PROBING SUPERSYMMETRY IN RARE B DECAYS†

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Abstract

We determine the ability of future experiments to observe supersymmetric contributions to the rare decays \( B \to X_s \gamma \) and \( B \to X_s l^+ l^- \). A global fit to the Wilson coefficients which contribute to these decays is performed from Monte Carlo generated data. This fit is then compared to supersymmetric predictions for several different patterns of the superpartner spectrum.

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We determine the ability of future experiments to observe supersymmetric contributions to the rare decays $B \rightarrow X_s \gamma$ and $B \rightarrow X_s l^+ l^-$. A global fit to the Wilson coefficients which contribute to these decays is performed from Monte Carlo generated data. This fit is then compared to supersymmetric predictions for several different patterns of the superpartner spectrum.

The first conclusive observation of penguin mediated processes, the exclusive $B \rightarrow K^{*} \gamma$ and inclusive $B \rightarrow X_s \gamma$, by CLEO has placed the study of rare $B$ decays on new ground. These flavor changing neutral current (FCNC) transitions provide an essential opportunity to test the Standard Model (SM) and offer a complementary strategy in the search for new physics by probing the indirect effects of new particles and interactions in higher-order processes. With the expected high luminosity of the $B$-Factories presently under construction (and the associated advanced detector technology), radiative $B$ decays will no longer be rare events, and the exploration of FCNC transitions can continue by probing decay modes with even smaller predicted branching fractions. The cleanest rare decay which occurs at a rate accessible to these machines is $B \rightarrow X_s l^+ l^-$. In fact, experiments at $e^+ e^-$ and hadron colliders are already closing in on the observation of the exclusive modes $B \rightarrow K^{(*)} l^+ l^-$ with $l = e$ and $\mu$, respectively. Once this decay is observed, the utilization of the kinematic distributions of the $l^+ l^-$ pair, such as the lepton pair invariant mass distribution and forward backward asymmetry, and the tau polarization asymmetry in $B \rightarrow X_s \tau^+ \tau^-$, together with $B(B \rightarrow X_s \gamma)$ will provide a stringent test of the SM. In this talk we determine the ability of $B$-Factories to probe possible supersymmetric contributions to these decays.

Softly broken supersymmetry (SUSY) is a decoupling theory, thus making it a challenge to search for its effects through indirect methods. However, a promising approach is to measure observables where supersymmetry and the SM arise at the same order in perturbation theory. In this case the SUSY contributions do not suffer an extra $\alpha/4\pi$ reduction compared to the SM amplitudes. The relative ratio between the lowest order SM amplitudes and those of supersymmetry could then be $O(1)$ if $\tilde{m} \approx M_W$. Rare $B$-decays could then provide such an opportunity for discovering indirect effects of supersymmetry.

The effective field theory for $b \rightarrow s$ transitions which incorporates QCD corrections is governed by the Hamiltonian

$$\mathcal{H}_{\text{eff}} = \frac{-4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) \mathcal{O}_i(\mu),$$

where the $\mathcal{O}_i$ are a complete set of renormalized operators of dimension six or less which mediate $b \rightarrow s$ transitions and are catalogued in, e.g., Ref. 7. The $C_i$ represent the corresponding Wilson coefficients which are evaluated perturbatively at the electroweak scale where the matching conditions are imposed and then evolved down to the renormalization scale $\mu \approx m_b$. The expressions for $C_i(M_W)$ are given by the Inami-Lim functions.

For $B \rightarrow X_s l^+ l^-$ this formalism leads to the physical decay amplitude (neglecting $m_s$)

$$\mathcal{M} = \frac{\sqrt{2}G_F^2 a_{\pi}}{\pi} V_{tb} V_{ts}^* \left[ C_9^{\text{eff}} \bar{s}_L \gamma_\mu b_L \bar{\ell}_R^\mu \ell + C_{10} \bar{s}_L \gamma_\mu b_L \bar{\ell}_\gamma \gamma_5 \ell \right.\left. - 2 C_7^{\text{eff}} m_b \bar{s}_L i \sigma_{\mu \nu} \frac{q^\nu}{q^2} b_R \bar{\ell}_R^\mu \ell \right],$$

where $q^2$ represents the momentum transferred to the lepton pair. We incorporate the NLO analysis for this decay which has been performed in Buras et al. where it is stressed that a scheme independent result can only be obtained by including the leading and next-to-leading logarithmic corrections to $C_9(\mu)$ while retaining only the leading logarithms in the remaining Wilson coefficients.
QCD bremsstrahlung corrections \( \mathcal{O}_9 \), yielding an effective value \( C_9^{eff} \). The effective value for \( C_7^{eff}(\mu) \) refers to the leading order scheme independent result. The operator \( \mathcal{O}_{10} \) does not renormalize. The numerical estimates (in the naive dimensional regularization (NDR) scheme) for these coefficients are displayed in Table 1. The reduced scale dependence of the NLO versus the LO corrected coefficients is reflected in the deviations \( \Delta C_9(\mu) \lesssim \pm 10\% \) and \( \Delta C_7^{eff}(\mu) \approx \pm 20\% \) as \( \mu \) is varied. We find that the values of the coefficients are much less sensitive to the remaining input parameters, with \( \Delta C_9(\mu) \), \( \Delta C_7^{eff}(\mu) \lesssim 3\% \), varying \( \alpha_s(M_Z) = 0.118 \pm 0.003 \) \cite{1406}, and \( m_t^{phys} = 175 \pm 6 \text{GeV} \) \cite{1406}. The resulting inclusive branching fractions (which are computed by scaling the width for \( B \to X_s \ell^+ \ell^- \) to that for \( B \) semileptonic decay) are found to be \( (6.25^{+1.04}_{-0.93}) \times 10^{-6} \), \( (5.73^{+0.75}_{-0.78}) \times 10^{-6} \), and \( (3.24^{+0.41}_{-0.54}) \times 10^{-7} \) for \( \ell = e, \mu \), and \( \tau \), respectively, taking into account the above input parameter ranges, as well as \( B_{sl} \equiv B(B \to X_s \nu) = (10.23 \pm 0.39)\% \) \cite{1406}, and \( m_c/m_b = 0.29 \pm 0.02 \).

The operator basis for the decay \( B \to X_s \gamma \) contains the first eight operators in the effective Hamiltonian of Eq. \( \mathcal{H} \). The leading logarithmic QCD corrections have been completely resummed, but lead to a sizeable \( \mu \) dependence of the branching fraction and hence it is essential to include the next-to-leading order corrections. In this case, the calculation involves several steps, requiring NLO corrections to both \( C_7^{eff} \) and the matrix element of \( \mathcal{O}_7 \). For the matrix element, this includes the QCD bremsstrahlung corrections \( b \to s \gamma + g \), and the NLO virtual corrections which have recently been completed \cite{1406}. Summing these contributions to the matrix elements and expanding them around \( \mu = m_b \), one arrives at the decay amplitude

\[
\mathcal{M}(b \to s \gamma) = \frac{4G_F V_{tb}^* V_{ts}}{\sqrt{2}} D(s\gamma|\mathcal{O}_7(m_b)|b)_{tree},
\]

with

\[
D = C_7^{eff}(\mu) + \frac{\alpha_s(m_b)}{4\pi} \left( C_i^{(0)eff}(\mu) \gamma_i^{(0)} \log \frac{m_b}{\mu} \right. \\
\left. + C_i^{(0)eff} r_i \right).
\]

Here, the quantities \( \gamma_i^{(0)} \) are the entries of the effective leading order anomalous dimension matrix, and the \( r_i \) are computed in Greub et al. \cite{1406}, for \( i = 2, 7, 8 \). The first term in Eq. \( 3 \) \( C_7^{eff}(\mu) \), must be computed at NLO precision, while it is consistent to use the leading order values of the other coefficients. For \( C_7^{eff} \) the NLO result entails the computation of the \( \mathcal{O}(\alpha_s) \) terms in the matching conditions \cite{1406}, and the renormalization group evolution of \( C_7^{eff}(\mu) \) must be computed using the \( \mathcal{O}(\alpha_s^2) \) anomalous dimension matrix. Preliminary NLO results for these anomalous dimensions have recently been reported \cite{1406}, with the conclusion being that in the NDR scheme the NLO correction to \( C_7^{eff}(\mu) \) is small. Therefore, a good approximation for the inclusive width is obtained by employing the leading order expression for \( C_7^{eff}(\mu) \), with the understanding that this introduces a small inherent uncertainty in the calculation. We then find the branching fraction (again, scaling to semileptonic decay)

\[
B(B \to X_s \gamma) = (3.25 \pm 0.30 \pm 0.40) \times 10^{-4},
\]

where the first error corresponds to the combined uncertainty associated with the value of \( m_t \) and \( \mu \), and the second error represents the uncertainty from \( \alpha_s(M_Z), B_{sl}, \) and \( m_c/m_b \). This is well within the range observed by CLEO \cite{1406} which is \( B = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4} \) with the 95\% C.L. bounds of \( 1 \times 10^{-4} < B(B \to X_s \gamma) < 4.2 \times 10^{-4} \).

Measurements of \( B(B \to X_s \gamma) \) alone constrain the magnitude, but not the sign, of \( C_7^{eff}(\mu) \). We can write the coefficients at the matching scale in the form \( C_i(M_W) = C_i^{SM}(M_W) + C_i^{new}(M_W) \), where \( C_i^{new}(M_W) \) represents the contributions from new interactions. Due to operator mixing, \( B \to X_s \gamma \) then limits the possible values for \( C_i^{new}(M_W) \) for \( i = 7, 8 \). These bounds are summarized in Fig. \ref{fig:1}. Here, the solid bands correspond to

| \( \mu = m_b/2 \) | \( \mu = m_b \) | \( \mu = 2m_b \) |
|---|---|---|
| \( C_7^{eff} \) | -0.371 | -0.312 | -0.278 |
| \( C_9 \) | 4.52 | 4.21 | 3.81 |
| \( C_{10} \) | -4.55 | -4.55 | -4.55 |
the constraints obtained from the current CLEO measurement, taking into account the variation of the renormalization scale $m_b/2 \leq \mu \leq 2m_b$, as well as the allowed ranges of the other input parameters. The dashed bands represent the constraints when the scale is fixed to $\mu = m_b$. We note that large values of $C_{9\text{new}}(M_W)$ are allowed even in the region where $C_{7\text{new}}(M_W) \approx 0$.

Measurement of the kinematic distributions associated with the final state lepton pair in $B \rightarrow X_s \ell^+ \ell^-$ as well as the rate for $B \rightarrow X_s \gamma$ allows for the determination of the sign and magnitude of all the Wilson coefficients for the contributing operators in a model independent fashion. We have performed a Monte Carlo analysis in order to ascertain how much quantitative information will be obtainable at future $B$-factories and follow the procedure outlined in Ref. 5. For the process $B \rightarrow X_s \ell^+ \ell^-$, we consider the lepton pair invariant mass distribution and forward-backward asymmetry for $\ell = e, \mu, \tau$, and the tau polarization asymmetry for $B \rightarrow X_s \tau^+ \tau^-$. A three dimensional $\chi^2$ fit to the coefficients $C_{7,9,10}(\mu)$ is performed for three values of the integrated luminosity, $3 \times 10^7$, $10^8$, and $5 \times 10^8$ $B\bar{B}$ pairs, corresponding to the expected $e^+e^-$ $B$-factory luminosities of one year at design, one year at an upgraded accelerator, and the total accumulated luminosity at the end of the programs. The $95\%$ C.L. allowed regions as projected onto the $C_9(\mu) - C_{10}(\mu)$ and $C_{7\text{eff}}(\mu) - C_{10}(\mu)$ planes are depicted in Figs. 2(a-b), where the diamond represents the central value for the expectations in the SM. We see that the determinations are relatively poor for $3 \times 10^7$ $B\bar{B}$ pairs and that higher statistics are required in order to focus on regions centered around the SM.

We analyze the supersymmetric contributions to the Wilson coefficients in terms of the quantities

$$R_i = \frac{C_i^{\text{susy}}(M_W)}{C_i^{\text{SM}}(M_W)} - 1 = \frac{C_i^{\text{new}}(M_W)}{C_i^{\text{SM}}(M_W)},$$

where $C_i^{\text{susy}}(M_W)$ includes the full SM plus superpartner contributions. $R_i$ is meant to indicate the relative fraction difference from the SM value.

Supersymmetry has many potential sources for flavor violation. The flavor mixing angles among the squarks are $a$ priori separate from the CKM angles of the SM quarks. We adopt the viewpoint here that flavor-blind (diagonal) soft terms at the high scale are the phenomenological source for the soft scalar masses, and that the CKM angles are the only relevant flavor violating sources. The spectroscopy of the supersymmetric states is quite model dependent and here
we analyze two possibilities. The first is the familiar minimal supergravity model. In this case all the supersymmetric states follow from a common scalar mass and a common gaugino mass at the high scale. The second possibility is to relax the condition of common scalar masses and allow them to take on uncorrelated values at the low scale while still preserving gauge invariance.

We begin by searching over the full parameter space of the minimal supergravity model. We generate these models by applying common soft scalar and common gaugino masses at the boundary scale. The tri-scalar $A$ terms are also input at the high scale and are universal. The radiative electroweak symmetry breaking conditions yield the $B$ and $\mu^2$ terms as output, with a sign$(\mu)$ ambiguity left over. (Here $\mu$ refers to the Higgsino mixing parameter.) We also choose $\tan \beta$ and restrict it to a range which will yield perturbative Yukawa couplings up to the GUT scale. We have generated thousands of solutions according to the above procedure. The ranges of our input parameters are $0 < m_0 < 500$ GeV, $50 < m_{1/2} < 250$ GeV, $-3 < A_0/m_0 < 3$, $2 < \tan \beta < 50$, and we have taken $m_t = 175$ GeV. Each supersymmetric solution is kept only if it is not in violation with present constraints from SLC/LEP and Tevatron direct sparticle production limits, and it is out of reach of LEP II. For each of these remaining solutions we now calculate $R_{7-10}$. Our results are shown in the scatter plots of Fig. 3 in the $(a)$ $R_7 - R_8$ and $(b)$ $R_9 - R_{10}$ planes. The diagonal bands represent the bounds on the Wilson coefficients as previously determined from our global fit. We see that the current CLEO data on $B \rightarrow X_s \gamma$ already places significant restrictions on the supersymmetric parameter space.

The first thing to note from the figure is that large values of $R_7$ and $R_8$ are generated, and that they are very strongly correlated. These large effects arise from models with $|\mu| \lesssim 400$ GeV. This is because light charged Higgsinos (or rather light charginos with a large Higgsino fraction) are necessary in order to obtain a large effect on the Wilson coefficients. We see that the values of $R_9$ and $R_{10}$ are bounded by about 0.04, which is small compared to the range for $R_7$. The main reason for these smaller values is the dependence on the bottom Yukawa $\lambda_b \propto 1/\cos \beta$. $R_7$ also has a contribution directly dependent on this $1/\cos \beta$ Yukawa factor, however the other multiplicative terms associated with $\lambda_b$ are the large top Yukawa and a large kinematic loop factor. $R_9$ and $R_{10}$ do not have such additional factors due the chirality structure of these operators and the requirement that leptons and sleptons only couple via $SU(2)$ and $U(1)$ gauge couplings. These conditions, along with the correlations between the mass spectra dictated by minimal supergravity relations, render the minimal supergravity contributions to $R_{9,10}$ essentially unobservable.

We now adopt our second, more phenomenological, approach. The maximal effects for the parameters $R_i$ can be estimated for a superparticle spectrum independent of the high scale assumptions. However, we still maintain the assumption that CKM angles alone constitute the sole source of flavor violations in the full supersymmetric lagrangian. We will focus on the region $\tan \beta \lesssim 30$. The most important features which result in large contributions are a light $t_1$ state present in the SUSY spectrum and at least one
light chargino state. For the dipole moment operators a light Higgsino is most important. A pure higgsino and/or pure gaugino state have less of an effect than two mixed states when searching for maximal effects in $C_9$ and $C_{10}$ and we have found that $M_2 \simeq 2\mu$ is optimal.

Fig. 4 displays the maximum contribution to $R_{9,10}$ versus an applicable SUSY mass scale. The other masses which are not plotted (\(\tilde{t}_1, \tilde{t}_L\), etc.) are chosen to be just above the reach of LEP II or the Tevatron, whichever gives better bounds. We see that the maximum size of $R_{9,10}$ is much larger than what was allowed in the minimal supergravity model. This is due to the lifted restriction on mass correlations. Light sleptons, sneutrinos, charginos, and stops are allowed simultaneously with mixing angles giving the maximal contribution to the $R_i$’s. However, we find that the maximum allowed values for $R_{9,10}$ are still much less than unity. Earlier we determined that $B$ factor data would be sensitive to $\Delta R_9 \gtrsim 0.3$ and $\Delta R_{10} \gtrsim 0.08$ at the highest luminosities, and so the largest SUSY effect would give a $1-2\sigma$ signal in $R_{9,10}$, hardly enough to be a compelling indication of physics beyond the standard model.

Given the sensitivity of all the observables it is instructive to narrow the focus to $C_7(M_W)$. There exists the possibility that one eigenvalue of the stop squark mass matrix might be much lighter than the other squarks due to the large top Yukawa and the mixing term $m_t (A_t - \mu \cot \beta)$ in the stop mass matrix. We then present results for $C_7(M_W)$ in the limit of one light squark, namely the $\tilde{t}_1$, and light charginos. We allow the $\tilde{t}_1$ to have arbitrary components of $\tilde{t}_L$ and $\tilde{t}_R$ since cross terms can become very important. This is especially noteworthy in the high $\tan \beta$ limit. We note that the total supersymmetric contribution to $C_7(M_W)$ will depend on several combinations of mixing angles in both the stop and chargino mixing matrices and cancellations can occur for different signs of $\mu$.

The first case we examine is that where the lightest chargino is a pure Higgsino and the lightest stop is purely right-handed: $\chi^+_1 \sim H^\pm$, $\tilde{t}_1 \sim \tilde{t}_R$. The resulting contribution to $R_7$ is shown as a function of the $\tilde{t}_R$ mass in Fig. 4 (dashed line) for the case of chargino masses out of reach of LEP II ($m_{\chi^+_1} \gtrsim M_W$). Note that the SUSY contribution to $C_7(M_W)$ in this limit always adds constructively to that of the SM. Next we examine the limit where the only light chargino is a pure Wino. The effects of a light pure Wino are generally small since (i) it couples with gauge strength rather than the top Yukawa, and (ii) generally supersymmetric models do not yield a light $\tilde{t}_L$ necessary to couple with the Wino. This contribution to $R_7$ is shown in Fig. 4 (dotted line). As expected, we see that this contribution is indeed small. Now we discuss our third limiting case. As mentioned above, what we mostly expect in minimal supergravity models is a highly mixed $\tilde{t}_1$ state; here, for large effects, it is crucial that there be substantial $\tilde{t}_R$ and $\tilde{t}_L$ contributions to $\tilde{t}_1$. We find that in this case large $\tan \beta$ solutions ($\tan \beta \gtrsim 40$) can yield greater than $O(1)$ contributions to $R_7$ even for SUSY scales of 1 TeV! Low values of $\tan \beta$ can also exhibit significant enhancements. This is demonstrated for $\tan \beta = 2$ in Fig. 4 (solid line). Note that in this case large contributions are possible in both the
Figure 5: Contributions to $R_7$ in the different limits described in the text. The top solid line is the charged $H^\pm/t$ contribution versus $m_{H^\pm}$. The bottom solid line is the $\tilde{\chi}_1^\pm/t_1$ contribution versus $m_{\tilde{\chi}_1^\pm}$ where both the chargino and stop are maximally mixed states with $\mu < 0$. The dashed line is the $H^\pm/t_R$ contribution, and the dotted line represents the $\tilde{W}^\pm/t_1$ contribution. These two lines are both shown as a function of $m_{\tilde{\chi}_1^\pm}$ mass. All lines are for $\tan \beta = 2$ and $m_t = 175$ GeV. We have set all other masses to be just above the reach of LEPII.

In this talk we have studied the effects of supersymmetry to the FCNC observables concerning $b \to s$ transitions, and we have seen that deviations from the standard model could be detected with supersymmetric masses even at the TeV scale. The large $\tan \beta$ enhancements in the $b \to s$ processes are unique; it is thus possible that the first distinct signs of supersymmetry could come from deviations in rare $B$ decays. One, of course, would like direct confirmation of such a deviation, if observed at $B$-Factories, and collider programs could provide it.

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