The moisture regime calculation of single-layer enclosing structures on the basis of the discrete-continuum method application

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Abstract. The paper examines a number of moisture regime mathematical models. Engineering method for unsteady-state moisture regime calculation using discrete-continuum method based on Gagarin’s moisture potential was proposed. A new formula for moisture potential determination in any section of single-layer enclosing structure, at any time, under continuous control for temperature distribution, was derived. Separate methods of moisture potential theory: Gagarin’s method, Kozlov’s method, and new discrete-continuum method – were compared. Single-layer enclosing structure made of aerated concrete D400 with inner and outer plastering was used as a research sample. It is shown that the proposed discrete-continuum method gives moisture distribution results for the enclosing structure similar to moisture distribution results calculated by Gagarin’s unsteady-state method. Results can be obtained by a finite formula without numerical method. It was found out that the discrete-continuum method takes moisture transfer response rate into account.

1. Introduction

Enclosing structure moisture regime is one of important area of focus in construction. Enclosing structure over-wetting can result in premature failure of the structure [1,2,3,4]. Moisture amount in construction material influences on building heat loss [5,6,7,8]. Development of moisture state calculation methods for enclosing structures [9,10,11,12,13,14], and also further control of obtained results by means of building on-site inspections under operating conditions [15,16] and laboratory experiments on moisture diffusion coefficients [17,18] is one of the most important research areas.

Let’s examine the existing moisture transfer model [9] assuming that total moisture flow in the single-layer enclosing structure thickness is a sum of liquid and vapor moisture flows:

\[ g_{tot} = g_v + g_L, \]

where \( g_{tot} \) – total moisture flow, \( kg / (m^2 \cdot s) \), \( g_v \) – vapor moisture flow, \( kg / (m^2 \cdot s) \), \( g_L \) – liquid moisture flow, \( kg / (m^2 \cdot s) \).

Gagarin’s moisture potential, which takes vapor and liquid moisture into account, is known:
\[ F(w,t) = E_t(t) \cdot \varphi(w) + \frac{1}{\mu} \int \beta(\zeta) d\zeta, \] (2)

where \( F \) – moisture potential, \( Pa; \) \( E_t \) – maximum water vapor tension, \( Pa; \) \( \varphi \) – relative air humidity; \( \mu \) – vapor permeability coefficient, \( kg/(m \cdot s \cdot Pa); \) \( \beta \) – moisture conductivity coefficient, \( kg/(m \cdot s \cdot kg/kg), \) depending on humidity.

Thermal conductivity differential equation describes temperature distribution of a single-layer enclosing structure:

\[ \frac{c \cdot \gamma_0}{\tau} \frac{\partial t}{\partial \tau} = \frac{\partial^2 t}{\partial x^2}, \] (3)

where \( t \) – temperature, °C; \( \tau \) – time, s; \( x \) - coordinate, m; \( \gamma_0 \) – enclosing structure dry material density, \( kg/m^3; \) \( c \) – specific thermal capacity of material, \( J/(kg \cdot °C). \)

Boundary condition of heat exchange between outside air and enclosing structure outer surface is set as follows:

\[ -\lambda \left. \frac{\partial t}{\partial x} \right|_{x=1} = \alpha_{ext} (t_{ext} - t_1), \] (4)

where \( t_{ext} \) – outside air temperature, °C; \( t_1 \) – temperature of enclosing structure section which contacts with outside air, °C; \( \alpha_{ext} \) – heat exchange coefficient between outside air and enclosing structure surface, \( W/(m^2 \cdot °C). \)

Boundary condition of heat exchange between inside air and enclosing structure inner surface is set as follows:

\[ \lambda \left. \frac{\partial t}{\partial x} \right|_{x=0} = \alpha_{int} (t_m - t_N), \] (5)

where \( t_m \) – inside air temperature, °C; \( t_N \) – material temperature near enclosing structure surface which contacts with inside air, °C; \( \alpha_{int} \) – heat exchange coefficient between inside air and enclosing structure surface, \( W/(m^2 \cdot °C). \)

Moisture transfer differential equation describes moisture potential distribution of a single-layer enclosing structure:

\[ \gamma_0 \cdot \frac{1}{\mu} \frac{\partial \varphi(w)}{\partial w} + \frac{\partial F(w,t)}{\partial t} = \frac{\partial^2 F(w,t)}{\partial x^2}. \] (6)

Boundary condition of moisture exchange between outside air and structure surface is set as follows:

\[ -\mu \left. \frac{\partial F}{\partial x} \right|_{x=1} = \beta_{ext} (F_{ext} - F_1), \] (7)

where \( F_{ext} \) – outside air moisture potential equal to partial pressure of outside air water vapor \( Pa; \) \( F_1 \) – moisture potential of the enclosing structure section which contacts with outside air, \( Pa; \) \( \beta_{ext} \) – moisture exchange coefficient of outside air and enclosing structure surface, \( kg/(m^2 \cdot s \cdot Pa). \)

Boundary condition of moisture exchange between inside air and enclosing structure inner surface is set as follows:

\[ \mu \left. \frac{\partial F}{\partial x} \right|_{x=0} = \beta_{int} (F_m - F_N), \] (8)

where \( F_m \) – inside air moisture potential equal to partial pressure of inside air water vapor \( Pa; \) \( F_N \) – moisture potential of the enclosing structure surface material which contacts with inside air, \( Pa; \) \( \beta_{int} \) – moisture exchange coefficient of inside air and enclosing structure surface, \( kg/(m^2 \cdot s \cdot Pa). \)
As heat transfer speed is several orders greater than moisture transfer speed, stationary temperature distribution can be examined for single-layer enclosing structure temperature distribution without significant loss of accuracy:

\[ \frac{\partial^2 T}{\partial x^2} = 0. \]  \hspace{1cm} (9)

Unsteady-state moisture regime calculation by equations (2), (4) – (9) requires numerical method application using computer, thus, it is difficult to be used in engineering practice. Kozlov’s quasi-stationary method, based on moisture potential steady-state distribution during a month, is more applicable for engineering calculation:

\[ \frac{\partial^2 F}{\partial x^2} = 0. \]  \hspace{1cm} (10)

Equations (9) and (10) are much more applicable in engineering design practice, however this method is less practicable as compared to unsteady-state calculation.

The problem is the development of unsteady-state moisture regime engineering calculation method for single-layer enclosing structure using discrete-continuum approach.

2. Materials and methods

“Relative potential capacity coefficient” has been proposed, which is determined by formula:

\[ \xi_{F0} = \frac{d w}{d \phi_F}, \]  \hspace{1cm} (11)

where \( \phi_F \) – “relative elasticity of moisture potential”

“Relative elasticity of moisture potential” is determined by formula:

\[ \phi_F = \phi + \frac{1}{\mu_0} \int \beta(\sigma) d \sigma / E_i. \]  \hspace{1cm} (12)

“Coefficient of material hygrothermal properties» has been introduced:

\[ \kappa = \frac{\mu}{\gamma_0 \cdot \xi_{F0}}. \]  \hspace{1cm} (13)

Moisture transfer modified differential equation has been proposed taking into account rearrangements (11) – (13):

\[ \frac{\partial F(w,t)}{\partial \tau} = \kappa(w,t) \cdot E_i(t) \frac{\partial^2 F(w,t)}{\partial x^2}. \]  \hspace{1cm} (14)

We assume that inside air moisture potential is constant during the whole period, and outside air moisture potential varies continuously during a month as a linear function:

\[ F_{\text{ext}} = m \cdot \tau + n., \]  \hspace{1cm} (15)

where \( m \) – boundary conditions slope ratio within a month; \( n \) – boundary conditions graph rise within a month, Pa;

Using discrete-continuum approximation [19, 20] of equation (14) and taking into account equations (7), (8), (15), a system of simultaneous equations can be written as:

\[
\begin{align*}
\frac{\partial F_1}{\partial \tau} &= \frac{\kappa}{h^2} \cdot E_{i1} \cdot (-1 + \frac{\beta_{\text{ext}}}{\mu} \cdot h) \cdot F_1 + F_2 + \frac{\kappa}{h^2} \cdot E_{i1} \cdot \frac{\beta_{\text{ext}}}{\mu} \cdot h \cdot F_{\text{ext}}, \\
\frac{\partial F_i}{\partial \tau} &= \frac{\kappa}{h^2} \cdot E_{i-1} \cdot (F_{i-1} - 2 \cdot F_i + F_{i+1}), i = 2, 3, 4, ..., N-1, \\
\frac{\partial F_N}{\partial \tau} &= \frac{\kappa}{h^2} \cdot E_{iN} \cdot (-1 + \frac{\beta_{\text{ext}}}{\mu} \cdot h) F_N + \frac{\kappa}{h^2} \cdot E_{iN} \cdot \frac{\beta_{\text{ext}}}{\mu} \cdot h \cdot F_{\text{ext}}, \\
F(x,0) &= u_i(x), 0 \leq x \leq l
\end{align*}
\]  \hspace{1cm} (16)

Simultaneous equations (16) represent a Cauchy problem in matrix form:
\[
\bar{F}_x = E_i \cdot A \cdot \bar{F} + p \cdot \tau \cdot \bar{L} + \bar{B},
\]
\[
\bar{F}(0) = \bar{F}_0, 0 \leq x \leq l
\]

where \( p = \kappa \cdot E_{i1} \cdot \beta_{et} \cdot h \cdot ml (h^2 \cdot \mu) \).

\[
A = \frac{\kappa}{h^2} \left[ \begin{array}{cccc}
-(1 + \beta_{et} \cdot h \cdot ml) & 1 & 0 & 0 \\
-2 & 1 & 0 & 0 \\
0 & \ddots & \ddots & \ddots \\
0 & 0 & 1 & -(1 + \beta_{et} \cdot h \cdot ml)
\end{array} \right] \cdot E_0 = \begin{pmatrix}
E_{i1} & 0 & 0 & 0 \\
0 & E_{i2} & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & E_{i(n-1)} \\
0 & 0 & 0 & 0 & E_{i0}
\end{pmatrix}
\]

Problem solution (17) can be written as:

\[
\bar{F} = p \cdot \left( (E_i \cdot A)^{-2} \cdot e^{\xi_i \cdot \tau} - \tau \cdot (E_i \cdot A)^{-1} - (E_i \cdot A)^{-2} \cdot \bar{L} + (E_i \cdot A)^{-3} \left( e^{\xi_i \cdot \tau} - E \right) \cdot \bar{B} + e^{\xi_i \cdot \tau} \cdot \bar{F}_0 \right),
\]

where \( E \) is unit matrix.

3. Results and discussion
Calculation results based on solution (18) were compared with results obtained by Gagarin’s unsteady-state method and Kozlov’s engineering method. Initial data for calculation: single-layer enclosing structure made of 0.6 m thick aerated concrete, built in Moscow. Inside air temperature 20°C and relative air humidity 50% was maintained constant during a year. Comparison of moisture regime calculation results obtained by three different methods of moisture potential theory for January is given in Figure 1.

The figure shows that moisture distribution in single-layer enclosing structure obtained by means of the proposed discrete-continuum method gives quantitative and qualitative results similar to moisture distribution obtained by means of Gagarin’s unsteady-state method. It should be noted that this distribution is based on expression, which makes calculation easier.

![Figure 1. Comparison of moisture potential theory methods for moisture regime. calculation of single-layer enclosing structure in January (1 – moisture distribution in enclosing structure according to Gagarin’s unsteady-state method, 2 – moisture distribution in enclosing structure according to discrete-continuum method, 3 – moisture distribution in enclosing structure according to Kozlov’s engineering method).](image-url)
4. Conclusion

Unsteady-state moistening regime determination method with discrete-continuum approach is proposed for single-layer enclosing structures. Results of moistening regime calculation by means of the new method are close to results obtained by existing Kozlov’s unsteady-state method, but in the first case, moisture regime is calculated on equation (18) without numerical method, thus, the proposed method is applicable in engineering practice.

References

[1] Wu Z, Wong H S and Buenfeld N R 2017 Cement and Concrete Research 98 136–54
[2] Liu Z C, Hansen W and Wang F Z 2018 Construction and Building Materials 158 181–8
[3] Georget F, Prevost J H and Huet B 2018 Cement and Concrete Research 104 1–12
[4] Zvicevicius E, Raila A, Cipliene A, Cerniauskiene Z, Kadziuliene Z and Tilvikiene V 2018 Renewable Energy 119 185–92
[5] Hoseini A and Bahrami M 2017 Journal of Building Engineering 13 107–15
[6] Suchorab Z, Barnat-Hunek D and Sobczuk H 2011 Ecolog. Chem. Eng. S 18 L1 111–20
[7] Belkharchouche D and Chaker A 2016 Int. J. Hydrogen Energy 41 Issue 17 7119–25
[8] Jin H Q, Yao X L, Fan L W, Xu X and Yu Z T 2016 Int. J. Heat Mass Transfer 92 589–602
[9] Galbraith G H, Guo J S and McLean R C 2000 Building Research and Information 28 245–59
[10] Lal S, Lucci F, Defraeye T, Poulikakos L D, Partl M N, Derome D and Carmeliet J 2018 Int. J. Therm. Sci. 123 86–98
[11] Skerget L, Tadeu A and Ravnik J 2017 Eng. Anal. Bound. Elem 74 24–33
[12] Vavrovic B 2014 Advanced Materials Research 855 97-101
[13] Tang Y C, Min J C and Wu X M 2018 Int. J. Heat Mass Transfer 116 371–6
[14] Arfvidsson J and Claesson J 2000 Build. Env. 35 Iss 6 519–36
[15] Eklund J A, Zhang H, Viles H A and Curteis T 2013 Int. J. Architec. Heritage 7 L6 207–24
[16] Sass O and Viles H A 2010 Two-dimensional resistivity surveys of the moisture content of historic limestone walls in Oxford, UK: implications for understanding catastrophic stone deterioration Limestone in the Built Environment: Present Day Challenges for the Preservation of the Past ed. B J Smith, M Gomez-Heras, H A Viles and J Cassar SP 331 (London: Geological Society) pp 237–49
[17] Nizovtsev M I, Sterlyagov A N and Terekhov V I 2009 Effect of material humidity on heat- and moisture-transfer processes in gas-concrete Concrete Materials: Properties, Performance and Applications (NY: Nova Science Publishers) pp 397–429
[18] Perre P, Pierre F, Casalinho J and Ayoub M 2015 Drying Technology 33 1068–75
[19] Sidorov V N and Matskevich S M 2016 Discrete-analytical solution of the unsteady-state heat conduction transfer problem based on the finite element method Proc. IDT 2016 Rzeszow, Poland 5-7 July 2016 (IEEE) pp 241–44
[20] Sidorov V N and Matskevich S M 2016 Key Engineering Materials 685 211–6