Sociophysics Simulations

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Abstract: Check if we need one

Introduction:

Physicists since a long time have tried to apply their skills to fields outside physics, with
more or less success. Econophysics is a big fashion at present, though partly based on the
wrong belief that economists did not make empirical observations of financial markets or made
no Monte Carlo simulations where rational judgement is replaced by random decisions. Also
Frederick Soddy ventured into economics after his 1921 Nobel prize in chemistry. Quantum
mechanics co-inventor Erwin Schrödinger more than half a century ago wrote a book asking
“What is life?”, long before today’s interests in biophysics and bioinformatics. Sociophysics
has been around for at least three decades, with or without that name [1, 2, 3, 4]. The
present review summarizes some of the more recent simulations in sociophysics and is clearly
biased by the personal preferences and experience of its author. In June 2002, I visited the
first conference devoted only to Sociophysics, organized by F. Schweitzer and K.G. Treutsch,
www.ais.fhg.de/frank/. We ignore here car traffic simulations [5, 6], scale-free networks
[7], social percolation [8] and active Brownian particles [9] since they were reviewed recently.
Instead we look at the models of Bonabeau et al [10], Sznajd [11] and similar consensus models
[12, 13]. We concentrate on simple models which take about one page of Fortran program,
available from stauffer@thp.uni-koeln.de. We thus update similar summaries published
before [14, 15].

Hierarchies:

How come someone is born into nobility, and others are members of the proletariat. Some
scientists are given tenure, and others have to leave academia. The elites of all countries and all
times has always some explanations, like the Grace of God having them put into the upper levels
of society. Statistical Physicists, of course, assume these hierarchies to arise from randomness.
(The illusion that everything is random is a professional disease ”morbus Boltzmann” among
these physicists, just as silicosis = black lung affects mine corkers.)

If nobility is connected with ownership of the land, then it cannot develop easily in a nomadic
society, while sedentary societies may have ground property. The peasants then can become
slaves of the nobility owning the piece of territory on which the peasants work. Thus sedentary
societies with agricultural fields may develop stronger hierarchies than nomadic tribes with
just a few light goods to be carried around. The Hollywood movie “Dances with Wolves” is an
example how the industrialized world may imagine beautiful nomadic paradise to have been.
The problem therefore is to develop a model giving rise to strong hierarchies at high population densities and weak hierarchies at low population densities. Such a model, with a sharp first-order phase transition at some critical density, was proposed by Bonabeau et al. [10] and followed-up by others [18, 19]. People diffuse on a square lattice to nearest neighbours (Margittai Neumann neighbourhood, not Moore neighbourhood). When one person tries to move to a place already occupied by another person, the two have a fight, the winner takes the contested place, and the loser moves to (or stays at) the other place. Initially, the probability to win or lose is 50 percent. But after some time, memories of past fights and their outcomes accumulate and are stored in the history $h(i)$ for each person $i$, where $h$ is the number of victories minus the number of defeats of this person. After each iteration, $h$ is diminished by, say, 10 percent of its current value to take into account that memories fade away.

The probability $q$ for $i$ to win against $k$ is assumed as

$$q = 1/(1 + \exp(\sigma [h(k) - h(i)])$$

(1)
like a Fermi function: More victories in the past mean a higher chance to win now. Here $\sigma$, the
inverse width of the transition at Fermi level, is the rms fluctuation in the winning probabilities:

$$\sigma = (\langle q^2 \rangle - \langle q \rangle^2)^{1/2}$$

(2)

and thus is zero if everybody is equal ($q = 1/2$), and is $1/\sqrt{6}$ if the values for $q$ are distributed
homogeneously between zero and unity. Thus $\sigma$ measures the inequality in this society. And this
inequality re-enters into the probabilities to win and lose, thus enhancing existing inequalities.
(For the first ten Monte Carlo steps per site, $\sigma = 1$ to allow a build-up of hierarchies.)

The simulations show either a $\sigma$ going after some initial positive values rapidly to zero with
time (density below 32 percent), or a $\sigma$ staying at or above 0.25 for density above 32 percent.
This first-order phase transition actually has a complicated history [10, 18, 19], and only the
present status is summarized here. Fig.1 shows the distribution of $q$ values near the end of a
simulation with and without hierarchy.

Even in a strongly hierarchical society, revolutions can happen, and then a new hierarchy
builds up. Thus averaging for one individual over very long time would give an average winning
probability $q$ near 1/2, for all people. The proper way to distinguish between hierarchical
and non-hierarchical societies is thus the above snapshot method, where the inequality $\sigma$ is
determined from all $q$ values at one given moment, via eq.(2).

F. Schweitzer, private communication, has criticized the symmetry built into this model,
where the probability to belong to the leaders (high $q$) is the same as the one to belong to the
followers (low $q$). A monarchy needs one king with many subjects. This is only one of many
questions which are still open for further modelling.

Consensus:

A democracy needs both a stable government and a viable opposition. But for a selection
comitee trying to fill a professor position at the university it is much nicer if finally a consensus
is estabished about who are the best candidates. Thus consensus may be a good or a bad thing,
depending on the application. This section deals with some models of opinion formation where,
depending on details, a complete consensus is or is not found [20].

Imagine that the possible opinions of a large set of people $i = 1, 2, \ldots, N$ are described by
a real number $S$ between zero and unity; it politics this correponds to the traditional left-right
classification. Now these people talk to each other and try to convince each other of their
opinions. Normally people with vastly different opinions will not agree on any compromise but
when the opinions are simular then a compromise is possible. For example, if two people $i$ and
$k$ have similar opinions $S_i$ and $S_k$ such that their difference $|S_i - S_k|$ is smaller than some small
limit $\epsilon$ ("bounded confidence"), then a reasonable compromise is that both accept the average
$(S_i + S_k)/2$ as their new opinions. This rule can be generalized to several people agreeing within
$\epsilon$ with opinion $S_i$, or to a vector of several opinion variables per person, instead of merely one
$S_i$ [13, 12].
With simultaneous updating of 625 initially random opinions, [13] found dozens of final opinions for \( \epsilon = 0.01 \), two opinions (“polarization”) for \( \epsilon = 0.15 \), and one opinion (“consensus”) for \( \epsilon = 0.25 \). Thus, the more tolerant people are, the higher is the chance for consensus, certainly a plausible result showing that the model is reasonable. Similar results were found by the French group [12], giving about \( 1/(2\epsilon) \) different final opinions in a similar model. In both cases, everybody could interact with everybody, like at a long conference reception where nobody can sit and thus everybody walks around a lot.

In the opposite extreme, people sit on a lattice and talk only with their nearest neighbours. And in between is the case of slow diffusion when after every chat with a neighbour everybody tries to move to an empty neighbour place, similar to [1]. Computationally it is simpler to model opinions as discrete integers \( S = 1, 2, 3, \ldots q \) such that only neighbouring opinions \( S \pm 1 \) can influence opinion \( S \). Thus geometric space is a square lattice, and opinion space is a one-dimensional chain. The role of the previous parameter \( \epsilon \) is now played by \( 1/q \).

This discrete dynamics was used in particular for the Sznajd model [11] where, however, opinion \( S_i \) is not influenced by its neighbours (as in Ising models [1, 3]), but instead it influence them. Thus in a simple version, one site \( i \) forces its opinion \( S_i \) onto all those nearest neighbours on the lattice which have the opinion \( S_i \pm 1 \). Information thus flows from inside out, instead of the usual information flow from outside inwards. The people are now more like missionaries trying to convince others, not negotiators looking for a compromise or opportunists accepting the majority opinion of their neighbours (voter models).

Most of the research on Sznajd models uses, however, the original principle of “united we stand, divided we fall”. Then two neighbours with the same opinion convince their six neighbours on the square lattice. If the two people in the middle have divided opinions, they do not convince anybody. Similarly, the children obey perhaps their parents of both parents agree with each other; if mother says something different from father, the children are more free to do what they want. This model, which has also been simulated in one and three dimensions as well as on the triangular lattice, always leads to a consensus when \( q = 2 \), i.e. when all opinions are similar to each other in the sense of the above \( S \pm 1 \) rule. (If we allow \( q > 2 \) and relax the bounded confidence restriction \( S \pm 1 \) by letting a pair convince all neighbours irrespective of the difference in opinion, then again always a consensus is found.) Since the Sznajd model was reviewed in [16] we now summarize mainly the more recent results.

The difference between simultaneous and random sequential updating of opinions [13, 12] is quite crucial for the Sznajd model [21]. If simultaneously two pairs of neighbours tell me that I should vote the way they want, and these two pairs have different opinions, then I am frustrated and do not change my opinion. This frustration effect makes a consensus very difficult, one needs a very large initial majority for one opinion to find a consensus for this opinion (\( q = 2 \), square lattice). Thus formal committee meetings have less chance of success than informal encounters spread over a longer time interval.
What if the Sznajd agents diffuse slowly on a half-filled square lattice under bounded confidence, i.e. $S$ convinces only $S \pm 1$, with $q > 2$ opinions. For $q = 5$, usually most people at the end will have adopted the centrist opinion 3, some the extremist opinions 1 and 5, but opinions 2 and 4 have died out completely. More interesting are $q = 4$ opinions: One of the more centrist opinions, like 2, wins over nearly everybody at the end, after having eaten up all neighbouring opinions 1 and 3. A small opposition of opinion 4 usually remains left. Thus if you want to stay with the winner, observe the evolution of votes: The one who is leading half-way through the race will get most votes at the end, the one who is on second place at half-time will get nothing, while the third-ranked opinion has a small set of followers at the end. The fourth-ranked opinion dies out soon. Fig. 2 shows two typical examples on a $101 \times 101$ square lattice, with 4 and 5 opinions. (For clarity the opinions in the $q = 4$ case are plotted not at 1, 2, 3, 4 but at 1.5, 2.5, 3.5 and 4.5; the 39 votes at 4.5 are barely visible on this scale.)

Figure 2: Final distribution of votes for $q = 4$ (solid line, $x$) and $q = 5$ (dashed line, $+$) on a half-filled $101 \times 101$ Sznajd lattice. For $q = 4$ the opinions are shifted to the right by 1/2 for clarity.

Thus far opinions were exchanged only among neighbours. There are, however, also mass media which try to influence us through advertising. This can be simulated [22] by assuming
for $q = 2$ that at every iteration after the exchange with neighbours every person is flipped into opinion 1, if opinion 1 is the one which advertises. Then for large lattices already a small amount of advertising is sufficient to convince the whole square lattice, if initially the two opinions were shared equally often.

Physicists like to have a Hamiltonian (usually meaning just energy) for their models; for Sznajd this was achieved only in one dimension [23]. Physicists also are accustomed that long range forces proportional to $1/\text{distance}^2$ facilitate phase transitions. The Sznajd model with interactions only between nearest neighbours on the square lattice has such a phase transition for $q = 2$: That opinion which initially has a tiny majority, at the end gets all votes. The introduction of long-range forces [24] does not change this; in fact, $z = 1$ seems to make the transition less pronounced than $z = 4$.

**Summary:**

In the above examples, the whole human being is reduced to a simple number, which represents her opinion (consensus models) or his history (hierarchies). This is, of course, a great simplification, and cognitive scientists may dislike it. Actually, more complicated models of human behaviour have been simulated extensively as neural networks, and one could apply these neural network techniques onto the above models: Does a person recognize before a fight the enemy through associative memory? How much external influence is needed to move from one intrinsic fixed point of the neural network to another and thus to change opinion (consensus models). The above models ignore these details just as Kepler’s laws how the Earth circles the Sun ignore the whole structure of the Earth. Clearly, the Earth is not point-like, Kepler knew that, and geographers endanger their employment if they treat the Earth as a point mass. But for the purpose of describing celestian motions, the model of a point-like Earth is good and lead to the development of theoretical mechanics by Newton and others later.

Humans, in contrast to the Earth, should have intelligence and thus the above analogy with the Earth may be inappropriate. We make our own decision whether we smoke, drink beer, or go on a diet; and all these decisions may influence our health and age of death. Clearly, we do not make these decisions randomly. Nevertheless, averaged over a large population, experts have constructed life tables, which entered into health and life insurance business and seem to work reasonably even though they assume humans as dying randomly. The first life tables were constructed centuries ago by Halley, for whom a famous ”comet” is named. In a similar sense, the above computer simulations may give us information on averages over many people, but not the fate of one specific person. For example, the Sznajd model was used to simulate the distribution of votes among candidates in Brazilian elections in general, but could not predict how many votes one named candidate in one specific election got.

I hope this small selection of examples encourages the readers to enter this field and to invent their own simulation models. Thanks are due to A. Maksymowicz and K. Kulalkowski of AGH in Kraków, Poland for hospitality when this review was drafted there.
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