Effect of baffle size and thermal boundary conditions on mixed convection flow in a channel with cavity

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Abstract.
A numerical investigation is performed to explore the mixed convection in a channel-cavity containing an adiabatic baffle. The bottom portion of the channel-cavity imposes two different types of heating. The remaining portions of the channel-cavity is adiabatic. The finite difference method is used to solve the governing equations for diverse grouping of relevant constraints. The averaged Nusselt number, averaged bulk temperature and the Drag force are also calculated. It is detected that the averaged energy transport raises on rising the length of the baffle. It is also established that the sinusoidal heating provides higher heat transfer than linearly heating case.

Keywords: Mixed convection, channel, open cavity, baffle.

1. Introduction
Over the past decades, researchers have been interesting on the fundamental problem of convective flow and energy transport in various geometries like channels and cavities because of its applications in many fields [1]. Manca et al. [2] numerically explored mixed convection in a channel with an open cavity. Brown and Lai [3] numerically deliberated combined heat and mass transfer in a horizontal channel with an open cavity. Leong et al. [4] executed the analysis of combined convection in a horizontal channel with an open cavity. Saeidi and Khodadadi [5] numerically explored the forced convection in a square cavity with inlet and outlet ports. Rahman et al. [6] numerical explored the magnetic field influence on mixed convection in a horizontal channel-cavity with bottom heating. The pertinent parameters were strongly affected the convective stream and energy transport. Rahman et al. [7] numerically investigated the combined convection in a channel within a cavity which has a partially or fully heated on left side. They explored that higher energy transfer was found for partial heater at higher Ra. Sharma et al. [8] numerically discovered mixed convection in a grooved channel in the centrally placed an adiabatic baffle from the top wall. They perceived that notable augmentation of heat transfer is found in the presence of baffle in the mixed convection regime.

The effect of various thermal boundary conditions are explored by several researches recently [8-20]. The effect of sinusoidal heating on convective flow in a porous enclosed box is examined by Janagi et al. [8], Cheong et al. [9] and Sivasankaran and Bhuvaneswari [10]. The magneto-convection in a box with sinusoidal heating on vertical sidewall is explored by Sivasankaran et al. [11] and Bhuvaneswari et al. [12]. Recently, sinusoidal heating is used to study the heat transfer problems. The influence of sinusoidal thermal condition on stream pattern and energy transport in an inclined box is investigated by Sivasankaran et al. [13] and Cheong et al. [14]. Number of studies on the influence of non-uniform heating in enclosed spaces are reported recently [15-18]. They explored from those studies that non-uniform heating provides heat transfer augmentation in the system. Bhuvaneswari et al. [19] and Sivasankaran et al. [20] examined the convective flow in cavities with linearly heated wall.

There is no study reported on the literature to explore the effect of boundary conditions and baffle size on mixed convection in a channel-cavity. Hence, the present study investigates the effect of thermal boundary conditions and length of the adiabatic baffle on convective flow in a channel with open cavity.
2. Mathematical Modeling

We consider the two-dimensional, laminar, steady, incompressible flow in a channel with open cavity of length $L$ and width $L/2$ as shown in figure 1. The bottom portion of the cavity inside the channel imposes two different types of heating, viz., linearly heating and sinusoidal heating. The action of gravitational force towards the downward direction is considered. The Boussinesq approximation is valid.

The governing equations for the present model are:

\[ u_x + v_y = 0 \]  
\[ uu_x + vv_y = -\frac{1}{\rho_0} p_x + v [\nabla^2 u] \]  
\[ uv_x + vv_y = -\frac{1}{\rho_0} p_y + v [\nabla^2 v] + g\beta(\theta - \theta_c) \]  
\[ uT_x + vT_y = \alpha (\nabla^2 T) \]

where $(u, v)$ are velocities, $g$, $p$, $\rho_0$, $\beta$, $\alpha$, $T$ are gravitational acceleration, pressure, density, thermal expansion coefficient, kinematic viscosity, thermal diffusivity, and temperature respectively.

The boundary & initial settings are

At inlet: $u = U_0$, $v = 0$, $T = 0$

At outlet: $\frac{\partial u}{\partial x} = 0$, $v = 0$, $\frac{\partial T}{\partial x} = 0$, $p = 0$

At bottom wall: $u = 0$, $v = 0$, $T = T_h(x) = \begin{cases} \sin \left( \frac{\pi x}{L} \right) (T_h - T_c) + T_c & \text{case 1} \\ \frac{x(T_h - T_c)}{L} + T_c & \text{case 2} \end{cases}$

Other walls: $u = 0$, $v = 0$, $\frac{\partial T}{\partial n} = 0$

where ‘$n$’ is the distances either $x$ or $y$ direction acting normal to the surface. The above equations are non-dimensionalized by using the following dimensionless quantities $(X, Y) = (x, y)/L$, $(U, V) = (u, v)/U_0$, $P = p/(\rho_0 U_0^2)$, $\theta = (T - T_c)/(T_h - T_c)$. The dimensionless form of governing system for the present problem are specified as follows:

\[ \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = 0 \]  
\[ U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \]  
\[ U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + Gr \frac{\partial \theta}{\partial X} \]  
\[ U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = -\frac{1}{Re Pr} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \]
The dimensionless parameters are, \( \text{Pr} = \frac{V}{\alpha} \), Prandtl number, \( Gr = \frac{g \beta (\theta_0 - \theta_L) L^4}{\nu^3} \), Grashof number, \( Re = \frac{U_o L}{\nu} \), Reynolds number and \( Ri = \frac{Gr}{Re^2} \), Richardson number. The stream function is estimated by using \( U = \Psi_Y \) and \( V = -\Psi_X \). The dimensionless boundary conditions for the present problem are:

At inlet: \( U = 1, V = 0, \theta = 0 \)
At outlet: \( \frac{\partial U}{\partial X} = 0, V = 0, \frac{\partial \theta}{\partial X} = 0, P = 0 \)
At bottom wall: \( U = 0, V = 0, \theta = \theta_b(X) = \begin{cases} \sin(\pi X) & \text{case 1} \\ X & \text{case 2} \end{cases} \) \hspace{1cm} (10)
Other walls: \( U = 0, V = 0, \frac{\partial \theta}{\partial n} = 0 \)

The physical quantities of the present investigation are defined here. The average Nusselt number at the heated surface is calculated as \( Nu = -\int_0^1 \frac{\partial \theta}{\partial Y} dX \), the drag force, \( D = -\int_0^1 \frac{\partial U}{\partial Y} dX \) and the average temperature of the fluid is defined as \( \theta_{avg} = \frac{\int \theta dV}{V} \), where \( V \) is the cavity volume.

3. Numerical procedure

The governing mathematical model is solved numerically by using the finite difference technique. The algebraic equations obtained in tri-diagonal structure are solved by iterative method. In order to determine the proper grid size for this study, a grid independence test is conducted with \( Ri=1 \) and \( Re=316.2 \). The various types of mesh are considered for the grid independence study. The average transport rates are determined using Trapezoidal rule.

4. Results and discussion

![Figure 2. Streamlines for different baffle lengths with Ri=1](image_url)
The numerical simulations are prepared for various length of baffle and two kinds of thermal boundary conditions with different values of Grashof and Reynolds numbers. The value for Prandtl number is taken as 0.71. Figure 2 shows the stream pattern for various length of baffle and two cases of thermal conditions. It is evidently seen that the baffle influences much on stream pattern. The recirculation zones are formed on both sides of the baffle and size of the eddy is strongly depends on the length of the baffle. The flow influence inside the cavity is less in the absence of the baffle. The presence of the baffle induces the stream inside the cavity and results in the convective transport. Figure 3 shows the corresponding isotherms for various length of baffle and two cases of thermal conditions. The difference between the thermal boundary conditions is clearly seen from the isotherms. The thermal boundary layers are formed along the bottom wall of the cavity. In the absence of baffle, the stream is not vigorous flowing inside the cavity and heat transfer is feeble. The baffle helps the flow to move inside the cavity and it affects the heat transport. The heat transfer is enhanced on raising the length of the baffle because the flow is more induced inside the cavity by the presence of baffle and it results the high-energy transport inside the cavity.

Case 1       Case 2

L_B=0

L_B=0.25

L_B=0.5

L_B=0.75

Figure 3. Isotherms for different baffle lengths with Ri=1

Figures 4 and 5 show the influence of Richardson number on flow and thermal fields with L_B=0.5. The size of the recirculating eddy is small in the forced convection regime. The size of the recirculating eddy is increased on rising the values of Ri and it occupies the majority of area in the buoyant convection regime. The circulating eddies in the cavity is disappeared in the buoyant convection regime. Figure 6(a-f) shows the local Nusselt number for various length of baffle, different Ri and two cases of thermal boundary conditions. The profiles of local Nusselt number clearly shows the direct impact of imposed thermal boundary conditions on the bottom wall of the cavity. The higher local energy transport is occurred near the middle of the wall for sinusoidal heating case (case 1) whereas higher local energy transport is happened about Y=3/4 for linearly heating case (case 2). Further comparing these two cases local energy transport is higher in the sinusoidal heating case for all values of Ri and L_B. There is a small difference in local heat transfer rate for all values of L_B when L_B≤0.5. When L_B=0.75, the local heat transfer rate is suddenly boosted up for all values of Ri and both heating cases. Nevertheless, the attained peak value is not in the same place for both heating cases.
Figure 4. Streamlines for different Ri with LB=0.5

Figure 5. Isotherms for different Ri with LB=0.5

Figure 7(a-b) demonstrated the drag force for different values of Ri and length of baffle with two kinds of thermal conditions. The drag force gradually increases with Richardson number when LB≤0.5 for sinusoidal heating case. The drag force gradually increases with Richardson number when LB≤0.25 for linearly heating case, but, it behaves nonlinearly with Ri for LB=0.5. It diminishes steeply until Ri=10 and it raises for both heating cases. Figure 8(a-b) portrayed the average temperature of the system for diverse values of Ri and length of baffle with two kinds of thermal conditions. The average temperature rises with Richardson number for both cases. The average temperature is high in the absence of baffle. Figure 9(a-b) described the averaged heat transfer rate for various values of Ri and length of baffle with two types of thermal conditions. The averaged energy transport diminishes gradually until Ri=10 and it increases for case 1 whereas it diminishes gradually until Ri=1 and it increases for case 2 when LB≤0.5. However, the averaged energy transport diminishes steeply on rising the values of Ri. It is also observed that the averaged energy transport rises by raising the length of the
baffle. On comparing the two thermal boundary conditions, sinusoidal heating provides higher energy transport than the linearly heating case for all parameters considered.

Figure 6. Local Nusselt number for various baffle length for case 1 (a, c, e) and case 2 (b, d, f)

5. Conclusions
The effect of length of adiabatic baffle and two kinds of thermal boundary conditions on convective flow and heat transfer in a channel with open cavity is examined numerically. The results are analysed for various combinations of baffle length and Ri for sinusoidal and linearly heating cases. The following outcomes are observed. The average temperature inside the system is high in the absence of baffle. The drag force gradually increases with Richardson number when $L_B \leq 0.5$. But, the drag force diminishes as $L_B$ increases. For $L_B = 0.75$, the drag force is significantly lower compared to other cases. The heat transfer is highest for $L_B = 0$ in both cases, and decreases as $L_B$ increases.
steeply until Ri=10 when LB=0.75. The averaged energy transport raises on growing the length of the baffle. The sinusoidal heating provides higher heat transfer rate than linearly heating case.

![Figure 7](image1.png)  
**Figure 7.** Drag force vs. Ri for various LB values

![Figure 8](image2.png)  
**Figure 8.** Average temperature vs. Ri for various LB values

![Figure 9](image3.png)  
**Figure 9.** Averaged Nusselt number vs. Ri for various LB values

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