Convective fluid flows in a horizontal channel with evaporation: analytical and experimental investigations

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Abstract. Heat- and mass transfer processes in a two-layer system of the liquid and gas are studied with respect to evaporation at interface. The stationary convective flows of two immiscible viscous incompressible fluids filling an infinite channel and being under action of the transverse gravitation field are studied analytically. Mathematical modeling of the flows is carried out with the help of the Navier-Stokes equations in Boussinesq approximation. The Dufour and Soret effects are taken into consideration in the gas-vapor phase. In the two-dimensional case the exact solutions of special type are constructed under condition of a given specific gas flow rate. Comparison of the analytical results with results of the physical experiments with the "liquid-gas" system like "ethanol-air" are presented.

1. Introduction

Investigations of the nonisothermal flows of fluids, complicated by the interfaces, are stimulated by necessity of explanation of results of the scientific and technological experiments aimed at the study the vortex fluid structures in the open cavities and in the moving layers, of the experiments directed to the study the film motion under action of the gas flows [1]. The study of exact solutions of a special type allows one to consider the influence of various physical factors on fluid flows [2-8]. Much attention is paid to investigation of flows in a two-layer system accompanied by evaporation. One of the first examples of the solutions of the problems with mass transfer at the interface has been obtained in [4]. A number of works are devoted to the investigation of convective flows with a thermocapillary interface under the action of a co-current gas flow and longitudinal temperature gradients (see [5] and [6-8]). In [7] the results of the comparison of analytical data obtained using the exact solutions with experimental data are shown. The results of study of the stability of the two-layer flows with evaporation on the thermocapillary interface are presented in [9-11] (see also [12]).

In the present paper the analytical and experimental investigations of the liquid flows accompanied by a gas flux with evaporation at a thermocapillary interface are performed. In the upper layer a process of vapor diffusion is taken into account. To characterize the flow topology the examples of the velocity profiles and temperature distributions for the system "ethanol and air" are presented.
2. Mathematical model of the process

Let two immiscible viscous incompressible fluids (the liquid and the gas-vapor mixture) fill the infinite horizontal layers of depth equal to \( l \) and \( h \), correspondingly. The upper and lower walls of the channel are the fixed impermeable boundaries. A common gas-liquid thermocapillary interface, remaining to be undeformed one, is given here by \( y = 0 \). The coordinate system is chosen so that the gravity acceleration vector is directed opposite to the \( Oy \) axis. The system of differential equations based on the Oberbeck-Boussinesq approximation of the Navier-Stokes equations [13] is used to model the flows of the liquid and gas and to find the unknown functions. Concentration of the vapor understood as a passive impurity in the gas layer is determined with the help of the diffusion equation. The Soret and Dufour are taken into consideration in the mathematical model [13, 14].

Let the solution of the system of equations be the solution of the Ostroumov-Birikh type [2, 3] in the form:

\[
\begin{align*}
  u_i &= u_i(y), \quad v_i = 0, \quad p'_j = p'_j(x, y), \quad T_i = (a^1_i + a^2_i y)x + \partial_i(y), \quad C = (h_1 + b_2 y)x + \phi(y). \\
\end{align*}
\]

Here \( u_i, v_i \) are the velocity components, \( T_i \) is the temperature, \( p'_j \) is the modified pressure and \( C \) is the vapor concentration. The coefficients \( a^j_i \) and \( b_j \) \( (j = 0, 1) \) (the longitudinal gradients of temperature and concentration) are determined with the help of the boundary conditions for concentration and temperature. The functions \( u_i \) are defined as fourth-degree polynomials, \( \partial_i \) and \( \phi \) are the polynomials of the seventh degree, \( p' \) is the eighth degree polynomial on the transverse coordinate \( y \) (see [7]).

Let us formulate the boundary conditions for the unknown functions. On the thermocapillary interface the kinematic and dynamic conditions must be satisfied [14]. The kinematic condition is fulfilled automatically, the dynamic condition is written taking into account the thermocapillary effect. The conditions of the velocity and temperature continuity are set at the interface. The heat transfer condition and the mass balance equation at the interface should be written with respect to the Dufour and Soret effects. The saturated vapor concentration will be determined with the help of the relation that is a result of the Clapeyron-Clausius equation and Mendeleev-Clapeyron equation for ideal gas [6, 7, 14].

On the lower and upper solid impermeable walls of the flow domain the no-slip conditions should be fulfilled. The boundaries \( y = -l \) and \( y = h \) are heated linearly with respect to the longitudinal coordinate:

\[
\begin{align*}
  T_{1}|_{y = -l} = A_1 x + \vartheta^-, \quad T_{2}|_{y = h} = A_2 x + \vartheta^+, \\
\end{align*}
\]

here \( A_1, A_2, \vartheta^-, \vartheta^+ \) are constant. The vapor concentration on the upper boundary is assumed to be equal to zero.

The gas flow rate in the upper layer is given and defined by the relation:

\[
Q = \int_0^h \rho_2 u_2(y)dy,
\]

\( \rho_2 \) is the density of the gas.

Note that the condition of continuity of temperature at the interface \( y = 0 \) entails the equality of the temperature gradients \( a^1_i = a^2_i = A \). Conditions on free and solid boundaries determine the relationship among longitudinal temperature gradients \( A, A_1 \) and \( A_2 \). One of the longitudinal gradients is chosen arbitrarily.

3. Results of analytical calculations. Comparison with physical experiments

Consider the two-layer system of liquid and gas such as "ethanol - air". The values of the physicochemical parameters can be found in [15]. We suppose that the Dufour coefficient \( \delta \) is equal to \( 10^5 \) (see [10]); the Soret coefficient takes the value \( -5 \cdot 10^3 \) (it corresponds to the normal thermocapillary effect, see [14]). The concentration coefficient of density \( \gamma \) is equal to \( -0.62 \).
Figure 1 shows the velocity profiles and temperature distributions (presented in Celsius degrees) for different values of the gas flow rate in the upper layer of the system, obtained with the help of the analytical formulas. The longitudinal temperature gradient on the interface $A$ is taken equal to -20 K/m, the height of the gas layer of $h$ is equal to $h = 0.3 \cdot 10^{-2}$ m, and the height of the liquid $l = 0.3 \cdot 10^{-2}$ m. Calculations show that the increase of the gas flow rate entails quantitative changes of the velocity (an increase in the liquid velocity), the qualitative character of the velocity profiles is conserved. The temperature distribution in the system quantitatively changes with increasing values of $Q$. The maximum value of temperature also grows.

Figure 2 presents the velocity profiles and the distributions of temperature by the values of the temperature gradients on the interface equal to 20 K/m. In the case, when the longitudinal temperature gradient changes the sign to the opposite one, the velocity values are changed weakly. The character of the temperature distribution in the system changes significantly, the zone of the greatest cooling of the liquid moves in the negative direction of the Ox axis.

Figure 3 presents the experimental and analytical results demonstrating the values of the evaporative mass flow rate at the interface by the different values of the longitudinal temperature gradients on the lower solid boundary. The experimental data are presented in one system of measurements. By comparison of the experimental and analytical results of investigations of the dependence of the evaporative mass flow rate $M$ on the gas flow rate it has been found that the results can have a similar quantitative and qualitative character. The rate of evaporation increases with increasing of the gas flow rate $Q$, but according to the experimental data this dependence of $M$ on $Q$
has a nonlinear form. Figure 3 shows a rather good correlation of the analytical and experimental results for the value range of $Q$ between 0.0003 and 0.0005 (kg/(m s)) in the case of presented values of the longitudinal temperature gradient $A$.

![Figure 3. Dependence of the evaporation rate on the gas flow rate. Here $h = l = 0.3 \cdot 10^{-2}$ m, (1 – experimental data; 2 – calculations $A = -20$ K/m; 3 – $A = -50$ K/m)](image)

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