Can the Majoron be gauged away?

Luis N. Epele,¹ Huner Fanchiotti,¹ Carlos García Canal,¹ and William A. Ponce²

¹Laboratorio de Física Teórica, Departamento de Física,
Universidad Nacional de La Plata, C.C. 67-1900, La Plata Argentina.
²Instituto de Física, Universidad de Antioquia, A.A. 1226, Medellín, Colombia.

Abstract

In a recent paper [Phys. Rev. D73, 075005 (2006)], the authors presented the lepton number violation mechanisms in the econimic version of the 3-3-1 model, without explicitly pointing where the pseudo Goldstone Majoron lies. In this comment we clarify this point and show an extended version of the model where the Goldstone Majoron becomes a gauge away would be Majoron.

PACS numbers: 11.30.Fs, 12.60.Cn, 14.80.Mz
Masses for neutrinos require physics beyond the standard model (SM) connected either to the existence of right handed neutrinos and/or to the breaking of the Barion minus Lepton (B−L) symmetry. Besides, Majorana masses for neutrinos require the violation of the lepton number; violation that can be spontaneous implying the existence of the so called Majoron [1], or either explicit implying the existence of a pseudo Goldstone Majoron.

Interesting extensions of the SM are based on the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ (3-3-1 for short) in which the weak sector of the SM is extended to $SU(3)_L \otimes U(1)_X$. Several models for this gauge structure have been constructed so far, being the most popular the original minimal model [2] and the so called model with right handed neutrinos [3]. An amusing variant of this last model is the so called “economic 3-3-1 model” [4, 5, 6] in which the spontaneous symmetry breaking and the generation of masses for all the particles in the model is done by a minimal set of only two Higgs scalar triplets.

One, among the many outstanding features of 3-3-1 models, is the appropriate explanation of neutrino masses and oscillations. The particular analysis done for the minimal version of the model, and for the model with right handed neutrinos, carried at the expense of violating the lepton number L by two units, implies in some cases the existence of a ruled out Majoron [1]. But the economic version of the model presents special features as discussed in Refs. [5, 6] and in what follows.

The fermion content of the economic model is given by the following 3-3-1 anomaly free structure [4]: $\psi^T_{Ll} = (\nu^0_l, l^-, \nu^c_l)_L \sim (1, 3, -1/3)$, $l^+_L \sim (1, 1, 1)$, $Q^T_{3L} = (d_i, u_i, D_i)_L \sim (3, 3^*, 0)$, $Q^T_{3L} = (u_3, d_3, U)_L \sim (3, 3, 1/3)$, where $l = e, \mu, \tau$ is a family lepton index, $\nu^c_l$ stands for the right-handed neutrino field and $i = 1, 2$ for the first two quark families. The right handed quark fields are $u^c_{aL} \sim (3^*, 1, -2/3)$, $d^c_{aL} \sim (3^*, 1, 1/3)$, $D^c_{aL} \sim (3^*, 1, 1/3)$ and $U^c_L \sim (3^*, 1, -2/3)$, where $a = 1, 2, 3$ is the quark family index and there are two exotic quarks with electric charge $-1/3$ ($D_i$) and another with electric charge $2/3$ ($U$).

The Gauge Boson sector of the model includes, besides $W^\pm$ and $Z^0$ as in the SM, five more particles: $K^\pm$, $K^0$, $\bar{K}^0$ (which carry U and V isospin respectively) and an extra Boson $Z^0$ related to a second diagonal weak neutral current. The scalar sector consists of only two scalars triplets (instead of three as in the original model with right handed neutrinos [3]) which are: $\chi^T = (\chi^0_1, \chi^0_2, \chi^0_3) \sim (1, 3, -1/3)$ and $\rho^T = (\rho^+_1, \rho^0_2, \rho^+_3) \sim (1, 3, 2/3)$, with the following Vacuum Expectation Values (VEV) $\langle \chi \rangle^T = (v_1, 0, V)$ and $\langle \rho \rangle^T = (0, v, 0)$, where $v_1 < v \sim 10^2$ GeV$< V \sim$ TeV.
The lepton number $L$ is not a good quantum number in the context of this model because, $\nu_L^0$, the third component in the triplet $(1, 3, -1/3)$, is identified with the antiparticle of $\nu_L^0$, which in turn implies that $L$ does not commute with $SU(3)_L \otimes U(1)_X$. The lepton number assignments for this model are $\nu^-_L$, $\nu^0_L$, $\nu^+_L = 1, L(u_aL, u^c_aL, d_aL, d^c_aL, W^{\pm}, Z^0, Z'^0) = 0$; $L(K^+, K^0, U_L, D_{iL}) = -2 = -L(K^-, K^0, D_iL, U_L^c)$. Then, the Yukawa coupling constants imply $L(\chi^2, \chi^1) = 2$, $L(\rho^+\rho^-, \rho^0, \chi^3) = 0$. Notice that $D_i^{-1/3}$ and $U^{2/3}$ are bileptons and $K^+, K^-, K^0$ and $K^0$ are bilepton gauge bosons.

A global symmetry $\mathcal{L}$ of the full Lagrangean, not broken by the VEV of the scalars, which commutes with the $SU(3)_L \otimes U(1)_X$ Gauge group, can be constructed via the equation:

$$L - \frac{2\lambda_8}{\sqrt{3}} = \mathcal{L}I_3,$$

which implies the following assignments: $\mathcal{L}(\psi_{iL}) = 1/3$, $\mathcal{L}(Q_{iL}) = 2/3$, $\mathcal{L}(Q_{3L}, \rho) = -2/3$, $\mathcal{L}(\chi) = 4/3$, $\mathcal{L}(D_{iL}) = -2$, $\mathcal{L}(U_L^c) = 2$, $\mathcal{L}(l_L^c) = 2$ and $\mathcal{L}(u_a^c, d_a^c) = 0$ (the Gauge Bosons are neutral under $\mathcal{L}$).

Since the scalar sector is very simple now, the model is highly predictable. As a matter of fact, the full scalar potential consist only of the following six terms:

$$V(\chi, \rho) = \mu_1^2|\chi|^2 + \mu_2^2|\rho|^2 + \kappa_1|\chi^\dagger|^2 + \kappa_2|\rho^\dagger|^2 + \kappa_3|\chi^\dagger\rho|^2,$$

$$+ \kappa_4|\chi^\dagger|^2 + h.c..$$

(2)

A simple calculation shows that both, $\mathcal{L}$ and the lepton number $L$ are conserved by $V(\chi, \rho)$ and also by the full Lagrangean, except for the Yukawa interactions $\mathcal{L}^Y = \mathcal{L}^Y_{LNC} + \mathcal{L}^Y_{LNV}$. They induce masses for the fermions as follows:

$$\mathcal{L}^Y_{LNC} = h^U_{ij}\chi^iQ_{3L}Cu_j^c + h^D_{ij}\chi^jQ_{iL}CD_j^c + h^\rho_{ij}\rho^jQ_{iL}Cu_j^c + h^\rho_{jL}\psi_{iL}Cu_j^c + h_{iL}\rho\psi_{iL}^c \chi^{iL} + h.c.$$

(3)

$$\mathcal{L}^Y_{LNV} = h^\rho_{ij}\rho^iQ_{3L}Cu_j^c + h^\rho_{jL}\psi_{iL}Cu_j^c + h_{iL}\rho\psi_{iL}^c \chi^{iL} + h.c.$$

(4)

where the subscripts LNC and LNV indicate lepton number conserving and lepton number violating term respectively. As a fact, $\mathcal{L}^Y_{LNV}$ violates explicitly $\mathcal{L}$ and $L$ by two units.
After spontaneous breaking of the gauge symmetry, the scalar potential \( V(\chi, \rho) \) develops the following lepton number violating terms:

\[
V_{\text{LNV}} = v_1 [\sqrt{2} H_\chi (\kappa_1 |\chi|^2 + \kappa_3 |\rho|^2)] \\
+ v_1 \kappa_4 [\rho_1^+ (\chi^\dagger \rho) + \rho_1^- (\rho^+ \chi)],
\]

(5)

where we have defined as usual \( \chi_1^0 = v_1 + (H_\chi + i A_\chi)/\sqrt{2} \). \( H_\chi \) and \( A_\chi \) are the so called CP even and CP odd (scalar and pseudoscalar) components, and for simplicity we are taking real VEV.

Notice that the lepton number violating part in (5) is trilinear in the scalar fields, and as expected, \( V_{\text{LNV}} = 0 \) for \( v_1 = 0 \). From the former expression we can identify \( A_\chi \) as the only candidate for a Majoron in this model.

The minimization of the scalar potential has been done in full detail in Refs. [4]. For that purpose two more definitions were introduced: \( \rho_2^0 = v + (H_\rho + i A_\rho)/\sqrt{2} \) and \( \chi_3^0 = V + (H_\chi' + i A_\chi')/\sqrt{2} \). An outline of the main results in Ref. [4], important for the present discussion, is:

- The three CP odd pseudoscalars \( A_\chi \), \( A_\chi' \) and \( A_\rho \), the would be Goldstone bosons, are eaten up by \( Z, Z' \) and \((K^0 + \overline{K}^0)/\sqrt{2}\), the real part of the neutral bilepton gauge boson.

- Out of the three CP even scalars, \( (v_1 H_\chi' - VH_\chi)/\sqrt{v_1^2 + V^2} \) becomes a would be Goldstone boson eaten up by \( i(K^0 - \overline{K}^0)/\sqrt{2} \), the imaginary part of the neutral bilepton gauge boson which picks up \( L=2 \) via \( H_\chi \). The other two CP even scalars become the SM Higgs boson and one extra Higgs boson with a heavy mass of order \( V \) respectively.

- In the charged scalar sector \((\rho_{1\pm}^\pm, \chi_{2\pm}^\pm, \rho_{3\pm}^\pm)\) there are four would be Goldstone bosons, two of them are \((V\chi_{2\pm} - v \rho_{3\pm}^\pm)/\sqrt{V^2 + v^2}\) with \( L=\pm2 \), eaten up by \( K^\pm \), and other two with \( L=0 \) eaten up by \( W^\pm \).

- Two charged scalars remain as physical states.

Counting degrees of freedom tells us that there are in \( \chi \) and \( \rho \) twelve ones namely, three neutral CP even, three neutral CP odd and six charged ones. Eight of them are eaten up by the eight gauge bosons \( W^\pm, K^\pm, K^0, \overline{K}^0, Z, \) and \( Z' \). Four scalars remains as physical states, one of them being the SM Higgs scalar.
Since L is explicitly broken in the context of this model by the Yukawa term $L_{LNV}^Y$, the result is that the would be pseudo Goldstone Majoron $A_\chi$, the only CP odd electrically neutral scalar with $L=2$, is eaten up by $(K^0 + \bar{K}^0)/\sqrt{2}$, the real part of the bilepton gauge boson.

A variant of this model was introduced in Ref. [8] where the authors considered the fermion mass spectrum under a $Z_2$ discrete symmetry which discards all the LNV interactions in the Yukawa potential($L_{LNV}^Y = 0$). For this variant of the model, $L$ is conserved through the entire Lagrangean, the lepton number $L$ is only spontaneously violated by $V_{LNV}$ in Eq. (5) and the would be Majoron $A_\chi$ is gauged away, eaten up by $(K^0 + \bar{K}^0)/\sqrt{2}$. Notice that being $L$ a good quantum number now, the spontaneous violation of SU(3)$_L$ implies the spontaneous violation of $L$ via Eq. (1), something that it is now allowed because the fermion sector for $L$ is vectorlike and thus non-anomalous. As attractive as the model is by itself, it is in someway incomplete. In fact, as a consequence of the $Z_2$ discrete symmetry used, new zero VEV scalar fields must be added in order to reproduce a consistent mass spectrum.[8]. The quark mass spectrum without making use of the $Z_2$ symmetry is analyzed in Ref. [6].

In this comment, we have identified a model in which the Majoron, coming from the spontaneous violation of the lepton number, is gauged away. This unusual mechanism that we have noticed here by the first time, has its origin in the symmetry breaking process and relation (1).

[1] G.Gelmini and M.Roncadelli, Phys.Lett. B 99,411(1981); H.Giorgi, S.Glashow and S.Nussinov, Nucl. Phys. B 193, 297 (1981).
[2] F.Pisano and V.Pleitez, Phys. Rev. D 46, 410 (1992); P.H.Frampton, Phys. Rev. Lett. 69, 2889 (1992).
[3] R.Foot, H.N.Long and T.A.Tran, Phys. Rev. D 50, R34 (1994); H.N.Long, Phys. Rev. D 54, 4691 (1996).
[4] W.A.Ponce, Y.Giraldo, and L.A.Sánchez, Phys. Rev. D 67, 075001 (2003); P.V.Dong, H.N.Long, D.T.Nhung and D.V.Soa, Phys. Rev. D 73, 035004 (2006).
[5] P.V.Dong, H.N.Long and D.V.Soa, Phys. Rev. D 73, 075005 (2006).
[6] P.V.Dong, T.T.Huong, D.T.Huong and H.N.Long, Phys. Rev. D 74, 053003 (2006).
[7] M.B.Tully and G.C.Joshi, Phys.Rev.D 64, 011301(R)(2001).

[8] D.A.Gutiérrez, W.A.Ponce and L.A.Sánchez, Int. J. of Mod. Phys. A 21, 2217 (2006).