The Higgs boson from an extended symmetry

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Abstract

The variety of ideas put forward in the context of a “composite” picture for the Higgs boson calls for a simple and effective description of the related phenomenology. Such a description is given here by means of a “minimal” model and is explicitly applied to the example of a Higgs-top sector from an $SO(5)$ symmetry. We discuss the spectrum, the ElectroWeak Precision Tests, B-physics and naturalness. We show the difficulty to comply with the different constraints. The extended gauge sector relative to the standard $SU(2) \times U(1)$, if there is any, has little or no impact on these considerations. We also discuss the relation of the “minimal” model with its “little Higgs” or “holographic” extensions based on the same symmetry.
1 Introduction

The first thorough exploration of the energy range well above the Fermi scale, $G^{-1/2}$, made possible by the Large Hadron Collider, may require a dramatic revision of the Standard Model (SM) of elementary particles. This is actually very likely to be the case if the Higgs boson is a naturally light fragment of the spectrum of whatever theory accounts for the fundamental interactions at any scale above $G^{-1/2}$. To the point that one wonders whether one should not have already seen, through the ElectroWeak Precision Tests (EWPT), at least some indirect manifestation of the required extension of the SM. This very consideration is in fact at the same time a source of concern and, in absence of more crucial information, one of the guidelines in trying to foresee what the LHC will discover.

Without even trying to list the different theoretical directions that have been taken to address this problem, whose relevance will be judged by the forthcoming LHC experiments themselves, here we concentrate our attention on the option that the Higgs boson emerges as a remnant in one way or another of an (approximate or spontaneously broken) extended symmetry. This is in fact a rather general framework in itself, with many more specific realizations: the Higgs as a Pseudo-Goldstone Boson, the little Higgs, the composite Higgs, the Higgs as $A_5$, the intermediate Higgs, the twin Higgs, et cetera. In turn this variety calls for an effective and, to some extent, unified way to describe the related relevant phenomenology. Steps in this direction have been recently made in Ref. [1] and [2]. In this work we focus our attention on the low energy description of the relevant dynamics, as dictated only by consideration of the approximate symmetry of the Higgs-top system, since we believe this to be the most important element in judging the consistency with the current data and in determining the LHC phenomenology. We look for a simple and, at the same time, accurate description of this dynamics.

A special aspect that emerges from these considerations is the following. In most of the specific realizations alluded to above, the Higgs boson is thought to emerge as the low energy remnant of some kind of strong dynamics, hence the common qualification of “composite Higgs”. While this is certainly an interesting possibility, actually forced in many specific realizations by the consistency with the EWPT, we think that it makes also sense to consider the approximate symmetry of the Higgs-top system as a simple extension of the SM only, remaining in the perturbative regime. In principle therefore it is the experiment that should decide between the elementary or the composite option, leaving open, for the time being, the question of what will provide the necessary cutoff.

For concreteness we describe in the following a specific example based on the $SO(5)$ symmetry as a minimal extension of the $SO(4)$ symmetry of the Higgs potential in the SM. This is obtained by adding to the usual Higgs doublet a fifth real component and by equally extending the left handed top-bottom doublet to a five-plet of $SO(5)$. The $SO(5)$ symmetry is allowed to be broken by soft terms, of unspecified origin, apart from the standard gauge interactions and the Yukawa couplings other than the top one. The simple explicit nature of the corresponding Lagrangian allows a straightforward discussion of the resulting phenomenology, with a special focus on the key issue, as already mentioned, of the EWPT and of B-physics. Alternative and relatively more complex ways to extend the top-bottom sector in an approximately $SO(5)$ invariant way are also described.

Depending on its parameters, our “minimal” model can either be viewed as describing a per-
turbatively coupled Higgs boson or as the low energy description of a strongly coupled theory at the naturalness cutoff. To illustrate this dual role we consider the properties of a “little Higgs” or a “holographic” extension. In the last case, this is precisely the model already discussed in [3,4], whereas the little Higgs extension corresponds to the “deconstructed” version of the same model.

2 Minimal model at strong coupling

Motivated by minimality and by the requirement of including the custodial symmetry, we consider in the following a model based on the $SO(5)$ symmetry, although the approach followed here is relevant for any model with a Higgs boson arising from an extended symmetry. As we will explain in more detail in Section 4, the model gives a low energy description of any theory in which the ElectroWeak Symmetry Breaking (EWSB) sector has the $SO(5)$ global symmetry partly gauged by the SM ElectroWeak group. Both for substantial and for phenomenological reasons we first discuss the “strong coupling” case, where the coupling that controls the $SO(5) \rightarrow SO(4)$ breaking is large. Later we will consider extending this coupling to the perturbative regime.

2.1 EWSB sector

The low-energy description of the EWSB sector of our models is the sigma-model with $SO(5)$ global symmetry broken spontaneously to $SO(4)$. Its dynamics is described by a scalar five-plet $\phi$ subject to a constraint

$$\phi^2 = f^2, \quad (2.1)$$

where $f$ is the scale of the $SO(5) \rightarrow SO(4)$ breaking, which is assumed to be somewhat higher than the EWSB scale $v = 175$ GeV. The cutoff of this model is

$$\Lambda \simeq \frac{4\pi f}{\sqrt{N_g}}, \quad (2.2)$$

where $N_g = 4$ is the number of Goldstones. One interpretation is that $\Lambda$ is the compositeness scale of $\phi$, although other UV completions may be imagined (see Sections 5, 6 below).

The SM electroweak group $G_{SM} = SU(2)_L \times U(1)$ gauges a part of the $SO(5)$. More precisely, we pick a fixed subgroup $SO(4) \equiv SU(2)_L \times SU(2)_R \subset SO(5)$, acting on $\phi \equiv (\text{the first 4 components of } \phi)$ and gauge $SU(2)_L$ and the $T_3$ generator of $SU(2)_R$. The kinetic Lagrangian of $\phi$ has thus the form

$$L_{\text{kin}} = \frac{1}{2}(D_{\mu}\phi)^2, \quad D_{\mu}\phi = \partial_{\mu}\phi - i(W^a_{\mu}T^a_L + B_{\mu}T^3_R)\phi. \quad (2.3)$$

The direction of $\phi$ chooses the angle of alignment between the residual $SO(4)$ subgroup of the $SO(5) \rightarrow SO(4)$ breaking and the $SO(4)$ inside which $G_{SM}$ lives. For $\phi = (0,0,0,f)$ there is no EWSB and the W and Z bosons are massless. On the other hand, $\bar{\phi}^2 = f^2$ corresponds to maximal EWSB. In general, we can construct the usual $SU(2)$ Higgs doublet out of $\bar{\phi}$:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}. \quad (2.4)$$
The $W$ boson mass will be related to the Vacuum Expectation Value (VEV) of $\vec{\phi}$ by the standard relation

$$m_W^2 = \frac{g^2 v^2}{2}, \quad v^2 = \langle |H|^2 \rangle = \frac{1}{2} \langle \vec{\phi}^2 \rangle. \quad (2.5)$$

We will describe the dynamics fixing the VEV of $\vec{\phi}$ by a potential which includes, apart from the $SO(5)$ symmetric term enforcing the constraint (2.1), the most general soft-breaking terms up to dimension 2 and consistent with the gauge symmetry:

$$V = V_0 f^2 \delta (\phi^2 - f^2) - Af^2 \vec{\phi}^2 + Bf^3 \phi_5. \quad (2.6)$$

There may be several sources of these soft-breaking terms (e.g. the gauge interactions in (2.3) break the $SO(5)$ symmetry explicitly, and will generate the $\vec{\phi}^2$ term); their precise origin is left unspecified. We will treat the dimensionless coefficients $A$ and $B$ as free parameters within their typical ranges consistent with Naturalness as discussed below.

The potential (2.6) gives a VEV to $\vec{\phi}$ provided that $A > 0$ and

$$\langle \vec{\phi}^2 \rangle = f^2 \left[ 1 - \left( \frac{B}{2A} \right)^2 \right] > 0. \quad (2.7)$$

This relation shows that to have $v \ll f$ will require finetuning the ratio $B/2A$ to 1. This finetuning can be quantified by the usual logarithmic derivative:

$$\Delta = \frac{A}{v^2} \frac{\partial v^2}{\partial A} \simeq \frac{f^2}{v^2}. \quad (2.8)$$

For $f = 500$ GeV (the benchmark value used throughout this paper) we have $\Delta \simeq 8$ which corresponds to a $\sim 10\%$ finetune, and, from (2.2), to $\Lambda \simeq 3$ TeV.

The Higgs particle in this model has a mass

$$m_h = 2 \sqrt{Av}. \quad (2.9)$$

Its coupling to the weak gauge bosons, and in fact to any other SM particles, will be suppressed with respect to the SM by a factor

$$\cos \alpha = \left( 1 - \frac{2v^2}{f^2} \right)^{1/2}. \quad (2.10)$$

This suppression has its origin in the wavefunction renormalization which takes place when expanding the kinetic Lagrangian (2.3) around a point with $\vec{\phi} \neq 0$. Alternatively, it can be viewed as a consequence of the fact that the Higgs particle is an admixture of an $SU(2)$ doublet $\vec{\phi}$ and a singlet $\phi_5$. At the LHC, such a Higgs boson will have the VBF (Vector Boson Fusion) production cross section suppressed by $(\cos \alpha)^2 \simeq 0.75$ for $f = 500$ GeV. This effect could be observable, since the VBF cross section is expected to be measured with $\sim 5\%$ error [5].

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1 This estimate will apply e.g. under the assumption that $B$ is distributed uniformly in the range $|B| < 2A$. If the typical range of $B$ is larger, the finetuning will be larger.
As a consequence of the reduced coupling of the Higgs particle to the gauge bosons, the longitudinal \( WW \) scattering amplitude grows in this model as

\[
\mathcal{A}(W_L W_L \to W_L W_L) = -\frac{G_s}{\sqrt{2}} (\sin \alpha)^2 (1 + \cos \theta),
\]

where \( s \) is the square of the center-of-mass energy, and \( \theta \) is the scattering angle. This growth can be used to give an alternative estimate for the cutoff of the model. Indeed, the amplitude (2.11) would saturate the unitarity bound at

\[
s_c = \frac{s_c^{SM}}{(\sin \alpha)^2},
\]

where \( s_c^{SM} = (1.2 \text{ TeV})^2 \) is the analogous bound in the Higgsless SM\(^2\). For \( f = 500 \text{ GeV} \) we have \( \sqrt{s_c} = 2.4 \text{ TeV} \), which is not far from \( \Lambda \simeq 3 \text{ TeV} \) from (2.2).

### 2.2 ElectroWeak Precision Tests

It is straightforward at this point to compute the modifications introduced in the EWPT relative to the SM, which arise at one loop level due to the modified couplings of the Higgs boson to the gauge bosons. Since these couplings are weaker than standard, the Higgs exchange regulates the logarithmic divergence in the gauge boson self-energies only partially. The resulting modification is easy to write down in the heavy Higgs approximation, in which the electroweak parameters \( \hat{S}, \hat{T} \) in the SM are given by

\[
\hat{S}, \hat{T} = a_{S,T} \log m_h + b_{S,T},
\]

where \( a_{S,T}, b_{S,T} \) are constants. In this model, in the same approximation we will have

\[
\hat{S}, \hat{T} = a_{S,T}[(\cos \alpha)^2 \log m_h + (\sin \alpha)^2 \log \Lambda] + b_{S,T},
\]

which amounts to replace \( m_h \) in the SM by an effective mass

\[
m_{\text{EWPT,eff}} = m_h(\Lambda/m_h)^{\sin^2 \alpha}.
\]

This modification, numerically important for low \( f \), has been typically overlooked in the previous studies of the composite Higgs models.

On top of this effect we also expect possible contributions from physics at the cutoff, which can only be estimated by means of proper higher dimensional operators. We do not expect any such contribution for \( \hat{T} \) due to the custodial \( \text{SO}(4) \) contained in the \( \text{SO}(5) \). There will in general be, however, contributions to \( \hat{S} \), which can be estimated as\(^4\)

\[
\delta \hat{S}|_{\Lambda} \sim \frac{g^2 v^2}{\Lambda^2} \simeq 1.4 \times 10^{-3} \left( \frac{3 \text{ TeV}}{\Lambda} \right)^2.
\]

The study of concrete examples of partial UV completion (see Section\(^6\)) shows that this estimate is trustworthy, including its sign.

\(^2\)More precisely, the SM bound is obtained by imposing the relation \( |a_0| < 1/2 \) for the 0th partial wave amplitude for elastic scattering of the state \( (2W^+_L W^-_L + Z_L Z_L)\)/\( \sqrt{3} \).

\(^3\)We use parameters \( \hat{T}, \hat{S} \) which are proportional to the Peskin-Takeuchi parameters: \( \hat{T} = \alpha_{\text{EM}} T, \hat{S} = \frac{g^2}{\log S} S \).

\(^4\)This estimate generally applies in models without T-parity.
2.3 Naturalness

The finetuning estimated in eq. (2.8) will be the only source of finetuning in this model if, as we assume, the parameters $A$ and $B$ take typical values as consistent with the UV-sensitive contributions from various couplings breaking the $SO(5)$ symmetry. The parameter $B$ is the only one which breaks the $\phi_5 \rightarrow -\phi_5$ symmetry\(^5\) and will be renormalized multiplicatively; there is no naturalness constraint on its value.

The parameter $A$ is renormalized, first of all, by gauge boson loops:

$$\delta A_{\text{gauge}} = -\frac{3(2m_W^2 + m_Z^2)}{16v^2} \left( \frac{\Lambda}{2\pi f} \right)^2 \simeq -0.13.$$ 

In this estimate we assumed that the loop is cutoff by $\Lambda$, which is reasonable if the Higgs boson is composite, see Fig. 1.

![Figure 1: Quadratically divergent gauge-boson contributions to the renormalization of $A$. If the Higgs boson is composite, the scalar form factor cuts off the divergence in the first diagram. The second diagram is related to the first one by gauge invariance, and hence will be cut off at a comparable scale.](image)

Furthermore, if the Yukawa coupling of top is as in the SM, $A$ is also renormalized by the top quark loop:

$$\delta A_{\text{top}} = \frac{3m_t^2}{4v^2} \left( \frac{\Lambda_{\text{top}}}{2\pi f} \right)^2 \simeq 0.7 \left( \frac{\Lambda_{\text{top}}}{3 \text{ TeV}} \right)^2 \quad (2.15)$$

Although we used $\Lambda_{\text{top}} = 3 \text{ TeV}$ as reference, it is worth pointing out that a priori there is no reason to identify $\Lambda_{\text{top}}$ with the compositeness scale of the sigma-model $\Lambda$ given by (2.2). Anyhow, contribution (2.15) is the dominant one and provides a typical expected value of the $A$ parameter.

Via (2.9), $A \simeq 0.7$ corresponds to a 300 GeV Higgs boson. Notice that we have improved naturalness. Remember that in the SM we have $\Lambda_{\text{top}}^{\text{SM}} \simeq 400 \text{ GeV}$ for $m_h = 115 \text{ GeV}$ without finetuning ($\Delta = 1$). Here we have increased the Higgs mass and also allowed a finetuning $\Delta \sim f^2/v^2$. Thus it is not surprising that we can raise the scale of physics expected to cutoff the top loop by a factor $\sqrt{\Delta(m_h/115 \text{ GeV})}$ up to about 3 TeV.

For selfconsistency, we can also estimate the size of the quartic term $\frac{\kappa}{4} \bar{\phi} \phi^3$ omitted from (2.6). The top loop will generate a term

$$\kappa = \frac{3}{16\pi^2} \lambda_t^4 \log \frac{\Lambda^2}{m_t^2} \simeq 0.1.$$ 

Such a small coupling, if present, would be negligible for the present discussion. In particular, it would not influence in any significant way the minimization of the potential, and the quadratically

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\(^5\)This symmetry may be broken by possible Yukawa interactions, see Section 3
divergent contribution to the $A$ parameter induced by its presence,

\[ \delta A_{\kappa} \approx -\frac{5\kappa}{8} \left( \frac{\Lambda}{2\pi f} \right)^2 \approx -0.06 \]

is negligible compared to $\delta A_{\text{top}}$.

As a provisional conclusion, the model as it stands so far is hard to defend because of the EWPT. For $f = 500$ GeV we will have $(\sin \alpha)^2 \approx 0.25$, $m_{\text{EWPT,eff}} \approx 250 \div 500$ GeV for $m_h = 115 \div 300$ GeV. The combination of (2.13), (2.14) leads therefore to an embarrassing comparison with the experimental constraints on the electroweak parameters $\hat{S}, \hat{T}$. (See Fig. 2 the tip of the arrow marked ‘from cutoff’).

One obvious way to make the model consistent with the EWPT is to increase $f$. For example, for $f = 1$ TeV we will have $(\sin \alpha)^2 \approx 0.25$, $m_{\text{EWPT,eff}} \approx 145 \div 360$ GeV for $m_h = 115 \div 300$ GeV, and $\Delta S \approx 0.04$ from (2.14). In principle, this is consistent (at the border of the $2\sigma$ ellipse) for $m_h$ close to the direct lower bound. However, the finetuning price of $f = 1$ TeV from (2.8) is $\sim 3\%$, which in our opinion is starting to get uncomfortably large. Because of this we would like to stick to $f = 500$ GeV, and pursue another strategy to improve the EWPT consistency. Namely, we will add a new sector to the model which provides an extra positive contribution to $\hat{T}$. In [8], we solved a similar problem by enlarging the scalar sector of the SM.

![Figure 2: The minimal model in the ST plane, including the contributions (2.13) (‘from scalars’) and (2.14) (‘from cutoff’). The dashed arrow shows an extra positive contribution to $T$ needed to make the model consistent with the data. In Section 3.2 we discuss if such $\delta T > 0$ may come from an extended 3rd generation. Experimental contours taken from the LEPEWWG ST plot [9].](image)

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6Recall that the MSSM requires $\sim 5\%$ finetuning to increase the Higgs mass above the direct lower bound.
3 Third generation fermions

There are two principal motivations for extending the \( SO(5) \) symmetry to the top sector. First of all, the dominant UV sensitivity of the \( A \) parameter given by the top loop, Eq. (2.15), will be reduced from quadratic to logarithmic in the extended model. Second, we have seen above that the model begs for an extra contribution to \( \hat{T} \), and it is natural to ask if the physics which regulates the top loop can be simultaneously responsible for this contribution. An additional motivation is that we would like to find an effective four-dimensional (4d) description of existing 5d ‘composite Higgs’ models \[3\], \[4\], in which the \( SO(5) \) symmetry is naturally present in the fermion sector from the very beginning.

3.1 A minimal model

The minimal way to extend the \( SO(5) \) symmetry to the top Yukawa coupling is to enlarge the left-handed top-bottom doublet \( q_L \) to a vector \( \Psi_L \) of \( SO(5) \), which under \( SU(2)_L \times SU(2)_R \) breaks up as \((2, 2) + 1\). The full fermionic content of the third quark generation will be

\[
\Psi_L = (q, X, T)_L; \quad t_R, b_R, X_R, T_R
\]

where \( q_L, X_L, X_R \) are \( SU(2)_L \)-doublets, while all the other fields are singlets. We have introduced the right-handed states needed to preserve parity in the strong and electromagnetic interactions and to give mass to the new left-handed fermions. The \( q_L = (t_L, b_L) \) and \( t_R, b_R \) have the standard \( SU(2)_L \times U(1) \) quantum numbers, while the \( SO(5) \) symmetry fixes the hypercharges of the new vector-like states\[3\]: \( Y(X_{L,R}) = 7/6, Y(T_{L,R}) = 2/3 \).

The Yukawa Lagrangian of the third quark generation consists of an \( SO(5) \) symmetric mass term for the top and of three symmetry-breaking mass terms:

\[
\mathcal{L}_{\text{top}} = \lambda_1 \bar{\Psi}_L \phi t_R + \lambda_2 f \bar{T}_L T_R + \lambda_3 f \bar{T}_L t_R + m_X \bar{X}_L X_R + \text{h.c.}
\] (3.2)

The coupling \( \lambda_2 \) and the mass \( m_X \) are soft-breaking terms; at one loop they generate logarithmically divergent contributions to the \( A \) parameter in (2.6). The coupling \( \lambda_3 \) breaks \( \phi_5 \to -\phi_5 \) symmetry and generates quadratically divergent \( B \). Thus \( B \sim A \) can be natural for \( \lambda_3 \sim 1/(4\pi)^2 \). The Yukawa coupling that generates the bottom mass is taken conventional, i.e. explicitly breaking, like the gauge couplings, the \( SO(5) \) symmetry.

Since explicitly

\[ \bar{\Psi}_L \phi = \bar{q}_L H^c + \bar{X}_L H + \bar{T}_L \phi_5, \]

after the EWSB \( \mathcal{L}_{\text{top}} \) becomes to leading order in \( H \)

\[
\mathcal{L}_{\text{top}} = \lambda_1 \bar{q}_L H^c t_R + \lambda_1 \bar{X}_L H t_R + (\lambda_1 + \lambda_3) f \bar{T}_L t_R + \lambda_2 f \bar{T}_L T_R + m_X \bar{X}_L X_R + \text{h.c.}
\] (3.3)

To zeroth order in \( v \), mass matrix diagonalization is achieved by the field rotation:

\[
T_R \to \cos \chi T_R - \sin \chi t_R, \quad t_R \to \cos \chi t_R + \sin \chi T_R,
\]

\[
\tan \chi = \lambda_1'/\lambda_2, \quad \lambda_1' = \lambda_1 + \lambda_3.
\]

\[7\)The hypercharge of the components of \( \Psi_L \) is given by \( Y = T^3_R + 2B \), with \( B \) the baryon number. We can take \( T^3_R \) of \( q_L, X_L \) as \(-1/2 \) and \(+1/2 \) respectively, whereas \( T_L \) has \( T^3_R = 0 \).
Table 1: The masses and compositions of the physical states in terms of the fields appearing in (3.4), denoted by zero superscript. The mixing parameters are $\epsilon_R = \frac{m_t}{m_X}$ and $\epsilon_L = \frac{\lambda_T v}{m_T}$.

under which

$$
\mathcal{L}_{\text{top}} \to \bar{q}_L H c (\lambda_t t_R + \lambda_T T_R) + \bar{X}_L H (\lambda_t t_R + \lambda_T T_R) + m_T \bar{T}_L T_R + m_X \bar{X}_L X_R + \text{h.c.}
$$

(3.4)

The mass and the composition of the physical top quark and of the three new quarks $T$, $X_{2/3}$ and $X_{5/3}$ in terms of the fields appearing in (3.4), in the relevant limit $m_T, m_X \gg m_t$, are given in Table 1.

Notice that the bottom quark in this model is, at tree level, completely standard, which is just a consequence of the absence of states which could mix with it.

### 3.2 Fermionic loop corrections

The new fermions will give rise to relevant loop corrections, both to the parameters in the potential (2.6) and to several directly observable quantities. Here we concentrate on the contributions to the electroweak parameters and to B-physics.

#### 3.2.1 The $\rho$-parameter

Unlike the scalar sector, in the fermion sector every modification of $\hat{T}$, $\hat{S}$ relative to the SM dies out as $v^2/m^2$, where $m$ is a mass of the new colored states. Nevertheless, since $\hat{T}$, or $\delta \rho$, in the SM from the top-bottom loops is about 1%, the extra correction to $\hat{T}$ from the heavy fermions may be significant, whereas they are negligible in the case of $\hat{S}$. For ease of exposition, we give approximate analytic formulae for the corrections to $\hat{T}$ starting from (3.4) with the term $\bar{X}_L H \lambda_T T_R$ neglected. This allows to treat separately the contributions from $T$ and $X$, which are mixed with the top via the parameters $\epsilon_L = \lambda_T v/m_T$ and $\epsilon_R = m_t/m_X$ respectively (See Table 1). We have checked numerically that, in the region of interest, these approximations are defendable and can correctly guide the physical discussion of the various effects.
To leading order in $v^2/m_T^2$, the extra contributions to $\hat{T}$ are given by

\[
\delta \hat{T}_T \simeq \hat{T}^\text{SM}_{\text{top}} \left[ 2 \varepsilon_L^2 \left( \log \frac{m_T^2}{m_t^2} - 1 + \frac{\lambda_T^2}{2 \lambda_t^2} \right) \right],
\]

(3.5)

\[
\delta \hat{T}_X \simeq \hat{T}^\text{SM}_{\text{top}} \left[ -4 \varepsilon_R^2 \left( \log \frac{m_X^2}{m_t^2} - \frac{11}{6} \right) \right],
\]

(3.6)

where

\[ \hat{T}^\text{SM}_{\text{top}} = \frac{3}{32 \pi^2} \frac{m_t^2}{v^2} \simeq 0.009. \]

Taking into account the starting point of Fig. (2), the negative contribution to $\hat{T}$ from (3.6) makes an $X$-particle lighter than about 1.5 TeV unacceptable. On the contrary, a $T$-fermion singlet mixed with the left handed top can give the desired positive contribution to $\hat{T}$.

### 3.2.2 $Z \to b\bar{b}$ and $b \to s l\bar{l}$

Since the top quark mixes with states with different $SU(2)_L$ quantum numbers, see Table 1, we can expect small deviations from the SM in effects involving the bottom quark. In the discussion below we will neglect the mixing between $t_R$ and $X_R$, since the $X$ quark is necessarily quite heavy ($m_X \gtrsim 1.5$ TeV), we do not expect this mixing to lead to observable constraints. Instead, we will concentrate on the mixing between $t_L$ and $T_L$, since the EWPT suggest that this mixing may be significant.

To have a meaningful unified discussion of all effects, we must first of all introduce the flavor structure in our model. The most natural way to do this is to assume that the mechanism described in Section 3, with the softly broken $SO(5)$ symmetric Yukawa coupling for the top, is operational in the up quark sector of all three generations. For simplicity we also assume that the matrices $\lambda_1$, $\lambda_2$ and $\lambda_3$ are simultaneously diagonalizable. On the other hand, the Yukawa couplings generating the down quark masses are taken like in the SM. Decoupling the very heavy $X$ quarks, the relevant Yukawa Lagrangian is

\[
\mathcal{L}_{\text{Yuk}} = \bar{q}_L H^c \lambda^u u_R + \bar{q}_L H^c \lambda^t T_R + \bar{T}_L m_T T_R + \bar{q}_L H V \lambda^d d_R + \text{h.c.}
\]

(3.7)

where we went to the basis in which $\lambda^u$, $\lambda^d$, $\lambda^t$, $m_T$ are diagonal, and $V$ is the unitary CKM matrix. The discussion of the previous subsection remains unchanged with an obvious meaning of the symbols.

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8This result can also be found in [11], eq. (42),(43). We are grateful to José Santiago for pointing out a numerical error in the first version of the paper.

9The approximate expressions (3.5), (3.6) are reasonably accurate for $m_T \gtrsim 500$ GeV, $m_X \gtrsim 1$ TeV. Numerical analysis shows that $\delta T_X$ grows even more negative for $m_X < 1$ TeV. (In particular, (3.6) does not apply for $m_X \lesssim 400$ GeV, when the RHS of (3.6) becomes positive.) This behavior can be traced to the opposite sign of $T_3(X_{2/3})$ with respect to the top. Notice that for $m_X = 0$ Lagrangian (3.2) is custodially-symmetric, which implies that in this limit the contribution of $X$ to the $\rho$-parameter should exactly compensate the standard top contribution.
As it happens in the SM, one loop exchanges of the top and of the heavier \( T \) modify the couplings of the \( Z \) to the down quarks as

\[
\left(-\frac{1}{2} + \frac{\sin^2 \theta_w}{3} + A_{bb}\right) \frac{g}{\cos \theta_w} Z \bar{b}_L \gamma^\mu b_L \quad \text{and} \quad A_{bs} \frac{g}{\cos \theta_w} Z \bar{b}_L \gamma^\mu s_L.
\]

In the large \( m_t \) limit, the SM values

\[
A_{bb}^{SM} = \frac{\lambda_t^2}{32\pi^2}, \quad A_{bs}^{SM} = V_{ts} V_{tb}^* A_{bb}^{SM}
\]

are corrected in the model under consideration by the same relative factor

\[
\frac{A}{A_{SM}} = 1 + 2\epsilon_L^2 \left( \log \frac{m_T^2}{m_t^2} + \frac{\lambda_T^2}{2\lambda_t^2} \right) \quad (3.8)
\]

The experimental constraints on \( A_{bb} \) from the LEP precision measurements of \( R_b = \Gamma(Z \to bb)/\Gamma(Z \to \text{had}) \) is\(^\text{10}\)

\[
A_{bb} / A_{bb}^{SM} = 0.88 \pm 0.15. \quad (3.9)
\]

The current constraint on \( A_{bs} \) coming from the data on \( B \to X_s l^+ l^- \) decays is\(^\text{13}\)

\[
A_{bs} / A_{bs}^{SM} = 0.95 \pm 0.20. \quad (3.10)
\]

A comparison of (3.5) and (3.8) shows that the required increase of \( \hat{T} \) in the SM by about 30–40% will induce an analogous effect in both \( A_{bb} \) and \( A_{bs} \) which looks hardly consistent with Eqs (3.9) and (3.10). Future measurements of the branching ratio \( B(B_s \to \mu^+ \mu^-) \) at the LHCb experiment are expected to reduce the error in (3.10) to 10% level. A similar effect will also be present for the \( b \to s \gamma \) process, which agrees with the SM at 10% level of error\(^\text{14}\).

3.2.3 \( B\bar{B} \) mixing

In a fully analogous way the box diagrams with exchanges of the top and of the heavier \( T \)-quark modify the \( B\bar{B} \) mixing both in the \( B_d \) and in the \( B_s \) systems. Still neglecting corrections vanishing like \((m_W/m_t)^2\), the effective \( \Delta B = 2 \) Lagrangian will have the form, at the top mass scale,

\[
\mathcal{L}^{eff} = C_d (\bar{b}_L \gamma^\mu d_L)^2 + C_s (\bar{b}_L \gamma^\mu s_L)^2 \quad (3.11)
\]

with

\[
C_{d,s} = C_{d,s}^{SM} \left[ 1 + 2\epsilon_L^2 \left( \log \frac{m_T^2}{m_t^2} + 1 + \frac{\lambda_T^2}{2\lambda_t^2} \right) \right] \quad (3.12)
\]

in terms of the SM coefficients \( C_{d,s}^{SM} \). Again, if one wants to produce the desired \( \hat{T} \) from \( T \)-quark exchanges, an increase in \( C_{d,s} \) is required which may be however at the level of the current 25–30% uncertainty of the lattice calculations of the relevant matrix elements\(^\text{15}\).

\(^{10}\)We used the measured value \( R_b = 0.21629 \pm 0.00066 \)\(^\text{12}\) as well as the theoretical estimate \( R_b = 0.21578 \pm 0.0001 - 0.99(A_{bb} - A_{bb}^{SM}) \).


4 Third generation quarks: alternatives

As we have seen in the previous section, in the model with the minimal fermion content getting a positive contribution to $T$ of the necessary size seems impossible without generating at the same time contributions to B-physics observables exceeding the experimental constraints. One may wonder if this problem could be solved at the price of extending the 3rd generation sector even further. Below we will discuss two possible extensions, which have a common feature that the Yukawa Lagrangian of the 3rd generation consists of two parts

$$\mathcal{L}_{\text{top}} = \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{BSM}}$$

(4.1)

where the $\mathcal{L}_{\text{BSM}}$ involves only the new, Beyond-Standard-Model, fermionic fields, while $\mathcal{L}_{\text{int}}$ couples bilinearly the SM fermions to a subset of BSM fields, $Q, T$, with appropriate quantum numbers:

$$\mathcal{L}_{\text{int}} = \lambda_1 f \bar{q}_L Q_R + \lambda_2 f \bar{T}_L t_R + \text{h.c.}$$

4.1 Fermions in the spinorial

An alternative description of the top Yukawa coupling in an $SO(5)$-symmetric way is through the spinor, $\chi$, rather than the vector, $\Psi$, of $SO(5)$ (see e.g. [16] for a related model). In this case the full fermionic content in (3.1) gets replaced by

$$\chi_{L,R} = (Q, B, T)_{L,R}; \quad q_L, t_R, b_R$$

where, under $SU(2)_L \times U(1)$,

$$Q_L, Q_R, q_L = 2_{1/6}, \quad B_L, B_R, b_R = 1_{-1/3}, \quad T_L, T_R, t_R = 1_{2/3}.$$ 

The Yukawa Lagrangian has the form (4.1) with

$$\mathcal{L}_{\text{BSM}} = y_{\chi} \bar{\chi}_{L,R} \phi_{\Gamma} \chi_{R} + m_Q \bar{Q}_L Q_R + m_T \bar{T}_L T_R + m_B \bar{B}_L B_R + \text{h.c.},$$

(4.2)

where, in terms of the $SO(5)$ $\Gamma$-matrices,

$$\chi&\phi_{\Gamma} \chi = f (\bar{Q}_L Q - \bar{B}_L B - \bar{T}_L T) + \sqrt{2} (\bar{Q}_L H^c T_R + \bar{Q}_L H B_R - \bar{T}_L H Q_R - \bar{B}_L H^c Q_R).$$

From these equations it is straightforward to obtain the spectrum and the composition of all the colored states (4 more than normal). Without doing this here explicitly, we limit ourselves to notice that the physical left-handed b-quark becomes an admixture of doublet and singlet $B_L$, at first order in $v/f$, which is phenomenologically problematic for low $f$.

4.2 Extended model with fermions in the fundamental

The 3rd quark generation in such a model includes, apart from the SM fields $q_L, t_R, b_R$, 5 new quarks organized in a Dirac fiveplet $\Psi_{L,R} = (Q, X, T)_{L,R}$ of the $SO(5)$ symmetry. The Yukawa Lagrangian has the form (4.1) with

$$\mathcal{L}_{\text{BSM}} = y_1 \bar{\Psi} \phi T_R + y_2 \bar{T}_L \phi^\dagger \Psi_R + m_Q \bar{Q}_L Q_R + m_X \bar{X}_L X_R + m_T \bar{T}_L T_R + \text{h.c.}$$
Since $b_L$ in this model can mix only with $Q^d_L$, which has the same quantum numbers, the Z-boson coupling of the physical left-handed bottom quark will be standard at tree level\textsuperscript{11}.

It would be interesting to compute the one-loop contributions to the $T$ parameter in this model explicitly, and determine if there are regions of the parameter space consistent with the EWPT and the B-physics constraints. We do not expect, however, that the situation will be significantly better than for the minimal fermionic content. The basic reason is that in both cases the major source of positive $\delta T$ is the mixing of $t_L$ with the singlet $T_L$, and it is precisely this mixing which, at one-loop level, led to unacceptably large contributions to $Z \to b\bar{b}$ and $B\bar{B}$ mixing observables in Section 3.2.

5 Perturbative minimal model

As already mentioned, a natural interpretation of the model described so far is in terms of a “composite” picture for the Higgs boson, produced by an unspecified strong dynamics at $\Lambda$. As an alternative, however, it makes sense to consider also the case in which the entire model is fully perturbative up to a suitable cut-off scale. To this end we replace the potential (2.6) with

$$V = \lambda (\phi^2 - f^2)^2 - Af^2 \phi^2 + Bf^3 \phi_5, \quad (5.1)$$

where the coupling $\lambda$ is somewhat greater than $A$ and $B$ but always perturbative. The explicit discussion of this case proceeds along parallel lines to the ones followed so far for the strong coupling, with one more parameter present, which is usefully taken as $A/\lambda$. By requiring that the one loop correction to the squared mass of $\phi$ from the symmetric coupling $\lambda$ does not exceed the tree level term (a weaker condition can be easily implemented), one obtains the new cutoff scale

$$\Lambda_{\text{nat}} \simeq \frac{4\pi f}{\sqrt{N+2}} \simeq 4.7 f, \quad (5.2)$$

where $N = 5$, i.e. $\Lambda_{\text{nat}} \simeq 2.4$ TeV for $f = 500$ GeV. This constraint on the cutoff dominates over every other consideration.

There are some significant differences with respect to the strongly interacting case in the EWSB sector, both in the spectrum and in the couplings. The connection of $v$ to $f$ is now given by

$$\langle |H|^2 \rangle \equiv v^2 = \frac{f^2}{2} \left[ 1 + \frac{A}{2\lambda} - \left( \frac{B}{2A} \right)^2 \right] > 0,$$

hence a modified finetuning relation

$$\Delta = \frac{A}{v^2} \frac{\partial v^2}{\partial A} \simeq \frac{f^2}{v^2} (1 + z), \quad z = \frac{3A}{4\lambda}, \quad (5.3)$$

which requires $z$ to be somewhat smaller than unity in order not to worsen the finetuning. More importantly, the scalar spectrum now contains two scalar particles below the cutoff

$$h = \cos \alpha \, \phi_3 + \sin \alpha \, \phi_5, \quad \sigma = -\sin \alpha \, \phi_3 + \cos \alpha \, \phi_5, \quad (5.4)$$

\textsuperscript{11}Incidentally, for $m_Q = m_X$ the BSM sector of the model has $O(4) = SU(2)_L \times SU(2)_R \times P_{LR}$ symmetry, which is known to protect the $Zb_L \bar{b}_L$ coupling \textsuperscript{10}. 

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Figure 3: \((f = 500 \text{ GeV})\) The masses of the light and heavy scalars in the perturbative minimal model, Eq. (5.5). The region above the dashed line corresponds to \(z > 1/2\) and is relatively disfavored by Naturalness, see Eq. (5.3).

whose masses, given by

\[
m^2 = 4\lambda f^2 \left(1 + z \pm \left[1 + 2z \left(1 - 4v^2/3f^2\right) + z^2\right]^{1/2}\right)
\]

are shown in Fig. 3 as functions of \(A\) and \(\lambda\) (for \(f = 500 \text{ GeV}\)). Note that \(\lambda = 3\) makes \(m_\sigma\) exceed \(\Lambda_{\text{nat}}\). In the same way the mixing angle and the effective value of \(m_{\text{EWPT,eff}}\), to be used as in Section 2.2 to determine the corrections to \(\hat{S}, \hat{T}\), are given in Fig. 4.

An important consequence of the presence of the \(\sigma\) particle below the cutoff is that the growth of the longitudinal \(W W\) cross section as in (2.11) is actually cutoff at \(\sqrt{s} \simeq m_\sigma\), where the constant behavior sets in. Finally the description of the third-generation quarks and of their consequences for the EWPT are unchanged after the following identification between the \(f\) parameters of the models at perturbative and strong coupling:

\[
f_{\text{strong}} = f_{\text{pert}} \left(1 + A^2/\lambda\right)^{1/2}.
\]

We conclude that in a large range of couplings, \(\lambda = 0.5 \div 3\), the perturbative model gives a simple extension of the strong coupling model of Section 2. Their cutoffs (5.2) and (2.2) have a different interpretation (in the former case, new physics should cutoff the quadratic divergence destabilizing the \(f\) scale, while in the latter it should restore unitarity of the longitudinal \(W W\) scattering, see (2.12)), however numerically they are both close to 3 TeV. The finetuning needed to get \(v \ll f\) is comparable. Consistency with the EWPT could be even better than in the strong coupling case, since the contribution to the \(S\) parameter from the cutoff, eq. (2.14), need not be present if the physics which cuts off the quadratic divergence of the \(\phi\) potential enters only at loop level\(^{12}\). In this case a relatively smaller \(\Delta T \simeq 0.1 - 0.2\) could suffice to restore the consistency of

\(^{12}\)In strong coupling case, we expect vectorial resonances at the cutoff, which contribute to the \(S\) parameter at tree level, see examples in Section 6.
Figure 4: ($f = 500$ GeV) The mixing angle $\alpha$ between the heavy and light scalars, see Eq. (5.4), and the effective EWPT mass as defined by (2.13) with $\Lambda$ replaced by $m_\sigma$. The region above the dashed line has the same meaning as in Fig. 3.

the EWPT fit. Such a $\Delta T$ could be produced by the extended 3rd generation, eq. (3.6), without exceeding experimental constraints in B physics discussed in Section 3.2.2 (although giving effects which could be observable in the future).

Finally, it could be nontrivial to distinguish the strongly coupled and perturbative model at the LHC. Even in the most favorable case $m_\sigma \simeq 1$ TeV, $(\sin \alpha)^2 \simeq 0.2$ (the lower left corner of the $\lambda, A$ plane in the plots), the production cross section of the $\sigma$ particle will be only $\sim 10$ fb, which makes its observation challenging if not impossible.

6 Non-minimal models

6.1 General picture

As we have already mentioned above, and stress again now, the model described in Section 2.1 can be considered as a low energy description of any model in which the EWSB sector has an $SO(5)$ global symmetry partly gauged by $G_{SM}$. In all such theories one can introduce an effective dimensionless fiveplet field $\phi_{eff}$, $\phi_{eff}^2 = 1$, which specifies the alignment angle. Symmetry considerations imply that the symmetry breaking term in the effective action for the SM gauge fields has to have the form

$$\mathcal{L}_{EWSB,eff} = \frac{1}{2} \eta_{\mu\nu} \Pi(p^2) \phi_{eff} A_\mu A_\nu \phi_{eff}, \quad \eta_{\mu\nu} = \eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}, \quad \eta_{\mu\nu}^\perp = \eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}, \quad (6.1)$$

where $A_\mu$ is an auxiliary $SO(5)$ gauge field with all of the components except for the SM gauge fields set to zero:

$$A_\mu = W_\mu^a T_L^a + B_\mu T_R^3. \quad (6.2)$$

The self-energy $\Pi(p^2)$ depends on the theory under consideration; to compute it, it is enough to consider the perfect alignment case $\phi = (0, 0, 0, 0, 1)$. This observation is the essence of the
so-called method of matching effectively applied in [3, 4] in the case of 5d models. By comparing with Section 2.1, we can identify the effective sigma-model scale:

\[ f^2 = \Pi(0), \]

(6.3)

so that the weak scale (or the W mass) is given in terms of the misaligned VEV

\[ \phi_{\text{eff}} = (\varepsilon, 0, 0, 0, \sqrt{1 - \varepsilon^2}) \]

(6.4)

by the same relation as (2.5):

\[ v^2 = \frac{1}{2} \varepsilon^2 f^2. \]

(6.5)

For nonzero \( \Pi'(0) \), the Lagrangian (6.1) also describes the kinetic mixing between \( W_3 \) and \( B \), i.e. the \( \hat{S} \) parameter. For the same \( \phi_{\text{eff}} \) as in (6.5), we have [3]

\[ \hat{S} = \frac{g^2 \varepsilon^2}{4} \Pi'(0) = \Pi'(0) \frac{m_W^2}{f^2}. \]

(6.6)

One can imagine that in a general class of models the low energy effective potential for \( \phi_{\text{eff}} \) will have the form of a quadratic polynomial \( f^4 (A_{\phi_{\text{eff}}}^2 + B_{\phi_{\text{eff}},5}) \), analogous to (2.0), with parameters \( A \) and \( B \) functions of more fundamental parameters of the theory. Assuming that \( A \) and \( B \) scan their typical ranges, the finetuning estimate (2.8) will apply generally to all models of this sort. This means that two different models with the same \( f \) will likely have the same level of finetuning, and at this point can be meaningfully compared with respect to other criteria, such as consistency with the EWPT.

To illustrate the above general points, we consider two concrete examples of extended models which can be efficiently described by Eqs. (6.1)-(6.6). We then compare their ‘performance’ with the minimal models of Sections 2.1 and 5.

### 6.2 Two-site deconstructed model ("Little Higgs")

The EWSB sector of this model consists of a real scalar \( 5 \times 5 \) matrix field \( \Sigma \), of a scalar fiveplet \( \Phi \), and of an \( SO(5) \) gauge field \( X_\mu \) which acts on \( \Phi \) and, from the right, on \( \Sigma \). The \( \Sigma \) is assumed to take a VEV, \( \langle \Sigma \rangle = f_0 1 \). There is a global \( SO(5) \) symmetry acting on \( \Sigma \) from the left. If \( \Phi \) also takes a VEV, \( \Phi^2 = F^2 \), this \( SO(5) \) global symmetry is broken to \( SO(4) \). We put the SM gauge group inside an \( SO(4) \) subgroup of \( SO(5) \) acting on \( \Sigma \) from the left (without loss of generality, we assume that this \( SO(4) \) acts on the first 4 lines of \( \Sigma \)). At this point we see that this model fits the general framework of the previous Section, and thus we expect that the effective symmetry breaking lagrangian will have the form (6.1) with \( \phi_{\text{eff}} = \langle \Phi / |\Phi| \rangle \).

We can obtain an equivalent description of the same model by going to a different gauge in which \( \Phi = (0, 0, 0, 0, F) \). In this gauge the Goldstone degrees of freedom are contained in the matrix field \( \Sigma \) subject to the constraint \( \Sigma \Sigma^t = f_0^2 1 \). In what follows we concentrate on the interesting limiting case \( F \gg f_0 \). In this case the model simplifies, since only the \( SO(4) \) subgroup of the \( X_\mu \) gauge bosons survives. Thus the model has an extended gauge group \( G_{SM} \times [SU(2)_1 \times SU(2)_2] \),
with the two new SU(2)’s assumed to have the same coupling $g_s$. The EWSB comes from the kinetic Lagrangian of the $\Sigma$ field:

$$\mathcal{L}_{\text{kin}} = \frac{1}{2} \text{Tr} \left( D_\mu \Sigma \right)^2,$$

$$D_\mu \Sigma = \partial_\mu \Sigma + i(W^a_\mu T^a_L + B_\mu T^3_R)\Sigma - i\Sigma(W^a_\mu T^a_L + W^a_2 T^a_R)$$

This model can be obtained as a two-site deconstruction of the 5d model described below, which explains its name.

Constructions of this type (although perhaps not this precise one) were extensively discussed in the literature on Little Higgs models \[^{17}\] because they realize the so-called collective symmetry breaking mechanism, which removes, at the one-loop level, the quadratically divergent contribution to the Higgs mass parameter from the coupling to the gauge bosons.\[^{13}\] The SM gauge boson loop is canceled by a loop of new heavy gauge bosons present in the theory.

In principle, as we have seen in Section 2.3, the sensitivity of the Higgs mass parameter to the gauge boson loop is subdominant to the typical top-loop contribution, even when the latter is cutoff by extra fermionic states. In practice this means that naturalness considerations do not require the presence of states regulating the gauge-boson loop below $2 \div 3$ TeV, which weakens the case for their observation at the LHC. Nevertheless, let us go ahead and analyze the two-site model in some detail. The low energy effective action for the SM gauge bosons will have the form

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2} \frac{\eta_{\mu\nu}}{g_0^2} \left( \frac{p^2}{g_0^2} W_\mu W^\mu + \frac{p^2}{g_0^2} B_\mu B_\nu + \Pi_0(p^2) \text{Tr}[A_\mu A_\nu] - \Pi(p^2) \phi^A \phi^A \right).$$  \hspace{1cm} (6.7)

Here the last two terms appear when integrating out the $W_{1,2}$ at tree level. In agreement with the above discussion, they can be written in an SO(5)-covariant form using the auxiliary field notation (6.2). The effective SU(2)$_L \times$ U(1) coupling constants $g, g'$ at low energy are given by (see (6.4))

$$\frac{1}{g^2} - \frac{1}{g_0^2} = \frac{1}{g'^2} - \frac{1}{g_0'^2} = \delta ,$$  \hspace{1cm} (6.8)

$$\delta = \Pi_0'(0) - \Pi'(0) \frac{\epsilon^2}{4}$$

The symmetric and symmetry-breaking formfactors $\Pi_0$ and $\Pi$ are evaluated by an explicit calculation to be:

$$\Pi_0 = \frac{p^2}{g_0^2} \left( 1 - \frac{p^2}{g_0^2 f_0^2} \right)^{-1}, \quad \Pi = 2 f_0^2 \left( 1 - \frac{p^2}{g_s^2 f_0^2} \right)^{-1}.$$

This gives $\delta = g_s^{-2}(1 - \epsilon^2/2)$ in (6.8). The couplings $g_0, g'_0$, and $g_s$ have to be adjusted so that $g$ and $g'$ take their SM values.

Using the general formulas (6.3) and (6.6), we can also compute the $\hat{S}$ parameter of the model:

$$\hat{S} = \frac{m_W^2}{m_{W'}^2} \simeq 1.6 \times 10^{-3} \left( \frac{2 \text{TeV}}{m_{W'}} \right)^2$$  \hspace{1cm} (6.9)

\[^{13}\]The Little Higgs models in a strict sense of the term also contain a mechanism to generate the Higgs quartic coupling at tree level. Such a mechanism is absent in the model under discussion.
where

\[ m_{W'} \simeq g_s f_0 \]

is the mass of the lightest among the new heavy gauge bosons present in the theory (these masses can be found as extra zeros of the SM gauge bosons self-energies occurring at \( p^2 > 0 \)). Physically the nonzero \( \hat{S} \) parameter appears because of tree-level mixing between the SM gauge bosons and these new vector states. Going back to the discussion in Section 2.2, we see that the EWPT do not allow the heavy gauge bosons below about 2 TeV.

In the above discussion we did not have to make any assumption about the dynamics which causes \( \phi_{\text{eff}} \) to assume a misaligned VEV (6.4). Rather generally, one can describe such dynamics in terms of a soft symmetry breaking potential for \( \Sigma \) of the form consistent with the gauge symmetry of the model:

\[ A f_0^2 \text{Tr}(\Sigma_1 \Sigma^t_1) + B f_0^3 \Sigma_{55} = \text{diag}(1, 1, 1, 1, 0). \]

In this case, the finetuning of the model can be estimated by (2.8) via the effective sigma-model scale of the model \( f \), found from (6.3) to be

\[ f^2 = 2 f_0^2. \]

Finally, let us estimate the cutoff of the model. This can be done imagining a completion into a linear sigma-model with the \( SO(5) \times SO(5) \) symmetric potential

\[ V = \lambda \text{Tr}(\Sigma \Sigma^t - f_0^2 1)^2. \]

Demanding that the contribution from self-interaction loops to \( f_0^2 \) does not exceed its low-energy value, we get the cutoff \((N = 5)\)

\[ \Lambda = \frac{4\pi f_0}{\sqrt{2N + 1}} \simeq 3f_0. \quad (6.10) \]

In the limit of strong \( \lambda \) we should use e.g. unitarity bounds to properly set the cutoff, but we expect that the bounds obtained this way will be not far from (6.10), as it happened in the \( SO(5)/SO(4) \) case, see Sections 2.1 and 5. For \( f = 500 \) GeV we get \( \Lambda \simeq 1.5 \) TeV, which is a rather low value. In particular, the heavy gauge bosons, in order to be consistent with the EWPT, should have masses exceeding the cutoff of the theory.

### 6.3 5d model

The model [3, 4] is defined in flat 5d spacetime compactified on an interval \( 0 \leq y \leq l \). It has \( SO(5) \) gauge symmetry in the bulk which is broken to \( G_{\text{SM}} \) at \( y = 0 \) (the so-called UV brane) and to \( SO(4) \) at \( y = l \) (IR brane). The model thus fits the general scheme of Section 6.1. The Lagrangian is

\[
\mathcal{L} = \mathcal{L}_0 \delta(y) + \mathcal{L}_5 + \mathcal{L}_l \delta(y - l),
\]

\[
\mathcal{L}_0 = -\frac{1}{4g_0^2} (W_\mu^a)^2 - \frac{1}{4g_0^2} B_{\mu
u}^2,
\]

\[
\mathcal{L}_5 = -\frac{M}{4} A_{MN}^2,
\]

\[
\mathcal{L}_l = \frac{1}{2} (\partial_\mu \Phi - A_\mu \Phi)^2, \quad \Phi^2 = F^2.
\]
Here $A_M$ is an $SO(5)$ gauge field in 5d, which on the UV brane has only $G_{SM}$ nonzero boundary values (5.2). The parameter $M$ with dimension of mass is related to the 5d gauge coupling constant by $M = 1/g_5^2$. The alignment parameter is $\phi_{\text{eff}} = \Phi/|\Phi|$, the same as in the previous model. In what follows we consider the limit when $F$ is much bigger than any other scale in the theory, so that $L$ effectively enforces boundary conditions $A_\mu \phi_{\text{eff}}\big|_l = 0$.

The original model [3, 4] was formulated in the AdS space, with the purpose of resolving the Hierarchy Problem up to the Planck scale. Since we are interested only in the Little Hierarchy Problem and in the LHC phenomenology, we here consider a simpler version in flat 5d space. As is well known [18], the curvature of the AdS space can be mimicked by the kinetic terms for the SM gauge fields on the UV brane contained in $L_0$.

After integrating out the bulk, the low-energy effective Lagrangian for the SM gauge bosons takes again the form (6.7). The formfactors are however different; they are given by $(p \equiv \sqrt{p^2})$

$$
\Pi_0 = pM \tan pl, \quad \Pi = 2pM[\tan pl + (\tan pl)^{-1}]
$$

Applying the general formulas (6.3), (6.6), (6.8), we have

$$
\frac{1}{g^2} = \frac{1}{g_0^2} + ML \left(1 - \frac{\varepsilon^2}{3}\right),
$$

$$
f^2 = \frac{2M}{l},
$$

$$
\hat{S} = \frac{2}{3}(m_W l)^2 \simeq 2 \times 10^{-3} \left(\frac{1.5 \text{ TeV}}{l^{-1}}\right)^2
$$

(6.11)

Of interest is the maximal possible value of $M$, because it controls the energy cutoff of the 5d theory

$$
\Lambda_{\text{NDA}} = \frac{24\pi^3 M}{N_c}, \quad N_c = 5.
$$

(6.12)

Using $l = (1.5 \text{ TeV})^{-1}$ and $f = 500 \text{ GeV}$ (which are the maximal values affordable without compromising too much with EWPT or Naturalness), we have $M \simeq 80 \text{ GeV}$, corresponding to

$$
\Lambda_{\text{NDA}} \simeq 12 \text{ TeV},
$$

The heavy vector resonances have masses found from the equation

$$
\frac{p^2}{g_0^2} + \Pi_0(p^2) = 0.
$$

Since we are in the regime $ML \ll g_0^{-2}$, the first few resonance masses are well approximated by

$$
m_W \simeq \frac{\pi}{2l} n \simeq (2.4 \text{ TeV})n, \quad n = 1, 3, 5 \ldots
$$

(6.13)

It is instructive to rewrite (6.11) in terms of the lightest resonance mass as

$$
\hat{S} = \frac{\pi^2 m_W^2}{6 m_W^2} \simeq 1.6 \frac{m_W^2}{m_W^2}.
$$

(6.14)
The analogous relation in the original AdS model \[3\] had a slightly larger coefficient:

\[
\hat{S}_{\text{AdS}} = \frac{27\pi^2}{128} \frac{m_W^2}{m_W^2} \simeq 2.1 \frac{m_W^2}{m_W^2}. \tag{6.15}
\]

Comparing (6.9), (6.14), (6.15) with (2.14), we see that the latter estimate works quite well in all three cases, provided that we identify the cutoff with the mass of the first resonance.

### 6.4 Comparison and appraisal

Let us conclude this Section with a comparison of the two extended models with the two minimal models of Sections 2.1 and 5. The models can only be meaningfully compared at the same level of finetuning, which in practice means at the same value of the effective sigma-model scale \(f\).

In the two-site model, consistency with the EWPT pushes the heavy gauge bosons above the cutoff. In such a situation it is hard to see any gain in introducing the extra gauge bosons in the first place. The calculability of the theory gets completely lost. In particular, there is no reason to single out the heavy gauge boson contribution to \(\hat{S}\), Eq. (6.9), among contributions of other states present at the cutoff.

At the first glance, the situation in the 5d model case is more favourable, since the ratio of the cutoff and the lightest resonance mass is \(\Lambda_{\text{NDA}}/m_W' \simeq 5\). However, the first resonance mass is exactly equal, rather than being smaller, to the energy scale (2.12) at which the WW scattering in the effective sigma-model description exceeds unitarity. In view of this, it appears to us that the claims about improved calculability in this model have to be substantiated better than appealing to the NDA estimate (6.12), which could be too optimistic. This could be done, e.g., by computing the one-loop correction to the tree-level result (6.11) for the \(\hat{S}\) parameter, and demonstrating explicitly that this correction is small. Notice also, from (6.13), that the resonances are not equally spaced, so that already the 3rd resonance mass equals \(\Lambda_{\text{NDA}}\).

### 7 Conclusions

The variety of ideas put forward in the context of a “composite” picture for the Higgs boson calls for a simple and, at the same time, effective description of the related phenomenology. In this paper we attempted to give such a description, and applied it to the potentially relevant example of a Higgs-top sector from an \(SO(5)\) symmetry.

Our starting point is the simple observation that much of the important phenomenology at relatively low energies should be captured by an approximate \(SO(5)\)-invariant Lagrangian obtained by suitably extending the SM Higgs doublet and the left handed top-bottom doublets: the minimal way is by a real 5-plet for the Higgs field and again a 5-plet of Weyl spinors for the top doublet, one for each colour, complemented by the three right-handed partners of the extra components. Other less economic extensions of the third generation quark-doublet may be considered as well. The \(SU(2) \times U(1)\) gauge group of the SM is left untouched.

We believe that this approach is effective in capturing the relevant phenomenological features of any model based on the same symmetry up to LHC energies. In particular this makes possible
to study in a simple and precise way the impact of these models on the EWPT as well as on the modified top-bottom couplings to the gauge bosons. On the basis of this analysis, we conclude that the minimal $SO(5)/SO(4)$ model with up to 10% finetuning may be valid up to about 2.5 TeV, but it has problems in complying with the EWPT and B-physics constraints. The minimal example that we have analyzed cannot accommodate the required positive extra contribution to $\delta \rho$ from fermion loops without introducing at the same time unobserved modifications of the SM in B-physics. It remains to be seen if this is possible at all in more extended versions of the 3rd generation quarks. In any event we do not expect in the spectrum a relatively light $SU(2)$ doublet of hypercharge 7/6.

All of these considerations do not depend on possible extensions of the gauge sector. Such extensions, however, can be and have actually been attempted. Our results apply to them as well, once the symmetry of the EWSB sector and the description of the top Yukawa coupling are made explicit. Some restrictions of the parameter space can arise. It is interesting to ask, on the other hand, which other phenomena may be expected and, especially, if extending the gauge sector allows to enlarge the domain of the “minimal” model. To this end we have considered both a “little Higgs” two-site extension and a “holographic” extension of the $SO(5)/SO(4)$ model. From our results we hardly see any improvement in the “little Higgs” case, whereas naive dimensional analysis suggests an extended range of validity for the “holographic” model. It is questionable, however, whether calculability is at the same time maintained. In our view to assess this issue would require further investigations.

In part for these reasons we have also considered and defended a purely perturbative version of the “minimal” model, without any gauge extension, up to a suitable cutoff, emphasizing the differences with respect to the strongly coupled case. We noticed that the EWPT consistency could be better in the perturbative case, if the new physics at the naturalness cutoff contributes to the $S$ parameter only at loop level. We believe that the issue of the perturbative versus “composite” nature of the Higgs boson should be left as an open (and nontrivial) question for the experiment to decide.

There are several possible directions for further work along these lines, both from the point of view of the “minimal” models and/or of their connections with “non-minimal” models. Other symmetries than $SO(5)$ can be considered, the obvious case being $SU(3)$ (or $SO(6)$). Much of the phenomenological analyses can be made more precise and explicit. The restrictions arising on the parameter space of the minimal model from interesting extensions may be useful to study.\footnote{A relevant study has been recently preformed in \cite{19} for warped holographic extensions, although the modification of the Higgs contributions to $T$ and $S$ with respect to the SM, eq. (2.13), has not been taken into account, and the constraints from $BB$ mixing and $b \to s\gamma$ have not been imposed. The issue of calculability has not been addressed.}

Last but not least, one may try to address the issue of (partially) UV completing the “minimal” model, either by giving a close look at the calculability of existing proposals or by exploring totally new directions.

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