Abstract. In the recent years we have learned that light quarks play a
crucial role in QCD-like theories, transforming it to many different phases. We
review what is known about them, both from lattice and non-lattice approaches.
A particularly simple mechanism of the QCD chiral restoration phase transition is
discussed first: it suggests that it is a transition from randomly placed tunneling
events (instantons) at low \( T \) to strongly localized tunneling-anti-tunneling pairs
at high \( T \). Many features of the transition found on the lattice can be explained
in this simple picture. Very relevant for RHIC, this approach predicts a strong
non-perturbative interaction between quarks above the phase transition. It also
predicts that QGP-like phase sets in at zero temperature, provided few more light
quark flavors are added to QCD. Finally, we also discuss possible experimental
signatures of the QCD phase transition. One issue is CERN dilepton data, possibly
related with “dropping” masses of \( \rho, A_1 \) mesons. Another is direct manifestation of
a softness of EOS (smallness of pressure/energy density) in the phase transition
region in flow and even the global lifetime of the system.

1 Introduction

My topic is to review our current understanding of critical phenomena in
QCD (and its close relatives). But before that, let me comment on a some-
what different type of critical phenomena discussed in one (late) session of
the workshop, the so called “self-organized criticality” exemplified by the famous sand piles. Not being an expert in this field, I still dare to suggest that the RHIC summer workshop itself is a perfect example of this phenomenon. Indeed, only several years ago many of the participant of RHIC project (experimentalists, accelerator people and even most of the theorists) would not even listen to a talk with such title as mine. (This is commonly described by exponentially decaying correlations, with rather short correlation length.) Now, with common goals and concerns about directions future experiments at RHIC will follow, we are all well inside “one correlation length”. This is clearly a phase transition in its own right.

Returning to QCD, let me start with the comment that during the last few years we have learned about many new phases, which the gauge theories may have. Of course, the one we are going to look for experimentally is still the *chirally symmetric Quark-Gluon Plasma* (QGP), in which the charge is screened rather than confined. Recent unexpected findings suggest that it may have completely new features, such as preserve some hadronic modes as a bound states.

However, theory predicts some other phases which has taught us many new lessons. In particular, it seems that QGP may exist even at zero temperature in QCD with about 5-7 light fermions. Even more flavored QCD, with 7-16 light quarks, is expected to be in even stranger *conformal* phase, an extended relative of a condition commonly studied only at the second order phase transition points. Similar phases are proven to exist in supersymmetric extension of QCD, in which there were recently a significant advances due to Seiberg and Witten.

In addition, we have learned about some unwanted phases, such as (i) the *Aoki phase* appearing in lattice studies with Wilson fermions (see Ukawa review) and (ii) the *Stephanov phase* appearing in quenched lattice simulations with the non-zero chemical potential.

As emphasized by T.D.Lee in his opening talk, before thinking about “small” (the particles) one should first clearly understand “large” (the phase one actually is in): as we will see it is a very good advice to these lattice calculations indeed.

The bottom line of this talk is the crucial (but still not quite well understood) role of light fermions, generating all these phases of QCD. In particular, compare the phase transition found in “quenched” calculations ($N_f = 0$) and those with $N_f = 2 – 3$ dynamical quarks. The former case has a “de-
confinement" phase transition at high $T_c \approx 260 MeV$, while the latter show a “chiral restoration” transition at much lower $T_c \approx 150 MeV$ \footnote{By changing the quark masses from light to heavy continuously, it was shown that these two transitions are indeed different phenomena, separated by a large gap in which there is no transition at all.} Unlike at normal (T=0) conditions, at $T \approx T_c$ physical quantities are very sensitive to such little details as the mass value of the strange quark. Therefore we are still not quite sure about the order of the transition: the latest lattice results \footnote{For example, the MIT bag model literally predicts QGP formation in heavy ion collisions at unrealistically small (BEVALAC/GSI) energies.} incline again toward the 1-st order in the real world.

But do we understand why the transitions happen? Is there a simple picture which can explain its microscopic mechanism? Can we build a working model, reliable enough to provide some guidance in delicate questions relevant to experimental observables?

Qualitative explanations of why the QCD phase transition takes place are often done in an over-simplified way, emphasizing the “overlapping” hadrons in a bag-model-type picture. But such pictures give all numbers and physics wrong. Pure gluodynamics is an especially good example: at $T_c \approx 260 MeV$ the density of glueballs is negligible since even the lightest one has a mass of about 1.7 GeV! Looking at the low-T side of this transition it is impossible to tell why it happens. The same is qualitatively true for the chiral restoration: on the hadronic side the matter is still relatively dilute. But the reason why the phase transition happens is very simple: just two very different phases happen to have the same free energy. The lesson: one cannot understand a QCD phase transition without a quantitative model for both phases.

The energetics of the “deconfinement” and “chiral restoration” transitions is entirely different, suggesting different physics. The former has huge latent heat, few GeV/fm$^3$, so that probably all of the non-perturbative vacuum energy density (proportional to the “gluon condensate”) is “melted” in it. This is not the case for “chiral restoration” in QCD: large portion of the gluon condensate should actually survive it \footnote{For example, the MIT bag model literally predicts QGP formation in heavy ion collisions at unrealistically small (BEVALAC/GSI) energies.}. What is this remaining “hard glue” or “epoxy”, as Gerry Brown called it? Why, unlike “soft glue” does it not produce a quark condensate? Can it affect quark interactions and hadronic masses?

The major “non-lattice” approaches to the problem include: (i) models based on particular effective Lagrangians, like the sigma model or chiral
Effective Lagrangians models [5, 6]; (ii) QCD sum rules at finite temperature/density [7]; (iii) the interacting instanton liquid model (IILM) [2, 8].

Effective Lagrangians are very powerful tools at low temperatures: with parameters fixed by data one can indeed accurately account for effects due to non-zero occupation factors. However, they are clearly “one-sided”, unable to deal with QGP. In principle, QCD sum rules are not limited to a description of only hadronic phase: the structural changes can be adequately described by VEV of different operators, or “condensates”. However, since in practice those are unknown, people use simplifications. The most popular one is “vacuum dominance”, reducing average values of any quark operators to powers of $\langle \bar{q}q \rangle$, therefore missing non-perturbative effects above $T_c$.

Among theory issues discussed intensely during the last couple of years [9, 10, 11, 12] is the fate of the U(1) chiral symmetry. It cannot be exactly restored, as suggested in [10], but its violation certainly is dramatically reduced at $T \approx T_c$. In practice it means that $\eta_{\text{non-strange}}$ (a combination of $\eta, \eta'$) and isovector scalar (we call $\delta$) may be nearly as light as a pion.

One very important issue (which should have been discussed more) is what happens with the QCD phase transition for larger number of quark flavors $N_f$. Several phase transition lines are expected there, the lowest probably being the chiral symmetry restoration at $T=0$.

2 A mechanism for the chiral phase transition

The main point of this talk is that due to recent developments it becomes increasingly clear what is the microscopic mechanism underlying this phase transition. It is rearrangement of instantons, from relatively random liquid at low $T$ to a gas of instanton anti-instanton “molecules”. These $\hat{I}I$ molecules are the “epoxy” mentioned above: at $T > T_c$ they indeed do not create the quark condensate but contribute to the gluon one. Furthermore, they generate new type of the inter-quark interaction, and may even create hadronic states, even above $T_c$!  

Several events lead to these developments: (i) Demonstration that “in-

Note a similarity to Kosterlitz-Thouless transition in $O(2)$ spin model in 2 dimensions: there are paired topological objects (vortices) in one phase and random liquid in another.
stanton vacuum” explains the QCD correlation functions and hadronic spectroscopy at T=0 [13, 2]; (ii) Lattice confirmation of instanton liquid parameters and of the dominant role in general [14, 15]; (iii) As shown in [16] instantons cannot be screened at T < T_c; (iv) Confirmation of this statement on the lattice [17]; (v) Discovery of polarized $\bar{I}I$ molecules at $T \approx T_c$ [19]; (vi) Numerical simulations [2] and analytic studies [8] of the phase transition in the interacting instanton ensemble.

It is very easy to explain what happens at $T \approx T_c$ in this approach. Recall that finite temperature is described in Euclidean space-time by periodic boundary conditions, with the Matsubara period $1/T$. So a rising T means a decreasing box, and when it fits to the size of one $\bar{I}I$ molecule, one gets a “geometric” transition. In Fig. 1 from [2] one can see it clearly. (The plots show projections of a four dimensional box into the z axis-imaginary time plane. Instantons and anti-instanton positions are indicated by + and – symbols. The lines correspond to strongest fermionic “bonds”.) Notice that molecules are strongly “polarized” in the time direction and that they are separated by half Matsubara box in time $\Delta \tau = 1/(2T)$.

In a series of recent numerical simulations [2] it was found that like QCD, the instanton model has second order transition for $N_f = 2$ massless flavors, but a weak first order one for QCD with physical masses. Furthermore, the thermodynamic parameters, the spectra of the Dirac operator, the T-dependence of the quark condensate and various susceptibilities, the screening masses are all consistent with available lattice data.

3 Phases of QCD with more quark flavors

In this section we discuss further the role of quarks in QCD, adding more flavors to it. If we add too much of them, namely $N_f > 33/2$ (here and below we imply 3 colors), the asymptotic freedom is lost and we get uninteresting field theory with a charge growing at small distances, basically a theory as

4Previously considered scenario based on the “instanton suppression” does not in fact work until rather high T, well in the QGP domain, and thus it cannot be the reason for the phase transition.

5Thus one can even find the approximate phase transition temperature. The molecule fits onto the torus if $4\rho \approx 1/T_c$, and with known instanton size $\rho \approx 0.35$ fm one gets $T \approx 150$ MeV, close enough to the observed one.
bad as QED! So our “most flavored” QCD (with 16 flavors, and work all the way down from it (see fig 3). The phase we are in there is actually rather simple one, known as the Banks-Zaks conformal domain \([25]\). It has the infrared fixed point at small coupling \(g_\ast^2/16\pi^2 = -b/b' \ll 1\) (\(b, b'\) are the one and two-loops coefficients of beta function). It happens in the perturbative domain, so the charge is small both at small and large distances. There are no particles in this phase, and all correlators decay as powers of the distance. In this phase the non-perturbative phenomena like instantons are exponentially suppressed, \(\exp(-const/g_\ast^2)\). However, as one decrease the fermion number, the fixed point \(g_\ast^2\) moves to larger values and eventually disappears. Lattice simulations of multi-flavor QCD were recently reported in \([26]\). These authors studied QCD with up to 240 flavors. Studying the sign of the beta function in the weak and strong coupling domains, they confirmed the existence of an infrared fixed point as low as at \(N_f = 7\).

It is not known which phase we find next, at \(N_f = 5 - 6\), most probably it is the so called Coulomb phase (basically QGP).

The results of the interacting instanton model are summarized by Fig. 2(b-d) show how one singular point at \(N_f = 2\) develops into the discontinuity line

Figure 1: Typical instanton configurations for temperatures \(T= 76\) and 158 MeV.
Figure 2: Phase diagram (temperature-quark mass plane) of the instanton liquid for different numbers of quark flavors, $N_f=2,3$ and $5$. The open squares indicate non-zero chiral condensate, while solid ones indicate that it is zero. The dashed lines show the approximate location of the discontinuity line.

for $N_f = 3$. The value of $T_c$ goes down with increasing $N_f$ and at $N_f = 5$ one finds that the chiral symmetry is restored even at $T=0$, provided quarks are light enough.

New lattice results have been presented at this meeting by R.Mawhinney for QCD with $N_f = 4$. Details can be found in his (and N.Christ’s) talk, and I should only say that when they have extrapolated the measured masses to quark masses $m \to 0$, they have found a dramatic significant drop in chiral symmetry breaking effects, such as $\pi - \sigma, \rho - a_1, N - N^*(1/2^-)$ splittings, very much unlike the $N_f = 0-3$ studied before. It suggests that chiral restoration is nearby, very similarly to what was found in the instanton calculations.

Finally, we return to Figure 3 and explain its rhs, showing similar phase diagram for the $N=1$ SUSY QCD based on [27]. Not going into details, let me only mention that the vacuum is now definitely dominated by instanton-

\footnote{Note that $N_f = 4$ case is missing. It is because I have found the condensate to be small and comparable to finite-size effects. In order to separate those one should do calculations in different boxes, which is time consuming.}
antiinstanton molecules, and their contribution can be calculated without problems (in QCD subtraction of perturbation theory is a great one). There are two phases which are impossible in QCD: a case without the ground state (molecule force the system toward infinite Higgs VEV) and also a funny situation with chiral symmetry unbroken but confinement (hadrons exist but are degenerate in parity). It is amusing that chiral symmetry is restored at $N_f = N_c + 1$, similar to what was found in QCD in the instanton model. Also note, that in this case the existence of the Coulomb phase is a proven fact.

4 Inter-quark interaction and hadrons near and above $T_c$

It is well known that the nucleon effective mass in nuclear matter is reduced, and there are also indications that $m_\rho$ does the same [37]. Among approaches predicted “dropping masses” at high $T$ the simplest (and the most radical) one is Brown-Rho scaling. According to it all hadronic masses get their scale from $\langle \bar{q}q \rangle$, and therefore vanish at $T \to T_c$. This idea is supported by the QCD sum rules, provided the “vacuum dominance” approximation is used. However it is not so in a vacuum made of the instanton molecules, because they generate non-zero average values for some fermionic operators, even when $T > T_c$ and $\langle \bar{q}q \rangle = 0$. Those operators can be treated as new non-perturbative inter-quark interaction, which can be described by the following (Fierz symmetric, color-singlet only) Lagrangian [22]

$$L_{mol\ sym} = G \left\{ \frac{2}{N_c} \left[ (\bar{\psi}\tau^a\psi)^2 - (\bar{\psi}\tau^a\gamma_5\psi)^2 \right] - \frac{1}{2N_c} \left[ (\bar{\psi}\tau^a\gamma_\mu\psi)^2 + (\bar{\psi}\tau^a\gamma_\mu\gamma_5\psi)^2 \right] + \frac{2}{N_c} (\bar{\psi}\gamma_\mu\gamma_5\psi)^2 \right\} + L_8, \quad (1)$$

which has Nambu-Jona-Lasinio-type form and the coupling constant is proportional to density of molecules $G = \int n(\rho_1, \rho_2) d\rho_1 d\rho_2 \frac{1}{27 T^4} (2\pi\rho_1)^2 (2\pi\rho_2)^2$. Here, $n(\rho_1, \rho_2)$ is the tunneling probability for the IA pair and $T_{IA}$ is the corresponding overlap matrix element, $\tau^a$ is a four-vector with components $(\vec{\tau}, 1)$. The effective Lagrangian (1) was determined by averaging over all possible
molecule orientations. Near the phase transition, molecules are polarized and all vector interactions are modified according to $(\bar{\psi}\gamma_\mu\Gamma\psi)^2 \to 4(\bar{\psi}\gamma_0\Gamma\psi)^2$.

Numerical simulations and analytic studies have been used to calculate both spatial and temporal correlation function. The former exponentially decay with the so called “screening masses”: their T-dependence for a number of hadronic channels is shown in Fig.(4)(b) show overall agreement with lattice ones. Especially important is strong attraction in scalar-pseudoscalar channels, shifting these masses down from their high-T asymptotic, $M/\pi T = 2$. This attraction is also clearly seen in temporal correlation functions as well. For example, the pion correlators at several T are shown in Fig.(4)(a). They all have upward enhancement, which is absent in other channels (such as $\rho$). Fit of the correlator suggests that pion survives the phase transition and exist at $T > T_c$ as a (non-Goldstone) massive bound state. Most other hadrons “melt” into constituent quarks. Those have no effective mass but effective energy (or “chiral mass”) above $T_c$. Unfortunately, accuracy of the calculations done so far does not allow to get information about the fate of vector and axial mesons, the major players in the next section.

Unfortunately, the temporal correlators are very difficult to get on the lattice: there are only few points in time direction. The best data we can compare with definitely show attraction in the pion channel at $T > T_c$, which is absent in vector ones.

5 The $U(1)_A$ restoration?

The fate of the $U(1)_A$ anomaly at finite temperature has received a lot of attention lately. A number of authors have emphasized that a (partial) $U(1)_A$ restoration may be the case, with rather dramatic observable consequences. Indeed, recall that at T=0 the $\eta' - \pi$ mass difference is larger than all other meson mass splittings, and any tendency for it to shrink would therefore strongly affect the whole spectroscopy.

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7Those are directly analogous to the BCS theory of superconductivity, but with the instanton-induced interactions like (1).

8An interesting situation arises for $N_f \geq 3$ massless flavors. In this case, the anomaly does not affect the $\eta'$ correlation function above $T_c$ [21]. If chiral symmetry is unbroken, extra zero modes cannot be absorbed by the condensate, and the 't Hooft
It is well known that this splitting and the $U(1)_A$ anomaly are related with the topological charge, or well isolated instantons. It was argued in [9] that those should be very rare in the ensemble at $T > T_c$, because all instantons are “paired” into molecules. The density of isolated instantons above $T_c$ is small $O(m^{N_f})$, but due to zero modes the quark propagators contribute a factor $1/m$, and some results are finite in the chiral limit. So, a significant drop in the strength of the $U(1)_A$ anomaly around $T_c$ is expected, although to non-zero value. It was further suggested in [9] that instead of dealing with $\eta'$ (which involve a complicated flavor mixing pattern), it is more convenient to look at a difference between $\pi$ and its $U(1)_A$ partner isovector scalar $\delta$.

Such measurements have been done, both on the lattice and in the instanton model. All of them observe that the difference of $\pi - \delta$ susceptibilities indeed drops dramatically at $T_c$, indicating a move toward the $U(1)_A$ restoration.

However, the calculations disagree about what exactly is its value at $T > T_c$. The problem is the $m \to 0$ limit, which is very difficult to do in practice. Then the conclusion is sensitive to extrapolations: MILC collaboration [32] has fitted it quadratically $\chi_\pi - \chi_\delta = C_1 + m^2 C_2$ and gets a non-zero $C_1$, but extensive results from the Columbia group (see their data in N.Christ’s talk) have shown that the linear fit is much better, and it leads to $\chi_\pi - \chi_\delta \to 0$.

In the instanton calculations [2] the value is clearly non-zero (although it is not accurately determined yet). So, it seems that here we (for the first time) see a serious qualitative disagreement with lattice simulations at $T > T_c$. This can be seen explicitly from the form of the Dirac spectrum. In the one we have calculated there are still quasi-zero modes due to isolated instantons, while in Columbia ensemble those seem to be completely absent. Why this happens remains unclear.

The problem of $\eta\eta'$ mixing at finite $T$ was studied in more detail in [30]. The most important conclusions is that there is a tendency towards ideal mixing, a separation of strange and non-strange component. However, contrary to others, it is shown that for $T > T_c$ the anomaly only can operate in the non-strange sector, with $\bar{u}u$ transition into $\bar{d}d$ being proportional to $m_s$. It is claimed that non-strange component may be rather heavy.

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vertex only contributes to $2N_f$-point correlators. For $N_f = 3$ this means that the $\eta'$ and $\pi$ are degenerate above $T_c$, but the singlet and non-singlet $\Lambda$ are split.

Another way to explain it is to say, that for $m=0$ the individual instantons are absent in the vacuum, but can be created by the operators themselves.
In summary, a clear tendency toward diminishing of the role of U(1) anomaly at $T \approx T_c$ is observed, but quantitative results for $T > T_c$ are still missing. If (as Columbia data suggests) it is vanishingly small, we would have 8 rather than 4 massless modes (for $N_f = 2, m = 0$), and understand why it moved away from O(4) indices to the first-order ones. However, the instanton calculations show that it is not so small.

6 The phase transition at finite baryon density: the old and the new puzzles

First attempts to introduce the non-zero chemical potential associated with baryon number have been done in *quenched* approximation a decade ago [34]. It was found that a very strange thing (referred as the “old puzzle”) happens. At small $\mu$ as expected, nothing depends on it, till some threshold $\mu_c$ is reached. But instead of finding a threshold to be around the constituent quark mass $\mu_c \approx m_N/3$ [35], they have found it at $\mu_c \approx m_\pi/2$. Those are not too different if quark bare mass is large, but if it goes to zero the latter value vanishes. That means that in the chiral limit some states with non-zero baryonic number exist (and can be excited) at arbitrarily small $\mu$. There are no such states in the real world, so why lattice data have found them?

For ten years it was not clear why, but the phenomenon was not a numeric artifact, and it has been since then seen in many other calculations. The resolution of the puzzle was recently made by Stephanov [35]. He has pointed out, that it is “quenching” the QCD partition function which one should blame. Interestingly enough, it turns out that elimination of the quarks can be done in two very different ways. In both one writes in the bosonic partition function the factor $[\det(i\hat{D} + i\mu\gamma_0 + im)]^{N_f}$ coming from the integration over fermions, and then takes $N_f \to 0$. This factor is real at $\mu = 0$ and complex otherwise, so it can be written as a modulus and a phase. Stephanov has shown that the usual quenching corresponds to the limit, in which one keeps modulus and ignores the phase[36].

10By tradition, lattice people just put $\mu$ in the Dirac operator without extra coefficients, which means that in their units quarks have baryonic charge 1, not 1/3.

11One can do this also without the limit, at finite $N_f$: such approach was suggested and studied separately by Gocksch [36].
Putting the absolute value of the determinant means that anti-quarks have a conjugate operator, with the opposite sign of the chemical potential $\mu \rightarrow -\mu$: so instead of having the baryonic charge -1 they also have it equal to 1, like quarks. Then mesons $\bar{q}_1 \Gamma q_2$ may have the baryon charge 2, and the massless excitations excited at small $\mu$ are nothing else but such pions. Those exist in the Stephanov (unwanted) phase, in which the baryon number is spontaneously broken by the quark condensate, which also obtains the baryon charge.

Another talk we had on the lattice data at finite baryon density have been given by M.Lombardo. This time it is based on rather complicated algorithm and strong coupling limit, but it is unquenched and includes the determinant as is, with its phase.

A good news is the excitation to non-zero baryonic density happens around the expected threshold value. But the bad news is the excitation starts about 30% lower it and is more smooth compared to what one expects on the basis of the temperature value involved. So, we still see some unusual baryonic states being excited, like some nuclei or nuclear matter with unexpectedly large binding energy. I do not know any explanation of this puzzle.

7 “Dropping masses” in the experiment?

Already at QM93, CERN dilepton experiments have provided first preliminary indications for a signal, significantly exceeding the theoretical expectations (e.g.\cite{38}). Later CERES has found dramatic excess of dileptons at $M_{e^+e^-} < m_\rho$. In spite of multiple attempts by theorists to explain it by “conventional sources”, it was found to be impossible\cite{13}. A list of “unconventional” explanations (more or less in a chronological order) include: (i) dropping $m_\rho$\cite{39,40}; (ii) pion occupation numbers at low momenta are very high\cite{41}; (iii) a very long-lived fireball\cite{38}; (iv) dropping $m_{\eta' }$\cite{11,12}; (v) a

\footnote{In fact Stephanov has found boundaries of this phase and many interesting details about distributions of Dirac eigenvalues at non-zero $\mu$, both in the wrong (no phase) and right (with the phase) ensembles, using a particular model of chiral restoration based on random matrix approach. Unfortunately, we have no time to discuss it here.}

\footnote{In fact, I have never before seen that rather involved calculations by several groups agree so well, as far as the shape of spectra of $M_{e^+e^-}$ is concerned.}
modified pion dispersion curve [42]; (vi) dropping $m_{A_1}$.

I will only discuss the $\rho - A_1$ story, with brief remarks about others. (ii) The observed low-$p_t$ excess of pions (over thermal spectra) can indeed lead to a dramatic increase in low-mass dilepton yield. However, it is most probably due to resonance decays or effects of collective potentials. Both are late-stage phenomena, which hardly affect the early-stage dilepton production. (iii) Even if the long-lived fireball would appear at 200 GeV/A (which is hardly possible) it was found to produce about the same $M_{e^+e^-}$ spectrum as the usual space-time scenarios, so it does not work. (iv) Above we have indeed suggested that $m_{\eta'}(T)$ should drop more than any other mass: however in order to account for CERES data one should increase yield of “escaping $\eta'$” by too huge a factor. (In connection to U(1) restoration issue, however, it would be extremely interesting to measure the $\eta'$ yield, though.)

The “dropping $m_{\rho}$” idea is very well known, and it seems to be about the only one which can explain the dilepton data. It was studied in details by Li,Ko and Brown [40] in the cascade model based on Walecka-type model with attraction mediated by a scalar field. With much simpler hydro-based approach we have also verified, that one can get a very good description of CERES data by making the rho mass T-dependent, without any changes in standard thermal rate formulae or in space-time evolution.

A point I want to make though is that “dropping $m_{\rho}(T)$” are not consistent with chiral symmetry restoration, unless modifications of its chiral partner $a_1$ follow. Relation between the two were made e.g. in the contents of Weinberg-type sum rules [14]. Both states should become identical at $T_c$, so $m_{\rho}(T_c) = m_{A_1}(T_c)$. Important role of $a_1$ for production of photons and dileptons was discussed in [52, 38, 43, 46]; we have used rather general expressions recently derived in [47]. Possible scenarios of how chiral restoration may proceed are shown in the $m_{\rho} - m_{A_1}$ plane in Fig.5(a). For example, path G corresponds to ref.3, while C corresponds to Brown-Rho scaling. The corresponding dilepton spectra are shown in Fig.5(b): the variant D, with $m_{\rho}(T_c) = m_{A_1}(T_c) \sim (1/2)m_{\rho}(0)$, does the best job, and Fig.5(c) we show contribution of separate stages in this scenario. The $A_1$ contribution is mainly at small $M_{e^+e^-}$, where CERES acceptance is low and background.

14 In order to explain why $a_1$ is important, let us go “backward in time”: it is the first hadronic resonance which may be excited in a collision of a photon, real or virtual, with a pion.
rather high. Still, with some effort one may be able to dig out the $A_1$ contribution, using other information (especially the $p_t$ distribution). Note finally, that none of the scenarios mentioned above happen to violate current upper limit on the direct photons are set by WA80 [48].

8 Flow, the “softness” of the EOS, and the fireball lifetime

Our last topic deals with a more straightforward approach to observable signal of the QCD phase transition: instead of hunting for “dropping masses” we look instead at the effect of “dropping pressure”. The ratio $p/\epsilon$, pressure normalized to the energy density, is shown in Fig.6(a) we show how this ratio depends on $\epsilon$. The existence of the “softest point”, the minimum of this ratio, is clearly seen. Our second point: in those (hydro relevant) plot the curves with and without baryon density look similar in many models: compare the two curves in Fig.6.

Effect of the “softness” of EOS at $T \approx T_c$ on transverse radial flow was discussed for a long time [53, 54]. More radical (and more controversial) idea was proposed in [49], where it was suggested that it can affect the longitudinal expansion and thus the global lifetime of the excited system in some energy window. In this workshop we had a detailed talk by D.Rischke, where he has shown that the effect of “softness” of EOS increases the lifetime at RHIC energies by about factor 2.

It was shown in [49] that, due to softness of the EOS, there exists a window of collision energies in which the secondary acceleration of matter is completely impossible. Thus, if stopping does occur, a very long-lived fireball should be formed.

To study this idea semi-quantitatively, a simple 1-fluid relativistic hydrodynamic model was used [10]. Our lattice-based EOS shown (in unusual form) in Fig.6(a) have smooth crossover in a narrow temperature interval $\Delta T \sim 5$

\footnote{The issue was strongly oversimplified: in fact, transverse velocity is the integral over the whole history, and initial “softness” is only part of the story. Our hydro studies have shown, that the mean flow velocity is not sensitive to EOS at early stages, although there are modifications of the shape of its distribution.}

\footnote{The major uncertainties are in the initial conditions: we assumed that half of the total energy is stopped.}
MeV at $T_c = 160$ MeV the energy density jumps by about an order of magnitude.

As the collision energy is scanned down, from 200 A GeV (SPS) to 10 A GeV (AGS), we found three radically different scenarios: (I) At SPS one starts in the QGP phase, therefore longitudinal explosion quickly restore ultra-relativistic longitudinal motion, even if stopping a la Landau takes place, and expansion resembles the Bjorken picture, (see Fig.5 (a)): (II) As the initial energy density passes the softest point, the QGP phase disappears and so does rapid longitudinal expansion. Instead we find a slow-burning fireball. For the heavy nuclei and initial conditions we discuss, the burning front moves mostly in the compressed longitudinal direction (see Fig.6 (b)). The total lifetime of the fireball appears to be nearly as long as predicted for RHIC, but of course it is kind of upper bound on the effect. (III) At still lower energies burning process becomes more spherical, and ordinary expansion is developed. The main result, shown in Fig.6(b), is a significant peak in a lifetime.

Remarkably enough, these completely different scenarios lead to not-so-different spectra! With little fitting of the initial conditions, they can be made consistent with available data. Thus, one-particle spectra alone is not enough, and details about the space-time picture has to come from analysis of flow and HBT.

How one can test these unusual predictions and try to locate the “softest point” experimentally? First of all, it seems that it is above the AGS domain\footnote{Preliminary results from the E802 AGS experiment reported studies of HBT which indicate significant growth of lifetime for the most central Au Au collisions\cite{51}.}, so one should rather scan downward at SPS. The following ideas are discussed in literature: (i) Look for maximal lifetime (or minimum of the HBT parameter $\lambda$)\cite{49}; (ii) Look for the minimum of the “directed” flow in the collision plane\cite{50}; (iii) Look for the nearly isotropic distribution of dileptons, produced in the long-lived fireball\cite{49}.

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Figure 3: Schematic phase diagram of zero T QCD (a) and supersymmetric QCD (b) as a function of the number of colors $N_c$ and the number of flavors $N_f$.

Squares show where lattice calculations have found the infrared fixed point, dots are where the instanton ensemble is purely “molecular”, with the unbroken chiral symmetry.
Figure 4: (a) Temporal correlation function for the pion (divided by the one corresponding to free massless quarks) versus distance (in fm). Solid triangles and squares are for $T = 1., 1.13T_c$, open triangles, squares and hexagons for $T.43, 0.6, 0.86T_c$, respectively. (b) T-dependence of the screening masses (given in units if $\pi T$) calculated from the IILM. Note that at chiral restoration the partners ($\sigma, \pi$), ($\rho h o, a_1$) become identical.
Figure 5: (a) Possible scenarios of chiral restoration in $m_\rho - m_{a_1}$ plane, (b) the dilepton spectra corresponding to them (CERES data are shown with background from hadronic decays subtracted); (c) the most favorable case D is shown in details, with contribution from different stages. The dominant one is clearly the long-dashed one, or hadronic part of the mixed phase.
Figure 6:  
(a) The EOS in hydro-relevant coordinates for resonance gas and QGP. The solid line is for zero baryon number, the dashed line is for $\mu_b = 0.54 GeV$. The minimum is the “softest point” discussed in the text. 
(b) Solid line (and left scale) show the lifetime in the center, the dashed line (and the right scale) shows the space-time volume occupied by the mixed phase ($T \approx T_c$).
Figure 7: Space-time picture of AuAu collisions at SPS energies (a) and at the “softest point” (b). Solid lines correspond to fixed energy density while the short-dash lines are contours of fixed longitudinal velocity. Q,M,H stands for Quark, Mixed and Hadronic phases. Note qualitative difference between the two figures, as well as different time scales.
