Single Round-trip Hierarchical ORAM via Succinct Indices

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ABSTRACT

Access patterns to data stored remotely create a side channel that is known to leak information even if the content of the data is encrypted. To protect against access pattern leakage, Oblivious RAM is a cryptographic primitive that obscures the (actual) access trace at the expense of additional access and periodic shuffling of the server’s contents. A class of ORAM solutions, known as Hierarchical ORAM, has achieved theoretically optimal logarithmic bandwidth overhead. However, to date, Hierarchical ORAMs are seen as only theoretical artifacts. This is because they require a large number of communication round-trips to locate (shuffled) elements at the server and involve complex building blocks such as cuckoo hash tables.

To address the limitations of Hierarchical ORAM schemes in practice, we introduce Rank ORAM; the first Hierarchical ORAM that can retrieve data with a single round-trip of communication (as compared to a logarithmic number in previous work). To support non-interactive communication, we introduce a compressed client-side data structure that stores, implicitly, the location of each element at the server. In addition, this location metadata enables a simple protocol design that dispenses with the need for complex cuckoo hash tables.

Rank ORAM requires asymptotically smaller memory than existing (non-Hierarchical) state-of-the-art practical ORAM schemes (e.g., Ring ORAM) while maintaining comparable bandwidth performance. Our experiments on real network file-system traces demonstrate a reduction in client memory, against existing approaches, of a factor of 100. For example, when outsourcing a database of 17.5TB, required client-memory is only 290MB vs. 40GB for standard approaches.

CCS CONCEPTS

• Security and privacy → Management and querying of encrypted data.

KEYWORDS

ORAM, Succinct data structures

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1 INTRODUCTION

In remote storage settings, where a client outsources their data to a server, encryption is an important mechanism for protecting data content. Unfortunately, encryption alone cannot protect against all vulnerabilities. For instance, in order in which a client accesses its outsourced data, known as the access pattern, can leak sensitive information. Access pattern vulnerabilities have been demonstrated in a number of domains. These include, but are not limited to, leakage through page fault patterns in secure processors [37, 41, 43], through SQL query patterns on encrypted outsourced databases [20, 21] and through search patterns, resulting in query recovery attacks, in searchable encryptions [6, 20].

To mitigate access pattern leakage, Goldreich and Ostrovsky introduced the notion of Oblivious RAM (ORAM) [14]. An ORAM scheme transforms a sequence of original (referred to as virtual) accesses into a sequence of physical accesses that is independent of the original sequence. This transformation eliminates information leakage in the access trace. Ultimately, an adversary must not be able to distinguish between the access patterns produced by an ORAM on an arbitrary pair of input sequences of the same length.

The transformation induced by an ORAM obfuscates the original access pattern by performing both additional dummy accesses and periodic shuffling of the server’s contents. Since these operations incur bandwidth overhead, the principal aim of ORAM research is to minimize this bandwidth overhead while maintaining adequate privacy. However, it is known that one cannot achieve performance better than a logarithmic overhead (in terms of the number of outsourced blocks) due to the established lower bound [14, 23].

A class of solutions known as Hierarchical ORAMs (HierarchicalORAM) [1, 2] has achieved optimality in terms of bandwidth. The HierarchicalORAM was first introduced by Goldreich and Ostrovsky [14] and has enjoyed a long line of improvements [7, 15, 16, 22, 24, 30, 32, 42] and variations [3, 11, 25] over the past two decades. However, these constructions have poor concrete bandwidth performance and require multiple round-trips of communication per access. As a result, they have not been adopted in practice. Further, the processes of rebuilding and

Throughout this work “concrete” refers to the actual bandwidth cost, unobscured by complexity notation.
shuffling the server’s contents are often complex and have been shown to be error prone [17, 22].

In this paper, we address the limitations of HierarchicalORAM schemes in practice. We construct RankORAM, the first HierarchicalORAM that performs a single round-trip access without the help of server-side computation. The main insight of our construction is that knowledge of when a block was last accessed enables non-interactive queries to a Hierarchical ORAM. However, storing this recency information for all data blocks is expensive and may be prohibitive in memory-constrained environments. We overcome this challenge with a novel succinct data structure, called historicalMembership, that keeps recency information in a compressed format at the client. HistoricalMembership builds on work on approximate recency queries by Holland et al. [19] and exploits the periodic rebuilds of Hierarchical ORAMs.

RankORAM not only supports low latency, but additionally, permits simplified rebuild operations. For example, it is standard practice for Hierarchical ORAMs to employ oblivious cuckoo hash tables as the core data structure. However, constructing oblivious cuckoo hash tables is expensive in practice. HistoricalMembership enables RankORAM to replace the cuckoo hash tables with simpler permutated arrays. Thus, the use of HistoricalMembership, as an additional structure at the client, induces a trade-off between client memory and bandwidth efficiency. Notably, larger client memory can be supported in cloud computing applications where the service can rely on the client memory, either RAM or disk, of a desktop computer [9].

The following theorem captures the performance of RankORAM.

**Theorem 1.1.** RankORAM is an oblivious RAM that stores a database of $n$ blocks of $B$ bits, requires $O(n + \sqrt{n} \cdot B)$ bits of private memory, performs access in a single round-trip, rebuild in an amortized constant number of round-trips and observes an amortized bandwidth overhead of $4 \log n$ blocks.

ORAM schemes can be categorized into two types: classical and extended. Classical ORAM schemes, which include the class of HierarchicalORAM, assume a client memory allocation of $O(B)$ bits (i.e., a constant number of blocks) and no server side computation. In contrast an extended ORAM scheme relaxes one or both of these assumptions. RankORAM provides several performance advantages with respect to both types of ORAM, as summarised in Tables 1 and 2. Compared to classical ORAM schemes, RankORAM is the only non-interactive protocol (see Table 1). Though it requires more memory on the client, the allocation is small when compared to the size of the outsourced database in practice (e.g., 290MB for a 17.5TB database). In comparison to extended ORAM schemes, including popular tree-based constructions [40], RankORAM achieves a state-of-the-art bandwidth overhead with a client-memory allocation smaller than comparable approaches (see Table 2). Notably, relative to Partition ORAM [39] (PartitionORAM) and Ring ORAM [35] (RingORAM), we reduce the memory allocation at the client by a factor of $O(\log n)$ — making it the first single-round trip ORAM to achieve that without server side computation. In comparison to the recursive Path ORAM, an interactive and small memory predecessor of RingORAM, we reduce the bandwidth overhead by a factor of 4 and the number of round trips by log $n$.

In summary, our contributions are as follows:

- We propose RankORAM, the first HierarchicalORAM that achieves the combination of a single round-trip per access, a constant number of round-trips for rebuilds and a low concrete bandwidth overhead (Table 1).
- RankORAM is supported by a novel client-side data structure, historicalMembership, that compresses the location metadata of blocks at the server; historicalMembership can be used to reduce the number of round-trips per access for any hierarchical ORAM solution.
- The client memory allocation of RankORAM is asymptotically smaller than prior ORAMs that achieve state-of-the-art latency and concrete bandwidth overhead (Table 2). RankORAM is the first ORAM to achieve, without server side computation, a single round trip of communication and $O(n)$ bits of client memory.
- Our experiments in Section 8, conducted on real network file system traces, demonstrate, for a 4KB block size, a reduction in memory for RankORAM by a factor of 100 against a non-compressed structure.

We continue with related work (Section 2) and some preliminaries (Section 3). A technical overview of our solution is presented in Section 4. It sketches the core ideas that drive our algorithms and highlights how HistoricalMembership enables the performance gains achieved by RankORAM. The low-level details of RankORAM and HistoricalMembership are presented, respectively, in Section 5 and Section 6. Section 7 provides an assessment of the performance and security of RankORAM. Finally, Section 8 contains an experimental evaluation of RankORAM against prior state-of-the-art solutions.

## 2 RELATED WORK

### Hierarchical ORAMs

RankORAM belongs to the class of HierarchicalORAM schemes that have been extensively studied in the literature. A HierarchicalORAM distributes the outsourced data across levels that increase exponentially in size. Each level is implemented as an oblivious hash table. All the variations of HierarchicalORAM depend on the implementation of this primitive. The amortized bandwidth overhead of HierarchicalORAM is determined by the cost of the rebuild (offline bandwidth) and the cost of an access (online bandwidth). In the original proposal by Goldreich and Ostrovsky, to store a database of $n$ blocks of $B$ bits, at level $l$, the hash table contains $2^l$ buckets of $O(\log n)$ depth. When accessing a bucket obliviously, a linear scan is performed. The scheme’s amortized bandwidth cost of $O(\log^3 n)$ is dominated by the rebuild phase.

Subsequent improvements were achieved by changing the hashing primitive to an oblivious cuckoo hash table [7, 15, 16, 32]. With cuckoo hashing, the lookup time is constant. These schemes incur an amortized $O(\log^2 n/\log \log n)$ bandwidth cost that is dominated by a rebuilding phase, which relies on expensive oblivious sorting. Patel et al., with PanORAMa, provide a cuckoo construction algorithm that does not rely on oblivious sorting [30]. They assume that the input to the construction algorithm is randomly shuffled and

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2Oblivious sorting in $O(B)$ bits is expensive in practice and is required in many ORAM schemes for the rebuilding phase. For a discussion on the trade-offs between client memory and bandwidth, see Holland et al. [18].
Table 1: Asymptotical comparison of classical ORAM solutions with RankORAM. ORAMs are parameterized as outsourcing n blocks of B bits each. RankORAM is the only solution that achieves a single round-trip per access, while all other solutions in the table require $\Omega(\log n)$ round-trips.

|            | client storage | bandwidth | hash table | HierarchicalORAM |
|------------|---------------|-----------|------------|------------------|
| Square Root [14] | $O(B)$       | $O(\sqrt{n})$ | Permuted array | ✓                |
| Hierarchical [14]  | $O(B)$       | $O(\log^2 n)$ | Balls-in-bins | ✓                |
| Cuckoo [7, 15]   | $O(B)$       | $O(\log^2 n/\log n)$ | Cuckoo | ✓                |
| Chan et al. [7]  | $O(B)$       | $O(\log^2 n/\log n)$ | Two tier | ✓                |
| PanORAMa [30]    | $O(B)$       | $O(\log n \cdot \log n)$ | Cuckoo | ✓                |
| OptORAMa [1]     | $O(B)$       | $O(\log n)$ | Cuckoo | ✓                |
| RankORAM         | $O(n + \sqrt{n} \cdot B)$ | $4 \log n$ | Permuted array | ✓                |

Table 2: Comparison of extended ORAM schemes with low concrete bandwidth and/or a single round-trip of communication per access. Compared to state-of-the-art approaches, which, as noted in the table, do not use server-side computation, RankORAM reduces client memory by a factor of $O(\log n)$. RingORAM is a variation of recursive PathORAM that stores additional metadata at the client. In addition, RankORAM, RingORAM and PartitionORAM require a constant number of round-trips for rebuilds.

|            | client storage | bandwidth | single round-trip access | server comp. |
|------------|---------------|-----------|---------------------------|--------------|
| SR-ORAM [42]  | $O(B)$       | $O(\log n)$ | ✓                        | ✓            |
| TWORAM [13]   | $O(\log n)\cdot(1 - B)$ | $O(\log n)$ | ✓                        | ✓            |
| BucketORAM [11] | $O(\log n)\cdot(1 - B)$ | $O(\log n)$ | ✓                        | ✓            |
| PathORAM [40] | $O(\log n)\cdot(1 - B)$ | $16 \log n$ | ✓                        | ✓            |
| RingORAM [35] | $O(n \log n + \sqrt{n} \cdot B)$ | $3 \log n$ | ✓                        | ✓            |
| PartitionORAM [39] | $O(n \log n + \sqrt{n} \cdot B)$ | $3 \log n$ | ✓                        | ✓            |
| RankORAM      | $O(n + \sqrt{n} \cdot B)$ | $4 \log n$ | ✓                        | ✓            |

Concerning server block locations explicitly at the client. The metadata is stored in position maps, occupying $\Theta(n \log n)$ bits using an array, and allows accesses to be executed with a single round-trip of communication. Both PartitionORAM and RingORAM achieve state-of-the-art total bandwidth overhead of $3 \log n$, which can be reduced further following optimizations, using server-side computation, proposed by Dautrich et al. [9].

These schemes can be adapted to smaller memory environments by recursively storing the metadata in a sequence of smaller ORAMs at the server. The technique was first presented in [36]. This comes at the cost of increased bandwidth and latency. The performance of this class of ORAMs is summarized in Table 2. In comparison to state-of-the-art, RankORAM uses $O(\log n)$ less memory and obtains a comparable bandwidth overhead.

**Compressing metadata.** The position maps for both PartitionORAM and RingORAM, with multiple types of metadata per block, occupy $\Theta(n \log n)$ bits. As $n$ increases, this term begins to dominate the client memory allocation. To alleviate this burden, Stefanov et al. [39] compress the position map with a method, which we call compressedCounters, that is designed for sequential workloads, but has a worst-case memory allocation of $O(n \log n)$ bits. The theoretical properties of different client-side data structures are summarized in Table 3.

In contrast to this line work, some ORAMs [5, 34] are designed to reduce space utilization at the server, not the client. As they focus on a problem orthogonal to ours, we do not include these works in our comparison. However, it is worth noting that the use...
of permuted arrays in hierarchical ORAM achieves state-of-the-art space utilization at the server (≈ 50%).

3 PRELIMINARIES

Fixing notation, we consider the setting where a client outsources $n$ blocks, each of $B$ bits, to untrusted storage.

3.1 Performance Metrics

For measuring performance we consider three key parameters: client memory, bandwidth overhead and the number of round-trips (latency). The size of the client memory measures the amount of storage, both temporary and permanent, required to execute an ORAM scheme. The bandwidth overhead refers to the number of blocks, possibly amortized, exchanged between the client and server per virtual access. It represents the multiplicative overhead of moving from a non-oblivious to an oblivious storage strategy. The number of round trips counts the rounds of communication between the client and server per virtual access.

3.2 Definitions

Security. We adopt the standard security definition for ORAMs. Intuitively, it states that the adversary should not be able to distinguish between two access patterns of the same length. In other words, the adversary should learn nothing from observing the access pattern.

Definition 3.1 (Oblivious RAM [39]). Let
\[ \mathcal{g} = \{(\text{op}_1, a_1, \text{data}_1), \ldots, (\text{op}_m, a_m, \text{data}_m)\} \]
denote a sequence of length $n$, where $\text{op}_j$ denotes a read($a_j$) or write($a_j, \text{data}_j$). Specifically, $a_j$ denotes the logical address being read or written and $\text{data}_j$ denotes the data being written. Let $A(\mathcal{g})$ denote the (possibly randomized) sequence of accesses to the remote storage given the sequence of data requests $\mathcal{g}$. An ORAM construction is deemed secure if for every two data-request sequences, $\mathcal{g}$ and $\mathcal{z}$, of the same length, their access patterns $A(\mathcal{g})$ and $A(\mathcal{z})$ are, except by the client, computationally indistinguishable.

Oblivious Shuffle. A key primitive of oblivious RAM solutions, including RankORAM, is oblivious shuffle. It implements the following functionality.

Definition 3.2 (Functionality: Array Shuffle). Let $\mathcal{P}$ denote a set of permutations. On input array $U$, of key-value pairs, and permutation $\pi \in \mathcal{P}$, the Array Shuffle outputs the array $V = \text{shuffle}(\pi, U)$, where $V[i] = (k, o)$ and $\pi(k) = i$.

We assume that the permutation function, $\pi$, is given to the algorithm in a form that allows for its efficient evaluation. For example, it could be provided as a seed to a pseudo-random permutation. Oblivious algorithms preserve the input-output behaviour of a functionality and produce an access pattern that is independent of the input. We now define the notion of oblivious algorithm.

Definition 3.3 (Oblivious Algorithm). Let $A(M^\mathcal{F}(x))$ denote the access pattern produced by an algorithm $M$ implementing the functionality $\mathcal{F}$ on input $x$. The algorithm $M$ is oblivious if, for every two distinct inputs, $x_1$ and $x_2$, of the same length, except to the client, their access patterns, $A(M^\mathcal{F}(x_1))$ and $A(M^\mathcal{F}(x_2))$, respectively, are computationally indistinguishable.

An oblivious shuffle implements functionality Definition 3.2 and does not reveal anything about the input permutation through its access pattern. This is because two executions on any distinct pair of input permutations should be indistinguishable.

3.3 Hierarchical ORAM

The Hierarchical ORAM contains a hierarchy of oblivious hash tables $T_0, \ldots, T_L$, with $L = \log n$. In the words of Goldreich and Ostrovsky [14], the ORAM consists of “a hierarchy of buffers of different sizes, where essentially we are going to access and shuffle buffers with frequency inversely proportional to their sizes”. The hash table abstraction contains a look-up query and a construction algorithm. For the construction algorithm to be oblivious, by Definition 3.3, the input blocks must be placed in the table without leaking their locations through the access pattern.

A Hierarchical ORAM has the following structure. The hash table $T_l$ stores $2^l$ data blocks. Next to each table, a flag is stored to indicate whether the hash table is full or empty. When receiving a request to an address, $x$, the ORAM operation involves both an access and rebuild phase:

1. access: Access all non-empty hash tables in order and perform a lookup for address $x$. If the item is found in some level $l$, perform dummy look ups in the non-empty tables of $T_{l+1}, \ldots, T_L$. If the operation is a read, then store the found data and place the block in $T_0$. If the operation is a write, ignore the associated data and update the block with the fresh value.

2. rebuild: Find the smallest empty hash table $T_l$ (if no such level exists, then set $l = L$). Merge the accessed item and all of $\{T_j\}_{j \leq l}$ into $T_l$. Mark levels $T_0, \ldots, T_{l-1}$ as empty. A block is never accessed twice in the same hash table in between rebuilds at a given level. This invariant is crucial to the security of the scheme. As each block is always retrieved from a different location, the sequence of hash table probes produced by Hierarchical ORAM appears random to an adversary.

Note that access is interactive since the client does not know which level a block belongs to. That is, the client has to query the levels sequentially until the target block is found. This requires a round trip per level and increases the latency of the protocol. Our
client-side data structure in RankORAM is designed to remove this cost.

3.4 Indexed Dictionary

Our client-side data structure is built on set-membership structures that support the following operations on the set $S$:

- $S$.index($r$) = return the $r$th smallest item in $S$ (1)
- $S$.rank($x$) = $|\{y | y \in S, y < x\}$ (2)

A data structure that supports these operations, in addition to a membership query ($x \in S$), is called an indexed dictionary [33]. For example, for the set $S = \{2, 5, 7, 9\}$, the query functions evaluate as $S.rank(7) = 3$ and $S.index(3) = 7$.

4 RANKORAM DATA STRUCTURES

RankORAM is an ORAM solution that addresses the limitations preventing the adoption of Hierarchical ORAMs in practice. It differs from prior instantiations of HierarchicalORAM by storing block metadata at the client. This results in two crucial improvements: first, we reduce the number of round-trips of communication per access; and second, we can store blocks at the server in a simple and efficient data structure. The latter improvement leads to a bandwidth and latency efficient rebuild phase.

At a high level, RankORAM follows the template of HierarchicalORAM (Section 3.3) with the addition of a client-side data structure. RankORAM builds on the observation that, in order to achieve a single round-trip, the client needs to know where a block is located, a priori, to avoid searching for it interactively at the server. To this end, we design a compact data structure called historicalMembership to encode this information at the client. In particular, historicalMembership computes, for each block, its current level and its hash table position within the level. Subsequently, courtesy of this metadata, historicalMembership informs the protocol of where to retrieve an accessed block and of where to retrieve dummies at the server. Thus, all the level probes can be batched in a single non-interactive request to the server. A naive way of instantiating historicalMembership requires $\Theta(n \log n)$ bits. Instead we design a succinct data structure that requires only $O(n)$ bits and supports efficient update and lookup operations.

In addition, historicalMembership allows us to improve the design of server-level storage and instantiate each level with an efficient permuted array. This allows RankORAM to dispense with the (expensive) cuckoo hash table. Hash tables are needed in HierarchicalORAM to enable the efficient retrieval of elements in the domain $[1, n]$ from a table of size smaller than $n$ (that is, size $2^l$ at level $l$). historicalMembership compactly stores this mapping from the block domain to the hash table indices. Therefore, a permuted array suffices as the level hash table. Rebuilding a permuted array relies on a single call to an oblivious shuffle, making the rebuild phase not only simpler than constructing a cuckoo hash table, but also more efficient.

The remainder of this section details the two core data structures of RankORAM. We outline our client-side data structure historicalMembership, which enables the performance improvements enjoyed by RankORAM. Afterwards, we describe our oblivious hash table implementation. In the subsequent Section we describe how these data structures are used during the access and rebuild phases.

4.1 Client Data Structure

RankORAM combines HierarchicalORAM with a client-side data structure historicalMembership. Let $S_l = \{a \mid (a, c) \in T_l\}$ denote the set of logical addresses at level $l$ in a HierarchicalORAM. historicalMembership maintains each set $S_l$, for $l \in [L - 1]$, in a compressed indexed dictionary (as defined in Section 3.4). As access patterns typically have low entropy, for example, access patterns in file systems are highly sequential [29], the sets are compressible. historicalMembership, comprised of the collection of indexed dictionaries $S = \{S_0, S_1, \ldots, S_{L-1}\}$, supports the following two functions:

- level($x, S$) = $\min\{i \mid (x, \emptyset) \in T_i, i \in \{1, \ldots, L\}\}$ (3)
- position($x, S$) = $\begin{cases} S_{\text{level}(x)}.\text{rank}(x) & \text{if level}(x) < L \\ x & \text{if level}(x) = L \end{cases}$ (4)

The function level denotes the level a block belongs to and is calculated in a sequence of membership queries starting from $S_0$ and progressing through the hierarchy. The function position($x$) denotes the rank of address $x$ within its current level. The level functionality is observed in prior work [35, 39], where it is implemented in an array (called the position map) mapping element addresses to a collection of auxiliary information. The position functionality is novel to this work and is used to map addresses into hash table locations. An example of the position function for a given instance of RankORAM is given in Figure 1.

When a rebuild happens at the server ($T_l \leftarrow \bigcup_{i=1}^{L-1} T_i$), historicalMembership is updated accordingly: $S_l \leftarrow \bigcup_{i=1}^{L-1} S_i$ and $S_l \leftarrow \emptyset$, $\forall i \in \{1, \ldots, L - 1\}$. Thus, one requirement for the choice of indexed dictionary is that it supports efficient merging. Further, as each element belongs to exactly one level, we do not need to store $S_L$. To evaluate Equation (3), if the element is not a member of $\{S_l\}_{l \leq L-1}$, it must be a member of $S_L$. Further, we do not need rank information at level $L$. This is covered in more detail in Subsection 4.2.

The level information is a function of block recency. Consequently, historicalMembership can be used to estimate the recency of a block. This technique is similar to that used by Holland et al. to estimate recency in small memory and with an arbitrary amount of relative error [19]. Our work differs in the low-level details, such as the implementation of the component dictionaries, and with the additional requirement that we need to support rank queries at each level.

To instantiate historicalMembership, any indexed dictionary can be used. For our result, we use run-length encoding [4] and achieve a memory allocation of $O(n)$ bits. This is the worst-case allocation and run-length encoding will compress any entropy in the access trace. A naive approach, of storing the level and position information in an array, would require $O(n \log n)$ bits. Notably, run-length codes can be merged efficiently while keeping stored information in a compressed state. The technical details are provided in Section 6.

historicalMembership can be used by existing HierarchicalORAM solutions that employ other hash tables to reduce round-trips of communication. However, we note that historicalMembership
cannot be used to encode the metadata arrays for RingORAM and PartitionORAM, as these arrays do not exhibit temporal locality. For example, both constructions store arrays mapping addresses to random numbers. These numbers not only contain high entropy, but are fixed for a given address between accesses. In contrast, the level data stored in historicalMembership is a function of block recency and changes with each rebuild.

4.2 Permuted Array as Oblivious Hash Table

RankORAM implements the level-wise hash table at the server using a permuted array. Fixing notation, let $T_l$ refer to the array, located at the server, that stores the elements of level $l$ and let $n_l := 2^l$. The array has length $2n_l$ and stores at most $n_l$ real elements and, consequently, at least $n_l$ dummy elements. For levels $l < L$, we utilize the rank information to assign each element in level $l$ to a unique index in the domain $[n_l]$. The rank information is available through the indexed dictionaries that form historicalMembership. Elements are then permuted, according to the permutation $\pi_l : [2n_l] \rightarrow [2n_l]$, using their ranks:

$$T_l[\pi_l(S_l, \text{rank}(a))] = (a, \text{data}). \quad (5)$$

The rank can be interpreted as the unpermuted index of the item. For level $L$, we do not need to worry about mapping the address space onto a smaller domain and we can proceed by simply permuting the address space:

$$T_L[\pi_L(a)] = (a, \text{data}). \quad (6)$$

Thus, each block can be retrieved with the position function. When a level is rebuilt, a new permutation function is generated with fresh randomness.

PartitionORAM places blocks in $\sqrt{n}$ small Hierarchical ORAMs each of size $\sim\sqrt{n}$. There are some similarities between our construction and the component ORAMs of the PartitionORAM, which also utilize permuted arrays. The key difference between our construction and the component ORAMs of the PartitionORAM is that, with the rank information compactly encoded in $S_l$, we can access blocks at the server directly through $\pi_l$ (see Equation (5)). In other words, we are able to map the set $S_l$ onto the domain of $\pi_l$ without collisions. Without rank information, PartitionORAM is required to store the offsets explicitly. Through our experiments, we demonstrate that our approach leads to significant savings in client memory.

5 RANKORAM OPERATIONS

We now describe the RankORAM implementation of the HierarchicalORAM template. This covers the access and rebuild operations. In addition, we detail the implementation of an evict operation that is used within a rebuild to remove untouched blocks from expired hash tables.

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3Stefanov et al. provide a collection of pseudorandom permutation functions that are fast and suitable for small domains [38]
5.1 Access Phase

The access algorithm, formalized in Algorithm 1, is similar to that of prior Hierarchical ORAMs. The primary difference is that the procedure begins by determining the block metadata (the level and position of the data block) stored at the client. The position function is used to find the index of the block within its level (line 7). With the level information, we know, a priori, which level the block is stored at and which levels receive a dummy probe. Thus, membership queries to all hash tables can be accumulated, as a set containing one hash table index per non-empty level, and sent to the server as a single request (lines 6–10). The server retrieves all requested blocks in one batch, resulting in one round-trip of communication.

A dummy counter is used to ensure that the scheme only returns untouched blocks from a given hash table instance. Recall that this constraint is necessary for the obliviousness of the scheme. When a dummy is retrieved (line 9), we increment the counter (Line 10) so that an untouched dummy is retrieved at the next dummy request. An example of the access procedure is presented in Figure 1.

5.2 Rebuild Phase

A rebuild enforces the invariant that a block is never retrieved more than once from a given hash table instance. This ensures that the sequence of hash table probes appears random to the adversary. The rebuild carries out the invariant by periodically moving accessed blocks up the hierarchy into fresh hash table instances. Consequently, a block at level \( l < L \) has recency less than \( 2 \cdot 2^l \).

Given that the hash tables are implemented as permuted arrays, the rebuild function is straightforward. We update historicalMembership and subsequently rearrange the server’s memory. For a rebuild into level \( l \), we merge the compressed dictionaries in levels \( \{0, \ldots, (l - 1)\} \) to obtain the dictionary \( S_l \). The rebuild at the server involves a single oblivious shuffle. The input array is the concatenation of the untouched blocks (including dummies) in levels \( \{0, \ldots, (l - 1)\} \). An evict operation (Algorithm 3) is executed to construct the array of untouched blocks. The input array is padded with dummy blocks\(^4\) to the width of the output array. We generate a new pseudo-random permutation, \( \pi_l \), (Line 2) and the input permutation for the oblivious shuffle is the composition of functions \( \pi_l = \text{rank} \circ (\cdot, S_l) \). Any oblivious shuffle algorithm can be used to perform this step [28, 31].

The procedure concludes by updating some client-side data; the dictionaries at levels \( \{0, \ldots, (l - 1)\} \) are deleted; the dummy counter is set for level \( l \); and the set of non-empty (or occupied) levels is adjusted (line 10). The rebuild procedure is presented in Algorithm 2. The function msb (Line 2) computes the most significant bit of the input.

RankORAM can use any prior oblivious shuffle during a rebuild. However, in our context, we have dummy blocks in both the input and output arrays. A given dummy block can be placed in any vacant location in the output array and we can exploit this fact to gain performance improvements relative to general shuffling algorithms. Consequently, we construct a variation of the cacheShuffle [31], named shortQueueShuffle, that leverages a shuffling instance where dummies are placed in the output array. For comparison, with the standard cacheShuffle, permitting level \( l \) would cost \( 9 \cdot 2^l \) blocks of bandwidth. With shortQueueShuffle, we reduce this cost to \( 7 \cdot 2^l \). The details of shortQueueShuffle are presented in Appendix B and its performance is summarized below.

Definition 5.1 (Dummy shuffle functionality). On an input array of length \( n \) and a permutation function \( \pi : [2n] \rightarrow [2n] \), the dummy shuffle functionality outputs an array of length \( 2n \) with the input elements placed according to \( \pi \) and the remaining locations filled with dummies.

Lemma 5.2. shortQueueShuffle is an oblivious dummy shuffling algorithm. On an input of length \( n \), shortQueueShuffle completes in \( 7n \) blocks of bandwidth, requires \( \sqrt{n} \) round-trips of communication and uses \( O(\sqrt{n} \cdot B) \) bits of private memory.

5.3 Eviction procedure

Hash table eviction (used in rebuild phase) involves removing all untouched blocks from the hash table in order from lowest index to highest index. This procedure can be carried out by the server. For completeness, and as we do not allow additional server power, we provide two efficient ways for the client to perform eviction. We could store a bitmap, locally at the client, that indicates the eviction by requesting the server to delete every block retrieved in Line 14 in Algorithm 1.

---

\(^4\)The dummy blocks are indexed so that their encryptions are indistinguishable from real blocks.

---

Algorithm 2: Rank ORAM Rebuild

1. define rebuild()
2. \( l \leftarrow \text{msb}(\text{count}) + 1 \)
3. \( \pi_l \leftarrow \text{a pseudo random permutation on the domain } [2^l] \)
4. \( S_l \leftarrow S_0 \cup \cdots \cup S_{l-1} \)
5. \( I \leftarrow \text{evict}(0) \parallel \cdots \parallel \text{evict}(l - 1) \)
6. \( I \leftarrow I \parallel [n_l\text{ dummy blocks}] \)
7. \( T_l \leftarrow \text{shuffle}(\pi_l(\text{rank} \cdot S_l), I) \)
8. for \( i \in \{0, \ldots, l - 1\} \) do
9. \( S_i \leftarrow \emptyset \)
10. dummyCnt\(_l\) \( \leftarrow |S_l| \)
11. occupiedLevels \( \leftarrow \{I\} \cup \text{occupiedLevels} \setminus \{0, \ldots, l - 1\} \)
12. return
6 HISTORICAL MEMBERSHIP WITH RUN-LENGTH ENCODING

We now provide an encoding for the indexed dictionaries that are the components of historicalMembership. In the ORAM setting the encoding must obtain an efficient worst-case compression. This allows the client to safely allocate, a priori, a compact allotment of memory to the data structure. Otherwise, the adversary could use the size of the memory allocation at the client as another side-channel to infer that the sequence of accesses belongs to a subset of the possible access patterns. An alternative method for compressing run-length encoded strings [4] and a search procedure over them.

Elusive addressing. In the ORAM setting the auxiliary structure consists of an array of addresses $A$, where $A[i]$ denotes the virtual address stored at the beginning of segment $i$, and an array of pointers $P$, where $P[i]$ points to $A[i]$ in the run-length code. Notably, $A$ is utilized, in membership and rank, for the fast identification of the correct segment. Combined, these arrays support forward access to the beginning of each segment.

Query algorithms. Let $W = \Theta(\log n)$ denote the width of every segment. To execute a fast membership $(a, S)$ query, the procedure performs a binary search on $A$ to realize the correct segment, that is, the index $i$ such that $a \in [A[i], A[i+1))$. With $P[i]$, the procedure then jumps to the correct segment of the run-length code in a single probe. The query is completed with a linear scan of the segment. Pseudo code is provided in Algorithm 5 in Appendix C. The query rank $(a, S)$ can be decomposed into two parts:

$$\text{rank}(a, S) = \text{rank}(A[i], S) + \text{rank}(a, S[i]),$$

where $S[i]$ is the segment that contains the neighborhood of $a$. As above, the index $i$ is determined through a binary search on $A$. As the segments have equal cardinality, $\text{rank}(A[i], S) = i \cdot W$. The local query on the segment $S[i]$ is computed through a linear scan. The index $(i, S)$ query is computed as follows. The procedure first identifies the correct segment as $j = \lfloor i/W \rfloor$. It then performs a local index query on segment $j$ for input $i' = i - j \cdot W$. This can be executed through a linear scan of the segment. The run-time cost of the algorithms are summarized in the following.

**Lemma 6.1.** membership, rank and index queries on rle$(S_i)$ take $O(\log n)$ time.

Proof. The code is divided into segments of width $O(\log n)$. The rank and membership operations involve two stages; (1) locate the correct segment in the code (as specified by the auxiliary structure); (2) scan the segment in search of the block address. The auxiliary array $A$ contains an ordered sequence of $O(|S_i|/\log n)$ block addresses that correspond to the starting address of each segment. Thus the forward pointer to the correct segment can be located, through a binary search on $A$, in $o(\log |S_i|)$ time. The subsequent linear scan can be executed in time proportional to the length of the segment. Thus, both rank and membership require $O(\log n)$ time to complete. The index query does not require binary search on the auxiliary structure and can locate the correct segment in constant time.

#### Algorithm 3: Rank ORAM hash table eviction: the procedure removes all blocks that were not touched during access at level $l$

```plaintext
1 define evict(l)
2   E ← [] // empty array of length $n_l$
3   for $i \in \{0, 1, \ldots, 2 \cdot n_l - 1\}$ do
4     $r ← \pi^l_j(i)$
5     if $r < |S_l|$ then
6       // $r$ represents a real element
7       $a ← \text{index}(r, S_l)$
8       $l' ← \text{level}(a, S)$
9       if $l' = l$ then
10          // $a$ is untouched
11          $E ← E \parallel T[i]$
12       else if $r ≥ \text{dummyCnt}$ then
13          // $r$ represents an untouched dummy
14          // index
15          $E ← E \parallel T[i]$
16   return $E$
```

given encodings of the sets $S_1$ and $S_2$, one can enumerate $S_1 \cup S_2$, in order, in $O(|S_1| + |S_2|)$ time and with a working space of $O(1)$ words. For example, to merge rle$(S_1)$ and rle$(S_2)$, we extract, iteratively, the smallest key from the front of its corresponding code and place it in the new code. This means that the keys in $S_1 \cup S_2$ are enumerated in order, without uncompressing the codes.

### 6.2 Auxiliary data structure

The efficiency of the level (Equation (3)) and position (Equation (4)) functions depend, respectively, on the efficiencies of the membership and rank queries on the component dictionaries. To support these functions efficiently, we supplement each code with an auxiliary structure of forward pointers that is constructed as follows. We divide the run-length encoding rle$(S_i)$ into $O(|S_i|/\log n)$ segments of equal cardinality of order $\Theta(\log n)$. The auxiliary structure consists of an array of addresses $A$, where $A[i]$ denotes the virtual address stored at the beginning of segment $i$, and an array of pointers $P$, where $P[i]$ points to $A[i]$ in the run-length code. Notably, $A$ is utilized, in membership and rank, for the fast identification of the correct segment. Combined, these arrays support forward access to the beginning of each segment.
time. It also requires a linear scan of the segment and completes in $O(\log n)$ time.

6.3 Performance of historicalMembership

With the auxiliary structure established, we conclude by evaluating the performance of historicalMembership. We begin with the memory allocation.

Lemma 6.2. historicalMembership, compressed with run-length codes, requires $O(n)$ bits of memory.

Proof. With respect to space- and time-efficiency, the worst-case occurs when the levels are full, that is, when $|S_l| = 2^l, \forall l \in [L]$. At level $l$ each array in the auxiliary structure has $|S_l|/\Theta(\log n)$ entries of $O(\log n)$ bits. The auxiliary structure therefore occupies $O(|S_l|)$ bits. Thus, the set $\{\text{rle}(S_l)\}_{l \in [L]}$, following Inequality (7), for some constant $c > 0$, uses

$$\sum_{l=1}^{L-1} c|S_l| \log(n/|S_l|) + O(|S_l|) \leq c \sum_{l=1}^{L-1} 2^l \cdot \log(n/2^l) + \sum_{l=1}^{L-1} O(2^l)$$

$$= c \left( \sum_{l=1}^{L} 2^l \cdot (L - 1) \right) + O(n)$$

$$\leq c \cdot 2^{L+1} + O(n)$$

$$= O(n) \text{ bits.}$$

The update time is dominated by the cost of merging the run-length codes. The choice of code is important: merging should be efficient and avoid decompressing the codes in a manner that would trespass over the memory bound.

Lemma 6.3. On a database of size $n$, historicalMembership supports updates in $O(\log^2 n)$ amortized time.

Proof. Following the schedule of HierarchicalORAM, each update involves inserting an address into $S_0$ and performing a merge. For a merge into level $l$, in order to stay within our memory bounds, we extract the addresses from $S_0 \cup S_1 \cup \cdots \cup S_{l-1}$, in sorted order, and build the run-length code, on the fly, without resorting to a plain form representation. To do this, we retrieve one address from the front of each code and place it in a list of length $l$. Then, for $|S_l|$ rounds, we retrieve the smallest address from the list (deleting any duplicates) and place it in the next position of rle($S_l$) by computing the corresponding run-length. When an address is removed from the list, we retrieve the next address from the front of the corresponding code by adding the next run-length to the removed address.

The list has length $l$ and we perform one scan per round. Inserting a new run-length into $S_l$ and extracting an address from a code both take constant time. Therefore, a merge takes $O(l \cdot |S_l|)$ time.

The auxiliary structure can be built with one scan of the code. For $W = \Theta(\log n)$, we read $W$ run-lengths at a time. When we reach the beginning of each segment, we place both the starting address and offset in the corresponding auxiliary arrays. This process completes in $|S_l|$ time.

As a merge occurs every $|S_l|$ updates, the amortized cost of merging into level $l$ is $O(l)$. Aggregated across all levels $\{0, 1, \ldots, L-1\}$ (we do not perform a merge into level $S_L$), this leads to an amortized update cost of

$$\sum_{l=0}^{L-1} O(l) = O(L^2) = O(\log^2 n).$$

The query functions are listed in Equations (3) and (4). Their runtime costs are a function of the cost of evaluating the membership and rank primitives on the component dictionaries.

Lemma 6.4. On a database of size $n$, historicalMembership admits position and level queries in $O(\log^2 n)$ time.

Proof. The level($x$) function sequentially probes, from bottom to top, the dictionaries $S = \{S_0, S_1, \ldots, S_L\}$ with membership queries. In the worst-case, that is, when $x \in S_L, L = \log(n)$ membership queries are preformed. As each query takes $O(\log n)$ time, by Lemma 6.1, the level function is evaluated in $O(\log^2 n)$ time in the worst-case. The position($x$) function requires one additional rank query on $S_{\text{level}(x)}$ at a cost of $O(\log n)$ time.

7 RANKORAM PERFORMANCE AND SECURITY

In this section we first describe two optimizations to reduce both the online and offline bandwidth of RankORAM. We then evaluate the overall performance of the protocol.

7.1 Optimizations

We begin with a modification to the hierarchical ORAM template (see Section 3.3). As the oblivious shuffle requires $O(\sqrt{n} B)$ bits of memory, we can afford to store a collection of the smaller levels at the client (a common optimization found in PartitionORAM and RingORAM). To stay within the memory bound, we trim the server side structure and store levels 1 to $L/2$ at the client. This reduces the access bandwidth by a half.

In addition, to save bandwidth, the evict and shuffle subroutines of the rebuild can be intertwined. evict(0) || \cdots || evict(\log(n) - 1) and retrieve untouched blocks or generate fresh dummies as the shuffle algorithm requires them.

7.2 Performance

Lemma 7.1 (Round-trips). In RankORAM, an access requires one round-trip of communication and a rebuild requires an amortized constant number of round-trips of communication.

Proof. Algorithm 1 details an access. There is one transmission of hash table probes in Line 12 and one transmission of the required blocks in Line 16. This represents a single round-trip of communication.

For rebuild, first note that the for-loop of the evict operation can be executed in a sequence of batches of size $\sqrt{n}$. We generate $\sqrt{n}$ untouched indices at the client and download the corresponding blocks in a single-round of communication (the blocks can be
subsequently written to the server in a single batch). Similarly, By Lemma 5.2, the shortQueueShuffle requires $O(\sqrt{n})$ round-trips of communication. As a rebuild at level $l$ occurs every $2^l$ updates, each level completes its rebuild in an amortized $O(\sqrt[4]{n^2/2^l}) = 2^{-l/2}$ round-trips of communication. Summing across all log $n$ levels, the total cost of rebuilding RankORAM is (amortized)

$$\sum_{l=0}^{\log n} 2^{-l/2} = O(1)$$

round trips of communication. \hfill \Box

**Lemma 7.2 (Bandwidth).** The amortized bandwidth overhead for RankORAM is $4 \log n$.

**Proof.** We reduce bandwidth by storing levels 1 to $L/2$ at the client (see Subsection 7.1). Therefore, for online bandwidth, RankORAM downloads, $L/2 = \log n/2$ blocks from the server per access. Similarly, for offline bandwidth, we only need to account for the amortized cost of rebuilding levels $(L/2+1)$ to $L$. For $l > (L/2 + 1)$, a level rebuild costs $7n_l$ blocks of bandwidth by Lemma 5.2. Further, a rebuild occurs every $n_l$ updates. Thus, the amortized bandwidth cost of maintaining a level stored at the server is 7 blocks. As there are $L/2$ levels stored at the server, the amortized offline bandwidth overhead is $7 \cdot L/2 = 3.5 \log n$. Therefore, total bandwidth is $4 \log n$. \hfill \Box

**Lemma 7.3 (Client total work).** Per each combined access and rebuild operation, the cost incurred by historicalMembership is amortized $O(\log^4 n + C(B) \cdot \log n)$, where $C(B)$ is the cost of encrypting/decrypting a block of $B$ bits.

**Proof.** During the access phase, Algorithm 1, the client initially probes historicalMembership with level and position queries (Lines 2 and 3). By Lemma 6.4, both queries require $O(\log^2 n)$ time.

During the rebuild phase, Algorithm 2, we update historicalMembership (Line 4) and use historicalMembership to perform eviction (Line 5). By Lemma 6.3, the former procedure requires amortized $O(\log^2 n)$ time. A rebuild at level $l$ evict operations. Each eviction procedure, Algorithm 3, at level $l$, executes $2 \cdot n_l$ level queries. As a rebuild at level 1 occurs every $n_l$ accesses, by Lemma 6.3, the amortized time cost incurred by historicalMembership is $O(l \cdot \log^3(n))$. Summing over levels $\{1, \ldots, \log n - 1\}$, the total cost is

$$\sum_{l=1}^{L-1} O(l \cdot \log^3 n) = O(L^2 \log^3 n) = O(\log^4 n).$$

This runtime dominates the cost incurred by historicalMembership during the access phase.

During interactions with the server, the client encrypts and decrypts $O(\log n)$ blocks, amortized, giving the additional cost of $O(C(B) \cdot \log n)$, where $C(B)$ is the time cost of encrypting/decrypting a block of $B$ bits. \hfill \Box

When compared to a traditional HierarchicalORAM scheme, $O(\log^4 n)$ represents the additional cost incurred by the use of historicalMembership. This cost is added to a logarithmic number of encryption/decryption operations per each $B$-bit size block. In practice, the latter is likely to be the dominating cost incurred by the client, while still being lower than the bandwidth time. Recall that the encryption/decryption costs are inherent to all ORAM solutions.

**7.3 Security**

In Algorithms 1-3, the red lines indicate operations that involve communication with the server. To demonstrate the obliviousness of RankORAM, we need to show that its access pattern is independent of the input sequence.

**Lemma 7.4.** RankORAM is oblivious

**Proof.** We follow the standard security argument for HierarchicalORAM constructions since RankORAM adopts the same template (Section 3.3). Recall that the security of accesses relies on the following invariant: a block is accessed only once in a given hash table instance. Then as long as each block is located at a location independent of the block content and address, any block retrieved from a level appears as a random index access to the adversary [14]. As blocks are mapped to locations using pseudorandom functions, we only need to show that RankORAM satisfies the invariant on one-time-retrieval for real and dummy accesses. For real accesses, when a specific address is retrieved it is always inserted into the top of the hierarchy and a rebuild occurs. Therefore, when it is next accessed, the block will be located in a new hash table instance. Similarly, all dummy fetches are “fresh” and retrieve an untouched physical address determined by the pseudo random permutation and a unique counter.

To complete the proof, we need to establish the security of the rebuild method. A rebuild involves two interactions with the server. First, we construct the input array for the shuffle through the evict routine (Algorithm 3). This algorithm combines all the elements that were not accessed by the client and hence are known to the adversary already. The number of these untouched indices is deterministic and is fixed in size ($n_l$ for level $l$). The order in which they are accessed is irrelevant and independent of the block content. Second, we perform a shuffle using an oblivious shuffle that is data-independent based on its security definition.

Given that we use pseudorandom functions, it follows that the access patterns for an arbitrary pair of input sequences are computationally indistinguishable. \hfill \Box

Now we have all the components of Theorem 1.1. Security is given by Lemma 7.4. The memory cost is incurred by the underlying shuffling algorithm (Lemma 5.2) and the client-side data structure (Lemma 6.2). Finally, the bandwidth cost is secured by Lemma 7.2.

**8 EXPERIMENTS**

In this paper we have presented RankORAM; a Hierarchical ORAM scheme based on a novel client-side data structure, historicalMembership. RankORAM trades off client memory to achieve bandwidth efficiency. We focus the experimental evaluation on comparing the overhead of methods for storing and maintaining metadata at the client. We have implemented two baseline approaches, arrayMap and compressedCounters, whose properties are summarized in Table 3. The former is the standard for storing the metadata of ORAM solutions [8, 9, 35, 39]. The latter has not been used in an ORAM but mentioned in the context of PartitionORAM [39] albeit
Table 4: Problem and database sizes for datasets on commercial cloud traces.

| Workload | n   | Database size |
|----------|-----|---------------|
| Tencet   | 4,370,466,280 | 17.5 TB |
| K5cloud  | 1,065,643,040 | 4.3 TB |

without an instantiation. Hence, our implementation is based on our instantiation described in Appendix A. This implementation is of independent interest. Not that, unlike historicalMembership, compressedCounters requires server-side computation.

For each data structure, we measure peak client memory and the update time as these measurements depend on the access patterns. The update time measures the total time per access. This would include any query costs that are required to execute the access. We refer the reader to Table 2 for the bandwidth costs of these schemes (compressedCounters can be used with both PartitionORAM and RingORAM) as the bandwidth performance is determined only by the size of the input (otherwise it would leak information).

In order to evaluate the performance of historicalMembership and baseline approaches in practice, we use real and synthetic workloads. The real workloads come from two separate commercial cloud storage network traces. The first trace, provided by Zhang et al. [44], is collected on Tencent Cloud Block Storage over a 6 day period. The average block size is 4KB. The second trace, provided by Oe et al. [27], is collected on the Fujitsu K5 cloud service. The properties of the traces are summarized in Table 4.

In addition, we have generated synthetic workloads based on uniform and Zipfian distributions. With the Zipfian, or skewed, datasets, we vary the size of the problem instance n, from $2^{21}$ to $2^{29}$ and the skew parameter $\phi \in \{1.1, 1.2, 1.3, 1.4, 1.5\}$, where 1.1 represents low skew and 1.5 represents high skew. The synthetic datasets allow us to test the schemes on average and worst-case access scenarios that ORAM is designed to protect. We use two block sizes, 64 bytes and 4KB, simulating the size of a cache line and a page size, respectively.

Recall that large-client ORAM schemes, including RankORAM, utilize client memory to temporarily store and shuffle $O(\sqrt{n})$ blocks. To this end, we also measure the amount of temporary memory required for reshuffles, which we refer to as blockBuffer, and compare it to the memory requirements of the index data structures stored at the client (i.e., arrayMap, compressedCounters and historicalMembership). Intuitively, the size of blockBuffer is the minimum memory requirement of these ORAMs. Hence, the use of compressed methods, such as compressedCounters and historicalMembership, would be justified only if (1) the memory allocation of arrayMap significantly exceeds the allocation of the blockBuffer; and (2) the compression methods produce memory allocations less than or equal to the allocations of the blockBuffer. Our experiments demonstrate when this is the case.

Prior work has stated that, for a sufficiently large block size, in practice, the memory allocation of the block cache significantly exceeds that of the metadata array. However, even for 4KB blocks, experimental work has demonstrated that this is not always the case [8].

8.1 Experimental setup

All code is written in C++. We simulate the client and server on a single machine. The server is simulated by an interface that abstracts array management: all data is stored and retrieved on disk (representing the server) and the data structures are stored in RAM (representing the client).

Each workload is executed on a HierarchicalORAM to generate a dataset of accesses and rebuilds. The client-side data structures are evaluated on these datasets through the metrics of client-memory and update time. This approach of simulating the accesses and rebuilds at the client allows for a more accurate calculation of update time.

8.2 Real data

The results on cloud traces are displayed in Table 5. The size of the blockBuffer, with $B = 4$KB, is 540MB for Tencent and 260MB for K5cloud. Notably, the size of blockBuffer is significantly smaller than the memory allocation for arrayMap, including a factor 80 difference on the Tencent cloud. This indicates that, particularly for large n, the arrayMap represents the significant component of client memory. historicalMembership outperforms arrayMap and compressedCounters in client memory. On the Tencent dataset, historicalMembership encodes the block metadata in 0.53 bits per block and reduces client memory by a factor of 135 against the baseline arrayMap. On the K5cloud dataset, compressedCounters encodes the block metadata 9.8 bits per block and attains a memory allocation that is larger than historicalMembership by a factor of 10. The encoding is larger than the 2 bits per block hypothesized by Stefanov et al. [39] and demonstrates its sensitivity to the access pattern. Recall that, unlike historicalMembership, compressedCounters has poor worst-case behaviour (see Table 3).

Both compression techniques, historicalMembership and compressedCounters, obtain memory allocations comparable to the blockBuffer. Further, their update times are markedly smaller than a standard network latency of 30-40ms. Thus, the experiments demonstrate the feasibility of both these techniques in practice.

8.3 Synthetic Data

The results on synthetic data are displayed in Figure 2. Both plots contain lines that approximate the size of the blockBuffer for $B = 64$ bytes (long dashed) and $B = 4$KB (dashed). Figure 2a expresses the effect of the problem size for skew parameter $\phi = 1.2$. As expected, the arrayMap is proportional to $O(n \log n)$ and grows notably faster than the blockBuffer. Both historicalMembership and compressedCounters grow linearly with the database size. For historicalMembership, this is in line with the theoretical bounds. In contrast, for compressedCounters, this indicates that the worst-case bounds do not hold when there is moderate skew in the access pattern. Further, for an Intel SGX secure processor, with an Enclave Page Cache of 96MB and block size $B = 64$ bytes (matching a typical processors cache line), Figure 2a demonstrates that RankORAM can be executed, on these access patterns, entirely in private memory for $n \leq 2^{27}$. In comparison, compressedCounters fits in private memory for $n < 2^{23}$.

To illustrate the effect of skew on the compression techniques, Figure 2b plots client memory against the skew parameter. The
Table 5: Performance of data structures on real cloud traces with block size $B = 4$KB. The memory required for the rebuild phase (i.e., the size of blockBuffer) is 540MB (0.54 GB) for the Tencent dataset and 260MB (0.26 GB) for the K5cloud dataset. historicalMembership outperforms all competitors in terms of client-memory size and requires less memory than blockBuffer. It is also faster than compressedCounters on both test cases. Note that the arrayMap is the standard used in implementations for PartitionORAM and RingORAM [8]

| dataset   | data structure    | client memory (GB) | update time ($\mu$ seconds) |
|-----------|-------------------|--------------------|-----------------------------|
| Tencent   | arrayMap          | 39.33              | 0.1                         |
|           | compressedCounters| 0.56               | 14.4                        |
|           | historicalMembership | 0.29             | 10.6                        |
| K5cloud   | arrayMap          | 9.60               | 0.1                         |
|           | compressedCounters| 1.30               | 8.1                         |
|           | historicalMembership | 0.13             | 2.7                         |

Figure 2: The size of client memory across several ORAM implementations for synthetic workloads of varying size and skew. Dashed lines correspond to the size of blockBuffer (temporary client memory needed during reshuffle) for block sizes of 64 bytes and 4 KB. Note that the size of arrayMap, HierarchicalORAM and compressedCounters are independent of the block size.

database size is fixed at $n = 2^{27}$. Both historicalMembership and compressedCounters, on highly skewed access patterns, reduce client memory by a factor of 100 against the baseline arrayMap. However, the performance of compressedCounters degrades significantly as the amount of skew decreases. To test this further, we conduct a separate experiment on a uniformly distributed access pattern, which is the worst-case. In this instance compressedCounters obtained a memory allocation larger than arrayMap (1.5 GB for compressedCounters and 1.2 GB for arrayMap when $n = 2^{27}$). In contrast, historicalMembership, still outperformed arrayMap by a factor of 12 when the access pattern was uniformly distributed.

On all instances of synthetic data, historicalMembership requires a lower memory allocation than the blockBuffer (the memory required for the rebuild phase). At the same time, for larger values of $n$, the arrayMap obtains a memory allocation that exceeds the blockBuffer by a factor of at least 10. This testifies to the efficacy of our approach.

One has to be careful when deploying historicalMembership and compressedCounters in practice as varying memory requirements between the skews could introduce an additional side-channel revealing the type of access pattern. To this end, reserving memory for the worst-case is advisable. For such cases, historicalMembership would be preferred due to an order of magnitude smaller memory requirements in the worst case.

9 CONCLUSIONS

We have presented the first protocol for Hierarchical ORAM that can retrieve the accessed block in a single-round of communication without requiring server computation. Our construction, RankORAM, exploits a larger client memory allocation, relative to prior work, to achieve improved bandwidth and latency performance. The foundation of RankORAM is a novel client-side data structure, historicalMembership, that maintains a compact representation of the locations of the blocks at the server. Significantly, historicalMembership can be used in any Hierarchical ORAM to reduce the number of round trips of communication, per access, from $\log n$ to one. Further, with historicalMembership levels at the server can be stored as permuted arrays, avoiding complex hash tables and allowing fast and practical oblivious shuffle algorithms to be used for rebuilds.

Compared to state-of-the-art passive solutions, PartitionORAM and RingORAM, we reduce client memory by a logarithmic factor, while maintaining comparable bandwidth and latency performance. The standard for passive ORAMs is to use an array to store position maps at the client. Our experiments, on real network file system traces, demonstrate a reduction in client memory by a factor of a 100 compared to the array approach and by a factor of 10 compared to closest related work.
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A COMpressing POSITION MAPS

Stefanov et al., with PartitionORAM, were the first to suggest compressing client-side metadata to support a client-memory efficient ORAM protocol [39]. Their method constitutes a key baseline for historicalMembership. However, the authors omit details for implementing the approach. To supplement their work, and to provide a suitable comparison for empirical evaluation, we provide those details here. We begin with an overview of their concept.

PartitionORAM partitions the database into $\sqrt{n}$ smaller Hierarch-Oram of size $\sim \sqrt{n}$. Each block is randomly assigned to a point in the partition. The position map of PartitionORAM stores the following pieces of metadata for a block: (1) the partition number; (2) the level number; and (3) the hash table offset. The partition numbers are selected uniformly at random and thereby have high entropy. Consequently, to achieve a more compressible position
map, the count of each block is stored, instead of the partition number, and used as input to a pseudorandom function. For example, let $\text{ctr}_i$ be the count of block $i$ and PRF denote the function. Then block $i$ is assigned to partition $\text{PRF}(i \mid \text{ctr}_i)$. On each access, the count is incremented and the pseudorandom function generates a fresh, and seemingly random, partition number. Unlike recency, the count is fixed between accesses. Therefore, historicalMembership cannot be used by PartitionORAM to generate partition numbers. The advantage of storing the counts is that they are highly compressible for sequential access patterns. This is demonstrated by Opera et al. [29], who outline a compression method designed to leverage sequentiality.

Further, Stefanov et al. note that, if all levels are full, each block has probability $2^{L-I}$ of being in level $l$. Thus, the level information has low entropy and is highly compressible. No compression algorithm is nominated. Lastly, the hash table location metadata is dispensed with by uploading the blocks, including dummies, with random “aliases”. Then, during retrieval, the client requests blocks by their alias and the server finds the block on the clients behalf. Consequently, server-side computation is introduced. We refer to this combined approach of compressing metadata as compressed-Counters. It requires two data structures: one for compressing the block counters; and one for compressing the level information. We now provide an instantiation of both.

### A.1 Data structures

A method for compressing counters involves storing counter intervals instead of a separate count for each block [29]. A counter interval stores a single count for each interval of consecutive blocks with the same access count. The data structure contains two arrays. An index array $\text{ind}$ stores the starting index for each interval and a counter array $\text{ctr}$ stores the count for each interval. Thus the count for an index $i \in [\text{ind}[j], \text{ind}[j+1])$ has count $\text{ctr}[j]$. The challenge is to keep the arrays compact under a dynamic workload. We want to avoid resizing the array at each update. Thus, we divide each array into segments of width $\lfloor \frac{1}{2} Z, Z \rfloor$ for some parameter $Z$. Each segment is implemented with a dynamic array that we resize in accordance with its current capacity. Thus, to keep the memory allocation tight, at most one segment is resized after each access. The segments are stored in the leaves of a balanced binary search tree (we use an AVL tree [12]). When the combined cardinality of adjacent segments falls below $Z$, we merge the segments. Inversely, when the cardinality of a segment exceeds $Z$, we split the segment. The mechanics of the tree implementation keep the structure balanced. The parameter $Z$ invokes a trade-off; small $Z$ results in fast updates, as the smaller segment requires less shifting of elements and reallocation of memory, but incurs a large tree structure. On the other hand, large $Z$ induces slow updates but a small auxiliary tree structure.

The level information can be stored in any dynamic string implementation, such as a wavelet tree [26]. We adopt a run-length encoding, similar to the constituent dictionaries in Section 6. To maintain a dynamic run-length code, we apply the same method for maintaining counter intervals. The code is split into segments, implemented with dynamic arrays, and stored in the leaves of a balanced binary search tree.

### A.2 Performance

For completeness, we provide a theoretical bound for compressed-Counters. The method is suited for sequential access patterns and has poor worst-case behaviour. This could limit its application in memory constrained environments under dynamic workloads with changing distributions. In the worst-case, if the memory allocation exceeds the application bounds and this scenario is observable to the adversary, it introduces an additional side-channel.

**Lemma A.1.** For a database of size $n$, for $Z = \Theta(\log n)$, to store the block frequencies on a workload of length $n \cdot \text{poly}(n)$, compressed-Counters requires $O(n \log n)$ bits.

**Proof.** We construct a pathological workload that induces the theoretical bound. Consider an access pattern $A = \langle 1, 3, 5, \ldots, n-1 \rangle$ that is executed $c = \text{poly}(n)$ number of times. $A$ contains $n$ distinct counter intervals: $[0,0), (1, c), (2, 0), (3, c), \ldots$. As the values of the frequencies are polynomial in size, each interval requires $O(\log n)$ bits to store. Thus, the total memory allocation for the counter intervals is $O(n \log n)$ bits.

As each segment contains $\Theta(\log n)$ intervals, the auxiliary binary search tree contains $O(n / \log n)$ nodes and, with $O(\log n)$ bit pointers, occupies $O(n)$ bits.

The memory allocation of the compressed level information can be computed from its entropy. Let $X_i$ be a random variable that denotes the current level of block $i$. We assume that the access pattern is uniformly distributed, as this is the worst-case for run-length codes. If all levels are filled, then

$$\Pr[X_i = l] = 2^{L-I}.$$  

Thus, the entropy of $X_i$ is

$$H(X_i) = \sum_{i=0}^{L-1} \log_2(2^{L-I}) = \sum_{i=1}^{L} i \cdot 2^{-i} < 1. \quad (8)$$

As we calculate the run lengths $R = (R_1, R_2, \ldots)$ from the block levels $X = (X_1, \ldots, X_n)$, it holds that

$$H(R) \leq H(X) \leq \sum_{i=1}^{n} H(X_i) = n \cdot H(X_i) < n,$$

by Equation (8). Thus, the run length encoding can be stored in less than $n$ bits. Similar to the counter intervals, the auxiliary tree, with $Z = \Theta(\log n)$, has $O(n / \log n)$ nodes and occupies $O(n)$ bits. □

### A.3 Parameter Tuning

Performance of compressedCounters depends on a parameter $Z$ that we tune as follows and use for the main experiments in Section 8.

Recall that the core of the compressedCounters data structure is a dynamic array, for the counters, and a dynamic string of run lengths, for the levels. Both dynamic structures are split into segments of size $\lfloor \frac{1}{2} Z, Z \rfloor$, for a parameter $Z$, and the segments are stored in the leaves of a balanced binary search tree. To infer the effect of $Z$ on performance we tested compressedCounters on a synthetic dataset $(n = 2^{24}$ and $\phi = 1.2$ as described in Section 8.3) for values $Z \in \langle 20, 200, 2000, 20000 \rangle$. The results are displayed in Figure 3. The test demonstrates a clear trade-off between client memory and update time. For $Z = 20$ updates are fast, as only a small segment
Through \( \pi \) arranges the real blocks according to the chunk with any remaining blocks in its corresponding queue; combines is half the departure rate. Subsequently, the client, in consecutive rounds, downloads each chunk in the temporary array; combines in the correct chunk in the temporary array, that is, a chunk that

Through \( \pi \) takes as input an array of

For a

Algorithm 4: shortQueueShuffle oblivious dummy shuffle method. The algorithm takes an input a permutation \( \pi : [2n] \rightarrow [2n] \) and an array \( I \) of length \( n \) stored remotely at the server.

- \( \phi \leftarrow \text{random permutation to assign array indicies to buckets} \)
- Initialize temporary arrays \( T_1, \ldots, T_{\sqrt{n}} \) at the server
- Initialize the queues \( Q_1, \ldots, Q_{\sqrt{n}} \) at the client
- \( R \leftarrow \{ \phi(j) \mid j \in \{ (i-1) \cdot \sqrt{n}, \ldots, i \cdot \sqrt{n} - 1 \} \} \)
- \( I_i \leftarrow I[R] \)
- \( d \leftarrow [\pi(x)/\sqrt{n}] \)
- \( Q_{d} \cdot \text{push}(x) \)
- \( T \leftarrow \text{download } T_j \text{ from the server} \)
- \( T \leftarrow T \cup Q_j \)
- Remove dummies from \( T \), order elements according to \( \pi \) and insert “fresh” dummies.

\[ \text{repeat} \]
\[ \text{if } Q_j \text{ is non-empty then} \]
\[ \text{Write element from } Q_j \text{ to } T_j \]
\[ \text{else} \]
\[ \text{Write dummy element to } T_j \]
\[ \text{until Twice} \]

- Initialize output array \( O \) from the server
- \( T \leftarrow \text{download } T_j \text{ from the server} \)
- \( T \leftarrow T \cup Q_j \)
- Remove dummies from \( T \), order elements according to \( \pi \) and insert “fresh” dummies.

\[ O\{ (i-1) \cdot \sqrt{n}, \ldots, i \cdot 2\sqrt{n} - 1 \} \leftarrow T \]

return \( O \)

B SHUFFLING IN RANKORAM

For a rebuild into level \( l \), our algorithm, named shortQueueShuffle, takes as input an array of \( n_l \) untouched blocks (possibly including dummies) and produces an output array of length \( 2n_{l} \). The output array contains a random shuffling of the untouched blocks plus an additional \( n_l \) dummy blocks. This is a variation of the functionality of Definition 3.2.

For the sake of generalization, for the remainder of the exposition, we set \( n \coloneqq n_l \) and \( \pi \coloneqq \pi_l\left(S_l, \text{rank}\right) \). Similar to cacheShuffle [31], shortQueueShuffle uniformly at random assigns the indicies of the input array into \( \sqrt{n} \) buckets each of equal size \( C \approx \sqrt{n} \). Let \( I_1, \ldots, I_{\sqrt{n}} \) denote the buckets of the input indices. The client also initializes a temporary array at the server, divided into \( \sqrt{n} \) chunks of size \( 2C \). Let \( T_1, \ldots, T_{\sqrt{n}} \) denote the chunks of the temporary array.

The client initializes, in private memory, the queues \( Q_1, \ldots, Q_{\sqrt{n}} \). Through \( \sqrt{n} \) rounds, the client performs the following operations. For round \( j \):

1. Download the input chunk \( I_j \) into private memory.
2. For each real block \( x \in I_j \), let \( d \leftarrow [\pi(x)/\sqrt{n}] \), and place \( x \) in queue \( Q_d \).
3. In \( \sqrt{n} \) rounds, for each queue \( Q_k \), place two blocks in \( T_k \). If the queue is empty, place dummy blocks.

At the conclusion of this subroutine, all untouched blocks are either in the correct chunk in the temporary array, that is, a chunk that contains its final destination index, or the correct queue. The routine is named “short queue shuffle” as the arrival rate for each queue is half the departure rate. Subsequently, the client, in consecutive rounds, downloads each chunk in the temporary array; combines the chunk with any remaining blocks in its corresponding queue; arranges the real blocks according to \( \pi \); fills empty spaces with fresh dummies; and uploads the shuffled chunk to the output array. Pseudocode is provided in Algorithm 4.

The procedure is oblivious as the access pattern at the initial downloading of the input buckets does not depend on the input and the remaining accesses are identical for all inputs of the same length. However, if the combined size of the queues becomes \( \omega(\sqrt{n}) \), we exceed our memory threshold and the algorithm fails. Fortunately, this happens only with negligible probability. We summarize performance with the following Lemma.

**Lemma 5.2.** shortQueueShuffle is an oblivious dummy shuffling algorithm. On an input of length \( n \), shortQueueShuffle completes in \( 7n \) blocks of bandwidth, requires \( \sqrt{n} \) round-trips of communication and uses \( O(\sqrt{n} \cdot B) \) bits of private memory.

Due to its similarities to Lemma 4.2 in [31] and space constraints, the proof is placed in the full version of the work. Informally, the algorithm requires \( \sqrt{n} \) round-trips as each step can be iterated over each upload/download of \( \sqrt{n} \) blocks. For example, the step where blocks are iteratively placed in the temporary array segments can be completed in a single batch. For the bandwidth, the first component of the algorithm downloads \( n \) addresses and uploads \( 2n \) addresses. The second component, in a sequence of rounds, downloads the
full temporary array of length \(2n\) and uploads it to the output array. This leads to a total bandwidth cost of \(7n\).

C  ALGORITHM 5

**Algorithm 5:** Query algorithms for a run-length code dictionary \(D\). The code is split into segments of size \(W\) and supported by the auxiliary arrays \(A\), where \(A[i]\) denotes the virtual address at the beginning of segment \(i\), and \(P\), where \(P[i]\) denotes a pointer to segment \(i\).

1. **define** membership\((a, D)\)
   2. \(i \leftarrow \text{return index of binary search on } A \text{ with input } a\)
   3. \(\text{// } a \in [A[i], A[i+1])\)
   4. \(\text{ptr } \leftarrow P[i]\)
   5. \(\text{temp } \leftarrow \text{return address at location ptr in } D\)
   6. **while** \(a \leq \text{temp}\) **do**
   7. \(\text{if } a = \text{temp then}\)
   8. \(\text{return true}\)
   9. \(\text{else }\)
   10. \(\text{// move to the next element in } D\)
   11. \(\text{ptr } \leftarrow \text{ptr } + 1\)
   12. \(\text{temp } \leftarrow \text{return address at location ptr in } D\)
   13. **return** false

12. **define** index\((i, D)\)
   13. \(j \leftarrow \lfloor i/W \rfloor\)
   14. \(i' \leftarrow i - j \cdot W\)
   15. \(\text{ptr } \leftarrow P[j]\)
   16. \(\text{ptr } \leftarrow \text{ptr } + i' \text{// iterate through } i' \text{ prefix codes}\)
   17. **return** address at location ptr in \(D\)