Factorial moment studies of Cherenkov images

A. Razdan, D. K. Koul, M. K. Koul, S. K. Charagi
Nuclear Research Laboratory, Bhaba Atomic Research Centre, Mumbai 400 005, India

Presenter: A. Razdan (akrazdan@apsara.barc.ernet.in), ind-razdan-A-abs1-og27-poster

In this paper we study factorial moments of simulated Cherenkov images of TACTIC type γ-ray telescope for cosmic γ-ray and proton segregation. A comparative and supplementary studies between factorial moments and Hillas parameters using ANN is discussed.

1. Motivation

In this exploratory simulation studies, we use factorial moments as a tool to segregate γ-ray and proton initiated showers. Extensive Air Showers (EAS) are produced when VHE/UHE primary photons, protons and other high Z nuclei enter atmosphere from the top and produce Cherenkov radiation which can be studied by imaging technique or by measuring lateral distribution. In the Hillas parameterization approach, image is approximated as an ellipse and parameters like image “shape” and “orientation” are calculated. Hillas parameters like length, width, alpha, distance, size etc. are essentially set of second order moments. Among these parameters “width” and “alpha” are known to be very powerful parameters for segregation of γ-ray initiated cherenkov photon from cosmic ray background. Using this approach, present day γ-ray telescope have been able to reject cosmic ray background events up to 99.5 % level, while retaining upto 50 % of the γ-ray events from a point source. Hillas parameters are found to be good classifiers for small images (close to telescope threshold energies) but fail for large images (of higher primary energy) as too many pixels are part of the image. Again for GeV energy region telescopes like MAGIC and MACE, Hillas parameters alone are not very attractive. A loss of 50 % of actual image due to parameterization is also a big constraint. Hence the idea of trying factorial moments as a possible tool is a subject matter of this paper.

2. Global factorial moments

A given image is divided into M equal bins of width d. If \( n_m \) is the number of particles in the mth bin, quantity \( s_i \) can be defined as

\[
(s_i)_m = n_m(n_m - 1).......(n_m - i + 1)
\]

and calculated for each bin. The standard moments are defined as

\[
F_i^s(M) = \frac{\{\frac{1}{M} \sum_{m+1}^M (s_i)_m > M^i\}}{<N>^i}
\]

The normalized factorial moments are

\[
F_i^n(M) = \frac{\frac{1}{M} \sum_{m+1}^M (s_i)_m M^i}{N(N-1).......(N-i+1)}
\]

where N is the total number of photoelectrons in the image. Factorial moment has a power law dependence

\[
<F_i^n(M)> \sim \left(\frac{1}{d}\right)^\alpha
\]
3. Simulation strategy

For the present studies simulations were carried out using CORSIKA [1](version 5.6211) along with EGS4, VENUS,GHEISHA codes for Cherenkov option. Simulated data is generated for 208 elements of TACTIC [2] configuration, each element of the size 4m × 4m. Simulated data corresponds to Mt.Abu altitude (1300 m) and appropriate magnetic field. Cherenkov databases correspond to wavelength band of 300-450 nm. Simulated databases have been subjected to atmospheric absorption. For realistic simulation studies of TACTIC array, photoelectron content of each pixel is modified to include shot noise contribution from night sky background. A noise profile based on actual experimental conditions is superimposed on each image.

In this studies 15800 cherenkov images each corresponding to γ-ray (1 TeV) and proton (2 TeV) initiated showers are considered. The innermost pixels of the camera generate low level gamma ray threshold triggers using 3NCT (Nearest neighbour non-collinear triplets) topological trigger configuration [3]. Only those im-

---

Figure 1. ANN output with Hillas parameters as input
ages are selected which satisfy 3NCT trigger configuration with the condition that there are minimum of 6 photoelectrons in each pixel. By applying 3NCT criteria a total of 5300 $\gamma$-ray images and 2407 proton images are obtained. Hillas parameters and factorial moments have been obtained for these chosen images.

Cherenkov images of $\gamma$-ray showers are mainly elliptical in shape, hence compact. However, the cherenkov images of hadronic showers are mostly irregular in shape. For all cherenkov images, shape parameters like length, width and distance and orientation parameters like $\alpha$, azwidth and Miss are calculated. It is observed that width and $\alpha$ parameters have strong segregation capability. For all cherenkov images factorial moments $F_i^n$ of order $i$ ($i=2,3,4,5,6$) have been calculated. It is observed that factorial moment of fifth and sixth order are very powerful parameters to segregate $\gamma$-rays from protons. We have used Artificial neural network (ANN) for event classification. Figure 1 shows ANN output with only Hillas parameters (width and $\alpha$) as ANN inputs. Figure 2 displays ANN output with fifth and sixth order factorial moments as ANN inputs. Only those images with fifth order and sixth order moments greater than 4000 have been considered. Figure 3 depicts ANN output with hillas parameters and factorial moments (5th and 6th order only) together as ANN inputs.

Figure 2. ANN output with factorial moments as input
4. conclusion

It is clear that factorial moments are very powerful parameters and combine very well with hillas parameterization for segregation of γ-rays from protons.

![Figure 3. ANN output with Hillas parameters and factorial moments as input](image)

References

[1] D.Heck et. al. FZKA-report, 1998 Forchungszentrum Karlshru, Germany
[2] C.L.Bhat et. al. "Towards a Major Atmospheric Cerenkov Detector-III" Universal Academic Press 1994, ed. T.Kifune, p.207
[3] N.Sathybama et. al. Intl. Symp. on high energy gamma ray astronomy, year 2000, Heidelberg, Germany