QCD soft gluon exponentiation: YFS MC Approach

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We develop and prove the theory of the QCD extension of the YFS Monte Carlo approach to higher order QED radiative corrections. As a corollary, a new approach to quantum gravity by one of us (B.F.L.W.) is illustrated. Semi-analytical results and preliminary explicit Monte Carlo data are presented for the processes $p \bar{p} \rightarrow t \bar{t} + X$ at FNAL energies. We comment briefly on the implications of our results on the CDF/D0 observations and on RHIC/LHC physics.

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1. Introduction

The problem of soft gluon resummation is well known\cite{1,2} and some of its many phenomenological applications are also: the FNAL $t \bar{t}$ production cross section higher order corrections (the current situation\cite{3,4} has the experimental cross section $6.2^{+1.2}_{-1.0}$ pb to be compared with a theoretical prediction\cite{5} of $5.1^{+0.5}_{-0.7}$ pb) and the attendant soft gluon uncertainty in the extracted value of $m_{t} = 0.1743 \pm 0.0051$ TeV, where $\Delta m_{t} \sim 2-3$ GeV of the latter error could be due to soft gluon uncertainties; RHIC hard scattering polarized $pp$ scattering processes, etc. For the LHC/TESLA/LC, the requirements on the corresponding theoretical precisions will be even more demanding and the QCD soft $n(G)$ MC exponentiation which we discuss in the following will be an important part of the necessary theory – YFS\cite{2} exponentiated $\mathcal{O}(\alpha_{s}^{2})L$ corrections realized on an event-by-event basis.

The results which we present also will allow us to investigate from a different perspective some of the outstanding theoretical issues in perturbative QCD, such as the treatment of phase space, no-go theorems for the soft regime, etc. – see ref.\cite{11}.

For definiteness, we will use the process in Fig. 1, $\bar{Q}(p_{1})Q(q_{1}) \rightarrow t(p_{2})\bar{t}(p_{1}) + G_{1}(k_{1}) \cdots G_{n}(k_{n})$, as our proto-typical process. Extension of the methods we develop to other related processes will be immediate – one of us (B.F.L.W.) has realized a new approach to quantum gravity as a by-product. Although we shall use the older EEX formulation of YFS MC exponentiation as defined in ref.\cite{9}, the realization of our results via the the newer CEEX formulation of YFS exponentiation in ref.\cite{10} is also possible and is in progress\cite{12}.

After a brief review of the QED case, we prove our result for the QCD case and conclude with some illustrative results.

2. Review of YFS Theory: An Abelian Gauge Theory Example

The Abelian gauge theory example of QED has been worked-out and realized in many applications for SLC/LEP1 and LEP2 physics in the MC’s YFS2, YFS3, BHLUMI, BHWIDE, KO-
Figure 1. The process $\bar{Q}Q \rightarrow t \bar{t} + n(G)$. The four-momenta are indicated in the standard manner; $q_1$ is the four-momentum of the incoming $Q$, $q_2$ is the four-momentum of the outgoing $t$, etc., and $Q = u, d, s, c, b, G$.

RLZ, K\'C MC, YFSWW3, YFSZZ and KoralW by S. Jadach et al. in refs. [9,10,12]. For example, for the process $e^+(p_1)e^-(q_1) \rightarrow f(p_2)f(q_2) + n(\gamma)(k_1, \ldots, k_n)$, renormalization group improved YFS theory [13] gives

$$d\sigma_{\text{exp}} = e^{2\alpha \text{Re}B + 2\alpha B} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{n} \frac{d^3k_j}{k_0^2} \int \frac{d^4y}{(2\pi)^4} e^{iy(p_1 + q_1 - p_2 - q_2 - \sum_j k_j) + D_{\text{QCD}}}$$

where the YFS infrared functions $\tilde{B}$, $B$ and the IR finite function $D$ are known [13]. For example, the YFS hard photon residual $\beta_i$ in [13], $i = 0, 1, 2$, are given in refs. [13] for BHLUMI 4.04 realizing the YFS exponentiated exact hard photon residuals $\bar{D}_{\text{IR}}$ finite function

$D_{\text{YFS}}$ theory [13] gives

$$M^{(n)} = \text{exp}(\alpha_s B_{\text{QCD}}) \sum_{n=0}^{\infty} m_j^{(n)},$$

where $\alpha_s(Q)B_{\text{QCD}}$ is the usual YFS integral of $S_{\text{QCD}}(k)$ and $m_j^{(n)}$ do not contain the virtual IR singularities in the product $S_{\text{QCD}}(k_1) \cdots S_{\text{QCD}}(k_n)$.

For the respective real IR singularities, we proceed as in ref. [8] so that, for the real emission factor $S_{\text{QCD}}(k)$ defined in ref. [8], upon effecting the analogous YFS expansion in $S_{\text{QCD}}(k)$ and summing on $n$, we arrive at the “YFS-like” result

$$d\tilde{\sigma}_{\text{exp}} = \int \frac{d^4y}{(2\pi)^4} e^{iy(p_1 + q_1 - p_2 - q_2 - \sum_j k_j) + D_{\text{QCD}}}$$

where the IR finite functions $SU M_{\text{IR}}(QCD)$, $D_{\text{QCD}}$ are defined in ref. [6] and the $\beta_i$ are the QCD hard gluon residuals [6].

Each order in $\alpha_s$ must make an infrared finite contribution to $d\tilde{\sigma}_{\text{exp}}$. Since both $SU M_{\text{IR}}(QCD)$ and $d\tilde{\sigma}_{\text{exp}}$ are IR finite. If $\beta_0^{(t)} = \bar{\beta}_0^{(t)} + D\bar{\beta}_0^{(t)}$, where $\bar{\beta}_0^{(t)}$ is completely free of IR divergences, then the IR finiteness just noted allows us to conclude that the contributions from the $\{D\bar{\beta}_0^{(t)}\}$ cancel in $d\tilde{\sigma}_{\text{exp}}$ so that
we arrive at the new result

\[ d\sigma_{\text{exp}} = \sum_n d\sigma^n = e^{\text{SUM}\text{ln}(QCD)} \sum_{n=0}^{\infty} \prod_{j=1}^{n} \frac{d^3k_j}{k_j^2} \int \frac{d^4y}{(2\pi)^4} e^{i\psi(p_1+q_1-p_2-q_2-\sum k_j)} D_{\text{QCD}} \]

\[ \beta_n(k_1, \ldots, k_n) \frac{d^3p_1}{p_1^2} \frac{d^3p_2}{p_2^2} q_2^2. \]

where the new hard gluon residuals \( \beta_n(k_1, \ldots, k_n) = \sum_{t=0}^{\infty} \beta_n^{(t)}(k_1, \ldots, k_n) \) are completely free of IR divergences. Earlier arguments in ref. [14] were insufficient to derive the analog of (3).

There are many consequences of (3) and the theory which underlies it [11]. We illustrate some of them in the next section. In addition, one of us (B.F.L.W.) thanks Prof. S. Bethke for the support and kind hospitality of the MPI, Munich, during the final stages of this work. The authors also thank Profs. G. Altarelli and Prof. Wolf-Dieter Schlatter for the support and kind hospitality of CERN while a part of this work was completed. These same two authors also thank Profs. F. Gilman and W. Bardeen of the former SSCL for their kind hospitality while this work was in its development stages.

4. **YFS Exponentiated QCD Corrections to \( t\bar{t} \) Production at High Energies**

We have realized the result (3) via semi-analytical methods and via MC methods [8]. Here we illustrate these realizations.

For the process \( pp \to t\bar{t} + X \) at FNAL energies, we use a semi-analytical realization of (3) together with the standard formula structure function formula (3) for \( \sigma(t\bar{t} + X) \), where the DGLAP synthesisization procedure presented in ref. [16] is applied to avoid over-counting resummation effects already included in the structure function DGLAP evolution. In the MC realization, we employ the MC methods of ref. [8] to get the MC ttp1.0 as the respective event-by-event simulation. For the semi-analytical analysis, we have found in ref. [7] the normalization \( n(G) \) effect \( (r_{\text{exp}}^{\text{nls}} \text{ is the ratio of the exponentiated and Born cross sections}) \) \( r_{\text{exp}}^{\text{nls}} = 1.086, 1.103, 1.110 \) for \( \alpha_s \) evaluated at the scales \( \sqrt{s}, 2m_t, m_t \), respectively, which implies [8] that the corrections for \( \mathcal{O}(\alpha_s^n, n \geq 2) \) give \( 0.006 - 0.008 \) of the NLO cross section, in agreement with ref. [13]. This is an important cross check of our methods.

For the MC data from ttp1.0, which are preliminary, show in Fig. 2 where we see in the gluon transverse momentum distribution that \( <p_{\perp} > \) is indeed large at FNAL and its effect must be taken into account in precision studies of the top quark production and decay systematics.

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