Acceleration Response Analysis of the SCR rigid body swing under wave and x-direction motion

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Abstract. Under the action of waves or current, the rigid body oscillation from the suspension point to touch down point (TDP) is a real problem of steel catenary riser (SCR). The influence of the phenomenon on the acceleration in cross flow of riser is not to be ignored. On the basis of the slightness beam with large deformation model coupling the wave force model and the rigid body rotation model, the impact of rigid body swing on the cross flow of structure’s acceleration is researched under the wave action and x direction motion. Numerical calculation manifests that the maximum impact of rigid body swing on structure acceleration is approximately 20%. Rigid body rotation is differ from the wave action and the top motion reducing with the depth of the water. Rigid body rotation increases with the radius vector S. The relative reduction calculation shows that between 10th and 140th, the structural response decreased by about 30%, and between 140th-seabed, by about 15%. The normalized calculation results from each node show that between 10th and 140th, the structural response decreases with the increase of the x-direction load frequency or the decrease of amplitude. Between the 140th-seabed, the structural response is increasing. It is expected that some rational and pragmatic suggestions can be provided for the computation of transverse flow of SCR. With the help from the team, the work of the article can be carried out. It is desired that, this article will help the team.

1. Introduction
Lou min et al. [1] used the similarity theory to scale the actual riser model into the test model for the offshore riser in shallow water, and measured the dynamic response compared it with Ansys finite element numerical simulation results of the riser under the action of fluid flow in the pipe and wave load outside the pipe. Based on linear wave theory, Fan baiming et al.[2] used spectral analysis to determine stress spectrum and calculate fatigue life. Based on the theory of isolated wave mKdV, linear wave theory and vortex-excited load theory, Jiang wujie[3] calculated the Morsion formula, considering the internal flow, and studied the dynamic response problem under the action of internal isolated wave and non-uniform flow of the tension riser. Liu bitao[4] et al. proposed a numerical simulation of the interaction between internal isolated waves and deep-sea riser. The flow field was carried out by numerical tank. The structure was carried out by shell theory, and the structural response of the flow situation was transmitted in real time. Li yan et al.[5] used the three-dimensional centralized mass model to calculate the time domain of the riser.

Based on Cable3D [6-7], this paper studies the influence of the rigid body swing on the structure. Lin Wang et.al[8] studied Severe Slugging Flow dynamic response of a riser. Peng Li et.al[9]
analyzed the risers based on FBG Sensing Technology. Decheng Wan and Yu Duanmu[10] did a recent review of long slender flexible risers in deep sea.

Since the swing axis of the SCR model rigid body oscillating system consists of the two points of the top end point and the touch down point, the top end point and the bottom touch point change will change the swing axis of the SCR model, and the changing swing axis will make further influence on the SCR model, and thus affect the rigid body rotation performance of the SCR model. In this paper, the influence of rigid body swing under the x-direction motion and wave motion is researched.

2. SCR large deflection slender beam model [6-7]

2.1 Basic Equations for SCR

The figure below shows the global coordinate system in which the equation of motion of the riser is located. The shape of the beam is represented by the vector \( \tilde{r}(s,t) \), which is a function of the length of the arc \( s \) after the deformation of the beam and the time \( t \). And \( \tilde{e}_x, \tilde{e}_y, \tilde{e}_z \) represent the unit vector of the curve following coordinate system, the direction of which corresponds to the global coordinate \( x, y, z \). \( \tilde{n}, \tilde{b}, \tilde{t} \) represent the unit vector of the normal, the minor, and the tangent at any point on the curve \( s \).

![Fig. 1. Following coordinate of large deflection slender beam](image)

According to the conservation of momentum and momentum, the equilibrium equation for the length of the beam can be obtained.

\[
F' + q = \rho \ddot{\tilde{r}}(s,t) \quad (1)
\]

\[
\tilde{M}' + r' \times F + m = 0 \quad (2)
\]

In the formula:

- \( F \) ---- Internal force of the beam section;
- \( q \) ---- Distribution force per unit length of the beam;
- \( \rho \) ---- Beam unit length quality;
- \( \tilde{M} \) ---- Internal moment of beam section;
- \( m \) ---- External torque of unit length on the beam.

According to Bernoulli-Euler theory, the moments in the beam section \( \tilde{M} \) (including bending moments and torque) can be expressed as:

\[
\tilde{M} = r' \times (Br'') + Hr' \quad (3)
\]

Deriving:
\[ \bar{M}' = r' \times (Br^*)' + Hr' + Hr^* \]  

In the formula, \( B \) is the bending stiffness and \( H \) is the torque.

Assumed \( H = 0 \) \( m = 0 \), substitute \( \bar{M}' \) into (2):

\[ F = \lambda r' - (Br^*)' \]  

\( \lambda \) -----Lagrangian operator.

substitute (5) into (1), equation of motion for large deflection slender beams:

\[ \rho \dddot{r} + (Br^*)'' - (\lambda r')' = q \]  

2.2 Load condition

Gravity of the unit length riser:

\[ q_i(s,t) = -\rho_i \ddot{g} A_i \]  

Inertia force:

\[ q'_i(s,t) = \rho_f A_f C_{Mh} N(\ddot{u} - \ddot{r}) + \rho_f A_f C_m T(\dot{u} - \dot{r}) \]  

Drag force:

\[ q''_i(s,t) = \frac{1}{2} \rho_f D_f C_{Dh} N(u - \dot{r}) |N(u - \dot{r})| \]

\[ + \frac{1}{2} \rho_f D_f C_{Dh} T(u - \dot{r}) |T(u - \dot{r})| \]  

In the formula:

\( C_{Mh}, C_{Mf} \) - Normal and tangential additional mass coefficients; \( C_{Dh}, C_{Df} \) - Normal and tangential drag coefficient \( \rho_i, \rho_f, \rho_i \) - Internal fluid density, and Seawater density; In-tube fluid density \( A_i, A_f \) Cross-sectional area and Inner diameter area

Out-of-tube F-K force:

\[ q^{F-K}_f(s,t) = \rho_f (\ddot{g} + \ddot{u}) A_f + (P_f A_f r')' \]  

In-tube F-K force:

\[ q^{F-K}_i(s,t) = -\rho_f (\ddot{g} + \ddot{u}) A_i - (P_f A_f r')' \]  

\( D_f \) - Outer diameter (hydrodynamic diameter), \( u, \dot{u} \) - Sea speed and acceleration \( P_f \) - Seawater pressure, \( P_i \) - In-tube fluid pressure.

\( T \) and \( N \) are vector transformation matrix:

\[ T = r'^T r' \]  

\[ N = I - T \]  

\( I \) - Unit matrix.

Then substituting formulas (7)–(13) into formula (6) to obtain the vibration control equation of steel catenary riser:

\[ M\dddot{r} + (Br^*)'' - (\lambda r')' = q \]  

\( M \)—Mass matrix:

\[ M = (\rho_f A_i + \rho_i A_i) I + \rho_f A_f C_{Mh} N + \rho_f A_f C_{Mh} T \]
Load matrix:

\[
q = (\rho_f A_f - \rho_i A_i - \rho_f A_i) g e_y + \rho_f A_f (I + C_{Mh} N + C_{Mh} T) \ddot{u} + \frac{1}{2} \rho_f D_f C_{Dn} N (u - \dot{r}) ||N(u - \dot{r})||
\]

\[
+ \frac{1}{2} \rho_f D_f C_{Dn} T (u - \dot{r}) ||T(u - \dot{r})||
\]

\[
\approx (T + P_f A_f - P_i A_i) - B K^2
\]  

(16)

(17)

2.3 SCR Riser bottom boundary condition

In addition to the fixed constraints at both ends, the steel catenary riser also has constraints on the interaction between the streamline segment and the seabed, including the normal elastic plastic deformation support and suction of the seafloor and the tangential friction and the resistance of the groove.

In the static analysis, the seafloor normal constraint is simulated by a spring system, and the dynamic analysis is simulated by a spring-damping system. The distribution constraint model of the spring is:

\[
\left\{ \begin{array}{l}
\frac{S}{D} \{D - (r_{y} - D_{bmn})\}, D - (r_{y} - D_{bmn}) > 0 \\
0, D - (r_{y} - D_{bmn}) \leq 0
\end{array} \right.
\]

(18)

In the formula: \( D \) - SCR Hydrodynamic radius, \( D_{bmn} \) - Longitudinal coordinates of the seabed, \( S \) - Wet weight per unit length.

The seafloor friction model can be expressed as:

\[
q_{Frict} = \left\{ \begin{array}{l}
C_f \cdot f \cdot \frac{S}{1 + \varepsilon}, D - (r_{y} - D_{bmn}) > 0 \\
0, D - (r_{y} - D_{bmn}) \leq 0
\end{array} \right.
\]

(20)

\[
C_f = \begin{cases} 
-1, \nu_t > C_v \\
-\frac{\nu_t}{C_v}, \nu_t \leq C_v \\
1, \nu_t < C_v
\end{cases}
\]

(21)

\( f \) - Coefficient of friction, \( \nu_t \) - Tangential speed, \( C_v \) - Tangential speed limit.

The distributed damping force model can be expressed as follows:

\[
q^{Dmao} = \left\{ \begin{array}{l}
-C_e \cdot \dot{r} \cdot e_y, D - (r_{y} - D_{bmn}) > 0 \\
0, D - (r_{y} - D_{bmn}) \leq 0
\end{array} \right.
\]

(22)

Critical damping coefficient - \( C_e = 2\sqrt{\rho S / D} \).

2.4 SCR Rigid Rotation Model.

According to the theory of momentum moment, the riser rotation equation is:
\[(m + m_a)s^2 \ddot{a}_r + c_a s^2 \dot{a}_r + mgc_1sa_r = q_s \sqrt{s_1^2 + s_2^2} + q_sc_2s_3, \quad (23)\]

\( m, m_a \) - the quality of the riser per unit length and the added mass, \( q_s, q_z \) - environmental load projection for riser, \( \alpha, \dot{\alpha}, \ddot{\alpha} \) - riser angular displacement, angular velocity and angle acceleration.

When equation (23) is added to (14), the following equation can be obtained:

\[ M\ddot{r} + (Br^*)'' - (\lambda r')' = q + mg - (m + m_a)\dddot{r} - c_a \dddot{r}, \quad (24)\]

2.5 Wave force model

The Morison equation is a formula for calculating the wave load perpendicular to the rigid column of the seabed. If it is a cylinder, moving in waves with \( \dot{x} \) and acceleration \( \dddot{x} \), the equation is:

\[ f_H = \frac{1}{2} C_D \rho A (u_x - \dot{x}) |u_x - \dddot{x}| + C_M \rho \frac{\pi D^2}{4} \frac{\partial u_x}{\partial t} - C_M \rho \frac{\pi D^2}{4} \dddot{x}, \quad (25)\]

In the formula, \( A \) - The projected area of the unit cylinder perpendicular to the wave direction; \( \rho \) - Seawater density; \( \bar{V}_0 \) - Unit column height drainage volume; \( C_m \) - Additional quality factor; \( C_M \) - Mass coefficient; \( C_D \) - Drag coefficient perpendicular to the axis of the cylinder; \( u_x \) - Water level horizontal velocity at any height of the column; \( u_x' \) - Water point acceleration at any height of the cylinder.

Add (25) as a coupling term to (14):

\[ M\ddot{r} + (Br^*)'' - (\lambda r')' = q + mg - (m + m_a)\dddot{r} - c_a \dddot{r} + f_H, \quad (24)\]

2.6 SCR Top motion boundary condition

The X-direction movement of the top of the SCR will affect the riser motion. In the calculation and analysis, by adding the X-direction motion to the calculation file, the rigid body swing effect is superimposed, and the structure is changed in the X-direction movement of the top suspension point to make the rigid body swing axis change, and the structure transverse flow direction acceleration amplitude change characteristics.

3. SCR wave rigid body response under linear motion

3.1 Basic parameter.

Applying a linear motion of X to the top suspension point of the SCR model simulates the floating platform response. This section only considers the wave action, and does not consider the influence of the current action. The wave incident direction is the X direction. Table 1 shows the top motion parameters for each operating condition.

| Table 1. SCR parameters[7] | |
|--------------------------|------------------|
| Calculation parameter    | Numerical value  |
| SCR outer diameter (m)   | 0.355            |
| SCR inner diameter (m)   | 0.305            |
Modulus of elasticity (Gpa) | 207
Lift coefficient | 0.7
Drag coefficient | 1.2
Mass coefficient | 1.0
Hydrodynamic parameters (m) | 0.355

Table 2. Motion parameters of working conditions[7]

| No | amplitude/m | Period/s | Frequency (Hz) | Angular frequency (Hz) |
|----|-------------|----------|----------------|------------------------|
| 1  | 3           | 10.8     | 0.093          | 0.58148                |
| 2  | 2           | 9.9      | 0.101          | 0.6343                 |
| 3  | 1           | 9.0      | 0.111          | 0.69778                |

3.2 Response analysis in z direction.
Figure 2-5, Figure 6-9 and Figure 10-13 are the results of the calculation of working condition 1-3.
Fig. 4: Acceleration response of the 140th node of the riser in case of working condition 1.

Fig. 5: Acceleration response of the 200th node of the riser of working condition 1.

Fig. 6: Acceleration response of the 10th node of the riser at operating condition 2.
Fig. 7: Acceleration response of the 80th node of the riser at operating condition 2

Fig. 8: Acceleration response of the 140th node of the riser at operating condition 2

Fig. 9: Acceleration response of the 200th node of the riser at operating condition 2
Fig. 10 Acceleration response of the 10th node of the riser at operating condition 3.

Fig. 11 Acceleration response of the 80th node of the riser at operating condition 3.

Fig. 12 Acceleration response of the 140th node of the riser at operating condition 3.
Fig. 13 Acceleration response of the 200th node of the riser at operating condition 3

Table 3 Calculated values of acceleration of each working condition node

| No | node  | Cab (m/s²) | Csw (m/s²) | growth rate |
|----|-------|------------|------------|-------------|
| 1  | 10th  | 0.097      | 0.096      | -1.03%      |
| 1  | 80th  | 0.062      | 0.063      | 1.61%       |
| 1  | 140th | 0.027      | 0.028      | 3.70%       |
| 1  | 200th | 0.014      | 0.011      | -21.43%     |
| 2  | 10th  | 0.087      | 0.086      | -1.15%      |
| 2  | 80th  | 0.061      | 0.063      | 3.28%       |
| 2  | 140th | 0.03       | 0.029      | -3.33%      |
| 2  | 200th | 0.017      | 0.015      | -11.76%     |
| 3  | 10th  | 0.077      | 0.078      | 1.30%       |
| 3  | 80th  | 0.055      | 0.057      | 3.64%       |
| 3  | 140th | 0.031      | 0.032      | 3.23%       |
| 3  | 200th | 0.019      | 0.016      | -15.79%     |

Table 4 Acceleration of each working condition node relative to working condition 1 acceleration reduction calculation value

| No | node  | Cab (m/s²) | Relative reduction | Csw (m/s²) | Relative reduction |
|----|-------|------------|--------------------|------------|--------------------|
| 1  | 10th  | 0.097      | 36.08%             | 0.096      | 34.38%             |
| 1  | 80th  | 0.062      | 36.08%             | 0.063      | 36.46%             |
| 1  | 140th | 0.027      | 13.40%             | 0.028      | 17.71%             |
| 1  | 200th | 0.014      | 14.43%             | 0.011      | 11.46%             |
| 2  | 10th  | 0.087      | 29.89%             | 0.086      | 26.74%             |
| 2  | 80th  | 0.061      | 35.63%             | 0.063      | 39.53%             |
| 2  | 140th | 0.03       | 14.94%             | 0.029      | 16.28%             |
| 2  | 200th | 0.017      | 19.54%             | 0.015      | 17.44%             |
| 3  | 10th  | 0.077      | 28.57%             | 0.078      | 26.92%             |
| 3  | 80th  | 0.055      | 31.17%             | 0.057      | 32.05%             |
Note: Cab is the wave and top x-direction movement acceleration response, and Csw is acceleration response under wave and top x-direction movement superimposed rigid body swing.

Figure 2-5 shows the graphs in case 1, the nodes 10th, 80th, 140th and 200th calculated as 0.097 m/s², 0.062 m/s², 0.027 m/s² and 0.014 m/s². It shows that as the depth of water increases, the structural cross-flow response acceleration is bated in the wave action and the X-direction motion, as shown in Figure 14-16. Without taking the current into account, as the depth of water increases, the structure is bated by the wave action and the top motion, and the response acceleration value is weakened.

Under the action of wave and top motion placed on rigid body oscillation, the calculated results of nodes 10th, 80th, 140th and 200th are 0.096 m/s², 0.063 m/s², 0.028 m/s² and 0.011 m/s². It shows that as the depth of water increases, the structural cross-flow acceleration response is bated under the action of the wave and top motion placed on rigid body swing, as shown in Figure 14-16. It is worth noting that after the rigid body swing is placed on, the structural velocity change trend is similar to that under the action of the wave and the top, and the structure is affected by the rigid body swing without exceeding the wave and top motion loads.

The rigid body oscillating effect and the wave and top effect are simply regarded as linear superposition, and the nodes 10th, 80th, 140th and 200th are calculated to increase by -1.03%, 1.61%, 3.70% and -21.43%. As the depth of water increases, the rigid body oscillating effect first increases and then decreases, and as the shaft diameter S increases, it's positively correlated with S. The rest of the conditions have a similar situation, as shown in figures 6-9, 10-13 and Table 3-4.

It is worth adding that as shown in Table 4, as the water depth increases, the attenuation or attenuation of each node interval decreases from 10th to 80th, and the 80th-140th attenuation decreases sharply or strongly with respect to the top node response. After 140th, the range is weaker.

3.3 Response analysis at the same mass point

Figure 14-16 Structural acceleration response with node variation in working condition 1-3 and Figure 1720 are 10th 200th structural acceleration response of node as a function of operating conditions.
Fig. 15 Case 2 Structural acceleration response as a function of node

Fig. 16 Case 3 Structural acceleration response as a function of node

Fig. 17 Structural acceleration response of node 10th as a function of operating conditions
Table 5 Acceleration of each working condition node relative to the working condition 1 acceleration normalized value

| No | node  | Cab normalized value | Csw normalized value |
|----|-------|----------------------|----------------------|
| 1  | 10th  | 0.097                | 1.00                 |
| 2  | 10th  | 0.087                | 0.90                 |
| 3  | 10th  | 0.077                | 0.79                 |
| 1  | 80th  | 0.062                | 1.00                 |
| 2  | 80th  | 0.061                | 0.98                 |
Comparing the nodes 10th, 80th, 140th and 200th working conditions 1-3 Figs. 17-20 and Table 5, as the amplitude of motion decreases, the response acceleration of the 10th and 80th nodes decreases, and 140th and 200th increase greatly. The 200th rigid body swing amplitude of the structure at the touchdown node is different from the other body acceleration amplitude. The reason may be that the bottom response is complicated. In the middle node, the rigid body swing amplitude is larger than the no rigid body swing amplitude, and there are also mutual increase or decrease.

### 4. Conclusion

Under the action of waves or current, the rigid body oscillation from the suspension point to touchdown point is a real problem of steel catenary riser in calculating lateral flow response of the SCR. This article does not consider the role of water flow. From the perspective of the linear load motion of the structure and the top X direction, the complex response problem of the structure after the coupled rigid body oscillation is studied.

Under the action of wave load, the transverse flow acceleration of the structure is gradually reduced with the water depth. After coupling the top X direction, as the water depth increases, this phenomenon of decreasing with the increase of water depth still exists, indicating that the linear motion of the top X direction does not affect the structural velocity beyond the influence of the wave load.

The structure is affected by the rigid body oscillation, which is usually an increase in the lateral flow acceleration of the structure. In this paper, after superimposing the linear motion in the X direction, the structural response appears as the bottom contact area, and the structural response does not increase the cross-flow acceleration response, and the other parts of the structure appear to increase or cancel with the water depth response.

In this paper, the decrease of structural response in depth direction is studied from the point of view of increasing response value relative to water depth. The calculation shows that between 10th and 140th, the structural response decreased by about 30%, between 140th-seabed, by about 15%, and the upper response decreased by about twice as much as the lower one.

In this paper, normalization coefficient is used to analyze the working conditions of each node. The calculation results from each node show that between 10th and 140th, the structural response decreases with the increase of the x-direction load frequency or the decrease of amplitude. Between the 140th-seabed, the structural response is increasing.

This relatively obvious change in 140th may be related to vector diameter S.

Thanks to the support of the national nature science foundation of China (51239008, 51739010, 51679223), Thank to Texas A&M university, professor Zhang Jun’s support. Thanks for all the help from all.

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