General expression of pressure angle and principal curvature of globoidal indexing cam mechanism

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Abstract. Given the lack of general mathematical model for the parameters of the globoidal indexing cam (GIC) mechanism, a method of modeling the contour for the GIC based on the unified mathematical expression is proposed by applying the envelope theory and the matrix transformation of the homogeneous coordinates. Taking the globoidal cams with cylindrical, conical, and hyperbolic meshing turret-rollers as the research object, a unified mathematical model of profile model for the GIC is derived. On this foundation, the general analytic expression of pressure angle and principal curvature is further obtained, which provides a theoretical basis for the precise design and manufacture of the GIC and provides a mathematical basis for the fatigue strength analysis of the GIC material.

1. Introduction
Globoidal indexing cam (GIC) is widely used in various intermittent motion mechanisms, which can meet the needs of substantial rigidity, massive torque, and accurate positioning. The pressure angle is one of the critical parameters affecting the performance, the force, and the structural size of this kind of mechanism. Therefore, the pressure angle of the camber cam mechanism must be analyzed. Besides, to prevent curvature interferences between the cam surface and the roller surface in the GIC mechanism and the root cutting phenomena when the cam is processed, it is necessary to analyze the principal curvature of the curved conjugate surface of the GIC. All along, scholars at home and abroad have done a lot of research on the profile equation, the pressure angle, and the principal curvature of GIC. A general contour equation of GIC is derived using the theory of generating method and the conjugate principle of the spatial surface [1-2]. A general equation for curved surfaces of GIC is established based on the meshing theory and differential geometry of spatial mechanisms [3]. A contour equation of globoidal cam of steel ball roller is determined using the approach of rotation transformation tensor, and the analytical expression of pressure angle is derived [4-5].
Furthermore, expressions of pressure angle and curvature of cylindrical roller follower globoidal cam mechanism are also derived from the principle of spatial meshing [6-7]. Besides, the pressure angle of globoidal cam mechanism with spherical cone roller is calculated and analyzed [8]. These studies are significant, but the derivation processes are complex and geometric intuition is not reliable. At the same time, different roller shapes and the angle between the camshaft and the rotating axis of the turret result in disunity of the final mathematical expressions in these references. This increases the difficulty and workload of formula deduction to some extent and further complicates the development of the globoidal cam software. At present, there has not been any study on the general expression of the pressure angle and the principal curvature of the GIC mechanism using the theory of envelopes for a one-parameter family of surfaces.

In this paper, based on the theory of envelopes for a one-parameter family of surfaces [9-12], a unified treatment of homogeneous coordinate transformation is performed for different roller shapes (cylindrical, conical, and hyperbolic ones) of the GIC and different angles between the camshaft and the turret rotating axis. The cam profile of the GIC is also derived via the same procedure. Moreover, a general expression of pressure angle and principal curvature of GIC mechanism is then established based on the unified mathematical model.

2. The Contour Surface Equation of the Globoidal Cam

The rotational angular displacement $\phi_c$ of the GIC is regarded as the parametric variable of the surface family. The outer surface equation of the roller and the curved cam profile is described by the roller angle $\theta$ and the roller height $h$. The unified mathematical model of the contour for the GIC is established using the theory of envelopes.

2.1 A unified mathematical expression of the surface equation of the outer surface of the rollers

The curve of the GIC mechanism is shown in figure 1. The mechanism consists of three parts, namely a globoidal cam, a turret, and a roller. In this paper, three kinds of rollers are meshed with the globoidal cam to generate the cam profile. As shown in figure 2, (a) is a cylindrical roller, (b) is a conical roller, and (c) is a hyperbolic roller. A coordinate system $(XYZ)_1$ is established. Specifically, $r$ is the bottom radius of the roller ($h=0$), and $L$ is the height of the roller. Here $\theta$ and $h$ represent roller rotation angle and height respectively while $\gamma$ represents the angle between $Z_1$ axis and the generating line of the ruled surface for the conical roller and the hyperbolic roller.

Let $R_{cy}$, $R_{co}$, and $R_{hy}$ represent the surface equations of the cylindrical, the conical, and the hyperbolic rollers in the $(XYZ)_1$ coordinate system respectively. Assuming that the unit direction vectors of $x$, $y$, and $z$ axes are $i$, $j$, and $k$ respectively, the surface equations of these rollers are expressed in the coordinate system $(XYZ)_1$ as follows:

$$R_{cy} = r \cos \theta i + r \sin \theta j + hk$$

$$R_{co} = (r + L \tan \gamma) \cos \theta i + (r + L \tan \gamma) \sin \theta j + hk$$

$$R_{hy} = (r \cos \theta - L \tan \gamma \sin \theta) i + (r \sin \theta + L \tan \gamma \cos \theta) j + hk$$

We can unify Eqs. (1), (2), and (3) as follows:

$$R = [(r + W_1 L \tan \gamma) \cos \theta - W_2 L \tan \gamma \sin \theta]i + [(r + W_1 L \tan \gamma) \sin \theta + W_2 L \tan \gamma \cos \theta] j + hk$$

where
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\[ W_1 = 0, \ W_2 = 0 \quad \text{for cylindrical roller} \]
\[ W_1 = 1, \ W_2 = 0 \quad \text{for conical roller} \]
\[ W_1 = 0, \ W_2 = 1 \quad \text{for hyperbolic roller} \]

**Figure 1.** Diagram of a GIC mechanism

(a) Cylindrical roller (b) Conical roller (c) Hyperbolic roller

**Figure 2.** Diagrammatic sketch of the driven rollers

**Figure 3.** Coordinate system

2.2 **Matrix transformation of homogeneous coordinates**

The matrix transformation of homogeneous coordinates can be used to make the spatial positional relationship between the GIC and the follower more intuitionistic [13-14]. As shown in figures 1 and 3, the necessary design parameters and corresponding coordinate systems are given. Figure 3 shows the two fixed coordinate systems \((XYZ)_f\) and \((XYZ)_2\) with four relevant moving coordinate systems \((XYZ)_c\), \((XYZ)_b\), \((XYZ)_1\), and \((XYZ)_2\). Among them, the rotation axis of the globoidal cam is \(Z_f\), and the rotation axis of the turret is \(Z_2\). Angle \(\beta\) is defined between the positive axis \(Y_f\) and the negative one \(Z_f\). The
coordinate origins \( O_t \) and \( O_f^2 \) coincide at the turret center, while \( O_1 \) and \( O_f^1 \) coincide at the globoidal cam center, respectively. Furthermore, the coordinate origins \( O_1 \) and \( O_2 \) are located at the center of the bottom plane of the roller (when \( h=0 \)). \( Z_1 \) is collinear with \( Z_2 \) and \( X_f^1 \) is collinear with \( X_f^2 \). The rotational angular displacements of the globoidal cam and turret are \( \phi_c \) and \( \phi_t \), respectively. Positive \( \phi_c \) and \( \phi_t \) are measured counterclockwise for \( Z_f^1 \) and \( Z_f^2 \), respectively. The distance between the globoidal cam and turret axes is \( b \), while that between the turret center and the roller top surface is \( \delta \) (when \( h=L \)). Here \( \alpha \) is the half-angle between the axes of two adjacent rollers.

In the homogeneous coordinates, the coordinate transformation is described by 4×4 matrices, according to the coordinate system and set parameters. The detailed transformation process is as follows:

The transformation matrices providing the transformation between the following coordinate systems are as follows:

- from \((XYZ)_1\) to \((XYZ)_2\):
  \[
  T_1 = \begin{bmatrix}
  \cos \beta & -\sin \beta & 0 & 0 \\
  \sin \beta & \cos \beta & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1 \\
  \end{bmatrix}
  \]  

- from \((XYZ)_2\) to \((XYZ)_t\) :
  \[
  T_2 = \begin{bmatrix}
  \sin \alpha & 0 & -\cos \alpha & (\delta + L) \cos \alpha \\
  \cos \alpha & 0 & \sin \alpha & -(\delta + L) \sin \alpha \\
  0 & -1 & 0 & 0 \\
  0 & 0 & 0 & 1 \\
  \end{bmatrix}
  \]  

- from \((XYZ)_t\) to \((XYZ)_f^2\) is:
  \[
  T_3 = \begin{bmatrix}
  \cos \phi_t & -\sin \phi_t & 0 & 0 \\
  \sin \phi_t & \cos \phi_t & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1 \\
  \end{bmatrix}
  \]  

- from \((XYZ)_f^2\) to \((XYZ)_f^1\):
  \[
  T_4 = \begin{bmatrix}
  -1 & 0 & 0 & b \\
  0 & -\sin \beta & -\cos \beta & 0 \\
  0 & -\cos \beta & \sin \beta & 0 \\
  0 & 0 & 0 & 1 \\
  \end{bmatrix}
  \]  

- from \((XYZ)_f^1\) to \((XYZ)_c\):
  \[
  T_5 = \begin{bmatrix}
  \cos \phi_c & \sin \phi_c & 0 & 0 \\
  -\sin \phi_c & \cos \phi_c & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1 \\
  \end{bmatrix}
  \]
2.3 Establishment of the general expression for profile equation of GIC

Equation (4) can be further reduced as follows:

\[
R_i = [(r + W_1 \tan \gamma) \cos \theta - W_2 \tan \gamma \sin \theta] i + [(r + W_1 \tan \gamma) \sin \theta + W_2 \tan \gamma \cos \theta] j + h k
\]

\[
= [A_1 \cos \theta - A_2 \sin \theta] i + [A_1 \sin \theta + A_2 \cos \theta] j + h k
\]

(10)

where \( A_1 = r + W_1 \tan \gamma \) and \( A_2 = W_2 \tan \gamma \) for \( 0 \leq \theta < 2\pi \) and \( 0 \leq h < L \).

In homogeneous coordinates, the outer surface parameter equation \( R_1 \) of the driven roller is expressed in a four-order matrix form as follows:

\[
R_1 = \begin{bmatrix}
A_1 \cos \theta - A_2 \sin \theta \\
A_1 \sin \theta + A_2 \cos \theta \\
h \\
1
\end{bmatrix}
\]

(11)

The rotational angular displacement \( \phi \) of GIC is the input motion parameter of the mechanism. The rotational angular displacement \( \phi \) of the turret is the output motion parameter of the mechanism. It is assumed that the law of motion of \( \phi \) varies with \( \phi \), where \( \phi = \phi (\phi) \) has been given in advance. The parametric equation of the GIC contour surface is as follows:

\[
r_r = (\theta, h, \phi) = T_r T_r T_r T_r R_1
\]

\[
= \{A_1 \cos (\theta + \beta) \sin (\phi - \alpha) \cos \phi - \cos (\theta + \beta) \sin (\phi - \alpha) \cos \phi + \sin (\theta + \beta) \cos (\phi - \alpha) \cos \phi \}
\]

\[
\sin (\theta + \beta) \sin \phi - \cos (\theta + \beta) \sin \phi \}
\]

\[
- (\delta + L - h) [\cos (\phi - \alpha) \cos \phi + \sin (\theta + \beta) \sin (\phi - \alpha) \cos \phi \]
\]

\[
+ h \cos (\phi - \alpha) \sin \phi \}
\]

\[
- (\delta + L - h) [\cos (\phi - \alpha) \sin \phi - \sin (\theta + \beta) \sin (\phi - \alpha) \cos \phi \]
\]

\[
+ h \sin (\phi - \alpha) \cos \phi \}
\]

\[
= -[A_1 \cos (\theta + \beta) \cos \phi \cos (\phi - \alpha) + \sin (\theta + \beta) \sin \phi - A_2 \sin (\theta + \beta) \sin \phi]
\]

\[
+ (\delta + L - h) \cos (\phi - \alpha) \cos \phi \]

(12)

for \( 0 \leq \theta < 2\pi \) and \( \phi \) is an independent cam motion parameter.

\[
\frac{\partial^2 r}{\partial \theta \partial h} = \left\{ \begin{array}{l}
\{-A_1 \sin \beta \sin \phi \cos (\theta + \beta) + A_2 \sin \beta \sin \phi \sin (\theta + \beta) - (A_1 + A_2 A_1) \cos \phi \}
\cos (-\phi + \alpha) + [-A_1 \cos \phi \cos (\theta + \beta) + A_2 \\
\cos \phi \sin (\theta + \beta) + (A_1 + A_2 A_1) \sin \beta \sin \phi \}
\sin (-\phi + \alpha) + \cos \beta \sin \phi (A_2 \cos (\theta + \beta) + A_1 \sin (\theta + \beta)) \}
\end{array} \right.
\]

\[
+ [-A_1 \sin \beta \cos \phi \cos (\theta + \beta) + A_2 \sin \beta \cos \phi \sin (\theta + \beta) + (A_1 + A_2 A_1) \sin \phi \}
\cos (-\phi + \alpha) + [-A_1 \sin \phi \cos (\theta + \beta) - A_2 \\
\sin \phi \sin (\theta + \beta) + (A_1 + A_2 A_1) \cos \phi \}
\sin (-\phi + \alpha) + \cos \beta \cos \phi (A_2 \cos (\theta + \beta) + A_1 \sin (\theta + \beta)) \}
\end{array} \right.
\]

\[
+ [-A_1 \cos \beta A_1 \cos (\theta + \beta) - A_2 \sin (\theta + \beta) \}
\cos (-\phi + \alpha) + (A_1 + A_2 A_1) \sin (-\phi + \alpha)
\sin \beta (A_1 \cos (\theta + \beta) + A_2 \\
\sin (\theta + \beta)) \}
\end{array} \right.
\]

(13)

where \( A_1 = W_1 \tan \gamma \) and \( A_2 = W_2 \tan \gamma \)

The contour equation of GIC should satisfy both Eq. (12) and the following one:
\[
\frac{\hat{r} \times \hat{r}}{\hat{r} \times \hat{r}} = \frac{\partial \hat{r}}{\partial \theta} \times \frac{\partial \hat{r}}{\partial h}
\]

\[
\{[A_1 \sin \beta + A_2 \cos \beta (-A_1, -A_2, A_3, \delta + L - h)] \cos (\theta + \beta) + [-A_1 \sin \beta + A_2 \cos \beta (-A_1, -A_2, A_3, \delta + L - h)] \sin (\theta + \beta) \} \cos (\phi + \alpha) - \sin \beta b (A_1, A_2, A_3) \sin (\phi + \alpha) + [A_1 (-A_1, -A_2, A_3, \delta + L - h)] \sin \beta - A_2 \cos \beta \phi' A_1 (-A_1, -A_2, A_3, \delta + L - h)] \cos (\theta + \beta) + \sin (\theta + \beta) [A_1 (-A_1, -A_2, A_3, \delta + L - h)] \sin \beta - A_2 \cos \beta \phi' A_1 (-A_1, -A_2, A_3, \delta + L - h)] = 0
\]

where \( \phi' = \frac{d \phi}{d \phi} \).

From Eq. (14), we can obtain:

\[
\theta = 2 \arctan \left[ \frac{-H \pm \sqrt{H^2 + I^2 - J^2}}{J - I} \right] - \beta
\]

where

\[
H = [-A_1^2 A_1 + A_2 (\delta + L - h) - A_1 A_3] \cos \beta \cos (\phi + \alpha) - A_2 \cos \beta - \sin \beta \cos (\phi + \alpha) + [A_1^2 A_2 - A_1 (\delta + L - h)]
\]

\[
I = [-A_1^2 A_1 + A_2 (\delta + L - h)] \cos \beta \cos (\phi + \alpha) - A_2 \cos \beta - \sin \beta \cos (\phi + \alpha) + [-A_1^2 A_1 + A_2 (\delta + L - h) - A_1 A_3]
\]

\[
J = (A_1 A_2 + A_2 A_3) \sin \beta \sin (\phi + \alpha)
\]

Equation (15) can be substituted into Eq. (12) to eliminate \( \theta \) and yield the general expression of the GIC profile equation:

\[
r_v = r_v(h, \phi)
\]

Two values for \( \theta \) can be obtained from Eq. (15). They represent the outline surface profile coordinates of two sides of the globoidal cam groove, respectively.

3. Pressure angle and principal curvature analysis of GIC mechanism

Based on the general equation of the GIC profile, general analytic expressions of the pressure angle and the principal curvature of the surface of the cam mechanism can be easily derived. This will lay a foundation for further studies of the machining error, the dynamic characteristics, and the design of the globoidal cam mechanisms.

3.1 The general expression of the pressure angle of GIC

The pressure angle of the globoidal cam mechanism refers to the angle between the moving direction of the driven part and the normal contour of the cam surface at the conjugate point of contact between the globoidal cam and the outer surface of the driven roller. As shown in figure 4, let the common unit normal vector of the cam and the driven roller at the contact point be \( n \), the pressure angle at this point be \( \psi \), and the unit direction vector of the driven part be \( p \). Here \( n \) is defined by the following formula:

\[
n = \frac{\hat{r} \times \hat{r}}{\hat{r} \times \hat{r}} = \frac{B_1 i + B_2 j + B_3 k}{\left[ (A_1^2 + 1) A_2^2 + 2 A_1 A_2 A_3 A_4 + A_2^2 \left( A_4^2 + 1 \right) \right]^{1/2}}
\]
where

\[ B_1 = [-\sin \beta \sin \phi \cos (\theta + \beta) A_1 + \sin \beta \sin \phi \sin (\theta + \beta) A_2 - \cos \phi \left(A_1 A_2 + A_1 A_4 \right) \cos (-\phi + \alpha) + [-\cos \phi A_1 \cos (\theta + \beta) + \cos \phi A_4 \sin (\theta + \beta) + \sin \phi A_2 \sin (\theta + \beta)] \sin (-\phi + \alpha) + \cos \phi A_4 \cos (\theta + \beta) ] \sin (-\phi + \alpha) + [-\sin \phi A_1 \cos (\theta + \beta) - \sin \phi A_2 \sin (\theta + \beta) + \cos \phi \left(A_1 A_2 + A_1 A_4 \right) \sin (-\phi + \alpha) + \cos \phi A_4 \cos (\theta + \beta) A_2 + \sin (\theta + \beta) A_4] \]

\[ B_2 = [-\sin \beta \cos \phi \cos (\theta + \beta) A_1 + \sin \beta \cos \phi \sin (\theta + \beta) A_2 + \sin \phi \left(A_1 A_2 + A_1 A_4 \right) \cos (-\phi + \alpha) + [-\sin \phi A_1 \cos (\theta + \beta) - \sin \phi A_2 \sin (\theta + \beta) + \cos \phi \left(A_1 A_2 + A_1 A_4 \right) \sin (-\phi + \alpha) + \cos \phi A_4 \cos (\theta + \beta) A_2 + \sin (\theta + \beta) A_4] \]

B_3 = -\cos \beta \cos (\theta + \beta) A_1 - \sin (\theta + \beta) A_2 \cos (-\phi + \alpha) + \cos \beta \left(A_1 A_2 + A_1 A_4 \right) \sin (-\phi + \alpha) - \sin \beta \left[\cos (\theta + \beta) A_2 + \sin (\theta + \beta) A_4\right]

\[ \omega_{t} = [0, 0, \omega_t, 1]^T \]

After a series of coordinate transformations, the following equation can be derived:

\[ W_t = \begin{bmatrix} \cos \phi_c & \sin \phi_c & 0 & 0 \\ -\sin \phi_c & \cos \phi_c & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -\sin \beta & -\cos \beta & 0 \\ 0 & \cos \beta & \sin \beta & \omega_t \\ 0 \end{bmatrix} = \omega_t (-\sin \phi_c \cos \beta i - \cos \phi_c \cos \beta j + \sin \beta k) \]

Thus, \( p \) can be further derived via Eq. (22).

\[ p = \frac{W_t[r_{MC}]}{|W_t[r_{MC}]|} = \frac{W_t[r_C - r_M]}{|W_t[r_C - r_M]|} \]
\[ C_i + C_j + C_k = C_i + C_j + C_k = \left( \omega^2 \left[ (A_i^2 - A_j^2) \cos(\theta + \beta)^2 - 2\cos(\theta + \beta) \sin(\theta + \beta) A_i A_j + A_i^2 + (\delta + L - h)^2 \right] \right)^{1/2} \]  
\hspace{1cm} (22)

where

\[
\begin{align*}
C_i &= -\omega \left[ \cos \phi A_i \cos(\theta + \beta) + \cos \phi A_i \sin(\theta + \beta) + \sin \phi \sin \phi \left( \delta + L - h \right) \cos(-\phi + \alpha) + \sin \phi \sin \phi \cos(\theta + \beta) A_i \\
&- \sin \phi \sin \phi \sin \phi \left( \delta + L - h \right) \sin(-\phi + \alpha) \right] \\
C_j &= -\omega \left[ -\sin \phi A_i \cos(\theta + \beta) - \sin \phi A_i \sin(\theta + \beta) + \sin \phi \cos \phi \left( \delta + L - h \right) \cos(-\phi + \alpha) - \sin \phi \cos \phi \cos(\theta + \beta) A_i + \sin \phi \cos \phi \sin(\theta + \beta) A_i + \sin \phi \cos \phi \sin(\theta + \beta) A_i \right] \\
C_k &= -2 \omega \cos \beta \left( \cos(-\phi + \alpha) \left( \delta + L - h \right) + \sin(-\phi + \alpha) \right) \left( \cos(\theta + \beta) A_i - \sin(\theta + \beta) A_i \right) \right]
\end{align*}
\]

Therefore, the general expression of pressure angle can be deduced via Eq. (23).

\[
\cos \psi = n^p \cdot P \left( \omega \left[ \cos(\theta + \beta) A_i - \sin(\theta + \beta) A_i \right] \right) = \left[ \left( A_i^2 + 1 \right) A_i^2 + 2A_i A_j A_i + A_j^2 \left( A_i^2 + 1 \right) \right]^{1/2} \left( \omega^2 \left[ (A_i^2 - A_j^2) \cos(\theta + \beta)^2 - 2\cos(\theta + \beta) \sin(\theta + \beta) A_i A_j + A_i^2 + (\delta + L - h)^2 \right] \right)^{1/2}
\]  
\hspace{1cm} (23)

It can be seen from Eq. (23) that the pressure angle of the GIC is a complex function of the motion law of the follower and the geometric size of the globoidal cam mechanism, so the pressure angle can be adjusted by improving the motion law of the follower or the geometric size of the globoidal cam.

### 3.2 The general expression of principal curvature of GIC

The principal curvature of the profile surface of the globoidal cam must be calculated and analyzed to avoid the phenomena of over cutting and turning distortion when the cam is used. The principal curvature, \( k \), can be evaluated by the following equation based on differential geometry [15] together with Eq. (16).

\[ k^2 - 2Qk + S = 0 \]  
\hspace{1cm} (24)

where

\[
\begin{align*}
P_1 &= r_c \cdot r_c, \quad P_2 = r_c \cdot r_c, \quad P_3 = r_c \cdot r_c, \quad P_4 = r_c \cdot r_c, \quad P_5 = r_c \cdot r_c, \quad P_6 = r_c \cdot r_c, \\
P_7 &= r_c \cdot r_c, \quad P_8 = r_c \cdot r_c, \quad P_9 = r_c \cdot r_c, \quad P_{10} = r_c \cdot r_c, \quad P_{11} = r_c \cdot r_c, \quad P_{12} = r_c \cdot r_c, \\
Q &= P_1 P_2 - 2P_1 P_3 + P_1 P_4, \quad S = P_1 P_3 - P_2^2
\end{align*}
\]

Similar to the pressure angle general expression, the principal curvature of the globoidal cam profile is also a complex function of the motion law of the follower and the geometric size of the globoidal cam mechanism. Therefore, the principal curvature can be similarly adjusted by improving the motion law of the follower or the geometry of the globoidal cam.

### 4. Model verification

To verify the validity of the pressure angle mathematical model, the Mathematica numerical simulation tool is used. The parameters of the GIC mechanism are as follows: \( r=12.05 \text{ mm} \), \( L=15 \text{ mm} \), \( b=70 \text{ mm} \), \( \alpha=30 \text{ deg} \), \( \delta=31 \text{ mm} \), \( \beta=0 \text{ deg} \), and \( \gamma=0 \text{ deg} \). The number of rollers, \( z \), is 6 and the dividing
period, \( \tau \), is 270 deg, and the rest period, \( \phi_0 \), is 90 deg. The motion law of the turret is an improved sine curve motion relationship, as shown in Eqs. (25), (26), and (27). The distribution of pressure angle along the contact line of the globoidal cam mechanism obtained by the above parameters is calculated as shown in figure 5 (only the stress surface is listed). Figure 5 shows that the pressure angle \( \psi \) of the roller varies with the rotational angular displacement \( \phi \) of cam and the contact point depth \( h \) of the roller surface at different contact points. Thus, the distribution of the pressure angle conforms to the motion law of the GIC mechanism. Therefore, the numerical results verify the validity of the pressure angle mathematical model.

\[
\phi = h \left[ 0.43990 \frac{\phi_t - \phi_d}{\tau} - 0.035014 \sin(4\pi \frac{\phi_t - \phi_d}{\tau}) \right], \quad 0 < \frac{\phi_t - \phi_d}{\tau} \leq \frac{1}{8} \tag{25}
\]

\[
\phi = h \left[ 0.28005 + 0.43990 \frac{\phi_t - \phi_d}{\tau} - 0.035014 \sin\left(\frac{4}{3} \pi \frac{\phi_t - \phi_d}{\tau} + \frac{\pi}{3}\right) \right], \quad \frac{1}{8} < \frac{\phi_t - \phi_d}{\tau} \leq \frac{7}{8} \tag{26}
\]

\[
\phi = h \left[ 0.56010 + 0.43990 \frac{\phi_t - \phi_d}{\tau} - 0.035014 \sin(4\pi \frac{\phi_t - \phi_d}{\tau}) \right], \quad \frac{7}{8} < \frac{\phi_t - \phi_d}{\tau} \leq 1 \tag{27}
\]

\[\text{Figure 5. The distribution of pressure angle along the contact line of the bearing surface of the GIC}\]

5. Conclusions

✓ Based on the theory of envelopes and the matrix transformation of homogeneous coordinates, the coordinate transformation process is described in detail. An equation of the curved surface of the GIC is established from a unified mathematical model, and general expressions of the pressure angle and the principal curvature of the mechanism are derived. The model can directly and accurately reflect the distribution of pressure angle along the contact line of GIC.

✓ The method used in the derivation process has a strong geometric intuition. The whole process is simple, straightforward, and requires no complex motion analyses. The final general expression applies to different roller shapes and different angles between the camshaft and the rotating axis of the turret.

✓ The unified expressions of the mathematical model of the pressure angle and the principal curvature can help verify the correctness of the design parameters and motion laws of the globoidal cam and distinguish whether the conjugate surfaces interfere or not. At the same time, they have specific practical values for the digital design and manufacture of the GIC mechanism.
and provide a theoretical basis for the fatigue strength analysis of the globoidal cam material.

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