A large-scale one-way quantum computer in an array of coupled cavities

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We propose an efficient method to realize a large-scale one-way quantum computer in a two-dimensional (2D) array of coupled cavities, based on coherent displacements of an arbitrary state of cavity fields in a closed phase space. Due to the nontrivial geometric phase shifts accumulating only between the qubits in nearest-neighbor cavities, a large-scale 2D cluster state can be created within a short time. We discuss the feasibility of our method for scale solid-state quantum computation.

A quantum computer (QC) will exhibit advantages over its classical counterpart only when a large number of qubits can be manipulated coherently, hence a useful QC must allow control of large quantum systems, composed of thousands or millions of qubits \cite{1}. Many architectures of QC’s based on scalable physical systems, such as ion traps \cite{2,3,4,5}, optical lattices \cite{6}, semiconductor \cite{7}, have been widely investigated. There are two well-known models for quantum computation, i.e., the quantum circuit model and measurement-based model. A class of measurement-based models of quantum computation proposed by Raussendorf and Briegel \cite{8}, is the so-called cluster-state model, or one-way quantum computer. Ref \cite{8} has shown that two- and three-dimensional (2D and 3D) cluster states can be used as universal resource for quantum computation via local, single-qubit projective measurements and feedforward.

Recently, coupled cavity arrays \cite{9,10,11,12,13,14,15,16,17,18,19,20,21} have emerged as a fascinating alternative for simulating quantum many-body phenomena and realizing quantum computing. In particular, theoretical works have shown that the Mott-superfluid phase transition of polaritons \cite{10,16,17,18}, the Heisenberg spin chains \cite{19,20}, and fractional quantum hall state \cite{21} can be realized in the coupled cavity arrays. There are a variety of technologies have been employed for realizing these systems, such as microwave circuit cavities \cite{22,23}, microtoroidal cavity arrays \cite{24,25}, photonic crystal defects \cite{26}.

In this work, we propose a scaling method for one-way quantum computation with spin-$\frac{1}{2}$ physical qubits in a 2D array of coupled cavities \cite{9,21}. We find that when all the qubits are simultaneously prepared in a spin state $|\downarrow\rangle$ or $|\uparrow\rangle$, after coherent displacements of the quantum state $|\Psi\rangle$ of cavity fields in a closed phase space, only the qubits in nearest-neighbor cavities can fast accumulate a nontrivial geometric phase shift, leading to creation of a large 2D cluster state within a very short time. Since the individual addressability of a qubit in the coupled cavity array is easily performed, the 2D cluster state serves as an effective resource for one-way quantum computation.

First we give a brief review of the geometric phase shift due to displacement along an arbitrary path \cite{27,28}. An arbitrary quantum state $|\Psi\rangle$ of a harmonic oscillator can be coherently displaced in the phase space. The effect of two sequential displacements $D(\alpha)$ and $D(\beta)$ is additive up to a phase factor:

$$D(\alpha)D(\beta) = D(\alpha + \beta)\exp[i\operatorname{Im}(\alpha\beta^*)].$$

For a path $P$ consisting of $N$ short straight sections $\Delta\alpha_i$, $i \in \{1, N\}$. The total operation is given by

$$D_{\text{total}} = D(\Delta\alpha_N) \cdots D(\Delta\alpha_1) = \sum_{i=1}^{N} D(\Delta\alpha_i) \exp\{i \operatorname{Im}[\sum_{i=2}^{N} \Delta\alpha_i (\sum_{j=1}^{i-1} \Delta\alpha_j^*)]\}. \quad (2)$$

Going to the limit of infinitesimal steps by replacing $\Delta\alpha_i$ with $d\alpha$ yields:

$$D_{\text{total}} = D[\int (d\alpha/dt)dt] \exp(i\gamma), \text{ with } \gamma = \operatorname{Im}[\int \alpha^*(d\alpha/dt)dt]. \quad (3)$$

If the path $P$ is closed, then:

$$D_{\text{total}} = D(0) \exp(i\gamma), \text{ and } \gamma = \operatorname{Im}\int_{P} \alpha^*(d\alpha/dt)dt. \quad (4)$$

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The phase $\gamma$ is referred to as the geometric phase, which is independent of the quantum state $|\Psi\rangle$. The fidelity of this geometric phase gate due to the displacements, might be significantly higher than that of the dynamical ones, as demonstrated in recent experiment in the context of trapped ions [27]. In the next part of this paper, we will show that this geometric phase can be used for preparation of arbitrary large 2D cluster state in principle. Finally, we discuss the feasibility of our scheme.

As sketched in Fig. 1, our model consists of a 2D ($M \times N$) array of cavities that are coupled via exchange of photons with a spin-$\frac{1}{2}$ physical qubit in each cavity. The transition of two spin states $|\uparrow\rangle_m,n, |\downarrow\rangle_m,n$ of the qubit at the site $\{m,n\}$ couples to the cavity mode with the standard Jaynes-Cummings type interaction $H_1 = \sum_{m=1}^{M} \sum_{n=1}^{N} (a_{m,n}g_{m,n} |\uparrow\rangle_m,n + a^{\dagger}_{m,n}g_{m,n} |\downarrow\rangle_m,n)$, where $a_{m,n}$ and $a^{\dagger}_{m,n}$ are creation and annihilation operators for the cavity mode at the site $\{m,n\}$, $g_{m,n}$ is the coupling strength. The Hamiltonian that describes the photons in the cavity modes is $H_{cav} = \sum_{m=1}^{M} \sum_{n=1}^{N} \delta a^\dagger_{m,n}a_{m,n} + J \sum_{m=1}^{M} \sum_{n=1}^{N} (a_{m,n}a^\dagger_{m,n+1} + a_{m,n+1}a^\dagger_{m,n} + H.c.)$, where $\delta$ denotes the detuning of the cavity mode from the transition of two spin states, $J$ is the tunneling rate of photons. $H_{cav}$ can be diagonalized via the Fourier transform: $a_{L,K} = \sqrt{MN} \sum_{m=1}^{M} \sum_{n=1}^{N} e^{i(\omega_{L,K} t + Lm + Kn)} a_{m,n}$, with $L = \frac{2\pi l}{N}$ and $K = \frac{2\pi k}{M}$ ($l = 0, 1, 2, ...M - 1, k = 0, 1, 2, ...N - 1$), to give $H_{cav} = \sum_{L} \sum_{K} \omega_{L,K} a^\dagger_{L,K}a_{L,K}$ with $\omega_{L,K} = \delta + 2J \cos L + 2J \cos K$. Then $H_1$, switched to an interaction picture with respect to $H'_{cav}$, can be rewritten as

$$H'_1 = \frac{1}{\sqrt{MN}} \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{L} \sum_{K} g e^{-i(\omega_{L,K} t + Lm + Kn)} a_{L,K} |\uparrow\rangle_m,n (|\downarrow\rangle_m,n + H.c.)].$$

(5)

Simultaneously we apply a classical field to each qubit, the interaction Hamiltonian is described by $H_{cla} = \sum_{m=1}^{M} \sum_{n=1}^{N} \Omega_{m,n}\sigma_{m,n}$, where $\Omega_{m,n}$ is the Rabi frequency of the classical field and $\sigma_{m,n}$ is the Pauli operator for the qubit at the site $\{m,n\}$. In the strong driving regime $2\Omega_{m,n} \gg g_{m,n}$, $\omega_{L,K}$, we can realize a rotating-wave approximation and eliminate the terms that oscillate with high frequencies, and obtain a new Hamiltonian [29]

$$H''_1 = \frac{1}{\sqrt{MN}} \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{L} \sum_{K} \sigma^{x}_{m,n} \sum_{L} \sum_{K} \left[ g e^{-i(\omega_{L,K} t + Lm + Kn)} a_{L,K} + g e^{i(\omega_{L,K} t + Lm + Kn)} a^{\dagger}_{L,K} \right],$$

(6)

where we have assumed that $g_{m,n} = g$. The Hamiltonian in Eq. (6) is an analogy to that for the high-speed gates with trapped ions simultaneously interacting many vibrational modes [28], except that the periodic phase factor $e^{-i(Lm + Kn)}$ is dependent of the site for qubit in the array, which is key importance, as shown below, for fast preparation of the cluster states in parallel.
FIG. 2: (Color online) The geometric phase shift $\Gamma$ as a function of $\delta/g$, assuming that $M = N = 19$, $J = 0.1g$, $g\tau = 3$.

We define a new operator $J_X = \sum_{m=1}^{M} \sum_{n=1}^{N} [\sigma^x_{m,n} e^{i(Lm + Kn)}]$. The time-evolution operator for the Hamiltonian in Eq. (6), based on Eq. (3), can be expressed as

$$U(\tau) = \sum_{L} \sum_{K} \exp \left( \frac{iJ_X^{*} \beta_{L,K} \mathcal{A}_{L,K} - J_X^{*} \beta_{L,K}^{*} \mathcal{A}_{L,K}}{M N \omega_{L,K}} \right) \exp \left( i\gamma J_X^{*} J_X \right),$$

with

$$\beta_{L,K} = \frac{g}{\sqrt{M N \omega_{L,K}}} (1 - e^{i\omega_{L,K} \tau}),$$

and

$$\gamma = \sum_{L} \sum_{K} \frac{2g^2}{MN \omega_{L,K}} \left[ \tau - \frac{\sin(\omega_{L,K} \tau)}{\omega_{L,K}} \right].$$

Then we consider the state evolution under the operators in Eq. (7) and $S = \prod_{m,n} \sigma^z_{m,n}$ (i.e., single-qubit operation $\sigma^z$ to each qubit) by turns

$$U'(t) = S U(\tau) S U(\tau) = \prod_{m,n} \sigma^z_{m,n} \sum_{L} \sum_{K} \exp \left( \frac{iJ_X^{*} \beta_{L,K} \mathcal{A}_{L,K} - J_X^{*} \beta_{L,K}^{*} \mathcal{A}_{L,K}}{M N \omega_{L,K}} \right) \exp \left( i\gamma J_X^{*} J_X \right) \prod_{m,n} \sigma^z_{m,n} \sum_{L} \sum_{K} \exp \left( \frac{iJ_X^{*} \beta_{L,K}^{*} \mathcal{A}_{L,K} - J_X^{*} \beta_{L,K} \mathcal{A}_{L,K}}{M N \omega_{L,K}} \right) \exp \left( i\gamma J_X^{*} J_X \right),$$

where we have used the commutation relation $[S, J_X^{*} J_X] = 0$ and anticommutation relations $\{S, J_X\} = 0$, $\{S, J_X^{*}\} = 0$. The operator in Eq. (10) is equivalent to

$$U'(t) = \exp \left[ \sum_{m' > m, n' > n} \frac{i\Gamma \sigma^z_{m,n} \sigma^z_{m', n'}}{M N} \right],$$

up to an overall phase factor, where $\Gamma = 4\gamma \cos[L(m' - m) + K(n' - n)]$.

The geometric phase shift $\Gamma$ has the following significant characters: (i) when $\delta = 0$, $\Gamma$ has a feasible value, while $\delta \gg g, J, \Gamma \to 0$. In Fig. 2, we plot $\Gamma$ a function of $\delta/g$, from it we can see that when $\delta \gg 10g$, $\Gamma$ is approximate to 0. (ii) Interestingly, if and only if $m' - m = \pm 1$ and $n' - n = 0$, or $m' - m = 0$ and $n' - n = \pm 1$, $\Gamma$ has a feasible value,
otherwise, $\Gamma \to 0$, which is an analogy to nearest-neighbor interaction in optical lattices [6]. In Fig. 3, we plot $\Gamma$ versus the interaction time $\tau$ in units of $1/g$, with different $\{m' - m, |n' - n|\}$. (ii) A scheme suitable for such solid-state system, where the photons in the cavities have a long coherence time, is most suitable for such solid-state system, where the photons in the cavities have a long coherence time, the effective preparation of large-scale 2D cluster states can be achieved within a short time. We show the feasibility of our method for various practical systems. It seems that our scheme is most suitable for such solid-state system, where the photons in the cavities have a long coherence time, effective preparation of large-scale 2D cluster states can be achieved within a short time.

In order to generate the cluster states, the initial state of all qubits in coupled-cavity array should be prepared in the superposition of two eigenstates of $\sigma^x_{m,n}$, for example the spin state $\{|\uparrow\rangle\}$. The time evolution of the qubits under the operator in Eq. (11), when $4\Gamma = \pi$, the state of the qubits is equivalent to a 2D cluster state. From Fig. 3, we see that the required time for preparation is in the order of $1/g$ with $\Gamma = 0.25\pi$. We note that some theoretical schemes [9,11,14] have been proposed for construction of 1D or 2D cluster states in coupled-cavity array. Besides geometric phase shifts, our method, in principle, is suitable for preparation of arbitrary large 2D cluster states in parallel, which provides the possibility to implement scale quantum computation with their coherence times.}

Now we address the experimental feasibility of the proposed schemes. First, we show that our method for solid-state qubit trapped in a 2D array of circuit cavities [8], in which solid-state qubits such as Cooper pair boxes (CPB) and quantum dots (QD) are strongly coupled to circuit cavities [22,23,31,32], while the microwave photons have small loss rates. The tunneling rate $J$ of photons between neighboring circuit cavities has a feasible value about $100 MHz$, and the qubit frequency can be tuned in a large range. Typically, for CPBs interacting with the circuit cavities [22,23], the coupling strength is $g \sim 2\pi \times 50 MHz$, the photon lifetime is $T_c \sim 1/\kappa_c \sim 20\mu s$, and the dephasing time of the two spin states $\{\langle\uparrow\rangle_{m,n}, \langle\downarrow\rangle_{m,n}\}$ is $T_a \sim 1\mu s$. The required time for preparation of arbitrary large-qubit cluster state, in principle, is $T \sim 0.01\mu s$ [33], which is much shorter than $T_c, T_a$. For double-quantum-dot qubits trapped in the circuit cavities [31,32], the coupling strength is $g' \sim 2\pi \times 125 MHz$, the photon decay time $T'_c \sim 50\mu s$, the spin dephasing time and charge relaxation time is about $T'_a \sim 1\mu s$. The required time for preparation of a large-qubit cluster state is $T' \sim 5\mu s \ll T'_c, T'_a$. For these solid-state qubits, as shown above, the required time for preparation of cluster state is smaller than microwave-photon coherence time, by about three orders of magnitude. Therefore the cavity loss can be neglected in our situation. The dephasing of the qubits themselves is the dominant source of decoherence. After the solid-state qubits prepared in the cluster state, they can be stored in the molecular ensembles [23], which serve as a quantum memory with a long coherence time. Second, for toroidal micro-cavities [21,24], in which the achievable parameters are predicted to be $g'' \sim 2.5 \times 10^8 Hz$, spontaneous emission rate of the high energy is $\kappa_e = 1.6 \times 10^7 Hz$, the photon decay time $T''_c \sim 1/\kappa_e \sim 1/(0.4 \times 10^5 Hz) = 25\mu s$, and the tunneling rate $J \sim 1.6 \times 10^6 Hz$. For suppressing atomic spontaneous emission, two stable low levels, which are coupled efficiently by a Raman process, are used as two spin states $\{\langle\uparrow\rangle_{m,n}, \langle\downarrow\rangle_{m,n}\}$. Thus the effective strength $g'' \sim 1 \times 10^8 Hz$, and the effective energy relaxation time $T''_a \sim 100/\kappa_e \sim 6\mu s$. The required time for preparation of cluster state is $T'' \sim 0.08\mu s \ll T''_c, T''_a$. In conclusion, we have provided an method to a large-scale one-way quantum computer with spin-$\frac{1}{2}$ physical qubits in a 2D array of coupled cavities. After coherent displacements of the quantum state of cavity fields in a closed phase space, only the qubits in nearest-neighbor cavities can accumulate a nontrivial geometric phase shift, which is of importance for our scheme. We show the feasibility of our method for in various practical systems. It seems that our scheme is most suitable for such solid-state system, where the photons in the cavities have a long coherence time, effective preparation of large-scale 2D cluster states can be achieved within a short time.
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