The abundance of brown dwarfs

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ABSTRACT

The amount of mass contained in low-mass objects is investigated anew. Instead of using a mass–luminosity relation to convert a luminosity function to a mass function, I predict the mass–luminosity relation from assumed mass functions and the luminosity functions of Jahreiss & Wielen and Gould, Bahcall & Flynn. Comparison of the resulting mass–luminosity relations with data for binary stars constrains the permissible mass functions. If the mass function is assumed to be a power law, the best-fitting slope lies either side of the critical slope, \( \alpha = -2 \), below which the mass in low-mass objects is divergent, depending on the luminosity function adopted. If these power-law mass functions are truncated at \( 0.001 M_\odot \), the contribution to the local density from stars lies between 0.013 and 0.10 \( M_\odot \) \( \text{pc}^{-3} \) depending on the mass at which the mass function is normalized and the adopted value of \( \alpha \). Recent dynamical estimates of the local mass density rule out stellar mass densities above \( 0.05 M_\odot \text{pc}^{-3} \). Hence, power laws steeper than \( \alpha = -2 \) that extend down to \( 0.001 M_\odot \) are allowed only if one adopts an implausible normalization of the mass function. If the mass function is generalized from a power law to a low-order polynomial in \( \log(M) \), the mass in stars with \( M < 0.1 M_\odot \) is either negligible or strongly divergent, depending on the order of the polynomial adopted.

Key words: stars: luminosity function, mass function.

1 INTRODUCTION

The abundance in galaxies of low-mass objects is of fundamental importance because we know that near the Sun at least half of the mass density of the Galaxy is made up of stars fainter than \( M_V = 10 \), and uncertainty in the expected mass-to-light ratio of the Galactic disc is dominated by uncertainty in the number of stars with \( M_V \approx 16 \), which are extremely hard to detect despite being intrinsically numerous.

The traditional way to determine the density of the lowest-mass stars is the use of a wide-field proper-motion survey to pick up faint but nearby stars, for which photometry and perhaps parallaxes can be obtained. More recently, two alternative strategies have become available: (i) searches for gravitational microlensing events, and (ii) narrow-field surveys to identify extremely red stars.

Microlensing surveys detect stars through their gravitational fields rather than their radiation, so microlensing surveys should provide a powerful probe of the mass contained in low-mass stars. Unfortunately, there are two large problems. First, to obtain even the projected density of deflectors along lines of sight towards the survey stars, one must know how the deflectors are distributed along the line of sight. Secondly, to determine the mass function of the deflectors one requires a model of their kinematics. In consequence of these difficulties, there is no consensus regarding the nature of the deflectors that have caused the observed microlensing events towards either the Galactic Centre or the Magellanic Clouds.

The work with the Hubble Space Telescope (HST) of Gould, Bahcall & Flynn (1997, hereafter GBF) and Gould, Flynn & Bahcall (1998) pushes the strategy of counting extremely red stars to its ultimate form, in which the limiting magnitude of the survey becomes extremely faint and the field becomes very narrow. Consequently, most objects detected are of low luminosity and rather distant (~ 2 kpc) (as are objects detected by microlensing surveys), and it is a non-trivial task, that involves the adoption of a large-scale model of the Galaxy, to infer the local luminosity function from the data. Despite these difficulties, the luminosity function of the Galactic disc is now well determined at \( M_V \approx 13 \) and is usefully constrained down to \( M_V \approx 19 \) – see Fig. 1.

This paper is concerned with the problem of converting a known luminosity function into a mass function. The conventional procedure involves the adoption of some mass–luminosity relation. The mass–luminosity relation for cool stars is complex and hard to determine either theoretically or observationally, while we expect, a priori, that the mass function is simple. Therefore, following Kroupa, Tout & Gilmore (1990) I assume plausible mass functions and use measured luminosity functions to infer mass–luminosity relations. Comparison of these inferred relations with the data for binary stars clarifies the amount of mass that may
Mazzitelli (1994) write that the theoretical determined values of $MV$ these considerations imply that any empirical determination of $L$ in the range $2$. The problem with this approach is that $M_*(M)$ relation does depend critically on theoretical $T_{\text{eff}}$, and the derivative of this relation will be even more uncertain than the relation itself.

3 AN ALTERNATIVE APPROACH

The mass function, $\xi(M)$, is the outcome of a chaotic process, which involves a wide range of densities, temperatures, velocities and magnetic-field strengths. Consequently, it is much more likely to be featureless than either the luminosity–mass relation, $M_*(M)$, or the luminosity function, $\Phi(M_V)$, which is determined by $\xi(M)$ and $M_*(M)$. Hence, it makes sense to assume a simple functional form for $\xi(M)$ and then to use the observed luminosity function and equation (1) to predict $M_*(M)$ and to compare this prediction with the relevant observational data. From equation (1) we have (Kroupa et al. 1990)

$$\int_{M_1}^{M_2} dM \xi = - \int_{M_1}^{M_2} dM_V \Phi.$$  (2)

With this equation, I use a generalization of the traditional power-law mass function, namely

$$\xi = \exp \left( \sum_{n=0}^{N} \alpha_n \mu^n \right).$$  (3a)

where

$$\mu = \ln(M/M_\odot).$$  (3b)

For $N = 1$ and $\alpha_1 = -2.35$ this coincides with the Salpeter mass function. For $N > 1$ the mass function is not a simple power law and a characteristic mass is implied by each of the coefficients $\alpha_n$ for $n > 1$.

Fig. 1 shows the two luminosity functions that I have used: the full curve is a spline fit to the luminosity function of Jahreiss & Wielen (1997, hereafter JW) that employs Hipparcos parallaxes. An alternative luminosity function is shown by the dashed curve,
which deviates from the full curve to pass through the points of GBF.  

Fig. 2 shows four curves $M_*(M)$ that one obtains for the power-law case, $N = 1$, alongside empirical data points from Popper (1980) and Henry & McCarthy (1993). The two full curves are for $\alpha_1 = -2.1$; the upper curve is for the JW luminosity function, while the lower full curve is for the GBF luminosity function. The two dotted curves are for $\alpha_1 = -1.8$. The normalizations of $\Phi$ and the integration constants in equation (2) have been chosen to force all curves to coincide at $M = 1.6 M_\odot$ and $0.09 M_\odot$. The dashed curve, which is from Brewer et al. (1993), is the result of fitting stellar models to observed subdwarfs. Only one of these four curves can be said to be an inadequate fit to the data, namely the lower full curve, which is for $\alpha_1 = -2.1$ and the GBF luminosity function. In particular, for $M_V \lesssim 4$ the upper full curve ($\alpha_1 = -2.1$ and the JW luminosity function) fits the data better than the upper dotted curve, which assumes the same luminosity function and $\alpha = -1.8$.

Formally, none of the curves shown in Fig. 2 provides a satisfactory fit to the data – the lowest value of $\chi^2$ per degree of freedom is 10.2, which is attained for $\alpha_1 = -2.1$ with the JW luminosity function. However, there is probably no plausibly smooth curve that would give an acceptable fit to the data, because at a given value of $M_V$ there are observational points that lie far outside the error bars of other observational points, probably because there is, in reality, no single mass–luminosity relation: age and metallicity need to be taken into consideration.

Although there is no compelling case for going beyond the simplest case $N = 1$, it is interesting to see what can be achieved by increasing $N$ to 2 and 3. Fig. 3 shows the result of minimizing $\chi^2$ with respect to the $\alpha_n$ in equation (3a) in the cases $N = 2$ (full curve) and $N = 3$ (dotted curve). Both fits are for the JW luminosity function and give significantly smaller values of $\chi^2$ per degree of freedom than any power law: $\chi^2$ per degree of freedom is 4.1, 3.1 for $N = 2, 3$, respectively, while the best-fitting power law has $\alpha_1 = -2.06$ and $\chi^2$ per degree of freedom 9.0. (The best power law with the GBF luminosity function has $\alpha_1 = -1.76$ and $\chi^2$ per degree of freedom = 13.3.)

Fig. 4 shows the mass functions that underlie the fits of Fig. 3. These are remarkably similar for $0.1 \lesssim (M/M_\odot) \lesssim 3$, which is the entire range of masses over which the luminosity function contains information, and are tangent to the best-fitting power law at $M = 0.4 M_\odot$. The negative curvature of these curves clearly implies that the empirical mass–luminosity function is better fitted by a mass function that at small masses turns down below the best-fitting power law.

4 DISCUSSION

The luminosity function at $M_V \geq 13$ is controversial. Fig. 1 illustrates this point by showing in addition to the JW points, values derived by Reid, Hawley & Gizis (1995) from a kinematically selected sample and by GBF and Gould, Flynn & Bahcall (1998) from photometrically selected samples of stars observed with the HST. The data of Gould, Flynn & Bahcall, which are for spheroid stars, only extend to $M_V = 13.5$ and are compatible with the JW points. The points of Reid et al. and GBF go fainter and agree well down to $M_V = 14.5$. GBF remark that had they been able to detect more secondaries in binaries, their data would move up towards the Reid et al. points, which include faint companions. At $M_V = 14.5$, the GBF and Reid et al. data imply that $\Phi$ is a factor of 3 smaller than the value of JW. If the GBF luminosity function were to be preferred to that of JW, Fig. 2 would rule out a simple power law with a slope $\alpha_1 < -2$ that predicts divergent integrated mass in low-mass objects. If the JW luminosity function is correct, such values of $\alpha_1$ are not favoured over values such as $\alpha_1 = -1.8$ that place only finite mass in low-mass bodies.

The luminosity of a stellar population such as the Galactic disc is dominated by stars with zero-age masses between 0.9 $M_\odot$ and 3 $M_\odot$. Therefore, two stellar populations that have the same density of stars at $M = 2 M_\odot$ and above will have very nearly the same luminosity, regardless of their mass functions. Suppose two such populations have power-law mass functions for $M < 2 M_\odot$, one of slope $\alpha_1 = -2.1$ and the other of slope $\alpha_1 = -1.8$, and that in both cases the power law is truncated at $M = 0.001 M_\odot$.

Then the mass, and hence the mass-to-light ratio $\Upsilon$, of the population with $\alpha_1 = -2.1$ will be larger than that of the other population by a factor of 2.9. Hence the small differences between acceptable curves in Fig. 2 are associated with considerable differences in $\Upsilon$.

From table 3.13 of Binney & Merrifield (1998, hereafter BM) we take that a $2 M_\odot$ main-sequence star has $M_V = 1.9$, and that at this mass $(dM_V/d\ln M) = 3.5$. From table 3.16 of BM we take
that there are $4.8 \times 10^{-4}$ stars per pc$^3$ with $1.4 < M_V < 2.4$. Hence, with equation (1) a mass function that is normalized to produce the observed density of $2 M_\odot$ stars is

$$\xi = 8.5 \times 10^{-4} \left( \frac{M}{2 M_\odot} \right)^{-a} M_\odot^{-1} \text{pc}^{-3}. \quad (4)$$

Integrating this down to 0.001 $M_\odot$ we find that the local mass density in main-sequence stars is $0.039 M_\odot$ pc$^{-3}$ if $a = -2.1$ and $0.013 M_\odot$ pc$^{-3}$ if $a = -1.8$. As table 3.16 of BM gives the V-band luminosity density near the Sun as $0.053 L_\odot$ pc$^{-2}$, the corresponding mass-to-luminosity ratios are $Y_V = 0.72$ and $Y_V = 0.25$. These numbers are usefully compared with the mass densities determined from the mass-to-light ratios in the Hipparcos catalogue by Crézé et al. (1998) and Holmberg & Flynn (1999), namely $0.076 \pm 0.015$ and $0.098 \pm 0.011$ $M_\odot$ pc$^{-3}$, respectively. Of these values $0.04 M_\odot$ pc$^{-3}$ may be contributed by gas, and between 0.007 (Gould, Bahcall & Flynn 1996) and $0.015 M_\odot$ pc$^{-3}$ (Crézé et al. 1998) may be contained in remnants. Hence while the main-sequence density obtained with $a = -2.1$ is consistent, within the wide uncertainties, with other measures of the content of the solar neighbourhood, that obtained with $a = -1.8$ is on the low side.

The solar-neighbourhood mass function is well known to have a slope steeper than $a = -3$ for masses in excess of $M_\odot$ (Scalo 1986; Kroupa, Tout & Gilmore, 1993). Consequently, a power-law mass function with $a \sim -2$ yields a significantly larger mass density when it is normalized to give the observed density of main-sequence stars with $M = M_\odot$ than when it is normalized at $M = 2 M_\odot$. Specifically, if we proceed as above to normalize at $M_V = 4$, we find the mass function to be

$$\xi = 7.2 \times 10^{-3} \left( \frac{M}{1.16 M_\odot} \right)^{-a} M_\odot^{-1} \text{pc}^{-3}, \quad (5)$$

and the density in stars with masses between 2 and 0.001 $M_\odot$ becomes $0.10 M_\odot$ pc$^{-3}$ for $a = -2.1$ and $0.042 M_\odot$ pc$^{-3}$ for $a = -1.8$. Hence, with this normalization of the mass function, the predicted density in stars is clearly excessive for $a = -2.1$ and just about acceptable for $a = -1.8$. Because the errors on the value of the luminosity function are smallest near $M_\odot$, normalization near this mass is preferable to normalization at $2 M_\odot$ so long as one is not interested in the luminosity density of the disc. Hence, recent determinations of the local mass density do suggest that it is unlikely that a slope steeper than $a = -2$ continues down to 0.001 $M_\odot$.

5 CONCLUSIONS

A priori we expect the mass function to be a smooth function, while we have every reason to believe that the mass–luminosity relation is far from smooth. Indeed, the whole concept of a mass–luminosity relation strictly speaking fails for an inhomogeneous population such as that of the solar neighbourhood because luminosity depends on age and metallicity in addition to mass. In these circumstances the optimum procedure is to assume a simple functional form for the mass function and to determine the parameters in the fitting function by calculating the mass–luminosity relation that is obtained by combining it with the observed luminosity function.

This has been done using binary data from Popper (1980) and Henry & McCarthy (1993) and two recent luminosity functions: that obtained by JW from Hipparcos parallaxes, and that obtained by GBF from HST observations. For both luminosity functions, acceptable fits to the data can be obtained with a mass function that is a pure power law with exponent near the critical value, $-2$, at which the mass implied at small masses diverges. The JW luminosity function favours an exponent $a_1 = -2.06$ while the GBF function favours $a_1 = -1.76$. If one continues a power-law mass function that is normalized to yield the observed density of $2 M_\odot$ stars down to $M = 0.001 M_\odot$, one obtains a local stellar mass density $\rho_0 = 0.039 M_\odot$ pc$^{-3}$ if $a_1 = -2.1$, and $\rho_0 = 0.013 M_\odot$ pc$^{-3}$ if $a_1 = -1.8$. Local stellar densities that are larger by a factor of $\sim 3$ are obtained when the mass functions are normalized to yield the observed abundance of solar-mass stars. The values inferred from the dynamics of Hipparcos stars by Crézé et al. (1998) and Holmberg & Flynn (1999) lie in the range $0.05$ to $0.02 M_\odot$ pc$^{-3}$ depending on the rather uncertain values of the local densities of the interstellar medium (ISM) and remnants. In so far as normalization of the mass function at $\sim M_\odot$ is safer than normalization at a higher mass, slopes steeper than $a = -2$ and extending down to 0.001 $M_\odot$ are ruled out by recent dynamical determinations of the local mass density.

Significantly better fits to the observed mass–luminosity relation can be obtained by using a two- or three-parameter mass function rather than a power law. Such functions provide a better fit by virtue of their ability to have a slope that decreases between $M_\odot$ and $0.1 M_\odot$. If a two-parameter function is employed, negligible mass is contained in stars below the hydrogen-burning limit. By contrast, a three-parameter function predicts a huge amount of mass in low-mass stars. Consequently, nothing can be securely inferred about the amount of mass in such stars from currently available luminosity functions. The only real constraint is imposed by the dynamics of nearby stars.

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