EPR-Bell Tests with Unsharp Observables
and Relativistic Quantum Measurement

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1 Introduction

In this contribution I will review the analysis of the Einstein-Podolsky-Rosen argument \cite{19}, Bell’s inequalities \cite{5} and of associated experiments for spins in terms of positive operator valued measures (in short: POVMs). Specifically, I will explore the relation between the Clauser-Horne-Shimony-Holt (CHSH) \cite{16} inequality and a fundamental classicality property of observables – their coexistence; this leads to the question whether for macroscopic systems a relatively ‘small’ amount of unsharpness may suffice to ensure (and explain) the practical impossibility of exhibiting nonclassical features such as those represented by Bell-type inequalities.

I will present a derivation of Bell’s inequalities for unsharp spins which follows a reconstruction by Mittelstaedt and Stachow \cite{32} of the original EPR argument. In this treatment, the Bell inequalities follow from a conjunction of two assumptions, (unsharp) reality and locality, applied in the context of the quantum mechanics of an entangled pair of spins. Since the reality assumption can be consistently incorporated into the quantum formalism, it is locality that is incompatible with the latter. However, a contradiction only arises when the degree of unsharpness of the spins is not too high; otherwise the nonlocality of quantum mechanics cannot be detected with such observables. The contradiction can be resolved if the locality assumption is weakened so as to allow for a benign form of nonlocality: one has to accept that (unsharp) objectification can occur over spacelike distances or between dynamically separated parts of a system. Note that this argument is not about the supplementation of quantum mechanics with hidden variables but exhibits only the inevitability and nature of quantum nonlocality. But it does raise the question of a consistent description of the process of measurement for extended, entangled systems and for localised measurements in spacelike separated regions of spacetime.

A note on terminology may be in place. The term ‘Bell inequality’ refers, strictly speaking, to the inequality originally exhibited by Bell for pair probabilities associated with a triple of spin observables. Bell’s argument made explicit use of the strict correlation between certain pairs of observables in the spin singlet state and thereby ignored the unavoidable experimental imprecisions. In order
to provide an derivation of Bell’s theorem without any recourse to quantum mechanical properties while taking into account experimental imperfections, Clauser, Horne, Shimony and Holt considered the case of a quadruple of spins, one pair each pertaining to one of the two particles involved. The ensuing inequality is known as ‘CHSH inequality’. I will follow the widespread practice of referring to the latter as (Bell-)CHSH or simply Bell inequality.

2 Bell-CHSH inequalities, joint probabilities and coexistence

The role of CHSH inequalities as classicality conditions has been systematically studied by Pitowski [36] and by Beltrametti and Maczynski in the early 1990s [6, 7, 8]. This was preceded by the observation due to Fine [20, 21] that a full set of Bell or CHSH inequalities is equivalent to the existence of triple or quadruple joint probability distributions. The concept of POVM as a joint observable for EPR-Bell observables was considered by Abu-Zeid and deMuynck in 1984 [1], with the conclusion that the violation of Bell inequalities reflects the nonexistence of such joint observables in the case of noncommuting sharp spin observables. The issue of formulating and exploring the meaning and role of Bell-type inequalities for unsharp spins has to my knowledge been addressed first by Busch in 1985 [11]; this was taken up and generalised by Kar and Roy in 1996 [28]. This part of my contribution will draw on the valuable review of Kar and Roy [29].

In this section I will exhibit the relationship between operator Bell inequalities and coexistence, showing that in the EPR context the latter implies the former but not conversely. This stands in contrast to the situation discussed by Fine and others, who showed that a set of Bell inequalities forms a necessary and sufficient condition for a family of pair probabilities to be embeddable into a quadruple joint probability. To explain the reason for this discrepancy, it will be helpful to briefly review Fine’s theorem.

2.1 Fine’s theorem

In an EPR-Bell experiment on a correlated pair of spin 1/2 systems, one measures pairs of random variables \( \{a_k, a_{\bar{k}}\}, \{b_\ell, b_{\bar{\ell}}\} \), where \( k \in \{1, 2\}, \bar{k} \in \{\bar{1}, \bar{2}\} \) and \( \ell \in \{3, 4\}, \bar{\ell} \in \{3, 4\} \) label two variables (spin observables) of system A and B, respectively. This gives rise to sets of frequencies which are to approach probabilities provided in a theoretical model of the experiment:

\[
p_1, \ p_{\bar{1}}, \ p_2, \ p_{\bar{2}}, \ p_3, \ p_{\bar{3}}, \ p_4, \ p_{\bar{4}},
\ p_{13}, \ p_{1\bar{3}}, \ p_{14}, \ \ldots, \ p_{23}, \ \ldots, \ p_{24}, \ \ldots, \ p_{2\bar{1}}.
\]  

(1)

Fine’s theorem establishes a set of Bell-CHSH inequalities as a necessary and sufficient condition for this set of probabilities to be embeddable into a single classical probability model, that is, for the existence of a quadruple joint probability measure such that the single and pair probabilities arise as marginals.
**Theorem 1** For a system of probabilities to be embeddable into a quadruple joint probability distribution \( \{p_{1234}, p_{1243}, \ldots, p_{1234}\} \) it is necessary and sufficient that the following set of Bell-CHSH inequalities holds:

\[
\begin{align*}
0 & \leq p_{13} + p_{14} - p_{24} + p_{23} \leq 1, \\
0 & \leq p_{14} + p_{13} - p_{23} + p_{24} \leq 1, \\
0 & \leq p_{23} + p_{24} - p_{14} + p_{13} \leq 1, \\
0 & \leq p_{24} + p_{23} - p_{13} + p_{14} \leq 1, \\
\end{align*}
\]

or equivalently:

\[
\begin{align*}
0 & \leq p_1 + p_4 - p_{13} - p_{14} - p_{24} + p_{23} \leq 1, \\
0 & \leq p_1 + p_3 - p_{13} - p_{14} - p_{23} + p_{24} \leq 1, \\
0 & \leq p_2 + p_4 - p_{23} - p_{14} - p_{24} + p_{13} \leq 1, \\
0 & \leq p_2 + p_3 - p_{13} - p_{23} - p_{24} + p_{14} \leq 1. \\
\end{align*}
\]

We sketch the first steps of the proof. We introduce short-hands for the sought-for 4-probabilities:

\[
\begin{align*}
p_{1234} &= a, & p_{1234} &= e, & p_{1234} &= k, & p_{1234} &= p, \\
p_{1234} &= b, & p_{1234} &= f, & p_{1234} &= \ell, & p_{1234} &= q, \\
p_{1234} &= c, & p_{1234} &= g, & p_{1234} &= m, & p_{1234} &= r, \\
p_{1234} &= d, & p_{1234} &= h, & p_{1234} &= n, & p_{1234} &= s.
\end{align*}
\]

Next we use these to reproduce the pair probabilities:

\[
\begin{align*}
p_{13} &= a + b + e + f, & p_{13} &= k + \ell + p + q, \\
p_{13} &= c + d + g + h, & p_{13} &= m + n + r + s. \\
p_{14} &= a + c + e + g, & p_{14} &= k + m + p + r, \\
p_{14} &= b + d + f + h, & p_{14} &= \ell + n + q + s. \\
p_{23} &= a + b + k + \ell, & p_{23} &= e + f + p + q, \\
p_{23} &= c + d + m + n, & p_{23} &= g + h + r + s. \\
p_{24} &= a + c + k + m, & p_{24} &= e + g + p + r, \\
p_{24} &= b + d + \ell + n, & p_{24} &= f + h + q + s.
\end{align*}
\]

We have to establish a minimum subset of \( \{a, b, \ldots, s\} \) such that all other numbers can be expressed in terms of these and the given marginality relations. Start with \( a, b, c, d \) assumed given. This yields:

\[
\begin{align*}
e + f &= p_{13} - a - b, & p + q &= p_{13} - k - \ell = p_{13} - p_{23} + a + b, \\
g + h &= p_{13} - c - d, & r + s &= p_{13} - m - n = p_{13} - p_{23} + c + d, \\
e + g &= p_{14} - a - c, & p + r &= p_{14} - k - m = p_{14} - p_{24} + a + c, \\
f + h &= p_{14} - b - d, & q + s &= p_{14} - \ell - n = p_{14} - p_{24} + b + d, \\
k + \ell &= p_{23} - a - b, & p + q &= p_{23} - e - f = p_{23} - p_{13} + a + b, \\
m + n &= p_{23} - c - d, & r + s &= p_{23} - g - h = p_{23} - p_{13} + c + d, \\
k + m &= p_{24} - a - c, & p + r &= p_{24} - e - g = p_{24} - p_{14} + a + c, \\
\ell + n &= p_{24} - b - d, & q + s &= p_{24} - f - h = p_{24} - p_{14} + b + d.
\end{align*}
\]

3
Next, consider \( e, k, p \) given:

\[
\begin{align*}
\ f & = p_{13} - a - b - e \\
\ g & = p_{14} - a - c - e \\
\ h & = p_{13} - c - d - (p_{14} - a - c - e) = p_{13} - p_{14} + a + e - d \\
\ \ell & = p_{23} - a - b - k \\
\ m & = p_{24} - a - c - k \\
\ n & = p_{23} - c - d - (p_{24} - a - c - k) = p_{23} - p_{24} + a + k - d \\
\ q & = p_{13} - p_{23} + a + b - p \\
\ r & = p_{14} - p_{24} + a + c - p \\
\ s & = p_{33} - p_{13} + c + d - (p_{14} - p_{24} + a + c - p) \\
& = p_{33} - p_{13} - p_{14} + p_{24} + d + p - a
\end{align*}
\]

As a check, we can see that

\[
a + b + \cdots + r + s = p_3 + p_5 = 1. \tag{8}
\]

The task is to ensure that all numbers \( a,b,\ldots,r,s \) are nonnegative. Hence:

\[
\begin{align*}
\ a \geq 0, \\
\ b \geq 0, \\
\ c \geq 0, \\
\ d \geq 0, \\
\ e \geq 0, \\
\ f \geq 0, \\
\ g \geq 0, \\
\ h \geq 0, \\
\ k \geq 0, \\
\ \ell \geq 0, \\
\ m \geq 0, \\
\ n \geq 0, \\
\ p \geq 0, \\
\ q \geq 0, \\
\ r \geq 0, \\
\ s \geq 0.
\end{align*}
\tag{9}
\]

Inserting the expressions for the pair probabilities into the Bell inequality (2) and using the positivity (9) readily confirms the validity of the Bell inequality, given the existence of the quadruple joint probabilities (4-jpd). This constitutes the necessity part of the proof. Next one wants to see that a sufficient set of Bell inequalities ensures the existence of a 4-jpd. Thus one has to ensure that numbers \( a,b,c,d,e,k,p \geq 0 \) can be found such that (6) holds and all remaining numbers \( f,g,h,\ell,m,n,q,r,s \), which are determined by the first seven numbers, are nonnegative.

The nine inequalities \( f,g,h,\ell,m,n,q,r,s \geq 0 \) can be organised as follows, using (7):

\[
\begin{align*}
\ p_{14} - p_{13} + d & \leq a + e \leq \min \{p_{13} - b, p_{14} - c\} \\
\ p_{24} - p_{23} + d & \leq a + k \leq \min \{p_{23} - b, p_{24} - c\} \\
\ p_{13} + p_{14} - p_{23} - p_{24} + d & \leq p - a \leq \min \{p_{13} - p_{23} + b, p_{14} - p_{24} + c\}
\end{align*}
\tag{10}
\]

This system, together with the inequalities \( a,b,c,d,e,k,p \geq 0 \), leads eventually to a set of inequalities for \( b,c \) and \( d \), hence these numbers must lie in the intersection of a number of intervals. The condition that these intervals are nonempty finally entails the CHSH inequalities. Then one can choose \( h, c, d \geq 0 \) to lie in their respective intervals, and this enables one to choose \( a,e,k,p \geq 0 \) satisfying (10), which ensure the nonnegativity of the remaining nine constants.
2.2 Coexistence and Bell-CHSH inequalities for spin $\frac{1}{2}$

In recent years there has been increasing interest in the use of POVMs for tests of Bell-type inequalities as an indication of nonlocal quantum correlations (e.g., \cite{4, 24, 37, 40, 42}). There are nonseparable mixed states for which the Bell-CHSH inequalities are violated not for the usual pairs of sharp spins but only for suitable families of unsharp observables. This situation is one illustration of the fact that optimisation of information gain in measurements can under certain conditions only be achieved with POVMs that are no PVMs. A comprehensive introduction to the topic of POVMs and their application in quantum foundations and experiments can be found in the monograph \cite{13}.

We will only be concerned with POVMs whose domains are finite Boolean algebras, which can be represented as power sets of finite value spaces $\Omega = \{1, 2, \ldots, N\}$, $\Sigma = 2^{\Omega}$. Thus the definition of the full POVM follows from the additivity if only the map $i \mapsto E_i := E(\{i\})$ is given. Hence in the sequel we will simply refer to the POVM $E : X (\in 2^{\Omega}) \mapsto E(X)$ in terms of set $\{E_1, E_2, \ldots, E_N\}$.

The set of POVMs is known to contain noncommuting subsets that can be measured jointly, that is, their ranges can be contained in the range of one common POVM. Such families of POVMs are called coexistent. It has been shown that pairs or triples of unsharp spin observables are coexistent if their degree of unsharpness is large enough \cite{12}. Let us consider spin $\frac{1}{2}$ POVMs generated by effects of the form

$$E(n, \lambda) := \frac{1}{2} (I + \lambda n \cdot \sigma),$$

where $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ denotes the vector of Pauli spin matrices, $n$ is a unit vector in $\mathbb{R}^3$ denoting a point on the unit sphere $S^2$, and $\lambda \in [0, 1]$. The eigenvalues are $\frac{1}{2}(1 \pm \lambda)$, and the spectral projections are $P_n := \frac{1}{2}(I \pm n \cdot \sigma)$. Thus,

$$E(n, \lambda) = \frac{1}{2} (1 + \lambda) P_n + \frac{1}{2} (1 - \lambda) P_{-n}.$$

From this representation it is evident that the POVM $\{E(n, \lambda), E(-n, \lambda)\}$ is a smeared version of the PVM $\{P_n, P_{-n}\}$. This is the formal sense in which the former represents an unsharp spin.

A pair of sharp spin observables is noncommutative if their respective vectors $n_1, n_2$ are not collinear. Such pairs have no joint observable. But two unsharp spin observables can be coexistent. Necessary and sufficient conditions for this to happen are as follows \cite{12}:

**Theorem 2** A pair of unsharp spin observables $a = \{E(n_1, \lambda), E(-n_1, \lambda)\}$, $a' = \{E(n_2, \lambda), E(-n_2, \lambda)\}$ is coexistent if and only if

$$\lambda \|n_1 + n_2\| + \lambda \|n_1 - n_2\| \leq 2. \quad (11)$$

The term in brackets has maximal value $2\sqrt{2}$, which is assumed for $n_1 \perp n_2$. Hence this coexistence condition is satisfied for all pairs of directions $n_1, n_2$ if and
ensured by the condition
\[ \lambda \leq \frac{1}{\sqrt{2}} =: \lambda_2. \]

A joint observable can be given explicitly:
\[ E_{k\ell} = \frac{1}{4} \left( 1 + \frac{1}{2} n_k \cdot n_{\ell} \right) I + \frac{1}{4} \lambda (n_k + n_{\ell}) \cdot \sigma, \quad k \in \{1, \overline{1}\}, \quad \ell \in \{2, \overline{2}\}. \]

Here we use \( n_1 = -n_1, \ n_2 = -n_2 \). It is easily verified that the marginality properties are satisfied:
\[
\begin{align*}
E_{12} + E_{12} &= E (n_1, \lambda), \quad E_{12} + E_{1\overline{2}} = E (-n_1, \lambda), \\
E_{12} + E_{1\overline{2}} &= E (n_2, \lambda), \quad E_{1\overline{2}} + E_{1\overline{2}} = E (-n_2, \lambda).
\end{align*}
\]

By studying eigenvalues, is easy to see that positivity of all four effects \( E_{k\ell} \) is ensured by the condition \( \lambda \leq \lambda_2 = 1/\sqrt{2} \). It is less straightforward to formulate necessary and sufficient conditions for triples or quadruples of unsharp spins to be coexistent.

In the EPR experiment for spins, we are dealing with effects of the form \( E_{13} = E_1 \otimes E_3, \) etc. The Bell-CHSH inequalities, written as operator inequalities for the POVMs \( \{E_{13}, E_{1\overline{3}}, E_{1\overline{3}}, E_{\overline{1}3}\}, \ \{E_{14}, E_{1\overline{4}}, E_{1\overline{4}}, E_{\overline{1}4}\}, \ \{E_{23}, E_{2\overline{3}}, E_{2\overline{3}}, E_{\overline{2}3}\}, \ \{E_{24}, E_{2\overline{4}}, E_{2\overline{4}}, E_{\overline{2}4}\} \) (with \( E_1 = E_{13} + E_{1\overline{3}} = E_{14} + E_{1\overline{4}}, \ ) etc.), e.g.,
\[
0 \leq E_{13} + E_{14} - E_{24} + E_{23} \leq 1,
\]
are necessary conditions for the existence of a quadruple joint observable
\[ \{E_{1234}, E_{123\overline{4}}, \ldots, E_{\overline{1}2\overline{3}\overline{4}}\} \]
(with \( E_{ijkl} \geq 0, \sum E_{ijkl} = I \) such that \( E_{13} = E_{1234} + E_{123\overline{4}} + E_{1\overline{2}34} + E_{\overline{1}2\overline{3}4}, \) etc.

One can follow the whole line of argument presented in the preceding subsection to deduce a collection of operator inequalities which are all necessary for the construction of such a joint observable. However, sufficiency is not warranted as the set of effects (positive operators bounded above by \( I \)) is not linearly ordered, so that operator inequalities \( A \leq B, \ C \leq D \) do not by themselves ensure that there exists an operator \( X \) such that \( A \leq X \leq B, \ C \leq X \leq D \). (In fact the pairs \( A, B \) and \( C, D \) could be supported on mutually orthogonal subspaces, so \( O \leq X \leq C, \ X \leq D \) implies \( X = 0 \), which is ruled out unless \( A = B = O \).) Hence the condition of coexistence is stronger than the set of operator Bell inequalities.

This can also be seen by the fact that the existence of such a quadruple joint observable implies that the marginals \( \{E_{12}, E_{1\overline{2}}, E_{1\overline{2}}, E_{\overline{1}2}\}, \ \{E_{34}, E_{\overline{3}4}, E_{\overline{3}4}, E_{\overline{3}4}\} \) exist and constitute POVMs on the subsystems 1 and 2, respectively. In fact they are joint observables for \( \{E_1, E_1\}, \ \{E_2, E_2\}, \) and \( \{E_3, E_3\}, \ \{E_4, E_4\}, \) respectively. Thus we conclude that the quadruple coexistence entails that \( \lambda \leq \lambda_2 \). Conversely, this condition on \( \lambda \) is also sufficient to ensure coexistence of all four observables. In fact we have the following.
**Theorem 3** Observables \( a = \{ E_1, E_1 \} \), \( a' = \{ E_2, E_2 \} \), \( b = \{ E_3, E_3 \} \), and \( b' = \{ E_4, E_4 \} \) are coexistent if, and only if, the pairs \( a, a' \) and \( b, b' \) are coexistent, that is, exactly when the following holds:

\[
\lambda \|n_1 + n_2\| + \lambda \|n_1 - n_2\| \leq 2, \quad \lambda \|n_3 + n_4\| + \lambda \|n_3 - n_4\| \leq 2.
\]

In that case, if \( \{ E_{ij} : i = 1, \bar{1}, j = 2, \bar{2} \} \) and \( \{ E_{k} : k = 3, \bar{3}, \ell = 4, \bar{4} \} \) are joint observables for \( a, a' \) and \( b, b' \), respectively, then the set \( \{ E_{ij} \otimes E_{k} \} \) constitutes a joint observable for \( a, a', b, b' \).

For the proof we only need to verify the sufficiency of the two inequalities: if they are given, then the previous theorem ensures that \( a, a' \) as well as \( b, b' \) are coexistent. Hence joint observables \( \{ E_{ij} \} \) for \( a, a' \), and \( \{ E_{k} \} \) for \( b, b' \) exist. But then it is easy to see that the set \( \{ E_{ij} \otimes E_{k} \} \) constitutes a POVM and that its range contains \( a, a', b, b' \). Hence all four observables are coexistent.

Given a joint quadruple observable for \( a, a', b, b' \), it follows from Fine’s theorem that the pair probabilities must satisfy the Bell-CHSH inequalities for all quantum states. We now proceed to show that the coexistence condition, which only concerns the pairs \( a, a' \) and \( b, b' \) is in fact stronger than Bell’s inequalities.

With a slight misuse of notation we write

\[
a = n_1 \cdot \sigma, \quad a' = n_2 \cdot \sigma, \quad b = n_3 \cdot \sigma, \quad b' = n_4 \cdot \sigma,
\]

and use the shorthand \( E(a) := E(n_1, \lambda) \), \( E(-a) := E(-n_1, \lambda) \), etc. We introduce a generalised Bell operator:

\[
\tilde{B}_\lambda := E(a) \otimes E(-b) + E(-a) \otimes E(b') - E(a') \otimes E(b') + E(a') \otimes E(b).
\]

The operator Bell-CHSH inequalities then assume the form

\[
O \leq \tilde{B}_\lambda \leq I,
\]

or equivalently

\[
-2I \otimes I \leq \lambda^2 B \leq 2I \otimes I,
\]

where \( B \) is the standard Bell operator \([3]\):

\[
B = a \otimes (b + b') + a' \otimes (b' - b).
\]

We recall that

\[
B^2 = 4I \otimes I + [a, a'] \otimes [b, b']
\]

\[
= 4 \{ I \otimes I - (n_1 \times n_2) \cdot \sigma \otimes (n_3 \times n_4) \cdot \sigma \},
\]

from which it follows that

\[
\|B\| = 2 [1 + |n_1 \times n_2| \cdot |n_3 \times n_4|]^{1/2} \leq 2\sqrt{2},
\]

where the upper (‘Cirel’son’ \([17]\)) bound occurs at \( n_1 \perp n_2, n_3 \perp n_4 \). Thus we obtain:
Theorem 4 All operator Bell-CHSH inequalities are fulfilled for arbitrary \( a = \{ E_1, \bar{E}_1 \}, a' = \{ E_2, \bar{E}_2 \}, b = \{ E_3, \bar{E}_3 \}, b' = \{ E_4, \bar{E}_4 \} \) if and only if
\[
\lambda \leq \lambda_{CHSH} = \frac{1}{\sqrt{2}}.
\]
This condition is obviously weaker than the coexistence condition \( \lambda \leq 1/\sqrt{2} \). Hence for \( 1/\sqrt{2} < \lambda \leq 1/\sqrt{2} \) there exist quadruples \( a, a', b, b' \) which are not coexistent but do satisfy the Bell-CHSH inequalities.

Finally we consider the Bell-CHSH inequalities for unsharp spin observables in the singlet state,
\[
\Psi = \frac{1}{\sqrt{2}} \left( \psi_n \otimes \psi_{-n} - \psi_{-n} \otimes \psi_n \right),
\]
where \( \psi_n \) denotes a normalised eigenvector of \( n \cdot \sigma \) associated with eigenvalue +1. We have
\[
p_{ij} = \langle \Psi | E(n_i, \lambda) \otimes E(n_j, \lambda) | \Psi \rangle = \frac{1}{4} \left( 1 - \lambda^2 n_i \cdot n_j \right) = (1 - 2\varepsilon) \frac{1}{2} \sin^2 \left( \frac{1}{2} \theta_{ij} \right) + \varepsilon,
\]
\[
\varepsilon = \frac{1}{2} (1 - \lambda^2).
\]
One of the Bell-CHSH inequalities then assumes the form
\[
f := |n_1 \cdot n_3 + n_1 \cdot n_4 - n_2 \cdot n_3 + n_2 \cdot n_4| \leq 2 (1 - 2\varepsilon)^{-1} =: F.
\]
The term denoted \( f \) assumes its maximum value
\[
f = f_{max} = 2\sqrt{2} \quad \text{at} \quad \theta_{13} = \theta_{14} = \theta_{24} = \frac{1}{4} \pi, \theta_{23} = \frac{3}{4} \pi.
\]
Then
\[
f_{max} \leq F \iff \varepsilon \geq \frac{1}{2} \left( 1 - \frac{1}{\sqrt{2}} \right) =: \varepsilon_{CHSH} \iff \\
\lambda \leq \frac{1}{\sqrt{2}} = \lambda_{CHSH}.
\]
Hence in order to ensure that Bell’s inequalities are satisfied in the singlet state for all possible choices of spin directions, it is necessary and sufficient to have \( \lambda \) less or equal to the CHSH value previously established.

To summarise, the representation of measurement inaccuracies in terms of unsharp spin observables shows that the quantum mechanical violation of Bell-CHSH inequalities is a robust phenomenon in that small inaccuracies, represented by means of small values of the unsharpness parameter \( \varepsilon \), do not blur the violation. It
is only with sufficient large unsharpness that Bell inequalities are always satisfied. Coexistence of the unsharp observables involved will ensure that the unsharpness is indeed large enough for this to happen. Considering that coexistence is a feature characteristic of observables in the macroscopic domain, this raises the question as to what relative degrees of unsharpness is required to guarantee coexistence and thus validity of Bell inequalities in the case of macroscopic observables. A study of coexistence conditions for systems with higher-dimensional Hilbert spaces appears to be rather nontrivial and challenging, but it is highly desirable as a contribution towards an operational understanding of the classical limit problem.

3 EPR Argument for Unsharp Measurements

I now present a version of the EPR-Bell argument that is due to Mittelstaedt and Stachow [32], who developed it in an abstract quantum language. My reformulation will be in terms of Hilbert space quantum mechanics, and I will consider a modification that allows one to take into account unsharp spin measurements [11]. This will confirm that the EPR-Bell argument is indeed robust against small inaccuracies.

It should be noted that the argument to be presented is not a no-hidden-variable argument; it is rather a demonstration of the (in-)compatibility of quantum mechanics with certain interpretational ideas, such as a criterion of reality and a property of locality. The criterion of (unsharp) reality is of the form

\[ (R) \equiv \{(R_1) \rightarrow (R_2)\} \]

where

(R₁) Property \(\text{[unsharp property]}\) \(P\) of system \(S\) can be predicted \(\text{[almost]}\) with certainty, without changing \(S\) \(\text{[much]}\).

(R₂) \(P\) corresponds to an element of \(\text{[unsharp]}\) reality.

The assumption of locality is of the form

\[ (L) \equiv \{(L_1) \rightarrow (L_2)\} \]

where

(L₁) Systems \(S_1\) and \(S_2\) are separated far enough from each other so that any interaction between them is negligible.

(L₂) A measurement on \(S_1\) does not change \(S_2\).

3.1 Quantum mechanics vs. reality and locality

The argument then goes as follows.
1. A system \( S_1 + S_2 \) consisting of two spin-1/2 particles is given in the singlet state \( \Psi \).

2. The spatial wave packets of \( S_1, S_2 \) are such that (L) is satisfied.

3. A \textit{an unsharp} measurement of \( n \cdot \sigma^{(1)} \) is made on \( S_1 \) with outcome +1, say. Then due to the strict anticorrelation between \( n \cdot \sigma^{(1)} \) and \( n \cdot \sigma^{(2)} \) encoded in the singlet state \( \Psi \), the value of \( P = n \cdot \sigma^{(2)} \) for \( S_2 \), to be obtained in a \textit{an unsharp} measurement, can be predicted \textit{almost} with certainty.

4. Assumption (L), together with (L) from 2., give (L2). Hence the measurement considered in 3. does not change \( S_2 \) in any way.

5. The conclusions of 3. (nondisturbance of \( S_2 \)) and (L2) from 4. (property (L2)) entail (R1) for \( S_2 \) and \( n \cdot \sigma^{(2)} \).

6. Assumption of (R) together with (R1) from 5. leads to the conclusion (R2) for \( n \cdot \sigma^{(2)} \) of \( S_2 \). That is, \( n \cdot \sigma^{(2)} \) is an element of \textit{an unsharp} reality for \( S_2 \).

7. Due to the nondisturbance [step 4.], the value of \( n \cdot \sigma^{(2)} \) must have been definite all along, irrespective of whether or not the measurement on \( S_1 \) is made or not.

8. Since \( n \cdot \sigma^{(1)} \) could be any spin observable of \( S_1 \), conclusion 7. must hold for all \( n \cdot \sigma^{(2)} \).

9. By symmetry, exchanging the roles of \( S_1 \) and \( S_2 \), all \( n \cdot \sigma^{(1)} \) of \( S_1 \) must have definite values, too.

10. If in any ensemble of such pairs \( S_1 + S_2 \), the subsystem observables \( n \cdot \sigma^{(1)} \) and \( n \cdot \sigma^{(2)} \) have definite values, there must then exist joint probabilities for \( a = E(n_1, \lambda), a' = E(n_2, \lambda), b = E(n_3, \lambda), b' = E(n_4, \lambda) \). Hence Bell’s inequalities must be satisfied. This contradicts the predictions of quantum mechanics where violations of Bell-CHSH inequalities must occur.

To summarise, we have a contradiction:

\[
\{(QM) \& (R) \& (L) \rightarrow (Bell) \rightarrow \neg (QM)\} \equiv \land.
\] (12)

Here ‘(QM)’ stands for ‘quantum mechanics is correct’, ‘(Bell)’ for ‘Bell-CHSH inequalities hold’, and ‘\( \neg (QM) \)’ for ‘quantum mechanics is false’. As the correlations observed in experiments agree with the predictions of quantum mechanics and do show violations of quantum mechanics, at least one of the assumptions (R) and (L) must be rejected. I shall argue that (R) can be naturally incorporated into quantum mechanics, so that (L) cannot be maintained.
3.2 Reality condition and Lüders measurements

EPR [19] regard their reality condition as a sufficient but not necessary criterion. They explicitly refer to the possibility that there may be many other ways of ascertaining the presence of elements of reality. They also mention that eigenstates of a given observable represent conditions under which the value of that observable can be predicted with certainty, without changing the system: the knowledge of the eigenstate suffices. Now, in the EPR experiment, some act of measurement must be carried out on one subsystem, and thus on the total system, in order to be able to use the known correlations to predict the value of an observable of the other system. If one maintains that this measurement act may affect the total system in some way, the question arises as to what exactly this effect could be.

From the quantum theory of measurement one knows that every observable admits a multitude of possible measurement schemes, along with many different ways in which the measured system is changed as a result of the measurement [14]. These state changes, which are conditional on the measurement outcome, are described by the concept of state transformer (or instrument). A state transformer is a state transformation valued measure on some measurable space. Here a state transformation is a linear, positive, trace-norm contractive map on the set of trace class operators. For discrete (sharp) observables there exists a distinguished class of measurements, the so-called ideal measurements, characterised by the property that their induced state transformer acts in a minimally disturbing way on the system. More precisely, a state transformer is associated with discrete observable \( A = \sum a_i P_i \) if

\[
\text{tr} [I_i(\rho)] = \text{tr} [\rho P_i] \quad \text{for all } \rho, i,
\]

where \( I_i \) is the state transformation associated with measurement outcome \( a_i \). Such a state transformer is called ideal if, whenever \( \text{tr} [\rho P_i] = 1 \) then \( I_i(\rho) = \rho \) for all states \( \rho \). It is known that ideal state transformers are exactly those of the form introduced by Lüders [30],

\[
I_{A,L,i}(\rho) = P_i \rho P_i =: \rho_{A,i}^L.
\]

The associated non-selective state transformation is given by the (trace preserving) Lüders map,

\[
I_A^L(\rho) = \sum P_i \rho P_i =: \rho_A^L.
\]

Ideal measurements are therefore also called Lüders measurements. Now one can use Lüders measurements to ‘look’ at a system to ascertain the value of the measured observable. If the system is already in an eigenstate, one will obtain the corresponding value as the outcome, without changing the state of the system. Hence a Lüders measurement enables one in this case to determine the value of observable \( A \) without changing the system. This corresponds exactly to our ‘classical’ notion of a definite property (or element of reality): if we are able to determine the value of a physical quantity just by ‘looking’ at the system, without changing
it, then we would conclude that this value must have been definite all along (or
at least immediately prior to the measurement).

If the system is not in an eigenstate, it will be in the state
\( P_i \rho P_i \) when the outcome was \( a_i \). Hence the probability for a repeated measurement of \( A \) to obtain the same outcome is equal to unity. Lüders measurements are in fact repeatable.

In the case of a discrete unsharp observable, \( E = \{E_1, E_2, \ldots, E_N\} \), the appropriate generalisation of a Lüders state transformer is given by the following:

\[
I_{E,L,i}(\rho) = E_1^{1/2} \rho E_1^{1/2} =: \rho_{L,i}^E
\]

and the sum of these terms constitutes the non-selective Lüders map,

\[
I_{E,L}(\rho) = \sum E_1^{1/2} \rho E_1^{1/2} =: \rho_L^E.
\]

These state transformations are almost non-disturbing (ideal) in the following sense.

**Theorem 5** For a positive operator \( E \) (with \( O \leq E \leq I \)), and for \( \varepsilon \in [0, \frac{1}{2}] \), if \( \text{tr} [\rho E] \geq 1 - \varepsilon \) then

\[
\left\| \rho - \frac{E_1^{1/2} \rho E_1^{1/2}}{\text{tr} [\rho E]} \right\|_1 \leq 2 (\varepsilon + \sqrt{\varepsilon}) \quad \text{and} \quad \text{tr} \left[ \frac{E_1^{1/2} \rho E_1^{1/2}}{\text{tr} [\rho E]} \right] \geq \text{tr} [\rho E] \geq 1 - \varepsilon.
\]

That is, whenever a (sharp or unsharp) property is approximately real in state \( \rho \) then this state does not change much, and the ‘degree’ of reality of the property is preserved. This justifies the concept of ‘unsharp’ element of reality introduced above. To conclude, there seems to be no difficulty with the reality condition; its premise can even be strengthened (the condition thereby weakened) by allowing the word ‘predicted’ to be replaced with ‘ascertained’. To ascertain a value without changing the system is exactly what the Lüders measurement allows one to do.

### 3.3 Resolution of the EPR-Bell contradiction

A study of the state changes for \( S_1 \) and \( S_2 \) in the EPR-Bell experiment will show a way to resolve the contradiction, provided that one accepts the state changes due to measurements as real, autonomous physical processes. Formally, a Lüders measurement changes the initial singlet state into a mixture,

\[
P[\Psi] \longrightarrow P[\Psi]^{n-\sigma \otimes I} = \left( E(n, \lambda)^{1/2} \otimes I \right) P[\Psi] E(n, \lambda)^{1/2} \otimes I + \left( E(-n, \lambda)^{1/2} \otimes I \right) P[\Psi] E(-n, \lambda)^{1/2} \otimes I.
\]

This final mixed state corresponds to the situation where it is known that the measurement has taken place but the result is not yet known. In other words, an ignorance interpretation with respect to the given components applies after the
measurement. Hence the transition from the pure state to the mixture is referred to as the \textit{unsharp} objectification of the measured observable. The reading of the outcome, once it will be possible, enables the observer to decide which of the component states is actually the final state of the system.

The state change of subsystem $S_2$ is obtained by taking partial traces:

$$\text{tr}_{S_1} [P [\Psi]] = \frac{1}{2} I^{(2)} \rightarrow \text{tr}_{S_1} [P [\Psi_{n, \sigma \otimes I}^{(2)}]] = \frac{1}{2} I^{(2)}.$$  

Hence it appears as if the measurement on system $S_1$ does not change the state of $S_2$. However, it must be noted that the reduced state before the measurement arises from a pure state, so that an ignorance interpretation with respect to any convex decomposition would contradict the nonobjectivity of all observables $I \otimes E (n', \lambda)$. By contrast, after the measurement the objectification of $I \otimes E (n, \lambda)$ is inherited by that of $E (n, \lambda) \otimes I$. To see which components of $\frac{1}{2} I^{(2)}$ the ignorance interpretation can be applied to after the measurement has taken place, we have to calculate the partial traces with respect to $S_1$ of the component states of the final mixture of $S_1 + S_2$:

$$\rho_{L,+}^{(2)} := \text{tr}_{S_1} [E (n, \lambda)^{1/2} \otimes I P [\Psi] E (n, \lambda)^{1/2} \otimes I] = \frac{1}{2} E (-n, \lambda),$$

$$\rho_{L,-}^{(2)} := \text{tr}_{S_1} [E (-n, \lambda)^{1/2} \otimes I P [\Psi] E (-n, \lambda)^{1/2} \otimes I] = \frac{1}{2} E (n, \lambda).$$

The probability for the event represented by $E (n, \lambda)$ to occur in the final state $\rho_{L,+}^{(2)}$ is

$$\text{tr} [E (n, \lambda)^2] = \frac{1}{4} (1 + \lambda)^2 + \frac{1}{4} (1 - \lambda)^2 = \frac{1}{2} (1 + \lambda^2).$$

Hence as a result of the measurement, the probability to obtain this outcome has increased from the value $\frac{1}{2}$ before the measurement.

The EPR-Bell contradiction \cite{EPR} can now be resolved by accepting that the state of $S_2$ does not remain unchanged but is modified as a result of the objectification of $E (n, \lambda) \otimes I$, which induces the objectification of $E (n, \lambda)$ for $S_2$. In fact if such objectification \textit{at a distance} (a term coined by Mittelstaedt \cite{Mittelstaedt}) is allowed to take place, this amounts to a weakening of the locality assumption,

$$(L) \rightarrow (L)_w \equiv \{ (L_1) \rightarrow (L_2)_w \},$$

where the conclusion is weakened to read:

$$(L_2)_w \text{ A measurement on } S_1 \text{ does not change } S_2, \text{ except (possibly) for the } \text\{unsharp\} \text{ objectification of some property of } S_2.$$  

In this way the EPR-Bell argument breaks down as one can no longer conclude that the value of $n \cdot \sigma (2)$ must have been definite even before or without any measurement on $S_1$. Furthermore, weak locality still ensures that measurements
on $S_1$ do not lead to superluminal signals from $S_1$ to $S_2$, simply because the mixed state operator of $S_2$ before the measurement, which does not allow an ignorance interpretation, is the same as the state operator after the measurement (which does allow an ignorance interpretation). This means that no conflict with relativity can arise in case these two systems are observed in spacelike separated regions.

4 EPR Experiment and Relativistic Quantum Measurement

At this point one might be tempted to lean back in relief – were it not for the issue of the uneasy coexistence between quantum mechanics and relativity touched upon with the last remark. In the context of the present approach the problem of the compatibility between quantum mechanics and relativity emerges in the form of (at least) two questions.

Q1. Can a consistent covariant description be given of collections of local measurements performed in different space time regions?

Q2. Can the concept of relativistically local measurements be formulated in a way that is compatible with quantum mechanics?

I believe that an affirmative answer to the first question can be justified while the second question is largely open. This view will be explained in the next two subsections.

4.1 Schlieder’s theory of covariant collapse

An answer to Q1. was formulated with great care by Schlieder [38] in the framework of relativistic quantum theory. As this work, which was written in German, has hardly received the attention it deserves, I will describe Schlieder’s approach in some detail.

Schlieder starts with the observation that (local) measurements induce state changes so that a system cannot be described by one single state (even though described in the Heisenberg picture), which would pertain globally to all of Minkowski space time $M$. Instead, a system’s history is (probabilistically) determined by the set of local measurements performed on it and is thus to be described by an associated set of (Heisenberg) states $\rho_j$ which pertain to different parts $M_j$ of a partition of $M$. Hence the objective history of a system is described as follows:

$$\{\rho_j (M_j)\} \text{ with } \bigcup_j M_j, \ M_j \cap M_k = \emptyset \text{ for } j \neq k.$$  

A cover $\{M_j\}$ of $M$ is called an $M$-cover, and the set $\{\rho_j (M_j)\}$, which describes the history of the system, is called $M$-chart. Measurements are idealised as taking place in space time points. Now Schlieder refers to the EPR experiments for pairs of spin-1/2 particles and for entangled $K$ meson pairs, discussed in the paper
of Bohm and Aharonov [10], taking them as evidence for the fact that quantum measurements entail state changes at spacelike separations from the measurement region. He emphasises that these state changes are objective and not just a representation of improved knowledge about the system.

In order to compare descriptions of the same system given by different observers, and to assess their consistency, one must assume that a unitary representation of the (inhomogeneous) Lorentz group is implemented into the Hilbert space theory of the system under consideration. Schlieder then goes on to show that consistency cannot be achieved if every observer were to assume that the state change due to a measurement at \( x^* \) occurs in his or her hyperplane simultaneous to \( x^* \). Hence he proposes that the ‘influence region’ for a single measurement localised at point \( x^* \) should be taken to be the complement \( B(x^*) \) of the (closed) backward light cone of \( x^* \). This gives rise to an invariant \( \mathcal{M} \)-cover \( \mathcal{M}_1 = B(x^*), \mathcal{M}_2 = \mathcal{M} \backslash B(x^*) \).

An observer will now ascribe state descriptions which will depend on his location relative to the ‘information domain’, the region in which the outcome of the measurement at \( x^* \) can be known to him. Accepting the requirement of Einstein causality at the level of classical communication, one finds that the information domain of the measurement at \( x^* \) is the (closed) forward light cone \( F(x^*) \) of \( x^* \). This concept gives rise to another covering of \( \mathcal{M} \), called \( \mathcal{N} \)-cover, here with \( \mathcal{N}_1 = \mathcal{M} \backslash F(x^*), \mathcal{N}_2 = F(x^*) \).

The crucial point now is to specify what state changes are to be used by any given observer, and to see whether consistency can be achieved. Schlieder argues that the Lüders state transformer provides an appropriate means of describing the state changes due to measurements. I doubt that this choice is necessary, but it can be adopted as a convenient and simple model. It amounts to the restriction of the totality of measurements of a given local observable \( A \) to a particular subclass.

Now we are ready to give Schlieder’s prescription. Let \( O(u) \) denote an observer at space time point \( u \), where \( u \) runs through a timelike worldline. In the present case of one single measurement at \( x^* \), one obtains two \( \mathcal{M} \)-charts representing \( O(u) \)’s state assignments, according to whether \( u \in \mathcal{N}_1 \) or \( u \in \mathcal{N}_2 \):

\[
\begin{align*}
\mathcal{N}_1 & : \{ \rho(M_1), \rho^{A_j}_L(M_2) \} \\
\mathcal{N}_2 & : \{ \rho(M_1), \rho^{A_j}_L(M_2) \}
\end{align*}
\]

Here it is assumed that the outcome of the measurement is \( a_j \). It is important to observe that the state change from \( M_1 \) to \( M_2 \) is that from a (possibly) pure quantum state to a Lüders mixture equipped with an ignorance interpretation, in accordance with the objectification-at-a-distance.

Now let us consider the EPR situation with two measurements of \( a \) and \( b \) at spacelike separated points \( x^* \) and \( y^* \), respectively. The initial state is \( \rho \), which in our case of interest is the singlet state. We assume that all observers are informed of the measuring programme. Let us denote the complement of a subset \( \mathcal{M}_k \) of \( \mathcal{M} \) as \( \mathcal{M}_k^c \). We then have the following \( \mathcal{M} \)-cover characterising the various influence
Note that due to the commutativity of \( a \) descriptions are frame independent. If the values of the measurements of \( a \) and \( b \), say, then the value assignment to \( a \) and \( b \) in the influence regions are as follows:

\[
\begin{align*}
\mathcal{M}_1 &= \mathcal{B}(x^*) \cap \mathcal{B}(y^*) , \quad \mathcal{M}_2 = \mathcal{B}(x^*) \cap \mathcal{B}(y^*)^c , \\
\mathcal{M}_3 &= \mathcal{B}(x^*)^c \cap \mathcal{B}(y^*) , \quad \mathcal{M}_4 = \mathcal{B}(x^*)^c \cap \mathcal{B}(y^*)^c .
\end{align*}
\]

Similarly one has an \( \mathcal{N} \)-cover representing the domains of equal information:

\[
\begin{align*}
\mathcal{N}_1 &= \mathcal{F}(x^*)^c \cap \mathcal{F}(y^*)^c , \quad \mathcal{N}_2 = \mathcal{F}(x^*) \cap \mathcal{F}(y^*)^c , \\
\mathcal{N}_3 &= \mathcal{F}(x^*)^c \cap \mathcal{F}(y^*) , \quad \mathcal{N}_4 = \mathcal{F}(x^*) \cap \mathcal{F}(y^*) .
\end{align*}
\]

Again let \( O(u) \) denote an observer at point \( u \) of his timelike worldline. The \( \mathcal{M} \)-chart given by \( O(u) \) reads as follows:

\[
\begin{align*}
u \in \mathcal{N}_1 & : \left\{ \rho(\mathcal{M}_1), \rho^b_{L}(\mathcal{M}_2), \rho^a_{L}(\mathcal{M}_3), [\rho^a_{L}]^b_{L}(\mathcal{M}_4) \right\} , \\
u \in \mathcal{N}_2 & : \left\{ \rho(\mathcal{M}_1), \rho^b_{L}(\mathcal{M}_2), \rho^a_{L,j}(\mathcal{M}_3), [\rho^a_{L,j}]^b_{L}(\mathcal{M}_4) \right\} , \\
u \in \mathcal{N}_3 & : \left\{ \rho(\mathcal{M}_1), \rho^b_{L,k}(\mathcal{M}_2), \rho^a_{L}(\mathcal{M}_3), [\rho^b_{L,k}]^a_{L}(\mathcal{M}_4) \right\} , \\
u \in \mathcal{N}_4 & : \left\{ \rho(\mathcal{M}_1), \rho^b_{L,k}(\mathcal{M}_2), \rho^a_{L,j}(\mathcal{M}_3), [\rho^a_{L,j}]^b_{L}(\mathcal{M}_4) \right\} .
\end{align*}
\]

Note that due to the commutativity of \( a \otimes I \) and \( I \otimes b \), the state operators \( [\rho^a_{L}]^b_{L} \), \( [\rho^b_{L}]^a_{L} \), \( [\rho^a_{L,j}]^b_{L} \), \( [\rho^b_{L,j}]^a_{L} \), \( [\rho^a_{L,k}]^b_{L} \), and \( [\rho^b_{L,k}]^a_{L} \) are all identical. Similarly nonselective Lüders operations for \( a \) commute with selective operations for \( b \), and vice versa. Hence the net result of such sequential state changes is (time) order independent, so that the state descriptions are frame independent.

The description given by observers with \( u \in \mathcal{N}_4 \), who have complete information about the outcomes of the measuring programme, represents the objective history of the system. If the values of the measurements of \( a \) and \( b \) are \( a_+^{(1)} \) and \( b_+^{(2)} \), say, then the value assignment to \( a \) and \( b \) in the influence regions are as follows:

\[
a, b \text{ indefinite (} \mathcal{M}_1 \text{) ; } b_+^{(1)} \land b_+^{(2)} (\mathcal{M}_2) ; \quad a_+^{(1)} \land a_+^{(1)} (\mathcal{M}_3) ; \quad a_+^{(1)} \land b_+^{(2)} (\mathcal{M}_4) .
\]

Schlieder’s proposal was discarded by Hellwig and Kraus [27] as unnecessarily complicated, and replaced with a simpler prescription that they regarded as physically equivalent: the simplification consists in avoiding the use of mixtures with ignorance interpretation and describing instead the collapse as the transition to final state conditional on the outcome. This proposal has been challenged by Aharonov and Albert [2, 3]. They concluded that a covariant description of collapses is not possible as this would preclude the measurability of nonlocal observables, which they demonstrated by means of an example. Their proposal was to define hyperplane-dependent state descriptions. A similar approach was taken by Dieks [3] in the context of a modal interpretation which treats measurement as a dynamic process and introduces ‘collapse’ only as an effective description, not as a physical process.
In contrast to the Hellwig and Kraus approach of ignoring the intermediate stage of objectification, Mittelstaedt and Stachow [31, 32] took seriously Schlieder’s distinction between the intermediate stage of quantum mechanical objectification (at a distance) and the actual collapse into an eigenstate, and used it to provide a consistent relativistic account of the EPR experiment, including a proof of relativistic causality (no-superluminal-signalling). The crucial point lies in making a difference between quantum mechanical objectivity, or value definiteness, which can spread to spacelike distances, and relativistic nonobjectivity, which pertains until the observers enter the forward lightcone (causal future) of the measurement event. I believe that question 1 has been answered affirmatively in this way.

However, the proposed resolution of the EPR-Bell contradiction, which makes explicit use of state collapse, cannot be regarded as entirely satisfactory until a comprehensive theory of measurement dynamics will have been found. Work on this problem has led to attempts to reformulate dynamical reduction models so as to take into account the requirements of relativistic covariance. A review of these developments was recently published by Ghirardi [22] who puts particular emphasis on Aharonov and Albert’s contribution. Interestingly, their criticism of the Hellwig-Kraus reduction rule has not gone unchallenged either: in [34] an interpretation is offered of the Aharonov-Albert nonlocal measurement in terms of the Hellwig-Kraus theory.

4.2 Local measurements and the localisation problem

The second question concerns the task of understanding and explaining, in terms of quantum mechanics, the localisation of ‘local’ measurement events. Any attempt to formulate localisation observables in a relativistic quantum theory seems to lead to peculiar difficulties. First, any concept of spatial localisation (with respect to a hyperplane) in the spirit of the Euclidean-covariant Newton-Wigner position operator or the more rigorous Mackey-Wightman projection valued measures inevitably appear to lead to causality problems, as elaborated by Schlieder [39] (violation of weak, or macro-causality) and Hegerfeldt (violation of strong, or micro-causality). A recent review can be found in [26]. This problem is not alleviated by allowing the localisation observables to be POVMs [15, 25].

The instantaneous spreading of wave packets, which corresponds to the immediate infinite delocalisation of a quantum particle in the passage from one hyperplane, where it was localised in a bounded region, to an infinitesimally later (or earlier) hyperplane, has the effect of blurring the distinction between the two spin-1/2 particles in the singlet state. This makes it difficult to see how one could ascribe definite properties to either of the particles, as is crucial for the EPR-Bell argument [35]. A possible way to deal with this objection may be offered by the observation that the probability for measuring the ‘wrong’ particles (i.e. those whose wave packets are concentrated in the respective local measurement regions a and b but which happen to be observed coincidentally at b and a, respectively) is extremely small if a and b are at spacelike, macroscopically large distances. Hence the error probability would be very small and the ‘unsharp’ version of the
EPR argument presented above would seem to apply without difficulties. The problem seems to be much more serious in the case of pairs of identical particles, such as photons or protons, whose combined state vector is subject to the (anti-)symmetrisation rule for indistinguishable particles.

An alternative, more formal approach to the problem of defining a physically reasonable covariant localisation observable consists in constructing realisations of the relativistic canonical commutation relations (RCCR); in this case it turns out that the natural candidates for position and spin observables do not commute with each other. Hence if a strictly localised measurement were to be made of the spins of the two particles, these measurements could not be sharp measurements. More generally, any local spin measurement would amount to performing a joint position and spin measurement, which could only be represented by means of a POVM which is not a PVM.

One may argue that spatial localisation is not at issue; what really matters is the fact that the spin measurements are localised in spacetime regions. The problem of defining a quantum mechanical concept of spacetime localisation has been addressed only very recently [23, 41]. The result is again that the localisation is necessarily unsharp in that the associated covariant POVM is no PVM.

Whatever localisation concept will ultimately prove viable, it will be necessary to scrutinise the EPR-Bell argument in the light of a quantum theoretical representation of the localisation of local measurements. We leave this as an open problem.

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