UAV formation obstacle avoidance control method based on artificial potential field and consistency

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Abstract. As a typical multi-agent formation, UAV formation is playing an increasingly powerful role in the civilian and military fields. Obstacle avoidance, as an important technology in controlling formation, determines the application prospects of UAVs. This paper studies the time-varying formation of UAVs with interactive topology to avoid obstacles, aiming to improve the ability of UAV formations to deal with complex environments while traveling. Firstly, a repulsive force field is reasonably introduced based on the existing control scheme, and an improved distributed time-varying formation control scheme based on artificial potential field is proposed. Then combined with the basic idea of model predictive control, an obstacle avoidance strategy in which UAV obstacle avoidance and formation shaping are carried out simultaneously is proposed. Finally, a time-varying formation simulation experiment containing four UAVs was carried out to verify the validity of the results.

Key words: Time-varying formation, interactive topology, artificial potential field, obstacle avoidance, model prediction.

1. Introduction
In recent years, the formation-controlling technology of multi-agent system has drawn wide attention and there are many research on this topic. Moreover, it can be widely applied. It plays indispensable role in fields like surveillance [1], spacecraft attitude tracking [2], ocean observation [3] and has promising prospects in the formation control of agent in intelligent robots [4] [5], unmanned ground vehicles (UGV) [6] and Autonomous Underwater Vehicles (AUV) [7]. With the development of communication and other technologies, unmanned aerial vehicles (UAV) [8], as a typical representative of agents, are used to solve more and more complex tasks and have obvious advantages in execution efficiency, stability, and robustness. Three typical formation control methods are based on leader-follower method [9], virtual structure [10] and behavior [11]. As the most used formation-controlling technology at present, the leader-follower method stipulates the hierarchy of formation control problems. Its basic idea is that a UAV acts as a leader to sail along a path that is predetermined or temporarily set. Other members in the formation as followers make corresponding actions based on the distance and orientation information relative to the leader to achieve formation control. The virtual structure-based method regards the formation as a rigid structure, and the behavior-based method divides the formation control tasks into simple ones and integrates them, such as obstacle avoidance, following and
maintaining formation, and make different responses according to different environmental information to realize formation movement control. This method is easy to realize distributed control. Distributed control is the result of classification according to the way of information exchange between agents. In addition, there are two formation control methods: centralized control and decentralized control. Since distributed control does not have a high command of the bandwidth of the communication, its good adaptability to the environment, good robustness, simplicity, and ease of engineering realization are advantages to the reorganization and separation of formations. It is the mainstream form of information interaction in formations at present.

Based on the leader-follower method, the virtual structure-based method, and the behavior-based method, the system has become mature and complete so far, which has been widely used to solve various UAV formation control problems. But respective flaws still exist. For example, the leader-follower-based approach relies too much on the leader, and the robustness is poor. The virtual structure-based approach needs to always maintain a rigid structure and lacks flexibility and environmental adaptability, especially when it comes to obstacle avoidance, the behavior-based approach can neither have a clear definition of the overall behavior of the formation system nor strictly prove its convergence mathematically. With the consistency theory proved by Jadbabaie and used by Ren and Beard, R.W to extend its network topology structure to the directed graphs [12], the consistency theory [13] [14] [15] in recent years has witnessed continuous development and application. It is found that it can be used well in formation-controlling problems and the key is whether to find quantitative information and topological structures that converge within a limited time. The essay [16] studies the time-varying formation control problem of the UAV swarm system with switching interactive topology, analyzes the necessity and sufficiency for its realization. It proposes a consistent formation-controlling method based on the commonly used Lyapunov function method and algebraic Riccati equation technology, which well solves the problem of formation keeping and tracking in formation-controlling, but it needs to be pointed out that it does not take obstacle avoidance into consideration.

Obstacle avoidance [17] is actually a local path planning problem. In an environment with increasing complexity, it is difficult for us to obtain information about all obstacles in advance and plan a path accordingly, which brings huge security risk for the UAV movements. Therefore, a local path planner is needed to avoid these unknown obstacles. Some common local path planning methods include rapid exploration of random trees, artificial potential fields, fuzzy logic algorithms, neural networks [18]. As a real-time path planning method, the artificial potential field method has a simple model, low computational cost, good real-time performance, gaining a significant advantage in obstacle avoidance.

Based on the essay [16], this paper also uses the dual-loop formation-controlling model to design a time-varying formation-controlling method for the UAV swarm system for the outer loop controller that drives the UAV to move to the desired position. It combines the basic ideas of model predictive control and artificial potential field to avoid obstacle of the UAV formation while keeping the formation as much as possible. Finally, to verify the theoretical results, the simulation results of the time-varying formation are given.

The rest of this article is organized as follows. In the second chapter, the statement of the problem is shown. The third chapter introduces the method of time-varying formation control in the reference and the theoretical result of modification after considering obstacle avoidance. The fourth chapter gives the simulation results. The fifth chapter is the conclusion of the whole work.

2. Problem Formulation
Set a formation consisting of N drones, the collection uses \( A = \{1, 2, \ldots, N\} \) contains the numbers of all drones. For each of the drones, the reference essay [16] adopts a dual closed-loop control structure, in which the inner loop controller stabilizes the posture, and the outer loop controller adjusts the movement trajectory of the drone. This paper considers the overall performance in the interactive process between the drone swarm and environment so only the outer loop controller is studied. Figure 1 depicts the structure diagram of the double closed-loop formation control model, where \( \varphi(t); \Omega(t); M(t); \xi(t) \) represent the expected motion state, expected posture, control torque, and actual state of motion. The
motion state refers to a quantity that contains both the position and speed of the drone. For each drone \(i\), \(i \in \{1, 2, \ldots, N\}\), suppose \(x_i(t) \in \mathbb{R}^n\) and \(v_i(t) \in \mathbb{R}^n\) represent the position and velocity vector of the drone respectively. Then the dynamic equation of the outer ring is expressed as follows:

\[
\begin{align*}
\dot{x}_i(t) &= v_i(t) \\
\dot{v}_i(t) &= u_i(t)
\end{align*}
\]

Among them, \(u_i(t) \in \mathbb{R}^n\) represents the input control vector. For the convenience of calculation, unless specifically specified, \(n = 1\) is commonly recognized. In fact, we can also generalize to higher-dimensional forms through Kronecker product.

The interactive topological structure between drones in the formation can be described by an undirected graph \(G = \{V, E, W\}\), where \(V = \{v_1, v_2, \ldots, v_n\}\) means that drone \(i\) is used as the node of the node set of \(v_i\); \(E \subseteq \{(v_i, v_j) : v_i, v_j \in V\}\) represents the information exchange channel between different drones \(i\) and \(j\) with \(e_{ij} = (v_i, v_j)\) as the edge set of edges; \(W = [w_{ij}] \in \mathbb{R}^{n \times n}\) represents the corresponding lead matrix. Since it is an undirected graph, the matrix \(W\) is symmetric. For drones \(i\) and \(j\), \(i, j \in \{1, 2, \ldots, N\}\), if \(e_{ij} = (v_i, v_j)\) exists in \(E\), we consider these two drones are adjacent.

For the convenience of subsequent calculations, we further define the Laplacian matrix \(L\) as \(L = D - W\), where \(D = \sum_{j=1}^{N} w_{ij}\) is the degree matrix. In addition, it is stipulated that if there is at least one path between any two nodes, the undirected graph \(G\) is said to be connected. This article assumes that all drones can interact with other drones to for information exchanges and then obtain information about the whole formations. So, all interactive topologies structures used later are considered to be connected.

For the convenience and generality of the calculation, we assume that the obstacles in the environment where the drone is located are all static and regular circles, and the maximum distance from the geometric center of the obstacle to the edge is \(r_b\). To ensure the security of flight, the drone keeps a distance of at least \(d_b\) from the obstacle and give the influence distance value \(\varepsilon\) of the repulsion field in advance, then the minimal security distance from the drone to the geometric center of the obstacle \(l_{min}\) is \(l_{min} = r_b + d_b + \varepsilon\). This paper assumes that there is a prediction time \(T_p > 0\). When the drone determines that it will collide with an obstacle within the prediction time, that is, when the condition \(l < l_{min}\) is met, where \(l\) is the distance of the geometric center of the drone and the obstacle, obstacle avoidance operation will be performed. The basic idea of artificial potential field \([19]\) is used in obstacle avoidance operation. Figure 2 depicts the setting of obstacles and the corresponding repulsion field in this paper. The formula of the repulsion field \(U_{re}\) generated by the obstacle \(X_{bar}\) is as follows:

\[
U_{re} = \begin{cases} 
\frac{1}{2} K_{re} \left( \frac{1}{d} - \frac{1}{d_{bar}} \right)^2, & d \leq d_{bar} \\
0, & d > d_{bar}
\end{cases}
\]

\[
d = l - r_b - d_b
\]

\[
l = \|x_i - x_{bar}\|
\]
Among them, $K_{rr}$ represents the repulsive force coefficient, $d$ represents the shortest distance between the drone and the safe area around the obstacle, and $d_{bar}$ represents the farthest influencing distance of the obstacle repulsive force field, that is, $\varepsilon$ that is given above to ensure the continuity of the repulsive force field, $x_i$ represents the current position vector of the drone, $x_{bar}$ represents the position vector of the geometric center of the obstacle. The formula shows that if the distance between the drone and the safety range around the obstacle exceeds $d_{bar}$, then the obstacle has no repulsive force on the drone.

Fig. 2 Schematic diagram of obstacle settings and repulsion field

3. Control Law Description

For $\forall i \in \{1, 2, \ldots, N\}$, define $\varphi_i(t) = [\varphi_{ix}(t), \varphi_{iy}(t)]^T$ and $\varphi(t) = [\varphi_1^T(t), \varphi_2^T(t), \ldots, \varphi_N^T(t)]^T \in \mathbb{R}^{2n}$ where $\varphi_i(t)$ is considered to be continuously differentiable and $\varphi(t)$ is time-varying, $\xi_i(t) = [x_i(t), v_i(t)]^T$, $B_1 = [1, 0]^T$, $B_2 = [0, 1]^T$, then the following equation holds for drone $i$:

$$\dot{\xi}_i(t) = B_1B_2^T\xi_i(t) + B_2u_i(t)$$

(5)

Reference essay [16] makes the following definition:

Definition 1. If $r(t) \in \mathbb{R}^2$ exists and meets the following condition:

$$\lim_{t \to \infty}(\xi_i(t) - \varphi_i(t) - r(t)) = 0 \quad (i = 1, 2, \ldots, N)$$

Then, it is considered that drone formation (5) can realize time-varying formation and $r(t)$ is named as formation reference function.

Also, a time-varying formation-controlling idea targeted at drone $i$ is mentioned:

$$u_i(t) = K_1(\xi_i(t) - \varphi_i(t)) + K_2\sum_{j \in N_i^i} w_{ij} \left((\xi_j(t) - \varphi_j(t)) - (\xi_i(t) - \varphi_i(t))\right) + \varphi_{iv}(t)$$

(6)

Among them, $K_1 \in \mathbb{R}^{1 \times 2}$ and $K_2 \in \mathbb{R}^{1 \times 2}$ are both constant gain matrix, $N_i^i$ represents the neighbor set of drone $i$ at time $t$, that is, the drone set that have direct information exchanges with drone $i$. $w_{ij}$ is an element of the matrix $w_{ij}$ of the interactive topology structure at time $t$. It has been proved that this control plan has good convergence.

Make $\xi(t) = [\xi_1^T(t), \xi_2^T(t), \ldots, \xi_N^T(t)]^T$, $\varphi_k(t) = [\varphi_{1x}^T(t), \varphi_{2x}^T(t), \ldots, \varphi_{Nx}^T(t)]^T$ and $\varphi_{iv}(t) = [\varphi_{1v}^T(t), \varphi_{2v}^T(t), \ldots, \varphi_{Nv}^T(t)]^T$, combine the control plan (6) and the motion pattern (5) of the single drone, the drone formation can be expressed as following:

$$\dot{\xi}(t) = \left(I_N \otimes (B_2K_1) + B_1B_2^T\right)\xi(t) - \left(I_N \otimes (B_2K_2) - L_t \otimes (B_2K_2)\right)\varphi(t) + (I_N \otimes B_2)\varphi_{iv}(t).$$

(7)
Where $L_t$ refers to the matrix $L$ of the interactive topology of the drone formation at time $t$. Some necessary theorems and explanations in the reference paper [16] need to be given below:

Theorem 1. Drone formation with switching interactive topology (7) can realize time-varying formation when and only when the following conditions are established:

1. For all $t \in [1, 2, \ldots, N]$
   \[
   \lim_{t \to \infty} \left( (\varphi_{iv}(t) - \varphi_{jv}(t)) - (\varphi_{ix}(t) - \varphi_{jx}(t)) \right) = 0, j \in N^i_t. \tag{8}
   \]

2. The switched linear system shown below is stable gradually:
   \[
   \dot{\theta}(t) = (I_{N-1} \otimes (B_2 K_1 + B_1 B_2^T)) - A_t \otimes (B_2 K_2) \theta(t), \tag{9}
   \]

Where $\theta(t)$ is the state function of the system represented by (9).

Where $A_t$ is a diagonal matrix, let $\lambda_i^t (i = 1, 2, \ldots, N)$ be the eigenvalues of the Laplacian matrix $L_t$ at time $t$. $\lambda_1^t \leq \lambda_2^t \leq \cdots \leq \lambda_N^t$ is generally hypothesized, then $A_t = \text{diag}(\lambda_1^t, \lambda_2^t, \ldots, \lambda_N^t)$. We also know that $\lambda_1^t = 0$ and its feature vector $\vec{u}_1 = 1/\sqrt{N}$, where $1_N$ is an $N$-dimensional column vector with 1 as the element. Let $\lambda_{m_{\min}} = \min\{\lambda_m^t (\forall m \in S; i = 2, 3, \ldots, N)\}$, where $S$ is the set of all possible topological structures in the movement of all formations with switched interactive topologies.

Theorem 2. If the condition (i) in Theorem 1 is met, then the drone formation (5) can achieve the desired time-varying formation $\varphi(t)$ when $K_2 = (2\lambda_{m_{\min}})^{-1}B_2^T P$ is satisfied, where $P$ is the positive definite solution of the following algebraic Riccati equation:

\[
P(B_2 K_1 + B_1 B_2^T) + (B_2 K_1 + B_1 B_2^T)^T P - PB_2 B_2^T P + I = 0. \tag{10}
\]

As for the determination of the constant gain matrix $K_1$ and $K_2$ in the scheme (6), there is a rigorous proof in the reference essay [16]. Combining with the theorem given above, here only the final determination steps are shown as follows:

Step 1: Check whether the desired formation form $\varphi(t)$ meets the feasibility condition (8) of time-varying formation convergence, if it is satisfied, the formation continues, otherwise $\varphi(t)$ is reselected again.

Step 2: Select the characteristic value of $B_2 K_1 + B_1 B_2^T$ through the desired movement mode of the formation reference function to obtain the appropriate $K_1$ value.

Step 3: Solve the algebraic Riccati equation (10) to obtain the value of $P$, and then obtain the value of $K_2$ according to $K_2 = (2\lambda_{m_{\min}})^{-1}B_2^T P$.

It should be pointed out that in the control scheme (6), only the formation problem is studied, and the obstacle avoidance problem is not considered. This paper makes some improvements to the original scheme based on the idea of artificial potential field. Combining the repulsion field formula (2)(3)(4) and $F = -\nabla U$, the general expression of repulsion can be obtained as follows:

\[
F_{re} = \begin{cases}
K_{re} \left( \frac{1}{d_{bar}} - \frac{1}{d} \right) \frac{1}{d^2} x_t x_{bar}, & d \leq d_{bar} \\
0, & d > d_{bar}
\end{cases} \tag{11}
\]

Therefore, for drone $i (i = 1, 2, \ldots, N)$, $F_{rel}(t) = \sum_{k \in M_i^t} F_{relk}(t)$, where $M_i^t$ means the set of obstacles that drone $i$ examines to be collided up to time $t$. $F_{relk}(t)$ is the repulsive force of obstacle $k$ to drone $i$. Considering that the completed obstacles may have repeated obstacle avoidance problems, in order to reduce the amount of calculation, they can be marked to further improve the algorithm. Assuming that $F_{rel}(t)$ is the matrix form of the vector $F_{rel}(t)$, for example, when considering the two-dimensional plane movement of the drone formation, $F_{rel}(t) = [F_{reix}(t), F_{reiy}(t)]^T$, where $F_{reix}(t)$ and $F_{reiy}(t)$ are the horizontal and vertical components of $F_{rel}(t)$, respectively. Accordingly, the time-varying formation control scheme (6) for drone $i$ is modified as follows:
\[ u_i(t) = K_t(\xi_i(t) - \varphi_i(t)) + K_z \sum_{j \in \mathcal{N}} w_{ij} (\xi_j(t) - \varphi_j(t)) - (\xi_i(t) - \varphi_i(t)) \]

\[ + \varphi_{ip}(t) + F_{rei}(t) \quad (12) \]

Make \( F_{re}(t) = [F_{re1}^T(t), F_{re2}^T(t), \ldots, F_{reN}^T(t)]^T \), then the modification of drone formation (7) is as follows:

\[ \xi(t) = (I_N \otimes (B_4K_1 + B_3B_2^T) - L_i \otimes (B_2K_2))\xi(t) - (I_N \otimes (B_4K_1) - L_i \otimes (B_2K_2))\varphi(t) + (I_N \otimes B_2)(\varphi_{ip}(t) + F_{re}(t)) \quad (13) \]

In fact, in consideration of maintaining a certain formation shape during obstacle avoidance, form (13) will not be used. The simple summary is that the drones in the drone formation that need to perform obstacle avoidance will independently perform obstacle avoidance actions and are replaced by a virtual drone at the same time. There is no need for drones that perform obstacle avoidance and these virtual drones to converge to the desired trajectory in form (7). When one drone completes the obstacle avoidance, it can return to the formation. The above operations are updated in real time. Suppose that at time \( t \), \( R(t) \) is the set of numbers of all drones that need to perform obstacle avoidance actions, \( N(t) = A - R(t) \) is the set of all the remaining drones, and \( \xi_{ri}(t) \) is the state of virtual drones corresponding to drone \( i (i = 1, 2, \ldots, N) \), \( \xi_h(t) = [\xi_{h1}(t), \xi_{h2}(t), \ldots, \xi_{hN}(t)]^T \) is called the mixed formation state matrix, where \( \forall i \in R(t) \), \( \xi_{hi}(t) = \xi_i(t) \), and \( \forall j \in N(t) \), \( \xi_{hj}(t) = \xi_j(t) \). The specific obstacle avoidance strategy based on the artificial potential field at a certain moment is as follows:

Step 1: According to the mixed formation state \( \xi_h(t_0) \) at the moment of \( t_0 \) and the expected formation state \( \varphi(t_0) \) and form (7), the predicted formation change \( \xi(t_0) \) is obtained. Then it can be predicted that the formation state \( \xi_p(t_0 + T) \) at the moment of \( t_0 + T \), where \( T \) is the sampling interval.

Step 2: Let \( R(t_0) = R(t_0 - T) \), \( N(t_0) = N(t_0 - T) \), judge whether there is a drone in \( N(t_0) \) that meets the obstacle avoidance condition according to \( \xi_p(t_0 + T) \), that is, \( l_{ik} < l_{mink} \) which is mentioned above, where \( l_{mink} \) is the minimum safe distance of obstacle k, and \( l_{ik} \) is the distance of the geometric center between the drone \( i (i = 1, 2, \ldots, N) \) and the obstacle \( k \). Record the number of the drone that needs to perform obstacle avoidance operations and add it to the set \( R(t_0) \).

Step 3: Repeat the operations of the first and second steps, continue to predict the next formation state according to the predicted information of the formation state, and obtain \( \xi_p(t_0 + 2T) ; \ldots ; \xi_p(t_0 + cT) \) at each prediction time \( t_0 + iT \), \( N(t_0) \) is obtained from \( N(t) = A - R(t) \), until \( cT \) reaches the predicted time \( T_p \) or it is found that all drones need to perform obstacle avoidance operations, that is, the prediction is stopped when \( R(t_0) = A \).

Step 4: For all \( i \in R(t_0) \), according to the control scheme (12) and form (5), the real state change \( \dot{\xi}_i(t_0) \) of the drone \( i \) after the obstacle avoidance action, and then the motion state \( \varphi_i(t_0 + T) \) of drone \( i \) after conducting avoidance actions and the motion state of the corresponding virtual drone \( \xi_{ri}(t_0 + T) = \xi_{pi}(t_0 + T) \), for \( \forall j \in N(t_0) \) after noting the obstacle avoidance action, the motion state \( \xi_{j}(t_0 + T) = \xi_{pj}(t_0 + T) \) of drone \( j \), the mixed formation state \( \xi_h(t_0 + T) \) at the next moment is obtained.

Step 5: According to \( \xi_{ri}(t_0 + T) , i \in R(t_0) \), judge whether to let the drone \( i \) that is avoiding obstacles return to the formation. The condition for return is that the distance between the virtual drone \( i \) and the obstacle \( k \) is greater than the minimal safe distance, that is, \( l_{rik} > l_{mink} \), \( k \in M_{t_0}^i \), where \( l_{rik} \) represents the distance between the virtual drone \( i \) and the obstacle \( k \). If drone \( i \) returns, then \( \xi_{hi}(t_0 + T) = \xi_i(t_0 + T) \), that is, the information of real motion state of drone \( i \) replaces its corresponding virtual drone information and pay attention to update \( R(t_0) \) and \( N(t_0) \).

The above is the operation that the drone formation needs to perform at \( t_0 \). Starting from time \( 0 \) and continuing to execute the above cycle, compared to the solution (6)’s added repulsion force that only
works to avoid obstacles and does not affect the convergence result of the final movement, it can achieve a better formation avoidance effect.

4. Simulation Results
Assuming that the drone formation is moving in the horizontal plane \( n = 2 \), that is, the flying height of each drone remains constant. Select the sampling time \( T = 0.01s \) and the prediction time \( T_p = 10s \). It is worth noting that the movement of each drone along the X-axis and Y-axis in the plane is decoupling. The Kronecker product can be used to extend the above control scheme to two dimensions. For \( \forall i \in \{1, 2, \ldots, N\} \), \( \varphi_i(t) = [\varphi_{1xX}(t), \varphi_{1yX}(t), \varphi_{1xY}(t), \varphi_{1yY}(t)]^T \), \( \varphi_{1}(t) = [\varphi_{1xX}(t), \varphi_{1yX}(t), \varphi_{1xY}(t), \varphi_{1yY}(t)]^T \), \( \xi_i(t) = [\xi_{1xX}(t), \xi_{1yX}(t), \xi_{1xY}(t), \xi_{1yY}(t)]^T \), \( \xi_{1xX}(t) = [\xi_{1xX}(t), \xi_{1yX}(t), \xi_{1xY}(t), \xi_{1yY}(t)]^T \), \( u_i(t) = [u_{1xX}(t), u_{1yX}(t)]^T \), \( F_{rel}(t) = [F_{relX}(t), F_{relY}(t)]^T \), where \( \varphi_{1xX}(t), \varphi_{1yX}(t), \varphi_{1xY}(t), \varphi_{1yY}(t), \xi_{1xX}(t), \xi_{1yX}(t), \xi_{1xY}(t), \xi_{1yY}(t) \) and \( u_{1xX}(t), u_{1yX}(t) \) refer to the desired position, desired speed, actual position, actual speed, virtual position, virtual speed, input control, and repulsive force of the drone \( i \) along the X-axis and Y-axis. At the same time, for the sake of simplicity, there is only one interactive topology during the movement of the formation and it is \( 0 - 1 \) weighted.

Considering the time-varying formation with four drones \( (N = 4) \), the desired state of motion is set as follows:

\[
\varphi_i(t) = \begin{bmatrix}
  r_1 \cos \left( \omega_1 t + \frac{(i-1)\pi}{2} \right) + h_x(t) \\
  -\omega_1 r_1 \sin \left( \omega_1 t + \frac{(i-1)\pi}{2} \right) + h_y(t) \\
  r_2 \sin \left( \omega_1 t + \frac{(i-1)\pi}{2} \right) + h_{xx}(t) \\
  \omega_2 r_1 \cos \left( \omega_1 t + \frac{(i-1)\pi}{2} \right) + h_{yy}(t)
\end{bmatrix} (i = 1, 2, 3, 4),
\]

Where \( \omega_1 = 0.05\text{rad/s}, r_1 = 10\text{m}, \) and each component of \( h(t) \) and \( h_p(t) \) have the following form:

\[
h_x(t) = \begin{cases} 
    r_2 (1 - \cos(\omega_2 t)), & t \leq \pi/(2\omega_2) \\
    r_2 + v(t - \pi/(2\omega_2)), & t > \pi/(2\omega_2)
\end{cases},
\]

\[
h_y(t) = \begin{cases} 
    r_2 \sin(\omega_2 t), & t \leq \pi/(2\omega_2) \\
    r_2, & t > \pi/(2\omega_2)
\end{cases},
\]

\[
h_{xx}(t) = \begin{cases} 
    v(t - \pi/(2\omega_2)), & t \leq \pi/(2\omega_2) \\
    v, & t > \pi/(2\omega_2)
\end{cases},
\]

\[
h_{yy}(t) = \begin{cases} 
    r_2 \sin(\omega_2 t), & t \leq \pi/(2\omega_2) \\
    0, & t > \pi/(2\omega_2)
\end{cases}.
\]

Where \( \omega_2 = 0.05\text{rad/s}, r_2 = 15\text{m}, v = 0.75\text{m/s} \). Here, it can be considered that as the four drones are moving around a certain reference point in a certain regularity (a circular motion here), the reference point is also moving, and the position and speed information of the movement are \( h(t) \) and \( h_p(t) \), where \( h_X(t), h_{VX}(t) \) and \( h_Y(t), h_{VY}(t) \) are the components of \( h(t) \) and \( h_p(t) \) along the X-axis and Y-axis respectively. Assume that the interactive topology structure of the formation is shown in Figure 3.
It can be verified that the condition (i) in Theorem 1 is satisfied, that is, the drone system with this topology structure can finally realize stable formation movement under the control of scheme (6). The design reference formation mode $r(t)$ is in a stable state, select $K_1 = I_2 \otimes [-0.6, -0.8]$, it can be found that the characteristic value of $B_2K_1 + B_1B_2^T$ is $-0.4 + 0.6633j, -0.4 + 0.6633j, -0.4 - 0.6633j$ and $-0.4 - 0.6633j$, where $j^2 = -1$, which means that the reference formation mode $r(t)$ is stationary when the formation reaches the desired state of motion stably. According to the interactive topology structure in Figure 3, the non-zero eigenvalues of the Laplacian matrix can be calculated to be 3.4142 and 0.5858, and it is easy to know that $\lambda_{\text{min}} = 0.5858$. According to Theorem 2, $P = I_2 \otimes \begin{bmatrix} 1.4618 & 0.5662 \\ 0.5662 & 0.8651 \end{bmatrix}$, and then $K_2 = I_2 \otimes [0.4833, 0.7384]$.

Set the initial state of four drones respectively as $\xi_1(0) = [-9.76, 0.42, -0.56, -0.06]^T, \xi_2(0) = [0.24, -0.16, 10.78, 0.19]^T, \xi_3(0) = [10.57, -0.02, 0.05, 0.23]^T$ and $\xi_4(0) = [0.13, -0.08, -9.93, -0.23]^T$. Set four obstacles: $x_{bar1} = [15, 25]^T, x_{bar2} = [25, 5]^T, x_{bar1} = [20, 17]^T, x_{bar1} = [80, 20]^T$ and $r_{b1} = 2m, r_{b2} = 1m, r_{b3} = 3m, r_{b4} = 3m$. Set the security distance $d_h = 1m$, the distance of repulsion field impact $\varepsilon = 3m$ and the repulsion coefficient $K_r = 2$. Finally, the drone formation movement is simulated for 200s, and the formation state trajectory in the simulation is shown in Figure 4 among which the colors representing the four drones are red, green, blue, and purple in the order of labeling. The initial state of the trajectory is marked by a circle, and the final state is marked by a cross, a hexagon, a diamond, and an upward pointing triangle. The solid line between the initial state and the final state represents the actual trajectory of the drone. The dashed line with the same color represents the expected trajectory, and the black solid line represents the trajectory of the reference point $h(t)$. The physical range of obstacles is marked in black, and the gray circle around each obstacle represents the area within its safe distance.
Fig. 4 State trajectories of four UAVs in simulation. (a) Positions and (b) velocities.
It is required that the drone not enter the gray area. Figure 5 depicts the position and velocity errors of the four drones. The error calculation method is as follows:

\[ \text{error}_x = \| x_i(t) - x_{\phi}(t) \|_2 \]
\[ \text{error}_v = \| v_i(t) - v_{\phi}(t) \|_2 \]

Among them, \( x_i(t) \) and \( v_i(t) \) are the actual position and velocity vector of drone \( i \) at time \( t \), and \( x_{\phi}(t) \) and \( v_{\phi}(t) \) are its expected position and velocity vector at the same time. It can be found that drones can well perform the obstacle avoidance tasks during the movement and the formation of drones can be maintained during the obstacle avoidance, and finally realize the desired time-varying formation.

![Fig. 5 Formation error of four UAVs in simulation. (a) Positions and (b) velocities.](image)

5. Conclusion
Based on the essay [16], the problem of time-varying UAV formation with interactive topology to avoid obstacles is studied. The repulsion factor is added to the original control scheme, and a distributed time-varying formation-controlling method based on artificial potential field is improved. The scheme, combined with the basic idea of model prediction and control, adding the step of prediction and generation of virtual UAVs, and gives an obstacle avoidance strategy in which UAV obstacle avoidance and formation shaping are carried out simultaneously. A time-varying formation simulation experiment involving four UAVs was carried out to verify the theoretical results. On the whole, this control scheme takes into account the impact of certain environmental factors on formation-controlling and has more prospects of application. For the practical and efficient realization of the scheme, it is necessary to further consider the problems of input control saturation, communication delay and interruption.

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