Test of primary model predictions by EAS size spectra

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Abstract. High statistical accuracy of experiments KASCADE and ANI allowed to obtain approximations of primary energy spectra and elemental composition in the "knee" region. Obtained results point out to the correctness of QGSJET interaction model and 2-component model of primary cosmic ray origin. The testing of primary energy spectra was carried out in the framework of method (Ter-Antonyan and Haroyan, 2000).

1 Introduction

Absolute differential EAS size spectra around the knee measured at different atmosphere depths and different zenith angles are not explained yet from the point of view of a single $A - A_{Air}$ interaction model and a single model of primary energy spectra and elemental composition. Such an attempt has been made in work (Ter-Antonyan and Haroyan, 2000) based on an unified analysis of KASCADE, AKENO, EAS-TOp and ANI EAS size spectra. The results of approximations of primary energy spectra by rigidity-dependent steepening spectra pointed to the correctness of QGSJET interaction model and two-component composition of primary proton spectrum in the knee region.

Here, on the basis of KASCADE (Glasstetter et al., 1999) and ANI (Chilingarian et al., 1999) EAS size spectra the multi-component model of primary cosmic ray origin (Biermann, 1993) has been tested in the framework of method (Ter-Antonyan and Haroyan, 2000).

2 Method

The testing of primary energy spectra was carried out in 0.03–500 PeV primary energy range using $\chi^2$-minimization (Ter-Antonyan and Haroyan, 2000)

$$\min\{\chi^2/\xi\} \equiv \min\left\{\frac{1}{\xi}\sum_{i}^{m}\sum_{j}^{n}\frac{(f_{i,j} - F_{i,j})^2}{\sigma_f^2 + \sigma_F^2}\right\} \quad (1)$$

where

$$f_{i,j} \equiv \frac{\partial I(E_e, \overline{\theta}, t)}{\partial N_{e,i}^*} \quad (2)$$

is detectable EAS size spectra measured in $i = 1, \ldots, m$ size intervals and $j = 1, \ldots, n$ zenith angular intervals (see Fig.1, symbols), $E_e$ is an energy threshold of detected EAS electrons, $N_{e,i}^*(E > E_e)$ is the estimation value of EAS sizes obtained by detected electron lateral distribution functions at observation level ($t$);

$$F_{i,j} \equiv \sum_A \int_{E_{min}}^{E_e} \frac{\partial \Omega_A}{\partial E_0} W_\theta(E_0, A, N_{e,i}^*) dE_0 \quad (3)$$

are the expected EAS size spectra at $\partial \Omega_A/\partial E_0$ energy spectra of primary nuclei ($A = 1, \ldots, 59$);

$$\xi = m \cdot n \cdot p - 1$$

is a degree of freedom at $p$ number of unknown parameters.

The function $W_\theta$ in expression (3) is determined in general case as

$$W_\theta \equiv \int_{\theta_1}^{\theta_2} \int_0^\infty \frac{\partial \Omega(E_0, A, \theta, t) P_0}{\partial N_e} \frac{\partial \Psi(N_e)}{\partial N_e^*} \sin \theta d\theta dN_e \quad (4)$$

where $\partial \Omega/\partial N_e$ is an EAS size spectrum at the observation level ($t$) for given $E_0, A, \theta$ parameters of a primary nucleus and depends on $A - A_{Air}$ interaction model;

$$\Delta_\theta = \cos \theta_1 - \cos \theta_2;$$

$$P_0 \equiv \frac{1}{X \cdot Y} \int \int D(N_e, E_0, A, \theta, x, y) dxdy \quad (5)$$

is a probability to detect an EAS by scintillation array at EAS core coordinates $|x| < X/2, |y| < Y/2$ and to obtain estimations of EAS parameters ($N_{e,i}^*, s$ - shower age, $x^*, y^*$ - shower core location) with given accuracies;

$$\partial \Psi/\partial N_e^*$$

is a distribution of $N_{e,i}^*(N_e, s, x, y)$ for given EAS size ($N_e$).

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In most of EAS experiments the EAS cores are selected in $P \simeq 1$ range providing a log-Gaussian form of a measuring error $(\partial \Psi / \partial N_e)$ with an average value $\ln(N_e \cdot \delta)$ and RMSD $\sigma_N$, where $\delta$ involves all transfer factors (an energy threshold of detected EAS electrons, $\gamma$ and $\mu$ contributions) and slightly depends on $E_0$ and $A$. In these cases, one may standardise the measured EAS size spectra to the EAS size spectra at the observation level

$$\partial I(\theta, \bar{r}, t) / \partial N_e \simeq \eta \partial I(E_0, \bar{r}, t) / \partial N_e,$$

(6)

where $\eta = \delta^{(\gamma_e - 1)} \exp\{(\gamma_e - 1)^2 \sigma_N^2 / 2\}$ and $\gamma_e$ is the EAS size power index.

Taking the above into account, the expected EAS size spectra (3) can be estimated according to the expression

$$F_{i,j} = \eta \sum_{k=1}^{k_{\text{max}}} \int_{E_{\text{min}}}^{\infty} \frac{\partial \Omega(A_k, E_0, \bar{r}, t)}{\partial E_0} dE_0$$

(7)

where the sum is performed into limited number ($k_{\text{max}}$) of nuclear groups.

2.1 2-component primary energy spectra

Energy spectra of primary nuclei ($A \equiv 1 - 59$) according to the multi-component model of primary cosmic ray origin (Biermann, 1993) are presented in 2-component form:

$$\frac{\partial \Omega}{\partial E_A} = \Phi_A \left( \delta_{A,1} \frac{\partial \Omega_1}{\partial E_A} + \delta_{A,2} \frac{\partial \Omega_2}{\partial E_A} \right)$$

(8)

where the first component (ISM) is derived from the explosions of normal supernova into an interstellar medium with expected rigidity-dependent power law spectra (Biermann, 1993)

$$\frac{d \Omega_1}{d E_A} = \left\{ \begin{array}{ll}
E_A^{-\gamma_1} & : E_A < E_{\text{ISM}} \\
0 & : E_A > E_{\text{ISM}}
\end{array} \right.$$  

(9)

and the second component (SW) is a result of the explosions of stars into their former stellar winds with expected rigidity-dependent power law spectra (Biermann, 1993)

$$\frac{d \Omega_2}{d E_A} = \left\{ \begin{array}{ll}
E_A^{-\gamma_2} & : E_A < E_{\text{SW}} \\
E_{\text{SW}}^{-\gamma_2}(E_A/E_{\text{SW}})^{-\gamma_3} & : E_A > E_{\text{SW}}
\end{array} \right.$$  

(10)

where $\Phi_A$ is a scale factor ($E$ in TeV units) from approximations (Wiebel-Sooth and Biermann, 1998);

$$E_{\text{ISM}} = R_{\text{ISM}} \cdot Z, \quad E_{\text{SW}} = R_{\text{SW}} \cdot Z$$

(11)

are the corresponding rigidity-dependent cut-off energies of ISM-component and knee energies of SW-component; $R_{\text{ISM}}$ and $R_{\text{SW}}$ are model parameters of magnetic rigidities of corresponding components and $Z$ is the charge of $A$ nucleus.

The values of model predictions (Biermann, 1993) for spectral parameters are:

$$\gamma_1 = 2.75 \pm 0.04, \quad \gamma_2 = 2.67 \pm 0.03, \quad \gamma_3 = 3.07 \pm 0.1$$

and rigidities $R_{\text{ISM}} \simeq 120$ TV, $R_{\text{SW}} \simeq 700$ TV at factors of uncertainty $\sim 2$.

The fractions of each component ($\delta_{A,i} \equiv \delta_i(E_A, A)$, $i = 1, 2$) are determined according to

$$\delta_{A,1} = 1 - \delta_{A,2}$$

(12)

$$\delta_{A,2} = (2ZR_{\text{ISM}})^{\left(\gamma_2 - \gamma_{A,0}\right)}$$

(13)

at $\gamma_{A=1,0} = 2.75$ and $\gamma_{A>1,0} = 2.66$ (Biermann, 1993). The expressions (12,13) are consequences of normalization of (8-10) to approximation of balloon and satellite data (Wiebel-Sooth and Biermann, 1998) at $E_A = 1$ TeV.

Thus, minimizing $\chi^2$-functional (1) on the basis of measured values of $\partial I(\bar{r}, t) / \partial N_{e,i}$ and corresponding expected EAS size spectra (7) at given $m$ zenith angular intervals and $n$ EAS size intervals one may evaluate parameters of primary spectra for given ($k_{\text{max}}$) nuclear groups. Evidently, the accuracies of solutions for spectral parameters strongly depend on the number of measured intervals ($m \cdot n$), statistical errors and correctness of $\partial \Omega(E_0, A, \theta, t) / \partial N_e$ determination in the framework of a given interaction model. Moreover, the value of $\chi^2$ points out a reliability of applying primary model.
Differential EAS size spectra $\partial \Omega(E_0, A, \theta, t)/\partial N_e$ for given $E_0 = 0.032, 0.1, \ldots, 100$ PeV, $A = 1, 4, 12, 16, 28, 56, t = 0.5, 0.6, \ldots, 1$ Kg/cm$^2$, $\cos \theta = 0.8, 0.9, 1$ were calculated using CORSIKA562 (NKG) EAS simulation code (Heck et al., 1998) at GGSJET (Kalmykov and Ostapchenko, 1993) interaction model. Intermediate values are calculated using 4-dimensional log-linear interpolations. Estimations of errors of expected EAS size spectra $\partial \Omega/\partial N_e$ at fixed $E_0, A, \theta, t$ parameters did not exceed 3 $\pm$ 5%.

Unknown parameters in minimization (1) were:
- $E_{ISM}$ - cut-off energy of ISM proton component;
- $E_{SW}$ - knee energy of proton SW-component;
- $\gamma_3$ - power index after the knee ($E_{SW}$);
- $\eta$ - a systematic shift (discrepancy) from expression (6).

Moreover, power indices $\gamma_{1,2}$ were changed too in the range of the model uncertainty and the relative uncertainties of expected spectra ($\sigma_F$) in the $\chi^2$-minimization (1) we set equal to 3%.

3 Results

The testing of primary model predictions by minimizations (1) were carried out on the basis of KASCADE (Glasstetter et al., 1999) ($t = 1020$ g/cm$^2$) and ANI (Chilingarian et al., 1999) ($700$ g/cm$^2$) EAS size spectra at 5 zenith angular intervals. The KASCADE EAS size spectra (symbols) and corresponding expected spectra by 2-component model (lines) are shown in Fig. 1. The obtained primary all-particle energy spectrum and energy spectra of $P$, $He$, $O$, $Fe$ nuclear groups are presented in Fig. 2 in comparison with CASA-BLANCA (Fowler et al., 2000) and AGASA (Yoshida et al., 1995) measurements. The obtained values of approximation parameters of primary energy spectra are:

- $\gamma_1 = 2.78 \pm 0.03$; $\gamma_2 = 2.65 \pm 0.03$; $\gamma_3 = 3.28 \pm .07$;
- $E_{ISM} = 210 \pm 60$ TeV; $E_{SW} = 1900 \pm 100$ TeV;
- $\eta = 1.03 \pm 0.03$ at $\chi^2 = 1.22$.

Obtained fractions of the primary stellar wind (SW) component

$$\Delta_{SW}(E_A, A) = \delta_{A,2} \frac{\partial \gamma_2/\partial E_A}{\partial \gamma_3/\partial E_A}$$

versus energy at different primary nuclei are presented in Fig. 3. The results are extrapolated up to 0.1 TeV primary energy range.

The testing of 2-component model of CR origin was also carried out by ANI EAS size spectra (Chilingarian et al., 1999) measured at the mountain level. The results are shown in Fig. 4 and correspond to $\chi^2 = 1.05$ and systematic shift $\eta = 1.14$. The systematic underestimations of KASCADE ($\eta = 1.03$) and ANI ($\eta = 1.14$) EAS size spectra (see expression (6)) one may explain by energy thresholds of detected EAS electrons ($E_e \sim 3$ MeV for KASCADE and $E_e \sim 10$ MeV for ANI experiments).

The values of spectral parameters obtained by approximations of ANI data agree with corresponding parameters obtained from approximations of KASCADE data except for cut-off energy $E_{ISM} = 460 \pm 100$ TeV at ANI data analysis.
4 Conclusion

Predictions of 2-component model of the cosmic ray origin and QGSJET high energy interaction model allow us to explain the measured EAS size spectra in the knee region with accuracy better than 10%.

All model parameters of 2-component primary energy spectra, obtained from EAS data agree with theoretical predictions (Biermann, 1993) in the frames of standard errors with the exception of $\gamma_3$ parameters. Obtained spectral slopes ($\gamma_3 = 3.28 \pm 0.07$) after the knee of SW-component are significantly steeper than 2-component model predictions (Biermann, 1993), $3.07 \pm 0.1$ and this result requires of further investigations.

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