Modelling effects of public health educational campaigns on drinking dynamics

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ABSTRACT
This paper deals with the global property of a drinking model with public health educational campaigns. With the help of Lyapunov function, global stability of equilibria of the model is derived. The alcohol-free equilibrium is globally asymptotically stable and the alcohol problems are eliminated from population if \( R_0 \leq 1 \). A unique alcohol present equilibrium is globally asymptotically stable if \( R_0 > 1 \). Furthermore, the basic reproductive \( R_0 \) for the model is compared with the basic reproductive number \( R_1 \) for the absence of public health educational campaigns. We conclude that the public health educational campaigns of drinking individuals can slow down the drinking dynamics. Some numerical simulations are also given to explain our conclusions.

1. Introduction

The drinking behaviours of college students had posed significant public health concerns for several generations. Alcohol abuse by college students led to a range of negative consequences. For example, college students who abused alcohol were more likely to have a lower grade point average, drive under the influence of alcohol, engage in vandalism, have unplanned and/or unprotected sex and cause injury to themselves and/or others [8,10,16]. Although there had been many attempts to reduce the problem, alcohol abuse by college students had persisted and in some cases increased over the past several decades [17].

Mathematical models could mimic the process of drinking and provided useful tools to analyse the spread and control of drinking behaviour. Several different mathematical models for drinking had been formulated and studied [2,6,7,9,12,13,18]. Giuseppe and Brian [6] developed a two-stage (four component) model for youths with serious drinking problems and their treatment. The youths with alcohol problems were split into two classes, namely those who admitted to having a problem and those who did not. The stability of all the equilibria was analysed. Mubayi et al. [13] introduced a simple framework where drinking was modelled as a socially contagious process in low and high-risk connected environments.
Individuals were classified as light, moderate (assumed mobile), and heavy drinkers. Moderate drinkers provided the link between both environments. The focus there was on the effect of moderate drinkers, measured by the proportion of their time spent in ‘low’ versus ‘high-’ risk drinking environments, on the distribution of drinkers. Lee et al. [12] introduced a mathematical model of drinking that incorporated the impact of relapse, and was analysed primarily under the impact of two time-dependent controls put in place over a finite time horizon. Xiang et al. [18] presented a quit drinking model taking into account of permanent quit drinker compartment and relapse, and global stability of equilibria was obtained. Huo and Song [9] introduced a more realistic two-stage model for binge drinking problem, where the youths with alcohol problems were divided into those who admitted the problem and those who did not admit. Mathematical analyses established that the global dynamics of the model were determined by the basic reproduction number. For the other mathematical models for drinking, we referred to [2,7] and the references therein.

It was well known that the prevalence of any epidemic was strongly dependent on the social behaviour of individuals in a population. It was noted that behavioural change played a very important role in the spread of disease [4]. Del Valle et al. [3] studied the effects of education, temporary vaccination and treatment on HIV transmission in a homosexually active population. They suggested that public health educational campaigns could be an efficient option for reducing the spread of disease. Such type of models had been widely discussed in recent decades [1,14,15].

Motivated by above works, in this paper, we consider a drinking model with public health educational campaigns. The organization of this paper is as follows. In the next section, a drinking model with public health educational campaigns is formulated. In Section 3, the basic reproduction number and the existence of equilibria are investigated. The global stability of the alcohol free and alcohol present equilibria is proved in Section 4, and some numerical simulations are given in Section 5. In the last section, we give some brief discussions.

2. The model formulation

In this section, we introduce a drinking model with public health educational campaigns. To describe how alcoholism spreads, and how reformed alcoholics relapse, and the effect of the public health educational campaigns, the drinking population is subdivided into five groups: susceptible drinkers $S(t)$, who consume alcohol in moderation and do not accept the public health education, but may develop problems with alcohol; educated drinkers $E(t)$, who consume alcohol in moderation and accept the public health education; alcoholics $A(t)$, who have drinking problems or addictions; and temporarily recovered drinkers $R(t)$, former alcoholics who have entered treatment and are abstaining from alcohol; quit drinkers $Q(t)$, who permanently quit drink. The total number of population at time $t$ is given by

$$N(t) = S(t) + E(t) + A(t) + R(t) + Q(t).$$

The population flow among those compartments is shown in the following diagram (Figure 1).
Figure 1. Transfer diagram of the model (1).

The transfer diagram leads to the following system of ordinary differential equations:

\[
\begin{align*}
\dot{S} &= q\mu\Lambda - \beta SA - (p + \mu)S, \\
\dot{E} &= (1 - q)\mu\Lambda + pS - \beta\sigma EA - (\mu + \varepsilon)E, \\
\dot{A} &= \beta SA + \beta\sigma EA + \delta R - (\mu + a_1 + \gamma)A, \\
\dot{R} &= \gamma A - (\delta + \xi + \mu + a_2)R, \\
\dot{Q} &= \xi R + \varepsilon E - \mu Q.
\end{align*}
\] (1)

As a part of the general population, the drinking population cannot be isolated at all. Peer pressure could also be a key factor to recruit new drinkers from the general population. For simplicity, we assume that at any moment time, new recruits enter the population at a rate \(\mu\Lambda\), which is a constant. A proportion \(q(0 < q < 1)\) of these individuals is assumed to be uneducated and the complementary proportion \(1 - q\), move to the educated susceptible class \(E\). The susceptibles \(S\) are also educated at a constant rate \(p\) into the \(E\) class of educated individuals. Susceptible individuals turn to drink at a time-independent rate \(\beta\). A susceptible individual move into the \(A\) class of drink individuals through peer pressure. However, educated individuals turn to drink at a rate \(\sigma\beta\), where \(\sigma\) is the overall effectiveness of the public health educational campaigns, that is the factor by which the average turn rate of educated individuals is reduced relatively to the turn rate of uneducated individuals. In this context, \(0 < \sigma < 1\), the range does not include 0 and 1 because 0 implies that education is completely effective and 1 implies that education is useless. \(\gamma\) is the rate coefficient of transfer from alcoholics compartment to temporarily recovered drinkers. \(\delta\) is the rate coefficient at which temporarily recovered drinkers enter into alcoholics compartment. \(\xi\) is the rate coefficient of transfer from temporarily recovered drinkers to quit drinkers. \(\varepsilon\) is the rate coefficient of transfer from educated individuals to quit drinkers. \(\mu > 0\) is the natural death rate. \(a_1, a_2\) are death rates due to excessive drinking. Note that when the model (1) without public health educational campaigns \((q = 1, p = \varepsilon = \sigma = E = 0)\), it reduces to the model that has been studied in [18]. Next, we only consider the model with public health educational campaigns.

It is important to show positivity and boundedness for the system (1) as they represent populations. Similarity to the proof of [9,18], we have the following lemma.

Lemma 2.1: If \(S(0) > 0\), \(E(0) > 0\), \(A(0) > 0\), \(R(0) > 0\), \(Q(0) > 0\), the solutions \(S(t), E(t), A(t), R(t), Q(t)\) of system (1) are positive for all \(t > 0\).
**Proof**: If the conclusion does not hold, then at least one of $S(t), E(t), A(t), R(t), Q(t)$ is not positive. Thus, we have one of the following five cases.

(1) there exists a first time $t_1$ such that

$$S(t_1) = 0, S'(t_1) < 0, E(t) \geq 0, A(t) \geq 0, R(t) \geq 0, Q(t) \geq 0, \quad 0 \leq t \leq t_1,$$

(2) there exists a first time $t_2$ such that

$$E(t_2) = 0, E'(t_2) < 0, S(t) \geq 0, A(t) \geq 0, R(t) \geq 0, Q(t) \geq 0, \quad 0 \leq t \leq t_2,$$

(3) there exists a first time $t_3$ such that

$$A(t_3) = 0, A'(t_3) < 0, S(t) \geq 0, E(t) \geq 0, R(t) \geq 0, Q(t) \geq 0, \quad 0 \leq t \leq t_3,$$

(4) there exists a first time $t_4$ such that

$$R(t_4) = 0, R'(t_4) < 0, S(t) \geq 0, E(t) \geq 0, A(t) \geq 0, Q(t) \geq 0, \quad 0 \leq t \leq t_4,$$

(5) there exists a first time $t_5$ such that

$$Q(t_5) = 0, Q'(t_5) < 0, S(t) \geq 0, E(t) \geq 0, A(t) \geq 0, R(t) \geq 0, \quad 0 \leq t \leq t_5.$$

In case (1), we have

$$S'(t_1) = q\mu \Lambda > 0,$$

which is a contradiction with $S'(t_1) < 0$.

In case (2), we have

$$E'(t_2) = (1 - q)\mu \Lambda + pS(t_2) > 0,$$

which is a contradiction with $E'(t_2) < 0$.

In case (3), we have

$$A'(t_3) = \delta R(t_3) \geq 0,$$

which is a contradiction with $S'(t_3) < 0$.

In case (4), we have

$$R'(t_4) = \gamma A(t_4) \geq 0,$$

which is a contradiction with $R'(t_4) < 0$.

In case (5), we have

$$Q'(t_5) = \xi R(t_5) + \varepsilon E(t_5) \geq 0,$$

which is a contradiction with $Q'(t_5) < 0$. Thus, the solutions $S(t), E(t), A(t), R(t), Q(t)$ of system (1) remain positive for all $t > 0$. 

Summing equations in Equation (1) yields

\[(S + E + A + R + Q)' = \mu[\Lambda - (S + E + A + R + Q)] - a_1A - a_2R\]
\[\leq \mu[\Lambda - (S + E + A + R + Q)],\]

then it follows that \(S(t) + E(t) + A(t) + R(t) + Q(t) \leq \Lambda\), so the set
\[\Omega = \{(S, E, A, R, Q) \in \mathbb{R}_+^5 : S + E + A + R + Q \leq \Lambda\}\]
is positively invariant for Equation (1). Therefore, we will consider the global stability of Equation (1) on the set \(\Omega\). ■

3. The basic reproduction number and existence of equilibria

The model has an alcohol-free equilibrium \(P_0(S_0, E_0, 0, 0, Q_0)\), where
\[S_0 = \frac{q\mu\Lambda}{(p + \mu)}, \quad E_0 = \frac{\mu\Lambda[p + (1 - q)\mu]}{(\mu + \epsilon)(p + \mu)}, \quad Q_0 = \frac{\epsilon\Lambda[p + (1 - q)\mu]}{(\mu + \epsilon)(p + \mu)}.\]  (2)

In the following, the basic reproduction number of system (2) will be obtained by the next generation matrix method formulated in [5].

Let \(x = (A, R, Q, S, E)^T\), then system (1) can be written as
\[\frac{dx}{dt} = F(x) - V(x),\]  (3)
where
\[F(x) = \begin{pmatrix} \beta SA + \beta \sigma EA \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad V(x) = \begin{pmatrix} (\mu + a_1 + \gamma)A - \delta R \\ (\delta + \xi + \mu + a_2)R - \gamma A \\ \mu Q - \xi R - \epsilon E \\ \beta SA + (p + \mu)S - q\mu\Lambda \\ \beta \sigma EA + (\mu + \epsilon)E - pS - (1 - q)\mu\Lambda \end{pmatrix}.\]  (4)

The Jacobian matrices of \(F(x)\) and \(V(x)\) at the alcohol-free equilibrium \(P_0\) are, respectively,
\[D_F(E_0) = \begin{pmatrix} F_{2 \times 2} & 0 \\ 0 & 0 \end{pmatrix}, \quad D_V(E_0) = \begin{pmatrix} V_{2 \times 2} & 0 & 0 & 0 \\ 0 & -\xi & \mu & 0 & -\epsilon \\ \beta S_0 & 0 & 0 & p + \mu & 0 \\ \beta \sigma E_0 & 0 & 0 & -p & \mu + \epsilon \end{pmatrix}.\]  (5)

where
\[F = \begin{pmatrix} \beta S_0 + \beta \sigma E_0 & 0 \\ 0 & 0 \end{pmatrix}, \quad V = \begin{pmatrix} \gamma + \mu + a_1 & -\delta \\ -\gamma & \xi + \delta + \mu + a_2 \end{pmatrix}.\]  (6)

The basic reproduction number, denoted by \(R_0\) is thus given by
\[R_0 = \rho(FV^{-1}) = \frac{\beta(\xi + \delta + \mu + a_2)(S_0 + \sigma E_0)}{(\gamma + \mu + a_1)(\xi + \delta + \mu + a_2) - \delta \gamma}.\]  (7)

The quantity \(R_0\) is the alcoholics generation number. It measures the average number of new alcoholics generated by a alcoholics in a population of susceptible drinkers and
educated drinkers when $\delta = 0$ and $\gamma = 0$. Using Theorem 2 from van den Driessche and Watmough [5], the following result is established.

**Theorem 3.1:** The alcohol-free equilibrium $P_0$ is locally asymptotically stable for $R_0 < 1$ and unstable for $R_0 > 1$.

The alcohol present equilibrium $P^* (S^*, E^*, A^*, R^*, Q^*)$ of system (1) is determined by equations

\begin{align*}
q \mu \Lambda - \beta SA - (p + \mu)S &= 0, \\
(1 - q) \mu \Lambda + pS - \beta \sigma EA - (\mu + \epsilon)E &= 0, \\
\beta SA + \beta \sigma EA + \delta R - (\mu + a_1 + \gamma)A &= 0, \\
\gamma A - (\delta + \xi + \mu + a_2)R &= 0, \\
\xi R + \epsilon E - \mu Q &= 0.
\end{align*}

The first two equations in (8) lead to

\begin{align}
S &= \frac{q \mu \Lambda}{(p + \mu + \beta A)}, \\
E &= \frac{[pq + (1 - q)(\mu + p + \beta A)] \mu \Lambda}{(\mu + \epsilon + \beta \sigma A)(p + \mu + \beta A)}. 
\end{align}

From the forth equation in (8), we have

\begin{equation}
R = \frac{\gamma A}{\xi + \delta + \mu + a_2}. \tag{10}
\end{equation}

Substituting Equations (9) and (10) into the last equation in (8) gives

\begin{align}
Q &= \frac{\xi \gamma A(\mu + \epsilon + \beta \sigma A)(p + \mu + \beta A) + \epsilon [pq + (1 - q)(\mu + p + \beta A)] \mu \Lambda (\delta + \xi + \mu + a_2) - \delta \gamma}{\mu (\mu + \epsilon + \beta \sigma A)(p + \mu + \beta A)(\delta + \xi + \mu + a_2)}. \tag{11}
\end{align}

For $A \neq 0$, substituting Equation (10) into the third equation in (8) gives

\begin{equation}
S + \sigma E = \frac{1}{\beta} \frac{(\gamma + \mu + a_1)(\delta + \xi + \mu + a_2) - \delta \gamma}{\delta + \xi + \mu + a_2}. \tag{12}
\end{equation}

Substituting Equation (9) into (12) yields

\begin{align}
H(A) &= \frac{q(\mu + \epsilon + \beta \sigma A) + \sigma [pq + (1 - q)(\mu + p + \beta A)]}{(\mu + \epsilon + \beta \sigma A)(p + \mu + \beta A)} \\
&\quad - \frac{(\gamma + \mu + a_1)(\delta + \xi + \mu + a_2) - \delta \gamma}{\beta \mu \Lambda (\delta + \xi + \mu + a_2)} \\
&= \frac{\beta \sigma A + q(\mu + \epsilon) + \sigma [p + (1 - q)\mu]}{(\mu + \epsilon + \beta \sigma A)(p + \mu + \beta A)} - \frac{(\gamma + \mu + a_1)(\delta + \xi + \mu + a_2) - \delta \gamma}{\beta \mu \Lambda (\delta + \xi + \mu + a_2)} \\
&= 0. \tag{13}
\end{align}
Direct calculation shows
\[
\frac{H'(A)}{\beta} = -\frac{(\beta \sigma A)^2 + 2\beta \sigma A[q(\mu + \varepsilon) + \sigma[p + (1 - q)\mu]] + \sigma_0}{[(\mu + \varepsilon + \beta \sigma A)(p + \mu + \beta A)]^2} < 0,
\]
where
\[
\sigma_0 = (p + \mu)[p + (1 - q)\mu]\sigma^2 + pq(\mu + \varepsilon)\sigma + q(\mu + \varepsilon)^2,
\]
then function \( H(A) \) is decreasing for \( A > 0 \). Since \( (\mu + \varepsilon + \beta \sigma A)(p + \mu + \beta A) > \beta A(\beta \sigma A + \sigma p + \varepsilon + \mu) \), and it follows from \( 0 < \sigma < 1 \), \( 0 < q < 1 \) that \( \beta \sigma A + q(\mu + \varepsilon) + \sigma[p + (1 - q)\mu] < \beta \sigma A + \sigma p + \varepsilon + \mu \), then
\[
H(A) < \frac{1}{\beta A} - \frac{(\gamma + \mu + a_1)(\delta + \xi + \mu + a_2)}{\beta \mu \Lambda(\delta + \xi + \mu + a_2)}. (16)
\]
Thus,
\[
H(0) = \frac{q(\mu + \varepsilon) + \sigma[p + (1 - q)\mu]}{(\mu + \varepsilon)(p + \mu)} - \frac{(\gamma + \mu + a_1)(\delta + \xi + \mu + a_2) - \delta \gamma}{\beta \mu \Lambda(\delta + \xi + \mu + a_2)}(R_0 - 1),
\]
\[
H(\Lambda) < \frac{1}{\beta \Lambda} - \frac{(\gamma + \mu + a_1)(\delta + \xi + \mu + a_2) - \delta \gamma}{\beta \mu \Lambda(\delta + \xi + \mu + a_2)} = -\frac{\gamma(\xi + \mu + a_2) + a_1(\xi + \delta + \mu + a_2)}{\beta \mu \Lambda(\xi + \delta + \mu + a_2)} < 0
\]
Therefore, by the monotonicity of function \( H(A) \), for Equation (13) there exists a unique positive root in the interval \((0, \Lambda)\) when \( R_0 > 1 \); there is no positive root in the interval \((0, \Lambda)\) when \( R_0 \leq 1 \). Since the set \( \Omega = \{(S, E, A, R, Q) \in R^5_+ : S + E + A + R + Q \leq \Lambda \} \) is positively invariant for Equation (1), the equilibrium solution \( P^*(S^*, E^*, A^*, R^*, Q^*) \) are also in \( \Omega \), we have \( S^* + E^* + A^* + R^* + Q^* \leq \Lambda \). We summarize this result in Theorem 3.2.

**Theorem 3.2:** For system (1), there is always the alcohol-free equilibrium \( P_0(S_0, E_0, 0, 0, Q_0) \). When \( R_0 > 1 \), system (1) also has a unique alcohol present equilibrium \( P^*(S^*, E^*, A^*, R^*, Q^*) \), where
\[
S^* = \frac{q\mu \Lambda}{(p + \mu + \beta A^*)},
\]
\[
E^* = \frac{[pq + (1 - q)(\mu + p + \beta A^*)]\mu \Lambda}{(\mu + \varepsilon + \beta \sigma A^*)(p + \mu + \beta A^*)},
\]
\[
R^* = \frac{\gamma A^*}{\xi + \delta + \mu + a_2},
\]
\[
Q^* = \frac{\xi \gamma A^*}{\mu(\delta + \xi + \mu + a_2)} + \frac{\varepsilon[pq + (1 - q)(\mu + p + \beta A^*)]\Lambda}{(\mu + \varepsilon + \beta \sigma A^*)(p + \mu + \beta A^*)},
\]
and \( A^* \) is the unique positive root of equation \( H(A) = 0 \).
4. Global stability of equilibria

**Theorem 4.1:** For system (1), the alcohol-free equilibrium $P_0$ is globally stable if $R_0 \leq 1$; the alcohol present equilibrium $P^*$ is globally stable if $R_0 > 1$.

**Proof:** Since the variable $Q$ does not appear explicitly in the first four equations in (1), then the dynamics of Equation (1) is the same as the following system:

\[
\begin{align*}
\dot{S} &= q\mu\Lambda - \beta SA - (p + \mu)S, \\
\dot{E} &= (1-q)\mu\Lambda + pS - \beta\sigma EA - (\mu + \varepsilon)E, \\
\dot{A} &= \beta SA + \beta\sigma EA + \delta R - (\mu + a_1 + \gamma)A, \\
\dot{R} &= \gamma A - (\delta + \xi + \mu + a_2)R.
\end{align*}
\]  

(19)

We only need to prove stability of equilibria of (19). First, we prove the stability of alcohol-free equilibrium of Equation (19). For the alcohol-free equilibrium $P_0 = (S_0, E_0, A_0, R_0, 0, 0)$ of Equation (19), $S_0, E_0$ satisfy equations:

\[
\begin{align*}
q\mu\Lambda - (p + \mu)S &= 0, \\
(1-q)\mu\Lambda + pS - (\mu + \varepsilon)E &= 0,
\end{align*}
\]  

(20)

then Equation (19) can be rewritten as follows:

\[
\begin{align*}
S' &= S \left[ q\mu\Lambda \left( \frac{1}{S} - \frac{1}{S_0} \right) - \beta A \right], \\
E' &= E \left[ (1-q)\mu\Lambda \left( \frac{1}{E} - \frac{1}{E_0} \right) + p \left( \frac{S}{E} - \frac{S_0}{E_0} \right) - \beta\sigma A \right], \\
A' &= \beta A \left( S_0 + \sigma E_0 + (S - S_0) + \sigma (E - E_0) \right) + \delta R - (\gamma + \mu + a_1)A, \\
R' &= \gamma A - (\xi + \delta + \mu + a_1)R,
\end{align*}
\]

(21)

Define the Lyapunov function

\[
V_1 = \left( S - S_0 - S_0 \ln \frac{S}{S_0} \right) + \left( E - E_0 - E_0 \ln \frac{E}{E_0} \right) + A + \frac{\delta}{\xi + \delta + \mu + a_2} R
\]  

(22)

The derivative of $V_1$ is given by

\[
V'_1 = (S - S_0) S' S + (E - E_0) E' E + A' + \frac{\delta}{\xi + \delta + \mu + a_2} R'
\]

\[
= (S - S_0) \left[ q\mu\Lambda \left( \frac{1}{S} - \frac{1}{S_0} \right) - \beta A \right]
\]

\[
+ (E - E_0) \left[ (1-q)\mu\Lambda \left( \frac{1}{E} - \frac{1}{E_0} \right) + p \left( \frac{S}{E} - \frac{S_0}{E_0} \right) - \beta\sigma A \right]
\]

\[
+ \beta A \left( S_0 + \sigma E_0 + (S - S_0) + \sigma (E - E_0) \right) + \delta R - (\gamma + \mu + a_1)A
\]

\[
+ \frac{\delta}{\xi + \delta + \mu + a_2} [\gamma A - (\xi + \delta + \mu + a_2)R]
\]

\[
= \frac{(\gamma + \mu + a_1)(\xi + \delta + \mu + a_2) - \delta\gamma}{\xi + \delta + \mu + a_2} (R_0 - 1)A + F(S, A),
\]  

(23)
where
\[
F(S, A) = q\mu \Lambda (S - S_0) \left( \frac{1}{S} - \frac{1}{S_0} \right) + (1 - q)\mu \Lambda (E - E_0) \left( \frac{1}{E} - \frac{1}{E_0} \right) \\
+ p(E - E_0) \left( \frac{S}{E} - \frac{S_0}{E_0} \right) = q\mu \Lambda \left( 2 - \frac{S}{S_0} - \frac{S_0}{S} \right) \\
+ (1 - q)\mu \Lambda \left( 2 - \frac{E}{E_0} - \frac{E_0}{E} \right) + pS_0 \left( 1 + \frac{S}{S_0} - \frac{E}{E_0} - \frac{SE_0}{S_0E} \right).
\]  
(24)

Denote \( x = S/S_0, y = E/E_0, \) then
\[
F(S, A) = q\mu \Lambda \left( 2 - x - \frac{1}{x} \right) + (1 - q)\mu \Lambda \left( 2 - y - \frac{1}{y} \right) + pS_0 \left( 1 + x - y - \frac{x}{y} \right) \\
= : \bar{F}(x, y).
\]  
(25)

Applying Equation (19) to function \( \bar{F}(x, y) \) yields
\[
\bar{F}(x, y) = 2q\mu \Lambda + 2(1 - q)\mu \Lambda + pS_0 - q\mu \Lambda \frac{1}{x} - (1 - q)\mu \Lambda \frac{1}{y} - pS_0 \frac{x}{y} \\
- \mu S_0x - (1 - q)\mu \Lambda y - pS_0 y \\
= \mu S_0 \left( 2 - x - \frac{1}{x} \right) + (1 - q)\mu \Lambda \left( 2 - y - \frac{1}{y} \right) + pS_0 \left( 3 - \frac{1}{x} - y - \frac{x}{y} \right).
\]  
(26)

Since the arithmetical mean is greater than, or equal to the geometrical mean, then, \( 2 - x - 1/x \leq 0 \) for \( x > 0 \) and \( 2 - x - 1/x = 0 \) if and only if \( x = 1; 2 - y - 1/y \leq 0 \) for \( y > 0 \) and \( 2 - y - 1/y = 0 \) if and only if \( y = 1; 3 - 1/x - y - x/y \leq 0 \) for \( x, y > 0 \) and \( 3 - 1/x - y - x/y = 0 \) if and only if \( x = y = 1. \) Therefore, \( \bar{F}(x, y) \leq 0 \) for \( x, y > 0 \) and \( \bar{F}(x, y) = 0 \) if and only if \( x = y = 1. \) Therefore, when \( R_0 < 1, V_1 \leq 0, \) and the equality holds only for \( S = S_0, E = E_0, \) and \( A = 0; \) when \( R_0 = 1, V_1 \leq 0, \) and the equality holds only for \( S = S_0, E = E_0. \) Since the disease-free equilibrium \( P_0 \) is the only invariant set of Equation (19) contained entirely in \( ((S, E, A, R) \in \Omega : dV_1/dt = 0), \) hence by the asymptotic stability theorem [11] the disease-free equilibrium \( P_0 \) is globally stable on \( \Omega. \)

Next, we will prove stability of the alcohol present equilibrium of Equation (19). For the alcohol present equilibrium \( P^*(S^*, E^*, A^*, R^*) \) of Equation (19), \( S^*, E^*, A^*, R^* \) satisfy equations
\[
q\mu \Lambda - \beta SA - (p + \mu)S = 0, \\
(1 - q)\mu \Lambda + pS - \beta \sigma EA - (\mu + \varepsilon)E = 0, \\
\beta SA + \beta \sigma EA + \delta R - (\mu + a_1 + \gamma)A = 0, \\
\gamma A - (\delta + \xi + \mu + a_2)R = 0.
\]  
(27)

which will be used many times in the following inference.
By applying Equation (27) and denoting

\[ x = \frac{S}{S^*}, \quad y = \frac{E}{E^*}, \quad z = \frac{A}{A^*}, \quad u = \frac{R}{R^*}, \]

we have

\[ x' = x \left[ \frac{q \mu \Lambda}{S^*} \left( \frac{1}{x} - 1 \right) - \beta A^* (z - 1) \right], \]

\[ y' = y \left[ \frac{(1 - q) \mu \Lambda}{E^*} \left( \frac{1}{y} - 1 \right) + \frac{p S^*}{E^*} \left( \frac{x}{y} - 1 \right) - \beta \sigma A^* (z - 1) \right], \]

\[ z' = z \left[ \beta S^* (x - 1) + \beta \sigma E^* (y - 1) + \frac{\delta R^*}{A^*} \left( \frac{u}{z} - 1 \right) \right], \]

\[ u' = u \left[ \frac{\gamma A^*}{R^*} \left( \frac{z}{u} - 1 \right) \right]. \]

Define the Lyapunov function

\[ V_2 = S^* (x - 1 - \ln x) + E^* (y - 1 - \ln y) + A^* (z - 1 - \ln z) + \frac{\delta}{\xi + \delta + \mu + a_2} R^* (u - 1 - \ln u). \]

The derivative of \( V_2 \) is given by

\[ V'_2 = S^* \frac{x - 1}{x} x' + E^* \frac{y - 1}{y} y' + A^* \frac{z - 1}{z} z' + \frac{\delta}{\xi + \delta + \mu + a_2} R^* \frac{u - 1}{u} u' \]

\[ = (x - 1) \left[ \frac{q \mu \Lambda}{S^*} \left( \frac{1}{x} - 1 \right) - \beta S^* A^* (z - 1) \right] \]

\[ + (y - 1) \left[ (1 - q) \frac{\mu \Lambda}{E^*} \left( \frac{1}{y} - 1 \right) + \frac{p S^*}{E^*} \left( \frac{x}{y} - 1 \right) - \beta \sigma E^* A^* (z - 1) \right] \]

\[ + (z - 1) \left[ \beta S^* A^* (x - 1) + \beta \sigma E^* A^* (y - 1) + \frac{\delta R^*}{A^*} \left( \frac{u}{z} - 1 \right) \right] \]

\[ + \frac{\delta \gamma A^*}{\xi + \delta + \mu + a_2} (u - 1) \left( \frac{z}{u} - 1 \right) \]

\[ = q \mu \Lambda (x - 1) \left( \frac{1}{x} - 1 \right) - \beta S^* A^* (x - 1) (z - 1) + (1 - q) \mu \Lambda (y - 1) \left( \frac{1}{y} - 1 \right) \]

\[ + p S^* (y - 1) \left( \frac{x}{y} - 1 \right) - \beta \sigma E^* A^* (y - 1) (z - 1) + \beta S^* A^* (z - 1) (x - 1) \]

\[ + \beta \sigma E^* A^* (z - 1) (y - 1) + \delta R^* (z - 1) \left( \frac{u}{z} - 1 \right) + \frac{\delta \gamma A^*}{\xi + \delta + \mu + a_2} (u - 1) \left( \frac{z}{u} - 1 \right) \]
\[ p \mu \Lambda = q \mu \Lambda \left( 2 - x - \frac{1}{x} \right) + (1 - q) \mu \Lambda \left( 2 - y - \frac{1}{y} \right) + pS^* \left( x - y - \frac{x}{y} + 1 \right) \]

\[ + \delta R^* \left( u - z - \frac{u}{z} + 1 \right) + \frac{\delta \gamma A^*}{\xi + \delta + \mu + a_2} \left( z - u - \frac{z}{u} + 1 \right) \]

\[ = \frac{\delta \gamma A^*}{\xi + \delta + \mu + a_2} \left( 2 - \frac{u}{z} - \frac{z}{u} \right) + pS^* \left( x - y - \frac{x}{y} + 1 \right) \]

\[ + q \mu \Lambda \left( 2 - x - \frac{1}{x} \right) + (1 - q) \mu \Lambda \left( 2 - y - \frac{1}{y} \right) \]

\[ = \frac{\delta \gamma A^*}{\xi + \delta + \mu + a_2} \left( 2 - \frac{u}{z} - \frac{z}{u} \right) + F(x, y), \tag{31} \]

where

\[ F(x, y) = pS^* \left( x - y - \frac{x}{y} + 1 \right) + q \mu \Lambda \left( 2 - x - \frac{1}{x} \right) + (1 - q) \mu \Lambda \left( 2 - y - \frac{1}{y} \right) \]

\[ = 2q \mu \Lambda + 2(1 - q) \mu \Lambda + pS^* - (q \mu \Lambda - pS^*) x - [(1 - q) \mu \Lambda + pS^*] y \]

\[ - q \mu \Lambda \frac{1}{x} - (1 - q) \mu \Lambda \frac{1}{y} - pS^* \frac{x}{y}. \tag{32} \]

Using the equilibrium relation

\[ q \mu \Lambda = \beta S^* A^* + \mu S^* + pS^*. \tag{33} \]

We can rewrite \( F(x, y) \) as

\[ F(x, y) = \beta S^* A^* \left( 2 - x - \frac{1}{x} \right) + \mu S^* \left( 2 - x - \frac{1}{x} \right) \]

\[ + (1 - q) \mu \Lambda \left( 2 - y - \frac{1}{y} \right) + pS^* \left( 3 - \frac{1}{x} - y - \frac{x}{y} \right) \tag{34} \]

Similarly, we can rewrite \( V'_2 \) as

\[ V'_2 = \frac{\delta \gamma A^*}{\xi + \delta + \mu + a_2} \left( 2 - \frac{u}{z} - \frac{z}{u} \right) + \beta S^* A^* \left( 2 - x - \frac{1}{x} \right) + \mu S^* \left( 2 - x - \frac{1}{x} \right) \]

\[ + (1 - q) \mu \Lambda \left( 2 - y - \frac{1}{y} \right) + pS^* \left( 3 - \frac{1}{x} - y - \frac{x}{y} \right). \tag{35} \]

Since the arithmetical mean is greater than, or equal to the geometrical mean, then, \( 2 - x - 1/x \leq 0 \) for \( x > 0 \) and \( 2 - x - 1/x = 0 \) if and only if \( x = 1; 2 - y - 1/y \leq 0 \) for \( y > 0 \) and \( 2 - y - 1/y = 0 \) if and only if \( y = 1; 3 - 1/x - y - x/y \leq 0 \) for \( x, y > 0 \) and \( 3 - 1/x - y - x/y = 0 \) if and only if \( x = y, 2 - u/z - z/u \) for \( z, u > 0 \) and \( 2 - u/z - z/u = 0 \) if and only if \( u = z \). Therefore, \( V'_2 \geq 0 \) for \( x, y, z, u > 0 \) and \( V'_2 = 0 \) if and only if \( x = y, z = u = x \), which corresponds to set \( \Omega' = \{(S, E, A, R) : S = S^*, E = E^*, A/A^* = R/R^* \} \subset \Omega \). It is easy to see that the maximum invariant set of system (1) on the set \( \Omega' \) is the singleton \( P^* \). Thus, the alcohol present equilibrium \( P^* \) is globally asymptotically stable if \( R_0 > 1 \) by LaSalle’s invariance principle [11]. \( \blacksquare \)
5. Numerical simulation

In this section, some numerical results of system (1) are presented for supporting the analytic results obtained above. The model parameters are taken as: The value for $\mu = 0.25\text{day}^{-1}$ may be obtained from census data. We here take $\mu = 0.25\text{day}^{-1}$ (which is taken from [6]). For the probability (rate) $\beta = 0.3\text{person}^{-1}\text{day}^{-1}$, there are many data available on binge drinking. We here take $\beta = 0.3\text{person}^{-1}\text{day}^{-1}$ (which is taken from [6]). Other parameters are estimated as follows $q = 0.6$, $\Lambda = 3.496\text{day}^{-1}$, $\xi = 0.4\text{day}^{-1}$, $a_1 = a_2 = 0.01\text{day}^{-1}$, $\delta = 0.1\text{day}^{-1}$, $\sigma = 0.2$, $\varepsilon = 0.2\text{day}^{-1}$.

First, we choose $p = 0.9$, $\delta = 0.1\text{day}^{-1}$, numerical simulation gives $R_0 = 0.6866 < 1$, then the alcohol-free equilibrium $P_0$ is globally asymptotically stable (Figure 2).

Second, we choose $p = 0.5$, $\delta = 0.875\text{day}^{-1}$, numerical simulation gives $R_0 = 1$, then the disease-free equilibrium $P_0$ is globally asymptotically stable (Figure 3).

At last, we choose $p = 0.3$, $\delta = 0.8\text{day}^{-1}$, numerical simulation gives $R_0 = 1.2149 > 1$, the alcohol present equilibrium $P^\ast$ is globally asymptotically stable (Figure 4).

Figure 2. When $R_0 < 1$, the alcohol-free equilibrium $P_0$ is globally asymptotically stable.

Figure 3. When $R_0 = 1$, the alcohol-free equilibrium $P_0$ is globally asymptotically stable.
Figure 4. When $R_0 > 1$, the alcohol present equilibrium $P^*$ is globally asymptotically stable.

6. Discussion

We have formulated a drinking model with public health educational campaigns and liner relapse. By means of the next generation matrix, we obtained their basic reproduction number, $R_0$, which play a crucial role. The global stability of system (1) has been proved by using the Lyapunov function. When $R_0 \leq 1$, all solutions converge to the alcohol-free equilibrium, that is, the alcohol problems disappear eventually; when the $R_0 > 1$, the alcohol present equilibrium is globally stable, that is, the alcohol problems will persist in the population and the number of problem drinkers tends to a positive constant.

In this paper, we considered the effect of public health educational campaigns on the drinking dynamic. The basic reproductive number is given by

$$R_0 = \frac{\rho (FV^{-1})}{(\gamma + \mu + a_1)(\xi + \delta + \mu + a_2) - \delta \gamma}.$$ \hspace{1cm} (36)

where

$$S_0 = \frac{q\mu \Lambda}{(p + \mu)}, \quad E_0 = \frac{\mu \Lambda [p + (1 - q)\mu]}{(\mu + \epsilon)(p + \mu)}. \hspace{1cm} (37)$$

However, if the model does not have the effect of the public health education campaigns, then the basic reproductive number is

$$R_1 = \frac{\beta \Lambda (\xi + \delta + \mu + a_2)}{(\gamma + \mu + a_1)(\xi + \delta + \mu + a_2) - \delta \gamma}. \hspace{1cm} (38)$$

It is similar to the basic reproductive number in [18]. From Equation (38), we know that if $\delta$ increases, then $R_1$ will increase.

For system (1), $\delta$ reflects the relapse. Direct calculation shows that $\partial R_0 / \partial \delta > 0$, then decreasing the relapse coefficient is helpful to reduce the alcohol abuse. On the other hand, $\partial R_0 / \partial p < 0$ implies that increasing $p$ has positive effect on alcohol abuse control, then increasing the value of $p$, that is, increasing public health education campaigns. Figure 5 shows the relation among the basic reproduction number $R_0$, the relapse coefficient $\delta$, and the education rate $p$. From Figure 5, we find that even if $\delta$ is large, the basic reproduction
Figure 5. The relationship among $R_0, p, \delta$.

number $R_0$ of the model can be less than unit by increasing public health education campaigns. This means that public health education campaigns is one of the effective measure to control the alcohol problems.

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