Jones Matrix Holography with Metasurfaces

Aun Zaidi*, Noah A. Rubin, Ahmed Dorrah, Zhujun Shi, and Federico Capasso
John A. Paulson School of Engineering and Applied Sciences, Harvard University, Cambridge, Massachusetts 02138, USA
*azaidi01@g.harvard.edu

Abstract: We propose a new class of computer generated holograms whose far fields possess designer-specified polarization response. Using form-birefringent metasurfaces, we demonstrate holograms whose far-fields implement parallel polarization analysis and custom waveplate-like behavior. © 2021 The Author(s)

1. Introduction

Adolf Lohmann, a pioneer of the computer generated hologram (CGH), once remarked in one of his papers [1] that “a hologram is not really a total recording, since only one amplitude and one phase are recorded, which would be adequate if light were a scalar wave.” Indeed, light’s polarization is often ignored in the study of holography and diffractive optics. However, recent advances in technologies, such as metasurfaces [2, 3], permitting precise control of polarization over wavelength-scales, have sparked new interest in the field of polarization controlled holography. Metasurfaces, composed of arrays of engineered subwavelength nanostructures, are the specific focus and implementation medium of this work.

It is well known that in the paraxial regime, the Fourier transform \( \mathcal{F} \), links the ‘near-field’ (incident field times the aperture function) with the far-field, a desired phase and/or amplitude distribution many wavelengths away. Often, a hologram (referring to the transforming media and not the image) is described by its spatially-varying, complex-valued aperture transmission function \( t(x, y) \). Most approaches to holography involving metasurfaces have either been scalar, by implementing \( t(x, y) \), or vectorial, by implementing \( t_x(x, y) \) and \( t_y(x, y) \) independently for \( X \) and \( Y \) polarizations respectively. The most general picture, as the one we use in our work, handles polarization by describing the hologram instead by a \( 2 \times 2 \) Jones matrix transfer function \( \mathbf{J}(x, y) \).

2. Concept

We start by describing the metasurface as spatially-varying Jones matrices \( \mathbf{J}(x, y) \) [4]. Assuming plane wave incidence, the far-field Jones matrix transformations, analogous to the scalar case, are given by a Fourier transform \( \mathbf{A}(k_x, k_y) = \mathcal{F}\{\mathbf{J}(x, y)\} \), where the Fourier transform now distributes over all four elements of the matrix. \( \mathbf{A}(k_x, k_y) \) essentially gives the polarization dependent behavior of each plane wave \( (k_x, k_y) \) in the far field. Now if a far-field polarization-dependent response described by some \( \mathbf{A}(k_x, k_y) \) is desired, a ‘Jones matrix hologram’ implementing it is given by the inverse Fourier transform:

\[
\mathbf{J}(x, y) = \mathcal{F}^{-1}\{\mathbf{A}(k_x, k_y)\}
\]

The Jones matrix \( \mathbf{J}(x, y) \) can be decomposed using the matrix polar decomposition as:

\[
\mathbf{J} = \mathbf{H}\mathbf{U}
\]

where \( \mathbf{H} \) is Hermitian (lossy, polarizer like) and \( \mathbf{U} \) is unitary (lossless, waveplate-like). Now note that a dielectric metasurface is entirely composed of transmissive nanopillars performing unitary (and symmetric) transformations. Meanwhile, in general, \( \mathbf{J}(x, y) = \mathcal{F}^{-1}\{\mathbf{A}(k_x, k_y)\} \) of a desired field \( \mathbf{A}(k_x, k_y) \) will not simply be unitary but a product of Hermitian and unitary transformations. So, how can we inverse design a unitary and symmetric only \( \mathbf{J}(x, y) \) for a desired far-field response \( \mathbf{A}(k_x, k_y) \)? We realize that this problem is analogous to the problem of designing far-field intensity holograms, with phase-only media such as liquid crystals and metasurfaces. Using this analogy, we adapt and innovate upon an already existing scalar phase-retrieval algorithm known as Gerchberg Saxton [5] to deal with matrix distributions instead of scalar distributions. Our new iterative algorithm allows us to design Jones matrix holograms using metasurfaces, whose far-fields exhibit designer-specified polarization behavior.
3. Results

An experimental demonstration of one particular design is shown in Fig. 1. The fabricated metasurface is illuminated with collimated laser light (\(\lambda = 532\) nm) of variable polarization. The angular spectrum (far-field) that results is imaged onto a CMOS sensor using a relay setup. Images are acquired for many input polarization states (without saturation), permitting a full polarimetric characterization of the grating’s response.

Fig. 1. Illustrative example of a polarization-analyzing hologram: (a) Illustration of the experimental illumination and desired far-field response. (b) Actual measured response of the hologram recorded on a CCD sensor. Scale bar shows the cone angle subtended by the far-field.

Fig. 1(b) depicts the far-field produced by the metasurface hologram for six incident polarization states, each of which is denoted in the bottom left corner of its image by a white label. As can be seen, each incident polarization state prompts the strongest response in the region of the hologram corresponding to itself. For example, \(|45°⟩\) prompts the hologram to direct most power to the drawing of diagonal, linearly polarized light, while the image of anti-diagonal polarization is dark with a gradient in-between, while drawings of \(|x⟩, |y⟩, |R⟩, and |L⟩\) are all about half as bright. When circular polarization is incident, all linear polarizations are about equally bright (and half as bright as the image of the incident circular state). The intensity of each polarization depiction is proportional to the projection of the incident polarization state onto the depicted state in accordance with Malus’ Law. This metasurface demonstration is an example of what can be achieved with our new algorithm. In our complete work, we also demonstrate other specifically designed polarization holograms, including wave-plate like holograms where the far-field retardance (and eigen-axes) can be similarly controlled on a pixel level.

As we show in our work, a treatment based on the Jones calculus enables the design and analysis of these holograms without specification of the incident polarization state. This work may find application in a variety of areas. A Jones matrix hologram could add custom polarization-dependence to an optical system’s point spread function, either to address systematic polarization aberrations \([6]\) in precision imaging systems or to enable wholly new functionalities.

References

1. A. W. Lohmann, “Reconstruction of Vectorial Wavefronts,” *Applied Optics*, vol. 4, no. 12, p. 1667, 1965.
2. A. Arbabi, Y. Horie, M. Bagheri, and A. Faraon, “Dielectric metasurfaces for complete control of phase and polarization with subwavelength spatial resolution and high transmission,” *Nature Nanotechnology*, vol. 10, no. 11, pp. 937–943, 2015.
3. J. P. B. Mueller, N. A. Rubin, R. C. Devlin, B. Groever, and F. Capasso, “Metasurface polarization optics: independent phase control of arbitrary orthogonal states of polarization,” *Physical Review Letters*, vol. 118, p. 113901, 2017.
4. N. A. Rubin, G. D’Aversa, P. Chevalier, Z. Shi, W. T. Chen, and F. Capasso, “Matrix Fourier optics enables a compact full-Stokes polarization camera,” *Science*, vol. 365, no. 6448, p. eaax1839, 2019.
5. R. W. Gerchberg and W. O. Saxton, “A practical algorithm for the determination of phase from image and diffraction plane pictures,” *Optik*, vol. 35, no. 2, pp. 237–246, 1972.
6. R. A. Chipman, W.-S. T. Lam, and G. Young, *Polarized Light and Optical Systems*. CRC Press, 2019.