Abstract

If dark energy is a form of quintessence driven by a scalar field $\phi$ evolving down a monotonically decreasing potential $V(\phi)$ that passes sufficiently below zero, the universe is destined to undergo a series of smooth transitions: the currently observed accelerated expansion will cease; soon thereafter, expansion will come to end altogether; and the universe will pass into a phase of slow contraction. In this paper, we consider how short the remaining period of expansion can be given current observational constraints on dark energy. We also discuss how this scenario fits naturally with cyclic cosmologies and recent conjectures about quantum gravity.

Introduction. In the $\Lambda$CDM model, dark energy is assumed to be due to a positive cosmological constant, in which case the current period of accelerated expansion will endure indefinitely into the future [1]. An alternative is that the current vacuum is metastable and has positive energy density. If it is separated by an energy barrier from a true vacuum phase with zero or negative vacuum density, then accelerated expansion will be ended by the nucleation of a bubble of true vacuum that grows to encompass us. Until that moment, cosmological observations will be indistinguishable from the $\Lambda$CDM picture. Without extreme fine tuning, the time scale before a bubble will nucleate [2] and pass our location can be exponentially many Hubble times in the future (see, for example,
Refs. [3, 4]). (Here and throughout, ‘Hubble time’ refers to $H_0^{-1} \approx 14$ Gy where $H_0$ is the current Hubble expansion rate.) Also, the ultra-relativistic bubble wall will likely destroy all observers in its path, so there will be no surviving witnesses to the end of accelerated expansion [2].

A third possibility, to be considered here, is that the dark energy is a form of quintessence due to a scalar field $\phi$ evolving down a monotonically decreasing potential $V(\phi)$ [5]. In this case, since the current value of $V(\phi_0)$ is extraordinarily small today as measured in Planck mass units, there is a wide range of forms for $V(\phi)$ that pass through zero and continue to large negative values where $V(\phi) \ll -V(\phi_0)$. In this case, the equations of motion of Einstein’s general theory of relativity dictate that the universe is destined to undergo a remarkable series of smooth transitions [6, 7].

First, as the positive potential energy density decreases and the kinetic energy density comes to exceed it, the current phase of accelerated expansion ends and smoothly transitions to a period of decelerated expansion. Next, as the scalar field continues to evolve down the potential, the potential energy density becomes sufficiently negative that the total energy density ($\propto H^2(t)$) and, consequently, the Hubble parameter $H(t)$, reaches zero. Expansion ($H > 0$) stops altogether and smoothly changes to contraction ($H < 0$). More precisely, the transition is to a phase of slow contraction [6, 7] in which the Friedmann-Robertson-Walker (FRW) scale factor $a(t) \propto |H|^\alpha$ where $\alpha < 1/3$.

In this paper, we consider how soon these transitions could begin. That is, what is the minimal time, beginning from the present ($t = t_0$), before expansion ends and contraction begins given current observational constraints on dark energy and without introducing extreme fine-tuning? One might imagine the answer is several Hubble times given how well $\Lambda$CDM is claimed to fit cosmological data.

The Q-SC-CDM model. Q-SC-CDM refers to cold dark matter (CDM) models with a phase of quintessence-driven (Q) accelerated expansion transitioning in the future to decelerated expansion and subsequently to slow contraction (SC),
where all phases are dominated by a scalar field \( \phi(x, t) \) evolving down a potential \( V(\phi) \).

The series of continuous transitions can be understood by tracking the total cosmic equation-of-state \( \varepsilon_{\text{TOT}}(t) \), including both matter and dark energy densities:

\[
\varepsilon_{\text{TOT}} \equiv \frac{3}{2} \left( 1 + \frac{p_{\text{TOT}}}{\rho_{\text{TOT}}} \right) \equiv \frac{3}{2} \left( \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \frac{\dot{\rho}_M}{\rho_M} \right),
\]

where \( p_{\text{TOT}} \) and \( \rho_{\text{TOT}} \) are the total pressure and energy density, respectively; \( p_Q \equiv \frac{1}{2} \dot{\phi}^2 - V \) is the scalar field pressure; \( \rho_Q \equiv \frac{1}{2} \dot{\phi}^2 + V \) is the scalar field energy density; and \( \rho^0_M \) is the current (pressureless) matter density. (Dot represents the derivative with respect to FRW time.)

As \( V(\phi) \) approaches zero from above, \( \varepsilon_{\text{TOT}} \) grows to be greater than one, which marks the end of accelerated expansion (\( \ddot{a} > 0 \)) and the beginning of decelerated expansion (\( \ddot{a} < 0 \)) according the Friedmann equation:

\[
\frac{\ddot{a}}{a} = \frac{4\pi G}{3} (1 - \varepsilon_{\text{TOT}}) \rho_{\text{TOT}},
\]

where \( G \) is Newton’s constant. The value of \( \varepsilon_{\text{TOT}} \) continues to rise as \( V(\phi) \) passes below zero until \( V(\phi) \) becomes sufficiently negative that \( \rho_{\text{TOT}} \) reaches zero. According to the Friedmann constraint,

\[
H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho_{\text{TOT}},
\]

the expansion rate \( H(t) \) also reaches zero at that point. (Spatial curvature is negligible today and throughout these stages.) The Friedmann equations above combined with the equation-of-motion for the scalar field

\[
\ddot{\phi} + 3H \dot{\phi} = -V_{,\phi}
\]

dictate that the field continues to evolve down its potential and that \( H \) continues to decrease. This means that \( H \) passes through zero; \( i.e., \) the universe necessarily begins to contract. (Note that Eq. (3) ensures that \( \rho_{\text{TOT}} \) cannot become negative; rather \( \rho_{\text{TOT}} \) increases from zero once contraction begins.) For the
Figure 1: The Q-SC-CDM scalar field potential in Eq. (5), \( V(\phi) \) (in units of \( H_0^2m_{Pl}^2 \)) vs. \( \phi/m_{Pl} \) with \( M = 1.7m_{Pl} \) and \( m = 0.1m_{Pl} \). As described in the text, the field is fixed by Hubble friction near \( \phi_b \) until around redshift \( z = 3 \) (\( t = t_b = -0.8H0^{-1} \)); it then evolves to \( \phi = 0 \) today (\( t = t_0 \)); continues to evolve to \( \phi_{dec} > 0 \), at which time (\( t_{dec} \)) accelerated expansion turns to decelerated expansion; and then \( \phi \) evolves further until \( V(\phi) \) becomes sufficiently negative (at \( t = t_{con} \)) that the Hubble parameter \( H \) passes through zero, the expansion phase ends and slow contraction begins.

steep potentials of interest in this paper that minimize the time until expansion ends, \( \varepsilon_{TOT} \) becomes \( \gg 3 \), corresponding to \( a(t) \propto |H|^\alpha \) where \( \alpha \approx 1/\varepsilon_{Q} \ll 3 \), the condition for slow contraction.

Notably, the entire sequence of transitions from accelerated expansion to slow contraction would be sufficiently smooth and slow that observers could safely survive and bearing witness to each stage.

To determine the minimal time before these transitions could occur, we consider Q-SC-CDM potentials of the form

\[
V(\phi) = V_0 e^{-\phi/M} - V_1 e^{\phi/m}. \tag{5}
\]

The initial conditions and parameters \( V_{0,1} > 0 \) are chosen such that \( \phi \) evolves from \( \phi \leq 0 \) in the past (\( t \leq t_0 \)) to \( \phi > 0 \) in the future, as illustrated in Fig. 1. The first (positive) potential term dominates during the quintessence-
driven accelerated expansion phase (which includes the past and part of the future; and the second (negative term) dominates beginning at some time in the future.

The initial value of the scalar field at the beginning of the dark energy dominated phase, $\phi = \phi_b$, is uniquely determined once the parameters are chosen such that $\Omega_m^0$ and $\Omega_{DE}^0$, the ratios of the present dark energy and matter densities to the critical density, are in accord with current observational limits. More precisely, extrapolating the Friedmann equations and the equation of motion for $\phi$ back in time beginning from $\phi_0 = 0$, one finds that the scalar field becomes frozen by Hubble friction (the $3H\dot{\phi}$ term in Eq. (4)) as matter dominates over dark energy, which is what sets the value of $\phi_b$.

This form for $V(\phi)$ is well-suited to determining the shortest time before the end of expansion. The shortest time – i.e., the fastest evolution of $\phi$ – corresponds to the steepest potential – the largest allowed value of $|V,\phi/V|$ or smallest values of $M$ and $m$ in Eq. (5) – both consistent with observations and without encountering extreme fine-tuning. As shown in Ref. [8], a positive exponential potential with $M \approx 1.7m_{Pl}$, is the steepest potential compatible (to within $2\sigma$) with current observations, where $m_{Pl} = 1/\sqrt{8\pi G}$ is the reduced Planck mass. The negative potential term is negligible in the past, so $m$ is not constrained by observations. However, $m/M$ can be viewed as the figure-of-merit for judging fine-tuning; we therefore confine our study to values of $10^{-2} < m/M < 1$.

A worked example. Fig. 1 illustrates the case where $m = 0.1m_{Pl}$. The potential parameters $V_0 = 2.1H_0^2m_{Pl}^2$ and $V_1 = 0.28H_0^2m_{Pl}^2$ are chosen such that $\Omega_m^0 = 0.29$ and $\Omega_{DE}^0 = 0.71$, within current observational limits [9] [10].

For the negative exponential potential term in Eq. (5) that dominates by the time $H$ reaches zero and contraction begins, there is an attractor solution with $a(t) \propto |H|^\alpha$ where $\alpha = 2(m/m_{Pl})^2$; for the worked example with $m = 0.1m_{Pl}$, $\alpha = 0.02 \ll 1/3$, the signature of slow contraction.

Fig. 2a compares the evolution of of the Hubble parameter $H(t)$ as a func-
Figure 2: (a) The Hubble parameter $H(t)$ for the best-fit ΛCDM model (dotted) and Q-SC-CDM (solid) models. (b) A plot of $1/\sqrt{\varepsilon_{\text{TOT}}}$ vs. $t$ for the Q-SC-CDM model depicted in Figs. 1 and Fig. 2a. Unshaded regions are periods of decelerated expansion. The light grey shaded region is the phase of accelerated expansion ($H > 0$ and $\varepsilon_{\text{TOT}} < 1$ beginning about redshift $z = 0.75$). The dark shaded region is the phase slow contraction ($H < 0$ and $\varepsilon_{\text{TOT}} > 3$) that begins at $t = t_{\text{con}}$.

The evolution of the total cosmic equation-of-state $\varepsilon_{\text{TOT}}(t)$ is shown in Fig. 2b. According to the Friedmann equation (2), $\ddot{a} \propto (1 - \varepsilon_{\text{TOT}})$, so, when
Figure 3: The predicted luminosity-redshift relations (distance modulus vs. z) for the Q-SC-CDM model shown in preceding figures compared to supernovae observations [11], a fit to within 2σ.

$H > 0$, acceleration corresponds to $\varepsilon_{\text{TOT}} < 1$ (light shade region in the middle) and deceleration corresponds to $\varepsilon_{\text{TOT}} > 1$ (unshaded regions). From the figure, one can observe when quintessence first dominates sufficiently for accelerated expansion to begin and when the accelerated expansion ends in the future. During contraction ($H < 0$), $\varepsilon_{\text{TOT}} \gg 3$, corresponding to slow contraction (dark shaded region). The value of $\varepsilon_{\text{TOT}}$ rapidly asymptotes to $\frac{1}{2}(m_{\text{Pl}}^2/m^2) = 50 \gg 3$ after contraction begins.

Fig. 3 shows the predicted luminosity-redshift relation for the Q-SC-CDM model compared to current supernovae observations [11], demonstrating that the goodness of fit is 2σ. The distance modulus is the conventional way of parameterizing the apparent luminosity of an object at redshift $z$; for standard candles, the modulus is equal to $5 \log_{10}[d_p(z)(1+z)/(10 \text{ pc})]$ where $d_p(z)$ is the luminosity distance. (Because the evolution of $H(t)$ is so similar in the past to the $\Lambda$CDM model, the Q-SC-CDM model fits no better or worse; so it does not alleviate or exacerbate the current ‘$H_0$ problem’ [12].)

Results and Discussion. For the worked example in Fig. 2) with $m_{\text{Pl}}/m = 10$, $t_{\text{dec}} - t_0 = 0.1H_0^{-1}$ and $t_{\text{con}} - t_0 = 0.27H_0^{-1}$ – less than a Hubble time and on the order of billions of years. For steeper potentials ($m_{\text{Pl}}/m > 10$), the minimal
times are predicted to be even sooner, as shown in Fig. 4. For each value of $m$, the potential parameters are chosen to ensure fits to current observational constraints on $\varepsilon_{\text{TOT}}$, $\Omega^0_{m}$ and $\Omega^0_{DE}$ at the $2\sigma$ level.

These minimum time intervals before the end of acceleration and the end of expansion are each strikingly soon, cosmologically speaking. In fact, they can be compared to geologic timescales. For $m_{Pl}/m = 10$, the minimum time remaining before the end of expansion, for example, is roughly equal to the period since life has existed on Earth; for $m_{Pl}/m = 50$, the time interval remaining before the end of acceleration is less than the time since the Chicxulub asteroid brought an end to the dinosaurs.

Yet, curiously, we could not detect the oncoming dramatic cosmic events given the best-available observations today. The problem is that accurate cosmological measures of the expansion rate and other cosmological parameters are based on observations of the cosmic microwave background, baryon acoustic oscillations and distant objects, like supernovae, whose detected light was emitted far in the past, whereas, as we have shown, the transitions to deceleration and
slow contraction may all occur within a small fraction of a Hubble time. For this reason, it is a challenge to detect the end of contraction even when the time is nigh. Improvements in measuring $\varepsilon_{\text{TOT}}$ and especially its time variation would be a generic approach. In the example in Fig. 2 for example, the prediction is that $1/\sqrt{\varepsilon_{\text{TOT}}}$ has already reached a maximum and is beginning to decrease because the $\dot{\phi}$ has begun to increase significantly. As $\dot{\phi}$ increases, there may be a rich set of additional observable effects to pursue, depending on the couplings of $\phi$ to other fields [13, 14, 15, 16].

Notably, Q-SC-CDM connects naturally with the recently proposed cyclic model of the universe [17] in which the big bang is replaced by a non-singular classical bounce that connects a previous period of slow contraction to a subsequent period of radiation-, matter- and dark energy-dominated expansion. Slow contraction is an essential element because it is responsible for making the universe homogeneous, isotropic and spatially flat and for setting the background conditions needed to generate a nearly scale invariant spectrum of density perturbations [18]. By construction, a cyclic model demands that each dark energy phase, including the current one, comes to an end and transitions smoothly to the next phase of slow contraction in order to set the large-scale properties of the universe for the cycle to come.

Q-SC-CDM provides all the necessary conditions and can easily be incorporated as part of each cycle; see, for example, Ref. [17]. In this case, there is an interesting connection between the results presented here and the ‘why now?’ problem. The ‘why now’ problem is the mystery of explaining why dark energy only began to dominate recently, just as the galaxies like our Milky Way formed and planets capable of supporting life first evolved. In a cyclic universe with a very short time between now and the end of expansion, the issue seems less mysterious because there is only a very limited period within each cycle with the conditions needed for life to emerge and that is about ‘now’ in the current cycle.

Q-SC-CDM also dovetails with recent conjectures about quantum gravity constraints on dark energy. The so-called ‘swampland conjectures’ [19, 20, 21]
rule out the possibility that dark energy is a cosmological constant or that the energy density is associated with a metastable phase, and only allow the possibility considered here – that dark energy is due to a quintessence field with a monotonically decreasing $V(\phi)$. It does not require that $V(\phi)$ pass below zero, but that is allowed and occurs for a wide range of parameters. The swampland conjectures also place a quantitative constraint on how long the current period of accelerated expansion might last based on the condition that ultraviolet sub-Planckian fluctuations should never expand to scales larger than the Hubble radius. The upper bound to the end of expansion according to these conjectures is about 2.4 trillion years or about 160 Hubble times. The lower bound derived here based on independent reasoning is consistent.

Curiously, three different theoretical developments point to the same outcome: the end of expansion could occur surprisingly soon.

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References

[1] M. S. Turner, Found. Phys. 48, 1261 (2018), 2109.01760.
[2] S. R. Coleman, Phys. Rev. D 15, 2929 (1977), [Erratum: Phys.Rev.D 16, 1248 (1977)].
[3] G. Degrassi, S. Di Vita, J. Elias-Miro, J. R. Espinosa, G. F. Giudice, et al., JHEP 1208, 098 (2012), 1205.6497.
[4] V. Branchina, E. Messina, and M. Sher, Phys. Rev. D 91, 013003 (2015), 1408.5302.
[5] R. R. Caldwell, R. Dave, and P. J. Steinhardt, Phys. Rev. Lett. 80, 1582 (1998), astro-ph/9708069.

[6] P. J. Steinhardt and N. Turok, Phys. Rev. D65, 126003 (2002), hep-th/0111093.

[7] A. Ijjas and P. J. Steinhardt, Class. Quant. Grav. 35, 135004 (2018), 1803.01961.

[8] P. Agrawal, G. Obied, P. J. Steinhardt, and C. Vafa, Phys. Lett. B784, 271 (2018), 1806.09718.

[9] N. Aghanim et al. (Planck), Astron. Astrophys. 641, A6 (2020), [Erratum: Astron. Astrophys. 652, C4 (2021)], 1807.06209.

[10] S. Aiola et al. (ACT), JCAP 12, 047 (2020), 2007.07288.

[11] C. L. Steinhardt, A. Sneppen, and B. Sen, Astrophys. J. 902, 14 (2020), 2005.07707.

[12] W. L. Freedman, Astrophys. J. 919, 16 (2021), 2106.15656.

[13] K. V. Berghaus, P. W. Graham, D. E. Kaplan, G. D. Moore, and S. Rajendran, Phys. Rev. D 104, 083520 (2021), 2012.10549.

[14] P. Agrawal, G. Obied, and C. Vafa, Phys. Rev. D 103, 043523 (2021), 1906.08281.

[15] I. M. Bloch, C. Csáki, M. Geller, and T. Volansky, JHEP 12, 191 (2020), 1912.08840.

[16] C. Csáki, R. T. D’Agnolo, M. Geller, and A. Ismail, Phys. Rev. Lett. 126, 091801 (2021), 2007.14396.

[17] A. Ijjas and P. J. Steinhardt, Phys. Lett. B795, 666 (2019), 1904.08022.

[18] W. G. Cook, I. A. Glushchenko, A. Ijjas, F. Pretorius, and P. J. Steinhardt, Phys. Lett. B 808, 135690 (2020), 2006.01172.
[19] H. Ooguri and C. Vafa, Nucl. Phys. B766, 21 (2007), hep-th/0605264.

[20] G. Obied, H. Ooguri, L. Spodyneiko, and C. Vafa (2018), 1806.08362.

[21] E. Palti, Fortsch. Phys. 67, 1900037 (2019), 1903.06239.

[22] A. Bedroya and C. Vafa, JHEP 09, 123 (2020), 1909.11063.