Spectral energy distribution and generalized Wien’s law for photons and cosmic string loops

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Abstract

Physical objects with energy $u_\omega(l) \sim l^{3w}$, with $l$ a characteristic length and $w$ a dimensionless constant, lead to an equation of state $p = \omega \rho$, with $p$ the pressure and $\rho$ the energy density. Special entities with this property are, for instance, photons ($u = \hbar c/l$, with $l$ the wavelength) with $w = 1/3$, and some models of cosmic string loops ($u = (e^l/\alpha G)l$, with $l$ the length of the loop and $\alpha$ a numerical constant), with $w = -1/3$. Here, we discuss some features of the spectral energy distribution of these systems and the corresponding generalization of Wien’s law, which in terms of $l$ has the form $T l^3 m p = constant$, being $l m p$ the most probable size of the mentioned entities.

Keywords: photons, cosmic string loops, statistical mechanics, Wien’s law, dark energy

(Some figures may appear in colour only in the online journal)

1. Introduction

The future of the universe strongly depends on its contents. Before 1998, it was believed that expansion was slowed by gravitational force, but the question remained whether this expansion would last forever or whether it would arrive at a maximum value, after which it would follow an accelerated contraction. In 1998, observation of distant galaxies led to the striking conclusion that the expansion was accelerating. This was attributed to a so-called dark energy, distributed over the whole space, with a repulsive interaction, leading to such acceleration.

Since then, the search for candidates for dark energy has been a stimulus to recent thermodynamics of exotic systems that had not attracted the interest of researchers before the discovery of cosmic acceleration and the need for systems with sufficiently negative pressure [1–7]. Dark energy is believed to contain 70 percent of the energy of the whole universe and to make the cosmic expansion to accelerate. According to general relativity, the latter fact requires that $p < -(1/3) \rho$, $\rho$ being the energy density and $p$ the pressure. Thus, much interest has been invested on systems that obey $p = \omega \rho$, with $w$ a dimensionless constant, as the simplest possibilities for dark energy. Here we focus our attention on the cases with $w = 1/3$ and $w = -1/3$, corresponding respectively a gas of photons and to a gas of string loops, and that exhibit an interesting duality relation between themselves [8–10].

Other physical entities of this family could be, for instance, small cosmic membranes of size $l$, with energy proportional to the area—that is, to $l^2$, corresponding to $w = -2/3$. Note that these kinds of entities are only particular cases of a much wider family of string entities having more general expressions for $u(l)$—that is, not only a simple potential law—as vortices in superfluids, macromolecules in solutions, dislocations in solids, color-flux filaments in quark-gluon plasmas, or magnetic flux tubes in type-II superconductors [11]. Here we do not aim at a general
analysis of these systems but to a comparison of Wien’s law for photons and for cosmic string loops relating the most probable characteristic length to temperature. For photons, it reduces to the well-known form of Wien’s law for electromagnetic radiation \((T\lambda_{mp} = \text{constant})\), while for cosmic string loops it leads to a significantly different result \((T/l_{mp} = \text{constant})\). Such generalized law would provide a simple link between the geometrical features of those objects and the temperature of the corresponding system. The physical motivation is to explore the thermodynamic contribution of these physical ingredients in the future expansion of the universe.

In section 2 we summarize some previous thermodynamic results, in section 3 we present the Ehrenfest adiabatic theorem and its general relation with the spectral distribution, and in section 4 we apply them to our systems. Section 5 is devoted to conclusions and comments.

2. Previous thermodynamic results

In [8–10], we considered a family of hypothetical physical entities characterized by a length \(l\) and an energy \(u_w(l)\), given by

\[
u_w(l) = C_w l^{-3w} \quad \text{with} \quad C_w = \frac{h \epsilon}{a^2} \left(\frac{c^3}{\hbar G}\right)^{\frac{1}{3-2w}}.
\]

In particular, photons correspond to \(w = 1/3\) (with \(l\) the wavelength and \(u = hc/l\)) and cosmic strings to \(w = -1/3\) \((u = (c^3/a^2G)l\), with \(l\) the length), other possibilities are, for instance, cosmic dust, with \(w = 0\) (no characteristic length, usually taken as dots), or cosmic membranes, with energy proportional to their area, with \(l\) the lateral size and \(w = -2/3\). In (2.1), \(a\) is a numerical constant that may depend on the model of loop, and whose value, yet uncertain, lies in the range \([1, 10^3]\), according to data based on observations of the energy peaks of astronomical flares from gamma ray bursts and from active galactic nuclei [12–14].

These systems lead in a direct way to \(p = w\rho\), and to a vanishing chemical potential [8–10], under two further physical hypotheses—namely, (a) that their absolute temperature \(T\) may be related to the average value of the internal energy as \(k_B T = \langle u(l) \rangle \) denoting the average value over the length distribution function of the objects) and (b) that the average separation between these entities is proportional to their average size.

According to (b), in an expansion at constant energy, these loops aggregate to form longer and more separated loops, which leads to a decrease of entropy. Thus, \((dS/dV)_U\) is negative, and this yields a negative pressure. This would be otherwise had we considered loops whose length would have stayed constant and become more separated. A dilute gas of these latter kind of loops would have the same equation as a system of points, instead of yielding a negative pressure.

Thus, both features (a) and (b) are relevant for the definition of the systems considered here, which may be considered as mathematical models, independently of their actual physical existence.

In [8–10] we focused our attention on the explicit determination of the thermodynamic functions of these systems, as, for instance, \(U(T, V), S(T, V), S(U, V), p(U, V), F(T, V),\) and so on being \(U, S, F, T,\) and \(V\) internal energy, entropy, Helmholtz free energy, temperature, and volume, respectively. In particular, it was shown that

\[
u_w(T) = \frac{U_w(T, V)}{V} = A_w T^{(1+w)/w},
\]

with \(A_w\) a constant, and the entropy

\[
S_w(T, V) = (1 + w) B_w T^{1/w},
\]

with \(B_w\) another constant. Note that in an adiabatic expansion the temperature of the system decreases for \(w > 0\) and increases for \(w < 0\). This is so because for \(w > 0\), pressure is positive and the expansion work is negative, whereas for \(w < 0\), pressure is negative and expansion work is positive.

Furthermore, in [9] we have considered in detail the duality relations between electromagnetic radiation \((w = 1/3)\) and cosmic string loops \((w = -1/3)\), while in [10] it was extended to systems with \(w \neq \pm 1/3\). Here, we go a step beyond and explore in more depth the thermodynamic clues on the spectral energy distribution—that is, \(\rho_w(T, \nu)\) describing the energy density distribution among the several modes of frequency \(\nu\) at a given temperature \(T\). This is more than an academic exercise, as this information is essential for a deeper physical understanding of these systems for a comparison with other kinds of systems.

3. Adiabatic theorem and spectral energy distribution

In this presentation we follow the lines by Lima and Alcaniz [15] in their analysis of the same problem we are interested in—namely, to explore the spectral energy distribution of systems with \(p = w\rho\) using thermodynamic methods. However, our approach is very different, since we have concrete (although hypothetical) models for our systems, which provide us information on the dispersion relation between frequency and characteristic lengths. As a consequence, our conclusions about the form of Wien’s law in terms of temperature and wavelength for different values of \(w\) will be very different from those obtained in [15], although its expression in terms of temperature and frequency will be the same.

Here, instead of the internal energy \(U_w(T, V)\) of the system in a volume \(V\) at temperature \(T\), we wish to find thermodynamic constraints on the spectral distribution \(U_w(T, V, \nu)\) or the spectral energy density.
\( \rho_v(T, \nu) = U_v(T, V, \nu)/V \). Then, we may write:

\[
U_v(T, V) = \int U_v(T, V, \nu) d\nu = \int \rho_v(T, \nu) V d\nu
\]  

(3.1)

with \( U_v(T, V, \nu) \) being the total energy contained in a small interval band of frequencies \( \Delta \nu \) (between \( \nu - (\Delta \nu)/2 \) and \( \nu + (\Delta \nu)/2 \)) and \( \rho_v(T, \nu) \) being the energy density per unit volume. Our main concern is to find the form of \( \rho_v(T, \nu) \).

For later use, we note that (3.1) may also be written in a discrete form as

\[
U_v(T, V) = \sum_{i=0}^{\infty} \rho_v(T, \nu_i) V \Delta \nu_i
\]  

(3.2)

where \( \nu_i \) is an infinite collection of frequencies, each with its corresponding small frequency slot \( \Delta \nu_i \). This may be more pedagogical for our presentation.

We start our analysis by using the adiabatic theorem proved by Ehrenfest in 1917 [16], which states that for any reversible adiabatic change of volume inside an enclosure at temperature \( T \) at thermodynamic equilibrium, the internal energy in any frequency slot of width \( \Delta \nu_i \) centered in frequency \( \nu_i \), divided by frequency \( \nu_i \) is constant—that is:

\[
\frac{\rho_v(T, \nu_i) V \Delta \nu_i}{\nu_i} = \frac{\rho_v(T, \nu_{i+1}) V \Delta \nu_{i+1}}{\nu_{i+1}}.
\]  

(3.3)

Here, it means that we consider an enclosure containing the system defined by (2.1) at temperature \( T_i \) and focus our attention on a band of frequencies \( \Delta \nu_i \) centered on frequency \( \nu_i \). Assume that temperature \( T_i \) changes to temperature \( T_{i+1} \) in an adiabatic (and reversible) volume change from \( V_i \) to \( V_{i+1} \). The frequency band being considered will change from \( \nu_i \) and \( \Delta \nu_i \) to \( \nu_{i+1} \) and \( \Delta \nu_{i+1} \), and the corresponding energy contained in these bands will be \( \rho_v(T, \nu_i) \Delta \nu_i \) and \( \rho_v(T, \nu_{i+1}) \Delta \nu_{i+1} \), respectively. Initially, this result was shown for harmonic oscillators, and later it was generalized to other kinds of periodic or quasi-periodic systems [16] (see also [17] for a detailed historical overview). Here, we use it for oscillators with dispersion relations given by (4.1); therefore, the theorem is applicable.

Thus, according to (3.3) and taking into account (2.3) and that entropy is constant in an adiabatic and reversible process, then one has for the corresponding frequency bands \( \nu_i \) and \( \nu_{i+1} \) (for all possible \( i \)):

\[
\frac{\nu_{i+1}\rho_v(T, \nu_i) \Delta \nu_i}{\nu_i \rho_v(T, \nu_{i+1}) \Delta \nu_{i+1}} = \frac{V_i}{V_{i+1}} = \frac{T_i^{\nu_{i+1}}}{T_{i+1}^{\nu_i}}
\]  

(3.4)

On the other side, according to (2.2) and (3.2), the energy density should change as

\[
\sum_{i=0}^{\infty} \rho_v(T, \nu_i) \Delta \nu_i = \left( \frac{T_i}{T_{i+1}} \right)^{1+\nu_i}
\]  

(3.5)

where summation refers to the slots \( \Delta \nu_i \) in which the frequency domain can be divided. Equation (3.5) can be also written using (3.3) and (3.4) as

\[
\sum_{i=1}^{\infty} \left( \frac{\nu_{i+1} T_i}{\nu_i T_{i+1}} - 1 \right) \rho_v(T, \nu_{i+1}) \Delta \nu_{i+1} = 0
\]  

(3.6)

which, because of the arbitrary choice of the ensemble \( \nu_i \) and the corresponding \( \nu_{i+1} \) and their corresponding slots \( \Delta \nu_i \) and \( \Delta \nu_{i+1} \), implies

\[
\frac{\nu_i}{T_i} = \frac{\nu_{i+1}}{T_{i+1}}
\]  

(3.7)

for each value of \( i \). We emphasize that (3.7) follows from (3.6) because the collection of values \( \nu_i \) may be taken in many arbitrary different forms (if only a definite choice of the collection of values \( \nu_i \) were taken, then this property would not follow). On the other hand, this does not mean that \( \nu_i/T \) is a constant at a given \( T \), because of course \( \nu_i \) may take many different arbitrary values. It simply means that for every choice of \( \nu_i \), when temperature changes in an adiabatic reversible form, the concrete value of \( \nu_i \) is changed according to (3.7).

From here follows also that \( \Delta \nu_i/T_i = \Delta \nu_{i+1}/T_{i+1} \). Thus, one has from (3.4)

\[
\frac{\rho_v(T, \nu_i) \Delta \nu_i}{\rho_v(T, \nu_{i+1}) \Delta \nu_{i+1}} = \frac{T_i}{T_{i+1}} = \frac{\nu_{i+1}}{\nu_i}
\]  

(3.8)

From (3.8) and taking into account (3.7), one may write
\( \rho_v (T, \nu) \) as

\[
\rho_v (T, \nu) = a_v \nu'^n \Phi \left( \frac{T}{\mu} \right), \tag{3.9}
\]

where \( a_v \) is a dimensionless constant, and \( \Phi \left( \frac{T}{\mu} \right) \) is a function to be determined, but that does not vary in adiabatic processes because its argument, \( T/\mu \), remains constant in such changes for each \( \nu \), according to (3.7) (all the frequencies of the mode change as \( T \) in the adiabatic process). Of course, \( T/\mu \) at a given \( T \) but different \( i \) is not a constant. This may be considered as the generalized Wien spectrum, because for electromagnetic radiation, it was Wien who proposed this kind of functional form. He proposed an approximate expression of exponential form for \( \Phi \left( \frac{T}{\mu} \right) \), whose exact form for electromagnetic radiation (\( w = 1/3 \)) was finally obtained by Planck in 1900—namely (see figure 1),

\[
\rho_{\text{electric}} (T, \nu) = \frac{8\pi h \nu^3}{c^4} e^{\frac{h\nu}{kT}} - 1, \tag{3.10}
\]

with \( k_B \) Boltzmann’s constant.

In the next section we turn our attention to the family of systems defined by (2.1), which generalizes the thermodynamics of electromagnetic radiation to a wider family of systems.

4. Spectral energy distribution for photons, cosmic string loops, and related physical objects

To apply the ideas of the preceding section to our family of systems (2.1), we need information on their characteristic frequency. In the case of electromagnetic radiation, this is clear, because \( \omega = c / \lambda \), with \( \lambda \) the wavelength and \( c \) the speed of light in vacuum. To do this in a more general way for all the systems in (2.1), we consider (2.1) as expressions for the energy quanta in terms of \( l \) with the Einstein-Planck expression of the energy of the quanta in terms of \( \nu \)—namely, \( u (\nu) = h\nu \). Then, combining (2.1) with this expression, we obtain

\[
\nu = c \left( \frac{c^3}{\hbar a^2 G} \right)^{1/3n} l^{-3w}. \tag{4.1}
\]

For photons this just yields \( \nu = c/l \) (\( l \) being \( \lambda \)). For string loops this yields

\[
\nu = \frac{c^4}{\hbar a^2 G} l, \tag{4.2}
\]

which gives the characteristic frequency of a loop of length \( l \).

Thus, the general form of energy spectral distribution for these systems will be of the kind (3.9). In principle, the form of \( \Phi \left( \frac{T}{\mu} \right) \) depends on \( w \). It is tempting to assume that for all of them an analogous of the Planck distribution holds in such a way that

\[
\rho_v (T, \nu) = \frac{a_v \nu^{1/w}}{e^{h\nu/kT} - 1} \tag{4.3}
\]

But this is only a guess that turns out to be untenable for \( w < 0 \). Indeed, it is known that for \( w > 0 \), the systems are thermodynamically stable, but for \( w < 0 \), they have negative specific heat and therefore are unstable and require a continuous input to keep themselves in existence (in cosmology, this input could be the cosmic expansion itself). Therefore, one may expect that the form of \( \Phi \left( \frac{T}{\mu} \right) \) may be considerably different for \( w > 0 \) and for \( w < 0 \). The situation \( w = 0 \), corresponding to cosmic dust, implies constant energy, independent of the length. Usually, it takes \( l = 0 \) as its characteristic length—namely, point-like dust—and the characteristic temperature \( T \) takes zero, in order that \( VT^{1/3} \) may be finite.

In particular, the dynamics of strings and string loops have been studied in different systems, going from cosmic strings [18–21] to quantized vortex loops in superfluid helium [22–26]. The results indicate that in a number of situations one should expect a potential distribution law. The detailed analysis of vortices is much more complicated than that of the simple entities we are considering in this paper [22–26]. Anyway, as a simple illustration, we will consider for the length probability distribution the form [22]

\[
n (l) d l = B_s \frac{l^n}{l_{\text{min}}^{n+q}} d l \tag{4.4}
\]

with \( n (l) d l \) the number of loops of length comprised between \( l \) and \( l + d l \) per unit volume; \( B_s \) a dimensionless normalization constant, \( q \) a characteristic exponent, and \( l_{\text{min}} \) the minimum length (otherwise, for a vanishing minimum length, the distribution function would be divergent).

But here we aim to relate (4.4) to the spectral energy distributions (3.9). To do so, we need to express the energy distribution to relate \( l \) to the frequency \( \nu \) and to introduce temperature \( T \), which is not evident, because (4.4) differs very much from the usual forms of equilibrium statistical distribution functions, for which the temperature is well identified. In references [8, 9, 27], we used for temperature the definition

\[
k_B T = \langle u (l) \rangle, \tag{4.5}
\]

with \( \langle \ldots \rangle \) standing for the average value of the energy over the distribution of lengths of the objects. In reference [8], we showed that using distribution (4.4), one obtains

\[
k_B T = \frac{q - 1}{q - 2 + (1 + 3w)u} (l_{\text{min}}^n) \tag{4.6}
\]
The energy density distribution in terms of $T$ and $l$ will be
\[ \rho_{\chi}(T, l) dl = u_{\chi}(l)n(l) dl, \] (4.7)

We may convert it to $\rho_{\chi}(T, \nu) d\nu$, taking into account the relation (4.1) between $l$ and $\nu$. Since $u(\nu) = h \nu$, we have
\[ \rho_{\chi}(T, \nu) d\nu = -\frac{1}{3w} f^{(q-1)/3w} B_{q} C_{q-1}^{e^{-1}/l} \nu^{(q-1)/3w} \left( \frac{l}{\nu} \right)^{L-q} d\nu, \] (4.8)

where we have taken into account that $d\nu = -3w \frac{c_{\text{w}}}{\hbar} l^{(1+3w)} dl$.

Equation (4.8) for $w = -1/3$ becomes
\[ \rho_{-1/3}(T, \nu) d\nu = \hbar^{2-q} B_{q} C_{q-1}^{e^{-1}/l} \left( \frac{l}{\nu} \right)^{L-q} d\nu. \] (4.9)

Note that this can be used only for $w < 0$; otherwise, the distribution function would correspond to negative values of the probability.

Introducing (4.6) into (4.8), we get
\[ \rho_{\chi}(T, \nu) d\nu = B_{q}^{\nu} \nu^{L+q} \Phi_{q}^{\nu}(T/\nu) d\nu, \] (4.10)

with $B_{q}^{\nu} = -\frac{1}{3w} f^{(q-1)/3w} B_{q} C_{q-1}^{e^{-1}/l} \left( \frac{l}{\nu} \right)^{L-q} \left( \frac{1}{kq_{g} - l+3w} \right)^{(4-q)/3w}$ and
\[ \Phi_{q}^{\nu}(T/\nu) = \left( \frac{\nu}{T} \right)^{L-q/3w}. \] (4.11)

The expression of the energy density (4.10) for $w = -1/3$ is plotted in figure 2 at three different values of temperature for a direct comparison with the same expression for $w = 1/3$ plotted in figure 1. Then, although the distribution (4.4) does not seem to have any relation to the energy spectral distribution (3.9), we have seen here that relating $l$ to energy, energy to frequency, and introducing a suitable definition for temperature, we can obtain for the systems with $w < 0$ an energy spectral distribution function that has indeed the form required by thermodynamic arguments. Though this could seem natural, this is not so, because a thermodynamic formalism for systems with $w < 0$ is scarcely known; thus, our result provides an additional argument in favor of the internal consistency of the thermodynamic analysis of systems (2.1).

5. Conclusions and remarks: generalized Wien’s law for cosmic string loops and electromagnetic radiation

Wien’s law characterizes an important feature of the thermodynamics of electromagnetic radiation, stating that the most probable wavelength changes inversely with the absolute temperature. If cosmic string loops indeed play a possible role as constituents of the universe, it is logical to ask how the most probable length of the loops will change with temperature. This has been the main topic of this paper, and our conclusion is that, in contrast with electromagnetic radiation, the most probable length changes proportionally with the absolute temperature.

In more detailed terms, in this paper we continued to develop the study of the thermodynamical properties of physical objects with energy $u_{\chi}(l)$, which scales as $l^{-3w}$, with $l$ a characteristic length and $w \in [-1, 1]$, with emphasis on photons ($w = 1/3$) and cosmic string loops ($w = -1/3$).

In section 3 it has been recalled that the adiabatic theorem shows that $T/\nu$ is a constant in adiabatic reversible processes. From this relation it follows, for instance, that the spectral distribution of $\nu$ should have the general form (3.9) and that $T/l_{\nu}$ is a constant, where $l_{\nu}$ is the most probable value of the frequency $\nu$. In the case of photons (where $l = \lambda$ is the wavelength and $w = 1/3$), we get the well-known Wien’s law, with $\lambda_{\nu}T = \text{const.}^{[28]}$.

In section 4, we showed that from $T/l_{\nu}$, one cannot conclude in general that $\lambda T$ is a constant, as was done in reference [15], because this is a result for photons (or, more generally, for systems with $u(l)$ proportional to $l^{-1}$). In view of (4.1), instead, it is seen that $T/l_{\nu}$ is a constant
\[ T/l_{\nu} = \text{const.}, \] (5.1)

where $l_{\nu}$ is the most probable value of the length $l$.

For electromagnetic radiation ($3w = 1$), this yields $T/l_{\nu} = \text{const.}$ However, for cosmic string loops ($3w = -2$), the corresponding form is $T/l_{\nu} = \text{const.}$ Note also that for cosmic membranes ($3w = -2$), (5.1) yields $T/l_{\nu} = \text{const.}$ Thus, having some detailed information on the dispersion relation of the systems with $w$ different from $-1/3$ is essential to go from the general form $T/\nu = \text{const.}$ to the several particular forms summarized in $T/l_{\nu} = \text{const.}$ in terms of the characteristic length. Since in reference [15] this information was lacking, it was assumed that the form $T/l_{\nu} = \text{const.}$ was valid for systems with all values of $w$. In fact, the behavior (5.1) is consistent with expansion (2.3) for the entropy. In an expansion, electromagnetic radiation cools as $(RT) = \text{const.}$ (R being the length scale factor of the
container), whereas the system of cosmic string loops becomes hotter, as $R/T = \text{constant}$.

It is worth noting that systems with $w < 0$ do not seem to follow Planck statistics (nor Bose–Einstein nor Fermi–Dirac ones). This may be related to the fact that they are not stable equilibrium systems but are steady states kept by an external input, which makes them depend strongly on the particular dynamics reflected in (4.11) through the value of the exponent $q$. This makes it even more surprising that even for these systems, the energy spectral distribution may be written in the general form (3.9).

A topic of discussion in connection to systems with negative pressure is about the possibility or impossibility of the so-called phantom dark energy. In the present family of models, phantom dark energy with $w < -1$ would lead to a negative entropy. Equation (4.6) gives some more requirements related to the distribution function (4.4), which says to have $T > 0$, the absolute value of $3w$ should be less than $q - 1$, $q$ being the exponent in (4.4). The situation with $w = -1$ would then require that $q > 4$, but not much is known about the actual value of $q$. We also point out that in superfluid turbulence, the exponent $q$ describing the length probability distribution of quantized vortices is $q = 5/2$; in the hypothetical case that $q$ also had this value (this is only meant for the sake of a concrete illustration), the values of $w$ consistent with $T > 0$ would be $w > -1/2$. Thus, for the family of systems studied here, the admissible values of $w$ are related to their spectral properties (as, for instance, (4.4) or (4.11)) in a restrictive way.

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