Affinity Paths and Information Diffusion in Social Networks

José Luis Iribarren*

Instituto de Ingeniería del Conocimiento, Universidad Autónoma de Madrid, 28049 Madrid, Spain

Esteban Moro1

Instituto de Ciencias Matemáticas CSIC-UAM-UC3M-UCM,
Departamento de Matemáticas & GISC, Universidad Carlos III de Madrid, 28911, Leganés (Madrid),
Instituto de Ingeniería del Conocimiento, Universidad Autónoma de Madrid, 28049 Madrid, Spain

Abstract

Widespread interest in the diffusion of information through social networks has produced a large number of Social Dynamics models. A majority of them use theoretical hypothesis to explain their diffusion mechanisms while the few empirically based ones average out their measures over many messages of different content. Our empirical research tracking the step-by-step email propagation of an invariable viral marketing message delves into the content impact and has discovered new and striking features. The topology and dynamics of the propagation cascades display patterns not inherited from the email networks carrying the message. Their disconnected, low transitivity, tree-like cascades present positive correlation between their nodes probability to forward the message and the average number of neighbors they target and show increased participants' involvement as the propagation paths length grows. Such patterns not described before, nor replicated by any of the existing models of information diffusion, can be explained if participants make their pass-along decisions based uniquely on local knowledge of their network neighbors affinity with the message content. We prove the plausibility of such mechanism through a stylized, agent-based model that replicates the Affinity Paths observed in real information diffusion cascades.

Key words: Word-of-Mouth, Viral Marketing, Information Diffusion, Social Networks, Complex Systems

1. Introduction and Background

The discovery of quantitative laws in the collective properties of large numbers of people, for example the birth and death rates or crime frequencies, was one of the factors pushing the development of statistics and led scientists and philosophers to call for some quantitative understanding on how such precise regularities stem from the apparently erratic behavior of individuals. Hobbes, Laplace, Comte, Stuart Mill and many others shared, to a different extent, this line of thought (Ball, 2004).

The question to investigate was how the interactions between social agents create order in their behavior from an initially disordered state. The basic premise was that agents’ repeated interactions should make people more similar since the information exchanges involved led to higher degrees of homogeneity in values, thoughts or preferences. The dynamic nature of the information diffusion, the poor understanding of the human behavior causes and the fact that the agents interactions take place in the thick of complex social networks, made the Social Dynamics problem largely untractable for a long time.

The appearance of new social phenomena related to the Internet (Social Media, Collaborative Filtering, Social Tagging...) whose interactions can be captured in large databases and the tendency of social scientists to move toward the formulation of simplified models and their quantitative analysis, have ushered in an era of scientific research in the field of Social Dynamics (Lazer et al., 2009). Several key questions have been posed: What favors the homogenization process? What hinders it? What are the fundamental interaction mechanisms fostering the adoption of innovations, the spreading of rumors, the evolution towards a dominant opinion or the emergence of trends and fashions?

Initially, the difficulty in obtaining micro-level data on the diffusion of information between individuals, the absence of suitable mathematical algorithms to rigorously analyze the phenomena and the calculation complexity involved in simulations...
with large real networks limited theoretical advancements to the construction of population average diffusion models based on master or differential equations. Those models were in general borrowed from mathematical epidemiology (Hethcote, 2000) since it was assumed that information would propagate just like diseases do. However information diffusion research has deeply evolved since step-by-step tracking of interactions through electronic media made detailed diffusion data plentiful (although not necessarily accessible or easy to gather).

The development of the science of complex systems and advancements in the computerized treatment of Social Network Analysis methods have spurred the emergence of a “new” science of networks (Watts, 2004) which provides more robust tools for the scientific treatment of social dynamics processes. As a result scientists realized that information spreading mechanisms vary with the type of information which spawned a rush to develop the appropriate model for each. According to their algorithmic approach those models can be categorized as population-average or network-based. The population-average models assume fully-mixed or homogeneous substrate networks and describe the agents’ social dynamic behavior at the aggregate level through differential or master equations. Examples of those are the seminal “two-step influence model” of information diffusion by Katz and Lazarsfeld (1955), the rumor diffusion model of Daley and Kendall (1965), the innovations adoption model of Bass (1969), its stochastic version by Niu (2002), the minority spreading opinion formation model of Galam (2002), the innovation diffusion model with influencers and imitators of Van den Bulte and Joshi (2007) or the percolation-based product lifecycle model of Frenken et al. (2008). On the other hand, network-based models include the influence of the underlying social network topology by way of agent-based stochastic algorithms. Some examples of them are the classic innovation adoption “threshold model” of Granovetter (1978), the model of diffusion of technological innovations with upgrading costs of Guardiola et al. (2002), the fads and fashion formation model of Bettencourt (2002), models on the impact of the structural characteristics of a network on innovations diffusion (Jackson and Yariv, 2005; Liu et al., 2005), the stochastic model for opinion formation of Sznajd-Weron (2005) or the network variant of the Daley-Kendall rumor model by Nekovee et al. (2007).

However, this profusion of theoretical models was mainly justified by plausibility arguments and Social Dynamics models based on empirical data are still scarce. A few examples are the referral networks study of Vilpponen et al. (2006) which found that the structure of electronic communication networks is different from that of the traditional interpersonal communication ones, the chain-letter diffusion research of Liben-Novell and Kleinberg (2008) whose strikingly long and narrow spreading chains were attributed to a new mechanism involving asynchronous response times of the forwarders or the study on information diffusion through blogs of Gomez-Rodriguez et al. (2010) which found a core periphery structure in the blogosphere news diffusion network. Nevertheless, all these studies could only trace the propagation of messages with varying content and are unable to discriminate the propagation of individual content items. As a result, none of them could study the impact of the information content on the diffusion processes. While the lack of insight into the content impact would be expected of past century information diffusion research, its absence in more recent literature can only be explained because propagation data at the individual level, being usually proprietary because of its economic value or usage restrictions, is kept under tight wraps and results very hard to obtain.

Our research addresses such shortcomings. Unlike the works cited that study information propagation through the aggregate effect of propagating messages of varying content, ours tracked the precise paths of a viral marketing campaign fixed and invariable message as it spread through an email social network. The message content remained identical through the propagation. This allowed us to scrutinize the individuals’ reactions to a particular message instead of just averaged out behavior over diverse information items. By discriminating all factors impacting the participants’ spreading patterns from the message content we were able to detect the effects produced by the latter. We found that the message diffusion cascades evolve through a branching process that presents some characteristic and unique patterns unexamined until now although some literature (Leskovec et al., 2007; Watts and Peretti, 2007) has shown an inkling of them. We noticed a steady increase in the spreaders’ activity parameters as the message gets deeper in the propagation cascades. This surprising pattern can not be observed in empirical experiments collecting propagation data of varying content messages. It can be explained if the cascades growth stems from a mechanism based on the affinity between the message content and the preferences of those receiving it and not on the receiving node neighbors’ status or on the underlying social network structure used in many of the current models. We test and validate that hypothesis through a stylized agent-based propagation model. The rest of the article is organized as follows: First we describe the data obtained from our empirical research on real viral marketing campaigns and the control parameters of their messages propagation. Second, we present our findings on the structure and growth patterns of the information cascades. Third we introduce the message affinity propagation model and compare its predictions with the empirical results. The article ends with our conclusions.

2. Word-Of-Mouth diffusion research

We tracked and measured the “word-of-mouth” diffusion of viral marketing campaigns ran in eleven European markets which offered incentives to current subscribers of an IT company online newsletter to promote new subscriptions through recommendation emails to friends and colleagues. The campaigns were entirely web based: banner ads, emails, search engines and the company homepage drove participants into the campaign site. In it, participants accessed a referral form to register themselves and enter the addresses of those to whom they recommended subscribing the newsletter. The submission of this form triggered a personalized, but otherwise identical, rec-
However, participants with cookies disabled could send multiple referrals to the same person. Thus 183 referrals (0.76% of total) were discarded.

The incentive offered to recommenders was the possibility of winning laptop computers in a lottery to be held at the end of the campaign period. Aside from the obvious goal of increasing participation, the incentive mission was twofold: Firstly, discourage indiscriminate referrals to prevent spamming-like behavior and, secondly, ensure legal cover for the tracking of sender-receiver data. To accomplish such requirement, participants’ email client prevented sending multiple referrals between same nodes were discarded. In combination with invalid or undeliverable email addresses, loops and multiple referrals between same nodes were discarded. In compliance with the sponsor rigorous policy, all personal information was codified and masked to guarantee the participants’ privacy protection.

### Campaigns propagation data set

Table 1 presents the summary data set of the campaigns propagation network. $s_{\text{max}}$ is the largest cascade size by its number of nodes. Quantities in **All markets** may not add up to the sum of their column because network partition removes inter-country links. The number of Seed Nodes ($N_s$) may not coincide with that of cascades due to cascades merging with one another during the propagation or because, sometimes, a Seed Node can not be identified (for example in the case of recommendation reciprocity between two nodes). Results in some countries are aggregated in homogeneous markets for statistical significance. Nordic includes DK, FI, NO and SE.

| Market       | $N$  | $N_s$ | $N_v$ | $N_p$ | Arcs | Casc. | $s_{\text{max}}$ |
|--------------|------|-------|-------|-------|------|-------|------------------|
| France       | 11,758 | 3,247 | 524   | 7,987 | 8,593 | 3,248 | 139              |
| DE+AT        | 7,943  | 1,760 | 567   | 5,616 | 6,239 | 1,750 | 146              |
| Spain        | 5,260  | 855   | 505   | 3,900 | 4,454 | 843   | 122              |
| Nordic       | 2,509  | 530   | 176   | 1,803 | 2,004 | 524   | 34               |
| UK+NL        | 2,111  | 521   | 107   | 1,483 | 1,618 | 518   | 25               |
| Italy        | 1,602  | 323   | 108   | 1,171 | 1,324 | 319   | 41               |
| **All markets** | 31,183 | 7,225 | 2,002 | 21,956| 24,207| 7,188 | 146              |

Table 1

**Campaigns propagation data set**: Count of Total Nodes ($N$), Seed Nodes ($N_s$), Viral Nodes ($N_v$), Passive Nodes ($N_p$), Total directed links (Arcs), and Total of Independent Cascades (Casc.) measured on the campaigns propagation network. $s_{\text{max}}$ is the largest cascade size by its number of nodes. Quantities in **All markets** may not add up to the sum of their column because network partition removes inter-country links. The number of Seed Nodes ($N_s$) may not coincide with that of cascades due to cascades merging with one another during the propagation or because, sometimes, a Seed Node can not be identified (for example in the case of recommendation reciprocity between two nodes). Results in some countries are aggregated in homogeneous markets for statistical significance. Nordic includes DK, FI, NO and SE.

### Cascades Network structural metrics

Here we examine differences and similarities between the Cascades Network topology and that of the reported email networks through which they propagate. Table 2 shows the Cascades Network structural parameters measured without considering links direction. The cumulative distribution function (c.d.f) of the undirected network total degree $k$ is a power-law $P(k) \sim k^{-2.8}$ whose significant probability of very connected nodes evidences higher heterogeneity than the exponential degree distributions found in some email networks (Guimerà et al., 2003; Newman et al., 2002). However, their heterogeneity is less marked than that of the email network studied by Ebel et al. (2002) whose power-law degree distribution (p.d.f) of exponent $\gamma_k = 1.81$ is fatter tailed. Additionally, email networks present positive correlations between the nodes degree at either end of an edge, a property called degree assortativity and measured, according to Newman (2002), by the Pearson correlation coefficient.
coefficient. For example, the degree correlation coefficient in the email network of Guimerà et al. (2003) is \( p_k = +0.188 \), indicating of a correlated network. The equivalent for the Cascades Network \( p_k = -0.001 \) shows total uncorrelation. Besides, in networks with skewed node degree distributions and degree correlations, such as the email networks, the average connectivity of the network \( \bar{\ell} \) is typically lower than that of the nearest neighbors of a node \( \bar{k}_{nn} \). For example in the Guimerà et al. (2003) email network, the ratio \( \bar{k}_{nn} / \bar{k} \) is approximately 2. Such phenomenon is responsible for the first neighbors of a node having in average more contacts than such node or, quoting Feld (1991), for the fact that “your friends always have more friends than you do.” Interestingly, this feature is more marked in the Cascades Network whose \( \bar{k}_{nn} / \bar{k} \) ratio ranges from 2.24 in UK+NL to 4.24 in Spain.

Another difference between the Cascades Network and the email networks through which they propagate lies in their transitivity, a property typical of acquaintance networks whereby two individuals with a common friend are more likely than average to know each other. The Clustering coefficient \( C \), defined as the fraction of all triangles found in the network relative to the total number of triads\(^4\) measures the transitivity. Table 2 shows that our Cascades Networks with a Clustering coefficient \( C = 4.8 \times 10^{-3} \) for the graph of All markets are highly intransitive yet ten times more transitive than an equivalent random network of the same size and connectivity. In any case, a very low value compared to the range \( C [0.15 - 0.60] \) found in social or email networks (Newman and Park, 2003). Probabilistic considerations show the logic of such feature: since the Cascades Network percolates its underlying email network only partially, the dyadic closure that builds clustering in the former must be just a fraction of the one in the latter. As a result our campaigns viral diffusion cascades, like the one in Fig. 1, are almost pure trees.

\(^4\) A triad is a group of three nodes connected by two links

### Table 2

| Market       | \( \bar{\ell} \) | \( \sigma_k \) | \( \bar{k}_{nn} \) | \( C \)   | \( C_{\text{rand}} \) | \( \bar{C} \) | \( g_{\text{max}} \) |
|--------------|------------------|----------------|-------------------|----------|----------------------|---------------|------------------|
| France       | 1.46             | 1.594          | 3.99              | 0.0000   | 0.00012              | 2.164         | 8                |
| DE+AT        | 1.57             | 2.027          | 5.59              | 0.0049   | 0.00020              | 2.671         | 7                |
| Spain        | 1.69             | 2.383          | 7.17              | 0.0054   | 0.00032              | 3.287         | 9                |
| Nordic       | 1.60             | 1.575          | 4.07              | 0.0077   | 0.00064              | 2.243         | 5                |
| UK+NL        | 1.53             | 1.364          | 3.43              | 0.0112   | 0.00073              | 2.026         | 5                |
| Italy        | 1.65             | 1.918          | 5.22              | 0.0234   | 0.00103              | 2.229         | 6                |
| All markets  | 1.55             | 1.568          | 4.97              | 0.0048   | 0.00005              | 2.671         | 9                |

The last distinctive property of email networks, the Small World or low average shortest path length (Boccaletti et al., 2006), although seemingly present since \( \bar{\ell} = 2.67 \) (Table 2) and lower than that of email networks \( \bar{\ell}_{\text{rand}} \sim 3.5 \) (Eckmann et al., 2004; Guimerà et al., 2003) is not comparable with those due to the nature of the Cascades Network that, split in many disconnected components, limits paths calculation to reachable pairs of nodes which necessarily yields lower values. The distribution of those cascades size \( s \), like the total degree, is a very skewed power-law whose c.d.f. exponent is \( \gamma = 1.35 \). With largest cascade size \( s_{\text{max}} = 146 \) nodes, mean size \( \bar{\ell} = 4.33 \), and \( \sigma_s = 5.27 \), the cascade in Fig. 1 is 25 times more likely to appear in our campaigns than in percolation through a random network\(^5\).

In consequence, the viral Cascades Network topology lacks all the four key features of email networks (fat tailed node degree distribution, nodes degree correlations, high clustering and the Small World property) and can not be formally characterized as a social network. This is quite logical since the viral propagation cascades of diffusion processes far from saturation, such as ours, overlay just sections of the underlying email network and, as a result, can only unveil a small portion of it. Paraphrasing Liben-Nowell and Kleinberg (2008) in their study of chain-letters propagation, it is as if “the progress of the viral messages had a type of stroboscopic effect serving to briefly light up the structure of the global email network.” Unfortunately, not having any details on the topology of the email network substrate, we can not judge the extent of its influence on the Cascades Network topology.

\(^5\) The tail of the cascade size distribution in large random networks near the transition to the giant component goes as \( n_s^c \sim s^{-5/2} \) (Albert and Barabási, 2002) and the probability of a cascade of size 122 is \( \sim 6.1 \times 10^{-6} \).
A markovian model of a population where each individual in generation $g$ produces in generation $g + 1$ a random number of individuals extracted from the same probability distribution.

\[ \lambda = \frac{N_v}{N - N_s} \]  
(2)

and both parameters combine to yield the Basic Reproductive Number $R_0$ or average number of secondary recommendations produced by reached nodes as

\[ R_0 = \lambda \tau_v \]  
(3)

This number is widely used in mathematical epidemiology (Hethcote, 2000) to determine the moment when a disease outbreak becomes a self-sustaining epidemic. Thus, if $R_0 \geq 1$ the spreading process reaches the Tipping-point\(^7\) an elusive goal that none of our campaigns attained. Table 3 presents the propagation dynamic parameters and cascades average size $\overline{\tau}$ of our campaigns and their predicted value $\overline{\overline{\tau}}$ for the infinite propagation limit given by the Galton-Watson Branching model as

\[ \overline{\overline{\tau}} = 1 + \frac{\tau_v}{1 - R_0}, \quad R_0 \leq 1 \]  
(4)

where $\tau_v$ is the average number of messages sent by Seed Nodes and $R_0$ the viral propagation Basic Reproductive Number. The last column in Table 3 shows the remarkable accuracy of the cascades average size predicted by the Galton-Watson Branching model versus the empirical values.

### 3. Patterns of the information cascades growth

Despite the Galton-Watson model statistically accurate description of the distribution of cascades at a global level, a detailed study of the Cascades Network growth, reveals patterns indicating that viral messages spreading dynamics is quite peculiar. Firstly, we present a node level analysis showing the correlation in the spreading activity of a node with that of its active offspring down the message propagation tree. Secondly, we conduct a generation level analysis on the probability of the nodes becoming active as a function of their ordinal position in the message diffusion path which shows that viral messages diffusion propensity increases with distance from the Seed Node. Both findings lead to a striking prediction corroborated by the measurements on our viral campaigns: The viral messages diffusion dynamic parameters at the population level are correlated, a fact that has not been observed in other social dynamics processes such as innovations adoption, rumors spreading or opinions propagation. Note that both findings are incompatible with the assumptions in the Galton-Watson model in which the branching mechanism is homogeneous both at the social network level and within the cascades.

---

### Table 3

Cascades growth dynamic parameters: Transmissibility ($\lambda$), Fanout Coefficients of Seed ($\tau_s$) and Viral ($\tau_v$) nodes, Standard error of the Viral Nodes Fanout coefficient ($\overline{\tau}_v$, SEM), Basic Reproductive Number for secondary spreaders ($R_0$) and average Cascade size ($\tau$) by market as measured in the campaigns. In the last two columns $\overline{\tau}$ is the average Cascade size predicted by the Galton-Watson Branching model Eq. (4) and % Dev. the deviation of that prediction from the actual measurements.

| Market    | $\lambda$ | $\tau_s$ | $\tau_v$ | $\tau_s$ SEM | $R_0$ | $\tau$ | $\overline{\tau}$ | % Dev. |
|-----------|-----------|----------|----------|--------------|-------|-------|-------------------|-------|
| France    | 0.062     | 2.21     | 2.50     | 0.1023       | 0.154 | 3.62  | 3.61              | -0.22 |
| DE+AT     | 0.092     | 2.48     | 3.06     | 0.1155       | 0.281 | 4.54  | 4.45              | -2.04 |
| Spain     | 0.115     | 3.16     | 3.45     | 0.1909       | 0.400 | 6.24  | 6.23              | -0.20 |
| Nordic    | 0.089     | 2.82     | 2.91     | 0.1836       | 0.259 | 4.79  | 4.81              | +0.31 |
| UK+NL     | 0.067     | 2.49     | 2.87     | 0.2398       | 0.236 | 4.08  | 4.09              | +0.15 |
| Italy     | 0.084     | 2.87     | 2.80     | 0.2301       | 0.236 | 5.02  | 4.76              | -5.20 |
| All markets | 0.083   | 2.51     | 2.96     | 0.065        | 0.246 | 4.34  | 4.33              | -0.30 |

---

\[ P_{L_{\alpha \beta}}(r_s) = \frac{H_{\alpha \beta}}{\beta + r_v} \]  
(1)

whose parameters for the All markets network take the values $H_{\alpha \beta} = 11.6$, $\alpha = 2.83$ and $\beta = 10.96$ using Maximum Likelihood Estimation.

We can visualize the cascades of a viral propagation process growing through successive layers, or generations, as nodes reached in one generation resend the message to nodes in the next generation. The latter nodes constitute the off-spring of the earlier ones in an evolution of the propagation trees whose node-level dynamics is well described by the Galton-Watson Branching model\(^6\) (Harris, 2002). Two parameters fully de-
finite networks and is usually replaced by the average degree of the nearest neighbors of a node in a base two logarithmic binning. The linear fit positive slope (0.69) shows correlation between the spreading activity of a node and that of its active offspring in the propagation tree: the more active a node is, the more active its nearest neighbors in average are.

3.1. Correlated spreading of active nodes

The first distinctive pattern of the viral messages Cascades Network growth is the marked positive correlation of the spreading activity between Viral Nodes and their active off-spring. In undirected networks, the nodes total degree correlation is given by the conditional probability \( P(k | k') \) of a node of degree \( k \) pointing to a node of degree \( k' \). This function is very noisy in finite networks and is usually replaced by the average degree of the nearest neighbors of \( k \)-degree nodes \( \tau_{nn}(k) = \sum_{k'} k' P(k | k') \) (Boccaletti et al., 2006). When \( \tau_{nn}(k) \) is an increasing function of the degree \( k \) the nodes tend to connect to others of similar connectivity and such network, called assortative, displays positive node total-degree correlations.

However the active nodes network is directed and instead one should study its out-degree correlation defined as the tendency of nodes to connect with others that have similar out-degrees to themselves. Its formal metric is the out-assortativity coefficient\(^8\) but considering throughout only the active nodes throughout a simplified analysis of the average out-degree of the active nearest neighbors \( \langle \tau_r \rangle_{ann} \) of nodes of out-degree \( r \geq 1 \) presented in Fig. 2 suffices to prove that, in terms of the number of recommendations sent in our campaigns, the more active a node is the more prolific in average its progeny is. We studied the out-degree spreading pattern of active nodes in our campaigns (Seed Nodes excluded) and found that the activity of a node \( (r_v) \) correlates with that of its active nearest neighbors. Such correlation implies that the average number of recommendations sent by the active nearest neighbors of a node \( \langle \tau_r \rangle_{ann} \) grows with the number of recommendations \( r_v \) that it has sent. The slope of the linear regression of \( \langle \tau_r \rangle_{ann}(r_v) \) is +0.69 indicating strong out-degree correlation. The actual values of \( \langle \tau_r \rangle_{ann} \) range between 1 and 31.33, the mean of their distribution is 2.48 and its standard deviation 2.08.

This very peculiar feature of viral messages diffusion has not been observed on any other type of propagation processes in social networks. We can hypothesize two different explanations of it. One, the increased spreading activity of the active children of a node is a reflection of the out-degree correlation present in the substrate email network. Lacking any data on such network for our campaigns this hypothesis is impossible to verify. Besides, the out-degree positive correlation in the substrate email network merely means that its nodes tend to link to others of similar out-degree but does not by any means indicate that the number of recommendations made by active participants, hence the interest in participating in the campaign, should be a growing function of the number of recommendations made by their parent in the cascade. The other possible explanation, which we adopt, is that the intrinsic mechanism whereby participants in viral marketing campaigns forward the messages involves the sender selecting targets among those of her contacts perceived to be the most receptive to the content of the message being passed-along. The iteration of these target filtering decisions through several generations of senders would lead, in a process akin to targeted search, to focusing the message on groups of individuals genuinely interested on it. Those, in turn, would also be in average more active than their ancestors. The fact that this mechanism has not been observed in other types of information diffusion, such as referral networks (Vilponnen et al., 2006), e-commerce recommendations (Leskovec et al., 2007) or email chain-letters (Liben-Nowell and Kleinberg, 2008) may indicate either that the phenomenon is specific of viral marketing messages or that those authors analysis did not isolate the content factor.

3.2. Diffusion acceleration with path length

The second characteristic of viral spreading dynamics appears when measuring the probability of the nodes becoming active spreaders as a function of their position in the propagation tree. Thus, the Transmissibility by generation \( \lambda_g \) in our campaigns grows in correlation with the ordinal \( g \) representing the individuals’ location in the message propagation path. As shown in Table 4 for the All markets data, \( \lambda_g \) increases steadily with the generation \( (\rho(g) | \lambda_g = 0.908) \) with parallel growth of the Reproductive Number by generation

\[
R_g = \lambda_g \langle \tau_r \rangle_g = \frac{N_{g+1}}{N_g}
\]

where \( N_g \) is the total number of individuals reached at generation \( g \). Besides, there is a growth trend for \( \langle \tau_r \rangle_g \), the Fanout by generation which is visible in our campaigns (Table 4) whose ratio through generations \( \langle \tau_r \rangle_{g+1} / \langle \tau_r \rangle_g \) positively correlates with the generation number \( (\rho = 0.4) \). Those properties of messages diffusion were detected, but not studied, by Watts and Peretti (2007) or Leskovec et al. (2007) as shown in Fig. 3.

---

\(^8\) A convoluted combination of the probability distributions of a link going out of a node of out-degree \( r_v \), of a link going into a node of out-degree \( r_v' \) and the joint probability of links to go from a node of out-degree \( r_v \) to another of out-degree \( r_v' \) (Piraveenan et al., 2009)
along with our campaigns measurements. As before, we posit that such pattern is due to “preferential forwarding,” defined as the spreaders’ propensity of passing a message preferentially to neighbors they presume to have more interest, or affinity, for it. Such mechanism results in an increase of the recipients propensity to pass the message along. As a consequence, the message follows network paths such that the Transmissibility by generation \( \lambda_g \) increases as the propagation progresses. We denominate Affinity Paths to the chains of individuals with similar or increasing affinity for the message. They imply some knowledge by message spreaders of their immediate neighbors interests, a local awareness with global impact that leads to a different class of propagation than that of other Social Dynamics processes. Its consciously driven spreading mechanism causes messages to progress through paths presenting the homophily\(^9\) properties typical of social networks (McPherson et al., 2001). This phenomenon has been observed in the web where, according to Singla and Richardson (2008) “there is correlation between preferences and behavior of an individual and those of others in its immediate circle”.

3.3. Dynamic Parameters correlation

As a result of the previous two properties the parameters \( \lambda \) and \( \tau_v \) are correlated. Let us consider the relationship between the Fanout Coefficient and the generation parameters in Table 4

\[
\tau_v = \frac{\sum_{g=2}^{N_g} N_g}{\sum_{g=1}^{N_g} \lambda_g N_g} = \frac{1 - P_g(1)}{\sum_{g=1}^{N_g} \lambda_g P_g}
\]

where \( P_g(1) = N_g / \sum_{g=1}^{N_g} N_g = N_g \tau_v / (N - N_g) \) is the probability of an individual to have received the message from a Seed Node. Since \( \sum_{g=1}^{N} \lambda_g P_g = \lambda \) one obtains the important expression \( \lambda \tau_v = 1 - P_g(1) \) which means that for \( \lambda \) and \( \tau_v \) to increase simultaneously one must reduce the probability \( P_g(1) \) of finding nodes in the first generation or, equivalently, grow longer cascades. Thus, a growing \( \lambda_g \) yields longer paths and causes a parallel growth of \( \tau_v \). Our campaigns show that the average shortest path length \( \bar{\tau}_v \) of the diffusion cascades and the dynamic parameters are strongly correlated: \( \rho(\bar{\tau}_v \lambda) = 0.88 \) and \( \rho(\bar{\tau}_v \lambda) = 0.89 \). An increase of the Transmissibility \( \lambda \) grows the paths length and the average number of recommendations made \( \tau_v \) as well. Plotting the dynamic parameters for various markets (Fig. 4) their correlation was found to be very strong with a Pearson coefficient \( \rho(\lambda \tau_v) = 0.92 \). The values of \( \lambda \) and \( \tau_v \) by country from Table 3 fit to the decreasing exponential \(^{10}\)

\[
\tau_v = 1 + b (1 - e^{-c \lambda})
\]

which for \( \lambda \ll 1 \), and through a MacLaurin series expansion of \( e^{-c \lambda} \), turns into \( \tau_v = 1 + a \lambda \) \((a = bc)\). One can consider the slope \( a \) of this “response line” as the message “fitness” with respect to each market. The exponential decrease for large \( \lambda \) in Eq. (7) is due to the substrate network nodes clustering which limits propagation through saturation and finite size effects.

In principle this correlation between Fanout Coefficient and Transmissibility should invalidate the Galton-Watson model used in Section 2.3, because that model assumes that those parameters are uncorrelated. However, this is not the case since most of the participants in the campaign appear at very low generation numbers and thus the phenomena observed here is only a significant correction affecting a small fraction of participants.

\(^9\) The tendency of individuals to associate and bond with similar others.

\(^{10}\) Y intercept set to 1 since \( \tau_v \to 1 \) as \( \lambda \to 0 \) because fit is on active nodes.

Table 4

| \( g \) | \( N_g \) | \( P_g \) | \( (N_g)_g \) | \( \lambda_g \) | \( (\tau_v)_g \) | \( R_g \) | SEM |
|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 18,032 | 0.7527 | 1,398  | 0.0775 | 2.891 | 0.224 | 0.0056 |
| 2     | 4,042  | 0.1687 | 393    | 0.0972 | 3.239 | 0.315 | 0.0120 |
| 3     | 1,273  | 0.0531 | 139    | 0.1092 | 2.784 | 0.304 | 0.0228 |
| 4     | 387    | 0.0162 | 40     | 0.1034 | 3.150 | 0.326 | 0.0621 |
| 5     | 126    | 0.0053 | 20     | 0.1587 | 3.550 | 0.564 | 0.1804 |
| 6     | 71     | 0.0030 | 8      | 0.1127 | 2.125 | 0.239 | 0.0612 |
| 7     | 17     | 0.0007 | 3      | 0.1765 | 2.000 | 0.353 | 0.1765 |
| 8     | 6      | 0.0003 | 1      | 0.1667 | 4.000 | 0.667 | 0.0   |
| 9     | 4      | 0.0002 | 0      | 0      | 0     | 0     | N/A   |

Fig. 3. Diffusion acceleration with path length: Reproductive Number by generation \( R_g \) in viral messages propagation. Solid circles with error bars correspond to our IT newsletter campaign. Other data sets (no error bars available): Oxygen Network advocacy portal collecting contributions for hurricane Katrina relief (squares); Tide Coldwater campaign for an energy-efficient washing detergent (empty circles); StopTheNRA, an appeal for gun control launched by the father of a Columbine shootings victim (upward triangles) per Watts and Peretti (2007); referrals in e-commerce (downward triangles) per Leskovec et al. (2007).
The Galton-Watson Branching model used in Section 2.3 explains well the message being forwarded. Furthermore, the model propagation nodes are in one of the following three states: epidemic model on networks (Pastor-Satorras and Vespignani, 2007) built a model proving that incoming activity from the substrate email network they run upon are correlated and that such correlation of neighbors to send it to, typically made in a single act by each forwarding individual, are correlated and that such correlation emerges as a function only of their affinity with the content of the message being spread.

4. The Message Affinity Model (MAM)

The correlation between the messages propagation dynamic parameters $\lambda$ and $c$ and the independence of the nodes spreading activity from the substrate email network they run upon are intriguing properties of the viral marketing diffusion processes. Watts and Dodds (2007) built a model proving that information propagation can happen independently of the underlying social network structure and concluded that “large cascades of influence are driven not by the influential but by a critical mass of easily influenced individuals.” However, their model does not explain the dynamic parameters correlation nor the increase with the generation of the nodes propensity of becoming spreaders. We posit that both features are due to the fact that the decisions of forwarding a viral message and of the number of neighbors to send it to, typically made in a single act by each forwarding individual, are correlated and that such correlation emerges as a function only of their affinity with the content of the message being spread.

The agent-based Message Affinity Model (MAM) incorporates that mechanism by assigning to the substrate network nodes a propensity value representing their affinity with the message being forwarded. Furthermore, the model propagation rules combine a variant of the states transition steps of the SIR epidemic model on networks (Pastor-Satorras and Vespignani, 2001) with the stochastic evolution of a pseudo-markovian $^{11}$ Galton-Watson Branching model. At any step, the network nodes are in one of the following three states:

- **Susceptible ($S$)**: Node has not received the message
- **Informed ($I$)**: Node is propagating the message
- **Refractory ($R$)**: Node does not spread the message anymore

Unlike the SIR model, MAM does not use a global probability for the nodes states transitions. Instead, they stem from the aggregate decisions that result from the interplay between the node pass-along propensity and the message “fitness” to diffuse. Drawn from a continuous probability density function $p(a)$, the Affinity $a_n \in [0, 1]$ of a node represents its propensity to engage in spreading the message. The message fitness to trigger the node activations is represented by their Affinity Threshold $A_T \in [0, 1]$, the lowest $a_n$ value for which such message can push the node into the Informed state: low threshold messages are capable of activating more nodes and are, as a result, forwarded more often than high threshold ones. The process starts by turning a random fraction of the substrate network nodes into the Informed state while leaving all others Susceptible. From that point onwards the following rules govern the stochastic propagation:

(i) **Susceptible** nodes touched by the message become Informed if their Affinity is higher than the message threshold ($a_n > A_T$) and Refractory otherwise, while, if touched, Informed or Refractory nodes stay unchanged.

(ii) An Informed node $n$ forwards a number of messages $(r_n)_n = (a_n - A_T) \times r$, with $r$ drawn from a PL distribution. The neighbors receiving those messages are:

(a) those with highest $a_n$ with probability $(a_n - A_T)$
(b) chosen randomly with probability $1 - (a_n - A_T)$

(iii) Informed nodes become Refractory immediately after spreading the message and the process ends when no Informed nodes are left

The quantity $a_n - A_T$ embodies the interplay between the participants interests and the message content. The choice in Rule (ii) of the neighbors that will receive the message represents the evaluation Informed nodes make, based on their local knowledge, of their neighbors' affinity. It implies that local knowledge grows with the Affinity: nodes of high $a_n$ are more likely to choose targets with the highest propensity to pass the message while those with low $a_n$ will mostly choose their targets randomly. $A_T$ may vary by individual but, without loss of generality, we take it constant including all variations in $p(a)$.

4.1. MAM Simulation Results

Here we present the result of Monte Carlo simulations of viral messages propagation ran with the MAM model and show that they replicate the patterns observed in real processes. The simulations ran on two substrate networks with the same degree distribution but different structure: the real email network of a Spanish university (Guimera et al., 2003) and a synthetic configuration model network built with the Molloy and Reed method (Callaway et al., 2001). They differ in their Clustering Coefficient $C_{email} = 0.22$ vs. $C_{conf} = 0.014$ and in the fact that the email network node degrees are correlated while the
configuration network ones are not. Their nodes Affinity, with correlation between nearest neighbors, was drawn from a uniform distribution. The Cascades Network resulting from the propagation of messages with Affinity Threshold between 0.6 and 0.97 were averaged over 15K cascades with 500 different allocations of the substrate nodes Affinity.

The simulations generate graphs with a large number of disconnected components that, like those in the real campaigns, feature distributions of Eq. (1) type for both their viral nodes activity $P(r_v)$ and cascades size $P(s)$. The exponents $\gamma_s$ and $\gamma_r$ of their power-laws are in the range 1 - 3 depending on the values of the model parameters nodes Affinity ($a_{nn}$) and message Affinity Threshold ($A_T$) used. Besides, the average cluster size of the graphs obtained in the simulations follows closely the branching model predictions as shown in Fig. 5. It plots the average size $\bar{s}$ of the propagation network components obtained with different values of the message Affinity Threshold versus their reproductive number $R_0$ for each. The lines are not a fit to the data but the prediction $\bar{s} = \frac{\lambda_0}{(1 - \lambda_{ss})}$ given by Eq. (4). Notice their remarkable agreement and the fact, shown in the inset, that when the effect of Seed Nodes is removed by plotting $(\bar{s} - 1)/\bar{s}$, the results for the simulations on both substrate networks match exactly. This indicates that as our model predicts, for processes running well below the Tipping-point the impact of the substrate network in the cascades average size or the dynamic parameters of the propagation is very low.

The plot of the Cascades Network dynamic parameters in the main panel of Fig. 6 and their fit to Eq. (7) shows how MAM accurately replicates their correlation pattern. This proves that the viral messages propagation patterns are independent of the substrate network structure for low $\lambda$. However their $\bar{T}_c$ values diverge as $\lambda$ grows because the email network clustering and degree correlations accelerate saturation effects and curtail propagation. The diffusion acceleration with path length presented in Fig. 3 and typical of viral messages propagation is also properly replicated with MAM. The inset of Fig. 7 presents the evolution of $\lambda_{ss}$ with $g$ for simulations on the real email network (empty symbols) compared with that of the real campaigns (full circles).

5. Conclusions and Discussion

We tracked and analyzed the structure and growth dynamics of the propagation network created by the diffusion of a content-controlled message in real viral marketing campaigns driven through email forwarding. The resulting Cascades Network, formed by almost pure trees of very low clustering, shows two striking dynamical patterns not observed so far in other Social Dynamics processes like rumor spreading, innovations adoption or email chain-letters. First, there is positive correla-
tion between the spreading nodes activity level as measured by their out-degree and that of their active off-spring and, second, the propensity of nodes reached by the message to becoming spreaders, the Transmissibility $\lambda$, grows with those nodes depth in the propagation path. These novel properties can only be detected by scrutinizing the propagation of messages of fixed and identical content. The scarcity of such type of data may explain why they have remained unobserved until now. The discovered patterns have two remarkable consequences. On the one hand, the dynamic parameters Transmissibility and Fanout Coefficient for a given message across different markets are correlated. On the other, the topology of the email network underlying the propagation has limited influence on the Cascades Network although its features are compatible with the structure of the substrate email network that conditions their formation.

Our explanation of all those peculiarities stems from the mechanism driving the messages propagation which involves the affinity of the campaign participants with the content of the message. Participants would make a simultaneous and conscious decision of spreading it or not and to whom which leads to a positive correlation between the probability of becoming a spreader after receiving the message and the average number of messages forwarded. This decision would result from a single intrinsic property of the nodes in the substrate network, their affinity with the message being passed-along. Besides, the dynamic parameters by generation $\lambda_g$ and $(\tau_g)$ tend to grow with $g$ since the choice of targets to forward the message to is based on the participants’ awareness of their neighbors’ affinity with it. Such mechanism steers the message through paths of increased affinity termed Affinity Paths.

This hypothesis is tested through an agent-based model (MAM) that replicates the patterns discovered and validates the proposed Affinity-driven information diffusion mechanism. It combines a stochastic branching process with propagation rules that create cascades of touched nodes by taking the substrate network nodes message awareness through a sequence of Susceptible, Informed and Reluctant states. The MAM uses just two control parameters: the Affinity distribution $p(a)$ of the substrate network nodes to assign them an affinity value between 0 (message is not sent) and 1 (message will certainly be forwarded) and the Affinity Threshold $A_T$ representing the message fitness to be passed-around. As the model runs through a substrate network list of edges, the interplay between $A_T$ and the nodes Affinity generates cascades with all the expected features while providing a glimpse into the substrate network topology. The empirical analysis and the theoretical model validate our conclusion that the mechanism driving viral marketing messages propagation results from the affinity between the campaign participants’ preferences and the messages content. In fact, the viral cascades features depend more on the individuals’ reaction to the message than on the substrate network topology. However, we could not verify this conclusion empirically since the structure of our campaigns substrate network being unknown, a comparison between the Cascades Network and the substrate email network was impossible. Also, MAM does not replicate the merging of cascades that occurs near the Tipping-point as it assumes that Seed Nodes are planted in a boundless network and far apart of each other to avoid propagation clashing. Finally, MAM only runs on undirected and fully connected networks.

References

Albert, R., Barabási, A.-L., 2002. Statistical mechanics of complex networks. Rev. Modern Phys. 74, 47–97.

Ball, P., 2004. Critical Mass: How One Thing Leads to Another. Farrar, Strauss and Giroux, London, UK.

Bass, F. M., 1969. A new product growth model for consumer durables. Management Science 15, 121–227.

Bettencourt, L. M. A., 2002. From boom to bust and back again: the complex dynamics of trends and fashions. cond-mat/0212267.

Boccaletti, S., Latora, V., Moreno, Y., Chavez, M., Hwang, D.-U., 2006. Complex networks: Structure and dynamics. Physics Reports 424, 175–308.

Callaway, D. S., Hopcroft, J. E., Kleinberg, J. M., Newman, M. E. J., Strogatz, S. H., 2001. Are randomly grown graphs really random? Phys. Rev. E 64, 041902.

Daley, D. J., Kendall, D. G., 1965. Stochastic rumours. IMA Journal of Applied Mathematics 1(1), 42–55.

Ebel, H., Mielsch, L.-I., Bornholdt, S., 2002. Scale-free topology of e-mail networks. Phys. Rev. E 66, 035103(R).

Eckmann, J.-P., Moses, E., Sergi, D., 2004. Entropy of dialogues creates coherent structures in e-mail traffic. Proc. Natl. Acad. Sci. USA 101, 14333–14337.

Feld, S., 1991. Why your friends have more friends than you do. American Journal of Sociology 96, 1464–1447.

Frenken, K., Silverberg, G., Valente, M., 2008. A percolation model of the product lifecycle. Tech. rep., United Nations University - UNU-MERIT.

Galam, S., 2002. Modelling rumors: the no plane pentagon french hoax case. Physica A 320, 571–580.

Gomez-Rodriguez, M., Leskovec, J., Krause, A., 2010. Inferring networks of diffusion and influence. In: The 16th ACM SIGKDD Conference on Knowledge Discovery and Data Mining (KDD).

Granovetter, M., 1978. Threshold models of collective behavior. American Journal of Sociology 83 (6), 1420–1443.

Guardiola, X., Díaz-Guilera, A., Pérez, C., Arenas, A., Llas, M., 2002. Modelling diffusion of innovations in a social network. Physical Review E 66, 026121.

Guimerà, R., Danon, L., Díaz-Guilera, A., Giralt, F., Arenas, A., 2003. Self-similar community structure in a network of human interactions. Phys. Rev. E 68, 065103(R).

Harris, T. E., 2002. The Theory of Branching Processes. Springer-Verlag, Berlin.

Hethcote, H. W., 2000. The mathematics of infectious diseases. SIAM Review 42 (4), 599–653.

Jackson, M., Yariv, L., 2005. Diffusion on social networks. Economie Publique 16, 3–16.

Katz, E., Lazarsfeld, P. F., 1955. Personal influence; the part played by people in the flow of mass communications. Glen-
Lazer, D., Pentland, A., Adamic, L., Aral, S., Barabási, A.-L., Brewer, D., Christakis, N., Contractor, N., Fowlie, J., Gutmann, M., Jebara, T., King, G., Macy, M., Roy, D., Alstyne, M. V., 2009. Computational social science. Science 323 (5915), 721–723.

Leskovec, J., Adamic, L., Huberman, B., 2007. The dynamics of viral marketing. ACM Transactions on the Web 1, 1.

Liben-Nowell, D., Kleinberg, J., 2008. Tracing information flow on a global scale using internet chain-letter data. Proc. Natl. Acad. Sci. USA 105 (12), 4633–4638.

Liu, B.-S.-C., Madhavan, R., Sudharshan, D., 2005. Diffunet: The impact of network structure on diffusion of innovation. European Journal of Innovation Management 8 (2), 240–262.

McPherson, M., Smith-Lovin, L., Cook, J. M., 2001. Birds of a feather: Homophily in social networks. Annual Review of Sociology 27, 415–444.

Nekovee, M., Moreno, Y., Bianconi, G., Marsili, M., 2007. Theory of rumour spreading in complex social networks. Physica A 374, 457–470.

Newman, M. E. J., 2002. Assortative mixing in networks. Phys. Rev. Lett. 89, 208701.

Newman, M. E. J., Forrest, S., Balthrop, J., 2002. Email networks and the spread of computer viruses. Phys. Rev. E 69, 026113.

Newman, M. E. J., Park, J., 2003. Why social networks are different from other types of networks. Phys. Rev. E 68, 036122.

Niu, S.-C., 2002. A stochastic formulation of the bass model of new-product diffusion. Review of Marketing Science Working Papers 1 (4), 1.

Pastor-Satorras, R., Vespignani, A., May 2001. Epidemic dynamics and endemic states in complex networks. Phys. Rev. E 63 (6), 066117.

Piraveenan, M., Prokopenko, M., Zomaya, A., 2009. Assortative mixing in directed biological networks. IEEE Transactions on Computational Biology and Bioinformatics.

Singla, P., Richardson, M., 2008. Yes, there is a correlation - from social networks to personal behavior on the web. In: Proceeding of WWW'2008.

Sznajd-Weron, K., 2005. Sznajd model and its applications. Act. Phys. Pol. B 36 (8).

Van den Bulte, C., Joshi, Y. V., 2007. New product diffusion with influentials and imitators. Marketing Science 26 (3), 400–421.

Vilpponen, A., Winter, S., Sundqvist, S., 2006. Electronic word-of-mouth in online environments: exploring referral network structure and adoption behavior. Journal of Interactive Advertising 6 (2).

Watts, D. J., 2004. The "new" science of networks. Annual Review of Sociology 30, 243–270.

Watts, D. J., Dodds, P. S., 2007. Influentials, networks and public opinion formation. Journal of Consumer Research 34, 441–458.

Watts, D. J., Peretti, J., 2007. Viral marketing for the real world. Harvard Business Review F0705A.