Dynamic multi-objective optimization of battlefield support resources based on Interactive Genetic Algorithm

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Abstract. With the deterioration of the international environment and the intensification of military conflicts, the scale, type, direction and emphasis of military activities are constantly changing, which requires the corresponding adjustment of maintenance and support resources to ensure that the wartime support tasks are met in various battlefield environments. Based on the background that the support demand and battlefield situation have changed and the original resource support scheme can not meet the needs of wartime support tasks, this paper readjusts the support resources dynamically to ensure the strong support of combat forces, constructs a dynamic multi-objective optimal allocation model of battlefield support resources, formulates corresponding strategies, and presents a heuristic algorithm for solving the model.

1. Introduction
Systematic warfare under the condition of informationization puts forward high requirements for the highly coordinated use of various arms and equipment. Large-scale equipment system has greatly increased the demand for battlefield support resources. With the sudden and uncertain increase of war, battlefield environment and task requirements are constantly changing. Under the circumstance of limited support resources and capability, how to efficiently and quickly meet the needs of support resources for various types of weapons and equipment, so that equipment support departments can accurately predict the consumption of support resources, formulate more scientific support schemes, and enhance the battlefield support capability is a hot issue that needs to be solved urgently.

2. Model objectives and model assumptions
Battlefield environment is complex, there are many factors to consider and fighter planes are fleeting, so the dynamic allocation of battlefield support resources needs to consider multiple objectives. This paper is divided into three sub-objectives:

- The maximum satisfaction of support after the optimal allocation of resources.
- The shortest time to complete the optimal allocation of resources.
- The minimum number of support resources after the optimal allocation.

The hypothesis of the model is as follows:

- The location and resource allocation of each support resource point are known before deployment adjustment.
- Each safeguard resource point has different opening time according to its safeguard level, without considering the time consumption of closing the safeguard resource point.
- The support response time between the combat unit and the support resource point is determined and known.
- The transfer time of each guaranteed resource point is known.
- The time to build new support resource points at alternative sites is known.
Dynamic adjustment of the number of support resource points before and after deployment is known.

Only in a certain range of alternatives to consider the location of new security resources.

Next stage, each demand point ensures that the demand information is known.

The existing security resource points are open.

The allocation of resources and transportation capacity of existing security agencies will remain unchanged when their positions change, and there will be no adjustment of the security resources and transportation capacity among the various security agencies.

Only one safeguard resource point can be set up for each alternative site.

Dynamic Multi-objective Optimal Distribution Model

Assuming that the number of guaranteed resource points remains unchanged, the model is constructed as follows:

\[
\begin{align*}
\text{min } f_1(z) &= - \sum_{i \in I} \sum_{q \in Q} w_i z_{iq} F_i(t_{iq}) G_q(e_{iq}) \\
\text{min } f_2(z) &= \max \left\{ d_{kj} z_{kj} \right\} \\
\text{min } f_3(z) &= \sum_{k \in K} z_{kj} \\
\text{subject to} & \quad z_j' = \sum_{k \in K} z_{kj} , \quad \forall j \in J \\
& \quad \sum_{i \in I} \sum_{q \in Q} w_i z_{iq} F_i(t_{iq}) G_q(e_{iq}) \geq \epsilon^* \\
& \quad \sum_{j \in J} x_{jq} \leq N_{iq} , \quad \forall i \in I , q \in Q \\
& \quad e_{iq} = \sum_{j \in J} x_{jq} / N_{iq} , \quad \forall i \in I , q \in Q \\
& \quad \sum_{i \in I} x_{jq} \leq b_{jq} z_{jq}' , \forall j \in J , q \in Q \\
& \quad \sum_{i \in I} \theta_q x_{jq} t_{ji} \leq c_{jq}' z_{jq}' , \forall j \in J \\
& \quad y_{jq} = \text{sgn}(x_{jq}), \quad \forall i \in I , j \in J , q \in Q \\
& \quad t_{iq} = \max(t_{jq'}, y_{jq}), \quad \forall i \in I , q \in Q \\
& \quad z_j , z_j' \in [0,1] , \quad \forall j \in J \\
& \quad b_{jq}' = \sum_{k \in K} b_{ik} z_{ik} , \quad \forall j \in J , q \in Q \\
& \quad c_{jq}' = \sum_{k \in K} c_{ik} z_{ik} , \quad \forall j \in J \\
& \quad x_{jq} \in \mathbb{Z}^+ \cup \{0\} , \forall i \in I , j \in J , q \in Q \\
& \quad \sum_{j \in J} z_{jq}' = \rho' 
\end{align*}
\]

Formula (1) is to maximize the satisfaction of timely arrival of support resources to combat tasks under the new distribution scheme of support resources. Formula (2) is used to minimize the transition time of guaranteed resource distribution. Formula (3) denotes the number of guaranteed resource points.
points that minimize the adjustment position. Formula (4) indicates whether the adjusted guarantee resource point J is open or not. Formula (5) requires that the satisfaction degree of support after adjustment must meet the minimum requirement of the support system. Formula (6) constraints the supply-demand relationship of resource q at demand point i. Formula (7) is the resource satisfaction rate formula of demand point i to resource q. Formulas (8) and (9) indicate that the security resource points set up meet the resource and capacity constraints after adjustment and deployment. Formula (10) is the computational expression of the safeguard relationship. Formula (11) obtains the guaranteed response time of resource q for demand point i. Formula (12) is the 0-1 constraint of location decision variables. Formula (13) and Form (14) indicate that after adjustment of deployment, the capacity and resources of the original guaranteed resource point are transferred to a new location. Formula (15) is a non-negative integer constraint of resource allocation in the guarantee process. Formula (16) is a constructive constraint on the number of guaranteed resource points.

Aiming at the particularity of dynamic distribution of battlefield support resources, this paper adopts the interactive programming method to solve the problem[1-3]. In the process of solving the problem, it provides objective and convenient information for decision-makers to compare non-inferior solutions, so that decision-makers can make decisions clearly.

Firstly, the constraint satisfaction model is constructed by the constraint method, and the multi-objective programming model is transformed into a single-objective programming model[4-6]. Let $z = (z_{i1}, \ldots, z_{pj})$ be the decision variable to ensure the transfer of resource points in the model. $f_1(z), f_2(z), f_3(z)$ correspond to the three sub-goals of the model respectively: the adjusted guarantee satisfaction, the adjusted time and the number of guarantee resource points. Let $f_1(z)$ be the main target. Target $f_2(z), f_3(z)$ is constrained so that the two objectives are constrained to $\alpha$ and $\beta$ respectively. Adding the original constraints, the modified model is shown below.

$$
\begin{align*}
\min & \quad f_1(z) \\
\text{s. t.} & \quad f_2(z) \leq \alpha \\
& \quad f_3(z) \leq \beta \\
& \quad z \in Z
\end{align*}
$$

Z is a set of constraints consisting of original constraints.

3. Solution of multi-objective programming model based on interactive genetic algorithms

3.1. Coding strategy
Chromosomes are coded in real numbers, $x_i^t$ represents the i individual of the t generation, $i \in \{1,2,\ldots,n\}$, the number of gene digits is h, and the number of guaranteed resource points is (2 3 5 6 9). The number of guaranteed resource points is unchanged. Gene coding $x_i^t = (2,3,4,7,9)$ represents the location of guaranteed resource points at the 2,3,9 positions of candidate points, the location of guaranteed resource points at the 5 positions of candidate points is shifted to the 4 positions, and the location of guaranteed resource points at the 6 positions of candidate points is shifted to the location of guaranteed Point 7.

3.2. Fitness function
The fitness function takes the target W, and after the coding is determined, the location scheme, transportation capacity and resource allocation are determined.

3.3. Initial population
The initial population is generated by a combination of stochastic and heuristic methods. According to the formula (18), The alternative points of the adjustment time from the original guaranteed resource
point to the new allocation point within A constitute the alternative set for each gene locus adjustment. Random selection of $\gamma$ loci ($\gamma \leq \beta$) from $h$ loci. Each selected gene site is randomly selected as an alternative point in the alternative set, which ensures that all feasible solutions are generated and improves the fitness of the initial population.

### 3.4. Selection and crossover operators

In this paper, the proportional selection method is used to realize the selection operation. Single-point crossover method was used to combine parents and produce offspring. Reasonable selection of initial population will not lead to the solution of unsatisfactory formula (18) when crossing, but it may lead to unsatisfactory constraint formula (19) after crossing. For the infeasible solution, the repair strategy is adopted to transform the infeasible solution into the feasible solution. The specific method is: if the number of adjustment points exceeds $c$, $c$ loci will be randomly selected from the loci where the location changes and placed in the initial position before the adjustment. In this way, the crossed individuals can satisfy the two constraints of adjusting time and adjusting number at the same time.

### 3.5. Mutation operator

According to the mutation probability, the random variation of gene loci in the corresponding alternative set ensures that the solution satisfying formula (18) is generated, and the infeasible solution is repaired according to the repair strategy mentioned above.

Based on the above ideas, an interactive genetic algorithm based on constraint satisfaction is designed for this model, which is an interactive iterative process. The specific steps are as follows:

- **Step 1:** The improved genetic algorithm is used to solve the minimum and maximum values of each sub-target, and the minimum $z^{1*}, z^{2*}, z^{3*}$, minimum $f_1^*, f_2^*, f_3^*$ and maximum $f_1^*, f_2^*, f_3^*$ of the three sub-targets are obtained respectively.

$$
\begin{align*}
    f_1^* &= f_1(z^{1*}) = \min_{z \in Z} f_1(z), \\
    f_2^* &= f_2(z^{2*}) = \min_{z \in Z} f_2(z), \\
    f_3^* &= f_3(z^{3*}) = \min_{z \in Z} f_3(z) \\
    f_1^* &= \max_{z \in Z} f_1(z), \\
    f_2^* &= \max_{z \in Z} f_2(z), \\
    f_3^* &= \max_{z \in Z} f_3(z)
\end{align*}
$$

- **Step 2:** Check the minimum points, if $z^* = z^{1*} = z^{2*} = z^{3*}$, output the optimal solution $f_1(z^*)$ and the optimal value $z^*$, otherwise turn to step 3;

- **Step 3:** Taking $f_1(z)$ as the main target, $\alpha$ and $\beta$ take larger relaxed values to construct the initial constraint set.

- **Step 4:** Construct the constraint satisfaction model according to the constraint set and solve it according to the improved genetic algorithm.

- **Step 5:** Satisfaction discrimination. If the decision maker is not satisfied with the target value $f_2(z), f_3(z)$, the $\alpha$ and $\beta$ values are modified or the main objective function is changed, the constraint set is updated, and step 4 is changed. If all the objectives are satisfied, the satisfactory solution is output and the algorithm is finished.

An interactive genetic algorithm process based on constraint satisfaction is shown in Figure 1.
4. Example Analysis
According to the pre-operational plan, the existing equipment support organizations are deployed in the theatre, and five resource support points are formed to provide support for the combat units. With the progress of the war, according to the confrontation between the enemy and the enemy and the distribution of operations, the demand for support is gradually clear. The operational departments have known the forecast of the demand for support in the next stage, and the emphasis of support has changed. The demand for support in the battlefield has changed from support area 1 to support area 2, which forms the demand situation of battlefield support as shown in Figure 2.
There are 21 security demand points, 9 alternative points and 5 security resource points.

Table 1 Importance of Requirement Points

| Demand point | Demand point 1 | Demand point 2 | Demand point 3 | Demand point 4 | Demand point 5 | Demand point 6 | Demand point 7 |
|--------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Importance degree | 0.8            | 0.9            | 0.9            | 0.9            | 1.0            | 0.6            | 0.5            |

| Demand point | Demand point 8 | Demand point 9 | Demand point 10 | Demand point 11 | Demand point 12 | Demand point 13 | Demand point 14 |
|--------------|----------------|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Importance degree | 0.6            | 1.0            | 0.5             | 1.0             | 1.0             | 0.4             | 0.6             |

| Demand point | Demand point 15 | Demand point 16 | Demand point 17 | Demand point 18 | Demand point 19 | Demand point 20 | Demand point 21 |
|--------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Importance degree | 0.8            | 0.7             | 0.6             | 1.0             | 1.0             | 0.5             | 1.0             |

In order to adapt to the change of current battlefield situation, it is necessary to dynamically adjust the distribution of the original support resources by using the limited battle gap time. Reasonable allocation of different combat units to the corresponding support agencies can optimize the effectiveness of support. According to the above model, an interactive genetic algorithm based on constraint satisfaction is used to solve the problem. The algorithm is programmed in MATLAB 2010. The computing environment is Intel Pentium Dual-Core E5200 2.52GHz and the memory is 2.00GB.

Find the extremum under a single target. The genetic algorithm is used to obtain:

\[
\begin{align*}
\max f_1 & = 17.3 & \min f_1 & = 12.2 & \max f_2 & = 91 & \min f_2 & = 17 & \max f_3 & = 5 & \min f_3 & = 1
\end{align*}
\]

Construct a constraint satisfaction model to solve the problem. The calculation process is shown in Table 2. Table A→B indicates that the support force is transferred from alternative point A to alternative point B. If A and B are the same, the position will remain unchanged.

Table 2 Interactive genetic algorithm process based on constraint satisfaction

| Interaction times | Dynamic optimization distribution decision scheme | target 1 | target 2 | target 3 | \( \alpha \) | \( \beta \) |
|-------------------|--------------------------------------------------|----------|----------|----------|-----------|---------|
| 1                 | 1→7, 3→6, 5→5, 6→9, 8→4                         | 17.3     | 91       | 4        | 91        | 5       |
| 2                 | 1→7, 3→3, 5→5, 6→9, 8→8                         | 15.5     | 80       | 2        | 91        | 4       |
| 3                 | 1→5, 3→9, 5→2, 6→6, 8→4                         | 14.6     | 36       | 4        | 40        | 4       |
| 4                 | 1→1, 3→9, 5→2, 6→6, 8→4                         | 14.2     | 36       | 3        | 40        | 3       |
Table 2 shows that as the preferences of the decision maker are determined step by step, the solution approaches the optimal solution under the preferences. For the decision maker, scheme 4 is the optimal solution. However, these dynamic distribution schemes belong to Pareto optimum solution. For different decision makers, they may choose different schemes. If they pursue the maximization of battlefield support efficiency, they will choose scheme 1. If they pursue the minimum number of support resource points for adjusting deployment while taking into account support efficiency, they will choose scheme 2. If they want to adjust deployment time as small as possible, they will choose scheme 2. Option 3: The compromise chooser will choose option 4. At this time, the security efficiency, the adjustment time and the number of adjustment resources are in a more balanced state.

Figures 3 to 6 show the dynamic optimal distribution scheme under different decision preferences.

5. Work summary and outlook
As battlefield posture and support requirements change, the original distribution scheme of support resources can not meet the task needs, in order to maintain the efficient operation of the support network, it is necessary to deploy the support resources dynamically. Aiming at “the greatest satisfaction of support, the shortest adjustment time and the least adjustment of the number of support resources”, the number of support resources points is constant, reduced and numbered respectively. An
interactive programming method based on constraint satisfaction is proposed to solve the dynamic multi-objective optimal distribution model of battlefield support resources with increasing quantity. By exchanging information between decision makers and optimization models, this method fully reflects decision preferences of decision makers and improves the scientificity and feasibility of decision-making.

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