Simulating finite-time quantum isothermal processes with generic superconducting quantum circuit

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The finite-time isothermal process is fundamental in quantum thermodynamics yet complicated with combination of changing control parameters and the interaction with the thermal bath. Such complexity prevents the direct application of the traditional thermodynamics measurement of the relevant quantities. In this paper, we provide a discrete-step method to separate the work done and the heat exchange in the isothermal process by decomposing the process into piecewise adiabatic and isochoric processes. The piecewise control scheme makes it possible to simulate the whole process on a generic quantum computer, which provides a new platform to experimentally study quantum thermodynamics. We implement the simulation on ibmqx2 to show the $C/\tau$ scaling of the extra work in the finite-time isothermal process.

I. INTRODUCTION

Quantum thermodynamics, originally believed as an extension of classical thermodynamics, has sharpen our understanding toward the fundamental aspects of thermodynamics [1–6]. Along with the theoretical progresses, experimental tests and validation of the underline principles are relevant in the realm. Simulation of the quantum thermodynamic phenomena [7–10], as one of the experimental efforts, has been intensively explored with specifically designed system for specific purpose, e.g., the trapped ion for testing the Jarzynski equation [11], the BEC system for quantum work extraction [12], and the superconducting qubit for the shortcut to adiabatic [13]. These designed systems often have limited functions to test specific quantum thermodynamic properties. Simulation with generic quantum computing system shall offer a universal system to demonstrate essential quantum thermodynamic phenomena.

Yet, simulation of quantum thermodynamics with the universal quantum computer remains a challenge mainly due to the lack of the flexibility to physically tune control parameters and the difficulty to measure the work extraction. In quantum thermodynamics, the work extraction, as a fundamental quantity [14–16], requires the ability to tune the control parameters. Such requirement is achievable in the specifically designed system, e.g., the force shift in the trapped ion [11, 17], the trapped frequency in BEC [12] and the transition frequency in the superconducting system [13]. However, in the universal optimized quantum computer, e.g. IBM quantum computer (ibmqx2), the operations are limited for the users and such tuning is unfortunately not available. Additional problem is the measurement of the work extraction, which can be easily measured in classical thermodynamics by recoding the control parameters and measuring the conjugate quantities. Such measurement remains a challenge in the quantum region [18].

In this paper, a proposal is given and experimentally demonstrated to simulate the basic non-equilibrium thermodynamics process with quantum computer by introducing a virtual way to tune the control parameters, i.e. without physically tuning the parameters. The dynamics are realized using quantum gates according to the parameter changes. As a demonstration, we realize the simulation on ibmqx2 [19] for the isothermal process, which is a fundamental process for quantum heat engine cycle, yet complicated with both the changing control parameters and the interaction with a thermal bath. To implement the simulation using, we consider the discrete-step method to approach the quantum isothermal process: the isothermal process is divided into series of elementary processes, each consisting of the adiabatic process and the isochoric process. In the adiabatic process, the parameter adjusting is performed virtually and the unitary evolution is implemented with quantum gates. In the isochoric process, the effect of the bath contact is performed with quantum channel simulation [20–25] with the assist of ancillary qubits for quantum dynamics simulation in open quantum systems[26–28]. With this approach, we achieve the simulation of the isothermal process on the universal quantum computing system without flexibility to directly adjust physical parameters such as the energy levels.

The simulation with universal quantum computer brings clear advantages in our proposal. First, the arbitrary change of the control parameters is archived by the virtual tuning via the simulation of corresponding dynamics, avoiding the difficulty in tuning the actual physical system. In turn, the parameters can be controlled to follow an arbitrary designed functions. Second, we can realize the immediate change of environment parameters, such as the temperature. The effect of the bath is reflected through the state of the auxiliary qubit, which can be controlled flexibly using quantum gates.
This paper is organized as follows. In Sec. II, we introduce the discrete-step method to approach the quantum isothermal process. The isothermal process is approached with series of the adiabatic and the isochoric processes. In Sec. III, we present circuit realization of one elementary process. Two methods, the hybrid simulation and the fully quantum simulation, are designed to simulate the quantum thermodynamic process using quantum circuits. In Sec. IV, we show an application of our simulation scheme. The ibmqx2 simulation results of the discrete isothermal process are compared with the numerical results. The conclusion is given in Sec. V.

II. DISCRETE-STEP METHOD TO QUANTUM ISOTHERMAL PROCESS

In quantum thermodynamics, the generic evolution of the concerned system can be considered as a quantum evolution with the changing Hamiltonian while in contact with a thermal bath. The interplay between work and heat during the general process has limited the characterization of the quantum thermodynamic cycle on the microscopic level, where the classical method to measure the work, via force and distance, is not applicable [18]. In the limit of timescale of tuning the control parameter far larger than the thermal bath response time, the evolution is the thermodynamic adiabatic process, where the heat exchange with the bath is neglected and the internal energy changes due to the work done through the changing control field. Another extreme case with unchanged control parameters is known as the isochoric process, where the internal energy changes solely by the heat exchange with the thermal bath. Therefore, the work and heat can be separated clearly in the adiabatic and isochoric process, and be obtained directly by measuring the internal energy change.

One method to simulate the generic quantum thermodynamic process while keeping the separation of the work and heat, is to use the discrete-step method with a series of adiabatic and isochoric processes [29, 30]. In Fig. 1, the discrete-step method is illustrated with the minimal model, a two-level system with the energy spacing \( \omega(t) \) between the ground state \( |g\rangle \) and the excited state \( |e\rangle \). Such two-level system can be physically realized with a qubit, which is a elementary unit of the quantum computer. For the clarity of the later discussion, we use the term “two-level system” to denote the simulated system, and the term “qubit” as the simulation system hereafter without specific mention. The state of the two-level system is represented by the density matrix \( \rho_s(t) \) of the system qubit, and the effect of the thermal bath is simulated by the assist of some ancillary qubits. Initially, the system qubit is prepared to the thermal state \( \rho_s(0) \) with the corresponding energy \( \omega_0 \) and the temperature \( T \). The evolution of the open quantum system can be implemented with single-qubit and two-qubit quantum gates on the superconducting quantum computer. The internal energy of the two-level system is determined by

\[
E(t) = \omega(t)p_e(t),
\]

with the population of the excited state \( p_e(t) = \langle e | \rho_s(t) | e \rangle \).

For the system to be simulated, we use the discrete-step method to approach the finite-time isothermal process for the two-level system. The discrete isothermal process contains \( N \) steps of elementary processes with the total time consuming \( \tau + \tau_{adi} \), where \( \tau \) (\( \tau_{adi} \)) denotes the time consuming in the isochoric (adiabatic) process. Each elementary process is composed by an adiabatic process and an isochoric process. We set equal time consuming for every elementary process \( \delta \tau = (\tau + \tau_{adi})/N \), with \( \tau/N \) (\( \tau_{adi}/N \)) for each isochoric (adiabatic) process.

In the adiabatic process, the system is isolated from the bath and evolves under the time-dependent Hamiltonian. Such process is described by a unitary evolution with the control time as \( \tau_{adi}/N \). The performed work is determined by the change of the internal energy at the initial and the final time. For a generic adiabatic process, the unitary evolution of the system can be simulated with the single-qubit gate acted on the system qubit. In this paper, we consider the adiabatic process as the instant quench with zero time consuming \( \tau_{adi} = 0 \), occurred at time \( t_j - 1 = (j - 1)\delta \tau, \ j = 1, 2, ..., N \). As the result of the quench, the energy of the excited state is shifted from \( \omega_{j-1} \) to \( \omega_j \), while the density matrix \( \rho_s(t_{j-1}) \) remains unchanged after the quench. At the initial time \( t_0 = 0 \), the energy is quenched from \( \omega_0 \) to \( \omega_1 \) after the initial state preparation. The performed work for the quench at time \( t_{j-1} \) reads

\[
W_j = (\omega_j - \omega_{j-1})p_e(t_{j-1}).
\]

(1)

To obtain the performed work for the simulated finite-time quantum isothermal process, we only need to obtain
the diagonal element $p_s(t_{j-1}) = \langle e| \rho_s(t_{j-1}) | e \rangle$ via measuring the state population of the system qubit at the beginning of each isochoric process.

In the isochoric process of the $j$-th elementary process ($t_{j-1} < t \leq t_j$), the two-level system is brought into contact with the thermal bath at the temperature $T$. The evolution is given by the master equation

$$
\dot{\rho}_s = -i[H_j, \rho_s] + \gamma_0 N_j \mathcal{L}(\sigma_+) [\rho_s] + \gamma_0 (N_j + 1) \mathcal{L}(\sigma_-) [\rho_s],
$$

with

$$
\mathcal{L}(\sigma) [\rho_s] = \rho_s \sigma^+ \frac{1}{2} \sigma^- \rho_s - \frac{1}{2} \rho_s \sigma^\dagger \sigma.
$$

Here, $H_j = \omega_j |e\rangle \langle e|$ is the Hamiltonian of the system during the period $t_{j-1} < t \leq t_j$, $N_j = 1/[\exp(\beta \omega_j) - 1]$ is the average photon number with the inverse temperature $\beta = 1/(k_B T)$, and $\sigma_+ = |e\rangle \langle g|$, $\sigma_- = |g\rangle \langle e|$ is the transition operator. In this process, the change of the internal energy is induced by the heat exchange with the thermal bath, and no work is performed. During the whole discrete isothermal process, the work is only performed at the time $t_j$.

### III. Simulation with Quantum Circuits

In this section, we first show the simulation of one elementary process with quantum gates in the circuit. The simulation is formulated for the adiabatic and the isochoric processes as follows.

#### I. Adiabatic process:

In the superconducting quantum computer, e.g., IBM Q system, the tuning of the physical energy levels of qubits is unavailable for the users. Usually, the parameters of the system are fixed at the optimal values to possibly reduce noise and error induced by decoherence and imperfect control. The Hamiltonian of the simulated two-level system is modulated as $H(t) = \omega(t) |e\rangle \langle e|$, where

$$
\omega(t) = \omega_j, \quad t_{j-1} < t \leq t_j \quad \text{for} \quad j = 1, 2, ..., N.
$$

We will show that the shifted energy $\omega(t)$ of the simulated two-level system only affects the transition rate induced by the thermal bath. In the simulation, the thermal transition is simulated through the quantum channel simulation with the assist of ancillary qubits, and the transition rate can be flexibly tuned by the single-qubit gates on the ancillary qubits. Therefore, we do not have to physically tune any parameters of the quantum computer, and just algorithmically modulate the simulated thermal transition instead. We propose a virtual tuning of the energy levels with details explained as follows.

In the virtual process, we need to simulate the unitary evolution of the adiabatic process with single-qubit gates on the system qubit. For the adiabatic process, i.e. the quench, the state of the system does not evolve in the short period. We don’t have to do anything with the physical system, and just pretend that the energy of the simulated system is tuned from $\omega_j$ to $\omega_j + \delta \omega$ in the $j$-th adiabatic process. This virtual tuning of the energy is reflected by the modulation of the thermal transition rate in the simulation of the isochoric process.

#### II. Isochoric process:

The simulation of the isochoric process is to simulate the dynamics governed by the master equation in Eq. (2). The dynamical evolution of the isochoric process can be simulated with the generalized amplitude damping channel (GADC)

$$
\rho_s(t_j) = \mathcal{E}_{GAD}^{(j)}[\rho_s(t_{j-1}) e^{iH_j \delta \tau}],
$$

where $\mathcal{E}_{GAD}^{(j)} = p_+^{(j)} \mathcal{E}_+^{(j)} + p_-^{(j)} \mathcal{E}_-^{(j)}$ is divided into two sub-channels, the amplitude damping channel

$$
\mathcal{E}_{+}^{(j)}[\rho_s] = M_0^{(j)} \rho_s M_0^{(j)} \dagger + M_1^{(j)} \rho_s M_1^{(j)} \dagger,
$$

and the amplitude pumping channel

$$
\mathcal{E}_{-}^{(j)}[\rho_s] = M_2^{(j)} \rho_s M_2^{(j)} \dagger + M_3^{(j)} \rho_s M_3^{(j)} \dagger.
$$

The corresponding Kraus operators are $M_0^{(j)} = \cos \theta_j |e\rangle \langle e| + |g\rangle \langle g|$, $M_1^{(j)} = \sigma_- \sin \theta_j$, $M_2^{(j)} = |e\rangle \langle e| + \cos \theta_j |g\rangle \langle g|$ and $M_3^{(j)} = \sigma_+ \sin \theta_j$. The coefficient $p_+^{(j)} = 1/[\exp(\beta \omega_j) + 1]$ ($p_-^{(j)} = 1 - p_+^{(j)}$) shows the probability of excitation (de-excitation) of the two-level induced by the thermal bath. The evolution time of the $j$-th elementary process is encoded in the control parameter $\theta_j$ via the relation

$$
\cos \theta_j = \exp\left(-\frac{\gamma_0 \delta \tau e^{\beta \omega_j} + 1}{2 e^{\beta \omega_j} - 1}\right).
$$

The quasi-static discrete isothermal process with infinite evolution time can be realized by setting $\theta_j = \pi/2$, where the system reaches thermal equilibrium at the end of each isochoric process.

For the initial thermal state $\rho_s(0) = e^{-\beta H(0)}/\text{Tr}(e^{-\beta H(0)})$, the off-diagonal term remains zero throughout the whole discrete isothermal process in the current control scheme. In this situation, the evolution by Eq. (5) is simplified as

$$
\rho_s(t_j) = \mathcal{E}_{GAD}^{(j)}[\rho_s(t_{j-1})].
$$

For an initial state with non-zero non-diagonal term, the coherence does not affect the diagonal term of the density matrix. This comes from the fact that the diagonal term
and the non-diagonal term of the density matrix satisfy respective differential equations by Eq. (2).

Figure 2 shows the quantum circuit to simulate the isochoric process. The two sub-channels $\rho_{s,j}^{(i)}$ and $\rho_{s,j}^{(f)}$ are realized with an ancillary qubit initially prepared in the ground state [25]. The circuits for these two sub-channels are illustrated in Fig. 2(a). The meaning of each gate is explained at the bottom of Fig. 2. Such simulation circuits are extensively studied in the field of quantum computing and quantum information that we will not explain the setup in detail [21].

For the probabilities $p_{s,j}^{(i)}$ and $p_{s,j}^{(f)}$ in selecting the two sub-channels, we design two methods; the hybrid simulation and the fully quantum simulation, to achieve the random selection, as shown in Fig. 2(b). The former uses one ancillary qubit for each elementary process with the assist of a classical random number generator (CRNG), which can be realized using a quantum computer with fewer qubits. The latter utilizes fully quantum circuit with two ancillary qubits for each elementary process, which can be realized using a quantum computer with adequate qubits.

1. Hybrid simulation of isochoric process with classical random number generator (CRNG)

With the limited number of qubits, it is desirable to save the unnecessary usage of qubits. To simulate the quantum channel for the system qubit, one ancillary qubit is inevitably used to simulate the non-unitary evolution of the open quantum system [23]. In this design, one qubit is used to represent the two-level system, and one more qubit is needed for adding one elementary process. Therefore, it requires $N + 1$ qubits to simulate the $N$-step isothermal process.

In the hybrid simulation, the CRNG is used to select the sub-channel $\rho_{s,j}^{(f)} = \rho_{s,j}^{(i)}$ or $\rho_{s,j}^{(f)}$ for the isochoric process in the $j$-th elementary process, as shown in Fig. 2(b). $l$ denotes the $l$-th simulation of the discrete isothermal process. For each isochoric process, the CRNG generates a random number $r_{s,j}^{(f)} \in [0, 1]$ with uniform distribution. The sub-channel $\rho_{s,j}^{(f)}$ is selected as $\rho_{s,j}^{(i)}$ ($\rho_{s,j}^{(f)}$) when the random number satisfies $r_{s,j}^{(f)} \leq p_{s,j}^{(i)}$ ($r_{s,j}^{(f)} > p_{s,j}^{(f)}$).

2. Fully quantum simulation of isochoric process

For the system with adequate qubits, the selection of the two sub-channels can be achieved with an additional ancillary qubit, which encodes the classical probability, as shown in Fig. 2(c). In each step, one more ancillary qubit is used, prepared to the super-position state $\cos(\alpha_j/2) |0\rangle + i \sin(\alpha_j/2) |1\rangle$ through the $R_x(\alpha_j)$ gate with $\cos(\alpha_j/2) = |p_{s,j}^{(f)}|^{1/2}$. Since two ancillary qubits are needed for adding one elementary process, this method requires $2N + 1$ qubits to simulate the $N$-step isothermal process.

In Table I, we summarize the simulation procedure for the adiabatic and the isochoric processes. The correspondence between the parameters in the simulated system and the simulation is listed in the table.

In the current simulation scheme, we solve the problem of separating work and heat. The unitary evolution of the adiabatic process requires isolation from the environment, while the isochoric process needs the contact with environment. Switching on and off the interaction
The unitary evolution is realized with the virtual tuning on the system Hamiltonian.

The circuit of every selection with population decreases from a true thermal state without coherence can be initially of the thermal state. With another ancillary qubit, not affect the final state population so long as the initial state population is implemented on ibmqx2. With the five qubits, it is feasible to simulate repetitive quantum channels on ibmqx2. With the five qubits, it is feasible to simulate repetitive quantum channels on ibmqx2. In fully quantum simulation, the fixed circuit is implemented repetitively, and the excited state population \( p_e(t_j) \) is obtained by measuring the state of the system qubit. The performed work for the simulated system is given by

\[
W = \sum_{j=1}^{N} (\omega_j - \omega_{j-1}) p_e(t_{j-1}).
\]  

The performed work \( W \) of the whole process is the average of the microscopic work \( W^{[l]} \). Figure 3(c) shows the fully quantum simulation realized on ibmqx2. With the five qubits, at most two-step isothermal process can be realized with the fully quantum simulation, since the qubit resetting process is unavailable on ibmqx2. In fully quantum simulation, the fixed circuit is implemented repetitively, and the excited state population \( p_e(t_j) \) is obtained by measuring the state of the system qubit. The performed work for the simulated system is given by

\[
W = \sum_{j=1}^{N} (\omega_j - \omega_{j-1}) p_e(t_{j-1}).
\]  

Since ibmqx2 does not allow the user to reset the state of the qubit, each elementary process needs new additional ancillary qubit(s). If it is available to reset the ancillary qubit, two (three) qubits are enough to complete the simulation with the hybrid simulation (fully quantum simulation) by resetting the ancillary qubit(s) at the end of each isochoric process. This control scheme is realized in Ref. [25] to simulate repetitive quantum channels on a single qubit.

### Table I. The discrete isothermal process to be simulated and the two simulation methods, the hybrid simulation and the fully quantum simulation.

| Parameters         | Time Duration: \( \delta \tau = t_{j+1} - t_j \) | Temperature: \( T \) |
|--------------------|-----------------------------------------------|-----------------------|
| **Discrete isothermal process** | \( U[R(t)], \ t \in [\tau_i, \tau_{i+1}] \) | Generalized amplitude damping \( \phi_{GAD}^{(l)} \) with the classical random number generation |
| **Isochoric process** | System relaxation in Eq. 2 | Generalized amplitude damping \( \phi_{GAD}^{(l)} \) with an additional qubit at the state \( \cos(\alpha_j/2) | 0 \rangle + i \sin(\alpha_j/2) | 1 \rangle \) |
| **Adiabatic process** | \( \cos^2 \) | \( \cos \theta_j = \exp(-\frac{2\delta \tau \omega_j^2}{1 - \frac{1}{e^{2\alpha_j}}}) \) |
| **Simulation**       | Hybrid simulation with CRNG | Fully quantum simulation |

With the simulation of the elementary process, we can put together the elementary process to simulate the effect of the environment. The advantage of quantum channel simulation over the real coupling to the environment is its flexibility to control parameters, e.g., the temperature, the coupling strength, et. al.

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In the hybrid simulation, the sub-channel \( \mathcal{O}^{[l]} \) of each elementary process can be selected as either the amplitude damping \( \mathcal{E}^{(l)} \) or the pumping one \( \mathcal{G}^{(l)} \). For a \( N \)-step isothermal process, there are \( 2^N \) selections of the sub-channels \( \{ \mathcal{O}^{[l]}_1, \mathcal{O}^{[l]}_2, \ldots, \mathcal{O}^{[l]}_j, \ldots, \mathcal{O}^{[l]}_N \} \) for the whole process. The circuit of every selection with \( N = 2, 3 \) and 4 is implemented on ibmqx2. For each selection, the excited state population \( p_e^{[l]}(t_j) \) at each step is obtained by repeatedly implementing the circuit and measuring the state of the system qubit. The work in each selection, namely the microscopic work is given by

\[
W^{[l]} = \sum_{j=1}^{N} (\omega_j - \omega_{j-1}) p_e^{[l]}(t_{j-1}).
\]

The performed work \( W \) of the whole process is the average of the microscopic work \( W^{[l]} \).

IV. TESTING \( C/\tau \) SCALING OF EXTRA WORK

One possible application of the thermodynamic simulation is to test the \( C/\tau \) scaling of the extra work. In equilibrium thermodynamics, the work done for an ideal isothermal process is equal to the change of the free energy \( \Delta F \). The ideal isothermal process requires infinite control time to ensure the equilibrium at every
moment and avoid irreversibility. For a real isothermal process, the irreversibility arises and the extra work is needed to complete the process in finite time. For a fixed control scheme, it is proved that the extra work decreases inverse proportional to the control time at the long time limit \[32\]. Such \(C/\tau\) scaling has been verified for the compression of dry air in experiment \[33\].

The thermodynamic simulation with the quantum circuit provides an experimental proposal to study quantum thermodynamics. We demonstrate the scaling behavior of the extra work in finite-time isothermal process can be observed with the current experimental proposal. The parameters of the simulated system can be arbitrarily chosen. Here, the parameters of the simulated two-level systems are chosen as \(\gamma_0 = 1\) and \(\beta = 1\) for convenience. The energy level spacing is modulated from \(\omega_0 = 1\) to \(\omega_N = 2\) in \(N\) steps of elementary processes.

In Fig. 4, the \(C/N\) scaling of the extra work is shown with the simulation results using the IBM quantum computer with different time consuming \(\delta\tau = 0.5\) (blue dashed curve) and 10 (red solid curve). For large step number \(N\), it is observed that the extra work is inverse proportional to the step number as \(\overline{W} - \Delta F \propto C/N\). The free energy difference of the final and the initial state, namely the performed work in the ideal isothermal process is

\[
\Delta F = \omega_N - \omega_0 - k_B T \ln \frac{1 + e^{\beta \omega_N}}{1 + e^{\beta \omega_0}},
\]
The average work is obtained by summing the work in each selection with the corresponding probability $P(K_j) = \prod_{j} P_{K_j}^{(j)} (K_j = \uparrow \text{ or } \downarrow)$. If the random selections of the sub-channels are possible, $P(K_j)$ should be determined by the CRNG. Yet, here it is not implemented in the experiment but calculated with $P_{K_j}^{(j)}$, since the random selection of the two sub-channels is not feasible on ibmqx2.

Figure 5 (c) and (d) shows the excited state population of the system qubit for the fully quantum simulation of the two-step isothermal process on ibmqx2. The time consuming of each isochoric process is set as $\delta \tau = 0.5$ in (c) and $\delta \tau = 10$ in (d). By measuring the state of the system qubit at each step, the excited state population $p_e(t_j)$ with $t_j = 0$, $\delta \tau$ and $2\delta \tau$ is obtained by implementing 40960 shots in of the corresponding circuits. Compared to that of the numerical result (gray bar), the excited state population $p_e(t)$ of the ibmqx2 result (blue bar) is larger due to the noise in the quantum computer. At the end $t = 2\delta \tau$ of the process, the most quantum gates are used, and the absolute error reaches about 0.05.

The performed work of the ibmqx2 results for both the hybrid simulation and the fully quantum simulation are listed in Table II. In Fig. 4, the extra work in the ibmqx2 results is larger than that of the numerical result since the error of the excited state population is accumulated with the longer circuit. The error might mainly comes from the two-qubit gates, since the error rate of the two-qubit gates (from 1.2e-2 to 3.1e-2) is much greater than that of the single gate (from 3.0e-4 to 1.3e-3) [19]. The computing accuracy can be improved by using either quantum error correction or quantum mitigation [34]. Limited to the precision of operation on ibmqx2, the results deviate from the theoretical expectation values.

In the simulation, the Hamiltonian is commutative at different steps $[H(t_j), H(t_l)] = 0$, and the adiabatic process is considered as a quench with zero time consuming $\delta \tau_{\text{adi}} = 0$. The current proposal can be generalized to the discrete isothermal process with finite-time adiabatic processes. The finite-time effect of the adiabatic process will also lead to extra cost of the performed work for the case with the non-commutative Hamiltonian at different time [35]. For a generic adiabatic process, the unitary evolution of the two-level system should be simulated with the single-qubit gates on the system qubit.
V. CONCLUSION

We show a proposal to simulate the finite-time isothermal process of two-level system using the superconducting quantum computer. Two methods, the hybrid simulation and the fully quantum simulation, are given to realize the generalized amplitude damping channel. Assisted by the classical random number generator or the quantum superposition, the hybrid simulation or the fully quantum simulation can simulate a $N$-step isothermal process with $N + 1$ or $2N + 1$ qubits, respectively.

We have used the quantum computer of IBM (ibmqx2) to demonstrate the simulation of the discrete isothermal process. The discrete isothermal process has been realized for four steps with the hybrid simulation and two steps with the fully quantum simulation. If more steps of elementary processes can be realized in the experiment, the $C/\tau$ scaling of the extra work can be observed using the thermodynamic simulation on the universal quantum computer.

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