Network Topology of the Austrian Airline Flights

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The information of the Austrian airline flights was collected and quantitatively analyzed by the concepts of complex network. It displays some features of small-world networks, namely large clustering coefficient and small average shortest-path length. The degree distributions of the networks reveal power law behavior with exponent value of 2 ~ 3 for the small degree branch but a flat tail for the large degree branch. Similarly, the flight weight distributions show power-law behavior for the small weight branch. Furthermore, we found that the clustering coefficient C, 0.206, of this flight network is greater than that of a random network, 0.01, which has the same numbers of the airports (N) and mean degree ((k)), and the diameter D, 2.383, of the flight network is significantly smaller than the value of the same random network, 18.67. In addition, the degree-degree correlation analysis shows the network has disassortative behavior, i.e. the large airports are likely to link to smaller airports. Furthermore, the clustering coefficient analysis indicates that the large airports reveal the hierarchical organization.

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Network behaviors emerge across many interdisciplinary sciences and attract the interests from many researchers in different research fields. Network is usually a set of items, which we will call vertices or sometimes nodes, with connections between them, called edges. Systems with the form of networks are distributed over the world. Examples include the Internet, the World Wide Web (WWW), social networks of friends, networks of business relations between companies, neutral network, metabolic networks, food webs, distribution networks such as blood vessels or postal delivery routes, networks of citations between papers, networks of paper collaborators, network of publication download frequency, and traffic transportation networks and many others. Even in microscopic scale, such as in nuclear fragmentation produced in heavy ion collisions, the hierarchical power-law distribution of nuclear fragments emerges by Ma’s nuclear Zipf-type plots around the nuclear liquid gas phase transition, which also shows a similar character to the scale free network.

In a pioneering work of Barabasi and Albert, they found that the degree of node of Internet routes, URL (universal resource locator) - linked networks in the WWW satisfies the power-law distribution, also called as the scale-free networks. The degree distribution of a scale-free network is a power law,

\[ P_k \sim k^{-\gamma}, \]

where \( \gamma \) is called the degree exponent. In addition, there are other two main topological structures of complex networks: random-graph models and small-world networks. The research on random graphs was initiated by Erdős and Rényi in the 1950s. It has shown that the degree distribution of a random graph is of Poisson type, \( P(k) = e^{-\langle k \rangle}/k! \). Small-world networks are somehow in between regular networks and random ones.

In this work we study the flight network affiliated with Austrian Airline company to shed light on understanding the real topological structure and inherent laws of flight network design by a specific airline company. Some features of the flight network will be compared with those of the above three categories of networks. To this end, we would like to check the similarities and differences among possibly different networks. Some studies have been performed for the flight networks, such as for international transportation airport networks by the foreign colleagues, as well as the US and China flight networks by Cai’s group. Some interesting features have been demonstrated for such flight networks, such as the small-world property: high clustering coefficient, small diameter and hierarchical structure. However, our present work is different in motivation and results. The previous flight network involved in a whole national or international airport networks, which did not care about the detailed information of the flights which were operated by a specific airline company. These national- or world-wide flight networks are large scale, but they are the result from collective role by the various Airline company networks. Therefore, it is of interesting to survey a particular airline flight network instead of a whole national or international- wide flight network. Based upon this motivation, we will investigate a smaller network which was composed by the flights affiliated with a specific airline in the present work. As an example, we have investigated the flight network of a central European airline company, Austrian Airline. The flight information is available in the web page, http://www.ana.com/

In the flight network, the airports can be represented by the vertices and the flights connecting two airports by edges. In the previous studies, some features of the structure of flight networks have been recognized: (1) the network is directional. All the flights are directed, sorted as outgoing and incoming. (2) the network has weight. To reflect how busy a certain line is, it is important to record the exact number of flights be-
between any given airport \(i\) and \(j\), even to record the seat numbers available in different flights. (3) the network may be a little different day by day in a whole week. Hence, the weekly flight information partially involves the information on evolution of the flight network. Our data contain a whole week information of around \(N \sim 134\) airports and 9560 flights. The detailed numbers of the airports and flights are listed in Table I. For the number of flights, it is the largest on Monday and the smallest on Saturday.

The paper is organized as follows. First we present a sample of the flight network in Friday and its degree distribution. Then we give the results of the flight weight distributions, of the clustering coefficient, of the diameter and of the assortative coefficient, respectively. Finally a summary is given.

The vertex degree distribution function \(P_k\) gives the probability that a randomly selected vertex has exactly \(k\) edges. Figure 1 shows a topological structure of the Austrian airline flight network on Friday, where each airport is expressed by a node and the flights are connected by the lines between two nodes. The Vienna airport is the dominative airport operated by the Austrian airlines, which has naturally the largest amount of edges. There are several major airports, such as Paris, Frankfurt etc, which have frequent flights to connect with other small airports operated by the Austrian Airlines. For comparison, the Erdős and Rényi-type random network structure which has the same vertices \(N = 136\) and mean degree \(\langle k \rangle = 1.31\) is also plotted in the figure. The obvious different topological structure is there.

Three kinds of degree distributions, namely \(P_k(in)\), \(P_k(out)\) and \(P_k(all)\) are shown in Figure 2. \(P_k(in)\) and \(P_k(out)\) represent the frequencies of incoming and outgoing of flights, respectively. \(P_k(all)\) is used when we do not distinguish outgoing and incoming flights, i.e. it is just the degree number which is regardless whether the flight is outgoing, incoming, or both of them. Note that the present degree distribution is not cumulative distribution as done in Ref. [5, 6]. Even though the statistical fluctuation could keep large in degree distribution in comparison with the cumulative distribution, the distribution can give the direct probability how many Austrian Airline flights are coming or taking off. Two branches are seen in Figures 2(a) and 2(b): the first one follows the power-law \(P_k \sim k^{-\gamma}\) when \(k < 7\) and the second one is the flat tail distribution when \(k \geq 7\), which is basically related to some largest airports which serve for Austrian Airlines. This behavior can be partially attributed to different mechanisms between small airports and large airports. For an example, they have different growth rates since the construction of small airports or the flight line extension to small airports by the Austrian Airlines is much easier and faster than that of large airports. In the following, we can extract the exponents of the degree distribution for small airports which Austrian Airline flights cover. When \(k < 7\), the mean weekly value of \(\gamma_{in}\), \(\gamma_{out}\) and \(\gamma_{all}\) correspond to 2.61, 2.63 and 2.47; Exponents in each day in Figure 2 are listed in Table I. The average degree of the flight network is given by \(\langle k \rangle = \frac{1}{N} \sum_i k_i\). The average \(\langle k \rangle_{all} = 1.30\). That means each airport is linked to 1.3 other airports for the flights affiliated with Austrian Airlines. Similarly, \(\langle k_{in} \rangle = 1.279\) and \(\langle k_{out} \rangle = 1.277\). In details, \(\langle k \rangle\) on each day are listed in Table I.
As shown in Figure 1, the topological structure of flight network is significantly different from those of random graphs. In a random graph of the type studied by Erdős and Rényi, each edge is present or absent with equal probability, and hence the degree distribution is binomial or Poisson distribution in the limit of large graph size. Real-world networks are mostly found to be very unlike the random graph in their degree distributions. The degrees of the vertices in most network are highly right-skewed. This is the case of the present flight network. From the exponents $\gamma$ of different days in a week as shown in Table I, we can find that exponents $\gamma_{in,out}$ and $\gamma_{all}$ on Saturday are the largest and on Monday are basically the smallest. Similarly, the mean degrees of flights on Saturday and Monday are significantly different: it is the smallest on Saturday and the largest on Monday, which is in consistent with the largest value of $\gamma$ on Saturday and the smallest on Monday. This is also not contradicted with the difference of total day-flight number between Saturday and Monday as shown in the same table. In other words, Monday is the busiest flight transportation day and Saturday is the most unoccupied flight transportation day for the Austrian Airline. This can be partially related to the behavior of human business travel.

Since the flight network involves in transportation flux, the weight is important and can reflect some information of the whole network. As shown in Fig. 3, the flight weight distribution in a week has a power-law distribution in the small weight branch,

$$P_n \sim n^{-\gamma_{flight}},$$

where $n$ is the exact number of flights between any given airport $i$ and $j$. The outgoing network exponents, $\gamma_{flight-out}$, of different days in a week are shown in Table I. The mean exponent of a week is 1.33. Again, there is a different value for working days and weekend. The exponent $\gamma_{flight-out}$ is around 1.2 in working days but it shifts dramatically to larger values on Saturday and Sunday (see Table I). Again, the value of Saturday is the largest and the one of Monday is the smallest. The larger $\gamma_{flight-out}$ means the steep slope, which results in
the smaller mean weight. Therefore, the values of larger $\gamma_{\text{flight-out}}$ can be attributed to the declining flight number on weekends.

To have a visual feeling of the Table, we take the outgoing flight network as an example to make some correlation plots which are shown in Fig. 4. Each point represents the data of a day in one week. Fig. 4(a) shows the correlation of total flight numbers ($M_{\text{flight}}$) and the power-law exponents of the weight outgoing flight distributions ($\gamma_{\text{flight-out}}$); (b) the relationship between $\langle k_{\text{out}} \rangle$ and $\gamma_{\text{out}}$; (c) the correlation between $\langle k_{\text{in}} \rangle$ and $\gamma_{\text{in}}$; (d) the relationship between $\langle k_{\text{out}} \rangle$ and $\gamma_{\text{flight-out}}$; (e) the correlation between $\gamma_{\text{in}}$ and $\gamma_{\text{out}}$; (f) the correlation between $\gamma_{\text{out}}$ and $\gamma_{\text{flight-out}}$.

The solid lines represent the linear fits and dashed lines for exponential decay fits. Each point represents a day in a week (the

TABLE I: Comparison of relevant variables: (1) numbers of the airports $N$; (2) the numbers of flights $M$; (3) $\gamma_k$, which represents the exponent of first segment of degree distribution; (4) $\langle k \rangle$, the average degree; (5) $\gamma_{\text{flight}}$, the exponents of weight flight distribution; (6) the clustering coefficient $C$ of the system; (7) the assortative coefficient.

| $N_{\text{airport}}$ | Mon | Tue | Wed | Thu | Fri | Sat | Sun |
|----------------------|-----|-----|-----|-----|-----|-----|-----|
| $M_{\text{flight}}$  | 1518| 1421| 1418| 1296| 1471| 1158| 1275|
| $\gamma_{\text{in}}$ | 2.319| 2.476| 2.399| 2.507| 2.312| 2.622| 2.443|
| $\gamma_{\text{out}}$| 2.319| 2.509| 2.410| 2.506| 2.413| 2.653| 2.424|
| $\gamma_{\text{all}}$| 2.331| 2.319| 2.478| 2.519| 2.495| 2.649| 2.457|
| $\langle k_{\text{in}} \rangle$ | 1.587| 1.248| 1.256| 1.221| 1.221| 1.206| 1.212|
| $\langle k_{\text{out}} \rangle$ | 1.576| 1.230| 1.271| 1.215| 1.281| 1.108| 1.256|
| $\gamma_{\text{flight-out}}$ | 1.120| 1.234| 1.218| 1.224| 1.258| 1.724| 1.543|
| $C$                  | 0.202| 0.204| 0.195| 0.206| 0.242| 0.180| 0.210|
| $r$                  | -0.529| -0.515| -0.519| -0.517| -0.517| -0.562| -0.543|
Quantitatively, this inherent tendency to cluster can be expressed by the clustering coefficient \( C \). For a selected vertex \( i \) of the network, it has \( k_i \) edges which we call the nearest neighbours of \( i \). In this case, the maximum possible edges among \( k_i \) neighbours are \( k_i(k_i - 1) = 2 \). If we use \( N_{\text{real}} \) to denote the number of edges that actually exist, the clustering coefficient of vertex \( i \) can be written as

\[
C_k = \frac{N_{\text{real}}}{k_i(k_i - 1)/2}
\]

and the clustering coefficient of the entire network is defined as \( C = \frac{1}{N} \sum_i C_k \). The clustering coefficient \( C \) of the Austrian Airline flight network in a week is 0.206. We also calculate \( C \) on each day (see Table 1). To look for the difference, we complete \( C \) for our flight network with that of a random graph which has the same \( N \) and \( \langle k \rangle \). In such a random graph, the clustering coefficient is \( C_{\text{rand}} = \langle k \rangle / N = p = 0.01 \) where \( p \) is the connection probability. Thus, \( C \) in our flight network is much larger than that in a random graph. Fig. 5 shows the scattering plots of the clustering coefficient for un-directional flight network of each day in a week as a function of the vertex degree. Similar to the degree distribution (Fig. 2), there are two segments: a nearly flat distribution for small \( k \) \((<7)\) and a power-law decay with the exponent \( \sim 1.7 \) for \( k \geq 7 \). The small-\( k \) branch corresponds to the majority of airports with a few links to other airports, each such airport \( i \) has a clustering coefficient close to 1. The high-\( k \) airports include many large airports, and thus, their neighbors are not necessarily linked to each other, resulting in a smaller \( C_k \). A power-law decay of high-\( k \) branch indicates that a hierarchical organization \([28]\) for large airports, in contrast to the \( k \) independent \( C_k \) predicted by the scale-free networks as in small-\( k \) branch.

The average shortest-path length between any two airports in the system can be characterized by so-called ”diameter” in small-world networks \([20]\), which is defined as

\[
D = \frac{1}{N(N-1)} \sum_{i \neq j} d_{ij},
\]

where \( d_{ij} \) is the minimum number of edges traversed from vertex \( i \) to vertex \( j \). The diameter of the flight network reflects the average number of least possible connections between any two airports. The corresponding probability distribution of the shortest path lengths of the whole flight network, namely 1, 2, 3, 4 and 5, is shown in Fig. 6 i.e. numerically 0.018, 0.641, 0.278, 0.050 and 0.013, respectively. The line is just fourth order polynomial fit. This implies that from airport \( i \) to \( j \), there will be basically not more than three connections (the shortest-path length of 1 means a direct flight) where the probability is smaller than 10\%. The diameter of our flight network is \( D = 2.383 \), which means that on the average there will be 1.383 connections from airport \( i \) to \( j \). Using the same approach, we compare the \( D \) of our flight network with that of the random graph. The diameter of the random graph is \( D_{\text{rand}} = \ln(N)/\ln(\langle k \rangle) = 18.67 \) \([23]\). In other word, the diameter of our flight network is significantly smaller than the one of the random graph with the same nodes and mean degree.

Many networks show assortative mixing on their degrees, i.e., a preference for high-degree vertices to attach to other high-degree vertices, while others show disassortative mixing—high-degree vertices attach to low-degree ones. Quantitatively, the degree-degree correlation coefficient (also called assortative coefficient) can be written as

\[
r = \frac{\frac{1}{M} \sum_{i \neq j} j_i k_i - \frac{1}{N} \sum_i j_i \sum_{j \neq i} k_j}{\frac{1}{N} \sum_{i \neq j} j_i k_i + \frac{1}{N} \sum_i j_i \sum_{j \neq i} j_i}.
\]

where \( j_i \) and \( k_i \) are the degrees of the vertices at the ends of the \( i \)th edge, with \( i = 1, \ldots, M \). As Newman showed, the values of \( r \) of the social networks have significant assortative mixing. By contrast, the technological and biological networks are all disassortative \([29]\). In this work, we also check the coefficient \( r \) and we list those values of each day in Table 1. As we expected the values are all negative, which means the flight network is disassortative. In other word, the large airports are likely to link
to smaller airports. This fact is in agreement with many technological and biological networks [29]. The value of Saturday shows the largest.

In summary, our analysis demonstrates that the Austrian Airline flight network displays the small-world property: high clustering coefficient and small diameter. The clustering coefficient $C \approx 0.206$ is greatly larger than that of a random network (0.01) with the same $N$ and $\langle k \rangle$ while the diameter $D \approx 2.383$ of the flight network is significantly smaller than the value of the same random network (18.67). The degree distributions for smaller airports show the power-law behavior with an exponent 2.47 for undirected flight networks. Also the flight weight distributions have power-law distributions with exponent of approximately 1.33. Further, the network shows disassor-
tative behavior which indicates that the large airports are likely to link to smaller airports. In addition, the power-law decay behavior of the clustering coefficient against the degree for high-$k$ nodes reflects that the large airports reveal the hierarchical organization. In a whole week, the power-law exponents of the degree distribution and the flight weight distribution show different values day-by-day, especially between Monday and Saturday. The smallest exponent for Monday corresponds to the busiest flight transportation day and the largest exponent on Saturday corresponds to the most unoccupied flight transportation day for the Austrian Airline.

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