Orbital angular momentum of the proton and intrinsic five-quark Fock states

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Abstract

The orbital angular momentum ($L_q$) of the proton is studied by employing the extended constituent quark model. Contributions from different flavors, namely, up, down, strange, and charm quarks in the proton are investigated. Probabilities of the intrinsic $q\bar{q}$ pairs are calculated using a $^3P_0$ transition operator to fit the sea flavor asymmetry $I_a = \bar{d} - \bar{u} = 0.118 \pm 0.012$ of the proton \cite{1}. Our numerical results lead to $L_q = 0.158 \pm 0.014$, in agreement with $4/3I_a = 0.157 \pm 0.016$, and consistent with findings based on various other approaches.
I. INTRODUCTION

In the late 1980’s, the European Muon Collaboration (EMC) published experimental results [2] on the spin asymmetry in polarized deep inelastic scattering, providing unexpected evidence that the sum of the spins of the quarks add up only to a fraction of the proton’s total spin. That finding being in contrast to the Gell-Mann–Zweig quark model [3], in which the spin of the proton is totally generated by the spins of the three valence quarks, gave rise to the proton spin ”crisis”.

Since then, efforts aiming at uncovering the spin structure ”puzzle” of the nucleon have triggered a significant number of measurements using various facilities. In order to emphasize the context of the present work, we start with the Ji’s sum rule [4], according to which the nucleon spin can be decomposed as,

\[ J_N = \sum_{f=u,d,s,c...} \left( \frac{1}{2} \Delta \Sigma_q + L_q \right) + J_g, \]  

where \( \frac{1}{2} \Delta \Sigma_q \) is the contribution from the intrinsic quark spin, \( L_q \) the quark orbital angular momentum (OAM) and \( J_g \) the gluon total angular momentum.

In Eq. (1), the sum over quarks flavors goes beyond the naive constituent quark model (CQM), embodying higher Fock states, namely, in addition to the conventional nucleon structure with three constituent quarks (\( |qqq\rangle; q \equiv u, d \)), one introduces higher Fock five–quark components \( |qqqQ\bar{Q}\rangle \), with quark–antiquark pairs \( Q\bar{Q} \equiv uu, dd, ss, cc... \).

Need for the \( Q\bar{Q} \) components in the nucleon was emphasized in 1990’s by measurements of the \( \bar{d} - \bar{u} \) flavor asymmetry and the ratio \( \bar{u}/\bar{d} \) performed by the New Muon [5], E772 [6], NA51 [7], HERMES [8], and FNAL E866/NuSea [1] collaborations. Latest results from each one of the experimental groups using various facilities: BNL, CERN, DESY, Jlab, and SLAC are given in chronological order in [9]. In spite of healthy set of experimental data and intensive theoretical investigations, the question is still open; for recent reviews, see e.g. [10–17].

Actually, genuine higher Fock states in the baryons’ wave functions constitute a pertinent nonperturbative source of the intrinsic quark–antiquark components [18]; to be distinguished from the extrinsic pairs arising from gluon splitting in perturbative QCD and contributing to \( J_g \). The well known non-vanishing \( \bar{d} - \bar{u} \) flavor asymmetry measured [1], with high enough accuracy, provided stringent constraints on the role played by the virtual \( Q\bar{Q} \) pairs in the
nucleon. Moreover, while the CQM predicts also a vanishing value for the OAM, the $Q\bar{Q}$ components lead to $L_q \neq 0$. Actually, the contribution of the OAM to the spin of proton was found to be comparable to that of the sea quark ($\approx 30\%$ each) \cite{19}, and much larger than that of the gluons \cite{20}.

The present work is devoted to studying the proton’s OAM, which continues to be investigated via various formalisms; see review papers, e.g. \cite{12,14-17}.

In phenomenological approaches, based on meson-baryon degrees of freedom, the intrinsic $Q\bar{Q}$ pairs, sea quarks, are handled as meson-cloud surrounding the baryon \cite{23-26}. Accordingly, the traditional constituent quark model was extended to take into account the Fock components via pionic fluctuations, and hence generating the measured $\bar{d} - \bar{u}$ flavor asymmetry, and OAM in the nucleon. The most commonly used configurations embody $N\pi$ and $\Delta\pi$ Fock components in the proton. In this frame, Garvey \cite{25} obtains $L_q = 0.147 \pm 0.027$. In \cite{27}, relationship between the OAM and the sea flavor asymmetry of the proton in different models was investigated.

Bijker and Santopinto performed a calculation within the Unquenched Quark Model (UQM) \cite{28}, based on a quark model with continuum components, to which quark–antiquark pairs are added perturbatively employing a $^3P_0$ model \cite{29}. Fixing $J = 1/2$, they found $L_q = 0.162$. Lorcé and Pasquini studied the Wigner distributions in the light-cone constituent quark model (LCCQM) \cite{30}, reaching to a comparable value, $L_q = 0.126$. Lattice QCD calculations is an ongoing long endeavor, see e.g. \cite{31,32}. Recently, Alexandrou et al. released the results of a calculation \cite{33} of the quark and gluon contributions to the proton spin, using an ensemble of gauge configurations with two degenerate light quarks with mass fixed to approximately reproduce the physical pion mass. They found the OAM carried by the quarks in the nucleon to be $L_q = 0.207 \pm 64 \pm 45$. Another recent LQCD calculation by Yang \cite{32} lead to smaller central value $L_q = 0.10 \pm 9$, but due to the size of the uncertainties, results from the two investigations turn out to be compatible with each other.

The theoretical frame of the present work is based on an extended chiral constituent quark model (E$\chi$CQM), complemented with the $SU(6) \otimes O(3)$ symmetry breaking effects. Recently, the intrinsic sea flavor content including $\bar{u}$, $\bar{d}$, $\bar{s}$ and $\bar{c}$ in the nucleon were investigated employing our formalism, within which all the possible five-quark Fock components in the nucleon wave function were taken into account \cite{34,35}, coupling between the three- and five-quark components was assumed to be via $^3P_0$ quark-antiquark pair creation mech-
anism [29], and the coupling strength was fixed by fitting [34, 35] the sea flavor asymmetry of the proton [1]. The corresponding obtained pion-nucleon, strangeness-nucleon [36] and charm-nucleon sigma terms [35] were found to be reasonably consistent with predictions by the Lattice QCD and chiral perturbation theory.

Analogous to the meson-cloud description for the nucleon, the five–quark components in the baryons’ wave functions naturally contribute to the OAM of the proton, required by the angular momentum conservation law. Consequently, in the present work we study the contributions to the proton’ OAM from different quark flavors, by taking into account all possible five—quark Fock components, based on the results obtained in [34, 35].

The present manuscript is organized in the following way: in section II, we present our theoretical formalism which includes the wave functions and couplings between three- and five–quark components, and extract the contributions to the proton’s OAM from relevant five–quark configurations. We report on our numerical results in section III and proceed to comparisons with the outcomes of the other approaches briefly presented above. Section IV contains summary and conclusions.

II. THEORETICAL FRAME

As shown in [34, 35], considering possible pentaquark components, wave function of the proton can be expressed as follows,

$$|\psi_p\rangle = \frac{1}{\sqrt{N}} \left( |uud\rangle + \sum_{i,n_r,l} C_{in,l} |uud(qq\bar{q}), i, n_r, l\rangle \right),$$

(2)

where the first term is the conventional wave function for the proton with three constituent quarks, and the second one a sum over all possible higher Fock components with a $qq\bar{q}$ pairs, namely, the light, strange, and charm quark–antiquark pairs. Different possible orbital–flavor–spin–color configurations of the four–quark subsystems in the five–quark system are numbered by $i$; $n_r$ and $l$ denote the inner radial and orbital quantum numbers, respectively, as discussed in [34], the orbital quantum number $l$ in the present case can only be 1, and contributions from the configurations with $n_r \geq 1$ should be negligible, if one takes the coupling between three- and five–quark components to be via the $^3P_0$ mechanism, within
which the transition operator can be written as,
\[
\hat{T} = -\gamma \sum_j \mathcal{F}_{i,j}^{00} C_{OFSC} \sum_m \langle 1, m; 1, -m|00\rangle \chi_{j,5}^{1,m} \mathcal{Y}_{j,5}^{1,-m} (\vec{p}_j - \vec{p}_5) b^\dagger(\vec{p}_j) d^\dagger(\vec{p}_5). \tag{3}
\]

In the above equation, \(\hat{T}\) has units of energy, so that \(\gamma\) is (in natural units) a dimensionless constant of the model. \(\mathcal{F}_{i,j}^{00}\) and \(C_{i,j}^{00}\) are the flavor and color singlet of the quark–antiquark pair \(Q_i\bar{Q}_i\) in the five–quark system, and \(C_{OFSC}\) is an operator to calculate the orbital-flavor-spin-color overlap between the residual three–quark configuration in the five–quark system and the valence three–quark system. \(\chi_{j,5}^{1,m}\) is a spin triplet wave function with spin \(S=1\) and \(\mathcal{Y}_{j,5}^{1,-m}\) is a solid spherical harmonics referring to the quark and antiquark in a relative \(P–\)wave. \(b^\dagger(\vec{p}_j)\) and \(d^\dagger(\vec{p}_5)\) are the creation operators for a quark and antiquark with momenta \(\vec{p}_j\) and \(\vec{p}_5\), respectively. The operator \(\hat{T}\), expressed in second-quantization form, can then be applied in the Fock space. The coefficient \(C_{in,l}\) for a given five–quark component can be related to the transition matrix element between the three– and five–quark configurations of the studied baryon,
\[
C_{in,l} = \frac{\langle uud(i\bar{q}), i, n_r, l|\hat{T}|uud\rangle}{M_p - E_{in,l}}, \tag{4}
\]
where \(M_p\) is the physical mass of the proton, and \(E_{in,l}\) the energy for a corresponding five–quark component. In order to estimate the energy splitting for different pentaquark configurations, we employ the chiral constituent quark model in which the hyperfine interaction between quarks takes the following form:
\[
H_h = -\sum_{i<j} \vec{\sigma}_i \cdot \vec{\sigma}_j \left[ \sum_{a=1}^3 V_a(r_{ij}) \lambda^a_i \lambda^a_j + \sum_{a=4}^7 V_K(r_{ij}) \lambda^a_i \lambda^a_j + V_\eta(r_{ij}) \lambda^8_i \lambda^8_j + \sum_{a=9}^{12} V_D(r_{ij}) \lambda^a_i \lambda^a_j \right. \\
+ \left. \sum_{a=13}^{14} V_{Ds}(r_{ij}) \lambda^a_i \lambda^a_j + V_{qe}(r_{ij}) \lambda^{15}_i \lambda^{15}_j \right], \tag{5}
\]
where \(\lambda^a_i\) denotes the \(SU(4)\) Gell-Mann matrix acting on the \(i^{th}\) quark, \(V_M(r_{ij})\) is the potential of the \(M\) meson-exchange interaction between \(i^{th}\) and \(j^{th}\) quark, as extensively discussed in [37].

Accordingly, there are 17 different pentaquark configurations (Table II) forming the Fock components in the proton wave function. Those 17 configurations are classified into four different categories according to the orbital and spin symmetry of the four-quark subsystem. As shown in Table II orbital symmetry for the four-quark subsystem of five–quark components in the proton can be either the mixed symmetric [31]_X, or the completely symmetric
TABLE I: Categories (2\textsuperscript{nd} line) and associated configurations (lines 3-8) for five-quark components.

| i | Category / Config. | i | Category / Config. | i | Category / Config. | i | Category / Config. |
|---|------------------|---|------------------|---|------------------|---|------------------|
| I / [31]x[22]S | II / [31]x[31]S | III / [4]x[22]S | IV / [4]x[31]S |
| 1 | [31]x[4]FS[22]F[22]S | 5 | [31]x[4]FS[31]F[31]S | 11 | [4]x[31]FS[211]F[22]S | 14 | [4]x[31]FS[211]F[31]S |
| 2 | [31]x[31]FS[211]F[22]S | 6 | [31]x[4]FS[31]2F[31]S | 12 | [4]x[31]FS[31]F[22]S | 15 | [4]x[31]FS[22]F[31]S |
| 3 | [31]x[31]FS[31]F[22]S | 7 | [31]x[31]FS[211]F[31]S | 13 | [4]x[31]FS[31]2F[31]S | 16 | [4]x[31]FS[31]F[31]S |
| 4 | [31]x[31]FS[31]2F[22]S | 8 | [31]x[31]FS[22]F[31]S | 17 | [4]x[31]FS[31]2F[31]S |
| 9 | [31]x[31]FS[31]2F[31]S |
| 10 | [31]x[31]FS[31]2F[31]S |

\[ |uud(qq), i, 0, 1; +1/2\rangle = \sum_{abcde} \sum_{M',M} C_{J^M}^{\frac{1}{2}L} C^{[14]}_{1m, s_{z}} C^{[31]}_{[31]a[211]a} C^{[31]}_{[FS]c} C^{[FS]c}_{[F]c[31]c} [31]x_{m}(b) \]

\[ |uud(qq), i, 0, 1; +1/2\rangle = \sum_{abcde} \sum_{s_{z}, mm'} C_{1s_{z}, jm}^{\frac{1}{2}L} C_{1m', s_{z}^{'}}^{[14]} C^{[31]}_{[31]a[211]a} C^{[31]}_{[FS]c} C^{[FS]c}_{[F]c[31]c} [211]c_{a} \]

\[ \bar{Y}_{1m'} \bar{\bar{x}}_{s_{z}'} \varphi(\{\bar{r}_{q}\}) \ i = 11, \cdots, 17, \] respectively. Here [F], [S] and [211] correspond to the flavor, spin and color state wave functions, denoted by their relevant Weyl tableaux; \( \bar{Y}_{1m'} \) and \( \bar{\bar{x}}_{s_{z}'} \) refer to the orbital and spin states, respectively. Considering the flavor symmetry of the four-quark subsystem, \([31]_{F}\) limits the quark-antiquark pair in the pentaquark configurations to be \( u\bar{u} \) or \( d\bar{d} \), while \([31]_{F}^{2}\) and \([211]_{F}\) rule out the pentaquark configurations with light quark-antiquark pair.

At this point, we discuss the OAM possibly arising from each of the four categories in Table I. In category I, spin symmetry of the four-quark subsystem is \([22]_{S}\), which leads to the spin quantum number \( S = 0 \). It is straight forward to show that the projections of the quark orbital angular momentum arising from all the four configurations are the same:

\[ \langle uud(q\bar{q}), i, 0, 1; +1/2 | \hat{L}_{qz} | uud(q\bar{q}), i, 0, 1; +1/2 \rangle = 2/3 C_{m_{z}f}^{2}/N, \ i = 1, \cdots, 4. \] 

Note that we have taken the notation,

\[ \hat{L}_{qz} = \sum_{f} \hat{L}_{f+} + \sum_{f} (\hat{L}_{f} + \hat{L}_{f}^{z}), \]
where $\hat{l}_f$ and $\hat{l}_{\bar{f}}$ are the OAM operators for the quark and antiquark with flavor $f$, respectively, and the sum runs over the flavors $u, d, s,$ and $c$.

The four configurations in category I contribute differently to the proton sea flavor asymmetry. Taking the flavor $SU(3)$ symmetry for light and strange quarks, and neglecting the five-quark components with $c\bar{c}$ pair in the proton, then respective contributions to $I_a = \bar{d} - \bar{u}$ due to the four configurations in category I read,

$$I_{a,1} = \frac{2}{3} C_{1n,t}^2 / \mathcal{N}, \quad I_{a,2} = 0, \quad I_{a,3} = -\frac{1}{3} C_{3n,t}^2 / \mathcal{N}, \quad I_{a,4} = 0,$$

(10)

Here, we have labeled the contribution from the $i$th five-quark configuration as $I_{a,i}$, and hereafter we will take the same convention for the other configurations.

In category II, the spin symmetry of the four-quark subsystem is $[31]_S$, which leads to the spin quantum number $S = 1$. Coupling between spin $S = 1$ and orbital angular momentum $L = 1$ of the four-quark subsystem leads to the total angular momentum $J_4$ equal to 0 or 1. In the present work, we take $J_4 = 0$ because of the lower energy. Then, one finds that the projections of the quark orbital angular momentum arising from all the configurations in category II vanish:

$$\langle uud(q\bar{q}), i, 0, 1; +1/2 | \hat{L}_{qz} | uud(q\bar{q}), i, 0, 1; +1/2 \rangle = 0, \quad i = 5, \cdots, 10. \quad (11)$$

For the six configurations in category II, respective contributions to the proton sea flavor asymmetry are,

$$I_{a,5} = -\frac{1}{3} C_{5n,t}^2 / \mathcal{N}, \quad I_{a,6} = I_{a,7} = I_{a,10} = 0, \quad I_{a,8} = \frac{2}{3} C_{8n,t}^2 / \mathcal{N}, \quad I_{a,9} = -\frac{1}{3} C_{9n,t}^2 / \mathcal{N}, \quad (12)$$

according to the flavor structure of the corresponding configuration.

In categories III and IV, the orbital wave function for the four-quark subsystem is $[4]_X$, namely, the orbital angular momentum of the four-quark subsystem is $L = 0$. And the antiquark is in its first orbitally excited state in the present case. Therefore, contributions to the proton angular momentum by configurations in categories III and IV should be from the antiquark,

$$\langle uud(q\bar{q}), i, 0, 1; +1/2 | \hat{L}_{qz} | uud(q\bar{q}), i, 0, 1; +1/2 \rangle = \frac{2}{3} C_{in,t}^2 / \mathcal{N}, \quad i = 11, \cdots, 13, \quad (13)$$

$$\langle uud(q\bar{q}), i, 0, 1; +1/2 | \hat{L}_{qz} | uud(q\bar{q}), i, 0, 1; +1/2 \rangle = 0, \quad i = 14, \cdots, 17. \quad (14)$$
Moreover, it is straightforward to show that,

$$I_{a,11} = 0, \ I_{a,12} = -1/3C_{12n_r,l}^2/N, \ I_{a,13} = 0,$$

$$I_{a,14} = 0, \ I_{a,15} = 2/3C_{15n_r,l}^2/N, \ I_{a,16} = -1/3C_{16n_r,l}^2/N, \ I_{a,17} = 0.$$  \hspace{1cm} (15)

Accordingly, the projection of the proton OAM reads,

$$p\langle \psi; +1/2|\hat{L}_{qz}|\psi; +1/2\rangle_p = \frac{2}{3N} \left( \sum_{i=1,4} C_{in_r,l}^2 + \sum_{i=11,13} C_{in_r,l}^2 \right),$$  \hspace{1cm} (17)

and the flavor asymmetry of the proton takes the following form:

$$I_a = \bar{d} - \bar{u} = \frac{2}{3N} \left( C_{1n_r,l}^2 + C_{8n_r,l}^2 + C_{15n_r,l}^2 \right) - \frac{1}{3N} \left( C_{3n_r,l}^2 + C_{5n_r,l}^2 + C_{9n_r,l}^2 + C_{12n_r,l}^2 + C_{16n_r,l}^2 \right).$$  \hspace{1cm} (18)

It is obvious that in the present approach, projections of the OAM and flavor asymmetry of the proton are not equivalent to each other. Finally, one has to note that the flavor asymmetry $I_a$ given in (18) is obtained by neglecting the five-quark components with charm quark-antiquark pair and taking $SU(3)$ flavor symmetry for light and strange quarks. In any case, since probabilities for the five-quark components with strange and charm quark-antiquark pairs in the nucleon should not be significantly large, one can expect that projection of the OAM should be slightly larger than the flavor asymmetry according to Eqs. (17) and (18).

### III. NUMERICAL RESULTS AND DISCUSSION

To get the numerical results, one has to determine the probabilities for all the light, strangeness and charmness components in the proton, as discussed in Refs. [34-36]. They depend on the coupling strengths $V$ for Goldstone boson exchanges, the degenerated energy $E_0$ for different pentaquark configurations, when differences between the light, strange and charm quark constituent masses, flavor $SU(4)$ symmetry breaking effects and hyperfine interactions between quarks are not included, and the general orbital overlap factor $V \propto \langle uud|q\bar{q}, i, n_r, l|\hat{T}|uud \rangle$. Same as in [34], here the parameters $V$ for Goldstone boson exchange model are taken to be the empirical values $[37]$. $E_0 = 2127$ MeV is also an empirical value $[34]$, and $V$ was determined by fitting $[34, 35]$ the sea flavor asymmetry of the proton $I_{exp}^a = 0.118 \pm 0.012$ $[1]$, resulting in,

$$V = 572 \pm 47 \text{ MeV}.$$  \hspace{1cm} (19)
With the parameters given above, one obtains the probabilities for the five–quark Fock components in the proton wave function; the numerical values were reported in [35].

As discussed in Sec. II, the pentaquark configurations in categories II and IV cannot contribute to the projection of the OAM, since the total angular momentum $J_4 = 0$ for the four–quark subsystem in category II and $J_5 = 0$ for the antiquark in category IV.

In our previous studies on the strangeness magnetic form factor of the proton [22] and the nonperturbative strangeness suppression [22], which successfully reproduced the relevant data, all four categories intervene. But, in the present case, only the pentaquark configurations in categories I and III contribute to the OAM. Accordingly, the expectation values for the projection of the OAM of different flavors reads,

$$\langle \hat{l}_{f+f} \rangle_q = \langle uud(q\bar{q}), i, 0, 1; +1/2 | \hat{l}_{f+f} | uud(q\bar{q}), i, 0, 1; +1/2 \rangle,$$

\[ (20) \]

with $i = 1, \cdots 4; 11, \cdots 13$, $f = u, d, s, c$ denoting contributions from different flavors, and the subscript $q = l, s, c$ denoting contributions from the light, strangeness and charmness components in the proton (Table II). In addition, the corresponding probabilities for the five–quark Fock components are also listed in columns $P^i_l$, $P^i_s$ and $P^i_c$ in Table II. Accordingly, the OAM per flavor reads,

$$\langle \hat{l}_{f+f} \rangle = \sum_{i=1}^{4} \left( P^i_l \langle \hat{l}_{f+f} \rangle^i_l + P^i_s \langle \hat{l}_{f+f} \rangle^i_s + P^i_c \langle \hat{l}_{f+f} \rangle^i_c \right) + \sum_{i=11}^{13} \left( P^i_l \langle \hat{l}_{f+f} \rangle^i_l + P^i_s \langle \hat{l}_{f+f} \rangle^i_s + P^i_c \langle \hat{l}_{f+f} \rangle^i_c \right).$$

\[ (21) \]

**TABLE II:** Contributions to the projection of the proton’s OAM from different flavors for quark and anti-quark components $\langle \hat{l}_{f+f} \rangle_q^i$, per five–quark Fock configuration $i$, with probability $P^i_q$ (in %).

|     | light                      | strangeness              | charmness          |
|-----|----------------------------|--------------------------|--------------------|
| i   | $P^i_l$ | $(\hat{l}_{u+d})^i_l$ | $(\hat{l}_{d+d})^i_l$ | $(\hat{l}_{s+s})^i_l$ | $(\hat{l}_{c+c})^i_l$ | $P^i_s$ | $(\hat{l}_{u+d})^i_s$ | $(\hat{l}_{d+d})^i_s$ | $(\hat{l}_{s+s})^i_s$ | $(\hat{l}_{c+c})^i_s$ | $P^i_c$ | $(\hat{l}_{u+d})^i_c$ | $(\hat{l}_{d+d})^i_c$ | $(\hat{l}_{s+s})^i_c$ | $(\hat{l}_{c+c})^i_c$ |
| 1   | 14.62 (1.21) | 0.333 | 0.333 | 0 | 0 | 0.98 (8) | 0.333 | 0.167 | 0.167 | 0 | 0.04 (0) | 0.333 | 0.167 | 0 | 0.167 |
| 2   | 0 | 0 | 0 | 0 | 0 | 0.36 (3) | 0.250 | 0.208 | 0.208 | 0 | 0.03 (1) | 0.250 | 0.208 | 0 | 0.208 |
| 3   | 1.65 (14) | 0.500 | 0.167 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4   | 0 | 0 | 0 | 0 | 0 | 0.26 (2) | 0.361 | 0.097 | 0.208 | 0 | 0.03 (1) | 0.361 | 0.097 | 0 | 0.208 |
| 11  | 0 | 0 | 0 | 0 | 0 | 0.85 (8) | 0 | 0 | 0.667 | 0 | 0.09 (1) | 0 | 0 | 0 | 0.667 |
| 12  | 4.14 (37) | 0.444 | 0.222 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 13  | 0 | 0 | 0 | 0 | 0 | 0.65 (6) | 0 | 0 | 0.667 | 0 | 0.09(0) | 0 | 0 | 0 | 0.667 |
Accordingly, contributions to the projection of the OAM of the proton from different flavors are as follows:

\[
\langle \hat{L}_{u+\bar{u}} \rangle = 0.081 \pm 0.007, \\
\langle \hat{L}_{d+\bar{d}} \rangle = 0.063 \pm 0.006, \\
\langle \hat{L}_{s+\bar{s}} \rangle = 0.013 \pm 0.001, \\
\langle \hat{L}_{c+\bar{c}} \rangle \simeq 0.001 \pm 0.000.
\]

(22)

Contributions from up and down quarks to the projection of the proton’s OAM are roughly in the same range, Table III, while \(\langle \hat{L}_{d+\bar{d}} \rangle\) is slightly smaller, and those from the strange and charm quarks are much smaller. In total, one gets,

\[
L_q \equiv \langle \hat{L}_{qz} \rangle \equiv \langle \hat{L}_{u+\bar{u}} \rangle + \langle \hat{L}_{d+\bar{d}} \rangle + \langle \hat{L}_{s+\bar{s}} \rangle + \langle \hat{L}_{c+\bar{c}} \rangle = 0.158 \pm 0.014, \\
\]

(23)

and then the relation between the orbital angular momentum and the sea flavor asymmetry, as expected, reads,

\[
\langle p, +1/2 | \hat{L}_{qz} | p, +1/2 \rangle \simeq 4/3 I_a.
\]

(24)

As briefly presented in Introduction, the quark contributions to the proton OAM and the spin structure of the nucleon have been intensively investigated, using different approaches. In Table III we compare our numerical results to those recently reported within other approaches.

| Approach [Ref.] | \(\langle \hat{L}_{u+\bar{u}} \rangle\) | \(\langle \hat{L}_{d+\bar{d}} \rangle\) | \(\langle \hat{L}_{s+\bar{s}} \rangle\) | \(\langle \hat{L}_{c+\bar{c}} \rangle\) | \(L_q \equiv \langle \hat{L}_{qz} \rangle\) |
|-----------------|----------------|----------------|----------------|----------------|----------------|
| E\(\chi\)CQM [Present work] | 0.081(7) | 0.063(5) | 0.013(2) | 0.001(0) | 0.159(14) |
| UQM [28] | – | – | – | – | 0.162 |
| LCCQM [30] | 0.071 | 0.055 | – | – | 0.126 |
| \(\pi\)-Cloud [25] | – | – | – | – | 0.147 |
| LQCD [31] | -0.107(40) | 0.247(38) | 0.067(21) | – | 0.207(78) |
| LQCD [32] | -0.14(4) | 0.20(3) | 0.04(2) | – | 0.10 (9) |

In the naive quark model, since all the constituent quarks in the proton are in their ground states, the projections of the OAM due to both up and down quarks are zero.

In [28], the nucleon orbital angular momentum is investigated using the unquenched quark model (UQM), within which the effects of the quark–antiquark pairs including \(u\bar{u},\)


$dd$ and $ss$ are taken into account, and the quark–antiquark pairs creation is assumed to be via a $^3P_0$ mechanism. Their findings show that the quark–antiquark pairs have sizable contributions to the proton OAM. Their numerical result, and ours are in (almost) perfect agreement, although contributions per flavor are not given in [28].

Within the meson-cloud picture, as discussed in Sec. II if one only considers the $N\pi$ and $\Delta\pi$ Fock components in the proton, projection of the OAM of the proton should be equal to the proton flavor asymmetry $\bar{d} - \bar{u}$, as studied in [25], i.e. $L_q \sim 0.147$, consistent with our result within $1\sigma$.

The quarks contribution to the OAM was also obtained from the Wigner distribution for unpolarized quarks in the longitudinally polarized nucleon. The formalism is applied in the light-cone constituent quark model (LCCQM), leading to compatible value with ours, within $2 - 3\sigma$.

Numerical values for $L_q$ within the Lattice QCD calculations were reported. Here, a caution is in order: in the present model contributions to the proton OAM are exclusively due to the intrinsic sea content $qq$, while LQCD approaches embody also extrinsic quark–antiquark pairs arising from the gluon splitting in perturbative QCD regime ($g \to q\bar{q}$). Nevertheless, in Table III we report results from two approaches [32, 33]. The first remark is that contributions per flavor for light quarks are very different from our values, as well as from those obtained within LCCQM. For the strangeness components, discrepancies are around $2\sigma$. However, given the rather large uncertainties in the LQCD results, the sum over all contributions turns out to be consistent, within $1\sigma$, with all other values reported in Table III. Accordingly, a meaningful comparison would require separating in the LQCD calculations contributions from intrinsic and extrinsic quark–antiquark pairs, and reducing significantly the uncertainties, which is a huge task.

IV. SUMMARY AND CONCLUSIONS

To summarize, in the present work we investigate the OAM of the proton by taking into account all the possible light, strangeness and charmness five–quark Fock components in the wave function of proton. Coupling between three– and five–quark components was dealt with via the $^3P_0$ quark–antiquark pairs creation mechanism, model parameters are empirical values [34, 37]. The only adjusted parameter, $V$ in Eq. (19), for Goldstone boson exchange
model, was determined by fitting\cite{34,35} the experimental data for sea flavor asymmetry $I_a = \bar{d} - \bar{u} = 0.118 \pm 0.012$ of the proton\cite{1}. This ensemble allowed us postdicting, on the one hand the strangeness magnetic moment $\mu_s$ and the strangeness magnetic moment $G^s_M$ of the proton\cite{21}, and on other hand shedding a light\cite{22} on the measured\cite{39} quark–antiquark ratios $r_\ell = u\bar{u}/d\bar{d}$, $r_s = s\bar{s}/d\bar{d}$, and the strangeness content of the proton $\kappa_s = 2s\bar{s}/(u\bar{u} + d\bar{d})$.

In the present work, we studied the complete set of the 17 five–quark configurations, falling in four categories and showed that only 7 configurations in two of the categories contribute to the OAM. Accordingly, the proton OAM carried by quarks turns out be $L_q = 0.158 \pm 0.014$ in our model, in perfect agreement with $4/3I_a = 0.157 \pm 0.016$, as expected. Contributions from the up and down quarks and antiquarks are dominant ones and comparable to each other, while those from strange and charm quarks and antiquarks are rather small.

We proceeded to comparisons between our results and recent findings within other approaches. Perfect agreement was obtained with the result coming from the unquenched quark model\cite{28}. The meson-cloud picture, embodying the $N\pi$ and $\Delta\pi$ Fock components in the proton, leads to a value\cite{25} consistent with ours within $1\sigma$. That is also the case with respect to the LQCD\cite{32,33}, albeit with rather large uncertainties. Light-cone constituent quark model’s outcome is compatible with ours, within $2 - 3\sigma$.

In conclusion, our determination of the proton’s OAM falls reasonably well in the range of values reported by other authors, underlining the crucial role played by intrinsic five–quark components in the proton.

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[1] R. S. Towell et al. [FNAL E866/NuSea Collaboration], Improved measurement of the anti-d/anti-u asymmetry in the nucleon sea, Phys. Rev. D 64, 052002 (2001).

[2] J. Ashman et al. [European Muon Collaboration], A Measurement of the Spin Asymmetry and Determination of the Structure Function g(1) in Deep Inelastic Muon-the proton Scattering, Phys. Lett. B 206, 364 (1988); An Investigation of the Spin Structure of the proton in Deep Inelastic Scattering of Polarized Muons on Polarized the protons, Nucl. Phys. B 328, 1 (1989).

[3] M. Gell-Mann, A Schematic Model of Baryons and Mesons, Phys. Lett. 8, 214 (1964); G. Zweig, An SU(3) model for strong interaction symmetry and its breaking, CERN Reports TH401 and 412 (1964).

[4] X. D. Ji, Lorentz symmetry and the internal structure of the nucleon, Phys. Rev. D 58, 056003 (1998).

[5] P. Amaudruz et al. [New Muon Collaboration], The Gottfried sum from the ratio F2(n) / F2(p), Phys. Rev. Lett. 66, 2712 (1991); M. Arneodo et al. [New Muon Collaboration], A Reevaluation of the Gottfried sum, Phys. Rev. D 50, R1 (1994).

[6] P. L. McGaughey et al. E772 Collaboration, Limit on the anti-d / anti-u asymmetry of the nucleon sea from Drell-Yan production, Phys. Rev. Lett. 69, 1726 (1992).

[7] A. Baldit et al. [NA51 Collaboration], Study of the isospin symmetry breaking in the light quark sea of the nucleon from the Drell-Yan process Phys. Lett. B 332, 244 (1994).

[8] K. Ackerstaff et al. [HERMES Collaboration], The Flavor asymmetry of the light quark sea from semi-inclusive deep inelastic scattering, Phys. Rev. Lett. 81, 5519 (1998).

[9] P. L. Anthony et al. [E155 Collaboration], Measurements of the Q**2 dependence of the proton and neutron spin structure functions g(1)**p and g(1)**n, Phys. Lett. B 493, 19 (2000); B. Adeva et al. [Spin Muon (SMC) Collaboration], Spin asymmetries for events with high p(T) hadrons in DIS and an evaluation of the gluon polarization, Phys. Rev. D 70, 012002 (2004); A. Airapetian et al. [HERMES Collaboration], Precise determination of the spin structure function g(1) of the proton, deuteron and neutron, Phys. Rev. D 75, 012007 (2007); A. Adare et al. [PHENIX Collaboration], Inclusive double-helicity asymmetries in neutral-pion and etameson production in p̅ + p̅ collisions at √s = 200 GeV, Phys. Rev. D 90, 012007 (2014); L. Adamczyk et al. [STAR Collaboration], Precision Measurement of the Longitudinal Double-
spin Asymmetry for Inclusive Jet Production in Polarized Proton Collisions at $\sqrt{s} = 200$ GeV, Phys. Rev. Lett. 115, 092002 (2015); R. Fersch et al. [CLAS Collaboration], Determination of the proton spin structure functions for $0.05 < Q^2 < 5 \text{GeV}^2$ using CLAS, Phys. Rev. C 96, 065208 (2017); C. Adolph et al. [COMPASS Collaboration], Leading-order determination of the gluon polarisation from semi-inclusive deep inelastic scattering data, Eur. Phys. J. C 77, 209 (2017).

[10] S. E. Kuhn, J.-P. Chen and E. Leader, Spin Structure of the Nucleon - Status and Recent Results, Prog. Part. Nucl. Phys. 63, 1 (2009).

[11] M. Burkardt, C. A. Miller and W. D. Nowak, Spin-polarized high-energy scattering of charged leptons on nucleons, Rept. Prog. Phys. 73, 016201 (2010).

[12] C. A. Aidala, S. D. Bass, D. Hasch and G. K. Mallot, The Spin Structure of the Nucleon, Rev. Mod. Phys. 85, 655 (2013).

[13] W. C. Chang and J. C. Peng, Flavor Structure of the Nucleon Sea, Prog. Part. Nucl. Phys. 79, 95 (2014).

[14] E. Leader and C. Lorcé, The angular momentum controversy: What’s it all about and does it matter?, Phys. Rept. 541, 163 (2014).

[15] M. Wakamatsu, Is gauge-invariant complete decomposition of the nucleon spin possible?, Int. J. Mod. Phys. A 29, 1430012 (2014).

[16] K. F. Liu and C. Lorcé, The Parton Orbital Angular Momentum: Status and Prospects, Eur. Phys. J. A 52, 160 (2016).

[17] A. Deur, S. J. Brodsky and G. F. De Téramond, The Spin Structure of the Nucleon, arXiv:1807.05250 [hep-ph].

[18] S. J. Brodsky, C. Peterson and N. Sakai, Intrinsic Heavy Quark States, Phys. Rev. D 23, 2745 (1981); S. J. Brodsky, A. Kusina, F. Lyonnet, I. Schienbein, H. Spiesberger and R. Vogt, A review of the intrinsic heavy quark content of the nucleon, Adv. High Energy Phys. 2015, 231547 (2015).

[19] R. Bijker and E. Santopinto, Spin and flavor content of octet baryons, J. Phys. Conf. Ser. 322, 012014 (2011).

[20] S. J. Brodsky and S. Gardner, Evidence for the Absence of Gluon Orbital Angular Momentum in the Nucleon, Phys. Lett. B 643, 22 (2006).

[21] C. S. An and B. Saghai, Strangeness magnetic form factor of the proton in the extended chiral
quark model, Phys. Rev. C 88, no. 2, 025206 (2013).

[22] C. S. An and B. Saghai, Intrinsic light and strange quark–antiquark pairs in the proton and nonperturbative strangeness suppression, Phys. Rev. D 95, no. 7, 074015 (2017).

[23] F. Myhrer and A. W. Thomas, A possible resolution of the proton spin problem, Phys. Lett. B 663, 302 (2008); A. W. Thomas, Interplay of Spin and Orbital Angular Momentum in the proton, Phys. Rev. Lett. 101, 102003 (2008).

[24] F. Huang, F. G. Cao and B. Q. Ma, Phenomenological analysis of nucleon strangeness and meson-nucleon sigma terms, Phys. Rev. D 76, 114016 (2007).

[25] G. T. Garvey, Orbital Angular Momentum in the Nucleon, Phys. Rev. C 81, 055212 (2010).

[26] M. Alberg and G. A. Miller, Taming the Pion Cloud of the Nucleon, Phys. Rev. Lett. 108, 172001 (2012).

[27] E. R. Nocera and E. Santopinto, Can sea quark asymmetry shed light on the orbital angular momentum of the proton?, arXiv:1611.07980 [hep-ph].

[28] R. Bijker, E. Santopinto and E. Santopinto, Unquenched quark model for baryons: Magnetic moments, spins and orbital angular momenta, Phys. Rev. C 80, 065210 (2009).

[29] A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Naive quark pair creation model of strong interaction vertices, Phys. Rev. D 8, 2223 (1973); Naive quark pair creation model and baryon decays, Phys. Rev. D 9, 1415 (1974).

[30] C. Lorcé and B. Pasquini, Quark Wigner Distributions and Orbital Angular Momentum, Phys. Rev. D 84, 014015 (2011); C. Lorcé, B. Pasquini, X. Xiong and F. Yuan, The quark orbital angular momentum from Wigner distributions and light-cone wave functions, Phys. Rev. D 85, 114006 (2012).

[31] H. W. Lin et al., Parton distributions and lattice QCD calculations: a community white paper, Prog. Part. Nucl. Phys. 100, 107 (2018).

[32] Y. B. Yang, A Lattice Story of the proton Spin, arXiv:1904.04138 [hep-lat].

[33] C. Alexandrou, M. Constantinou, K. Hadjiyiannakou, K. Jansen, C. Kallidonis, G. Koutsou, A. Vaquero Avilés-Casco and C. Wiese, Nucleon Spin and Momentum Decomposition Using Lattice QCD Simulations, Phys. Rev. Lett. 119, 142002 (2017).

[34] C. S. An and B. Saghai, Sea flavor content of octet baryons and intrinsic five-quark Fock states, Phys. Rev. C 85, 055203 (2012).

[35] S. Duan, C. S. An and B. Saghai, Intrinsic charm content of the nucleon and charmness-nucleon
sigma term, Phys. Rev. D 93, 114006 (2016).

[36] C. S. An and B. Saghai, Pion- and strangeness-baryon $\sigma$ terms in the extended chiral constituent quark model, Phys. Rev. D 92, no. 1, 014002 (2015).

[37] L. Y. Glozman and D. O. Riska, The Spectrum of the nucleons and the strange hyperons and chiral dynamics, Phys. Rept. 268, 263 (1996). L. Y. Glozman and D. O. Riska, The Charm and bottom hyperons and chiral dynamics, Nucl. Phys. A 603, 326 (1996), Erratum: [Nucl. Phys. A 620, 510 (1997)].

[38] C. S. An, D. O. Riska and B. S. Zou Strangeness spin, magnetic moment and strangeness configurations of the proton, Phys. Rev. C 73, 035207 (2006).

[39] M. Mestayer et al. [CLAS Collaboration], Strangeness suppression of $q\bar{q}$ creation observed in exclusive reactions, Phys. Rev. Lett. 113, 152004 (2014).