A comment on black hole state counting in loop quantum gravity

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There are two ways of counting microscopic states of black holes in loop quantum gravity, one counting all allowed spin network labels $j, m$ and the other involving only the labels $m$. Counting states with $|m| = j$, as done in a recent Letter, does not follow either.

Loop quantum gravity has yielded detailed counts of microscopic quantum states corresponding to a black hole. A start was made in [1] in the direction of quantizing a black hole characterized by an isolated horizon. The quantum states arise when the cross sections of the horizon are punctured by spin networks. The spin quantum numbers $j, m$, which characterize the punctures, can label the quantum states. The entropy is obtained by counting states that are consistent with a fixed area of the cross section [1] and a total spin projection constraint. An estimation of the entropy was carried out in [2] counting only $m$-labels (pure horizon states) – see also [3]. In [4], the $j$-labels were also recognized as characterizing states. Both approaches follow discussions in [1]. Unlike these approximate estimations, [5] has used exact numerical methods, counting $j, m$-labels as in [4]. Recently, [6] has also attempted exact calculations using some number theory. [4] presents two calculations, one of which counts $j, m$-labels, but the other counts only states having $|m| = j$, in a bid to follow [2]. Unfortunately, as explained below, this prescription is not in general equivalent to the rule of counting all horizon states or $m$-labels: it was reached only approximately for large black holes [2, 5]. Consequently this counting of states in [6] is invalid.

We use units such that $4\pi\gamma L_p^2 = 1$, where $\gamma$ is the so-called Barbero-Immirzi parameter involved in the quantization and $L_p$ the Planck length. Setting the area $A$ of the horizon equal to an eigenvalue of the area operator, we write

$$ 2 \sum_{j,m} s_{j,m} \sqrt{j(j+1)} = A, \quad (1) $$

where $s_{j,m}$ is the number of punctures carrying spin quantum numbers $j, m$ and obeying the spin projection constraint

$$ \sum_{j,m} ms_{j,m} = 0. \quad (2) $$

Consider for definiteness a small black hole with $A = 4\sqrt{6}$. This corresponds to 2 punctures each with $j = 2$. Each puncture in principle has 5 allowed values for $m$, but not all the 25 states obey (2), which is satisfied only if $m_2 = -m_1$, so that there are 5 states. These 5 states have different $j, m$-labels and therefore the number of states in the $j, m$-counting of [4] is 5. This is of course what the $j, m$ calculation of [6] yields. But in fact the $j$-values of the two punctures being the same, the states have different $m$-labels, so that the number of states in the pure $m$-label counting envisaged in [1] is also 5. On the other hand, the number of states which the $|m| = j$ calculation of [6] gives is only 2, namely the states with $m_2 = -m_1 = \pm 2$.

Consider next the situation $A = 2\sqrt{2} + 2\sqrt{6}$. Here, there are 2 punctures with $j = 1, 2$. For [2] to be satisfied, $m_2$ cannot be larger than 1 in magnitude, so that there are only 3 combinations of $m$ possible. As the $j$ values may be interchanged, there are 6 states in the $j, m$ counting prescription. However, the $m$ counting prescription yields only 3 because the $j$ values are not to be taken into consideration here. On the other hand, setting $|m| = j$, as in [6], leads to no state at all because $\pm 2$ and $\pm 1$ cannot cancel.

In short, in considering small black holes, or any black hole which can be treated exactly, the $|m| = j$ method of [6] may not count all states with distinct $m$-labels. It gives a severely reduced estimate except in special cases involving $j = \frac{1}{2}$ or for large area [2, 3]. In general, to get the correct number of horizon states ($m$ counting), one has to use the formula $\left( \sum_{m} s_{m} \right)^2$, where $s_{m} \equiv \sum_{j} s_{j,m}$, [2] for each allowed set $\{m\}$ and find the sum.

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