THE SYSTEMATIC ERROR TEST FOR PSF CORRECTION IN WEAK GRAVITATIONAL LENSING SHEAR MEASUREMENT BY THE ERA METHOD BY IDEALIZING PSF

YUKI OKURA1 AND TOSHIFUMI FUTAMASE2

1 RIKEN Nishina Center, Japan; yuki.okura@riken.jp
2 Department of Astrophysics and Atmospheric Science, Kyoto Sangyo University, Japan

Received 2016 May 10; revised 2016 June 11; accepted 2016 June 24; published 2016 August 18

ABSTRACT

We improve the ellipticity of re-smeared artificial image (ERA) method of point-spread function (PSF) correction in a weak lensing shear analysis in order to treat the realistic shape of galaxies and the PSF. This is done by re-smearing the PSF and the observed galaxy image using a re-smearing function (RSF) and allows us to use a new PSF with a simple shape and to correct the PSF effect without any approximations or assumptions. We perform a numerical test to show that the method applied for galaxies and PSF with some complicated shapes can correct the PSF effect with a systematic error of less than 0.1%. We also apply the ERA method for real data of the Abell 1689 cluster to confirm that it is able to detect the systematic weak lensing shear pattern. The ERA method requires less than 0.1 or 1 s to correct the PSF for each object in a numerical test and a real data analysis, respectively.

Key words: gravitational lensing: weak

1. INTRODUCTION

It is now widely recognized that weak gravitational lensing is a unique and powerful tool to obtain mass distribution in the universe. Coherent deformation of the shapes of background galaxies carries not only the information of intervening mass distribution but also the cosmological background geometry and thus the cosmological parameters (Mellier 1999; Schneider 2006; Munshi et al. 2008).

In fact, weak lensing studies have revealed the averaged mass profile for a galaxy cluster (Okabe et al. 2013; Umetsu et al. 2014) and detected the cosmic shear; weak lensing by large-scale structure is expected to be useful for studying the property of dark energy. However, the signal of cosmic shear is very weak and it is difficult to obtain useful constraints on the dark energy. Currently, several surveys have just started that plan to measure the cosmic shear accurately enough to constrain the dark energy property, such as the Hyper Suprime-Cam on Subaru (http://www.naoj.org/Projects/HSC/HSCP/Project.html), EUCLID (http://sci.esa.int/euclid), and LSST (http://www.lsst.org).

Since the signal of cosmic shear is very small, these surveys plan to observe a huge number of background galaxies to reduce statistical error. This means that any systematic errors in the lensing analysis must be controlled to be smaller than the statistic error, a roughly 1% ∼ 0.1% error is required for the systematic error. For this purpose, many methods (Bernstein & Jarvis 2002; Refregier 2003; Kuijken 2006; Miller et al. 2007; Kitching et al. 2008; Melchior 2011) have been developed and tested with simulations (Heymans et al. 2006; Massey et al. 2007; Bridle et al. 2010 and Kitching et al. 2012). Although there has been great progress, it seems that no fully satisfying method is available yet.

One of the systematic errors comes from the smearing effect by atmospheric turbulence and imperfect optics. This effect is described by the point-spread function (PSF) and we need to correct the effect very accurately in order to study the dark energy property. Previous approaches of PSF correction adopted some sort of approximation for the form of PSF, which prevents an accurate correction in some cases. Recently, we proposed a new approximation free method of PSF correction called the ERA method (ERA1: Okura & Futamase 2014, ERA2: Okura & Futamase 2015) based on the E-HOLICs method (Okura & Futamase 2011, 2012, 2013), which is a generalization of the KSB method (Kaiser et al. 1995) and uses an elliptical weight function to avoid the expansion of the weight function in measuring ellipticity. The method makes use of the artificial image constructed by re-smearing the observed image to have the same ellipticity with the lensed image. We have confirmed by numerical simulation that the method is free from systematic error, but is restricted to the case that PSF has relatively simple forms. In this paper, we generalize the ERA method to be applicable for a realistic situation, namely for complicated shapes of PSF and galaxies. Then we show that the improved ERA method has no systematic error by numerical simulation, and can be applied for real data of Abell 1689 taken by the Subaru Suprime Camera.

This paper is organized as follows. In Section 2, we explain our definitions and notations used in this paper. In Section 3, we explain the idea of PSF correction with idealizing in the ERA method. In Section 4, we show simulation tests for this method with several types of galaxy and PSF images, and then we apply the method to real data. Finally, we summarize our method and provide a discussion in Section 5.

2. DEFINITIONS AND NOTATIONS

In this section, we introduce the definitions and notations that we used in this paper. The use of a bold symbol, i.e., angular position on celestial sphere, reduced shear and so on, means it is a complex number. The basics of weak lensing also can be seen in Bartelmann & Schneider (2001).

2.1. Notations of the Brightness Distributions

In this paper, many brightness distributions are used to explain the ERA method. Table 1 summarize our notation and the definition of the brightness distributions. The corresponding distributions in Fourier space are written with the same character with a hat. More details of the definition are explained when these are used.
2.2. The Idea of Zero Plane and Zero Image

In this section, we introduce the idea of the zero plane and the zero image. The idea of the zero plane is that the intrinsic ellipticity of the source comes from a (virtual) source with zero ellipticity (called the zero image) in the virtual plane (the zero plane).

Suppose we have the reduced shear due to lensing and the intrinsic ellipticity written as of lensing and the intrinsic ellipticity, respectively, as $g^L$ and $g^C$, respectively, then the relationship between the displacements in the zero plane $\beta$ and the source plane $\beta$ and the lens plane $\theta$ are described as

$$
\beta = \beta^* - g^L \beta^* \quad (1)
$$

$$
\beta = \theta - g^C \theta^* \quad (2)
$$

where $g^C$ is combined shear, which is defined as

$$
g^C = g^L + g^L \theta^* \quad (3)
$$

Figure 1 shows the relation between zero, source, and image plane.

This combined shear has information of the intrinsic ellipticity and the lensing reduced shear. Since the intrinsic ellipticities are random (where we do not consider intrinsic alignment due to the galaxy cluster tidal field), the lensing reduced shear can be obtained by removing the information of intrinsic ellipticity as

$$
\frac{g^C - g^L}{1 - g^L g^L} = g^T = 0. \quad (4)
$$

This shows that we can obtain the lensing reduced shear in two steps. The first step is to obtain the combined shear from each object (Equation (2)) and the second step is to obtain the lensing reduced shear by averaging (Equation (4)).

In this paper, we consider these two steps to be combined into one, so we consider only the relationship between the zero plane and the lens plane, and we use $\beta$ as $\beta$ and $g^C$ as $g$ for notational simplicity.

2.3. Ellipticity for the Simulation Test

In this section, we define the ellipticity we used for the following sections. As we will explain in Section 3, the PSF correction in the ERA method does not depend on the definition of the ellipticity for object image, but we need to adopt at least one definition for the real analysis and the simulation test.

The ellipticity used in this paper is defined by moments of the images. The complex moments of the measured image are denoted as $Z^N_M$ and measured as

$$
Z^N_M(A, \epsilon_W) = \int d^2\theta A(\theta) W(\theta, \epsilon_W) \quad (5)
$$

except 0th moments defined as

$$
Z^0_0(A, \epsilon_W) = \int d^2\theta \theta^2 A(\theta) W(\theta, \epsilon_W) \quad (6)
$$

where $W$ is a weight function, which is a function of displacement from the centroid $\theta$ and ellipticity $\epsilon_W$, the subscript $N$ is the order of moments and $M$ indicates the spin number, and $\phi_0$ is the position angle at $\theta$.

The ellipticities are defined as a spin-2 combination of the quadrupole moments or a combination of the 0th moments with normalization.

$$
\epsilon_{2nd} = \frac{Z_2^2}{Z_0^2} A_{l,cw} \quad (8)
$$

$$
\epsilon_{2nd} = \frac{\epsilon_{2nd}}{|\epsilon_{2nd}|^2} (1 - \sqrt{1 - |\epsilon_{2nd}|^2}) \quad (9)
$$

$$
\epsilon_{0th} = \frac{Z_0^2}{Z_0^2} A_{l,cw} \quad (10)
$$

$$
\epsilon_{0th} = \frac{\epsilon_{0th}}{1 + |\epsilon_{0th}|^2} \quad (11)
$$

We refer to $\epsilon_{0th}$ and $\epsilon_{0th}$ as the “0th ellipticity” and $\epsilon_{2nd}$ and $\epsilon_{2nd}$ as the “2nd ellipticity.” If the profile of the measured image is simple, for example, an elliptical Gaussian, the 0th- and 2nd-ellipticities have the same value ($\epsilon_{0th} = \epsilon_{2nd}$), but because a real image has a complex form, these ellipticities
usually have different values. The ellipticity of the weight function \( e_w \) should be zero in the zero plane, so it is \( e_{\text{2nd}} \) or \( e_{\text{0th}} \) in the image plane. By selecting so, we can obtain the ellipticity of the object without any approximation. The relationship between the ellipticities and the reduced shear is obtained as follows.

\[
e = g \quad g < 1 \tag{12}
\]

\[
e = \frac{1}{g^*} \quad g > 1. \tag{13}
\]

for \( e_{\text{0th}} \) and \( e_{\text{2nd}} \). More details can be seen in ERA1 and ERA2.

3. THE BASICS OF PSF CORRECTION IN THE ERA METHOD

In this section, we present the basics of the ERA method.

One of the methods of measuring the ellipticity of a galaxy from a smeared image is to reconstruct the galaxy image by deconvolution. Suppose that \((\text{GAL}, G(\theta))\) is the galaxy image without the effect of PSF. This image is smeared by a point-spread function \((\text{PSF}, P(\theta))\), which can be measured from star images, and thus the image \((\text{SMD}, S(\theta))\) we actually observe is related by convolution as

\[
\hat{S}(k) = \hat{G}(k)\hat{P}(k). \tag{14}
\]

Then, the deconvolved galaxy \((\text{DGAL}, D(\theta))\) is defined as

\[
\hat{D}(k) = \frac{\hat{S}(k)}{\hat{P}(k)}. \tag{15}
\]

If there is no noise on the GAL and PSF, the DGAL coincides with GAL, so the brightness distribution of GAL can be obtained as \(G(\theta) = D(\theta)\). However, in the real observation, we have not only signal \(A(\theta)\) but also noise \(N(\theta)\), e.g., Poisson noise of the sky count; therefore, the GAL and PSF we observed can be decomposed

\[
P(\theta) = P_0(\theta) + P_N(\theta) \tag{16}
\]

\[
S(\theta) = S_0(\theta) + S_N(\theta), \tag{17}
\]

and so DGAL is written as

\[
\hat{D}(k) = \frac{\hat{S}(k)}{\hat{P}(k)} = \frac{\hat{S}_0(k) + \hat{S}_N(k)}{\hat{P}_0(k) + \hat{P}_N(k)} = \hat{G}(k) + \hat{S}_N(k)/\hat{P}_N(k). \tag{18}
\]

It is difficult to use DGAL for shear analysis, because DGAL could diverge at some \(k\) where \(\hat{P}_N(k) \sim -\hat{P}_0(k)\); therefore, DGAL does not have an analyzable shape.

The basic idea of the ERA method is re-smearing PSF and SMD by the re-smearing function \((\text{RSF}, R(\theta))\) to idealize the PSF in the following sense. “Re-smearing” GAL and PSF by some RSF is defined as

\[
\hat{P}^r(\theta) \equiv \hat{P}(\theta)\hat{R}(\theta) \tag{19}
\]

\[
\hat{S}^r(\theta) \equiv \hat{S}(\theta)\hat{R}(\theta) = \hat{G}(k)\hat{P}(k)\hat{R}(k) = \hat{G}(k)\hat{P}^r(\theta), \tag{20}
\]

where \(P^r(\theta)\) and \(S^r(\theta)\) are re-smeared PSF \((\text{REPSF}, P^r(\theta))\) and re-smeared galaxy \((\text{RESMD}, S^r(\theta))\), respectively. These two equations mean that the PSF shape can be chosen arbitrarily because RESMD can be written as a convolution of GAL and REPSF, and REPSF is obtained by the re-smearing too. Therefore, REPSF is a new PSF and the shape can be set arbitrary. It is obvious that the simpler PSF is the better for shear analysis, so the idealized PSF \((\text{IPSF}, I(\theta))\) should be defined as an elliptical simple function, e.g., a simple elliptical Gaussian, then RSF is obtained as

\[
\hat{R}(k) = \hat{I}(k)\hat{P}(k), \tag{21}
\]

where we must be careful with the divergence, but it can be avoided by noticing the arbitrariness of the IPSF. We discuss this point later. Finally, PSF can be idealized as

\[
\hat{P}^r(\theta) = \hat{I}(k) \tag{22}
\]

\[
\hat{S}^r(\theta) = \hat{G}(k)\hat{I}(k). \tag{23}
\]

Next, we explain how to choose IPSF in order to get the ellipticity of GAL in a simple case. Let GAL have one ellipticity, that is the ellipticity without radial dependence. Then IPSF is a simple function and has the same ellipticity as GAL, and the RESMD also has the same ellipticity as GAL, because the three images can be made by affine transformation with the same shear from three circular images. Therefore, IPSF will be chosen in such a way that it causes RESMD to have the same ellipticity as GAL. In this re-smearing, we do not mention the size of IPSF. It is a free parameter in our case because we need only the ellipticity of GAL, so the ellipticity of RESMD, and sizes of GAL and RESMD are not important in this simple situation. One of the important things to note is that the way ellipticity is defined is irrelevant in our method. We can use any definition of ellipticity if we know the relation between the ellipticity and gravitational shear. More details can be seen in ERA1.

In a general case, GAL does not have a simple shape and it has radially dependent ellipticity; therefore, we need to consider the situation in which the re-smearing procedure averages over the radially dependent ellipticity. The ellipticity of the observed galaxies consists of the intrinsic ellipticity of the galaxy and gravitational shear. The intrinsic ellipticity has a radial dependence in general but shear has no radial dependence, so the re-smearing merges only intrinsic ellipticity. This means that the intrinsic ellipticity has a different value for different re-smearing sizes. A reasonable choice for the size of IPSF would be the PSF size. However, this choice might cause the divergence we mentioned above. One of the techniques to avoid the divergence is to use an RSF larger in size than PSF. Therefore, it seems natural to use IPSF, which has a size as small as possible but larger than PSF. The problem now is that the size depends on the PSF of each exposure, and causes different intrinsic ellipticity in each exposures. Based on the above consideration, we choose a fixed size of IPSF for multi-epoch exposures that is slightly larger than the maximum PSF size of all exposures. It will obviously depend on optics, seeing, and so on.

The maximum value \(\mu_{\text{max}}\) of the ratio \(\mu\) between IPSF and PSF in Fourier space will be used to check the divergence

\[
\mu = \left(\frac{\hat{I}(k)}{\hat{I}(k_0)}\right)\left(\frac{\hat{P}(k)}{\hat{P}(k_0)}\right) \tag{24}
\]

in all \(k\) except \(k_0\), where \(k_0\) is a position of the peak of the function.
If some objects have $\mu_{\text{max}}$ much larger than one, it may be due to pixel noise or some other noise, we should remove these objects in the shot. However, if many objects have max much larger than one, it is probably due to a bad choice of the IPSF size. The choice depends on how large the noise of the object images is. This situation will be studied in detail in future works. In the following simulation test, we use the size of IPSF 1.5 times larger than the measured Gaussian best size. Then we test how the precision in the ellipticity measurement changes with the IPSF size.

In the above, we discussed that the size of the IPSF is not directly related with PSF size. However, the size should depend directly on the ellipticity of GAL. The lensing shear is one type of affine (linear) transformations, and the transformation relates shapes between with/without shear distortion. This is very useful, especially in simulation the test, because we can obtain the same intrinsic ellipticity from images with/without lensing shear. Let us consider the IPSF $I^0(\beta)$ in the zero plane. In the zero plane, GAL has zero ellipticity by the definition; therefore, the IPSF is a circular function. The IPSF $I(\theta)$ in the lens plane should have a profile that is transformed by shear distortion from $I^0(\beta)$, so

$$I(\theta) = I^0(\beta).$$  \hspace{1cm} (25)

For example, if we use an elliptical Gaussian for IPSF, then it is described in the zero plane as

$$I^0(\beta) = \exp\left(-\frac{\beta_0^2}{\sigma^2}\right),$$ \hspace{1cm} (26)

the IPSF in the lens plane should be

$$I(\theta) = \exp\left(-\frac{\theta_0^2}{(1 + |g|^2)\sigma^2} \cdot \delta\right),$$ \hspace{1cm} (27)

$$\delta = \frac{2g}{1 + |g|^2}. \hspace{1cm} (28)$$

So, in this situation, by using an IPSF with an ellipticity dependent size with the factor $\sqrt{1 + |g|^2}$, we can obtain the same intrinsic ellipticity in PSF correction for different lensing shear. We use this IPSF size determination in the following simulation test.

One of the methods to determine the ellipticity of IPSF is to use an iteration like

$$e_{\text{IPSF}}^{i+1} = e_{\text{IPSF}}^i + e_{\text{RESMD}}^i - e_{\text{REPSF}}^i \hspace{1cm} (29)$$

for the $i$th iteration, where the ellipticity of SMD can be used for initial ellipticity $e_{\text{IPSF}}^0$.

4. SIMULATION TEST

In this section, we present a simulation test of precision and analysis speed in the shear measurement by the ERA method. In this test, we analyze the ellipticity or lensing shear from several simulated objects with different situations to investigate what makes the systematic error. The profile of IPSF is an elliptical Gaussian, the size is 1.5 times larger than the size of the corresponding PSF as Gaussian. Ellipticity of images are defined by the quadrupole moment (2nd ellipticity) and 0th moment (0th ellipticity). Then we also test the time required to analyze objects that have realistic sizes. All simulated images have no pixel noise to investigate the systematic error only from the PSF correction. The systematic error from pixel noise will be investigated in future works.

4.1. Test with Several PSF Models

In this section, we consider the most simple situation where galaxies and PSFs have sizes large enough to ignore pixelization. Then galaxies have only one profile (Gaussian or Sérsic) and thus one ellipticity, but we consider several types of PSFs with complex shapes. Although the shapes of GAL and PSF in this test are not realistic, it is useful to investigate systematic errors only from PSF shape.

We use Gaussian ($\text{Type} = G$) and Sérsic ($\text{Type} = S$) profiles for the galaxy images with ellipticity $= [0.4, 0.0]$, this ellipticity is the true ellipticity in this simulation, and 100 pixel Gaussian radius for Type G and corresponding Gaussian size for Type S. Then we assumed four types of PSFs, the first is a circular Gaussian ($\text{Type} = C$), the second is a highly elliptical Gaussian ($\text{Type} = E$), the third is a double Gaussian ($\text{Type} = D$), the forth is a combination of three Gaussians with position shift ($\text{Type} = T$). Table 2 shows the parameters of the simulated images and Figure 2 show the simulated images with linear contours.

The precision is quantified by systematic error $\Delta \epsilon$ which is defined as

$$\Delta \epsilon_X \equiv \frac{\epsilon_X^{\text{corrected}} - \epsilon_X^{\text{true}}}{\epsilon_X^{\text{true}}},$$ \hspace{1cm} (30)

where $\epsilon_X^{\text{corrected}}$ is the PSF corrected ellipticity we use for measuring shear and $\epsilon_X^{\text{true}}$ is the true ellipticity of the galaxy, X is “2nd” for 2nd ellipticity or “0th” for 0th ellipticity’. Table 3 shows results of the precision test. The systematic errors in the PSF corrections are smaller than 0.1%. Therefore, one can conclude that the systematic error in PSF correction for the simple shape of the galaxy can be ignored even if the PSF has a complicated shape.

4.2. Test with the Pixelized Galaxy

Next, we test systematic error and analysis speed for the realistic size of the pixelized image. We selected SMD ($\text{ID} = G$) and PSF ($\text{ID} = C$), which are used in Section 4.1. We rescale them to GAL, which has a Gaussian size of 2.0 or 4.0 pixels, so that the SMD size and PSF size are 50 pixels and 25 times the scale down with pixelization. Table 4 shows the systematic error $\Delta \epsilon$ of the pixelized images.
We can see that the measurement by using 0th ellipticity has a systematic error of the order of 0.1% compared with very small errors by 2nd ellipticity. We guess that the larger systematic error comes from the center and surrounding pixels, because in such a region brightness distribution fluctuates rapidly over pixel scale. This does not necessarily mean that the 0th ellipticity cannot be used for cosmic shear measurement, because 0th ellipticity has a higher, approximately 1.5 times, signal-to-noise ratio than 2nd ellipticity, so 0th ellipticity has less systematic error from pixel noise, and so it may be useful for shear measurement from faint galaxies. One of the ideas that solves this systematic error from pixelization is to reduce weight at the center and surrounding pixels. We will study the weight and other methods to correct the systematic error in a forthcoming publication.

Table 3
The Systematic Error in Shear Measurement

| Galaxy ID | Type  | PSF | $r_{PSF}$ (pixels) | $r_{IPSF}$ (pixels) | $\Delta \epsilon_{2nd}$ | $\Delta \epsilon_{0th}$ |
|-----------|-------|-----|-------------------|---------------------|------------------------|------------------------|
| G         | C     | 100.0 | 150.0 | 1.343e-7 | 8.764e-8 |
| G         | E     | 100.0 | 350.0 | -2.425e-6 | -2.349e-6 |
| G         | D     | 90.2 | 150.0 | -2.973e-7 | -4.256e-7 |
| G         | T     | 150.0 | 275.0 | -3.260e-6 | -9.045e-6 |
| S         | C     | 100.0 | 150.0 | -6.765e-8 | 5.201e-7 |
| S         | E     | 100.0 | 350.0 | 2.483e-8  | -3.050e-7 |
| S         | D     | 90.2 | 150.0 | 5.747e-7  | -3.376e-7 |
| S         | T     | 150.0 | 275.0 | -8.324e-7 | -1.757e-6 |

Note. Radius of the PSF is the best radius of the weight function for measuring PSF shape with elliptical Gaussian weight.

We can see that the measurement by using 0th ellipticity has a systematic error of the order of 0.1% compared with very small errors by 2nd ellipticity. We guess that the larger systematic error comes from the center and surrounding pixels, because in such a region brightness distribution fluctuates rapidly over pixel scale. This does not necessarily mean that the 0th ellipticity cannot be used for cosmic shear measurement, because 0th ellipticity has a higher, approximately 1.5 times, signal-to-noise ratio than 2nd ellipticity, so 0th ellipticity has less systematic error from pixel noise, and so it may be useful for shear measurement from faint galaxies. One of the ideas that solves this systematic error from pixelization is to reduce weight at the center and surrounding pixels. We will study the weight and other methods to correct the systematic error in a forthcoming publication.

Table 4 also shows the analysis speed $T$, which is the average time over the time analyzing 1000 of the same images. We can see that the analysis speed depends on the radius of objects, but in any cases, the speed is about 0.1 s or shorter. This speed is for simulated images, so it is expected that the speed becomes longer for real object images due to pixel noise and the size variation of galaxies. We test the analysis speed time for real objects in Section 4.4.

Table 4
The Unit of the Analysis Speed Is Seconds/Object

| Radius of GAL | $r_{PSF}$ (pixels) | $r_{IPSF}$ (pixels) | $\Delta \epsilon_{2nd}$ | $\Delta \epsilon_{0th}$ | $T_{2nd}$ | $T_{0th}$ |
|---------------|-------------------|-------------------|------------------------|------------------------|----------|----------|
| 2.0 pixels    | 2.0               | 3.0               | 9.580e-7               | -2.221e-3              | 0.024    | 0.028    |
| 4.0 pixels    | 4.0               | 6.0               | 9.705e-7               | -1.236e-4              | 0.072    | 0.119    |
4.3. Test with Double Elliptical Galaxy and Spiral Galaxy

Next, we test the systematic error in the measurement for images with radially dependent ellipticity. Since the ellipticity changes with radius, we cannot define a unique ellipticity for such an image. In order to test the precision of the measurement for such images, we consider four galaxy images with four different intrinsic ellipticities whose directions are 90° different from each other and distort them by the same amount of shear $\gamma_{\text{true}} = [0.1, 0.0]$. Then we measure the shear $\gamma_{\text{measured}}$ by averaging the combined shear of these four images, then we define the systematic error ratio $\Delta \gamma_X \equiv \gamma_{\text{measured}} - \gamma_{\text{true}}$, as

$$
\Delta \gamma_X = \frac{\gamma_{\text{measured}} - \gamma_{\text{true}}}{\gamma_{\text{true}}},
$$

(31)

First, we test five data sets of galaxies that have two ellipticities with a Gaussian profile core and a Sérsic profile tail. The Gaussian core has a 10 pixels Gaussian radius and the Sérsic tail has a 50 pixel radius as the corresponding Gaussian radius; then, PSF is a circular Gaussian with $r_{\text{PSF}} = 50$ pixels and IPSF has an elliptical Gaussian with $r_{\text{IPSF}} = 75$ pixels. Figure 5 shows the original direction images of each of the five data sets and a PSF with the same size scale. Each image set has ID = [00, 02, 20, 22, 2R2], where the first (second) number means the intrinsic shear of Gaussian (Sérsic) and “0” (“2”) means 0.0(0.2), and “R” means that the direction of ellipticity of the Sérsic rotates 45° from the direction of ellipticity of the Gaussian profile. Next, we test a galaxy with two spiral arms. Figure 4 shows the simulated spiral galaxy image, PSF image, and the smeared spiral galaxy image with the same size scale. We can see that the smeared galaxy image has a radially dependent ellipticity. In this test, PSF has a circular Gaussian profile with $r_{\text{PSF}} = 64$ pixels, so we set $r_{\text{IPSF}} = 96$ pixels for IPSF. Figure 5 shows the systematic error in the shear measurement. We can see that the systematic error is smaller than 0.1% in all cases; this means that the PSF effect can be corrected precisely enough even if the GAL has radially dependent ellipticity.

Then, we test the size dependence of IPSF for the systematic error. We measured shear in the above situation ID = “20” except that IPSF size which is selected in the range between half and twice the PSF size. As Figure 5 shows, the result is precise enough precision if IPSF size is selected from higher than PSF size. This test is done under the ideal situation, so the result may change in a more realistic situation, e.g., with pixel noise. However, it is important to test intrinsic systematic error in the selecting size of IPSF. The test with pixel noise will be studied in the future.

Finally, we test the situation with different PSF shapes. In this test, we use four galaxy images with ID = “02” with
different rotations. Same as in the above test and distorted by the same shear \([0.1, 0.0]\), and these four galaxy images are smeared by four PSFs; these images are 50 pixels in size but of different shapes. One of the PSFs has ID = “C,” so it is circular, and the others have ID = “D” with different directions. IPSF is an elliptical Gaussian with a fixed size of 75 pixels. The last row of Table 5 shows the result, and the systematic error ratio is smaller than 0.1\%. This means that the PSF correction has enough precision for galaxies with radially dependent ellipticity and PSF variances.

| Galaxy ID | \( \Delta \gamma_{\text{GAL}} \) | \( \Delta \gamma_{\text{2nd}} \) | \( \Delta \gamma_{\text{0th}} \) |
|-----------|-----------------|-----------------|-----------------|
| 00        | 3.254e-8        | -7.225e-7       | 1.029e-5        |
| 02        | 1.455e-8        | -5.661e-5       | -1.067e-4       |
| 20        | -1.230e-7       | -7.182e-7       | 1.031e-5        |
| 22        | -9.077e-9       | -8.600e-7       | -6.371e-6       |
| 2R2       | 3.087e-7        | -4.277e-7       | 1.115e-5        |
| spiral    | -6.286e-6       | -8.332e-5       | -1.259e-4       |
| multi     | -1.536e-8       | -3.387e-5       | -5.264e-5       |

Table 6

| Selected Number | Number Analyzed | \( T_{\text{all}} \) | \( T_{\text{succeeded}} \) |
|-----------------|-----------------|-----------------|-----------------|
| 2nd             | 2180            | 2163            | 0.0499          | 0.0475          |
| 0th             | 2144            | 2143            | 0.0924          | 0.0906          |

4.4. Test with Real Data

Now we apply our method of PSF correction for real data and test analysis speed with a real observed image. We used a 3000 \times 3000\) square pixel image including the A1689 galaxy cluster, then approximately 1500 objects were selected as galaxies. The analysis time for each object can be seen in Table 6. The fourth column of the table is the total time over the succeeded number, so it includes time for rejection for the rejected objects. The third column of the table is the mean of the analysis time of only objects succeeded in PSF correction, so it does not include time for the rejection, this is meaningful as re-analyzing time. The mean size of galaxies in which we succeeded in analyzing shear is 2.46 pixels. The time required for analyzing shear from real data is roughly two or three times longer than simulated images with 2.0 radius images. We think

Figure 4. Simulated images of spiral galaxies (left), PSF (center), and smeared galaxy (right), with same size scale. The smeared galaxy has radially dependent ellipticity.

Table 5

The Systematic Error in the Shear Measurement from the Double Elliptical Galaxies

Figure 5. Systematic error ratio by each \( r_{\text{PSF}} \) with PSF size = 50 pixels. The dashed line is the error ratio in 2nd (0th) ellipticity.
this is a reasonable time because of the radius distribution, non-simple shape of the galaxies, pixel noise on the images, and so on, which affect real data. As the last test, we reconstructed the weak lensing convergence map of the Abell 1689 galaxy cluster. The aim of this test is to check if the ERA method can obtain the systematic shear pattern, and we do not intend to determine the mass distribution of the cluster very accurately. For this aim, it is enough to select the background galaxies simply from their sizes and magnitude and signal-to-noise ratio. The image data is 100 arcmin$^2$ in size and has approximately seven selected background galaxies per 1 arcmin$^2$. The Figure 6 shows the reconstructed convergence, the steps of the contours means 0.5 signal-to-noise ratio, and the lowest contour means 2.0. We can see that the convergence has a peak in the center of the cluster as expected. Thus the ERA method can be applied to real data and can be successfully obtained by the systematic weak lensing shear pattern.

5. SUMMARY
We previously developed a new method of PSF correction in the weak gravitational shear analysis called the ERA method without adopting any approximation to PSF. In this paper, we improve the ERA method by using an ideal PSF in order to treat more realistic PSF and galaxy shape. The correction re-smears PSF and SMD again by RSF for measuring PSF corrected ellipticity with the following steps

1. Determine profile of IPSF $I(\theta)$, e.g., an elliptical Gaussian function.
2. Decide the size of IPSF by considering PSF sizes in multi-exposures, and it must be fixed in the analysis.
3. Calculating RSF $R(\theta_{\text{PSF}}, e_{\text{PSF}}; \theta)$ from IPSF with a trial ellipticity $e_{\text{PSF}}$ and PSF.
4. Re-smear the SMD and the PSF by the RSF.
5. Compare the ellipticity of RESMD and REPSF. If the differences of the two ellipticities are small enough, it is the ellipticity of GAL. If the differences are larger than the precision you want, try re-smearing again, modifying the ellipticity of the IPSF.

We tested the ERA method with several types of simulated galaxies and star images. We consider not only simple shapes such as a simple Gaussian and Sérsic, but also more complicated shapes, such as a galaxy with radially dependent ellipticity, a spiral galaxy, a highly elliptical PSF, and a PSF with a pointing error. The results show that our method is able to correct PSFs with systematic errors under than 0.1% in any situation we consider. We also applied the ERA method to real data with the Abell 1689 cluster to check the analysis speed and to confirm that the systematic shear pattern can be obtained. It turned out that the ERA method can analyze real objects faster than 0.1 s for 2nd ellipticity and faster than 1 s for 0th ellipticity.

In this paper, we have not considered the systematic error caused by the pixelization effect and pixel noise. For a more realistic treatment and accurate estimation of cosmic shear, we
cannot neglect these effects and we will come back to them in a future study.

This work is supported in part by a Grant-in-Aid for Scientific Research from JSPS (No. 26400264 for T.F.).

REFERENCES

Bartelmann, M., & Schneider, P. 2001, PhR, 340, 291
Bernstein, G. M., & Jarvis, M. 2002, AJ, 123, 583
Bridle, S., Balan, S., Bethge, M., et al. 2010, MNRAS, 405, 2044
Heymans, C., Van Waerbeke, L., Bacon, D., et al. 2006, MNRAS, 368, 1323
Kaiser, N., Squires, G., & Broadhurst, T. 1995, ApJ, 449, 460
Kitching, T. D., Miller, L., Heymans, C. E., van Waerbeke, L., & Heavens, A. F. 2008, MNRAS, 390, 149
Kitching, T. D., Balan, S. T., Bridle, S., et al. 2012, MNRAS, 423, 3163
Kuijken, K. 2006, A&A, 456, 827
Massey, R., Heymans, C., Bergé, J., et al. 2007, MNRAS, 376, 13
Melchior, P. 2011, MNRAS, 412, 1552
Mellier, Y. 1999, ARA&A, 37, 127
Miller, L., Kitching, T. D., Heymans, C., Heavens, A. F., & van Waerbeke, L. 2007, MNRAS, 382, 315
Munshi, D., Valagesas, P., Van Waerbeke, L., & Heavens, A. 2008, PhR, 462, 67
Okabe, N., Smith, G. P., Umetsu, K., Takada, M., & Futamase, T. 2013, ApJL, 769, 35
Okura, Y., & Futamase, T. 2011, ApJ, 730, 9
Okura, Y., & Futamase, T. 2012, ApJ, 748, 112
Okura, Y., & Futamase, T. 2013, ApJ, 771, 37
Okura, Y., & Futamase, T. 2014, ApJ, 792, 104 (ERA1)
Okura, Y., & Futamase, T. 2015, A&A, 576, 63 (ERA2)
Refregier, A. 2003, MNRAS, 338, 35
Schneider, P. 2006, Part 1: Introduction to Gravitational Lensing and Cosmology (Heidelberg: Springer)
Umetsu, K., Medezinski, E., Nonino, M., et al. 2014, ApJ, 795, 163U