Four-Dimensional Planck Scale is Not Universal in Fifth Dimension in Randall-Sundrum Scenario

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Abstract

It has recently been proposed that the hierarchy problem can be solved by considering the warped fifth dimension compactified on $S^1/Z_2$. Many studies in the context have assumed a particular choice for an integration constant $\sigma_0$ that appears when one solves the five-dimensional Einstein equation. Since $\sigma_0$ is not determined by the boundary condition of the five-dimensional theory, $\sigma_0$ may be regarded as a gauge degree of freedom in a sense. To this time, all indications are that the four-dimensional Planck mass depends on $\sigma_0$. In this paper, we carefully investigate the properties of the geometry in the Randall-Sundrum model, and consider in which location $y$ the four-dimensional Planck mass is measured. As a result, we find a $\sigma_0$-independent relation between the four-dimensional Planck mass $M_{Pl}$ and the five-dimensional fundamental mass scale $M$, and remarkably enough, we can take $M$ to the TeV region when we consider models with the Standard Model confined on a distant brane. We also confirm that the physical masses on the distant brane do not depend on $\sigma_0$ by considering a bulk scalar field as an illustrative example. The resulting mass scale of the Kaluza-Klein modes is on the order of $M$.

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1 Introduction

The vast gap between the electroweak scale and the Planck scale, known as the ‘hierarchy problem’, remains as a major mystery in particle physics. Recently, it has been suggested that large compactified extra dimensions may provide a solution to the hierarchy problem.\cite{1} The relation between the four-dimensional Planck scale $M_{\text{Pl}}$ and the higher dimensional scale $M$ is given by $M_{\text{Pl}}^2 = M^{n+2} V_n$, where $V_n$ is the volume of the extra compactified dimensions. If $V_n$ is large enough, $M$ can be on the order of several TeV. Unfortunately, this scenario alone does not completely solve the problem. The original hierarchy can be translated into another hierarchy between $M$ and the compactification scale $r_c^{-1} = V_n^{-1/n}$.

In Ref.\cite{2}, Randall and Sundrum (RS) proposed an alternative scenario based on an extra dimension compactified on $S^1/Z_2$ with three-branes located at two boundaries. Standard Model fields are assumed to be confined on a distant three-brane, while gravitons propagate in the five-dimensional bulk. The background metric takes the form

$$ds^2 = G_{MN} dx^M dx^N = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2,$$

where $x^M = (x^\mu, y)$ is the coordinate of the five-dimensional spacetime. By solving the five-dimensional Einstein equation, the function $\sigma(y)$ is found.

$$\sigma(y) = k|y| + \sigma_0,$$

where $k$ is the curvature scale related to the five-dimensional cosmological constant $\Lambda$. It was then shown that the effective Planck mass in four dimensions is given by the formula

$$M_{\text{Pl}}^2 = \frac{M^3}{k} (1 - e^{-2\pi kr_c}) e^{-2\sigma_0},$$

where $r_c$ is the radius of the fifth dimension.

Two points are to be noted here. The first point is that according to the above formula (3), the four-dimensional Planck mass is ‘universal’; it takes the same value on both branes located at different boundaries. On the other hand, it was argued that mass scales on the distant brane located at $y = \pi r_c$ are rescaled by the warp factor $e^{-\pi kr_c}$. It is this difference that generates the hierarchy

$$\frac{M_W}{M_{\text{Pl}}} \sim \frac{M e^{-\pi kr_c}}{M} \ll 1.$$
The second point to be noted is that the formula (3) apparently contains the integration constant $\sigma_0$, which is left undetermined by the boundary condition of the five-dimensional theory. Moreover, a certain kind of symmetry under the exchange of two boundaries is not manifest in this formula (3), as is pointed out in Ref.[4].

In this paper, we argue that these two points are intimately related. We carefully discuss the induced metric and the brane coordinates, and point out that the value of the four-dimensional Planck mass differs for hidden and visible brane; that is, it is non-universal. As a result, we find that the four-dimensional Planck mass does not depend on $\sigma_0$, and exchanging-symmetry is manifest.

This paper is organized as follows. After a review of the RS model in §2, we present our formula for the Planck mass in §3. In §4, we apply our prescription for a bulk scalar field as an example and examine the masses of its Kaluza-Klein (KK) modes. We show that these masses are also independent of $\sigma_0$. As a result, in the $k > 0$ scenario, we can adjust $M$ to the TeV region since the Planck mass as the gravitational coupling constant on the distant brane should be set to $10^{18}$ GeV, and we find that the mass of the KK modes is on the order of $M$. Section 5 is devoted to conclusion and discussion.

2 RS model

First, we review the derivation of the four-dimensional Planck mass within the scenario of Ref.[4].

The background metric of the model takes the form

$$ds^2 = G_{MN}dx^M dx^N = e^{-2\sigma(y)}\eta_{\mu\nu} dx^\mu dx^\nu + dy^2,$$

where $x^M = (x^\mu, y)$ is the coordinate of the five-dimensional spacetime, and the fifth dimension is compactified on $S^1/Z_2$ with radius $r_c$. The fundamental region of the fifth dimension is given by $0 \leq y \leq \pi r_c$. A set of three-branes is located at each fixed point $y = y_i$ of $S^1/Z_2$. The brane at $y_0 = 0$ is called a ‘hidden’ brane, and the brane at $y_1 = \pi r_c$ is called ‘visible’. Here and hereafter, we use the subscripts $i = 0, 1$ for quantities at $y = 0, \pi r_c$, respectively.

The action is

$$S = S_{\text{gravity}} + S_{\text{brane}},$$

$$S_{\text{gravity}} = \int d^4x \int_{-\pi r_c}^{\pi r_c} dy \sqrt{-G} \left\{-\Lambda + 2M^3 R\right\},$$

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\[ S_{\text{brane}} = \sum_{i=0,1} \int d^4x \sqrt{-g^{(i)}} \left\{ \mathcal{L}_{(i)} - V_{(i)} \right\}, \quad (6) \]

where \( g^{(i)}_{\mu\nu}(x) = G_{\mu\nu}(x, y = y_i) \) is the induced metric on the \( i \)-th brane, and the brane tensions \( V_{(i)} \) are subtracted from the three-brane Lagrangians.

With the metric (5), the five-dimensional Einstein equation reduces to two differential equations for \( \sigma(y) \) (using \( \sigma' = \partial_y \sigma \)):

\[ (\sigma'(y))^2 = \frac{-\Lambda}{24M^3}, \quad (7) \]
\[ \sigma''(y) = \sum_{i=0,1} \frac{V_{(i)}}{12M^3} \delta(y - y_i). \quad (8) \]

The solution to (7) is given by

\[ \sigma(y) = k|y| + \sigma_0, \quad (9) \]

with

\[ k = \pm \sqrt{-\frac{\Lambda}{24M^3}}. \quad (10) \]

Here, \( \sigma_0 \) is the integration constant which is not determined by the boundary condition of the five-dimensional theory.\(^\dagger\) Conventionally, this integration constant \( \sigma_0 \) is omitted by saying that it just amounts to an overall constant rescaling of \( x^\mu \). For any value of \( \sigma_0 \), the consistency of the solution (9) with the second equation (8) requires that the brane tensions and bulk cosmological constant are related by

\[ k = \frac{V_{(0)}}{24M^3} = -\frac{V_{(1)}}{24M^3}. \quad (11) \]

Following to Ref.[2], we consider a fluctuation \( h_{\mu\nu} \) of the Minkowski spacetime metric \( \eta_{\mu\nu} \), and replace \( \eta_{\mu\nu} \) by a four-dimensional metric \( g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \). Then we have

\[ S_{\text{eff}} = \int d^4x \int_{-\pi r_c}^{\pi r_c} dy 2M^3 e^{-2\sigma(y)} \sqrt{-g} R(g) + \cdots \equiv 2M_{\text{Pl}}^2 \int d^4x \sqrt{-g} R(g) + \cdots, \quad (12) \]

\(^\dagger\)The integration constant \( \sigma_0 \) might be determined by the fundamental theory in higher dimensions.\([1]\)
where $R(\mathcal{F})$ denotes the four-dimensional scalar curvature constructed from $\mathcal{F}_{\mu\nu}$. This gives the formula (3) for the ‘universal’ four-dimensional Planck mass\footnote{We use the normalization of Ref.\[2\].}

$$M_{\text{Pl}}^2 = \frac{M^3}{k}(1 - e^{-2\pi kr_c})e^{-2\sigma_0}.$$ \hfill (13)

We stress that this expression for the Planck mass depends on the integration constant $\sigma_0$. With the choice $\sigma_0 = 0$, as in Ref.\[2\], the above expression reduces to

$$M_{\text{Pl}}^2 = \frac{M^3}{k}(1 - e^{-2\pi kr_c}).$$ \hfill (14)

With this choice, one is forced to take $M$ to be of the order of $M_{\text{Pl}} \sim 10^{18}$ GeV when considering $k > 0$.

As pointed out in Ref.\[4\], we are free to choose the $y$-independent constant $\sigma_0$ in Eq. (9). The particular choice $\sigma_0 = -\pi kr_c/2$ was made so as to meet the requirement that the expression for $M_{\text{Pl}}$ is manifestly invariant with the respect to the change $k \rightarrow -k$, which amounts to exchanging the role of the two boundaries. Then, the four-dimensional Planck mass can be written

$$M_{\text{Pl}}^2 = \frac{2M^3}{k}\sinh(2\pi kr_c).$$ \hfill (15)

As was noted in Ref.\[4\], however, it is almost certainly true that a change of $\sigma_0$ has no net physical effect and must not change the values of four-dimensional physical quantities. Therefore the physical quantities should have the above exchanging-symmetry \textit{without choosing} $\sigma_0$. In other words, physical quantities, including Planck mass, should have the following two properties. First, they are independent of $\sigma_0$. Second, they are not affected by the above brane-exchanging.

In the next section, we present a prescription that naturally realizes these two properties.

3 \hspace{1em} \textbf{Four-dimensional effective Planck mass}

As stated above, the choice of $\sigma_0$ has no net physical effect, and it must not change the values of physical quantities. That is, all the physical quantities, including the Planck mass, must be independent of $\sigma_0$. In this sense $\sigma_0$ may be regarded as a gauge
degree of freedom. In particular, the $\sigma_0$-independence of the four-dimensional Planck mass may be understood by the following argument. Observe that $\sigma_0$ determines the ratio of the length scales of the fifth-dimensional direction and four-dimensional direction at $y = 0$. Therefore, after we have integrated over the full fifth dimension when calculating $M_{Pl}$, the freedom of this ratio will be invisible in the effective theory.

To find the four-dimensional Planck mass more carefully, it is important to make it clear in which location $y$ the four-dimensional Planck mass is measured. To this end, we need to reconsider the choice of the brane coordinate and the induced metric.

We first recall the general situation. When the $i$-th brane with the brane coordinate $\xi^\mu_i$ is embedded into five-dimensional spacetime by $x^\mu = x^\mu(\xi_i)$ and $y = y_i$, the induced metric on it is given by

$$g^{(i)}_{\mu\nu}(\xi_i) = \frac{\partial x^M}{\partial \xi^\mu_i} \frac{\partial x^N}{\partial \xi^\nu_i} G_{MN}(x = x(\xi_i)\), \ y = y_i). \quad (16)$$

In the discussion given in §2, the implicit choice $x^\mu = \xi^\mu_i$ was made so that $g_{\mu\nu}^{(i)} = G_{\mu\nu}(y = y_i)$.

When the fine-tuning conditions are satisfied, the branes are flat and the induced metrics generally take the form $g^{(i)}_{\mu\nu} = e^{-2\alpha} \eta_{\mu\nu}$, with the $x^\mu$-independent constant $\alpha$. In view of Eq. (14), the corresponding brane coordinates are uniquely determined by $\xi^\mu_i = e^{\alpha - \sigma_i} x^\mu$ (up to a Poincaré transformation). Therefore, when discussing a four-dimensional effective theory on the brane, one should use the correct set of the induced metric and brane coordinate as

$$\left( g_{\mu\nu}^{(i)} = e^{-2\alpha} \eta_{\mu\nu}, \ \xi^\mu_i = e^{\alpha - \sigma_i} x^\mu \right). \quad (17)$$

This is the point of our treatment. With this correct set, we can determine the relation of the five-dimensional scale $M$ and the four-dimensional Planck scale $M_{Pl(i)}$ on the $i$-th brane by integrating over the fifth dimension as

$$S_{eff} = 2M^3 \int d^4x \int_{-\pi r_c}^{\pi r_c} dy e^{-2\sigma(y)} \sqrt{-g} R(g; x) + \cdots$$

$$= 2M_{Pl(i)}^2 \int d^4\xi_i \sqrt{-g^{(i)}} R(g^{(i)}; \xi_i) + \cdots, \quad (18)$$

where $R(g^{(i)}, \xi_i)$ is the four-dimensional scalar curvature constructed from the induced metric $g^{(i)}_{\mu\nu}$ and the coordinate $\xi^\mu_i$. We note that, following Ref.[2], we define
$M_{\text{Pl}}$ as a coefficient of the Einstein-Hilbert (EH) term. This is a proper definition of graviton self-couplings contained in the EH term. Alternatively, we can determine $M_{\text{Pl}}$ from the graviton coupling to the matter stress tensor. We confirmed that both methods yield the same result, given below.

We now determine the relation between $M_{\text{Pl}}$ and the five-dimensional quantities $M$, $k$, and $r_c$ by using the set $(17)$. For definiteness, let us choose $\alpha = 0$ so that the induced metric is precisely Minkowskian (as is usual in field theory in flat space-time). This means that we choose $(\eta_{\mu\nu}, \xi_{\mu} = e^{-\sigma_i} x^\mu)$ as the induced metric and brane coordinate. With this choice, we change the integration variables to $\xi_{\mu} = e^{-\sigma_i} x^\mu$ and contract the indices with $g^{(i)}_{\mu\nu} = \eta_{\mu\nu}$. We then obtain

$$S_{\text{eff}} = \int d^4 \xi_i \int_{-\pi r_c}^{\pi r_c} dy \, 2M^3 e^{-2(\sigma_i - \sigma)} \sqrt{-g^{(i)}} R(g^{(i)}; \xi_i) + \cdots$$

$$\equiv 2M^2_{\text{Pl}(i)} \int d^4 \xi_i \sqrt{-g^{(i)}} R(g^{(i)}; \xi_i) + \cdots, \quad (19)$$

where $\sigma_i = \sigma(y_i)$. It follows that

$$M^2_{\text{Pl}(i)} = \int_{-\pi r_c}^{\pi r_c} dy M^3 e^{-2(\sigma(y_i) - \sigma)}$$

$$= M^3 \int_{-\pi r_c}^{\pi r_c} dy e^{-2k|y|}$$

$$= \frac{M^3}{k} (1 - e^{-2\pi kr_c}) e^{2(\sigma_i - \sigma_0)}. \quad (20)$$

With (20), it is clear that $M_{\text{Pl}(i)}$ is independent of $\sigma_0$. Explicitly, we find

$$M^2_{\text{Pl}(0)} = \frac{M^3}{k} (1 - e^{-2\pi kr_c}) \quad (21)$$

on the brane at $y = 0$ and

$$M^2_{\text{Pl}(1)} = \frac{M^3}{k} (e^{2\pi kr_c} - 1) \quad (22)$$

on the brane $y = \pi r_c$. Note that these expressions are transformed into each other by exchanging $k$ with $-k$. Thus our results naturally possess the two properties stated in the previous section.

We note that our expression (21) for the brane at $y = 0$ coincides with Eq. (14), which is derived by simply neglecting $\sigma_0$. Therefore in this case, we have explicitly
confirmed the naive expectation that $\sigma_0$ may be absorbed by the rescaling of $x^\mu$, since our expression (21) takes account of $\sigma_0$ by using the correct set $(\eta_{\mu\nu}, \xi_{i=0} = e^{-\sigma_0}x^\mu)$ of the induced metric and brane coordinate.

The same is not true for the expression (22), however. When we consider the scenario of Ref.[2], in which Standard Model fields are assumed to be confined on the brane at $y = \pi r_c$ with a negative tension, the naive expectation is no longer correct, and the original expression (14) should be modified to our (22). The origin of the discrepancy can be understood as follows. If one tries to absorb $\sigma_0$ by the rescaling $\xi_{i=1} = e^{-\sigma_0}x^\mu$ as in the $y = 0$ case, the induced metric $\eta_{\mu\nu}$ cannot be used, since $(\eta_{\mu\nu}, \xi_{i=1} = e^{-\sigma_0}x^\mu)$ is not the correct set of the induced metric and brane coordinate. The correct set is $(\eta_{\mu\nu}, \xi_{i=1} = e^{-\sigma_1}x^\mu)$. Using this correct set, one finds that the Planck scale at $y = \pi r_c$ is given by our formula (22).

The most important aspect of the RS model is that it gives rise to a localized graviton field.[3] Our results (21) and (22) can naturally be understood from this fact; the small Planck scale $M_{\text{Pl}(0)}$ arises because of the localized graviton in the fifth dimension near the brane of positive tension, while the large Planck scale $M_{\text{Pl}(1)}$ arises because of the small overlap of the graviton with the brane of negative tension.[3]

A striking feature of our results (21) and (22) is that the relative size of four and five-dimensional Planck scales crucially depends on the location at which $M_{\text{Pl}}$ is measured. In the model in which Standard Model fields are confined on the positive tension brane at $y = 0$, as in Ref.[3], we have the relation (21), from which $M$ is of the same order as $M_{\text{Pl}(0)} \sim 10^{19}$ GeV, that is, $M \sim M_{\text{Pl}(0)}$. This conclusion is the same as that in the original proposal, of course. In the model in which Standard Model fields are confined on the negative tension brane at $y = \pi r_c$, as in Ref.[2], however, we now have our modified relation (21), which gives

$$M_{\text{Pl}(1)}^2 \approx \frac{M^3}{k} e^{2\pi kr_c}.$$  \hspace{1cm} (23)

We see that the fundamental mass scale $M$ becomes much smaller than the Planck scale, unlike in the original proposal.[2] As a result, it is perfectly possible that the fundamental scale $M$ lies in the TeV region. We need to use the expressions (21) and (22) of the four-dimensional Planck mass properly, depending on the model employed.

3To be precise, $M_{\text{Pl}(1)}$ should be identified as the Planck scale in models with the Standard Model confined on the brane at $y = \pi r_c$, and $M_{\text{Pl}(0)}$ should be identified as that in models with the SM confined at $y = 0$. In any case, we have only one Planck scale in a given model.
4 Physical mass scale

In order to check from another viewpoint that the physical quantities are independent of $\sigma_0$, we consider a massless bulk scalar field as an illustrative example, and examine the masses of the KK modes. Extensions to the case of a massive bulk scalar field and a bulk gauge field are straightforward.

The action is given by

$$S_{\text{scalar}} = -\frac{1}{2} \int d^5x \sqrt{-G} G^{MN} \partial_M \Phi \partial_N \Phi$$

$$= \frac{1}{2} \int d^5x e^{-2\sigma} \Phi(\Box + e^{2\sigma} \partial_y e^{-4\sigma} \partial_y) \Phi,$$

(24)

where $\Box \equiv \eta^{\mu\nu} \partial_\mu \partial_\nu$. The KK mode expansion is

$$\Phi(x, y) = \sum_{n \geq 0} \varphi_n(x) \chi_n(y),$$

(25)

where the mode functions $\chi_n(y)$ are chosen to satisfy

$$\frac{d}{dy} \left( e^{-4\sigma} \frac{d}{dy} \right) \chi_n(y) = -M_n^2 e^{-2\sigma} \chi_n(y)$$

(26)

with mass eigenvalues $M_n$. The solutions to this equation are related to Bessel functions $J_2$ and $Y_2$ of order two.[5] We should be careful with Eq. (26) however, since it still contains $\sigma_0$. As the orthonormality condition for $\chi_n$, let us take

$$\int_{-\pi r_c}^{\pi r_c} dy e^{-2\sigma} \chi_m \chi_n = \delta_{mn} e^{-2\sigma_i}.$$

(27)

Then we have

$$S_{\text{scalar}} = \frac{1}{2} \sum_{n \geq 0} \int d^4x \ e^{-2\sigma_i} \varphi_n(\Box - M_n^2) \varphi_n.$$

(28)

From our point of view, the action should be described by using the $i$-th brane coordinate $x_i^\mu = e^{-\sigma_i} x^\mu$. Then, the four-dimensional volume element $d^4x$ and the differential operator $\Box$ are replaced by $d^4x_i = d^4x e^{-4\sigma}$ and $\Box_i = \Box e^{2\sigma_i}$, respectively:

$$S_{\text{scalar}} = \frac{1}{2} \sum_{n \geq 0} \int d^4x_i \varphi'_n \left( \Box_i - e^{2\sigma} M_n^2 \right) \varphi'_n.$$

(29)
We find that canonically-normalized fields in four dimensions are \( \varphi'_n(x_i) = \varphi_n(x) \), and the physical masses of the KK modes are given by

\[
M^2_{n(i)} = e^{2\sigma_i} M^2_{n(i)} .
\]

To clarify the physical meaning of (30), let us rewrite (26) as

\[
\frac{d}{dy} \left( e^{-4ky} \frac{d}{dy} \right) \chi_n = - \left( M^2_n e^{2\sigma_0} \right) e^{-2ky} \chi_n \equiv -m^2_n e^{-2ky} \chi_n .
\]

With \( m^2_n \) regarded as an eigenvalue, this equation is independent of \( \sigma_0 \). This implies that \( m^2_n \) depends on the parameters \( k \) and \( r_c \), but not on \( \sigma_0 \); \( m^2_n = m^2_n(k, r_c) \).

Therefore the \( \sigma_0 \) dependence cancels out in Eq. (30) as

\[
M^\prime_{n(i)} = M_n e^{\sigma_i} = m_n e^{(\sigma_i - \sigma_0)} .
\]

The mass spectrum of the KK modes is determined by the boundary condition at \( y = \pi r_c \), \( (d/dy) \chi_n (y = \pi r_c) = 0 \). In particular, we are interested in the mass scale \( M'_{n(1)} \), as measured on the visible brane. To this end, note that Eq. (33) is precisely the same equation that treated in Ref.[5], where it was shown that

\[
m_n \sim k e^{-\pi kr_c} = k e^{-(\sigma_1 - \sigma_0)}. \]

Therefore the mass scale of the KK modes is estimated as

\[
M'_{n(1)} \sim k \sim M \ll M_{Pl(1)}. \]

We thus confirm that the mass scale of the KK modes is significantly smaller than the Planck scale \( \sim 10^{18} \) GeV. We remark, however, that our interpretations is quite different from the usual one,

\[
m_n \ll k \sim M \sim M_{Pl}. \]

5 Conclusions

The most important point in our treatment is to make the brane coordinate transformation, by which the induced metric on each brane becomes the Minkowski metric \( \eta_{\mu \nu} \). When measured with such brane coordinates, the four-dimensional Planck mass
becomes different on each brane. We found that the four-dimensional Planck mass, when calculated by this procedure, does not depend on the integration constant $\sigma_0$. We showed that the masses of the KK modes are also independent of $\sigma_0$ for a massless bulk scalar. This holds for any kinds of fields. Moreover, the exchanging-symmetry is manifest in our expression for the Planck masses. On the brane with negative tension, the Planck mass is always much larger than on the brane with positive tension. This fact is interpreted as reflecting the smallness of the gravitational coupling constant, that is, the small overlap of the graviton with the brane with negative tension. When we identify the brane of negative tension with the visible brane, the fundamental scale $M$ can be significantly smaller than the Planck mass, and the masses of the KK modes are of the same order as $M$ in the massless bulk scalar. We can summarize the above statement as follows. When we estimate the values of physical quantities, we must multiply by the warp factor corresponding to their mass dimension to the values in the bulk. This rule is not exceptional to the Planck mass.

The most striking result is that the fundamental scale $M$ in the five-dimensional theory can be significantly smaller than that was supposed so far. For instance, it can lie in the TeV region. Given this, it will be interesting to find direct evidence for the extra dimension in the Randall-Sundrum-type scenario. High-energy accelerator experiments in the near future might directly prove the existence of the extra dimension.

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