The impact of the new Earth gravity model EIGEN-CG03C on the measurement of the Lense-Thirring effect with some existing Earth satellites

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Abstract

The impact of the latest combined CHAMP/GRACE/terrestrial measurements Earth gravity model EIGEN-CG03C on the measurement of the Lense-Thirring effect with some linear combinations of the nodes of some of the existing Earth’s artificial satellites is presented. The 1-sigma upper bound of the systematic error in the node-node LAGEOS-LAGEOS II combination is 3.9% (4% with EIGEN-GRACE02S, ∼ 6% with EIGEN-CG01C and ∼ 9% with GGM02S), while it is 1% for the node-only LAGEOS-LAGEOS II-Ajisai-Jason-1 combination (2% with EIGEN-GRACE02S, 1.6% with EIGEN-CG01C and 2.7% with GGM02S).

1 Introduction

In this brief note we release an update about the impact of the Newtonian part of the terrestrial gravitational potential on the measurement of the general relativistic gravitomagnetic Lense-Thirring effect [1] with some linear combinations [2, 3, 4, 5, 6, 8, 7] of the orbital residuals of the nodes Ω and the perigees ω of certain existing Earth’s artificial satellites. The Earth gravity model EIGEN-CG03C [9], recently released by the GeoForschungsZentrum, is considered here.

The gravity field combination model EIGEN-CG03C is an upgrade of EIGEN-CG01C [10]. The model is based on the same CHAMP mission and surface data (0.5 x 0.5 deg. gravimetry and altimetry), but takes into account almost twice as much GRACE mission data. Instead of 200 days

[1]See on the WEB http://www.gfz-potsdam.de/pb1/op/grace/results/index_RESULTS.html for retrieving the model
now 376 days out of February to May 2003, July to December 2003 and February to July 2004 have been used.

Let us recall that one of the most important sources of systematic error in measuring the Lense-Thirring precessions with the data from the Earth’s satellites is represented by the much larger classical effects induced by the static and time–varying parts of the even zonal harmonic coefficients \( J_\ell, \dot{J}_\ell \ell = 2, 4, 6, \ldots \) of the multipolar expansion of the terrestrial gravitational potential. Indeed, the accuracy with which such parameters are currently known does not yet allow to analyze only one satellite. The linear combination approach, proposed for the first time by Ciufolini [2] and, subsequently, extended and generalized by Iorio [3, 4, 5, 6, 7], consists in suitably combining the orbital elements of different satellites in order to disentangle, by construction, the Lense-Thirring effect from the largest number of even zonals as possible.

## 2 Some linear combinations

### 2.1 The node-node-perigee combination with the LAGEOS satellites

The first combination proposed and analyzed with some of the latest pre-CHAMP/GRACE Earth gravity models involves the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II [2]. It cancels out \( J_2 \) and \( J_4 \) but is heavily affected by the non-gravitational perturbations (direct solar radiation pressure, Earth’s albedo, thermal effects as the solar Yarkovsky-Schach and the terrestrial Yarkovsky-Rubincam forces) which act on the perigee of the LAGEOS-type satellites.

### 2.2 The node-node combination with the LAGEOS satellites

Later, the improvements in our knowledge of the terrestrial gravitational field due to the first CHAMP and, especially, GRACE models allowed to explore different alternatives. The most viable, from the point of view of the reduction of the non-gravitational perturbations and of the relative easiness of the data-analyzing process, is the \( J_2 \)-free combination\(^2\) of the nodes of LAGEOS and LAGEOS II [3, 4, 5, 6] which has recently been used in a test with real data over 11 years [12]. It has the great advantage of discarding the perigee of LAGEOS II. However, it is affected by \( J_4, J_6, J_8, \ldots, \dot{J}_4, \dot{J}_6 \) whose

\(^2\)The idea of only using the nodes of the LAGEOS satellites in view of the results from the GRACE mission was qualitatively put forth for the first time in [11].
impact on the total available accuracy is still rather controversial [13, 14]. It is so mainly because the error due to the static part of the geopotential even zonals \( J_4, J_6, J_8, \ldots \) is still rather model-dependent ranging from \( \sim 4\% \) to \( \sim 9\% \) at 1-sigma level. Moreover, the effect of the secular rates \( \dot{J}_4, \dot{J}_6 \), which could induce an additional \( \sim 11\% \) bias of the Lense-Thirring effect thus increasing the total error to \( 19-24\% \) at 1-sigma level, is important as well.

2.3 The node-only LAGEOS-LAGEOS II-Ajisai-Jason-1 combination

A combination involving the nodes of LAGEOS, LAGEOS II, Ajisai and the altimeter satellite Jason-1 has been proposed in [7, 8]. It cancels out \( J_2, J_4, J_6 \) along with their secular variations \( \dot{J}_2, \dot{J}_4, \dot{J}_6 \). As a consequence, the systematic error due to the remaining even zonal harmonics is smaller (\( \sim 1-2\% \)) and less model-dependent than that of the node-node LAGEOS-LAGEOS II combination. Another important point is that GRACE should better improve the mid-high degree even zonal harmonics which are just the most relevant for such a combination. Thus, it is likely that the error of gravitational origin will be further reduced well below the 1% level with the forthcoming, more robust solutions from GRACE. Instead, it might not be so for the node-node LAGEOS-LAGEOS II combination because of the fact that it is affected by the low-degree zonals \( J_4, J_6 \) for which the future improvements by GRACE should be less relevant. The weakest point of the use of Jason-1 is represented by the non-gravitational perturbations acting on such a spacecraft of complex shape and attitude. Indeed, the Jason’s area-to-mass ratio \( S/M \), to which the non-gravitational forces are proportional, is almost two orders of magnitude larger than that of the LAGEOS satellites. On the other hand, the coefficient with which Jason-1 enters the combination is 0.068, i.e., just two orders of magnitude smaller than the coefficients weighing LAGEOS (1) and LAGEOS II (0.347). Moreover, no secular or sinusoidal perturbations with long periods should affect the node of Jason-1. Finally, the orbital maneuvers which periodically are performed are mainly, although not entirely, in-plane, while the node is related to the out-of-plane component of the orbital path. Thus, the implementation of such a combination, although certainly more difficult than the analysis of the observables involving LAGEOS and LAGEOS II, should deserve attention from the geodesist’s community.
Table 1: Percent systematic error $\frac{\delta \mu}{\mu}$ of even zonals in the measurement of the Lense-Thirring effect with various linear combinations of the nodes $\Omega$ and the perigees $\omega$ of some existing geodetic and altimeter satellites due to the uncancilled even zonal harmonics of the geopotential according to the calibrated sigmas of the EIGEN-CG03C Earth gravity model. The quoted errors are 1-sigma level upper bounds calculated by linearly adding over degree $\ell$ the absolute values of the combined mismodelled precessions. No covariance matrix was used. The 1-sigma root-sum-square errors are quoted in round brackets. L=LAGEOS, L II=LAGEOS II, Aji=Ajisai, Jas=Jason-1, Str=Starlette, Stl=Stella.

| Combination | $\frac{\delta \mu}{\mu}$ (%) |
|-------------|-----------------------------|
| $\Omega^L - \Omega^L II - \omega^L II$ | 0.6 (0.4) |
| $\Omega^L - \Omega^L II$ | 3.9 (3.0) |
| $\Omega^L - \Omega^L II - \Omega^Aji - \Omega^Jas$ | 1.0 (0.5) |
| $\Omega^L - \Omega^L II - \Omega^Aji - \Omega^Str - \Omega^Stl$ | 19.9 (6.2) |

2.4 The node-only LAGEOS-LAGEOS II-Ajisai-Starlette-Stella combination

A combination involving only the nodes of the currently best tracked geodetic satellites LAGEOS, LAGEOS II, Ajisai, Starlette and Stella was also proposed [3, 4]. However, it turned out to be not competitive with the other implemented or proposed combinations because of the larger number of even zonal harmonics of high degree to which the other low-altitude satellites like Starlette and Stella are sensitive.

3 The results

The results obtained with EIGEN-CG03C are shown in Table 1. It should be noted that the 1-sigma upper bounds of the error in the node-node LAGEOS-LAGEOS II combination are $\sim 6\%$ with EIGEN-CG01C, $4\%$ with the GRACE-only GFZ EIGEN-GRACE02S solution [15] and $\sim 9\%$ with the Center for Space Research (CSR/UT) GRACE-only GGM02S model [16].

In regard to the combination involving the nodes of LAGEOS, LAGEOS II, Ajisai and Jason-1, EIGEN-CG01C yields $1.6\%$, EIGEN-GRACE02S $2\%$ and GGM02S $2.7\%$. The combination of the nodes of LAGEOS, LAGEOS II, Ajisai, Starlette and Stella still remains far from the achievable levels.
obtainable with the combination of the nodes of LAGEOS and LAGEOS II and the combination of the nodes of LAGEOS, LAGEOS II, Ajisai and Jason-1.

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