Thermal Leptogenesis Scenarios in the Restrictive Left-Right Symmetric Model

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We investigate thermal leptogenesis scenarios in the left-right symmetric extension of the standard model. Imposing the realization of $D$-parity below the GUT scale and grand unification make our model more restrictive and improve its predictive capability. In this case, a $D$-parity odd singlet plays a critical role. This singlet may cause a very large mass hierarchy of the $SU(2)_{L,R}$ triplet scalars.

We test our model by investigating baryogenesis via leptogenesis. Our model has two sources for the lepton number asymmetry in the universe, the heavy right-handed neutrinos $N_i$ ($i=1, 2, 3$) and the $SU(2)_L$ triplet scalar $\Delta_L$. Leptogenesis scenarios can be categorized according to these mass scales. If the light neutrinos are Majorana and have a hierarchical mass spectrum, we can obtain a successful result in leptogenesis through $N_1$-decay. However, we find that the normal mass hierarchy of the light neutrinos is inconsistent with leptogenesis through $\Delta_L$-decay in the SM. In order to obtain successful thermal leptogenesis through $\Delta_L$-decay, we need to introduce additional Higgs doublets. This result suggests the two-Higgs doublet model with an $SU(2)_L$ triplet scalar.

§1. Introduction

The $SO(10)$ gauge theory is regarded as a strong possibility for the grand unification theory (GUT). The first important feature of this theory is the unification of three gauge interactions. Because of this feature, the Gell-Mann–Nishijima relation can be rewritten as $Q = I^3_L + I^3_R + (B - L)/2$; i.e. the electric charge can be quantized and related to classically familiar charges, the baryon number $B$ and the lepton number $L$. Here $I^3_L$ and $I^3_R$ denote the third components of the left-handed and right-handed isospins, respectively. The second important feature of this theory is matter unification. In contrast to $SU(5)$ GUT, in the $SO(10)$ gauge theory, each quark and lepton corresponds to a 5-bit eigenstate of the Cartan subalgebra:

$$\nu_{eL} = \{\downarrow\downarrow\downarrow; \uparrow\downarrow\}, \quad u^c_r = \{\downarrow\uparrow\uparrow; \downarrow\uparrow\}, \quad u^b_L = \{\uparrow\uparrow\downarrow; \uparrow\downarrow\}, \quad u^q_L = \{\uparrow\uparrow\uparrow; \uparrow\uparrow\},$$

$$\nu_{eL}^c = \{\uparrow\uparrow\uparrow; \uparrow\uparrow\downarrow\},$$

$$d^c_L = \{\uparrow\uparrow\downarrow; \downarrow\uparrow\}, \quad d^b_L = \{\uparrow\downarrow\uparrow; \downarrow\uparrow\}, \quad d^q_L = \{\downarrow\uparrow\uparrow; \downarrow\uparrow\},$$

where $\uparrow$ and $\downarrow$ in the kets denote the eigenvalues of the five generators of the Cartan subalgebra: The first three arrows are for $SO(6) \simeq SU(4)_c$, and last two are for $SO(4) \simeq SU(2)_L \times SU(2)_R$. Consequently, by adding the three right-handed neutrinos, we can obtain an economic picture of matter in the universe.

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Unfortunately, however, in the $SO(10)$ gauge theory the GUT scale physics is often highly suppressed. For this reason, we focus on the left-right symmetric extension of the standard model (SM) as the low-energy effective theory of the $SO(10)$ theory. The left-right symmetric model (LR)$^3$ is one of the oldest extensions of the SM. Because the LR model, like the $SO(10)$ theory, naturally introduces right-handed neutrinos, it is compatible with recently obtained evidence of nonzero neutrino masses. Generally, this model requires $SU(2)$ triplet scalars in order to reproduce the very small neutrino masses.$^4$ These triplets may provide clear evidence of the utility of the LR model. The LR models studied in the literature have more free parameters and are less predictive. We can obtain more restrictive scenarios by considering the LR with $D$-parity and grand unification.

In order to approach ultra high energy scales, we consider baryogenesis via leptogenesis in the universe.$^5$ The WMAP collaboration obtained the baryon-to-photon number ratio with high precision:

$$\eta_{\text{CMB}}^C = \frac{n_B}{n_\gamma} = \frac{n_b - n_{\bar{b}}}{n_\gamma} = (6.14 \pm 0.25) \times 10^{-10}. \quad (2)$$

Also, a second determination of $\eta_B$ can be obtained from nucleosynthesis, i.e. the abundances of the light elements D, $^3$He, $^4$He and $^7$Li:$^7$

$$\eta_{\text{BBN}}^B = (2.6 - 6.2) \times 10^{-10}. \quad (3)$$

The full $SO(10)$ GUT framework has been extensively studied.$^{11}$ We analyze whether the observed values of $\eta_B = \mathcal{O}(10^{-10})$ can be reproduced in order to investigate our restrictive minimal LR model. Also, through this analysis we investigate whether we are able to detect the $SU(2)_L$ triplet scalar particles in the accelerator experiments as evidence of the utility of the LR model.

This paper is organized as follows. In §2.1, we briefly review the LR extended model and formulate our model. As remarked above, in order to make our model more restrictive and predictive, we impose D-parity restoration and some grand unification like that of $SO(10)$. Here we regard a GUT as describing the situation in which all the gauge couplings come together at a high energy scale (§2.2) and the known quark and lepton fields are unified (§2.3). Note that this GUT condition does not necessarily imply $SO(10)$ GUT. In §2 we show that the hierarchical structure of the GUT scale and the $D$-parity restoration scale is essential. In §2.3 we derive the neutrino mass spectrum. We succeed in obtaining a hierarchical heavy neutrino spectrum by means of the normal hierarchy of light neutrinos and a few assumptions. We study baryogenesis through leptogenesis for certain specific scenarios in §3.

## §2. Models

### 2.1. Building more constrained models

We consider the left-right symmetric breakdown of grand unification. The simplest possibility for grand unification of this type is $SO(10)$, and its minimal sets for the Higgs multiplets are listed below:
• **GUT → G_{3221} × D (→ G_{3221}) → G_{321}**
  This requires the set of scalars 210, 126 ⊕ 126 and 10 of the SO(10) group. An SU(4)c adjoint representation (15, 1, 1) ∈ 210 breaks SO(10).

• **GUT → G_{422} × D (→ G_{422}) → G_{3221} → G_{321}**
  This scenario requires 54, 45, 126 ⊕ 126 and 10. An SU(4)c adjoint representation (15, 1, 1) ∈ 45 breaks G_{422}.

• **GUT → G_{422} × D (→ G_{422}) → G_{421} → G_{321}**
  Although in this breaking chain, the essential set of scalars is the same as that above, i.e. 54, 45, 126 ⊕ 126 and 10, the SU(2)_R adjoint representation (1, 1, 3) ∈ 45 partially breaks the right-handed isospin symmetry.

Here G_{3221}, G_{422}, G_{421} and G_{321} denote the LR gauge group SU(3)c × SU(2)L × SU(2)_R × U(1)_B−L, the Pati-Salam (PS) group SU(4)c × SU(2)L × SU(2)_R,^8 SU(4)c × SU(2)_R and the SM gauge group SU(3)c × SU(2)_L × U(1)_Y, respectively. Also, the numbers in the brackets are the PS quantum numbers. In order to obtain more predictive models, we consider Michel’s conjecture. Generally, in SO(10) grand unification, quarks and leptons are assigned to three 16-dimensional spinor representations, as listed in Eq. (1), and the gauge fields are in a 45-dimensional adjoint representation.

It should be noted that the PS-singlet in 210 is axial under D-parity. However, that in 54 is not. The D-parity is a Z₂-symmetry within the SO(10) symmetry, which acts as a left-right transformation for the SU(2)_L × SU(2)_R representations. This difference is important in our model building. Furthermore, because the first scenario requires the fewest breaking steps, it is likely to be the most constrained. Considering the minimal LR model and using the GUT realization as a boundary condition at a higher energy scale, we find that this is indeed the case. We present the details in the next subsection.

### 2.2. Numerical ansatz

First, we present all the required scalar representations underlined above:

\[
\sigma = (1, 1, 1, 0) \in 210, \\
\Delta_L = (1, 3, 1, +2) = \begin{pmatrix}
\Delta_L^+ / \sqrt{2} \\
\Delta_L^0 \\
-\Delta_L^- / \sqrt{2}
\end{pmatrix} \in 126, \\
\Delta_R = (1, 1, 3, +2) = \begin{pmatrix}
\Delta_R^+ / \sqrt{2} \\
\Delta_R^0 \\
-\Delta_R^- / \sqrt{2}
\end{pmatrix} \in 126, \\
\Phi = (1, 2, 2, 0) \in 10.
\]

Here we have presented the G_{3221} quantum numbers. The bidoublet Φ corresponds to the two Higgs doublets; i.e. the SM doublet H and the extra doublet H'. This model is known as the minimal renormalizable LR model with spontaneously broken D-parity. It is depicted as

\[
G_{3221} \times D \xrightarrow{(σ)} G_{3221} \xrightarrow{(Δ_R)} G_{321} \xrightarrow{(Φ)} QCD \times QED,
\]
and the required vacuum expectation values (vevs) are given by

$$
\langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix}, \quad \langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \quad \langle \Phi \rangle = \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{pmatrix}.
$$

(9)

Here we ignored the relative phase between $\kappa_1$ and $\kappa_2$. Phenomenologically, the relations $v_R \gg \kappa_+ \gg v_L$ are required, where $\kappa_+$ is the standard electroweak breaking vev, $\kappa_+^2 \equiv \kappa_1^2 + \kappa_2^2$. In this model, according to the extreme value analysis of the Higgs potential, we have the following relations:

$$
v_L \sim -\frac{\beta \kappa_+^2}{2 M \eta_P} v_R\tag{10}
$$

and

$$
M^2_{\Delta_L} = \mu^2_\Delta - (M \eta_P + \gamma \eta_P^2),
$$

$$
M^2_{\Delta_R} = \mu^2_\Delta + (M \eta_P - \gamma \eta_P^2),
$$

(11, 12)

where each coupling is defined as follows:

$$
V \ni M \sigma \left[ \text{Tr} \left( \Delta_L^\dagger \Delta_L \right) - \text{Tr} \left( \Delta_R^\dagger \Delta_R \right) \right]
+ \gamma \sigma^2 \left[ \text{Tr} \left( \Delta_L^\dagger \Delta_L \right) + \text{Tr} \left( \Delta_R^\dagger \Delta_R \right) \right]
+ \beta_1 \left[ \text{Tr} \left( \phi \Delta_R \phi \Delta_L \right) + \text{Tr} \left( \phi \Delta_R \phi \Delta_L \right) \right]
+ \beta_2 \left[ \text{Tr} \left( \phi \Delta_R \phi \Delta_L \right) + \text{Tr} \left( \phi \Delta_R \phi \Delta_L \right) \right]
+ \beta_3 \left[ \text{Tr} \left( \phi \Delta_R \phi \Delta_L \right) + \text{Tr} \left( \phi \Delta_R \phi \Delta_L \right) \right]
+ \beta_4 \left[ \text{Tr} \left( \phi \Delta_R \phi \Delta_L \right) + \text{Tr} \left( \phi \Delta_R \phi \Delta_L \right) \right],
$$

(13a)

$$
\beta_{ab} = \begin{pmatrix} \beta_1 & \beta_2 \\ \beta_3 & \beta_4 \end{pmatrix}.
$$

(13b)

In Eq. (10), $\beta$ denotes the typical size of the dimensionless couplings $\beta_i$ ($i = 1, 2, 3, 4$), and $\eta_P$ denotes the vacuum expectation value of the $\sigma$ field potential, which is defined as

$$
\eta_P \equiv \langle \sigma \rangle = \frac{\mu_\sigma}{\sqrt{2 \lambda_\sigma}}, \quad V_\sigma = -\mu_\sigma^2 \sigma^2 + \lambda_\sigma \sigma^4.
$$

(14)

A relation like Eq. (10) represents the vev seesaw mechanism. We see that Eqs. (11) and (12) are not symmetric, in spite of the left-right symmetric framework. This results from the fact that the scalar $\sigma$ is an axial scalar under the $D$-parity. We obtain two important results from these relations. First, according to Eq. (10), the very large magnitude of the $D$-parity breaking scale $\eta_P$ results in the smallness of the vev of $\Delta_L$. Below we report the results of the numerical verification of this fact. Second, let us consider Eqs. (11) and (12). If the terms $M \eta_P$ and $\gamma \eta_P^2$ are of
the same order, they cancel each other, and then we have $M_{\Delta R}^2 \sim \mu^2$. If this is the case, the squared mass of $\Delta_L$ would be given by $M_{\Delta R}^2 - 2\gamma^2 \eta_P^2$. Hence, it would be possible that the left-handed triplet $\Delta_L$ is much lighter than the right-handed triplet $\Delta_R$:

$$M_{\Delta L}^2 \ll M_{\Delta R}^2.$$  \hspace{1cm} (15)

Below we take this possibility into account. The singlet $\sigma$ may cause a very large mass hierarchy between the $SU(2)_L$ and $SU(2)_R$ triplet scalars. As discussed in greater detail below, in this paper we consider a mass spectrum characterized by the following:

$$100^2 \text{[GeV]}^2 \lesssim M_{\Delta L}^2 \lesssim M_{\Delta R}^2.$$  \hspace{1cm} (16)

Since it is absolutely imperative for leptogenesis to generate the lepton asymmetry, we are interested in determining the point of which the $SU(2)_R \times U(1)_{B-L}$ symmetry breaks down. To this end, we solve the two-loop renormalization equations for the gauge couplings. As remarked above, in order to obtain more restrictive models, we impose two boundary conditions. The first boundary condition is the restoration of the $D$-parity. This means that above a high energy scale, the $SU(2)_R$ gauge coupling evolves along with the $SU(2)_L$ one. The second boundary condition is grand unification, which is essential to include $U(1)$ gauge groups. This constraint suggests that the hypercharge and $B-L$ charge are normalized as follows:

$$\tilde{Y} = \sqrt{\frac{3}{5}} Y, \quad \tilde{V} = \sqrt{\frac{3}{2}} \frac{B-L}{2}.$$  \hspace{1cm} (17)

Then at the GUT scale, we have

$$\alpha_{s}(M_{\text{GUT}}) = \alpha_{L}(M_{\text{GUT}}) = \alpha_{R}(M_{\text{GUT}}) = \alpha_{V}(M_{\text{GUT}}).$$  \hspace{1cm} (18)

Before presenting the results, let us discuss the number of free input parameters. In this renormalization group analysis, we have four input parameters, $M_{Z_R}$, $\theta_R$, $m_{H'}$, and $M_{\Delta_L}$. Here, $m_{H'}$ represents the mass of the second doublet Higgs, and the mixing angle $\theta_R$ is defined by

$$\left( \begin{array}{c} B_0^0 \\ Z_0^0_R \end{array} \right) = \left( \begin{array}{cc} \cos \theta_R & \sin \theta_R \\ -\sin \theta_R & \cos \theta_R \end{array} \right) \left( \begin{array}{c} B_0^0 \\ W_0^0_R \end{array} \right).$$  \hspace{1cm} (19)

Here we denote the two mass eigenstates by $B^0$ and $Z_R^0$. $B^0$ mediates the $U(1)_Y$ gauge interaction. We assume that $\Delta_R$ exists at the same energy scale as $Z_R$, and therefore we do not treat $M_{\Delta_R}$ as a free input parameter. At this stage, high-precision measurements of $\alpha_2(M_W)$ would help a great deal. Because the current observational error of the electroweak gauge coupling is very small, the allowed range of the mixing angle $\theta_R$ of the neutral $SU(2)_R$ boson $W_R^0$ and $U(1)_{B-L}$ boson $B^0$ is extremely narrow. Consequently, we can determine $\cos \theta_R$ almost uniquely, and thus we can obtain a strong constraint on the $Z_R^0$ ($\simeq Z_R^0$) boson mass. In contrast to the usual left-right symmetric models, the consequence of this argument is that the set of three parameters, $\{M_{Z_R}, m_{H'}, M_{\Delta_L}\}$, provides unique values of the $SU(2)_R \times U(1)_{B-L}$ breaking scale, $v_R$, the $D$-parity scale, $\eta_P$, and the GUT scale, $M_{\text{GUT}}$. $v_R$ can be
calculated from $\alpha_2(Q)$, $\alpha_Y(Q)$, $M_{Z_R}$ and $\theta_R$ with a Newton–Raphson-like method. Note that we can exclude the regions $M_{\Delta_L} > M_{Z_R}$ ($= M_{\Delta_R}$) and $M_{\Delta_L} < 100$ GeV. The former comes from Eq. (15), and the latter is concluded from experimental results. As a direct consequence of Eq. (15), the two triplets $\Delta_L$ and $\Delta_R$ do not mix. Then, note that the experimentally detected doubly-charged Higgs boson $h^{++}$ can be identified as pure $\Delta^{++}_L$. Dilepton detection puts limits on the mass of the doubly-charged Higgs boson $h^{++}$. The decay modes of $\mu\mu$, $ee$ and $e\mu$ yield the results $M_{h^{++}} > 136$, 133 and 115 GeV$^{17}$ respectively. Also, experiments searching for the long-lived doubly-charged Higgs yield the relation $M_{h^{++}} > 134$ GeV$^{18}$ Thus we can exclude the latter region.

We present the solutions of the renormalization equations in Fig. 1. Here we assume $m_{H'}$ to be 200 GeV or $M_{Z_R}$. We find that, in either case, we have $10^{10} \lesssim M_{Z_R} \lesssim 10^{12}$ GeV. However we also find that our model does not constrain the value of $M_{\Delta_L}$.

Here we give some illustrations. First we consider the heavier $SU(2)_L$ triplet. We start with Fig. 2, where we set $m_{H'} = M_{Z_R}$ and $M_{\Delta_L} = 6 \times 10^9$ GeV. In this case, the low-energy effective theory is the standard model (SM). Figure 2 reveals to us that $v_R = 8.3 \times 10^9$ GeV and $\eta_P \gtrsim 10^{11}$ GeV. Next, we consider Fig. 4, which can be obtained by setting $m_{H'} = 200$ GeV and $M_{\Delta_L} = 1 \times 10^{10}$ GeV. This case leads to the two Higgs doublet model (2HDM) at low energy scales. We obtain

![Fig. 1. The allowed regions for grand unification.](https://academic.oup.com/ptp/article-abstract/117/6/1099/1917443)
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$v_R = 1.9 \times 10^{10}$ GeV and $\eta_P \gtrsim 10^{11}$ GeV. These two parameterizations are located close to the upper ends of Fig. 1.

Next we consider the cases of the lighter triplet. These are located close to the lower ends of Fig. 1. First, we depict the case of $m_{H'} = M_{Z_R}$ and $M_{\Delta_L} = 100$ GeV in Fig. 3. We obtain $v_R = 2.6 \times 10^{11}$ GeV and $\eta_P \gtrsim 10^{12}$ GeV. Then, at the low energy scale, we have the SM with an $SU(2)_L$ triplet. Finally, let us consider the case in which both $H'$ and $\Delta_L$ are near the electroweak scale. We depict the case in which $m_{H'} = 200$ GeV and $M_{\Delta_L} = 100$ GeV in Fig. 5. We obtain $v_R = 1.3 \times 10^{12}$ GeV and $\eta_P \gtrsim 10^{13}$ GeV, and we find that the low-energy theory becomes 2HDM with an $SU(2)_L$ triplet.

Now we make an important comment on these results. A commonly-observed feature is that the values of $\eta_P$ for the computed $D$-parity breaking scales are much higher than the obtained $SU(2)_R \times U(1)_{B-L}$ breaking scales. We find that the values of $\eta_P/v_R$ are larger than $O(10)$, with a conservative estimate. This is mainly attributable to the mass threshold effects at $v_R$. Applying this observation to Eq. (10) gives a small value of $v_L$. We use this argument below. Then, we can use the calculations as guidelines for considering leptogenesis scenarios.

Fig. 2. SM. $m_{H'} = M_{Z_R}$ and $M_{\Delta_L} = 6.0 \times 10^9$ GeV.

Fig. 3. SM with an $SU(2)_L$ triplet. $m_{H'} = M_{Z_R}$ and $M_{\Delta_L} = 100$ GeV.

Fig. 4. 2HDM. $m_{H'} = 200$ GeV and $M_{\Delta_L} = 1.0 \times 10^{10}$ GeV.

Fig. 5. 2HDM with an $SU(2)_L$ triplet. $m_{H'} = 200$ GeV and $M_{\Delta_L} = 100$ GeV.
2.3. Neutrino mass spectrum

Before solving the sets of Boltzmann equations for leptogenesis, we need information concerning the neutrino mass spectrum. Generally, the leptonic Yukawa coupling is given by

\[
\mathcal{L}_{\text{Yukawa}} = Y_{ij} \ell_i L_j \Phi + \bar{Y}_{ij} \ell_i L_j \Phi + \text{h.c.} + Y_{\Delta ij} \left[ (\ell_i L_i)^c \ell_j L_j \Delta_L + (\ell_i R_i)^c \ell_j R_j \Delta_R \right] + \text{h.c.}
\]  

The left-right symmetry shows that the Dirac-Yukawa couplings \(Y\) and \(\tilde{Y}\) and the Majorana-Yukawa coupling \(Y_{\Delta}\) are hermitian and symmetric, respectively, in the family space: \(Y_{ij} = Y_{ji}^\dagger, \tilde{Y}_{ij} = \tilde{Y}_{ji}^\dagger\) and \(Y_{\Delta ij} = Y_{\Delta ji}\). The symmetry breaking (8) results in the following mass matrix of the light neutrinos:

\[
m_\nu = m_\nu^I + m_\nu^I = Y_{\Delta} v_L - M_D M_R^{-1} M_D^T. \quad M_R^{-1} = \frac{Y_{\Delta}}{v_R}^{-1}.
\]  

The generation of the mass suppressed by the very large \(M_R\) is called the type-I seesaw mechanism,\(^{19}\) while the mass originating from the very small \(v_L\) is the type-II seesaw mass. As remarked in the last subsection, in all cases, we find \(\eta_P / v_R > \mathcal{O}(10)\). This result verifies the validity of assuming the hierarchical structure \(v_L \ll \max(\kappa_1, \kappa_2) \ll v_R \ll \eta_P\) [see Eq. (10)]. Let us assume that the type-I mass dominates in the effective neutrino mass matrix (21):

\[
m_\nu \simeq m_\nu^I = -M_D M_R^{-1} M_D^T.
\]  

The light neutrino mass matrix \(m_\nu\) can be written as

\[
m_\nu = U^* m_\nu^{\text{diag}} U^\dagger,
\]  

where \(U\) denotes the light neutrino mixing matrix, which is given by \(U = U_{\text{PMNS}} P\):

\[
U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} = \begin{pmatrix} c_{12} c_{13} \\ -s_{12} c_{23} - c_{12} s_{13} s_{23} e^{i\delta} \\ s_{12} s_{23} - c_{12} s_{13} s_{23} e^{i\delta} \end{pmatrix} \begin{pmatrix} s_{12} c_{13} & c_{12} c_{23} - s_{12} s_{13} s_{23} e^{i\delta} & s_{13} e^{-i\delta} \\ -c_{12} s_{23} - c_{12} s_{13} s_{23} e^{i\delta} & c_{12} c_{23} - s_{12} s_{13} s_{23} e^{i\delta} & c_{13} s_{23} \\ s_{12} s_{23} - c_{12} s_{13} s_{23} e^{i\delta} & c_{12} s_{23} - c_{12} s_{13} s_{23} e^{i\delta} & c_{13} c_{23} \end{pmatrix},
\]  

\[
P = \text{diag}(e^{i\alpha}, e^{i\beta}, 1).
\]  

Here we use the notation \(c_{12} = \cos \theta_{12}, s_{12} = \sin \theta_{12}\), and so on. We identify \(\theta_{12}\) and \(\theta_{23}\) as the solar neutrino mixing angle \(\theta_\odot\) and the atmospheric neutrino mixing angle \(\theta_{\text{atm}}\), respectively. Then we substitute the following neutrino oscillation data\(^{20}\) into \(U_{\text{PMNS}}\):

\[
\Delta m^2_\odot = \begin{pmatrix} 7.9 \pm 0.3 \\ 7.1 - 8.9 \end{pmatrix} \times 10^{-5}, \quad \sin^2 \theta_\odot = \begin{pmatrix} 0.30^{+0.02}_{-0.02} \\ 0.24 - 0.40 \end{pmatrix},
\]  

\[
|\Delta m^2_{\text{atm}}| = \begin{pmatrix} 2.5^{+0.20}_{-0.25} \\ 1.9 - 3.2 \end{pmatrix} \times 10^{-3}, \quad \sin^2 \theta_{\text{atm}} = \begin{pmatrix} 0.50^{+0.08}_{-0.07} \\ 0.38 - 0.64 \end{pmatrix},
\]
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where the lower values represent the 95% confidence intervals. Below, we concentrate on the normal hierarchical mass spectrum of the light neutrinos, i.e. \( m_1 \ll m_2 \ll m_3 \). Thus, we identify \( \Delta m^2_\odot \) and \( |\Delta m^2_{\text{atm}}| \) as \( m_2^2 - m_1^2 \) and \( m_3^2 - m_2^2 \simeq m_3^2 - m_1^2 \), respectively. Reactor neutrino oscillation experiments provide the following results:20,21)

\[
\sin^2 \theta_{13} < \begin{cases} 
0.027(0.048) & \text{CHOOZ + atm. + LBL,} \\
0.033(0.071) & \text{solar + KamLAND,} \\
0.020(0.041) & \text{3-\( \nu \) global data.}
\end{cases}
\]

(28)

Here, the values in parentheses are the 95% upper confidence limits. Although we have five input parameters, \( m_1, \theta_{13}, \delta, \alpha \) and \( \beta \), the lightest mass eigenvalue \( m_1 \) and the reactor neutrino angle \( \theta_{13} \) are highly constrained. Hereafter, we assume that \( \theta_{13} = 0.02 \). Then we have three free input parameters, \( \delta, \alpha \) and \( \beta \), which are all \( CP \)-violating phases.

In order to obtain the neutrino mass spectrum from these experimental data, we make one more assumption. First, we use the up-down unification relation, i.e. the neutrino mass Dirac mass matrix22)

\[
M_D = \frac{m_t}{m_b} M_\ell = \frac{m_t}{m_b} \text{diag}(m_e, m_\mu, m_\tau).
\]

(29)

Then, we use the basis in which the charged lepton mass matrix \( M_\ell \) is diagonal and real-valued. The GUT realization supports this assumption: The leptons belong to some large representation of the GUT group with the quarks. Thus, it seems natural that the lepton Dirac mass spectrum would be proportional to the quark Dirac mass spectrum. Combining the above considerations, we obtain

\[
M_R = \frac{m_t^2}{m_b^2} M_\ell m_\nu^{-1} M_\ell = \frac{m_t^2}{m_b^2} \begin{pmatrix} m_e & m_\mu & m_\tau \end{pmatrix} U_{\text{PMNS}} P^2 \begin{pmatrix} m_1 & m_2 & m_3 \end{pmatrix} U_{\text{PMNS}}^T \begin{pmatrix} m_e & m_\mu & m_\tau \end{pmatrix}.
\]

(30)

This gives the mass eigenvalues and phases \( M_i = |M_i| e^{i\phi_i/2} \) \( (i = 1, 2, 3) \). In the following, for simplicity we assume maximal \( CP \)-violation, and thus we set the Dirac and Majorana \( CP \)-phases \( \delta, \alpha \) and \( \beta \) as \( \delta = \alpha = \pi/2 \) and \( \beta = \pi \). As a result, we obtain the following values for \( 0.0001 \leq m_1 \leq 3 \text{ eV} \):

\[
\begin{align*}
1.70 \times 10^8 & \lesssim M_1 \lesssim 1.55 \times 10^{10} \text{ GeV}, \\
3.52 \times 10^{10} & \lesssim M_2 \lesssim 1.96 \times 10^{12} \text{ GeV}, \\
2.03 \times 10^{13} & \lesssim M_3 \lesssim 5.25 \times 10^{14} \text{ GeV}.
\end{align*}
\]

(31)

According to Ref. 25), we have two choices:

1. Before the leptogenesis mechanism begins to act, there exists no initial \( N_1 \)-abundance. In the case of the hierarchical neutrino mass spectrum \( m_1 \ll m_2 \ll m_3 \), which we consider now, the relation \( M_1 \gtrsim 2.4 \times 10^9 \text{ GeV} \) is required.
2. If the GUT interactions progress rapidly enough, $N_1$ is in thermal equilibrium in this temperature region. Then, the relation $M_1 \gtrapprox 4.9 \times 10^8$ GeV is compatible with the hierarchical neutrino mass spectrum. Summarizing these two constraints, in the following, we consider the case $4.9 \times 10^8 \lesssim M_1 \lesssim 1.55 \times 10^{10}$ GeV. Finally, we are ready to compute the generation of the net lepton and baryon numbers.

§3. Thermal leptogenesis

Our framework provides two sources of lepton asymmetry, the lightest heavy neutrino $N_1$ and the $SU(2)_L$ triplet scalar representation $\Delta_L$. The produced $CP$-asymmetries are defined as

$$\epsilon_{N_j} \equiv \frac{1}{8\pi} \sum_k \text{Im}[(Y^\dagger \tilde{Y})_{jk}(\tilde{Y}^\dagger Y)_{jk}]f(x_k),$$

$$\epsilon_{\Delta} \equiv 2\frac{\Gamma(\Delta_L \rightarrow \ell_i \ell_j) - \Gamma(\Delta_L \rightarrow \ell_i \ell_j)}{\Gamma(\Delta_L \rightarrow \ell_i \ell_j) + \Gamma(\Delta_L \rightarrow \ell_i \ell_j)},$$

where the symbol $h$ denotes the SM Higgs doublet $H$ or $H'$. First, let us consider the $N$-decaying contribution $\epsilon_{N_j}$. This can be computed as

$$\epsilon_{N_j} = \frac{1}{8\pi} \sum_k \text{Im}[(Y^\dagger \tilde{Y})_{jk}(\tilde{Y}^\dagger Y)_{jk}]f(x_k),$$

$$f(x) = \sqrt{x} \left\{ 1 - (1 + x) \ln \left( 1 + \frac{1}{x} \right) + \frac{1}{1 - x} \right\},$$

where $x_k \equiv M_k^2/M_j^2$. We can replace the Yukawa couplings by the mass matrix:

$$\epsilon_{N_j} = \frac{1}{8\pi \kappa^2_j (M_D^\dagger M_D)_{jj}} \sum_{k\neq j} \text{Im}[(M_D^\dagger M_D)_{jk}]^2 f(x_k).$$

As remarked in the previous section, the heavy neutrino mass spectrum is hierarchical. This suggests that while the heavier $N_2$ and $N_3$ decay, the lightest $N_1$ is still in equilibrium. In other words, the lepton number asymmetry generated by $N_2$ and $N_3$ decay processes should be canceled by the lepton number violating scatterings, due to the presence of $N_1$. Then, we find that only the $N_1$ decay processes are dominant.
in the asymmetry $\epsilon_{N_j}$:

$$\sum_{N_j} \epsilon_{N_j} \simeq \epsilon_1 = \frac{3}{16\pi^2} \sum_{k=2,3} \text{Im}[(M_D^\dagger M_D)_{11}^2 M_1 M_k].$$  (37)

Next, we consider the $N_j$ decay, including the virtual $\Delta_L$, as typified by Fig. 7:

$$\epsilon_{N_j}^\Delta = -\frac{1}{2\pi} \sum_{k,l} \sum_{a,b} \text{Im}[Y_{kj}^{*}\tilde{Y}_{lj} Y_{\Delta lk}^* \beta_{ab} v_R g(x_j)],$$  (38)

$$g(x_j) = 1 - \frac{M_2^2}{M_j^2} \ln \left( 1 + \frac{M_j^2}{M_2^2} \right).$$  (39)

Substituting the couplings for the mass matrices, we find

$$\epsilon_{N_j}^\Delta = -\frac{1}{2\pi v_L M_j (M_D^\dagger M_D)_{jj}} \sum_{k,l} \sum_{a,b} \text{Im}[(M_D)_{ij}^* (M_D)_{kj}^* (m_\nu^\beta_{ab} v_R) g(x_j)].$$  (40)

Our last task is to compute the $\Delta_L$ decay contribution, such as that depicted in Fig. 8. This is given by

$$\epsilon_\Delta = \frac{1}{8\pi} \sum_k \sum_{a,b} M_k \sum_{i,j} \left| Y_{\Delta ij} \right|^2 \frac{\text{Im}[Y_{kj}^{*}\tilde{Y}_{lj} Y_{\Delta ij}]}{M_2^2 + \sum_{a,b} |\beta_{ab}|^2 v_R^2} \ln \left( 1 + \frac{M_2^2}{M_k^2} \right).$$  (41)

3.1. SM-type scenario

First, we consider the parametrization depicted in Fig. 2, where $M_{\Delta_L} = 6 \times 10^9$ GeV. This parametrization requires a very large mass hierarchy in the doublet Higgs sector, namely $m_H \ll m_{H'}$. If we employ the smallest value of $M_1$ from Eq. (31), we find that lepton asymmetry is effectively generated in the temperature range

$$100 \text{ GeV} \sim T_{EW} < T_{sph}^{SM} < M_1.$$  (42)
According to Fig. 2, the surviving gauge symmetry in this region is that of the SM. Then, in addition to the sphaleron processes, all the interactions in the ordinary SM are in thermal equilibrium. These equilibrium interactions cause the chemical potentials of all the particles to be interdependent. Furthermore, because the universe must be neutral, the conserved charges, i.e. the third component of the left-handed isospin \( I_L^3 \) and the hypercharge \( Y \), must be zero. \( I_L^3 \) and \( Y \) are given by

\[
I_L^3 = \frac{g_L^2 T^3}{6} \left\{ \frac{1}{2} \sum_i \left( \sum_{\text{color}} (\mu_{u_L^i} - \mu_{d_L^i}) + (\mu_{\nu_L^i} - \mu_{e_L^i}) \right) + (\mu_{h^+} - \mu_{h^0}) + 4\mu_{W^+} \right\},
\]

\[
Y = \frac{g_Y^2 T^3}{6} \left[ \sum_i \left( \sum_{\text{color}} \left( \frac{1}{3}(\mu_{u_L^i} + \mu_{d_L^i}) + \frac{4}{3} \mu_{u_R^i} - \frac{2}{3} \mu_{d_R^i} \right) - (\mu_{\nu_L^i} + \mu_{e_L^i}) - 2\mu_{e_R^i} \right) \right] + 2(\mu_{h^+} + \mu_{h^0}).
\]

In the temperature region (42), because the \( SU(2)_L \) gauge interactions of up-type quarks are in thermal equilibrium, all \( u_L^i \) are sufficiently mixed. Therefore, we cannot distinguish the chemical potentials of up-type quarks, and hence we have \( \mu_{u_L^i} \equiv \mu_{u_L^i} \) \((i = 1, 2, 3)\). Similarly, we can ignore the generation index of down-type quarks. Then we obtain \( \mu_{d_L^i} \equiv \mu_{d_L^i} \) \((i = 1, 2, 3)\). The equilibrium Yukawa interactions lead to \( \mu_{u_R^i} \equiv \mu_{u_R^i} \) and \( \mu_{d_R^i} \equiv \mu_{d_R^i} \) \((i = 1, 2, 3)\). Similarly, for the lepton sector we obtain \( \mu_{\nu_L^i} \equiv \mu_{\nu_L^i} \), \( \mu_{e_L^i} \equiv \mu_{e_L^i} \), and \( \mu_{e_R^i} \equiv \mu_{e_R^i} \). Consequently, we find \( \mu_B = \mu_{W^0} = \mu_{W^+} = 0 \). Because \( SU(2)_L \times U(1)_Y \) gauge interactions are in thermal equilibrium, this is exactly what we expected. Also, we obtain the well-known formula

\[
B = \frac{28}{79} (B - L) = -\frac{28}{51} L.
\]

Since \( M_1 < M_\Delta \), using Eq. (10), we can write Eq. (40) as

\[
\sum_{N_j} \epsilon_{N_j}^A \simeq \epsilon_{N_1}^A = -\frac{3 M_1 \eta_{pl}}{8 \pi \kappa_+^2 M_1 (M_D^\dagger M_D)_{11}} \sum_{k,l} \text{Im} [(M_D)^{i_1 j_1} (m_{pl}^{i_1 j_1})_{lk} (M_D^*)_{k1}].
\]

Here, we consider the case that the dominant \( CP \)-asymmetry comes from the \( N_1 \) decay, i.e. \( \epsilon \sim \epsilon_{N_1} \). Then, the net lepton number is determined by the ordinary \( N_1 \) decay process and the lepton number violating scattering. This situation has been extensively studied. The Boltzmann equation (BE) for the \( N_1 \) abundance is written

\[
\frac{d \tilde{Y}_{N_1}}{dz} = -z \frac{\langle \Gamma_{N_1} \rangle_z}{H(z = 1)} \left( \tilde{Y}_{N_1} - \tilde{Y}_{N_1}^{\text{eq}} \right),
\]

where \( \tilde{Y}_{N_1} \equiv g_s Y_{N_1} \) and \( \tilde{Y}_{N_1}^{\text{eq}} \equiv g_s Y_{N_1}^{\text{eq}} \). The variable \( Y_I \) is called the yield variable for the net particle \( I \) number; and it is defined as

\[
Y_I \equiv \frac{n_I}{s} = \frac{n_i - n_{\bar{i}}}{s},
\]
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Fig. 9. Evolutions of $Y_{N_1}(z)$, $Y_{N_1}^{eq}(z)$ and $Y_{B-L}(z)$ without an initial $N_1$ abundance.

Fig. 10. Evolutions of $Y_{N_1}(z)$, $Y_{N_1}^{eq}(z)$ and $Y_{B-L}(z)$ with an initial $N_1$ in equilibrium.

where $n_I$ and $s$ denote the net particle number density and the entropy density, respectively. In Eq. (47) we have used the dimensionless evolution parameter $z = M_1/T$. There $\langle \Gamma_{N_1}(z), H(z) \rangle$ and $Y_{N_1}^{eq}(z)$ are the Maxwell-Boltzmann averaged total decay rate of $N_1$, the Hubble parameter and the equilibrium yield variable of the particle species $i$, respectively. The net lepton number density $n_L = n_\ell - n_\bar{\ell}$ evolves according to the following:

$$
\frac{d\tilde{Y}_L}{dz} = z \frac{\langle \Gamma_{N_1}(z) \rangle}{H(z = 1)} \left( \tilde{Y}_{N_1} - \tilde{Y}_{N_1}^{eq} \right) - \frac{12\zeta(3)}{\pi^2} \frac{\langle \Gamma_{N_1}(z) \rangle}{H(z = 1)} \left( 2\tilde{Y}_L + \tilde{Y}_{h_1} \right) \left( \frac{\pi^2}{2\zeta(3)} \frac{\langle \Gamma_{N_1} \rangle}{g_s T^3} \tilde{Y}_{N_1}^{eq} + \frac{45}{2\pi^2} \frac{2\zeta (3)}{\pi^2 g_s S} \langle \sigma' |v| \rangle \right),
$$

where $\tilde{Y}_L$ is defined by $g_s/\epsilon \cdot n_L/s$, and $\langle \sigma' |v| \rangle$ denotes the thermal averaged cross section of the $\Delta L = 2$ scatterings in the thermal bath. Furthermore, because $H$ always appears with a lepton $\ell$, the BE for the net Higgs number density $n_{h_1} \equiv n_H - n_{\bar{H}}$ is identical to that for $n_L$. Now we consider a lepton-symmetric universe with no doublet: $n_L(z \rightarrow 0) = n_{h_1}(z \rightarrow 0) = 0$. For example, in order to obtain successful baryogenesis, we choose $m_1 \sim 0.0020197$ eV, $m_2 = 0.0086557$ eV and $m_3 = 0.049245$ eV. This choice gives

$$
M_1 = 3.6892 \times 10^8 \text{ GeV}, \quad M_2 = 1.0103 \times 10^{10} \text{ GeV}, \quad M_3 = 1.2725 \times 10^{14} \text{ GeV},
$$

and $|\epsilon_{N_1}| \approx 1.2135 \times 10^{-8}$. Then we find $\eta_B = 5.8556 \times 10^{-10}$ (see Fig. 9). We plot the time evolution in Fig. 9. The abundance of $B-L$ changes sign during leptogenesis. This value of $M_1$ is lower than the lower bound in the case of no initial $N_1$ abundance.

Next, we assume that $N_1$ has already been in equilibrium for $z \rightarrow 0$. The mass spectrum (50) gives $\eta_B = 5.8556 \times 10^{-10}$. In Eq. (50), $M_1$ is near the lowest value in the case of an initially thermal $N_1$ abundance. We plot the time evolution in Fig. 10.
3.2. 2HDM-type scenario

Now let us expand our investigation one step further, considering the parametrization shown in Fig. 4, where $M_{\Delta L} = 1 \times 10^{10}$ GeV. In contrast to the previous case, here we need no fine-tuning in the mass spectrum of doublets. Owing to the relation $M_1 < M_{\Delta L}$, lepton asymmetry is produced in the range

$$100 \text{ GeV} \sim T_{\text{EW}} < T_{\text{sph}}^{2\text{HDM}} < M_1.$$  \hspace{1em} (51)

Because, in addition to the SM interactions, the extra Dirac-Yukawa interactions are also in thermal equilibrium, the baryon conversion ratio is modified only slightly. The second Higgs doublet $H'$ behaves like a copy of $H$ with regard to the chemical potential. Then, ignoring the family index, as above, we can replace Eqs. (43) and (44) by

$$I^3_L = \frac{g^2_L T^3}{6} \left\{ \frac{1}{2} \sum_i \left( \sum_{\text{color}} (\mu_{u_L}^i - \mu_{d_L}^i) + \left( \mu_{\nu_L}^i - \mu_{e_L}^i \right) \right) + 2(\mu_{h^+} - \mu_{h^0}) + 4\mu_{W^+} \right\},$$  \hspace{1em} (52)

$$Y = \frac{g^2_Y T^3}{6} \left[ \sum_i \left( \sum_{\text{color}} \left( \frac{1}{3} (\mu_{u_L}^i + \mu_{d_L}^i) + \frac{4}{3} \mu_{u_R}^i - \frac{2}{3} \mu_{d_R}^i \right) - (\mu_{\nu_L}^i + \mu_{e_L}^i) - 2 \mu_{e_R}^i \right) + 4(\mu_{h^+} + \mu_{h^0}) \right],$$  \hspace{1em} (53)

respectively. Using these equations, we obtain

$$B = \frac{8}{23} (B - L) = -\frac{8}{15} L.$$  \hspace{1em} (54)

Since the doublet Higgs bosons do not have $B-L$ charges, we find that the baryon conversion factor $C_{\text{sph}}^{2\text{HDM}} = 8/23 \approx 0.348$ is about the as same as $C_{\text{sph}}^{\text{SM}} = 28/79 \approx 0.354$.

This parametrization leads to the result that the expected $CP$-asymmetry is double that in the previous SM case (37). Therefore, this situation relaxes the constraint on $m_1$ for successful leptogenesis. Because $H'$ behaves like a copy of $\mathcal{P}$, we can introduce $Y_\Phi \equiv Y_{h1} + Y_{h2} = 2Y_{h1}$, where $Y_{h2}$ denotes the net second Higgs number yield variable: $Y_{h2} \equiv (n_{H'} - n_{H'})/s$. For example, let us consider $m_1 = 10^{-4}$ eV. We present the results in Figs. 11 and 12. We obtain $\eta_B = 9.5634 \times 10^{-9}$ and $1.6560 \times 10^{-9}$, respectively.

3.3. Leptogenesis through the $\Delta L$ decay process

In the following, we consider the case in which the dominant $CP$-asymmetry comes from $\Delta L$ decay, i.e. $\epsilon \sim \epsilon_{\Delta L}$, and we consider the type-I seesaw mechanism (22) (The case of the type-II seesaw mechanism is studied in Ref. 26). We can imagine our configuration in the case $M_{\Delta L} \preceq M_1$. As discussed above, Fig. 1 shows that the mass of $\Delta_L$ does not affect whether the gauge unification is realized. Let us assume the SM with a sufficiently light triplet (SM+Δ), $M_{\Delta L} \sim \mathcal{O}(M_W)$. Figure 1 allows
\[\Delta L\] to exist at the electroweak scale, while the hierarchical relation \(m_H \ll m_{H'}\) requires a fine-tuning. In the region \(T > M_{\Delta L}\), the following interactions are in thermal equilibrium.\(^{28}\)

(a) Gauge interactions of triplets:
\[
\begin{align*}
\Delta_{LL}^{-} + \Delta_{LL}^{0} &\leftrightarrow W_{L}^{-}, & \mu_{\Delta^{-}} &= \mu_{\Delta^{0}} + \mu_{W^{+}}, \\
\Delta_{LL}^{--} + \Delta_{LL}^{+} &\leftrightarrow W_{L}^{-}, & \mu_{\Delta^{--}} &= \mu_{\Delta^{+}} + \mu_{W^{+}}.
\end{align*}
\] (55a, 55b)

(b) Majorana-Yukawa couplings of leptons:
\[
\begin{align*}
\Delta_{L}^{0} &\leftrightarrow \bar{\nu}^{i} + \bar{\nu}^{j}, & \mu_{\Delta^{0}} &= -2\mu_{\nu}, \\
\Delta_{L}^{+} &\leftrightarrow \bar{\nu}^{i} + \bar{\nu}^{j}, & \mu_{\Delta^{+}} &= -\mu_{\nu} - \mu_{e^{L}}, \\
\Delta_{L}^{++} &\leftrightarrow \bar{e}^{L} + \bar{e}^{L}, & \mu_{\Delta^{++}} &= -2\mu_{e^{L}}.
\end{align*}
\] (56a, 56b, 56c)

(c) Cubic interactions between two doublets and a triplet:
\[
\begin{align*}
\Delta_{L}^{0} &\leftrightarrow h^{0} + h^{0}, & \mu_{\Delta^{0}} &= 2\mu_{h^{0}}, \\
\Delta_{L}^{+} &\leftrightarrow h^{0} + h^{+}, & \mu_{\Delta^{+}} &= \mu_{h^{0}} + \mu_{h^{+}}, \\
\Delta_{L}^{++} &\leftrightarrow h^{+} + h^{+}, & \mu_{\Delta^{++}} &= 2\mu_{h^{+}}.
\end{align*}
\] (57a, 57b, 57c)

Note that the above interactions (c) explicitly violate the lepton number. These interactions prevent the appearance of a dangerous Majoron. The inclusion of these interactions is motivated by astrophysical observations and the \(Z^{0}\) total width data obtained at LEP. Further, neutralness under the \(SU(2)_{L}\) and \(U(1)_{Y}\) gauge symmetries is guaranteed by the relations
\[
0 = r_{L}^{3} \propto \frac{1}{2} \sum_{i} \left( \sum_{\text{color}} (\mu_{v_{L}^{i}} - \mu_{d_{L}^{i}}) + (\mu_{v_{L}^{i}} - \mu_{e_{L}^{i}}) \right) + (\mu_{h^{+}} - \mu_{h^{0}})
\]
Also, the global neutralness conditions are given by

$$-2\mu_{\Delta^0} + 2\mu_{\Delta^+} + 4\mu_{W^+},$$

(58)

$$0 = Y \propto \sum_{i} \left\{ \sum_{\text{color}} \left( \frac{1}{3} (\mu_{u'_L} + \mu_{d'_L}) + \frac{4}{3} \mu_{u'_R} - \frac{2}{3} \mu_{d'_R} \right) - (\mu_{\nu'_L} + \mu_{\nu'_L}) - 2\mu_{e'_R} \right\}$$

$$+ 2(\mu_{h^+} + \mu_{h^0}) + 4(\mu_{\Delta^0} + \mu_{\Delta^+} + \mu_{\Delta^{++}}).$$

(59)

The sphaleron anomaly processes produce baryon number asymmetry in the temperature region

$$100 \text{ GeV} \sim T_{\text{EW}} < T_{\text{sph}}^{\text{SM}+\Delta} < M_{\Delta_L}.$$  

(60)

If the mass of the second doublet $H'$ is also on the electroweak scale, the effective theory becomes the 2HDM with a light triplet (2HDM+$\Delta$). This configuration requires no fine-tuning of $m_H$ or $m_{H'}$. In this case, baryogenesis occurs in the region

$$100 \text{ GeV} \sim T_{\text{EW}} < T_{\text{sph}}^{2\text{HDM}+\Delta} < M_{\Delta_L}.$$  

(61)

In addition to (a), (b) and (c), the following interactions are also in thermal equilibrium.

(d) Cubic interactions between two second doublets and a triplet:

$$\Delta^0 \leftrightarrow h^0 + h^0, \quad \mu_{\Delta^0} = 2\mu_{h^0},$$

(62a)

$$\Delta^+ \leftrightarrow h^0 + h^+, \quad \mu_{\Delta^+} = \mu_{h^0} + \mu_{h^+},$$

(62b)

$$\Delta^{++} \leftrightarrow h^+ + h^+, \quad \mu_{\Delta^{++}} = \mu_{h^+}.$$  

(62c)

Also, the global neutralness conditions are given by

$$0 = T_{L}^3 \propto \frac{1}{2} \sum_{i} \left( \sum_{\text{color}} (\mu_{u'_L} - \mu_{d'_L}) + (\mu_{\nu'_L} - \mu_{\nu'_L}) \right) + 2(\mu_{h^+} - \mu_{h^0})$$

$$-2\mu_{\Delta^0} + 2\mu_{\Delta^+} + 4\mu_{W^+},$$

(63)

$$0 = Y \propto \sum_{i} \left\{ \sum_{\text{color}} \left( \frac{1}{3} (\mu_{u'_L} + \mu_{d'_L}) + \frac{4}{3} \mu_{u'_R} - \frac{2}{3} \mu_{d'_R} \right) - (\mu_{\nu'_L} + \mu_{\nu'_L}) - 2\mu_{e'_R} \right\}$$

$$+ 4(\mu_{h^+} + \mu_{h^0}) + 4(\mu_{\Delta^0} + \mu_{\Delta^+} + \mu_{\Delta^{++}}).$$

(64)

As in the case of leptogenesis through $N_1$ decay, $\Delta_L$ must decouple from the thermal bath at high temperature. This decoupling condition is given by $\langle \Gamma_{\Delta_L} \rangle_{T=M_{\Delta L}} < H(T=M_{\Delta L})$. Before solving the BEs, let us investigate this condition with an order estimation. The tree-level total decay rate of the triplet scalar $\Delta_L$ is given by

$$\langle \Gamma_{\Delta_L} \rangle_{T=M_{\Delta L}} = \frac{K_1(T=M_{\Delta L})}{K_2(T=M_{\Delta L})} \frac{M_{\Delta L}}{8\pi} \left( \sum_{i,j} |Y_{\Delta ij}|^2 + \sum_{a,b} |\beta_{ab}|^2 \frac{v_R^2}{M_{\Delta L}^2} \right),$$

(65)

where $K_i(x)$ denotes the modified Bessel functions. Also, the Hubble parameter at temperature $T$ can be written

$$H(T) = \sqrt{\frac{4\pi^3 g_*}{45}} \frac{T^2}{M_p},$$

(66)
where $M_P = 1.22 \times 10^{19}$ GeV. If $\sum_{i,j} |Y_{\Delta ij}|^2 \gtrsim \sum_{a,b} |\beta_{ab}|^2 v_R^2/M_{\Delta L}^2$ and $\beta_{ab} = O(1)$, the decoupling condition and our RG analysis suggest that $M_{\Delta L}$ is larger than at least $6.60 \times 10^{19}$ GeV (SM+\Delta) or $1.24 \times 10^{22}$ GeV (2HDM+\Delta). This clearly contradicts the relation $M_{\Delta L} \lesssim M_1$, according to Eq. (31). For this reason, we conclude that the term proportional to $v_R^2/M_{\Delta L}^2$ gives the dominant contribution, then we have

$$\langle \Gamma_{\Delta_L} \rangle_{T=M_{\Delta L}} \simeq \frac{K_1(T = M_{\Delta L})}{K_2(T = M_{\Delta L})} \frac{v_R^2}{8\pi M_{\Delta L}},$$

(67)

which implies that

$$\sum_{i,j} |Y_{\Delta ij}|^2 = \frac{1}{v_R^2} \sum_{i,j} |M_{Rij}|^2 \sim \frac{M_3^2}{v_R^2} \ll \frac{v_R^2}{M_{\Delta L}^2}.$$  

We thus obtain

$$M_{\Delta L} \lesssim v_R^2/M_3.$$  

(68)

Our RG analysis suggests that 100 GeV $< M_{\Delta L} < O(10^{5-10})$ GeV for the SM+\Delta and $100 \text{ GeV} < M_{\Delta L} < O(10^{12-14})$ GeV for the 2HDM+\Delta. Here let us evaluate the consistency of the above relation and our model. In addition to the decoupling of $\Delta L$, at temperature $T = M_{\Delta L}$, the W boson also must be out of equilibrium. This gives another decoupling condition, $\langle \Gamma_W \rangle_{T=M_{\Delta L}} \lesssim H(T = M_{\Delta L})$. According to Ref. 13), we have $M_{\Delta L} \gtrsim 4.8 \times 10^{10}$ GeV. The above can be summarized as follows:

$$M_{\Delta L} = O(10^{10}) \text{ GeV} \quad \text{for the SM+\Delta},$$

(69a)

$$M_{\Delta L} = O(10^{10-14}) \text{ GeV} \quad \text{for the 2HDM+\Delta}.$$  

(69b)

Apparently Eq. (69a) is a strong constraint. However, when we substitute this and $v_R = O(10^9)$ GeV to Eq. (68), we obtain $M_3 \lesssim O(10^8)$ GeV. This upper bound of $M_3$ contradicts the hierarchical neutrino mass spectrum (31). Therefore, we conclude that leptogenesis through the $\Delta_L$ decay process is impossible in the case of a hierarchical neutrino mass spectrum in the SM+\Delta. By contrast, in the 2HDM+\Delta, Eq. (69b) is consistent with the hierarchical spectrum (31). In this case, we can obtain a reasonable value of $\eta_B$ if $m_1 \ll 10^{-4}$ eV.

§4. Conclusion and unresolved issues

We analyzed the LR model with D-parity restoration and grand unification. We succeeded in constructing the restrictive models with few free input parameters. We found that the allowed region in the $(M_{Z_R}, M_{\Delta L})$ parameter space is highly constrained in our model. In this work, we focused on the nonsupersymmetric versions, and studied two specific cases, $M_{\Delta L} = 200$ GeV and $M_{\Delta L} = M_{Z_R} \approx M_{\Delta R}$. We found that the thermal $N_1$-leptogenesis scenario is successful in both the SM and the 2HDM. In addition, we found that the thermal $\Delta_L$-leptogenesis scenario accords with the hierarchical neutrino spectrum only in the 2HDM+\Delta, while thermal leptogenesis through $\Delta_L$-decay is incompatible with the hierarchical neutrinos in the SM.
Here, it should be noted that though we can construct more strongly constrained models using numerical analysis of RGEs, we have no guiding principle for $\mathcal{O}(M_{\Delta_L})$. We realized our original goal of building constrained LR models.

Our next step is to investigate within the framework of finite-temperature field theory. In the literature, thermal leptogenesis scenarios are obtained using the usual zero-temperature field theory. By including thermal corrections, we can investigate truly “thermal” leptogenesis processes. In Ref. 29) the thermal $N_1$-leptogenesis in the SM and the MSSM is analyzed using finite-temperature field theory. We are presenting in the process of recomputing our models using the Keldysh (real time) formalism.29) This approach can yield more faithful predictions, also not only leptogenesis scenarios but also the doublet Higgs sector. It can also be applied to the PS models and the $SO(10)$ GUT models. Another approach is to introduce supersymmetry. Although the supersymmetric extension of LR models (LRSUSY)31) is compatible with $R$-parity conservation, unfortunately the LRSUSY models have many parameters. In particular, especially the soft breaking couplings make it difficult to carry out predictive analysis. Furthermore, supersymmetry provides a new possibility for leptogenesis, i.e. Affleck-Dine leptogenesis. We are also interested in investigating the competition between thermal leptogenesis32) and Affleck-Dine leptogenesis34) with the LRSUSY models. Even if existing models generate sufficient baryon asymmetry, the gravitino and reheating problems33) remain to be solved. We believe that our study is a first step toward developing predictive models. These bottom-up approaches could provide information concerning the Majorana couplings and the scalar four-point couplings.

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**References**

1) F. Wilczek and A. Zee, Phys. Rev. D 25 (1982), 553.
2) C. S. Aulakh and A. Girdhar, Int. J. Mod. Phys. A 20 (2005), 865.
3) R. N. Mohapatra, G. Senjanović and M. D. Tran, Phys. Rev. D 28 (1983), 546.
4) J. F. Gunion, J. Grifols, A. Mendez, B. Kayser and F. Olness, Phys. Rev. D 40 (1989), 1546.
5) J. Gluza and M. Zra/ek, Phys. Rev. D 51 (1995), 4695.
6) M. Czakon, Acta Phys. Polon. B 30 (1999), 3365.
7) Super-Kamiokande Collaboration, S. Fukuda et al., Phys. Rev. Lett. 85 (2000), 3999, and references therein; Phys. Rev. Lett. 86 (2001), 5656, and references therein.
8) SNO Collaboration, Q. R. Ahmad et al., Phys. Rev. Lett. 87 (2001), 071301.
9) A. Aguilar et al., Phys. Rev. D 64 (2001), 112007.
10) LSND Collaboration, G. B. Mills, Nucl. Phys. B (Proc. Suppl.) 91 (2001), 198, and references therein.
11) M. Fukugita and Y. Yanagida, Phys. Lett. B 174 (1986), 45.
12) David N. Spergel et al., Astrophys. J. Suppl. 148 (2003), 175.
13) M. Kusakabe, T. Kajino and G. J. Mathews, Phys. Rev. D 74 (2006), 023526.
14) J. C. Pati and A. Salam, Phys. Rev. D 10 (1974), 275.
15) L. Michel, Rev. Mod. Phys 52 (1980), 617.
16) T. Kibble, G. Lazarides and Q. Shafi, Phys. Rev. D 26 (1982), 435.
17) D. Chang, R. N. Mohapatra and M. K. Parida, Phys. Rev. Lett. 52 (1984), 1072.
11) T. Fukuyama and N. Okada, J. High Energy Phys. 11 (2002), 011.
   T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, Eur. Phys. J. C 42 (2005), 191; J. Math. Phys. 46 (2005), 033505.
   T. Fukuyama, T. Kikuchi and T. Osaka, J. Cosmol. Astropart. Phys. 06 (2005), 005.
12) D. Chang and N. Mohapatra and M. K. Parida, Phys. Rev. Lett. 52 (1984), 1072.
13) N. Sahu and U. Sarkar, Phys. Rev. D 74 (2006), 093002.
14) D. Chang, R. N. Mohapatra and M. K. Parida, Phys. Rev. Lett. 52 (1984), 1072.
15) R. N. Mohapatra and G. Senjanović, Phys. Rev. D 23 (1981), 165.
   N. G. Deshpande, J. F. Gunion, B. Kayser and F. Olness, Phys. Rev. D 44 (1991), 837.
16) P. Adhya, D. Rai Chaudhuri and A. Raychaudhuri, Eur. Phys. J. C 19 (2001), 183.
17) OPAL Collaboration, Phys. Rev. Lett. B 295 (1992), 347; Phys. Lett. B 526 (2002), 221.
   DELPHI Collaboration, Phys. Lett. B 552 (2003), 127.
   L3 Collaboration, CERN-EP/2003-060.
18) OPAL Collaboration, CERN-EP/2003-041.
19) P. Minkowski, Phys. Lett. B 67 (1977), 421.
   T. Yanagida, in Proceedings of the Workshop on the Baryon Number of the Universe and Unified Theories, Tsukuba, Japan, 13-14 Feb 1979.
   M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity (North Holland, Amsterdam, 1979).
   S. Glashow, NATO Adv. Study Inst. Ser. B Phys. 59 (1979), 687.
20) T. Schwetz, Physica Scripta T127 (2006), 1, and references therein.
21) M. Maltoni, T. Schwetz, M. A. Tórtola and J. W. F. Valle, Phys. Rev. D 68 (2003), 113010.
   M. Maltoni, T. Schwetz, M. A. Tórtola and J. W. F. Valle, New J. Phys. 6 (2004), 122.
   S. Goswami and A. Y. Smirnov, Phys. Rev. D 72 (2005), 053011.
22) K. S. Babu, A. Bachri and H. Aissaoui, Nucl. Phys. B 738 (2006), 76.
23) T. Hambye and Goran Senjanović, Phys. Lett. B 582 (2004), 73.
   T. Hambye, M. Raidal and A. Strumia, Phys. Lett. B 632 (2006), 667.
24) M. Flanz and E. A. Paschos and U. Sarkar, Phys. Lett. B 345 (1995), 248 [Errata: 382 (1996), 447].
   L. Covi, E. Roulet and F. Vissani, Phys. Lett. B 384 (1996), 169.
   M. Flanz, E. A. Paschos, U. Sarkar and J. Wess, Phys. Lett. B 389 (1996), 693.
   A. Pilaftsis, Phys. Rev. D 56 (1997), 5431.
   W. Buchmüller and M. Plümacher, Phys. Lett. B 431 (1998), 354.
25) R. Barbieri, P. Creminelli, A. Strumia and N. Tetradis, Nucl. Phys. B 575 (2000), 61.
26) S. Antusch and S. F. King, Phys. Lett. B 597 (2004), 199.
27) M. A. Luty, Phys. Rev. D 45 (1992), 455.
   E. K. Akhmedov, M. Blennow, T. Hallgren, T. Konstandin and T. Ohlsson, hep-ph/0612194.
   P. Hosteins, S. Lavignac and C. A. Savoy, Nucl. Phys. B 755 (2006), 137.
28) K. Hasogawa, Phys. Rev. D 70 (2004), 054002.
29) G. F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, Nucl. Phys. B 685 (2004), 89.
30) U. Kraemmer and A. Rebhan, Rep. Prog. Phys. 67 (2004), 351.
31) R. M. Francis, M. Frank and C. S. Kalman, Phys. Rev. D 43 (1991), 2369.
   R. N. Mohapatra and A. Rašin, Phys. Rev. D 54 (1996), 5835.
   C. S. Aulakh, K. Benakli and G. Senjanović, Phys. Rev. Lett. 79 (1997), 2188.
   C. S. Aulakh, A. Melfo and G. Senjanović, Phys. Rev. D 57 (1998), 4174.
32) M. Frank, Phys. Rev. D 70 (2004), 036004.
33) M. Yu. Khlopov and A. D. Linde, Phys. Lett. B 138 (1984), 265.
   F. Balestra, G. Piragino, D. B. Pontecorvo, M. G. Sapozhnikov, I. V. Falomkin and M. Yu. Khlopov, Sov. J. Nucl. Phys. 39 (1984), 626.
   M. Yu. Khlopov, Yu. L. Levitan, E. V. Sedelnikov and I. M. Sobol, Phys. Atom. Nucl. 57 (1994), 1393.
34) I. Affleck and M. Dine, Nucl. Phys. B 249 (1985), 361.