Yukawa coupling corrections to the decay $H^+ \rightarrow W^+ A^0$

A. Akeroyd$^{a}$, A. Arhrib$^{b,c,d}$ and E. Naimi$^{c,d}$

a: Departamento de Física Teórica, IFIC/CSIC, Universidad de Valencia, Burjassot 46100, Valencia, Spain

b: Departamento de Física–CENTRA, Instituto Superior Técnico Av Rovisco Pais 1, 1096 Lisboa Codex, Portugal

c: Département de Mathématiques, Faculté des Sciences et Techniques B.P 416, Tanger, Morocco

d: UFR–High Energy Physics, Physics Departement, Faculty of Sciences PO Box 1014, Rabat–Morocco

Abstract

We compute the fermionic radiative contributions to the decay $H^+ \rightarrow W^{+(*)} A^0$ in the framework of models with two Higgs doublets (2HDM), for the case of an on–shell and off–shell W. We show that, in the majority of the cases, current measurements of the $\rho$ parameter suggest $M_{H^\pm} \geq M_A$ and such decays could invalidate current charged Higgs searches or aid detection in the region $M_{H^\pm} \approx M_W$. We find that the radiative corrections may approach 50% for small values of $\tan \beta$. 
1. Introduction

The search for the Higgs boson ($\phi^0$) of the Standard Model (SM) is one of the major challenges for present and future colliders. In recent years there has been growing interest in the study of extended Higgs sectors with more than one Higgs doublet. The simplest extension is the Two Higgs Doublet Model (2HDM), and such a structure is required for the Minimal Supersymmetric Standard Model (MSSM). Models with two (or more) Higgs doublets predict the existence of charged Higgs bosons, and their discovery would be conclusive evidence of an extended Higgs sector. In the 2HDM extension of the SM, from the 8 degrees of freedom initially present in the 2 Higgs doublets, only 5 remain after the electroweak symmetry breaking and should be manifested as physical particles: 2 charged Higgs scalars ($H^\pm$), 2 CP–even scalars ($h^0$ and $H^0$) and one CP–odd scalar ($A^0$). Accurate predictions for the branching ratios (BR) of these particles are required in order to facilitate the searches and in this paper we consider the radiative corrections to the decay $H^\pm \rightarrow A^0 W^*(\gamma)$. In the non–supersymmetric 2HDM (hereafter to be called simply 2HDM) the masses $M_A$ and $M_{H^\pm}$ may be taken as free parameters and so one may consider both the case of an off–shell and on–shell $W$. This is in contrast to the MSSM in which $M_A$ and $M_{H^\pm}$ are correlated and the two body decay is never allowed. We shall show that current measurements of the $\rho$ parameter strongly suggest $M_{H^\pm} \geq M_A$ for $M_{H^\pm} \geq 100$ GeV.

Recently it has been shown that the decay $H^\pm \rightarrow A^0 W^*(\gamma)$ may be dominant or even close to 100% in the 2HDM (Model I) over a wide range of parameter space relevant at LEP–II. This would affect current charged Higgs searches at LEP–II and the Tevatron which only assume the decays $H^\pm \rightarrow \tau \nu$, $\tau$ and $cs$. We therefore feel it important to calculate the fermionic radiative corrections to this potentially strong tree–level process. An additional use of the three–body decay would be the possibility of detection in the difficult $M_{H^\pm} \approx M_W$ region, which is considered marginal if $H^\pm$ decays conventionally to two fermions. Although a thorough analysis is beyond the scope of this paper, the three–body decay would give rise to high multiplicity signatures of more than 4 jets, with a possibility of detection above the strong $WW$ background. We note that it is possible to have the Model II type structure and weaken the above bound on $M_{H^\pm}$ in a 2HDM which relaxes natural flavour conservation (NFC) or a general model with $N(\geq 3)$ doublets. In this paper we are concerned with the 2HDM which imposes NFC. Limits on $M_{H^\pm}$ from the Tevatron are $\beta$ dependent since one requires a significant BR($t \rightarrow H^+ b$) in order to obtain a visible signal. In the 2HDM with the Model II type structure this BR can be significant for small ($\leq 1$) or large ($\geq 40$) values of $\beta$. For the Model I type structure it is only possible at low $\tan \beta$.

Current mass bounds from LEP–II for the $A^0$ of the MSSM force $M_{H^\pm} \geq 110$ GeV in this model, thus taking $H^\pm$ out of the LEP–II discovery range. In addition, a recent analysis of the MSSM charged Higgs contributions to $b \rightarrow s \gamma$ requires $M_{H^\pm} \geq 110$ GeV, a limit valid in both the MSSM and its simplest extension by adding a Higgs singlet superfield (NMSSM). Therefore from the point of view of charged Higgs phenomenology at LEP–II one may consider the 2HDM (Model I) but not more popular extended structures. We will present results for the case of $W$ on–shell and off–shell for charged Higgs masses.
of interest at LEP–II and the LHC. The paper is organized as follows. In Section 2 we introduce our notation and the models in question. In Section 3 we evaluate the fermionic one loop corrections for the case of an on–shell and off–shell \( W \), while Section 4 displays the counterterms. In Section 5 we present our results, and Section 6 contains our conclusions.

2. Notation, couplings and lowest order results

2.1 Notation and relevant couplings

In this paper we will use the following notation and conventions. The momentum of the charged Higgs boson \( H^+ \) is denoted by \( p_{H^+} \) (\( p_H \) is incoming), \( p_W \) is the momentum of the \( W^+ \) gauge boson and \( p_A \) the momentum of the CP-odd \( A^0 \) (\( p_W \) and \( p_A \) are outgoing).

The relevant part of the lagrangian describing the interaction of the \( W^\pm \) with \( H^\pm \) and \( A^0 \), comes from the covariant derivative and is given by:

\[
\mathcal{L} = \frac{e}{2 s_W} W^\pm _\mu (H^- \stackrel{\leftrightarrow}{\partial} \theta) A^0 + \text{h.c.} \quad (2.1)
\]

This interaction is model independent (SUSY or non–SUSY) and it depends only on standard parameters: electric charge (\( e \)) and Weinberg angle (\( s_W = \sin \theta_W \)).

As we are concerned with the fermionic one loop corrections, we will give hereafter the relevant couplings. In the 2HDM there exist four different ways to couple the Higgs fields to matter (we assume natural flavour conservation [14]). The two most popular are: Model I: The quarks and leptons couple only to one of the 2 Higgs doublet exactly as in the minimal standard model. Model II: To avoid the problem of Flavor Changing Neutral Currents (FCNC), one assumes that one of the 2 Higgs fields couples only to down quarks (and charged leptons) and the other one couples to up quarks (and neutral leptons). Model type II is the pattern found in the MSSM.

In general, the couplings of the charged Higgs boson \( H^\pm \), Goldstone \( G^\pm \), CP–odd \( A^0 \) and the gauge boson \( W^\pm \) to a pair of fermions are:

\[
\begin{align*}
H^+ u \bar{d} &= Y_{ud}^L \frac{(1 - \gamma_5)}{2} + Y_{ud}^R \frac{(1 + \gamma_5)}{2}, \\
G^+ u \bar{d} &= G_{ud}^L \frac{(1 - \gamma_5)}{2} + G_{ud}^R \frac{(1 + \gamma_5)}{2}, \\
A^0 u \bar{u} &= Y_{uu} \gamma_5, \\
A^0 d \bar{d} &= Y_{dd} \gamma_5, \\
W^+ u \bar{d} &= -i \frac{g V_{ud}}{\sqrt{2}} \gamma_5 \frac{(1 - \gamma_5)}{2} \quad (2.2)
\end{align*}
\]

Where:

\[
\begin{align*}
Y_{ud}^L &= \frac{g V_{ud} m_u}{\sqrt{2} M_W \tan \beta}, & Y_{ud}^R &= -\frac{g V_{ud} m_d}{\sqrt{2} M_W \tan \beta} \quad \text{Model I} \\
Y_{uu}^L &= \frac{g m_u}{2 M_W \tan \beta}, & Y_{dd}^L &= \frac{gm_d}{2 M_W \tan \beta} \quad \text{Model I} \\
Y_{uu}^R &= -\frac{g m_u}{2 M_W \tan \beta}, & Y_{dd}^R &= -\frac{m_d g \tan \beta}{2 M_W} \quad \text{Model II} \\
G_{ud}^L &= \frac{g m_u V_{ud}}{\sqrt{2} M_W}, & G_{ud}^R &= -\frac{g m_d V_{ud}}{\sqrt{2} M_W} \quad (2.3)
\end{align*}
\]
$V_{ud}$ is the Kobayashi–Maskawa matrix element. It is worth noting that Models I and II are not very different for the top–bottom loop corrections at low $\tan \beta$ because the term $m_t/\tan \beta$ will dominate and it is common to both types.

2.2 Lowest order results

The lowest–order Feynman diagram for the two body decay $H^+ \to A^0 W^+$ and for the three body decay $H^+ \to A^0 W^* \to A^0 f f'$ are depicted in the following figure:

![Figure 1](image)

Figure 1

In the Born approximation, the decay amplitude of the charged Higgs into on–shell CP–odd Higgs boson $A^0$ and the gauge boson $W^+$ (Fig.1.a) can be written as:

$$\mathcal{M}^0(H^+ \to W^+ A^0) = \epsilon_\mu \Gamma^\mu_0 \quad \text{where} \quad \Gamma^\mu_0 = i \frac{e}{2s_W} (p_H + p_A)_\mu$$

(2.4)

Here $\epsilon$ is the W polarization vector. We then have the following decay width:

$$\Gamma^0_\text{on} = \frac{\alpha}{16s_W^2 M_W^2 M_{H^\pm}^2} \lambda^2 (M_{H^\pm}, M_A, M_W^2)$$

(2.5)

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + yz + xz)$ is the familiar two–body phase space function. Note that in the MSSM the two–body decay of the charged Higgs boson into $W^+ A^0$ is kinematically not allowed.

Below threshold, and taking into account that the virtual $W^*$ decays into a pair of fermions $f f'$ ($f \neq t$) (Fig.1.b) which we will take to be massless, the Dalitz plot density for this three–body decay $H^+ \to A^0 W^* \to A^0 f f'$ is given by [13]:

$$\frac{d\Gamma^0_{\text{off}}}{dx_1 dx_2} = \frac{9 \alpha^2}{32\pi s_W^4} M_{H^\pm} \frac{[(1 - x_1)(1 - x_2) - \kappa_A]}{\left[(1 - x_1 - x_2 - \kappa_A + \kappa_W)^2 + \kappa_W \gamma_W\right]}$$

where

$$\kappa_{A,W} = \frac{M_{A,W}^2}{M_{H^\pm}^2}, \quad \gamma_W = \frac{\Gamma_W^2}{M_{H^\pm}^2},$$

$\Gamma_W$ is the total width of the W gauge boson and $x_i = 2E_i/M_{H^\pm}$ are the scaled energies of the massless fermions in the final state. We note that in the non–SUSY 2HDM null–searches at LEP in the $e^+ e^- \to h^0 A^0, h^0 Z$ channels eliminate regions in the $M_A, M_h$ plane [4], [4]. The excluded region does not have a simple shape, and there are still areas which allow $M_A + M_h \leq 90$ GeV. Thus $M_A$ may be taken as light as 10 GeV. This is in contrast to the MSSM in which one can derive individual lower limits on the masses, of $M_h \geq 70.7$ GeV and $M_A \geq 71.0$ GeV [4]. Therefore the off–shell decay in the 2HDM can be relevant even for a small $M_{H^\pm} (\leq 80$ GeV) in range at LEP–II.
3. Fermionic radiative corrections.

We have evaluated the fermionic radiative corrections to $H^+ \to W^+ A^0$ (for both the on–shell and off–shell $W$) at the one loop level. This set of corrections is Ultra–Violet (UV) divergent. The UV singularities are treated by dimensional regularization \cite{17} in the on–mass–shell renormalization scheme.

The typical Feynman diagrams for the virtual corrections of order $\alpha$ are drawn in figure 2. These include the vertex correction (Fig.2.a1, Fig.2.a2 ), $W^+ – W^–$ self–energy (Fig.2.a3) and the mixed $W^+ – G^–$ self–energy (Fig.2.a4). Note that diagrams 2.a3 and 2.a4 are not to be considered if the gauge boson $W$ is on–shell. These contributions have to be supplemented by the counterterm renormalizing the vertex $H^+ A^0 W^–$ (Fig.2.c1), the counterterm for the off–shell $W$ gauge boson self–energy (Fig.2.c2) and by the counterterm for the mixing $W – G$ (Fig 2.c3). These Feynman diagrams are generated and computed using FeynArts and FeynCalc \cite{18, 19} packages. We also use the fortran FF–package \cite{20} in the numerical analysis. Note that in the general 2HDM, the vertices $W^+ A^0 G^–$, $W^+ G^0 H^–$ and $A^0 H^+ H^–$ are not present, and so the mixing $G^+ – H^–$, $G^0 – A^0$ and $W^+ – H^–$ does not give any contribution to our process.

The one loop amplitude $M^1$ can be written as:

$$M^1(H^+ \to W^+ A^0) = \epsilon^*_\mu \Gamma^\mu$$ (3.1)

Using Lorentz invariance, $\Gamma^\mu$ can be projected as:

$$\Gamma^\mu = \frac{e}{2s_W} (\Gamma_H p_H^\mu + \Gamma_W p_W^\mu)$$ (3.2)

$\Gamma_H$ and $\Gamma_W$ can be cast as follow:

$$\Gamma_W = \Gamma_{W, H}^{\text{vertex}} + \Gamma_{W, H}^{W^+ W^–} + \Gamma_{W, H}^{W^+ G^–} + \delta \Gamma_{W, H}^{\text{vertex}} + \delta \Gamma_{W, H}^{W^+ W^–} + \delta \Gamma_{W, H}^{W^+ G^–}$$ (3.3)

$$\Gamma_H = \Gamma_{W, H}^{\text{vertex}} + \Gamma_{W, H}^{W^+ W^–} + \delta \Gamma_{W, H}^{\text{vertex}} + \delta \Gamma_{W, H}^{W^+ W^–}$$ (3.4)

Where $\Gamma_{W, H}^{\text{vertex}}$, $\Gamma_{W, H}^{W^+ W^–}$ and $\Gamma_{W, H}^{W^+ G^–}$ are respectively the contribution of the two vertices, the contribution of the self–energy of the $W$ and the contribution of the mixed $W^+ G^–$ self–energy; $\delta \Gamma_{W, H}^{\text{vertex}}$, $\delta \Gamma_{W, H}^{W^+ W^–}$ and $\delta \Gamma_{W, H}^{W^+ G^–}$ are the counterterms needed to remove the UV divergences contained in $\Gamma_{W, H}^{\text{vertex}}$, $\Gamma_{W, H}^{W^+ W^–}$ and $\Gamma_{W, H}^{W^+ G^–}$. In what follows, we write the above one loop corrections explicitly. The expressions for the counterterms can be found in Section 4.

3.1 Vertex with u–u–d exchange: Fig.2.a1

The amplitude of the u–u–d quarks contribution to $H^+ A^0 W^+$ vertex is given by

$$\Gamma_{H}^{u u d} = N_C \frac{\alpha}{2\pi \sqrt{2M_{H^\pm}^2}} Y_{uu} \left( (m_D^2 - 3M_{H^\pm}^2 + m_U^2)Y_{ud}^L B_0(M_{H^\pm}^2, m_D^2, m_U^2) + \right.$$

$$\left. (m_D^2 - m_U^2)Y_{ud}^R B_0(0, m_D^2, m_U^2) - 2M_{H^\pm}^2 \left\{ (m_U^2 Y_{ud}^L + m_D Y_{ud}^R)C_0 + Y_{ud}^L p_W^2 C_1 - 2C_{00} + (M_A^2 + M_{H^\pm}^2 - p_W^2)C_{12} - 2M_A^2 C_{22} \right\} \right)$$ (3.5)

$$\Gamma_{W}^{u u d} = N_C \frac{\alpha}{\pi \sqrt{2}} Y_{uu} \left( Y_{ud}^L B_0(M_{H^\pm}^2, m_D^2, m_U^2) + m_U (m_D Y_{ud}^L + m_D Y_{ud}^R)C_0 - \right.$$

$$\left. Y_{ud}^L \left\{ (M_A^2 - M_{H^\pm}^2)C_1 + M_A^2 C_2 + 2C_{00} + (M_A^2 - M_{H^\pm}^2 + p_W^2)C_{11} + (3M_A^2 - M_{H^\pm}^2 + p_W^2)C_{12} + 2M_A^2 C_{22} \right\} \right)$$ (3.6)
with \(A_0, B_0, C_i\) and \(C_{ij}\) the Passarino-Veltman functions [21] which we define in Appendix A. \(N_C = 3\) for quarks and 1 for leptons. All the \(C_i\) and \(C_{ij}\) have the same arguments: 

\( (p_W^2, M_{H^\pm}^2, M_A^2, m_U^2, m_D^2, m_U^2)\)

### 3.2 Vertex with d-d-u exchange: Fig.2.a_2

The amplitude of this diagram can be obtained from the above one just by making the following replacement:

\[
\Gamma^{ddu}_{H,W} = \Gamma^{wud}_{H,W}[m_U \leftrightarrow m_D, \ Y^{uL}_u \leftrightarrow Y^{dL}_d, \ Y_{uu} \rightarrow Y_{dd}]
\]

The total contribution of vertex is:

\[
\Gamma^{\text{vertex}}_{W,H} = \Gamma^{wud}_{W,H} + \Gamma^{ddu}_{W,H}
\]

### 3.3 \(W^+ - W^-\) self–energy: Fig.2.a_3

The contribution of \(W\) self–energy Fig.2.a_3 evaluates to

\[
\Gamma^{WW}_H = \frac{N_C \alpha}{2 \pi s_W^2 (p_W^2 - M_W^2)} \left\{ A_0(m_U^2) + m_D^2 B_0(p_W^2, m_D^2, m_U^2) - 2 B_{22}(p_W^2, m_D^2, m_U^2) \\
+ p_W^2 B_1(p_W^2, m_D^2, m_U^2) \right\}
\]

\[
\Gamma^{WW}_W = - \frac{N_C \alpha}{4 \pi s_W^2 (p_W^2 - M_W^2)} \left\{ A_0[m_U^2] + m_D^2 B_0(p_W^2, m_D^2, m_U^2) - 2 B_{22}(p_W^2, m_D^2, m_U^2) \\
+ p_W^2 B_1(p_W^2, m_D^2, m_U^2) + 2(M_{H^\pm}^2 - M_A^2)(B_1(p_W^2, m_D^2, m_U^2) + B_{21}(p_W^2, m_D^2, m_U^2)) \right\}
\]

### 3.4 \(W–G\) mixing: Fig.2.a_4

In accordance with Lorentz invariance, the mixing self–energy \(W–G\) is proportional to \(p_W^\mu\) and evaluates to

\[
\Gamma^{WG}_W = \frac{N_C \alpha (M_{H^\pm}^2 - M_A^2)}{4 \pi m_W^2 s_W^2 (p_W^2 - M_W^2)} \left\{ m_D^2 B_0(p_W^2, m_D^2, m_U^2) + m_U^2 B_1(p_W^2, m_D^2, m_U^2) \right\}
\]

\[
\Gamma^{WG}_H = 0
\]

### 4. On–mass–shell Renormalization.

The parameters entering the tree–level amplitude in eq.\([2.3]\) are all standard model parameters \((e\text{ and }s_W)\). This fact will render the one loop renormalization rather simple, in the sense that all non–standard parameters appearing first at the one loop level (like \(\tan \beta\)), will not get renormalized. This is in contrast to the calculation in [22] for the process \(H^+ \rightarrow hW^+\) which explicitly contains the factor \(\cos^2(\beta - \alpha)\) at tree–level. Therefore renormalization conditions related to the definition of \(\tan \beta\) are not explicitly needed here. We will need, however, to renormalize the electric charge, the Weinberg angle, charged Higgs wave–function, CP–odd Higgs wave function and \(W\) gauge boson wave function. In our case the \(W^\pm\) gauge boson mixes with the Goldstone boson \(G^\pm\), by virtue of the
Laurentz invariance of the self–energy; therefore $\Sigma_{\mu}^{G+W}$ is proportional to $p_{\mu}^W$ and so if the $W$ is on-shell the mixing would have a vanishing contribution but in the off–shell case we have to take this mixing into account.

In what follows we will follow to an extent the on–shell–renormalization developed by R. Santos et al [22] which is the generalization to the 2HDM of the I. Aoki et al on–shell renormalization scheme [23, 24]. The crucial point in this scheme is that all fields and masses are renormalized after the diagonalization of the bare mass matrices. Another important point in this scheme is that the gauge fixing is written in terms of the renormalized parameters and fields and as a consequence it does not contain any counterterm.

4.1 Vertex $H^+ A^0 W^+$ counterterm

To obtain the renormalized vertex $W^+ A^0 H^+$ vertex we have to make the following substitutions in eq. (2.1):

\[
W_\mu \rightarrow Z_W^{1/2}W_\mu \\
H^\pm \rightarrow Z_{H^\pm}^{1/2}H^\pm \\
A^0 \rightarrow Z_{A^0}^{1/2}A^0 \\
e \rightarrow Z_e e = (1 + \delta Z_e)e \\
M_W^2 \rightarrow M_W^2 + \delta M_W^2 \\
M_Z^2 \rightarrow M_Z^2 + \delta M_Z^2
\]  

Note that in the on–shell scheme, the Weinberg angle is defined as: $s_W^2 = 1 - \frac{M_Z^2}{M_W^2}$. Therefore the counterterm of $s_W$ is completely fixed by the counterterm of the $W$ and $Z$ boson masses and is given by:

\[
\frac{\delta s_W}{s_W} = \frac{1}{2} \frac{e_W}{s_W} \left( \frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2} \right) 
\]

Setting $Z^{1/2} = 1 + \frac{1}{2} \delta Z$, one obtains the following counterterm:

\[
\delta \mathcal{L} = \frac{e}{2s_W} W^+_{\mu}(H^- \epsilon^{\mu \nu} A^0) \left( \frac{1}{2} \delta Z_W + \frac{1}{2} \delta Z_{A^0} + \frac{1}{2} \delta Z_{H^+H^-} + \delta Z_e - \frac{\delta s_W}{s_W} \right)
\]

In the on–mass–shell scheme the counterterms can be fixed by the following renormalization conditions:

- On–shell condition for the charged Higgs boson $H^\pm$, CP–odd $A^0$ and the $W$ and $Z$ gauge Bosons. We choose to identify the physical mass with the corresponding parameter in the renormalized lagrangian, and require the residue of the propagator to have its tree–level value, i.e.,

\[
\delta M^2 = \text{Re} \Sigma(M^2) \quad \text{and} \quad \delta Z = -\frac{\partial \Sigma(k^2)}{\partial k^2}\bigg|_{k^2=M^2}
\]

where $\Sigma(k^2)$ is the bare self–energy of the $H^\pm$, $A^0$ or $W$.

- the electric charge $e$ is defined as in the minimal standard model [24, 25].
Tadpoles are renormalized in such a way that the renormalized tadpoles vanish:
\[ T_h + \delta t_h = 0, \quad T_H + \delta t_H = 0. \]
These conditions guarantee that \( v_{1,2} \) appearing in the renormalized lagrangian are located at the minimum of the one loop potential.

Using these renormalization conditions and as is shown in [24], the renormalization constant of the electric charge and counterterm of gauge boson mass are given by:
\[
\delta Z_e = -\frac{1}{2} \delta Z_{\gamma\gamma} + \frac{1}{2} \frac{s_W}{c_W} \delta Z_{\gamma Z} = \frac{1}{2} \frac{\partial \Sigma_{\gamma\gamma}^0 (k^2)}{\partial k^2} |_{k^2=0} \delta Z_{\gamma\gamma} + \frac{s_W}{c_W} \frac{\Sigma_{\gamma Z}^0 (0)}{M_Z^2} \]  
(4.7)
\[
\delta m_W^2 = \Sigma_{T}^{WW} (M_W^2) \quad \text{and} \quad \delta m_Z^2 = \Sigma_{T}^{ZZ} (M_Z^2) \]  
(4.8)

Where \( \Sigma_{T}^{WW}, \Sigma_{T}^{ZZ}, \Sigma_{T}^{\gamma\gamma} \) are respectively the \( W, Z \) and photon self-energies depicted in Fig. 2. The self-energy appears in the \( W \) gauge fixing term generated from the covariant derivative, one finds the following counterterm for the mixing \( \delta Z_{\gamma Z} \) of the Goldstone and its mixing with charged Higgs boson will be fixed at \( k^2 = 0 \).

### 4.2 Counter term for the W self–energy and the mixing W–G

One obtains the counterterm for the \( W \) self–energy by substituting eqs (1.1) and (1.3) in the \( W \) lagrangian:
\[
\delta (W^\mu W^\nu) = i (g^{\mu\nu} - p_W^\mu p_W^\nu/p_W^2) (\delta m_W^2 + (M_W^2 - p_W^2) \delta Z_W) + i \frac{p_W^\mu p_W^\nu}{p_W^2} (\delta m_W^2 + M_W^2 \delta Z_W) \]  
(4.9)

All the counterterms appearing in \( \delta (W^\mu W^\nu) \) are fixed by the renormalization conditions fixed above eqs. (1.6) and (1.8).

As we have mentioned above, \( W^+ \) boson and \( G^+ \) goldstone mix. To treat this mixing, R. Santos et al [22] have considered the mixing of \( G^+–H^- \) which they have renormalized in the following way:
\[
H^\pm \rightarrow Z_{H^+}^{1/2} \pm H^\pm + Z_{G^+}^{1/2} G^\pm \]  
(4.10)
\[
G^\pm \rightarrow Z_{G^+}^{1/2} G^\pm + Z_{H^+}^{1/2} H^\pm \]  
(4.11)

At the one loop level \( Z_{ii}^{1/2} = 1 + 1/2 \delta Z_{ii} \) and \( Z_{ij}^{1/2} = \delta Z_{ij} \) where \( \delta Z_{ij} = \mathcal{O}(\alpha) \). These four renormalization constants together with the counterterm mass of the charged Higgs bosons are fixed by imposing the on–shell condition (mass located at the pole of the propagator and residue equal to one) and the vanishing mixing both for \( \Sigma_{G^+} (k^2) \) self–energy at \( k^2 = M_H^2 \) and \( \Sigma_{H^+} (k^2) \) self–energy at \( k^2 = 0 \). Note that the Goldstone boson receives its renormalized mass from the gauge fixing lagrangian. Before introducing this lagrangian the Goldstone boson is massless, and so the renormalization conditions imposed on the propagator of the Goldstone and its mixing with charged Higgs boson will be fixed at \( k^2 = 0 \).

At the one loop level the renormalization constants \( \delta Z_{H^+ H^+} \) and \( \delta Z_{G^+ G^+} \) are given by
\[
\delta Z_{H^+ H^+} = -\frac{\partial \Sigma_{H^+ H^+} (k^2)}{\partial k^2} |_{k^2=M_H^2} \quad \text{and} \quad \delta Z_{G^+ G^+} = -\frac{\partial \Sigma_{G^+ G^+} (k^2)}{\partial k^2} |_{k^2=0} \]  
(4.12)

Performing the replacement (1.1) and (1.3) in the W gauge fixing term \( i M_W \partial^\mu W^+ \gamma^\nu \), generated from the covariant derivative, one finds the following counterterm for the mixing \( W^+–G^- \):
\[
\delta (W^\mu G^-) = i \frac{p_W^\mu}{p_W^2} M_W (\delta m_W^2 - \frac{1}{2} (\delta Z_W + \delta Z_{G^+ G^+})) \]  
(4.13)
This completes the set of counterterms needed for our study. The renormalization constants of the wave function and the mass counterterms are given in the appendix B.

4.3 Back to counter–terms form factors

After the short discussion in section 4.2 about the on–shell renormalization we are using, we are now able to give the expressions of the counterterms $\delta \Gamma_{W,H}^{\text{vertex}}$, $\delta \Gamma_{W,H}^{W,W}$, $\delta \Gamma_{W}^{W,G}$ defined in eq. (3.4)

$$
\delta \Gamma_{W}^{\text{vertex}} = -(\delta Z_e - \frac{\delta s_W}{s_W} + \frac{1}{2}(\delta Z_{H+H} + \delta Z_{A0} + \delta Z_W))
$$

$$
\delta \Gamma_{H}^{\text{vertex}} = -2\delta \Gamma_{W}^{\text{vertex}}
$$

$$
\delta \Gamma_{W}^{W,W} = \{(m_{H}^2 + p_{W}^2 - m_{W}^2)\delta Z_W - \delta M_{W}^2 - M_{W}^2 \delta Z_{W})/\left(p_{W}^2 - M_{W}^2\right)
$$

$$
\delta \Gamma_{H}^{W,W} = 2\{(m_{H}^2 + (m_{W}^2 - p_{W}^2)\delta Z_W)/(p_{W}^2 - M_{W}^2)
$$

$$
\delta \Gamma_{W}^{W,G} = \frac{1}{2}(m_{H}^2 - M_{A}^2)\{(\delta M_{W}^2/M_{W}^2 + \delta Z_W + \delta Z_{G+G+})/(p_{W}^2 - M_{W}^2)
$$

5. Numerical results and discussion

In the previous section we have summarized the analytical formulae for the fermionic $O(\alpha)$ radiative correction to the decay $H^+ \rightarrow W^+ A^0$. In this section we focus on the numerical analysis. We take the following experimental input for the physical parameters [26]:

- the fine structure constant: $\alpha = e^2/4\pi = 1/137.03598$.
- the gauge boson masses: $M_Z = 91.187 \text{ GeV}$, $M_W = 80.41 \text{ GeV}$ and $\Gamma_W = 2.06 \text{ GeV}$
- the input lepton masses: $m_e = 0.511 \text{ MeV}$, $m_\mu = 0.1057 \text{ GeV}$, $m_\tau = 1.784 \text{ GeV}$
- for the light quark masses we use the effective values which are chosen in such a way that the experimentally extracted hadronic part of the vacuum polarizations is reproduced [27]:

$$
m_d = 47 \text{ MeV} \quad m_u = 47 \text{ MeV} \quad m_s = 150 \text{ MeV} \quad m_c = 1.55 \text{ GeV} \quad m_b = 4.5 \text{ GeV}
$$

For the top quark mass we take $m_t = 175 \text{ GeV}$. In the on–shell scheme we consider, $\sin^2 \theta_W$ is given by $\sin^2 \theta_W \equiv 1 - \frac{M_t^2}{M_Z^2}$, and this expression is valid beyond tree–level.

In the on–shell case it can be shown that the interference term $2\text{Re}\bar{M}_0 \cdot M^1$, found from squaring the one loop corrected amplitude $|\mathcal{M}_0^1 + \mathcal{M}^1|^2$, is equal to $\Gamma_H |\mathcal{M}_0|^2$. Hence the one loop corrected width $\Gamma_{on}^1$ can be written as $\Gamma_{on}^1 = (1 + \Gamma_H)\Gamma_{on}^0$, with $\Gamma_H$ being interpreted as the fractional contribution to the tree–level width. In the off–shell case, and taking the final state fermions to be massless, $2\text{Re}\bar{M}_0^1 \cdot M^1$ is again equal to $\Gamma_H |\mathcal{M}_0|^2$, although $\Gamma_H$ now has a dependence on $E_1$ and $E_2$ and thus cannot be factorized out of the phase space integral. Therefore we define the fractional contribution to the tree–level width as $\delta \Gamma_{off}$, with:

$$
\Gamma_{off}^1 = (1 + \delta \Gamma_{off})\Gamma_{off}^0
$$
Since $\Gamma_W$ does not contribute to the corrected matrix element it is evident that the $W^+G^+$ mixing has a vanishing contribution and is given in Section 3.3 for completeness.

We now briefly consider the constraints on the masses of the Higgs bosons that can be extracted from current precision measurements of $\rho^0$, defined by:

$$\rho^0 = \frac{M_W^2}{\rho M_Z^2 \cos^2 \theta_W}$$ \hspace{1cm} (5.1)

Here $\rho$ in the denominator contains all purely SM radiative corrections, while $\rho^0 \equiv 1$ in the absence of new physics. In the 2HDM there are extra contributions to $\rho^0$ and Ref. [28] shows that $-0.0017 \leq \delta \rho^0 \leq 0.0027$ at the 2$\sigma$ level. Imposing this condition and using the formulae in Ref. [28] we plot in Fig. 3 the allowable values of $M_{H\pm}$ and $M_A$. We vary all Higgs masses up to 500 GeV and respect the current experimental lower limits for 5000 randomly chosen values. In Fig. 3 the triangles (points) disallow (allow) the decay $H^{\pm} \rightarrow AW^\ast$. From the figure we can clearly see that for $M_{H\pm} \geq 100$ GeV the vast majority of the allowed parameter space satisfies $M_{H\pm} \geq M_A$, thus implying that the decay $H^{\pm} \rightarrow AW^{(\ast)}$ will be open for $M_{H\pm}$ of interest at the LHC and the Tevatron. For $M_{H\pm} \leq 100$ GeV (i.e. the LEP–II range) it is easier to find $M_{H\pm} \leq M_A$.

5.1 On–shell W gauge boson

We now present our results for the case of the $W$ boson being on–shell. There are three unknown parameters which determine the magnitude of the one loop corrected width $\Gamma_{on}$: $M_{H\pm}$, $M_A$ and $\tan \beta$. This is in contrast to the decay $H^{\pm} \rightarrow hW$ in which the mixing angle $\alpha$ and the mass of the heavier CP–even Higgs Boson ($H$) enter the calculation [22]. We stress that this latter analysis only considered the top–bottom loops, while we include all the fermions corrections and find that the light fermion loops are not entirely negligible. Moreover, there can be significant interference among the various contributions, both destructive and constructive. We consider both Model I and Model II, which have effectively identical results at small $\tan \beta$, although differ at large $\tan \beta$.

Let us discuss first the effect of a relatively light charged Higgs ($M_{H\pm} < 250$ GeV) and a very light CP–odd ($M_A \approx 35$ GeV) on $\Gamma_H$. In Fig. 4 we plot $\Gamma_H$ in Model II as a function of $M_{H\pm}$ for several values of $\tan \beta$. We note first that for a fixed value of $\tan \beta$, $\Gamma_H$ is insensitive to the variation in $M_A$ when $M_{H\pm}$ is varied from 120 to 260 GeV. The peaks correspond to the opening of the decay $H^+ \rightarrow t\bar{b}$. For small $\tan \beta$ and $M_{H\pm} < 170$ GeV the correction is rather small ($\approx 2\%$); when $M_{H\pm} > 180$ GeV one can reach a correction of 10%. In the case where $\tan \beta$ is large, the effect comes exclusively from the bottom quark mass and is around 10%.

In Fig. 5 we plot $\Gamma_H$ as a function of $M_A$, taking $M_{H\pm} = 570$ GeV and 3 small values of $\tan \beta$. Since we are not considering large $\tan \beta$ this plot is relevant for both Model I and II. For $M_A$ less than 300 GeV or heavier than 360 GeV the effect is about 5%. When $M_A$ becomes close to $2m_t$ a sharp peak appears and this corresponds to the opening of the channel $A^0 \rightarrow t\bar{t}$, the maximal effect in this case being around 50%. For $M_A$ away from this threshold value ($M_A \approx 330 \rightarrow 345$ GeV) and for small $\tan \beta$ one can have a correction of about $-14\% \rightarrow -41\%$. As $\tan \beta$ increases one quickly approaches a horizontal line at 3.3%. These effects are explained as follows: the $t\bar{t}b$ loop correction is proportional to $Y_{uu}$ and dominates the $b\bar{b}t$ loop correction at small $\tan \beta$ because $m_t \gg m_b$. Since $Y_{uu}$ is proportional to $1/\tan \beta$ we can explain the $\tan \beta$ dependence in Fig. 5. As $\tan \beta$
increases the contribution of the $ttb$ loop weakens rapidly and the dominant contribution to the corrected width becomes that of the renormalized $e$ and $s_W$, giving a fixed value of $\Gamma_H \approx 3.3\%$ which is very insensitive to $\tan \beta$ (note that the $bbt$ loop in Model II is proportional to $\tan \beta$ – see below). We do not notice an obvious correlation between $M_{H^\pm}$ and $\Gamma_H$; for the optimal case considered of $\tan \beta = 0.5$ and $M_A \approx 330$ GeV, varying $M_{H^\pm}$ from 450 GeV to 800 GeV causes $\Gamma_H$ to fall from $-18\%$ to $-27\%$.

In Fig. 6 we plot $\Gamma_H$ in Model II as a function of $\tan \beta$ for $\tan \beta \geq 20$. In Model I all the fermion loops decouple as $\tan \beta$ increases and one has $\Gamma_H \approx 3.3\%$ for $\tan \beta \geq 4$. In Model II the $bbt$ loop dominates with increasing $\tan \beta$ and for $\tan \beta \geq 20$ the value of $\Gamma_H$ starts to differ from the corresponding value in Model I. Again one can find sizeable negative corrections, with the largest occurring for smaller $M_A$ i.e. the closer $M_A$ is to $2m_b$, the more on–shell the virtual $b$ quarks are.

In Fig. 7 we show graphically the relative magnitude of the sum of the heavy quark loops, $ttb$ and $bbt$, compared to the sum of the remaining fermion loops ($\Gamma_{light}$). Since we plot only low values of $\tan \beta$ the $ttb$ contribution dominates the $bbt$ loop and so we label the sum of the $ttb$ and $bbt$ contributions as $\Gamma_{ttb}$. One can see that $\Gamma_{light}$ is of comparable strength to the heavy quark loops unless $M_A$ is close to $2m_t$. In addition there can be constructive or destructive interference, which is shown in Fig. 7 by the sign of the ratio.

### 5.2 Off–shell W gauge boson

We now consider the case of the W gauge boson being off–shell. Since the decay $H^\pm \to AW^*$ is possible for a light $H^\pm$ in range at LEP–II we shall present results for $M_{H^\pm} = 80$ GeV, which is also in the mass region considered problematic for detection channels which make use of the conventional decays $H^\pm \to \tau \nu_\tau$, $cs$. As is mentioned in the introduction, charged Higgs bosons of Model II are excluded from the LEP–II discovery range by precision measurements of $b \to s\gamma$. Our discussion will therefore be focussed on Model I. In the massless fermion final state limit, the $WW$ self–energy is the only additional contribution to the one loop corrected width for the off–shell decay\[^1\]. The $WW$ self–energy is the standard diagram and does not depend on $\tan \beta$. Hence all the $\tan \beta$ dependence is contained in the vertex contribution and in the case of Model I is enhanced when $\tan \beta$ is small.

In Fig. 8 we plot the magnitude of the one loop corrections, $\delta \Gamma_{off}$, as a function of small $\tan \beta$ for two values of $M_A$. We can see that for $\tan \beta \geq 2$ one approaches a fixed value ($\approx 2\%$) for $\delta \Gamma_{off}$ – this is to be interpreted (as before) as the fermion loops decoupling, leaving a $\tan \beta$ independent value which comes from the $WW$ self–energy and from the renormalized $e$ and $s_W$ in the vertex contribution counterterms. For low $\tan \beta$ the one loop corrections are pulled negative. Very large corrections of up to $-90\%$ are possible for exceptionally small ($\approx 0.1$) values of $\tan \beta$, although such values are strongly disfavoured by measurements of $R_b$ which require $\tan \beta \geq 1.8$ (95% c.l) for $M_{H^\pm} = 85$ GeV\[^2\].

\[^1\]Note that for the W being off–shell, there are extra contributions coming from box diagrams which will be considered in ref. \[^3\].
6. Conclusions

We have computed the Yukawa coupling corrections to the decay $H^+ \rightarrow A^0 W^+$ in the case of an on–shell and off–shell W gauge boson. We have included in our analysis both top–bottom contributions and light fermion contributions, the latter being non–negligible and may interfere destructively or constructively with the former. Restrictions on the possible values of the Higgs boson masses from considering the $\rho$ parameter were also included and found to give in the majority of the cases $M_{H^{\pm}} > M_A$. In the on–shell case, we studied the sensitivity of the Yukawa corrections to $\tan \beta$, and found similar effects for small $\tan \beta$ in both Model I and Model II which can reach 50% for $m_A \approx 2m_t$. For large $\tan \beta$, in Model I all the fermions corrections decouple and reach a constant value 3.3% for $\tan \beta > 4$; in Model II, the top mass effect is suppressed while the bottom mass effect is increased for $\tan \beta > 20$, allowing sizeable corrections of 10% or greater. For the case of the W gauge boson being off–shell, the charged Higgs bosons in the LEP–II range and $\tan \beta$ not too small, the corrections are rather small and do not surpass 2%.

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Appendix A: Passarino Veltman functions

Let us recall the definitions of scalar and tensor integrals [21] we use:

A.1 One point function:

The one point function is defined by:

$$A_0(m_0^2) = \frac{(2\pi\mu)^{4-d}}{i\pi^2} \int d^d q \frac{1}{[q^2 - m_0^2]}$$

$\mu$ is an arbitrary renormalization scale.

$$A_0(m_0^2) = m_0^2 [1 + \Delta_0] + \mathcal{O}(d - 4) \quad (A.1)$$

The UV divergences are contained in $\Delta_0$ which is given by

$$\Delta_i = \frac{2}{4-d} - \gamma_E + \log(4\pi) + \log \left(\frac{\mu^2}{m_i^2}\right)$$

$\gamma_E$ is the Euler-Mascheroni constant.

Note that in dimensional regularization $A_0(0) = 0$. 
A.2 Two point functions:

The two points functions are defined by:

\[
B_{0,\mu,\nu}(p^2, m^2_0, m^2_1) = \frac{(2\pi\mu)^{4-d}}{i\pi^2} \int d^n q \frac{1, q_{\mu,\nu}}{[q^2 - m^2_0][(q + p_1)^2 - m^2_1]}
\]

\[
B_0 = \frac{1}{2}(\Delta_0 + \Delta_1) - \int_0^1 dx \ln \frac{x^2 k^2 - x(k^2 - m^2_0 + m^2_1) + m^2_1 - i\epsilon}{m_0 m_1}
\]

(A.3)

the derivative of \(B_0\) function is defined as:

\[
B'_0[X, m^2_1, m^2_2] = \frac{\partial}{\partial p^2} B_0[p^2, m^2_1, m^2_2] |_{p^2 = X}
\]

note that \(A_0\) can be expressed in term of \(B_0\)

\[
A_0(m^2) = m^2 + m^2 B_0(0, m^2, m^2)
\]

using Lorentz invariance, we have:

\[
B_\mu = p_1^\mu B_1
\]

\[
B_{\mu\nu} = p_1^\mu p_1^\nu B_{21} + g_{\mu\nu} B_{22}
\]

A.3 Three point functions:

The three point functions are defined as:

\[
C_{0,\mu,\nu}(p_1^2, p_2^2, m^2_0, m^2_1, m^2_2) = \frac{1}{i\pi^2} \int d^n q \frac{1, q_{\mu,\nu}}{[q^2 - m^2_0][(q + p_1)^2 - m^2_1][(q + p_2)^2 - m^2_2]}
\]

where \(p^2_{12} = (p_1 + p_2)^2\). Using Lorentz invariance, \(C_\mu\) and \(C_{\mu\nu}\) can be written as:

\[
C_\mu = p_1^\mu C_1 + p_2^\mu C_2 \tag{A.4}
\]

\[
C_{\mu\nu} = g_{\mu\nu} C_{00} + p_1^\mu p_1^\nu C_{11} + p_2^\mu p_2^\nu C_{22} + (p_1^\mu p_2^\nu + p_2^\mu p_1^\nu) C_{12} \tag{A.5}
\]

Appendix B: Renormalization constants

Hereafter we give all the renormalization constants necessary to compute the counterterms defined in eqs [4.14]:

B.1 gauge bosons self–energies

Let \(i\epsilon \gamma^\mu (V_L \frac{1-\gamma_5}{2} + V_R \frac{1+\gamma_5}{2})\) the general coupling of the gauge bosons \(V_\mu\) to a pair of fermions \(f\) and \(f'\). The coefficient of \(-g_{\mu\nu}\) of the self–energy of the gauge boson \(V_\mu\) is given by:

\[
\Sigma_{TT}^{VV}(p^2) = -\frac{N_c \alpha}{2\pi} \left\{ (V_L^2 + V_R^2)[A_0(m^2_f) - 2B_{22}(p^2, m^2_f, m^2_f) + p^2 B_1(p^2, m^2_f, m^2_f)] + m_f [m_f (V_L^2 + V_R^2) - 2m_f V_L V_R B_0(p^2, m^2_f, m^2_f)] \right\}
\]

(B.1)
The renormalization constant of the electric charge is given by:

$$\delta Z_e = -\frac{1}{2} \frac{\delta Z_{\gamma\gamma}}{\delta k^2} \bigg|_{k^2=0} = -\frac{1}{2} \frac{\partial \Sigma_T^{\gamma\gamma}(k^2)}{\partial k^2} \bigg|_{k^2=0}$$

The mass counterterms for the gauge boson $W$ and $Z$ are given by:

$$\delta M_W^2 = \Sigma_T^{WW}(p^2 = M_W^2), \quad \delta M_Z^2 = \Sigma_T^{ZZ}(p^2 = M_Z^2)$$

### B.2 Wave functions renormalization

The wave function renormalization constants for the $W$ gauge boson can be obtained from the self-energy as:

$$\delta Z_W = -\frac{\partial \Sigma_T^{WW}(k^2)}{\partial k^2} \bigg|_{k^2=M_W^2}$$

The renormalization constants of the charged Higgs, CP–odd Higgs and the Goldstone boson wave function are given by:

$$\delta Z_{H^+H^+} = \frac{N_C}{4\pi} \frac{\alpha}{2\pi} \{ Y_{ud}^2 + Y_{ud}^R \} \delta B_0(M_{H^+H^+}, m_d^2, m_u^2)$$

$$+ \{ m_d^2 + m_u^2 - M_{H^+H^+} \} \{ (Y_{ud}^L + Y_{ud}^R) + 4m_d m_u Y_{ud}^L Y_{ud}^R \} \delta B'_0(M_{H^+H^+}, m_d^2, m_u^2)$$

$$\delta Z_{A^0} = -\frac{N_C}{2\pi} \{ Y_{ud}^2 \} \delta B_0(M_{A^0}, m_d^2, m_d^2)$$

$$+ \{ m_d^2 + m_u^2 - M_{A^0} \} \{ (Y_{ud}^L + Y_{ud}^R) + 4m_d m_u Y_{ud}^L Y_{ud}^R \} \delta B'_0(M_{A^0}, m_d^2, m_d^2)$$

$$\delta Z_{G^+G^+} = \frac{\alpha N_C}{8\pi M_W^2 s_W} \{ -2m_d^2 + m_u^2 \} \delta B_0(0, m_d^2, m_u^2)$$

$$+ \{ m_d^2 - m_u^2 \} \{ (4m_d m_u Y_{ud}^L Y_{ud}^R) \} \delta B'_0(0, m_d^2, m_u^2)$$
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Figure Captions

Fig. 1 Feynman diagrams for the Born approximation to the decay $H^+ \rightarrow A^0 W^{\pm*}$, W on–shell (1.a), W off–shell (1.b).

Fig. 2 Feynman diagrams for the one loop corrections to the decay $H^+ \rightarrow A^0 W^{\pm*}$: vertex (2.a1 and 2.a1), WW self–energy (2.a3), W–G mixing (2.a4). Charged Higgs boson, Goldstone boson and CP–odd self–energies (2.b1,2,3). (2.b4,5,6) WW, ZZ and $\gamma\gamma$ self–energies. (2.c1) is the vertex counterterm, (2.c2) W self–energy counterterm and (2.c5) is the W–G mixing counterterm.

Fig. 3 Scatter plot of values of $M_{H^\pm}$ and $M_A$ consistent with measurements of $\rho^0$. Triangles disallow the decay $H^\pm \rightarrow AW^{(*)}$.

Fig. 4 $\Gamma_H$ as a function of $M_{H^\pm}$ (Model II) for $M_A = 35$ GeV, $\tan\beta = 0.5, 1.0, 4$ and 70.

Fig. 5 $\Gamma_H$ as a function of $M_A$ (Models I and II) for $\tan\beta = 0.6, 1.0$ and 2.5.

Fig. 6 $\Gamma_H$ as a function of $\tan\beta$ (Model II) for $\tan\beta \geq 20$.

Fig. 7 $\Gamma_{t\bar{t}b}/\Gamma_{light}$ as a function of small $\tan\beta$ for several values of $M_A$ (Models I and II).

Fig. 8 $\delta\Gamma_{off}$ as a function of small $\tan\beta$ and for $M_A = 15, 40$ GeV (Model I).
Figure 2
Figure 3
Figure 4

Model II

$M_A = 35$ GeV

$tan\beta = 4$
$tan\beta = 1$
$tan\beta = 0.5$
$tan\beta = 70$
Figure 5

$M_{H^+} = 570 \text{ GeV}$

Models I and II

$tan\beta = 0.6$

$M_A$ (GeV)

200 220 240 260 280 300 320 340 360 380 400
Figure 6

Model II

$M_{H^+} = 440$ GeV

$M_A = 300$

$M_A = 220$

$M_A = 150$

$M_A = 90$

$M_A = 60$
Figure 7

$\Gamma_{tb}/\Gamma_{light}$

$M_A = 80$

$M_A = 250$

$M_A = 320$

$M_A = 340$

$M_{H^+} = 570$ GeV

Models I and II

$\tan\beta$
Figure 8

\[ \delta \Gamma_{\text{eff}} \]

- \( M_A = 15 \text{ GeV} \)
- \( M_A = 40 \text{ GeV} \)

Model I
\[ M_{H^+} = 80 \text{ GeV} \]

\[ \tan \beta \]