Decision making based on a bicriteria approach taking into account the stochasticity of criteria functions

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Abstract. Determining the rational parameters of the structure and technology of railway transport facilities activity is one of the most important tasks in the field of operational management. Analysis of existing regulatory documents and literature sources has shown that the choice of technical and technological parameters of railway stations is made with insufficient accuracy and may lead to incorrect decisions. In order to reduce the negative consequences and the risk of choosing ineffective alternatives, a method for refining the Pareto set is proposed based on the stochasticity of criterion functions caused by the use of simulation. To confirm the effectiveness of the proposed method, the paper presents the results of choosing the most rational option for improving the processing capacity of an extracurricular railway station.

1. Introduction

Nowadays, a problem of economic efficiency enhancement of railway transport enterprises is becoming more and more relevant. The solution of this problem is closely connected with the determination of correct parameters of station facilities with the impermissibility of excessive and insufficient capacity. Inaccuracies in calculations, in their turn, can lead to huge economic losses from the excess or insufficiency of the capacity of railway transport enterprises [1,2].

However, despite the risk of economic losses, the choice of parameters of structural elements and operation technology of transport facilities is made with insufficient accuracy.

The most important issue is that the decision-making process for choosing the most effective parameters of the structure and technology of the transport facility remains unregulated. According to the Methodological recommendations for evaluating the effectiveness of investment projects: “Methods for choosing investment projects are an informal procedure, since they require simultaneous consideration of many quantitative and qualitative factors of a social and political, economic and technical nature. Therefore, the choosing of projects can not be carried out on the basis of a single – however complex – formal criterion, and requires almost non-algorithmized expert assessments”. Within the framework of Methodological recommendations on the structure of sections of investment justification and requirements for their content (including the calculation of economic efficiency) for investment projects of JSC “Russian Railways” there is an analogous formulation: “The option of an investment project that has the best indicators of economic efficiency can be chosen for its implementation at the investment stage. When choosing the best option for implementing an investment project from two (several) parity options, additional arguments may be used”.

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Thus, within the framework of existing regulatory documents, there is no unified algorithm for choosing an investment project among a number of alternative options. In addition to indicators of comparative economic efficiency, additional criteria can be used, but the decision-making process in multi-criteria conditions remains outside the scope of recommendations.

Therefore, the purpose of the study is to develop a methodology for more precise determination of structure elements parameters based on solution of multicriteria problem.

2. Materials and Methods

The complexity of multi-criteria choice problems lies in the fact that there is not always an option that would be simultaneously the best for all criteria at once: if one of the criteria option has the best values, then the other, as a rule, it will be far from the best. According to the Edgeworth-Pareto principle, the chosen option should be Pareto-optimal, but in most multi-criteria problems, the Pareto set is quite wide, which makes the process of choosing a rational solution difficult. For this reason, the problem of narrowing the Pareto set arises, associated with the choice of a particular Pareto-optimal variant as the most effective [3].

To date, there are many different approaches to solving this problem, which can be divided into 3 groups: 1 group assumes a choice based on a generalized criterion, 2 group - a choice with the help of an “artificial” preference relation, 3 group – interactive choice procedures [3,4].

The most traditional approach is based on the use of a generalized criterion. A special case of this approach is linear convolution of criteria, in which the choice of options is reduced to determining the coefficients of linear convolution, which are often interpreted as some “scales” (or “coefficients of importance”) appropriate criterion. This method of solving the problem of multi-criteria choice is extremely simple, but it does not stand up to criticism from the standpoint of the current level of development of the theory of decision-making. The most important drawback is that there is no precise definition of the “scales” of the criteria. Thus, linear convolution refers to approaches that may lead to far from the best final options of choice [5].

In general, instead of linear convolution of criteria, a generalized criterion is used. The maximum point of the generalized criterion on the set of solutions is declared the “best” option [4].

The methods based on the application of the generalized criterion include the procedures of the multi-criteria utility theory MAUT [6], as well as methods related to target programming [7]. All disadvantages related to linear convolution of criteria also apply to all methods of choice based on a generalized criterion.

The second group includes methods according to which the decision-maker (DM) is asked to select a particular binary relation from the set already available to researchers. After setting the preference relation, the final choice can be made on a set of non-dominant options. This group includes such widespread methods as ELECTRE, MACBETH, and PROMETHEE [7]. Despite a good understanding of the properties of the ready-made “artificial” preference relation, these methods have serious drawbacks. First, a significant disadvantage is that ready-made “artificial” binary relations rarely satisfy the DM in full. Second, pair comparison can lead to a violation of transitivity. Third, incomparability may be the most appropriate conclusion from comparing certain pairs. In this case, the method stops working [4].

The approaches included in the third group are based on an iterative procedure for searching for the “best” solution using certain information obtained from the DM at each step of the calculation. The procedure is designed in such a way that the task of searching for the most effective option is achieved by trial and error method [4]. The methods of the group under consideration suggest using very specific information for each iteration, which is not easy to get from the DM. The excessive complexity of the questions addressed to the DM leads to inaccuracies that accumulate with each step and can significantly affect the final version and lead to the adoption of far from the best solution.

Thus, all existing methods for solving multi-criteria problems have disadvantages. To obtain the most accurate and effective parameters of the structure and technology of the transport facility activity and reduce possible negative consequences and errors in decision-making, it is suggested to use a
combined approach. The approach requires first narrowing the Pareto set as much as possible, and then determining the final choice using the convolution method for the original set of criteria, but on a new, narrower Pareto set. The more we can narrow down the Pareto set, the fewer possible errors there may be.

3. Results of the Research

3.1. Formalization of the decision-making task to coordinate the parameters of the structure and technology of railway stations activity

To implement the proposed approach, a mathematical model of the decision-making situation under many criteria has been developed to determine the rational parameters of the structure and technology of railway stations activity.

\[
\langle X, Z, f, M \rangle
\]

where \( X \) – a set of possible solutions; \( Z \) – a set of vector estimates; \( f \) – a vector criterion; \( M \) – a selection mechanism.

To achieve the goal of the study, namely, to develop a methodology for more accurate calculation of the parameters of the structure and technology of road transport systems activity, it is necessary to analyze in detail the meaning of the model elements.

The task of making a decision is to choose the best parameters of the structure and technology of the transport object activity.

To implement the choice function, a set of alternatives is formed \( X \). The effectiveness of a particular variant \( x \in X \) is characterized by the values of particular criteria \( f_i \), that form the vector criterion \( f = (f_1, \ldots, f_n) \). The criterion \( f_i \) is a function defined on \( X \) and taking values from a set \( Z_i \), called the criterion scale.

The choice mechanism can be represented as a model:

\[
\langle \delta, \pi \rangle
\]

where \( \delta \) – a set of information that allows us to compare options; \( \pi \) – a selection rule.

Rational behavior of the DM is characterized by compliance with the Edgeworth-Pareto rule, so the formal method for solving the choice problem under the vector criterion is to find a set \( X_o \) of Pareto-optimal alternatives, i.e., such alternatives that do not dominate each other and are equal in this sense. The solution \( x_1 \) is preferable to a solution \( x_2 \) if two conditions are met:

\[
\forall i : [f_i(x_1) \leq f_i(x_2)] \\
\exists i_0 : [f_{i_0}(x_1) < f_{i_0}(x_2)]
\]

The dominance ratio gives preference to one alternative over the second only if the first one is better than the second one by all criteria. If the preference for at least one criterion differs from the preference for another, then such alternatives are recognized as incomparable. As a result of pairwise comparison of alternatives, the worst alternatives according to all criteria are discarded, and all remaining incomparable alternatives are accepted. If all the maximum achievable values of particular criteria do not belong to the same alternative, then the accepted alternatives form a Pareto set \( X_o \) and the choice of alternatives ends there.
3.2. Development of a method for specifying the Pareto set based on uncertainty accounting

To make effective decisions, the DM must have a comprehensive set of information, which requires a full-fledged study of the object under consideration on a simulation model [8,9], within which technical and technological indicators of its operation are determined for each \( x \in X \). In addition to the natural performance of the transport facility, the DM must take into account the economic component of the decisions made. The analysis showed that the choice of structural and technological solutions should be made on the basis of two criteria: “Delays in technological operations caused by the structure” and “Reduced construction and maintenance costs” that make up the vector criterion \( f = (f_1, f_2) \).

As already noted in the second section of the paper, in order to reduce the possible negative consequences associated with the application of criteria convolution, it is proposed, first, to narrow the Pareto set as much as possible. This requires a more detailed study of alternatives. Detailed research means taking into account the natural uncertainty of the results, which is associated with the use of simulation. In contrast to strict mathematical models, the simulation model does not provide an exact match between the input information and the result (the response of the system).

Since the relationship between alternatives and outcomes is not deterministic, it is necessary to study the dispersion characteristics of the calculation results for each of the considered alternatives in order to narrow the Pareto set. To do this, we need to calculate, construct, and evaluate the ellipses of dispersion.

The calculation algorithm consists of four consecutive stages: calculating the numerical characteristics of sample populations based on selected criteria, correlation analysis, direct construction and estimation of dispersion ellipses, and single-factor analysis of variance. Standard methods are used which can be found in [10,11], to construct dispersion ellipses and perform correlation and single-factor analysis of variance.

This algorithm takes into account the uncertainty of the results of calculating criteria, which is associated with the use of simulation modeling, allows us to narrow the Pareto set as much as possible, which significantly reduces the likelihood of making erroneous decisions.

3.3. Decision making based on the convolution of criteria

During the linear convolution of criteria, low values are compensated for one criteria, and high values are compensated for another. To get more uniform results, it is most rational to use the ideal point method, which allows us to “pull” the indicators of the criteria to the best level more evenly. To implement the method, we must set the ideal point \( X_o \), i.e. the object with the best values for all criteria. In the study of railway stations, it is advisable to use the results already obtained, so the ideal point will be the one that gives the maximum for each of the particular criteria from the set of acceptable solutions \( X \).

After determining the coordinates of the ideal point, we can apply the ideal point method, which is expressed in the formula:

\[
W_{id}(x) = \sqrt{w_i \sum_{i=1}^{n} ((f_i(x) - f_i(x_o))^2)} \rightarrow \min, \ x \in X
\]

where \( w_i \) is the weight coefficient of the importance of \( i \)-th criterion.

To determine the importance of criteria, it is proposed to use the method of direct expert assessments. For this purpose, a group of experts in the field of operational management is selected, which determines the significance of the criteria.

Thus, the ideal point method allows us to achieve more uniform results compared to other methods of convolution of criteria, and the expert evaluation method provides relevant information about the importance of the criteria under consideration.
4. Discussing the Results
To test the Pareto set refinement method and demonstrate the operation of the proposed algorithm, a large number of experiments were conducted in the simulation system on the model of the Osentsy railway station. In order to achieve the research goal, we considered the situation with an increase in processing volumes up to 850 vans per day.

To increase the processing capacity of the station, a set of alternative structural and technological solutions has been formed, consisting of 11 options (table 1).

Table 1. Values of sample averages for the criteria “Delays in technological operations caused by the structure” and “Reduced construction and operating costs”.

| No of options | Delays in technological operations caused by the structure, h. | Reduces construction and operating expenses, million rubles. |
|---------------|------------------------------------------------------------|----------------------------------------------------------|
| 1             | 69.72                                                      | 753.164                                                  |
| 2             | 74.36                                                      | 777.791                                                  |
| 3             | 66.42                                                      | 890.593                                                  |
| 4             | 89.83                                                      | 610.803                                                  |
| 5             | 67.94                                                      | 792.853                                                  |
| 6             | 64.67                                                      | 816.580                                                  |
| 7             | 76.37                                                      | 754.1721                                                 |
| 8             | 61.78                                                      | 918.217                                                  |
| 9             | 86.14                                                      | 634.440                                                  |
| 10            | 64.56                                                      | 840.247                                                  |
| 11            | 68.90                                                      | 815.126                                                  |

To choose the most rational solution, the Pareto set is formed. As a result of pairwise comparison of alternatives, options No 2, No 3, No 7, and No 11 turned out to be ineffective according to Pareto, and therefore they are excluded from further research. Thus, the set of non-dominated solutions \( X_o \) includes alternatives No 1, No 4, No 5, No 6, No 8, No 9 and No 10.

At the next stage, the generated Pareto set was refined using the ellipses of dispersion (table 2) and single-factor analysis of variance.

Table 2. The parameters of the ellipses of dispersion.

| No of option | Angle of inclination | \( \sigma_1 \) | \( \sigma_2 \) | \( \chi \sigma_1 \) | \( \chi \sigma_2 \) |
|--------------|----------------------|----------------|----------------|---------------------|---------------------|
| 6            | -18.12               | 0.66           | 1.49           | 1.61                | 3.64                |
| 9            | 0                    | 0.71           | 2.55           | 1.75                | 6.25                |
| 10           | -20.43               | 0.74           | 2.05           | 1.81                | 5.01                |
| 11           | -25.38               | 0.62           | 1.49           | 1.51                | 3.65                |
| 13           | -18.51               | 0.59           | 1.64           | 1.45                | 4.01                |
| 15           | 0                    | 0.93           | 1.75           | 2.28                | 4.29                |
| 16           | -21.56               | 1.11           | 1.84           | 2.71                | 4.52                |

Since the ellipses of dispersion do not intersect, the considered alternatives differ significantly and must be considered as substantially different variants. However, the confidence intervals for the mean square deviation according to the criterion “Delays in technological operations caused by structure” for alternatives No 4 and No 9 have common parts. The situation is similar with alternatives No 1, No 5, No 6, and No 10, so it is advisable to test the hypothesis of equality of averages in the group. The results of a single-factor analysis of variance showed that the sample means for the criterion “Delays in technological operations caused by the structure” differ slightly in both cases under consideration, and it is possible to compare alternatives for the criterion “Reduced construction and operating costs”.
As a result of using the developed methodology, it was determined that the variants No 5, No 6, No 9, No 10 are ineffective.

Despite the significant narrowing of the Pareto set, the analysis of the uncertainty of the criteria values did not allow us to determine the only effective option, which requires the convolution of the criteria. To select the most rational option for increasing the processing capacity of a railway station, we use the ideal point method. For its application, the weight coefficients for each of the selected criteria are determined, and these criteria are reduced to a single dimension and scale. The results of the criteria convolution on the initial Pareto set and using the Pareto set refinement method are shown in tables 3 and 4.

Table 3. The results of convolution of criteria in the initial Pareto set.

| No of variant | 1  | 4  | 5  | 6  | 8  | 9  | 10 |
|---------------|----|----|----|----|----|----|----|
| $W_{id}$      | 0.105 | 0.280 | 0.107 | 0.103 | 0.148 | 0.244 | 0.114 |

Table 4. The results of convolution of criteria using the Pareto set refinement method.

| No of variant | 1  | 4  | 8  |
|---------------|----|----|----|
| $W_{id}$      | 0.105 | 0.280 | 0.148 |

Based on the results presented in the tables, the proposed method for refinement of the Pareto set allowed avoiding making an incorrect decision (table 3 – option No 6), which would have been made when applying the convolution of criteria on the initial Pareto set.

Thus, an additional narrowing of the Pareto set qualitatively increases the efficiency of decisions made.

5. Conclusions

Thus, a combined approach to decision-making related to the choice of technical and technological solutions at railway stations, namely the application of the proposed method for the refinement of the Pareto set in addition to the method of convolution of the criteria allowed us to further explore alternative solutions and to narrow down the Pareto set non-dominated alternatives, thus reducing the risk of choice an inefficient solution.

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