K-Core Decomposition on Super Large Graphs with Limited Resources

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ABSTRACT
K-core decomposition is a commonly used metric to analyze graph structure or study the relative importance of nodes in complex graphs. Recent years have seen rapid growth in the scale of the graph, especially in industrial settings. For example, our industrial partner runs popular social applications with billions of users and is able to gather a rich set of user data. As a result, applying K-core decomposition on large graphs has attracted more and more attention from academics and the industry. A simple but effective method to deal with large graphs is to train them in the distributed settings, and some distributed K-core decomposition algorithms are also proposed. Despite their effectiveness, we experimentally and theoretically observe that these algorithms consume too many resources and become unstable on super-large-scale graphs, especially when the given resources are limited. In this paper, we deal with those super-large-scale graphs and propose a divide-and-conquer strategy on top of the distributed K-core decomposition algorithm. We evaluate our approach on three large graphs. The experimental results show that the consumption of resources can be significantly reduced, and the calculation on large-scale graphs becomes more stable than the existing methods. For example, the distributed K-core decomposition algorithm can scale to a large graph with 136 billion edges without losing correctness with our divide-and-conquer technique.

CCS CONCEPTS
• Mathematics of computing → Graph algorithms; • Theory of computation → Divide and conquer; • Computer systems organization → Distributed architectures;

KEYWORDS
K-core Decomposition, Divide-and-Conquer, Large-scale Graphs

1 INTRODUCTION
Background and Motivation. Relationships between individuals or entities can be captured as graphs where each node (a.k.a. vertex) represents an individual or entity, and each edge represents the corresponding connection or relation. Capturing graph structure of data is useful in many applications, such as targeted advertising [41, 43], knowledge distillation [45, 46], data annotation [47–49], and protein analysis [17, 39]. Each node in a graph often exhibits a crucial property—the importance or utility of a node depends on the number of connections between it and other nodes. Especially in social networks, the engagement of a node is more likely to happen if its most neighbors are engaged. To this end, the concept of K-core is introduced to estimate the coreness of a node. A K-core subgraph of graph G is the largest induced subgraph where every vertex has a degree of at least k.

The most prevailing way to estimate the coreness of a node is by the K-core decomposition, which has widespread adoption in many areas, such as network structure visualization, Internet topology exploration, and community-related problems [1, 7, 38]. Furthermore, it works as a fundamental building block for a variety of problems, such as finding the approximation for the densest subgraph problem or the densest at-least-k-subgraph problem [24]. Consequently, K-core decomposition has become a popular topic in both academic and industrial communities. Notably, driven by the explosive surge of data volume and increasing complexity of social networks, many efforts have been devoted to accomplishing the K-core decomposition in a distributed manner [29, 32, 33].

One of our industrial partners runs popular social applications with billions of users and can extract various graphs, which are then utilized to mine the user relationships for other applications,
such as user profiling and machine learning tasks. To enhance the model performance, our industrial partner attempts to process larger graphs, i.e., more nodes and more edges. Therefore, distributed k-core decomposition has become a widely-used application in its production pipeline.

**Challenges.** Although the increment in data volume is beneficial for mining more precise user information, significant performance degradation is encountered. To summarize, we observe two severe issues when processing large graphs.

First, since the production cluster is shared by many tasks, the maximal resources allowed for each task are limited, which becomes insufficient when the scale of graphs increases. Take a real case in our industrial partner as an example, where the graph is composed of 2.2 billion nodes and 136 billion edges. It takes several terabytes of memory to store such a huge graph. To the best of our knowledge, the existing distributed graph processing engines keep the entire graph in memory throughout the execution. Therefore, none of these existing works can support k-core decomposition for such a large graph within the resource limitation.

Second, even for graphs that can be supported with the resource constraints, the performance is undesirable when the graphs are enlarged due to the increase in “communication amount”. During the execution, the k-core decomposition algorithm iteratively updates the coreness value of each node according to its neighbors. In the distributed setting, every update triggers the communication between different workers to ensure that all neighbors can retrieve up-to-date information. We call the total number of communicated updates as the communication amount. Undoubtedly, when there are tremendous edges, the number of updated nodes increases correspondingly, leading to non-trivial communication overhead and thus dampening the efficiency.

**Summary of Contributions.** As discussed above, the undesirable performance mainly comes from the large scale of graphs. To tackle these challenges, we develop a divide-and-conquer strategy, namely DC-kCore, to support extremely large graphs under certain resource constraints and accelerate the total running time cost. To the best of our knowledge, this is the first work that applies the divide-and-conquer strategy for the k-core decomposition problem. The major contributions of this work are summarized below.

- **The Divide Step: Graph Division.** To support extremely large graphs under limited resources, we propose to divide the graphs into different parts, where each part is a subgraph of the original graph. We devise a brand new graph division strategy that matches the coreness computation characteristics, rather than randomly distributing the original graphs in the vertex- or edge-centric manner. Specifically, since the nodes with small coreness values have zero impact on the computation for nodes with large coreness values, we propose dividing the graphs by a given degree threshold. By doing so, different subgraphs can be processed individually, reducing the peak resource requirement.

- **The Conquer Step: Subgraph Decomposition.** After an original graph is divided into multiple subgraphs, we introduce to apply k-core decomposition on each subgraph individually. However, since the edges connected between different subgraphs are cut off, part of the neighbor information has been lost. To ensure the correctness of subgraph decomposition, we generate the external information, which can be reckoned as the summary of neighbor information between different subgraphs. A novel algorithm that decomposes the subgraph with the help of external information is developed. Compared with decomposing the original graphs directly, the utilization of external information can reduce the communication amount to a large extent, boosting the overall performance.

**Deployment and Evaluation on Real-World Applications.** We implement the proposed divide-and-conquer strategy, namely DC-kCore, on top of Apache Spark and the Angel parameter server platform [22, 44]. DC-kCore has been widely deployed in the production pipeline of our industrial partner. There are apps adopting DC-kCore to execute KOL (key opinion leader) analysis, or using it to detect the merchant from massive Payment networks. Besides, it is also used in fraud-detection for risk control. Comprehensive experiments are conducted to evaluate the effectiveness of DC-kCore. First, empirical results show that DC-kCore is more efficient than the existing distributed k-core decomposition implementations and supports a much larger scale of graphs with up to 136 billion edges. To the best of our knowledge, this is the first work supporting such a large graph scale. Second, we empirically verify that DC-kCore is also efficient for small- or medium-scale of graphs since it can reduce the communication amount significantly by summarizing the neighbor information. Moreover, we evaluate the sensitivity of DC-kCore in terms of the number of subgraphs or choices of division strategies, shedding light on the potential improvement on a wider range of applications.

## 2 RELATED WORK

The definition of K-core is first brought out by Serdman[37] to characterize the cohesive regions of graphs. There is a considerable amount of research[10, 20, 28, 31, 34] done to study the existence of non-empty k-core in a random graph.

Thanks to the well-defined structure of k-core, it plays an important role in the analysis of the structure of certain types of networks[12] and generating graphs with specific properties[6]. There exist many graph problems such as maximal clique finding[4], dense subgraph discovery[2], and betweenness approximation[18] using k-core decomposition as a subroutine.

The first k-core decomposition algorithm was proposed by Batagelj and Zaveršnik (BZ)[5]. In their work, the coreness of each node is generated by recursively removing nodes of degree less than k. It requires random access to the entire graph. Therefore, in order to ensure good efficiency, the entire graph should be stored in memory while processing. As the scale of the graph increases, memory cannot support the calculation of the whole graph. As a result, k-core decomposition algorithm using the combination of memory and external memory was developed. The first K-core decomposition algorithm (EMCore) using secondary storage was proposed by Cheng et al. in [9]. EMCore uses a strategy based on the BZ algorithm which requires $O(k_{\text{max}})$ scans of the graph, where $k_{\text{max}}$ is the largest coreness value of the graph. [23] optimized the implementation of EMCore and can handle networks of billions of edges using a single consumer-level machine and can produce excellent approximations in only a fraction of the execution time.
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Table 1: Notations

| Symbols | Definitions |
|---------|-------------|
| $N(v)$ | Neighbor set of node $v$ |
| $\text{deg}(v)$ | Number of neighbors of node $v$ (a.k.a., degree) |
| $G_k(V_k, E_k)$ | The $k$-core subgraph of $G$ (abbr. $G_k$) |
| $G_k(V_k, E_k)$ | The rest part of $G$ by removing $G_k$ (abbr. $G_k$) |
| $\mathcal{N}_G(v)$ | Neighbor set in $G_k$ of node $v$ |
| $\text{deg}_{G_k}(v)$ | Number of neighbors in $G_k$ of node $v$ |
| $\text{coreness}(v)$ | the coreness value of node $v$ |
| $\mathcal{E}(v)$ | Number of neighbors in $G_k$ of node $v \in G_k$ |

[13] computes an approximation of nodes coreness by sampling edges into subgraphs. The computation of each subgraph follows a way of peeling nodes.

With the increasing graph scale, the idea of distributed K-core decomposition was first proposed in [32], where node index was introduced. This work has been implemented by Mandal et al.[29] on Spark. The realization of k-core decomposition on FSGraph is also based on [32]. This algorithm was further extended to dynamic graphs by Aridhi et al.[3]. Moreover, there are I/O efficient algorithms[9, 40] aiming at handling large-scale graphs which cannot fit into memory proposed, based on the distributed algorithm proposed by Montresor et al.[32]. There is also another distributed algorithm using MapReduce[11, 33].

K-core decomposition is also studied on various kinds of networks, such as on directed[16] and weighted[15] network. Moreover, Li et al.[25] deal with the problem of k-core maintenance in large dynamic graphs. There are many works that deal with k-core decomposition on dynamic graphs[19, 30, 36]. And the k-core decomposition in the streaming scenario was first discussed in [35].

3 PRELIMINARIES

3.1 Problem Definition

A network or graph is denoted by $G(V, E)$, where $V$ is the set of vertices (nodes) and $E$ is the set of edges (links). We use the symbol $n$ to represent the number of nodes ($n = |V|$) and the symbol $m$ to represent the number of edges ($m = |E|$). Given a node $v \in V$, the set of its neighbors are denoted by $\mathcal{N}_G(v)$, i.e., the set of all nodes adjacent to $v$. We use $\text{deg}_G(v)$ to denote the degree (number of neighbors) of node $v$ ($\text{deg}_G(v) = |\mathcal{N}_G(v)|$).

Given an undirected and unweighted graph $G(V, E)$, the concept of K-core decomposition is considered as the following definitions.

**Definition 1 (K-core Subgraph).** Let $H(C, E(C)$ be a subgraph of $G(V, E)$, where $C \subseteq V, E(C) = \{(u, v) \in E : u \in C \land v \in C\}$. $H$ is defined to be the K-core subgraph of $G$, denoted by $G_k$, if it is the maximal subgraph of $G$ satisfying the following property: $\forall u \in C : \text{deg}_H(u) \geq k$.

**Definition 2 (Coreness Value).** A node $v \in V$ has coreness value $\text{coreness}(v) = k$, if it belongs to the $k$-core subgraph but not to the $(k+1)$-core subgraph.

For the example shown in Figure 1, the graph $G$ has 15 nodes, and each k-core subgraph is surrounded by a circle. Given an undirected and unweighted graph $G(V, E)$, the goal of **k-core decomposition** is to find all the k-core subgraphs ($G_k$)$_{k=0}^{k_{max}}$, where $k_{max} = \max\{\text{coreness}(v) | \forall v \in V\}$. Obviously, this problem is equivalently to find the coreness values for all nodes in $G$.

3.2 Distributed K-core Decomposition

Owing to the incredible surge of data volume, it is impossible to process the entire graph on a single machine. Thus, several works were developed to accomplish the k-core decomposition in a distributed manner. The first distributed k-core decomposition algorithm was reported by Montresor et al. [32]. The essential idea is to update the coreness values by calculating the node index iteratively. Due to space constraint, we refer interested readers to [27] for more details on how to estimate the coreness values via the node index. As shown in Algorithm 1, in each iteration, each node receives the estimated coreness values of its neighbors from the previous iteration, estimates its own coreness value, and sends the new estimation to its neighbors for the next iteration. The estimation will converge to the exact coreness value, which is proven by Montresor et al. [32], and the decomposition accomplishes when the estimated coreness values for all nodes become stable.

Mandal and Al Hasan [29] implement the distributed k-core decomposition algorithm on top of the GraphX platform, which is a popular distributed graph processing engine based on Apache Spark. However, since the resilient distributed datasets (RDDs) of Spark
are immutable, the implementation in GraphX needs to reconstruct the RDDs frequently and suffers from heavy communication cost. So, the efficiency is insufficient for many industrial-scale graphs.

PSGraph [21] is a graph processing platform that supports extremely large-scale graph. PSGraph is also built upon Spark in order to be production-friendly. However, unlike GraphX, PSGraph leverages the power of parameter server to update the estimated coreness values so that it does not need to reconstruct the graph data repeatedly. Therefore, PSGraph is much more efficient than GraphX and supports much larger scale of graphs with up to billions of edges. Nevertheless, since it stores the entire graph during the \(k\)-core decomposition task, extremely huge graphs cannot be supported under the resource limitation of our productive environment.

4 OUR DIVIDE-AND-CONQUER STRATEGY

In this section, we introduce a brand new solution on large scale \(k\)-core decomposition, namely \(DC-k\)Core.

4.1 Overview

As introduced in Section 1, processing a huge graph in one shot is sub-optimal, either due to the resource limitation or the performance degradation. Hence, we propose to split the graph into different parts and process each part individually. Figure 2 depicts the overview of our work. Given a pre-defined threshold \(k\), we divide the original graph into two parts, one for the nodes with coreness values not less than \(k\) and the other for the remaining nodes. To ensure the correctness, some external information is generated to aid the decomposition, as described later. Finally, the coreness values computed by the two subgraphs are merged together.

4.2 The Dividing Step

4.2.1 The Exact-Divide Strategy. According to Definition 1, nodes with coreness values smaller than \(k\) will never affect the calculation of \(k\)-core subgraph. Thus, if we extract the \(k\)-core subgraph \(G_k\) out of \(G\) and execute decomposition on \(G_k\) directly, we can get part of the coreness values of \(G\). So, given a threshold \(k\), the Exact-Divide strategy divides the original graph into two parts — the \(k\)-core subgraph \(G_k\) and the rest of the graph \(G_{\bar{k}}\), and apply decomposition to the two subgraphs individually in the conquer step.

4.2.2 The Rough-Divide Strategy. As aforementioned, with the Exact-Divide strategy, the graph \(G\) is divided into \(G_k\) and \(G_{\bar{k}}\), which...
Then the two parts are processed individually, as described below.

For the first part, i.e., $G_k$, we observe that since it is the superset of $G_k$, and according to Definition 1, nodes with coreness less than $k$ have zero impact on the calculation of the $k$-core subgraph, decomposing $G_k$ will yield a superset of the results obtained by decomposing $G_k$. As a result, there are two steps to achieve the $k$-core subgraph $G_k$: (i) apply decomposition on $G_k$, and (ii) remove the nodes with coreness values smaller than $k$ in $G_k$. Moreover, the coreness values of all nodes in $G_k$ are achieved simultaneously.

For the rest of the original graph, we adopt a similar procedure as Exact-Divide. In other words, the external information of all nodes in $G_k$ are generated to aid the computation. However, unlike Exact-Divide, the external information generation of the Rough-Divide strategy requires that we have already accomplished the processing of $G_k$ so that we can find out all nodes for $G_k$. So, the two parts are processed sequentially. In practice, such a sequential processing is consistent with many real-world scenarios — for whom does not have enough resources, different parts are expected to be processed one by one to reduce the peak amount of resources.

We conclude this section with an example of the Rough-Divide strategy in Figure 5. Initially, we extract a rough subgraph $G_k^*$, which contains all nodes and edges in $G_k$. Then we decompose $G_k^*$ individually. As analyzed above, once the decomposition of $G_k^*$ has been accomplished, the coreness values of all nodes that is no less than $k$ can be determined, which make up to be $G_k$. Finally, we can extract the rest part $G_{k_k}$ and decompose it in a similar way as the Exact-Divide strategy. As we will evaluate in Section 5, thanks to the acceleration in subgraph extraction, the Rough-Divide strategy is more efficient than the Exact-Divide strategy.

4.3 The Conquer Step

Following the dividing step, the conquer step treats each subgraph as a sub-task and decompose it individually. As the decomposition of $G_k$ or $G_k^*$ will not be influenced by the nodes in $G_k$, we focus on the decomposition of $G_k$ with the external information, including the correctness analysis and implementation details.

4.3.1 Theoretical Analysis

Now we explain how the external information could help the computation of coreness values for nodes in $G_k$. By counting the number of neighbors in $G_k$, the upper bound of $E(v)$ is $\text{coreness}(v)$, which is summarized in Corollary 1.

**Corollary 1.** Given $G(V, E, G_k(V_k, E_k))$ and $G_{k_k}(V_{k_k}, E_{k_k})$, the upper bound of $E(v)$ is $\text{coreness}(v)$.

**Proof.** By contradiction. Given a node $v \in V_k$, assume that $E(v) > \text{coreness}(v)$, which means $v$ has at least $\text{coreness}(v) + 1$ neighbors belonging to $G_k$, and since $v \notin G_k$, thus $k > \text{coreness}(v)$, that means according to Definition 2, the coreness value of $v$ is at least $\text{coreness}(v) + 1 \neq \text{coreness}(v)$, a contradiction. □

External information for a node takes the same effect as if the missing edges are still connected. For nodes with coreness $k$, only the number of neighbors with coreness larger or equal to $k$ matters. Based on Corollary 1, we formalize the correctness of Exact-Divide via Corollary 2.

**Corollary 2.** Given $G(V, E, G_k(V_k, E_k))$ and the external information $E$, coreness value of $v \in V_k$, $\text{Core}_k(v)$, generated by executing a bottom-up algorithm on $G_k$ will converge to the correct coreness $C(v)$ of $v$ in $G$.

**Proof.** According to Corollary 1, we have $E(v) < k$. And the coreness values of all nodes in $G_{k_k}$ are smaller than $k$. The proof is by induction on the coreness $C(v)$ of node $v \in V_{k_k}$.

- $C(v) = 0$. In this case, $v$ is isolated in both $G$ and $G_{k_k}$, and the external information is $E(v) = 0$. The coreness value $\text{Core}_k(v)$ is initialized by $\text{deg}_{G_k}(v) + E(v) = 0$. Thus $C(v) = 0 = \text{Core}_k(v)$.
- $C(v) = 1$. By contradiction, assume that $C(v) = 1$ but $\text{Core}_k(v) > 2$. We have $E(v) \leq 1$.

If $E(v) = 1$, node $v$ has only one neighbor belongs to the $k$-core subgraph. However, since $\text{Core}_k(v) > 2$, which means that there exists a neighbor of $v$ in $G$, whose coreness is less than $k$ but greater than 1, which leads to $C(v) > 1$, a contradiction.
If $E(v) = 0$, node $v$ has no neighbor belongs to the $k$-core subgraph. Since $\text{Core}_k(v) \geq 2$, which means that there exist two neighbors of $v$ in $G$, whose coreness is less than $k$ but greater than 1, which leads to $C(v) > 1$, a contradiction.

- **Induction step** ($1 < C(v) < k$). By contradiction, assume that there is a node $v$ satisfying $C(v) = s$ and $\text{Core}_k(v) > s$ for all iterations. We have $E(v) \leq s$ and $\text{Core}_k(v) > s$, which means that there exists at least one neighbor of $v$ in $G$, whose coreness is less than $k$ and larger than $s$. Therefore, $v$ have at least $s + 1$ neighbors belonging to the $(s+1)$-core subgraph, i.e., $C(v) \geq s+1$, which forms a contradiction.

Now that we can correctly compute the coreness values for all subgraphs, thus the final results can be achieved by merging all the results of subgraphs.

### 4.3.2 Implementation

Since the calculation of coreness values for nodes in $G_k$ requires the external information generated from $G_k$ directly applying decomposition on $G_k$ via Algorithm 1 will yield incorrect results. As a result, we develop Algorithm 2, which estimates the coreness values with the help of external information. The essential idea is to complement $E(v)$ to the estimation for the edges connected to $G_k$. Specifically, if node $v$ has no neighbors in $G_k$, we denote $E(v) = 0$ for simplicity.

Similar as `PSGraph`, we implement `DC-kCore` on top of Apache Spark and the Angel parameter server. As shown in Figure 6, our distributed $k$-core decomposition has the following steps:

1. **Data Loading.** The graph is loaded and distributed into several partitions in the vertex-centric manner. To be specific, for each partition, we co-located the external information of corresponding nodes in the same RDD partition.
2. **Pull Coreness.** In each iteration, we sample a batch of nodes and pull the estimated coreness values of their neighbors from the parameter servers.

![Figure 6: The overview of our implementation of distributed $k$-core decomposition.](image)

**Algorithm 2** Procedure to estimate the coreness value for a node $v$ with external information in one iteration. We denote $E(v) = 0$ if node $v$ has no neighbors in $G_k$.

**Input:**
- $\mathcal{N}(v)$: The neighbors of node $v$;
- $\mathcal{E}(v)$: The external information of node $v$;

**Output:**

1. **Estimate Coreness.** Based on the estimated coreness values of neighbors and the external information, we estimate the coreness values for all nodes in $\mathcal{N}(v)$ from the previous iteration;
2. **Cores** ← $\mathcal{Cores}.sorted.reverse$;
3. $i \leftarrow 0$;
4. $C_v \leftarrow \mathcal{E}(v) + \mathcal{Cores}.length$;
5. **while** $i < \mathcal{Cores}.length$ **do**
6. if $\mathcal{Cores}(i) \geq \mathcal{E}(v) + i + 1$ **then**
7. $i \leftarrow i + 1$;
8. **else**
9. $C_v \leftarrow \mathcal{E}(v) + i$;
10. **break** while;
11. **return** $C_v$;

(3) **Estimate Coreness.** Based on the estimated coreness values of neighbors and the external information, we estimate the coreness values for all nodes in the current batch via Algorithm 2.

(4) **Push Coreness.** For nodes whose estimated coreness values are different from the previous iteration, we push the updated estimation to parameter servers.

(5) **Update Coreness.** Upon receiving the newly estimated coreness, the parameter servers update the estimation in place for the next iteration.

Step (2)-(5) repeat iteratively until the estimation of all nodes become stable, which finalizes the $k$-core decomposition procedure.

### 5 EVALUATION IN THE REAL WORLD

As introduced in Section 1, DC-$k$Core has been integrated into the production pipeline of our industrial partner and works as a powerful graph analytics toolkit for many real-world applications. In this section, we conduct experiments on both public graph dataset and some super-large-scale use cases to evaluate DC-$k$Core empirically. There are algorithms deal with large-scale graphs with secondary storage [9][23], but our method takes memory as the only storage, the comparison with these algorithms is unnecessary.

#### 5.1 Experimental Setup

**Datasets.** As summarized in Table 2, three graph datasets are used in our experiments, including one public graph dataset and two industrial-scale graph datasets. The `com-friendster` dataset is a public directed graph dataset with 1.8B edges from Friendster, a social networking site where users can make friends with each other. In this dataset, each node represents an user and each edge represents the social relationship between the two connected users. WX-15B and WX-136B are two graphs generated from a payment app of our industrial partner for risk control and advertising. The number of edges of these two industrial-scale datasets are up to 15
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Table 2: Description of datasets.

| Dataset      | #Nodes      | #Edges      | max k |
|--------------|-------------|-------------|-------|
| com-friendster | 65,608,366  | 1,806,067,135 | 304   |
| WX-15B        | 646,408,482 | 15,179,911,593 | 401   |
| WX-136B       | 2,226,845,928 | 136,588,315,957 | 1,179 |

The experimental results verify our theoretical analysis empirically. As a result, our divide-and-conquer strategy will not harm the correctness of the k-core decomposition tasks.

5.3 Scalability and Efficiency

The scalability is extremely important to large-scale graphs [8, 14]. To evaluate the scalability and efficiency of the proposed method, we compare Spark-kCore, PSGraph, and DC-kCore, on all datasets, and record the total running time in Table 3. For DC-kCore, we divide the dataset com-friendster by the degree threshold of 80, WX-15B by 100, and WX-136B by 250, respectively. (Note: if a graph is divided by the degree threshold of t, it means nodes with coreness values larger than t and nodes with coreness values not larger than t are calculated separately in two subgraphs.)

Table 3: Comparison of end-to-end running time. We use the symbol “-” to indicate that the competitor fails to support the given workload.

| Dataset      | com-friendster | WX-15B | WX-136B |
|--------------|---------------|--------|---------|
| Spark-kCore  | 1.0h          | -      | -       |
| PSGraph      | 21min         | 1.9h   | -       |
| DC-kCore     | 21min         | 1.4h   | 27.5h   |

The scalability is significantly improved by applying the divide-and-conquer strategy. DC-kCore can support all the three datasets.

5.4 Interpretability

Extensive experiments have demonstrated that our method is able to improve both the efficiency and scalability of k-core decomposition. In order to anamolize its effectiveness, we record various factors during the execution, including the time cost of decomposition and amount of communication, and analyze the results in this section.
Figure 7: Time cost of decomposition of PSGraph and DC-kCore. For DC-kCore, we show the time cost of decomposing the two subgraphs $G_k$ and $G_{!k}$, respectively.

Figure 8: Communication amount of PSGraph and DC-kCore. For DC-kCore, we show the communication amount of decomposing the two subgraphs $G_k$ and $G_{!k}$, respectively.

Figure 9: Time cost in seconds (at log scale) for the generation of the rough $k$-core subgraph $G_k$ and the exact $k$-core subgraph $G_{!k}$. Generating $G_k$ is $3.7 - 14.3\times$ faster than $G_{!k}$.

decomposing $G_k$ is much smaller than $G_{!k}$. Eventually, by breaking the original graph into smaller ones, our divide-and-conquer strategy reduces the communication overhead so that it is more efficient that decomposing the original graph directly.

5.5 Comparison of Dividing Strategies

In Section 4, we propose two dividing strategies, i.e., Exact-Divide and Rough-Divide. To compare their efficiency, we record the time cost of subgraph extraction and show the results in Figure 9.

Each group on the X-axis represents the dataset and the degree threshold for subgraph extraction. For example, “10billion+100” stands for the generation of rough or exact 100-core subgraph for the WX-15B dataset, “100billion+250” for the 250-core subgraph of WX-15B, and “cf+80” for the 80-core subgraph of com-friendster, respectively. In short, the Rough-Divide strategy is $3.7 - 14.3\times$ faster than the Exact-Divide strategy in terms of subgraph extraction. For instance, for “WX-136B+250”, the rough approach saves 6.5 hours compared with the exact venue. As a result, the Rough-Divide strategy is more suitable for huge graphs in practice.

5.6 Influence of the Number of Divided Parts

As discussed in Section 4, by applying the divide-and-conquer strategy recursively, a graph can be divided into more than two parts. To evaluate the efficiency improvement of dividing into multiple parts, and to study the overhead of subgraph extraction and external information generation, we conduct experiments to vary the number of parts from 2 to 4 on the com-friendster and WX-15B datasets. Specifically, for the com-friendster dataset, we set the dividing thresholds as 80, 100, and 150, respectively, whilst for the WX-15B dataset, the dividing thresholds are set as 80, 100, and 180, respectively.

Communication Amount on Multi-parts. We first analyze the impact on the total amount of communication when the number of dividing parts varies. As shown in Figure 10, the total amount of communication of DC-kCore is always smaller than PSGraph, regardless of the number of parts, which shows that the divide-and-conquer strategy can indeed reduce the communication overhead.
Figure 10: Total amount of communication of PSGraph and DC-k-Core. For DC-k-Core, we consider dividing into 2-4 parts (subgraphs) and report the sum of communication amount of all parts.

Figure 11: Time cost of preprocessing in minutes, including the time cost of subgraph extraction and external information generation. When the number of parts increases, the preprocessing overhead enlarges.

For com-friendster, the total communication amount decreases when the number of parts increases. For WX-15B, although increasing the number of parts from 2 to 3 causes a slight increase in communication, dividing it into 4 parts achieves the smallest communication amount. It is consistent with our analysis in Section 5.4 that the reduction in the number of edges brought by graph division can significantly alleviate the communication overhead.

Preprocessing Cost. Next, we record the time cost of preprocessing in Figure 11, including the extraction of subgraphs and the generation of external information. The results show that when we divide the original graph into more parts, the overhead of preprocessing increases in the meantime. For instance, for the WX-15B dataset, the preprocessing cost becomes 2.5× larger when we increase the number of parts from 2 to 4. It shows that there exists a trade-off between the communication amount and preprocessing overhead — a large number of parts will lead to smaller communication amount but heavier preprocessing cost, and vice versa. It would be an interesting topic to investigate how to configure the optimal number of parts for a given workload. We will leave it as our future work.

6 Conclusion and Future Works

In this paper, we propose a novel divide-and-conquer strategy for the k-core decomposition of huge graphs under limited resource constraints. To reduce the resource requirement, we devise graph division strategies to break the huge graph into smaller subgraphs. To ensure the correctness, we develop an algorithm that decomposes the subgraph with the help of external information. Empirical results show that our work is able to support much larger scale of graphs and is more efficient than the existing works.

As a possible future direction, we would like to generalize the divide-and-conquer strategy to more graph processing algorithms to develop a robust framework for industrial-scale graph datasets.

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