Neutron Electric Dipole Moment with Domain Wall Quarks

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We present preliminary results for nucleon dipole moments computed with domain wall fermions. Our main target is the electric dipole moment of the neutron arising from the $\theta$ term in the gauge part of the QCD lagrangian. The calculated magnetic dipole moments of the proton and neutron are in rough accord with experimental values.

1. Introduction

One of the most intriguing aspects of QCD is that it allows a T and P-odd, gauge invariant interaction term, the so called $\theta$ term. The presence of such a term has the profound effect that the Strong interactions violate CP. One of the best ways to monitor this is by searching for the neutron electric dipole moment, $d_N$. In the Standard Model, the CP-odd phase of the CKM mixing matrix can also trigger a non-vanishing value for $d_N$, but this can not occur at one loop order (in the Weak interaction) and is consequently estimated to be $\leq 10^{-30}$ e·cm, many orders of magnitude below the current experimental bound $^{11}$, $d_N = |\bar{d}_N| < 6.3 \times 10^{-26}$ e·cm. Using this experimental bound along with various estimates of $d_N$ due to the $\theta$ term then invariably implies that the CP-odd parameter in the QCD action, $\theta \leq 10^{-10}$, is exceedingly and indeed unnaturally small. Since there is no good reason for this number to be so different from unity, its minuteness requires extraordinary fine-tuning. This is often termed the Strong CP problem.

To translate the above experimental bound to a constraint on the fundamental $\theta$ parameter requires evaluation of nucleon matrix elements, therein lattice QCD enters.

There have been several past attempts to calculate $d_N$, in the continuum and on the lattice.

1. V. Baluni $^2$ computed $d_N$ in the frame-work of the MIT bag model obtaining $d_N \simeq 8.2 \cdot 10^{-16}\theta e\cdot cm$.

2. Crewther et al. $^3$, using an effective chiral lagrangian found $d_N \propto \theta M_u^2 \ln(M_u^2) \simeq 5.2 \cdot 10^{-16}\theta e\cdot cm$.

3. Pospelov and Ritz $^4$, using QCD sum rules techniques, found $d_N = 1.2 \times 10^{-16}\theta e\cdot cm$.

4. Aoki and Gocksch $^5$ were the first to try a pioneering lattice calculation of the $d_N$ in the quenched approximation.

A non-zero $d_N$ is induced by the $\theta$-term in the QCD gauge action.

$$S_\theta = \int d^4x \ i\theta \frac{g^2}{32\pi^2} \text{tr} \left[ G(x) \hat{G}(x) \right] = i\theta Q. \ (1)$$

where $Q$ is the topological charge of the QCD vacuum. Note the factor of $i$ in the $\theta$ term makes the action complex and therefore difficult to handle in lattice simulations for arbitrary $\theta$. If $\theta$ is small, as discussed below, the difficulty can be avoided.

Using the axial anomaly, one can replace the CP violating gauge action above with the fermionic action, $S'_\theta = -i\theta \overline{m} \int d^4x P(x)$ where $P(x) = \bar{u}(x)\gamma_5 u(x) + \bar{d}(x)\gamma_5 d(x) + \bar{s}(x)\gamma_5 s(x)$ and $\overline{m}^{-1} = m_u^{-1} + m_d^{-1} + m_s^{-1}$. In $^6$ $d_N$ was evaluated by extracting the spin-up and spin-down neutron masses from the two-point functions obtained by adding to the action $S'_\theta$ and a term corresponding to a constant background electric field oriented in
a fixed spatial direction. Since $\theta$ is small, it is sufficient to consider a single insertion of $S_\nu$ in the 2-point functions.

Since the operator insertion in diagram (a) of [5] is manifestly CP-conserving, its contribution to a CP-odd physical observable (such as electric dipole moment) must vanish. Contributions to $d_N$ therefore must come from diagram (b) of [5] which was ignored in that calculation.

More detailed reasoning for the fact that $d_N$ resides only in fig(b) of [5] was given shortly thereafter by [6]; a significant extension of that analysis appeared recently in [7]. As stressed in [6], the quenched approximation does not justify ignoring diagram (b) of [5].

2. Our computational strategy

There are two important elements to our computational strategy:

(1) Compute the matrix elements of the electromagnetic current between nucleon states, $\langle p', s | J^\mu | p, s \rangle = \bar{u}(p', s) \Gamma_\mu(q^2) u(p, s)$, where

$$
\Gamma_\mu(q^2) = \gamma_\mu F_1(q^2) + i \sigma_\mu q^\nu \frac{F_2(q^2)}{2m} \quad (2)
$$

and use projectors to obtain linear combinations of $F_1$ and $F_2$,

$$
G_E(q^2) = F_1(q^2) + \frac{q^2}{(2m)^2} F_2(q^2) \quad (3)
$$

$$
G_M(q^2) = F_1(q^2) + F_2(q^2) \quad (4)
$$

and $F_3(q^2)/2m$. By forming ratios of $G_M(q^2)$ and $F_3(q^2)$ with $G_E(q^2)$ and taking $q^2 \to 0$, we find both magnetic and electric dipole moments, respectively: $e F_1(0) + a_\mu/2m$ and $d_N = e F_3(0)/2m$ (note, $G_E(0) = F_1(0) = 1$ and 0 for the proton and neutron, respectively, and $a_\mu \equiv F_2(0)$ is the anomalous magnetic moment).

In the above $q^2 \leq 0$, and $m$ is the nucleon mass.

(2) Expand $\langle p', s | J^\mu | p, s \rangle$ to lowest order in $\theta$ and compute $F_3(q^2)$ in each topological sector $\nu$, and then average over all sectors with weight $Q_\nu$.

A disadvantage of this method is that unlike the background-electric-field-method used in [5], our method does not allow a direct calculation of the electric dipole moment, i.e. the value of the form-factor at $q^2 = 0$ since on a finite lattice only the form factor $F_1$ in Eq. 2 can be computed at $q^2 = 0$ [9]. Our method requires extrapolation of the form factors to $q^2 = 0$ from non-vanishing values of $q^2$.

3. Remarks on the quenched case

The QCD partition function in the presence of explicit CP violation is $Z(\eta, \eta) = \int D A_\mu \det[D(m) + i \not\partial \gamma_5] e^{i \eta D(m) - i \eta \gamma_5}$. Setting $\det[D(m)] = 1$, we lose CP violating physics. However, if $\theta$ is small, $\det[D(m) + i \not\partial \gamma_5] = \det[D(m)] [1 + i \not\partial \gamma_5 (\gamma_5 D(m)^{-1})] + \mathcal{O}(\theta^2)$, and we quench as usual by setting $\det[D(m)] = 1$. Considering disconnected insertions of $P = \text{tr}(\gamma_5 D(m)^{-1})$, it seems clear that one can compute the disconnected diagram (b) of [5] using quenched gauge configurations and obtain a non-zero result.

4. The chiral limit

The spectral decomposition of $D^{-1}(m)$ leads to

$$
\sum_{f=1}^{N_f} \text{Tr} \left[ \gamma_5 D^{-1}(m_f) \right] = \frac{n_+ - n_-}{m} = \frac{Q}{m} \quad (5)
$$

for $N_f$ flavors and $n_+$ and $n_-$ the number of right- and left-handed zero modes of $D(m)$. If we trade $Q$ for a disconnected insertion of $-\not\partial P$, $d_N$ will vanish in the chiral limit only if $(\det D(m))^{N_f}$ vanishes, and thus $d_N$ can not vanish in the quenched chiral limit (recall that $\det D(m) \sim m$ for $Q \neq 0$, and contributions to $d_N$ vanish for $Q = 0$).

5. Numerical Results

In Table 1 we summarize results for the ratios of neutron and proton magnetic to proton electric form factors which become the dipole moment in question in the limit $q^2 \to 0$. These results were computed on 280 $N_f = 2$, $m_{sea} = 0.02$, domain wall fermion configurations (separated by 10-15 trajectories) with $m_{cal} = 0.04$ and 0.08 [10]. The lattice size is $16^3 \times 32$, $L_s = 12$, and the...
inverse lattice spacing in the \( m_{sea} = 0 \) limit is \( a^{-1} \approx 1.7 \) GeV. We have averaged over time slices 14-17 and (equivalent) permutations of momenta, \( \vec{p} = (1, 0, 0), (1,1,0), \) and \((1,1,1)\). \( Q \) was computed by integrating the topological charge density after APE smearing the gauge fields (20 sweeps with ape weight 0.45) \[11\]. We are also investigating computing the topological charge from the index defined from the domain wall fermion Dirac operator (strictly valid in the limit \( L_s \to \infty \)).

Considering the crude extrapolations just described, these are roughly consistent with the experimental values \( a_{\mu}^p = 1.79 \) and \( a_{\mu}^N = -1.91 \) (and of course \( d_N \sim 0 \)). The error estimates are statistical uncertainties only. In physical units, \( d_N = -7.4(18.0) \times 10^{-16} \, \theta \, \text{e-cm} \), consistent with the model calculations mentioned above. We note that \( a_{\mu} \) increases in magnitude for both the proton and neutron as \( m_{val} \) decreases. The electric dipole moment of the neutron does not show any significant dependence on the quark mass, within (relatively large) statistical errors. In the near future, we will strive to reduce the statistical error on our determination of \( d_N \) which has already yielded an interesting first-principles bound on the magnitude of \( d_N \).

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