Radiative muon (pion) pair production in high energy electron-positron annihilation process

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Process of muon (pion) pair production with small invariant mass in the electron–positron high–energy annihilation, accompanied by emission of hard photon at large angles, is considered. We find that the Dell–Yan picture for differential cross section is valid in the charge–even experimental set–up. Radiative corrections both for electron block and for final state block are taken into account.

I. INTRODUCTION

Initial state radiation processes in high energy electron-positron annihilation processes with creation of a hadronic system provide the laboratory for studying the hadron states created by virtual photon. Special interest is paid to the case when the invariant mass of hadron system $\sqrt{s_1}$ is small compared with the center–of–mass total energy $\sqrt{s} = 2\varepsilon$. Here such interesting physical quantities as the form factor of pion and nucleon can be investigated. The process of radiative annihilation into muon and pion pairs, considered here, plays a crucial role as a normalization one as well as the ones, providing an essential background process in studying the hadron creation. Description of its differential cross section with a rather high level of accuracy (better than 0.5%) is the goal of our paper.

We specify the kinematics of radiative muon pair creation process

$$e_-(p_-) + e_+(p_+) \rightarrow \mu_-(q_-) + \mu_+(q_+) + \gamma(k_1),$$

(1)

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as follows:

\[ p_\pm^2 = m^2, \quad q_\pm^2 = M^2, \quad k_1^2 = 0, \]
\[ \chi_\pm = 2k_1 p_\pm, \quad \chi'_\pm = 2k_1 q_\pm, \quad s = (p_- + p_+)^2, \quad s_1 = (q_- + q_+)^2, \]
\[ t = (p_- - q_-)^2, \quad t_1 = (p_+ - q_+)^2, \quad u = (p_- - q_+)^2, \quad u_1 = (p_+ - q_-)^2. \]

(2)

where \( m \) and \( M \) are the electron and muon (pion) masses, respectively. Throughout the paper we will suppose

\[ s \sim -t \sim -t_1 \sim -u \sim -u_1 \sim \chi_\pm \sim \chi'_\pm \gg s_1 \sim M^2 \gg m^2. \]

(3)

Situation where \( s \gg s_1 \gg M^2 \) is allowed too.

We will systematically omit the terms of order \( M^2/s \) and \( m^2/s_1 \) compared with ones of order of magnitude of unity. In \( \mathcal{O}(\alpha) \) radiative corrections we drop also terms suppressed by the factor \( s_1/s \). A kinematical diagram of the process under consideration is drawn in Fig. 1.

![Kinematical diagram](image)

Fig. 1: Kinematical diagram for a radiative event with a small invariant mass \( s_1 \).

In this paper we will consider only the charge–even part of the differential cross section, which can be measured in an experimental set–up blind to charges of the created particles. A detailed study of the charge–odd part of the radiative annihilation cross section in general kinematics will be presented elsewhere.

Our paper is organized as follows. The next Section is devoted to the Born–level cross section. Radiative corrections to the final and initial states are considered in Sect. IIII. In Conclusion we give some numerical illustration and discuss the resulting precision. Some details of calculations and useful formulae are given in Appendix.
II. THE BORN–LEVEL CROSS SECTION

Within the Born approximation, the matrix element of the initial state emission process has the form:

\[ M_B = \frac{(4\pi\alpha)^{3/2}}{s_1} \tilde{v}(p_+) \left[ \gamma_\rho \hat{p}_- - \hat{k}_1 + m \right] \hat{\epsilon} + e \hat{\epsilon} \left[ \gamma_\rho \hat{p}_+ + \hat{k}_1 + m \right] u(p_-) J^\rho \]

with

\[ J^\rho = \bar{u}(q_-) \gamma^\rho u(q_+) \]

for muon pair production, and

\[ J_{\pi}^\rho = (q_- - q_+)\rho F_{\text{str}}(s_1) \]

for the case of charged pions, \( F_{\text{str}}(s_1) \) is the pion strong–interaction form factor.

The corresponding contributions to the cross section is

\[ \frac{d\sigma_B}{d\Omega} = \frac{\alpha^3}{8\pi^2 s s_1} R^j, \quad R^j = B^{\rho\sigma} i^{(0)j}_{\rho\sigma}, \quad i^{(0)j}_{\rho\sigma} = \sum J^j_{\rho}(J^j_{\sigma})^*, \quad j = \mu, \pi, \]

\[ B_{\rho\sigma} = B_g g_{\rho\sigma} + B_{11}(p_- p_-)_{\rho\sigma} + B_{22}(p_+ p_+)_{\rho\sigma}, \]

\[ B_g = -\frac{(s_1 + \chi_+)^2 + (s_1 + \chi_-)^2}{\chi_+ \chi_-}, \quad B_{11} = -\frac{4s_1}{\chi_+ \chi_-}, \quad B_{22} = -\frac{4s_1}{\chi_+ \chi_-}, \]

where we used the short notation \((qq)_{\rho\sigma} = q_\rho q_\sigma, (pq)_{\rho\sigma} = p_\rho q_\sigma + q_\rho p_\sigma\). For the case with the muon final state

\[ i^{(0)\mu}_{\rho\sigma} = 4 \left[ (q_+ q_-)_{\rho\sigma} - g_{\rho\sigma} \frac{s_1}{2} \right]. \]

For the case of pions,

\[ i^{(0)\pi}_{\rho\sigma} = |F_{\text{str}}(s_1)|^2 (q_- - q_+)_\rho (q_- - q_+)_\sigma. \]

Here we put also the formulae for Born cross section in the case when only terms of order \((m^2/s)\) are omitted and hard photon are emitted on large angle \(\theta\)(see [9]):

\[ \frac{d\sigma_B}{d\Omega} = \frac{\alpha^3 |\vec{q}_1| \omega}{2\pi^2 s 2\varepsilon - \omega + \omega \cos \theta} A, \]

\[ A = \frac{t^2 + t_1^2 + u^2 + u_1^2}{s s_1} [W_{\text{even}} + W_{\text{odd}}], \]

\[ W_{\text{even}} = \frac{s}{\chi_+ \chi_-} + \frac{s_1}{\chi'_{+} \chi'_-} + O \left( \frac{M^2}{s} \right), \]

\[ W_{\text{odd}} = -\frac{t_1}{\chi_+ \chi'_+} - \frac{t}{\chi_- \chi'_-} + \frac{u}{\chi_- \chi'_+} + \frac{u_1}{\chi_+ \chi'_-} + O \left( \frac{M^2}{s} \right), \]
for muon pair production, and for pion pair production

\[ \frac{d\sigma}{d\Gamma} = \frac{\alpha^3}{4\pi^2s} \frac{tu + t_1u_1}{ss_1} [W_{\text{even}} + W_{\text{odd}}]. \]  

(11)

Functions \( W_{\text{even}} \) and \( W_{\text{odd}} \) give rise to the charge–even and charge–odd contributions to the cross section. In quantity \( W_{\text{even}} \) the term \( s/(\chi_+\chi_-) \) is the most important in our kinematic region. In what follows, only the even part will be taken into account.

In our kinematics some simplification takes place:

\[ B_g = -\frac{21 + c^2}{1 - c^2}, \quad B_{11} = B_{22} = -\frac{16s_1}{s^2(1 - c^2)}. \]  

(12)

with

\[ c = \cos(\vec{p}_-\vec{q}_-) = \cos \theta. \]  

(13)

We will suppose that the emission angle of hard photon lies outside the narrow cones around the beam axis \( \pi - \theta_0 < \theta_1 < \theta_0 \), with \( \theta_0 \ll 1, \theta_0\epsilon \gg M \). For initial state radiative process (in paper \cite{2} the scattering channel was considered). The phase volume of the final particles is

\[ d\Gamma = \frac{d^3q_-}{\varepsilon_-} \frac{d^3q_+}{\varepsilon_+} \frac{d^3k_1}{\omega_1} \delta^4(p_+ + p_- - q_+ - q_- - k_1). \]  

(14)

For the case of small invariant mass of created pair \( s_1 \ll s \) it can be rewritten as (see fig.):

\[ d\Gamma = \pi^2 dx_- dc ds_1, \]  

(15)

with

\[ x_\pm = \frac{\varepsilon_\pm}{\varepsilon}, \quad x_+ + x_- = 1, \quad \omega = \frac{1}{2}\sqrt{s}, \quad \chi_\pm = \frac{s(1 \pm c)}{2}, \]

\[ t = -\frac{sx_-(1 - c)}{2}, \quad t_1 = -\frac{sx_+(1 + c)}{2}, \quad u = -\frac{sx_+(1 - c)}{2}, \quad u_1 = -\frac{sx_-(1 + c)}{2}. \]

In the limiting case of small invariant mass of the created pair omitting terms of the order \( s_1/s \) and \( M^2/s \), the Born level cross section takes a simple form:

\[ d\sigma_B^{(\mu)}(p_-, p_+; k_1, q_-, q_+) = \frac{\alpha^3(1 + c^2)}{ss_1(1 - c^2)} \left[ 2\sigma + 1 - 2x_-x_+ \right] dx_- dc ds_1, \]  

(16)

\[ d\sigma_B^{(\pi)}(p_-, p_+; k_1, q_-, q_+) = \frac{\alpha^3(1 + c^2)}{ss_1(1 - c^2)} \left[ -\sigma + x_-x_+ \right] dx_- dc ds_1, \]

\[ \frac{1}{2}(1 - \beta) < x_- < \frac{1}{2}(1 + \beta), \quad \beta = \sqrt{1 - \frac{4M^2}{s_1}}, \quad \sigma = \frac{M^2}{s_1}. \]

Here \( \beta \) is the velocity of a pair component in the center–of–mass reference frame of the pair.
III. RADIATIVE CORRECTIONS

Radiative corrections (RC) can be separated into three gauge–invariant parts. They can be taken into account by the formal replacement (see (7)):

\[
\frac{R^j_{\rho\sigma}}{s_1^2} \rightarrow \frac{K^j_{\rho\sigma} J^j_{\rho\sigma}}{s_1^2 |1 - \Pi(s_1)|^2}
\]

(17)

where \(\Pi(s_1)\) describes the vacuum polarization of the virtual photon (see Appendix); \(K^j_{\rho\sigma}\) is the initial–state emission Compton tensor with RC taken into account; \(J^j_{\rho\sigma}\) is the final state current tensor with \(O(\alpha)\) RC.

First we consider the explicit formulae for RC due to virtual, soft, and hard collinear final state emission. As concerning RC to the initial state for the charge–blind experimental set–up considered here, we will use the explicit expression for the Compton tensor with heavy photon \(K^j_{\rho\sigma}\) calculated in the paper of one of us [2] for the scattering channel and apply the crossing transformation. Possible contribution due to emission of an additional real photon from the initial state will be taken into account too. In conclusion we give the explicit formulae for cross section, consider separate the kinematics of collinear emission, and estimate the contribution of higher orders of perturbation theory (PT).

A. Corrections to the final state

The third part is related with lowest order RC to muon and pion current

\[
J_{\rho\sigma} = i^{(v)}_{\rho\sigma} + i^{(s)}_{\rho\sigma} + i^{(h)}_{\rho\sigma}.
\]

(18)

The virtual photon contribution \(i^{(v)}_{\rho\sigma}\) takes into account the Dirac and Pauli form factors of muon current

\[
J^{(v)}_{\rho} = \bar{u}(q–)[\gamma_{\rho}F_1(s_1) + \frac{\hat{q}\gamma_{\rho} - \gamma_{\rho}\hat{q}}{4M}F_2(s_1)]u(q+), \quad q = q_+ + q_–, \quad s_1 = q^2.
\]

(19)

We have

\[
B^{\rho\sigma}i^{(v)}_{\rho\sigma} = B_g \sum |J^{(v)}_{\rho}|^2 + B_{11} \left[ \sum |p–J^{(v)}_{\rho}|^2 + \sum |p+J^{(v)}_{\rho}|^2 \right].
\]

(20)

Here \(\Sigma\) means sum over muon spin states and

\[
\sum |J^{(v)}_{\rho}|^2 = \frac{\alpha}{\pi} \left[-8(s_1 + 2M^2) f^{(\mu)} - 12s_1 f_2^{(\mu)} \right],
\]

\[
\sum |J^{(v)}_{\rho} p_\pm|^2 = \frac{\alpha}{\pi} s^2 (1 \pm c) (x_+ x_– f_1^{(\mu)} - \frac{1}{4} f_2^{(\mu)}),
\]

(21)
see Appendix for details. For the pion final state we have

\[B^{\rho \sigma} i^{(\nu \pi)}_{\rho \sigma} = 2 B^{\rho \sigma} i^{(0\pi)}_{\rho \sigma} f_{\pi}^{QED},\]

\[B^{\rho \sigma} i^{(0\pi)}_{\rho \sigma} = |F^{\text{str}}(s_1)|^2 \left[ (4M^2 - s_1)B_y + \frac{1}{8}s^2B_{11}(x_+ - x_-)^2(1 + c^2) \right]. \quad (22)\]

The explicit expression for the electromagnetic form factor of pion is given in Appendix.

The soft photon correction to the final state currents reads

\[\Delta_{1'2'} = \frac{\alpha}{\pi} \Delta_{1'2'} (s_1), \quad \Delta i^{(s\mu)}_{\rho \sigma} = \frac{\alpha}{\pi} \Delta i^{(0\mu)}_{\rho \sigma},\]

\[\Delta_{1'2'} = -\frac{1}{4\pi} \int \frac{d^3k}{\omega} \left( \frac{q_+ - q_-}{q_+ k - q_- k} \right)^2 \Bigg|_{\omega \Delta \varepsilon} \left( \frac{1 + \beta^2}{2 \beta} \ln \frac{1 + \beta}{1 - \beta} - 1 \right) \ln \frac{\Delta \varepsilon^2 M^2}{\varepsilon + \varepsilon - \lambda^2} \]

\[+ \frac{1 + \beta^2}{2 \beta} \left[ -g - \frac{1}{2} \ln^2 \frac{1 + \beta}{1 - \beta} - \ln \frac{1 + \beta}{1 - \beta} \ln \frac{1 - \beta^2}{4} - \frac{\pi^2}{6} - 2 \text{Li}_2 \left( \frac{\beta - 1}{\beta + 1} \right) \right],\]

\[g = 2 \beta \int_0^1 \frac{dt}{1 - \beta^2 t^2} \ln \left( 1 + \frac{1 - t^2 (x_+ - x_-)^2}{4 x_+ x_-} \right) = \ln \left( \frac{1 + \beta}{1 - \beta} \right) \ln (1 + z - z/\beta^2),\]

\[+ \text{Li}_2 \left( \frac{1 - \beta}{1 + \beta / r} \right) + \text{Li}_2 \left( \frac{1 - \beta}{1 - \beta / r} \right) - \text{Li}_2 \left( \frac{1 + \beta}{1 - \beta / r} \right) - \text{Li}_2 \left( \frac{1 + \beta}{1 + \beta / r} \right),\]

\[\beta = \sqrt{1 - \frac{4M^2}{s}}, \quad \varepsilon = \frac{1}{2} \left( \sqrt{x_+} - \sqrt{x_-} \right)^2, \quad r = |x_+ - x_-|, \quad \Delta = \frac{\Delta \varepsilon}{\varepsilon}.\]

The contribution of an additional hard photon (with momentum \(k_2\)) emission by the muon block, provided \(s_1 = (q_+ + q_- + k_2)^2 \sim s_1 \ll s\) can be found by the expression

\[B^{\rho \sigma} i^{(h\mu)}_{\rho \sigma} = \frac{\alpha}{4\pi^2} \int \frac{d^3k_2}{\omega_2} B^{\rho \sigma} \sum J^{(\gamma)}_{\rho}(J^{(\gamma)}_{\sigma})^* \Bigg|_{\omega_2 \Delta \varepsilon}, \quad (24)\]

with

\[\sum |J^{(\gamma)}_{\rho}|^2 = 4Q^2(s_1 + 2k_2q_- + 2k_2q_+ + 2M^2) - 8 \frac{(k_2q_-)^2 + (k_2q_+)^2}{(k_2q_-)(k_2q_+)};\]

\[Q = \frac{q_- - q_+ - k_2}{q_- - q_+};\]

and

\[\sum |J^{(\gamma)}_{\rho}p_{\pm}|^2 = -8Q^2(q_- p_{\pm})(q_+ p_{\pm}) + 8(p_{\pm}k_2) \left( Qq_+ \frac{p_+ q_+ - p_- q_-}{q_+ k_2} - Qq_- \frac{p_- q_- - p_+ q_+}{q_- k_2} \right)\]

\[+ 8(p_{\pm}k_2) \left( \frac{p_+ q_- + p_- q_+}{q_+ k_2} \right) + 8(p_{\pm}Q)(p_+ q_+ - p_- q_-) - 8 \frac{(k_2p_{\pm})^2}{(k_2q_+)(k_2q_-)}. \quad (25)\]
For the case of charged pion pair production the radiative current tensor has the form

\[ i^{(h\pi)}_{\rho\sigma} = -\frac{\alpha}{4\pi^2} \int \left| F_{\pi}^{\text{str}}(s_1) \right|^2 \frac{d^3k_2}{\omega_2} \left[ \frac{M^2}{\chi^2_{2-}}(Q_1Q_1)_{\rho\sigma} + \frac{M^2}{\chi^2_{2+}}(Q_2Q_2)_{\rho\sigma} - \frac{q_+q_-}{\chi_{2+}\chi_{2-}}(Q_1, Q_2)_{\rho\sigma} \right. \\
+ \frac{1}{\chi_{2-}}(Q_1, q_-)_{\rho\sigma} + \frac{1}{\chi_{2+}}(Q_2, q_+)_{\rho\sigma} \right] \bigg|_{\omega_2 > \Delta \varepsilon}, \]

(26)

One can check that the Bose symmetry and the gauge invariance condition is valid for the pionic current tensor. Namely it is invariant regarding the permutation of the pion momenta operation and turns to zero after conversion with 4-vector \( q \).

The total sum of RC to the muon current does not depend on \( \Delta \varepsilon/\varepsilon \).

**B. Corrections to the initial state**

Let us now consider RC to the Compton tensor with RC, which describe virtual corrections to the initial state. In our kinematical region it will be convenient to rewrite the tensor explicitly extracting large logarithms. We will distinguish two kinds of large logarithms:

\[ l_s = \ln \frac{s}{m^2}, \quad l_1 = \ln \frac{s}{s_1}. \]

(27)

We rewrite the Compton tensor \([17]\) in the form:

\[ K_{\rho\sigma} = \left( 1 + \frac{\alpha}{2\pi} \rho \right) B_{\rho\sigma} + \frac{\alpha}{2\pi} \left[ \tau_1 g_{\rho\sigma} + \tau_{11}(p_+p_-)_{\rho\sigma} + \tau_{22}(p_+p_+)_{\rho\sigma} - \frac{1}{2} \tau_{12}(p_-p_+)_{\rho\sigma} \right], \]

(28)

with \( \tau_i = a_i l_1 + b_i \) and

\[ a_{11} = -\frac{2s_1}{\chi_+\chi_-} \left[ \frac{2b^2}{\chi_+\chi_-} + \frac{4s^2 + b\chi_-}{a^2} - \frac{b^2(2c - \chi_-)}{c^2\chi_-} - \frac{2s + \chi_+}{\chi_+} \right], \]

(29)

\[ b_{11} = \frac{2}{\chi_+\chi_-} \left[ -s_1(1 + \frac{s^2}{\chi_+^2})G_- - s_1 \left( 2 + \frac{b^2}{\chi_-^2} \right) G_+ - \frac{s_1b^2(2c - \chi_-)}{c^2\chi_-} \ln \frac{s}{\chi_+} \right. \\
- \frac{s_1}{\chi_+}(2s + \chi_+) \ln \frac{s}{\chi_-} - \frac{4(s^2 + b\chi_-)}{a} - 4s - 2s_1 - \chi_+ - \frac{b^2}{c} \bigg], \]

(30)
\begin{align}
  a_{12} &= -\frac{2s_1}{\chi_+\chi_-} \left[ -\frac{4ss_1}{a} + \frac{8(\chi_+\chi_- - s^2)}{a^2} - \frac{4s}{a} + 4ss_1 \left( \frac{1}{c\chi_-} + \frac{1}{b\chi_+} \right) \right] \\
  &+ (2ss_1 + 4\chi_+\chi_-) \left( \frac{1}{c^2} + \frac{1}{b^2} \right), \\
  b_{12} &= \frac{2}{\chi_+\chi_-} \left[ \frac{2s_1}{\chi_+^2} (sc - \chi_-\chi_+) G_- + \frac{2s_1}{\chi_-^2} (sb - \chi_-\chi_+) G_+ \\
  &+ s_1 \left( \frac{2ss_1 + 4\chi_-\chi_+}{c^2} \right) \ln \frac{s}{\chi_+} + s_1 \left( \frac{2ss_1 + 4\chi_-\chi_+}{b^2} \right) \ln \frac{s}{\chi_-} \\
  &+ \frac{8(s^2 - \chi_+\chi_-)}{a} - 2s \left( \frac{\chi_+}{c} + \frac{\chi_-}{b} \right) + 2s_1 + 10s \right],
\end{align}

(31)

\begin{align}
  a_g &= -2s \left( \frac{s_1}{\chi_+\chi_-} - \frac{2}{a} \right) + \frac{c}{\chi_+} \left( \frac{3s}{b} - 1 \right) + \frac{b}{\chi_-} \left( \frac{3s}{c} - 1 \right), \\
  b_g &= -\frac{1}{\chi_+\chi_-} \left( \frac{ss_1 + 2sb + \chi_-^2}{\chi_-} \right) G_- - \frac{1}{\chi_-\chi_+} \left( \frac{ss_1 + 2sc + \chi_+^2}{\chi_+} \right) G_+ \\
  &- \frac{c}{\chi_+} \left( \frac{3s}{b} - 1 \right) \ln \frac{s}{\chi_-} - \frac{b}{\chi_-} \left( \frac{3s}{c} - 1 \right) \ln \frac{s}{\chi_+} + \frac{2s^2 - \chi_+^2 - \chi_-^2}{2\chi_+\chi_-}, \\
  a &= -(\chi_+ + \chi_-), \quad b = s_1 + \chi_-, \quad c = s_1 + \chi_+,
\end{align}

(32)

\begin{align}
  G_- &= -\ln^2 \frac{\chi_-}{s} + \frac{2\pi^2}{3} - 2Li_2 \left( 1 - \frac{s_1}{s} \right) - 2Li_2 \left( 1 + \frac{s_1}{\chi_-} \right), \\
  G_+ &= -\ln^2 \frac{\chi_+}{s} + \frac{2\pi^2}{3} - 2Li_2 \left( 1 - \frac{s_1}{s} \right) - 2Li_2 \left( 1 + \frac{s_1}{\chi_+} \right), \\
  \tau_{22}(\chi_-, \chi_+) &= \tau_{11}(\chi_+, \chi_-).
\end{align}

Infrared singularity (the presence of \textit{photon mass} \(\lambda\) in \(\rho\)) is compensated by taking into account soft photon emission from the initial particles:

\begin{equation}
  \frac{d\sigma}{d\sigma^\text{soft}} = \frac{d\sigma}{d\sigma^0} \Delta_{12},
\end{equation}

(35)

\begin{equation}
  \Delta_{12} = \frac{1}{4\pi} \int \frac{d^3k}{\omega} \left. \frac{2}{(p_+ + k_+ - p_- + k_-)} \right|_{\omega \leq \Delta\varepsilon} = 2(l_s - 1) \ln \frac{m\Delta\varepsilon}{\lambda\varepsilon} + \frac{l_s^2 - \pi^2}{3},
\end{equation}

as a result the quantity \(\rho\) in formulae (28) will be

\begin{equation}
  \rho \to \rho_\Delta = (4 \ln \frac{\Delta\varepsilon}{\varepsilon} + 3)(l_s - 1) - 3l_1 + \frac{2\pi^2}{3} - \frac{3}{2}.
\end{equation}

(36)

Cross section of two hard photon emission for the case when one of them is emitted collinearly to the incoming electron or positron can be obtained by means of the quasi–real electron method [7]:

\begin{equation}
  \frac{d\sigma_{\gamma\gamma, \text{coll}}}{dx_- dc ds_1} = dW_{p_-}(k_3) \frac{d\tilde{\sigma}_{B}(p_- (1 - x_3), p_+; k_1, q_+, q_-)}{dx_- dc ds_1} \\
  + dW_{p_+}(k_3) \frac{d\tilde{\sigma}_{B}(p_-, p_+ (1 - x_3); k_1, q_+, q_-)}{dx_- dc ds_1},
\end{equation}

(37)
with
\[
dW_p(k_3) = \frac{\alpha}{\pi} [(1 - x_3 - \frac{x_3^2}{2}) \ln(\varepsilon \theta_0)^2 - (1 - x_3)] \frac{dx_3}{x_3}, \quad x_3 = \frac{\omega_3}{\varepsilon}, \quad x_3 > \frac{\Delta \varepsilon}{\varepsilon}. \tag{38}
\]

Here we suppose that the polar angle \(\theta_3\) between the directions of the additional collinear photon and the beam axis does not exceed some small value \(\theta_0 \ll 1, \varepsilon \theta_0 \gg m\).

The \textit{boosted} differential cross section \(d\tilde{\sigma}_B^\mu(p_-x, p_+y; k_1, q_+, q_-)\) with reduced momenta of the incoming particles reads (compare with Eq. (16))
\[
\begin{align*}
\frac{d\tilde{\sigma}_B^\mu}{dx \cdot dc ds_1} &= \frac{\alpha^3(1 + 2\sigma - 2\nu_+\nu_-)(x_+^2(1 - c)^2 + x_2^2(1 + c)^2)}{s_1 s x x_2(1 - c^2)(x_1 + x_2 + c(x_2 - x_1))}, \\
\frac{d\tilde{\sigma}_B^\pi}{dx \cdot dc ds_1} &= \frac{\alpha^3(\nu_- (1 - \nu_-) - \sigma)(x_+^2(1 - c)^2 + x_2^2(1 + c)^2)}{s_1 s x^2 x_2^2(1 - c^2)(x_1 + x_2 + c(x_2 - x_1))}, \\
\nu_- &= \frac{x_2}{y_2}, \quad y_2 = \frac{2x_1 x_2}{x_1 + x_2 + c(x_2 - x_1)}. \tag{39}
\end{align*}
\]

In a certain experimental situation, an estimate of the contribution of the additional hard photon emission outside the narrow cones around the beam axes. It can be estimated by
\[
\frac{d\sigma^j}{dx \cdot dc ds_1} = \frac{\alpha}{4 \pi^2} \int d^3k_3 \left[ \frac{\varepsilon^2 + (\varepsilon - \omega_3)^2}{\varepsilon \omega_3} \right] \left\{ \frac{1}{k_3 p_-} \frac{d\sigma_B^j(p_-(1 - x_3), p_+; k_1, q_+, q_-)}{dx \cdot dc ds_1} + \frac{1}{k_3 p_+} \frac{d\sigma_B^j(p_-(1 - x_3); k_1, q_+, q_-)}{dx \cdot dc ds_1} \right\} \bigg|_{\theta_3 \geq \theta_0, \Delta \varepsilon < \omega_3 < \omega_1}, \quad x_3 = \frac{\omega_3}{\varepsilon}. \tag{40}
\]

It is a simplified expression for the two–photon initial state emission cross section. Deviation for the case of large angles emission of our estimation from the exact quantity is small. It does not depend on \(s\) and slightly depends on \(\theta_0\). For \(\theta_0 \sim 10^{-2}\) we have
\[
\frac{\pi}{\alpha} \left| \frac{\int (d\sigma^j_{\gamma\gamma, \text{noncoll}} - d\sigma^j_{\gamma\gamma, \text{noncoll} \text{exact}})}{\int d\sigma^j_B} \right| \lesssim 10^{-1}. \tag{41}
\]
C. Master formula

By summing all contributions for charge–even set–up we can put the cross section of radiative production in the form:

\[
\frac{d\sigma^j(p_+, p_-; k_1, q_+, q_-)}{dx\_dc\_ds_1} \left( 1 + \frac{\alpha}{\pi} K^j \right) + \frac{\alpha}{2\pi} \int_\Delta \left( \frac{1 + (1 - x)^2}{x} \ln \frac{\theta_0^2}{4} + x \right) \frac{dx}{dx\_dc\_ds_1} \left[ D(x_1, l_s) D(x_2, l_s) \left( 1 + \frac{\alpha}{\pi} K^j \right) + \frac{\alpha}{2\pi} \int_\Delta \left( \frac{1 + (1 - x)^2}{x} \ln \frac{\theta_0^2}{4} + x \right) \frac{dx}{dx\_dc\_ds_1} \right]
\]

\[
D(x, l_s) = \delta(1 - x) + \frac{\alpha}{2\pi} P^{(1)}(x)(l_s - 1) + \ldots, 
\]

\[
P^{(1)}(x) = \left( \frac{1 + x^2}{1 - x} \right)^j, \quad j = \mu, \pi.
\]

The boosted cross sections $d\tilde{\sigma}$ is defined above in Eq. (39). The lower limits of the integrals over $x_{1,2}$ depend on experimental conditions.

The structure function $D$ include all dependence on the large logarithm $l_s$. The so–called $K$-factor reads

\[
K^j = \frac{1}{R^j} B^{\lambda_\sigma} \left( i^{(v)j}_{\lambda_\sigma} + i^{(s)j}_{\lambda_\sigma} + i^{(b)j}_{\lambda_\sigma} \right) + \frac{\pi^2}{3} - \frac{3}{4} + R^{(j)}. \tag{43}
\]

It includes the RC which connected with final state interaction.

Quantities $R^{(j)}$ include the ”non-leading” contributions from the initial state radiation. Generally, they are rather cumbersome expressions for the case $s_1 \sim s$. For the case $s_1 \sim M^2 \ll s$ we obtain

\[
R^{(u)} = -\frac{1 - c^2}{16(1 + c^2)} \left\{ -2 \frac{1 + c^2}{1 - c^2} - 32 \frac{5 + 2c + c^2}{1 - c^2} \ln^2 \left( \frac{2}{1 + c} \right) + 4 \frac{5 - c}{1 + c} \ln \left( \frac{2}{1 + c} \right) \right\} + \frac{2}{(1 - 2x_+x_-) + 2\sigma} \left[ 4x_+x_- - 2 \frac{1 + c^2}{1 - c^2} + 4\sigma \right] + (c \rightarrow -c), \tag{44}
\]

\[
R^{(s)} = \frac{1 - c^2}{16(1 + c^2)} \left\{ 4 \frac{5 + 2c + c^2}{1 - c^2} \ln^2 \left( \frac{2}{1 + c} \right) - 4 \frac{5 - c}{1 + c} \ln \left( \frac{2}{1 + c} \right) \right\} + \left( \frac{1 + c^2}{1 - c^2} + 2 \right) + (c \rightarrow -c). \tag{45}
\]

Here we see the remarkable phenomena: the cancellation of terms containing $\ln(\frac{s}{s_1})$. In such a way only one kind of large logarithm $\ln(s/m^2)$. This fact is the sequence of renormalization group.
IV. CONCLUSION

In paper [5], the RC to one–photon annihilation of $e\bar{e}$ to hadrons was calculated beyond the leading logarithmic (LL) approximation. In paper [2], the heavy photon Compton tensor was calculated for the scattering channel.

We had considered the charge–even contributions, which correspond to the charge-blind experimental set–up. The charge–odd contribution to the cross section under consideration appears starting from the Born level [see Eq. (10)]. In the kinematics discussed here, the odd part of the cross section is suppressed by the factor $s_1/s \ll 1$ with respect to the even one.

In papers [4], the inclusive event selection set-up was considered for radiative pion pair production process. In such a set-up the hard photon is emitted preferably in direction close to the beam axes, unlike the kinematics considered here.

We see that cross section in our quasi $2 \to 2$ kinematics can be written down in the form of cross–section of Drell–Yan process. It is not a trivial fact as well as that two types of large logarithms are present in the problem. Possible background from peripherical process $e\bar{e} \to e\bar{e}\mu\bar{\mu}$ is negligible in our kinematics: it is suppressed by factor $\frac{\alpha s}{\pi s}$ and can be eliminated if the registration of the primary hard photon (see Eq. (1)) is required by experimental conditions for event selection.

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Appendix

The one–loop QED form factors of muon and pion are

\[ \text{Re } F_1^\mu(s_1) = 1 + \frac{\alpha}{\pi} f_1^\mu(s_1), \quad \text{Re } F_2^\mu(s_1) = \frac{\alpha}{\pi} f_2^\mu(s_1) \]

\[ f_1^\mu(s_1) = \left( \ln \frac{M}{\lambda} - 1 \right) \left( 1 - \frac{1 + \beta^2}{2} l_\beta \right) + \frac{1 + \beta^2}{2} \left( -\frac{1}{4} l_\beta^2 + l_\beta \ln \frac{1 + \beta}{2\beta} + \frac{\pi^2}{3} \right) \]

\[ + 2 \text{Li}_2 \left( \frac{1 - \beta}{1 + \beta} \right) - \frac{1}{4\beta} l_\beta, \]

\[ f_2^\mu(s_1) = -\frac{1 - \beta^2}{8\beta} l_\beta, \]

\[ \text{Re } F_\pi^{QED}(s_1) = 1 + \frac{\alpha}{\pi} f_\pi^{QED}(s_1), \]

\[ f_\pi^{QED}(s_1) = \left( \ln \frac{M}{\lambda} - 1 \right) \left( 1 - \frac{1 + \beta^2}{2} l_\beta \right) + \frac{1 + \beta^2}{2} \left( -\frac{1}{4} l_\beta^2 + l_\beta \ln \frac{1 + \beta}{2\beta} + \frac{\pi^2}{3} \right) \]

\[ + 2 \text{Li}_2 \left( \frac{1 - \beta}{1 + \beta} \right), \quad \beta^2 = 1 - \frac{4M^2}{s_1}, \quad l_\beta = \ln \frac{1 + \beta}{1 - \beta}. \quad (A.1) \]

We put here the expressions for leptonic and hadronic contributions into the vacuum polarization operator \( \Pi(s) \):

\[ \Pi(s) = \Pi_l(s) + \Pi_h(s), \]

\[ \Pi_l(s) = \frac{\alpha}{\pi} \Pi_1(s) + \left( \frac{\alpha}{\pi} \right)^2 \Pi_2(s) + \left( \frac{\alpha}{\pi} \right)^3 \Pi_3(s) + \ldots \]

\[ \Pi_h(s) = \frac{s}{4\pi\alpha} \left[ \text{PV} \int_{4m^2_{\mu,\tau}}^{\infty} \frac{d\sigma^{e^+e^-\to\text{hadrons}}(s')}{s' - s} d\sigma^{e^+e^-\to\text{hadrons}}(s) \right]. \quad (A.2) \]

The first order leptonic contribution is well known [1]:

\[ \Pi_1(s) = -\frac{1}{3} L - \frac{5}{9} + f(x_\mu) + f(x_\tau) - i\pi \left[ \frac{1}{3} + \phi(x_\mu) \Theta(1 - x_\mu) + \phi(x_\tau) \Theta(1 - x_\tau) \right], \quad (A.3) \]

where

\[ f(x) = \begin{cases} \frac{-5}{9} - \frac{x}{3} + \frac{1}{6}(2 + x)\sqrt{1-x} \ln \left| \frac{1 + \sqrt{1-x}}{1 - \sqrt{1-x}} \right| & \text{for } x \leq 1, \\ \frac{-5}{9} - \frac{x}{3} + \frac{1}{6}(2 + x)\sqrt{x-1} \arctan \left( \frac{1}{\sqrt{x-1}} \right) & \text{for } x > 1, \end{cases} \]

\[ \phi(x) = \frac{1}{6}(2 + x)\sqrt{1-x}, \quad x_{\mu,\tau} = \frac{4m^2_{\mu,\tau}}{s}. \]

In the second order it is enough to take only the logarithmic term from the electron contribution

\[ \Pi_2(s) = \frac{1}{4}(L - i\pi) + \zeta(3) - \frac{5}{24}. \quad (A.4) \]
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