Quantum Statistics and Entanglement Problems

L. E. H. Trainor
Department of Physics, University of Toronto, Toronto, Canada M5S 1A8

Charles J. Lumsden
Institute of Medical Science, University of Toronto, Toronto, Canada M5S 1A8
(Dated: December 23, 2021)

Interpretations of quantum measurement theory have been plagued by two questions, one concerning the role of observer consciousness and the other the entanglement phenomenon arising from the superposition of quantum states. We emphasize here the remarkable role of quantum statistics in describing the entanglement problem correctly and discuss the relationship to issues arising from current discussions of intelligent observers in entangled, decohering quantum worlds.

PACS numbers: 03.65.Ta, 03.65.Ud, 03.65.Yz 05.30.Fk, 87.19.Bb, 01.70+w
Keywords: entanglement, spin, quantum statistics, decoherence, einselection, Many Worlds Interpretation

INTRODUCTION

In the Copenhagen interpretation of quantum mechanics and the theory of measurement associated with it, one is required a priori to assume the existence of a classical measuring apparatus. To many physicists this has appeared to be a circular argument because the classical apparatus itself is composed of quantum particles, so why is not a quantum mechanical description possible “all the way down”? Recently great attention has been devoted to this problem [1, 2, 3, 4, 5, 6] through the supposition of environmental influences, such as the assumption of a global, all encompassing Schrödinger wave function. In this theory, classical mechanics is recovered from a quantum description by means of decoherence and einselection [7]. It is generally recognized that a basic problem to be resolved in this process is the one arising from the superposition principle of quantum mechanics and its resulting entanglement complication.

The purpose of this letter is to stress the complications for the interpretations in quantum mechanics of many-particle wave functions, first studied seriously by Pauli [8], and to re-emphasize the role of quantum statistics, in distinction to dynamics, in the quantum entanglement problem.

PAULI ENTANGLEMENT

Both in the light of Pauli’s remarks and with an eye toward recognizing the importance of the Bell Theorem [9] in showing that EPR (Einstein, Podolsky and Rosen) were unjustified in claiming [10] that any complete description of reality must include a locality assumption, we use a simple case of a two-particle wave function to illustrate the two points made in the previous paragraph.

First let us consider a general, two-particle wave function (non-identical particles) \( \psi(x_1, x_2) \), isolated with coordinates \( x_1 \) and \( x_2 \) with time evolution implicit. In the interest of exploring EPR separability, we suppose that during some portion of the two-particle history, this wave function separates into a product of two single particle wave functions, one dependent on \( x_1 \), the other on \( x_2 \):

\[
\psi(x_1, x_2) = \phi(x_1)\chi(x_2)
\]

and we enquire into the interpretation of \( \phi(x_1) \). We do this by multiplying both sides of Eq. (1) by \( \chi^*(x_2) \) and integrating over the coordinates of \( x_2 \) (assuming normalized single particle wave functions) to obtain

\[
\phi(x_1) = \int \psi(x_1, x_2)\chi^*(x_2)dx_2
\]

This familiar procedure makes it clear that even in product form, the wave function for particle 1 is entangled with that of particle 2, and that the criterion for locality required by EPR is not realized since a measurement on particle 2 at any time changes \( \chi^*(x_2) \) and thus the integral in Eq. (2), so that \( \phi(x_1) \) is not a true description of particle 1 independently of what happens to particle 2.

The entanglement situation is even more interesting, however, when one considers two identical fermions, since then one has the Pauli requirement that \( \psi(x_1, x_2) \) in Eq. (1) be replaced by

\[
\Psi_A(x_1, x_2) = \frac{1}{\sqrt{2}}[\phi(x_1)\chi(x_2) - \phi(x_2)\chi(x_1)]
\]

We see that entanglement is profoundly enhanced by the Pauli principle since the simplifying procedure leading to Eq. (2) no longer holds. We further note that any generalization to Eq. (3) such as

\[
\psi(x_1, x_2) = \sum_{i,j} C_{ij}\phi_i(x_1)\phi_j(x_2)
\]

where the \( C_{ij} \) are \( N \)-numbers, will have similar properties, particularly when the requirement of antisymmetry is imposed.
In the case of two identical bosons the same argument leads to similar results except a plus sign replaces the minus sign on the right hand side of Eq. (3), with

$$\Psi_S(x_1, x_2) = \frac{1}{\sqrt{2}} [\phi(x_1) \chi(x_2) + \phi(x_2) \chi(x_1)].$$

(5)

Supersymmetry has no consequence for these arguments since we live in a world where supersymmetry has already been broken.

**GLOBAL ENTANGLEMENT**

Insufficient attention has been paid to entanglement due to statistics, particularly in view of the current interest expressed in a global Schrödinger wave function and pointer states. Decoherence due to the “environment” is used to suggest that classical systems can be deduced from overarching quantum systems, thus avoiding the need in the Copenhagen interpretation for a separate classical world (the apparatus and observer) in the measurement process, joined somehow by the punctate collapse of the wave packet.

It is worth emphasis that the symmetrization and antisymmetrization of wave functions for identical bosons, respectively fermions, is not inherent in the dynamics of the Schrödinger wave function theory, but must be imposed outside the dynamical theory itself. There is no dynamical procedure in this picture, for example, that tells two electrons that they must antisymmetrize at some point in their mutual histories. The Pauli principle itself has the effect of entangling the wave functions of all electrons in the universe. This entanglement can often be ignored because the electrons “outside” the system are separated well beyond their dynamical range of interaction with electrons “inside” the system. Second quantization, which is designed to deal with many particle systems, does not alleviate this situation, since the use of commutators, respectively anticommutators, leads to results in consonance with the first quantization picture in these respects, namely that once particles come within the dynamical range of their interactions, the symmetrization/antisymmetrization behavior must be recognized and taken into account.

This aspect of quantum mechanics is well illustrated in the experiments described by Mott and Massey in which electrons in a monochromatic beam were scattered off hydrogen atoms in their ground state, and the angular distributions observed. The results were analyzed and explained by Trainor and Wu and by Corinaldesi and Trainor. TW showed that to obtain the experimental results antisymmetrization of the incident and target electrons had to be taken into account. This could be done by antisymmetrization of the electron pairs in their initial states, with the antisymmetrization then maintained throughout the analysis. CT subsequently established that in the Born-Oppenhemier approximation all of the required interaction matrices could be calculated exactly. The Schrödinger dynamics only gives the correct results when antisymmetrization of the beam electrons and the target electrons is imposed on the formation of the initial states. An electron in the incoming beam could have arrived from Japan and collided with a hydrogen electron in American for all the system cares.

**PAULI, BELL, AND MWI: NO SAFE HARBORS**

Once it became clear from Bell’s theorem and the experimental verification of quantum mechanics that EPR’s assumption was incorrect — reality did not demand local behavior for a complete theory — there were influential attempts to minimize the impact of these results. Ballentine, for example, proposed that the statistical interpretation of quantum mechanics brought it into closer accord with an EPR-type reality than could be realized from a single particle point of view. This is not the case, however. Kunstatter and Trainor showed that in the statistical interpretation, the results of Bell and Aspect also required EPR-like locality assumptions to be abandoned.

Another important attempt to contain the non-locality issue was made by Page using the Many Worlds Interpretation. KT argued, however, that the ideas used to describe observers in quantum measurement theory are largely intuitions based on what cognitive scientists and philosophers of mind now term “folk psychology” and thus highly problematic in their own right: “do intelligent observers exist in quantum mechanics?” The nature and role of conscious observers in quantum mechanics, particularly as raised by Wigner, has continued to plague and challenge interpretation theories. The challenges are even greater for theories based on the proposition that there is a global Schrödinger equation for the Universe as a whole, and that it is “quantum mechanics all the way down”, particularly since there is no accepted quantum (or any other) theory of consciousness. This point is emphasized, for example, in the recent exchange between Tegmark and Hagan et al. on quantum decoherence issues relating to the role of neuronal microtubules as basic quantum computational elements in a theory of self awareness, subjectivity, and memory in the human brain.

The recent paper of Zeh proposing that, when worlds divide in the Everett’s theory, minds also divide is a bold and speculative attempt to deal with such questions as decoherence and MWI, but leaves untouched those raised by KT on whether intelligent behavior as such is even describable by the quantum theories we now have. Indeed, to conjecture in the affirmative is to assert
a locality hypothesis of a different sort within quantum mechanics, namely that mind, self-awareness, and perception as phenomena may be inferred from the theory, i.e. traced to a finite (and thus circumscribed or epistemically “local”) set of assumptions (the only kind of axiom set we currently know how to reason with). Since local realism — EPR-like locality requirements on either the single or quantum statistical levels — has fared so poorly to date, it seems reasonable to conjecture that no safe local harbors exist at all within quantum theory, and that entanglement and decoherence point to problems about observers and the Universe whose solution will entail further significant changes in our understanding of quantum mechanics.

* Correspondence: ltrainor@physics.utoronto.ca; Dept. of Physics, University of Toronto, 180 St. George St., Toronto, Canada M5S 1A8

[1] H. D. Zeh, quant-ph/0204088.
[2] H. D. Zeh, www.iworld.de/~ej/deco/essays.html.
[3] M. Tegmark and H. S. Shapiro, Phys. Rev. E 50, 2538 (1994).
[4] E. Joos, quant-ph/9908008.
[5] W. H. Zurek, quant-ph/9805065.
[6] M. Tegmark and J. A. Wheeler, Sci. Am. 284(Feb), 68 (2001).
[7] W. Pauli, *De Allgemeinen Prinzipien der Wellenmechanik. Handbuch der Physik, 2nd ed., Vol.24/1 reprint*, (J.W. Edwards Publisher, Ann Arbor, 1957).
[8] J. S. Bell, Physics 1, 195 (1964).
[9] A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. 49, 1804 (1982).
[10] A. Einstein, B. Podolski and N. Rosen, Phys. Rev. 47, 779 (1935).
[11] N. F. Mott and H. Massey, *The Theory of Atomic Collisions*, 3rd ed. (Clarendon Press, Oxford, 1965), pp.520–560.
[12] L. E. H. Trainor and T. Y. Wu, Phys. Rev. 89, 273 (1953).
[13] E. Corinaldesi and L. E. H. Trainor, Il Nuovo Cimento 9, 940 (1952).
[14] L. Ballentine, Rev. Mod. Phys. 42, 358 (1970).
[15] G. Kunstatter and L. E. H. Trainor, Am. J. Phys. 52, 598 (1984).
[16] D. N. Page, Phys. Lett. A 91, 57 (1982).
[17] H. Everett III, Rev. Mod. Phys. 29, 454 (1957).
[18] G. Kunstatter and L. E. H. Trainor, Phys. Lett. A 103, 32 (1984).
[19] P. Smith Churchland, *Neurophilosophy: Toward a Unified Science of Mind/Brain*, (MIT Press, Cambridge MA, 1986).
[20] E.P. Wigner, in *The Scientist Speculates*, ed. I. J. Good, (Heinemann, London, 1962).
[21] M. Tegmark, Phys. Rev. E 61, 4194 (2000).
[22] S. Hagan, S. R. Hameroff and J. A. Tuszynski, quant-ph/0005025.
[23] H. D. Zeh, quant-ph/9908084.