Estimation of state-of-charge using cubature Kalman filter based on online model

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Abstract. The parameters of batteries are time-varying and non-linear in different working conditions, so the method using FFRLS is proposed to estimate the parameters of batteries online. Based on online estimation of battery parameters, the method using CKF is employed to estimate SOC instead of traditional EKF. During the change from nonlinear to linear, EKF only keep the first order accuracy, however, CKF can keep the third order accuracy. An experiment has been carried out to evaluate the performances of the proposed methods. Compared with the traditional EKF, CKF based on online model has better SOC estimation accuracy.

1. Introduction

Since batteries are complicated electrochemical devices with a distinct nonlinear behavior depending on various internal and external conditions, their monitoring is a challenging task [1]. Accurate estimation of State of Charge (SOC) is a key state to Battery Management System (BMS) which is also used for an important reference of electric vehicle control. With an accurate indication of the battery SOC, the user ensures that the battery is not over-charged or under-discharged [2].

Online method to estimate battery parameters makes the battery model more adaptive which contributes to the accurate SOC estimation. FFRLS is easy to be implemented. In [3], FFRLS is used to estimate the parameters of battery model and the open circuit voltage. According to the SOC-OCV relationship, calculate SOC. However some types of batteries have a platform zone in the relationship of SOC-OCV, the method may cause a large error. In [4], FFRLS is also used to estimate the parameters of battery model and adaptive EKF is used to estimate SOC. During the change from nonlinear to linear, EKF only keep the first order accuracy. CKF is used to estimate SOC in [5][6], however, they do not use online battery parameter identification method.

This paper aims to develop a method based on the online battery model. We propose an idea that using CKF to estimate SOC based on online battery model parameters identification. The equations of battery model are transformed into difference equations which are used to identify the parameters of battery model online. FFRLS method is employed to estimate the parameters of the difference equation. Then the parameters of batteries can be derived through the relations between the parameters of the difference equation and the parameters of battery model. On the strength of online identification of model parameters, CKF is employed to estimate SOC, which can keep the third order accuracy. Moreover, experiments are performed. Experiments show that the method can improve SOC estimation performance. SOC estimation accuracy using CKF algorithm based on online model is better than that uses EKF.
2. Model of Battery

2.1. Online Model of Battery
An equivalent circuit model of battery shows in figure 1. Under different working conditions, parameters of battery model have different values. The modelling technique choice is a trade-off between capturing cell dynamics and computational demand [7]. By online parameters identifying, 1RC network-based model of battery has good performance in [8].

The state equations of equivalent circuit model is:

\[
\begin{align*}
\dot{U}_t &= -\frac{1}{C_1 R_1} U_t + \frac{1}{C_1} I_t \\
U_t &= U_{\infty} - U_t - I_t R_0
\end{align*}
\]  
(1)

The voltage of RC network is:

\[
U_{t,k+1} = e^{-\frac{\Delta t}{C_1 R_1}} U_{t,k} + \left(1 - e^{-\frac{\Delta t}{C_1 R_1}}\right) R_1 I_{t,k}
\]  
(2)

Terminal voltage of battery is:

\[
U_{t,k} = U_{\infty,k} - U_{t,k} - I_{t,k} R_0
\]  
(3)

Open circuit voltage (OCV) is the function of SOC as we can see in figure 2. The method is given in [9]. With MATLAB cftool, we get the relationship between OCV and SOC as in equation (4).

\[
OCV = 1234.66 \times SOC^9 - 6088.5 \times SOC^8 + 12689.99 \times SOC^7 - 14552.51 \times SOC^6 + 10017.37 \times SOC^5
- 4241.22 \times SOC^4 + 1090.95 \times SOC^3 - 163 \times SOC^2 + 13.15 \times SOC + 3.28
\]  
(4)

2.2. Online Parameter Estimation of the Battery
First, transform the system equations into the form of least squares:

\[
U_t(k) = (1 - a_1) U_{\infty}(k) + a_1 U_t(k-1) + a_2 I_L(k) + a_3 I_L(k-1)
\]  
(5)

Define \( y(k) = U_t(k) \), and

\[
\Phi(k) = \begin{bmatrix} 1 & U_t(k-1) & I_L(k) & I_L(k-1) \end{bmatrix} \\
\theta(k) = \begin{bmatrix} (1 - a_1) U_{\infty}(k) & a_1 & a_2 & a_3 \end{bmatrix}^T
\]  
(6)

Thus,

\[
y(k) = \Phi(k) \cdot \theta(k)
\]  
(7)

The process of iteration is:
\[ K(k) = P(k-1)\Phi^T(k)\left[\Phi(k)P(k-1)\Phi^T(k) + \mu\right]^{-1} \] (8)

\[ \hat{\theta}(k) = \hat{\theta}(k-1) + K(k)\left[ y(k) - \Phi(k)\hat{\theta}(k-1) \right] \] (9)

\[ P(k) = \frac{1}{\mu}[1 - K(k)\Phi(k)]P(k-1) \] (10)

The parameter \( a_i (i = 1, 2, 3) \) is used to calculate the parameters \( R \) and \( C \) in battery model. And the relations between \( a_i (i = 1, 2, 3) \) and parameters in battery model are as follows:

\[
\tau = \frac{a_i T + T}{2 - 2a_i}, \quad R_0 = \frac{T(a_1 - a_2)}{T + a_i}, \quad R_i = -\frac{(a_i + a_2)(T + 2\tau)}{2T} - R_0, \quad C_i = \frac{\tau}{R_i} \] (11)

Where \( T \) is sample time.

Then, the value of \( R \) and \( C \) are put into the battery model. Curves of battery voltage and model voltage under some current conditions are displayed in figure 3. Voltage error between battery and model is displayed in figure 4.

![Figure 3. Battery and model voltage.](image)

![Figure 4. Voltage error.](image)

Maximum error between battery and model is less than 0.01V. It is obvious that FFRLS has a good performance to identify parameters of battery model.

3. **Estimate SOC Based on Online Model**

The ability of CKF to deal with the nonlinear system is stronger than Extended Kalman Filter (EKF). EKF is only effective for nonlinear systems with approximate linearity, and then truncation error is introduced [10]. The calculation method of CKF is not complex. It calculates the posterior probability density function according to the cubature points. However, EKF should transform a complex Jacobian matrix. CKF can also avoid truncation errors.

Steps are as follows:

- **Initialization**
  - Initial mean \( \bar{x}_0 \), posterior error covariance \( P_0 \) with a random state vector \( x_0 \), process noise covariance \( Q \), and measurement noise covariance \( R_0 \): \n
\[
\bar{x}_0 = E[ x_0 ] \quad , \quad P_0 = E\left[ (x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T \right] \] (12)

- **Time update**
  - Factorize the error covariance:

\[
S_{k-1} = \text{chol}(P_{k-1}) \] (13)

Where \( \text{chol}(\bullet) \) represents a Cholesky decomposition. That is to say:

\[
P_{k-1} = S_{k-1}S_{k-1}^T \] (14)
Calculate the cubature points

\[ x_{k-1}^{(i)} = S_{k-1} x_{k-1}^{(i)} + \tilde{x}_{k-1} \quad i = 1, 2, ..., 2n \]  

(15)

Where \( n \) is the number of state variables and \( \xi^{(i)} \) is the set of standard cubature points, which is given by

\[ \xi^{(i)} = \begin{cases} \sqrt{n}[1]^{(i)} & i = 1, 2, ..., n \\ -\sqrt{n}[1]^{(i)} & i = n + 1, n + 2, ..., 2n \end{cases} \]  

(16)

Where \([1]\) represents the identify matrix and \([1]^{(i)}\) denotes its \( i \)-th column vector. The state matrix is 2-th column vector, so \( n = 2 \):

\[ [1] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]  

(17)

Calculate the cubature point that is propagated through the equation of state

\[ \tilde{x}_{k | k-1}^{(j)} = f \left( x_{k-1}^{(i)}, u_{k-1} \right) \]  

(18)

Predict the state:

\[ \tilde{x}_{k | k-1} = \frac{1}{2n} \sum_{i=1}^{2n} \tilde{x}_{k | k-1}^{(i)} \]  

(19)

Predict covariance:

\[ P_{k | k-1} = \frac{1}{2n} \sum_{i=1}^{2n} \left( x_{k | k-1}^{(i)} - \tilde{x}_{k | k-1} \right) \left( x_{k | k-1}^{(i)} - \tilde{x}_{k | k-1} \right)^T + Q_{k-1} \]  

(20)

Where \( Q_{k-1} \) is the process noise covariance matrix at time step \( k-1 \).

• Measurement update

Factorize the error covariance:

\[ S_{k | k-1} = \text{chol} \left( P_{k | k-1} \right) \]  

(21)

Calculate the cubature points:

\[ x_{k | k-1}^{(i)} = S_{k | k-1} x_{k | k-1}^{(i)} + \tilde{x}_{k | k-1} \quad i = 1, 2, ..., 2n \]  

(22)

Calculate the cubature point that is propagated through the equation of measurement

\[ \tilde{y}_{k | k-1}^{(j)} = h \left( x_{k | k-1}^{(i)}, u_{k} \right) \]  

(23)

Predict the measurement:

\[ \tilde{y}_{k | k-1} = \frac{1}{2n} \sum_{i=1}^{2n} \tilde{y}_{k | k-1}^{(i)} \]  

(24)

Calculate the estimated covariance:

\[ P_{k | k} = \frac{1}{2n} \sum_{i=1}^{2n} \left( y_{k | k-1}^{(i)} - \tilde{y}_{k | k} \right) \left( y_{k | k-1}^{(i)} - \tilde{y}_{k | k} \right)^T + R_{k-1} \]  

(25)
\[ P_{i|k-1} = \frac{1}{2n} \sum_{i=1}^{2n} (x_{i|k-1}^T - \bar{x}_{i|k-1})(y_{i|k-1} - \bar{y}_{i|k-1})^T \]  

(26)

Where \( R_{k-1} \) is the measurement noise covariance matrix at time step \( k-1 \).

Calculate the Kalman gain:

\[ K_k = P_{i|k-1} \left( P_{i|k-1}^T \right)^{-1} \]  

(27)

Update the predicted state

\[ \hat{x}_k = \bar{x}_{i|k-1} + K_k \left( y_k - \bar{y}_{i|k-1} \right) \]  

(28)

Where \( y_k \) is the measured output at time step \( k \).

Update the error covariance:

\[ P_k = P_{i|k-1} - K_k P_{i|k-1} K_k^T \]  

(29)

4. Experiments results

In order to compare the performance between CKF and EKF, they are both employed basing on online parameters identification.

SOC reference is calculated by ampere-hour based on current sensor with high precision and OCV correction. If initial SOC of two algorithms is not accurate, the results are showed in figure 5 and figure 6.

![Figure 5. Results of EKF and CKF in inaccurate initial SOC.](image1)
![Figure 6. Estimation error of EKF and CKF in inaccurate initial SOC.](image2)

At the end of SOC estimation, final error of EKF is 4.243% and CKF is 0.05%, respectively. Maximum error of EKF is 5.18% and CKF is 1.39%, respectively.

If initial SOC of two algorithms is accurate, the results are showed in figure 7 and figure 8.

![Figure 7. Results of EKF and CKF in accurate initial SOC.](image3)
![Figure 8. Estimation error of EKF and CKF in accurate initial SOC.](image4)

At the end of SOC estimation, final error of EKF is 4.2% and CKF is 0.04835%. Maximum error of EKF is 5.2% and CKF is 1.4 9%.

5. Conclusion

In this paper, the method to estimate SOC using CKF based on online model parameters identification is proposed. Online parameters identification using FFRLS solve the problem that battery is time-
varying and non-linear in different working conditions. By this way, it is unnecessary to test values under different conditions which saves a lot of time and makes the battery model more adaptive. Experiments show that FFRLS has a good performance to identify parameters of battery model. In comparison of EKF, CKF can keep the third order accuracy. However, EKF can only keep the first order accuracy. So CKF has a better performance in estimating SOC than EKF. Experiments have proved. Both the algorithms FFRLS and CKF are not complex. Therefore, we believe that the proposed method is very suitable for applications in Battery Management System.

Acknowledgement
This work is supported by the Primary Research and Development Plan of Shandong Province(2016ZDJS03A04), the Fundamental Research Funds for the Central Universities(HIT.NSRIF.20170) and the Natural Science Foundation of Shandong Province(ZR2017MEE011).

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