Analysis of Comparison of Calculation Methods for Downdrag of Single Pile

Yizhao Wang1, Liangliang Liu2*
1 School of Civil and Resource Engineering, University of Science and Technology Beijing, Beijing, Beijing,100083, China
2 College of Civil Engineering, Tongji University, Shanghai, Shanghai,200092, China
*Corresponding author’s e-mail: liuliangliang27@163.com

Abstract. The problem of negative skin friction has acquired great importance for the circles of civil engineering over that last few decades. It aims to understand the mechanisms of pile-soil interaction, define more adequate analytic methods for the load transferand ensure the bearing capacity of pile foundation. But for the complexity of problems, there are some issues that have not been well solved in engineering practice. In order to avoid the failure of pile body, the uneven settlement of pile foundation and other engineering disasters, the researches of downdrag have been carried out since 1965. And engineers have proposed a number of calculation formulas for the downdrag of single pile, yet the calculation results differ sharply on account of the respective applicability and limitation of these formulas. This paper introduces several common calculation methods of downdrag on single pile and recommends diverse calculation situations in self-weight collapsible loess area. In this paper, some calculation methods based on limit analysis method will be compared from different angles and the applicable conditions, advantages and disadvantages, etc. are also discussed to provide reference for practical engineering design.

1. Introduction
For some reason, the settlement deformation of soil around pile is greater than that of pile body, accordingly, downward frictional resistance appears in the entire lateral surface or partial surface of pile, which is called negative skin friction (NSF). Due to the negative surface frictional resistance of pile, the negative skin friction drags down the pile and a part of the weight of soil around pile will be transferred to pile body, which forms the downdrag. This load may be large enough to increase settlement of pile foundation or uneven settlement and lead to damage or destruction to the buildings (Teh et al. 1995; Chen et al. 2009)[1-2].

As a result of the load transferred to the pile, the vertical effective stress in soil decreases with the decrease of soil mass. Then two functions arise: 1) The load on pile or pier increases; 2) The effective stress at the end of the pile height is reduced, so the ultimate bearing capacity is also reduced. Consequently, the downdrag exerted on the pile can be the same with load on pile top, as the external load or surcharge, and can be a negative bearing capacity. The long time observation experiment of negative skin friction and the stress of pile have revealed that negative skin friction or downdrag would not disappear once produced. It is a long term load, but also can be regarded as a constant load on the pile.

The negative surface frictional resistance is the unit value of negative skin friction on pile side, while the downdrag is the value of axial force on pile section. In order to ascertain the ultimate bearing
capacity of pile, or to determine the design load and safety degree of pile, it is necessary to comprehend the effect of the downdrag on pile. The differences of the size and distribution mode of pile top load and ground surcharge, strength and deformation property of soil around pile and pile end soil, relative displacement of pile and soil, the pore water pressure dissipation condition of layered soil, all of these have effects on the size and distribution of the negative skin friction. Some practical experience methods and other manageable calculation methods are the current popular methods. In this paper, these methods based on limit analysis method are compared from different angles and the applicable conditions, advantages and disadvantages, etc. are discussed.

2. The Method of Total Stress

Terzaghi and Peek (1967) considered that the negative skin friction exerted on the pile is not greater than the shear strength of the soil, which means \(0 < q_n < \tau\). The downdrag \(Q_n\) acted on a single pile can be calculated as:

\[
Q_n = u \int_{0}^{L_n} q_n \, dz
\]  

(1)

\[
q_n = \alpha S_u
\]  

(2)

Where \(Q_n\) = downdrag of a single pile; \(u\) = perimeter of pile section; \(L_n\) = depth of the neutral point; \(q_n\) = negative skin friction. In the above formula; \(S_u\) = the average undrained shear strength of clay; \(\alpha\) = an empirical adhesion factor along the length of the pile; \(Q_n\) is the maximum possible value which means the situation that the negative skin friction can be fully played. The following methods are similar to the formula mentioned above[3].

In the “Code for design of building foundation structure” of Japan, the calculation formula of the downdrag is as follows:

\[
Q_n = \lambda u \int_{0}^{L_n} q_n \, dz
\]  

(3)

Before formula (1), a coefficient \(\lambda\) termed shape coefficient of pile tip is added and \(Q_n\) is considered to be the maximum axial force at the neutral point.

In the “Code for harbor building on soft foundation” of the Soviet Union, the calculation formula is:

\[
Q_n = u \sum_{i=1}^{n} q_{n_i} l_i
\]  

(4)

Where \(u\) = perimeter of pile section; \(q_n\) = negative skin frictions; \(l_i\) = thickness of the ith layer. And

\[
q_n = K_0 m_0 q_a
\]  

(5)

Where \(K_0\) = homogeneity coefficient of sinking soil layer (check the relevant table); \(m_0\) = work condition coefficient of pile (\(l_i < 4 m, m_0 = 0.4; l_i > 4 m, m_0 = 0.8\)); \(q_a\) = standard value of the negative skin friction.

In the “Code for design of building foundation” of national standard of China, the calculation formula applied to the multiple layered of the downdrag is as follows:

\[
Q_n = \eta_n u \sum_{i=1}^{n} q_{n_i}^m l_i
\]  

(6)

Where \(\eta_n\) = pile group effect coefficient of the negative skin friction (\(\eta_n = 1\) in term of single pile foundation); \(u\) = perimeter of pile section; \(n\) = number of soil layers above neutral point; \(q_{n_i}^m\) = standard value of the negative skin friction with the weighted average based on soil thickness above the neutral point of the ith layer; \(l_i\) = thickness of the ith layer.

The standard value of the negative skin friction above neutral point of the ith layer for single pile can be calculated in the following formula:

\[
q_{n_i}^m = \epsilon m_1 \sigma_i'
\]  

(7)

When the filling and self-weight collapse loess bring about collapse deformation, underconsolidated soil layer generates consolidation or groundwater level declines; \(\sigma_i' = \sigma_i'\). And
\[ \sigma'_i = \sum_{e=1}^{i-1} y_e \Delta Z_e + \frac{1}{2} y_i \Delta Z_i \] 

(8)

Where \( q_{nl} \) = standard value of the negative skin friction of the ith layer, when the calculated value is greater than the standard value of positive friction force, the latter can be accommodated to do the calculations. \( \varepsilon_{ni} \) = negative friction coefficient of the ith layer; \( \sigma'_i \) = average vertical effective stress of the ith layer; \( \sigma_{nl} \) = average vertical effective stress of the ith layer due to the self-weight of soils; \( \gamma \) = bulk density of soil; \( \Delta Z \) = soil thickness.

The aforementioned multiple calculation formulas of downdrag are identical in essence and all of these methods develop based on the method of Terzaghi and Peek. The first two methods combine the negative skin friction of pile with the shear strength of soil, which means the negative skin friction of pile can be approximately determined by the contrast with the shear strength of soil. Moreover, the method that the shear strength of soil is measured by indoor and outdoor soil tests is simpler and more convenient. And this method is still used in practice. Canadian Geotechnical Society recommends that \( \alpha \) value is in the range of 0.5~1.0 and the undrained shear strength \( S_u \) is specified as the strength after consolidation under new load.

The third method with the help of the vertical effective stress to calculate the negative skin friction takes into account the property of different soil layer and introduces the negative friction coefficient. Compared with the first two methods, the third one is more reasonable and effective and it is also the most frequently used calculation method of downdrag in China. In fact, concerning the computation of the negative skin friction, this method is closer to the following based on the effective stress theory.

3. The Method of Effective Stress

In this method, firstly, the effective stress method is used to calculate negative skin friction. Then calculation depth is identified regarding the different situation. Afterward, the total downdrag is acquired by the quadrature method. The negative skin friction can occur under three conditions in this method such as the following:

- If a fill of clay soil is placed over a granular soil layer into which a pile is driven, the fill will gradually consolidate. This consolidation process will exert a downward drag force on the pile during the period of consolidation.
- If a fill of granular soil is placed over a layer of soft clay, it will induce the process of consolidation in the clay layer and thus exert a downward drag on the pile.
- Lowering of the water table will increase the vertical effective stress on the soil at any depth, which will induce consolidation settlement in clay. If a pile is located in the clay layer, it will be subjected to a downdrag (Braja 2004)[5].

3.1 Clay fill over granular soil

On the basis of testing results, Bjerrum et al. (1965, 1969) presented the effective stress method to calculate the negative skin stress on the pile

\[ q_{nl} = K' \sigma_0' \tan \delta' \] 

(9)

Where \( K' = K_0 = 1 - \sin \phi' = \) earth pressure coefficient; \( \sigma_0' = \gamma' z \) = vertical effective stress at any depth \( z \); \( \gamma' = \) effective unit weight of fill; \( \delta' = \) soil-pile friction angle \( \approx 0.5 \phi' - 0.7 \phi' \) [6].

Hence, the total downdrag on a pile is

\[ Q_n = \int_0^{H_f} (uK' \gamma' \tan \delta') \, zdz = \frac{uK' \gamma' H_f^2 \tan \delta'}{2} \] 

(10)

Where \( H_f \) = height of the fill. If the fill is above the water table, the effective unit weight, \( \gamma_f' \) should be replaced by the moist unit weight.

On the basis of testing results, Bjerrum et al. (1965, 1969) presented the effective stress method to calculate the negative skin stress on the pile.
\[ q_n = K' \sigma'_0 \tan \delta' \]  

(11)

Where \( K' = K_0 = 1 - \sin \theta' \) = earth pressure coefficient; \( \sigma'_0 = \gamma'_f z \) = vertical effective stress at any depth \( z \); \( \gamma'_f \) = effective unit weight of fill; \( \delta' \approx 0.5 \theta' - 0.7 \theta' \).

Hence, the total downdrag on a pile is

\[ Q_n = \int_0^{H_f} (uK' \gamma'_f \tan \delta') z \, dz = \frac{uK' \gamma'_f H_f^2 \tan \delta'}{2} \]  

(12)

Where \( H_f \) = height of the fill. If the fill is above the water table, the effective unit weight, \( \gamma'_f \) should be replaced by the moist unit weight.

3.2 Granular soil fill over clay

In this case, Vesic (1977) indicated that the negative skin stress on the pile may exist from \( z = 0 \) to \( z = L_1 \), which is also referred to as the neutral depth. The neutral depth may be given as (Bowles 1982)

\[ L_1 = \frac{L - H_f}{L_1} \left( \frac{L - H_f}{2} + \frac{\gamma'_f H_f}{\gamma'} \right) - \frac{2 \gamma'_f H_f}{\gamma'} \]  

(13)

Where \( \gamma'_f \) and \( \gamma' \) = effective unit weights of the fill and the underlying clay layer, respectively [7-8].

Once the value of \( L_1 \) is determined, the downdrag is obtained in the following manner: The unit negative skin friction at any depth from \( z = 0 \) to \( z = L_1 \) is

\[ q_n = K' \sigma'_0 \tan \delta' \]  

(14)

Where \( K' = K_0 = 1 - \sin \theta' \); \( \sigma'_0 = \gamma'_f H_f + \gamma' z \); \( \delta' \approx 0.5 \theta' - 0.7 \theta' \). Hence, the total downdrag is

\[ Q_n = \int_0^{L_1} u \, q_n \, dz = \int_0^{L_1} u \, K' (\gamma'_f H_f + \gamma' z) \tan \delta' \, dz \]

\[ = \left( uK' \gamma'_f H_f \tan \delta' \right) L_1 + \frac{L_1^2 uK' \gamma'_f \tan \delta'}{2} \]  

(15)

Through the practical experience for many years, the effective stress method has referenced and practical evaluation for offering an effective method in the research of the calculation of downdrag and is widely adopted by engineering and technical personnel. Howbeit the calculated value of downdrag, which is made by way of the effective stress method, corresponds to the actual value only in the upper part of pile body, while it is very different from the actual value in the lower part. And the negative skin friction of the pile foundation calculated by the effective stress method is the limit value, the result of which is usually conservative.

4. The Method of Different Stress

The use of sections to divide the text of the paper is optional and left as a decision for the author. Where the author wishes to divide the paper into sections the formatting shown in table 2 should be used.

Zeevaert (1959) made several assumptions before the research on negative skin friction of pile foundation [9]. 1) The pile is a rigid body and the pile end soil is rigid layer; 2) The negative skin frictional is a function of the surcharge on pile and it is distributed according to earth pressure at rest; 3) The downdrag acted on the pile equals the decrease of vertical stress in soil around pile. And then the following formula is obtained

\[ Q_n = A (\sigma_{oz} - \sigma_{uz}) \]  

(16)

Where \( Q_n \) = downdrag of single pile at depth \( z \); \( A \) = sectional area of settlement soil column shared by a pile in pile group; \( \sigma_{oz} \) = initial vertical effective stress in foundation at depth \( z \);

\[ \sigma'_{oz} = p + \gamma' Z \]  

(17)
Where $p = \text{distributed load on the ground}$; $\gamma' = \text{effective unit weight of soil around pile}$; $\sigma'_{uz} = \text{vertical effective stress in foundation at depth z when the negative skin friction is fully played after pile sinking}$;

$$\sigma'_{uz} = \gamma' \left(1 - e^{-mz}\right) + \sigma'_0 e^{-mz}$$  \hspace{1cm} (18)

$$m = \frac{1}{A} u K_o \tan \varphi'$$  \hspace{1cm} (19)

$$\gamma^* = \gamma' - \frac{1}{A} u C'$$  \hspace{1cm} (20)

Where $u = \text{perimeter of pile section}$; $K_o = \text{static earth pressure coefficient of soil around pile}$; $C' = \text{effective cohesion of soil around pile}$; $\varphi' = \text{internal friction angle of soil}$; $e = \text{void ratio of soil}$.

The above formula is used in the calculation of downdrag for a single pile in pile group and it does not need to involve the specific numerical value of the negative skin friction of pile but requires to calculate the changes of the vertical effective stress before and after pile sinking which means the calculation method is relatively simple. Nevertheless, it is not only a lack of consideration for the depth of neutral point, but is lack of the differentiation between side pile and corner pile in pile group. In addition, because of the three assumptions made to simplify the pile-soil interaction mechanism, the method is only applicable to the situation where pile length is shorter and pile stiffness is larger in end bearing pile.

Afterwards, Zeevaert (1972) presented that the effective division area should be discussed in pile group to obtain more accurate results for the estimation of downdrag, in the case of considering side pile and corner pile[10]. The effective division area, also defined as the calculation range of negative skin friction on a pile in pile group, is changed with the depth and the arrangement of pile. According to the position of each pile in pile group, the equivalent radius of pile can be determined. Then we can calculate approximately the downdrag applied on pile or pier with the equivalent diameter of $2\tilde{r}_0$. The equilibrium equation can be expressed as

$$\frac{d(NF)}{dz} dz = 2\pi \tilde{r}_0 S_{oz} dz$$  \hspace{1cm} (21)

Suppose $2\pi \tilde{r}_0 = \tilde{\omega}$, then integrate, the following equation can be written as

$$(NF)_d = \tilde{\omega} \int_0^d S_{oz} dz$$  \hspace{1cm} (22)

The ultimate shear strength of reconsolidated soil close to pile-soil interface at the depth of $Z$ is

$$S_{oz} = K (\sigma_{oz} - \Delta \sigma_z)$$  \hspace{1cm} (23)

Where $K = \text{friction coefficient of pile and soil}$. So it can get the following formula:

$$(NF)_d = \tilde{\omega} K \int_0^d \sigma_{oz} dz - \tilde{\omega} K \int_0^d \Delta \sigma_z dz$$  \hspace{1cm} (24)

Where $\int_0^d \sigma_{oz} dz$ — initial stress area; $\int_0^d \Delta \sigma_z dz$ — reduced vertical stress area in soil because of the effect of NF on pile or pier, which is nearly equal to

$$\int_0^d \Delta \sigma_z dz = \frac{1}{3} \Delta \sigma_d$$  \hspace{1cm} (25)

Suppose $\tilde{\alpha}$ is effective division area and assume that it does not vary with the depth

$$\tilde{\alpha} = \pi (n \tilde{r}_0)^2$$  \hspace{1cm} (26)

So we can obtain the express of the calculation formula of downdrag

$$(NF)_d = \frac{\tilde{\omega} K}{4 + \frac{\omega K}{\tilde{\alpha}}} \int_0^d \sigma_{oz} dz$$  \hspace{1cm} (27)
For a single pile or pier, the value of $a_\text{th}$ can be substituted by $\pi (12r_0)^2$; for pile group, $a_\text{th}$ is the effective division area of each pile. As far as utility and accuracy are concerned, the above formula, which makes up for the deficiency that the downdrag of single pile cannot be obtained if some assumptions are not made on the load distribution of soil around pile in the formula (14), can be used for a single pile. And the precondition is that it needs to gain the radius of equivalent soil column based on the position of each pile in the foundation.

5. Conclusion and Recommendation

Through the analysis and comparison of the above methods, the following results are obtained:

1) All kinds of calculation methods have their own advantages and disadvantages. In the design of pile foundation engineering, a comprehensive consideration of various calculation methods and the results of the optimization as a design reference can be one of the methods that engineers can use.

2) Several common calculation methods, the effects of time effect and relative displacement of pile-soil on the downdrag, among other factors, have not been considered. Therefore, the establishment of the calculation methods reflects the relationship between the relative displacement of pile-soil, construction effect and the negative skin friction, for the purpose of more accurate calculation of the downdrag, can be the target of deep research.

3) In general, the negative skin friction does not occur in the whole soft soil layer. And the magnitude of negative skin friction influences the settlement of pile foundation, which reduces the negative skin friction in turn. So repeatedly, it will eventually reach a dynamic balance, which makes it difficult to estimate the location of neutral point in engineering applications and it needs to be studied further.

References

[1] Teh C, Wong K S. (1995)Analysis of downdrag on pile groups. Geotechnique, 45(2): 191~207.

[2] Chen R P, Zhou W H, Chen Y M. (2009)Influences of soil consolidation and pile load on the development of negative skin friction of a pile. Computers and Geotechnics, 36:1265~1271.

[3] Terzaghi P. (1967)Soil Mechanics in Engineering Practice (2nd ed). New York: John Willey and Sons Press.

[4] Johannesson I J, Bjerrum L. (1965)Measurements of the Compression of a Steel Pile to Rock due to Settlement of the Surrounding Clay. Cahiers De Recherche, 2: 261~264.

[5] Braja MDas. (2004)Fundamental of Geotechnical Engineering. (2nd ed) Thomson Press, 14.14:517~519.

[6] Bjerrum L, Johannessen, I J. Eide O. (1969)Reduction of Negative Skin Friction on Steel Piles to Rock. Soil Mech & Fdn Eng Conf Proc /Mexico,2: 27~34.

[7] Vesic A S. Design of pile foundations. Nchrp Synthesis of Highway Practice. 1977:68.

[8] Bowles J E. (1982) Foundation Design and Analysis. McGraw-Hill, New York.

[9] Zeevaert L. (1959)Reduction of point bearing capacity because of negative friction. Proc. First Pan American Conference on Soil Mechanics and Foundation Engineering,3: 1145~1152.

[10] Zeevaert L. (1972)Foundation Engineering for Difficult Subsoil Conditions. Van Nostrand.