GLUON RADIATION AND TOP WIDTH EFFECTS∗†

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ABSTRACT
The large width of the top quark influences the way gluons are radiated in top events, giving rise to interesting interference effects in top production and decay. We discuss top width effects in soft gluon radiation in $e^+e^- \rightarrow t\bar{t}$ at high energies and near $t\bar{t}$ threshold.

1. Introduction

The mass of the top quark is large enough that top decays to a real $W$ and a $b$ quark with a width $\Gamma$ that increases with $m_t$ and is typically in the GeV range. This implies a weak lifetime comparable to strong interaction timescales, which in turn gives rise to interesting new effects involving the interplay between top’s strong and weak interactions. For example, a heavy top quark might decay too quickly to form bound states when it is produced, leaving little or no resonant structure in the threshold region (which, among other things, makes it difficult to measure $\Gamma$). Here we are interested in the perturbative side of the strong–weak interplay: how the top quark’s large decay width influences the way gluons are radiated. More detailed discussion of the work presented here can be found in [1] and [2]; for reviews of theoretical and experimental issues in top physics, see [3].

We will discuss soft radiation in $e^+e^- \rightarrow t\bar{t}$ at high energies and near $t\bar{t}$ threshold, by sketching out the general formalism and showing some examples of gluon distributions which are sensitive to $\Gamma$. We are motivated in part by the following questions, which we will return to at the end.

• Does the large top width suppress radiation off the top quark, i.e., does the gluon distribution in top events look as though the $b$ quarks were produced directly rather than through top decay?

• Do gluons radiated in the production and decay stages interfere?

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• At/near the $t\bar{t}$ threshold, can the top width affect the gluon distribution, even though the $t$ and $\bar{t}$ are produced at rest and do not radiate?

• What does “large width” mean in the context of perturbative QCD? In non-perturbative QCD, large means large compared to $\Lambda_{QCD}$. What is the appropriate dimensionful quantity to which we should compare $\Gamma$ here?

• Can width-dependent effects be exploited to obtain a measurement of $\Gamma$ from soft gluon distributions?

2. Soft Gluon Radiation in $e^+e^- \to t\bar{t}$: General Formalism\textsuperscript{[1]}

Consider $e^+e^- \to t\bar{t} \to W^+W^-b\bar{b}$ with emission of a (single) gluon. The gluon can be radiated by any one of the quarks, and we must include the diagrams for all four possibilities. In the limit of soft gluons the matrix element $\mathcal{M}$ factorizes and we can write $\mathcal{M} \sim \mathcal{M}^{(0)} J \cdot \epsilon$, where $\mathcal{M}^{(0)}$ is the zeroth-order matrix element and $J^\mu$ and $\epsilon^\mu$ are respectively the gluon current and polarization. This factorization allows us to define a gluon emission probability density, which is just the gluon part of the differential cross section normalized to the zeroth-order cross section $\sigma_0$, i.e.,

$$dN \equiv \frac{1}{\sigma_0 d\sigma_g} = \frac{d\omega}{\omega} \frac{d\Omega}{4\pi} \frac{C_F \alpha_s}{\pi} \mathcal{R},$$

where $\omega$ and $\Omega$ denote the gluon energy and solid angle. $\mathcal{R}$ is given by integrating the square of the current over the top-quark virtualities:

$$\mathcal{R} \equiv \omega^2 \left( \frac{M \Gamma}{\pi} \right)^2 \int dq_1^2 dq_2^2 \left[ -J \cdot J^* \right].$$

The full expression for $\mathcal{R}$ can be found in \[1\] (see also \[4\]); we will give some numerical examples below.

The important point is that the current $J^\mu$ can be decomposed in a gauge-invariant way into terms that correspond to radiation at the production and decay stages. Thus, in $\mathcal{R}$ we can identify unambiguously contributions from gluon emission in $t\bar{t}$ production, $t$ and $\bar{t}$ decay, and their interference. $\Gamma$ enters $J^\mu$ via the top quark propagators. After the integration in Eq. 2, $\Gamma$ drops out of the pure–production and pure–$t$ (or $\bar{t}$) decay terms and all of the width dependence is in the production–decay and $t$ decay–$\bar{t}$ decay interference. It is also worth noting that the magnitudes of these interference terms increase with increasing $\Gamma$ and vanish as $\Gamma \to 0$.

It will be useful to keep in mind two limiting cases. Our intuition tells us that if the top width is large, the top quark decays before it has time to radiate, and the gluon distribution is identical to that for a $b$ and $\bar{b}$ produced directly (assuming the same final state kinematics); as $\Gamma \to \infty$, the interference cancels all contributions that involve emission off the top quark lines. On the other hand, if the width is small, the interference is small and the radiation in the production and decay stages can be treated independently.\textsuperscript{[\textsection]}

\textsuperscript{[\textsection]}Thus the way the top quark is treated in Monte Carlo simulations usually corresponds to one or the other limiting value of $\Gamma$.\textsuperscript{[\textsection]}
Figure 1: Soft gluon distribution in $e^+e^- \rightarrow t\bar{t}$ for c.m. energy 1 TeV, $m_t = 140$ GeV, $\omega = 5$ GeV and $\phi = 0^\circ$. $\theta$ is the $t$–$g$ angle. (a) $\theta_b = 180^\circ$. (b) $\theta_b = 90^\circ$.

3. Collisions at High Energies

At high collision energies, gluons radiated at the top production and decay stages interfere destructively for most configurations, and the decay–decay interference is negligible. Thus the effect of the top width is to suppress the gluon distribution. In addition, it turns out that the interference is largest for large $b$–$t$ angular separations.

This is illustrated in Figure 1, where we show the gluon emission probability as a function the angle $\theta$ of the gluon with respect to the top-quark direction. We vary the top width, and take $m_t = 140$ GeV, collision energy 1 TeV, and gluon energy $\omega = 5$ GeV. Because in the soft limit we neglect the gluon momentum in the kinematics, the $t$ and $\bar{t}$ are produced back–to–back. For simplicity we also choose the $b$ and $\bar{b}$ back–to–back. In Fig. 1(a) $\theta_b = 180^\circ$, that is, the $t$ decays to a backward $b$. The different curves correspond to different values of the top width $\Gamma$. As $\Gamma$ increases, the peaks in the gluon distribution are suppressed and eventually disappear. We see also that for the SM width ($\Gamma = 0.7$ GeV at lowest order) the gluon distribution is much closer to the zero-width curve than that for $\Gamma = \infty$.

Similar behavior appears in Fig. 1(b), where we show the slightly more likely configuration where the $b$’s come off at right angles from their parent $t$’s ($\theta_b = 90^\circ$). There is more structure in the distributions because this configuration is less symmetric than in Fig. 1(a). As a result, the gluon distribution not only gets suppressed but also changes shape as the width increases and wipes out all traces of the top quarks at $\theta = 0^\circ$ and $180^\circ$.

Not shown is the case where $\theta_b = 0^\circ$; while it is by far the most likely configuration (because a high energy top quark tends to decay to a collinear $b$), there is very little interference and the width has virtually no effect on the distribution. That this must be true becomes clear when we think in terms of moving color charges: QCD is flavor-blind, so the decay entails only a slight change in speed, but not direction, of

\[\Gamma\] appears as a parameter here, and so using the lowest order value is adequate for our purposes. Corrections to the top width are discussed elsewhere.\[^3\]

[^3]: Corrections to the top width are discussed elsewhere.
Figure 2: Soft gluon distribution in $e^+e^- \rightarrow tt$ for $m_t = 140$ GeV, near $t\bar{t}$ threshold, with gluon perpendicular to $b\bar{b}$ plane; $\theta_{12}$ is the $b$-$\bar{b}$ angle.

the color source.

4. Collisions Near $t\bar{t}$ Threshold[2]

It is perhaps more interesting to consider the $t\bar{t}$ threshold region, which is of particular importance to top physics.[3] One may ask whether the size of the width influences the radiation of gluons for top pairs produced near threshold. It might seem unlikely, because the $t$ and $\bar{t}$ are produced nearly at rest and only the $b$’s radiate. However, the top width does affect the distribution by determining to what extent the radiation from the $b$’s is coherent or independent. This can be understood by considering the same limiting cases as above. As $\Gamma \rightarrow \infty$, the top lifetime approaches 0, the $b$ and $\bar{b}$ appear instantaneously, and they radiate coherently. But if $\Gamma$ is very small, the top lifetime is large, the $b$ and $\bar{b}$ appear at very different times and they radiate independently, with no interference. Thus $\Gamma$ regulates the interference between gluons radiated by the $b$ and $\bar{b}$ and, again, in the limit of large top width the radiation looks as though the $b$ and $\bar{b}$ were produced directly.

This can be made quantitative by taking the threshold limit of the expression for $\mathcal{R}$ in Eq. 2.[1] If $v$ is the velocity of the $b$ (or $\bar{b}$), $\theta_{1(2)}$ is the angle between the $b$ ($\bar{b}$) and the gluon, and $\theta_{12}$ is the angle between the $b$ and $\bar{b}$, then we obtain

$$\mathcal{R} = \frac{v^2 \sin^2 \theta_1}{(1 - v \cos \theta_1)^2} + \frac{v^2 \sin^2 \theta_2}{(1 - v \cos \theta_2)^2} + 2\chi \frac{v^2 (\cos \theta_1 \cos \theta_2 - \cos \theta_{12})}{(1 - v \cos \theta_1)(1 - v \cos \theta_2)},$$

(3)

where $\chi \equiv \frac{\Gamma^2}{\Gamma^2 + \omega^2}$. The first two terms correspond to independent emission by the $b$ and $\bar{b}$, and the last term is the interference, regulated by the top width through $\chi$. Note that $0 \leq \chi \leq 1$ and the extreme values of $\chi$ correspond to the limits of independent ($\Gamma = 0$) and coherent ($\Gamma \rightarrow \infty$) emission. Note also that $\chi$ depends not on $\Gamma$ alone but on its ratio to the gluon energy $\omega$; hence $\omega$ determines the relevant scale for $\Gamma$. Furthermore, from the form of $\chi$ it is clear that we have maximum sensitivity to the top width when $\Gamma$ and $\omega$ are comparable.

*The same result can be obtained from a semi-classical derivation based on the motion of color sources and the quantum-mechanical arguments of the previous paragraph; see [2].
Figure 3: Soft gluon emission probability near $t\bar{t}$ threshold ($m_t = 140$ GeV) integrated over $5 \leq \omega \leq 10$ GeV, $0 \leq \phi \leq 360^\circ$, and (a) $0 \leq \theta \leq \theta_{12}$ and (b) $\theta_{12} \leq 180^\circ$; angles are measured from the $b$ direction. $\Gamma = 0.7, 3, 5$ GeV for the dotted, dashed, and dashed–dotted lines.

Let us look at some examples. Since the interference term contains the width dependence, and it is also the only term that depends on the relative orientation of the $b$ and $\bar{b}$, it will be useful to consider how the gluon distributions vary with $\theta_{12}$. A particularly simple radiation pattern is that for gluons emitted perpendicular to the $b\bar{b}$ plane, in which case $\theta_1 = \theta_2 = \pi/2$ and $R \propto (1 - \chi \cos \theta_{12})$. Figure 2 shows the distribution for various values of $\chi$. The $\chi = 0$ distribution is flat and exhibits no interference effects. As $\chi$ increases, $\theta_{12}$ dependence is induced and we see that the interference is destructive for small $\theta_{12}$ and constructive for large $\theta_{12}$.

For a 5 GeV gluon, our 140 GeV top quark with a width of 0.7 GeV gives $\chi \approx 0.02$, much closer to the independent emission case than the large width limit. If the width were as large as $\omega$, or, conversely, if we could detect gluons with energies of about 1 GeV, we would see that the radiation pattern matched neither the large nor the small width limit, and that the distribution was very sensitive to the exact value of $\Gamma$.

Interference between gluons gives rise to so-called angular ordering behavior: After integration over the azimuthal angle about the $b$ quark direction, emission is enhanced between the $b$ and $\bar{b}$ and suppressed outside them. Here “between” and “outside” refer to the gluon polar angle $\theta$ (with respect to the $b$ direction) respectively less or greater than $\theta_{12}$. This effect can be seen in Figure 3, where we show (partly integrated) distributions for gluons (a) between and (b) outside the $b\bar{b}$ pair for $\Gamma$ ranging from 0 to $\infty$.

Finally, we show in Figure 4 the total gluon distribution as a function of $\theta_{12}$, integrated over all angles and energies from 5 to 10 GeV. We see, again, no $\theta_{12}$ dependence for $\Gamma = 0$, and, as $\Gamma$ increases, the interference — destructive for small $b\bar{b}$ angles and constructive for large $b\bar{b}$ angles — changes the shape of the curve. As in the differential distributions, the SM case ($\Gamma = 0.7$) GeV gives a result much closer to that for $\Gamma = 0$ than infinity. Because the top width is so small, interference between the $b$ and $\bar{b}$ is almost completely suppressed, contrary to the large-width expectation of coherent emission as if the $b$ and $\bar{b}$ were produced directly.
5. Conclusions

We conclude by answering explicitly the questions raised in the Introduction.

- **Does the top width suppress radiation off the top quark?** No; unless \( \Gamma \) is extremely large, \( t \) and \( \bar{t} \) emission contribute substantially to the gluon distribution.

- **Do gluons radiated in the production and decay stages interfere?** Yes, and the interference is destructive at high energies, leading to a suppression of radiation and sometimes a change in the shape of the distribution.

- **At/near the \( t\bar{t} \) threshold, can the top width affect the gluon distribution?** Yes; the top width suppresses the interference between the \( b \) and \( \bar{b} \) by a factor \( \chi \equiv \frac{\Gamma^2}{\Gamma^2 + \omega^2} \).

- **What does “large width” mean in the context of perturbative QCD, i.e., what is the appropriate quantity to which we should compare \( \Gamma \)?** The gluon energy \( \omega \) sets the scale for the width. As we have seen, although a width of 0.7 GeV is large compared to \( \Lambda_{QCD} \), it is not large compared to accessible gluon energies. Therefore, contrary to what we might expect, a 140 GeV top quark has a small width for PQCD purposes.

- **Can width-dependent effects be exploited to measure \( \Gamma \) from soft gluon distributions?** There is maximum sensitivity to the width when it is comparable to the gluon energy. If the minimum accessible gluon energy is around 5 GeV, then the best prospects for a measurement are for \( \Gamma \) also a few GeV or higher, \( i.e. \), for an anomalously large width (for “typical” top masses around 140-150 GeV), or for a very heavy top quark. In general, the larger the top width, the better the prospects for using soft gluon radiation to measure it.

6. References

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