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Tendon-driven Underactuated Hand Design via Optimization of Mechanically Realizable Manifolds in Posture and Torque Spaces

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Abstract—Grasp synergies represent a useful idea to reduce grasping complexity without compromising versatility. Synergies describe coordination patterns between joints, either in terms of position (joint angles) or force (joint torques). In both of these cases, a grasp synergy can be represented as a low-dimensional manifold lying in the high-dimensional joint posture or torque space. In this paper, we use the term Mechanically Realizable Manifolds to refer to the subset of such manifolds (in either posture or torque space) that can be achieved via mechanical coupling of the joints in underactuated hands. We present a method to optimize the design parameters of an underactuated hand in order to shape the Mechanically Realizable Manifolds to satisfy a pre-defined set of desired grasps. Our method guarantees that the resulting synergies can be implemented in a physical underactuated hand, and will enable the resulting hand to both reach the desired grasp postures and achieve quasistatic equilibrium while loading the grasps. We demonstrate this method on three concrete design examples derived from a real use case, and evaluate and compare their performance in practice.

Index Terms—underactuated hands, Mechanically Realizable Manifolds, synergies

I. INTRODUCTION

GRASP synergies refer to the correlation of multiple degrees-of-freedom (DoFs) in a hand, providing an effective tool to realize versatile grasping in a simple fashion. The idea of grasp synergies originates in studies of human hands [1], but is also adopted in many robotic applications. For example, synergies can be used in the planning or control algorithms for robotic hands (e.g., [2]–[9]). Synergies can also be embedded into the mechanical design of underactuated hands, moving some of the control intelligence to the hardware. The latter idea is the main focus of this paper.

Underactuated hands are gaining increasing attention in academia and industry. In particular, multi-fingered underactuated hands sit between two categories at opposite ends of the complexity spectrum (parallel jaw grippers vs. fully actuated dexterous hands), and can combine some of the advantages of both (simplicity and versatility). Taking advantage of synergies, multi-fingered underactuated hands can perform different grasps with a small number of actuators and control inputs. At the same time, these hands can conform to objects by virtue of differential or breakaway mechanisms, therefore they do not require careful grasp synthesis, and are robust to perception error.

In joint posture space, a synergy of a (fully- or underactuated) hand can be represented by a low-dimensional manifold, along which the point of hand configuration can slide. In joint torque space, a low-dimensional torque coordination scheme can be physically realized by underactuated mechanisms. These manifolds are parameterized based on mechanical specifications, and can be altered by changing values of the design parameters to exhibit different shapes corresponding to different hand behaviors.

We note that realizing an arbitrary manifold using mech-
A common place to apply the idea of postural synergies for robotic hands is in the planning or control for fully-actuated dexterous hands. Researchers presented various studies in this category, though the term for synergies may be different, e.g., “eigengraspers” or “eigenpostures”. Planning in the low-dimensional subspace can significantly reduce the computation complexity of the grasp search. For example, Rosell et al. [2] studied the motion planning problem of a hand-arm system in a reduced-dimensional synergy space. Ciocarlie and Allen [3] discussed the use of low-dimensional postural subspace in the automated grasp synthesis, and proposed a planner which takes advantage of reduced dimensionality and can be fast enough to run in real-time. Moreover, the technique of synergy is also adopted in the control of robotic hands, meaning the joints are commanded in a coupled fashion. For example, the work from Wimbock et al. [4] showed a synergy-level impedance controller for a multi-finger hand. One specific idea in this category of robot hand control is to use the low-dimensional synergies as a human-robot interface for teleoperation, learning by demonstration or prosthetics, to reduce the required communication bandwidth between the human and the robot. For teleoperation, this method has been shown in the studies from Gioioso et al. [5] and Meeker et al. [6]. For hand prosthetics, Matrone et al. [7] [8] as well as Tsoli and Jenkins [9] presented the aforementioned idea and developed working prototypes. However, these studies mostly consider postural control without counting for grasping force equilibrium. Besides, most of these studies are limited to anthropomorphic hands in which the synergies can be extracted from human data.

Since the methods above implement synergies in software, in contrast, we can transfer this idea by coupling joints together in hardware. This leads to the mechanical realization of pre-defined synergies. For example, Brown and Asada [10] designed a mechanical implementation of PCA results for a hand using pulley-slider systems to realize inter-finger coordination. Xu et al. [11] [12], Li et al. [13], Chen et al. [14], Xiong et al. [15] also proposed several studies to enable hardware synergies for anthropomorphic hands, based on different types of mechanisms such as gears, continuum mechanisms, cams, and sliders. These studies only consider postural behaviors without the notion of force, so they do not have a guarantee...
for grasp stability. To deal with this issue, a series of works from Gabicciniet al. [16], Practitichetto et al. [17], Grioli et al. [18], and Catalano et al. [19], presented the concept of “soft synergies” and “adaptive synergies” and provided the models and tools for the underactuated hands with such features to account for force generation and force equilibrium. They also used this theory in the design and control of a multi-finger dexterous hand (the Pisa/IIT hand). Though all studies above present feasible ways to implement synergies mechanically, they only take anthropomorphic hands into account, whereas we aim to discover synergies for a broader range of hands (both anthropomorphic and non-anthropomorphic). We also aim to implement the synergistic behavior only by altering tendon routes and spring parameters, without the need for additional mechanisms such as sliders, gears, differentials, etc.

To fulfill certain requirements of an underactuated hand design, one common way is to select parameters via optimization. There is a lot of literature in this category, for example, Birglen et al. [20] presented an optimization study in detail for linkage-driven underactuated hands. Dollar and Howe [21] optimized the kinematic configuration and joint stiffnesses to maximize successful grasp range and minimize contact forces. Ciocarlie and Allen [22] formulated the parameter design problem of a certain underactuated gripper as a globally convex quadratic programming. Saliba and Silva [23] used a quasi-dynamic analysis for the optimization of an under-actuated gripper. Dong et al. [24] optimizes the dimensions and tendon routes of a tendon-driven hand using Genetic Algorithm. Ciocarlie et al. [25] optimized the Velo gripper to achieve both fingertip grasp and enveloping grasp using a single actuator. Compared to existing design optimization works, the uniqueness of this work stems from the idea of optimizing the mechanisms of a highly underactuated hand in order to satisfy a specific set of desired grasps. Using an analogy to data fitting problems, the “data points” here are the target grasps, and the “fitting function” is the Mechanically Realizable Manifold. We minimize the deviation between the “data points” and the “fitting function”.

The preliminary work of this study is shown in [26]. In this paper, we extend the concept of previously reported “Mechanically Realizable Manifold” also to the domain of joint torques, and present a more generic optimization framework, a more comprehensive design procedure, more efficient algorithms, more design examples, and in-depth analysis and discussions.

III. PROBLEM FORMULATION

Our overall goal is to design highly underactuated hands able to perform a set of versatile grasps in a stable fashion. We formalize our problem as follows.

First of all, since the design space of a robot hand is very large, we narrow down our problem by requiring a hand model with pre-defined dimensions and kinematics; the unknowns are the (under-)actuation parameters which determine the hand behavior when driven by motors. The given hand kinematics specifies the number of fingers and links, the shape and dimensions of the fingers, the tendon connectivity pattern (specifying which tendons drive which joints), etc. The unknown parameters include tendon moment arms, restoring spring parameters, etc. We point out that we are not presenting a method to conduct the initial design for hand dimensions or kinematics, but only the (under-)actuation parameters that determine hand behavior. However, for different pre-specified kinematic design options, the results computed from our optimization algorithms can be used as performance metrics to compare them, as we will discuss in Section VI.

In addition, we assume we have collected a set of stable simulated grasps as desired grasps (such as the ones shown in Fig. 1(a) and Fig. 2) using the hand model with pre-defined kinematics. These grasps do not account for underactuation, i.e. the joints are considered independent and the hand can exhibit arbitrary poses as needed. All of these grasps are required to have force-closure, i.e. there exists a set of contact forces that produce zeros resultant wrench on the object while satisfying friction constraints.

Our goals are as follows. We aim to design the underactuation mechanism for the hand to conduct the set of desired grasps using (many) fewer actuators than joints. This is equivalent to two simultaneous requirements: even with constraints due to underactuation, the hand needs to both reach the desired postures, and apply the needed forces to load the grasps stably. Using the concept of underactuation manifolds, we thus aim to optimize the actuation parameters under real-world constraints, in order to (1) shape the Mechanically Realizable Posture Manifold to approach the desired grasping poses in the pre-contact phase, and also (2) optimize the Mechanically Realizable Torque Manifold to provide the joint torques as close to equilibrium as possible in the post-contact phase.

In our case, the actuation parameters we wish to optimize are: the tendon moment arms (or the pulley radii) in the joints, the restoring spring stiffnesses, and the spring preloads (defined as the spring flexion when joint angles are zeros). Fig. 2 shows two different cases where (a) shows the parameterization for the design using pulleys and (b) shows the design with tendon via-points.

The optimization needs to obey certain constraints. For example, restoring spring stiffnesses are constrained by the physical dimensions of the allowed mounting space, and they are only available in discrete series of values offered by the manufacturer. For another example, restoring spring torques should be less than the maximum values the actuator can
overcome, but larger than the torques to support the weight of link in any configuration. The ability to deal with real-world constraints is one of the advantages of our method compared to the direct implementation of synergies in [10] - [15].

IV. DESIGN METHOD
A. Problem Decomposition

The aforementioned optimization is a multi-objective problem in the same design space: we need to optimize for both pre-contact kinematic behaviors and post-contact force generation behaviors, and both behaviors are related to all actuation parameters listed above. While formulating some weighted combination of these objectives is possible, it would require arbitrarily assigned weights, which we would like to avoid; besides, it is also a high-dimensional optimization if we search all parameters simultaneously. It would thus be beneficial if we could split the optimization in an appropriate way.

Our insight is that we can solve our problem by decomposing it into three parts:

- The optimization of Mechanically Realizable Torque Manifold
- The optimization of Mechanically Realizable Postural Manifold for inter-tendon behaviors
- The optimization of Mechanically Realizable Postural Manifold for intra-tendon behaviors

We explain this decomposition as follows.

First of all, we can split the problem into the optimizations for pre-contact and post-contact behaviors, by assuming that the hand is in equilibrium in both of these conditions. (If the result indicates this assumption cannot be achieved for any grasp in our set, we have measures to exclude such grasps and conduct the optimization again, as shown at the end of Subsection IV-B.) For a joint (either single DoF or multi-DoF (e.g. universal joint)), equilibrium just before the contacts are made can be expressed as \( t_{pre,j} \times \mathbf{r}_j + \mathbf{\tau}_s(q) = \mathbf{0} \) (1), which models the general case that there are multiple tendons connecting to a joint. \( t_{pre,j} \), \( \mathbf{r}_j \), \( \mathbf{\tau}_s(q) \) are the vectors of tendon moment arm, pre-contact tension, joint value and joint spring torque. \( p \) is the number of tendon connected to this joint.

Once additional torque is applied and the grasp is loaded, equilibrium can be expressed as \( \sum_{j=1}^{p} t_{post,j} \times \mathbf{r}_j + \mathbf{\tau}_s(q) = \mathbf{0} \) (2), where the \( t_{post,j} \) is the post-contact tendon tension.

Combining (1) and (2) shows that the net joint torque is only affected by the moment arms but not spring parameters, shown in (3).

\[
\begin{align*}
\mathbf{\tau}_{net} &= \sum_{j=1}^{p} \left( t_{post,j} - t_{pre,j} \right) \times \mathbf{r}_j \\
&= \sum_{j=1}^{p} t_{net,j} \times \mathbf{r}_j
\end{align*}
\] (3)

Therefore, we can optimize for the post-contact grasp equilibrium first (which results in zero post-contact movement as the contact forces are increased), and then optimize the rest of parameters for the pre-contact kinematic behaviors.

Furthermore, the optimization for the pre-contact stage has two aspects: the inter-tendon kinematic behavior and the intra-tendon kinematic behavior. In the case of inter-tendon kinematics, different tendons are driven by the same motor, which exhibit no differential behavior: if one tendon stops (due to contact on a driven link), then all tendons on the same motor stop as well (as we do not use differential mechanisms such as floating pulleys [21]). However, in the case of intra-tendon kinematics, multiple joints are driven by the same tendon, which do exhibit a differential-like behavior: assuming a proximal joint is stopped, more distal joints can continue to move as tendons can slide on the via points or around idlers on the blocked joints. Thus, the inter-tendon optimization ensures the coordination between the kinematic chains on different tendons are as desired, and the intra-tendon optimization ensures the correlation of individual joints connected on a same tendon are as desired. We select different groups of design parameters in different optimizations: the former determines the tendon moment arms, and the latter determines the spring parameters.

The decomposition of the problem is summarized in Fig. 3.

In the first step, we alter the tendon moment arms to optimize the Mechanically Realizable Torque Manifold, in order to create post-contact equilibrium. As the optimization in this step has an infinite number of minima, which we will explain in the next subsection, we pick one that also solves the second step: the optimization of Mechanically Realizable Postural Manifold for inter-tendon behaviors. At this point, we can fully determine the optimal tendon moment arms. After that, we optimize the spring parameters (stiffnesses and preloads) to make sure that the Mechanically Realizable Postural Manifold for each tendon passes closely to the desired grasps.
B. Optimization of Mechanically Realizable Torque Manifold

The objective of this optimization is to have the underactuated hand and object as close to equilibrium as possible in the post-contact phase. The variables we alter are the tendon moment arms in all joints. We will explain at the end of this subsection that there are infinite number of solutions, so the optimal tendon moment arms will be determined in the next step with an extra objective.

1) Grasp Analysis: To form an objective for the optimization, an evaluation of grasp stability is needed. Here we employ a well-established grasp analysis formulation [22]. We use the (linearized) model of point-contact with friction. For contact \( k \), contact wrench \( c_k \) can be expressed as linear combinations of \( \beta_k \) — the amplitudes of the frictional and normal components, related by a matrix \( D_k \), as shown in [4]. Equations (5) and (6) model the effect that contact force must be constrained inside the friction cone (or pyramid). The details about the construction of matrices \( D_k \) and \( F_k \) can be found in [27].

\[ c_k = D_k \beta_k \quad (4) \]
\[ F_k \beta_k \leq 0 \quad (5) \]
\[ \beta_k \geq 0 \quad (6) \]

In general, a grasp is stable if the following conditions are satisfied:

- **Hand equilibrium**: the active joint torques are balanced by contact forces.
  \[ J^T c = J^T D \beta = \tau_{eq} \quad (7) \]

- **Object equilibrium**: the resultant object wrench is zero.
  \[ Gc = GD \beta = 0 \quad (8) \]

- **Friction constraints**: the contact forces are constrained inside the friction cone.
  \[ F \beta \leq 0 \quad (9) \]
  \[ \beta \geq 0 \quad (10) \]

In these formulations, \( J \) is the contact Jacobian, \( G \) is the Grasp Map matrix, \( D \) and \( F \) are block-diagonal matrices constructed by \( D_k \) and \( F_k \) respectively, \( c \) and \( \beta \) are stacked vectors constructed by \( c_k \) and \( \beta_k \) respectively, and \( \tau_{eq} \) is the desired joint torque vector to create hand equilibrium.

This grasp stability analysis [7] - [10] can be turned into an optimization problem by switching either one condition to an objective. We further discuss the optimization formulation in the next subsection.

2) Optimization Formulation: The overview of our formulation is as follows: we incorporate a dual-layer optimization framework, where the inner layer is an optimization to calculate a quality metric of a specific grasp given a certain set of design parameters, and the outer layer is a search over the parameters for all considered grasps.

In our problem, since the hand is underactuated, the joint torques are not independent. Instead, the actually generated net torque \( \tau_{net}^{gen} \) follows the relationship:

\[ \tau_{net}^{gen} = At_{net} \quad (11) \]

where \( A \) is the Actuation Matrix, which is a function of the tendon moment arms \( r_1, r_2, \cdots, r_m \) (\( m \) is the number of DoFs), and may also be configuration-dependent. \( t_{net} \) is the net tendon tension vector comparing to the tension just before touch.

For a set of given tendon moment arms and a given grasp pose, we wish to find the contact force magnitudes \( \beta \) and the net tension in each tendon \( t_{net} \), which solve [7]–[11]. We change the search for the exact solution to an optimization problem by turning hand equilibrium (7) from a constraint into an objective function. The unbalanced joint torque vector \( \Delta \tau_{post} \) is shown in [12] (subscript \( post \) meaning “post-touch”). We use the norm of this vector \( \| \Delta \tau_{post} \| \) as the stability metric, and a lower value is considered better.

\[ \Delta \tau_{post} = \tau_{eq} - \tau_{net}^{gen} = J^T D \beta - At_{net} \quad (12) \]

The inner layer problem - to find the minimal norm of unbalanced torques for one given grasp - is a convex Quadratic Program (QP) as follows, shown in [13] - [18].

find:

\[ x = \begin{bmatrix} \beta \\ t_{net} \end{bmatrix} \quad (13) \]

minimize:

\[ \| \Delta \tau_{post} \|^2 = \| Qx \|^2 = x^T Q^T Qx \quad (14) \]

where \( Q = [J^T D - A] \)

subject to:

\[ GD O x = 0 \quad (15) \]
\[ F O x \leq 0 \quad (16) \]
\[ x \geq 0 \quad (17) \]
\[ [1 \cdots 1] [J^T D O] x = 1 \quad (18) \]

The constraints [15]–[17] are extended versions of [3]–[10] and the last one [18] prevents the trivial solution where all contact forces and joint torques are zeros, by constraining the sum of joint torques to be one. In this way, the calculated grasp stability metric \( \| \Delta \tau_{post} \| \) is a normalized unitless torque.

Since the aforementioned inner layer can give a stability metric for a specific desired grasp, the outer-layer is a global search over the tendon moment arms for all considered grasps using the inner layer results. The objective function for the global search is an overall metric using the root of squared sum of all grasps’ quality metrics, shown in [20]. The outer-layer global optimization is formulated as:
search:

\[ r_1, r_2, \cdots, r_m \]  

minimize:

\[
\sum_{i=1}^{n} \left( \frac{\| \Delta \tau_{\text{post},i} \|^2}{f_{\text{trq}}(r_1, r_2, \cdots, r_m)} \right)^{1/2}
\]  

subject to:

\[ r_i \in [r_{lb}, r_{ub}], \quad i = 1, 2, \cdots, m \]  

save:

\[
f_{\text{trq}}^{\min} = \min(f_{\text{trq}}(r_1, r_2, \cdots, r_m))
\]  

where \( \| \Delta \tau_{\text{post},i} \| \) is the individual stability metrics computed by the QP in (13) - (18) for each simulated desired grasp, \( n \) is the number of grasps, \( r_{lb} \) and \( r_{ub} \) are the lower and upper bounds of tendon moment arms. 

Though the inner-layer is convex, the outer-layer is not, and is not trivial to be reformulated as a convex problem. Therefore we decided to use a stochastic global search. The optimizer we choose is the Covariance Matrix Adaptation Evolutionary Strategy (CMA-ES) [28][29]. It is a stochastic, derivative-free algorithm for black-box global optimization, in which covariance matrix of the distribution of the candidate solutions is updated adaptively in each generation. This method learns a stochastic second-order approximation of the objective, and drives the candidate solutions to the optimum, even when the function is ill-conditioned.

We aim to find a Mechanically Realizable Manifold that minimizes the unbalanced torque \( \| \Delta \tau_{\text{post},i} \| \) for all grasps. In practice, we exclude from this optimization the grasps where the \( \| \Delta \tau_{\text{post},i} \| \) is found to be significantly worse (i.e. larger) than others, and then we iterate again and solve the optimization problem for the rest of grasps. In this way, we can design the hand to perform as well as possible on the grasps possible to create, instead of attempting to also satisfy equilibrium for impossible grasps. Moreover, the number of grasps excluded in this way is an important metric of the hand’s overall capabilities to achieve our design goals. All our results, presented later in the paper, will thus report both the number of excluded grasps, and the values of all optimization objectives (computed over the grasps that have been kept).

Here we explain why the minimum is not unique. Let us assume we have found a set of optimal tendon moment arms, with a certain \( x \) (thus a certain set of \( t_{\text{net}} \)) in (13). As long as the QP (13) - (18) can find an \( x \) (and thus \( t_{\text{net}} \)) that can keep \( A t_{\text{net}} \) the same when we alter the tendon moment arms (one possible way is to scale \( x \) or \( t \) while scaling entries in \( A \)), the metric of the inner layer (the QP (13) - (18)) remains minimal, then the objective function value of the outer layer (the global search (20) - (21)) remains minimal. Therefore there are other optimal solutions for the tendon moment arms.

The optimization of Mechanically Realizable Torque Manifold is summarized as Fig. 4. After this step, we can find a Mechanically Realizable Torque Manifold, on which the joint torque distribution, determined by the tendon moment arms, is as close to equilibrium as possible for all the considered grasps. We will pick the one set of values for tendon moment arms from the non-unique and equally good solutions with an extra objective in the next subsection.

C. Optimization of the Mechanically Realizable Posture Manifold for Inter-tendon Kinematic Behaviors

In this step we aim to get a coordinated finger movement among different tendons. As the hand moves from a reference starting pose into a grasp pose, each tendon must travel a specific amount, dictated by the movement of each joint as well as the tendon’s moment arm around each joint. (This assumes that tendons are inextensible, and also are not allowed to become slack as they lose force transmission abilities.) However, tendon travel must also be in accordance to the travel of the motor that the tendon is connected to, a problem which becomes non-trivial for the case of multiple tendons connected to the same motor.

We again translate this requirement into an optimization problem. For each of the desired grasp poses, we minimize the differences between required and actual tendon travel for all tendons connected to the same motor, with motor travel angle(s) as a free variable.

This optimization is only related to the tendon moment arms, which are picked from the set of equally good solutions in the previous optimization, and can be fully determined with the extra objective in this step.

We note that the pool of desired grasps is different from the previous subsection; we also include the fully open configuration. For each grasping pose in the original grasp pool, we add an opening pose, so the opening and closing poses are always in pairs. In this way, we can ensure the hand can actually open, instead of moving between desired grasping poses.

We also follow the dual-layer optimization framework as the previous subsection. The inner layer minimizes the norm of an error vector whose entries are the difference between the tendon travel the motor collects and the one the grasp requires. The outer layer is also a stochastic global optimization to minimize the overall metric for all grasps, with the constraint that the objective function in the previous optimization needs to reach its minimum.

In the inner layer, the error vector can be expressed as (23), where the matrix \( M \) is a motor-tendon connection matrix.
whose entries can be either motor pulley radius \( r_1, r_2, \ldots, r_m \) (meaning the corresponding tendon is connected to the corresponding motor) or zero (meaning not connected), \( \theta_{mot} \) is a vector of angles that the motors moved, and \( s \) is the tendon travel vector whose entries are the travel of the corresponding tendon required by the joint values in the desired grasp (compared to zero positions).

\[
e = M\theta_{mot} - s \tag{23}
\]

Minimizing the norm of the error vector \( \|e\| \) is a QP over the angles the motors moved \( \theta_{mot} \), with no constraints. It is shown in the QP \((24)\) \((25)\) below.

**find:**

\[
\minimize: \quad \theta_{mot} \tag{24}
\]

\[
\|e\|^2 = \theta_{mot}^T M\theta_{mot} - 2s^T M\theta_{mot} + s^T s \tag{25}
\]

The outer layer, which is a global optimization over the tendon moment arms to minimize the overall metric \( f_{inter}(r_1, r_2, \ldots, r_m) \) (which is the root of squared sum of inner layer results), has to obey the constraint that the result of previous optimization (of Mechanically Realizable Torque Manifold) needs to take its minimum value. The constraint is handled by giving a high penalty if the constraint is violated, and more penalty if the candidate solutions are farther away from the feasible region. The outer layer optimization is as follows:

**find:**

\[
r_1, r_2, \ldots, r_m \tag{26}
\]

**minimize:**

\[
f_{inter}(r_1, r_2, \ldots, r_m) = \left( \sum_{i=1}^{n} \|e_i\|^2 \right)^{1/2} \tag{27}
\]

**subject to:**

\[
f_{trq}(r_1, r_2, \ldots, r_m) = f_{trq}^{min} \tag{28}
\]

\[
r_i \in [r_{lb}, r_{ub}], \quad i = 1, 2, \ldots, m \tag{29}
\]

where \( \|e_i\| \) is the individual inner layer metric for each grasp calculated from the QP \((24) - (25)\), and \( f_{trq}^{min} \) is the minimal function value saved from the previous step. We also incorporate CMA-ES for the global search.

At this point, the tendon moment arms \( r_1, r_2, \ldots, r_m \) are fully determined. The optimization of Mechanically Realizable Posture Manifold for inter-tendon kinematic behaviors can be summarized as Fig. 5.

**D. Optimization of Mechanically Realizable Posture Manifold for Intra-tendon Kinematic Behaviors**

The goal of this optimization is to coordinate the joint movements in a same tendon to have the Mechanically Realizable Posture Manifold close to the desired grasp poses. We import the tendon moment arms from the previous optimization, and the remaining parameters to optimize are: the spring stiffnesses and the spring preloads.

We translate the goal of quasi-statically reaching desired grasps to the one of minimizing the pre-contact unbalanced spring torques if the hand is posed in the desired grasp configurations. Lower unbalanced torque means the Mechanically Realizable Posture Manifold is closer to the desired grasp. We emphasize that in this part we only consider the equilibrium of the hand itself, without the object.

The unbalanced spring torque vector can be calculated as \((30)\), where the matrix \( A \) is the aforementioned Actuation Matrix, and \( \tau_{spr} \) is a vector of spring torques calculated by given spring parameters and given poses (shown in \((31)\)). We note that here the \( t \) is the absolute tendon tension, which is different from the \( t_{net} \)

\[
\Delta \tau_{pre} = At - \tau_{spr} \tag{30}
\]

\[
\tau_{spr} = [K_1(q_1 + q_{01}), \ldots, K_m(q_m + q_{0m})]^T \tag{31}
\]

We wish to find the \( t \) vector resulting in a minimum norm of unbalanced joint torques, which is also a convex QP as shown below.

**find:**

\[
t \tag{32}
\]

**minimize:**

\[
\|\Delta \tau_{pre}\|^2 = t^T A^T At - 2\tau_{spr}^T At + \tau_{spr}^T \tau_{spr} \tag{33}
\]

**subject to:**

\[
t \geq 0 \tag{34}
\]

We still incorporate the dual-layer optimization. In the outer layer, we use CMA-ES to find the global optimum of the overall metric \( f_{intra}(K_1, \ldots, K_m, q_{01}, \ldots, q_{0m}) \), as shown in \((35) - (37)\). In practice, since the intra-tendon kinematic behaviors are only related to the joint along that tendon, we can perform optimization only for those joints separately at a time, in order to reduce the search dimensions. It is also needed to note that the spring stiffnesses can only take discrete values offered by the manufacturer, so we incorporate the integer handling in CMA-ES.

**find:**

\[
K_1, K_2, \ldots, K_m, q_{01}, q_{02}, \ldots, q_{0m} \tag{35}
\]
Fig. 6. Summary of the optimization for Mechanically Realizable Posture Manifold for intra-tendon kinematic behaviors. The algorithm consists of two layers: the outer layer optimization search over spring parameters while the inner layer optimization gives a metric for the outer layer.

\[
\text{minimize:} \quad f_{\text{inter}}(K_1, \ldots, K_m, q_{01}, \ldots, q_{0m}) = \left( \sum_{i=1}^{n} \| \Delta \tau_{\text{pre}, i} \|^2 \right)^{\frac{1}{2}} \\
\text{subject to:} \\
K_i \in \mathcal{K}, \ i = 1, 2, \ldots, m \\
q_{0i} \in [q_{0l}, q_{0u}], \ i = 1, 2, \ldots, m \\
\]

where the \( \| \Delta \tau_{\text{pre}, i} \| \) is the individual metric for each grasp from the inner layer optimization \((32) - (34)\), and \( \mathcal{K} \) is the set of discrete spring stiffnesses provided by the manufacturer.

The optimization of Mechanically Realizable Posture Manifold for intra-tendon kinematic behaviors can be summarized as Fig. 6.

E. Summary

Fig. 7 is the recap of our method. The design process starts from pre-specified desired grasps and hand kinematics, then goes through the aforementioned three steps for different aspects of hand behaviors: the optimization of Mechanically Realizable Torque Manifold, as well as the Mechanically Realizable Posture Manifold for inter- and intra-tendon behaviors. Finally, it result in a set of optimal actuation parameters for both posture shaping and force generation.

V. DESIGN CASES AND EVALUATIONS

To implement and test the proposed optimization framework, we completed three concrete design examples of three-finger underactuated hands, for the Astrobee robot [30] in the International Space Station (ISS). The Astrobee is a cube-shaped free-flying assistive robot, designed to help the astronauts for in-cabin monitoring and many other tasks. We aim to enable it to do object retrieval and manipulation by mounting a simple arm and a versatile hand to its payload bay.

Our method is suitable for this design task for several reasons. First of all, a highly synergistic underactuated but versatile hand is needed for this application because of the limited onboard space and control signals. Second, the objects in the ISS are known and relatively unchanged, which means we have a given set of objects, and including them in our simulated grasp set may have a good chance of good performance in practice. Third, the available room to store the hand inside the Astrobot is given, so the dimensions of the hand can be specified beforehand, which is also required by our method.

A. Design Case I: Single-motor Hand with Roll-pitch Fingers

The first design case is a three-finger single-motor hand with roll-pitch finger configuration (we define rolling axes perpendicular to the palm). The hand has altogether eight joints: two joints (proximal and distal joints) on the thumb, and three joints (the finger roll joint, proximal joint and distal joint) on the opposing two fingers. The hand models with kinematic configuration and tendon connectivity pattern are shown in Fig. 8.

1) Grasp Collection: The hand model is built in GraspIt! Simulator [31] without considering underactuation, and 21 desired grasps are created for 15 commonly-used ISS objects selected according to a case study with a domain expert, mainly including food (such as can) and tools (such as screwdriver). All of the desired grasps are shown in Fig. 9. All grasps have force-closure property, checked with the Ferrari-Canny \(\epsilon\)-metric [32] by \(\epsilon > 0\).

2) Optimization of the Mechanically Realizable Torque Manifold: In this step, we optimize the joint pulley radii (tendon moment arms) \(r_{tp}, r_{td}, r_{fr}, r_{fp}, r_{fd}\), where the subscripts \(t\) and \(f\) represent thumb and finger, and \(r\), \(p\), and \(d\) represent
the roll, proximal and distal joints (we consider the two fingers are just mirrored versions of each other). These parameters are illustrated in Fig. 2(a).

The actuation scheme we designed is also shown in Fig. 8, where each finger is actuated by one tendon, and all tendons are rigidly connected to the actuator. We note that the finger tendons wrap around the roll joint pulleys, and then the plane of routing rotated 90 degrees and the tendons go to the proximal and distal joints in the fingers.

The vector of generated net joint torque in (11) \( t_{\text{net}}^{\text{gen}} \in \mathbb{R}^3 \), the vector \( t_{\text{net}} \in \mathbb{R}^3 \) (each element represents the tension on one tendon), and the Actuation Matrix has the specific form of

\[
A = \begin{bmatrix}
    r_{tp} & r_{td} & -r_{fr} & r_{fp} & r_{fr} \\
    r_{fp} & r_{fr} & r_{fp} & r_{fr} & r_{fd} \\
    -r_{fr} & r_{fd} & r_{fp} & r_{fr} & r_{fd}
\end{bmatrix}
\]  

(39)

In global optimization \( 19 - 22 \), the range of pulley radii is set to 2 mm to 12 mm, so that the pulley is large enough to be manufacturable but small enough to be put into the joints. Using an cutoff unbalanced torque of 0.1 (unitless normalized torque), one outlier grasps is excluded in this step. The convergence tolerance is set to \( 10^{-10} \) for inner layer QP and \( 10^{-6} \) for outer layer CMA-ES. The above conditions are set the same for this design case and also for Design Case II and III. We note again that in this step, there are non-unique solutions, so the pulley radii are not fully determined yet. The minimum objective function value is recorded for the next step.

The computation time on a commodity desktop computer (quad-core 3.4 GHz CPU) is 10 minutes, using the cvxopt (for QP) and pycma (for CMA-ES) packages implemented in python.

3) Optimization of the Mechanically Realizable Posture Manifold for Inter-tendon Kinematic Behaviors: As discussed in the subsection IV-C, we need to pick a unique solution of pulley radii \( (r_{tp}, r_{td}, r_{fr}, r_{fp}, r_{fd}) \) from the non-unique solutions from the previous step. The goal is to minimize the differences of actual and required tendon travel for different tendons.

The motor-tendon connection matrix \( M \) in (23) has a specific form of

\[
M = [r_{mot}, r_{mot}, r_{mot}]^T
\]  

(40)

Besides, the tendon travel vector \( s \) has a specific form of

\[
s = A^T \theta
\]  

(41)

For the outer layer, first, the candidate solutions need to satisfy the constraint that the previous optimization takes its minimal value with a tolerance of \( 10^{-3} \), and then the CMA-ES algorithm finds the optimal set of pulley radii with function value convergence tolerance of \( 10^{-3} \). No outlier grasp is found in this step with a cutoff length error of 2 mm.

The optimal pulley radii we got are shown in Table I. The computation time is 30 minutes.

4) Optimization of the Mechanically Realizable Posture Manifold for Intra-tendon Kinematic Behaviors: In this step, we optimize spring stiffnesses \( K_{tp}, K_{td}, K_{fr}, K_{fp}, K_{fd} \) and spring preload angles \( \theta_{tp}, \theta_{td}, \theta_{fr}, \theta_{fp}, \theta_{fd} \), where the subscripts have the same meaning as previous. These parameters are also illustrated in Fig. 2.

In practice, since the pre-contact kinematic behaviors of each finger is independent from every other one, we can search for each finger separately, and thus reduce the search dimensionality.

The spring stiffnesses are limited by the physical dimensions allowed in the mounting area in the joints. Also, they can only take discrete numbers offered by the manufacturer. In this design, the available stiffnesses are 10 discrete values from 2.25 Nmm/rad to 19.25 Nmm/rad.

The convergence tolerance is set to \( 10^{-6} \) for inner layer QP and \( 10^{-3} \) for outer layer CMA-ES. We note again that in this step, there are non-unique solutions, so the pulley radii are not fully determined yet. The minimum objective function value is recorded for the next step.

Fig. 9. All desired grasps built in GraspIt! simulator for Design Case I. All grasps have force-closure property.

TABLE I

| Parameter | Value |
|-----------|-------|
| \( r_{tp} \) | 12.0 |
| \( r_{td} \) | 4.6 |
| \( r_{fr} \) | 4.0 |
| \( r_{fp} \) | 11.8 |
| \( r_{fd} \) | 4.5 |

| Parameter | Value |
|-----------|-------|
| \( K_{tp} \) | 5.94 |
| \( K_{td} \) | 2.25 |
| \( K_{fr} \) | 3.60 |
| \( K_{fp} \) | 19.25 |
| \( K_{fd} \) | 7.56 |

| Parameter | Value |
|-----------|-------|
| \( \theta_{tp} \) | 4.71 |
| \( \theta_{td} \) | 3.93 |
| \( \theta_{fr} \) | 4.34 |
| \( \theta_{fp} \) | 4.71 |
| \( \theta_{fd} \) | 3.78 |
In addition, the spring preload angles are limited between the maximum allowed torsional angles by the datasheet and the minimum torsion angles to provide enough restoring torques over the entire range of motion. In this design case, the preload angles range from $\pi/4$ to $7\pi/4$ radians for the roll joints, $\pi/4$ to $3\pi/2$ radians for the proximal joints, and 0 to $3\pi/2$ radians for the distal joints.

In this step, there are three outlier grasps excluded using a cutoff unbalanced spring torque of 2 Nmm for the thumb and 5 Nmm for fingers.

The optimal spring parameters are shown in Table I. The computation time is 5 minutes.

B. Design Case II: Dual-motor Hand with Roll-pitch Fingers

In the previous design, all eight joints are actuated by a single motor, but it is interesting to see the benefits of having an additional motor controlling part of the hand motion separately. Therefore, we propose a variation of the previous design: a dual-motor hand with roll-pitch finger configuration, where one motor is in charge of the flexion of all fingers, and the other motor is in charge of the finger rolling. The hand kinematic configuration is shown in Fig. 10.

Here, the tendon tension vector in (11) $t_{act} \in \mathbb{R}^5$ (the first three element are the tensions on three tendons going to the thumb and fingers, the last two are the forces on the roll transmission connected to the second motor). The Actuation Matrix $A$ in (11) has the specific form of

$$ A = \begin{bmatrix} r_{tp} & r_{rd} & -r_{fr} \\ r_{fp} & r_{fd} & r_{fr} \end{bmatrix} $$

The motor-tendon connection matrix $M$ in (23) has a specific form of

$$ M = \begin{bmatrix} r_{mot1} & r_{mot1} \\ r_{mot2} & r_{mot2} \end{bmatrix} $$

and the tendon travel vector $s$ in (23) is also $A^T \theta$, where $\theta$ is the vector of joint angles in a desired grasp configuration.

All design details are the same as Design Case I, with the exception of the roll DoF not having a spring, and thus no associated spring parameters. There are altogether four grasps excluded using the same criteria. The optimal pulley radii and spring parameters are shown in Table I.

C. Design Case III: Dual-motor Hand with Pitch-yaw Fingers

The third design case is an underactuated hand with pitch-yaw fingers. The pitch-yaw two-DoF proximal joint is realized by a universal joint with a three-tendon parallel mechanism. The back tendon of the joint is connected to a spring with preload. The front two tendons are actively controlled, besides, they are not terminated in the proximal joint, but connected to distal joint pulleys.

The two actively controlled tendons in a finger are connected to different motors. Meanwhile, the symmetric tendons
TABLE IV
OPTIMIZATION METRICS (IN UNITLESS TORQUE, MM, NMM FOR COLUMN 1, 2 AND 3)

| Optimization of: | Mechanically Realizable Torque Manifold | Mechanically Realizable Postural Manifold (inter-tendon) | Mechanically Realizable Postural Manifold (intra-tendon) |
|------------------|----------------------------------------|---------------------------------------------------------|---------------------------------------------------------|
| Design Case I    | 0.13                                   | 5.82                                                    | 8.22                                                    |
| (1 grasp excluded)| (0 grasps excluded)                    | (3 grasps excluded)                                     | (14 grasps excluded)                                     |
| Design Case II   | 0.08                                   | 3.05                                                    | 2.82                                                    |
| (1 grasp excluded)| (0 grasps excluded)                    | (3 grasps excluded)                                     | (14 grasps excluded)                                     |
| Design Case III  | 0.33                                   | 6.02                                                    | 14.06                                                   |
| (0 grasps excluded)| (0 grasps excluded)                    | (14 grasps excluded)                                    | (14 grasps excluded)                                    |

The motor-tendon connection matrix $M$ in (45) has a specific form of

$$M = \begin{bmatrix}
    r_{mot} & 0 \\
    0 & r_{mot} \\
    0 & 0 \\
    0 & 0 
\end{bmatrix}$$  \hspace{1cm} (45)

And the tendon travel vector $s$ in (46) is

$$s = \begin{bmatrix}
    2(\theta_{fp}r_{fp}+\theta_{fd}r_{fd}) \\
    \theta_{fp}r_{fp}+\Delta l_{fp11} \\
    \theta_{fd}r_{fd}+\Delta l_{fp12} \\
    \theta_{fd}r_{fd}+\Delta l_{fp21} \\
    \theta_{fd}r_{fd}+\Delta l_{fp22} 
\end{bmatrix}$$  \hspace{1cm} (46)

where the $\Delta l$’s are the tendon length changes between zero-configuration and grasp configuration. We present the details of the derivation of the above matrices in the Appendix.

There are 14 grasps excluded using the same criteria. The optimal pulley radii and spring parameters are shown in Table III.
D. Numerical Evaluation

All the metrics provided by our optimization framework for all three design cases, as well as the number of excluded grasps in each design step and for each case, are summarized in Table IV.

In addition to the numerical metrics, it can also be informative to visualize the Mechanically Realizable Torque Manifold and Postural Manifold. However, these manifolds are high-dimensional. Therefore, we plot the thumb in two-dimensions and the fingers in three-dimensions separately.

The Mechanically Realizable Torque Manifolds for all three design cases are shown as Fig. 12. Each column shows one design case, and each row shows the plots for thumb or finger. The red dots represent the considered grasps, while the gray ones represent the excluded ones. The blue lines or planes are the Mechanically Realizable Torque Manifolds, and the orange lines or planes are the least-square fitting of red dots based on PCA. We note that, for each grasp, there are infinitely many solutions for equilibrium torques; among these, we chose to display on the plots (as a red or gray dot) the equilibrium torques closest to the Mechanically Realizable Torque Manifold.

Similarly, the Mechanically Realizable Posture Manifolds for all the design cases are shown in Fig. 13. The red dots represent the considered grasps, while the gray ones represent the excluded grasps. The lines or surfaces in blue are the Mechanically Realizable Posture Manifolds, and the lines or planes in orange are the PCA-based manifolds.

E. Construction of the Hands and Experimental Evaluations

We physically constructed prototypes for Design Case I and II, shown in Fig. 14 and Fig. 15.

In Design Case I, all joints are actuated by a single motor. Fig. 14 (and the accompanying multimedia attachment) demonstrates the finger trajectories, in which the hand first closes toward the center, making a spherical grasp posture and then a pinch grasp posture. Continuing to close, the fingertips do not collide but rather pass each other (due to motion in the roll degree of freedom). Finally, the hand creates an enveloping grasp. Fig. 16 (a) - (f) show several grasps using this hand, displaying the versatility of the hand. We can see the hand can perform stable pinch grasps (c)(f), spherical grasps (a)(d), and power grasps (b)(e).

In Design Case II, the tendons of three fingers are connected to the pulley of the main motor, and the roll joints are actuated by a smaller motor via gear transmission (which shares the same mathematical expression as the antagonistic tendon transmission in Fig. 10 (b)). Therefore, the closing and rolling motion are controlled separately. Shown in Fig. 15, the hand can close the fingers towards the center to pinch small objects, or close the fingers in a parallel fashion so the finger can pass each other and make an enveloping grasp. Fig. 16
Fig. 14. Finger closing trajectory and the types of grasp along the trajectory of Design Case I.

Fig. 15. Finger trajectory of Design Case II. The first row shows the non-parallel closing, resulting in a pinch grasp. The second row shows the parallel-finger closing, resulting in an enveloping grasp. Unlike the Design Case I, the finger roll angle can be actively controlled in Design Case II, which leads to the different finger trajectories.

Fig. 16. Grasp examples using prototype hands. (a) - (f): Design Case I, (g) - (l): Design Case II. (g) - (l) show some example grasps of the resulted hand of Design Case II. We note that this design can perform pinch grasps very small objects such as (g) and (l).

As for Design Case III, since the optimization result indicates that the performance is worse than the other two, we did not build it physically.

VI. DISCUSSION

The results for all three design cases demonstrate that our method is effective: the proposed optimization framework can indeed shape the Mechanically Realizable Torque and Postural Manifolds to fit the desired grasps.

The comparison with PCA results in Fig.12 and Fig.13 also illustrates the performance of our method. Assuming linear manifolds, the PCA-based manifolds explain the most variance of the desired grasps in theory. However, due to the mechanical constraints, those theoretically optimal manifolds cannot be reached exactly. In contrast, the proposed method calculates a manifold which attempts to approach the PCA result as much as possible under physical constraints. In addition, the proposed method can also find nonlinear manifolds that make use of mechanism characteristics. For example, Design Case III shows the Mechanically Realizable Manifolds which are not limited to the linear domain.

The numeric values of objective functions, as well as the number of grasps excluded because they are beyond the resulted hand’s capability, provide evaluations of a design. Even though the problem of initial kinematic design is out of the scope of our work (we require a pre-specified kinematic configuration), one can use these metrics calculated in our method to compare different kinematic designs, which provides insights for the choice of kinematics:

- Comparing the roll-pitch single motor (Design Case I) with dual-motor design (Design Case II), the results match our intuition: With the same kinematics, the dual-motor design has more versatility in terms of the postures, and has more capability of force generation to create stability, due to the partially decoupling of some joints.
- Comparing the roll-pitch configurations (Design Case I and II) with the pitch-yaw configuration (Design Case III), we can see that the pitch-yaw design is worse, especially for the intra-tendon optimization. From this result we can conclude that such a combination of pitch-yaw proximal joints with the certain tendon routing scheme in Fig.11 has very limited capability.

The dual-layer optimization framework combining the non-convex stochastic global search in the outer-layer and the
convex optimization in the inner-layer is a useful formulation. Except for some particular kinematic designs, it is not possible to formulate the parameter selection problem as a global convex program similar to [22]. In contrast, the dual-layer framework is capable of dealing with various kinematic configurations and actuation methods.

VII. CONCLUSION

In this paper, we proposed the concept of Mechanically Realizable Manifolds, both in joint torque and joint angle space. We presented a dual-layer framework to optimize the Mechanically Realizable Manifolds for underactuated hands, in order to achieve desired grasp ability in terms of posture shaping and force generation. We conducted three design cases using this theory, compared the results quantitatively among the cases, constructed prototypes of two cases that showed better capabilities and verified their performance in practice.

We believe that the underactuated synergistic behavior is part of the general intelligence of robotic hands, and the idea of combining hardware-embedded and software-based intelligence is a promising path towards versatile grasping. Our method of Mechanically Realizable Manifolds is a step on this path, building a bridge between the dexterous hand behaviors and the underactuated hardware design. For future work, we aim for progressively more general approaches to modeling both hardware-embedded hand behavior and software-based control strategy, which could then be used for deriving co-optimization frameworks. Ultimately, we are aiming to design robotic hands with an informed distribution of intelligence over hardware and software, resulting in more dexterity and less complexity.

APPENDIX

DERIVATION OF THE MATRICES IN DESIGN CASE III

First of all, the inverse kinematics (i.e. calculating the tendon length from angles) for two-DoF proximal joint need to be solved. Fig. 17 shows the two-DoF joint and the attached coordinate frames. The tendon lengths can be solved using coordinate transforms. For example, the length of the tendon connecting $A_1$ and $B_1$ can be calculated as:

$$
\|O_A B_1 A_1\| = \|O_A O A_1 - O_A R_O O_R_B O_B B_1\| \tag{47}
$$

where the prescripts are the coordinates the vector is described in, and the rotation matrix $O_A R_O$ represents the transform from coordinate $\{O_A\}$ to $\{O\}$.

Next, we show the derivation of the Actuation Matrix, which relates the tendon lengths and the net joint torques. Here we use the torque of pitch DoF $\tau_p$, as an example:

$$
\tau_p = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \cdot \\
\underbrace{O_A O A_1 \times \frac{O_A B_1 A_1}{\|O_A B_1 A_1\|} t_1}_{\|O_A B_1 A_1\|} \tag{48}
$$

$$
+ \frac{O_A O A_2}{\|O_A B_2 A_2\|} t_2 \tag{50}
$$

$$
+ \frac{O_A O A_3}{\|O_A B_3 A_3\|} t_3 \tag{51}
$$

Following the assumption that the post-contact movement is negligible, and the reasoning shown in [1] [2] and [3], the net joint torque can be expressed as:

$$
\tau_{p,net} = O_A \rho_{1t_1,net} + O_A \rho_{2t_2,net} \tag{52}
$$

where $t_1$, $t_2$, $t_3$ are the magnitude of tendon forces. We denote:

$$
O_A \rho = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \cdot \\
\left[ O_A O A_1 \times \frac{O_A B_1 A_1}{\|O_A B_1 A_1\|} O_A O A_2 \times \frac{O_A B_2 A_2}{\|O_A B_2 A_2\|} O_A O A_3 \times \frac{O_A B_3 A_3}{\|O_A B_3 A_3\|} \right] \tag{49}
$$

The moment arms $O_A \rho_{p1}$ and $O_A \rho_{p2}$ in (51) are the entries $\rho_{fp11}$ and $\rho_{fp12}$ of the Actuation Matrix $A$ shown in (44). We can construct the entire Actuation Matrix in a similar way. Finally, we give some details in the tendon shortening vector $s$ in (46). The $\Delta l$s are the tendon length changes between zero-configuration and grasp configuration. For example:

$$
\Delta l_{fp11} = l_{1(\text{grasp})} - l_{1(\text{zero})} \tag{53}
$$

where $l_{1(\text{grasp})}$ and $l_{1(\text{zero})}$ can be solved by the aforementioned joint inverse kinematics using the pitch and yaw angles.

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