On the shape and evolution of a cosmic ray regulated galaxy-wide stellar initial mass function.

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\section*{ABSTRACT}
In this paper, we present a new derivation of the shape and evolution of the integrated galaxy-wide initial mass function (IGIMF), incorporating explicitly the effects of cosmic rays (CRs) as regulators of the chemical and thermal state of the gas in the dense cores of molecular clouds. We predict the shape of the IGIMF as a function of star formation rate (SFR) and CR density, and show that it can be significantly different with respect to local estimates. In particular, we focus on the physical conditions corresponding to IGIMF shapes that are simultaneously shallower at high-mass end and steeper at the low-mass end than a Kroupa IMF. These solutions can explain both the levels of \(\alpha\)-enrichment and the excess of low-mass stars as a function of stellar mass, observed for local spheroidal galaxies. As a preliminary test of our scenario, we use idealized star formation histories to estimate the mean IMF shape for galaxies of different \(z = 0\) stellar mass. We show that the fraction of low-mass stars as a function of galaxy stellar mass predicted by these mean IMFs agrees with the values derived from high-resolution spectroscopic surveys.

\textbf{Key words:} stars: mass function - galaxies: evolution - galaxies: fundamental parameters - galaxies: stellar content

\section*{1 INTRODUCTION}
In recent years, a wealth of observations have challenged the notion of a universal stellar initial mass function (IMF) in early-type galaxies (ETGs), pointing to possible variations as a function of galaxy properties such as stellar mass \((M_\star)\) or velocity dispersion \((\sigma)\). Dynamical studies (see e.g. Treu et al. 2010; Cappellari et al. 2012; Dutton et al. 2013) suggest a systematic excess of the mass-to-light ratios derived using integral field stellar kinematics with respect to the values estimated from photometry assuming a Milky-Way (MW)-like (e.g. Kroupa or Chabrier) IMF. The excess is found to increase with \(\sigma\). It is important to stress that the dynamical analysis is not able to disentangle (see e.g. Tortora et al. 2016) between a ‘top-heavy’ and a ‘bottom-heavy’ scenario, where the excess of stellar mass is due to an enhanced fraction of either stellar remnants (from exploding Supernovae) or of low-mass stars.

Spectroscopy can provide an alternative and more specific approach to study IMF variations, by taking advantage of spectral features, such as, e.g., the NaI doublet at \(\lambda\lambda 8183, 8195\) Å (Faber & French 1981; Schiavon et al. 1997, hereafter NaI8200), the TiO1 and TiO2 (Trager et al. 1998) features (\(\lambda\lambda 6000, 6300\) Å), and the Wing-Ford FeH band (Wing & Ford 1969; Schiavon et al. 1997) at 9, 900 Å. These features depend on the IMF, because of their sensitivity to stellar surface gravity and/or effective temperature, and bear information on the ratio between the total number of dwarf \((m_\star \lesssim 0.5M_\odot)\) and more massive stars \((m_\star \gtrsim 0.5M_\odot)\). In order to extract this information from high signal-to-noise ratio spectra, a comparison with predictions from stellar population synthesis models is required. Using this technique, several studies have reported an excess of low- relative to high-mass stars in the IMF of massive ETGs, i.e. a bottom-heavy IMF. The slope at the low-mass end becomes increasingly steeper (in some

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cases “super”-Salpeter, i.e. with a dwarf-to-giant ratio exceeding that of a Salpeter IMF at increasing galaxy velocity dispersion or stellar mass (Cenarro et al. 2003; Conroy & van Dokkum 2012; Ferreras et al. 2013; La Barbera et al. 2013; Spinelli et al. 2014). In addition, radial IMF gradients in local ETGs have been measured (Martín-Navarro et al. 2015; La Barbera et al. 2014). In 2013, La Barbera et al. suggested that the dwarf-enhanced population is mostly confined to the galaxy innermost regions (but see also Alton et al. 2017). It is worth noting that for old systems, such as ETGs, the study of unresolved stellar populations through their integrated light is sensitive only to long-lived stars, i.e. it does not constrain the high-mass end slope of the IMF (i.e. above \( \sim 1M_\odot \)). Moreover, IMF-sensitive features are mostly sensitive to the dwarf-to-giant ratio in the IMF (La Barbera et al. 2013, hereafter LB13), although the IMF shape might be constrained in detail, by combining features at different wavelengths (Conroy & van Dokkum 2012; La Barbera et al. 2016; Lyubenova et al. 2016).

Possible IMF variations as a function of galaxy SFR activity have been often proposed in the literature. Based on the analysis of optical colours and H$_\alpha$-line strengths for galaxies in the Galaxy And Mass Assembly (GAMA) survey, Gunawardhana et al. (2011) found evidence for a flatter IMF high-mass slope in systems with higher star formation rate. From a theoretical point of view, strongly star-forming regions are expected to have the largest deviations with respect to a universal, MW-like, IMF (see e.g. Klessen et al. 2005). Weidner & Kroupa (2005) proposed a derivation of the integrated galaxy-wide stellar IMF (IGIMF), based on a limited number of physically and observationally motivated axioms. It is possible to reformulate each of these as a function of SFR, and thus predict the IMF shape as a function of this key physical property of galaxies. The original IGIMF approach postulates the universality of the IMF inside individual molecular clouds (MCs), with the variability being driven by additional assumptions on the distribution of molecular clouds in the galaxy and by physical considerations on the mass of the most massive star that can form in a given MC. These assumptions translate into an invariant shape for the low-mass end of the IGIMF, which is at variance with results obtained from the spectra of ETGs (but see Jerabkova et al. in preparation, for a recent update of the IGIMF framework). A different approach has been taken by Papadopoulos et al. (2011, PP11 hereafter), who consider the role of cosmic rays (CRs), associated with SNe and stellar winds, in regulating star formation in MCs. CRs are assumed to be very effective in altering the thermal and chemical properties of the inner - UV shielded - regions of MCs, eventually changing the relation between gas density and temperature in MC cores. The characteristic Jeans mass of young stars (\( M_J \)) forming in each MC is thus affected. PP11 discuss numerical solutions to the thermal and chemical equations describing the evolution of the interstellar medium in this scenario (see also Thi et al. 2009). These numerical solutions provide an estimate for \( M_J \) as a function of cluster core density \( \rho_c \) and CR ionization rate.

Although featuring a different evolution of the IMF shape (in the IGIMF framework the evolution is mainly tied to the high-mass end, whereas in the CR approach to the knee of the mass function), both models predict similar variations in strongly star-forming objects, i.e. the IMF should become 'top-heavier', at the high mass end, than in the local neighbourhood at increasing SFR or CR energy density. In previous work, we tested the impact of the IGIMF (Fontanot et al. 2017; F17 hereafter) and CR regulation (Fontanot et al. 2018; F18 hereafter) on the evolution of the physical and chemical properties of galaxies, as predicted by the semi-analytic model GAEA (Galaxy Evolution and Assembly - Hirschmann et al. 2016). We have shown that in both frameworks we reproduce a mass and mass-to-light ratio excess (Cappellari et al. 2012; Conroy et al. 2013) with respect to a MW-like IMF, which we interpret as driven by the mismatch between intrinsic physical properties and those derived from synthetic photometry under the assumption of a universal IMF. In addition, in both scenarios we are able to reproduce, at the same time, the observed increase for the [\( \alpha/Fe \)] ratio as a function of galaxy stellar mass, and the mass-metallicity relation of ETGs. Matching both observables represents a known problem for hierarchical models implementing a universal IMF (De Lucia et al. 2017). In particular, reproducing the [\( \alpha/Fe \)]-\( M_\star \) relation requires a balancing between the relative number of Type-II and Type-Ia supernovae, contributing to most of the \( \alpha \)-elements and iron, respectively. Assuming a 'top-heavy' IMF at increasing SFR helps matching the observations, as it implies a larger fraction of Type-II SNe in starbursts (associated with massive galaxies). However, that is in contrast with the finding of a 'bottom-heavy' IMF in massive galaxies. On the other hand, a 'bottom-heavy' IMF with a slope steeper than Salpeter over the entire stellar mass range implies a low fraction of Type-II supernovae and this is in contrast with the observed metallicity and abundance ratios of massive galaxies. These considerations are even more puzzling in light of recent studies finding IMF radial trends in ETGs, with a steepening of the IMF low-mass end slope in the innermost, most metal-rich, galaxy core regions. In order to reconcile these apparently contradicting results, Weidner et al. (2013) and Ferreras et al. (2013) have proposed a time-varying scenario, where the low- and high-mass ends of the IMF vary independently over different time scales, with the top-heavy regime dominating the first (bursty) phase of star formation, followed by a bottom-heavy regime. The implications of such a variable IMF for ETG chemical evolution have been explored, e.g., in De Masi et al. (2018). However, a physical explanation for this time-varying scenario, possibly related to the complex physics of gas fragmentation in the dense core regions of massive galaxies at high redshift, is still lacking.

In the present work, we combine the IGIMF and PP11 approaches to derive a new formulation for the IGIMF that takes into account explicitly the effect of CR heating on the IMF of individual MCs. We present this new combined derivation, that we dubbed CR-IGIMF, in Sec. 2, we show its basic properties in Sec. 3 and we then discuss its results and their implications in Section 4.

## 2 COSMIC RAYS REGULATED IGIMF

In this paper, we present a derivation of the IGIMF (\( \varphi_{\text{IGIMF}} \)) based on an approach similar to Weidner & Kroupa (2005, see also Kroupa et al. 2013 for a review). We integrate the IMF associated with individual clouds (\( \varphi_c(m) \)), weighted by the mass function of individual MCs (\( \varphi_{\text{CL}}(M_{\text{CL}}) \)):

\[
\varphi_{\text{IGIMF}}(m) = \int_{M_{\text{min}}}^{M_{\text{max}}} \varphi_c(m \leq m_{\text{max}}(M_{\text{CL}})) \varphi_{\text{CL}}(M_{\text{CL}}) dM_{\text{CL}}(1)
\]

The key quantities involved in the integration are the maximum value of the mass of a star cluster (\( M_{\text{max}}^{\text{Cl}} \)) and the largest stellar mass (\( m_{\text{max}} \)) forming in a given cluster. We set the mass of the smallest star cluster to \( M_{\text{min}}^{\text{Cl}} = 5M_\odot \), following evidences from the Taurus-Auriga complex (Kroupa & Bouvier 2003). As in F17, \( M_{\text{max}}^{\text{Cl}} \) and \( m_{\text{max}} \) can be defined as a function of the instantaneous SFR using the following axioms:
\[\log M_{\text{cl}}^{\text{max}} = 0.746 \log SFR + 4.93;\]  
\[\log m_{\text{br}}^{\text{max}} = 2.56 \log M_{\odot} \times \left[3.82^{0.17} + (\log M_{\odot})^{0.17}\right]^{0.17} = 0.38;\]  
\[\varphi_{\text{cl}}(M_{\text{cl}}) \propto M_{\text{cl}}^{-\beta};\]  
\[\beta = \begin{cases} 2 & \text{SFR} < 1 M_{\odot}/\text{yr} \\ -1.06 \log \text{SFR} + 2 & \text{SFR} \geq 1 M_{\odot}/\text{yr} \end{cases}\]  
\[\alpha_3 = \begin{cases} 2.35 & \rho_{\odot} < 9.5 \times 10^4 M_{\odot}/\text{pc}^3 \\ 1.86 - 0.43 \log (\rho_{\odot}) & \rho_{\odot} > 9.5 \times 10^4 M_{\odot}/\text{pc}^3 \end{cases}\]  
\[\log \rho_{\odot} = 0.61 \log M_{\odot} + 2.85.\]

Using stellar cluster data, Weidner et al. (2004) derived Eq. 2 to describe the dependence of \(M_{\text{max}}^{\text{cl}}\) on the instantaneous SFR (see also Kroupa et al. 2013 for an analytic derivation). As in F17, we impose \(M_{\text{max}}^{\text{cl}} \lesssim 2 \times 10^7 M_{\odot}\). Eq. 3 represents a fit to the numerical solution to the problem of finding the maximum stellar mass forming in a cluster of mass \(M_{\text{cl}}\) (Pflamm-Altenburg et al. 2007), under the hypothesis that it contains exactly one \(m_{\text{br}}^{\text{max}}\) star and using the canonical IMF. Eq. 3 describes the assumed functional shape for the star cluster mass function. Allowed values for \(\beta\) (Eq. 5) are chosen based on local observations that suggest \(\beta = 2\) (Lada & Lada 2003), and on \(z \lesssim 0.35\) data from the GAMA survey (Gunawardhana et al. 2011) that require a flattening of \(\beta\) at high-SFRs. Eq. 5 takes into account possible variations of the high-mass end slope \(\alpha_3\) from the reference choice \(\alpha_3 = 2.35\) (see Kroupa et al. 2013, for a review). As in F17, in this work we assume \(\alpha_3\) depends on \(\rho_{\odot}\), as proposed by Marks et al. (2012). This relation has the advantage of being independent of other parameters (such as metallicity), that have been shown to also correlate with \(\alpha_3\) (see e.g. Martin-Navarro et al. 2013). Finally, Eq. 7 describes the relation between \(\rho_{\odot}\) and \(M_{\odot}\) as derived by Marks & Kroupa (2012).

Using the equations above, and assuming a given \(\varphi_{\text{cl}}(m)\) for individual MCs, it is then possible to construct the IGIMF corresponding to an individual SF event. In the original Weidner & Kroupa (2005) framework, the IMF associated with individual MCs has a canonical broken power-law shape with three slopes (as in Kroupa 2002; see Eq. 1 in F17). In this paper, we use a slightly different approach by considering a variable inner break at \(m_{\text{br}}\):

\[\varphi_{\text{cl}}(m) = \begin{cases} m_{\text{br}}^{\alpha_1} & m_{\text{br}} \leq m < m_{\text{low}} \\ m_{\text{low}}^{\alpha_2} & m_{\text{low}} \leq m < m_1 \\ m_1^{\alpha_3} & m_1 \leq m \leq m_{\text{max}} \end{cases}\]

where the parameters \(m_{\text{br}} = 0.1, m_1 = 1.0, \alpha_1 = 1.3\) and \(\alpha_2 = 2.35\) are fixed. The position of the break at \(m_{\text{br}}\) is derived using the approach of PP11, assuming that it corresponds to the characteristic Jeans mass (\(M_J^*\)) of young stars in a MC with core density given by Eq. 7 and affected by the CR energy density \(U_{\text{CR}}\). Whereas in F18 we fixed the knee to the value of \(M_J^*\) for a typical density \(\rho_{\odot} = 10^5 \text{cm}^{-3}\), in the present paper we use a different approach, considering a grid of values \(m_{\text{br}} = M_J^*(\rho_{\odot}, U_{\text{CR}})\).

For practical reasons, we use the numerical solutions to the chemical and thermal equations for the CR regulated ISM presented in Fig. 4 of PP11. These numerical experiments provide indeed interesting insights on the key physical dependencies. In particular, \(m_{\text{br}}\) increases with \(U_{\text{CR}}\); this is mainly due to the higher CR heating associated with the increased energy density. At fixed \(U_{\text{CR}}\), \(m_{\text{br}}\) is predicted to decrease with \(\rho_{\odot}\). This behaviour is not trivial, as high density regions are expected to be generally hotter, and the Jeans mass increases with temperature and decreases with density. Hence, in the numerical treatment of PP11, the dependence on temperature appears sub-dominant with respect to that on density. The effect is important in our analysis, as it allows changes in the low-mass end slope of the IGIMF (as discussed below). In the following, we use the same notation as in PP11, and consider \(U_{\text{CR}}\) values relative to the corresponding MW one (\(U_{\text{MW}}\)).

### 3 PROPERTIES OF THE CR-IGIMF

We can now compute the IGIMF shape as a function of both SFR and \(U_{\text{CR}}/U_{\text{MW}}\). It is important to keep in mind that the original PP11 approach focuses on individual MCs, and SFR and \(U_{\text{CR}}/U_{\text{MW}}\) are treated as independent variables (\(U_{\text{CR}}\) represents an external field). Our combined approach, instead, considers both SFR and \(U_{\text{CR}}\) as “global” variables, given by an average over SF regions. Therefore, in our implementation, SFR and \(U_{\text{CR}}/U_{\text{MW}}\) are not completely independent variables. In Fig. 4 we show some representative IMF shapes. Each panel refers to a different value of \(U_{\text{CR}}/U_{\text{MW}}\), and three different SFR values (100, 1, and 0.01 \(M_{\odot} \text{ yr}^{-1}\)) each normalized to 1 \(M_{\odot}\). Fig. 4 shows that the CR-IGIMF framework features a variety of IMF shapes, while for a SF galaxy with MW-like energy density field it predicts an IMF shape that is in good agreement with a Kroupa-like IMF (green solid line in the middle-upper panel). At fixed \(U_{\text{CR}}/U_{\text{MW}}\), the high-mass end becomes shallower at increasing SFR, as in the original IGIMF approach. We thus expect that a model implementing the CR-IGIMF should be able to reproduce the observed trend in the [o/Fe]-stellar mass relation of ETGs (see e.g. Thomas et al. 2010). At the low-mass end, the situation is more complex, and depends on \(U_{\text{CR}}\). For CR densities much higher than in the MW, the low-mass end slope is constant, whereas for \(U_{\text{CR}}/U_{\text{MW}} \lesssim 10\) there is a clear trend for a steepening of the IMF at increasing SFR. By construction (see above), the minimum and maximum slopes for the low-mass end of the CR-IGIMF are \(\alpha_1\) and \(\alpha_2\). In our approach, \(\alpha_1\) and \(\alpha_2\) assume a fixed value (1.3 and a Salpeter-like slope of 2.35). This is a conservative choice, which complies with both local observations of individual clouds and with theoretical calculations of the fragmentation of giant MCs (Hennebelle & Chabrier 2008). However, we note that there is no physical reason for these slopes to be the same for MCs evolving in physical conditions very different from those in the MW. Indeed, variations in the IMF shape of individual MCs have been considered in the original IGIMF framework (see e.g. Yan et al. 2017). In particular, Eq. 8 follows an empirical calibration connecting the IMF high-mass end slope and the observed...
properties of local MCs (Marks et al. 2012). Nonetheless, as shown explicitly in Papadopoulos et al. (2011), the impact of $U_{CR}$ on $M_J^*$ is such that it can also affect the IMF shape of individual clouds for $m_\star \lesssim 1M_\odot$. Our approach accounts for this effect, thus assuming that CRs can affect the IMF shape on a wider stellar mass range than in the original IGIMF formulation.

For a MW-like $U_{CR}$ environment, high-SFR events correspond to a steeper low-mass end slope and to a shallower high-mass end slope with respect to a Kroupa IMF, at the same time. Therefore, our implementation of the CR-IGIMF should explain the enhanced fraction of low-mass-to-giant stars inferred from IMF-sensitive features in the spectra of ETGs (Conroy & van Dokkum 2012, La Barbera et al. 2013, Sarzi et al. 2017), while qualitatively preserving most of the results discussed in F17 and F18. Analysing these aspects in detail requires a self-consistent theoretical model including explicitly the effects of the CR-IGIMF. In the present work, we present a preliminary analysis focusing on the mass fraction of low-mass stars ($f_{dg}$), defined as the fraction of mass in stars with $m_\star < 0.6M_\odot$, with respect to the total mass in stars $m_\star < 1.0M_\odot$.

Figure 1. Composite shapes for the Cosmic Ray regulated IGIMF. Different panels refer to different CR energy densities, as labelled in the lower–left corner of each panel. Red dashed, green solid and blue dotted lines correspond to SFR=100, 1, and 0.01 $M_\odot$ yr$^{-1}$, respectively. In all panels, the thin solid line shows the canonical Kroupa (2001) IMF.
The value \( f_{dg} \) represents the dwarf-to-giant ratio, that is the main quantity constrained by IMF-sensitive spectral features in the spectra of \( z \sim 0 \) ETGs. Using a large SDSS sample of ETGs, Ferreras et al. (2013) and LB13 constructed 18 stacked galaxy spectra, covering a velocity dispersion range from \( \sim 100 \) to \( \sim 300 \text{ km/s} \). The authors found a systematic steepening of the low-mass end as a function of galaxy velocity dispersion, and parameterized the IMF by either a single power-law (“unimodal”) or a low-mass tapered (“bimodal”) distribution. We consider here results of the “2SSP+XFe” fitting method of LB13, where spectral indices are used to fit stellar population models including two single stellar populations (SSPs), and accounting for the effect of non-solar abundance ratios. As shown in LB13, both unimodal and bimodal models fit equally well the data, but provide very different mass-to-light ratio estimates. The two IMF parameterizations provide, however, very similar dwarf-to-giant fractions. Computing \( f_{dg} \) from the LB13 “2SSP+XFe” results, we find values ranging between 0.6 and 0.95. These values refer to the inner core regions of ETGs, as observed by the SDSS fiber aperture spectra. For galaxies in the highest mass bin considered in LB13, this aperture corresponds to about 1/4 of a galaxy’s effective radius. We compute \( f_{dg} \) for individual CR-IGIMFs, using a grid of 42 independent realizations, with seven SFR levels (with \( \log(SFR/M_\odot \text{ yr}^{-1}) = -3, -2, -1, 0, 1, 2, 3 \)) and six \( U_{CR}/U_{\text{MW}} \) values (0.2, 1, 10, 100, 1000, 10000, as in Papadopoulos et al. (2011)). On this CR-IGIMF grid, \( f_{dg} \) ranges between 0.6 and 0.8, as shown in Fig. 2. The reason for this range of values is that, by construction (see Eq. 3), no CR-IGIMF in our approach can have a slope steeper than \( \alpha_2 = 2.35 \) (i.e. a Salpeter IMF), or shallower than \( \alpha_1 = 1.3 \). For single power-law IMFs, these slopes correspond to \( f_{dg} = 0.84 \) \( (\alpha_2) \) and \( 0.6 \) \( (\alpha_1) \), respectively. The Figure shows a clear trend for an increase of \( f_{dg} \) with SFR at fixed \( U_{CR} < 100 \). Viceversa, \( f_{dg} \) decreases with increasing \( U \), at fixed SFR. In particular, for CR density values larger than 100 times that of the MW, the low-mass end slope of the IMF is constant, with ratios below the expectation for a Kroupa IMF. In order to get \( f_{dg} \) values as high as 0.95 (found for the core regions of the most massive ETGs in the LB13 sample), one should postulate that also the intrinsic IMF for individual MCs should assume “super”-Salpeter slopes under specific physical conditions (e.g. for high-metallicity/density environments; see Martín-Navarro et al. (2015)).

4 RESULTS AND DISCUSSION

The crucial aspect we want to test in the present work is if the CR-IGIMF scenario, once embedded in a realistic galaxy formation context, can match the observed trends. To this purpose, we use mean star formation histories (SFHs) predicted by the GAEA semi-analytic model (Hirschmann et al. 2016) for \( z = 0 \) galaxies in four different stellar mass bins, namely \( M_\star \sim 10^{10}, 10^{11.5}, 10^{10.5}, \) and \( 10^{9.25} M_\odot \), respectively. These SFHs have been extracted from the reference GAEA realisation, and from runs implementing variable IMF approaches (F17 and F18). Galaxies more massive than \( \sim 10^{10.5} \) are typically characterized by an early peak of SFR, followed by a smooth decay. The epoch, height and width of the peak depend on the final galaxy stellar mass, with more massive galaxies having an earlier, narrower, and higher peak (see also...

Figure 2. Fraction of dwarf-to-giant stars, \( f_{dg} \), for individual CR-regulated IGIMFs (see the text for details). Different lines correspond to different CR densities, as labelled in the upper–left corner of the Figure. The horizontal black dashed line shows the \( f_{dg} \) value for a MW-like Kroupa IMF.

Figure 3. The expected fraction of dwarf-to-giant stars, \( f_{dg} \), is plotted against galaxy velocity dispersion, for mock SFHs extracted from semi-analytic models. Red bullets with errorbars show the observational constraints from LB13, after correcting \( f_{dg} \) to an aperture of one effective radius by assuming IMF radial gradients as in La Barbera et al. (2017). Blue stars are predicted values of \( f_{dg} \) for mean SFHs extracted from GAEA, in reference stellar mass bins (i.e. \( M_\star \sim 10^{12}, 10^{11.5}, 10^{10.5}, \) and \( 10^{9.25} M_\odot \), respectively), and assuming a fixed \( U_{CR}/U_{\text{MW}} = 1 \). Grey shaded areas correspond to 1-, 2- and 3-\( \sigma \) levels around the mean \( f_{dg} \) versus \( \sigma \) relation for the mocks galaxy sample extracted from the PP11 run, while green diamonds correspond to the predictions for mean SFHs in our reference mass bins (see the text for details).
Figure 4. Same as in Fig. 1 but different panels refer to different SFR levels, as labelled in the lower–left corner of each panel. Yellow solid, magenta dashed and cyan dotted lines correspond to $U_{\text{CR}}/U_{\text{MW}} = 1$, 10, 100, respectively. In all panels, the thin solid line shows the canonical Kroupa IMF.

De Lucia et al. (2006). Lower mass galaxies have almost constant SFHs. For the mean SFH in each mass bin, we compute the mass-weighted global IMF using our CR-IGIMF grid, and then the corresponding value of $f_{\text{dg}}$. It is worth noting that in our estimate for $f_{\text{dg}}$ we only consider the $< 1\, M_\odot$ mass range, where the mass weighted IMF coincides with the present day mass function. Moreover, observational constraints are closer to luminosity-weighted IMFs. However, for very old stellar populations (i.e. massive ETGs), we do not expect this difference to change our conclusions significantly. In Fig. 4 we compare these $f_{\text{dg}}$ for $U_{\text{CR}}/U_{\text{MW}} = 1$ (blue stars - for each mass bin we plot separately the values corresponding to the mean SFHs extracted from the three GAEA realizations considered), with the observed values. We correct the observed values of $f_{\text{dg}}$ from LB13 to an aperture of one effective radius, whose properties we consider to be comparable to SAM predictions. To this aim, we assume an IMF radial profile with the same shape as that recently derived for one massive ETG by La Barbera et al. (2017). For each mass bin, the profile is rescaled in the central region, in order to match the $f_{\text{dg}}$ value within the SDSS fiber aperture, as detailed in LB13. The correction, which is largest at highest mass, and negligible at lowest mass brings all $f_{\text{dg}}$ values from LB13 in the range from $\sim 0.72$ to $\sim 0.8$ as illustrated in Fig. 5 (see red dots with error bars, corresponding to the 18 stacked spectra of LB13). Remarkably, the aperture correction brings the observed $f_{\text{dg}}$ values within the range predicted by the CR-IGIMF.

For model galaxies, we estimate $\sigma$ using the stellar mass–$\sigma$ relation derived by Zahid et al. (2016), and compute stellar masses from the SFHs under the hypothesis of a Kroupa IMF, consistently with the adopted mass–$\sigma$ relation. We checked that our conclusions hold if we estimate the actual stellar mass associated with the adopted SFHs, i.e. using the actual IMF shape and appropriate mass fraction locked into stellar remnants for each varying IMF. Model predictions with $U_{\text{CR}}/U_{\text{MW}} = 1$ reproduce well the trend of increasing $f_{\text{dg}}$ with $\sigma$. We verified that this applies also to models with fixed $U_{\text{CR}}/U_{\text{MW}} \lesssim 10$. We finally relax the hypothesis of a uniform $U_{\text{CR}}/U_{\text{MW}}$ for our toy SFHs, and we consider predictions from the GAEA run implementing the PP11 approach (as defined in F18). In this run, we are able to track at the same time the evolution of SFR and $\Sigma_{\text{SFR}}$ for each model galaxy. As in F18, we assume $U_{\text{CR}}/U_{\text{MW}} = \Sigma_{\text{SFR}}/\Sigma_{\text{MW}}$, i.e. that the SFR density is a good proxy for $U_{\text{CR}}$ over the star-forming disc. We then compute the mean SFRs and SFR densities as a function of cosmic time in our reference mass bins. We also extract individual SFHs and SFR density histories for $\sim 380000$ individual $z = 0$ model galaxies from the same realisation. This information allows us to follow the time evolution of our model galaxies in the CR-IGIMF library. Fig. 5 shows the resulting $f_{\text{dg}}$ as a function of $\sigma$. Green diamonds mark average values corresponding to the four reference mass bins. The grey areas correspond to the 1-, 2-, 3-$\sigma$ levels around the mean relation for the mock galaxies sample extracted from the PP11 run, that nicely match the (aperture-corrected) trend of increasing $f_{\text{dg}}$ with $\sigma$ obtained by LB13. Interestingly, the trend is predicted to flatten at the highest stellar mass bin probed by the F18 models, although the number of mock galaxies in this range is small. This is mainly due to the fact that more massive galaxies in F18 form preferentially in strong compact starburst, with high SFR and $\Sigma_{\text{SFR}}$, that correspond to lower $f_{\text{dg}}$. To better highlight this effect, we show in Fig. 6 the different CR-IGIMF shapes at fixed SFR, as a function of $U_{\text{CR}}/U_{\text{MW}}$. In our toy SFHs a massive galaxy is characterized by an early large peak of SF: in these conditions (upper panels in Fig. 6) our CR-IGIMF scenario consistently predicts a shallower high-mass end slope. The low-mass end slope, that is mainly responsible for setting $f_{\text{dg}}$ strongly depends on the SFR density (that we use as a proxy for $U_{\text{CR}}$). Therefore, an accurate estimate for the CR energy density is a crucial element to correctly predict the shape of the CR-IGIMF. The late-time evolution of a massive galaxy is associated with low-SFR and low-SFR density, that correspond to shallower low-mass end slopes (bottom panel of Fig. 6).

In order to further compare predictions from our CR-IGIMF model to observations, we compute synthetic model spectra corresponding to the F18 mean SFHs in the reference mass bins of $M_* = 10^{9.25}$ and $10^{11.5}$ ($\sigma \sim 60$ and $\sim 280$ km/s). We consider the MILES$^5$ stellar library and the Padova 2000 isochrones$^6$. Since we want to highlight the effect of a varying IMF, rather than other stellar population properties, for both mass bins, we consider the same (old) age of $\sim 10$ Gyr, and solar metallicity. Fig. 7 shows, as thick red curve, the ratio between these toy model spectra for the

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4 It is worth noting that this approach is consistent with recent findings that lowest mass galaxies have shallower gradients compared to the most massive ones (Martín-Navarro et al. 2013; Parikh et al. 2013).

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6 In order to obtain the desired values of metallicity, gravity, and effective temperature, we interpolate MILES spectra using the same algorithm as in Vazdekis et al. (2010). We then weight the interpolated spectra using the mass–$V$-band luminosity relation, rather than using empirical relations (as in Vazdekis et al. 2010). Therefore, the resulting SSPs should be regarded as simple toy models, and not full MILES SSP SEDs.
Such a model would allow us to carry out a more detailed comparison with observational data, including a direct comparison of synthetic spectra for our mock galaxies with observed ones. Another aspect that deserves further investigation is that of IMF radial gradients in ETOs. We expect the central regions (i.e. those showing a stronger signal for a “bottom-heavy” IMF) to have larger SFRs and SFR densities, with respect to the galaxy outskirts. Naively, in the current CR-IGIMF implementation, we would tend to associate slightly lower SFH values to these regions, in contrast to some recent observational results.

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van Dokkum et al. 2017, Sarzi et al. 2017. Again, a self-consistent version of GAEA implementing the CR-IGIMF is clearly needed to study this aspect, taking into account the different star-forming regions associated with the hierarchical assembly of massive ETOs and their individual IMFs. Moreover, as mentioned above, one should consider that specific physical properties (e.g. metallicity - see Martin-Navarro et al. 2013) may affect the IMF of individual MCs in the core regions of massive ETOs, hence requiring further modifications to the CR-IGIMF framework.

The results presented in this work are meant to show the capabilities of the CR-IGIMF approach in a galaxy evolution framework. Our models predict \( f_{\text{dg}} \) values, over a spatial scale comparable to that of galaxy sizes, that are consistent with the observed ones. This demonstrates that the CR-IGIMF scenario is able to capture key features of the IMF low- and high-mass end, that have been elusive so far. Forthcoming work will focus on the development of theoretical models self-consistently including both the effect of CR-IGIMF on model galaxy evolution and the synthesis of realistic mock spectra. The present work shows that these tools are of fundamental importance for a interpreting the wealth of observational data in favour of a variable IMF.
