I. INTRODUCTION

While a qubit is considered to be a building block for quantum information processing, the actual quantum computer invariably involves complex states of multiple qubits [1]. The transition from one to two qubits is of fundamental importance because it is the two-qubit system for which we can have entangled states and hence a nontrivial quantum advantage for information processing [2, 3]. The manipulation of two-qubit states is qualitatively more difficult than that for a single qubit. As a matter of fact, the dynamics of a single qubit finds a classical analog in polarization optics [4], and it is only when we create entangled states of two qubits, do the nontrivial quantum aspects emerge [3]. It may appear that moving from two qubits to several qubits is merely a matter of detail. However, this is not the case and new quantum aspects emerge for a three-qubit system, which is the simplest system for which the concept of multi-partite entanglement can be introduced. Unlike the two-qubit case, the maximally entangled states of three qubits are not equivalent up to local unitary transformations and instead fall into two inequivalent classes, namely the GHZ and W classes of states [3].

The fact that maximally entangled two-qubit states are equivalent up to local unitaries, leads to a simple canonical form for two-qubit pure states. Any two-qubit pure state can be written in the canonical form \( |\psi\rangle = \cos \theta |00\rangle + e^{i \phi} \sin \theta |11\rangle \), \( 0 \leq \theta \leq \pi/4 \), \( \phi \) being the entanglement parameter. The coefficients of the canonical form contain all information about non-local properties of the state. A canonical form for three qubits turns out to be nontrivial and involves a combination of GHZ and W states. It has been shown that all pure states of a system of three qubits are equivalent under local unitary transformations to a canonical state with five independent non-zero components [5]. Other canonical forms of three qubits have also been proposed, all of which have five non-zero coefficients [3–11].

For two qubits, when the state is not separable, the reduced density operators for individual qubits are typically noisy and the mixedness of the reduced states is a measure of the entanglement of the original two-qubit state. While the one-qubit reduced states have information about the amount of entanglement in the two-qubit pure state, they do not uniquely determine the state. On the other hand, it turns out that almost every three-qubit pure state is completely determined by its two-qubit reduced density matrices and there is no more information in the full quantum state than what is already contained in the three two-qubit reduced states [12]. It is indeed somewhat surprising that even when nontrivial multi-partite entanglement is present, the “parts” can determine the “whole”. The only exceptions to the above hypothesis are the generalized GHZ states, and no set of their reduced states can uniquely determine such entangled states. These results have been generalized to \( n \)-party correlations and it was shown that all the information in almost all generic \( n \)-party pure states can be captured in the set of reduced states of just over half the parties [13–14]. While the original scheme to reconstruct three-qubit pure states was given by Linden et al. [12], an explicit prescription to completely characterize a generic three-qubit pure state using an optimal number of reduced two-party measurements was described by Diosi et al. [12].

There have been several experimental implementations of tripartite-entangled W and GHZ states using different physical resources [16, 21]. GHZ and W states have
been used as a resource in a quantum prisoner’s dilemma game [21], to simulate the violation of Bell-type inequalities [22], in quantum erasers [23, 24] and complementarity measurements [25], quantum key distribution [26], quantum secret sharing [27] and quantum teleportation [28]. In the context of NMR quantum computing, GHZ and W states have been generated on a one-dimensional Ising chain [24, 30], their decoherence properties studied [31], and their ground state phase transitions investigated in a system with competing many-body interactions [32, 33].

This work has two main results: (a) We prescribe a scheme to create generic states of three qubits and implement it on an NMR quantum computer. The complete class of separable, biseparable and maximally entangled three-qubit states can be generated using our scheme; (b) We experimentally demonstrate the reconstruction of generic three-qubit states from their two-qubit reduced marginals. The material in this paper is organized as follows: Section II describes the NMR implementation of a generic state with a nontrivial five-parameter set, and the implementations of the GHZ and W-states as special cases of the general scheme. The density matrices of all the states are reconstructed by using an optimal set of NMR state tomography experiments. Section III describes the three-qubit state reconstruction from their two-party reduced states for a generic state and for the W-state. By comparing the state tomographs obtained from the two-qubit marginals and by a full tomography of the three-qubit state we demonstrate that, reduced two-qubit density matrices are indeed able to capture all information about the full three-qubit state. Section IV contains some concluding remarks.

II. NMR IMPLEMENTATION

The three-qubit system that we use for NMR quantum information processing is the molecule trifluoroiodoethylene dissolved in deuterated acetone. The three qubits were encoded using the $^{19}$F nuclei. The Hamiltonian of the three-qubit system in the rotating frame is given by

$$H = \sum_{i=1}^{3} \nu_i I_{iz} + \sum_{i<j,i=1}^{3} J_{ij} I_{iz} I_{jz}$$

(1)

where $\nu_i$ are the Larmor frequencies of the spins and $J_{ij}$ are the spin-spin coupling constants. The coupling constants recorded are $J_{12} = 69.8$ Hz, $J_{23} = -129.0$ Hz, and $J_{13} = 47.5$ Hz. Decoherence is not a major issue in this system, with average fluorine longitudinal $T_1$ relaxation times of 5.0 seconds and $T_2$ relaxation times of 1.0 seconds respectively. The structure of the three-qubit molecule as well as the equilibrium NMR spectrum obtained after a $\pi/2$ readout pulse are shown in Fig. 1. The resonance lines of each qubit are labeled by the corresponding states of the other two coupled qubits. All experiments were performed at room temperature on a Bruker Avance III 400 MHz NMR spectrometer equipped with a z-gradient BBO probe. The three fluorine nuclei cover a very large bandwidth of 68 ppm. Standard shaped pulses (of duration 400$\mu$s) were hence modulated to achieve uniform excitation of all the three qubits by exciting smaller bandwidths simultaneously at different offsets. Individual qubits were addressed using low power ‘Gaussian’ shaped selective pulses of 265$\mu$s duration. Before implementing the entangling circuits, the system was first initialized into the $|000\rangle$ pseudopure state by the spatial averaging technique [34], with the density operator given by

$$\rho_{000} = \frac{1 - \epsilon}{8} I_8 + \epsilon |000\rangle \langle 000|$$

(2)

with a thermal polarization $\epsilon \approx 10^{-5}$ and $I_8$ being an $8 \times 8$ identity matrix. The experimentally created pseudopure state $|000\rangle$ was tomographed with a fidelity of 0.99. All experimentally generated states were completely characterized by performing NMR state tomography [35]. A modified tomographic protocol was used [36], wherein a set of operations defined by $\{III, IIX, IXI, XII, IYY, YI, YII, YYI, YXY, XIX, XXX, XYY\}$ is performed on the system before recording the signal. Here $X(Y)$ denotes a single spin operator and $I$ is the identity operator. These operators can be implemented by applying the corresponding spin selective $\pi/2$ pulses. This reduced set of 11 operations is sufficient to determine all the 63 variables for our system of three qubits. As a measure of the fidelity of the experimentally reconstructed density

FIG. 1. Molecular structure, NMR parameters and $^{19}$F equilibrium spectrum of trifluoroiodoethylene. The three fluorine spins in the molecule are marked as the corresponding thermal equilibrium state. The resonance lines of each qubit are labeled by the corresponding logical states of the other two qubits in the computational basis.

Spin 1 Spin 2 Spin 3

|11⟩ |10⟩ |01⟩ |00⟩ |10⟩ |00⟩ |11⟩ |01⟩ |

$\omega_F$ (in ppm) 25.5 25.0 0.0 $-37.0$

$^{19}$F spectrum is obtained after a $\pi/2$ read pulse on the thermal equilibrium state. The resonance lines of each qubit are labeled by the corresponding logical states of the other two qubits in the computational basis.
matrices, we use [37]:

\[
F = \frac{\text{Tr}(\rho_{\text{theory}}^{\dagger}\rho_{\text{theory}})}{\sqrt{\text{Tr}(\rho_{\text{theory}}^{\dagger}\rho_{\text{theory}})^2}} \frac{\text{Tr}(\rho_{\text{expt}}^{\dagger}\rho_{\text{expt}})}{\sqrt{\text{Tr}(\rho_{\text{expt}}^{\dagger}\rho_{\text{expt}})^2}}
\] (3)

where \(\rho_{\text{theory}}\) and \(\rho_{\text{expt}}\) denote the theoretical and experimental density matrices respectively.

A. Generic state implementation

(a)

\[
|0\rangle \rightarrow U_{\alpha} |0\rangle
\]

(b)

\[
|0\rangle \rightarrow R_{12}(\beta) |0\rangle
\]

\[
|0\rangle \rightarrow R_{13}(\gamma) |0\rangle
\]

\[
|0\rangle \rightarrow R_{23}(\delta) e^{\phi} |0\rangle
\]

\[
T O M O G R A P H Y
\]

FIG. 2. (a) Quantum circuit showing the specific sequence of implementation of the controlled-rotation, controlled-NOT, controlled-controlled-NOT and controlled-controlled-phase gates required to construct a generic state and (b) NMR pulse sequence to implement a general three-qubit generic state; \(\tau_{ij}\) is the evolution period under the \(J_{ij}\) coupling. The 180° pulses are represented by unfilled rectangles. The other pulses are labeled with their specific flip angles and phases. The last pulse (gray shaded) on the first qubit is a transition-selective 180° pulse on the \(|011\rangle\) to \(|111\rangle\) transition and the last pulse (gray shaded) on the third qubit is another transition-selective pulse of angle \(\phi\) on the \(|110\rangle\) to \(|111\rangle\) transition.

The canonical (generic) state for three qubits proposed in [7] is given by:

\[
|\psi\rangle = a_1 |000\rangle + a_2 |001\rangle + a_3 |010\rangle + a_4 |100\rangle + a_5 e^{i\phi} |111\rangle
\]

where the \(a_i\) are real parameters (four moduli and one phase). The normalization condition leads to reduction of one parameter and hence the state has five independent non-zero, real parameters (four moduli and one phase). The state is symmetric under permutations of the qubits and the five components which are invariant under local unitaries (single-qubit operations) are the minimal number of non-local parameters required to completely specify the state. Any three-qubit state up to local unitaries, can hence be written in the form given in Eqn. (4). We base our experimental construction on this canonical form and will henceforth refer to it as the generic three-qubit state.

The sequence of gates with four real parameters \(\alpha, \beta, \gamma, \delta\) representing the amplitude parameters \(a_1 \cdots a_5\) and the phase \(\phi\) and their effect is detailed below:

\[
|000\rangle \rightarrow U_{12}^{\alpha} \cos \alpha |000\rangle + \sin \alpha |100\rangle
\]

\[
CROT_{12}^{\beta} \cos \alpha |000\rangle + \sin \alpha \cos \beta |100\rangle + \sin \alpha \sin \beta |110\rangle
\]

\[
CNOT_{21}^{\gamma} \cos \alpha |000\rangle + \sin \alpha \cos \beta |100\rangle + \sin \alpha \sin \beta |010\rangle
\]

\[
CROT_{13}^{\delta} \cos \alpha |000\rangle + \sin \alpha \cos \beta \cos \gamma |100\rangle
\]

\[
CNOT_{31}^{\epsilon} \cos \alpha |000\rangle + \sin \alpha \cos \beta \sin \gamma |100\rangle + \sin \alpha \sin \beta |010\rangle
\]

\[
CROT_{23}^{\eta} \cos \alpha |000\rangle + \sin \alpha \cos \beta \cos \gamma |100\rangle
\]

\[
CNOT_{32}^{\zeta} \cos \alpha |000\rangle + \sin \alpha \cos \beta \sin \gamma |100\rangle + \sin \alpha \sin \beta |010\rangle
\]

\[
CROT_{13}^{\theta} \cos \alpha |000\rangle + \sin \alpha \sin \beta |011\rangle + \sin \alpha \sin \beta |111\rangle
\]

\[
CCN_{34}^{\phi} \cos \alpha |000\rangle + \sin \alpha \cos \beta \cos \gamma |100\rangle
\]

\[
\Phi_{12}^{\psi} \cos \alpha |000\rangle + \sin \alpha \cos \beta \sin \gamma |100\rangle + \sin \alpha \sin \beta |010\rangle + \sin \alpha \sin \beta |111\rangle
\]

(5)

where \(U_{\alpha}^{i}\) are a complete set of separable, non-entangling transformations belonging to the \(SU(2)\) group which implements a rotation by an arbitrary angle \(\alpha\) on the \(i\)th qubit, leading to a generalized superposition state of the qubit. The global phase is not detectable in NMR experiments and is thus ignored throughout in gate implementation; \(CROT_{ij}^{\theta}\) implements a controlled rotation by an arbitrary angle \(\theta\), with the \(i^{th}\) qubit as control and \(j^{th}\) as target; \(CN_{ij}\) implements a controlled-NOT gate, with the \(i^{th}\) qubit as control and \(j^{th}\) as target;
CCN$_{ij,k}$ implements a controlled-controlled-NOT (Toffoli) gate on the $k$th qubit i.e. it flips the state of qubit $k$, if and only if both qubits $i$ and $j$ are in the $|1\rangle$ state; $P^h_{ij,k}$ is a controlled-controlled-phase shift gate with $i,j$ as control qubits and $k$ being the target qubit. The state thus obtained has five variables: $\alpha \in [0,\pi], \beta \in [0,\pi/2], \gamma \in [0,\pi/2], \delta \in [0,\pi/2]$ and $\phi \in [0,2\pi]$. The quantum circuit for generic state construction is given in Fig. 2(a). The circuit consists of a single-qubit rotation gate, followed by several two-qubit controlled-rotation and controlled-NOT gates, a three-qubit controlled-controlled NOT (Toffoli) gate, and finally a controlled-controlled phase gate that introduces a relative phase in the $|111\rangle$ state. The NMR pulse sequence to construct the generic three-qubit state starting from the pseudopure state $|000\rangle$ is given in Fig. 2(b). The evolution intervals have been carefully dovetailed so that wherever possible, some of the controlled-rotation or controlled-NOT gates can be applied in parallel. Refocusing pulses are used in the middle of all the J-evolution intervals to eliminate undesirable evolution due to other J-couplings. The 180$^\circ$ pulses are represented by unfilled rectangles, while the other pulses are labeled with their specific flip angles and phases. The controlled-controlled NOT (Toffoli) and controlled-controlled phase gates are implemented using a transition-selective pulse (gray-shaded in Fig. 2(b)) on the $|011\rangle$ to $|111\rangle$ transition of flip angle $\pi$ and on the $|110\rangle$ to $|111\rangle$ transition of flip angle $\phi$ respectively. To demonstrate our general method to create generic three-qubit states, we implement our scheme to create a state with a nontrivial structure. We chose a state in which all the terms in the generic state expression given in Eqn. 4 are involved in a nontrivial way. We have chosen $\phi = 0$, $\alpha = 135^\circ$ and $\beta = \gamma = \delta = 60^\circ$. This set of parameters leads to the creation of the generic state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|000\rangle + \frac{\sqrt{3}}{4}|100\rangle + \frac{\sqrt{3}}{4}|001\rangle + \frac{\sqrt{3}}{4}|010\rangle + \frac{3}{4}|111\rangle) \quad (6)$$

This state is related to the same state but with all positive coefficients, by local unitaries. We have deliberately generated the state given in Eqn. (6) so that the tomographs of the state have negative entries, which are easier to depict pictorially. The tomograph corresponding to this state is shown in Fig. 3 wherein the experimentally tomographed state (Fig. 3(b)) is compared with the theoretically expected state (Fig. 3(a)). The fidelity of the experimentally tomographed state (by the definition given in Eqn. 8) in this case is 0.92.

Our method is quite general and can be used to construct any generic state of the three-qubit system. Given that the relaxation times for our system are quite long and the qubits are well separated in frequency space, it is also possible to perform single-qubit operations to transform the state further. It is difficult to achieve a nontrivial value of the parameter $\phi$, as the gate involves transition-selective pulses in the three-qubit state space. We tried to implement a generic state with a random value for $\phi$, however we were not able to achieve good fidelities.

B. GHZ state implementation

Generalized GHZ states are a special case of the generic state given in Eqn. 4, corresponding to the parameter values $\alpha = \alpha, \beta = \gamma = \delta = 90^\circ, \phi = 0$. The last two controlled-controlled gates are hence redundant and the circuit given in Fig. 2 simplifies to a single-qubit rotation followed by controlled-NOT gates (equivalent to controlled rotation gates, with a rotation angle of $\theta = \pi$).

An arbitrarily weighted GHZ kind of entangled state can hence be prepared from the initial pseudopure state $|000\rangle$ by the sequence of operations

$$|000\rangle \xrightarrow{U^2} \cos \frac{\alpha}{2} |000\rangle + \sin \frac{\alpha}{2} |010\rangle$$
$$\xrightarrow{\text{CNOT}^{21}} \cos \frac{\alpha}{2} |000\rangle + \sin \frac{\alpha}{2} |100\rangle$$
$$\xrightarrow{\text{CNOT}^{23}} \cos \frac{\alpha}{2} |000\rangle + \sin \frac{\alpha}{2} |110\rangle \quad (7)$$

For $\alpha = 90^\circ$, the above sequence leads to a pure GHZ state $|\psi_{\text{GHZ}}\rangle$:

$$|\psi_{\text{GHZ}}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \quad (8)$$

FIG. 3. The real (left) and imaginary (right) parts of the (a) theoretical and (b) experimental density matrices for the three-qubit generic state, reconstructed using full state tomography. The values of the parameters are $\alpha = 135^\circ, \beta = \gamma = \delta = 60^\circ$. The rows and columns encode the computational basis in binary order, from $|000\rangle$ to $|111\rangle$. The experimentally tomographed state has a fidelity of 0.92.
The quantum circuit and the NMR pulse sequence used to create an arbitrary GHZ-like entangled state beginning from the pseudopure state $|000\rangle$ and ignoring overall phase factors are given in Fig. 4. The CNOT$_{12}$ and CNOT$_{13}$ in the circuit are controlled-NOT gates with qubit 1 as the control and qubit 2 (3) as the target. Since the target qubits are different in both these cases, these gates commute and can be applied in parallel, leading to a reduction in experimental time. For our system $\tau_{13} > \tau_{12}$, where $\tau_{ij}$ denotes the evolution period under the $J_{ij}$ coupling term. Hence, during the period $\tau_{12}$, both qubits 2 and 3 evolve under the the $J$-couplings $J_{12}$ and $J_{13}$. After this combined evolution, the system is allowed to evolve during another evolution period $\tau_d = \tau_{13} - \tau_{12}$ (Fig. 4(a)), such that evolution in this period is governed solely by the $J_{13}$ coupling term. The state generated experimentally (Fig. 5(b)) was tomographed and lies very close to the theoretically expected state (Fig. 5(a)) with a computed fidelity of 0.97.

C. W-state implementation

Generalized W-states are another special case of the generic state given in Eqn. 4, corresponding to the parameter values $\alpha = \pi, \beta, \gamma \in (0, \pi/2), \delta = 0, \phi = 0$, leading to the state $|\psi\rangle = \cos \gamma \cos \beta |100\rangle + \sin \gamma \cos \beta |001\rangle + \sin \beta |010\rangle$. The circuit for generalized W-states is given in Fig. 6(a) and can be constructed by the sequential operation of the gates:

\[
|000\rangle \xrightarrow{U^\dagger_\alpha} |100\rangle \\
\xrightarrow{\text{CROT}^{\beta}_{12}} \cos \beta |100\rangle + \sin \beta |110\rangle \\
\xrightarrow{\text{CNOT}_{21}} \cos \beta |100\rangle + \sin \beta |010\rangle \\
\xrightarrow{\text{CROT}^{\gamma}_{13}} \cos \gamma \cos \beta |100\rangle + \sin \gamma \cos \beta |101\rangle + \sin \beta |010\rangle \\
\xrightarrow{\text{CNOT}_{31}} \cos \gamma \cos \beta |100\rangle + \sin \gamma \cos \beta |001\rangle + \sin \beta |010\rangle
\]

The first gate in the circuit, namely a rotation by $\pi$ on the first qubit, can be avoided by starting the implementation on a different initial state. We hence begin with the pseudopure state $|100\rangle$ as the initial state in our experiments. We also avoid implementing the second gate in the circuit in Eqn. 4, namely the controlled-rotation CROT$^{\beta}_{12}$ gate, and instead implement the much simpler $U^\dagger_{23}$ gate on the second qubit, which in this case yields the same result. For $\beta = 2 \sin^{-1}(1/\sqrt{3})$ and $\gamma = 45^\circ$, the circuit leads to implementation of the standard W-state.
upto a phase factor

\[ |\psi_W\rangle = \frac{1}{\sqrt{3}} (|001\rangle + |010\rangle + |100\rangle) \] (10)

One can get rid of the extra phase factor by a single-qubit unitary gate. The NMR pulse sequence for the creation of an arbitrary W-like entangled state beginning from the pseudopure state \(|100\rangle\) and ignoring overall phase factors, is given in Fig. 6(b). The experimentally reconstructed density matrix (Fig. 7(b)) matches well with the theoretically expected values (Fig. 7(a)), with a computed state fidelity of 0.96.

![Quantum circuit and NMR pulse sequence](image)

FIG. 6. (a) Quantum circuit and (b) NMR pulse sequence to experimentally implement the W-state.

III. THREE-QUBIT STATE RECONSTRUCTION FROM TWO-PARTY REDUCED STATES

Linden et al. discovered a surprising fact about multi-party correlations, namely, that “the parts determine the whole for a generic pure state” [12, 13]. For three qubits, this implies that all the information in a generic three-party state is contained in its three two-party reduced states, which then uniquely determine the full three-party state. The only states that do not have this property are the generalized GHZ states. This is an important result which sheds some light on how information is stored in multipartite entangled states. In a related work, Diosi et al. [15] presented a tomographic protocol to completely characterize almost all generic three-qubit pure states, based only on pairwise two-qubit detectors.

![Real and imaginary parts of density matrix](image)

FIG. 7. The real (left) and imaginary (right) parts of the (a) theoretical and (b) experimental density matrices for the W state, reconstructed using full state tomography. The rows and columns encode the computational basis in binary order, from \(|000\rangle\) to \(|111\rangle\). The experimentally tomographed state has a fidelity of 0.96.

![Real and imaginary parts of density matrix](image)

FIG. 8. The real (left) and imaginary (right) parts of the three-qubit density matrix for the W state, reconstructed from two sets of the corresponding two-qubit reduced density matrices. The rows and columns encode the computational basis in binary order, from \(|000\rangle\) to \(|111\rangle\). The tomographed state has a fidelity of 0.97.

In this paper we describe the first experimental demonstration of this interesting quantum mechanical feature of three-qubit states. We use the same algorithm delineated by Diosi et al. [15], to reconstruct three-qubit states from their two-party reduced states. Let us consider a three-qubit pure state \(\rho_{ABC} = |\psi_{ABC}\rangle \langle \psi_{ABC}|\), with \(\rho_{AB}, \rho_{BC}, \rho_{AC}\) being its two-party reduced states. The single-qubit reduced states \(\rho_A, \rho_B\) and \(\rho_C\) can be further obtained from the two-party reduced states. Since \(\rho_{ABC}\) is pure,
\(\rho_A\) and \(\rho_{BC}\) share the same set of eigen values, and can be written as

\[
\rho_A = \sum_i p^i_A |i\rangle \langle i|
\]

\[
\rho_{BC} = \sum_i p^i_A |BC; i\rangle \langle BC; i|
\]  \(11\)

where \{\{i\}\} are the eigenvectors of \(\rho_A\) with eigenvalues \{\(p^i_A\)\}, and \{\{BC; i\}\} are the eigenvectors of \(\rho_{BC}\) with eigenvalues \{\(p^i_A\)\}. The three-qubit states compatible with \(\rho_A\) and \(\rho_{BC}\) are

\[
|\psi_{ABC}; \alpha\rangle = \sum_i e^{i\alpha_i} \sqrt{p^i_A} |i\rangle \otimes |i; BC\rangle
\]  \(12\)

Using a similar argument, the set of three-qubit pure states obtained from \(\rho_{AB}\) and \(\rho_C\) is given by

\[
|\psi_{ABC}; \gamma\rangle = \sum_k e^{i\gamma_k} \sqrt{p^k_C} |k; AB\rangle \otimes |k\rangle
\]  \(13\)

where \{\{k\}\} are the eigenvectors of \(\rho_C\) with eigenvalues \{\(p^k_C\)\} and \{\{k; AB\}\} are the corresponding eigenvectors of \(\rho_{AB}\). Since the pure state \(|\psi_{ABC}\rangle\) is compatible with both \(\rho_{AB}\) and \(\rho_{BC}\), we can determine the values of \(\alpha_i\) and \(\gamma_k\) such that \(|\psi_{ABC}; \alpha\rangle = |\psi_{ABC}; \gamma\rangle\). We thus obtain almost all three-qubit pure states from any two of their corresponding two-party reduced states. The set \((\rho_{AB}, \rho_{AC})\) or the equivalent set \((\rho_{AC}, \rho_{BC})\) can be used to reconstruct \(\rho_{ABC}\).

![Real Imaginary](image)

FIG. 9. The real (left) and imaginary (right) parts of the three-qubit density matrix for the generic state, reconstructed from the two-qubit reduced density matrices. The parameter set includes \(\alpha = 135^0, \beta = \gamma = \delta = 60^0\). The rows and columns encode the computational basis in binary order, from \{000\} to \{111\}. The tomographed state has a fidelity of 0.91.

The two-party reduced states \(\rho_{AB}, \rho_{BC}\) and \(\rho_{AC}\) were computed by performing partial state tomography. The set of tomography operations performed to experimentally reconstruct all three two-party reduced states include: \{III, IIX, IY, XIX\} to reconstruct \(\rho_{AB}\); \{III, IIX, IY, IXX\} to reconstruct \(\rho_{BC}\) and \{III, IIX, IY, XIX\} to reconstruct \(\rho_{AC}\). Almost any three-qubit pure state \(\rho_{ABC}\) (except those belonging to the generalized GHZ class) can be determined by choosing any two sets from the above. The three-party state \(\rho_{ABC}\) reconstructed using the \((\rho_{AB}, \rho_{AC})\) set of two-party reduced states was compared with the same state reconstructed using complete tomography, and the results match well. The reconstructed density matrix for the W-state is shown in Fig. 8 computed from two sets of the corresponding two-qubit reduced density matrices. The tomographed state has a fidelity of 0.97, which matches well with the fidelity of the original three-qubit density matrix of the W-state (Fig. 7(b)). As another illustration of reconstructing the whole state from its parts, the reconstructed density matrix of the experimentally generated generic state with a parameter set: \(\alpha = 135^0, \beta = \gamma = \delta = 60^0\), is shown in Fig. 9. The two-party reduced states were able to reconstruct this three-qubit state with a fidelity of 0.91, which compares well with the full reconstruction of the entire three-qubit state given in Fig. 8(b).

IV. CONCLUDING REMARKS

We have proposed and implemented an NMR-based scheme to construct a generic three-qubit state from which any general pure state of three-quibits (including separable, biseparable and maximally entangled states) can be constructed, up to local unitaries. Full tomographic reconstruction of the experimentally generated states showed good fidelity of preparation and we have achieved a high degree of control over the state space of three-qubit quantum systems. Generating generic three-qubit states with a nontrivial phase parameter remains a challenge, and numerical optimization of pulse profiles might be a promising direction to explore in this context. It has been previously shown that in a system of three qubits, no irreducible three-party correlations exist and that all information about the full quantum state is completely contained in the three two-party correlations. We have demonstrated this important result experimentally in a system of three qubits. The three-qubit density operator \(\rho_{ABC}\) is obtained by complete quantum state tomography and compared with the same three-qubit state reconstructed from tomographs of the two-party reduced density operators given by \(\rho_{AB}, \rho_{BC}\) and \(\rho_{AC}\). It is expected that our experiments will pave the way for an understanding of how information is stored in multi-partite entangled systems.

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