Gauged $B - 3L_\tau$, low-energy unification and proton decay

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Abstract

We point out that if there is a gauged $B - 3L_\tau$ symmetry at low energy, it can prevent fast proton decay. This may help building models with theories with extra dimensions at the TeV scale. For purpose of illustration we present an explicit model with large extra dimensions. The Higgs required for a realistic fermion masses and mixing are included. The problem of neutrino masses are solved with triplet Higgs scalars. The proton remains stable even after the $B - 3L_\tau$ symmetry breaking.

The minimal standard model of electroweak and strong interactions is based on the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$. Apart from this gauge symmetry, it has accidental global symmetries, which are:

a) the baryon number $B$;

b) the generational lepton numbers $L_A$, where $A$ stands for the charged leptons $e$, $\mu$ and $\tau$.

These symmetries are anomalous. A common strategy for venturing to gauge theories beyond the standard model is to gauge some linear combination of these symmetries which is anomaly-free. Thus we have obtained models with gauged $B - L$ quantum number $\mathbf{1}$, which is the only non-anomalous global symmetry of the standard model that treats all fermion generations on the same footing. In addition, there are also “non-universal” models where the generational universality is violated by gauging quantities of the form $L_A - L_B$.

In the recent literature, a different type of non-anomalous global symmetries of the standard model and the prospects of gauging such symmetries has been studied $\mathbf{2}$. These are symmetries of the form $B - 3L_A$. For phenomenological reasons, the $B - 3L_\tau$ symmetry

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has been considered most seriously and a simple model with a U(1) symmetry based on this quantum number has been constructed [2, 3, 4]. We point out that in models of low energy unification this symmetry can play an important role.

In recent times an interesting possibility is emerging, in which it is possible to have a TeV scale unification. It has been proposed [5] that if there are compact extra dimensions with very large radius, in which only gravity propagates and the ordinary particles are confined in the usual 3 + 1 space-time dimensions, then it is natural to have a very low effective Planck scale. In these theories gravity will get unified with other gauge interactions at a few TeV scale. All the gauge interactions, namely, the strong and the electroweak interactions will also get unified at a few TeV. There is another class of theories [6] with small extra dimensions, where the extra dimensions are warped, giving rise to a low energy unification. The hierarchy problem disappears in this latter class of theories.

In any realistic model with low unification scale, there are a few generic problems which need to be solved. The most important of them are the reasons for the astounding stability of the proton and the minuteness of neutrino masses. In 4-dimensional unification models, both proton decay rate and neutrino masses are suppressed by inverse powers of a high scale, e.g., the unification scale. This requires large mass scales in the theory, $10^{15}$ GeV or higher. Low energy unification models do not have such high scales. The purpose of the present article is to show how these two problems may be addressed in the context of a low energy unification model. We point out that in the non-universal gauge model with a $B-3L_\tau$ symmetry, it is possible to prevent proton decay altogether, without any constraint on the possible unification scale, and without resorting to any mechanism originating from the extra dimensions. Neutrino mass, on the other hand, is suppressed because of intricacies of a higher dimensional theory, as we will show later.

We first discuss proton stability. The simplest baryon number breaking operators allowed by the gauge symmetry and the fermion content of the standard model have dimension six [7, 8]. In 4-dimensional spacetime, they are therefore suppressed by the inverse square of some large mass scale $M$. Consequently, one needs $M \gtrsim 10^{15}$ GeV so that proton lifetime becomes large enough to be consistent with experiments. In theories with large extra dimensions, $M$ is of the order of a few TeV, and so one must find a mechanism to prevent fast proton decay.

There are a few suggestions to ensure proton stability at levels beyond predicted by the above argument. For example, in 4-dimensional unification models, it was shown that if baryon number is a part of the gauge symmetry in the unified model, the effective operators for proton decay have very high dimensions and so the scale can be much lower [9]. Other suggestions involve physics from extra dimensions [10].

We now discuss what happens in the model at hand. To begin with, we write the representations of all known fermion fields under $\text{SU}(2)_L \times \text{U}(1)_Y \times \text{U}(1)_{B-3L_\tau}$. We omit the $\text{SU}(3)_c$ representation for the sake of brevity. The index $i$ runs over all generations of fermions, whereas $a$ runs only over the first two.

$$
q_{iL} = (2, \frac{1}{3}, \frac{1}{2}), \quad u_{iR} = (1, \frac{2}{3}, \frac{1}{2}), \quad d_{iR} = (1, -\frac{1}{3}, \frac{1}{2}),
$$
$$
\psi_{aL} = (2, -1, 0), \quad \ell_{aR} = (1, -1, 0),
$$
$$
\psi_{\tau L} = (2, -\frac{3}{2}, -3), \quad \tau_R = (1, -1, -3).
$$

(1)
Table 1: Scalar multiplets with integral \(B - 3L_\tau\) which can couple to fermions. We have not included couplings involving right-handed neutrinos.

| Name   | Representation | Yukawa couplings |
|--------|----------------|------------------|
| \(\phi\) | \((2,\frac{1}{2},0)\) | \(\bar{q}_iLd_jR, \bar{\psi}_{aL}\ell_bR, \bar{\psi}_{\tau L}\tau_R\) |
| \(\bar{\phi}\) | \((2,-\frac{1}{2},0)\) | \(\bar{q}_iL\) |
| \(\xi_0\) | \((3,1,0)\) | \(\psi_{aL}\psi_{bL}\) |
| \(\xi_3\) | \((3,1,3)\) | \(\psi_{aL}\psi_{\tau L}\) |
| \(\xi_6\) | \((3,1,6)\) | \(\psi_{\tau L}\psi_{\tau L}\) |
| \(\chi_0^+\) | \((1,1,0)\) | \(\psi_{aL}\psi_{bL}, (a \neq b)\) |
| \(\chi_3^+\) | \((1,1,3)\) | \(\psi_{aL}\psi_{\tau L}\) |
| \(L_0^{++}\) | \((1,2,0)\) | \(\ell_{aR}\ell_{bR}\) |
| \(L_3^{++}\) | \((1,2,3)\) | \(\ell_{aR}\tau_R\) |
| \(L_6^{++}\) | \((1,2,6)\) | \(\tau_R\tau_R\) |

In this model anomaly cancellation requires introduction of right-handed neutrinos. In models with \(B - L\) gauged symmetry \(^1\) all three generations of right-handed neutrinos are required. In the gauged \(B - 3L_\tau\) models, only the right-handed tau-neutrino, \(\nu_{\tau R}\), is necessary in this regard. However, to keep the analysis general, one may also add \(\nu_{e R}\) and \(\nu_{\mu R}\), which will be gauge singlets. Thus the list shown in Eq. (1) should be augmented by the following ones in order to obtain all fermions in the model:

\[
\nu_{e R} = (1,0,0), \quad \nu_{\mu R} = (1,0,0), \quad \nu_{\tau R} = (1,0,-3). \tag{2}
\]

With this fermion content, it will be possible to discuss the question of baryon number violation by looking into the effective higher dimensional operators. We perform this analysis in two steps. In the first step, we disregard the effects of \(U(1)_{B-3L_\tau}\) breaking and discuss the nature of baryon number violation in the theory with unbroken \(U(1)_{B-3L_\tau}\) symmetry. In the next stage, effects of breaking of this symmetry is brought in.

Any operator which breaks baryon number should have at least three quarks, so the minimal operator must contain the combination \(QQQ\), where \(Q\) is any quark multiplet, left handed or right handed. This combination has \(B - 3L_\tau = 1\) and is not consistent with the symmetry. If we add leptons, we can change the \((B - 3L_\tau)\) values in units of three and can never form an invariant. The lowest dimensional \((B - 3L_\tau)\) invariant operator with non-zero baryon number is

\[
Q^0L_\tau, \tag{3}
\]

where \(L_\tau\) is a leptonic multiplet of the third generation, i.e., any one of the set \(\psi_{\tau L}\), \(\tau_R\) and \(\nu_{\tau R}\). This operator has a coefficient \(1/\mathcal{M}^{11}\), where \(\mathcal{M}\) is the scale of new physics. This provides a huge suppression for any baryon number violating process. Further, this breaks baryon number by 3 units, and therefore cannot induce proton decay. \(|\Delta B| = 1\) operators are impossible to construct, as commented above, and so proton is completely stable in
this model. Simplest baryon number violating processes arising out of the operator in Eq. (3) are, for example,
\[ n + n \rightarrow \bar{n} + \bar{\nu}_\tau, \]
\[ n + p \rightarrow \bar{n} + \bar{\tau}^+. \] (4)

To understand if the proton remains stable even after the gauged \( B - 3L_\tau \) is broken, let us include the Higgs scalars in our discussion. As we will discuss later, the neutrino masses and mixing in theories of extra dimensions may require a few different Higgs multiplets beyond the standard model Higgs multiplet \( \phi \). So we shall demonstrate the stability of proton with a rather generalized Higgs content.

The standard model Higgs multiplet will be neutral under \( U(1)_{B - 3L_\tau} \). If our model contained only this Higgs multiplet, it would not have been any different from the standard model, except that the \( \tau \) leptons would not have interacted with other leptons, which might have led to some inconsistency. In order to break \( U(1)_{B - 3L_\tau} \), we need some other Higgs multiplets. We restrict ourselves to multiplets which have integral \( B - 3L_\tau \) quantum numbers, and which can couple to fermions. Table 1 gives a list of all such multiplets, along with fermion bilinears with which they can couple. Clearly, if we restrict ourselves to scalar multiplets shown in this table, there will be no proton decay even after all symmetry breaking. The reason is simple. Baryon number will remain an accidental symmetry of the model even after introducing all Yukawa couplings, and it will not be broken by any vacuum expectation value of scalars.

Let us now discuss a scenario of fermion masses and mixing in this gauged \( B - 3L_\tau \) model, including the question of neutrino masses. The quark masses and mixing and the charged lepton masses come from the Yukawa interactions with the usual standard model Higgs scalar \( \phi \),

\[ \mathcal{L}_Y = \sum_{i,j} f^{(u)}_{ij} \bar{q}_i \tilde{u}_j \phi \beta + \sum_{i,j} f^{(d)}_{ij} \bar{q}_i \tilde{d}_j \phi \beta + \left[ \sum_{a,b} f^{(\ell)}_{ab} \bar{\psi}_a \tilde{\ell}_b \phi \beta + f^{(\tau)} \bar{\psi}_\tau \tilde{\tau}_R \phi \beta \right] \phi \beta + \text{h.c.}. \]

The up and down quark masses and mixing are not constrained in the model with this simplest choice of Higgs scalar. In the leptonic sector, the charged current mixing matrix will depend on the neutrino mass matrix. For a realistic neutrino mass matrix with appropriate mixing, the \( B - 3L_\tau \) symmetry has to be broken and the Higgs scalars should transform non-trivially under \( B - 3L_\tau \).

To solve the problem of hierarchy between the three generations of fermions, we consider the thick wall scenario [11]. The basic assumption is that the particles in our brane are not confined to a point. Instead, they have a Gaussian profile. The Gaussian width for the scalar particle is much larger than all other fermions, so that all the fermions have complete overlap with the scalar. However, the overlaps between the profiles of left and right-handed particles are not maximal. This would then introduce a suppression factor, which can then explain the hierarchy between the three generations of fermions.

Consider a five dimensional example, with co-ordinates \( z = \{ x, y \} \), where \( x \) stands for the 4-dimensional co-ordinates. For simplicity we assume that the scalar field \( \phi \) is same
over the entire thick wall and falls off sharply outside the wall, but any fermion $\Psi$ has a Gaussian profile in the extra dimension $y$, centered around $y_0$:

$$\Psi(z) = A e^{-\mu^2(y-y_0)^2} \psi(x),$$  \hspace{1cm} (6)

where $\psi$ is a normalized four-dimensional massless left-handed fermion; $A = (2\mu^2/\pi)^{1/4}$ is the normalization and $\mu$ is the slope of the profile. A generic Yukawa coupling term in the action $\mathcal{A}$ will be

$$\mathcal{A}_Y = \int d^5z \sqrt{L} \bar{\Psi}_i \Psi_j \Phi = \int d^4x \lambda \bar{\psi}_i \psi_j \phi,$$  \hspace{1cm} (7)

where $L$ is the wall thickness and the four-dimensional Yukawa coupling is given by

$$\lambda = \int dy \, \Lambda \, A e^{-\mu^2(y-y_i)^2} A e^{-\mu^2(y-y_j)^2} = \Lambda e^{-\mu^2(y_i-y_j)^2/2}.$$  \hspace{1cm} (8)

The only assumption here is that wall thickness $L$ is much larger than the Gaussian width $\mu^{-1}$.

Depending on the positions $y_i$ and $y_j$ of the left and right handed particles, the four dimensional Yukawa couplings could then have a hierarchy. A large hierarchy between the three generations of fermions could thus be generated by localizing the fields at a small distance in units of $\mu^{-1}$, within the thick wall.

Since the present results on neutrinos indicate only three Majorana neutrinos and preferably no sterile neutrinos, we decouple the right-handed neutrinos from our discussion using the same thick wall mechanism. We assume that the profile of the left and the right-handed neutrinos have very little overlap and hence the Dirac mass terms are almost zero. Later we introduce a Majorana mass term for the right-handed neutrinos at the scale of $B - 3L\tau$ breaking, which makes the right-handed Majorana neutrinos to be heavy with very little mixing with the left-handed neutrinos. For the rest of the discussions we shall thus ignore the right-handed neutrinos and their couplings with the left-handed neutrinos. For completeness, we shall now show how left-handed neutrinos could have small masses in this model.

The smallness of the neutrino masses is usually explained by introducing a large lepton number violating scale $M_L$ in the denominator and allowing an effective higher dimensional operator which allows a Majorana mass of the neutrino. In theories with TeV scale gravity, the neutrino masses and mixing requires new physics since there are no large lepton number violating scale in the theory. There are a few suggestions to solve this problem and we shall be discussing here one such solutions in some detail [12].

We work in a model of large extra dimensions, in which our four dimensional world (3-brane $\mathcal{P}$ at $y = 0$) is localized at the origin of a higher $(n > 4)$ dimensional space. The usual fermions, gauge bosons and the Higgs doublets propagate only in our 3-brane and are blind to all extra dimensions, while only gravity propagates in the higher dimensional bulk. The extra dimensions are compact and have a large radius $r$, but since the overlap of the gravitons with the particles in our brane is extremely small, the scale of gravity could be as low as a few TeV. For phenomenological reasons we assume the scale of gravity and compactification of the extra dimension is about 100 TeV, which is the fundamental scale in the theory.
We include triplet Higgs scalars with different $B - 3L_\tau$ charges, $\xi_0, \xi_3$ and $\xi_6$ in our 3-brane, which couples to the leptons

$$L_\xi = \sum_{a,b} f_{ab} \xi_0 \bar{\psi}_a \psi_b + \sum_a f_{a\tau} \xi_3 \bar{\psi}_a \psi_\tau + f_{\tau\tau} \xi_6 \bar{\psi}_\tau \psi_\tau + \text{h.c.},$$

as seen in Table I. The scalar potential for these fields $\xi_i$, $i = 0, 3, 6$ are so chosen that they do not acquire any vacuum expectation value (VEV) even when the $B - 3L_\tau$ symmetry is broken. This will ensure that total lepton number $L = L_e + L_\mu + L_\tau$ is conserved in our brane. If there are no interactions of these fields with any bulk matter, then the total lepton number $L$ will not be broken at any time.

We consider one extra $SU(2)_L$ doublet scalar $\kappa \equiv (2, 1/2, -3)$. The Yukawa coupling of this field is given by

$$L = \sum_a f^{(\nu)}_{a\tau} \bar{\nu}_a L \nu_\tau R \kappa + \text{h.c.},$$

This will determine the total lepton number of $\kappa$ to be $L = 0$. Since $L$ is conserved in our brane, the interactions of the type $\xi \phi \phi$, $\xi_3 \kappa \phi$ and $\xi_6 \kappa \kappa$ are all absent. All trilinear interactions of the triplet Higgs scalars will now be forbidden by the total lepton number $L$ conservation in our brane.

We now introduce a singlet scalar

$$\sigma \equiv (1, 0, 0)$$

which can propagate in the bulk. This singlet carries lepton number $L = 2$, but it does not have any gauge interactions. In the present scenario the total lepton number $L$ is broken in another distant 3-brane ($\mathcal{P}'$ at $y = y_*$), when the singlet scalar

$$\eta \equiv (1, 0, 0)$$

acquires a VEV. This scalar $\eta$ carries lepton number $L = 2$. Due to the interaction of this field $\eta$ with the bulk scalar $\sigma$, the lepton number violation in the brane $\mathcal{P}'$ will be communicated to our brane through the interaction of the bulk scalar $\sigma$ with fields in our brane.

Lepton number is conserved everywhere before the field $\eta$ acquires vev. The lepton number conserving interactions of the field $\sigma$ are given by,

$$S_\sigma = \int_\mathcal{P} d^4x \left[ h_1 \xi_0^\dagger(x) \phi(x) \phi(x) \sigma(x, y = 0) + h_2 \xi_3^\dagger(x) \phi(x) \kappa(x) \sigma(x, y = 0) \\
+ h_3 \xi_6^\dagger(x) \kappa(x) \kappa(x) \sigma(x, y = 0) \right] + \int_{\mathcal{P}'} d^4x' \mu^2 \eta^\dagger(x') \sigma(x', y = y_*) \right].$$

Lepton number violation from $\mathcal{P}'$ is transmitted to our brane $\mathcal{P}$ through the shined values of $\langle \sigma \rangle$,

$$\langle \sigma(x, y = 0) \rangle = \Delta_n(r) \langle \eta(x, y = y_*) \rangle,$$

where $\langle \eta \rangle$ acts as point source and $\Delta_n(r)$ is the Yukawa potential. For an interesting choice with $m_\sigma r \ll 1$ and $n = 2$ the asymptotic form of the profile of $\sigma$ is [12]

$$\langle \sigma \rangle \approx \frac{\Gamma(n-2)}{4\pi^{n/2}} \frac{M_*}{(M_* r)^{n-2}}.$$
This will then induce an effective vev for the triplet fields $\xi(x)$ in our brane,

$$
\langle \xi_0 \rangle = h_1 \frac{\langle \sigma \rangle \langle \phi \rangle \langle \phi \rangle}{M_{\xi_0}^2},
$$

$$
\langle \xi_3 \rangle = h_2 \frac{\langle \sigma \rangle \langle \phi \rangle \langle \kappa \rangle}{M_{\xi_3}^2},
$$

$$
\langle \xi_6 \rangle = h_3 \frac{\langle \sigma \rangle \langle \kappa \rangle \langle \kappa \rangle}{M_{\xi_6}^2}.
$$

(16)

The resultant neutrino mass matrix comes out to be

$$
M_\nu = \begin{pmatrix}
    f_{ab} \langle \xi_0 \rangle & f_{a\sigma} \langle \xi_3 \rangle \\
    f_{a\tau} \langle \xi_3 \rangle & f_{\tau\tau} \langle \xi_6 \rangle
\end{pmatrix}
$$

(17)

with $a, b = e, \mu$. The Majoron in this scenario is a singlet and its couplings to the charged fermions is suppressed by a factor $m_\nu \sqrt{G_F}$. For appropriate choice of parameters this model could explain the required neutrino masses and mixing.

The above scenario of large extra dimensions with the $B - 3L_\tau$ gauge symmetry can now explain the neutrino masses without any large scales in the theory, in addition to explaining the quark and charged lepton masses and mixing. We have now introduced three triplet Higgs, which carry $B - 3L_\tau$ quantum numbers 0, 3 and 6. This again ensures that there is no proton decay in the theory. No new physics is required from extra dimensions to prevent fast proton decay if the $B - 3L_\tau$ symmetry is gauged, as argued earlier in the paper.

We now turn to another possibility with supersymmetry and R-parity violation. R-parity is usually imposed to prevent fast proton decay in models with supersymmetry. In models with gauged $B - 3L_\tau$ symmetry all baryon number violating and R-parity violating renormalizable terms are all absent. As a result, in supersymmetric models with gauged $B - 3L_\tau$ symmetry, there is no need to impose R-parity. Proton decay is naturally suppressed.

In conclusion, we point out that in models with gauged $B - 3L_\tau$ symmetry, proton decay is automatically suppressed. We constructed one model of extra large dimensions with gauged $B - 3L_\tau$ and without any large scale in the theory, which can explain the neutrino mass problem. Proton decay is suppressed without any additional input. In R-parity violating supersymmetric models also there is no problem of proton decay, if the $B - 3L_\tau$ symmetry is gauged.

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