Design of Mathematical Model of Main Working Parts of Grain Combine Harvester

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Abstract: In order to improve the working efficiency of the grain combine harvester, this paper decomposes the harvester into three parts: the power system, the walking system and the operating system. The paper determines the matching relationship between engine power and driving speed, corn harvesting speed, peeling speed and threshing speed. This paper determines the mathematical model of the main working parts by establishing a matching relationship. Based on the mathematical model, this paper designs the harvester simulation model by using Matlab and simulink software tools. In this paper, the effects of engine power simulation parameters on the speed, threshing and harvesting speed of the harvester in the mathematical model are studied. The results show that the harvester power increases, the walking speed of the harvester increases, the harvesting speed increases, the speed of corn peeling and threshing increases, and the loss of corn in threshing and clearing increases. The mathematical model of the harvester is very high and belongs to the large inertia system.

1. Introduction
In this paper, the structure of the grain combine harvester is deeply studied, and the harvester is reasonably decomposed into three parts: the power system, the walking system and the operating system. The paper determines the matching relationship between engine power and driving speed, corn harvesting speed, peeling speed and threshing speed. By studying the working principle, structure and characteristics of subsystems, this paper introduces reasonable assumptions to simplify the system structure and constructs the mathematical models of each subsystem[1]. Based on the mathematical model of the harvester, this paper designs the computer simulation model of the harvester using Matlab and simulink software tools, and simulates the transient characteristics of the combine harvester[2]. It is proved that the construction method and model structure of the mathematical model of this paper can be applied to all types of full-feeding combine harvesters, and the research on semi-feeding combine harvesters also has certain reference value[3].

2. Grain combine harvester system structure and basic working principle
In this paper, the nine-row grain combine harvester newly developed by Shandong Jin Dafeng is used. The harvesting machine's header can realize the function of automatic line-to-line adjustment. The threshing adopts the vertical axial flow roller structure, and the harvester is hydraulically driven. The structure of the harvester has a lot of novelty. The research in this paper has guiding significance for manufacturing harvesters in the same industry. The grain harvester adopts HST to realize the adjustment of walking speed, the adjustment of harvesting speed, the adjustment of corn threshing
speed, and the change of speed by adjusting the HST ratio. The V-belt transfers the power of the engine to the cleaning mechanism. Grain combine harvester system, the system is divided into three parts: power system, walking system and operating system. The operation of the travel system and the operating system relies on the power system to provide basic power and constitute the load of the power system. The combined harvest system load model is shown in figure 1.

Figure 1. Combine harvester system load model

3. Mathematical model establishment of main working parts of grain combine harvester

The grain combine harvester uses a gasoline engine as the power source, and the equation of motion of the engine indicates:

$$J_e \omega_e \frac{d \omega_e}{dt} + \frac{F_g \omega_e}{M_0} \phi = - \eta - \alpha_N$$  \hspace{1cm} (1)

$$m_n z_0 \frac{d^2 \eta}{dt^2} + f_n z_0 \frac{dn}{dt} + \frac{F_n z_0}{2F_r} \eta = \phi$$  \hspace{1cm} (2)

In the middle: \(J_e\) is the unit's moment of inertia; \(\omega_e\) is the engine output angular speed reference when the balance condition; \(M_0\) is the diesel engine main torque; \(\phi\) is the engine output angular speed relative deviation; \(\eta = \frac{\Delta Z_a}{z_0}\) is the relative deviation of the slip ring displacement \(a\); \(z_0\) is the balance condition Slip ring reference position. \(\alpha_N = \frac{\Delta M_e}{M_e}\) is the relative deviation of the negative interception outside the engine; \(m_n\) is the equivalent mass of each moving part of the centrifugal sensing element converted to the slip ring, including the moving mass of the fuel injector rod and the oil supply mechanism; \(F_r\) is the restoring force of the centrifugal element; \(f_n\) is the liquid friction coefficient of the centrifugal sensing element, including the fuel pump toothed rod and the oil supply mechanism; \(F_g\) is the static stability value of the unit; \(F_n\) is the static stability value of the centrifugal sensing element.

Take: \(T_a = \frac{J_e}{M_0}\) is the time constant of the unit acceleration, \(T_g = \frac{F_g \omega_e}{M_0}\) is the time constant of the unit speed recovery, \(T_r^2 = \frac{m_n z_0}{2F_r}\) is the time constant of the governor's centrifugal sensing element, and \(T_k = \frac{f_n z_0}{2F_r}\) is the time constant of the liquid friction.

The load speed regulation characteristics of the diesel engine obtained by Equations 1 and 2 are:

$$T_a T_r \frac{d^2 \phi}{dt^2} + \left( T_a T_k + T_g T_r^2 \right) \frac{d^2 \phi}{dt^2} + \left( T_g T_k + \delta_n T_n \right) \frac{d \phi}{dt} + \left( \delta_n T_g + 1 \right) \phi = T_r^2 \frac{d^2 \alpha_N}{dt^2} + T_k \frac{d \alpha_N}{dt} - \alpha_N \delta_n$$  \hspace{1cm} (3)
The governing model of the engine is established by using the formula 3. The formula takes the load torque as the input and the rotational speed as the output, and determines the engine speed according to the total load torque of the system such as walking and operation. In order to control the speed of the harvester, the harvester installs a current amplification version on the HST joystick, and uses the controller to output a current to control the rotation angle of the motor to realize the adjustment of the swashplate angle of the HST variable pump. Ignore the moment of inertia of the variable pump swashplate. The swashplate angle formula is:

$$\alpha = \int_0^t i_s f_e \alpha_0 dt$$  \hspace{1cm} (4)

In the formula: $i_s$ is the ratio of the joystick to the variable pump swashplate. The measured range of the joystick is about 30°, and the swashplate rotation range is approximately 90°, take $i_s = 3$; $f_e$ is the step pulse frequency; $\alpha_0$ is the step angle of each pulse into the motor. The displacement ratio of the variable pump is controlled by the angle of the swashplate, ignoring the deformation of the swashplate itself, and the displacement ratio is:

$$\varepsilon_p = \frac{tg(\alpha)}{tg(\alpha_{max})}$$  \hspace{1cm} (5)

In the formula: $\alpha_{max}$ is the maximum angle of the swashplate.

The HST model first determines the variable pump swashplate angle and its displacement ratio according to the stepper motor transmission characteristic formula, and determines the output speed of the HST according to the variable pump displacement ratio and the load torque, and combined with the input engine speed.

### 3.1 Mathematical Modeling of Hydraulic Drive of Combine Harvester

The maximum output torque of the hydraulic transmission is limited by the maximum working pressure of the system and is affected by the mechanical efficiency of the hydraulic motor. The static output torque formula is:

$$M_{HO} = \frac{(p_m - p_0)V_m \eta_m}{2\pi}$$  \hspace{1cm} (6)

In the formula: $p_m$ is the motor input pressure, MPa; $p_0$ is the motor return pressure, MPa; $\eta_m$ is the motor mechanical efficiency, %.

To facilitate modeling, you must first simplify the following important assumptions:

1. The moment of inertia of the internal gear and shaft of the transmission is much smaller than the moment of inertia of the load on the output shaft of the transmission, and its influence can be ignored.

2. The torsional stiffness of the gear and shaft is large enough, and its mechanical response speed is much faster than the response speed of the hydraulic system, and its influence on the overall response speed of the transmission can be ignored.

3. The transmission output shaft is only affected by the slowly varying load torque and the inertia load torque.

4. Correct the output shaft fixed value load torque, ignoring the influence of the internal mechanical efficiency of the transmission.

Based on the above simplified conditions, HST can establish kinematic equations:

$$V_{p_{max}} \frac{\omega_{Hi}}{i_{Hi}} \varepsilon_p = V_m i_H Z \omega_{HO} + \frac{c_s (V_{p_{max}} + V_m)}{u} \Delta P + \frac{V_H \Delta P}{\beta_e} dt$$  \hspace{1cm} (7)

### 3.2 Combined Harvester Header Walking Mathematical Model

The HST output shaft connects the wheels through the variable box and drives the vehicle to travel. The walking speed is:

$$v = 2\pi r_d \omega_{HO} i_d$$  \hspace{1cm} (8)

In the formula: $v$ is the vehicle travel speed; $r_d$ is the drive wheel radius; $i_d$ is the travel system gear train ratio.

In this paper, according to the formula, the walking speed of the harvester is determined according to the output speed and the ratio in the HST.
3.3 Combine harvester header harvesting mathematical model

According to the working process of the header, the crop harvesting speed model of the header can be obtained\(^8\). To facilitate the modeling, the following assumptions are introduced:

1. The moment of inertia of the gear and shaft of the header drive system is much smaller than that of the chain, the picking roller and the auger, and the influence can be neglected.

2. The rigidity of the header operating system is large enough, and the mechanical system does not deform during the operation.

3. The header drive system is sufficiently efficient to ignore its friction loss.

The crop harvesting speed of the header is determined by the field crop density, harvester walking speed, cutting width and cutting height. Ignore the time required for the crop to move on the header, and the amount of crop output from the header is the amount of harvest. The harvesting speed of the combine harvester is:

\[
q_c = D_c V_w
\]  \(\text{(9)}\)

In the formula: harvesting speed, \(\text{kg} / \text{s}\); field crop density, \(\text{kg} / \text{m}^2\); \(v\) is the harvester walking speed; \(w\) is the cutting width.

The walking mechanism model is based on the formula 9, and the harvesting speed of the harvester is determined according to the input speed and the ratio.

3.4 Combine harvester threshing mathematical model

The grain combine harvester uses the longitudinal axial flow for threshing\(^6\). The threshing process is a very complicated mechanical process\(^7\). Before the establishment of its mathematical model, some necessary simplifications must be made, assuming:

1. During the normal threshing process, the grain is continuously fed evenly;

2. The grain is placed in close continuous flow with the concave plate or the cover plate, ignoring the relative sliding between the grains;

3. The extracted kernel passes through the concave plate at the instant of threshing, and its speed is equal to the circumferential speed of the drum;

4. Most of the grain is released shortly after the crop enters the drum, ignoring the time required for the grain to escape.

According to the above assumption, the threshing space is equivalent to a single-input dual-output system, the input amount is the feed amount 1, the crop-grass ratio is \(\Delta\), the output 1 is the grain output, and the output 2 is the corn cob output. According to the internal nails of the drum and the movement law of the grain, a mathematical model of the threshing grain movement in the semi-concave axial flow threshing drum can be established:

\[
r^2 \frac{d^2 \theta}{dt^2} = (k_r r k_r - f_r \sin \phi_r) r \left( \frac{d \theta}{dt} \right)^2 - k_r k_r(R_t + r) \omega_r \frac{d \theta}{dt} + g(\cos \theta - f_r \sin \phi_r \sin \theta) + k_v k_v \omega_r^2
\]  \(\text{(10)}\)

\[
\frac{d^2 z}{dt^2} = (k_r k_r - f_r \sin \phi_r) r \left( \frac{d \theta}{dt} \right)^2 - k_r k_r(R_t + r) \omega_r \frac{d \theta}{dt} - f_r \cos \phi_r \sin \theta + k_v k_v R_t \omega_r^2
\]  \(\text{(11)}\)

\[
\phi_r = \tan^{-1} r \frac{d \theta}{dt}
\]  \(\text{(12)}\)

\[
r^2 \frac{d^2 \theta}{dt^2} = -rf_c \sin \beta_r \left( \frac{d \theta}{dt} \right)^2 + g (1 - k_r \cos \beta_r) \cos \theta - f_c \sin \beta_r \sin \theta
\]  \(\text{(13)}\)

\[
\frac{d^2 z}{dt^2} = -rf_c \cos \beta_r \left( \frac{d \theta}{dt} \right)^2 + g (k_r \cos \theta - f_c \sin \theta)
\]  \(\text{(14)}\)

\[
k_r = \frac{d N_r N_r}{d \theta} \left( \frac{e}{1-e} + \frac{\Delta u(\pi)}{1+\Delta} \right)
\]  \(\text{(15)}\)

\[
k_r = \cos \delta + f_r \sin \delta
\]  \(\text{(16)}\)

\[
k_r = \sin \delta - f_r \cos \delta
\]  \(\text{(17)}\)

\[
k_r = \cos \beta_r + f_r \sin \beta_r
\]  \(\text{(18)}\)

\[
k_r = \sin \beta_r - f_r \cos \beta_r
\]  \(\text{(19)}\)
In the formula: \( r, \theta, z \) is the cylindrical coordinate of the grain point in the threshing space; \( \phi \) is the helix angle of the grain moving along the concave plate; \( \beta \) is the spiraling blade lifting angle; \( R_t \) is the radius of the nail on the drum; \( \omega \) is the angular velocity of the drum; \( g \) is gravity acceleration; \( f_s, f_c \) is the friction coefficient between grain and concave plate and cover plate; \( k_{rt}, k_{r2}, k_{r3}, k_{r4} \) is the constant related to the geometrical parameters of the drum; \( \Delta \) is the ratio of grass to valley; \( \mu(z) \) is the separation rate of grain along the \( z \)-axis in the axial flow threshing space; \( e \) is the speed growth coefficient; \( 10 \) is the coefficient of friction between the grain and the nail and the spiral guide; \( f_v, f_t \) is the angle of action of the nail to the grain; \( \delta = \frac{\pi}{2} - \phi \) is the diameter of the nail; \( N_r, N_c \) is the number of spiral heads and the number of columns; \( L_p \) is the pitch of the spiral.

It can be seen that the grain dynamics model is a second-order nonlinear differential equation system. The hollow of the model is the rate of axial movement of the grain in the drum, thereby obtaining the time required for the grain to pass through the drum:

\[
\tau = \int_0^1 \frac{dt}{dz} \, dz
\]  

(20)

After time \( x \), the corn cob reaches the exit 2, and the output of the corn cob is:

\[
q_{o2} = \frac{q_{r}}{1+\Delta} \delta(t - \tau)
\]  

(21)

The crop kernel portion is detached from the stem during the axial movement of the crop and is output from the drum through the gravior screen. The output of grain per unit time is:

\[
q_{o1} = \frac{q_{r} \Delta}{1+\Delta}
\]  

(22)

In summary: the peeling speed of corn is:

\[
v_2 = \frac{q_{o2}}{\tau}
\]

The threshing speed of corn is:

\[
v_1 = \frac{q_{o1}}{\tau}
\]

4. Simulation of mathematical model of important working parts during the operation of grain combine harvester

In this paper, the Simulink simulation model of the combine harvester is used to carry out simulation experiments to study the basic characteristics of the combine harvester and to verify the usability of the simulation model\(^8\).

4.1 Harvester steady speed simulation of the driving speed

In this paper, the engine throttle is kept at the same rated position, the HST transmission ratio is adjusted, and the steady-state walking speed of the harvester under different excitations is tested. The simulation results are shown in figure 2.

Figure 2. Steady-state simulation results of the harvesting speed of the harvester

In this paper, the HST variable pump swashplate position is adjusted, and the walking speed of the harvester under different HST transmission ratio conditions is tested. The steady state is obtained.
when the system reaches a stable state for a long time, and the corresponding walking speed is read. It can be seen from the simulation results in figure 2 that the position of the swashplate is increased and the walking speed of the harvester is increased. As the walking speed increases, the feeding load of the harvester gradually increases under the condition of a certain concave crop density, causing an increase in the engine load torque and a decrease in the engine speed. Therefore, when the vehicle speed is lower, it has a linear relationship with the swashplate position. When the ratio is increased to a certain extent, the curve gradually bends downward to exhibit a certain saturation characteristic.

4.2 Steady-state simulation of the threshing drum speed

In this paper, the engine throttle valve is kept at the same rated position, the HST transmission ratio is adjusted, and the steady-state drum speed of the harvester is tested. The simulation results are shown in figure 3.

![Figure 3. Steady-state simulation results of the threshing drum rotation speed](image)

This paper adjusts the position of the HST variable pump swashplate and tests the speed of the harvester threshing drum under different HST transmission ratio conditions. It can be seen from the simulation results in Fig. 3 that the position of the swashplate is increased, and the speed of the threshing drum of the harvester is increased first and then decreased. When the position of the swashplate of the HST variable pump is 50, the rotation speed of the threshing drum is the highest. The peeling speed, the threshing speed and the rotation speed of the threshing drum are positively correlated. The peeling speed and the threshing speed are the highest at the time of the HST variable pump swashplate position at 50.

4.3 Harvest speed steady state simulation

In this paper, the engine throttle valve is kept at the same rated position, the HST transmission ratio is adjusted, and the harvesting speed of the harvester header is tested. The simulation results are shown in figure 4.
It can be seen from the simulation results in Fig. 4 that the position of the swash plate is increased, the harvesting speed of the harvester is increased, and the feeding load of the harvester is gradually increased to increase the loss rate.

Summary: From the mathematical model of the harvester and computer simulation test, the overall characteristics of the harvester have the following basic characteristics:

1. The model order is very high, belonging to the large inertia system, especially the engine and the threshing drum have higher order and larger time constant.
2. The harvester power increases, the walking speed of the harvester increases, the harvesting speed increases, the speed of corn peeling and threshing increases, and the loss of corn in threshing and clearing increases.
3. There are plurality of parameters in the system that vary with the working time, and the major influences include the total mass of the harvester and the coefficient of rubbing of the threshing drum.

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References
[1] Liu Chunya. Experimental study on the cleaning device of peanut harvester [D]. Henan University of Science and Technology, 2018.
[2] Pang Jing, Li Yaoming, X Lizhang, Ye Xiaofei, Hu Biyou. Construction method of load measurement system for crawler full feed combine harvester [A]. China Agricultural Machinery Society. 2012 China Agricultural Machinery Society International Academic Conference Proceedings [C]. China Agricultural Machinery Society: China Agricultural Machinery Society, 2012: 5.
[3] Yang Ranbing. Design and experimental study of main equipment of 4HQL-2 peanut combine harvester [D]. Shenyang Agricultural University, 2009.
[4] Wei Wei. Research on improvement of power transmission system of multifunctional agricultural work machine [D]. Jiangsu University, 2017.
[5] Qin Yun. Research on load control system of combine harvester [D]. Jiangsu University, 2012.
[6] Liu Linhe. Design and research of single vertical axial flow corn threshing test bench and its control system [D]. Jilin University, 2018.
[7] Sheng Ping. Design and simulation of 4YZ-4B corn combine harvester and construction of virtual training system [D]. Shandong University, 2017.
[8] Ye Yi, Liu Xingbo, Lin Juntang, Chang Jianguo, Chen Yong. Design of hydrostatic drive system for corn combine harvester[J]. Agricultural Machinery Usage & Maintenance, 2016(07):13-14.