Theory of neutrinoless double-beta decay

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Received 24 November 2011, in final form 26 April 2012
Published 7 September 2012
Online at stacks.iop.org/RoPP/75/106301

Abstract
Neutrinoless double-beta decay, which is a very old and yet elusive process, is reviewed. Its observation will signal that the lepton number is not conserved and that the neutrinos are Majorana particles. More importantly it is our best hope for determining the absolute neutrino-mass scale at the level of a few tens of meV. To achieve the last goal certain hurdles must be overcome involving particle, nuclear and experimental physics.

Nuclear physics is important for extracting useful information from the data. One must accurately evaluate the relevant nuclear matrix elements—a formidable task. To this end, we review the sophisticated nuclear structure approaches which have recently been developed, and which give confidence that the required nuclear matrix elements can be reliably calculated employing different methods: (a) the various versions of the quasiparticle random phase approximations, (b) the interacting boson model, (c) the energy density functional method and (d) the large basis interacting shell model. It is encouraging that, for the light neutrino-mass term at least, these vastly different approaches now give comparable results.

From an experimental point of view it is challenging, since the lifetime is long and one has to fight against formidable backgrounds. One needs large isotopically enriched sources and detectors with high-energy resolution, low thresholds and very low background.

If a signal is found, it will be a tremendous accomplishment. The real task then, of course, will be the extraction of the neutrino mass from the observations. This is not trivial, since current particle models predict the presence of many mechanisms other than the neutrino mass, which may contribute to or even dominate this process. In particular, we will consider the following processes:

(i) The neutrino induced, but neutrino-mass independent contribution.
(ii) Heavy left and/or right-handed neutrino-mass contributions.
(iii) Intermediate scalars (doubly charged, etc).
(iv) Supersymmetric (SUSY) contributions.

We will show that it is possible to disentangle the various mechanisms and unambiguously extract the important neutrino-mass scale, if all the signatures of the reaction are searched for in a sufficient number of nuclear isotopes.

(Some figures may appear in colour only in the online journal)

This article was invited by P-H Heenen.
1. A brief history of double-beta decay

A brief history of double-beta decay (DBD) is presented.

1.1. The early period

DBD, namely the two-neutrino DBD (2νββ-decay)

\[(A, Z) \rightarrow (A, Z+2) + e^- + e^- + \bar{\nu}_e + \nu_e, \quad (1)\]

was first considered in the publication [1] of Maria Goeppert-Mayer in 1935. It was Eugene Wigner who suggested this problem to the author of [1] about one year after the Fermi weak interaction theory appeared. In the work of Maria Goeppert-Mayer [1] an expression for the 2νββ-decay rate was derived and a half-life of 10^{17} years was estimated by assuming a Q-value of about 10 MeV.

Two years later (1937) Ettore Majorana formulated a new theory of neutrinos, whereby the neutrino ν and the antineutrino \(\bar{\nu}\) are indistinguishable, and suggested antineutrino induced β^- decay for experimental verification of this hypothesis [2]. Giulio Racah was the first to propose testing Majorana’s theory with real neutrinos by chain of reactions

\[(A, Z) \rightarrow (A, Z+1) + e^- + \nu, \quad (2)\]

\[\nu + (A', Z') \rightarrow (A', Z' + 1) + e^-, \quad (2)\]

which is allowed in the case of the Majorana neutrino and forbidden in the case of the Dirac neutrino [3]. In 1939, Wolfgang Furry for the first time considered neutrinoless DBD (0νββ-decay),

\[(A, Z) \rightarrow (A, Z+2) + e^- + e^-, \quad (3)\]

a Racah chain of reactions with virtual neutrinos \((A, Z+1) \equiv (A', Z')\) [4]. Here A, A' are the nuclear mass numbers and Z, Z' the charges of the nuclei involved. The available energy \(\Delta\) is equal to the Q-value of the reaction, i.e. the mass difference of the ground states of the two atoms involved.

In 1952 Henry Primakoff [5] calculated the electron–electron angular correlations and electron energy spectra for both the 2νββ-decay and the 0νββ-decay, producing a useful tool for distinguishing between the two processes.

At that time nothing was known about the chirality suppression of the 0νββ-decay. It was believed that due to a considerable phase-space advantage, the 0νββ-decay mode dominated the DBD rate. From 1950 this phenomenon was exploited in early geochemical, radiochemical and counter experiments [6]. It was found that the measured lower limit on the ββ-decay half-life far exceeds the values expected for this process, \(T^\text{lim}_{1/2} \sim 10^{12}–10^{13}\) years. In 1955 the Raymond
Davis experiment [7], which searched for antineutrinos from the reactor via nuclear reaction $^{37}\text{Cl} \rightarrow ^{37}\text{Ar} + e^-$, produced a zero result. The above experiments were interpreted as proof that the neutrino was not a Majorana particle, but a Dirac particle. This prompted the introduction of the lepton number (LN) to distinguish the neutrino from its antiparticle. The assumption of LN conservation allows the $\nu\beta\beta$-decay but forbids the $0\nu\beta\beta$-decay, in which LN is changed by two units.

In 1949 Fireman reported the first observation of the $\beta\beta$-decay of $^{124}\text{Sn}$ in a laboratory experiment [8], but he disclaimed it later [9]. The first geochemical observation of the $\nu\beta\beta$-decay, with an estimated half-life of $T_{1/2}(^{130}\text{Te}) = 1.4 \times 10^{21}$ years, was announced by Ingram and Reynolds in 1950 [10]. Extensive studies have been made by Gentner and Kirsten [11, 12] and others [13, 14] on such rare-gas isotopes as $^{82}\text{Kr}$, $^{128}\text{Xe}$ and $^{130}\text{Xe}$, which are $\beta\beta$-decay products of $^{82}\text{Se}$, $^{128}\text{Te}$ and $^{130}\text{Te}$, respectively, obtaining half-lives of around $10^{21}$ y for $^{130}\text{Te}$.

### 1.2. The period of scepticism

Shortly after its theoretical formulation by Lee and Yang, parity violation in the weak interaction was established by two epochal experiments. In 1957 Wu et al discovered asymmetry in the angular distribution of the $\beta$-particles emitted relative to the spin orientation of the parent nucleus $^{60}\text{Co}$. A year later Goldhaber et al [15] discovered that the neutrinos are polarized and left-handed by measuring the polarization of a photon, moving back-to-back with the neutrino produced by the de-excitation of a $^{152}\text{Eu}$ nucleus after K-capture. In 1958 the seemingly confused situation was simplified in the form of the vector–axial vector (V-A) theory of weak interactions, describing maximal parity violation in agreement with the available data. In order to account for the chiral symmetry breaking of the weak interaction, only left-handed fermions participate and the mediating particles must be vectors of spin 1, which are left-handed in the sense that they couple only to left-handed fermions.

The maximal parity violation is easily realized in the lepton sector using the two-component theory of a massless neutrino, proposed in 1957 by L. Landau, T D Lee, C N Yang and A Salam. This idea was first developed by H Weyl in 1929, but it was rejected by Pauli in 1933 on the grounds that it violates parity. In this theory, neutrinos are left handed and antineutrinos are right handed, leading automatically to the V-A couplings.

With the discovery of parity violation, it became apparent that the Majorana/Dirac character of the electron neutrino was still in question. The particles that participate in the $0\nu\beta\beta$-decay reaction at nucleon level are right-handed antineutrino $\bar{\nu}_e$ and left-handed neutrino $\nu_e$:

$$n \rightarrow p + e^- + \bar{\nu}_e, \quad \nu_e^L + n \rightarrow p + e^-.$$  \hspace{1cm} (4)

Thus even if the neutrino is a (massless) Majorana particle, since the first neutrino has the wrong helicity for absorption by a neutron, the absence of the $0\nu\beta\beta$-decay implies neither a Dirac electron neutrino nor a conserved LN.

The requirement that both LN conservation and the $\gamma_5$ invariance of the weak current had to be violated in order for the $0\nu\beta\beta$-decay to occur, discouraged experimental searches.

### 1.3. The period of grand unified theories

The maximal violation of parity (and of charge-conjugation) symmetry is accommodated in the standard model (SM), which jointly describes weak and electromagnetic interactions. This model was developed largely upon the empirical observations of nuclear beta decay during the latter half of the past century. Despite the phenomenological success of the SM, the fundamental origin of parity violation has not been understood. In spite of the fact that the SM represents the simplest and the most economical theory, it has not been considered as the ultimate theory of nature. It was assumed that, most likely, it describes a low-energy approximation to a more fundamental theory.

With the development of modern gauge theories during the last quarter of the previous century, perceptions began to change. In the SM it became apparent that the assumption of LN conservation led to the neutrino being strictly massless, thus preserving the $\gamma_5$-invariance of the weak current. With the development of grand unified theories (GUTs) of the electroweak and strong interactions, the prejudice has grown that LN conservation was the result of a global symmetry, not a gauge symmetry, and had to be broken at some level. In other words modern GUTs and supersymmetric (SUSY) extensions of the SM suppose that such conservation laws of the SM may be violated to some small degree. The LN may only appear to be conserved at low energies because of the large grand unified mass scale $A_{GUT}$ governing its breaking. Within the proposed see-saw mechanism one expects the neutrino to acquire a small Majorana mass of a size $\sim (\text{light mass})^2/A_{GUT}$, where ‘light mass’ is typically that of a quark or charged lepton. The considerations of the sensitivity of the $0\nu\beta\beta$-decay experiments to neutrino mass $m_\nu \sim 1\text{eV}$ became the genesis of a new interest in DBD. Thus the interest in $0\nu\beta\beta$-decay was resurrected through the pioneering work of Kotani and his group [16], which again brought it to the attention of the nuclear physics community.

Neutrino masses require either the existence of right-handed neutrinos or violation of the LN so that Majorana masses are possible. Hence, one is forced to go beyond the minimal models again, whereby LF and/or LN violation can be allowed in the theory. A good candidate for such a theory is the left-right symmetric model of grand unification inaugurated by Salam, Pati, Mohapatra and Senjanović [17–19] and especially models based on $\text{SO}(10)$, which were first proposed by Fritzsch and Minkowski [20] with their supersymmetric versions [21–23]. The left–right symmetric models, representing generalization of the $SU(2)_L \otimes U(1)$ SM, predict not only that the neutrino is a Majorana particle, meaning that it is up to a phase identical with its antiparticle, but automatically predicts that the neutrino has a mass and a weak right-handed interaction.

In the left–right symmetric models the LN conservation is broken by the presence of the Majorana neutrino mass. The LN violation is also inherent in those SUSY theories whereby $R$-parity, defined as $R_p = (-1)^{3B+L+2S}$, with $S$, $B$, and $L$ being the spin, baryon and LN, respectively, is no longer a conserved quantity.

The $0\nu\beta\beta$-decay, which involves the emission of two electrons and no neutrinos, has been found as a powerful tool to study the LN conservation. Schechter and Valle proved that,
if the $0\nu\beta\beta$-decay takes place, regardless of the mechanism causing it, the neutrinos are Majorana particles with non-zero mass [24, 25]. It was recognized that the GUTs and R-parity violating SUSY models offer a plethora of the $0\nu\beta\beta$-decay mechanisms triggered by exchange of neutrinos, neutralinos, gluinos, leptoquarks, etc [26–28].

The experimental effort concentrated on high $Q_{\beta\beta}$ isotopes, in particular on $^{48}$Ca, $^{76}$Ge, $^{82}$Se, $^{96}$Zr, $^{100}$Mo, $^{116}$Cd, $^{130}$Te, $^{136}$Xe and $^{150}$Nd [29–31]. In 1987 the first actual laboratory observation of the two-neutrino DBD ($2\nu\beta\beta$-decay) was carried out for $^{82}$Se by Moe and collaborators [32], who used a time projection chamber. Within the next few years, experiments employing counters were able to detect the $2\nu\beta\beta$-decay of many nuclei. In addition, experiments searching for the signal of the $0\nu\beta\beta$-decay pushed by many orders of magnitude the experimental lower limits for the $0\nu\beta\beta$-decay half-life of different nuclei.

1.4. The period of massive neutrinos—the current period
Various early measurements of neutrinos produced in the sun, in the atmosphere, and by accelerators suggested that neutrinos might oscillate from one ‘flavor’ (electron, muon and tau) to another, expected as a consequence of non-zero neutrino mass. Non-zero neutrino mass can be accommodated by fairly straightforward extensions of the SM of particle physics. Since 1998 we have had convincing evidence for the existence of neutrino masses due to SuperKamiokande [33], SNO, [34] KamLAND [35] and other experiments.

Thus neutrino oscillations have supplied additional information for constructing GUTs of physics. It has also provided additional input for cosmologists and opened new perspectives for observation of the $0\nu\beta\beta$-decay.

So far the $2\nu\beta\beta$-decay has been recorded for eleven nuclei ($^{48}$Ca, $^{76}$Ge, $^{82}$Se, $^{96}$Zr, $^{100}$Mo, $^{116}$Cd, $^{128}$Te, $^{130}$Te, $^{150}$Nd, $^{136}$Xe, $^{238}$U) [29–31]. In addition, the $2\nu\beta\beta$-decay of $^{105}$Mo and $^{156}$Nd to $0^+$ excited state of the daughter nucleus has been observed and the two-neutrino double electron capture process in $^{138}$Ba has been recorded. Experiments studying $2\nu\beta\beta$-decay are currently approaching a quantitatively new level, where high-precision measurements are performed not only for half-lives but for all other observables of the process. As a result, a trend is emerging toward thorough investigation of all aspects of $2\nu\beta\beta$-decay, and this will furnish very important information about the values of nuclear matrix elements (NMEs), the parameters of various theoretical models, and so on. In this connection, one may expect advances in the calculation of NMEs and in the understanding of the nuclear-physics aspects of DBD (figure 1).

Neutrinoless DBD has not yet been confirmed. The strongest limits on the half-life of the $0\nu\beta\beta$-decay were set in Heidelberg–Moscow [36], NEMO3 [37, 38], CUORICINO [39] and KamLAND-Zen [40] experiments:

$$T^{\nu\beta\beta}_{1/2}(^{76}\text{Ge}) \geq 1.9 \times 10^{25} \text{ y}, \quad T^{\nu\beta\beta}_{1/2}(^{100}\text{Mo}) \geq 1.0 \times 10^{24} \text{ y}, \quad T^{\nu\beta\beta}_{1/2}(^{130}\text{Te}) \geq 3.0 \times 10^{24} \text{ y}, \quad T^{\nu\beta\beta}_{1/2}(^{136}\text{Xe}) \geq 5.7 \times 10^{24} \text{ y}. \quad (5)$$

There is also a claim for an observation of the $0\nu\beta\beta$-decay of $^{76}$Ge with a half-life of $T^{0\nu\beta\beta}_{1/2} = 2.23^{+0.44}_{-0.31} \times 10^{25}$ years [41, 42]. One of the goals of the upcoming GERDA experiment [43] is to put this claim to the test by improving the sensitivity limit of the detection by more than an order of magnitude. The next generation of experiments, which will be using several other candidate nuclei, will eventually be able to achieve this goal as well [31].

1.5. The period of Majorana neutrinos?
There is a hope that the period of Majorana neutrinos is not far off. This period should begin by a direct and undoubtable observation of the $0\nu\beta\beta$-decay. It would establish that neutrinos are Majorana particles, and a measurement of the decay rate, when combined with neutrino oscillation data and a reliable calculation of NMEs, would yield insight into all three neutrino-mass eigenstates.

2. An overview
The question of neutrino masses and mixing is one of the most important issues of modern particle physics. It has already been discussed in a number of excellent reviews [44–48] and its relevance to the $0\nu\beta\beta$-decay will also be briefly discussed in this report.

Today, seventy five years later, $0\nu\beta\beta$-decay (3) continues to be one of the most interesting processes. The experimental status and prospects regarding this process will be reviewed in section 8. The corresponding non-exotic $2\nu\beta\beta$ (1) has been observed in many systems; see section 8.

If the neutrinos are Majorana particles other related processes in which the charge of the nucleus is decreased by two units may also occur, if they happen to be allowed by energy and angular-momentum conservation laws, e.g.

$$(A, Z) \rightarrow (A, Z-2) + e^+ + e^- (0\nu \text{ positron emission}).$$

(6)

Here the available energy is $\Delta = Q - 4m_e c^2$, i.e. everything else being equal, it is somewhat kinematically disfavored
compared with the usual two-electron emission for which the available energy $\Delta$ is equal to the $Q$-value.

Electron–positron conversion:

$$(A, Z) + e_1^- + e_2^- \rightarrow (A, Z - 2) + e^+ \quad (0\nu \text{ electron–positron conversion}).$$

$$\Delta = Q - 2m_e c^2 - \varepsilon_b,$$

where $\varepsilon_b$ is a binding energy of the absorbed atomic electron.

The resonant neutrinoless double electron capture ($0\nu$ECEC),

$$(A, Z) + e_1^- + e_2^- \rightarrow (A, Z - 2)^{++}, \quad (\text{resonant } 0\nu \text{ double electron capture}),$$

was first considered a very long time ago by Winter [50]. It is always allowed whenever (7) is. This reaction was later expounded in more detail [51,52] as a two-step process: in the first step the two neutral atoms $(A, Z)$ and the excited $(A, Z - 2)$, are admixed via the lepton number violating (LNV) interaction. In the second step the $(A, Z - 2)$ atom and possibly the nucleus de-excite. The available energy is $\Delta = Q - B_{0\nu}$, $B_{0\nu}$ being the energy of two electron holes in the atomic shells of the daughter nucleus.

Non-resonant decays to nuclear ground states are in some cases possible. Then, in addition to x-rays one has various decay modes with the emission of a single $\gamma$, a pair of $\gamma$s, internal electron–positron pair formation and the emission of an electron by internal conversion [53], i.e.

$$(A, Z) + e_1^- + e_2^- \rightarrow (A, Z - 2) + X,$$

$$X = \gamma, \quad 2\gamma, \quad e^+e^-, \quad e^-e^-.$$  \hspace{1cm} (9)

The lifetime expected was very long, since the above mixing amplitude was tiny compared with the energy difference of the two atoms involved. It has recently, however, been gaining in importance [54, 55] after ion Penning traps [56] made it possible to accurately determine the $Q$-values, which gave rise to the presence of resonances. This, in turn, could lead to an increase in the width by many orders of magnitude; see section 12 for details.

Another LNV process, not hindered by energy conservation in any nuclear system, involves the neutrinoless bound muon capture [57,58],

$$(A, Z) + \mu_b^- \rightarrow (A, Z - 2) + e^+ \quad (0\nu \text{ muon–positron conversion}).$$

The best experimental limit on the muon–positron conversion branching ratio has been established at PSI [59] for the $^{48}$Ti nuclear target. The muonic analog of neutrinoless DBD [60,61],

$$(A, Z) + \mu_b^- \rightarrow (A, Z - 2) + \mu^+ \quad (0\nu \text{ muon–muon conversion}),$$

has never been searched for experimentally. For a bound muon in an atom energy conservation of the process is very restrictive. It is satisfied only for three isotopes, namely $^{44}$Ti, $^{72}$Se and $^{82}$Sr.

The above processes are expected to occur whenever one has LNV interactions. LN, being a global quantity, is not sacred, but it is expected to be broken at some level. In short, these processes pop up almost everywhere, in every theory. On the other hand, since the neutrinos have to be Majorana particles if LN violating interactions exist, all the above processes can, in principle, decide whether or not the neutrino is a Majorana particle, i.e. it coincides with its own antiparticle. This is true even if these processes are induced not by intermediate neutrinos but by other mechanisms as we will see below.

Neutrinoless DBD (equation (3)) seems to be the most likely to yield the information [44, 62–68] we are after. For this reason we will focus our discussion on this reaction, but will pay some attention to resonant neutrinoless double electron capture, which has recently been revived [51,52, 54, 55, 69–71], since its observation seems to be a realistic possibility [54, 55]. We will only peripherally discuss the other less interesting processes [51].

From a nuclear physics point of view [65,66, 72–76], calculating the relevant NMEs is indeed a challenge. First, almost all nuclei, which can undergo DBD, are far from closed shells and some of them are even deformed. Thus faces a formidable task. Second, the NMEs are small compared with a canonical value, like the one associated with the matrix element to the (energy non-allowed) double Gamow–Teller resonance or a small fraction of some appropriate sum rule. Thus, effects which are normally negligible become important here. Third, in many models the dominant mechanism for $0\nu\beta\beta$-decay does not involve intermediate light neutrinos, but very heavy particles. Thus one must be able to cope with the short-distance behavior of the relevant operators and wave functions (see section 10 for details).

From the experimental point of view it is also very challenging to measure perhaps the slowest process accessible to observation, particularly if one realizes that even if one obtains only lower bounds on the lifetime for this $0\nu\beta\beta$-decay, the extracted limits on the theoretical model parameters may be comparable, if not better, and complementary to those extracted from the most ambitious accelerator experiments.

The recent discovery of neutrino oscillations [77–80] has given the first evidence of physics beyond the SM and in particular indicates that the neutrinos are massive particles. The oscillations were able to show that the neutrinos are massive particles if LN violating interactions exist, all the above processes can, in principle, decide whether or not the neutrino is a Majorana particle, i.e. it coincides with its own antiparticle. This is true even if these processes are induced not by intermediate neutrinos but by other mechanisms as we will see below.

• Whether the neutrinos are Majorana or Dirac particles. It is obviously important to proceed further and decide on this important issue. Neutrinoless DBD can achieve this, even if, as we have mentioned, there might be processes that dominate over the conventional intermediate neutrino mechanism of $0\nu\beta\beta$-decay. It has been known that whatever the LN violating process, which gives rise to $0\nu\beta\beta$-decay, it can be used to generate a Majorana mass for the neutrino [24]. This mechanism, however, may not be the dominant mechanism for generating the neutrino mass [82].
The scale of the neutrino masses. These experiments can measure only mass squared differences. This task can be accomplished by astrophysical observations or via other experiments involving low-energy weak decays, like triton decay or electron capture, or the $0νββ$-decay. It seems that for a neutrino mass in the meV ($10^{-3}$ eV) region, the best process for achieving this is the $0νββ$-decay. The extraction of neutrino masses from such observations will be discussed in detail and compared with each other later (see section 4).

The neutrino hierarchy. They cannot at present decide which scenario is realized in nature, namely the degenerate, the normal hierarchy (NH) or the inverted hierarchy (IH). They may be able to distinguish between the two hierarchies in the future by observing the wrong sign muons in neutrino factories [83, 84].

For details on such issues see a recent review [85].

The study of the $0νββ$-decay is further stimulated by the development of GUTs and supersymmetric models (SUSY) representing extensions of the $SU(2)_L \otimes U(1)_Y$ SM. The GUTs and SUSY offer a variety of mechanisms which allow the $0νββ$-decay to occur [86].

The best known mechanism leading to $0νββ$-decay is via the exchange of a Majorana neutrino between the two decaying neutrinos [44, 62–66, 87]. Nuclear physics allows us to study the light ($m_ν \ll m_ν$) and heavy ($m_ν \gg m_ν$) neutrino components separately. In the presence of only left-handed currents and for the light intermediate neutrino components, the obtained amplitude is proportional to a suitable average neutrino mass, which vanishes in the limit in which the neutrinos become Dirac particles. On the other hand, in the case of heavy Majorana neutrino components the amplitude is proportional to the average of the inverse of the neutrino mass, i.e. it is again suppressed. In the presence of right-handed currents one can have a contribution similar to the one above for heavy neutrinos but involving a different (larger) inverse mass with some additional suppression due to the fact that the right-handed gauge boson, if it exists, is heavier than the usual left-handed one.

In the presence of right-handed currents it is also possible to have interference between the leptonic left and right currents, $J_L - J_R$ interference. In this case the amplitude in momentum space becomes proportional to the four-momentum of the neutrino and, as a result, only the light neutrino components become important. One now has two possibilities. First, the two hadronic currents have a chirality structure of the same kind, i.e. $J_L - J_R$. One can then extract from the data a dimensionless parameter $\lambda$, which is proportional to the square of the ratio of the masses of the L and R gauge bosons, $\kappa = (m_L / m_R)^2$. Second, the two hadronic currents are left-handed, which can happen via the mixing $\epsilon$ of the two bosons. The relevant LNV parameter $\eta$ is now proportional to this mixing $\epsilon$. Both of these parameters, however, also involve the neutrino mixing. They are, in a way, proportional to the mixing between the light and heavy neutrinos.

In gauge theories one has, of course, many more possibilities. Exotic intermediate scalars may mediate $0νββ$-decay [44]. These are not favored in current gauge theories and will not be further discussed. In superstring inspired models one may have singlet fermions in addition to the usual right-handed neutrinos. Not much progress has been made on the phenomenological side of these models and they are not going to be discussed further.

In recent years supersymmetric models have been taken seriously and semirealistic calculations are taking place. In standard calculations one invokes universality at the GUT scale, employing a set of five independent parameters, and uses the renormalization group equation to obtain all parameters (couplings and particle masses) at low energies. Hence, since such parameters are in principle calculable in terms of the five input parameters, one can use experimental data to constrain the input parameters. One can then use the $0νββ$-decay experiments to constrain the $R$-parity violating couplings, which cannot be specified by the theory [26, 88–95]. Recent review papers [31, 65, 66, 68] also give a detailed account of some of the latest developments in this field.

From the above discussion it is clear that one has to consider the case of heavy intermediate particles. One thus has to tackle problems related to the very short-ranged operators in the presence of the nuclear repulsive core. If the interacting nucleons are point-like one gets negligible contributions. We know, however, that the nucleons are not point-like, but that they have a structure described by a quark bag with a size that can be determined experimentally. It can also be accounted for by a form factor, which can be calculated in the quark model or parametrized in a dipole shape with a parameter determined by experiment. This approach, first considered by Vergados [96], has now been adopted by almost everybody. The resulting effective operator has a range somewhat less than the inverse of the proton mass (see section 4).

Another approach in handling this problem consists of considering particles other than the nucleons present in the nuclear soup. For $0^+ \rightarrow 0^+$ the most important of such particles are the pions. One may thus consider the DBD of pions in flight between nucleons, such as

$$\pi^- \longrightarrow \pi^+ + e^- + e^-, \quad n \longrightarrow p + \pi^+ + e^- + e^-.$$ (12)

Recognition of this contribution first appeared as a remark by the genius of Pontecorvo [97] in the famous paper in which he suggested that the ratio of the lifetimes of the $^{126}$Te and $^{130}$Te isotopes, which merely differ by two neutrons, is essentially independent of nuclear physics. He did not perform any estimates of such a contribution. Such estimates and calculations were first performed by Vergados [98] in the case of heavy intermediate neutrinos, i.e. vector and axial vector currents. This approach was found to yield results of the same order as the nucleon mode with the above recipe for treating the short-range behavior. It was revived by the Tuebingen group [26–28, 93] in the context of $R$-parity violating interactions, i.e. scalar, pseudoscalar and tensor currents arising out of neutralino and gluino exchange, and it was found to dominate.

In yet another approach one may estimate the presence of six quark clusters in the nucleus. Then, since the change of charge takes place in the same hadron there is no suppression due to the short nature of the operator, even if it is a $\delta$-function.
One only needs a reliable method for estimating the probability of finding these clusters in a nucleus [99].

All the above approaches seem reasonable and lead to quite similar results. The matrix elements obtained are not severely suppressed. This gives us a great degree of confidence that the resulting matrix elements are sufficiently reliable, allowing DBD to probe very important physics.

The other recent development is the better description of nucleon current by including momentum-dependent terms, such as the modification of the axial current due to PCAC and the inclusion of weak-magnetism terms. These contributions have been considered previously [87, 100], but only in connection with the extraction of the $\eta$ parameter mentioned above. Indeed these terms were very important in this case since they compete with the p-wave lepton wave function, which, with the usual currents, provides the lowest non-vanishing contribution. Since in the mass term only s-wave lepton wave functions are relevant such terms have hitherto been neglected.

It was recently found [101], however, that for light neutrinos the inclusion of these momentum-dependent terms reduces the NME by about 25%, independently of the nuclear model employed. On the other hand, for heavy neutrinos the effect can be larger and it depends on the nuclear wave functions. The reason for expecting them to be relevant is that the average momentum $\langle q \rangle$ of the exchanged neutrino is expected to be large [102]. In the case of a light intermediate neutrino the mean nucleon–nucleon separation is about 2 fm which implies that the average momentum $\langle q \rangle$ is about 100 MeV/c. In the case of a heavy neutrino exchange the mean inter-nucleon distance is considerably smaller and the average momentum $\langle q \rangle$ is supposed to be considerably larger.

Since $0\nu\beta\beta$ decay is a two-step process, in principle, one needs to construct and sum over all the intermediate nuclear states, a formidable job indeed in the case of the SM calculations. Since, however, the average neutrino momentum is much larger compared with the nuclear excitations, one can invoke closure using some average excitation energy (this does not apply in the case of $2\nu\beta\beta$ decays). Thus one needs to construct only the initial and final $^0$ nuclear states. This is not useful in quasiparticle random phase approximation (QRPA), since one must construct the intermediate states anyway. In any case, it was explicitly shown, taking advantage of the momentum space formalism developed by Vergados [103], that this approximation is very good [104, 105]. The same conclusion was reached independently by others [106] via a more complicated technique relying on coordinate space.

 Granted that one takes into account all the above ingredients in order to obtain quantitative answers for the LNV parameters from the results of $0\nu\beta\beta$-decay experiments, it is necessary to evaluate the relevant NMEs with high reliability. The most extensively used methods are the large basis interacting shell-model (ISM) calculations, (for a recent review see [65]) and QRPA (for a recent review see [65, 66]). The ISM is forced to use few single-particle orbitals, while this restriction does not apply in the case of QRPA. The latter suffers, of course, from the approximations inherent in the RPA method. Hence, a direct comparison between them is not possible.

The shell-model calculations have a long history [75, 76, 107–111] in DBD calculations. In recent years it has lead to large matrix calculations in traditional as well as Monte Carlo types of formulations [72–74, 112–114]. For a more complete set of references as well as a discussion of the appropriate effective interactions see [65]).

There have been a number of QRPA calculations covering almost all nuclear targets [115–126]. These involve a number of collaborations, but the most extensive and complete calculations in one way or another include the Tuebingen group. We have also seen some refinements of QRPA, such as proton–neutron pairing and the inclusion of renormalization effects due to Pauli principle corrections [127, 128]. Other less conventional approaches, such as operator expansion techniques, have also been employed [66].

Recently, calculations based on the projected Hartree–Fock–Bogoliubov (PHFB) method [129], the interacting Boson model (IBM) [130] and the energy density functional (EDF) method [131] entered the field of such calculations. The above schemes, in conjunction with the other improvements mentioned above, offer some optimism in our efforts for obtaining NMEs accurate enough to allow us to extract reliable values of the lepton-violating parameters from the data.

As we have mentioned, neutrinoless DBDs are concerned with the fundamental properties of neutrinos. These properties arise out of interactions involving high-energy scales, which are of great interest from the viewpoint of particle physics and cosmology. On the other hand, DBD processes are nuclear rare decays in the low-energy scale, which are studied experimentally by low-energy and low-background nuclear spectroscopy, as given in the review papers [30, 31].

DBDs are low-energy second-order weak processes with $Q_{\beta\beta} \approx 2–3$ MeV. Decay rates of $2\nu\beta\beta$-decay within the SM are of the order of $10^{-20}$ yr$^{-1}$, and the rates of $0\nu\beta\beta$-decay beyond the SM are even many orders of magnitude smaller than $2\nu\beta\beta$-decay rates, depending on the $Q_{\beta\beta}$ value and the effective Majorana neutrino mass $\langle m_\nu \rangle$ (for definition see equation (33)) in the case of the light neutrino-mass process. Then the $0\nu\beta\beta$-decay half-lives are of the orders of $T_{1/2}^{\nu\beta\beta} \approx 10^{27}$ yr and $10^{29}$ yr in cases of the IH mass of $\langle m_\nu \rangle \approx 30$ meV and the NH mass of $\langle m_\nu \rangle \approx 3$ meV, respectively.

For experimental studies of such rare decays, large detectors with ton-scale DBD isotopes are needed to obtain $0\nu\beta\beta$-decay signals in the case of the IH $\nu$ mass. Here the signals are very rare and are as low as $E_{\beta\beta} \approx 2–3$ MeV. Background (BG) signal rates, however, are huge in the energy region of $E_{\beta\beta} \lesssim 3$ MeV. Thus it is crucial to build ultra-low BG detectors to find the rare and small $0\nu\beta\beta$-decay signals among huge BGs in the low-energy region. We are going to review this (see section 8) in the case of most of the nuclear targets of experimental interest [30, 31, 132] ($^{76}$Ge, $^{82}$Se, $^{96}$Zr, $^{100}$Mo, $^{116}$Cd, $^{128}$Te, $^{130}$Te, $^{136}$Xe, $^{150}$Nd).

3. The neutrino-mass matrix in various models

Within the SM of elementary particles, with the particle content of the gauge bosons $A_\mu \cdot Z_\mu$ and $W^{\pm,0}_\mu$ the Higgs scalar isodoublet $\Phi = (\phi^0, \phi^-)$ (and its conjugate $\Phi^\ast$), and the fermion fields arranged in
• isodoublets: \((u_{\alpha L}, d_{\alpha L})\) and \((v_{\alpha L}, e_{\alpha L})\) for quarks and leptons respectively, and
• isosinglets: \(u_{\alpha R}, d_{\alpha R}\) and \(e_{\alpha R}\),

where \(\alpha\) is a family index taking three values, the neutrinos are massless. They cannot obtain mass after the symmetry breaking, like the quarks and the charged leptons, since the right-handed neutrino is absent.

### 3.1. Neutrino masses at tree level

The minimal extension of the SM that would yield mass for the neutrino is to introduce an isosinglet right-handed neutrino. Then one can have a Dirac mass term arising via coupling of the leptons and Higgs as follows:

\[
h_{a,\beta}(\bar{\nu}_{aL}, \bar{\nu}_{aL})(\phi^0) v_{\beta R} \rightarrow h_{a,\beta}(\bar{\nu}_{aL}, \bar{\nu}_{aL})\left(\frac{v}{\sqrt{2}}\right) v_{\beta R} \quad \text{or} \quad m_{a,\beta} = h_{a,\beta} \frac{v}{\sqrt{2}}. \quad (13)
\]

Thus one can have

\[
\mathcal{M} = (\bar{v}_L, \bar{v}_L') \left(\begin{array}{cc} 0 & m_D^0 \\ (m_D^0)^T & 0 \end{array}\right) \left(\begin{array}{c} v^R \\ v^R \end{array}\right). \quad (14)
\]

In the above expression, as well as in analogous expressions below, we have explicitly indicated not only the \(6 \times 6\) matrix, but also the states on which this matrix acts and \(m^0\) is the above \(3 \times 3\) matrix. One then obtains six Majorana eigenvectors which pair-wise can be associated with opposite eigenvalues. Their sum and their difference may equally well be selected as physical states which correspond to three Dirac neutrinos and their charged conjugate (antineutrinos). This is fine within the above minimal extension. In GUTs, however, one is faced with the problem that these neutrinos are going to be very heavy with a mass similar to that of up quarks, which is clearly unacceptable. Such a model is therefore inadequate\(^8\). Moreover, such neutrinos viewed as Majorana with opposite CP eigenvalues or as Dirac particles cannot contribute to \(0\nu\beta\beta\) decay.

The next extension is to introduce a Majorana-type mass involving the isosinglet neutrinos and an additional isosinglet Higgs field, which can acquire a large vacuum expectation value, an idea essentially put forward by Weinberg [134] a long time ago. Thus the neutrino-mass matrix becomes

\[
\mathcal{M} = (\bar{v}_L, \bar{v}_L') \left(\begin{array}{cc} 0 & m_D^0 \\ (m_D^0)^T & 0 \end{array}\right) \left(\begin{array}{c} v^R \\ v^R \end{array}\right). \quad (15)
\]

Thus, provided that the Majorana mass matrix has only very large eigenvalues, one obtains an effective Majorana \(3 \times 3\) matrix:

\[
\mathcal{M}_\nu = -\bar{v}_L (m_D^0)^T M_R^{-1} m_D^0 v^R, \quad (16)
\]

which can provide small neutrino masses provided that the eigenvalues of the matrix \(M_R\) are sufficiently large. \(M_R\) can be arbitrarily large, since the new scale, associated with the vacuum expectation of the isosinglet, does not affect the low-energy scale arising from the vacuum expectation value of the standard Higgs particles. This is the celebrated see-saw mechanism, or more precisely, the type I see-saw mechanism, since, as we will see below, there exist other see-saw types (for a summary see, e.g. Abada et al [135]).

Thus with the above mechanism the neutrino flavors become admixed, the resulting eigenstates are Majorana particles and LNV interactions, like \(0\nu\beta\beta\) decay, become possible.

Other extensions of the SM are possible, which do not require the addition of the right-handed neutrinos but additional exotic scalars or fermions [47, 48], e.g.

- An isosinglet \(\Delta\) of Higgs scalars whose charge decomposition is \(\delta^-\), \(\delta^0\), \(\delta^+\).

Then this leads to the coupling:

\[
(h_T)_{a,\beta}(\bar{\nu}_{aL}, \bar{\nu}_{aL})(\delta^-)(-\delta^0) \left(\begin{array}{c} e^\nu_{\beta R} \\ -e^\nu_{\beta R} \end{array}\right).
\]

This, after the isosinglet acquires a vacuum expectation value becomes

\[
(h_T)_{a,\beta}(\bar{\nu}_{aL}, \bar{\nu}_{aL})(\delta^-)(-\delta^0) \left(\begin{array}{c} 0 \\ -v_\Delta/\sqrt{2} \end{array}\right) \left(\begin{array}{c} e^\nu_{\beta R} \\ -e^\nu_{\beta R} \end{array}\right)
\]

yielding the neutrino Majorana mass matrix

\[
m^M_{a,\beta} = (h_T)_{a,\beta} \frac{v_\Delta}{\sqrt{2}} \bar{\nu}_{aL} e^\nu_{\beta R} \quad (17)
\]

If, for some reason, the introduction of such an isosinglet, acquiring a small vacuum expectation value, is not preferred, the Majorana mass matrix can be obtained assuming that the isosinglet \(\Delta\) possesses a cubic coupling \(\mu_\Delta\) with two standard Higgs doublets [44, 136–138] (see figure 2). Then one finds an effective Majorana neutrino mass from equation (17) via the substitution

\[
\frac{v_\Delta}{\sqrt{2}} \rightarrow \frac{v^2 \mu_\Delta}{2 m_\Delta^2},
\]

where \(v/\sqrt{2}\) is the vacuum expectation value of the standard Higgs doublet (see equation (13)) and \(m_\Delta\) is the mass of \(\delta^0\). This is sometimes referred to as see-saw mechanism II [47, 48].

As we will see later this mechanism may lead to a new contribution to neutrinoless DBD via the direct decay of \(\delta^-\) into two electrons.

---

\(^8\) There may exist light Dirac neutrinos in theories formulated in extra dimensions, see, e.g., the recent review by Smirnov [133]. If these neutrinos do not couple to the usual leptons they are of little interest to us. If they do and it so happens that the standard neutrinos are Majorana, they also become Majorana, except in the case of very fine tuning.

---

Figure 2. The tree level contribution to the neutrino mass mediated by an isosinglet scalar.
• An isotriplet of fermions with hypercharge zero $(\Sigma^+, \Sigma^0, \Sigma^-)$. In this case, the leptons couple to the isotriplet via the Higgs doublet (see [139, 140] and references therein):

$$\begin{align*}
(h_\Sigma)_a(\bar{\nu}_a, \bar{e}_a) &= \left(\frac{\Sigma^0_R}{\sqrt{2}}, \frac{\Sigma^+_R}{\sqrt{2}}, -\frac{\Sigma^-_R}{\sqrt{2}}\right) \left(\phi^0, \phi^+, \phi^\dagger\phi\right) \\
&\rightarrow (h_\Sigma)_a(\bar{\nu}_a, \bar{e}_a) = \left(\frac{\Sigma^0_R}{\sqrt{2}}, \frac{\Sigma^+_R}{\sqrt{2}}, -\frac{\Sigma^-_R}{\sqrt{2}}\right) \left(v_\Sigma^0\right) \\
&= (h_\Sigma)_a \left(\frac{1}{\sqrt{2}}\bar{\nu}_a, \Sigma^0_R + \frac{1}{\sqrt{2}}\bar{e}_a \Sigma^+_R\right).
\end{align*}$$

Then a subsequent coupling of the isotriplet to the leptons yields an effective Majorana coupling of the form

$$m^M_{\alpha\beta} = -(h_\Sigma)_a(h_\Sigma)_b \frac{v^2}{2} \frac{1}{m^a}$$  \hspace{1cm} (18)

where $m^a$ is the mass of the neutral component of the isotriplet (see figure 3 and [139]). It is sometimes referred to as see-saw mechanism III. This mechanism, however, by itself cannot constitute a viable neutrino-mass generator since it leads to two eigenstates with zero mass. This can be circumvented [139, 140], but then the model becomes more complicated.

3.2. Neutrino masses at the loop level

There are many ways to obtain neutrino masses at the one-loop level [141], which have been nicely summarized by Smirnov [133]. We will only discuss one such case, which arises in the presence of $R$-parity violating supersymmetry, which leads to a viable neutrino-mass spectrum [142], in the sense that it can yield three massive neutrinos, if one includes not only the tree level contribution arising from the bilinear terms [143], but both types of one-loop contributions [144, 145] shown in figure 4. This is interesting, since in such models, as we will see below, one can have particles other than neutrinos contributing to $0\nu\beta\beta$-decay. The above Majorana matrices are symmetric in general complex matrices.

3.3. SUSY, GUTs and family symmetries

In many models, such as the standard see-saw, the smallness of neutrino mass requires the existence of a heavy mass scale. The coexistence of two mass scales can naturally be accommodated in supersymmetry. In minimal supersymmetric extensions of the SM [146] one can construct the see-saw matrix of equation (15), see, e.g., the review [46]. Furthermore, this can be extended to larger symmetries, e.g. two commuting symmetries, a grand unified symmetry $G_{\text{GUT}}$ and a family symmetry $G_f$. The family symmetry could be continuous, such as $SU(3)$, $SU(2)$ or $U(1)$, or discrete $Z_k$, $S_3$ or $S_5 \times S_5$, etc. We are not going to further elaborate on such situations, which have been summarized in recent reviews [46, 147, 148] to which we direct the interested reader. We should also mention that $U(1)$ flavor symmetries arise naturally in superstring inspired models [149–151], in particular for the heterotic string and the 4D fermionic constructions. D-brane models have also paved the way for completely new structures [152–155] and in particular a very interesting formulation D-brane inspired mass textures [156].

All the models considered in this section lead to a light effective neutrino mass acting jointly or separately. Almost all of them involve parameters which can be adjusted to fit phenomenology. It is not clear which one, if any, is ultimately going to be the theoretically preferred one.

3.4. Neutrino mixing

We have seen above that in general the neutrino-mass matrix is a complex symmetric matrix. It can, however, be diagonalized by separate left and right unitary transformations, which can take the form [44]:

$$\begin{align*}
S_L &\leftrightarrow \begin{pmatrix} v^0_L, v^e_L \\ v^\alpha_L \end{pmatrix} = \begin{pmatrix} S^{(11)} & S^{(12)} \\ S^{(21)} & S^{(22)} \end{pmatrix} \begin{pmatrix} v^i_L \\ N^i_L \end{pmatrix}, \\
S_R &\leftrightarrow \begin{pmatrix} v^0_R, v^e_R \\ v^\alpha_R \end{pmatrix} = \begin{pmatrix} S^{(11)*} & S^{(12)*} \\ S^{(21)*} & S^{(22)*} \end{pmatrix} \begin{pmatrix} v^i_R \\ N_R^i \end{pmatrix}
\end{align*}$$  \hspace{1cm} (19)

where we have added the superscript 0 to stress that they are the states entering the weak interactions. $S^{(ij)}$ are 3 $\times$ 3 matrices with $S^{(11)}$ and $S^{(22)}$ approximately unitary, while $S^{(12)}$ and $S^{(21)}$ are very small. $(v^i_L, N^i_L)$ and $(v^i_R, N^i_R)$ are the eigenvectors from the left and right, respectively. Thus the neutrino mass in the new basis takes the form

$$M_v = \sum_{j=1}^3 \left( m_j \bar{v}^i_{jL} v_{jR} + m_j \bar{N}^i_{jL} N^i_{jR} \right) + H.C.$$.  \hspace{1cm} (20)

This matrix can be brought into standard form by writing $m_j = |m_i| e^{-i\omega_j}$, $M_j = |M_i| e^{-i\Phi_j}$ and defining:

$$\begin{align*}
v_j &= v^i_{jL} + e^{-i\omega_j} v^\alpha_{jL} \\
N_j &= v^i_{jL} + e^{-i\Phi_j} N^i_{jR}
\end{align*}$$

Then

$$M_v = \sum_{j=1}^3 \left( m_j |\bar{v}_j v_j| + |M_j| |\bar{N}_j N_j| \right)$$  \hspace{1cm} (21)

Note, however, that

$$\begin{align*}
v^c &= v^i_{jR} + e^{i\omega_j} v^\alpha_{jL} = e^{i\omega_j} \left( v^i_{jL} + e^{-i\omega_j} v^\alpha_{jL} \right) = e^{i\omega_j} v^i_{jL} \\
N^c &= N^i_{jR} + e^{i\Phi_j} N^i_{jR} = e^{i\Phi_j} \left( v^i_{jR} + e^{-i\Phi_j} N^i_{jR} \right) = e^{i\Phi_j} N^i_{jR}.
\end{align*}$$  \hspace{1cm} (22)
i.e. they are Majorana neutrinos with the given Majorana phases. Furthermore,  
\[ v_L = v'_L, \quad v_R = e^{-i\Phi}v'_R, \quad N_L = N'_L, \quad N_R = e^{-i\Phi}v'_R. \]

The second of equations (19) can now be written as  
\[ S_R \leftrightarrow (v_R^0, v_R^0) = \left( \frac{(S^{(1)})^*}{(S^{(2)})^*} \right) \left( e^{i\Phi}v_R^0 \right) \left( e^{i\Phi}N_R \right) \]  
(23)

where \( e^{i\Phi} \) and \( e^{iH} \) are diagonal matrices containing the above Majorana phases. The neutrinos interact with the charged leptons via the charged current (see below). So the effective coupling of the neutrinos to the charged leptons involves the mixing of the electrons \( S^e \). Thus the standard mixing matrix appearing in the absence of right-handed neutrinos is  
\[ U_{\text{PMNS}} = U = U^{(11)} = (S^e)^* S^{(1)} \]  
(24)

The other entries are defined analogously:  
\[ U^{(ij)} = (S^e)^* S^{(ij)}, \quad (ij) = (12), (21), (22). \]  
(25)

In particular the usual electronic neutrino is written as:  
\[ v_{\nu e_L} = \sum_j \left( U^{(1j)}_{e_j} v_j + U^{(12)}_{e_j} N_j \right) \]  
(26)

\[ v_{\nu e_R} = \sum_j \left( U^{(2j)}_{e_j} v_j + U^{(22)}_{e_j} N_j \right). \]  
(27)

In other words the left-handed neutrino may have a small heavy component, while the right-handed neutrino may have a small light component. Note also that the neutrinos appearing in weak interactions can be Majorana particles in the special case that all Majorana phases are the same.

The Pontecorvo–Maki–Nakagawa–Sakata neutrino mixing matrix \( U_{\text{PMNS}} \) is parametrized by  
\[ U_{\text{PMNS}} = R_{23} \tilde{R}_{13} R_{12}, \]  
(28)

where the matrices \( R_{ij} \) are rotations in \( ij \) space, i.e.  
\[ R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{12} & s_{12} \\ 0 & -s_{12} & c_{12} \end{pmatrix}, \quad R_{13} = \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix}, \]

\[ R_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \]  
(29)

where \( c_{ij} = \cos (\theta_{ij}), \quad s_{ij} = \sin (\theta_{ij}). \)  
(30)

\[ \theta_{12}, \theta_{13} \text{ and } \theta_{23} \text{ and three mixing angles and } \delta \text{ is the CP-violating phase. Then, we obtain} \]

\[ U_{\text{PMNS}} = \begin{pmatrix} c_{12} c_{13} & -c_{12} s_{13} & c_{13} s_{23} \\ s_{12} c_{13} & c_{12} c_{23} - s_{12} s_{23} & -c_{13} s_{23} \\ -s_{12} s_{13} & c_{12} s_{23} + s_{12} c_{23} & c_{13} c_{23} \end{pmatrix}. \]  
(31)

If neutrinos are Majorana particles, \( U_{\text{PMNS}} \) in equation (31) is multiplied by a diagonal phase matrix \( P \), which contains two additional CP-violating Majorana phases \( \alpha_1 \) and \( \alpha_2 \):  
\[ P = \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\beta}). \]  
(32)

4. The elusive absolute scale of the neutrino mass

With the discovery of neutrino oscillations quite a lot of information regarding the neutrino sector has become available (for recent reviews see e.g. [85, 157]). More specifically we know the following.

- The mixing angles \( \theta_{12} \) and \( \theta_{23} \) and we have both a lower and an upper bound on the small angle \( \theta_{13} \).
- We know the mass squared differences:  
  \[ \Delta_{\text{SUN}}^2 = \Delta_{12}^2 = m_2^2 - m_1^2 \]
  \[ \Delta_{\text{ATM}}^2 = |\Delta_{23}^2| = |m_3^2 - m_2^2| \]

  entering the solar and atmospheric neutrino oscillation experiments. Note that we do not know the absolute scale of the neutrino mass and the sign of \( \Delta_{23}^2 \).

For determination of an absolute scale of the neutrino mass the relevant neutrino oscillation parameters are the MINOS value \( \Delta m_{\text{SUN}}^2 = (2.43 \pm 0.13) \times 10^{-3} \text{ eV}^2 \) [158], the global fit value \( \Delta m_{\text{SUN}}^2 = (7.6 \pm 0.20) \times 10^{-5} \text{ eV}^2 \) [81], the solar-KamLAND...
value \(\tan^2\theta_{12} = 0.452^{+0.035}_{-0.030}\) [159] and the recent Daya Bay observation \(\sin^22\theta_{13} = 0.092 \pm 0.016\) (stat. ± 0.005 (syst)) with a significance of 5.2 standard deviations [160]. We note that the non-zero value of mixing angle \(\theta_{13}\) was already observed by the T2K (0.04 < \sin^2\theta_{13} < 0.34) [161], the DOUBLE CHOOZ \(\sin^22\theta_{13} = 0.085 \pm 0.029\) (stat. ± 0.042 (syst) (68% CL)) [162] and the RENO \(\sin^22\theta_{13} = 0.103 \pm 0.013\) (stat. ± 0.011 (syst)) [163] collaborations.

Based on the above we have the following scenarios:

- Normal spectrum (NS), \(m_1 < m_2 < m_3\):

\[
\Delta m^2_{\text{SUN}} = m^2_2 - m^2_1, \quad \Delta m^2_{\text{ATM}} = m^2_3 - m^2_1
\]

\[m_0 = m_1, \quad m_2 = \sqrt{\Delta m^2_{\text{SUN}} + m^2_0}, \quad m_3 = \sqrt{\Delta m^2_{\text{ATM}} + m^2_0}\]

- Inverted spectrum (IS), \(m_3 < m_1 < m_2\):

\[
\Delta m^2_{\text{SUN}} = m^2_3 - m^2_1, \quad \Delta m^2_{\text{ATM}} = m^2_3 - m^2_2
\]

\[m_0 = m_3, \quad m_2 = \sqrt{\Delta m^2_{\text{ATM}} + m^2_0}, \quad m_1 = \sqrt{\Delta m^2_{\text{ATM}} - \Delta m^2_{\text{SUN}} + m^2_0}\]

The absolute scale \(m_0\) of neutrino mass can in principle be determined by the following observations:

- Neutrinoless DBD.

As we shall see later (section 5) the effective light neutrino mass \(m_\nu\) extracted in such experiments is given as follows [44, 164]:

\[
\langle m_\nu \rangle = \sum_k \left| U_{e k}^{(11)} \right|^2 m_k = c_{12}^2 c_{13}^2 e^{2i\alpha_1} m_1 + c_{13}^2 s_{12} e^{2i\alpha_2} m_2 + s_{13}^2 m_3. \quad (33)
\]

- The neutrino mass extracted from ordinary beta decay, e.g. from triton decay [165–167]:

\[
\langle m_\nu \rangle_{\text{decay}} = \sum_k \left| U_{e k}^{(11)} \right|^2 m_k = c_{12}^2 c_{13}^2 m_1 + c_{13}^2 s_{12} m_2 + s_{13}^2 m_3. \quad (34)
\]

assuming, of course, that the three neutrino states cannot be resolved.

- From astrophysical and cosmological observations (see e.g. the recent summary [168]):

\[
m_\nu = \sum_k m_k \leq m_{\text{astro}} \quad (35)
\]

The current limit \(m_{\text{astro}}\) depends on the type of observation [168]. Thus CMB primordial gives 1.3 eV, CMB+distance 0.58 eV, galaxy distribution and lensing of galaxies 0.6 eV. On the other hand the largest photometric red shift survey yields 0.28 eV [169]. For the purposes of illustration we will take a world average of \(m_{\text{astro}} = 0.71\) eV.

The above mass combinations can be written as follows:

(i) NH, \(m_1 \ll m_2 \ll m_3\).

In this case we have

\[
\Delta m^2_{\text{SUN}} = m^2_2 - m^2_1, \quad \Delta m^2_{\text{ATM}} = m^2_3 - m^2_1.
\]

Thus

- Triton decay:

\[
\langle m_\nu \rangle_{\text{decay}} = \sqrt{c_{12}^2 c_{13}^2 (\Delta m^2_{\text{SUN}} + m_0^2) + s_{13}^2 (\Delta m^2_{\text{ATM}} + m_0^2)}.
\]

(36)

- Cosmological bound:

\[
m_\nu = m_1 + \sqrt{\Delta m^2_{\text{SUN}} + m_0^2} + \sqrt{\Delta m^2_{\text{ATM}} + m_0^2}.
\]

(37)

- 0νββ decay:

\[
\langle m_\nu \rangle_{0\nu\beta\beta} = c_{12}^2 c_{13}^2 m_1 e^{2i\alpha_1} + s_{13}^2 c_{13}^2 e^{2i\alpha_2} \sqrt{\Delta m^2_{\text{SUN}} + m_0^2} + s_{13}^2 \sqrt{\Delta m^2_{\text{ATM}} + m_0^2},
\]

(38)

where \(\alpha_1\) and \(\alpha_2\) (\(\alpha_2 = \delta\)) are Majorana CP violating phases of the elements \(U_{e1}\) and \(U_{e2}\) with \(U_{e j} = |U_{e j}| e^{i\alpha_j} (j = 1, 2)\).

By assuming NH, i.e. that \(m_1\) is negligibly small \(m_1 \ll \sqrt{\Delta m^2_{\text{SUN}}}, m_2 \approx \sqrt{\Delta m^2_{\text{SUN}}}, m_3 \approx \sqrt{\Delta m^2_{\text{ATM}}}, \) we obtain

\[
|\langle m_\nu \rangle| \approx \sqrt{\Delta m^2_{\text{SUN}} + s_{13}^2 \Delta m^2_{\text{ATM}}} \approx 4 \times 10^{-3}\text{eV}. \quad (39)
\]

(ii) IH, \(m_3 \ll m_1 < m_2\). In this case we have

\[
\Delta m^2_{\text{SUN}} = m^2_2 - m^2_1, \quad \Delta m^2_{\text{ATM}} = m^2_2 - m^2_3.
\]

- Triton decay:

\[
\langle m_\nu \rangle_{\text{decay}} = \left( s_{12}^2 m_3 + s_{12}^2 c_{13} (\Delta m^2_{\text{ATM}} + m_3^2) + c_{12}^2 c_{13}^2 (\Delta m^2_{\text{ATM}} - \Delta m^2_{\text{SUN}} + m_3^2) \right)^{1/2}.
\]

(40)

- Cosmological bound:

\[
m_\nu = m_3 + \sqrt{\Delta m^2_{\text{ATM}} + m_3^2} + \sqrt{\Delta m^2_{\text{SUN}} + m_3^2}.
\]

(41)

- 0νββ decay:

\[
\langle m_\nu \rangle_{0\nu\beta\beta} = s_{13}^2 m_3 + c_{12}^2 s_{12}^2 e^{2i\alpha_2} \sqrt{\Delta m^2_{\text{ATM}} - \Delta m^2_{\text{SUN}} + m_3^2} + c_{12}^2 e^{2i\alpha_2} \sqrt{\Delta m^2_{\text{SUN}} + m_3^2}.
\]

(42)

Since in IH scenario \(m_3\) is negligibly small \(m_1 \approx m_2 \approx \sqrt{\Delta m^2_{\text{ATM}}}\) and \(m_3 \ll \sqrt{\Delta m^2_{\text{ATM}}}, \) we find

\[
|\langle m_\nu \rangle| \approx \sqrt{\Delta m^2_{\text{ATM}}} c_{13} (1 - \sin^2 2\theta_{12} \sin^2 \alpha_{12})^{1/2}.
\]

(43)

where \(\alpha_{12} = \alpha_2 - \alpha_1\). The phase difference \(\alpha_{12}\) is the only unknown parameter in the expression for \(|\langle m_\nu \rangle|\).

From (43) we obtain the following inequality [170]:

\[
1.5 \times 10^{-2}\text{eV} \leq |\langle m_\nu \rangle| \leq 5.0 \times 10^{-2}\text{eV}. \quad (44)
\]
Quasi-degenerate (QD) spectrum, $m_{\nu}$

In the upper part of figure 6. The NH allowed region for 0.23 eV for the NS and IS. It is assumed that $\Delta m_{atm}^2 = (2.43 \pm 0.13) \times 10^{-3} \text{eV}^2$ [158], $\Delta m_{solar}^2 = (7.65^{+0.05}_{-0.03}) \times 10^{-5} \text{eV}^2$ [81], $\tan^2 \theta_{12} = 0.452^{+0.035}_{-0.033}$ [159] and $\sin^2 2\theta_{13} = 0.092 \pm 0.016$ [160].

The neutrino-mass limits in eV as a function of mass of the lowest eigenstate $m_0$ also in eV, extracted from cosmology (left panel) and triton decay (right panel). From the current upper limit of 2.2 eV of the Mainz and Troitsk experiments we deduce a lowest neutrino mass of 2.2 eV both for the NS and IS. From the astrophysical limit value of 0.71 eV the corresponding neutrino mass is extracted is about 0.23 eV for the NS and IS. It is assumed that $\Delta m_{atm}^2 = (2.43 \pm 0.13) \times 10^{-3} \text{eV}^2$ [158], $\Delta m_{solar}^2 = (7.65^{+0.05}_{-0.03}) \times 10^{-5} \text{eV}^2$ [81], $\tan^2 \theta_{12} = 0.452^{+0.035}_{-0.033}$ [159] and $\sin^2 2\theta_{13} = 0.092 \pm 0.016$ [160].

The above results are exhibited in figure 5 for the tritium $\beta$-decay and cosmological limits as a function of the lowest neutrino mass, and in figure 6 for the case of the $0\nu\beta\beta$-decay both for the NS and the IS scenarios. The allowed range values of $|\langle m_0 \rangle|$ as a function of the lowest mass eigenstate $m_0$ is exhibited. For the values of the parameter $\sin^2 2\theta_{13}$ new Double Chooz data are used [162]. The IH allowed region for $|\langle m_0 \rangle|$ is presented by the region between two parallel lines in the upper part of figure 6. The NH allowed region for $|\langle m_0 \rangle| \approx$ few meV is compatible with $m_0$ smaller than 10 meV. The quasi-degenerate spectrum can be determined if $m_0$ is known from future $\beta$-decay experiments KATRIN [165, 166] and MARE [167] or from cosmological observations. The lowest value for the sum of the neutrino masses, which can be reached in future cosmological measurements [171–173], is about (0.05–0.1) eV. The corresponding values of $m_0$ are in the region where the IS and the NS predictions for $|\langle m_0 \rangle|$ differ significantly from each other.

From the most precise experiments on the search for $0\nu\beta\beta$-decay [36, 39, 40] using the NMEs of [174] the following stringent bounds were inferred (see table 1)

$$|\langle m_0 \rangle| < (0.20–0.32) \text{ eV (}^{76}\text{Ge)},$$

$$< (0.33–0.46) \text{ eV (}^{130}\text{Te}),$$

$$< (0.17–0.30) \text{ eV (}^{136}\text{Xe}).$$

These bounds were obtained using the $0\nu\beta\beta$-decay NMEs of [174] calculated with Brueckner two-nucleon short-range correlations. There is however, a claim of the observation of the $0\nu\beta\beta$-decay of $^{76}\text{Ge}$ made by some of the participants of the Heidelberg–Moscow collaboration [42]. Their estimated value of the effective Majorana mass (assuming a specific value for the NME) is $|\langle m_0 \rangle| \approx$ 0.4 eV. This result will be checked by an independent experiment relatively soon. In the new germanium experiment GERDA [43, 175], the Heidelberg–Moscow sensitivity will be reached in about one year of measuring time.
In future experiments, CUORE [176], EXO [177, 178], MAJORANA [179], SuperNEMO [180], SNO+ [181], KamLAND-Zen and others [6, 31, 182], a sensitivity

\[
|\langle m_\nu \rangle| \simeq \text{a few } 10^{-2} \text{ eV}
\]

is expected to be reached, which is the region of the IH of neutrino masses. In the case of the normal mass hierarchy \(|\langle m_\nu \rangle|\) is very small in order to be probed in the 0\nu\beta\beta-decay experiments of the next generation.

Recently, however, for the explanation of the reactor antineutrino anomaly [185], a light sterile neutrino has been introduced with mass squared difference:

\[
\Delta m_{24}^2 = |m_2^2 - m_4^2| \approx \Delta m_{14}^2 = |m_1^2 - m_4^2| \geq 1.5 \text{ (eV)}^2.
\]

which couples to the electron neutrino with a mixing angle:

\[
sin^2 2\theta_{14} = 0.14 \pm 0.08 \quad (95\% \text{ C.L.}).
\]

On the other hand, in a recent global analysis more than one sterile neutrino is needed [186], with somewhat smaller mass squared differences, but similar couplings. Due to such a mixing, even if their couplings are of the usual Dirac type, the resulting mass eigenstates are of the Majorana type due to their coupling to the usual neutrino.

The \(U(4 \times 4)\) neutrino mixing matrix in the presence of one sterile neutrino with a small mixing becomes [187]

\[
U = R_{34} \tilde{R}_{34} R_{14} R_{23} \tilde{R}_{13} R_{12} P.
\]

It depends on six mixing angles \((\theta_{14}, \theta_{24}, \theta_{34}, \theta_{12}, \theta_{13}, \theta_{23})\), three Dirac \((\delta_{24}, \delta_{14}, \delta_{13})\) and three Majorana \((\alpha_1, \alpha_2, \alpha_3)\) CP-violating phases entering the diagonal \(P\) matrix:

\[
P = \text{diag} \left( e^{i\alpha_1}, e^{i\alpha_2}, e^{i(\alpha_1+\alpha_3)}, e^{i\delta_1} \right).
\]

Similarly, one can parametrize the \(U(5 \times 5)\) mixing matrix for two sterile neutrinos as (10 mixing angles and 5+4 CP-violating phases)

\[
U = R_{45} \tilde{R}_{45} R_{35} \tilde{R}_{35} R_{24} R_{23} \tilde{R}_{14} R_{13} R_{12} P,
\]

where \(P = \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i(\alpha_1+\alpha_3)}, e^{i(\alpha_2+\alpha_4)}, e^{i\delta_1})\).

In the presence of one sterile neutrino, the effective neutrino mass in the 0\nu\beta\beta-decay is given by [187]

\[
|\langle m_\nu \rangle|_{3+1} = |c_{13}^2 c_{14}^2 c_{15} c_{23}^2 e^{2i\alpha_1} m_1 + c_{13}^2 c_{14}^2 c_{23}^2 e^{2i\alpha_2} m_2 + c_{13}^2 c_{14}^2 c_{23}^2 e^{2i\alpha_3} m_3 + c_{13}^2 c_{14}^2 c_{23}^2 e^{2i\alpha_4} m_4|.
\]

We note that from three additional angles \(|\langle m_\nu \rangle|\) depends on only one of them, namely \(\theta_{14}\). If there are two sterile neutrinos we end up with

\[
|\langle m_\nu \rangle|_{3+2} = |c_{12}^2 c_{13}^2 c_{14}^2 c_{15} c_{23}^2 e^{2i\alpha_1} m_1 + c_{12}^2 c_{13}^2 c_{14}^2 c_{23}^2 e^{2i\alpha_2} m_2 + c_{12}^2 c_{13}^2 c_{14}^2 c_{23}^2 e^{2i\alpha_3} m_3 + c_{12}^2 c_{13}^2 c_{14}^2 c_{23}^2 e^{2i\alpha_4} m_4 + c_{12}^2 c_{13}^2 c_{14}^2 c_{23}^2 e^{2i\alpha_5} m_5|.
\]

Due to the extra terms in equations (54) and (55) and their couplings, depending on the extra majorana phases, the sterile could dominate, increase or deplete \(|\langle m_\nu \rangle|\). By assuming best fit values for \(|U_{e4}|\), \(m_4^2\) and \(|U_{e5}|, m_5^3\) from reactor antineutrino data [186] the allowed range values of \(|\langle m_\nu \rangle|\) as a function of the lowest mass eigenstate \(m_0\) in the presence of one and two sterile neutrinos is shown in figure 7.

5. The intermediate Majorana neutrino mechanism in 0\nu\beta\beta decay

To proceed further in our study of the neutrino mediated 0\nu\beta\beta-decay process it is necessary to study the structure of the
Figure 8. The neutrino-mass contribution at the nuclear level in the presence of left-handed currents for light intermediate neutrino (a) and heavy neutrino (b).



The neutrino-mass contribution at the nuclear level in the presence of left-handed currents for light intermediate neutrino (a) and heavy neutrino (b).

effective weak beta decay Hamiltonian. In general it has both left-handed and right-handed components. Within the $SU(2)_L \times SU(2)_R \times U(1)$ at low energies it takes the current–current form

$$\mathcal{H}^{\mu} = \frac{G_F}{\sqrt{2}} 2 \left[ (\bar{e}_L g \gamma_\mu v_{\ell R}^0) \left( J_{\mu}^{\ell} + e J_{\mu}^{R} \right) + (\bar{e}_R g \gamma_\mu v_{\ell R}^0) \left( \epsilon J_{\mu}^{\ell} + \epsilon J_{\mu}^{R} \right) + h.c. \right].$$  \quad (56)

Here, $\epsilon$ is the mixing of $W_L$ and $W_R$ gauge bosons

$$W_L = \cos \epsilon W_1 - \sin \epsilon W_2, \quad W_R = \sin \epsilon W_1 + \cos \epsilon W_2$$  \quad (57)

where $W_1$ and $W_2$ are the mass eigenstates of the gauge bosons with masses $M_{W_1}$ and $M_{W_2}$, respectively. The mixing is assumed to be small, $\sin \epsilon \approx \epsilon$, $\cos \epsilon \approx 1$, and $m_{W_1} \approx m_{W_1}$, $m_{W_2} \approx m_{W_2}$. $\kappa$ is the mass squared ratio

$$\kappa = \frac{m_{W_1}^2}{m_{W_2}^2}.$$  \quad (58)

$J_{\mu}^{\ell}$ is the standard hadronic current in the V–A theory:

$$J_{\mu}^{\ell} = \sum_i \bar{u}_p (i) \times \left[ g_\mu \gamma^\mu + ig_\mu \sigma_{\mu\nu} q^\nu \gamma_5 - g_\mu \gamma^\mu \gamma_5 q^\nu - g_\mu \gamma^\mu \gamma_5 q^\nu \right] u_n (i).$$  \quad (59)

where $u_p (i)$ and $u_n (i)$ are the spinors describing the proton and neutron $i$. $m_p$ is the nucleon mass and $q^\mu$ is the momentum transfer. $g_\nu \equiv g_\nu (q^2)$, $g_M \equiv g_M (q^2)$, $g_A \equiv g_A (q^2)$, and $g_{\Lambda} \equiv g_{\Lambda} (q^2)$ are, respectively, the vector, weak-magnetism, axial-vector and induced pseudoscalar form factors.

We will see below that it is necessary to consider also the right-handed current $J_{\mu}^{R}$ of the form V+A,

$$J_{\mu}^{R} = \sum_i \bar{u}_p (i) \times \left[ g_\mu \gamma^\mu + ig_\mu \sigma_{\mu\nu} q^\nu + g_\Lambda \gamma^\mu \gamma_5 + g_{\Lambda} q^\mu \gamma_5 \right] u_n (i).$$  \quad (60)

even though normally one expects its contribution to ordinary beta decay to be suppressed by a factor $\kappa$ or $\epsilon$. Some relations among form factors in $J_{\mu}^{\ell}$ and $J_{\mu}^{R}$ are considered because the strong and electromagnetic interactions conserve parity.

$\epsilon_{\ell} (\bar{e}_R)$ and $v_{\ell R}^0 (v_{\ell R}^0)$ are field operators representing the left- (right-)handed electrons and electron neutrinos in a weak interaction basis in which the charged leptons are diagonal. We have seen above that the weak neutrino eigenstates can be expressed in terms of the propagating mass eigenstates [44] (see equations (26) and (27)). The mass eigenstates $\nu_k, N_k$ satisfy the Majorana condition:

$$\nu_k \xi_k = C \nu_k, N_k \Xi_k = C \bar{\Xi}_k,$$

\quad where $C$ denotes the charge conjugation and $\xi$, $\Xi$ are phase factors, which guarantee that the eigenmasses are positive ($\xi_k = e^{i\alpha_k}$, $\Xi_k = e^{i\phi_k}$; see equation (22)).

Before proceeding further we should mention that in the context of the above interaction neutrinoless DBD is a two-step process. The neutrino is produced via the lepton current in one vertex and propagates in the other vertex. If the two current helicities are the same one picks out of the neutrino propagator the term

$$\frac{m_j}{q^2 - m_j^2} \rightarrow \left\{ \frac{m_j}{q^2}, \quad m_j^2 \ll q^2 \right\}$$  \quad (61)

where $q$ is the momentum transferred by the neutrino. In other words the amplitude for a light neutrino becomes proportional to its mass, but for a heavy neutrino it becomes inversely proportional to its mass.

If the leptonic currents have opposite chirality one picks out of the neutrino propagator the term

$$\frac{q}{q^2 - m_j^2} \rightarrow \frac{q}{q^2}, \quad m_j^2 \ll q^2,$$  \quad (62)

i.e. in the interesting case of the light neutrino the amplitude does not explicitly depend on the neutrino mass. The kinematics becomes different than that for the mass term.

5.1. The Majorana neutrino-mass mechanism

This mechanism is the most popular and most commonly discussed in the literature (see figure 8). From this figure
we can see the read-off of the couplings and the phases. We have also seen that for currents of the same chirality one picks out the mass of the propagating neutrino. Thus the lepton violating parameter is defined as $\langle m_\nu \rangle / m_e$ with $\langle m_\nu \rangle$ given by equation (33).

We will consider only $0^+ \rightarrow 0^+_f$ transitions. Then both outgoing electrons are in the $s_{1/2}$ state. Thus for the ground-state transition, restricting ourselves to the usual light left-handed neutrino-mass mechanism, we obtain the following expression for the $0\nu\beta\beta$-decay inverse half-life:

$$[T^{0\nu}_{1/2}]^{-1} = G_{01} \left| \frac{\langle m_\nu \rangle}{m_e} \right|^2 \left| M_{0\nu}^{0\nu} \right|^2.$$  

Extraction of $\langle m_\nu \rangle$ from the above lifetime will have a wide range of implications in physics, for example it can constrain the baryon asymmetry in the universe [188] $|Y_B|$ etc. Another less popular possibility is the mass contribution arising from DBD in the presence of right-handed currents (see figure 9) or heavy neutrinos in general [189]. The above expression can be generalized to include many mass mechanisms [44, 62–66, 87] as follows:

$$[T^{0\nu}_{1/2}]^{-1} = G_{01} \left| \frac{\langle m_\nu \rangle}{m_e} \right|^2 \left| M_{0\nu}^{0\nu} \right|^2 + \sum_k \left[ (U_{1k}^{(12)})^2 \sum_j \frac{m_p}{M_k} \right].$$  

The coefficient $\tilde{C}_l$ is negligible, because these terms do not interfere with the leading order due to the different helicity of the two electrons. Here ... indicate other non-traditional modes (SUSY, etc). The NMEs entering the above expression are given in units of $M_G T$ and are denoted [63] by $\chi$.

$$X_L = \frac{\langle m_\nu \rangle}{m_e} M_{0\nu}^{0\nu} + \eta^L N M_{0\nu}^{0\nu}, \quad X_R = \eta^R M_{0\nu}^{0\nu},$$  

where the NMEs $M_{0\nu}^{0\nu}$ and $M_{0\nu}^{0\nu}$ will be discussed later (see section 9). The subscripts L(R) indicate left- (right-)handed currents, respectively. The LN non-conserving parameters, i.e. the effective neutrino mass $\langle m_\nu \rangle$ given by equation (33) and $\eta^L, \eta^R$, are given as follows [44]:

$$\eta^L_N = \sum_k \left( U_{1k}^{(12)} \right)^2 \frac{m_p}{M_k}, \quad \eta^R_N = \frac{\langle m_\nu \rangle}{m_e} \chi_{\gamma} + \frac{\langle m_\nu \rangle}{m_e} \chi_{\gamma} + \frac{\langle m_\nu \rangle}{m_e} \chi_{\gamma},$$  

with $m_p (m_e)$ being the proton (electron) mass. $G_{01}$ is the integrated kinematical factor [51, 53, 63, 87]. The NMEs associated with the exchange of light and heavy neutrinos must be computed in a suitable nuclear model. The ellipses ($\ldots$) mean that equation (64) can be generalized to the mass term resulting from any other intermediate fermion.

At this point we should stress that the main suppression in the case of light neutrinos comes from the smallness of neutrino masses. In the case of the heavy neutrino it comes not only...
Figure 10. The Feynman diagrams at the nucleon level when the leptonic currents are of opposite chirality leading to the dimensionless LNV parameters $\langle \lambda \rangle$ ((a) and (b)) and $\langle \eta \rangle$ (c) and (d)) of $0\nu\beta\beta$-decay. Note that in (a) the process proceeds via the right-handed vector boson, while in (b) through the mixing of the left- and right-handed bosons.

The above expression for the lifetime is now modified to yield [45, 190]

$$[T_{1/2}^{0\nu}]^{-1} = C_{01}^{0\nu} |M_{GT}^{0\nu}|^2 \left\{ |X_L|^2 + |X_R|^2 - \tilde{C}_1 X_L X_R + \ldots + \tilde{C}_2(\langle \lambda \rangle |X_L| \cos \psi_1 + \tilde{C}_3(\langle \eta \rangle |X_L| \cos \psi_2 + \tilde{C}_4(\langle \lambda \rangle)^2 + \tilde{C}_5(\langle \eta \rangle)^2 + \tilde{C}_6(\langle \lambda \rangle)(\langle \eta \rangle) |\cos(\psi_1 - \psi_2) + \text{Re}(\tilde{C}_2(\langle \lambda \rangle) X_R + \tilde{C}_3(\langle \eta \rangle) X_R) \right\} ,$$

(69)

where $X_L$ and $X_R$ are defined in equation (65), $\psi_1$ and $\psi_2$ are the relative phases between $X_L$ and $\lambda$ and $X_L$ and $\eta$, respectively. The coefficient $C_{ij}$, representing the mixing between the left- and right-handed currents, is kinematically suppressed [191]. The ellipses \{\ldots\} indicate contributions arising from other particles, e.g., intermediate SUSY particles or unusual particles which are predicted by superstring models or exotic Higgs scalars, etc (see section 6).

Many NMEs appear in this case, but they are fairly well known and they are not going to be reviewed here in detail (see, e.g., [44, 62–66]). For the reader’s convenience we will only briefly indicate in our notation [87]) the additional NMEs not encountered in the mass mechanism. These are $\chi_{F\nu}^\ast$, $\chi_{GT\nu}^\ast$, $\chi_{R}$, $\chi_{1\pm}$, $\chi_{2\pm}$, $\chi_{F\nu}^\ast$, $\chi_{GT\nu}^\ast$, $\chi_{T}$, $\chi_{P}$ where

$$\chi_{F\nu} = \left( \frac{g_{F\nu}}{g_A} \right)^2 \frac{M_{F\nu}}{M_{GT}} .$$

(70)
excellent recent reviews [65, 66, 192]. These reviews also relevant for neutrinoless DBD, we refer once again to existing which do not DBD [195].

The approximation schemes [193, 194], but only in connection with examined in some exactly soluble models, e.g. SO(8), or better care and further exploration is necessary. It has only been the p–n pairing is incorporated. This point needs special (see below).

There seem to be significant changes in the NMEs when the p–n pairing is incorporated. This point needs special care and further exploration is necessary. It has only been examined in some exactly soluble models, e.g. SO(8), or better approximation schemes [193, 194], but only in connection with the $2\nu\beta\beta$-decay, or shell-model calculations, but for systems which do not DBD [195].

Returning back to the question of the availability of NMEs relevant for neutrinoless DBD, we refer once again to existing excellent recent reviews [65, 66, 192]. These reviews also provide a more detailed description of the nuclear models employed.

5.3. Another neutrino-mass independent mechanism (majoron emission)

It is well known that in some theories LN is associated with a global, not a local, symmetry. When such theories are broken spontaneously, one encounters physical Nambu–Goldstone bosons, called majorons. These bosons only couple to the neutrinos. So in any model which gives rise to a mass term for the light neutrino (mass insertion in the neutrino propagator), one may naturally have a competing majoron–neutrino–antineutrino coupling. Such a mechanism is shown at the quark level in figure 11. The majoron, which couples to the left-handed neutrinos, comes from the neutral member of the isorotplet. Such a multiplet, however, cannot easily be be accommodated theoretically. Hence this type of majoron is not present in the usual models. On the other hand, there is a majoron $\chi^0$, the imaginary part of an isosinglet scalar, which couples to the right-handed neutrino with a coupling $g_{ij}^0$. This gives rise to the mechanism shown in figure 12 at the nucleon level. The right-handed neutrino, however, has a small component of light neutrinos (see equation (27)).
with \( g = \sum_{i,j} U^{(21)}_{ij} g_{ij} \). Note that even if \( g_{ij} \) takes natural values, the coupling \( g_{ij} \) is very small due to the smallness of the mixing matrix \( U^{(21)} \). Thus the effective coupling \( g \) is very small. Hence, even though we do not suffer in this case from the suppression due to the smallness of the mass of the neutrino, the Majoron emission mechanism is perhaps unobservable. There exist, however, exotic models, which, in principle, may allow Majoron emission with a large coupling, like the bulk majoron [196] and others [197, 198], which we are not going to pursue theoretically any further. In any case it is straightforward to extract the limits on the effective coupling \( g \), since the NMEs are the same as in the light neutrino-mass mechanism. Note, however, that the spectrum of the summed energy of the two electrons is continuous and the kinematical function is different.

6. Mechanisms in 0νββ decay not involving intermediate neutrinos

6.1. The direct decay of doubly charged particles to leptons

Such a candidate is the isodoublet scalar, which can generate Majorana neutrino mass as see-saw mechanism II. The doubly charged charged (doubly charged Higgs, present in left–right symmetric models. The coupling to the quarks is now achieved via the right-handed gauge bosons.

Figure 13. We show the direct decay of the isodoublet into two leptons (a). The coupling to the quarks is achieved via the charged Higgs isodoublet in models where it survives the Higgs mechanism (e.g. SUSY). In (b) we show the direct decay into two leptons of an isosinglet doubly charged Higgs, present in left–right symmetric models. The coupling to the quarks is now achieved via the right-handed gauge bosons.

6.2. The R-parity violating contribution to 0νββ decay

In SUSY theories R-parity is defined as

\[
R = (-1)^{3B+L+2s},
\]

with \( B = \text{baryon}, L = \text{LNs} \) and \( s \) the spin. It is +1 for ordinary particles and −1 for their superpartners. R-parity violation has recently been seriously considered in SUSY models. It allows additional terms in the superpotential given by

\[
W = \epsilon_i L^H H_{2\alpha} + \lambda_{ijk} L^i U^j H^D + \lambda_{ijk} U^i D^j H^L + \lambda_{ijk} L^i U^j D^k H^L,
\]

where a summation over the flavor indices \( i, j, k \) and the isospin indices \( a, b \) is understood (\( \lambda_{ijk} \) is antisymmetric in the indices \( i \) and \( j \)). The last term has no bearing in our discussion, but we will assume that it vanishes due to some discrete symmetry to avoid very fast proton decay. The first term is an LNV bilinear and, since it cannot be rotated away, it can lead to neutrinoless DBD. The \( \lambda s \) are dimensionless couplings not predicted by the theory. The couplings are assumed to be given in the basis in which the charged fermions are diagonal. In the above notation \( L, Q \) are isodoublet and \( E^c, D^c \) isosinglet chiral superfields, i.e. they represent both the fermion and the scalar components.

The above R-parity violating superpotential can lead to Majorana neutrino masses without the need for introducing the right-handed neutrino and invoking the see-saw mechanism [95, 142]. One can then have contributions to neutrinoless DBD in the usual way via intermediate massive neutrinos as discussed above.

6.2.1. The contribution arising from the bilinears in the superpotential

The first term in the superpotential, equation (82), can cause mixing between the neutrino and the neutralinos as soon as the s-neutrino develops a vacuum expectation value \( v_R \). The above LNV parameter associated with figure 13 is given by

\[
\eta_{\Delta H} = \frac{2\sqrt{2} \lambda_{ijk} m^2_{\nu_R} m_{\nu_R} \epsilon_{\nu_R}}{m_{\nu_R} m_{\nu_R} m_{\nu_R} G_F}, \quad \eta_{\Delta W} = \frac{\omega_{\nu_R} m^2_{\nu_R} m_{\nu_R} \epsilon_{\nu_R}}{m_{\nu_R} m_{\nu_R} m_{\nu_R} G_F}. \quad (80)
\]

Taking \( \epsilon_{\nu_R} = 1 \), a natural value, and \( \lambda_{ijk} \approx 1 \) TeV, \( m_{\nu_R} = m_{\nu} = 100 \text{ GeV} \) and \( m_{\nu_R} = 10 \text{ MeV} \), one finds [44] \( \eta_{\Delta H} \approx 3 \times 10^{-8} \). Similarly taking \( v_R = m_{w_R} = m_{\nu_R} = m_{\nu_R}/\sqrt{3} = 10 m_{w_R} \), we find \( \eta_{\Delta W} \approx 10^{-6} \).

Note that the term \( \eta_{\Delta H} \) adds to \( X_L \) of equation (64), while \( \eta_{\Delta W} \) adds to the \( X_R \).

6.2.2. The contribution arising from the bilinears in the superpotential

The first term in the superpotential, equation (82), can cause mixing between the neutrino and the neutralinos as soon as the s-neutrino develops a vacuum expectation value. As a result it can directly lead to neutrinoless DBD [94, 95, 199] via the effective \( W \)-charged lepton-neutralino interaction:

\[
\mathcal{L} = -\frac{g}{\sqrt{2}} \bar{\nu}_L \gamma^\mu \lambda_{\nu \nu} H^L \frac{\epsilon_{\nu_R}}{\sqrt{3}} \lambda_{\nu \nu}^0 \lambda_{\nu \nu}. \quad (83)
\]
Figure 14. The $R$-parity violating contribution to $0
u\beta\beta$ decay mediated by neutralinos arising from the bilinear terms in the superpotential. For comparison we give the neutrino-mediated process of figure 8 expressed at the quark level.

where $\kappa_\alpha$ is a dimensionless quantity, associated with each of the four neutralinos, which arises due to neutrino neutralino mixing [94]. This term then gives rise to a diagram analogous to that of figure 8 with the intermediate particle now being the neutralino, which is heavy with mass $M_{\tilde{\chi}_0}^2$ and leads to a short-ranged operator. For the reader’s convenience this is shown in figure 14 at the quark level. One thus obtains an analogous LNV parameter:

$$\eta_{\tilde{\chi}_0}^\perp \rightarrow \eta_{\tilde{\chi}_0}^\perp = \sum_n \kappa_n^2 \frac{m_{\tilde{\chi}_0}}{M_{\tilde{\chi}_0}}. \quad (84)$$

6.2.2. The contribution arising from the cubic terms in the superpotential. It has also been recognized quite some time ago that the second and third terms (cubic terms) in the superpotential could lead to neutrinoless $B$ [26, 88, 89]. Typical diagrams at the quark level are shown in figure 15. Note that as intermediate states, in addition to the sleptons and squarks, one must consider the neutralinos, four states of which are linear combinations of the gauginos and higgsinos, and the colored gluinos (supersymmetric partners of the gluons). Whenever the process is mediated by gluons a Fierz transformation is needed to lead to a colorless combination. The same thing is necessary whenever the fermion line connects a quark to a lepton. As a result one gets at the quark level not only scalar (S) and pseudoscalar (P) couplings, but tensor (T) couplings as well. This must be contrasted to the V and A structure of the traditional mechanisms. One must therefore face the problem of how to transform these operators from the quark to the nucleon level.

6.2.3. The LNV parameters from the cubic terms without intermediate neutrinos. As we have mentioned, the effective LNV parameter arising from the bilinear terms in the superpotential is analogous to that arising from the heavy intermediate neutrinos, and thus it will not be further discussed. We will concentrate on the cubic terms in the superpotential [26, 66, 200–202]. Then the effective LNV parameter arising from these terms, assuming that the pion-exchange mode dominates, as the authors of [26, 66] find, can be written as

$$\eta_{\text{SUSY}} = (\lambda_{111}^\perp)^2 \frac{1}{2} (\chi_{PS} \eta_{PS} + \eta_T), \quad (85)$$

with $\eta_{PS}(\eta_T)$ associated with the scalar and pseudoscalar (tensor) quark couplings given by

$$\eta_{PS} = \eta_{\tilde{\chi}_0} \bar{e} + \eta_{\tilde{\chi}_0} \tilde{e} + \eta_{\tilde{\chi}_0} \tilde{e} + 7 \eta_{\tilde{\chi}_0}^2, \quad (86)$$

$$\eta_T = \eta_{\tilde{\chi}_0} \bar{e} + \eta_{\tilde{\chi}_0} \tilde{e} - \eta_{\tilde{\chi}_0}^2 \eta_{\tilde{\chi}_0} \tilde{e}. \quad (87)$$

These authors find $\chi_{PS} = \sqrt{5}/3$, but, as we shall see below, this depends, in general, on ratios of NMEs. For the diagram of figure 15(a) one finds

$$\eta_{\tilde{\chi}_0} = \frac{2 \pi \alpha}{(G_F m_W^2)^2} \left( \kappa_d^2 \left( \frac{m_p}{m_{\tilde{\chi}_0}} \right)^2 \right) \cdot \quad (88)$$

For the diagram of figure 15(b) one finds

$$\eta_{\tilde{\chi}_0} = \frac{\pi \alpha}{2(G_F m_W^2)^2} \left( \kappa_d^2 \left( \frac{m_p}{m_{\tilde{\chi}_0}} \right)^2 \right) \cdot \quad (89)$$

$$\eta_{\tilde{\chi}_0} = \frac{\pi}{6} \alpha_s \left( \frac{m_p}{m_{\tilde{\chi}_0}} \right)^2 \left( \kappa_d^2 \right) \cdot \quad (90)$$

For the diagram of figure 15(c) one finds

$$\eta_{\tilde{\chi}_0} = \frac{\pi}{12} \alpha_s \left( \frac{m_p}{m_{\tilde{\chi}_0}} \right)^2 \kappa_d^2 \kappa_u \cdot \quad (91)$$

$$\eta_{\tilde{\chi}_0} = \frac{\pi}{12} \alpha_s \left( \frac{m_p}{m_{\tilde{\chi}_0}} \right)^2 \kappa_d^2 \kappa_u \cdot \quad (92)$$

where

$$\kappa_X = \left( \frac{m_W}{m_X} \right)^2, \quad X = \tilde{e}_L, \tilde{u}_L, \kappa_d \cdot \quad (93)$$

$$\langle \tilde{e}_L, f \tilde{e}, \tilde{f} \tilde{e} \rangle = \sum_{i=1}^4 \epsilon_{\tilde{e}_L, f} \epsilon_{\tilde{e}, \tilde{f}} \frac{m_p}{m_{\tilde{e}_L}}, \quad (94)$$

where $\epsilon_{\tilde{e}_L, f}$ and $\epsilon_{\tilde{e}_L, f}$ are the couplings of the ith neutralino to the relevant fermion and sfermion. These are calculable (see, e.g., [44]). Thus, ignoring the small Yukawa couplings coming via the Higgsinos and taking into account only the gauge couplings, we find

$$\epsilon_{\tilde{e}_L, \tilde{e}} = \frac{Z_{\tilde{e}_L} + \tan \theta_W Z_{\tilde{u}_L}}{\sin \theta_W}, \quad (95)$$

$$\epsilon_{\tilde{e}_L, \tilde{e}} = \frac{Z_{\tilde{e}_L} + (\tan \theta_W / 3) Z_{\tilde{u}_L}}{\sin \theta_W}, \quad (96)$$

$$\epsilon_{\tilde{e}_L, \tilde{e}} = -\frac{Z_{\tilde{u}_L}}{3 \cos \theta_W}, \quad (97)$$

where $Z_{\tilde{u}_L}, Z_{\tilde{e}_L}$ are the coefficients in the expansion of the $B, W_3$ in terms of the neutralino mass eigenstates. Note that in this convention some of the masses $m_{\tilde{e}_L}$ may be negative.

We should mention here that, if the gluino exchange is dominant, the LNV parameter $\eta_{\text{SUSY}}$ simplifies and becomes $\eta_{\tilde{e}_L}$ with $\eta_{\tilde{e}_L}$ given in section 9.
6.2.4. The case of light intermediate neutrinos. It is also possible to have light neutrino mediated 0νββ decay originating from R-parity violating interactions. In this case one has the usual β decay vertex of the V–A type in one vertex and the sfermion mediated vertex of the S–P type, in the other end [95, 199, 200] (see figure 16). The LN violation is achieved via the mixing between the isodoublet and isosinglet sfermions. The simplest diagram, which involves intermediate sleptons, can arise from the following interactions:

\[ \mathcal{L}_{\text{QIL}} \rightarrow \lambda_{111} \sum_k V_{e_k}^L \bar{u}_L d_R \tilde{\ell}_k^* \]  

(98)

for the d-quark vertex and

\[ \mathcal{L}_{\text{LL}} \rightarrow \lambda_{111} \sum_{j,k} V_{e_k}^R U_{e_j} \bar{v}_{jL} e_k^* \]  

(99)

for the neutrino vertex. In the above expression, \( V^L(V^R) \) are the mixing matrices which express the doublet (singlet) selectrons in terms of the mass eigenstates. \( U \) is the usual neutrino mixing matrix. The effective s-lepton propagator is

\[ P = \sum_{k=1}^3 \frac{V_{e_k}^L V_{e_k}^R}{M_{\tilde{\ell}_k}^2} \approx \sum_{k=1}^3 \frac{V_{e_k}^L V_{e_k}^R \delta M_{\tilde{\ell}_k}^2}{M_{\tilde{\ell}_k}^2}, \quad \delta M_{\tilde{\ell}_k}^2 = M_{\tilde{\ell}_k}^2 - M_{\tilde{\ell}_k}^2. \]  

(100)
The last equation follows from the orthogonality condition on the mixing matrices and the fact that the splitting $\delta M^2_{\ell}$ is small compared with the average s-lepton mass $M^2_{\ell}$. If, further, the mixing between generations can be ignored we obtain

$$P = \frac{\sin 2\theta_{\ell}}{2} \Delta M^2_{\ell}.$$  

(101)

Combining the above results with the usual V–A coupling one gets:

$$\mathcal{M} = \left(\frac{G_F}{\sqrt{2}}\right)^2 \Lambda_{\ell} \bar{u}_{\ell} \gamma^\mu \frac{k_{u}^{\alpha}}{k^2} \gamma^\nu \epsilon^*_{\ell} \epsilon_{\ell} \gamma_\mu \gamma_{\nu} d_L.$$  

(102)

with

$$\Lambda_{\ell} = \sqrt{\frac{\gamma}{G_F}} (\lambda_{111}^2)^{\frac{3}{2}} \frac{\sin 2\theta_{\ell}}{2} \Delta M^2_{\ell}.$$  

(103)

In the case of squark exchange of figure 16 the above expressions become

$$\mathcal{L}_{LQD} \rightarrow \lambda_{111}^2 \sum_k V^L_{dk} \bar{u}_{L} \epsilon_k \bar{d}_k.$$  

(104)

for the u-quark vertex and

$$\mathcal{L}_{LQD} \rightarrow \lambda_{111}^2 \sum_k V^R_{dk} \bar{u}_{L} \gamma_\mu \gamma_{\nu} d_K \bar{d}_k.$$  

(105)

for the neutrino vertex. Combining them we obtain

$$\mathcal{M} = \left(\frac{G_F}{\sqrt{2}}\right)^2 \Lambda_{\ell} \bar{u}_{\ell} \gamma^\mu \frac{k_{u}^{\alpha}}{k^2} \gamma^\nu \epsilon^*_{\ell} \epsilon_{\ell} \gamma_\mu \gamma_{\nu} d_L,$$  

(106)

with

$$\Lambda_{\ell} = \sqrt{\frac{\gamma}{G_F}} (\lambda_{111}^2)^{\frac{3}{2}} \frac{\sin 2\theta_{\ell}}{2} \Delta M^2_{\ell}$$  

(107)

in a rather obvious notation.

In the case of $\bar{e}$ and $\bar{d}$ contributions, in the context of perturbation theory, one can simplify the above expressions using explicitly the coupling between the singlet and the doublet sfermions of the lower charge. In this case

$$\frac{\sin 2\theta_{\ell}}{2} \Delta M^2_{\ell} = (\mu + A \tan \beta)m_\epsilon, \quad x = e, d.$$  

(108)

Before proceeding further we have to perform a Fierz transformation:

$$\bar{u}_{L} \epsilon^*_{k} \gamma^\mu \frac{k_{u}^{\alpha}}{k^2} \gamma^\nu \epsilon_{\ell} \gamma_\mu \gamma_{\nu} d_R = \frac{1}{2} \left[ \bar{u}_{L} \gamma^\mu \gamma_{\nu} d_R \epsilon_k \epsilon_{\ell} + \bar{u}_{L} \gamma^\mu \gamma_{\nu} d_R \epsilon_{\ell} \epsilon_k \right] + \bar{u}_{L} \gamma^\mu \gamma_{\nu} d_R \epsilon_k \epsilon_{\ell} - \frac{1}{8} \bar{u}_{L} \gamma^\mu \gamma_{\nu} d_R \epsilon_k \epsilon_{\ell}.$$  

(109)

We must now go to the nucleon level and perform a Fourier transform to the coordinate space. For $0^+ \rightarrow 0^+$ transitions the space component yields

$$\mathcal{M} = \left(\frac{G_F}{\sqrt{2}}\right)^2 \left((f_{\Lambda})^2 \times \left( -\lambda \left[M_T + M_{GT} + r_F M_F \right] \right) \times \frac{\Delta_{\ell}}{2} \left(M_{GT} + \left(1 - \frac{2\Delta_{\ell}}{\Lambda_{\ell}}\right) \frac{1}{f_{\Lambda}} \frac{M_F}{2} \right) \right)$$

$$\left(\sin \theta_{\ell} (1 + \gamma_5)\epsilon^* \epsilon \right) \times \frac{\Lambda_{\ell}}{2} \left(M_{GT} + \left(1 - \frac{2\Delta_{\ell}}{\Lambda_{\ell}}\right) \frac{1}{f_{\Lambda}} \frac{M_F}{2} \right) \right) \left(\sin \theta_{\ell} (1 + \gamma_5)\epsilon^* \epsilon \right)$$  

(110)

with

$$\lambda = \Lambda_{\ell}/96, \quad r_F = \frac{3}{4} \frac{\Lambda_{\ell}}{N_{\Lambda}} \left(\frac{\Delta_{\ell} - 1}{\Lambda_{\ell}}\right).$$  

(111)

The parameter $\lambda$ as well as the quantities $M'$ and $\tilde{M}$ have the same meaning as in the mass-independent contribution in the conventional approach (see section 5.2). Note, however, that in the present mechanism there is no term analogous to the $\eta$ of section 5.2. We should stress that this novel mechanism can lead to transitions $J^* = 0^+$ and $J \neq 0$. Hence, contrary to conventional wisdom, from the observation of such transitions one cannot definitely infer the existence of right-handed currents.

7. Handling the short-range transition operators

We have seen that there exist many mechanisms contributing to neutrinoless DBD, involving the exchange of only heavy particles. This results in short-range transition operators.

7.1. The mode involving only nucleons

If the nucleons are treated as point-like particles, then the effective transition operator essentially behaves like a $\delta$ function in the inter-nucleon distance. Thus their contribution vanishes, due to the presence of a nuclear hard core. This can be cured, if the nucleons can be treated as extended objects. This can be carried out by introducing into the nucleon current a nucleon form factor [96], e.g. like a dipole shape with a characteristic mass $m_{\Lambda} \approx 850$ MeV. This can also be accomplished if one utilizes a quark model for the nucleon [203].

- V–A theories.
  - In this case the approach has become pretty standard, i.e. the spin isospin structure is similar to that for the light neutrino-mass term except for the radial part [44], which now becomes:
    $$\frac{m_{\Lambda}^2}{m_{\Lambda}^2 m_{\nu}^2} \left(\frac{R_0}{F_N(x_{\Lambda})} \times \frac{x_{\Lambda}}{x_{\nu}} m_{\nu} m_{\Lambda} \right),$$

  - and we are not going to discuss them further. We only mention that it can also proceed via the two-pion mode with the nuclear operator associated with $\sigma_{2r} = 0.1$ (see next section).

- S, PS theories.
  - In the scalar case we find that the operator is spin independent and has the same radial part as in the previous case.
  - In the case of the pseudo-scalar part we find that the operator becomes
    $$\Omega_{PS} = \frac{m_{\Lambda}^2}{m_{\Lambda}^2 m_{\nu}^2} \left[ \frac{1}{3} \left(\frac{m_{\Lambda}}{2m_{P}^2/3} \right)^2 \sum_{k \neq \ell} \kappa_{s1}(k) \kappa_{s1}(\ell) \right]$$
    $$\times \left( \frac{m_{\Lambda}}{2m_{P}^2/3} \right)^2 \left(\frac{m_{\Lambda}}{2m_{P}^2/3} \right)^2 \left(\frac{m_{\Lambda}}{2m_{P}^2/3} \right)^2 \left(\frac{m_{\Lambda}}{2m_{P}^2/3} \right)^2 \left(\frac{m_{\Lambda}}{2m_{P}^2/3} \right)^2$$
    $$\times \frac{dF_N(x_{\Lambda})}{dx_{\Lambda}} + \frac{dF^2_N(x_{\Lambda})}{dx_{\Lambda}^2}$$

(112)
where $T(\hat{\chi}_A, \sigma_k, \sigma_i)$ is the tensor component.
This process can also proceed via the two-pion mode (see next section). In this case one can use the LNV parameter $\eta_{\Delta H}$ with NME as given in the next section with $\alpha_{2\pi} = 0.20$.

7.2. The pion mode in R-parity induced $0\nu\beta\beta$ decay

Even though the pion model (12) may be important in other cases when the intermediate particles are heavy, giving rise to short-range operators, in this section we will elaborate a bit further on its application in the extraction of the R-parity violating parameters associated with the processes discussed above. The NMEs can now be calculated using the effective transition operators

$$ME_k = \left(\frac{m_A}{m_p}\right)^2 \alpha_{\chi_2} \frac{m_p}{m_e} \left[M_{\text{GT}}^{0+} + M_{\text{T}}^{0+}\right],$$

(113)

where the two above matrix elements are the usual GT and T matrix elements with the additional radial dependence given by

$$F_{\text{GT}}^{0+} = e^{-x}, \quad F_{\text{T}}^{0+} = (3 + 3x + x^2) e^{-x}/x,$$

$$F_{\text{GT}}^{0-} = (x - 2) e^{-x}, \quad F_{\text{T}}^{0-} = (1 + x) e^{-x}.$$  

(114)

(115)

In the case of the pion-exchange mechanism, in particular for $1\pi$ exchange, it is important to include the nucleon form factors [202, 204]. We are not going to elaborate further on this point. The complete expressions for the transition operators are given in section 9.

We will, instead, concentrate on the coefficients $\alpha_{2\pi}$ and $\alpha_{1\pi}$. We will begin by considering the elementary particle treatment [27].

- The coupling coefficients $\alpha_{2\pi}$. One finds

$$\alpha_{2\pi} = \frac{1}{6} f_A \left(\frac{m_\pi}{m_p}\right)^4.$$  

(116)

Obtained under the factorization approximation [27]:

$$\langle \pi^+ | \mathcal{F}_P | \pi^- \rangle = \frac{5}{3} \langle \pi^+ | \mathcal{F}_P | 0 \rangle \langle 0 | \mathcal{F}_P | \pi^- \rangle,$$

$$\langle 0 | \mathcal{F}_P | \pi^- \rangle = m_p^2 h_{\pi}$$

with the parameter $h_{\pi}$ given by

$$h_{\pi} = 0.668 \sqrt{\frac{2}{i \frac{m_\pi}{m_q + m_\pi}}}. $$

(117)

(118)

Thus using the current quark masses these authors [27] find $\alpha_{2\pi} = 0.20$.

- The coupling coefficients $\alpha_{1\pi}$. One finds

$$\alpha_{1\pi} = -\frac{1}{36} \frac{1}{f_A^2} \left(\frac{m_\pi}{m_p}\right)^4.$$  

(119)

The necessary parameters were obtained using the factorization approximation in the case of $1 - \pi$ mode

$$\langle p | \mathcal{F}_P | n \pi^- \rangle = \frac{5}{3} \langle p | \mathcal{F}_P | n \rangle \langle n | \mathcal{F}_P | \pi^- \rangle,$$

$$\langle p | \mathcal{F}_P | n \rangle = f_p \approx 4.41.$$  

(120)

The matrix element $\langle 0 | \mathcal{F}_P | \pi^- \rangle$ was given above (see equation (117)). Thus these authors [27] find

$$\alpha_{1\pi} = -4.4 \times 10^{-2}.$$  

(121)

In order to provide a check of the approximations involved in the above treatment and to explore the new possibilities appearing in the microscopic treatment, e.g. the role played by the non-local terms or the three possibilities entering in figure 18, which are not distinguished in the standard treatment, we will consider the quark structure of the pion and the nucleon [203]. Thus we will evaluate the relevant amplitude by making a non-relativistic expansion of the hadronic current employing a constituent quark mass equal to 1/3 of the nucleon mass. Furthermore, for the pion and the nucleon internal relative quark wave functions we will employ harmonic oscillator wave functions, adjusting the size parameters to fit related experiments [203]. Then we will compare this amplitude to that obtained by more standard techniques [93].

(a) The $1\pi$ mode. Let us begin with the second process of equation (12) (see figure 17(b)), which is further analyzed at the quark level in figure 18. In this case it is clear that the amplitude must be of the PS type only. The tensor contribution cannot lead to a pseudoscalar coupling at the nucleon level. Such a coupling is needed to be combined with the usual pion nucleon coupling in the other vertex to obtain the relevant operator for a $0^+ \rightarrow 0^-$ decay.

Let us begin with figure 18(a), which is simply the decay of the pion into two leptons with a simultaneous change of a neutron to a proton by the relevant nucleon current. Then one finds that, if the non-local terms, which lead to new type operators not studied until now, are ignored, the ‘direct’ diagram makes no contribution.

The ‘exchange’ contribution, figure 18(b), in which the produced up quark of the meson is not produced from the
Figure 18. The pion mediated $0\nu\beta\beta$-decay in the so-called $1\pi$ mode. In (a) the quarks of the pion are spectators, i.e. the heavy intermediate heavy fermion $f$ is exchanged between the other two quarks. In (b) one of the interacting quarks is in the pion. In (c) the $q\bar{q}$ pair is produced by the weak interaction, while in (a) and (b) the $q\bar{q}$ is produced by the strong interaction out of the vacuum in the context of a multigluon exchange (a la $3p_0$ mode) indicated by $\times$. $f$ stands for an intermediate neutral fermion (heavy Majorana neutrino, gluino or neutralino).

'vacuum' but comes from the initial nucleon, is a bit more complicated. The result is

$$c_{1\pi} = 1.37 f_{1\pi}^{\text{con}}(x), \quad \alpha_{1\pi} = 0.071 f_{1\pi}^{\text{con}}(x) \approx 0.071, \quad (122)$$

which is in size almost a factor of two larger than that obtained in elementary particle treatment [27] (see equation (121)). Note, however, that our results depend on the pion size parameter. Finally figure 18(c), in which the $q\bar{q}$ is produced by the weak interaction, leads to the expression:

$$c_{1\pi} = \frac{20\sqrt{3}\sqrt{\pi}m_\pi}{3g_s b^3_{NN}} f_{1\pi}^A(x) \approx 3.4 \times 10^{-4} f_{1\pi}^A(x)$$

(123)

with

$$f_{1\pi}^A(x) = x^{3/2}, \quad x = \frac{b_N}{b_\pi}. \quad (124)$$

The corresponding coefficient that must multiply the NME for $x = 2$ is

$$\alpha_{1\pi} = c_{1\pi} f_{2\pi}^A f_{2\pi}^A = 5.0 \times 10^{-3} \quad (125)$$

(b). The $2\pi$ mode. The first process of equation (12) is described by figure 17(c) and is further illustrated in figure 19 at the quark level [203]. In figure 19, $f$ stands for an intermediate fermion, heavy Majorana neutrino, neutralino or gluino.

In the case of $V$–$A$ theories we find [203]

$$\alpha_{2\pi} = \frac{2}{3g_A^2} f_{2\pi}^A C_{2\pi}, \quad c_{2\pi} = \frac{1}{\sqrt{2\pi}} \frac{16}{b^3_{NN} m^2_\pi m_\pi} \quad (126)$$

The actual value critically depends on the harmonic oscillator size parameter. For $b_\pi = 0.5$ fm we obtain $\alpha_{2\pi} \approx 0.1$.

In supersymmetry one encounters scalar, pseudoscalar and tensor interactions. For the pseudoscalar interaction the situation is a bit more complicated [203]. One finds

$$c_{2\pi} = \frac{1}{\sqrt{2\pi}} \frac{4}{b^3_{NN} m^2_\pi m_\pi} \left( \frac{1}{4} (\kappa_d^2 + \kappa_u^2) \right), \quad \kappa_d = \frac{1}{2m_d b_N}, \quad \kappa_u = \frac{1}{2m_u b_N}. \quad (127)$$

Taking $m_u = m_d = (1/3)m_\pi$, $b_N = 1$ fm and $b_\pi = 0.5$ fm we find that $\alpha_{2\pi}$ is much smaller than the value obtained in the elementary particle treatment [93], i.e. we find $\alpha_{2\pi} = -0.05$. 

Figure 19. The $0\nu\beta\beta$-decay of pions in flight ($2\pi$ mode of figure 17) illustrated at the quark level. $f$ stands for an effective exchange of a heavy Majorana fermion (heavy neutrino or, as in $R$-parity violating supersymmetry, a neutralino, gluino, etc). The arcs around the pion line merely indicate that the pion is a bound state of two quarks.
A larger value can be obtained, if one uses the current quark masses. For $m_s \approx 2m_d = 10 \text{ MeV}$ one finds $\delta m_2 \approx -1.3$, but then the validity of non-relativistic expansion may be questionable.

8. Experimental aspects of DBDs

8.1. Progress of DBD experiments

8.1.1. Experimental aspects of neutrinoless DBDs. Neutrinoless DBDs are concerned with fundamental properties of neutrinos and weak interactions, which bear some signatures of the high-energy scale, and are of great interest from the viewpoint of particle physics and cosmology, as has been described in previous sections. On the other hand, DBD processes are nuclear rare decays in the low-energy scale, which are studied experimentally by low-energy and low-background nuclear spectroscopy, as given in review papers [30, 31, 62, 63].

Since DBDs are low-energy second-order weak processes, the decay rates of $\nu\beta\beta$ within the SM are of the order of $10^{-20}$ y$^{-1}$, and the rates of $\nu\beta\beta$-decay beyond the SM are even smaller than those of $\nu\beta\beta$-decay rates, depending on the assumed light neutrino mass in the case of the Majorana neutrino mediated process. Then the expected $\nu\beta\beta$ half-lives are of the order of $T_{1/2}^{\nu\beta\beta} \approx 10^{27}$ y and $10^{39}$ y in the case of the IH mass of 30 meV and the NH mass of 3 meV, respectively.

For experimental studies of such rare decays, large detectors with ton-scale DBD isotopes are needed to obtain $\nu\beta\beta$-decay signals in the case of the IH neutrino mass. Here, the signals are very rare and occur as low as $E_{\nu\beta\beta} \approx 2–3$ MeV. BG signal rates, however, are huge in the energy region of $E_B \leq 3$ MeV. Thus it is crucial to build ultra-low BG detectors to find the rare and small $\nu\beta\beta$-decay signals among huge BGs in the low-energy region.

In spite of this, DBDs have several unique features that make it realistic to search for the low-energy ultra-rare $\nu\beta\beta$ signals among huge BGs.

(i) Since $\beta\beta$ half-lives of the order of $T_{1/2}^{\beta\beta} \simeq 10^{19}–10^{21}$ y are 10 orders of magnitude longer than the age of the Earth, the $\beta\beta$ isotopes are available as almost stable isotopes on Earth, and it is possible to get ton-scale $(10^{20})$ $\beta\beta$ isotopes in order to observe some rare $\nu\beta\beta$-decay events with $T_{1/2}^{\nu\beta\beta} \approx 10^{27}$ y.

(ii) There are even–even nuclei, where the DBDs are allowed due to the pairing interaction, but single $\beta$-decays are energetically forbidden. Using such DBD nuclei, one can be free from huge single $\beta$ BGs, which would be larger than $\beta\beta$ rates by factors around $10^{30}$.

(iii) The $\nu\beta\beta$-decay process with a virtual Majorana neutrino exchange between two nucleons in a nucleus is significantly enhanced because the nucleons are close to each other in the nucleus. Then it is feasible to access the small neutrino masses of the orders of $\delta m_{\text{SUN}}$ (solar $\nu$)–$\delta m_{\text{SM}}$ (atmospheric $\nu$).

(iv) High-energy resolution and/or correlation studies of $\beta\beta$ rays select low-energy rare $\beta\beta$ signals from huge BG events. There are several DBD nuclei to be studied to confirm ultra-rare $\nu\beta\beta$-decay events and possible DBD processes.

Then $\beta\beta$ experiments study neutrino properties in nuclei, which are around $10^{-15}$ m in diameter. Thus the $\beta\beta$ nuclei are regarded as excellent femto ($10^{-15}$) laboratories with a large filtering power to reject single $\beta$ and other RI BG signals and a large enlargement factor to enhance the $\nu\beta\beta$-decay signal of the neutrino physics interest. In the nuclear femto laboratory, two nucleons collide with each other. The luminosity is around $L \approx 10^{48} \text{ cm}^{-2} \text{ s}^{-1}$ in a single DBD isotope. Then the summed luminosity for one ton $(10^{20})$ isotopes is around $10^{76} \text{ cm}^{-2} \text{ s}^{-1}$. The cross section for exchange of the Majorana neutrinos with IH mass of 40 meV is of the order of $10^{-83} \text{ cm}^2$. Then the event rate is around 2–3 per year. The huge luminosity of the DBD femto collider enables one to search for the ultra-rare $\nu\beta\beta$ event and the very small neutrino mass [49].

8.1.2. Progress of DBD experiments. Progress of DBD experiments is well described in review papers [30, 31, 62, 63, 205] and references therein. Here brief remarks on progress are given.

Early experimental studies of $\beta\beta$-decays were made by geochemical methods by measuring the number of the $\beta\beta$-decay products. They are inclusive measurements of both the $2\nu\beta\beta$ and $0\nu\beta\beta$-decay rates to the ground and excited states. In most realistic cases, the geochemistry methods give the $2\nu\beta\beta$-decay rate to the ground state because $0\nu\beta\beta$-decay rates are much reduced due to the small neutrino mass and decays to excited states are disfavored due to the small phase-space volume.

Advanced mass spectroscopy was used to measure the number of $\beta\beta$ product isotopes accumulated for a long geological period of around $10^{6–7}$ years in old ores, and to evaluate the long half-lives of the order of $10^{20}$ y. Extensive studies have been made on such rare-gas isotopes as $^{83}$Kr, $^{128}$Xe and $^{130}$Xe, which are $\beta\beta$-decay products of $^{82}$Se $^{128}$Te and $^{130}$Te, respectively, as described in the review papers. Among them three groups have obtained half-lives of around $10^{21}$ for $^{130}$Te [11–14].

Direct counter experiments of $\beta\beta$ decays are exclusive measurements to identify $0\nu\beta\beta$-decay signals. They have been made by coincidence counter measurements of two $\beta$ rays or by two $\beta$ measurements with tracking chambers, as described in the review papers.

High-sensitivity counter experiments were conducted using detectors as $\beta\beta$ sources. Der Mateosian and Goldhaber obtained limits on the $^{48}$Ca $0\nu\beta\beta$-decay rate using large CaF$_2$ scintillators [206]. Stringent limits on the $^{76}$Ge $0\nu\beta\beta$-decay rate were obtained with high-energy-resolution Ge semiconductor detectors by Fiorini et al [207, 208]. Coincidence measurements of two $\beta$ tracks with spark and streamer chambers were made to obtain limits on the $^{48}$Ca and $^{82}$Se $0\nu\beta\beta$ rays [209–212].

In the 1980s, the Ge experiments for $^{76}$Ge $0\nu\beta\beta$-decays were much improved by the Milano group [213] and Avignone et al [214]. Ejiri et al used the ELEGANT III with a Ge detector surrounded by Nal scintillators to limit the $^{76}$Ge $0\nu\beta\beta$-decays to the ground and excited states [215–217].
The first measurement of the $2\nu\beta\beta$ rays by the direct counting method was made on $^{82}$Se by Elliott et al [32]. The observed rate agrees with the $\beta\beta$ rate measured previously by the geochemical method [218, 219]. The first measurement of the $2\nu\beta\beta$-decay rate by the direct counting method alone on $^{100}$Mo, where no geochemical method was made beforehand, was carried out by the ELEGANT group (Ejiri et al) [220, 221].

So far, high-sensitivity counter experiments with the mass sensitivities in the order of eV and sub-eV have been made on several $\beta\beta$-decay nuclei, and half-lives of $2\nu\beta\beta$-decay rates on many nuclei have been measured by direct counting methods, as reported in the review papers [30, 31, 205]. Recent experiments are discussed in section 8.3.

8.2. Methods and detectors for DBD experiments

8.2.1. Methods for DBD experiments. DBDs proceed normally through the $2\nu\beta\beta$-decay process within the SM. Transition rates of the $0\nu\beta\beta$-decay processes beyond SM are much rarer than those of the $2\nu\beta\beta$-decay process in most cases. It is thus necessary to separate experimentally the $0\nu\beta\beta$-decay processes from the $2\nu\beta\beta$-decay process.

Geochemical methods count the number of the decay product isotopes in ores of DBD nuclei for a geological time process from the $2\nu\beta\beta$-decay rate by the direct counting method alone on $^{100}$Mo, where no geochemical method was made beforehand, was carried out by the ELEGANT group (Ejiri et al) [220, 221].

It is thus necessary to separate experimentally the $0\nu\beta\beta$-decay processes from the $2\nu\beta\beta$-decay process.

Direct counting methods have been extensively used for measuring various DBD processes. The $0\nu\beta\beta$-decay processes are identified from the sum energy spectrum of $E_{\beta\beta} = E(\beta_1) + E(\beta_2)$ for two $\beta$ rays, as shown in figure 2 of [30]. They show a sharp peak characteristic of the two-body kinematics at $E_{\beta\beta} \approx Q_{\beta\beta}$, while $2\nu\beta\beta$ shows a continuum spectrum characteristic of the four-body kinematics. Neutrinoless DBD processes are identified from the three-body kinematics.

The $0\nu\beta\beta$-decay processes due to the left-handed weak current and the right-handed current (RHC) are experimentally identified by measuring the energy and angular correlations of the two $\beta$ rays, as shown in figure 4 of [30].

The left-handed weak current $0\nu\beta\beta$ process includes several modes such as the light $v$ exchange, the heavy $v$ exchange, the SUSY particle exchange and others, as discussed in the previous sections. Relative contributions of these modes to the $0\nu\beta\beta$-decay rate may be investigated by observing several DBD isotopes as well as those for the ground and excited states, provided that the matrix elements are properly evaluated. Then experimental studies of several DBD isotopes are necessary.

8.2.2. Sensitivity of DBD experiments. DBD event rates are so low that DBD experiments are necessarily carried out using high-sensitivity detectors at low-background underground laboratories.

In the case of the $0\nu\beta\beta$-decay process with the light Majorana neutrino exchange, the transition rate $\Gamma_{0\nu\beta\beta}$ per 1 ton (t) of DBD isotopes is expressed in terms of the nuclear sensitivity $S_n$ and the effective mass of the light Majorana neutrinos of $|\langle m_1 | \rangle|$ as

$$\Gamma_{0\nu\beta\beta} = |\langle m_1 | \rangle|^2 S_n.$$  (128)

The nuclear sensitivity is written as

$$S_n = (78 \text{ meV})^{-1}|M_{0\nu\beta\beta}|^2 G_{0\nu\beta\beta}(0.01 A)^{-1},$$  (129)

where $M_{0\nu\beta\beta}$ is the NME, $A$ is the mass number, and $G_{0\nu\beta\beta}$ is the phase-space factor in units of $10^{-14} \text{ y}^{-1}$.

The mass sensitivity $|\langle m_1 | \rangle|$ is defined as the minimum mass to be measured by the $0\nu\beta\beta$-decay experiment. It is expressed in terms of the detector sensitivity $D$ as follows:

$$|\langle m_1 | \rangle| = S_n^{-1/2} D^{-1/2}, \quad D = (\epsilon N_T) (\delta)^{-1},$$  (130)

where $\epsilon$ is the $0\nu\beta\beta$ peak efficiency, $N$ is the number of the DBD isotopes in unit of ton, $T$ is the run time in unit of year and $\delta$ is the minimum number of counts required for the peak identification with 90% CL (confidence level). It is given as $\delta \approx 1.6 + 1.7 (BNT)^{1/2}$ with $B$ being the BG rate $\text{y}^{-1}$ and $\delta \approx 2.3$ for $BNT \geq 2$ and $\leq 2$, respectively.

The nuclear sensitivity $S_n$ is proportional to the phase-space factor $G_{0\nu\beta\beta}$ and $|M_{0\nu\beta\beta}|^2$. Thus DBD nuclei with large $G_{0\nu\beta\beta}$ and large $M_{0\nu\beta\beta}$ are selected for their high nuclear sensitivity. DBD detectors with a high efficiency $\epsilon$, a large amount ($N$) of isotopes, and a small BG rate ($B$) are used for the high detector sensitivity.

8.2.3. DBD detectors. Neutrinoless $\beta^+\beta^-$ decays of $(A, Z) \rightarrow (A, Z + 2)$ are studied by measuring two $\beta^+$ rays, while neutrinoless $\beta^+\beta^+$ decays of $(A, Z) \rightarrow (A, Z - 2)$ proceed through $\beta^+\beta^+$, $\beta^+\text{EC}$ and $\text{EC EC } \gamma$, where EC (electron capture) is detected by measuring the x-ray, and $\beta^+$ by measuring the $\beta^+$ and the two 511 keV annihilation $\gamma$ rays.

High-sensitivity DBD experiments require DBD isotopes with high nuclear sensitivity $S_n$, i.e. the large $Q_{\beta\beta}$ and the large phase-space factor $G_{0\nu\beta\beta}$, as given by equation (129).

There are several such DBD isotopes of $\beta^+\beta^-$ decays. However, no $\beta^+\beta^+$ nuclei with large $S_n$ are available, although BG rates for the $\beta^+\beta^+\text{EC}$ are quite small by measuring both $\beta^+$ and the annihilation $\gamma$ rays and/or the x-ray. Accordingly, most high-sensitivity DBD experiments are concentrated on the $0\nu\beta^\beta$ decays with the large $Q_{\beta\beta}$ and $G_{0\nu\beta\beta}$. Thus hereafter, we discuss mainly $\beta^+\beta^-$ decays. The large $Q_{\beta\beta} \approx 3 \text{ MeV}$ helps reduce BG rates since most BGs from natural RI are below 3 MeV.

The $0\nu\beta^\beta$-$\beta^-$ experiments with the IH (30 meV) mass sensitivity are carried out using low-BG ($B \approx 1 \text{t}^{-1} \text{y}^{-1}$) detectors and ton-scale ($N \approx 0.5-1$) DBD isotopes with the large NME of $|M_{0\nu\beta\beta}| \approx 3$ (see table 3) and the large phase-space factor of $G_{0\nu\beta\beta} \approx 5$ in units of $10^{-14} \text{y}^{-1}$ for a long ($T \approx 2-4$ year) run time. On the other hand one needs DBD isotopes of around $N \approx 50-100$ ton and ultra-low-BG detectors with $B \approx 0.01 \text{t}^{-1} \text{y}^{-1}$ to reach the NH (2-4 meV) mass sensitivity.

Possible DBD isotopes to be used for high-sensitivity $0\nu\beta^\beta-\beta^-$ experiments are given in table 1. Among them, $^{82}$Se,
have been carried out on several isotopes such as 48Ca, 76Ge, 116Cd, and 130Te. Among them, 76Ge experiments (Heidelberg–Moscow, IGEX) with Ge semiconductors [36, 227–230] and the 130Te experiment (CUORICINO) with 41 kg TeO2: cryogenic bolometers [231, 232] have good energy-resolution and give stringent limits on the absolute value of effective Majorana neutrino mass of the order of 0.3–0.5 eV. Currently, the most stringent limit on $|m_\nu|$ comes from the lower limit on the $T_{1/2}^{\beta\beta}(136Xe)$ measured in KamLAND-Zen experiment [40] (see table 1).

Spectroscopic detectors (ELEGANT V, NEMO III) have been used for isotopes such as 82Se, 100Mo, 116Cd and other isotopes with large $Q_{\beta\beta}$ values [38, 233–235]. They are $\beta$-ray tracking detectors with $\beta\beta$ sources separated from detectors. NEMO III provides stringent limits on the $0\nu\beta\beta$ half-lives for 82Se, 100Mo, and other isotopes [38, 234, 235].

Recently, a claim for the $0\nu\beta\beta$-decay peak, corresponding to the effective Majorana neutrino mass of 0.32 eV, was made by part of the Heidelberg–Moscow collaboration [41, 42]. The result depends on the off-line analysis method. It should be checked by future GERDA/MAJORANA experiments with lower BG rates.

Neutrino-mass sensitivities of the CUORICINO and NEMO III detectors are limited to around 300–500 meV because of the limited $\beta\beta$ isotopes of 11–7 kg and the large BG rates. Thus future experiments with higher mass sensitivity are necessary to prove/disprove the Heidelberg–Moscow claim and to search for the Majorana neutrino in the lower mass regions.

The neutrino oscillation studies have given strong impact to high-sensitivity studies of $\beta\beta$ experiments since the effective mass suggested is of the order of $\sqrt{\delta m^2} \sim 2–50$ meV, which next-generation $\beta\beta$ detectors can access if the $\nu\nu$ are Majorana particles. Future experiments with higher mass sensitivities are in progress (see below).

8.3.2. Two-neutrino DBDs. The $2\nu\beta\beta$-decay is a process fully consistent with the SM of electroweak interaction. The inverse half-life of $2\nu\beta\beta$-decay is free of unknown parameters on the particle physics. It can be expressed as a product of an accurately known phase-space factor $G^{2\nu}(E_0, Z)$, which includes the fourth power of $g_A$, and the double Gamow–Teller transition matrix element $M^{2\nu}(A, Z)$, which is a quantity of second order in the perturbation theory:

$$\left(T_{1/2}^{2\nu}\right)^{-1} = G^{2\nu}(E_0, Z) |M^{2\nu}(A, Z)|^2.$$  
(131)

Here $M^{2\nu}$ includes nuclear effects due to the nuclear residual interactions, the nuclear medium and the nuclear quenching. It is obtained from the experimental decay rate. Experimental studies of $2\nu\beta\beta$-decay half-lives have been made on some nuclei by geochemical methods, and several nuclei by direct counting methods [241]. The $2\nu\beta\beta$-decay has been observed so far in twelve nuclides ($^{48}$Ca, $^{76}$Ge $^{82}$Se, $^{96}$Zr, $^{100}$Mo, $^{116}$Cd, $^{128}$Te, $^{130}$Te, $^{138}$Ba, $^{136}$Xe, $^{150}$Nd and $^{238}$U) and in two excited states [241]. Recent NEMO III experiments provide high-statistic spectroscopic studies of the $2\nu\beta\beta$-decay rates [38, 240]. Spectroscopic measurements of two $\beta$ rays are
Table 1. Limits on neutrinoless DBDs $T_{1/2}^{0\nu–\exp}$ (claim for evidence is denoted in [42]). $Q_{\beta\beta}$: $Q$-value for the $0^+ \rightarrow 0^+$ ground-state transition. $G^{0\nu}$: kinematical (phase space volume) factor ($g_A = 1.25$ and $R = 1.2$ fm $A^{1/3}$). $(m_1)$: the upper limit on the effective Majorana neutrino mass, deduced from $T_{1/2}^{0\nu–exp}$ by assuming the ISM [236] $(g_{A}^{\text{ISM}} = 1.25$, UCOM src), the EDF [131] $(g_{A}^{\text{EDF}} = 1.25$, UCOM src), the (R)QRPA $(1.00 \leq g_{A}^{\text{RQRPA}} \leq 1.25$, the modern self-consistent treatment of src), and the IBM-2 [130] $(1.00 \leq g_{A}^{\text{IB}} \leq 1.25$, Miller–Spencer src), NMEs (see section 10). src means short-range correlations.

| Isotope | $A$ (%) | $Q_{\beta\beta}$ (MeV) | $G^{0\nu}$ ($10^{-14}$ y) | $T_{1/2}^{0\nu–exp}$ ($10^{24}$ y) | NME (eV) | $|\langle m_1 \rangle|_{\text{eV}}$ | Future experiments |
|---------|---------|------------------------|---------------------------|------------------------------------|----------|---------------------|------------------|
| $^{48}$Ca | 0.19 | 4.276 | 7.15 | 0.014 [237] | ISM | 19.1 | CANDLES |
| $^{76}$Ge | 7.8 | 2.039 | 0.71 | 19 [36, 227, 228] | ISM, EDF | 0.51, 0.31 | GERDA |
| $^{76}$Ge | 7.8 | 2.039 | 0.71 | 22 [42] | ISM, EDF | 0.47, 0.29 | — |
| $^{76}$Ge | 7.8 | 2.039 | 0.71 | 16 [229, 230] | ISM, EDF | 0.55, 0.34 | MAJORANA |
| $^{82}$Se | 9.2 | 2.992 | 3.11 | 0.36 [38, 234, 235] | ISM, EDF | 1.88, 1.17 | SuperNEMO |
| $^{100}$Mo | 9.6 | 3.034 | 5.03 | 1.0 [38, 234] | EDF | 0.46 | MOON |
| $^{116}$Cd | 7.5 | 2.804 | 5.44 | 0.17 [238] | EDF | 0.46 | MOON |
| $^{130}$Te | 34.5 | 2.529 | 4.89 | 3.0 [231, 232, 239] | EDF | 0.52, 0.27 | MAO-Re |
| $^{136}$Xe | 8.9 | 2.467 | 5.13 | 5.7 [40] | EDF | 0.44, 0.23 | COBRA |
| $^{150}$Nd | 5.6 | 3.368 | 23.2 | 0.018 [38, 240] | EDF | 4.68 | CUORE |

Table 2. The $2
\nu\beta\beta$ matrix elements $|M^{2\nu}|$ deduced from the measured half-life $T_{1/2}^{2\nu}$ [40, 241]. $g_A = 1.269$ is assumed.

| Nucleus | $T_{1/2}^{2\nu}$ years | $|M^{2\nu}|$ (MeV)$^{-1}$ |
|---------|----------------------|-------------------------|
| $^{48}$Ca | $4.4^{+0.6}_{-0.5} \times 10^{19}$ | $0.046^{+0.003}_{-0.003}$ |
| $^{76}$Ge | $1.5^{+0.1}_{-0.0} \times 10^{21}$ | $1.13^{+0.03}_{-0.02}$ |
| $^{82}$Se | $4.9^{+0.8}_{-0.7} \times 10^{19}$ | $0.099^{+0.004}_{-0.004}$ |
| $^{96}$Zr | $2.5^{+0.4}_{-0.3} \times 10^{19}$ | $0.091^{+0.004}_{-0.004}$ |
| $^{100}$Mo | $7.1^{+0.5}_{-0.4} \times 10^{18}$ | $0.23^{+0.004}_{-0.004}$ |
| $^{100}$Mo$^{\text{Mo*}}$ | $5.9^{+0.8}_{-0.6} \times 10^{20}$ | $0.31^{+0.01}_{-0.01}$ |
| $^{116}$Cd | $2.8^{+0.2}_{-0.1} \times 10^{19}$ | $0.128^{+0.003}_{-0.003}$ |
| $^{128}$Te | $1.9^{+0.2}_{-0.14} \times 10^{24}$ | $0.047^{+0.002}_{-0.002}$ |
| $^{150}$Te | $6.8^{+0.1}_{-0.1} \times 10^{20}$ | $0.034^{+0.001}_{-0.001}$ |
| $^{136}$Xe | $2.3^{+0.1}_{-0.1} \times 10^{21}$ | $0.018^{+0.001}_{-0.001}$ |
| $^{150}$Nd | $8.2^{+0.1}_{-0.1} \times 10^{18}$ | $0.061^{+0.002}_{-0.002}$ |
| $^{150}$Nd$^{\text{Nd*}}$ | $1.3^{+0.2}_{-0.1} \times 10^{20}$ | $0.045^{+0.003}_{-0.003}$ |

useful to reduce BG rates and energy correlations of two $\beta$ rays are used to identify the $2\nu\beta\beta$ mechanism.

The measurement of $2\nu\beta\beta$ decay rates gives us information about the product of the squared effective axial-vector coupling constant and $2\nu\beta\beta$-decay matrix elements. The values obtained by the counter experiments are presented in table 2. They are sensitive to nuclear-spin isospin correlations. The observed values for $M^{2\nu}$ are used to investigate the nuclear structure and the nuclear interactions associated with the $0\nu\beta\beta$-decays.

8.3.3. High-sensitivity experiments. In the case of the inverted mass hierarchy, $\beta\beta$ detectors with the IH mass sensitivity of $(m_\nu) \approx 20–50$ meV can be used to study the $0\nu\beta\beta$-decay, while in the case of the NH one needs higher sensitivity detectors with $(m_\nu) \approx 2–4$ meV. Several groups are now working for next-generation $\beta\beta$ experiments with the IH mass sensitivities of $20–50$ meV, as discussed in the reviews [30, 31, 183].

Experimental proposals for new experiments have been made on several $\beta\beta$ isotopes, as listed in table 1. They are mostly $0^+–0^+$ experiments because of large kinematical (phase-space) factors. DBD experiments with different isotopes and different methods are indispensable to confirm and identify the $0\nu\beta\beta$-decay event and the $0\nu\beta\beta$-decay mechanism. Some of them are briefly described below.

$^{76}$Ge experiments with low-BG high resolution $^{76}$Ge detectors are of special interest for proving or disproving the Heidelberg–Moscow claim of the large $0\nu\beta\beta$-decay peak [42], and for further high-sensitivity ton-scale experiments.

**GERDA**: this is aimed at high-energy resolution studies of $^{76}$Ge $0\nu\beta\beta$-decays using high-purity $^{76}$Ge detectors to check the Heidelberg–Moscow claim and the possible Majorana neutrinos in the QD region at LNGS (Gran Sasso). GERDA uses 18 kg $^{76}$Ge detectors in phase I, and adds 20 kg detectors in Phase II. The Ge detectors are immersed in high-purity liquid.
Ar in order to avoid BG contributions from cryostats [175].
GERDA is now running.

**MAJORANA**: the MAJORANA demonstrator uses 40 kg Ge detectors at the Sanford underground lab. To test/investigate the half-life of the Heidelberg–Moscow claim and the QD mass regions and to prove the feasibility for a future ton-scale IH (10^{27} y, 20–40 meV) experiment. The Ge detectors are PPC (P-type point contact) detectors with excellent PSA (pulse shape analysis). They are cooled using an ultra-pure electro-formed Cu cryostat [179]. The BG goal is 4 \times 10^{-1} y^{-1}, which scales to 1 \times 10^{-1} y^{-1} for the 1 ton experiment.

The enriched detectors will be on-line in 2013–2014.

These detectors can also be used to study DM and neutrino scattering in the low-energy region. The GERDA and the Majorana collaboration will be merged for one ton-scale future experiment by selecting the best techniques developed and tested by GERDA and MAJORANA. Recent developments are given in the report [242].

There are other experimental plans for QD masses. Among them, CANDLES is for 48Ca \beta\beta-decays with an array of CaF2 crystals [243], which is based on the ELEGANT VI experiment with CaF2. The Q_{\beta\beta} is large, but the natural abundance of 48Ca is only 0.2%. Thus efficient isotope enrichment is crucial.

Several groups are working for future experiments with IH mass sensitivities of around (m_\nu) \approx 20–50 meV. DBD isotopes required are those with large Q_{\beta\beta} = 2.5–3 MeV and G^{00} = 3–5 \times 10^{-14}/y to get large nuclear sensitivities of the order of (S_\text{N})^{-1/2} = 15–20 meV. The experiments use large-scale low-BG detectors with ton-scale enriched isotopes and B \approx 1 \times 10^{-1} y^{-1}.

**MOON (Molybdenum Observatory Of Neutrinos):** this is an extension of ELEGANT V [233]. It is a hybrid \beta\beta and solar \nu experiment with 10^4Mo to study the Majorana \nu masses with QD-IH mass sensitivities of 100–20 meV and low-energy solar vs [244–248]. DBDs to both the ground and the 1.132 MeV excited 0^+ states are studied to confirm the 0\nu\beta\beta events and to study the 0\nu\beta\beta-decay mechanism. MOON can be used for supernova neutrinos as well [249].

Detectors under consideration are the following: (A) the super-module of PL plate and fiber scintillators [245–247] and (B) the cryogenic bolometer of ZnMoO4. The PL scintillator module (A) is used for spectroscopic study of two \beta\beta-ray energy and angular correlations to identify the 0\nu\beta\beta-decay process. Here one module consists of a plate (PL) scintillator for the \beta energy and two sets of X–Y fiber scintillator planes for the vertex identification, between which a thin 10^4Mo film is interleaved. The energy resolution is \sigma \approx 2.2% at E = Q_{\beta\beta} to reduce the 2\nu\beta\beta-decay tail in the 0\nu\beta\beta-decay window. The half-life (mass) sensitivity is 3 \times 10^{20} y (45 meV) with 480 kg 10^4Mo for 5 years. Prototype detectors were built to show the energy resolution as required [245, 246].

The ZnMoO4 bolometer (B), which is under discussion with the Milano–Rome group, is for calorimetric study of the sum of two \beta-ray energy. The high-energy resolution (\Delta E \approx 5 keV), the particle identification by scintillation and/or pulse-shape analysis and the high efficiency (\epsilon \approx 0.8) make high-sensitivity study possible. Thus it is good to start with the detector B, and proceed to A to confirm the 0\nu\beta\beta-decay process by two \beta-ray measurement. The half-life (mass) sensitivities are \times 10^{20} y (120 meV) for 3 y run with 12 kg 10^4Mo and 7 \times 10^{20} y (30 meV) for 5 y run with 220 kg 10^4Mo.

**SuperNEMO**: the goal of SuperNEMO is to reach a sensitivity of 10^{26} y, which corresponds to the IH mass of 40–110 meV [250]. The detector consists of huge tracking chambers and scintillation detectors with 100 kg of \beta\beta isotopes of 150Nd or 82Se to search for the \nu mass below 0.1 eV. The efficiency is 30% and the resolution is 4% in FWHM. The BG impurities are reduced to be 208Tl < 2 and 214Bi < 10 in units of \mu Bq/kg. It plans to use 20 modules, each module with 5 kg \beta\beta isotopes. The first module is a demonstrator with 7 kg of 82Se. The mass sensitivity of the demonstrator with 15 kg y is 210–570 meV, while that of the full detector array with 500 kg y is 53–145 meV, depending on the NME. It is based on NEMOIII, and thus it is crucial to improve the energy resolution and the efficiency.

**AMoRE (Advanced Molybdenum-based Rare Process Experiment)**: large volume CaMoO4 crystals with enriched material have been developed to study the 0\nu\beta\beta-decays of 10^4Mo and to search for cold dark matter [251]. Pilot experiments of 1 kg with scintillation technique and Cs(I) active veto are in preparation. In order to improve the energy resolution, cryogenic CaMoO4 detectors are being developed. To avoid BGs from the 2\nu\beta\beta-decays of 48Ca, depletion of Ca in 48Ca \lesssim 0.001% is made using ALSIS (advanced laser stable isotope separation). Additional light sensor and time constant of phonon signal are effective to select signals. The goal of AMoRE is to study 0\nu\beta\beta decays of 10^4Mo in the region of IH mass of 50 meV (3 \times 10^{26} y) by using 100 kg CaMoO4 cryogenic detectors.

**COBRA**: this uses a large number of high-energy resolution CdZnTe semiconductors at room temperature [252]. The modular design makes coincidence measurements possible to reduce BG rates. The crystal includes several \beta\beta isotopes to be studied. The collaboration now tests 64 CZT 1 cm^3 detectors at LNGS. The goal is to study the Majorana neutrino in the IH mass region by measuring the 0\nu\beta\beta from 116Cd with Q_{\beta\beta} = 2.804 MeV. The detector is composed of 64 K crystals with 0.42 ton Cd isotopes enriched in 116Cd. Reduction of BGs from RI impurities inside and around detectors are important. Pixelisation (semiconductor tracker) could be a major step forward.

**CUORE (Cryogenic Underground Observatory for Rare Event)**: this is an expansion of CUORICINO. It is a high-energy-resolution bolometer array to measure the 130Te 0\nu\beta\beta decays with Q = 2.529 MeV at LNGS [239]. It uses natural TeO2 crystals with the natural 130Te abundance of 34%. It consists of 988 TeO2 crystals with the net 130Te mass of 203 kg. The detector array has been under construction since 2005.

The experiment will be 20 times more massive than CUORETINO, have better energy resolution of 5 keV, high granularity and thus low-BG rates. The CUORICINO BG rate, which is around 0.16 keV^{-1} kg^{-1} y^{-1} \times 10^25/5 keV^{-1} y^{-1} ton^{-1} of 130Te, is expected to be reduced to 0.02–0.01 keV^{-1} kg^{-1} y^{-1} in CUORE. Then, in cases of the BG rates of B = 0.01–0.001 keV^{-1} kg^{-1} y^{-1} \times 2 \times 10^25/2 \times 2 \times 10^25/5 keV^{-1} y^{-1} ton^{-1} of 130Te), the half-life and the \nu
mass sensitivities are expected to be around 2.1–6.5 × 10^{26} y and 50–25 meV, which depend on the NME. The first CUORE tower is CUORE-0.

**EXO (Enriched Xenon Observatory):** the $\beta\beta$ experiment of $^{136}$Xe with $Q = 2.467$ MeV [253] is made using the laser tagging technique to select the residual nuclei of $^{136}$Ba to suppress all kinds of RI BGs. The energy resolution of around $\sigma \sim 2\%$ is achieved by measuring both the ionization and scintillation signals. The 1 ton enriched Xe detector with the energy resolution of $\sigma = 1.6\%$ gives the $\nu$-mass sensitivity of 50–70 meV for a 5 y run. The 10 ton Xe detector with the improved energy resolution of $\sigma = 1\%$ will give a sensitivity of 11–15 meV. The 200 kg $^{136}$Xe liquid detector is used at WIPP to study the $2\nu\beta\beta$-decay and the quasi-degenerate $\nu$-mass, as the first step without the Ba tagging. EXO observed the $2\nu\beta\beta$ half-life of 2.1 × 10^{21} [254]. The key point of this experiment is the tagging efficiency of the $^{136}$Ba nuclei.

**KamLAND-Zen (Kamioka Large Anti-Neutrino Detector Zenon):** this studies the $^{136}$Xe DBD by means of the KamLAND detector with the 1 kton liquid scintillator at Kamioka [40, 255]. A mini balloon 3.2 m in diameter is set at the center for the $^{136}$Xe-loaded liquid scintillator. It includes $^{136}$Xe isotopes around 400 kg. The energy resolution and the vertex resolution are 6.8%/√E and 12.5 cm/√E. The collaboration measured the $2\nu\beta\beta$ half-life [40], and hopes to reach a sensitivity of around 50 meV.

**NEXT (Neutrino Experiment with a Xe TPC):** A Xe TPC with 100–150 kg enriched $^{136}$Xe is used at LSC [256]. It is a low BG and good E-resolution TPC with separate readout planes for tracking and energy. The NEXT-100 sensitivity for a 5 y run is about 5.9 × 10^{15} y (better than 100 meV).

**Borexino with $^{136}$Xe:** The sensitivity with 2 ton $^{136}$Xe is about 100 meV [257].

**DCBA (Drift Chamber Beta-ray Observatory):** this uses a tracking chamber in a magnetic field to study $^{150}$Nd $\beta\beta$ decays [258]. The $\beta$ energy is obtained by the $\beta$-ray trajectories. DCBA-T3 is now under construction. Good energy resolution and efficient enrichment of $^{150}$Nd isotopes are necessary.

**SNO+ (Sudbury Neutrino Observatory +):** this uses the 1 kton scintillation detector with 0.1% Nd isotopes to study QD-IH $\nu$ masses using natural (5.6%) and enriched (50%) $^{150}$Nd isotopes [259]. The mass sensitivities are 100 meV with the natural (5.6% $^{150}$Nd 56 kg) and 40 meV with enriched isotopes (50% enriched $^{150}$Nd 500 kg). The collaboration is trying to find a realistic way of Nd isotope separation, which is of great interest to study the HI neutrino mass. It aims at the scintillator filling at the beginning of 2013.

**DBD experiments for NH mass:** higher sensitivity DBD experiments for NH masses of $\langle m_\nu \rangle = 2–4$ meV require a large amount of high nuclear-sensitivity ($S_N \approx 15$ meV) DBD isotopes of the order of $\approx 10–50$ tons, and extremely low-BG detectors with $B \approx 0.1–0.01$ t^{-1} y^{-1}. DBD isotopes to be studied are $^{82}$Se, $^{100}$Mo and $^{136}$Xe. High-energy-resolution cryogenic detectors such as ZnSe, ZnMoO$_4$ with pulse shape analysis and/or scintillation signals [223, 224] may be used to search for the NH masses by $0\nu\beta\beta$-decay experiments on $^{82}$Se and $^{100}$Mo.

### 8.4. Experimental studies of DBD matrix elements

#### 8.4.1. Experimental probes for DBD matrix elements

NMEs ($M^{\nu\beta\beta}$) for $0\nu\beta\beta$-decay are crucial for extracting the effective Majorana $\nu$ mass and other parameters, relevant to particle physics models beyond the SM, from $0\nu\beta\beta$ experiments, while NMEs ($M^{\nu\beta\beta}$) for $2\nu\beta\beta$-decay can be derived experimentally from the observed $2\nu\beta\beta$-decay half-lives. Extensive calculations of $M^{\nu\beta\beta}$ have been made in terms of QRPA, RQRPA, shell model and so on, as given elsewhere in the theoretical sections.

Most $\beta\beta$ strengths are located in $\beta\beta$ (double Gamow–Teller) giant resonances, i.e. in the high-excitation region [132]. Thus the $\beta\beta$ matrix elements get very small in comparison with single-particle estimates and are sensitive to nuclear structures, nuclear spin–isospin correlations, nuclear deformations, nuclear interactions, nuclear medium effects on the weak coupling constant $g_A$ and others. The theoretical evaluations for them are hard. Experimental studies of nuclear structures and nuclear interactions, which are relevant to $2\nu\beta\beta$- and $0\nu\beta\beta$-decay matrix elements, are very interesting to get reliable evaluations for them [30, 49, 260, 261].

NMEs of $M^{\nu\nu\tau\tau}$ and $M^{\nu\nu\mu\mu}$ are expressed in terms of the successive single $\beta$ processes through intermediate ($J^P$) states. Among the intermediate states, low-lying single-particle–hole states play dominant roles [260, 262]. The single $\beta$ matrix elements are given by spin–isospin responses for $Q_{TSLJ} = \tau^i [i^J r Y_L \times \sigma^s]$. They are studied experimentally using hadron, photon and lepton probes, as shown in figure 20.

**Lepton probes.** Nuclear weak responses for low-lying intermediate states are obtained from single $\beta$ decay rates and EC rates. However, they are limited to $\beta^\pm$ decays from the ground state in the intermediate nucleus. The decays are mostly allowed Gamow–Teller (GT) decays with $\tau^\pm \sigma$, and are first forbidden decays with $\tau^\pm ir Y_1$ and $\tau^\pm [i^J r Y_L \times \sigma^s]$ in some nuclei.

Muon capture reactions of ($\mu^-, \nu_\mu$) are used to get the $\beta^+$ strengths in the intermediate nucleus [261, 263]. Excitation energies and angular momenta involved in this reaction are $E = 0–50$ MeV and $0^+$, $1^+$ and $2^+$. The capture cross section is quite large, and most muons are stopped and captured in the case of medium heavy nuclei. Then the $\tau^+$ weak strength distribution in the intermediate nucleus is derived by measuring the decaying neutrons and $\gamma$-rays from excited states and radioactive isotopes produced by the muon capture.

One direct way to get the weak responses is to use $\nu$ beams. Since $\nu$ nuclear cross sections are as small as $\sigma = 10^{-40}–12$ cm$^2$, one needs high-flux $\nu$ beams and large detectors [30, 49, 132]. Low-energy $\nu$ beams with $E \leq 100$ MeV can be obtained from pion decays. Intense pions are produced by nuclear interaction with GeV protons. Weak decays of stopped $\pi^+$ give low-energy neutrinos as

$$\pi^+ \rightarrow \mu^+ + \nu_\mu, \quad \mu^+ \rightarrow e^- + \bar{\nu}_e + e^+, \quad (132)$$

where $\nu_\mu$ and $\bar{\nu}_e$ are well separated by the decay time. Intense 1 GeV protons from SNS at ORNL provide intense neutrinos around 10^{15} per second [264] and the J-PARC booster synchrotron with 3 GeV protons produces neutrinos around $3 \times 10^{14}$ per second [265].
Nuclear spin isospin responses for weak interactions. They are studied by $\nu$ probes via weak interactions, EM ($\gamma$) probes via EM interactions and by nuclear probes via strong interactions [49, 30, 132].

**Figure 20.** Nuclear spin isospin responses for weak interactions. They are studied by $\nu$ probes via weak interactions, EM ($\gamma$) probes via EM interactions and by nuclear probes via strong interactions [49, 30, 132].

### Photon probes

Weak responses for $\beta^+ \rightarrow e^+ + \nu_e$ decays are studied using photo-nuclear ($\gamma$, X) reactions through isobaric analog states (IASs) as shown for the first time by $(p, \gamma)$ reactions [266, 267]. The $\beta$ and $\gamma$ matrix elements are related as

$$\langle f|g_\nu m^\beta|i\rangle \approx \frac{g_\nu}{e} (2T)^{1/2} \langle f|e m^\nu|\text{IAS}\rangle,$$

where $|\text{IAS}\rangle = (2T)^{-1/2} |T^{-1}\rangle$, $T$ is the isospin of the parent nucleus and $m^\beta$ and $m^\nu$ are analogous $\beta$ and $\gamma$ transition operators. Thus one can obtain the $\beta$ matrix element for $|i\rangle \rightarrow |f\rangle$ by observing the analogous $\gamma$ absorption $|f\rangle \rightarrow |\text{IAS}\rangle$ through the IAS of $|i\rangle$, where $|f\rangle$ and $|i\rangle$ are the final state and the intermediate state in the $\beta^+$-decay, as shown in figure 21. These photo-nuclear reactions through IAS are used to get the $\beta^+$ matrix elements to excited states in the intermediate nucleus.

In medium heavy nuclei, IAS is located on the E1 giant resonance (GR) in the high-excitation region. Accordingly IAS shows up as a sharp isobaric analog resonance (IAR), and the photo-nuclear reaction includes IAR, GR and the interference term as given by [268]

$$\frac{d\sigma(\gamma, N)}{d\Omega} = k|A(I)J_f|^2 + \Sigma_j |A(G)J_j|^2 + 2Re(A(I)J_f A(G)J_j \phi)\rangle,$$

where $A(I)J_f$, $A(G)J_j$ and $\phi$ are the IAR and GR amplitudes and the relative phase at IAR. Then one can get the phase of the matrix element from the interference.

Laser electron photons, which are obtained from laser photons scattered off GeV electrons, are used for the photo-nuclear reaction. The polarization of the photon can be used to study E1 and M1 matrix elements separately [261]. High-intensity $\gamma$-ray sources, new SUBARU and other electron synchrotrons with $E_e = 1–3 \text{ GeV}$ provide laser electron photons used for the photo-nuclear reaction.

### Nucleon and nuclear probes

Nuclear charge-exchange reactions with nuclear (hadron) probes are used to study nuclear spin–isospin responses. Charge-exchange spin–nonflip reactions are used for vector weak responses, while charge-exchange spin–flip reactions are used for axial-vector weak responses [30, 260]. Extensive studies of charge exchange reactions have been made to get GT(1 +) responses with $\tau^\pm \sigma$. The reactions studied are $(p, n)$ (n, p), $(d, 2\text{He})$, $(3\text{He}, t)$ (t, 3He) and $(7\text{Li}, 7\text{Be})$ at IUCF, KVI, MSU, RCNP, Triumf and others [269–280]. Medium-energy projectiles with $E_i/A = 0.1–0.3 \text{ GeV}$ are used for studying $\tau_\sigma$ responses because of the relatively large spin–isospin interaction ($V_{\tau_\sigma}$) and the small distortion interaction ($V_0$) at medium energy.

The charge-exchange reaction with medium-energy projectiles is mainly due to the central isospin and spin–isospin interactions. Then the cross section with the transferred momentum ($q$) and energy ($\omega$) is given as

$$\sigma(q, \omega) = K(E_i, \omega) \exp(-\frac{1}{2}q^2\langle r^2\rangle)N^D_{\alpha}|J_\alpha|^2 B(\alpha),$$

where $K(E_i, \omega)$, $N^D_{\alpha}$, $J_\alpha$, and $B(\alpha)$ are the kinematical factor, the nuclear distortion factor, the volume integral of the spin–isospin interaction and the nuclear spin–isospin response,
respectively. \(\alpha\) denotes the isospin and spin channel; \(\alpha = F\) for isospin Fermi and \(\alpha = GT\) for spin–isospin GT.

The cross section at \(0^\circ\) for \(q \approx 0\) and \(\omega \approx 0\) is corrected for the kinematical and normalization factors, and is expressed as

\[
\frac{d\sigma_\omega(0)}{d\Omega} = \frac{1}{K(E_\omega, 0)N^D_a} |J_\omega|^2 B(\alpha), \quad \alpha = F, \ GT. \quad (136)
\]

The proportionality of the cross section at the forward angle of \(\theta \sim 0^\circ\) to the \(\tau\) response \(B(\omega)\) has been studied for charge-exchange reactions with medium-energy light projectiles. In fact, the proportionality is good for spin–flip reactions with \(B(\text{GT}) \geq 0.1\), but some deviation from the proportionality is found in the reactions with smaller \(B(\text{GT})\) due to contributions from tensor/non-central interactions.

Charge-exchange \((^3\text{He}, t)\) reactions (ChER) relevant to the \(\beta\beta\)-decays were studied at RCNP using the 420 MeV \(^3\text{He}\) beam and the high-energy resolution beam line and spectrometer system [269, 270, 274–280]. The beam stability and the beam line system have been improved to give the fantastic energy-resolution of \(\Delta E/E \approx 5 \times 10^{-5}\) and \(\Delta E \approx 25\) keV, and ChERs have been studied for several \(\beta\beta\) nuclei.

The GT strengths are mostly located at the GR resonances in the high-excitation region, and accordingly, the strengths for the low-lying states are small. In the cases of \(^{96}\text{Zr}, ^{100}\text{Mo}\) and \(^{116}\text{Cd}\), where valence neutrons and valence protons are in different major shells, there is only one strong GT state (ground state) of \((g7/2)^n(g9/2)^p\). Thus the \((^3\text{He}, t)\) reaction shows a strong major shell effect, i.e., there is only one strong GT state (ground state). The SSD hypothesis looks fine in the case of nuclei such as \(^{100}\text{Mo}\), where there is only one \(J^\pi = 1^+\) state.

Recently the NMEs \(M_{GT}^{(1)}\) have been shown to be expressed in terms of single \(\beta\) matrix elements via Fermi-surface quasi particle states (FSQP) [262, 283, 284],

\[
M_{GT}^{(1)} = \sum_{k}\frac{M(k)(\Delta(k))^{-1}}{M_{GT}^{(1)}(k)}, \quad M(k) = M_{GT}^{(1)}(k)M_{GT}^{(-1)}(k), \quad (138)
\]

where \(\sum\) is the over the FSQP (low-lying) states \(k\) in the intermediate nucleus, \(\Delta(k)\) is the energy denominator and the matrix elements of \(M_{GT}^{(1)}(k)\) and \(M_{GT}^{(-1)}(k)\) are the experimental single GT matrix elements deduced from charge-exchange reactions, \(\beta\) decay rates and EC rates. Thus it includes no adjustable parameters. The possible deviation of \(g_A\) from 1.26 is embedded in the observed matrix elements of \(M_{GT}^{(2)}\) and similarly in \(M_{GT}^{(1)}(k)\) and \(M_{GT}^{(-1)}(k)\).

In the quasiparticle representation, the experimental matrix elements are given by

\[
M_{GT}^{(1)}(k) = k_A^{\text{eff}} m(j_k, J_k)P_i(k), \quad M_{GT}^{(1)}(k) = k_A^{\text{eff}} m(j_k, J_k)P_f(k), \quad (139)
\]

where \(m(j_k, J_k)\) is the single-particle matrix element for the GT transition between single particle \(j_k\) and \(J_k\) states with \(j_k\) and \(J_k\) being the neutron and proton spins, and \(P_i(k) = U_P(j_k)U_A(j_k)\) and \(P_f(k) = U_P(j_k)U_A(j_k)\) are the pairing reduction factors for transitions from the initial ground state to the \(4\)th intermediate state and from this state to the final state, respectively. The effective coupling constants, \(k_A^{\text{eff}}\) and \(k_A^{\text{eff}}\), in units of \(g_A\), represent the nuclear core effects such as the spin–isospin correlations, the short-range correlations and the nucleus, in addition to the quenching effect, while the nuclear surface (shell) effects are given by the pairing factors of \(P_i(k)\) and \(P_f(k)\). In fact, values for \(k_A^{\text{eff}}\) and \(k_A^{\text{eff}}\) do not depend much on individual states, and thus one can evaluate the single \(\beta\) matrix elements, if not available experimentally, using the \(k_A^{\text{eff}}\) and \(k_A^{\text{eff}}\) for other states in neighboring nuclei and the calculated values for \(m(j_k, J_k)P_i(j_f)\).

As shown in figure 22, the observed \(2\nu\beta\beta\)-decay matrix elements are well reproduced by the sum of the matrix elements through the low-lying FSQP \(1^+\) states in intermediate nuclei.

In general there are several low-lying \(1^+\) states, and thus \(2\nu\beta\beta\)-decay proceeds not only through the lowest state but also other FSQP states. The product of the matrix elements is \(M_{GT}^{(1)}(k)M_{GT}^{(-1)}(k) = k_A^{\text{eff}} k_A^{\text{eff}} m(j_k, J_k)P_i(k)P_f(k)\), which is positive. Thus contributions from the FSQP states do not cancel each other, but are constructive.
The \( 0\nu\beta\beta \) transition operator is a two-body operator. It is shown that the matrix element is approximately given by the sum of the matrix elements for successive processes via intermediate states \([30, 132]\). Then the matrix elements for the excited 0\(^+\) state in \(^{100}\)Ru are plotted at \( A = 102 \) \([283, 284]\).

Since the \( 0\nu\beta\beta \)-decay process is a virtual neutrino exchange between two nucleons in the nucleus, intermediate states with higher spins are involved. They are studied by measuring angular distributions of charge-exchange reactions to intermediate \( 1^\pm \), \( 2^\pm \) states and photo-excitations to IAS of the intermediate \( 1^\pm \) states \([261]\). Charge-exchange reactions and photo-excitations of IAS are under progress by Muenster, MSU, NC, RCNP and other groups to study nuclear structures relevant to \( 2\nu\beta\beta \) and \( 0\nu\beta\beta \) processes.

**8.5. Two-neutrino DBD and bosonic neutrino**

Neutrinos may possibly violate the spin-statistics theorem, and hence obey Bose statistics or mixed statistics despite having spin half. A violation of the spin-statistics relation for neutrinos would lead to a number of observable effects in cosmology and astrophysics. In particular, bosonic neutrinos might compose all or part of the cold cosmological dark matter (through bosonic condensate of neutrinos) and simultaneously provide some hot dark matter. A change of neutrino statistics would have an impact on the evolution of supernovae and on the spectra of supernova neutrinos. The idea of bosonic neutrinos has been proposed independently in \([285]\), where cosmological and astrophysical consequences of this hypothesis have been studied.

If neutrinos obey at least partly the Bose–Einstein statistics, the Pauli exclusion principle (PEP) is violated for neutrinos. The parameter \( \sin^2\chi \) can be introduced to characterize the bosonic (symmetric) fraction of the neutrino wave function \([286]\). A smooth change of \( \sin^2\chi \) from 0 to 1 transforms fermionic neutrinos into bosonic ones. The assumption of violation of the PEP leads to a number of fundamental problems, which include loss of positive definiteness of energy, violation of the CPT invariance and, possibly, of the Lorentz invariance as well as of the unitarity of the S-matrix. No satisfactory and consistent mechanism for the Pauli exclusion principle violation has been proposed so far.

The LN conserving \( 2\nu\beta\beta \)-decay can be used to study the statistical properties of neutrinos \([286]\). In the presence of bosonic neutrinos two contributions to the amplitude of the decay from diagram with permuted neutrino momenta have relative plus sign instead of minus in the Fermi–Dirac case.

The PEP violation strongly changes the rates of the \( 2\nu\beta\beta \)-decays and modifies the energy and angular distributions of the emitted electrons. The effect of bosonic neutrinos is different for transitions to 0\(^+\) ground states and 2\(^+\) excited states. In figure 23 the energy spectra of two electrons for different values of the bosonic-fraction \( \sin^2\chi \) is presented for the \( 2\nu\beta\beta \)-decay of \(^{100}\)Mo to ground state of final nucleus. With increase of \( \sin^2\chi \) the spectra shift to smaller energies. We note that substantial shift occurs only when \( \sin^2\chi \) is close to 1.0. Pure bosonic neutrinos are excluded by the present data \([286]\). In the case of partly bosonic (or mixed-statistics) neutrinos the analysis of the existing data allows one to put the conservative upper bound \( \sin^2\chi < 0.6 \) \([286]\).

**9. Effective transition operators**

The subject of interest is the LNV parameters associated with the exchange of light and heavy Majorana neutrinos and with \( R \)-parity breaking SUSY mechanisms.

By assuming the dominance of a single mechanism determined by the LNV parameter \( \eta_{\chi} \) the inverse value of the \( 0\nu\beta\beta \)-decay half-life for a given isotope \((A, Z)\) can be
written as

\[ \frac{1}{T_{0+}} = |\eta_0|^2 |M_{0+}^{\nu}|^2 G_0^{\nu}(E_0, Z). \]  

(140)

Here, \( G_0^{\nu}(E_0, Z) \) and \( M_{0+}^{\nu} \) are, respectively, the known phase-space factor (\( E_0 \) is the energy release) and the NME, which depends on the nuclear structure of the particular isotopes \((A, Z)\), \((A, Z + 1)\) and \((A, Z + 2)\) under study.

The phase-space factor \( G_0^{\nu}(E_0, Z) \) includes the fourth power of unquenched axial-vector coupling constant \( g_A \) and the inverse square of the nuclear radius \( R^{-2} \), compensated by the factor \( R \) in \( M_{0+}^{\nu} \). The assumed value of the nuclear radius is \( R = r_0 A^{1/3} \) with \( r_0 = 1.1 \) fm or \( r_0 = 1.2 \) fm in different publications. The implicit radius and \( g_A \) dependences in \( G_0^{\nu}(E_0, Z) \) and NME and the problem of the correct use of them were discussed in [289].

The NME \( M_{0+}^{\nu} \) is defined as

\[ M_{0+}^{\nu} = \left( \frac{g_A^{\text{eff}}}{g_A} \right)^2 M_{0+}^{\nu}. \]  

(141)

Here, \( g_A^{\text{eff}} \) is the quenched axial-vector coupling constant.

This definition of \( M_{0+}^{\nu} \) [290, 291] allows one to display the effects of uncertainties in \( g_A^{\text{eff}} \) and to use the same phase factor \( G_0^{\nu}(E_0, Z) \) when calculating the \( 0\nu\beta\beta \) decay rate.

Before we proceed with the discussion of the NMEs we will summarize the various types of transition operators entering the neutrinoless DBD process. We recall that in the various particle models the lightest particle exchanged between the two nucleons participating in this process is either much lighter than the electron or much heavier than the proton. It is thus possible to separate the particle physics parameters from those of nuclear physics. Furthermore, the nature of this exchanged particle dictates the form of the transition operators. The LNV parameters of interest together with corresponding NMEs are briefly presented below.

9.1. Transition operators resulting from light neutrino exchange

In the case of light neutrino-mass mechanism of the \( 0\nu\beta\beta \) decay we have

\[ \eta_0 = \left| \frac{m_\alpha}{m_e} \right|^2. \]  

(142)

The NME associated with the light Majorana neutrino exchange \( M_{0+}^{\nu} \) consists of the Fermi (F), Gamow–Teller (GT) and tensor (T) parts as

\[ M_{0+}^{\nu} = -M_{0+}^{\nu}\left( \frac{g_A^{\text{eff}}}{g_A} \right)^2 + M_{0+}^{\nu} - M_{0+}^{\nu}. \]

(143)

where

\[ S_{kl} = \frac{3}{2}(\vec{\alpha}_k \cdot \vec{r}_{kl})(\vec{r}_l \cdot \hat{e}_k) - \sigma_{kl}, \quad \sigma_{kl} = \vec{\alpha}_k \cdot \vec{\sigma}_l. \]  

(144)

The radial parts of the exchange potentials are

\[ H_{F,GT,T}(r_{kl}) = \frac{2}{\pi} R \int_0^{\infty} J_{0,0,2}(q r_{kl}) h_{F,GT,T}(q^2) dq. \]  

(145)

where \( R \) is the nuclear radius and \( \bar{E} \) is the average energy of the virtual intermediate states used in the closure approximation. The closure approximation is adopted in the calculation of the NMEs relevant for \( 0\nu\beta\beta \)-decay with the exception of the QRPA. The functions \( h_{F,GT,T}(q^2) \) are given by [292]

\[ h_F(q^2) = \frac{2}{3} f_2(q^2) \left( \frac{g_A}{g_A^{\text{eff}}} \right)^2 \frac{q^2}{4 m_p^2} \]  

(146)

Here, \( f_2(q^2) \) is the usual dipole approximation is adopted:

\[ f_2(q^2) = 1/(1 + q^2/M_V^2), \quad f_4(q^2) = 1/(1 + q^2/M_A^2), \quad M_V = 850 \text{ MeV} \text{ and } M_A = 1086 \text{ MeV}. \]

\[ g_A = 1.254 \text{ is assumed and the difference in magnetic moments of proton and neutron is } (\mu_p - \mu_n) = 4.71. \]

The above definition of \( M_{0+}^{\nu} \) includes contribution of the higher order terms of the nucleon current and for the induced pseudoscalar the Goldberger–Treiman PCAC relation, \( g_\pi(q^2) = 2m_p g_A(q^2)/(q^2 + m_\pi^2) \), was employed [101].

9.2. Transition operators resulting in the heavy neutrino exchange mechanism

We assume that the neutrino-mass spectrum includes heavy Majorana states \( N_k \) with masses \( M_k \) much larger than the typical energy scale of the \( 0\nu\beta\beta \) decay. These heavy states can mediate this process as well as the previous light neutrino exchange mechanism. The difference is that the neutrino propagators in this case can be contracted to points and, therefore, the corresponding effective transition operators are local unlike in the light neutrino exchange mechanism with long range internucleon interactions. The corresponding LNV parameters are \( \eta_0^{\text{eff}} \) and \( \eta_0^{\text{eff}} \).

Separating the Fermi (F), Gamow–Teller (GT) and tensor (T) contributions we write down for the NME

\[ M_0^{\nu} = M_0^{\text{F}} + M_0^{\text{GT}}(N) + M_0^{\text{F}}(N). \]

(147)

where \( S_{kl} \) and \( \sigma_{kl} \) are given in equation (144). The radial parts of the exchange potentials are

\[ H_{F,GT,T}(r_{kl}) = \frac{2}{\pi} R \int_0^{\infty} J_{0,0,2}(q r_{kl}) h_{F,GT,T}(q^2) dq. \]  

(148)
9.3. Transition operators resulting from the \( R \)-parity breaking SUSY mechanism

Assuming the dominance of gluino exchange, we obtain for the LNV parameter the following simplified expression

\[
\eta_{\nu} = \frac{\pi \alpha_s}{6} \frac{\lambda^{2}_{11}}{G_F m^{4}_{\tilde{d}} m^{4}_{\tilde{g}}} \left[ 1 + \left( \frac{m^{2}_{\tilde{d}}}{m^{2}_{\tilde{g}}} \right)^2 \right]^{1/2}. \tag{149}
\]

Here, \( G_F \) is the Fermi constant, \( \alpha_s \approx g^2/(4\pi) \) is SU(3)\(_c\) gauge coupling constant. \( m_{\tilde{d}} \) and \( m_{\tilde{g}} \) are the masses of the u-squark, d-squark and gluino, respectively.

We should mention again that for these types of interactions the pion-exchange mechanism (1\( \pi \) and 2\( \pi \)) discussed in section 7.2 dominates over the conventional two-nucleon mechanism. Thus, denoting the 0\( \nu \beta \beta \)-decay NME associated with gluino and neutralino exchange as \( M_{\nu}^{\text{GT}} \), we have [28, 93]

\[
M_{\nu}^{\text{GT}} = c^{1\pi} \left( M_{\text{GT}}^{1\pi} - M_{T}^{1\pi} \right) + c^{2\pi} \left( M_{\text{GT}}^{2\pi} - M_{T}^{2\pi} \right) \tag{150}
\]

with

\[
c^{1\pi} = \frac{\sqrt{2}}{9} \frac{f_{\pi}^2 m^4_{\pi}}{m^4_p m_\pi (m_\pi + m_d)} \frac{g_s F_P}{g^{2}_\lambda},
\]

\[
c^{2\pi} = \frac{1}{18} \frac{f_{\pi}^2 m^4_{\pi}}{m^4_p m_\pi (m_\pi + m_d)^2} \frac{g^2}{g^2_\lambda}. \tag{151}
\]

Here, \( g_s \) and \( F_P \) stand for the standard pion–nucleon coupling constant (\( g_s = 13.4 \)) and the nucleon pseudoscalar constant (we take the bag model value \( F_P \approx 4.41 \) from [204]), respectively. \( f_{\pi} \approx 0.668 m_\pi \) and \( m_\pi \) is the mass of pion. \( m_d \) and \( m_\pi \) denote current quark masses. The partial NMEs of the \( R \)\(_d\) SUSY mechanism for the 0\( \nu \beta \beta \) process are

\[
M_{\text{GT}}^{1\pi} = (0_{i}^0 | \sum_{k=1} \tau_{k}^i \tau_{1}^i H^{1\pi}_{GT}(r_{kl}) | 0_{i}^0),
\]

\[
M_{T}^{1\pi} = (0_{i}^0 | \sum_{k=1} \tau_{k}^i \tau_{1}^i H^{1\pi}_{T}(r_{kl}) | 0_{i}^0) \tag{152}
\]

with

\[
H^{1\pi}_{GT,T}(r_{kl}) = -\frac{2}{\pi^2} \int_{0}^{\infty} j_{0,2} (q r_{kl}) \frac{q^4/m^4_{\pi}}{1 + q^2/m^2_{\pi}} f_{\lambda}^2(q^2) \frac{d q^2}{q^2}.
\]

The two-nucleon exchange potentials are expressed in momentum space as the momentum dependence of normalized to unity nucleon form factors \( (f_{\lambda}(q^2)) \) is taken into account.

9.4. Transition operators resulting from the squark–neutrino mechanism

In the case of the squark–neutrino mechanism [202], due to the chiral structure of the R-parity breaking \( R \)\(_d\) SUSY interactions, the amplitude of 0\( \nu \beta \beta \)-decay does not vanish in the limit of zero neutrino mass unlike the ordinary Majorana neutrino exchange proportional to the light neutrino mass. Instead, the squark–neutrino mechanism is roughly proportional to the momentum of the virtual neutrino, which is of the order of the Fermi momentum of the nucleons inside of nucleus \( p_F \approx 100 \) MeV. This is a manifestation of the fact that the LNV necessary for 0\( \nu \beta \beta \)-decay is supplied by the \( R \)\(_d\) SUSY interactions instead of the Majorana neutrino-mass term and therefore this mechanism is not suppressed by the small neutrino mass. The corresponding SUSY LNV parameter is defined as

\[
\eta_{\tilde{q}} = \sum_{k} \frac{\lambda^{2}_{11}}{2 \sqrt{2} G_F} \sin(\frac{2\pi}{d} k) \left( \frac{1}{m^2_{\tilde{d}(k)}} \right) \tag{154}
\]

Here we use the notation \( d(k) = d, s, b \). This LNV parameter vanishes in the absence of \( \tilde{q}_L - \tilde{q}_R \) mixing when \( \theta^d = 0 \).

At the hadron level we assume dominance of the pion-exchange mode. Then, the NME associated squark–neutrino mechanism can be written as a sum of GT and tensor contributions [202]

\[
M^{0\nu}_{\tilde{q}} = M^{GT}_{\tilde{q}} - M^{T}_{\tilde{q}}. \tag{155}
\]

The exchange potentials are given by

\[
H^{0\nu}_{GT,T}(r_{kl}) = \frac{2}{\pi^2} \int_{0}^{\infty} j_{0,2} (q r_{kl}) h_k^\lambda (q^2) \frac{d q^2}{q^2(q + E)} \tag{156}
\]

with

\[
h_k^\lambda (q^2) = -\frac{1}{6} f_{\lambda}^2(q^2) \frac{m^4_{\pi}}{m_\pi (m_\pi + m_d)} \frac{q^2}{(q^2 + m^2_{\pi})^2}. \tag{157}
\]

10. 0\( \nu \beta \beta \)-decay NMEs

Interpreting existing results as a measurement of \( |\langle m_{n}\rangle| \) and planning new experiments depends crucially on the knowledge of the corresponding NMEs that govern the decay rate. The NMEs for 0\( \nu \beta \beta \)-decay must be evaluated using tools of nuclear structure theory. There are no observables that could be directly linked to the magnitude of 0\( \nu \beta \beta \)-decay NMEs and, thus, could be used to determine them in an essentially model independent way. A reliable calculation of NMEs will be of help in predicting which are the most favorable nuclides to be employed for 0\( \nu \beta \beta \)-decay searches.

The evaluation of the NMEs can be separated into two steps.

- The evaluation of the transition matrix elements between the two interacting particles (two-body ME).
- Each particle is assumed to occupy a set of single-particle states, determined by the assumed model. The spin as well the orbital structure of the operator as has been discussed in sections 5 and 6. The operators discussed in section 5 are long range, except when the intermediate neutrino is heavy leading to short-range transition operators. In section 6, except for the case of equation (110), all operators are short range. The way of dealing with the short-range operators has been discussed in section 7. Taking all these into account the effective transition operator have been constructed in section 9.

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• The second step involves the construction of the many-body wave functions.

One needs the wave functions of the initial and final nuclear systems. If closure is not employed, as in the case of QRPA, one also needs the wave functions of all the virtual (intermediate) states allowed by the assumed nuclear model. Some many-body features arising from the nuclear medium are the following: (i) the renormalization effects on the \( g_A \) coupling and the modification of the nucleon currents, which have already been discussed in section 9 and (ii) the short-range correlations, which will be discussed below. The main techniques of the construction of the many-body wave functions will be reviewed in this section. We remind the reader that in some cases information about the nuclear ME can be extracted from experiments (see section 8.4).

The calculation of the \( 0\nu\beta\beta \)-decay NMEs is a difficult problem because ground and many excited states (if closure approximation is not adopted) of open-shell nuclei with complicated nuclear structure have to be considered. In the last few years the reliability of the calculations has considerably improved. Five different many-body approximate methods have been applied for the calculation of the \( 0\nu\beta\beta \)-decay NME:

(i) The ISM [236, 293–296].

The ISM allows one to consider only a limited number of orbits close to the Fermi level, but all possible correlations within the space are included. Proton–proton, neutron–neutron and proton–neutron (isovector and isoscalar) pairing correlations in the valence space are treated exactly. Proton and neutron numbers are conserved and angular-momentum conservation is preserved. Multiple correlations are properly described in the laboratory frame. The effective interactions are constructed starting from monopole corrected \( G \) matrices, which are further adjusted to describe sets of experimental energy levels. The Strasbourg–Madrid codes can deal with problems involving basis of \( 10^3 \) Slater determinants, using relatively modest computational resources. Good spectroscopy for parent and daughter nuclei is achieved. Due to the significant progress in shell-model configuration mixing approaches, there are now calculations performed with these methods for several nuclei.

(ii) QRPA [115, 116].

The QRPA has the advantage of large valence space but is not able to comprise all the possible configurations. Usually, single-particle states in the Wood–Saxon potential are considered. One is able to include for each orbit in the QRPA model space also the spin–orbit partner, which guarantees that the Ikeda sum rule is fulfilled. This is crucial to describe correctly the Gamow–Teller strength. The proton–proton and neutron–neutron pairings are considered. They are treated in the BCS approximation. Thus, proton and neutron numbers are not exactly conserved. The many-body correlations are treated at the RPA level within the quasiboson approximation. Two-body \( G \)-matrix elements, derived from realistic one-boson exchange potentials within the Brueckner theory, are used for the determination of nuclear wave functions.

(iii) IBM [130].

In the IBM the low-lying states of the nucleus are modeled in terms of bosons. The bosons are in either \( L = 0 \) (s boson) or \( L = 2 \) (d boson) states. Thus, one is restricted to \( 0^+ \) and \( 2^+ \) neutron pairs transferring into two protons. The bosons interact through one- and two-body forces giving rise to bosonic wave functions.

(iv) The PHFB method [129].

In the PHFB wave functions of good particle number and angular momentum are obtained by projection on the axially symmetric intrinsic HFB states. In applications to the calculation of the \( 0\nu\beta\beta \)-decay NMEs the nuclear Hamiltonian was restricted only to quadrupole interaction. The PHFB is restricted in its scope. With a real Bogoliubov transformation without parity mixing one can describe only neutron pairs with even angular momentum and positive parity.

(v) The EDF method [131].

The EDF is considered to be an improvement with respect to the PHFB. The density functional methods based on the Gogny functional are taken into account. The particle-number and angular-momentum projection for mother and daughter nuclei is performed and configuration mixing within the generating coordinate method is included. A large single-particle basis (11 major oscillator shells) is considered. Results are obtained for all nuclei of experimental interest.

The differences among the listed methods of NME calculations for the \( 0\nu\beta\beta \)-decay are due to the following reasons:

(i) The mean field is used in different ways. As a result, single-particle occupancies of individual orbits of various methods differ significantly from each other [297].

(ii) The residual interactions are of various origins and renormalized in different ways.

(iii) Various sizes of the model space are taken into account.

(iv) Different many-body approximations are used in the diagonalization of the nuclear Hamiltonian.

Each of the applied methods has some advantages and drawbacks, whose effect in the values of the NME can be sometimes explored. The advantage of the ISM calculations is their full treatment of the nuclear correlations, which tends to diminish the NMEs. In contrast, the QRPA, the EDF, and the IBM underestimate the multipole correlations in different ways and tend to overestimate the NMEs. The drawback of the ISM is the limited number of orbits in the valence space and as a consequence the violation of the Ikeda sum rule and underestimation of the NMEs.

In table 3, recent results of the different methods are summarized. The presented numbers have been obtained with the unquenched value of the axial coupling constant \( g_A^{\text{eff}} = g_A = 1.25 \),\(^9\) Miller–Spencer Jastrow short-range correlations

\(^9\) A modern value of the axial-vector coupling constant is \( g_A = 1.269 \). We note that in the referred calculations of the \( 0\nu\beta\beta \)-decay NMEs the previously accepted value \( g_A^{\text{eff}} = g_A = 1.25 \) was assumed.
The NME of the $0\nu\beta\beta$-decay $M_{0\nu}^\text{th}$ calculated within the framework of different approaches: ISM [236, 296], QRPA [184, 291, 299–301], projected Hartree–Fock Bogoliubov approach (PHFB, PQO2 parametrization) [129], EDF [131] and interacting boson model (IBM) [130]. QRPA(TBC) and QRPA(J) denote QRPA results of Tuebingen–Bratislava–Caltech (TBC) and Jyvaskyla (J) groups, respectively. The Miller–Spencer Jastrow two-nucleon short-range correlations are taken into account. The EDF results are multiplied by 0.80 in order to account for the difference between UCOM and Jastrow [292]. $g_A^\text{eff} = g_A = 1.25$ and $R = 1.2\text{fmA}^{1/3}$ are assumed.

| Transition | ISM [236, 296] | QRPA (TBC) [291, 184] | QRPA (J) [299, 300, 301] | IBM-2 [130] | PHFB [129] | EDF [131] |
|------------|----------------|-----------------------|--------------------------|--------------|-------------|------------|
| $^{48}\text{Ca} \to ^{46}\text{Ti}$ | 0.61, 0.57 | 4.92 | 4.72 | 5.47 | 3.70 | 1.91 |
| $^{76}\text{Ge} \to ^{76}\text{Se}$ | 2.30 | 4.39 | 2.77 | 4.41 | 3.39 | |
| $^{82}\text{Se} \to ^{82}\text{Kr}$ | 2.18 | 1.22 | 2.45 | 2.78 | 4.54 | |
| $^{96}\text{Zr} \to ^{96}\text{Mo}$ | 1.27 | 3.64 | 2.91 | 3.73 | 6.55 | 4.08 |
| $^{100}\text{Mo} \to ^{100}\text{Ru}$ | 3.86 | | | | | |
| $^{110}\text{Pd} \to ^{110}\text{Cd}$ | | | | | | |
| $^{116}\text{Cd} \to ^{116}\text{Sn}$ | 2.99 | 3.17 | | | | |
| $^{124}\text{Sn} \to ^{124}\text{Te}$ | 2.10 | 2.65 | | | | |
| $^{128}\text{Te} \to ^{128}\text{Xe}$ | 2.34 | 3.97 | 2.54 | 3.89 | 3.30 | |
| $^{130}\text{Te} \to ^{130}\text{Xe}$ | 2.12 | 3.56 | 3.56 | 4.06 | 4.36 | 4.12 |
| $^{136}\text{Xe} \to ^{136}\text{Ba}$ | 1.76 | 2.30 | 2.54 | 3.38 | | |
| $^{150}\text{Nd} \to ^{150}\text{Sm}$ | 3.16 | 2.32 | 3.16 | 1.37 | | |

Figure 24. The $0\nu\beta\beta$-decay half-lives of nuclei of experimental interest for $|\langle m_e \rangle| = 50$ meV and NMEs of different approaches. The Miller–Spencer Jastrow two-nucleon short-range correlations are considered. The axial-vector coupling constant $g_A$ is assumed to be 1.25.

From table 3 we can draw the following conclusions:

(i) The ISM values of NMEs, with the exception of the NME for the double magic nucleus $^{48}\text{Ca}$, practically do not depend on the nucleus. They are significantly smaller, by about a factor 2–3, when compared with NMEs of other approaches.

(ii) The largest values of NME are obtained in the IBM ($^{76}\text{Ge}$ and $^{128}\text{Te}$), PHFB ($^{100}\text{Mo}$, $^{130}\text{Te}$ and $^{150}\text{Nd}$), QRPA ($^{150}\text{Nd}$) and EDF ($^{48}\text{Ca}$, $^{82}\text{Se}$, $^{116}\text{Cd}$, $^{116}\text{Sn}$ and $^{136}\text{Xe}$) approaches.

(iii) NMEs obtained by the QRPA(TBC) and IBM methods are in good agreement (with the exception of $^{150}\text{Nd}$).

(iv) In the case of $^{130}\text{Te}$ all discussed methods, with the exception of the ISM, give practically the same result.

(v) The disagreement between IBM-2 and ISM is particularly troublesome, because IBM-2 is a truncation of the shell-model space to the S and D pair space and, in the limit of spherical nuclei, IBM-2 and ISM should produce the same results.

(vi) The disagreement between the QRPA(TBC) and QRPA(J) results is not large but it needs to be clarified.

Comparing $0\nu\beta\beta$-decay NMEs calculated by different methods gives some insight into the advantages or disadvantages of different candidate nuclei. However, matrix elements are not the only relevant quantities (see section 8 for the nuclear-sensitivity factor). Experimentally, half-lives are measured or constrained, and the effective Majorana neutrino mass $m_\nu$ is the ultimate goal. For $|m_\nu|$ equal to 50 meV the calculated half-lives for double $\beta$-decaying nuclei of interest are presented in figure 24. We see that the spread of half-lives for a given isotope is up to the factor of 4–5.

It is worth noting that due to the theoretical efforts made over the last years the disagreement among different NMEs is now much less severe than it was about a decade before. Nevertheless the present-day situation with the calculation of $0\nu\beta\beta$ NMEs cannot be considered as completely satisfactory. Further progress is required and it is believed that the situation will improve with time. Accurate determination of the NMEs, and a realistic estimate of their uncertainty, is of great importance. NMEs need to be evaluated with uncertainty of less than 30% to establish the neutrino-mass spectrum and CP-violating phases of the neutrino mixing.
10.1. Uncertainties in calculated NMEs

The improvement of the calculation of the $0\nu\beta\beta$-decay NMEs is a very important and challenging problem. The uncertainty associated with the calculation of the $0\nu\beta\beta$-decay NMEs can be diminished by suitably chosen nuclear probes. Complementary experimental information from related processes such as charge-exchange and particle transfer reactions, muon capture and charged-current (anti)neutrino–nucleus reactions is very relevant. A direct confrontation of nuclear structure models with data from these processes improves the quality of nuclear structure models (see section 8). The constrained parameter space of nuclear models is a promising way to reduce uncertainty in the calculated $0\nu\beta\beta$-decay NMEs.

Steady progress in nuclear structure approaches is gradually leading to a better understanding and to a reduction of the differences among their results. However, even in the most refined approaches, the estimates of $M_{\nu}$ remain affected by various uncertainties, whose reduction is of great importance.

(i) QRPA calculation of NMEs. Due to its simplicity the QRPA is a popular technique to calculate the $0\nu\beta\beta$-decay NMEs. One of the most important factors of the QRPA calculation of the $0\nu\beta\beta$-decay NMEs is the way the particle–particle strength of the nuclear Hamiltonian $g_{pp}$ is fixed. The Jyvaskyla group performing the calculation within the QRPA claims that $g_{pp}$ to the $2\nu\beta\beta$-decay rates the uncertainty associated with variations in $M_{\nu}$ calculated for the $0\nu\beta\beta$-decay NMEs can be significantly eliminated [290–302]. In particular, the results obtained in this way are essentially independent of the size of the basis, the form of different realistic nucleon–nucleon potentials, or whether QRPA or renormalized QRPA (taking into account the Pauli exclusion principle) is used. This new way of fixing parameter space was criticized by the Jyvaskyla group in a series of papers maintaining the role of single $\beta$-transitions. It was claimed that careful study of single $\beta$ and $2\nu\beta\beta$ observables points to serious shortcomings of the adopted procedure [303, 304]. These objections were refuted in [290, 291]. In recent publications [299–301] the Jyvaskyla group also adopted the procedure for fixing $g_{pp}$ proposed by the TBC group [302].

Usually, two variants of the QRPA are considered. The standard QRPA, which is based on the quasiboson approximation, and the renormalized QRPA (RQRPA) [127, 174, 290], which takes into account the Pauli exclusion principle. Further improvement is achieved within the self-consistent QRPA (SRQRPA) [297, 305, 306] by conserving the mean particle number in the correlated ground state. The restoration of the Pauli exclusion principle and of particle-number conservation leads to a reduction of the $0\nu\beta\beta$-decay NMEs [174, 290, 291, 297].

There is some controversy about the importance of the tensor $M_T$ contribution to $M_{\nu}^{0\nu}$. According to the ISM [236] the tensor term is small, a fact understood by the small model space adopted. The Jyvaskyla group performing the calculation within the QRPA claims that $M_T$ is negligible [299, 300]. $M_T$ was neglected in the PHFB [129] and EDF [131] calculations of the $0\nu\beta\beta$-decay NMEs. In contrast, results of the IBM-2 [130] and the QRPA(TBC) [101] calculations show that $M_T$ cannot be neglected and its absolute value can be up to 10% of $M_{\nu}^{0\nu}$.

(ii) The closure approximation. The $0\nu\beta\beta$-decay matrix elements are usually calculated using the closure approximation for intermediate nuclear states. Within this approximation energies of intermediate states $(E_n - E_i)$ are replaced by an average value $\bar{E} \approx 10$ MeV, and the sum over intermediate states is taken by closure, $\sum_n |n\rangle\langle n| = 1$. This simplifies the numerical calculation drastically. The calculations with exact treatment of the energies of the intermediate nucleus were achieved within the QRPA-like methods [174, 290–292, 302]. The effect of the closure approximation was studied in detail in [307]. It was found that the differences in NMEs are within 10%. The dependence of the NMEs on the average energy of the intermediate states $\bar{E}$ was studied within the nuclear shell model. By varying $\bar{E}$ from 2.5 to 12.5 MeV the variation in the NME was obtained to be less than 5% [296].

(iii) The two-nucleon short-range correlations and finite nucleon size. The physics of finite nucleon size (FNS) and two-nucleon short-range correlations (SRC) is different. Both reduce magnitude of the $0\nu\beta\beta$-decay NME by competing with each other. The importance of each of them depends on the type of SRC and involves form-factor parameters.

The FNS is taken into account via momentum dependence of the nucleon form factors. For the vector and weak-magnetism (axial-vector) form factors the usual dipole approximation is considered with a cut-off parameter $M_V = 850$ MeV ($M_A = 1086$ MeV), which comes from electron scattering (neutrino charged-current scattering) experiments. The form factors suppress high-momentum exchange. We note that in the limit of point-like nucleon ($M_{VA} \rightarrow \infty$) the weak-magnetism contribution to the $0\nu\beta\beta$-decay would be divergent.

The SRCs are included via the correlation function $f(r)$, that modifies the relative two-nucleon wave functions at short distances:

$$\Psi_{al}(r) \rightarrow [1 + f(r)]\Psi_{al}(r),$$

where $f(r)$ can be parametrized as [174]

$$f(r) = -ce^{-ar^2}(1 - br^2).$$

Previously, Miller–Spencer Jastrow SRC ($a = 1.1$ fm$^{-2}$, $b = 0.68$ fm$^{-2}$, $c = 1.0$) have been added into the involved two-body transition matrix elements, changing two neutrons into two protons, to achieve healing of the correlated wave functions. A suppression of $M_{\nu}^{0\nu}$ by 20% was found [130, 174, 236, 296]. However, recent work has questioned this prescription [174, 299, 300].

The two-nucleon short-range correlations were studied within the coupled clusters method (CCM) in [174]. The Jastrow function fit for $T = 1$ channel reported for Argonne V18 and Bonn-CD NN interactions set of parameters ($a = 1.59$ fm$^{-2}$, $b = 1.45$ fm$^{-2}$, $c = 0.92$) and ($a = 1.52$ fm$^{-2}$, $b = 1.88$ fm$^{-2}$, $c = 0.46$), respectively [174]. These correlation functions were confirmed by exploiting the construction of an effective shell-model operator for $0\nu\beta\beta$-decay of $^{82}$Se [308]. The notable differences between the
results calculated with Miller–Spencer Jastrow and CCM SRC are about 20%–30% [174, 296]. The previous results with Miller–Spencer treatment of SRC certainly overestimate the quenching due to short-range correlations. Of course, the results obtained with the CCM SRC are preferable. We note that in the case of Bonn-CD CCM SRC the 0νββ-decay NMEs are slightly increased [174].

The two-nucleon short-range correlations were treated also through the unitary-correlation operator method (UCOM) [299–309], which has the advantages of wave function overlap preservation and a range of successful applications [310]. The drawback of this approach, when applied to the 0νββ-decay, is that it violates some general properties of the Fermi and Gamow–Teller matrix elements [174].

Recently, a question of many-body short-range correlations in the evaluation of the 0νββ-decay NMEs was addressed within a simple model [311]. The existing calculations include long-range many-body correlations in model dependent (ISM, QRPA, PHFB, EDF, IBM) nuclear wave functions but allow only two particles to be correlated at short distances. There are some indications that it is not sufficient.

(iv) The effect of deformation. The nuclei undergoing DBD, which are of experimental interest, are spherical or weakly deformed nuclei with the exception of 150Nd, which is strongly deformed. It was found in [312] that deformation introduces a mechanism of suppression of the 2νββ-decay matrix element which gets stronger when deformations of the initial and final nuclei differ from each other [312, 313]. A similar dependence of the suppression of both M2ν and M0ν matrix elements on the difference in deformations has been found in the PHFB [314, 315] and the ISM [236]. The NMEs have a well-defined maximum when the deformations of parent and daughter nuclei are similar, and they are quite suppressed when the difference in the deformations is large. The ISM results suggest that a large mismatch of deformation can reduce the matrix elements by factors as large as 2–3. Within the IBM–2 the effects of the deformation are introduced through the bosonic neutron–proton quadrupole interaction. For weakly deformed nuclei the effect is a reduction by about 20%.

The QRPA calculation of the 0νββ-decay NMEs requires a construction of all states of the intermediate nucleus, even if the closure approximation is considered. The results were obtained in the spherical limit, which is a significant simplification. Recently, the proton–neutron deformed QRPA with a realistic residual NN interaction was developed [184, 316–318]. This approach was applied in the case of 76Ge, 150Nd and 160Gd and led to the conclusion that the 0νββ-decay of 150Nd, to be measured soon by the SNO+ collaboration, provides one of the best probes of the Majorana neutrino mass [184, 318].

(v) The occupancies of individual orbits. The occupancies of valence neutron and proton orbits determined experimentally represent important constraints for nuclear models used in the evaluation of the 0νββ-decay NME. For the 76Ge and 76Se they have been extracted by accurate measurements of one nucleon adding and removing transfer reactions by Schiffer and collaborators [319, 320]. The main motivation to study these nuclei was the fact that they are the initial and final states of 0νββ-decay transitions. These measurements offer a possibility to compare these experimental results with the theoretical occupations and, if necessary, detect which modifications would be required in the mean field or the effective interaction in order to obtain improved agreement with the experiment.

In a theoretical study [297] measured proton and neutron occupancies were used as a guideline for a modification of the effective mean field energies, which resulted in a better description of these quantities. The calculation of the 0νββ-decay NME for 76Ge performed with an adjusted Woods-Saxon mean field combined with the self-consistent QRPA (SRQRPA) method [297], which conserves the mean particle number in correlated ground state, led to a reduction of M2ν by 20%–30% when compared with the previous QRPA values.

In the ISM the variation of the NME for 0νββ-decay of 76Ge was studied after the wave functions were constrained to reproduce the experimental occupancies of the two nuclei involved in the transition. It was found [321] that in the ISM description the value of the NME is enhanced about 15% compared with previous calculations. This diminishes the discrepancies between the ISM and the QRPA approaches.

The role of occupancies of the single-particle orbitals in the standard QRPA calculation of M2ν to ground and 0+ states of final nucleus was studied also in [322, 323]. Unlike the treatment of [297], whereby occupancies in respect to the correlated SRQRPA ground state were considered, the occupancies were evaluated at the level of uncorrelated BCS ground state. The basic features of the ground and excited state decays were found to be quite different.

(vi) The axial-vector coupling constant gA. It is well known that the calculated strengths of Gamow–Teller β-decay transitions to individual final states are significantly larger than the experimental ones. That effect is known as the axial-vector quenching. To account for this, it is customary to quench the calculated GT matrix elements up to 70%. Formally, this is accomplished by replacing the true value of the coupling constant gA = 1.269 (the previous gA = 1.254 was considered) by a quenched value gA eff = 1.0. The origin of the quenching is not completely known. This effect is assigned to the A-isobar admixture in the nuclear wave function or to the shift of the GT strength to higher excitation energies due to the short-range tensor correlations. It is not clarified yet whether a similar phenomenon exists for other multipoles, in addition to J = 1+.

Quenching is very important for the DBD because gA eff appears to the fourth power in the decay rate. If it occurs also for the 0νββ-decay, it could significantly reduce the 0νββ-decay half-life by as much as a factor of 2–3. The axial-vector coupling constant gA eff or, in other words, the treatment of quenching, is also a source of differences in the calculated 0νββ-decay NMEs. M2ν is a function of squared gA eff, which appears by vector and weak-magnetism terms following the definition of equation (141). In [324] three independent lifetime data (2νββ-decay, EC, β-decay) were accurately reproduced in the QRPA by means of
two free parameters \( (g_{\nu}, s_A^{\text{eff}}) \), resulting in an overconstrained parameter space. The general trend in favor of \( s_A^{\text{eff}} < 1 \) was confirmed. This novel possibility to reconcile QRPA results with experimental data, which deserves further discussions and tests, warrants a reconsideration of the quenching problem from a new perspective.

As was manifested above, nuclear NMEs for \( 0\nu\beta\beta \)-decay are affected by relatively large theoretical uncertainties. Within the QRPA approach, it was shown that, within a given set of nuclei, the correlations among NME errors are as important as their size [325]. This represents a first attempt to quantify the covariance matrix of the NMEs, and to understand its effects in the comparison of current and prospective \( 0\nu\beta\beta \)-decay results for two or more nuclei. It would be useful if other theoretical groups in the \( 0\nu\beta\beta \) field could present ‘statistical samples’ of NME calculations as well, in order to provide independent estimates of (co)variances for their NME estimates. A covariance analysis like the one proposed in [325] represents a useful tool to estimate correctly current or prospective sensitivities to effective Majorana neutrino mass \( (m_\nu) \).

Currently, the uncertainty in calculated \( 0\nu\beta\beta \)-decay NMEs can be estimated up to factor of 2 or 3 depending on the considered isotope, mostly due to differences between the ISM results and the results of other approaches (QRPA, PHFB, EDF, IBM) and also due to unknown value of \( s_A^{\text{eff}} \).

10.2. Anatomy of NMEs

The anatomy of the \( 0\nu\beta\beta \)-decay NME was performed in [236, 292]. \( M_{0\nu}^{0\nu} \) was decomposed on the angular momenta and parities \( J^\pi \) of the pairs of neutrons that are transformed into protons with the same \( J^\pi \). It was found that the final value of \( M_{0\nu}^{0\nu} \) reflects two competing forces: the like particle pairing interaction that leads to the smearing of Fermi levels and the residual neutron–proton interaction that, through ground-state correlations, admixes ‘broken-pair’ (higher-seniority) states. The function \( C_0^{0\nu}(r) \) that describes the dependence of the \( M_{0\nu}^{0\nu} \) on internucleon distances \( r \),

\[
M_{0\nu}^{0\nu} = \int_0^\infty C_0^{0\nu}(r) \, dr,
\]

was a subject of interest. It was shown that the above competition implies that only internucleon distances \( r < 2–3 \text{ fm} \) contribute to \( M_{0\nu}^{0\nu} \) [292]. The maximum value of \( C_0^{0\nu}(r) \) occurs around \( r = 1 \text{ fm} \), which means that almost the complete value of \( M_{0\nu}^{0\nu} \) comes from contributions of decaying nucleons that are close to each other. This distance corresponds to a neutrino momentum of \( q \approx 200 \text{ MeV} \), twice as large as was expected before. This finding, which explains a small spread of results for different nuclei, was confirmed also by the ISM [236] and similar behavior for \( C_0^{0\nu}(r) \) was obtained also within the PHFB [129]. The QRPA and ISM functions \( C_0^{0\nu}(r) \) differ only by a scaling factor, which is expected to be related to the ratio of the average number of pairs in both calculations.

The largest component of \( M_{0\nu}^{0\nu} \) is the GT part. We have

\[
M_{0\nu}^{0\nu} = M_{0\nu}^{0\nu, \text{GT}} (1 + \chi_F + \chi_T),
\]

where \( \chi_F \) and \( \chi_{GT} \) are matrix element ratios that are smaller than unity and, presumably, less dependent on the details of the applied nuclear model. In [326] it was shown that \( M_{0\nu}^{0\nu, \text{GT}} \) is related to the closure \( 2\nu\beta\beta \)-decay NME \( M_{0\nu}^{2\nu} \). That relation is revealed when these matrix elements are expressed as functions of the relative distance between the pair of neutrons that are transformed into a pair of protons. We have

\[
C_{0\nu}^{0\nu}(r) = H_{0\nu}(r, E) C_{0\nu}^{2\nu}(r),
\]

where \( H(r, E) \) is the neutrino exchange potential in nucleus and \( C_{0\nu}^{2\nu}(r) \) is defined as

\[
M_{0\nu}^{2\nu} = \int_0^\infty C_{0\nu}^{2\nu}(r) \, dr.
\]

While the matrix element \( M_{0\nu}^{2\nu} \) gets contributions only from \( 1^+ \) intermediate states, the function \( C_{0\nu}^{2\nu}(r) \) gets contributions from all intermediate multipoles.

Equation (162) represents the basic relation between the \( 0\nu\beta\beta \)– \( 2\nu\beta\beta \)-decay modes. An analysis of this relation allowed one to explain the contrasting behavior of \( M_{0\nu}^{0\nu, \text{GT}} \) and \( M_{0\nu}^{2\nu} \) when \( A \) and \( Z \) are changed, namely that \( M_{0\nu}^{0\nu, \text{GT}} \) changes slowly and smoothly unlike \( M_{0\nu}^{2\nu} \), which has pronounced shell effects [326].

In [327] a connection of the Fermi \( 0\nu\beta\beta \)-decay NME \( M_{0\nu}^{0\nu} \) with an energy-weighted double Fermi transition matrix element was presented. It is argued that \( M_{0\nu}^{0\nu} \) can be reconstructed, if the isospin-forbidden Fermi transition between the ground state of the final nucleus and the isobaric analog state in the intermediate nucleus can be measured, e.g. by means of \( (n, p) \) charge-exchange reactions. By knowing \( M_{0\nu}^{0\nu} \) one can evaluate \( M_{0\nu}^{0\nu} \) by assuming an approximate relation \( M_{0\nu}^{0\nu} / M_{0\nu}^{2\nu} \approx -2.5 \), which follows from the QRPA calculations [174].

11. Distinguishing the \( 0\nu\beta\beta \)-decay mechanisms

Many extensions of the SM generated Majorana neutrino masses offer a plethora of \( 0\nu\beta\beta \)-decay mechanisms. Among these we should mention the exchange of heavy neutrinos, the exchange of SUSY superpartners with \( R \)-parity violation, leptoquarks, right-handed W bosons, or Kaluza–Klein excitations, among others, which have been discussed in the previous sections or can be found in the literature [68].

An unambiguous detection of \( 0\nu\beta\beta \)-decay will prove that the total LN is broken in nature and neutrinos are Majorana particles. However, after neutrino oscillations have established that the neutrinos are massive, as we have already mentioned, the observation of \( 0\nu\beta\beta \)-decay is expected to play a crucial role in determining the neutrino-mass scale. This prospect generates the questions: what is the mechanism that triggers the decay? What happens if several mechanisms are active for the decay?

11.1. Dominance of a single mechanism

Usually, the \( 0\nu\beta\beta \)-decay is discussed by assuming that one mechanism at a time dominates. Then the half-life in a given
nucleus $i \equiv (A, Z)$ can be written as
\[
(T_{1/2}^{0\nu}(i))^{-1} = |\eta_\nu|^2 \left| M_i^{0\nu}(i) \right|^2 G_i^{0\nu}(i).
\] (164)

Here, $\eta_\nu$, $M_i^{0\nu}$, $G_i^{0\nu}(A, Z)$ are the LNV parameter ($\kappa$ denotes a given mechanism of the $0\nu\beta\beta$-decay), associated NME and kinematical factor, respectively. The calculation of $M_i^{0\nu} = (g^{\text{eff}}_\lambda / g_A)^2 M_i^{\text{SM}}$ in some cases depends also on $g^{\text{eff}}_\lambda$ allows one to deduce a constraint on $\eta_\nu$ from the measured lower bound on the $0\nu\beta\beta$-decay half-life. The definition of $M_i^{0\nu}$ in (164) allows one to display the effects of uncertainties in $g^{\text{eff}}_\lambda$ and to use the same phase factor $G_i^{0\nu}$ when calculating the $0\nu\beta\beta$-decay rate [290, 291].

In connection with the neutrino oscillations, much attention is attracted to the light neutrino-mass mechanism of the $0\nu\beta\beta$-decay ($\eta_\nu = (m_\nu)/m_e$) (see section 4). Small neutrino masses and neutrino mixing are commonly considered as a signature of physics beyond the SM. Several beyond the SM mechanisms of neutrino-mass generation were proposed. The most viable and plausible mechanism is the famous seesaw mechanism, which is based on the assumption that the total LNV $L$ is violated at a scale much larger than the electroweak scale.

The $0\nu\beta\beta$-decay is ruled by the light Majorana neutrino-mass mechanism in the case of the standard seesaw mechanism of neutrino-mass generation, which is based on the assumption that the LN is violated at a large ($10^{15}$ GeV) scale. In [170] it was shown that if $0\nu\beta\beta$-decay is observed in future experiments sensitive to the effective Majorana mass in the inverted mass hierarchy region, then a comparison of the derived ranges with measured half-lives will allow us to probe the standard see-saw mechanism (see section 3), assuming that future cosmological data establish the sum of the neutrino masses to be about 0.2 meV.

A primary purpose of type I see-saw, see section 3, which is the simplest extension of the SM, is to account for light neutrino masses in a renormalizable gauge model. Only heavy sterile neutrino states are added to the spectrum of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ theory. These heavy states might lead to measurable effects also for the $0\nu\beta\beta$-decay. The possible contribution of sterile neutrino dominated Majorana mass eigenstate $v_h$ with mass $m_h$ to the $0\nu\beta\beta$-decay was examined in [328]. From the most stringent lower bound on the $0\nu\beta\beta$-decay half-life of $^{76}$Ge, upper limits on the neutrino mixing matrix element $|U_{eh}|^2$ in a wide region of values of $m_h$ (below and above TeV scale) were derived. It was assumed that the value of $|U_{eh}|^2$ is significantly smaller than the current limit on this quantity ($|U_{eh}|^2 \ll 0.2$–0.3 eV).

Recently, the $0\nu\beta\beta$-decay associated with the exchange of virtual sterile neutrinos, that mix with ordinary neutrinos and are heavier than 200 MeV, was revisited [329]. The dominant sterile neutrino contributions in $0\nu\beta\beta$ process provide a way to overcome the conflict between cosmology and the claim for evidence of the $0\nu\beta\beta$-decay by Klapdor and collaborators [41, 42].

There is a possibility that the total LN is violated at TeV scale [331–334], which is accessible at the Large Hadron Collider. The Large Hadron Collider can determine the right-handed neutrino masses and mixings. In [332] it was manifested that the discovery of left–right (LR) symmetry at the Large Hadron Collider would provide a strong motivation for $0\nu\beta\beta$ searches. By exploiting the LR model with type-II see-saw (see section 3) it was shown that the exchange of heavy neutrinos may dominate the $0\nu\beta\beta$-decay rate depending on the mass of right-handed charged gauge bosons and the mixing of right-handed neutrinos [332] (see equation (67)). A complementary study of lepton-flavor violating processes (e.g. $\mu \rightarrow e\gamma$), which can provide constraints on masses of right-handed neutrinos and doubly charged scalars, is of great importance [332, 335].

The LR symmetric models [17, 18, 336] are popular models of particle physics due to restoration of parity at high-energy scale and because they can naturally account for the smallness of neutrino masses. They allow not only the light and heavy neutrino-mass mechanisms of the $0\nu\beta\beta$-decay but also those associated with effective neutrino-mass independent parameters ($\eta$) and ($\lambda$), see section 5.2. As was shown already, there is an exchange of light neutrinos between two $\beta$-decaying nucleons in the nucleus in this case.

The three terms ($m_\nu$, $\eta$) and ($\lambda$) in the $0\nu\beta\beta$-decay rate show different characteristics in the angular correlations and energy spectrum [63]. By knowing the single electron energy spectrum and the angular correlation of the two electrons with sufficient accuracy, one could distinguish between decays due to coupling to the left-handed and right-handed hadronic currents [63]. This possibility was studied in the context of $^{76}$Ge and SuperNEMO (isotopes under consideration for SuperNEMO are $^{82}$Se, $^{150}$Nd and $^{48}$Ca [180, 337]) detectors. In [338–341] the expected pulse shapes to be observed for the $0\nu\beta\beta$-decay events in a big $^{76}$Ge detector have been calculated starting from their Monte Carlo calculated time history and spatial energy distribution. The conclusion was that with the spatial resolution of a large size Ge detector for the majority of $0\nu\beta\beta$ events it is not possible to differentiate between the contributions of $m_\nu$ and the right-handed weak current parameters ($\eta$) and ($\lambda$). In contrast, the SuperNEMO experiment [180, 337] has a unique potential to measure the decay electron’s angular and energy distributions and thus to disentangle these possible mechanisms for $0\nu\beta\beta$ decay [180, 342]. We note that the planned experiment SuperNEMO, which allows the measurement of $0\nu\beta\beta$-decay in several isotopes to both the ground and excited states, is able to track the trajectories of the emitted electrons and determine their individual energies. We should mention here that a measurement to both states can also be useful in reducing the background [343]. Other planned experiments that will be able to measure the energy and angular distributions are EXO [177], MOON [344] and COBRA [345].

There is motivation to consider the $0\nu\beta\beta$-decay rate in a general framework, parameterizing the new physics
contributions in terms of all effective low energy currents allowed by Lorentz invariance [201, 346], e.g. within the effective field theory [347]. This approach allows us to separate the nuclear physics part of the $0\nu\beta\beta$-decay from the underlying particle physics model, and derive limits on arbitrary lepton number violating theories. A general Lorentz-invariant effective Lagrangian for leptonic and hadronic charged weak currents was used to perform a comparative analysis of various $0\nu\beta\beta$-decay long-range mechanisms in [348, 349]. It was shown that measuring the angular correlations of emitted electrons in the $0\nu\beta\beta$-decay together with the ability to observe these decays in several nuclei would help significantly in identifying the dominant mechanism underlying this process.

There is a class of $0\nu\beta\beta$-decay mechanisms, which one cannot distinguish from each other kinematically. The light ($\eta_\nu$) and heavy ($\eta^{L}_N$, $\eta^{R}_N$) Majorana neutrino mass, the trilinear $R$-parity breaking mechanisms—both the short-range mechanism ($\eta^L_\nu$) with the exchange of heavy superpartners (gluino and squarks and/or neutralinos and selectron) [27, 88, 89, 350] and the long-range mechanism ($\eta^R_\nu$) involving both the exchange of heavy squarks and light neutrino [202] (called squark–neutrino mechanism), constitute such a group. A discussed possibility to distinguish between these mechanisms is a comparison of results for $0\nu\beta\beta$-decay in two or more isotopes [351–353].

Under the assumption of the dominance of a single mechanism of the $0\nu\beta\beta$-decay the LNV parameter $\eta_\nu$ drops out in the ratio of experimentally determined half-lives for two different isotopes. This ratio depends on the mechanism of the $0\nu\beta\beta$-decay due to NMEs and kinematical factors, but is free of the LNV parameter. Thus it can be compared with the theoretical prediction for different mechanisms. In addition, it is assumed that in the ratio of NMEs theoretical uncertainties are reduced due to cancellations of systematic effects. Relative deviations of half-life ratios for various new physics contributions, which were normalized to the half-life of $^{76}\text{Ge}$ and compared with the ratio in the light neutrino-mass mechanism, were studied in [353]. It was found that the change in ratios of half-lives varies from 60% for supersymmetric models up to a factor of 520 for extra-dimensional and LR-symmetric mechanisms. It is concluded that complementary measurements in different isotopes would be strongly encouraged [353, 354].

Another possibility to distinguish between the various $0\nu\beta\beta$-decay mechanisms is a study of the branching ratios of $0\nu\beta\beta$-decays to excited $0^+$ states [351, 355] and $2^+$ states [64, 356] and a comparative study of the $0\nu\beta\beta$-decay and neutrinoless electron capture with emission of positron ($0\nu EC\beta^+$) [357]. Unfortunately, the search for the $0\nu EC\beta^+$-decay is complicated due to small rates and the experimental challenge to observe the produced x-rays or Auger electrons, and most double beta experiments of the next generation are not sensitive to electron tracks or transitions to excited states.

11.2. NMEs of exotic mechanisms

Recently, great interest was paid to the calculation of NMEs associated with the light neutrino-mass mechanism and ground-state to ground-state transition. Less progress was achieved in the calculation of NMES of exotic $0\nu\beta\beta$-decay mechanisms.

Experimental studies of transitions to excited $0^+$ and $2^+$ final states\(^{11}\) allow us to reduce the background by gamma–electron coincidences. Drawbacks are lower $Q$ values and suppressed NMES. The theoretical studies of the corresponding nuclear transitions were performed within the ISM [236], Hartree–Fock–Bogoliubov [356] and QRPA [323, 355, 358, 359] approaches. In the ISM the $0\nu\beta\beta$-decays of $^{48}\gamma\text{Ca}$, $^{76}\gamma\text{Ge}$, $^{124}\gamma\text{Sn}$, $^{130}\gamma\text{Te}$ and $^{136}\gamma\text{Xe}$ to $0^+$ final state were found at least 25 times more suppressed with respect to the ground-state to ground-state transition in the case of light neutrino-mass mechanism. A similar conclusion was drawn for the $0\nu\beta\beta$-decays of $^{76}\gamma\text{Ge}$, $^{82}\gamma\text{Se}$, $^{100}\gamma\text{Mo}$ and $^{136}\gamma\text{Xe}$ to the excited collective $0^+$ state suggesting a suppression 10–100 larger than that of the transition to ground state [323, 355, 358]. In addition to light neutrino mass, right-handed current [358, 359] and $R$-parity breaking mechanisms [355] were also considered.

Quite the opposite is claimed in a different study [359], namely it was found that the transition rate of the $0\nu\beta\beta$-decay of $^{76}\gamma\text{Zr}$ to the first excited $0^+$ state is favored by the enhanced transition matrix elements attributed to the monopole-vibrational structure of this state.

The $0\nu\beta\beta$-decay of $^{76}\gamma\text{Ge}$ and $^{100}\gamma\text{Mo}$ to $2^+$ final state was investigated for light neutrino mass and right-handed current mechanisms by taking into account recoil corrections to the nuclear currents in [356]. The initial $0^+$ and final $2^+$ nuclear states were described in terms of the Hartree–Fock–Bogoliubov type wave functions, which were obtained by a variation after particle-number and angular-momentum projection [360]. By the numerical calculation of relevant NMES, it was found that the relative sensitivities of $0^+ \rightarrow 2^+$ decays to $(m_\nu)$ and $(\eta)$ are comparable to those of $0^+ \rightarrow 0^+$ decays. At the same time it was noted that the $0^+ \rightarrow 2^+$ decay is relatively more sensitive to $(\lambda)$. We remind the reader that the observation of $0^+ \rightarrow 2^+$ transition does not establish the presence of right-handed currents. For a more complete analysis one should consider not only the right-handed currents, but supersymmetric contribution as well (see the comment at the end of section 6).

The right-handed current mechanisms are associated with many different NMES. Within the nuclear shell model they were evaluated just for $0\nu\beta\beta$-decay of $^{48}\gamma\text{Ca}$ to the final ground state [73]. The VAMPIR approach was exploited to calculate them in the case of $^{76}\gamma\text{Ge}$ [360]. Many calculations of NMES related to right-handed current mechanisms were performed within the QRPA and for all nuclei of experimental interest [190, 358, 359, 361, 362]. However, they do not include recent improvements concerning the fixing of parameter space of nuclear Hamiltonian [290–292, 302] and concerning the description of two-nucleon short-range correlations [174].

There is revived interest in heavy neutrino mass ($\eta^{L,R}_N$) and $R$-parity breaking supersymmetric ($\eta_\nu$) mechanisms

\(^{11}\) As we have already mentioned, transitions to non-zero angular-momentum final states can only occur via the leptonic $\mu$, $j_\mu$ interference term associated with the $(\lambda)$ and $(\eta)$ parameters. In this section we will refer to it as the right-handed current contribution.
of the $0\nu\beta\beta$-decay. The NMEs governing these mechanisms were calculated only within the QRPA [28, 101] with the exception of the PHFB calculation for the heavy neutrino-mass mechanism [363], which, however, neglects the role of induced hadron currents.

Recently, NMEs $M_{\alpha \nu}^{\nu}$ (light neutrino-mass mechanism), $M_{\alpha N}^{\nu}$ (heavy neutrino-mass mechanism), $M_{\alpha \lambda}^{\nu}$ (trilinear $R$-parity breaking SUSY mechanism) and $M_{\alpha \eta}^{\eta}$ (squark mixing mechanism) were calculated for the $0\nu\beta\beta$-decay of $^{76}\text{Ge}$, $^{82}\text{Se}$, $^{100}\text{Mo}$, $^{130}\text{Te}$ and $^{136}\text{Xe}$ within the SRQRPA [191, 330]. Unlike in previous calculations the particle–

The LNV parameters $\langle m_\nu \rangle$, $\eta_\eta$, $\eta_L$ and $\eta_R$ deduced from current lower bounds on the half-life ($T_{1/2}^{\nu\beta\beta}$) of $0\nu\beta\beta$-decay for $^{76}\text{Ge}$ [36], $^{82}\text{Se}$, $^{100}\text{Mo}$ [38], $^{130}\text{Te}$ [39] and $^{136}\text{Xe}$ [40] are shown in table 5. The SRQRPA NMEs of table 4, in particular those evaluated with CD-Bonn potential and $g_\lambda$ were considered. We see that upper limits on $|\langle m_\nu \rangle|$ and $|\eta_\eta|$ from CUORICINO ($^{130}\text{Te}$) [39] and KamLAND-Zen ($^{136}\text{Xe}$) [40] experiments are already comparable to those from the Heidelberg–Moscow ($^{76}\text{Ge}$) experiment [36]. The running KamLAND-Zen experiment is slightly more sensitive to the $0\nu\beta\beta$-decay signal than the already finished Heidelberg–Moscow experiment in the case of the gluino exchange mechanism.

Table 4. NMEs $M_{\alpha \nu}^{\nu}$ (light Majorana neutrino-mass mechanism), $M_{\alpha N}^{\nu}$ (heavy Majorana neutrino-mass mechanism), $M_{\alpha \lambda}^{\nu}$ (trilinear $R$-parity breaking SUSY mechanism) and $M_{\alpha \eta}^{\eta}$ (squark mixing mechanism) for the $0\nu\beta\beta$-decays of $^{76}\text{Ge}$, $^{82}\text{Se}$, $^{100}\text{Mo}$, $^{130}\text{Te}$ and $^{136}\text{Xe}$ within the self-consistent renormalized quasiparticle random phase approximation (SRQRPA). $R = 1.1\text{ fm } \Lambda_{11}^{1/2}$ is assumed.

| Nucleus  | NN pot. | $g_\lambda^\text{eff}$ | $|M_{\alpha \nu}^{\nu}|$ | $|M_{\alpha N}^{\nu}|$ | $|M_{\alpha \lambda}^{\nu}|$ | $|M_{\alpha \eta}^{\eta}|$ |
|----------|---------|------------------|-----------------|-----------------|-----------------|-----------------|
| $^{76}\text{Ge}$ | Argonne | 1.25  | 5.44 | 265 | 700 | 718 |
|          | CD-Bonn | 1.00  | 4.39 | 196 | 461 | 476 |
|          | 1.25  | 5.82 | 412 | 596 | 728 |
| $^{82}\text{Se}$ | Argonne | 1.25  | 5.29 | 263 | 698 | 710 |
|          | CD-Bonn | 1.00  | 4.18 | 193 | 455 | 465 |
|          | 1.25  | 5.66 | 408 | 594 | 720 |
| $^{100}\text{Mo}$ | Argonne | 1.25  | 4.79 | 260 | 690 | 683 |
|          | CD-Bonn | 1.00  | 3.91 | 192 | 450 | 449 |
|          | 1.25  | 5.15 | 404 | 589 | 691 |
| $^{130}\text{Te}$ | Argonne | 1.25  | 4.18 | 240 | 626 | 620 |
|          | CD-Bonn | 1.00  | 3.34 | 177 | 406 | 403 |
|          | 1.25  | 4.70 | 385 | 540 | 641 |
| $^{136}\text{Xe}$ | Argonne | 1.25  | 2.75 | 160 | 426 | 418 |
|          | CD-Bonn | 1.00  | 2.19 | 117 | 277 | 271 |
|          | 1.25  | 3.36 | 172 | 460 | 459 |
|          | 1.00  | 2.61 | 125 | 297 | 297 |

The $\eta_\lambda$ and $\eta_\eta$ parameters are related to the $R_\eta$-couplings $\lambda_{111}^{\nu}$ and products of the trilinear $R_\eta$-couplings $\lambda_{111}^{011}$ ($k = 1, 2, 3$), respectively. The current limits on them, presented in table 5, have been derived under the conventional simplifying assumptions. We assumed all the squark masses and the trilinear soft SUSY breaking parameters $A_\eta$ to be approximately equal to a common SUSY breaking scale $\Lambda_{\text{SUSY}}$. Thus we approximately have [202]

$$\lambda_{111}^{011} \lambda_{111}^{011} \leq \epsilon_k \frac{1}{\sqrt{T_{1/2}^{\nu\beta\beta} G_{\text{et al.}}^{011}}} \frac{1}{|M_{\alpha \eta}^{\eta}|} \left( \frac{\Lambda_{\text{SUSY}}}{100 \text{ GeV}} \right)^3$$

with $\epsilon_k = (1.8 \times 10^{31}; 94.2; 3.9)$ calculated for the current quark masses $m_d = 9 \text{ MeV}$, $m_s = 175 \text{ MeV}$ and $m_u = 4.2 \text{ GeV}$. In the case of gluino and neutralino $R_\eta$ SUSY mechanisms of the $0\nu\beta\beta$-decay we obtain

$$\lambda_{111}^{011} \leq \frac{1}{\sqrt{T_{1/2}^{\nu\beta\beta} G_{\text{et al.}}^{011}}} \frac{1}{|M_{\alpha \eta}^{\eta}|} \left( \frac{m_{\tilde{q}}}{100 \text{ GeV}} \right)^2 \left( \frac{m_{\tilde{g}}}{100 \text{ GeV}} \right)^{1/2}$$

$$\lambda_{111}^{011} \leq \frac{1}{\sqrt{T_{1/2}^{\nu\beta\beta} G_{\text{et al.}}^{011}}} \frac{1}{|M_{\alpha \eta}^{\eta}|} \left( \frac{m_g}{100 \text{ GeV}} \right)^2 \left( \frac{m_{\tilde{X}}}{100 \text{ GeV}} \right)^{1/2}$$

with $m_{\tilde{q}} \simeq m_{\tilde{g}} \simeq m_{\tilde{e}} \simeq m_{\tilde{X}} = \Lambda_{\text{SUSY}}$. $m_{\tilde{q}}$, $m_{\tilde{g}}$, $m_{\tilde{e}}$ and $m_{\tilde{X}}$ are masses of squark, gluino, selectron and neutralino, respectively. This approximation is well motivated by the constraints from the flavor changing neutral currents.

It goes without saying that the calculated NMEs of light neutrino-mass and exotic mechanisms of the $0\nu\beta\beta$-decay depend on the assumption about the nuclear model. In order to improve their reliability and reliability of the upper limits
on the $0\nu\beta\beta$-decay LNV parameters, further investigations are necessary.

### 11.3. Two or more competing mechanisms

There is a general consensus that a measurement of the $0\nu\beta\beta$-decay in one isotope does not allow us to determine the underlying physics mechanism. Complementary measurements in different isotopes are very important especially for the case there are competing mechanisms of the $0\nu\beta\beta$-decay.

In the case of coexisting mechanisms with identical phase-space factors, equation (164) is generalized as

$$
\left( T_{1/2}^{0\nu(i)} \right)^{-1} = G^{0\nu}(i) \sum_k |\eta_k M^0_k(i)|^2.
$$

(167)

Here, $g_A^{\text{eff}} = g_A$ is assumed. The parameters $\eta_k$ may take either sign leading to constructive or destructive interference in the decay amplitude, if CP conservation is assumed. In the general case of CP violation they include complex phases. By exploiting the fact that the associated NMEs are target dependent, given definite experimental results on a sufficient number of targets, in principle one can determine or sufficiently constrain all LNV parameters including the light neutrino–mass term.

In [330] up to four coexisting mechanisms for the $0\nu\beta\beta$-decay, mediated by light Majorana neutrino exchange ($\eta_L$), heavy Majorana neutrino exchange ($\eta_H$), R-parity breaking supersymmetry ($\eta_\lambda$), and squark–neutrino ($\eta_{\tilde{q}N}$) were considered. Both, constructive or destructive interference in the decay amplitude and the $0\nu\beta\beta$-decay in four different candidate nuclei ($^{76}$Ge, $^{82}$Se, $^{100}$Mo, $^{130}$Te) with NMEs given in table 4 were assumed. It was found that, unfortunately, current NME uncertainties appear to prevent a robust determination of the relative contribution of each mechanism to the decay amplitude, even assuming accurate measurements of $0\nu\beta\beta$-lifetime.

Another important feature in analysis of two or more competing mechanisms was pointed out in [364]. For example, in the case of two active mechanisms represented by the LNV parameters $\eta_L = (m_\nu)/m_\nu$ and $\eta_R$, assuming the measurement of the $0\nu\beta\beta$-lifetime of two isotopes ($i = ^{76}$Ge, $^{130}$Te) and CP conservation, one obtains four sets (phases $++$, $+-$, $-+$, $-$) of two linear equations:

$$
\frac{\pm 1}{\sqrt{T_{1/2}^{0\nu}}(i) G^{0\nu}(i)} = \frac{|m_\nu|}{m_e} M_\nu(i) + \eta_L M^R(i). 
$$

(168)

It was found that this improved analysis leads to completely different results compared with those of one mechanism at a time. By making the additional assumption that the $0\nu\beta\beta$-decay of $^{76}$Ge was measured with half-life given in [41, 42] the two different solutions for $|m_\nu|$ are plotted as a function of $\xi$, where

$$
\xi = \frac{|M_\nu^{0\nu}(^{130}\text{Te})|}{|M_\nu^{0\nu}(^{76}\text{Ge})|} \sqrt{\frac{T_{1/2}^{0\nu}(^{130}\text{Te}) G^{0\nu}(^{130}\text{Te})}{T_{1/2}^{0\nu}(^{76}\text{Ge}) G^{0\nu}(^{76}\text{Ge})}}.
$$

(169)

in figure 25. The parameter $\xi$ represents the unknown half-life of the $0\nu\beta\beta$-decay of $^{130}$Te. We note that for $\xi = 1$ the solution
for active only light neutrino-mass mechanism is reproduced and that $\xi = 0$ means non-observation of the $0\nu\beta\beta$-decay for a considered isotope. By glancing at figure 25 the obtained results allows us to conclude:

(i) One of the solutions leads to small values of $|\langle m_0 \rangle|$, when all mechanisms add up coherently. This is compatible also with inverted ($m_1 < 50$ meV) or normal ($m_1 \approx$ few meV) hierarchy of neutrino masses.

(ii) The second solution allows quite large values of $|\langle m_0 \rangle|$, even larger than 1 eV. It can be excluded by cosmology and tritium $\beta$-decay. However, if the claim for evidence is ruled out by running GERDA [43, 365, 366], EXO [177] and future experiments [31], the values of two solutions will become smaller and perhaps it will no longer be possible to exclude this solution.

(iii) There is a possibility that the non-observation of the $0\nu\beta\beta$-decay for some isotopes could be in agreement with a value of $|\langle m_0 \rangle|$ in the sub-eV region.

(iv) The obtained results are sensitive to the accuracy of measured half-lives and to uncertainties in calculated NMEs.

Other possibilities of getting information about the different LNV parameters in the case of competing $0\nu\beta\beta$-decay mechanisms were discussed in [191]. First, two competitive mechanisms, namely light left-handed Majorana neutrino exchange and heavy right-handed Majorana neutrino exchange, were considered. As the interference term is negligible the $0\nu\beta\beta$-decay half-life for a given isotope is written as

$$\left(T_{1/2}^{0\nu}(i)G^{0\nu}(i)\right)^{-1} = |\eta_i M^{0\nu}(i)|^2 + |\eta_N M^R(i) N^{0\nu}(i)|^2,$$

(170)

where the index $i$ denotes the isotope. As we have mentioned in section 5.2, the interference between the left- and right-handed currents is small. Given a pair of nuclei, solutions for $|\eta_1|^2$ and $|\eta_N|^2$ can be found by solving a system of two linear equations. From the ‘positivity’ conditions ($|\eta_1|^2 > 0$ and $|\eta_N|^2 > 0$) it follows that the ratio of half-lives is within the range [191]

$$\frac{G^{0\nu}(i)|M^{0\nu}(i)|^2}{G^{0\nu}(j)|M^{0\nu}(j)|^2} \leq \frac{T_{1/2}^{0\nu}(i)}{T_{1/2}^{0\nu}(j)} \leq \frac{G^{0\nu}(i)|M^{0\nu}(i)|^2}{G^{0\nu}(j)|M^{0\nu}(j)|^2}.$$

(171)

Surprisingly, physical solutions are possible only if the ratio of the half-lives, in particular of three considered isotopes $^{76}$Ge, $^{100}$Mo and $^{130}$Te, takes values in very narrow intervals [191].

The $0\nu\beta\beta$-decay can be triggered also by two competitive mechanisms whose interference contribution to the decay rates is non-negligible. As an example the light Majorana neutrino-mass and gluino exchange mechanisms were considered in [191]. We have

$$\left(T_{1/2}^{0\nu}(i)G^{0\nu}(i)\right)^{-1} = |\eta_i M^{0\nu}(i)|^2 + |\eta_N M^R(i) N^{0\nu}(i)|^2 + 2 \cos \alpha |\eta_N| |\eta_i||M^{0\nu}(i)||M^R(i)|^2.$$

(172)

Here, $\alpha$ is the relative phase of $\eta_i$ and $\eta_N$. From (172) it is possible to extract the values of $|\eta_i|^2$, $|\eta_N|^2$ and $\cos \alpha$, setting up a system of three equations with these three unknowns as input the data on the half-lives of three different nuclei. Results of [191] show that using prospective upper bounds on the absolute scale of neutrino masses, stringent constraints on some of new physics mechanisms, which interfere destructively with light neutrino-mass mechanism, can be found or these scenarios can even be excluded.

12. Resonant neutrinoless double electron capture

As has already been mentioned in section 2, the resonant $0\nu$ECEC was already considered by Winter [50] in 1955 as a process that would demonstrate the Majorana nature of neutrinos and the violation of the total LN. The two asterisks denote the possibility of leaving the system in an excited nuclear and/or atomic state. The energy excess given by the $Q$-value of the initial atom is carried away by emission of x-rays (or Auger electrons) as the daughter atom has two electron holes and by emission of a single or few photons due to de-excitation of the final nucleus.

The possibility of a resonant enhancement of the $0\nu$ECEC in the case of a mass degeneracy between the initial and final atoms was pointed out by Bernabéu, De Rujula and Jarlskog as well as by Vergados about 30 years ago [51, 52]. The half-life of the process was estimated by considering non-relativistic atomic wave functions at nuclear origin, simplified evaluation of corresponding NME and assuming that the degeneracy parameter $\Delta = M_{A,Z} - M_{A,Z}^{\ast}$, being the difference of masses of the initial and final excited atoms with masses $M_{A,Z}$ and $M_{A,Z}^{\ast}$, varies from zero to 10 keV (representing the accuracy of atomic mass measurement at that time). The range of $\Delta$ induced uncertainty of about 5 orders in magnitude in calculated $0\nu$ECEC half-life. A list of promising isotopes based on the degeneracy requirement associated with arbitrary nuclear excitation was presented. The $^{112}$Sn $\rightarrow^{112}$Cd resonant $0\nu$ECEC transition was identified as a good case.

In 2004 Sujkowski and Wycech [69] and Lukaszuk et al. [368] analyzed the resonant $0\nu$ECEC process for nuclear $0^+ \rightarrow 0^+$ transitions accompanied by emission of a single photon. By assuming $|\langle m_0 \rangle| = 1$ eV and $\sigma$ error in the atomic mass determination the resonant $0\nu$ECEC rates of six selected isotopes were calculated by considering the perturbation theory approach. The lowest $0\nu$ECEC half-life was found for $^{152}$Gd.

The main limitation in identifying promising isotopes for experimental search of $0\nu$ECEC has been poor experimental accuracy of measurement of $Q$-values, which until recently were known with uncertainties of 1–10 keV only [369]. The resonance enhancement can increase the probability of capture by many orders of magnitude. Therefore, accurate mass difference measurements are of great importance in order to narrow down the possibilities. Progress in precision measurement of atomic masses with Penning traps [56, 370, 371] has revived interest in the old idea of the resonance $0\nu$ECEC capture. Recently, accuracy of $Q$-values at around 100 eV was achieved [55, 372–382], which has already allowed some isotopes to be excluded from the list of the most promising candidates (e.g. $^{112}$Sn and $^{164}$Er) for searching for the $0\nu$ECEC.

Recently, significant progress has been achieved also in theoretical description of the resonant $0\nu$ECEC [54, 70, 383]. A new theoretical framework for the calculation of resonant
0νECEC transitions, namely the oscillation of stable and quasi-stationary atoms due to weak interaction with violation of the total LN and parity, was proposed in [54, 70]. The 0νECEC transition rate near the resonance is ofBreit–Wigner form,
\[ \Gamma_{ab}^{0νECEC} (J^π) = \frac{\left| V_{ab}(J^π) \right|^2}{\Delta^2 + \frac{1}{4} \Gamma_{1ab}^2} \Gamma_{1ab}, \]  
(173)
where \( J^π \) denotes angular momentum and parity of final nucleus. The degeneracy parameter can be expressed as \( \Delta = Q - B_{ab} - E_{f} - E_{g} \), \( Q \) stands for a difference between the initial and final atomic masses in ground states and \( E_{f} \) is the excitation energy of the daughter nucleus. \( B_{ab} = E_{a} + E_{b} + E_{C} \) is the energy of two electron holes, whose quantum numbers \((n, j, l)\) are denoted by indices \( a \) and \( b \), and \( E_{C} \) is the interaction energy of the two holes. The binding energies of single electron holes \( E_{ab} \) are known with an accuracy of a few eV [384]. The width of the excited final atom with the electron holes is given by
\[ \Gamma_{ab} = \Gamma_{a} + \Gamma_{b} + \Gamma^{*}. \]  
(174)
Here, \( \Gamma_{a,b} \) is one-hole atomic width and \( \Gamma^{*} \) is the de-excitation width of daughter nucleus, which can be neglected. Numerical values of \( \Gamma_{ab} \) are about up to a few tens of eV [385].

For light neutrino-mass mechanism and favorable cases of capture of s1/2 and p1/2 electrons the explicit form of LNV amplitude associated with nuclear transitions \( 0^+ \rightarrow J^π = 0^{\pm} \), 1\(^{\pm}\) is given in [54]. By factorizing the electron shell structure and NME one obtains
\[ V_{ab}(J^π) = \frac{1}{4\pi} G^2_p m_e q_{0} \frac{g^2_{\nu}}{R} (F_{ab}) M^{0νECEC}(J^π). \]  
(175)
Here, \( (F_{ab}) \) is a combination of averaged upper and lower bispinor components of the atomic electron wave functions [54] and \( M^{0νECEC}(J^π) \) is the NME. We note that by neglecting the lower bispinor components \( M^{0νECEC}(0^+) \) takes the form of the 0νββ-decay NME for ground-state to ground-state transition after replacing isospin operators \( t^- \) by \( t^* \).

There is a straightforward generalization of the LNV potential \( V_{ab}(0^+) \) in (175) for the heavy neutrino exchange, the trilinear \( R \)-parity breaking with gluino and neutralino exchange and squark–neutrino mechanism. It is achieved by replacements \( \eta_{1} = (m_{1})/m_{Z} \) with \( \eta_{1} (k = N, \lambda', \tilde{q}) \) and \( M_{0}^{0νECEC}(0^+ \nu) \) with \( M_{0}^{0νECEC}(0^+) \). The 0νECEC leading to final states different from 0\(^{+}\), possible only in the presence of weak right-handed currents due to the leptonic \( J_{L} - J_{R} \) interference, has been discussed in [383].

New important theoretical findings with respect to the 0νECEC were achieved in [54]. They are as follows: (i) Effects associated with the relativistic structure of the electron shells reduce the 0νECEC half-lives by almost one order of magnitude. (ii) The capture of electrons from the \( np_{1/2} \) states is only moderately suppressed in comparison with the capture from the \( ns_{1/2} \) states, unlike in the non-relativistic theory. (iii) For light neutrino-mass mechanism selection rules appear to require that nuclear transitions with a change in the nuclear spin \( J \geq 2 \) are strongly suppressed. We note that, if right-handed currents are considered, selection rules are modified allowing also \( J \neq 0^+ \) [383]. (iv) New transitions due to the violation of parity in the 0νECEC process were proposed. For example, nuclear transitions \( 0^+ \rightarrow 0^\pm, 1^\pm \) are compatible with a mixed capture of \( s^- \) and \( p^- \) wave electrons. (v) The interaction energy of the two holes \( E_{C} \) has to be taken into account by evaluating a mass degeneracy of initial and final atoms. (vi) Based on the most recent atomic and nuclear data and by assuming \( M^{0νECEC}(J^π) = 6 \) the 0νECEC half-lives were evaluated and the complete list of the most appropriate isotopes for further experimental study was provided. Some isotopes such as \( ^{156}\text{Dy} \) have several closely lying resonance levels. A more accurate measurement of the \( Q \)-value of \( ^{156}\text{Dy} \) by the Heidelberg group confirmed the existence of multiple-resonance phenomenon for this isotope [379]. (vii) In the unitary limit some 0νECEC half-lives were predicted to be significantly below the 0νββ-decay half-lives for the same value of \( (m_{\nu}) \). The probability of finding resonant transition with low 0νECEC half-life was evaluated. (vii) The process of the resonant neutrinoless double electron production (0νEPEP), i.e. neutrinoless DBD to two bound electrons, namely
\[ (A, Z) \rightarrow (A, Z + 2)^{\pm} + e^- + e^+, \]  
(176)
was proposed and analyzed. This process was found to be unlikely as it requires that a \( Q \)-value is extremely fine tuned to a nuclear excitation. The two electrons must be placed into any of the upper non-occupied electron shells of the final atom leaving only restricted possibility to match to a resonance condition.

A detailed calculation of the 0νECEC of \( ^{152}\text{Gd}, ^{164}\text{Er} \) and \( ^{180}\text{W} \) associated with the ground-state to ground-state nuclear transitions was performed in [55, 386, 387]. Improved measurements of \( Q \)-value for these transitions with accuracy of about 100 eV [380–382, 386] were considered. The NMEs of \( ^{152}\text{Gd} \rightarrow ^{152}\text{Sm}, ^{164}\text{Er} \rightarrow ^{164}\text{Dy} \) and \( ^{180}\text{W} \rightarrow ^{180}\text{Hf} \) transitions were calculated within spherical and deformed QRPA [386, 387]. The obtained results exclude 164Er and \( ^{180}\text{W} \) from the list of prospective candidates to search for the 0νECEC. The 0νECEC half-life of \( ^{152}\text{Gd} \) is 2–3 orders of magnitude longer than the half-life of 0νββ decay of \( ^{76}\text{Ge} \) corresponding to the same value of \( (m_{\nu}) \) and is the smallest known half-life among known 0νECEC transitions at present.

The transition of \( ^{106}\text{Cd} \) to an excited state of \( ^{106}\text{Pd} \) with the nuclear excitation energy of 2717.59 keV was calculated in [388] by making the assumption that this is the 0\(^{+}\) state. However, it was noted in [54] that, as long as this level \( \gamma \)-decays by 100% into the 3\(^{+}\) state at 1557.68 keV, this possibility is excluded.

There is also increased experimental activity in the field of the resonant 0νECEC [71, 389–394]. The resonant 0νECEC has some important advantages with respect to experimental signatures and background conditions. The de-excitation of the final excited nucleus proceeds in most cases through a cascade of easy to detect rays. A two- or even higher-fold coincidence setup can cut down any background rate right from the beginning, thereby requiring significantly less active or passive shielding [54]. A clear detection of these \( \gamma \) rays would already signal the resonant 0νECEC without any doubt, as there are no background processes feeding those particular
Table 6. A comparison of the neutrinoless DBD and the resonant neutrinoless double electron capture for light neutrino-mass mechanism. The LNV amplitude $V_{ab}(J^\pi)$ is given in equation (175).

| \(0\nu\beta\beta\text{-decay}\) | \(0\nu\text{ECEC}\) |
|-----------------------------|-----------------------------|
| Definition | \((A, Z) \rightarrow (A, Z + 2) + e^- + e^-\) | \((A, Z) + e^-_c + e^-_c \rightarrow (A, Z - 2)^{**}\) |
| Formalism | Perturbation field theory | Oscillation of atoms \([54, 70]\) |
| Half-life | \(\frac{1}{T_{1/2}} = \left| \frac{(m_e)}{m_e} \right|^2 G^{0\nu}(M^{0\nu}(J^\pi))^2\) | \(\ln 2 \frac{T_{1/2}^{0\nu\text{ECEC}}}{T_{1/2}^{0\nu\beta\beta}} = -\frac{\Gamma_1^{0\nu\beta\beta} m_e}{\Gamma_1^{0\nu\text{ECEC}}} (M_{A,Z} - M_{A,Z-2})^2 + \frac{1}{4} \Gamma_{ab}^{0\nu\beta\beta} (A, Z)\) |
| Nucl. trans. | \(0^+ \rightarrow 0^+, 2^+\) | Mass difference \(< 10^{-25} - 10^{-27} \) y |
| Fav. at. syst. | Large \(Q\)-value (3–4 MeV) | Many orders in magn. |
| Uncert. in \(T_{1/2}^{0\nu}\) | Factor \(\sim 4–9\) due to calc. of NME | X-rays or Auger el. plus nucl. de-excitation |
| Exp. sign. | Peak at end of sum of two cl. energy spectra | \(>10^{19} - 10^{20}\) y |
| \(T_{1/2}^{0\nu-\exp}\) | \(>10^{19} - 10^{20} \) y | Small exper. yet |
| Exp. act. | Const. of \((0.1–1\text{ ton})\) exp. with sensitivity to inverted hierarchy of neutrino masses | 2\(\nu\beta\beta\text{-decay}\) upon resolution of exp. |
| Background | 2\(\nu\beta\beta\text{-decay}\) upon resolution of exp. | 2\(\nu\text{ECEC}\) is strongly suppressed |

A comparison of the \(0\nu\text{ECEC}\) with the \(0\nu\beta\beta\text{-decay}\) is presented in table 6. It is maintained that these two LNV processes are quite different and at different levels of both theoretical and experimental investigation. Precise measurements of \(Q\)-values between the initial and final atomic states, additional spectroscopic information on the excited nuclear states (energy, spin and parity) and reliable calculation of corresponding NMEs are highly required to improve predictions of half-lives of the resonant \(0\nu\text{ECEC}\). It is expected that the accuracy of 10 eV in the measurement of atomic masses will be achievable in the near future. The electron binding energy depends on the local physical and chemical environment. An interesting question is whether it is possible and, if so, how to manage the atomic structure in such a way as to implement the degeneracy of the atoms and create conditions for the resonant enhancement, as discussed in a recent work \([54]\).

13. Concluding remarks

In this review we discussed in some detail the LN violating neutrinoless DBD and other similar transitions, involving various nuclear isotopes for which ordinary beta decay and electron capture are forbidden or highly suppressed. Both theoretical and experimental aspects were considered.

We have seen that this is a process with a long and interesting history with important implications for physics and cosmology, but its observation is still elusive. It is an exotic process, which requires physics beyond the SM. At present a complete theory is missing and, thus, to motivate and guide the experiments we examined a number of reasonable viable models, beyond the SM, in particular in connection with the neutrino-mass matrix and mixing (see sections 3 and 4). Such models predict that LN violation, and consequently neutrinoless DBD, must occur at some level, implying that

nuclear levels. It is worth noting that LN conserving ECEC with emission of two neutrinos,

\[(A, Z) + e^-_c + e^-_c \rightarrow (A, Z - 2)^{**} + v_e + v_e, \quad (177)\]

is strongly suppressed due to almost vanishing phase space \([51, 52, 54]\). The ground-state to ground-state resonant \(0\nu\text{ECEC}\) transitions can be detected by monitoring the x-rays or Auger electrons emitted from excited electron shells of the atom. This can be achieved, e.g., by calorimetric measurements.

Till now, the most stringent limits on the resonant \(0\nu\text{ECEC}\) were established for \(^{74}\text{Se}\) \([393]\), \(^{106}\text{Cd}\) \([392]\) and \(^{112}\text{Sn}\) \([390]\). The ground state of \(^{74}\text{Se}\) is almost degenerate with the second excited state at 1204 keV in the daughter nucleus \(^{74}\text{Ge}\), which is a \(2^+\) state \([395]\). The \(2\gamma\)-ray cascade has been searched for using the low-radioactivity detector setup at the Comenius University in Bratislava and 3 kg of natural selenium. A lower limit for the half-life of \(T_{1/2}^{0\nu\text{ECEC}} > 4.3 \times 10^{19} \) y was determined \([393]\), which is slightly larger than the value reported in \([389]\). The resonant \(0\nu\text{ECEC}\) transition to the \(0^+_2\) excited state in \(^{112}\text{Cd}\) (1871.0 keV) has been investigated in an experiment performed with natural tin in the Modane Underground Laboratory. A lower bound on half-life of \(0.92 \times 10^{20} \) y was established. It is worth noting that a new mass measurement \([375]\) has excluded complete mass degeneracy for a \(^{112}\text{Sn}\) decay and has therefore disfavored significant resonant enhancement of the \(0\nu\text{ECEC}\) mode for this transition. Within the TGV experiment in Modane \([392]\) interest has also arisen in the \(0\nu\text{ECEC}\) resonant decay mode of \(^{106}\text{Cd}\) (KL-capture) to the excited 2741 keV state of \(^{106}\text{Pd}\). The spin value of this final state was unknown and it was assumed to be \(J = (1, 2)^+.\) After measurements had begun a new value for the spin of the 2741 keV level in \(^{106}\text{Pd}\) of \(J = 4^+\) was adopted, but, following recent theoretical analysis \([54]\), this channel is now disfavored. Nevertheless the most stringent limit on the \(0\nu\text{ECEC}\) half-life of 1.1 \(\times 10^{20} \) y was reported \([392]\).
the neutrinos are Majorana particles. These models, however, cannot provide a precise determination of the parameters involved, such as the absolute scale of the neutrino mass. Hence they must be extracted from the experiments, if and when reliable accurate results become available (see section 4 for the neutrino mass). The observed values may, then, be used to differentiate between such models and, hopefully, lead to the ultimate theory.

In order to achieve this goal first such processes must be definitely observed. Then the obtained results must be analyzed by considering the various mechanisms implied by the above models, see sections 5 and 6 for mechanisms involving intermediate neutrinos and other particles respectively. This, however, can only be done if the corresponding NMEs are evaluated with high precision, accuracy and reliability. We have seen that this is a formidable task, since the nuclei that can undergo DBD have rather complicated structure.

The evaluation of the NMEs involves two steps. In the first step the effective transition operators for each mechanism are derived (see section 9). Special attention must be paid to the proper treatment of these operators at short distances (short-range correlations, nucleon current corrections, inclusion of hadrons other than nucleons, etc). The second step consists of selecting the proper nuclear model for constructing the wave functions involved in the evaluation of the NMEs. Practically all models available in the nuclear theory artillery have been employed. The most prominent are the large basis shell model, the various refinements of the quasi-particle random phase approximation (QRPA) and the interacting boson model (IBM). The essential features of these models and the numerical values of the obtained NMEs have been summarized in section 10. We have seen that great progress has been made in this direction in recent years and it is encouraging that the NMEs obtained with these vastly different nuclear models tend to converge.

We have discussed in section 8 the ongoing, planned and future experiments. We have witnessed great progress in tackling the various background problems, improving the energy resolution and preparing large masses of the needed isotopes. It is thus expected that half-lives of the order of $10^{26}\text{y}$ can be achieved and, consequently, a sensitivity of a few tens of meV for the average neutrino mass can be reached. This may be sufficient to differentiate between the normal and inverted hierarchy scenarios (see section 4). Furthermore, we have seen that various nuclear charge changing nuclear reactions can be employed in an effort to experimentally extract useful information or provide checks for the NMEs.

It is clear that the observation of neutrinoless DBD will be a great triumph for physics and experimental physics in particular. It will demonstrate that the neutrinos are Majorana particles and there exist LNV interactions in the universe. This, however, will not be the end of the story. The data should be analyzed in such a way to determine the mechanism responsible for this process and, in particular, to extract the most important parameter, which is the scale of the neutrino mass. Great progress in this direction has recently been made as briefly exposed in section 11. In order to unambiguously accomplish this goal, however, the accuracy of the NMEs must be further improved.

Finally, recent developments toward the accurate determination of atomic masses, as well as the evaluation of inner shell atomic wave functions and energies, have stimulated interest in experiments involving the resonant neutrinoless double-electron capture, see section 12. This new process, if observed, especially in the case that it leads to negative parity final nuclear states, will greatly facilitate the analysis of determining the dominant mechanism involved in neutrinoless DBD.

Acknowledgments

The work of one of the authors (JDV) was supported in part by UNILHC PITN-GA-2009-237920 and the DIBOSON Thalis project. FŠ acknowledges the support by the VEGA Grant agency under the contract No 1/0876/12. The authors express their sincere thanks to Rastislav Dvornicky and Rastislav Hodak for the preparation of some of the figures.

Note added in proof. While our article was in press a paper has appeared by the EXO collaboration [396]. It reported that no signal has appeared in a search for neutrinoless double-beta decay of $^{136}\text{Xe}$ with an exposure of 32.5 kg-yr and a background of $1.5 \times 10^{-3}\text{kg}^{-1}\text{keV}^{-1}\text{y}^{-1}$ in the $\pm \sigma$ region of interest. This implies a lower limit on the half-life, $T^{\beta\beta \nu}_0 \geq 1.6 \times 10^{25}\text{y}$ at 90% CL, corresponding to an effective Majorana mass of less than $140\text{–}380\text{meV}$, depending on the nuclear matrix element.

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