Stability of the Coexistence Phase of Chiral Superconductivity and Noncollinear Spin Ordering with a Nontrivial Topology and Strong Electron Correlations

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It has been shown that quantum charge and spin fluctuations in a strongly correlated 2D system with a triangular lattice, significantly renormalizing the magnetic order parameter, do not destroy the coexistence phase of chiral \( \mathbf{d} + \mathbf{i}\mathbf{d} \) superconductivity and 120° spin ordering. The region of realization of nontrivial topology determined by the topological index \( \tilde{N}_3 \) holds. It has been shown that edge states for the topologically nontrivial phase include a Majorana mode. The spatial structure of this mode has been determined. It has been found that spin and charge fluctuations shift the critical electron densities at which quantum topological transitions occur. It has been shown that an increase in the intersite Coulomb repulsion reduces the number of such transitions.

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1. INTRODUCTION

Several superconducting systems allowing Majorana edge states have recently been proposed. Among them are superconductors with chiral \( p \)-wave symmetry [1, 2], interfaces between a superconductor and a topological insulator [3, 4], and systems with the spin–orbit interaction and induced superconductivity [5–9]. The greatest experimental progress was achieved for InAs and InSb semiconductor nanowires epitaxially coated by superconducting aluminum. A quantized conductance peak was observed in them at zero bias voltage with an increase in an external magnetic field [10].

It has recently been found that Majorana modes can occur in materials allowing the coexistence of singlet superconductivity and a long-range magnetic order [11, 12]. Such a scenario is promising because of the possibility of appearance of Majorana modes in a solid that does not have the spin–orbit interaction and is not subjected to an external magnetic field. Within such a concept, the existence of Majorana modes was demonstrated in superconducting systems with helical magnetic ordering (e.g., HoMo_6S_8 and ErRh_4B_4) [11].

It is known that edge and Majorana modes appear in a system with open boundaries if the ground state of this system with periodic boundary conditions corresponds to a phase with nontrivial topology. These phases are classified in terms of the topological invariant. In simple cases, where the Hamiltonians of 1D or 2D superconducting systems with broken time reversal symmetry (class D systems according to the classification proposed in [13]) are described by quadratic forms in secondary quantization operators, topological invariants are the \( Z_2 \) invariant (Majorana number [2]) or \( Z \) invariant, respectively. A relation between two invariants by the example of noncentrosymmetric superconductors was described in [14]. A transition between phases with different values of the topological invariant is induced when the gap in the elementary excitation spectrum is closed [15].

Lu and Wang [12] predicted that Majorana modes can be observed in 2D systems with a triangular lattice in the coexistence phase of chiral \( d_1 + id_2 \) superconductivity and a stripe magnetic order. However, the further analysis [16] showed that chiral superconductivity cannot coexist with stripe ordering of spins, but coexists with a magnetic order corresponding to a 120° structure. Conditions for appearance of Majorana modes in this coexistence phase were determined in [17]. The \( Z_2 \) and \( Z \) invariants were calculated with a quadratic Hamiltonian and it was shown that topologically nontrivial regions with an odd value of the \( Z \) invariant are in good agreement with the found regions of existence of Majorana modes.

1 Supplementary materials are available for this article at https://doi.org/10.1134/S0021364019110158 and are accessible for authorized users.
The problem of electron correlations in topological phases has become of great interest recently because correlations can change the topological classification [18]. Meanwhile, the classification holds in systems with an even dimension, which are described by the $Z$ invariant (e.g., class D systems, which includes the mentioned systems with the triangular lattice) [19].

The topological classification of systems with the interaction can be based on a universal method, where the topological index is defined in terms of Green’s functions [20]. The $Z$ invariant for 2D systems, where the excitation spectrum has an energy gap expressed in terms of the matrix Green’s function [20]. In [15], this invariant is denoted as $\tilde{N}_3$. This invariant allowed demonstrating a nontrivial topology of systems with the quantum Hall effect [20] and $^3$He-A [21] and $^3$He-B [22] phases of liquid helium. It was shown later [23] that the Green’s function at zero frequency is sufficient to describe topological phases.

The development of a topological classification of strongly correlated materials with the triangular lattice in the coexistence phase of chiral superconductivity and noncollinear spin ordering requires the preliminary analysis of the stability of such a phase with respect to charge and spin fluctuations. The reason is that the role of quantum fluctuations in the mechanism of destruction of the ordered phase increases significantly in the case of reduced dimension and the frustrated character of exchange bonds in the triangular lattice.

In this work, we show that the magnetic order parameter is strongly renormalized in the experimentally studied range of doping of sodium cobaltate, but the structure of spin ordering holds. For the coexistence phase of chiral superconductivity and the $120^\circ$ spin structure, we find the Green’s functions and topological characteristics of this phase in terms of the $\tilde{N}_3$ invariant. Changes in the topological properties of systems at an increase in the density and Coulomb repulsion of electrons are analyzed. A decrease in the number of topological transitions at an increase in the Coulomb interaction parameter is revealed. Solving the system of equations for Green’s functions with open boundary conditions along one direction of the 2D lattice, we demonstrate the structure of the Majorana mode in the topologically nontrivial region.

2. MODEL

The coexistence phase of chiral superconductivity and $120^\circ$ spin ordering taking into account spin and charge fluctuations in the presence of strong electron correlations is studied within the $t-J-V$ model. For definiteness, we consider electron-doped systems such as the Na$_x$CoO$_2$ superconductor [24]. The Hamiltonian of such a system in the atomic representation has the form

$$H = \sum_{\sigma} \varepsilon \cdot \sigma X_{\sigma}^2 + \sum_{J} (2\varepsilon + U - 2\mu) X_{J}^{22}$$
$$+ \sum_{J_{m}} J_{m} X_{J_{m}}^{22} + \frac{V}{2} \sum_{J} n_{J} n_{J+\delta}$$
$$+ \sum_{J_{m}} J_{m} \left( X_{J_{m}}^{\uparrow\downarrow} X_{J_{m}}^{\downarrow\uparrow} - X_{J_{m}}^{\uparrow\uparrow} X_{J_{m}}^{\downarrow\downarrow} \right),$$

where $\varepsilon$ is the bare electron energy, $\mu$ is the chemical potential, $U$ is the intra-atomic Coulomb repulsion, $J_{m}$ is the intensity of electron hoppings, $V$ is the parameter of the intersite Coulomb interaction, $n_{J} = X_{J}^{\uparrow\downarrow} + X_{J}^{\downarrow\uparrow} + 2X_{J}^{22}$ is the operator of the number of electrons at the site, and $J_{m}$ is the exchange interaction parameter.

3. CONDITIONS OF EXISTENCE OF GAPLESS FERMION EXCITATIONS IN THE COEXISTENCE PHASE

As known [15], topological transitions in systems with a nontrivial topology are possible when elementary fermion excitations calculated for periodic boundary conditions become gapless. The excitation spectrum in the noncollinear magnetic phase is gapless on the Fermi contour (lines in the 2D Brillouin zone) at any doping level. Gapless excitations in the superconducting phase with the chiral $d_{1} + id_{2}$ symmetry of the order parameter occur only at separate points of the Brillouin zone (nodal points) whose positions depend on the electron density [25].

The conditions for the existence of gapless excitations in the coexistence phase can be easily obtained using the expression for the fermion spectrum [26]

$$E_{1,2} = \frac{1}{2} \left( 2\lambda_{p}^{2} + \xi_{p}^{2} + |\Delta_{p}|^{2} + |\Delta_{-p}|^{2} \right)$$
$$+ R_{p} R_{-p} + \lambda_{p}^{1/2}. $$

Here,

$$\lambda_{p} = \frac{1}{4} \left( \xi_{p}^{2} - \xi_{-p}^{2} + |\Delta_{p}|^{2} - |\Delta_{-p}|^{2} \right)$$
$$+ R_{p} R_{-p} \left( \xi_{p}^{2} + \xi_{-p}^{2} + |\Delta_{p}|^{2} + |\Delta_{-p}|^{2} \right)^{1/2},$$

$\xi_{p} = \varepsilon + U - \mu + J_{0} (1 - n/2) + V_{0} n + n_{p}/2 = \xi_{0} + n_{p}/2$;

$n = \{n_{J} \}$ is the electron density per site; $J_{0}$ and $J_{Q}$ are the Fourier transforms of the exchange integral for the vector (0, 0) and magnetic structure vector $Q$, respectively; $V_{0} = 6V$; $R_{p} = M(t_{p} - J_{0})$; $R_{-p} = M(t_{p} - Q - J_{0})$; $M$ is the amplitude of the inhomogeneous magnetic order parameter determining the spin structure as...
\[ (S_f) = M(\cos(Qf), -\sin(Qf), 0); \Delta_p \text{ is the superconducting order parameter expressed in terms of } d_1 + id_2 \text{ and } p_1 + ip_2 \text{ chiral invariants.} \]

According to Eq. (2), the nodal points of the spectrum in the coexistence phase are determined by the equations

\[ \text{Im}(\Delta_p \Delta_{p+Q}^*) = 0, \]
\[ \xi_p \Delta_{p+Q} - \xi_p \Delta_p = 0, \]
\[ R_p R_{p+Q} - \xi_p^2 R_{p+Q} - \text{Re}(\Delta_p \Delta_{p+Q}^*) = 0. \]

The 120° spin order on the triangular lattice is specified by the vector \( Q = (2\pi/3, 2\pi/3) \). Here and below, the coordinates of wave vectors are presented in the basis of reciprocal lattice unit vectors. In this case, conditions \( \Delta_p = \Delta_{p+Q} = 0 \), under which Eqs. (3) and (4) are valid, are satisfied at the center of the hexagonal Brillouin zone (\( \Gamma = (0, 0) \) point) and at its edges (\( K = Q \) and \( K' = -Q \) points). The parameters determining the occurrence of gapless excitations at the \( \Gamma \) point are found from the equation

\[ M(n) = \begin{pmatrix} -\bar{\mu}(n, M) - 3n/2(t_1 - 2t_2 + t_3) \\ -3(t_1 - 2t_2 + t_3) + 3(J_1 - 2J_2) \end{pmatrix} \]

Here, \( \bar{\mu}(n, M) = \mu(n, M) - J_0(1 - n/2 - V_0n); t_1, t_2, \text{ and } t_3 \) are the hopping integrals for three coordination spheres; and \( J_1 \) and \( J_2 \) are the exchange coupling constants between the nearest and next-nearest spins, respectively.

Under the condition

\[ M^2(n) = \begin{pmatrix} -\bar{\mu}(n, M) - 3n/2(t_1 - 2t_2 + t_3) \\ -3(t_1 - 2t_2 + t_3) + 3(J_1 - 2J_2) \end{pmatrix} \]

\[ \times \begin{pmatrix} -\bar{\mu}(n, M) + 3n(t_1 - 2t_2 + t_3) \\ 6(t_1 + t_2 + t_3) + 3(J_1 - 2J_2) \end{pmatrix} = 0, \]

the gapless spectrum of fermion excitations occurs at the \( \Gamma \) and \( K \) points simultaneously.

The presented conditions for the appearance of the gapless spectrum of fermion quasiparticles and, thereby, conditions for topological transitions at the variation of the electron density show that it is necessary to obtain the expression for \( M \) renormalized by spin and charge fluctuations. This problem is solved in the next section.

### 4. Renormalization of the Amplitude \( M \)

Magnets with the 120° ordering of localized spins were theoretically considered in [27–31]. The electronic ensemble on the triangular lattice was studied within the Hubbard model in the mean-field approximation [32], as well as in the slave-boson representation [33]. Phase diagrams were obtained, indicating the possibility of existence of spin and charge ordering.

It is substantial that the ground state with 120° spin ordering holds at doping near half filling. Using the Monte Carlo method, the authors of [34] demonstrated the possibility of the coexistence phase of superconductivity and 120° spin ordering near \( n = 1.1 \).

To simplify the procedure of determination of renormalizations for \( M \), we transform the Hamiltonian as

\[ H \rightarrow \tilde{H} = UHU^+, \]

where

\[ U = \prod_i [\exp(i\frac{\pi}{2}S_f^i)] \exp[i\theta_i S_f^i]), \quad \theta_i = -Qf, \]

which corresponds to the transition to the coordinate system where the \( z \) axis is aligned with \( (S_f) \) at each site. The transformation laws for operators have the form

\[ S_f^x \rightarrow \tilde{S}_f^x = \cos(\theta_f)S_f^x - \sin(\theta_f)S_f^y, \]
\[ S_f^y \rightarrow \tilde{S}_f^y = \cos(\theta_f)S_f^y + \sin(\theta_f)S_f^x, \]
\[ S_f^z \rightarrow \tilde{S}_f^z = -S_f^z, \]

\[ X_f^{\sigma\sigma} \rightarrow \tilde{X}_f^{\sigma\sigma} = \sum_{\sigma, \tau} X_f^{\sigma\tau} / 2 - \eta_\sigma (X_f^{\tau\sigma} + X_f^{\sigma\tau}) / 2, \]

where \( X_f^{\tau\sigma} = S_f^\tau S_f^\sigma \) and \( \eta_\sigma = +1 \) and \( -1 \) for \( \sigma = \uparrow \) and \( \downarrow \), respectively.

After the separation of mean field corrections, the transformed Hamiltonian has the form

\[ \tilde{H} = \sum_{f, \sigma} \tilde{\xi}_{\sigma} X_f^{\sigma\sigma} + \sum_f (2\varepsilon + U + 2nV_0 - 2\mu) X_f^{22} \]
\[ + \sum_{f, m} \cos Q_2 f - m X_f^{\sigma\sigma} X_m^{\sigma\sigma} \]
\[ - i \sum_{f, m} \sin Q_2 f - m X_f^{\sigma\sigma} X_m^{\sigma\sigma} \]

\[ + \sum_{f, m} (1 + \cos Q_2 f - m) X_f^{\uparrow\uparrow} X_m^{\uparrow\uparrow} \]
\[ + \sum_{f, m} (1 - \cos Q_2 f - m) \left( X_f^{\uparrow\downarrow} X_m^{\uparrow\downarrow} + X_f^{\downarrow\uparrow} X_m^{\downarrow\uparrow} \right). \]

We present only those terms describing the exchange interaction that lead to fluctuation corrections in the one-loop approximation. In this case, \( \tilde{\xi}_{\sigma} = \varepsilon - \mu - (1 - n/2)J_0 + nV_0 - \eta_\sigma h_0 \), where \( h_0 = -MJ_0 \).

From the condition of completeness of single-ion states and the relation between the electron density and occupation numbers, the amplitude of the magnetic order parameter can be obtained in the form \( M = n/2 - 1 + N_\uparrow \) convenient for calculations. Here, \( N_\uparrow = \)}
\( \langle X_{\uparrow}^{\uparrow} \rangle \) is the occupation number of a state with a spin projection of 1/2. To determine this number, the diagrammatic form of perturbation theory in the atomic representation is used. Figure 1 shows the diagrammatic representation for \( N_{\uparrow} \) including the first contributions from spin and charge fluctuations. In the second diagram, \( \alpha \) denotes the type of elementary excitation (root vector [35, 36]). If \( \alpha = (\uparrow \downarrow) \), the diagram specifies contributions from spin fluctuations. The second root vector can have two values \( \beta = (\uparrow \downarrow), (\downarrow \uparrow) \) [36, 37]. Contributions from charge fluctuations are described by two terms because the fermion root vector is \( \alpha = (\uparrow \uparrow) \) and \( \beta \) can have two values (\( \uparrow \downarrow \) and (\( \downarrow \uparrow \)). Using the diagrammatic technique [35–37], we obtain the following expression for \( M \) in the limit \( T \to 0 \):

\[
M(n) = \frac{n}{2} - \frac{1}{2} \sum_{q} \left( A_{q} / 2 - J_{0} \right) - \frac{1}{2} \sum_{p} \left( f_{1p} + f_{2p} \right) - M \sum_{p} \left( J_{0} - t_{p}^{2} \right) \left( f_{1p} - f_{2p} \right),
\]

where \( A_{q} = J_{q} + (J_{q} - J_{0} + J_{0} + q) / 2 \) and \( \gamma_{q} \) is related to the spectrum of spin-wave excitations as

\[
\omega_{q0} = 2M \gamma_{q} = 2M \sqrt{\left( J_{q} - J_{0} \right) \left( J_{q} - J_{0} + J_{q} + q \right) / 2 - J_{0}}
\]

and \( f_{jp} \equiv f(\varepsilon_{jp} / T) \) are the Fermi–Dirac functions. The branches of the fermion spectrum are given by the expressions

\[
\varepsilon_{1,2p} = \varepsilon_{0} + nt_{p}^{2} / 2 + \sqrt{(nt_{p}^{2} / 2)^{2} + R_{p-Q/2} R_{p+Q/2}}.
\]

and \( t_{p} = (t_{p-Q/2} \pm t_{p+Q/2}) / 2. \)

When deriving Eq. (11), we take into account the relation of \( \langle X_{\uparrow}^{\uparrow} \rangle \) to the electron Green’s functions \( G_{\alpha z,\alpha}(p, i\omega_{n}) \). This leads to the following equation for finding the chemical potential:

\[
n - 1 = \frac{n}{4} \sum_{p} \left( f_{1p} + f_{2p} \right) + M^{2} \sum_{p} J_{0} - t_{p}^{2} \left( f_{1p} - f_{2p} \right).
\]

According to Eq. (11), a decrease in the magnetization caused by spin fluctuations is independent of the electron density. At half filling (\( n = 1 \)), when hoppings do not contribute, the magnetization of the 120° structure is given by the known expression [27]. The inclusion of antiferromagnetic exchange \( J_{2} \) between next–nearest neighbors results in frustrations and reduces \( M \).

The density dependence of the magnetization at the parameters \( J_{1} = 0.5t_{1} \) and \( J_{2} = 0.02t_{1} \) is shown by the solid line in Fig. 2. It is seen that hoppings near half filling lead to a stronger decrease in the magnetization with increasing density as compared to the trivial result \( 1 - n / 2 - C \) (shown by the dashed line in Fig. 2), where \( C \) is the contribution from spin fluctuations. The vertical straight lines indicate the densities \( n_{1} = 1.014 \) and \( n_{2} = 1.033 \) at which gapless excitations occur in the coexistence phase. The specificity of these densities is manifested in the energy spectrum of fermion states for noncollinear spin ordering. As seen in Fig. 3, states near the K* point are filled at densities \( 1 < n < n_{1} \). At densities \( n_{1} < n < n_{2} \), in addition to the mentioned filling, states near the \( \Gamma \) and K points are filled (see Fig. 4). Contributions to the magnetization that appear in the process of such density evolution are responsible for a kink in the dependence \( M(n) \) at \( n = n_{1} \). The filling of the upper band with the minimum at the K* point begins above the density \( n_{1} \). Its spectrum is described by the expression \( \varepsilon_{2p} \). In this case, arising contributions to the magnetization result in a decrease in the slope of the dependence \( M(n) \) and the appearance of the kink at the point \( n = n_{2} \).

Figures 5 and 6 shows the fermion spectrum at the densities \( n = n_{1} \) and \( n = n_{2} \), respectively. This spectrum determines the possibility of gapless excitations in the
coexistence phase. The listed effects show that the behavior of the magnetization and related characteristics can indicate the existence of gapless excitations and, correspondingly, topological transitions.

Figure 7 shows the density dependences of the amplitude $\Delta_{21}$ describing pairings caused by the exchange interaction in the first coordination sphere. The corresponding self-consistent equations are presented in [26]. In the next section, we show that a topological transition with a change in the topological invariant occurs in the presence of gapless excitations in the coexistence phase of superconductivity and $120^\circ$ ordering.

5. TOPOLOGICAL INvariant $\hat{N}_3$ AND MAJORANA MODES

To solve the problem of a nontrivial topology of the coexistence phase of superconductivity and noncollinear magnetism at strong electron correlations, we use the method based on the analysis of the integer-valued topological invariant $\hat{N}_3$ [15]

$$\hat{N}_3 = \frac{\varepsilon_{\text{avk}}}{24\pi^2} \times \int_{-\pi}^{\pi} d\omega \int_{-\pi}^{\pi} d\zeta \zeta_{\text{avk}} \text{Tr} \left( \hat{G} \partial_{\mu} \hat{G}^{-1} \hat{G} \partial_{\nu} \hat{G}^{-1} \hat{G} \partial_{\lambda} \hat{G}^{-1} \right).$$

$$\varepsilon_{\text{avk}} \left( \hat{G} \partial_{\mu} \hat{G}^{-1} \hat{G} \partial_{\nu} \hat{G}^{-1} \hat{G} \partial_{\lambda} \hat{G}^{-1} \right).$$
Here, the repeated indices $\mu, \nu, \lambda = 1, 2, 3$ imply summation, $\varepsilon_{\mu\nu\lambda}$ is the Levi-Civita symbol, $\partial_i = \partial / \partial k_i$, $\partial_3 = \partial / \partial \omega$, and $\hat{G}(i\omega_n, \mathbf{k})$ is the matrix Green’s function whose poles determine the spectrum of elementary fermion excitations of the system (details are presented in the supplementary material).

The value $\tilde{N}_3 = 0$ corresponds to the topologically trivial phase. In a topologically nontrivial phase, $\tilde{N}_3 \neq 0$. Transitions between phases with different $\tilde{N}_3$ values are topological transitions.

The calculation of the number $\tilde{N}_3$ shows that the coexistence phase of superconductivity and $120^\circ$ spin ordering is topologically nontrivial with $\tilde{N}_3 \neq 0$. The coexistence phase at $V = 0$ occurs in a fairly wide density range (see Fig. 7), but a particular $\tilde{N}_3$ value can be different. The sequence of changes in $\tilde{N}_3$ upon an increase in the density of fermions is as follows:

$$
(N_3 = -1) \xrightarrow{n = m} (N_3 = 3) \xrightarrow{n = m} (N_3 = 2).
$$

(14)

Topological transitions occur at the same parameters at which the bulk spectrum of elementary excitations becomes gapless. It is substantial that these conditions for existence of topological transitions are independent of the magnitude of the superconducting order parameter.

As seen in Fig. 7, different numbers of topological transitions occur in the coexistence phase depending on the parameter $V$. Indeed, at $V = 0$, two such transitions occur at the densities $n_1$ and $n_2$; as $V$ increases to $V = 0.3t_1$, the transition at $n = n_1$ disappears, but the topological transition at the density $n_2$ holds; and topological transitions are absent at $V = 0.6t_1$.

To determine the structure of the Majorana mode, we use a method similar to that used for models disregarding interactions [17]. We consider a system with the triangular lattice containing a finite number ($N_1$) of sites along the direction of the translation vector $a_1$, whereas periodic boundary conditions are imposed along the $a_2$ direction (cylindrical geometry). According to the solution of the system of equations in the coordinate–momentum representation, the low-energy “quasiparticle” Green’s function can be represented in the form

$$
(i\omega_m - \varepsilon_{jk_0}) G_{\alpha, \downarrow}(k_2; n'; i\omega_m) = (S^\dagger)_{j\alpha}.
$$

(15)

where $\varepsilon_{jk_0}$ are the branches of the excitation spectrum with $j = 1, 2, \ldots, N_1$, and $S$ is the transformation matrix diagonalizing the initial matrix of the system of equations.

The relation between the Green’s function found from Eq. (15) and initial Green’s functions in the coordinate–momentum representation makes it possible to determine the operators of elementary excitations for the coexistence phase in the cylindrical geometry in terms of the Hubbard fermion operators:

$$
\alpha_{jk_2} = \sum_{l=1}^{N_1} \alpha_l X_{k_2, l, \uparrow} + \beta_{j} X_{k_2, -Q_2, \downarrow, \downarrow} + \gamma_{j} X_{k_2, Q_2, \uparrow, \uparrow}.
$$

(16)

According to this definition and reasons of symmetry [17], the Majorana mode in the cylindrical geometry occurs at $K_2 = -K_2 + Q_2 + G$, i.e., at $K_2 = -Q_2 = -2\pi/3$, when the excitation energy is zero, as seen in Fig. 8.

The dependences of several branches of the excitation spectrum of the system on the quasimomentum $k_2$ at the density $n = 1.025$ and $V = 0$ are shown in Fig. 8, where the other parameters are the same as those used for Figs. 2 and 7. The dashed line denotes the boundary of the bulk excitation spectrum at peri-
After the topological transition to the region with \( \tilde{N}_3 = 2 \) for densities \( n > n_2 \), a gap for the branch of edge states opens in the excitation spectrum of the finite system at \( K_2 = -Q_2 \) and the Majorana mode is absent. The Majorana mode also occurs at \( K_2 = -Q_2 \) in the coexistence phase with \( N_3 = -1 \) in the narrow density range \( n < n_1 \). In this region, in contrast to the region with \( N_3 = 3 \), the branch of edge states is formed only near \( K_2 = -Q_2 \). However, such a topological phase is of low practical interest because the superconducting gap is very narrow even disregarding the intersite Coulomb interaction.

6. CONCLUSIONS

The results obtained in this work show that strong electron correlations significantly renormalize the spin structure parameter but do not destroy the coexistence phase of chiral superconductivity and noncollinear magnetic ordering. It has been found that a nontrivial topology also holds in this case, which is important for the formation of Majorana modes in this phase at open boundary conditions. The nontrivial topology has been proven using the topological invariant \( \tilde{N}_3 \) calculated in terms of the Green’s functions determined by means of the Hubbard operators. The atomic representation has made it possible not only to correctly describe the effects of strong electron correlations but also to study the structure of the Majorana mode for the strongly correlated coexistence phase of chiral superconductivity and noncollinear spin ordering by introducing Majorana operators in the atomic representation. The diagrammatic technique for the Hubbard operators has allowed the calculation of contributions from spin and charge fluctuations to the macroscopic characteristic of the magnetic structure. Particular calculations near half filling within the \( t-J-V \) model have demonstrated a change in the topological invariant \( \tilde{N}_3 \) at the variation of the electron density. It has been found that the character of a change in \( \tilde{N}_3 \) depends on the intersite Coulomb interaction. An important conclusion has been made from the form of the dependence of \( \tilde{N}_3 \): depending on the model parameters and external conditions, either edges states which are Majorana bound states or edge states which do not belong to the Majorana type can be formed in the coexistence phase. The transition between these two regimes occurs as a quantum topological transition in density.

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Fig. 9. (Color online) Coefficients \( A_l = \text{Re} (u_{1l} + z_{1l}) \) and \( B_l = \text{Im} (u_{1l} + z_{1l}) \) versus the site number for the density \( n = 1.025 \) and \( N_3 = 400 \).
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