Finally, experimental results are discussed. The theoretical framework of SU(1,1) atom interferometry has been realised with microwave circuit QED demonstrated in optics. This year marks the 125th anniversary of the Mach–Zehnder interferometer [1]. When transferring the basic idea of two path interference from light to massive particles, Feynman famously concluded that this is a ‘phenomenon which has in it the heart of quantum mechanics [2].’ Nowadays, the atomic analogue of Mach–Zehnder interferometry is an indispensable instrument in fundamental research as well as technology [3, 4]. Its prime example is the atomic clock [5].

These routinely employed interferometers use passive beam splitters during which a conserved number of particles is redistributed among two modes. In contrast, an active interferometer uses amplifying elements for beam splitting and recombination. Consequently, the total particle number in the corresponding interferometry modes becomes subject to change. Such an interferometer is described within the framework of the SU(1,1) group [6]. Due to the intrinsic amplification they provide benefits for detecting phase shifts of weak signals. Furthermore, the exploitation of entanglement [7] generated during quantum amplification allows reaching phase sensitivities better than those of classical passive interferometers [8].

Originally devised for optics [6], the essential idea of using two nonlinear processes with phase accumulation in-between has been realised with microwave circuit QED [9], spinor Bose–Einstein condensates [10], nonlinear optical fibres [11], Raman scattering [12] and four-wave mixing [13] in vapour gases. Improved phase sensitivities have been demonstrated in optics [14] and with atoms [8].

The centrepiece of SU(1,1) interferometry is the process of parametric amplification (PA) which is used as an active beam splitter. We devote the first part of this manuscript to detail its experimental implementation. We then present a theoretical framework of SU(1,1) transformations while contrasting it to the well known passive SU(2) interferometer. Finally, experimental results are discussed. The findings reported on in this manuscript build on the work of [8].

2. Experimental system: spinor Bose–Einstein condensate

Bose–Einstein condensates are a well established system to study nonlinear spin dynamics [15], interferometry [3], sensing [4], quantum metrology [16], and quantum information [17].

We experimentally realise the SU(1,1) transformations within the spin degree of freedom of a 87Rb Bose–Einstein condensate. For this, the atoms are tightly trapped in an optical standing wave potential...
such that their external degrees of freedom are frozen out. In this single-spatial mode, dynamics is restricted to the spin degree of freedom.

For PA we employ spin exchange [15] among three hyperfine levels. For this purpose a large population of \( N_f \approx 500 \) atoms in \( n_F = 0 \) acts as a particle reservoir (pump mode). Mediated by coherent collisions of two pump atoms, the initially empty modes \( m_F = \pm 1 \) get populated in a pairwise manner (figure 1(a)) such that the total \( n_F \) remains conserved. This process is described by the Hamiltonian \( \hat{H} = \hbar \kappa \left( \hat{a}^\dagger \hat{a}^\dagger + \text{h.c.} \right) \). Here \( \hat{a}^\dagger \) \( (\hat{a}) \) are the creation operators for mode \( \uparrow \) \( (\downarrow) \) \( (\uparrow\downarrow) \) and \( 2\pi \hbar \) denotes Planck’s constant. The effective nonlinear coupling strength \( \kappa = gN_f \) is enhanced by the number of pump atoms and arises microscopically as a result of different s-wave collision channels [18] which is parametrised by the scattering length difference \( \Delta a \). Then \( g \propto \Delta a \int d^3\Psi(\mathbf{x})^4 \) where \( \Psi(\mathbf{x}) \) is the condensates spatial mode function. This Hamiltonian remains valid as long as the pump mode acts as an unlimited reservoir and therefore excludes all other mode populations by far. In this undepleted pump approximation, both, the sign and the magnitude of the nonlinear coupling strength \( \kappa \) can be controlled via the pump mode: while its population defines the nonlinearity’s magnitude, the phase of the pump mode controls the phase of \( \kappa \). In practice, we transfer the pump atoms into an ancilla spin state that does not participate in PA to change the nonlinearity’s magnitude. We also use such state transfers to imprint a dynamical phase onto the pump mode.

Atomic populations of the individual spin components are detected via absorption imaging (figure 1(a)) with an uncertainty (one s.d.) of 4 atoms [19]. All lattice sites are independent with respect to the spin dynamics and used to increase the statistical sample size. For quantitative analysis we postselect experimental runs with 450–500 atoms per lattice site such that the nonlinearity \( \kappa \) is fixed on the 10% level. To accurately estimate mean values and their variances we repeat the experiment typically a few hundred times.

All experiments start with empty modes \( \uparrow \) \( (\downarrow) \) \( (\uparrow\downarrow) \) (vacuum state). The characteristic mode correlations subsequently generated by PA become manifest in figure 1(b). Here, the single-shot spin populations line up on the diagonal \( N_\uparrow = N_\downarrow \). Only small deviations from this line occur and are mainly caused by detection infidelities. This becomes apparent when comparing this width to the measured initial vacuum state (grey), whose isotropic extension is exclusively due to detection noise. Along the diagonal, however, the quantum state shows excess noise, i.e. fluctuations in total atom number \( N_\uparrow + N_\downarrow \). The population of the modes shows distinctive skewed distributions as witnessed by the histograms. The solid line is a fit to the theoretical expectation of a thermal-like distribution which also takes into account detection noise by Gaussian convolution.

A state with similar mean atom number produced by a passive beam splitter is shown in panel (c) (green). Here a resonant three-level rf-pulse couples the pump mode to \( \uparrow \) \( (\downarrow) \) \( (\uparrow\downarrow) \), which thereby populates these modes. The resulting state is much more concentrated around its mean atom number and its fluctuations are isotropic. In this case, the state’s extension is caused by both, detection noise and atomic shot noise as a consequence of Poissonian statistics. The fluctuations orthogonal to the diagonal, i.e. in particle number difference \( N_\uparrow - N_\downarrow \), are therefore enlarged compared to PA. This becomes even more pronounced when considering a state with larger mean atom number (purple).

Figure 1. Experimental realisation of parametric amplification. (a) We use spin mixing among three hyperfine levels in a spinor Bose-Einstein condensate. The atoms are trapped in a one-dimensional standing wave potential with additional transverse harmonic confinement. Atom numbers of the three spin states are obtained by state and lattice site resolving absorption imaging. A typical absorption image with counting regions indicated by ellipses is shown. (b) Atom number correlations of the two modes \( \uparrow \) \( (\downarrow) \) \( (\uparrow\downarrow) \) as a result of parametric amplification. The spread orthogonal to the diagonal is dominated by detection noise. The initial vacuum state is shown in grey. The mode populations after parametric amplification resemble a thermal-like state as shown by the respective histogram. (c) Atom number correlations after passive beam splitting. The green points depict a state with similar mean atom number to the state produced by parametric amplification. The mode populations are Gaussian as indicated by the histograms. Their width is a combination of detection noise and atomic shot noise. The contribution of the latter dominates for state with larger mean atom number (purple).
3. Theoretical description

We now discuss the interferometric sequence for both, active and passive beam splitting. For this we consider the two modes $|\uparrow\rangle$ and $|\downarrow\rangle$ to be populated by indistinguishable bosons. Passive interferometers can be described in a pictorial manner by resorting to the Bloch–sphere representation. In a similar fashion the sequence of an active interferometer can be described on a series of hyperbolic cones. Here, we will review both representations.

3.1. Active SU(1,1) interferometer

To describe the active interferometer and PA we introduce three operators [6]:

$$\hat{K}_z = \frac{1}{2}(\hat{a}_+^\dagger \hat{a}_- + \hat{a}_-^\dagger \hat{a}_+ + 1) = (\hat{N}_1 + \hat{N}_2 + 1)/2$$

encodes the number of atoms shared in both modes,

$$\hat{K}_x = \frac{1}{2}(\hat{a}_+^\dagger \hat{a}_- + \hat{a}_+ \hat{a}_-)$$

and

$$\hat{K}_y = \frac{1}{2i}(\hat{a}_-^\dagger \hat{a}_+ + \hat{a}_+ \hat{a}_-)$$

describes the coherent and pairwise creation and destruction of particles. These three operators belong to the SU(1,1) group description. As a prominent example, Ramsey’s method of separate oscillatory fields falls into this class. This is the atomic analogue of the Mach–Zehnder interferometer known in optics (schematically drawn in figure 2(d)) [23].

We introduce pseudo-spin operators whose $z$-direction is given by the atomic population difference,

$$\hat{J}_z = (\hat{a}_+^\dagger \hat{a}_- - \hat{a}_-^\dagger \hat{a}_+)/2 = (\hat{N}_1 - \hat{N}_2)/2.$$  

The corresponding coherences, i.e., $x$- and $y$-direction of the collective spin are given by

$$\hat{J}_x = (\hat{a}_+^\dagger \hat{a}_+ + \hat{a}_-^\dagger \hat{a}_-)$$

and

$$\hat{J}_y = (\hat{a}_+^\dagger \hat{a}_- - \hat{a}_-^\dagger \hat{a}_+)/2i.$$
These spin operators satisfy the defining commutator relation of the SU(2) algebra, i.e. $[\hat{J}_x, \hat{J}_z] = i\hat{J}_y$ and cyclic permutations thereof.

The conserved total number of particles $N$ is encoded in the fixed spin-length $\hat{J}_z^2 = \hat{J}_x^2 + \hat{J}_y^2 + \hat{J}_z^2$ which defines the surface of a sphere. As the angular momentum operators are generators of rotations, the action of these can be visualised on a generalised Bloch-sphere.

The interferometric sequence of such an SU(2) interferometer is built up by three rotations (shown in figure 2(d) from top to bottom). Starting with all atoms in state $\ket{1}$ (south pole of the Bloch sphere), a 90° rotation about the $y$-axis is performed to generate a phase sensitive linear superposition of $\ket{1}$ and $\ket{0}$. In this state the collective spin is aligned in the equatorial plane. Subsequent phase accumulation of $\varphi_1 - \varphi_2$ is described by a rotation about the $z$-axis. Finally, a readout rotation of 90° about the $y$-axis maps the accumulated phase onto the $J_x$ axis. By measuring the imbalance $2(\langle \hat{J}_x \rangle/N = \langle N_1 - N_0 \rangle/N = \cos(\varphi_1 - \varphi_2)$ the phase can be inferred (top, figure 2(c)). The state inside the interferometer has a vanishing imbalance (horizontal red line).

Similar to the SU(1,1) case, rotations about different axes can be implemented by phase changes $\varphi_{\text{rel}}$ of the linear coupling. When the first pulse rotates about the $y$-axis this phase change amounts to the substitution of $\hat{J}_y \rightarrow \cos \varphi_{\text{rel}} \hat{J}_y + \sin \varphi_{\text{rel}} \hat{J}_x$ for the final rotation.

### 3.3. Coherent states: fluctuations

The SU(1,1) coherent states, i.e. those states which are generated by linear combinations of the $\hat{K}$ operators acting onto vacuum, are given by $|r, \varphi\rangle = e^{-i\varphi \hat{K}_z} e^{-ir \hat{K}_x} |0\rangle |0\rangle$. This can be understood as PA of vacuum fluctuations [24,26]. These states satisfy $\langle \hat{K}_x \rangle = \cosh(2r)/2$. In terms of Fock states they are represented by

$$|r, \varphi\rangle = \frac{1}{\tanh(r)} \sum_{n=0}^{\infty} e^{in\varphi} \cosh(n) |n_1\rangle |n_0\rangle.$$  

This coherent superposition of twin-Fock states $|n_1\rangle |n_0\rangle$ is a highly entangled state as witnessed by this Schmidt decomposition. In quantum optics this state, known as two-mode squeezed vacuum, is the archetype of an entangled state and is routinely generated by parametric down conversion [17]. The individual modes do not have a mean field, i.e. $\langle n \mid \hat{a}_l \mid n \rangle = 0$ and similarly for mode $\ket{1}$ [27]. Therefore, the mode’s fluctuations become crucial: these characteristic fluctuations are thermal, meaning that compared to the mean mode populations, $\langle N_1 + N_0 \rangle = \langle N \rangle$ the variance reads $(\Delta N)^2 = \langle N(N-1) \rangle / \langle N \rangle$.

The SU(2) coherent states are generated by the $\hat{J}$ operators acting onto the lowest lying state: $|\theta, \varphi\rangle = e^{-i\theta \hat{J}_y} e^{-i\varphi \hat{J}_z} |0\rangle |N_0\rangle$. With this operation each atom is independently put into the same superposition state: $|\theta, \varphi\rangle = (\cos\theta/2 |1\rangle + e^{i\varphi} \sin\theta/2 |1\rangle) \otimes |N\rangle$. When measuring, each of the $N$ atoms is independently
distributed among the two modes like in a Bernoulli trial. These states are therefore not particle entangled.

Defining the success probability \( p = (\cos \theta + 1)/2 \) a possible representation in Fock states reads

\[
|\theta, \varphi\rangle = \sum_{n=0}^{N} e^{i\varphi} \sqrt{\binom{N}{n}} p^n (1 - p)^{N-n} |n\rangle_1 |N - n\rangle_2.
\]  

(7)

These SU(2) coherent states feature binomial fluctuations in population imbalance, i.e.

\[
(\Delta(N_1 - N_2))^2 = 4p(1 - p) \langle N \rangle,
\]

as well as individual mode populations. For equal populations of both modes we have \( (\Delta(N_1 - N_2))^2 = \langle N \rangle \).

This stark difference between binomial and thermal-like mode fluctuations is the subject of figure 1.

### 3.4. Phase dependence

The two classes of interferometers are sensitive to different phases. This is a consequence of the contrarian conservation laws: a fluctuating total number of particles is necessary to define the sum phase \( \phi = \phi_1 + \phi_2 \). In turn, a fluctuating number imbalances is needed to define the difference phase \( \phi = \phi_1 - \phi_2 \). Consequently, the SU(1,1) interferometer is sensitive to the sum phase while the difference phase is read out in the SU(2) interferometer.

All interferometers compare the phase accumulated during interrogation to the phase reference \( \phi_{\text{ref}} \) provided by the respective beam splitting mechanism. In a more complete picture of SU(1,1) interferometry, the relative phase shifts of PA (i.e. changes in boost direction) and the phase acquired during interrogation \( \phi_2 \) are combined. The output fringe is then given by \( \propto 1 + \cos(\phi_{\text{ref}} - \phi) \). Similarly, SU(2) interferometry yields a fringe \( \propto \cos(\phi_{\text{ref}} - \phi) \).

### 3.5. Hamiltonian in \( \hat{K} \)-representation

Within the undepleted pump approximation our experimental system of a quasi spin-1 condensate is described by the Hamiltonian \( \hat{H} = 2\hbar \kappa \hat{K}_x + 2\hbar \delta \hat{K}_z \). The detuning term \( \delta = \kappa + qB^2 + \delta_{\text{MW}} \) contains contributions of collisional shifts (\( \langle \kappa \rangle \)), Zeeman energy shifts (\( qB^2 \)) and level shifts due to microwave dressing (\( \delta_{\text{MW}} \)). As magnetisation conserving processes, all SU(1,1) transformations are unaffected by Zeeman energy shifts linear in magnetic field strength \( B \). Only its quadratic contribution constitutes a detuning term \( qB^2 \hat{K}_z \) with \( q \approx 2\pi \times 72 \text{ Hz G}^{-2} \). To achieve resonance \( \delta \approx 0 \) we use dispersive microwave dressing mainly affecting the pump mode [28].

By switching the phase of \( \kappa \rightarrow \kappa e^{i\phi_1} \), the second boost acts along a different direction as described by the Hamiltonian \( \hat{H} = 2\hbar \kappa (\cos \phi_{\text{ref}} \hat{K}_x + \sin \phi_{\text{ref}} \hat{K}_y) + 2\hbar \delta \hat{K}_z \). Experimentally, the phase \( \phi_{\text{ref}} \) of \( \kappa \) is controlled by phase imprints onto the pump mode. Changing the pump phase by \( \phi_0 \) yields twice the phase for \( \phi_{\text{ref}} = 2\phi_0 \), which is a consequence of the pairwise scattering during PA.

### 4. Experimental results

#### 4.1. Experimental sequence

We start with a Bose–Einstein condensate prepared in the \( |F, m_F\rangle = |1, 0\rangle \) state at a magnetic bias field of \( B \approx 0.9 \text{ G} \). To initialise the spin mixing we transfer all atoms to \( |2, 0\rangle \) by a microwave \( \pi \)-pulse. Thereby, the nonlinearity for PA is rapidly enhanced to \( \kappa \rightarrow 2\pi \times 20 \text{ Hz} \). Subsequently, we use dispersive microwave dressing to fulfil the energy resonance \( 2 \times |2, 0\rangle \leftrightarrow |2, 1\rangle + |2, -1\rangle \), i.e. \( \delta \approx 0 \). This realises a resonant SU(1,1) boost along the \( x \)-direction. Since the states \( |2, \pm 2\rangle \) are largely detuned due to the second order Zeeman shift \( 2\pi \times 180 \text{ Hz} \) and the associated coupling strength is weaker, the boost does not affect these states. To interrupt the boost at a particular point in time we stop applying microwave dressing and immediately transfer the pump atoms back to \( |1, 0\rangle \).

In other experiments spin exchange is often performed within the \( F = 1 \) manifold. Our reasons to use an effective three level system within the \( F = 2 \) manifold instead are twofold; first, the coupling strength for spin mixing in \( F = 2 \) is one order of magnitude larger than in \( F = 1 \). This allows for more robust and faster spin dynamics.

Secondly and more importantly, being able to quickly transfer the pump atoms into the \( F = 1 \) state allows us to effectively stop the nonlinear dynamics. Since the detuning change by deactivating the microwave dressing is too small, residual off-resonant spin dynamics is still present. Therefore, the transfer of the pump mode into a state that does not participate in spin exchange is crucial for fast and precise control over the nonlinear coupling. With the pump mode transferred the population in \( F = 2 \) is frozen. This is a result of the then diminished coupling strength for spin exchange and the correspondingly large detuning.

When the pump mode is in \( F = 1 \), spin dynamics is inhibited because the detuning \( \delta \approx 2\pi \times 60 \text{ Hz} \) is much larger than the relevant coupling strength for spin mixing within the \( F = 1 \) manifold. Therefore the modes
remains empty at all times and cannot act as a seed to classically speed-up spin exchange during the interferometric sequence. The second microwave pulse rotates the state back to \( |t\rangle \); \( |\pi\rangle \) is imprinted as detailed above. After this period, \( K_x \) and a geometric phase that is independent of the hold time. As we scan the phase via the hold time \( t_{\text{hold}} \), this phase imprint is composed of the dynamical phase \( \delta_{\text{MW}} \) after the first boost and microwave dressing is reapplied to realise the final state, both in \( |1, 1\rangle \) and \( |2, 0\rangle \), respectively. Then the first \( \pi \)-pulse (being only slightly detuned) rotates the state (almost completely) from the north to the south pole. During the subsequent hold time the state rotates about the \( z \)-axis at a rate given by the detuning of the microwave pulse, \( \delta_{\text{MW}} \). The second microwave pulse rotates the state back to the north pole. Both \( \pi \)-pulses perform rotations about an identical axis on the Bloch-sphere. As a result, the initial and final state, both in \( |2, 0\rangle \) differ by a phase imprint. This phase imprint is composed of the dynamical phase \( \delta_{\text{MW}} t_{\text{hold}} / \hbar \) and a geometric phase that is independent of the hold time. As we scan the phase via the hold time the geometric phase merely leads to a fixed phase offset. Experimentally we use holding times in \( F = 1 \) of a few milliseconds, see figure 6.

We first consider a single period of PA. While boosting from the vacuum state, the mean population grows nonlinearly described by \( \langle N_1 + N_2 \rangle \equiv \langle N \rangle = 2 \langle \hat{K}_z(t) \rangle - 1 = 2 \sinh^2(\pi t) \) (black line in figure 3(a)). Experimentally, we find good agreement for evolution times ranging up to \( \approx 15 \) ms (grey points). For larger evolution times pump depletion successively reduces the coupling strength and thereby limits further growth.

Additionally to this single boost, time traces of \( \langle N \rangle \) during the entire interferometric sequence are shown in colour (figure 3(a)). Here, the first boost is stopped at \( t_1 = 8 \) ms. By varying the holding time \( t_{\text{hold}} \) of the pump atoms in \( F = 1 \) (not shown in figure 3(a)) a dynamical phase shift \( \varphi_{\text{ref}} \) is imprinted as detailed above. After this holding time the pump atoms are transferred back to \( |2, 0\rangle \) and microwave dressing is reapplied to realise the final SU(1,1) readout boost.

For phases \( \varphi_{\text{ref}} \approx \pi \) the two boosts act in opposite direction such that the population dynamics is reversed (red trace). For phase \( \varphi_{\text{ref}} \approx 0 \) both boosts are in similar direction which is equal to the single resonant boost (grey). In the symmetric case of both boosts having the same duration \( t_1 = t_2 \) (highlighted by the dashed box) the phase dependent fringe is recovered (figure 3(b)) as indicated by the common plot markers. The solid line is a sinusoidal fit. Remarkably, compared to the average atom number \( n \) inside the interferometer (indicated by the horizontal grey line) the size of the output fringe is nonlinearly enhanced, meaning that the fringe maximum is at \( 2n(n + 2) \).

### 4.2. Fringe enhancement and noise suppression

The fringe enhancement is investigated by symmetrically altering the duration of both SU(1,1) boosts between 6–8 ms as shown in figure 4. Increasing the evolution times we find dramatically magnified output fringes.

![Figure 3](image-url)
Ideally, at the minimum perfect reversibility to vacuum is expected for all evolution times (within the undepleted pump approximation). Experimentally, however, we find a residual average population of ≈0.5–1.5 atoms per mode (see inset into dashed area) which is most likely caused by a non-perfect matching of both SU(1,1) boosts (see below). For the shortest evolution time the reversibility to vacuum is best.

The measured fluctuations $(\Delta N)^2$ of the phase dependent output signal are shown in figure 4(b). The excess variance is distinctive for the SU(1,1) coherent states. Its theory prediction, $(\Delta N)^2 = \langle N \rangle (\langle N \rangle + 2)$ is indicated by the solid lines. These predictions take as input only the fitted mean fringe of panel a. For the largest evolution time the deviations at the maxima (dashed) result from pump depletion. As a function of phase $\varphi_{\text{ref}}$, the relationship between mean fringe and the corresponding variance leads to a non-sinusoidal dependence; the variance fringe consists of two Fourier components $\propto \cos \varphi_{\text{ref}}$ and $\propto \cos^2 \varphi_{\text{ref}}$ yielding a flattened behaviour in vicinity of the minimum (inset into dashed area) [8]. The contribution of detection noise is independently measured (≈33 atoms$^2$) and is subtracted.

### 4.3. Phase sensitivity

The interplay between fringe amplification and suppressed fluctuations at the minimum (dark fringe) allows for precise phase measurements. The phase sensitivity of the interferometer can be extracted by Gaussian error propagation on the mean atom number, $(\Delta \varphi)^2 = \frac{(\Delta N)^2}{\langle N \rangle}$ [6, 29]. To compare the phase sensitivity of this SU(1,1) interferometer with the typical reference of passive SU(2) interferometer, one has to define a common measure of the resource. Phase sensitivity is usually expressed in terms of the phase sensing mean atom number $\langle N \rangle$ [7, 30]; therefore, only the atoms experiencing the phase shift $\varphi_a$ are counted as an expensive resource. Classical probe states allow measuring at best at the so-called Standard Quantum Limit, i.e. $(\Delta \varphi)^2 = 1/\langle N \rangle$ [7]. The SU(1,1) interferometer allows surpassing this limit by exploiting the entangled SU(1,1) coherent states as demonstrated in [8, 14]. Best phase sensitivity is found in close vicinity to $\varphi = \pi$. Ideally, at the dark fringe a phase sensitivity at the ultimate Heisenberg limit is predicted, $(\Delta \varphi)^2 = (\langle N \rangle (\langle N \rangle + 2))^{-1}$.

The amplification of the readout boost magnifies the signal, thereby generating larger slopes of the phase sensitive output signal $d\langle N \rangle/d\varphi$. However, posteriori amplification of classical states cannot enhance phase sensitivity since it does not differentiate between signal and noise and treats both likewise [31]. In contrast, the SU(1,1) interferometer features an improved signal-to-noise ratio because it harnesses both, the entanglement present at the probe stage generated by the first boost, as well as the second further (de-)amplifying boost [29]. Best phase sensitivities are reached for matched durations of both boosts [8].

### 4.4. Non-balanced interferometry

The improved phase sensitivity relies crucially on the entanglement-enabled suppression of fluctuations [13, 32] by the second SU(1,1) boost. To investigate this, we keep the first boost’s duration fixed at $t_1 = 8$ ms and vary the length of the readout boost. Perfect reversibility is only expected in the symmetric situation when both boosts have equal duration such that complete cancellation is reached at phase $\pi$. Figure 5(a) shows the fringes of $\langle N \rangle$ for unsymmetrical evolution times of $t_1 = 8$ ms and the second boost ranging between $t_2 = 4$–10 ms. By omitting the second boost entirely we probe the state inside the interferometer (shown in red) which shows no phase dependence as expected. Based on this state, the final boost either amplifies or absorbs the population in

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**Figure 4.** Fringe enhancement and noise suppression. (a) Output fringes for the symmetric case of both SU(1,1) boosts having the same duration (indicated by colour). The atom number is nonlinearly amplified during the second SU(1,1) boost yielding strongly increased fringe heights. Solid lines are sinusoidal fits. Suppression at the minimum (inset) works best for short evolution times. (b) The variance shows a non-sinusoidal phase dependence with a flattened behaviour around the minimum at phase $\pi$ (inset). Solid lines are theory predictions within the undepleted pump approximation. For long evolution times pump depletion limits the variance growth at the maxima (dashed lines).
Using longer readout pulses $t_3 > t_f$ amplify the output signal even further than the symmetric case. This amplification comes at the expense of incomplete suppression in the minimum. (b) Fluctuations at the fringe minimum for different durations of the readout boost. We find a pronounced minimum close to the symmetric case $t_3 \approx t_f$ but shifted towards shorter durations for the second boost.

4.5. Sensitivity to collisional shifts

The detuning term $\delta$ contains collisional shifts via $\delta = gN_{0}$. Therefore, the nonlinear coupling strength not only parametrises the strength of the SU(1,1) boosts but also implies energy shifts during phase interrogation. By choosing experimental runs with vastly different pump atom numbers $N_0$ we can quantify this collisional shift and thereby measure the nonlinear coupling strength $\kappa$ as a function of atom number. Figure 6(a) shows the SU(1,1) interferometry fringes obtained when post-selecting atom numbers between $N_0 = 300$ (red) and $N_0 = 600$ (pink). The fringes dephase by $\approx 2\pi$ after holding times exceeding 50 ms (right panel of figure 6(a)).

Fitting the frequency of the fringes shows a square root like atom number dependence (figure 6(b)). This behaviour is expected for a trapped mesoscopic Bose–Einstein condensate for that the mode function overlap $\int d^3x |\Psi(x)|^4$ lies in a cross over regime between that for single-particle wave functions and that within the Thomas-Fermi approximation. While in the former case interactions are neglected which leads to a constant mode overlap such that $\kappa \propto N$, the latter neglects kinetic energy resulting in a mode overlap $\propto N^{-3/2}$ leading to $\kappa \propto N^{1/2}$.

4.6. Detection requirements

The fluctuations in readout direction ($\hat{K}_z$) are at all stages of the SU(1,1) interferometer super-Poissonian. Furthermore, the SU(1,1) coherent states are completely determined by their mean atom number corresponding to the single boost parameter $r$ of the SU(1,1) coherent states. Since the states feature broad histograms without any detailed finer structure, the requirements on detector resolution are significantly relaxed. To leverage the improved phase sensitivity, only mean values and no higher moments need to be recorded at the detector [29, 33].

This stands in contrast to using highly entangled states with linear readout. When, for instance, the two-mode squeezed vacuum state is fed into a passive SU(2) interferometer, single-atom resolved measurements such as parity detection need to be performed [34]. For the atom number parity, the single shot outcome is assigned +1 when an even number of atoms is detected at one of the output ports, and −1 for an odd atom number, respectively. Such parity detection schemes are characterised by a threshold behaviour: if the detection noise is too large to determine the parity in a single shot, averaging many realisations cannot restore the output signal at all. The uncertainty obtained with the SU(1,1) interferometer, on the other hand, can be reduced by averaging no matter how large the detection noise is. This immunity towards detection noise [35–39] is enabled by the final boost which constitutes a nonlinear readout.
We detailed the first demonstration of an active interferometer with atoms. From a more general point of view, the interferometric sequence constitutes a nonlinear time reversal sequence [8] which is a particular useful form of nonlinear readout [35–38]. Due to its high degree of experimental control we believe that this nonlinear readout can be used to characterise and detect entanglement present in condensates involving many spatial modes [40].

Within the undepleted pump approximation a large portion of the atoms provides the nonlinear mechanism without being used for phase estimation. Surpassing the limitation arising due to the small fraction of sensing atoms has been the subject of recent theoretical investigations: [41] predicts improved phase sensitivities also when severely depleting the pump mode, while [42] suggests to use a combination of passive and active beam splitting.

For some applications, however, the uneven partitioning of atoms constitutes an advantage: interferometry between the large pump mode and the sparsely populated but highly sensitive probe fields might allow to investigate fundamental questions about phase diffusion in Bose–Einstein condensates [43].

The coupling strength for spin-exchange within the \( F = 1 \) manifold has an inverted sign as compared to \( F = 2 \). Therefore, by using one period of spin-exchange within \( F = 1 \) and another within \( F = 2 \) (after swapping all three involved states) the Hamiltonian can be inverted without relying on the validity of the undepleted pump approximation. This extended time reversal might allow to study out-of-time-order correlations [44, 45] and the associated scrambling of quantum information [46].

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