Quantized Precoding for Multi-Antenna Downlink Channels with MAGIQ

Andrei Nedelcu*, Fabian Steiner*, Markus Staudacher*, Gerhard Kramer*, Wolfgang Zirwas†, Rakash Sivasiva Ganesan†, Paolo Baracca‡, Stefan Wesemann‡

*Institute for Communications Engineering, Technical University of Munich, Germany
†Nokia Bell Labs, Munich, Germany, ‡Nokia Bell Labs, Stuttgart, Germany

Abstract—A multi-antenna, greedy, iterative, and quantized (MAGIQ) precoding algorithm is proposed for downlink channels. MAGIQ allows a straightforward integration with orthogonal frequency-division multiplexing (OFDM). MAGIQ is compared to three existing algorithms in terms of information rates and complexity: quantized linear precoding (QLP), SQUID, and an ADMM-based algorithm. The information rate is measured by using a lower bound for finite modulation sets, and the complexity is measured by the number of multiplications and comparisons. MAGIQ and ADMM achieve similar information rates with similar complexity for Rayleigh flat-fading channels and one-bit quantization per real dimension, and they outperform QLP and SQUID for higher order modulation.

I. INTRODUCTION

Massive multi-input multi-output (massive MIMO) foresees the deployment of large antenna arrays at the base station to serve many users that each have a small number of antennas [1]. The gains of multi-user MIMO include improved power and spectral efficiencies, and simplified signal processing. The gains are often stated for a large number of base station antennas and a large number of user equipments (UEs) when the ratio N/K is held constant.

A practical implementation for large N and K is challenging. A naive deployment uses N individual radio frequency (RF) chains, with the accompanying high hardware cost and power consumption. In particular, it seems impractical to use a massive number of high-resolution analog-to-digital and digital-to-analog converters (ADCs/DACs), along with linear but low efficiency power amplifiers. Two potential solutions are as follows. First, hybrid-beamforming uses analog beamformers in the RF chain of each antenna, whereas the digital baseband processing is shared among different RF chains. Second, one reduces the resolution of the ADCs/DACs, perhaps even to one bit so that simple comparators are used for both the in-phase and quadrature components of the signals. This permits using non-linear power amplifiers.

A. Uplink and Downlink

We consider the low-resolution quantizer approach. There are numerous studies on the uplink with either a linear detector, e.g., matched filter (MF), zero forcing (ZF), Wiener filter (WF), or a non-linear detector, e.g., approximate message passing (AMP) [4]–[6]. For example, it is known that even for one-bit quantization at the base station antennas, the UEs can communicate with higher order modulation formats if N is chosen large enough.

Recently, the downlink has received attention [7]–[12]. The authors of [7] use the Bussgang approximation to design quantized linear precoders (QLPs). The paper [8] introduces a lookup-table based precoder for quadrature phase-shift keying (QPSK) that minimizes the uncoded bit error rate (BER) at the UE. The paper [9] introduces a hybrid RF architecture that combines a constrained and a conventional MIMO array. A greedy knapsack-like algorithm is used to minimize the mean square error (MSE) between the desired and constructed UE signal points.

The authors of [10] describe nonlinear approaches based on semidefinite relaxation, 2 norm relaxation and sphere decoding. For a reasonable performance and complexity trade-off, they recommend 2 norm relaxation, which is named SQUID [10, Sec. IV]. Another approach is described in [11], where the authors extend the framework of alternating direction of multipliers (ADMM), and they report slight improvements over SQUID. The precoding problem for coarsely quantized, frequency-selective channels and its integration with OFDM was considered in [12], where the authors use linear precoding and a frequency domain approach.

B. Contributions and Organization

Our work is inspired by the greedy approach of [9]. We introduce a multi-antenna, greedy, iterative, and quantized (MAGIQ) precoding algorithm for downlink channels. The algorithm decomposes a high dimensional nonlinear and non-convex problem into low dimensional sub-problems that can be solved efficiently. We compare MAGIQ’s performance to QLP, SQUID, and the ADMM-based algorithm [7], [10]–[12].

We consider both low-order and higher-order modulation formats (4,16,64-quadrature amplitude modulation (QAM)). As a key performance metric, we compute a lower bound on the information rate of each UE by using the mismatched decoding framework [13], [14]. We thus take modulation constraints into account, rather than assuming idealized Gaussian signaling. For the frequency-selective case, our approach operates in the time domain and avoids switching between domains to enforce the discrete alphabet constraint. A related problem was presented in the context of precoding for frequency selective channels with constant envelope continuous modulation in [15].
This paper is organized as follows. In Sec. II, we describe the system model, the precoding problem for a \( K \)-user downlink channel with coarsely quantized transmit symbols and the QLP approaches. In Sec. III we present the MAGIQ precoding algorithm. Sec. V presents simulation results.

II. PRELIMINARIES

A. System Model

Consider the downlink of a multi-user MIMO channel with \( N \) transmit antennas and \( K \) UEs that each have a single antenna. A discrete time, frequency selective, baseband channel has a finite impulse response (FIR) filter between each pair of transmit and receive antennas. We collect the received signals \( y_k[t] \) of user \( k, \ k = 1, \ldots, K \), at time \( t \) into the \( K \)-dimensional column vector

\[
y[t] = \sum_{l=0}^{L-1} H[l] x[t-l] + z[t]
\]

where the noise \( z[t] \) is a circularly-symmetric, complex, Gaussian, random, column vector with a scaled identity covariance matrix, i.e., we have \( z \sim CN(0, \sigma^2I) \). The transmit column vector \( x[t] = [x_1[t], x_2[t], \ldots, x_N[t]]^T \) has entries taken from a discrete alphabet \( \mathcal{X} \) that has \( 2^b + 1 \) values where \( b \) bits encode the phase. More precisely, we choose

\[
\mathcal{X} = \{0, \exp(j 2\pi q/2^b)\}
\]

with \( q = 0, 1, \ldots, 2^b - 1 \). For the common definition of SNR we choose the transmit power to be unity and define the transmit signal-to-noise ratio (SNR) as \( \text{SNR} = 1/\sigma^2 \). However, MAGIQ does not output constant power precoding vectors, but with power at most unity. The channel impulse response matrix is

\[
H[l] = \begin{pmatrix}
h_{11}[l] & h_{12}[l] & \cdots & h_{1N}[l] \\
h_{21}[l] & h_{22}[l] & \cdots & h_{2N}[l] \\
\vdots & \vdots & \ddots & \vdots \\
h_{K1}[l] & h_{K2}[l] & \cdots & h_{KN}[l]
\end{pmatrix}
\]

for \( l = 0, 1, \ldots, L-1 \), where \( h_{kn}[l], l = 0, 1, \ldots, L-1, \) is the channel impulse response from the \( n \)-th antenna at the base station to the \( k \)-th UE. We study a Rayleigh fading frequency selective channel with a uniform power delay profile, i.e., we choose \( E\left[|h_{kn}[l]|^2\right] = 1/\ell \) for \( \ell = 1, \ldots, L \), where the individual \( h_{kn}[l] \sim CN(0, 1/\ell) \) are i.i.d. circularly-symmetric, complex Gaussian random variables. We further assume a block fading channel model, i.e., the channel realization remains constant for the duration of a frame, and the instantaneous realizations \( H[l], l = 0, 1, \ldots, L-1, \) are known at the transmitter.

We remark that the constraint (2) effectively permits per-symbol antenna selection. The idea of joint precoding and antenna selection also appeared in [16], but our algorithm selects antennas without an extra metric that enforces sparsity.

B. Flat Fading Channels

We first consider flat fading channels (\( L = 1 \)) for which the channel model (1) is

\[
y = H x + z.
\]

Let \( u_k \) be the noise-free complex symbol that we would like to generate at the \( k \)-th UE for \( k = 1, \ldots, K \). Let \( u \) be the column vector with these \( K \) symbols. Consider the precoding optimization problem

\[
\min_{x, \alpha} \|u - \alpha H x\|_2^2 + \alpha^2 K \sigma^2
\]

s.t. \( x \in \mathcal{X}^N \)

\[
\alpha > 0.
\]

The factor \( \alpha \) permits a trade-off between noise enhancement (consequence of the ZF solution for high SNR) and the received signal power which is more important at low SNR and maximized by the MF solution [17]. For a fixed value of \( \alpha \), the problem (5) represents a classic nonlinear integer least-squares problem [18].


C. Quantized Linear Precoding

QLP approximates the solution of (5) by \( x = Q(Pu) \), where \( P \in \mathbb{C}^{N \times K} \) is a precoding matrix and \( Q(\cdot) \) is a quantization function with range \( \mathcal{X}^N \) that operates entry-wise. Common choices for \( P \) are MF and ZF precoders, namely the respective

\[
P = \frac{1}{\alpha_{MF}}H^H \quad \alpha_{MF} = \text{tr}(HHH^H),
\]

\[
P = \frac{1}{\alpha_{ZF}}H^H(HHH^H)^{-1} \quad \alpha_{ZF} = \text{tr}((HHH^H)^{-1}).
\]

QLP is conceptually simple, but it does not perform well for higher order modulations and intermediate ranges of \( N \), as shown below.

III. MAGIQ PRECODING

A. Frequency-Flat Channels

Algorithm 1 outlines MAGIQ for frequency-flat channels (\( L = 1 \)). The inner loop evaluates the cost function

\[
F(x, \alpha) = \| u - \alpha Hx \|^2_2 + \alpha^2 K \sigma^2
\]

and selects (without replacement) an antenna index \( n \) and a precoded symbol \( x_n \) from \( \mathcal{X} \) for such that (8) is minimized. That is, each iteration selects the best coordinate in a greedy fashion. After forming a precoded vector \( x \), the factor \( \alpha \) is chosen to make the gradient of (5) with respect to \( \alpha \) become zero. The factor \( \alpha \) is thus a function of the channel, the target symbol \( u \), as well as the precoding vector \( x \).

The algorithm then performs iterations by updating \( x \) and \( \alpha \) until it reaches a stopping criterion or a maximum number of iterations. Simulations presented in Sec. V show that the algorithm converges to a good local minimum with a small number of iterations, and that the quality of the local minimum is at least as good as the minima obtained by the SQUID and ADMM algorithms proposed in [10], [11].

B. Frequency Selective Channels

We now consider frequency selective channels (\( L > 1 \)). The precoder puts out a string of column vectors \( x[1], \ldots, x[L] \), each with alphabet \( \mathcal{X}^N \), where \( T \) is at most the coherence time of the channel. In practice, \( T \) is chosen to balance the need for accurate channel state information (CSI) and quality-of-service (QoS) requirements.

Consider the \( K \)-dimensional column vectors \( u[1], \ldots, u[T] \) that we would like to generate at the \( K \) UEs. We state our optimization problem as follows.

\[
\begin{align*}
\min_{x[1], \ldots, x[T], \alpha} & \quad \sum_{t=1}^{T} \left\| u[t] - \alpha \sum_{l=0}^{L-1} H[l]x[t-l] \right\|^2_2 + \alpha^2 TK \sigma^2 \\
\text{s.t.} & \quad x[t] \in \mathcal{X}^N, \quad t = 1, \ldots, T \\
& \quad \alpha > 0.
\end{align*}
\]

The optimization problem (9) suggests a time domain approach rather than the frequency domain approach of [19]. The main advantage of the former approach is that one does not need to switch between domains to enforce the discrete alphabet constraint (2). The cost of each such switch is a length \( N \) discrete Fourier transform (DFT).

C. Precoding for OFDM

Fig. I shows how OFDM can be combined with MAGIQ. The frequency domain vectors \( \hat{U}[\cdot] \) corresponding to the \( K \) users are converted to the time domain vectors \( u[\cdot] \) by a length \( T \) DFT:

\[
\begin{align*}
\hat{U}[t] &= \left[ \hat{U}_1[t], \ldots, \hat{U}_K[t] \right]^T \\
u[t] &= \left[ u_1[t], \ldots, u_K[t] \right]^T
\end{align*}
\]

For the simulations, we generated the frequency domain symbols \( \hat{U}_k[t], \quad k = 1, \ldots, K, \quad t = 1, \ldots, T, \) uniformly from the constellations QPSK, 16-QAM, and 64-QAM. Each UE performs single user OFDM demodulation followed by a hard or soft decision detection. MAGIQ is flexible with respect to shaping constraints, constellation size, number of users, number of sub-carriers, and channel models.

D. Algorithm Description

For frequency selective channels, the vector \( x[t] \) of transmit symbols at time \( t \) should be chosen as a function of the transmit symbols at other time instances due to the channel memory, i.e., \( x[t] \) influences the channel output at times \( t + 1, t + 2, \ldots, t + L - 1 \). However, a joint optimization over strings of length \( T \) seems difficult because of the exponential increase in the size of the constraint space.

We approach the problem by splitting the joint optimization into a set of sub-problems with reduced complexity, see Algorithm 2. This approach is related to coordinate descent algorithms that have been successful in, e.g., compressed

---

**Algorithm 1: MAGIQ for frequency-flat channels**

1. **Inputs:** \( u, H, S = \{1, \ldots, N\}, \text{err}_{\min} \)
2. **Initialize:** \( x = 0, \alpha = 1 \)
3. **for** \( \text{iter} = 1 : I \) **do**
   4. \( \text{loopIdx} = 1 \)
   5. **while** \( (\text{err}_{\text{loopIdx}} > \text{err}_{\min}) \vee (\text{err}_{\text{loopIdx}} < \text{err}_{\text{loopIdx}-1}) \) \( \vee (\text{loopIdx} \leq N) \) **do**
   6. \( (x_n^*, n^*) = \arg\min_{x_n \in X, \alpha \in S} F(x, \alpha) \)
   7. \( (x_1, \ldots, x_n^*, \ldots, x_N)^T = (x_1, \ldots, x_n^*, \ldots, x_N)^T \)
   8. \( S \leftarrow S \setminus \{n^*\} \)
   9. \( \text{loopIdx} \leftarrow \text{loopIdx} + 1 \)
   10. \( \text{err}_{\text{loopIdx}} = \| u - \alpha H x_\alpha \|^2_2 + \alpha^2 K \sigma^2 \)
11. **end**
12. **end**
13. \( \alpha = \text{Re}(u^H x_\alpha) \)
14. **Output** \( x_\alpha \)
Algorithm 2: MAGIQ for frequency selective channels

1: Input: $H[t]$, $u[t]$, $t = 1, \ldots, T$
2: Initialization: $x[0] = 0$, $t = 1, \ldots, T$
3: for $t = 1 : T$
4:     $x[t] = \arg \min \bar{x} \sum_{l=0}^{L-1} \log \| x[t-l] \|^2 + \alpha^2 T K \sigma^2$
5:     $\alpha = \frac{\sum_{l=0}^{L-1} \log \| x[t-l] \|^2}{\sum_{l=0}^{L-1} \| x[t-l] \|^2} + \alpha^2 T K \sigma^2$
6: end
7: Output: $x[t]$, $t = 1, \ldots, T$, $\alpha$

sensing [20]. We perform a two-fold coordinate-wise splitting of the problem stated in (9). First, we solve the precoding problem for one time coordinate $t$ at a time, starting at time 1 and ending at time $T$. Under this formulation, we replace the cost function [8] with:

$$G(x[1], \ldots, x[t-1], x[t], x[t+1], \ldots, x[T])$$

$$= \sum_{t=1}^{T} \left\| u[t] - \alpha \sum_{l=0}^{L-1} H[l] x[t-l] \right\|^2 + \alpha^2 T K \sigma^2$$

(11)

where

$$\bar{u}[t] = u[t] - \alpha \sum_{l=1}^{L-1} H[l] x[t-l].$$

The last line in (11) has the same form as the cost function in [8]. We again split this problem in a coordinate-wise fashion and solve it with Algorithm 1. Algorithm 2 then does multiple iterations over the frame until prescribed convergence criteria are met.

Simulations show that the performance over frequency selective channels is very close to the one achieved by the best known precoders over frequency flat channels, with as little as 5 iterations.

E. Information Rates

We use the mismatched decoding framework [13], [14] to calculate a lower bound on information rates. Consider a channel $p_{Y|X}(\cdot|x)$ with input $X \in \mathcal{X}$ and channel output $Y$. A lower bound to the mutual information

$$I(X; Y) = \mathbb{E} \log_2 \left( \frac{p_{Y|X}(Y|X)}{\sum_{a \in \mathcal{X}} p_{Y|X}(Y|a) P_X(a)} \right)$$

is given by the generalized mutual information (GMI)

$$\max_{s \geq 0} \mathbb{E} \log_2 \left( \frac{q_{Y|X}(Y|X)^s}{\sum_{a \in \mathcal{X}} q_{Y|X}(Y|a)^s P_X(a)} \right).$$

The lower bound is clearly tight if the true channel $p_{Y|X}$ is the same as the auxiliary channel $q_{Y|X}$, but often $p_{Y|X}$ is difficult to characterize, e.g., due to interference, and hence we resort to simpler channels $q_{Y|X}$.

To evaluate the performance of different precoding strategies, we proceed as follows.

1) Perform Monte Carlo simulations of the selected precoding strategy and collect $S$ pairs of input and output samples $(x_i^T, y_i^T)$ for each of the $K$ users.

2) Choose a Gaussian auxiliary channel $Y = h \cdot X + Z$ with channel law

$$q_{Y|X}(y|x; h, \sigma^2_q) = \frac{1}{\pi \sigma^2_q} e^{-\frac{|y-h \cdot x|^2}{\sigma^2_q}}. \quad (15)$$

The parameters $h \in \mathbb{C}$ and $\sigma^2_q \in \mathbb{R}^+$ are obtained by maximum-likelihood (ML) estimation from the sample sequences. A Gaussian auxiliary channel is reasonable, and it is right choice when $N \to \infty$ by the weak law of large numbers.

3) Estimate the GMI as the empirical mean

$$R_s \approx \max_{s \geq 0} \frac{1}{S} \sum_{s=1}^{S} \sum_{a \in \mathcal{X}} \log_2 \left( \frac{q_{Y|X}(y_i|x_i)^s}{\sum_{a \in \mathcal{X}} q_{Y|X}(y_i|a)^s P_X(a)} \right). \quad (16)$$

IV. COMPLEXITY ANALYSIS

This section studies the complexity of MAGIQ over flat fading channels and compares it with SQUID [10, Sec. IV] and ADMM [11]. We start with a worst case analysis of Algorithm 1. Suppose the while loop is evaluated the maximum number $N$ times. The first execution of the while loop evaluates [8] for each coordinate $n$ and each symbol from $X$. This means that the product $H x$ must be performed $|X|$ times, resulting in $N K |X|$ multiplications. The norm evaluation requires another $N K |X|$ multiplications and $N K^2 |X|$ additions. Finally, evaluating the minimum requires $N |X|$ comparisons. We remark that these terms do not change inside the while loop. The further loop passes are thus dominated by additions and the minimum operation, giving a total worst case of $\sum_{i=1}^{N-1} \log_2 (|N - i| |X|)$ comparisons. The worst case number of multiplications is thus $2 N K |X|$ with a memory with $N K |X|$ entries.

The worst case complexity seems to be large, but the average complexity of MAGIQ can be substantially reduced. For example, one can initialize the algorithm with a good starting point. The results presented below are based on initializing MAGIQ with the QLP solution of the MF, which adds only $N K$ multiplications. One can further reduce the number of coordinate updates by updating a coordinate only if the cost reduction is significant. The significance level can be translated into a threshold for the norm update.

For example, Fig. 2 shows the empirical cumulative distribution function (CDF) of the total number of iterations of MAGIQ with the aforementioned fine tuning for a total number of $I = 6$ iterations for 16-QAM, over the whole SNR range $-10$ dB to $14$ dB. The worst case number of loop passes is therefore $N \cdot I = 128 \cdot 6 = 768$, but the average number of loop passes is an order of magnitude lower.
Fig. 2. Empirical CDF of the number of iterations with $N = 128$ and $I = 6$. The average CDF in thick black curve.

Fig. 3. GMI rates for $N = 128$ and $K = 16$. than the worst case. Fig. 3 shows the GMI for 64-QAM for MAGIQ with and without the thresholding operation. We chose this particular example because it exhibits the largest gap in performance between the two options. The reduction in computational complexity comes at a price. However, even at the aggressive levels we have set it in this example, there is only a 0.5 dB gap at a spectral efficiency of 5.4 bpcu, corresponding to a code rate of 0.9, which is a reasonable operating point for a coded system. However, in the range 3 bpcu to 4.5 bpcu (corresponding to code rates of 0.5 to 0.75) the gap is insignificant.

The ADMM algorithm, in its least complex implementation (denoted as IDE2 in [1]) has a complexity of $4NK + 3N$ multiplications for the first iteration, and another $2NK + N$ multiplications for each new iteration. The reason is that initially computed quantities can be cached and then used as memory calls in later iterations. For SQUID there are $NK^2 + K^3 + 3NK$ multiplications in the pre-processing phase. After pre-processing, there are $N^2 + 2N$ multiplications per iteration. For the case where the precoding factor $\alpha$ is updated we define for ADMM and SQUID $I$ outer iterations which correspond to number of times $\alpha$ is updated and $J$ inner iterations for the computation of $x$ with a fixed value of $\alpha$.

Unlike MAGIQ that has a variable number of loop passes that depends on stop criteria, ADMM and SQUID have fixed complexity once the number of iterations is fixed. The per-sample complexity of MAGIQ for frequency selective channels is the same as that of MAGIQ for flat fading channels, but the metric is more involved due to convolutions. The metric computation for each of the $NK|\mathcal{X}|$ terms requires $L$ multiplications for the convolution. This results in a total of $NK|\mathcal{X}|L$ multiplications to retrieve the channel outputs. For the norm again $NK|\mathcal{X}|L$ multiplications are required. This means that the per-sample complexity of MAGIQ is increased by a factor equal to the channel length for the first iteration. In subsequent iterations the already computed convolutions can be reused by caching.

V. NUMERICAL RESULTS

We first study the frequency-flat case. We compare the performance of different precoding schemes by means of their GMI for massive MIMO setting with $K = 16$ UEs and $N = 128$ antennas at the base station As a reference, we show the ZF solution with infinite precision ADCs, i.e., $Q(\cdot)$ in Sec. II-C is the identity function. We also include the performance of the MF and ZF QLP schemes, and the performance of the SQUID and ADMM algorithms. For the simulations, MAGIQ uses $I = 3$, SQUID uses $I = 50$ iterations, and ADMM uses $I \cdot J = 10 \cdot 10 = 100$ iterations. Going beyond these numbers did not result in any further improvements.

We further improve SQUID by applying an iterative update for $\alpha$ (SQUID-\(\alpha\)) with $J = 8$ outer iterations. For MAGIQ and ADMM, we use one bit per real dimension, i.e., we set $b = 2$ in [2]. The precoding solution for SQUID uses phase modulation only, i.e., there is no antenna selection.

We display the achievable rates for QPSK, 16-QAM and 64-QAM in Figs. 4, 5 and 6 respectively. For QPSK, SQUID, ADMM, and MAGIQ show similar performance over the whole SNR range. The QLP approaches perform poorly at high SNR, and the MF solution even saturates below the maximum rate of 2 bpcu. For 16-QAM and 64-QAM, QLP performs even worse. Both MAGIQ and ADMM show similar performance for higher order modulation formats. For 16-QAM, SQUID with adaptive $\alpha$ is competitive with MAGIQ and ADMM, but it does not reach the maximum rate at high SNR for 64-QAM.

Fig. 7 compares the performance of MAGIQ with $K = 16$ users and 16-QAM to ZF with infinite precision for different numbers $N$ of antennas. Observe that the MAGIQ curve lies
### TABLE I
COMPUTATIONAL COMPLEXITY IN MULTIPLICATIONS AND COMPARISONS

| Algorithm            | Total No. of multiplications                      | Total No. of comparisons |
|----------------------|--------------------------------------------------|--------------------------|
| SQUID                | $I \cdot (NK^2 + K^3 + 3NK) + I \cdot J \cdot (N^2 + 2N)$ | $I \cdot J \cdot (N + 1) \log(N)$ |
| SQUID Num. example Fig.5 | $7 \times 10^6$                              | $3.6 \times 10^7$         |
| ADMM                 | $I \cdot (4NK + 3N) + I \cdot J \cdot (2NK + N)$ | $I \cdot J \cdot \log(N)$ |
| ADMM Num. example Fig.5 | $5.08 \times 10^7$                            | $700$                    |
| MAGIQ (worst case)   | $I \cdot (2NK|X|) + NK + J \cdot 0$              | $I \cdot J \cdot \sum_{n=1}^{N} \log((N - n)|X|)$ |
| MAGIQ Num. example Fig.5 | $6.34 \times 10^4$                            | $1.07 \times 10^7$       |
| MAGIQ (average, $I = 3, J_{average} = 62$) Fig.2,3 | $6.34 \times 10^4$ | $568$ |

![Graph](image1)

**Fig. 4.** GMI rates for QPSK, $N = 128$, and $K = 16$.

![Graph](image2)

**Fig. 5.** GMI rates for 16-QAM, $N = 128$, and $K = 16$.

![Graph](image3)

**Fig. 6.** GMI rates for 64-QAM, $N = 128$, and $K = 16$.

![Graph](image4)

**Fig. 7.** GMI rates with infinite resolution ZF and different numbers of antennas for $K = 16$ and 16-QAM.

between the ZF performance for $N = 74$ and $N = 94$ antennas. A modest 1.5 to 1.7-fold increase of the number of antennas thus compensates for quantizing with $b = 2$. 

![Graph](image5)
The frequency selective case is considered in Fig. 8 for a channel with $L = 13$ taps, $K = 12$ users, and OFDM symbols with $T = 64$. We evaluate the symbol error rate (SER) for 16-QAM and 64-QAM and different numbers $N$ of antennas. We observe that 2-3 additional antennas suffice to compensate for the loss induced by the frequency selective channels. This suggests that MAGIQ is well-suited for orthogonal frequency-division multiplexing (OFDM) based systems.

REFERENCES

[1] T. Marzetta, “Noncooperative Cellular Wireless with Unlimited Numbers of Base Station Antennas,” IEEE Trans. Wireless Commun., vol. 9, no. 11, pp. 3590–3600, Nov. 2010.

[2] H. Q. Ngo, E. Larsson, and T. Marzetta, “Energy and Spectral Efficiency of Very Large Multiuser MIMO Systems,” IEEE Trans. Commun., vol. 61, no. 4, pp. 1436–1449, Apr. 2013.

[3] A. F. Molisch, V. V. Ratnam, S. Han, Z. Li, S. L. H. Nguyen, L. Li, and K. Haneda, “Hybrid beamforming for massive MIMO: A survey,” IEEE Commun. Mag., vol. 55, no. 9, pp. 134–141, 2017.

[4] C. Risi, D. Persson, and E. G. Larsson, “Massive MIMO with 1-bit ADC,” arXiv:1404.7736 [cs, math], Apr. 2014.

[5] C. K. Wen, C. J. Wang, S. Jin, K. K. Wong, and P. Ting, “Bayes-Optimal Joint Channel-and-Data Estimation for Massive MIMO With Low-Resolution ADCs,” IEEE Trans. Signal Process., vol. 64, no. 10, pp. 2541–2556, May 2016.

[6] S. Jacobsson, G. Durisi, M. Coldrey, U. Gustavsson, and C. Studer, “Throughput Analysis of Massive MIMO Uplink With Low-Resolution ADCs,” IEEE Trans. Wireless Commun., vol. 16, no. 6, pp. 4038–4051, Jun. 2017.

[7] A. K. Saxena, I. Fijalkow, and A. L. Swindlehurst, “Analysis of One-Bit Quantized Precoding for the Multituser Massive MIMO Downlink,” IEEE Trans. Signal Process., vol. 65, no. 17, pp. 4624–4634, Sep. 2017.

[8] H. Jedda, J. A. Nossek, and S. Shamai, “Information rates and error exponents of compound channels with application to antipodal signaling in a fading environment,” AEU: Archiv für Elektronik und Übertragungstechnik, vol. 47, no. 4, pp. 228–239, 1993.

[9] A. Ganti, A. Lapidoth, and I. E. Telatar, “Mismatched decoding revisited: General alphabets, channels with memory, and the wide-band limit,” IEEE Trans. Inf. Theory, vol. 46, no. 7, pp. 2315–2328, Nov. 2000.

[10] T. Marzetta, “Noncooperative Cellular Wireless with Unlimited Numbers of Base Station Antennas,” IEEE Trans. Wireless Commun., vol. 9, no. 11, pp. 3590–3600, Nov. 2010.

[11] S. Jacobsson, G. Durisi, M. Coldrey, and C. Studer, “Linear Precoding with Low-Resolution DACs for Massive MU-MIMO-OFDM Downlink,” arXiv:1709.04846 [cs, math], Sep. 2017.

[12] G. Kaplan and S. Shamai, “Information rates and error exponents of compound channels with application to antipodal signaling in a fading environment,” AEU: Archiv für Elektronik und Übertragungstechnik, vol. 47, no. 4, pp. 228–239, 1993.

[13] A. Ganti, A. Lapidoth, and I. E. Telatar, “Mismatched decoding revisited: General alphabets, channels with memory, and the wide-band limit,” IEEE Trans. Inf. Theory, vol. 46, no. 7, pp. 2315–2328, Nov. 2000.

[14] S. Jacobsson, O. Castaño, C. Jeon, G. Durisi, and C. Studer, “Non-linear Phase-Quantized Constant-Envelope Precoding for Massive MU-MIMO-OFDM,” ArXiv e-prints, Oct. 2017.

[15] T. Tong Wu and K. Lange, “Coordinate descent algorithms for lasso penalized regression,” ArXiv e-prints, Mar. 2008.