Research and Application of Optimal Water Resources Allocation Based on Improved Multi-dimensional Dynamic Programming

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Abstract. To solve the problem of the multi-dimensional dynamic programming model which including three state variables and three decision variables in each stage, we have been using discrete differential dynamic programming (DDDP) algorithm. In this paper, considering the particularity of each variable of the objective function is independent in the same stage and the variables of the constraint condition is increasing step by step, we propose a one-dimensional dynamic programming recursive optimization method to obtain the initial solution of the DDDP algorithm.

1. Introduction
The joint operation of reservoirs is a complex optimization problem with multi-objective, multi-stage and nonlinear characteristics. It is one of the key topics in research at home and abroad in recent years. Intelligent algorithms and dynamic programming have been widely applied in this topic and have achieved certain results. For example, Bai Xiaoyong et al. use the artificial fish swarm algorithm to solve the initial test trajectory required for discrete differential dynamic programming (DDDP) to obtain a better and more stable solution in the optimal operation of the daily regulating reservoir; Cheng Chuntian et al. use the improved fine-granularity in parallel with DDDP algorithm to solve the long-term optimal operation problem of the cascade hydropower stations in the Lancang River. The above research is mainly aimed at the optimal operation of cascade hydropower stations. However, there are still relatively few studies on the joint operation of multiple lakes in the plain river network area to realize the optimal allocation of water resources in the water supply system. Especially when it comes to solving the optimal solution of multidimensional problems, there is still some space for exploration.

According to the particularity of the objective function in the dynamic programming model studied in which the variables are independent and the constraints of the variables increase step by step, this paper proposes a step-by-step recursive optimization of the one-dimensional dynamic programming of the initial strategy of obtaining the DDDP algorithm. method. It is applied to a large-scale inter-basin water transfer example from the Yangtze River, and compared with the conventional DDDP algorithm from the optimization results of the model, the calculation efficiency and accuracy of the program.

2. Dynamic programming algorithm for a class of multidimensional problems
Research questions:
In the above model, the objective function corresponds to the sum of the mutually independent functions $\varphi(x_i), \eta(y_i), \zeta(z_i)$ formed by the three variables $x_i, y_i, z_i$ in each $i$ stage, and the decision variables are $(3n)$. There are three constraints, of which the first constraint involves only the variable $x_i$, the second constraint involves the variables $x_i$ and $y_i$, and the third constraint involves the variables $x_i, y_i$ and $z_i$, forming a complex constraint that increases variables step by step.

For this kind of problem, the model can be understood as multi-dimensional dynamic programming. As each $i$-stage includes three state variables (represented by $\lambda_{ij}$, where $j=1, 2, 3$) and three decision variables ($x_i, y_i, z_i$), it can be solved by the traditional DDDP algorithm. Wherein, each state variable $\lambda_{ij}$ can be discrete in a corresponding feasible domain according to a certain step, and the specific calculation is:

1. First, set an initial strategy $\{x_1, y_1, z_1\}$ that satisfies the constraints. Thus, according to each state transition equation, a state sequence $\{\lambda_{ij}\}$ that satisfies the constraint is obtained. The initial test trajectory can be obtained. At the same time, the corresponding initial value $f_1$ of the objective function is calculated. Among them, the choice of the initial strategy is particularly salient.

2. Select the increment $\pm \Delta\lambda_{ij}$ around the test trajectory, and form a decision corridor C of three-dimensional dynamic programming including 27 (i.e. $3^3$) state points per phase by $\{\lambda_{ij} \pm \Delta\lambda_{ij}\}$. For the value of the increment $\Delta\lambda_{ij}$, a large incremental value is generally selected at the initial stage of the calculation, and then gradually decreased during the iteration, thereby speeding up the convergence of the decision.

3. In the corridor C, the conventional dynamic programming method is adopted to find the optimal objective function value $f_2$. Inversion is performed to find the optimal strategy $\{x_2, y_2, z_2\}$ and the optimal trajectory $\{\lambda_{ij}\}_2$. Since this optimization is not carried out in the entire decision feasible domain, the strategy and trajectory sought are not the global optimal solution, but merely improve the strategy and trajectory.

4. The improved trajectory $\{\lambda_{ij}\}_2$ of the above optimization is used as the test trajectory for the next iteration. A new corridor is established to gradually find the optimal objective function value $f_k$ ($k = 3, 4, 5, \ldots$). If the relative error of the two target values before and after is less than the predetermined allowable error, the value of the state increment $\Delta\lambda_{ij}$ is decreased. Then, a smaller decision corridor C is created around the new test trajectory and the above steps are repeated to continue iterative optimization. Afterwards, the state increment value of each stage is continuously reduced until it is smaller than a prescribed value determined according to the specific problem. The state increment value for each phase is then continuously reduced until it is less than a specified value determined by the particular problem. At the same time, when the relative error satisfying the objective function value is less than the set allowable error, the calculation is ended.

5. Considering that the problem of solving is not necessarily a convex programming, whether the initial strategy is appropriate or not will directly affect the optimization result and convergence speed of the decision. Therefore, in view of the particularity of the objective function in the above model in which the variables are independent and the constraints of the variables increase step by step, this paper proposes an optimal method for obtaining the initial strategy. That is to say, the variable $x_i$ is
taken as the research object to establish a one-dimensional dynamic programming model. The objective functions and constraints are as follows:

① First layer model

\[
\max Z = \sum_{i=1}^{n} \varphi_i(x_i) \] 
\[
\text{s.t. } \sum_{i=1}^{n} h_{1i}(x_i) \leq b_1 
\]

A set of optimal solutions \( \{x_i\}, i=1,2,3,\ldots,n \) is obtained by solving the one-dimensional dynamic programming model. Based on the known \( x_i \), the variable \( y_i \) is conducted as the research object to establish a one-dimensional dynamic programming model. The objective functions and constraints are as follows:

② Second layer model

\[
\max Z = \sum_{i=1}^{n} \left[ \varphi_i(x_i) + \eta_i(y_i) \right] 
\]
\[
\text{s.t. } \sum_{i=1}^{n} h_{2i}(x_i, y_i) \leq b_2 
\]

A set of optimal solutions \( \{y_i\} \) is obtained by solving the model. Then, based on the known \( x_i \) and \( y_i \), the variable \( z_i \) is taken as the research object, and the one-dimensional dynamic programming model is also established. The objective functions and constraints are as follows:

③ Third layer model

\[
\max Z = \sum_{i=1}^{n} \left[ \varphi_i(x_i) + \eta_i(y_i) + \zeta_i(z_i) \right] 
\]
\[
\text{s.t. } \sum_{i=1}^{n} h_{3i}(x_i, y_i, z_i) \leq b_3 
\]

A set of optimal solutions \( \{z_i\} \) is obtained by solving the model, and \( x_i, y_i \) and \( z_i \) have all been obtained. As the initial strategy \( \{x_i, y_i, z_i\} \) of the above multidimensional dynamic programming model calculation, the DDDP is used to continue the iterative calculation to obtain the optimal solution of the above multidimensional model.

3. Case analysis of water resources optimization configuration model for complex water supply systems

3.1. Project Overview

For a large-scale inter-basin water transfer project that draws water from the Yangtze River, its joint optimization operation is a complex optimization problem involving multiple lakes and multiple targets. In considering the water resources analysis and management operation of the water supply system, on the one hand, it is necessary to solve the problem of water use in the water receiving area along the system, so that the water supply of the whole supply system is minimized. On the other hand, it is also required to take into account the supply priorities of the water users and the local water use benefits of the transfer area. At the same time, it is needed to achieve the goal of less water pumping and less water in the water supply system on the premise of meeting the water user requirements as much as possible. The operation of the water transfer project is shown in Figure 1.

In the figure, 1, 2, and 3 are the lake section numbers. It is the natural net amount of water after deducting the evaporation leakage loss for each interval in the t-th period; \( D_t \) is the amount of water that is self-flowing from the upstream during the t-th period. \( P_t \) is the amount of water drawn by the downstream step pump station during the t-th period. \( R_t \) is the actual amount of water provided to the water users in the water receiving area in each interval of the t-th period. \( P_{Ot} \) is the amount of water
pumped into the upstream by the step pumping station in the t-th period; DOt is the amount of water flowing into the downstream from the flowing water in the t-th period.

3.2. Establishment and solution of mathematical models

3.2.1. Objective function. In view of the above-mentioned water supply system with complexity and timeliness, this paper takes the pumping/disposing water volume $X_{jt}$ (ie $PI_{jt}$ or $PO_{jt}$) of each lake as the decision variable, and the water storage capacity $\lambda_{jt}$ (ie $V_{jt}$) at the end of each period of each lake as the state variable. The total pumping capacity of the water supply system is the minimum objective function. In this way, the stages are divided in chronological order to establish a mathematical model for dynamic programming of multi-lake joint operation.

$$\min f = \sum_{t=1}^{T} \left( \sum_{j=1}^{3} PI_{jt} + \sum_{j=0}^{3} PO_{jt} \right)$$

In the equation, $T$ is the number of the calculation period, and a hydrological year is used as the calculation period, which means $T=36$. $j$ is the number of lake intervals, where $j=0$ refers to the source of the water supply system. $V_{jt}$ and $V_{jt+1}$ are the initial and final water storage capacities of the lake during the $t$-th period. The rest of the parameters are the same as before.

3.2.2. Five constraints.

(1) Water balance constraint: the water balance equation of each lake.

$$V_{jt+1} = V_{jt} + I_{jt} + DI_{jt} + PI_{jt} - R_{jt} - PO_{jt} - DO_{jt}$$  (13)

(2) Lake state constraint: Each lake is considered by the annual adjustment reservoir. The storage capacity of the lake at each time is not lower than the lower limit of the lake (that is, the dead water level corresponds to the storage capacity) and is not higher than the upper limit of the lake (that is, the water level corresponding to the storage capacity during the flood season or non-flood period).

(3) Pumping station capacity constraint: The pumping capacity of the cascade pumping stations in each interval must be less than or equal to the designed pumping capacity of the pumping station.

(4) External water constraint: The actual external water adjustment in each interval of each section must be less than or equal to the maximum of the external water adjustment in the corresponding downstream.

(5) Water supply guarantee constraint: The actual water supply and external water transfer in each interval meet the requirements of designed the water supply guarantee rate.
3.2.3. Model solving. In the above model, since the river network between the lakes does not consider the storage function, there is a certain quantitative relationship between the pumping amount $PI_j^t$ of any lake in the objective function and the lake pumping amount $PO_j^t$ of the next step. Therefore, the model is a dynamic programming problem with three decision variables and three state variables in each stage. At the same time, the decision variables of the objective function in the same stage are independent of each other, and the three lake water balance constraint equations are recursively. The dynamic programming optimization algorithm can be used to solve the problem. The programming language is implemented in Fortran software.

3.3. Analysis of results

3.3.1. Calculation results. This optimization method is used to optimize the dispatching analysis of the water supply system of 50%, 75% and 95% incoming water frequency. The results are compared with conventional operation data, as shown in Table 1.

| Scheduling method         | Water frequency | Total water supply (Billion m$^3$) | Total pumping volume (Billion m$^3$) | Total water abandonment (Billion m$^3$) |
|---------------------------|-----------------|-----------------------------------|--------------------------------------|----------------------------------------|
| Regular scheduling        | 50%             | 136.92                            | 201.28                               | 140.79                                 |
|                           | 75%             | 147.93                            | 279.27                               | 57.23                                  |
|                           | 95%             | 125.17                            | 351.28                               | 4.20                                   |
| Optimized scheduling      | 50%             | 137.23                            | 149.94                               | 133.94                                 |
|                           | 75%             | 148.22                            | 184.40                               | 27.27                                  |
|                           | 95%             | 126.21                            | 323.09                               | 0.00                                   |

It can be seen from the above table that with the optimized operation method, the total water supply of system ① is slightly higher than that of the conventional operation under each incoming water frequency. At the same time, it is also possible to provide a distribution system for the water users in each interval to avoid serious water shortage in a few periods, so as to achieve reasonable operation of water volume in space and time. Under each incoming water frequency of system ②, the total pumping amount and the amount of discarded water are significantly lower than the conventional operation, which reduces the unnecessary pumping cost and obtains obvious benefits.

3.3.2. Calculation accuracy and efficiency. In this paper, the conventional DDDP algorithm is used to set an initial pumping (discarded) water decision, and the iterative selection is performed by programming calculation. Since the initial decision number is as high as 108 (that is, $36 \times 3$) each time, which is only set due to empirical assumptions, so it must bring about a tedious calculation time. Through the above example calculation, the following reasons are summarized:

① The arbitrarily set initial strategy often results in the long-term repetitive cycle selection of the dynamic programming model into the target solution. The number of selected corridors is so high that the program will not terminate when the calculation error is always unable to meet the allowable error. This leads to increased calculation time. Only by sacrificing the accuracy of the calculation can the program be ended.

② However, considering the validity of the calculation results, iterative calculations need to be performed by setting different initial strategies and experimental trajectory schemes multiple times. The calculation results are compared with each other to select a better target value. As a result, the calculation time is bound to continue to increase on the basis of the previous. The more times an initial strategy is given, the longer the calculation will take.

③ In order to further improve the accuracy of the calculation, it is also necessary to reduce the discrete step of the water storage state of the three storage lakes from the initial 100 million m$^3$ to 0.01 billion m$^3$, or even to a smaller one. At the same time, the number of discrete points of water storage increases by a hundredfold, and the calculation time increases by the same ratio, which makes the
calculation difficult to achieve the best results. In the end, only one target value of a relative optimization corridor can be obtained. Simultaneously, the initial strategy of the DDDP algorithm is searched by the step-by-step recursive optimization method of the one-dimensional dynamic programming proposed in this paper. That is to say, the optimal pumping (discarded) water decision of the first-level lake is first obtained as the input condition of the second-level lake. Then, the optimal pumping (discarded) water decision for the second-level lake is obtained as the input condition for the third-level lake. Finally, the optimal pumping (discarded) water decision for the third-level lake is obtained, and the first two-level lake decision constitutes the initial strategy of the multi-dimensional model of the water supply system with DDDP algorithm. Since the decision to obtain the initial pumping (discarded) water is derived from the optimal solution of the one-dimensional recursion model step by step, combined with the actual constraints of the water transfer project operation, it is finally possible to find the target value in the approximate optimal corridor in the model and to iterate continuously in the gradual improvement of the corridor. Eventually, by comparing with the results obtained by the conventional DDDP algorithm, the optimization of the target value is improved by more than 10%; From the calculation time, the repeated calculations caused by the conventional algorithm are avoided, and the calculation efficiency and precision are significantly improved, so that the optimal solution of the entire water supply system operation model is quickly obtained.

4. Conclusion

Based on the characteristics of the joint operation project of water supply lakes in plain river network, this paper establishes a dynamic programming mathematical model of multi-lake joint operation. The model is generalized and a multi-dimensional dynamic programming problem with universal application significance is constructed. At the same time, combined with the particularity that the objective function in the model is independent of each other and the variables of the constraint increase step by step in the same stage, a one-dimensional step-by-step recursive optimization of dynamic programming is proposed to obtain the initial strategy of the DDDP algorithm. Through the case analysis, on the one hand, the problem that the conventional DDDP algorithm always falls into the local optimal solution is solved. Under the premise of ensuring the total water supply, the calculation results lower the total pumping capacity and discarded water of the system and reduce the engineering operation cost, realizing the optimal water allocation. On the other hand, the model with fast calculation speed, high optimization efficiency and novel solution has high practical value, which enables to provide a basis for quickly seeking optimal planning and optimization operation scheme for complex water supply projects.

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