Supernova pointing by neutrino matter oscillation

A Burgmeier¹, K Scholberg² and R Wendell²
¹ Institut für experimentelle Kernphysik, Karlsruhe Institute of Technology, Wolfgang-Gaede-Str. 1, 76131 Karlsruhe, Germany
² Department of Physics, Duke University, Durham, NC 27708, USA
E-mail: burgmeier@ekp.uni-karlsruhe.de

Abstract. A core-collapse supernova will emit a neutrino burst that can be detected on Earth. If the neutrinos travel through the Earth before reaching the detector they oscillate via interaction with Earth’s matter, yielding oscillations in the neutrino energy spectrum. The frequency of these oscillations in energy is correlated with the pathlength traveled in the Earth and therefore contains information on the supernova location. For this technique to be useful for pointing, good energy resolution, well-known oscillation parameters and high statistics are required. This method is inferior to pointing with elastic scattering in a water Cherenkov detector but could be applied for scintillator-type detectors which have better energy resolution but weak intrinsic pointing capabilities. By the time a nearby supernova happens the requirements might well be fulfilled, and if no water Cherenkov detector is running at that time it may provide the only possibility to gain directional information. The pointing quality can be further improved by the combination of measurements from multiple detectors and also by taking relative timing into account.

1. Introduction
More than 99 % of the energy released in a core-collapse supernova is radiated away by neutrinos [1, 2]. As the neutrinos arrive at Earth between a few hours and a few days before the optical signal, it is desirable to obtain directional information from the neutrino signal [3]. Water Cherenkov detectors can reconstruct the direction of the incident neutrino in elastic scattering reactions; however this will happen only for a small fraction of all events since the majority will be inverse beta decay [4].

The technique described both here and in more detail in [5] provides another way to point to the supernova. If the supernova neutrinos travel through Earth matter before reaching a detector, and if oscillation parameters are so that matter oscillations are allowed [6] (see also table 1) then the oscillations can be seen in the energy spectrum of a given neutrino flavor. Given this, conclusions can be drawn about the distance traveled in Earth matter by determining the neutrino energy spectrum. Information can be obtained by a single detector alone (unlike timing triangulation) but the method is more powerful when combining data from multiple detectors.

2. Basic method
The primary reaction in water Cherenkov and scintillator detectors is inverse beta decay $\bar{\nu}_e + p \rightarrow e^+ + n$ which is only sensitive to electron anti-neutrinos. Therefore we only consider
Table 1. Effect of unknown oscillation parameters on matter oscillation, from [6].

| $\sin^2 \theta_{13}$ | Normal hierarchy | Inverted hierarchy |
|------------------------|------------------|-------------------|
| $\lesssim 10^{-3}$     | $\nu_e$ and $\bar{\nu}_e$ | $\nu_e$ and $\bar{\nu}_e$ |
| $\gtrsim 10^{-3}$      | $\bar{\nu}_e$   | $\nu_e$           |

$\bar{\nu}_e$ in this study, and we assume that oscillation parameters allow matter oscillations for $\bar{\nu}_e$. The so-called “Garching” supernova model predicts the neutrino flux to be of the form [7, 8]

$$F_0(E) = \frac{\Phi_0}{E_0} \Gamma(1+\alpha) \left( \frac{E}{E_0} \right)^{\alpha} \left( 1 + \frac{\alpha}{(\alpha+1)} \frac{E}{E_0} \right)^{-1} e^{-(\alpha+1) \frac{E}{E_0}}. \quad (1)$$

In accordance with [9] we choose the parameters to be $\alpha = 3$, $E_{\bar{\nu}_e} = 15$ MeV, $E_{\bar{\nu}_x} = 18$ MeV and $\Phi_{\bar{\nu}_e}/\Phi_{\bar{\nu}_x} = 0.8$. The neutrino interaction cross section was assumed to be proportional to $E^2$, so the events observed per energy goes with $f(E) = \sigma F \propto F_0(E) \cdot E^2$.

The oscillation probabilities have been computed numerically assuming Earth density to be piecewise constant according to the PREM model [10] and assuming that neutrinos arrive at the Earth in pure mass eigenstates. The $\bar{\nu}_e$ survival probability depends on $\sin^2 \left( \frac{L}{E} \right)$ which, for $L > 0$, leads to bumps in the $\bar{\nu}_e$ spectrum originating from mixing with the slightly different $\bar{\nu}_x$ spectrum. As the peaks of these oscillations are equidistant in inverse energy $y = 12.5/E$ they lead to a peak in the modulus squared of the Fourier transform (power spectrum) $G_{\sigma F}$.

![Energy spectrum](image1)
![Energy spectrum](image2)

Figure 1. Top: Energy spectrum (left) and power spectrum (right) for no matter oscillation. Bottom: Energy spectrum (left) and power spectrum (right) for $L = 6,000$ km.
\[
G_{\sigma F}(k) = \left| \int_{-\infty}^{\infty} dy \ e^{iky} f(y) \right|^2.
\]

(2)

Figure 1 compares the original energy spectrum and the power spectrum for the cases of no matter oscillations and for a neutrino path of \(L = 6,000\) km through Earth matter. In the second case the power spectrum clearly yields a peak whose position in \(k\) depends on \(L\), the pathlength traveled.

Figure 2 shows the peak for different pathlengths \(L\). Its position in \(k\) depends roughly linearly on \(L\). For \(L \lesssim 2,500\) km the peak disappears in the low frequency peak that results from the general shape of the spectrum. For high \(L \gtrsim 10,000\) km multiple peaks occur due to the greater matter density in the Earth’s core in comparison to its mantle [9]. For a measured energy spectrum the idea is to measure the peak position and find \(L\) from it which, if not zero, gives a ring in the sky as a possible supernova location.

3. Degrading effects

In practice various effects will degrade the quality of pointing. The main effects we are considering in this study are finite statistics and finite energy resolution of the detector. We take these effects into account by selecting a certain number of events from the primary distribution as shown in figure 2 and then smear out every event using a Gaussian whose width is chosen according to the energy resolution of a typical water or scintillator detector according to figure 3.

We choose a rather large number of 60,000 events to begin with as the pointing quality decreases rather rapidly for fewer events. For each set of selected events we determine \(k\) of the peak via the following peak finding algorithm\(^1\): we choose the peak \(k\) so that the integral from \(k - \Delta k / 2\) to \(k + \Delta k / 2\) is maximized for \(\Delta k = 4\) and we require \(k > 40\) to avoid being shadowed by the low \(k\) region. By doing this many times for each \(L\) we obtain a distribution of the peak position for a certain detector type.

Figure 4 shows these distributions for idealized water and scintillator detectors. The \(L\) dependence is minimal for Cherenkov detectors and so we concentrate on scintillator detectors in the rest of this study, however it is worth noting that Cherenkov detectors are already capable of very accurate pointing via elastic scattering for large numbers of events.

\(^1\) The technique could likely be enhanced by using a more sophisticated algorithm, for example one that makes use of the effect that peaks at higher \(k\) tend to be lower and wider, or by matching against template power spectra.
Figure 4. Distribution of the peak position for a detector with perfect energy resolution (top left), a water Cherenkov detector (top right) and a scintillator detector (bottom left). The picture on the bottom right shows the distribution of the peak height for a perfect detector. The peak height can be used to differentiate between the “no oscillation”, “mantle only” and “mantle and core” regions.

Figure 5. Neyman construction for 68 % C.L. (green) and 90 % C.L. (red). By drawing a horizontal line one can read off the allowed region in $L$ for a measured peak $k$.

The distributions, when normalized, can be interpreted as the likelihood function $\mathcal{L}(L; k)$ for a given $L$ to be true when a peak at $k$ is observed. $\mathcal{L}$ allows us to do a Neyman construction for a given confidence level from which we can obtain allowed values of $L$ for a measured $k$ [11]. Figure 5 shows such a Neyman construction. As can be seen nearly every $k$ includes the low $L$ region, and since $L = 0$ means half the sky it is desirable to be able to distinguish that case. In this case the peak is much smaller (see the bottom right plot in figure 4) since there is only
random noise but no physical peak, so including the peak height \( h \) in the Neyman construction can help to solve this issue.

Figure 6. Example skymaps with allowed regions in red. The true supernova position is indicated by a black star and Pyhääläni, Finland has been chosen as the detector location. The large region in the right picture is because \( L = 0 \) could not be excluded.

Finally the allowed \( L \) values can be mapped to allowed regions in the sky for a given detector location. Assuming a detector in Finland, figure 6 shows two example skymaps in equatorial coordinates.

4. Multiple detectors

Figure 7. Example skymaps for two detectors (left) and three detectors (right) with 60,000 events each. The detectors are located in Finland, Hawaii and South Dakota.

Using the Neyman construction method it is straightforward to combine the result of multiple detectors. Instead of a single \( L \) we have an \( \{L_i\} \) tuple which represents the length of the path in Earth matter between the supernova and detector \( i \). Note that the set of valid \( \{L_i\} \) values depends on the location of the detectors relative to each other. Also \( k \) and \( h \) are replaced by a tuple of \( \{(k_i, h_i)\} \) pairs, one for each detector. The Neyman construction yields allowed \( \{L_i\} \) which can again be used to generate skymaps. The multi-dimensional integrals have to be computed using Monte Carlo techniques. In figure 7 this is shown for two and three detectors.

The mean sky coverage of the allowed region for a supernova at fixed declination gives an overall estimate of the pointing quality. Figure 8 shows this quantity, averaged over right ascension, for one, two and three detectors.

Another way to combine results of multiple detectors is by using relative timing information [12]. Given the high statistics that we require for this technique, this gives a good additional constraint on the allowed region. Figure 9 shows a two-detector skymap with information from relative timing between the two detectors superimposed. We assume \( \delta(\Delta t) \approx 1 \) ms.
5. Conclusions
We demonstrated a new technique to locate a supernova via its neutrino signal. The method is feasible for scintillator-type detectors with good energy resolution. Very high statistics are required. A 50 kt scintillator detector can see enough events for a supernova at 5 kpc distance from the Earth. Also the as yet unknown oscillation parameters must allow matter oscillation for $\bar{\nu}_e$. The method naturally allows combination of the data of multiple detectors and can be further improved by taking relative timing information into account.

Elastic scattering in a water detector is still a better method for supernova pointing, however, if such a detector is not running when a supernova occurs the presented technique may be an effective alternative.

References
[1] Barger V, Marfatia D and Wood B P 2002 Phys. Lett. B\textbf{547} 37–42 (Preprint hep-ph/0112125)
[2] Scholberg K 2007 (Preprint astro-ph/0701081)
[3] Scholberg K 2008 Astron. Nachr. \textbf{329} 337–339 (Preprint 0803.0531)
[4] Ikeda M \textit{et al.} (Super-Kamiokande) 2007 Astrophys. J. \textbf{669} 519–524 (Preprint 0706.2283)
[5] Scholberg K, Burgmeier A and Wendell R 2010 Phys. Rev. D\textbf{81} 043007 (Preprint 0910.3174)
[6] Dighe A S, Keil M T and Raffelt G G 2003 JCAP \textbf{0306} 006 (Preprint hep-ph/0304150)
[7] Raffelt G G, Keil M T, Buras R, Janka H T and Rampp M 2003 Kanazawa 2003, Neutrino oscillations and their origin 380–387 (Preprint astro-ph/0303226)
[8] Keil M T, Raffelt G G and Janka H T 2003 Astrophys. J. \textbf{590} 971–991 (Preprint astro-ph/0208035)
[9] Dighe A S, Kachelriess M, Raffelt G G and Tomas R 2004 JCAP \textbf{0401} 004 (Preprint hep-ph/0311172)
[10] Dziewonski A and Anderson D 1981 Phys. Earth, Planet, Interiors \textbf{25} 297
[11] Amsler C \textit{et al.} (Particle Data Group) 2008 Phys. Lett. B\textbf{667} 1
[12] Beacom J F and Vogel P 1999 Phys. Rev. D\textbf{60} 033007 (Preprint astro-ph/9811350)