Is the Abraham electromagnetic force physical? — Comment on “Possibility of measuring the Abraham force using whispering gallery modes”

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Abstract

In a recent paper [I. Brevik and S. Ellingsen, Phys. Rev. A 81, 063830 (2010)], a conventional general electromagnetic force definition is used for calculations of Abraham radiation torque produced by the whispering gallery mode in a micrometer-sized dielectric spherical resonator. However in this paper, we would like to indicate that this conventional force definition is flawed; namely the physical existence of the Abraham-term force in the definition is questionable.

In principle, the correctness of $\mathbf{f} = \mathbf{f}^\text{AM} + \mathbf{f}^\text{A}$ as a general EM force definition cannot be legitimately affirmed by enumerating specific examples, no matter how many; however, the correctness can be directly negated by finding specific examples, even only one. In the following, such a specific example is given to show why the conventional EM force definition $\mathbf{f} = \mathbf{f}^\text{AM} + \mathbf{f}^\text{A}$ is flawed.

An electromagnetic plane wave, although not practical, is a simplest strict solution of Maxwell equations, and it is often used to explore most fundamental physics. For example, Einstein used a plane wave to develop his special theory of relativity and derived the well-known relativistic Doppler formula in free space [5]. Thus if the general force definition $\mathbf{f} = \mathbf{f}^\text{AM} + \mathbf{f}^\text{A}$ is correct, it must withstand the test of a monochromatic plane wave in a non-dispersive, lossless, isotropic uniform medium.

Suppose that the EM fields are given by $(\mathbf{E}, \mathbf{B}, \mathbf{D}, \mathbf{H}) = (\mathbf{E}_0, \mathbf{B}_0, \mathbf{D}_0, \mathbf{H}_0) \cos \Psi$ for the plane wave, where $\mathbf{E}_0$, $\mathbf{B}_0$, $\mathbf{D}_0$, and $\mathbf{H}_0$ are the constant amplitude vectors, and $\Psi = \omega t - \mathbf{k}_w \cdot \mathbf{x}$ is the phase function with $\omega$ the frequency and $\mathbf{k}_w$ the wave vector. Since the medium is uniform, $\mathbf{f}^\text{AM} = 0$ holds, and the general EM force $\mathbf{f} = \mathbf{f}^\text{AM} + \mathbf{f}^\text{A}$ is reduced to

$$
\mathbf{f} = \mathbf{f}^\text{A} = \frac{n^2 - 1}{c^2} \frac{\partial}{\partial t} (\mathbf{E} \times \mathbf{H}).
$$

From Maxwell equations, the momentum conservation equation is given by

$$
\frac{\partial}{\partial t} \left( \frac{\mathbf{E} \times \mathbf{H}}{c^2} \right) = -\nabla \cdot \mathbf{A},
$$

where the forces associated with the material deformation are not included; $\mathbf{f}^\text{AM}$ is the Abraham-Minkowski (term) force, and $\mathbf{f}^\text{A}$ is the Abraham (term) force. Note: For a uniform medium, Eq. (1) is the same as the one given in [3]. For a non-magnetic medium (relative permeability $\mu_r = 1$) without any sources, $\mathbf{f}^\text{AM}$ and $\mathbf{f}^\text{A}$ are, as shown by Brevik and Ellingsen [1, 4], given by

$$
\mathbf{f}^\text{AM} = -\frac{1}{2} \varepsilon_0 \mathbf{E}^2 \nabla n^2_d,
$$

and

$$
\mathbf{f}^\text{A} = \frac{n^2 - 1}{c^2} \frac{\partial}{\partial t} (\mathbf{E} \times \mathbf{H}),
$$

where $\varepsilon_0$ is the vacuum permittivity constant, $n_d = (\mu_r \varepsilon_r)^{1/2}$ is the refractive index with $\varepsilon_r$ the relative permittivity, and $c$ is the vacuum light speed.

In the idea proposed by Brevik and Ellingsen, the dielectric medium is assumed to be uniform ($\nabla n_d = 0$), and thus $\mathbf{f}^\text{AM} = 0$ holds; $\mathbf{f} = \mathbf{f}^\text{AM} + \mathbf{f}^\text{A} = \mathbf{f}^\text{A}$ is thought to form possibly measurable Abraham radiation torque in a micrometer-sized dielectric spherical resonator operating at the whispering gallery mode.
where the stress tensor $\hat{T}_A$ is given by

$$\hat{T}_A = \beta_{ph}^2 [-(E \cdot D + HB) + \hat{l} \cdot \left(\frac{1}{2}(D \cdot E + B \cdot H)\right)], \quad (6)$$

with $|\beta_{ph}| = 1/\sqrt{n^2}$ the absolute phase velocity normalized to the light speed $c$, and $\hat{l}$ the unit tensor.

Inserting $E \times H = E_0 \times H_0 \cos \omega t$ into Eq. (5), we indeed have $f = f^A \neq 0$ (except for those discrete points); thus $f \neq 0$ looks like a “force”, but that is not true. This can be seen from the following analysis.

Inserting Eq. (5) into Eq. (4) we have $f = (n^2_\omega - 1)\nabla \cdot \hat{T}_A \neq 0$.

From Eq. (6), we know that $\hat{T}_A \propto \cos^2 \omega t$ is a “pure travelling-wave” stress tensor, and thus the Abraham momentaums flowing into and out from a differential box are usually different, resulting in $\nabla \cdot \hat{T}_A \neq 0 \Rightarrow f \neq 0$, but there is no net momentum left in the box on time average ($< \nabla \cdot \hat{T}_A > = 0 \Rightarrow f > = 0$).

From this we can see that $f \neq 0$ is resulting from the attribution of the “pure travelling wave” of tensor $\hat{T}_A$. This “pure travelling-wave” attribute will not produce any “force” effect on the medium. This phenomenon can be clearly understood through Einstein’s light-quantum hypothesis: photons are the carriers of light momentum and energy. Since the dielectric medium is assumed to be a non-dispersive, lossless, isotropic uniform medium, all the photons move uniformly at the dielectric light speed $c/n_d$, and they do not have any momentum exchanges with the medium.

Since $f = f^A + f^A = f^A \neq 0$ does not represent a force for a plane wave, $f = f^A + f^A$ cannot pass the plane-wave test, and $f = f^A + f^A$, as a general EM force definition, is flawed. Unfortunately, this flawed EM force definition is widely accepted in the community [1, 4, 6, 7, 8], and it is argued that the $f^A$-term “simply fluctuates out when averaged over an optical period in a stationary beam”, but “it is in principle measurable” [6].

In summary, we have shown that the conventional general EM force definition $f = f^A + f^A$ is flawed. Specifically speaking, the Abraham term $f^A = (n^2_\omega - 1)\partial/\partial t(E \times H)/c^2$ is not a “physical EM force” at all for a plane wave.

Appendix A. Abraham momentum conservation equation

Isotropic medium is a special case of anisotropic media. Below we will show that Eq. (5) is also valid for an anisotropic medium.

Statement. If a plane wave propagates in a lossless, non-dispersive, non-conducting, uniform anisotropic medium, the Abraham momentum conservation equation can be written as

$$\frac{\partial}{\partial t} \left(\frac{E \times H}{c^2}\right) = -\nabla \cdot \hat{T}_A, \quad (A.1)$$

where the Abraham stress tensor is given by

$$\hat{T}_A = \beta_{ph}^2 \left[-(E \cdot D + HB) + \hat{l} \cdot \left(\frac{1}{2}(D \cdot E + B \cdot H)\right)\right]. \quad (A.2)$$

\(\text{Proof.}\) For a monochromatic plane wave with a phase function of $\Psi = \omega t - k_w \cdot \mathbf{x}$, Maxwell equations are simplified into

$$\omega \mathbf{B} = k_w \times \mathbf{E}, \quad \omega \mathbf{D} = -k_w \times \mathbf{H}, \quad (A.3)$$

where the EM fields are given by $\mathbf{E} = (E_0, B_0, D_0, H_0) \cos \omega t$, with $E_0, B_0, D_0,$ and $H_0$ the real constant vectors. The frequency $\omega$ and the wave vector $k_w$ are real because the medium is assumed to be non-conducting and lossless.

From Eq. (A.3) we have

$$\mathbf{D} \times \mathbf{B} = \left(\frac{\mathbf{D} \cdot \mathbf{E}}{\omega}\right) k_w. \quad (A.5)$$

By making cross products of $k_w \times (\omega \mathbf{B} = k_w \times \mathbf{E})$ and $k_w \times (\omega \mathbf{D} = -k_w \times \mathbf{H})$ from Eq. (A.3), with vector identity $a \times (b \times c) = (a \cdot c)b - (a \cdot b)c$ taken into account, we have

$$\mathbf{E} = (\mathbf{n} \cdot \mathbf{E}) \mathbf{n} - \mathbf{v}_{ph} \times \mathbf{B}, \quad (A.6)$$

$$\mathbf{H} = (\mathbf{n} \cdot \mathbf{H}) \mathbf{n} + \mathbf{v}_{ph} \times \mathbf{D}, \quad (A.7)$$

where $\mathbf{n} = k_w/|k_w|$ is the unit wave vector, and $\mathbf{v}_{ph} = (\hat{n}/|\hat{n}|)/c$ is the phase velocity. The refractive index for anisotropic media is defined as $n_d = |k_w|/|\omega/c|$, with $c$ the vacuum light speed, and thus the phase velocity also can be written as $\mathbf{v}_{ph} = (\hat{n}/|\hat{n}|)(c/n_d)$.

By making inner products of $\mathbf{H} \cdot (\omega \mathbf{B} = k_w \times \mathbf{E})$ and $\mathbf{E} \cdot (\omega \mathbf{D} = -k_w \times \mathbf{H})$ from Eq. (A.3), with $\mathbf{H} \cdot (k_w \times \mathbf{E}) = (\mathbf{k} \times \mathbf{H})$ taken into account we obtain $\mathbf{E} \cdot \mathbf{D} = \mathbf{B} \cdot \mathbf{H}$.

From Eqs. (A.5) and (A.7), and Eq. (A.3) we obtain

$$\frac{\mathbf{E} \times \mathbf{H}}{c^2} = \beta_{ph}^2 \frac{\omega}{\omega} \cos^2 \Psi \times \left[\mathbf{v}_{ph} \cdot (\mathbf{E} \cdot \mathbf{D}) - \mathbf{v}_{ph} \cdot (\mathbf{H} \cdot \mathbf{B})\right], \quad (A.8)$$

where $\beta_{ph} = \mathbf{v}_{ph}/c$ is the normalized phase velocity.

With the help of $\nabla \cdot (a \times b) = (\nabla \cdot a) b - a \cdot (\nabla b)$ we obtain

$$\nabla \cdot \left[\mathbf{v}_{ph} \times (\mathbf{E} \cdot \mathbf{D})\right] = 2 \cos \Psi \sin \Psi \left(\mathbf{D} \cdot \mathbf{E} \cdot \mathbf{D} \cdot \mathbf{E}\right) k_w. \quad (A.9)$$

By use of $\mathbf{D} \cdot \mathbf{k}_w = 0$ from Eq. (A.4), $\mathbf{B} \cdot \mathbf{H} = \mathbf{E} \cdot \mathbf{D}$, and $a \times (b \times c) = (a \cdot c)b - (a \cdot b)c$, we have

$$\nabla \cdot \left[\frac{1}{2}(\mathbf{D} \cdot \mathbf{E} + \mathbf{B} \cdot \mathbf{H})\right] = 2 \cos \Psi \sin \Psi \left(\mathbf{D} \cdot \mathbf{E} \cdot \mathbf{D} \cdot \mathbf{E}\right) k_w. \quad (A.10)$$

Inserting Eqs. (A.8), (A.9), and (A.10) into Eq. (A.1), we find the left- and right-hand sides of Eq. (A.1) are equal. Thus Eq. (A.1) is confirmed.

References

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