Endogenous versus exogenous origins of financial rallies and crashes in an agent-based model with Bayesian learning and imitation

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Abstract

We present a simple agent-based model to study how the proximate triggering factor of a crash or a rally might relate to its fundamental mechanism, and vice versa. Our agents form opinions and invest, based on three sources of information, (i) public information, i.e. news, (ii) information from their “friendship” network, promoting imitation and (iii) private information. Agents use Bayesian learning to adapt their strategy according to the past relevance of the three sources of information. We find that rallies and crashes occur as amplifications of random lucky or unlucky streak of news, due to the feedback of these news on the agents’ strategies into collective transient herding regimes. These ingredients provide a simple mechanism for the excess volatility documented in financial markets. Paradoxically, it is the attempt for investors to learn the level of relevance of the news on the price formation which leads to a dramatic amplification of the price volatility due to their collective search for the “truth”. A positive feedback loop is created by the two dominating mechanisms (Bayesian learning and imitation) which, by reinforcing each other, result in rallies and crashes. The model offers a simple reconciliation of the two opposite (herding versus fundamental) proposals for the origin of crashes within a single framework and justifies the existence of two populations in the distribution of returns, exemplifying the concept that rallies and crashes are qualitatively different from the rest of the price moves.

Key words: stock market, crash, rallies, bubble, herding
1 Introduction

Stock market crashes are momentous financial events that are fascinating to academics and practitioners alike. According to the consecrated academic view that markets are efficient, only the revelation of a dramatic piece of information can cause a crash, yet in reality even the most thorough post-mortem analyses are typically inconclusive as to what this piece of information might have been.

It is often observed that crashes occur soon after a long run-up of prices, referred to as a bubble. A crash is thus often the burst of the bubble. According to the rational expectation theory of bubbles [Blanchard 1979, Blanchard and Watson 1982], bubbles are spontaneous deviations of the price from fundamental value, and the associated apparent anomalous returns are nothing but the required remuneration for rational investors to stay invested in the market in the presence of growing crash risks. Thus, the price has to increase with the crash hazard rate and vice versa, so that it becomes more and more probable for a crash to occur towards the end when the bubble price peaks. For instance, in the specification of Johansen et al. [Johansen et al. 1999c], it is the increasing crash hazard rate determined by the dynamics of noise traders which drives the price run-up. Contrarily, in the specification of Sornette et al. [Sornette and Andersen 2002], positive feedbacks operating on the price by agents following momentum strategies drive it up, therefore concomitantly pushing up the crash hazard rate.

There is a vast empirical literature aiming more generally at diagnosing the presence of bubbles (see [Camerer 1989, Adam and Szafarz 1992] for surveys of this literature) and at characterizing their underlying origin(s) and mechanism(s) (see e.g. [Kaufman 2001, Shefrin 2005, Shiller 2000, Sornette 2003]). However, there is still no consensus in the academic community on what is really a bubble and what are its characteristic properties. Bubbles do not seem to be fully explained by bounded rationality [Levine and Zajac 2007], speculation [Lei et al. 2001] or the uncertainty in the market [Smith et al. 1988]. It is also not clear what is the nature and limits of the predictability of its burst time, if any. Finally, there is no really satisfactory theory of bubbles, which both encompasses its different possible mechanisms and adheres to reasonable economic principles (no or limited arbitrage, equilibrium or quasi-equilibrium with only transient deviations, bounded rationality, and so forth). Indeed, the literature, which is too large to be explicated here, is still uncertain on even how to define a bubble, because an exponentially growing price can always be argued to result from some fundamental economic factor. This is related to the problem that the fundamental price is not directly observable, giving no strong anchor to understand observed prices. How then is it possible to ascertain with some level of confidence that a market is or is not in a bubble regime? How does this question impact the expectations and
Coming back to crashes, most approaches to explaining crashes search for possible mechanisms or effects that operate at very short time scales (hours, days, or weeks at most). Here, we build on the radically different hypothesis summarized in (Sornette, 2003) that the underlying cause of the crash should be found in the preceding months and years, in the progressively increasing build-up of a characteristic that we refer to as “market cooperativity,” which expresses the growth of the correlation between investors’ decisions leading to stronger effective interactions between them as a result of several positive feedback mechanisms. The hypothesis is that this increased market cooperativity is translated into accelerating ascent of the market price (the bubble). According to this “critical” point of view, the proximal triggering factor for price collapse should be clearly distinguished from the fundamental factor. A crash occurs because the market has entered an unstable phase toward the culmination of a bubble and any small disturbance or process may reveal the existence of the instability. Think of a ruler held up vertically on your finger: this very unstable position will lead eventually to its collapse, as a result of a small (or an absence of adequate) motion of your hand or due to any tiny whiff of air. The collapse is fundamentally due to the unstable position and not the effect that triggered it; the instantaneous cause of the collapse is “the tree hiding the forest,” the forest being the fundamental cause lying in the maturation of the unstable state. In the same vein, the growth of the sensitivity and of the instability of the market close to such a critical point might explain why attempts to unravel the proximal origin of the crash have been so diverse. Essentially, anything can trigger the avalanche once the system is ripe.

What is then the origin of the maturing instability? A follow-up hypothesis underlying our present work is that, in some regimes, there are significant behavioral effects underlying price formation leading to the concept of “bubble risks.” This idea is probably best exemplified in the context of financial bubbles, such as the recent Internet episode culminating in 2000 or the real-estate bubble in the USA peaking in 2006. Many studies have suggested that bubbles result from the over-optimistic expectation of future earnings (see, for instance, (Sheffrin, 2005)), and others have argued contrarily for rational explanations (e.g. (Garber, 2000)). History provides a significant number of examples of bubbles driven by unrealistic expectations of future earnings followed by crashes. The same basic ingredients have been documented to occur repeatedly (Sornette, 2003). According to this view, fuelled by initially well-founded economic fundamentals, investors develop a self-fulfilling enthusiasm by an imitative process or crowd behavior that leads to the building of castles in the air, to paraphrase Malkiel (Malkiel, 1990). We borrow from the literature emphasizing the importance of imitation and herding via positive feedbacks due to technical as well as behavioral mechanisms, leading to self-organized
cooperativity and the development of possible endogenous instabilities. Accordingly, this literature suggests that most crashes have fundamentally an endogenous, or internal, origin and that exogenous, or external, shocks only serve as triggering factors. As a consequence, the origin of crashes is probably much more subtle than often thought, as it is constructed progressively by the market as a whole, as a self-organizing process. In this sense, the true cause of a crash could be termed a systemic instability.

Based on these ideas, the present paper adds to the literature by providing a detailed analysis of how the proximate triggering factor of a crash might relate to its fundamental mechanism in terms of a global cooperative herding mechanism. This allows us to propose a possible reconciliation of the two opposite view points described above within a coherent framework. In particular, we rationalize the finding of Cutler et al. (Cutler et al., 1989) that exogenous news are responsible for no more than a third of the variance of the returns and that major financial crises are not preceded by any particular dramatic news. The tendency of our agents to gather all possible information and to adapt to the recent past makes them develop a transient phase of herding with positive feedbacks, nucleated by random occurrences of runs of positive exogenous news. One could thus summarize our findings as follows: rallies and crashes occur due to random lucky or unlucky streaks of news that are amplified by the feedback of the news on the agents’ strategies into collective transient herding regimes. As a bonus, these ingredients provide a simple mechanism for the excess volatility documented in financial markets (Shiller, 1981).

In a nutshell, our multi-period many agent-based model is designed as follows. At each successive time step $t$, each investor forms an opinion on the next-period value of a single stock traded on the market. This opinion is shaped by weighting and combining three sources of information available at time $t$: (i) public information, i.e., news, (ii) information from their “friendship” network, promoting imitation and (iii) private information. In addition, we assume that the agents adapt their strategy, i.e., the relative importance of the public, network and private sources of information, according to how well they performed in the past in predicting the next-time step valuation. This ingredient in the model is motivated by the general observation that investors often over-trade, due to over-reaction to the recent past. This point is well-illustrated by the anecdotes reported by a broker in his insider analysis of the behavior of the most active traders of his company (Guyon, 1965). This broker was utterly mystified as to why, after a full cycle of rise and fall after which stocks were valued just where they were at the start, all his clients have lost money. His clients felt fearful during bear markets and so traded in and out constantly. This observation performed in the first decade of the twentieth century is still applicable today. Since even the forward-looking agents are per force using information on the past, their adaptive steps are essentially influenced by the past and therefore, one could argue, adapted to the past.
It thus comes as no great surprise that their adaptation may actually lead them to lose money on average, similarly to the investors in (Guyon, 1965), providing another example of the mechanism of “illusion of control” discovered in the context of minority games and Parrondo games (Satinover and Sornette, 2007a,b, 2008).

This paper is organized into four sections. In Section 2, the detailed working of the model is presented. The results are shown and discussed in section 3 and section 4 concludes.

2 The model

2.1 General set-up

We consider a fixed number of \( N \) agents who are buying or selling a single stock, which can be considered to be the market portfolio, that is traded on an organized market coordinated by a market maker. At each time step, the agents have the possibility to either trade or to stay out of the market. We focus on the question of understanding the detailed dynamics underlying the development of bubbles and crashes. We thus limit the realism of our model to a set of simple ingredients.

The set-up of our model builds on that of Zhou and Sornette (Zhou and Sornette, 2007), which focused on herding and on the role of “irrational” misattribution of price moves to generate most of the stylized facts observed in financial time series. The main differences between the present model and that of Zhou and Sornette (Zhou and Sornette, 2007) are:

1. The presence of liquidity constraints;
2. The possibility for agents to exit the market and be solely invested in bonds;
3. The adaptation of the agents’ strategies are based on (bounded) rationality, i.e., our agents adapt both their propensity to imitate and their reliance on incoming news. In contrast, Zhou and Sornette (Zhou and Sornette, 2007) have considered adaptation only of the propensity of agents to imitate. As a consequence of our different and more general rule, we find that bubbles can emerge without the need for an “irrational” misattribution.
4. The definition of a successful trade. In (Zhou and Sornette, 2007), a trader is successful if he betted in the same direction as the immediately following price movement (corresponding to a so-called “majority game”). With such a rule, the trader had an immediate feedback to adapt his decision making process. On the other hand, our agents make a successful investment if the price of the stock they bought increases after their purchase,
so one time step later. In this way, the investors get their payoff a time step later compared with the agents in (Zhou and Sornette 2007) and, as a consequence, do not influence directly (but only after a lag) the realized return that determines their profit or loss.

Insofar as the information that forms the basis of the agents’ decision is limited to the three sources discussed above (friends, news, idiosyncratic) and to the realized returns, our agents act rationally, i.e. they use all information available to them to maximize their profits. In addition, they try to learn and adapt by weighting the importance of the three sources of information according to their past performance. Since they use backward looking learning strategies with finite time horizons, our agents have bounded rationality, which reflects limits in competence, resources and available time.

One limit of the model is to exclude a mechanism that has been documented in the past to provide a significant fuel for the expansion of bubbles, namely the increasing access to financial liquidity (Caginalp et al., 2001), due for instance to changing margin requirements for loans or to the flow of foreign capital. Sornette and Zhou (Sornette and Zhou 2004) in particular document a remarkable quantitative correlation between the flux of foreign investment in the US market and the growing ICT bubble that culminated in 2000. Via the lowering of the leading interest rates, this mechanism in terms of increasing access to liquidity has probably also been a significant component of the recent real-estate bubble in the USA (Zhou and Sornette 2006, 2008). While liquidity is probably a catalyst in the development of bubbles, it may not be the nucleating and triggering factor. We thus investigate in the present model whether bubbles and crashes can develop even in situations where liquidity is constrained. This is done by endowing initially each of our agents with a finite amount of cash, and by tracking the wealth (in bonds and stocks) of each agent. Thus, the finite liquidity ensures that the stock price is mean-reverting around a fundamental value. Our model describes the dynamics of the price deviations from the fundamental value.

2.2 Three sources of information

At each time step, the agents examine and weight their available information to form their own private opinion on what the future price variation will be. Based on their private opinion, they decide whether to trade or remain inactive. The information they use has three different origins.

A first source of information is the private information, \(\epsilon_i(t)\), which may reflect the unique access to information not available publicly or the idiosyncratic subjective view of the particular agent on how the stock will perform in
the future. The private information \( \epsilon_i(t) \) is different for every agent, is taken uncorrelated across agents, and changes with every time step, \( \epsilon_i(t) \sim N(0,1) \) i.i.d. This does not mean that the agents change their mind at every time step but only that they re-evaluate their trading decision at each time step.

A second source is the public information, \( \text{news}(t) \). The public information includes the economic, financial and geo-political news that may influence the future economic performance of the stock. All agents have access to this public information but, as we shall see, may decide to weight differently its importance. To capture the ideas that the public news, \( \text{news}(t) \), is fully informational with no redundancy (Chaitin, 1987), we take \( \text{news}(t) \) to be generated as a white Gaussian noise, uncorrelated with the private information \( \epsilon_i(t) \) of the agents.

The third source of information is provided by the expected decisions of other agents. With limited access to information and finite computing power (bounded rationality), it can be shown to be optimal to imitate others (Roehner and Sornette, 2000). In our model, agents gather information on the opinions of their “neighbors” in their social network and incorporate it as an ingredient into their trading decision.

This third source of information leads to an opinion formation dynamic described by models derived from the Ising model. Many works borrow concepts from the theory of the Ising models and of phase transitions to model social interactions and organization, e.g. (Callen and Shapero, 1974; Montroll and Badger, 1974). In particular, Orléan (Orléan, 1984, 1986, 1989a,b, 1991, 1995) has captured the paradox of combining rational and imitative behavior under the name “mimetic rationality,” by developing models of mimetic contagion of investors in the stock markets which are based on irreversible processes of opinion forming. The Ising model is one of the simplest models describing the competition between the ordering force of imitation or contagion and the disordering impact of private information or idiosyncratic noise that promotes heterogeneous decisions (McCoy and Wu, 1973).

### 2.3 Opinion formation

Using the three sources of information described in the previous section, we postulate the following opinion formation algorithm for agent \( i \) at time step \( t \):

\[
\text{opinion}_i(t) = c_{1i} \cdot \sum_{j=1}^{J} k_{ij}(t-1) \cdot E_i[s_j(t)] + c_{2i} \cdot u(t-1) \cdot \text{news}(t) + c_{3i} \cdot \epsilon_i(t),
\] (1)
where $\epsilon_i(t)$ represents the private information of agent $i$, $\text{news}(t)$ are the public information, $J$ is the number of neighbors that agent $i$ polls for their opinion and $E_i[s_j(t)]$ is the expected action of the neighbor $j$ estimated by agent $i$ at time $t$. Expression (1) embodies our hypothesis that an agent forms her opinion based on a combination of the three sources of information. She weights these three sources according to her own preference, quantified by the coefficients $c_{1i}$, $c_{2i}$ and $c_{3i}$, which are different from agent to agent. To model the heterogeneity of preferences among agents with respect to the importance of the three sources of information, we endow each agent $i$ at the beginning of the dynamics with a set of three fixed values $c_{1i}$, $c_{2i}$ and $c_{3i}$ (frozen disorder), chosen randomly from three uniform distributions over the respective intervals $[0, C_1]$, $[0, C_2]$ and $[0, C_3]$.

To account for learning and adaptation, each agent $i$ can modify the weight she attributes to each source of information, according to the factors $u(t)$ for the public news and $k_{ij}(t)$ for the predicting power of the her neighbor $j$. Following the Bayesian learning literature [Kim and Verrecchia 1991a, Kandel and Pearson 1995, Veronesi 2000], we assume that, in order to update these beliefs, agents are strongly driven by their prior beliefs and by how relevant was the new information in explaining recent price changes. In Bayesian learning, the precision of how information lead to precise forecasts is of particular importance. Assuming that (1) agents represent their prior belief as well as the likelihood function of their forecasts by Gaussian distributions, and (2) agents use linear predictors, Bayesian learning leads to the following updates of the weight factors $u(t)$ and $k_{ij}(t)$:

\[
\begin{align*}
    u(t) &= \alpha \cdot u(t - 1) + r(t) \cdot \text{news}(t - 1) \cdot \frac{1 - \alpha}{\sigma_r}, \\
    k_{ij}(t) &= \alpha \cdot k_{ij}(t - 1) + r(t) \cdot E_i[s_j(t - 1)] \cdot \frac{1 - \alpha}{\sigma_r}.
\end{align*}
\]  

(2)  

(3)

Given that $0 < \alpha < 1$ and $0 < \sigma_r$, these expressions mean that the correct prediction of the sign of the realized stock return $r(t)$ from a given information source tends to reinforce the weight of that source of information, all the more so the larger the return (scaled by its volatility $\sigma_r$) and the larger the strength of the signal. We account for a finite memory over approximately $1/\ln(\alpha)$ time steps through the use of the factor $0 < \alpha < 1$. In the limit of $\alpha \to 0$, agents update their belief in the quality of each information source just on the basis of the last time step performance. In the other limit $\alpha \to 1$, the remote past as well as the present and all intermediate times remains equally relevant. Equations (2) and (3) are used to update $u$ and $k_{ij}$ at each time step.

Related models have also been developed in which agents’ successful forecasts reinforce the use of the strategies as the basis of the forecasts (Lux and Marchesi 1999, 2000, Brock and Hommes 1999, Kirman and Teyssiére 2002). Such
models have been found to generate swings in opinions, regime changes and long memory. An essential feature of these models is that agents are wrong for some of the time, but, whenever they are in the majority they are essentially right. Thus they are not systematically irrational (Kirman 1997).

Finally, for simplicity and because we do not believe that significant differences will be obtained with other networks for the specific question we investigate here, we put our agents on a virtual square lattice with $J = 4$ neighbors each, with periodic boundary conditions.

2.4 Trading decision

**When:** To account for heterogeneity in risk aversion among agents, we assume that different agents treat differently their opinion, once formed. Some agents will be satisfied by a slightly positive view on the stock to accept the risk of investing in the market. Other agents may require a much stronger convergence between the three information sources before deciding to invest. We thus assume that each agent $i$ is endowed with a fixed random threshold ‘opinion-th$_i$', so that she decides to invest when her opinion $\text{opinion}_i(t)$ becomes larger in absolute value than her threshold opinion-th$_i$. The parameter opinion-th$_i$ thus plays the role of a risk-aversion parameter, the larger opinion-th$_i$ is, the larger the risk aversion of the agent. We take these risk aversion parameters opinion-th$_i$ to be distributed among agents uniformly in the interval $[0, \text{Max-opinion-th}]$.

**Buy or sell:** The sign of the trade is simply determined by the sign of the agent’s opinion: $\text{opinion}_i(t) > 0 \to \text{‘buy’}$ and $\text{opinion}_i(t) < 0 \to \text{‘sell’}$. We implement borrowing and short-sale constraint, i.e., investors can only buy if they have the cash, and they can only sell when they have a stock in their portfolio. As discussed in section 2.1, we exclude here the mechanism for bubble formation in terms of liquidity growth and easy access to borrowed money. And, we account for the possible influence of short-sale constraints, in the spirit of (Miller 1977; Chen et al. 2002; Ofek and Richardson 2003).

**Position size:** At a given time $t$, the portfolio of a given agent $i$ is made of an amount $\text{cash}_i(t)$ of cash and of the number $\text{stock}_i(t)$ of the single stock traded on our artificial market. When an agent decides to buy, she uses a fixed fraction $g$ of her cash. When an agent decides to sell, she sells a fixed fraction $g$ of her stocks. The direction $s_i(t)$ of her decision and her action $a_i(t)$ (in units of number of stock shares) is thus given by:
- if opinion\(_i(t) > |\text{opinion-th}_i|\) : \(s_i(t) = +1\) (buying) 
  \[a_i(t) = g \cdot \frac{\text{cash}_i(t)}{\text{price}(t-1)}\]

- if opinion\(_i(t) < -|\text{opinion-th}_i|\) : \(s_i(t) = -1\) (selling) 
  \[a_i(t) = g \cdot \text{stocks}_i(t)\]

When an agent is buying, her order size \(a_i(t)\) is determined by her available cash and by the stock share price \(p(t-1)\) at the previous time step \(t-1\). The price she will have to pay to actually realize her order will be the new price \(p(t)\). This new price is determined by the price clearing mechanism that aggregates the excess demand after all the traders have submitted theirs actions.

### 2.5 Price clearing condition

Once all the agents have put their orders, the new price of the stock is determined by the following equations:

\[
r(t) = \frac{1}{\lambda \cdot N} \sum_{i=1}^{N} s_i(t) \cdot a_i(t) \tag{4}
\]

\[
\log [\text{price}(t)] = \log [\text{price}(t-1)] + r(t) \tag{5}
\]

where \(\lambda\) represents the relative impact of the excess demand upon the price, i.e. the market depth. In expression (4), we assume a linear market impact function, as a rough approximation at time scales significantly larger than the tick-per-tick time scales for which nonlinear impact functions are observed (Plerou et al., 2002).

It is important to stress that expressions (4,5) with (1) imply that the agents are not taking into account their own impact, either in deciding on investing, in sizing their position, or in adapting the relative weights of the three information sources. We believe this is not a serious problem for our goal to describe bubbles and their subsequent corrections.

Expressions (4,5) also imply the existence of a market maker, who accepts all the agents’ trades and who has an unlimited amount of cash and stocks at his disposal. We show below that the market maker on average gets a significant return for his service, while limiting the growth of his inventory of stock shares. We do not need to add any other ingredient or strategic behavior on the part of the market maker to obtain realistic results for his wealth and inventory. It is in the nature of the aggregation of the agents’ decisions in our model, that agents who try to get the best out of their sources of information overall
consistently lose against the market maker.

2.6 Cash and stock positions

We assume a frictionless market with no transaction fees. Once the return and the new price are determined by the market clearing equations (4,5), the cash and number of stocks held by each agent $i$ are updated according to

$$\text{cash}_i(t) = \text{cash}_i(t - 1) - a_i(t)p(t)$$

$$\text{stocks}_i(t) = \text{stocks}_i(t - 1) + a_i(t).$$

As we stressed above, the price paid to buy or obtained in the redemption when selling is the value $p(t)$ obtained by the aggregation of all orders through the market clearing conditions. This takes into account the real effect that too many buy orders by other agents make the realization of a buy order expensive. Similarly, too many sell orders by other agents make the realization of a sell order unfavorable, yielding a smaller-than-expected cash value. Our model thus takes correctly into account the qualitative nature of stock markets that during the placement of orders it is better to be in the minority (Challet et al., 2005).

2.7 Comparison with related models

Two lines of research are closely related to our model. The first one aims at developing a theory of “convention” (Orléan, 1984, 1986, 1989a,b, 1991, 1995), which emphasizes that even the concept of “fundamental value” may be a convention established by positive and negative feedbacks in a social system. A first notable implementation by Topol (Topol, 1991) proposes a model with an additive learning process between an ‘agent-efficient’ price dynamics and a mimetic contagion dynamics. Similar to our own set-up, the agents of Topol (Topol, 1991) adjust their bid-ask prices by combining the information from the other buyers’ bid prices, the other sellers’ ask prices and the agent’s own efficient price corresponding to his knowledge of the economic fundamentals. Topol (Topol, 1991) shows that mimetic contagion provides a mechanism for excess volatility. Another implementation by Wyart and Bouchaud (Wyart and Bouchaud, 2007) of the concept of convention shows that agents who use strategies based on the past correlations between some news and returns may actually produce by their trading decisions the very correlation that they postulated, even when there is no a priori economic basis for such correlation. The fact that agents trade on the basis of how the information forecasts the return is reminiscent of our model, with however several important differences.
The first important conceptual change is that Wyart and Bouchaud (Wyart and Bouchaud, 2007) do not take into account the effect of imitation through the social network. The second difference is in the correlation that the agents calculate to adapt their strategies. In (Wyart and Bouchaud, 2007), the agents’ payoffs are controlled by the level of correlation between the news and the return resulting immediately from their aggregate action based on that news (this takes into account the agents’ own impact). Our agents’ payoffs are determined by the correlation between their information and the return one time step later, which is the first time step that defines if the action was profitable or not.

The second line of research closely related to our model is known as “information cascades.” “An informational cascade occurs when it is optimal for an individual having observed the action of those ahead of him, to follow the behavior of the preceding individual without regard to his own information” (p. 994, Bikhchandani et al., 1992)). In these models, the agents know that they have only limited information and in order to complement it, they look around and turn to their neighbors for tips. The reason why agents imitate is that they think the other agents together have more information. Bikhchandani et al. showed that the fact that agents use the decisions of other agents to make their own decision will lead with probability 1 to an informational cascade (Bikhchandani et al., 1992) under conditions where the decisions are sequential, one agent after another, and irreversible. This model was later generalized by A. Orlean (Orléan, 1995) into a non-sequential version. Even under these more relaxed conditions, informational cascades were found to still be possible.

The main differences between the “information cascade” model and ours are the following. Informational cascades models assume that the “truth” exists, that there is a true fundamental price or a correct choice to be made. The agents are trying to guess what is this correct choice or what is the real fundamental value using both their private information and the information they obtain from the choices made by other agents. In our model, the agents are also using the decisions of their neighbors to determine how to act but the reason is different. Our agents are not so much interested in the fundamental value of the stock, but more in its future directions. They analyze if their neighbors performed well in the past, i.e. if they did predict well the future returns. If they did not, the agents tend to ignore what their neighbors are doing and concentrate on the news and their own private information to shape their trading decision. In times when their neighbors did successfully predict future returns (better than the news), they give more weight to their judgment and tend to ignore the news, which translates into strengthened imitation. Another major difference is that, in the informational cascade framework, which choice actually was the right one (i.e. what was the fundamental value of the stock) is exogenously given. The agents have no influence on the outcome,
whereas in our model the outcome, whether selling or buying a stock was the right choice, is endogenously emerging from the aggregated choices of all agents. There is no a priori right or wrong answer, it is decided during the process.

Our model is better suited for the description of financial crises, during which fundamental values do not change much and appear to be of minor relevance. The prices can drop and/or shoot up dramatically without any significant change of the fundamental economic outlooks. Just because many traders were selling their stocks and the price actually went down, those traders seem to have predicted well the returns, so other traders will jump on the bandwagon (believing that the first traders are revealing an information that they do not possess themselves) and sell their stocks too, which will make the price go down even further. In that extended sense, an information cascade emerges.

3 Results of the model

3.1 General properties

Our model is an idealized “test tube” representation of a financial market, aiming to obtain an understanding of how the interplay of news, herding and private information concur in the formation of rallies and crashes. Given the simplifications put into the model, we do not aim at reproducing faithful statistical characteristics of realistic price dynamics. We first point out a few properties of the dynamics of the prices, that derive straightforwardly from the model set-up.

Because we model a closed system, the price has an upper and lower bound. This is easy to see from the fact that agents always invest a ratio of their present cash into stocks. Thus, a local maximum value of the price is attained as soon as all the agents have use all their money to buy stocks. When no agents are left to buy and push the price further up, the price reaches a high point and can only go down afterwards. The specific high point is path dependent due to the non-zero-sum game mechanism associated with the price clearing condition. The high points are bounded from above by the absolute maximum price that would be obtained in the hypothetical case where the agents rebound from a previous minimum by all being buyers at the same time. A symmetric situation describes selling orders, that tend to push the price down. Since our model describes deviations from a hidden reference fundamental value, the existence of the upper and lower bounds for the price dynamics can be thought of as expressing a mean-reversal process in which the total price does not deviate too much from this fundamental price (i.e.,
the observed price is co-integrated with the fundamental price).

Moreover, this phenomenon is associated with a progressive exhaustion of price appreciation during run-ups, resulting from the decreasing buying powers of cash-restricted buyers: the larger the stock price, the smaller the number of stock shares that can be acquired per unit of cash. In this way, our set-up eliminates the possibility of forming exponentially or super-exponentially growing bubbles as for instance documented in \(\text{(Sornette 2003)}\). It puts rather the emphasis on the dynamics of the nucleation of the bubbles and on their corrections, i.e., the nucleation of their reversal. Symmetrically, when the price is low, a few buyers can move it significantly with moderate order sizes.

In our simulations, we fix the number of agents in the system to \(N = 2500\), the market depth to \(\lambda = 0.25\), the maximal individual conviction threshold to \(\text{Max-opinion-th} = 2.0\), the initial amount of cash and stocks held by each agent to \(\text{cash}_i(0) = 1\) and \(\text{stocks}_i(0) = 1\), and the memory discount factor to \(\alpha = 0.95\), corresponding to a characteristic time of \(1 / \ln(\alpha) \approx 20\) time steps. Setting \(C_1 = C_2 = C_3 = 1.0\), figure 1 shows a typical realization of the time evolution of the price \(p(t)\), of the one-time-step return \(r(t)\) and of the average over all agents of the weight factors \(\langle u(t) \rangle\) and \(\langle k_{ij}(t) \rangle\) used by the agents to assess the relevance of the three information sources. The time series of one-time-step returns exhibit clear evidence of clustered volatility. The middle right panel shows the distribution of returns with clear evidence of a non-Gaussian fat tail structure. The lower right panel shows the absence of correlation between returns together with the presence of non-negligible correlation of the volatility (here measured as the absolute value of the returns).

While the information source \(\text{news}(t)\) in 1 are i.i.d. white noise, the dynamics of \(u(t)\) are given by 2 are far from white noise. One of the reason for this structure is that expression 2 has an AR(1) component with the autoregressive coefficient \(\alpha = 0.95\) which ensures a rather large long-term memory. If the other terms in 2 were noise, this AR(1) component would just have the effect of reddening the noise, changing the white spectrum into a \(1/f^2\) spectrum. But the middle left panel of figure 1 exhibits clustered positive and negative spikes which are very different from the patterns expected from a simple AR(1) process. These spikes reflect the collective herding of agents through their propensity \(k_{ij}(t)\) to imitate: the lower left panel shows that the average \(\langle k_{ij}(t) \rangle\) over all agents of the propensity to imitate also moves concomitantly with strong spikes. It is interesting to note that the average weight \(u(t)\) that agents attribute to the economic news can become at some times strongly negative, reflecting a contrarian interpretation of the news. This reflects the fact that the same kind of news can have a different impact on the price, depending on the circumstances and present dominating convention \(\text{(Orléan 1995)}\). The lower left panel shows that the average propensity to imitate remains positive.
at all times, showing that the agents find consistently some value in imitating their neighbors. This is not very surprising, given the fact that the obtained results are indeed the result of the aggregation of the group of investors. This is also warranted because there is no explicit contrarian strategies that the investors may activate.

The crucial parameters of our model are the parameters $C_1, C_2, C_3$, which control the level of heterogeneity of the innate agents preferences for the three different types of information. Changing these parameters changes the way the agents behave in ways that we now explore systematically.

### 3.2 $C_1$-dependence

Each agent is endowed with a fixed individual preference level controlling how much she takes into account the information coming from the actions of their neighbors. This level is different from agent to agent, and is drawn from a uniform distribution in the interval $[0, C_1]$. Thus, the parameter $C_1$ sets the maximal innate propensity to imitate, the larger $C_1$ is, the larger are the average imitation and the herding effects.

Figure 2 plots the evolution of the price for three different values of $C_1$, all other parameters, including the seed of the random number generator, remaining the same. For zero propensity to imitate ($C_1 = 0$), some price spikes can be observed, which are generated by the news only, whose influence can be amplified by the positive feedback resulting from learning that tends to increase the relevance that investors attribute to news after a lucky run of news of the same signs. For $C_1 = 2.0$, one can observe that these peaks are amplified due to the imitation now also contributing to the agents’ actions. For $C_1 = 4.0$, a qualitatively different price evolution appears. For such large values of the average propensity to imitate, the price is driven to its extremes, its dynamics being slowed down by the agents’ finite cash and stock portfolio reaching their boundaries. We show below that this reveals an underlying on-off activation of the phase transition underlying the Ising model.

To better illustrate the effect of increasing $C_1$, figure 3 shows the average weight factor $\langle k_{ij}(t) \rangle$ used by the agents to assess the relevance of the information source stemming from their neighbors for the same parameters and realization of the noise as in figure 2. In periods of high $\langle k_{ij}(t) \rangle$, agents are very much influenced by their neighbors and herding follows. During these periods, large price fluctuations are associated with extreme behavior of $\langle k_{ij}(t) \rangle$. Increasing $C_1$, agents are by default more susceptible to the decisions taken by their neighbors, making them more likely to act in the same way and more prone to synchronising their actions. Consequently, since the price dynamics is
governed by the aggregate demand, the neighbors’ information is more likely
to be a good predictor of the return and the agents adapt to that situation by
giving this information even more weight via an increase of their $k_{ij}(t)$-values.
Due to this positive feedback loop, a qualitatively different regime appears
upon increasing $C_1$.

The existence of a bifurcation beyond which a new regime appears is docu-
mented in figure 1. Its left panel shows the 10 largest values of $\langle k_{ij}(t) \rangle$ picked
out of a pool of 40 runs, each of more than 15000 time steps, simulated with
the same parameters $C_2 = C_3 = 1$ for each value of $C_1$. One can observe
a rather abrupt transition occurring at around $C_1 = 2$. Naively, one would
surmise that this transition is akin to a “first-order phase transition” to use
the language of physics or to a “sub-critical bifurcation” to use the language
of mathematics. But, given the definition (3), $\langle k_{ij}(t) \rangle$ can be considered a
proxy for the underlying control parameter; then for a pure Ising model in
two dimension, it is known that the order parameter exhibits a continuous
transition at some critical value of $C_1$ with a small exponent known exactly
since Onsager to be $\beta = 1/8$. In the presence of quenched disorder of the
random field type, a continuous phase transition is believed to still be present
but with a still smaller value of the exponent $\beta \approx 0.02$ (Cao and Machta,
1993). The same qualitative results hold for Ising models with disorder in the
coupling coefficients as long as their average is strictly positive. In our case, in
the presence of the learning feedback loops, we cannot ascertain the nature of
the transition. We just note that it would be extremely difficult just based on
numerical simulations to distinguish between a continuous critical transition
with a small exponent $\beta$ and a first-order abrupt transition. For our purpose,
we just record the observation that there is an abrupt change of regime as
a function of $C_1$. The existence of this change of regime explains the radical
difference of properties shown in figures 2 and 3 from $C_1 = 0, 2$ to $C_1 = 4$.
The jump in $\langle k_{ij}(t) \rangle$ at $C_1 \approx 2$ is mirrored by a similar transition in the values
of the largest possible drawdowns and drawups as a function of $C_1$ (see top
right plot in Fig. 4). This reveals a second regime of very large price moves
for $C_1 > 2$, that are characteristics of the existence of “outliers/kings,” i.e.,
market events that are fundamentally different from the rest (Johansen and
Sornette, 1998, 2001) (see (Sornette, 2003), chapter 3 for a general introduc-
tion on how the concept of “outliers/kings” reveal different transient market
phases).

Having plotted the dependency of the extreme values of $\langle k_{ij}(t) \rangle$ on $C_1$ for the
model with i.i.d. news, we also show in the left panel of figure 4 a dashed line
which represents the mean outcome of the same observable performed when
the news exhibit a persistence described by a fractional Brownian noise with
Hurst exponent equal to 0.55. Under this situation where the news exhibit
a positive long-range auto-correlation, the qualitative picture of the change
of regime is not altered. The only noticeable difference is that the critical
value of $C_1$ at which the transition occurs has shifted to a smaller value. This results from the fact that sequences of news having the same sign are now more likely to occur and a weaker intrinsic imitation propensity is sufficient to lead to the strong herding regime. For higher values of the Hurst exponent, the critical value of $C_1$ will only shift further to the right. Apart from this shift, the results reported here for i.i.d. news hold for news with persistence and, in particular, so does the main result on the existence of strong market moves amplified by the interplay between imitation and Bayesian learning (the larger the auto-correlation of the news, the stronger the effect).

Below $C_1 = 2$, agents do not give the information provided by their neighbors enough importance to trigger a feedback loop that lead to synchronized actions. For $C_1 < 2$, the market is approximately efficient and the price fluctuates rather closely around its fundamental value. The price fluctuations, which are induced by the news modeled as a white Gaussian noise, exhibit however characteristics very different from white Gaussian noise. First, the learning process can amplify the occurrence of runs of news of the same sign, leading to amplified price fluctuations. Second, a small but nevertheless present propensity to imitate further amplifies the price fluctuations in response to the news input.

A first interesting conclusion can be drawn. The excess volatility, documented since Shiller (Shiller, 1981), appears here through a simple mechanism. Investors who do not fully believe that news (such as dividends) is the only source of relevant information may be led to try to learn how well the news predicts the market returns. A straightforward learning algorithm with reinforcement leads unavoidably to a strong reinforcement of the variability of prices, due to the memory of the learning procedure amplifying the random occurrence of runs of news of the same sign. Paradoxically, it is the attempt of investors to learn the level of relevance of the news on the price formation which leads to a dramatic amplification of the price volatility due to their collective search for the “truth.” This may be thought of as another embodiment of the “illusion of control” found in the Minority and the Parrondo games (Satinover and Sornette 2007a, 2008), according to which sophisticated strategies are found to under-perform simple ones.

For $C_1 > 2.0$, $\langle k_{ij}(t) \rangle$ exhibits extreme values, resulting in large price deviations from the fundamental price, as shown in figure 2. In this regime, the market is in an easily excitable state, in which the agents are uncertain on how the market will perform in the future. They listen to the news but infer that there might be a crash coming or a new market with immense profit opportunities. If they can see that some of their colleagues make money consistently on the market, they conclude that their colleagues might know more than they do and start imitating them. By following them, they amplify the departure of the price from its fundamental value and so influence other traders who have
remained on the sideline. The resulting price dynamics can be characterized as a crash or a boom.

Another interesting characteristic of the herding regime $C_1 > 2.0$ is that it is very difficult to infer this regime when looking at the properties of the price outside those transient episodes of booms or crashes. Indeed, outside these special moments of exuberance, the market behaves normally as in the regime $C_1 < 2$. Bubbles and crashes do not belong to the normal regular dynamics of the model, they are only experienced when certain conditions are fulfilled, they are outliers/kings in the sense of Johansen and Sornette (Johansen and Sornette, 1998, 2001). The right panel of figure 4 indeed shows the appearance of an extremely fat tail in the distribution of $\langle k_{ij}(t) \rangle$ over the ensemble of different realizations as a function of time for $C_1 = 4$, while the bulk of the distribution remains approximately the same as for the smaller values of $C_1$. This is the evidence for the existence of a second class of transient regimes, the booms and crashes, coexisting with the more normal dynamics of the prices.

We thus come to our second important conclusion: the present model provides a simple mechanism for the existence of two populations in the distribution of prices, exemplifying the concept that booms and crashes are qualitatively different from the rest of the price moves. The second population of boom-crash outliers/kings appears when the innate propensity to herd reaches a threshold above which a self-reinforcing positive feedback loop starts to operate intermittently.

In the next subsection, we take a closer look at the conditions under which such outliers/kings occur.

### 3.3 News Impact

To clarify the mechanisms leading to the previous described dynamics, we now present some details on the microscale dynamics of the model.

#### 3.3.1 Dispersed regime: $C_1 < 2$

Figure 5 shows a typical realization with $C_1 = C_2 = C_3 = 1.0$, which should be compared with a realization with $C_1 = 3$ and $C_2 = C_3 = 1.0$ whose characteristics are illustrated in figures 6 and 7.

Figure 5 is focused at the dynamics of the key variables around a specific episode occurring around the time $t = 4800$. Around this time, the price crashes suddenly and then rebounds rapidly, and then relaxes slowly to its pre-existing level (zero level = fundamental value). One can see that the averages
over all agents of the weight factors $\langle u(t) \rangle$ and $\langle k_{ij}(t) \rangle$ used by the agents to assess the relative relevance of the three information sources exhibit sharp increases occurring at the same time. Actually, one can trace the origin of this burst to the random occurrence of a sequence of same signed news, shown in the lower right panel of figure 5. Recall that we assume that the news are independently and identically distributed. Thus the dip structure in the lower right panel of figure 5 is purely “bad luck”, i.e. a stream of small bad news impact here the market. The response of the agents to these run of bad news develops as follows. The observation of $\text{news}(t)$ gives the agents an information about the next return $r(t+1)$, but in order to profit from this insight, the agents have to act before $t+1$, i.e. they use the $\text{news}(t)$ to buy or sell at time $t$. Therefore, a burst of activity, which has its origin in the news, can only occur if the sign of the news is, by chance, the same for several time steps as it is the case from $t = 4799$ to 4809. As a result of their continuous reassessment of the relevance of the news in predicting the stock price, the investors amplify this bad news and this triggers a transient herding run to the exit (the crash) followed by a fast reassessment, the rally, and then a slow cooling off when the subsequent news comes back to the standard noisy featureless characteristics.

Let us report minutely the micro dynamics of the model to better understand this burst of activity. At $t = 4799$, the news turns out to be negative, which suggests to the agents that the price may drop from $t = 4800$ to $t = 4801$. Some agents may sell, starting to bias slightly the price downward. If the price has dropped previously, the agents have started to upgrade the weight they attribute to the news in their learning process. Then at $t = 4800$, the news is by chance again negative. Having put some more weight on the news, more agents decide to sell some of their stock, whose collective action results in a negative return. This negative return has a larger positive impact on the weight $u$ because $\text{news}(4799)$ was a good predictor. The exponential growth of the weight $u$ continues as long as the sequence of negative news goes on, further amplifying the impact of the news on the agents’ decision and therefore on the price. Note that the average weight $\langle k_{ij} \rangle$ of the propensity to imitate also exhibits a fast acceleration followed by a slower decay. This is due to the fact that the agents find that imitation is also a good predictor of the returns, since a majority of agents are now herding by following the news. By this process, there is an amplification of the response of the whole herd to the exogenous news. When the run of bad news stops, it takes about $\approx 1/\ln(\alpha) \approx 20$ times steps for $u$ to relax back to its previous value. In this example, the maximum of $u(t)$ occurs at $t = 4807$. At $t = 4808$, $u$ decreases lightly due to the small amplitude of the news at $t = 4807$. Once $u$ has reached a certain level, the product $u(t-1) \cdot r(t)$ and $\text{news}(t)$ always have the same sign, which means that the news totally dominates the determination of the returns. At $t = 4810$, $u$ exhibits a sharp decrease, which is due to the fact that the news changed sign from $t = 4809$ to 4810. Because the agents are in regime where they overweight so much the relevant of the news, they massively buy...
making the return positive at $t = 4810$. But the update of $u(4810)$ is done with $\text{news}(4809)$, which was negative, so that the news did not predict well the future return. After that, $u$ decreases exponentially fast because the news resuming its usual random switching signs is no longer a good predictor of the return, i.e. the market recovers its average level of efficiency.

This case study illustrates that the occurrence of bursts of price variations is nothing but the amplification of runs of same-sign news, which leads to an exponential growth of the news weighting factor $u$, which itself increases dramatically the sensitivity of the agents to all future news. This heightened sensitivity lasts over a characteristic scale determined by the coefficient $\alpha$ governing the memory of the learning process \[2\]. The Bayesian learning process of agents’ adaptation to the three different sources of information, together with the random lucky or unlucky occurrence of runs of news of the same quality, is at the origin of the occurrence of strong rallies and crashes in the price forming process. In our model, rallies and crashes reflect a combination of randomness and amplification by learning and some herding.

### 3.3.2 Herding regime: $C_1 > 2$

We illustrate the strong imitation regime in figure 6 for $C_1 = 3.0$. A large price drop is observed as a result of a run of negative news lasting slightly longer than usual. The mechanism at the origin of this burst of activity is similar to that described in the previous section, except that the large value of the average innate propensity to imitate makes the role of herding dominant. The burst of activity starts with the random occurrence of a run of same-sign news. As a consequence, the predictive powers of both the news and the imitation term exhibit a streak of success leading to the fast growth the weight factors $u$ and $\langle k \rangle$ through the Bayesian learning process used by the agents. When the sequence of same-sign news is sufficiently long (and it does not need to be that long for $C_1$ significantly larger than the threshold value 2, only a few time steps), the level reached by the imitation weight $\langle k \rangle$ becomes so large that the agents are strongly imitating each other, pushing the price down (up) if the news is negative (positive). In the regime $C_1 > 2$, it is the growth of the imitation weight $\langle k \rangle$ which controls the dynamics of the crash or of the rally, in contrast with the regime $C_1 < 2$ where the news weight $u$ is the dominating variable.

A consequence of the strong herding propensity for $C_1 > 2$ is that, once a price rally or a crash is started, the dominating impact of the herd both in the Bayesian learning and in their price impact makes the price trend self-reinforcing and basically independent of the sign of the news. This explains the very large amplitude of the crash shown in figure 6 which is typical of this regime. However, the rally or crash becomes progressively exhausted as the
traders saturate their position: during a crash, they all strive to sell, to get all in cash. At first, their sell positions are large which moves the price significantly. When their portfolios are almost out of stock, their selling pressure become negligible and the downward price trend levels off. But their Bayesian learning process also implies that they progressively lose their previous insight on the predictive power of imitation, since \( r(t) \cdot E_i[s_{j(t-1)}] \) becomes small. As a consequence, their imitation weights \( k_{ij} \) decreases, leading to less and less herding, smaller collective action and thus smaller and smaller volatility. It may then happen that some agents, because of some positive private information, will buy some stocks. At this point, most of the agents have a large amount of cash, while the price per share is very low, so that they will get a large quantity of shares for \( g \cdot \text{cash}_i(t) \). This results in a large increase of the price. This will lead to the end of the first trend (negative in the example shown in figures 6 and 7) and the price will bounce back to its fundamental value. Figure 7 exemplifies this process by showing a magnification of figure 6.

In this regime \( C_1 > 2 \), large price fluctuations can occur (and the largest ones do occur) without any significant news, in agreement with reported evidence in the literature (Cutler et al. 1989). Our bubbles and crashes are indeed nucleated by news, but the magnitude of the news has nothing to do with the magnitude of the resulting returns. In contrast with the Efficient Market Hypothesis according to which large price variations should be only due to large exogenous shocks, our model illustrates a process based on an endogenous amplification of small news. We have illustrated two mechanisms which can simultaneously reinforce each other: Bayesian learning/adaptation and imitation. This reconciles the two opposite (herding versus fundamental) proposals for the origin of crashes within a single framework.

### 3.4 Wealth evolution

Figure 8 shows the time evolution of the agents’ and market maker’s cash and share holding in the “dispersed regime” \( C_1 < 2 \), over a very long time interval of \( 10^5 \) time steps. The main message of this figure is that the traders lose on average over the long time, even if they enjoy short streaks of gains. Correspondingly, the market gains over the long run. This occurs without the deployment of any strategy of his part and results only from the fact that the price rises when there is more demand than supply and vice versa, with a one-time step delay that the agents are unable to arbitrage. Note that every stock that is not in the agents’ hands belongs to the market maker (agents trade fractions of their belongings with an initial endowment of one monetary unit of cash and one stock share).
Figure 9 shows the impact of a crash on the wealth distribution of the agents. The wealth of an agent at a given time $t$ is defined by the sum of their cash at that time plus the number of shares they own multiplied by the value of the share at $t$. One can observe a dramatic change in the distributions of wealth before and after the crash. The main characteristics of the agents which governs the good versus bad performance through the occurrence of a crash or a rally is the opinion threshold \( \text{‘opinion-th’} \), which reflects the agents’ risk aversion. Only when her opinion is sufficiently polarized so as to pass this threshold will an agent decide to enter a trade. As defined in section 2.4, these thresholds are distributed uniformly in the interval \([0, \text{Max-opinion-th}]\), where Max-opinion-th has been fixed to the value 2 throughout the paper. Therefore, some traders have low opinion thresholds, making them trade on the basis of small information biases. Other traders at the other end of the spectrum of ‘opinion-th’ values have strong risk-aversions, so that they require a strong convergence between the exogenous news, their idiosyncratic information and the insight coming from their neighbors, before deciding to enter a trade. Traders with low thresholds are more reactive than more prudent agents (with higher opinion-th) who require much stronger signals before entering into trade. As a crash develops, the more risk-averse agents adjust their position with a larger delay and thus suffer the most during times of rapid changes, compared with low-threshold traders. The inset on the left in figure 9 shows that the distribution of wealth of agents with large opinion threshold is narrow and concentrated around their initial wealth, simply because they tend to be conservative and do not enter the market much. In contrast, the agents with low ‘opinion-th’ are more broadly distributed, reflecting their varied investment performances. After a crash, the agents with low ‘opinion-th’ are even more broadly distributed, with some agents having made significant gains or losses. In contrast, all agents with large ‘opinion-th’ have lost a lot of money during the crash because their cautious attitude led them to react too slowly to the dropping price, thus taking losses by selling too low too late and by buying too high too late during the ensuing price rebound. The same dynamics are observed during a boom of the price.

We find that the distribution of wealth after many such crashes and rallies do not evolve appreciably compared with that shown in the right inset of figure 9 developing a fat tail, approximately described by an exponential fall-off, in line with empirical observations reported by V.M. Yakovenko and collaborators (Dragulescu and Yakovenko 2001; Prange and Yakovenko 2004).
4 Conclusion

In this paper, we have addressed two major questions in finance:

- Why do rallies/bubbles and crashes exist?
- How to they emerge?

This is done by using an agent based model where the actions of the trading agents are determined by their anticipation of the future price changes. This anticipation is formed by consolidating three different sources of information: private information, public information (news) and information from their neighbors in their network of professional acquaintance. To rate these different sources of information, our agents are endowed with a learning capacity. They can dynamically weight the importance of the different sources to their opinion formation based on the recent past predicting performance. In this way, our agents are scanning the influx of information in search of a reasonable good predictor for future returns and are constantly adapting their strategy to the environment to adapt their behavior if an opportunity arises.

We find that two regimes appear, depending on how strong the agents are influenced by their neighbors on average (controlled by the parameter $C_1$). In the regime of small $C_1$’s, the low herding regime, the agents are sometimes more influenced by the news and sometimes more by their neighbors, but due to the small amount of trust they put into their neighbors by default, they do not get carried away in over-imitating their neighbors if the latter, for a short time interval, seem to be good predictors. The returns are mostly driven by the global and idiosyncratic news. The resulting market is approximately efficient for small $C_1$.

We find that the return distribution is however quite different from the one describing the exogeneous news. Our simple agents are able to transform the string of independent news distributed normally (both for the global and idiosyncratic news) into a return distribution with fatter than exponential tails, showing a clear sign of excess volatility. Also clustered volatility and a non-zero autocorrelation in the absolute returns are observed while the returns themselves remain uncorrelated, in agreement with absence of arbitrage opportunities (at least at the linear correlation level). These different properties show that our simple model can reproduce some important stylized facts of the stock market, and can motivate the possibility to test its prediction in other market regimes.

By increasing $C_1$ to be larger than a certain critical value, the system enters a second regime where the agents give on average more importance to their neighbors’ actions than to the other pieces of information. When increasing the awareness of their neighbors’ actions, the agents are more likely to syn-
chronize their actions, which increases the probability that the direction of the return results in the predicted direction, which then again increases their trust in these good predicting neighbors. Due to this positive feedback loop, the average coefficients $\langle k_{ij} \rangle$ (the dynamic trust of agent $i$ in agent $j$) can surpass a critical value and the agents’ opinions are dominated by only this information term, resulting in series of consistently large same-signed returns. Because the agents are always trying to maximize their returns, it is rational for the agents to follow the majority in these herding times and to “surf” the bubble or the crash. This regime is characterized by large deviations from the fundamental price resulting from a synchronization of the agents’ actions due to their local adaptation of their strategy to the mood of the market. Not only is it rational to follow the herd, we also showed that the agents who are early imitators of the herd in the early stage of a crash/boom are those who will accumulate the most wealth and will belong to the wealthiest among all the agents after the market has returned to its normal behavior. This is the contribution to the ‘why’ of the ‘crash/bubble problem’, the ‘how’ is treated in the following paragraph.

We have determined that the origin of these large deviations from the fundamental price nucleate from the news. A random occurrence of a sequence of same signed news pushes the price in one direction and starts the synchronization process of the agents. This situation is quite realistic as economic news often exhibits persistence, so that the traders get overconfident (string of good news) or too pessimistic (series of bad news) on the future of the economy (as occurs during a financial crisis, such as the subprime crisis for example) which makes the market go preferably in one direction. Once the market has reacted by a significant drop in some index (in the case of a crisis), traders might seek advice from some of their professional colleagues. If those colleagues tell them that they will reduce their exposure to that market and they trust these colleagues, they will follow their advice, resulting in the same kind of behavior produced by our model.

However by increasing $C_1$ to a high value, we do not change the average behavior and properties of the resulting dynamics of the model. Outside of these large price variations occurring during rare bubbles and crashes, the dynamics looks similar to that documented in the low $C_1$ regime, i.e. like an efficient market. The difference is that, with large $C_1$, there is a larger probability for a crash/boom to happen, with drastically different statistical characteristics, showing the extraordinary character of these crashes, as proposed by Sornette et al. (Johansen and Sornette 1998, 2001; Sornette 2003) based on the empirical analysis of drawdowns.

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Fig. 1. This figure shows a typical realization of the major observables of the system. These observables are the time evolution of the price $p(t)$ (upper left panel), the one-time-step return $r(t)$ with clear evidence of clustered volatility (upper right panel), and the average over all agents of the weight factors $\langle u(t) \rangle$ (middle left panel) and $\langle k_{ij}(t) \rangle$ (lower left panel). The middle right panel shows the distribution of returns: the linear-log scales would qualify a Gaussian distribution as an inverted parabola, a double-exponential as a double tent made of two straight lines; in contrast, one can observe a strong upward curvature in the tail of this distribution, qualifying a fat-tail property compatible with a stretched exponential or power law. The lower right panel shows the absence of correlation between returns together with the presence of non-negligible correlation of the volatility (here measured as the absolute value of the returns). Note the positive value of the correlation of the volatility up to a time about 25 time steps, followed by a small negative value up to 80 time steps. The time scale of the correlation of volatility is set by the memory factor $\alpha = 0.95$ corresponding to a characteristic time scale of 20 time steps. These results are obtained for $C_1 = C_2 = C_3 = 1.0$, and frozen weights attributed by the agents to the three information sources drawn out of a uniform distribution from 0 to $C_1, C_2, C_3$, respectively.
Fig. 2. Evolution of the price for $C_1 = 0.0, 2.0, 4.0$ and $C_2 = C_3 = 1.0$ with the same random seed for the three realizations, resulting in the same realisation of the news for the different runs (shown in the bottom panel). Note the much larger scale of the y-axes for $C_1 = 4.0$. 
Fig. 3. Time evolution of the average over all agents of the weight factor $\langle k_{ij}(t) \rangle$ used by the agents to assess the relevance of the information source stemming from their neighbors, for $C_1 = 0.0, 2.0, 4.0$ and $C_2 = C_3 = 1.0$ with the same random seed for three realizations.
Fig. 4. Top Left: Values of the 10 largest peaks in $\langle k \rangle (t)$ for $C_1$ ranging from 0 to 4, illustrating the existence of a sharp transition between two regimes with the news being uncorrelated Gaussian noise (i.e. fractional Brownian noise with an Hurst exponent of 0.5). The dashed line is the average of the 10 largest peaks in $\langle k \rangle (t)$ realized with correlated news (fractional Brownian noise with an Hurst exponent of 0.55). Top Right: Values for the 10 largest drawdown and drawups with uncorrelated (crosses) and correlated (circles) noise for the news showing the same behavior as for the largest peaks in $\langle k \rangle$. Bottom: Normalized histogram of $\langle k_{ij}(t) \rangle$ for different values of $C_1$. 

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Fig. 5. Time series of several characteristic variables showing the response to the news for $C_1 = 1$ (dispersed regime) and $C_2 = C_3 = 1.0$. Upper left panel: a portion of the price time series with a crash following by a rally. Upper right panel: a magnification of the upper left panel around the crash-rally. Middle left panel: the time series of the returns. Middle right panel: the weight $u$ of the news showing a fast growth over the time interval in which the news are all negative, followed by a decay over a time scale given by $1/\ln(\alpha) \approx 20$ time steps. Lower left panel: The average weight $\langle k_{ij} \rangle$ of the propensity to imitate also exhibits a fast acceleration followed by a slower decay. Lower right panel: the time series of news, generated as a white noise, which can nevertheless exhibit runs of same-sign values.
Fig. 6. Evolution of the same observables as in figure 5 for a realization with $C_1 = 3$ (herding regime) and $C_2 = C_3 = 1.0$. 
Fig. 7. Magnification of the realization of the crash shown in figure 6. Top to bottom: plots of the price, return, activity, news weight factor and average imitation factor, as a function of time.
Fig. 8. Evolution of the average number of shares and cash of the agents and of the market maker for $C_1 = C_2 = C_3 = 1.0$. With the market clearing condition implemented here, the market maker makes money on the long run.
Fig. 9. Price evolution for a system with $C_1 = 3.0$, $C_2 = C_3 = 1.0$ around a crash. The two insets show the histograms of the agents’ wealth before (left inset) and after (right inset) the crash. The three curves correspond to agents with different values of their opinion threshold parameter ‘opinion-th’, as defined in section 2.4, which controls their risk aversion: continuous line (all agents); dotted line (agents with high ‘opinion-th’, i.e. high risk aversion); thick dashed line (agents with low ‘opinion-th’, i.e. low risk aversion).