The Inflationary Gravity Waves in light of recent Cosmic Microwave Background Anisotropies data.

Alessandro Melchiorri\(^b\) and Carolina J. Ödman\(^\sharp\)

\(^b\) Astrophysics, Denys Wilkinson Building, University of Oxford, Keble road, OX1 3RH, Oxford, UK
\(^\sharp\) Astrophysics Group, Cavendish Laboratory, Cambridge University, Cambridge, U.K.

One of the major predictions of inflation is the existence of a stochastic background of cosmological gravitational waves (GW). These gravitational waves can induce significant temperature anisotropies in the Cosmic Microwave Background (CMB) on the angular scales recently probed by the Archeops experiment. Here, we perform a combined analysis of Archeops together with information from other CMB experiments and/or cosmological datasets, in order to constrain the amplitude of the GW background. We find that, for a scale-invariant GW background, the ratio of tensor/scalar perturbations at the CMB quadrupole is now constrained to be \( r \leq 0.43 \) at 95\% c.l., while the bound on the spectral index of primordial density fluctuations is \( n_S = 0.97^{+0.10}_{-0.09} \). We discuss the implications for future GW detections through CMB polarization measurements.

I. INTRODUCTION

The last years have seen spectacular advances in our ability to confront the inflationary scenario of structure formation to observational data. The “multiple peaks” observed in the Cosmic Microwave Background (CMB) angular power spectrum (\([28], [17], [23], [32], [35]\)) are indeed providing strong supporting evidence for the inflationary predictions of a flat universe and of a primordial background of scale-invariant adiabatic perturbations (see e.g. \([39], [29]\)). More recently, the new CMB results from the Archeops experiment (\([1]\)) have confirmed and refined the present observational status, sampling angular scales between those probed by the COBE satellite and the latest high precision datasets. Again, flatness, adiabaticity and scale invariance are in agreement with the data (\([2]\)).

It has been argued that the next and probably most conclusive evidence for inflation would be the detection of a stochastic background of Gravity Waves (GW) (see e.g. \([7], [44]\)). Two types of spacetime metric fluctuations are indeed naturally produced during inflation: density perturbations (scalar modes), which form the “seeds” of structure formation, and gravity waves (tensor modes) (\([16]\)).

The GW background, if detected, would also provide valuable information on the inflationary scenario. In particular, in most inflationary models (and certainly in the simplest ones), the amplitude of the GW background is proportional to the square of the energy scale of inflation (see e.g. \([8]\)). Furthermore, a complementary measurement of the “tilt” of the GW perturbations (and of the scalar as well) can give direct information up to the second derivatives of the inflaton potential, shedding light on the physics at \( \sim 10^{16} \text{GeV} \) (see e.g. \([19]\)).

The GW background leaves an imprint on the CMB anisotropies at large scales through the Sachs-Wolfe effect. On scales smaller than the horizon at recombination, however, unlike the anisotropies generated by scalar fluctuations, those generated by GW damp like fluctuations in a fluid of massless bosons (see e.g. \([11]\)). Since the theoretical spectrum, normalized to COBE, is a linear sum of the scalar and tensor components, if there is a relevant contribution from GW this would lower the predicted amplitude of the acoustic peaks on sub-degree angular scales.

With the advent of the new CMB peaks detections, many authors have therefore addressed the question of the GW’s contribution (see e.g. \([26], [21], [39], [12], [24], [40], [18]\)). However, despite the different scale dependence, robust constraints on tensor modes remain difficult to obtain. The decrease in the amplitude of the acoustic oscillations induced by GW can indeed be compensated by an increase in one of the unconstrained parameters of the model, like, for example, the spectral index of scalar fluctuations \( n_S \). Therefore, some form of ‘cosmic degeneracy’ arises in the tradeoff between these two (and more) parameters (see \([26], [12]\)) and only weak constraints on the GW background were obtained.

In this context, and before more accurate polarization data become available (see discussion below), the new results on intermediate angular scales, as recently provided by Archeops, can offer an interesting opportunity.

As we illustrate in Fig.1, this spectral region has a particular sensitivity to a GW contribution. In the figure, we plot two theoretical power spectra. The models have identical power on sub-degree scales and on COBE scales (considering cosmic variance), but different tensor contributions, parametrized by a tensor over scalar ratio of the angular power spectrum quadrupole \( r = C_T^2 / C_S^2 \) (see e.g. \([21]\)).

As we can see, while the two models are degenerate on scales \( \ell \geq 200 \), the degeneracy is broken on larger angular scales (see the bottom panel), mostly in the region sampled by Archeops. Both increasing \( n_S \) and adding tensors change the rate of growth of the scalar modes from the Sachs-Wolfe plateau towards the first
FIG. 1. Best-fit models to recent CMB data with and without GW contribution (Top Panel). The Archeops data points are shown as open circles. In the Bottom panel we plot the % difference between the two degenerate models together with the cosmic variance limit (dashed line) averaged in bins of $\Delta \ell = 10$.

peak and this can in principle be used to constrain the GW background.

It is therefore extremely timely to analyze the Archeops data allowing the possibility of a GW contribution in order to see if the amplitude of this background can now be better constrained than in the past.

Furthermore, the GW background produces a unique statistical signature in the polarization of the CMB by inducing a curl component (\cite{33}, \cite{20}), often defined as $B$ mode, while scalar (but also tensor) perturbations produces a gradient component ($E$ mode). Given the large number of future and ongoing CMB polarization experiments, it is interesting to forecast from the present CMB temperature data the expected amplitude of the $B$ modes and/or if the $E$ modes produced by tensors can be distinguished from those produced by scalar perturbations only.

We pursue this investigation in the present Rapid Communication as follows: in Section II we illustrate our analysis method. In section III we present our results. Finally, in section IV, we discuss our findings.

II. ANALYSIS: METHOD

As a first step, we consider a template of flat, adiabatic, $\Lambda$-CDM scalar and tensor spectra computed with CMBFAST (\cite{36}), sampling the various parameters as follows: $\Omega_{cdm} h^2 \equiv \omega_{cdm} = 0.05, ...0.25$, in steps of 0.02; $\Omega_b h^2 \equiv \omega_b = 0.009, ...0.024$, in steps of 0.003, $\Omega_{\Lambda} = 0.5, ...0.95$, in steps of 0.05. Our choice of the above parameters is motivated by the Big Bang Nucleosynthesis bounds on $\omega_0$ (both from $D$ \cite{6} and $^4He + ^7Li$ \cite{9}), from supernovae (\cite{14}) and galaxy clustering observations (see e.g. \cite{38}).

Variations in the tensor and scalar spectral indices, $n_S$ and $n_T$ are not computationally relevant. However, we restrict our analysis to relevant inflationary values $n_S = 0.7, ..., 1.3$ and we fix $n_T = 0$ (see discussion below for different values of $n_T$).

Furthermore, the value of the Hubble constant is not an independent parameter, since $h = \sqrt{(\omega_{cdm} + \omega_b)/(1 - \Omega_{\Lambda})}$. We also include the further top-hat prior $h = 0.7 \pm 0.2$ (\cite{13}) and we consider only models with age $t_0 > 11$ Gyrs.

We allow for a reionization of the intergalactic medium by varying the compton optical depth parameter $\tau_c$ in the range $\tau_c = 0.0, ...0.45$ in steps of 0.05. We note here that high values of $\tau_c$ are in severe disagreement with recent estimates of the redshift of reionization $z_{re} \sim 6 \pm 1$ (see e.g. \cite{15}) which points towards $\tau_c \sim 0.05 - 0.10$. On the other hand, if the reported CBI excess at $\ell = 3000$ is due to Sunyaev-Zeldovich effect, then this would favour values $\tau_c \sim 0.3$ (\cite{3}).

For the CMB data, we use the recent results from the BOOMERanG-98, DASI, MAXIMA-1, CBI, VSA and Archeops experiments. The power spectra from these experiments were estimated in 19, 9, 13, 14, 10 and 16 bins respectively (for the CBI, we use the data from the MOSAIC configuration, \cite{10}), spanning the range $2 \leq \ell \leq 1500$. We also use the COBE data from the RAPDACK compilation (\cite{34}).

For the CBI, DASI, MAXIMA-I and VSA experiments we use the publicly available correlation matrices and window functions. For the Archeops and BOOMERanG experiments we assign a flat interpolation for the spectrum in each bin $\ell(\ell+1)C_{\ell}/2\pi = C_B$, and we approximate the signal $C_B$ inside the bin to be a Gaussian variable. The likelihood for a given theoretical model is defined by $-2 \ln L = (C_B^{th} - C_B^{ex})M_{BB'}(C_B^{th} - C_B^{ex})$ where $M_{BB'}$ is the Gaussian curvature of the likelihood matrix at the peak.

We consider 5%, 10%, 4%, 5%, 3.5% and 5% Gaussian distributed calibration errors (in $\Delta T$) for the Archeops, BOOMERanG-98, DASI, MAXIMA-1, VSA, and CBI experiments respectively and we include the beam uncertainties by the analytical marginalization method presented in (\cite{4}).

Finally, we parametrize the GW contribution by the tensor over scalar quadrupole ratio $r = C_T^{BB}/C_S^{BB}$ and we rescale the sum spectrum by a prefactor $C_{10}$, assumed to be a free parameter, in units of $C_{10}^{CMB}$.

III. ANALYSIS: RESULTS

The main results of our analysis are plotted in Fig.2. In the left top panel we plot the likelihood contours in
While the inclusion of the Archeops data has little effect on the $n_S - r$ (Top Panel, Left), $n_S - \tau$ (Top Panel, Right), $r - \tau$ (Bottom Panel) planes for the models considered in our analysis (see text). The line contours are confidence levels without the Archeops data.

We do this in the 3 panels of Fig.3, where we plot the envelope of the minima and maxima polarization signal. In the right top panel of Fig.2, we plot the likelihood contours in the $n_S - \tau_c$ plane. As we can see, the present CMB constraint on $\tau_c$ is rather weak, with $\tau_c < 0.25$ ($\tau_c < 0.36$) at 68% C.L. (95% C.L.). It is interesting to note that the inclusion of the ARCHEOPS datapoints has little effect.

Finally, in the bottom panel of Fig.2, we plot the likelihood contours in the $r - \tau_c$ plane. An increase in $\tau_c$ or $r$ produces a similar damping on the small/intermediate angular scales. It is interesting to notice that the present data is allowing just a well defined amount of small-scale damping. Values of $\tau_c \sim 0.3$ are in disagreement with the presence of a tensor component. If $\tau_c > 0.2$ then $r < 0.05$ at 68% C.L.

To each theoretical model in the likelihood planes produced in Fig.2, is possible to associate a theoretical polarization power spectrum and translate the confidence contours into an expected maximum and minimum polarization signal.

As we can see from the center panel of Figure 3, the level of the $B$-modes, is expected to be of $\sim 0.2 \mu K$, at maximum. The signal is out of the reach of most of the current polarization experiment like DASI or POLATRON which are sensitive to few $\mu K$. Near future experiments like B2K or QUEST, will probably have enough sensitivity to have a statistical $B$-mode detection. However, the $B$-signal in the angular region sampled by these experiments ($\ell > 50$), can be contaminated by a foreground component due to the conversion of $E$ modes to $B$ modes from gravitational lensing (see Fig. 3) ([41]). Higher-order correlations will be necessary to map the cosmic shear and subtract this contribution to the $B$ mode ([30]).

Tensor perturbations produce $E$ modes as well. However, the amplitude of the $E$ tensor modes is predicted to be generally much smaller than those from the scalar modes (see bottom panel). A window of opportunity may appear in the temperature-polarization ($<TE>$) cross-correlation spectra, where, at $\ell \sim 50$, the amplitude from tensor can be larger than those from scalar modes, leaving a possible detectable excess for experiments like QUEST or B2K.

In order to cross-check if any information can be obtained on $n_T$ we performed the analysis on just one cosmological model defined by $\Omega_A = 0.7$, $\omega_b = 0.022$, $\Omega_m = 1$, $\tau_c = 0.04$. We then considered tensor contributions by varying the scalar and tensor spectral indices independently: $n_S = 0.7, ..., 1.3$ and $n_T = -0.3, ..., 0.0$, step 0.01. We found that the ten-
sor spectral index is not constrained by the present data, but that a value of \( n_T = 0 \) is preferred.

**IV. CONCLUSIONS**

In this Rapid Communication we have presented new constraints on the stochastic background of gravitational waves from recent microwave anisotropy data. Thanks to Archeops, our results improve the constraints on tensor modes from previous analyses (see e.g. [40], [21]).

In the framework of models we considered, we found (at 95% C.L.) \( r < 0.43 \) and \( n_S = 0.97^{+0.10} \, ^{-0.12} \). The energy scale of inflation \( E_{inf} \) can be related to tensor by \( E_{inf}^4 = 0.65C_2^T m_{Pl}^4 \). The above bound translates therefore in \( E_{inf} \leq 1.6 \times 10^{18}\text{GeV} \).

When comparing with the results presented in [2], a part from the different template of theoretical models considered, our analysis differs mainly in the following points: we assumed the low-\( \ell \) Archeops bins as gaussian distributed, we included the COBE data using the RADPACK compilation, we have a strong upper limit on \( \omega_b < 0.025 \) from BBN and, finally, we numerically computed the models with \( \tau_c > 0 \) (while in [2] an analytical formula was used).

The GW background induces a unique signature in the polarization of the CMB by producing a curl component, not present in the case of scalar perturbations. In the set of models considered (and under the assumption of a bayesian method of statistical analysis) we found that the maximum expected level of \( B \) modes allowed by current data is of about \( \sim 0.2\mu K \), which can be partially attainable by near future experiments and severely contaminated by lensing \( E \rightarrow B \) conversion.

The \( E \) modes expected from gravity waves are lower than the \( E \) modes expected from scalar perturbations. However, the tensor \( <TE> \) cross-correlation might be larger at \( \ell \sim 50 \).

In this context, deviations in the \( <TE> \) cross-correlation scalar spectrum at \( \ell \sim 50 \) can possibly offer competitive information with respect to \( B \) modes search.

All our predictions derived from the temperature data are consistent with the recent claim of detection of polarization \( E \) modes from the DASI experiment ([22]).

As a final remark, we want to stress that the results presented here have been obtained under the assumption of a theoretical framework. The bounds on the polarization spectrum must be considered just as an indication of what future observations may detect.

In particular, we just considered \( n_T = 0 \) in the main analysis and we looked at the effect of having \( n_T \) as low as \( -0.3 \). For inflation, only values \( n_T \leq 0 \) can be considered and we checked that varying \( n_T \) has little effect on the final results. Spectra with “blue” \( (n_T > 0) \) spectral indices can be produced in Ekpyrotic ([37]) or Pre-Big Bang (see e.g. [27]) scenarios. However, extremely blue spectra \( (n_T \sim 2) \) are excluded by constraints on the GW energy density background from timing milli-second binary pulsars [31]. Allowing for extra primordial perturbation modes like isocurvature, will probably tighten our constraints on GW, since the shape of CDM scalar isocurvature modes is similar to those from adiabatic tensor modes. However, considering the most general initial conditions scheme and including cross correlations, will certainly enlarge our constraints ([5]). Including curvature \( (\Omega_{tot} \neq 1) \) would relax our bounds on \( r \) (see e.g. [39]). Non-flat models in agreement with the CMB data are in general closed models, which, a part from a few exceptions ([25]), are difficult to obtain from inflation. Finally, including a different model for dark energy like quintessence would change the large scale anisotropy through the Integrated Sachs Wolfe effect (see e.g. [47]), affecting our constraints as well.

Even if the results presented here do not hint for a presence of GW background, the data is still consistent with a sizable tensor contribution. It will therefore be the duty of future and ongoing experiments to scrutinize this fundamental prediction from inflation.

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