Using algorithms for costs optimization between two or more harbours

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Abstract. This paper’s main objective is to analyse and emphasize the mathematical solutions for resolving an essential problem which rises in maritime domain, from economical perspectives, considering the minimum costs of transport. The possibility of finding the minimum cost between two or more harbours represents an advantage for the crew and the employer, and by doing this it helps at planning the voyage in advance. For a rigorous mathematical study, each source must carry to more than one harbours and each destination must be supplied with more than one resources. Taking into consideration the minimum costs, on each route the ship should be able to carry a limited quantity, and also in case of more intermediary points, in order to fulfill the requests. Into this paper we will present different methods for finding the minimum shipping costs between two or more harbors.

Keywords: minimum costs, mathematical solutions, source, destination, route, shipping.

1. Introduction

It is well knowing that one of important problem which rises from economical aspects in maritime domain is that of obtaining good price for delivering goods.

Taking in account this aspect the purpose of this paper is to analyse and emphasize the mathematical solutions for solving essential problems like those of the determination of the minimum sailing cost between two or more harbours.

The possibility of finding the minimum cost between two or more harbours represents an advantage for those who sail, primary by helping in an efficient mode those responsible for voyage planning.

In almost every concrete situation in naval practice raises the issue of delivering a cargo from some harbours, named sources, to another harbours, named destinations. For doing this it must be taken into consideration a sailing cost.

Delivering cargo to destinations could be done directly from sources or through other points, named intermediate points, thus obtaining a series of complex connections between sources and/or between destinations. The totality of sources, routes, destinations and other intermediate points is named transport network.

For this type of problem to require a rigorous mathematical study, every source must carry to more harbours and every destination must be provided from more sources. Another problem is the
determination of a minimum cost between two or more harbours, considering that on every route must be carried a limited quantity, and also in case of more intermediate points.

The notions presented until now can be arranged through a mathematical model transport problem.

2. Theoretical considerations

The mathematical model of transport problem, as it is well known[1],[4], is part of modeling problem of linear programming. Given the large number of variables, solving a transport problem with the simplex algorithm is generally less efficient. Therefore, special techniques are used to solve transport problems[1], [2]. This paper is dedicated to presenting some of these techniques in a simple way.

Being given m departure harbours \( A_i \)\((i=1, 2, ..., m)\), where an omogenous product is available in \( a_1, a_2, ..., a_m \) quantities and n dispatching harbours \( B_j \)\((j=1, ..., n)\), where that product is necessary in \( b_1, b_2, ..., b_n \) quantities, and \( x_{ij} \) are cargo quantities which are to be delivered from departure harbours \( A_i \) to dispatching harbours \( B_j \), and the result must be the economical route.

The characteristics of a navigation problem where it is necessary to find a minimum cost are:

- every source must provide at least one destination and every destination must be provided from at least one source;
- it can exist source-destination pairs between which a transfer can’t be made (blocked routes);
- there aren’t limitation regarding the carried quantity on each route;
- the available quantities are known in each source and also the quantities required in each destination;
- every route has associated a cost which does not depend on the reversal route.

The objective of this problem is finding those quantities which must be carried on each route so they fulfill the destinations requirements, depending on each source quantity, with the minimum cost possible.

If we note with \( x_{ij} \) the quantity which will be carried from source \( i \) to destination \( j \), then we have the following problem to solve:

\[
\text{(min)} f = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}
\]

There are three possible cases:

\[
\sum_{i=1}^{m} a_i > \sum_{i=1}^{m} b_i \\
\sum_{i=1}^{m} a_i < \sum_{i=1}^{m} b_i \\
\sum_{i=1}^{m} a_i = \sum_{i=1}^{m} b_i
\]

In the first case, the problem has the best solution, and the quantity which exceeds the request remains at the sources, and that is represented by the deviation variables from first \( m \) restrictions. These quantities could be regarded as a request from a fictive consumer and considering the fact that the quantities aren’t carried nowhere, the costs on this fictive route are zero. Adding this consumer to the table, the request can be represented by the following equation: \( \sum_{i=1}^{m} A_i = \sum_{j=1}^{n} B_j \). By doing this we will have a type three problem and these steps can be repeated for type two problem.

The third type of this navigation problem is called a balanced navigation problem. Further we have presented methods for finding initial solution for a practical possible problem.
2.1. Finding an initial solution

For solving this transport problem we start from an initial basic solution, which we find as it follows:

First a variable \( x_{ij} \) is given the following value: \( x_{ij} = \min_{ij} \{a_i, b_j\} \). If \( a_i < b_j \) then \( x_{ij} = a_i \). We cut the i line and \( b_j \) will be replaced with \( b_j^{(2)} = b_j - a_i \). If \( a_i > b_j \), then \( x_{ij} = b_j \). We cut the j column and \( a_i \) will be replaced with \( a_i^{(1)} = a_i - b_j \). By doing this we obtain a reduced table that differs from the initial one with a line or a column. The procedure is repeated until all the dispatch harbours have received the quantities requested[1], [2].

Next, we analyse more particular cases, derived from this general one.

2.1.1. North-West method. We choose the variable from the north-west table corner, situated on the first line and first column and we note it with \( x_{11} \). If \( \min \{a_1, b_1\} = a_1 \), then \( x_{11} = a_1 \) and \( x_{12} = x_{13} = ... = x_{1n} = 0 \). The result is that we cut the first line and we find \( x_{21} \) from the following relation: \( x_{21} = \min \{a_2, b_1, a_1\} \). If \( \min \{a_1, b_1\} = b_1 \), then \( x_{1j} = b_1 \) and \( x_{21} = x_{31} = ... = x_{m1} = 0 \). We eliminate the first column and we find \( x_{12} \) from the following relation: \( x_{12} = \min \{a_1 - b_1, b_2\} \). This algorithm is repeated until we successively find the \( x_{ij} \) values situated on the first line and column that hasn’t been eliminated[1],[3].

2.1.2. Minimum line cost of the matrix. We choose the \( x_{ij} \) variable that is situated on the first line which corresponds the minimum cost cell: \( c_{ik} = \min \{c_{i1}, c_{i2}, ..., c_{in}\} \), \( k = 1, m \). Then \( x_{1k} \) is found with the following relation: \( x_{1k} = \min \{a_1, b_k\} \). If \( a_1 < b_k \Rightarrow x_{1k} = a_1 \). \( x_{11} = x_{12} = ... = x_{1k} = x_{1k+1} = ... = x_{1n} = 0 \). The first line is eliminated and the procedure is repeated with the second line. If \( a_1 > b_k \Rightarrow x_{1k} = b_k \), \( x_{2k} = x_{3k} = ... = x_{mk} = 0 \), the k column is cut and the algorithm is continued by choosing \( x_{ij} \) which is found on the first line corresponding the minimum cost remained after the column k is cut. This algorithm is repeated until all the \( x_{ij} \) values found on the first line have been determined. The same steps are made for the following matrix lines[1],[3].

2.1.3. Minimum column cost of the matrix. We choose \( x_{ij} \) variable situated on the first column which corresponds to \( c_{ij} \) cell where the cost is minimum: \( c_{kj} = \min \{c_{i1}, c_{i2}, ..., c_{in}\} \), \( k = 1, n \). Then we find \( x_{k1} = \min \{a_k, b_1\} \). After that, two cases are distinguished: first one if \( a_k < b_1 \Rightarrow x_{k1} = a_k \); then \( x_{k2} = x_{k3} = ... = x_{kn} = 0 \), line k is cut and the algorithm continues by choosing \( x_{ij} \) variable which is situated on the first column, that corresponds to the minimum cost after the line k is eliminated and second one if \( a_k > b_1 \Rightarrow x_{k1} = b_1 \); then \( x_{11} = x_{21} = ... = x_{k-1,1} = x_{kn,1} = ... = x_{m1} = 0 \) and the first column is cut. The algorithm is repeated with the second column and similar steps are repeated with the following columns[1], [3].

2.1.4. Minimum matrix cost. We choose \( x_{ij} \) variable which corresponds the cell with the minimum \( c_{ij} \) cost is minimum. If: \( c_{mn} = \min_{ij} \{c_{ij}\} \), we find \( x_{mn} \) with the following relation: \( x_{mn} = \min \{a_m, b_n\} \). The procedure is repeated similarly until all the \( x_{ij} \) values have been determined[3].

2.1.5. Maximal differences method. There are two main steps to take into consideration when using this method. First one: for every line and column we calculate the difference between the two minimum costs of the routes and we find the maximum of these differences (Observation: the difference can be 0 if the minimum is multiple) and second one: from all corresponding routes on the lines and columns of the matrix we choose the minimum route cost (Observation: if the minimum is multiple we will choose randomly). The algorithm is repeated until all the \( x_{ij} \) variables have been found[3].

In the next section we have presented the possible practical problem that we have considered like a good case study.
The practical problem

In this section, we have emphasized the methods described above by using them in a practical and possible problem, more precisely we have considered four nautical至关重要ness (suppliers are ADM ROMANIA LOGISTICS SRL – F1, AMATIMAR AGENT SRL – F2, BRIGHT MARITIME – F3, AMS FREIGHT&LOGISTICS – F4) which have quantities of corn, as it follows:

- Corn quantities of F1 are 196Mm, F2 1170Mm, F3 3262Mm, F4 3362Mm.
- Costs for transporting the corn between two or more ports, calculated with approximation. So, for supplier one (F1): $c_{11}=180$, $c_{12}=180+0,01(1170-500)=187$, $c_{13}=180+0,01(1574-500)=191$, $c_{14}=180+0,01(3262-500)=208$, $c_{15}=180+0,01(3362-500)=209$, $c_{16}=180+0,01(4458-500)=220$;
- Analogue, for supplier two (F2): $c_{21}=180$, $c_{22}=194$, $c_{23}=194$, $c_{24}=215$, $c_{25}=217$, $c_{26}=230$;
- for supplier three (F3): $c_{31}=180$, $c_{32}=185$, $c_{33}=189$, $c_{34}=202$, $c_{35}=203$, $c_{36}=210$; and for supplier four (F4): $c_{41}=180+0,01(196-100)=181$, $c_{42}=180+0,01(190-100)=190$, $c_{43}=180+0,01(1574-100)=195$, $c_{44}=180+0,01(3262-100)=210$, $c_{45}=180+0,01(3362-100)=212$, $c_{46}=180+0,01(4458-100)=223$;

If we note with $x_{ij}$, the product quantity which will be sailed from F$i$, suppliers, $i=1,2,3,4$, to harbours $P_j$, $j=1,2,3,4,5,6$, then we will obtain the following mathematical model:

\[
\begin{align*}
\sum_{j=1}^{6} x_{1j} &= 100,000 \\
\sum_{j=1}^{6} x_{2j} &= 100,000 \\
\sum_{j=1}^{6} x_{3j} &= 100,000 \\
\sum_{j=1}^{6} x_{4j} &= 100,000 \\
\end{align*}
\]

In the table below, we have presented the solution using table forms where $(a_i)$ represent available quantities and $(b_j)$ the necessary quantities. For finding an initial solution by using the North-West method, we proceed as it follows:

\[
\begin{align*}
x_{11} &= \min \{20,000,40,000\} = 20,000 \Rightarrow x_{12} = x_{13} = x_{14} = x_{15} = x_{16} = 0 \\
x_{21} &= \min \{40,000,40,000-20,000\} = 20,000 \Rightarrow x_{31} = x_{41} = 0 \\
x_{22} &= \min \{40,000-20,000,10,000\} = 10,000 \Rightarrow x_{23} = x_{24} = 0 \\
x_{23} &= \min \{40,000-20,000-10,000,25,000\} = 10,000 \Rightarrow x_{23} = x_{25} = x_{26} = 0
\end{align*}
\]
\[ x_{33} = \min\{30.000, 25.000 - 10.000\} = 15.000 \Rightarrow x_{43} = 0 \]

\[ x_{34} = \min\{30.000 - 15.000, 15.000\} = 15.000 \Rightarrow x_{35} = x_{36} = x_{44} = 0 \]

\[ x_{45} = \min\{50.000 - 25.000, 30.000 - 20.000\} = 10.000 \Rightarrow x_{25} = 0; \]

\[ x_{36} = \min\{40.000 - 15.000, 20.000, 20.000\} = 5.000 \Rightarrow x_{46} = 15.000 \]

Minimum function value for the initial solution found is:
\[(\text{min}) f = (180 \cdot 20.000) + (180 \cdot 20.000) + (215 \cdot 15.000) + (230 \cdot 50.000) + (185 \cdot 10.000) + (203 \cdot 20.000) + (195 \cdot 25.000) + (212 \cdot 10.000) + (223 \cdot 15.000) = 27.825.000 \text{ S} \]
Now applying minimal column cost of the matrix we obtained:

\[ x_{11} = \text{min}\{20.000,40.000\} = 20.000 \Rightarrow x_{12} = x_{13} = x_{14} = x_{15} = x_{16} = 0; \]
\[ x_{21} = \text{min}\{40.000 - 20.000,40.000\} = 20.000 \Rightarrow x_{31} = x_{41} = 0; \]
\[ x_{32} = \text{min}\{30.000,10.000\} = 10.000 \Rightarrow x_{22} = x_{42} = 0; \]
\[ x_{33} = \text{min}\{30.000 - 10.000,25.000\} = 20.000 \Rightarrow x_{34} = x_{35} = x_{36} = 0; \]
\[ x_{34} = \text{min}\{25.000 - 20.000,40.000 - 20.000\} = 5.000 \Rightarrow x_{44} = 0; \]
\[ x_{44} = \text{min}\{50.000,15.000\} = 15.000 \Rightarrow x_{34} = 0; \]
\[ x_{45} = \text{min}\{50.000 - 15.000,30.000\} = 30.000 \Rightarrow x_{25} = 0; \]
\[ x_{46} = \text{min}\{50.000 - 30.000 - 15.000,20.000\} = 5.000 \Rightarrow x_{26} = 15.000. \]

Minimum function value for the initial solution founded is:

\[ \text{(min)f} = (180 \cdot 20.000) + (180 \cdot 20.000) + (194 \cdot 5.000) + (230 \cdot 15.000) + (185 \cdot 10.000) + (189 \cdot 20.000) + (210 \cdot 15.000) + (212 \cdot 30.000) + (223 \cdot 5.000) = 27.875.000 \] $ \]

The steps that we made are represented in the table below:

| P_1 | P_2 | P_3 | P_4 | P_5 | P_6 | a_i |
|-----|-----|-----|-----|-----|-----|-----|
| F_1 | 180 | 187 | 191 | 208 | 209 | 220 | 20.000 |
|     | 20.000 | 0 | 0 | 0 | 0 | 0 |
| F_2 | 180 | 189 | 194 | 215 | 217 | 230 | 40.000 |
|     | 20.000 | 0 | 5.000 | 0 | 0 | 0 | 15.000 |
| F_3 | 180 | 185 | 189 | 202 | 203 | 210 | 30.000 |
|     | 0 | 10.000 | 20.000 | 0 | 0 | 0 |
| F_4 | 181 | 190 | 195 | 210 | 212 | 223 | 50.000 |
|     | 0 | 0 | 0 | 15.000 | 30.000 | 5.000 |
| F_5 | 40.000 | 10.000 | 25.000 | 15.000 | 30.000 | 20.000 | 140.000 |

Using the minimum matrix cost for finding an initial solution, we proceed as it follows:

\[ x_{11} = \text{min}\{20.000,40.000\} = 20.000 \Rightarrow x_{12} = x_{13} = x_{14} = x_{15} = x_{16} = 0; \]
\[ x_{21} = \text{min}\{40.000 - 20.000,40.000\} = 20.000 \Rightarrow x_{31} = x_{41} = 0; \]
\[ x_{32} = \text{min}\{30.000,10.000\} = 10.000 \Rightarrow x_{22} = x_{42} = 0; \]
\[ x_{33} = \text{min}\{30.000 - 10.000,25.000\} = 20.000 \Rightarrow x_{34} = x_{35} = x_{36} = 0; \]
\[ x_{34} = \text{min}\{25.000 - 20.000,40.000 - 20.000\} = 5.000 \Rightarrow x_{44} = 0; \]
\[ x_{44} = \text{min}\{50.000,15.000\} = 15.000 \Rightarrow x_{34} = 0; \]
\[ x_{45} = \text{min}\{50.000 - 15.000,30.000\} = 30.000 \Rightarrow x_{25} = 0; \]
\[ x_{46} = \text{min}\{50.000 - 30.000 - 15.000,20.000\} = \text{min}\{5.000,20.000\} = 5.000 \Rightarrow x_{26} = 15.000. \]

Minimum function value for the initial solution founded is:

\[ \text{(min)f} = (180 \cdot 20.000) + (180 \cdot 20.000) + (194 \cdot 5.000) + (230 \cdot 15.000) + (185 \cdot 10.000) + (189 \cdot 20.000) + (210 \cdot 15.000) + (212 \cdot 30.000) + (223 \cdot 5.000) = 27.875.000 \] $ \]

Because all of those elements that we have determined are identical with those represented in the table before, we considered that it is not necessary to repeat the table.

Finally using the maximal differences method firstly we calculated the differences on the lines and columns:

\[ c_{11} - c_{21} = 0; \quad c_{22} - c_{12} = 2; \quad c_{13} - c_{33} = 2; \quad c_{14} - c_{34} = 6; \quad c_{15} - c_{35} = 6; \quad c_{16} - c_{36} = 10; \]
\[ c_{42} - c_{41} = 7; \quad c_{22} - c_{21} = 9; \quad c_{32} - c_{31} = 5; \quad c_{42} - c_{41} = 9. \]

Then we represented the following differences table:
We chose the sixth column ($P_6$) because it corresponds the maximal difference and we calculate the cell which corresponds the minimum cost, so:

$x_{36} = \min \{30.000, 20.000\} = 20.000 \Rightarrow x_{16} = x_{26} = x_{46} = 0$

The algorithm is repeated and it will result:

$x_{21} = \min \{40.000, 40.000\} = 40.000 \Rightarrow x_{22} = x_{23} = x_{24} = x_{25} = x_{26} = x_{31} = x_{32} = x_{33} = x_{41} = x_{42} = x_{43} = x_{44} = x_{45} = x_{46} = 0$

$x_{42} = \min \{50.000, 25.000\} = 20.000 \Rightarrow x_{43} = x_{44} = x_{45} = x_{46} = 0$

$x_{41} = \min \{30.000, 20.000, 15.000\} = 10.000 \Rightarrow x_{31} = x_{32} = x_{33} = x_{41} = x_{42} = x_{43} = x_{44} = x_{45} = x_{46} = 0$

$x_{45} = \min \{50.000, 30.000\} = 30.000 \Rightarrow x_{43} = 5.000; x_{44} = 5.000$

All this are represented in the table below:

| $F_i$ | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $c_{in}$ |
|-------|-------|-------|-------|-------|-------|-------|---------|
| $F_1$ | 180   | 187   | 191   | 208   | 209   | 220   | 7       |
| $F_2$ | 180   | 189   | 194   | 215   | 217   | 230   | 9       |
| $F_3$ | 180   | 185   | 189   | 202   | 203   | 210   | 5       |
| $F_4$ | 181   | 190   | 195   | 210   | 212   | 223   | 9       |
| $c_{mj}$ | 0 | 2   | 2.000 | 6.000 | 6.000 | 10.000 |         |

The minimum function value for the initial solution found is:

$\text{min} f = 191 \cdot 20.000 + 180 \cdot 40.000 + 202 \cdot 10.000 + 210 \cdot 20.000 + 190 \cdot 10.000 + 195 \cdot 5.000$

$+ (210 \cdot 5.000) + (212 \cdot 30.000) = \textbf{23.525.000 \$}$

**Conclusion**

From a mathematical and economic point of view it can be concluded that using the maximal differences method the cost obtained is better than those obtained using other methods.

We also observed, that firstly, it would be good, to use the minimum matrix cost, that requires attention only for the cost selection.

By applying these mathematical methods, we can easily conclude that it is very important to know mathematical theory in various domains, one concrete example that we used is maritime navigation, because doing so the profit and strategy of the company is maximized.

The solution offered by this paper might help companies to work to obtain the increasing or decreasing of cost for their transportation process in according to the relationship between the demand and supply of quantities of goods.
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