Photon-pair generation in random nonlinear layered structures

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Nonlinearity and sharp transmission spectra of random 1D nonlinear layered structures are combined together to produce photon pairs with extremely narrow spectral bandwidths. Indistinguishable photons in a pair are nearly unentangled. Also two-photon states with coincident frequencies can be conveniently generated in these structures if photon pairs generated into a certain range of emission angles are superposed. If two photons are emitted into two different resonant peaks, the ratio of their spectral bandwidths may differ considerably from one and two photons remain nearly unentangled.

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I. INTRODUCTION

After the first successful experiment generating photon pairs in the nonlinear process of spontaneous parametric down-conversion in a nonlinear crystal and demonstration of its unusual temporal and spectral properties more than 30 years ago [1], properties of the emitted photon pairs have been addressed in detail in numerous investigations [2]. The key tool in the theory of photon pairs has become a two-photon spectral amplitude [3, 4, 5, 6] that describes a photon pair in its full complexity. At the beginning the effort has been concentrated on two-photon entangled states with anti-correlated signal- and idler-field frequencies that occur in usual nonlinear crystals. New sources able to generate high photon fluxes have been discovered using, e.g., periodically-poled nonlinear crystals [7, 8], four-wave mixing in nonlinear structured fibers [9, 10], nonlinear planar waveguides [11] or nonlinearity in cavities [12]. Chirped periodically-poled crystals opened the door for the generation of signal and idler fields with extremely wide spectral bandwidths [13, 14]. Photon pairs with wide spectral bandwidths can also be generated in specific non-collinear geometries [15]. Very sharp temporal features are typical for such states that can be successfully applied in metrology (see [16] for quantum optical coherence tomography). Later, even two-photon states with coincident frequencies have been revealed. They can be emitted provided that the extended phase matching conditions (phase matching of the wave-vectors and group-velocity matching) are fulfilled [17, 18, 19]. Also other suitable geometries for the generation of states with coincident frequencies have been found. Nonlinear phase matching for different frequencies at different angles of propagation of the interacting fields can be conveniently used in this case [20, 21, 22].

Or wave-guiding structures with transverse pumping and counter-propagating signal and idler fields can be exploited [23, 24, 25, 26, 27, 28, 29, 30, 31, 32]. The last two approaches are quite flexible and allow the generation of states having two-photon amplitudes with an arbitrary orientation of main axes of their (approximately) gaussian shape. These approaches emphasize the role of a two-photon state at the boundary between the two mentioned cases. This state is, a bit surprisingly, unentangled and, at the first sight, should not be very interesting. The opposed is true [31, 32] due to the fact that the signal and idler photons are completely indistinguishable and perfectly synchronized in time for pulsed pumping. Non-collinear configurations of bulk crystals of a suitable length and pump beam with a suitable waist can also be used to generate such state [15, 33]. These states with completely indistinguishable photons are very useful in many quantum-information protocols (e.g., in linear quantum computing) that rely on polarization entanglement.

It has been shown that spectral entanglement affects polarization entanglement and causes the lowering of polarization-entanglement performance in quantum-information protocols. For example, the role of spectral entanglement in polarization quantum teleportation has been analyzed in detail in [34]. We note that this analysis is also appropriate for quantum repeaters, quantum relays or quantum repeaters. If two photons are identical and without any mutual entanglement, the polarization degrees of freedom are completely separated from the spectral ones and in principle the best possible performance of quantum information protocols is guaranteed. In practice, mode mismatches both in spectral and spatial domains may occur and degrade the performance. It has been shown in [35] that Gaussian distributed photons with large bandwidths represent states with the best tolerance against errors. Under these conditions high visibilities of interference patterns created by a simultaneous detection of in principal many photons are expected.

As shown in this paper, nonlinear layered struc-
tures with randomly positioned boundaries are a natural source of unentangled photon pairs. If the downconverted photons are generated under identical conditions (i.e. into the same transmission peaks) they are indistinguishable and thus ideal for quantum-information processing with polarization degrees of freedom. Moreover their spectra are very narrow and temporal wave packets quite long and so their use in experimental setups is fault tolerant against unwanted temporal shifts \[35\]. Also because the down-converted photons are generated into localized states with high values of electric-field amplitudes higher photon-pair generation rates are expected. This is important for protocols exploiting several photon pairs. These states with very narrow spectra can also be useful in entangled two-photon spectroscopy. Superposition of photon pairs generated under different emission angles is possible and two-photon states coincident in frequencies can be engineered this way.

We note that the use of high electric-field amplitudes occurring in localized states in random structures for the enhancement of nonlinear processes has been studied in \[39\]. Enhanced nonlinear processes has been studied in \[39\] and this theory is used in our calculations. We note that a suitable choice of vector and photonic properties can result in the generation of photon pairs with a two-photon spectral amplitude antisymmetric in the signal- and idler-field frequencies \[42\]. Some of the properties of photon pairs generated in layered structures are common with these of photon pairs originating in nonlinear crystal super-lattices composed of several nonlinear homogeneous pieces \[43\].

In further considerations, we assume fixed polarizations of the signal and idler fields and restrict ourselves to a scalar model. Then the perturbation solution of Schrödinger equation for the signal- and idler-fields wave-function results in the following two-photon wave-function \[43\]:

\[
|\psi_{s,i}(t)\rangle = \int_0^\infty d\omega_s \int_0^\infty d\omega_i \phi(\omega_s, \omega_i) \\
\times \hat{a}_s^\dagger(\omega_s)\hat{a}_i^\dagger(\omega_i) \exp(i\omega_s t) \exp(i\omega_i t) |\text{vac}\rangle.
\]

The two-photon spectral amplitude \(\phi(\omega_s, \omega_i)\) giving a probability amplitude of generating a signal photon at frequency \(\omega_s\) and its idler twin at frequency \(\omega_i\) is determined for given angles of emission and polarizations of the signal and idler photons. Creation operator \(\hat{a}_m^\dagger(\omega_m)\) adds one photon at frequency \(\omega_m\) into field \(m\) \((i = s, i)\) and \(|\text{vac}\rangle\) means the vacuum state for the signal and idler fields. Both photons can propagate either forward or backward outside the structure as a consequence of scattering inside the structure. Here we analyze in detail the case when both photons propagate forward and note that similar behavior is found in the remaining three cases. More details can be found in \[39, 40\].

The two-photon spectral amplitude \(\phi(\omega_s, \omega_i)\) can be decomposed into the Schmidt dual basis with base functions \(f_{s,n}\) and \(f_{i,n}\) \[43, 44\] in order to reveal correlations (entanglement) between the signal and idler fields:

\[
\phi(\omega_s, \omega_i) = \sum_{n=1}^\infty \lambda_n f_{s,n}(\omega_s)f_{i,n}(\omega_i);
\]

\(\lambda_n\) being coefficients of the decomposition. Entropy \(S\) defined as \(45\):

\[
S = -\sum_{n=1}^\infty \lambda_n^2 \log_2 \lambda_n^2
\]

is a suitable quantitative measure of spectral entanglement between the signal and idler fields. Symbol \(\log_2\) stands for logarithm with base 2. An additional measure of spectral entanglement can be expressed in terms of the

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**II. SPONTANEOUS PARAMETRIC DOWN-CONVERSION AND CHARACTERISTICS OF PHOTON PAIRS**

Spontaneous parametric down-conversion in layered media can be described by a nonlinear Hamiltonian \(\hat{H}\) in the formalism of quantum mechanics:

\[
\hat{H}(t) = \epsilon_0 \int_V d\mathbf{r} [E_p^{(+)}(r,t)\mathbf{E}_s^{(-)}(r,t)\mathbf{E}_i^{(-)}(r,t) + \text{h.c.}].
\]

(1)

Vector properties of the nonlinear interaction are characterized by a third-order tensor of nonlinear coefficients \(\mathbf{d}\) \[38\]. Symbol \(E_p^{(+)}\) denotes a positive-frequency pump-field electric-field amplitude, whereas negative-frequency electric-field amplitude operators \(E_m^{(-)}\) for \(m = s, i\) describe the signal and idler fields. Symbol \(\epsilon_0\) is permittivity of vacuum, \(V\) interaction volume, h.c. stands for a hermitian conjugated term, and : means tensor multiplication with respect to three indices. The electric-field amplitudes of the pump, signal, and idler fields can be conveniently decomposed into forward- and backward-propagating monochromatic plane waves when describing scattering of the considered fields at boundaries of a layered structure \[32, 40, 41\]. Vector properties of the electric-field amplitudes are taken into account using decomposition into TE and TM modes. A detailed theory has been developed in \[39\] and this theory is used in our calculations. We note that a suitable choice of vector and photonic properties can result in the generation of photon pairs with a two-photon spectral amplitude antisymmetric in the signal- and idler-field frequencies \[42\]. Some of the properties of photon pairs generated in layered structures are common with these of photon pairs originating in nonlinear crystal super-lattices composed of several nonlinear homogeneous pieces \[43\].

Conclusions.

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*References*:

1. [33]...
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The two-photon spectral amplitude \( \phi(\omega_s, \omega_i) \) describes completely properties of photon pairs. Number \( N \) of the generated photon pairs as well as signal-field energy spectrum \( S_s(\omega_s) \) can simply be determined using the two-photon spectral amplitude \( \phi \) \cite{39}:

\[
N = \int_0^\infty d\omega_s \int_0^\infty d\omega_i |\phi(\omega_s, \omega_i)|^2, \tag{6}
\]

\[
S_s(\omega_s) = \hbar \omega_s \int_0^\infty d\omega_i |\phi(\omega_s, \omega_i)|^2. \tag{7}
\]

Fourier transform \( \phi(t_s, t_i) \) of the spectral two-photon amplitude \( \phi(\omega_s, \omega_i) \),

\[
\phi(t_s, t_i) = \frac{1}{2\pi} \int_0^\infty d\omega_s \int_0^\infty d\omega_i \sqrt{\frac{\omega_s \omega_i}{\omega_s^0 \omega_i^0}} \phi(\omega_s, \omega_i) \times \exp(-i\omega_s t_s) \exp(-i\omega_i t_i), \tag{8}
\]

is useful in determining temporal properties of photon pairs. Symbol \( \omega_s^0 \) in Eq. \( 5 \) denotes the central frequency of field \( m, m = s, i \). This Fourier transform is linearly proportional to the two-photon temporal amplitude \( A \) defined along the expression

\[
A(t_s, t_i) = \langle \psi_s(t_s) | \hat{E}_i^+(0, t_s) \hat{E}_s^+(0, t_i) | \psi_{s,i}(t_0) \rangle \tag{9}
\]

and giving a probability amplitude of detecting a signal photon at time \( t_s \) together with its twin at time \( t_i \). We have \cite{39}:

\[
A(t_s, t_i) = \frac{\hbar \sqrt{\omega_s^0 \omega_i^0}}{4\pi c \epsilon_0 B} \phi(t_s, t_i), \tag{10}
\]

where \( c \) is speed of light in vacuum and \( B \) transverse area of the fields. Photon flux \( \mathcal{N} \) of, e.g., the signal field is determined using a simple formula (valid for narrow spectra) \cite{39}:

\[
\mathcal{N}_s(t_s) = \hbar \omega_s^0 \int_{-\infty}^{\infty} dt_i |\phi(t_s, t_i)|^2. \tag{11}
\]

Direct measurement of temporal properties of photon pairs is impossible because of short time scales needed and so temporal properties may be detected only indirectly. Time duration of the (pulsed) down-converted fields as well as entanglement time can be experimentally addressed using Hong-Ou-Mandel interferometer. Normalized coincidence-count rate \( R_n^{\text{HOM}} \) in this interferometer is derived as follows \cite{4}:

\[
R_n^{\text{HOM}}(\tau_t) = 1 - \tilde{\rho}(\tau_t), \tag{12}
\]

\[
\tilde{\rho}(\tau_t) = \frac{1}{R(0, 0)} \text{Re} \left[ \int_0^\infty d\omega_s \int_0^\infty d\omega_i \omega_s \omega_i \phi(\omega_s, \omega_i) \phi^*(\omega_i, \omega_s) \exp(i\omega_s \tau_t) \exp(-i\omega_i \tau_t) \right]. \tag{13}
\]

Normalization constant \( R(0, 0) \) occurring in Eq. \( 13 \) is given in Eq. \( 15 \) below. Relative time delay \( \tau_t \) between the signal and idler photons changes in the interferometer.

A preferred direction of correlations between the signal- and idler-field frequencies can be determined from the orientation of coincidence-count interference fringes in Franson interferometer \cite{23} that is characterized by normalized coincidence-count rate \( R_n^F \) in the form:

\[
R_n^F(\tau_s, \tau_t) = \frac{1}{4} + \frac{1}{8R(0, 0)} \text{Re} \{ 2R(\tau_s, 0) + 2R(0, \tau_t) + R(\tau_s, \tau_t) + R(\tau_s, -\tau_t) \}. \tag{14}
\]

The function \( R \) in Eq. \( 14 \) is defined as follows:

\[
R(\tau_s, \tau_t) = \int_0^\infty d\omega_s \int_0^\infty d\omega_i \omega_s \omega_i |\phi(\omega_s, \omega_i)|^2 \times \exp(i\omega_s \tau_s) \exp(i\omega_i \tau_t). \tag{15}
\]

Time delay \( \tau_m \) \((m = s, i)\) corresponds to a relative phase shift between two arms in the path of photon \( m \).

III. PROPERTIES OF RANDOM 1D LAYERED STRUCTURES

We consider a 1D layered structure composed of two dielectrics with mean layer optical thicknesses equal to \( \lambda_0/4 \) \((\lambda_0 = 1 \times 10^{-6} \text{ m is chosen})\). Such structure, for example, can be fabricated by etching a crystal made of LiNbO\(_3\) and filling free spaces with a suitable material (SiO\(_2\)). The optical axis of LiNbO\(_3\) is parallel to the planes of boundaries and coincides with the direction of fields’ polarizations that correspond to TE modes. This geometry exploits the largest nonlinear coefficient of tensor \( \mathbf{d} \) \((d_{zzz})\). The fields propagate as ordinary waves with the dispersion formula \( n^2(\omega) = 4.9048 + 0.11768/(354.8084 - 0.0475\omega^2) - 9.6398\omega^2 \) \cite{47}. The second material (SiO\(_2\)) is characterized by its index of refraction \( n(\omega) = 1.45 \). A random structure is generated along the following algorithm:

1. The number \( N_{\text{elem}} \) of elementary layers of optical thicknesses \( \lambda_0/4 \) is fixed. Then the material of each elementary layer is randomly chosen.

2. At each boundary between two materials, an additional random shift of the boundary position governed by a gaussian distribution with variance \( \lambda_0/40 \) is introduced.

The randomness given by a random choice of the material of each layer is crucial for the observation of features typical for random structures. The randomness introduced by gaussian shifts of boundary positions is only additional and does not modify considerably properties of a random structure. It also describes errors occurring in the fabrication process. We note that the same generation algorithm has been used in \cite{36, 37} where second-harmonic generation in random structures has been addressed.
The process of spontaneous parametric down-conversion can be efficient in these structures provided that the signal and idler fields are generated into transmission peaks (corresponding to spatially localized states) and the structure is at least partially transparent for the pump field. This imposes restrictions to the allowed optical lengths of suitable structures. They have to have such lengths that the down-converted fields (usually degenerate in frequencies) are localized whereas there occurs no localization at the pump-field wavelength. This is possible because the shorter the wavelength the larger the localization length for a given structure. We assume the pump-field (signal-, idler-field) wavelength \(\lambda_p\) (\(\lambda_s, \lambda_i\)) in the vicinity of \(\lambda_0/2\) (\(\lambda_0\)).

Numerical simulations for the considered materials and wavelengths have revealed that the best suitable numbers \(N_{\text{elem}}\) of elementary layers lie between 200 and 400. In general, the greater the contrast of indices of refraction of two materials the smaller the number of needed layers. Widths of transmission peaks corresponding to localized states in a random structure may vary by several orders of magnitude. As an example, probability distribution \(P_{\Delta \lambda_s}\) of widths of transmission peaks for an ensemble of structures with \(N_{\text{elem}} = 250\) elementary layers (optical lengths of these structures lie around \(6 \times 10^{-5}\) m) is shown in Fig. 1a. Comparison with the distributions appropriate for \(N_{\text{elem}} = 500\) and \(N_{\text{elem}} = 750\) in Fig. 1a demonstrates that the longer the structure the narrower transmission peaks can be expected. Localization optical length at \(\lambda_0\) \([37, 49]\) in the direction perpendicular to the boundaries lies around \(22 \times 10^{-6}\) m for the considered structure lengths. As for the simulation, we have randomly generated several tens of thousands structures in order to have around \(3 \times 10^4\) transmission peaks for the statistical analysis. Transmission peaks can be easily distinguished from their flat surroundings for which intensities lower than one percent of the maximum peak intensity are typical.

Widths of intensity transmission peaks have been determined as FWHM. Simulations have revealed that peaks with very small intensity transmissions prevail, but there occur also peaks with intensity transmissions close to one. Such peaks are useful for the generation of photon pairs because of the presence of high electric-field amplitudes inside the structure. Photon pairs can be also generated under nonzero angles of emission. The greater the radial angle \(\theta_s\) of signal-photon emission the narrower the transmission peaks as documented in Fig. 1b. Also the localization lengths (projected into the direction perpendicular to the boundaries) shorten with an increasing radial angle \(\theta_s\) of signal-photon emission. Localization length equal to \(9.5 \times 10^{-6}\) m (\(2.5 \times 10^{-6}\) m) has been determined at the angle of signal-photon emission \(\theta_s = 30\) deg (60 deg).

Fabrication of such structures is relatively easy due to allowed high tolerances. A typical sample has several transmission peaks at different angles \(\theta_s\) of emission for a given frequency of the signal field. If we assume that also the idler field is tuned into the same transmission peak the (normally incident) pump field has to have twice the frequency corresponding to this peak. In non-collinear geometry the transverse wave-vectors of the signal and idler fields have to have the same magnitudes and opposed signs in order to fulfill phase-matching conditions in the transverse plane. If the angle \(\theta_s\) of signal-photon emission increases a given transmission peak survives increasing its central frequency \(\omega_0^s\). Moreover the dependence of central frequency \(\omega_0^s\) on radial emission angle \(\theta_s\) can be considered to be linear in a certain interval of angles \(\theta_s\). This property leads to wide tunability in frequencies.

IV. PROPERTIES OF THE EMITTED PHOTON PAIRS

We assume a normally incident pump beam forming a TE wave and generation of the TE-polarized signal and idler fields into the same transmission peak. Thus both emitted photons have the same central frequencies and the central frequency of the pump field is twice this frequency, i.e. the conservation law of energy is fulfilled. If a structure is pumped by a broadband femtosecond pulse, photon pairs are generated into a certain range of emis-

![Graph](image-url)
sion angles. If a signal photon occurs at a given radial angle $\theta_s$, its idler twin is emitted at its radial angle $\theta_i = -\theta_s$ and wave-vectors of both photons and the pump field lie in the same plane (i.e., their azimuthal angles $\psi_s$ and $\psi_i$ coincide) as a consequence of phase matching conditions in the transverse plane. The central frequency $\omega_0^s$ of a signal photon depends on the angle $\theta_s$ of signal-photon emission, as illustrated in Fig. 2 showing the signal-field intensity spectrum $S_{\text{rel}}^s$. Width of the spectrum $S_s$ coincides with the width of intensity transmission peak because the transmission peaks are very narrow and so all frequencies inside them have nearly the same conditions for the nonlinear process. This means that linear properties of the photon-band-gap structure have a dominant role in the determination of spatial properties of the generated photon pairs. Localization of the signal and idler fields inside the considered photonic-band-gap structure leads to the enhancement of photon-pair generation rate up to 5000 times (in the middle of transmission peak) as can be deduced from Fig. 2. This is in accordance with the finding that the process of second-harmonic generation can be enhanced by 3 or 4 orders of magnitude [37] in these structures. This enhancement is at the expense of dramatic narrowing of the range of the allowed frequencies of emitted photons. We estimate from 10 to $10^3$ generated photon pairs into the whole emission cone per 100 mW of the pump power depending on the width of transmission peak. The wider the transmission peak the higher number of pairs is expected. We note that extremely narrow down-converted fields can be obtained also from parametric pumping of $^{87}$Rb atoms [51].

Two-photon spectral amplitudes $\phi(\omega_s, \omega_i)$ for different angles $\theta_s$ of signal-photon emission are very similar in their shape; they differ in their central frequencies. We note that the central frequencies of the signal and idler photons are the same $(\omega_0^s = \omega_0^i)$ for a given angle $\theta_s$ of emission. A typical shape of the two-photon spectral amplitude $\phi(\omega_s, \omega_i)$ resembling a cross in its contour plot (see Fig. 3) reflects the fact that the signal and idler fields are nearly perfectly separable. This is confirmed by Schmidt decomposition of the amplitude $\phi(\omega_s, \omega_i)$ in which only the first mode is important ($\lambda_1 = 1.00$). Entropy $S$ of entanglement for the two-photon spectral amplitude $\phi$ in Fig. 3 equals 0.00 and cooperativity parameter $K$ is 1.00. The spectral dependence of the first three mode functions in the decomposition is shown in Fig. 4.

A typical temporal two-photon amplitude $A(t_s, t_i)$ spreads over tens or hundreds of ps reflecting narrow frequency spectra of the signal and idler fields. Its contour plot resembles a droplet [59] that originates in the zig-zag movement of the emitted photons inside the structure that delays the occurrence time of photons at the output plane of the structure. Both photons have the same wave-packets due to identical emission conditions as can be verified in Hong-Ou-Mandel interferometer showing the visibility equal to one.
Assuming the normally incident pump beam the signal-field intensity profile at a given frequency in the transverse plane is nonzero around a circle due to the rotational symmetry of the photonic-band-gap structure around the pump-beam propagation direction. This symmetry assumes an appropriate rotation of polarization base vectors. However, values of intensity change around the circle depending on fields’ polarizations (defined by analyzers in front of detectors) and properties of nonlinear tensor \( \mathbf{d} \).

Correlations between the signal and idler fields in the transverse plane are characterized by a correlation area that is defined by the probability of emitting an idler photon in a given direction [defined by angular declinations \( \Delta \theta \) (radial direction) and \( \Delta \psi \) (azimuthal direction) from the ideal direction of emission] provided that the direction of signal-photon emission is fixed. Correlation area is in general an ellipse with typical lengths in radial \((\sigma \Delta \theta)\) and azimuthal \((\sigma \Delta \psi)\) directions. These angles are very small for plane-wave pumping, typically of the order of \(10^{-3} - 10^{-4}\) rad. However, focusing of the pump beam can increase their values considerably as shown in Fig. 5 where the pump-beam diameter \(a\) varies from 30 \(\mu\)m up to 1 mm. Spread of the correlation area in radial direction is smaller compared to that in azimuthal direction, because emission of a photon is more restricted in radial direction (for geometry reasons) by narrow bands of the photonic structure. Release of strict phase-matching conditions in the transverse plane caused by a focused pump beam (with a circular spot) affects radial and azimuthal angles of emission in the same way.

We note that spreading of the signal- and idler-field intensity profiles caused by pump-beam focusing has been experimentally observed in [51] for a type-II bulk crystal.

V. TWO-PHOTON STATES COINCIDENT IN FREQUENCIES

Superposition of photon pairs with signal photons emitted under different radial emission angles \(\theta_s\) results in states coincident in frequencies. This spectral beam combining of fields from different spatial modes and with different spectral compositions can be achieved using an optical dispersion element like a grating [52]. This technique has already been applied in the construction of a source of photon pairs that uses achromatic phase matching and spatial decomposition of the pump beam [20,21].

There is approximately a linear dependence between the central frequencies \(\omega_0 = \omega_0^s\) and radial angle \(\theta_s\) of signal-field emission for sufficiently large values of \(\theta_s\) assuming a normally incident pump beam. The resultant two-photon spectral amplitude \(\Phi_M(\omega_s, \omega_i)\) after superposing photon pairs from \(M\) equidistantly positioned pinholes present both in the signal and idler beams can be approximately expressed as:

\[
\Phi_M(\omega_s, \omega_i) = \sum_{n=0}^{M-1} \exp(i\varphi n)\phi(\omega_s + n\Delta \omega, \omega_i + n\Delta \omega),
\]

(16)

where \(\Delta \omega\) is the difference between the signal-field central frequencies of fields originating in adjacent pinholes. Phase \(\varphi\) determines the difference in phases of two-photon spectral amplitudes at the central signal- and idler-field frequencies coming from adjacent pinholes.

The two-photon spectral amplitude \(\phi(\omega_s, \omega_i)\) in Eq. (16) belongs to a two-photon state coming from the first pinhole. Schmidt decomposition of the two-photon spectral amplitude \(\Phi_M\) describing photon pairs coming from \(M\) pinholes shows that there are nearly \(M\) independent modes [determined by the value of cooperativity parameter \(K\) introduced in Eq. (5)]. These modes are collective modes, i.e. they have non-negligible values in areas of frequencies associated with every pinhole (see Figs. 6 and 7 for \(M = 2\) pinholes). Such states are perspective for the implementation of various quantum-information protocols.

The Fourier transform \(\Phi_M(t_s, t_i)\) defined in Eq. (8) can be approximately expressed as follows:

\[
\Phi_M(t_s, t_i) \approx \sum_{n=0}^{M-1} \exp[i(\varphi - \Delta \omega(t_s + t_i))n]\phi(t_s, t_i),
\]

(17)

where

\[
\phi(t_s, t_i) = \frac{1}{2\pi} \int_{0}^{\infty} d\omega_s \int_{0}^{\infty} d\omega_i \phi(\omega_s, \omega_i) \exp(i\omega_s t_s) \times \exp(i\omega_i t_i)
\]

(18)

means the Fourier transform of a two-photon amplitude associated with one pinhole. The expression in Eq. (17) indicates that interference of photon-pair amplitudes originating in different pinholes creates interference fringes in the sum \(t_s + t_i\) of the occurrence times of signal

![Figure 5](image-url)

**FIG. 5:** Spread of the correlation area in radial (\(\sigma \Delta \theta\), solid line) and azimuthal (\(\sigma \Delta \psi\), solid line with *) directions of idler-photon emission as a function of pump-beam diameter \(a\). The transmission peak in the structure with 250 elementary layers occurs at \(\theta_s^0 = 26.3\) deg, i.e. the central radial idler-photon emission angle \(\theta_s^i\) equals -26.3 deg. Logarithmic scale on the \(x\) axis is used.
and idler photons. This is caused by positive correlations in the signal- and idler-field frequencies. The analysis of the sum in Eq. (17) shows that the greater the number $M$ of pinholes, the narrower the range of allowed values of the sum $t_s + t_i$ $\lim_{M \to \infty} \sum_{n=0}^{M-1} \exp(i\pi n) \approx \delta(x)$. Comparison of shapes of the two-photon temporal amplitudes $A_M(t_s, t_i)$ derived in Eq. (10) for $M = 2$ and $M = 8$ pinholes shown in Fig. 8 reveals this tendency for localization in time domain. These features originating in positive correlations of the signal- and idler-field frequencies are experimentally accessible by measuring the pattern of coincidence-count rate $R^{c}_{\text{F}}$ in Franson interferometer for sufficiently large values of signal- and idler-photon delays $\tau_s$ and $\tau_i$. A (nearly separable) two-photon state from one pinhole creates a chessboard tilted by 45 degrees (see Fig. 9a). If amplitudes from several pinholes are included typical fringes oriented at 45 degrees become visible in coincidence-count patterns given by rate $R^{c}_\text{F}$ (compare Figs. 9a and 9b). The greater the number $M$ of pinholes, the better the fringes are formed.

Superposition of photon-pair amplitudes (with a suitable phase compensation) from a given range of signal-field emission angles $\theta_s$ (and the corresponding range of idler-field emission angles $\theta_i$) as defined by rectangular apertures gives a two-photon spectral amplitude $\Phi_M$ for $M = 2$ is plotted in Fig. 6. Positions of the edge pinholes for $M = 8$ coincide with those for $M = 2$ and the remaining six pinholes are equidistantly distributed in between them. Normalization is such that $\int dt_s \int dt_i |A_M(t_s, t_i)|^2 = 1$.
larger the spectral width of the signal and idler fields the larger the cooperativity parameter $K$. This means that the number of effective independent modes can be easily changed just by changing the height of rectangular apertures. This makes this source of photon pairs extraordinarily useful. Both emitted photons are perfectly indistinguishable providing visibility equal to one in Hong-Ou-Mandel interferometer. Positive correlation in the signal- and idler-field frequencies gives coincidence-count interference fringes in Franson interferometer tilted by 45 degrees for sufficiently large values of the signal- and idler-field delays $\tau_s$ and $\tau_i$.

VI. SPECTRALLY NON-DEGENERATE EMISSION OF PHOTON PAIRS

The simplest way for the observation of spectrally non-degenerate emission of a photon pair is to consider collinear geometry and exploit a random structure with two different transmission peaks. Pumping frequency is then given as the sum of central frequencies of these transmission peaks into which signal and idler photons are emitted. The generation of a suitable structure using the algorithm and geometry presented in Sec. III in this case is by an order of magnitude more difficult compared to that providing photon pairs degenerated in frequencies. Structures having two peaks with considerably different bandwidths of the transmission peaks are especially interesting. Spectral bandwidths of the signal and idler fields differ accordingly. Such states are interesting in some applications, e.g., in constructing heralded single-photon sources.

The two-photon spectral amplitude $\phi(\omega_s, \omega_i)$ has a cigar shape prolonged along the frequency of the field with a larger spectral bandwidth (say the signal field). On the other hand, the two-photon temporal amplitude $A(t_s, t_i)$ is broken into several islands along the axis giv-
FIG. 12: Photon fluxes of the signal ($N_s$, solid line) and idler ($N_i$, solid line with *) fields for a structure with $N_{\text{elem}} = 250$ layers having two different transmission peaks in collinear geometry. The ratio of their intensity bandwidths equals cca 4. The structure is pumped by a pulse 250 fs long.

FIG. 13: Coincidence-count rate $R_{\text{HOM}}$ in Hong-Ou-Mandel interferometer as a function of relative time delay $\tau_l$ for the two-photon state used in Fig. 12. 

ing the idler-photon detection time $t_i$. Phase modulation of the two-photon spectral amplitude $\phi(\omega_s, \omega_i)$ with faster changes along the frequency $\omega_i$ and slower changes along the frequency $\omega_s$ is responsible for this behavior. Photon fluxes of the signal and idler fields are then composed of several peaks as documented in Fig. 12. This feature is reflected in an asymmetric dip in coincidence-count rate $R_{\text{HOM}}$ in Hong-Ou-Mandel interferometer, that also shows oscillations at the difference of the central signal- and idler-field frequencies. Asymmetry of the interference dip shown in Fig. 12 can be related to the shape of spectral two-photon amplitude $\phi$. Origin of these effects lies in delays caused by the zig-zag movement of the generated photons inside the structure.

VII. CONCLUSIONS

Nonlinear random layered structures in which an optical analog of Anderson localization occurs have been analyzed as a suitable source of photon pairs with narrow spectral bandwidths and perfect indistinguishability. Spectral bandwidths in the range from 1 nm to 0.01 nm are available for different realizations of a random structure. Photon pairs with the same signal- and idler-field bandwidths as well as with considerably different bandwidths can be generated. Random structures are flexible as for the generated frequencies and emission angles. Two-photon states with coincident frequencies and variable spectral bandwidth can be reached if two-photon amplitudes of photon pairs generated into different emission angles are superposed. Also two-photon states with signal- and idler-field spectra composed of several peaks and characterized by collective spectral mode functions are available. All these states are very perspective for optical implementations of many quantum information protocols. Possible implementation of these sources into integrated optoelectronic circuits is a great advantage.

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