An Energy Efficient Authentication Scheme using Chebyshev Chaotic Map for Smart Grid Environment

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Abstract—Smart grid (SG) is an automatic electric power transmission network with bidirectional flows of both energy and information. As one of the important applications of smart grid, charging between electric vehicles has attracted much attention. However, authentication between vehicle users and an aggregator may be vulnerable to various attacks due to the usage of wireless communications. Although several authentication schemes are proposed for smart grid environments with privacy protection, some of them still impose security issues such as anonymity absence. In addition, the existing authentication schemes have not thoroughly tackled the charging peak in their design, so these schemes cannot guarantee the requirement in terms of low energy consumption in smart grid environments. In order to reduce the computational costs yet preserve required security, the Chebyshev chaotic map based authentication schemes are proposed. However, the security requirements of Chebyshev polynomials bring a new challenge to the design of authentication schemes based on Chebyshev chaotic maps. To solve this issue, we propose a practical Chebyshev polynomial algorithm by using a binary exponentiation algorithm based on square matrix to achieve secure and efficient Chebyshev polynomial computation. We further apply the proposed algorithm to construct an energy-efficient authentication and key agreement scheme for smart grid environments. Compared with state-of-the-art schemes, the proposed authentication scheme effectively reduces the computational and communication costs by adopting the proposed Chebyshev polynomial algorithm. Furthermore, the ProVerif tool is employed to analyze the security of the proposed authentication scheme. Our experimental results justified that our proposed authentication scheme can outperform state-of-the-art schemes in terms of the computational overhead while achieving privacy protection.

Index Terms—Smart grid environment, Authentication, Key agreement, Chebyshev Chaotic Maps, Privacy protection.

I. INTRODUCTION

SMART grids create a widely distributed network of automated energy delivery by using two-way flows of electricity and information [1]. Compared with the traditional grid infrastructure, smart grids are more efficient, secure and reliable. As one of the important applications of smart grids, the vehicle-to-grid (V2G) network has attracted more and more attention due to the electric vehicles that are promising in reducing pollution and integrating renewable resources [2]. As shown in Figure 1, the V2G network contains three entities – power grid, aggregator (AGT), and electric vehicles (EVs). The power grid generates electricity from new renewable sources, such as solar and wind, and then sends the resulting electricity to the charging stations.

As an intermediary between the power grid and the EVs, the aggregator monitors and collects the current state of the EVs, optimizes and adjusts the EV charging plan, and minimizes the charging energy cost of electric vehicles. When an electric vehicle owner (EVO) intends to charge vehicle battery, he/she first logs in to the system using a smartcard and credentials. Then the EV will send a request to the AGT to establish communication between them. After that, EVO can complete the payment by interacting with the charging station.

However, the wireless communication technology adopted in V2G networks may cause various security threats [4, 5]. User private and sensitive information (e.g. the user identity, location and route of the vehicle) may be compromised by several attacks, such as impersonation attacks, message modification attacks, and etc. In practice, an attacker may utilize public channels to invade user privacy information and impersonate users to obtain services through V2G networks. Furthermore, a malicious station may impersonate others and charge more from users. Therefore, a security mechanism should be provided to protect the user privacy information during the charging process in smart grid environments.

Fig. 1. the System Model of the Vehicle-to-Grid Network.
Several authentication schemes have been explored to achieve secure communication \cite{3,6,11}. In order to provide high security, public-key cryptography has been applied in the design of the authentication scheme for secure vehicle charging services in smart grid environments. Despite the fact that the public-key cryptography solutions enhance the security, these existing authentication schemes still face another challenge, i.e., energy limitation. The EV charging increases the electric loads, potentially amplifying peak demand or creating new peaks in electricity demand \cite{8,10}. One possible scenario is that many EV owners go to charging stations after work to recharge their EVs, creating a charging peak in a short period of time. These charging activities can seriously affect smart grid systems, which causing network losses and increasing the likelihood of blackouts \cite{12}. Furthermore, when the charging requirements received by charging stations increase significantly, it may cause congestion and then greatly affects the convenience of users. However, most of the existing authentication schemes do not fully consider the charging peak in their design and fail to achieve a delicate balance between security and performance due to the usage of time-consuming operations such as expensive bilinear operations. Therefore, how to design an energy efficient authentication scheme for smart grid environments remains a challenging work.

A. Motivation

Although several authentication schemes are proposed for smart grid environments, some of them still impose security issues such as anonymity and perfect forward secrecy. In addition, to avoid the congestion caused by charging peak and provide more security and convenient services to EVOs, the authentication scheme should be lightweight. However, most existing authentication schemes have to rely on time-consuming operations to ensure security which may not be suitable for smart grid environments.

Chebyshev polynomials cryptosystem, as a promising method, is expected to reduce computational costs while preserving high security. However, the public key algorithm based on Chebyshev polynomial to deal with real numbers is not secure \cite{13}. To solve this problem, the Kocarev et al. \cite{14} extended Chebyshev polynomials from real fields to finite fields and finite rings, making the public key algorithm more secure and practical. Later, Chen et al. \cite{15} proved that the Chebyshev polynomials $T_n(x)$ mod $N$ is safe when modulus $N$ is a strong prime number satisfying $N-1=2p_1$ and $N+1=2p_2$, where $p_1$ and $p_2$ are also prime numbers. Therefore, modulus $N$ should be carefully selected to ensure that Chebyshev polynomials can produce sequences of sufficient period to resist violent attacks. However, in the existing Chebyshev polynomials algorithms such as the algorithms adopted in \cite{16,17}, the parameter $n$ of Chebyshev polynomials $T_n(x)$ mod $N$ is constructed with small primes which brings a new security challenge to the design of authentication schemes based on Chebyshev chaotic maps.

These issues inspire us to design a practical Chebyshev polynomials algorithm and then apply it to construct an energy efficient authentication scheme for smart grid environments.

B. Contribution

In this study, we focus on solving the energy limitation issue with privacy protection during authentication and key negotiation process in smart grid environments. The main contributions of our work are summarized as follows:

1) To solve the security challenge of Chebyshev polynomials algorithm in authentication scheme design, we propose a practical Chebyshev polynomial algorithm that adopts a binary exponentiation algorithm based on square matrix to achieve secure and efficient Chebyshev polynomial computation. The proposed algorithm guarantees the security requirements that are proved by Chen et al \cite{15}. Also, our experimental results also justified that the proposed algorithm is an efficient Chebyshev polynomial algorithm, thus it can be applied in our designated authentication scheme to achieve both security and efficiency.

2) Based on the proposed Chebyshev polynomial algorithm, we further construct an energy efficient authentication scheme for smart grid environments. The proposed scheme achieves fast mutual authentication and key agreement with anonymity. Furthermore, we employ an automatic verifier named ProVerif to analyze the security of the proposed scheme. The security analysis demonstrates that our proposed authentication scheme can resist known attacks.

3) The proposed authentication scheme is a lightweight authentication scheme since only the efficient Chebyshev polynomials and hash functions are adopted during the authentication and key negotiations process. The performance analysis shows that the proposed authentication scheme is more efficient in comparison with the state-of-art schemes.

C. Organization

The rest of this paper is organized as follows. Section II describes the related work. The mathematical background of Chebyshev polynomial is described in Section III. In Section IV our proposed Chebyshev polynomial algorithm is introduced in detail. The proposed scheme is presented in detail in Section V. In Section VI, the security of the proposed scheme is analyzed. The performance of the proposed scheme is discussed in Section VII. And the paper is concluded in Section VIII.

II. RELATED WORK

In recent years, various authentication and key agreement schemes using public-key cryptography are proposed for smart grids. To protect the private data transmitted in the smart grid, Wu and Zhou \cite{13} proposed a key management scheme using public-key cryptosystem and symmetric key cryptosystem. However, Xia and Wang \cite{19} demonstrated that their scheme \cite{18} was vulnerable to man-in-the-middle attacks. Although another key distribution scheme was proposed by Xia and Wang \cite{19} to overcome the weakness of \cite{13}, the new scheme \cite{19} suffered from impersonation attacks \cite{20}. Later, Tsai and Lo \cite{21} presented an anonymous key distribution scheme using identity-based signature. However, their scheme \cite{21} failed to resist privileged-insider attacks \cite{22}.

To further enhance the security and reduce the computational costs, elliptic curve cryptography (ECC) is adopted in
the design of authentication schemes. Based on ECC, He et al. [23] proposed a key distribution scheme for smart grids which achieved users anonymously and reduced the computational costs. Odelu et al. [24] also presented a key agreement scheme using ECC and analyzed their scheme with the CK-adversary model. But their scheme [24] fails to resist impersonation attacks and man-in-the-middle attacks, as pointed out in [25]. Then an ECC based self-certified key distribution scheme was proposed by Abbaspour et al. [26] to realize higher security. But their scheme requires the use of a tamper-proof security module. Later, Mahmood et al. [27] presented an ECC-based authentication scheme for the smart grid. However, their scheme fails to provide user anonymity since the user identity is transmitted directly over the public channel without protection. Recently, Kumar et al. [28] employed ECC, symmetric encryption and hash function to construct an authentication and key agreement scheme. Although ECC based scheme reduces the computational costs, the EC point multiplication operations involved in these schemes are still time-consuming operations for smart grid environments.

Chebyshev polynomials as a lightweight operation are also adopted in the design of authentication schemes. Chaotic maps based authentication schemes have been applied to a variety of environments, such as smart grids [29], multi-server environment [30,31], isolated smart meters [32] and point-of-care systems [33]. Recently, Abbaspour et al. [3] proposed a privacy preserving authentication scheme for vehicle to grid connections using Chebyshev chaotic maps. Their scheme reduces the computational cost in theory. However, they only execute each cryptography operation independently on the devices and then calculate a theoretical time as the execution time of their scheme. Obviously, it is not appropriate to take the theoretical execution time as the actual execution time of the authentication scheme.

III. PRELIMINARIES

In this section, we review the basic concepts of Chebyshev chaotic and the corresponding difficult problems associated with it.

**Definition 1 (Chebyshev Chaotic Map):** Let \( n \) be an integer and \( x \in [-1, 1] \), the Chebyshev polynomial is defined as (1) or (2) [34,37].

\[
T_n(x) = \cos(n \cos^{-1}(x)) \tag{1}
\]

\[
T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x); n \geq 2, T_0(x) = 1, T_1(x) = x \tag{2}
\]

**Definition 2 (Semigroup Property):** One of the most important property of Chebyshev polynomial is the semigroup property, which is shown as

\[
T_u(T_v(x)) = T_{uv}(x) = T_v(T_u(x)) \tag{3}
\]

Zhang [38] demonstrate that the semigroup property of Chebyshev polynomials also holds, when Chebyshev polynomial domain is defined on intervals \((-\infty, +\infty)\). The enhanced Chebyshev polynomial is defined as (4), where \( p \) is a large prime number

\[
T_n(x) = (2xT_{n-1}(x) - T_{n-2}(x)) \mod p, n \geq 2, x \in (-\infty, +\infty) \tag{4}
\]

**Definition 3 (Chaotic Map-Based Discrete Logarithm Problem (CMBDLP)) [37,39,40]:** Given \( x \) and \( y \), it is almost impossible to find the integer \( v \), such that \( T_v(x) = y \). The probability that an adversary \( A \) can solve the CMBDLP is defined as \( Adv_A^{CMBDLP}(p) = Pr[A(x, y) = v : v \in Z_p^*, y = T_v(x) \mod p] \).

**Definition 4 (CMBDLP Assumption) [37,39,40]:** For any probabilistic polynomial time-bounded adversary \( A \), \( Adv_A^{CMBDLP}(p) \) is negligible, that is, \( Adv_A^{CMBDLP}(p) < \varepsilon \).

**Definition 5 (Chaotic Map-Based Diffie-Hellman Problem (CMBDHP)) [37,39,40]:** Given \( x, T_u(x) \) and \( T_v(x) \), it is almost impossible to find \( T_{uv}(x) \). The probability that a polynomial time-bounded adversary \( A \) can solve the CMBDHP is defined as \( Adv_A^{CMBDHP}(p) = Pr[A(x, T_u(x) \mod p, T_v(x) \mod p = T_{uv}(x) \mod p : u, v \in Z_p^*] \).

**Definition 6 (CMBDHP Assumption) [37,39,40]:** For any probabilistic polynomial time-bounded adversary \( A \), \( Adv_A^{CMBDHP}(p) \) is negligible, that is, \( Adv_A^{CMBDHP}(p) < \varepsilon \).

IV. CHEBYSHEV POLYNOMIAL ALGORITHM

In this section, we describe the proposed Chebyshev polynomial algorithm in detail. In order to satisfy the security requirements and reduce the time complexity of Chebyshev polynomial algorithm, we adopt a binary exponentiation algorithm based on square matrix to compute Chebyshev polynomials. Furthermore, we employ the following matrices instead of recursive relationships to define Chebyshev polynomials.

\[
\begin{bmatrix}
T_{n+1}(x) \\
T_n(x)
\end{bmatrix} =
\begin{bmatrix}
2x & -1 \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
T_n(x) \\
T_{n-1}(x)
\end{bmatrix} \mod q
\]

\[
\begin{bmatrix}
T_{n+1}(x) \\
T_n(x)
\end{bmatrix} =
\begin{bmatrix}
2x & -1 \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
T_n(x) \\
T_{n-1}(x)
\end{bmatrix} \Rightarrow
\begin{bmatrix}
2x & -1 \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
T_1(x) \\
T_0(x)
\end{bmatrix} \mod q
\]

Fig. 2. Flow chart of Chebyshev polynomial algorithm.

The \( n \)th power modulus \( q \) of the matrix \( \begin{bmatrix} 2x & -1 \\ 1 & 0 \end{bmatrix} \) can be solved by the binary power algorithm in polynomial time.
The detailed steps of the proposed algorithm to compute $T_n(x)$ mod $n$ are shown in Figure 2, where $[n]$ represents the integer part of $n$.

To meet the security requirements, in our proposed Chebyshev polynomial algorithm, $n$ is a large prime number, and modulus $N$ is a strong prime number satisfying $N=2p_1$ and $N+1=2p_2$. Therefore, our proposed algorithm is safe according to [13]. Furthermore, we have implemented the Chebyshev polynomial operations on Intel Intel Celeron CPU G3900T to investigate the practicability of our proposed algorithm. Table I illustrates that the proposed algorithm is an efficient Chebyshev polynomial algorithm. Furthermore, from Table I we can conclude that the execution time of Chebyshev polynomial increases as the number of bits of parameter $n$ increases. The above conclusion also shows that the calculation method of execution time adopted in [3] is inappropriate.

### Table I

| Execution Times of Chebyshev Polynomial |
|-----------------------------------------|
| The bits of parameter $n$ | Execution time  |
|---------------------------|-----------------|
| 128bits                   | 0.187385ms      |
| 160bits                   | 0.235239ms      |
| 256bits                   | 0.381572ms      |
| 512bits                   | 0.754054ms      |

### V. OUR PROPOSED SCHEME

Based on the proposed Chebyshev polynomial algorithm, we construct a lightweight authentication scheme that aims to reduce computational costs with high security. There are four phases in the proposed scheme, system setup phase, registration phase, login phase and authentication phase. In the system setup phase, the trusted authority (TA) generates system parameters and publishes public information. During the registration phase, TA completes registration of the electric vehicle (EV) with a smartcard and the aggregator (AGT) in a secure channel. Then in the authentication phase, the EV and AGT authenticate each other and establish session keys for future communications. Some notations used throughout the rest paper are described in Table II. And the procedures of the proposed scheme are presented in detail as follows.

### A. System Setup Phase

In the system setup phase, the trusted authority (TA) selects system parameters and generates its key pairs. Meanwhile, TA registers each aggregator (AGT) before their deployment in the network. The steps of this phase are given below.

**Step 1**: The trusted authority TA first chooses a large prime number $p$, and then selects a high entropy random integer $x$ as the seed of Chebyshev polynomial.

**Step 2**: The trusted authority TA selects a high entropy random integer $k$, and computes its corresponding public key $pubTA$ as in (1). Next, the TA generates its identity $IDTA$ and calculates a pseudo identity for itself using the value $IDTA$ and its private key $k$ as in (2).

$$pubTA = T_k(x) \mod p$$

$$RIDA = h(IDTA)[k]$$

**Step 3**: The trusted authority TA chooses a collision-resistant hash functions $h() : \{0, 1\}^* \rightarrow \{0, 1\}^t$. Then it publishes the system parameters $p, x$, $pubTA$, $h()$ and keeps its private key $k$ secretly.

### B. Registration Phase

When an electric vehicle $EV_i$ wants to access the aggregator $AGT_j$, it needs to perform the following registration process. In this phase, a smartcard is issued for each electric vehicle owner and the communication channels are supposed to be secure.

**Step 1**: The electric vehicle owner (EVO) freely chooses his/her identity $ID_i$ and password $PW_i$. Then, it selects a high entropy random integer $r_1$ and calculates its pseudo identity $RIDA$ as in (3). It also adopts its identity $ID_i$ and password $PW_i$ to compute $RPWi$ as in (4). After that, EVO sends message $\{RIDA, RPWi\}$ to the trusted authority TA through a secure channel. After receiving the message, the trusted authority TA chooses a high entropy random integer $r_u$ and then uses this integer, EVOs pseudo identity $RIDA$, public key $pubTA$ and private key $k$ to obtain $Q_i$ as in (5). Then, the TA further calculates the signature $s_i$ as in (6) via the computed value $Q_i$, pseudo identity $RIDA$, random integer $r_u$ and private key $k$. Next, the TA adopts its private key $k$ and the computed value $RPWi$ to generate $Y$ as in (7). After that, it writes $\{RIDA_T, Y, Q_i, s_i\}$ into the smartcard and delivers the smartcard to the EVO in a secure way. Upon receiving the smartcard, EVO adopts $Y$ stored in the smartcard and its identity $ID_i$ to compute $I$ as in (8). And then it computes the private key $k_i$ as in (9) using its privacy information $\{ID_i, PW_i, r_1\}$ and the signature $s_i$ of the TA. Then, the EVO stores the information $\{RIDA_i, k_i\}$ and replaces the $Y$ with $I$ in the memory of his/her smartcard. Finally, the memory of the smartcard contains $\{RIDA_T, RIDA_i, I, Q_i, s_i, k_i\}$. After this step, the EV/EVO finishes the registration at the TA.

$$RIDA_i = h(r_1 \oplus ID_i)$$

$$RPWi = h(ID_i \oplus PW_i)$$

$$Q_i = TRIDA_i, T_h(r_u \oplus k)(pubTA)$$

$$s_i = h(RIDA_i || Q_i)h(r_u \oplus k)$$

$$Y = TRPW_i, T_k(x)$$

$$I = T_i, I$$

$$k_i = h(r_1 \oplus ID_i || PW_i)$$

$$s_i$$
Step 2: Firstly, the aggregator $AGT_j$ generates its identity $ID_j$, and selects a high entropy random integer $r_j$. Then it calculates its pseudo identity $RID_j$ as in (10) and sends it to the trusted authority $TA$ in a secure channel. After receiving the message from the aggregator $AGT_j$, the TA chooses a high entropy random integer $r_s$ and computes $Q_j$ as in (11) using this integer, the public key $pubTA$ and private key $k$. Then the TA further adopts the computed value $Q_j$, random integer $r_s$ and its private key $k$ to calculate the signature $s_j$ as (12). After that, it sends the message $\{RIDTA, Q_j, s_j\}$ to the aggregator $AGT_j$ through a secure channel. Subsequently, the aggregator $AGT_j$ computes its private key $k_j$ as (13) using the signature $s_j$ received from the TA. Finally, the aggregator $AGT_j$ stores the information $\{RIDTA, RID_j, Q_j, s_j, k_j\}$ in its memory secretly. When this step is finished, the registration process of the aggregator $AGT_j$ on the TA is completed.

\[
\begin{align*}
RID_j &= h(ID_j \oplus r_j) \\
Q_j &= T_{RID_j}h(r_s \oplus k)(pubTA) \\
s_j &= h(RID_j || Q_j)h(r_s \oplus k)k \\
k_j &= h(ID_j \oplus r_j)s_j
\end{align*}
\]

C. Login Phase

In this phase, the registered EVO makes a login request to the aggregator $AGT_j$.

Step 1: The EVO first inserts the smartcard in the $EV_i$ and inputs his/her identity $ID_i$ and password $PW_i$.

Step 2: Then the electric vehicle $EV_i$ computes $I_0$ using the inputted identity $ID_i$ and password $PW_i$ as (14) to check whether the value of $I_0$ is equal to the value of $I$. If true, the electric vehicle $EV_i$ chooses a high entropy random integer $r_i$ and calculates $C_1$ as (15) via the private key $k_i$ of the $EV_i$. Finally, the electric vehicle $EV_i$ sends the login request $\{C_1\}$ to the corresponding aggregator $AGT_j$ via a public channel.

\[
\begin{align*}
I_0 &= T_{ID}(h(ID_i || PW_i)(pubTA)) \\
C_1 &= T_{r_ik_i}(x)
\end{align*}
\]

D. Authentication and key agreement Phase

After receiving a login request from electric vehicle $EV_i$, some messages need to transmit between the electric vehicle $EV_i$ and the accessed aggregator $AGT_j$ to achieve the mutual authentication and key agreement. In this phase, the $EV_i$ and the $AGT_j$ perform the following steps:

Step 1: When the aggregator $AGT_j$ receives the login request $\{C_1\}$, it chooses a high entropy random integer $r_2$ and calculates $C_2$ as (16). And the aggregator $AGT_j$ further computes $X$ as (17) and $C_3$ as (18). Finally, it generates an authentication message $Auth_u$ as (19) and sends message $\{C_2, C_3, Auth_u\}$ to the $EV_i$ via a public channel.

\[
\begin{align*}
C_0 &= T_{r_2}(Q_j) \\
X &= T_{r_2k_i}(C_1) = T_{r_1r_2k_ik_i}(x) \\
C_3 &= h(RID_j || Q_j) \oplus h(pubTA) || RIDTA \\
Auth_u &= h(X || RIDTA || C_1 || C_2)
\end{align*}
\]

Step 2: After receiving the corresponding message $\{C_2, C_3, Auth_u\}$ from the aggregator $AGT_j$, the electric vehicle $EV_i$ computes $h(RID_j || Q_j)$ as (20) by adopting the received message $C_3$, the public key $pubTA$ and the pseudo identity $RIDTA$. Then it uses the computed value $h(RID_j || Q_j)$, the received message $C_2$ and the EVOs private key $k_i$ to obtain $X'$ as (21). Next, the electric vehicle $EV_i$ computes $Auth_u$ as (22) and compares it with the received authentication message $Auth_u$. If they are equivalent, it computes $I_1$ as (23) using the private key $k_i$ and then it can obtain $C_4$ as (24). After that, the electric vehicle $EV_i$ uses the computed value $X'$ and $C_4$ to generate its authentication message $Auth_u$ as (25). Finally, the electric vehicle $EV_i$ computes the session key $SK_{ij}$ as (26) and sends the message $\{C_4, Auth_u\}$ to the aggregator $AGT_j$.

\[
\begin{align*}
& h(RID_j || Q_j) = C_3 \oplus h(pubTA) || RIDTA \\
& X' = T_{r_1k_i}(T_{r_1k_i}(C_4)) = T_{r_1k_i}(h(RID_j || Q_j)r_2(Q_j)) \\
& Auth_u = h(X || RIDTA || C_1 || C_2) \\
& I_1 = h(RID_i \oplus k_i) \\
& C_4 = h(RID_j || Q_j) \oplus X' \oplus I_1 \\
& Auth_u = h(X' || C_4) \\
& SK_{ij} = h(RIDTA || X' || I_1)
\end{align*}
\]

Step 3: Upon receiving the message from the electric vehicle $EV_i$, the aggregator $AGT_j$ first computes $Auth_u$ as (27). And then it checks whether the equation $Auth_u = Auth_u$ holds. If true, it computes $I'_1$ as (28) using the received message $C_4$ and the previous calculated value $X$, and then obtains the shared session key $SK_{ij}$ as (29).

\[
\begin{align*}
& Auth_u = h(X || C_4) = h(T_{r_2k_i}(C_1)) || C_4 \\
& I'_1 = h(RID_j || Q_j) \oplus C_4 \oplus X \\
& SK_{ij} = h(RIDTA || X || I'_1)
\end{align*}
\]

Finally, the electric vehicle $EV_i$ and the aggregator $AGT_j$ achieve mutual authentication and key negotiation. The login and authentication also are shown in Figure 3.

Suppose that the $EV_i$ and the $AGT_j$ are legal. The $SK_{ij}$ is the session key generated by the $EV_i$ and the $SK_{ij}$ is the session key computed by the $AGT_j$. Now we prove that the equation $SK_{ij} = SK_{ij}$ is held in our proposed scheme.

Proof: $SK_{ij} = h(RIDTA || X || h(RID_i \oplus k_i))$

Proof: $SK_{ij} = h(RIDTA || X || h(RID_i \oplus k_i))$

VI. Security Analysis

In this section, we adopt an automatic verifier named ProVerif to analyze the security of the proposed scheme. Moreover, we discuss the possible attacks in Section VI-B.
A. Automatic Formal Verification of Security Using ProVerif

In this section, we demonstrate the security of our proposed scheme using a widely accepted automatic protocol verifier named ProVerif [41]. ProVerif can be utilized to verify the correspondence assertions, observational equivalences and reachability properties. Specifically, we can validate the resistance of cryptographic protocols against impersonation attacks, modification attacks, and replay attacks by launching injective correspondence assertion queries. Moreover, by using observational equivalence queries, some security properties such as identity guessing attacks can be verified via ProVerif. Furthermore, by making reachability queries, both the anonymity feature and the secrecy of the session key can be checked. Significantly, ProVerif can also be used to verify the perfect forward secrecy of the protocol by leaking some parameters. So, we employed ProVerif tool to implement our proposed authentication scheme and the authentication phase of the EVi and the AGTj are shown in Figure 4.

Figure 5 indicates the results from the ProVerif. From Fig. 5, the results (1)-(2) show that the adversary cannot obtain the private key $k_i$ of the EVi and the private key $k_j$ of the AGTj. Results (3)-(4) demonstrate the secrecy of the session keys $SK_{ij}$ and $SK_{ji}$. Results (5)-(6) prove the anonymity of the EVi and the AGTj. Results (7)-(8) are the results of two injective correspondence assertions which guarantees that the mutual authentication between the EVi and the AGTj is valid. In addition, injectivity allows the EVi and the AGTj to check the freshness of received messages which can resist replay attacks. Therefore, the results (1)-(8) prove that the proposed scheme provides session key security, anonymity, mutual authentication and can resist replay attacks.

Moreover, we also conducted experiments to demonstrate that our proposed scheme provides perfect forward secrecy. In our experiments, the private keys of the EVi and the AGTj are transmitted over the public channel $c$, which means long-term keys are leaked to the adversary. As the results (1)-(2) of Figure 6 shows that both “not attacker($k_i[]$)” and “not attacker($k_j[]$)” are false, which proves that the adversary has obtained the $k_i$ and the $k_j$. However, the results of “not attacker ($SK_{ij}[]$)” and “not attacker ($SK_{ji}[]$)” are still true. It demonstrates that even if the $k_i$ and the $k_j$ are leaked, the
session key $SK_{ij}(SK_{ji})$ cannot be compromised. Therefore, the proposed scheme provides perfect forward secrecy.

So, malicious electric vehicle $EV_a$ cannot impersonate a legal electric vehicle $EV_i$ to communicate with the $AGT_j$. On the other hand, when a registered aggregator $AGT_b$ becomes a malicious attacker and try to impersonate another legitimate aggregator $AGT_j$, he/she needs to obtain $AGT_j$'s private key $k_j$. However, without the knowledge of $AGT_j$'s private key $s_j$, the $AGT_b$ cannot calculate $k_j$ correctly. Therefore, our scheme can resist insider impersonation attacks.

### Offline password guessing attacks with without smartcards

Assume that an adversary $A$ obtains all the messages transmitting between the $EV_i$ and the $AGT_j$ and tries to launch an offline dictionary attack to get $EV_i$'s password. To obtain the $PW_i$, the adversary $A$ first needs to extract $k_i$ from $C_1$ which is equivalent to solve an instance of CMBDLP. Even if the adversary $A$ gets the $k_i$, he/she still cannot obtain $PW_i$ without the knowledge of $EV_i$'s $ID_i$, $r_i$, and $s_i$. Therefore, the adversary $A$ cannot launch offline dictionary attacks without smartcards successfully.

Suppose that an adversary compromises all the private information $\{RID_{TA}, RID_i, Q_i, s_i, k_i, I\}$ stored in the smartcard of the $EV_i$ and performs offline dictionary attacks with smartcards. Compared with offline dictionary attacks without smartcards, the additional information known by the adversary $A$ in this attack is the information $\{RID_{TA}, RID_i, Q_i, s_i, k_i, I\}$ stored in the smartcard. According to the above discussion, the adversary $A$ cannot obtain the $EV_i$'s $PW_i$ using $k_i$. Furthermore, when the adversary $A$ tries to extract $PW_i$ from $I = T_{ID_i}(T_h(ID_i@PW_i)(x))$, he/she will face the CMBDLP. Even if the adversary $A$ solves the CMBDLP, without knowing the $EV_i$'s $ID_i$ and the $TAs$ private key $k_i$, he/she still cannot guess $PW_i$ correctly. Thus, the proposed scheme can resist offline password guessing attacks with without smartcards.

### VII. Performance Analysis

In this section, we will compare the security features, computational costs and communication costs of our proposed scheme with other related schemes [3, 22, 24, 27, 28].

#### A. Comparison of Security Features

The security features of our proposed scheme and the other five related schemes [3, 22, 24, 27, 28] are compared in Table III. As shown in Table III, Mahmood et al.'s scheme [22] fails to provide user anonymity. Odelu et al.'s scheme [24] is vulnerable to impersonation attacks and man-in-the-middle attacks. Although Wazid et al.'s scheme [22] and Kumar et al.'s scheme [28] are successful against common attacks, they involve time-consuming operations. Moreover, the related schemes [3, 22, 24, 27, 28] don't provide automatic formal verification of security. According to Table III, our proposed scheme can resist various attacks and provide more security features in comparison with the other five related schemes [3, 22, 24, 27, 28].

#### B. Computational Cost

In this subsection, we compare the computational costs of our proposed scheme and other five related schemes. In our experiments, we adopt OpenSSL library [42], GMP library [43] and PBC Library [44] to simulate these schemes on
two Ubuntu 16.04 virtual machines with an Intel Intel(R) Celeron(R) CPU G3900T 2.60 GHz processor, 4GB of RAM. The simulation results are shown in Table IV. The notation $T_h$, $T_e$, $T_s$, $T_h$, $T_m$, $T_a$ and $T_e$ denote the time for executing a bilinear pairing operation, a modular exponentiation operation, a symmetric key encryption/decryption operation, a one-way hash function operation, a scalar multiplication operation of an elliptic curve, a point addition operation of an elliptic curve, a symmetric key encryption/decryption operation and a Chebyshev polynomial operation, respectively.

As shown in Table IV, the Odelu et al. scheme requires performing four point multiplication operations of elliptic curve, fourteen hash operations, five modular exponentiation operations and two bilinear pairing operations to complete the authentication. Then, the execution time is given by $4T_m + 14T_h + 5T_e + 2T_a$ and the actual simulation time was 23.973 ms. From Table IV, the computational costs of Odelu et al. scheme are much higher than other related schemes and our scheme. That because Odelu et al. scheme involves heavyweight operations: bilinear pairing operations. In addition, from Table IV, the execution time of Wazid et al. scheme [22], Mahmood et al. scheme [27], and Kumar et al. scheme [28] is 15.347 ms, 17.436 ms and 16.296 ms respectively. Compared with Odelu et al. scheme [24], these schemes reduce the computational costs effectively by avoiding the use of bilinear pairing operations.

Furthermore, the proposed scheme requires to perform five Chebyshev polynomial operations and six hash operations on the EV side, and needs to execute two Chebyshev polynomial operations and five hash operations on the AGT side. Then the total execution time is given by $7T_e + 11T_h$ and the actual simulation time is 3.731 ms. As shown in Table IV, the computational costs of our scheme and Abbasinezhad-Mood et al. scheme [3] are 3.731 ms and 5.18 ms, which are much lower than other related schemes [22, 24, 27, 28]. According to Table IV, our scheme and Abbasinezhad-Mood et al. scheme [3] outperforms the related schemes in terms of the computational overhead. That is because efficient Chebyshev polynomial operations are adopted in our scheme and Abbasinezhad-Mood et al. scheme [3].

As shown in Figure 7, the proposed scheme achieves the best performance, taking only 3.731 ms in total. And on the AGT side, our scheme also achieves the lowest computational costs, which only takes 1.118 ms. Compared with other related schemes, our proposed scheme reduces the computational costs up to 28.0%, 75.7%, 84.4%, 78.6% and 77.1% respectively. Therefore, the proposed authentication scheme is an energy efficient authentication scheme and is suitable for smart grid environments.
REFERENCES

[1] X. Fang, S. Misra, G. Xue, and D. Yang, “Smart grid: the new and improved power grid: A survey,” IEEE Communications Surveys & Tutorials, vol. 14, no. 4, pp. 944–980, 2012.

[2] V. Raveendran, S. Kanaran, S. Shanthisree, and M. G. Nair, “Vehicle-to-grid ancillary services using solar powered electric vehicle charging stations,” in 2019 4th International Conference on Recent Trends on Electronics, Information, Communication & Technology (RTEICT), 2019.

[3] D. Abbasinezhad-Mood, A. Ostad-Sharif, S. M. Mazinani, and M. Nikooghadam, “Provably-secure escrowless chebyshev chaotic map-based key agreement protocol for vehicle to grid connections with privacy protection,” IEEE Transactions on Industrial Informatics, vol. PP, no. 99, pp. 1–1.

[4] Z. Wan, N. Xiong, and N. Ghani, “Adaptive unequal protection for wireless video transmission over ieee 802.11e networks,” Multimedia Tools and Applications, vol. 72, no. 1, pp. 541–571, 2014.

[5] C. Lin, Y. He, and N. Xiong, “An energy-efficient dynamic power management in wireless sensor networks,” in Fifth International Symposium on Parallel & Distributed Computing, 2006.

[6] A. Irshad, M. Usman, S. A. Chaudhry, H. Naqvi, and M. Shafiq, “A provably secure and efficient authenticated key agreement scheme for energy internet-based vehicle-to-grid technology framework,” IEEE Transactions on Industry Applications, vol. 56, no. 4, pp. 4425–4435, 2020.

[7] S. A. Chaudhry, A. Albeshri, N. Xiong, C. Lee, and T. Shon, “A privacy preserving authentication scheme for roaming in ubiquitous networks,” Cluster Computing, vol. 20, no. 2, pp. 1–14, 2017.

[8] L. Kelly, A. Rowe, and P. Wild, “Analyzing the impacts of plug-in electric vehicles on distribution networks in british columbia,” in Electrical Power & Energy Conference, 2010.

[9] L. Xiong, N. Xiong, C. Wang, X. Yu, and M. Shuai, “An efficient lightweight authentication scheme with adaptive resilience of asynchronization attacks for wireless sensor networks,” IEEE Transactions on Systems, Man, and Cybernetics: Systems, pp. 1–13, 2019.

[10] K. Clement-Nyns, E. Haesen, and J. Driesen, “The impact of charging plug-in hybrid electric vehicles on a residential distribution grid,” IEEE Transactions on Power Systems, vol. 25, no. 1, pp. 371–380, 2010.

[11] G. Xu, W. Zhou, A. K. Sangaiah, Y. Zhang, X. Zheng, Q. Tang, N. Xiong, K. Liang, and X. Zhou, “A security-enhanced certificateless aggregate signature authentication protocol for invanets,” IEEE Network, vol. 34, no. 2, pp. 22–29, 2020.

[12] S. Deilami, A. S. Masoum, P. S. Moses, and M. A. S. Masoum, “Real-time coordination of plug-in electric vehicle charging in smart grids to minimize power losses and improve voltage profile,” IEEE Transactions on Smart Grid, vol. 2, no. 3, pp. 456–467, 2011.

[13] P. Bergamo, P. D’Arco, A. De Santis, and L. Kocarev, “Security of public-key cryptosystems based on chebyshev polynomials,” IEEE Transactions on Circuits and Systems I: Regular Papers, vol. 52, no. 7, pp. 1382–1393, 2005.

[14] L. Kocarev, J. Makraduli, and P. Amato, “Public-key encryption based on chebyshev polynomials,” 2005, pp. 497–517.

[15] F. Chen, X. Liao, T. Xiang, and H. Zheng, “Security analysis of the public key algorithm based on chebyshev polynomials over the integer ring zn,” Information Sciences, vol. 181, no. 22, pp. 5110–5118, 2011.

[16] D. Xiao, X. Liao, and S. Deng, “A novel key agreement protocol based on chaotic maps,” Information Sciences, vol. 177, no. 4, pp. 1136–1142, 2007.

[17] X. Wang and J. Zhao, “An improved key agreement protocol based on chaos,” Communications in Nonlinear Science & Numerical Simulation, vol. 15, no. 12, pp. 4052–4057, 2010.

[18] D. Wu and C. Zhou, “Fault-tolerant and scalable key management for smart grid,” IEEE Transactions on Smart Grid, vol. 2, no. 2, pp. 375–381, 2011.

[19] J. Xia and Y. Wang, “Secure key distribution for the smart grid,” IEEE Transactions on Smart Grid, vol. 3, no. 3, pp. 1437–1443, 2012.
[20] Park, H. J., Kim, M., Kwon, and D., “Security weakness in the smart grid key distribution scheme proposed by xia and wang,” IEEE Transactions on Smart Grid, vol. 4, no. 3, pp. 1613–1614, 2013.

[21] J. L. Tsai and N. W. Lo, “Secure anonymous key distribution scheme for smart grid,” IEEE Transactions on Smart Grid, vol. 7, no. 2, pp. 906–914, 2016.

[22] M. Wazid, A. K. Das, N. Kumar, and J. J. P. C. Rodrigues, “Secure three-factor user authentication scheme for renewable-energy-based smart grid environment,” IEEE Transactions on Industrial Informatics, 2017.

[23] D. He, L. Wang, H. Wang, and M. K. Khan, “Lightweight anonymous key distribution scheme for smart grid using elliptic curve cryptography,” IET Communications, 2016.

[24] V. Odelu, A. K. Das, M. Wazid, and M. Conti, “Provably secure authenticated key agreement scheme for smart grid,” IEEE Transactions on Smart Grid, vol. 9, no. 3, pp. 1900–1910, 2018.

[25] C. Yuwen, M. Jos-Fernn, C. Pedro, and L. Lourdes, “An anonymous authentication and key establish scheme for smart grid: Fauth,” Energies, vol. 10, no. 9, p. 1354, 2017.

[26] D. Abbasinezhad-Mood and M. Nikooghadam, “An anonymous ecc-based self-certified key distribution scheme for the smart grid,” IEEE Transactions on Industrial Electronics, vol. 65, no. 10, pp. 7996–8004, 2018.

[27] K. Mahmood, S. A. Chaudhry, H. Naqvi, S. Kumari, X. Li, and A. K. Sangaiiah, “An elliptic curve cryptography based lightweight authentication scheme for smart grid communication,” Future Generation Computer Systems, vol. 81, no. APR., pp. 557–565, 2018.

[28] P. Kumar, A. Gurtov, M. Sain, A. Martin, and P. H. Ha, “Lightweight authentication and key agreement for smart metering in smart energy networks,” IEEE Transactions on Smart Grid, vol. 10, no. 4, pp. 4349–4359, 2019.

[29] M. Qi and J. Chen, “Two-pass privacy preserving authenticated key agreement scheme for smart grid,” IEEE Systems Journal, pp. 1–7, 2020.

[30] S. Chatterjee, S. Roy, A. K. Das, S. Chattopadhyay, N. Kumar, and A. V. Vasilakos, “Secure biometric-based authentication scheme using chebyshev chaotic map for multi-server environment,” IEEE Transactions on Dependable and Secure Computing, vol. 15, no. 5, pp. 824–839, 2018.

[31] H. Wang, D. Guo, Q. Wen, and H. Zhang, “Chaotic map-based authentication protocol for multiple servers architecture,” IEEE Access, vol. 7, pp. 161 340–161 349, 2019.

[32] D. Abbasinezhad-Mood and M. Nikooghadam, “Efficient anonymous password-authenticated key exchange protocol to read isolated smart meters by utilization of extended chebyshev chaotic maps,” IEEE Transactions on Industrial Informatics, vol. 14, no. 11, pp. 4815–4828, 2018.

[33] L. Zhang, H. Luo, L. Zhao, and Y. Zhang, “Privacy protection for point-of-care using chaotic maps-based authentication and key agreement,” Journal of Medical Systems, vol. 42, no. 12, 2018.

[34] L. Kocarev and Z. Tasev, “Public-key encryption based on chebyshev maps,” in International Symposium on Circuits & Systems, 2003, pp. III–28–III–31.

[35] L. Kocarev, “Chaos-based cryptography: a brief overview,” IEEE Circuits and Systems Magazine, vol. 1, no. 3, pp. 6–21, 2001.

[36] M. S. Farash and M. A. Attari, “An efficient and provably secure three-party password-based authenticated key exchange protocol based on chebyshev chaotic maps,” Nonlinear Dynamics, vol. 77, no. 1-2, pp. 399–411, 2014.

[37] T. Lee, “Provably secure anonymous single-sign-on authentication mechanisms using extended chebyshev chaotic maps for distributed computer networks,” IEEE Systems Journal, vol. 12, no. 2, pp. 1499–1505, 2018.

[38] L. Zhang, “Cryptanalysis of the public key encryption based on multiple chaotic systems,” Chaos Solitons & Fractals, vol. 37, no. 3, pp. 669–674, 2008.

[39] H. F. Zhu, Y. F. Zhang, Y. Xia, and H. Y. Li, “Password-authenticated key exchange scheme using chaotic maps towards a new architecture in standard model,” International Journal of Network Security, vol. 18, no. 2, pp. 326–334, 2016.

[40] Islam and S. K. Hafizul, “Provably secure dynamic identity-based three-factor password authentication scheme using extended chaotic maps,” Nonlinear Dynamics, vol. 78, no. 3, pp. 2261–2276, 2014.

[41] K. B. Bruno Blanchet, “Proverif: Cryptographic protocol verifier in the formal model,” http://prosecco.gforge.inria.fr/personal/bblanche/proverif/.

[42] 2018.[Online].Available:https://www.openssl.org/.

[43] 2016.[Online].Available:https://gmplib.org/.

[44] 2019.[Online].Available:https://crypto.stanford.edu/pbc/