Modelling the Tapping Process in Submerged Arc Furnaces Used in High Silicon Alloys Production

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The tapping process is an important step in the silicon and ferrosilicon production process. Tapping is simply how to transfer the melt from the furnace into the ladle. The tapping process has always been a challenging industrial operation where the metal flow rate is influenced by many different phenomena. In this present work we present a model for the tapping of the submerged arc furnaces. Using the model the effects of furnace crater pressure, metal height and permeability of the different internal zones have been studied. The model is based on computational fluid dynamics (CFD) where the geometry is taken from industrial furnace geometry. The internal zones with individual permeabilities are defined based on information from furnace excavations. From the model we extract new information about the process and explain phenomena which control the tapping flow rate. It was found a very good agreement between the model predictions and industrial measurements.

KEY WORDS: submerged arc furnace; tapping process; furnace crater pressure; bed permeability; computational fluid dynamics (CFD).

1. Introduction

High silicon alloys are industrially produced in the submerged arc furnaces. The furnace casing is made of sheet steel, the lower part lined with hard blocks of strongly calcined carbon and the upper part with firebrick. The furnace contains three electrodes which are situated at the corners of a regular triangle and they are submerged into the charge materials. Conversion of the electrical energy into heat in the bulk of the charge materials results in carbothermic reduction of the ore. As the smelting process proceeds the molten ferrosilicon together with process gases mainly composed of SiO and CO are produced. Ferrosilicon melt is accumulated over the furnace bottom and process gases move through charge particles towards the furnace top where they are sucked into the furnace off-gas system. The melt is after tapping from the furnace taphole refined or alloyed before the final casting.

If tapping due to any reason fails the furnace process must be stopped and the tapping must be completed before the process can proceed. Tapping flow rate and hence tapping time are the most important operational parameters. In practice the melt inside the furnace is never tapped completely. It means that some melt is always remaining inside the furnace when the taphole is closed. The tapping flow rate and tapping time are by large controlled by the conditions inside the furnace as well as different operational decisions. Therefore identification and investigation of the parameters affecting the tapping process is of significant industrial importance.

In the present work the main issues which impact the tapping process are considered to be the furnace crater pressure, the metal height over the furnace bottom and the permeability of the packed beds in different zones inside the furnace.

The full 3D flow model of the two-phase gas–liquid flow inside the furnace is using full scale industrial furnace geometry. The permeabilities of the zones inside the furnace were tuned based on the results of furnace excavations, data available from the literature and operational information. In order to validate the results of the model several industrial tests have been done and compared to the model results.

2. The Issues Affecting the Tapping Process

Industrially it is very important to control the tapping process, in particular represented by the metal flow rate and tapping time. The underlying physical phenomena affecting the tapping flow rate are believe to be the furnace crater pressure, metal height and the permeability of the packed beds inside the furnace.

2.1. Furnace Crater Pressure

Inside the furnace different chemical reactions take place and as the result of the reactions process gases mainly composed of SiO and CO are generated. The reactions can be formally treated as a combination of gross three reactions, Eqs. (1) and (3).
The process gases produced by these reactions, flow towards the furnace top by passing through semi-porous regions of charge materials. The results of the furnace excavations and different scientific investigations regarding silicon and ferrosilicon furnaces\(^3\)–\(^5\) have shown that in the region around and beneath the electrode tips, very close to furnace bottom, a cavity is formed. Formation of these cavities is mainly due to the fact that conversion of electric energy to heat happens inside these cavities through electric arcs. The size of these cavities increases as the production process in the furnace proceeds.\(^1\) During the process these cavities are filled with the process gases. The production of process gases together with the reduced permeability of charge materials around these cavities leads to creation of very high pressures in the furnace crater zone. A simplified representation of the conditions inside the furnace is shown in Fig. 1.

The gas pressure inside the furnace crater zone has been measured and reported in some previous works.\(^6\),\(^7\) Results from industrial measurements show that the furnace crater pressure has a dynamic nature. Several parameters affect the furnace crater pressure.\(^1\),\(^2\) The dominating parameters are the type of charge materials, rate of chemical reaction in the crater zone, electric load, electrode heights, reduced permeability of the charge materials due to condensation of process gases in the upper part of charge materials, while secondary effects, such as softening and melting of charge particles in the regions near by the crater zone, affects the local permeabilities. Avalanches, released naturally or by the stoking cause significant variations in the crater pressure.\(^6\) Figure 2 shows the time variation of the furnace crater pressure and its relation with the furnace electric load in a silicon producing furnace.\(^6\)

### 2.2. Permeability of the Packed Beds in the Furnace

Excavations of the submerged arc furnaces\(^3\)–\(^5\) show that inside the furnaces there are different zones with different physical properties. Due to different chemical reactions, uneven heat distribution, softening and melting of charge particles and condensation of process gases in the bulk of charge materials, formation of these zones is inevitable.

During tapping the melt is drained from a porous bed which lies on the furnace bottom. The bed is mainly composed of the unreacted charge materials and solid SiC which are produced by the furnace process.\(^1\)

The physical properties of each zone in the furnace, such as porosity and the particles size, are scaled to give the expected flow resistance for each zone of the charge. The porosity distribution in the charge material is influenced by proper chemical reactions happening in each zone. Endothermic and exothermic reactions determine the heat distribution and hence the porosity distribution in the charge packed bed. Reduced permeability in the charge materials creates higher resistance for the flow of process gases and hence increases the crater pressure. On the other hand, the permeability at the furnace bottom bed has a crucial effect on the melt flow rate from the taphole. Both these phenomena underline how the permeability of the packed beds can affect the tapping process.

### 2.3. Metal Height in the Furnace

The metal height in the furnace is expected to be a function of the melt production rate and the porosity of the packed bed at the furnace bottom. According to the Bernoulli’s equation for fluid flow the metal height will influence the tapping flow rate from any vessel, including our furnace.

Knowledge of the metal height in the furnace is always a challenging issue. If the level of the melt inside the furnace becomes too high the liquid silicon comes in contact with the molten quartz according to Eq. (4).\(^2\)

\[
\text{SiO}_2 (s, l) + 2C (s) = \text{Si} (s) + 2\text{CO} (g) \quad (1)
\]

\[
\text{SiO}_2 (s, l) + C (s) = \text{SiO} (g) + \text{CO} (g) \quad (2)
\]

\[
\text{Fe}_2\text{O}_3 (s, l) + 3C (s) = 2\text{Fe} (l) + 3\text{CO} (g) \quad (3)
\]

\[
\text{SiO} (s, l) + \text{C} (s) = \text{SiO} (g) + \text{CO} (g) \quad (4)
\]

This reaction takes place at temperatures above 1800°C and is strongly endothermic. Reaction (4) may disturb the process in three ways.\(^2\) Firstly, the reaction will normally increase production of silica-dust and hence lower the silicon yield. Secondly, this will have an impact on the carbon balance. An increase in the SiO-loss from the furnace will reduce the amount of carbon needed by the process.
Thirdly, the huge amount of energy needed for Reaction (4) may reduce the process temperature in the hot zone and increase the need for SiO circulation in the furnace, which is very unfavorable for the total process performance. Therefore it is unfavorable to have a very high metal height in the furnace before start of tapping. As a result the optimal time intervals between the taps are affected by the metal height inside the furnace.

3. Development of a Model for Tapping Process

In order to model the flow of gas and liquids out from the heart of the furnace we need a multidimensional multiphase model that can handle the flow of gas and liquid though porous materials. In addition we need to keep track of the gas–liquid interface as it is sharp. As the models concepts already were available in commercial Fluent code we used Fluent’s Volume-Of-Fluid (VOF) model to keep track of the gas–liquid interface.

In order to simplify the model we have limited the analyses to isothermal but high temperature conditions.

3.1. Model Governing Equations

As the gas–liquid interface is tracked by the VOF model –k

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \] .................(5)

\[ \frac{\partial \gamma \vec{u}}{\partial t} + \nabla \cdot (\gamma \rho \vec{u}) \] = \gamma \nabla \left( \rho + \frac{2}{3} \rho k \right) + \gamma \rho g \beta (T - T_{\text{sat}}) - \gamma \vec{R} \] .............(6)

Where \( \gamma \) is the fluid fraction and \( \rho \) is the fluid bulk density, with \( k \) representing gas and liquid respectively.

\[ \rho = \sum_k \gamma_k \rho_i \] \[ \mu = \sum_k \gamma_k \mu_i \] .................(7, 8)

The scalar equation for the propagation of the liquid fraction has similar form as Eq. (5).

The effective and turbulent viscosities are defined as follows:

\[ \mu_{\text{eff}} = \mu + \mu_t \] .................(9)

\[ \mu_t = \rho C_{\mu} \frac{k^2}{\varepsilon} \] .................(10)

The model is a generalization of Darcy’s Law, commonly used for flows in porous media, and of Reynolds-averaged Navier–Stokes equations, with an eddy viscosity accounting for the turbulent effects. The last term in Eq. (6) represents the resistance to flow in porous media. Based on the well-known Ergun’s equation, the resistance force for flow through a bed of particles is given by:

\[ \vec{R} = \frac{150 \mu}{D_p^2} \left( \gamma - 1 \right)^2 \vec{e} + \frac{1.75 \rho}{D_p} \left( \gamma - 1 \right) \right] \frac{\vec{e}}{\varepsilon} \] .......(11)

The defined resistance in the Ergun’s equation for each component can be written as:

\[ \vec{R} = \left( \frac{\mu}{\alpha} \vec{e} + \frac{1}{2} \rho \varepsilon \right) \] .................(12)

\[ \alpha = \frac{D_p^2}{150} \left( \gamma - 1 \right)^3 \] .................(13)

\[ C_2 = \frac{3.5}{\gamma^3} \] .................(14)

In order to model the turbulent flow of the gas and liquid phases available in the furnace, a modified version of the \( k-\varepsilon \) model for fluid flow in porous beds is applied. In this turbulence model an extra source term due to turbulence production due to large solid particles is added to both the kinetic energy and the dissipation rate model equations. The turbulent viscosity is then determined from these modified turbulent kinetic energy and dissipation rates. Accordingly, the model allows unified treatments for the flow of process gas and silicon melt over the entire furnace volume.

3.2. Geometry and Inside Conditions of the Furnace

Geometry of the furnace used in the modelling has been chosen based on an industrial ferrosilicon furnace. The schematic of the furnace geometry and a part of the computational grid used in the modelling process are shown in Figs. 3(a) and 3(b).

The furnace diameter is about 8 m, the furnace height is 3 m and the taphole diameter is 10 cm. Inside the model different zones have been defined. The physical properties of each zone have been selected based on the results of furnace excavation and the data available in the literature. Over the furnace bottom, where the melt is accumulated, three different zones have been defined. Difference in physical properties of these zones is directly related to the uneven temperature distribution in the existent packed bed. The flow resistance is designed to increase as we move out from the furnace centre and approach the taphole. High flow resistance is created partially by a high solids fraction. Hence, the volume fraction of the melt is highest in the furnace centre and decreases by increasing the distance from the furnace centre. The porosity both in the upper and lower parts of the furnace ranges between 10% up to 60% depending on the position of the packed beds. While studying the effect of metal height and furnace crater pressure on the tapping flow rate the porosity and permeability distribution in the lower part of the furnace is the same in all simulations. The 2D view of the defined zones in the model is presented in Fig. 4.
The liquid phase in the model is a special ferrosilicon alloy with the density of 4300 kg/m³. The gas phase is a mixture of process gases made of SiO and CO gas which in the furnace operating temperature is 0.23 kg/m³.

3.3. Numerical Method and Boundary Conditions

In order to solve the set of governing model equations the commercial CFD software Fluent 6.3.26 was used. The furnace operates at atmospheric pressure condition. Therefore the boundary conditions at the furnace top and at the taphole outlet were defined as constant pressure boundaries at atmospheric pressure. The fluid properties are based on the furnace operating temperature, which is around 1800°C, while local temperatures in the crater zone may reach levels above 2000°C. Over the internal surface of the furnace walls we apply a no-slip condition for the velocity of both liquid and gas.

In order to simulate the generation of process gases due to chemical reactions in the crater zone, a mass source term for gas production was defined inside the crater zone. The production rate of the melt which is around 70 ton/d for the considered ferrosilicon furnace in this study is introduced via defining a mass source in the metal zone of the model and it is fixed in all case studies. The numerical value for the mass source terms of process gas and melt was based on information from industrial operation.

4. Results

Simulations have been performed for a large number of operational conditions. This allows the investigation of how furnace crater pressure, metal height and the permeability impact the tapping flow rate, the total tapping weight and the tapping time. The results from the model are presented in different sections, according to the phenomena under examination. In addition to the model results, for sake of comparison and validation, the results of industrial measurements are also presented.

4.1. Investigation of the Effect of Crater Pressure

Measurements of the crater pressure in the silicon and ferrosilicon producing furnaces have been reported in the literature.6,7) The crater pressure can be measured by making a hole inside and parallel to the electrodes. As the hole extends all the way to the electrode tip pressure contact with the crater zone is obtained. In this manner the crater pressure can be measured at the top of the electrode where temperature is low. The measured furnace crater pressure shows a dynamic behavior due to multiple interacting mechanisms. However, still we can define representative average values. In the furnace under consideration in this study the reported crater pressure ranges from 30 mbar up to 200 mbar. The results of the model for the tapping flow rate in the case where the initial metal height inside the furnace is 10 cm is presented in Fig. 5.

Results show that the tapping flow rate is initially quite high, depending on the crater pressure. After a while the tapping rate decreases significantly and reaches to a level which is almost the same in all cases. The results in Fig. 5 show that the increased crater pressure leads to increased tapping flow rate and decreased waiting time for the drop in tapping flow.

The comparison between the result of the CFD model and the industrial measurements for both the tapping flow rate and the total tapping weight versus tapping time is presented in Fig. 6.

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The drop in flow rate drop during tapping is an interesting phenomenon which is a result of the melt flow pattern inside the furnace, caused by the crater pressure.9) When the tapping flow rate drops gas starts to emerge from the taphole. After this initiation of two phase flow from the tap hole, the gas flow rate increases as the tapping process pro-
ceeds. Still the gas mass flow is very small compared to the melt flow.

Figure 7 shows the comparison of the model results for the total tapping weight with initial metal height of 10 cm versus the industrial measurements. The points in the graph show the total tapping weight before tapping flow rate drops.

As it can be seen from Fig. 7 that a high crater pressure results in less metal tapped, even if the metal flow rate is higher and tapping time shorter. The consequence of this is that for an operation of the furnace at higher crater pressures the time intervals between taps will have to be reduced in order to keep a constant maximum metal height.

In order to show that the increased flow rate at the start of tapping and significant flow rate drop are due to existence of the high pressure crater zone a simple study was done. In this case the initial metal height inside the furnace was considered to be 12 cm but there was no additional pressure due to process gases released from the crater zone. In fact, the only parameter, driving the flow, was the metal height. The result of this simulation is shown in Fig. 8.

This result contains two important points. The first point is that the tapping flow rate is very low, even with higher initial metal height than the cases shown in Fig. 5. In fact, to have such high tapping flow rates without the crater pressure the initial metal height must be up to 60 cm. Such high metal height is neither acceptable nor possible from the industrial point of view. The second point is that when only the hydrostatic pressure due to the metal height drives the flow, the flow rate drop observed in the industrial measurements can not be explained. These two points substantiate that the crater pressure due to limited flow permeability is the cause of a significant change in tapping rate, appearing after a certain mass of metal is tapped.

### 4.2. Investigation of the Effect of Metal Height

In order to investigate the effect of metal height on the tapping parameters, different metal heights have been studied. The range considered for the metal height starts from...
very low level around 4.5 cm up to 12 cm, the latter being higher than the taphole level. The effect of metal height on the tapping flow rate is for a crater pressure of 90 mbar presented in Fig. 9.

It is observed that the increased metal height leads to increased tapping flow rate. In addition we see an increase in the tapping time before the flow rate drops. It is obvious that the effect of metal height on the tapping flow rate is not as significant as the effect of furnace crater pressure is.

The most important result from the investigation of the metal height effect is that in all different range of metal heights, the tapping rates before the metal flow rate drops are virtually the same. It means that the metal height is not the factor which determines at which tapping rate the metal flow rate will drop. In other words, it is the furnace crater pressure which determines at which tapping rate the metal flow rate drops. The effect of metal height is to determine when the tapping flow rate drops. It is obvious that high metal height and corresponding large metal mass will lead to larger tapping time. Furthermore, we see that for a given crater pressure the metal mass tapped before reduction in tapping rate is a clear indication of the total amount of metal in the furnace.

Figure 10 shows the same results as Fig. 9, but now as the total tapped weight. It is seen how the total tapping weight from the furnace increases with increased metal height.

### 4.3. Investigation of the Combined Effect of Crater Pressure and Metal Height

Industrial measurements prove that the furnace crater pressure has a dynamic behavior during the smelting process. In the industrial operation the metal production rate is not constant and different phenomena can change the production rate and hence the metal height in the furnace varies by time. Therefore investigation of the most important parameters over a larger span of values is necessary. In this research we have investigated the furnace crater pressure range from 45 to 180 mbar and for the metal height range from 4.5 to 12 cm. The tapping time is also a key parameter in modeling the tapping process and therefore we explore how tapping time is affected by crater pressure and metal height. Note that from now on we define the tapping time as the time until the tapping flow rate suddenly drops.

The reason for this definitions is that after gas break through in the taphole the tapping rate drops down to very low level which is almost the same in all cases, and has a low metal flow rate that is in good agreement with the industrial measurements.

Figure 11 shows the effect of furnace crater pressure on the average tapping flow rate and the total tapping weight for different metal heights inside the furnace. It is observed that furnace crater pressure has a positive effect on the tapping flow rate for different metal heights but the total tapping weight from the furnace is lower in the cases with higher crater pressure.

The simulation results can alternatively be presented as
the average tapping flow rate or the total tapping weight, both versus metal height (see Fig. 12).

As it is seen from Fig. 12(a) increase in the metal height leads to increased tapping flow rate for all crater pressures. Furthermore, and as expected, the total mass of tapped metal increases as the metal height and hence metal volume in the furnace is increased, as observed from Fig. 12(b).

As tapping time expresses the time we can tap before gassing from the taphole starts, tapping time is the most important operational parameter. The effects of metal height and furnace crater pressure on the tapping flow rate and the total tapping weight as a function of time is presented in Fig. 13.

Figure 14 represents the percentage of the melt which is tapped from the furnace inside as a function of tapping time for different crater pressures and metal heights. As it can be seen when the crater pressure is lower the amount of the metal tapped from the furnace is higher and the tapping time is longer. The general picture is that short tapping times correspond to low metal height and high crater pressures. However, in these cases with the shortest tapping times the results are telling that more than 90% of the metal remains inside the furnace when gassing from the taphole is initiated.

4.4. Prediction of the Crater Pressure and the Metal Height in the Furnace Using the Results of CFD Model

Comparison of the results of the model with the industrial measurements for the total tapping weight before the flow rate decrease leads to a very interesting and valuable result about prediction of the metal height inside the furnace. Figure 15 shows both the results of the CFD model and industrial measurements. It is observed that the range of industrial data falls between the metal height of 6 cm and 9 cm predicted by the model. A point that should be noticed is that the metal height considered in the modeling and the metal volume are not linearly related as the furnace bottom contains a packed bed of solid particles which displaces the melt and has a given porosity distribution.

The same method can be applied for the furnace crater
Figure 16 shows the comparison between the model results and the industrial measurements. Comparison of the industrial measurements with the model results support that variations in tapping rate observed industrially may be caused by the dynamic behavior of the furnace crater pressure. From Fig. 15 we see that the industrially observed variations in tapping rate can be explained by large with industrially observed variations in crater pressures while the initial metal level before tapping starts is around 8 cm. From Fig. 16 we can conclude that metal height variations cannot explain the industrially observed variations in tapping rate and tapping times. Based on these results it is likely that the model will be able to give indications of the furnace crater pressure by having just the information about the total tapping weight.

4.5. Investigation of the Effect of Bed Permeability

The packed bed at the furnace bottom has two important effects. Firstly it influences the metal height for a given metal volume. Secondly the packed bed creates a resistance against the flow of melt towards the taphole and leads to reduced tapping flow rate from the furnace. In fact the melt should drain through this packed bed during tapping process.

In order to investigate the effect of the backed bed properties on the tapping process, two different beds with different permeability (and hence resistance) were considered at the furnace bottom. The permeability difference was considered to be only due to different particles size inside the beds and not due to porosity change. The first bed is the main bed of the model and in the second bed the permeability has been reduced down to 40% of the base case. The comparison of the tapping parameters between the low and the high permeable beds for the crater pressures ranging between 90 mbar up to 180 mbar is presented in Fig. 17. The initial metal height in all these cases is constant and fixed at 12 cm.

As it can be seen from Fig. 17 the tapping flow rate decreases by decreasing the bed permeability. The tapping time is also influenced by the bed permeability although in the cases with higher crater pressure the difference in the tapping times is very small. It is clear that the effect of bed permeability on the total tapping weight is insignificant as the crater pressure increases.

Figure 18 shows that how the bed permeability affects the average tapping rate from the furnace and the total tapping weight before the flow rate drops in different range of crater pressures. It is obvious that the higher the bed permeability the higher the average tapping flow rate.

Investigation of the effect of bed permeability on the average tapping flow rate and the total tapping weight versus tapping time for different range of furnace crater pressures has been presented in Fig. 19.

As it can be seen from this figure as the crater pressure decreases both the tapping time difference and the propor-
ditional difference of the average tapping flow rate increases. The results show that the tapping flow rate is higher in the cases when the bed permeability is higher. It is also observed that both the tapping time difference and the proportional difference of the total weight of the melt tapped from the furnace increases as the furnace crater pressure decreases. The total tapping weight is higher in the case with more permeable bed.

5. Conclusions

A comprehensive model for the tapping process in the submerged arc furnaces used for production of high silicon alloys has been developed. Different important issues which can affect the tapping process have been investigated. The most important parameters affecting the tapping are furnace crater pressure, metal height inside the furnace and permeability of different packed beds inside the furnace.

The model results show that the furnace crater pressure affects the tapping process through influencing the tapping flow rate and the fraction of the metal that can be tapped before gassing from the taphole is initiated. The crater pressure is the factor which determines the tapping flow rate by the start of tapping as well as the tapping rate just before flow rate drops. Higher furnace crater pressure leads to higher tapping flow rate from the furnace at the start of tapping process and at the same time it causes the faster tapping flow rate drop.

The metal height affects the tapping flow rate and the total tapping weight before flow rate drops. The effect of metal height on the tapping flow rate is not as significant as the effect of furnace crater pressure. A trivial result is that the metal height, directly related to the total metal volume, is the key factor which determines when tapping flow rate drops.

The change of permeability in different zones inside the furnace influences the process gas flow resistance and pressure drop as the gas flows from the crater to the top of the charge. A low permeability in the charge results in a high pressure inside the crater zone. The permeability of the packed bed formed over the furnace bottom, particularly near the taphole zone, affects the tapping flow rate. Reduced permeability of the bottom packed bed causes reduced tapping flow rate and hence longer tapping time. In addition low permeability result in reduces yield of the tapped metal.

The results of the model are in very good agreements with the industrial measurements and the model is able to explain the variations in industrial tapping rates as a result of variations in crater pressure.

Nomenclature

\[ \rho : \text{ Fluid density (kg/m}^3\text{)} \]
\[ \gamma : \text{ Porosity of the bed} \]
\[ u, v : \text{ Fluid velocity (m/s)} \]
\[ \mu : \text{ Dynamic viscosity (Pa} \cdot \text{s)} \]
\[ \mu_t : \text{ Turbulent viscosity (Pa} \cdot \text{s)} \]
\[ p : \text{ Pressure (Pa)} \]
$k$: Turbulent kinetic energy (m$^2$/s$^2$)
$\varepsilon$: Turbulent dissipation rate (m$^2$/s$^3$)
$g$: Gravity acceleration (9.81 m/s$^2$)
$T$: Temperature (K)
$\beta$: Thermal coefficient of volumetric expansion (K$^{-1}$)
$D_p$: Particle diameter (m)
$C_k$: Turbulent model constant
$\alpha$: Permeability of the packed bed
$C_2$: Inertial resistance
$\kappa$: Liquid and gas phase index

REFERENCES

1) A. Schei, J. Tuset and H. Tveit: Tapir Forlag, (1998).

2) H. Tveit, T. Halland, K. I. Landro, S. T. Johansen and B. Ravary: Proc. Silicon for the Chemical Industry, Loen, Norway, (2002).
3) A. Schei: Proc. Tidsskrift Kjemi Bergvesen, Metallurgi 27, Trondheim, Norway, (1967).
4) Y. Otani, M. Saito, K. Usui and N. Chino: Proc. 6th Int. Cong. of Electro Heat, Brighton, England, (1968).
5) E. H. Myrhaug: PhD Thesis, NTNU, Trondheim, Norway, (2003).
6) S. T. Johansen, H. Tveit, S. Grådahl, A. Valderhaug and J. Byberg: Proc. 8th INFACON Conf., Beijing, China, (1998).
7) H. T. Ingason, J. Halfdanarson and J. A. Bakken: Proc. 7th INFACON Conf., Trondheim, Norway, (1995).
8) A. Nakayama and F. Kuwahara: J. Fluids. Eng., 121 (1999), 427.
9) M. Kadkhodabeigi, H. Tveit and S. T. Johansen: J. Prog. Comput. Fluid Dyn., 10 (2010), No. 5–6, 374.
10) S. Ergun: J. Chem. Eng. Prog., 48 (1952), No. 2, 89.