IMPROVED HARD-THERMAL-LOOP EFFECTIVE ACTIONS \textsuperscript{a}

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Hard thermal loop effective actions furnish the building blocks of resummed thermal perturbation theory, which is expected to work as long as the quantities under consideration are not sensitive to the nonperturbative (chromo-)magnetostatic sector. A different breakdown of perturbation theory occurs whenever external momenta are light-like, because the hard thermal loops themselves develop collinear singularities. By additionally resumming asymptotic thermal masses for hard modes these singularities are removed while leaving the gauge invariance of the effective action intact.

1 Review of Hard-Thermal-Loop Resummation

Precisely at (and below) those energy/momentum scales where perturbation theory of quantum field theories at non-zero temperature\textsuperscript{1} predicts new effects from collective behaviour like screening, plasma frequencies, Landau damping etc., the conventional loop expansion ceases to be reliable. The infrared singularity of the Bose-Einstein distribution

\begin{equation}
  n_B(k) = \frac{1}{e^{k/T} - 1} \sim \frac{T}{k} \quad \text{for } k \to 0
\end{equation}

causes an increasing sensitivity to the infrared as the loop order increases. Eventually, this gives rise to infrared divergences which inhibit further refinements of the results or which even may prevent one from determining the leading terms.

However, the very same nontrivial screening masses, plasma frequencies, and Landau damping effects in most cases provide the remedy for perturbation theory, because once taken into account they provide a cut-off to otherwise infrared divergent loop integrals.

One of the simplest cases where one can observe this mechanism of self-healing is a massless scalar field theory with quartic self-interactions $g^2 \phi^4$. At finite temperature, one finds for the thermal contribution to the one-loop scalar

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self-energy a simple constant proportional to $T^2$,

$$\Pi_{T > 0} - \Pi_{T = 0} = g^2 T^2 = m_{th}^2$$

which can be interpreted as a thermal mass for the scalars in a heat bath. It is not a mass in the usual sense, though. For instance it does not contribute to the trace of the energy-momentum tensor which a mass usually does. But it does modify the infrared behaviour of scalar particles by introducing a mass gap to their dispersion laws. Clearly, this mass term becomes important for momenta of the order $gT$. At and below this scale, it is at least as important as the usual kinetic term, and the effective propagator becomes $[k_0^2 - \overline{k}^2 - m_{th}^2]^{-1}$ instead of the massless one. As a consequence, infrared divergences in higher loop diagrams become cut off at the scale of $m_{th}$ and lead to an effective loop expansion parameter $g^2 T/m_{th} \sim g$.

Including the thermal mass into the scalar propagator corresponds to a reorganization of perturbation theory. Overcounting is avoided by writing

$$\mathcal{L} = \mathcal{L} - \frac{1}{2} m_{th}^2 \phi^2 + \frac{1}{2} m_{th}^2 \phi^2$$

and treating the thermal mass term as both a contribution to the classical action and as a one-loop counter-term. Alternatively, one may view $\mathcal{L}_{eff}$ as (the leading terms of) the effective low-energy action obtained after integrating out the hard modes with momenta $\gtrsim T$. These are indeed solely responsible for the appearance of a thermal mass for soft modes.

Turning to gauge theories now, a well-known collective phenomenon at finite temperature or density is Debye screening of electric fields. In a static situation one finds an effective Lagrangian with a mass term for the zero components of the vector potential,

$$\mathcal{L}_{eff} = \mathcal{L} + \frac{1}{2} m_D^2 A_0^2$$

with $m_D^2 = \frac{1}{3} e^2 T^2$ in the high-temperature limit. The necessity for a resummation of this mass term has been noted in the context of QED first by Gell-Mann and Brueckner. Otherwise infra-red divergences appear at three-loop order of a perturbative evaluation of the partition function.

In nonabelian gauge theories like QCD there is similarly a screening mass for chromo-electrostatic modes. The leading-order term differs only by the replacement of $e^2$ by $g^2 (N + N_f)$ for gauge group $SU(N)$ and $N_f$ fermions. However, because of the self-interactions of the gluons, Debye screening is
not sufficient for curing the infrared problem of perturbation theory. The (chromo)magnetostatic modes are unscreened at the scale $gT$ and are therefore expected to cause divergences beyond three loop order in the partition function. If an infrared cut-off effectively arises at the next lower scale $g^2T$, this will not cure perturbation theory as it did above: the effective expansion parameter $g^2T/m_{th}$ is then of order 1. In what follows we shall always be concerned only with that part of (resummed) perturbation theory where one is still below those loop orders where this presumably insurmountable barrier to perturbation theory appears.

If one is interested in computing thermal effects in nonstatic quantities, it turns out that taking into account Debye screening is not sufficient. The leading contribution to the gauge bosons’ self-energy is a nontrivial function of the temporal and spatial components of the momentum vector involving Legendre functions of the second kind and so represents a nonlocal contribution to an effective Lagrangian. Moreover the loss of Lorentz invariance (through the distinguished rest frame of the plasma) leads to two independent structure functions in the polarization tensor,

$$
\Pi^{\mu\nu}(q_0, q) = \Pi_t(q_0, q) P_t^{\mu\nu} + \Pi_\ell(q_0, q) P_\ell^{\mu\nu}
$$

where $P_t^{\mu\nu}$ are two symmetric tensors that are transverse with respect to the 4-momentum $Q = (q_0, \vec{q})$. One can be chosen to be transverse also with respect to 3-momentum and will be denoted as $P_t$; the second, $P_\ell$, will be taken to be orthogonal to the former.

In nonabelian gauge theories, $\Pi^{\mu\nu}$ does not necessarily have to be transverse, but one can always find gauges where it does. Assuming transversality, the functions $\Pi_{t,\ell}$ are given by

$$
\Pi_\ell = -\frac{Q^2}{q^2} \Pi_{00}, \quad \Pi_t = \frac{1}{2}(\Pi_{\mu\mu} - \Pi_{\ell\ell})
$$

in the rest frame of the plasma. The leading terms in the high-temperature expansion of the one-loop diagrams for the self-energy are relevant for the infrared domain $\omega, q \ll T$ and are given explicitly by

$$
\Pi_{\mu\mu} = \frac{e^2 T^2}{3},
$$

$$
\Pi_{00} = \frac{e^2 T^2}{3} \left(1 - \frac{\omega}{2q} \log \frac{\omega + q}{\omega - q}\right),
$$

where in the imaginary-time formalism an analytic continuation $Q_0 = 2\pi nT \rightarrow \omega + i\epsilon$ has been made. In the case of QCD the result, to leading order, is the same up to the substitution $e^2 \rightarrow g^2(N + N_f)$.
Corresponding to the two components in the self-energy tensor, there are now two different “mass-shells” in the (Dyson resummed) photon/gluon propagator, which are shown in fig. 1a.

Since quadratic scales have been used in this figure, a Lorentz invariant mass shell would be given by a straight line parallel to the light-cone which is represented by a dashed line. The full dispersion law can no longer be given in closed form but has to be obtained by numerically solving for poles in the propagator. For positive $q^2$, one finds propagating modes above the plasma frequency $\omega_{pl} = m = eT/3$. The spatially transverse branch, which corresponds to radiative modes, approaches asymptotically an ordinary mass-shell with mass $m_\infty = eT/\sqrt{6}$. The longitudinal branch describes a collective mode which has no analogue in the vacuum. Its residue decays exponentially fast with increasing $q$, for which it approaches the light-cone.

For frequencies below $\omega_{pl}$, the only poles in the propagator occur for imaginary values of $q$ (negative values of $q^2$ in fig. 1a) which correspond to screening effects. The two branches show different screening lengths, and in the static limit only the longitudinal one has a finite (Debye) screening length, whereas the transverse branch has a vanishing static screening mass.

Another feature worth mentioning is that the polarization tensor has a large imaginary part for space-like momenta. Its physical interpretation is given by the effect of Landau damping, which corresponds to the possibility of absorption of soft modes by the hard constituents of the plasma.

In fig. 1b, the dispersion laws for fermionic matter is given. Again one
finds a second, collective mode, which technically arises because the loss of Lorentz invariance leads to two independent structure functions in the fermion self-energy. In the high-temperature limit these are determined by

\[
\frac{1}{4} \text{tr} \ Q\Sigma = \frac{e^2 T^2}{8}, \tag{9}
\]

\[
\frac{1}{4} \text{tr} \ \gamma^0 \Sigma = \frac{e^2 T^2}{16q} \log \frac{\omega + q}{\omega - q}. \tag{10}
\]

(In the case of QCD, the result differs only by the replacement \( e^2 \rightarrow g^2 N^2 - \frac{1}{2N} \), as far as the leading terms given here are concerned.)

The resummed fermion propagator has the form

\[
S_F(\omega, q) = \frac{1}{2} \left( \gamma_0 + \frac{\gamma}{q} \right) D_+(\omega, q) + \frac{1}{2} \left( \gamma_0 - \frac{\gamma}{q} \right) D_-(\omega, q). \tag{11}
\]

Both branches have a common frequency \( \omega = M = eT/2 \) at \( q = 0 \). The (+)-branch tends asymptotically to \( M_\infty = \sqrt{2M} \). The (−)-branch on the other hand, which is sometimes termed "plasmino", approaches the light-cone with exponentially vanishing residue, pointing at its nature of a collective mode. It has a curious dip in its dispersion law which is reminiscent of rotons in superfluidity. It is anomalous also in that it has a reversed sign in the ratio of helicity and chirality, which comes from the fact that it describes the behaviour of hole (rather than particle) excitations in the plasma.

Notice that contrary to the bosonic case, there are no poles in the propagator for \( q^2 < 0 \), corresponding to the impossibility of a screening of fermion numbers. But as in the bosonic case, there is Landau damping for space-like momenta, since soft fermions can be annihilated by hard ones or can be transformed into hard ones by absorption of hard gauge bosons.

Evidently, the hard-thermal-loop modifications of the basic propagators are such that they can no longer be described by a simple effective mass term. The corresponding effective action is instead a highly nonlocal object, and it is moreover nonpolynomial in the fields, because there are also (nonlocal) vertex corrections that have to be treated on a par with the tree-level vertices once we are considering momenta of the order \( eT \). It turns out that all one-loop diagrams that have contributions proportional to \( T^2 \) in the high-temperature limit are of the same order of magnitude as tree-level vertices when \( \omega, q \sim eT \). Such hard-thermal-loop vertices exist for two fermions together with any number of gauge bosons and, in the nonabelian case, for any number of gluons in interaction. They happen to vanish in the static limit, where the effective action becomes local.
Remarkably, this nonlocal, nonpolynomial effective action can be given in a rather compact form involving an additional integration over the directions of an auxiliary light-like vector. Taylor and Wong have shown that this effective action can be written as

\[ \Gamma[A] = -m_D^2 \left( \frac{1}{2} \int d^4x A_0^a A_0^a + \frac{d\Omega}{4\pi} W(A \cdot P) \right) \]  

(12)

where

\[ W(A \cdot P) = \text{tr} \int d^4x \partial_0 (A \cdot P) f(\frac{1}{P^2} [iA \cdot P, \ast]) \frac{1}{P^2} A \cdot P \]  

(13)

with \( f(z) = 2 \sum_{n=0}^{\infty} \frac{z^n}{n!} \) and \( P = (1, \vec{P}), P^2 = 0 \). The average over the latter is the remnant of having integrated out the hard massless modes of the plasma constituents.

An alternative form which makes the gauge invariance of this effective action manifest has been given by Braaten and Pisarski and reads

\[ \Gamma[A] = -\frac{1}{4} m_D^2 \int \frac{d\Omega_{\vec{P}}}{4\pi} \int d^4x F_{a\mu}^\ast [P^2, D(A)]_{\mu \nu} \frac{P_\nu P_\rho}{P^2} F_{b\mu}^\rho + iM^2 \int \frac{d\Omega_{\vec{P}}}{4\pi} \int d^4x \bar{\psi} P \cdot D(A) \psi \]  

(14)

where we have now included also the fermionic part.

Hard-thermal-loop resummation means that one has to use resummed propagators and resummed nonlocal vertices in loop calculations. For example, the calculation of the production rate of virtual soft photons (soft dileptons) involves evaluating the imaginary part of the diagrams given in fig. 2. Its building blocks can be cut in a number of different ways. Cutting a soft line gives contributions from the quasi-particle poles as well as from Landau damping, which is given by the cut of the self-energy. Moreover, hard-thermal-loop vertices have a number of different cuts, as depicted in fig. 3. In these figures, hard and soft momenta are indicated by thin and heavy lines, respectively.

This approach has been applied successfully to a number of other problems, too, and even in those cases where the quantities under consideration turns out to be sensitive to the magnetic mass scale, it is usually possible to extract a leading logarithmic correction.

A quite different difficulty has been encountered recently in attempts to calculate the production rate of soft real photons from a quark-gluon plasma.
2 Collinear singularities

It seems to make sense physically to take the thermodynamic large-volume limit only with respect to strongly interacting matter and to assume that particles without strong interactions can escape immediately.

This is the rationale behind the above-mentioned calculation of the production rate of virtual photons which decay into dilepton pairs. Similarly one may consider real photons which are produced by the electrically charged quarks within the plasma.

Technically the simplest case is the one of hard real photons. Because of the external hard momentum that has to be routed through the diagrams, there is at most one soft propagator involved that needs to be dressed. In particular, there are no complicated hard-thermal-loop vertices to be taken into account. The relevant diagrams are shown in fig. 4.

This calculation has been performed in Ref. 15 with the result, when the energy of the photon $E \sim T$,

$$E \frac{dW_{HRP}}{d^3q} \propto e^2g^2T^2\log\frac{cT}{gT},$$

(15)

where $c$ is a calculable constant. Here resummation has replaced a log of the bare quark mass by log($gT$), thereby screening the collinear singularity that would appear in the limit of vanishing (or negligible) bare quark masses.

The analogous calculation for soft real photon production instead involves the diagrams of fig. 2, but in this case it turns out that the resummed result still diverges for massless quarks as the light-cone is approached,

$$E \frac{dW_{SRP}}{d^3q} \propto e^2g^3T^2\log\frac{cT}{gT}\log\frac{c''T^2}{Q^2}.$$  

(16)

The by now standard hard-thermal-loop resummation method appears to break down here, but in a novel way, for the magnetostatic sector is innocuous.

![Figure 2: One-loop resummed contributions to soft photon production](image-url)
this time. The difficulties are rather rooted in the hard thermal loops themselves. They, too, exhibit collinear singularities when one of the external soft momenta approaches the light-cone.

For example \( \Pi_{00} \) as given in (8) diverges logarithmically when \( |\omega| \to q \). Before carrying out the integration over hard momenta \( \Pi_{00} \) reads

\[
\Pi_{00}(Q) = -2e^2 \int \frac{d^3p}{(2\pi)^3} n'(p) \left\{ 1 - \frac{Q_0}{Q_0 - \not{q}/\not{p}} \right\}
\]

so this indeed comes from a collinear singularity, which for light-like \( Q \) arises when the soft momentum \( \not{q} \) becomes parallel to the hard momentum \( \not{p} \).

This singularity of \( \Pi_{00} \) looks harmless inasmuch as the longitudinal plasmon mass-shell, which is determined by \( \Pi_{00} \), never reaches the light-cone, although it does approach it exponentially. However, a problem similar to the one in the soft-photon production rate appears if one wants to calculate next-to-leading order corrections to the plasmon dispersion law in the vicinity of the light-cone. In Ref. 16 this has been done in the case of scalar electrodynamics.

\[
\text{Im} \left( \sum \text{terms} \right)
\]

Figure 4: One-loop resummed contributions to hard photon production
with the result that

$$\delta \Pi_{00}(Q) \sim \frac{em^2q}{\sqrt{Q^2}}.$$  \hfill (18)

This is even more singular than the leading, hard-thermal-loop result, so sufficiently close to the light-cone, namely when $Q^2/q^2 \ll e^2$, we have that $\delta \Pi_{00} \gg \Pi_{00}^{\text{HTL}}$, and the resummed perturbation theory clearly is breaking down.

It is precisely the singularity of $\Pi_{00}$ at the light-cone that is causing this trouble here, and this singularity is possible only by the masslessness of the hard modes. However, hard modes also acquire thermal masses through interactions with the plasma constituents. Usually, they are completely negligible, because they are suppressed by powers of the coupling constant when compared to the hard momentum scale. But this is no longer true in the vicinity of the light-cone. Indeed, including the thermal mass of the scalar particles renders $\Pi_{00}$ regular, and leads to

$$\Pi_{00}^{\text{leading}}(Q^2 = 0) = \frac{e^2}{\pi^2} \int_0^\infty dp \rho n(p) \left( 1 - 2 \log \frac{2p}{m_{\text{th}}} \right) = -\frac{e^2T^2}{3} \left( \log \frac{2T}{m_{\text{th}}} + \frac{1}{2} - \gamma + \frac{\zeta'(2)}{\zeta(2)} \right).$$  \hfill (19)

Computing now the next-to-leading order term gives

$$\delta \Pi_{00}(Q^2 = 0) = \frac{e^2Tm_{\text{th}}}{2\pi}$$

which because of $m_{\text{th}} \propto eT$ is smaller than the leading order result (19), as required for a sensible perturbative scheme.

With $\Pi_{00}$ being regular at $Q^2 = 0$, it turns out that there is a qualitative change of the plasmon dispersion law. Instead of being asymptotic to the light-cone, it now pierces the light-cone at a finite value of $q^2 = q_f^2 \sim \frac{1}{4}e^2T^2 \log(1/e)$. The failure of (standard) hard-thermal-loop resummation is just a reflection of the incapability of any perturbation theory to describe qualitative changes. Hence the necessity to modify the starting point, namely, the hard thermal loops themselves.

There is in fact nothing wrong with the existence of space-like plasmon modes: their group velocity remains smaller than 1 throughout. A similar phenomenon has already been found in 1961 by Tsytovich \cite{Tsytovich} in the case of non-ultrarelativistic quantum electrodynamics (i.e., with temperatures $T < m_e$).

Actually, the importance of these modes is reduced by the corresponding residue having dropped significantly in the vicinity of the light-cone. Moreover,
for space-like momenta, strong Landau damping sets in, quickly rendering them over-damped.

The on-set of Landau damping is usually discontinuous. With thermal masses for the hard modes taken into account, this is in fact smoothed out as well. The imaginary part of $\Pi_{00}$ sets in as

$$\text{Im } \Pi_{00}(Q^2 < 0) \bigg|_{q - \omega \ll \varepsilon q} = \frac{e^4 T^2 q}{8\pi (q - \omega)} e^{-\epsilon \sqrt{\frac{q}{8\pi (q - \omega)}}}$$

(21)

that is, starting from zero with all its derivatives vanishing.

3 Improved hard-thermal-loop effective action

In the above example of scalar electrodynamics, the collinear singularities within the hard thermal loops were removed by a resummation of the thermal mass of scalar particles for both soft and hard modes. In more complicated gauge theories we have seen that one is usually confronted with more than one dispersion law for a given species. However, it turns out that additional collective modes such as the longitudinal plasmon and the plasmino do not contribute to the hard thermal loops. Whereas the additional modes would have zero mass asymptotically, the transverse vector boson branch and the $(+)$-branch of the fermionic modes both tend to a mass hyperboloid with asymptotic masses $m_\infty$ and $M_\infty$, respectively. Therefore we can expect that the collinear singularities in hard thermal loops on the light-cone will again be cut off once these asymptotic masses are taken into account.

Our improved hard thermal loop action can be obtained almost exactly as usually with the only difference that the (hard) gluon propagator reads

$$D^{\mu\nu}(P) = \frac{1}{P^2 - m_\infty^2} P^{\mu\nu}_t + \frac{1}{P^2} (P^{\mu\nu} + \alpha \frac{P^\mu P^\nu}{P^2})$$

(22)

where we have used a general covariant gauge, and the fermion propagator

$$S_F(P) = \frac{P}{P^2 - M_\infty^2}.$$  

(23)

In the expression $\bar{q}q/p$ comes from the first term in the expansion of the energy difference $\omega(q) - \omega(q - \tilde{q})$ for $|\tilde{q}| \ll |q|$. However, for light-like $Q$ and collinear 3-momenta, the next term in the expansion becomes important, and we have $\omega(q) - \omega(q - \tilde{q}) \approx \bar{q}q/p - \bar{q}q m_\infty^2 / 2p^3$ if the hard modes involved are gluons (when the loop diagram is made from quarks, one just has to replace $m_\infty$ by $M_\infty$).
Since this extra contribution is only needed in the vicinity of the light-cone and when the 3-momenta are close to collinearity, the same regularization is obtained by replacing \( Q_0 \to (1 + m^2_{\infty} / 2\tilde{p}^2)Q_0 \) in the critical denominators. Because the potential collinear singularities are only logarithmic, the numerators can be simplified as usually in hard thermal loops. This holds true also for vertex functions as long as exceptional momentum configurations are avoided.

As a consequence, we find that the structure of the hard-thermal-loop effective action is largely preserved. In the version of Taylor and Wong (12) we have to modify the averaging over the directions of \( \tilde{p} \) according to

\[
\int \frac{d\Omega_{\tilde{p}}}{4\pi} f(P) \to \frac{3}{\pi^2} \int_0^\infty d\alpha \frac{\alpha^2 e^\alpha}{(e^\alpha - 1)^2} \int \frac{d\Omega_{\tilde{p}}}{4\pi} f(P(\alpha))
\]

\[
P = (1, \tilde{p}) \to P(\alpha) = (\sqrt{1 + \frac{m^2_{\infty} / T^2}{\alpha^2}}, \tilde{p})
\]

with \( \tilde{p}^2 = 1 \). These modifications indeed do not spoil any of the properties that are crucial for the gauge invariance of the effective action.

A similar modification applies for the manifestly gauge invariant form of the effective action due to Braaten and Pisarski (14). It differs from (24) in that

\[
\frac{3}{\pi^2} \int_0^\infty d\alpha \frac{\alpha^2 e^\alpha}{(e^\alpha - 1)^2} \cdots \to \frac{6}{\pi^2} \int_0^\infty d\alpha \frac{\alpha e^\alpha}{e^\alpha - 1} \cdots
\]

and furthermore in that \( P_0(\alpha) \) can no longer be simplified to 1 when it appears in numerators. This is because in the form (14) there are individual contributions which prior to the modification had collinear singularities that were stronger divergent than only logarithmically, and so more care has to be exerted when working with the manifestly gauge invariant version.

Another complication occurs when external fermion lines are involved. Then two different massive propagators occur inside hard thermal loops and the dominant term that shifts the pole causing collinear singularities is proportional to the difference of the bosonic and fermionic asymptotic masses and is independent of the soft momentum. The modification to be applied to the hard-thermal-loop effective action is then

\[
\int \frac{d\Omega_{\tilde{p}}}{4\pi} \cdots \to \frac{2}{\pi^2} \int_0^\infty d\alpha \frac{\alpha e^\alpha}{e^{2\alpha} - 1} \int \frac{d\Omega_{\tilde{p}}}{4\pi} \left[ \frac{\frac{m^2_{\infty} - M^2_{\infty}}{2\alpha T} - c.c.}{iP \cdot D(A) + \frac{m^2_{\infty} - M^2_{\infty}}{2\alpha T}} \right]
\]

where \( P \) remains a null vector, \( P^2 = 0 \). The manifest gauge invariance of the fermionic hard-thermal-loop effective action is clearly left intact by this replacement.
4 Open problems

Although the collinear singularities are removed by a resummation of asymptotic thermal masses for the hard modes that yield the hard-thermal-loop effective action, it is not clear that the correspondingly improved hard-thermal-loop effective action leads to a resummed perturbation theory where higher loop orders are suppressed by higher powers of the coupling.

In fact, in Ref. 19 it has recently been shown that in the case of the soft-real-photon production rate certain resummed two-loop diagrams involving both hard and soft internal momenta (shown in fig. 5) are even enhanced by a factor $1/g^2$ compared to the contributions of soft one-loop diagrams.

This need not signal a complete breakdown of perturbation theory, for the one-loop diagrams are unnaturally small because of a statistical suppression factor. But it certainly raises the question whether there could be even more enhanced higher-loop diagrams from stronger and stronger would-be collinear singularities.

If this is the case, one would probably require additional resummations beyond those of asymptotic thermal masses. A candidate would be the anomalously strong damping of hard modes from interactions with soft ones, whose damping constant $\gamma$ is of the order $g^2T\log(1/g)$.

Although formally of higher order than asymptotic thermal masses, such a damping term would indeed be more effective in cutting off the collinear singularities, since

$$\frac{Q^2}{q^2} \rightarrow \frac{Q^2\gamma}{q^2} \sim g \gg \frac{m^2}{T^2} \sim g^2.$$  \hspace{1cm} (27)

(The damping contributed by purely hard self-interactions on the other hand is of the order $\gamma \sim g^4T$ and so is less important than the asymptotic thermal mass.)

But in contrast to a mass resummation it is generally not sufficient to modify only the propagators, when damping is to be taken into account. Including damping only into the propagators violates the Ward identities. In Abelian
theories the latter imply $\Pi_{\mu\nu}Q^{\nu} \equiv 0$, but one finds e.g.

$$\Pi_{0\nu}(Q_0, 0)Q^{\nu} \bigg|_{\vec{q}=0} = \frac{2i\gamma Q_0}{Q_0 + 2i\gamma} e^{2T^2} 3.$$  \hspace{1cm} (28)

Correcting also the vertices restores gauge invariance, but it turns out that $\gamma$ gets eliminated not only from $\Pi_{\mu\nu}Q^{\nu}$, but from all the components of $\Pi_{\mu\nu}$!

Moreover, damping effects from interactions with soft modes cannot be incorporated into an effective action which summarizes the effects of integrating out the hard modes only. It is therefore much more difficult to find a consistent scheme. Instead, one would have to solve the hierarchy of Schwinger-Dyson equations.

Because of all these difficulties, the status of hard-thermal-loop resummation for processes involving light-like momenta is still rather unclear and remains a challenging problem.

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