1. Introduction

In the last few years a precise correspondence between certain black holes and quantum states in string theory has been found. In particular, the degeneracies of bound states of D-branes have been calculated and for large charges the result agrees with the Bekenstein-Hawking area law for the black hole entropy \[ S \propto A \]. The configurations that allow for the precise arguments are BPS saturated, i.e. they are supersymmetric black holes with zero temperature. It is interesting to consider also the near-BPS configurations because they exhibit non-trivial thermodynamical properties. There is agreement between string theory and the macroscopic black hole properties in this case \[ \text{near-BPS} \] also, but the reasoning is less securely founded in the microscopic theory. It is therefore important that the counting of microscopic states can be supplemented with comparisons of dynamical processes. The rate of Hawking decay and the precise energy dependence of the radiation in the semi-classical theory is in agreement with that in the string model. These results give strong support to the identification of the microscopic structure of black holes with an effective string also in the near-BPS case; the classical black holes “look” like strings.

The correspondence between the black holes and the effective strings is only possible when the semi-classical results take a very specific form. Thus, the logic can be turned around and we can use the semi-classical calculations as an efficient diagnostic test for the microscopic properties of the non-extreme black holes which are currently inaccessible by other methods in string theory. One might have expected that general black holes behaved very differently from strings but, as we shall see, even the general non-extreme black holes “look” surprisingly like strings. This does not imply that we already understand the precise connection between an effective string theory and non-extreme black holes, but it suggests the existence of a precise description, and some of its features can already be discerned. The situation is reminiscent of the early works on the BPS case which took classical black holes as starting points and found that these solutions exhibited properties that were in striking agreement with expectations from the microscopic string theory.

Subsequently, the D-brane description provided a foundation for those observations.
This contribution is based on work presented in [11–14]. It is organized as follows. In sec. 2 it is explained how some features of an underlying string description can be read off directly from the classical geometry. In sec. 3 we make these ideas explicit in the context of the statistical mechanics of the most general five dimensional black holes. We show that previously known results are recovered in the BPS and near-BPS limits; and we discuss some tests of the extension to the general non-extreme case. The four dimensional case is more involved than the five-dimensional one but, as discussed in [14], most results nevertheless carry over with only minor modifications.

As explained above the most detailed test of the correspondence between black holes and strings is that the spectrum of the Hawking radiation agrees in the two descriptions. In sec. 4 we develop this approach by discussing the features of the field equation of a minimally coupled scalar. In sec. 5 we use these results to find the grey-body factors of the black holes and highlight their string theory interpretation. This provides a test of the geometric picture presented in sec. 2. In sec. 6 we summarize the successes and the limitations of the string description of non-extreme black holes.

2. Thermodynamics of Strings and Geometry of Black Holes

A characteristic feature of strings is that their spectrum divides into two distinct sectors associated with the right (R) and left (L) moving excitations, respectively. The space of states therefore decomposes into a direct product of two spaces. Consequently the entropy, calculated from the degeneracies of these states, is the sum of two contributions:

\[ S = S_R + S_L. \]  

(1)

The condition for this division to be meaningful is that the coupling between the R- and L-moving modes is much weaker than the coupling within each sector. Under the same condition two independent temperatures can be introduced; \( T_{R,L} \) for the R- and L-moving modes, respectively. The temperature of the combined system is related to that of its parts as:

\[ T_H^{-1} = \frac{1}{2} (T_R^{-1} + T_L^{-1}). \]  

(2)

Other thermodynamic potentials similarly split into R- and L-moving contributions: there is one potential for each of the two kinds of modes, and a combined potential for the full system. In sum weakly coupled string theory gives rise to two distinct sets of thermodynamic variables.

The thermodynamics of black holes has a different starting point than the statistical mechanics of strings described above; the thermodynamic variables are identified with geometrical features of space-time rather than specific microscopic states. For example the Bekenstein-Hawking (BH) entropy is related to the area of the outer event horizon \( A_+ \) as:

\[ S = \frac{A_+}{4G_N}, \]  

(3)

and the Hawking temperature is related to the surface acceleration at the outer event horizon \( \kappa_+ \) as:

\[ T_H = \frac{\kappa_+}{2\pi}. \]  

(4)

Here \( G_N \) is the Newton’s constant.

In a microscopic description of black holes as weakly coupled effective strings the geometrically defined thermodynamic variables (1) and (2) must each be divided into two terms, as in the string system (1) and (2). The general rotating black holes of toroidally compactified string theory strongly suggest how this should be done [13], as we explain in the subsequent section. Here we present our proposal in geometrical terms, rendering it independent of the specific dimensionality of space-time and the compactification of the string theory. It is also invariant under coordinate changes as well as duality transformations.

The idea is that general black holes have two sets of thermodynamic variables associated with the two event horizons, the inner and the outer one. Specifically, the R– and L–moving entropies are:

\[ S_{R,L} = \frac{1}{2} (\frac{A_+}{4G_N} \mp \frac{A_-}{4G_N}), \]  

(5)
where $A_{\pm}$ are the areas of the inner and outer event horizons; and the R– and L– moving temperatures:

$$\frac{1}{T_{R,L}} = \frac{2\pi}{\kappa_{+}} \pm \frac{2\pi}{\kappa_{-}}, \quad (6)$$

where $\kappa_{\pm}$ are the surface accelerations of the inner and outer event horizons. These geometric definitions of the temperatures are consistent with the result of the first law of thermodynamics applied to the R– and L– moving entropies independently: $T_{R,L}^{-1} = 2(\frac{\partial S_{R,L}}{\partial M})_{\bar{Q}_{1},\bar{J}_{2}}$. It is an unfamiliar idea that the inner event horizon plays any role at all, as it is effectively isolated from an outside observer. The ultimate justification of this idea comes from its application: the calculations presented in the following sections indeed support that, at least formally, the “contributions” from the inner and the outer horizon appear on equal footing.

3. General Black Holes in String Theory — the Five-Dimensional Example

As the working example we consider the most general five-dimensional black hole in toroidally compactified string theory. It depends on the ADM mass, $M$, three $U(1)$ charges $Q_{i}$, and two angular momenta $J_{R,L}$. We parameterize these physical variables by the non-extremality parameter $\mu$, the three boost parameters $\delta_{i}$, and the (bare) angular momentum parameters $l_{1,2}$ introduced through:

$$M = \frac{1}{2} \mu \sum_{i=1}^{3} \cosh 2\delta_{i}, \quad (7)$$

$$Q_{i} = \frac{1}{2} \mu \sinh 2\delta_{i} \quad ; \quad i = 1, 2, 3, \quad (8)$$

$$J_{R,L} = \frac{1}{4} \mu (l_{1} \pm l_{2}) (\prod_{i=1}^{3} \cosh \delta_{i} \mp \prod_{i=1}^{3} \sinh \delta_{i}) \quad (9)$$

Units are such that the gravitational coupling constant in five dimensions is: $G_{5} = \frac{1}{2} (\alpha')^{4} g^{2}/(R_{1} R_{2} R_{3} R_{4} R_{5}) = \frac{1}{2}$, where $R_{i}$ are the radii of the compact tori, the string coupling is normalized so that under S-duality $g \to g^{-1}$, and $\alpha'$ is the Regge slope. The three $U(1)$ charges of the generating solution can be chosen to be coupled to D1-branes, D5-branes, and Kaluza-Klein charge, respectively. In this case they are related to quantized (integral) charges $n_{i}$ through:

$$Q_{1} = \frac{n_{1} R_{1}}{g \alpha'} \quad (D1 - \text{branes}) \quad (10)$$

$$Q_{2} = \frac{n_{2} R_{1} R_{2} R_{3} R_{4} R_{5}}{g (\alpha')^{3}} \quad (D5 - \text{branes}) \quad (11)$$

$$Q_{3} = \frac{n_{3}}{R_{1}} \quad (\text{KK} - \text{charge}) \quad (12)$$

These assignments will be assumed for definiteness, but many other choices are equivalent by duality. Indeed, duality transformations in the maximal compact subgroup of the full duality group generate the most general black hole when they act on the generating solution defined by (7–9) (This procedure was made explicit in [13].)

From the explicitly known solutions the areas of the inner and outer horizons can be read off [13]. The entropies calculated using (6) become:

$$S_{R,L} = 2\pi \sqrt{\frac{1}{4} \mu^{2} (\prod_{i} \cosh \delta_{i} \mp \prod_{i} \sinh \delta_{i})^{2} - J_{R,L}^{2}} \quad (13)$$

According to our interpretation these expressions should be identified with the entropies of the R– and L– moving modes of the underlying string theory.

The R– and L– moving temperatures are similarly calculated from the surface accelerations at the inner and outer event horizons, using eq. (6). They are:

$$\frac{1}{T_{R,L}} = \frac{\pi \mu^{2} (\prod_{i} \cosh \delta_{i} \mp \prod_{i} \sinh \delta_{i})}{\sqrt{\frac{1}{4} \mu^{2} (\prod_{i} \cosh \delta_{i} \mp \prod_{i} \sinh \delta_{i})^{2} - J_{R,L}^{2}}} \quad (14)$$

The physical content of these formulae becomes clearer in various limiting cases that we consider in the following.

The BPS limit:

The extremal case corresponds to the limit where $\delta_{i} \to \infty$. This limit is only regular when

2In fact the explicit generating solution given in [13] employs three charges in the NS-NS sector: electric winding and momentum charges, and a charge associated with the dualized anti-symmetric tensor field.

3For a review of general black hole solutions in toroidally compactified string theory see: [13].
also $J_R \to 0$. The BPS mass is given by the sum of the three charges:

$$M = Q_1 + Q_2 + Q_3 ;$$

so the black hole can be interpreted as a marginal bound state of three kinds of objects. The degeneracy of this composite object is given by the (exponential of) the entropy \([19]\):

$$S = 2\pi \sqrt{Q_1 Q_2 Q_3 - J_L^2} = 2\pi \sqrt{n_1 n_2 n_3 - J_L^2} .$$

In the intermediate step the quantization conditions (10–12) on the charges were used. The moduli cancel out \([9,10,20]\) so that the entropy can also be interpreted directly in terms of the underlying constituents. Note that the $R$–moving contribution indeed vanishes in the geometric interpretation because the two horizons coincide.

The string theory calculation that leads to (16) takes as its starting point the superconformal field theory (SCFT) with the target space \([1]\):

$$C = (T^4)^{n_1 n_2} / \Sigma_{n_1 n_2} ,$$

and level $n_3$. Here $\Sigma_k$ is the permutation group of $k$ objects. It acts on the product manifold in an orbifold construction and introduces twisted sectors that contribute fractionally to the momentum \([2]\). In the black hole limit it is the sector with the maximal fractionation that provides the most important contribution. The contribution of this sector can be captured by the effective level:

$$N_L = n_1 n_2 n_3 - J_L^2 ,$$

of a superstring with target space $T^4$, or more generally a SCFT with the central charge $c = 6$ ($\hat{c} = 4$). Indeed, \([16]\) is recovered using $S \simeq 2\pi \sqrt{\frac{c}{6}} N$, valid at large $N$. Although the concept of an effective level is approximate in general it becomes precise in the black hole regime. It is useful because it captures the symmetry between the three charges required by duality.

The angular momentum is the $U(1)$ component of the local $SU(2)$ world-sheet current of the $N = 4$ SCFT \([19]\). The projection onto the sector with a specific angular momentum is multiplication by an operator with the appropriate $U(1)$ world-sheet charge, and the scaling dimension (conformal weight) of this operator is responsible for the effective subtraction of $J_L^2$ in eq. \([18]\).

**Extreme Kerr-Newman limit:**

A related analysis can be applied to the extreme Kerr-Newman type black hole solutions. This limit is achieved by taking $(l_1 - l_2)^2 \to \mu$. Then $S_R = 0$, again, and thus $S = S_L$ which takes the form:

$$S = 2\pi \sqrt{n_1 n_2 n_3 + J_R^2 - J_L^2} .$$

Although this solution is far from the BPS-limit the BH entropy is again moduli independent; and the microscopic entropy can be modeled \([24]\) by an effective string with $c_R = c_L = 6$. The R-moving sector contributes to the angular momentum component $J_R$ but the excitation level is $N_R = 0$.

**The dilute gas limit:**

In this limit two of the boosts are large, say, $\delta_{1,2} \gg 1$ \([3,6]\). The corresponding two charges are conventionally chosen as the D1-brane and D5-brane charges. These charges act as backgrounds while the third charge signifies a deviation from the BPS-limit: it couples to both R- and L–moving momentum carrying waves. The effective levels can be found from the excess energy over the BPS-limit, assuming that the background branes fractionate the contributions to the levels, as in the BPS-limit, and that the important degrees of freedom are excited versions of the effective $c = 6$ string that appear in the BPS-limit:

$$N_{R,L} = Q_1 Q_2 \frac{1}{4} \mu e^{\pi \delta_3} - J_{R,L}^2 .$$

The corresponding entropies indeed agree with eq. \([13]\) in the limit where two boosts are large. This verifies the role of the inner horizon in the dilute gas limit.

**One large boost:**

The case where only one of the boosts is large, say, $\delta_1 \gg 1$, can be modeled similarly \([23,26]\): the
charge that corresponds to the large boost acts as a background that is inert, except that it induces the fractionation. The excess energy, $\Delta M$, is distributed among the states in the spectrum in a way that is similar to the fundamental string with both momentum and winding charge. Considering first the standard relation of perturbative string theory:

$$N_{R,L}^{\text{pert}} = \Delta M^2 - (Q_2 \pm Q_3)^2 = \mu^2 \cosh^2(\delta_2 \mp \delta_3)$$

(21)

We also take into account the fractionation and the projection onto a specific angular momentum sector. Then the effective levels become:

$$N_{R,L} = Q_1 \mu^2 \cosh^2(\delta_2 \mp \delta_3) - J_{R,L}^2.$$  

(22)

The corresponding entropies reproduce eq. (13) in the limit where one boost is large, thus verifying the role of the inner horizon in the regime of one large boost.

Note that this case includes general non-extreme black holes in the infinite momentum frame. This may be relevant for the description of black holes in the framework of M(atrix)-theory.\footnote{In [14] this result was inferred from the absorption cross-section. The derivation of $\mathcal{L}$ given here was found independently in [28].}

**The general case:**

The general expression for the BH entropy\footnote{In [14] this result was inferred from the absorption cross-section. The derivation of $\mathcal{L}$ given here was found independently in [28].} can be accounted for quantitatively by a non-critical string with central charge $c_R = c_L = 6$, excited to the effective levels:

$$N_{R,L} = \frac{1}{2} \mu^2 (\prod_i \delta_i + \prod_i \sinh \delta_i) - J_{R,L}^2.$$  

(23)

An important test of this idea is that it reduces to the limits described above when an appropriate number of boosts $\delta_i$ are large. However, in the general case the mass spectrum is not known from other considerations. Thus, unlike the previous special cases we cannot use this knowledge to justify the effective levels\footnote{In [14] this result was inferred from the absorption cross-section. The derivation of $\mathcal{L}$ given here was found independently in [28].}. Instead our interpretation provides the spectrum in the black hole regime.

As an alternative test of the proposal use $Q_1 Q_2 Q_3 = n_1 n_2 n_3$ to find:

$$N_R - N_L = n_1 n_2 n_3 + J_L^2 - J_R^2.$$  

(24)

Since the angular momenta have been normalized so that $J_R^2 - J_L^2$ is quantized as integers, the expression above is independent of moduli and it is an integer. This result lends support to the hypothesis that $N_{R,L}$ are individually quantized.

The length of the effective string can be derived from thermodynamics alone. Indeed, non-interacting gases in one dimension satisfy:

$$S = \frac{\pi c}{6} T \mathcal{L},$$  

(25)

where $\mathcal{L}$ is the “volume” — length — of the gas. Thus, using (13) for $S$ and (14) for $T$, the length $\mathcal{L}$ of the effective string becomes:

$$\mathcal{L} = 2\pi \mu^2 (\prod_i \cosh^2 \delta_i - \prod_i \sinh^2 \delta_i).$$  

(26)

This length scale is independent of the angular momenta, which provides another test of the model. Namely, in string theory angular momenta are implemented as projections on the Hilbert space of states and do not affect the length of the string. Moreover, it is satisfying that the same string length $\mathcal{L}$ is found in the R– and L–moving sectors. $\mathcal{L}$ reduces to $\mathcal{L} = 2\pi n_1 n_2 R$ in the BPS-limit but the general expression (26) is also valid for non-BPS black holes. The fact that the effective string length $\mathcal{L}$ increases with the size of the black hole is potentially important for the issue of information loss. Indeed, for large black holes the scale $\mathcal{L}$ is much larger than the Planck scale, where string effects are usually assumed to become important.

These considerations also apply to the static cases with one or more $\delta_i = 0$, including the Schwarzschild solution with all $\delta_i = 0$. However, in these examples $S_R = S_L$ and $T_R = T_L = T_H$; so the characteristic string features of the thermodynamics are absent. In the geometric interpretation this is a consequence of the degenerate limit of the vanishing inner horizon area. However, this presumably does not imply any limitation in the string description. A plausible interpretation is simply that, in this case, there is an equilibrium between the two sectors.
4. The Wave Equation — Minimally Coupled Scalar Field

A detailed test of the correspondence between black holes and strings is the spectrum of Hawking radiation. As preparation for this calculation we consider general properties of the wave equation for a minimally coupled scalar field:

\[ \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi) = 0 \, . \]  

(27)

It is straightforward (but tedious) to insert the metric given in \[13\]. The equation turns out to be separable. We can therefore write the wave function as:

\[ \Phi = \Phi_0(r) \chi(\theta) e^{-i\omega t + im_\phi \phi + in_\chi \psi} \, , \]  

(28)

where \( \phi \) and \( \psi \) are the two azimuthal angles. We also introduce a dimensionless variable \( x \) that is related to the standard radial coordinate \( r \) through:

\[ x = \frac{r^2 - \frac{1}{4}(r_+^2 + r_-^2)}{\frac{1}{4}(r_+^2 - r_-^2)} \, . \]  

(29)

In this coordinate system the outer and inner event horizons are at \( x = \frac{1}{2} \) and \( x = -\frac{1}{2} \), respectively, and the asymptotically flat region is at \( x = \infty \). Then the radial part of the wave equation becomes \[13\]:

\[ \partial_x (x^2 - \frac{1}{4}) \partial_x \Phi_0 + \frac{1}{4} [x \Delta \omega^2 + M \omega^2 - \Lambda + \frac{1}{x - \frac{1}{2}} (\frac{\omega}{\kappa_+} - \frac{\Omega_R}{\kappa_+} - \frac{\Omega_L}{\kappa_+})^2 ] \Phi_0 = 0 \, . \]  

(30)

\[ - \frac{1}{x + \frac{1}{2}} (\frac{\omega}{\kappa_-} - \frac{\Omega_R}{\kappa_-} + \frac{\Omega_L}{\kappa_-})^2 ] \Phi_0 = 0 \, . \]

In spite of its generality this equation is no more complicated than similar ones that have been considered in various special cases \[13,29,22\].

We first discuss the various terms in eq. (30). The variable \( \Delta \) is defined by \( \Delta = r_+^2 - r_-^2 \); so at very large \( x \) the equation becomes:

\[ \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial}{\partial r} + \omega^2) \Phi_0 = 0 \, . \]  

(31)

This is the radial part of the Klein-Gordon equation in flat space. Thus, the term \( \frac{1}{4} x \Delta \omega^2 \) is simply the energy of the perturbation in the absence of the black hole.

The term \( \frac{1}{2} M \omega^2 \) can be interpreted as the Coulomb-type screening due to the gravitational field. At large \( x \) this term is suppressed relative to flat space by one power of \( x \sim r^2 \) as expected for a Coulomb potential in five dimensions.

The variable \( \Lambda \) is the eigenvalue of the angular Laplacian. It takes the form:

\[ \Lambda = n(n + 2) + \mathcal{O}(\omega^2) \, , \]  

(32)

where the corrections \( \mathcal{O}(\omega^2) \) are due to the rotation of the background and are discussed in \[13\]. This term is also suppressed by one power of \( x \sim r^2 \), as expected for an angular momentum barrier.

The terms considered so far are manifestations of the long range fields and the flat asymptotic space. The remaining two terms diverge at the outer (\( x = \frac{1}{2} \)) and inner (\( x = -\frac{1}{2} \)) horizons, respectively, and so are specific for black hole backgrounds. The modes close to the outer horizon become (taking \( m_R = m_L = 0 \), and thus ignoring the effects of angular velocities, \( \Omega_R, \Omega_L \), for simplicity):

\[ \Phi_0 \sim (x - \frac{1}{2}) \frac{\partial}{\partial x} e^{-i\omega t} \, . \]  

(33)

The branch-cut around \( x = \frac{1}{2} \) is tantamount to the existence of Hawking radiation with temperature \( T_H = \frac{\kappa}{2\pi} \), a well known result found in several different ways in the seventies.

The modes close to the inner horizon similarly become:

\[ \Phi_0 \sim (x + \frac{1}{2}) e^{-i\omega t} \, . \]  

(34)

The significance of these modes is less clear because the inner horizon is not expected to have any effects on the asymptotic observers. However, there is a striking parallel between the two horizons. It is therefore natural to suspect that the branch-cut around \( x = -\frac{1}{2} \) nevertheless has a thermal interpretation. As explained in sec. 3 the inner horizon temperature \( T_- = \frac{\kappa}{2\pi} \) combines with the Hawking temperature, \( T_H \), and forms two new temperatures \( T_{R,L}^{-1} = T_H^{-1} \pm T_-^{-1} \) which can be attributed the R– and L–moving string modes, respectively.
Hidden Supersymmetry:

For rotating black holes there are not sufficiently many conserved quantities that the separation of variables can be guaranteed. It is therefore a surprise that we were able to do so. An analogous surprise is well known in the context of four-dimensional Kerr-Newman black holes. There the separation of variables is a consequence of conserved fermionic charges that are related to the more familiar conserved bosonic charges by the supersymmetry algebra \[ [30] \]. It is believed that the significance of the supersymmetry in this context is different from its role in particle physics. However, it would be interesting to reexamine this question in light of the relation between black holes and superstrings.

String symmetries:

The wave equation in the region close to the horizons has a simple form that may be universal, as the identical equation appears in both four and five dimensions \[ [14] \]. We now express this structure mathematically.

We introduce the dimensionless Rindler time \( \tau \) that is a regular time-like coordinate close to the outer horizon. The monodromy around the coordinate singularity is encoded by an imaginary period \( 2\pi iT \). Analogously, we introduce an “inner-horizon Rindler time” \( \sigma \). This coordinate is space-like, as the signature is changed close to the inner horizon, and it has an imaginary period \( 2\pi iT \). With these auxiliary variables the radial equation \[ [31] \] (without the flat space term \( \frac{1}{x} \Delta \omega^2 \)) becomes the eigenvalue equation of the operator:

\[
\mathcal{H}_r = -\partial_x (x^2 - \frac{1}{2}) \partial_x - \frac{1}{x^2} \partial^2_x + \frac{1}{x^2 + \frac{1}{2}} \partial^2_{\sigma} \tag{35}
\]

with the eigenvalue \( \frac{1}{2}(\Lambda - M\omega^2) \equiv h(h - 1) \). The significance of the variable \( h \) (as a conformal dimension of an effective string vertex) will be explained in the next section. The operator \[ [32] \] is the quadratic Casimir of the group \( SL(2, \mathbb{R})_R \times SL(2, \mathbb{R})_L \). Its appearance shows that the propagation in the black hole background is in fact the motion on this group manifold. A similar description has previously appeared in the context of exact conformal field theories that describe two-dimensional black holes, but the relation with the present results is not clear (see e.g., \[ [33] \]).

The compact generators \( R_3 \) and \( L_3 \) of the two \( SL(2, \mathbb{R})_{R,L} \) groups are diagonal and their eigenvalues are:

\[
R_3 = \frac{1}{2}(\partial_\tau + \partial_\sigma) = \frac{\omega}{\pi T_R}, \tag{36}
\]

\[
L_3 = \frac{1}{2}(\partial_\tau - \partial_\sigma) = \frac{\omega}{\pi T_L}. \tag{37}
\]

Note that the natural variables of the group are the sum and the difference of \( \tau \) and \( \sigma \), rather than \( \tau \) and \( \sigma \) themselves. The auxiliary variables \( \tau \) and \( \sigma \) are localized “times” close to each of the two horizons. Thus, in a definite sense, it is the sum and the difference of the two horizons that are singled out by the group structure. This result is satisfying because it is precisely these combinations of the surface accelerations that we assign microscopic significance.

The wave equation exhibits an obvious symmetry between inner and outer horizon terms that can be expressed in terms of the group generators as an automorphism of the algebra that takes \( R_3 \rightarrow R_3 \) and \( L_3 \rightarrow -L_3 \). In string theory the T-duality symmetry acts in precisely this way.

It is intriguing that symmetries that are closely associated with string theory are realized explicitly through the wave equation in the general black hole background. However, we must emphasize their precise significance remains unclear.

5. The Greybody Factors

We now consider the solutions of the wave equation discussed in sec. \[ [1] \] following \[ [12, 23, 13] \]

At large \( x \) we consider the radial equation in flat space and we include the effects of the long range fields from the black hole. The solution to this approximate equation is a Bessel function. Next we consider the equation with only the unperturbed energy \( \frac{1}{2} \Delta x \omega^2 \) omitted. Then the solution is a hypergeometric function. These wave functions with validity in some region of space can, under conditions discussed later, be combined to form an approximate solution that is valid throughout. The character of the re-

\[ ^5 \text{We would like to thank G. Gibbons for pointing this out.} \]
result depends on the value of the potential terms \( \frac{1}{8}(M \omega^2 - \Delta) = h(h - 1) \) in the region where the two partial solutions are both valid.

**Matching on zero potential:**

We first consider the situation where the potential vanishes in the “matching” region, i.e. \( h = 1 \). In this case the S-wave absorption cross-section becomes:

\[
\sigma_{\text{abs}}^{(0)}(\omega) = \frac{\pi^2 L \omega}{2} \left( \frac{e^{\omega/T_H} - 1}{(e^{\frac{\pi\omega}{T_H}} - 1)(e^{\frac{\pi\omega}{T_L}} - 1)} \right). \tag{38}
\]

Using the formula \( ST_H = 2 \pi L T_R T_L \) it can be shown that the absorption cross-section approaches the area of the outer horizon \((A_+)^{\frac{1}{2}}\) as \( \omega \rightarrow 0 \). This is the universal low-energy result \( \sigma_{\text{abs}}^{(0)} \) (which also applies to rotating black holes \( \text{(iii)} \)).

From the absorption cross-section the emission rate follows using detailed balance:

\[
\Gamma_{\text{em}}^{(0)}(\omega) = \frac{\sigma_{\text{abs}}(\omega)}{\omega} = \frac{\pi^2 L \omega}{2} \left( \frac{e^{\frac{\omega}{T_H}} - 1}{(e^{\frac{\pi\omega}{T_H}} - 1)(e^{\frac{\pi\omega}{T_L}} - 1)} \right) \tag{39}
\]

It is important to note that the Bose distribution with the Hawking temperature canceled out. Therefore the final result depends only on the quantities \( T_{R,L} \) and \( L \) that have significance in the effective string description. In fact there is an explicit microscopic interpretation of this formula: the emission is the result of a two-body process \( \text{(iii)} \). The Bose distributions are phase space factors of the R– and L–moving quanta propagating on the string. Moreover, the amplitude for the annihilation of two quanta colliding head-to-head on a string of length \( L \) can be calculated, using only the Nambu-Goto form of the string action. The result of this calculation is identical to eq. \( \text{(38)} \). Thus the agreement between the space-time calculation and the microscopic interpretation involves the functional dependence on the energy, and all numerical factors agree as well.

The microscopic two-body interpretation of the emission can be employed as a model whenever the form of the semi-classical result is of the type \( \text{(iii)} \). A sufficient condition for this is the low energy requirement \( M \omega^2 \ll 1 \). This implies \( (r_+^2 - r_0^2) \omega^2 \ll 1 \); thus the wave length of the probe is so large that the target cannot be positively identified as a black hole. On the other hand, the condition also implies \( \omega \ll 1 \). This presumably implies that the target cannot be unambiguously identified as a string either.

(i) An important special case is the dilute gas limit \( \text{(iii)} \), defined in sec. \( \text{B} \) as \( \delta_{1,2} \gg 1 \). In this case the low-energy condition \( M \omega^2 \ll 1 \) is satisfied for frequencies \( \omega \sim T_R \sim T_L \). Therefore the calculation is sensitive to the Bose distribution factors in eq. \( \text{(39)} \). This verifies in detail that the string temperatures have been correctly identified in the dilute gas regime.

(ii) Another interesting example is the regime of rapidly spinning black holes \( \text{(iii)} \), which is obtained by tuning the bare angular momenta \( l_{1,2} \), defined in eq. \( \text{(9)} \), so that \( l_2 = 0 \) and \( \mu - l_1^2 = \mu \varepsilon^2 \ll \mu \). As in the dilute gas case, the low energy condition is satisfied for \( \omega \sim T_R \sim T_L \) and the Bose factors are significant. However, now there are no conditions on the boosts \( \delta_i \); so the functional dependence of the temperatures \( T_{L,R} \) and the string length \( L \) on all three boosts \( \delta_i \) is tested in detail.

(iii) As a final example, consider the limit of very low energies where the universal absorption cross-section \( \sigma_{\text{abs}}^{(0)} = A_+ \) is valid for all black holes. The two-body model still applies \( \text{(iii)} \) but the test afforded by the calculation of the emission rates is weaker because it does not involve the functional dependence on \( \omega \). However, the agreement still involves the dependence on the independent black hole parameters \( \mu, \delta_i, \) and \( l_{1,2} \). This result thus provides evidence that, at low energies, the effective string model applies to all black holes.

**Matching on non-zero potential:**

The two-body annihilations considered so far are the simplest decay processes, but more complicated ones give important additional information. Recall that one of the conditions for the validity of the two-body form of the emission rate is that the potential vanishes in the region where the

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6In \( \text{B} \) two additional constraints were given. They are however automatically satisfied, given this condition.
asymptotic solution and the near horizon solution are matched. This condition can be replaced with the milder assumption that the two solutions can be matched in a region where the potential is any constant \[29\]. In this case the final result for the emission rate into the n-th partial wave becomes:

\[
\Gamma_{\text{em}}^{(n)}(\omega) = \frac{4\pi(n+1)^2}{\omega^3T^2(2h)} \left( \frac{\ell \omega^2}{8\pi} \right)^{2h-1} \times \Gamma(h - \frac{\omega}{2\pi T})^2 \Gamma(2h-1) \times G_{T_h}^R \left( \frac{\omega}{2} \right) G_{T_L}^h \left( \frac{\omega}{2} \right) \frac{d^4k}{(2\pi)^4},
\]

where:

\[
G_{T}^h \left( \frac{\omega}{2} \right) = (2\pi T)^{2h-1} e^{-\frac{\pi T^2}{2h}} \Gamma(h - \frac{\omega}{2\pi T})^2 \Gamma(2h-1).
\]

In the effective string description \(G_{T_h}^R(z)\) has an interpretation as the Fourier transform of the canonically normalized thermal Green’s functions for a conformal field with the scaling dimensions \(h_R = h_L = h\):

\[
G_{T_h}^R(z) = \left( \frac{\pi T_R}{\sinh \pi T_R z} \right)^{2h},
\]

and similarly for \(G_{T_L}^h\) with \(R \rightarrow L\) and \(z \rightarrow \bar{z}\). When \(h = 1\) the general result (40) reduces to the two-body case (29). For arbitrary \(h\) the emission rate can still be interpreted in the framework of an effective \(c_R = c_L = 6\) SCFT with \((4,4)\) supersymmetry that also accounts for the entropy 29. In this model the operator that is responsible for the emission has dimension \(h_R = h_L = h\) so that it accounts for the Green’s functions in the emission rate. In this way the eigenvalue \(h(h-1)\) of the quadratic Casimir of the \(SL(2,R) \times SL(2,R)\) group is related to the conformal dimension in the effective string interpretation.

The general conditions for validity of the emission rate (40) are given in 13. Here we consider the sufficient condition that the partial wave number \(n\) satisfies \(M\omega^2 \ll n^2\). For the various black holes with \(M\omega^2 \ll 1\), considered above, this holds for all \(n\). More importantly, for typical frequencies \(\omega \sim T_R \sim T_L\) the condition is valid for arbitrary black holes, as long as \(n \gg 1\). The condition \(M\omega^2 \ll n^2\) can be written in terms of the impact parameter \(b\) as \(M \ll b^2\). Thus the probe sees the entire black hole as one entity. However, this does not imply that the black hole appears point-like; indeed, it “looks” like a string.

The condition \(M\omega^2 \ll n^2\) gives \(h = \frac{n}{2} + 1\) so that the emission vertex operator has a free conformal field theory realization of the schematic form:

\[
V \sim : \partial X(z) \bar{\partial} X(\bar{z}) [S^a(z)]^n [\bar{S}^\hat{a}(\bar{z})]^n : ,
\]

where \(:\) denotes the normal ordering of the operators, \(X\) are coordinates of the string in the internal directions, and \(S^a, \bar{S}^\hat{a}\) are world-sheet fields with conformal dimensions \((h_R, h_L) = (1/2, 0)\) and \((h_R, h_L) = (0, 1/2)\), respectively, and whose indices \((a, \hat{a})\) specify quantum numbers of the space-time spinors 7. The emission is therefore interpreted as a many-body process that involves 1 boson and \(n\) fermions in both \(R\)- and \(L\)-moving sectors. Lorentz invariance implies that the coupling of this vertex operator to the outgoing field involves \(n\) derivatives acting on the outgoing field. This gives rise to a factor of \(\omega^{2n}\) in the rate and, remembering the normalization \(\omega^{-1}\) of the outgoing wave, the complete frequency dependence of (40) can be accounted for qualitatively 29. Thus, for large partial wave numbers, the microscopic description in terms of an effective string model accounts for the emission rates in an arbitrary black hole background.

An important unresolved problem remains the calculation of the overall numerical coefficient in (40). However, this issue is not specific to the non-extreme case. It is expected that this coefficient is calculable in the BPS-limit and that it agrees with the classical result. If this is borne out it is will also be possible to model the general non-extreme black hole.

The vertex operators (43) are fermionic when \(n\) is odd. Therefore the phase-space factors associated with the initial state are expected to be

7 The world-sheet fields \(s^a, \bar{s}^\hat{a}\) that transform as spinors under the \(SO(4) \sim SU(2)_R \times SU(2)_L\) current algebra have the respective conformal dimensions \((1/4, 0)\) and \((0, 1/4)\). Thus we can write \(S^a \sim s^a \chi, \bar{S}^\hat{a} \sim \bar{s}^\hat{a} \bar{\chi}\) where the fields \(\chi, \bar{\chi}\) have conformal dimensions \((1/4, 0)\) and \((0, 1/4)\). Presumably \(\chi, \bar{\chi}\) are associated with the internal degrees of freedom and might reflect the \(SL(2,R)_R \times SL(2,R)_L\) symmetry of the scattering equation.
of the Fermi-Dirac type. In this case the Green’s functions (41) are proportional to gamma functions with arguments whose real parts are half-integral, which indeed give factors of the Fermi-Dirac form \( (e^\omega/2T + 1)^{-1} \). This is an interesting example where the black hole “looks” like a string with fermionic degrees of freedom.

6. Conclusion

We conclude by summarizing the accomplishments and the shortcomings of the effective string model for non-extreme black holes, starting with the former:

- The model relates the thermodynamic variables in the effective (weakly coupled) string description directly to geometrical features of the black hole space-time.
- The extreme and near-extreme limits are in agreement with the model. This result provides a strong motivation that a general non-extreme black hole can be modeled by an effective string model as well.
- Two-body processes can be accounted for quantitatively. Specifically this result gives a microscopic interpretation of the universal low-energy absorption cross-section.
- Many-body processes can be understood qualitatively.

Despite these successes it is appropriate to conclude with some caution. The understanding of non-extreme black holes presented here leaves much room for improvement:

- The detailed connection with a specific fundamental string theory, along with the detailed description of the underlying SCFT, is not clear.
- The description of the string spectrum is limited to the black hole regime because we employ an approximate notion of an effective level.
- Current models of black holes in string theory are not sensitive to the nontrivial causal structure of black hole space-times.

It is not clear whether these obstacles can be overcome with further developments of the ideas and techniques that are presently known.

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