Distributed Design of Sensor Fault-Tolerant Control for Preserving Comfortable Indoor Conditions in Buildings

Panayiotis M. Papadopoulos * Vasso Reppa *
Marios M. Polycarpou * Christos G. Panayiotou *

* KIOS Research and Innovation Center of Excellence
Department of Electrical and Computer Engineering
University of Cyprus, Nicosia, 1678, Cyprus
(e-mail: papadopoulos.panagiotis@ucy.ac.cy, reppavasso@gmail.com,
mpolycar@ucy.ac.cy, christosp@ucy.ac.cy).

**Abstract:** This paper proposes a distributed fault-tolerant control (FTC) scheme that can preserve thermal comfort conditions in a multi-zone building despite the presence of faulty temperature sensors. The proposed methodology exploits the networked structure of a Heating, Ventilation and Air-Conditioning (HVAC) system controlling the temperature of physically interconnected zones in order to design a distributed FTC control scheme comprised of a set of dedicated control agents. For each control agent, two adaptive bounds on the tracking error are derived, taking into account: (i) healthy sensor measurements and (ii) a single sensor fault. Each adaptive bound constitutes a condition that allows the selection of an appropriate local control gain such that the thermal comfort conditions are satisfied. By utilizing the decisions of a sensor fault diagnosis scheme, the controller gain can be reconfigured to compensate the effects of sensor faults. The proposed methodology is illustrated by simulating a sensor fault in a 3-zone HVAC system.

**Keywords:** distributed control, fault tolerance, building automation, sensor faults.

1. INTRODUCTION

The Heating, Ventilation and Air Conditioning (HVAC) system is an important component in the smooth and efficient operation of buildings. Firstly, the HVAC system accounts for a large percentage of the energy consumed, reaching 40% of the total energy in commercial buildings and 30% in non-commercial buildings (Pérez-Lombard et al., 2008). In addition, the HVAC system is responsible for ensuring the comfort of occupants and therefore its normal operation is crucial for their productivity and health.

There are several works focusing on the design of energy efficient HVAC systems, either during the design phase or at the operation phase (Hyvärinen and Kärki, 1996). Some of the feedback control approaches that have been used for reducing energy consumption of HVAC systems include: (i) determining the optimal set points (Lu et al., 2005; Vakiloroaya et al., 2013), and (ii) determining the control inputs using model predictive control (Aswani et al., 2012; Ma et al., 2015).

The potential of energy waste in HVAC systems has significant impact on the outdoor environment as well as on the operational cost of buildings. Moreover, it has significant impact on the indoor conditions of buildings and the comfort of the occupants. Most works so far have focused on reducing the waste of energy in HVAC systems, while there are only few works that investigate the preservation of thermal comfort either under normal (healthy) operation (Liang and Du, 2005) or under faulty conditions (Liu and Dexter, 2001; Gunes et al., 2015).

The HVAC system is a complex system comprised of a range of electromechanical components and building zones that may be physically interconnected. One of the major causes of uncomfortable indoor conditions in buildings is the unmeasured disturbances. To reduce the effect of unmeasured disturbances, often new sensors need to be installed. A distributed control framework can provide the ability to install new sensors easily, since there is no need for redesigning the overall control scheme (Chen et al., 2010; Moroşan et al., 2010; Ma et al., 2011).

In a distributed monitoring and control architecture, sensor data is critical since it is not only used locally but it is also exchanged in the network of the distributed control agents. Thus, a single sensor fault can impact not only the local control agent, but also its neighbors due to the exchange of information. While the primary task is to detect and isolate sensor faults (Thumati et al., 2011; Beghi et al., 2013; Mulumba et al., 2015; Papadopoulos et al., 2015a, 2017), it is important to handle as quickly as possible the consequences of any sensor faults.

* This work was supported by the European Research Council under the ERC Advanced Grant ERC-2011-AdG-291508 and by the European Union’s Horizon 2020 research and innovation programme under grant agreement No 739551 (KIOS CoE).
Fault-tolerant control (FTC) schemes can compensate fault effects in control systems by deploying the appropriate remedial actions, classified as (i) fault accommodation and (ii) control reconfiguration (Blanke et al., 2016). Fault accommodation accounts for adjusting the parameters of the controllers to compensate the effects of faults, while for performing control reconfiguration the inputs and outputs of the controller are changed to reduce the effect of faults. Moreover, fault-tolerant control schemes can be categorized as data-driven (Jain et al., 2016) and model-based (Darure et al., 2016; Papadopoulos et al., 2015b). In order to learn the fault characteristics, data-driven methodologies require huge amount of data, while on the other hand model-based methodologies need an analytical model that characterizes the behavior of the system.

The goal and contribution of this paper is to design a distributed fault-tolerant control (FTC) scheme for preserving thermal conditions in the presence of sensor faults. The objective is to obtain the necessary conditions for the control gains not only to stabilize the local controlled subsystem but also to achieve thermal comfort for both healthy and faulty conditions. The computation of the appropriate control gains takes into consideration bounds on the measurement noise, modeling uncertainty and sensor faults. During the healthy operation of the HVAC system, the distributed sensor FTC scheme utilizes the computed control gains obtained to achieve thermal comfort. By employing a sensor fault diagnosis scheme the distributed sensor FTC scheme is activated when a sensor fault is detected and isolated and then its local control gain is reconfigured in order to achieve thermal comfort in the presence of the sensor fault despite the possible propagation of the sensor fault effects. In previous works of the authors (Reppa et al., 2014; Papadopoulos et al., 2015b), the faulty sensor outputs were reconstructed without changing the structure of the controller. Here on the other hand the goal is to change the control gain in order to compensate the effects of sensor faults.

The paper is organized as follows: Section 2 provides the description of the HVAC system and the conditions for thermal comfort in buildings. Section 3 presents the distributed FTC scheme. Simulation results are presented in Section 4 to illustrate the efficacy of the proposed distributed FTC scheme. Section 5 provides some concluding remarks.

2. PROBLEM FORMULATION

2.1 HVAC system description

The multi-zone HVAC system can be regarded as a network of \( N + 1 \) interconnected subsystems \( \Sigma^1, \Sigma^2, \ldots, \Sigma^N \), where \( \Sigma^i \) represents the temperature dynamics of the storage tank and \( \Sigma^0 \) represents the temperature dynamics of the i-th building zone. An example of a 3-zone HVAC system is illustrated in Fig. 1(a). Based on the physical model presented in (Papadopoulos et al., 2017), the 3-zone HVAC system that describes the temperature dynamics of the storage tank can be expressed as

\[
\Sigma^0: \begin{align*}
    \dot{T}_{st}(t) &= A^0 T_{st}(t) + g^0(T_{st}(t)) u_{st}(t) \\
    &+ h^0(T_{st}(t), T_{z}(t), u(t)) + \eta^0(T_{pl}(t)) + r^0(t)
\end{align*}
\]

where \( T_{st} \in \mathbb{R} \) models the local state of subsystem \( \Sigma^0 \) and \( u_{st} \in \mathbb{R} \) denotes its local control input. The variable \( r^0(t) \in \mathbb{R} \) models unknown disturbances affecting the water temperature dynamics. The vector \( T_z \in \mathbb{R}^{N} \) is the interconnection vector that includes the states of neighboring subsystems (temperatures of all building zones), where \( T_{z} \) is the air temperature of the interconnected building zone \( i \) (i.e. state of subsystem \( \Sigma^i \)), and \( u \in \mathbb{R}^{N} \) where \( u \) is the control input of subsystem \( \Sigma^i \). The terms \( g^0 \in \mathbb{R} \) and \( h^0 \in \mathbb{R} \) describe the nonlinear local and interconnection dynamics of the subsystem \( \Sigma^0 \), respectively, and are defined as

\[
\begin{align*}
    g^0(T_{st}) &= \frac{U_{st,max}}{C_{st}} P_s(T_{st}), \\
    h^0(T_{st}, T_{z}, u) &= \frac{a_{sz}}{C_{st}} \sum_{i=1}^{N} U_{i,max}(T_{st} - T_{zi}) u_i,
\end{align*}
\]

with \( A^0 = -\frac{a_{sz}}{C_{st}}, U_{st,max} \) is the rated capacity of the heat pump and \( \eta^0(T_{pl}) \) denotes known exogenous input with \( \eta^0(T_{pl}) = \frac{a_{sz}}{C_{pl}} T_{pl} \), where \( T_{pl} \) is the temperature of the plenum (duct) in the storage tank. Each subsystem \( \Sigma^i, i \in \{1, \ldots, N\} \) is interconnected with subsystems \( \Sigma^* \) and \( \Sigma^j, j \in K_i \) (see Fig. 1(b)) with \( K_i = \{ j : a_{zi,j} \neq 0 \} \) where \( a_{zi,j} \) is the inter-zone heat loss coefficient between i-th and j-th zone due to the presence of walls. The subsystem \( \Sigma^0 \) is described by

\[
\Sigma^0: \begin{align*}
    \dot{T}_{zi}(t) &= A^1(T_{zi}(t) + g^1(T_{st}(t), T_{zi}(t)) u_i(t) \\
    &+ h^1(T_{zi}(t), T_{K_i}(t)) + \eta^1(T_{i1}(t), T_{amb}(t)) + r^1(t)
\end{align*}
\]

where \( T_{K_i}(t) = [T_{zi}(t) : j \in K_i] \) is the vector of length \( \text{card}(K_i) \) (where \( \text{card}(\cdot) \) denotes the cardinality of a set), where each element corresponds to the state \( T_{zi} \) of the neighboring subsystem \( \Sigma^j, j \in K_i \). The variable \( r^1(t) \in \mathbb{R} \) models the unknown system disturbances of the subsystem \( \Sigma^i \), and \( A^0 = \frac{a_{zi}}{C_{zi}} - \frac{1}{C_{zi}} \sum_{j \in K_i} a_{zi,j} \). The terms \( g^1 \in \mathbb{R} \) and \( h^1 \in \mathbb{R} \) denote the local and interconnection nonlinear dynamics of the subsystem \( \Sigma^i \), respectively, described by

\[
\begin{align*}
    g^1(T_{st}, T_{zi}) &= g^0(T_{st} - T_{zi}), \\
    h^1(T_{zi}, T_{K_i}) &= \frac{1}{C_{zi}} \sum_{j \in K_i} a_{zi,j} T_{zi} + p^1 \left( \sum_{j \in K_i} \text{sgn}(T_{zi} - T_{zi}) \right) \\
    &\times A_{dz,ij} \max(T_{zi}, T_{zi}) \sqrt{|T_{zi} - T_{zi}|} \\
    \eta^1(T_{i1}, T_{amb}) &= \frac{a_{z,zi}}{C_{zi}} T_{i1} - \frac{hA_{zi}}{C_{zi}} T_{amb},
\end{align*}
\]

with \( p^1 = U_{i,max} a_{zi,j} \cdot p^1 = \rho a_{zi} C_a \sqrt{C_a - C_z} \), where \( U_{i,max} \) is the is maximum water flow rate in the i-th fan-coil unit, \( C_a \) is the heat capacity of the i-th zone, \( T_{i1} \) is the wall temperature of the i-th zone, \( T_{amb} \) is the ambient temperature, \( a_{zi,j} \) is the effectiveness of the fan-coil, \( A_{zi,j} \) is the area of the walls in the i-th zone and \( A_{dz,ij} \) is the area of the door connecting i-th and j-th zone. The constant \( C_a \) is the specific heat at constant pressure \( C_a = 1.004 \text{ kJ/kgK} \), \( C_z \) is the specific heat at constant volume \( C_z = 0.717 \text{ kJ/kgK} \) and \( \rho a_{zi} \) is the air density \( \rho a_{zi} = 1.22 \text{ kg/m}^3 \).

A general representation of a model-based distributed control scheme is shown in Fig. 1(c) for a 3-zone HVAC system.
The controller 
by the following control laws, i.e.,

\[ u_{st}(t) = s(t), u_i(t) = s_i(t) \]

\[ \text{sat}(u(t)) = \begin{cases} 0, & u(t) < 0 \\ u(t), & u(t) \in [0, 1] \\ 1, & u(t) > 1 \end{cases} \]

Note that the saturation in (4) is an outcome of the physical constraints of the system.

The water temperature of \( \Sigma^* \) (storage tank) is measured by the sensor \( S^* \), i.e.,

\[ S^* : y^*(t) = T_{st}(t) + n^*(t) \]

where \( y^* \in \mathbb{R} \) is the sensor output and \( n^* \in \mathbb{R} \) is the measurement noise. The output of the sensor \( S^{(i)} \) used to measure the air temperature of subsystem \( \Sigma^{(i)} \) is expressed as

\[ S^{(i)} : y^{(i)}(t) = T_{zi}(t) + n^{(i)}(t) + f^{(i)}(t) \]

where \( y^{(i)} \in \mathbb{R} \) is the sensor output and \( n^{(i)} \in \mathbb{R} \) is the measurement noise. The signal \( f^{(i)} \in \mathbb{R} \) denotes a permanent sensor fault (Reppa et al., 2016). The unknown terms satisfy the following assumptions:

**Assumption 1:** For all \( t \geq 0 \), the modeling uncertainties \( r^*(t), r^{(i)}(t) \) and noise measurements \( n^*(t), n^{(i)}(t) \) are uniformly bounded such that \( |r^*(t)| \leq \mathcal{P}^* \), \( |r^{(i)}(t)| \leq \mathcal{P}^{(i)} \), \( |n^*(t)| \leq \mathcal{N}^* \), and \( |n^{(i)}(t)| \leq \mathcal{N}^{(i)}, i \in \mathcal{N} \).

**Assumption 2:** For all \( t \geq t_f^{(i)} \), where \( t_f^{(i)} \) is the time instant that fault has occurred, the permanent sensor fault is bounded \( f^{(i)}(t) \in [f^{(i)}_{\min}, f^{(i)}_{\max}] \), for all \( i \in \mathcal{N} \).

### 2.2 Thermal Comfort Conditions in Buildings

In this section we discuss the criteria for indoor thermal comfort with respect to the air temperature in zones. The Predicted Percentage of Dissatisfied (PPD) is an index that establishes a quantitative prediction of the percentage of thermally dissatisfied people (Fanger, 1970) and is described by

\[ PPD = 100 - 95e^{-(0.3535(\text{PMV}(T_{zi}))^2 + 0.2179(\text{PMV}(T_{zi}))^2)} \]

The acceptable range of thermal comfort recommended by ANSI/ASHRAE (2010) for an interior space is \( \text{PMV} \in [-0.5, 0.5] \) that leads to \( PPD = 10\% \), i.e. 10% of the occupants is predicted to be dissatisfied. Based on this acceptable value of \( PPD \), we can determine a thermal comfort set \( T_{ci} \), derived by solving (5) with \( PPD = 10\% \), i.e.,

\[ T_{zi} \in T_{ci} = [T_{zi}^{\min}, T_{zi}^{\max}] \]

where \( T_{zi}^{\min} \) and \( T_{zi}^{\max} \) are the minimum and maximum values of air temperature that provide indoor thermal comfort conditions in zone \( i \).

The objective of this work is to design a distributed control scheme that satisfies the desired thermal comfort conditions described by (6) for all zones of the HVAC building system in the presence of healthy and faulty sensor measurements.

### 3. DISTRIBUTED SENSOR FAULT-TOLERANT CONTROL DESIGN

The design of the proposed distributed FTC is realized taking into account Assumptions 1 and 2, and the occurrence of a sensor fault affecting a single zone only. The distributed feedback control scheme is constructed by the control agents \( C^* \) and \( C^{(i)} \) that generate the control signals \( u_{st}^* \) and \( u_i^* \), defined in (2) and (3). The goal of
Based on the isolation signal uncertainties only, and can be proved (see Section 3.2) that under healthy conditions described by (11) satisfies
\[\bar{x}(t) \in \left[\bar{x}(t), \bar{x}(t)\right], \forall t \geq T_f.
\]

Taking into account Assumption 2, it can be proved (see Section 3.3) that the tracking error under faulty conditions is designed such that
\[\bar{x}(t) \in \left[\bar{x}(t), \bar{x}(t)\right] + T_{z_i}^\text{min}, -T_{z_i}^\text{min} + T_{z_i}^\text{max}.
\]

In the next section, we analyze the tracking error \(\bar{x}(t)\) defined in (10).

In the sequel, the dependence of the signals on time (e.g. \(x(t)\)) will be dropped for notation brevity.

### 3.2 Analysis of the tracking error

In this section the tracking error \(\bar{x}(t)\) is obtained analytically by applying the proposed distributed control scheme presented in (7)–(8) to (1). Hence, the dynamics of the tracking error \(\bar{x}(t)\) are computed as
\[\dot{\bar{x}}(t) = \dot{\bar{x}}(t) - \dot{\bar{x}}(t) = A(t) \bar{x}(t) + g(t) \bar{x}(t) \bar{u}(t) + h(t) \bar{x}(t) + \bar{h}(t) \bar{x}(t) + r(t).
\]

Adding and subtracting the terms \(g(t) \bar{x}(t) \bar{u}(t)\) and \(h(t) \bar{x}(t)\) on (14) results in
\[\dot{\bar{x}}(t) + A(t) \bar{x}(t) = \bar{x}(t) + g(t) \bar{x}(t) \bar{u}(t) + r(t).
\]

Assuming that \(a_i \in [0,1]\), we have \(u_i = u_i^c\). After some mathematical manipulations and by adding and subtracting the term \(K(t)\), the dynamics of tracking error \(\bar{x}(t)\) are computed by
\[\dot{\bar{x}}(t) = A(t) \bar{x}(t) + \bar{g}(t) \bar{u}(t) + \bar{h}(t) \bar{x}(t) + K(t) \bar{x}(t) - \bar{y}(t) + r(t),
\]

where \(\bar{g}(t) = g(t) \bar{x}(t) - \bar{y}(t)\) and \(\bar{h}(t) = h(t) \bar{x}(t)\) with
\[
\bar{g}(t) = \sigma(t) \left( \bar{n}(t) - n^s(t) + f(t) \right),
\]
\[
\bar{h}(t) = \mu(t) \left( T_{z_i}^n - \mu(t) \bar{y}(t) + y(t) \right) + \sum_{j \in K_i} \bar{u}(j) + f(t),
\]
and
\[\mu(t) \in \sigma(t) \left( \bar{n}(t) - n^s(t) + f(t) \right)\]

### 3.3 Tracking error under healthy conditions

Assuming healthy conditions, we have \(f(t) = 0, i \in N\). The healthy sensor measurements are given by
\[\bar{y}^H(t) = T_{st}^H + n^s(t),\]
\[\bar{y}(t) = T_{st} + n(t),\]
\[\bar{y}^H(t) = T_{K_i}^H + n^s(t),\]
\[\bar{y}^H(t) = T_{K_i} + n^s(t),\]
\[\bar{y}(t) = T_{K_i} + n(t),\]
\[\bar{y}^H(t) = T_{K_i} + n^s(t),\]
where $T^H$, $T^H_i$ are the water temperature and air temperature of the $i$-th zone under healthy conditions, respectively, and $T_{K_i}$ is a vector collects the air temperatures of the $|K_i|$ zones under healthy conditions. Using (15)–(17) with $f(i) = 0$, the dynamics of $\overline{x}_i(t)$ are given by

$$\begin{align*}
\dot{\overline{x}}_i(t) &= \left(A(i) - K_{H_i}(i)\right)\overline{x}_i(t) + \sigma(i)\left(n(i) - n^s\right) u^H_i \\
&+ h(i)(T_{z_i}, T_{K_i}) - h(i)(y^H_i, y^{K_i}) + r(i) - K_{H_i}n_i.(18)
\end{align*}$$

The bound on the tracking error is computed assuming healthy sensor measurements. The solution of (18) is given by

$$\begin{align*}
\overline{x}_i(t) &= e^{\left(A(i) - K_{H_i}(i)\right)t}\overline{x}_i(0) \\
&+ \int_0^t e^{\left(A(i) - K_{H_i}(i)\right)(t-\tau)}\sigma(i)\left(n(i)(\tau) - n^s(\tau)\right) u^H_i(\tau)d\tau \\
&+ \int_0^t e^{\left(A(i) - K_{H_i}(i)\right)(t-\tau)} \left(-K_{H_i}n_i(\tau) + r(i)(\tau)\right)d\tau \\
&- h(i)(T_{z_i}(\tau), T_{K_i}(\tau)) - h(i)(y^H_i(\tau), y^{K_i}(\tau))d\tau.
\end{align*}$$

Equivalently, (12) can be expressed as

$$|\overline{x}_i(t)| \leq \overline{x}(i).$$

Applying the triangular inequality on (19) and based on Assumption 1, $\overline{x}_i(t)$ results in

$$\begin{align*}
\overline{x}_i(t) &= e^{\left(A(i) - K_{H_i}(i)\right)t}\overline{x}_i(0) - \left(1 - e^{\left(A(i) - K_{H_i}(i)\right)t}\right)A(i) - K_{H_i}(i) - \sum_{j \in K_i} \sigma(j)\left(n(j) - n^s(j)\right) u^H_j(\tau)d\tau \\
&+ \sigma(i)\left(n^s + \overline{\pi}(i)\right) \int_0^t e^{\left(A(i) - K_{H_i}(i)\right)(t-\tau)} u^H_i(\tau)d\tau \\
&+ \sigma(i)\left(n^s + \overline{\pi}(i)\right) \int_0^t e^{\left(A(i) - K_{H_i}(i)\right)(t-\tau)} \sum_{j \in K_i} A_{ij} u^H_j(\tau)(y^H_i(\tau) - y^H_j(\tau))d\tau.
\end{align*}$$

where $|\overline{x}_i(t)| \leq \overline{x}(i)$ with $\overline{x}(i) = T_{z_i}(0) - \overline{x}_{\text{ref}}(0)$ and the function $\overline{\pi}(i)$ is such that

$$|\overline{\pi}(T_{z_i}, T_{z_j}) - \overline{\pi}(y^H_i, y^H_j)| \leq \overline{\pi}(y^H_i, y^H_j).$$

whose computation is given in the Appendix of (Papadopoulos et al., 2017). In order to sustain stability and thermal comfort in zone $i$ under healthy conditions, $K_{H_i}$ should be selected such that satisfies the following conditions for all $t \geq 0$

$$\begin{align*}
A(i) - K_{H_i}(i) < 0, \\
u^H_i \in [0, 1], \\
|\overline{x}_i(t), \overline{x}_i(t)| \leq -x_{\text{ref}}(i) + T_{z_i}^{\min}, -x_{\text{ref}}(i) + T_{z_i}^{\max}.
\end{align*}$$

3.4 Tracking error under local sensor faults

For the following analysis on the tracking error $\overline{e}(i)$ a local bias sensor fault $f(i)$ is considered. The remainder sensor measurements are healthy with $f^H(i) = 0$. The sensor measurements under a local fault in the sensor $S(i)$ are given by

$$\begin{align*}
y^s_i &= T_{z_i} + n^s_i, \\
y^i &= T_{z_i} + n^i + f(i), \\
y^K_i &= T_{K_i} + n^K_i.
\end{align*}$$

where the sensor fault can be written as

$$f(i) = \begin{cases} 
0, & t < t_\text{ref}(i) \\
f_0(i) + \overline{f}(i), & t \geq t_\text{ref}(i),
\end{cases}$$

where $f_0(i)$ is the constant offset of the sensor fault $f(i)$, $\overline{f}(i)$ is the deviation of the offset $f_0(i)$ from the actual fault value $f(i)$ and $t_\text{ref}(i)$ is the time instant that fault has occurred.

Based on Assumption 2, the offset $f_0(i)$ can be described by

$$f_0(i) = 0.5\left(f_{\text{min}}(i) + f_{\text{max}}(i)\right),$$

and the deviation $\overline{f}(i)$ satisfies

$$|\overline{f}(i)| \leq 0.5\left(f_{\text{max}}(i) - f_{\text{min}}(i)\right) = \overline{f}(i).$$

The dynamics of the tracking error $\overline{e}(i)$ can be expressed as

$$\begin{align*}
\dot{\overline{e}}(i) &= \left(A(i) - K_{F}(i)\right)\overline{e}(i) + \left(g^H(i)(T_{z_i}, T_{z_j}) - g^H(y^s_i, y^H_i)\right) u_i \\
&+ h(i)(T_{z_i}, T_{K_i}) - h(i)(y^H_i, y^{K_i}) - \left(g^H(i)(T_{z_i}, T_{z_j}) - g^H(y^s_i, y^H_i)\right) u_i - h(i)(T_{z_i}, T_{K_i}) + h(i)(y^H_i, y^{K_i}) \\
&- K_{F}(i)\left(f(i) + n^i\right) + g^H(i)(y^s_i, y^H_i)(u_i - u^c_i) + r(i).
\end{align*}$$

where $\overline{e}(i)(t_\text{ref}(i)) = \overline{e}_i(t_\text{ref}(i))$. By designing the controller such that $u^c_i = [0, 1]$ for all $t \geq t_\text{ref}(i)$ the last term of (27) is equal to zero. Using (16), the dynamics of the tracking error can be described by

$$\begin{align*}
\dot{\overline{e}}(i) &= \left(A(i) - K_{F}(i)\right)\overline{e}(i) - K_{F}(i)f_0(i) - K_{F}(i)\overline{f}(i) - K_{F}(i)n(i) \\
&+ \sigma(i)\left(f_0(i) + \overline{f}(i)\right) u_i + \sigma(i)\left(n(i) - n^s_i\right) u_i + r(i) \\
&+ h(i)(T_{z_i}, T_{K_i}) - h(i)(y^H_i, y^{K_i}).
\end{align*}$$

The bounds on the tracking error are computed under the scenario of a single bias sensor fault occurring locally at the subsystem $S(i)$. For $\overline{e}(i) = \overline{e}(i) - \left(A(i) - K_{F}(i)\right)^{-1}K_{F}(i)f_0(i)$, (28) can be expressed as

$$\begin{align*}
\overline{e}(i) &= \left(A(i) - K_{F}(i)\right)\overline{e}(i) - K_{F}(i)\overline{f}(i) + \sigma(i)\left(f_0(i) + \overline{f}(i)\right) u_i + \sigma(i)\left(n(i) - n^s_i\right) u_i \\
&- K_{F}(i)n(i) + r(i) + \sigma(i)\left(n(i) - n^s_i\right) u_i \\
&+ h(i)(T_{z_i}, T_{K_i}) - h(i)(y^H_i, y^{K_i}).
\end{align*}$$

The solution of (29) is given by
\[ z_F^{(i)} = \left( A^{(i)} - K_F^{(i)} \right) \left( t-t_i^{(i)} \right) \left( f_o^{(i)} \right) \]
\[ - K_F^{(i)} \int_{t_i^{(i)}}^{t} e^{\left( A^{(i)} - K_F^{(i)} \right) \left( t-t' \right)} \left( f_o^{(i)} \right) \left( n_i^{(i)}(t') \right) \right) dt \]
\[ + \sigma \left( f_o^{(i)} \right) \int_{t_i^{(i)}}^{t} e^{\left( A^{(i)} - K_F^{(i)} \right) \left( t-t' \right)} \left( u_i(t') \right) dt \]
\[ + \sigma \int_{t_i^{(i)}}^{t} e^{\left( A^{(i)} - K_F^{(i)} \right) \left( t-t' \right)} \left( n_i^{(i)}(t') \right) dt \]
\[ + \sigma \int_{t_i^{(i)}}^{t} \left( A^{(i)} - K_F^{(i)} \right) \left( t-t' \right) \left( n_i^{(i)}(t') - n^*(t') \right) \left( u_i(t') \right) dt \]
\[ + \int_{t_i^{(i)}}^{t} e^{\left( A^{(i)} - K_F^{(i)} \right) \left( t-t' \right) \sum_{j \neq i} a_{ji} C_{i, j} \left( n_i^{(i)}(t') \right) dt \]
\[ + \int_{t_i^{(i)}}^{t} e^{\left( A^{(i)} - K_F^{(i)} \right) \left( t-t' \right) \sum_{j \neq i} A_{di,j} \left( \mu_i^{(i)}(T_{si}(t), T_{ti}(t)) \right) dt \]
\[ - \mu_i^{(i)}(y_i^{(i)}(t), y_i^{(i)}(t)) + r^{(i)}(t) \right) \]
\[ \text{where } z_F^{(i)} = \left( A^{(i)} - K_F^{(i)} \right) \left( t-t_i^{(i)} \right) \left( f_o^{(i)} \right) \]
\[ - K_F^{(i)} \int_{t_i^{(i)}}^{t} e^{\left( A^{(i)} - K_F^{(i)} \right) \left( t-t' \right)} \left( f_o^{(i)} \right) \left( n_i^{(i)}(t') \right) \right) dt \]
\[ + \sigma \left( f_o^{(i)} \right) \int_{t_i^{(i)}}^{t} e^{\left( A^{(i)} - K_F^{(i)} \right) \left( t-t' \right)} \left( u_i(t') \right) dt \]
\[ + \sigma \int_{t_i^{(i)}}^{t} e^{\left( A^{(i)} - K_F^{(i)} \right) \left( t-t' \right)} \left( n_i^{(i)}(t') \right) dt \]
\[ + \sigma \int_{t_i^{(i)}}^{t} \left( A^{(i)} - K_F^{(i)} \right) \left( t-t' \right) \left( n_i^{(i)}(t') - n^*(t') \right) \left( u_i(t') \right) dt \]
\[ + \int_{t_i^{(i)}}^{t} e^{\left( A^{(i)} - K_F^{(i)} \right) \left( t-t' \right) \sum_{j \neq i} a_{ji} C_{i, j} \left( n_i^{(i)}(t') \right) dt \]
\[ + \int_{t_i^{(i)}}^{t} e^{\left( A^{(i)} - K_F^{(i)} \right) \left( t-t' \right) \sum_{j \neq i} A_{di,j} \left( \mu_i^{(i)}(T_{si}(t), T_{ti}(t)) \right) dt \]
\[ - \mu_i^{(i)}(y_i^{(i)}(t), y_i^{(i)}(t)) + r^{(i)}(t) \right) \]
Figures 3(a) and 3(c) illustrate the results with $K_H^{(2)} = 60$, while Figures 3(b) and 3(d) present the results by applying the distributed sensor FTC presented in (9). Figures 3(a) and 3(b) show the tracking error $\mathcal{E}^{(2)}$ (blue line), the bounds on the tracking error given in (20)-(21) before sensor fault isolation and (33) after sensor fault isolation (magenta line) and the interval derived from the comfort bounds presented in (13) (blue dashed line). Figures 3(c) and 3(d) present the reference temperature $x_{\text{ref}}^{(2)}$ (black dashed line), the air temperature $T_{z_2}$ (red dotted line), the sensor measurements $y^{(2)}$ (green line) and the thermal comfort interval $T_2$ (blue dashed lines) given in (36).

Based on Figures 3(a)–(d) we notice that before the fault occurrence $t \leq t_f^{(2)} = 0.5$ hours, the tracking error $\mathcal{E}^{(2)}$ and the air temperature $T_{z_2}$ do not violate the comfort bounds (blue dotted line) after a transient period. Furthermore, the lower and upper bound of the tracking error (magenta line) converge inside the comfort bounds which indicates that the temperature will be inside the comfort bounds regardless of the noise measurements of $S^{(2)}$ and the modeling uncertainty of zone 2. For $t > t_f^{(2)} = 0.5$ hours, in Fig. 3(a), the tracking error (blue line) and its lower and upper bounds (magenta lines) are outside the thermal comfort bounds (blue dashed lines) and in Fig. 3(c), the air temperature (red dotted line) violates also the comfort bounds $T_2$ (blue dashed lines) since the healthy control gain $K_H^{(2)} = 60$ is used for this simulation.

On the contrary, in Figures 3(b) and 3(d), the FTC scheme compensates the effects of the sensor fault $f^{(2)}$, since in Fig. 3(b) the tracking error $\mathcal{E}^{(2)}$ and the lower and upper bound of the tracking error (magenta line) converge inside the interval derived from the comfort bounds (blue dashed lines), and in Fig. 3(d) the air temperature $T_{z_2}$ (red dotted line) converges inside the thermal comfort bounds (blue dashed lines) in presence of the sensor fault.

5. CONCLUSION AND FUTURE WORK

This work presents a distributed sensor FTC control scheme for maintaining indoor thermal comfort conditions in a multi-zone HVAC system with faulty measurements. Through the analysis of the tracking error, analytical conditions are derived with respect to the controller gain for preserving stability and indoor thermal comfort, taking into account bounds on uncertainty, measurement noise and sensor faults. Based on the decision of a fault diagnosis scheme to detect and isolate the occurrence of a single sensor fault, the controller gain of the local control scheme is reconfigured to satisfy the analytical conditions. The methodology was evaluated in the presence of a single sensor fault for a 3-zone HVAC building system.

Future work will involve the optimal design of the control gains under both healthy and faulty conditions, and the comparison of the proposed FTC scheme to existing ones that perform model predictive control.

REFERENCES

ANSI/ASHRAE (2010). Standard 55-2010: Thermal Environmental Conditions for Human Occupancy.
ASHRAE.
Aswani, A., Master, N., Taneja, J., Krioukov, A., Culler, D., and Tomlin, C. (2012). Energy-Ecient Building HVAC Control Using Hybrid System LBMPC. In Proceedings of 4th IFAC Nonlinear Model Predictive Control Conference, 496–501.

Beghi, A., Cechinato, L., Corso, L., Rampazzo, M., and Simmimi, F. (2013). Process history-based Fault Detection and Diagnosis for VAVAC systems. In Proceedings of the IEEE International Conference on Control Applications, 1165–1170.

Blanke, M., Kimnaert, M., Lunze, J., and Staroswiecki, M. (2016). Diagnosis and Fault-Tolerant Control. Springer Berlin Heidelberg.

Chen, J., Cao, X., Cheng, P., Xiao, Y., and Sun, Y. (2010). Distributed Collaborative Control for Industrial Automation With Wireless Sensor and Actuator Networks. IEEE Transactions on Industrial Electronics, 57(12), 4219–4230.

Darure, T., Yame, J.J., and Hamelin, F. (2016). Fault-Fanger, P.O. (1970). Thermal comfort. Analysis and

Chen, J., Cao, X., Cheng, P., Xiao, Y., and Sun, Y. (2010). Distributed Collaborative Control for Industrial Automation With Wireless Sensor and Actuator Networks. IEEE Transactions on Industrial Electronics, 57(12), 4219–4230.

Darure, T., Yame, J.J., and Hamelin, F. (2016). Fault-adaptive Control of VAV damper stuck in a Multizone Building. In Proceedings of the IEEE Conference on Control and Fault-Tolerant Systems (SysTol), 170–176.

Fanger, P.O. (1970). Thermal comfort. Analysis and applications in environmental engineering. Copenhagen: Danish Technical Press.

Farrell, J.A. and Polycarpou, M.M. (2006). Adaptive approximation based control: Unifying neural, fuzzy and traditional adaptive approximation approaches, volume 48. Wiley-Interscience.

Gunes, V., Peter, S., and Givargis, T. (2015). Improving Energy Efficiency and Thermal Comfort of Smart Buildings with HVAC Systems in the Presence of Sensor Faults. In Proceedings of the IEEE International Conference on Embedded Software and Systems, 945–950.

Hyvärinen, J. and Kärki, S. (1996). Building Optimization and Fault Diagnosis Source Book. Technical Research Centre of Finland, VTT Building Technology.

Jain, T., Yame, J.J., and Sauter, D. (2016). Data-driven fault-tolerant control for energy efficiency in a multi-zone building. In Proceedings of the IEEE International Conference on Control, Automation, Robotics and Vision (ICARCV), 1–6.

Li, J. and Du, R. (2005). Thermal comfort control based on neural network for HVAC application. In Proceedings of IEEE Conference on Control Applications, 819–824. IEEE.

Liu, X.F. and Dexter, A. (2001). Fault-tolerant supervisory control of VAV air-conditioning systems. Energy and Buildings, 33(4), 379–389.

Lu, L., Cai, W., Xie, L., Li, S., and Soh, Y.C. (2005). HVAC system optimization – in-building section. Energy and Buildings, 37(1), 11–22.

Ma, Y., Anderson, G., and Borrelli, F. (2011). A distributed predictive control approach to building temperature regulation. In Proceedings of the American Control Conference, 2089–2094.

Ma, Y., Matusko, J., and Borrelli, F. (2015). Stochastic Model Predictive Control for Building HVAC Systems: Complexity and Conservatism. IEEE Transactions on Control Systems Technology, 23(1), 101–116.

Moroşan, P.D., Bourdais, R., Dumur, D., and Buisson, J. (2010). Building temperature regulation using a distributed model predictive control. Energy and Buildings, 42(9), 1445–1452.

Mulumba, T., Afshari, A., Yan, K., Shen, W., and Norford, L.K. (2015). Robust model-based fault diagnosis for air handling units. Energy and Buildings, 86, 698–707.

Papadopoulos, P.M., Reppa, V., Polycarpou, M.M., and Panayiotou, C.G. (2015a). Distributed Adaptive Estimation Scheme for Isolation of Sensor Faults in Multi-zone HVAC Systems. In Proceedings of 9th IFAC Symposium on Fault Detection, Supervision and Safety for Technical Processes, 1146–1151.

Papadopoulos, P.M., Reppa, V., Polycarpou, M.M., and Panayiotou, C.G. (2015b). Distributed Adaptive Sensor Fault Tolerant Control for Smart Buildings. In Proceedings of 53th IEEE Conference on Decision and Control, 3143–3148.

Reppa, V., Papadopoulos, P., Polycarpou, M.M., and Panayiotou, C.G. (2017). Distributed Diagnosis of Actuator and Sensor Faults in HVAC Systems. In Proceedings of 20th IFAC World Congress, 4293–4293.

Pérez-Lombard, L., Ortiz, J., and Pout, C. (2008). A review on buildings energy consumption information. Energy and buildings, 40(3), 394–398.

Panayiotou, C.G. (2014). A distributed virtual sensor scheme for smart buildings based on adaptive approximation. In Proceedings of IEEE International Joint Conference on Neural Networks, 99–106.

Papadopoulos, P.M., Reppa, V., Polycarpou, M.M., and Panayiotou, C.G. (2016). Sensor Fault Diagnosis. Foundations and Trends in Systems and Control, 3(1-2), 1–248.

Thumati, B.T., Feinstein, M.A., Fonda, J.W., Turnbull, A., Weaver, F.J., Calkins, M.E., and Jagannathan, S. (2011). An Online Model-based Fault Diagnosis Scheme for HVAC Systems. In IEEE International Conference on Control Applications, 70–75.

Vakiloroaya, V., Ha, Q., and Samali, B. (2013). Energy-efficient HVAC systems: Simulation-empirical modelling and gradient optimization. Automation in Construction, 31, 176–185.