Unification for Yukawas and its implications

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Abstract. The supersymmetric finite threshold effects are studied in the presence of non minimal soft terms that can correct the problematic mass ratios of the light generations in the minimal SU(5) GUT. We show that with large soft $A$–terms, one can achieve simple unification for lighter generations without additional Higgs multiplet, while having sfermions lighter than 1 TeV. The presence of such large $A$–terms will distort the sfermion mass spectrum upon running from GUT scale down to the electroweak scale making it distinct from the universal SUSY breaking scenarios, especially in the first two generations. The implications of these splittings are studied in $K$ and $D$ meson oscillations and in rare processes $D^+ \to \pi^+ \nu \bar{\nu}$ and $K^+ \to \pi^+ \nu \bar{\nu}$, and in the latter case the effect is found to be important.

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THE SUSY THRESHOLD EFFECTS ON THE QUARK MASSES

The appeal for the supersymmetry (SUSY) is enforced by the fact that the gauge couplings unify nicely while the $b$ and the $\tau$ Yukawas unify at a reasonably good level. These successes are not extended to the lighter generations: $m_d/m_s$ is an order of magnitude larger than $m_e/m_\mu$ instead of being equal if unified values are assumed at the grand unification (GUT) scale. The most pursued solutions to this so called problem of “wrong GUT ratios” include the introduction of new Higgs multiplets such as $45$ in SU(5) [1] or higher dimensional Planck mass suppressed operators.

Another option is due to the idea that the unification for the Yukawas of the lighter generations is obscured by the SUSY finite threshold effects at the electroweak scale [2]. In particular, the gluino–squark loop induced correction to the mass of the $i$th generation down–type quark is given by

$$ (\delta m_d)_i \simeq -\frac{2\alpha_s}{3\pi} (m_{LR}^d) m_{\tilde{g}} I \left( m_{\tilde{Q}_i}^2, m_{\tilde{d}_i}^2, m_{\tilde{g}}^2 \right), $$  

$$ m_{LR}^d = A_d v_d - Y_d v_u. $$

The most of the studies on these corrections have been in the limit of large $\tan \beta$ [3]. In the case of universal soft terms one has $m_{LR}^d = m_d (A_0 - \mu \tan \beta)$, which leads to universal change

$$ \frac{\delta m_{d_i}}{m_{d_i}} \simeq \frac{\alpha_s}{3\pi} \frac{(A_0 - \mu \tan \beta) m_{\tilde{g}}}{m_{d_i}^2}. $$

Here we approximated the loop integral as $I \left( m_{\tilde{Q}_i}^2, m_{\tilde{d}_i}^2, m_{\tilde{g}}^2 \right) \simeq 1/(2m_{d_i}^2)$. This expression for the leading SUSY QCD effect clearly shows that if the soft masses are universal so
are the induced relative changes in the quark masses. On the other hand, the needed change for the $d$ and $s$ quark masses are very different. If the down–type quark Yukawa couplings are set to that of the charged leptons at the GUT scale, the value of the down quark mass before the correction is less than its experimental value by $\sim 1.5$ MeV while that of the $s$–quark is greater by $\sim 0.16$ GeV at $M_{SUSY} = 1$ TeV. Thus it seems inevitable to depart from a universal assumption for the soft terms. Indeed the scans over the universal soft parameters by several studies (See for example [4]) have found no solution for the wrong GUT ratios.

Looking at the formula we see that there are two option available: choose either (i) different squark masses or (ii) non minimal $A$–terms that are not proportional to the Yukawa couplings. The first option requires $m_\tilde{d}/m_\tilde{u} \simeq 2.3$ for inducing $\delta m_d = -3.5$ MeV and $\delta m_s = -0.16$ GeV which will unlikely survive the constraint from the neutral Kaon mixing unless the squarks are quite heavy, in the range of TeVs. This leaves us with the second option if one wants have sfermions spectrum within the reach of the LHC.

To induce the needed corrections using the $A$–terms, the numerical values for them have to be much larger than the most SUSY breaking scenarios such as mSUGRA. There is a constraint on the $A$–terms from the stability of the vacuum

\[
(A_d)_{ij} \leq 1.75 \sqrt{\frac{1}{3} \left( m_{\tilde{Q}_i}^2 + m_{\tilde{d}_j}^2 + m_{H_d}^2 + \mu^2 \right)}.
\]

When satisfied it guarantees the stability of our vacuum at cosmological time scale [5] and allows much larger parameter space than the severe constraint [6] of absolute stability.

**LARGE $A$–TERMS FOR UNIFICATION AND THEIR IMPLICATIONS**

The presence of large $A$–terms will have several observable consequences due to their effects on the RGE running of the soft masses. Here we display several examples of large $A$–terms for various choices of $\tan \beta$, which lead to the SUSY finite corrections that give the correct low energy $d$ and $s$ quark masses. The corrections to the charged lepton masses are small at least by a factor $3\alpha'/(8\alpha_S) \simeq 0.03$ compared to the quarks and found to be at the level of less than a few percent. For this reason, the initial values for the Yukawa couplings of the down–type quarks are set to that of the charged leptons at the GUT scale. We assume the soft terms satisfy $SU(5)$ boundary condition at the GUT scale along with the gauge and the Yukawa couplings. The soft masses are chosen to have universal forms and the $A$–term of the up sector is given by $A_u = a_0 Y_u$. On the other hand, we let the $A$–terms of the down sector have large values that are not proportional to their respective Yukawa couplings. To avoid the FCNC constraints, we keep them in diagonal form in the basis where the corresponding Yukawa matrix is diagonal:

\[
(A_5)_{ij} \equiv (A_\ell)_{ij} = (A_d)_{ij} = a_i \delta_{ij} \neq a_0 Y_d.
\]
TABLE 1. The choices for the $\mu$–term and relevant soft trilinear $A$–terms at low energy and the induced change to the down–type quark masses.

| $\tan \beta$ | 5   | 10  | 15  | 20  |
|--------------|-----|-----|-----|-----|
| $\mu$ (GeV)  | 500 | 550 | 580 | 850 |
| $A_d$ (GeV)  | 3.5 | 6.4 | 9.2 | 16.6|
| $A_s$ (GeV)  | -280| -460| -760| -900|
| $A_b$ (GeV)  | -900| -950| -800| -228|
| $\delta m_d$ (MeV) | 1.50 | 1.43 | 1.55 | 1.69 |
| $\delta m_s$ (GeV)   | -0.170 | -0.167 | -0.158 | -0.156 |
| $\delta m_b$ (GeV)   | -0.730 | -0.732 | -0.697 | -1.0 |

TABLE 2. The mass splittings in $K$ and $D$ meson systems due to the SUSY effects.

| $\tan \beta$ | 5   | 10  | 15  | 20  |
|---------------|-----|-----|-----|-----|
| $\Delta M_D \times 10^{14}$ GeV$^{-1}$ | $1.44 \times 10^{-2}$ | 0.120 | 0.59 | 1.42 |
| $\Delta M_K \times 10^{15}$ GeV$^{-1}$ | $1.59 \times 10^{-2}$ | 0.105 | 0.706 | 1.55 |

In Ref. [4], the large $A$–terms were used also for generating the Cabibbo mixings in addition to correcting the wrong GUT ratio. This led to heavy sfermions $\geq 4.4$ TeV mainly to avoid the constraint from $\mu \rightarrow e\gamma$. Since our primary concern is not a possible origin of flavor structure but the unified common values for the Yukawas, we abandon that choice in favor of the form given in Eq. (5). Here we choose the following values for the soft masses $\{m_{1/2}, m_{10}^2, m_3^2\} = \{-210(-230), 560(580), 523(542)\}$ GeV for $\tan \beta = 5, 10(15, 20)$ at the GUT scale. The corresponding $A$–terms that induce the needed corrections to the down–type quarks are given in Table 1. In the numerical calculation, we have included the subleading neutralino and chargino corrections. See Ref. [7] for details. Our choices for the soft parameters indeed give the finite contributions that lead to quark masses in agreement with their experimentally determined values. For $\tan \beta = 20$, we also show, as an example, the case where the correction to the $b$–quark mass is somewhat larger than what one needs. Such cases could be easily compatible with experiment if, upon embedding to a concrete GUT model, the right–handed neutrino effect on the $\tau$ Yukawa RGE running is included.

Large $A$–terms in the first two generation will split the squark masses during the RGE running from the GUT scale to the electroweak scale. This could lead to excessive neutral meson mixings. We have calculated the induced Kaon and $D$–meson mass differences and they, as shown in Table 2, have been found to be at a safe level. Here the leading contributions are from the chargino and the gluino boxes for Kaon and the $D$–meson respectively. As for $B$ and $B_s$ mesons, the induced mass differences are expected to be small at low $\tan \beta$ values we have chosen.

We have looked into rare decays $D^+ \rightarrow \pi^+ \bar{\nu}\nu$ and $K^+ \rightarrow \pi^+ \bar{\nu}\nu$ since they do not have large long distance contributions. In the $D^+$ meson case the effect is, although $\sim 10^4$
times larger than the SM prediction, still far from the reach of BESIII. On the other hand, we find that the effect from the squark mass splitting could be quite important for $K^+$ and in some cases could be somewhat larger than the experimental results. The numerical results are given in Table 3. This contribution could be checked when the experimental precision is refined in future experiments [8].

**CONCLUSIONS**

In this talk I have presented the solution to the wrong GUT ratios through finite SUSY corrections and several of its experimental consequences. With large $A$–terms one can achieve the minimal Yukawa unification with sub TeV sfermion masses. Through RGE running the squark masses will split due to the large $A$–terms making the spectrum distinct from the widely studied universal scenarios. The charm and strange meson oscillations and their rare decays are studied. In particular, $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ process has been found to have a substantial SUSY contribution.

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