On the Convergence of Inexact Gradient Descent With Controlled Synchronization Steps

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Abstract—We develop a gradient-like algorithm to minimize a sum of peer objective functions based on coordination through a peer interconnection network. The coordination admits two stages: the first is to constitute a gradient, possibly with errors, for updating locally replicated decision variables at each peer and the second is used for error-free averaging for synchronizing local replicas. Unlike many related algorithms, the errors permitted in our algorithm can cover a wide range of inexactnesses, as long as they are bounded. Moreover, we do not impose any gradient boundedness conditions for the objective functions. Furthermore, the second stage is not conducted in a periodic manner, like many related algorithms. Instead, a locally verifiable criterion is devised to dynamically trigger the peer-to-peer coordination at the second stage, so that expensive communication overhead for error-free averaging can significantly be reduced. Finally, the convergence of the algorithm is established under mild conditions.

Index Terms—Distributed optimization, inexact algorithms.

I. INTRODUCTION

GRADIENT descent and its variants often lend themselves fully amenable to parallel and distributed algorithms, which are highly desirable in large-scale optimization problems [1]. As a result, solution methods for many problems of recent interest are predominantly based on such gradient-like algorithms [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14]. Broadly speaking, those algorithms developments are twofold [15]: a) a federated setting where a central controller (CC) intervenes for decision variable update [2], [3], [4], [5], [6], [7], [8]; b) a peer-to-peer (PP) setting where subsystems (SSs), each with its replicated decision variable, perform locally the update through some peer interconnection network, often modeled by a connected graph [9], [10], [11], [12], [13], [14]. In this setting, the algorithm relies on neighbors specified by the graph and does not rely on a CC like in the federated setting. As such, it appears that PP setting is more appealing than the CC setting due to many reasons, such as higher scalability and inherently decentralized collection of Big Data sets, among others [1], [15].

In the context of a PP setting, a more fundamental concern is that the distributed algorithms usually undergo inevitable inexact conditions, e.g., unreliable and often limited communication capabilities [1], [15], [16]. Thus, unlike the inexactnesses under CC settings [17], [18], [19], [20], [21], [22], those under PP settings influence the optimality, convergence, and effective implementation of algorithms. Consequently, there is an appeal to design effective algorithms under PP setting [23], [24], [25], [26], [27], [28], [29], [30], [31], [32], [33]. Algorithms in [23], [24], [25], [26], [27], [28] are based on distributed subgradient methods due to [13]. Some of these methods consider quantization models [23], [24], [25] and others consider event-triggered models [26], [27], [28], so as to reduce the communication burden between SSs. The gradient boundedness of underlying objective functions, although a restriction, has been considered in [23], [24], [25], [26], [27], [28], a technical assumption that enables convergences. The errors introduced in [23], [24], [25], [26], [27], [28] can be viewed as controllable, in the sense that they are at the disposal of the algorithm. For example, quantization models in [23], [25] are chosen to be unbiased, a favorable condition for convergence. However, a peer interconnection network can often admit errors that are not at the disposal of the algorithm, e.g., wireless links [34], §9, limiting the applicability of developments in [23], [24], [25], [26], [27], [28].

Works in [29], [30], [31], [32], [33] rely on PP coordination to constitute a gradient, in contrast to common federated settings where primal variables are coordinated instead. Then the resulting gradients are for updating their locally replicated decision variables. They are persuaded again under quantization settings (e.g., [29], [30]) and event-triggered settings (e.g., [31], [32]). Hybrid variants have also been considered by some authors, e.g., [33]. Similar to the developments noted in the preceding discussion, errors introduced in [29], [30], [31], [32], [33] are also controlled by the algorithms. For example, the quantization models in [29] and [30] are chosen so that the errors are diminishing and unbiased, respectively. Moreover, the authors in [31], [33] have specific impositions on the gradient boundedness.

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It is worth noting that many algorithms in either of the setting federated or PP (e.g., [6], [7], [8], [31], [32], [33]) have considered an averaging step performed at periodic or predefined epochs to enable the consistency of the locally replicated decision variables. Depending on the context, this entails periodic communication through the CC or through the PP interconnection network. From a communication overhead point of view, however, such an overhead for periodic communication seems like a restriction. This may be avoided by dynamically choosing the averaging epochs for synchronization.

In this letter, we develop an algorithm that relies on PP coordination to constitute a gradient for updating locally replicated decision variables associated with a problem of minimizing the sum of peer objective functions. The algorithm iterates two stages. The first is used to exchange gradients possibly with errors. We have no restrictions on the errors of local gradient estimates, except that they are bounded. As a result, our modeling can handle errors beyond those of classic quantization models with restrictions, such as diminishing and unbiasedness. For instance, a cheap low-bit quantization can be used throughout the algorithm iterates under the first stage. The second stage is used to error-free averaging for synchronizing local replicas. In this respect, unlike other related algorithms, we do not rely on periodic communication over the PP network. Instead, a locally verifiable criterion is devised to dynamically trigger the averaging step, only when necessary. This has the advantage of minimizing expensive communication overhead. Throughout this letter, we consider the PP network to be fully connected. Subsequently, the convergence of the algorithm is established and is shown to be linear.

II. PROBLEM FORMULATION

Consider \( N \) peers or subsystems which solve the problem

\[
\text{minimize } f(x) = \sum_{i=1}^{N} f_i(x)
\]

where \( x \in \mathbb{R}^n \) and \( f_i : \mathbb{R}^n \rightarrow \mathbb{R}, \; i \in \mathcal{N} \triangleq \{1, \ldots, N\} \), be a function satisfying the following standard assumption:

**AS 1:** The objective function \( f_i, i \in \mathcal{N} \), is strongly convex with constant \( \ell_i > 0 \) and is \( L_i \)-smooth, i.e., \( \nabla f_i \) is Lipschitz continuous with the constant \( L_i > 0 \).

A commonly used iterative algorithms for solving problem (1) is the gradient descent (GD) algorithm \( x^{(k+1)} = x^{(k)} - \gamma \sum_{j=1}^{N} \nabla f_j(x^{(k)}) \), where \( k \in \mathbb{Z}^+ \triangleq \{0, 1, 2, \ldots \} \) is the iteration index and \( \gamma \) is the step size. In contrast, here we assume a setting where each subsystem \( i \) performs locally the variable GD update of its own copy \( x_i^{(k+1)} \) of \( x^{(k)} \). This setting facilitates a distributed implementation of GD and thus each SS relies on a communication with SS \( j \) to get a rough measurement of \( \nabla f_j(x_j^{(k)}) \) as specified below:

**AS 2:** \( \forall i, j \in \mathcal{N}, \text{s.t. } i \neq j, \) gradient measurement \( h_{ij}^{(k)} \in \mathbb{R}^n \) received by \( i \)-th SS from \( j \)-th SS at \( k \)-th iteration is given by

\[
h_{ij}^{(k)} = \nabla f_j(x_j^{(k)}) + e_{ij}^{(k)}
\]

where \( e_{ij}^{(k)} \in \mathbb{R}^n \) is a error such that \( \|e_{ij}^{(k)}\| \leq \epsilon \) with \( \| \cdot \| \) denoting the Euclidean norm.

The parameters \( e_{ij}^{(k)} \) model measurement errors, noises, quantization errors due to compression, among others. However, note the upper bound condition on \( e_{ij}^{(k)} \) in **AS 2**, where \( \epsilon \) can be thought of as the worst-case characteristic of errors throughout the algorithm. Under **AS 2**, the gradient \( \nabla f(x_i^{(k)}) \) is distorted, which in turn admits the following iterate:

\[
x_i^{(k+1)} = x_i^{(k)} - \gamma \sum_{j=1}^{N} h_{ij}^{(k)}, \; i \in \mathcal{N}.
\]

Strictly speaking, the local variables updates should be consistent in the sense that \( \forall k \in \mathbb{Z}^+, \forall i, j \in \mathcal{N}, \) \( x_j^{(k)} = x_i^{(k)} \). However, (3) with distinct SSs do not admit at least a weaker form of the consistency, called **synchrony** given by

\[
\forall i, j \in \mathcal{N}, \; i \neq j, \; x_i^{(m)} = x_j^{(m)}
\]

where \( m \) is an iteration index of practical interest, e.g., the iteration index at the termination. Thus, the main challenge in this research is to establish the convergence properties of (3), while maintaining the synchrony. This challenge is taken up next, where the iterate (3) is integrated with potential SS coordination to yield an algorithm with guaranteed convergence.

III. ALGORITHM DEVELOPMENT

Let us first focus on establishing the evolutionary characteristics of (3) to set the stage for our subsequent developments.

A. Evolutionary Characteristics of (3)

From (3), (2), together with some standard algebraic manipulations as shown in the supplemental material, it can be shown that, under **ASs 1** and **2** and for \( \gamma \in (0, 1/\sum_{j=1}^{N} L_j) \),

\[
\|\nabla f\left(x_i^{(k)}\right) - h_i^{(k)}\| \leq 2\epsilon N (k + 1/2), \; i \in \mathcal{N}
\]

where \( h_i^{(k)} \triangleq \sum_{j=1}^{N} h_{ij}^{(k)} \). The inequality (5) indicates that, in the worst case, the norm of the difference between \( \nabla f(x_i^{(k)}) \) and its local representation \( h_i^{(k)} \) diverges as \( k \rightarrow \infty \). Thus, it is of paramount importance to control such growth for establishing convergence of iterates of the form (3). To this end, it is customary to rely on SS coordination possibly through an error-free communication medium. However, error-free communications are usually more expensive. Therefore, unlike the commonly considered periodic SS coordination [33], we seek to reduce the communication overhead by dynamically choosing the coordination epochs, so as to make it still possible to ensure convergences of the underlying sequences. As such, we consider a relative deviation of the gradient of the objective function \( f \) and its measurement from the standpoint of \( i \)-th SS, i.e.,

\[
e_i^{(k)} \triangleq \frac{\|\nabla f(x_i^{(k)}) - h_i^{(k)}\|}{\|\nabla f(x_i^{(k)})\|}, \; i \in \mathcal{N}.
\]

Let us next discuss how the preceding concept can be integrated into devise our algorithm. In this respect, the most crucial step is to identify an epoch at which the SS coordination is to be triggered. In other words, each SS needs a locally verifiable characterization of the iterates \( k \) for which \( e_i^{(k)} \) is sufficiently.
Algorithm 1: Inexact GD with IndComp–IntSync.

Input: \( x_j^{(0)} = x_i^{(0)} \) \( \forall i,j \in \mathcal{N}, \epsilon \geq 0, r \in (0, \sqrt{L}/(\sqrt{L} + \sqrt{\ell})) , \) \( s = 0, k = 0 \)

1: repeat
2: \( r \in (0, \sqrt{L}/(\sqrt{L} + \sqrt{\ell})) , \) \( s = 0, k = 0 \)
3: compute \( x_i^{(s+k+1)} \) from (3), \( k \leftarrow k + 1 \)
4: until \( \forall i \in \mathcal{N}, k - 1 > r ||h_i^{(s+k-1)}||(2\epsilon N) - 1/2 \)
5: if \( k \neq 1 \) then
6: \( s \leftarrow s + k - 1, k \leftarrow 0 \)
7: \( \bar{h}_i \leftarrow \frac{1}{N} \sum_{j=1}^{N} x_j^{(s+k-1)} \)
8: else
9: \( s \leftarrow s + k, k \leftarrow 0 \)
10: end if
11: until a stopping criterion true

small, despite the dependence of \( e_i^{(k)} \) on global information \( \nabla f(\cdot) \). As such, we rely on the condition

\[
k < r||h_i^{(k)}||(2\epsilon N) - 1/2 \Rightarrow e_i^{(k)} \leq r/(1 - r) \tag{6}
\]

where \( r \in (0, 1) \) is a design parameter, suitably chosen based on the strong convexity constants and the Lipschitz constants of the objective functions. The condition (6) follows from (5), together with that ||\( h_i^{(k)} || - \|

The evolution of the sequence \( \{f(x_i^{(s+k+1)}) - f(x^*)\} \) when the algorithm is in state 2 is established by the following result.

**Lemma 2:** Let \( \mathcal{AS} \) 1 and 2 hold, \( s \in \mathbb{Z}^+ \) be any iteration index at which the algorithm is in state 2. Moreover, suppose the inner loop of Algorithm 1 repeats only once, where the iteration index is \( s \). Then

\[
\limsup_{k \to \infty} \|x_i^{(k)} - x^*\| \leq \sqrt{\frac{\ell^2 N^2}{\ell^2 - L \ell^2}}
\]

where \( L = \sum_{j=1}^{N} L_j, \ell = \min_{j \in \mathcal{N}} \ell_j, r = \sqrt{\frac{1}{\ell}} - 1 \), and \( x^* = \arg \min_{x} f(x) \).

It is not difficult to see that the Proposition holds even if \( x_i^{(k)} \) is set as \( x_i^{(k)} = \frac{1}{k} \sum_{s=1}^{k} x_i^{(s)} \) for all \( k \in \mathbb{Z}^+ \). Note that until the termination of the algorithm [cf. step 12], the inner loop is in either of the following states: 1) it repeats more than once 2) it repeats only once. Thus, the proof of the Proposition is simply based on the characterization of the evolution of the sequence \( \{f(x_i^{(s+k+1)}) - f(x^*)\} \) when the algorithm is in either of the states. To this end, we shall require the following results, the proofs of which are given in supplementary material.

**Lemma 1:** Let \( \mathcal{AS} \) 1 and 2 hold, \( s \in \mathbb{Z}^+ \) be any iteration index at which synchrony is imposed, \( i \in \mathcal{N}, r \in (0, 1) \). Moreover, suppose the inner loop of Algorithm 1 repeats for \( s \) and \( k \in \mathbb{Z}^+ \), for some \( \kappa \geq 2 \). Then for \( k \in \{0, 1, \ldots, \kappa - 1\} \)

\[
f \left( x_i^{(s+k+1)} \right) - f(x^*) \leq q \left( f \left( x_i^{(s+k)} \right) - f(x^*) \right) \tag{7}
\]

where \( q = (1 + \gamma L \ell^{2}) - 2 ) \) is a positive constant.

**Lemma 1** characterizes the evolution of the sequence \( \{f(x_i^{(s+k+1)}) - f(x^*)\} \) when the algorithm is in state 1. Consequently, the recursive application of (7), together with the Jensen’s inequality yields

\[
f \left( x_i^{(s+k)} \right) - f(x^*) \leq q^\kappa \left( f \left( x_i^{(s)} \right) - f(x^*) \right) \tag{8}
\]

The evolution of the sequence \( \{f(x_i^{(s+k+1)} - f(x^*)\} \) when the algorithm is in state 2 is established by the following result.

**Lemma 2:** Let \( \mathcal{AS} \) 1 and 2 hold, \( s \in \mathbb{Z}^+ \) be any iteration index at which the algorithm is in state 2. Moreover, suppose the inner loop of Algorithm 1 repeats only once, where the iteration index is \( s \). Then

\[
f \left( x_i^{(s+1)} \right) - f(x^*) \leq (1 - \gamma \ell) \left( f \left( x_i^{(s)} \right) - f(x^*) \right) + \frac{\gamma^2 \ell^2 N^2}{2} \tag{9}
\]

The inequality (9) holds even if \( x_i^{(s+1)} \) from the inner loop of Algorithm 1 is set as \( x_i^{(s+1)} = \frac{1}{N} \sum_{j=1}^{N} x_j^{(s+1)} \).

Finally, the following Lemma asserts that the algorithm necessarily switches to state 2 from state 1.

**Lemma 3:** Let \( \mathcal{AS} \) 1 and 2 hold. Moreover, suppose \( \forall i \in \mathcal{N}, r||h_i^{(0)} || \geq \epsilon N, \) and thus, the algorithm starts at state 1, where \( r \in (0, \sqrt{L}/(\sqrt{L} + \sqrt{\ell})) \). Then \( \exists \bar{s}, \bar{k} \in \mathbb{Z}^+ \) such that Algorithm 1 switches to state 2 from state 1, where \( \bar{s} \) is an iteration index at which the synchrony is imposed and \( \bar{k} \) is a local iteration index within the inner loop.

Having armed with the above results, we are now ready to give the proof of Proposition 1.

**Proof of Proposition 1:** From Lemma 1 and (8), for any consecutive sequence of state 1, starting at some global iteration index \( n \in \mathbb{Z}^+ \) and ending at \( n + k \in \mathbb{Z}^+ \), we have

\[
f \left( x_i^{(n+k)} \right) - f(x^*) \leq q^k \left( f \left( x_i^{(n)} \right) - f(x^*) \right) \tag{10}
\]
\[ q^k \left( f \left( x_i^{(n)} \right) - f ( x^* ) \right) + \frac{\gamma^2 N^2}{2} \sum_{j=0}^{k-1} q^j. \] (11)

Similarly, recursively applying (9) in Lemma 2 for any consecutive sequence of state 2, starting at some global iteration index \( n \in \mathbb{Z}^+ \) and ending at \( n + k \in \mathbb{Z}^+ \), together with that \( 1 - \gamma \ell \leq q_0 \), we again have an equivalent form of (11). Moreover, the algorithm necessarily switches to state 2 from state 1, cf. Lemma 3. Thus, from (11), \( \forall k \in \mathbb{Z}^+ \), we have
\[
f \left( x_i^{(k)} \right) - f ( x^* ) \leq q^k \left( f \left( x_i^{(0)} \right) - f ( x^* ) \right) + \frac{\gamma^2 N^2}{2} \sum_{j=0}^{k-1} q^j.
\]

Noting that \( q < 1 \), we take the limit as \( k \to \infty \) to yield Part 1. Part 2 follows from Part 1 and [35, eq. 10, §1.4]. Finally, Part 3 follows from Part 1 and [35, eq. 35, §1.1]. □

IV. NUMERICAL RESULTS

Let us first verify the convergence results of Proposition 1. To this end, we consider problem (1) with quadratic \( f \)'s, i.e., \( f_i(x) = x^T B_i^T B_i x + c_i^T x \), where \( B_i^T B_i \in S^n_+ \), \( c_i \in \mathbb{R}^n \), and \( S^n_+ \) is the positive definite cone. The entries of \( B_i \) and \( c_i \) are generated from a normal distribution. Note that \( \ell_i \) and \( L_i \) are determined by \( B_i \). We let \( N = 4 \), \( n = 10 \), \( \gamma = 1/(2L) \), and \( r = 0.03 \). Only the results related to Proposition 1-(1) is presented, since those related to Proposition 1-(2) and (3) behave similarly.

Compared to the numerical results of Proposition 1, we also consider another algorithm which we refer to as the inexact-GD with distributed synchrony (IGDDS), i.e., Algorithm 1 with \( r = 0 \) and \( \epsilon = 0 \). We also consider another algorithm which we refer to as inexact-GD with distributed synchrony (IGDDS), i.e., Algorithm 1 with \( r = 0 \) and \( \forall i \in \mathcal{N}, \epsilon_i^{(k)} = \epsilon^{(k)} \) [cf. (2)]. In this respect, the 

Fig. 1(a) shows the error \( f ( x_i^{(s+k)} ) - f ( x^* ) \) vs global iteration index \( s+k \) for different \( \epsilon \), cf. solid lines. Results are averaged over 1000 initializations \( x^{(0)} \), whose entries are normally distributed. Plots agree with Proposition 1-(1), i.e., the smaller the \( \epsilon \), the smaller the error of the optimality. Results with IGDDS are given in non-solid lines. Convergence rates and the suboptimality obtained by Algorithm 1 and IGDDS seem almost identical. This is expected because the convergence rate of Algorithm 1, i.e., \( (1 - \gamma L \hat{r}^2 - \gamma \ell) \) and that of IGDDS, i.e., \( (1 - \gamma \ell) \) are almost identical when \( \gamma L \hat{r}^2 \ll 1 - \gamma \ell \). This condition is always realizable in practice, e.g., we have \( \gamma L \hat{r}^2 = 0.0005 \) and \( 1 - \gamma \ell = 0.9986 \) in our simulation. A similar comparison holds for the suboptimality as well.

It is important to note that Algorithm 1 does not require synchrony in every iteration as IGDDS. Therefore, for a fair comparison of Algorithm 1 and IGDDS in terms of communication overhead, it is instructive to plot the error versus the number of IntSync steps \( m \), where \( s_m m \in \mathbb{Z}^+ \) is the iteration index of \( s \) within Algorithm 1 at which the \( m \)-th synchrony is imposed.

Fig. 1(b) shows the error \( f ( x_i^{(s_m)} ) - f ( x^* ) \) vs \( m \) with Algorithm 1, see thick solid lines. Results related to IGDDS are also plotted, see the non-solid lines. Clearly, there is a shift of the plots with IGDDS towards the right relative to the plots with Algorithm 1. Therefore, for all considered \( \epsilon \) values, the number of IntSync steps \( m \) required to obtain a specified error with Algorithm 1 is smaller than with IGDDS. Moreover, if the number of IntSync steps \( m \) is fixed, the error with Algorithm 1 can be on the order of magnitude smaller than with IGDDS. This is useful in practice, because the cost of the error-free communication required for IntSync can be reduced with Algorithm 1 than with IGDDS. The benefits become greater as \( \epsilon \) decreases. Finally, we plot results due to GD, see the thin solid line in Fig. 1(b). Results show that still the Algorithm 1 can benefit from less expensive InComp steps. For example, in 150 IntSync steps, Algorithm 1 manages to yield an error significantly less than that from GD despite the value of \( \epsilon \). Clearly, GD outperforms Algorithm 1 if \( m \) is sufficiently large, since there are no inexactnesses. Thus, the results suggest if there is a choice for less expensive communication for IntSync, or a choice for allowing some inexactnesses, one can operate Algorithm 1 in a way where there is a trade-off between the error and IntSync steps \( m \).

V. CONCLUSION

A gradient-like algorithm with guaranteed convergence has been developed to minimize a sum of peer objective functions through a peer-to-peer interconnection network. Peer coordination can usually admit communications with bounded errors, however with some infrequent error-free synchronization epochs, which are dynamically triggered. Our algorithm can be attractive in many distributed applications, under inexact communication settings, such as decomposition with dual-subgradient methods and distributed learning systems with in-network computing capabilities, among others.
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