Small scale microwave background fluctuations from cosmic strings

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Abstract

The Cosmic Microwave Background (CMB) fluctuations at very small angular scales (less than 10') induced by matter sources are computed in a simplified way. The result corrects a previous formula appearing in the literature. The small scale power spectrum from cosmic strings is then calculated by a new analytic method. The result compares extremely well with the spectrum computed by numerical techniques (when the old, incorrect, formula is used). The upper bound on the string parameters derived from OVRO data is re-examined, taking into account the non-Gaussian nature of stringy perturbations on small scales. Assuming a conventional ionization history, the bound is $\gamma G\mu < 11 \times 10^{-6}$, where $\gamma^2$ is the number of horizon lengths of string per horizon volume. Current simulations give $\gamma^2 = 31 \pm 7$.

Subject headings: cosmology: cosmic microwave background — cosmic strings

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1 Introduction

It is now widely accepted that cosmic microwave background (CMB) fluctuations test the physics of the very early universe. With the recent flurry of experiments on various angular scales (Meyer, Cheng & Page 1991; Smoot et al. 1992; Gaier et al. 1992; Meinhold et al. 1993; Ganga, Cheng, Meyer & Page 1993; Schuster et al. 1993; Gundersen et al. 1993), we are becoming more able to eliminate theories of the origin of these fluctuations. There are two main contenders in the competition to explain the origin of these fluctuations: quantum fluctuations in the metric during inflation (see for example Brandenberger, Feldman & Mukhanov 1992), and various kinds of semiclassical dynamics after phase transitions in field theories (Kibble 1976; Zel’dovich 1980; Vilenkin 1980; Turok 1989; Barriola & Vilenkin 1989; Bennett & Rhie 1990). Both are able to generate a more or less Harrison-Zel’dovich spectrum of density fluctuations, and both produce CMB fluctuations which are consistent with the COBE observations (Smoot et al. 1992), as far as the theoretical uncertainties allow (Peebles 1982; Bond & Efstathiou 1987; Bennett, Stebbins & Bouchet 1992; Bennett & Rhie 1993; Pen, Spergel & Turok 1993). The physics is quite different in each case, and each suffers in different ways from theorists’ prejudices. However, it is to be hoped that future measurements of the spectrum of CMB fluctuations on a wide variety of angular scales will be able to distinguish experimentally between theories.

This paper is concerned with calculating the very small angular scale fluctuations from cosmic strings (Bouchet, Bennett & Stebbins 1988). Strings are perhaps the most venerable of the theories based on the dynamics of field theories during and after phase transitions. Despite predating the inflationary scenario, the theory has suffered from analytic intractability and is consequently comparatively undeveloped. Early work on string seeded galaxy perturbations (Brandenberger & Turok 1986; Traschen, Branden-
berger & Turok 1986) was based on the first numerical simulations (Albrecht & Turok 1989) whose detailed results have not all been confirmed by the two subsequent groups to work on the subject (Bennett & Bouchet 1989; Bennett & Bouchet 1990; Allen & Shellard 1990a; Allen & Shellard 1990b). What is lacking is a good analytic understanding of the evolution of the string network, although progress is being made in this regard (Kibble & Copeland 1990; Copeland, Kibble & Austin 1992; Embacher 1992a; Embacher 1992b). It is the intention of this work to try and create an analytic approach to the calculation of CMB fluctuations from strings. Although the results presented apply only to fluctuations on very small angular scales (less than about 10'), these are precisely the scales on which strings make their most distinctive contribution.

The philosophy behind the current approach is quite simple. It is that statistical measures of the string network itself can be used to calculate the statistics of the CMB fluctuations. One can in fact guess the form of simple string correlation functions on general grounds, backed up by some intuition gained from numerical simulations. In this paper, Stebbins’ (1988) formula for small angle CMB anisotropies is rederived in a more direct way, with an error in his and previous versions of this work corrected (Stebbins 1993). Then, the two-point string correlators are used to calculate the very high frequency end of the power spectrum of CMB fluctuations. The result is compared to a numerical computation of the spectrum by Bouchet, Bennett and Stebbins (1988), using the old anisotropy formula. In view of the approximations made, it is gratifying that the current approach reproduces both the shape and the amplitude of the spectrum very well. Limits on the string linear mass density are then re-examined. The results can only be trusted on very small angular scales, for which the best experimental limits currently available are derived from recent VLA observations (Fomalont et al. 1993) and from OVRO (Readhead et al. 1989; Myers, Readhead & Lawrence 1993). The best experimental geometry for finding string is the recent RING experiment at OVRO (Myers,
Readhead & Lawrence (1993), which consists of 96 overlapping double difference fields in a circle of radius $\sim 2^\circ$ around the North Celestial Pole. However, sky coverage has been increased at the expense of sensitivity, so the best limits on the r.m.s. fluctuations still come from the NCP observations (Readhead et al. 1989). Other experiments at larger scales have been used in the past to constrain the string scenario, although in view of the current uncertainty surrounding the predictions of strings these should not be regarded as reliable.

The plan of the paper is as follows. Firstly, I present the rederivation of Stebbins’ (1988) formula, for small angle microwave anisotropies. I then use this formula to derive an expression for the CMB power spectrum, subject to some plausible assumptions. These assumptions are justified by comparison to the numerical work of Bouchet, Bennett and Stebbins (1988), henceforth referred to as BBS. I then use the derived correlation function to obtain an upper limit on a certain combination of string parameters. To do this a revised Bayesian analysis is performed on OVRO data, which models the non-Gaussian statistics of strings. The bound, presented in (5.19), is expressed as one on $\gamma G \mu$, where $\gamma^2$ is the number of horizon lengths of string per horizon volume, for $\gamma$ is not well determined as yet. Furthermore, this combination appears in all calculations of string-induced perturbations, and so bounding it is more useful than simply bounding the string tension.

## 2 CMB anisotropies from moving strings

In this section I derive the formula for the temperature pattern induced in the CMB by a string moving in front of it. A Minkowksi space background is used, which limits the applicability to fluctuations inside the horizon at decoupling. Stebbins (1993) has corrected his original formula (Stebbins 1988), which was also incorrectly derived in
earlier versions of this work. We are now, happily, in agreement.

Imagine a large box of length \( b \) and cross-sectional area \( A \). In this box is some string with spacetime coordinates \( X^\mu(\sigma, \tau) \), where \( \sigma \) and \( \tau \) are respectively spacelike and timelike worldsheet coordinates. Consider the gravitational effect of the string on a set of photons passing through this box with momentum \( p_\mu = (E; 0, 0, E) \). Their unperturbed geodesics are

\[
Z^\mu = x^\mu + \lambda p^\mu. \tag{2.1}
\]

We would like to know what their energies are at \( x^\mu = (t_0; x, z_0) \) as a result of the string’s gravitational field. To first order in linear theory we have

\[
\delta p_\mu(x) = -\frac{1}{2} \int_{\lambda_1}^{\lambda_0} d\lambda h_{\nu\rho,\mu}(Z(\lambda)) p^\nu p^\rho, \tag{2.2}
\]

where \( h_{\mu\nu} \) is the perturbation around the Minkowski metric \( \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1) \).

To go further it is very convenient to choose the harmonic gauge, or

\[
h^{\mu\nu} = h^{\nu\mu}, \quad h^{\mu} = 0. \tag{2.3}
\]

where \( h = h^{\nu} \). In this gauge the first order field equations become

\[
\partial^2 h_{\mu\nu} = 16\pi G(T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T), \tag{2.4}
\]

where \( T_{\mu\nu} \) is the stress tensor. The string has its own gauge freedom, which we restrict by choosing the conformal gauge, or

\[
\dot{X} \cdot \dot{X} = 0, \quad \dot{X}^2 + \dot{X}^2 = 0, \tag{2.5}
\]

where the dot and the prime denote differentiation with respect to \( \tau \) and \( \sigma \) respectively. In this gauge the stress tensor is given by

\[
T^{\mu\nu} = \mu \int d\tau d\sigma (\dot{X}^\mu \dot{X}^\nu - \dot{X}^\mu X^\nu) \delta^{(4)}(x - X(\sigma, \tau)). \tag{2.6}
\]
One way to proceed from here is to compute $h_{\mu \nu}$ from $T_{\mu \nu}$ with a retarded Green’s function, differentiate to get $h_{\nu \rho, \mu}$, and finally integrate with respect to the photon affine parameter (Stebbins 1988). However, a great simplification is afforded by instead calculating $\nabla^2 \delta p_{\mu}$, where $\nabla$ is the partial derivative with respect to the transverse coordinates $x$. This is done in a roundabout fashion, using the fact that $(\partial_t - \partial_z)f(Z) = E^{-1}df/d\lambda$:

$$\nabla^2 \delta p_{\mu} = (\partial_t^2 - \partial_z^2 - \partial^2)\delta p_{\mu}$$
$$= -\frac{1}{2} \int_{\lambda_1}^{\lambda_0} d\lambda [(\partial_t + \partial_z)(\partial_t - \partial_z) - \partial^2](h_{\nu \rho, \mu} p^\nu p^\rho)$$
$$= \left[ \frac{1}{2E}(\partial_t + \partial_z)h_{\nu \rho, \mu} p^\nu p^\rho \right]^{\lambda_0}_{\lambda_1} - 8\pi G \partial_{\mu} \int_{\lambda_1}^{\lambda_0} d\lambda T_{\nu \rho} p^\nu p^\rho. \quad (2.7)$$

For temperature fluctuations we are interested in the variation on the photon energy $\delta p_0 \equiv \delta E$. For this component we may use a cunning identity due to Stebbins (1993), whose proof is reproduced in the Appendix. Writing $\hat{p}^\mu = p^\mu/E$, it is

$$\hat{p}^\nu T_{\nu,0} = -\nabla^i_\perp T_{i\rho} - \frac{1}{E} \frac{d}{d\lambda} \hat{p}^i T_{i\rho}. \quad (2.8)$$

Here, $\nabla^i_\perp = (\delta^{ij} - \hat{p}^i \hat{p}^j)\partial_j$ is the transverse derivative. Thus we have

$$\nabla^2 \frac{\delta E}{E} = 8\pi G \int_{\lambda_1}^{\lambda_0} E d\lambda \nabla^i_\perp T_{i\rho} \hat{p}^\rho$$
$$+ \frac{1}{2} \left[ (\partial_t + \partial_z)\partial_t \hat{h} - 16\pi G \hat{p}^i \hat{p}^\rho T_{i\rho} \right]^{\lambda_0}_{\lambda_1}. \quad (2.9)$$

The terms in the square brackets are fluctuations on the bounding surfaces of our imaginary box. Those at the observing time are negligible, since we are far from any sources, but if the initial surface is the decoupling time, there will be important fluctuations present. Our justification for dropping these is the finite thickness of the last scattering surface, which acts to smear out fluctuations on scales greater than about 10', assuming that $z_{dec} \simeq 1000$.

We now evaluate (2.9) for string sources. It is customary in the string literature to choose the temporal gauge $X^0 = \tau$; that is, identify the worldsheet time with the
background time coordinate. In that case we find (using the fact that $\delta T/T = \delta E/E$)

$$\nabla^2 \frac{\delta T}{T} = -8\pi G\mu \int d\sigma \left( \frac{\dot{X} \cdot \hat{p}}{(\dot{X} \cdot \hat{p})^2} \right) \cdot \nabla \delta^{(2)}(x - X),$$

(2.10)

where worldsheet variables are evaluated at $t_r$, given by $x^+ = X^+(\sigma, t_r)$, or $t_r = t + z - X^3(\sigma, t_r)$. Once again, the reader is cautioned that this formula, again due to Stebbins (1993), corrects Stebbins’ (1988) result, and earlier versions of this work.

The expression is simplified if instead we use the light cone gauge

$$X^+(\sigma, \tau) = \tau.$$  \hspace{1cm} (2.11)

The time parameter $\tau$ then labels the intersections with a set of null hyperplanes with the worldsheet, which is in hindsight an intuitive thing to do, since our photon geodesics $Z^\mu = x^\mu + \lambda p^\mu$ form just such a set. Then we find

$$\nabla^2 \frac{\delta T}{T} = -8\pi G\mu \int d\sigma \dot{X} \cdot \nabla \delta^{(2)}(x - X),$$

(2.12)

where quantities are now evaluated at $\tau = x^+$.

These is a nice result: it says that in Minkowski space the temperature distortions caused by moving strings depend only on the apparent positions of the strings and their light cone gauge transverse peculiar velocities, and not on the entire history of the worldsheet. In using this gauge we will need to exercise care in taking results from numerical simulations, where correlation functions are always measured in the temporal gauge.

3 Power spectrum

In this section an expression is derived for the power spectrum of temperature fluctuations in terms of the two point correlators of the transverse coordinates of the string, $X^A(\sigma)$. The basic two point functions are

$$\langle \dot{X}^A(\sigma) \dot{X}^B(\sigma') \rangle, \quad \langle \dot{X}^A(\sigma) \dot{X}^B(\sigma') \rangle, \quad \langle \dot{X}^A(\sigma) \dot{X}^B(\sigma') \rangle.$$  \hspace{1cm} (3.1)
The angle brackets denote an average over an ensemble of strings in our imaginary box. The starting assumption is that the string ensemble is a Gaussian process: that is, all correlators can be calculated in terms of the two point functions. This has not been tested directly, but the results can be regarded as justifying the means.

We now make some more justifiable assumptions about the ensemble: (i) rotation, reflection and translation invariance of the transverse coordinates; and (ii) worldsheet reflection and translation invariance. Then we can reduce the number of correlation functions to four:

\[
\langle \dot{X}^A(\sigma) \dot{X}^B(\sigma') \rangle = \frac{1}{2} \delta^{AB} V(\sigma - \sigma'),
\]

(3.2)
\[
\langle \dot{X}^A(\sigma) \dot{X}^B(\sigma') \rangle = \frac{1}{2} \delta^{AB} M_1(\sigma - \sigma') + \frac{1}{2} \epsilon^{AB} M_2(\sigma - \sigma'),
\]

(3.3)
\[
\langle \dot{X}^A(\sigma) \dot{X}^B(\sigma') \rangle = \frac{1}{2} \delta^{AB} T(\sigma - \sigma').
\]

(3.4)

For later convenience two other correlators will be defined:

\[
\Gamma(\sigma - \sigma') = \int_{\sigma}^{\sigma'} d\sigma_1 \int_{\sigma}^{\sigma'} d\sigma_2 T(\sigma_1 - \sigma_2) \equiv \langle (X^A(\sigma) - X^A(\sigma'))^2 \rangle,
\]

(3.5)
\[
\Pi(\sigma - \sigma') = \int_{\sigma}^{\sigma'} d\sigma_1 M_1(\sigma_1 - \sigma) \equiv \langle (X^A(\sigma) - X^A(\sigma')) \dot{X}^A(\sigma) \rangle.
\]

(3.6)

V and T are symmetric in their argument, while M_1 and M_2 are antisymmetric. It turns out that we will not need the mixed correlator M_2.

The two point correlation function of the temperature fluctuations is defined to be

\[
C(\mathbf{r}) = \langle \delta T(\mathbf{x}) \delta T(\mathbf{x} + \mathbf{r}) \rangle / T^2
\]

(3.7)

where again the angle brackets denote an average over the string ensemble. In terms of the two dimensional Fourier transform

\[
\delta_k = \int d^2x \frac{\delta T}{T}(\mathbf{x}) e^{i \mathbf{k} \cdot \mathbf{x}},
\]

(3.8)

we have

\[
C(\mathbf{r}) = \frac{1}{A} \int \frac{d^2k}{(2\pi)^2} |\delta_k|^2 e^{-i \mathbf{k} \cdot \mathbf{r}}.
\]

(3.9)
The power spectrum is the ensemble average of $|\delta_k|^2$. The Fourier transform of the temperature fluctuation formula (2.12) is

$$-k^2\delta_k = i8\pi G \mu k^A \int d\sigma \dot{X}^A(\sigma) e^{i\mathbf{k} \cdot \mathbf{X}(\sigma)}. \quad (3.10)$$

Thus, normalising for the moment to unit box area $A$,

$$P(k) = \left(8\pi G \mu \right)^2 \frac{k^A k^B}{k^4} \int d\sigma d\sigma' \left\langle \dot{X}^A(\sigma) \dot{X}^B(\sigma') e^{i\mathbf{k} \cdot (\mathbf{X}(\sigma) - \mathbf{X}(\sigma'))} \right\rangle. \quad (3.11)$$

With our assumptions about the string correlation functions the ensemble average can be reduced to

$$P(k) = \frac{1}{4} \left(8\pi G \mu \right)^2 \frac{1}{k^2} \int d\sigma d\sigma' \left( V(\sigma - \sigma') + \frac{1}{2} k^2 \Pi^2(\sigma - \sigma') \right) e^{-k^2 \Gamma(\sigma - \sigma')/4}. \quad (3.12)$$

At this point one should check the Gaussian approximation by measuring $V$, $M_1$ and $T$ in one’s favourite string simulation, computing $P(k)$ from (3.12), and then comparing with $|\delta_k|^2$ computed by a direct Fourier transform of the right hand side of (3.10) averaged over a set of string configurations $\{\dot{X}(\sigma), X(\sigma)\}$. The latter procedure was adopted by BBS using a string simulation with the code of Bennett and Bouchet (Bennett & Bouchet 1989; Bennett & Bouchet 1990). Unfortunately, there is very little correlation function data available, so we will have to content ourselves with making some educated guesses for the forms of $V$, $M_1$ and $T$, based on visual inspection of the simulations.

First, however, we must make the connection between the Minkowski space formalism and the real behaviour of photon geodesics in an expanding universe. Suppose the universe is flat with background metric $g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}$, where $\eta$ is the conformal time. Our calculations are valid for comoving momenta and comoving coordinates, and scaled metric $h_{\mu\nu} = g_{\mu\nu}/a^2$, provided we can neglect terms of order $h_{\mu\nu}\dot{a}/a$ in comparison to $\dot{h}_{\mu\nu}$. This means that our approximation can only be trusted for perturbations on scales inside the causal horizon at the time of decoupling of matter and radiation, $\eta_{\text{dec}}$. Bear-
ing that in mind, we would like to infer things about our light cone gauge correlation functions in a comoving box of area $A$ and length $\eta_0 - \eta_{dec}$.

Let us outline what is believed about the three dimensional two point correlators,

\[
\langle \dot{X}^i(\sigma) \dot{X}^i(\sigma') \rangle = \frac{1}{2} G^+(\sigma - \sigma'),
\]

(3.13)

\[
\langle \dot{X}^i(\sigma) \dot{X}^i(\sigma') \rangle = \frac{1}{2} G^-(\sigma - \sigma'),
\]

(3.14)

\[
\langle \dot{X}^i(\sigma) \dot{X}^i(\sigma') \rangle = \frac{1}{4} \Omega(\sigma - \sigma'),
\]

(3.15)

where Embacher’s (Embacher 1992a; Embacher 1992b) notation has been echoed. The correlators are all dimensionless functions, and so must have a scale $\xi$ in them if they are to be non-trivial. The evidence from the numerical simulations is that this correlation length is proportional to the most important physical scale in the expanding universe, the horizon size $\eta$. Let $s = (\sigma - \sigma')/\xi(\eta)$. Then it is found that

\[
\xi^2 \int_0^s ds_1 \int_0^s ds_2 G^+(s_1 - s_2) \sim \xi^2 s^{2\nu}.
\]

(3.16)

The exponent $\nu$ is a function of scale:

\[
\nu(s) \to \begin{cases} \frac{1}{2} & \text{if } s \gg 1, \\ 1 & \text{as } s \to 0. \end{cases}
\]

(3.17)

That is, on large scales, the network behaves as a Brownian random walk, while on very small scales it is approximately straight. There is numerical evidence for an “intermediate fractal” region for $s \approx 1$ (Bennett & Bouchet 1990; Allen & Shellard 1990b), where the exponent slowly interpolates between 1 and 0.5.

The velocity correlation function $G^-$ starts out at $2\bar{v}^2$, twice the mean square velocity, and vanishes rapidly for $s \gg 1$ as befits a random walk. There is also a constraint

\[
\int_0^\infty ds G^-(s) = 0
\]

(3.18)

which arises because there can be no coherent velocities in the network above the horizon scale. Thus $G^-(s)$ must go negative somewhere. The form of the mixed correlator $\Omega$
is also subject to the same integral constraint, and must also vanish at \( s = 0 \) by its antisymmetry. The inferred forms of the correlators are displayed in Figure 1.

The light cone gauge correlators contain string coordinates evaluated at all times between \( \eta_{\text{dec}} \) and \( \eta_0 \), and thus have contributions from the equal time temporal gauge correlators at all times in this interval, as well as from unequal time ones. However, most of the string in the box is near the initial time surface, so we will assume it is the correlation length of the string network at \( \eta_{\text{dec}} \) that sets the scale for the light cone gauge correlators.

To check that this is true, we note that the physical length density of the string is proportional to \( \xi^{-2} \), a result of having of order one segment of length \( \xi \) in a volume \( \xi^3 \). If we write \( \xi = \eta/\gamma \), where \( \gamma \) is the conformal time, then \( \gamma^2 \) is the number of physical horizon lengths of string per horizon volume \( \eta^3 \), and the comoving length of string in comoving volume \( Ad\eta \) is

\[
dL_c = \left( \frac{\gamma^2}{a(\eta)\eta_0^2} \right) Ad\eta, \quad (3.19)
\]

where \( \eta_0 \) is today’s conformal time. In the radiation era, \( a(\eta) = (\eta/\eta_0)^2 \), so

\[
dL_c = \gamma^2 \frac{d\eta}{\eta^2}. \quad (3.20)
\]

Thus most of the string is near \( \eta_{\text{dec}} \), and it is justifiable to assume that the light cone gauge correlators \( T, V \) and \( M_1 \) have the same general form as \( G^+, G^- \) and \( \Omega \) respectively, with scale \( \xi(\eta_{\text{dec}})/a(\eta_{\text{dec}}) \). The exponent \( \nu \) is likely to be greater than 0.5, for the apparent position of the string need not be precisely Brownian: the nearer sections of string are straighter.

We are now in a position to derive the asymptotic behaviour of \( P(k) \) as \( k\xi \) gets both large and small. Let us examine the first term in the power spectrum \( (3.12) \),

\[
P_V(k) = (8\pi G\mu)^2 \frac{1}{4k^2} \int d\sigma_+d\sigma_- V(\sigma_-) e^{-k^2\Omega(\sigma_-)/4}, \quad (3.21)
\]
where \( \sigma = \sigma \pm \sigma' \). For \( k \xi \gg 1 \) we find

\[
P_V(k) \simeq (8\pi G\mu)^2 \frac{1}{2k^2} L v^2 \left( \frac{4\pi}{X^2} \right) \frac{1}{k'},
\]

(3.22)

where \( L \) is the total length of string in the box. For \( k \xi \ll 1 \) we define \( \xi \) such that \( \Gamma(\sigma) \to \xi |\sigma| \) at large \( \sigma \), and find

\[
P_V(k) \simeq (8\pi G\mu)^2 \frac{1}{2k^2} (L\xi) v^2 u_1 k^2 \xi^2,
\]

(3.23)

where \( u_1 \) is a constant related to a moment of \( V \):

\[
u_1 = -\frac{1}{4v^2} \int |s| V(s) ds.
\]

(3.24)

The contribution to the power spectrum from the mixed correlator \( M_1 \) is

\[
P_M(k) = (8\pi G\mu)^2 \frac{1}{8} \int d\sigma_+ d\sigma_- \Pi^2(\sigma_-) e^{-k^2 \Gamma(\sigma_-)/4}.
\]

(3.25)

\( \Pi^2 \) goes as \( \sigma_-^4 \) at small \( \sigma_- \), so

\[
P_M(k) \sim (8\pi G\mu)^2 (L\xi)(k\xi)^{-5}.
\]

(3.26)

For small \( k \xi \), we define another constant \( i_1 = -\int |s| \Pi^2(s) ds / 4v^2 \) to obtain

\[
P_M(k) = \frac{1}{4} (8\pi G\mu)^2 (L\xi)i_1 k^2 \xi^2.
\]

(3.27)

Thus the mixed correlator contributes little at the asymptotes of the spectrum, and the form of \( k^2 P(k) \) is now clear. It rises as \( k^2 \) for \( k \ll \xi^{-1} \), and falls of as \( k^{-1} \) for high spatial frequencies. This general form is to be expected: at low resolution, the temperature pattern is uncorrelated, and so the power spectrum vanishes at low spatial frequency, while at high resolution we see a collection of randomly oriented edges.

At this point it must be stressed that the function \( P(k) \) that we have been computing in this section does not represent the only contribution to the fluctuations in the apparent
temperature. The other contributions include Doppler shifting from scattering off moving electrons at decoupling, and Sachs-Wolf contributions from metric fluctuations in the last scattering surface and from decaying scalar perturbations created by the motion of the strings as they enter the horizon between decoupling and the present time. Furthermore, the calculation is wrong for $k < \eta_{\text{dec}}^{-1}$, which is a consequence of using the Minkowski space formalism.

One could imagine fixing up this latter problem by computing the contributions to the power spectrum $\delta P(k, \eta)$ from different times, cutting off the spectrum for $k < \eta^{-1}$, and then adding them up for all times since $\eta_{\text{dec}}$. This approach was taken by Bennett, Stebbins and Bouchet (1992) to compute the fluctuations on COBE scales. This effectively assumes that there are no correlations between the string positions and velocities at one time and another, which perhaps not very safe in view of the correlations manifest in their numerical data (see Section 4). It also neglects the scalar perturbations induced by the strings as they fall within the horizon. One should not necessarily regard the fluctuations computed thus as a lower bound, for the temperature anisotropies caused by the scalar modes may well be correlated with those from the discontinuities across the string. A related approach in position space was adopted by Perivolaropoulos (1993). The correlation function was taken to be an incoherent sum of contributions made each time the universe doubles in size. The individual contributions were modelled by a cosine (which is essentially a geometrical factor arising from assuming that the strings are perfectly straight within a horizon distance), plus a sharp cut-off at the horizon. This form for the correlation function is however rather different from that derived in Section 5.

Doppler scattering occurs within the last scattering surface, and is a result of velocity perturbations to the electrons. These cannot be coherent above the horizon size at decoupling, which corresponds to an angular scale of about $2^\circ$. They are also expected
to be smoothed off on a scale corresponding to the thickness of the last scattering surface, which is about $\delta z \simeq 150$ for the conventional ionization history, where $z_{\text{dec}} \simeq 1100$. This corresponds to an angular scale of around $10'$. Thus we can trust our power spectrum only on scales below this, or $k_{\eta_{\text{dec}}} \gg 0.1$.

4 Comparison with numerical simulations

The BBS power spectrum was obtained by solving a Fourier space version of the temperature fluctuation formula for several null slices taken from a single run of a matter era string simulation (Bennett & Bouchet 1990), in which the scale factor at the end of the slice was twice its initial value. However, the incorrect formula was used, namely

$$\nabla^2 \delta T = -8\pi G\mu \int d\sigma \mathbf{u} \cdot \nabla \delta^{(2)}(\mathbf{x} - \mathbf{X}),$$

(4.1)

with $\mathbf{u} = (1 - (\dot{X} \cdot \dot{\mathbf{p}})^2/\dot{X} \cdot \dot{\mathbf{p}}^2) \dot{X}$ (all quantities being evaluated in the temporal gauge). For the purposes of testing techniques espoused in this paper, the incorrect formula must be used to compare the theoretical prediction with the numerical result. BBS fitted the power spectrum with a function with four parameters: two for the asymptotic power laws, one for the position of the maximum, and one for the overall amplitude. Their fit is expressed in the integrated form

$$\int_{2\pi/\lambda}^{\infty} \frac{d^2k}{(2\pi)^2} P(k) = (6G\mu)^2 \left( \frac{\lambda/\lambda_h}{(0.6)^{1.7} + (\lambda/\lambda_h)^{1.7}} \right)_{0.7},$$

(4.2)

where $\lambda_h$ is the comoving horizon size at decoupling. This corresponds to low and high $k$ behaviour of $k^{1.7}$ and $k^{-1.2}$ respectively. The exponents are not significantly different from the predicted values of 2 and -1. To illustrate this, the numerical spectrum has been compared to the function

$$F(k) = k^2 P(k)/(G\mu)^2 2\pi,$$

(4.3)
with
\[ F(k) = \frac{a_0(k/k_h)^2}{[a_1^2 + (k/k_h)^2]^{3/2}}, \] (4.4)
k_h being \(2\pi/\lambda_h\). Trial and error gave \(a_0 = 65 \pm 5\) and \(a_1 = 1.8 \pm 0.2\). Figure 2 shows \(k^2 P(k)/2\pi\) for the numerical spectrum, the BBS fit, and the two parameter fit (4.3, 4.4). The errors shown are the standard deviations of twelve null surfaces in the same simulation, taken at four different times over an expansion factor of \(\sqrt{2}\), in three orthogonal directions. They are clearly highly correlated, which makes \(\chi^2\) fitting pointless.

Not only does the theory predict the form of the spectrum at the high and low frequency ends from the little information that we have about correlation functions, it also predicts the normalisation. The theoretical asymptote of \(F\) is, when we return the normalizing area \(A\) to its rightful place,
\[ F_{th}(k) = 16\sqrt{\pi \bar{v}^2} \left( \frac{L\lambda_h}{\bar{v}} \right) \left( \frac{k_h}{k} \right), \] (4.5)
where \(\bar{v}^2 = \langle \dot{X}^2 \rangle\), which we would like to compare to the numerical fit \(a_0(k_h/k)\). Here we run into a problem engendered by using the light cone gauge, for the numerical simulations do not directly give \(\bar{v}, \bar{\dot{v}}\) or \(L\). We have to convert to the temporal gauge, in which
\[ \bar{v}^2 \rightarrow \langle u^2(\tau_r) \rangle, \quad \bar{\dot{v}} \rightarrow \sqrt{\langle \dot{X}^2(\tau_r) \rangle}, \] (4.6)
The mean square velocity \(V^2 = \langle \dot{X}^4 \rangle\) in the matter era in the Bennett and Bouchet simulations is \(0.37 \pm 0.02\) (Bennett & Bouchet 1990). Since the temperature fluctuations depend only on the transverse components, \(\langle u^2 \rangle\) must be strictly less than \(\frac{2}{3}V^2\). The exact figure cannot be calculated with the information at hand, but we can make a crude guess by replacing \((\dot{X}^3)^2\) and \((\ddot{X}^3)^2\) by their mean values, and ignoring all other correlations. We also face the problem of evaluating terms like \(\langle (\dot{X} \cdot \ddot{p})^{-m} \rangle\), with \(p = 2, 4\). We cannot consistently use a Gaussian approximation here, since these expectation values diverge.
due to the contribution at $X^3 = -1$, where the string has a cusp moving towards the observer. However, we should not expect a Gaussian distribution for quantities like $\dot{X}^3$ which are constrained to be less than or equal to 1. Instead, we shall do the simplest thing, which is a series expansion in $\langle (\dot{X}^3)^2 \rangle$, so that

$$\langle (\dot{X} \cdot \hat{p})^{-m} \rangle \simeq 1 + \frac{1}{2}m(m+1)\langle (\dot{X}^3)^2 \rangle.$$ \hspace{1cm} (4.7)

We can evaluate the required correlators with one more piece of information, the temporal gauge constraint $\langle (\dot{X}^i)^2 \rangle + \langle (\dot{X}^i)^2 \rangle = 1$. Then

$$\langle (\dot{X}^3)^2 \rangle = \frac{1}{3}(1 - V^2), \quad \langle (\dot{X}^3)^2 \rangle = \frac{1}{3}V^2, \hspace{1cm} (4.8)$$

$$\langle \dot{X}^2 \rangle = \frac{2}{3}(1 - V^2), \quad \langle \dot{X}^2 \rangle = \frac{2}{3}V^2. \hspace{1cm} (4.9)$$

Thus we find

$$\langle u^2 \rangle = \frac{2}{3}V^2 \left(1 - \frac{2}{3}(1 - V^4) + \frac{1}{3}(1 - V^2)^2(1 + \frac{10}{3}V^2)\right). \hspace{1cm} (4.10)$$

Putting in the numerical value of $V^2$, we have

$$\bar{v}^2 = 0.18 \pm 0.01, \hspace{1cm} (4.11)$$

$$\bar{t}^2 = 0.42 \pm 0.02. \hspace{1cm} (4.12)$$

The total projected comoving length of string per unit area is

$$\left(\frac{L}{A}\right) = \lambda^{-1}_h \left[1 - (z/z_{dec})^{1/2}\right] \gamma^2 \hspace{1cm} (4.13)$$

where $\gamma^2 = \rho_s \lambda^2_h / \mu z_{dec}$ is the number of horizon lengths of string per horizon volume (Copeland, Kibble & Austin 1992), which is $31 \pm 7$ in the matter era simulations of Bennett and Bouchet (Bennett & Bouchet 1990). The BBS spectra have $z/z_{dec} = 1/2$, so

$$L \lambda_h / A \simeq 9.1 \pm 2.1. \hspace{1cm} (4.14)$$
Thus the asymptotic theoretical spectrum is

\[ F_{th}(k) \simeq (72 \pm 17)(k_h/k), \tag{4.15} \]

which is in remarkably good agreement with the numerical spectrum. The assumptions that have been made amount to approximating the source of the small scale fluctuations by uncorrelated straight segments of moving string, which we see reproduces the spectrum very well. The error quoted in (4.15) is statistical, and mostly due to fluctuations in the string density between different simulations.

Lacking data for \( V(s) \), no prediction can be made for the low frequency end. Instead a hostage to fortune can be created by inferring a value for \( u_1 \) through comparison of the low frequency end of the fitted spectrum (4.4) with its theoretical form (3.23). We find

\[ u_1 = \sqrt{4\pi/\xi k_h a_1}^3. \tag{4.16} \]

The correlation length \( \xi \) can be estimated from Fig. 2 of Bennett and Bouchet (1989) to be about 0.2\( \lambda_h \) in the matter era. Thus the prediction for \( u_1 \) is

\[ u_1 \simeq 0.1, \tag{4.17} \]

whose confirmation (or otherwise) will have to await more detailed measurements of string correlation functions.

5 Comparison with observation at small angular scales

The best experiment which picks out the small scales where the theoretical spectrum can be trusted is OVRO (Readhead et al. 1989; Myers, Readhead & Lawrence 1993), which is a double difference experiment, with FWHM 108'' and beam throw 7'.15. There are also some recent VLA observations (Fomalont et al. 1993), which are of two regions about 7' across with up to 10'' resolution. However, the the VLA does not currently have as
good sensitivity as OVRO. The current theoretical uncertainties in the spectrum above 10′ make any bounds derived from larger scale experiments unreliable, and it is difficult to even quantify the level of unreliability.

The correlation function of the temperature fluctuations from strings is given by

$$C(r) = (G\mu)^2 \int \frac{d^2k}{2\pi k^2} F(k)e^{i\mathbf{k} \cdot \mathbf{r}}$$  \hfill (5.1)

This is observed by apparatus with a finite resolution. If $B(x)$ denotes the beam response, then the observed temperature distribution is

$$T_{\text{obs}}(x) = T * B = \int d^2y T(x + y) B(y),$$ \hfill (5.2)

and so the observed correlation function must also be convolved with the beam response:

$$C_{\text{obs}} = C * (B * B).$$ \hfill (5.3)

The OVRO beam response is well approximated by a Gaussian (Readhead et al. 1989),

$$B * B = \frac{1}{4\pi r_0^2} \exp\left(-r^2/4r_0^2\right),$$ \hfill (5.4)

where $r_0 = 0'.4247$. We denote the correlation function smeared on a scale $r_0$ by $C(r_0, r)$. The experiment measures the difference in temperature between a central beam ($T_M$) and the average of two flanking ones ($T_{R1}$ and $T_{R2}$), separated by $r_s = 7'.15$. The signal is therefore a temperature difference

$$\Delta T = T_M - \frac{1}{2}(T_{R1} + T_{R2}),$$ \hfill (5.5)

and so the the mean square temperature fluctuation measured by this type of experiment is

$$\langle(\Delta T/T)^2\rangle = \frac{3}{2}C(r_0, 0) - 2C(r_0, r_s) + \frac{1}{2}C(r_0, 2r_s).$$ \hfill (5.6)
There is a scale length $\zeta = 1/k_h a_1$ in the power spectrum, which at $\sim 0.1 \lambda_h \simeq 10'$ is about half the three dimensional correlation length of the string network, as defined before Eq. (3.23). Defining a dimensionless wavenumber $\kappa = k\zeta$, we find

$$C(r_0, r) = \int \frac{d\kappa}{\kappa} F(\kappa) J_0(\kappa r / \zeta) \exp(-\kappa^2 r_0^2 / \zeta^2),$$  \hfill (5.7)

where $J_0$ is the zeroth order Bessel function. We take the power spectrum to be

$$F(k) = \frac{A_0}{a_1} \frac{(k\zeta)^2}{[1 + (k\zeta)^2]^{3/2}}.$$  \hfill (5.8)

We must now give the spectrum its correct normalization, using (2.9). Hence we have

$$A_0 = 16\sqrt{\pi} \frac{1}{t} \langle (\dot{X} - (\dot{X} \cdot \hat{p})/(\dot{X} \cdot \hat{p}) \dot{X})^2 \rangle [1 - (z/z_{\text{dec}})^2] \gamma^2,$$  \hfill (5.9)

which we can evaluate as before, including all the string between the last scattering and the present day. The quantity in the angle brackets turns out to be $0.37 \pm 0.02$, so we arrive at

$$A_0 = (16.2 \pm 1.3) \gamma^2.$$  \hfill (5.10)

The form (5.8) is very convenient, because an approximation to the integration, valid when $(r_0 / \zeta)^2 \ll 1$, can be found in tables. Plugging in this function into the expression for $C(r_0, r)$, we find

$$C(r_0, r) \simeq C(0, r) + \frac{1}{2} C''(0, r)(r_0 / \zeta)^2,$$  \hfill (5.11)

where

$$C(0, r) = \frac{A_0}{a_1} \exp(-r / \zeta).$$  \hfill (5.12)

Thus

$$\langle (\Delta T / T)^2 \rangle (G\mu)^{-2} \simeq \frac{3}{2} \frac{A_0}{a_1} \left( 1 - \frac{4}{3} e^{-r_s / \zeta} + \frac{1}{3} e^{-2r_s / \zeta} \right) + O(r_0^2 / \zeta^2).$$  \hfill (5.13)

For small $r_s / \zeta$, as is relevant in a reionized universe, the right hand side is a factor $r_s / \zeta$ down on the total mean square fluctuation. One can interpret this factor as the probability that the beam pattern will straddle a string.
Putting in the experimental value for $r_s$, the theoretical value of $A_0$, and the fitted value of $a_1$, we find that the expected r.m.s. temperature fluctuations for OVRO are, with $z_{\text{dec}} = 1100$,
\[
\langle (\Delta T/T)^2 \rangle \simeq (6.3 \pm 0.5)(\gamma G\mu)^2.
\] (5.14)

We recall that the error is a statistical one corresponding to $1\sigma$ fluctuations in mean square string velocity. When $z_{\text{dec}} \ll 1100$, the formula (5.13) gives
\[
\langle (\Delta T/T)^2 \rangle = A_0(2\pi r_s/\lambda_h)(G\mu)^2 \simeq (7.1 \pm 0.6)(\gamma G\mu)^2(z_{\text{dec}}/1100)^{12}.
\] (5.15)

The OVRO NCP 95% confidence limit on the residual sky variance is (Readhead et al. 1989) $\Delta T_{\text{sky}} < 58 \mu K$, or $\Delta_{\text{sky}} = \Delta T_{\text{sky}}/T_{\text{sky}} < 2.1 \times 10^{-5}$ with $T_{\text{sky}} = 2.73$. This was obtained using a Bayesian analysis with a uniform prior on $\Delta_{\text{sky}}$, assuming that the sky fluctuations were uncorrelated between the fields and Gaussian. The first assumption is good for string sources too, since the NCP fields are separated by $30'$ and the scale length of the correlation function at short distances is $\zeta \simeq 10'$. However, the non-Gaussian nature of string-induced fluctuations on small scales means that we must repeat the analysis, using a stringy probability distribution $P_s$ for $\Delta T$. This is well approximated by an exponential function (Gott et al. 1990; Bennett, Bouchet & Stebbins 1993). The theoretical analysis of Moessner, Perivolaropoulos and Brandenberger (1994) also seems to support this contention. Thus the probability density for measuring $\Delta$ in an observation with noise $\sigma$ is
\[
P(\Delta, \sigma) = [2\sqrt{\pi}\sigma\Delta_{\text{sky}}]^{-1} \int d\Delta' \exp(-\sqrt{2}|\Delta - \Delta'|/\Delta_{\text{sky}} + \Delta'/2\sigma^2).
\] (5.16)

The likelihood function for the seven uncontaminated measurements $\{\Delta_i, \sigma_i\}$, which can be found in Table 4 of Readhead et al (1989), is then
\[
L(\{\Delta_i, \sigma_i\}|\Delta_{\text{sky}}) = \prod_i P(\Delta_i, \sigma_i)
\] (5.17)
This function, normalized to a maximum value of 1, is plotted in Figure 3. Assuming a uniform prior, as do Readhead et al (1989), it gives the Bayesian probability density for $\Delta_{\text{sky}}$. From it we can infer that

$$p(\Delta_{\text{sky}} < 2.8 \times 10^{-5}) = 0.95,$$

(5.18)

which is conventionally interpreted as a 95% confidence limit. We note that this limit is significantly looser than the bound on Gaussian fluctuations. This is because $P_s$ is strongly peaked near $\Delta T = 0$.

We can now bound $\gamma G\mu$: using our 95% confidence limit (5.18), and the central theoretical prediction (5.14), we obtain (for a universe without reionization)

$$\gamma G\mu < 11 \times 10^{-6}.$$  

(5.19)

We cannot rigorously bound the string tension, for the probability distribution of $G\mu$ depends crucially on how the probability distribution of $\gamma$, say $P_\gamma$, behaves near $\gamma = 0$. The probabilities for $\Delta$ and $\gamma$ are independent, so

$$P(G\mu) = \int_0^\infty P_s(6.3\gamma G\mu)P_\gamma(\gamma)\gamma d\gamma,$$

or

$$= \frac{1}{(6.3G\mu)^2} \int_0^\infty P_s(\Delta)P_\gamma(\Delta/6.3G\mu)\Delta d\Delta. \quad (5.20)$$

If $P_\gamma$ does not vanish sufficiently fast as $\gamma \to 0$, then $P(G\mu)$ will not converge swiftly enough to have a variance. Since we do not know $P_\gamma$, it is probably not worth doing more than estimating a bound for $G\mu$. The $2\sigma$ statistical lower bound on $\gamma$ is 4.1, so at a kind of 90% confidence level, we can say $G\mu < 3 \times 10^{-6}$. One is entitled to question the accuracy of the estimates of the correlation functions. However, the estimation method reproduced the numerical power spectrum to within about 10%, which corresponds to about 5% in the calculation of $\gamma G\mu$. Thus for safety we should perhaps quote $\gamma G\mu < 12 \times 10^{-6}$. A more precise bound will have to await a thorough reanalysis of the BBS simulations.
(Bennett, Bouchet & Stebbins 1993). However, this is still very stringent, and depends rather weakly on the decoupling redshift: the bound is raised by a factor $(1100/z_{\text{dec}})^{1/4}$.

We conclude by noting that the RING experiment in particular (Myers, Readhead & Lawrence 1993) deserves close examination, for there will be several fields containing strings (if the strings are there of course). This experiment does in fact report a signal, which if fitted to a Gaussian distribution results in an excess variance of around $100\mu K$. Although the authors do not entirely exclude contamination by point sources, it would be interesting to test the hypothesis that this signal is due to string.

6 Conclusions

In this paper a better analytic understanding of the CMB fluctuations produced by strings has been arrived at, at least on small angular scales (less than $10'$). There are a number of ingredients in the success of the approach, which unfortunately make its extension to larger scales difficult. Principally, the Minkowski space approximation results in a very simple formula for the projected temperature pattern, which depends only on the positions and velocities of the strings on the backward light cone of the observer. This can only be justified for fluctuations on angular scales less than a degree. An encouraging success is that the Gaussian approximation for strings, in which only the two point string correlators are used, reproduces accurately the numerical spectrum computed by BBS. Using the (corrected) theoretical spectrum, a limit (5.19) can be derived on $\gamma G\mu$ from observational data on small angular scales, where $\gamma$ is the number of horizon lengths of string per horizon volume. Because of the corrected formula, this translates to a more stringent limit than that given in Bennett, Bouchet & Stebbins (1989), even when statistical and theoretical uncertainties are properly accounted for. We also find that the correlation function on very small scales is approximately $C(r) \simeq C(0) \exp(-r/\zeta)$, with
\( \xi \simeq 10' \) (see Eq. 5.12). This is rather different from the Standard Cold Dark Matter form \( C(0)/(1 + r^2/2\alpha^2) \), with \( \alpha \simeq 10' \) (Bond & Efstathiou 1984). In experimental papers, a Gaussian autocorrelation function is often used to derive limits on temperature fluctuations, which is a reasonable approximation to the SCDM correlation function. However, when limits on cosmic strings are required, an exponential correlation function is to be preferred.

I am extremely grateful to Albert Stebbins for many useful discussions, and particularly for communicating the revised formula for temperature anisotropies. I am also grateful to him and his collaborators David Bennett and François Bouchet for making available their data. I thank also Paul Shellard for discussing his and Bruce Allen’s numerical results, and Ron Horgan for a factor \( \log_e 10 \). This work was supported by the SERC.

7 Appendix

Our purpose is here to establish the identity

\[
\hat{p}^\nu T_{\nu\rho,0}(Z) = -\nabla_\perp T_{\rho\nu}(Z) - \frac{1}{E} \frac{d}{d\lambda} \hat{p}^i T_{i\rho}(Z), \tag{A1}
\]

where \( Z^\mu = x^\mu + \lambda p^\mu \). used in Section 2 to derive the anisotropy formula. This we do using energy-momentum conservation, for

\[
\hat{p}^\nu T_{\nu\rho,0} = T_{\rho\nu,0} + \hat{p}^i T_{i\rho,0},
\]

\[
= T_{\rho,ij} + \hat{p}^i T_{\rho i,0}. \tag{A2}
\]

Now we use the fact that when acting on functions of \( Z \),

\[
\partial_t = \frac{1}{E} \frac{d}{d\lambda} - \hat{p}^i \partial_i. \tag{A3}
\]
Thus

\[ \hat{p}^\nu T_{\nu\rho,0} = T_{\rho i,j} + \hat{p}^i \hat{p}^j T_{\rho i,j} + \frac{1}{E \frac{d}{d\lambda}} \hat{p}^iT_{\rho i}, \]

\[ = (\delta^{ij} - \hat{p}^i \hat{p}^j) \partial_j T_{\rho i,j} - \frac{1}{E \frac{d}{d\lambda}} \hat{p}^i T_{\rho i}, \]

(A4)

which establishes the result, for \( \delta^{ij} - \hat{p}^i \hat{p}^j \) projects onto the transverse coordinates.
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9 Figure captions

Figure 1. The form of the three dimensional two-point string correlation functions $G^+(\sigma) = 2\langle \dot{X}^i(\sigma)\dot{X}^i(0) \rangle$ (Figure 1a); $G^-(\sigma) = 2\langle \dot{X}^i(\sigma)\dot{X}^i(0) \rangle$ (Figure 1b); and $\Omega(\sigma) = 4\langle \dot{X}^i(\sigma)\dot{X}^i(0) \rangle$ (Figure 1c).

Figure 2. Numerical and analytic forms of the power spectrum of CMB fluctuations induced at small angular scales by cosmic strings between $z_{\text{dec}}$ and $z_{\text{dec}}/2$. The spectrum is defined by $F(k) = k^2\langle |\delta_k|^2 \rangle / 2\pi (G\mu)^2$, and plotted against wavenumber in units of $k_h = 2\pi/\eta_{\text{dec}}$, where $\eta_{\text{dec}}$ is the horizon size at decoupling. The fitted functions are given in equations (4.2) and (4.4).

Figure 3. The likelihood function for the OVRO NCP measurements (excluding one contaminated field), using an exponential model for the probability distribution of string-induced sky fluctuations. This gives $p(\Delta T_{\text{sky}} < 77 \mu K) = 0.95$, which is a bound of $2.8 \times 10^{-5}$ on the fractional temperature anisotropy. This is about a factor of $4/3$ greater than the Gaussian limit.
Figure 1c
Cosmic string CMB power spectrum

- BBS spectrum
- 2 parameter fit
- BBS fit
