Axions and high-energy cosmic rays: 
Can the relic axion density be measured?

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Abstract. In a previous work we investigated the propagation of fast moving charged particles in a spatially constant but slowly time dependent pseudoscalar background, such as the one provided by cold relic axions. The background induces cosmic rays to radiate in the low-energy spectrum. While the energy loss caused by this mechanism on the primary cosmic rays is negligible, we investigate the hypothetical detection of the photons radiated and how they could provide an indirect way of verifying the cosmological relevance of axions. Assuming that the cosmic ray flux is of the form \( J(E) \sim E^{-\gamma} \) we find that the energy radiated follows a distribution \( k^{-2\gamma} \) for proton primaries, identical to the Galaxy synchrotron radiation that is the main background, and \( k^{-\frac{3\gamma}{2}} \) for electron primaries, which in spite of this sharper decay provide the dominant contribution in the low-energy spectrum. We discuss possible ways to detect this small diffuse contribution thereby leading to a potential direct detection of the cold axion background.

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1 Introduction

In this work we shall determine the flux of photons emitted by high-energy cosmic rays travelling through an extremely diluted pseudoscalar condensate oscillating in time. This background could be provided by cold relic axions resulting from vacuum misalignment in the early universe, which constitute at present a viable possibility to explain the dark matter density of the universe[1]. Detection of this radiation would constitute a strong indirect evidence of the existence of the axion background.

Provided that the reheating temperature after inflation is below the Peccei-Quinn transition scale[2], the axion collective field evolves in later times as

\[ a(t) = a_0 \cos m_a t, \quad k = 0 \quad (1.1) \]
\[ \rho \simeq a_0^2 m_a^2 \quad (1.2) \]
\[ \rho \simeq 10^{-30} \text{gcm}^{-3} \simeq 10^{-10} \text{eV}^4, \quad \rho_* \simeq 10^{-24} \text{gcm}^{-3} \simeq 10^{-4} \text{eV}^4. \quad (1.3) \]

The last figure refers to the presumed axion density in galactic halos, extending from 30 to 100 kpc[3].

The axion background thus provides an extremely diffuse concentration of a pseudoscalar condensate and one may be led to conclude that it is, except for its gravitational effects, totally irrelevant. However, the photon density associated to the microwave background radiation is also very low and yet it has an impact on ultra-high energy cosmic rays imposing the GZK cutoff[4]. Consequently it seems natural to investigate the effect of the axion background on the propagation of highly energetic charged particles.

In [5] it was seen that the time-varying axion background induces a small amount of Lorentz violation that makes possible the existence of ‘cosmic ray Bremsstrahlung’ processes, such as \( p \to p \gamma \).
or $e \to e\gamma$ (forbidden in a Lorentz invariant theory) provided that the initial particle of mass $m$ had an energy

$$E > E_{th} = \frac{2mm\gamma}{\eta}. \quad (1.4)$$

In the above expression $m\gamma$ is a medium-induced effective photon mass. Current bounds indicate that $m\gamma < 10^{-18}$ eV[6], but while we expect the value of $m\gamma$ not to be exactly zero it will likely be well below this experimental bound. In [5] we used as reference value $m\gamma \sim 10^{-18}$ eV. We shall return to this issue below.

The quantity $\eta$ is the parameter characterizing Lorentz violation. It appears in the following way. The interaction of photons with the axion background is described by the piece in the lagrangian

$$\Delta L = g_{\alpha\gamma\gamma}\frac{a}{2\pi f_a} F F = -g_{\alpha\gamma\gamma}\frac{a_0 m_a}{\pi f_a} \sin(m_a t) \epsilon^{ijk} A_i F_{jk} \quad (1.5)$$

Popular models such as DFSZ[7] and KSVZ[8] all give $g_{\alpha\gamma\gamma} \simeq 1$. Current observational bounds[1] indicate that the coupling and preferred mass range are

$$f_a > \mathcal{O}(10^7) \text{ GeV}, \quad 10^{-1}\text{eV} > m_a > 10^{-6} \text{ eV}, \quad (1.6)$$

although these bounds are based on a number of cosmological/astrophysical assumptions and are somewhat weak[9]. Direct experimental bounds on the axion couplings only indicate $f_a > 10$ TeV [10]. For pseudo-Goldstone bosons related to the strong CP problem[2] the approximate relation $f_a m_a \simeq f_\pi m_\pi$ should hold[11].

If the momentum of a particle propagating in this background is large, $p >> m_a$, it makes sense to treat the axion background adiabatically and consider $\eta$ approximately constant:

$$\Delta L = \frac{1}{4} \eta \epsilon^{ijk} A_i F_{jk}, \quad (1.7)$$

with $\eta = (\eta,0,0,0)$ where the “constant” $\eta$ will change sign with a period $1/m_a$. Numerically,

$$|\eta| \simeq a \frac{a_0 m_a}{f_a} = \alpha \sqrt{p} \simeq 10^{-20}\text{eV}, \quad (1.8)$$

or less. For the so called axion-like particles (see e.g. [12] for a recent proposal in connection with dark matter) $f_a$ is actually unrelated to the mass and thus not bound by astrophysical processes.

In the next section we quote the results on the kinematical limits concerning the process $p \to p\gamma$, possible in a Lorentz non-invariant background, which will be needed in the following discussion. In section 3 we compute the radiation probability and discuss the characteristics of the comics ray flux needed to determine the intensity of the radiation produced by the axion-induced Bremsstrahlung mechanism proposed. In section 4 we study the feasibility of the detection of the emitted radiation.
Our conclusions can be briefly summarized as follows. The dominant contribution to the radiation yield via this mechanism comes from electron (and positron) cosmic rays. If one assumes that the power spectrum of the cosmic rays is characterized by an exponent $\gamma$ then the produced radiation has a spectrum $k^{-\frac{\gamma + 1}{2}}$ for proton primaries, which becomes $k^{-\gamma}$ for electron primaries. The dependence on the key parameter $\eta \sim \sqrt{\frac{\rho}{f_a}}$ comes with the exponent $\eta^{1+\frac{\gamma}{2}}$ and $\eta^{2+\frac{\gamma}{2}}$ for protons and electrons, respectively. However for the regions where the radiation yield is largest electrons amply dominate. We have assumed that the flux of electron cosmic rays is uniform throughout the Galaxy and thus identical to the one observed in our neighbourhood, but relaxing this hypothesis could provide an enhancement of the effect by a relatively large factor. The effect for the lowest wavelengths where the atmosphere is transparent and for values of $\eta$ corresponding to the current experimental limit is of $\mathcal{O}(10^{-1})$ mJy. This is at the limit of sensitivity of antenna arrays that are already currently being deployed and thus a possibility worth exploring.

2 Summary of known results

We shall review here some of the results obtained in [5]. Consider the process

$$p(p) \rightarrow p(p-k)\gamma(k) \tag{2.1}$$

Let us first consider the case $m_\gamma = 0$. (see [5] for details). We denote $k = |k|$ and assume that $\eta > 0$ in what follows. The process is possible only for one polarization of the final photon, which gets reversed if $\eta < 0$ so there is no loss of generality in assuming a specific sign for $\eta$. For the threshold energy of the cosmic ray and the kinematical limits on the radiated photon we have

$$E_{th} = 0 \tag{2.2}$$

$$k_{min} = \eta, \text{ for } \cos \theta = -\eta/2p \tag{2.3}$$

$$k_{max} = \frac{E^2}{p + m_p^2/\eta}, \text{ for } \cos \theta = 1 \tag{2.4}$$

Note that $k_{max} \simeq E$ for $E \gg m_p^2/\eta$ and that $k_{max} \simeq \eta E^2/m_p^2$ for $E \ll m_p^2/\eta$.

Let us next consider the case $m_\gamma > 0$

$$E_{th} \simeq \frac{2m_\gamma m_p}{\eta} \tag{2.5}$$

$$k(\theta_{max}) \simeq \frac{2m_\gamma^2}{\eta} (1 - \frac{2m_\gamma^2}{E^2/\eta}) E_{th} \frac{2m_\gamma^2}{\eta}, \text{ for } \sin^2 \theta_{max} \rightarrow \frac{\eta^2}{4m_\gamma^2} \tag{2.6}$$

$\theta_{max}$ can be very small; photons are emitted in a narrow cone if $\eta < m_\gamma$ and more isotropically otherwise.
In the opposite extreme, for zero angle, there are two solutions

\[
k_+(0) \simeq \frac{E^2 \eta + m_p^2 - E \sqrt{E^2 \eta^2 - 4m_p^2 m_\gamma^2 + 2mpm_p^2 \eta}}{2p\eta + 2m_p^2} E \gg E_{th} \xrightarrow{\eta^2} \frac{E^2}{p + \frac{m_p^2}{\eta}} \tag{2.7}
\]

which is the same result obtained before, and

\[
k_-(0) \simeq \frac{E^2 \eta + m_p^2 - E \sqrt{E^2 \eta^2 - 4m_p^2 m_\gamma^2 + 2mpm_p^2 \eta}}{2p\eta + 2m_p^2} E \gg E_{th} \xrightarrow{\eta^2} \frac{m_\gamma^2}{\eta} \tag{2.8}
\]

\[k_-(0) < k(\theta_{\text{max}}) < k_+(0)\]

The rate of energy loss of the cosmic ray was also computed in [5]

\[
\frac{dE}{dx} = -\frac{1}{v} \int d\Gamma(Q)w(Q) \tag{2.9}
\]

\[
\frac{dE}{dx} = -\frac{\alpha}{2p^2} \int kdk[-\frac{1}{2}(m_\gamma^2 + \eta k) + p^2(1 - \cos^2 \theta)] \tag{2.10}
\]

There are two relevant limits

\[
E \ll \frac{m_p^2}{\eta} \rightarrow \frac{dE}{dx} = -\frac{\alpha \eta^2 E^2}{4m_p^2} \tag{2.11}
\]

\[
E \gg \frac{m_p^2}{\eta} \rightarrow \frac{dE}{dx} = -\frac{\alpha \eta}{3} E \tag{2.12}
\]

Notice that there are two key scales in this problem: \(E_{th} \simeq 2m_\gamma m_p/\eta\) and \(m_p^2/\eta\), the cross-over energy, where \(dE/dx\) changes behaviour; clearly \(m_p^2/\eta \gg E_{th}\). For energies \(E \gg m_p^2/\eta\)

\[
E(x) = E(0) \exp\left(-\frac{\alpha \eta}{3} x\right), \tag{2.13}
\]

giving a mean free path of \(O(1)\) pc. The ‘axion shield’ would indeed be very effective at such enormous energies. However due to the smallness of \(\eta\) the crossover scale \(m_p^2/\eta\) is many orders of magnitude larger than the highest energy rays measured and the above restriction on the mean free-path is not relevant. Even cosmic rays just below the GZK cut-off of \(10^{20}\) eV are well below the cross-over scale \(m_p^2/\eta\). In this regime, which is the relevant one, the expression for \(E(x)\) is

\[
E(x) = \frac{E(0)}{1 + \frac{\alpha \eta^2}{4m_p^2} E(0)x}, \tag{2.14}
\]

giving a much weaker suppression. From this expression and the fact that we detect (likely) extragalactic rays of large energy we can set at present the largely irrelevant bound

\[
\eta < 10^{-14}\ \text{eV} \tag{2.15}
\]

(recall that we expect \(\eta \sim 10^{-20}\) eV or less). It is peculiar to see that for extremely large distances \(E(x) \sim \frac{1}{x}\) independently of their primary energy but this regime is never reached even at the largest cosmic scales that are observable, so it remains a curiosity. The net effect of the oscillating pseudoscalar background on cosmic ray propagation is truly negligible.
3 Radiation yield

Let us turn to the radioemission due to the axion-induced Bremsstrahlung.

For primary protons, using $m_\gamma = 10^{-18}$ eV and $\eta = 10^{-20}$ eV as indicative values and the usual GZK cut-off, there would be electromagnetic activity in the region of the spectrum

$$10^{-16} \text{ eV}(0.024 \text{ Hz}, \lambda = 1.2 \times 10^7 \text{ km}) < k < 100 \text{ eV}(24 \text{ PHz}, \lambda = 12 \text{ nm}).$$

Before jumping prematurely to conclusions we have to estimate the energy yield which will in fact be quite small at high energies.

For primary electrons, which are much rarer in number but radiate more (see the expressions in the previous section), there would be activity in the range

$$10^{-16} \text{ eV} < k < 400 \text{ MeV},$$

assuming again $m_\gamma = 10^{-18}$ eV and $\eta = 10^{-20}$ eV and a cut-off similar to the one of protons. This last point is very questionable since the spectrum of electrons reaching the Earth seems to bend down around $\sim 10 \text{ TeV}$[13]; however the issue is still unclear. As we will see the intensities at such high frequencies are very low anyway so the uncertainties about the high-energy part of the electron cosmic ray spectrum are of little consequence. Note that $m_\gamma$ affects only the lower limit of the above ranges and that the kinematical limits on $k$ are proportional to $\eta$. We shall eventually set $m_\gamma = 0$.

In order to compute the radiation yield we shall need to estimate the number of cosmic rays and their differential emission rate into photons of wave vector $k$, $d\Gamma/dk$. This latter quantity was determined in [5]

$$\frac{d\Gamma}{dk} = \frac{\alpha}{8 k \omega} \left[ A(k) + B(k)E^{-1} + C(k)E^{-2} \right] \frac{E^2 \eta}{m^2} - k,$$

with

$$A(k) = 4(\eta k - m_\gamma^2), \quad B(k) = 4\omega(m_\gamma^2 - \eta k), \quad C(k) = -2m_\gamma^2 k^2 + 2\eta k^3 - m_\gamma^4 - \eta^2 k^2 + 2m_\gamma^2 \eta k,$$

where

$$\omega = \sqrt{m_\gamma^2 - \eta k + k^2},$$

and $m$ is the mass of the charged particle. Although given for an arbitrary value of $m_\gamma$ for completeness, it makes sense to set $m_\gamma = 0$ in the decay rate. The error is insignificant; $m_\gamma$ is only relevant in the kinematics.

Let us consider a surface element $dS_0$ in space and consider the number of photons radiated with wave vector $k$ by the cosmic rays crossing that surface element within a time interval $dt_0$ and
having an energy between $E$ and $E + dE$. The number of such cosmic rays (protons or electrons) per unit surface will be

$$d^3N = J(E) dE dS_0 dt_0.$$  \hspace{1cm} (3.6)

$J(E)$ is the usual cosmic ray flux; there is one for each type of cosmics. The $d^3N$ cosmic rays will eventually radiate at a time $t$, unrelated to $t_0$, and they will yield a number of photons with a given wave vector $k$ given by the usual differential decay formula

$$d^3N_\gamma = d^3N \frac{d\Gamma(E, k)}{dk} dk dt = J(E) \frac{d\Gamma(E, k)}{dk} dEdk dt_0 dS dt.$$ \hspace{1cm} (3.7)

$N_\gamma$ is dimensionless. $J(E)$ is expressed in units of eV$^{-1}$ m$^{-2}$ s$^{-1}$.

Now, assuming uniformity and isotropy of the cosmic rays we can safely assume that the flux is the same for any such surface element $dS$ (indeed we have already set $dS_0 = dS$ in the above expression), and for any time interval $dt_0$ and integrate over $t_0$ obtaining a factor $t(E)$ equivalent to the average lifetime of a cosmic ray of energy $E$. Therefore

$$\frac{d^3N_\gamma}{dk dS dt} = \int_{E_{th}}^{\infty} dE t(E) J(E) \frac{d\Gamma(E, k)}{dk} \quad E_{th} = \frac{2 m_p e m_\gamma}{\eta}. \hspace{1cm} (3.8)$$

Note that the units of $d^3N_\gamma/dk dS dt$ are the same as those of $J(E)$.

Observations indicate that cosmic rays exhibit an energy spectrum of the form

$$J(E) = N_i E^{-\gamma_i} \hspace{1cm} (3.9)$$

with $\gamma_i \simeq 3$. For protons we shall use the parametrization given in[14]. All energy units in what follows are given in electronvolts.

$$J_p(E) = \begin{cases} 
5.87 \cdot 10^{19} E^{-2.68} & 10^9 \leq E \leq 4 \cdot 10^{15} \\
6.57 \cdot 10^{28} E^{-3.26} & 4 \cdot 10^{15} \leq E \leq 4 \cdot 10^{18} \\
2.23 \cdot 10^{16} E^{-2.59} & 4 \cdot 10^{18} \leq E \leq 2.9 \cdot 10^{19} \\
4.22 \cdot 10^{49} E^{-4.3} & E \geq 2.9 \cdot 10^{19}
\end{cases} \hspace{1cm} (3.10)$$

For electrons (less well measured, but typically 1% of the proton flux)[13]

$$J_e(E) = \begin{cases} 
5.87 \cdot 10^{17} E^{-2.68} & E \leq 5 \cdot 10^{10} \\
4.16 \cdot 10^{21} E^{-3.04} & E \geq 5 \cdot 10^{10}
\end{cases} \hspace{1cm} (3.11)$$

Units are eV$^{-1}$ m$^{-2}$ s$^{-1}$ sr$^{-1}$ as stated. We shall consider in what follows that $E_{th} > 10^9$ eV as the flux of cosmics below that energy is likely to be influenced by local effects of the solar system. The previous parameterizations describe the flux of protons at all measured energies with good precision and roughly describes the one of leptons, which is more poorly known. The form of the
electron flux turns out quite relevant so our ignorance about the lepton flux is quite regrettable as it has a substantial impact in our estimation of the radiation yield.

Note that the above ones are values measured locally in the inner solar system. It is known that the intensity of cosmic rays increases with distance from the sun because the modulation due to the solar wind makes more difficult for them to reach us, particularly so for electrons. Therefore the above values have to be considered as lower bounds for the flux which may be up to $\sim 10$ times larger in the nearby interstellar medium. In addition, the hypothesis of homogeneity and isotropy hold for proton cosmic rays, but not necessarily for electron cosmic rays. Indeed because cosmic rays are deflected by magnetic fields they follow a nearly random trajectory within the Galaxy. Collisions of cosmic rays having large atomic number with the interstellar medium sometimes produce lighter unstable radioactive isotopes. By measuring their abundance we know that on average a hadronic cosmic ray spends about 10 million years in the galaxy before escaping into intergalactic space. This ensures the uniformity of the flux, at least for protons of galactic origin. On the contrary, electron cosmic rays travel for approximately 1 kpc on average before being slowed down and trapped. However, because $l \sim \sqrt{D(E)t}$ ($D(E)$ is the diffusion coefficient of the random walk) 1 kpc corresponds to a typical age of a electron cosmic ray $\sim 10^5$ yr$^{-1}$, a lot less than protons. In addition, the lifetime of an electron cosmic ray depends on the energy in the following way

$$t(E) \simeq 5 \times 10^5 \left( \frac{1 \text{ TeV}}{E} \right) \text{yr} = \frac{T_0}{E},$$

(3.12)

with $T_0 \simeq 2.4 \times 10^{40}$ if $E$ is measured in eV. To complicate matters further, it has been argued that the local interstellar flux of electrons is not even representative of the Galaxy one and may reflect the electron debris from a nearby supernova $\sim 10^4$ years ago$^{[16]}$.

To get an estimate we will replace in the integral $t(E) \rightarrow T$ and assume the value $T = 10^7$ yr for protons and use (3.12) for electrons. The measured photon energy flux $I(k)$ per unit wave vector, measured per unit surface per unit time and per sr will then be

$$I(k) = \frac{\omega(k)}{8\lambda k} \int_{E_{\text{min}}(k)>E_{\text{th}}}^{\infty} dE \frac{t(E)J(E) d\Omega}{dT}$$

$$= \frac{\alpha T}{8\lambda k} \int_{E_{\text{min}}(k)>th}^{\infty} dE \frac{t(E)N_i \left[ A(k)E^{-\gamma_i} + B(k)E^{-(\gamma_i+1)} + C(k)E^{-(\gamma_i+2)} \right]}{E_{\text{final}}^{\gamma_i-1} - E_{\text{initial}}^{\gamma_i-1}}$$

(3.13)

with $E_{\text{min}}(k) = \sqrt{m^2 c^2 \eta}$. In the previous expression the labels 'initial' and 'final' refer to the successive energy ranges where the different parameters $N_i$ and $\gamma_i$ are applicable. The above result applies to protons; for electrons the spectrum is is reduced by one additional power of the energy.
Numerically, it is straightforward to see that the whole contribution is dominated by the initial point $E_{\text{min}}(k)$. Furthermore only the term proportional to $A(k) = 4\eta k$ in the decay rate is numerically relevant. Then

$$I_p^\gamma(k) \simeq \frac{\alpha \eta T J_p(E_{\text{min}}(k)) E_{\text{min}}(k)}{\gamma_{\text{min}} - 1}. \quad (3.14)$$

and

$$I_e^\gamma(k) \simeq \frac{\alpha \eta T_0 J_e(E_{\text{min}}(k))}{2 \gamma_{\text{min}}}. \quad (3.15)$$

Energies are all expressed in eV. The value $\gamma_{\text{min}}$ is to be read from (3.10) and (3.11) depending on the range where $E_{\text{min}}(k)$ falls. The above approximate formula (3.14) and (3.15) reproduces the exact result within an accuracy that is sufficient for our purposes.

4 Results and discussion

We should now settle the discussion on the value of $m_\gamma$. The best observational limits on the effective photon mass come from measurements of the Jovean magnetic field of magneto-hydro dynamics of the solar wind. They are obviously measurements at very long wavelengths. Even more stringent (but not accepted as a direct limit by the Particle Data Group) is a $10^{-27}$ eV bound derived from the existence of the galactic magnetic field.

Theoretically we expect that the dominant contribution to the effective photon mass is induced by the electron density that is expected to be at most of the order of $n_e = 10^{-7}$ cm$^{-3}$. Photons would pick up a mass

$$m_\gamma^2 \simeq 4\pi \alpha \frac{n_e}{m_e}. \quad (4.1)$$

This expression gives $m_\gamma = 10^{-15}$ eV. However a first consideration is that the density of free electrons is of course not uniform, but significant only around some active regions with larger plasma densities. More importantly, because the density of free electrons is so low, it takes photons of very low momentum to ‘see’ a collective effect due to the density of electrons. Typically the distance for the collective effect of the electron plasma to induce an effective mass will have to be $>> n_e^{-\frac{1}{3}}$. Since we will typically be interested in photons with a shorter wavelength it seems safe to conclude that $m_\gamma$ has to be set to its fundamental value, namely zero.

If the above considerations hold the value assumed for $E_{\text{th}}$ comes not from kinematical considerations but from the practical need to ensure that all cosmic rays included in the determination of the radiation yield due to the axion-induced Bremsstrahlung have traveled a large distance and thus have had enough time to contribute to the electromagnetic yield. Cosmic rays from the solar system normally reach a maximum energy of 1 GeV, and very rarely 10 GeV[17]. We therefore take $E_{\text{th}} = 1$ GeV both for electrons and protons. In this way we can set, if the detection of the
effect is positive, a reliable lower bound on $\eta$. Since $E_{th} < E_{min}(k) = \sqrt{\frac{m_\chi^2}{\eta}}$ we are sure that photons with $k > 10^{-7}$ eV were radiated off cosmic rays not of solar origin. We take $k = 10^{-7}$ eV as the reference scale as this is approximately the minimum wave vector at which the atmosphere is transparent to electromagnetic radiation, even though the signal is higher for lower frequency photons. This corresponds to 30 MHz, a band in which an extensive antenna array (LWA) is already being commissioned[18]. In the same range of extremely low frequencies the Square Kilometer Array (SKA) project could cover a the range from 70 to 10,000 MHz with enormous sensitivity (see below) [19].

As a result of the previous considerations we expect the following measured intensities (flux densities) from the axion-induced Bremsstrahlung. First of all, the dominant contribution comes from electrons

$$I_e(k) \simeq 3 \times 10^2 \times \left( \frac{\eta}{10^{-20} \text{ eV}} \right)^{2.52} \left( \frac{k}{10^{-7} \text{ eV}} \right)^{-1.52} \text{m}^{-2} \text{s}^{-1} \text{sr}^{-1}.$$  \hfill (4.2)

For protons

$$I_p(k) \simeq 6 \times \left( \frac{T}{10^7 \text{ yr}} \right) \left( \frac{\eta}{10^{-20} \text{ eV}} \right)^{1.84} \left( \frac{k}{10^{-7} \text{ eV}} \right)^{-0.84} \text{m}^{-2} \text{s}^{-1} \text{sr}^{-1}.$$  \hfill (4.3)

In a way it is unfortunate that the dominant contribution comes from electron cosmic rays because they are still poorly understood. Note that $I(k)$ has the dimensions of energy per unit wave vector per unit surface per unit time. In radioastronomy the intensity, or energy flux density, is commonly measured in Jansky (1 Jy = $10^{-26}$ W Hz$^{-1}$ m$^{-2}$ sr$^{-1}$ $\simeq 1.5 \times 10^7$ eV eV$^{-1}$ m$^{-2}$ s$^{-1}$ sr$^{-1}$).

The expected overall intensity is shown in Figure 1 in a doubly logarithmic scale for a very wide range of wave vectors (many of them undetectable) and for the reference values for $T, \eta$ indicated in (4.2) and (4.3). It should be emphasized that this is really only a rough estimate of the background radiation provided by cosmic rays of galactic origin due to axion-induced Bremsstrahlung. We have assumed very conservatively that the flux measured in the inner solar system is a good representative of the average abundance of cosmic rays in the galaxy, but this is almost certainly an underestimate due to our peripheral position in the galaxy and the relatively short reach of electron cosmic rays.

In order to see whether this flux is measurable from the Earth or not one has to determine the diffuse noise perceived by the receiver in the appropriate wavelength, known identified background sources, and of course take into account the atmosphere transparency at that radiation wavelength. As it is well known[21], the atmosphere is transparent to radiation in the terrestrial microwave window ranging from approximately 6 mm (50 GHz, $2 \times 10^{-4}$ eV) to 20 m (15 MHz, $6 \times 10^{-8}$ eV), becoming opaque at some water vapor and oxygen bands and less transparent as frequency increases up to 1 THz. The current technology allows for radio detection from space up to 2 THz.
Figure 1. Total intensity $I_\gamma = I_p^\gamma + I_e^\gamma$ expected to be measured as a consequence of the axion Bremsstrahlung effect discussed here. Units are in $m^{-2} \cdot s^{-1} \cdot sr^{-1}$. The total yield is the external envelop and it is dominated by electrons for a wide range of frequencies. The figure is plotted using the exact formulae (solid line). The proton contribution is shown separately (dashed line). The approximate expressions discussed in the text are shown in dotted-dashed line (nearly invisible). For comparison the approximate galactic radio background (basically from electron synchrotron radiation) is shown[20]. Note that the radio background is not well measured at present below 10 MHz but there are indications suggesting a marked decrease below 3 MHz. In the 100 MHz region the axion induced signal is about nine orders of magnitude smaller than the background.

(e.g. with the Herschel Space Observatory[22]) but the low receiver sensitivity at frequencies in the submillimeter band ($> 300$ GHz) could be an issue. There are further considerable narrower windows in the near infrared region from 1 $\mu$m (300 THz, 1.2 eV) to around 10 $\mu$m (30 THz, 0.12 eV). This region can be explored by space missions. The atmosphere blocks out completely the emission in the UV and X-Ray region corresponding to $\lambda < 600$ nm ($k > 80$ eV), a region that is actively being explored by spaceborne missions.

If $\lambda > 2.5$ m (0.8 GHz, $3 \times 10^{-6}$eV), the galactic synchroton radiation noise increases rapidly dificulting the detection of any possible signal. Note however that while the power spectrum of the axion-related radiation from proton primaries is the same as the one from the synchrotron radiation they produce[23], the bulk of the Galaxy synchrotron radiation is due to electrons whose spectral power law describing the axion-induced Bremsstrahlung is different. In addition there would be a difference between the galactic and the axion based synchrotron emission anyway. In fact\footnote{We thank P. Planesas for pointing out this possibility to us}, in areas of high galactic latitude, where no local features superpose the broad galactic emission, the
measured spectral index is $\sim -0.5$ [24]. Instead, the axion induced effect has a power $\sim -1.5$ if we assume $\gamma \sim 3$.

The maximum observed values[25] for the intensities are: $10^4 \text{ m}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$ in the X-Ray region and up to from $10^{10}$ to $10^{14} \text{ m}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$ in the radio, IR and UV regions but the sensitivity of antenna arrays at very low frequencies such as the LWA[18] can be as low as 0.1 mJy $\simeq 10^3 \text{ m}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$ or even less. Of particular interest for our purposes is the sensitivity that can be reached in the SKA antenna. This can be estimated[26] assuming an integration time of 50 hrs at the lowest frequency to be 650 nJy. This is clearly several orders below the expected size of the effect, even assuming the worst possible case for the electron flux. Therefore, while the effect is below the sensitivity of existing antennas it will be within the reach of several projects in construction or under consideration$^2$.

Once it is clear that antennas can measure fluxes twelve orders below the dominant Galaxy synchrotron radiation in the galactic plane, it is obvious that sensitivity to the axion signal (‘only’ nine orders below the average galactic noise) is not an issue, the real difficulty is to disentangle the effect from the background or foreground. For this purpose the rather different power dependence should prove essential. The difference in power spectrum between the expected signal and the background is even more marked for regions of high galactic latitude as already mentioned. Good angular resolution will be essential too as observers looking for this signal will probably be interested in focusing their instruments in region with low magnetic fields$^3$, where synchrotron radiation will be at a minimum gaining several orders of magnitude in the signal-to-noise ratio$^4$.

While it is obviously beyond the scope of this paper (and the expertise of the authors) to present a definite proposal to measure the tiny axion-induced Bremsstrahlung predicted in this work, we do conclude that it is conceivably within the reach of a new generation of instruments specifically designed for exploration of the long wavelength region. We do not exclude that it can be found in the exploration of close extragalactic sources either. In both cases the main unknown is a detailed understanding of the nature and spectrum of electron cosmic rays, an issue worth investigating by itself for a variety of raisons.

Other comments pertinent here are the following. Firstly one should note that the effect discussed here is a collective one. This is at variance with the GZK effect alluded in the introduction

$^2$It may be worth noticing that the long standing project of setting up an antenna on the far side of the Moon[27] could reach sensitivities of $10^{-5}$ Jy or less, also providing enough sensitivity even for pessimistic values of the electron flux. Such an antenna would of course not be limited by the atmosphere opacity being sensitive -in principle- to even longer wavelengths.

$^3$Note that the Galaxy magnetic field varies by about three orders of magnitude from $\mu$G to mG

$^4$The synchrotron radiation depends quadratically on the magnetic field, hence a change of two orders of magnitude in the magnetic field represent a variation of four orders in the amount of the synchrotron radiation background
- the CMB radiation is not a coherent one over large scales. For instance, no similar effect exists for hot axions. A second observation is that some of the scales that play a role in the present discussion are somewhat non-intuitive (for instance the 'cross-over' scale $m_p^2/\eta$ or the threshold scale $m_\gamma m_p/\eta$). This is due to the non Lorentz-invariant nature of this effect. Also, it may look surprising at first that an effect that has such a low probability may give a small but not ridiculously small contribution. The reason why this happens is that the number of cosmic rays is huge. It is known that they contribute to the energy density of the Galaxy by an amount similar to the Galaxy’s magnetic field[28]. Finally we would like to comment that the calculations presented here in the limit where the oscillations are assumed to be adiabatic can be proven to be exact[29].

We hope that the present mechanism can shed some light on the presumed relevance of cold axions as a dark matter candidate.

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