Conic Programming Based Day-ahead Optimization Dispatch in Distribution Network Containing Large-scaled Distributed Generation and Electric Vehicle

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Abstract. Regarding day-ahead optimization dispatch in distribution network containing large-scaled distributed generations (DGs) and electric vehicles (EVs), this paper proposes an optimization dispatch method based on conic programming. A model for dispatch that can optimize the EV aggregator charging power and the output of DGs is established. The proposed model takes the constrains of EV charging energy and charging power into consideration. In addition, DG can curtail the active power in the model to guarantee the operation safety. Then nonlinear optimization dispatch model are transformed and relaxed into second-order cone programming model to be solved. The optimization dispatch method proposed in this paper is verified by the IEEE33 node test system.

1. Introduction
Fossil energy shortage and environmental issues are the major challenges of the world in the 21st century. DG and EV have incomparable advantages in alleviating energy crisis and reducing people's reliance on conventional fossil energy, which have attracted the attention of all countries. However, the DG fundamentally changes the characteristic of a conventional distribution network. Integration of large-scaled DGs to the radial distribution network and the load flow changes will inevitably affect the operating mode in distribution network [1]. As to EVs, these new electric loads will definitely trigger extreme surges in demand at rush hours [2], and therefore threaten the stability and security of the power grid. It is worth noting that EV is a highly promising resource for power balancing. Reasonable charging control of EVs not only shifts the peak load [3], but also absorbs surplus energies of DGs during peak outputs and hence increases the allowable capacity [4] of DGs in distribution network. Therefore, it is highly significant to realize optimization dispatch of active power distribution network containing large-scaled DGs and EVs.

In recent studies, more researches are carried on the real-time charging optimization which directly decides the charging scheduling of an EV [5,6,7]. The real-time charging optimization mainly aims at reducing the charging cost of each EV or aggregator within a day. But for the power grid, the EV load shifting after optimization control greatly affect the day-ahead market and dispatch, which distribution network operator should pay more attention to. A few papers have worked on this field. In [8-11], the different two-stage EV charging mechanisms involving with a day-ahead determination of the energy generation and real-time charging optimization are developed. The real-time charging strategy...
proposed in reference [8] can provide scheduling for each EV. In reference [9] the energy storage is utilized by the aggregator to mitigate the impact of uncertainty and inaccurate prediction in day-ahead stage. The maximum number of vehicles that can be charged both in the two stage is studied in reference [10]. Reference [11] includes most notably the technical constraints relating the aggregate battery state of charge of the vehicle fleet to the daily motion requirements and associated limitations on the provision of reserves and V2G services.

It is worth noting that reasonable strategy of the EV charging and DG output to achieve the economic operation in the distribution network system, is important to the dispatch center for the further power generation plan. Therefore, the optimization dispatch of EVs and DGs is the fundamental part for the economic operation in the whole power system. Compared with former researches, this paper takes the following aspects into consideration: the grid safety constrains, EV charging characteristics and DG regulation characteristics.

This paper proposes a model that can optimize the EV aggregator charging power and the output of DGs. And for efficient solution, the nonlinear part in this model is relaxed into second-order cone programming model, which can be conveniently solved by second-order cone programming and keep accuracy meanwhile. The optimization dispatch method proposed in this paper is verified by the IEEE33 node test system.

2. EV Charging Characteristics

The battery characteristic and driving habit of an EV are important to the charging model. In this paper, the EV charging load analysis [12] mainly focuses on the electric buses and private cars.

2.1. EV Charging Load

Based on the driving behaviour, the Monte Carlo sampling method make the day-ahead load curve accessible in the subsequent optimization dispatch research. The typical charging load curve of each type EV within one day is shown in figure 1.

![Figure 1. Charging Power Distribution of 1,000 EVs per type](image)

2.2. Stay Time Characteristics

The optimal charging characteristics are significantly affected by the EV parking behaviour, because the upper limit of optimal charging power must be consistent with the actual number of parking EVs. Based on the driving behaviour, the EV parking number of 1000 EVs per type during a day is calculated and shown in the figure 2.

![Figure 2. EV daily parking number of 1,000 EVs per type](image)

3. Problem Formulation
In this section, the optimal charging of EVs and the active power curtailment ability of DGs are utilized to establish a model for optimization dispatch in distribution network. This model aims to provide optimized strategy which satisfies the routine demands of EVs, make best use of DGs output and reduce the operating costs.

3.1. Optimization Objective

In the distribution network, the DGs represented by photovoltaic generation system should be preferentially used. When DGs affect safety of network, it is necessary to restrict the output of DGs. With the increase in number of EVs, the load peak is occurred when the conventional load is overlapped by the charging load under stochastic charging mode. The operation of network may face greater challenges. Therefore, it is necessary to optimize the EV charging and DG output.

In a distribution network containing large-scaled DGs and EVs, it is necessary to fully explore the dispatch ability and establish a model for the operational economy and safety. The control variables of the optimization dispatch model are the charging load of nodes which provides this service for EVs and the active power of DGs at various periods within a day. The optimization objective primarily includes two parts, as shown in (1), network loss and dispatch cost of DGs. Paper [13] proves that load variance and network loss are closely related. Optimizing network loss is equivalent to optimization the load variance which can achieve the peak load shifting, and it can guide more economical charging behaviour by the price mechanism. Consideration of the dispatch cost of DGs helps to preferentially utilize the renewable energy.

\[
\begin{align*}
\min f &= \Delta T \sum_{j=1}^{N_j} C_{\text{load}}(t) \sum_{j\in E} r_j (I_j(t)) + \sum_{j=1}^{N_j} C_{i,DG}(t) P_{i,DG}(t) \\
&\text{Where, } \Delta T \text{ is the time period, } N_T \text{ is the number of dispatch periods, } E \text{ is the branch set, } r_j \text{ and } I_j \text{ are the branch resistance and current, } C_{\text{load}}(t) \text{ and } P_{\text{load}}(t) \text{ are the unit loss cost and total loss during period } t; N_{DG} \text{ is the number of DGs; } C_{i,DG}(t) \text{ and } P_{i,DG}(t) \text{ are the unit power cost and restricted power of } i^{th} \text{ DG during period } t.
\end{align*}
\]

3.2. Constraints

The grid constrains contain power balance and operation safety. These constrains are formulated in Branch flow model [14], which are not described in detail here.

(1) EV charging energy

In order to ensure the EV use, the major battery charge should be complete within a certain period. It is assumed that the certain period for buses, private cars charging during work and after work are respectively 23:00–8:00, 8:00–19:00, and 18:00–8:00.

\[
\Delta T \sum_{i\in N_i} P_{i,\text{EV}}(t) = \Delta T \sum_{i=1}^{N_i} P_{i,\text{EV}}^0(t) \times 95\%
\]

Where, \(P_{i,\text{EV}}(t)\) and \(P_{i,\text{EV}}(t)\) are the charging power of node \(i\) during period \(t\) before and after optimization dispatch; \(N_T\) is the specific period.

Meanwhile, the charging load at various nodes should satisfy the basic charging needs of EVs.

\[
\Delta T \sum_{i=1}^{N_i} P_{i,\text{EV}}^0(t) = \sum_{i=1}^{N_{\text{EV}}} E_{j,c} \left( SOC_{j,c}^d - SOC_{j,c}^w \right)
\]

Where, \(N_{\text{EV}}\) is the number of EVs at node \(i\); \(E_{j,c}\) is the battery capacity of EV \(j\); \(SOC_{j,c}^d\) and \(SOC_{j,c}^w\) are the state of charge when EV\(_j\) leave and return.

(2) EV charging power

Charging power \(P_{i,\text{EV}}(t)\) of node \(i\) with period \(t\) should not exceed the sum of maximum possible charging load of EV parking at that time.

\[
\forall t, \forall i: \exists \left( 0 \leq P_{i,\text{EV}}(t) \leq N_{i,\text{EV}}(t) P_{i} \right)
\]

Where \(N_{i,\text{EV}}(t)\) is the number of EVs parking at node \(i\) during period \(t\).
(3) **DG constrains**

The photovoltaic generation represents DG in this paper. Its control variable is set as the ratio of the practical output to the maximum output, following the constraint shown below. It is worth noting that the output ratio of different DGs is keep consistent in this paper to guarantee the fairness.

\[ \forall (i) \exists (0 \leq \alpha_i (t) \leq 1) \]

Where, \(\alpha_i\) is the ratio of the practical active output to the maximum output for DG \(i\). And the relationship between \(\alpha_i\) and the variables in optimization objective is as follows:

\[ P_{i, DG} (t) = (1 - \alpha_i (t)) P_{i, DG}^{max} (t) \]

Where \(P_{i, DG}^{max}\) is the maximum active power output of DG \(i\).

4. **Effective Algorithm**

The optimization dispatch model based on Branch flow model is nonlinear. It can be transformed using second-order cone relaxation method [15]. If let \(\bar{I}_j = I_j^2\) and \(\bar{U}_j = U_j^2\), the optimal objective can be transformed to formula (7).

\[ \min f = \Delta T \sum_{i=1}^{N} \left( C_{loss} (t) \sum_{j \in \delta(i)} r_{ij} \bar{I}_j + \sum_{j \in \pi(i)} C_{i, DG} (t) P_{i, DG} (t) \right) \]

The constrains of power balance can be transformed to formula (7) - (9):

\[ \begin{align*}
 p_j (t) = \sum_{k \in \delta(j)} P_{jk} (t) - \sum_{i \in \pi(j)} \left( P_i (t) - \bar{I}_j (t) r_{ij} \right) + g_j \bar{U}_j (t) \\
 q_j (t) = \sum_{k \in \delta(j)} Q_{jk} (t) - \sum_{i \in \pi(j)} \left( Q_i (t) - \bar{I}_j (t) x_{ij} \right) + b_j \bar{U}_j (t) \\
 \bar{U}_j (t) = U_i (t) - 2 \left( P_i (t) r_{ij} + Q_i (t) x_{ij} \right) + \bar{I}_j (t) (r_{ij} + x_{ij}) \\
 \left| \frac{2P_i (t)}{2Q_i (t)} \right| \leq \bar{I}_j (t) + \bar{U}_j (t) \\
 \left| \bar{I}_j (t) - U_j (t) \right| \leq \bar{I}_j (t) + \bar{U}_j (t)
\end{align*} \]

Where \(\delta(j)\) is the branch end node set with \(j\) as the head node. \(\pi(j)\) is the branch head node set with \(j\) as the end node. \(P_{jk}\) (\(P_i\)) and \(Q_{jk}\) (\(Q_i\)) are the active and reactive power though branch \(jk\) (\(ij\)). \(p_j\) and \(q_j\) are injection active and reactive power of node \(j\). \(r_{ij}\) and \(x_{ij}\) are the resistance and reactance of branch \(ij\). \(g_j\) and \(b_j\) are conductance and conductivity of node \(j\).

The constrains of operation safety can be transformed to formula (11) and (12):

\[ U_j^{min} \leq \bar{U}_j (t) \leq U_j^{max} \]

\[ 0 \leq \bar{I}_j (t) \leq I_j^{max} \]

Where \(U_j^{min}\) and \(U_j^{max}\) are the voltage amplitude lower limit and upper limit of node \(j\). \(I_j^{max}\) is the current amplitude upper limit of branch \(ij\).

The constrains of EV and DG is linear, so they remain unchanged. Finally, the proposed optimization dispatch model is relaxed to a second-order cone programming model, which can be effectively solved by the algorithm of second-order cone programming.

5. **Case Study**

5.1. Test system

In this paper, IEEE 33 node test system is adopted to verify the proposed optimization dispatch method, with the detail load parameters given by [16]. The load in light load scenario is set to 1/3 of the peak load, while the load in heavy load scenario is set to be the peak load. In test region, the maximum irradiance is set at 1000W/m². The upper limit of voltage is 1.05 pu and the lower limit is...
0.95 pu. The number of time-frames \( N_t \) is set to 24. The PV systems are connected with the network at the node of 6, 12 and 28. The single system area is 3000 m\(^2\), with the photoelectric conversion efficiency is 14\% and maximum irradiance is 1000 W/m\(^2\).

5.2. **EV charging load**

The aggregators are the implement agent of optimization dispatch with a number of EVs. In the test system, the aggregators are set at various nodes for different EV types. In this paper, one aggregator at a specific node is only responsible for one specific EV type. The aggregator location and type are shown in table 1.

| node | EV type             | EV number |
|------|---------------------|-----------|
| 5    | Bus                 | 30        |
| 9    | Private car-during work | 40        |
| 15   | Bus                 | 30        |
| 20   | Private car-after work | 50        |
| 24   | Private car-during work | 40        |
| 31   | Private car-after work | 50        |

5.3. **Light Load scenario**

In the light load scenario, the DG output consumptive effect of EV is verified first. If the IEEE 33 system is only integrated with DG, the voltages in the time period of 12, 13, 15 and 16 are beyond the upper limit. So the DGs must be curtailed and the output ratios are respectively shown in table 2.

| Time Period | 12  | 13  | 15  | 16  |
|-------------|-----|-----|-----|-----|
| DG Output Ratio | 0.77 | 0.96 | 0.74 | 0.96 |
| Active Power of Curtailment per DG Unit (kW) | 51.07 | 7.89 | 47.69 | 5.70 |

When there are EV charging load in the test system, the daily load curve of each charging node is calculated by the random sampling method described above. Because of the consumptive effect of EV, the voltages do not exceed the limits even without the optimization dispatch. Compared to the optimal charging mode, the stochastic charging mode cannot balance the peak and valley load, leading to an uneconomic operation with larger network loss. The network loss comparison under the two charging modes is shown in table 3.

| Network loss under stochastic charging (kW) | Network loss under optimization charging (kW) | Network loss reduction ratio |
|--------------------------------------------|-----------------------------------------------|------------------------------|
| 277.61                                      | 247.58                                       | 10.82\%                      |

5.4. **Heavy Load scenario**

In the heavy load scenario, the EVs peak load with stochastic charging mode superposes on the residential peak load. It leads to a higher load peak and makes the voltage closed to the lower limits. While the optimal charging mode can effectively shift the load peak, improve the power quality and reduce the network loss. Figure 3 shows the voltage profile under two charging modes at 20:00 o’clock. Table 4 shows the network loss comparison under the two charging modes. The safety margin with respect to voltage drop limit should be enlarged when load level is high enough to make the voltage drop become a binding constraint. On the other hand, when the load level is low, minimising network losses would become a higher priority in optimization dispatch.
Figure 3. Voltage Profile Under Two Charging Modes at 20:00 O’clock in Heavy Load scenario

Table 4. Comparison of network losses under two charging modes

| Network loss under stochastic charging (kW) | Network loss under optimal charging (kW) | Network loss reduction ratio |
|--------------------------------------------|------------------------------------------|----------------------------|
| 2015.90                                    | 1918.5                                   | 4.70%                      |

Figure 4 show the node load with EV under two charging modes. Because the charging time of buses is in late night, which avoids the load peak, the effect of load shift is not obvious. But for private cars charging during work and after work, the charging load is shifted from morning and evening to the afternoon and late night.

Figure 4. Comparison of Node Load with EV Under Two Charging Modes

In this paper, the $C_{\text{evs}}(t)$ and $C_{\text{DG}}(t)$ are both adopted peak-valley price. The valley price lasts from 23:00 to 8:00. So for the three type charging behaviour, the charging fee of private car after work can obtain the maximum benefit, with the saving about 30 percentage. While the cost of other two type EVs just slightly decrease. Although the optimization objective is from the point of distribution system operation, the figure 5 shows that the decrease of network loss are consistent with the decrease of charging cost. Because both of the two objectives aim to realize the peak load shifting essentially under the peak-valley price mechanism.

Figure 5. Charging Cost Comparison Under Two Charging Modes
6. Conclusion
Regarding the optimization dispatch of distribution network containing large-scaled DGs and EVs, this paper proposes an optimization dispatch method based on second-order cone programming. A model that can optimize the node charging power of EVs and the output of DGs is established. Then the network loss in the optimization objective, as well as the nonlinear constrains are relaxed to a second-order cone programming model. The IEEE 33 test system verified that the optimization dispatch method containing optimal charging mode proposed in this paper can effectively reflects the energy and power constrains according to the actual driving habit of EV users. And the optimization solution can shift the peak load, consume the renewable energy and reduce the network loss, guaranteeing the safety and economy of the network. Meanwhile, the algorithm of second-order cone programming can quickly and accurately solve the relaxed model containing large-scaled DGs and EVs.

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