LARGE MASS DIPHOTONS FROM
RELATIVISTIC HEAVY ION COLLISIONS

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Abstract

We evaluate the production of large mass diphotons from quark annihilation at BNL
RHIC and CERN LHC energies from central collisions of gold nuclei. The collision
is assumed to lead to either a thermally and chemically equilibrated quark gluon
plasma, or a free-streaming quark gluon gas having an identical initial entropy, or a
chemically equilibrating quark gluon system, with the same entropy at $T = T_c$. We
also obtain an estimate of hard photon pairs from initial state quark annihilation and
find that the thermal production dominates the yield up to $M \approx 4$ GeV at RHIC,
and up to 6 GeV at LHC. A simulation study of decay versus thermal diphotons is
presented.

Key-Words: Relativistic heavy ion collisions, quark gluon plasma, dileptons,
diphotons, thermal mass, quark annihilation, pion annihilation, hydrodynamics,
free-streaming, chemical equilibrium, prompt photon pairs, transverse expansion,
mixed event analysis.

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Relativistic heavy ion collisions are expected to lead to a confirmation of the QCD phase transition from hadronic matter to quark matter and an ephemeral formation of quark gluon plasma (QGP). Analysis of the S and Pb induced collisions at the CERN SPS have brought the search for QGP to an interesting stage. Many of the proposed signatures of QGP have been observed, though, most of them can also be explained to some extent in a purely hadronic picture. It is expected that the issue will be clearly resolved when we have results from BNL RHIC and CERN LHC, as much larger initial temperatures are likely to be attained there.

Dileptons have long been considered as excellent probes of the early stages of relativistic heavy ion collisions. Large mass dileptons are likely to have their origin in the hot and dense stage of QGP and their detection would provide valuable information about this exotic state of matter. We suggest that an experimental detection of large mass diphotons can possibly provide a valuable confirmation of the results obtained from the measurement of dileptons. The theoretical understanding of the basic rates as well as the evolution of the interacting system has improved considerably since the early studies in this direction, necessitating a re-evaluation of the importance of diphoton measurements. Thus, we have utilized the thermal mass of the annihilating quarks while evaluating the rates, which is more appropriate for quarks in a plasma. We have also considered evolution mechanisms which encompass the entire range of possible scenarios describing the expansion of the QGP. We give, as far as we know, the first estimate of hard QCD diphotons from the colliding nuclei. We also present a feasibility study of these measurements by performing a simulation of the likely measurements.

Large mass diphotons from the QGP are produced from quark-antiquark annihilation. The cross section is given by: \[ \sigma \propto \frac{1}{\sqrt{s}} \]
\[ \sigma_{q\bar{q}}(M) = 2\pi\alpha^2 N_c (2S + 1)^2 \sum_q \frac{e_q^4}{M^2 - 4m_q^2} \left[ 1 + \frac{4m_q^2}{M^2} - \frac{8m_q^4}{M^4} \right] \ln \left\{ \frac{M^2}{2m_q^2} \left[ 1 + \left( 1 - \frac{4m_q^2}{M^2} \right)^{1/2} \right] - 1 \right\} \]

\[ - \left[ 1 + \frac{4m_q^2}{M^2} \right] \left[ 1 - \frac{4m_q^2}{M^2} \right]^{1/2}, \quad (1) \]

\[ M \gg m_q \quad 2\pi\alpha^2 N_c (2S + 1)^2 \sum_q \frac{e_q^4}{M^2} \ln \left\{ \frac{M^2}{2.718 m_q^2} \right\}. \quad (2) \]

In the above \( N_c = 3, \ S = 1/2 \) and \( e_q \) is the charge of the quark. This cross section diverges as \( m_q \to 0 \). The early works on diphotons [1] mostly used \( m_q = 5 \text{ MeV} \), which grossly overestimates the rates. We argue that as quarks in a heat bath acquire a thermal mass \( \tilde{m} \) given by \( \tilde{m} = \sqrt{\frac{2\pi\alpha_s/3}{T}} \), this singularity can be regularized by making the natural choice, \( m_q = \tilde{m} \). One can easily check that choosing \( m_q = 5 \text{ MeV} \), instead, will increase the basic cross-section by a factor of \( \sim 3-5 \). Increasing \( m_q \) from \( \tilde{m} \) to \( 2\tilde{m} \), say, will decrease the cross-section by less than 50%. We shall see later that the explicit appearance of \( T \) in the basic cross-section here provides us with an additional probe for the evolution of the temperature [3].

As remarked earlier, we investigate the production of thermal diphotons in three different scenarios, which essentially encompass the entire range of evolution dynamics one can imagine and thus give us the upper and the lower limits of the diphoton yield. As a first step, we consider the formation of a QGP in a thermodynamic (thermal as well as chemical) equilibrium at an initial time \( \tau_0 \) and initial temperature \( T_0 \) in a central collision of two gold nuclei. This case has been studied in great detail in the literature. One can use the condition of isentropic expansion to relate the initial conditions to the particle multiplicity density \( (dN/dy) \) [4]:

\[ T_0^3 \tau_0 = \frac{2\pi^4}{45\zeta(3)\pi R^2 a_Q} \frac{dN}{dy}. \quad (3) \]

For a QGP consisting of u, d, and s quarks, and gluons, \( a_Q = 47.5\pi^2/90 \). \( R \) is the initial transverse dimension of the system. Assuming a rapid thermalization, one may
get an upper limit on the temperature by taking $\tau_0 = 1/3T_0$. Taking $dN/dy = 1735$ for central collision of Au nuclei at RHIC we get an initial temperature of 478 MeV, while the corresponding numbers at LHC are 5624 and 860 MeV respectively. As we are interested in large mass photon pairs, we include only the emission from the QGP phase, i.e., till the system reaches the phase transition temperature $T_c$. We further neglect the transverse expansion which is known to affect the large invariant mass distribution only marginally. The proper time $\tau_q$, when the temperature drops to $T_c$, is obtained from the Bjorken relation $\tau_q = \tau_0 (T_0/T_c)^3$. The invariant mass spectrum for diphotons will then be,

$$\frac{dN}{dM^2dy} = \frac{\pi R^2 M^3}{2(2\pi)^4} \int_{\tau_0}^{\tau_q} \sigma(M,T)(1 - 4m_q^2/M^2)T(\tau)K_1(M/T)\tau \, d\tau.$$  \hspace{1cm} (4)$$

At the other extreme, one may imagine that the system is formed at a proper time $\tau_0$ with a temperature $T_0$ after which the constituents free-stream away from the collision zone. One can evaluate the yield of diphotons from such a gas, in a manner similar to Ref. [5], as

$$\frac{dN}{dM^2dy} = \pi R^2 \frac{M^2 \sigma(M,T_0)T_0^2 \tau_0^3}{2^7 \pi^3} \ln \left( \frac{R}{\tau_0} \right) \exp \left( -\frac{M}{T_0} \right) \left( 1 + 2.19 \frac{T_0}{M} \right).$$  \hspace{1cm} (5)$$

It is quite likely that the quark gluon system may be neither in thermal equilibrium nor in chemical equilibrium when produced initially. Even though a thermal (kinetic) equilibrium may be attained quickly, the system may still be away from chemical equilibrium, which may evolve with passage of time due to a number of partonic reactions. Biró et al. [10], for example, have studied the evolution of chemical equilibration with initial conditions determined from the HIJING model [11]. In order to have a meaningful comparison with the two descriptions above, we use the time variations of quark fugacity and temperature obtained by Strickland [12], with the stipulation that the particle multiplicity density at the end of the QGP phase is equal to the values of $dN/dy$ used here. Now the diphoton spectrum is obtained from an expression similar to Eq. 4 with the additional introduction of the quark fugacity $(\lambda_q)$, taken from the Fig. 1 of Ref. [12].
In Fig. 1 we give our results for the invariant mass distribution of the large mass photon pairs for RHIC energies. A number of observations are in order here. While the slope of the free-streaming description is essentially determined by $T_0$ (see Eq. 5), the temperature drops from $T_0$ to $T_c$ in the hydrodynamic description, thus leading to an effectively steeper slope. Further, the yield for the equilibrating plasma is smaller due to the appearance of the square of the quark-fugacity in the expression and a more rapid cooling. Similar results are seen for the LHC energies as well (see Fig. 2).

We must add here that the results for the equilibrated hydrodynamic evolution and those for the free-streaming are obtained by using $\tau_0 = 1/3T_0$ (see above), while those for the non-equilibrium hydrodynamics have $\tau_0 = 0.3$ fm/c. One may get yet another estimate for these measurements by starting the equilibrated hydrodynamic evolution from $\tau_0 = 0.3$ fm/c (see Figs. 1 & 2). Now the results for small $M$ get quite similar to that for the non-equilibrium hydrodynamics. This looks surprising at first. Recall that our evolution scenarios are tailored to lead to a given final entropy (multiplicity). This ensures that during the later stages the conditions of the equilibrium and the non-equilibrium hydrodynamic evolution scenarios are nearly identical, leading to the similarity of these predictions at low $M$.

The diphoton production from the hard QCD annihilation of quarks in the colliding nuclei at $y = 0$ is given by:

$$\frac{dN}{dM^2dy} = \frac{2\alpha^2}{sM^2N_c} T_{AB} \left[ \ln \left( \frac{M^2 - p_c^2}{p_c^2} \right) - \left( 1 - 2p_c^2/M^2 \right) \right] \times \sum_q e_q^A \left[ q^{N_p}(x, M^2)\bar{q}^{N_t}(x, M^2) + \bar{q}^{N_p}(x, M^2)q^{N_t}(x, M^2) \right],$$

where $x = M/\sqrt{s}$ and $\sqrt{s}$ is the c.m. energy of the colliding nucleons. $N_p$ and $N_t$ denote nucleons in the projectile and target respectively. We have introduced an arbitrary cut-off on the momentum transfer $p_c$ (=2 GeV), so that pQCD remains applicable for such cases and used the MRSD−′ set of parametrizations [13] for the quark structure functions. $T_{AB}$ (= 293.4 fm$^{-2}$) is the nuclear thickness function at zero impact parameter, corresponding to central Au+Au collisions. The result
of this analysis for the RHIC and LHC energies are also given in Figs. 1 and 2, respectively. We find that thermal pairs dominate the spectrum up to 4 GeV at RHIC and up to 6 GeV at LHC.

What additional information will large mass diphotons provide which we shall not already have from the measurement of dileptons? To explore this, we have evaluated the invariant mass distribution for dileptons as well as diphotons produced from a thermally and chemically equilibrated QGP, which expands and cools to a temperature $T_c$ (160 MeV), goes into a mixed phase where a phase transition occurs, and when all of the QGP has converted into hadrons, again expands and cools till freeze-out temperature $T_f$ (100 MeV) is reached. We consider a boost invariant longitudinal expansion without and with, a cylindrically symmetric, transverse expansion. We include the annihilation of pions in the hadronic matter and annihilation of quarks in the QGP as sources of dileptons \[7\] and diphotons \[1, 6\].

In Fig. 3 we have plotted the ratio of diphotons to dileptons as a function of $M$ at RHIC and LHC energies. Firstly, we note that the structure seen in this ratio has its origin in the pion form-factor leading to the dileptons. The minimum at $M \approx 0.8$ GeV comes from the $\rho$ peak in the pion form-factor and the rapid rise there-after reflects the sharp decrease in this. The maximum around 1.4 GeV corresponds to the region where the (structure-less) quark annihilation process starts dominating. The differences in the $1+1$ and the $3+1$ dimensional cases are mostly confined to lower masses, which have their origin in the late stages of the evolution when the transverse expansion of the system is truly large.

It is easy to verify that if we ignore the $T$ dependence of the thermal mass of the quarks appearing in Eq. 1, a universal curve would be obtained for this ratio at large masses at all energies. The temperatures likely to be attained at LHC are larger than those at RHIC. Thus, in a purely longitudinal expansion these ratios at larger masses are smaller at LHC (recall the factor $\sim \ln(M^2/2.7m_q^2)$ in expression for cross-section given by Eq. 2). The transverse expansion of the system leads to a more rapid cooling of the plasma and thus higher temperatures are retained only
for a shorter duration. This will further increase these ratios, as indeed we see from Fig. 3. Thus we note that large mass diphotons sensitively reveal the details of the dynamics of the evolution of the plasma, especially when studied together with dileptons. The predicted variation, if observed, will also confirm our understanding of the temperature dependence of the thermal mass of quarks.

Will it be possible at all to isolate diphotons from the huge background of photons coming from the decay of $\pi^0$ mesons? It is quite clear that the background of decay photons can be arbitrarily reduced by choosing a large enough $p_T$. The search for diphotons will have to start with an accurate reconstruction of the decay photons so that any new source of photon pairs could be identified. We utilize the method of mixed-event analysis pioneered by the WA80 collaboration. We illustrate the situation at LHC energies and approximate the $p_T$ distribution of pions $\sim \exp(-p_T/T)$, with $T \simeq 300$ MeV. The invariant mass spectrum for the photon-pairs from the decay of these pions in a sample of ten million central events is shown in Fig. 4(a). Similarly the invariant mass spectrum for the mixed events was also generated. This constitutes the background (see, e.g., Ref. [14]). A subtraction of the later from the invariant mass spectrum for the real events is shown in Fig. 4(b). The $\pi^0$ peak is seen very clearly. The diphotons from our calculations are seen to stand out clearly against this background. This simulation holds out the hope that diphotons could be isolated if high statistics data are available. It is important to note that the structure of the subtracted background is quite different from the structure of the diphoton signal. Very similar results were obtained by fitting the background (e.g., in Fig. 4a) to a polynomial and subtracting the fitted spectrum from the invariant mass distribution [14].

In brief, we have evaluated the large mass invariant mass spectrum of thermal diphotons for a number of plausible scenarios in relativistic heavy ion collisions. An estimate of prompt diphotons is also made. It is shown that a comparison of diphotons and dileptons from heavy ion collisions can lead to an interesting confirmation of the temperature dependence of the thermal mass of quarks. Finally a simulation
of decay versus thermal diphoton production at LHC energies is performed which shows that such measurements could indeed be feasible, once high statistics data are available.

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Figure Captions:

Figure 1: Invariant mass distribution of photon pairs from $q\bar{q}$ annihilation for the Au+Au system at RHIC energies in a fully equilibrated QGP (Eq. Hydro), a free-streaming gas of quarks and gluons, and a chemically equilibrating (Noneq. Hydro) quark gluon system. Invariant mass distribution of hard photon pairs (Hard) from initial state $q\bar{q}$ annihilation for momentum transfer greater than 2 GeV is also shown.

Figure 2: Same as Fig. 1 for LHC energies.

Figure 3: Ratio of diphoton and dilepton invariant mass distribution for a hadronizing QGP at RHIC and LHC energies. Sources are $q\bar{q} \rightarrow \gamma\gamma$ and $q\bar{q} \rightarrow \mu^+\mu^-$ in the QGP, and $\pi^+\pi^- \rightarrow \gamma\gamma$ and $\pi^+\pi^- \rightarrow \mu^+\mu^-$ in the hadronic phase. The solid and dashed lines indicate calculations with and without transverse expansion.

Figure 4: (a) Distribution of invariant mass generated from photon pairs of the same events at LHC energies. (b) Invariant mass distribution of thermal diphotons and the mixed-event subtracted background of photon pairs. The difference is normalized so that the integral from $M = 0.2$ to 3.5 GeV vanishes. The hatched bar shows the $\pi^0$ peak.
Au+Au @ RHIC

- $\tau_i = \frac{1}{3} T_i$
- $\tau_i = 0.3 \text{ fm}/c$

Eq Hydro
Noneq Hydro
Free Streaming
Au+Au @ LHC

- $\tau_i=1/3T_i$ Equilibrated Hydro
- $\tau_i=0.3$ fm/c Equilibrated Hydro
- Noneq Hydro
- Free Streaming

$dN_{\gamma\gamma}/dM^2 dy$ (GeV$^{-2}$)

M (GeV)
