Adaptive finite-time event-triggered command filtered control for nonlinear systems with unknown control directions

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Abstract In this article, we study an adaptive finite-time event-triggered command filtered control problem based on output feedback for a class of nonstrict-feedback nonlinear systems with unknown control directions. Firstly, the unknown nonlinear continuous functions in the systems are approached by fuzzy logic systems, and the coordinate transformations and Nussbaum function technique are introduced to settle the problem caused by the unknown control directions in the systems. Then, in order to reduce the communication burden between the controller and the actuator, a finite-time adaptive event-triggered control algorithm is designed by means of backstepping technique. It is proved that the tracking and observer errors are adjusted around zero with a small neighborhood in a finite time and all the signals in the closed-loop systems are bounded. In addition, the command filtering technology based on error compensation system with fractional power is constructed to avoid the complexity explosion issue in the backstepping design process. Finally, the simulation examples are included to verify the availability and superiority of the control approach.

Keywords Nonstrict-feedback systems · Unknown control directions · Adaptive fuzzy control · Finite-time tracking control · Event-triggered control

1 Introduction

In modern industrial control production, nonlinear systems are ubiquitous, such as mechanical systems [1], electrical systems [2], robot systems [3], aerospace systems [4] and so on. Therefore, the research on nonlinear systems has received increasing attention in recent decades. At present, the backstepping technique is one of the most resultful instruments to construct controllers for high-order nonlinear systems. Thus, the combination of backstepping technique and adaptive control is frequently utilized to settle the control issue of nonlinear systems with uncertain parameters [5–7]. In addition, fuzzy logic systems (FLSs) [8] and neural networks (NNs) [9] as universal approximators can approximate or identify the nonlinear specialities of the systems. On the strength of this property, FLSs and NNs have been diffusely applied in the adaptive backstepping control design process of nonlinear systems, and many research achievements have been acquired. To mention a few, an observer-based fuzzy adaptive control strategy was introduced in [10] for switched nonlinear systems with a nonstrict-feedback structure. In [11], an adaptive control framework was constructed for a category of high-order stochastic systems with time-varying delay by combining NNs with backstep-
ping technique. In [12], a robust fuzzy adaptive control scheme was established for multi-input multi-output (MIMO) nonlinear stochastic Poisson jump diffusion systems with the aid of $H_{\infty}$ control theory and backstepping technique.

It is important to note that the repetitive differentiations of the virtual control signals in the procedures of above backstepping control design can lead to the issue of complexity explosion. To settle this matter effectively, dynamic surface control (DSC) technology has been developed in [13], in which the first-order filters have been introduced in the process of backstepping design. Subsequently, a DSC technology-based robust adaptive neural network control framework was proposed in [14] for pure-feedback time-delay systems with quantized input. An adaptive fuzzy tracking control way was established in [15] for an uncertain nonlinear system with input saturation by combining backstepping method with DSC technology, and the designed controller was applied to missile-target interception systems. However, the errors caused by the filters are not taken into account in the DSC technology, which reduces the control performance of the systems. Fortunately, the command filtering technology [16] has also settled the complexity explosion issue in backstepping procedure, in which the error compensation system has been constructed to make up for the shortcomings of DSC technology. For instance, both output feedback and state feedback adaptive fuzzy control approaches were presented in [17] for uncertain MIMO nonlinear arbitrary switched systems with a nonstrict-feedback form with the help of backstepping method and command filtering technology. In [18], an adaptive neural tracking control scheme was introduced for a MIMO system with input saturation and unknown control directions by utilizing command filtering technology. However, the authors in [16–18] only considered the asymptotic convergence rate.

In the control of practical engineering systems, the convergence is a key index to reflect control performance. The asymptotic stability of the systems can only ensure the convergence of the systems when the time tends to infinity. Compared with the infinite-time control strategy, the finite-time control method has the merits of higher tracking precision, better anti-disturbance performance and faster convergence. Therefore, the finite-time control has drawn more and more attention with interesting achievements [19–23] in recent years. Specifically, the authors in [19,20] constructed two finite-time control frameworks for nonlinear systems by employing the backstepping technique and FLSs. Subsequently, the state observers were designed in [21–23], and the output feedback finite-time control schemes were established based on the backstepping technique. Furthermore, the effects of complexity explosion in backstepping procedures were eliminated by means of DSC or command filtering technologies.

On the flip side, event-triggered control (ETC) has important theoretical and practical significance because it can reduce the heavy communication burden and save communication resources in network control systems. In recent years, ETC has become one of the hot research subjects in the field of control, and a series of important research findings have been obtained. For instance, two ETC schemes based on input-to-state stability (ISS) hypothesis were proposed in [24,25]. However, it is not easy to test the ISS assumption in uncertain nonlinear systems. In order to avoid the ISS assumption, the controllers and event-triggered mechanisms were designed cooperatively in [26–28], and several adaptive ETC methods were constructed. Unfortunately, there are few research achievements on the problem of adaptive finite-time output feedback event-triggered command filtered control for nonlinear systems with unknown control directions at present. This motivates the research of this article.

The main contributions in this article are listed as follows:

(1) In this paper, the FLSs are utilized to approximate the unknown nonlinear functions in the systems, which eliminates the linear growth conditions required by the nonlinear terms in [29,30]. Thus, the conservatism of the control algorithm is reduced. Furthermore, different from the error compensation systems designed in [17,27], a novel fractional power-based error compensation system is constructed in this paper to decrease the errors caused by the command filter in finite time. The advantage of this design is that the control performance of the whole closed-loop system can be improved by adjusting the parameters of the error compensation system more appropriately while reducing the computational complexity of the systems.

(2) The collaborative design of finite-time adaptive controller and event-triggered mechanism avoids the ISS assumption needed in the existing literature.
The finite-time event-triggered controller designed in this article cannot merely save communication resources and reduce communication burden, but increase control efficiency in actual control systems compared with the nonfinite-time controller designed in [31].

(3) The control schemes designed in [18, 26, 27] are only suitable for system models with available states, while the adaptive output feedback finite-time control approach constructed in this article can be applied for controlled systems with unmeasurable states. In addition, a class of nonstrict-feedback nonlinear systems with unknown control directions is considered in this paper, which is more general than the systems considered in [19, 23]. Therefore, the control strategy developed in this article can be more applicable for practical system models.

2 Problem formulation and preliminaries

2.1 System descriptions

Take the following nonlinear systems with a nonstrict-feedback structure into consideration:

\[
\begin{aligned}
\dot{x}_i &= r_i x_{i+1} + f_i(x) + d_i(t), \quad 1 \leq i \leq n - 1 \\
\dot{x}_n &= r_n u + f_n(x) + d_n(t),
\end{aligned}
\]

(2.1)

where \( x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n \) denotes the system state; \( u \in \mathbb{R} \) and \( y \in \mathbb{R} \) represent the control input and the system output, respectively; \( f_i(x) \) are unknown smooth nonlinear functions; \( r_i \) are unknown constants; \( d_i \) denote the unknown external disturbances. In this article, it is supposed that the system state variables \( x_i(t), i = 2, \ldots, n \) are not available, only the system output \( y \) is measurable.

The control goal of this article is to establish an adaptive finite-time event-triggered controller, which can guarantee the boundedness of all the signals and ensure that the output \( y \) follows the appointed signal \( y_d \) in a finite time. To accomplish the control goal, we put forward the following assumptions and lemmas.

Assumption 2.1 [22] The desired signal \( y_d \) and its first derivative \( \dot{y}_d \) are considered to be bounded.

Assumption 2.2 [32] There are unknown constants \( D_i > 0, i = 1, 2, \ldots, n \), such that \( |d_i(t)| \leq D_i \).

Assumption 2.3 [33] It is supposed that the sign of \( r_i \) is unknown, and there are two constants \( \underline{r} > 0 \) and \( \bar{r} > 0 \) such that \( \underline{r} < |r_i| < \bar{r} \).

Assumption 2.4 [34] There exist constants \( \mu_j \), such that

\[
|f_j(X_1) - f_j(X_2)| \
\leq \mu_j \|X_1 - X_2\|, \quad \forall X_1, X_2 \in \mathbb{R}^l, \quad j = 1, \ldots, n.
\]

Remark 2.1 In [5, 28, 35], the tracking signal \( y_d \) and its \( i \)th order derivatives \( y_d^{(i)} \), \( i = 1, 2, \ldots, n \) are required to be bounded, while it is only necessary to know the boundedness of \( y_d \) and \( \dot{y}_d \) in Assumption 2.1 of this paper. Therefore, Assumption 2.1 reduces the conservatism of the control algorithm in practical applications. In addition, Assumptions 2.2-2.4 are also common constraint conditions, which makes it possible to devise an adaptive tracking controller.

Lemma 2.1 [36] For any \( v \in \mathbb{R} \) and \( \epsilon > 0 \), it follows that

\[
0 \leq |v| - v \tanh(v/\epsilon) \leq 0.2785\epsilon.
\]

Lemma 2.2 (Young’s inequality) For any \( x \in \mathbb{R} \) and \( \zeta > 0 \), the following relationship holds:

\[
xy \leq \frac{x^2}{2\zeta} + \frac{\zeta y^2}{2}.
\]

Lemma 2.3 [37] For \( p > 0, q > 0 \) and \( \nu(w, \zeta) \), the following relationship holds:

\[
|w|^p |\zeta|^q \leq \frac{p\nu(w, \zeta)|w|^{p+q}}{p + q} + \frac{q\nu(w, \zeta)^{-\frac{p}{q}} |\zeta|^{p+q}}{p + q}.
\]

Now, the first-order Levant differentiator [38] is proposed

\[
\Phi_1 = \varrho_1,
\]

\[
\varrho_1 = -L_1|\Phi_1 - \alpha_r|^2 \text{sign}(\Phi_1 - \alpha_r) + \Phi_2,
\]

\[
\Phi_2 = -L_2 \text{sign}(\Phi_2 - \varrho_1),
\]

where \( \Phi_1 \) and \( \Phi_2 \) denote command filter states, \( \alpha_r \) represents the input signal, \( L_1 \) and \( L_2 \) are parameters to be designed.

Lemma 2.4 [38] In the absence of input noises, select the appropriate parameters \( L_1 \) and \( L_2 \), the following equations are true after a finite time of a transient process

\[
\Phi_1 = \alpha_{r0}, \quad \varrho_1 = \dot{\alpha}_{r0},
\]

where \( \alpha_r = \alpha_{r0} \), and the corresponding solutions are finite-time stable.
Lemma 2.5 [38] If input noise satisfies $|\alpha_i - \alpha_{i0}| \leq \epsilon$, for scalars $\phi_1 > 0$ and $v_1 > 0$, the following relationships hold in finite time

$$|\Phi_1 - \alpha_{i0}| \leq \phi_1 \epsilon = \sigma_1,$$

$$|\phi_1 - \alpha_{i0}| \leq v_1 \epsilon^2 = \sigma_2.$$  \hfill (2.4)

For the controlled system (2.1), we introduce the coordinate transformations as follows

$$\eta_i = \frac{x_i}{\Gamma_i}, 1 \leq i \leq n,$$  \hfill (2.5)

in which $\Gamma_i = \prod_{j=i}^{n} r_j$.

From (2.1) and (2.5), it can be deduced that the new states $\eta_i, i = 1, 2, \ldots, n$ satisfy

$$\begin{align*}
\dot{\eta}_i &= \eta_{i+1} + \psi_i(\eta) + d_i(t), 1 \leq i \leq n-1 \\
\dot{\eta}_n &= u + \psi_n(\eta) + d_n(t), \\
y &= x_1 = \Gamma_1 \eta_1,
\end{align*}$$  \hfill (2.6)

where $\eta = [\eta_1, \eta_2, \ldots, \eta_n]^T, x = \Lambda \eta$ with $\Lambda = \text{diag}(\Gamma_1, \Gamma_2, \ldots, \Gamma_n), \psi_i(\eta) = f_i(x)/\Gamma_i, d_i(t) = d_i(t)/\Gamma_i$.

According to Assumptions 2.2 and 2.3, it is easy to conclude that there are scalars $d_i > 0, i = 1, 2, \ldots, n$ such that $|d_i'(t)| \leq \bar{d}_i$.

2.2 Fuzzy logic systems

Unknown nonlinear continuous functions in the systems are usually handled via the fuzzy logic systems, which can be described as follows.

IF-THEN rules:

$R^l$: If $x_1$ is $B^l_1$ and $x_2$ is $B^l_2$ and $\ldots$ and $x_n$ is $B^l_n$, then $y$ is $W^l, l = 1, 2, \ldots, K$,

where $x = [x_1, x_2, \ldots, x_n]^T$ is the FLSs input, $y$ denotes the FLSs output, $\mu_{B^l_j}(x_j)$ and $\mu_{W^l}(y)$ represent the fuzzy membership functions on fuzzy sets $B^l_j$ and $W^l$, respectively, and $K$ denotes the number of fuzzy rules.

Based on the fuzzy rules of the system, the FLS is formulated by

$$y(x) = \frac{\sum_{j=1}^{K} \theta_l \prod_{j=1}^{n} \mu_{B^l_j}(x_j)}{\sum_{j=1}^{K} \prod_{j=1}^{n} \mu_{B^l_j}(x_j)}.$$  \hfill (2.7)

in which $\theta_l = \max_{y \in R} \mu_{W^l}(y)$.

Let

$$\varphi_l(x) = \frac{\prod_{j=1}^{n} \mu_{B^l_j}(x_j)}{\sum_{l=1}^{K} \prod_{j=1}^{n} \mu_{B^l_j}(x_j)}.$$  \hfill (2.8)

Define $\varphi(x) = [\varphi_1(x), \varphi_2(x), \ldots, \varphi_K(x)]^T$ and $\theta = [\theta_1, \theta_2, \ldots, \theta_K]^T$. Then, the FLS is redescribed as follows

$$y(x) = \theta^T \varphi(x).$$  \hfill (2.9)

Lemma 2.6 [35] Let $\psi(x)$ be a continuous function on a compact set $\Omega$, then for $\forall \epsilon > 0$, there is an FLS (2.9) such that

$$\sup_{x \in \Omega} |y(x) - \theta^T \varphi(x)| \leq \epsilon.$$  \hfill (2.10)

2.3 Nussbaum function

At present, Nussbaum function is often utilized to solve the problem of unknown control directions. For an even continuous function $N(\varsigma)$, if the following properties are satisfied:

$$\lim_{s \to \infty} \sup_{N(\varsigma) \neq 0} \frac{1}{s} \int_0^s N(\varsigma) d\varsigma = \infty,$$

$$\lim_{s \to \infty} \inf_{N(\varsigma) \neq 0} \frac{1}{s} \int_0^s N(\varsigma) d\varsigma = -\infty,$$

then $N(\varsigma)$ is a Nussbaum function. For instance, the continuous functions $e^{s^2} \cos(\frac{\varsigma^2}{2})$ and $\varsigma^2 \cos \varsigma$ fall into the category of Nussbaum functions. The properties of the Nussbaum function are described by the following lemma.

Lemma 2.7 [39] Define smooth functions $V(\cdot)$ and $\varsigma(\cdot)$ over $[0, t_f]$ with $V(t) \geq 0, \forall t \in [0, t_f]$. $N(\varsigma)$ is a smooth Nussbaum function. If the following relationship is satisfied:

$$V(t) \leq e^{-bt} \int_0^t (p(s)N(\varsigma) + 1) \varsigma e^{bs} ds + g,$$

in which $b$ is a positive constant, $g$ is the suitable scalar, and $p(t)$ represents a bounded continuous function, then $V(t), \varsigma(t)$ and $\int_0^t (p(s)N(\varsigma) + 1) \varsigma e^{bs} ds$ remain bounded on $[0, t_f]$.

3 Controller design and stability analysis

In this section, an observer-based adaptive finite-time event-triggered command filtered control scheme is developed with the help of backstepping method and command filtering technology.
3.1 State observer design

The novel system (2.6) is obtained from the original system (2.1) by coordinate transformations (2.5). Since the states \( x_i \) in the original system (2.1) are assumed to be unmeasurable, it is obvious that the state variables \( \eta_i, i = 1, 2, \ldots, n \) in the novel system (2.6) are also unmeasurable. Thus, it is necessary to establish an observer to estimate the unavailable states in the nonlinear systems. Before designing the state observer, the systems (2.6) are redescribed as

\[
\dot{\eta} = A\eta + \sum_{i=1}^{n} B_i [\Psi_i(\eta) + \Delta \Psi_i + d_i] + Bu, \tag{3.1}
\]

where \( \Psi_i(\eta) = \psi_i(\eta) + k_i \eta_i, \Delta \Psi_i = \Psi_i(\eta) - \Psi_i(\hat{\eta}), \) \( \hat{\eta} = [\hat{\eta}_1, \hat{\eta}_2, \ldots, \hat{\eta}_n]^T \) is the estimate of \( \eta, \) and

\[
A = \begin{bmatrix}
-k_1 \\
-k_2 \\
. \\
. \\
-k_n \\
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
& \cdots & \cdots & \cdots \\
0 & \cdots & 1 & 0 \\
\end{bmatrix}, \quad \eta = [\eta_1, \eta_2, \ldots, \eta_n]^T, \quad B = \begin{bmatrix} 0 \\
\vdots \\
0 \end{bmatrix}.
\]

Select the vector \( k \), such that \( A \) belongs to the strict Hurwitz matrix category. Thus, for any given matrix \( Q = Q^T > 0 \), there always exists a symmetric matrix \( P > 0 \) such that

\[
A^T P + PA = -Q. \tag{3.2}
\]

According to Lemma 2.6, the nonlinear terms \( \Psi_i(\hat{\eta}) \) in the system (3.1) are handled by the following FLSs

\[
\Psi_i(\hat{\eta}) = \theta_i^T T \phi_i(\hat{\eta}) + \epsilon_i, \quad i = 1, 2, \ldots, n, \tag{3.3}
\]

where \( \theta_i^* \) denote the optimal parameter vectors, and \( \epsilon_i \) represent the minimum approximation errors, which satisfy \( |\epsilon_i| \leq \epsilon_i^* \) with \( \epsilon_i^* \) are positive constants.

From (3.1) and (3.3), one has

\[
\dot{\hat{\eta}} = A\hat{\eta} + \sum_{i=1}^{n} B_i \theta_i^{* T} \phi_i(\hat{\eta}) + \epsilon' + \Delta \Psi + Bu, \tag{3.4}
\]

where \( \Delta \Psi = [\Delta \Psi_1, \ldots, \Delta \Psi_n]^T, \epsilon' = [\epsilon'_1, \ldots, \epsilon'_n]^T \) with \( \epsilon_i = \epsilon_i + d_i \) and \( |\epsilon_i| \leq \bar{\epsilon}_i = \bar{\epsilon}_i + \bar{d}_i. \)

To estimate the unavailable states in the system, we design the following fuzzy state observer

\[
\dot{\hat{\eta}} = A\hat{\eta} + \sum_{i=1}^{n} B_i \hat{\theta}_i^T \phi_i(\hat{\eta}) + Bu, \tag{3.5}
\]

where \( \hat{\theta}_i, i = 1, 2, \ldots, n \) are the estimations of \( \theta_i^* \).

Define observer error vector as \( \tilde{\eta} = \eta - \hat{\eta} = [\tilde{\eta}_1, \tilde{\eta}_2, \ldots, \tilde{\eta}_n]^T. \) Then, according to (3.4) and (3.5), we have

\[
\dot{\tilde{\eta}} = A\tilde{\eta} + \sum_{i=1}^{n} B_i \bar{\theta}_i^T \phi_i(\tilde{\eta}) + \epsilon' + \Delta \Psi, \tag{3.6}
\]

where \( \bar{\theta}_i = \theta_i^* - \hat{\theta}_i \) denote the parameter estimate errors.

For the observation error system (3.6), we select

\[
V_0 = \tilde{\eta}^T P \tilde{\eta}, \tag{3.7}
\]

From (3.2) and (3.6), computing derivative of \( V_0 \) yields

\[
\dot{V}_0 = -\tilde{\eta}^T Q \tilde{\eta} + 2\tilde{\eta}^T P \sum_{i=1}^{n} B_i \bar{\theta}_i^T \phi_i(\tilde{\eta}) + \epsilon' + \Delta \Psi \tag{3.8}
\]

By applying Young’s inequality, Assumption 2.4 and \( 0 < \phi_i^T (\hat{\eta}) \phi_i(\hat{\eta}) \leq 1 \), it follows that

\[
2\tilde{\eta}^T P \Delta \Psi \leq ||\tilde{\eta}||^2 + \|P\|^2 \sum_{i=1}^{n} \mu_i^2 ||\tilde{\eta}||^2, \tag{3.9}
\]

\[
2\tilde{\eta}^T P \sum_{i=1}^{n} B_i \bar{\theta}_i^T \phi_i(\tilde{\eta}) \leq n ||\tilde{\eta}||^2 + \|P\|^2 \sum_{i=1}^{n} \bar{\theta}_i^T \bar{\theta}_i, \tag{3.10}
\]

\[
2\tilde{\eta}^T P \epsilon' \leq ||\tilde{\eta}||^2 + \|P\|^2 ||\bar{\epsilon}||^2, \tag{3.11}
\]

with \( \bar{\epsilon} = [\bar{\epsilon}_1, \bar{\epsilon}_2, \ldots, \bar{\epsilon}_n]^T. \)

Substituting (3.9)-(3.11) into (3.8), it can be obtained that

\[
\dot{V}_0 \leq -\lambda_0 ||\tilde{\eta}||^2 + \|P\|^2 \sum_{j=1}^{n} \bar{\theta}_j^T \bar{\theta}_j + M_0, \tag{3.12}
\]

where \( \lambda_0 = \lambda_{\min}(Q) - n - 2 - \|P\|^2 \sum_{i=1}^{n} \mu_i^2, M_0 = \|P\|^2 \|\bar{\epsilon}\|^2, \) and \( \lambda_{\min}(Q) \) is the minimum eigenvalue of matrix \( Q. \)

3.2 Event-triggered controller design

In this part, the virtual control signals and the finite-time event-triggered controller based on state observer and command filtering technology will be given. To facilitate the establishment of subsequent controller, the following transformations are utilized

\[
\chi_1 = y - y_d, \tag{3.13}
\]
\[ \chi_i = \hat{\eta}_i - \eta_{i,c}, i = 2, \ldots, n, \quad (3.14) \]

where \( \chi_1 \) is the tracking error, \( \eta_{i,c} \) denote the outputs of the command filter. Then, the command filter can be constructed as 
\[ \Phi_{i,1} = \Phi_{i,1}, \]
\[ \Phi_{i,2} = -L_{i,2}\text{sign}(\Phi_{i,2} - \eta_{i,1}), i = 1, 2, \ldots, n - 1, \quad (3.15) \]
in which \( \alpha_i \) are the inputs, \( L_{i,1} \) and \( L_{i,2} \) are positive constants, \( \eta_{i+1,c}(t) = \Phi_{i,1}(t) \) and \( \dot{\eta}_{i+1,c}(t) = \Phi_{i,1}(t) \) represent the outputs.

The error compensating signals \( \Upsilon_i, i = 1, 2, \ldots, n \) are designed as
\[ \dot{\Upsilon}_1 = -c_1 \Upsilon_1 + \hat{\Gamma}_1(\eta_{2,c} - \alpha_1) + \hat{\Gamma}_1 \Upsilon_2 - s_1 \Upsilon_1^{\beta}, \]
\[ \dot{\Upsilon}_2 = -c_2 \Upsilon_2 + (\eta_{3,c} - \alpha_2) - \hat{\Gamma}_1 \Upsilon_1 + \Upsilon_3 - s_2 \Upsilon_2^{\beta}, \quad (3.16) \]
\[ \dot{\Upsilon}_i = -c_i \Upsilon_i + (\eta_{i+1,c} - \alpha_i) - \Upsilon_{i-1} + \Upsilon_{i+1} - s_i \Upsilon_i^{\beta}, \]
\[ \dot{\Upsilon}_n = -c_n \Upsilon_n - \Upsilon_{n-1} - s_n \Upsilon_n^{\beta}, \]
in which \( c_i > 0, s_i > 0 \) and \( 0 < \beta < 1 \) are design constants, \( \Upsilon_0 = 0 \), and \( \hat{\Gamma}_1 \) is a known constant that satisfies \( |\hat{\Gamma}_1| \leq \Gamma_1 \).

Then, the compensated tracking errors are defined as follows
\[ \check{\theta}_i = \chi_i - \Upsilon_i, i = 1, 2, \ldots, n. \quad (3.17) \]

By adopting backstepping technique, the virtual control signals are established as follows
\[ \alpha_1 = N(\varsigma) \left( c_1 \chi_1 + \frac{(\tau + 8) \hat{\Gamma}_1^2 + \tau + 6}{4} \check{\theta}_1 \right. \]
\[ \left. + \frac{1}{2} k_1^2 \hat{\eta}_1^{\gamma} \check{\theta}_1 - \hat{\eta}_d + \hat{\theta}_1^T \varphi_1(\hat{\eta}_1) + s_1 \check{\theta}_1^{\beta} \right), \]
\[ \alpha_2 = -c_2 \chi_2 - \hat{\Gamma}_1 \chi_1 + k_2 \hat{\eta}_1 \]
\[ - \hat{\theta}_2^T \varphi_2(\hat{\eta}_2) - \frac{1 + \tau}{2} \check{\theta}_2 + \eta_{2,c} - s_2 \check{\theta}_2^{\beta}, \quad (3.18) \]
\[ \alpha_i = -c_i \chi_i - \chi_{i-1} + k_i \hat{\eta}_1 - \hat{\theta}_1^T \varphi_i(\hat{\eta}_i) \]
\[ - \frac{1 + \tau}{2} \check{\theta}_i + \hat{\eta}_{i,c} - s_i \check{\theta}_i^{\beta}, \]
\[ \alpha_n = -c_n \chi_n - \chi_{n-1} + k_n \hat{\eta}_1 - \hat{\theta}_n^T \varphi_n(\hat{\eta}_i) \]
\[ - \frac{1 + \tau}{2} \check{\theta}_n + \hat{\eta}_{n,c} - s_n \check{\theta}_n^{\beta}, \]

where \( \hat{\eta}_i = [\hat{\eta}_1, \hat{\eta}_2, \ldots, \hat{\eta}_i]^T \), \( \tau > 0 \) and \( k_i > 0 \) are design parameters. \( N(\varsigma) \) is a Nussbaum function, in which the adaptive law of parameter \( \varsigma \) is constructed as
\[ \dot{\varsigma} = \vartheta_1 \left( c_1 \chi_1 + \frac{(\tau + 8) \hat{\Gamma}_1^2 + \tau + 6}{4} \check{\theta}_1 \right. \]
\[ \left. + \frac{1}{2} k_1^2 \hat{\eta}_1^{\gamma} \check{\theta}_1 - \hat{\eta}_d + \hat{\theta}_1^T \varphi_1(\hat{\eta}_1) + s_1 \check{\theta}_1^{\beta} \right) \].

The adaptive laws of \( \hat{\theta}_j \) are designed as follows
\[ \dot{\hat{\theta}}_j = \kappa_j \hat{\theta}_j \varphi_j(\hat{\eta}_j) - \sigma_j \hat{\theta}_j, j = 1, 2, \ldots, n, \quad (3.20) \]
where \( \kappa_j > 0 \) and \( \sigma_j > 0 \) are design parameters.

The finite-time event-triggered controller can be established as
\[ \xi(t) = -(1 + \pi) \left( \alpha_n \tanh \left( \frac{\alpha_n \alpha_n}{\epsilon} \right) + \bar{a} \tanh \left( \frac{\alpha_n \bar{a}}{\epsilon} \right) \right), \quad (3.21) \]
\[ u(t) = \xi(t_q), \forall t \in [t_q, t_{q+1}), \quad (3.22) \]

with the event-triggered mechanism is selected as
\[ t_{q+1} = \inf \{ t \in R | |\alpha(t)| \geq \pi |u(t)| + a \}, \quad (3.23) \]
where \( \alpha(t) = \xi(t) - u(t) \) represents the measurement error, \( \epsilon > 0, a > 0, 0 < \pi < 1 \) and \( \bar{a} > a/(1 - \pi) \) are design parameters. \( t_q, q \in \mathbb{Z}^+ \) denotes the controller update time.

**Remark 3.1** Whenever (3.23) is triggered, the time is flagged as \( t_{q+1} \) and the control signal \( u(t_{q+1}) \) is used in the closed-loop system. During \( t \in [t_q, t_{q+1}) \), the control value remains a constant, i.e., \( \xi(t_q) \).

### 3.3 Stability analysis of control system

In this part, based on the virtual control signals, actual controller and parameter adaptive laws designed above, we will analyze the stability of the whole closed-loop system. The following theorem is provided to summarize the main result of this paper.

**Theorem 3.1** Consider the system (2.1) with a nonstrict-feedback structure under Assumptions 2.1-2.4 and Lemmas 2.1-2.7. The adaptive finite-time event-triggered control laws (3.21)-(3.23), along with fuzzy state observer (3.5), the command filter (3.15), the compensating signals (3.16), the virtual control signals (3.18) and the parameter adaptive laws (3.20), the following conclusions can be guaranteed:

1) All the signals in the closed-loop system are bounded.
2) The observer and tracking errors are adjusted around zero with a small neighborhood in a finite time.

3) There is a time \( t' > 0 \) such that \( t_{q+1} - t_q \geq t' \), \( \forall q \in z^+ \). That is, the Zeno behavior is avoided.

Proof: The desired conclusions can be achieved recursively by the following steps.

**Step 1.** According to (3.13) and (3.17), computing derivative of \( \dot{\vartheta}_1 \) yields

\[
\dot{\vartheta}_1 = \dot{y} - \dot{\vartheta}_d - \dot{\vartheta}_1 \\
= \Gamma_1(\eta_2 + \varPsi_1(\eta)) - k_1 \eta_1 + d'(t) - \dot{\vartheta}_d - \dot{\vartheta}_1 \\
= \Gamma_1(\tilde{\eta}_2 + \varchi_2 + \eta_2.c + \theta_1^T \varphi_1(\tilde{\eta})) \\
- k_1 \eta_1 + \Delta \varPsi_1 + \varepsilon_1' \\
- \dot{\vartheta}_d + \theta_1^T \varphi_1(\tilde{\eta}_1) - \theta_1^T \varphi_1(\vartheta_1) \\
+ \theta_1^T \varphi_1(\vartheta_1) - \dot{\vartheta}_1. \\
\tag{3.24}
\]

Select the Lyapunov function as follows

\[
V_1 = V_0 + \frac{1}{2} \dot{\vartheta}_1^T + \frac{1}{2 \kappa_1} \dot{\vartheta}_1^T \varphi_1. \\
\tag{3.25}
\]

According to (3.24) and (3.25), computing derivative of \( V_1 \) yields

\[
\dot{V}_1 = \dot{V}_0 + \vartheta_1[G_1(\tilde{\eta}_2 + \varchi_2 + \alpha_1) \\
+ (\eta_2.c - \alpha_1)) + \theta_1^T \varphi_1(\tilde{\eta}) \\
+ \Delta \varPsi_1 - k_1 \eta_1 + \varepsilon_1' - \dot{\vartheta}_d \\
+ \theta_1^T \varphi_1(\tilde{\eta}_1) - \theta_1^T \varphi_1(\vartheta_1) \\
+ \theta_1^T \varphi_1(\vartheta_1) - \dot{\vartheta}_1] - \frac{1}{\kappa_1} \dot{\vartheta}_1^T \varphi_1. \\
\tag{3.26}
\]

By using Young’s inequality, there holds

\[
\dot{\vartheta}_1 \Gamma_1(\tilde{\eta}_2 + \varPsi_1(\eta) - k_1 \eta_1 + k_1 \eta_1 + \varepsilon_1') \\
\leq 2 \tilde{\vartheta}_1^T \Gamma_1^2 + \frac{1}{2} \| \tilde{\vartheta}_1 \| ^2 + \frac{1}{2} \kappa_1 \| \vartheta_1 \| ^2 \\
+ \frac{1}{2} \varphi_1^2 + \frac{1}{2} \kappa_1 \varphi_1^2 + \frac{1}{2} \Gamma_1^2. \\
\tag{3.27}
\]

\[
\dot{\vartheta}_1(\Gamma_1 \theta_1^T \varphi_1(\tilde{\eta}) - \theta_1^T \varphi_1(\vartheta_1)) \\
\leq \frac{\tau (\Gamma_1^2 + 1)}{4} \theta_1^T + \frac{\tau}{2} \| \theta_1 \| ^2, \\
\tag{3.28}
\]

\[
\dot{\vartheta}_1(\Gamma_1 - \Gamma_1)(\varchi_2 + (\eta_2.c - \alpha_1)) \\
\leq \frac{3}{2} \vartheta_1^T + \frac{1}{2} (\Gamma_1 - \Gamma_1)^2 (\vartheta_1^2 + \vartheta_1^2 + \varphi_1^2), \\
\tag{3.29}
\]

in which \( \varphi_1 > 0 \) is a constant.

Putting (3.27)-(3.28) into (3.26), one has

\[
\dot{V}_1 \leq \dot{V}_0 + \vartheta_1(\Gamma_1 \alpha_1 + \nabla_1(\chi_2 + (\eta_2.c - \alpha_1)) \\
- \dot{\vartheta}_d + \theta_1^T \varphi_1(\tilde{\eta}) \\
- \nabla_1 + \frac{(r + \vartheta_1^2 + r + \varphi_1^2)}{4} \\
+ \frac{1}{2} \varphi_1^2 \varphi_1^2 + \frac{2}{r} \| \varphi_1 \| ^2 \\
+ \frac{1}{2} \kappa_1 \| \varphi_1 \| ^2 + \frac{3}{2} \varphi_1^2 + \frac{1}{2} (\Gamma_1 - \Gamma_1)^2 \\
(\vartheta_1^2 + \vartheta_1^2 + \varphi_1^2). \\
\tag{3.30}
\]

Then, substituting \( \alpha_1, \dot{\vartheta}_1, \dot{\vartheta}_d \) and (3.12) into (3.30), it follows that

\[
\dot{V}_1 \leq - \lambda_1 \| \tilde{\vartheta}_1 \| ^2 + \| \varphi_1 \| ^2 + \sum_{j=1}^n \| \vartheta_j \| ^2 - s_1 \vartheta_1^2 - s_1 \vartheta_1^{2+1} \\
+ s_1 \vartheta_1 \varphi_1^2 + (\Gamma_1 N(\chi) + 1) \varphi_1 + \frac{s_1}{\kappa_1} \varphi_1^2 \varphi_1 \varphi_1. \\
\tag{3.31}
\]

\[
\dot{\vartheta}_2 = \dot{\vartheta}_2 - \dot{\varphi}_2 - \dot{\varphi}_2 \\
= \dot{\vartheta}_3 - k_2 \dot{\vartheta}_3 + \theta_2^T \varphi_2(\tilde{\eta}_2) - \dot{\varphi}_2 - \dot{\varphi}_2 \\
= \varphi_3 + \eta_3.c - k_2 \dot{\vartheta}_3 - \theta_2^T \varphi_2(\tilde{\eta}_2) \\
- \theta_2^T \varphi_2(\tilde{\eta}_2) + \theta_2^T \varphi_2(\tilde{\eta}_2) + \theta_2^T \varphi_2(\tilde{\eta}_2) - \dot{\varphi}_2. \\
\tag{3.32}
\]

Select the Lyapunov function as follows

\[
\dot{V}_2 = V_1 + \vartheta_2(\varphi_3 + \eta_3.c - k_2 \dot{\vartheta}_3 - \theta_2^T \varphi_2(\tilde{\eta}_2) \\
+ \theta_2^T \varphi_2(\tilde{\eta}_2) - \theta_2^T \varphi_2(\tilde{\eta}_2) + \theta_2^T \varphi_2(\tilde{\eta}_2) + \theta_2^T \varphi_2(\tilde{\eta}_2) - \dot{\varphi}_2. \\
\tag{3.33}
\]

According to (3.32) and (3.33), the derivative of \( V_2 \) satisfies

\[
\dot{V}_2 = \dot{V}_1 + \vartheta_2(\varphi_3 + \eta_3.c - k_2 \dot{\vartheta}_3 - \theta_2^T \varphi_2(\tilde{\eta}_2) \\
+ \theta_2^T \varphi_2(\tilde{\eta}_2) - \theta_2^T \varphi_2(\tilde{\eta}_2) + \theta_2^T \varphi_2(\tilde{\eta}_2) + \theta_2^T \varphi_2(\tilde{\eta}_2) - \dot{\varphi}_2. \\
\tag{3.34}
\]

\( \odot \) Springer
By using Young’s inequality, there holds
\[ -\vartheta_2 \dot{\theta}_2 \psi_2(\hat{\eta}) \leq \frac{1}{2} \dot{\theta}_2^2 + \frac{1}{2} \vartheta_2^T \dot{\theta}_2, \] (3.35)
\[ \vartheta_2 (\theta_2^T \varphi_2(\hat{\eta}) - \theta_2^T \varphi_2(\hat{\eta}_2)) \leq \frac{\tau}{2} \vartheta_2^2 + \frac{2}{\tau} \|\vartheta_2^*\|^2. \] (3.36)

Putting (3.35) and (3.36) into (3.34), one can get
\[ \dot{V}_2 \leq -\lambda_1 \|\hat{\eta}\|^2 + \|P\|^2 \sum_{j=1}^{n} \dot{\theta}_j^T \dot{\theta}_j \]
\[ - c_1 \dot{\theta}_1^2 - s_1 \dot{\theta}_1^{\beta+1} + s_1 \dot{\theta}_1 \Gamma_1^\beta \]
\[ + (\Gamma_1 N(\xi) + 1) \dot{\xi} + \Gamma_1 \dot{\theta}_1 \]
\[ + \sigma_1 \dot{\theta}_1^T \dot{\theta}_1 + \vartheta_2 \chi_3 + \alpha_2 - \dot{\eta}_{2,c} \]
\[ + (\eta_{3,c} - \alpha_2) - k_2 \hat{\eta}_1 + \dot{\theta}_2 \varphi_2(\hat{\eta}_2) \]
\[ = \sum_{j=1}^{n} \dot{\theta}_j^T \dot{\theta}_j \]
\[ + \frac{1}{2} \vartheta_2^T \dot{\theta}_2 + \frac{1}{\kappa_2} \vartheta_2^T [k_2 \vartheta_2 \varphi_2(\hat{\eta}_2) - \dot{\theta}_2] \]
\[ + \frac{2}{\tau} \|\vartheta_2^*\|^2 + \Gamma_1 \]
\[ + \frac{1}{2} (\Gamma_1 - \Gamma_1)^2 (\vartheta_2^2 + \vartheta_2^2). \] (3.37)

Then, substituting \( \alpha_2, \hat{\gamma}_2 \) and \( \hat{\theta}_2 \) into (3.37), it follows that
\[ \dot{V}_2 \leq -\lambda_1 \|\hat{\eta}\|^2 + \|P\|^2 \sum_{j=1}^{n} \dot{\theta}_j^T \dot{\theta}_j \]
\[ - \sum_{j=1}^{n} c_j \dot{\theta}_j^2 - \sum_{j=1}^{n} s_j \dot{\theta}_j^{\beta+1} \]
\[ + \sum_{j=1}^{n} s_j \dot{\theta}_j \Gamma_1^\beta + (\Gamma_1 N(\xi) + 1) \dot{\xi} \]
\[ + \sum_{j=1}^{n} \sigma_j \dot{\theta}_j^T \dot{\theta}_j + \vartheta_2 \varphi_2(\hat{\eta}_2) \]
\[ + \frac{1}{2} \vartheta_2^T \dot{\theta}_2 + \Gamma_1 \]
\[ + \frac{1}{2} (\Gamma_1 - \Gamma_1)^2 (\vartheta_2^2 + \vartheta_2^2), \] (3.38)

where \( M_2 = M_1 + \frac{2}{\tau} \|\vartheta_2^*\|^2 \).

**Step 3** (3 ≤ i ≤ n - 1). Select the Lyapunov function as follows
\[ V_i = V_{i-1} + \frac{1}{2} \dot{\theta}_i^2 + \frac{1}{2\kappa_i} \dot{\theta}_i^T \dot{\theta}_i. \] (3.39)

According to (3.39), computing derivative of \( V_i \) yields
\[ \dot{V}_i = \dot{V}_{i-1} + \dot{\vartheta}_i [\chi_{i+1} + \eta_{i+1,c} - k_i \hat{\eta}_i - \dot{\theta}_i \varphi_i(\hat{\eta})] \]
\[ + \vartheta_i^T \varphi_i(\hat{\eta}) - \vartheta_i^T \varphi_i(\hat{\eta}_i) + \vartheta_i^T \varphi_i(\hat{\eta}_i) \]
\[ + \vartheta_i \varphi_i(\hat{\eta}_i) - \vartheta_i \varphi_i(\hat{\eta}_{i,c}) - \dot{\gamma}_i - \dot{\gamma}_i] \]
\[ = \sum_{j=1}^{n} \dot{\theta}_j^T \dot{\theta}_j \]
\[ - \sum_{j=1}^{n} c_j \dot{\theta}_j^2 - \sum_{j=1}^{n} s_j \dot{\theta}_j^{\beta+1} \]
\[ + \sum_{j=1}^{n} s_j \dot{\theta}_j \Gamma_1^\beta + (\Gamma_1 N(\xi) + 1) \dot{\xi} \]
\[ + \sum_{j=1}^{n} \sigma_j \dot{\theta}_j^T \dot{\theta}_j + \vartheta_i \varphi_i(\hat{\eta}_i) \]
\[ = \sum_{j=1}^{n} \dot{\theta}_j^T \dot{\theta}_j \]
\[ + \frac{1}{2} \vartheta_i^T \dot{\theta}_i + \Gamma_1 \]
\[ + \frac{1}{2} (\Gamma_1 - \Gamma_1)^2 (\vartheta_i^2 + \vartheta_i^2). \] (3.40)

By using Young’s inequality, there holds
\[ -\vartheta_i \dot{\theta}_i^T \varphi_i(\hat{\eta}) \leq \frac{1}{2} \dot{\theta}_i^2 + \frac{1}{2} \vartheta_i^T \dot{\theta}_i, \] (3.41)
\[ \vartheta_i (\theta_i^T \varphi_i(\hat{\eta}) - \theta_i^T \varphi_i(\hat{\eta}_i)) \leq \sum_{j=1}^{i} \vartheta_i \sigma_j \dot{\theta}_j^T \dot{\theta}_j + \vartheta_i \varphi_i(\hat{\eta}_i) \]
\[ + \sum_{j=1}^{i} \sigma_j \dot{\theta}_j^T \dot{\theta}_j + \vartheta_i \varphi_i(\hat{\eta}_i) \]
\[ = \sum_{j=1}^{i} \dot{\theta}_j^T \dot{\theta}_j \]
\[ - \sum_{j=1}^{i} c_j \dot{\theta}_j^2 - \sum_{j=1}^{i} s_j \dot{\theta}_j^{\beta+1} \]
\[ + \sum_{j=1}^{i} s_j \dot{\theta}_j \Gamma_1^\beta + (\Gamma_1 N(\xi) + 1) \dot{\xi} \]
\[ + \sum_{j=1}^{i} \sigma_j \dot{\theta}_j^T \dot{\theta}_j + \vartheta_i \varphi_i(\hat{\eta}_i) \]
\[ = \sum_{j=1}^{i} \dot{\theta}_j^T \dot{\theta}_j \]
\[ + \frac{1}{2} \vartheta_i^T \dot{\theta}_i + \Gamma_1 \]
\[ + \frac{1}{2} (\Gamma_1 - \Gamma_1)^2 (\vartheta_i^2 + \vartheta_i^2). \] (3.42)

Putting (3.41) and (3.42) into (3.40), it yields
\[ \dot{V}_i \leq -\lambda_1 \|\hat{\eta}\|^2 + \|P\|^2 \sum_{j=1}^{n} \dot{\theta}_j^T \dot{\theta}_j \]
\[ - \sum_{j=1}^{i-1} c_j \dot{\theta}_j^2 - \sum_{j=1}^{i-1} s_j \dot{\theta}_j^{\beta+1} \]
\[ + \sum_{j=1}^{i-1} s_j \dot{\theta}_j \Gamma_1^\beta + (\Gamma_1 N(\xi) + 1) \dot{\xi} \]
\[ + \sum_{j=1}^{i-1} \sigma_j \dot{\theta}_j^T \dot{\theta}_j + \vartheta_i \varphi_i(\hat{\eta}_i) \]
\[ = \sum_{j=1}^{i-1} \dot{\theta}_j^T \dot{\theta}_j \]
\[ + \frac{1}{2} \vartheta_i^T \dot{\theta}_i + \Gamma_1 \]
\[ + \frac{1}{2} (\Gamma_1 - \Gamma_1)^2 (\vartheta_i^2 + \vartheta_i^2). \] (3.43)

Then, substituting \( \alpha_i, \hat{\gamma}_i \) and \( \hat{\theta}_i \) into (3.43), one has
\[ \dot{V}_i \leq -\lambda_1 \|\hat{\eta}\|^2 + \|P\|^2 \sum_{j=1}^{n} \dot{\theta}_j^T \dot{\theta}_j \]
\[ - \sum_{j=1}^{i} c_j \dot{\theta}_j^2 - \sum_{j=1}^{i} s_j \dot{\theta}_j^{\beta+1} \]
\[ + \sum_{j=1}^{i} s_j \dot{\theta}_j \Gamma_1^\beta + (\Gamma_1 N(\xi) + 1) \dot{\xi} \]
\begin{align}
&+ \sum_{j=1}^{i} \frac{\sigma_j}{\kappa_j} \hat{\theta}_j + \theta_i \partial_{i+1} \\
&+ \sum_{j=2}^{i} \frac{1}{2} \bar{\theta}_j \dot{\theta}_j + M_i + \frac{1}{2} (\Gamma_1 - \bar{\Gamma}_1)^2 (\vartheta_2^2 + \vartheta_2^2),
\end{align}

(3.44)

where $M_i = M_{i-1} + \frac{\gamma}{2} \| \theta_n \|^2$.

**Step II.** From (3.23), one can conclude that the following equation holds

\begin{align}
\xi(t) = (1 + e_1(t) \pi) u(t) + e_2(t) a, t \in [t_q, t_q+1],
\end{align}

(3.45)

where $|e_1(t)| \leq 1$ and $|e_2(t)| \leq 1$ represent time-varying parameters. Thus, we can further deduce that

\begin{equation}
\dot{u}(t) = \xi(t)/(1 + e_1(t) \pi) - e_2(t) a/(1 + e_1(t) \pi).
\end{equation}

Select the Lyapunov function as follows

\begin{equation}
V_n = V_{n-1} + \frac{1}{2} \theta_n^2 + \frac{1}{2 \kappa_n} \bar{\theta}_n \dot{\theta}_n.
\end{equation}

(3.46)

According to (3.46), computing derivative of $V_n$ yields

\begin{equation}
\dot{V}_n = \dot{V}_{n-1} + \theta_n \left( \frac{\xi(t) - e_2(t) a}{1 + e_1(t) \pi} + \bar{\theta}_n \varphi_n(\hat{\theta}_n) - \bar{\theta}_n \varphi_n(\hat{\theta}_n) \right)
\end{equation}

(3.47)

By using Young’s inequality, there holds

\begin{equation}
- \theta_n \bar{\theta}_n \varphi_n(\hat{\theta}_n) \leq \frac{1}{2} \theta_n^2 + \frac{1}{2} \bar{\theta}_n \dot{\theta}_n.
\end{equation}

(3.48)

From $|e_1(t)| \leq 1$ and $|e_2(t)| \leq 1$, we can deduce that

\begin{equation}
\frac{\theta_n \xi(t)}{1 + e_1(t) \pi} \leq \frac{\theta_n \xi(t)}{1 + \pi}, \quad \frac{e_2(t) a}{1 + e_1(t) \pi} \leq \frac{a}{1 - \pi}.
\end{equation}

(3.49)

Putting (3.21), (3.48) and (3.49) into (3.47), it yields

\begin{equation}
\dot{V}_n \leq \dot{V}_{n-1} - \theta_n \left( \alpha_n \tan \left( \frac{\partial_n a}{\varepsilon} \right) + \tilde{a} \tan \left( \frac{\partial_n \tilde{a}}{\varepsilon} \right) \right)
\end{equation}

\begin{equation}
+ \left| \frac{\theta_n a}{1 - \pi} \right| + \theta_n \left[ -k_n \hat{n}_1 \hat{\eta}_n + \bar{\theta}_n \varphi_n(\hat{\theta}_n) - \hat{n}_c \right]
\end{equation}

\begin{equation}
+ \frac{1}{2} \bar{\theta}_n \dot{\theta}_n + \frac{1}{2 \kappa_n} \bar{\theta}_n \dot{\theta}_n + \frac{1}{\kappa_n} \bar{\theta}_n \varphi_n(\hat{\theta}_n) - \hat{n}_c \right].
\end{equation}

(3.50)

According to Lemma 2.1, (3.50) can be reexpressed as

\begin{equation}
\dot{V}_n \leq \dot{V}_{n-1} - \theta_n \left[ -k_n \hat{n}_1 + \bar{\theta}_n \varphi_n(\hat{\theta}_n) - \hat{n}_c \right] + \frac{1}{2} \theta_n \dot{\theta}_n
\end{equation}

\begin{equation}
\hat{n}_n + \theta_n \left[ k_n \partial_n \varphi_n(\hat{\theta}_n) - \hat{n}_c \right] + \frac{1}{2} \theta_n \dot{\theta}_n
\end{equation}

\begin{equation}
- \theta_n \hat{\theta}_n + \frac{\theta_n a}{1 - \pi} + 0.557 \epsilon.
\end{equation}

(3.51)

Then, substituting $\alpha_n$, $\hat{\gamma}_n$ and $\hat{\theta}_n$ into (3.51), it follows that

\begin{equation}
\dot{V}_n \leq - \lambda_1 \| \hat{n}_n \|^2 + \| P \|^2 \sum_{j=1}^{n} \bar{\theta}_j \dot{\theta}_j
\end{equation}

\begin{equation}
- \sum_{j=1}^{n} c_j \theta_j^2 - \sum_{j=1}^{n} s_j \theta_j^{\beta+1}
\end{equation}

\begin{equation}
+ \sum_{j=1}^{n} s_j \theta_j^{\beta+1} \left( \Gamma_1 N(\gamma) + 1 \right) \xi + \sum_{j=1}^{n} \bar{\theta}_j \dot{\theta}_j
\end{equation}

\begin{equation}
+ \sum_{j=2}^{n} \frac{1}{2} \bar{\theta}_j \dot{\theta}_j + M_n + \frac{1}{2} (\Gamma_1 - \bar{\Gamma}_1)^2 (\vartheta_2^2 + \vartheta_2^2),
\end{equation}

where $M_n = M_n-1 + 0.557 \epsilon$.

For the compensating system (3.16), we select the Lyapunov function as follows

\begin{equation}
W = \frac{1}{2} \sum_{j=1}^{n} \gamma_j^2.
\end{equation}

(3.53)

Then, computing derivative of $W$ yields

\begin{equation}
\dot{W} = \gamma_1 \dot{\gamma}_1 + \gamma_2 \dot{\gamma}_2 + \cdots + \gamma_n \dot{\gamma}_n
\end{equation}

\begin{equation}
= - c_1 \gamma_1^2 + \hat{\gamma}_1 \gamma_2 - s_1 \gamma_2^{\beta+1}
\end{equation}

\begin{equation}
+ \hat{\gamma}_1 \gamma_2 - s_2 \gamma_2^{\beta+1}
\end{equation}

\begin{equation}
\gamma_2 \gamma_3 - s_2 \gamma_3^{\beta+1}
\end{equation}

\begin{equation}
\gamma_3 \gamma_4 - s_3 \gamma_4^{\beta+1}
\end{equation}

\begin{equation}
\gamma_4 \gamma_5 - s_4 \gamma_5^{\beta+1}
\end{equation}

\begin{equation}
\gamma_5 \gamma_6 - s_5 \gamma_6^{\beta+1}
\end{equation}

\begin{equation}
\cdots (\cdots) - c_n \gamma_n^2 + \gamma_1 \gamma_{n-1} - s_{n-1} \gamma_{n-1}^{\beta+1}
\end{equation}

\begin{equation}
= \sum_{j=1}^{n} c_j \gamma_j^2 - \sum_{j=1}^{n} s_j \gamma_j^{\beta+1} + \sum_{j=2}^{n-1} \gamma_j (\gamma_{j+1} - \gamma_j - \gamma_j^2).
\end{equation}

(3.54)

From Lemmas 2.4-2.5, it can be concluded that the relation $|\eta_{j+1} - \alpha_j| \leq \alpha_j$ holds in a finite time $T_j$.

For $t \geq \max \{T_j\}$, we can deduce that

\begin{equation}
\dot{W} \leq - \sum_{j=1}^{n} c_j \gamma_j^2 - \sum_{j=1}^{n} s_j \gamma_j^{\beta+1}
\end{equation}

\begin{equation}
+ \sum_{j=1}^{n} a_j \gamma_j^2 + \sum_{j=2}^{n} \frac{1}{2} \gamma_j^2 + \frac{1}{2} \bar{\gamma}_j \gamma_j^2.
\end{equation}

(3.55)
Choose the Lyapunov function as follows
\[ V = V_n + W. \] (3.56)

According to (3.52) and (3.55), computing derivative of \( V \) yields
\[
\dot{V} \leq - \lambda_1 \| \tilde{\eta} \|^2 + \| P \|^2 \sum_{j=1}^{n} \tilde{\theta}_j^T \tilde{\theta}_j \\
- \sum_{j=1}^{n} \zeta_j \tilde{\theta}_j^2 - \sum_{j=1}^{n} s_j \tilde{\theta}_j^{\beta+1} \\
+ \sum_{j=1}^{n} s_j \tilde{\theta}_j \beta_j^\gamma_j + \sum_{j=1}^{n} \frac{\sigma_j}{\kappa_j} \tilde{\theta}_j^T \tilde{\theta}_j \\
+ (\Gamma_1 N(\varsigma) + 1) \tilde{\varsigma} + \sum_{j=2}^{n} \beta \tilde{\theta}_j^T \tilde{\theta}_j - \sum_{j=1}^{n} \tilde{c} \tilde{\varsigma}_j^2 \\
- \sum_{j=1}^{n} s_j \tilde{\gamma}_j^{\beta+1} + \sum_{j=1}^{n-1} \omega_{j,1}^2 + M_n,
\] (3.57)
in which \( \tilde{c} = \min(\varsigma_1, c_2 - \frac{1}{2}(\Gamma_1 - \Gamma_1)^2, c_j, 3 \leq j \leq n) \) and \( \tilde{c} = \min(\varsigma_1 - \frac{\rho_j^2}{2}, c_2 - \frac{1}{2} - \frac{1}{2}(\Gamma_1 - \Gamma_1)^2, c_j - \frac{1}{2}, 3 \leq j \leq n). \)

By using Young’s inequality, there holds
\[
\frac{\sigma_j}{\kappa_j} \tilde{\theta}_j^T \tilde{\theta}_j \leq - \frac{\sigma_j}{2\kappa_j} \tilde{\theta}_j^T \tilde{\theta}_j + \frac{\sigma_j}{2\kappa_j} \| \theta_j^\gamma_j \|^2.
\] (3.58)

Applying Lemma 2.3, it follows that
\[
\tilde{\theta}_j \beta_j \tilde{\gamma}_j \leq \frac{1}{\beta + 1} \tilde{\theta}_j^{\beta+1} + \frac{\beta}{\beta + 1} \tilde{\gamma}_j^{\beta+1}.
\] (3.59)

Substituting (3.58) and (3.59) into (3.57), one has
\[
\dot{V} \leq - \lambda_1 \| \tilde{\eta} \|^2 - R \left( \lambda_{\min}(Q) \| \tilde{\eta} \|^2 \right)^{\frac{\beta+1}{2}} \\
+ R \left( \lambda_{\min}(Q) \| \tilde{\eta} \|^2 \right)^{\frac{\beta+1}{2}} \\
- \sum_{j=1}^{n} \zeta_j \tilde{\theta}_j^2 - \sum_{j=1}^{n} \frac{\beta s_j}{\beta + 1} \tilde{\theta}_j^{\beta+1} - \sum_{j=1}^{n} s_j \tilde{\theta}_j + \sum_{j=1}^{n} \frac{\sigma_j}{\kappa_j} \| \theta_j^\gamma_j \|^2 \\
+ (\Gamma_1 N(\varsigma) + 1) \tilde{\varsigma} - \sum_{j=1}^{n} \frac{1}{2} \left( \frac{\sigma_j}{\kappa_j} - 2 \| P \|^2 - 1 \right) \tilde{\theta}_j^T \tilde{\theta}_j \\
- R \sum_{j=1}^{n} \left( \frac{1}{2\kappa_j} \tilde{\theta}_j^T \tilde{\theta}_j \right)^{\frac{\beta+1}{2}} \\
- \sum_{j=1}^{n} \frac{1}{2} \left( \frac{\sigma_j}{\kappa_j} - 2 \| P \|^2 - 1 - \left( \beta + 1 \right) \frac{s_j}{\kappa_j} \| \theta_j^\gamma_j \|^2 + \sum_{j=1}^{n-1} \omega_{j,1}^2 \right) \\
+ M_n + \frac{(n + 1)(1 - \beta)}{2},
\] (3.60)

where \( R \) is a positive design constant.

By utilizing Lemma 2.3, let \( w = 1, \xi_1 = \lambda_{\min}(Q) \| \tilde{\eta} \|, \xi_2 = \frac{1}{2\kappa_j} \tilde{\theta}_j^T \tilde{\theta}_j, p = \frac{1 - \beta}{2}, q = \frac{1 + \beta}{2} \) and \( v = \frac{1}{\bar{\eta}} \), it can be deduced
\[
R \left( \lambda_{\min}(Q) \| \tilde{\eta} \|^2 \right)^{\frac{\beta+1}{2}} \\
\leq \frac{1 - \beta}{2} + \frac{(\beta + 1)R^{\frac{\beta}{2}}}{2} \lambda_{\min}(Q) \| \tilde{\eta} \|^2.
\] (3.61)

From Lemma 2.7, it can be obtained that \( (\Gamma_1 N(\varsigma) + 1) \tilde{\varsigma} \) is bounded, namely, there is a constant \( h \) such that
\[
| (\Gamma_1 N(\varsigma) + 1) \tilde{\varsigma} | \leq h.
\] (3.63)

Putting (3.61)-(3.63) into (3.60), one can obtain
\[
\dot{V} \leq - \rho \| \tilde{\eta} \|^2 - R \left( \lambda_{\min}(Q) \| \tilde{\eta} \|^2 \right)^{\frac{\beta+1}{2}} \\
- \sum_{j=1}^{n} \zeta_j \tilde{\theta}_j^2 - \sum_{j=1}^{n} \frac{\beta s_j}{\beta + 1} \tilde{\theta}_j^{\beta+1} \\
- \sum_{j=1}^{n} \frac{\sigma_j}{\kappa_j} \| \theta_j^\gamma_j \|^2 + \sum_{j=1}^{n-1} \omega_{j,1}^2 \\
+ M_n + \frac{(n + 1)(1 - \beta)}{2},
\]

where \( \rho = \lambda_1 - \frac{(\beta + 1)R^{\frac{\beta}{2}}}{2} \lambda_{\min}(Q) \).

Therefore, we have
\[
\dot{V} \leq - \Xi_1 V - \Xi_2 V^{\frac{\beta+1}{2}} + \Xi_3,
\] (3.65)
in which
\[
\Xi_1 = \min\left\{ \frac{\rho}{\lambda_{\min}(Q)}, 2\zeta, 2\tilde{c}, \sigma_j - 2\kappa_j \| P \|^2 \right\},
\]
\[
\Xi_2 = \min\left\{ \frac{\beta s_j}{\beta + 1} \frac{\beta+1}{2}, \frac{s_j}{\beta + 1} \frac{\beta+1}{2}, R \right\},
\]
\[
\Xi_3 = \min\left\{ \frac{\beta s_j}{\beta + 1} \frac{\beta+1}{2}, \frac{s_j}{\beta + 1} \frac{\beta+1}{2}, R \right\}.
\]
\[ \Xi_3 = M_R + \sum_{j=1}^{n} \sigma_j \frac{\| \theta_j^* \|^2 + h}{2k_j} + \sum_{j=1}^{n-1} \omega_{j,1}^2 + \frac{(n+1)(1-\beta)}{2}. \]

By utilizing Lemma 2 and Corollary 1 in [40], it can be obtained that all the signals in the system are bounded in finite-time and \( \eta_1 \) and \( \Upsilon_1 \) can converge to

\[ |\eta_1| \leq \min \left\{ \sqrt{\frac{2 \Xi_3}{(1-\theta_0) \Xi_1}}, 2 \left( \frac{\Xi_3}{(1-\theta_0) \Xi_2} \right)^{\frac{\eta_1}{2}}, \right\}, \]

\[ |\Upsilon_1| \leq \min \left\{ \sqrt{\frac{2 \Xi_3}{(1-\theta_0) \Xi_1}}, 2 \left( \frac{\Xi_3}{(1-\theta_0) \Xi_2} \right)^{\frac{\eta_1}{2}}, \right\}, \tag{3.66} \tag{3.67} \]

where \( 0 < \theta_0 < 1 \), and the settling time is

\[ T^* = \max \left\{ \frac{1}{\theta_0 \Xi_1 (1-\frac{\beta+1}{2})} \ln \frac{\theta_0 \Xi_1 V^{\frac{\beta+1}{2}}(0) + \Xi_2}{\Xi_2 (1-\frac{\beta+1}{2})}, \frac{1}{\theta_0 \Xi_2} \ln \frac{\Xi_1 V^{\frac{\beta+1}{2}}(0) + \theta_0 \Xi_2}{\Xi_2 (1-\frac{\beta+1}{2})} \right\}. \]

According to (3.66)-(3.67) and \( \chi_1 = \eta_1 + \Upsilon_1 \), we can obtain that \( |\chi_1| \leq 2 \min\{((2 \Xi_3)/(1-\theta_0) \Xi_1))^{1/2}, \sqrt{2}((2 \Xi_3)/(1-\theta_0) \Xi_2))^{1/(\beta+1)} \} \) in finite time \( T^* \). That is, the tracking and observer errors are adjusted around zero with a small neighborhood in finite time \( T^* \).

Next, we will demonstrate Zeno behavior does not occur. That is, for any \( q \in \mathbb{Z}^+ \), there exists a time \( t' > 0 \) such that \( t_{q+1} - t_q \geq t' \). For this purpose, according to \( o(t) = \xi(t) - u(t) \), one has

\[ \frac{d}{dt} |o| = \frac{d}{dt} (o \times o) = \text{sign}(o) \dot{o} \leq |\dot{\xi}|. \]

From (3.18) and (3.21), it can be obtained that \( \xi \) is differentiable. The relationship \( |\dot{\xi}| \leq \iota \) holds, where \( \iota > 0 \) is a constant. By noting that \( o(t_q) = 0 \) and \( \lim_{t \to t_{q+1}} o(t)| = \pi |u(t)| + a \), one can be concluded that \( t' \geq \frac{\pi |u(t)| + a}{|\dot{\xi}|} \). Thus, the Zeno behavior is successfully avoided.

This completes the proof.

**Remark 3.2** The Zeno phenomenon means that the trigger event may be triggered countless times in a limited time, which leads to the instability of the systems. A common method to avert the Zeno phenomenon is to assure that the time interval between any two adjacent trigger events is positive. That is, for any \( q \in \mathbb{Z}^+ \), \( t_{q+1} - t_q > 0 \) holds. Based on the above proof, it follows that \( t_{q+1} - t_q \geq \frac{\pi |u(t)| + a}{|\dot{\xi}|} \). Hence, the Zeno phenomenon does not occur.

**Remark 3.3** According to \( |\chi_1| \leq \min\{((2 \Xi_3)/(1-\theta_0) \Xi_1))^{1/2}, \sqrt{2}((2 \Xi_3)/(1-\theta_0) \Xi_2))^{1/(\beta+1)} \} \) and the definitions of \( \Xi_1, \Xi_2 \), and \( \Xi_3 \), it can be concluded that the tracking error in the system mainly depends on the control parameters \( c_1, s_1, \sigma_1 \text{ and } \beta \). The system tracking error can be adjusted to be smaller by increasing \( c_1, s_1, \sigma_1 \) and reducing \( \beta \). However, the values of \( c_1 \) and \( s_1 \) should not be too large; otherwise, the control value will be too large, which is not suitable for practical application. Therefore, we need to adjust the control design parameters properly to acquire better control action in practical engineering.

The block diagram of the above-mentioned adaptive finite-time event-triggered control scheme is displayed in Fig. 1.
4 Simulation example

In this section, we provide two simulation examples to verify the feasibility and superiority of the constructed control framework.

**Numerical Example 1:** Take the following second-order nonstrict-feedback nonlinear system into consideration

\[
\begin{align*}
\dot{x}_1 &= r_1 x_2 + \frac{0.5 x_1^2}{r_1^2 x_2^2} + d_1(t), \\
\dot{x}_2 &= r_2 \mu + \sin(x_1 x_2) + d_2(t), \\
y &= x_1,
\end{align*}
\]

where \( r_1 = -2 \) and \( r_2 = -1 \). The bounded disturbances in the system are given as \( d_1(t) = 0.01 \sin(t + 0.5) \) and \( d_2(t) = 0.01 \cos(2t + 1) \). The desired reference signal is \( y_d = 0.5 \sin t \).

By utilizing the coordinate transformations \( \eta_1 = \frac{x_1}{r_1 r_2} \) and \( \eta_2 = \frac{x_2}{r_2} \), we can derive the following nonlinear system which is equivalent to system (4.1):

\[
\begin{align*}
\dot{\eta}_1 &= \eta_2 + \frac{0.5 r_1 r_2 \eta_1^2}{r_2^2} + d_1'(t), \\
\dot{\eta}_2 &= u(t) + \frac{\sin(r_1^2 \eta_1 \eta_2)}{r_2} + d_2'(t), \\
y &= r_1 r_2 \eta_1,
\end{align*}
\]

where \( d_1'(t) = \frac{0.01 \sin(t + 0.5)}{r_1 r_2} \) and \( d_2'(t) = \frac{0.01 \cos(2t + 1)}{r_2} \).

Next, we will conduct simulation research on the system (4.2).

To deal with the nonlinear terms, we choose the following fuzzy membership functions

\[
\begin{align*}
\mu_{B_1} &= e^{-\frac{(\eta_2 + 0)^2}{2}}, \\
\mu_{B_2} &= e^{-\frac{(\eta_2 + 0)^2}{2}}, \\
\mu_{B_3} &= e^{-\frac{(\eta_2 + 2)^2}{2}}, \\
\mu_{B_4} &= e^{-\frac{(\eta_2 + 0)^2}{2}}, \\
\mu_{B_5} &= e^{-\frac{(\eta_2 - 2)^2}{2}}, \\
\mu_{B_6} &= e^{-\frac{(\eta_2 - 0)^2}{2}}, \\
\mu_{B_7} &= e^{-\frac{(\eta_2 - 2)^2}{2}}, \\
\mu_{B_8} &= e^{-\frac{(\eta_2 - 0)^2}{2}},
\end{align*}
\]

The fuzzy state observer is constructed as

\[
\begin{align*}
\dot{\hat{\eta}}_1 &= \hat{\eta}_2 - k_1 \hat{\eta}_1 + \hat{\theta}^T_1 \psi_1(\hat{\eta}_1, \hat{\eta}_2), \\
\dot{\hat{\eta}}_2 &= u(t) - k_2 \hat{\eta}_1 - \hat{\theta}^T_2 \psi_2(\hat{\eta}_1, \hat{\eta}_2).
\end{align*}
\]

The adaptive finite-time event-triggered controller is designed as

\[
\begin{align*}
\alpha_1 &= N(\xi) \left( c_1 x_1 + \frac{(t+8)^2+76}{4} \vartheta_1 + \frac{1}{4} k_1^2 \vartheta_1 \vartheta_1 - \dot{y}_d \right), \\
\alpha_2 &= -c_2 x_2 - \Gamma_1 x_1 + k_2 \dot{\eta}_1 - \hat{\theta}^T_2 \psi_2(\hat{\eta}_1, \hat{\eta}_2), \\
\xi(t) &= -(1 + \pi) \left( \tilde{\alpha}_2 \tanh \left( \frac{\vartheta_2 \vartheta_2}{e} \right) + \tilde{\alpha} \tanh \left( \frac{\vartheta_2 \vartheta_2}{e} \right) \right), \\
u(t) &= \xi(t), q \in z^+,
\end{align*}
\]

with the adaptation laws of the parameters are

\[
\begin{align*}
\dot{\hat{\theta}}_1 &= k_1 \dot{\vartheta}_1 \psi_1(\hat{\eta}_1) - \sigma_1 \hat{\theta}_1, \\
\dot{\hat{\theta}}_2 &= k_2 \dot{\vartheta}_2 \psi_2(\hat{\eta}_1, \hat{\eta}_2) - \sigma_2 \hat{\theta}_2,
\end{align*}
\]

and Nussbaum function \( N(\xi) \) is selected as \( N(\xi) = e^{e^2 \cos \left( \frac{\pi}{4} \xi \right)} \).

The initial conditions are selected as \( \eta_1(0) = 0.01, \eta_2(0) = -0.2, \hat{\eta}_1(0) = 0.1, \hat{\eta}_2(0) = 0.15, \xi(0) = 0.3, \tilde{\alpha}_1(0) = [0, 0, 0, 0, 0, 0, 0, 0]^T, \tilde{\alpha}_2(0) = [0, 0, 0, 0, 0, 0, 0, 0]^T \).

The control design parameters in the Numerical Example 1 are shown in Table 1.

Figures 2–6 depict the simulation results. By utilizing the developed control design scheme in this paper and the one in [41], the responses of the outputs \( y \) and the appointed signal \( y_d \) are exhibited in Fig. 2, and the comparison curves of the tracking errors are plotted in Fig. 3. From Figs. 2 and 3, it can be seen that the system output \( y \) can follow the specified signal \( y_d \), and the finite-time control approach proposed in this article has higher tracking precision than the nonfinite-time control method developed in [41]. Figures 4 and 5 describe the responses of fuzzy adaptive parameters \( \hat{\theta}_i, i = 1, 2 \), parameter \( \xi \) and Nussbaum gain \( N(\xi) \), respectively.

The time intervals \( t_{q+1} - t_q \) of triggering events are indicated in Fig. 6. From Fig. 6, we can observe that the Zeno behavior is successfully avoided. Based on the above simulation results, it can be obtained that the adaptive finite-time control scheme developed in this article is effective.

**Practical Example 2:** Consider the pendulum model in [42], which is described by the following dynamic equations:

\[
ml\ddot{\phi} + kl\dot{\phi} + mg \sin \phi = u,
\]

where \( \phi \) denotes the acute angle between the vertical axis and the rod; \( \dot{\phi} \) stands for the angular velocity of the rod; \( m \) is the mass of the bob; \( l \) is the length of the rod; \( k \) denotes an unknown frictional coefficient; and
Table 1 Control parameters in the Numerical Example 1

| Parameters | Value | Parameters | Value | Parameters | Value | Parameters | Value |
|------------|-------|------------|-------|------------|-------|------------|-------|
| $c_1$      | 20    | $s_2$      | 50    | $L_{1,1}$  | 20    | $a$        | 1     |
| $c_2$      | 20    | $\kappa_1$| 0.1   | $L_{1,2}$  | 30    | $\tau$    | 2     |
| $k_1$      | 0.4   | $\kappa_2$| 0.1   | $\pi$      | 0.5   | $\beta$   | 3/5   |
| $k_2$      | 0.1   | $\sigma_1$| 1     | $\tilde{a}$| 3     | $\tilde{\Gamma}_1$| 3     |
| $s_1$      | 36    | $\sigma_2$| 1     | $\epsilon$| 10    |           |       |

$g$ represents the acceleration of gravity. Define transformations $\eta_1 = ml\phi$ and $\eta_2 = ml\dot{\phi}$, then the system (4.3) is redescribed as

$$
\begin{align*}
\dot{\eta}_1 &= \eta_2, \\
\dot{\eta}_2 &= u - mg \sin(\eta_1) - \frac{k}{m} \eta_2, \\
y &= \eta_1.
\end{align*}
$$

Fig. 2 $y$ and $y_d$ for the developed control approach in this article and the one in [41]

Fig. 3 The tracking errors for the developed control approach in this article and the one in [41]

Fig. 4 Fuzzy adaptive parameters

Fig. 5 The trajectories of $\varsigma$ and $N(\varsigma)$
To deal with the nonlinear terms, we choose the following fuzzy membership functions

\[
\begin{align*}
\mu_{B_j}^1 &= e^{-\frac{(\hat{\eta}_j + 1.5)^2}{2}}, \\
\mu_{B_j}^2 &= e^{-\frac{(\hat{\eta}_j + 1)^2}{2}}, \\
\mu_{B_j}^3 &= e^{-\frac{(\hat{\eta}_j + 0.5)^2}{2}}, \\
\mu_{B_j}^4 &= e^{-\frac{(\hat{\eta}_j - 0.5)^2}{2}}, \\
\mu_{B_j}^5 &= e^{-\frac{(\hat{\eta}_j - 1)^2}{2}}, \quad j = 1, 2.
\end{align*}
\]

The appointed reference signal is \( y_d = 0.5(\sin t + \sin(0.5t)) \). The adaptive finite-time event-triggered controller, the fuzzy state observer, the parameter adaptive laws and Nussbaum function are the same as those in Numerical Example 1. The initial conditions are chosen as \( \eta_1(0) = 0.2, \eta_2(0) = -0.1, \hat{\eta}_1(0) = 0.2, \hat{\eta}_2(0) = 0.1, \zeta(0) = 0.1, \hat{\theta}_1(0) = [0.1, 0, 0, 0, 0, 0, 0, 0]^T, \hat{\theta}_2(0) = [0, 0.2, 0, 0, 0, 0, 0, 0]^T \). The control design parameters in the Practical Example 2 are shown in Table 2.

Figures 7–14 depict the simulation results. By utilizing the developed control design scheme in this article and the one in [41], Fig. 7 exhibits the responses of the system outputs \( y \) and the desired signal \( y_d \), and Fig. 8 plots the comparison curves of the tracking errors. From Figs. 7 and 8, it can be seen that the output \( y \) of the system (4.4) can track the specified signal \( y_d \), and the finite-time control approach developed in this article has higher tracking precision than the nonfinite-time control approach developed in [41]. Figures 9 and 10 indicate the responses of state variables \( \eta_1, \eta_2 \) in the system (4.4) and their estimations \( \hat{\eta}_1, \hat{\eta}_2 \), respectively. According to Figs. 9 and 10, it is concluded that the observer errors \( \tilde{\eta}_1 \) and \( \tilde{\eta}_2 \) are adjusted around zero with a small neighborhood in a finite time. Figures 11 and 12 describe the responses of adaptive parameters \( \hat{\theta}_1, \hat{\theta}_2, \zeta \)

| Table 2 Control parameters in the Practical Example 2 |
|-----------------------------------------------------|
| Parameters | Value | Parameters | Value | Parameters | Value | Parameters | Value |
|------------|-------|------------|-------|------------|-------|------------|-------|
| \( m \)    | 1/4   | \( k_1 \)  | 0.5   | \( \sigma_1 \) | 10    | \( a \)    | 1     |
| \( l \)    | 4     | \( k_2 \)  | 3     | \( \sigma_1 \) | 10    | \( \epsilon \) | 10    |
| \( k \)    | 1/4   | \( s_1 \)  | 12    | \( L_{1,1} \) | 50    | \( \tau \)  | 2     |
| \( g \)    | 10    | \( s_2 \)  | 20    | \( L_{1,1} \) | 100   | \( \beta \) | 91/100 |
| \( c_1 \)  | 15    | \( \kappa_1 \) | 1    | \( \pi \)   | 0.2   | \( \bar{\Gamma}_1 \) | 1.2   |
| \( c_2 \)  | 20    | \( \kappa_2 \) | 1    | \( \bar{\theta} \) | 2     |
Adaptive finite-time event-triggered command filtered control for nonlinear systems

Fig. 8 The tracking errors for the developed control approach in this article and the one in [41]

Fig. 9 The trajectories of $\eta_1$ and $\hat{\eta}_1$

and $N(\xi)$. Figure 13 provides the response of the finite-time event-triggered controller $u$. Figure 14 indicates the time intervals $t_{q+1} - t_q$ of triggering events. From Figs. 13 and 14, it is observed that the event-triggered control method can lighten the communication burden and save communication resources. With the help of the above simulation results, the validity and superiority of the developed control scheme in this article can be explained.

Remark 4.1 From Figs. 3 and 8, it can be seen that the tracking errors are adjusted around the origin with a small neighborhood in both finite-time and nonfinite-time control schemes. In spite of this, one can observe that the finite-time control method has the merits of faster transient performance and higher tracking precision compared with the nonfinite-time control method in [41].

5 Conclusion

In this article, an adaptive finite-time output feedback event-triggered command filtered control approach has been established for a category of nonstrict-feedback nonlinear systems with unknown control directions. In the procedure of control design, the event-triggered mechanism was adopted to reduce the communication burden between the controller and the actuator. The finite-time command filter was applied to avoid
the ‘explosion of complexity’ caused by the backstepping method, and an error compensation system with fraction power was constructed to compensate for the filtering errors between the command filter and the virtual control signals. By means of the command filtering technology and the event-triggered mechanism, an adaptive finite-time event-triggered controller was designed. The controller can ensure that the tracking and observer errors can be adjusted around zero with a small neighborhood in a finite time and all the signals in the closed-loop system are bounded. Finally, two simulation examples have been provided to explain the effectiveness and superiority of the design approach. In this paper, the approximation characteristic of the FLSs was utilized to deal with the unknown nonlinear terms in the systems, which can only guarantee the semi-global stability of the closed-loop system, but cannot achieve global stabilization. In future work, we will consider the stochastic disturbances in the systems and attempt to design a fixed-time event-triggered control strategy to solve the global stabilization problem of stochastic nonlinear systems.

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Data availability All data generated or analyzed during this study are included in this published article.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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