estimations suggest that the $\alpha$ decay process dominantly occurs even if one neutron emission channel opens in low excitation energy.

In summary, we have shown that various chemical-bonding-like structures appear in the excite states of $^{12}\text{Be}$. Individual states are produced by exciting the $\alpha$ and neutron degrees of freedom. We also extend our study to the heavier system of $^{28}\text{Ne}$ and discussed the formation of the ionic $\alpha + ^{24}\text{O}$ configuration. According to the present estimation of the width for the $\alpha$ and neutron decays, the $\alpha + ^{24}\text{O}$ state is expected to have a large $\alpha$ decay width nevertheless one neutron emission opens in the lower excitation energy. A new experiment with coincident measurements of $\alpha$ and neutron is strongly desired.
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