Student engagement and learning with Quantum Composer

Shaeema Zaman Ahmed,∗ Carrie A. Weidner,† Jesper H. M. Jensen, Jacob F. Sherson, and H. J. Lewandowski

1Department of Physics and Astronomy, Aarhus University, 8000 Aarhus C, Denmark
2Department of Physics, University of Colorado, Boulder, CO 80309, USA
3JILA, National Institute of Standards and Technology and University of Colorado, Boulder, CO, 80309, USA

Knowledge of quantum mechanical systems is becoming more important for many science and engineering students who are looking to join the emerging quantum workforce. To better prepare a wide range of students for these careers, we must seek to develop new tools to enhance our education in quantum topics. We present initial studies on the use of one of these such tools, Quantum Composer, a 1D quantum simulation and visualization tool developed for education and research purposes. In particular, we conducted five think-aloud interviews with students who worked through an exercise using Quantum Composer that focused on the statics and dynamics of quantum states in single- and double-harmonic well systems. Our results show that Quantum Composer helps students obtain the correct answers to the questions posed, but additional support is needed to facilitate the development of student reasoning behind these answers. In addition, we find that students explore familiar and unfamiliar problems in similar ways, indicating that Quantum Composer is a useful tool for exploring systems that students have not seen before.

I. INTRODUCTION

Due to the recent attention on quantum technologies and their economic impact as a result of the so-called second quantum revolution [1], efforts are increasingly focused on educating the next generation of individuals who will make up the backbone of the quantum workforce. As such, current research is increasingly focusing on how higher education institutions can provide the preparation required for people to be successful in this emerging industry and to bring quantum technologies out of academic labs and into society [2].

To be able to meet this demand for a quantum literate workforce, the community needs to develop new quantum curricula at all levels [2–4], effective methods of teaching quantum mechanics to address common student difficulties in learning the relevant material [5, 6], and tools that allow students to visualize and explore quantum systems (e.g., those described in Refs. [7–9]). If implemented effectively, these efforts will allow students to gain the skills necessary to contribute to the development and deployment of quantum technologies.

Here, we present a study of a new quantum software tool known as Quantum Composer (or simply Composer) [10]. Composer is an interactive and flexible flow-based programming tool designed to allow users to simulate and visualize the static and dynamic properties of one-dimensional quantum systems, including systems that are not readily analytically tractable. This study, which is an extension of preliminary work [11], was designed to identify how students used Composer to explore questions posed about systems they had seen before and systems they were unfamiliar with. Previous work suggested that students used the visualizations contained within Composer to develop their understanding of a problem. The work presented here aims to probe the use of Composer more deeply.

In particular, we conducted think-aloud interviews with five students as they worked through an exercise designed to guide them through the statics and dynamics of quantum states in single- and double-harmonic well systems. Our goal was to perform an initial exploratory study of how students used Composer in an interview setting before probing how Composer could be used in a classroom environment in future studies. In particular, our study sought to answer the following four research questions:

• RQ1: Did Composer help students determine correct answers and reasoning of problems in simple 1D quantum systems?

• RQ2: What Composer features were most used by students?

• RQ3: How was the use of Composer features connected with outcomes?

• RQ4: How did students use Composer to explore an unknown system as compared to a system they have worked with previously?

This paper is organized as follows: Section II places our study in the context of the larger field of quantum education research and existing quantum simulation tools, as well as providing a brief description of Composer itself. This is followed by a description of the methods we used for the study in Sec. III. Our results are presented in Sec. IV and discussed in Sec. V. Section VII concludes with some ideas for future research.

∗ These two authors contributed equally.
† cweidner@phys.au.dk
II. BACKGROUND

A. Student understanding of quantum mechanics concepts

Given the non-intuitive nature of quantum mechanics, physics education researchers have studied extensively where common gaps in knowledge or misconceptions arise. In order to develop an exercise that adequately tested the use of Composer for its capabilities, its limitations, and student learning, we drew from this literature and selected a subset of topics that our study could address. This is briefly summarized below, with an emphasis placed on the elements that featured components relevant to our work, that is, the statics and dynamics of stationary and superposition bound states (i.e., not scattering states).

Early efforts to categorize student misconceptions in quantum mechanics were done by Styer [12] and Singh [13]. These studies focused on how quantum states are represented, evolve in time, and are measured. More comprehensive reviews of these student difficulties were recently compiled by Singh and Marshman [5] and Krijtenburg-Lewerissa et al. [6]. In particular, Singh found students had difficulties with answering questions about the time-dependence of systems; this result is confirmed by a later study by Cataloglu and Robinett [14] that focused on testing conceptual and visual understanding in quantum mechanics. Singh found that this difficulty persists even beyond the introductory level and into graduate courses [15]. This study and another by Zhu and Singh [16] also suggest that students face challenges with sketching the shape of the wavefunction. In later work, Emigh et al. indicated that students fail to see the difference between the time evolution of wavefunctions versus probability densities, and they struggle to interpret a wavefunction’s time-dependent phase factor(s) [17]. Student difficulties in interpreting complex exponentials were also identified by Wan et al. [18]. Similarly, Emigh et al. noted that students tended to misinterpret the physical meaning of the real and imaginary components of the wavefunction. This is supported by other work done by McKagan et al. [19] and Passante and Kohnle [20], and both of these studies were framed in the context of visualization tools and their potential to assist student learning.

B. Quantum visualization and simulation tools

In general, studies have shown that visualization and simulation tools help facilitate student learning in quantum mechanics in a multitude of ways: developing scientific understanding by making connections with pre-existing knowledge [14], building mental models [8], and developing visual understanding of quantum physics concepts [21, 22]. Such tools also present numerical calculations, allowing students to visualize complex phenomena and systems beyond what is analytically tractable, including multidimensional systems and time-dependent behavior [14].

The use of computer-based visualization and simulation tools in quantum mechanics education has its roots in a series of books published in 1995-6 by the Consortium for Upper Level Physics software, two of which focus on quantum mechanics [23, 24]. Other such software-textbook combinations have been published [25–27]. More recently, the Physlet applets were developed by Belloni et al. [28], and similar tutorial-style units with integrated simulations have been developed [29, 30]. There also exist other online resources for interested students and instructors [31–33]. Additionally, one can simulate quantum phenomena through the use of text-based programming platforms like MATLAB, Python, and C++, and several quantum-specific libraries for these platforms are available, among them those described in Refs. [34–36].

However, the three main research-based quantum visualization tools that are being actively developed (besides Composer) are the PhET simulations developed at the University of Colorado Boulder [37], the quantum interactive learning tutorials (QuILTs) developed at the University of Pittsburgh [38], and the quantum mechanics visualization simulations (QuVIS) developed at St. Andrews University [39]. Unlike Composer, each of these tools is modular in the sense that they present individual simulation modules typically addressing a carefully chosen subset of concepts. They have all been demonstrated to enhance learning of quantum mechanics at the undergraduate level and are freely available online.

The first of these tools, the PhET interactive simulations, is a suite of over 150 simulations covering multiple STEM fields, of which 21 are currently categorized under quantum phenomena. These cover topics like band structure, the structure of the hydrogen atom, and the Stern-Gerlach experiment. Extensive research has gone into the development and use of these simulations, both in general [40–43] and specifically for quantum mechanics [9, 19].

The second tool, the QuILTs, has been described in detail in Ref. [7]. The QuILTs are a set of 14 Java applets designed to help students use computer-based visualization tools coupled with curriculum elements like tutorials, tests, and homework assignments to tackle the challenging topics identified through research [5, 13]. Student engagement with these tutorials is covered in detail in DeVore et al. [44]. They suggest that such tools can be powerful in focused, one-on-one interview sessions, but, in some cases, they may be used more superficially (i.e., less effectively) in unsupervised self-study sessions. QuILT topics are applicable for a broad range of educational levels from the introductory to the advanced undergraduate, including expectation values in quantum mechanics [45], the double-slit experiment [46], and degenerate perturbation theory [47].

Finally, the QuVIS project has developed numerous research-based simulation applets, typically accompanied by an activity sheet, for students at the high school to ad-
advanced undergraduate level [8, 22]. Topics covered by the QuVIS project include the time-dependence of states [20], two-level systems [48], perturbation theory [49], and single-photon superposition states [50], among many others.

C. Quantum Composer

In this study, we investigate the student use of a new quantum simulation tool: Quantum Composer. Rather than focusing on individual, curated simulations like the three projects mentioned earlier, Composer is a flexible and interactive tool that enables educators and students to build and simulate one-dimensional quantum systems through a ‘drag-and-drop’ visual programming and execution interface. Composer is described in detail in Ref. [10] and relevant elements are discussed briefly here.

The interface consists of a simulation environment where elements are dragged-and-dropped and connected together sequentially, as shown in Fig. 1. These elements are referred to as nodes with interactive capabilities, e.g., entering scalars to create an arbitrary linear combination or defining a potential function. Composer also consists of a collection of visualization nodes, such as the State Comparison Plot that displays single eigenstates and superposition states, both in static and time-dependent configurations. The plots have checkboxes that enable the user to select which information they would like to see on the plot. For instance, one can visualize the real part, imaginary part, and the probability density of any wavefunction either simultaneously or one at a time by selecting the relevant checkbox. Composer is also capable of time-evolving a state given an initial state and potential. Once a simulation is built, it can be saved and loaded as a Flowscene, which can be used by students for guided exploration.

This investigation extends previous developments on the use and impact of visualization and simulation tools in physics education research. For the purpose of this study, we built two Flowscenes for the exercise covering harmonic single- and double-well systems (Fig. 1), respectively. Students answered questions related to these systems based on an exercise (cf. Sec. III C) given to them. Their actions and thought processes during the exercise were captured through a think-aloud protocol (cf. Sec. III B) and these data were analyzed to investigate the use and impact of Composer on student learning of quantum mechanics.

III. METHODS

A. Research context

This study took place at Aarhus University (AU), the second-largest university in Denmark, with approximately 38,000 students enrolled in bachelor’s, master’s, and PhD programs. The student population is predominantly white and native to Denmark.

Students recruited for the study were enrolled in a second-year undergraduate quantum mechanics for nanoscientists course in Fall 2019 (that semester, 31 students were enrolled in the course). The quantum course was taught in English, and the study was in English, but all of the students who volunteered for our study spoke English as a second language. Students were recruited in-class and via messages posted on the course’s online message board, and volunteers were asked to give two hours of their time. Participants were compensated with a gift card worth 500 Danish kroner (about 75 USD at the time of the interview). The think-aloud interviews took place in the weeks following the final exam, and all interviews were conducted by S.Z.A. Students were given one hour to complete the exercise, after which they were asked some basic questions on their background, thoughts about Composer, and gender information. Participants were told that a response to the gender question was optional, but all students responded. Three men and two women participated in the interviews.

In addition to instruction in the quantum mechanics for nanoscientists course, four of the five students reported also having had some introductory quantum mechanics in a physical chemistry course taken concurrently with the quantum course. In addition to courses covering quantum topics, nanoscience students typically take courses covering mechanics, thermodynamics, calculus, electrodynamics, waves, and optics prior to their quantum course.

The quantum course was taught using Griffiths’ textbook [51] and included seven homework assignments based on the book. Several of the problems required students to use programming tools like MATLAB. Three times throughout the course, the students were given additional activities built around Composer Flowscenes (see Fig. 1); these activities were designed by C.A.W., S.Z.A., and J.H.M.J. None of the activities were graded, nor mandatory. The Composer activities were designed so that students did not have to build anything from scratch in Composer (although they were shown how this could be done). The first set of activities explored the infinite and finite square well, the second set investigated superposition and expectation values, and the third discussed time-independent perturbation theory. None of the Composer activities explored time dependence, but time-dependence was covered in the course. Before the first Composer activity was given in the course, students were given a 15-minute presentation on how to use Composer. For all Composer activities, one of the authors was present for approximately 30 minutes to answer questions. Aside from the Composer elements in the quantum course, students did not report significant simulation-based activities in past courses.
FIG. 1. (Color online) A screenshot of the *flowscene* for the double-well potential used by the interview participants. The blue dotted lines separate the time-independent parts of the simulation from the time-dependent dynamics that take place in the *Time evolution* loop. All parameters shown in white boxes could be changed by the students. Students could observe the behavior of the system under study during the (a) the time-independent parts of the scenario (orange, dashed box within “statics”) and (b) the time-dependent parts of the scenario (green, dashed box within “dynamics”). The plots in (b) updated continuously if the student initiated time evolution by pressing the play button (not shown). The *flowscene* for the single-well potential was similar to what is shown here.

B. Think-Aloud Interviews

In order to examine whether and how students provide answers and reasoning to problems in the quantum mechanics exercise for this study, a think-aloud protocol was used. The think-aloud protocol involves speaking aloud the thoughts that come to the interviewee’s mind while performing the task at hand [52]. This methodology is used to provide rich verbal data describing how one tackles a problem-solving task, which provides inferences about the steps taken to solve the problem [53]. Therefore, for the purpose of our study, this methodology was considered to be appropriate. The potential of this methodology has been explored for education action-research [54] and has been applied to different educational research studies as well [21, 55, 56].

In the context of this study, the think-aloud protocol was used to collect data on how students work through the different components of the exercise. Interviews were conducted with five students, and during each interview, the student volunteer verbalized their thoughts aloud while working through the exercise. During this process, the students’ voices and computer screens were recorded. Additionally, the students’ sketches and notes were also documented during the interview. We recorded each student working through the exercise for 55-65 minutes, followed by answers to background questions about Composer and demographics for approximately 5-10 minutes. Before the start of the interview, the students provided their written consent to this study and were briefed about the think-aloud format. Throughout the interview, the interviewer intervened in only two cases: either to remind the student to talk aloud if there were long periods of silence or if the student encountered technical problems with using Composer, e.g., not being able to find the *Play* button or zooming in and zooming out on the screen.

C. Interview protocol and exercise

The exercise that the students worked through in the think-aloud interview covered statics and dynamics in two separate scenarios using single- and double-harmonic well potentials, as shown in Fig. 2 (see Supplemental Material for the full exercise prompts). The quantum course covered the quantum harmonic oscillator extensively, but did not cover the double-well system at all. As such, the exercise was designed to have students begin with something they were familiar with before moving into a
near-extension exercise, wherein they had to consider a situation with both familiar and unfamiliar aspects.

For the interviews, we provided previously constructed flowscenes for the participants to work with. To minimize confusion, we built a separate flowscene for each of the two parts of the exercise (single- and double-wells), the latter of which is shown and described briefly in Fig. 1. Additionally, we used the Scope feature within Composer to hide unnecessary details of the simulation [10]. In this way, we hoped to encourage students to focus on the important aspects of the simulation and avoid unnecessary cognitive overhead that would hinder their exploration [57].

The exercise was designed in the following way. The first part included a single-well potential and was designed to use a Predict, Explore, Revise framework. That is, before using Composer, students were asked to Predict (using either words, equations, or sketches) specific aspects of the system under study. After making their predictions, the students were then prompted to Explore the same system in Composer and finally, they were asked to Revise their answer and reasoning if needed. The think-aloud protocol was chosen because it would elicit student reasoning about the problems at hand as they worked through the different phases of the exercise. Thus, even if students were not asked after every question to explicitly vocalize their reasoning, we still hoped to capture their thinking because of the chosen methodology. These key words were also used in the second part of the exercise using the double-well system, but, due to the participants’ lack of previous experience with the double-well potential, this part started with exploration of the system behavior. Beyond this initial exploration, students were asked to use the Predict, Explore, Revise framework for the remainder of the exercise.

For the first part of the exercise, students began by considering the eigenstates of a single harmonic well. The exercise asked them to sketch the real part, imaginary part, and probability density of the ground and first excited states evolved in time, after which they went through the same Explore and Revise phases.

After considering single eigenstates, students then considered the superposition state

\[ \psi_{\text{lin}}(x,t) = \frac{1}{2} (\psi_0(x)e^{-iE_0 t/\hbar} + \psi_1(x)e^{-iE_1 t/\hbar}) \]

with the initial state at \( t = 0 \) given by

\[ \psi_{\text{lin}}(x) = \sqrt{\frac{1}{2}} (\psi_0(x) + \psi_1(x)) \].

First, students considered the static case, and as before, they were asked to Predict the real part, imaginary part, and probability density of \( \psi_{\text{initial}}(x) \). However, they were also asked to Predict whether or not the probability density of the state would evolve in time and why. Again, students were asked to Explore and Revise their predictions using Composer. This marked the end of the single-well exercise. The students then went on to work with the double-well system and the new flowscene.

As stated before, students were simply asked to Explore the double-well system when they started working with it. That is, students were asked to vary the spacing between the two wells between 0 and 1, where 0 gave a single harmonic well and 1 corresponded to maximally-separated wells. While doing so, they were asked to Explore how the energy levels of the system changed with well separation. Subsequently, they were asked to visualize the eigenstates in Composer and Explore how they changed with well separation. This was done to prepare the students for the next part of the exercise where we resumed the Predict, Explore, Revise framework.

At this point in the exercise, students were asked to set the well spacing to 0.75 and consider a superposition state of the form

\[ \psi_{\text{double}}(x) = c_0 \psi^D_0(x) + c_0 \psi^D_1(x) \]

where \( \psi^D_0(x) \) and \( \psi^D_1(x) \) refer to the ground and first excited states of the double-well system, respectively. We then asked students to Predict the values of \( c_0 \) and \( c_1 \) that resulted in a state localized to the left and right wells, respectively. They were then asked to go through the usual Explore and Revise steps. Note that this exercise did not ask students to consider normalization, as Composer could be set to auto-normalize the linear combinations they produced.

The final part of the exercise asked students to consider a state localized to the left well. They were asked to Predict how the state would evolve in time, as well as how

\footnote{1 In the units used in Composer, \( \hbar = m = 1 \) and we have set the harmonic oscillator frequency to \( \omega = 10 \), so the characteristic harmonic oscillator length is \( a_{\text{HO}} = 1/\sqrt{10} \). This can be used to set a scale for the well separation. Note that if students used well separations greater than one, their results were misleading, as dynamics were occurring near the boundaries of the simulation. This was coded as Using Composer incorrectly in our analysis, cf. Sec. III D.}

\footnote{2 The answer being \( (c_0,c_1) = (1,1) \) for the left well and \( (1,-1) \) for the right well, if one ignores normalization.}
FIG. 2. A simplified flowchart of the exercise with the questions numbered as discussed in the Results section and the Predict, Explore, Revise phases associated with each question. Students began by exploring the single-well system (blue), then moved on to the double-well system (green). In general, the exercise asked students to consider statics, then dynamics, of single eigenstates before moving on to superposition states. The exercise was designed to get progressively more challenging, as students are first exposed to a familiar system (single-well), then an unfamiliar one (double-well).

this evolution would change as the potential well separation was changed. After another round of exploration and revision, they were asked to Explore what happens if the wells were very far apart from one another. After this final exploration, the exercise ended, and the students were asked the background and demographics questions described in Sec. III A.

The original exercise included a Compare section in the single-well part of the exercise, directly after the static eigenstate problem. This part of the exercise asked students to discuss whether or not \( \psi_0(x) \) and \( \psi_1(x) \) were orthogonal, but in retrospect, this was determined to be confusing and difficult to determine in the flowscene provided to the students.\(^3\) Therefore, this part of the interview was not coded or considered in the analyses reported on in Sec. IV. The next section covers the coding scheme and analysis.

\(^3\) This type of exploration is not difficult to do in Composer, but the required nodes were not present in what the students used.

D. Coding scheme

After all interviews were completed, the audio was transcribed by humans via an online transcription service [58]. The video, audio, and transcripts were used in combination in the coding process. In this context, coding refers to categorizing and labeling the students’ vocalizations and on-screen actions. Sections of the interview were examined (including audio, video, and transcripts), and the audio-visual data was coded using the NVivo software package in one-minute increments. The audio transcripts and student sketches were used as supporting information during the coding process. The process of creating and refining the codebook was done collaboratively by C.A.W. and S.Z.A., with input and consistency checks from H.J.L.

The final codebook is shown in Tab. I. To begin, the codebook was divided into three parts, corresponding to the Predict, Explore, and Revise phases of the exercise. We began with an a priori codebook of main codes (in
bold in Tab. I) that were driven by the exercise protocol. From the data obtained during iterative rounds of interview coding, we determined the emergent codes and subcodes (unbolded in the table) that best captured the students’ behavior during the exercise. Initially, students’ behavior during the **Explore** and **Revise** phases was coded separately, but after the first coding iteration, these were combined as students would typically verbalize their revision in the **Explore** phase. Given this, we describe the codes contained in Tab. I in what follows.

In our final codebook, the **Part of Exercise** code category was used simply to label which part of the exercise the student was working with for future reference when doing data analysis (see Supplemental Material for the exercise.). As students were working through the exercise, they used sketches, math, and Composer to explain their responses. Hence, the **Tool/Representation** codes denoted the tool(s) and/or representation(s) being used as they were working through the exercise. When using Composer, students engaged with the tool and explored the physical system under study through visualizations (plots) and interactions like changing plot parameters or values in relevant fields. This exploration was coded under the **Exploration** code category divided into the **Visualization** and **Interactions** subcodes.

The **Visualization** subcodes were used to identify when a student was using a visualization within Composer to answer a question, e.g., describing what was going on within a plot or watching the system evolve in time, and the subcodes were applied based on which functions were visible on the plots (e.g., the real part of the wavefunction) when the interaction was occurring. In order for this code to be applied, we required that the interaction be obvious, e.g., the student was moving the mouse around the plot while discussing its contents or the student was describing the wavefunction’s behavior as it evolved in time or the student changed parameters (like the spacing between the wells in the double-well system). The **Interaction** code was applied when students were actively interacting with Composer elements and changing the parameters set in the program.

As students worked through the exercise and used Composer in the **Explore/Revise** phase, the answers to the questions and reasoning about those answers were coded as **Outcomes**. Depending on whether this was related to either learning something new, remembering something they learned before, or confirming something they already knew, these **Outcomes** were coded as **Discovery**, **Recollection**, and **Reinforcement** respectively. **Discovery** outcomes were coded in two scenarios: when students’ prediction was absent or incorrect and they had to revise their response after using Composer and when students demonstrated clear evidence of new conceptual or mathematical understanding. **Recollection** was coded if their prediction was incorrect and using Composer helped them to recall the correct relevant physics concepts from their previous knowledge, e.g., explicitly mentioning that they now remembered something from their course. Finally, **Reinforcement** was coded when the students’ prediction was correct and the exploration in Composer con-

| Main code          | Subcode                      |
|--------------------|------------------------------|
| **Part of Exercise** | **Single-well**               |
| **Double-well**    | **Single eigenstate**        |
| **Superposition state** | **No time evolution**   |
| **Time evolution** | **Tool/Representation**      |
| **Math**           | **Sketches**                 |
| **Composer**       |                              |

| **Exploration** | **Visualization** | **Real part** |
|-----------------|-------------------|--------------|
|                 | **Imaginary part**|              |
|                 | **Static plot**   | **Dynamic plot** |
|                 |                   | **Eigenstates** |
|                 |                   | **Energy levels** |
| **Interaction** |                   | **Reading eigenvalues** |
|                 | **Change plot parameters** | **Play time evolution** |
|                 | **Set linear combination** | **Set number of eigenstates** |
|                 | **Set distance between wells** |

| **Outcomes**   | **Answer** | **Reasoning** |
|----------------|------------|---------------|
| **Discovery**  | **Answer** | **Reasoning** |
| **Recollection** | **Answer** | **Reasoning** |

| **Understanding of Composer** | **Orienta** | **Asking for help** | **Receiving help** | **Using Composer incorrectly** |
|-------------------------------|-------------|---------------------|-------------------|--------------------------------|
|                               | **Answer**  | **Reasoning**       | **Answer**        | **Reasoning**                 |

**TABLE I.** Coding scheme used in the study to code one-minute intervals of each think-aloud interview covering the **Predict** and **Explore-Revise** phases of the exercise. The codes denoted by an * are the relevant Composer features for the single-well system as discussed in Section IVB.
firmed their prediction. Each outcome was coded under one of two subcodes: the Answer and Reasoning subcodes were used to differentiate if the student simply stated the answer (describing explicitly what was happening in a given system) or attempted to ascribe some physical reasoning to the system’s behavior (describing why something was happening), respectively. Finally, in situations where students were trying to understand the workings and features of Composer itself (e.g., how to play the simulation), this was coded under Understanding of Composer, including situations where students were simply using Composer incorrectly, e.g., not pressing the Play button to begin time evolution. Importantly, if the student was using Composer incorrectly, we did not code an Outcome.

Once the final codebook was agreed upon after multiple iterations of individual-researcher coding, discrepancy reduction, and codebook modifications by C.A.W. and S.Z.A., all five interviews were coded collaboratively coded by C.A.W. and S.Z.A., with input from H.J.L. In addition to coding the interviews (using audio, video, and transcript data), we documented the accuracy of students’ responses to the questions in the Predict phase to capture whether a prediction and the reasoning surrounding it were correct (including partially correct), completely incorrect, or not present. This documentation and the students’ notes and sketches were used as supporting information during the coding process and subsequent data analysis.

IV. RESULTS

We address our four main research questions (outlined in Sec. I) in separate subsections. Sec. IV A discusses RQ1, where we present the results on the comparison of students’ responses in the Predict and Explore/Revise phases, including quotes from the interviews in order to provide insight into how Composer facilitates student learning. In Sec. IV B, we present results related to RQ2 by showing the most commonly used features of Composer. This is followed by Sec. IV C, where we discuss how the student outcomes relate to the Composer features they used. Lastly, in Sec. IV D, we compare the use of Composer in the single-well exercise (previously covered in the quantum course) and the double-well exercise (new to students). All exercise questions referred to in this section are labelled and briefly described in the flowchart in Fig. 2.

A. Impact of Composer on student answers and reasoning

Here, we address RQ1: does Composer help students determine correct answers and associated reasoning of problems in simple 1D quantum systems? We present the numbers of correct/incorrect students’ responses (answers and reasoning) for all of the questions in both the Predict and Explore/Revise phases of Q1.1. The columns represent the real and imaginary parts of the wavefunction and the probability density. The question consisted of two parts in the Predict phase, where students were asked to draw the ground and the first excited state in the static case (at $t = 0$). In the Explore/Revise phase, they were asked to compare their predictions for both the ground and excited states to their results from Composer and to explain any new reasoning. For each phase, we combine the results for the ground and excited states. Thus, for the five students, the maximum number of correct answers or reasoning is 10.

| Predict | Explore/Revise |
|---------|---------------|
| Re($\psi$) | Im($\psi$) | $|\psi|^2$ | Re($\psi$) | Im($\psi$) | $|\psi|^2$ |
| Answer | | | | | |
| Correct | 10 | 4 | 10 | 10 | 10 | 10 |
| Incorrect | 0 | 2 | 0 | 0 | 0 | 0 |
| Not present | 0 | 4 | 0 | 0 | 0 | 0 |

| Reasoning | | |
|---|---|---|
| Correct | 0 | 1 | 4 | 2 | 0 | 4 |
| Incorrect | 0 | 0 | 0 | 0 | 0 | 0 |
| Not present | 10 | 9 | 6 | 8 | 10 | 6 |

We categorized students’ responses to the exercise questions as ‘correct’ if they were entirely or partially correct, ‘incorrect’ if the responses were completely incorrect, or ‘not present’ if no response was provided. This categorization, when applied during the Predict phase indicates the measure of students’ understanding of the concept before using Composer, and when applied in the Explore/Revise phase indicates if Composer facilitated any enhancement of their knowledge. Therefore, a comparison of the responses between the two phases can indicate, in a very simplistic way, whether Composer was at all effective in helping students to answer the questions posed in the exercise. As mentioned in Sec. III, the questions concerning single eigenstates covered the ground and first excited states, but since the conceptual understanding behind these states is similar, the data for both states have been combined in Tabs. II and III.

Overall, the data from the Predict phase in Tab. II suggest that students provided mostly correct answers regarding the shape of the real part of the wavefunction and probability density. However, the shape of the imaginary part of the wavefunction was more challenging for students, as they tended to either answer incorrectly or provided no answer at all. For example, Student 4 stated

"I can’t actually remember...about the imaginary part. I don’t remember if there is a imaginary part, so I’ll just skip that and maybe think about it later."

Students typically provided no reasoning during either phase for any of the questions. This suggests that students may have known the answers, but may not necessarily have understood the reasoning behind their an-
TABLE III. Number of students’ responses provided during the Predict and Explore/Revise phases of Q1.2. The columns represent the real and imaginary parts of the wavefunction and the probability density. This question was similar to Q1.1, except it asked students to consider the time evolving case (t ≥ 0). For each phase, we combine the results for the ground and excited states. Thus, for the five students, the maximum number of correct answers or reasoning is 10.

|       | Predict | Explore/Revise |
|-------|---------|----------------|
| Q1.2  | Re(ψ)   | Im(ψ) | | Re(ψ)   | Im(ψ) | | Re(ψ) | Im(ψ) |
|       | ψ|^2 |        | ψ|^2 |        | |
| Answer |         |         |         |         |         |         |         |         |         |
| Correct| 5       | 8       | 8       | 9       | 8       | 9       | |
| Incorrect| 4   | 2       | 2       | 0       | 0       | 0       | |
| Not present| 1 | 0       | 0       | 1       | 2       | 1       | |
| Reasoning|        |         |         |         |         |         |         |         |         |
| Correct| 5       | 8       | 7       | 5       | 4       | 4       | |
| Incorrect| 1   | 2       | 1       | 0       | 0       | 0       | |
| Not present| 4 | 0       | 2       | 5       | 6       | 6       | |

swers. For example, in the Explore/Revise phase of Q1.1, the data in Tab. II show that even though all students answered correctly for all components after using Composer, very few provided any reasoning. This could be partially due to students whose predictions were correct simply moving on to the next part of the exercise, as the exercise text only asked for revision if students’ original predictions were incorrect.

We now move on to the next question in the exercise, which involved time evolution. From Tabs. II and III, the data show that students provided more incorrect answers in Q1.2 as compared to Q1.1 in the Predict phase. This could be because the question considered the time evolution of eigenstates, and studies suggest that students face difficulties with this concept [21]. Conversely, students tended to answer correctly for |ψ|^2 in both cases, and they either provided correct reasoning or no reasoning at all. Unlike for the previous question, most students provided correct answers and reasoning for the imaginary part of the wavefunction in Q1.2. This could be due to the fact that they were given the mathematical representation of the wavefunction in the exercise. An example of this is where Student 1 states

So, as t increases, the exponential factor gets smaller. That’s imaginary. Okay. I’ve only got the time and the exponential factor. We’re introducing an imaginary part, so the imaginary part won’t be zero anymore, I guess, at least.

On the other hand, more students tended to make incorrect predictions for the time evolution of the real part of the wavefunction as compared to the time-independent case. Many students predicted the real part to be static in time. This could be because they would often fail to consider the complex time-dependent exponential, and also would not see the explicit dependence of the static wavefunction on t, even though it was later provided in the exercise text (written as in Eq. (3)) for the time-dependent case. However, after using Composer, more students provided correct answers to these questions, as seen in Tab. III. The data for Explore/Revise-Re(ψ) show that the number of correct answers increased almost two-fold compared to the Predict-Re(ψ) phase. Moreover, the number of incorrect answers have reduced from 4 to 0 in the Explore/Revise phase. In some cases, students did not provide answers for the Explore/Revise phase. This was observed to be because of two reasons: either the questions were overlooked or the students did not attempt the question after using Composer incorrectly. To elucidate why some students’ responses changed from the Predict to the Explore/Revise phases, we present some quotes. For example, Student 3 initially provided the incorrect answer that the real part of the first-excited state will not evolve in time, but after using Composer, the student said

Of course it’s changing in time as well, the real part. Yeah. Because now we have a time dependency. Yeah. I didn’t think about that. That’s why before we didn’t have one, and would always be stationary...So this is the same. So I think my explanation for the density, probability density might still hold up, but for the real part, I actually didn’t even think about the time dependency of the imaginary part working here on the real part. So it was actually nice seeing that. It had an influence because sometimes when I’m just looking at the equations it doesn’t give you the interpretation of the wave functions.

Here, the student realized that the real part is time-dependent after looking at the dynamic State Comparison Plot in Composer. Additionally, the student mentioned that it was not obvious earlier that the real part would evolve in time, even though one may know the equations. The student was able to connect the visualizations of the wavefunctions shown in Composer with the underlying mathematics describing the physics of the system.

In some cases, we found that students would use something they recalled with the help of Composer to rethink an incorrect prediction and subsequently vocalize a correct answer later in the exercise. For example, Student 2 initially did not predict that the probability density of the ground state would be stationary in time. Using Composer helped them to recall what they had learned in class, as shown below.

Oh, I need the probability density...and here we go. Let’s see. There’s something wrong...why isn’t that moving more?...Oh, okay. So this part, I thought earlier would be moving. [It] doesn’t move because it’s the probability. But it makes sense, actually, from the theory that I have learned.

Later in the exercise, when asked to predict the dynamics of the first excited state, Student 2 indicates a
ear superposition state, the probability density for the static case (t = 0) of an equal linear superposition state, \( \psi_{in}(x) = \left[ \psi_0(x) + \psi_1(x) \right] / \sqrt{2} \). Here, there is only one state to draw, so the maximum number of responses is equal to the number of students (5).

| Predict | Explore/Revise |
|---------|---------------|
| Q1.3    |               |
| \( \text{Re}(\psi) \) | \( \text{Im}(\psi) \) | \( |\psi|^2 \) | \( \text{Re}(\psi) \) | \( \text{Im}(\psi) \) | \( |\psi|^2 \) |
| Correct | 3 4 2 5      | 3 5 |
| Incorrect | 2 1 3 0 | 0 0 |
| Not present | 0 0 0 0 | 2 0 |

| Reasoning |
|-----------|
| Correct | 3 4 2 3 0 3 |
| Incorrect | 0 1 2 0 0 0 |
| Not present | 2 0 1 2 5 2 |

[Table IV]

Thus, the student has developed an improved understanding in the earlier part of the exercise and applied this to make a correct prediction in another part of the exercise.

We now move from considering eigenstates of the single-well system to considering the superposition state in Eq. (2), first in the static (t = 0) case. From Tab. IV, the data indicate that students provided correct and incorrect answers almost equally, with the exception of the imaginary part of the wavefunction. In this case, most students provided correct answer and reasoning. This is a change from the numbers in Q1.1 shown in Tab. II, where roughly half of the students did not answer correctly and most provided no reasoning. As the questions were similar in nature and differed only in the system considered (eigenstate vs. superposition state), students may have applied what they had visualized or learned from the earlier questions. This is supported from the think-aloud interviews, for instance, as stated by Student 3:

And the imaginary part, yeah, so it also is constant at time equals zero. Okay. I’ll just revise. So I think it’s getting better, because I now have the knowledge from Composer, how they actually evolve, now I was able to answer more correctly than before. So I think it’s working as it should.

After working with the static superposition, students were asked how the initial superposition state in Eq. (3) would evolve in time. Their responses are tabulated in Table V, and these data imply that there is an almost an equal likelihood of students providing correct and incorrect answers in the Predict phase. In the Explore/Revise phase, the data show that all students provided correct answers after using Composer. However, only two of them provided correct reasoning, and three did not provide any reasoning.

The shift to more correct answers was aided by Composer in the following example, where Student 4 is looking at the Dynamic plot and states:

So, I can see the green one is the probability density, and the blue one is the wavefunction. And, yeah, I can see that I was quite wrong, and it also makes sense because I can’t just add the probability like before because that’s not how it works. For the wavefunctions, I have to be able to draw it properly, and I can see it doesn’t quite look like that. And then, I just have to use this to make the norm square and to get the right answer, so I was quite wrong...I have learned that I just can’t look at the probability density from the two eigenstates and then add it together, and...yeah. That’s also makes quite much sense because when I have to find the norms but of linear combinations, I have to multiply each component with each other. And then, it’s just not adding them together.

Here, the student compared their prediction to the output from Composer and noticed a discrepancy. This caused them to recall from previous experiences that they need to sum the wavefunctions and not the probability densities. Thus, Composer facilitated their learning by making them confront the conflicting answers.

Overall, during the Predict phase, students provided answers with varying levels of correctness for all the questions (Q1.1-Q1.4), and they often struggled with providing reasoning, even in situations where they provide correct answers. Then, in the Explore/Revise phase, there is
Outcomes were also coded using the Explore/Revise rect or incorrect during the
ing around these types of quantum systems.
not be a sufficient stand-alone tool for developing reason-
after using Composer. This suggests that Composer may
the number of students providing correct reasoning even
reasoning (answer) component for each outcome.

generally an increase in the number of students answering correctly. However, there is no substantial increase in the number of students providing correct reasoning even after using Composer. This suggests that Composer may not be a sufficient stand-alone tool for developing reasoning around these types of quantum systems.

In addition to categorizing students’ responses as correct or incorrect during the Explore/Revise phase, their responses were also coded using the Outcomes codes listed in Tab. I. Figure 3 shows the percentage of all student responses that were coded by each Outcomes for questions Q1.1 - Q1.4. The figure indicates that the majority of outcomes were coded as Discovery or Reinforcement. Recollection was rarely coded, indicating that students almost never explicitly discussed drawing from material they had learned in the course when resolving incorrect predictions. This suggests that although Composer allowed students to confirm or refute their predictions, they typically did not then express aloud the connections to concepts covered in class. Moreover, these data also indicate that for all Outcomes the number of instances coded for answers is approximately twice the number of instances coded for reasoning. This supports our earlier analysis that Composer might help students to provide correct answers, but often their reasoning is still incorrect or absent.

B. Composer features

In this section, we address RQ2: what Composer features were most used by students? We present results from the interviews on the use of Composer features during the Explore/Revise phase of the exercise on the single-well system.

We quantified the use of Composer features in two ways: the number of instances coded and the total duration coded (in minutes) for the different features. For example, if we code the use of a Composer feature for three consecutive minutes, this is counted as one instance, but three minutes duration. In this section, we consider six subcodes under the Exploration code (denoted by * in Tab. I), as these are the relevant features used during the single-well system exercise.

For each of the relevant Composer feature codes, we present the cumulative number of instances and minutes that were coded in all five interviews in Fig. 4. The graph indicates that students engage regularly with most of the relevant features, except Set number of eigenstates, where only two students used this feature a few times. This is largely due to the fact that this value did not need to be changed to complete the exercise. All other features were used by all five students, although for varying number of instances and amount of time. When comparing the number of instances and minutes for the use of visualization features, the data show that the number of instances are around half of the number of minutes. This suggests that whenever students used these plots, they used them continuously for a number of consecutive minutes, whereas for the other features, the number of instances is closer to the number of minutes. As such, students are continuously engaging with the visualization features, but they tend to set quantities in more discrete intervals.

C. Connecting Composer use with Outcomes

In this section, we address RQ3: how was the use of Composer features connected with outcomes? The analysis was performed using coding stripes as shown in Fig. 5. The colored stripes in the figure correspond to a single main code, each characterized by a color. This type of data visualization can assist in determining patterns and discerning the interplay between the use of Composer features and the outcomes obtained. In our analysis, we consider the broad categories of Interactions, Visualizations, and Outcomes.

The coding stripes shown in Fig. 5 suggest that students almost always engaged with the Interaction and Visualization features at the same time. Outcomes also tended to occur simultaneously with the Interactions and Visualizations, but not always at the end of the time segment as one might initially suspect. This suggests that students revised their understanding during the use of Composer, as Outcomes are coded when the students communicated their understanding. However, there are a few scenarios where Outcomes are not coded concurrently with use of Composer. For example, looking at Student 1’s data for the single-well system exercise, outcomes are absent between minutes 8-16, even though the student engaged with Composer through its interaction and visualization features. The video data indicate that Student 1 was engaging with Composer to understand how the software worked during this time, but they were
FIG. 4. Bar charts showing (a) the number of separate instances and (b) the cumulative number of minutes coded for each student’s use of various Composer Interactions and Visualizations during the Explore/Revise phase of the single-well system exercise. The different colored bars in the charts show (a) how often or (b) for how long each student performed each Interaction or made use of each Visualization, and the stacked sum of all bars shows the total number summed across all students.

using the software incorrectly. Specifically, they were trying to look at the time evolution of a state without pressing the Play button that would allow dynamics to commence. This resulted in an absence of an outcome, as the student was making inferences about the system based on incorrect information. Thus, instead of coding for an Outcome, this was coded as the student Using Composer Incorrectly. Interactions like these also indicate the presence of a learning curve with Composer in that students must familiarize themselves with the tool before they can adequately use it to answer questions.

D. Comparison between single-well and double-well system exercises

In this section, we address our final research question, RQ4: how do students use Composer to explore a new system as compared to a system they have worked with previously? That is, the single-well system was covered extensively in the course, but the double-well system was not addressed at all.

We start by comparing students’ explorations in Composer for the single-well and double-well systems. In Fig. 5, we present the coding stripes for both halves of the exercise, where the single-well system is coded to the left of the black vertical line and the double-well to the right. Both parts of the exercise show a similar pattern of interplay of Interactions, Visualizations and Outcomes. This suggests that even if the students are exploring a new problem, they engage with Composer in a similar way. For example, when considering the double-well system, we see that Outcomes are still coded concurrently with Interactions and Visualizations. There are slightly longer periods of time devoted to exploring with Composer during the double-well as compared to the single-well. This is not unexpected as students lacked familiarity with this quantum system, and it reflects the exercise
FIG. 5. Coding stripes of think-aloud interviews of all five students are shown for the Interactions (I, blue), Visualizations (V, purple) and the Outcomes (O, dark red) codes. The black vertical line demarcates the two halves of the exercise; the stripes to the left (right) of the vertical line correspond to the single (double) well system. All students except Student 2 finished within 55 minutes.

design, which asked students to Explore the double-well system before making any predictions. This, however, does indicate that students were still able to investigate unfamiliar problems with Composer and, therefore, it could be used as an exploratory, and not just a confirmatory, tool.

To further bolster the claim that students were able to use Composer in similar ways for both types of systems, we present the duration of the Interactions and Visualization codes for all questions in the single-well and the double-well exercise (Fig. 6). The stacked bar chart shows that for all questions, both codes were applied for roughly equal duration, regardless of what part of the exercise is considered.

In addition to comparing the patterns of how students used Composer between different types of questions, it is interesting to note to what extent students were able to report correct answers and reasoning for this new system. We examine two of the questions for the double-well system that had complete Predict-Explore-Revise components. Q2.3 asked students to determine the coefficients in the wavefunction to localize the state in either the left or right well. Then, Q2.4 asked them about the time evolution of those states. Note again that students were asked to Explore how the energy levels and eigenstates changed with the well separation in Q2.1 and Q2.2, respectively, before moving to the questions presented here, so they had some idea of how the system’s eigenstates behaved in the static case.

As we can see from the data reported in Tab. VI for Q2.3, the number of correct answers doubled after using Composer, but students’ expression of correct reasoning did not change much. We note, however, that, as with
the other questions, students provided no incorrect reasoning after using Composer. For Q2.4, we see that most students reported a correct answer in both the Predict and Explore/Revise phases, even though this was a system unfamiliar to them. Because of the small numbers and the number of correct initial predictions, there is little insight to be gained from this particular question. However, there is evidence that Composer helped students to understand what was happening, as Student 4 states:

If I move [the wells] closer then they will oscillate quicker between the left well and the right well. Okay. So, did this agree with what I predicted? Nope, it did not at all. Yeah. So I think I’ve already rethink (sic) my understanding of the, what Composer is showing me. And I think it makes sense to what is actually happening, which again is a good tool. Seeing how it actually works because it’s hard getting an understanding of wave functions, oscillating through time, when you almost only look at the stationary state.

Overall, we see that, when considering a new problem, students engage with Composer in a similar way as compared to a known problem based on these very simple measures and that using Composer continues to increase the number of correct answers reported by students. This is encouraging because it indicates that students familiar with Composer may be able to use it productively when faced with unfamiliar problems.

V. DISCUSSION

These results show that Composer assists in student learning of quantum mechanics by allowing them to visualize the correct behavior of a given system and facilitates students’ ability to make connections between what they learned in their coursework with the visualizations obtained from Composer. As they went through the exercise, almost all students were able to express the correct answers after using Composer, even if they provided incorrect or no answers at all in the Predict phase. This holds true for all questions for the single- and double-well parts of the exercise. In questions regarding the time-evolution of single eigenstates and superposition states, students explicitly mentioned that the visualization in Composer helped them to think about the complex time-dependent exponential, an element they would typically forget to consider during the Predict phase of the exercise. Therefore, we believe that Composer could be particularly beneficial in addressing questions concerning time evolution of states, as it allows students to visualize the real part, imaginary part, and the probability density for any state. Moreover, features like the State Comparison Plot (coded as Dynamic plot) help students compare the behavior of a quantum state in the static and dynamic cases. This assists students in understanding the differences between the two cases, although Composer alone is not necessarily well-suited to helping students to express their reasoning behind their answers. However, our data show that students do not express false reasoning after using Composer.

This study shows that only sparse moments of stu-
dent reasoning happen during the Explore/Revise phase. We think that this may be due to the fact that Composer was not developed to focus on student reasoning, but more on simulating and visualizing a wide array of quantum mechanical systems [10]. This suggests that Composer is best used in conjunction with a textbook, exercise, lecture, or other tool that focuses on elucidating the reasoning behind the physical behavior of the system under study. For example, student reasoning may be better developed in an exercise where students work out a problem using mathematical representations and check their answers in Composer. This can help them to build insight into problems where they connect what they see in Composer with what they have worked out mathematically. Additionally, these exercises should help students develop better reasoning through guiding questions that connect the math to the visualizations [59].

Regarding students’ exploration in Composer, this study demonstrates that students mostly interact and visualize simultaneously, which resulted in new learning or revision of their previous knowledge. The simultaneous interplay of Interactions, Visualizations, and Outcomes shown through coding stripes may be an attribute of the rapid feedback that Composer provides. This study also demonstrates that students’ explorations occur in roughly the same manner, independent of whether they are exploring a familiar or new quantum mechanical system. This could be an indication that when using Composer, students may be less intimidated when they explore new problems. Therefore, Composer can be useful when exploring problems outside the typical set of analytically-solvable problems contained within an introductory or advanced quantum mechanics curriculum that are relevant for technological applications or research. We do note, however, that there were cases where students had trouble finding the play icon or using the linear combination node. This informs future design changes in the introduction of students to Composer or to the interface itself. Generally speaking, a comprehensive introduction is likely a prerequisite to let the user gain familiarity with Composer before diving in for exploration [10].

VI. LIMITATIONS

One of the main limitations of this study is that the inferences drawn may not be generalizable with respect to claims about enhanced learning, as the study covered think-aloud interviews of only five students from one course at one university. This university was also the one where Composer was developed. Additionally, there existed a language barrier as English was not the first language of the interviewees, and this may have hindered the efficacy of the think-aloud protocol and our ability to discern their thinking. Finally, we note that as this was the first such study using a brand new visualization tool, the data collected do not allow us to probe in-depth the exact nature of how students used Composer to facilitate their learning, but allow us to make only broad claims.

VII. CONCLUSION AND OUTLOOK

The aim of this study was to probe whether Quantum Composer can be used as a tool to assist students’ learning of quantum mechanics in two different kinds of problems: single-well and double-well systems. The exercise covered questions related to the static and dynamic case of the real part, imaginary part, and probability density of single eigenstates and superposition states. The study used a think-aloud protocol with five students, where we found that all students could determine or revise answers to problems in quantum mechanics by changing parameters and visualizing the system in Composer. Overall, student learning was aided by exploration in Composer, in particular, with regards to their ability to provide the correct answers to questions posed. This work bolsters previous studies on the impact of visualization tools in learning quantum mechanics. From the results presented, we recommend that Quantum Composer be used as an accompanying tool to other course materials or environments for teaching and learning one-dimensional quantum mechanics.

As this was just our first exploratory study of the benefits and limitations of using Composer to aid in student learning of 1D quantum mechanical systems, there are many opportunities for more in-depth research studies. Future work will include a larger number of student participants with different demographics and educational backgrounds. Additionally, we could examine clickstream-type data to better understand how exactly students interact with all of Composer’s features. We could also create new types of exercises that help develop and elicit student reasoning around these topics. Given Composer’s unique flexibility, we could also study how students and educators create and/or modify flowscenes, as they illustrate or explore different concepts. Ultimately, we hope to be able to not just refine Quantum Composer so that it is more useful for students, but we also hope to understand the best way to incorporate Composer in quantum mechanics courses for all science and engineering majors, including those focused on a career in quantum information science and technology.

ACKNOWLEDGMENTS

The authors would like to thank M. Murdrich for his support regarding the use of Composer in the course. We acknowledge funding from the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie QuSCo grant agreement No. 765267 and the ERC Proof-of-Concept grant PQTEI. Additional support was provided by the US National Science Foundation (PHY-1734006 and QLCI Award OMA-
[1] I. H. Deutsch, Harnessing the power of the second quantum revolution, PRX Quantum 1, 020101 (2020).
[2] M. F. J. Fox, B. M. Zwickl, and H. J. Lewandowski, Preparing for the quantum revolution: What is the role of higher education?, Phys. Rev. Phys. Educ. Res. 16, 020131 (2020).
[3] H. K. E. Stadermann, E. van den Berg, and M. J. Goedhart, Analysis of secondary school quantum physics curricula of 15 different countries: Different perspectives on a challenging topic, Phys. Rev. Phys. Educ. Res. 15, 010130 (2019).
[4] C. D. Aiello, D. D. Awschalom, H. Bernien, T. Brower-Thomas, K. R. Brown, T. A. Brun, J. R. Caram, E. Chitambar, R. D. Felice, K. M. Edmonds, M. F. J. Fox, S. Haas, A. W. Holleitner, E. R. Hudson, J. H. Hunt, R. Joynt, S. Koziol, M. S. Larsen, H. J. Lewandowski, D. T. McClure, J. Palsberg, G. Passante, K. L. Pudenz, C. J. K. Richardson, J. L. Rosenberg, R. S. Ross, M. Saffman, M. Singh, D. W. Steuerman, C. Stark, J. M. Thijssen, N. Vamvakas, J. Whitfield, and B. M. Zwickl, Achieving a quantum smart workforce, Quantum Science and Technology (2021).
[5] C. Singh and E. Marshman, Review of student difficulties in upper-level quantum mechanics, Phys. Rev. ST Phys. Educ. Res. 11, 020117 (2015).
[6] K. Krijtenburg-Lewerissa, H. J. Pol, A. Brinkman, and W. R. van Joolingen, Insights into teaching quantum mechanics in secondary and lower undergraduate education, Phys. Rev. Phys. Educ. Res. 13, 010109 (2017).
[7] C. Singh, Interactive learning tutorials on quantum mechanics, American Journal of Physics 76, 400 (2008), https://doi.org/10.1119/1.2837812.
[8] A. Kohlme, D. Cassetari, T. J. Edwards, C. Ferguson, A. D. Gillies, C. A. Hooley, N. Korolkova, J. Llama, and B. D. Sinclair, A new multimedia resource for teaching quantum mechanics concepts, American Journal of Physics 80, 148 (2012), https://doi.org/10.1119/1.3657800.
[9] S. B. McKagan, K. K. Perkins, M. Dubson, C. Malley, S. Reid, R. LeMaster, and C. E. Wieman, Developing and researching PhET simulations for teaching quantum mechanics, American Journal of Physics 76, 406 (2008), https://doi.org/10.1119/1.2885199.
[10] S. Z. Ahmed, J. H. M. Jensen, C. A. Weidner, J. J. Sørensen, and J. F. Sherson, Quantum Composer: Quantum simulation and visualization tool for education and research, American Journal of Physics 89, 307 (2021).
[11] C. A. Weidner, S. Z. Ahmed, J. H. M. Jensen, J. F. Sherson, and H. J. Lewandowski, Investigating student use of a flexible tool for simulating and visualizing quantum mechanics, in Physics Education Research Conference 2020, PER Conference (Virtual Conference, 2020) pp. 563–568.
[12] D. F. Styer, Common misconceptions regarding quantum mechanics, American Journal of Physics 64, 31 (1996).
[13] C. Singh, Student understanding of quantum mechanics, American Journal of Physics 69, 885 (2001), https://doi.org/10.1119/1.1365404.
[14] E. Cataloglu and R. W. Robinett, Testing the development of student conceptual and visualization understanding in quantum mechanics through the undergraduate career, American Journal of Physics 70, 238 (2002).
[15] C. Singh, Student understanding of quantum mechanics at the beginning of graduate instruction, American Journal of Physics 76, 277 (2008), https://doi.org/10.1119/1.2825387.
[16] G. Zhu and C. Singh, Surveying students’ understanding of quantum mechanics in one spatial dimension, American Journal of Physics 80, 252 (2012), https://doi.org/10.1119/1.3677653.
[17] P. J. Emigh, G. Passante, and P. S. Shaffer, Student understanding of time dependence in quantum mechanics, Phys. Rev. ST Phys. Educ. Res. 11, 020112 (2015).
[18] T. Wan, P. Emigh, G. Passante, and P. Shaffer, Student understanding of period in introductory and quantum physics courses, in Physics Education Research Conference 2016, PER Conference (Sacramento, CA, 2016) pp. 380–383.
[19] S. B. McKagan, K. K. Perkins, and C. E. Wieman, Deeper look at student learning of quantum mechanics: The case of tunneling, Phys. Rev. ST Phys. Educ. Res. 4, 020103 (2008).
[20] G. Passante, B. P. Schmermerhorn, S. J. Pollock, and H. R. Sadaghiani, Time evolution in quantum systems: A closer look at student understanding, European Journal of Physics 41, 015705 (2019).
[21] G. Passante and A. Kohlme, Enhancing student visual understanding of the time evolution of quantum systems, Phys. Rev. Phys. Educ. Res. 15, 010110 (2019).
[22] A. Kohlme, M. Douglass, T. J. Edwards, A. D. Gillies, C. A. Hooley, and B. D. Sinclair, Developing and evaluating animations for teaching quantum mechanics concepts, European Journal of Physics 31, 1441 (2010).
[23] D. Brandt, J. R. Hiller, and M. J. Moloney, Modern Physics Simulations: The Consortium for Upper-Level Physics Software (Wiley, Hoboken, NJ, 1995).
[24] J. R. Hiller, I. D. Johnson, and D. F. Styer, Quantum Mechanics Simulations: The Consortium for Upper-Level Physics Software (Wiley, Hoboken, NJ, 1995).
[25] J. R. Hiller, I. D. Johnston, and I. Johnson, Quantum Mechanics Simulations (John Wiley and Sons, Inc., 1995).
[26] B. Thaller, Visual Quantum Mechanics (Springer-Verlag New York, 2000).
[27] B. Thaller, Advanced Visual Quantum Mechanics (Springer-Verlag New York, 2005).
[28] W. Christian, F. Belloni, Mario nd Esquemeb, B. A. Mason, L. Barbato, and M. Riggsee, The Physlet approach to simulation design, The Physics Teacher 53, 419 (2015).
[29] D. A. Zollman, N. S. Rebello, and K. Hogg, Quantum mechanics for everyone: Hands-on activities integrated with technology, American Journal of Physics 70, 252 (2002).
[30] R. Müller and H. Wieser, Teaching quantum mechanics on an introductory level, American Journal of Physics 70, 200 (2002).
[31] http://www.falstad.com/mathphysics.html, retrieved 5/30/2020.
[32] L. T. Escalada, N. S. Rebello, and D. Zollman, Student explorations of quantum effects in LEDs and luminescent devices, The Physics Teacher 42, 10.1119/1.1664385 (2004).
[33] http://www.quantum-physics.polytechnique.fr (2019), retrieved 5/30/2020.

[34] J. Johansson, P. Nation, and F. Nori, QuTiP: An open-source Python framework for the dynamics of open quantum systems, Computer Physics Communications 183, 1760 (2012).

[35] J. Johansson, P. Nation, and F. Nori, QuTiP 2: A Python framework for the dynamics of open quantum systems, Computer Physics Communications 184, 1234 (2013).

[36] J. J. Sørensen, J. H. M. Jensen, T. Heinzel, and J. F. Sherson, QEngine: A C++ library for quantum optimal control of ultracold atoms, Comp. Phys. Comm. 243, 135 (2019).

[37] https://phet.colorado.edu/ (2021), retrieved 3/25/2021.

[38] https://www.physport.org/curricula/QuILTs/ (2021), retrieved 3/25/2021.

[39] https://www.st-andrews.ac.uk/physics/quvis/index.html (2021), retrieved 3/25/2021.

[40] C. Wieman, W. Adams, P. Loeblein, and K. Perkins, Teaching physics using PhET simulations, The Physics Teacher 48, 10.1119/1.3361987 (2002).

[41] W. K. Adams, S. Reid, R. LeMaster, S. B. McKinigan, K. K. Perkins, M. Dubson, and C. E. Wieman, A Study of Educational Simulations Part I - Engagement and Learning., Journal of Interactive Learning Research 19 (2008).

[42] W. K. Adams, S. Reid, R. LeMaster, S. McKinigan, K. Perkins, M. Dubson, and C. E. Wieman, A Study of Educational Simulations Part II - Interface Design, Journal of Interactive Learning Research 19 (2008).

[43] N. S. Podolefsky, K. K. Perkins, and W. K. Adams, Computer simulations to classrooms: tools for change, in AIP Conference Proceedings, Vol. 1179 (2009).

[44] S. DeVore, E. Marshman, and C. Singh, Challenge of engaging all students via self-paced interactive electronic learning tutorials for introductory physics, Phys. Rev. Phys. Educ. Res. 13, 010127 (2017).

[45] E. Marshman and C. Singh, Investigating and improving student understanding of the expectation values of observables in quantum mechanics, Eur. J. Phys. 38, 10.1088/1361-6044/aa6d34 (2017).

[46] R. Sayer, A. Maries, and C. Singh, Quantum interactive learning tutorial on the double-slit experiment to improve student understanding of quantum mechanics, Phys. Rev. Phys. Educ. Res. 13, 010123 (2017).

[47] C. Keebaugh, E. Marshman, and C. Singh, Improving student understanding of corrections to the energy spectrum of the hydrogen atom for the Zeeman effect, Phys. Rev. Phys. Educ. Res. 15, 010113 (2019).

[48] A. Kohnle, C. Baily, A. Campbell, and N. Korolkova, Enhancing student learning of two-level quantum systems with interactive simulations, American Journal of Physics 83, 560 (2015).

[49] A. Kohnle and G. Passante, Characterizing representational learning: A combined simulation and tutorial on perturbation theory, Phys. Rev. Phys. Educ. Res. 13, 020131 (2017).

[50] A. Kohnle, C. Baily, and S. Ruby, Investigating the influence of visualization on student understanding of quantum superposition, in Physics Education Research Conference 2014, PER Conference (Minneapolis, MN, 2014) pp. 139–142.

[51] D. J. Griffiths and D. F. Schroeter, Introduction to Quantum Mechanics, Third Edition (Cambridge University Press, 2018).

[52] J. S. M.W. van Someren, Y. F. Barnard, The think aloud method: a practical approach to modelling cognitive processes (Academic Press, London, 1994).

[53] M. E. Fonteyn, B. Kuipers, and S. J. Grobe, A description of think aloud method and protocol analysis, Qualitative Health Research 3, 430 (1993), https://doi.org/10.1177/104973239300300403.

[54] J. Cowan, The potential of cognitive think-aloud protocols for educational action-research, Active Learning in Higher Education 20, 219 (2019), https://doi.org/10.1177/1469787417735614.

[55] L. Ríos, B. Pollard, D. R. Doumas-Frazer, and H. J. Lewandowski, Using think-aloud interviews to characterize model-based reasoning in electronics for a laboratory course assessment, Phys. Rev. Phys. Educ. Res. 15, 010140 (2019).

[56] D. Cotton and K. Gresty, Reflecting on the think-aloud method for evaluating e-learning, British Journal of Educational Technology 37, 45 (2006), https://berajournals.onlinelibrary.wiley.com/doi/pdf/10.1111/j.1467-8535.2005.00521.x.

[57] N. S. Podolefsky, K. K. Perkins, and W. K. Adams, Computer simulations to classrooms: tools for change, in AIP Conf. Proc., Vol. 1179 (2009) p. 233.

[58] https://www.rev.com (2021), retrieved 4/19/2021.

[59] B. R. Wilcox, M. D. Caballero, D. A. Rehn, and S. J. Pollock, Analytic framework for students’ use of mathematics in upper-division physics, Phys. Rev. ST. Phys. Educ. Res. 9, 020119 (2013).