Critical Temperature in Nanoscopic Superconductors

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Abstract. Recently, we have studied the properties of nano-scale superconductors different from that of bulk superconductors. One of the properties is the critical temperature of normal-superconducting transition. We evaluate the critical temperature with two method by solving Bogoliubov-de Gennes equation. The former method is the general approximation calculation, which is good approximation in bulk superconductors. The latter is the direct calculation of the $T$-dependence of the order parameter $\Delta$ by solving the Bogoliubov-de Gennes equation self-consistently. These methods lead to the much different critical temperature and the true critical temperature is higher than we expect in the bulk superconductors.

1. Introduction

It has been known that the properties of nano-scaled superconductors differ from bulk superconductors. Especially, vortex states have been studied widely by using Ginzburg-Landau equation [1,2] and Bogoliubov-de Gennes equation [3,4]. The multi-vortex state is a special vortex configuration of several vortices. In the giant vortex state, there is a vortex which has multiple flux quanta $n\Phi_0$ ($n = 2, 3, \cdots$), where $\Phi_0 = hc/2e$ is a flux quanta. The symmetry of shape of the superconductor is important for these vortex structures.

We have researched the superconducting state of the giant vortex state by using microscopic Bogoliubov-de Gennes (BdG) equation [4]. In this calculation, we must have evaluated the superconducting-normal transition temperature $T_c$ because the giant vortex appears near critical temperature. We have calculated the critical temperature $T_c^a$ in the same way as we evaluate critical temperature of bulk superconductors [5]. In Ref. [6,7] this critical temperature is consistent with the temperature at the order parameter $\Delta = 0$. The giant vortex have, however, appeared higher temperature than $T_c^a$. Moreover, the order parameter $\Delta$ at $T_c^a$ has been compared to $\Delta|T=0$.

The decrease of system size leading to the increase of $T_c$ have been shown in [7], though this effect looks very small. We must find the origin leading to the difference between $T_c^a$ and the true $T_c$. 
2. Two microscopic evaluation of critical temperature

Microscopic superconducting states are described by the BdG equation [8]. We can evaluate the critical temperature of nano-sized superconductors by using the two method of solving the BdG equation. (The general BdG equation with external field is refereed to Ref.[3].)

We consider the case at the almost critical temperature of no external field. Therefore, we can ignore the vector potential. In this case, the BdG equation becomes

\[
\begin{align*}
\left(-\frac{\hbar^2}{2m_e} \nabla^2 - \mu\right) u_n (r) + \Delta (r) v_n (r) &= E_n u_n (r), \\
\left(-\frac{\hbar^2}{2m_e} \nabla^2 - \mu\right) v_n (r) + \Delta^* (r) u_n (r) &= E_n v_n (r),
\end{align*}
\]

where \( u_n, v_n \) and \( E_n \) are the electron-like wavefunction, the hole-like wavefunction and the energy eigenvalue of \( n \)-th quasi-particle state, respectively. \( \Delta \) is the order parameter, and \( \mu \) is the chemical potential which is determined by the electron number conservation. The order parameter is obtained by

\[
\Delta (r) = g \int \left[ \sum_n \left( 1 - 2f (E_n) \right) u_n (r) v_n^* (r) \right] dr,
\]

where \( g \) is the interaction constant and \( f (E_n) \) is Fermi distribution function of the argument of \( E_n \). We set the boundary conditions as follows:

\[
u = v = 0.
\]

2.1. General approximation method

We consider the superconducting state of the critical temperature as \( \Delta (r) \to 0 \). This method is the general method leading to the famous result \( \Delta |_{T=0}/T_c = 1.764 \) [5], we can set the order parameter a minimal constant \( \overline{\Delta} \). Moreover, we set \( v_n (r) = O(\Delta^*) \simeq \overline{\Delta} v_n (r) \) for \( E_n > 0 \). Therefore, eq. (1) becomes

\[
\begin{align*}
\left(-\frac{\hbar^2}{2m_e} \nabla^2 - \mu\right) u_n (r) &= E_n u_n (r), \\
\left(-\frac{\hbar^2}{2m_e} \nabla^2 - \mu + E_n\right) v_n (r) &= -u_n (r).
\end{align*}
\]

We can solve eq. (4a) as the eigenvalue equation and subsequently eq. (4b) as the simultaneous equation. In the same way, we set \( u_n (r) = O(\Delta) \simeq \overline{\Delta} u_n (r) \) for \( E_n \leq 0 \), and we get the BdG equation as follow:

\[
\begin{align*}
\left(-\frac{\hbar^2}{2m} \nabla^2 - \mu - E_n\right) u'_n (r) &= -v_n (r), \\
\left(-\frac{\hbar^2}{2m} \nabla^2 - \mu\right) v_n (r) &= E_n v_n (r).
\end{align*}
\]

We solve these equations (4) (5) so that we get the eigen states of the limitation \( \Delta \to 0 \). The limitation form of eq. (2) is given by

\[
1 = g \int \left[ \sum_{n \geq 0} \left( 1 - 2f (E_n) \right) u_n (r) v_n^* (r) + \sum_{n \leq 0} \left( 1 - 2f (E_n) \right) u_n (r) v_n^* (r) \right] dr.
\]

We solve the nonlinear equation for temperature included in \( f (E_n) \) and the temperature is defined as \( T_c^a \).
2.2. Self-consistent method

The other method is a very simple method. The way is to calculate the $T$-dependence of $\Delta$ by solving (1) and (2) self-consistently, though we need a huge number of calculation. This method can include the spatial distribution of the order parameter.

3. Results and discussion

We consider a square superconducting plate, of which size is $L \times L$ and the coherence length $\xi_0/L = 0.20, 0.25$. We set the order parameter $\Delta_0/E_c = 0.2$ at $T = 0$, the Fermi wave number $k_F\xi_0 = 3.0$.

The result of the approximation method is $T_c^a = 0.114$ for $\xi_0/L = 0.20$ and $0.111$ for $\xi_0/L = 0.25$. These show the $\Delta_0/T_c^a \sim 1.75$. Therefore, this is consistent with the general definition of the critical temperature.

Figure 1 shows the $T$-dependence of $\Delta$ in the self-consistent method. The temperature is normalized by $T_c^a$ of each $\xi_0/L$. The order parameters at $T = 0$ is approximately 0.28 which is as high as $\Delta_0/E_c$. $\Delta_0/E_c$ denote the constant or the average of the order parameter. The approximation method is the former and the self-consistent method has the spatial variation satisfactory for $\Delta = 0$ at the boundary. Therefore, the maximum of the order parameter is increased. The true critical temperature is approximately $1.49T_c^a$ in $\xi_0/L = 0.20$ and $1.60T_c^a$ in $\xi_0/L = 0.25$. The major difference of the two method is the spatial distribution of the order parameter. The characteristic length of the order parameter is $1/k_F\xi_0$. This leads to the increase of the critical temperature in the nano-scale superconductor. The two $\xi_0/L$ which have the same $1/k_F\xi_0$ shows that a smaller superconducting plate tends to have higher critical temperature.

Figure 1. $T$-dependence of $\Delta$ at the center of the superconducting plate with the coherence length $\xi_0/L = 0.20$ and $\xi_0/L = 0.25$. 
4. Conclusion
We have solved the Bogoliubov-de Gennes equation and evaluated the two type of the critical
temperature. The critical temperature $T_{c}$ of the approximation method is inconsistent with the
true critical temperature. As the plate size became smaller, the critical temperature became
higher. Though the effect is utilized in applications, we need to solve the Bogoliubov-de Gennes
equation and to find the critical temperature. Avoiding the difficulty, we must make good
approximation with the critical temperature in the nano-scale superconductor.

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