DARK MATTER PREDICTIONS WITH NON-UNIVERSAL
SOFT BREAKING MASSES

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The effect of non-universal SUSY soft breaking on predictions of dark matter detector event rates are surveyed for supergravity models with gravity mediated soft breaking. For universal soft breaking in the first two generations and \( \tan \beta \lesssim 20 \), non-universal effects can be characterized by four parameters (two for Higgs and two for third generation squarks) in addition to those of the minimal model (MSGM). These can increase or decrease event rates by a factor of 10-100 in the domain \( m_{\chi_1^0} \lesssim 60 \) GeV \( (\chi_1^0 = \text{lightest neutralino}) \) but produce generally small effects at higher masses. The value of the top mass and \( b \to s + \gamma \) branching ratio eliminates most of the parameter space for \( \mu < 0 \), causing event rates for \( \mu < 0 \) to be a factor \( \approx 100 \) smaller than for \( \mu > 0 \). A correlation between large (small) event rates and small (large) \( b \to s + \gamma \) branching ratio is observed. The effect of future satellite (MAP and PLANCK) precision determinations of cosmological parameters on predicted event rates is examined for examples of the \( \Lambda \)CDM and \( \nu \)CDM models. It is seen that these could sharpen the event rate predictions and also restrict the allowed gaugino mass ranges, thus influencing predictions for accelerator SUSY searches.

1 Introduction

The composition of the dark matter (DM) in the universe, which makes up 90% or more of the universe’s total matter, is one of the most important unresolved problems in astronomy. At present, dark matter has been observed only from its gravitational interactions and may consist of a number of different components. There may be baryonic (B) dark matter (e.g. machos), hot dark matter (HDM) which was relativistic at the time galaxy formation (possibly massive neutrinos) and cold dark matter (CDM) which was non-relativistic during galaxy formation. In addition a cosmological constant (\( \Lambda \)) may be present.

The amount of each type of DM can be specified by the parameter \( \Omega_i = \rho_i/\rho_c \) where \( \rho_i \) is the matter density of type “i” , \( \rho_c = 3H^2/8\pi G_N \) is the critical matter density to close the universe, \( H \) is the Hubble constant, \( H = h100 \) km/s/Mpc and \( G_N \) the Newtonian constant. Currently, measurements of \( h \) fall in the range

\[
0.5 \lesssim h \lesssim 0.75
\]
and different models give estimates for the cosmological CDM of

\[ 0.1 \leq \Omega_{CDM} h^2 \leq 0.4 \]  

The CDM that can be directly observed is that which exists locally in the Milky Way. This has been estimated to have a density of \( \rho_{DM}^{MW} \approx 0.3 \text{ GeV/cm}^3 \), impinging on the Solar System with velocity \( v_{DM} \approx 300 \text{ km/s} \).

The cold dark matter is of particular interest in that if it is of particle nature it is most likely an “exotic” particle, i.e. one not found in the Standard Model (SM). We consider here models of physics beyond the Standard Model based on supergravity grand unification with R-parity invariance, where supersymmetry (SUSY) breaking takes place at a scale near or above the GUT scale \( M_G \approx 1.5 \times 10^{16} \text{ GeV} \). This breaking occurs in a “hidden” sector and is transmitted (super)gravitationally to the physical sector. Such models automatically predict the existence of CDM in that the lightest supersymmetric particle (LSP) is absolutely stable and over most of the parameter space is the lightest neutralino, \( \chi_1^0 \). Thus the relic \( \chi_1^0 \) left over from the Big Bang would be the CDM seen today. Further, over a significant part of the SUSY parameter space, the amount of CDM predicted is in accord with what is observed astronomically, i.e. Eq.(2).

2 DM Detector Event Rates

Terrestrial experiments detect incident local (Milky Way) DM particles by their scattering by quarks in nuclear targets. We briefly review in this section the relevant formulae for prediction of detector event rates for SUSY models.

The analysis proceeds as follows. One first calculates the relic density of neutralino CDM which remain after neutralino annihilation in the early universe. One finds

\[ \Omega_{\chi_1^0} h^2 \approx 2.48 \times 10^{-11} \left( \frac{T_{\chi_1^0}}{T_\gamma} \right)^3 \left( \frac{T_\gamma}{2.73} \right)^3 \frac{N_f^{1/2}}{J(x_f)} \]  

where \( x_f = kT_f/m_{\chi_1^0} \) (\( T_f = \) neutralino freezeout temperature, \( N_f = \) number of degrees of freedom at freezeout), \( T_\gamma \) is the cosmic microwave background (CMB) temperature, \( (T_{\chi_1^0}/T_\gamma)^3 \) is the reheating factor and

\[ J(x_f) = \int_0^{x_f} dx \langle \sigma v \rangle(x) \text{ GeV}^{-2} \]

In Eq.(4), \( \sigma \) is the neutralino annihilation cross section (calculated from the SUSY model), \( v \) the relative velocity and \( \langle \cdots \rangle \) means thermal average. One
sees that $\Omega_{\chi_1^0} h^2$ scales inversely with the annihilation cross section, i.e. the more annihilation there is, the less relic $\chi_1^0$ remain. One restricts the SUSY parameter space so that Eq.(2) is obeyed, and also that the current SUSY parameter space bounds from LEP, the Tevatron and CLEO ($b\to s+\gamma$ decay) are satisfied.

One then calculates, in the restricted SUSY parameter space, the quantity $R$, the expected event rate of detector scattering events (per kilogram of detector per day) for the incident flux of Milky Way neutralinos $\chi_1^0$:

$$R = (R_{SI} + R_{SD}) \left[ \frac{\rho_{\chi_1^0}}{0.3 \text{ GeV/cm}^3} \right] \left[ \frac{\bar{v}_{\chi_1^0}}{320 \text{ km/s}} \right] \text{ events/kg d} \quad (5)$$

where

$$R_{SI} = \frac{16 m_{\chi_1^0} M_N^3 M_Z^4}{|M_N + m_{\chi_1^0}|^2} |A_{SI}|^2 \quad (6)$$

$$R_{SD} = \frac{16 m_{\chi_1^0} M_N}{|M_N + m_{\chi_1^0}|^2} \lambda^2 J(J + 1) |A_{SD}|^2 \quad (7)$$

where $M_N$ is the nuclear target mass and $J$ is its spin, $M_Z$ is the $Z$ boson mass and $\langle N| \sum \hat{S}_i |N \rangle = \lambda \langle N| \hat{J} |N \rangle$ with $\hat{S}_i$ the $i$th nucleon’s spin. In Eqs.(6) and (7), $A_{SI}(A_{SD})$ are the spin independent (spin dependent) scattering amplitudes. Note that for heavy nuclear targets, $R_{SI} \sim M_N$ while $R_{SD} \sim 1/M_N$ making the heavier targets generally more sensitive than lighter ones and $R_{SI}$ generally the dominant contribution for heavy targets. There are a number of uncertainties in the above analysis involving the strange quark contribution to the nucleon, the nature of nuclear form factors etc., making the theoretical predictions of $R$ uncertain to perhaps a factor $\approx 2$.

3 Soft Breaking: Universal Case

We consider here models where the GUT group $G$ breaks to the Standard Model group at $M_G : G \to SU(3) \times SU(2) \times U(1)$. The simplest supergravity model of this type, the minimal SUGRA model (MSGM) depends at $M_G$ on four extra soft breaking parameters and one sign to determine all the masses and interactions of the 32 SUSY particles. These new parameters are $m_0$ (the universal scalar soft breaking mass), $m_{1/2}$ (the universal gaugino mass), $A_0$ (the universal cubic soft breaking parameter), $B_0$ (the quadratic soft breaking parameter) and the sign of $\mu_0$, the Higgs mixing parameter in the effective potential term $\mu_0 H_i H_2$ (where $H_i, i = 1, 2$ are the two Higgs doublets). The renormalization group equations (RGE) then allow one to proceed downward
to lower energy scales where the soft breaking parameters trigger the breaking of $SU(2) \times U(1)$. In fact one may show that a necessary condition that electroweak breaking occur at a lower scale is that at least one soft breaking parameter and $\mu_0$ be non-zero at $M_G$, and this breaking will occur at the electroweak scale ($M_Z$) provided $m_t$ obeys $90 \text{ GeV} \lesssim m_t \lesssim 200 \text{ GeV}$. Thus the model automatically requires a heavy top quark and it is SUSY soft breaking at $M_G$ that gives rise to electroweak breaking at $M_Z$.

The conditions for electroweak breaking are

$$\mu^2 = \mu_1^2 - \mu_2^2 \tan^2 \beta \frac{1}{\tan \beta^2 - 1} - \frac{1}{2} M_Z^2; \quad \sin^2 \beta = -\frac{2B \mu}{\mu_1^2 + \mu_2^2 + 2\mu^2}$$

where $\mu^2, \mu_1^2, B$ are running parameters at $Q = M_Z$, $\mu_2^2 = m_{H^1}^2 + \sum_i$, $\tan \beta = \langle H_2 \rangle / \langle H_1 \rangle$ and $\sum_i$ are loop corrections. Thus radiative breaking allows one to determine $\mu^2$ and eliminate $B$ in terms of $\tan \beta$. The determination of $|\mu|$ greatly enhances the predictive power of the model. In the following we will restrict the parameter space examined to be in the domain

$$m_0, m_{\tilde{g}} \leq 1 \text{ TeV}, \quad 2 \leq \tan \beta \leq 25, \quad -7 \leq A_t / m_0 \leq 7$$

(9)

where $m_{\tilde{g}}$ is the gluino mass and $A_t$ is the $t$-quark $A$-parameter at the electroweak scale. (Eq.(9) satisfies usual “naturalness” conditions.) For most of the parameter space, $\mu^2 / M_Z^2 \gg 1$ which leads to “scaling” relations for the charginos ($\chi^\pm_i, i = 1, 2$) and neutralinos ($\chi^0_i, i = 1, ..., 4$):

$$2m_{\chi^0_i} \simeq m_{\chi^\pm_1} \simeq m_{\chi^\pm_2} \simeq \frac{1}{3} - \frac{1}{4} m_{\tilde{g}}; \quad m_{\tilde{g}} \simeq \frac{\alpha_3(M_Z)}{\alpha_G} m_{1/2}$$

(10)

$$m_{\chi^0_3} \simeq m_{\chi^0_4} \gg m_{\chi^0_1}$$

(11)

In addition, the four Higgs bosons, $h, H^0, A^0$ and $H^\pm$ obey in this domain the relations

$$m_{H^0} \simeq m_A \simeq m_{H^\pm} \gg m_h$$

(12)

and $m_h \lesssim 120 \text{ GeV}$.

Most of the SUGRA dark matter analysis has been done within the above framework of universal soft breaking. We examine next what modifications arise when non-universal soft breaking occurs.

4 Soft Breaking: Non-Universal Case

The MSGM model with universal soft breaking parameters, is of course the simplest SUGRA model, which in part is why it has been examined so extensively. However, there are a number of reasons why one might expect non-universal SUSY soft breaking to occur at $M_G$. Thus in general, non-universal
soft breaking will arise if in the Kahler potential, the interactions between the hidden sector fields (whose VEVs give rise to SUSY breaking) and the physical sector fields are not universal. Further, even if universality where to hold at a more fundamental level, e.g. at the Planck or string scales, running the RGE down to the GUT scale can produce significant non-universal soft breaking at $M_G$. Finally, we note that in the breaking of a higher rank GUT group down to the SM group at $M_G$, the $D$ terms can generate non-universalities.

Flavor changing neutral currents [FCNC] will be suppressed if the soft breaking masses of the first two generations are universal at $M_G$, and in the following we will assume that this is the case and let $m_0$ be their common mass. We then parametrize the Higgs and third generation squark and slepton masses at $M_G$ as follows:

$$m_{H_i}^2 = m_0^2 (1 + \delta_i); \quad m_{H_2}^2 = m_0^2 (1 + \delta_2)$$ (13)

$$m_{q_L}^2 = m_0^2 (1 + \delta_3); \quad m_{u_R}^2 = m_0^2 (1 + \delta_4); \quad m_{e_R}^2 = m_0^2 (1 + \delta_5)$$ (14)

$$m_{d_R}^2 = m_0^2 (1 + \delta_6); \quad m_{l_L}^2 = m_0^2 (1 + \delta_7)$$ (15)

where $q_L = (\tilde{u}_L, \tilde{d}_L)$ is the left squark doublet, $l_L = (\tilde{\nu}_L, \tilde{e}_L)$ the left slepton doublet etc. The $\delta_i$ represent the deviations from universality, and we will restrict these to obey $-1 \leq \delta_i \leq +1$. In addition, we denote the $A$ parameters at $M_G$ by $A_{t, b}^0$ and $A_{0, \tau}$ which also need not be universal.

For GUT groups containing an $SU(5)$ subgroup with matter embedded in the $10 + \bar{5}$ of $SU(5)$ (e.g. $SU(N), N \geq 5; \ SO(N), N \geq 10; \ E_6$) one has

$$\delta_3 = \delta_4 = \delta_5; \quad \delta_6 = \delta_7; \quad A_{0, b} = A_{0, \tau}$$ (16)

We note that for $\tan \beta \lesssim 20$, $\delta_3, \delta_6, \delta_7$ and $A_{0, b}, A_{0, \tau}$ do not enter significantly in the calculations, and we will set them to zero in the following.

To examine the significance of the non-universal soft breaking, we exhibit the 1-loop RGE expressions for $\mu^2$ of Eq.(8) for $Q = M_Z$.
where $t \equiv \tan \beta$, $D_0 \equiv 1 - (m_t/200 \sin \beta)^2$, $A_R \equiv A_t - 0.61 m_\beta$, $\alpha_G \equiv 1/24$, $S_0 = Tr Y m^2$ ($Y$ is hypercharge and $m$ are the soft breaking masses at $M_G$) and $C_\mu = \frac{1}{2} D_0 (1 - D_0) (H_3/F)^2 + e - g/t^2$ (with the RGE form factors $H$, $F$, $e$, $g$ given in Ibañez et al.\cite{Ibañez}). In Eq.(17), the $m_0^2$ term has been divided into a universal part and a non-universal part (dependent on the $\delta_i$). $D_0$ vanishes at the $t$-quark Landau pole and is generally small ($0 < D_0 \lesssim 0.25$). $A_R$ is the residue at the Landau pole (i.e. $A_{\mu t} = A_R/D_0 - (H_3/F) m_{1/2}$). Since $m_{1/2} \approx (\alpha_G/\alpha_3) m_\beta$, where $\alpha_G$ is the GUT coupling constant, the $C_\mu m_{1/2}^2$ term scales with $m_\beta^2$ or alternately by Eq.(10) with $(m_{\chi^0_1})^2$. $S_0$ vanishes for universal soft breaking masses (by hypercharge anomaly cancellation) but is non-zero for non-universal masses. (The $S_0$ term is generally small, usually only a few percent correction.)

5 Effects Of Non-Universal Soft Breaking On DM Event Rates

While many parameters enter into the DM event rate formula Eq.(5), $\mu^2$ plays a central role in determining $R$. This is because $\mu^2$ governs the interference between the Higgsino and gaugino parts of $\chi^0_1$ in the $\chi^0_1$ - quark scattering amplitude and it is this interference which gives rise to $R_{SI}$ (which we have seen for most detectors is the dominant part of $R$). One finds in general that increasing (decreasing) $\mu^2$ decreases (increases) the amount of interference and hence the size of $R$.

Since $D_0$ is small, one sees from Eq.(17) that $\delta_3$ and $\delta_4$ contribute oppositely to $\delta_2$ in $\mu^2$. Thus in evaluating the contribution of non-universal masses, it is necessary to consider both Higgs and squark masses since they produce effects of the same size. There are certain situations where the non-universal terms become relatively large compared to the universal contributions, greatly enhancing the non-universal effects. Thus the universal contribution of the Landau pole term (which is generally quite large) vanishes if the residue $A_R$ is zero, i.e. when $A_t \approx 0.61 m_\beta$. Also the $C_\mu m_{1/2}$ term is small for light $\chi^0_1$ (or light $\tilde{g}$) according to the scaling relations Eq.(10). In addition, for $t^2 \gg 1$ (i.e. $t \gtrsim 3$), the universal contribution to the $m_0^2$ term becomes small when $D_0 \approx 1/3$.

The above ideas allow one to understand qualitatively the detailed computer calculations of event rates\cite{Carena}. Fig.1 shows maximum and minimum event rates for the case $\delta_3 = 0 = \delta_4$. The solid curves are for universal soft breaking. From Eq.(17), the case $\delta_2 = -1 = -\delta_1$ (dotted curves) corresponds to increasing $\mu^2$, which reduces the event rates. This reduction can be as much as a factor of $O(10 - 100)$ for $m_{\chi^0_1} \lesssim 60 GeV$. The effect is largest for the minimum event rates, since these occur at small $\tan \beta$ (which by Eq.(17) would
Figure 1: Maximum and minimum event rates for a Xe detector as a function of \( m_{\chi_1^0} \) for \( \mu > 0 \), \( 0.1 \leq \Omega_{\chi_1^0} h^2 \leq 0.4 \) with \( \delta_3 = 0 = \delta_4 \) and \( \delta_2 = 0 = \delta_1 \) (solid), \( \delta_2 = 1 = -\delta_1 \) (dotted), \( \delta_2 = 1 = -\delta_1 \) (dashed).

magnify the effect). The reverse situation, \( \delta_2 = 1 = -\delta_1 \) (dashed curve) causes a decrease of \( \mu^2 \) and hence increase of event rates. The effects of non-universal soft breaking masses becomes small for heavy neutralinos, \( m_{\chi_1^0} \gtrsim 60 \) GeV, since then the \( C_\mu m_{1/2} \) becomes larger masking the non-universal effects.

Fig.2 shows a similar analysis with \( \delta_1, \delta_2 \) and \( \delta_3 \) set to zero. We see the curves with \( \delta_4 = 1 \) (dotted) resembles the \( \delta_2 = 1 = -\delta_1 \) curves of Fig.1 while the \( \delta_4 = -1 \) (dashed) resembles the \( \delta_2 = 1 = -\delta_1 \) curves of Fig.1. Fig.3 shows curves similar to Fig.2 with \( \delta_4 \) non-zero. These results follow from Eq.(17) where the effects of \( \delta_3 \) and \( \delta_4 \) were seen to be opposite to those of \( \delta_2 \), e.g. a negative \( \delta_4 \) simulating a positive \( \delta_2 \) etc.

As seen in Eq.(16), \( \delta_3 = \delta_4 \) for most GUT groups. Fig.4 examines this case with the choice \( \delta_3 = \delta_4 = -1 \). Here the squark non-universal masses add coherently in Eq.(17) with the Higgs non-universal masses for the case \( \delta_2 = 1 = -\delta_1 \) to reduce \( \mu^2 \). The maximum event rate for this case (dashed curve) which occurs for large \( \tan \beta \), is then greatly enhanced over a wide range of \( m_{\chi_1^0} \) rising to \( R \sim (1-10) \) events/kg d. This value is the current sensitivity of the NaI detectors, and so this case is at the edge of experimental observation. For the reverse situation, \( \delta_2 = -1 = -\delta_1 \) (dotted curve), the squark and Higgs non-universal effects mostly cancel in Eq.(17) and one indeed finds that these curves lie close to the universal one for most of the neutralino mass range, even though a large amount of non-universal soft breaking has occurred.
Figure 2: Same as Fig.1 with $\delta_1 = \delta_2 = \delta_3 = 0$ and $\delta_4 = 0$ (solid), $\delta_1 = +1$ (dotted), $\delta = -1$ (dashed).

Figure 3: Same as Fig.1 with $\delta_1 = \delta_2 = \delta_3 = 0$ and $\delta_4 = 0$ (solid), $\delta_3 = +1$ (dotted), $\delta_3 = -1$ (dashed).
The above discussion shows how squark and Higgs non-universal soft breaking masses can interact with each other, sometimes enhancing and sometimes canceling their effects depending on the sign of the non-universal terms.

6 The $t$–Quark Mass, $b \to s + \gamma$ Decay And The Sign Of $\mu$

While the value of $\mu^2$ is a crucial parameter in determining DM event rates, non-universal soft breaking masses also enter in other parameters which control the event rates in a more indirect way. Thus the stop mass matrix is given by

$$M_L^2 = \begin{pmatrix} m_{t_L}^2 & -m_t(A_t + \mu \cot \beta) \\ -m_t(A_t + \mu \cot \beta) & m_{t_R}^2 \end{pmatrix},$$

where

$$m_{t_L}^2 = m_{Q_L}^2 + m_t^2 + \left( \frac{1}{2} - \frac{2}{3} \sin^2 \Theta_W \right) M_Z^2 \cos 2\beta$$

$$m_{t_R}^2 = m_{U_R}^2 + m_t^2 + \frac{2}{3} \sin^2 \Theta_W M_Z^2 \cos 2\beta$$

Here:

$$m_{Q_L}^2 = \left[ \left\{ \frac{1 + D_0}{2} \right\} + \left\{ \frac{5 + D_0}{6} \delta_3 - \frac{1 - D_0}{6} (\delta_2 + \delta_4) \right\} \right] m_0^2$$

$$- \frac{1}{6} \left( 1 - D_0 \right) \frac{A_t^2}{D_0} + C_Q m_1^2 / 2 - \frac{1}{66} \left( 1 - \frac{\alpha_1(M_Z)}{\alpha_G} \right) S_0$$

Figure 4: Same as Fig.1 with $\delta_3 = -1 = \delta_4$ and $\delta_2 = 0 = \delta_1$ (solid), $\delta_2 = -1 = -\delta_1$ (dotted), $\delta_2 = 1 = -\delta_1$ (dashed).
\[
\begin{align*}
m_t^2 &= \left\{ D_0 \right\} + \left\{ \frac{2 + D_0 \delta_4 - \frac{1}{3} D_0 (\delta_2 + \delta_3)}{3} \right\} m_0^2 \\
&- \frac{1}{3} (1 - D_0) A_R^2 D_0 + C_U m_{1/2}^2 + \frac{2}{33} (1 - \frac{\alpha_3(M_Z)}{\alpha_G}) S_0
\end{align*}
\]

(22)

\(\text{and } C_Q, C_U \) are given in Ref.\[13\]. Note here \(\delta_3\) and \(\delta_4\) act oppositely to each other, though if the GUT condition \(\delta_3 = \delta_4\) is imposed, they still act oppositely to \(\delta_2\).

Since the top quark is heavy, i.e. \(D_0\) is small, the Landau pole term in Eq.(22) can drive \(\tilde{t}_1\), the light stop, tachyonic if \(A_R\) is large enough, eliminating such parameter points\[15\]. Since \(A_R \sim (A_t - 0.61m_0)\), regions where \(A_t\) is negative get eliminated, and one finds the allowed region of parameter space is restricted by \[13,16\]

\[
A_t/m_0 > -0.5
\]

(23)

The \(b \to s + \gamma\) decay also strongly restricts the SUSY parameter space\[13\]. The measured branching ratio for this process by CLEO is \[B(B \to X_s \gamma) = (2.32 \pm 0.67) \times 10^{-4}\]. The SM prediction, with NLO effects included is \(B(B \to X_s \gamma) = (3.48 \pm 0.31) \times 10^{-4}\) where the error in the theoretical calculations is now dominated by uncertainties in the input parameters, e.g. \(\alpha_3(M_Z), m_t, m_c/m_b, \) etc.\[3\] While these two results are statistically consistent, it is clear that there is a certain amount of tension between the data and the SM. The SUSY corrections to the SM reduce the SM branching ratio for \(A_t/\mu > 0\) and increase it or \(A_t/\mu < 0\). Since by Eq.(23) \(A_t\) is mostly positive, and the experimental branching ratio lies below the SM value, the current CLEO data already eliminates most of the \(\mu < 0\) part of the parameter space at the 95\% C.L. level. The part of the \(\mu < 0\) parameter space that survives is for small \(\tan \beta\), and so one finds in most of the remaining parameter space, the event rates for \(\mu < 0\) are a factor of 100 (or more) smaller than those of \(\mu > 0\) and this result is true with or without non-universal soft breaking masses\[3\].

A second feature involving the \(b \to s + \gamma\) decay is the existence of a correlation between the DM event rate \(R\) and the \(b \to s + \gamma\) branching ratio \(B\): large (small) event rates tend to correlate with small (large) branching ratio. This shows up most strongly if one considers the maximim \((R_{\text{max}})\) or minimum \((R_{\text{min}})\) event rate at fixed values of the neutralino mass. One finds (for universal soft breaking) that the part of the parameter space with \(B \leq 2 \times 10^{-4}\) corresponds to \(R_{\text{max}} \geq 0.1 \text{ events/kg d}\), and \(B \geq 3 \times 10^{-4}\) corresponds to \(R_{\text{max}} \leq 0.01 \text{ events/kg d}\). Similarly for \(R_{\text{min}} \leq 0.003 \text{ events/kg d}\), one finds \(B > 3 \times 10^{-4}\). The correlation is less strong for values of \(R\) in between \(R_{\text{max}}\) and \(R_{\text{min}}\), but still significant\[3\].
It is clear from the above that as the $b \rightarrow s + \gamma$ data becomes more precise, it will have a strong impact on DM predictions, and on restricting the supersymmetry parameter space in general.

7 Determination Of Cosmological Parameters

One of the major sources of uncertainty in dark matter event rate predictions is the lack of accurate knowledge of the basic cosmological parameters. Future satellite experiments MAP (scheduled for the year 2000) and PLANCK (planned for 2005) will be able to measure the angular power spectrum very accurately. Thus one defines the correlation function
\[
c(\theta) = \langle \frac{\Delta T(\hat{q}_1)}{T_0} \frac{\Delta T(\hat{q}_2)}{T_0} \rangle
\]
where $T_0 = 2.728 \pm 0.002^\circ K$ is the cosmic microwave background (CMB) temperature, $\Delta T(\hat{q}_i)$, $i = 1, 2$ are the deviations from $T_0$ in directions $\hat{q}_i$ and $\cos \theta = \hat{q}_1 \cdot \hat{q}_2$. Expanding in Legendre polynomials
\[
c(\theta) = \sum \frac{2l+1}{4\pi} c_l P_l(\cos \theta)
\]
these experiments will be able to determine the $c_l$ out to $l \approx 1000$. Since the $c_l$ are sensitive to the different cosmological parameters, it will be possible to determine the Hubble constant, the amount of dark matter, the cosmological constant etc. to an accuracy of a few percent. We consider here what such determinations might mean for DM predictions for two cosmological models.

7.1 $\Lambda$CDM Model

Current astronomical measurements suggest that while there is a large amount of CDM, its mean density is considerably less than the critical density $\rho_c$ and that models with a cosmological constant $\Lambda$ and $\Omega_{CDM} \approx 0.4$ are good, if not better fits to the current astronomical data. We consider here the $\Lambda$CDM model with cold and baryonic DM such that $\Omega_{total} = 1$ where $\Omega_{total} = \Omega_{CDM} + \Omega_B + \Omega_{\Lambda}$. We assume that the central values of the cosmological parameters measured by PLANCK and MAP are
\[
\Omega_{CDM} = 0.4, \quad \Omega_B = 0.05, \quad \Omega_{\Lambda} = 0.55, \quad h = 0.62
\]
(consistent with current determinations of these quantities). The accuracy with which each of these quantities can be measured by the PLANCK satellites has been estimated in Refs and one finds from this that
\[
\Omega_{CDM} h^2 = 0.154 \pm 0.017
\]
This is to be compared with the very broad window of Eq. (2) for the current estimates of $\Omega_{CDM} h^2$.

Assuming as before that the CDM is the relic neutralinos, this sharpening of the allowed region of $\Omega_{\chi_1^0} h^2$ significantly affects the DM event rate predictions. Fig.5 shows the maximum and minimum event rates as a function of $m_{\tilde{g}}$ for universal soft breaking ($\delta_i = 0$). (Recall $m_{\chi_1^0}$ scales with $m_{\tilde{g}}$ by Eq.(10).) Comparing with e.g. the solid curve of Fig.1, we see that the effect of Eq.(27) is to increase $R_{\text{min}}$ significantly and put an upper bound on $m_{\tilde{g}}$ (and hence on $m_{\chi_1^0}$). This latter effect arises from the fact that the $\chi_1^0$ annihilation cross section in the early universe is a decreasing function of $m_{\chi_1^0}$, and hence an upper bound on the relic density produces an upper bound on $m_{\chi_1^0}$. Fig.6 shows the effects of non-universal soft breaking on the event rates. Note that for $\delta_2 = 1 = -\delta_1$ there is also a minimum value of $m_{\tilde{g}}$ (and hence a minimum $m_{\chi_1^0}$) since for this case a lower value of $m_{\chi_1^0}$ would violate the lower bound on $\Omega_{\chi_1^0} h^2$ of Eq.(27). (The additional isolated points for $\delta_2 = 1 = -\delta_1$ arise from the fact that for this case $\mu^2$ is driven small (as can be seen from Eq.(17)) and so the scaling relations Eq.(10) no longer hold allowing $m_{\chi_1^0}$ to be sufficiently small so that the upper bound of Eq.(27) can be satisfied even though $m_{\tilde{g}}$ is large.) We also note that for $\delta_2 = 1 = -\delta_1$, almost the entire parameter space has now been eliminated for $\mu < 0$. 

Figure 5: Maximum and minimum event rates for a Xe detector for $\mu > 0$ as a function of $m_{\tilde{g}}$ for universal soft breaking. The solid (dashed) curves are the 1std (2std) range of Eq.(27).
Figure 6: Same as Fig.5 for 1 std range of Eq.(27) with $\delta_2 = 0 = \delta_1$ (solid), $\delta_2 = -1 = -\delta_1$ (dotted), $\delta_2 = 1 = -\delta_1$ (dashed) and $\delta_3 = 0 = \delta_1$. The isolated points are for $\delta_2 = 1 = -\delta_1$ ($R_{\text{max}}$ are empty squares, $R_{\text{min}}$ are solid squares).

The above discussion shows that the upper bound on $\Omega_{\chi_0} h^2$ of Eq.(27) for the $\Lambda$CDM model produces upper bounds on $m_{\tilde{g}}$ and by scaling, on the other gauginos. Thus (aside from the discrete points in Fig.6) the 1std (2std) gaugino upper bounds are $m_{\tilde{g}} \leq 520\,(560)\,\text{GeV}$, $m_{\chi_1^0} \leq 70\,(77)\,\text{GeV}$, $m_{\chi_1^\pm} \lesssim 150\,\text{GeV}$, and also the light Higgs obeys $m_h \lesssim 120\,\text{GeV}$. It is interesting to compare these bounds with what might be the expected reach of an upgraded Tevatron with $25\,\text{fb}^{-1}$ of data. Thus it is estimated that the gluino could be observed with a mass reach of $m_{\tilde{g}} \lesssim 450\,\text{GeV}$ for most of the parameter space, the chargino with $m_{\chi_1^\pm} \lesssim 150\,\text{GeV}$ for about $2/3$ of the parameter space and a Higgs for $m_h \lesssim 120\,\text{GeV}$. Hence the example of the $\Lambda$CDM model considered here would suggest that an upgraded Tevatron could examine a significant part of the SUSY parameter space allowed by the PLANCK satellite measurements of the cosmological parameters.

As discussed in Sec.6, there is a correlation between large (small) DM event rates $R$ and small (large) $b \to s + \gamma$ branching ratios $B$. This is exhibited in detail for the $\Lambda$CDM model in Fig.7 which shows a general scatter plot over the full parameter space of the model of Eq.(27). Recall that the current CLEO data is $B = (2.32 \pm 0.67) \times 10^{-4}$ while the SM prediction is $B = (3.48 \pm 0.31) \times 10^{-4}$. One sees from Fig.7 that if $B > 3 \times 10^{-4}$ (consistent with the SM predictions) then $R$ is small i.e. $R < 0.1\,\text{events/kg d}$ while if
$B < 3 \times 10^{-4}$ (suggested by the current CLEO data) then $R > 0.05 \, \text{events/kg d}$ for almost all parameter points. Thus one finds an interesting correlation between accelerator physics and cosmology.

7.2 $\nu$CDM model

If neutrinos do have masses in the $eV$ range, they would contribute significantly to the hot dark matter of the universe. As a second example of what the new satellite experiments might see, we consider the neutrino cold dark matter ($\nu$CDM) model and assume that the MAP and PLANCK satellites determine the following central values for the cosmological parameters:

$$\Omega_\nu = 0.2; \quad \Omega_{CDM} = 0.75; \quad \Omega_B = 0.05; \quad h = 0.62$$

(28)

Using the estimated accuracies that these quantities can be measured by the PLANCK satellite $^{23,24}$ one finds

$$\Omega_{CDM} h^2 = 0.288 \pm 0.013$$

(29)

As in the $\Lambda$CDM model, one finds a narrowing in the difference between the maximum and minimum event rates for $m_{\chi_1^0} \lesssim 60 \, GeV$. There is also an upper bound on the gaugino masses, which now is higher than in the $\Lambda$CDM model since $\Omega_{\chi_1^0} h^2$ is larger. One finds for the 1std (2std) bounds of Eq.(29) that $^2 m_{\tilde{g}} \lesssim 700(720) \, GeV$, $m_{\chi_1^0} \lesssim 95(100) \, GeV$ and $m_{\chi_1^+} \lesssim 200 \, GeV$. Thus
for this model, the LHC would be crucial in order to explore the full SUSY parameter space. The 1std window of Eq.(29) also shows an additional feature of gaps in the allowed range of $m_{\tilde{g}}$ (and $m_{\chi^0_1}$). While these gaps fill in at the 2std level, they illustrate the impact that astronomical measurements may have on accelerator physics searches.

8 Conclusions

We have considered here the effects of non-universal SUSY soft breaking on predictions of dark matter detection rates for supergravity models with $R$ parity where supersymmetry is broken at a scale $\gtrsim M_G$, the breaking being communicated by gravity to the physical sector. The lightest neutralino becomes the CDM candidate for almost the entire parameter space. For models which have universal soft breaking in the first two generations (to suppress FCNC processes), one generally needs nine additional parameters to describe the non-universal effects of the Higgs boson and third generation squark and slepton masses (which reduce to five parameters if one imposes the symmetry constraints, Eq.(16), of most GUT groups). For $\tan \beta \gtrsim 20$ only four of these, two Higgs masses and two squark masses, enter significantly. While this is still a major enlargement of the minimal SUSY parameter space, the extensive knowledge already obtained from studying SUGRA dark matter predictions for universal soft breaking have allowed one to determine in large measure the effects that such non-universal soft breaking produce. Thus the squark and Higgs non-universal effects can act to enhance or cancel each other (depending on their relative signs) and can increase or decrease event rates by a factor of $\approx 10 - 100$ in the domain $m_{\chi^0_1} \lesssim 60$ GeV ($m_{\tilde{g}} \lesssim 400$ GeV). However, the non-universal effects are generally small at higher masses.

SUGRA models can account for both the existence of CDM as well as accelerator SUSY phenomena, and interaction between the two sets of phenomena has begun to occur. Thus the fact that the top quark is heavy, and that the measured $b \rightarrow s + \gamma$ branching ratio lies about 1.6 std below the SM prediction eliminates most of the $\mu < 0$ and $A_t < 0$ parts of the parameter space at the 95\% C.L. This then leads to DM event rate predictions for the small remaining part of the parameter space with $\mu < 0$ to be $\approx 100$ times smaller than for $\mu > 0$. Further one finds a correlation between large (small) event rates and small (large) predicted $b \rightarrow s + \gamma$ branching ratio for SUGRA models. One expects that further $b \rightarrow s + \gamma$ data will play an important role in SUGRA dark matter event rate predictions.

Future satellite experiments by PLANCK and MAP as well as ballon and ground based experiments will be able to determine the basic cosmological
parameters with great accuracy (at the 1-10% level) and hence greatly reduce the uncertainties in DM predictions. To illustrate what might be expected from the PLANCK and MAP experiments, examples of the $\Lambda$CDM model and $\nu$CDM model were considered. The narrowing of the allowed window for $\Omega_\chi h^2$ generally reduces the spread between the maximum and minimum event rates for $m_\chi^0 \lesssim 60 \text{ GeV}$, and limits the maximum (and for some signs of non-universal soft breaking the minimum) values of gaugino masses. Thus astronomical measurements can have a significant impact on SUSY accelerator searches.

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References

1. For a review see G. Jungman, M. Kamionkowski and K. Greist, *Super-symmetric Dark Matter*, Phys. Rep. 267, 195 (1995).
2. A.H. Chamseddine, R. Arnowitt and P. Nath, Phys. Rev. Lett. 49, 970 (1982). For reviews see P. Nath, R. Arnowitt and A.H. Chamseddine, *Applied N=1 Supergravity* (World Scientific, Singapore, 1984); H.P. Nilles, Phys. Rep. 110, 1 (1984); R. Arnowitt and P. Nath, Proc. of VII J.A. Swieca Summer School ed. E. Eboli (World Scientific, Singapore, 1994).
3. E.W. Kolb and M.S. Turner, *The Early Universe* (Addison-Wesley, Redwood City, 1989).
4. L. Hall, J. Lykken and S. Weinberg, Phys. Rev. D 27, 2359 (1983); P. Nath, R. Arnowitt and A.H. Chamseddine, Nucl. Phys. B 227, 121 (1983).
5. K. Inoue et al. Prog. Theor. Phys. 68, 927 (1982); L. Ibañez and G.G. Ross, Phys. Lett. B 110, 227 (1982); L. Alvarez-Gaumé, J. Polchinski and M.B. Wise, Nucl. Phys. B 221, 495 (1983); J. Ellis, J. Hagelin, D.V. Nanopoulos and K. Tamvakis, Phys. Lett. B 125, 2275 (1983); L.E. Ibañez and C. Lopez, Nucl. Phys. B 233, 545 (1984); L.E. Ibañez, C. Lopez and C. Muños, Nucl. Phys. B 256, 218 (1985).
6. J. Ellis and F. Zwirner, Nucl. Phys. B 338, 317 (1990); R. Arnowitt and P. Nath, Phys. Rev. D 46, 3981 (1992).
7. R. Arnowitt and P. Nath, Phys. Rev. Lett. 69, 725 (1992). P. Nath and R. Arnowitt, Phys. Lett. B 289, 368 (1992).
8. For recent papers see R. Arnowitt and P. Nath, *Mod. Phys. Lett.* A **10**, 1275 (1995); P. Nath and R. Arnowitt, *Phys. Rev. Lett.* **74**, 4592 (1995); E. Diehl, G. Kane C. Kolda and J. Wells, *Phys. Rev.* D **52**, 4223 (1995); R. Arnowitt and P. Nath, *Phys. Rev.* D **54**, 2374 (1996); L. Bergstrom and P. Gondolo, *Astropart. Phys.* **5**, 263 (1996); J.D. Vergados, *J. Phys.* G **22**, 253 (1996); V.A. Bednyakov, S.G. Kovalenko, H.V. Klapdor-Kleingrothaus and Y. Ramachers, *Z. Phys.* A **357**, 339 (1997); J. Edsjö and P. Gondolo, [hep-ph/9704361]; V. Barger and C. Kao, [hep-ph/9704403]; H. Baer and M. Brhlik, [hep-ph/9706509]. Analyses with non-universal Higgs soft breaking masses are given in D. Matalliotakis and H.P. Nilles, *Nucl. Phys.* B **435**, 115 (1995); M. Olechowski and S. Pokorski, *Phys. Lett.* B **344**, 201 (1995); V. Berezinsky, A. Bottino, J. Ellis, N. Forrengo, G. Mignola and S. Scopel, *Astropart. Phys.* **5**, 1 (1996).

9. S.K. Soni and H.A. Weldon, *Phys. Lett.* B **126**, 215 (1983); V.S. Kaplunovsky and J. Louis, *Phys. Lett.* B **306**, 268 (1993).

10. N. Polonski and A. Pomerol, *Phys. Rev.* D **51**, 6532 (1995); R. Barbieri, L. Hall and A. Strumia, *Nucl. Phys.* B **445**, 219 (1995).

11. M. Drees, *Phys. Lett.* B **181**, 279 (1986); P. Nath and R. Arnowitt, *Phys. Rev.* D **39**, 2006 (1989); J.S. Hagelin and S. Kelley, *Nucl. Phys.* B **342**, 95 (1990); Y. Kawamura, H. Murayama and M. Yamaguchi, *Phys. Lett.* B **324**, 52 (1994).

12. S.Dimopoulos and H. Georgi, *Nucl. Phys.* B **206**, 387 (1981).

13. P. Nath and R. Arnowitt, [hep-ph/9701301].

14. A. Bottino, F. Donato, G. Mignola and S. Scopel, P. Belli and A. Incicchitti, *Phys. Lett.* B **402**, 113 (1997).

15. P. Nath, J. Wu and R. Arnowitt, *Phys. Rev.* D **52**, 4169 (1995).

16. J. Wu, R. Arnowitt and P. Nath, *Phys. Rev.* D **51**, 1371 (1995).

17. P. Nath and R. Arnowitt, *Phys. Lett.* B **336**, 395 (1994); F. Borzumati, M. Drees and M. Nojiri, *Phys. Rev.* D **51**, 341 (1995).

18. P. Nath and R. Arnowitt, *Phys. Rev. Lett.* **74**, 4592 (1995); R. Arnowitt and P. Nath, *Phys. Rev.* D **54**, 2374 (1996); H. Baer and M. Brhlik, *Phys. Rev.* D **55**, 3201 (1997).

19. M.S. Alam et al. (CLEO Collaboration), *Phys. Rev. Lett.* **74**, 2885 (1995).

20. A.J. Buras, A. Kwiatkowski and N. Pott, [hep-ph/9707482].

21. R. Arnowitt and P. Nath, unpublished.

22. http://map.gsfc.nasa.gov/ http://astro.estec.esa.nl:80/SA-general/Projects/Cobras/cobras.html.

23. A. Kosowsky, M. Kamionkowski, G. Jungman and D. Spergel, *Nucl.
Phys. Proc. Suppl. 51B, 49 (1996).
24. S. Dodelson, E. Gates and A. Stebbins, Astroph. J. 467, 10 (1996).
25. M.S. Turner, astro-ph/9703161; L. Krauss, astro-ph/9706227.
26. T. Kamon, J.L. Lopez, P. McIntyre and J.T. White, Phys. Rev. D 50, 5676 (1994); Report of the tev-2000 Study Group, eds. D. Amidei and R. Brock, FERMILAB-Pub-96/082.