Passivity-Based Stability Analysis of Electricity Market Trading with Dynamic Pricing

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Abstract: This paper deals with passivity-based stability analysis of electricity market trading with dynamic pricing. In the deregulated electricity market, power consumers and generators will participate in market trading as market players. For such a new kind of the market trading system, the dynamic pricing procedure has to be taken into account of the intermittent participation of power consumers and generators. Then, this paper discusses the stability of the electricity market trading system including power flow using passivity analysis. This paper also shows that the optimal power demand, supply and electricity prices are asymptotically stable and these values are derived in a distributed manner through market trading.

Key Words: smart grid, demand response, dynamic pricing, passivity.

1. Introduction

Demand response has been attracted more attentions from researchers to solve energy problems discussed all over the world, in which both power consumers and generators participate in market trading as maker players. Among many kinds of useful methods regarding demand response proposed until now, dynamic electricity pricing is expected to make a significant contribution to the efficient use of energy since this method derives the optimal electricity prices depending upon the time or season. Moreover, this pricing method also results in increased economic benefits for consumers compared to current fixed pricing. Thus, dynamic pricing is being studied from a variety of perspectives as a key element in future smart grids.

In order to derive appropriate electricity prices in the electricity market, we have to deal with the economic problems of power generation and consumer utility, but also the physical constraints on account of the transmission lines and generation capacity placed on the power networks. In particular, we have to keep power balance between power supply and demand including power flow at any time to maintain stable operation of power systems. Therefore, even in the deregulated electricity market with selfish market players, appropriate electricity prices should be decided to satisfy the above physical conditions regarding power network.

For this problem, there have been several studies regarding dynamic electricity pricing and some distributed power supply-demand management methods using these pricing methods [1]–[5]. Among them, a distributed market-based algorithm has been proposed in [4], in which power supply and demand of each electricity market player is decided in a distributed manner through market trading considering power flow among areas. Also, the paper [5] proposed a real-time power adjustment method using incentive prices for consumers in negawatt trading. In addition, in [6],[7], power supply and demand management methods are discussed to maximize the social welfare of the entire power network with some dynamic models which represent the behavior of consumers and generators. However, in the future electricity markets, numerous and varied power consumers and generators across multiple areas will be participating in market trading as market players. Therefore, dynamic electricity pricing is required to take into account intermittent participation in market trading by market players and power flow among areas.

This paper discusses a passivity-based stability analysis of electricity market trading based on dynamic electricity pricing including power flow. Here, passivity represents the energy balance relationship between the input and output of the system, and is used in many kinds of control problems, such as the stability analysis of teleoperation robots or the formation control for multi-agent systems [8]–[11]. In detail, [8] shows the stability of the system using the passivity framework, and [9] proves the global asymptotic stability of the equilibrium point using the Lyapunov stability theorem.

Also, the paper [12] shows the relationship between the gradient algorithm for convex optimization problem and power systems, and [13] discussed the stability of the electricity market trading system represented by the convex optimization problem based on passivity. Besides these literatures, [14]–[17] deal with the stability of gradient dynamics from a viewpoint of passivity. In particular, [17] proposed an integrated dynamic market mechanism, which combines real-time market and frequency regulation allowing market players, including renewable generators and flexible consumers. However, though [12] assumed that consumers have their upper bounds for their demand, the stability analysis discussed in the above literatures did not explicitly deal with the upper and lower bounds regarding those systems. Since this paper discusses the market trading system with intermittent participation of market player, we have to consider the situation in which various kinds of market
player participate in the market trading. Then, in this paper, we show that the decision making system of each market player is also strictly passive in the electricity market trading, even if each consumer and generator has its upper and lower bounds in their own systems. In addition, this paper also shows that, by using DC approximation, the decision making system regarding power flow in AC power grid is also strictly passive including its upper and lower bounds in each transmission line. We then show that the electricity market trading system using dynamic pricing with the above market players’ decision making systems becomes a passive system and analyze its stability including power flow among areas. Also, this paper shows further theoretical analysis of our existing result given in [18], and proves that each value determined in this electricity market trading system converges to their optimal ones that maximize the social welfare of the entire power network using the Lyapunov stability theorem.

This paper is organized as follows. In Section 2, we first describe the economic behavior of market trading by market players and of a power network model. Next, in Section 3, the social welfare maximization problem of the entire power network is discussed. In Section 4, we derive the decision making systems of each market player and shows that these systems are passive systems. Furthermore, we demonstrate that the optimal power demand and supply, the optimal power flow, and its optimal electricity prices which maximize the social welfare are asymptotically stable in the electricity market trading system designed with the above decision making systems, and thus, each decision variable converges to its optimal one through market trading. Finally, in Section 5, we verify the effectiveness of the proposed method through numerical simulation.

2. Problem Formulation

Figure 1 shows a model of a power network and the electricity market trading discussed in this study. Suppose that multiple power consumers and generators are present in the areas connected by transmission lines, and that these market players and the ISO serving as market managers are engaged in the trading of power supply and demand via electricity prices. Additionally, consumers and generators in each area are not required to constantly participate in electricity market trading, and can participate intermittently at different times. Note that, in the rest of this paper, \( \mathcal{A} \) denotes a set of areas in this power network, \( |\mathcal{A}| \) represents the number of these areas, and similarly \( |X| \) denotes the number of elements of an arbitrary set \( X \).

2.1 Consumer

This subsection describes an economic behavior model for determining the consumer power demand. Here, let \( \mathcal{L}_l \) be a set of consumers in the area \( l \) participating in market trading, and \( P_{li}^l, i \in \mathcal{L}_l \) be the demand of each consumer in the area \( l \). In addition, let \( v_i'(P_{li}^l), i \in \mathcal{L}_l \) be the utility functions expressing the monetary satisfaction of consumers with the following assumption:

**Assumption 1** The utility function \( v_i'(P_{li}^l) \) is in \( \mathbb{C}^2[0, \infty) \) and is monotonically increasing and strictly concave.

The consumer welfare function \( W_{li}^l(P_{li}^l, \lambda^l) \) is defined as follows using the above utility function and price \( \lambda^l \):

\[
W_{li}^l(P_{li}^l, \lambda^l) := v_i'(P_{li}^l) - \lambda^l P_{li}^l \quad \forall i \in \mathcal{L}_l \quad \forall \lambda^l \in \mathcal{A}.
\] (1)

As a result, the economic behavior model for determining consumer power demand can be expressed as follows:

\[
\begin{align*}
\max_{P_{li}^l} & \quad W_{li}^l(P_{li}^l, \lambda^l) \\
\text{s.t.} & \quad P_{li}^{l\min} \leq P_{li}^l \leq P_{li}^{l\max},
\end{align*}
\] (2)

where \( P_{li}^{l\min} \) and \( P_{li}^{l\max} \) are the upper and lower bounds, respectively on the consumer demand.

2.2 Generators

Next, this subsection discusses an economic behavior model of the generators. Here, let \( \mathcal{G}_l \) be a set of generators in an area \( l \) participating in market trading, and \( P_{Gi}^l, i \in \mathcal{G}_l \) be the power supply of each generator in the area \( l \). In addition, \( c_i^l(P_{Gi}^l) \), \( i \in \mathcal{G}_l \) is the cost functions of the power generating facilities in possession of the generators with the following assumption:

**Assumption 2** The cost function \( c_i^l(P_{Gi}^l) \) is in \( \mathbb{C}^2[0, \infty) \) and is monotonically increasing and strictly convex.

Similar to the case regarding consumers, using the above cost function and price \( \lambda^l \), the generator welfare function \( W_{Gi}^l(P_{Gi}^l, \lambda^l) \) is defined as follows:

\[
W_{Gi}^l(P_{Gi}^l, \lambda^l) := \lambda^l P_{Gi}^l - c_i^l(P_{Gi}^l) \quad \forall i \in \mathcal{G}_l \quad \forall \lambda^l \in \mathcal{A}.
\] (4)

Thus, the economic behavior model for determining the power supply of a generator is given by

\[
\begin{align*}
\max_{P_{Gi}^l} & \quad W_{Gi}^l(P_{Gi}^l, \lambda^l) \\
\text{s.t.} & \quad P_{Gi}^{l\min} \leq P_{Gi}^l \leq P_{Gi}^{l\max},
\end{align*}
\] (5)

where \( P_{Gi}^{l\min} \) and \( P_{Gi}^{l\max} \) are the upper and lower bounds, respectively on the power generation by the generator.

2.3 AC Power Grid Model

Finally, this subsection shows a power grid model used in this paper. In this study, the following assumptions are introduced regarding the power networks:

**Assumption 3** The power grid is assumed to satisfy the following properties: i) The resistance loss in the transmission grid is negligible. ii) The voltage of each node is approximately equal to 1 p.u.. iii) The voltage phase difference between each node is sufficiently small.

With these three assumptions, the active power from the area \( l \) to the area \( k \), which is denoted by \( P_{lk} \), is given by

\[
P_{lk} = B_{lk}(\theta_l - \theta_k),
\] (7)
where \( \theta_1 \) and \( \theta_2 \) are the voltage phase angles of the nodes in the areas \( l \) and \( k \), respectively and \( B_{lk} \) is the susceptance of transmission lines connecting the two sets of nodes.

Here, let \( \mathcal{A}_l \) be a set of the areas adjacent to the area \( l \), which means that the areas in \( \mathcal{A}_l \) directly connect to the area \( l \) via transmission lines. Then, the active power flow equation in the area \( l \) using the power demand of the consumers and the power supply of the generators in the area \( l \) can be expressed as follows:

\[
P^l_G - P^l_L = \sum_{k \in \mathcal{A}_l} B_{lk}(\theta_l - \theta_k),
\]

where \( P^l_G := \sum_{i \in \mathcal{G}_l} P^l_{Gi} \) and \( P^l_L := \sum_{i \in \mathcal{L}_l} P^l_{Li} \).

Consequently, when all of the locational active power flow equation given in (8) are integrated into one equation, the active power flow equation throughout an entire power network becomes

\[
P^G + B\theta = P_L,
\]

where \( P^G \in \mathbb{R}^{|\mathcal{G}|} \), \( P_L \in \mathbb{R}^{|\mathcal{L}|} \) and \( \theta \in \mathbb{R}^{|\mathcal{A}|} \) are respectively the following:

\[
P^G = [P^1_G, \cdots, P^{|\mathcal{G}|}_G]^T \quad P_L = [P^1_L, \cdots, P^{|\mathcal{L}|}_L]^T \quad \theta = [\theta_1, \cdots, \theta_{|\mathcal{A}|}]^T.
\]

In addition, \( B \in \mathbb{R}^{|\mathcal{A}| \times |\mathcal{G}|} \) is a matrix which summarizes the susceptance \( B_{lk} \) of each transmission line.

### 3. Social Welfare Maximization Problem

This section considers the problems of maximizing the social welfare of the entire power network. Here, let us define a welfare function for an entire power network, using the vectors combining the power demand and supply of the market players, \( P_L \) and \( P_G \), as follows:

\[
\mathcal{W}(P_L, P_G, \theta) := \sum_{l \in \mathcal{A}} \left( \sum_{i \in \mathcal{L}_l} \psi_l(P^i_L) - \sum_{i \in \mathcal{G}_l} \zeta_l(P^i_G) - f_l(\theta_l) \right),
\]

where, \( f_l(\theta_l) \) is the cost function for the change of the voltage phase angle. In order to represent that the physical and monetary cost increases as the change of these angles increase, we introduce the following assumption to this cost function:

**Assumption 4** The cost function \( f_l(\theta_l) \) is in \( C^2[-\pi, \pi] \) and is strictly convex.

Then, the social welfare maximization problem of this power network is given by

\[
\max_{P_L, P_G, \theta} \mathcal{W}(P_L, P_G, \theta) \quad \text{s.t.} \quad P^l_G + B\theta = P^l_L, \\
\quad P^l_{Gi} \geq 0 \forall i \in \mathcal{G}_l \forall l \in \mathcal{A}, \\
\quad P^l_{Li} \geq 0 \forall i \in \mathcal{L}_l \forall l \in \mathcal{A}, \\
\quad P^l_{Li} \geq P^l_{Gi} \forall i \in \mathcal{L}_l \forall l \in \mathcal{A}, \\
\quad P^l_{Gi} \leq P^l_{Li} \forall i \in \mathcal{G}_l \forall l \in \mathcal{A}, \\
\quad P_{lk}^l \geq 0 \forall (l, k) \in \mathcal{E}.
\]

\[\text{(11)}\]

where \( \mathcal{E} \) represents the set of groups of areas connected by transmission lines, and \( P^l_{Gi}^{\min} \) and \( P^l_{Gi}^{\max} \) are the upper and lower bounds, respectively on the power flow in individual transmission lines. Here, let \( \lambda_0 = [\lambda_0^1 \cdots \lambda_0^{|\mathcal{A}|}]^T \in \mathbb{R}^{|\mathcal{A}|} \) be a Lagrange multiplier for the equality constraint (12), then a dual problem partially related to the above optimization problem becomes

\[
\min_{\lambda_0} \max_{P_L, P_G, \theta} \mathcal{W}(P_L, P_G, \theta) + \lambda_0^T \left( P_G + B\theta - P_L \right)
\]

\[\text{s.t.} \quad (13) - (15). \]

\[\text{(16)}\]

In addition, by applying a dual decomposition to the above dual problem, the optimization problems (11)–(15) can be decomposed into optimization problems for each consumer, generator, and the ISO, which implies that the Lagrange multiplier \( \lambda^l_0 \) equals the electricity price \( \hat{\lambda}^l \). Note that, in the rest of this paper, the notations for these variables are unified, and both are represented using \( \lambda^l \), \( l \in \mathcal{A} \). Additionally, with the dual problem defined in (16) and each of the optimization problems discussed in the following section, the optimal solution for each variable is represented using a respective superscript (-\( l \)).

### 4. Passivity-Based Stability Analysis of Dynamic Electricity Pricing

This section discusses a passivity-based analysis of the stability of an electricity market trading system involving participation of the power consumers, generators, and the ISO pursuing maximization of benefits for themselves. As described in Section II, the consumers and generators in each area participating in the electricity market trading decide the demand \( P^{l*}_L \), \( i \in \mathcal{L}_l \), \( l \in \mathcal{A} \) or power supply \( P^{l*}_G \), \( i \in \mathcal{G}_l \), \( l \in \mathcal{A} \), and then the ISO uses this information to update the price \( \lambda \). In addition, we suppose that the ISO decides the power flow among areas, and thus, the ISO uses the price information \( \lambda \) to decide the voltage phase angle \( \theta \) for each area.

Furthermore, in the following discussion, we denote the positive projection \( (f(x))^+ \) for any function \( f(x) \), \( x(\geq 0) \) to be as follows [8]:

\[
(f(x))^+ := \begin{cases} 
(f(x)) & \text{if } x > 0, \text{ or } x = 0 \text{ and } f(x) \geq 0, \\
0 & \text{if } x = 0 \text{ and } f(x) < 0.
\end{cases}
\]

\[\text{(17)}\]

### 4.1 Decision Making in Market Trading by Market Players

This subsection shows the dynamic models for deciding the power demand or power supply by consumers and generators, respectively in each area, deciding the inter-area power flow, and updating the electricity prices by the ISO.

#### 4.1.1 Consumers

First, consider the case of the consumers. By applying dual decomposition to (16), and with the Lagrange multipliers for the upper and lower bounds on each amount of demand, which are denoted by \( \mu^l_0 \) and \( \mu^l_1 \), we obtain the following equation:

\[
\min_{x, \mu^l_0 \geq 0, \mu^l_1 \geq 0} \mathcal{W}_L^l(\mu^l_0, \mu^l_1),
\]

\[\text{where } \mathcal{W}_L^l(\cdot) := \psi_l(P^l_L - \lambda^l_0 P^l_L) - \lambda^l_0 (P^l_{Gi} - P^l_{Li}) - \mu^l_1(P^l_{Gi} - P^l_{Li}), \quad \mu^l_0 \geq 0
\]

\[\text{(19)}\]

Now, suppose that the initial value of \( P^l_L \) satisfies the condition (13) and those of \( \mu^l_0 \) and \( \mu^l_1 \) are 0. Then, by using a term in which the right side of (19) is partially differentiated for each variable, the demand decision system for each consumer is given by

\[
\frac{d}{dt} \begin{bmatrix} P^l_L \\ \mu^l_0 \\ \mu^l_1 \end{bmatrix} = f^l_0(P^l_L, \mu^l_0, \mu^l_1, \lambda^l_0), \quad \lambda^l_0 = P^l_L
\]

\[\text{(20)}\]
where
\[
  f^d_i(L) = \begin{bmatrix}
    K^d_i (\nu^d_i(P^L_i) - \lambda^i + \mu^i - \mu^i_{\text{max}}) \\
    K^d_i (P^\text{min}_i - P^L_i) \\
    K^d_i (P^L_i - P^\text{max}_i)
  \end{bmatrix},
\]
and, \(K^d_i\), \(K^d_i\), and \(K^d_i\) are all positive gains.

Here, let us define the difference between an arbitrary variable \(x\) from its optimal solution \(x^*\) as \(\tilde{x} := x - x^*\). Then, the following lemma is obtained regarding the above consumer’s demand decision system.

**Lemma 1** Suppose that Assumptions 1-4 hold. Then, the consumer demand decision system (20) is a strictly passive system from input \(-\lambda^i\) to output \(\tilde{P}^L_i\).

**Proof.** Let us consider the following candidate of the positive definite function:
\[
  V^d_i := \frac{1}{2} \begin{bmatrix}
    (\nu^d_i)^2 & (\tilde{\mu}^i)^2 \\
    (\tilde{\mu}^i)^2 & (\tilde{\mu}^i)^2
  \end{bmatrix}.
\]

Then, the time derivative of the above positive function along a trajectory is given by
\[
  \frac{d}{dt} V^d_i = \frac{1}{K^d_i} \tilde{P}^L_i \frac{d}{dt} P^L_i + \frac{1}{K^d_i} \tilde{\mu}^i \frac{d}{dt} \tilde{\mu}^i + \frac{1}{K^d_i} \tilde{\mu}^i \frac{d}{dt} P^L_i,
\]
\[
  = P^L_i (\nu^d_i(P^L_i) - \lambda^i + \mu^i) + \tilde{\mu}^i (P^\text{min}_i - P^L_i) + \tilde{\mu}^i (P^L_i - P^\text{max}_i),
\]
\[
  \leq P^L_i (\nu^d_i(P^L_i) - \lambda^i + \mu^i) + \tilde{\mu}^i (P^\text{min}_i - P^L_i) + \tilde{\mu}^i (P^L_i - P^\text{max}_i).
\]

In addition, from the saddle point theorem, the following inequality holds regarding the dual problem (18) and (19):
\[
  \tilde{W}^d_i(P^L_i, \tilde{x}^i, \mu^i_{\text{max}}, \mu^i_{\text{min}}) \leq \tilde{W}^d_i(P^L_i, \tilde{x}^i, \mu^i_{\text{max}}, \mu^i_{\text{min}}).
\]

Then, by partially differentiating (19) with \(\tilde{x}^i, \mu^i_{\text{min}}\) and \(\mu^i_{\text{max}}\) for \(P^L_i\), and substituting its optimal solution \(P^\text{min}_i\) to this differentiated equation, the following equation holds:
\[
  v^d_i(P^L_i) - \lambda^i + \mu^i_{\text{min}} - \mu^i_{\text{max}} = 0.
\]

Additionally, since the optimal solution \(P^\text{min}_i\) satisfies the condition (13), the inequality \(P^\text{min}_i \leq P^L_i \leq P^\text{max}_i\) holds. Then, (22) becomes as follows:
\[
  \frac{d}{dt} V^d_i \leq P^L_i (v^d_i(P^L_i) - v^d_i(P^L_i)) \leq P^L_i (\nu^d_i(P^L_i) - \lambda^i + \mu^i) + \tilde{\mu}^i (P^\text{min}_i - P^L_i) + \tilde{\mu}^i (P^L_i - P^\text{max}_i).
\]

Above, the utility function \(v^d_i(P^L_i)\) is a strictly concave function, so at \(P^L_i \neq 0\), \(P^L_i (v^d_i(P^L_i) - v^d_i(P^L_i))\) becomes negative. Consequently, the consumer’s demand decision system (20) is a strictly passive system from input \(-\lambda^i\) to output \(\tilde{P}^L_i\).

**4.1.2 Generators**

Note that this and the other following results basically use the idea given in [8], which did not include the constraint condition of the upper and lower bounds regarding each system. Then, in order to consider the intermittent participation of market players in the electricity market trading, we extend this idea and show that the passivity of the decision making system regarding each market player is also hold even if each of them has their own upper and lower bounds. Here, as described before, the demand decision system in (20) is obtained by differentiating the consumer’s welfare function with its constraint conditions in (19). Therefore, this system represents the economic behavior of consumers to seek their optimal demand which maximizes their profit according to their own utility functions and electricity prices informed from the ISO. As a result, it can be reasonable to suppose that each consumer determines or updates their own demand according to this strictly-passive demand decision system (20) in electricity market trading.

Next, let us deal with the generators. Similar to the case of the consumers, the following equation is obtained regarding generators by applying dual decomposition to (16):
\[
  \begin{align*}
    \min_{x^i, \mu^i, \tilde{\mu}^i_{\text{min}}, \tilde{\mu}^i_{\text{max}}} & \quad \tilde{W}^g_i(\mu^i, \lambda^i, \mu^i, \mu^i_{\text{max}}) \\
    \text{subject to} & \quad \mu^i, \tilde{\mu}^i_{\text{min}} \geq 0, \quad \tilde{\mu}^i_{\text{max}} \\
    & \quad \lambda^i(P^g_i - \mu^i_{\text{max}}) - \mu^i_{\text{min}}(P^g_i - P^g_i) - \mu^i_{\text{max}}(P^g_i - P^g_i),
  \end{align*}
\]

where \(\mu^i_{\text{min}}\) and \(\mu^i_{\text{max}}\) are the Lagrange multipliers on the upper and lower bounds, respectively on each amount of power supply. Here, suppose that the initial value of \(P^g_i\) satisfies the condition (14) and those of \(\mu^i_{\text{min}}\) and \(\mu^i_{\text{max}}\) are 0. Then, the supply decision system for each generator is given by
\[
  \frac{d}{dt} P^g_i = f^g_i(P^g_i, \mu^i_{\text{min}}, \mu^i_{\text{max}}, \lambda^i), \quad \lambda_i = -P^g_i,
\]
where
\[
  f^g_i(P^g_i, \mu^i_{\text{min}}, \mu^i_{\text{max}}, \lambda^i) = \begin{bmatrix}
    K^g_i (\lambda^i - c^g_i(P^g_i) + \mu^i_{\text{min}} - \mu^i_{\text{max}}) \\
    K^g_i (P^\text{min}_i - P^g_i) \\
    K^g_i (P^g_i - P^\text{max}_i)
  \end{bmatrix},
\]
and, \(K^g_i\), \(K^g_i\), and \(K^g_i\) are all positive gains. Then, the following lemma holds:

**Lemma 2** Suppose that Assumptions 1-4 hold. Then, the power supply decision system of the generator (28) is a strictly passive system from input \(-\lambda^i\) to output \(\tilde{P}^g_i\).

**Proof.** Let us consider the following candidate of the positive definite function:
\[
  V^g_i := \frac{1}{2} \begin{bmatrix}
    (\tilde{\mu}^i)^2 & (\tilde{\mu}^i)^2 \\
    (\tilde{\mu}^i)^2 & (\tilde{\mu}^i)^2
  \end{bmatrix}.
\]

Then, the time derivative of the above positive function along a trajectory is given by
\[
  \frac{d}{dt} V^g_i = \frac{1}{K^g_i} P^g_i \frac{d}{dt} P^g_i + \frac{1}{K^g_i} \tilde{\mu}^i \frac{d}{dt} \tilde{\mu}^i + \frac{1}{K^g_i} \tilde{\mu}^i \frac{d}{dt} P^g_i,
\]
\[
  = P^L_i (\lambda^i - c^g_i(P^g_i) + \mu^i_{\text{min}} - \mu^i_{\text{max}}) + \tilde{\mu}^i (P^\text{min}_i - P^g_i) + \tilde{\mu}^i (P^g_i - P^\text{max}_i).
\]
\[
\dot{V}_{th}(t) = \sum_{(l,k) \in E} \left( \sum_{k \in A} \tilde{\mu}_l B_{lk}(\tilde{\theta}_l - \tilde{\theta}_k) + \sum_{k \in A} \tilde{\mu}_k B_{lk}(\tilde{\theta}_l - \tilde{\theta}_k) \right).
\]

In the above equation,
for some \( l \in \mathcal{A} \). Therefore, the power flow decision system (36) is a strictly passive system from input \(-\lambda\) to output \(-\theta\).

Also, we discuss the problem of electricity prices decided by the ISO. Here, a system for update of electricity prices by the ISO is denoted as follows:

\[
\frac{d}{dt} \lambda = -K_A \left( P_G + \bar{\theta} - P_L \right), \quad y = \lambda, \tag{42}
\]

where \( K_A > 0 \) is a control gain for updating a price. In addition, we set the initial value of \( \lambda \) as 0. Then, the following lemma holds:

**Lemma 4** Suppose that Assumptions 1-4 hold. The price updating system (42) is a passive system from input \(-\left( \bar{P}_G + \bar{\theta} - \bar{P}_L \right)\) to output \( \lambda \).

**Proof.** We consider the following positive definite function:

\[
V_l = \frac{1}{2} \sum_{i=1}^{n} \lambda^2 \frac{P_l^i}{K_l}, \tag{43}
\]

The time derivative of the above function along a trajectory is given by

\[
\frac{d}{dt} V_l = \sum_{i=1}^{n} \lambda^2 \frac{d}{dt} \lambda^i = - \sum_{i=1}^{n} \lambda^2 \left( B_l^i P_l^i + B_l^i \theta_l + \sum_{k \in \mathcal{A}} B_l^i \theta_k - P_l^i \right) = -\lambda^2 (P_G + \bar{\theta} - P_L). \tag{44}
\]

Then, following equation holds:

\[
P_G^* + \bar{\theta}^* - P_L^* = 0. \tag{45}
\]

Then, (44) becomes as follows:

\[
\frac{d}{dt} V_l = -\lambda^2 (\bar{P}_G + \bar{\theta} - \bar{P}_L) \tag{46}
\]

Thus, the price updating system of an ISO (42) is a passive system from input \(-\left( \bar{P}_G + \bar{\theta} - \bar{P}_L \right)\) to output \( \lambda \). \( \square \)

### 4.2 Passivity-Based Stability Analysis

In the previous subsections, we have shown the decision making systems regarding each market player. Then, this subsection discusses the passivity-based stability analysis of the electricity market trading system using dynamic electricity pricing in which consumers, generators, and the ISO in different areas are participating. Here, as mentioned below Lemma 1, it is reasonable to design the electricity market trading system with the decision making systems of each market player described in the previous subsections. Then, we design the electricity market trading system based on dynamic pricing with these systems as shown in Figs. 2 and 3. Note that, in Fig. 3, \( D_l \in \mathbb{R}^{nA} \) is a \(|\mathcal{A}|\) column vector, wherein the \( i \)th element is 1 and the other elements are 0.

Then, we have the following theorem regarding this electricity market trading system based on dynamic electricity pricing including the above power flows:

**Theorem 1** Suppose that Assumptions 1-4 hold. The electricity market trading system based on dynamic electricity pricing with the demand decision system of a consumer (20), power supply decision system of a generator (28), and the ISO’s power flow decision system (36) and the price updating system (42) is presented in Figs. 2, 3; then, each solution of the electricity market trading system with the market participants’ decision making systems (20), (28), (36) and the price updating system (42) converges to the optimal solutions \( P_L^*, P_G^*, \theta^* \) for the optimization problems (11)-(15) and the optimal Lagrange multiplier \( \lambda^* \).

**Proof.** According to Lemma 1, the demand decision system of a consumer (20) is a strictly passive system from input \(-\lambda\) to output \( P_L^* \), and similarly according to Lemma 2, the power supply decision system of a generator (28) is also a strictly passive system from input \(-\lambda\) to output \( P_G^* \). Consequently, as shown in Fig. 3, systems that connect them in parallel are also strictly passive because of the property of passive systems [19], and additionally, in the system with \( D_l \) or \( D_l \), passivity is preserved. In addition, according to Lemma 3, the power flow decision systems managed by the ISO is a strictly passive system from input \(-\lambda\) to output \(-\theta\), and according to Lemma 4, the price updating system managed by the ISO is a passive system from input \(-\left( \bar{P}_G + \bar{\theta} - \bar{P}_L \right)\) to output \( \lambda \). Consequently, passivity is preserved in the electricity market trading system in Fig. 2.

Then, let us consider the following positive definite function:

\[
V = \sum_{i \in \mathcal{A}} \sum_{l \in \mathcal{L}} V_{l,l}^i + \sum_{i \in \mathcal{A}} \sum_{l \in \mathcal{L}} V_{G_l}^i + V_{p_l} + V_{D_l}. \tag{47}
\]

The time derivative of the above function along a trajectory is:

\[
\frac{d}{dt} V = \sum_{i \in \mathcal{A}} \sum_{l \in \mathcal{L}} \frac{d}{dt} V_{l,l}^i + \sum_{i \in \mathcal{A}} \sum_{l \in \mathcal{L}} \frac{d}{dt} V_{G_l}^i + \frac{d}{dt} V_{p_l} + \frac{d}{dt} V_{D_l} \leq \sum_{i \in \mathcal{A}} \left( P_l^i \left( V_l^i(P_l^i) - V_l^i(P_l^i) \right) - \lambda^* P_l^i \right)
\]

Where \( \lambda^* \) is the optimal solution of the optimization problem (11).
Based on the above, we can prove \( P_L^l \rightarrow 0, P_G^l \rightarrow 0, \bar{\theta}_i \rightarrow 0 \). Therefore, each solution of the electricity market trading system with the market participants’ decision making systems (20), (28), and (36) and the price updating system (42) converges to the optimal solutions \( P_L^*, P_G^*, \theta^* \) for the optimization problems (11)–(15) and the optimal Lagrange multiplier \( \lambda^* \). □

5. Simulation Verification

This section shows the results of numerical simulation to verify the effectiveness of the method proposed in this paper.

5.1 Simulation Conditions

In this simulation, we used the power grid model with two areas which are connected by transmission lines, and suppose that the respective number of consumers and generators in each area participating in market trading are \( |L| = 3, |G| = 1 \) \( \forall l \in A \). Additionally, in order to verify the validity of the proposed method even in cases in which the number of players involved in market trading had changed, we performed numerical simulation under the participation conditions shown in Fig. 4. This figure shows whether consumers 1 to 3 in each area are participating in electricity market trading by assigning the value 1 for indicating participation in market trading and 0 corresponding to non-participation.

On the other hand, we established the utility functions for each consumer, the cost functions for generators, and the cost functions for power flow using a logarithmic and a quadratic function as shown below:

\[
\begin{align*}
\ell_l^i(P_L^l) &= d_i^l \log(P_L^l + 1) \forall i \in L, \forall l \in A, \\
c_i^l(P_G^l) &= b_i^l P_G^l \forall i \in G, \forall l \in A, \\
f_l(\theta_l) &= \varepsilon \theta_l^2 \forall l \in A.
\end{align*}
\] (49)

Here, the value of each coefficient is decided by referencing literature [4], and the value of each gain is \( K_{L_i} = K_{G_i} = K_{\bar{G}_i} = 1.0 \times 10^3 \forall i \in A, K_{\alpha} = 1.0 \times 10^{-3}, K_{\beta_{L_i}} = K_{\beta_{G_i}} = K_{\beta_{\bar{G}_i}} = K_{\beta_{\bar{G}_i}} = 1.0 \forall i \in A, K_{\mu_{L_i}} = K_{\mu_{G_i}} = 1.0 \forall (l, k) \in E \). In addition, the upper and lower bounds of each variable are established as follows, provided that the units are all kW:

![Time schedule of consumers’ participation.](image)

Fig. 4 Time schedule of consumers’ participation.

![Power demand in Area 1](image)

(a) Power demand in Area 1

![Power demand in Area 2](image)

(b) Power demand in Area 2

![Power supply](image)

(c) Power supply

![Power flow](image)

(d) Power flow

![Power balance](image)

(e) Power balance

![Price](image)

(f) Price

Fig. 5 Results of electricity market trading with intermittent participation of consumers with power flow.
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