Recent results on moments of parton distribution functions

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We report on recent results for the pion and nucleon matrix element of the twist-2 operator corresponding to the average momentum of non-singlet quark densities. We discuss finite size effects for the nucleon matrix element and present first preliminary results for the non-perturbative renormalisation from full dynamical simulations.

1. INTRODUCTION

Phenomenological fits to experimental data provide results for moments of parton distribution functions which allow a direct comparison with lattice calculations. In a recent paper\textsuperscript{1} we were able to compute a continuum limit value of the lowest moment of a twist-2 operator, corresponding to the average momentum of non-singlet quark densities, in pion states. In obtaining the renormalisation group invariant matrix element, we have controlled important systematic errors, such as non-perturbative renormalisation\textsuperscript{2}, finite size effects\textsuperscript{3} and effects of a non-vanishing lattice spacing. The crucial limitation of our calculation was the use of the quenched approximation.

In the scope of our current investigations is the comparison of finite size effects for pion and nucleon matrix elements. In addition we concentrate on computing non-perturbatively the evolution of the twist-2 operators, using the Schrödinger functional finite-size technique, on $N_f = 2$ dynamical configurations provided by the ALPHA collaboration\textsuperscript{5}.

2. FINITE SIZE EFFECTS

Finite size effects for nucleon matrix elements have not been studied in detail before, but might influence considerably the precise determination of moments of parton distribution functions from lattice QCD. We have investigated the effects of limited lattice extent at $\beta=6.0$ ($a \approx 0.093$ fm) with the non-perturbatively improved clover fermion action and lattice sizes $(12 - 32)^3 \times 32$.

The twist-2 non-singlet operator for a flavour doublet of fermions belongs to two irreducible representations of the lattice $H(4)$ group

$$O_{12}(x) = \frac{1}{4} \bar{\psi}(x) \gamma_1 \bar{D}_2 \gamma^3 \psi(x)$$

$$O_{44}(x) = \frac{1}{2} \bar{\psi}(x) \left[ \gamma_4 \bar{D}_4 - \frac{1}{3} \sum_{k=1}^3 \gamma_k \bar{D}_k \right] \gamma^3 \psi(x).$$

For the study of finite size effects (FSE) we concentrate solely on the $O_{44}$ operator, since it can be computed at zero external momentum and thus provides a reliable signal. In particular we use Schrödinger functional (SF) boundary conditions\textsuperscript{6} for our computation, which allows the extraction of the matrix element from one large plateau around the middle of the time extent $T$ of the lattice. The correlation function of the matrix element $f_M$ is obtained by inserting the $O_{44}$ operator between two SF nucleon states $S$ and $S'$ at the time boundaries $t=0$ and $T$ and suitable normalisation with the boundary-to-boundary correlator $C_1$

$$f_M(x_0/L) = -\frac{1}{2} \sum_x \langle S' O_{44}(x) S \rangle \sim e^{-m_N T} \langle N | O_{44} | N \rangle \{1 + \ldots\}$$

$$C_1 = -\frac{1}{2} \langle S'S \rangle \sim e^{-m_N T}.$$  

The bare nucleon matrix element is defined by

$$\langle x \rangle_{u-d}^{\text{bare}} = \frac{2 \kappa}{m_N} \langle N | O_{44} | N \rangle = \frac{2 \kappa}{m_N} f_M(x_0/L)/C_1.$$  

In order to obtain the infinite volume limit nucleon mass and matrix element we performed a purely phenomenological fit $X(L) = c_0 + c_1/L^{3/2} \exp(-c_2 L)$, where $X = m_N a$ or $\langle x \rangle_{u-d}$. Please note that a power-law fit ansatz would describe the data almost equally well. In figs.\textsuperscript{1} and
R_{m_N}(L) = \frac{(m_N(L) - m_N)}{m_N}

\kappa = 0.1332  
0.1335  
0.1338

Figure 1. Finite size effects for \( m_N a \)

\[ R_{\langle x \rangle u-d}(L) = \frac{\langle \langle x \rangle u-d(L) - \langle x \rangle u-d \rangle}{\langle x \rangle u-d} \kappa = 0.1332  
0.1335  
0.1338

Figure 2. Finite size effects for nucleon \( \langle x \rangle u-d \)

3. \( N_f = 2 \) CONTINUUM STEP SCALING FUNCTION

The strategy used to compute the non-perturbative evolution of the operators \( O_{44} \) and \( O_{12} \) has been described in detail in our study for the quenched case [2]. The evolution from initially large \( L/\mu \) to small \( L/\mu \) is obtained by applying the so called step scaling function (SSF) in the Schrödinger functional scheme. Once the perturbative regime is safely reached one continues the evolution in perturbation theory computing the scale and scheme independent RGI matrix element. The connection with experimental results can be obtained by making the adequate perturbative evolution of the RGI matrix element in the \( \overline{\text{MS}} \) scheme.

The renormalisation conditions for the local \( O_{44} \) and \( O_{12} \) operators are given by

\[ O_R(\mu) = Z^{-1}(a\mu)O(a), \quad O_R(\mu = L^{-1}) = O^{(0)} \]

The correlators to compute the Z factor are

\[ f_O(x_0/L) = - \frac{1}{2} \sum_{x,y,z} \langle O(x) S_q(y,z) \rangle \]

\[ f_1 = - \frac{1}{2} \sum_{u,v,y,z} \langle S'_q(u,v) S_q(y,z) \rangle \]

where \( S_q \) and \( S'_q \) are suitable quark sources to probe the operators \( O \). With this definition the renormalisation constants are obtained by

\[ Z(a/L, \mu) = c \frac{f_O(x_0/L)}{\sqrt{f_1}}; \quad c = \frac{\sqrt{f_1^{(0)}}}{f_O^{(0)}(x_0/L)} \]

In order to map the \( L \) dependence recursively we use the SSF, rigorously defined on the lattice by

\[ \sigma Z_O = \lim_{a \to 0} \Sigma Z_O(u, a/L) \]

\[ \Sigma Z_O(u, a/L) = Z_O(u, 2L/a) \frac{Z_O(u, L/a)}{Z_O(u, L/a)}; \quad u = \overline{g}_{SF}(L) \]

The values of \( \beta \) corresponding to a fixed running coupling for \( N_f = 2 \) dynamical \( O(a) \)-improved
Wilson fermions are available in [4,5]. We have computed the SSF at six values of the renormalised coupling $\bar{g}^2_{SF}(L) = 0.98$ to 3.33 for the lattice sizes $L/a = 6, 8, 12$, which includes the determination of $Z$ factors on the corresponding $L/a$ and $2L/a$ lattices. The continuum limit has been obtained by a weighted average of several independent runs on the larger $L/a = 8, 12$ (and $2L/a$) lattices using the HMC with one or two pseudo-fermion fields [7]. In fig. 3 we show the $N_f = 2$ continuum running of $\sigma_{Z_{44}}$ in comparison with the previously obtained quenched evolution [2]. In addition we plot the universal 1-loop (dotted) and 2-loop order ($N_f = 0$, dashed) results of perturbation theory. The corresponding plot for the running of $\sigma_{Z_{12}}$ shows a very similar behaviour.

Complemented by a computation of the pion or nucleon matrix element for $N_f = 2$ dynamical fermions, the SSF allows to obtain the RGI matrix element completely non-perturbatively. The whole strategy is summarised by the following formula:

$$\langle O \rangle_{RGI} = \lim_{a \to 0} \frac{\langle O \rangle(a)}{Z_O(a, \mu_0)} \times \sigma_{Z_O}(\mu/\mu_0, \bar{g}^2(\mu)) \mathcal{F}_{SF}(\bar{g}^2(\mu))$$

where we use the $n = 6$ SSF computed at $\mu_n$

$$\sigma(\frac{\mu}{\mu_0}, \bar{g}^2(\mu)) = \sigma(\frac{\mu_1}{\mu_0}, \bar{g}^2(\mu_1)) \cdots \sigma(\frac{\mu_n}{\mu_{n-1}}, \bar{g}^2(\mu_n))$$

to jump from the non-perturbative scale $\mu_0$ to the perturbative (ultraviolet) scale $\mu$. At this point one can try to do the perturbative matching using

$$\mathcal{F}_{SF}(\bar{g}^2(\mu)) = \left[ (\bar{g}^2(\mu))^{\frac{-2}{\alpha_0}} \times \exp \left\{ - \int_0^{\bar{g}(\mu)} dx \left[ \frac{\gamma(x)}{\beta(x)} - \frac{\gamma_0}{2b_0} \right] \right\} \right]$$

computed with 3-loop $\beta$ and 2-loop $\gamma$ functions. If the perturbative matching has been successful the quantity

$$\sigma^{UV}_{INV}(\mu_0) = \sigma(\mu/\mu_0, \bar{g}^2(\mu)) \mathcal{F}_{SF}(\bar{g}^2(\mu))$$

should be independent from the ultraviolet scale $\mu$. Indeed we found in the quenched case that the last steps give a very nice plateau [2]. The determination of $\sigma^{UV}_{INV}$ for $N_f = 2$ dynamical fermions requires a new computation of the 2-loop anomalous dimension $\gamma_{SF}^{1}$ in the Schrödinger functional scheme, which is currently work in progress.

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