Light radiation pressure upon a wrinkled membrane – parametrization of an optically orthotropic model

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Abstract. In this paper, the problem of representing the light pressure force upon the surface of a thin wrinkled film is discussed. The common source of wrinkles is the shear deformation of the membrane sample. The optical model of such a membrane is assumed to be optically orthotropic and an analytic equation for infinitesimal light pressure force is written. A linear regression model in the case of wrinkle geometry, where a surface element can have different optical parameters, is constructed and the Bayesian approach is used to calculate the parameters of this model.

1. Model of optically orthotropic surface
In this section, we describe the model of light radiation pressure upon a surface with wrinkles. We assume that the optical parameters of such a surface can be represented in as follows: there are two perpendicular directions in which the optical parameters are different.

Let us introduce the local coordinate frame $Ox_1x_2x_3$ on an infinitesimal element $dA$ of the surface whose optical parameters are assumed to be orthotropic. The unit vectors of the local coordinate frame are $\hat{e}_1, \hat{e}_2, \hat{e}_3$. Let $\hat{n}$ be a unit vector of local normal to the surface element $dA$, $\hat{n} = \hat{e}_3$. The area element $dA$ is located on the plane $Ox_1x_2$. For any vector, e.g., $\hat{r}$, we use the direction angles $(\beta, \theta)$ in the local frame as follows:

• $\beta \in [0, \pi/2]$ is the angle between the vectors $\hat{r}$ and $+x_3$.
• $\theta \in [0, 2\pi]$ is the angle between the axis $Ox_1$ and the projection of $\hat{r}$ on the plane $Ox_1x_2$, counterclockwise around $Ox_3$.

Let $\hat{s}$ be a unit vector from the light source to $dA$. In the formulas given below, $q_0$ is a light flux, $c$ is the light speed in a vacuum, $\sigma$ is the Stephan–Boltzmann constant, and $T$ is the temperature of $dA$.

Let $\epsilon'_m(\beta, \theta)$ be the directional emissivity in the orthotropic model [1]:

$$
\epsilon'_m(\beta, \theta) = [\epsilon_1 \cos^2(\theta - \theta_m) + \epsilon_2 \sin^2(\theta - \theta_m)] \cos \beta,
$$

where $\theta_m$ is an angle in the plane $Ox_1x_2$, which determines the direction in which the emissivity is equal to $\epsilon_1$. In the perpendicular direction, the emissivity is equal to $\epsilon_2$. 


The equation for the emission pressure from the orthotropic surface is given by the formula
\[ dF_s = -\frac{\epsilon_1 B_m \sigma T^4}{c} \mathbf{n}, \]  
(1)
where the modified Lambertian coefficient \( B_m \) is
\[ B_m = \frac{\epsilon_1 + \epsilon_2}{3\epsilon_1}. \]

The equation for the light pressure force of the light diffusely reflected from an orthotropic surface can be written in vector form where
\[ F = \mathbf{n}. \]  

The bidirectional reflectivity for the diffuse reflection is
\[ \rho_{1m}''(\beta, \theta, \beta^R, \theta^R) = 2\pi(1-s)[\rho_1 \cos^2(\theta_m - \theta^R) + \rho_2 \sin^2(\theta_m - \theta^R)] \cos \beta^R, \]
where \( \beta^R, \theta^R \) is the orientation vector for the reflected light flux.

The optically orthotropic model for the specularity \( s \) becomes
\[ s = s_1 \cos^2(\theta_m - \theta^R) + s_2 \sin^2(\theta_m - \theta^R). \]

The equation for the light pressure force of the light diffusely reflected from an orthotropic surface can be written in vector form
\[ dF_{R1} = \frac{q_0}{c} \rho(1-s) B_\rho (\mathbf{n} \cdot \mathbf{s}) \mathbf{n}, \]
(3)
where
\[ B_\rho = \frac{(4 - 3s_1 - s_2) \rho_1 + (4 - s_1 - 3s_2) \rho_2}{12(1 - s_1) \rho_1}. \]

The bidirectional reflectivity for the specular reflection is
\[ \rho_{2s}'' = 2\pi \delta(\beta^R - \beta_0) \delta(\theta^R - \theta_0 - \pi)[\rho_1 \cos^2(\theta_m - \theta^R) + \rho_2 \sin^2(\theta_m - \theta^R)][s_1 \cos^2(\theta_m - \theta^R) + s_2 \sin^2(\theta_m - \theta^R)]. \]

The unit vector of orientation of the reflection axes in local frame is
\[ \mathbf{m} = \cos \theta_m \mathbf{e}_1 + \sin \theta_m \mathbf{e}_2. \]

The light radiation pressure upon \( dA \) from a specularly reflected light flux is
\[ dF_{R2m} = \frac{q_0}{c} \frac{(\mathbf{n} \cdot \mathbf{s}) s - 2(\mathbf{n} \cdot \mathbf{s})^2 \mathbf{n}}{[1 - (\mathbf{n} \cdot \mathbf{s})^2]^2} \{\rho_1 (\mathbf{m} \cdot \mathbf{s})^2 + \rho_2 [(\mathbf{m} \times \mathbf{s}) \cdot \mathbf{n}]^2\} \{s_1 (\mathbf{m} \cdot \mathbf{s})^2 + s_2 [(\mathbf{m} \times \mathbf{s}) \cdot \mathbf{n}]^2\}. \]

The bidirectional reflectivity for the back reflection is
\[ \rho_{2b}'' = 2\pi \delta(\beta^R - \beta_0) \delta(\theta^R - \theta_0 - \pi)[\rho_1 \cos^2(\theta_m - \theta^R) + \rho_2 \sin^2(\theta_m - \theta^R)][s_1 \cos^2(\theta_m - \theta^R) + s_2 \sin^2(\theta_m - \theta^R)] \frac{s_1 \cos^2(\theta_m - \theta^R) + s_2 \sin^2(\theta_m - \theta^R)}{\cos \beta^R} k, \]
where \( k \) is an empirical parameter.

The light pressure from the back reflection is
\[ dF_{R2b} = \frac{q_0}{c} \frac{k \mathbf{s}}{[1 - (\mathbf{n} \cdot \mathbf{s})^2]^2} \{\rho_1 (\mathbf{m} \cdot \mathbf{s})^2 + \rho_2 [(\mathbf{m} \times \mathbf{s}) \cdot \mathbf{n}]^2\} \{s_1 (\mathbf{m} \cdot \mathbf{s})^2 + s_2 [(\mathbf{m} \times \mathbf{s}) \cdot \mathbf{n}]^2\}. \]

Table 1. Summary of the posterior distribution for different simulations for surface with waves with different optical parameters [1].

| Case          | $\rho_1$ | $\rho_2$ | $s_1$ | $s_2$ | $k$  |
|---------------|----------|----------|-------|-------|------|
| $\rho = 0.5$, $s = 0$ | 0.041    | 0.121    | 0.806 | 0.524 | 3.480 |
| $\rho = 0.5$, $s = 0.5$ | 0.057    | 0.074    | 0.218 | 0.448 | 4.450 |
| $\rho = 0.5$, $s = 1$   | 0.047    | 0.211    | 0.094 | 0.296 | 2.330 |
| $\rho = 1$, $s = 0$    | 0.104    | 0.369    | 0.677 | 0.331 | 2.440 |
| $\rho = 1$, $s = 0.5$  | 0.291    | 0.476    | 0.179 | 0.222 | 1.870 |
| $\rho = 1$, $s = 1$    | 0.749    | 0.729    | 0.803 | 0.822 | 0.186 |

2. Parametrization of optical parameters

As has already been shown by many authors, both analytically [2, 3], numerically [4–6], and experimentally [7], the surface of a wrinkled thin membrane can be represented according to the sine or cosine law. The common source of wrinkles in practical applications is the shear deformation of the membrane. Another source is the uniaxial stretching. The magnitude of wrinkles can be calculated by the formula [2]

$$x'_3 = \delta \sin \frac{2\pi x'_2}{\lambda},$$

where $\delta = \delta(x'_1, x'_2)$ is a smooth field of wrinkle amplitude over the wrinkled domain $\mathcal{D}$ and $\lambda = \lambda(x'_1, x'_2)$ is a smooth field of wrinkle wavelength over the wrinkled domain $\mathcal{D}$.

In [1], we used the following parameters of the wrinkled surface: $x'_1$, $x'_2 \in [-0.5, 0.5]$, $x'_3 = \delta[\cos(8\pi x'_2) - 1]$. There were four waves of wavelength $\lambda = \text{const} = 0.25 \text{m}$, and amplitude $\delta = \text{const} = 0.2 \text{m}$, and the global frame was shifted along $+x'_3$ by $\delta/2$. The optical parameters of this surface were uniform and could be specular, diffuse or specular-diffuse with the corresponding specularity coefficient: $\rho_0 \in \{0.5, 1\}$ is the surface reflectivity, $s_0 \in \{0, 0.5, 1\}$ is the surface specularity, $B_0 = 2/3$ is the Lambertian coefficient of the front side; there are six combinations in the whole. We calculated the light radiation pressure vector for 189 combinations of the incident light flux and obtained parameterized values for the optical parameters using the Bayesian approach by Markov Chain Monte Carlo simulation [8] based on Metropolis-Hastings sampling [9] for statistical toolkit JAGS [10] for $R$ programming language.

The summary of the posterior distribution for optical parameters is represented in table 1.

In this paper, we introduce the following linear regression model for optical parameters of initially optically homogeneous wrinkled surface:

$$\begin{align*}
\rho_1 &\sim A_1 \rho + A_2 s + A_3, \\
\rho_2 &\sim B_1 \rho + B_2 s + B_3, \\
s_1 &\sim C_1 \rho + C_2 s + C_3, \\
s_2 &\sim D_1 \rho + D_2 s + D_3, \\
k &\sim E_1 \rho + E_2 s + E_3.
\end{align*}$$

For this model, the inverse link function is a normal distribution with corresponding standard deviation $\sigma_A$, $\sigma_B$, $\sigma_C$, $\sigma_D$, and $\sigma_E$.

The prior distribution for all parameters of the model is a normal distribution with mean 0 and deviation 1. For standard deviation of the link function, the prior distribution is exponential with parameter 0.1.
Figure 1. Posterior distribution for the parameter $A_1$ of linear regression model.

Figure 2. Posterior distribution for the parameter $A_2$ of linear regression model.

Table 2. Summary of posterior distribution for linear regression model of parameters of optically orthotropic surface. Limits are 99% Higher Density Interval.

| Parameter | Value         | Note     |
|-----------|---------------|----------|
| $A_1$     | 0.523$^{+1.137}_{-1.410}$ | figure 1 |
| $A_2$     | 0.293$^{+0.947}_{-1.075}$  | figure 2 |
| $A_3$     | $-0.319^{+1.202}_{-0.971}$ | figure 3 |
| $\sigma_A$ | 0.260$^{+0.830}_{-0.179}$  | figure 4 |
| $B_1$     | 0.726$^{+0.764}_{-1.098}$  |          |
| $B_2$     | 0.217$^{+0.684}_{-0.668}$  |          |
| $B_3$     | $-0.322^{+0.908}_{-0.665}$ |          |
| $\sigma_B$ | 0.134$^{+0.541}_{-0.955}$  |          |
| $C_1$     | 0.311$^{+1.601}_{-1.091}$  |          |
| $C_2$     | $-0.216^{+1.396}_{-1.264}$ |          |
| $C_3$     | 0.324$^{+1.374}_{-1.107}$  |          |
| $\sigma_C$ | 0.443$^{+1.107}_{-0.289}$  |          |
| $D_1$     | 0.096$^{+1.374}_{-1.236}$  |          |
| $D_2$     | 0.131$^{+1.089}_{-1.181}$  |          |
| $D_3$     | 0.299$^{+1.162}_{-1.162}$  |          |
| $\sigma_D$ | 0.330$^{+0.890}_{-0.224}$  |          |
| $E_1$     | 0.424$^{+2.236}_{-2.004}$  |          |
| $E_2$     | 0.085$^{+2.346}_{-2.175}$  |          |
| $E_3$     | 1.200$^{+2.100}_{-2.240}$  |          |
| $\sigma_E$ | 2.090$^{+4.300}_{-1.407}$  |          |

We calculated the posterior distribution using the previously mentioned MCMC approximation. The summary of this approximation is represented in table 2. The examples of posterior distribution for the parameters $A_1$, $A_2$, $A_3$, and $\sigma_A$ are represented in figures 1–4, where HDI stands for the higher probability density interval, the distribution median is represented by a short vertical red line. The exact values of the probability density function are irrelevant since we use only relative probabilities and likelihoods.
Figure 3. Posterior distribution for the parameter $A_3$ of linear regression model.

Figure 4. Posterior distribution for the parameter $\sigma_A$ of linear regression model.

Conclusions

We presented an optically orthotropic model of optical parameters for the surface of thin film structure with wrinkles. The variation of wrinkle parameters leads to the variation of parameters of reflected flux. This dependence can be parametrized using linear regression for an optically orthotropic surface parameters.

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