High profile students’ growth of mathematical understanding in solving linear programming problems

Utomo1, TA Kusmayadi1, I Pramudya1

1Department of Mathematics Education, Sebelas Maret University, Surakarta, Indonesia
E-mail: utomo@student.uns.ac.id

Abstract. Linear program has an important role in human’s life. This linear program is learned in senior high school and college levels. This material is applied in economy, transportation, military and others. Therefore, mastering linear program is useful for provision of life. This research describes a growth of mathematical understanding in solving linear programming problems based on the growth of understanding by the Piere-Kieren model. Thus, this research used qualitative approach. The subjects were students of grade XI in Salatiga city. The subjects of this study were two students who had high profiles. The researcher generally chose the subjects based on the growth of understanding from a test result in the classroom; the mark from the prerequisite material was ≥ 75. Both of the subjects were interviewed by the researcher to know the students’ growth of mathematical understanding in solving linear programming problems. The finding of this research showed that the subjects often folding back to the primitive knowing level to go forward to the next level. It happened because the subjects’ primitive understanding was not comprehensive.

1. Introduction

Program linear is one of the obligatory materials for competency tests in mathematics class of senior high school students grade XI [5]. Mathematically, the program linear is a part of applied mathematics, with the model of mathematics which is made up of equation or inequality linear, regional work level unit containing a program development for solving various problems [14]. Linear program is widely applied to help solve economic problem, industry, military, and social [16]. Therefore, it is important for the students to comprehend the material well that it will be used in workplaces.

Understanding is an important aspect in learning mathematics [12]. Understanding is also the center of developing curriculum in every country [10]. The importance of understanding in learning makes many curricula suggest to create mathematics learning by understanding [12]. In addition, understanding is the point of learning mathematics in school [17]. The importance of understanding makes education experts try to define it. One of the experts is Pirie-Kieren.

Understanding is a level that is not linear and it will take place continuously. Understanding will never stop as long as the learning level takes place and has eight levels of growth such as primitive knowing (PK), image making (IM), image having (IH), property noticing (PN), formalising (F), observing (Ob), structuring (S), and inventising (I) [12,15].
Piere-Kieren defined the description of each level of growth of mathematical understanding; the first level is primitive knowing. The growth of mathematical understanding process starts from this level. This level also becomes the background knowledge of mathematical understanding which is used to construct an understanding about a certain concept. In this research, the students could write the information of the problem. The second level is image making. On this level, the students can make a difference from the previous knowledge and use it in a new way. In this research, the students could write the relation and arrange the information from the problem. The third level is image having. On this level, the students can use the mental illustration about a certain topic without doing any activity that triggers the illustration. In this research, the students could create examples, write constraint and goal functions from the information. The fourth level is property noticing. On this level, the students can manipulate or combine the aspects from an image to construct a characteristic in a context. In this research, the students could develop that the maximum and minimum values can be determined from the goal function. The fifth level is formalising. On this level, the students can create an abstract of mathematical concept based on the characteristic that appears. This research shows that the students can give true meaning to all values that have been obtained, and formalize or confirm how the optimum value is true. The sixth level is observing. On this level, the students can reflect and coordinate formal activities so they are able to be used in some problems that they face as well as express them as theorems. The students also ask about how a question about concepts can be related to each other and how to find patterns in order to define ideas as algorithm or theorem. In this research, the students could say that a linear programming problem has an optimum value at a critical point. The seventh level is structuring. This level happens when the students think about a formal observation as the theory and say a logical argument in form of evidence. In this research, the students could say that a linear programming problem has a limited area of set completion so the optimum value is at the critical point. The eighth level is inventising. On this level, the students can think about a formal observation as the theory and they can get an understanding about a concept as well as ask questions, and then construct a new concept.

There are much research, because of the importance in learning process, in order to improve students' mathematical understanding in the last period [7]. Research that are related to understanding are as followed: Investigating the use of initial knowledge about how to teach logarithm function [4], Illustrating the theory of Pirie-Kieren in the context of language and exam contest [6], The use of Pirie-Kieren's understanding model to investigate the role of definition in proving the theorem [11], and Investigating the increase of students' mathematical understanding that uses learning model of problem posing [3].

In Indonesia, students' understanding about the linear program material is still low. This statement is supported by data based on the results of Senior High School (Science Stream) National Examination in 2015/2016. The results of the examination show that students’ absorption in solving daily problems which are related to the linear program is only 52.70%. This shows that the ability of students in solving mathematics problems is still low.

The description above shows that the importance of mathematical understanding in learning process is no exception for linear program material. For that reason, the researcher is interested to investigate the students' growth of mathematical understanding in solving the linear programming problems based on the growth of the mathematical understanding by Pirie-Kieren.
2. Method

This research aims to find out the students growth of mathematical understanding in solving the linear programming problems based on the theory of Pirie-Kieeren. This research used the descriptive qualitative method. This research was conducted in SMA Negeri 1 Salatiga (Grade XI) whereas the subjects of the research were 2 students. The researcher generally chose the subjects based on the growth of understanding in the classroom from the test result, the mark obtained from the prerequisite material (≥ 75) and teachers’ feedback.

The instruments of this research were test worksheets. The students’ answers are strengthened by interviews that were used to know the growth of subjects’ understanding. The data was obtained from the students’ test answers and interviews. The data analysis technique was reducing the data, presenting the data and conclusion. Reducing data is selecting, focusing and simplifying the data obtained. Presenting the data arranges the information in narration and the reducing result is used to get the conclusion.

3. Result and Discussions

At the beginning, I gave a test to the students. It was used to find out students’ growth of mathematical understanding toward the problem given by the researcher. The subjects of this research were a male student and a female student. In choosing the subject for this research, the researcher took a look at the prerequisite material which was about inequality, by drawing on an inequality graphic whose value was ≥ 75.

In subject S1 the value of prerequisite material was 75 and for the S2 was 87. I also generally chose the subject based on the students’ growth of mathematical understanding in the classroom in solving the problem that was given by the teacher and also feedback from the teacher.

3.1. Result

3.1.1. Subject 1

1. Tuliskan apa saja yang diketahui!

\[
\begin{align*}
\text{Rot A} & \quad \text{Rot B} \\
50 \text{ gr mentega} & \quad 100 \text{ gr mentega} \\
60 \text{ gr tepung} & \quad 20 \text{ gr tepung} \\
\text{Harga paket Rot A} & \quad \text{Rp 20.000} \\
\text{Harga paket Rot B} & \quad \text{Rp 25.000}
\end{align*}
\]

Figure 1. S1’s answer for No.1

The initial process done by S1 was writing the information that could be found in the problem. S1 wrote the information about bread A, bread B and the price of the package of each bread. S1 had not written the availability of the butter and the flour (Figure 1). According to Pirrie-Kieren, this level means that the student is in primitive knowing level. S1 also had the prerequisite material to learn the linear program. The prerequisite material was linear inequalities and drawing the linear inequalities’ graphics. The student’s answer was strengthened by an interview which was done by the researcher with S1. Below is the interview result:
R : “What can you find from the problem given?”
S1 : “There are bread A and bread B. For bread A, there are 50 gr of butter and 60 gr of flour and there are 100 gr of butter and 20 gr of flour for bread B. Then, the price of package of bread A is Rp20.000,00 and the price of package of bread B is Rp25.000,00”
R : “Do you think it is enough to know that information only?”
S1 : “(nodding) yes, it is enough”

Figure 2. S1’s answer for No.2

The next step was finding out the relation of the information that was known from the problem. From the written constraints, they show that the student can write the example of the information (Figure 2). When S1 was able write the example, it gave a strong clue that S1 moved from the primitive knowing level to the image making level. It was also strengthened by the result of the interview below:

P : “Okay. I want to ask. So, what is x?”
S : “The butter of bread A”
P : “Is it correct if x is the butter of bread A? Please try to understand carefully, you wrote x in making a package of bread A, it needs 50 gr of butter and 60 gr of flour. So, what is x? Is it butter or what?”
S : “(the student was still reading and smiling)”

The following step was writing the constraints. The constraints were 50 + 100y ≤ 3,5 with 60x + 20y ≤ 2,2 and x ≥ 0 with y ≥ 0. The S1’s growth of mathematical understanding moved from the image making level to the image having level. S1 faced difficulties when S1 wrote 50x + 100y ≤ 3,5 and 60x + 20y ≤ 2,2. Therefore, she needed to go back from the image having level to the primitive knowing level. It was strengthened by the interview result of the researcher with S1 as follows:

P : “According to you, what is the mathematical form of that kind of problem?”
S : “The mathematical form is determined by the type, the first is 50x+100y≤3,5. It means that 50 gr of bread A and 100 gr of butter. Then, the butter means the butter.”

Figure 3 shows S1’s flow of thought. S1 also had a concept that will be used to solve the problem given even though it was not correct. According to Pierre-Kieren, S1’s answer shows the level from image having level to property noticing level. S1 was able to determine the intersection of the line with the x and y axes. S1 did not change the inequality of 50x + 100y ≤ 3,5 into the equation at the beginning. That constraint still needs to be fixed, so the result was y ≤ 350 which
is not correct. In the second constraint, $S1$ still did the same thing and the result was $x \leq 44$. $S1$ also did the same thing while determining 17 intersections of two lines, so the intersections of two lines were $x \leq \frac{17}{500}$ and $y \leq \frac{18}{100} \times \frac{1}{100}$. $S1$ also made the linear inequality graphics and determined 100 areas of set completion even though they were incorrect. The constraints faced by $S1$ made $S1$ went back from the property noticing level to the primitive knowing level again.

Based on the data obtained by $S1$, I can make a scheme from stages of the student’s growth of mathematical understanding based on Pirie-Kieren model. The scheme of $S1$’s student growth of mathematical understanding is shown in Figure 4.

![Figure 3. S1’s answer for No. 3](image-url)
Figure 4. Pirie-Kieren’s model of S1’s growth of mathematical understanding

Details of Figure 4:

- PN : Primitive Knowing
- IM : Image Making
- IH : Image Having
- PN : Property Noticing
- F : Formalising
- Ob : Observing
- St : Structuring In: Inventising

------------------- : Student’s growth of mathematical understanding which is incorrect
------------------- : Student’s growth of mathematical understanding which is correct

3.1.2 Subject 2

Figure 5. S2’s answer for No.1

The initial level done by S2 was writing the information that it could be found in the problem. S2 wrote the information about bread A, bread B and the prices of the package of each bread. S2 had not written the availability of the butter and the flour. According to Pirrie-Kieren, this level means that student is in primitive knowing level. S2 also had the prerequisite material to learn the linear
program. The prerequisite material was linear inequalities and drawing the linear inequalities’
graphics.

The next process was finding out the relation of the information from the problem. It was
showed by S2 in Figure 5. S2 could make a good example of this. According to Pirie-Kierin, what
was done by S2, there is a movement from the primitive knowledge level to the image making
level. Then, S2 wrote the constraints that S1 found in the problem. The constraints were $100m +
20t \geq 0$ and $50m + 60t \geq 0$. The level done by S2 was a movement from the image making
level to the image having level. While writing the constraints, S2 faced difficulty in writing so S2
needed to be back from the image having level to the primitive knowing level. S1’s student growth
of mathematical understanding suits with Pirie–Kierin model which moves into the higher level.
The level is property noticing. S2 was able to write the representative of the functions of the goal
from the problem given correctly. S2 could determine the intersection from the line even it was not
correct. The S2’s answer were $100m + 20(0) = (100, 0)$ and $100(0) + 20t = (0, 20)$. The
other intersections were $50m + 60(0) = (50, 0)$ and $50(0) + 60t = (0, 60)$. The student had
not known how to draw the graphic function from the constraints. Based on the data obtained by S2,
the researcher can make a scheme from the stages of student’s growth of mathematical
understanding based on Pirie-Kierin model. The scheme of student’s growth of mathematical
understanding is shown in the Figure 6.

![Figure 6. S2’s answer for No.3](image-url)
Figure 7. S2’s growth of mathematical understanding by Pirie – Kieren model

Details of Figure 7:

| PN   | IM   | IH   | PN   | F    | Ob   | St   | In   |
|------|------|------|------|------|------|------|------|
| PK   | IM   | IH   | PN   | F    | Ob   | St   | In   |

---

The subject S1 and S2’s growth of mathematical understanding were started from the primitive knowing level until the property noticing level and the process had not run well. Both of the subjects wrote the information from the problem incompletely which means that the primitive knowing level had not yet been fulfilled. It was based on the answers and strengthened by interview results.

3.2. Discussions

I will discuss further about subject S1. S1’s growth of mathematical understanding occurred in several stages. Those levels will be elaborated further from the subject’s answers. S1’ answer shows that when S1 wrote the information from the problem (primitive knowing level), S1 had not written the information completely therefore researcher should ask again. One of the student’s mistakes was the student did not understanding the problem well [2]. The next level done by S1 was seeing the relation between the information in the problem (image making level), so S1 could determine the variables (image having level). It could be done by the student well. The next level (property noticing level) was writing the S1’s constraints. S1 still faced difficulties because S1 forgot to adjust the unit of the problem at the beginning. S1 needed to come back to the primitive knowing level in order to continue to the higher level. Furthermore, one of the student’s mistakes was modeling the problem into mathematical form incorrectly [2]. S1 had also determined the
intersection point of the x and y axes. However, S1 had not changed the constraints (inequality) into an equation. Most students are still confused with the operation that they are going to use [13]. S1 also drew the graphics from the constraints, but S1 faced difficulties again when S1 determined areas of set completion. This is the same with a research conducted by Cahyani [1]. She stated that students’ mistake is the mistake when the students draw the areas worthy of linear programming problems.

S1’s growth of mathematical understanding apparently must have often been returning to the lower level (primitive knowing) to expand S1’s understanding. It is suitable with the Pieri – Kieren’s theory that is when someone faces the condition where he/she cannot answer directly related to the problem, he/she can return to the lower level. It is similar to what expressed by [9]. She stated that when someone finds a mistake that can be solved immediately, he/she must fold back to a deeper level.

Growth of mathematical understanding of the subjects toward the mistakes given is suitable with Pirie – Kieren theory. The growth of mathematical understanding started with primitive knowing, image making, image having and property noticing even though it was not correct. It means that the subjects must master the prerequisite material before learning new materials. The previous researchers stated that college students still often return to the previous understanding level in order to understand the concept of limit [17] and also folding back is an important activity to the growth of understanding [8].

4. Conclusion
Growth of mathematical understanding of high profile students occurred in some steps. The first step (primitive knowing step) was started since the subjects have initial knowledge about inequality, drawing graphics of linear inequality, after reading the problem, the subjects wrote the information again from the problem. The second step (image making step) happened when the subjects wrote examples (image having step). The fourth (property noticing step) happened when then subjects determined the constraints, and the maximum value of the goal’s function.

Growth of mathematical understanding in solving the problem of linear programming problems is based on Pirie – Kieren theory on the primitive knowing level until property noticing level. When the students are on the higher level, the students should still return to primitive knowing level. It is because the students’ primitive knowing level had not been fulfilled. In other words, the students had not mastered the prerequisite material to solve the programming linear problems. Therefore, the students should be given questions to strengthen their understanding toward a growth of mathematical understanding level.

Hence, in teaching mathematics materials in classroom, it is very important for the teachers to give a deep reinforcement of the prerequisite material before starting new material. Furthermore, the process of learning mathematics should pay attention to each layer of understanding by Pirie-Kieren so that the process of students’ understanding runs well.

Acknowledgments
The authors would like to thank the references for their suggestions and Sebelas Maret University, Surakarta Indonesia for finding this research in the academic years of 2017.
References

[1] Cahyani D, Ismail Y, and Yahya, L 2015 Identifikasi Kesalahan Siswa dalam Menyelesaikan Soal Cerita Matematika pada Materi Program Linier (Gorontalo: Prodi Matematika Universitas Negeri Gorontalo)

[2] Fitria T N 2013 Analisis Kesalahan Siswa dalam Menyelesaikan Soal Cerita Berbahasa Inggris pada Materi Persamaan dan Pertidaksamaan Linier Satu Variabel. MATHeunesa 2 (1)

[3] Ghanny and Ferdinanto 2014 Meningkatkan Kemampuan Pemahaman Matematis Siswa melalui Problem Posing. Journal Euclid 1 I-59

[4] Kastberg 2002 Understanding mathematical concepts: The case of The Logarithmic Function Georgia (Disertasi of The University of Georgia in Partial)

[5] Kemedikbud 2016 Lampiran Permedikbud Nomor 24 Tahun 2016 Tentang KI KD Kurikulum 2013 (Jakarta: Kementerian Pendidikan dan Kebudayaan RI)

[6] Manu S 2005 Growth of Mathematical Understanding in a Bilingual Context: Analysis and Implications. Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education 3 289-296

[7] Martin L, LaCroix L, and Fownes L 2005 Folding back and Growth of Mathematical Understanding in Workplace Training. ALM International Journal 1 19-35

[8] Martin L 2008 Folding Back and Dynamical Growth of Mathematical Understanding: Elaborating The Pirie-Kieren Theory. The Journal of Mathematical Behavior 27 64 – 85

[9] Mell D E 2003 Models and theories of mathematical understanding: comparing pirie and kieren’s model of the growth of mathematical understanding and APOS theory USA: CBMS Issues in Mathematical Education 12

[10] Mousley J 2005 What Does mathematics Understanding Look Like?. Paper presented at the the Annual Conference held at RMIF, Melbourne 7 – 9 Juli.

[11] Paremeswara R 2010 Expert Mathematicians’ Approach to Understanding Definitions. The mathematics Educator 2010 20(1) 43 – 51

[12] Pirie S and Kieren T 1994 Growth in Mathematical Understanding: How We Can Characterize it and How We can Represent it. Educational Studies in Mathematics 9 160–190

[13] Priawan I M, Abbas N, and Nurwan 2015 Pemecahan Masalah Matematis pada Materi Persamaan dan Pertidaksamaan Linier Satu Variabel di Kelas VII SMP Negeri 1 Batuda (Gorontalo: Prodi Pendidikan Matematika Universitas Gorontalo)

[14] Rahmi and Mulia S 2016 Buku Ajar Program linier (Yogyakarta: Deepublish).

[15] Siswono T Y E 2009 Pengembangan Model Pembelajaran Matematika Berbasis Pengajuan dan Pemecahan Masalah untuk Meningkatkan Kemampuan Berpikir Kreatif Siswa (Departemen Pendidikan Nasional: Makalah Simposium Pusat Penelitian)

[16] Sri Mulyono 2005 Riset operasi (Jakarta: Fakultas Ekonomi UI)

[17] Susiswo 2014 Folding Back Mahasiswa dalam Menyelesaikan Masalah Limit berdasarkan Pengetahuan Konseptual dan Pengetahuan Prosedural. Prosiding Seminar Nasional TEQIP (Teachers Quality Improvement Program) dengan tema “Membangun Karakter Bangsa melalui Pembelajaran Bermakna TEQIP”, 1st December 2014 in Malang of University