Vector rogue waves on a double-plane wave background

LI-CHEN ZHAO(a), LIANG DUAN, PENG GAO and ZHAN-YING YANG(b)

School of Physics, Northwest University - Xi’an, 710069, China and Shaanxi Key Laboratory for Theoretical Physics Frontiers - Xi’an, 710069, China

received 30 September 2018; accepted in final form 10 February 2019
published online 21 March 2019

PACS 05.45.Yv – Solitons
PACS 42.65.Tg – Optical solitons; nonlinear guided waves
PACS 02.30.Ik – Integrable systems

Abstract – We study the dynamics of rogue waves on a double-plane wave background in two-component coupled systems. The results indicate that many different patterns can still be evolved from resonant perturbations with the two plane waves background. The obtained vector rational solutions can be decomposed into two rational solutions located on the two plane waves separately. This enables us to investigate the superposition of several well-known fundamental rogue wave patterns, mainly including eye-shaped, anti-eye-shaped, and four-petaled one. The explicit conditions for different possible superpositions are clarified by a phase diagram. The analysis indicates that the obtained rational solution described patterns admit many different profiles, in contrast to the ones reported before. The studies can be extended to investigate other localized waves on double-plane wave backgrounds and even more plane waves involved cases in multi-component coupled systems.

Introduction. – A rogue wave (RW) depicts a unique event that seems to appear from nowhere and disappear without a trace, and it appears in a variety of different contexts [1–7]. A rational solution (RS) of nonlinear dynamical systems has been suggested to describe RW phenomenologically [8–10]. There can be some notable deviations from the RS in real physical cases. For example, recent experiments demonstrated a notable deviation from the Peregrine breather solution due to a significant asymmetric widening of the spectrum [11]. It should be also noted that some RSs cannot be used to describe RWs at all, such as rational W-shaped solitons [12]. This comes from the fact that the modulational instability (MI) transits to be a modulational stability in the parameter regimes [12]. But most of RSs in the MI regime can still be served as a prototype in the modelling of RWs [13–16]. According to the component number, RSs can be classified into scalar RSs [17–25], and vector RSs [26–32]. Most of the previous RSs described by RWs are excited on one-plane wave background. But RWs usually do not just admit one-simple-plane wave background in complex systems [33,34]. Then, if the background field contains more plane waves, can the RSs still exist? And would the RW-like characters be kept? We would like to study these questions through revisiting vector RSs, since scalar systems usually do not admit a plane wave superposition background, but vector systems could admit a plane wave superposition form.

Vector RSs have been studied widely on a one-plane wave background [27–32]. It has been shown that vector RSs admit more abundant excitation patterns than the scalar ones. For examples, anti-eye-shaped and four-petaled patterns were reported in vector systems [30–32]. We still choose a standard two-component coupled nonlinear Schrödinger equation (NLSE) to discuss the vector RS dynamics, because of its wide applications in physical systems [35,36]. We note that the coupled NLSE can admit a double-plane wave background (DPWB). This provides possibilities to investigate RWs on a DPWB in detail through deriving RSs on the background.

In this paper, we obtain vector RWs on a DPWB with the aid of the Darboux transformation method. The results demonstrate that RW-like characters can still be evolved from the resonant perturbations with the two-plane waves background. The obtained vector RSs can be decomposed into two RSs located on the two plane waves, respectively. This enables us to investigate the superposition of several well-known fundamental wave patterns (the
eye-shaped, anti-eye-shaped, and four-petaled ones). The detailed analysis indicates that localized wave patterns on the two plane waves are related and the superposition patterns cannot be chosen arbitrarily. It is impossible to obtain the superposition of two anti-eye-shaped RWs or two four-petaled RWs on the DPWB. Their possible combinations are clarified clearly by a phase diagram. Furthermore, high-order vector RSs are investigated on the DPWB.

The paper is organized as follows. In the next section, we describe the theoretical model and present a general DPWB form. In the third section, we present the fundamental RW on the DPWB, and clarify explicit conditions for different possible superpositions of localized wave patterns by showing a phase diagram. In the fourth section, we further demonstrate that high-order RWs can exist on the DPWB, which admit abundant wave patterns, in contrast to the ones reported before. Finally, we summarize the results and give some discussions in the last section.

The coupled nonlinear Schrödinger equations and double-plane wave backgrounds. – We begin with the well-known two-component coupled NLSE in dimensionless form,

\begin{align}
    i\dot{q}_1 &= \frac{1}{2} q_{1,xx} + \frac{1}{2} (|q_1|^2 + |q_2|^2) q_1 = 0, \\
    i\dot{q}_2 &= \frac{1}{2} q_{2,xx} + \frac{1}{2} (|q_1|^2 + |q_2|^2) q_2 = 0.
\end{align}

The coupled NLSE model can describe the dynamics of matter waves in quasi-1-dimensional two-component Bose-Einstein condensate [35], the evolution of optical fields in a two-mode or polarized nonlinear fiber [36], and even the vector financial systems [37]. The model is the well-known integrable Manakov model [38]. The vector RSs have been studied widely on plane wave backgrounds, for which there is only one-plane wave background in each component [27–31]. The vector RSs demonstrate many nontrivial properties compared with the scalar RSs of NLSE. They admit two more fundamental wave patterns, mainly including the anti-eye-shaped, and four-petaled ones. We would like to derive RSs on the DPWB, as previous works done for RW on one-plane wave background in [17,19–25,27–31].

It is noted that the coupled NLSE also admits the following DPWB solution except for the well-known one-plane wave background in each component. The generic DPWB solution can be written as

\begin{align}
    q_{1F} &= H_1(x,t) + H_2(x,t), \\
    q_{2F} &= H_1(x,t) - H_2(x,t),
\end{align}

where \(H_i = a_i e^{\theta_i} + a_i^* 2i(x_R+k_i x)/(x_R+k_i x + 2i\lambda^2 - 1) e^{\theta_i}, A_i = (x_R+k_i)^2 + \chi_R^2, \theta_i = i(k_i x + (a_i^2 - k_i^2/2|\theta_i|^2))/i, i = 1, 2.\) The parameter \(\chi_R = \text{Re}(\chi), \chi = \text{Im}(\chi) (\text{where } \chi \text{ is a root for the equation } 1 + \sum_{i=1}^2 a_i^2 = 0).\) The solution enables us to investigate RW-like dynamics on a DPWB. The rational solutions on DPWB are distinct from the ones on one-plane wave background, and the derivations are much more complicated than the ones reported before. As an example, we show one case in fig. 1. It is shown that the backgrounds demonstrate both spatial and temporal oscillation behaviors. The RS described localized wave structure is distinct from the ones reported before [27–31]. The superposition RS’ maximal amplitude in fig. 1 is about 1.8 times the background amplitude in both the two components (see fig. 1(c) and (d)). It is seen that the duration of the visible localized peak (time duration for the visible amplitude peak above the background) is about 0.7 in scaling units, and the spatial width of the high peak is about 0.8 in scaling units. The duration and spatial width of the localized wave peak depend on the background amplitude, and they can be reduced greatly by increasing the background amplitudes. These characters agree with the striking character of RW phenomena. But the localized wave peak value on the DPWB is much smaller than the scalar RS with an identical background density value, whose peak value always admits 3 times background amplitude. Moreover, the localized wave peak on DPWB depends on the relative wave vector \(k_2 - k_1\) and the ratio of two background amplitudes \(a_2/a_1\). The highest amplitude peak value of the fundamental RS on DPWB can be 3 times the background amplitude as the well-known scalar fundamental RS [18–20], when wave vectors \(k_1 = k_2\) and

\[a_1 = a_2.\]
Vector rogue waves on a double-plane wave background

![Figure 1](image1.png)

**Fig. 1:** The dynamics of the fundamental rational solution on a double-plane wave background. (a) Localized wave pattern in component $q_1$ and (b) localized wave in component $q_2$. They are shown by $|q_1,2|$. (c) and (d): the profiles of waves (shown by $|q_1,2|^2$ along the time and spatial direction) in the two components, respectively. Based on the phase diagram in fig. 2, we can know that the localized wave pattern in each component is the result of the linear superposition of an eye-shaped and anti-eye-shaped one on the two plane waves separately. The parameters are $a_1 = a_2 = 1$, $k_1 = 0$, $k_2 = 2$.

![Figure 2](image2.png)

**Fig. 2:** A phase diagram for localized wave patterns on a double-plane wave background. (a) Localized wave structure types on one-plane wave background $a_1 e^{i(k_1 x - k_1^2 t + \phi_1 t)}$ and (b) localized wave structure types on the other plane wave background $a_2 e^{i(k_2 x - k_2^2 t + \phi_2 t)}$. “ES”, “AES”, and “FP” denote eye-shaped, anti-eye-shaped, and four-petaled structure for the localized wave pattern, respectively. This phase diagram enables one to know the conditions for superpositions of two of the three fundamental structures. Moreover, we can see that the maximal amplitudes depend on relative wave vector $k_2 - k_1$ and the ratio of two background amplitudes $a_2/a_1$. The parameters are $a_1 = 1$, $k_1 = 0$.

the background amplitudes $a_1 = a_2$. The duration of the highest peak is identical to the scalar ones with identical background amplitudes. The highest peak will be lower than the scalar RSs for the cases with $k_1 \neq k_2$. Moreover, the maximal amplitude of the RS described wave also depends on the local background amplitude value where the highest peak locates, since the backgrounds admit stripe patterns. Nextly, let us discuss which types of spatial-temporal pattern can emerge, based on the obtained RS solution.

The obtained vector RS can be decomposed into two RSs located on the two plane waves separately. The RS on each plane wave background had been discussed clearly. They mainly admit three well-known fundamental spatial-temporal wave patterns, mainly including the eye-shaped, anti-eye-shaped, and four-petaled ones. This enables us to investigate the linear superposition of two of the three well-known fundamental wave patterns. It has been shown that the RS describing the wave structure depends on the plane wave properties [32,40]. The anti-eye-shaped pattern described by the RS and MI properties in the vector NLSE had been observed in a two-mode nonlinear fiber [41,42]. Therefore, it is meaningful to clarify under which conditions the superposition of two fundamental wave patterns can be observed for possible experimental observations.

Recently, baseband MI or resonant perturbation in MI was found to play an essential role in RS excitations [13–15]. Through Fourier analysis, we can see that each perturbation is resonant with the plane wave background on which it exists. This agrees well with the fact that the resonant perturbation in the MI regime induces RS excitation [15]. Furthermore, the underlying mechanism was uncovered very recently for forming different spatial-temporal structures of fundamental RSs [43]. It was suggested that both imaginary and real parts of the dispersion form for weak perturbations played important roles in determining the RS described spatial-temporal patterns. Since $\chi$ is a root of a fourth-order algebraic equation, there are two MI branches which are different from one MI branch for scalar NLSE [15]. This character shows that there are two different RS described wave patterns on an identical background. Namely, the two fundamental localized wave structures come from the two branches of MI dispersion form on a certain background. This provides possibilities to investigate nonlinear superpositions of different localized wave patterns on one-plane wave background [44,45]. Therefore, we plot the conditions for different fundamental patterns according to one MI branch. The other MI branch can be addressed similarly. Through analyzing the extreme points of RSs on a spatial-temporal distribution plane [43], we summarize the conditions for these different fundamental patterns according to one MI branch (as shown in fig. 2). It should be noted that the perturbation signals for RSs on the two plane waves are connected by the condition $1 + \sum_{i=1}^{2} a_i^2 \chi_{i}^2 = 0$. Therefore, we plot the localized wave patterns on the two plane waves separately (fig. 2(a) and (b)). The phase diagram is plotted in a two-parameter space which are the relative amplitude $a_2$ and wave vector $k_2$ with fixed $a_1 = 1$ and $k_1 = 0$. The settings of $a_1 = 1$ and $k_1 = 0$ do not lose generality, because of the scalar transformation invariance.
and the reference can be chosen to be one-component background.

It is seen that an eye-shaped localized wave always exists at least on one plane wave. The parameter domain for a four-petaled pattern is much smaller than the one for the eye-shaped one and anti-eye-shaped one. This explains well why the four-petaled pattern was not shown in the two-component case [27], and it was demonstrated lately by varying the relative frequency between the two components [32]. From the phase diagram, we can see that it is possible to obtain the superposition of the eye-shaped one with the eye-shaped, anti-eye-shaped or four-petaled ones. But it is impossible to obtain the superposition of the anti-eye-shaped one with the anti-eye-shaped one or four-petaled one in this two-component case. Moreover, the maximal amplitudes of RS depend on the wave pattern type chosen for the linear superposition, and they vary with the relative wave vector between the background, or the ratio of two plane wave amplitudes. It is found that the superposition of two eye-shaped ones admits the highest peak value when the relative wave vector is zero and the background amplitudes are equal. Moreover, we emphasize that the results are discussed for the integrable focusing-focusing NLSE case. Similar discussions can be made for other complex cases, such as the nonintegrable case, the defocusing-defocusing, and mixed cases.

The phase diagram can be used to clarify the explicit conditions for investigating different superpositions of fundamental RSs. For an example, fig. 1 is plotted with \(a_1 = a_2 = 1, k_1 = 0, k_2 = 2\) according to the identical MI branch for fig. 2. Based on the phase diagram, we can know directly that the RS in fig. 1 represents the results of the linear superposition of an eye-shaped and anti-eye-shaped ones. If we want to investigate the superposition of an eye-shaped and a four-petaled pattern, we can choose the background conditions \(a_2 = 1.5\) and \(k_2 = 1.5\). The dynamical processes in the two components are shown in fig. 3(a) and (b). The profiles of them are shown in fig. 3(c) and (d). It is seen that the duration of the visible wave peak is about 0.5 in scaling units, and the spatial width of the high peak is about 0.6 in scaling units. The duration values are smaller than the ones in fig. 1, since the background amplitude is larger. Especially, fig. 3(d) shows clearly that the maximal amplitude of the RS also depends on the local background amplitude value where the highest peak locates. The localized wave peak is almost invisible in component \(q_2\), because its location background amplitude value is very small. But the RS amplitude peak in the component \(q_1\) is much higher and about twice the background amplitude. The value is less than the peak value of scalar fundamental RS with the same background amplitude.

After the scalar fundamental RS was observed experimentally in many different physical systems [7–10], many scalar high-order RSs were excited successfully in a water wave tank, mainly including the wave triplets for the second-order one [46] and up to the fifth-order ones [47].

Fig. 3: The dynamics of rational solutions on a double-plane wave background. (a) Localized wave pattern in component \(q_1\) and (b) localized wave in component \(q_2\). They are shown by \(|q_1,2|^2\). (c) and (d): the profiles of waves (shown by \(|q_1,2|^2\) along the time and spatial direction) in the two components, respectively. Based on the phase diagram in fig. 2, we can see that the localized wave pattern in each component is the results of the linear superposition of an eye-shaped and a four-petaled one on the two plane waves separately. Especially, it also shows clearly that the maximal amplitude of the rational solution depends on the local background amplitude value where the highest peak locates. The parameters are \(a_1 = 1, a_2 = 1.5, k_1 = 0, k_2 = 1.5\).

Those results suggest that analytic high-order RSs are meaningful physically and can be realized experimentally [48]. Therefore, we further investigate the dynamics of high-order vector RSs on the DPWB.

High-order vector rogue waves on a double-plane wave background. – The high-order vector RSs had been derived in two-component coupled systems [44,45,49]. The vector RSs including multi-ones and high-order ones had been derived and investigated in detail [49]. Similarly, we can derive the multi-RSs and high-order ones on the DPWB by performing the methods in [49]. The compact form for these high-order solutions can be written as \(q_1-h = H_{1-h}(x,t) + H_{2-h}(x,t), q_2-h = H_{1-h}(x,t) - H_{2-h}(x,t)\), where \(H_{1-h}\) is the high-order RW solution on the corresponding plane wave background. Because the related expressions of these high-order RSs are too complicated, we do not show them in detail. For examples, we show some cases to demonstrate that high-order RSs on DPWB admit very abundant wave patterns. Four distinctive localized waves on the background are shown in fig. 4, and six on the DPWB are shown in fig. 5. Since the multi-RSs or high-order RSs are nonlinear superpositions of the fundamental ones, we do not show their cross-sections anymore. These results indicate that the perturbations with high-order resonance in MI can excite
Vector rogue waves on a double-plane wave background

Fig. 4: One case of four localized waves pattern on a double-plane wave background. (a) and (b): wave dynamics in the component $q_1$ and component $q_2$, respectively. They are shown by $|q_1, q_2|$. It is shown that a multi-rational solution can be also exist on the double-plane wave background. The parameters for backgrounds are $a_1 = a_2 = 1/2, k_1 = 3/5, k_2 = -3/5$.

Fig. 5: One case of six localized waves pattern on a double-plane wave background. (a) and (b): localized waves in the component $q_1$ and component $q_2$, respectively. They are shown by $|q_1, q_2|$. It is shown that a multi-rational solution can be also exist on the double-plane wave background. The parameters for backgrounds are $a_1 = a_2 = 1/2, k_1 = 3/5, k_2 = -3/5$.

Fig. 6: The numerical simulation from the initial condition given by the exact rational solution at $t = -5$. It is seen that the dynamical processes agree with the processes in fig. 1 described by the exact solution. The background parameters are identical to the ones in fig. 1.

Conclusion and discussion. – We discuss dynamics of vector RWs on a general DPWB with the aid of the Darboux transformation method. The results indicate that RW-like patterns can still be excited successfully from resonant perturbations with the two-plane waves background [15]. The fundamental vector RS on DPWB can be decomposed into two RSs located on the two plane waves separately. This enables us to investigate the superposition of several well-known fundamental localized wave patterns. The explicit conditions for different possible superpositions are clarified by a phase diagram. The studies would inspire more studies to investigate other localized waves on DPWB and even more plane waves involved cases, and provide possibilities to generate the RS described localized waves on multi-plane wave backgrounds in experiments.

The anti-eye-shaped wave pattern and MI character in a two-component coupled NLSE had been observed in a two-mode nonlinear fiber [41,42,50]. The phase and intensity modulation techniques can be performed to prepare the initial conditions given by the RSs on the DPWB. Very recently, experimental and numerical results showed that the universal mechanism that yielded the RS described excitation structure was highly robust and could be observed over a broad range of parameters [51]. Many previous numerical simulations and experiments also indicated that RS described dynamics could still be observed even with initial conditions which admit obvious derivations from the ideal initial conditions given by RSs [15,41,43,51]. We also simulate these dynamical processes described by analytical solutions numerically. The simulation results also show that the dynamical processes presented above are robust to numerical error or small deviations. As an example, we show the robustness of the dynamics process of fig. 1 in fig. 6. Therefore, these results could be helpful for experimental observations on RSs on multi-plane wave backgrounds.

Very recently, it was shown that the integrable two-component coupled NLSE admitted $SU(2)$ symmetry, and proper linear combinations of the solutions of the integrable coupled NLSE were also the solutions of the coupled NLSE [52]. The above RSs can be also obtained by the linear transformation with $SU(2)$ symmetry. Moreover, it is emphasized that the multi-component coupled NLSE admits a similar symmetry, which can be used to construct many different nonlinear waves on multi-plane wave backgrounds.

This work is supported by National Natural Science Foundation of China (Contact Nos. 11775176, 11405129), China Scholarship Council, Guangdong Natural Science Foundation (Contact No. 2017A030313008), a special research project of the Education Department of Shaanxi Provincial Government (Contract No. 16JK1763), the Key Innovative Research Team of Quantum Many-Body System.
REFERENCES

[1] Khairif C., Pelinovsky E. and Slunyaev A., Rogue Waves in the Ocean (Springer, Heidelberg) 2009.
[2] Osborne A. R., Nonlinear Ocean Waves and the Inverse Scattering Transform (Elsevier, New York) 2010.
[3] Ruban V., Kodama Y., Ruderman M., Dudley J., Grimshaw R., Mcintock P. V. E., Onorato M.,
Khairif C., Pelinovsky E., Soomere T., Lindgren G., Akhmediev N., Slunyaev A., Solli D.,
Ropers C., Jalali B., Dias F. and Osborne A., Eur. Phys. J. ST, 185 (2010) 5.
[4] Akhmediev N. and Pelinovsky E., Eur. Phys. J. ST, 185 (2010) 1.
[5] Khairif C. and Pelinovsky E., Eur. J. Mech. B/Fluids, 22 (2003) 603.
[6] Pelinovsky E. and Khairif C., Extreme Ocean Waves (Springer, Berlin) 2008.
[7] Onorato M., Residori S., Bortolozzo U., Montina A. and Arecchi F. T., Phys. Rep., 528 (2013) 47.
[8] Kibler B., Fatome J., Finot C., Millot G., Dias F., Genty G., Akhmediev N. and Dudley J. M., Nat.
Phys., 6 (2010) 790.
[9] Chabchoub A., Hoffmann N. P. and Akhmediev N., Phys. Rev. Lett., 106 (2011) 204502.
[10] Baily H., Sharma S. K. and Nakamura Y., Phys. Rev. Lett., 107 (2011) 255005.
[11] Shemer L. and Alperovich L., Phys. Fluids, 25 (2013) 051701.
[12] Zhao L. C., Li S. C. and Ling L. M., Phys. Rev. E, 89 (2014) 023210; Phys. Rev. E, 93 (2016) 032215.
[13] Baronio F., Conforti M., Degasperis A., Lombardo S., Onorato M. and Wabnitz S., Phys.
Rev. Lett., 113 (2014) 044101.
[14] Baronio F., Chen S., Greplu P., Wabnitz S. and Conforti M., Phys. Rev. A, 91 (2015) 033804.
[15] Zhao L. C. and Ling L., J. Opt. Soc. Am. B, 33 (2016) 850.
[16] Mu G., Qin Z. and Grimshaw R., SIAM J. Appl. Math., 75 (2015) 1.
[17] Osborne A. R., Mar. Struct., 14 (2001) 275.
[18] Akhmediev N., Ankiewicz A. and Taki M., Phys. Lett. A, 373 (2009) 675.
[19] Akhmediev N., Ankiewicz A. and Taki M., Phys. Lett. A, 373 (2009) 675.
[20] Akhmediev N., Ankiewicz A., Soto-Crespo J. M. and Dudley John M., Phys. Lett. A, 375 (2011) 541.
[21] Ohta Y. and Yang J. K., Proc. R. Soc. A, 468 (2012) 1716.
[22] Guo B. L., Ling L. M. and Liu Q. P., Phys. Rev. E, 85 (2012) 026607; Stud. Appl. Math., 130 (2013) 317.
[23] He J. S., Zhang H. R., Wang L. H., Porfizian K. and Fokas A. S., Phys. Rev. E, 87 (2013) 052914.
[24] Ling L. M. and Zhao L. C., Phys. Rev. E, 88 (2013) 045201.
[25] Kedziora D. J., Ankiewicz A. and Akhmediev N., Phys. Rev. E, 88 (2013) 013207.
[26] Bludov Y. V., Konotop V. V. and Akhmediev N., Eur. Phys. J. ST, 185 (2010) 109.
[27] Zhao L. C. and Liu J., J. Opt. Soc. Am. B, 29 (2012) 3119.
[28] Baronio F., Degasperis A., Conforti M. and Wabnitz S., Phys. Rev. Lett., 109 (2012) 044102.
[29] Guo B. L. and Ling L. M., Chin. Phys. Lett., 28 (2011) 110202.
[30] Zhao L. C. and Liu J., Phys. Rev. E, 87 (2013) 013201.
[31] Baronio F., Conforti M., Degasperis A. and Lombardo S., Phys. Rev. Lett., 111 (2013) 114101.
[32] Zhao L. C., Xin G. G. and Yang Z. Y., Phys. Rev. E, 90 (2014) 022918.
[33] Shin H. J., Phys. Rev. E, 88 (2013) 032919.
[34] Cousins W. and Sapsis T. P., Phys. Rev. E, 91 (2015) 063204.
[35] Kevrekidis P. G., Frantziskakis D. and Carretero-Gonzalez R., Emergent Nonlinear Phenomena in Bose-Einstein Condensates: Theory and Experiment (Springer, Berlin, Heidelberg) 2009.
[36] Agrawal G. P., Nonlinear Fiber Optics, 4th edition (Academic Press) 2007.
[37] Yan Z. Y., Phys. Lett. A, 375 (2011) 4274.
[38] Makarov S. V., Sov. Phys.-JETP, 38 (1974) 248.
[39] Zhao L. C., Ling L., Qi J. W., Yang Z. Y. and Yang W. L., Commun. Nonlinear Sci. Numer. Simul.,
49 (2017) 39.
[40] Wu Y., Zhao L. C. and Lei X. K., Eur. Phys. J. B, 88 (2015) 297.
[41] Frisquet B., Kibler B., Morin P., Baronio F., Conforti M., Millot G. and Wabnitz S., Sci. Rep.,
6 (2016) 20785.
[42] Frisquet B., Kibler B., Fatome J., Morin P., Baronio F., Conforti M., Millot G. and Wabnitz S.,
Phys. Rev. A, 92 (2015) 053854.
[43] Ling L., Zhao L. C., Yang Z. Y. and Guo B., Phys.
Rev. E, 96 (2017) 022211.
[44] Zhao L. C., Guo B. and Ling L., J. Math. Phys., 57 (2016) 043508.
[45] Chen S. and Mihalache D., J. Phys. A: Math. Theor., 48 (2015) 215202.
[46] Chabchoub A. and Akhmediev N., Phys. Lett. A, 377 (2013) 2590.
[47] Chabchoub A., Hoffmann N., Onorato M., Slunyaev A., Sergeeva A., Pelinovsky E. and Akhmediev N., Phys. Rev. E, 86 (2012) 056601.
[48] Erkintalo M., Hammani K., Kibler B., Finot C., Akhmediev N., Dudley J. M. and Genty G., Phys.
Rev. Lett., 107 (2011) 253901.
[49] Ling L., Guo B. and Zhao L. C., Phys. Rev. E, 89 (2014) 041201(R).
[50] Baronio F., Frisquet B., Chen S., Millot G., Wabnitz S. and Kibler B., Phys. Rev. A, 97 (2018) 013852.
[51] Tikan A., Billet C., El G., Tovbis A., Bertola M., Sylvestre T., Gustave F., Randoux S., Genty G.,
Suret P. and Dudley J. M., Phys. Rev. Lett., 119 (2017) 033901.
[52] Kartashov Y. V. and Konotop V. V., Phys. Rev. Lett., 118 (2017) 194010.