Shock related unsteadiness of axisymmetric spiked bodies in the supersonic flow

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Shock related unsteadiness over axisymmetric spiked body configurations is experimentally investigated at a freestream supersonic Mach number of 2.0 at zero degree angle of attack. Three different forebody configurations mounted with a sharp spike-tip ranging from blunt to streamlined (flat-face, hemispherical, and elliptical) are considered. Steady and unsteady pressure measurements, short exposure shadowgraphy, shock footprint analysis from $x - t$ plots, and identification of dominant spatiotemporal modes through modal analysis are carried out to explain the unsteady flow physics. The present investigation tools are validated against the well-known events of ‘pulsation’ associated with the flat-face case. The hemispherical case is characterized by the formation of a separated free shear layer and associated localized shock oscillations. The cycle of charging and ejection of fluid mass from the recirculation zone, confined between the separated shear layer and the spiked body, is identified to drive flow unsteadiness. Such an event triggers the out-of-phase motion between the separated and reattachment shocks. In the elliptical case, the overall flow field resembles that of the hemispherical case, except with dampened unsteadiness. The cone angle ($\lambda$) of the recirculation region is found to be responsible for the intensity of charging and ejection of fluid mass, thereby the intensity of out-of-phase shocks motion. In the latter case, $\lambda$ is observed to be smaller and exhibits a reduction in shock unsteadiness. Based on the gather results and understanding, a modified spike-tip geometry is proposed to almost completely eliminate the out-of-phase shock motion, and it is indeed shown to exhibit the least level of shock-related unsteadiness.

Key words: aerospie, supersonic flow, shock related unsteadiness

1. Introduction

Flow unsteadiness is generally observed ahead of a variety of axisymmetric forebodies, flying especially at supersonic and hypersonic speeds (Stewartson 1950; Clemens & Narayanaswamy 2014). Configurations like mixed compression inlets (Howlett & Hunter 1986; Gao et al. 2018), double cones (Jagadeesh et al. 2003; Gomes-Fernandes 2013), forward-facing cavities, (Badiger & Saravanan 2018), axially positioned cavities, (Charwat et al. 1961), wall protrusions (Estruch-Samper & Chandola 2018), and spiked/aerodisk forebodies (Mair 1952; Maull 1960; Panaras & Drikakis 2009) are known

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to exhibit severe flow unsteadiness. The unsteadiness is seen throughout a wide range of flow Reynolds numbers (\(Re\)) corresponding to laminar as well as turbulent states (Guenther & Reding 1977; Ahmed & Qin 2010; Sahoo et al. 2016). In general, the strongest form of shock-related unsteadiness is termed as ‘buzzing’ (Motycka & Murphy 1965; Farahani et al. 2019). It is primarily driven by the inviscid unsteady shock phenomena (D’souza et al. 1972) and is known to be devastating to the vehicle structure. Another form of unsteadiness is mostly dominated by the viscous shockwave boundary layer interactions (SWBLI), which are found to be less intense in comparison with the former (Ericsson 1967; Tumuklu et al. 2018). However, these interactions could not be neglected in practice, as their occurrence is still a threat to the vehicle’s structural integrity and flight control.

In the middle of the last century (Alexander 1947; Jones 1952; Mair 1952), significant studies had been conducted to design space launch vehicles, which can fly at a very high speed with minimal drag. Spiked axisymmetric forebodies of specific forebody base diameter (D) were found to be more efficient and practical (Piland & Putland 1954), among the several proposed configurations. The forebody shapes were ranging from a simple flat-face cylinder to a more sophisticated ogives or rounded-off cylinders (Jones 1952), as shown in Figure 1. With the attachment of a spike, those forebody shapes had achieved a reduced drag of up to 68% (Das et al. 2013). Under particular flow and geometrical conditions, spiked bodies can exhibit ‘pulsation’ type of flow unsteadiness, which is equivalent to the ‘buzz’ phenomena in a typical supersonic inlet (Farahani et al. 2019). Even though flying configurations were realized through several experimental (Guenter & Reding 1977) and computational (Paskonov & Cheranova 1984) studies, the exact flow physics that govern the intensity of unsteady shock-laden flow field is still a topic of interest. After some efforts, many researchers have found (see Table 1) that there exist a variety of parameters influencing the unsteady flow field around the spiked bodies including Reynolds number (\(Re_D\)), freestream Mach number (\(M_\infty\)), spike length to forebody diameter ratio or slenderness ratio (\(l/D\)), spike diameter to forebody diameter ratio or fineness ratio (\(d/D\)), and the shapes of spike and forebody themself. In the review paper of Ahmed & Qin (2011), many such parametric studies had been listed from both experiments and computations. In the last two decades, investigators have reported the usage of novel active and passive control techniques on the spiked bodies to achieve stable flow with superior heat transfer and drag reduction capabilities. Yan et al. (2019) list out many such studies in their review article very elaborately, and all the results are presented from the engineering viewpoint considering mainly drag and heat transfer reduction. However, to the best of our knowledge, the existence of varying unsteady flow field is not reasoned out with sufficient clarity.

In the beginning, researchers (Mair 1952; Beastall & Turner 1957; Bogdonoff & Vas 1959; Maull 1960) had been pondering to find a way to cope with the extent of observed unsteadiness, especially in the spiked flat-face body as it is known to exhibit a very violent form of shock-related unsteadiness. The associated strongest forms of unsteadiness were then identified as ‘pulsation’ and ‘oscillation’ (Kabelitz 1971; Antonov & K 1974; Kenworthy 1978), based on the spike geometrical aspects as sketched in Figure 2. Initially, it had been stated that the two forms of unsteadiness were observed to be completely relying on the geometrical parameters of the spiked bodies itself from a simple high-speed schlieren and shadowgraph images captured at higher frame rates (fps). Later, unsteady pressure transducers were mounted on the spiked bodies to measure the intensity of flow unsteadiness (Calarese & Hankey 1985) through wall static pressure fluctuations. Despite providing minimal information on the driving flow physics, unsteady pressure data revealed dominant temporal details.

Seeking a numerical approach was considered to be an ideal way to understand the un-
Figure 1. Typical schematic showing the basic flow feature encountered at a given instant over a spiked body of different forebody configurations (a) spiked flat-face body (blunt), (b) spiked hemispherical body (less blunt/streamlined), and (c) spiked elliptical body (streamlined) at a supersonic freestream Mach number ($M_\infty > 1$). Flow is from left to right.
Table 1. Some of the relevant list of experiments and numerical simulations performed in the past and present studies at different flow conditions, model configurations, and investigation techniques.

| References | Forebody | Spike | \([l/D]\) | \([d/D]\) | \(M_\infty\) | \(Re_D \times 10^6\) | \(D (\text{mm})\) | Methods | Tools |
|------------|----------|-------|----------|----------|----------|-----------------|-----------------|---------|-------|
| Mair (1952) | F, H | ST | 0 – 2.1 | 0.1 | 1.96 | 0.165 | 12.7 | E | S, P |
| Beastall & Turner (1957) | F | ST, CA | 0 – 4.6 | 0.16 | 1.5, 1.6, 1.8 | 1.125 | 38.1 | E | S, P |
| Bogdonoff & Vas (1959) | F, H | ST | 0 – 8.0 | 0.1 | 14 | 0.365 | 12.7 | E | S, P |
| Maull (1960) | F, H | ST | 0 – 4 | 0.06 | 6.8 | 0.085 | 12.7 | E | S |
| Antonov et al. (1976) | F | ST | 0 – 5 | 0.06 | 2.1 – 6 | 0.07 – 1.6 | N.a | E | S |
| Guenther & Reding (1977) | HC | FA | 0.86 | 0.062 | 1.6 – 3.5 | 4.825 – 7.046 | 795.1 | E | S, P |
| Kenworthy (1978) | F, C | ST | 0 – 2.5 | 0.06 | 2.21, 6 | 0.12, 0.13 | 46, 20 | E | S, P |
| Shang et al. (1982) | CF | BT | 0.14 – 0.62 | 0.14 | 3 | 0.62 | 88.65 | N | UL |
| Calaresu & Hankey (1985) | CF | BT | 0.14 – 0.62 | 0.14 | 3 | 0.62 | 88.65 | E | S, P |
| Mikhail (1991) | F | FT | 0.68 – 1.06 | 0.177 | 1.72 | 2.322 | 64.77 | N | UR |
| Khlebnikov (1995) | F, C | ST, FA | 0.35 – 2.02 | 0.06 | 3 | 0.8 | 60 | E | T, P |
| Yamauchi et al. (1995) | H | ST | 0.5, 1.0, 2.0 | 0.1 | 2.01, 14.15, 6.8 | 0.14 – 14 | 76.2 | N | SL |
| Mehta (2002) | H | ST | 0.5, 1.0, 2.0 | 0.1 | 6.8 | 0.14 | 76.2 | N | UR |
| Feszy et al. (2004a) | F | ST | 2 | 0.065 | 6 | 0.13 | 20 | N | UL |
| Feszy et al. (2004b) | F | ST | 1 | 0.065 | 2.21 | 0.12 | 46 | N | UL |
| Jagadeesh et al. (2005) | BC | ST, IA | 1 | 0.052 | 6.99 | 0.123 | 50 | E, N | S, UL |
| Zapryagaev & Kavun (2005) | F | ST | 0.55 – 1.45 | 0.16 | 6.01 | 0.65 | 50 | E | S, P |
| Zapryagaev & Kavun (2007) | F | ST | 1 | 0.16 | 6.01 | 0.65 | 50 | E | S, P |
| Panaras & Drikakis (2009) | F, H, CF, C | ST, BT | 0.5 – 2.5 | 0.12 | 3.0 – 6.8 | 0.12 – 0.394 | 20 – 88.65 | N | UL |
| Ahmed & Qin (2011) | H | ST, HA | 0.5 – 2.5 | 0 – 0.4 | 6 | 0.037 | N.a | N | UL |
| Sahoo et al. (2016) | H | ST, BT, IA | 0.75, 1.15 | 0.133 | 2.0 | 0.35 | 15 | E, N | S, P, UL |
| Balakalyani & Jagadeesh (2017) | F | ST | 0.7, 1.8 | 0.07 | 6 | 0.414 | 70 | E | S, P |
| Xue et al. (2018) | H | FA | 1 | 0.07 | 4.5 | 0.5 | 70 | N | DES |
| Konstantin (2018) | F, ST, FT | 0.5 – 1.5 | 0.065 | 2.22 | 1 | 23 | N | UR |
| Sahoo et al. (2019) | F, H | ST, HA | 1 | 0.06 | 2.0 | 2.159 | 50 | E, N | S, P, UL |
| Present Studies | F, H, EL | ST | 1 | 0.06 | 2.0 | 2.159 | 50 | E | S, P, M |

All the experiments or simulations were performed at zero degree angle of attack. In addition, the considered models are axisymmetric in nature and exhibited unsteady flow features; F – flat-faced; H – hemispherical; C – conical; CF – cone frustum; HC – hemispherical cone; BC – blunt cone; EL – elliptical; ST – sharp tip; BT – blunt tip; FT – flat tip; CA – conical aerodisk; FA – flat aerodisk; IA – inverted conical aerodisk; HA – hemispherical aerodisk; E – experiments; N – Numerical simulations; S – shadowgraph or schlieren; P – steady or unsteady pressure; U(S)L – unsteady(steady) laminar Navier-Stokes simulations; UR – unsteady Reynolds-averaged Navier-Stokes simulations; DES – Detached Eddy Simulations; n.a – not available; T – surface tufts; M – Modal analysis.
Figure 2. Typical schematic showing the different modes of unsteadiness around a spiked body configuration based on the spike length \((l)\) and model base-body diameter \((D)\) in a supersonic flow \((M_\infty > 1)\): (a) Oscillation mode of unsteadiness \((1.5 < l/D < 2.5)\) and (b) Pulsation mode of unsteadiness \((0.5 < l/D < 1.5)\). Model forebody semi-cone angle \((\epsilon)\) for the illustrated example is 90°, which corresponds to a flat-face (blunt) forebody. Flow is from left to right. In the oscillation mode, the shock motion is dominant in the lateral direction whereas, in the pulsation mode, it is observed to be in along the axis.

steady flow physics, as the first experimental observations of these phenomena happened only in the laminar flow regime at a higher Mach number (Maul 1960). Unsteady laminar compressible Navier-Stokes equations were solved numerically to resolve the flow field around the spiked bodies both spatially and temporally (Karlovskii & Sakharov 1986; Mikhail 1991). At a lower Reynolds number \((Re_D \approx 0.1 \times 10^6)\), for certain geometrical configurations, ‘pulsation’ and ‘oscillation’ phenomena were seen independently. In some cases, it had been reported (Calarese & Hankey 1985; Feszty et al. 2002) switching between the two forms in the same run as the spike length was varied. Recently, the unsteady flow field around the spiked bodies has also been studied for higher Reynolds numbers using appropriate turbulence modeling (Konstantin 2018; Xue et al. 2018). The reported results are in agreement with the proposed mechanism of ‘pulsation’ and ‘oscillation’ provided by Feszty et al. (2000, 2002), and Feszty et al. (2004a,b). However, in the experiments at higher Reynolds numbers \((Re_D \approx 1–7 \times 10^6)\), researchers (Beastall & Turner 1957) reported the absence of ‘oscillation’ form of unsteadiness for a variety of physical parameters that had been tested. In the pioneering work of Moeckel (1951), the author systematically reported the occurrence of a range of flow unsteadiness as the model diameter was gradually increased. He reported the existence of the unsteady flow field for particular geometrical configurations using the separated shock strength and flow deflection angle. However, his study was just confined to a blunt wall protrusion having a sharp corner.

Few of the literature (Kenworthy 1978) had already made comments about the gradual reduction of flow unsteadiness as the forebody geometry is changed. It is not surprising to see the influence of forebody shape changes, as it determines the net drag force acting on the body, re-entry, and heat transfer capabilities at higher Mach number regimes, and payload bearing capacities. However, for most of the space missions, the payload storage in the forebody is given priority, which always requires a higher volume. Reducing the volume in the forebody region leads to the allocation of extra space somewhere else in the vehicle, thereby increasing the net weight of the vehicle itself. For example, the overall
forebody volume in comparison with a simple flat-face cylindrical forebody is reduced by
22% and 33% for hemispherical and elliptical forebodies, respectively.

In addition, in a few of the cases (Mair 1952; Maull 1960), the pulsation form
of unsteadiness vanishes abruptly as the forebody shape changes to a hemisphere. In
the early days, Jones (1952) reported the importance of longer spikes and rounded-off
or streamlined forebodies in his studies to achieve the desirable aerodynamic design.
However, the author had not considered the effects of flow unsteadiness for the utilized
configurations as his experiments were only time-averaged. Later, few of the researchers
had attempted to study the unsteady flow field around the rounded-off edges. One
particular case to mention is the work of Charwat et al. (1961), where the authors had
observed an abrupt switch in the unsteadiness as the flat-face model was rounded-off
or streamlined progressively along the corners. In the recent paper of Balakalyani &
Jagadeesh (2019), the influence of the spike-shape and shoulder-rounding on the modes
of unsteadiness have been studied in a blunt spiked body through unsteady pressure
measurements in a hypersonic shock-tunnel. Similarly, in the recent work of Sahoo
et al. (2019), supersonic wind tunnel experiments of flat-face and hemispherical forebody
shapes with a spike reveal a significant drop in the intensity of shock-related unsteadiness.
However, in both of the research works, the flow physics behind the intensity of shock-
related unsteadiness, especially over the rounded or streamlined forebody configurations
have not been discussed in detail.

Thus, it can be seen that a collection of engineering solutions for the existing unsteady
flow problem is already available. From the literature, it is clear that for a given
configuration and flow conditions, the existence of a specific form of unsteadiness is
widely known for the spiked bodies. However, to the best of our knowledge, the flow
physics that leads to the observation of such flow unsteadiness as the geometrical and
flow parameters are varied in a spiked body configuration, especially in the cases of
rounded-off blunt or streamlined forebody are far from being fully explained. This is
our main motivation to experimentally investigate the shock-related unsteadiness in the
axisymmetric spiked bodies for a range of configurations, which can trigger the unsteady
flow field around the spiked bodies at different intensities.

In Table 1, a comprehensive set of literature that has reported about the flow unsteadi-
ness on a variety of geometrical configurations and flow conditions are listed out. Most
of the research works are numerical studies on simplistic configurations at low Reynolds
number and higher Mach number \(M_\infty > 4\). Hence, we have also selected a generic
case of sharp spike-tip flat face (ST-FF) and sharp-spiked hemispherical forebody (ST-
HF) primarily for experimental studies at higher Reynolds number \(Re_D \sim 2 \times 10^6\)
and lower Mach number \(M_\infty \sim 2\). At first, with the initial model configurations,
shock-related unsteadiness associated with the spiked flat-faced forebody (pulsation) is
studied. Then, to examine the extent of forebody rounding effects on the shock-related
unsteadiness, hemispherical (ST-HF) and elliptical (ST-EF) forebodies are considered.
Finally, a simple hemispherical spike/aerodisk tip on a hemispherical forebody (HA-HF)
is used to demonstrate a reduction in the shock-related unsteadiness based on the findings
from the previous studies.

Following the introduction, a detail description of the experimentation procedure is
given. It includes details about the facility, geometrical aspects of the selected configu-
rations, and a brief layout of the measurement methodology. In the third section, the
data analysis procedures to extract the dominant spatiotemporal modes from the modal
analysis of shadowgraph images are briefly described. The fourth section carries the
discussion on uncertainties observed in the experiments and data analysis. In the results
and discussions section, drag and volumetric efficiency of the adopted test models are
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Figure 3. Schematic of the supersonic blow-down type wind tunnel with a ‘Z-type’ high-speed short-exposure shadowgraph imaging (Settles 2001) setup to study the unsteadiness in different spiked-body configurations at a supersonic freestream Mach number ($M_\infty = 2.0$). Flow is from left to right.

presented first. The observed shock-related unsteadiness on the considered models is then addressed using the unsteady pressure measurement data. Then, the dominant energetic and dynamic spatiotemporal contents present in these configurations are described, and the unsteady shock-laden flow field is explained. Before reaching the conclusions, through the understanding of unsteady flow physics, an appropriate configuration is suggested to reduce/control the shock-related unsteadiness observed in a typical axisymmetric spiked body configuration. In the supplementary, high-speed shadowgraph videos\(^\dagger\) are provided to visually appreciate the varying unsteady flow field around the axisymmetric spiked bodies.

2. Experimentation

All the experiments are performed in the blow-down supersonic wind tunnel facility located in the Wind Tunnel Complex of Technion-Israel Institute of Technology, Israel. The details of the experimental facility, the measurement methodology, and the operating conditions adopted in the present work are provided in the following sub-sections.

\(^\dagger\) Filename of the videos given for different spiked forebody configurations in the supplementary: ‘video1.avi’ (Flat-face), ‘video2.avi’ (Hemisphere), and ‘video6.avi’ (Elliptical). Captions for each of the video is available in the ‘VideoCaptions.txt’ file.
2.1. Supersonic Blow-down Facility

The supersonic wind tunnel having a test section size of 400 mm (width) × 500 mm (height) is designed with a variable throat nozzle (convergent-divergent) to generate a freestream flow of Mach number ($M_\infty$) ranging from 1.6 to 3.5. The high-pressure air required to run the supersonic tunnel is taken from the compressed air storage. The storage facility consists of 48 balloons having a compressed air storage volume of $0.6 \text{ m}^3$/balloon. These balloons can be charged up to a maximum storage pressure of 200 bar. The storage facility is charged using two units of 5-stage piston-type air compressors, operating in parallel. Each compressor has a rated power of 400 kW and has an air discharge capacity of $0.5 \text{ m}^3/s$. Before storing the compressed air, moisture and impurities are removed using appropriate filter systems. The pneumatic control valve and ON/OFF gate valve are used to regulate the compressed air flow entering the settling chamber. Freestream flow stagnation conditions are measured at this location. Then, the compressed air is passed through the convergent-divergent (C-D) nozzle corresponding to a particular $M_\infty$. At the end of the C-D nozzle, freestream flow conditions are established in the constant area test section of length 1 m. Optical quality glass windows (BK-7) (550 mm in length and 300 mm in height) are mounted for flow visualization studies on the sidewalls of the test section. Suitable sting arrangements are made inside the test section to mount the models and the force and pressure measurement systems. The air is then discharged into the ambient through a constant area duct of similar size as that of the test section. A typical schematic explaining the layout of the supersonic blow-down type wind tunnel facility is shown in Figure 3.

2.2. Model geometry

A cylindrical model with a base diameter (D) of 50 mm and an overall length of 1.5D has been adopted for all the configurations that are investigated in the present work. For the elliptical forebody configuration, an ellipse with a semi-major axis length ($a$) of 0.75D and semi-minor axis length ($b$) of 0.5D has been employed. A sharp spike having a semi-cone angle of $10^\circ$ and a stem diameter ($d$) of 6 mm (0.12D) has been used. The spike length ($l$) is chosen to be equal to that of the base body diameter ($D$). The cross-sectional area of the test model results in only 0.98% blockage in the test section, provided the angle of attack is kept at $0^\circ$. Figure 4 shows the primary geometrical details of the spiked body configurations that are used in the present investigation. Any points on the model surface at a given $xy$–plane is measured in terms of a segment length parameter ($S$) along the model surface from the stagnation point of the blunt body. Any points on the model surface at a given $yz$–plane is represented by an azimuthal angle $\phi$ as shown in Figure 4b. A parameter called the geometrical shape factor ($\xi$) is proposed to distinguish between the blunt and streamlined forebodies. The parameter ($\xi$) is defined as the ratio of vertical distance ($H_S$) measured from the forebody axis to the point where $S \approx 0.5D$ along the forebody base of radius ($D/2$) as shown in Figure 4. In case of the flat-face spiked body configuration $\xi_F = 2H_S/D \approx 1$. For the hemispherical and elliptical spiked body configurations $\xi_H = 2H_S/D \approx 0.92$ and $\xi_E = 2H_S/D \approx 0.84$, respectively. The definition of $\xi$ comes handy in the discussions made in the subsequent sections.

2.3. Measurement methodology

The wave drag associated with the three different models is calculated using the strain gauge based drag measurement techniques. The magnitude of the shock-related unsteadiness around the model is measured using an unsteady pressure transducer mounted in the forebody. At first, shadowgraph is used as a qualitative imaging technique.
Figure 4. Schematic showing the primary geometrical parameters required to construct different spiked body configurations: (a) spiked body, and (b) spike. Origin is always at the spike tip and it is positioned in such a way that it always faces the flow. Azimuthal location of an arbitrary point anywhere on the model surface is represented using the circumferential angle ($\phi$). Distance of any point along the model surface in the axial direction is given by $S$. Flow is from left to right.

Later, the light intensity fluctuations from the shadowgraph images are used to get information regarding the extent of shock-related unsteadiness through rigorous image processing and data analysis (see section 3). Quantitative information like the shock trajectories from the $x-t$ plots, and the spatiotemporal modes from the modal analysis are then obtained. The details of all these measurement procedures are given briefly in the following sub-sections.

2.3.1. **Shadowgraph flow visualization**

The flow past spiked body configurations is visualized using the standard Z-type shadowgraph technique (Settles 2001). Shadowgraph imaging captures the density variations in the shock-laden flow field. The image intensity is a direct representation of the double spatial derivative of the density in the flow field. A high-intensity plasma light (white light) source of 25 W is used to produce the required light intensity for shadowgraph imaging. The light is allowed passing through a slit which forms the point light source for shadowgraphy. Planar mirrors are used to deflect the light to the parabolic mirrors and the camera. As shown in Figure 3, a parallel beam of light to pass through the test section is formed using the parabolic reflecting mirrors of 310 mm diameter and 2750 mm focal length. A Phantom V211, monochrome, 12-bit high-speed camera is utilized to record the shadowgraph images at a reduced resolution of $256 \times 160$ pixels with a pixel resolution of 0.5 mm/pixel. The unsteady flow field around all the configurations is captured at a frame rate of 43000 Hz with the least exposure time of 2 $\mu$s. For exceptional cases of imaging, a maximum frame rate of 140740 Hz is achieved at a resolution of $128 \times 64$ pixels. The refracted light is captured using a lens system in front of the camera. A 40 mm diameter achromatic doublet lens having an aperture of $f/2.5$ and a focal length of 100 mm is used to focus the light on the camera sensor.

Typical instantaneous shadowgraph images for different spiked body configurations are shown in Figure 5. Some of the critical flow features like weak leading edge shock, separation shock, shear layer, recirculation region, and reattachment shock could be identified from it. The flow field around the flat-face spiked body configuration (Fig-
Figure 5. Typical instantaneous shadowgraph images showing the basic flow features observed around different spiked body configurations at a supersonic freestream Mach number ($M_\infty = 2.0$). Dominant flow features: 1. Weak leading edge shock; 2. Separation shock (SS); 3. Shear layer; 4. Recirculation region; 5. Reattachment shock (RS); 6. Refracted shock; Thickness of the shock seen inside the red dotted boxes qualitatively represents the shock strength. Flow is from left to right. High-speed shadowgraph videos for each of the cases are available in the supplementary (flat-face: ‘video1.avi’, hemispherical: ‘video2.avi’, elliptical: ‘video6.avi’).

Figure 5a) stands out completely different from the flow field that is observed around the hemispherical (Figure 5b) and elliptical (Figure 5c) spiked body configurations. Also, the qualitatively estimated shock strength from the shadowgraph images (through the light intensity variations) shows that the detached shock in-front of the flat-face spiked body configuration is the strongest. The shock strength in the other cases is found to decrease from the hemisphere to ellipse, as $\xi$ is decreasing (see the colored boxes in Figure 5). The flow field around the hemispherical (Figure 5b) and elliptical (Figure 5c) spiked body configurations exhibit similarity in terms of overall flow features. However, they have varying shock intensities, as well as different locations of separating and reattaching shocks.

2.3.2. Drag force measurements

An in-house built six components strain gauge balance has been used for the drag measurements. A DC voltage of 18 V is used to excite the balance having a capacity to measure the axial and normal force of 50 kg and 250 kg, respectively. Only the axial force has been considered in the present investigation as the model is mounted at only 0° angle of attack. More details about the data acquisition system are given in the next sub-section.

2.3.3. Steady and Unsteady pressure measurements

Steady-state pressure measurements are carried out using the low response time Honeywell®-Model FP2000 type configurable pressure transducers. The transducers measure the gauge pressure and have a response time of 2 ms. Two different range (sensitivity) of pressure transducers are used—50 psi (1000 $\mu$V/psi) and 100 psi (500 $\mu$V/psi). The data acquired from these sensors are sampled at 5 kHz with an accuracy of ±0.1% from the measured value. Pressure taps (~1 mm in diameter) are made at several locations along the model forebody between 0.15 < [S/D] < 0.9 (see Figure 10a), and they are connected to the sensors using an integral polyurethane 1.5 m cable passing through the model interior. Unsteady pressure transducers (Kulite®: XCL-100-50A of head size ~2.6 mm), having a maximum frequency response of 250 kHz are used to measure the pressure fluctuations generated on the spiked forebody. The location of the
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Figure 6. Typical pressure signals observed during the supersonic wind tunnel operation at a supersonic freestream Mach number of $M_\infty = 2.0$: (a) static pressure signal (unfiltered & filtered) and, (b) unsteady pressure fluctuations observed in the spiked flat-face body at $[S/D] = 0.25$ and $\phi = 90^\circ$. The unsteady pressure data (circles) are compared with the computation values from the work of Feszty et al. (2004a) and also with the experiments of Kenworthy (1978).

unsteady pressure transducers are at $[S/D] = 0.25$ for the flat-face, $[S/D] = 0.4$ for the hemispherical and $[S/D] = 0.39$ for the elliptical forebody configurations (see Figure 10b-d). A sampling rate of 50 kHz for a total period of 2 seconds has been used for the final experiments after considering the effects of aliasing. Both types of transducers are flush-mounted near the reattachment shock corresponding to an azimuthal location of $\phi = 90^\circ$ during different experimental runs.

2.3.4. Data acquisition system

The electrical output of all the measured parameters (forces, pressures, and angle of attack) is acquired using a National Instruments data acquisition system (NI-DAQ). The module includes 32 sequential measuring channels. The sampling rate for all the analog channels is 250 kHz. The acquired signals are filtered, amplified, and transformed into digital data. The digital data are then transferred to a PC to evaluate the tunnel operating conditions, aerodynamic coefficients, and pressure coefficients. The obtained pressure histogram is further processed using a Matlab program to extract the power spectral density (PSD) and other useful unsteady statistics. The filtered and unfiltered static pressure data measured at the tunnel wall (see $P_\infty$ in Figure 3) during a typical test is plotted in Figure 6a. All the experimental data have been acquired during the steady test time, as indicated in Figure 6a. Experimental validation has been carried out by repeating the unsteady experimental results of Kenworthy (1978) and the computational results of Feszty et al. (2004a) for the case of a spiked flat-face body. As shown in Figure 6b, the present results are well compared with the previous ones.

2.4. Operating conditions

All the experiments in the present investigation are carried out at a freestream Mach number of $M_\infty = 2.0$ with a settling chamber pressure of $P_0 = 3.5$ bar. The freestream Reynolds number based on the base diameter ($D$) of the model is $Re_D = 2.16 \times 10^6$. Necessary wind tunnel calibration for the Mach number has been conducted before starting the experiments. The freestream static pressure fluctuation intensity $\sqrt{(P_\infty - \bar{P}_\infty)^2}$ in the supersonic tunnel, obtained from the unsteady pressure measurements, is 1.57%. Table 2 shows the achieved tunnel operating conditions during the experiments.
Table 2. Flow conditions achieved in the test section of the supersonic blow-down type wind tunnel during the present investigation along with measurement uncertainties given in percentage about the measured values.

| Quantities                      | Values                                      |
|---------------------------------|---------------------------------------------|
| Total Pressure ($P_0$)          | 347910.52 ± 5% (Pa)                         |
| Total Temperature ($T_0$)       | 294.46 ± 2% (K)                             |
| Freestream Mach number ($M_\infty$) | 2.01 ± 1%                                  |
| Freestream Temperature ($T_\infty$) | 163.59 ± 2% (K)                           |
| Freestream Pressure ($P_\infty$) | 43908.88 ± 5% (Pa)                         |
| Freestream Velocity ($U_\infty$) | 515.15 ± 2% (m/s)                          |
| Dynamic Viscosity ($\mu_\infty$) | $1.044 \times 10^{-5}$ ± 2% (Pa s)         |
| Freestream Density ($\rho_\infty$) | 0.935 ± 5% (kg/m$^3$)                     |
| Reynolds number ($Re_D = U_\infty D/\nu_\infty$, D = 50 mm) | $2.159 \times 10^6$ ± 5%                        |

3. Data analysis

Firstly, the obtained shadowgraph images from the procedure mentioned in Section 2.3.1 are subjected to a rigorous image pre-processing routines using Matlab programs before starting the data analysis. During the acquisition, image intensity saturation is avoided to monitor the fluctuations at all spatial points. Due to the non-uniformity and defects present in either the optical windows or the mirrors, the obtained images may possess spatial intensity variations. Besides, the density variations along the light path inside the laboratory environment cause spatiotemporal light intensity fluctuations. These anomalies need to be sorted out. Else the analysis picks up these features as the dominant ones. Many investigators (e.g. Kutz et al. (2016); Prothlin et al. (2016)) have followed some standard image pre-processing routines to handle such kind of images. In the present study, the time series subtraction filter and the image intensity normalization filter used by Karthick et al. (2017) in the acetone planar laser-induced fluorescence (acetone-PLIF) studies in a typical supersonic confined jet flow problem are used to prepare the shadowgraph images for the modal analysis. Besides, the images are also corrected for perspective distortions and vignetting effects arising due to the telephoto type lens system in the high-speed camera. Later, the corrected images are calibrated based upon the physical scale of the model geometry for proper representation.

In the present paper, data-driven analysis techniques like Proper Orthogonal Decomposition (POD) (Lumley 1970, 1981; Berkooz et al. 1993; Meyer et al. 2007) and Dynamic Mode Decomposition (DMD) (Schmid et al. 2011; Schmid 2010), are used to identify the dominant energetic and dynamic contents in the flow field. The authors have used the classic decomposition tool called ‘method of snapshots’ (Sirovich 1987; Berkooz et al. 1993) to identify the dominant modes from the shadowgraph images. Traditionally these methods are used on vector fields (Meyer et al. 2007) obtained from PIV analysis. However, it has also been utilized in the scalar fields (Berry et al. 2017; Schmid et al. 2011). The methodology employed in the present study is adopted and expanded from the procedure described in the work of Mohan et al. (2016), and Mohan & Gaitonde (2014, 2017). For simplicity and better understanding, a simplified mathematical routine is described here.
\[ R = \{ r_1, r_2, ... r_n \}, \quad (3.1) \]
\[ \overline{R} = \frac{1}{n} \sum_{i}^{n} r, \quad (3.2) \]
\[ R' = R - \overline{R} = \{ r'_1, r'_2, ... r'_n \}. \quad (3.3) \]

Each snapshot of size \( i \times j \) (in pixels) carrying the light intensity fluctuations represents the line of sight integrated variations in the double derivative of the flow field density \( \sim \nabla^2 \rho \), directly. The obtained two-dimensional snapshot is represented as a column vector \( \mathbf{R} \) of size \( i \times j \). A large matrix \( \mathbf{R} \) based on the total number of snapshots \( n \) is thus formed by converting and stacking the temporally evolving images in succession. The final size of the snapshot matrix is \( k \times n \). After utilizing the image pre-processing routines, the instantaneous images \( \mathbf{R} \) are used to create a time-averaged image \( \overline{\mathbf{R}} \). A sequence of images carrying the fluctuation field alone \( \mathbf{R}' \) are then computed. The steps are briefly described in equation 3.1-3.3. The value of \( n \) in the current studies is only 1000 as no significant difference was found by increasing \( n \) from 1000 to 10000. The snapshots are captured at an image resolution of \( 256 \times 160 \) pixels at 43000 Hz with 2 \( \mu s \) light exposure (see Figure 5). Thus, the spatial and spectral resolution in the observed dynamic and energetic spatiotemporal modes is 0.5 mm/pixel and 43 Hz, respectively.

\[ R'_1 = \{ r'_1, r'_2, ... r'_{n-1} \}, \quad (3.4) \]
\[ R'_2 = \{ r'_2, r'_3, ... r'_n \}, \quad (3.5) \]
\[ R'_2 = A R'_1. \quad (3.6) \]

Two time-lagged snapshot matrices \( ( \mathbf{R}'_1, \mathbf{R}'_2 ) \) are constructed by only considering snapshots between 1 to \( n - 1 \) and 2 to \( n \) in order to study the time dynamics (see equation 3.4 and 3.5). The evolution of a time-dependent system of variables like \( \mathbf{R} \) in terms of a linear propagator \( A \) is represented through time-lagged matrices \( ( \mathbf{R}'_1, \mathbf{R}'_2 ) \) as stated in equation 3.6. Solving for \( A \) is computationally expensive and hence, it has been approximated using a companion matrix \( \tilde{B} \). Later, one of the time-lagged matrix \( ( \mathbf{R}'_1 ) \) is rewritten through a singular value decomposition (SVD) as stated in equation 3.7 and 3.8 Schmid (2010).

\[ \mathbf{R}'_2 \approx \tilde{B} \mathbf{R}'_1, \quad (3.7) \]
\[ \mathbf{R}'_2 \approx \mathbf{U} \mathbf{S} \mathbf{V}^H \tilde{B}. \quad (3.8) \]

In the above stated equations, equation 3.8 carries the energetic spatiotemporal modes obtained from the singular value decomposition of the time-lagged matrix \( \mathbf{R}'_2 \) (Schmid 2010). Hence, the SVD step can be considered as a POD operation, since the decomposition results in the formation of \( \mathbf{U} \) containing the spatial information \( \Phi_n \), \( \mathbf{V} \) having the temporal information \( a_{\phi_n}(t) \), and \( \mathbf{S} \) has the energy contents \( \lambda \) (eigenvalues). It has to be noted that the singular values \( ( \mathbf{S} ) \) and the eigenvalues \( ( \lambda ) \) are related by \( \mathbf{S}^2 = \lambda \) and hence the percentage of energy contents in each mode has to be calculated accordingly (Taira et al. 2017). While using shadowgraph images, the word energy is, in general, a misnomer, as the eigenvalues carry only the square of light intensity fluctuations, unlike the more conventional use of the square of velocity fluctuations obtained from velocimetry data. However, for the consistent usage with the existing literature, the terminology is withheld in this manuscript.
After performing the decomposition, the approximated matrix ($\tilde{B}$) is evaluated as stated in equation 3.9 and subjected to an eigenvalue problem formulation like in the case of a POD analysis (Meyer et al. 2007) as given by equation 3.10. In equation 3.10, $\Delta$ represents the eigenvector and $\Lambda$ contains the eigenvalues. The DMD spatial modes ($\Theta$) are then computed as shown in equation 3.11. The DMD spectral contents ($f_\Theta$) are accessed by taking the imaginary part of the products of $f_s/2\pi$ and $\log(\Lambda)$, where $f_s$ is the sampling rate at the time of image acquisition (see equation 3.12). The DMD amplitudes ($\alpha_\Theta$) are obtained by taking the pseudo inverse of any one of the DMD modes ($\Theta$) and projecting it on to the corresponding fluctuating snapshot ($r'$) from the respective time-lagged column matrix ($R'_2$) as stated in equation 3.13. In equation 3.13, the first DMD spatial mode ($\theta_1$) and the first fluctuating snapshot ($r'_1$) are used to compute the amplitude ($\alpha_\Theta$). However, such a routine for a larger matrix will be computationally expensive. For a more efficient method to compute the DMD amplitudes, the readers are referred to the paper of Jovanovic et al. (2014).

$$\Theta = \{\theta_1, \theta_2, \ldots, \theta_{n-1}\} = R'_2 VS^{-1} \Delta,$$  
(3.11)

$$f_\Theta = \Im\left(\frac{f_s \log(\Lambda)}{2\pi}\right),$$  
(3.12)

$$\alpha_\Theta = \theta_1^+ r'_1.$$  
(3.13)

With the DMD components $\Theta$, $f_\Theta$ and $\alpha_\Theta$, unwanted image noise or irrelevant modes (spatiotemporal artefacts) from the snapshots are identified. Higher values of $\alpha_\Theta$ for lower $f_\Theta$ in the DMD spectrum generally corresponds to parasite reflections from the scratches on the viewing window, and they are removed after visualizing the corresponding DMD components. The time-lagged matrix is approximated and reconstructed using the remaining DMD components for the considered time period. The time evolution term ($t$) and the eigenvalue term ($\Lambda$) are used along with the other DMD components to reconstruct the approximated time-lagged matrix ($\tilde{R}'_2$) as described in equation 3.14 and 3.15. More details about the DMD based image noise removal is available in the book of Kutz et al. (2016). Similar image noise filtering routines to those described in the above mentioned book, are carried out in the present analysis. The difference between the unfiltered and filtered operator-based time-averaged image for the flat-face spiked configuration is demonstrated in Figure 7.

$$t = \left\{\frac{1}{f_s}, \frac{2}{f_s}, \ldots, \frac{n-1}{f_s}\right\},$$  
(3.14)

$$\tilde{R}'_2 = \Theta \left[\exp\left(f_s \log(\Lambda) t\right) \alpha_\Theta\right].$$  
(3.15)

In the unfiltered normalized time-averaged image (Figure 7a), scratches in the viewing window, readout lines from the camera sensor (light horizontal lines), and the dynamic flow events around the body are visible with similar light intensity. Hence, the actual flow events around the body are not distinguished. However, while performing a normalized operator-based time-averaging (Figure 7b), where the normalized difference between $\bar{R}$ and $R_{rms}$ is considered, dynamic flow events around the flat-face spiked configuration is
distinctly visible. The window scratches that cause light intensity fluctuations are still present in the imaging frame, irrespective of the operator-based time-averaging routines. After performing DMD based image noise filtering (see Figure 7c), it is evident that only the dominant flow events around the body of interest are existing and all the other noises are filtered out, including the window defects. In the present studies, all the images are subjected to a thorough DMD based image noise filtering routines, before they are considered for any analysis.

Few of the POD and DMD parameters are compared both qualitatively and quantitatively with the experiments to validate the data analysis techniques. The energy contained in each of the modes ($S^2$) for all the spiked body configurations obtained from the POD analysis is plotted in Figure 8a using equation 3.8. The first ten modes
Figure 8. (a) The energy contents (in %) in each mode observed for the different spiked body configurations obtained from the POD analysis; (b) The associated cumulative POD energy contents for the different spiked body configurations; (c) A typical spectral contents for the different spiked body configurations obtained from the DMD analysis; (d) Comparison of a simple reconstruction of POD time coefficients for the first mode ($a_{\Phi_1}(t/T)$), and DMD temporal signal ($a_{\Theta}(t/T)$), with a static pressure signal measured at a particular location ($\frac{P}{P_{\infty}}_{S/D=0.25}$) for the flat-face spiked forebody configuration.

$(n = 10)$ are found to be dominant, and they influence the overall flow behavior. The rest of the modes contain only a little fraction of the energy in each of the three cases. The first few modes of the flat-face spiked body configuration contain a larger fraction of energy in comparison with the other configurations. In Figure 8b, the cumulative energies $\left(\sum_{1}^{n-1} S^2 \right)$ associated with each of the modes $(n)$ are shown. 40% of the cumulative energy associated with the flat-face spiked body is contained in the first ten modes $(n = 10)$, whereas the other cases require at least $n = 100$ (hemispherical spiked body) and $n = 200$ (elliptical spiked body) to represent the same amount of energy.

The drastic difference in the cumulative energy between the flat-face and the other
cases is obvious while seeing Figure 5, and the corresponding videos† given in the supplementary. The close resemblance between the flow features in Figure 5b and Figure 5c explains the similar order of $n$ to represent 40% of the cumulative energy in the hemispherical and elliptical cases, respectively. The DMD components $\alpha_\Theta$ and $f_\Theta$ are plotted for all the cases in Figure 8c. The flat-face case exhibits periodic fluctuations, whereas the other cases are results of broadband spectra. Comparing the wall-static pressure signals at $[S/D] = 0.25$ for the flat-face case ($[P/P_\infty]$) with the corresponding dominant POD signal ($a_{\Phi 1}(t/T)$) and the decomposed DMD signal ($a_{\Theta}(t/T)$) shows good agreement as seen in Figure 8d, thus validating the present analysis. More observations and discussions on the flow field analysis using the POD and DMD techniques are given elaborately in Section 5.3.

4. Uncertainty

Each experiment is repeated 5 times to ensure repeatability, and the reported data are the ensemble average of the 5 runs. However, built-in limitations of the instruments in terms of accuracy and precision in obtaining the measured quantities and the propagating errors in the derived quantities lead to uncertainty, which is inevitable in the experimental and analysis procedures. In the unsteady pressure measurement, the obtained values carry an uncertainty of around ±5%. The uncertainty in the coefficient of drag measurement is found to be around ±5%. The uncertainties in the measurement of the other derived quantities in the present work are further listed in Table 2. In the modal analysis also, five sets of 1000 images are acquired from different runs, and they are processed individually to access the consistency in the final results. The reported values are the ensemble average from the five sets of modal analysis performed for individual cases. The uncertainty in representing the spatial and temporal features are mainly dependent on the resolution itself (i.e., 0.5 mm/pixel & 43 Hz). Also, the spatial features are, in general, smeared due to the exposure limitations of the camera. Hence, there is an error associated with the exact representation of the spatial structure. However, for a 2 µs exposure, the smearing effect is found to be minimal, as reported in the work of Rao & Karthick (2019). The total uncertainty involved in the spatial mode representation includes the error propagation in the image processing routines and the calibration procedure. It has been estimated to be around ±4%. The recommendations of Rao & Karthick (2019) is followed to avoid aliased signals in the calculated spectral characteristics from the modal analysis. The total uncertainty in the temporal mode representation is identified to be around ±3%.

5. Results and Discussion

The results obtained from the experiments conducted over all the spiked body configurations are presented in this section. Firstly, the drag coefficients are measured to emphasize the influence of change in forebody shapes. Spectral characteristics from the unsteady pressure measurements are later obtained to quantify the dynamic loads while changing the forebody shapes. Lastly, different types of shock-related unsteadiness along with dominant spatiotemporal modes observed with forebody modifications are discussed in detail. Besides, at the end of the last section, a simple configuration aiming to alleviate the shock-related unsteadiness is proposed and verified.

† Filename of the videos given for different spiked forebody configurations in the supplementary: ‘video1.avi’ (Flat-face), ‘video2.avi’ (Hemisphere), and ‘video6.avi’ (Elliptical). Captions for each of the video is available in the ‘VideoCaptions.txt’ file.
Table 3. Tabulation of different parameters including the drag coefficient \((c_d)\) (with and without spike), the pressure loading \((\zeta)\), and the pressure fluctuation intensity \((\kappa)\) observed for different spiked forebody configurations with varying geometric shape factor \((\xi)\) at a freestream supersonic Mach number \((M_\infty)\) of 2.0.

| Cases                  | [S/D] | \(c_d\) (±5%) | \(\zeta\) (±5%) | \(\kappa\) (±5%) |
|------------------------|-------|----------------|----------------|-----------------|
| Flat face \((\xi \approx 1)\) | 0.25  | 1.51           | 1.06           | 4.84            | 0.452           |
| Hemisphere \((\xi \approx 0.92)\) | 0.4   | 0.8            | 0.52           | 3.47            | 0.111           |
| Ellipse \((\xi \approx 0.84)\)    | 0.39  | 0.61           | 0.42           | 3.29            | 0.087           |

5.1. Drag reduction

Streamlined forebody offers lower drag than the blunt-body. In the present study, the drag measurements are carried out using the strain-gauge type force-balance system (see Sec. 2.3.2), which measures only the overall drag coefficient \((C_d)\). However, for all practical purposes, the forebody drag coefficient \((c_d)\) is preferred. Since the model is at zero degree angle of attack, a pressure sensor kept in the base portion of the model is used to estimate the base drag coefficient, \(C_{d,base}\) (Greenwood 1966). Finally, the forebody drag coefficient \((c_d)\) is calculated by subtracting the base drag coefficient \((C_{d,base})\) from the overall drag coefficient \((C_d)\). The forebody drag coefficient \((c_d)\) for all configurations are measured with and without spike at \(M_\infty = 2.0\). The values are tabulated in Table 3. The elliptical spiked body configuration, which has the lowest \(\xi\) provides the minimum drag coefficient \((c_d)\) in comparison to the hemispherical and flat-face spiked body configurations. The mounting of a spike, reduces significantly the associated drag of all configurations. Nevertheless, the spike, as is elaborated in the upcoming sections, causes severe flow unsteadiness.
5.2. Unsteady pressure spectrum around the forebodies

A normalized operator-based time averaging ($\|\bar{R} - R_{rms}\|$) is applied to the shadowgraph images in order to study the key dynamic features around the different spiked forebody configurations qualitatively as shown in Figure 9. The darker zones in the images mark the locations of dynamic flow events. In Figure 9a associated with a flat-face spiked body, darker regions are observed, suggesting severe shock related unsteadiness. In Figure 9b-c associated with the hemispherical and elliptical forebodies have different intensities of darkness, especially near the interface of the reattachment shock and the separated shear layer. Steady and unsteady pressure sensors are mounted near those regions to know the level of shock-related unsteadiness on these spiked body configurations, as shown in Figure 10. Locations and values of mean wall-static pressure coefficients for both the spiked and unspiked bodies are shown in Figure 10a, whereas the normalized unsteady pressure spectrum for each of the spiked configurations are shown in Figure 10b-d. The location of the unsteady pressure sensor is selected after identifying the maximum value of the pressure coefficient from Figure 10a for each of the configurations except the flat-face, as shown in the bottom-left portion of the schematic in Figure 10c-d.

A non-dimensional parameter called pressure loading ($\zeta$) is defined to quantify the net pressure load acting on the respective forebody due to the shock laden flow field. Similarly, the unsteady flow features near the reattachment shock of the forebody are represented through a parameter called pressure fluctuation intensity ($\kappa$). The values for pressure loading ($\zeta$) and pressure fluctuation intensity ($\kappa$) are obtained from equations 5.2 and 5.1, respectively.

$$\zeta = \frac{P_{rms}}{P_{\infty}},$$

$$\kappa = \frac{P_s}{P_{rms}}.$$  

where,

$$P_{rms} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} P_i^2}, \quad P_s = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (P_i - \bar{P})^2}, \quad P_i = \bar{P} + P', \quad \bar{P} = \frac{1}{n} \sum_{i=1}^{n} P_i.$$

The measured unsteady pressure values in terms of $\zeta$ and $\kappa$ are also tabulated in Table 3. From the table, it can be seen that $\zeta$ is larger for the flat-face ($\zeta = 4.84$) than the hemispherical ($\zeta = 3.47$) and the elliptical ($\zeta = 3.29$) counterpart. While analyzing for the shock-related flow unsteadiness, the flat-face spiked body configuration shows a maximum $\kappa$ of 45%, whereas the hemispherical and elliptical spiked body configurations show only 11% and 8.6%, respectively. The observed values of $\zeta$ and $\kappa$ for each of the configurations with varying $\xi$ are also in agreement with the instantaneous shadowgraph images shown in Figure 5 and also from the shocks and shear layers intensity observed in the operator-based time-averaged shadowgraph images shown in Figure 9.

The existence of higher $\zeta$ and $\kappa$ for the flat-face spiked configuration can be explained through the previous works of Kenworthy (1978) and Feszty et al. (2004a) on similar spiked forebodies. Accordingly, the pulsation mode of unsteadiness around the spiked body of large bluntness is the primary reason. The pulsation mode of unsteadiness consists of three phases termed as ‘collapse’, ‘inflation’, and ‘withhold’ (Feszty et al. 2004a). These three phases collectively cause the entire shock system ahead of the flat-face spiked body to move forth and back periodically, as shown in Figure 2b, 5a, and 9a. Hence, the
Figure 10. (a) Pressure coefficient variation along the forebody curvature length \([S/D]\) for the hemispherical and the elliptical forebody configurations (with and without sharp spike tip) obtained through the time-averaged wall-static pressure measurements. Power spectral density obtained from the unsteady pressure measurements for the different forebody configurations (with and without sharp spike tip): (b) Flat-face at \([S/D] = 0.5\), (c) Hemisphere at \([S/D] = 0.5\), and (d) Ellipse at \([S/D] = 0.4\). The location where the unsteady pressure measurements are carried out are also shown in the small snippet in each of the graphs for the spiked body configurations at a supersonic freestream Mach number \((M_\infty = 2.0)\).

Unsteady pressure spectra obtained around the flat-face spiked forebody at the time of pulsation mode show a dominant peak at 1338.3 Hz \((St = fU_\infty / D = 0.13)\) with harmonics (see Figure 10b). The obtained Strouhal number \((St)\) is in-fact a bit lower than the value of 0.183, obtained from the empirical relation (see equation 5.3) given by Kenworthy (1978) for the same \([l/d]\).

\[
St = 0.25 - 0.067 \left( \frac{l}{D} \right). \tag{5.3}
\]

However, in the cases of hemispherical and elliptical spiked forebodies, the shock oscillations are localized at the points of separation and reattachment of the shear layer, as shown in Figure 9b-c, which result in lower \(\zeta\) and \(\kappa\). The unsteady pressure spectra in these two cases have no discrete frequencies like in the case of flat-face spiked configuration but only broadband (see Figure 10c-d). In Figure 10c, the power spectral
density of the unsteady pressure signal in the hemispherical spiked body configuration reveals a broadband spectrum between 2000-7000 Hz (blue line) in comparison to the no-spike case (orange line). This range of amplified frequencies is most likely associated with the shear layer instabilities. A similar broadband spectrum is also observed in the same frequency range for the elliptical spiked body configuration but with a slightly lower amplitude, as shown in Figure 10d.

Thus, from the initial studies carried out through shadowgraph imaging, and pressure measurements (both steady and unsteady), the blunt (flat-face) and the streamlined (hemispherical and elliptical) spiked bodies are proven to exhibit different forms of unsteadiness. More details on the shock-related unsteady modes specific to each of the configurations are discussed in the next section.

5.3. Shock related unsteady modes

Two methods are sought to understand and extract the dominant shock-related unsteady modes for the configurations under consideration: a. $x-t$ shock trajectory analysis and b. Modal analysis. Further, the steady and unsteady pressure measurements from the previous sections are used as supplements to validate the findings from the approaches mentioned above.

5.3.1. Shock pulsation in the flat-face spiked body configuration

The geometrical details of the flat-face spiked body configuration used for the present study are shown in Figure 4a. Figure 11a-I shows the time-averaged flow field obtained from the sequence of instantaneous shadowgraph images. The two extreme positions of the pulsating shock, along with the weak leading-edge shock, are observed. A complete cycle of shock pulsation including all the three different phases (collapse, inflation, and withhold), as mentioned by Kenworthy (1978) and Feszty et al. (2004a) are represented in the instantaneous shadowgraph images in Figure 11b. Few of the frames are skipped to express the pulsation cycle, briefly. In the image, the timestamps for the representative instantaneous shadowgraph images are indicated in which $\Delta t = 1/f_s$, where $f_s$ is the sampling rate of the camera. Unless otherwise noted, $f_s = 43000$ Hz for all the configurations. All the phases are segregated and presented in Figure 12 without skipping any frames to describe the complete pulsation cycle in detail. The ‘phase of collapse’ is captured between $t$ to $t+9\Delta t$, where the shock system approaches towards the forebody (see Figure 12a). Between $t+12\Delta t$ to $t+18\Delta t$, the ‘phase of inflation’ is captured. In this phase, the two shock systems are seen to merge and start inflating radially upstream (see Figure 12b). Finally, between $t+21\Delta t$ to $t+26\Delta t$, the ‘phase of withhold’ is observed, where the inflated shock stands at the tip of the spike before it starts to collapse again to repeat the cycle (see Figure 12c).

In Figure 12a, the strong shock wave starts to accelerate downstream from the leading edge of the spike ($S_{1in}$) towards the flat-face body till $[x/D] \approx 0.6$. A $x-t$ diagram is constructed by stacking the line segments of instantaneous shadowgraph images. It is essentially a temporal footprint of the dominant flow structures (in our case, it is the shock). The velocity ($u/U_\infty$) and location ($x/D$) of $S_{1in}$ are easily extracted using the $x-t$ diagram shown in Figure 13. The acceleration of $S_{1in}$ is seen in the initial frames from $t$ to $t+2\Delta t$ (Figure 12a) and also in the $x-t$ diagram (Figure 13c), thus initiating the phase of ‘collapse’. When $S_{1in}$ passes through the spike stem, the supersonic free stream flow ($U_\infty$) creates a weak shock (WS) from the sharp spike tip ($t+\Delta t$). The recirculating flow from the previous cycle (seen as turbulent structures downstream $S_{1in}$) convects along the downstream at a supersonic velocity just like $S_{1in}$. As $S_{1in}$ accelerates towards the forebody, the convecting recirculation region (CRR) gets compressed in front of the
Figure 11. (a) Time-averaged shadowgraph image showing some of the flow features: 1. Weak leading edge shock, 2. Pulsating shock wave’s extreme locations, and 3. Expansion fan. Glass window defect before image processing is marked as 4. (b) Instantaneous shadowgraph images at different time interval showing the cycles involved in the pulsation mode of shock unsteadiness at a supersonic freestream Mach number ($M_\infty = 2.0$). Flow is from left to right. (Corresponding video file is available in the supplementary under the name ‘video1.avi’)

forebody (from $t$ to $t+3\Delta t$). From Figure 13c, it could be seen that the velocity reaches to a freestream value of $[u/U_\infty] \approx 1$ at $[x/D] \approx 0.6$. Such higher velocities result in the formation of a strong detached shock ($S_{2\text{out}}$) in front of the flat-face forebody (see frames from $t+4\Delta t$ to $t+6\Delta t$). The produced detached shock wave ($S_{2\text{out}}$) gains strength and moves upstream gradually till $[x/D] \approx 0.6$ at a $[u/U_\infty] \approx -0.1$ (see Figure 13c) due to rapid compression of CRR by $S_{1\text{in}}$ against $S_{2\text{out}}$. As $S_{1\text{in}}$ and $S_{2\text{out}}$ move against each other, a weak compression front (WF) is formed in between the space around $[x/D] \approx 0.6$ ($t+6\Delta t$ and $t+7\Delta t$). Expansion fan (EF) is seen near the shoulder of the sharp spike tip after $S_{1\text{in}}$ passes through it ($t+7\Delta t$). The flow in front of $S_{2\text{out}}$ separates due to $\beta - \theta - M$ criteria in deflecting the flow downstream of $S_{2\text{out}}$. The deflected flow passes
Figure 12. Instantaneous shadowgraph images at different time intervals showing the detail features of (a) collapse, (b) inflation and (c) withhold during the pulsation mode of unsteadiness over a spiked flat-face forebody configuration at a supersonic freestream Mach number \( M_\infty = 2.0 \). Flow is from left to right. Dominant flow features: \( S_{1_{in}} \)-shock from the leading edge of the spike moving towards the body, WS-weak leading edge shock, \( S_{2_{out}} \)-shock moving away from the forebody, WF-weak compression front, EF-expansion fan, SF-shock foot, FSF or \( S_{1_{in}} \)-front shock foot, RSF or \( S_{2_{in}} \)-rear shock foot, SLI-shear layer interface, TP-triple point of SF, VSTP-vortex shedding from the triple point of SF, RS-refracted shock, TV-torroidal vortex formed from the TP, VTP-vertically moving TP, IRR-inflating recirculation region, TVR-torroidal vortex ring, LTP-locus of TP and trace of RSF about the axis, CRR-convecting recirculation region along the flow direction.
around the forebody and enters into the freestream. The flow separation results in the formation of lambda shock foot (SF, $t+8\Delta t$). Later, $S_{2\text{out}}$ collides with the incoming $S_{1\text{in}}$ (from $t+7\Delta t$ to $t+9\Delta t$) and thus, ending the phase of ‘collapse’.

The phase of ‘inflation’ begins after the head-on collision of the two shocks $S_{1\text{in}}$ and $S_{2\text{out}}$ ($t+10\Delta t$). The head-on collision leads to the growth of SF with a triple point (TP), a front shock foot (FSF), and a rear shock foot (RSF) (from $t+10\Delta t$ to $t+11\Delta t$). The shock wave boundary layer interaction (SWBLI) leads to the inflation of FSF, and the flow later evolves in the same way as observed in the shock tubes (Kryukov & Ivanov 2018). After $S_{1\text{in}}$ moves through the lambda shock foot system and hits $S_{2\text{out}}$, it gets refracted. The refracted shock (RS) hits the flat-face forebody ($t+12\Delta t$) causing it to produce harmonics in the unsteady pressure spectra (see Figure 10b). The FSF is considered to be the dominant shock and continues to move upstream towards the spike tip as $S_{1\text{out}}$ with decreasing velocity (Figure 13c). A shear layer interface (SLI) exists between the separated flow region and FSF ($t+14\Delta t$). The vortices shedding from the TP (VSTP) interacts with RS and starts to feed the flow below SLI (from $t+13\Delta t$ to $t+19\Delta t$) and inflates FSF or $S_{1\text{out}}$. However, RSF hangs between TP and SLI ($t+18\Delta t$). The inflating
Figure 14. Typical instantaneous snapshots during one of the cyclic events showing the expanding locus of the TP and trace of RSF (LTP). Due to flow turbulence, these events happen in a non-axisymmetric way, resulting in the visualization of LTP as another ring besides from TV at oblique angles (See the caption of Figure 12 for the abbreviations). The time stamps mentioned in the images are marked as $t'$ instead of $t$ as these images are not used in the analysis.

recirculating region (IRR) is seen as turbulent structures in $t+16\Delta t$. The clockwise VSTP forms a toroidal vortex (TV), and the eye of TV is seen in the toroidal vortex ring (TVR) as dark spots (see frame $t+17\Delta t$). During this process, the formed TP starts to rush in the vertical (VTP) direction (from $t+15\Delta t$ to $t+19\Delta t$).

Due to the line of sight integrated shadowgraph imaging, the locus of TP and RSF (LTP) is seen as a vertical line ($t+18\Delta t$); however, it is just an axisymmetric ring. Due to some fluctuations, the axisymmetric flow nature is perturbed in a few of the images. During that time, captured images reveal the ring-like LTP more clearly (Figure 14). Later, LTP and RSF are considered to be evolving as $S_2^\text{in}$. The phase of inflation comes to an end when FSF or $S_1^\text{out}$ is distinguishable from lambda shock foot and turns to a bow shock at the leading edge of sharp spike tip ($t+19\Delta t$).

During the phase of ‘withhold,’ the curvature of $S_1^\text{out}$ changes further, and it nearly becomes a normal shock (from $t+20\Delta t$ to $t+29\Delta t$). The flow fed below the SLI keeps inflating the recirculating region, and it holds $S_1^\text{out}$ at the leading edge of the sharp spike tip. Owing to the change in curvature of $S_1^\text{out}$, $S_2^\text{in}$ is pushed towards the forebody (frame $t+22\Delta t$ to $t+29\Delta t$). The charging of flow by VSTP continues until the height of SLI exceeds the value of D/2 (frame $t+24\Delta t$) from the axis. Once $S_2^\text{in}$ reaches near the forebody and SLI crosses the forebody height (D/2) with a zero slope, the trapped IRR is pushed back to the freestream. The recirculating region starts to convect (CRR) along the freestream direction, and $S_1^\text{out}$ can no longer be held at the sharp spike tip (Figure 13c). The withheld $S_1^\text{out}$ at the spike tip begins to collapse and starts accelerating towards the forebody as $S_1^\text{in}$. Thus, the pulsation cycle repeats. In this phase, the velocity gain profile of $S_1^\text{in}$ and $S_2^\text{in}$ is of a similar trend (Figure 13c). The value of $S_2^\text{in}$, however, reaches to higher velocities ($[u/U_\infty] \approx 1.2$ at $[x/D] \approx 1.4$) more than that of the freestream for a shorter duration since it has just crossed the expansion corner.

In the $x - t$ diagram of Figure 13, the three events of the pulsation cycle are marked based on the explanations mentioned earlier. In the ‘inflation’ zone, the trace of $S_1^\text{out}$ is seen. At ‘withhold,’ $S_1^\text{out}$ could be seen to stay at a constant place for some time, and during ‘collapse,’ $S_1^\text{in}$ is seen to accelerate towards the body. In the supplementary, a video of the high-speed shadowgraphy depicting the pulsation mode of unsteadiness captured over the flat-face spiked body configuration in supersonic flow is given for reference in the supplementary (‘video1.avi’).

Modal decomposition is performed on the obtained shadowgraph images using the procedure mention in Section 3. The dominant dynamic $[\Theta_1(x/D, y/D)]$ and energetic
[\Phi_1(x/D,y/D)] spatial mode are observed to be the same as shown in Figure 15a-(i) and 15b-(i). The dominant spatial modes have persistent spatial structures that are well correlated with the time averaged shadowgraph image (see Figure 11a). Such correlation is expected as the dynamic and energetic contents are extracted from the single time lagged matrix (see equation 3.4-3.5), by performing single value decomposition (see equation 3.8) and eigenvalue decomposition (see equation 3.10). In addition, the temporal coefficients of the dominant POD mode \((a_{\Phi_1(t/T)})\), reconstructed temporal signal from the DMD analysis \((a_{\Theta}(t/T))\), and the normalized unsteady pressure signal for the flat-face spiked body configuration \((P/P_\infty)\) are also shown to be closely matching in Figure 8d. Thus, the spatiotemporal modes from the modal analysis are verified and validated.

From Figure 15a-b, the spatiotemporal modes from DMD and POD analyses are very similar, except little grains in DMD spatial modes. The dynamic contents are sensitive to the spatial resolution and the image noise. Hence, the DMD spatial modes given in Figure 15a are grainy. However, POD spatial modes (Figure 15b) are energy based orthogonal projections. Hence, they are not sensitive to such temporal noises. For further discussions, only energetic spatial modes are used. The first spatial mode \((\Phi_1(x/D,y/D))\), represents the time-averaged shadowgraph image (see Figure 11a). The dominant temporal content \((a_{\Phi_1}(t/T))\) for \(\Phi_1(x/D,y/D)\) is observed at a fundamental frequency \((f_1[a_{\Phi_1}(t/T)])\) of 1548 Hz from Figure 15c-(i). There are at least three harmonics present \((f_1[a_{\Phi_1}(t/T)] = 3096 \text{ Hz} , f_2[a_{\Phi_1}(t/T)] = 4644 \text{ Hz} , f_3[a_{\Phi_1}(t/T)] = 6063 \text{ Hz})\), however, they have lower amplitudes. Also, the unsteady pressure spectra obtained at \([S/D]=0.25\) for the flat-face case in Figure 10b compares well with the above findings.

The analysis of the first four energetic spatial modes \((\Phi_1(x/D,y/D)\) to \(\Phi_4(x/D,y/D))\) gives information about the position of the shocks and its progression over a given time period (Figure 15). The modal analysis gives the correlation parameters based on the shock motion. The red and blue contour levels provide positive and negative correlations. By observing the color levels of \(\Phi_1(x/D,y/D)\), it can be seen that \(S1\) and \(S2\) are moving exactly in opposite directions. The actual direction could not be ascertained using a single mode, but the sense of relative direction between the shocks can always be identified. The first and last snapshot of the collapse phase in Figure 12a, \(S1\), and \(S2\) are observed at their extreme forward locations \((\text{S1}_{\text{out}}\) and \(\text{S2}_{\text{out}}\)). However, the spectra of the POD coefficients in Figure 15c-(i) contain the same frequencies as observed from the unsteady pressure measurements (Figure 10b). Thus the dominant energetic spatial mode could be inferred to represent the complete cycle itself. Both DMD (Figure 8c) and POD spectrum (Figure 15c-(i)), confirm the existing of a dominant spectral peak around 1550 Hz. In Figure 13d, from \(x-t\) diagram, the dominant event is also observed to be around 1550. While looking at Figure 13b, the corresponding spectral line on the \(x-t\) diagram is also found to be existing between the beginning and ending of a single pulsation cycle. Thus the dominant spatial mode \(\Phi_1(x/D,y/D)\) from POD analysis is pulsation.

In the second dominant energetic spatial mode \((\Phi_2(x/D,y/D))\), the terminal phase of ‘collapse’ is represented (Figure 15b-(ii)). The shock \(S1\) has refracted through \(S2\), and it might be approaching the flat-faced forebody. While looking at the temporal content, the first harmonic \((f_1[a_{\Phi_2}(t/T)] = 3096 \text{ Hz})\) is reaching an amplitude equivalent to that of the fundamental \((f_1[a_{\Phi_2}(t/T)] = 1548 \text{ Hz})\). For better understanding, the \(x-t\) diagram from the shadowgraph images (Figure 13) is sought. It can be seen that shock \(\text{S2}_{\text{out}}\) moves away from the body at a low speed, while the approaching shock \(\text{S1}_{\text{in}}\) comes at a higher velocity. These two shocks interact and refract at about \([x/D] \approx 0.7\), which is captured as the second dominant mode in Figure 15b-(ii). The shock refractions are attributed to the popping up of the first harmonic \((f_1)\) at twice the dominant frequency.
Shock related unsteadiness of axisymmetric spiked bodies in the supersonic flow

Figure 15. Comparison of the first four dominant (a) dynamic and (b) energetic spatial modes obtained from the DMD ($\Theta_{1-4}(x/D, y/D)$) and the POD ($\Phi_{1-4}(x/D, y/D)$) analysis for the flat spiked body configuration at a supersonic freestream Mach number ($M_\infty = 2.0$). Flow is from left to right. (c) Spectral analysis of the POD temporal coefficients ($a_{\Phi_{1-4}}(t)$) for $\Phi_{1-4}(x/D, y/D)$. (The spectra from the DMD analysis are given in Figure 8c)

$(f_f)$ in Figure 15c-(ii). Calarese & Hankey (1985) had also reported the occurrence of similar harmonics; however, it was not reasoned out clearly.

Similarly, the third dominant energetic spatial mode ($\Phi_3(x/D, y/D)$) picks up a stage during the phase of ‘inflation’ with toroidal vortex (TV) formation and the inflation of $S_{1_{out}}$ shock (Figure 15b-(iii)). In this case, the temporal mode yields a dominant frequency, which is the first harmonic ($f_1[a_{\Phi_3}(t/T)] = 3096$ Hz) as shown in Figure 15c-(iii). It makes sense, as the refracted shock yields the formation of a toroidal vortex system, and the $S_{2_{in}}$ shock completely changes its structure into a lambda shock system.

The fourth energetic spatial mode ($\Phi_4(x/D, y/D)$), represent the beginning and the terminal phase of ‘withhold’ (Figure 15b-(iv)). The shock is standing at the tip of the spike ($S1$) and the temporal mode ($a_{\Phi_4}(t/T)$) yields a spectral content equivalent of the first dominant energetic spatial mode ($\Phi_1(x/D, y/D)$) as shown in Figure 15c-(iv). The other energetic spatiotemporal modes, carry only minimal fluctuations about the mean. Also, from the energy contents given in Figure 8a-b, it can be seen that the first four modes only carry most of the energy contents ($>30\%$). Hence, we consider them to be sufficient to represent the entire flow field.

Thus, the present analysis confirms most of the findings from the previous investigation by Feszty et al. (2004a) through a $x - t$ diagram and provided some more insights about the unsteady spatiotemporal modes. Consequently, the present investigation methodology can be used in the next cases, to access the flow physics associated with shock unsteadiness.
5.3.2. **Strong shock interactions in hemispherical spiked body configuration**

Looking at the spectra of Figure 10c, we notice that the discrete frequencies associated with the flat-face spiked forebody have vanished for the hemispherical spiked forebody configuration under the same freestream flow conditions. This observation is consistent with the previous reports of Mair (1952) and Maull (1960). However, the underlying reason behind the broadband spectra of Figure 10c has not been clearly explained. In the present study, we have already shown that the reduction of the pressure loading ($\zeta$) in the hemispherical spiked body configuration is significant due to the change in the geometrical shape factor ($\xi$).

In order to explain the accompanied reduction of the pressure fluctuation intensity ($\kappa$) (Table 3), time-resolved shadowgraph images are analyzed qualitatively. The results are presented in Figure 16 which includes both the time-averaged (Figure 16a) and instantaneous (Figure 16b) shadowgraphy. From Figure 16a, weak leading-edge shock, separation shock, shear layer, and reattachment shock around the forebody are observed. The flow history represented in Figure 16b through instantaneous snapshots indicates the presence of shock-related unsteadiness, especially at the point of shock separation (SP) on the spike stem and the reattachment point (RP) near the forebody shoulder.

The evolution of large scale structures (KH instabilities) from the SP and the movement of the RS (see Figure 16b) resemble the charging and ejection of fluid mass from the recirculation bubble as seen in the case of a simple forward-facing step (Estruch-Samper & Chandola 2018) in a supersonic flow. It is also similar to the shock-wave turbulent boundary layer interaction (SWTBLI) problems such as those induced by compression ramps, reflected shocks, protrusions, and fins (Clemens & Narayanaswamy 2014). Knowledge of these canonical forms of SWTBLI flow problems gives better insights to the present case. These events are highly non-linear and broadband in nature due to multi-scales involved in the unsteady process, which further explains the broadband signal seen in Figure 10c. In addition, the majority of the above-said SWTBLI problems arise due to either the upstream influence (Plotkin 1975; Andreopoulos & Muck 1987; Brusniak & Dolling 1994) or the downstream influence (Pirozzoli & Grasso 2006; Piponniau et al. 2009). For the upstream influence to be dominant, the incoming boundary layer should be of a considerable thickness to influence the oscillation of the separation shock foot. The downstream influence is attributed to the recirculation bubble dynamics, which drive the oscillation of both the separation and reattachment shock foot (see Figure 1 of Clemens & Narayanaswamy (2014)).

Very recently, in the paper of Estruch-Samper & Chandola (2018), the authors have shown that in the problem of SWTBLI interactions, the free shear layer drives the shock-related unsteadiness. They have shown that the volume inside the recirculation region increases once the recirculation bubble is charged with the freestream fluid. Therefore, it achieves a pressure value that is sufficiently larger than the static pressure existing behind the RS. At this point, the authors have shown, using their unsteady pressure measurement data, that the RS is pushed upstream of the body, and the SS is pushed towards the body. At this moment, the excess fluid mass inside the recirculation bubble is reportedly ejected to the freestream. However, for the range of protrusion height used in their experiments, they have concluded that the free shear layer thickness increases with protrusion height for a given flow condition. The longer separation length ($L_S$) leads to the formation of a thicker free shear layer near the RP, and it has been concluded that such a free shear layer interaction drives the shock-related unsteadiness. Also, in their study, the authors realized that the separation angle ($\lambda$) associated with $L_S$ does not change with protrusion height.
Figure 16. (a) Time-averaged shadowgraph image; (b) Instantaneous shadowgraph images at different time intervals showing the unsteady motion of the shock wave in a spiked hemispherical body configuration at a supersonic freestream Mach number ($M_\infty = 2.0$). Flow is from left to right. Some dominant features: 1. Weak-leading edge shock, 2. Separation shock (SS), 3. Separated shear layer, 4. Recirculation bubble, 5. Reattachment shock (RS); SP - Separation point, KH - Kelvin-Helmholtz structures, SKL - Shocklets, RP - Reattachment point, $M_E$ - Ejected mass. (Corresponding video file is available in the supplementary under the name ‘video2.avi’).
Figure 17. Simple sectional image analysis routines performed for the hemispherical spiked body configuration with a sharp spike-tip at a supersonic freestream Mach number ($M_\infty = 2.0$). (a) Yellow-line represents the considered section in a typical instantaneous image; (b) Time evolution of the sectional intensity scan revealing the dominant features in the flow — 1. Weak leading-edge shock in-front of the sharp spike-tip, 2. Separated shock, 3. Oscillating reattachment shock near the hemispherical forebody; (c) Spectral analysis (normalized) of the sectional images evolving in time ($x-t$ diagram) revealing the dominant frequency contents in the flow. Flow is from left to right.

In our present study, the boundary layer thickness until the SP over the spike stem, is very thin for $[l/D] = 1$. Consequently, it does not provide any significant upstream influence on the shock dynamics. Hence, the only phenomena that might be driving the shock-related unsteadiness are the downstream influence. As shown in Figure 16b, a detached shock pulse or shocklet (SKL) is seen to be generated near the SS. These SKLs propagate downstream along with the large scale (KH) structures from SP to RP and interacts with the reattachment shock (RS), severely. During the formation and convection of the large scale structures, part of the freestream fluid mass is entrained into the recirculation bubble. As the large scale structures convect along with the separated shear layer from the SP to the RP, they also grow in size. These structures transport the
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Figure 18. Instantaneous shadowgraph images at different time intervals captured at a frame rate of $f_s = 140470$ Hz showing mass ejection across the oscillating reattachment shock of a spiked hemispherical body configuration at a supersonic freestream Mach number ($M_\infty = 2.0$). Golden-yellow circle is shown to track the movement of a fluid mass ejecting out. Flow is from left to right. Dominant flow features: 1. Kelvin-Helmholtz (KH) structures, 2. Separation shock, 3. Reattachment shock. (Corresponding video files are available in the supplementary under the name ‘video4.avi’ and ‘video5.avi’)

mass, momentum, and energy of the flow from the freestream to the recirculation zone and form a visible axisymmetric cone around the spike stem.

The SKLs are carried along with the large scale structures (as seen in Figure 16b) just like the wavefronts carried by the structures in the convecting free shear layer of a supersonic jet (Papamoschou 1995; Lee et al. 1991). Feet of SKLs are inside the separation bubble, and it cannot be seen in the present images due to the line of sight integration. However, it can be seen from the video provided in the supplementary (‘video3.avi’), which is based on our accompanied numerical studies†. When the large scale structures along with the SKLs hit the RS, the foot of the SKL hits the forebody and reflects forth and back. When another SKL comes in contact with the wall or the reflected SKL from the wall, the velocity of the reflecting and refracting SKLs changes drastically. At this point, the large scale structures also hit the RS and thus, displacing the shock slightly. During such an event, the charged fluid mass from the recirculation bubble is ejected ($M_E$) to the freestream. In the meantime, reflected waves from the forebody wall further propagate upstream towards the SP. While doing so, the reflected wave interacts with the incoming SKLs and perturbs the SP. A perturbed SP produces a large scale structure that gets amplified then due to the KH instability present in the free shear layer. Thus, the unsteadiness is maintained.

A simple $x-t$ diagram reveals many of the unsteady events happening in the hemispherical spiked body configuration (see Figure 17). The line along which the shock footprints are obtained with respect to time is marked in Figure 17a. The shock footprints are shown in Figure 17b and the spectral analysis of Figure 17b is given in Figure 17c. Weak leading edge shock location, SP, and RP are seen in Figure 17b. The motion of SKLs, along with the large scale structures, are seen as inclined lines with a slope of $[1/u_c]$. The slope of the line directly provides the convective speed ($u_c$), and information about the frequencies of the SKLs carried along with the large scale structures. Also, at the SP,

† The DES results from the numerical studies are given in a qualitative manner here, just to appreciate the observations from the present analysis. Details about the computation methods are beyond the scope of the current paper and hence, it is not discussed at this point. Some vital information about the numerical studies are given in the video caption itself.
the fluctuation of the shock footprint is minimal. Whereas, it becomes substantial near the RP (region-3 marked with yellow dashed lines in Figure 17b). The reason is due to the impingement of the growing large scale structures near the RS. Performing spectral analysis on the shock footprint reveals the presence of broadband spectra near the SP and RP (Figure 17c).

The charging and ejection of fluid mass are also seen in the recirculation region represented in the sequence of shadowgraph images captured at a higher frame rate ($f_s = 140740$ Hz), as shown in Figure 18. A small semi-transparent golden-yellow circle is marked around a turbulent structure that is initially inside the recirculation region. As time progresses, the turbulent structure is ejected out into the freestream flow. The RS is pushed ahead of the forebody, and the SS is slightly moved towards the body in the opposite direction (see also Estruch-Samper & Chandola (2018)). Also, the large-scale structures produced along with the shear layer due to the KH instability are seen in the second frame of Figure 18. Findings from the $x - t$ plots given in Figure 17 are also consistent with the above statements. Videos showing these phenomena at a frame rate of both $f_s = 43000$ (‘video2.avi’) Hz and $f_s = 140740$ (‘video5.avi’) Hz are given for reference in the supplementary.
The movement of the separation and reattachment shock in the opposite phase during the charging-ejection sequence is shown more clearly in Figure 19a. The shocks system are seen to be moving in the opposite phase. The dashed and solid lines mark the positions of the separation and reattachment shocks on the forebody, respectively. The red and green color of the dashed/solid lines marks the initial and final location of the respective shocks during the time of charging (Figure 19a-I) and ejection (Figure 19a-II). At the time of charging, the reattachment shock is attached to the forebody, and during mass ejection, it is observed to be detached. The actual magnitude and motion of the shock system in the opposite phase are further discussed in the upcoming section.

From the POD analyses, the dominant spatial mode ($\Phi_1(x/D, y/D)$) and the corresponding temporal coefficient ($a_{\Phi_1}(t/T)$) spectrum are extracted as shown in Figure 20. In Figure 20a, $\Phi_1(x, y)$ shows that the oscillations of the separation shock and the reattachment shock are out-of-phase motion with respect to each other. The alternating color contours in the separation shock (blue-red), and the reattachment shock (red-blue) visualizes the opposite phase of the shocks motion. A clear representation is given later in Figure 26a, where the magnitude of such out-of-phase motion is marked. The observation of shocks system moving in opposite phase from $\Phi_1(x/D, y/D)$, in turn, supports the existence of recirculation bubble dynamics with charging and ejection mechanism as proposed by Estruch-Samper & Chandola (2018) in their studies on the axisymmetric protuberance at supersonic speeds. From Figure 8a-b, it is also shown that there is no single dominant mode that constitutes a larger energy content. From Figure 8b, it is observed that the cumulative energy contained in each of the modes does not vary linearly (on a log scale) for the hemispherical case, unlike the flat-face spiked configuration.

Hence, it requires at least $\sim 70$ modes to represent $\sim 30\%$ of the energy in the flow, unlike $\sim 4$ modes for the flat-face case. The presence of a higher number of spatial modes to represent 30\% of energy contents emphasize the presence of turbulent structures at different scales (broadband). The spectral characteristics from $a_{\Phi_1}(t/T)$ (see Figure 20b) is also consistent with the findings from the unsteady pressure measurement in Figure 10c (blue color line showing pressure spectra at $S/D=0.4$) and also with the dynamic contents computed from the DMD analysis (see orange colour line in Figure 8b). In Figure 10c,
Figure 21. (a) Time-averaged shadowgraph image; (b) Instantaneous shadowgraph images at different time intervals showing the reduction in unsteady motion of the shock wave in a spiked elliptical body configuration at a supersonic freestream Mach number ($M_\infty = 2.0$). Dominant features: 1. Weak leading edge shock, 2. Separation shock, 3. Shear layer, 4. Recirculation region, 5. Reattachment shock, SS - Weakly oscillating separation shock, SP - Separation point, SKL - Weak shocklets, KH - Free shear layer with Kelvin-Helmholtz structures, RP - Free shear layer reattachment point on the forebody, RS - Mildly oscillating reattachment shock, $M_E$ - Ejected mass. Flow is from left to right. (Corresponding video file is available in the supplementary under the name ‘video6.avi’)

The shear layer frequencies are about 4000-7000 Hz. However, in Figure 8b, the peak is at 1000 Hz. This is because we show the dominant mode in Figure 8b. Asides $\Phi_1(x/D, y/D)$, the other spatial modes represent only the changes coming from the asymmetry of flow separation about the axis on the spike stem. Asymmetry is unavoidable due to the non-linear nature of the shocks system and the presence of helical modes, as seen in the previous work of Mair (1952). However, $\Phi_1(x/D, y/D)$ is not having any asymmetric effect because the dominant spatial mode is driven purely by the shocks motion.
Figure 22. Schematic showing the different parameters and their respective values like the deflection angle ($\theta$), shock angle ($\beta$) and Mach number ($M$) in certain regions of the flow as calculated from the gas dynamic tables for an axisymmetric cone at a supersonic freestream Mach number ($M_\infty = 2.0$). Flow is from left to right. Flow region: 1. freestream, 2. before separation shock, 3. after separation shock, 4. recirculation region.

5.3.3. Weak shock interactions in elliptical spiked body configuration

Until now, the decrement in $\zeta$ and $\kappa$ while reducing $\xi$ of the spiked forebody is reasoned. We have also shown from the unsteady pressure spectra (Figure 10b-c) that the hemispherical spiked body ($\xi \approx 0.92$) exhibits reduced unsteadiness in comparison to the flat-face. For the most streamlined spiked forebody configuration like the elliptical case ($\xi \approx 0.84$), unsteadiness is even lesser as evident from the spectrum (Figure 10d) and the smaller values of $\zeta$ and $\kappa$ (Table 3). However, the gross flow field looks similar between the hemispherical and the elliptical spiked forebody configurations, as shown in Figure 9b-c. In this section, when the forebody shape ($\xi$) changes from hemispherical to elliptical, the reason behind the damping of shock-related unsteadiness is investigated.

Figure 21 shows both the time-averaged (Figure 21a) and the instantaneous (Figure 21b) shadowgraph images taken during the steady test time. In the time-averaged shadowgraph images, the weak leading-edge shock, the separation shock (SS), the free shear layer, and the reattachment shock (RS) are seen clearly. In the instantaneous shadowgraph image series, the separation point (SP), weakly oscillating SS, weak shocklets (SKLs), free shear layer with large scale structures (KH instabilities), reattachment point (RP), and the weakly oscillating reattachment shock are seen. The term weak is used in comparison to the observations made in case of the spiked hemispherical forebody using shadowgraphy. In the supplementary, a high-speed shadowgraphy video of the spiked elliptical forebody configuration is given for reference (‘video6.avi’).

The time-averaged shadowgraph image in Figure 16a and Figure 19a show that the location of RP is observed at a different height ($y_{RP}$) for each of the cases with different reattachment length ($L$). However, the separation length ($L_S$) and $M_\infty$ have been the same for both the cases. In the elliptical case, RP shifts further downstream, thereby increasing $y_{RP}$ and $L$, in comparison to the hemispherical one. The values of $y_{RP}$ for both the cases are also in agreement with the empirical relationship given by Moeckel (1951) (see equation 18 in the reference). In the elliptical case, as $L$ increases, the cone angle ($\lambda$) that constitutes the recirculation zone decreases. Reduction in $\lambda$ leads to a lower separation shock angle ($\beta$) and a higher Mach number ($M_3$) behind the separation shock. For the same flow conditions in the hemispherical case, $M_3$ is therefore lesser due to higher $\lambda$. All the relevant variables are calculated using the simple gas dynamic relationships for supersonic flow past an axisymmetric cone at zero angle of attack (Sims 1964), as shown in Figure 22.

In an attempt to clarify the difference between the hemispherical and elliptical spiked forebodies associated unsteadiness, the recirculation zone (marked as region 4 in Figure 22) is assumed as a dead air region, and the air velocity is approximated as zero.
Figure 23. A schematic (top) and an instantaneous shadowgraph image (bottom) qualitatively showing the reduction in the large scale structures (KH instabilities) while changing the spiked forebody geometry from (a) hemispherical to (b) elliptical at a supersonic freestream Mach number ($M_{\infty} = 2.0$). Flow is from left to right.

The convective Mach number ($M_C$) is obtained by averaging the Mach numbers of the two streams across the free shear layer. For the elliptical case, $M_C$ is higher due to a larger $M_3$ in comparison to the hemispherical one. The growth rate of a compressible free shear layer thickness was also shown to increase when $M_C$ decreases (Slessor et al. 2000). Therefore, in the case of the elliptical spiked body, the growth rate of the free shear layer and the size of the roller structures are comparatively lower. Thus, $\lambda$, $L$, $M_3$, and $M_{\infty}$ are the important parameters influencing the intensity of shock-related unsteadiness in both the configurations.

In Figure 19b, the separation shock foot motion and the extremes of the reattachment shock oscillation are given qualitatively for the elliptical spiked body configuration. The shock strength in the elliptical case (see Figure 19b) is found to be weaker in comparison with the hemispherical one (Figure 19a). Looking at the intensity of shock motion in both cases, as shown in Figure 19, the unsteadiness is observed to be damped considerably in the elliptical case. As the gross flow features between the hemispherical and the elliptical cases are similar, charging and ejection of fluid mass from the recirculation region is also seen in the present case. However, from the schematics drawn in Figure 23 and the video given in the supplementary (‘video6.avi’) for the elliptical case, the size of the large scale structures are observed to be smaller in the free shear layer, and are expected to have higher frequency contents.

A simple shock footprint analysis using the $x - t$ diagram, as shown in Figure 24, sheds some more light on the flow physics in the elliptical case. The temporal footprint of the weak leading edge shock, SS and RS, are shown in Figure 24b. As shown in the previous case (Figure 17b), SKLs are also present in the current case between $[x/D] = 0.5$ and 1.0 (see Figure 17b). However, they are weak, and hence, a distinct trace of SKLs in the elliptical case is not present. Weak SKLs in the elliptical case result in weaker reflection from the model forebody. Hence, they exert only smaller perturbations to the incoming SKLs and SP. Smaller disturbance in SP is not amplified as much as it is in the hemispherical case, which leads to the production of relatively finer KH structures. These
structures comparatively grow at a lower rate when they make contact with the forebody at RP, which causes weaker oscillations of the RS. Hence, near the reattachment zone (region-3) marked in Figure 24b, the relative unsteadiness of the RS footprint is confined only to a distance of $[x/D] \sim 0.15$ (the distance between the yellow dashed vertical lines) for the elliptical case. However, the RS footprint fills up the space of $[x/D] \sim 0.25$ for the hemispherical one (see Figure 17b). The spectrum of Figure 24b, given in Figure 24c, is a broadband one just like the hemispherical case near the reattachment point. The spectrum of the unsteady pressure measurements shown in Figure 10d is also in agreement with the present findings.

In Figure 25, the dominant energetic spatial ($\Phi_1(x/D, y/D)$) and temporal modes for
the elliptical spiked body configuration is shown. $\Phi_1(x/D, y/D)$ is well correlated to the time-averaged image shown in Figure 21a. From Figure 8a, it requires at least 100 modes to represent $\sim 30\%$ of the energy contents in the flow field, which is comparatively higher than the hemispherical case ($\sim 70$ modes). The requirement for a larger number of modes in the elliptical case is a result of the reduction in KH scales in comparison to the hemispherical one. Hence, a broadband spectrum for a wider range of frequencies is seen in the temporal mode of Figure 25b. The unsteady pressure spectrum for the elliptical case shown in Figure 10d (blue line) is also in agreement with the above findings. The amplitude around $\sim 1000$ kHz is reduced relatively between the hemispherical and the elliptical cases (see blue lines in Figure 10c-d). The relative drop in amplitude suggests the presence of lower amplitude higher-frequency events in the free shear layer for the elliptical case. The corresponding DMD spectra ($a_\Theta(t/T)$) also show a slight drop in comparison with the hemisphere, confirming the presence of smaller scales (see Figure 8c). The time-averaged wall-static pressure coefficients (see Figure 10a) and the $x - t$ diagram (Figure 24) have already shown to be in agreement with the above statements. The difference in the intensity and amplitude of shock oscillation between the hemispherical and the elliptical spiked body configurations can be seen more clearly from the respective $\Phi_1(x/D, y/D)$ as shown in Figure 26. It shows the existence of the out-of-phase shocks motion in both cases; however, at a different strength. Such a behavior supports the charging and ejection of fluid mass associated with the recirculation region. The amplitude of shock oscillation decreases by almost 50% while changing the forebody shape ($\xi$) from the hemisphere to ellipse, as seen in Figure 26.

5.4. On the control of shock related unsteadiness in the spiked body configuration

So far, in the present research, the types of shock-related unsteadiness for different spiked forebody configurations have been explored. It has been shown that for a flat-face spiked forebody ($\xi \approx 1$) with $l/D = 1$ at $M_\infty = 2.0$, the shock-related unsteadiness is through the mode of pulsation. In the hemispherical forebody configuration ($\xi \approx 0.92$), the unsteadiness is driven by the separated free shear layer and localized shock oscillations. The cone angle ($\lambda$) of the recirculation region and the out-of-phase shocks motion along with the shocklets affect the size of the large scale structures (KH-instabilities)
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Figure 26. Dominant energetic spatial modes obtained from the POD analysis ($\Phi_1(x/D, y/D)$) for (a) the hemispherical and (b) elliptical spiked forebody configurations showing the difference in amplitude of out-of-phase shock oscillation at a freestream supersonic Mach number ($M_\infty = 2.0$). Flow is from left to right. Dominant features: 1. Separation shock (SS), 2. Reattachment shock (RS), 3. Separated free shear layer.

Figure 27. (a) Instantaneous shadowgraph image and (b) Time-averaged shadowgraph imaging of $|\bar{R} - R_{rms}|$ showing the change in flow features observed around a modified spiked body configuration with a hemispherical spike tip at a supersonic flow Mach number ($M_\infty = 2.0$). Flow is from left to right; Dominant flow features: 1. Detached shock ahead of the hemispherical spike tip, 2. Separation point (SP, with no separation shock), 3. Recirculation region (with larger volume), 4. Reattachment shock (RS). (Corresponding video file is available in the supplementary under the name ‘video7.avi’)

in the free shear layer. The magnitude of the shock oscillation at the SP and RP are significant. For the elliptical spiked forebody ($\xi \approx 0.84$), the mode of unsteadiness is similar to that of the previous case. However, the free shear layer is seen to have finer structures owing to lower $\lambda$ and less intense out-of-phase shock motions along with weaker shocklets. It results in the observation of finer large scale structures near the RP, and hence, the shock oscillation magnitude in particular at the RP is reduced, comparatively.

The present findings can be used to develop a spiked body configuration with a reduced level of shock-related unsteadiness. If the size of the large scale structures in the free shear layer could be brought down, the hemispherical spiked body configuration would itself be
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Figure 28. (a) Pressure coefficient variation along forebody curvature length \([S/D]\) for three different hemispherical forebody configurations (clean or no-spike: filled black squares, sharp spike tip: filled orange squares, hemispherical spike tip: filled blue squares) obtained through time-averaged wall-static pressure measurements; (b) Power spectral density obtained from the unsteady pressure measurement near the forebody location of the hemisphere mounted with different spiked tip configurations (hemisphere with sharp spike tip: solid orange line, hemisphere with hemispherical spike tip: solid blue line) at a supersonic freestream Mach number \((M_\infty = 2.0)\).

a right candidate for practical aerodynamic purposes. Hence, experiments are conducted on the existing hemispherical spiked body with a single modification: the sharp spike tip is changed into a hemispherical dome. The diameter of the hemispherical dome is twice the spike stem \((d)\). A typical instantaneous shadowgraph image and the operator based time-averaged shadowgraph image \((\bar{R} - R_{rms})\) are shown in Figure 27. Unlike the previous cases where the unsteady SS is prevalent at the SP, in the present case, the SS is very weak or eliminated. There is only a strong bow shock in-front of the hemispherical spike tip. Furthermore, from Figure 27, it seems that the gradient of the gray levels between the free shear layer and its surrounding is much reduced in comparison with that of the sharp-tip spiked-forebodies (Figure 9b-c). Darker levels suggest stronger activities of the large scale structures along the free shear layer. In the supplementary, a high-speed shadowgraphy video of the modified spike tip on a hemispherical forebody is provided (‘video7.avi’).

Mean wall-static pressure coefficients and unsteady pressure measurements are carried out on the modified hemispherical spike tip configuration to ascertain the shock dynamics (see Figure 28). The results are compared against the sharp spike tip case with the same hemispherical forebody. The peak wall static pressure coefficient (Figure 28a) is smaller and slightly shifted downstream for the hemispherical spike tip in comparison with the sharp one. Such behavior is purely due to the elevation of the separated free shear layer and the consequence of lowering \(\lambda\). The spectrum from the unsteady pressure signal obtained at \([S/D] = 0.4\) (blue line in Figure 28b), shows a significant drop in amplitude across almost the entire range of frequencies in comparison to the sharp spike tip case (orange line in Figure 28b). The reduction in amplitude, especially at the mid-frequency range, could be attributed to the fact that the large scale structures are finer in the separated free shear layer.
Figure 29. Instantaneous shadowgraph images of sharp spike tip, and hemispherical spike tip forebodies, showing the unsteadiness and shocks system that are formed at a supersonic freestream Mach number \( M_\infty = 2.0 \). Flow is from left to right. (a) Sharp spike-hemispherical forebody: \( c_d=0.52, \lambda=22^\circ \), (b) Sharp spike-elliptical forebody: \( c_d=0.42, \lambda=17^\circ \), (c) Hemispherical spike-hemispherical forebody: \( c_d=0.36, \lambda=17^\circ \), (d) Graph showing the variation of the pressure loading (\( \zeta \)) and pressure fluctuation intensity (\( \kappa \)) parameters for different forebodies with no spike (\( \zeta \pm 4\kappa \)), sharp spike (\( \zeta \pm 2\kappa \)), and hemispherical spike (\( \zeta \pm \kappa \)), (e) Graph showing variation of the pressure loading (\( \zeta \)) and pressure fluctuation intensity (\( \kappa \)) parameter for different forebody locations with sharp spike (\( \zeta \pm 2\kappa \)), and hemispherical spike (\( \zeta \pm 2\kappa \)).

Figure 29a-c show an instantaneous shadowgraph image for the hemispherical and elliptical forebody mounted with a sharp spike tip and a hemispherical forebody mounted with a hemispherical spike tip. In the images, the values of \( c_d \) and \( \lambda \) are added. In Figure 29d, values of \( \zeta \) and \( \kappa \) (marked as vertical error bars) for the above mentioned cases are plotted. Variation of \( \zeta \) and \( \kappa \) along the forebody surface at three different \([S/D]\) is shown in Figure 29e for the sharp and hemispherical spike tip cases with the hemispherical forebody. The shielding of the hemispherical forebody by the hemispherical spike tip from the freestream flow through the stronger bow shock reduces \( c_d \) by 10%. By monitoring \( \zeta \), as shown in Figure 29d-e, the modified spiked body exhibits a reduction in \( \zeta \) by 42% in comparison to the hemispherical spiked body with a sharp spike tip. Similarly, a reduction of \( \kappa \) from 11% to 10% is also observed for the case of the hemispherical spike tip comparison to the sharp spike case.

Just like in the previous cases, shock footprint analysis through a \( x - t \) diagram has been carried out to validate the findings (see Figure 30). From the analysis, the bow shock in-front of the hemispherical spike tip and the SP of the free shear layer are observed to be fairly steady. In the reattachment zone, the unsteadiness in the shock motion is contained...
Figure 30. Simple sectional image analysis routines performed for the hemispherical spiked body configuration with a hemispherical spike-tip at a supersonic freestream Mach number ($M_\infty = 2.0$). Flow is from left to right. (a) Yellow-line represents the considered section in a typical instantaneous shadowgraph image; (b) Time evolution of the sectional intensity scan revealing the dominant features in the flow: 1. Bow shock in-front of the hemispherical spike-tip, 2. Separated shear layer from the flat base of the hemispherical spike-tip, 3. Oscillating reattachment shock (RS) near the hemispherical forebody; (c) Spectral analysis (normalized) of the sectional images evolving in time ($x-t$ diagram) revealing the dominant frequency contents in the flow.

within $[x/D] \sim 0.12$ which is of the same order as that of the elliptical case ($[x/D] \sim 0.15$) and significantly smaller than the hemispherical case ($[x/D] \sim 0.25$) (see also Figures 17 and 24). The spectrum of the $x-t$ diagram (Figure 30b) is shown in Figure 30c, and it shows no dominant frequency at the point of free shear layer separation. Similarly, near the reattachment zone, a low amplitude broadband spectrum is observed. A simple modal analysis reveals that the dominant energetic spatial mode ($\Phi_1(x/D, y/D)$) shows variation only near the RS (see Figure 31a) and there are no distinct events near the SP. Also, the alternating color contours near the bow shock and SP are absent. Thus, it can be concluded that there is no out-of-phase shock motion, unlike in the previous
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Figure 31. (a) Dominant energetic spatial mode ($\Phi_1(x/D, y/D)$), and (b) the normalized spectrum of the temporal modes ($a_{\Phi_1}(t/T)$) obtained from the POD analysis of the instantaneous shadowgraph images. The configuration under study is the hemispherical forebody mounted with the hemispherical spike tip at a supersonic freestream Mach number ($M_\infty = 2.0$). Flow is from left to right.

cases (see Figure 26). The spectrum from the POD temporal coefficients ($a_{\Phi_1}(t/T)$) also reveal a broadband spectrum (see Figure 31b).

6. Conclusions

Experiments are conducted in a supersonic wind tunnel at a freestream Mach number of 2.0 to study the flow field around three representative spiked forebody configurations. The geometries are selected based upon the forebody geometrical shape factor ($\xi$): (a) Flat-face ($\xi \approx 1$), (b) hemispherical ($\xi \approx 0.92$), and (c) elliptical ($\xi \approx 0.84$) spiked forebody configurations. The flow physics that drives the shock-related unsteadiness is explored for the considered configurations in detail. Time-resolved short exposure shadowgraph images are captured to infer qualitative and quantitative details. Both steady and unsteady pressure measurements are carried out to investigate the flow further. A thorough shock-foot print analysis is executed using the $x-t$ plots to access the intensity and time scales of the shock-related unsteadiness. Modal analysis is performed on the time-resolved shadowgraph images to identify the dominant spatiotemporal modes in each case using Proper Orthogonal Decomposition (POD) and Dynamic Mode Decomposition (DMD) techniques.

Following are the major conclusions from the present study:

(i) In case of the flat-face spiked body configuration, for a given spike length to forebody diameter ratio ($[l/D] = 1$), the well known pulsation mode of unsteadiness (Feszty et al. 2004a) is observed (together with its weaker harmonics) which produce a significant pressure loading ($\zeta$) and pressure fluctuation intensity ($\kappa$) on the forebody. The ‘pulsation’ cycle consisting of three phases: collapse, inflation, and withhold, is described in details, using the $x-t$ diagram and modal analysis. The spatiotemporal modes from the modal analysis are captured for the first time using shadowgraph images, and they agree well with the previous findings.

(ii) In the case of the hemispherical spiked body configuration, the intensity of the shock-related unsteadiness is observed to be lower in comparison with the flat-face case
due to the absence of a dominant frequency. Shock unsteadiness, in this case, is dictated only by the separated free shear layer and the recirculation region. Shock footprint analysis through the $x - t$ diagram and modal analysis provide the magnitude of the shock oscillation near the separation (SP) and reattachment (RP) points. The charging and ejection of fluid mass from the recirculation region, coupled with the out-of-phase shocks motion, is the driving flow physics.

(iii) In the case of the elliptical spiked body configuration, the intensity of shock-related unsteadiness is observed to be the least. Nevertheless, the flow physics is found to be similar to that of the hemispherical case. The dampening of shock-related unsteadiness in this particular case is probed further through $x - t$ plots and modal analysis. The cone angle ($\lambda$) of the recirculation region is found to play an essential role in the unsteady shock dynamics. Values of $\lambda$ are lower in the elliptical case than the hemispherical one.

(iv) The flow physics: As $\lambda$ increases, the shocklets along the free shear layer become stronger. Stronger shocklets undergo multiple reflections and refractions near the wall. The resulted disturbances are deflected upstream by traveling inside the recirculation region and perturb the SS. The perturbed SS oscillates and further disrupts the separated free shear layer. Later, the disruptions get amplified due to the presence of KH instability in the separated free shear layer itself, and large scale structures are formed. Further downstream, they grow in size and impinge at the RP. The impingement perturbs the RS, and it oscillates with a magnitude proportional to the size of the large scale structures. The oscillation of SS and RS is observed to be out-of-phase. During this out-of-phase shock motion, the charged fluid mass by the large scale structures inside the recirculation region gets ejected back to the freestream along the RP.

(v) From the studies of three different forebody shapes, lower $\xi$, absence of out-of-phase shock motion, and lower $\lambda$ are found to be the desired characteristics to have the least level of shock-related unsteadiness. The existing spiked hemispherical forebody is modified by changing the sharp spike tip to a hemispherical spike tip. Through the same experimental methodology, it has been shown that the modified configuration has met the fore-mentioned criteria and shown to exhibit relatively a lower level of shock-related unsteadiness.

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