Adaptive Asymptotic Fault Tolerant Tracking of Uncertain Nonlinear Systems with Unknown Control Directions

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Received: / Accepted:

Abstract In the article, the issues of asymptotic adaptive tracking control for the uncertain nonlinear systems in the presence of actuator faults and unknown control directions are investigated. By using the properties of Nussbaum function and backstepping technique, the problems resulted from the unknown signs of the nonlinear control functions are circumvented successfully. Moreover, a new adaptive asymptotic tracking control method is presented with the fault tolerant control framework, which is capable of realizing zero-tracking performance. The stability of the controlled system is ensured through fractional Lyapunov stability analysis. Finally, the validity of the raised scheme is verified by a simulation example.

Keywords Uncertain nonlinear system · Fault tolerant control (FTC) · Adaptive asymptotic tracking control · Unknown control directions · Unknown control directions

1 Introduction

During the last few years, the tracking control issues of the nonlinear systems has gained much attention among researchers owing to its extensive applications in numerous fields, such as uncertain robot [1], a flexible manipulator [2], and autonomous vehicles [3]. Existing approaches to the analysis and design of nonlinear systems can be divided into two categories: Robust control and adaptive control. The former was shown to be effective for the nonlinear systems without uncertainties. However, if there exist uncertainties, particularly with parameter perturbations and unknown parameters, the problem of interest becomes challenging. Recent works related to nonlinear control have focused on the use of adaptive techniques to overcome this control problem from an online identification perspective. This method has been proven to not only be able to compensate the parameter uncertainties but has fast convergence to ensure a satisfactory system performance [4]-[12].

Actuators in modern dynamical systems are increasingly more vulnerable to various unpredictable faults that can degrade their performance or may even lead to catastrophic accidents [13]-[21]. Therefore, it is of practical significance to develop useful actuator fault compensation methods to ensure the essential stability and satisfactory performance of the controlled system with actuator faults. In particular, many efforts recently have been made to apply the advanced FTC techniques to settle the tracking control problems of uncertain nonlinear systems [22]-[34]. For example, a backstepping based FTC scheme was reported in [22] for the tracking problem of single-input single-output (SISO) parameter uncertain nonlinear systems. The same approach was applied for multi-input multi-output (MIMO) nonlinear systems in [23] by designing a direct failure compensation scheme. Such an approach was generalized to uncertain nonlinear systems for improving transient performance [24]. This problem was also investigated to solve the unknown failures of hysteretic actuators by
means of smooth adaptive control in. The first result on adaptive FTC for the uncertain nonlinear systems with infinite number of faults were addressed by the authors in [26] using backstepping techniques. This result is extended by the authors in [27] and [28] using decentralized adaptive backstepping control techniques. In [29], they also worked on the design of FTC against infinite number of actuator faults by applying tuning function approach. Besides, [33] develop an adaptive FTC that guarantees asymptotic tracking of the controlled dynamical system suffering from actuator faults and input quantization. The works in [34] further studied the FTC attack strategy and extended the results to the finite time control.

Noting that the required conditions in the above results on FTC still need to know the sign of the control coefficients, which indicates that these results on FTC are difficult to be applied to the uncertain nonlinear systems with unknown control directions. Thus, designing adaptive FTC without previous knowledge of the sign of unknown control functions is significant for keeping the scalability of control. Simultaneously, an hottest topic is to integrate both adaptive control and Nussbaum gain technique to realize the desired control objective. A low-complexity prescribed tracking performance FTC method was discussed for the nonlinear systems without the prior information of the sign of unknown control directions [35]. However, it can be emphasized that the aforementioned results studied only the loss of effectiveness fault and the actuator stuck fault is neglected. An exception is [36], which investigates FTC of nonlinear systems with unknown control directions and infinite number of unknown actuator faults. Nevertheless, it should be pointed out that the results in [36] are still rather restrictive. Roughly speaking, they are restricted to a certain class of nonlinear systems with strict output feedback structure and the methods cannot be suitable for more general nonlinear systems with time varying parameters. However, the existing FTC schemes can only ensure that the tracking error converges to the small compact set, whose size depends on appropriate selection of parameters and other unknown terms, making it impossible to make a priori selection of control design parameters for a random given steady-state performance. In many engineering systems, however, it is required to provide exact tracking performance with zero-error tracking. Therefore, it is worth further investigating on how to present a novel adaptive asymptotic FTC for the systems without the knowledge of the sign of unknown control functions.

The goal of the present work is the development of novel adaptive asymptotic control method for the uncertain nonlinear systems. The simultaneous existence of unknown control directions and actuator faults involves considered difficulties basically different from the closely related researches. The presented analysis consists of the following three main items:

1. The systems under research are classic and practical enough: large time varying parameter and unknown control directions are simultaneously first occurred in the nonlinear system with FTC structure. However, in the relative researches [35,36], large uncertainties are neglected or only the strict output feedback was studied.

2. Second, with the help of the bound estimate method and the Nussbaum function, a new adaptive design scheme is developed with appropriate design parameters. Particularly, the proposed adaptive controllers can effectively cope with the time-varying parameter uncertainties, the unknown sign of control functions and actuator faults.

3. The control objective is enhanced: the whole signals in the controlled system are constrained to be globally bounded and the difference between output and reference signal tends to zero asymptotically. On the contrary, in the researches [11]-[22] only the boundedness of states can be guaranteed.

2 Problem statement

The aim of this research is to settle the issues of adaptive asymptotic FTC for the following nonlinear systems:

\[
\begin{align*}
\dot{x}_i &= g_i(x)x_i + \theta^T(t)\phi_i(x) + d_i(t), \quad 1 \leq i \leq n-1, \\
\dot{x}_n &= g_n(x)\sum_{j=1}^m u_j(t) + \theta^T(t)\phi_n(x) + d_n(t), \\
y &= x_1
\end{align*}
\]

where \( x_i = [x_1, x_2, \ldots, x_i]^T \in \mathbb{R}^i, \) \( i = 1, 2, \ldots, n, \) and \( x = [x_1, x_2, \ldots, x_n] \in \mathbb{R}^n \) are the systems states; \( y \in \mathbb{R} \) is the output of the system; \( \theta(t) \in \mathbb{R}^m \) and \( g_i(x) \in \mathbb{R} \) are bounded time-varying parameters and unknown virtual control function; \( \phi_i(x) \in \mathbb{R}^i, i = 1, 2, \ldots, n \) are known smooth nonlinear functions; \( d_i(t) \) are bounded disturbances; \( u_j(t) \) denotes the fault control input. In this paper, we assume that there are \( q \) fault modes in the fault control input \( u_j(t) \). Then, the actuator fault is modeled as followed:

\[
u_j(t) = \rho_{j,q}v_j(t) + \omega_{j,q}(t), \quad t \in [T_{j,q}^s, T_{j,q}^e], \quad q = 1, 2, 3, \ldots
\]

where \( v_j(t) \) is a designed control signal, \( 0 \leq \rho_{j,q} \leq 1, T_{j,q}^s \) and \( T_{j,q}^e \) are the time at which the \( j \)th actuator fault begins and ends satisfying \( 0 \leq T_{j,1}^s < T_{j,1}^e \leq T_{j,q}^e < T_{j,q}^s \leq T_{j,q}^e \leq T_{j,1}^e \) for all \( j \) and \( q \).
$$T_{i,j}^{+} \leq T_{i,j}^{-} \leq \cdots \leq +\infty,$$

and $\varpi_{j,q}(t)$ are unknown, continuous and bounded. Note that (2) implies the four types of faults:

1. $\rho_{j,q} = 1$ and $\varpi_{j,q} = 0$. The fault-free case.
2. $0 < \rho_{j,q} \leq \bar{\rho}_{j,q} < 1$ and $\varpi_{j,q} = 0$. The partial loss of effectiveness fault.
3. $\rho_{j,q} = 1$ and $\varpi_{j,q} \neq 0$. The bias fault.
4. $\rho_{j,q} = 0$ and $\varpi_{j,q} \neq 0$. In this case, $v_{j}$ is independent from the control inputs $u_{j}$. This indicates the total loss of effectiveness (TLOE) fault.

Thus the corresponding closed-loop system is given by

$$\dot{x}_{i} = g_{i}(t)x_{i+1} + \theta^{T}(t)\phi_{i}(x_{i}) + d_{i}(t), 1 \leq i \leq n - 1,$$

$$\dot{x}_{n} = g_{n}(t)\sum_{j=1}^{m}(\rho_{j,q}v_{j}(t) + \varpi_{j,q}(t)) + \theta^{T}(t)\phi_{n}(x) + d_{n}(t),$$

$$y = x_{1}$$

(3)

**Remark 1** The FTC schemes in [25, 32, 29, 30, 33, 34, 22, 23, 27, 36, 24, 26, 31] assumed the sign of the control coefficients is known, which is infeasible in some applications. However, the systems in (1) assumes the sign of the control coefficients is unknown and thus does not have this restriction. Although FTC schemes with unknown control direction have been studied in [36, 35], all these results limited to a special class of nonlinear systems that have a strict output feedback form, time varying uncertainties are not considered. Besides, the aforementioned results only guarantee that the adaptive tracking error converges to a very small residual set, the asymptotic convergence cannot be ensured.

The main goal of this research are to propose an effective adaptive FTC tracking controller for the system (1) such that the whole signals of the controlled systems are globally bounded on $[0, +\infty)$, while the tracking error $y - y_{r}$ asymptotically converges to zero. For our subsequent analysis, we make the following assumptions.

**Assumption 1** There exists an unknown positive constant $\bar{\varpi}_{j,q}$ such that $|\varpi_{j,q}| \leq \bar{\varpi}_{j,q}$.

**Assumption 2** No more than $n - 1$ actuators are allowed to undergo TLOE at the same time.

**Assumption 3** The signs of $g_{i}(t)(i = 1, \ldots, n)$ are unknown, there exists the unknown constants $0 < g_{\min} < g_{\max} < \infty$ which satisfies $g_{\min} \leq |g_{i}(t)| \leq g_{\max}$ for all $t$.

**Remark 2** Assumption 1 states that the time-varying parameters are bounded. Assumption 2 is necessary to guarantee the controllability of system (1) in FTC literatures [12]-[26] Assumption 3 guarantees that $g_{i}(\cdot) \neq 0$ for all time, thus the controllability of system (1) is guaranteed.

To realize the main objective of this article, we introduce the following technical lemmas.

**Lemma 1** [38] If there exists a positive definite function $V(t)$ such that the following inequality satisfy

$$V(t) \leq \sum_{j=1}^{n} \int_{0}^{t} (k_{j}\varpi_{j}(x_{j}) - 1)\dot{x}_{j}(t)\,d\tau + c$$

with $c$ is a constant. Then, one can obtain the boundedness of $V(t)$ and $\sum_{j=1}^{n} \int_{0}^{t} (k_{j}\varpi_{j}(x_{j}) - 1)\dot{x}_{j}(t)\,d\tau$.

### 3 Adaptive Controller design

This section provides the process of controller design to guarantee the boundedness of the controlled systems and asymptotic convergence of the tracking error.

#### 3.1 Adaptive backstepping controller design

Following the backstepping approach, coordinate transformation is introduced for the $i$th subsystem

$$z_{1} = g_{1} - y_{r},$$

$$z_{i} = x_{i} - \alpha_{i-1}, i = 2, \ldots, n$$

where $\alpha_{i}$ represents the intermediate control law recursively designed later.

Before proceeding further, we define

$$\vartheta = \sup_{t \in [0, \infty)} \|\Theta(t)\|, \quad \rho_{i} = \sup_{t \in [0, \infty)} \|D_{i}\|, i = 1, \ldots, n$$

(6)

where $\Theta(t) = [\vartheta^{T}(t), g_{1}, \ldots, g_{n-1}]^{T} \in \mathbb{R}^{m+n-1}$, $D_{i} = [d_{1}, \ldots, d_{i-1}, d_{i}]^{T} \in \mathbb{R}^{i}$ with $i = 1, 2, \ldots, n - 1$ and $D_{n} = [d_{1}, d_{2}, \ldots, d_{n-1}, d_{n} + g_{n}\sum_{j=1}^{m} \varpi_{j,q}(t)]$. Denoting the estimate of the parameters $\vartheta$ and $\rho_{i}$ as $\hat{\vartheta}$ and $\hat{\rho}_{i}$, respectively, then the definition of estimation errors are $\hat{\vartheta} = \vartheta - \hat{\vartheta}$ and $\hat{\rho}_{i} = \rho_{i} - \hat{\rho}_{i}$.

**Step 1.** By (1) and (5), we can derive the time derivative of $z_{1}$ as

$$\dot{z}_{1} = g_{1}z_{2} + g_{1}\alpha_{1} + \Theta^{T}\zeta_{1} + d_{1} - y_{r}$$

(7)

where $\zeta_{1} = [\vartheta^{T}, 0, \ldots, 0]^{T} \in \mathbb{R}^{m+n-1}$.

Construct a novel Lyapunov function candidate as

$$V_{1} = \frac{1}{2}z_{1}^{2} + \frac{1}{2\gamma}\hat{\vartheta}^{2} + \frac{1}{2\lambda_{1}}\hat{\rho}_{1}^{2}$$

(8)

where $\gamma > 0$ and $\lambda_{1} > 0$ are known constants.
In view of (6) and (7), we have
\[
\dot{V}_1 = z_1 (g_1 \alpha_1 + \Theta^T \zeta_1 + d_1 - \dot{y}_r) + g_1 z_1 z_2
\]
\[
- \frac{1}{\gamma} \ddot{\theta} - \frac{1}{\lambda_1} \dot{\rho}_1
\]
\[
\leq z_1 g_1 \alpha_1 + g_1 z_1 z_2 - z_1 \dot{y}_r + \vartheta |z_1| |\zeta_1| + |z_1| |\rho_1|
\]
\[
- \frac{1}{\gamma} \ddot{\theta} - \frac{1}{\lambda_1} \dot{\rho}_1
\]
Thus, design the first virtual control input \( \alpha_1 \) as
\[
\alpha_1 = N (\chi_1) v_1
\]
\[
v_1 = k_1 z_1 + \beta_1 \ddot{\theta} + \xi_1 \dot{\rho}_1 - \dot{y}_r
\]
\[
\chi_1 = z_1 v_1
\]
with
\[
\beta_1 = \frac{z_1 \zeta_1^T \zeta_1}{\sqrt{z_1^2 \zeta_1^T \zeta_1 + \sigma_1^2}}
\]
\[
\xi_1 = \frac{z_1}{\sqrt{z_1^2 + \sigma_1^2}}
\]
(11)
where \( \sigma_1 \) satisfies \( \lim_{t \to -\infty} \int_0^t \sigma_1(s) \, ds \leq \sigma_1 \leq +\infty \).

From the definition of \( \dot{\chi}_1 \), one has
\[
\dot{V}_1 \leq -k_1 z_1^2 + (g_1 N (\chi_1) + 1) \dot{\chi}_1 + g_1 z_1 z_2
\]
\[
- z_1 \theta \beta_1 + \vartheta |z_1| |\zeta_1| - z_1 \xi_1 \rho_1 + |z_1| \rho_1
\]
\[
- \frac{1}{\gamma} \ddot{\theta} (\ddot{\theta} - \gamma z_1 \beta_1) - \frac{1}{\lambda_1} \dot{\rho}_1 (\dot{\rho}_1 - \lambda_1 z_1 \xi_1)
\]
(12)
According to the definition \( \beta_1 \) and \( \xi_1 \) in (11), it is obtained that
\[
-\vartheta z_1 \beta_1 + \vartheta |z_1| |\zeta_1|
\]
\[
- \frac{\vartheta z_1^2 \zeta_1^T \zeta_1}{\sqrt{z_1^2 \zeta_1^T \zeta_1 + \sigma_1^2}} + \vartheta |z_1| |\zeta_1|
\]
\[
\leq \dot{\sigma}_1
\]
\[
- z_1 \xi_1 \rho_1 + |z_1| \rho_1
\]
\[
= - \frac{z_1^2 \rho_1}{\sqrt{z_1^2 + \sigma_1^2}} + |z_1| \rho_1
\]
\[
\leq \sigma_1 \rho_1
\]
(13)
Choosing the first turning function \( \tau_1 \) for \( \dot{\theta} \) as
\[
\tau_1 = \gamma z_1 \beta_1 - \gamma \sigma_1 \dot{\theta}
\]
and adaptive law \( \dot{\rho}_1 \) as
\[
\dot{\rho}_1 = \lambda_1 z_1 \xi_1 - \lambda_1 \sigma_1 \dot{\theta}
\]
(15)
Then, substituting (13)-(15) into (12), we have
\[
\dot{V}_1 \leq -k_1 z_1^2 + (g_1 N (\chi_1) + 1) \dot{\chi}_1 + g_1 z_1 z_2
\]
\[
- \frac{1}{\gamma} \ddot{\theta} (\ddot{\theta} - \tau_1) + \sigma_1 (\ddot{\theta} \dot{\rho}_1 + \dot{\rho}_1 \dot{\rho}_1)
\]
\[
+ (\theta + \rho_1) \sigma_1
\]
(16)
**Step 2.** Taking the time derivative of \( z_2 \) in (5) gives
\[
\dot{z}_2 = g_2 z_3 + g_2 \alpha_2 + \Theta^T \phi_2 + d_2 + \eta_2
\]
\[
- \frac{\partial \alpha_1}{\partial x_1} (g_1 x_2 + \Theta^T \phi_1 + d_1) - \frac{\partial \alpha_1}{\partial \dot{\theta}} \dot{\theta}
\]
(17)
where
\[
\eta_2 = - \frac{\partial \alpha_1}{\partial y_r} \dot{y}_r - \frac{\partial \alpha_1}{\partial \dot{y}_r} \ddot{y}_r - \frac{\partial \alpha_1}{\partial \dot{\rho}_1} \dot{\rho}_1 - \frac{\partial \alpha_1}{\partial \chi_1} \dot{\chi}_1
\]
(18)
The Lyapunov function is selected as
\[
V_2 = V_1 + \frac{1}{2} z_2^2 + \frac{1}{2 \lambda_2} \rho_2^2
\]
(19)
Then, differentiate \( V_2 \) along the solution of system (17) to yield
\[
\dot{V}_2 \leq -k_1 z_1^2 + (g_1 N (\chi_1) + 1) \dot{\chi}_1
\]
\[
+ z_2 (g_2 z_3 + g_2 \alpha_2 + \Theta^T \phi_2 + \eta_2 - \frac{\partial \alpha_1}{\partial \dot{\theta}} \dot{\theta})
\]
\[
+ z_2 s_2^T D_2 \dot{\theta} - \frac{1}{\gamma} (\dot{\theta} - \tau_1) + \sigma_1 (\ddot{\theta} + \dot{\rho}_1 \dot{\rho}_1)
\]
\[
+ (\theta + \rho_1) \sigma_1 - \frac{1}{\lambda_2} \rho_2 \ddot{\theta}
\]
(20)
where
\[
\xi_2 = [\phi_2^T - \frac{\partial \alpha_1}{\partial x_1} \phi_1^T, z_1 - \frac{\partial \alpha_1}{\partial x_1} x_2, 0, \ldots, 0]^T \in \mathbb{R}^{m+n-1}
\]
\[
s_2 = [-\frac{\partial \alpha_1}{\partial x_1}]^T
\]
(21)
Design the second virtual control input \( \alpha_2 \)
\[
\alpha_2 = N (\chi_2) v_2
\]
\[
v_2 = k_2 z_2 + \beta_2 \ddot{\theta} + \xi_2 \dot{\rho}_2 + \eta_2 - \frac{\partial \alpha_1}{\partial \dot{\theta}} \dot{\theta}
\]
\[
\dot{\chi}_2 = z_2 v_2
\]
(22)
and turning function \( \tau_2 \)
\[
\tau_2 = \tau_1 + \gamma z_2 \beta_2
\]
(23)
with
\[
\beta_2 = \frac{z_2 \zeta_1^T \zeta_2}{\sqrt{z_2^2 \zeta_2^T \zeta_2 + \sigma_2^2}}
\]
\[
\xi_2 = \frac{z_2 s_2^T s_2}{\sqrt{z_2^2 s_2^T s_2 + \sigma_2^2}}
\]
(24)
where \( \sigma_2 \) satisfies \( \lim_{t \to -\infty} \int_0^t \sigma_2(s) \, ds \leq \sigma_2 \leq +\infty \).
According to the definition of $\dot{\chi}_2$, (20) can be transformed into

$$
\dot{V}_2 \leq - \sum_{j=1}^{i-1} k_j z_j^2 + \sum_{j=1}^{i-1} (g_j N (x_j) + 1) \dot{\chi}_j + g_2 z_2 \dot{z}_3
$$

$$
+ z_2 \theta (\dot{\beta}_2 + \dot{\theta}) \| z_2 \| - z_2 \xi_2 \rho_2 + \| z_2 s_j^T \| \rho_2
$$

$$
- \frac{1}{\gamma} \dot{\gamma} (\dot{\gamma} - \tau_2) - z_2 \frac{\partial \alpha_1}{\partial \theta} (\dot{\gamma} - \tau_2)
$$

$$
+ \sigma_1 (\dot{\theta} + \dot{\rho}_1 \dot{\rho}_1) - \frac{1}{\lambda_2} \dot{\rho}_2 (\dot{\rho}_2 - \lambda_2 z_2 \xi_2)
$$

$$
+ (\theta + \rho_1) \sigma_1
$$

(25)

According to the definition $\beta_2$ and $\xi_2$ in (24), it is obtained that

$$
- \dot{\theta} z_2 \beta_2 + \| z_2 \| \| \dot{\xi}_2 \| \leq \theta \sigma_2
$$

$$
- z_2 \xi_2 \rho_2 + \| z_2 \| \rho_2 \leq \rho_2 \sigma_2
$$

(26)

Choosing the adaptive law $\hat{\rho}_2$ as

$$
\dot{\hat{\rho}}_2 = \lambda_2 z_2 \xi_2 - \lambda_2 \sigma_2 \rho_2
$$

(27)

Substituting (26) and (27) into (25), we have

$$
\dot{V}_2 \leq - \sum_{j=1}^{i-1} k_j z_j^2 + \sum_{j=1}^{i-1} (g_j N (x_j) + 1) \dot{\chi}_j + g_2 z_2 \dot{z}_3
$$

$$
- \frac{1}{\gamma} \dot{\gamma} (\dot{\gamma} - \tau_2) - z_2 \frac{\partial \alpha_1}{\partial \theta} (\dot{\gamma} - \tau_2)
$$

$$
+ \sigma_1 \dot{\theta} + \sum_{j=1}^{i-1} \sigma_j \hat{\rho}_j \hat{\rho}_j + \sum_{j=1}^{i-1} (\theta + \rho_1) \sigma_j
$$

(28)

**Step i**, $\{2 \leq i \leq n - 1\}$. According to the derivative of $z_i$ in (5), there exists

$$
\dot{z}_i = g_i z_{i+1} + g_i \alpha_i + \theta^T \phi_i + d_i + \eta_i
$$

$$
- \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} (g_j x_{j+1} + \theta^T \phi_j + d_j) - \frac{\partial \alpha_{i-1}}{\partial \theta} \dot{\gamma}
$$

(29)

where

$$
\eta_i = \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y^{(j)}_r} y_r^{(j)} - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \rho_j} \rho_j - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \dot{x}_j
$$

(30)

Define

$$
V_i = V_{i-1} + \frac{1}{2} z_i^2 + \frac{1}{2 \lambda_i} \hat{\rho}_i^2
$$

(31)

In view of (31), differentiating $V_i$ yields

$$
\dot{V}_i \leq - \sum_{j=1}^{i-1} k_j z_j^2 + \sum_{j=1}^{i-1} (g_j N (x_j) + 1) \dot{\chi}_j + g_2 z_2 z_{i+1}
$$

$$
+ z_2 (g_i \alpha_i + \theta^T \xi_i + \eta_i - \frac{\partial \alpha_{i-1}}{\partial \theta} \dot{\gamma}) + z_i s_i^T D_i
$$

$$
- \frac{1}{\gamma} \dot{\gamma} (\dot{\gamma} - \tau_{i-1}) - \sum_{j=2}^{i-1} z_j \frac{\partial \alpha_{j-1}}{\partial \theta} (\dot{\gamma} - \tau_{j-1})
$$

$$
+ \sigma_1 \dot{\theta} + \sum_{j=1}^{i-1} \sigma_j \hat{\rho}_j \hat{\rho}_j + \sum_{j=1}^{i-1} (\theta + \rho_1) \sigma_j
$$

(32)

where

$$
\zeta_i = [\theta^T - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \theta^T - \frac{\partial \alpha_{i-1}}{\partial x_1} x_2, \ldots, - \frac{\partial \alpha_{i-1}}{\partial x_{i-2}} x_{i-1}]_T
$$

$$
\zeta_i = - \frac{\partial \alpha_{i-1}}{\partial x_{i-1}} x_i, 0, 0, \ldots, 0]_T \in \mathbb{R}^{m+n-1}
$$

$$
s_i = [- \frac{\partial \alpha_{i-1}}{\partial x_1}, \ldots, - \frac{\partial \alpha_{i-1}}{\partial x_{i-1}}, 1]_T
$$

(33)

Design the ith virtual control input $\alpha_i$

$$
\alpha_i = N (\xi_i) v_i
$$

$$
v_i = k_i z_i + \beta_i \dot{\gamma} + \xi_i \dot{\rho}_i + \eta_i - \sum_{j=2}^{i-1} z_j \frac{\partial \alpha_{j-1}}{\partial \theta} \gamma \beta_i
$$

(34)

and turning function $\tau_i$

$$
\tau_i = \tau_{i-1} + \gamma z_i \beta_i
$$

(35)

with

$$
\beta_i = \frac{z_i \zeta_i^T \zeta_i}{\sqrt{z_i^T \zeta_i \zeta_i + \sigma_i^2}}
$$

$$
\xi_i = \frac{z_i s_i^T s_i}{\sqrt{z_i^T s_i s_i + \sigma_i^2}}
$$

(36)

where $\sigma_i$ satisfies $\lim_{t \to \infty} \int_0^t \sigma_i(s) ds \leq \sigma_i \leq + \infty$. 

Subtracting \( \dot{x}_i \) into (32), we can get
\[
\dot{V}_i \leq -\sum_{j=1}^{i} k_j z_j^2 + \sum_{j=1}^{i} (g_j N(x_j) + 1) \dot{x}_j + g_i z_i \dot{z}_{i+1} - z_i \tilde{\beta}_i + \dot{\vartheta} \rho_i \| \zeta_i \| - z_i \xi_i \rho_i + |z_i \sigma_i^T | \rho_i \\
- \frac{1}{\gamma} \vartheta (\dot{\vartheta} - \tau_i) - \sum_{j=2}^{i} z_j \frac{\partial \alpha_{j-1}}{\partial \vartheta} (\dot{\vartheta} - \tau_i) \\
+ \sigma_1 \dot{\vartheta} + \sum_{j=1}^{i-1} \sigma_j \dot{\rho}_j \dot{\vartheta} + \sum_{j=1}^{i-1} (\vartheta + \rho_j) \sigma_j
\]
(37)

According to the definition of \( \dot{\beta}_i \) and \( \dot{\xi}_i \) in (36), it is obtained that
\[
-\vartheta z_i \beta_i + \vartheta \rho_i \| \zeta_i \| \leq \vartheta \sigma_i \\
- z_i \xi_i \rho_i + |z_i \sigma_i^T | \rho_i
\]
(38)

Choosing the adaptive law \( \hat{\rho}_i \) as
\[
\hat{\rho}_i = \lambda_i z_i \xi_i - \lambda_i \sigma_i \dot{\rho}_i
\]
(39)

Substituting (38) and (39) into (37), we have
\[
\dot{V}_i \leq -\sum_{j=1}^{i} k_j z_j^2 + \sum_{j=1}^{i} (g_j N(x_j) + 1) \dot{x}_j + g_i z_i \dot{z}_{i+1} \\
- \frac{1}{\gamma} \vartheta (\dot{\vartheta} - \tau_i) - \sum_{j=2}^{i} z_j \frac{\partial \alpha_{j-1}}{\partial \vartheta} (\dot{\vartheta} - \tau_i) \\
+ \sigma_1 \dot{\vartheta} + \sum_{j=1}^{i-1} \sigma_j \dot{\rho}_j \dot{\vartheta} + \sum_{j=1}^{i-1} (\vartheta + \rho_j) \sigma_j
\]
(40)

**Step n.** Finally, the time derivative of \( z_n \) in (5) gives
\[
z_n = g_n \sum_{j=1}^{n} (\rho_j u_j + x_j, t_j) + \vartheta^T \phi_n + d_n + \eta_n \\
- \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} (g_j x_{j+1} + \vartheta^T \phi_j + d_j) - \frac{\partial \alpha_{n-1}}{\partial \vartheta} \dot{\vartheta} 
\]
(41)

where
\[
\eta_n = \frac{\sum_{j=1}^{n} \frac{\partial \alpha_{n-1}}{\partial y_j} y_j}{\frac{\partial \alpha_{n-1}}{\partial \vartheta} \dot{\vartheta}} - \sum_{j=1}^{n} \frac{\partial \alpha_{n-1}}{\partial \rho_j} \dot{\rho}_j - \sum_{j=1}^{n} \frac{\partial \alpha_{n-1}}{\partial x_j} \dot{x}_j
\]
(42)

In addition, it also leads to the construction of the following Lyapunov function
\[
V_n = V_{n-1} + \frac{1}{2} z_n^2 + \frac{1}{2} \beta_n \rho_n^2
\]
(43)

Then, differentiate \( V_n \) along the solution of system (41) to yield
\[
\dot{V}_n \leq -\sum_{j=1}^{n-1} k_j z_j^2 + \sum_{j=1}^{n-1} (g_j N(x_j) + 1) \dot{x}_j \\
+ z_n (g_n \sum_{j=1}^{m} \rho_j u_j + \vartheta^T \zeta_n + \eta_n - \frac{\partial \alpha_{n-1}}{\partial \vartheta} \dot{\vartheta}) \\
+ z_n s_n^T D_n - \frac{1}{\gamma} \vartheta (\dot{\vartheta} - \tau_{n-1}) - \sum_{j=2}^{n-1} z_j \frac{\partial \alpha_{j-1}}{\partial \vartheta} (\dot{\vartheta} - \tau_{n-1}) \\
+ \sigma_1 \dot{\vartheta} + \sum_{j=1}^{n-1} \sigma_j \dot{\rho}_j \dot{\vartheta} + \sum_{j=1}^{n-1} (\vartheta + \rho_j) \sigma_j
\]
(44)

where
\[
\zeta_n = [\phi_n^T - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} x_{n-1}, \ldots, \frac{\partial \alpha_{n-1}}{\partial x_{n-2}} x_{n-1}, \ldots, \frac{\partial \alpha_{n-1}}{\partial x_1} x_n, 0, \ldots, 0]^T \in \mathbb{R}^{m+n-1} \\
\tau_n = [\frac{\partial \alpha_{n-1}}{\partial x_1}, \ldots, \frac{\partial \alpha_{n-1}}{\partial x_{n-1}}, 1]^T
\]
(45)

Design the actual control input \( u_j \) as
\[
u_n = N(x_n) v_n \\
u_n = k_n z_n + \beta_n \dot{\vartheta} + \xi_n \dot{\rho}_n + \eta_n - \sum_{j=2}^{n-1} z_j \frac{\partial \alpha_{j-1}}{\partial \vartheta} \gamma \beta_n
\]
(46)

\[
\dot{x}_n = z_n v_n \\
\text{and turning function } \tau_n
\]
(47)

with
\[
\beta_n = \frac{z_n \xi_n^T \xi_n}{\sqrt{z_n^2 s_n^T s_n + \sigma_n^2}} \\
\xi_n = \frac{z_n \xi_n^T s_n}{\sqrt{z_n^2 s_n^T s_n + \sigma_n^2}}
\]
(48)

where \( \sigma_n \) satisfies \( \int_{-\infty}^{t_n} \sigma_n(s) ds \leq \sigma_n \leq +\infty. \)
Substituting (53) into (52), we can get
\[
\dot{V}_n \leq -\sum_{j=1}^{n} k_j \dot{z}_j^2 + \sum_{j=1}^{n} (g_j N(x_j) + 1) \dot{x}_j \\
+ \left( g_n \sum_{j=1}^{m} \rho_j h N(x_n) + 1 \right) \dot{x}_n \\
- \frac{1}{\gamma} \nabla \dot{\varphi} (\dot{\varphi} - \tau_n) - \sum_{j=2}^{n} \frac{\partial x_j}{\partial \varphi} \frac{d x_j}{d \varphi} (\dot{\varphi} - \tau_n) \\
+ \sigma_1 \dot{\varphi} - \frac{1}{\lambda_n} \rho_n (\dot{\rho}_n - \lambda_n \dot{x}_n) \\
+ \sum_{j=1}^{n} \sigma_j \dot{\rho}_j \dot{x}_j + \sum_{j=1}^{n} (\varphi + \rho_j) \sigma_j 
\]
(49)

Then, similar to the derivation of (38), it can be deduced that
\[
-\varphi \dot{\varphi} \leq \varphi \sigma_n \\
-\dot{\varphi} \leq \varphi \sigma_n 
\]
(50)

Furthermore, the adaptive laws \(\dot{\rho}_n\) and \(\dot{\varphi}\) can be designed as
\[
\dot{\rho}_n = \varphi \dot{\rho}_n - \lambda_n \varphi \sigma_n \\
\dot{\varphi} = \varphi \sigma_n - \lambda_n \dot{\rho}_n 
\]
(51)

By completing the squares, one has
\[
\dot{\rho}_n \dot{\rho}_j \dot{x}_j = \varphi \dot{\rho}_n (\dot{\rho}_j - \dot{\rho}_j) = -\varphi \dot{\rho}_j \leq \varphi \sigma_j \\
\dot{\varphi} \dot{\varphi} = \varphi \sigma_n - \lambda_n \dot{\rho}_n 
\]
(53)

Substituting (53) into (52) yields
\[
\dot{V}_n \leq -\sum_{j=1}^{n} k_j \dot{z}_j^2 + \sum_{j=1}^{n} (g_j N(x_j) + 1) \dot{x}_j \\
+ \frac{\sigma_1 \varphi^2}{4} + \sum_{j=1}^{n} (\varphi + \rho_j) \sigma_j 
\]
(54)

3.2 Stability analysis

Based on the aforementioned analysis and design, the following major result was concluded.

**Theorem 1** If the uncertain nonlinear system (1) satisfies Assumptions 1-3, the adaptive controller is designed as (46) and the adaptive laws are given by (51), we can conclude that the whole signals of the controlled system are globally uniformly bounded and the actual output \(y\) can track the desired bounded reference signal \(y_r\) asymptotically.

**Proof.** Integrating both sides of (54) from from \(t_0\) to \(t\), we obtain
\[
V_n(t) \leq V_n(t_0) - \sum_{j=1}^{n} k_j \int_{t_0}^{t} \dot{z}_j^2 (s) ds \\
+ \sum_{j=1}^{n} \int_{t_0}^{t} (g_j N(x_j) + 1) \dot{x}_j + \frac{\varphi^2}{4} \int_{t_0}^{t} \sigma_j (s) ds \\
+ \sum_{j=1}^{n} \int_{t_0}^{t} (\varphi + \rho_j) \sigma_j (s) ds \\
\leq \sum_{j=1}^{n} \int_{t_0}^{t} (g_j N(x_j) + 1) \dot{x}_j d\tau + \Delta_0 
\]
(55)

With \(\Delta_0 = V_n(t_0) + \varphi^2 \sigma_1 + \sum_{j=1}^{n} (\varphi + \rho_j) \sigma_j + \sum_{j=1}^{n} \int_{t_0}^{t} (g_j N(x_j) + 1) \dot{x}_j d\tau\), we can obtain that \(\dot{\rho}_n\) is bounded. Noting that \(z_{i+1} = z_i - y_r\), one has the boundedness of \(z_i, \dot{z}_i, \ddot{z}_i\) for \(i = 1, 2, \ldots, n\). From (55), the boundedness of \(z_i, \dot{z}_i, \ddot{z}_i\) and the boundedness of \(z_{i+1}, \dot{z}_{i+1}, \ddot{z}_{i+1}\) are bounded on \([0, +\infty)\). In view of the definition of \(V_n(t)\), we can obtain that \(\dot{\rho}_n\) is bounded. By repeated iteration, it can be concluded that \(x_i, i = 3, 4, \ldots, n\) are all bounded on \([0, +\infty)\). With the boundedness of the close-loop system, it leads to the conclusion that the whole signals of the controlled system are bounded on \([0, +\infty)\).

Subsequently, we will demonstrate that \(\lim_{t \to \infty} z_1 = 0\), that is, \(y\) tracks \(y_r\) asymptotically.

From (55), one has
\[
\lim_{t \to \infty} \sum_{j=1}^{n} k_j \int_{t_0}^{t} \dot{z}_j^2 d\tau \leq \int_{t_0}^{t} (g_j N(x_j) + 1) \dot{x}_j d\tau + \Delta_0 \leq +\infty 
\]
(56)

Based on the boundedness of the whole signals of the controlled system, we can obtain that \(\dot{z}_j, \ddot{z}_j\) are bounded,
which implies $z_j(t)$ are uniformly continuous on $t$. Thus, by using the Barbalat Lemma [39], one has
\begin{equation}
\lim_{t \to \infty} z_j = 0, \quad j = 1, \ldots, n
\end{equation}
Therefore, we can conclude that the asymptotic convergence is achieved. □

Remark 3 To the literature of we are aware, the existing FTC schemes [25, 32, 29, 30, 33, 34, 22, 23, 27, 36, 24, 26, 31], which require that the signs of unknown control functions are known or the system is free from the uncertain parameters will be invalid when applied to the system (1) with time-varying parameters and unknown control directions. Theorem 3.1 is the first result on global asymptotic tracking of the uncertain nonlinear system (1) with unknown control directions and actuator failure. Based on Nussbaum function and bound estimation method, an adaptive FTC method was designed to achieve global asymptotic convergence of the tracking error for the system (1) and ensure the boundedness of the whole signals of the controlled systems.

4 Simulation Results

In this paper, we chose the following faults as
\begin{align}
&u_1^F(t) = \psi_{1,h}, t \in [3h, 3(h + 1)] \\
&u_2^F(t) = \psi_{2,h}u_2(t), t \in [3h, 3(h + 1)], h = 1, 3, 5, \ldots
\end{align}

Remark 4 From the observation of $\theta(t) = 1.05-0.1 \sin(20t)$, we can obtain that the controlled system are uncertain nonlinear system with time-varying parameters, which brings great difficulties to the process of controller design. Therefore, to solve the problems resulted from time-varying parameters, the bound estimation method were proposed in this paper.

Remark 5 The actuator failure (60) implies that the first actuator suffers stuck fault every 3 seconds and returns to normal next 3 seconds. The second experiences the partial loss of validity every 3 seconds and recovers next 3 seconds. And the parameters are designed as $\psi_{1,h} = 0.1$ and $\psi_{2,h} = 0.5$.

In this section, the initial values are selected as $x(0) = [0.2, 0.2]^T, \dot{\theta}(0) = 0.1, \ddot{\rho}(0) = [0.4, 0.2]^T, \chi(0) = [0.1, 0.1]^T$. The other parameters are designed as $k_1 = 0.1, k_2 = 0.1, \sigma_1 = 8e^{-0.01t}, \sigma_2 = 2e^{-0.01t}, \lambda_1 = \lambda_2 = 1, \gamma = 1$.

Fig. 1 State trajectories of $y_1$ and $y_2$.

The simulation results are shown as the following pictures in Figs. 1-6. Fig. 1 plots the performance of output variable $y$ and the desired reference signal $y_r$, from which we can see that the output of the controlled system can track the reference signal asymptotically. Fig. 2 shows the trajectories of states. Fig. 3 and Fig. 4 display the actual input and output of actuators. The parameters are plotted in Fig. 5 and the tracking errors are showed in Fig. 6. It is apparent that the whole signals are bounded from the observation of Figs. 2-5.

Generally, through the simulation results, we can conclude that the difference between output and reference signal converges to zero asymptotically while the whole signals of the controlled system are bounded.
5 Conclusion

This article has considered the adaptive asymptotic tracking problems for the uncertain nonlinear systems with unknown nonlinear functions and time-varying parameters. By utilizing the construction of positive smooth function and backstepping technique, an adaptive asymptotic tracking controller of uncertain nonlinear systems with unknown control directions and actuator faults have been designed successfully. The difficulties caused by unknown control directions and unknown uncertainties were settled by the Nussbaum function and bounds estimation method. Moreover, the presented scheme can ensure the boundedness of the whole signals and the performance of zero-error tracking. Finally, the validity of the presented scheme is demonstrated by a simulation example.

Acknowledgements

This work was supported in part by the Funds of National Science of China (Grant Nos. 61973146, 61773188), in part by the Doctoral Research Initiation of Foundation of Liaoning Province (No. 20180540047), and in part by the Distinguished Young Scientific Research Talents Plan in Liaoning Province (Nos. XLYC1907077, JQL201915402).
Declarations

Conflict of interest

The authors declare that they have no conflict of interest.

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