Algorithm for calculating flat-loaded shells of environmental structures

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Abstract. Structures made of thin-walled shells are widely distributed elements in the environmental protection systems. In this connection, it is necessary to develop their effective and refined calculation. The point of the shell of the environmental protection structures is considered with a plane load in the initial position, deformed after j loading steps (displacement vector) and adjacent after (j + 1) loading step. The displacement increments and their derivatives are taken as nodal unknowns. The displacement vectors of the inner point of a finite element are represented in the initial basis, and their components are approximated through nodal unknowns using Hermite polynomials of the 3rd degree. To determine the deformed state of the shell, the algorithm of the method of discrete continuation with respect to the parameter in the vicinity of a singular point is used, in which the increment of loading at the step is the desired parameter. Based on the obtained dependencies, a step-by-step calculation procedure is organized, which allows you to get correct results if there is a special point.

1. Introduction

There is a range of problems in the mechanics of a rigid deformable body that can be reduced to solving algebraic, differential or integral equations that contain an explicit parameter. This can be a temperature parameter, a load parameter, a geometric parameter, a design parameter, etc. [1]

The problem that occurs in geometrically nonlinear systems is already found in the simplest two-rod Mises truss, and consists in the fact that the analytical form of the solution can be obtained only in individual cases, if the ideal rods are considered and their longitudinal bending is not taken into account. If you complicate the problem by adding an additional condition, then it is not possible to solve the resulting system of equations analytically, and numerical methods are used to obtain the result.

The complexity of the problem increases when solving problems for determining the stress-strain state of structures in the form of arches and shells. In solving such problems, the finite element method shows itself well [2, 3, 4, 5], which is shown in a number of papers [6, 7, 8] regarding the determination of the stress-strain state of shells.

The method of continuation of the solution by the parameter is actively used together with numerical methods in scientific research in determining the stress-strain state of geometrically nonlinear systems.

In [9], the method of solving the continuous parameter solution in combination with the original variation of the finite-difference method and the nonlinear spectral problem is used to solve the geometrically nonlinear stability problem of a composite multilayer plate.

Flat shells are also proposed to be calculated using an algorithm based on the Bubnov-Galerkin
method and the method of continuation of the solution with respect to the parameter [10], while the results show good convergence, including with the solutions of other authors.

The results of studies of the stress-strain state of shells based on the Ritz method and the method of continuation by the best parameter are given in [11], where the authors use examples in comparison with the results of other researchers to prove the workability of the proposed method, and analyze the deformation of a square shell with a double curvature.

This article is devoted to the development of a geometrically nonlinear prismatic finite element of a shell geometry 2.

2. Materials and methods

Shell geometry. The position of an arbitrary point \( M' \) on the median surface of the shell is determined by the radius-vector \( \vec{R}' \)

\[
\vec{R}' = x\hat{i} + zk, 
\]

(1)

where \( x, z \) are coordinates of the Cartesian system; \( \hat{i}, \hat{k} \) are orthonormal vectors of the same basis.

The unit vectors of the local basis of an arbitrary point on the median surface are defined by the expressions

\[
\vec{g}'_s = \vec{R}'_s = x_s\hat{i} + z_s\hat{k},
\]

\[
\vec{g}'_j = \vec{g}'_s \times \vec{j} = -\vec{t}_s + 2k x_s. 
\]

(2)

The derivatives of the vectors of the local basis are determined by the differentiation (2)

\[
\vec{g}'_{s,s} = \vec{R}'_{s,s} = x_{s,s}\hat{i} + z_{s,s}\hat{k},
\]

\[
\vec{g}'_{s,j} = -\vec{t}_{s,s} + \hat{k}x_{s,s}. 
\]

(3)

Using (2), the derivatives of vectors of a local basis can be represented by a decomposition over vectors of the same basis

\[
\begin{bmatrix}
\vec{g}'_{s}\n
\end{bmatrix}_{2 \times 1} = \begin{bmatrix} l \end{bmatrix}_{2 \times 1} \begin{bmatrix}
\vec{g}'_{s}\n
\end{bmatrix}_{2 \times 1},
\]

(4)

where \( \begin{bmatrix} \vec{g}'_{s}\n
\end{bmatrix}_{2 \times 1} = \begin{bmatrix} \vec{g}'_{s,1}\vec{g}'_{s,3}\n
\end{bmatrix}; \)

\[
\begin{bmatrix}
\vec{g}'_{s}\n
\end{bmatrix}_{2 \times 1} = \begin{bmatrix}
\vec{g}'_{s,1}\vec{g}'_{s,3}\n
\end{bmatrix};
\]

\[
\begin{bmatrix}
\vec{g}'_{s}\n
\end{bmatrix}_{2 \times 1} = \begin{bmatrix} x_{s,s}x_s + z_{s,s}z_s \quad -x_{s,s}z_s + z_{s,s}x_s \quad -z_{s,s}x_s + z_{s,s}z_s \quad z_{s,s}x_s + x_{s,s}z_s\n
\end{bmatrix}. 
\]

The position of an arbitrary point of the shell \( M' \) located at a distance \( t \) from the median surface is determined by the radius vector

\[
\vec{R}' = \vec{R}'_0 + t\vec{g}'_3. 
\]

(5)

The vectors of the local basis of a point \( M' \) are determined by the differentiation (5)

\[
\vec{g}'_{s,0} = \vec{R}'_{s,0} + t\vec{g}'_{3,0} = \vec{g}'_{s,0} + t(1 + tl_{21}) + tl_{22}\vec{g}'_{3,0},
\]

\[
\vec{g}'_{s,3} = \vec{R}'_{s,3} = \vec{g}'_{3,0}. 
\]

(6)

In step loading, there are three positions of an arbitrary point of the shell are distinguished: \( M' = \) initial, \( M' = \) after \( j \) loading steps, \( M' = \) after the \( (j + 1) \) loading step.

The displacement vector \( \vec{W} \) of the point \( M' \) after \( j \) loading steps and the displacement vector \( \vec{w} \) after the \( (j + 1) \) loading step are determined by the components in the basis of the point \( M' \)

\[
\vec{W} = W_1\vec{g}'_1 + W_3\vec{g}'_3,
\]

\[
\vec{w} = w_1\vec{g}'_1 + w_3\vec{g}'_3. 
\]

(7)

Their derivatives in the basis of the point \( M' \) are determined by differentiating (7) taking into account

\[
\vec{W}_s = \vec{g}'_{s,0}W_1 + W_1l_{11} + W_1l_{21} + \vec{g}'_{3,0}(W_1l_{12} + W_1l_{22} + W_3l_{22}) = f_1\vec{g}'_1 + f_1\vec{g}'_3;
\]

\[
\vec{W}_t = W_1'\vec{g}'_1 + W_3\vec{g}'_3; 
\]

\[
\vec{w}_s = \vec{g}'_{s,0}W_1 + W_1l_{11} + W_1l_{21} + \vec{g}'_{3,0}(W_1l_{12} + W_3l_{22} + W_3l_{22}) = a_1\vec{g}'_1 + a_1\vec{g}'_3;
\]

\[
\vec{w}_t = W_1\vec{g}'_1 + W_3\vec{g}'_3. 
\]

(8)

2

(9)
The positions of points \( M' \) and \( M'^r \) are determined by the radius vectors
\[
R^t = R^{0t} + \bar{W} ;
\]
\[
R^{t'} = \bar{R}^t + \bar{w}.
\]
(10)
The vectors of their local bases are equal to the derivatives (10)
\[
\bar{G}^0 = \bar{R}^t = \bar{g}^0 + \bar{W}_s ;
\]
\[
\bar{G}^0 = \bar{g}^0 + \bar{W}_t .
\]
(11)
\[
\bar{G}^1 = \bar{R}^t = \bar{G}_1 + \bar{w}_s ;
\]
\[
\bar{G}^1 = \bar{G}_1 + \bar{w}_t .
\]
(12)
Deformations and stresses. Total deformations after \( j \) loading steps are determined by the relations of continuum mechanics [12]
\[
\bar{\epsilon}_{ij} = \frac{1}{2} \left( \bar{g}_{ij} - \bar{g}_{ij}^0 \right) = \frac{1}{2} \left[ \bar{c}_{ij} \bar{w}_s + \bar{c}_{ij} \bar{w}_t + \bar{w}_s \bar{w}_t \right] ,
\]
(13)
where the Greek indices \( \alpha, \beta \) take the values 1 and 3 (for a value of 1 \( \equiv s \), and for a value of 3 – \( \alpha \equiv t \).
Given (11-12) and (8) from (13), we can obtain the expressions
\[
\bar{\epsilon}_{ij} = \frac{1}{2} \left( \bar{g}_{ij} - \bar{g}_{ij}^0 \right) = \frac{1}{2} \left[ \bar{c}_{ij} \bar{w}_s + \bar{c}_{ij} \bar{w}_t + \bar{w}_s \bar{w}_t \right] .
\]
(14)
Deformations at the \( (j + 1) \) loading step are determined by the expressions
\[
\Delta \bar{\epsilon}_{ij} = \frac{1}{2} \left( \bar{g}_{ij} - \bar{g}_{ij}^0 \right) = \frac{1}{2} \left[ \bar{c}_{ij} \bar{w}_s + \bar{c}_{ij} \bar{w}_t + \bar{w}_s \bar{w}_t \right] .
\]
(15)
Using expressions (11 – 12) and (9), we obtain
\[
\Delta \bar{\epsilon}_{11} = c_1 w_1^2 + c_2 w_s + w(c_1 l_{11} + c_2 l_{12}) + w(c_1 l_{21} + c_2 l_{22}) + \frac{1}{2} w_s \bar{w}_s ;
\]
\[
\Delta \bar{\epsilon}_{33} = w_1^2 W_1^1 + w_t (1 + W_1^1) + \frac{1}{2} w_t \bar{w}_t ;
\]
\[
2 \Delta \bar{\epsilon}_{13} = w_1^1 c_1 + w_1 w_2 (1 + W_1^1) + + w_1^1 l_{11} W_1^1 + l_{12} (1 + W_1^1) + \frac{1}{2} w_s \bar{w}_s .
\]
(16)
Nonlinear terms at the loading step can be ignored, and relations (16) can be represented in matrix form
\[
\{ \Delta \bar{\epsilon} \} = [L] \{ \bar{w} \}, \quad \{ \Delta \bar{\epsilon} \} = [L] \{ \bar{w} \}.
\]
(17)
where \( \{ \Delta \bar{\epsilon} \}^T = \{ \Delta \bar{\epsilon}_{11} \Delta \bar{\epsilon}_{33} 2 \Delta \bar{\epsilon}_{13} \} ; \{ w \}^T = \{ w^1 w \} ; [L] \) is a matrix of algebraic and differential operators.
The relations between stresses and deformations are determined by the expressions [12, 13]
\[
\sigma^{ij} = \lambda \bar{\epsilon}_{ij} G^{ij} + 2 \mu G^{im} G^{jn} \bar{\epsilon}_{mn},
\]
(18)
where \( \lambda, \mu \) are the Lame parameters.
For an flat stress state, the following relations are obtained from (18):
\[
\bar{\epsilon}_{11} = \sigma^{11} \frac{1}{E} G_{11} G_{11} + \sigma_{33} \left[ 1 + \frac{\nu}{E} G_{13} G_{13} - \frac{\nu}{E} G_{33} G_{33} \right] + \sigma^{13} \frac{2}{E} G_{11} G_{13} ;
\]
\[
\bar{\epsilon}_{33} = \sigma^{11} \left[ \frac{1 + \nu}{E} G_{31} G_{31} - \frac{\nu}{E} G_{11} G_{33} \right] + \sigma^{13} \frac{1}{E} G_{33} G_{33} + \sigma^{33} \frac{1}{E} G_{13} G_{33} ;
\]
\[
2 \bar{\epsilon}_{13} = \sigma^{11} \frac{1}{E} G_{11} G_{31} + \sigma^{33} \frac{1}{E} G_{13} G_{33} + \sigma^{13} \frac{1}{E} G_{11} G_{33} .
\]
(19)
\[ +2\sigma^{13} \left[ \frac{1 + \nu}{E} G_{11} G_{33} - \frac{\nu}{E} G_{13} G_{13} \right], \]

where \( E \) is the elastic modulus of the material, and \( \nu \) is the Poisson’s ratio.

Relations (19) can be represented as
\[
\begin{align*}
\{ \epsilon \} &= [D] \{ \sigma \}, \\
\{ \epsilon \} &= [D] \{ \sigma \},
\end{align*}
\]
where \( \{ \epsilon \}^T = \{ \epsilon_{11}, \epsilon_{22}, 2\epsilon_{12} \}; \{ \sigma \}^T = \{ \sigma_{11}, \sigma_{22}, \sigma_{12} \} \).

The stresses for \( j \) loading steps and the stress increments at the \( (j+1) \) loading step are determined by the expressions
\[
\begin{align*}
\{ \sigma \} &= [C] \{ \epsilon \}; \{ \Delta \sigma \} = [C] \{ \Delta \epsilon \}, \\
\{ \sigma \} &= [C] \{ \epsilon \}; \{ \Delta \sigma \} = [C] \{ \Delta \epsilon \}.
\end{align*}
\]
(21)

where \([C] = [D]^{-1}\).

Finite element stiffness matrix at the \((j+1)\)-th loading step. We consider a three-dimensional finite element that has a XOZ cross-section in the XOZ plane in the form of an arbitrary quadrilateral with nodes \( i, j, k, \) and \( l \). The displacements and their first derivatives with respect to the \( S \) and \( t \) coordinates are chosen as nodal unknowns.

To perform numerical integration, we use a square with local coordinates \( \xi_1, \xi_2 \), varying within \(-1 \leq \xi_1, \xi_2 \leq 1\). Global coordinates \( S, t \) are related to their nodal values by bilinear functions of coordinates \( \xi_1, \xi_2 \)
\[
\lambda = \{ \phi \}^T \{ \xi_1, \xi_2 \}^T \{ \lambda_y \}, \quad \lambda = \{ \phi \} \{ \xi_1, \xi_2 \}^T \{ \lambda_y \},
\]
(22)

where \( \lambda \) is coordinates \( S, t; \{ \lambda_y \} = \{ \lambda^1 \lambda^2 \lambda^3 \lambda^4 \} - \) a string of node coordinate values \( \lambda \).

Differentation of (22) are determined by the derivatives of the global coordinates in the local system \( S_{\xi_1, S_{\xi_2}, T_{\xi_1}, T_{\xi_2}} \) and the local coordinates in the global system \( S_{\xi_1}, \xi_2, T_{\xi_1}, T_{\xi_2} \).

The vectors of nodal unknowns of a finite element in the local and global coordinate systems have the form
\[
\begin{align*}
\{ v_{\text{local}}^T \} &= \begin{bmatrix}
\{ w_{\xi_1}^T \} & \{ w_{\xi_2}^T \}
\end{bmatrix}^T, \\
\{ v_{\text{global}}^T \} &= \begin{bmatrix}
\{ w_1^T \} & \{ w_2^T \}
\end{bmatrix}^T
\end{align*}
\]
(23)

where
\[
\begin{align*}
\{ w_{\xi_1}^T \} &= \{ w^{11}_{i,j} w^{11}_{k,l} \}, \\
\{ w_{\xi_2}^T \} &= \{ w^{1j}_{i,j} w^{1j}_{k,l} \}.
\end{align*}
\]
(24)

The vectors \( \{ v_{\text{local}}^T \} \) and \( \{ v_{\text{global}}^T \} \) are written similarly to (24).

Based on (22), the matrix relation can be written between the vectors (23)
\[
\begin{bmatrix}
\{ v_{\text{local}}^T \} \\
\{ v_{\text{global}}^T \}
\end{bmatrix} = [R] \begin{bmatrix}
\{ R \} \{ w_{12}^T \}
\end{bmatrix}.
\]
(25)

Movements of the internal point of a finite element are approximated through nodal values by the relations
\[
\begin{align*}
\{ w^1 \} &= \{ \psi(\xi_1, \xi_2) \}^T \{ v_{\text{local}}^T \}; \\
\{ w^2 \} &= \{ \psi(\xi_1, \xi_2) \}^T \{ v_{\text{global}}^T \}; \\
\{ w^3 \} &= \{ \psi(\xi_1, \xi_2) \}^T \{ w_{\xi_1}^T \}; \\
\{ w^4 \} &= \{ \psi(\xi_1, \xi_2) \}^T \{ w_{\xi_2}^T \};
\end{align*}
\]
(26)

where the elements of a function of the form \( \psi \) are Hermite polynomials of the 3rd degree.
The relation (26) can be represented in matrix form
\[
\{v\} = \begin{bmatrix} w^1 \\ w \end{bmatrix} = \begin{bmatrix} A \\ v_1^{loc} \end{bmatrix};
\]
(27)

Using (26) of strain (17) at the loading step, we write in matrix form
\[
\{\Delta \varepsilon\} = \begin{bmatrix} L \\ A \end{bmatrix} \{v_1^{loc}\} = \begin{bmatrix} B \\ v_y^{loc} \end{bmatrix};
\]
(28)

To form the stiffness matrix at the loading step, the Lagrange functional is used, which expresses the equality of the work of external and internal forces at the loading step
\[
\Phi_L = \int_\gamma \left[ \{\sigma\}_T + \{\Delta \sigma\}_T \right] \{\Delta \varepsilon\} \, dV = \int_S \left[ \{v\}_T \left[ \{q\} + \{\Delta q\} \right] \right] \, dS = 0,
\]
(29)

where \(\{q\} = \{q = q_1, q_2\}; \{\Delta q\} = \{\Delta q_1, \Delta q_2\} - \) vectors of full and step loads.

Using relations (20), (25), and (28), equality (29) can be represented as
\[
\begin{bmatrix} K \end{bmatrix} \begin{bmatrix} v_y^{glob} \end{bmatrix} = \{\Delta f\} + \{R\},
\]
(30)

where \([K]\) = \(\int_\gamma [B]^T [C] [B] \, dV\) - stiffness matrix of the finite element in the \((j + 1)\)-th step of loading; 
\(\{\Delta f\} = \int_\gamma [A]^T \{\Delta q\} \, dS\) is the vector of nodal loads; 
\(\{R\} = \int_S [A]^T \{q\} \, dS \int_\gamma [B]^T \{\sigma\} \, dV\) - residual errors in the \((j + 1)\) step of loading.

3. Results and discussion

At the loading step, the stiffness matrix of a separate finite element and the vector of its nodal forces are formed. If a special point appears in the process of loading the structure during deformation, then the developed algorithm can be used in this sequence. Based on (30), the stiffness matrix of the calculated shell is formed at the loading step. Since in the calculations in the presence of a singular point, the increment of the external load parameter is an unknown quantity, the system of equations of the calculated shell will consist of \(m\) equations with \((m + 1)\) the number of unknowns and can be represented as
\[
[K_0]^{j+1} \{\Delta M_y\}^{j+1} = \Delta \lambda^{j+1} \{F\} + \{R^*\},
\]
(31)

where \(\{\Delta M_y\}^{j+1}\) - increments of nodal unknowns at the \((j + 1)\) loading step; \(\{F\}\) - nodal force vector; \(\Delta \lambda^{j+1}\) - increment of the external load parameter at the \((j + 1)\) loading step; \(\{R^*\}\) - discrepancy in the values of the external and internal forces for \(j\) loading steps.

The vectors of nodal unknowns in the space \(R_{m+1}\) at the \(j\)-th and \((j + 1)\) loading steps can be written by the expressions
\[
\begin{bmatrix} \Delta \lambda \end{bmatrix}^T = \begin{bmatrix} \{\Delta M_y\}^T \Delta \lambda \end{bmatrix};
\]
(32)

\[
\begin{bmatrix} \Delta \lambda^{j+1} \end{bmatrix}^T = \begin{bmatrix} \{\Delta M_y^{j+1}\}^T \Delta \lambda^{j+1} \end{bmatrix}.
\]

The components \(\{\Delta M_y\}\) are deferred in the \(m\)-directions, and the components \(\Delta \lambda^j\) are deferred in the \((m + 1)\)-th direction [1]

At the \((j + 1)\) step of loading, the vector of increments of unknowns \(\{\Delta \lambda^{j+1}\}\) is represented in the following form
\[
\{\Delta X^{j+1}\}_{1 \times (m+1)}^T = T \left\{ \{\Delta X^{j}\}_{1 \times (m+1)}^T + \{\Delta Z^{j+1}\}_{1 \times (m+1)}^T \right\},
\]

where the new vector of increments of unknowns has the form
\[
\{\Delta Z^{j+1}\}_{1 \times (m+1)}^T = \begin{pmatrix} \{\Delta Z_{w}^{j+1}\}_{1 \times m}^T \, \{\Delta Z_{\lambda}^{j+1}\}_{1 \times 1}^T \end{pmatrix},
\]

\(T\) is the assigned numeric parameter.

The components \(\{\Delta Z_{w}^{j+1}\}_{1 \times m}^T\) are shifted in the \(m\)-directions of the \(Rm + 1\) space \(R_{m+1}\), and the component \(\Delta Z_{\lambda}^{j+1}\) in the \((m + 1)\) direction.

From the comparison of (32) and (33), we can write the expressions
\[
\begin{align*}
\{\Delta M_{y}^{j+1}\}_{m \times 1}^T &= t \{\Delta M_{y}^{j}\}_{m \times 1}^T + \{\Delta Z_{w}^{j+1}\}_{m \times 1}^T; \\
\Delta \lambda^{j+1} &= t \Delta \lambda^{j} + \Delta Z_{\lambda}^{j+1},
\end{align*}
\]

where the unknown quantities at the loading step are the vector \(\{\Delta Z_{w}^{j+1}\}_{1 \times m}^T\) and the parameter \(\Delta Z_{\lambda}^{j+1}\).

Substituting (35) into (31) we can obtain a system of \(m\)-equations with \((m + 1)\) unknowns
\[
\begin{pmatrix} K_{0} \end{pmatrix}_{m \times m} \begin{pmatrix} t \{\Delta M_{y}^{j}\}_{m \times 1}^T + \{\Delta Z_{w}^{j+1}\}_{m \times 1}^T \end{pmatrix} = \begin{pmatrix} F \end{pmatrix}_{m \times 1} (t \Delta \lambda^{j} + \Delta Z_{\lambda}^{j+1}) - \{R^{*}\}_{m \times 1}^{j}.
\]

To get an additional equation to the system (36) the condition of orthogonality of vectors is used
\[
t \{\Delta X^{j}\}_{1 \times (m+1)} \{\Delta Z^{j+1}\}_{(m+1) \times 1} = 0,
\]

which, taking into account (33) and (34), will take the form
\[
\begin{pmatrix} \{\Delta M_{y}^{j}\}_{1 \times m}^T \end{pmatrix} \begin{pmatrix} \{\Delta Z_{w}^{j+1}\}_{1 \times m}^T \end{pmatrix} + \Delta \lambda^{j} \Delta Z_{\lambda}^{j+1} = 0.
\]

Equations (36) and (38) are resolved jointly.

From (38), the value of the load parameter at the \((j + 1)\) loading step is determined
\[
\Delta Z_{\lambda}^{j+1} = -\frac{1}{\Delta \lambda^{j}} \begin{pmatrix} \{\Delta M_{y}^{j}\}_{1 \times m}^T \end{pmatrix} \begin{pmatrix} \{\Delta Z_{w}^{j+1}\}_{1 \times m}^T \end{pmatrix}.
\]

Substituting (39) into (36) yields \(m\) equations with \(m\) unknowns
\[
\begin{pmatrix} K_{0} \end{pmatrix}_{m \times m} \begin{pmatrix} \{\Delta Z_{w}^{j+1}\}_{1 \times m}^T \end{pmatrix} = \begin{pmatrix} f \end{pmatrix}_{m \times 1},
\]

where \(\begin{pmatrix} K \end{pmatrix}_{1}^{j} = [K_{0}]^{j} + \frac{1}{\Delta \lambda^{j}} \{R^{*}\}_{1}^{j} \begin{pmatrix} \{\Delta M_{y}^{j}\}_{1 \times m}^T \end{pmatrix}^{T}\) is the structural rigidity matrix; \(\begin{pmatrix} f \end{pmatrix} = \begin{pmatrix} F \end{pmatrix} t \Delta \lambda^{j} - \{R^{*}\}^{j} - [K_{0}]^{j} \begin{pmatrix} \{\Delta M_{y}^{j}\}_{1 \times m}^T \end{pmatrix}\) is the vector of nodal forces in the vicinity of the singular point.

After determining the increment vector \(\{\Delta Z_{w}^{j+1}\}_{1 \times m}^T\), the value \(\Delta Z_{\lambda}^{j+1}\) is determined from relations (39), and by the formulas (35) we find the required values \(\{M_{y}^{j+1}\}_{1 \times m}^T\) and \(\Delta \lambda^{j+1}\), which are necessary for forming the structural stiffness matrix at the next loading step.

4. Conclusion

The developed algorithm makes it possible to calculate the displacements in the vicinity of a singular point using the method of parameter propagation and on the basis of the finite element method in the form of the displacement method when solving geometrically nonlinear problems for determining the stress-strain state of axially symmetric shell structures.
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