Interference of guiding polariton mode in “traffic” circle waveguides composed of dielectric spherical particles

I. Ya. Polishchuk

Department of Chemistry, Tulane University, New Orleans, LA 70118, USA
C Kurchatov Institute, Kurchatov Sq., 1, 123182 Moscow, Russia
Max-Planck-Institut für Physik Komplexer Systeme, D-01187

M. I. Gozman

Department of Chemistry, Tulane University,
New Orleans, LA 70118, USA and
C Kurchatov Institute, Kurchatov Sq., 1, 123182 Moscow, Russia

G. S. Blaustein and A. L. Burin

Department of Chemistry, Tulane University, New Orleans, LA 70118, USA

Abstract

The interference of polariton guiding modes propagating through ”traffic circle” waveguides composed of dielectric spherical particles is investigated. The dependence of intensity of the wave on the position of the particle was studied using the multisphere the Mie scattering formalism. We show that if the frequency of light belongs to the passband of the waveguide, electromagnetic waves may be considered as two optical beams running along a circle in opposite directions and interfering with each other. Indeed, the obtained intensity behavior can be represented as a simple superposition of two waves propagating around a circle in opposite directions. The applications of this interference are discussed.

PACS numbers:
I. INTRODUCTION

Low-dimensional aggregates of nanoparticles absorbing, scattering and guiding electromagnetic waves are attracting growing interest because they can be used in various micro- and nano-systems where optical energy is received, transferred and converted on a subwavelength scale \[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\]. If the material’s refractive index is sufficiently large, then these aggregates can possess long-lived quasistates, which are polariton modes representing the superposition of light and polarization oscillation of the material \[9, 12\]. These modes propagate within the aggregates, while their losses due to light emission are very small. Consequently, low-dimensional aggregates of particles can be used as waveguides for optical energy transport. This transferred energy can stimulate a number of processes such as the photoexcitation of a molecule or a chemical reaction \[7, 8\].

For example, if we excite the polariton mode within a linear chain of particles using a point harmonic source located near the end of the chain, then there exists a frequency domain, i.e., the passband, where almost all of the energy from the source will be guided by the chain \[14\]. Other modes in the related frequency domain can be treated as bound or guiding modes. To our knowledge, the first nano-waveguide capable of transferring optical energy a distance of 100 nm was constructed by Atwater and coworkers \[7, 8\] as a linear chain of spherical silver or gold particles. The use of metallic particles has an advantage in that the losses due to photon emission can be almost completely suppressed \[12\], yet the efficiency of energy transfer is suppressed due to optical energy absorption by free electrons.

Guiding systems of dielectric particles have negligible absorption losses, while radiative losses can also be avoided as dielectric materials have characteristically larger refractive indices with \(n_r > 2\) \[9, 12\]. The finite size of the guiding energy band in periodic arrays of particles results in the the formation of slow light modes at the band edge \[12\]. Therefore, it may be more convenient to use dielectric materials having very weak light absorption \[9, 10, 11, 12\]. Frequency pass bands of linear chains were investigated \[11, 12, 13, 14\] and it was found that the top of the frequency pass band corresponds to the top of the Brillouin band edge in quasi-momentum space, while the bottom of the frequency pass band is determined by the guiding mode criterion (see Eq. (14) below) \[9, 12\]. It was discovered that the closer the frequency of electromagnetic waves emitted by the source to the upper boundary of the pass band and the longer the chain, the more effective the waveguide and the lower the
amount energy is dissipated into the surroundings. It was shown that for \( N \to +\infty \), the rate of decay of electromagnetic waves decreases as \( \gamma(N) \sim N^{-3} \) [10, 11, 12].

Another example of a low-dimensional aggregate of particles possessing wave-guiding properties is the circular array of spherical particles. Because of its symmetry, one can expect that the highest quality factor can be attained for particles arranged in a circle. A circle has no sharp ends in contrast with a particle chain where the lifespan of polariton is limited to the time it takes to travel from one end of the chain to the other [10]. It has been demonstrated [9] that the bound modes in a circular array of spherical particles possess a quality factor that grows exponentially with the number of particles, just as in the circular arrays of cylinder-shaped antennas ([15, 16, 17, 18]).

As previously mentioned, waveguides comprised of spherical particles can be used to manipulate optical energy on a subwavelength scale. In addition, modes at the edge of the passband possess a vanishing group velocity which creates an interesting opportunity to realize the phenomenon of slow light [12]. The properties of polariton modes within passbands are quite similar to those of electrons within a wire. The question of interest is whether we can exploit several properties known about electron behavior in a wire for wave-guiding systems. In this manuscript, we investigate the effect of interference on the transport of optical energy. The geometry of the waveguide shown in Figs. 1, 2, 3 is similar to the one illustrating the Aharonov-Bohm effect in electronic systems. The sample geometry can be described as a "traffic circle" without an entrance or exit (Fig. 1) or with them (Figs. 2, 3).

The polariton is excited by a point source located near one of the aggregate particles as shown in Figs. 1, 2, 3. Practically, this source can be created by placing a dye molecule within an end particle (particle A) [1]. When the frequency of the source belongs to the passband, almost all of the optical energy generated by the source is transferred through the first chain [14]. After traveling through the first chain, an electromagnetic wave enters the circle at the junction between the circle and the first chain (particle B). It then splits into two beams, one of them propagating around the circle clockwise and the other counterclockwise. These two beams rejoin at the junction between the circle and the second chain (particle C) and travel through the second chain to particle D.

Actually, the beam propagating along the circle clockwise from particle B to particle C may pass through particle C from one segment of the circle to the other segment and continue traveling from particle C back to particle B. Analogously, the other beam propagating along
the circle counterclockwise from particle B to particle C also may pass through particle C and continue traveling back to particle B in the other segment of the circle. This recirculation in both the clockwise and counterclockwise directions remarkably enhance interference effects. Therefore, we investigate the interference of waves having frequencies corresponding to resonances of circular arrays where this enhancement is maximized.

To describe our expectations about the interference effect, we can suggest the following simple scenario perfectly that is also applicable to an electronic system. Each of two beams of light may be expressed as a wave $\exp \{ikx\}$ if the beam travels clockwise and $\exp \{ik(L - x)\}$ if the beam travels counterclockwise. Here, $k$ is the wave vector of the beam and it is defined by the frequency of the source, $x$ is the coordinate of a point on the circle (we suppose that for point B $x = 0$) and $L$ is the circumference of the circle. So, if these two beams interfere, the intensity of the field is given by the superposition of these two waves:

$$\exp \{ikx\} + \exp \{ik(L - x)\}.$$  \hfill (1)

FIG. 1: Traffic circle without chains composed of spherical particles.
This interference may have a number of interesting applications including, for instance, filtering and selective enhancement of a guiding signal. It is not clear though whether this simple interference model Eq. (1) is applicable to optical modes because of the strong spatial dispersion and essential non-local coupling of spherical particles composing the waveguide. In this manuscript, we show that despite the above-mentioned problems, our simple interference model Eq. (1) works quite well.

To ascertain that interference of polaritons in a circular array takes place, we have provided numerical investigations of a circle with and without attached chains. We have studied three particular cases: a circle without chains, a circle with chains situated diametrically opposed to each other and a circle with chains situated orthogonally to each other. For each of these three cases, we have investigated two situations: the presence and the absence of absorption.

The system under consideration is a traffic circle composed of \( N_C = 80 \) identical dielectric spherical particles (see Figs. 1, 2, 3). The radii of particles were chosen to be unity. The

![Traffic circle with collinear chains composed of spherical particles](image)

**FIG. 2:** Traffic circle with collinear chains composed of spherical particles
chains (linear arrays externally joined to the circle) are composed of same particles. The number of particles in these chains is \( N_1 = N_2 = 20 \). The angle between the chains is denoted \( \alpha \). If the chains are collinear, then one has \( \alpha = 0 \) Fig. 2 and if the chains are orthogonal to each other then one has \( \alpha = \pi/2 \) Fig. 3.

We introduce an oscillating magnetic dipole oriented orthogonally to the system plane as the source of electromagnetic waves. A magnetic dipole is chosen because it corresponds to the guiding band with the lowest frequency and, consequently, the best guiding properties [9]. Indeed, as shown in the mentioned work, the lowest frequency guiding mode originates from magnetic-like Mie resonance. If the system under consideration is a circle with chains, the source is situated near the end of one of the chains, as shown in Figs. 2, 3. If the circle without attached chains is considered Fig. 1 the source is situated outside of the circle near the first particle. In this manuscript, we consider GaAs particles having a refractive index \( n_r = 3.5 \). GaAs is a convenient to consider since it possesses one of the highest refractive indices amongst conventional optical materials and, consequently, it has better

![Diagram](image_url)

**FIG. 3:** Traffic circle with perpendicular chains composed of spherical particles
guiding properties (cf. [9]). The absorption effect was also probed and we discuss it when reporting the results of our calculations.

All calculations were performed using the multisphere Mie scattering formalism ([19, 20]). This formalism is introduced in Sec. II where we also discuss the calculations of the polariton dispersion needed to determine the resonant modes of the traffic circle. This formalism is used in Sec. III to investigate the interference of polaritons in different geometries. We investigate the intensity of the polariton around the circle in arrays shown in Figs. 1, 2, 3 as well as the intensity transferred through the waveguide with chains at arbitrary angles. The obtained dependence was compared with Eq. (I) representing the sum of two optical beams to verify if the dependence may be approximated with this function.

The dependence of the intensity of an electromagnetic wave scattered by the a spherical particle in the circle on an arbitrary point on the surface of this particle is demonstrated in Figs. 4, 5, 6, 7, 8. It was found that in all investigated cases the dependence may be approximated by Eq. (I).

In Sec. IV the results are summarized and their applications to various systems of interest are discussed.

II. MULTISPERE MIE SCATTERING FORMALISM.

A. General formalism

The multisphere Mie scattering formalism ([19, 20] (see also [9, 10, 11, 12, 13, 14]) has been developed to study the scattering of electromagnetic waves in aggregates of spherical dielectric particles. The electromagnetic field outside each particle may be represented as the sum of all incident waves falling on this particle (say, the $j$-th particle) and of diverging waves scattered by this particle: $E(r) = E_{inc}^j(r) + E_{sca}^j(r)$, $H(r) = H_{inc}^j(r) + H_{sca}^j(r)$. The incident wave $E_{inc}^j(r)$, $H_{inc}^j(r)$ may be expressed as a linear combination of spherical vector harmonics $N_{mn}^{(1)}(r)$, $M_{mn}^{(1)}(r)$ taken with respect to the center of $j^{th}$ particle

$$E_{inc}^j(r) = \sum_{n=1}^{+\infty} \sum_{m=-n}^{n} \left( p_{mn}^j N_{mn}^{(1)}(r - r_j) + q_{mn}^j M_{mn}^{(1)}(r - r_j) \right), \tag{2}$$

$$H_{inc}^j(r) = \sum_{n=1}^{+\infty} \sum_{m=-n}^{n} \left( q_{mn}^j N_{mn}^{(1)}(r - r_j) + p_{mn}^j M_{mn}^{(1)}(r - r_j) \right). \tag{3}$$
Analogously, the scattered wave $E_{\text{sca}}^j (r)$, $H_{\text{sca}}^j (r)$ is a linear combination of spherical vector harmonics $N_{mn}^{(3)} (r)$, $M_{mn}^{(3)} (r)$ corresponding to outgoing waves

$$E_{\text{sca}}^j (r) = \sum_{n=1}^{+\infty} \sum_{m=-n}^{n} i \left( a_{mn}^j N_{mn}^{(3)} (r - r_j) + b_{mn}^j M_{mn}^{(3)} (r - r_j) \right),$$

$$H_{\text{sca}}^j (r) = \sum_{n=1}^{+\infty} \sum_{m=-n}^{n} \left( b_{mn}^j N_{mn}^{(3)} (r - r_j) + a_{mn}^j M_{mn}^{(3)} (r - r_j) \right).$$

Here $r_j$ is the coordinate of center of the $j^{th}$ particle; indices $j = 1, \ldots, N$ enumerates all particles, $N$ is the total number of particles in the array; index $n = 1, \ldots, +\infty$ represents angular momenta of spherical vector harmonics, and index $m = -n, \ldots, n$ stands for the projection of angular momentum on the z-axis. The coefficients $a_{mn}^j$, $b_{mn}^j$, $p_{mn}^j$, $q_{mn}^j$ are partial amplitudes of scattered and incoming spherical waves at the $j^{th}$ sphere.

The incident wave falling on the $j$-th particle is the sum of the external waves emitted by the source and the waves scattered by all other particles. Because of the boundary conditions at each sphere, amplitudes of these waves obey the relationship

$$a_{mn}^j + \sum_{l=1 (l \neq j)}^{N} \sum_{\mu=1}^{+\infty} \sum_{\nu=-\mu}^{\mu} \left( A_{mn\mu\nu}^{lj} a_{\mu\nu}^l + B_{mn\mu\nu}^{lj} b_{\mu\nu}^l \right) = p_{mn}^j,$$

$$b_{mn}^j + \sum_{l=1 (l \neq j)}^{N} \sum_{\mu=1}^{+\infty} \sum_{\nu=-\mu}^{\mu} \left( A_{mn\mu\nu}^{lj} b_{\mu\nu}^l + B_{mn\mu\nu}^{lj} a_{\mu\nu}^l \right) = q_{mn}^j,$$

where $a_n$ and $b_n$ are Mie scattering coefficients depending on the size and the refraction index of particles. They originate from boundary conditions for an electric field between particles and air. Parameters $A_{mn\mu\nu}^{lj}$ and $B_{mn\mu\nu}^{lj}$ are vector translation coefficients that depend on the relative position of spheres $j$ and $l$. These coefficients express delayed multipole interactions of different spheres in the frequency domain [9]. These translation coefficients can be used to represent the outgoing wave scattered by the $l^{th}$ particle as a sum of spherical vector harmonics $N_{mn}^{(1)} (r)$, $M_{mn}^{(1)} (r)$ taken with respect to the $j^{th}$ particle as

$$M_{\mu\nu}^{(3)} (r - r_j) = \sum_{n=1}^{+\infty} \sum_{m=-n}^{n} \left( A_{mn\mu\nu}^{lj} M_{mn}^{(3)} (r - r_j) + B_{mn\mu\nu}^{lj} N_{mn}^{(3)} (r - r_j) \right),$$

$$N_{\mu\nu}^{(3)} (r - r_j) = \sum_{n=1}^{+\infty} \sum_{m=-n}^{n} \left( B_{mn\mu\nu}^{lj} M_{mn}^{(3)} (r - r_j) + A_{mn\mu\nu}^{lj} N_{mn}^{(3)} (r - r_j) \right).$$

To complete the setup of the problem Eq. [9], one has to determine the coefficients $p_{mn}^j$ and $q_{mn}^j$ for the incoming wave of the point magnetic dipole source having the coordinate $r_0$.
This external wave can be expressed as a linear combination of spherical vector harmonics \(N^{(3)}_{mn}(r), M^{(3)}_{mn}(r)\)

\[
E_{extr}(r) = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} i \left( P_{mn}N^{(3)}_{mn}(r - r_0) + Q_{mn}M^{(3)}_{mn}(r - r_0) \right),
\]

\[
H_{extr}(r) = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \left( Q_{mn}N^{(3)}_{mn}(r - r_0) + P_{mn}M^{(3)}_{mn}(r - r_0) \right).
\]

Here \(P_{mn}\) and \(Q_{mn}\) are partial amplitudes of an emitted wave characterizing the point source. As mentioned above, we chose the source to be the magnetic dipole parallel to the \(z\) direction and perpendicular to the \(x\)−\(y\) sample plane. Then the wave emitted by the source can be described by the parameters \(P_{mn}\) and \(Q_{mn}\) defined as \(P_{-11} = P_{01} = P_{11} = 0, Q_{-11} = Q_{11} = 0, \) and \(Q_{01} = 1.\)

The coefficients \(p^i_{mn}\) and \(q^i_{mn}\) are associated with coefficients \(P_{mn}\) and \(Q_{mn}\) by means of the relations

\[
p^i_{mn} = \sum_{\nu=1}^{\infty} \sum_{\mu=-\nu}^{\nu} \left( A^i_{mn\mu\nu}P_{\mu\nu} + B^i_{mn\mu\nu}Q_{\mu\nu} \right),
\]

\[
q^i_{mn} = \sum_{\nu=1}^{\infty} \sum_{\mu=-\nu}^{\nu} \left( B^i_{mn\mu\nu}P_{\mu\nu} + A^i_{mn\mu\nu}Q_{\mu\nu} \right),
\]

where vector translation coefficients are taken between the source and the center of the corresponding sphere. Thus, using Eqs. (11) and (12) one can calculate the right hand side of Eq. (6).

Eq. (6) has the form of a system of linear equations, with partial amplitudes \(a^j_{mn}\) and \(b^j_{mn}\) yet to be determined. Our main task is to calculate these partial amplitudes and to investigate their dependence on the number of spheres.

In this work, the dipolar approximation is used which is sufficiently accurate to characterize the system [9, 10, 12]. Using this approximation, Eq. (6) can be expressed as

\[
a_{m1}^j + \sum_{l=1}^{N} \sum_{\mu=1}^{1} \left( A_{m1\mu1}^j a_{\mu1}^l + B_{m1\mu1}^j b_{\mu1}^l \right) = B_{m1}^0, \]

\[
b_{m1}^j + \sum_{l=1}^{N} \sum_{\mu=1}^{1} \left( A_{m1\mu1}^j b_{\mu1}^l + B_{m1\mu1}^j a_{\mu1}^l \right) = A_{m1}^0. \]

The numerical solution of these equations is used later in Sec. III to investigate interference of waves in the circular traffic waveguide Figs. [11, 12, 13].
B. Determination of the Dispersion Law

Partial amplitudes $a^j_{mn}$ and $b^j_{mn}$ characterize interference of two polaritons running clockwise and counterclockwise. The parameter $k$ entering into Eq.(1) is the quasi-wave vector of a polariton and it is determined by the frequency $\omega$ of the wave emitted by the source. The relationship between $\omega$ and $k$ is the wave dispersion law which has to be determined for the analysis of our simulation data. Below we briefly described how we did that.

If the radius of the circle is large enough as in our case (a circle containing $N = 80$ particles), one can use the dispersion law for an infinite linear chain. Any frequency $\omega$ lying in the conducting band of an infinite linear chain corresponds to a certain value of the quasi-wave vector $k$.

The frequency domain of bound modes, or the pass band, is the domain of frequencies where losses due to photon emission are completely suppressed in the limit $N \to \infty$. This frequency domain exists where photon emission is forbidden by the momentum conservation law. To illustrate, consider a polariton within an infinite chain of particles parallel to the $z$-axis. According to the light dispersion law, we can define the wave vector of light in vacuum as $q = \omega c$, where $c$ is the speed of light. If an optical mode can exist outside of the chain as a free photon it still has the fixed projection of the wave vector to the $z$-axis given by $k$ so we should have $q_z = k$ due to translational symmetry. Then the perpendicular component of the electric field wave vector that describes the photon emission outside of the system can be expressed as $q_\perp = \sqrt{q^2 - k^2}$. For unbound modes, this projection is real, meaning that the optical energy can escape from the chain of particles, while for bound modes it is imaginary so that the mode intensity decays at distance $\rho$ from the chain as $e^{-\rho\sqrt{k^2-q^2}}$.

Thus modes with $\omega/c < k$ are bound, while the modes with $\omega/c > k$ are unbound. Besides that, according to the Bloch theorem the absolute value of $k$ cannot exceed the value $\pi/a$, where $a$ is the period of the chain. Therefore, bound modes can possess quasi-wave vectors $k$ from the interval

$$\frac{\omega}{c} < k < \frac{\pi}{a}.$$  \hspace{1cm} (14)

Certainly, this condition may be satisfied only if $\omega/c < \pi/a$. This guiding mode criterion is equivalent to the condition that the interparticle distance $a$ should be less than half the resonant wavelength: $a < \lambda/2$ [21].

To find the dispersion law for a polariton within an infinite chain [2, 3, 4, 5] one has
to calculate the eigenvalues of the frequency $\omega$ for the given quasi-wave vector $k$. The
eigenfrequencies can be computed using the condition of the existence of nontrivial solutions
of a homogeneous system of equations

$$\begin{align*}
\sum_{l=1(l\neq j)}^{N} \sum_{\mu=-\nu}^{\nu} \left( A_{mn\mu\nu}^{l} a_{\mu\nu}^{l} + B_{mn\mu\nu}^{l} b_{\mu\nu}^{l} \right) &= 0, \\
\sum_{l=1(l\neq j)}^{N} \sum_{\mu=-\nu}^{\nu} \left( A_{mn\mu\nu}^{l} b_{\mu\nu}^{l} + B_{mn\mu\nu}^{l} a_{\mu\nu}^{l} \right) &= 0.
\end{align*}$$

(15)

According to the Bloch’s theorem, the partial amplitude in an infinite periodic chain can be
represented by

$$\begin{align*}
a_{mn}^{j} &= a_{mn} e^{ikx}, \\
b_{mn}^{j} &= b_{mn} e^{ikx},
\end{align*}$$

(16)

where $k$ is the quasi-wave vector and $x = aj$ is the coordinate of the center of $j^{th}$ particle
in the chain (remember, we set $a = 1$).

Substituting Eq. (16) into Eq. (15), we get (cf. Refs. [3, 9])

$$\begin{align*}
\sum_{\nu=m}^{+\infty} \sum_{\nu=1}^{\infty} \left( A_{mn\mu\nu}^{l} a_{\mu\nu}^{l} + B_{mn\mu\nu}^{l} b_{\mu\nu}^{l} \right) &= 0, \\
\sum_{\nu=1}^{+\infty} \sum_{\nu=1}^{\infty} \left( A_{mn\mu\nu}^{l} b_{\mu\nu}^{l} + B_{mn\mu\nu}^{l} a_{\mu\nu}^{l} \right) &= 0.
\end{align*}$$

(17)

We used the momentum projection $m$ conservation law due to the rotational invariance of
the system.

In the dipolar approximation, Eq.(17) takes the form

$$\begin{align*}
\sum_{\nu=1}^{+\infty} \left( A_{mn\mu\nu}^{l} a_{\mu\nu}^{l} + B_{mn\mu\nu}^{l} b_{\mu\nu}^{l} \right) &= 0, \\
\sum_{\nu=1}^{+\infty} \left( A_{mn\mu\nu}^{l} b_{\mu\nu}^{l} + B_{mn\mu\nu}^{l} a_{\mu\nu}^{l} \right) &= 0.
\end{align*}$$

(18)

If $m = \pm 1$, the modes are called $t$-modes (meaning transverse modes with polarization
perpendicular to the chain), and if $m = 0$ the modes are called $l$-modes (meaning longitudinal
modes with polarization parallel to the chain). In this manuscript we are interested in $t$-modes.

As was discussed above, the source of electromagnetic waves in our problem of interest is
a magnetic dipole directed perpendicular to the plane of the circle. This source excites only
transverse electromagnetic waves. Therefore we are interested in equations for $m = 1$

$$\begin{align*}
\sum_{\nu=1}^{+\infty} \left( A_{mn\mu\nu}^{l} a_{\mu\nu}^{l} + B_{mn\mu\nu}^{l} b_{\mu\nu}^{l} \right) &= 0, \\
\sum_{\nu=1}^{+\infty} \left( A_{mn\mu\nu}^{l} b_{\mu\nu}^{l} + B_{mn\mu\nu}^{l} a_{\mu\nu}^{l} \right) &= 0.
\end{align*}$$

(19)
This system contains only two variables $a_{11}$ and $b_{11}$ and the dispersion law is defined by the following condition

$$
\left( \frac{1}{a_{1}(\omega)} + A_{1111}(\omega, k) \right) \left( \frac{1}{a_{2}(\omega)} + A_{1111}(\omega, k) \right) - (B_{1111}(\omega, k))^2 = 0. \tag{20}
$$

Eq. (20) can be solved only numerically. We used the Newton-Raphson method to find quasiwavevector for certain frequencies of interest following Ref. [9].

III. RESULTS

In this work three arrays of particles were considered, including the circle of particles without attached chains Fig. 1 and two structures with chains Figs. 2 and 3. Each of these structures is investigated for particles of ideal non-absorbing material. Our preliminary study of wave transmission through a traffic circle at various frequencies has demonstrated that it is most efficient for modes corresponding to resonances of a circular array (see Ref. [9]). These resonances takes place at frequencies corresponding to integer projections $q = -N/2, ..., N/2$ of a polariton’s angular momentum to the $z$-axis. These modes can be characterized by quantized quasi-wavevectors $k = 2\pi q/L$. The upper boundary of the pass band corresponds to $q = N/2$, i.e. $k = \pi$. For the circular array under consideration this upper boundary is given by $\omega = 0.8434$.

In this work we investigate a circular array composed by $N = 80$ particles. To characterize the interference, we chose two modes having angular momenta $q_1 = 38$ and $q_2 = 37$ and frequencies $\omega_1 = 0.8424$ and $\omega_2 = 0.8413$, respectively, located at the top of the passband ($q = 40$). Interference behavior is quite similar for other modes with $q \geq 30$. The mode at the top of the band characterized by $q = 40$ does not show any interference because the wave intensity does not change from sphere to sphere; though, the field changes its sign between neighboring particles.

The intensity of a polariton mode can be characterized by squared partial amplitudes $a_{mn}^j$ and $b_{mn}^j$. Since we are using the dipolar approximation, only coefficients $a_{mn}^j$ and $b_{mn}^j$ for $n = 1$ should be considered. The partial amplitudes $a_{01}^j$, $b_{-11}^j$ and $b_{11}^j$ are all equal to zero because of the planar structure of our system. Therefore, the intensity of a polariton within the $j$-th particle is characterized by three coefficients $a_{-11}^j$, $a_{11}^j$ and $b_{01}^j$. For the sake of simplicity, we define it as $I_j = | b_{01}^j |^2$. Using the multisphere Mie scattering formalism
we investigated the dependence of intensity of the wave on the \( j \)-th particle on the position of the particle. The position of the particle is defined by its number \( j \) and may be described as the length of the segment of the circle between the first particle and the \( j \)-th particle counted counterclockwise: \( x_j = (j - 1) \). The intensity of the wave should be compared with the one following from Eq. (11)

\[
I_j \propto |\exp\{ikx\} + \exp\{ik(L - x)\}|^2 / 4 = \cos^2(\pi q - kx) = \cos^2(2\pi qx/N). \tag{21}
\]

Since for guiding modes of interest one has \( q \approx N/2 \), this function strongly oscillates for non-integer \( x \) and is difficult to show in our graphs. Therefore, we compare it directly to our simulations in Fig. 4 only, while in all other graphs it is replaced with the function

\[
I_j \propto \cos^2(2\pi(N/2 - q)x/N), \tag{22}
\]

which coincides with Eq. (21) for all integer points \( x \) of interest. For chosen modes with \( q = 38 \) and \( q = 37 \), this function can be expressed as \( \cos^2(2 \cdot 2\pi x/40) \) and \( \cos^2(3 \cdot 2\pi x/40) \), respectively.

Below we report our calculations for the electromagnetic field within dielectric spheres for two selected modes for all three configurations shown in Figs. 1, 2, 3 and then investigate the effect of interference on the transmitted intensity.

### A. Circle without attached chains

Consider the intensity behavior in a circle without attached chains (Fig. 1). In Fig. 4 and Fig. 5 we show the results of numerical calculations of the wave intensity \( I_j \) depending on the number of particles for two different resonant source frequencies \( \omega_1 \) and \( \omega_2 \) defined previously. Solid curves on Figs. 1, 5 represent the data fit by functions Eqs. (21) and (22), respectively, with a numerical prefactor chosen as a scaling factor to make the fit intensity identical to the calculated one for \( j = 1 \). According to Figs. 4 and 5 at both frequencies \( \omega_1 \) and \( \omega_2 \), there is a perfect agreement between our calculations and our simple interference model Eqs. (21), (22). Thus guiding modes in a circular array excited by a point source can be treated using the interference of two optical beams propagating along a circle in opposite directions.

We verified that the weak absorption \( \text{Im} n_r < 0.01 \) does not qualitatively affect the interference behavior Eqs. (22), but remarkably reduces the wave intensity. This is because
guiding modes of circular arrays possess extremely high quality factors growing exponentially with the number of particles. Even absorption as weak as $\text{Im } n_r \sim 10^{-5}$ results in remarkable broadening of all resonances.

### B. Traffic circle with attached chains

Consider the traffic circle with attached chains Figs. 2, 3. The circle is composed of $N_C = 80$ particles, and the chains are composed of $N_1 = N_2 = 20$ particles. The source of electromagnetic waves is again a magnetic dipole oscillating normally to the plane of the circle. It is located close to the end of one of the chains (see Fig. 2, 3).

As in the previous case, we investigate the dependence of intensity of a polariton at particle $j$ on the number $j$. The influence of the chains on this dependence is examined. We begin our consideration with the system with the chains situated collinearly as in Fig. 2.

The results of calculation are presented in Fig. 6. Symbols denote the intensity $I_j$.

![High resolution representation of intensity of the wave depending on particle number $j$ within a circle without attached chains](image)

FIG. 4: High resolution representation of intensity of the wave depending on particle number $j$ within a circle without attached chains
calculated numerically, and solid lines show fit using the analytical function Eq. (22) with arguments $x_j = 2(j - 1)$. The squares and the upper solid line correspond to the frequency $\omega_1 = 0.8424$ and the triangles and the lower solid line correspond to the frequency $\omega_2 = 0.8413$.

It is clear that the symbols are located near the solid lines. Therefore, the dependence of intensity $I_j$ can be approximately expressed by the function Eq. (22), similarly to the traffic circle without attached chains. However, the agreement of the numerically calculated intensity with the analytical expression Eq. (22) is less accurate than in the case of the circle without attached chains. This is clear because guiding mode propagation in a circle with chains is much more complicated than in the circle without chains due to scattering at its junctions.

Consider a circle with chains forming right angles with each other. In Fig. 7, the results of numerical calculations of the absolute value of the intensity $I_j$ are shown for the two modes under consideration. In this case, similarly to the case of collinear chains, the dependence of intensity on the position of a particle in the circle may be well approximated by the function

![Graph](image-url)

**FIG. 5:** Intensity of a wave depending on particle number $j$ within a circle without attached chains
Eq. \eqref{22}. Therefore, the approximation of intensity by the superposition of two waves is acceptable for any position of the second chain.

The intensities for a circle without chains and for a circle with chains in the absence of absorption differ by six orders of magnitude. This effect is due to the reduction of the mode quality factor due to polariton scattering by the chains. Indeed, quality factors of resonant modes of circular arrays increase exponentially with the number of particles in the circle. However, quality factors of circles with chains behave similarly to those of finite linear chains where they increase with the number of particles according to the power law \cite{12}. Six orders of magnitude difference in intensity is also due to a difference in quality factors of resonant modes under consideration.

Another interesting phenomenon is that absorption in a circle with chains has a weaker influence on intensity. This is another consequence of the difference in quality factors.

![Intensity graph](image)

**FIG. 6:** Intensity $I_j$ versus particle number $j$ within a circle with collinear chains.
C. Transmitted intensity affected by interference

Traffic circle interference is characterized by its transmitted intensity. This is the intensity at the end of the second chain (particle D - see Figs. 2, 3). We investigate its dependence on junction position, which can be any particle in the circular array. Similarly to Figs. 2, 3 the second chain is placed in a radial direction perpendicular to the circle, while its starting point can any arbitrary particle within the circle. This dependence characterizes the interference effect on the output signal.

We expect that the dependence can still be approximated (at least roughly) by the function Eq. (22) assuming that the intensity at the end of the second chain should be proportional to the intensity of field at its origin. Since the intensity in the $j$-th particle is defined by the interference of polaritons and may be approximated by Eq. (22), one can assume that the intensity at the end of the chain behaves similarly.

In Fig. 8 the calculated dependence of the transmitted intensity $I_j$ on the junction position $j$ is shown. As shown previously, the intensities calculated numerically are marked

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig7.png}
\caption{Intensity $I_j$ versus particle number $j$ within a circle with perpendicular chains}
\end{figure}
by symbols and analytical fits by Eq. (22) are shown using solid lines. One can see that the dependence of partial amplitude \( b_{01} \) at the end of the second chain on the junction position can be approximated by the function Eq. (22). Yet this approximation is rough. For several positions of the junction the error can be as high as 30%. Still this approximation is convenient for a qualitative description of behavior of an electromagnetic field at the end of the second chain. The influence of absorption on the transmitted intensity of field at the end of the second chain is weak similarly to a circle without chains as considered previously.

IV. CONCLUSION

In this work we have investigated the interference of guiding modes in a traffic circle waveguide of dielectric spherical particles using the multisphere Mie scattering formalism at wave frequencies corresponding to two resonant guiding modes. For different sample geometries we demonstrated that the field distribution within the circle as well as the transmitted intensity may be well approximated by interference of two optical beams propagating
clockwise and counterclockwise around the circle.

The main conclusion of this work is that strong interference exists in the waveguides composed of particles and is quite similar to interference as seen in electronic systems. This interference can be used to control the output (transmitted) intensity through varying the system geometry and/or the mode frequency. These results must also be applicable to the subwavelength waveguides consisting of metal particles [7, 8] despite absorption by conducting electrons and can be used to narrow the transferring signal in the frequency domain.

It is difficult to apply our results directly to experiments with whispering gallery modes in arrays of polystyrene particles [1] because the dipolar approach is irrelevant for these modes having large angular momenta. However, interference effects might take place there because coupling of spheres is essentially local [2]. Another interesting application can be developed for the propagation of microwaves in arrays of coupled antennas [3, 15] which are very similar to our particle aggregates. Interference can be used in this application for frequency selective emission, absorption or transport of the microwave signal.

We do not discuss in detail frequency dependence of the transmitted intensity. Our preliminary study shows that this dependence is very complicated because the interference of clockwise and counter clockwise beams is superimposed with circular array resonances. This problem is still under investigation and we do not have a reasonable interpretation for it yet.

This work is supported by the Air Force Office of Scientific Research (Award no. FA 9550-06-1-0110). Authors acknowledge fruitful discussions with Vasilii Asratov, Olga Samoylova, Mark Sulkes and Arthur Yahjian.

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