Fractal gradation of filling material and the optimum grading parameter

Jincheng Xie¹, Dengpan Qiao¹ †, Jun Wang¹, Guangtao Li², Fei Huang¹, Hongsheng Sun²

¹Kunming University of Science and Technology, Yunnan province, China
²YUXI Mining company| Yunnan province, China

Abstract. In order to overcome the difficulty of choosing appropriate grading parameters of filling material for mines which are using filling mining method, the fractal theory is introduced, and a fractal equation of grading is built to represent gradation. Meanwhile taken Dahongshan copper mine as an example to discuss the relationship between fractal dimension and porosity, cementing strength under different parameters. A prediction model of cementing strength is built based on the experimental data, and the results show that the optimal grading parameters of Dahongshan copper mine is fractal dimension = 2.82. The optimized grading parameters of filling material in Dahongshan run steady, which greatly reduced the filling cost.

1. The fractal equation of grading of filling material

1.1. The fractal formula of grading of filling material

The backfilling materials used in filling is composed of multi-level particle size of waste rock and tailings. Fractal geometry is used to describe chaotic and irregular objects in nature, among them, there is some description of particle number and particle size distribution.

$$A(\delta) = A_0 (\frac{\delta}{\delta_{\text{max}}})^{E-D}$$

(1)

Among them, \(A(\delta)\) means quantity, \(A_0\) is the value of A when Graphics is plastic \((E = D)\), \(\delta_{\text{max}}\) is the maximum measured size. D is the dimension. When \(E=0\), A and \(\delta\) is the number of dot (such as cantor set); when \(E = 1\), A and \(\delta\) correspond to the length of Graphics; when \(E=2\), A and \(\delta\) correspond to the area and length of Graphics; when \(E=3\), A and \(\delta\) correspond to the volume and length of Graphics;

Using lines to express the grading of filling material (As shown in Fig.1).
Fig. 1 The schematic diagram of size of filling material

Obviously, in Fig. 1:

$$N_0 = \sum_{i=1}^{n} N_{i}$$

$N_0$ is the total number of all filling particles, from $N_{i}$ to $N_{n}$ represent different sieve pore size. Thus, in the equation $E=0$, the above formula become:

$$N(x) = N_1 \cdot (x/x_{max})^{-D} \tag{2}$$

Define the particle size of filling material negative cumulative distribution function as:

$$F(x) = \frac{N(x)}{N_0} \tag{3}$$

Where $N(x)$ represent the number of particle size of tailing material which is no more than $X$, $N_0$ is the total number of particles of filling material, $X$ represent the sieve pore size.

Put the formula (2) into formula (3):

$$F(x) = N_1 \frac{(x/x_{max})^{-D}}{N_0} \tag{4}$$

There is a max value ($F(x_{max}) = 1$) in the particle size of filling material negative cumulative distribution function when the screen size is equal to the largest size. So

$$F(x_{max}) = \frac{N_1}{N_0} = 1$$

$N_1 = N_0$

So, the formula (4) is equal to:

$$F(x) = (x/x_{max})^{-D} \tag{5}$$

Define the quality of distribution function as:

$$P(x) = \frac{M(x)}{N_0} \tag{6}$$
Where $M_0$ represent total mass, $M(x)$ represent the total mass of particle which their particle size is not more than X, $P(x)$ represent the screening frequency of negative cumulative distribution.

$V(x)$ and $dN(x)$ represent the volume and number in the interval $(x, x + dx)$.

For any 3D objects exist volume and size relations:

$V(x) = k \cdot x^2$

So

$M_0 \cdot dP(x) = k \cdot x^3 \cdot \rho \cdot dN(x) \quad (7)$

And

$dN(x) = N_0 \cdot dF(x), \quad dF(x) = (-D) \cdot x^{D_{max}} \cdot x^{-D-1} \cdot dx$

So

$P(x) = \frac{k \cdot \rho \cdot N_0 \cdot x^{D_{max}} \cdot x^{2-D}}{(3-D)M_0} + C \quad (8)$

Because, $P(x_{\text{max}}) = 1$, assume that $P(x_{\text{min}}) = a$, bring the above conditions to formula (8):

$P(x) = \frac{(1-a)(x_{\text{max}}^{2-D} - x_{\text{min}}^{2-D})}{(x_{\text{max}}^{2-D} - x_{\text{min}}^{2-D})} + 1 \quad (9)$

$x_{\text{max}}, x_{\text{min}}$ represent the maximum and minimum sieve pore size respectively, $a$ is the corresponding frequency of smallest sieve aperture. $D$ represent the dimension of filling material.

1.2. The fractal regression analysis of filling material

The west wing of Dahongshan copper mine have more than 70% orebody under medium thickness, the change of form and angle is very large, mining is difficult.

The negative cumulative distribution of particle size of waste rock and tailings in Dahongshan copper mine shown in Fig. 2.

![Fig. 2 The distribution of particle size of waste rock and tailings](image)

As shown in Fig. 2, the particle size of tailing which accounted for more than 70% under 0.05mm, is fine. If use tailings for cemented filling alone the superfine particles can easily lead to low cementing strength and even cracking. On the contrary, the particle size distribution of continuous degree of waste rock is better, adding a certain amount of waste rock in filling materials can effectively improve quality of filling material, at the same time reduce the consumption of cement in a certain extent.

Take Dahongshan copper mine as an example, put different mass ratio of waste rock into filling materials, and use formula (9) to regression analysis the particle size distribution of filling materials, the results are shown in table 1.
Table 1 Regression fitting results

| Waste rock mass ratio | Dimension | R²   | F-Value | Waste rock mass ratio | Dimension | R²   | F-Value |
|-----------------------|-----------|------|---------|-----------------------|-----------|------|---------|
| 1                     | 2.589     | 0.986| 2253    | 0.50                  | 2.938     | 0.946| 1481    |
| 0.95                  | 2.517     | 0.971| 1067    | 0.45                  | 2.977     | 0.938| 1409    |
| 0.90                  | 2.574     | 0.974| 1332    | 0.40                  | 3.016     | 0.929| 1331    |
| 0.85                  | 2.628     | 0.975| 1539    | 0.35                  | 3.055     | 0.919| 1247    |
| 0.80                  | 2.679     | 0.974| 1661    | 0.30                  | 3.094     | 0.908| 1157    |
| 0.75                  | 2.727     | 0.971| 1704    | 0.25                  | 3.133     | 0.895| 1064    |
| 0.70                  | 2.772     | 0.968| 1695    | 0.20                  | 3.173     | 0.879| 969     |
| 0.65                  | 2.816     | 0.963| 1658    | 0.15                  | 3.215     | 0.862| 875     |
| 0.60                  | 2.858     | 0.958| 1606    | 0.10                  | 3.257     | 0.842| 785     |
| 0.55                  | 2.898     | 0.952| 1546    | 0.00                  | 2.837     | 0.937| 481     |

As shown in table 1, the material of cemented filling which composed of waste rock and tailings in Dahongshan copper mine regression fitting correlation coefficient is close to 1, it means the application of formula (9) is quite wide in this kind of filling material. Meanwhile, along with the reduction of waste rock accounted, the dimension of filling material began to increase, it also conforms to the definition of the filling material which is the description of the dimension is the complexity of filling material. As the increasing of the tailings, the space which set up by coarse particles is filled with fine particles, this kind of phenomenon lead to the fractal structure of the entire graphics is more and more complicated and increasing the value of dimension.

The preponderance of filling structure reflects in the porosity of filling material. Figure 3 shows the relationship between dimension and porosity of filling material.

![The relationship of porosity and dimension](image)

As shown in Fig.3 the type of curve is a reverse parabola. With the increase of the dimension of filling material the porosity decreases to a turning point and then increased. The relationship of dimension and porosity also reveal the spatial structure of filling material, the skeleton formed by coarse particles was filled by fine particles gradually, and when the internal spaces filled up with particles, particles will overflow which lead to the increasing of porosity. Dimension, therefore, not only is the embodiment of the complexity of filling material, as well as the variation of the porosity. Therefore, the dimension has a great advantage for characterize the gradation of filling material.

2. The optimum grading parameter
Considering the actual production of Dahongshan copper mine, setting the compressive strength as the appraisal index performance of filling mechanics, design the experiment contains dimension, same water cement ratio, the mass concentration of the slurry and the cementation strength of filling body (as shown in table 2). The specimen was made by the size of 100mm×100mm×100mm, and the
temperature and humidity standard curing box was used for curing specimen by the conditions of temperature 28 °C and 85% humidity. Finally, the W5-874 hydraulic universal testing machine was used for testing the compressive strength of specimen by determination of the age for 3 days, 7 days and 28 days.

Table. 2 The orthogonal experiment scheme of cementation strength

| Level factors | Dimension | Slurry concentration (%) | water cement ratio |
|---------------|-----------|--------------------------|-------------------|
| 1(1)          | 2.589     | 80                       | 1.65              |
| 2(0.9)        | 2.574     | 79                       | 1.65              |
| 3(0.8)        | 2.679     | 78                       | 1.65              |
| 4(0.7)        | 2.772     | 77                       | 1.65              |
| 5(0.6)        | 2.858     | 76                       | 1.65              |
| 6(0.5)        | 2.938     | 75                       | 1.65              |
| 7(0.4)        | 3.016     | 74                       | 1.65              |
| 8(0.3)        | 3.094     | 73                       | 1.65              |
| 9(0.2)        | 3.173     | 72                       | 1.65              |
| 10(0.1)       | 3.257     | 71                       | 1.65              |
| 11(0)         | 2.837     | 70                       | 1.65              |

According to the experimental plan a scatter diagram (Fig.4) was draw to show the relationship between cementation strength of different age and dimension of filling material. The figure shows that cementation strength changing with dimensions which accordance with the above correlation analysis. There is a peak in the curve which shows the Curve is satisfy the shape of the parabolic equation.

![Fig. 4 Scatter diagram of dimensions and cementation strength](image)

(Slurry concentration 78%, water cement ratio=1.65)

Set up the prediction equation according to the above analysis which contains cementation strength, dimension (D):

\[ p(x) = AD^2 + BD + k \] (10)

\( p(x) \) represents the cementation strength, \( D \) represents the dimension of filling material, \( A, B, \) and \( g \) are regression fitting coefficient.

According to the constructed model (10), regression analysis software (OriginLab) is used to regression fitting the experimental data of different age. Fitting of related parameters and the regression coefficient shown in table 3.
Table. 3 regression fitting results of experimental data in different age

| Category | 3 Days  | 7 Days  | 28 Days |
|----------|---------|---------|---------|
| A        | -1.01   | -1.55   | -2.86   |
| B        | 5.73    | 8.74    | 16.71   |
| k        | -6.31   | -8.49   | 17.99   |
| R2       | 0.90    | 0.92    | 0.95    |
| F-value  | 62706   | 76798   | 62706   |

The result of fitting model is good, the values of $R^2$ are close to 1, which prove model prediction accuracy is high. Relationship of dimension and cementing strength of filling material is similar to the growth of the parabola: when value of water cement ratio, ages, Slurry concentration in a certain place, cement strength slowly increase with the increase of the dimension of the filling material, then peaked and then gradually decreased. The Fig4 shows that the maximum cementing strength appears at the time of Dimension equal to 2.2, all the peak value almost at the same dimension, that provide the optimal grading parameter is here (Dimension=2.2).

3. Conclusion
(1) According to the two basic characteristics of fractal theory and combined with analysis of the gradation of filling materials the fractal equation was deduced. It shown that Dimensions of filling material can fully revealed the relation of its internal space structure and thus can better characterize the grading of the filling material.

(2) Prediction model are used to get the optimum proportion of filling material which meet the requirement of Dahongshan copper mine, and the results consistent with actual situation.

Acknowledgments
This work was financially supported by the national natural science fund of china project (51164016).

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