Influence of Gravitation on Mass-Energy Equivalence Relation

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Abstract

We study influence of gravitational field on the mass-energy equivalence relation by incorporating gravitation in the physical situation considered by Einstein (Ann. Physik, 17, 1905, English translation in ref. [1]) for his first derivation of mass-energy equivalence. In doing so, we also refine Einstein’s expression (Ann. Physik, 35, 1911, English translation in ref. [3]) for increase in gravitational mass of the body when it absorbs $E$ amount of radiation energy.

1 Introduction

Different forms of mass-energy equivalence relation existed even before Einstein’s first derivation of the relation [1] and which have been reviewed along with other developments on the relation after the year 1905 (see Ref. [2] and references cited therein). These relations were obtained in the absence of gravitational field. In this paper, we show that the presence of gravitational field affects the mass-energy equivalence relation. In doing so, we include gravitational field in the physical situation considered by Einstein [1] and derive the general mass-energy equivalence relation which reduces to the well known Einstein’s relation in the absence of gravitation. We also suggest refinement to the expression, provided by Einstein [3], for increase in gravitational mass due to the absorption of radiation energy $E$.

2 Analysis

Consider a ‘stationary’ reference frame $S_s$ with coordinate axes $x, y, z$ and another moving reference frame $S_v$ having coordinate axes $x', y', z'$ which are par-

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allel to axes x, y, z, respectively. Let $S_v$ is moving along the $x$ axis with a constant translational velocity $v$ as measured in $S_s$. At time $t = 0$ measured in $S_s$, let the origins of $S_v$ and $S_s$ coincide. Also, consider these two reference frames in the homogeneous gravitational field whose potential is $\Phi_s$ and $\Phi_v$ as measured in $S_s$ and $S_v$, respectively. For the homogeneous gravitational field with acceleration of gravity $\gamma$ and lines of force of the gravitational field in the negative direction of the axes $z$ and $z'$, we have $\Phi_s = \Phi_v = \gamma h$. Here $h$ is distance along the $z$ and $z'$ axis from the origins of $S_s$ and $S_v$, respectively.

Consider a body $W$ of mass $M_s$ kept at rest in $S_s$ on the $z$ axis at a distance $h$ from the origin, i.e having coordinates $(0, 0, h)$. At some instance it emits in two opposite directions equal quantity of light having energy $L/2$ where $M_s$ and $L$ are measured in $S_s$. The conservation of energy principle for this situation in $S_s$ can be written as

$$E_0 = E_1 + L \frac{1 + \frac{\Phi_s}{c^2}}{\sqrt{1 - \frac{\Phi_s^2}{c^4}}}$$  \hspace{1cm} (1)

where $E_0$ and $E_1$ are, respectively, total energy of the body before and after the emission of the light as measured in $S_s$ and $c$ is speed of light in $S_s$. It should be noted that $c$ depends on the gravitational field, written as [3]

$$c \cong c_0 \left(1 + \frac{\Phi_s}{c^2}\right)$$  \hspace{1cm} (2)

where $c_0$ is speed of light in the absence of gravitational field, i.e. when $\Phi_s = 0$. The factor multiplying $L$ in the last term takes into account gravitation of the emitted energy $L$ [3] and which reduces to 1 in the absence of gravitational field i.e. $\Phi_s = 0$.

It should be noted that the last term can be approximated as

$$L \frac{1 + \frac{\Phi_s}{c^2}}{\sqrt{1 - \frac{\Phi_s^2}{c^4}}} \cong L + L \frac{\Phi_s}{c^2}$$  \hspace{1cm} (3)

when $c^2 >> \Phi_s$. Einstein [3] considered this approximate expression $L + L \frac{\Phi_s}{c^2}$, representing summation of emitted energy $L$ and gravitation of energy $L \frac{\Phi_s}{c^2}$, for his derivation of gravitation of energy and showed increase in gravitational mass equal to increase in inertia mass for any body when the body absorbs radiation energy. Here we should mention that if, instead of approximate expression of type $L + L \frac{\Phi_s}{c^2}$, we consider more accurate expression of type $L \left(\frac{1 + \frac{\Phi_s}{c^2}}{\sqrt{1 - \frac{\Phi_s^2}{c^4}}} - 1\right)$ in the Einstein’s analysis [3] on increase in gravitational mass of the body when it absorbs $E$ amount of radiation energy, we would obtain this increase in mass equal to

$$\frac{E}{\Phi_s} \left[\frac{1 + \frac{\Phi_s}{c^2}}{\sqrt{1 - \frac{\Phi_s^2}{c^4}}} - 1\right]$$  \hspace{1cm} (4)
instead of $E/c^2$ as suggested by Einstein [3].

Now, the conservation of energy principle for the body $W$ as observed from $S_v$ can be written as

$$H_0 = H_1 + \frac{L}{\sqrt{1 - v^2/c^2}} \left( \frac{1 + \Phi_v}{1 - \Phi_v c^2} \right)$$

(5)

where $H_0$ and $H_1$ are, respectively, total energy of the body before and after the emission of the light as measured in $S_v$. While writing Eq. (5), we have used expression

$$\frac{L}{\sqrt{1 - v^2/c^2}}$$

(6)

for emitted energy as measured in $S_v$. Subtracting Eq. (1) from Eq. (5) and using $\Phi_v = \Phi_s$ yield

$$(H_0 - E_0) - (H_1 - E_1) = L \left[ \frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right] \left( \frac{1 + \Phi_s}{1 - \Phi_s c^2} \right)$$

(7)

Now, the total energy of the body is summation of potential energy $P$, kinetic energy $K$ and energy related to internal state of the body which we refer here as internal energy $I$. We now write total energies ($E_0, E_1, H_0$ and $H_1$) before and after the emission in reference frames $S_s$ and $S_v$ in terms of potential, kinetic and internal energies.

In $S_s$,

$$E_0 = M_s \Phi_s + M_s I_s$$

(8)

$$E_1 = (M_s - m_s) \Phi_s + (M_s - m_s) I_s'$$

(9)

Here $M_s$ is mass of the stationary body $W$ before the emission, $m_s$ is decrease in mass of the body due to the emission, $I_s$ and $I_s'$ are internal energy per unit mass of the body before and after the emission, respectively, and all are measured in $S_s$ in which the body is stationary all the time.

As measured in $S_v$, we denote the mass of the moving body $W$ before the emission by $M_v$, rest mass of the body before the emission when at rest in $S_v$ by $M_v^*$, decrease in rest mass due to the emission by $m_v^*$, potential energy per unit mass by $\Phi_v$, internal energy per unit mass of the body before and after the emission by $I_v$ and $I_v'$, respectively. With these notations we can write total energy of the body before and after the emission as measured in $S_v$ as

$$H_0 = K_0 + M_v^* \Phi_v + M_v^* I_v,$$

(10)

$$H_1 = K_1 + (M_v^* - m_v^*) \Phi_v + (M_v^* - m_v^*) I_v'$$

(11)
where $K_0$ and $K_1$ represent kinetic energies before and after the emission, respectively. Substituting Eqs. (8)-(11) into Eq. (7), we obtain

$$K_0 - K_1 = L\left[\frac{1}{\sqrt{1-v^2/c^2}} - 1\right] \left[1 + \frac{\Phi_s}{c^2}\right] \sqrt{1 - \Phi_s^2/c^4}$$

where we have used the fact that $M_s = M_s^0, m_s = m_s^0, \Phi_s = \Phi_s$ and internal energy per unit mass of the body is same in $S_s$ and $S_v$, i.e. $I_v = I_s, I'_v = I'_s$. Further incorporation of expressions for kinetic energies $K_0$ and $K_1$ [4] yield

$$m_sc^2 = L\left[\frac{1 + \frac{\Phi_s}{c^2}}{\sqrt{1 - \Phi_s^2/c^4}}\right]$$

exhibiting the effect of gravitation (through $\Phi_s$) on the decrease in mass due to the emission of energy $L$. When $\Phi_s = 0$, this final Eq. (13) reduces to the well known mass-energy equivalence relation $L = m_sc^2$ derived by Einstein [1].

References

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[2] W. L. Fadner, Am. J. Phys. 56, 114 (1988); J. Stachel and R. Torretti, Am. J. Phys. 50, 760 (1982); *The EINSTEIN Myth and the IVES papers*, edited by D. Turner and R. Hazelett (The Devin-Adair Company, 1979)

[3] A. Einstein, in *The Principle of Relativity*, edited by W. Perrett and G. B. Jeffery (Dover Publications, Inc., New York, 1952), p. 99-108.

[4] A. Einstein, in *The Principle of Relativity*, edited by W. Perrett and G. B. Jeffery (Dover Publications, Inc., New York, 1952), p. 37-65.