High proton energies from cone targets: electron acceleration mechanisms

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Abstract. Recent experiments in the Trident laser facility (Los Alamos National Laboratory) have shown that hollow conical targets with a flat top at the tip can enhance the maximum energy of proton beams created during the interaction of an ultra-intense short laser pulse with the target (Gaillard S A et al 2011 Phys. Plasmas 18 056710). The proton energies that have been seen in these experiments are the highest energies observed so far in laser-driven proton acceleration. This is attributed to a new acceleration mechanism, direct light pressure acceleration of electrons (DLLPA), which increases the number and energy of hot electrons that drive the proton acceleration. This acceleration process of protons due to a two-temperature sheath formed at the flat-top rear side is very robust and produces a large number of protons per shot, similar to what is regularly observed in target normal sheath acceleration (Hatchett S P et al 2000 Phys. Plasmas 7 2076, Maksimchuk A et al 2000 Phys. Rev. Lett. 84 4108, Snively R A et al 2000 Phys. Rev. Lett. 85 2945) with flat foils. In this paper, we investigate

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the electron kinetics during DLLPA, showing that they are governed by two mechanisms, both of which lead to continuous electron acceleration along the inner cone wall. Based on our model, we predict the scaling of the hot electron temperature and ion maximum energy with both laser and target geometrical parameters. The scaling of \( T_{\text{hot}}^{\text{DLLPA}} = m_e c^2 a_0^2 / 4 \) with the laser strength parameter \( a_0 \) leads to an ion energy scaling that surpasses that of some recently proposed acceleration mechanisms such as radiation pressure acceleration (RPA), while in addition the maximum electron energy is found to scale linearly with the length of the cone neck. We find that when optimizing parameters, high proton energies suitable for applications can be reached using compact short-pulse laser systems with pulse durations of only a few tens to hundreds of laser periods.

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1. Introduction

The increase of maximum proton energy during laser-driven proton acceleration is of great interest for many applications such as fast ignition fusion [1] and tumor therapy [2]. Curved-wall hollow micro-cone targets, with a flat top at the tip, were recently shown to increase the proton acceleration compared to flat foil targets and reduced mass targets [3]. We attribute this increase in maximum proton energy to the formation of surface currents along the cone wall, which are observed when the laser is aligned tangentially to the inner cone wall, as seen, for example, in [4–6]. These currents are flowing toward the tip and are made up of high-energy electrons. Their energies predicted by particle-in-cell (PIC) simulations well exceed the ponderomotive energy [7]. This in turn can enhance the proton acceleration from the top as compared to regular flat foils [3]; see figure 1. The conditions necessary to create such currents are a high laser contrast, a high laser pulse intensity and the use of low-density, small Z-targets [5], while the mechanism responsible for the energy increase has remained a subject of debate. Sentoku et al [6] showed that for a certain class of experiments—i.e. sub-focal spot-sized inner cone neck diameter and straight inner cone walls at moderate laser intensity—the geometric micro-focusing of the laser light leads to higher laser intensity and thus higher electron energy. However, an enhancement of proton energies in experiments and of electron energies in PIC simulations was also found for cones with a neck diameter much larger than the laser focal waist, and the energies exceeded that expected from micro-focusing alone. Nakamura et al [8] found that a resonant acceleration of electrons oscillating in a self-created surface potential can
lead to higher electron energy in a long capillary with walls covered by preplasma when the laser is aligned at a resonant angle with respect to the walls.

In this paper we investigate, with the aid of two-dimensional (2D) PIC simulations, the electron dynamics in the case of cones with a curved neck (see figure 2) and a neck diameter that well exceeds the laser focal waist. We find that micro-focusing and resonant acceleration in this case are not efficient in generating hot electrons and cannot explain the simulated electron energies. Rather, the continuous, direct acceleration of electrons by the laser light is found to be dominant and easily scalable. We analyze in detail the interaction of the laser with the target in the case of grazing incidence, thereby identifying the mechanisms relevant for optimizing the cone geometry with respect to proton acceleration. Then we give analytical and empirical scaling laws for the electron energy with laser parameters and infer optimum geometric parameters for cone targets. The scaling laws show that compact laser sources for laser-driven particle acceleration can indeed create proton energies high enough to be interesting for applications.

2. Setup and methods

Our numerical simulations were performed with the two-dimensional, three-velocity component (2D3V) fully relativistic PIC code iPICLS2D [9], including collisions and ionization. The typical laser duration $\tau$ used in our simulations, if not stated otherwise, is $\tau = 100\omega_0^{-1}$ full-width at half-maximum (FWHM) with a Gaussian profile, where $\omega_0$ is the laser angular frequency. In the cases where we are interested in the electron dynamics and temperature scaling, a temporal profile with a flat top of $68\omega_0^{-1}$ and a Gaussian rise and fall of $16\omega_0^{-1}$ was used to provide sufficient duration with constant intensity. Throughout the text, the time $t$ will always be given relative to the time when the laser maximum reaches the front inner surface of the flat top. The spatial laser pulse profile was chosen to be Gaussian with a focal spot size of $w_0 = 2\lambda$, where $\lambda$ is the laser wavelength. The laser was linearly polarized, unless stated otherwise, with the electric field vector pointing in the $x$-direction and the magnetic field vector pointing...
in the $y$-direction, $E = E e_y$ and $B = B e_y$. We furthermore define the dimensionless field strengths $a = E / E_0 = eE / m_e c n_c$ and $b = B / B_0 = eB / m_e c n_c$. In the simulations presented here, unless stated otherwise, the laser strength parameter $a_0$ of a laser with intensity $I$, given by

$$a_0 = \frac{e}{2\pi m_e c^2} \sqrt{\frac{2I\lambda^2}{\varepsilon_0 c^3}} = \sqrt{\frac{2I}{n_c m_e c^3}},$$

was set to $a_0 = 8.5$, but other laser intensities were also used to study the scaling of the interaction processes with $a_0$. Here,

$$n_c = \frac{m_e c^3 \varepsilon_0}{e^2}$$

denotes the critical density of the cold plasma. Throughout this paper we will use a dimensionless unit system where $c = m_e = \omega_0 = e = 1$, which simplifies equations. For example, in the defined unit system it is simply $n_c = 1$.

The target geometry is shown in figure 2. It consists of a hollow cone, with curved walls of a typical radius of curvature of $10\lambda$ and a thickness of $5\lambda$. The separation distance between the walls is set to $15\lambda$, which is much larger than the laser focal spot size of $2\lambda$. At the tip of the cone a flat foil is mounted with a diameter of $90\lambda$ and thickness $5\lambda$. The target is composed of copper, which was fourfold pre-ionized in order to mimic the effect of prepulses and amplified

Figure 2. Initial density and composition of the cone target (not to scale). The cones are positioned $12.5\lambda$ from the left simulation box boundary and centered in the simulation box in the vertical direction. Its walls have a radius of curvature of $R = 10\lambda$ with an inner neck diameter of $15\lambda$; the top has a diameter of $90\lambda$. The thickness of all copper walls is $5\lambda$; the top is additionally covered with $2\lambda$ of hydrogen ions. The resulting position of the top front surface is $27.5\lambda$ from the left box border. In some simulations the neck was extended, as shown in the right figure, and the wall curvature was varied. The influence of changing the geometric properties is discussed in section 3.2.
spontaneous emission, while the flat top is additionally covered with a neutral proton–electron plasma layer of thickness 2λ.

In order to keep computational demands acceptable, we reduced the target electron density \( n_{e,0} \) (fully ionized) compared to realistic solid targets, where \( n_{e,0} \) typically amounts to a few hundreds of \( n_c \). For most of the simulations we set the density to \( n_{e,0} = 10 n_c \). For the simulations regarding the intensity scaling we set the density to \( n_{e,0} = 40 n_c \) for \( a_0 > 8.5 \) in order to prevent an artificial relativistically induced transparency that would occur for \( n_{e,0}/n_c \leq \gamma \) (\( \gamma \) is the relativistic Lorentz factor of the hot electrons). As long as \( n_{e,0} \gg \gamma n_c \), the reduced plasma density is expected not to alter the kinematics in the simulations compared to real solid densities, since the currents are all expected to be less than \( \gamma n_c c \). Additionally, resistive effects connected to collisions are negligible in the scope of this work, since the mean free path of electrons in the region of interest where the laser interacts with the plasma is much larger than the Debye length. Furthermore, we did not run the simulations using the full number of real particles but rather had to use macro-particles, treating several neighboring real particles as one, as is standard in PIC simulations for laser–matter interaction. Still, the dynamics of the macro-particles can be used to infer the dynamics of the real particles, thus allowing us to study the acceleration process on the particle level. The number of macro-ions per cell was set to 4, which results in 116 macro-electrons when fully ionized. This choice ensures that the macro-particle dynamics still closely resembles the single-particle dynamics. The simulation box volume of \( z \times x = 241.6 \lambda \times 120.8 \lambda \) was divided into 6000 \( \times \) 3000 cells, resulting in a cell size of \( \Delta z = \Delta x = 0.04 \lambda = 0.125 \times 2\pi c/\omega_{p,0} \) (\( \omega_{p,0} \) is the cold plasma angular frequency when the plasma is fully ionized). Correspondingly, the simulation time was discretized with steps of \( \Delta t = 0.04 \cdot T = 0.125 \times 2\pi/\omega_{p,0} \). Hence, there are at least eight cells per plasma wavelength and eight time steps per plasma period.

3. Results

Compared to regular flat foils, flat-top cone targets with circular walls have been shown in experiments to enhance the maximum energy of protons emitted behind the target [3] (figure 1). This has been attributed to the laser interaction with electrons along the inner cone wall. The higher electron energy observed in PIC simulations is the key factor leading to higher proton energies, since the accelerated electrons can cross the cone top and contribute to a quasi-target normal sheath acceleration (TNSA) process at the rear surface. This process is equivalent to the regular TNSA process [10] on flat foils but now with two-electron ensembles: the ponderomotively heated electrons from the top front surface and the more energetic electrons from the cone walls which are responsible for the increase in maximum proton energy.

The laser electric field pulls out electrons from the cone wall into the vacuum. Since the transverse electric field is oriented negatively (corresponding to an upward force on the electron) once every laser cycle, the resulting electron density modulation is also periodic with a period length of \( 1\lambda \). In figure 3 the energy distribution is plotted over the longitudinal dimension summed over a region of \( \pm 5\lambda \) around the laser axis, which is aligned grazingly along the inner cone wall. In panel (a), the laser polarization is aligned parallel to the wall surface (s-polarization), while in panel (b) it is perpendicular (p-polarization). While in the first case electrons acquire most of their energy close to the cone top inner surface—comparable to the case of a flat foil—in the latter case the electric field can pull out electrons from the wall into the vacuum region forming bunches of electrons separated by \( 1\lambda \). This should be seen in contrast.
Figure 3. Kinetic energy of electrons at a distance of ±5λ or less from the laser axis for grazing laser incidence. We compare the cases for (a) s-polarized and (b) p-polarized light. For this comparison, the electric field strength of the laser in (a) the y-direction and (b) the x-direction is plotted in gray. Electron energies are normalized to the maximum energy for the case of p-polarization; fields are normalized to their respective maximum value. Whereas for p-polarization the electrons are pulled out of the cone wall and form bunches that are accelerated towards the cone tip, for s-polarization the interaction along the wall is negligible and most electrons are accelerated only in the vicinity of the inner cone top surface located at 27.5λ. As expected, the latter case is comparable to the case of using a flat foil target.

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to the well-known 2ω₀ bunches created, for example, at oblique incidence on a plasma by the 2ω₀ v × B force. Although in the present case the Lorentz force still acts on the electrons at a frequency of 2ω₀, electrons that can be pushed forward by the Lorentz force are present only once every laser cycle, since they have to be pulled out of the cone wall by the laser electric field. Pushed forward by the Lorentz force, these electrons follow the laser direction for a long distance along the cone target inner wall surface until they cross the cone top surface, still showing the initial 1λ modulation in density, thereby forming bunches of energetic electrons. If the electrons remain in phase with the laser, they can gain much more energy than they could in the case of a flat foil target. In our simulations the maximum electron energy reaches up to 34 MeV with an average energy of 6 MeV, which is more than three times that of a flat foil of equal geometry as the cone top only (see figure 9). The electrons are kept from reentering the foil by a quasi-static magnetic field, self-created by the hot electron current and the cold return current inside the wall [6, 11] (figure 4), as long as the angle of incidence is small enough,

$$\alpha < \arccos \left( 1 - \frac{w}{R_β} \right).$$

Here, w denotes the width of the magnetic field region and $R_β/λ = \sqrt{γ^2 - 1}/2π⟨b⟩$. In our simulations the magnetic field extends about w ≈ 0.5λ inside the vacuum with an average magnitude of ⟨b⟩ ≈ 2, preventing even the most energetic electrons from reentering the foil for α < 25°. Other proposed mechanisms such as the geometric micro-focusing of the laser light inside the foil.

9 See section 3.1 for a detailed analysis of the electron acceleration mechanisms.
Figure 4. Schematic representation of electron confinement at the cone wall/vacuum interface. The black structure is a part of the cone target which is irradiated by the laser (red, electric field direction indicated by up/down arrows). The laser electric field extracts electrons from the cone wall at a separation of 1λ, which can then be forward accelerated as described in section 3.1 (black arrows). This current is balanced by a continuous return current inside the wall (white arrow), building up a quasi-static magnetic field at the surface. The inset shows the trajectory of an electron (black) injected in a homogeneous quasi-static magnetic field at an angle α, following a circular path with the cyclotron radius Rβ small enough to be fully confined laterally in the interaction region close to the cone wall surface.

cone [6] or the resonant acceleration of electrons along the cone surface [8, 12] cannot explain the electron dynamics and temperature seen in these simulations [3].

3.1. Electron acceleration at the cone wall surface

In this section, we will have a detailed look at how exactly the electrons gain their high energy in cones when the laser is aligned at grazing incidence. Figure 5(a) shows the longitudinal and transverse electric fields along the inner wall. We observe a longitudinal electric field which is shifted with respect to the transverse electric laser field by π/2. The electron bunches at the vacuum–wall interface (figure 5(b)), formed by the transverse electric field, are positioned with a phase shift of π, so the longitudinal electric field is a superposition of the laser field bent around the curved wall and the Coulomb repulsion forces created by the electron bunches. As we will show later, the electron acceleration in our case is not a resonant process [8] but rather a continuous acceleration. The possible acceleration scenarios for a surface electron in this case are sketched in figure 5(d), which shows the qualitative electron dynamics in the co-moving frame. An electron pulled out of the wall by the transverse electric laser field can gain forward momentum via the $v \times B$ force. It can then get caught in an accelerating $v \times B$ phase (moving upward (i) or downward (ii)) or into the longitudinal electric field region (iii). The resulting energy gain can be seen in figure 5(c), where a 2D histogram of the electron density in the $z$–$\gamma$ plane is shown. To eliminate the influence of the curved neck, here we used a cone with a prolonged straight neck (figure 2, right). As the electrons are guided along the inner wall

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Figure 5. Longitudinal (gray) and transverse (black) electric fields along the cone wall (a) and spatial distribution of electrons (b) when the laser is aligned tangentially to the cone wall ($\omega_0 = 8.5$, FWHM $100/\omega_0$ (Gaussian), $t = 0$). Hot electrons with energy exceeding 10 MeV, which is the maximum electron kinetic energy seen in the simulation of flat foils, are shown in red. The longitudinal electric field is phase shifted by $\pi/2$ with respect to the transverse laser field and the electron bunches at the surface are phase shifted by $\pi$ with respect to the transverse electric extracting force ($-eE$). Panel (c) shows the distribution of electrons in the $z$-$\gamma$ plane of a cone with a $15\lambda$ straight neck (logarithmic rainbow color scale; the gray lines in the sketch in the inset correspond to gray lines in the graph). (d) Qualitative electron dynamics in the frame co-moving with the laser phase: the electron bunches (blue) extracted by the transverse laser field are moving approximately in phase with the laser; the longitudinal forces on electrons are indicated by horizontal arrows (top: forces due to quasi-static longitudinal fields; middle: $v \times B$ forces (here: for electrons moving upward)).

An electron extracted from the wall initially has a velocity in the laser direction of $\beta_z \ll 1$, so it will be overtaken by the laser pulse (light line, (iv)). If $\beta_z \approx 1$, it can be continuously accelerated via transverse fields ($v \times B$, (i, ii)) and longitudinal fields (iii).
surface, their energy is found to be constantly increasing while they are moving toward the cone top. In fact, the energy gain $d\gamma/dz$ is larger in the extended neck region, where the laser is aligned perfectly parallel to the cone wall, than in the curved neck region, with electron energies reaching up to 5 MeV/\(\lambda\).

In order to quantitatively describe the mechanism, we will follow the trajectories in the PIC simulation for all electrons originating within a region of $\pm 1\lambda$ around the laser axis where we expect the most energetic electrons to originate from. The energy gain $d\gamma/dt$ of a single electron is given by

$$\frac{d\gamma}{dt} = \frac{p}{\gamma} \frac{dp}{dt}.$$  

Multiplying the Lorentz force equation with $p = \gamma \beta$, 

$$p \frac{dp}{dt} = p (a + c \beta \times b) = -pa,$$

and using $a = a_x e_x$ for the electric field of the laser wave, one obtains

$$\frac{d\gamma}{dt} = -a_x \beta_x$$

for the energy gain of an electron due to the transverse laser field. For large $a_0 \gg 1$, this energy is predominantly converted into forward momentum via the $v \times B$ force. Similarly, one can define

$$\frac{d\gamma_z}{dt} = -a_z \beta_z$$

as the energy gained by longitudinal fields. We define

$$\Gamma_z = -\int a_z \beta_z dt,$$

$$\Gamma_x = -\int a_x \beta_x dt$$

and we will calculate the corresponding values for each tracked electron. The first of the two integrals is a measure of the amount of energy gained by the electron due to longitudinal electric fields (trajectory (iii) in figure 5(d), in the following referred to as electron population ‘A’). The second integral is a measure of the amount of energy gained due to transverse electric fields, which for ultra-relativistic intensities is converted into forward momentum via $v \times B$ (trajectories (i) and (ii) in figure 5(d), electron population ‘B’). Since we are interested in the energy gain beyond the energy seen in a flat foil, we choose the time when the electron has an energy of more than 4 MeV as a lower limit of the integral, an energy that well exceeds the ponderomotive energy. The upper limit of the integrals is given by the time when the electron crosses the top inner surface and leaves the interaction region, which is at $z = 27.5 \lambda$.

We can now gain more insight into the acceleration mechanism by studying the trajectories and forces of the most energetic electrons of each group. For the most energetic yet representative electron of group ‘A’, figure 6(A) shows the trajectory (a), sources of energy gain (b) and the gain of energy over time (c). It can be seen that the electron is caught in an accelerating phase of the longitudinal electric field, after being extracted from the cone wall and performing a few oscillations during which it is slower than the laser phase velocity, while the contribution of the transverse field to its acceleration remains very small. For the most
Figure 6. Trajectories of the most energetic electron of group ‘A’ (A) or group ‘B’ (B) (a), its sources of energy gain (b) and the energy gain over its longitudinal propagation along the wall (c). Energy is continuously acquired mainly by longitudinal electrical fields (green). Laser and target parameters are described in the text, with the laser having a flat-top temporal profile and $a_0 = 8.5$. The cone neck was elongated to $l = 15\lambda$.

An energetic electron from group ‘B’, the same graphs are shown in figure 6(B). In this specific case, after being extracted at $z = 22.7\lambda$, the particle at first experiences a strong acceleration due to the longitudinal electric field. Later, the electric field becomes decelerating and the $v \times B$ acceleration due to the transverse electron velocity becomes dominant. At the end of the acceleration process, the net energy gain due to longitudinal fields even becomes negative. The particle is not oscillating but rather moves upwards monotonically and remains in phase with...
the laser. In order to demonstrate that for the majority of electrons the acceleration is indeed continuous, we introduce
\[
\Gamma_{[x]} = - \int |\beta_x| a_x \, dt
\]
and study the ratio
\[
\Phi \equiv \frac{\Gamma_{[x]}}{\Gamma_x}.
\]
The electric field strength of the laser \( a_x(t) \) is a periodic function with \( \langle a_x(t) \rangle = 0 \). In the case of resonant absorption, \( |\beta_x(t)| \) is also periodic and hence the integral \( \Gamma_{[x]} \) and \( \Phi \) vanish when integrating over many periods. In the case of an electron co-moving with the laser phase, \( \beta_x(t) \) is increasing monotonically; hence the integral \( \Gamma_{[x]} \) takes on a large value, and \( \Phi \) becomes \( \pm 1 \). Figure 7 shows the distribution of \( \Phi \) for all forward accelerated electrons of group 'B'. It can be seen that there are only a few electrons with \( \Phi \approx 0 \), but there are two distinct maxima around \( \Phi = \pm 1 \). This means that indeed most electrons are accelerated continuously, not by resonant energy transfer but by co-moving with the laser field.

The continuous acceleration of electrons leads to a significant increase of the hot electron temperature compared to a conventional flat foil consisting of the cone top only. Figures 8 and 9 show the spectra obtained from simulation with \( a_0 = 8.5 \) and \( \tau = 100 \omega_0^{-1} \) for a flat foil and a cone when the laser is aligned tangentially to the inner cone wall. We plot the distribution of energy of the individually tracked electrons at the respective time when they cross the flat-top front surface and leave the interaction with the laser, up to the time when the laser maximum reaches the cone top front surface. The resulting energy distribution is a direct imprint of the laser–electron interaction. We did not use the spectra of the electrons at a certain point in time, since they would be biased by a transfer of energy to ions while the electrons bounce back and forth across the flat top several times during the laser pulse due to the electro-magnetic fields building up at the target surfaces. The solid black line shows the spectrum including all electrons, while the thick dark gray lines show the spectra of electrons with \( \Gamma_{[x]} > \Gamma_{x} \) (dotted) and \( \Gamma_{z} > \Gamma_{x} \) (dashed). In the case of a flat foil, most of the electrons follow an exponential distribution with a scale length of 1.2 MeV. In the case of the cone, in the low-energy region the spectra also follow an exponential curve with approximately the same scale length. In that part, the spectrum is very similar to that of a flat foil, from which we can conclude that these are the electrons accelerated at the cone top front surface. For high energies \( E > 7 \) MeV, the electrons follow a second exponential curve with a significantly larger scale length close to 9 MeV. This part of the spectrum is dominated by the surface electrons accelerated via the two mechanisms described before. If we subtract the low-temperature electrons from the spectra (via fitting an exponential up to \( E < 4 \) MeV), we can estimate the spectral distribution of surface electrons for both of the electron sub-ensembles over the entire energy range. It turns out that the number of particles from groups 'A' and 'B' is approximately the same.

The electron acceleration depends on the geometric parameters of the cone (e.g. the wall radius of curvature and the neck length), and in the above discussion we used a wall curvature and neck length optimized for proton acceleration (\( R = 10 \lambda, l = 2 \lambda \)). In this case, we find that the temperature of electrons from group 'B' saturates and coincides with the temperature of electrons from group 'A'. Then, the acceleration length \( l_{acc} \), which can be defined as the length between the point where the curved cone wall approaches the laser axis by less than \( w_0 \) and the cone top, \( l_{acc} = \sqrt{R^2 - (R - w_0)^2} + l \), coincides with the dephasing length of a single electron.

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Figure 7. (a) The ratio $\Phi$ for electrons from group ‘B’ accelerated forward. The black line represents the distribution if the lower limit of the integral is set to the time when the electron energy exceeds 4 MeV, as defined in the main text. For comparison, the gray line shows the distribution of $\Phi$ when the lower limit is set to the starting time of the simulation. As can be seen, the electron distribution exhibits distinct maxima at $\Phi = \pm 1$ that represent continuous acceleration, while there is no distinct peak at $\Phi = 0$ that would represent resonant energy absorption. (b) $\Gamma_x$, $\Gamma_{|x|}$ and $\Phi$ for different combinations of synchronization of electron transverse motion and laser electric field. The first three rows represent forward acceleration (considered for the top figure); the last three rows represent backward acceleration. Laser and target parameters are the same as in figure 6.

In a plane wave
\[
\frac{l_{\text{deph}}}{\lambda} = \frac{a_0^2}{8}. \tag{3}
\]

From figure 10 it can be seen that an extension of the neck length does not change the temperature for electrons from group ‘B’ significantly. For electrons from group ‘A’ it leads to higher electron temperatures and higher maximum energy, but at the same time the number of hot electrons decreases, and hence an increased neck length is not beneficial for ion acceleration which depends on the temperature and the number of electrons (see section 3.2).

The temperature of electrons in the optimum case can be estimated by approximating the electron motion along the cone wall with the energy of a single initially resting electron in a
Figure 8. Electron spectrum of a flat foil after $t = 150 \alpha_0^{-1}$, irradiated by a laser with $\alpha_0 = 8.5$ and a temporal profile having a flat top (see the main text for details). The dashed (dotted) line shows the spectrum for electrons of group ‘A’ (‘B’) mainly accelerated via longitudinal (transverse) electric fields. Laser parameters are the same as in figure 6.

Figure 9. Electron spectrum of a cone where the laser is aligned grazing along a wall. The dashed (dotted) line shows the spectrum for electrons of group ‘A’ (‘B’). At low energies, the spectrum is dominated by electrons accelerated at the cone top with a temperature equal to that of a flat foil (see figure 8). Laser parameters are the same as in figure 6.

Plane electro-magnetic wave. This is given from the relativistic invariant $\gamma - p_z = 1$ by

$$\gamma(\varphi) = 1 + \frac{a(\varphi)^2}{2}. \quad (4)$$

Here we used $p_z = p_{z0}^2/2$. In general, the energy of an electron is determined by the laser phase $\varphi = t - x$ in which it is born and in which it leaves the laser (e.g. by going into an overcritical plasma region) and the average energy over all electrons is hence given by [13]

$$T_{e, hot} = \frac{\int_0^{2\pi} \gamma d\varphi}{\int_0^{2\pi} d\varphi}.$$
With $a(\varphi) = a_0 \sin \varphi$, it follows that

$$\langle \gamma_{\text{cone}} \rangle = 1 + \frac{a_0^2}{4}. \tag{5}$$

This estimate describes very well the average hot electron temperature seen in the simulations. In the case of the standard simulation parameters, equation (5) predicts 9 MeV in agreement with the spectrum shown in figure 9. We performed additional simulations with $a_0$ ranging from 1 to 20 and find that in all cases with $a_0 < \frac{n_{e0}}{n_c}$, equation (5) agrees very well with the PIC results (see figure 11).

Coming back to the comparison of our results obtained for grazing laser incidence with regular flat foils, we find that in the case of flat foils the average energy of the hot electrons is significantly less than what was observed for the case of laser grazing the cone wall, and is even less than the ponderomotive energy $T_{\text{hot, pond}} = \sqrt{1 + \frac{a_0^2}{2} - 1}$. \tag{6}

Indeed, only a few electrons follow the ponderomotive energy scaling. Figure 11 shows the average hot electron energy (black circles). Whereas for $a_0 < 1$ it approaches the ponderomotive energy
Figure 11. The scaling of electron energy with laser strength. Circles and squares show the average energy $T_e + 1$ of hot electrons from a flat foil and a cone with grazing laser incidence, respectively, as obtained from simulations run with $n_e = 10n_c$ for $a_0 < 8.5$ and $n_e = 40n_c$ for $a_0 \geq 8.5$. The cone wall radius was varied to reach the maximum electron temperature to account for the intensity-dependent dephasing length. The black dashed line for comparison shows the ponderomotive scaling equation (6), the gray line is the prediction acquired from equation (5) and the black line is an empirical fit equation (7) to the temperatures in flat foils.

scaling, for large $a_0$ its scaling can be described by [13]

$$T_e^{\text{hot}} \approx 1.6 \frac{a_0}{\ln(4a_0)} - 1,$$

which is significantly less than what was found for laser grazing cones (equation (5)).

3.2. Optimum cone target

The above results demonstrate efficient generation of energetic electrons in the case of laser grazing incidence on a curved cone target along the inner wall. In this section, we will analyze how improved electron acceleration will influence the acceleration of ions from the cone top based on geometric parameters (wall diameter and preplasma) and laser parameters (intensity and duration). The ion acceleration process at the cone top is TNSA-like. Hot electrons that have been created both at the front surface and along the cone wall travel through the top and exit at the rear, building up a quasi-static electric field. The ions, which due to their larger mass remain initially at rest, are then accelerated in this quasi-static field at the rear side of the cone top. We will compare the maximum achievable proton energies from cone targets to conventional flat foils of the same geometry as the cone top only, where the ion acceleration is governed by TNSA, as well as to the maximum proton energies predicted by an alternative, widely discussed acceleration mechanism, radiation pressure acceleration (RPA), using the results of [14]. RPA is highly promising for its predicted scaling of the maximum ion energy of up to $\epsilon_{\text{max}} \propto a_0^2$, even though the optimum experimental conditions are very difficult to realize (e.g. a flat-top laser pulse with a very sharp rising edge, circular polarization, a very high contrast and ultrathin foils).

To address the question of how geometric and laser parameters influence the ion acceleration from cone targets, we first discuss simulations varying the cone wall diameter and adding an exponential preplasma to the cone inner surfaces (figure 12). Figure 12(a) shows the

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Figure 12. Proton maximum energy from cones with laser grazing incidence normalized to the maximum energy from flat foils, $a_0 = 8.5$, (a) as a function of the cone wall radius without (black) and with preplasma (gray, scale length $0.6 \lambda$) and (b) as a function of preplasma scale length for a radius of $10 \lambda$ (gray) and $40 \lambda$ (black).

dependence of the maximum proton energy accelerated from a cone at grazing incidence as a function of the radius of curvature of the walls. As discussed before, the dephasing length of an electron in the laser field prevents the electrons from gaining more energy with increasing the acceleration length $l_{\text{acc}}$. For smaller than optimum wall radii the electron temperature and hence the proton energy is lower, because the electron acceleration length is less than what is necessary to reach the maximum energy. For larger radii, electrons dephase with the laser and are decelerated again, and the temperature remains constant. The density of electrons behind the top decreases due to the divergence of the electron beam, resulting in a reduced proton energy. Consequently, one expects an optimum radius of the cone walls where $l_{\text{acc}} = l_{\text{deph}}$,

$$R_{\text{opt}} = \frac{a_0^4}{128w_0[\lambda]} + \frac{w_0[\lambda]}{2}. \quad (8)$$

Indeed we observe a pronounced maximum near $R_{\text{opt}}$, which however is shifted to smaller radii; for example, for the laser strength $a_0 = 8.5$ and laser waist $w_0 = 2\lambda$ used in the simulation, we observe an optimum radius of $10\lambda$. To reach the maximum possible energy within 10%, we find that the radius must be within $\pm 4\lambda$ around the optimum. The smaller optimum radius can be explained by pump depletion and laser reflection.

Next we add an exponentially decreasing preplasma density gradient at the surface of the inner cone walls and the inner cone top with a scale length of $0.6\lambda$. The gray line in figure 12(a) represents the maximum energies normalized to the maximum energy from a flat foil with the same preplasma at the front surface. An important finding is that now the condition for the radius in order to reach the maximum possible energy within 10% is fulfilled up to much greater values, i.e. to radii more than $40\lambda$. This means that at the same time the laser depletion connected with the propagation through the preplasma along the cone wall does not degrade the proton acceleration. This is especially important experimentally where the preplasma can be controlled by the laser prepulse contrast and amplified spontaneous emission (ASE) level, since
Figure 13. Scaling of proton maximum energy with laser strength. Squares and circles show the maximum energies from cones at grazing laser incidence and flat foils (same geometry as the cone top), respectively, as obtained from simulations. Solid lines are the predictions acquired from equation (9) as explained in the text, while the dashed line shows for comparison the maximum ion energies expected from RPA at optimum laser and foil parameters using the results of [14].

It could allow one to lower the restrictions on the cone geometry. Also, instabilities in laser pointing would be more tolerable when preplasma is added. The optimum value for the wall radius remains unchanged and the relative proton energy increase at the optimum wall radius is nearly the same as that without any preplasma. The absolute energies are slightly increased as expected due to a more efficient laser absorption [15]. We note that there is an optimum preplasma scale length since for large scale lengths the laser depletion will be large and the laser will eventually not reach the cone top [16].

Figure 13 presents simulation results for different laser intensities at the respective optimum cone wall radius (and no preplasma). The gray circles show PIC results for a flat foil, and the black squares show the results for cone targets at grazing laser incidence. For intensities where the plasma is opaque, \( a_0 < n_e/n_c \), the cone targets show a significantly higher maximum proton energy that is up to more than three times the energy seen for flat foils. Following the discussion of the previous section, we can analytically estimate the proton energy enhancement. For long pulse durations \( \omega_p \tau \gg 2\pi \) the maximum proton energy can be approximated in the TNSA regime by a time-limited fluid model with [17]

\[
\epsilon_{\text{max}} \approx 2T_{\text{hot}} \left[ \ln \left( t_p + \sqrt{t_p^2 + 1} \right) \right]^2. \quad (9)
\]

Here, \( t_p \equiv \omega_p t_{\text{acc}}/\sqrt{2e} \) where \( t_{\text{acc}} \approx 1.3\tau \) [18], and \( \omega_p = \sqrt{Zm_e/m_i\omega_p} \) is the ion plasma frequency of the protons with mass \( m_i = 1836m_e \) and charge number \( Z = 1 \) and \( e \) is Euler’s number. The electron plasma frequency is given by \( \omega_e \equiv \omega_0\sqrt{n_{e,\text{hot}}} \), where the density of hot electrons can be estimated by an energy conservation argument [19] to

\[
\frac{n_{e,\text{hot}}}{n_c} \approx \frac{a_0^2}{\langle \gamma \rangle - 1} \frac{\eta w_0}{W}, \quad (10)
\]

where \( W \) is the hot electron spot size at the target rear side. In the case of cones with grazing laser incidence where \( \gamma \) scales with \( a_0 \) as given by equation (5), the hot electron density behind the target is \( n_{e,\text{hot}} = \text{const} \), and its maximum value is \( \eta n_c \). We can now evaluate equation (9) analytically with \( t_p = 1.3\tau a_0 \sqrt{m_i/m_{\text{hot}}} / 2Zm_e(\langle \gamma \rangle - 1)W \). Based on our PIC simulations, the laser absorption coefficient varies only a little with the intensity over the intensity region considered here and is
of the order of \( \eta_{\text{foil}} \approx \text{const} \approx 0.45 \); the average divergence is \( \alpha \approx 40^\circ \). Thus, for a fixed pulse duration, \( t_p \) is constant. The maximum proton energy predicted by equation (9) then scales as

\[
\epsilon_{\text{max}} \propto a_0^2
\]

as is indicated by the black line in figure 13. The maximum energies observed in our PIC simulations agree very well with the analytical values, exceeding the proton energy from flat foils significantly.

For a constant laser pulse energy, equation (9) predicts a slight increase of the proton maximum energy with decreasing pulse duration, saturating at \( \epsilon_{\text{max}} \approx 12 \text{ MeV} \) for \( \tau \ll \omega_0^{-1} \). Analytically, it can be easily found that for \( \tau < 150 \omega_0^{-1} \), the increase of proton energy with pulse duration is larger than \( \propto \tau \), while for larger pulse durations the proton energy increases more slowly. Combining the above, it follows that for a given laser pulse energy in the first region it would be more beneficial to optimize for a longer pulse duration, while in the latter region it would be better to optimize for a higher laser intensity.

Next we compare the proton energies from cone targets with those obtained from flat foils (gray circles in figure 13) of the same geometry as the cone top. From the PIC simulations, we again find that the laser absorption coefficient varies only a little with the intensity over the intensity region considered here and is of the order of \( \eta_{\text{foil}} \approx \text{const} \approx 0.25 \) for all laser intensities. Using equation (9) together with (10) and the flat foil temperature scaling (7), we obtain analytical flat foil estimates of the proton maximum energy that again agree well with the PIC results. The scaling of \( \epsilon_{\text{max}} \) with intensity at a fixed pulse duration for flat foils is different for short and long pulses. For short pulses (\( \tau \ll \sqrt{2e/(1.3 \omega_{\text{pi}})} \)) we have \( \ln \left( t_p + \sqrt{1 + t_p^2} \right) \approx t_p \) and hence it follows for a given pulse duration \( \epsilon_{\text{max}} \propto T_{\text{hot}} n_{e,\text{hot}} \). With \( n_{e,\text{hot}} \propto a_0^2 / T_{\text{hot}} \) we find \( \epsilon_{\text{max}} \propto a_0^2 \), in agreement with [20]. Even though this scaling is the same as that for cones, the cones still yield significantly higher-energy protons, for our parameters by a factor of more than 1.8. For long pulses, equation (9) shows a scaling of \( \epsilon \propto a_0^{3/2} \). This is much worse than that in the case of grazing cones, where the scaling remains at \( \epsilon_{\text{max}} \propto a_0^2 \). Interestingly, the factor of proton energy gain at constant pulse energy and laser strength \( 3 \leq a_0 \leq 30 \) peaks at an optimum pulse duration of \( 100 < a_0 \tau < 350 \), the optimum pulse duration varying less at higher \( a_0 \) (figure 14). Hence, at a given laser pulse energy there exists an optimum pulse duration and intensity for which the cone geometry gives the highest increase in proton energy compared to flat foils, and the cone geometry, consequently, should be especially beneficial for short-pulse laser systems.

For flat foils, the maximum proton energy is expected to increase with decreasing thickness [14, 21]. For ultrathin foils the laser strength can become strong enough for the laser pulse to drive all electrons out of the target, so that proton acceleration is no longer dominated by TNSA but rather RPA. The semi-analytic scaling of the maximum proton energy in this case is predicted to be better than that for TNSA [14]. In figure 13, the RPA predictions are plotted with a dashed line for comparison. While the proton energies predicted are significantly larger than those for the thick flat foil in the TNSA regime (solid line), they are only slightly higher than those in the case of the cone targets for \( a_0 < 20 \) and even smaller for yet higher laser strength. Considering the experimental difficulties for the RPA regime, the presented cone target geometry appears to be a very promising alternative.
Figure 14. The enhancement factor of proton maximum energy from cones compared to flat foils as a function of pulse duration \(\tau\) and laser strength \(a_0\), as obtained from equation (9), assuming a constant electron divergence and laser absorption as given in the main text. Dashed curves are iso-pulse-energy lines. For constant pulse energy, the enhancement peaks at a certain point, indicated by the strong black line (guide to the eye).

4. Conclusions

We have studied the acceleration kinematics of fast electrons close to the curved cone wall inner surface by 2D PIC simulations. When the laser spot size is smaller than the cone neck diameter, strong electron currents are created only when the laser is aligned grazing to the wall. Then, the laser electric field extracts electrons from the wall once every cycle. The Lorentz force and longitudinal electric fields accelerate the extracted electrons, forming energetic bunches following the inner cone wall towards the cone tip by self-generated fields. When reaching the tip these bunches add to the electron sheath that creates the electric field responsible for accelerating protons.

The main mechanism of electron acceleration along the wall is the continuous acceleration of electrons. Other mechanisms such as micro-focusing or resonant acceleration of surface electrons are found to be of minor importance and can be neglected. Consequently, using the temperature scaling derived for the continuously accelerated electrons along the wall alone, we were able to derive an analytic model that can accurately predict the proton maximum energy as seen in simulations. In our model, the electron temperature scaling with intensity can be described by a simple model based on the vacuum energy gain of free electrons in a plane electro-magnetic wave. From the electron dephasing length, an optimum value for the cone wall curvature radius with respect to electron maximum energy can be found.

The increase in electron density and temperature at the cone top due to the continuously accelerated electrons leads to a significant increase of the proton maximum energy, especially
for high laser intensities compared to flat foils in the TNSA regime. This increase is comparable to what is to be expected in the RPA regime and proton energies even surpass those predicted by RPA at high laser intensities. For a given laser pulse energy we find that there exists an optimum pulse duration for which the cone geometry is expected to give the greatest proton energy increase compared to flat foils.

Our findings show that the mechanism of hot electron generation discussed in this work can be used to achieve high proton energies at short laser pulse durations and high intensities with constraints on laser contrast and beam pointing stability achievable with existing laser systems. It therefore favors the use of compact laser systems that can provide for laser repetition rates high enough for applications such as tumor therapy, where both maximum proton energy and dose rate play a role. Since the mechanism of proton acceleration is essentially the same as in the TNSA we expect this scheme to provide for a robust source of energetic proton bunches with a large per-shot number of particles.

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