Application of Extension Analytic Hierarchy Process in Spanning Tree

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Abstract. In view of the problem how to choose the minimum spanning tree firstly, a minimum spanning tree was found by using broken circle. Secondly connected graph was reduced to obtain a reduced graph. Using broken circle method get all the minimum spanning tree, finally, it uses the extension analytic hierarchy process (EAHP), build the hierarchy, for all of the minimum spanning tree of outlier, aggregation coefficient, average path length, network sorting efficiency, optimal spanning tree, extension analytic hierarchy process provides the theoretical foundation for the screening of minimum spanning tree.

1. Introduction

The minimum spanning tree algorithm includes Kruskal algorithm, Prim algorithm, Sollin algorithm and loop breaking method. In general, more than one the minimum spanning tree was obtained there may be multiple schemes for the minimum spanning tree. YAN guojian proposed an algorithm to solve all minimum spanning trees by using the "breaking circle method. When to find the minimum spanning tree, get the number of the minimum spanning tree is not the only spanning tree structure is not the same, the connection between nodes is not the same, if the node is the number of the need to install a device, such as the number of switches, due to the difference of degree of node, the investment demand is different, the selection of the minimum spanning tree has the difference, for example in the logistics network, logistics node function, logistics nodes are heterogeneous, complete unit logistics cost is different also, it has been a new problem, how to choose from all of the minimum spanning tree optimal spanning tree, The analytic hierarchy process is used to evaluate all spanning trees, and the optimal spanning tree is obtained. Zhou yusheng applied the extension analytic hierarchy process to the evaluation of line lightning protection transformation, and obtained the best lightning protection measures for each base tower, which had good lightning protection effect. Wu tonghan applied the extension analytic hierarchy process to the research on the type determination of maintenance support equipment, and obtained the functional requirements of each combat equipment by setting a reasonable threshold, so as to achieve reasonable matching. Han lihong combined the extension analytic hierarchy process with the fuzzy comprehensive evaluation method and finally obtained the comprehensive evaluation value. An example proved the rationality and effectiveness of the method. Li rong applied the fuzzy analytic hierarchy process to the location of logistics center, and obtained that the maximum weight of traffic conditions was the location of logistics center, which provided a reliable theoretical basis. Yang yu established a comprehensive evaluation index system by using extenics matter-element analysis to establish an evaluation model for regional logistics site selection, which was proved to be scientifically reasonable by an example.
When calculating the minimum spanning tree, the number of minimum spanning trees obtained is not unique, the structure of the spanning tree is also different, and the connection between nodes is also different. If the node degree requires the installation of a certain device, the number of switches for water pipelines and natural gas pipelines is different due to different node degrees and investment requirements. For example, in the logistics network, the functions of logistics nodes are different, and the logistics nodes are different. Qualitative, the cost of completing unit logistics is also different, which brings up a new problem—how to select the optimal minimum spanning tree from all the minimum spanning trees. Evaluation, the highest score is the best spanning tree. Extension is a method discipline to solve contradictory problems. The theoretical pillar of extension is matter-element theory, and logical cells are primitives. Based on extension set theory and method, the judgment matrix conforming to the requirement of consistency is fused into extension.

2. Algorithm of Finding All Minimum Spanning Trees by Breaking Loop
Step 1: Use the breaking circle method to find the minimum spanning tree $T_0$.
Step 2: Identify the reduced graph
1. Let $P = \{p_1, p_2, \ldots, p_i\}$ be a connected set, $i = 1$
2. Add $p_i$ to $T_0$ to form a basic loop $f$, if $p_i$ is the only longest side in the basic loop $f$, remove $p_i$ from graph $G$, if not, leave $p_i$ in graph $G$.
3. $i = i + 1$, if $i$ is less than or equal to $k$, then return to (2)
4. Output the reduced graph $G'$
Step 3: Seek a fixed edge (retain the only shortest edge in the basic cut set).
1. Let $E = \{e_1, e_2, \ldots, e_n\}$ be the branch set of the minimum spanning tree $T_0$, then find the branch set of the branch $e_i$ in the reduced graph $G'$, $S = \{S_1, S_2, \ldots, S_n\}$.
2. If $e_i$ is the only shortest edge in the basic cut set $S_i$, then take $e_i$ as the fixed edge and remark $S_i$.
3. $i = i + 1$, if $i$ is less than or equal to $n$, return to (2)
Step 4: Find switching-in edge (if there is a remark in the basic cut set, it’s a fixed edge, if there is not remark, then solve swap-in edge in form it)
1. Let $H$ be a set of swap-in edge and $F$ be a set of swap-out edge, the initial $H$ and $F$ are empty, $i = 1$
2. If there is a remark in the basic cut set $S_i = \{p_1, p_2, \ldots, p_d\}$, then $e_i, e_i$ is a fixed edge, perform (8)
3. If there is no remark in the basic cut set $S_i$ of $e_i$, then $e_i$ is put into $F$.
4. Let $k = 1$
5. If $e_i \neq p_k$ and the weight $w(e_i) = w(p_k)$, $p_k$ is put into $H$;
6. $k = k + 1$
7. If $k < d$ (d is the length of $S_i$, that is the number of the elements), return to (5)
8. $i = i + 1$, if $i \leq n - 1$, return 2)

3. Extension Analytic Hierarchy Process
3.1. Construct Extension Judgment Matrix
When using EAHP method, establishes the hierarchy, for some factor at $k - 1$ layer and the matching of all factors at $k$ layer, the pairwise comparison, using a quantitative representation extension interval number the relative important degree among them, so as to construct an extension of interval judgment matrix $A$, which said $a_{ij} = \{a_{ij}^-, a_{ij}^+\}$ extension interval number, in order to make
quantitative judgment matrix of each element, the value of extension interval number \( (a^+ - a^-)/2 \) is used in AHP analytic hierarchy process (AHP) T.J.Saaty 1-9 scale integer. When using EAHP method, establishes the hierarchy, some factors, for the first layer and the matching of all factors, The pairwise comparison, using a quantitative representation extension interval number the relative important degree among them, so as to construct an extension of interval judgment matrix, which said a extension interval number, in order to make quantitative judgment matrix of each element, the value of extension interval number is used in AHP analytic hierarchy process (AHP) T.J.Saaty 1-9 scale integer.

Extension judgment matrix \( A = (a_{ij})_{n \times n} \) is positive reciprocal matrix:

\[
a_{ii} = 1, \quad a_{ji} = a_{ij}^{-1} = \left( \frac{1}{a_{ij}}, \frac{1}{a_{jj}} \right), \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, n
\]

(1)

3.2. Calculation of Comprehensive Extension Judgment Matrix and Weight Vector

Set \( a'_{ij} = (a_{ij}^{+}, a_{ij}^{-}) \) \((i, j = 1, 2, \ldots, n, t = 1, 2, \ldots, T)\) is the extension interval number given by an expert, according to the formula

\[
a'_{ij} = (a_{ij}^{+}, a_{ij}^{-}) \quad (i, j = 1, 2, \ldots, n, t = 1, 2, \ldots, T)
\]

(2)

The formula is the comprehensive extension interval of the \( k \) layer, and the extension judgment matrix of the \( k \) layer for an element of the \( k-1 \) layer is obtained. The steps for the weight of the matrix satisfying the consistency condition are:

(1) find the normalized eigenvector \( x^+, x^- \) corresponding to the maximum eigenvalue of \( A^+, A^- \).

(2) Calculated by the \( A^+ = (a_{ij}^+)_{n \times n}, \quad A^- = (a_{ij}^-)_{n \times n} \):

\[
k = \sum_{j=1}^{n} \frac{1}{\sum_{i=1}^{n} a_{ij}^+}, \quad m = \sum_{j=1}^{n} \frac{1}{\sum_{i=1}^{n} a_{ij}^-}
\]

(3) Find the weight vector

\[
S^k = \left( S^k_1, S^k_2, \ldots, S^k_n \right) = \left( kx^+, mx^- \right)
\]

(3)

3.3. Single Hierarchical Arrangement

Let \( a = (a^+, a^-), \quad b = (b^+, b^-) \) be two extension interval Numbers, \( V(a \geq b) \) is obtained by the following formula:

\[
V(a \geq b) = \frac{2(a^+ - b^-)}{(b^+ - b^-) + (a^+ - a^-)}
\]

By the formula of \( V\left(S^i_j \geq S^j_i\right)(i = 1, 2, \ldots, n, \quad i \neq j) \),

\[
P^k_{ji} = 1, P^k_{ij} = V\left(S^i_j \geq S^j_i\right) \quad (i = 1, 2, \ldots, n, \quad i \neq j)
\]

(4)
Where $P_{ih}^k$ represents the single ordering of the $i$ factor in the $k$ layer on the $h$ factor in the $k-1$ layer, and after normalization, $P_{ih}^k = \left( P_{ih1}^k, P_{ih2}^k, \cdots, P_{ihn_{k-1}}^k \right)^T$ represents the weight vector of the single ordering of the $h$ factor in the $k-1$ layer on each factor in the $k$ layer.

### 3.4. Overall Ranking Levels

When solve for $P_{ih}^k = \left( P_{ih1}^k, P_{ih2}^k, \cdots, P_{ihn_{k-1}}^k \right)^T$, $h = i, 1, 2, \cdots, n_{k-1}$, we get the $n_k \times n_{k-1}$ matrix

$$P^k = \left( P_{11}^k, P_{12}^k, \cdots, P_{1n_{k-1}}^k; \right.$$

$$P_{21}^k, P_{22}^k, \cdots, P_{2n_{k-1}}^k; \right.$$

$$\vdots; \right.$$

$$P_{n_{k-1}}^k, P_{n_{k-1}1}^k, \cdots, P_{n_{k-1}n_k}^k \right)$$

The ranking vector $W^{k-1} = \left( W_{1}^{k-1}, W_{2}^{k-1}, \cdots, W_{n_{k-1}}^{k-1} \right)^T$ of the general objective in layer $k-1$, Then, the composition order $W^k$ of all elements in $k$ layer to the general target can be calculated by the following formula:

$$W^k = \left( W_{1}^k, W_{2}^k, \cdots, W_{n_k}^k \right)^T = P^k W^{k-1}$$

(5)

generally speaking

$$W^k = P^k P^{k-1} \cdots P^2 W^2$$

(6)

Here $W^2$ is actually just single sort vector

### 4. Example

Taking connected graph 1 as an example, all the minimum spanning trees can be obtained by breaking loop method.

![Connected graph](image)

**Figure 1.** Connected graph

The reduced graph obtained by removing the single largest edge:
all minimum spanning trees is solved by breaking loop Algorithm:

The extension analytic hierarchy process was used to comprehensively evaluate the four evaluation indexes of each minimum spanning tree, namely, the ratio of isolated nodes, aggregation coefficient, average path length and network efficiency. The optimal minimum spanning tree was selected as the final preferred scheme.

Table 1. Factor values for each minimum tree

| Minimum spanning tree | ratio of isolated nodes | aggregation coefficient | average path length | network efficiency |
|-----------------------|-------------------------|-------------------------|---------------------|--------------------|
| Minimum spanning tree | 3/7                     | 7/2                     | 2.9                 | 0.5238             |
| Minimum spanning tree | 3/7                     | 41/12                   | 2.045               | 0.5532             |
| Minimum spanning tree | 2/7                     | 18/5                    | 2.714               | 0.5786             |
| Minimum spanning tree | 3/7                     | 41/12                   | 2.045               | 0.5532             |

Step 1: According to the requirements of the overall objective, the decision makers participating in the evaluation will score the pairwise comparison of each index to obtain the extension interval judgment matrix of the evaluation criteria, and calculate the extension comprehensive weight vector of the
comparison of each evaluation index to the target layer.

Table 2. Judgment matrix of extension interval between criterion layer and target layer

|   | $C_1$    | $C_2$    | $C_3$    | $C_4$    |
|---|----------|----------|----------|----------|
| $C_1$ | $<1,1>$  | $<0.14,0.15>$ | $<0.31,0.36>$ | $<0.32,0.34>$ |
| $C_2$ | $<6.9,7.1>$ | $<1,1>$  | $<4.8,5.2>$  | $<3.9,4.1>$  |
| $C_3$ | $<2.8,3.2>$ | $<0.19,0.21>$ | $<1,1>$  | $<1.9,2.1>$  |
| $C_4$ | $<2.9,3.1>$ | $<0.24,0.26>$ | $<0.48,0.53>$ | $<1,1>$  |

$$A^* = \begin{bmatrix} 1 & 0.24 & 0.38 & 0.59 \\ 2.83 & 1 & 1.33 & 1.54 \\ 1.33 & 0.38 & 1 & 1 \\ 0.85 & 0.28 & 0.5 & 1 \end{bmatrix} , A^* = \begin{bmatrix} 1 & 0.37 & 0.75 & 1.25 \\ 4.17 & 1 & 2.67 & 3.59 \\ 2.67 & 0.75 & 1 & 2 \\ 1.75 & 0.4 & 1 & 1 \end{bmatrix}$$

Calculated from (3) $x^-=(0.0623,0.6163,0.1833,0.1381)^T$ $x^+=(0.0624,0.6107,0.189,0.1379)^T$, $k=0.956$, $m=1.011$, and then $S_1=<0.059,0.063>$, $S_2=<0.589,0.0617>$, $S_3=<0.175,0.191>$, $S_4=<0.132,0.0139>$ and then $V(S_2 \geq S_1)=34.875$, $V(S_3 \geq S_1)=13.2$, $V(S_4 \geq S_1)=14.51$, Calculated from (4):

$$P_1=1, P_2=34.875, P_3=13.2, P_4=14.51$$

The single ranking of four evaluation indexes to the total objective is obtained by standardization. $P=(0.016,0.548,0.208,0.228)$.

Step 2: According to the factor values of each evaluation index of each program, the judgment matrix of each program relative to a certain evaluation criterion is obtained through pairwise comparison. Then the ranking weight of each scheme relative to each evaluation index is obtained

Table 3. Judgment matrix between scheme layer and criterion layer

|   | $C_1$    | $C_2$    | $C_3$    | $C_4$    |
|---|----------|----------|----------|----------|
| $<1,1>$ | $2.9,3.1>$ | $0.19,0.2$  | $0.48,0.53>$ | $<1,1>$  |
| $<0.32,0.34>$ | $1,1>$  | $0.14,0.15>$  | $0.31,0.36>$ | $<1,1>$  |
| $<4.9,5.1>$ | $6.8,7.2>$ | $<1,1>$  | $2.8,3.1>$  | $<1,1>$  |
| $<1.9,2.1>$ | $2.8,3.2>$ | $<0.32,0.36>$ | $<1,1>$  | $<1,1>$  |
| $<0.32,0.34>$ | $<1,1>$  | $0.19,0.2$  | $<0.32,0.34>$ | $<1,1>$  |
| $<0.48,0.53>$ | $<1,1>$  | $0.19,0.2$  | $<0.48,0.53>$ | $<1,1>$  |
| $<3.9,4.1>$ | $<1,1>$  | $1.8,2.2>$  | $<4.9,5.1>$ | $<1,1>$  |
| $<2.9,3.1>$ | $<3.8,4.2>$ | $<0.45,0.56>$ | $<1,1>$  | $<1,1>$  |

Table 4. Single sorting table of each scheme level at each quasi-test layer

|   | $u_1$ | $u_2$ | $u_3$ | $u_4$ |
|---|------|------|------|------|
| $C_1(0.016)$ | 0.1368 | 0.0653 | 0.5820 | 0.2159 |
| $C_2(0.548)$ | 0.0976 | 0.0620 | 0.5684 | 0.2721 |
| $C_3(0.208)$ | 0.1248 | 0.0778 | 0.4918 | 0.3056 |
| $C_4(0.228)$ | 0.1104 | 0.0577 | 0.5471 | 0.2848 |

Step 3: total hierarchical sorting
Table 5. General sorting table

| Minimum spanning tree $u_1$ | Minimum spanning tree $u_2$ | Minimum spanning tree $u_3$ | Minimum spanning tree $u_4$ |
|----------------------------|----------------------------|----------------------------|----------------------------|
| Total points               | 0.107                      | 0.064                      | 0.548                      | 0.281                      |
| rank                       | 3                          | 4                          | 1                          | 2                          |

The extension analytic hierarchy process EAHP used to comprehensively sort the ratio of isolated nodes, aggregation coefficient, average path length and network efficiency of all the minimum spanning trees to obtain the best spanning tree,

![Figure 4. The best minimum spanning tree](image)

5. Conclusion
This paper combines the broken circle method and the extended analytic hierarchy process to filter the best spanning tree. Among them, the idea of the broken circle method is to reduce the workload and improve the efficiency for all the minimum spanning trees. The tree provides a feasible and effective evaluation method. In future research, more network attributes can be considered as the basis for selecting the best spanning tree.

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