A Simplified Formulation for the Backward/Forward Sweep Power Flow Method

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Abstract

This paper describes a simplified formulation of the Backward/Forward (BW/FW) Sweep Power Flow applied to radial distribution systems with distributed generation under positive sequence modelling. Proposed formulation was applied in an illustrative test system.

Keywords: Backward/forward sweep, load flow, power flow, distribution system analysis

1 Introduction

Several Backward/Forward (BW/FW) sweep algorithms have been discussed in literature. In 1967, Berg presented a paper which can be considered as the source for the all variants of BW/FW sweep methods [1]. Later, a similar approach was presented in [2] based on ladder network theory. The BW/FW Sweep algorithms use the Kirchhoff laws. Different formulations can be found [3, 4, 6, 7, 8, 10, 11, 12, 13, 14, 15]. BW/FW sweep methods typically present a slow convergence rate but computationally efficient at each iteration. Using these methods, power flow solution for a distribution network can be obtained without solving any set of simultaneous equations. In this work, the standard BW/FW sweep power flow is reformulated in convenient form. An illustrative four-bus example is solved.

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2 The Method

The input data of this algorithm is given by node-branch oriented data used by most utilities. Basic data required is: active and reactive powers, nomenclature for sending and receiving nodes, and positive sequence impedance model for all branches.

In the following, the standard BW/FW sweep power flow method is written in matrix notation using complex variables. Branch impedances are stated as a vector $\mathbf{Z}$ corresponding to a distribution line model containing a series positive sequence impedance for line or transformer. Shunt impedances are not considered in this first approach. Fig. 1 shows a radial distribution network with $n + 1$ nodes, and $n$ branches and a single voltage source at the root node 0. Branches are organized according to an appropriate numbering scheme (list), which details are provided in [3].

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z}_{01} & \cdots & \mathbf{Z}_{ij} & \cdots & \mathbf{Z}_{mn} \end{bmatrix}$$ (1)

where,

$$\mathbf{Z}_{ij} = R_{ij} + jX_{ij} \quad i, j = 1, \ldots, n \quad i \neq j$$ (2)

Bus data is given by

$$\mathbf{S} = \begin{bmatrix} \overline{S}_1 \\ \vdots \\ \overline{S}_i \\ \vdots \\ \overline{S}_n \end{bmatrix} = \begin{bmatrix} P_1 + jQ_1 \\ \vdots \\ P_i + jQ_i \\ \vdots \\ P_n + jQ_n \end{bmatrix}$$ (3)
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where net nodal active and reactive powers are given by generated and demanded powers:

\[ P_i = P_{Gi} - P_{Di} \]  \hspace{1cm} (4)  \\
\[ Q_i = Q_{Gi} - Q_{Di} \]  \hspace{1cm} (5)

The numbering of branches in one layer begins only after all the branches in the previous layer have been numbered. Considering that initial voltages are known: voltage at substation is set \( V_0 = V_{ref} \) and an initial voltage vector is given by:

\[ V^0 = \left[ V^0_1 \hspace{0.5cm} \ldots \hspace{0.5cm} V^0_i \hspace{0.5cm} \ldots \hspace{0.5cm} V^0_n \right] \]  \hspace{1cm} (6)

The state of the system is reached solving two steps iteratively.

2.1 Step 1 - Backward Sweep

For each iteration \( k \), branch currents are aggregated from loads to origin:

\[ J^k = -T \cdot I^k \]  \hspace{1cm} (7)

The relationship between nodal currents \( I^k \) and branch currents \( J^k \) is set through an upper triangular matrix \( T \) accomplishing the Kirchhoff Current Laws (KCL). Each element \( T^k_i \) of \( I^k \) associated to node \( i \) is calculated as function of injected powers \( S_i \) and its voltage profile \( V^k_i \) as shown below:

\[ T^k_i = \frac{S^k_i}{V^k_i} \hspace{0.5cm} i = 1, \ldots, n \]  \hspace{1cm} (8)

2.2 Step 2 - Forward Sweep

Nodal voltage vector \( V \) is updated from the origin to loads according the Kirchhoff Voltage Laws (KVL), using previously calculated branch currents vector \( J \), branch impedances vector \( Z \):

\[ V^{k+1} = V_0 - T^T \cdot D_Z \cdot J^k \]  \hspace{1cm} (9)

where \( V_0 \) is a \( n \)-elements vector with all entries set at voltage at origin (swing node) \( V_0 \) and branch impedances \( D_Z \) is the diagonal matrix of vector \( Z \).

Using Eq. \( 7 \)

\[ V^{k+1} = V_0 + T^T \cdot D_Z \cdot T \cdot I^k \]  \hspace{1cm} (10)
Updated voltages can be updated using only one equation:

\[
V^{k+1} = V_0 + TRX \cdot I^k
\]  

(11)

where \( TRX = T^T \cdot D_Z \cdot T \)

2.3 Convergence

Updated voltages are compared with previous voltages in order to perform convergence check in.

\[
\varepsilon \leq |V_{i}^{k+1} - V_{i}^{k}| \quad i = 1, ..., n
\] 

(12)

3 Illustrative Example: Simply 4-node Network

To illustrate the proposed methodology, it is used the 4-node example shown in Fig. 2. Length of all sections is 1 mile. Load demand at nodes 2 and 3 are 2MW with \( \cos \varphi = 1.0 \).

![4-Node Network Topology](image)

Using the following bases \( S_B =10\text{MW} \) and \( V_B=12.47\text{kV} \), data and results are given in per unit. Loads are 0.2 in nodes 2 and 3. Reference voltage at node 0 is \( V_0 = 1 + j0 \) and initial voltages are set \( V^0 = \begin{bmatrix} 1 + 0j & 1 + 0j & 1 + 0j \end{bmatrix} \).

Branches are represented by:

\[
Z = R + jX = \begin{bmatrix} .0296 \\ .0296 \\ .0296 \end{bmatrix} + j \begin{bmatrix} .0683 \\ .0683 \\ .0683 \end{bmatrix}
\]

Network topology is represented through a 3x3 upper triangular matrix \( T \).
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\[
T = \begin{bmatrix}
1 & 1 & 1 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Then, \( D_Z \) is:

\[
D_Z = \begin{bmatrix}
.0296 + j.0683 & 0 & 0 \\
0 & .0296 + j.0683 & 0 \\
0 & 0 & .0296 + j.0683
\end{bmatrix}
\]

Solution reached at iteration 3 for \( \varepsilon = 10^{-4} \) and displayed in Table 1. Results are presented in per unit and degrees.

| \( V_0 \) | \( \theta_0 \) | \( V_1 \) | \( \theta_1 \) | \( V_2 \) | \( \theta_2 \) | \( V_3 \) | \( \theta_3 \) |
|---------|---------|---------|---------|---------|---------|---------|---------|
| 1.000   | 0.00    | 0.987   | -1.59   | 0.981   | -2.40   | 0.981   | -2.40   |

4 Conclusion

This paper describes a convenient formulation of the Backward/Forward (BW/FW) Sweep Power Flow applied to radial distribution systems with distributed generation. Proposed formulation was applied in an illustrative test system.

5 Nomencature

List of Symbols

- \( D_Z \): Diagonal matrix of branch impedance vector \( Z \)
- \( R \): Diagonal matrix of branch resistance vector \( \Re Z \)
- \( X \): Diagonal matrix of branch reactance vector \( \Im Z \)
- \( \varepsilon \): Convergence criteria
- \( I \): Current vector
- \( J \): Branch Current vector \( J \)
- \( n \): Number of nodes, excluding origin
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\[ \mathbf{P} \quad \text{Active Power Injected vector} \]
\[ \mathbf{Q} \quad \text{Reactive Power Injected vector} \]
\[ P_j \quad \text{Active Power Injected at node } j \]
\[ Q_j \quad \text{Reactive Power Injected at node } j \]
\[ P_{Dj} \quad \text{Active Power Demanded at node } j \]
\[ Q_{Dj} \quad \text{Reactive Power Demanded at node } j \]
\[ P_{Gj} \quad \text{Active Power Generated at node } j \]
\[ Q_{Gj} \quad \text{Active Power Generated at node } j \]
\[ R_{ij} \quad \text{Resistance between node } i \text{ and node } j \]
\[ S_{Dj} \quad \text{Apparent Power Demanded at node } j \]
\[ S_{Gj} \quad \text{Apparent Power Generated at node } j \]
\[ \mathbf{T} \quad \text{Triangular matrix} \]
\[ \mathbf{V} \quad \text{Voltage vector} \]
\[ X_{ij} \quad \text{Reactance between node } i \text{ and node } j \]
\[ \mathbf{Z} \quad \text{Branch Impedance vector } \mathbf{Z} \]
\[ \mathbf{Z}_{ij} \quad \text{Branch Impedance between node } i \text{ and node } j \]
\[ \mathbf{Z}^{ij} \quad \text{Impedance matrix between node } i \text{ and node } j \]

**Operators**

\[ T \quad \text{Transpose Matrix} \]
\[ D \quad \text{Diagonal Matrix} \]
\[ * \quad \text{Conjugate of a complex number} \]

**Sub-Indexes**

\[ i \quad \text{Associated to node } i \]
\[ j \quad \text{Associated to node } j \]
\[ k \quad \text{Associated to iteration } k \]

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