Topological flatness of local models  
in the ramified case

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Abstract. Local models are schemes defined in terms of linear algebra which  
can be used to study the local structure of integral models of certain Shimura  
varieties, with parahoric level structure. We investigate the local models for  
groups of the form $\text{Res}_{F/\mathbb{Q}_p} \text{GL}_n$ and $\text{Res}_{F/\mathbb{Q}_p} \text{GSp}_{2g}$ where $F/\mathbb{Q}_p$ is a totally  
ramified extension, as defined by Pappas and Rapoport, and show that they  
are topologically flat. In the linear case, flatness can be deduced from this.

1 Introduction

It is an interesting problem to define 'good' models of Shimura varieties over  
the ring of integers of the reflex field, or over its completion at some prime ideal.  
For Shimura varieties of PEL type, which can be described as moduli spaces of  
abelian varieties, it is desirable to define such a model by a moduli problem,  
too.

In their book [RZ], Rapoport and Zink define such models in the case of para-  
horic level structures. They also define the so-called local models which étale-  
locally around each point of the special fibre are isomorphic to the model of the  
Shimura variety, but which are defined purely in terms of linear algebra. They  
provide a very useful tool to investigate local properties of the corresponding  
models.

Rapoport and Zink conjectured that these models are flat, which is a property  
a good model certainly should have. The conjecture is true for Shimura vari-  
eties associated to unitary or symplectic groups that split over an unramified  
extension of $\mathbb{Q}_p$ (see [C1], [C2]). But Pappas [P] showed that it is in general  
false if the group splits only after a ramified base change. Often the models  
are not even topologically flat, i. e. the closure of the generic fibre is not even  
set-theoretically equal to the whole model. We follow the terminology of Pap-  
pas and Rapoport and call the local model associated to these models of the  
Shimura variety the naive local model.

For groups of the form $\text{Res}_{F/\mathbb{Q}_p} \text{GL}_n$, where $F/\mathbb{Q}_p$ is a totally ramified extension,  
Pappas and Rapoport [PRH] then defined a new local model, which they call the  

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local model. In the case of level structures corresponding to a maximal parahoric subgroup, it is essentially defined as the closure of the generic fibre in the old local model, so it is automatically flat. Pappas and Rapoport showed that this new model has several good properties; for example its special fibre is normal, with Cohen-Macaulay singularities, and over a finite extension of $\mathcal{O}_E$ admits a resolution of singularities. The definition of the local model in the maximal parahoric case gives rise to a definition of a new local model in the general parahoric case. It is not obvious anymore that these general local models are flat, too, and it is the purpose of this note to show that they are topologically flat. Together with the methods used in [G1] (Frobenius splitting of Schubert varieties in the affine flag variety), one can then infer flatness. We therefore obtain the following theorem:

**Theorem 1.1** Let $F/\mathbb{Q}_p$ be a (possibly ramified) finite extension. Then the local model associated to $\text{Res}_{F/\mathbb{Q}_p} \text{GL}_n$ is flat over $\mathbb{Z}_p$.

An essential ingredient of the proof is a combinatorial result of Haines and Ngô about the so-called $\mu$-admissible and $\mu$-permissible sets (see below for further details). This result, as it stands, relates to the Iwahori case. In section 7 we show how it can be used to prove the corresponding result in the general parahoric case.

In section 6 we show that the local model for groups of the form $\text{Res}_{F/\mathbb{Q}_p} \text{GSp}_{2g}$, $F/\mathbb{Q}_p$ totally ramified, is topologically flat, too. (In this case the naive local model and the local model coincide topologically.) But here it is more difficult to deduce the flatness from the topological flatness, and we can only make a conjecture in this direction. On the other hand, it seems reasonable to expect that in this case the naive local model is flat itself (and thus coincides with the local model).

In a subsequent article [PR2] Pappas and Rapoport define yet another model, which they call the *canonical model*. Loosely speaking, it is the image of a certain morphism from a twisted product of unramified local models to the naive local model. In particular it is always flat, because the unramified local models are flat, and in addition has other good properties. Nevertheless the flatness question for the original local model remains interesting; if the local model is flat, it must coincide with the canonical local model. On the one hand this shows that the special fibre of the canonical local model has the 'correct' combinatorial description, on the other hand one gets an interesting 'resolution' of the local model.

The results of Pappas and Rapoport together with the combinatorial results of Haines and Ngô, and in section 7 respectively, can be used to give a shorter (but less elementary) proof of the topological flatness, as came out of discussions with Haines. See the remark after proposition 5.1 for an outline.

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2 Definitions

We will use the notation of Pappas and Rapoport [PRI]. Let us repeat part of it:

Let $F_0$ be a field, complete with respect to a non-archimedean valuation. Let $\mathcal{O}_{F_0}$ denote its ring of integers, $\pi_0$ a uniformizer. We assume that the residue class field is perfect. We fix a separable closure $F_{0,\text{sep}}$ of $F_0$.

Let $F/F_0$ be a totally ramified extension of degree $e$ inside $F_{0,\text{sep}}$. Let $\mathcal{O}_F$ be the ring of integers of $F$, and let $\pi \in \mathcal{O}_F$ be a uniformizer which is the root of an Eisenstein polynomial $T^e + \sum a_k T^k$. For each embedding $F \to F_{0,\text{sep}}$ we choose an integer $0 \leq r_\varphi \leq d$. Associated to these data we have the reflex field $E$, a finite extension of $F_0$ contained in $F_{0,\text{sep}}$, which is defined by $\text{Gal}(F_{0,\text{sep}}/E) = \{ \sigma \in \text{Gal}(F_{0,\text{sep}}/F_0); \forall \varphi : r_\sigma \varphi = r_\varphi \}$.

Further, let $V = F^d$, $\Lambda_0 = \mathcal{O}_F^d \subset V$, and denote the canonical basis by $e_1, \ldots, e_d$.

Choose $I \subseteq \{1, \ldots, d\}$.

Let us recall the definition of the 'naive' local model $\mathcal{M}_{\text{naive}} = \mathcal{M}(e, d, (\Lambda_i)_{i \in I}, (r_\varphi))$ (of course $\mathcal{M}_{\text{naive}}$ depends on $F/F_0$, not just on $e$, so this is an abuse of notation which nevertheless seems useful):

It is defined over the ring of integers $\mathcal{O}_E$ of the reflex field $E$, and its $S$-valued points are the isomorphism classes of commutative diagrams

$$
\begin{array}{cccccc}
\Lambda_{i_0,S} & \to & \Lambda_{i_1,S} & \to & \cdots & \to & \Lambda_{i_{m-1},S} & \to & \Lambda_{i_0,S} \\
F_0 & \to & F_1 & \to & \cdots & \to & F_{m-1} & \to & F_0 \\
\end{array}
$$

where $\Lambda_{i,S}$ is $\Lambda_i \otimes \mathcal{O}_S$, and where the $F_i$ are $\mathcal{O}_F \otimes \mathcal{O}_{F_0,\text{sep}} \mathcal{O}_S$-submodules. We require that locally on $S$, the $F_i$ are direct summands as $\mathcal{O}_S$-modules and that we have the following identity of polynomials ('determinant condition'):

$$
\det_{\mathcal{O}_S}(T - \Pi|F_i) = \prod_{\varphi}(T - \varphi(\pi))^{r_\varphi}.
$$

Pappas and Rapoport show that this naive local model is almost never flat, even if $I$ consists of only one element. In this case, i.e. when $I$ consists of only one element, they define a new local model $\mathcal{M}_{\text{loc}}$ as the scheme-theoretic closure of the generic fibre in the naive local model, and show that this new local model...
has several good properties. In particular, its special fibre is reduced, and can be described as a union of Schubert varieties with the 'right' index set.

Based on this, they define a new Iwahori type local model $M^{\text{loc}}$ as the closed subscheme of the naive local model such that all projections to the parahoric local model map to the new local model. Below we will show that this local model is topologically flat. Furthermore, using the technique of Frobenius splittings, one can see that its special fibre is reduced. Thus $M^{\text{loc}}$ is flat.

3 A certain map between local models

It is clear that in the definition of the local model, we have not used the fact that the sequence $\Lambda_0 \rightarrow \cdots \rightarrow \Lambda_{n-1} \rightarrow \pi^{-1}\Lambda_0$ is a lattice chain — in fact we could make a completely analogous definition for any sequence of free $O_F$-modules. (Compare [G1], 'general schemes of compatible subspaces'.)

In particular, consider the following situation: Choose a partition $\{0, \ldots, d - 1\} = \coprod_{\alpha} I_\alpha$. We can then decompose the lattices $\Lambda_i$ as

$$\Lambda_i = \bigoplus_{\alpha} \Lambda_i^\alpha,$$

where $\Lambda_i^\alpha = \bigoplus_{j \in I_\alpha} O_F e_j^i$. Here the $e_j^i = \begin{cases} \pi^{-1}e_j & j \leq i \\ e_j & j > i \end{cases}$ denote the canonical $O_F$-basis of $\Lambda_i$.

Now choose $0 \leq r_\alpha \leq |I_\alpha|$ such that $\sum_\alpha r_\alpha^\alpha = r_\phi$.

This gives rise to a decomposition of our lattice chain. More precisely for each $\alpha$ we get a sequence

$$\Lambda_0^\alpha \rightarrow \cdots \rightarrow \Lambda_{n-1}^\alpha \rightarrow \pi^{-1}\Lambda_0^\alpha$$

of free $O_F$-modules. The difference between these sequences and the original lattice chain (apart from the rank of the lattices) is that now some of the maps may be the identity. Nevertheless, we can define a 'local model' $M^\alpha = M(e, d, (\Lambda_i^\alpha)_{i=0,\ldots,n-1}, (r_\alpha^\alpha)_\phi)$. It is isomorphic to some local model in the original sense (because as far as the local model is concerned, we can just omit the 'identity steps').

The reflex field associated to a tuple $(r_\alpha^\alpha)_\phi$ will in general be different from the one belonging to $(r_\phi)_\phi$. Thus the $M^\alpha$ will in general not be defined over the same ring as the naive local model $M/O_E$ associated to the $r_\phi$. To simplify the situation, we make the following additional assumption: whenever $r_\alpha = r_\psi$, we have $r_\alpha^\alpha = r_\psi^\alpha$ for all $\alpha$. Under this assumption, all $M^\alpha$ will be defined over $O_E$, or even over a smaller ring. If necessary we apply a base change, and in the following we consider all the $M^\alpha$ as $O_E$-schemes.
We then have a canonical map
\[ \prod_{\alpha} M(e, d, (\Lambda^\alpha_i)_{i=0,\ldots,n-1}, (r^\alpha_\varphi)_\varphi) \to M(e, d, (\Lambda_i)_{i=0,\ldots,n-1}, (r_\varphi)_\varphi) \]
\[ ((F^\alpha_i)_i)_\alpha \to (\bigoplus_{\alpha} F^\alpha_i)_i. \]

4 The case \( d = 1 \)

In this section, we will look at the (trivial) case where \( d = 1 \). We will use the following observations in section 5.

Let us give a description of the local model corresponding to a maximal parahoric subgroup (the standard local model in the sense of [PR1], section 2). (Actually, in this case this coincides with the Iwahori type local model.)

So, we have integers \( 0 \leq r_\varphi \leq 1, \varphi = 1,\ldots,e \). Let us assume, for notational convenience, that the \( r_\varphi \) are in descending order, such that they are completely determined by \( r = \sum r_\varphi \).

The local model in this case is just \( \text{Spec} O_E \), where \( E \) is the corresponding reflex field. The point in the special fibre is the subspace \( \mathcal{F} \) corresponding to the \( e \times r \)-matrix
\[ \overline{M} = \begin{pmatrix} I_r \\ 0 \end{pmatrix}; \]
the operator \( \Pi|\mathcal{F} \) has Jordan type \( (r) \). Let us denote the subspace corresponding to the \( R \)-valued point of the local model by \( \mathcal{F}(e, r) \). The description of this subspace in terms of matrices is the following. We have a matrix
\[ M(e, r) = M = \begin{pmatrix} I_r \\ (b_{ij})_{i=1,\ldots,e-r}^{j=1,\ldots,r} \end{pmatrix}, \]
such that
\[ \Pi M = MA, \]
for some matrix \( A \) with characteristic polynomial
\[ \det(T - A) = \prod (T - \varphi(\Pi))^{r_\varphi}. \]

Furthermore, the reduction of \( M \) modulo \( \pi \) is \( \overline{M} \).

Example. Assume that \( \pi = \pi_0^6 \). Now, if we let \( r = 2 \) and choose the \( r_\varphi \) such that the resulting characteristic polynomial is \( T^2 - \pi^2 \), we get
\[ M(6, 2) = \begin{pmatrix} 1 & 1 \\ \pi^2 & \pi^2 \\ \pi^4 & \pi^4 \end{pmatrix}. \]
5 Lifting of points

**Proposition 5.1** The local model $M^{\text{loc}}$ is topologically flat, i.e. the generic points of the irreducible components can be lifted to the special fibre.

*Remark.* The following easier, though less elementary, proof resulted from discussions with Haines. It uses the theory of the splitting model developed by Pappas and Rapoport.

In [PR2], Pappas and Rapoport define the *splitting model*, which is a twisted product of unramified local models, and which maps to the local model $M^{\text{loc}}$. By definition, the *canonical local model* $M^{\text{can}}$ is the image of this morphism.

It is not hard to see that the special fibre of $M^{\text{can}}$ consists of the Schubert cells corresponding to the elements of the $\mu$-admissible set. By the theorem of Haines and Ngô (and the generalization in section 7, respectively), this set coincides with the $\mu$-permissible set, which parametrizes the Schubert cells in the special fibre of $M^{\text{loc}}$.

Since the splitting model is flat by [G1], the canonical local model is flat, and thus the local model $M^{\text{loc}}$ is topologically flat.

We now give a more direct proof of the proposition.

*Proof.* In this section, we consider only the Iwahori case. It will be obvious though, that the proof carries over to the general parahoric case without any problems once the results of Haines and Ngô are generalized correspondingly. In section 7 we will give a proof of the more general statement which we need.

We can embed the special fibre of $M^{\text{loc}}$ into the affine flag variety for $GL_d$ in the standard way, see [G1].

By the definition of the local model and since the special fibers of the maximal-parahoric new local models have the right stratification, the special fibre is the union of the Schubert cells corresponding to the $\mu$-permissible alcoves. Here $\mu = (\mu_1, \ldots, \mu_d)$ is the dual partition to $(r_\varphi)_{\varphi}$.

In order to show that the local model associated to some dominant coweight $\mu$ is topologically flat, we need to have a good understanding of the set of irreducible components of its special fibre. This amounts to a combinatorial problem in the extended affine Weyl group, namely to relating the so-called $\mu$-permissible and $\mu$-admissible sets.

By recent work of Haines and Ngô [HN] we know that in this case (as in the minuscule case), the set of $\mu$-permissible alcoves coincides with the set of $\mu$-admissible alcoves. Thus the maximal elements (with respect to the Bruhat order), which correspond to the irreducible components of the special fibre, are just the conjugates of $\mu$ under the finite Weyl group, i.e. the permutations of $\mu$ under $S_d$.

Choose a permutation $\nu$ of $\mu$ and write $\nu = (\nu_1, \ldots, \nu_d)$. 
A key observation is the following lemma which came up in a discussion with T. Haines.

**Lemma 5.2** Let $R$ be a discrete valuation ring, let $s$ be the closed and $\eta$ the generic point of Spec $R$. Let $U$ be an $R$-scheme of finite type, and $x \in U_s$ a closed point of the special fibre which lies in a unique irreducible component $S$ of $U_s$.

Furthermore, assume that $U_\eta$ is irreducible, that $\dim S_s = \dim U_\eta$, and that $x$ can be lifted to a closed point of $U_\eta$.

Then the generic point of $S$ can be lifted to $U_\eta$.

**Proof.** Denote by $U'$ the scheme-theoretic closure of $U_\eta$ in $U$. Then $U'$ is flat over $R$, and $x \in U'_s$. Furthermore, $\dim \mathcal{O}_{U',x} = \dim \mathcal{O}_{U'_s,x} + 1$ (see [M], theorem 15.1), and $\dim \mathcal{O}_{U',x} \geq \dim U_\eta + 1$, since $x$ can be lifted to a closed point of $U_\eta$.

Thus $\dim \mathcal{O}_{U'_s,x} \geq \dim U_\eta = \dim S$, which implies that $S \subseteq U'$. $\square$

Although the lemma is easy to prove, it reduces the combinatorial difficulty of our task enormously.

The lemma shows that it really is enough to show that one suitably chosen closed point of the special fibre can be lifted to the generic fibre. We will choose the point $x = (F_i)_i$, where each $F_i$ is given by the $de \times r$-matrix

$$M = \begin{pmatrix}
I_{\nu_1} & 0_{e-\nu_1 \times \nu_2} & I_{\nu_2} & 0_{e-\nu_2 \times \nu_3} & \ldots & I_{\nu_d} & 0_{e-\nu_d \times \nu_d}
\end{pmatrix}.$$

It is clear that $x$ lies in the stratum corresponding to $\nu$. (Look at the Jordan type of $\Pi|\mathcal{F}_i$.)

Now how can we lift this point to the generic fibre? We will describe two different methods to do this. The first one is more elementary, the second one more conceptual.

### 5.1 First method (elementary)

To simplify the notation, let us assume that the $r_\varphi$ are in descending order. We will build the corresponding matrix (over $\mathcal{O}_E$) by putting together several matrices of the form $M(e,s)$. More precisely, consider the $\mathcal{O}_E$-valued point where each subspace $\mathcal{F}_i$ is given by the same matrix
\[
M = \begin{pmatrix}
M(e, \nu_1) & M(e, \nu_2) & \cdots & M(e, \nu_d)
\end{pmatrix}.
\]

Note that the coefficients of this matrix are indeed contained in \(\mathcal{O}_E\). Clearly, the reduction of \(M\) is \(\overline{M}\). We have to check that

- \(M\) describes a subspace that lies in the maximal-parahoric local model (determinant condition)
- The subspace \(\mathcal{F}_i\) is mapped to \(\mathcal{F}_{i+1}\).

But both conditions are clearly satisfied, since the \(M(e, \nu_i)\) satisfy the corresponding conditions and since there are no interactions between the blocks. Let us make that more precise.

To check the first condition, fix any \(i\). Note that \(\mathcal{F}_i\) is \(\Pi\)-invariant: this means that there is a matrix \(A\) such that \(\text{diag}(\Pi|_{\mathcal{F}_i}, \ldots, \Pi|_{\mathcal{F}_i})M = MA\), and we can simply take \(A = \Pi|_{\mathcal{F}_i} = \text{diag}(\Pi|_{\mathcal{F}(e, \nu_1)}, \ldots, \Pi|_{\mathcal{F}(e, \nu_d)})\). Now let us check the determinant condition; the characteristic polynomial of \(\Pi|_{\mathcal{F}_i}\) is just the product of the characteristic polynomials of the \(\Pi|_{\mathcal{F}(e, \nu_i)}\), so we get

\[
\prod_{i=1}^{d} \prod_{\varphi=1}^{\nu_i} T - \varphi(\Pi) = \prod_{\varphi=1}^{\nu} (T - \varphi(\Pi))^{r_{\varphi}},
\]

as it should be.

The second condition is easily checked, too. We just have to observe that

\[
\text{diag}(1, \ldots, 1, \Pi, 1, \ldots, 1)M = M \text{diag}(1, \ldots, 1, \Pi|_{\mathcal{F}(e, \nu_i)}, 1, \ldots, 1).
\]

Thus we have indeed found a lifting of our point to the generic fibre.

### 5.2 Second method (conceptual)

In this section, we will describe the \(R\)-valued point constructed in the previous section as a morphism from a product of trivial (i.e. \(\cong \text{Spec} \mathcal{O}_E\)) local models to the given local model, which comes from the situation studied in section 3.

As partition of \(I\), we will choose the partition into singleton sets: \(I_\alpha = \{\alpha\}\), \(\alpha = 0, \ldots, d - 1\). Furthermore we choose \(0 \leq r_{\varphi}^\alpha \leq 1\) such that

\[
\sum_{\alpha} r_{\varphi}^\alpha = r_{\varphi}, \quad \text{and} \quad \sum_{\varphi} r_{\varphi}^\alpha = \nu_\alpha.
\]
It is easy to see that such \( r^\alpha \) exist and that they are uniquely determined. Furthermore, whenever \( r_\varphi = r_\psi \), we have \( r^\alpha = r_\psi^\alpha \) for all \( \alpha \). Thus all \( M(e, d, (\Lambda_i^\alpha)_{i=0,\ldots,n-1}, (r_\varphi^\alpha)_{\varphi}) \) are defined over a subring of \( \mathcal{O}_E \), and we denote the base change to \( \mathcal{O}_E \) by \( M^\alpha \).

Now consider the map

\[
\prod_{\alpha} M_{\alpha} \rightarrow M(e, d, (\Lambda_i)_{i=0,\ldots,n-1}, (r_\varphi)_{\varphi})
\]

\[
((F^\alpha_i)_{i=\alpha}) \mapsto \left( \bigoplus_{\alpha} F^\alpha_i \right)_{i=\alpha}
\]

associated to these data. All the \( M^\alpha \) are just isomorphic to Spec \( \mathcal{O}_E \), so their product is Spec \( \mathcal{O}_E \) again.

It is not hard to check is that the image of the closed point under this map is the point described above.

## 6 The case of the symplectic group

Finally, let us consider the question of topological flatness of the local model for the symplectic group. Let us repeat, with slight notational modifications, the definition of the naive local model for the symplectic group given in [PR2]; cf. also [RZ]. To simplify the notation, we consider only the Iwahori case.

Consider a totally ramified extension \( F/F_0 \) of degree \( e \) as before. Let \( V = F^{2g} \) with basis \( e_1, \ldots, e_g, f_1, \ldots, f_g \), and denote by \( \langle \cdot, \cdot \rangle \) the standard symplectic pairing, i.e.

\[
\{e_i, e_j\} = \{f_i, f_j\} = 0, \quad \{e_i, f_{g-j}\} = \delta_{ij}.
\]

Let \( \delta \) be an \( \mathcal{O}_F \)-generator of the inverse different \( \mathcal{D}^{-1}_{F/F_0} \) (if \( F \) is tamely ramified over \( F_0 \), we can take \( \delta = \pi^{1-e} \)). Let \( \langle v, w \rangle = \text{Tr}_{F/F_0}(\delta \{v, w\}) \). This is a non-degenerate alternating form on \( V \) with values in \( F_0 \).

The standard lattice chain \( (\Lambda_i)_{i} \) is self-dual with respect to \( \langle \cdot, \cdot \rangle \).

Now let \( r_\varphi = g \) for all \( \varphi \). Associated to \( F/F_0 \), \( V \) and \( (r_\varphi) \varphi \) we have the naive local model for \( GL_{2g} \). In this case the reflex field is \( F_0 \) and \( \mu \) is just \( (e, \ldots, e, 0, \ldots, 0) \).

The naive local model for the symplectic group is the closed subscheme of \( M(e, 2g, (\Lambda_i)_{i}, (r_\varphi)_{\varphi}) \) consisting of 'self-dual' families of subspaces:

\[
M^{\text{naive}}_{GSp} = \{(F_i)_{i} \in M_{\text{loc}}; F_i \rightarrow \Lambda_{i,S} \cong \Lambda_{2g-i,S}^\vee \rightarrow F_{2g-i}^\vee \text{ is the zero map}\}.
\]

Here \( \cdot^\vee \) denotes the \( \mathcal{O}_S \)-dual \( \text{Hom}(\cdot, \mathcal{O}_S) \), and the isomorphism \( \Lambda_{i,S} \cong \Lambda_{2g-i,S}^\vee \) is the one given by the pairing.

Similarly, we define the local model \( M^{\text{loc}}_{GSp} \) as the closed subscheme of \( M^{\text{loc}} \) consisting of self-dual families of subspaces:

\[
M^{\text{loc}}_{GSp} = \{(F_i)_{i} \in M^{\text{loc}}_{\text{loc}}; F_i \rightarrow \Lambda_{i,S} \cong \Lambda_{2g-i,S}^\vee \rightarrow F_{2g-i}^\vee \text{ is the zero map}\}.
\]
Actually, we know that in this case the two local models for $GL_{2g}$ coincide topologically. Namely, it is enough to check this in the maximal parahoric case where the special fibre is irreducible, so that one just has to see that the generic and special fibres have the same dimension. Of course this implies that $M_{GSp}^{\text{naive}}$ and $M_{GSp}^{\text{loc}}$ are topologically the same, too. So in order to prove the topological flatness, we can work with either one.

**Proposition 6.1** The local model $M_{GSp}^{\text{loc}}$ is topologically flat, i.e. the generic points of the irreducible components of the special fibre of $M_{GSp}^{\text{loc}}$ can be lifted to the generic fibre. The same holds for $M_{GSp}^{\text{naive}}$.

**Proof.** Since the symplectic local model is defined as a subscheme of the linear local model, the strata of the special fibre of $M_{GSp}^{\text{loc}}$ correspond to the intersection $\text{Perm}(\mu) \cap \tilde{W}_{GSp}$ (in the Iwahori case).

The results of Haines and Ngô [HN] show that this set coincides with the $\mu$-admissible set for the symplectic group, and thus the irreducible components of the special fibre of the local model correspond to the conjugates of $\mu$ under the Weyl group of the symplectic group (considered as a subgroup of $S_{2g}$).

In section 7 we generalize Haines’ and Ngô’s results to the parahoric case. Since this is the only difference between the Iwahori case and the general parahoric case, until the end of this section we will assume that we are in the Iwahori case, to simplify notation.

As in the linear case, it is enough to show that for each irreducible component there is one (suitably chosen) point that can be lifted to the generic fibre.

In this case we cannot hope to find the lifting as the image of a map of a product of trivial local models for the symplectic group, because decomposing the lattice chain as we did in the linear case would not preserve the pairing. Nevertheless the proof of the proposition is even simpler in this case, because the $\mu$ we have to deal with is so special.

We consider the symplectic local model as a closed subscheme of the linear local model. Let $\nu$ be a conjugate of $\mu$ under the Weyl group of the symplectic group. It is enough to show that we can lift the point where each $F_i$ is given by the $2ge \times r$-matrix

$$M = \begin{pmatrix} I_{\nu_1} & \cdots & \cdots & I_{\nu_{2g}} \\ 0_{e-\nu_1 \times \nu_1} & \cdots & \cdots & 0_{e-\nu_{2g} \times \nu_{2g}} \end{pmatrix}$$

(which obviously lies inside $\overline{M}^{\text{sympl}}$) to the generic fibre (of $\overline{M}^{\text{sympl}}$).
Let us first lift this point to a point in the generic fibre of the linear local model $M^{\text{loc}}$. Afterwards we will show that the lifting actually lies in the symplectic model.

Since all the $\nu_i$ are either $e$ or 0, we can just lift this point by exactly the same matrix over $\mathcal{O}_E$. It is clear that these matrices do describe a point in the (linear) local model over $\mathcal{O}_E$, in particular that the determinant condition is satisfied.

But since $\nu$ is not an arbitrary permutation of $\mu$, but one under the Weyl group of the symplectic group, it can never happen that $\nu_i$ and $\nu_{2g-i+1}$ are both 1. In other words, if in the matrix above there is a unit matrix somewhere in rows $e(i-1)+1, \ldots, e(i-1)+e$, the columns $e(2g-i)+1, \ldots, e(2g-i)+e$ will entirely consist of 0's. Taking into account that all the $F_i$ are given by the same matrix, it is then clear that this point lies in the symplectic local model. □

We cannot prove that $M_{GSp_{2g}}$ (or $M^{\text{loc}}_{GSp_{2g}}$) is flat. But since $M_{GSp_{2g}}$ is topologically flat, we are led to the

**Conjecture 6.2** The naive local model $M^\text{naive}_{GSp_{2g}}$ is flat.

To prove this conjecture, it would be sufficient to show that the special fibre is reduced. This would follow if one could identify it with a union of Schubert varieties in some affine flag variety, and by the usual method using Frobenius splittings, it would be enough to do this for the local models which correspond to a maximal parahoric subgroup. But even that seems to be a difficult problem in commutative algebra.

## 7 The parahoric case

In this section we will provide the generalization of the results of Haines and Ngô about the $\mu$-admissible and the $\mu$-permissible sets which is needed to prove the topological flatness in the general parahoric case.

Although we are interested only in the cases of $GL_n$ and $GSp_{2g}$, the $\mu$-admissible and $\mu$-permissible sets can be defined for any split connected reductive group. We denote by $\tilde{W}$ the extended affine Weyl group, and by $\Omega$ its subgroup of elements of length zero. In other words, $\Omega$ is the stabilizer of the base alcove. Then $\tilde{W}$ is the semi-direct product of the affine Weyl group $W_{\text{aff}}$ and $\Omega$.

We fix a dominant coweight $\mu$, and denote by $\tau$ the unique element of $\Omega$ such that $\mu \in W_{\text{aff}}\tau$.

We denote by $\overline{a}$ the closure of the base alcove, and by $P_\mu$ the convex hull of the translates of $\mu$ under the finite Weyl group.

**Definition 7.1** (Kottwitz-Rapoport)

i) The $\mu$-permissible set $\text{Perm}(\mu)$ is the set of elements $x \in W_{\text{aff}}\tau$ such that $x(v) - v \in P_\mu$ for all $v \in \overline{a}$. 


ii) The $\mu$-admissible set $\text{Adm}(\mu)$ is the set of $x \in \tilde{\mathcal{W}}$ such that there exists $w \in W_0$ with $x \leq tw_\mu$.

It was shown by Kottwitz and Rapoport that these two sets coincide for $G = GL_n$ or $GSp_{2g}$ and minuscule $\mu$, and that the $\mu$-admissible set is always contained in the $\mu$-permissible set. Furthermore, we have the following theorem by Haines and Ngô:

**Theorem 7.2** (Haines and Ngô also show that in general the $\mu$-admissible and the $\mu$-permissible set do not coincide.)

(Haines and Ngô also show that in general the $\mu$-admissible and the $\mu$-permissible set do not coincide.)

ii) If $\mu$ is a multiple of the dominant minuscule coweight $(1^g,0^g)$ for $GSp_{2g}$, then $\text{Perm}(\mu) = \text{Adm}(\mu)$.

It is clear that the set of irreducible components of the special fibre of $M^{\text{loc}}$ is exactly the set of maximal elements (with respect to the Bruhat order) of the $\mu$-permissible set. The theorem says that these maximal elements are just the conjugates of $\mu$ under the finite Weyl group.

The theorem as it stands relates to the Iwahori case. To prove topological flatness in the general parahoric case, we need a generalized version which covers the parahoric case, too. Clearly the Iwahori case is the most difficult among all the parahoric cases, and as we will show, the general parahoric case can be deduced from the Iwahori case relatively easily.

### 7.1 $GL_n$

We use the notation of [KR]. Let us recall part of it:

Let $\mathcal{F}$ be a non-empty subset of $\mathbb{Z}/n\mathbb{Z}$, and denote by $I \subseteq \mathbb{Z}$ its inverse image under the projection $\mathbb{Z} \rightarrow \mathbb{Z}/n\mathbb{Z}$. A family $(v_i)_{i \in I}$, $v_i \in \mathbb{Z}^n$, is called a face of type $I$ if it satisfies the following conditions:

1. $v_{i+n} = v_i + 1$ for all $i \in I$,

2. $v_i \leq v_j$ for all $i, j \in I, i \leq j$,

3. $\sum(v_i) - \sum(v_j) = i - j$ for all $i, j \in I$.

We denote the set of faces of type $I$ by $\mathcal{F}_I$.

Clearly, a face of type $\mathbb{Z}/n\mathbb{Z}$ is simply an alcove. We denote by $\omega$ the standard alcove $\omega = (\omega_0, \ldots, \omega_{n-1})$, $\omega_i = (1^i,0^{n-i})$, $i = 0, \ldots, n-1$, as well as the corresponding face of type $I$, $\omega = (\omega_i)_{i \in I}$.
The extended affine Weyl group \( \tilde{W} \) acts transitively on the set \( F_I \). Taking \( \omega \) as a base point, we identify \( F_I \) with the set \( \tilde{W}/W_I \), where \( W_I \) is the stabilizer of \((\omega_i)_{i \in I}\) in \( \tilde{W} \).

If \( J \subseteq I \) is a non-empty subset, and \( J \subseteq \mathbb{Z} \) its inverse image under the projection \( \mathbb{Z} \rightarrow \mathbb{Z}/n\mathbb{Z} \), we have a \( \tilde{W} \)-equivariant surjection \( F_I \rightarrow F_J \), defined by \((v_i)_{i \in I} \mapsto (v_i)_{i \in J}\).

We can now adapt the definition above to the parahoric case:

**Definition 7.3 (Kottwitz-Rapoport)**

i) The \( \mu \)-permissible set \( \text{Perm}_I(\mu) \subseteq F_I \) is the set of elements \((v_i)_{i \in I}\) such that for all \( i \in I \), \( v_i - \omega_i \in P_\mu \).

ii) The \( \mu \)-admissible set \( \text{Adm}_I(\mu) \subseteq F_I \) is the image of \( \text{Adm}(\mu) \) under the surjection \( F_{\mathbb{Z}/n\mathbb{Z}} \rightarrow F_I \).

**Proposition 7.4** Let \( J \subseteq I \) be a non-empty subset. Then the restriction of the map \( F_I \rightarrow F_J \) to \( \text{Perm}_I \) is a surjection \( \text{Perm}_I \rightarrow \text{Perm}_J \).

**Proof.** It is clearly enough to prove the proposition for \( I = \mathbb{Z}/n\mathbb{Z} \). Given \((v_j)_{j \in J}\) we can then fill in the missing \( v_j \)'s step by step, and thus are reduced to the following lemma. \( \square \)

**Lemma 7.5** Let \( k, l \) be integers, \( k < l \leq k + n \), and let \( v_k, v_l \in \mathbb{Z} \). Assume that \( v_k - \omega_k \in P_\mu \), \( v_l - \omega_l \in P_\mu \), \( v_k - v_l \) is minuscule and \( \sum(v_l) - \sum(v_k) = l - k \).

Then there exists \( v_{k+1} \in \mathbb{Z} \) such that \( v_{k+1} - \omega_{k+1} \in P_\mu \), \( v_{k+1} - v_k \) and \( v_l - v_{k+1} \) are minuscule and \( \sum(v_{k+1}) - \sum(v_k) = 1 \).

**Proof.** For \( v \in \mathbb{Z}^n \) we have \( v \in P_\mu \) if and only if \( v_{\text{dom}} \leq \mu \), where \( v_{\text{dom}} \in W.v \) is dominant.

We denote the standard basis vectors of \( \mathbb{Z}^n \) by \( e_1, \ldots, e_n \). For \( m \in \mathbb{Z} \), let \( r \) be the unique integer with \( 1 \leq r \leq n \), \( r \equiv m \mod n \). We define \( e_m \) to be the basis vector \( e_r \).

We are looking for \( m, 1 \leq m \leq n \), such that \( v_{k+1} \) with

\[
v_{k+1} - \omega_{k+1} = (v_k - \omega_k) + e_m - e_{k+1}
\]

satisfies the conditions above, i.e. such that

1. \( v_l - v_{k+1} \) is minuscule, i.e. \( v_l(m) - v_k(m) = 1 \)
2. \( v_{k+1} - \omega_{k+1} \in P_\mu \)
(The other two conditions are satisfied automatically.)

Let \( k' \) be the unique integer in \{1, \ldots, n\} such that \( k' \equiv k + 1 \pmod{n} \).

1st case. \( v_l(k') - v_k(k') = 1 \)

In this case we can simply choose \( m = k' \).

2nd case. \( v_l(k') = v_k(k') \)

This case is more complicated. Let \( \sigma \in S_n \) be such that \( \sigma(v_k - \omega_k) \) is dominant, and among all such \( \sigma \), choose \( \sigma \) with \( \sigma^{-1}(k') \) maximal.

Let

\[
\tilde{m} = \max\{M; \sigma(v_l)(M) > \sigma(v_k)(M)\}
\]

and

\[
\bar{m} = \min\{M; \sigma(v_k - \omega_k)(M) = \sigma(v_k - \omega_k)(\tilde{m})\}
\]

Let \( m = \sigma(\tilde{m}) \). Obviously this implies \( v_l(m) - v_k(m) = 1 \). So all that remains to show is that \( v_{k+1} - \omega_{k+1} \in P_\mu \) with this choice of \( m \).

By the definition of \( \sigma \) and \( m \), \( \sigma(v_{k+1} - \omega_{k+1}) \) is dominant, too. Thus it is enough to show that \( \sigma(v_{k+1} - \omega_{k+1}) \preceq \mu \).

If \( \tilde{m} > \sigma^{-1}(k') \), then this is clear. So let us now consider the case \( \tilde{m} < \sigma^{-1}(k') \).

We write \( \mu = (\mu(1), \ldots, \mu(n)) \in \mathbb{Z}^n \). We have to show that

\[
\sum_{i=1}^{N} \sigma(v_{k+1} - \omega_{k+1})(i) \leq \sum_{i=1}^{N} \mu(i)
\]

for \( N = 1, \ldots, n - 1 \). (It is clear that for \( N = n \) we have equality since \( \sum_{i=1}^{n} \sigma(v_{k+1} - \omega_{k+1})(i) = \sum_{1}^{n} \sigma(v_k - \omega_k)(i). \))

- For \( N \geq \sigma^{-1}(k') \) and for \( N < \tilde{m} \), we have

\[
\sum_{i=1}^{N} \sigma(v_{k+1} - \omega_{k+1})(i) = \sum_{i=1}^{N} \sigma(v_k - \omega_k)(i) = \sum_{i=1}^{N} \mu(i),
\]

because \( v_k - \omega_k \in P_\mu \).

- Let \( \tilde{m} \leq N < \sigma^{-1}(k') \).

We have \( \sigma(v_k - \omega_k)(k') > \sigma(v_l - \omega_l)(k') \), and for all \( i > \tilde{m} \) we have \( \sigma(v_l)(i) = \sigma(v_k)(i) \) and thus \( \sigma(v_l - \omega_l)(i) \leq \sigma(v_k - \omega_k)(i) \). Since furthermore \( \sum_{i=1}^{n} \sigma(v_l - \omega_l)(i) = \sum_{i=1}^{n} \sigma(v_k - \omega_k)(i) \), we see that

\[
\sum_{i=1}^{N} \sigma(v_k - \omega_k)(i) < \sum_{i=1}^{N} \sigma(v_l - \omega_l)(i).
\]

So we have

\[
\sum_{i=1}^{N} \sigma(v_{k+1} - \omega_{k+1})(i) = \sum_{i=1}^{N} \sigma(v_k - \omega_k)(i) + 1 \leq \sum_{i=1}^{N} \sigma(v_l - \omega_l)(i) \leq \sum_{i=1}^{N} \mu(i).
\]
Finally, consider $\tilde{m} \leq N < \hat{m}$.

We know that $\sum_{i=1}^{N} \sigma(v_k - \omega_k)(i) \leq \sum_{i=1}^{\hat{m}} \mu(i)$, since $v_k - \omega_k \in P_\mu$, and we want to show that for $\tilde{m} \leq N < \hat{m}$ we even have $<$ here. For $N = \tilde{m}$ this is certainly true.

Now suppose we had $\sum_{i=1}^{N} \sigma(v_k - \omega_k)(i) = \sum_{i=1}^{\hat{m}} \mu(i)$ for some $N$, $\tilde{m} \leq N < \hat{m}$. This implies
\[ \sum_{i=N+1}^{\tilde{m}} \sigma(v_k - \omega_k)(i) < \sum_{i=N+1}^{\hat{m}} \mu(i), \]
and thus $\sigma(v_k - \omega_k)(\tilde{m}) < \mu(N + 1) \leq \mu(N)$ (because for $\tilde{m} \leq N < \hat{m}$ all $\sigma(v_k - \omega_k)(N)$ are equal). But then we get
\[ \sum_{i=1}^{N-1} \sigma(v_k - \omega_k)(i) > \sum_{i=1}^{N-1} \mu(i), \]
which is a contradiction.

The lemma is proved. \[\square\]

**Corollary 7.6** Let $\mu$ be a dominant coweight for $GL_n$, and let $I$ be the inverse image of a non-empty subset $\tilde{I} \subseteq \mathbb{Z}/n\mathbb{Z}$. Then $\text{Perm}_I(\mu) = \text{Adm}_I(\mu)$.

**Proof.** The $\mu$-admissible set is always contained in the $\mu$-permissible set. Since we know that in the Iwahori case the two sets coincide, and because we have surjections $\text{Perm}(\mu) \twoheadrightarrow \text{Perm}_I(\mu)$ (by the proposition) and $\text{Adm}(\mu) \twoheadrightarrow \text{Adm}_I(\mu)$ (obvious), the corollary follows. \[\square\]

**7.2 $GSp_{2g}$**

Now let $G = GSp_{2g}$. Since the proofs for the symplectic group are based on reductions to the linear case, we use a subscript $\cdot_G$ to denote data corresponding to the symplectic group; notation without subscript refers to the $GL_{2g}$-case, as in the previous section.

We denote by $\Theta : \mathbb{Z}^{2g} \rightarrow \mathbb{Z}^{2g}$ the automorphism given by $(x_1, \ldots, x_{2g}) \mapsto (-x_{2g}, \ldots, -x_1)$.

This automorphism acts on the root system of $GL_{2g}$, and the 'Theta-invariant part', denoted $R^{[\Theta]}$, is the root system of $GSp_{2g}$. See [HN], §§9, 10. In particular, the extended affine Weyl group $\tilde{W}_G$ for the general symplectic group is a subgroup of the extended affine Weyl group for the general linear group. Furthermore, by [HN] Proposition 9.6, the Bruhat order on $\tilde{W}_G$ is inherited from the Bruhat order on $\tilde{W}$.

The vectors $\eta_i = \frac{1}{2}(\omega_i + \omega_{2g-i})$, $i = 0, \ldots, g$, serve as 'vertices' of the base alcove for the symplectic group.
Let $\mu$ be a dominant coweight for $G$. We can consider the $\mu$-admissible set $\text{Adm}_G(\mu)$ and the $\mu$-permissible set $\text{Perm}_G(\mu)$ as subsets of $W$. We have
\[
\text{Adm}_G(\mu) = \{ x \in \tilde{W}; \ x \leq t_{w\mu} \text{ for some } w \in W_{0,G} \},
\]
\[
\text{Perm}_G(\mu) = \{ x \in \tilde{W}; \ x(\eta_i) - \eta_i \in P_{G,\mu} \text{ for all } i = 0, \ldots, g \}.
\]

Here we denote by $P_{G,\mu}$ the convex hull of the translates of $\mu$ under the finite Weyl group (of $\text{Sp}_{2g}$).

Now let $I \subseteq \mathbb{Z}/2g\mathbb{Z}$ be a non-empty symmetric subset, i.e. a non-empty subset such that its inverse image $I$ under the projection $\mathbb{Z} \rightarrow \mathbb{Z}/2g\mathbb{Z}$ satisfies $I = -I$. (Obviously there is a one-to-one correspondence between symmetric subsets of $\mathbb{Z}/2g\mathbb{Z}$ and subsets of $\{0, \ldots, g\}$.) We describe the set of $G$-faces of type $I$ as a subset of $F_{G,I}$: A $G$-face of type $I$ is a face $(v_i)_{i \in I}$ for $GL_{2g}$ such that there exists $d \in \mathbb{Z}$ with
\[
v_{2g-i} = \Theta(v_i) + (d^2)
\]
for all $i \in I$. Clearly a $G$-face of type $\mathbb{Z}/2g\mathbb{Z}$ is simply an alcove. We denote the set of $G$-faces of type $I$ by $F_{G,I}$.

We obtain a commutative diagram
\[
\begin{array}{ccc}
\tilde{W}_G/W_{G,I} \cong F_{G,I} \\
\downarrow \quad \quad \downarrow \\
\tilde{W}/W_I \cong F_I.
\end{array}
\]

Here $W_{G,I} = W_I \cap \tilde{W}_G \subseteq \tilde{W}_G$ is the stabilizer of $(\omega_i)_{i \in I}$ in $\tilde{W}_G$.

We can now define parahoric versions of the admissible and permissible sets.

**Definition 7.7**

i) The $\mu$-permissible set $\text{Perm}_{G,I}(\mu) \subseteq \tilde{W}_G/W_{G,I} \cong F_{G,I}$ is the set of elements $x\tilde{W}_{G,I}$ such that for all $i \in I \cap \{0, \ldots, g\}$, $x(\eta_i) - \eta_i \in P_{G,\mu}$.

ii) The $\mu$-admissible set $\text{Adm}_{G,I}(\mu) \subseteq F_{G,I}$ is the image of $\text{Adm}_G(\mu)$ under the surjection $F_{G,\mathbb{Z}/2g\mathbb{Z}} \rightarrow F_{G,I}$.

It turns out, however, that the set which naturally describes the strata occurring in the special fibre of a local model is not the $\mu$-permissible set but the intersection $\text{Perm}_I(\mu) \cap \tilde{W}_G$. The goal of this section is to show that actually
\[
\text{Adm}_{G,I}(\mu) = \text{Perm}_{G,I}(\mu) = \text{Perm}_I(\mu) \cap \tilde{W}_G.
\]

The key point is to show that the natural map $\text{Perm}(\mu) \cap \tilde{W}_G \rightarrow \text{Perm}_I(\mu) \cap \tilde{W}_G/W_{G,I}$ is surjective.
If \( \pi : \bar{J} \subseteq \bar{I} \) is a non-empty symmetric subset, we have a \( \tilde{W}_G \)-equivariant surjection \( F_{G,I} \rightarrow F_{G,J} \).

Now suppose \( \bar{J} \subseteq \bar{I} \subseteq \mathbb{Z}/2g\mathbb{Z} \) are non-empty symmetric subsets, such that \( \bar{I} = \bar{J} \cup \{ k + 1 \} \) for some \( k \in \bar{J} \) with \( k + 1 \notin \bar{J} \). (For an integer \( m \) we denote by \( \bar{m} \) its class in \( \mathbb{Z}/2g\mathbb{Z} \).) Let \( l \) be the smallest integer in \( \bar{J} \) which is greater than \( k \).

**Lemma 7.8** ([KR], Lemma 10.3) In the situation above we have a bijection
\[
\pi^{-1}(v) \rightarrow \left\{ w \in \mathbb{Z}^2; v_k \leq w \leq v_l, \sum (w) = \sum (v_k) + 1 \right\}, (w_i)_{i \in I} \mapsto w_{k+1}.
\]

**Proposition 7.9** Let \( \mu \) be a positive multiple of the dominant minuscule coweight \((i^g, 0^g)\) for \( G \). Let \( \bar{J} \subseteq \bar{I} \) be a non-empty symmetric subset. Then the natural map \( \text{Perm}_I(\mu) \cap \tilde{W}_G/W_{G,I} \rightarrow \text{Perm}_J(\mu) \cap \tilde{W}_G/W_{G,I} \) is surjective.

**Proof.** It is clearly enough to consider \( \bar{J} \subseteq \bar{I} \) as in the lemma. Let \( (v_j)_{j \in J} \in \text{Perm}(\mu)_{G,J} \). We would like to extend this face to a \( G \)-face of type \( I \) by defining suitable \( v_{k+1}, v_{-(k+1)} \in \mathbb{Z}^2 \). By the lemma above and lemma [7.5] we find a \( G \)-face \((w_i)_{i \in I} \) of type \( I \) which maps to \((v_j)_{j} \) under \( \pi \) and such that \( w_{k+1} - \omega_{k+1} \in P_{G,\mu} \).

We have to show that then \( w_{-(k+1)} - \omega_{-(k+1)} \) automatically holds, too. Now \( \mu \) is of the form \((d^g, 0^g)\) for some \( d \), and
\[
w_{-(k+1)} - \omega_{-(k+1)} = (d^{2g}) + \Theta(w_{k+1} - \omega_{k+1})
\]
since \((w_i)_{i \in I} \) is a \( G \)-face.

Since \( \mu \) has this special form, for a dominant coweight \( \lambda \) we have \( \lambda_{\text{dom}} \preceq \mu \) if and only if \( \lambda(i) \leq d \) for all \( i \) (and \( \sum(\lambda) = gd \)). If this holds for some \( \lambda \), it clearly holds for \((d^{2g}) - \lambda\) as well, so we are done. \( \square \)

**Corollary 7.10** Let \( \mu \) be a positive multiple of the dominant coweight for \( G \), and let \( \bar{T} \subseteq \mathbb{Z}/2g\mathbb{Z} \) be a non-empty symmetric subset. Then \( \text{Perm}_{G,I}(\mu) = \text{Adm}_{G,I}(\mu) \).

**Proof.** First, we have an inclusion \( \text{Perm}_{G,I}(\mu) = \text{Perm}_J(\mu) \cap \tilde{W}_G/W_{G,I} \). This is theorem 10.1 in [IN] for \( I = \mathbb{Z}/2g\mathbb{Z} \), and it is easy to see that this is a 'vertex-by-vertex' proof, i.e. it works for arbitrary \( I \).

Now recall that in addition we know that in any case the \( \mu \)-admissible set is contained in the \( \mu \)-permissible set.

Since \( \text{Adm}_{G,I}(\mu) \) and \( \text{Perm}_J(\mu) \cap \tilde{W}_G/W_{G,I} \) both coincide with the image of \( \text{Perm}(\mu) \cap \tilde{W}_G \), the corollary follows. \( \square \)
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