Constraint from $D\bar{D}$ Mixing in Left-Right Symmetric Models

Bhaskar Dutta and Yukihiro Mimura
Department of Physics, Texas A&M University, College Station, TX 77843-4242, USA
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We study the constraint arising from the recently observed $D\bar{D}$ mixing in the context of supersymmetric models with left-right symmetry. In these models, the supersymmetric contributions in the mixing amplitudes of $D-D$, $K-K$ and $B-B$ are all correlated. We compare the constraint from the $D\bar{D}$ mixing with the $K\bar{K}$ mixing and find that the $D\bar{D}$ mixing constrains the maximal supersymmetric contribution to the $B_s\bar{B}_s$ mixing amplitude. The maximal supersymmetric contribution can allow a large CP phase of $B_s\bar{B}_s$ mixing which agrees with the recent measurement of the CP asymmetry of $B_s \to J/\psi\phi$ decay.

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Recently, BaBar and Belle have observed signals of $D^0\bar{D}^0$ mixing [1]. The HFAG [2] interpretation of the current data gives us

$$x_D = 8.7^{+3.0}_{-3.4} \times 10^{-3}, \quad y_D = (6.6 \pm 2.1) \times 10^{-3},$$

(1)

where $x_D = \Delta M_D/\Gamma_D$ and $y_D = \Delta \Gamma_D/(2\Gamma_D)$. $\Gamma_D$ is the average decay width of two neutral $D$ meson mass eigenstates. The mass difference of $D^0\bar{D}^0$ is obtained as

$$\Delta M_D = (1.4 \pm 0.5) \times 10^{-11} \text{ MeV}.$$  

(2)

This new data can constrain new physics such as supersymmetry (SUSY) in the similar way as the traditional constraint from the $K\bar{K}$ mixing data [3].

In SUSY models, the flavor degeneracy is often assumed in squark and slepton mass matrices to suppress flavor changing neutral currents (FCNC) [4]. The flavor violation effects in the sfermion mass matrices can only come from the evolution of renormalization group equations (RGE). If this is the case, the flavor violation highly depends on the unification scenario of quarks and leptons. In the minimal extension of SUSY standard model (MSSM), the induced FCNCs from RGE effects are not large in the quark sector, but sizable effects can be generated in the lepton sector since the neutrino mixings are large [5]. In quark-lepton unified models, the loop effects due to the large neutrino mixings can induce sizable effects also in the quark sector. Therefore, it is important to investigate the FCNC effects to obtain a footprint of the unification models.

Left-right symmetric model construction is an interesting candidate to unify matter (including right-handed neutrino) in the gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [6]. The left-right parity is broken spontaneously, and the hypercharge arises from a linear combination of $U(1)_{B-L}$ and the $U(1)$ subgroup of $SU(2)_R$. This left-right symmetric branch can be easily unified in $SO(10)$ grand unified models. In the SUSY version of left-right symmetric models, the box diagrams for meson mixing ($K\bar{K}$, $B\bar{B}$ and $D\bar{D}$) can be enhanced by gluino contribution. Therefore, the newly observed $D\bar{D}$ mixing can be an important probe for left-right symmetric models. Further, in such models, $D\bar{D}$, $K\bar{K}$, $B_d\bar{B}_d$ and $B_s\bar{B}_s$ mixing amplitudes get correlated. This creates an interesting opportunity for cross-checking these models, since the most interesting observation from the sizable SUSY contribution will be the phase of $B_s\bar{B}_s$ mixing, which can be measured by $B_s \to J/\psi\phi$ decay.

The mass difference of $B_s\bar{B}_s$ (the absolute value of the mixing amplitude $M_{12}$) has been measured [7], and the measurement is consistent with the Standard Model (SM) prediction. Therefore, if there is a sizable SUSY contribution, the phase of $B_s\bar{B}_s$ mixing (argument of the amplitude) must be large. The CP asymmetry of $B_s$ decay is being measured and the current result is

$$2\beta_s = -0.70^{+0.47}_{-0.36} \text{ (rad)},$$

(3)

while the SM prediction is $\sim 0.03 - 0.04$. If this result holds in future, then it will indicate an existence of new physics.

The SUSY contribution to the $B_s\bar{B}_s$ mixing is related to the 23 off-diagonal elements of the squark mass matrices, which may be large since it can be related to the large atmospheric mixing. On the other hand, it is hard to predict the amount of the SUSY contribution to the $K\bar{K}$ and $D\bar{D}$ mixings due to cancellation. However, we can show that cancellations for both $K\bar{K}$ and $D\bar{D}$ mixings are not allowed simultaneously when the non-universal terms in squark mass matrices originate from left-right symmetric models. Consequently, the recent observation of the $D\bar{D}$ mass difference restricts the amount of SUSY contribution, and thus, it also restricts the phase of $B_s\bar{B}_s$ mixing. In this Letter, we will show how to obtain the constraint of SUSY contribution from the $D\bar{D}$ mixing, and study the correlation of the constrained phase of $B_s\bar{B}_s$ mixing to other measurements, e.g., phase of $B_d\bar{B}_d$ mixing, in left-right symmetric models.

In left-right symmetric models, the Lagrangian is invariant under the exchange $Q_L \leftrightarrow Q'_R$, where $Q_L$ is $SU(2)_L$ doublet and $Q'_R$ is $SU(2)_R$ doublet which contains conjugate of right-handed up- and down-type quarks $U^c$, $D^c$. As a result, Yukawa couplings are given
by symmetric matrices. The squark matrices are given at unification scale as

\[ M^u_F = m_0^2 \left( 1 - \kappa U_F \begin{pmatrix} k_1 & 0 \\ k_2 & 1 \end{pmatrix} U_F^\dagger \right), \]

(4)

where \( U_F \) is a unitary matrix and \( F \) denotes \( Q, U^c, D^c \). In the original basis where we respect the left-right symmetry, \( U_F \) is common for \( Q, U^c, D^c \). We note that the squark mass matrices are given in the notation: \((M^u_F)_{ij}Q_iQ_j^c + (M^d_F)_{ij}U_i^cU_j + (M^\nu_F)_{ij}D_i^cD_j^c\). The nonuniversal part of squark mass matrices can be generically parameterized by the \( \kappa \) term. We consider that the \( \kappa \) term is generated from a loop diagram in the form \( \alpha f \). Here, \( f \) is the quark Majorana coupling \( f(Q_{L}Q_{L}\Delta_{qq} + Q_{R}Q_{R}\Delta_{qq}^c) \), which can be unified into the neutrino Majorana coupling in a \( SO(10) \) model. In general, \( U_F \) is parameterized as \( U_F = PU_q \) where \( P \) is a diagonal phase matrix and \( U_q \) includes 3 mixing angles \( (\theta^u_{ij}) \) and 1 phase \( (\delta^u) \). We parameterize \( U_q \) in the basis where the down-type quark Yukawa matrix is diagonal. The mixing angles and a phase are parameterized in the same convention as the CKM matrix. If we consider the type II seesaw scenario in a \( SO(10) \) model, \( k_2 \) corresponds to the ratio of neutrino mass squared and \( U_F \) is the neutrino mixing matrix in the basis where the charged-lepton mass matrix is diagonal, and thus \( \theta^u_{ij} \) correspond to neutrino mixing angles when both charged-lepton and down-type quark mass matrices are simultaneously diagonalized. In general, \( \theta^u_{ij} \) are not necessarily exactly same as the neutrino mixings. The Yukawa matrices for up- and down-type quarks \( (Y_u \) and \( Y_d \)) are given as

\[ Y_u = V^{T}_{\text{CKM}} Y_u^{\text{diag}} P_u V_{\text{CKM}}, \quad Y_d = Y_d^{\text{diag}} P_d, \]

(5)

where \( P_{u,d} \) are diagonal phase matrices.

We can calculate the off-diagonal elements of the squark mass matrices \( \delta_{ij} \equiv (M^u_F)_{ij}/m_0^2 \) in the above notation.

\[ |\delta^d_{12}| \simeq \kappa \left| \frac{1}{2} k_2 \sin 2\theta^u_{13} \sin \theta^d_{13} \sin \theta^d_{12} + e^{i\delta^u} \sin \theta^d_{13} \sin \theta^d_{23} \right|, \]

(6)

\[ |\delta^d_{13}| \simeq \kappa \left| \frac{1}{2} k_2 \sin 2\theta^u_{13} \sin \theta^d_{23} - e^{i\delta^u} \sin \theta^d_{13} \cos \theta^d_{12} \right|, \]

(7)

\[ |\delta^d_{23}| \simeq \frac{1}{2} k_2 \sin 2\theta^d_{23}, \]  

(8)

where superscript \( d \) stands for that it is given in the basis where the down-type Yukawa matrix is diagonal. These quantities enter into the calculation of \( K-K, B_d-B_d \) and \( B_s-B_s \) mixing amplitudes.

When a flavor degeneracy is assumed at the unification scale and only the MSSM RGE is considered, the chargino diagram contribution dominates the SUSY contribution. However, if the flavor violation is induced by a loop diagram at the unification scale (as discussed before), the gluino diagram can generate the dominant contribution. This contribution to the mixing amplitude \( M^g_{12} \) can be written in the following mass insertion form

\[ \frac{M^g_{12}}{M^g_{11}} \simeq a (\delta^d_{LL})_{12} + (\delta^d_{RR})_{12} - \delta^d_{LL}(\delta^d_{RR})_{12}, \]

(9)

\((ji = 21, 31, 32 \) for \( K-K, B_d-B_d \) and \( B_s-B_s \) respectively) where \( a \) and \( b \) depend on squark and gluino masses, and \( \delta^d_{LL,RR} = (M^g_{22})_{LL,RR}/m^2 \) \((m \) is an averaged squark mass). The matrix \( M^g_{22} \) is a down-type squark mass matrix \( (Q, D^c)M^g_{22}(Q^c, D) \) in the basis where the down-type squark mass matrix is real (positive) diagonal. When squark and gluino masses are less than 1 TeV, \( a \sim O(1) \) and \( b \sim O(100) \). We also have contributions from \( \delta^d_{LL,RR} \), but we neglect them since they are suppressed by \((m_{0}/m_{\text{SU}}) \) \(^2 \). It is worth noting that the left-right symmetric boundary conditions give much larger SUSY contribution since both off-diagonal elements for \( LL \) and \( RR \) are large and \( b \gg a \) in the mass insertion formula.

When \( LL-RR \) contributions are dominant, the phases in \( P \) are cancelled due to \((M^g_{22})_{LL} = M^g_{22} \) and \((M^g_{22})_{RR} = (M^g_{22})^T \) (when we neglect the RGE effects). The phase of the mixing amplitude is generated from the phases in \( P_d \). Since there is no constraint for the phases in \( P_d \), the phase of the mixing amplitude is free. However, there are only two physical phases in \( P_d \) and therefore the phases of the SUSY contributions for \( K-K, B_d-B_d \) and \( B_s-B_s \) are correlated. We will show the impact of this correlation later.

The gluino contribution for the \( D-D \) mixing is obtained when we change \( d \) to \( \bar{u} \), but it needs to be written in the basis where the up-type quark Yukawa matrix is diagonal. The important quantity for the \( D-D \) mixing is \( \delta^d_{12} \) (in \( Y_u \) diagonal basis), which can be written as

\[ [V^*_{\text{CKM}}(\delta^d)Y^T_{\text{CKM}}]_{12} \sim \delta^d_{12} + V_{us}\delta^d_{22}, \]

(10)

up to the \( P_u \) phase (\( P_u \) phase gives just an overall phase of \( \delta^d_{12} \) and it is not important for the cancellation since the short-distance SM contribution of \( D-D \) is small.), and \( \delta^d_{22} \approx \kappa \sin^2 \theta^d_{23} \). Therefore, when \( \kappa \sin^2 \theta^d_{23} \) is large, both \( K-K (\delta^d_{12}) \) and \( D-D (\delta^d_{12}) \) SUSY contribution cannot be cancelled away simultaneously.

In Fig.1, we show the maximal value for \( \kappa \) allowed by the experimental results for \( K-K \) and \( D-D \) mixings as a function of \( \sin \theta^d_{13} \). We use \( \sin^2 \theta^d_{13} = 1/2 \), \( \tan^2 \theta^d_{12} = 0.4 \), \( k_2 = 0 \) and \( k_2 = 0.05 \). The SUSY parameters are chosen to be \( m_{0} = 1 \) TeV, \( m_{1/2} = 300 \) GeV (gaugino mass), \( A_0 = 0 \) (trilinear scalar coupling) and \( \tan \beta = 10 \) (ratio of Higgs vacuum expectation values). The phase \( \delta^d \) and the other phases are chosen to make the \( \kappa \) value maximal. In the usual convention, \( \sin \theta^d_{13} \) is positive since its negative value can be redefined by rephasing \( \delta^d \). But, in order to show the figure simply, we also use negative
sin $\theta_{12}^d$ as a convention. The $K\bar{K}$ ($\delta_{12}^d$) is cancelled at $\sin \theta_{13}^d \sim -\frac{1}{2} k_2 \sin 2\theta_{12}^d \cot \theta_{23}^d$ and $D\bar{D}$ ($\delta_{12}^\pi$) is cancelled at $\sin \theta_{13}^d \sim \pm \sin \theta_{23}^d V_{us}$. Due to the fact that phases in $P$ are free, $D\bar{D}$ ($\delta_{12}^\pi$) can be cancelled for both positive and negative $\theta_{13}^d$.

For most of Fig.1, the $D\bar{D}$ constraint, using the recent experimental result, is weaker than the $K\bar{K}$ constraint. However, the $D\bar{D}$ mixing is important at the $K\bar{K}$ cancellation region ($\delta_{12}^d \to 0$). As a result, the newly observed $D\bar{D}$ mixing can restrict the maximal SUSY contribution to the $B\bar{B}$ mixing. From Fig.1, we see that the maximal SUSY contribution is obtained at the $K\bar{K}$ cancellation region after satisfying the $D\bar{D}$ constraint.

We can classify the solution for fitting the of $K\bar{K}$ mixing amplitude in the following three cases, which are illustrated in Fig.2. The $K\bar{K}$ mixing amplitude is given as $M_{12} = M_{12}^{SM} + M_{12}^{SUSY}$. The mass difference is given as $\Delta M_{K} = 2|\Delta M_{12}|$, and the CP violation parameter $|\epsilon_K| = \text{Im} M_{12}/(\sqrt{2}\Delta M_{K})$. The SM predication for $M_{12}$ is in the fourth quadrant of the $M_{12}$-complex plane. The experimental measurement for $M_{12}$ is more accurate rather than the illustration in the Fig.2. However, the numerical value can have ambiguity from bag parameters and the charm quark mass. The experiment measures only the absolute values of real and imaginary parts of $M_{12}$. So the possible solutions to satisfy the experiment are the four separate regions as shown in the Fig.2. Solution A is given as $M_{12}^{SUSY} \sim -2M_{12}^{SM}$. In this solution, $M_{12}$ is in the second quadrant. Since $\text{Im} M_{12} \ll \text{Re} M_{12}$, $M_{12}$ lying in the third quadrant is almost same as solution A. In solution A, the $M_{12}^{SUSY}$ phase is almost $\pi$. In solution B, $M_{12}$ is in the first quadrant, and the $M_{12}^{SUSY}$ phase is about $\pi/2$. In solution C, $M_{12}$ is in the fourth quadrant. When $|M_{12}^{SUSY}| \ll |\text{Im} M_{12}^{SM}|$, the SUSY contribution is negligible in the $K$ system, and phase of $M_{12}^{SUSY}$ can be arbitrary. When $|M_{12}^{SUSY}| \sim \text{Im} M_{12}^{SM}$, the phase of $M_{12}^{SUSY}$ should be $0$ or $\pi$ in solution C. The solutions A, B, C which provide maximal value of $\kappa$ are shown in the Fig.1. In solutions B and C, the amount of cancellation of $\delta_{12}^d$ is larger than in solution A for a given $\kappa \sim 0.2$.

As noted, the phases of SUSY contributions for $K\bar{K}$, $B_s\bar{B}_d$, $B_s\bar{B}_s$ mixing amplitudes are related since the phases of $\delta_{12}^S$ is cancelled (up to small RGE modification) and only two physical phases in $P_t$ remain. Since all solutions A, B, C of the $K\bar{K}$ mixing provide restriction to the phases of the SUSY contributions, phases of the SUSY contribution for $B_{d,s}\bar{B}_{d,s}$ mixings are restricted for large SUSY contribution. We draw Fig.3 to show the correlation of the phases. We choose CKM parameters as $\sin 2\beta_{SM}^{SUSY} \simeq 0.77$ and $\sin 2\beta_{SM}^{SM} \simeq 0.04$. We use the same parameters for $\theta_{23}^d$, $\theta_{12}^d$, $k_2$ and SUSY mass parameters as we have used to draw Fig.1. We choose $\kappa = 0.2$ in each solution. As described, the phase of the SUSY contribution $M_{12}^{K\bar{K}}$ is almost $\pi$ in solution A. We choose the SUSY phases to be $2\pi/2$ and $\pi$ for solutions B and C, respectively. In the plot, we choose the absolute values of SUSY contributions to be same for both solutions B and C. Since the $B_{d,s}\bar{B}_{d,s}$ SUSY contribution is determined by $\delta_{23}^S$ (eq.(3)), the maximal values of $|\delta_{23}^S|$ is almost same in all three solutions. On the other hand, the $B_{d}\bar{B}_d$ SUSY contribution depends on $\theta_{12}^d$ and it is different for solutions A and B, C. One finds that $\sin 2\beta_{SM}^{eff}$ is smaller than the SM value when $\beta_{SM}^{eff}$ becomes positive in solutions A and C. Solution B gives us opposite result. This correlation is a consequence of the fact that there are only two physical phases for three different mixing amplitudes.

The global fit of the experimental data [11, 12] shows that $\sin 2\beta$ arising from the $V_{ub}$ measurement has a $2\sigma$ discrepancy from the $\sin 2\beta$ measurement from $B_d \to J/\psi K$ [11], $\sin 2\beta = 0.678 \pm 0.026$ [2]. Thus a negative SUSY contribution is favored for $\sin 2\beta_{SM}^{eff}$. The present data for $\beta_s$, eq.(4), favors negative value. As a result,
We have chosen \( \theta_{23}^\beta = \pi/4 \), which generates the maximal SUSY contribution to \( B_s^\ast \bar{B}_s \) for a given \( \kappa \). It is important that the \( D \bar{D} \) mixing constrains \( \kappa \sin^2 \theta_{23}^\beta \) at the \( K \bar{K} \) cancellation region, and thus, the \( D \bar{D} \) mixing data constrains the \( B_s^\ast \bar{B}_s \) mixing for a given \( \theta_{23}^\beta \). Naively, the SUSY contribution of \( B_s^\ast \bar{B}_s \) is proportional to \( (\kappa \sin 2\theta_{23}^\beta)^2 \). Thus, when the SUSY contribution saturates the observed \( D \bar{D} \) mixing, the maximal value of \( |\beta_s| \) becomes larger for smaller \( \theta_{23}^\beta \). Such a direction also decreases \( \sin 2\beta \), which is favored by the experimental result. So we see that the meson mixings are all related in left-right symmetric models. More accurate measurement of \( B_s^\ast \bar{B}_s \) phase will impose interesting constraint on the model.

If we consider a \( SO(10) \) model, the SUSY contribution of \( B_s^\ast \bar{B}_s \) also gets correlated to the \( \tau \rightarrow \mu \gamma \) decay amplitude [12], which is more important compared to the \( D \bar{D} \) constraint for small \( m_0 \) and large \( \tan \beta_H \). In the case of large \( m_0 \), however, the \( D \bar{D} \) constraint can be stronger than \( \tau \rightarrow \mu \gamma \). The \( \mu \rightarrow e \gamma \) decay amplitude is small due to the same cancellation condition for \( \delta_{12}^\beta \) (which reduces \( K \bar{K} \) mixing amplitude) and \( \delta_{12}^\beta \).

We assume that the left-right symmetry under the exchange of \( Q_L \leftrightarrow Q_R^\ast \). We can also consider the exchange \( Q_L \leftrightarrow (Q_R^\ast)^\ast \). In this case, the Yukawa matrices are Hermitian instead of symmetric matrices [13]. The phase matrices \( P_d \) and \( P_u \) become just signature matrices. However, the squark mass matrices satisfy \( M_2^\beta = M_3^\beta = (M_3^\beta)^\ast \) and therefore, the phases in \( P \) are not cancelled in the meson mixings. As a result, in the Hermitian Yukawa case, the phases of meson mixings are also correlated at the \( K \bar{K} \) cancellation region as in the symmetric Yukawa case.

In conclusion, we have studied the importance of \( D \bar{D} \) mixing in left-right symmetric models. We showed that the \( D \bar{D} \) mixing data constrains the phase of \( B \bar{B} \) mixing for given parameters in left-right symmetric models, and studied the correlation of the meson mixings. If we consider unified models without left-right symmetry such as \( SU(5) \), where only right-handed squark mixings can be large naively, the SUSY contribution is not very enhanced. Besides, since right-handed squark mixings are unknown, both \( K \bar{K} \) and \( D \bar{D} \) can be cancelled away separately, and therefore there is no constraint. Therefore, left-right symmetric models are very interesting candidates to investigate correlation among measurements especially when the SUSY contribution is maximal and the \( B_s^\ast \bar{B}_s \) phase is large. The improved result of this mixing phase will further shed light on the correlation of meson mixings and left-right models.

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