On Field Theories of Loops

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Abstract

We apply stochastic quantization method to real symmetric matrix models for the second quantization of non-orientable loops in both discretized and continuum levels. The stochastic process defined by the Langevin equation in loop space describes the time evolution of the non-orientable loops defined on non-orientable 2D surfaces. The corresponding Fokker-Planck hamiltonian deduces a non-orientable string field theory at the continuum limit.

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String field theory [1] is believed to be the most promising approach to investigate non-perturbative effect in string theories. Recently, non-critical string field theories have been proposed for $c = 0$ [2] and for $0 < c < 1$ [3]. Among these, some [2] are based on the transfer-matrix formalism [3] in dynamical triangulation of random surfaces [3]. While the other [3] is derived by using stochastic quantization method [8].

In the approach by stochastic quantization, introducing the loop variable $\text{tr} M^n$ [3] for hermitian matrix models and interpreting the fictitious time as a time coordinate, Jevicki and Rodrigues [3] showed that the Fokker-Planck hamiltonian (or loop space hamiltonian), in which the loop variable and the conjugate momentum are identified with the creation operator and the annihilation operator respectively, realizes the string field theories which are equivalent to the field theory derived by Ishibashi-Kawai [2]. Inspired by the work in Ref. [3], we apply stochastic quantization method to real symmetric matrix models [10] and show that it leads to a field theory of non-orientable (non-critical) strings. The stochastic process defined by the Langevin equation in loop space describes the time evolution of the non-orientable loops on non-orientable 2D surfaces. The corresponding Fokker-Planck hamiltonian is a loop space hamiltonian of non-orientable string field theories. At the equilibrium limit, it deduces the Virasoro constraint equation for the probability distribution functional. The continuum limit of the field theory of discretized non-orientable loops is taken for the simplest one-matrix case ($c = 0$) and deduces the continuum field theory of non-orientable strings.

Let us start with the Langevin equation for one matrix model,

\[
\Delta M_{ij}(\tau) = -\frac{\partial}{\partial M} S(M)_{ij}(\tau) \Delta \tau + \Delta \xi_{ij}(\tau),
\]

\[
S(M) = -\sum_{\alpha=0}^\infty \frac{g_\alpha}{\alpha + 2} N^{-\alpha/2} \text{tr} M^{\alpha+2},
\]

(1)

$M_{ij}$ denotes a real symmetric matrix. The fictitious time $\tau$ is discretized with the unit time step $\Delta \tau$. We consider the discretized version of time evolution $M_{ij}(\tau + \Delta \tau) \equiv M_{ij}(\tau) + \Delta M_{ij}(\tau)$, with the Langevin equation for convenience of stochastic calculus and for understanding the corresponding stochastic process precisely. The discretized
fictitious time development with $\Delta \tau$ precisely corresponds to the one step deformation in dynamical triangulation in random surfaces. In the following argument, the specific form of the action of the matrix model is not relevant. The correlation of the white noise $\Delta \xi_{ij}$ is defined by

$$< \Delta \xi_{ij}(\tau) \Delta \xi_{kl}(\tau) >_\xi = \Delta \tau (\delta_{il}\delta_{jk} + \delta_{ik}\delta_{jl}) .$$

(2)

It is uniquely determined from the requirement that (1) is transformed covariantly preserving the white noise correlation (2) invariant under the transformation $M \rightarrow U M U^{-1}$, where $U$ denotes orthogonal matrices for the real symmetric matrix models.

The basic field variables are loop variables $\phi_n = \text{tr}(M^n)N^{-1-\frac{n}{2}}$. Following to Ito’s stochastic calculus [11], we evaluate

$$\Delta \phi_n \equiv \phi_n(\tau + \Delta \tau) - \phi_n(\tau) ,$$

$$= n \text{tr}(\Delta MM^n-1)N^{-1-\frac{n}{2}} + \frac{1}{2} n \sum_{k=0}^{n-2} \text{tr}(\Delta MMk\Delta MM^{n-k-2})N^{-1-\frac{n}{2}} + O(\Delta \tau^{3/2}) .$$

(3)

The terms in R.H.S. should be of the order $\Delta \tau$, thus we obtain

$$\Delta \phi_n = \Delta \tau \frac{n}{2} \left\{ \sum_{k=0}^{n-2} \phi_k \phi_{n-k-2} + (n-1) \frac{1}{N} \phi_{n-2} \right\} + \Delta \tau \ n \sum_{\alpha=0} \ g_\alpha \phi_{n+\alpha} + \Delta \zeta_{n-1} ,$$

$$\Delta \zeta_{n-1} \equiv n \text{tr}(\Delta M M^{n-1})N^{-1-\frac{n}{2}} .$$

(4)

The correlation of the new noise variables appeared in (4) is given by

$$< \Delta \zeta_{m-1}(\tau) \Delta \zeta_{n-1}(\tau) >_\xi = \Delta \tau \frac{2}{N^2} nm < \phi_{m+n-2}(\tau) >_\xi ,$$

(5)

The new noise is not a simple white noise but includes the value of the loop variable itself. In a practical sense, it might be tedious to generate the noise variable. We notice that $\phi_{m+n-2}(\tau)$ in R.H.S. of eq.(5) does not include the white noise at $\tau$ but the series of noises up to the one step (fictitious time unit $\Delta \tau$) before. This means that the expectation value

$^1$ For an hermitian matrix $M_{ij}$ in (1), the nose correlation is $< \Delta \xi_{ij}(\tau) \Delta \xi_{kl}(\tau) >_\xi = 2 \Delta \tau \delta_{il}\delta_{jk} .
in R.H.S. should be defined with respect to the white noise correlation up to the fictitious time $\tau - \Delta \tau$.

We also notice $< \Delta \zeta_n(\tau) >_\xi = 0$ by means of Ito’s stochastic calculus. In the context of SQM approach, the property of the noise yields the Schwinger-Dyson equation by assuming the existence of the equilibrium limit at the infinite fictitious time, or equivalently, $\lim_{\tau \to \infty} < \Delta \phi_n(\tau) >_\xi = 0$. We have,

$$< \frac{n}{2} \sum_{k=0}^{n-2} \phi_k \phi_{n-k-2} + \frac{1}{2} (n - 1) \frac{1}{N} \phi_{n-2} + n \sum_{\alpha=0} g_\alpha \phi_{n+\alpha} >_\xi = 0$$

(6)

The order of the noise correlation (5), $1/N^2$, realizes the factorization condition in the large N limit. Therefore we obtain the S-D equation at large N limit for discretized non-orientable strings.

$$\frac{1}{2} \sum_{k=0}^{n-2} < \phi_k >_\xi < \phi_{n-k-2} >_\xi + \sum_{\alpha=0} g_\alpha < \phi_{n+\alpha} >_\xi = 0$$

(7)

This shows that the S-D equation for non-orientable strings takes the same form as that for orientable strings at large N limit. The correspondence at the large N limit is exact if we define the corresponding hermitian matrix model by replacing all the couplings, $g_\alpha \rightarrow 2g_\alpha$ in (1). As a consequence, the disc amplitude in non-orientable strings is exactly the same as that in orientable strings.

The geometrical meaning of the stochastic process described by the Langevin equation (4) is the following. The one step fictitious time evolution of a discretized loop, $\phi_n(\tau) \rightarrow \phi_n(\tau) + \Delta \phi_n(\tau)$, generates the splitting of the loop into two smaller pieces, $\phi_k$ and $\phi_{n-k-2}$. The process is described by the first term in R.H.S. of (4). In a field theoretical sense, it is interpreted as the annihilation of the loop $\phi_n$ and the simultaneous pair creation of loops, $\phi_k$ and $\phi_{n-k-2}$. The first term in R.H.S. of (4) preserves the orientation of these loops, while the second term, which is the characteristic term of the order of $1/N$ for non-orientable interaction, does not preserve the orientation. Since the new noise variables in (5), $\Delta \zeta_{n-1}$’s, are translated to “annihilation” operators in the corresponding Fokker-Planck hamiltonian, the factor 2 in the correlation (5) for the new noise variables comes from the sum of the orientation preserving and non-preserving merging interactions.
Namely, (5) describes the simultaneous annihilation of two loops \( \phi_m \) and \( \phi_n \) and the creation of a loop \( \phi_{m+n-2} \). The geometrical picture allows us to identify the power “\( n \)” of matrices in \( \phi_n \) to the length of the discretized non-orientable loop \( \phi_n \). We notice that, in each time step, the interaction process decreases the discretized loop length by the unit “2”. The process which comes from the original action of matrix models extends the length of discretized loops. These features are equivalent to the transfer-matrix formalism \([1]\) in dynamical triangulation of 2D random surfaces in which the one step deformation of a specified loop on a triangulated surface defines a discretized (proper) time evolution of the loop.

The definition of the F-P hamiltonian operator gives the precise definition of a field theory of second quantized non-orientable strings. In terms of the expectation value of an observable \( O(\phi) \), a function of \( \phi_n \)’s, the F-P hamiltonian operator \( \hat{H}_{FP} \) is defined by,

\[
< \phi(0)|e^{-\tau \hat{H}_{FP}}O(\phi)|0> \equiv <O(\phi_\xi(\tau))>_{\xi}.
\]

In R.H.S., \( \phi_\xi(\tau) \) denotes the solution of the Langevin equation (4) with the initial configuration \( \phi(0) \neq 0 \). The time evolution of R.H.S. is given by,

\[
<\Delta O(\phi(\tau))>_{\xi} = \sum_m \partial_m O(\phi(\tau)) \Delta \phi_m + \frac{1}{2} \sum_{m,n} \partial_m \partial_n O(\phi(\tau)) \Delta \phi_m \Delta \phi_n >_\xi + O(\Delta \tau^{3/2}) ,
\]

\[
\equiv -\Delta \tau <H_{FP}(\tau)O(\phi(\tau))>_{\xi} ,
\]

where \( \partial_n \equiv \frac{\partial}{\partial \phi_n} \). By substituting the Langevin equation (4) and the noise correlation (5) into (9), we obtain

\[
H_{FP}(\tau) = -\sum_{n>0} X_n n \pi_n ,
\]

\[
X_n \equiv \frac{1}{N^2} \sum_m m \phi_{m+n-2} \pi_n + \frac{1}{2} \sum_{k=0}^{n-2} \phi_k \phi_{n-k-2} + \frac{1}{2} (n-1) \frac{1}{N^2} \phi_{n-2} + \sum_{a=0} g_a \phi_{a+2}
\]

where \( \pi_n \equiv \frac{\partial}{\partial \phi_n} \). To define the operator formalism corresponding to eq.(8), we introduce \( \hat{\phi}_m \) and \( \hat{\pi}_m \) as the creation and the annihilation operators for the loop with the length \( n \),
respectively. Then we assume the commutation relation \([\hat{\pi}_m, \hat{\phi}_n] = \delta_{mn}\), and the existence of the vacuum, \(|0\rangle\), with \(\hat{\pi}_m|0\rangle = 0\) for \(m > 0\). In the representation, \(<Q > \equiv <0|e^{\sum_m Q_m \hat{\pi}_m}|0\rangle\) and \(|Q > \equiv \Pi_m \delta(\hat{\phi}_m - Q_m)|0\rangle\), the F-P hamiltonian operator \(\hat{H}_{FP}\) in (8) is given by replacing \(\phi_m \rightarrow \hat{\phi}_m\), and \(\pi_m \rightarrow \hat{\pi}_m\) in \(H_{FP}\) in (10) with the same operator ordering.

From the equality (8), the probability distribution function \(P(\phi, \tau)\), which is defined by \(<O(\phi(\tau)) > \equiv \int \Pi_n d\phi_n O(\phi)P(\phi, \tau)\), is given by,

\[P(\phi, \tau) = <\phi(0)|e^{-\tau \hat{H}_{FP}}|\phi > .\]  

(11)

The initial distribution, \(P(\phi, 0) = \Pi_m \delta(\phi_m - \phi_m(0))\), represents the initial value of the solution of the Langevin equation (4). Eq.(11) follows the Fokker-Planck equation for the probability distribution,

\[\Delta P(\phi, \tau) = +\Delta \tau \hat{H}_{FP}P(\phi, \tau),\]  

(12)

where \(\hat{H}_{FP}\) is the adjoint of \(H_{FP}\) in (10),

\[\hat{H}_{FP}(\tau) = -n\pi \sum_{n>0} \tilde{X}_n ,\]

\[\tilde{X}_n \equiv -\frac{1}{N^2} \sum_m m\pi_m \phi_{m+n-2} + \frac{1}{2} \sum_{k=0}^{n-2} \phi_k \phi_{n-k-2} + \frac{1}{2} (n-1) \frac{1}{N} \phi_{n-2} + \sum_{\alpha=0}^\Phi g_{\alpha} \phi_{\alpha+2} .\]  

(13)

In the context of stochastic quantization, the F-P hamiltonian (10) in loop space was found for hermitian matrix models. The remarkable observation was that it includes the Virasoro constraint [3]. Since the fictitious time evolution is generated by the noise essentially, the emergence of Virasoro constraint is traced to the noise correlations in eq.(5) which are equivalent to the insertion of matrices into the loop variable, \(M \rightarrow M + \Delta \tau M^{m-1}\), in \(\phi_n\). It generates the transformation \([-\Delta \tau L_{m-2}, \phi_n] = n \Delta \tau \phi_{m+n-2}\), which corresponds to the noise correlation (5). In fact, for real symmetric matrix models (non-orientable strings), \(L_n \equiv -N^2 X_{n+2}\) satisfies the Virasoro algebra without central
extension,
\[ [L_m, L_n] = (m - n)L_{m+n}. \quad (14) \]

We notice that, although \( \hat{H}_{FP} \) is not hermitian, one can define an hermitian F-P hamiltonian from \( \hat{H}_{FP} \) as a direct consequence of the fact that Ito’s stochastic calculus automatically picks up the Jacobian factor [3] (or more precisely, invariant measure) which is induced in the space of loop variables by the change of dynamical variables from a matrix to loop variables [4].

It is also worthwhile to note that the F-P equation (12) realizes the Virasoro constraint for the probability distribution. Namely, \( \tilde{L}_n \equiv N^2 \tilde{X}_{n+2} \) also satisfies the Virasoro algebra without central extension (14). Therefore, the F-P equation deduces a constraint equation for the distribution function even at the discretized level, justifying the generation of the partition function which satisfies the Virasoro constraint at the infinite fictitious time.

\[ \tilde{L}_n \lim_{\tau \to \infty} P(\phi, \tau) = 0, \quad \text{for } n = -1, 0, 1, \ldots . \quad (15) \]

For hermitian matrix models, the Virasoro constraint for the partition function (15) was found as the S-D equation [4]. In the continuum limit, it deduces the continuum version of the Virasoro constraints [5]. The expressions (8) and (11) also give a constraint on possible initial condition dependence of the expectation value and the partition function at the infinite fictitious time limit, such as, \( \lim_{\tau \to \infty} H_{FP}[\phi(0), \frac{\partial}{\partial \phi(0)}]P[\phi, \tau] = 0 \). This implies that these quantities may have the initial value dependence up to the solution of the Virasoro constraint.

Now we take the continuum limit. First we introduce a length scale “a” and define the physical length of the loop created by \( \phi_n \) with \( l = na \). Then we may redefine field variables and the fictitious time at the continuum limit as follows.

\[
\begin{align*}
G_{st} & \equiv N^{-2}a^{-D}, \\
d\tau & \equiv a^{-2+D/2} d\tau, \\
\Phi(l) & \equiv a^{-D/2} \phi_n, \\
\Pi(l) & \equiv a^{-1+D/2} \pi_n,
\end{align*}
\]
where we would like to keep the string coupling constant, $G_{st}$, finite at the double scaling limit \[16\]. For the existence of the smooth limit from the discretized fictitious time evolution to the “continuum” one, we require the condition, $D - 2 > 0$. The scaling dimensions of all the quantities in (16) have been determined except the scaling dimension of the string coupling constant, $D$, by assuming \[2,3\],

\[
\Delta \tau H_{FP} = d \tau H_{FP},
\]

\[
[\Pi(l), \Phi(l)] = \delta(l - l') \, .
\]

(17)

Then we obtain the continuum F-P Hamiltonian, $H_{FP}$, from $H_{FP}$ at the continuum limit,

\[
H_{FP}^{\text{non-or.}} = -\frac{1}{2} \int_0^\infty dl \left\{2G_{st} \int_0^\infty dl' \Phi(l + l')l'' \Pi(l')l\Pi(l) + \int_0^l dl' \Phi(l - l')\Phi(l')l\Pi(l) + \sqrt{G_{st}}l\Phi(l)l\Pi(l) + \rho(l)l\Pi(l) \right\} ,
\]

(18)

for non-orientable strings. By the redefinition (16), the F-P Hamiltonian (18) is uniquely fixed at the continuum limit except the cosmological term. To specify the explicit form of the cosmological term $\rho(l)$ in (18), we have to carefully evaluate the contribution which comes from the 3-point splitting interaction term and the matrix model potential, $a^{1-D}\sum_{\alpha=0} g_\alpha \phi_{n+\alpha}$, at the double scaling limit of the real symmetric matrix model. Here we remember that the S-D equation for non-orientable string at large $N$ limit takes exactly the same form as the orientable one under the suitable choice of the matrix model coupling constants. Since the continuum limit is taken by using the universal part of the disc amplitude \[2\], we naively expect $\rho(l)$ takes the same form as that for orientable strings.

To show explicitly this is indeed the case, we take the continuum limit in the real symmetric matrix model with the same procedure in Ref. \[3\]. We consider the simplest matrix potential given by, $g_0 = -1/2, g_1 = g/2, g_2 = g_3 = ... = 0$ in (1), which corresponds to
\( c = 0 \). Let us introduce the string field variable
\[
\phi(z) \equiv \sum_n z^{-1-n} \phi_n = \frac{1}{N} \text{tr} \frac{1}{z - MN^{-1/2}},
\]
\[
\Delta \zeta(z) \equiv \sum_n z^{-1-n} \Delta \zeta_n.
\] (19)

To take the continuum limit, we redefine the field variable,
\[
\phi(z) \equiv \frac{1}{2} (z - g z^2) + c_0 z_c^{-1} a^{3/2} \Phi(u),
\]
\[
\Delta \zeta(z) \equiv c_0 z_c^{-1} a^{3/2} \tilde{\rho}(u),
\] (20)

where we have introduced the “renormalized” parameters, \( z \equiv z_c e^{g u} \), and \( g \equiv g_c e^{-c_1 a^2 t} \), and the “continuum” fictitious time \( d\tau \equiv c_0 z_c^{-2} a^{3/2} \Delta \tau \), where \( z_c = 3^{\frac{1}{4}} \left( 3^{\frac{1}{2}} + 1 \right) \) and \( g_c = \frac{1}{2 \cdot 3^{\frac{1}{2}}} \) are the critical values and \( t \) denotes the cosmological constant. The constants \( c_0 \) and \( c_1 \) are chosen for convenience. The scaling dimension of all the quantities have been determined so that the string coupling, \( 1/G_{st} \equiv c_0^2 N^2 a^5 \), is fixed at the double scaling limit [16]. Then we have the following Langevin equation in continuum limit.

\[
\begin{align*}
\begin{align*}
\frac{d\Phi(u)}{d\tau} & = \frac{d\tilde{\rho}(u)}{d\tau} - \frac{1}{2} a^3 \left( \frac{1}{4} (z - g z^2)^2 + g z \right) , \\
\tilde{\rho}(u) & = 3 u^2 - \frac{3 t}{4} . 
\end{align*}
\end{align*}
\] (21)

We have only picked up the terms in the noise correlation which survive at the continuum limit. We notice, since \( \partial_u^2 (z - g z^2) = O(a^2) \), the field redefinition in (20) is irrelevant in the term of the order \( 1/(Na_5^{1/2}) \). \( \tilde{\rho}(u) \) in (21) is precisely the cosmological term appeared in the orientable string.

By the Laplace transformation,
\[
\Phi(u) = \int_0^\infty d\tau e^{-u\tau} \Phi(l) ,
\] (22)
we obtain the Langevin equation which is equivalent to the continuum F-P hamiltonian (18),
\[ d\Phi(l) = d\zeta(l) + \frac{1}{2}d\tau\{l\int_0^ldl'\Phi(l')\Phi(l-l') + \rho(l) + \sqrt{G_{st}}l^2\Phi(l)\} , \]
\[ <d\zeta(l)d\zeta(l')> = 2d\tau G_{st}ll' <\Phi(l+l')> , \]
\[ \rho(l) = 3\delta''(l) - \frac{3t}{4}\delta(l) , \]
(23)
for non-orientable string. It is consistent with the naive continuum limit of its discretized version (4) except the term \( \rho(l) \). By the same procedure, the Langevin equation for orientable string is given by
\[ d\Phi(l) = d\zeta(l) + d\tau\{l\int_0^ldl'\Phi(l')\Phi(l-l') + \rho(l)\} , \]
\[ <d\zeta(l)d\zeta(l')> = 2d\tau G_{st}ll' <\Phi(l+l')> , \]
(24)
in the hermitian matrix model with replacing the coupling constants, \( g_{\alpha} \to 2g_{\alpha} \), in (1).
As we have shown explicitly, the cosmological term \( \rho(l) \) takes the same form both for orientable and non-orientable strings. The field theory of non-orientable strings is also consistent with ref. [4] in transfer matrix formalism. The double scaling limit of the real symmetric matrix model has been studied in a quartic potential [10], while our result shows that it happens in the cubic potential as well. We notice that the continuum F-P hamiltonian includes the continuum Virasoro generator \( \mathcal{L}(l) \),
\[ \mathcal{L}(l) = -\{\int_0^\infty dl'\Phi(l+l')l'\Pi(l') + \frac{1}{2G_{st}}\int_0^ldl''\Phi(l-l')\Phi(l') + \frac{1}{2\sqrt{G_{st}}}d\Phi(l) + \frac{1}{2G_{st}}\rho(l)\} . \]
(25)
These generators satisfy the continuum Virasoro algebra, \([\mathcal{L}(l), \mathcal{L}(l')] = (l-l')\mathcal{L}(l+l')\).
Let us briefly comment on the multi-matrix model cases. We may start from a set of Langevin equations.
\[ \Delta M(p)_{ij}(\tau) = -\frac{\partial}{\partial M(p)}S(M)_{ij}(\tau)\Delta\tau + \Delta\xi_{ij}(\tau) , \]
\[ < \Delta \xi(p)_{ij}(\tau) \Delta \xi(p)_{kl}(\tau) > = 2 \Delta \tau \delta_{il} \delta_{jk}, \]

(26)

The index \( p \) specifies the \( p \)-th matrix \( M(p)_{ij} \) and the white noise \( \Delta \xi(p)_{ij} \). \( p \) runs from 1 to \( N_0 \), namely we consider \( N_0 \) matrices. The size of all the matrices is assumed to be \( N \).

The basic loop variable is typically of the form,

\[ tr(\Pi_\alpha M(p_\alpha)^{n_\alpha}) = tr(M(p_1)^{n_1} M(p_2)^{n_2} ... M(p_\alpha)^{n_\alpha} ...), \]

(27)

The Langevin equation for these variables takes the form similar to the one matrix model case. Namely, it includes only linear terms and bilinear terms of loop variables (annihilation of a loop and simultaneous creation of a pair of loops). The correlation of the noise variables is given by a linear combination of loops. It realizes the process where various pairs of loops are annihilated simultaneously and a loop is created. In this case, the colored noise correlation is still equivalent to the insertion of matrices. Therefore, if we identify the length of the loops by the power of the matrices included in the loop variable, we may conclude that there exists Virasoro constraint with respect to the loop length indices in the loop variables (string fields).

In conclusion, we have shown that the Langevin equation (or equivalently the corresponding F-P Hamiltonian) for real symmetric matrix models written by the loop variables defines the time evolution in the (non-critical) non-orientable string field theory at both discretized and continuum levels. The partition function for non-orientable strings satisfies the Virasoro constraint at the equilibrium limit in both discretized and continuum level. In the stochastic quantization viewpoint, since the fictitious time scaling dimension is given by \( \frac{D}{2} - 2 = \frac{1}{2} > 0 \) for \( c = 0 \), we expect that the discretized version of the loop space Langevin equation for real symmetric matrix models may provide a possible method for numerical calculation of non-orientable 2D random surfaces to sum up the topologies of surfaces.

Note
After the completion of this work, we found the works, refs. [17] [18] [19], where the second quantization of master fields are discussed by stochastic quantization method. The expectation value of master fields corresponds to eq. (9) in the present scheme.

Acknowledgements

The author would like to thank J. Ambjorn, H. B. Nielsen and Y. Makeenko for valuable discussions and comments. He also wishes to thank Y. Watabiki for discussions on the transfer matrix formalism and all members in high energy group at Niels Bohr Institute for hospitality.

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