THE POLARIZATION OF THE SOLAR Mg II h AND k LINES

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ABSTRACT

Although the h and k lines of Mg II are expected to be of great interest for probing the upper solar chromosphere, relatively little is known about their polarization properties which encode the information on the magnetic field. Here we report the first results of an investigation whose main goal is to understand the physical mechanisms that control the scattering polarization across these resonance lines and to achieve a realistic radiative transfer modeling in the presence of arbitrary magnetic fields. We show that the joint action of partial frequency redistribution (PRD) and quantum interference between the upper J-levels of the two lines produces a complex fractional linear polarization (Q/I) pattern with large polarization amplitudes in the blue and red wings, and a negative feature in the spectral region between the two lines. Another remarkable peculiarity of the Q/I profile is a conspicuous antisymmetric signal around the center of the h line, which cannot be obtained unless both PRD and J-state interference effects are taken into account. In the core of the k line, PRD effects alone produce a triplet peak structure in the Q/I profile, the modeling of which can also be achieved via the two-level atom approximation. In addition to the Hanle effect in the core of the k line, we also emphasize the diagnostic potential of the circular polarization produced by the Zeeman effect in the h and k lines, as well as in other Mg II lines located in their wings.

Key words: polarization – radiative transfer – scattering – Sun: chromosphere – Sun: surface magnetism – Sun: transition region

1. INTRODUCTION

It is through the Zeeman and Hanle effects that the magnetic fields of the solar atmosphere leave their fingerprints in the polarization of the emergent spectral line radiation (e.g., Casini & Landi Degl’Innocenti 2007). The art of “measuring” solar magnetic fields thus relies on the development of suitable diagnostic tools based on such effects. The Zeeman effect produces circular and linear polarization signals whose amplitudes are proportional to R and R2, respectively, with R the ratio between the Zeeman splitting and the Doppler line width. The Hanle effect is the modification of the linear polarization produced by scattering processes in a spectral line, caused by the presence of a magnetic field. This line width. The Hanle effect is the modification of the linear polarization of the radiative transfer computations by assuming that the line source function and the integrated mean intensity are equal to the Planck function, Auer et al. (1980) estimated values of Q/I of about 4% in the positive polarization maxima in the far wings, and of about 1.8% in the negative minimum between the two lines, for a line of sight (LOS) with μ = cos θ = 0.1 (with θ the heliocentric angle).

Another key physical ingredient for modeling the Mg II h and k lines is partial frequency redistribution (PRD). As shown by radiative transfer calculations carried out using a two-level model atom, PRD effects are expected to produce a conspicuous triplet peak Q/I signal in the core of the k line (e.g., Sampoorna et al. 2010; see also Figure 9 in Trujillo Bueno 2011, obtained in collaboration with Drs. M. Sampoorna and J. Štěpán). However, since the dispersion wings of these lines are significantly overlapped and J-state interference cannot be neglected, it is crucial to investigate the scattering polarization problem of the Mg II h and k system accounting for the joint action of both PRD and J-state interference effects. As we shall see below, our PRD with J-state interference approach to this complex
theoretical problem is not restricted to the line wings, so that our full radiative transfer calculations in model C of Fontenla et al. (1993, hereafter FAL-C model) allow us to provide information on the amplitude and shape of the \( Q/I \) signal throughout the whole profile. A correct modeling of the scattering polarization in the core of spectral lines is particularly important because it is precisely in this spectral line region where the Hanle effect operates. We also provide estimates on the circular polarization induced by the longitudinal Zeeman effect.

2. FORMULATION OF THE PROBLEM

A quantum mechanical derivation of the redistribution matrix for polarized radiation, for a two-level atom with unpolarized and infinitely sharp lower level, was carried out by Domke & Hubeny (1988) and Bommier (1997a, 1997b). In the atom rest frame (where the frequency and angular dependencies can be factorized), frequency redistribution is described by the linear combination of Hummer’s \( R_\beta \) and \( R_\alpha \) functions (see Hummer 1962), which describe purely coherent scattering and completely redistributed scattering, respectively. The corresponding branching ratios for the \( K \)th multipole component of the redistribution matrix are given by

\[
R_\beta : \alpha = \frac{\Gamma_R + \Gamma_I}{\Gamma_R + \Gamma_I + \Gamma_E}, \tag{1}
\]

\[
R_\alpha : \beta = \frac{\Gamma_R + \Gamma_I}{\Gamma_R + \Gamma_I + D^{(K)}}, \tag{2}
\]

where \( \Gamma_R, \Gamma_I, \) and \( \Gamma_E \) are the broadening widths of the upper level due to radiative decays, superelastic collisions (i.e., collisional de-excitation), and elastic collisions, respectively, while \( D^{(K)} \) is the depolarizing rate due to elastic collisions \( (D^{(0)} = 0) \). Note that the branching ratios given in Equations (1) and (2) do not include the factor \( (1 - \epsilon) \), with \( \epsilon = \Gamma_I/\Gamma_R + \Gamma_I \) the photon destruction probability (cf. Equations (54) and (55) of Bommier 1997b). Their sum is equal to 1 for the \( K = 0 \) multipole component \((\beta^{(0)} \equiv 1)\), while in the presence of depolarizing collisions it is generally smaller than 1 for \( K \neq 0 \).

As mentioned in Section 1, quantum interference between different \( J \)-levels is expected to produce significant observable effects on the scattering polarization signal across the Mg \( \text{II} \) \( h \) and \( k \) lines. An atomic model accounting for the contribution of \( J \)-state interference (such as the two-term model atom presented in Landi Degl’Innocenti & Landolfi 2004, hereafter LL04) is therefore needed for the investigation of this resonance doublet. Unfortunately, a rigorous quantum mechanical derivation of the redistribution matrix for such an atomic model, in the presence of elastic and inelastic collisions, has not been carried out yet. Nevertheless, the limit of coherent scattering \textit{in the atom rest frame} is a reasonable approximation for modeling the scattering polarization across the Mg \( \text{II} \) \( h \) and \( k \) lines. This can be clearly seen from Figure 1, where the branching ratio \( \alpha \) corresponding to the \( h \) line (the one corresponding to the \( h \) line is practically identical) is shown as a function of height in the FAL-C model of the solar atmosphere. As shown in the figure, \( \alpha \) is indeed equal to one at the heights where the line-core optical depth is unity for an observation at \( \mu = 0.1 \), and it is also very close to one at the heights where the optical depth in the wavelength interval between the two lines is unity.

A quantum mechanical derivation of the redistribution matrix for a two-term atom with unpolarized, infinitely sharp lower levels, in the limit of purely coherent scattering in the atom’s rest frame has been carried out by Landi Degl’Innocenti et al. (1997) within the framework of the metalevel theory. This redistribution matrix is suitable for investigating the scattering polarization across the Mg \( \text{II} \) \( h \) and \( k \) lines, since the lower level of these lines is the ground level (the hypothesis of an infinitely sharp lower level is thus suitable), and it has \( J_f = 1/2 \). Such redistribution matrix was obtained by neglecting collisions, and is valid in the atom’s rest frame. Following a derivation analogous to the one presented in Hummer (1962), we have calculated the corresponding expression in the observer’s frame, taking Doppler redistribution into account. An alternative derivation has been recently carried out by Smitha et al. (2011) starting from the Kramers–Heisenberg scattering formula.

We consider a two-term model atom for Mg \( \text{II} \), the lower term being composed of the ground level \( ^2S_{1/2} \) and the upper term of the upper levels of the \( h \) and \( k \) lines \( ^2P_{1/2} \) and \( ^2P_{3/2} \), respectively). Although a rigorous derivation of the collisional rates in a two-term atom has not been carried out yet, we included the effect of (isotropic) inelastic and superelastic collisions in the redistribution matrix, under the following assumptions. We first assumed that the relaxation rate due to superelastic collisions is the same for the two upper \( J \)-levels (see Section 7.13 of LL04 for the explicit expressions of such rate). We then assumed that such collisional relaxation rate is also the one for the interference between the two upper \( J \)-levels. Accordingly with the previous approximation, we calculated the inelastic collision transfer rates from the common lower level to the two upper levels. Since the lower level is unpolarized, these latter collisions only affect the populations of the upper \( J \)-levels, and the corresponding rates are identical to the case of a multi-level atom (see Section 7.13 of LL04). Collisional rates between pairs of \( J \)-levels pertaining to the same term have been neglected, as well as elastic collisions. Under these hypotheses, following the convention according to which primed quantities refer to the incident photon, while unprimed quantities to the scattered photon, indicating with \( \nu \) and \( \Omega \) the photon’s frequency

![Figure 1](image-url)
(in the observer’s frame) and propagation direction, we obtained

\[ R_{ij}(v', \Omega'; v, \Omega) = \frac{2L_u + 1}{2S + 1} \sum_k \sum_{J_u' J_u} (-1)^{J_u' - J_u} \times (2J_u + 1)(2J_u' + 1) \times \left[ L_u L_\ell 1 \right] \left[ J_u J_u' 1 \right] \left[ L_u L_\ell 1 \right] \left[ J_u' J_u 1 \right] \left[ K J_u J_u' 1 \right] \left[ J_u' J_u 1 \right] \left[ P^{(K)}(\Omega', \Omega) \right]_{ij} \times \frac{1}{\pi \Delta v_D^2 \sin \theta} \exp \left[ -\frac{(v' - v - v_{JaJb})^2}{4 \Delta v_D^2 \sin^2(\theta/2)} \right] \times \frac{1}{1 + \epsilon' + \frac{v_{JaJb}}{2\epsilon}} \left[ W \left( \frac{a}{\cos(\theta/2)} \frac{v_{JaJb} + v_{JaJb}'}{2\cos(\theta/2)} \right)^* \right]. \] (3)

where \([P^{(K)}(\Omega', \Omega)]_{ij}\) is the Kth multipole component of the scattering phase matrix \((i, j = 0, 1, 2, 3)\), \(\theta\) is the scattering angle, \(\Delta v_D\) is the Doppler width (assumed to be the same for the two lines), and \(\epsilon' = \Gamma_1/\Gamma = A_{at}/C_{at}\), with \(A_{at}\) the Einstein coefficient for spontaneous emission from the upper to the lower term and \(C_{at}\) the superelastic collision relaxation rate. The function \(W\) is defined as

\[ W(a, v) = H(a, v) + i L(a, v), \] (4)

where \(H\) is the Voigt function and \(L\) is the associated dispersion profile. The reduced frequencies \(v_{JaJb}\) and \(v_{JaJb}'\) are given by

\[ v_{JaJb} = \frac{(v_{JaJb} - v)}{\Delta v_D}, \quad v_{JaJb}' = \frac{(v_{JaJb} - v)}{\Delta v_D}, \] (5)

with \(v_{JaJb}\) the Bohr frequency between the levels \(J_a\) and \(J_b\). Although elastic collisions have been neglected (consistently with the assumption of purely coherent scattering in the atom rest frame), we took into account their broadening effect in the evaluation of the broadening constant \(\Gamma\) appearing in Equation (3), and of the damping parameter \(a = \Gamma/\Delta v_D\).

3. THE SCATTERING POLARIZATION PATTERN ACROSS THE Mg II H AND K LINES

In order to estimate, in the absence of magnetic fields, the amplitude and shape of the scattering polarization pattern across the Mg II doublet, we have developed a non-LTE radiative transfer code based on the angle-averaged expression (e.g., Rees & Saliba 1982) of the redistribution matrix described in the previous section, including the contribution of an unpolarized continuum.

The numerical method of solution will be explained in detail in a forthcoming publication; it is based on a direct generalization to the PRD case of the Jacobian iterative scheme presented in Trujillo Bueno & Manso Sainz (1999). The initialization of the iterative calculation was done using the self-consistent solution of the corresponding unpolarized problem, which we have obtained by applying Uitenbroek’s (2001) radiative transfer code. This code has also been used to compute the inelastic and elastic collisional rates, as well as the continuum total opacity (including the UV line haze contribution) and emissivity.

Figure 2. \(Q/I\) profile across the Mg II \(h\) and \(k\) lines, calculated in the FAL-C model atmosphere for an LOS with \(\mu = 0.1\). Solid line: two-term atom PRD solution with \(J\)-state interference. Dashed line: two-term atom PRD solution neglecting \(J\)-state interference. Dotted line: two-level atom PRD solution for the \(k\) line transition. The reference direction for positive \(Q\) is the parallel to the nearest limb. The lower panel shows in more detail the line core regions.

In Figure 2 we show the results of the following PRD calculations of the \(Q/I\) profile of the emergent radiation for an LOS with \(\mu = 0.1\): (1) the full two-term atom solution obtained by taking into account the impact of interference between the two excited \(J\)-levels, (2) the two-term atom solution obtained by neglecting \(J\)-state interference effects, and (3) the solution for the Mg II \(k\) line alone assuming a two-level atom with \(J_1 = 1/2\) and \(J_u = 3/2\).

The upper panel of Figure 2 shows the overall structure of the \(Q/I\) profile. Consider first the two-term atom solutions obtained by including (solid line) and neglecting (dashed line) \(J\)-state interference. Clearly, the impact of \(J\)-state interference is very important, since it leads to much larger polarization amplitudes in the wings (about 10% at around \(\pm 15\) Å from the \(h\) and \(k\) line centers, respectively) and to a significant negative polarization feature between the \(h\) and \(k\) lines with a maximum amplitude of about \(-2\%\). The two-level atom solution for the \(k\) line (dotted line) produces wing polarization signals which are also significantly smaller than those corresponding to the full solution.

The lower panel of Figure 2 shows the details of the \(Q/I\) profile in the core regions of the Mg II \(h\) and \(k\) lines. The three solutions perfectly agree in the core of the \(k\) line, with a line-center amplitude of about \(2\%\). However, it can be observed
that the solution obtained by neglecting J-state interference (dashed line) does not show either the small asymmetry in the two negative Q/I peaks around the k line core, or the clear antisymmetric Q/I feature that the combined action of PRD and J-state interference effects produces around the center of the h line (whose polarizability is zero).

Although the hypothesis of purely coherent scattering in the atom’s rest frame is a good approximation in the core of the lines and between them, frequency redistribution effects due to elastic collisions might not be completely negligible going toward the wings of the lines (see Figure 1). In order to have a qualitative idea of the impact of this physical ingredient, we carried out a calculation including the contribution of the R_{III} redistribution function, under the limit of complete frequency redistribution (CRD) in the observer’s frame. We derived the expression of R_{III} from the CRD theory presented in LL04, including the effect of inelastic and superelastic collisions as in the case of R_{II}. We used the branching ratios α and (β^{(k)} - α) defined in Equations (1) and (2) (strictly valid in the case of a two-level atom).\(^4\) We assumed \(D^{(g)} = 0\) (i.e., we neglected the depolarizing effect of elastic collisions). As expected, the contribution of R_{III} is negligible in the core of the lines, and very small between them. However, as shown in Figure 3, it modifies in an appreciable way the amplitude of the Q/I profile in the wings of the lines. Note that the solution corresponding to the approximation of purely coherent scattering in the observer’s frame is also shown in Figure 3 (see the filled circles).

4. CONCLUDING COMMENTS

As shown in this Letter, the physics that controls the scattering polarization of the Mg II h and k lines is the joint action of PRD and J-state interference effects, which produces a complex Q/I pattern with sizable polarization maxima in the wings and a negative polarization feature at wavelengths between the two line centers (see Figure 2). These Q/I wing features can also be understood using the approximation of coherent scattering in the observer’s frame (see the filled circles in Figure 3).\(^5\) Another remarkable feature of the Q/I profile we have obtained is a clear antisymmetric Q/I signal around the center of the h line (see Figure 2), which can only be found when taking into account the joint action of PRD and J-state interference effects. In the core of the k line PRD effects alone are responsible of the triplet peak structure of the Q/I profile (see Figure 2).

The reported linear polarization is sensitive to a magnetic field through the Hanle effect, which operates only in the core of the k line. Here magnetic fields weaker than 100 G are expected to produce significant changes in the line-center polarization amplitude. As shown in Figure 2, it is a good news that the Q/I profile around the core of the Mg II k line can be well modeled through the two-level atom approximation, since this will facilitate the development of Hanle-effect diagnostic tools. In addition to the Hanle effect in the k line, complementary information on the magnetic field of the solar chromosphere could be obtained by measuring the circular polarization produced by the Zeeman effect in all the Mg II lines that are located between 2790 Å and 2805 Å (see caption of Figure 1). The Stokes V signals of all these lines should be measurable. For example, a volume filling magnetic field of 100 G inclined by 45° with respect to the LOS produces V/I amplitudes of about 1% in all these Mg II lines.

Spectroscopy of the Mg II h and k lines with novel space telescopes like the Interface Region Imaging Spectrograph (see Title 2012) will provide precious information on temperatures, flows, and waves. However, the magnetic field information is encoded in the spectral line polarization, whose measurement with high spatial and/or temporal resolution requires the development of larger aperture telescopes such as SOLAR-C (see Shimizu et al. 2011). The results shown in this Letter for the h and k lines of Mg II, and in Trujillo Bueno et al. (2011, 2012) for the Lyα lines of H I and He I, strongly encourage the development of FUV and EUV spectro-polarimeters for the new generation of solar space telescopes. Additional results obtained in different atmospheric models, along with the details of the methods used in this investigation, will be presented in a forthcoming paper.

\(^4\) Note that the factor \((1 - ε)\) is included in our R_{II} and R_{III} redistribution functions.

\(^5\) It is interesting to note that the observations performed by Henze & Stenflo (1987) could not confirm the negative polarization minimum between the h and k lines. Although the statistical significance of such pioneering observations is very marginal, a forthcoming investigation will include the interaction with the continuum polarization processes in order to study its possible impact on the polarization amplitudes across these lines.
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REFERENCES

Auer, L., Rees, D. E., & Stenflo, J. O. 1980, A&A, 88, 302
Belluzzi, L., & Trujillo Bueno, J. 2011, ApJ, 743, 3
Bommier, V. 1997a, A&A, 328, 706
Bommier, V. 1997b, A&A, 328, 726
Casini, R., & Landi Degl’Innocenti, E. 2007, in Plasma Polarization Spectroscopy, ed. T. Fujimoto & A. Iwamae (Berlin: Springer), 249
Domke, H., & Hubeny, I. 1988, ApJ, 334, 527
Fontenla, J. M., Avrett, E. H., & Loeser, R. 1993, ApJ, 406, 319
Henze, W., & Stenflo, J. O. 1987, Sol. Phys., 111, 243
Hummer, D. G. 1962, MNRAS, 125, 21
Jefferies, J. T., & White, O. R. 1960, ApJ, 132, 767
Landi Degl’Innocenti, E., Landi Degl’Innocenti, M., & Landolfi, M. 1997, in Science with THÉMIS, ed. N. Mein & S. Sahal-Bréchot (Paris: Obs. Paris-Meudon), 59
Landi Degl’Innocenti, E., & Landolfi, M. 2004, Polarization in Spectral Lines (Dordrecht: Kluwer)
Rees, D. E., & Saliba, G. J. 1982, A&A, 115, 1
Sampoorna, M., Trujillo Bueno, J., & Landi Degl’Innocenti, E. 2010, ApJ, 722, 1269
Shimizu, T., Tsuneta, S., Hara, H., et al. 2011, Proc. SPIE, 8148, 81480B
Smitha, H. N., Sampoorna, M., Nagendra, K. N., & Stenflo, J. O. 2011, ApJ, 733, 4
Stenflo, J. O. 1980, A&A, 84, 68
Title, A. R. 2012, in IEEE Proc., The Interface Region Imaging Spectrograph, in press
Trujillo Bueno, J. 2011, in ASP Conf. Ser. 437, Solar Polarization 6, ed. J. Kuhn et al. (San Francisco, CA: ASP), 83
Trujillo Bueno, J., & Manso Sainz, R. 1999, ApJ, 516, 436
Trujillo Bueno, J., Štěpán, J., & Belluzzi, L. 2012, ApJ, 746, L9
Trujillo Bueno, J., Štěpán, J., & Casini, R. 2011, ApJ, 738, L11
Uitenbroek, H. 1997, Sol. Phys., 172, 109
Uitenbroek, H. 2001, ApJ, 557, 389