Poincaré covariant light-front spectral function
and transverse momentum distributions

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Abstract. In valence approximation the fermion correlator is simply
related to the light-front spectral function. Then the leading twist time-
reversal even transverse momentum distributions can be explicitly ob-
tained from the light-front wave function of the system and the twist-
three distributions are linear combinations of the transverse distributions
at leading twist.

Keywords: Poincaré covariance, light-front Hamiltonian dynamics, trans-
verse momentum distributions

1 Introduction

Transverse momentum distributions (TMDs) are a powerful tool to study hadron
structure [1]. Light-cone models have been used to study the three-dimensional
hadron structure, to disentangle contributions from different angular momentum
components and to investigate possible relations among the TMDs, with the aim
to offer a guide for the extraction of TMDs from experimental data [2,3].

In this paper a Poincaré covariant, light-front (LF) spin-dependent spectral
function is considered to investigate hadrons within the LF Hamiltonian dynam-
ics in valence approximation. We present both the most general expression for
the spin-dependent momentum distribution in terms of six scalar functions and a
linear relation between the LF spectral function and the fermion correlator. This
link implies approximate relations between the six time-reversal even (T-even)
TMDs, as well relations between the leading twist and the twist-three TMDs [4].

The LF spectral function, \( \mathcal{P}^\tau_{\mathcal{M},\sigma',\sigma}(\bar{\kappa}, \epsilon, S) \), was defined in Ref. [5], starting
from the LF wave function for a three-body system with spin 1/2, third com-
ponent \( \mathcal{M} \) and polarization vector \( S \). The energy \( \epsilon \) is the energy of a fully interacting
two-particle [23] subsystem and the variable \( \bar{\kappa} = (\kappa^\tau, \kappa_{\perp}) \) is the LF momentum
for particle 1 in the intrinsic reference frame of the cluster [1,(23)]. The spectral
function is defined through the overlaps between the LF wave function of the
system and the tensor product of a plane wave of momentum \( \kappa \) and the state
which describes the intrinsic motion of the two-particle spectator subsystem.
The mentioned tensor product allows one to take care of macrocausality and to introduce a new effect of binding in the spectral function. The LF spectral function, through the Bakamjian-Thomas construction of the Poincaré generators [6], allows one to embed the successful phenomenology for few-nucleon systems in a Poincaré covariant framework and to satisfy at the same time normalization and momentum sum rule. As a first test of our approach the EMC effect for $^3\text{He}$ is being evaluated. Preliminary results show encouraging improvements with respect to a convolution approach with a momentum distribution [7].

2 Light-Front spin-dependent spectral function and transverse momentum distributions

2.1 Spin-dependent momentum distribution

Integration of the LF spectral function on the intrinsic energy $\epsilon$ of the $(A - 1)$ system, gives the LF spin-dependent momentum distribution [5]

$$n_{\sigma'\sigma}(x, k_{\perp}; M, S) = \sum \int \frac{d\epsilon}{2(2\pi)^3} \frac{1}{1-x} \frac{E_S}{\kappa^+} \mathcal{P}_{M,\sigma'\sigma}(\kappa, \epsilon, S)$$

(1)

where $\kappa^+ = xM_0[1, (23)]$, with $M_0[1, (23)]$ the free mass of the cluster [1, (23)], and $E_S = \sqrt{4m^2 + 4mc + |\kappa|^2}$. Within the LF approach, the momentum distribution can be expressed through the three available independent vectors: i) the polarization vector $S$; ii) the unit vector $\hat{n}$ (identified with $\hat{z}$) which defines the $\pm$ LF components, $v^\pm = v^0 \pm \hat{n} \cdot \mathbf{v}$, and iii) the transverse (with respect to the $z$ axis) momentum component $k_\perp = p_\perp = \kappa_\perp$ of the momentum $p$

$$n_{\sigma'\sigma}(x, k_{\perp}; M, S) = \frac{1}{2} \{b_{0,M} + \sigma \cdot f_M(x, k_{\perp}; S)\}_{\sigma'\sigma}$$

(2)

where $f_M(x, k_{\perp}; S)$ is a pseudovector

$$f_M(x, k_{\perp}; S) = S b_{1,M} + \kappa_{\perp} (S \cdot \kappa_{\perp}) b_{2,M} + \kappa_{\perp} (S \cdot \hat{z}) b_{3,M} + \hat{z} (S \cdot \kappa_{\perp}) b_{4,M} + \hat{z} (S \cdot \hat{z}) b_{5,M}$$

(3)

The functions $b_{i,M}(x, k_{\perp})$ ($i = 0, 1, ..., 5$) and then $n_{\sigma'\sigma}(x, k_{\perp}; M, S)$ can be obtained from the LF wave function of the system in momentum space. It results that the spin-dependent momentum distribution is an integral on the relative intrinsic momentum $k_{23}$ of the interacting spectator pair [4]

$$n_{\sigma'\sigma}(x, k_{\perp}; M, S) = \frac{2(-1)^{M+1/2}}{(1-x)} \int dk_{23} \sum_L Z_{\sigma'\sigma}(x, p_{\perp}, k_{23}, L, S)$$

(4)

where $L$ is the orbital angular momentum of the contributions to the one-body off-diagonal density matrix (only the values $L = 0$ or $L = 2$ are allowed).
2.2 Fermion correlator and LF spectral function

The fermion correlator in terms of the LF coordinates is\footnote{[1]}

\[ \Phi_{\alpha,\beta}(p,P,S) = \frac{1}{2} \int d\xi^- d\xi^+ e^{ip\xi} \langle P, S, A| \bar{\psi}_\beta(0) \psi_\alpha(\xi)| A, S, P \rangle \]  

where \(| A, S, P \rangle \) is the A-particle state and \( \psi_\alpha(\xi) \) the particle field (e.g. a nucleon of isospin \( \tau \) in a nucleus, or a quark in a nucleon). A linear relation exists between the correlator in valence approximation, \( \Phi^{p \tau} \), and the spectral function\footnote{[4]}

\[ \Phi_{\alpha,\beta}^{p \tau}(p,P,S) = D \frac{2p^+}{2p^+} \sum_{\sigma \sigma'} \{ u_\alpha(\hat{p}, \sigma') \ P_{M, \sigma' \sigma}(\hat{\kappa}, \epsilon, S) \ \bar{u}_\beta(\hat{p}, \sigma) \} , \]  

where \( D = \frac{[(P^+)^2 - E_S]}{\{p^+ - m M_0[1, (23)]\}} \). Then, traces of \( \Phi^p \) can be expressed by traces of the spectral function:

\[ \text{Tr}(\gamma^+ \Phi^p) = D \text{Tr} \left[ \hat{P}_M(\hat{\kappa}, \epsilon, S) \right] \]  
\[ \text{Tr}(\gamma^+ \gamma_5 \Phi^p) = D \text{Tr} \left[ \sigma_\epsilon \ \hat{P}_M(\hat{\kappa}, \epsilon, S) \right] \]  
\[ \text{Tr}(\hat{p}_\perp \gamma^+ \gamma_5 \Phi^p) = D \text{Tr} \left[ \hat{p}_\perp \cdot \sigma \ \hat{P}_M(\hat{\kappa}, \epsilon, S) \right] \]  

The proper integration on \( p^- \) of Eqs. (7,8,9) and taking \( p^+ = xP^+ \) gives relations between the TMDs at leading twist and the functions \( b_i, M \) in Eqs. (2,3), viz

\[ f(x, |p_\perp|^2) = b_0 \quad \Delta f(x, |p_\perp|^2) = b_{1,M} + b_{5,M} \]  
\[ g_{1T}(x, |p_\perp|^2) = \frac{M}{|p_\perp|^2} b_{4,M} \quad \Delta_{1T} f(x, |p_\perp|^2) = \frac{1}{2} \left\{ 2 b_{1,M} + b_{2,M} \right\} \]  
\[ h_{1L}^L(x, |p_\perp|^2) = \frac{M}{|p_\perp|^2} b_{3,M} \quad h_{1T}^L(x, |p_\perp|^2) = \frac{M^2}{|p_\perp|^2} b_{2,M} \]  

Linear equations between transverse parton distributions were discussed in\footnote{[2]}

\[ \Delta f = \Delta_{1T} f + \frac{|p_\perp|^2}{2M^2} h_{1T}^L \quad g_{1T} = -h_{1L}^L \]  

From the explicit expressions of the functions \( b_{i,M} \) in terms of the wave function of the system, one finds that these equalities hold exactly in valence approximation when the contribution to the transverse distributions from the angular momentum \( L = 2 \) is absent. This implies a vanishing value of the orbital angular momentum of the particle in the system wave function\footnote{[4]}. On the contrary the quadratic relation presented in\footnote{[2]} does not hold in our approach, even if the contribution from the angular momentum \( L = 2 \) is absent, because of the presence of \( \int dk_{23} \) in the expressions of\footnote{[4]} the transverse momentum distributions.

By evaluating proper traces of both the correlator and the spectral function, one can also obtain the twist-three TMDs in terms of the functions \( b_{i,M} \) and relate twist-three and twist-two TMDs. In our approximation the same linear relations found in\footnote{[8]} hold, once the gluon contributions are disregarded. Obviously the T-odd TMDs vanish in valence approximation\footnote{[4]}. 

3 Conclusions and perspectives

The LF spin-dependent momentum distribution for a spin 1/2 system composed of three fermions (as $^3\text{He}$ or a nucleon in valence approximation) can be expressed through six functions $b_{i,M}$, that can be written in terms of the LF wave function of the system in momentum space. A simple relation exists between the fermion correlator in valence approximation and the LF spectral function. Then it follows that the TMDs are combinations of the functions $b_{i,M}$.

As a result we found that the linear relations proposed between the T-even twist-two TMDs hold in valence approximation whenever the contribution from the $L=2$ orbital angular momentum term in the one-body off-diagonal density matrix is absent, while the quadratic relation does not hold even in this case. Furthermore, in valence approximation the proper relations between the twist-three and the twist-two TMDs hold, once the gluon contributions are disregarded.

In the close future we will evaluate the transverse momentum distributions for a nucleon in $^3\text{He}$, that could be extracted from measurements of appropriate spin asymmetries in $^3\text{He}(e,e'p)$ experiments at high momentum transfer.

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