Computation of Thermodynamic and Hydrodynamic Properties of the Viscous Atmospheric Motion on the Rotating Earth in 2D Using Naiver-Stokes Dynamics

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Abstract In this article, we model Earth’s lower small-scale eddies motion in the atmosphere as a compressible neutral fluid flow on a rotating sphere. To justify the model, we carried out a numerical computation of the thermodynamic and hydrodynamic properties of the viscous atmospheric motion in two dimensions using Naiver-Stokes dynamics, conservation of atmospheric energy, and continuity equation. The dynamics of the atmosphere, governed by partial differential equation without any approximation, and without considering latitude-dependent acceleration due to gravity. The numerical solution for those governed equations was solved by applying the finite difference method with applying some sort of horizontal air mass density as a perturbation to the atmosphere at a longitude of $\Delta \lambda$. Based on this initial boundary condition with taking temperature-dependent transport coefficient into account, we obtain the propagation for each atmospheric parameter and presented in graphically as a function of geometrically position and time. All of the parameters oscillating with respect to time and satisfy the characteristics of atmospheric wave. Finally, the effect of the Coriolis force on resultant velocity were also discussed by plotting contour lines for the resultant velocity for different magnitude of Coriolis force, then we also obtain an interesting wave phenomena for the respective rotation of the Coriolis force.

Keywords: Naiver-Stokes Equations; Finite difference method; Viscous atmospheric motion; Viscous dissipation; convective motion.

1 Introduction

As stated in [1], it is very difficult to define the exact size, mass, and weight of the Earth’s atmosphere, but in many respects, our atmosphere is a very thin layer, compared to Earth’s radius. It is a critical system for life on our planet and together with the oceans, the atmosphere shapes Earth’s climate and weather patterns and makes some regions more habitable than others. The atmosphere consists of a mixture of ideal gases: although molecular nitrogen and molecular oxygen predominate by volume. Like other planetary atmospheres, the earth’s atmosphere figures centrally in transfers of energy between the sun and the planet’s surface and from one region of the globe to another; these transfers maintain thermal equilibrium and determine the planet’s climate [2].

As cited in reference [3], Atmospheric thermodynamics is the study of heat-to-work transformations (and their reverse) that take place in the earth’s atmosphere and manifest as the weather of climate. Hence it is involved in every atmospheric process, from the large-scale general circulation to the local transfer of radiative, sensible and latent heat between the surface and the atmosphere and the microphysical processes producing clouds [4]. As we stated above, the earth’s atmosphere is the fluid system envelope surrounding the planet, and the atmosphere is capable of supporting a wide spectrum of motions, ranging from turbulent eddies of a few meters to circulations having dimensions of

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the earth itself. Its motion is strongly influenced by different factors such as the effect of the rotation of the Earth, gravity of the earth, air pressure force and the viscosity of the fluid [5].

As suggested in reference [6], the atmosphere is governed by the laws of continuum mechanics and these can be derived from the laws of mechanics and thermodynamics governing a discrete fluid body by generalizing those laws to a continuum of such systems. In order to simulate these fluid flow and other related physical phenomena, it is necessary to describe the associated physics in mathematical form.

Nearly all the physical phenomena of interest to us in this finding are governed by principles of conservation and are expressed in terms of partial differential equations. Hence, Clauude-Louis Navier (1823) derived the equations of motion for a viscous fluid from molecular considerations, and George Stokes (1845) also derived the equations of motion for a viscous fluid in a slightly different form and the basic equations that govern fluid flow are now generally known as the Naiver-Stokes equations which arise from applying Isaac Newton’s second law to fluid motion [7]. According to [8], the dynamics that described by the Naiver-Stokes equations (NSE) of fluid dynamics, for a variable the density of incompressible ocean and compressible atmosphere expressing conservation of mass, momentum, and energy. All the atmospheric gases are constituents of air characteristics with their pressure, density, and temperature and these parameters vary with altitude, latitude, longitude, and related to each other by the equation of state [9] and these field variables are related to one another by the equation of state for an ideal gas.

Life is too short to solve every complex problem in detail, and the atmospheric and oceanic sciences abound with such a complex problem. To solve real-world problems we need to add water vapor as well as the equations of radiative transfer on Thermodynamical equation. All these make a complex system, and to make progress, we need to simplify where possible and eliminate unimportant effects from the model. Thus, during the last decades, direct numerical simulations (DNS) have been recognized as a powerful and reliable tool for studying the basic physics of turbulence, and numerous findings showed that the results obtained by DNS are in excellent agreement with experimental findings [10]. In reference, [11], one of the challenging equations in atmosphere phenomena is the Naiver-Stokes equations which can not be solved exactly. So, approximations and simplifying assumptions are commonly made to allow the equations to be solved approximately.

Recently, high speed computers have been used to solve such equations by replacing them with a set of algebraic equations using a variety of numerical techniques like finite difference method (FDM) [12, 13], the finite element method (FEM) [14] the finite volume method (FVM)[15], meshfree methods and boundary elements method [16]. Many authors in their finding considered the coefficient of viscosity as well as the thermal conductivity of the fluid as a constant parameter, and on many other studies fluid as incompressible for numerical computation of Navier-Stokes dynamics, and there is still no findings that have been done considering on this area (apply the temperature dependent viscosity and thermal conductivity and their influence on the accuracy of the final solution) for solving an atmosphere model, and we numerical compare on the two solutions through finding the relative error between on two mechanism.

The strategy intended to achieve our goal that is to compute the thermodynamic and hydrodynamic properties of the viscous atmospheric motion on the rotating Earth frame. The assumptions used in this analysis are the flow is simple a compressible neutral fluid, with considering temperature dependent transport coefficients of the fluid flow over small-scale motion. In order to get the distributions of the resultant velocity, pressure, density, and temperature of the fluid on our atmosphere, the governing equations have been derived based on the above-mentioned assumptions and given as follows in Section2 and 3, and the numerical results obtained are presented graphically and discussed. in section6.

2 The Laws of Thermodynamics of the Atmosphere

In this context, the first law of the thermodynamics which when applied to the moving fluid elements is consists of the net transfer of heat by fluid flow, net heat transfer by conduction, rate of internal heat generation, the rate of work transfer from the control volume to its environment. Hence, the first law of thermodynamics equation containing the viscous dissipation term Φ is given in equation (1)

\[
\rho \frac{De}{Dt} = \rho \dot{q} + k \nabla^2 T + \nabla k \nabla T - p \nabla \cdot \vec{u} + \Phi \tag{1}
\]
The way we have written (1) is intended to give a clear picture of the balances of work and energy in the flow field. Thus, the heat added by conduction, radiation and chemical reaction (the right-hand-side terms in the first line of (1)) is employed directly to increase the total internal energy. Viscous dissipation as well as compression work, written for clarity in separate lines in (1), are the mechanisms transforming mechanical energy into internal energy, with only the latter being reversible[17].

The natural coordinates in which to express our equations, when they are applied to the Earth, are spherical coordinates \((r, \phi, \lambda)\), where \(r\) is the distance from the center of the Earth, \(\phi\) is latitude and \(\lambda\) is longitude. The detailed derivation of the atmospheric equations of motion (1) in Cartesian coordinates was given in [1], and in spherical coordinate for an compressible fluid it becomes expressed as

\[
\rho c_v \left[ \frac{\partial T}{\partial t} + \frac{u}{r \cos \phi} \frac{\partial T}{\partial \lambda} + \frac{v}{r} \frac{\partial T}{\partial \phi} + w \frac{\partial T}{\partial z} \right] = 
\]

\[
k \left[ \frac{1}{r^2 \cos^2 \phi} \frac{\partial^2 T}{\partial \lambda^2} - \frac{\tan \phi}{r^2} \frac{\partial T}{\partial \phi} + \frac{2}{r} \frac{\partial T}{\partial z} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} \right] 
\]

\[
+ \left( \frac{1}{r \cos \phi} \right)^2 \frac{\partial \mu}{\partial \lambda} \frac{\partial T}{\partial \lambda} + \frac{1}{r \cos \phi} \frac{\partial \mu}{\partial \phi} \frac{\partial T}{\partial \phi} + \frac{\partial \mu}{r} \frac{\partial T}{\partial r} 
\]

\[
- \rho \left[ \frac{1}{r \cos \phi} \frac{\partial u}{\partial \lambda} + \frac{1}{r \cos \phi} \frac{\partial v}{\partial \phi} + 2w \frac{\tan \phi}{r} - \frac{u}{r} \frac{\partial w}{\partial r} \right] 
\]

\[
\rho \dot{q} + \Phi_t 
\]

Here, the coefficient of the thermal conductivity of the air and it’s viscosity \(\mu\) depends on the temperature based on experimental result and they are approximately by using Sutherland’s law[18]

\[
k_T = k_o \left( \frac{T}{T_o} \right)^3 \frac{T_o + S_k}{T + S_k} 
\]

\[
\mu_T = \mu_o \left( \frac{T}{T_o} \right)^3 \frac{T_o + S_\mu}{T + S_\mu} 
\]

Where, \(S_k\), &\(S_\mu\) are an effective temperature for coefficient of thermal conductivity and viscosity of the air, respectively and their value as mentioned in Sutherland law. And \(\mu\), \(\mu_o\) are dynamic viscosity at input temperature \(T\) and at reference temperature \(T_o\), respectively. In the atmospheric range of temperatures, the two specific heat capacity are given by \(c_p = c_{pd} (1 + 0.87 q_v)\) and \(c_v = c_{pv} (1 + 0.97 q_v)\) where \(q_v\) is the mass ratio of the vapor to the moist air \((q_v = \frac{m_v}{m_{total}})\). The dissipation term \(\Phi_t\) in spherical coordinate system for compressible fluid flow is given as

\[
\Phi_t = (2\mu - u) \left( \frac{1}{r \cos \phi} \frac{\partial u}{\partial \lambda} + \frac{1}{r} \frac{\partial v}{\partial \phi} + \frac{2w}{r} \frac{\tan \phi}{r} \right) 
\]

\[
+ (2\mu - u) \left( \frac{\mu}{r^2} \frac{\partial w}{\partial r} \right) + \mu \left( \frac{\mu}{r^2} \frac{\partial u}{\partial \phi} + \frac{1}{\cos \phi} \frac{\partial v}{\partial \lambda} \right)^2 
\]

\[
- \frac{4\mu}{r \cos \phi} \frac{\partial u}{\partial \lambda} \frac{\partial v}{\partial \phi} 
\]

3 The Navier-Stokes Equations for the moist air in Rotational Coordinate System

In this section, we introduce the basic fluid dynamical laws that govern our atmospheric flows. When we assume that the blob is instantaneous of cuboidal shape, with sides \(\delta x, \delta y\) and \(\delta z\) which is fixed in space and different type of forces exerted on this blob, then Newton’s second could be expressed as

\[
m_\text{blob} \frac{d\vec{U}}{dt} = \sum \vec{F} 
\]

Here, the vector \(\vec{F}\) is the sum of all the relevant forces exerted on the neutral fluid elements that include the pressure, viscous, Coriolis, and effective gravitational forces. Hence, the equations governing small-scale atmospheric motion
\[ \frac{\rho}{\rho} \frac{D \vec{U}}{Dt} = -\nabla p - 2\Omega \times U + \rho \vec{g} + \nabla \cdot \kappa \]  

(7)

In order to formulate the last term of (7), we use the following analytic expression for the stress tensor \( \kappa \) [19]:

\[ \kappa = \mu (\nabla \vec{v} + \nabla \vec{v}^\top) - \lambda \nabla \cdot \vec{v} \]

(8)

We have used Lamè’s coefficients of viscosity \( \lambda \) and \( \mu \), which will be treated as temperature-dependent.

\[ \nabla \cdot \kappa = \mu \nabla^2 \vec{v} + (\mu - \lambda) \nabla (\nabla \cdot \vec{v}) + \nabla \mu \cdot (\nabla \vec{v} + \nabla \vec{v}^\top) - (\nabla \cdot \vec{v}) \nabla \lambda \]

(9)

In Cartesian coordinator, the viscous forces along \( x \) and \( y \) components from equation (9) can be expressed as following in equations (10), and (11), respectively.

\[ \nabla \cdot \kappa_x = \mu \nabla^2 u + (\mu - \lambda) \frac{\partial}{\partial x} (\nabla \cdot \vec{v}) + 2 \left( \frac{\partial \mu}{\partial x} \frac{\partial \vec{u}}{\partial x} \right) + 2 \frac{\partial \mu}{\partial y} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) - \frac{\partial \lambda}{\partial x} (\nabla \cdot \vec{v}) \]

(10)

\[ \nabla \cdot \kappa_y = \mu \nabla^2 v + (\mu - \lambda) \frac{\partial}{\partial y} (\nabla \cdot \vec{v}) + 2 \left( \frac{\partial \mu}{\partial y} \frac{\partial \vec{v}}{\partial y} \right) + 2 \frac{\partial \mu}{\partial x} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - \frac{\partial \lambda}{\partial y} (\nabla \cdot \vec{v}) \]

(11)

In spherical coordinates with the components of the velocity vector given by \( (\vec{u} = u, v) \), the Naiver-Stokes equations are given by

- **\( \lambda \)-component of the momentum equation**

\[ \frac{\partial u}{\partial t} + \left( \frac{u}{r \cos \phi} \frac{\partial u}{\partial \lambda} + \frac{v}{r} \frac{\partial u}{\partial \phi} \right) + \frac{uw}{r} - \frac{uv}{r} \tan \phi + lw - fv \]

\[ + \frac{1}{r \rho \cos \phi} \frac{\partial p}{\partial \lambda} = \]

\[ \mu \left( \frac{4}{3r^2 \cos^2 \phi} \frac{\partial^2 u}{\partial \phi^2} - \frac{\tan \phi}{r^2} \frac{\partial u}{\partial \phi} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \phi^2} \right) \]

\[ + \frac{(\mu - \lambda)}{r \cos \phi} \frac{\partial}{\partial \lambda} (\nabla \cdot \vec{v}) - \frac{1}{r \cos \phi} \frac{\partial \lambda}{\partial \lambda} (\nabla \cdot \vec{v}) + \frac{2}{r^2 \cos^2 \phi} \frac{\partial u}{\partial \lambda} \left( \frac{\partial u}{\partial \lambda} \right) + \]

\[ \frac{2}{r^2 \cos \phi} \frac{\partial u}{\partial \phi} \left( \frac{\partial u}{\partial \phi} \right) + \frac{1}{r \cos \phi} \frac{\partial v}{\partial \lambda} \]

(12)

- **\( \phi \)-component of the momentum equation**

\[ \frac{\partial v}{\partial t} + \left( \frac{u}{r \cos \phi} \frac{\partial v}{\partial \lambda} + \frac{v}{r} \frac{\partial v}{\partial \phi} \right) + \frac{uw}{r} - \frac{uv}{r} \tan \phi + \]

\[ fu + \frac{1}{r \rho} \frac{\partial p}{\partial \phi} = \mu \left( \frac{4}{3r^2 \cos^2 \phi} \frac{\partial^2 v}{\partial \phi^2} + \frac{1}{r^2 \cos^2 \phi} \frac{\partial^2 v}{\partial \phi^2} \right) \]

\[ + \frac{(\mu - \lambda)}{r} \frac{\partial}{\partial \phi} (\nabla \cdot \vec{v}) - \frac{1}{r \cos \phi} \frac{\partial \lambda}{\partial \phi} (\nabla \cdot \vec{v}) + \frac{2}{r^2 \cos^2 \phi} \frac{\partial u}{\partial \phi} \left( \frac{\partial u}{\partial \phi} \right) + \]

\[ \frac{2}{r^2 \cos \phi} \frac{\partial v}{\partial \phi} \left( \frac{\partial v}{\partial \phi} \right) + \frac{1}{r \cos \phi} \frac{\partial v}{\partial \lambda} \]

(13)

Here, the latitudinal dependence of the Coriolis parameter is accounted for this work are \( f = 2\Omega \sin \phi \), \( l = 2\Omega \cos \phi \), respectively and \( \rho \) is the density of the fluid, \( p \) pressure, \( \vec{g} \) the effective gravitational acceleration, and the right side terms of equation (12)-(13) comes from the normal and shear stresses due to friction.
4 Application of the Dynamics

As cited in ref [20], the above both atmospheric energy and Naiver-Stokes equations are useful because they describe the physics of many phenomena of scientific and engineering interest. They may be used to model the weather, ocean currents, water flow in a pipe, and airflow around a wing. The Naiver-Stokes equations, in their full forms, help with the design of aircraft and cars, the study of blood flow, the design of power stations, the analysis of pollution, and many other things. Coupled with Maxwell’s equations, they can be used to model and study magneto-hydrodynamics.

The above description of the fluid system is not complete until we also provide a relation between density and pressure for the heterogeneous nature of the Earth’s lower atmospheric layer. For moist neutral air in the atmosphere behaves approximately as an ideal gas, and its density depends on the specific humidity and virtual temperature of the air, and so we write:

\[
p = \rho R d T_o (1 + 0.608 q_v)
\]  

(14)

We recall that global existence of solutions of the atmospheric energy and the Naiver-Stokes equations is known to hold in every space and time dimension [21]. However, theoretical understanding of the solutions to these equations in many finding is done by taking some standard techniques to estimate wind resources and they did not considered tiny errors at the smallest scales will ultimately have huge effects on the overall solution.

5 Numerical Simulation of the Primitive Equations

In this section, we propose finite-difference-central difference schemes, Runa Kutta fourth-order schemes, and 2D unstaggered grid methods were implemented for proving of existence of global solutions for the above governed equations. In all numerical solutions, the continuous partial differential equation is replaced with a discrete approximation, and the discretization of those equations are carried out with respect dimensional coordinates \( \lambda \) and \( \phi \) to convey the equations in finite difference form by approximating the functions and their derivatives in terms of central differences with applying the Courant-Friedrichs-Lewy stability criterion be obeyed[19], and the spherical form of this criterion we have used for numerical solve our model is given by

\[
\frac{\partial r^2}{\partial t} \leq \frac{1}{4}.
\]

As indicated in last section of the introduction, we considered the small scale neutral fluid motion of lower atmosphere as a compressible. Our flow is a shallow box of dimensions \( l_x = 45^\circ \) in the longitude direction, and \( l_y = 45^\circ \) in the latitude direction of the Earth. The grid space is \( \Delta \lambda = 1^{-1} \) in the longitude direction, and \( \Delta \phi = 1^{-1} \) in the latitude direction of the Earth, with these parameters, the angular speed of the rotation of Earth to be \( \Omega = 7.30 \times 10^{-5} \text{s}^{-1} \). We apply some sort of horizontal air mass density (\( \rho = 10 \text{kg/m}^3 \)) as a perturbation to the atmosphere at a longitude of \( 5 \Delta \lambda \), and some useful physical constants were taken from Appendix A of reference[1]. The computations have been performed using \( \Delta t = 1 \) to ensure the stability with the time step \( n_t = 50000 \), then the over all characteristics of the atmospheric variables would be brief discussed in the next section 6.
6 Results and Discussion

The result presented in this section is computed thermodynamic and hydrodynamic properties of the viscous atmospheric motion with restricting our attention to the lower part of the atmosphere as compressible neutral fluid. To see the effect of transport coefficients (viscosity and thermal conductivity) on the propagation of atmospheric resultant velocity, temperature, density, and pressure with respect to time, we perform a numerical simulation using a constant and temperature-dependent transport coefficient (viscosity and thermal conductivity). Taking these parameters into account, and based on our initial boundary condition, we obtain the propagation of resultant velocity, and temperature against time for temperature-dependent as well as for temperature-independent transport coefficient as shown in Figure 1a(left 1&2 column), and in Figure 1b(right 1&2 column), respectively. By comparing their magnitude, we obtain the change in resultant, and temperature from the corresponding two graphs, and presented in graphically as shown in the last end column of Figures 1a & 1b. Thus, from those corresponding figures, we have observed that there is a slight change in both resultant velocity and temperature with respect to time when we used temperature-dependent transport coefficient instead of temperature-independent transport coefficient.

Moreover, the propagation of density and pressure with respect to time is becomes as shown in Figure 2a(left column) and 2b(right column) for both temperature-dependent ($\mu(T)$, $K(T)$) and temperature-independent transport coefficient ($\mu$, $k$). Again here, their corresponding variation with respect to time is presented in graphically as shown in Figure 2a (left last column) and Figure 2b (right last column), respectively. Based on these results, a correction, should be always considered for the computation of thermodynamic and hydrodynamic properties of the viscous atmospheric motion, and for better accuracy, temperature-dependent thermal conductivity and coefficient of viscosity should be considered. Hence, we consider the temperature-dependent transport coefficient for numerical solving all of the governed equations that interpret our model.

Fig. 1: Propagation of resultant velocity and the temperature of moist air with respect to time. From top to bottom: resultant velocity(Fig.1a) and temperature (Fig.1b)
Depending on the above result, that is by taking temperature-dependent transport coefficient, we obtain the numerical solution for the governed equations and it is presented in graphically as shown in below Figures 3&4. From those figures, we have seen that all the atmospheric parameters profile behaves like a wave with respect to time, and the flow patterns of Figures are very close to those shown in[22]. The propagation of resultant velocity and the temperature have opposite in directions as we observe, in Figure3(3a , & 3b),and the reasons for this is currently is due to the dissipation of the fluid their is a transformation mechanical energy to the thermal and a vice verse relation.

When we apply the finite difference method on continuity equation, we obtain the propagation of density($\rho$) as function of geometrically positions and the time taken. Here, the propagation of density was presented graphically as shown in Figure4a with respect to time and longitude, and the propagation of atmospheric pressure as shown in Figure4b. Thus, both of the density and the pressure graph oscillate with same direction respect to time . Due to ideal gas relation, from Figures 3b, 4a, & 4b we have seen that the propagation of temperature is also in opposite direction to the propagation of density and pressure of the fluid.
Fig. 4: The distribution of density (Fig. 4a) and pressure (Fig. 4b) with respect to time and longitude.

When a sort of horizontal air mass density, \( \rho \approx 10 \text{kg/m}^3 \) occurred as a perturbation on the atmosphere at a specific longitude then this perturbation of the density change its propagation with time as shown in Figure 5 in the form of full three-dimensional surface. Thus, from the first figure 5a, we observed that propagation of density has a maximum amplitude at perturbation point, then its oscillation becomes decay to atmospheric density at sea level \( \rho = 1.225 \text{kg/m}^3 \) in both direction of longitude. When the time taken reaches to 100sec and 200sec, the propagation of density behavior like a random motion with increasing in amplitude as shown in figures (5b, & 5c). In similar fashion, using this perturbed density value, we also determine the propagation for the remaining of the atmospheric parameter along to the geometrically position (longitude, and latitude) at specific time taken (see appendices with detailed). Finally, we can put observed that the propagation of resultant velocity, temperature, and pressure behave like random motion with respect to longitude and latitude at different time moments.
Finally, it is interesting to note that the Coriolis effect does not appear in the radial component of Navier-Stokes equation which is related to two-dimensional velocity. Its effect appears on the equations for resultant velocity. Hence, by contrast the magnitude of the rate of momentum change due to Coriolis forces that exerted on the atmospheric compressible fluid, we obtain the resultant velocity field shown as a color plot with some contour lines for corresponding equation (12) - (13) at a different time of the simulation as shown in below figures 6 (left, and right columns) for small and large magnitude of Coriolis force.

Figure 5: The graph of density against longitude & latitude. From top to bottom: propagation of density at t=0sec (left column) and at t=100sec(right column), and t=200sec(left last column).

(c) The computed numerical air mass density at time = 200 sec
7 Summary and Conclusions

In this article, we propose an efficient computational strategy to deal with thermodynamic and hydrodynamic properties of the viscous atmospheric motion in two dimension with considering temperature-dependent viscous coefficient. The dynamics of the atmosphere, governed by partial differential equation without any approximation, and without considering latitude-dependent acceleration due to gravity. The numerical solution for those governed equations was solved by applying the finite difference method with applying some sort of horizontal air mass density as a perturbation to the atmosphere at a longitude of $\Delta \lambda$. Based on this initial boundary condition with taking temperature-dependent transport coefficient into account, we obtain the propagation for each atmospheric parameter and presented in graphically as a function of geometrically position and time. All of the parameters oscillating with respect to time and satisfy the characteristics of atmospheric wave.

Finally, the effect of the Coriolis force on resultant velocity were also discussed by plotting contour lines for the resultant velocity for different magnitude of Coriolis force, then we also obtain an interesting wave phenomena for the respective rotation of the Coriolis force. Generally, the above research result highly sensitive to the initial given value of the parameters, longitude and latitude grid size, and on time scale. Our future work shall be towards evolution of three dimensional governed equation taking compressible ionized fluid with latitudinal dependent acceleration due to gravity.
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8 Appendix

8.1 The distribution of atmospheric parameters with respect to geometrically position

| Longitude (Deg) | Latitude (Deg) | Resultant Velocity (m/s) |
|-----------------|----------------|--------------------------|
| 0               | 14.7091        | 0                        |
| 0.5             | 14.7091        | 0.5                      |
| 1               | 14.7091        | 1                        |
| 1.5             | 14.7091        | 1.5                      |
| 2               | 14.7091        | 2                        |
| 2.5             | 14.7091        | 2.5                      |
| 3               | 14.7091        | 3                        |

(a) The plots of $\vec{v}(t, \lambda)$ at $t = 100$sec

Fig. 7: The distribution of resultant velocity at 100sec (Fig. 7a), and 200sec (Fig. 7b) with respect to longitude, and latitude.

| Longitude (Deg) | Latitude (Deg) | Temperature (K) |
|-----------------|----------------|-----------------|
| 0               | 288.151        | 288.151         |
| 0.5             | 288.151        | 288.151         |
| 1               | 288.151        | 288.151         |
| 1.5             | 288.151        | 288.151         |
| 2               | 288.151        | 288.151         |
| 2.5             | 288.151        | 288.151         |
| 3               | 288.151        | 288.151         |

(b) The plots of $\vec{v}(t, \lambda)$ at $t = 200$sec

Fig. 8: The distribution of temperature at 100sec (Fig. 8a), and 200sec (Fig. 8b) with respect to longitude, and latitude.
Fig. 9: The distribution of pressure at 100sec(Fig.9a) ,and 200sec(Fig.9b) with respect to longitude, and latitude.