Manipulation of a weak signal pulse by optical soliton via double electromagnetically induced transparency

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Abstract

We propose a scheme to realize the manipulation of a weak signal pulse by ultraslow optical soliton in a coherent inverted-Y-type atomic system via double electromagnetically induced transparency (EIT). Based on Maxwell-Bloch equations, we derive nonlinear equations governing the spatial-temporal evolution of the probe and signal pulse envelopes. We show the giant enhancement of optical Kerr nonlinearity can be obtained under the condition of the double EIT, which results in the generation of a (2+1)-dimension optical soliton and can realize the manipulation of a weak signal pulse. Applying a far-detuned laser field to the system, we find that a weak signal pulse can be trapped by a (3+1)-dimension light bullet. In particular, the trajectories of the light bullet and trapped signal pulse can be manipulated and controlled by introducing a Stern–Gerlach gradient magnetic field. The results predicted here may not only open a route for the study of weak-light nonlinear optics but also have potential applications in the precision measurements and optical information processing and transmission.

1. Introduction

Optical trapping is a prospective technique for manipulating material objects [1]. The principle is that a high intensity laser light transfers some of its momentum to small particles and hence a force acts on the particles, which has significant applications in many research fields [2–5]. In recent years, much attention has been focused on soliton-radiation trapping (i.e., trapping light by light) in many nonlinear optical media through cross-phase modulation (CPM) effect, which have led to the observation of the Raman-induced frequency shifts and supercontinuum generation [6–11].

When the group velocities of both the optical soliton and the nonsolitonic pulse are matched, the nonlinearity of the soliton will affect the nonsolitonic pulse via CPM that can balance the dispersion and/or diffraction effects, and thus the phenomenon of trapping light by light occurs [12]. This phenomenon has been intensively investigated due to the rich nonlinear physics and important applications [13–15]. However, the mentioned works are utilized passive optical media, in which far-off resonance excitation schemes are employed for avoiding significant optical absorption. Moreover, for obtaining optical pulse trapping, very high light-intensity and ultrashort laser pulses are usually needed to generate nonlinearity strong adequately to balance dispersion and/or diffraction effects; furthermore, an active manipulation on the property of optical pulse trapping is not easy to realize in passive media because of the absence of energy-level structure and selection rules that can be used.

The disadvantages mentioned above can be overcome by the discovery of electromagnetically induced transparency (EIT) in a resonant atomic system at very low light level [16]. Through the giant enhancement of self-Kerr and cross–Kerr nonlinearities induced by EIT, there are many interesting researches paying attention to numerous nonlinear optical progresses, including temporal (spatial) optical solitons [17–19], Manakov/Thirring vector solitons [20–25], high-dimensional spatiotemporal optical solitons [26–30], efficient multiwave mixing [31], the bistable state [32], and so on. In addition, different from the mechanism of Manakov/Thirring...
model, trapping light by light here is that a soliton pulse formed by self-phase modulation (SPM) offers an external potential to a weak laser light pulse through CPM. The relevant works have been explored in a tripod-type atomic system via double EIT, in which a weak signal pulse is trapped by a probe optical soliton [33, 34].

In this work, we present a physical scheme for manipulating a weak signal pulse by (2+1)-dimension (2+1)D optical soliton and (3+1)D light bullet (LB). The system we consider is an inverted-Y four-level coherent atomic gas working under double EIT. Based on Maxwell-Bloch equations, we derive nonlinear equations governing the spatial-temporal evolution of the probe and signal pulse envelopes. We show that the optical Kerr nonlinearity of the system can be enhanced dramatically under the condition of double EIT. We also show that the optical pulse can induce an enough strong SPM effect to balance the diffraction effect, while the weak signal pulse relies on the CPM effect of the probe pulse to form a localized wave packet with their matched group velocities. To investigate the manipulation of a weak signal pulse by (3+1)D LB, we introduce a far-detuned laser field into the system to stabilize the ultraslow LB. In particular, when a Stern-Gerlach (SG) gradient magnetic field is applied to the system, the LB and trapped signal pulse will undergo a significant deflection.

Before proceeding, we note that the (3+1)D LBs have been recently predicted and investigated extensively both in experimental [35–38] and theoretical [39–43] work with various passive optical media. To generate such LBs, high light-intensity and ultrashort laser pulses are generally needed, and they are not easy to manipulate actively. Although the (3+1)D LBs can be formed in EIT-based system with low generation power [26–30], all of them only consider isolated LB and no interaction between LBs. A weak signal pulse trapped by a probe soliton has been explored deeply in a tripod-type atomic system via double EIT [33, 34], but no (3+1)D LBs are touched. In contrast, the trapping phenomenon presented here in our work are based on (3+1)D LB with inverted-Y atomic gas (even with Rydberg gas [44–46]), and the trajectories of the localized wave packets can be controlled through an SG gradient magnetic field. The research results predicted here may not only open a route for the study of weak-light nonlinear optics but also have potential applications in the precision measurements and optical information processing and transmission (e.g., design of all-optical switching at very low light level).

The article is arranged as follows. In section 2, the theoretical model under study is described. In section 3, the nonlinear envelope equations governing the evolution of the probe and signal pulses are derived. In section 4, the manipulation of a signal pulse by (2+1)D probe optical soliton is studied. In section 5, the manipulation of a signal pulse by (3+1)D probe light bullet and their trajectory control are investigated. Finally, in section 6 a summary of our main results obtained in this work is given.

2. Theoretical model

We consider a lifetime-broadened atomic gas with an inverted-Y-type four-level configuration as shown in figure 1(a). Here, \( \Omega_p \) is the half Rabi frequency of the probe field acting upon the \( |1\rangle \leftrightarrow |3\rangle \) transition; The half Rabi frequency \( \Omega_s \) represents a weak signal field coupling with the \( |2\rangle \leftrightarrow |3\rangle \) transition; The continuous-wave strong control field with half Rabi frequency \( \Omega_r \) couples to the \( |3\rangle \leftrightarrow |4\rangle \) transition. The inverted-Y-type system is composed by two ladder-type configurations, i.e., \( |1\rangle \leftrightarrow |3\rangle \leftrightarrow |4\rangle \) and \( |2\rangle \leftrightarrow |3\rangle \leftrightarrow |4\rangle \), sharing the excited-state levels \( |3\rangle \) and \( |4\rangle \). We assume that the atoms are initially prepared under an ultralow temperature to cancel...
Doppler broadening and collisions, and are distributed in the ground states [1] and [2]. A possible arrangement for LBs trapping a weak signal pulse and their trajectories manipulation is shown in figure 1(b).

For simplicity, we assume that the electric field vector of all the laser fields in the system reads
\[ E = \sum_{l=1}^{S} E_{l}(t) e_{l} e^{i(k_{l}z - \omega_{l}t)} + c.c., \]
for propagating along the z direction. Here \( e_{l} \) is the unit polarization vector (envelope) of the \( l \)th polarization component, and \( k_{l} = \omega_{l}/c \) is the wavenumber of the laser field before entering the atomic gas. The continuous-wave control field counterpropagating with the probe and signal fields is used to suppress the Doppler broadening. To investigate the trajectory control of the probe and signal pulses, an SG gradient magnetic field is applied to the system, i.e.,
\[ B(x, t) = e_{z}B_{x} \]
where \( e_{z} \) is the unit vector in the z direction and \( B \) indicates the transverse gradient. Thus a small but space-dependent Zeeman level shift \( \Delta E_{j, Zeeman} = \mu_{B}g_{j}I_{j}B_{x} \) for the level \( |j\rangle \) occurs. Here \( \mu_{B}, g_{j} \) and \( I_{j} \) are Bohr magneton, gyromagnetic factor, and magnetic quantum number of the level \( |j\rangle \), respectively.

According to the electric-dipole and rotating-wave approximations, the Hamiltonian of the system in the interaction picture reads
\[ \hat{H}_{\text{int}} = -\sum_{j=1}^{4} \hbar \Delta_{j}^{i} \langle j | \hat{J}_{j}^{i} | j \rangle - \hbar [\Omega_{4}(1) + \Omega_{1}(3) + \Omega_{2}(2) + \Omega_{4}(4) + H.c.] \]
Here \( \Delta_{j}^{i} = \Delta_{j} - \frac{\mu_{j}B_{x} + 2\alpha_{j}E_{a}(y/R_{1})I_{j}}{\hbar} \), with \( \mu_{j} = \mu_{B}(g_{j}I_{j}^{2} - g_{j}I_{j}^{2})/\hbar \) and \( \alpha_{j} = (\alpha_{j} - \alpha_{j})/\hbar \).
\[ \Omega_{4} = (e_{x} \cdot p_{4}) E_{4}/\hbar, \quad \Omega_{1} = (e_{x} \cdot p_{1}) E_{1}/\hbar, \quad \Omega_{2} = (e_{x} \cdot p_{2}) E_{2}/\hbar \]
where \( p_{i} \) is the electric-dipole moment element related to the states \( |j\rangle \) and \( |l\rangle \). \( \Delta_{1} = \omega_{2} - \omega_{21}, \Delta_{2} = \omega_{2} - \omega_{1}, \Delta_{3} = \omega_{1} - \omega_{21}, \Delta_{4} = \omega_{2} + \omega_{1} - \omega_{41} \) are the detunings in the relevant transitions, where \( \omega_{i} = (E_{i} - E_{j})/\hbar \) with \( E_{j} \) being the eigen energy of the state \( |j\rangle \).

The motion of atoms is governed by the Bloch equation \[ \frac{\partial \sigma}{\partial t} = -i/\hbar [\hat{H}_{\text{int}}, \sigma] - \Gamma \sigma, \]
where \( \sigma \) is a 4 × 4 density matrix in the interaction picture and \( \Gamma \) is a 4 × 4 relaxation matrix denoting the spontaneous emission and dephasing of the system. The explicit expression for density matrix elements \( \sigma_{ij} \) is given in appendix A.

The equation of motion for probe-field and signal-field Rabi frequencies \( \Omega_{4} \) and \( \Omega_{c} \) can be captured by the Maxwell equations under slowly varying envelope approximation [19], which is read as
\[ i \left( \frac{\partial}{\partial t} + \frac{1}{c} \frac{\partial}{\partial z} \right) \Omega_{4} + \frac{e}{2\omega_{c}} \left( \frac{\partial^{2}}{\partial x^{2}} + \frac{\partial^{2}}{\partial y^{2}} \right) \Omega_{4} + \kappa_{13} \sigma_{31} = 0, \]
\[ i \left( \frac{\partial}{\partial t} + \frac{1}{c} \frac{\partial}{\partial z} \right) \Omega_{c} + \frac{e}{2\omega_{c}} \left( \frac{\partial^{2}}{\partial x^{2}} + \frac{\partial^{2}}{\partial y^{2}} \right) \Omega_{c} + \kappa_{25} \sigma_{52} = 0, \]
where \( \kappa_{13} = N_{0}g_{j}[p_{4} \cdot e_{j}^{2}]/(2e_{0}c\hbar) \) and \( \kappa_{25} = N_{0}g_{j}[p_{3} \cdot e_{j}^{2}]/(2e_{0}c\hbar) \) with \( N_{0} \) the atomic concentration. In addition, we have assumed the continuous-wave control field is strong enough so that \( \Omega_{c} \) can be regarded as a constant during the evolution of the probe and signal pulses.

3. Nonlinear envelope equations and Giant Kerr effect

3.1. Asymptotic expansion and coupled nonlinear envelope equations

Because we are interested in the nonlinear evolution and the possible formation of optical solitons in the system, we employ the standard method of multiple scales [19] to derive nonlinear envelope equations of the probe and signal pulses based on the Maxwell-Bloch(MB) equations (4) and (5). To this end, we take the asymptotic expansion \( \sigma_{ij} = \sum_{n=0}^{N} e^{\alpha_{ij} t} \sigma_{ij}^{(n)}(j, l = 1, 2, 3, 4, \Omega_{4} = \sum_{n=1}^{\infty} e^{\alpha_{11} t} \Omega_{4}^{(n)}, \Omega_{c} = \sum_{n=2}^{\infty} e^{\alpha_{11} t} \Omega_{c}^{(n)} \)
we additionally assume \( E_{a}(y) = e^{iE_{a}^{(0)}(y/R_{1})}, \)
and \( B(x) = e^{iB_{x}x} \), and thus \( \bar{d}_{\delta} = d_{\delta}^{(0)} + e^{iE_{a}^{(0)}}d_{\delta}^{(0)} \), with \( d_{\delta}^{(0)} = \delta_{l} - \delta_{i} + i\gamma_{2} \) and \( d_{\delta}^{(2)} = \frac{1}{2} \alpha_{i}E_{a}^{(2)}(y/R_{1})^{2} - \mu_{B}B_{x} \). Here \( \sigma_{ij}^{(0)} \) is the initial population distribution prepared in the state \( |j\rangle \) \( (j = 1, 2) \), which is assumed as 1/2 for simplicity; \( \epsilon \) is a dimensionless small parameter characterizing the typical amplitude of the probe pulse. All the quantities on the right-hand sides of the expansion are considered as functions of the multiscale variables \( x_{i} = \epsilon x, y_{i} = \epsilon y, z_{i} = \epsilon^{i} z, \) and \( t_{\alpha} = \epsilon^{\alpha} t (\alpha = 0, 2) \).
We furthermore substitute the expansion into MB equations (4) and (5), and compare the coefficients of \(e^\alpha (\alpha = 1, 2, 3, \cdots)\), thus a set of linear but inhomogeneous equations for \(\sigma_j^{(0)}\) and \(\Omega_j^{(0)}\) can be obtained. At the first order (\(\alpha = 1\)), we obtain \(\Omega_j^{(1)} = F_1 e^{\phi} \) with \(\theta_j = K_j(\omega) z_0 - \omega \tau_0^4\) and \(F_1\) being a yet to be determined envelope function of the slow variables \(t_2, x_1, y_1, z_2\). The linear dispersion relation \(K_j(\omega)\) is given by \(K_j(\omega) = \omega/c + \kappa_{\omega}(\omega) + \frac{1}{2} \text{Im}(\Delta_{ij})/D_1\), with \(D_1 = [\Omega_j^{(0)} - (\omega + d_{ij}^{(4)})(\omega + d_{ij}^{(4)})].\) At the second order (\(\alpha = 2\)), we obtain \(\Omega_j^{(2)} = F_1 e^{\phi} \), where \(\theta_j = K_j(\omega) z_0 - \omega \tau_0, F_1\) the envelope function yet to be determined and \(K_j(\omega) = \omega/c + \kappa_{\omega}(\omega) + \frac{1}{2} \text{Im}(\Delta_{ij})/D_1\), with \(D_1 = [\Omega_j^{(0)} - (\omega + d_{ij}^{(4)})(\omega + d_{ij}^{(4)})].\) Note that the property of \(K_j(\omega)\) is similar to \(K_j(\omega)\), thus they have EIT transparency windows in the imaginary parts \(\text{Im}(K_0^p)\) and \(\text{Im}(K_0)\) for \(\kappa = 0\), i.e., double EIT [33].

At the third order (\(\alpha = 3\)), the divergence-free condition yields the nonlinear equation for the envelope \(F_1\) of the probe pulse:

\[
i \left( \frac{\partial}{\partial z_2} + \frac{1}{V_{g1}} \frac{\partial}{\partial t_2} \right) F_1 + \frac{\epsilon}{2\omega_p} \left( \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial y_1^2} \right) F_1 + W_1|F_1|^2 F_1 e^{-2a_k z_2} + V(x_0, y_1)F_1 = 0.
\]  

(6)

At the fourth order (\(\alpha = 4\)), we obtain the nonlinear equation for the envelope \(F_2\) of the signal pulse:

\[
i \left( \frac{\partial}{\partial z_2} + \frac{1}{V_{g2}} \frac{\partial}{\partial t_2} \right) F_2 + \frac{\epsilon}{2\omega_p} \left( \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial y_1^2} \right) F_2 + W_2|F_2|^2 F_2 e^{-2a_k z_2} + V(x_0, y_1)F_2 = 0.
\]  

(7)

Here we should clearly indicate that \(a_k = e^{-2} \text{Im}(K_{0^p})\) and \(V_{g1(2)} = [\partial K_{p(j0)}(\omega)/\partial \omega]^{-1}\) is the group velocity of the probe (signal) pulse; \(W_{1(2)}\) is the SPM coefficient and \(W_{2(3)}\) is the CPM coefficient; \(V_1 = M_1|E_1(\omega)|^2 + N_1 B_0\) \((j = 1, 2)\) is contributed by the far-detuned laser field and the SG gradient magnetic field. The explicit expressions of \(W_{1(2)}, W_{2(3)}, M_0, N_0\) and each-order approximation solutions are presented in appendix B.

For the convenience of later calculations, we introduce some dimensionless variables \(u_j = \epsilon F_j/U_0 e^{-\alpha_k z_2}, \tau = t/\tau_0, s = z/L_{\text{Diff}}, (\zeta, \eta) = R_{\text{sc}}^{-1}(x, y), \) and \(\lambda_j = V_{gj} \tau_0/L_{\text{Diff}}.\) Here \(L_{\text{Diff}} = \omega_p R_{\text{sc}}^2/\epsilon, \tau_0, \) and \(U_0\) are typical diffraction length, pulse duration, and Rabi frequency of the probe pulse, respectively. Thus equations (6) and (7) can be converted into the dimensionless form

\[
i \left( \frac{\partial}{\partial s} + \frac{1}{\lambda_j} \frac{\partial}{\partial \tau} \right) u_j + \frac{1}{2} \left( \frac{\partial^2}{\partial \xi_j^2} + \frac{\partial^2}{\partial \eta_j^2} \right) u_j + w_j|u_j|^2 u_j + V_j(\xi, \eta)u_j = -iA_j u_j,
\]  

(8)

\((j = 1, 2)\), where \(w_0 = W_{0j}/W_{1(2)}, A_{1(2)} = \text{Im}[K_{p(ij0)}]L_{\text{Diff}}, \) and \(V_j(\xi, \eta) = M_j|\phi_j|^2 + N_j \bar{\xi}\) with \(\vartheta = E_j(\eta)/E_0, M_j = M_j|E_0|^2 L_{\text{Diff}}, \) and \(N_j = N_j L_{\text{Diff}} R_i B.\) We additionally assume the solution with Gaussian envelope propagates in the \(z\) direction [19, 25], i.e.,

\[
u_j(\tau, \xi, s) = G_j(\tau, s) v_j(\tau, \xi, \eta),
\]  

(9)

with \(G_j(\tau, s) = \sqrt{\frac{i}{2}} e^{-i(\lambda_j \tau + s)}\) and \(\rho_0\) being a free real parameter. After integrating over the variable \(s\), equation (8) becomes

\[
i \frac{\partial}{\lambda_j} \frac{\partial}{\partial \tau} v_j + \frac{1}{2} \left( \frac{\partial^2}{\partial \xi_j^2} + \frac{\partial^2}{\partial \eta_j^2} \right) v_j + w_j|v_j|^2 v_j + V_j(\xi, \eta) v_j = -iA_j v_j,
\]  

(10)

3.2. Giant Kerr effect and ultrasmall matched group velocities

The model mentioned above can be easily realized by selecting realistic physical systems. One of them is the ultracold \(^{87}\text{Rb}\) atoms with the energy levels selected as [1] = \(5^2S_{1/2}, F = 1, m_F = 1\) (\(\sigma = -1/2\)),

\[|2\rangle = |5^2S_{1/2}, F = 2, m_F = 0\] (\(\sigma = 1/2\)),[3] = \(5^2P_{1/2}, F = 2, m_F = 0\) (\(\sigma = 1/2\)), and

\[|4\rangle = |5^2D_{3/2}, F = 3, m_F = 0\] (\(\sigma = 2/3\)) [48, 49]. Generally speaking, the coefficients in equation (10) are complex which is difficult to support the stable nonlinear localized solutions. Fortunately, when the system keeps in the condition of double EIT, then the real parts of these coefficients can be made much larger than their imaginary parts so that the nonlinear localized solutions are possible.

To this end, we now consider the third-order self- and cross-Kerr nonlinear optical susceptibilities \(\chi_{11}^{(3)}\) and \(\chi_{21}^{(3)}\), which are respectively proportional to the SPM coefficient \(W_{11}\) in equation (6) and CPM coefficient \(W_{21}\) in equation (7). The relation obeys

\[
\chi_{11}^{(3)} = \frac{2\epsilon}{\omega_p} \frac{|p_3|L^2}{\hbar^2}, W_{11},
\]

(11a)
The system parameters are given by \(\Gamma_1 \approx 1\) kHz, \(\Gamma_3 \approx 5.75\) MHz, \(\Gamma_4 \approx 0.4\) MHz, and \(|\psi_{0,\|}| \approx |\psi_{\perp,\|}| = 2.54 \times 10^{-7}\) C cm [48]. Additionally, the other realistic parameters are \(\Omega \approx 50.0\) MHz, \(\Delta_1 = 100\) kHz, \(\Delta_3 = -70.0\) MHz, \(\Delta_4 = -9.0\) MHz, and \(\kappa_{i3} \approx \kappa_{23} \approx 1.0 \times 10^{6}\) cm\(^{-1}\) s\(^{-1}\), we thus obtain \(\chi_{11}^{(3)} = (1.3 - 0.03i) \times 10^{-3}\) cm\(^2\) V\(^{-1}\) and \(\chi_{21}^{(3)} = (1.3 - 0.09i) \times 10^{-3}\) cm\(^2\) V\(^{-2}\), which possesses two obvious features. On the one hand, their real parts have the order of magnitude \(10^{-3}\) cm\(^2\) V\(^{-1}\), which is \(10^{12}\) times larger than the third-order nonlinear optical susceptibilities found in conventional nonlinear optical media [47]. On the other hand, the real parts are indeed much larger than their imaginary parts, which is due to the quantum destructive interference.

To obtain the nonlinear localized solutions, the system should satisfy \(L_{\text{Diff}} \equiv L_{\text{Nonl}}\), where \(L_{\text{Nonl}} = 1/|U_0|^2 |W|_{\|}\) is the typical nonlinearity length. When we take \(U_0 = 4.85\) MHz, \(R_1 = 25.0\) \(\mu\)m, and \(\tau_0 = 7.6 \times 10^{-7}\) s, thus \(L_{\text{Diff}} \equiv L_{\text{Nonl}} = 0.49\) cm. Furthermore, the typical linear absorption length \(L_a = 1/|\text{Im}(\xi_{p(0)})|\) is around 23.9 cm, which is much larger than \(L_{\text{Diff}}\) and \(L_{\text{Nonl}}\). As a result, one has the dimensionless coefficients \(\lambda_i = 1.0 + 0.03i\), \(\beta_i = 1.0 + 0.03i\), \(w_{i1} = 1.0 - 0.02i\), \(w_{i2} = 1.0 - 0.07i\), and \(A_i \approx 0.02\), and hence the corresponding terms in equation (10) can be indeed negligible safely. Meanwhile, the group velocities of the probe and signal pulses are \(V_{g1} \approx V_{g2} = 9.0 \times 10^{-5}\) c, which are much smaller than the light speed c in vacuum.

### 4. Manipulation of a signal pulse by (2+1)D soliton

We primarily investigate the manipulation of a signal pulse by (2+1)D probe soliton. To neglect the diffraction in the y direction, we assume the transverse radii of the two pulses satisfying the relation \(R_y \ll R_x\). In addition, Stark laser field and SG gradient magnetic field are not applied to the system, hence equation (10) can be reduced to

\[
\frac{\partial v_1}{\partial \tau} + \frac{1}{2} \frac{\partial^2 v_1}{\partial \xi^2} + w_{i1} |v_1|^2 v_1 = 0, \tag{12a}
\]

\[
\frac{\partial v_2}{\partial \tau} + \frac{1}{2} \frac{\partial^2 v_2}{\partial \xi^2} + w_{i2} |v_2|^2 v_2 = 0, \tag{12b}
\]

which is similar to [14, 33] but in the different physical model. We see that the solution \(v_1\) plays a role of ‘external potential’ for \(v_2\) in equation (12b). It is obvious that equation (12a) can support a single-soliton solution [50], which is written as

\[
v_1(\tau, \xi) = s_i \text{sech}[\sqrt{\kappa_{11}} \xi (\eta_0 \tau - \xi_0)] e^{i(\eta_0 \xi - n_0^2 \xi^2 - \vartheta_0)}, \tag{13}
\]

with \(s_i, \eta_0, \xi_0, \vartheta_0\) being free real parameters. Thus the trapped solution of \(v_2\) of equation (12b) is determined by the solution (13).

For convenience of the following calculations, we set \(\eta_0 = \xi_0 = \vartheta_0 = 0\) for the soliton solution (13), i.e., \(v_1(\tau, \xi) = s_i \text{sech}(\sqrt{\kappa_{11}} \xi) \exp(i \beta \tau/2)\). In this case, the solution form of \(v_2\) can be described as

\[
v_2(\tau, \xi) = s_i h(\xi) \exp(i \beta \tau) \text{sech}(\sqrt{\kappa_{21}} \xi), \tag{12b}
\]

which is similar to (12b) but in the different physical model. We see that the solution of \(v_2\) of equation (12b) is determined by the solution (13).

By the solution (13), we can obtain an eigenvalue equation - \(\frac{1}{\kappa_{11}} \beta^2 h + U(\xi) h = -\beta h\), after substituting the function of \(v_2\) into equation (12b). Here \(U(\xi) = -w_{i1} s_i^2 \text{sech}^2(\sqrt{\kappa_{11}} \xi)\), \(h(\xi)\), and \(-\beta\) are respectively the potential well, the eigenfunction, and the corresponding eigenvalue. One can analytically calculate two localized modes (i.e., even and odd modes) of the trapped signal pulse for this system. The even mode solution is read as

\[
h_{e}(\xi) = \text{sech}(\sqrt{\kappa_{11}} \eta_0^2 \xi), \tag{12b}
\]

with \(\eta_0 = (\sqrt{\kappa_{11}} \xi)/q_0\), where \(q_0 = \sqrt{w_{i1} \xi_0^2}, \beta = w_{i1} \xi_0^2 n_0^2/2\), and \(n_0 = (-1 + \sqrt{1 + 8w_{i1} w_{i1}/W_{11}})/2\). The odd mode behaves as

\[
h_{o}(\xi) = \text{sech}(\sqrt{\kappa_{11}} \eta_0^2 \xi) \text{tanh}(\eta_0^2 \xi), \tag{12b}
\]

with \(\beta = w_{i1} \xi_0^2 n_0^2/2, n_1 = n_0 - 1\). Based on the results in section 3.2, we can obtain \(n_0 = 1\) and \(n_1 = 0\), thus the spatial profiles (i.e., eigenfunction) of the even and odd modes are described as \(n_{0,1}(\xi) = \tan h(\xi)\). We see that the trapped signal pulse displays a bright soliton or a dark soliton under the interaction of the probe soliton pulse in the coherent atomic system as long as \(\xi_2 \ll \xi_1\).

Shown in figure 2 is the numerical result of the manipulation of a weak signal pulse by probe soliton pulse based on equations (12a) and (12b) by taking \(|\psi_{0}\|\) (for probe pulse) and \(|\psi_{2}\|\) (for signal pulse) as functions of \(\tau/\tau_0\) and \(\xi/R_1\). We obviously see that the launched signal pulse with the even mode (figure 2(a)) or odd mode (figure 2(b)) expands rapidly during propagation when the probe pulse is absent. As a result, no trapping of the signal pulse occurs. When a probe soliton pulse is launched into the system (shown in figure 2(c)) and its group velocity matches well with that of the signal pulse, the CPM effect induced by the probe soliton pulse thus plays a significant role for manipulating the weak signal pulse. In this case, the diffraction-induced broadening of the
signal pulse is compensated and both the probe and the localized signals propagate together stably; as shown in figures 2(d) and (e). In the numerical simulation, the amplitudes of the probe and signal pulses we consider are $\varsigma_1 = 1.0$ and $\varsigma_2 = 0.15$, respectively. The trapping phenomenon predicted here maybe have potential applications in quantum information processing, i.e., designing an all-optical switcher at very low light level [33].

5. Manipulation of a signal pulse by (3+1)D LB and their trajectory control

5.1. Manipulation of a signal pulse by (3+1)D LB

We now turn to explore the manipulation of a weak signal pulse by (3 + 1)D probe LB and their trajectory control. To find the analytical solutions, here we should consider a reasonable approximation: The potential wells of the optical lattice are assumed to be deep enough. The envelopes of probe and signal pulses are almost trapped in the wells in the $y$ direction, hence $\psi_j(\xi, \eta)$ can be approximated by $M_j - M_j\eta^2 + N_j\xi$. In this case, equation (10) becomes

$$\frac{\partial\psi_j}{\partial \tau} + \left(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2}\right)\psi_j + w_j|\psi|^2\psi_j + (M_j - M_j\eta^2 + N_j\xi)\psi_j = 0.$$  (14)

For simplicity, we firstly investigate the manipulation of a weak signal pulse by (3+1)D probe LB without SG gradient magnetic field (i.e., $N_1 = N_2 = 0$). Here we assume the solution of equation (14) has the form

$$\psi_j(\tau, \xi, \eta) = \phi_j(\tau, \xi)\psi_j(\eta)\exp[i(M_j - \sqrt{M_j/2})\tau],$$

where $\psi_j = \sqrt{2M_j/\pi}\exp(-\sqrt{M_j/2}\eta^2)$ is a normalized ground state solution for the eigenvalue problem $(\partial^2/\partial \eta^2 - 2M_j\eta^2)\psi_j = 2E_j\psi_j$ with $E_j = -\sqrt{M_j/2}$ being the eigenvalue [28]. Note that $\psi_1(\eta)$ and $\psi_2(\eta)$ have the same forms because of $M_1 \approx M_2$.

After integrating over the variable $\eta$, one obtains

$$\frac{i\partial\phi_1}{\partial \tau} + \frac{1}{2}\frac{\partial^2\phi_1}{\partial \xi^2} + g_1|\phi_1|^2\phi_1 = 0,$$  (15a)

$$\frac{i\partial\phi_2}{\partial \tau} + \frac{1}{2}\frac{\partial^2\phi_2}{\partial \xi^2} + g_2|\phi_2|^2\phi_2 = 0,$$  (15b)

with $g_j = \sqrt{2M_j/2\pi^2}w_j (j = 1, 2)$. We obviously see that equation (15a) has the same form with equation (12). Therefore, one can obtain the single-soliton solution $\phi_j$, i.e.,

$$\phi_j(\tau, \xi) = \varsigma_j \text{sech}\left[\varsigma_j(\xi - \eta_j\tau - \xi_0)\right]\exp\left[i\left[\eta_j(\xi - \eta_j^2 - \xi_0^2)/2 + \vartheta_0\right]\right],$$

with $\varsigma_j, \eta_0, \xi_0$, and $\vartheta_0$ being free real parameters. Finally, the solution of $\omega_1$ in equation (10) is

$$\omega_1 = \varsigma_j(2M_j/\pi)^{1/4}\text{sech}\left[\varsigma_j(\varsigma_j\eta_j\xi + \xi_0)\right]e^{-\sqrt{M_j/2}\eta_j^2/4 + \vartheta_0},$$  (16)

with $\Theta = (M_1 - \sqrt{M_j/2} + \varsigma_j^2\eta_j^2/2)\tau$. We see the nonlinear solution (16) is localized in three space and one time dimensions, i.e., the (3+1)D LB solution of the system. Here we have set $\eta_0 = \xi_0 = \vartheta_0 = 0$ for simplicity.

![Figure 2. Manipulation of a weak signal pulse by probe soliton pulse. Panel (a)(b)](image)
Using the method section 4, the trapped solution form of $u_2$ can be obtained by the (3+1)D probe LB given by $u_1$. The solution form of $\phi_2$ can be set as $\phi_2(\tau, \xi) = \omega_2 h(\xi) \exp(i\beta\tau)$, where $\omega_2$ and $\beta$ are the amplitude and propagation constants. Thus $h(\xi)$ has two localized modes: (1) the even mode solution $h_0(\xi) = \text{sech}(\sqrt{8\delta_1} \xi)$; (2) The odd mode solution $h_1(\xi) = \tanh(\sqrt{8\delta_1} \xi)$. As a result, the trapped signal pulse has the form $u_2 = \omega_2 \text{sech}(\sqrt{8\delta_1} \xi) \psi_2(\tau, s) e^{i\beta\tau}/2$ for even mode, or $u_2 = \omega_2 \tanh(\sqrt{8\delta_1} \xi) \psi_2(\tau, s)$ for odd mode. The consequence indicates that the manipulation of a signal pulse by (3+1)D probe LB can also be realized.

Shown in figure 3 is the numerical result of the manipulation of the weak signal pulse by a (3+1)D probe LB without the SG gradient magnetic field (i.e., $B = 0$) based on equation (14), by taking $|u_0|$ and $|u_2|$ as functions of $(x, y)/R_\perp$ and $z/\Delta$. For simplicity, we consider only the even mode (i.e., bright soliton) of the weak signal pulse. Figures 3(a)–(c) show the light intensity $|u_0|$ of the signal pulse under the diffraction effect solely absence of probe soliton for travelling to $z = 2\Delta$, $z = 4\Delta$, and $z = 6\Delta$, respectively. The consequence indicates that the manipulation of a signal pulse by (3+1)D probe LB can also be realized.

5.2. Trajectory control of the trapped signal pulse and (3+1)D probe LB

We then turn to investigate the trajectories of the trapped signal pulse and (3+1)D probe LB by an SG gradient magnetic field (i.e., $B \neq 0$). Thus equation (15) contains an external potential proportional to $\xi$ written as
The single-soliton solution can be read as

\[ \psi_1(\tau, \xi) = \varsigma_1 \text{sech}\left[ \sqrt{\beta_{11}} \phi_1(\xi - \eta_0 \tau + \xi_0) \right] e^{i (\eta_0 \xi - \eta_0 \xi_0 + \xi_1 \tau / 2 + \theta_0)}, \]  

where \( \varsigma_1, \eta_0, \xi_0, \) and \( \theta_0 \) are free real parameters. However, equation (17b) is transformed into another but uncomplicated form due to \( N_2 = 0 \). Finally, we can obtain the solution \( u_1 = \varsigma_1 \text{sech} \left[ \sqrt{\beta_{11}} \phi_1(\xi - \eta_0 \tau + \xi_0) \right] \) \( \psi_1(\eta) G_1(\tau, s) e^{\Theta}, \) with \( \Theta = [\eta_0 - \sqrt{\beta_{11}} / 2 + \xi_1 / 2 + N_1(\xi - \xi_0 / 2)] \) \( \tau \) and \( \eta_0 = \xi_0 = \theta_0 = 0. \) One can see that \((3+1)D\) probe LB moves along a parabolic trajectory. Because the analytical solution \( u_2 \) is not available, we just make a numerical calculation.

Shown in figure 4 are the deflection of the probe and signal pulses by numerically simulating equation (14) with the SG gradient magnetic field \( B = 50.3\, \text{mG cm}^{-1}. \) By taking \( |u_1| \) and \( |u_2| \) as functions of \((x, y)/R_L\) and \( z/L_{\text{Diff}}\). Figures (a)–(d) show the deflection of the probe and signal pulses when they travel to \( z = 2L_{\text{Diff}}, z = 4L_{\text{Diff}}, z = 6L_{\text{Diff}}, \) and \( z = 8L_{\text{Diff}}, \) respectively. The left spot and right spot in each panel respectively denote the probe LB pulse and the trapped signal pulse. We can see the following: (i) the weak signal pulse can be perfectly trapped by the LB because of the CPM effect induced by the probe pulse; (ii) the probe LB and the trapped signal pulses undergo deflection in the negative \( x \) direction by the contribution of the SG gradient magnetic field. Obviously, the trajectory of the trapped signal pulse and \((3+1)D\) probe LB can be steered by manipulating the SG gradient magnetic field. Such manipulation is useful for optical information processing, e.g., for the control of the behavior of all-optical switching.

In addition, we can determine the deflection angles of the \((3+1)D\) probe LB and trapped signal pulse. Due to \( N_2 = 0, \) the trapped signal pulse deflect along the trajectory of \((3+1)D\) probe LB. It is easy to obtain the propagating velocity of \((3+1)D\) probe LB by solution \( u_1, \) which is

\[ V = \left[ \begin{array}{c} \eta_0 \frac{V_0^2}{L_{\text{Diff}}^2} R_L \nu_{\gamma} \ 0 \ 0 \end{array} \right]. \]  

The expected deflection angle of the output probe LB is defined as the ratio \( V_x / V_z, \) where \( V_x = N_1 V_0^2 R_L \) \( t / L_{\text{Diff}}^2 \) and \( V_z = V_{\gamma} \) are the propagating velocities respectively along the \( x \) axis and the \( z \) axis.
Here we assume the probe LB passes through the atomic medium with length $L$ along the $z$ direction, thus the traveling time in the $z$ direction is $t = L/V_g$. As a result, the deflection angle of probe LB is

$$\theta = N_l R_s^2 \frac{L}{L_{Diff}} \cdot B.$$  \hspace{1cm} (21)

From the figure 4, we obtain deflection angle $(\theta_1, \theta_2, \theta_3, \theta_4) = -(1.02, 2.04, 3.06, 4.08) \times 10^{-3}$ rad for $L = (2L_{Diff}, 4L_{Diff}, 6L_{Diff}, 8L_{Diff})$ corresponding to figures 4(a)–(d), which is two orders of magnitude larger than that for linear polariton obtained in [51]. On the contrary, one can precisely measure the micro magnetic fields through the detection of the deflection angle of the LB. We expect that the significant deflection obtained here may have potential applications in optical magnetometry, quantum information processing, i.e., designing optical beam splitters.

6. Summary

In this work, we have proposed a physical scheme for manipulating a weak signal pulse by $(2+1)D$ optical soliton and $(3+1)D$ LB in a coherent inverted-Y four-level atomic system via double EIT. Based on MB equations, we have derived nonlinear equations governing the spatial-temporal evolution of the probe and signal pulse envelopes. We have shown that the optical Kerr nonlinearity of the system can be enhanced dramatically under the condition of double EIT. We have found that the probe pulse can induce an enough strong SPM effect to balance the diffraction effect, while the weak signal pulse relies on the CPM effect of the probe pulse to form a localized wave packet with matched group velocities; and thus the trapping light by light can be obtained. Furthermore, we have investigated the manipulation of a weak signal pulse by $(3+1)D$ LB by introducing a far-detuned laser field into the system to stabilize the LB. In particular, the trajectories of the LB and the trapped signal pulse can be manipulated when an SG gradient magnetic field is applied to the system. The research results predicted here may not only open a route for the study of weak-light nonlinear optics but also have potential applications in the precision measurements and optical information processing and transmission (e.g., design of all-optical switching at very low light level).

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Appendix A. The density-matrix elements equation

The explicit equations of motion for the density matrix elements $\sigma_{ij}$ are given by

\begin{align}
\dot{\sigma}_{11} + i(\Gamma_1 - \Omega_p^s)\sigma_{11} + i\Gamma_{12}\sigma_{12} + i\Gamma_{13}\sigma_{13} + \Omega_p^s\sigma_{31} - \Omega_p^s\sigma_{31} &= 0, \hspace{1cm} (1.1a) \\
\dot{\sigma}_{22} + i\Gamma_{12}\sigma_{22} - i\Gamma_{21}\sigma_{21} + i\Gamma_{23}\sigma_{23} + \Omega_p^s\sigma_{32} - \Omega_p^s\sigma_{32} &= 0, \hspace{1cm} (1.1b) \\
i\left(\frac{\partial}{\partial t} + \Gamma_3\right)\sigma_{33} - i\Gamma_{34}\sigma_{44} + \Omega_p^s\sigma_{31} + \Omega_p^s\sigma_{32} + \Omega_p^s\sigma_{43} - \Omega_p^s\sigma_{31} - \Omega_p^s\sigma_{32} - \Omega_p^s\sigma_{43} &= 0, \hspace{1cm} (1.1c) \\
i\left(\frac{\partial}{\partial t} + \Gamma_4\right)\sigma_{44} + \Omega_p^s\sigma_{43} - \Omega_p^s\sigma_{43} &= 0, \hspace{1cm} (1.1d) \\
i\left(\frac{\partial}{\partial t} + d_{21}\right)\sigma_{21} + \Omega_p^s\sigma_{31} - \Omega_p^s\sigma_{31} &= 0, \hspace{1cm} (1.1e) \\
i\left(\frac{\partial}{\partial t} + d_{31}\right)\sigma_{31} + \Omega_p^s(\sigma_{11} - \sigma_{33}) + \Omega_p^s\sigma_{21} + \Omega_p^s\sigma_{41} &= 0, \hspace{1cm} (1.1f) \\
i\left(\frac{\partial}{\partial t} + d_{32}\right)\sigma_{32} + \Omega_p^s(\sigma_{22} - \sigma_{33}) + \Omega_p^s\sigma_{21} + \Omega_p^s\sigma_{42} &= 0, \hspace{1cm} (1.1g) \\
i\left(\frac{\partial}{\partial t} + d_{41}\right)\sigma_{41} + \Omega_p^s\sigma_{31} - \Omega_p^s\sigma_{31} &= 0, \hspace{1cm} (1.1h)
\end{align}
\[
\left( i \frac{\partial}{\partial t} + d_{2t} \right) \sigma_{42} + \Omega_c \sigma_{32} - \Omega_\delta \sigma_{43} = 0, 
\]
\[
\left( i \frac{\partial}{\partial t} + d_{4s} \right) \sigma_{43} + \Omega_c (\sigma_{33} - \sigma_{44}) - \Omega_\delta^2 \sigma_{44} - \Omega_\delta^2 \sigma_{42} = 0, 
\]
where \( d_{ij} = \Delta_i - \Delta'_i + i\gamma_{ij} \) with \( \Delta'_i = \Delta_i - \mu_g (g^2_i m^2_i - g^2_i m^2_i) B_x / h + (\Omega_i - \alpha_i)|E_\omega|^2 / (2 \hbar) \). Dephasing rates are defined as \( \gamma_{ij} = (\Gamma_{ij} + \Gamma_{ji}) / 2 + \gamma_{ij}^{\text{col}} \), with \( \Gamma_{ij} = \sum_{E_j < E_i} \Gamma_{ij} \) denoting spontaneous emission rate from the state \(|j\rangle\) to all lower energy states \(|i\rangle\) and \( \gamma_{ij}^{\text{col}} \) denoting the dephasing rate reflecting the loss of phase coherence between \(|j\rangle\) and \(|l\rangle\).

### Appendix B. Solutions of the asymptotic expansion at each order

(i) **First-order approximation:**

\[
\Omega_p^{(1)} = F_1 e^{i \omega t}, 
\]
\[
\sigma_{31}^{(1)} = \frac{\omega + a_{41}^{(0)}}{D_1} F_1 e^{i \omega t}, 
\]
\[
\sigma_{41}^{(1)} = - \frac{\Omega_c a_{11}^{(0)}}{D_1} F_1 e^{i \omega t}, 
\]
where \( D_1 = |\Omega_c|^2 - (\omega + a_{31}^{(0)})(\omega + a_{41}^{(0)}) \).

(ii) **Second-order approximation:**

\[
\Omega_p^{(2)} = F_2 e^{i \omega t}, 
\]
\[
\sigma_{32}^{(2)} = \frac{\omega + a_{42}^{(0)}}{D_2} F_2 e^{i \omega t}, 
\]
\[
\sigma_{33}^{(2)} = \frac{\Omega_c a_{22}^{(0)}}{D_2} F_2 e^{i \omega t}, 
\]
\[
\sigma_{43}^{(2)} = - \frac{\Omega_c a_{33}^{(0)}}{D_2} F_2 e^{i \omega t}, 
\]
where \( D_2 = |\Omega_c|^2 - (\omega + a_{32}^{(0)})(\omega + a_{42}^{(0)}) \) and

\[
a_{31}^{(2)} = a_{32}^{(2)} = - \frac{1}{2} (a_{33}^{(2)} + a_{44}^{(2)}), 
\]
\[
a_{33}^{(2)} = \frac{a_{41}^{(0)}}{D_1} \left( \frac{\omega + a_{41}^{(0)}}{D_1} - \frac{\omega + a_{43}^{(0)}}{D_1} \right), 
\]
\[
a_{44}^{(2)} = \frac{G a_{33}^{(2)} + Q}{\Omega + G}, 
\]
\[
a_{43}^{(2)} = - \frac{\Omega_c}{\omega + a_{43}^{(0)}} \left( a_{34}^{(2)} - a_{33}^{(2)} - \frac{\sigma_{11}^{(0)}}{D_1} \right), 
\]
with

\[
G = \frac{|\Omega_c|^2}{\omega + a_{43}^{(0)}}, 
\]
\[
Q = \frac{|\Omega_c|^2}{D_1 (\omega + a_{43}^{(0)})^{(0)}} - \frac{|\Omega_c|^2}{D_1 (\omega + a_{43}^{(0)})^{(0)}}, 
\]

(iii) **Third-order approximation:** The solvability condition of \( F_1 \) requires

\[
\left( \frac{\partial}{\partial z_2} + \frac{1}{V_{g1}} \frac{\partial}{\partial t_2} \right) F_1 + \frac{\epsilon}{2 \Omega_p} \left( \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial y_1^2} \right) F_1 + W_0 |F_1|^2 F_1 e^{-2i B_x z_2} 
\]
\[
+ M_i |E_\omega|^2 F_1 + N_i B_x F_1 = 0, 
\]
\[
(iii) 
\text{Third-order approximation: The solvability condition of } F_1 \text{ requires} 
\]
\[
\left( \frac{\partial}{\partial z_2} + \frac{1}{V_{g1}} \frac{\partial}{\partial t_2} \right) F_1 + \frac{\epsilon}{2 \Omega_p} \left( \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial y_1^2} \right) F_1 + W_0 |F_1|^2 F_1 e^{-2i B_x z_2} 
\]
\[
+ M_i |E_\omega|^2 F_1 + N_i B_x F_1 = 0, 
\]
with
\[ W_{11} = \kappa_{13} \frac{\Omega^* a_{43}^{(2)} + (\omega + d_{41}^{(0)}) (a_{11}^{(2)} - a_{33}^{(2)})}{D_1}, \]  
(2.6a)
\[ M_1 = \kappa_{13} \frac{(\omega + d_{41}^{(0)}) \alpha_{31} + |\Omega_1|^2 \alpha_{41}}{4D_1^2}, \]  
(2.6b)
\[ N_1 = -\kappa_{13} \frac{(\omega + d_{41}^{(0)}) \mu_{31} + |\Omega_1|^2 \mu_{41}}{2D_1^2}. \]  
(2.6c)

And the third-order approximation solution reads
\[ \sigma^{(3)}_{21} = d_{21}^{(3)} F_1 F_2^* e^{i(\theta-\phi)}, \]  
(2.7)
with
\[ d_{21}^{(3)} = \frac{1}{\omega + d_{41}^{(0)}} \left( \omega + d_{42}^{(0)} \sigma_{22} - \frac{\omega + d_{41}^{(0)}}{D_1} \sigma_{11}^{(1)} \right). \]  
(2.8)

(iv) Fourth-order approximation: The solvability condition for $F_2$ requires
\[ i \left( \frac{\partial}{\partial z_2} + \frac{1}{V_{zz}} \frac{\partial}{\partial t_2} \right) F_2 + \frac{\epsilon}{2\omega} \left( \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial y_1^2} \right) F_2 + W_{21} |F_1|^2 F_2 e^{-2i\theta_2} + M_2 |a_1^{(1)}|^2 F_2 + N_2 B x_1 F_2 = 0, \]  
(2.9)
with
\[ W_{21} = \kappa_{23} \frac{\Omega^* a_{43}^{(2)} + (\omega + d_{41}^{(0)}) (a_{12}^{(2)} - a_{33}^{(2)})}{D_2}, \]  
(2.10a)
\[ M_2 = \kappa_{23} \frac{(\omega + d_{42}^{(0)}) \alpha_{32} + |\Omega_2|^2 \alpha_{42}}{4D_2^2}, \]  
(2.10b)
\[ N_2 = -\kappa_{23} \frac{(\omega + d_{41}^{(0)}) \mu_{32} + |\Omega_2|^2 \mu_{42}}{2D_2^2}. \]  
(2.10c)

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