Kinetic mixing, custodial symmetry and a lower bound on the dark $Z'$ mass.

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We show that in the general case of theories for physics beyond the standard model with an extra $U(1)$ factor subgroup, whenever the Higgs potential respects custodial symmetry, the protected relation $M_{W}^{2} = M_{Z}^{2} \cos^{2} \theta_{W}$ permits to rewrite the relevant parameters of the kinetic mixing in terms of the physical values of the primary parameters $\alpha, G_{F}, M_{Z}$ and of the mass of the $Z'$ boson. This procedure, combined with the result $\rho_{0}^{BSM} > 1$ from the fit to electroweak precision data, yields a lower bound $M_{Z'} > M_{Z}$ for the $Z'$ mass. As an example, we study the hidden dark matter scenarios and show that the above constraint holds and yields also a lower bound, $M_{d} \equiv g_{d} v_{d} \gtrsim M_{W} / \cos \theta_{W}$, for the natural dark sector scale $M_{d}$.

I. INTRODUCTION

The possible existence of new neutral gauge bosons has been present during decades mainly due to the prediction of string low energy phenomenology of the existence of the product of several $U(1)$ groups at low energies, whose specific content depend on the chosen path for the unification of fundamental interactions (for a review and a complete list of previous references see [1], [2]). One of the low energy effects of the possible existence of new neutral bosons is their kinetic mixing with the standard model (SM) neutral gauge bosons [3], [4], [5] and this possibility has recently gained renewed interest because in combination with other model-dependent mechanisms allows for weak interactions of dark matter with standard model particles. Indeed, the mystery of the interactions of dark matter with SM particles beyond gravity is presently one of the major challenges in high energy physics, and results of the intensive experimental search for signals of the more popular candidates for Weakly Interacting Massive Particles (WIMPs) yields no trace of them [6], [7], which has lead to explore alternative explanations for the nature of dark matter. Among these ideas, the bottom-up exploration of new neutral gauge bosons, now conceived as part of the mediators of dark matter gauge interactions leads naturally to the existence of kinetic mixing of the SM $U(1)_{Y}$ gauge boson with every abelian gauge boson in the dark gauge group $G_{D}$ [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32].

Our interest on the subject actually comes from the results on a new proposal for dark matter based on an unconventional $(1, 0) \oplus (0, 1)$ space-time structure posed in [33] (details on the formal aspects of the corresponding fields are given in [34], [35]). This is a framework where a consistent description of the dark matter relic density, upper bounds for dark matter annihilation into $b \bar{b}, \tau^{+} \tau^{-}, \mu^{+} \mu^{-}, \gamma \gamma$ and results from direct detection of dark matter is obtained. The gamma ray excess from the galactic center around $\omega = 3$ GeV can be explained in this framework only if the spin-one dark matter has a mass $M \approx M_{H}/2$ [36]. The intriguing sharp prediction $M \approx M_{H}/2$ and the fact that the leading terms are dimension-four lead us to explore the possibility that dark matter interactions have a gauge structure, which naturally brings into the scenario neutral dark gauge bosons. If the dark gauge group contains a factor $U(1)_{D}$ subgroup we have to consider the dimension-four term for kinetic mixing with the $U(1)_{Y}$ of the SM. The main result of the present work, however, is based on general assumptions and goes beyond this framework.

The kinetic mixing causes SM neutral particles to pick up a small coupling to the dark particles. Additional interactions are provided by the Higgs sector, since the dark Higgs fields can mix with the SM Higgs field to yield complementary Higgs portals to dark matter with small couplings. The specific effective interactions depend on the chosen path for the unification of fundamental interactions (for a review and a complete list of previous references see [1], [2]). One of the low energy effects of the possible existence of new neutral bosons is their kinetic mixing with the standard model (SM) neutral gauge bosons [3], [4], [5] and this possibility has recently gained renewed interest because in combination with other model-dependent mechanisms allows for weak interactions of dark matter with standard model particles. Indeed, the mystery of the interactions of dark matter with SM particles beyond gravity is presently one of the major challenges in high energy physics, and results of the intensive experimental search for signals of the more popular candidates for Weakly Interacting Massive Particles (WIMPs) yields no trace of them [6], [7], which has lead to explore alternative explanations for the nature of dark matter. Among these ideas, the bottom-up exploration of new neutral gauge bosons, now conceived as part of the mediators of dark matter gauge interactions leads naturally to the existence of kinetic mixing of the SM $U(1)_{Y}$ gauge boson with every abelian gauge boson in the dark gauge group $G_{D}$ [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32].

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The kinetic mixing causes SM neutral particles to pick up a small coupling to the dark particles. Additional interactions are provided by the Higgs sector, since the dark Higgs fields can mix with the SM Higgs field to yield complementary Higgs portals to dark matter with small couplings. The specific effective interactions depend on the setup of the dark sector. In the simplest case of $G_{D} = U(1)_{D}$, we have dimension-four operators for kinetic mixing and Higgs mixing to be addressed below.

In preparation for a more detailed phenomenological study of specific models for dark matter with conventional or unconventional space-time structures, we present here a simple and general result for the mass of the extra neutral physical boson to be denoted $Z'$ in the following: whenever the complete potential for the Higgs sector of the extended theory respects the custodial symmetry of the SM Higgs sector [27], [28], [29], the mass of the extra physical boson is bounded from below by the value of the $\rho_{0}^{BSM}$ parameter extracted from the fit to electroweak precision data (EWPD). This result is obtained changing the focus from the parameters of the specific model to the consequences for the masses of the physical gauge bosons dictated by the symmetries of the theory in combination with results from the fit to EWPD.

Our work is organized as follows: in the next section we address pure-kinetic-mixing to point out that in this case the $Z'$ is massless and to establish relations between the involved parameters that will allow us to fix sign ambiguities.
in the next sections. In Section III we study the case where the additional gauge boson has a mass term without
invoking a specific mechanism for it, to show that the mass of the physical \( Z' \) must satisfy \( M_{Z'} \gtrsim M_Z \). Section IV is
devoted to remark that this relation has its root in the custodial symmetry of the SM Higgs sector and to show that in
general whenever this symmetry holds the lower bound \( M_{Z'} \gtrsim M_Z \) holds and can be translated to lower bounds
on the extra \( U(1) \) natural mass scale. In Section V we apply this result to hidden dark matter scenarios finding a
lower bound to the natural mass scale in the dark sector. Our conclusions are given in Section VI.

II. PURE KINETIC MIXING

We start considering the case of pure kinetic mixing, i.e., when there is no mass term for the new gauge boson in
the Lagrangian. The kinetic Lagrangian for the gauge fields is

\[
\mathcal{L}_{\text{gauge}}^{K} = -\frac{1}{4}(\tilde{W}^{a\mu\nu} \tilde{W}_a^{\mu\nu} + \tilde{B}^{\mu\nu} \tilde{B}_\mu^{\nu\mu} + \tilde{V}^{\mu\nu} \tilde{V}_\mu^{\nu\nu} + 2\sin \chi \tilde{V}^{\mu\nu} \tilde{B}_\mu^{\nu\mu})
\]

where \( \tilde{W}^{a\mu\nu}, \tilde{B}^{\mu\nu} \) denotes the \( SU(2)_L \) and \( U(1)_Y \) strength tensors respectively and \( \tilde{V}^{\mu\nu} \) denotes the Abelian strength
tensor for the extra \( U(1) \) symmetry in the ultraviolet completion of the SM. In general we use a hat for all quantities
(fields and couplings) in the extended theory, to distinguish them from the SM or physical (measured) quantities.

The first effect of the kinetic mixing is to produce a non-canonical Lagrangian in the neutral sector. A properly
normalized Lagrangian requires a redefinition of the fields by the following

\[
\tilde{B}_\mu = \hat{B}_\mu - \tan \hat{\chi} \tilde{V}_\mu, \quad \tilde{V}_\mu = \sec \hat{\chi} \tilde{B}_\mu,
\]

such that

\[
\mathcal{L}_{\text{gauge}}^{K} = -\frac{1}{4}(\tilde{W}^{a\mu\nu} \tilde{W}_a^{\mu\nu} + \tilde{B}^{\mu\nu} \tilde{B}_\mu^{\nu\mu} + \tilde{V}^{\mu\nu} \tilde{V}_\mu^{\nu\nu}).
\]

Writing the \( SU(2)_L \otimes U(1)_Y \) covariant derivative in terms of the corresponding fields \( \tilde{B}_\mu, \tilde{V}_\mu \) fields we obtain

\[
D^\mu = \partial^\mu + igT^a \tilde{W}^{a\mu\nu} + ig' \frac{Y}{2} \tilde{B}^\mu - ig' \tan \frac{\chi}{2} \tilde{V}^\mu.
\]

The last term, shows that extra gauge boson acquires a non-vanishing hypercharge due to kinetic mixing and this
term, under upon spontaneous symmetry breaking (SSB) in the SM, generates mixing mass terms. We notice that
the \( \tilde{B}^\mu \) field has the same coupling to SM fields as the original \( \hat{B}^\mu \) field and in order to maintain the \( U(1)_{em} \) as an
unbroken symmetry with generator \( Q = T_3 + Y/2 \) we need to consider \( \tilde{B}_\mu \) as the part of the hypercharge field \( \hat{B}_\mu \)
that mixes with \( \tilde{W}_3 \). Indeed, using the Weinberg rotation

\[
\begin{pmatrix}
\hat{B} \\
\hat{W}_3
\end{pmatrix} =
\begin{pmatrix}
\cos \hat{\theta}_w & -\sin \hat{\theta}_w \\
\sin \hat{\theta}_w & \cos \hat{\theta}_w
\end{pmatrix}
\begin{pmatrix}
\hat{A} \\
\hat{Z}
\end{pmatrix}
\]

we get

\[
\hat{g}T_3 \hat{W}_3 + \frac{\hat{g}}{2} \hat{B} = e\hat{Q} \hat{A} + \frac{\hat{g}}{\hat{c}_w} (T_3 - \hat{s}_w^2 \hat{Q}) \hat{Z},
\]

where \( \hat{c} = \hat{g} \hat{s}_w = \hat{g}' \hat{c}_w \), \( Q = T_3 + Y/2 \) and we use the shorthand notation \( \hat{s}_w = \sin \hat{\theta}_w, \hat{c}_w = \cos \hat{\theta}_w \). Notice however
that \( \hat{Z} \) is not the physical \( Z \) boson. Indeed, in the unitary gauge, the Higgs sector yields the following mass terms for
the gauge bosons

\[
L_{\text{mass}}^{gb} = \frac{\hat{g}^2 \hat{v}^2}{4} \tilde{W}^{+\mu} \tilde{W}_\mu - \frac{1}{2} \left[ \frac{\hat{v}^2 \hat{g}^2}{4 \hat{c}_w^2} \hat{Z}^2 + \frac{\hat{v}^2 \hat{g}'^2 \tan \chi \tilde{V}^{\mu} \tilde{Z}_\mu + \hat{g}'^2 \tan \frac{\chi}{2} \tilde{V}^2}{4 \hat{c}_w^2} \right].
\]

The photon field is massless and the \( \tilde{W}^\pm \) bosons have a mass \( M_{W} = \hat{g} \hat{v}/2 \). The SM \( \tilde{Z} \) field has a mass

\[
M_{\tilde{Z}}^2 = \frac{\hat{g}^2 \hat{v}^2}{4 \hat{c}_w^2} = \frac{M_{W}^2}{\hat{c}_w},
\]

(8)
but the Abelian kinetic mixing produces non diagonal mass terms inducing a mixing between the $\hat{Z}$ field and the extra gauge vector $\hat{V}$. The neutral gauge boson mass terms can be rewritten as

$$\mathcal{L}_{\text{mass}}^{\text{neutral}} = \frac{1}{2} \frac{M_W^2}{c_w^2} \left( \hat{Z} + \hat{s}_w \tan \chi \hat{V} \right)^2,$$

suggesting that for pure kinetic mixing, only the physical $Z$ boson is massive (in the zero-mixing limit we recover the SM $Z$-mass term). This expectation is confirmed if we perform the rotation diagonalizing the neutral bosons mass Lagrangian. Indeed, defining the physical fields as

$$\begin{pmatrix} \hat{Z} \\ \hat{V} \end{pmatrix} = \begin{pmatrix} \cos \theta_z & -\sin \theta_z \\ \sin \theta_z & \cos \theta_z \end{pmatrix} \begin{pmatrix} Z' \\ V \end{pmatrix},$$

we get

$$\hat{Z} + \hat{s}_w \tan \chi \hat{V} = (c_z + s_z \hat{s}_w \tan \chi) Z + (-s_z + c_z \hat{s}_w \tan \chi) Z' = \frac{1}{c_z} Z,$$

where $s_z \equiv \sin \theta_z$, $c_z \equiv \cos \theta_z$. Here, we identify the combination in Eq. (9) with the physical $Z$ boson and from the $Z'$ term we obtain

$$\tan \theta_z = \hat{s}_w \tan \chi.$$

The physical $Z$ boson mass in the case of pure kinetic mixing is given by

$$M_Z^2 = \frac{M_W^2}{c_w^2 c_z^2}.$$

Summarizing this section, pure kinetic mixing yields a massless $Z'$ boson \textit{independently of the strength of the mixing, i.e., of the value of the parameter $\chi$}. We will show in the next section that this possibility is ruled out by the results of the global fit to the electroweak precision data for the ratio of physical quantities $M_W^2/c_w^2 M_Z^2$.

### III. BEYOND PURE KINETIC MIXING

If a new neutral abelian gauge boson is going to play a role for the physics beyond the SM it must have a source for its mass in the completing theory. There are many possibilities in the model building for this mass term but in order to maintain our results as general as possible we do not invoke a specific mechanism in this section and just assume that there is an additional mass $\frac{1}{2} M^2 V^2$ term in the Lagrangian such that now

$$\mathcal{L}_{\text{mass}}^{\text{neutral}} = \frac{1}{2} \left( \hat{Z} \hat{V} \right) \begin{pmatrix} M_Z^2 & \Delta \\ \Delta & M_V^2 \end{pmatrix} \begin{pmatrix} \hat{Z} \\ \hat{V} \end{pmatrix},$$

where

$$M_Z^2 = \frac{M_W^2}{c_w^2}, \quad M_V^2 = \frac{M_W^2}{c_w^2} \hat{s}_w^2 \tan^2 \chi + M^2, \quad \Delta = \frac{M_W^2}{c_w^2} \hat{s}_w \tan \chi.$$

The mass matrix in Eq. (14) is diagonalized by the rotation in Eq. (10), yielding the following relations

$$M_Z^2 = M_Z^2 c_z^2 + M_V^2 s_z^2 + 2 \Delta s_z c_z,$$

$$M_{Z'}^2 = M_Z^2 s_z^2 + M_V^2 c_z^2 - 2 \Delta s_z c_z,$$

$$\tan 2 \theta_z = \frac{2 \Delta}{M_Z^2 - M_V^2}.$$

Below we will find useful the converse relations

$$M_Z^2 = M_Z^2 c_z^2 + M_Z^2 s_z^2,$$

$$M_V^2 = M_Z^2 s_z^2 + M_Z^2 c_z^2,$$

$$\Delta = \frac{1}{2} \sin 2 \theta_z (M_Z^2 - M_Z^2).$$
Considering the whole chain of transformations we have the following relations among the physical fields and the original gauge fields

\[
\begin{pmatrix}
\bar{B} \\
\tilde{W}_3 \\
\tilde{V}
\end{pmatrix} =
\begin{pmatrix}
\bar{c}_w, & -\bar{s}_w c_z - \tan \chi s_z, & \bar{s}_w s_z - \tan \chi c_z \\
\bar{s}_w, & \bar{c}_w c_z, & -\bar{c}_w s_z \\
0, & \sec \chi s_z, & \sec \chi c_z
\end{pmatrix}
\begin{pmatrix}
\hat{A} \\
\hat{Z} \\
\hat{Z}'
\end{pmatrix},
\] (22)

The physical gauge fields couple to the SM fields through these matrix elements. Explicitly, if the SM fields do not carry the extra U(1) quantum numbers, the covariant derivative for the SM fields, written in terms of the mass eigenstates is

\[
D_\mu = \partial_\mu + i e Q A_\mu + i \frac{g}{c_w} \left[ c_z (T_3 - s_w^2 Q) - s_w s_z \tan \frac{Y}{2} \right] Z_\mu
+ i \frac{\hat{g}}{\sqrt{2}} (T^+ W^+_\mu + T^- W^-_\mu)
- i \frac{\hat{g}}{c_w} \left[ s_z (T_3 - s_w^3 Q) + \hat{s}_w s_z \tan \frac{Y}{2} \right] Z'_\mu,
\] (23)

where \( T^\pm = T^1 \pm i T^2 \). Additional terms appear in the case when some of the SM fields carry the quantum numbers of the extra U(1) symmetry, but we will show later that these details are irrelevant for the mass relations on which we are interested in here.

Notice that the kinetic mixing parameter \( \chi \) in Eq. (1) is on the root of the \( \tilde{Z} - \tilde{V} \) mixing, thus we have redundant information in the matrix elements of Eq. (22). On the other side, experimental searches naturally focus in the resonance effects of new particles and it would be convenient to translate the information of \( \chi \) (or \( \theta_z \)) to the physical \( Z' \) mass. We can do this using the key relation \( M_Z^2 = M_W^2/c_w^2 \) (see Eq. (15)) in Eq. (19) to write the mixing angle in terms of the masses of the physical neutral gauge bosons

\[
s_w^2 = \frac{\sigma (\tau - 1)}{\tau - \sigma}, \tag{24}
\]

\[
c_z^2 = \frac{\tau (1 - \sigma)}{\tau - \sigma}, \tag{25}
\]

where

\[
\tau \equiv \frac{M_W^2}{M_Z^2 c_w^2}, \quad \sigma \equiv \frac{M_Z^2}{M_W^2 c_w^2}.
\] (26)

These relations involve the \( Z \) and \( Z' \) masses which dictates the size of the mixing angle \( \theta_z \). Information on \( \chi \) is encoded in \( c_w^2 \) which we need to rewrite in terms of physical quantities. In this concern, in the interpretation of data it is conventional to use the best measured quantities which become the primary parameters, in this case the electromagnetic constant \( \alpha \), the Fermi constant \( G_F \), and the physical \( Z \) boson mass, \( M_Z \). The physical value of \( e = \sqrt{4\pi \alpha} \) is related to the following combinations of the physical electroweak couplings and weak angle: \( e \equiv g s_w \equiv g' c_w \) (notice that this is a definition of the physical combination of parameters motivated by their validity in the SM). The value of the Fermi constant is extracted from measurements of the muon life-time and its value is related to the physical weak angle by the SM-inspired relation

\[
G_F \equiv \frac{\pi \alpha}{\sqrt{2} M_Z^2 s_w^2}, \tag{27}
\]

in such a way that the experimental value for the physical weak angle is given in terms of the primary parameters by

\[
c_w^2 = \frac{\pi \alpha}{2 G_F M_Z^2}.
\] (28)

We remark that these are operative definition of the relations between the primary and derived parameters, motivated by the corresponding relations in the SM but independent of other relations required in their derivation within this framework. In particular, these relations are not linked to the relation \( M_W^2 = c_w^2 M_Z^2 \), valid at tree level in the SM.

In the extended theory, the structure of the photon interactions, the weak interactions of the \( \tilde{W}^\pm \) with fermions, and the mass term of the \( W^\pm \) have the same form as in the SM but with the respective “hatted” quantities. In consequence, we have \( e = \hat{e} = \hat{g} \hat{s}_w \), the \( \tilde{W}^\pm \) is diagonal with a physical mass \( M_W = \hat{M}_W \) related to the mass term of

the non-diagonal $\hat{Z}$ field as $M_{Z}^2 = c_w^2 M_{Z'}^2$. Similarly, calculating muon decay in the extended theory we can write the experimental value of the Fermi constant in terms of the corresponding parameters to obtain
\[
c_w^2 s_w^2 = \frac{\pi \alpha}{\sqrt{2} G_F M_{Z}^2}, \tag{29}\]
Combining Eqs. (28, 29) yields the following relations between the physical quantities (the measured $M_Z$ and the derived value of $\theta_w$) and the parameters in the extended theory
\[
M_{Z}^2 c_w^2 s_w^2 = M_{Z'}^2 c_w^2 s_w^2. \tag{30}\]
This relation allows us write
\[
\rho_0 \equiv \frac{M_{W}^2}{c_w^2 M_{Z}^2} = \frac{s_w^2}{s_w^2}. \tag{31}\]
Possible effects of physics beyond the SM are explored in the global fit to the electroweak precision data (EWPD) defining the $\rho_0^{BSM}$ parameter which measures the deviation of the SM value as \[40\]
\[
\rho_0^{BSM} = \frac{M_{W}^2}{c_w^2 M_{Z}^2 \hat{\rho}}. \tag{32}\]
where $\hat{\rho} = \cos \theta_w (M_{Z}^2)$; $M_{W}^2$, $M_{Z}^2$ denote the physical quantities as extracted from experiments; and $\hat{\rho} = 1.01019 \pm 0.00009$ is the conventional SM rho parameter in the $\overline{\text{MS}}$ scheme including radiative corrections. The global fit to the electroweak precision data yields \[40\]
\[
\rho_0^{BSM} = 1.00038 \pm 0.00020. \tag{33}\]
Concerning our analysis in this work, beyond the specific numerical value, the crucial result is that the above value yields $\rho_0 = \rho_0^{BSM} \hat{\rho} = 1.010573 \pm 0.000029 > 1$.

Using Eq. (31) we can write the ratios in Eq. (26) entirely in terms of physical quantities $M_W$, $\theta_w$, $M_{Z'}$ and $M_Z$
\[
\tau = \rho_0 c_w^2 = \frac{\rho_0 c_w^2}{s_w^2}, \quad \sigma = \sigma_0 c_w^2 = \frac{\sigma_0 c_w^2}{s_w^2}, \tag{34}\]
where
\[
\sigma_0 \equiv \frac{M_{W}^2}{c_w^2 M_{Z}^2}. \tag{35}\]
Using these relations, we rewrite Eq. (24) in terms of physical quantities $M_W$, $c_w$, $M_{Z'}$ and writing $\rho_0 \equiv 1 + \delta$ we get
\[
s_w^2 = \frac{(1 - 2 s_w^2 + c_w^2 \delta) \sigma_0 \delta}{(1 - \sigma_0 + \delta)(c_w^2 + \delta)}. \tag{36}\]
The condition $s_w^2 \geq 0$ and the result $\delta > 0$ from the global fit to EWPD yields $\sigma_0 < \rho_0$, i.e.
\[
M_{Z'} > M_Z. \tag{37}\]

We remark that in the derivation of this result we did not use information on $\Delta$ or $M_{\hat{V}}^2$. It arises solely from the key relation $M_{Z}^2 = M_{Z'}^2 c_w^2$, the conventional procedure for the diagonalization of the $\hat{Z}$, $\hat{V}$ mass matrix and the experimental fact that $\rho_0 > 1$. The lower bound for the $Z'$ mass in Eq. (37) will hold whenever these conditions are satisfied.

Notice that we have transferred the information of the parameters of the model to the physical quantities $M_W$, $M_{Z'}$, $c_w$ and $\delta$. This connection allows us to write the matrix elements in Eq. (22) in terms of these physical parameters, which translates to their couplings with other fields, thus it is important to make it explicit. With this aim, we combine Eqs. (24, 25) to obtain
\[
\tan^2 \chi = \frac{1}{s_w^2} \Delta \tau, \quad \tan^2 \theta_z = \Delta \tau \Delta \sigma, \tag{38}\]
where
\[ \Delta \tau \equiv \frac{\tau - 1}{\tau}, \quad \Delta \sigma \equiv \frac{\sigma}{1 - \sigma}. \quad (39) \]

Conversely, these relations allow us to write the physical masses in terms of the parameters of the model. From Eq. (38) we get
\[ (\Delta \tau)^2 = \hat{s}_w^2 \tan^2 \chi \tan^2 \theta_z , \quad (\Delta \sigma)^2 = \frac{\tan^2 \theta_z}{\hat{s}_w \tan^2 \chi}. \quad (40) \]

There is an ambiguity in the signs when we extract the square root in Eqs. (40). It can be fixed in the limit of pure kinetic mixing, worked out in Section I. In this case, which corresponds to \( M \to 0 \) we know that \( M_Z^2 = M_w^2/(c_w^2 v^2) \), \( M_{Z'}^2 = 0 \) and \( \hat{s}_w \tan \chi = \tan \theta_z \) which requires
\[ \Delta \tau = -\hat{s}_w \tan \chi \tan \theta_z, \quad \Delta \sigma = -\frac{\tan \theta_z}{\hat{s}_w \tan \chi}. \quad (41) \]

Solving for the masses of the physical bosons we get
\[ M_Z^2 = \frac{M_{W^+}^2}{c_w^2} \left( 1 + \hat{s}_w \tan \theta_v \tan \chi \right), \quad (42) \]
\[ M_{Z'}^2 = \frac{M_{W^+}^2}{c_w^2} \left( 1 - \hat{s}_w \tan \chi \tan \theta_z \right). \quad (43) \]

As a cross check it is convenient to write the last equation in terms of \( M \). From Eqs. (35,38) we get
\[ M_{Z'}^2 = M_{W^+}^2 \frac{\Delta \tau}{\Delta \sigma} + M^2. \quad (44) \]

On the other side, from Eqs. (20,24,25) we obtain
\[ M_{W^+}^2 = \frac{M_{W^+}^2}{c_w^2} \left( \frac{1}{\tau \sigma} + \frac{\Delta \tau}{\Delta \sigma} \right). \quad (45) \]

Combining this result with Eqs. (16,17,33) we arrive at
\[ M_{Z'}^2 = \tau M^2 = \frac{M^2}{(1 + \hat{s}_w \tan \chi \tan \theta_z)}. \quad (46) \]

In the (ruled out by EWPD) case \( M = 0 \) we recover our result for the \( Z' \) mass for pure kinetic mixing. Combining Eqs. (33,37,36) requires the extra \( U(1) \) group scale to satisfy
\[ M^2 > \frac{M_Z^2 (c_w^2 + \delta)}{c_w^2 (1 + \delta)^2}. \quad (47) \]

**IV. KINETIC MIXING AND CUSTODIAL SYMMETRY**

It is well known that the value \( M_Z^2 = M_{W^+}^2/c_w^2 \) is due to the fact that the SM Higgs sector has actually a \( SU(2)_L \otimes SU(2)_R \) symmetry which under SSB is broken down to its \( SU(2)_L \) subgroup, dubbed custodial symmetry in [39], under which the \( SU(2)_L \) gauge bosons transforms in the \( t = 1 \) representation. This remnant symmetry requires the triplet to have a common mass, thus the mass term for the \( W_3^\mu \) is given as \( M_{W_3}^2 = M_{W_3}^2 \). The non-vanishing vacuum expectation value of the Higgs field generates a non-diagonal mass term given by \( g g^T \langle \phi_0 | T_3 Y | \phi_0 \rangle |^2 W_3^\mu B_\mu \), which introduces the weak mixing angle to yield the \( SU(2)_L \) protected relation \( M_{W_3}^2 = M_{W_3}^2 c_w^2 = M_{W_3}^2 c_w^2 \).

The simplest version of kinetic mixing considers an extra \( U(1) \) symmetry which does not modify the SM part of the Higgs potential nor its spontaneous breakdown pattern. If the extra \( U(1) \) is a spontaneously broken gauge symmetry contributing to \( M_{Z'}^2 \), the most general renormalizable Higgs sector for the \( G_{SM} \otimes U(1) \) gauge theory is given by
\[ V(\phi, \Phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 + \bar{\mu}^2 \Phi^\dagger \Phi + \bar{\lambda} (\Phi^\dagger \Phi)^2 + 2\kappa \Phi^\dagger \Phi \phi^\dagger \phi, \quad (48) \]
where $\phi$ stands for the SM Higgs doublet and $\Phi$ denotes the complex Higgs field for the extra $U(1)$ symmetry. In this case we have also mixing in the Higgs sector, the $\Phi$ field may have $U(1)_Y$ charges and SM fields could carry the $U(1)$ charges yielding more complex expressions for the matrix elements in Eq. (22). In spite of these complications, the crucial point is that the Higgs potential in Eq. (45) respects the $SU(2)_V$ custodial symmetry of the SM.

In general, whenever custodial symmetry is preserved, the relation $M_Z^2 = M_W^2/c_w^2$ holds at tree level and this clever information imposes a lower bound on the $Z'$ mass. Indeed, in this case, the mass Lagrangian of the neutral sector is given by Eq. (13), where $M_Z^2 = M_W^2/c_w^2$ but now the values of $\Delta$ and $M_W^2$ depend on the details of the specific model. Nonetheless, Eqs. (12,20,21) are a consequence of the rotation matrix structure only and still will hold. Similarly, Eqs. (24,25) are obtained from Eq. (19) and the custodial symmetry protected relation $M_Z^2 = M_W^2/c_w^2$ thus they are still valid. In consequence, the lower bound in Eq. (77) is valid whenever the Higgs potential respects custodial symmetry.

Finally, either from Eq. (20) or from the trace invariance of the mass matrix, we get

$$M_\Phi^2 > M_Z^2 \left( 1 + \frac{\delta}{c_w^2 + \delta} \right),$$

which, taking into account the constraint in Eq. (37), must satisfy

$$M_\Phi^2 > M_Z^2 \left( 1 + \frac{\delta}{c_w^2} \right).$$

For specific models this result can be used to set lower bounds on the scales of the extra $U(1)$ gauge symmetry similar to Eq. (37). As an example, we consider the case of hidden dark matter scenarios in the next section.

V. KINETIC MIXING, CUSTODIAL SYMMETRY AND HIDDEN DARK MATTER SCENARIOS.

In hidden dark matter scenarios we have a dark sector with no SM charges and viceversa. For the sake of simplicity we consider a spontaneously broken $U(1)_d$ gauge symmetry for the dark sector but the argument works for a dark gauge group $G_d$ containing an abelian $U(1)_d$ subgroup. In this case, the Lagrangian can be written as

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_d + \mathcal{L}_{int},$$

where $\mathcal{L}_{SM}$ stands for the SM Lagrangian, $\mathcal{L}_d$ denotes the $U(1)_d$ gauge theory Lagrangian and $\mathcal{L}_{int}$ account for the dimension four (or higher if we consider it as an effective theory) terms constructed from products of singlet operators on both sides. Concerning the SM, the lowest dimension gauge singlet operators (not necessarily Lorentz invariant) are the $U(1)_Y$ stress tensor $B^{\mu\nu}$ and the Higgs operator $\phi^4$. On the dark side, the lowest dimension gauge singlets are the $U(1)_d$ stress tensor $\hat{V}^{\mu\nu}$ and the dark Higgs operator $\Phi \Phi \Phi$. The leading terms connecting the SM and dark sectors are dimension four and given by

$$\mathcal{L}_{int} = -\frac{\sin \chi}{2} B^{\mu\nu} \hat{V}_{\mu\nu} + 2 \kappa \phi^4 \phi^4 \Phi \Phi.$$

There could be additional terms depending on the matter content in the dark side, e.g., if there is a dark neutrino whose right component is a singlet of $U(1)_d$, then $\tilde{L} \phi^4 \nu_R^4$ is also dimension four, but these terms do not affect the gauge sector considered here. The Higgs Lagrangian reads

$$\mathcal{L}_{Higgs} = (D^\mu \phi)^4 D_\mu \phi + (D^\mu \Phi)^4 D_\mu \Phi - V(\phi, \Phi)$$

with the covariant derivative

$$D_\mu = \partial_\mu + i \frac{g}{\sqrt{2}} (T^+ W^+_\mu + T^- W^-_\mu) + i e A_\mu$$

$$+ i \left[ \frac{g}{c_w} (c_2 T_3 - s_w^2 Q) - s_w s_z \tan \frac{\chi}{2} \right] + g_d s_z \sec \chi \frac{Y_d}{2} \right] Z_\mu$$

$$- i \left[ \frac{g}{c_w} (s_z T_3 - s_w^2 Q) + s_w c_z \tan \frac{\chi}{2} \right] - g_d c_z \sec \chi \frac{Y_d}{2} \right] Z'_\mu,$$
where $Y_d/2$ denotes the generator of $U(1)_d$.

The Higgs potential is given in Eq. (48) which respects the SM custodial symmetry. The minimum conditions now read

$$\frac{\partial V}{\partial \phi_i} = 2\phi_i [\mu^2 + 2\lambda (\phi^\dagger \phi) + 2\kappa (\Phi^* \Phi)] = 0,$$

$$\frac{\partial V}{\partial \Phi_i} = 2\Phi_i [\tilde{\mu}^2 + 2\tilde{\lambda} (\Phi^* \Phi) + 2\kappa (\phi^\dagger \phi)] = 0.$$  (56)

In the unitary gauge, the spontaneously broken solutions are

$$\hat{\phi} = \left( \frac{0}{v + B} \right), \quad \hat{\Phi} = \frac{v_d + \tilde{S}}{\sqrt{2}},$$

where $\tilde{H}$ and $\tilde{S}$ have vanishing vacuum expectation value. The minimum conditions yield the following relations

$$\mu^2 + \lambda v^2 + \kappa v_d^2 = 0,$$

$$\tilde{\mu}^2 + \tilde{\lambda} v_d^2 + \kappa v^2 = 0.$$  (57)

The Higgs sector yields the following mass terms for the gauge bosons

$$M_Z^2 = \frac{M_W^2}{c_w^2},$$

$$\Delta = \frac{M_W^2}{c_w^2} s_w \tan \chi,$$

$$M_V^2 = M_W^2 \tan^2 \theta_w \tan^2 \chi + g_0^2 v_d^2 \sec^2 \chi.$$  (60)

The $Z$ mass has the expected value from custodial symmetry and we obtain the specific values for $\Delta$ and $M_V^2$ in terms of the parameters of the model which can be explicitly shown to satisfy Eqs. (49,50).

The natural dark matter mass scale in this model is $M_d^2 \equiv g_0^2 v_d^2$, which can be written in terms of the masses of the physical gauge bosons as

$$M_d^2 = \frac{M_W^2 s_w^2}{c_w^2} \left( 1 + \frac{1}{s_w^2} \Delta \right)^{-1} \frac{1}{\sigma \tau}.$$  (61)

Explicitly, in terms of the physical masses and mixing angle we get

$$M_d^2 = \frac{M_W^2 s_w^2}{s_w^2 - \rho (1 - \rho c_w^2)(1 - \sigma c_w^2)}.$$  (62)

The constraint in Eq. (47), valid for every model respecting custodial symmetry, yields the following lower bound for the dark scale in hidden dark matter scenarios

$$M_d^2 > \frac{M_W^2 s_w^2}{s_w^2 - \rho (1 - \rho c_w^2)^2} = \frac{M_W^2}{c_w^2} + O(\delta).$$  (63)

VI. CONCLUSIONS

In this work we show that in ultraviolet gauge completions of the standard model containing a $U(1)$ factor subgroup, whenever the standard model part of the Higgs potential respects custodial symmetry, the mass of the extra gauge boson entering the kinetic mixing is bounded from below by the value of the $\rho^{BSM} \equiv M_W^2/c_Z^2 M_Z^2 \rho$ extracted from the fit to the electroweak precision data [10]. Here $c_Z^2 \equiv \cos^2 \theta_w (M_Z^2)$ and $\rho$ denotes the standard model contributions including radiative corrections.

Starting from the analysis of the pure kinetic mixing case (no mass term for the extra gauge boson in the Lagrangian), we show that the kinetic mixing induces in turn a mixing of the SM $Z$ boson with the extra gauge boson but the latter remains massless. Next we analyze the case when the extra boson has originally a mass term in the Lagrangian generated by an unspecified mechanism. In this case, the relation $M_V^2 = M_Z^2 \cos^2 \theta_w$ makes possible to rewrite the mixing matrix elements of the neutral sector in terms of the measured values of $M_W$, $M_Z$, $\theta_w$ and the mass of the
physical new gauge boson $M_Z'$. This rewriting allows us to recover the pure mixing case in the $M \to 0$ limit. In the $M \neq 0$ case, combined with the fact that $\rho^{\text{BSM}} > 1$, this procedure yields the lower bound $M_{Z'} > M_Z$. A similar lower bound is obtained for the extra gauge boson mass scale $M$ in this case. We remind the connection of the relation $M_W^2 = M_Z^2 \cos^2 \theta_w$ to the custodial symmetry of the standard model Higgs potential and show that in gauge theories for physics beyond the standard model with gauge groups containing a new $U(1)$ group factor, whenever this symmetry is respected, the outlined procedure can be carried out and the mass of the physical neutral extra gauge boson must satisfy $M_{Z'} > M_Z$. Finally we apply these results to hidden dark matter scenarios when the dark sector in general has a gauge structure with a $U(1)_D$ factor subgroup. We show that the lower bound $M_{Z'} > M_Z$ holds and the natural dark matter scale $M_d = g_d v_d$ must satisfy the lower bound $M_d \gtrsim M_W/c_w$.

VII. ACKNOWLEDGMENTS

One of us (H.H.A.) acknowledges CONACyT-México for a scholarship to pursue her Ph. D.

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