Driven low density granular mixtures.

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Abstract

We study the steady state properties of a 2D granular mixture in the presence of energy driving by employing simple analytical estimates and Direct Simulation Monte Carlo. We adopt two different driving mechanisms: a) a homogeneous heat bath with friction and b) a vibrating boundary (thermal or harmonic) in the presence of gravity. The main findings are: the appearance of two different granular temperatures, one for each species; the existence of overpopulated tails in the velocity distribution functions and of non trivial spatial correlations indicating the spontaneous formation of cluster aggregates. In the case of a fluid subject to gravity and to a vibrating boundary, both densities and temperatures display non uniform profiles along the direction normal to the wall, in particular the temperature profiles are different for the two species while the temperature ratio is almost constant with the height. Finally, we obtained the velocity distributions at different heights and verified the non gaussianity of the resulting distributions.

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I. INTRODUCTION

Granular materials present a rich and intriguing phenomenology, which has attracted the interest of the scientific community since the nineteenth century [1]. However, in spite of its recent progress the theoretical study of granular gases, i.e. of fluidized granular particles, is certainly less advanced than that concerning ordinary molecular fluids. The reason for this state of affairs is the presence of dissipation due to inelastic collisions and of friction with the surroundings, which prevents these system to reach thermodynamic equilibrium. In fact, in order to render stationary a granular system one needs to inject continuously energy into the system. This can be done, for instance by shaking or vibrating the grains.

In the present paper we illustrate the results of a numerical investigation concerning the properties of a two component granular mixture, modeled, following an established tradition, by inelastic hard spheres (IHS) with different masses, restitution coefficients, radii, and subject to different forms of external drive. The physical motivation for our study stems from the fact that in nature most granular materials are polydisperse from the point of view of their sizes and/or of their physical and mechanical properties. The theoretical study of granular mixtures has attracted so far the attention of several researchers [2, 3, 4, 5]. These studies comprehend both freely cooling and uniformly heated granular mixtures and have been performed almost contemporaneously with laboratory experiments [6, 7]. The most striking outcome is the lack of energy equipartition, i.e. the presence of two different kinetic temperatures, one for each species.

In the present paper our interest will be concentrated on the stationary state obtained by applying an energy feeding mechanism, represented by a uniform stochastic driving force or by a vibrating wall. In the first case the work of Barrat and Trizac [4] and Garzó et al. [5] provides a simple theoretical account, based on the Boltzmann equation approach and on the Direct Simulation Monte Carlo method (DSMC) [8, 9], of the lack of energy equipartition.

The present work differs from the recently appeared papers dealing with granular mixtures in several aspects:

- the heat bath, which drives the system, is at finite temperature, since we have introduced a finite friction of the particles with the surroundings and we have considered the possibility of having spatially inhomogeneous states;
• in addition, we considered a second situation in which the particles subject to a vertical gravitational field receive energy anisotropically from the bottom vibrating wall of the container;

• in both cases we provide information about the presence of inhomogeneities in the system, e.g. density clustering and non-uniform density and temperature profiles.

In Sec. II we present the model fluid and two different mechanisms of energy supply. In sect III we discuss the first sub-model, the one with the heat bath, and obtain by means of an approximate analytic method an estimate of the partial temperature of each component. Subsequently we study the same sub-model with a DSMC algorithm. In sect. IV we study numerically the second sub-model (the one with gravity and vibrating wall) by means of DSMC simulations. Finally in the last section, V, we present our conclusions.

II. DEFINITION OF THE MODELS

We shall consider a dilute inelastic gas constituted of $N_1$ particles of mass $m_1$ and $N_2$ particles of mass $m_2$ subject to some kind of external driving (this will be specified in the following). We suppose that the interactions between the grains can be described by the smooth inelastic hard sphere model (IHS) \cite{10}, thus we specify only the radius of the spheres, their masses and the fraction of the kinetic energy dissipated at each collision. This can be done by defining three different restitution coefficients $\alpha_{ij}$, i.e. $\alpha_{11}, \alpha_{22}$, and $\alpha_{12} = \alpha_{21}$, which account for normal dissipation in collisions among particles of type $i$ and $j$. No internal degrees of freedom (e.g. rotations) are included.

One can describe the velocity changes induced by the instantaneous inelastic collisions of smooth disks labeled 1 and 2 of diameter $\sigma_1$ and $\sigma_2$ by the following equations:

\begin{align*}
  v_1' &= v_1 - \frac{1 + \alpha_{\kappa_1\kappa_2} \frac{m_{\kappa_2}}{m_{\kappa_1} + m_{\kappa_2}}} {2} \left( (v_1 - v_2) \cdot \hat{n} \right) \hat{n} \quad (1a) \\
  v_2' &= v_2 + \frac{1 + \alpha_{\kappa_1\kappa_2} \frac{m_{\kappa_1}}{m_{\kappa_1} + m_{\kappa_2}}} {2} \left( (v_1 - v_2) \cdot \hat{n} \right) \hat{n} \quad (1b)
\end{align*}

where $\hat{n} = 2(x_1 - x_2)/(\sigma_{\kappa_1} + \sigma_{\kappa_2})$ is the unit vector along the line of centers $x_1$ and $x_2$ of the colliding disks at contact and $\kappa_1, \kappa_2$ are the species (1 or 2) to whom particles
1 and 2 belong. An elementary collision conserves the total momentum and reduces the relative kinetic energy by an amount proportional to \((1 - \alpha_{\kappa_1 \kappa_2})/4\). The collision rule we have adopted excludes the presence of tangential forces, and hence the rotational degrees of freedom do not contribute to the description of the dynamics.

Since the particles suffer mutual collisions and lose kinetic energy, in order to achieve a steady state, one needs to supply from the exterior some energy. The energy source has been modeled in two different fashions.

In the first sub-model we have assumed that the particles, besides suffering mutual collisions experience a uniform stochastic force and a viscous damping\(^{11, 12}\). The presence of the frictional, velocity-dependent term in addition to the random forcing, not only is motivated by the idea of preventing the energy of a driven elastic system \((\alpha_{\kappa_1 \kappa_2} \rightarrow 1)\), to increase indefinitely, but also mimics the presence of friction of the particles with the container. Moreover a fluctuation dissipation relation is assumed between the viscous force and the intensity of the noise. Even in extended systems with small inelasticity the absence of friction may cause some problems of stability\(^{15}\). We shall be mainly interested in the stationary situation determined by the balance between the energy feeding mechanism and the dissipative forces due both to the inelasticity of the collisions and to the velocity-dependent frictional force.

In the second sub-model the grains are constrained to move on a frictionless inclined plane and the bottom boundary vibrates (as a thermal\(^{13}\) or deterministic\(^{14}\) oscillating wall) giving energy only to the particles bouncing on it. Periodic boundary conditions are assumed laterally.

Since we consider throughout only sufficiently low density systems successive binary collisions are effectively uncorrelated and Boltzmann equation can be used to describe the non equilibrium dynamics. In fact, by assuming the validity of the Boltzmann molecular chaos hypothesis, introduced to treat the collision term, it is straightforward to derive the governing equations for the probability density distribution function of each species. Moreover in this situation we are allowed to perform Direct Simulation Monte Carlo to integrate numerically the inhomogeneous Boltzmann equation of the sub-models.
III. UNIFORMLY HEATED SYSTEM

In order to see the effect of the heat bath let us consider the system in the absence of collisions. In this case, the evolution of the velocity of each particle is described by an Ornstein-Uhlenbeck process. If we require that the two components must reach the same granular temperature in the limit of vanishing inelasticity we have two different possibilities to fix the heat bath parameters:

\[ \partial_t x_i(t) = v_i(t) \]  

\[ m_i \partial_t v_i(t) = -\gamma v_i(t) + \sqrt{2\gamma T_b} \xi_i(t) \]  

\[ m_i \partial_t v_i(t) = -m_i \beta v_i(t) + \sqrt{2m_i \beta T_b} \xi_i(t) \]

where \( i = 1, 2 \) and \( T_b \) is the heat bath temperature and \( \xi(t) \) is a Gaussian noise with the following properties:

\[ < \xi_i(t) > = 0 \]  

\[ < \xi_i(t_1)\xi_j(t_2) > = \delta(t_1 - t_2)\delta_{ij} \]

The associated Fokker-Planck equations for the two cases are respectively:

\[ \partial_t f_i(r, v, t) = \frac{\gamma}{m_i} \nabla_v (v f_i(r, v, t)) + \frac{\gamma T_b}{m_i^2} \nabla^2_v f_i(r, v, t) + v \nabla_r f_i(r, v, t) \]  

\[ \partial_t f_i(r, v, t) = \beta \nabla_v (v f_i(r, v, t)) + \frac{\beta T_b}{m_i} \nabla^2_v f_i(r, v, t) + v \nabla_r f_i(r, v, t) \]

A. Spatially Uniform solutions

When we take into account collisions among particles equations (5) become two coupled Boltzmann equations modified by the presence of a diffusion term due to the thermal noise. We shall first consider a spatially homogeneous case in order to derive the temperature of each species in the homogeneous stationary state. This represents a sort of mean field
approximation. In fact, in the original Boltzmann equations the collisions are considered to occur only between particles at the same spatial location, whereas here these can occur between arbitrary pairs of particles regardless their spatial separation. In the present subsection we compute the second moments, \( < v_i^2 > \), of the distribution functions in order to determine the partial temperatures and their ratio. Although the method of derivation of the equations for the partial temperatures is not original, we present it in order to render the paper self-contained and because it shows the differences between the particular heat bath we employed and those chosen by other authors [4]. First, indicating by \( n_i = N_i/V \) the partial density of species \( i \), we notice that both eqs. (5) possess the same equilibrium solution:

\[
  f_i(v) = n_i \left( \frac{m_i}{2\pi T_b} \right)^{\frac{d}{2}} e^{-\frac{m_i v^2}{2T_b}}
\]

although the relaxation properties are different. Only upon adding the inelastic collision term the two species display different temperatures. The resulting Boltzmann equation for a granular mixture is [2, 3, 4]:

\[
  \partial_t f_i(v_1; t) = \sum_j J_{ij} [f_j f_i] + \frac{\xi_0^2}{2} \nabla^2 f_i + \beta_i \nabla_v \cdot (v_1 f_i)
\]

where we have used a compact notation to represent the two different choices of heat bath:

- **Case 1**

  \[
  \xi_{0i}^2 \rightarrow \frac{2\gamma T_b}{m_i^2}, \quad \beta_i \rightarrow \gamma m_i
  \]

- **Case 2**

  \[
  \xi_{0i}^2 \rightarrow \frac{2\beta T_b}{m_i}, \quad \beta_i \rightarrow \beta
  \]
and $J_{ij}[v_1|f_i, f_j]$ is the collision integral:

$$J_{ij}[v_1|f_i, f_j] \equiv \sigma_{ij}^2 \int dv_2 \int d\hat{\sigma} \Theta(\hat{\sigma} \cdot g_{12})(\hat{\sigma} \cdot g_{12})[\alpha_{ij}^2 f_i(v_1') f_j(v_2') - f_i(v_1) f_j(v_2)] \quad (10)$$

The primed velocities are pre-collisional states, which can be obtained by inverting eqs. (1).

Due to the presence of the heat bath terms the system reaches asymptotically a steady state, characterized by time independent pdf’s. By imposing the vanishing of the time derivatives we obtain:

$$\sum_j J_{ij}[v_1|f_i, f_j] + \xi_{0i}^2 \nabla^2 v_i f_i + \beta_i \nabla v_i \cdot (v_1 f_i) = 0 \quad (11)$$

or after integrating over $v_1$:

$$\sum_j \int dv_1 v_1^2 J_{ij}[v_1|f_i, f_j] + \frac{\xi_{0i}^2}{2} \int dv_1 v_1^2 \nabla^2 v_i f_i + \beta_i \int dv_1 v_1^2 \nabla v_i \cdot (v_1 f_i) = 0 \quad (12)$$

After simplifying the second and the third integral by integration by parts and using the normalization property $\int f_i dv_i = n_i$ we find:

$$\sum_j \int dv_1 v_1^2 J_{ij}[v_1|f_i, f_j] + n_i d\xi_{0i}^2 - 2\beta_i \int dv_1 v_1^2 f_i(v) = 0 \quad (13)$$

By defining the partial temperature:

$$n_i T_i = \frac{1}{d} \int dv_1 m_i v_1^2 f_i \quad (14)$$

we can rewrite eq. (13) in terms of the temperatures:

$$T_i = \frac{m_i}{2d\beta_i} \left( \frac{1}{n_i} \sum_j \int dv_1 v_1^2 J_{ij}[v_1|f_i, f_j] + d\xi_{0i}^2 \right) \quad (15)$$

Eq. (13) determines the partial temperatures once the $f_i$ are known. In practice one can obtain an estimate of $T_i$ by substituting two Maxwell distributions:

$$f_i(v) = n_i \left( \frac{m_i}{2\pi T_i} \right)^{\frac{d}{2}} e^{-\frac{m_i v^2}{2 T_i}}$$
and performing the remaining integrals (see [2, 4]) one gets:

\[
\frac{d\Gamma(d/2)}{m_i\pi^{(d-1)/2}}2\beta_i(T_b - T_i) = \sigma_{ii}^{d-1}m_i\frac{2(1 - \alpha_{ii}^2)}{m_i^{3/2}}T_i^{3/2} \\
+ \sigma_{ij}^{d-1}n_j\mu_{ji}\left[\mu_{ji}(1 - \alpha_{ij}^2)\left(\frac{2T_i}{m_i} + \frac{2T_j}{m_j}\right) + 4(1 + \alpha_{ij})\frac{T_i - T_j}{m_1 + m_2}\right] \left(\frac{2T_i}{m_i} + \frac{2T_j}{m_j}\right)^{1/2}
\]

(16)

where \(\mu_{ij} = m_i/(m_i + m_j)\).

By solving numerically the nonlinear system of eqs. (16) one obtains the steady values of the partial temperatures in the spatially homogeneous situation.

In figs. 1 and 2 we report the temperature ratio \(T_1/T_2\) as a function of a common restitution coefficient \(\alpha\), having chosen equal coefficients \(\alpha_{11} = \alpha_{22} = \alpha_{12} = \alpha\). Assuming identical concentrations, and a varying the mass ratio \(m_1/m_2\), we studied the cases 1 and 2.

![Diagram showing the temperature ratio \(T_1/T_2\) vs. \(\alpha\) for different mass ratios.](image)

**FIG. 1:** Granular temperature ratio \(T_1/T_2\) vs. \(\alpha\) obtained with the heat bath, case 1, homogeneous, using \(T_b = 1\), \(\gamma = 0.1\) and different mass ratios.

We notice that with the second recipe (case 2) the temperature ratio is an increasing function of the mass ratio \(m_1/m_2\): this is exactly the opposite of what happens for the case 1. The experimental observation [6] suggest that the trend of case 2 is the right one. In other words, in the case 2 both the friction term and the power supplied are proportional to the mass of the two species and such a property is probably true also in the experiments. This is why we shall use the recipe of case 2 in the following DSMC simulations.
FIG. 2: Granular temperature ratio $T_1/T_2$ vs. $\alpha$ using the second recipe for the heat bath, case 2, homogeneous, with $T_b = 1$, $\beta = 0.1$ and various mass ratios.

B. Non uniform solutions

In the present section we illustrate the results obtained simulating the system with the heat bath (with recipe 2, i.e. equation (3b)) by the so called Direct Simulation Monte Carlo according to the implementation described in [16].

At every time step of length $\Delta t$ each particle is selected to collide with a probability $p_c = \Delta t/\tau_c$ (where $\tau_c$ is an \textit{a priori} fixed mean free time established consistently with the mean free path and mean squared velocity) and seeks its collision partner among the other particles in a neighborhood of radius $r_B$, choosing it randomly with a probability proportional to their relative velocities. Moreover in this approximation the diameter $\sigma$ is no more explicitly relevant, but it is directly related to the choices of $p_c$ and $r_B$ in a non trivial way: in fact the Bird algorithm allows the particles to pass through each other, so that a rigorous diameter cannot be defined or simply estimated as a function of $p_c$ and $r_B$. In this section, to indicate the degree of damping, we give the time $\tau_b = 1/\beta$ instead of $\beta$. This is useful to appreciate the ratio between the mean collision time $\tau_c$ and the mean relaxation time due to the bath which indeed is $\tau_b$.

In the present section we chose $N_1 = N_2 = 500$ and $T_b = 1$, and equal restitution
FIG. 3: Granular temperature ratios $T_1/T_2$ vs. mass ratio $m_2/m_1$ for a binary mixture (DSMC simulation) with different values of $\alpha$, $N_1 = N_2 = 500$, $L^2 = 1000$, $T_b = 1$, $\tau_b = 10$, $\tau_c = 0.16$, and recipe 2

FIG. 4: Granular temperature ratios $T_1/T_2$ vs. restitution coefficient $\alpha$ for a binary mixture (DSMC simulation) with different values of $m_1/m_2$, $N_1 = N_2 = 500$, $L^2 = 1000$, $T_b = 1$, $\tau_b = 10$, $\tau_c = 0.16$, and recipe 2
coefficients for all collisions and $\tau_c = 0.16$. As illustrated in figures 3 and 4, the two components display different granular temperatures in agreement with the analytical predictions of the homogeneous Boltzmann equations. We also comment that the ratio $T_1/T_2$ is very sensitive to the mass ratio and much less to differences in the restitution coefficients and concentrations. This is in agreement with experimental observations.

At a finer level of description we consider the rescaled velocity pdf for different values of the inelasticity parameter and different mass ratios $m_2/m_1 = 2$. One sees that not only the deviations from the Gaussian shape become more and more pronounced as we increase the inelasticity parameter $(1 - \alpha)$, but also the shape of the two distributions differ appreciably in the tails even after velocity rescaling to make the two pdf’s have the same variance. Such a property is similar to the one already reported in [6]. One can also observe that the rescaled pdf of the lighter species has slightly broader tails. The mass ratio also controls the deviations from the Gaussianity of the velocity pdf’s. Whereas it is known that the departure from the Maxwell-Boltzmann statistics is triggered by the inelasticity of the collisions, the larger the inelasticity the stronger the deviation, to the best of our knowledge this is the first numerical evidence of the phenomenon, predicted within a Maxwell model in ref. [3]. Comparing figs. 5 and 6 one sees that the mass asymmetry enhances the non gaussianity of the pdf.

We have also studied the limits of low and high $\tau_b$, in figure 7, to show how the velocity distributions change. For values of the characteristic time of the heat bath, $\tau_b$, comparable with the collision time, $\tau_c$, the dynamics is essentially controlled by the stochastic acceleration term. This fact renders the two partial temperatures very close and makes the velocity distributions nearly Maxwellian. As $\tau_b$ increases we have observed that the energy dissipation due to the inelasticity makes the temperatures of the two species different. Moreover, the temperature ratio displays power law decreasing trend as a function of $\tau_b$ whose strength depends on the mass ratio (see figure 8).

In order to obtain information about the spatial structure of the mixture we have performed an analysis of the following correlation function, already introduced in the context of granular media by [11, 12, 17]:

$$C_{\alpha\beta}(r) = \frac{1}{N(N-1)} \sum_{i \neq j} \Theta(r - |x_i^{\alpha} - x_j^{\beta}|)$$

(17)
For a spatially homogeneous system we expect that $C_{\alpha \beta}(r) \approx r^{d_2}$ with $d_2 = d$ the dimension of the embedding space. However, in the cases we have studied, we found that $d_2 < d$ (see fig 9), a signature that the system tends to clusterize. The larger the value of $\tau_b$ the stronger the deviation.

IV. SYSTEM WITH GRAVITY AND HEATED FROM BELOW

We turn, now, to illustrate the results relative to an inelastic mixture subject to gravity and confined to a vertical plane of dimensions $L_x \times L_y$. In the horizontal direction $x$ we assumed periodic boundary conditions. Vertically the particles are confined by walls. Energy is supplied by the bottom wall vibrating stochastically or periodically according to the method employed in [16] for a one component system.

The vibration can have either a periodic character (as in ref. [18]) or a stochastic behavior with thermal properties (as in [13]). In the periodic case, the wall oscillates vertically with the law $Y_w = A_w \sin(\omega_w t)$ and the particles collide with it as with a body of infinite mass.
FIG. 6: Rescaled (to have variance 1) velocity distributions $P(v)$ vs. $v$ in the numerical experiment (DSMC simulation) with the thermal bath, for binary mixtures of particles with mass $m_1 = 0.5$ and $m_2 = 5$, with $N_1 = N_2 = 500$, $L^2 = 1000$, $T_b = 1$, $\tau_b = 10$, $\tau_c = 0.16$, recipe 2, and different values of $\alpha$ with restitution coefficient $\alpha_w$, so that the vertical component of their velocity after the collision is $v'_y = -\alpha_w v_y + (1 + \alpha_w)V_w$ where $V_w = A_w \omega_w \cos(\omega_w t)$ is the velocity of the vibrating wall. In the stochastic case we assume that the vibration amplitude is negligible and that the particle colliding with the wall have, after the collision, new random velocity components $v_x \in (-\infty, +\infty)$ and $v_y \in (0, +\infty)$ with the following probability distributions:

$$P(v_y) = \frac{v_y}{T_w} \exp\left(-\frac{v_y^2}{2T_w}\right) \quad (18)$$

$$P(v_x) = \frac{1}{\sqrt{2\pi T_w}} \exp\left(-\frac{v_x^2}{2T_w}\right) \quad (19)$$

In this model we assume that the particle do not feel any external (environmental) friction. In all simulation we have chosen $N_1 = N_2 = 100$, $g = 1$, $m_1 = m_2/2 = 1/4$, $L_x = L_y = L = \sqrt{N}$.

The Boltzmann equations for the partial distribution functions $f_\kappa(r, v; t)$ with $\kappa = 1, 2$ read
FIG. 7: Rescaled (to have variance 1) velocity distributions $P(v)$ vs. $v$ in the numerical experiment (DSMC simulation) with the thermal bath, for binary mixtures of particles with mass $m_1 = 0.5$ and $m_2 = 5$, with $N_1 = N_2 = 500$, $L^2 = 1000$, $\alpha = 0.2$, $T_b = 1$, different values for $\tau_b$, $\tau_c = 0.16$, with recipe 2, and different values of $\alpha$

$$\left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + g_i \frac{\partial}{\partial v_i} \right) f_\kappa(\mathbf{r}, \mathbf{v}, t) = \sum_\beta J_{\kappa\beta}(f_\kappa, f_\beta).$$

(20)

The simulation results show that the tendency of the grains to form clusters is enhanced by such a choice of driving mechanism with respect to the homogeneous heat bath of the previous section. In the latter the noise acting uniformly was more effective in breaking the clusters. In addition the gravitational force tends to group the particles in the lower portion of the container for not too large driving frequencies $[16]$. 

Fig 10 illustrates the partial density profiles and granular temperature profiles in the presence of a thermal wall of intensity $T_p = 20$. The density profiles differ slightly near the bottom wall where both present a maximum (see inset). The temperature profiles are different, putting again in evidence a strong lack of equipartition in the system. Interestingly, the temperature ratio is almost constant along the vertical direction, notwithstanding the partial profiles are non constant. We have also performed numerical simulations with a harmonically vibrating wall: as the maximum velocity of the vibrating wall (which, for
FIG. 8: Temperature ratios $T_1/T_2$ vs. the viscosity time $\tau_b$ in the numerical experiment (DSMC simulation) with the thermal bath for a binary mixture of particles with mass $m_1 = 0.5$ and $m_2 = 5$, with $N_1 = N_2 = 500$, $L^2 = 1000$, $\alpha = 0.2$, $T_b = 1$, different mass ratios, $\tau_c = 0.16$, with recipe 2, and different values of $\alpha$.

FIG. 9: Correlation function used to compute the correlation dimension $d_2$, $C(r)$ vs. $r$ (see text for definition), for a binary mixture (DSMC simulation) with $m_1 = 0.5$, $m_2 = 5$, $\tau_c = 0.16$, $T_b = 1$, $N_1 = N_2 = 500$, $L^2 = 1000$, $\alpha = 0.2$, with different values for $\tau_b$. 
FIG. 10: Profiles for density and granular temperature $n(y)$, $T(y)$ vs. $y$ for a binary mixture with $m_1 = 0.5$, $m_2 = 2$, on an inclined plane with a thermal bottom wall (DSMC simulation) with $T_w = 20$, $N_1 = N_2 = 100$, $L^2 = 200$, $\alpha = 0.6$, $\alpha_w = 1$, $g = 1$; the dashed lines indicate the ratios for both profiles.

As $\omega$ increases, the position of the density maximum raises indicating that gravity becomes less and less relevant. As far as the partial temperatures are concerned figure (11) illustrates the corresponding situation.

One sees that the temperatures (see inset) next to the wall attain their largest value, then drop to increase again. This indicates that the region far from the bottom is hotter because of the lower density of the gas and of the lower collision rate. As the wall temperature increases the temperature becomes more homogeneous far from the bottom due to the major homogeneity in density. We also notice a small segregation effect.

Finally, we present the velocity distributions measured in the system with a thermal wall. Since the temperature depends on the vertical coordinate $y$ we have computed the velocity pdf at different heights. The various pdf are plotted in figure (12) after a suitable rescaling. We observe that the distributions deviate appreciably from a Gaussian shape and display overpopulated high energy tails.

Interestingly the value obtained by our simulation for the temperature ratio, having
FIG. 11: Profiles for density $n(y)$ vs. $y$ and ratios between granular temperatures of the two species $T_1(y)/T_2(y)$ vs. $y$, for a binary mixture (DSMC simulation) with $m_1 = 0.5$ and $m_2 = 2$, on an inclined plane with an harmonic oscillating bottom wall with $A = 1$, different values of $\omega$, $N_1 = N_2 = 100$, $L^2 = 200$, $\alpha = 0.9$, $\alpha_w = \alpha g = 1$.

chosen $\frac{m_1}{m_2} = 0.3$ and a single restitution coefficient $\alpha = 0.93$. $\frac{T_1}{T_2} \approx 0.75$ is not too far from the value obtained experimentally by Feitosa and Menon [6] which is $0.66 \pm 0.06$. One should recall that in our “setup” only the lower wall supplies energy to the system, whereas in the experiment both walls vibrate.

V. CONCLUSIONS

To summarize we have studied the steady state properties of a granular mixture subject to two different classes of external drive.

In the case of an heat bath acting homogeneously on the grains, we have first obtained the temperatures of each species by employing an approximate analytic method. We have then solved numerically the equations by allowing the density and the temperatures to be spatially varying and analyzed the spatial correlations and the velocity distributions.

Finally, we turned to investigate the properties of the mixture in the presence of gravity
FIG. 12: Rescaled distributions (to have variance 1) of velocity $P(v)$ vs. $v$ for particles taken in stripes at different heights, for a binary mixture (DSMC simulation) with $m_1 = 0.5$, $m_2 = 2$, on an inclined plane with a thermal bottom wall with $T_w = 20$, $N_1 = N_2 = 100$, $L^2 = 200$, $\alpha = 0.6$, $g = 1$.

and inhomogeneous drive.

The present results not only confirm the predictions of different temperatures for the two species for all types of drive, but also show the existence of different shape functions for the velocity distributions and of overpopulated high energy tails in agreement with the findings based on Maxwell models [3].

In order to obtain a direct comparison with experiments we employed parameters comparable to those utilized in the experimental work of ref. [6] and found temperature ratios not too far from the ones there reported. In spite of this success, the agreement seems to be limited to the average properties, while it is clear from the observation of the density profiles computed numerically that a better treatment of the excluded volume effect is necessary in order to obtain a more realistic description of the system. Finally, in view of the approximations inherent to the Boltzmann approach, we have not tried to include neither the effect of rotations of the grains nor the friction with the lateral walls which might be both relevant [13].
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