Focusing of Drive and Test Bunches in a Dielectric Waveguide Filled with Inhomogeneous Plasma

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ABSTRACT: The paper presents the results of numerical PIC-simulation of accelerated (test) and drive bunches dynamics in a dielectric waveguide of the THz frequency range filled with radially inhomogeneous plasma. The wakefield was excited by the azimuthally-symmetric on-axis drive electron bunch in a quartz dielectric tube embedded in metal waveguide. A radial displacement of test bunch is absent too. At simulations of plasma, we used different transverse density profiles, viz., the density profile formed in the capillary discharge, and the radially nonuniform density profile with the vacuum channel along the waveguide axis. For all the cases under study the plasma density was low, so that the plasma frequency was lower than the fundamental dielectric mode frequency. The obtained PIC-simulation data have shown that the vacuum channel in the inhomogeneous plasma cylinder improves the accelerated bunch focusing. There is the optimum vacuum-channel size value, at which the focusing turns out to be the strongest. The improvement in the accelerated bunch focusing is accompanied by the decrease in the accelerating gradient as compared with the full plasma filling of the drift channel. The best acceleration takes place in the absence of plasma; however, in that case the accelerated bunch focusing does not occur.

KEYWORDS: Accelerator modelling and simulations (multi-particle dynamics; single-particle dynamics); Wake-field acceleration (laser-driven, electron-driven); Beam dynamics; Plasma generation (laser-produced, RF, x ray-produced)
1 Introduction

Cherenkov radiation generated by charged particles bunches in dielectric structures can be used to develop high-gradient wake accelerators of the high-energy range [1–5] and to obtain intense coherent radiation for industrial purposes, environment monitoring and medicine [6–14].

However, there is severe shortcoming of dielectric wakefield accelerators (DWA) namely the susceptibility of bunches to the beam breakup (BBU) instability [15, 16] peculiar to electron linear accelerators [17]. In DWA, the BBU instability results in the limitation of the achievable accelerating-field value with an increase in the drive bunch charge [18]. Besides, high charges cause the beam quality degradation. To control BBU instability it was proposed to use BNS damping [19] consisting in variation the frequency of betatron oscillations along bunch. Recently new advanced method of suppressing the BBU instability in DWA have been proposed [20]. It is consisted in combination an energy chirp of bunch together with profiled quadrupole focusing. The application of this formful method for stabilization of the BBU instability is limited by the existing quadrupole technology with limit magnetic field gradients on the order of 1 T/mm. Even with such strong magnetic fields, accelerating gradients do not exceed a few hundred MeV/m [18, 21]. In addition, the use of energy chirp to stabilize the BBU instability instabilities limits the applicability of the accelerator as a promising candidate for future colliders because of a deterioration in beam quality.

The BBU instability in dielectric waveguides is developing due to the excitation of higher-order modes by off-axis bunches [22, 23]. However even in the case of ideal alignment of bunches (on-axis bunches), or at the BBU instability suppression by methods mentioned above, there is one problem in collider and FEL devices that has been the subject of intensive research, namely, preserving emittance and required size of bunches when they move through the channel of a dielectric waveguide. In the simplest case, with axial injection of bunches the task is gone to transportation them through a channel of predetermined transverse dimensions. For an obtaining the collider-quality beams one can require an additional focusing of accelerated beam reducing a cross section of accelerated beams. These tasks can be resolved by conventional methods with using strong magnetic fields. There is alternative method to provide an focusing of electron bunches which is well-known in plasma wakefield-based accelerators [24, 25]. The plasma focuses drive bunch (plasma lens), but in the linear condition the peak of an accelerating field for accelerated
bunch corresponds to zero focusing field. This is because the longitudinal and transverse forces are created by the same monochromatic wave, a plasma wave. An idea arose to separate the focusing process and the acceleration process of witness bunch, that is, to excite two waves in the slowing structure, one would accelerate and the other would focus the bunch.

This suggestion has been recently examined in plasma dielectric wakefield accelerator (PDWA) structure, i.e. in the DWA structure with filling of drift channel by isotropic plasma [26]. The focusing in the structure occurs owing to the excitation of the plasma wakefield, and the acceleration occurs owing to the dielectric wave (the wave existing in a dielectric waveguide without plasma). In order to realize the conditions of an independence for the longitudinal and transverse forces, certain conditions on the plasma density are necessary. At first the plasma frequency have to be appreciably less the dielectric wave frequency. At this condition the dielectric wave frequency and amplitude don’t depend from the plasma frequency and thus the witness bunch acceleration is defined by the dielectric wave. Secondly, screening the space charge field has to be weak $\omega_p a/v_b \leq 1$ ($\omega_p$ is the plasma frequency, $v_b$ is a bunch velocity, $a$ is radius of the drift channel). At this condition the radial electric field of plasma wave is appreciably greater its longitudinal electric field [26, 28]. Thus, the acceleration is defined by longitudinal electric field of the dielectric wave and the focusing is defined by radial electric field of plasma wave. At this, to create a strong focusing does not require very dense plasma. In example [26] with the accelerating gradient $\sim 200 \text{ MeV/m}$, a plasma density of $\sim 4 \cdot 10^{14} \text{ cm}^{-3}$ provides a focusing force of the order of 300 MeV/m, which is equivalent to a focusing magnetic field gradient of $\sim 2 \text{ T/mm}$.

The distinctive feature of the PDWA that makes it attractive for colliders is the possibility of focusing both the electron and positron bunches [26]. Significantly different frequencies of the dielectric and plasma waves one allow to adjust the focusing phase of the accelerated bunch by changing the plasma density. Analytical studies and numerical simulations have confirmed the possibility of focusing the drive and accelerated bunches at filling the drift channel with plasma not only for the cylindrical DWA configuration [28, 29], but for the rectangular one as well [30].

The PDWA studies [26, 28, 29] were carried out for the plasma density, which is homogeneous in the transverse section of the drift channel, and can be generated by an external source. The other way of plasma creation, widely used in the studies of wakefield acceleration methods, is the capillary discharge [31–34]. In this case, at great times of the discharge onset, the nonuniform plasma density distribution is formed with the radial dependence obtained by numerical simulation of the discharge in ref. [35], which can be described by the parabolic relation for a considerable part of the channel section [36]. The PIC simulation of the test electron bunch acceleration with the model plasma-density dependences [35, 36] has shown that the capillary discharge plasma improves the focusing of accelerated bunches [37] in comparison with the case of the uniform plasma density in the section.

At present, in studies of the concept of the beam-driven plasma wakefield accelerator (PWFA), for improving the positron bunch transport it has been suggested that the hollow plasma channel should be used (see refs. [38, 39] and the references cited there). The hollow plasma channel, and in the limiting case the vacuum channel in plasma, aids in improving the transport of the electron bunch, too [27]. In the present study, we investigate by the PIC simulation the effect of the vacuum channel in the radially inhomogeneous plasma on focusing of the drive and accelerated electron bunches in the PDWA.
2 The statement of the problem

The wakefield structure under study represented a metallic cylindrical waveguide of radius $b$, into which the dielectric tube of inner radius $a$ was inserted tightly. The drift channel (region inside the dielectric tube $r \leq a$) was partially filled ($r_{p1} \leq r \leq a$) with plasma, having generally a nonuniform radial density profile.

The cylindrically-shaped, azimuthally-symmetric on-axis drive electron bunch of radius $r_b$ passed through the slow-wave structure along its axis and excited the wakefield. In a given delay time $t_{\text{del}}$, following the drive bunch, the test bunch, having the same transverse and longitudinal profile, was injected into the system along its axis, the test bunch charge being 10 times less than it was for the drive bunch. The delay time of the test bunch was chosen such that the bunch center could get simultaneously into the accelerating and focusing phase of the drive bunch wakefield. No transverse misalignment the witness bunch relative to the drive bunch, i.e. the witness bunch is on-axis one. Also we don’t account any jitter drive or witness bunch in our simulations. The initial sizes of the drive and accelerated bunches are equal. The structure with the drive and accelerated bunches is called here the plasma-dielectric wakefield accelerator (PDWA).

The initial parameters of the dielectric waveguide and drive bunch were chosen to be the same as in the studies of the wakefield excitation by the relativistic electron bunch in the PDWA with homogeneous plasma [26]. The parameters provided the wakefield excitation of the THz frequency range (the dielectric-wave mode wavelength $E_{01}$ in the absence of plasma in the transport channel was equal to $\sim 1$ mm). The parameter values of the waveguide, and also, of drive and accelerated bunches used in numerical simulation were the following: inner and outer dielectric-tube radii were equal to 0.5 mm and 0.6 mm, respectively; inner plasma-cylinder radius ($r_{p1}$) changed from 0 to 0.5 mm; waveguide length $L$ was 8 mm; permittivity was 3.75 (quartz); the energy of bunch electrons was 5 GeV with no in energy spread; drive an test bunch charges were 3 nC and 0.3 nC, respectively. Longitudinal profile of bunches was Gaussian with $\sigma_\parallel = 0.05$ mm, transverse profile of bunches was rectangular with diameter $2r_b = 0.8993$ mm. Paraxial plasma density ($r_{p1} = 0$) was $4.41 \times 10^{14}$ cm$^{-3}$. So everywhere in the plasma volume $n_b/n_p < 1/3$ for the drive bunch [26] and $n_b/n_p < 1/30$ for the witness bunch ($n_b$ is the density of bunch electrons and $n_p$ is plasma density), that is we have quasi-linear regime of wakefield excitation. Very small ratio $n_b/n_p$ for the witness bunch guarantees that repulsion of plasma electrons by witness bunch will be small and a perturbation of the wakefield excited by the drive bunch will be weak.

To describe the plasma density profile, we started from the profile of density decrease from the dielectric surface, generally realizable at the capillary discharge. It was determined numerically by Bobrova et al. [35]. At some point $r = r_{p1}$, the plasma density fell off to zero, forming the vacuum channel. In all the computations, the plasma density, extrapolated to the waveguide axis with the parabolic relation [36], was put to be $4.41 \times 10^{14}$ cm$^{-3}$, this corresponding to the plasma wavelength of $\sim 1.6$ mm. So effective longitudinal size of witness bunch $2\sigma_\parallel = 0.1$ mm is significantly less the wavelength of transverse wakefield, and head and tail of witness bunch are under the action of about the same focusing force.
3 Results of 2.5-dimensional PIC-simulation

In numerical simulation by means of our 2.5D PIC-code we investigated the wakefield topography and dynamics of electron bunches in their motion in the drift channel. The studies were performed for several variants with different inner plasma-cylinder radius $r_{p1}$ values varying in the range from 0 to 0.5 mm.

Figure 1 shows comparative shots of the force components acting on the test electron in the PDWA for the time $t = 26.69$ ps (the drive bunch has reached the structure end) at different $r_{p1}$ values. Note that the case a) corresponds to the absence of plasma in the drift channel, and the case d) — to the complete plasma filling of the said region. From now on the data shown for the case d) have been partially presented in our paper [37]. It can be seen that the cases b) to d) encourage one to choose the position of the accelerated bunch such as to accelerate and focus the bunch simultaneously.

Figure 1. Color maps and level lines for the transverse $F_r(r, z)$ (left) and longitudinal $F_z(r, z)$ (right) Lorentz force components acting on the test electron for the time $t = 26.69$ ps (the drive bunch has reached the waveguide end) at different $r_{p1}$ values: a) $r_{p1} = 0.5$ mm, b) $r_{p1} = 0.4$ mm, c) $r_{p1} = 0.2$ mm, d) $r_{p1} = 0$.

To illustrate the mechanism of the accelerated bunch focusing, figure 2 shows the phase space combined with the relations of longitudinal $F_z(z)$ and transverse $F_r(z)$ forces at $r = 0.45$ mm for...
the same time as given in figure 1 for different plasma density-radius dependences. The red color in the phase space (see figure 2) shows the electron energy of the drive bunch. The blue color shows the electron energies of the accelerated (test) bunch. The test bunch delay $\tau_{del} = 1.651$ ps was chosen such as to provide the bunch acceleration in the first local maximum of the longitudinal force $F_z$. For the cases b), c) and d) this delay also provided the transverse focusing by the transverse force $F_r$. This time was chosen basing on the simulation of the wakefield excitation at the absence of a test bunch (unloaded field) in the case of a channel completely filled with plasma and was the same for the cases a)–d).

As it follows from the plots shown in figures 1a and 2a, in the absence of plasma in the drift channel, the transverse force $F_r$ for the ultrarelativistic drive bunch is practically equal to zero. In the $r_{p1} = 0.4$ mm case, the plasma layer thickness makes 0.1 mm, and the electron bunches blow over the plasma only slightly (over 0.05 mm). At that, as it follows from figure 2b, the intense transverse force $F_r$ gets excited. The local minimum of this force, equal to $-0.55$ GeV/m, is observed in the region, where the test bunch is found, and this results in good focusing of the latter.

With a further decrease in $r_{p1}$, the plasma layer thickness also increases, and the increasing part of electrons of bunches are localized in plasma. Figures 2c and 2d illustrate the cases with $r_{p1} = 0.2$ mm and $r_{p1} = 0$, respectively. It can be seen that here the transverse focusing force $F_r$ also gets excited, but its minimum in the position of the test bunch is somewhat smaller, viz., $-0.37$ GeV/m and $-0.35$ GeV/m, respectively. As a result, the focusing of the bunch becomes weaker.

To demonstrate the test bunch focusing for the above-mentioned four plasma density cases, figure 3 shows the configuration space representing the position of the drive and test (witness) bunch electrons being at the periphery (having the maximum radius at the given $z$, $R_{max}(z)$) for the time $t = 26.69$ ps. The dependences $R_{max}(z)$ for the drive and the test bunches can be interpreted.
Figure 3. Configuration space representing the position of the bunch electrons being at the periphery for $t = 26.69$ ps at different $r_{p1}$ values: a) $r_{p1} = 0.5$ mm, b) $r_{p1} = 0.4$ mm, c) $r_{p1} = 0.2$ mm, d) $r_{p1} = 0$.

as envelopes of bunches at a fixed time, but they are only obtained without the assumption of non-intersection of particle trajectories. When the bunches move through the waveguide, their transverse profiles dynamically change, but their qualitative dependences on the longitudinal coordinate are mainly preserved.

As it follows from the plots given in figure 3, in the absence of plasma in the drift channel (case a), $r_{p1} = 0.5$ mm, no focusing of the test bunch is observed. The case b) with $r_{p1} = 0.4$ mm exhibits the strongest focusing of the test bunch. In the cases c) at $r_{p1} = 0.2$ mm and d) $r_{p1} = 0$ the test bunch focusing is weaker than in the case b). Note that in the case of full plasma filling of the drift channel (case d), the test bunch is focused worse than in the case of the vacuum channel presence (cases b) and c)).

It can be also seen in figure 3 that the drive bunch shows up the focusing only in its back part with the radius at the head part of the bunch remaining unchanged (see figure 3b, c and d). This is in agreement with the transverse force dependence $F_r(z)$ given in figure 2b, c and d.

The behavior of the radii of the drive and accelerated bunches $R_{\text{max}}$ with variation of the vacuum channel radius $r_{p1}$ from 0 to 0.5 mm for different waveguide lengths $L$: 8 mm, 16 mm and 24 mm at the times $t$: 26.69 ps, 53.38 ps and 80.07 ps (every time corresponds the time when the drive bunch has reached the structure end), respectively is shown in figure 4a. As it follows from the obtained dependences, the increase in the $r_{p1}$ from 0 up to about 0.4 mm causes the improvement in the test bunch focusing, whereas with a further increase in $r_{p1}$ the focusing gets worse. If the bunches are entirely in the vacuum channel, i.e., the plasma layer is outside the bunches, the focusing is absent. It should be noted that the radius of the vacuum channel $r_{p1} \approx 0.4$ mm also corresponds to the smallest energy spread of the accelerated bunch (see figure 4b). With an increase in the length of the accelerator structure, a decrease in the transverse size of the bunch occurs according to a harmonic law and at short distances can be interpolated by a quadratic dependence. The results of numerical simulation for the case of complete filling of the drift channel with plasma coincide with semi-analytical results [29], that confirms the correctness of carried out simulation.

To explain the behavior of the transverse size of the bunches, shown in figure 4a, let us analyze the behavior of the plasma electrons in the drift channel. Figure 5 shows the plasma electron densities $n_{pe}(r, z)$ for the time $t = 5.9$ ps at different $r_{p1}$ values.

As it appears from the plots in figure 5, the drive bunch electrons push out the plasma electrons to the periphery of the drift channel. As a consequence, the excess of plasma ions is formed behind the drive bunch. These ions attract the plasma electrons pushed out by the drive, and the last ones
Figure 4. a) Bunch radius $R_{\text{max}}$ for the waveguide length $L = 8$ mm at the time $t = 26.69$ ps (short dotted curve), $L = 16$ mm at $t = 53.38$ ps (long dotted curve) and $L = 24$ mm at $t = 80.07$ ps (continuous curve), and b) energy gain of test bunch (blue curve) and energy loss of drive bunch (red curve) versus the radius of the vacuum channel, $r_{p1}$, for $L = 24$ mm at the time $t = 80.07$ ps.

Figure 5. Plasma electrons density $n_{pe}(r, z)$ for $t = 5.9$ ps at $r_{p1} = 0.45$ mm (a), $r_{p1} = 0.4$ mm (b) and $r_{p1} = 0$ (c). The red and blue rectangles show the positions of the drive and test bunches.

Turn to the waveguide axis. Here it should be noted that the plasma ion density during the delay time of the test bunch $t_{\text{del}}$ remains practically the same.

In the case of full plasma filling of the drift channel, at $r_{p1} = 0$ (see figure 5c), the above described processes lead to the formation of the region with a reduced plasma electrons density, and hence, to the excess of plasma ions in the area, where the test bunch is present. The excess of ions not only directs the plasma electrons to the waveguide axis, but also focuses the test bunch electrons being on all radii of the bunch.

At $r_{p1} = 0.4$ mm (see figure 5b) in the area, where the test bunch is present, the plasma electrons are practically absent. Hence, the plasma ions focus the test bunch electrons being only on the radii from $r_{p1}$ to $r_b$, i.e., from 0.4 mm to 0.45 mm, this appearing, however, sufficient for attaining the maximum focusing of the bunch.
If the test bunch moves in the vacuum channel, i.e., when the plasma tube surrounds the electron bunches (see figure 5a), the excess of plasma ions as compared to the plasma electrons is observed outside the test bunch, and this has no effect on its focusing.

Our present numerical experiments have also permitted us to estimate the influence of the plasma tube dimensions on the energy variations of the accelerated and drive bunches. It can be seen in figure 2 that in the absence of plasma at \( r_{p1} = 0.5 \) mm (see figure 2a), the test bunch is accelerated more intensively than with the presence of plasma in the drift channel (see figure 2b, c and d). At the same time, at \( r_{p1} = 0.4 \) mm (see figure 2b), the accelerated electrons of the test bunch show a considerably smaller energy dispersion than at other \( r_{p1} \) values shown in figure 2. The above-stated regularities are obviously seen in figure 4b, which shows the energy changes of both the accelerated test bunch (blue curve) and the decelerated drive bunch (red curve) as functions of the vacuum channel radius \( r_{p1} \) varying from 0 to 0.5 mm for the time \( t = 26.69 \) ps. It can be also noticed from figure 4b that the smaller is the plasma tube thickness, the lower will be the energy losses and the electron energy dispersion of the driver bunch.

4 Conclusions

The carried out numerical PIC simulations of wakefield excitation, as well as of self-consistent on-axis accelerated (test) and driver bunches dynamics in the plasma-dielectric slow-wave structure with a nonuniform transverse profile of the plasma density, have demonstrated that the vacuum channel in the plasma improves focusing of the accelerated bunch. There exists the optimum vacuum channel radius, at which the focusing proves to be the best.

The improvement in the accelerated bunch focusing in the presence of the vacuum channel is accompanied by a few decrease in the gradient of energy variation as compared with the full plasma filling of the drift channel.

It should be noted that the optimal acceleration takes place if the plasma is absent in the transport channel (traditional DWA), however, in that case no focusing of the test bunch occurs.

We investigated the transportation of drive and accelerated bunches at small lengths of a plasma-dielectric waveguide only. The goal was to study the effect of inhomogeneity of the plasma and the vacuum channel on it on the focusing of on-axis bunches. At short travel distances the value of transverse bunch compression is not big, however it is significantly greater than in case of homogeneous plasma. When we lengthened the accelerator structure, the transverse size of the witness bunch decreased according to an approximately harmonic law. Our simulations were carried out with using azimuthally-symmetric on-axis drive and witness bunches. A key drawback of the DWA is the beam-breakup (BBU) instability. Whether the focusing force due to the excitation of a plasma wave is a cure to suppress BBU instability is the subject of further research. This requires the development of analytical studies and calculations at large lengths of plasma-dielectric accelerator.

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