Numerical Study on Deflection Behaviour of Concrete Beams Reinforced with GFRP Bars

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Abstract. Fiber-Reinforced Polymer (FRP) bars are gaining popularity as sustainable alternatives to conventional reinforcing steel bars in reinforced concrete applications. The production of FRP bars has lower environmental impact compared to steel reinforcing bars. In addition, the non-corroding FRP materials can potentially decrease the cost or need for maintenance of reinforced concrete structural elements, especially in harsh environmental conditions that can impact both concrete and reinforcement. FRP bars offer additional favourable properties including high tensile strength and low unit weight. However, the mechanical properties of FRP bars can lead to large crack widths and deflections. The objective of this study is to investigate the deflection behaviour of concrete beams reinforced with Glass FRP (GFRP) bars as a longitudinal main reinforcement. Six concrete beams reinforced with GFRP bars were modelled using the finite element computer program ANSYS. The main variable considered in the study is the reinforcement ratio. The deflection equations in current North American codes including ACI 440.1R-06, ACI 440.1R-15 and CSA S806-12 are used to compute deflections, and these are compared to numerical results. It was concluded in this paper that deflections predicted by ACI 440.1R-06 equations are lower than the numerical analysis results while ACI 440.1R-15 is in agreement with numerical analysis with tendency to be conservative. The values of deflections estimated by CSA S806-12 formulas are consistent with results of numerical analysis.

1. Introduction
FRP materials have been used for decades in the aeronautical, aerospace, and automotive industries in addition to numerous other applications. Fibres used for manufacturing composite materials have typically high tensile strength, toughness, and durability. GFRP reinforcing bars are made primarily of glass fibres, but interest in Basalt FRP fibres is on the rise. GFRP bars are non-corroding, therefore, can extend the life of reinforced concrete structures significantly and reduce maintenance, repair, and replacement costs. It was shown by Mohamed and Al Hawat [1] that chlorides in aggressive environments can penetrate through self-consolidating concrete reaching and damaging reinforcing steel, therefore, impacting the service life of reinforced concrete structures. It is therefore desirable to replace reinforcing steel with the environmentally friendly GFRP bars, especially in combination with sustainable concrete in which significant amount of cement is replaced with other sustainable cementitious materials such as fly ash and silica fume [2]. Deflection of concrete beams reinforced with GFRP is of particular interest to structural designers, and is typically a function of moment of inertia and material properties. The design of GFRP reinforced concrete beams is typically governed by serviceability limit state requirements. This is because the modulus of elasticity of GFRP bars is much smaller than that of steel, therefore, affects the deformation response of GFRP reinforced beams. For example, the elastic modulus of GFRP bars is between 20 and 25 percent of that in steel bars. Due to the low elastic modulus of GFRP bars, the deflection criterion may control the design of medium- and long-span beams reinforced with GFRP bars.
Yost et al. [3] conducted an experimental study and proposed a model for deflection calculation in beams reinforced with GFRP bars. The parameters in their study included concrete compressive strength, reinforcement ratio and shear span-to-depth ratio. The results of forty eight simply supported concrete beams reinforced with GFRP were compared with the ACI 440.1R-06 [4] model. The authors concluded that the ratio of gross to cracked section properties ($I_g/I_c$) of concrete reinforced with GFRP bars is between four and eight times higher than similar steel-reinforced concrete samples.

Bischoff [5] evaluated the commonly used equations to compute short-term deflection for steel and FRP reinforced concrete beams. Gilbert [6] stated that Branson’s [7] approach consistently underestimates short-term deflection of lightly reinforced members as well as members reinforced with low-modulus of elasticity FRP bars.

El-Nemre et al. [8] investigated the deflection of GFRP-reinforced concrete beams. The test matrix included nine beams reinforced with different types and ratios of GFRP bars. The test results were compared against the predicted values using the CSA S806-02 [9], ACI 440.1R-06 [4], ISIS Manual No.3 [10], and the ACI 440-H [11]. The tested beams showed consistently a cracking moment that is lower than the predicted values using the CSA S806-02 [9], ACI 440.1R-06 [4], and ISIS Manual No.3 [10] equations. The authors stated that the ACI 440.1R-06 and ACI 440-H [11] underestimated the deflection of all test specimens. The average deflection ratio $\delta_{exp}/\delta_{pred}$ for ACI 440.1R-06 and ACI 440-H was 1.20±0.28 and 1.15±0.19 with a corresponding COV of 24% and 16%, respectively.

Doo-Yeol Yoo et al. [12] experimentally investigated the flexural behaviour of four Ultra High Performance Fiber Reinforced Concrete (UHPFRC) beams. The beams were reinforced with various ratios of GFRP bars. The authors studied the service deflections and developed equations for cracking moment and moment of inertia at cracked section. The authors concluded that ACI 440.1R-06 [4] and modified-Branson’s models significantly overestimated the service deflections of UHPFRC beams reinforced with GFRP bars, primarily due to inappropriate estimation of the cracking moment. On the other hand, the modified-ACI 440.1R-15 [13] was consistent with the experimental results. This conclusion is based on the average ratio of the deflections at service load obtained from experiments and modified-ACI 440.1R-15 [13] model of 0.98.

2. Determination of the effective moment of inertia

Faza and Ganga Rao [14] proposed Eq. (1) for the effective moment of inertia of FRP reinforced concrete beams. This model was based on the assumption that concrete section between point loads is fully cracked and the end sections are partially cracked. Therefore, the cracking moment of inertia, $I_c$, was used in the middle third section of the beam, while the current Branson equation, $I_s$, is used in the end sections.

$$I_{mod} = \frac{23I_c I_e}{8I_c + 15I_e}$$

(1)

Based on experimental program, Benmokrane et al. [15] proposed Eq. (2) for the effective moment of inertia of flexural members reinforced with FRP bars.

$$I_e = \left(\frac{M_c}{M_a}\right)^3 I_g + 0.84 \left[1 - \left(\frac{M_c}{M_a}\right)^3\right] I_c \leq I_g$$

(2)

Where,

- $M_c$ is the cracking moment, $M_a$ is the applied moment, $I_g$ is the moment of inertia of the gross section, and $I_c$ is the moment of inertia of the cracked transformed concrete section.

Mota et al. [16] proposed Eq. (3) for the effective moment of inertia of concrete reinforced with FRP bars.
Habeeb and Ashour [17] tested six specimens to investigate the behaviour of continuous GFRP-reinforced concrete beams. They proposed Eq. (4) by introducing a post-cracking stiffness reduction factor, $\gamma_d$, into Branson’s equation. The authors suggested $\gamma_d = 0.6$ gives best results for continuous beams and $\gamma_d = 1.0$ offers best results for simply supported beams.

$$I_e = \left( \frac{M_{cr}}{M_a} \right)^5 I_g + \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^5 \right] I_{cr} \leq I_g$$  \hspace{1cm} (3)

Eq. (5) is the ACI 440.1R-06 [4] model for calculation of effective moment of inertia, $I_e$, which is a modified version of Branson’s equation [18]. The modification is done by introducing the parameter, $\beta_d$, which is proportional to the ratio between the actual and balanced reinforcement ratios as indicated by Eq. (6).

$$I_e = \left( \frac{M_{cr}}{M_a} \right)^3 \beta_d I_g + \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] \leq I_g$$  \hspace{1cm} (5)

$$\beta_d = \frac{1}{5} \left( \frac{\rho_f}{\rho_{fb}} \right) \leq 1$$  \hspace{1cm} (6)

ACI 440.1R-15 [13] uses Eq. (7) for the estimation of the cracked moment of inertia, $I_{cr}$, which is based on Bischoff et al. [19]. This formula includes an additional factor $\gamma$ to account for the variation in stiffness along the length of the member as shown in Eq. (8) which was introduced by Bischoff and Gross [20]. The deflection prediction entails calculating a uniform moment of inertia throughout the beam length.

$$I_e = \frac{I_{cr}}{1 - \gamma \left( \frac{M_{cr}}{M_a} \right)^2 \left[ 1 - \frac{I_{cr}}{I_g} \right]} \leq I_g$$  \hspace{1cm} (7)

$$\gamma = 1.72 - 0.72 \left( \frac{M_{cr}}{M_a} \right)$$  \hspace{1cm} (8)

Eq. (9) was used by Bischoff and Gross [20] to calculate the actual deflection of a simply-supported beams loaded by concentrated load.

$$\delta_{max} = \frac{PL^3}{24E_cI_e} \left[ 3L^2 - 4a^2 \right]$$  \hspace{1cm} (9)

Where,

$\delta_{max}$ is the mid-span deflection (mm), $L$ is the span (mm), $a$ is the shear span (mm), $E_c$ is the modulus of elasticity of concrete (MPa), $I_{cr}$ is the moment of inertia of cracked section transformed to concrete (mm$^4$), and $L$ is the span length (mm).

CAN/CSA S806-12 [21] recommends curvature integration along the span to determine the deflection of a concrete member at any point, assuming the section is fully cracked with no contribution of
tension stiffness in the beam’s cracked regions. Therefore, the moment–curvature relation of FRP-reinforced concrete members can be assumed to be bilinear, where the curvature $\Psi$ is given by $M_a/E_cI_g$ for the uncracked parts of the beam, followed by an increase in curvature at a constant moment value (transition from uncracked to cracked stage) and $M_a/E_cI_cr$ for the cracked part when the applied moment $M_a$ is higher than the cracking moment ($M_{cr}$).

CAN/CSA S806-12 [21] provides Eq. (10) for the deflection of simply supported members subjected to two-point loading where the cracked moment of inertia ($I_{cr}$) is calculated from Eq. (11).

\[
\delta_{\text{max}} = \frac{PL^3}{24E_cI_{cr}} \left[ 3\left(\frac{a}{L}\right)^3 - 4\left(\frac{a}{L}\right)^2 - 8 \left(1 - \frac{I_{cr}}{I_g}\right) \left(\frac{L_g}{L}\right)^3 \right]
\]

(10)

\[
I_{cr} = \frac{bd^3}{3}k^3 + n_f A_f d^2 (1-k)^2
\]

(11)

Where,
- $\delta_{\text{max}}$ is the mid-span deflection (mm),
- $L$ is the span (mm),
- $a$ is the shear span (mm),
- $E_c$ is the modulus of elasticity of concrete (MPa),
- $I_{cr}$ is the moment of inertia of cracked section transformed to concrete (mm$^4$),
- $L_g$ is the un-cracked length (mm),
- $k$ is the ratio of depth of neutral axis to reinforcement depth,
- $n_f$ is the ratio of modulus of elasticity of FRP bars to modulus of elasticity of concrete, and
- $P$ is the applied load (kN)

3. Cracking Moment

The cracking moment, $M_{cr}$, is predicted using Eq.(12).

\[
M_{cr} = \frac{f_c I_g}{\gamma_f}
\]

(12)

The concrete modulus of rupture, $f_c$, is calculated using Eq. (13) in accordance with ACI 440.1R [10] & [11] and using Eq. (14) in accordance with CAN/CSA S806-12 [21].

\[
f_c = 0.62\lambda \sqrt{f_{c'}}
\]

(13)

\[
f_c = 0.60\lambda \sqrt{f_{c'}}
\]

(14)

Where,
- $f_c$ is the modulus of rupture (MPa),
- $f_{c'}$ is the concrete compressive strength (MPa), and
- $\lambda$ is a factor accounting for concrete density. For normal weight concrete, $\lambda = 1$.

4. Finite Element Modelling

ANSYS R12.1 [22] computer program was used in this study for finite element modelling of reinforced concrete. SOLID65 element is used to model the plain concrete material, since it has a capability of both cracking in tension and crushing in compression. SOLID65 element is defined by 8 nodes with three degrees of freedom at each node; translations in the nodal x-, y-, and z-directions. The element material is assumed to be initially isotropic. In addition to tension cracking and crushing, SOLID65 incorporates plastic and creep behaviours. The LINK8 element used to model the reinforcing steel and GFRP bars. It is a uniaxial tension-compression member that can include nonlinear material properties. The element comprises two nodes with three degree of freedom at each one. Elastic-perfectly plastic representation is assumed for the reinforcing bars. To ensure the necessary conditions for transfer of load, Link8, element should be located between two or more SOLID65 elements, otherwise, the forces would be only transferred at the nodes of Solid65 and the element Link8 is not effective in representing a reinforcing bar. The elements are shown in Figure 1. The crack modelling adopted by ANSYS [22] program is the smeared crack representation. The model includes the shear transfer coefficient, $\beta_s$, which represents the
shear strength reduction factor for the subsequent loads, which induce sliding shear across the crack face. If the crack closes, then all compressive stresses normal to the crack plane are transmitted and only a shear reduction factor, $\beta_c$, for a closed crack is introduced. Typical shear transfer coefficients range from zero, representing a smooth crack, to 1.0, representing a rough crack. In the present analysis, $\beta_t$ was taken as 0.1 and $\beta_c$ was taken as 0.8.

5. Model Description
In this paper, six beams were modelled and analysed using ANSYS software. The beams were longitudinally reinforced with GFRP bars. The mean reinforcement ratio, $\rho_s\%$, varied from 0.37% to 1.19. Concrete compressive strength, $f'_c$, is 35MPa. To prevent shear failure of the beams and to insure flexural failure, all beams were reinforced with 8-mm-diameter shear ties spaced @ 150mm. Beams details are shown in Figure 2. Table 1 shows specifications of beams. Table 2 shows properties of GFRP reinforcing bars.

6. Results and discussions
Table 3 shows a comparison between midspan deflections of beams obtained from the finite element model and those predicted using Eqs. (5), (7) and (10). The accuracy of the calculated cracking moment, $M_{cr}$, is key to the accuracy of the deflection calculations. Mid-span deflection at Table 3 corresponds to a cracking moment $M_{cr} = 17.5$ kN.m. Table 3 shows that increasing the reinforcement ratio of GFRP reduced the deflection by 45% . The deflections of GFRP reinforced beams were predicted using ACI
440.1R-06, ACI 440. 1R-15, and CAN/CSA S806-12 equations. The predicted numerical deflection ratio, $\delta_{\text{pred}}/\delta_{\text{Num}}$ for ACI 440.1R-06 [4] is ranged from 0.62 to 0.82 which indicates the code underestimates the deflection. However, for ACI 440.1R-15, $\delta_{\text{pred}}/\delta_{\text{Num}}$ is slightly greater than 1.0 indicating the code accurately predicts the deflections. On the other hand, for CAN/CSA S806-12, $\delta_{\text{pred}}/\delta_{\text{Num}}$ ranged from 1.18 to 1.36, indicating the code conservatively overestimates deflections.

Table 1. Specifications of beams

| Beam | Diameter of bar (mm) | No. of bars | $A_s$ (mm$^2$) | $\rho_s$ (%) |
|------|----------------------|-------------|---------------|-------------|
| GFB1 | 13                   | 2           | 258.00        | 0.37        |
| GFB2 | 16                   | 2           | 402.10        | 0.56        |
| GFB3 | 19                   | 2           | 508.90        | 0.79        |
| GFB4 | 13                   | 3           | 387.00        | 0.55        |
| GFB5 | 16                   | 3           | 603.20        | 0.84        |
| GFB6 | 19                   | 3           | 763.40        | 1.19        |

Table 2. Properties of reinforcing bars

| Diameter (mm) | Tensile Strength (MPa) | Modulus of Elasticity, $E$ (GPa) | Ultimate Strain(%) |
|--------------|------------------------|----------------------------------|-------------------|
| GFRP 13      | 817                    | 48                               | 1.68              |
| GFRP 16      | 724                    | 46                               | 1.55              |
| GFRP 19      | 690                    | 46                               | 1.58              |

Table 3. Numerical and predicted deflection

| Beam | Numerical Model Deflection (mm) | ACI 440.1R-06 | ACI 440.1R-15 | CAN/CSA S806-12 |
|------|--------------------------------|---------------|---------------|-----------------|
|      | Deflection (mm) | $\delta_{\text{Pred}}/\delta_{\text{Num}}$ | Deflection (mm) | $\delta_{\text{Pred}}/\delta_{\text{Num}}$ | Deflection (mm) | $\delta_{\text{Pred}}/\delta_{\text{Num}}$ |
| GFB1 | 12.5                  | 10.2          | 0.82          | 12.8            | 1.03            | 15.4            | 1.23             |
| GFB2 | 9.7                   | 5.5           | 0.66          | 10.3            | 1.06            | 13.2            | 1.36             |
| GFB3 | 6.9                   | 4.3           | 0.62          | 7.2             | 1.04            | 8.6             | 1.25             |
| GFB4 | 10.3                  | 7.8           | 0.76          | 10.5            | 1.02            | 13.4            | 1.30             |
| GFB5 | 8.5                   | 5.9           | 0.69          | 9.2             | 1.08            | 10              | 1.18             |
| GFB6 | 5.6                   | 4.2           | 0.75          | 6.0             | 1.07            | 6.7             | 1.20             |

7. Conclusions

This study investigated the deflection of the GFRP-reinforced concrete beams. Six concrete beams reinforced with different ratios of GFRP bars modelled using the finite element software ANSYS. The numerical results of deflection calculation were compared to the predicted values using ACI 440.1R-06, ACI 440.1R-15, and CAN/CSA S806-12. Based on the numerical results following conclusions could be drawn:

- Increasing the GFRP reinforcement ratio, decreases the mid span deflection value at the same loading level.
- ACI 440.1R-06 underestimated the deflection of beams reinforced with GFRP bars as indicated by $\delta_{\text{pred}}/\delta_{\text{Num}}$ ratios ranging from 0.62 to 0.75. However ACI 440.1R-15 showed improved capability of predicting deflections as indicated by average $\delta_{\text{pred}}/\delta_{\text{Num}}$ ratios between 1.02 and 1.08.
- CAN/CSA S806-12 provided conservative overestimation of deflections as indicated by $\delta_{\text{pred}}/\delta_{\text{Num}}$ ratios ranging from 1.18 to 1.36.
Acknowledgement
The authors gratefully acknowledge the financial support of the Center on Sustainable Built Environment at Abu Dhabi University.

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