A Practical Method of Modelling and Simulation for Ball End Mill CNC Grinding

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Abstract. The spherical cutting edge on the ball end mill directly affects its cutting performance. This paper presents a practical method for end mill rake/clearance face modelling. The rake face is a developed ruled surface to fulfill the desired normal rake angle. Revolution surface of the bottom curve on the rake face avoids the interference between the wheel with the designed rake face while grinding. An equidistance curve of the cutting edge on the sphere is deduced by the geodesic curve in order to adjust the position of the cutting edge. With this new modelling method as a platform, it is very helpful for model modification and contour error compensation. The measurement results show that the average value of contour error on spherical cutting edge of the machined sample is 0.01mm inspected by the tool measuring instrument. Its validity is verified by good agreement between the results computed and the data measured.

1. Introduction

Ball end mills are extensively used in machining of various kinds of surfaces for mold, automobile and aviation manufacturing. The spherical cutting edge on the ball part of mill directly affects the cutting performance and the tool life. Recently, the efficient and accurate method of CNC grinding for the ball end mill has attracted considerable attention in industry. A simple method of the planar rake face is usually designed by grinding with the side face of 1A1 cylindrical wheel. But it can hardly adapt to today’s diverse processing requirements. More complicated structures of ball end mill products are explored. But they may inevitably rely on numeric approximation and controversial accuracy.

A model of end mill with helical flute was presented by Y Altintas [1] with envelope method. He Yaoxiong [2] calculated the position of the grinding wheel, based on the condition of the wheel’s circle and the cutting edge just in contact during the grinding process. The core diameter and avoiding interference are obtained by adjusting the wheel’s direction. It can also be used to design the taper ball end mill or other complex cutters with a generalized helix. Sometimes, undercutting and unintended removal of material on the mill flute are likely to occur. Chen Fang [3] proposed a method of using the spherical wheel to accurately grind the rake face avoiding interference. But, dressing a shaped diamond wheel is also difficult.

Contents of the next sections are divided into two parts. Firstly, Sections 2, 3 and 4 are about the theoretical base of the proposed modelling method for the rake/clearance face grinding. Secondly, Section 5 compares the contour error of the manufactured sample with the desired one.
2. Mathematical model of cutting edge

As shown in Figure 1, \{o: x, y, z\} is the workpiece coordinates with the origin o at ball centre, x-axis coinciding with the end mill axis vertically. The coordinates of an arbitrary point on the sphere is expressed as

\[ r(x, \phi) = [x, \sqrt{R^2 - x^2} \sin \phi, \sqrt{R^2 - x^2} \cos \phi] \]  

(1)

where \(R\) is the radius of sphere, \(\phi\) is the circumferential angle of the point projected on the yoz-plane.

The cutting edge is a helix with a constant lead on the sphere. The helix angle \(\beta\) is defined as an angle between the tangent of the helix and the tool’s axis. The helical parameter is written as \(p = l/2\pi\), where \(l\) is the pitch.

\[ p = Rl \tan \beta = -dx/d\phi \]  

(2)

Integrating (2) gives

\[ x = -\frac{R\phi}{\tan \beta} + R \]  

(3)

It satisfies \(x = R\), at \(\phi = 0\).

The equation of the cutting edge on the ball part is derived by substituting Equation (3) into Equation (1):

\[ r(\phi) = \left[ R(1 - \frac{\phi}{\tan \beta}), R \sqrt{1 - \left(1 - \frac{\phi}{\tan \beta}\right)^2} \sin \phi, R \sqrt{1 - \left(1 - \frac{\phi}{\tan \beta}\right)^2} \cos \phi \right] \]  

(4)

![Figure 1. Coordinate systems.](image)

The derivative of the tangent vector defines both the tangent vector of the cutting edge \(T\), the normal vector of the sphere \(N\). The vector cross product of \(T\) and \(N\) defines the binormal vector \(B = N \times T\). These three unit vectors are all mutually orthogonal and define a surface curve frame [4]. The plane definitions of normal rake angle are conveniently given by these above three vectors. In Figure 1, the reference plane \(P_r\) is represented by \(N\), and the cutting edge plane \(P_s\) is represented by \(B\).

The normal rake angle \(\gamma_n\) is the angle between the reference plane \(P_r\) and the tool rake face measured in the normal plane \(P_n\) as shown in Figure 1. The unit vector \(L_r\) expresses the direction of the rake face, defined by

\[ L_r = \frac{-N \cos \gamma_n - B \sin \gamma_n}{| -N \cos \gamma_n - B \sin \gamma_n |} \]  

(5)

In the normal plane \(P_n\), \(C\) is an arbitrary point on the cutting edge and \(K\) is the bottom point of the rake face. The radial depth of the rake face written as \(|CK| = h_r(x)\), is the width between the cutting edge and the bottom curve in the normal plane \(P_n\), being a function of \(x\). At the end mill tip this depth...
function $h_r(x)$ is zero and increases along the negative direction of $x$-axis. Therefore, the equation of the rake face of the ball end mill can be expressed as

$$S_r(\varphi, \nu) = r(\varphi) + \nu L_r$$

where $r(\varphi)$ is the radius vector of the selected point $C$ on the cutting edge, defined by Equation (4), $\nu$ is the parameter along $L_r$ direction.

Thus, the equation of the bottom curve can be also deduced from the curve of the cutting edge $r(\varphi)$

$$r_b(\varphi) = r(\varphi) + h_r L_r = [x_b, y_b, z_b]$$

3. Determination the wheel location

3.1 Determination the direction of grinding wheel

The rake face is ground by the side face and the flute is ground by the revolution surface of the 1A1 cylindrical grinding wheel (Figure 2), simultaneously. The direction of the wheel axis is denoted by the vector $I$, perpendicular to the wheel side face. In order to generate a line $CK$ on the rake face when the grinding wheel contacts with the workpiece at any instant, the vector $L_r$ should be in the side face of wheel. If the tangent vector of the cutting edge is in the side face of the wheel too, it will ensure that only one line $CK$ on the cutting edge can be ground without interference. From this analysis, the vector $I$, the wheel axis direction, is determined by

$$I = T \times L_r$$

Due to the vector $I$ is perpendicular to the tangent surface of the rake face, the vector $I$ and the normal vector rake face are parallel.
3.2 Determination the position of grinding wheel

A criterion to avoid interference between the grinding wheel and the bottom curve is presented [5]. The principal normal vector of the bottom curve must be projected to the plane of the wheel’s side face. But it is not sufficient to avoid interference because the bottom curve is a spatial curve. Two spatial curves intersection cannot be simplified by discrimination the intersection condition with their projection curves in a plane.

In 3.1, the direction of the wheel has been determined by Equation 8. The grinding wheel and the rake face may still interfere in many potential locations (Figure 4). An intuitive direct method of avoiding interference is presented in this section. It is summarized as follows: Firstly, as the bottom curve rotates around the x-axis, a revolution surface is generated inside the solid of ball end mill in Figure 4. If the grinding wheel does not move into the interior of the revolution surface, undercut will be avoided completely and the desired rake face width \( h_r(x) \) will be guaranteed too. Secondly, the cross section of the revolution surface in the plane of wheel side face determines a cross-section curve \( r_t \). If the circle of the wheel’s side face and the cross curve \( r_t \) has the same tangent line, the above mentioned interference will be avoided. The position of the grinding wheel is determined by the following relative vector calculation.

The equation of the revolution surface is expressed by

\[
S_b(\varphi, \psi) = \left[ x_b, \sqrt{(y_b + z_b)^2 \sin \psi}, \sqrt{(y_b + z_b)^2 \cos \psi} \right]
\]

where \( \psi \) is the circumferential angle of the revolution. \([x_b, y_b, z_b]\) is the coordinates of the bottom curve, given by (7). The unit revolution surface normal \( N_k \) is given by its partial derivatives,

\[
N_k = \frac{\partial S_b(\varphi, \psi)}{\partial \varphi} \times \frac{\partial S_b(\varphi, \psi)}{\partial \psi}
\]

The tangent vector of the cross-section curve \( r_t \) at point \( K \), denoted as \( T_m \), is perpendicular to the revolution surface normal \( N_k \). Due to the vector \( T_m \) is also in the tangent surface of the rake face, \( T_m \) must be perpendicular to the normal vector of the rake face. The tangent vector \( T_m \) can be represented by

\[
T_m = N_k \times I
\]

![Figure 5. Vector \( V_{KG} \).](image)

The vector from the point \( K \) to the centre of the grinding wheel \( G \) is denoted as \( V_{KG} \). In the side face of the wheel, the vector \( V_{KG} \) is perpendicular to the grinding wheel axis vector \( I \). It can be represented as
\[ V_{KG} = T_m \times I \]  

(12)

Therefore, the centre point \( G \) of the grinding wheel can be expressed as

\[ r_G(\varphi) = r_b(\varphi) + R_w \frac{V_{KG}}{V_{KG}} \]  

(13)

where \( R_w \) is the radius of the wheel. \( r_b(\varphi) \) is determined by Equation (7).

4. Clearance grinding

Normally, there are two or three faces jointed to construct the clearance face. As a sample, the major clearance face in the front that intersects the rake face to form the cutting edge is calculated. In the normal plane \( P_n \), the angle between the clearance face and the cutting edge plane \( P_s \) is the relief angle \( \alpha_n \). The cutting edge plane \( P_s \) is perpendicular to \( P_r \), and contains the cutting edge expressed by \( B \).

5. CNC Grinding for Ball End Mill

5.1 Machine tool coordinates system

The axis movement of CNC grinder is usually restricted by its configuration. In Figure 9, a typical 5-axis grinder PTG-6 is made by Star cutter Company in USA. Its two rotary axes, named as A-axis and B-axis, are confined to be horizontal. The effective operation is to rotate the workpiece around A-axis, so that the grinding wheel can not be inclined when grinding. It can be achieved by the coordinate system transformation.

In Figure 7, \( \{o_m: x_m, y_m, z_m\} \) is the machine tool coordinates system. \( \{o: x, y, z\} \) is the workpiece coordinates fixed on the end mill that is depicted in Figure 1. \( \{o: x, y, z\} \) is the moving coordinates
system attached to the grinder’s headstock. Firstly, to rotate the workpiece around A-axis with angle \( \varphi_m \), the grinding wheel’s vector is transformed from \([I_x, I_y, I_z]\) to \([I'_x, I'_y, I'_z]\). Then, let the component of \( I'_z \) to be zero. It is determined the angle \( \varphi_m \) in Equation (16)

\[
\begin{align*}
I'_x &= I_x \\
I'_y &= I_y \cos \varphi_m - I_z \sin \varphi_m \\
I'_z &= I_y \sin \varphi_m + I_z \cos \varphi_m = 0
\end{align*}
\]

So,

\[ \varphi_m = \arctan\left(-\frac{I_z}{I_y}\right) \]  

(17)

Lastly, the wheel is swigged around B-axis with angle \( \theta_B \). The workpiece is calibrated using standard probing, initially. In Figure 8, the length of chuck is denoted as \( f_c \) and the clamped length of workpiece is denoted as \( f_c \). The distance from the center of chuck to the center of rotary B-axis is \( Z_m \). The coordinates of 5-axis are shown directly.

A axis \[ \theta_A = \frac{\varphi_m}{\pi} \times 180^\circ \]

B axis \[ \theta_B = \arctan\frac{I'_y}{I'_z} \]

X axis \[ I'_x + (l_c - l_s) \sin \theta_B \]

Y axis \[ I'_z \]

Z axis \[ I'_x + (l_c - l_s) \cos \theta_B + (f_n + f_c - R) \]

Figure 7. Machine tool coordinate systems.

Figure 8. Configuration of machine tool.
5.2 Error compensation along the cutting edge

For a 5-axis machine tool, there are 30 position-dependent error parameters [6] and a variety of methods to reduce the machining errors [7]. When grinding a ball end mill, these machining errors cause two types of error: the first is the cutting edge entirely offset from the designed one; the latter is that the deflections in some local places.

In order to reduce the contour error in the first case, the geodesic of the surface can be used [4]. Especially, the geodesic of a sphere is a circle. According to the tangent vector of a curve on the sphere, the geodesic curve must go through the given point and be perpendicular to the tangent vector of the cutting edge. In the surface curve frame, assuming the point \( C' \) is the offset point from the selected point \( C \) with a deviation \( d \) in arc length parameters. The central angle \( \angle COC' \) is denoted as \( \theta=d/R \). Thus, the equal distance curve is obtained from

\[
\mathbf{r}(\varphi) = \mathbf{r}(\varphi) - RB \sin \theta - RN (1 - \cos \theta)
\]

(18)

Figure 9. Offset of major cutting edge.

5.3 Contour detection and local error compensation

Figure 10. Samples of ground ball end mill.
Local derivation more frequently arouse in ball end mill manufacturing. The error can be inspected by Zoller Genius 3m cutting tool measuring device. After rotating the end mill around the $x$-axis, the cutting edge profiles are projected in the $xoz$-plane (Figure 9). The error of the profile offset from the designed one is denoted as $\delta$. Its circumferential angle of the selected point is $\eta$, starting from the positive direction of $x$-axis. In order to determine the error of profile in the cutting edge, the measured coordinates $(\eta, \delta)$ should be transform to the workpiece coordinates $\{o: x, y, z\}$.

In Figure 10, assuming the point $Q$ on the cutting edge is the corresponding point of the measured point $P$, it can be deduced that its component in $x$-axis is $R \cos \eta$. From Equation (4),

$$R \sqrt{1 - \left(1 - \frac{\varphi_i}{\tan \beta}\right)^2 \cos \varphi_i} = R \cos \eta$$

(19)

From the calculated value of $\varphi_i$, the respective sequence of $(x_i, y_i, z_i)$ is interpolated by B-spline accordingly. Assuming that the point $C$ on the cutting edge has error $\delta$, the error in normal direction is expressed as $d$, 

$$d = \delta \cos \gamma_n$$

(20)

After the error of the points on the cutting edge is determined, the new cutting edge, denoted as $r_\delta$, with error compensation is

$$r_\delta(\varphi) = r(\varphi) - \delta \cos \gamma_n L_r$$

(21)

It is compensated in the negative direction of the inspected error to decrease the error.

A sample of ball end mill with diameter of $\Phi 14$ is ground. Its dimensions are inspected in Table 1. The maximum error in the two sides of ball end mill in Figure 11 is $0.08$mm. After error compensation, the maximum error decreases to be $0.01$, as shown in Table 2.

| Diameter/mm | D | 14 |
|-------------|---|----|
| Helix angle$/^\circ$ | $\beta$ | 30 |
| Pre-angle$/^\circ$ | $\gamma_n$ | 0 |
| Clearance angle$/^\circ$ | $\alpha_n$ | 10 |
| Flank width/ mm | $h_f$ | 0.2 |
| Cutting edge offset distance/mm | $d$ | 0.1 |
Tabel 2. Cutting edge error statistic.

|                          |       |
|--------------------------|-------|
| Average deviation/mm     | -0.01 |
| Inside the maximum deviation/mm | 0.043 |
| Outside the maximum deviation/mm | 0.039 |

6. Conclusion

A mathematical modeling of the rake/clearance face with the ruled surface is established by analytical method. Two types of error compensation are analysed to increase the machining precision. The maximum contour error of the mill profile is 0.01mm, which is less than the required in GB Standards. A CNC grinding software is developed as a platform for the ball end mill manufacturing by Microsoft Visual C#. It would be useful in the end mill design manual, embedding integrated information of geometric model, machining procedure, and cutting performance.

Appendix

Proof that the rake face is the envelope surface of the single parameter plane family of 1A1 wheel side face

In this appendix, it will be proved that the contact line between the grinding wheel and the rake face is the line CK in the normal plane Pn.

When the grinding wheel is moving along the cutting edge, the selected point C on the cutting edge must be in the side face of the grinding wheel. It will ensure that in every position, the grinding wheel will grind a point on the cutting edge. The single parameter plane family of the grinding wheel is formed by the wheel side face. The vector LR is represented by Equation (5) to fulfill the desired normal rake angle. Due to the grinding wheel axis vector I is perpendicular to direction of rake face LR and the tangent vector T, the wheel axis vector I is represented by Equation (8).

By the envelope theory, when the grinding wheel is moving along the cutting edge, its side face forms a single parameter plane family,

\[ U(s) \cdot (r - r_c(s)) = 0 \]  \hspace{1cm} (A1)

where \( r \) is the vector of points on the plane of the grinding wheel side face; \( r_c \) is the radius vector of the selected point C on the cutting edge of the grinding wheel; \( I \) is the vector of grinding wheel axis direction; \( U \) is a single parameter plane family formed by the side face of the grinding wheel.

The envelope \( \Sigma \) of the single parameter plane family \( U \) is expressed as

\[ \Sigma : \begin{cases} U(s) : & I(s) \cdot (r - r_c(s)) = 0 \\ U(s)' : & \frac{d[I(s) \cdot (r - r_c(s))]}{ds} = 0 \end{cases} \]  \hspace{1cm} (A2)

The derivative is shown below

\[ \frac{d[I(s) \cdot (r - r_c(s))]}{ds} = (k_n \sin \gamma_n - k_g \cos \gamma_n)T \cdot (r - r_c(s)) \]  \hspace{1cm} (A3)

In the differential geometry [8], the derivatives of the three orthonormal vectors \( N, B \) and \( T \) of the moving frame on surfaces,

\[ \frac{dN}{ds} = -k_n T + \tau_g B \]  \hspace{1cm} (A4)
\[ \frac{dB}{ds} = -k_g T - \tau_g N \]  \hspace{1cm} (A5)
\[ \frac{dT}{ds} = k_n N + k_g B \]  \hspace{1cm} (A6)
where $k_n$ is the normal curvature, $k_g$ is the geodesic curvature, and $\tau_g$ the is geodesic torsion of the curve.

Especially, in a spherical curve, the direction of the normal vector $N$ always crosses the centre of the sphere. Equation (A4), (A5), and (A6) can be written as

$$\frac{dN}{ds} = -k_n T$$  \hspace{1cm} (A7)

$$\frac{dB}{ds} = -k_g T$$  \hspace{1cm} (A8)

$$\frac{dT}{ds} = k_n N + k_g B$$  \hspace{1cm} (A9)

Substituting Equation (A7), (A8), and (A9) into (A3):

$$\frac{d[I(s) \cdot (r - r_C(s))]}{ds} = [dT \times L_r + T \times dL_r] \cdot (r - r_C(s)) = (k_n \sin \gamma_n - k_g \cos \gamma_n)T \cdot (r - r_C(s)) \hspace{1cm} (A10)$$

The contact line between the envelope surface $\Sigma$ and the curve family $U$ is perpendicular to the tangent vector $T$ in the cutting edge, so the contact line between the envelope surface $\Sigma$ and the plane family $U$ is the line $Ck$. All of these lines $Ck$ consist of the rake face and intersect with the clearance at the cutting edge.

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