J=0 T=1 Pairing Interaction Selection Rules

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Abstract

Wave functions arising from a pairing Hamiltonian $E(0)$, i.e., one in which the interaction is only between $J=0^+$ $T=1$ pairs, lead to magnetic dipole and Gamow-Teller transition rates that are much larger than those from an interaction $E(J_{\text{max}})$ in which a proton and a neutron couple to $J=2j$. With realistic interactions the results are in between the two extremes. In the course of this study we found that certain M1 and GT matrix elements vanish with $E(0)$. These are connected to seniority and reduced isospin isospin selection rules. We find the surprising result that the M1 strength to the “single j scissors” is larger for a $J=0$ $T=1$ pairing interaction than it is for Q.Q.

1 Introduction

We have recently performed single $j$ shell studies of both schematic and realistic interactions[1]. They ranged from the $J=0^+ T=1$ to the $J_{\text{max}} T=0$ interactions. In this work we will focus more on the experimental consequences of choosing a given interaction. In particular we study Gamow-Teller and isovector M1 matrix elements for transitions in Sc and Ti isotopes. Some of the problems have been addressed numerical in a previous publication [2], but here we will present analytical proofs.

1.1 The Interactions

For 2 particles in a single $j$ shell the states of even angular momentum $J$ have isospin $T=1$ and those of odd $J$ $T=0$. For convenience we define $E(J)$ as a two body interaction which is zero except when the 2 particles couple to $J$. Hence, we have the $J=0^+ T=1$ pairing interaction designated as $E(0)$ and the other extreme $E(J_{\text{max}})$ which acts only in the $T=0$ state with $J_{\text{max}}=2j$. The $T=0$ odd $J$ interaction acts only between a neutron and a proton. We only consider charge independent interactions in this work. For a “realistic” interaction in the $f_{7/2}$ shell we use the MBZE interaction [3]. This is based on the works of Bayman et al. and McCullen et al. [4,5] but with improved $T=0$ two-body matrix elements[4]. From $J=0$ to $J_{\text{max}}=7$ the matrix elements, which were obtained from experiment are:

\[
(0.0000, 0.6111, 1.5863, 1.4904, 2.8153, 1.5101, 3.2420, 0.6163)
\]

Although the $J=0^+$ matrix element is the most attractive; in MBZE one also has low lying $T=0$ levels with $J=1^+$ and $J=J_{\text{max}}=7^+$. Indeed, one main thrust of the old papers was that there was a large probability in say, an even-even nucleus that the protons and neutrons do not couple to zero. Indeed; it was shown in ref [6] that a much better overlap with the realistic interaction was obtained with a quadrupole-quadrupole interaction(Q.Q) than with the $J=0$ pairing interaction. We should here also mention the work on GT by Lawson[7] who invoked a K selection rule to explain why GT matrix elements decrease with neutron excess.
2 Wave Functions And Quantum Numbers for a $J=0 \ T=1$ Pairing Interaction of a Q.Q. Interaction

In this section we present energy levels and wave functions of $^{43}\text{Sc}$ and $^{44}\text{Ti}$ which have a $J=0$, a $J=0 \ T=0$, and a Q.Q interaction. The wave functions are presented as column vectors of probability amplitudes. To identify the higher isospin states we subtracted 3 MeV from all $T=0$, two-body matrix elements for the pairing interaction. Doing so does not affect the wave functions of the non degenerate states but it will remove degeneracies of states with different isospins. For Sc isotopes we indicate a star (*) for states with $T=3/2$. For $^{44}\text{Ti}$ we indicate a star for $T=1$ and two stars (**) for $T=2$.

Table 2.1 Energies(MeV) and Wave Functions of $^{43}\text{Sc}$ with a $J=0 \ T=1$ Pairing Interaction

| $I$  | $J_p$ | $J_n$ | 1.125 | 1.125 | 5.625* |
|------|-------|-------|-------|-------|--------|
|       | 3.5   | 2.0   | 0.4210| -0.4600| 0.7817 |
|       | 3.5   | 4.0   | 0.4695| 0.8479 | 0.2462 |
|       | 3.5   | 6.0   | 0.7761| -0.2633| -0.5730|

| $I$  | $J_p$ | $J_n$ | 0.000 | 1.125 | 1.125 | 4.875* |
|------|-------|-------|-------|-------|-------|--------|
|       | 3.5   | 0.0   | 0.8660| 0.000 | 0.000 | 0.500  |
|       | 3.5   | 2.0   | 0.2152| -0.8924| -0.1358| 0.3727 |
|       | 3.5   | 4.0   | 0.2887| 0.1565 | 0.8014| 0.500  |
|       | 3.5   | 6.0   | 0.3469| 0.4232 | -0.5826| 0.6009 |

| $I$  | $J_p$ | $J_n$ | 1.125 | 1.125 | 5.625* |
|------|-------|-------|-------|-------|--------|
|       | 3.5   | 2.0   | -0.1015| 0.9416| -0.3212|
|       | 3.5   | 4.0   | 0.4930 | 0.3280| 0.08058|
|       | 3.5   | 6.0   | 0.8641 | -0.0766| -0.4975|

Table 2.2 Energies(MeV) and Wave Functions of $^{44}\text{Ti}$ with a $J=0 \ T=1$ Pairing Interaction

| $I$  | $J_p$ | $J_n$ | 0.000 | 0.750** | 2.25 | 2.25 |
|------|-------|-------|-------|----------|----|----|
|       | 0.0   | 0.0   | 0.8660 | -0.5000 | 0.000| 0.000|
|       | 2.0   | 2.0   | 0.2152 | 0.3737  | 0.8863| 0.1712|
|       | 4.0   | 4.0   | 0.2887 | 0.5000  | -0.1244| -0.8070|
|       | 6.0   | 6.0   | 0.3469 | 0.6009  | -0.4461| 0.5652|

| $I$  | $J_p$ | $J_n$ | 1.500* | 2.250* | 2.250* |
|------|-------|-------|--------|--------|--------|
|       | 2.0   | 2.0   | 0.1992 | 0.4433 | 0.8740 |
|       | 4.0   | 4.0   | 0.4879 | -0.8183| 0.3038 |
|       | 6.0   | 6.0   | 0.8498 | 0.3659 | -0.3793|
| $I=2.0$ | $J_p$ | $J_n$ | 1.000 | 1.250 | 1.750 | 2.250 | 2.250 | 2.250 | 2.250 | 2.250 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 2.0 | 0.6455 | 0.7071 | -0.2887 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2.0 | 0.0 | 0.6455 | -0.7071 | -0.2887 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2.0 | 2.0 | -0.1205 | 0.0000 | -0.2694 | 0.6032 | 0.3665 | 0.0549 | 0.0618 | -0.3799 | 0.5134 |
| 2.0 | 4.0 | 0.1730 | 0.0000 | 0.3869 | 0.1407 | -0.4053 | -0.0033 | 0.2281 | -0.7442 | 0.1746 |
| 4.0 | 2.0 | 0.1730 | 0.0000 | 0.3869 | 0.6458 | 0.3665 | 0.0549 | 0.0618 | -0.3799 | 0.5134 |
| 4.0 | 4.0 | -0.0193 | 0.0000 | -0.0431 | 0.0193 | 0.1407 | -0.4053 | -0.0033 | 0.2281 | -0.7442 |
| 4.0 | 6.0 | 0.1403 | 0.0000 | 0.3138 | 0.3245 | -0.4415 | 0.0626 | 0.2281 | -0.7442 | 0.1746 |
| 6.0 | 4.0 | 0.1403 | 0.0000 | 0.3138 | 0.0626 | -0.0068 | 0.5991 | 0.0946 | -0.2318 | 0.5230 |
| 6.0 | 6.0 | 0.2292 | 0.0000 | 0.5125 | -0.2997 | 0.6964 | -0.2418 | -0.2013 | -0.0407 | 0.0973 |

Table 2.3 Energies(MeV) and Wave Functions of $^{43}$Sc with a Q.Q. Interaction

| $I=2.5$ | $J_p$ | $J_n$ | 1.000 | 1.250 | 1.750 | 2.250 | 2.250 | 2.250 | 2.250 | 2.250 |
|---|---|---|---|---|---|---|---|---|---|---|
| 3.5 | 2.0 | 0.5053 | 0.7817 | -0.3655 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3.5 | 4.0 | 0.2885 | 0.2462 | 0.9253 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3.5 | 6.0 | 0.8133 | -0.5730 | -0.1011 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

| $I=3.5$ | $J_p$ | $J_n$ | 1.000 | 1.250 | 1.750 | 2.250 | 2.250 | 2.250 | 2.250 | 2.250 |
|---|---|---|---|---|---|---|---|---|---|---|
| 3.5 | 0.0 | 0.7069 | -0.5000 | 0.4402 | 0.2376 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3.5 | 2.0 | 0.6864 | 0.3727 | -0.4393 | -0.4439 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3.5 | 4.0 | 0.1694 | 0.5000 | -0.1549 | 0.8350 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3.5 | 6.0 | 0.0216 | 0.6009 | 0.7676 | -0.2218 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

| $I=4.5$ | $J_p$ | $J_n$ | 1.000 | 1.250 | 1.750 | 2.250 | 2.250 | 2.250 | 2.250 | 2.250 |
|---|---|---|---|---|---|---|---|---|---|---|
| 3.5 | 2.0 | 0.9032 | -0.2847 | -0.3212 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3.5 | 4.0 | 0.4186 | 0.4186 | 0.8058 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3.5 | 6.0 | 0.0949 | 0.8623 | -0.4975 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

Table 2.4 Energies(MeV) and Wave Functions of $^{44}$Ti with a Q.Q. Interaction

| $I=0.0$ | $J_p$ | $J_n$ | 1.000 | 1.250 | 1.750 | 2.250 | 2.250 | 2.250 | 2.250 | 2.250 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.0 | 0.7069 | -0.5000 | 0.4402 | 0.2376 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2.0 | 2.0 | 0.6864 | 0.3727 | -0.4393 | -0.4439 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4.0 | 4.0 | 0.1694 | 0.5000 | -0.1549 | 0.8350 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 6.0 | 6.0 | 0.0216 | 0.6009 | 0.7676 | -0.2218 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
3 Assigning Quantum Numbers For J=0 T=1 Pairing

We first take advantage of the fact that we have the energies and wave functions of the J=0+ and J=1+ states form an explicit matrix digitalization. It is convenient to add a constant so that the states which are not collective are at zero energy. When this is done, the energies of the J=0+ states for 44Ti are:

\[ -2.25, -1.5, 0, 0 \text{ MeV} \quad (2) \]

and the energies of the 1+ states are:

\[ -0.75, 0, 0 \text{ MeV} \]

We then fit these with the Flower’s formula [6], as given in Talmi’s book[8]:

\[ E = C \left\{ \left( \frac{n-v}{4} \right) (4j + 8 - n - v) - T(T+1) + t(t+1) \right\} \]

\[ C \]

is most easily determined by the isospin splitting of the T=2 state at -1.5MeV relative to the -2.25 ground state (in the shifted energies). We set -0.75 = 2*3C. So C=-0.125 (In the g9/2 shell C=-0.10). The quantum numbers are: shown in tables 3.1 and 3.2.

Table 3.1 Quantum Numbers for 43Sc with a Pairing Interaction

| Energy | J   | T   | t   | v   |
|--------|-----|-----|-----|-----|
| 0      | 5/2 | 1/2 | 1/2 | 3   |
| 0      | 5/2 | 1/2 | 1/2 | 3   |
| 0      | 5/2 | 3/2 | 3/2 | 3   |
| -1.125 | 7/2 | 1/2 | 1/2 | 1   |
| -0.75  | 7/2 | 3/2 | 1/2 | 1   |
| 0      | 7/2 | 1/2 | 1/2 | 3   |
| 0      | 7/2 | 1/2 | 1/2 | 3   |
| 0      | 9/2 | 1/2 | 1/2 | 3   |
| 0      | 9/2 | 1/2 | 1/2 | 3   |
| 0      | 9/2 | 3/2 | 3/2 | 3   |

4
Table 3.2 Quantum Numbers for $^{44}$Ti with a Pairing Interaction

| Energy | T | t | v |
|--------|---|---|---|
| -2.25  | 0 | 0 | 0 |
| -1.5   | 2 | 0 | 0 |
| 0      | 0 | 0 | 4 |
| 0      | 0 | 0 | 4 |

| Energy | T | t | v |
|--------|---|---|---|
| -1.0   | 1 | 1 | 2 |
| 0      | 1 | 1 | 4 |
| 0      | 1 | 1 | 4 |
| -0.5   | 2 | 1 | 2 |

| Energy | T | t | v |
|--------|---|---|---|
| -1.25  | 0 | 1 | 2 |
| 0      | 0 | 0 | 4 |
| 0      | 0 | 0 | 4 |
| 0      | 0 | 0 | 4 |

| Energy | T | t | v |
|--------|---|---|---|
| 0      | 1 | 0 | 2 |
| 0      | 1 | 1 | 4 |
| 0      | 1 | 1 | 4 |
| 0      | 2 | 2 | 4 |

4 Results

The Gamow-Teller operator is $C\sigma_t$. The wave functions for the Scandium isotopes are of the form $\Sigma D(J_n\nu) [j_p, J_n] [j_p, J_n]^{J_n} = 0$. Here $D(J_n\nu)$ is the probability amplitude that the neutrons couple to $J_n$. The matrix element from McCullen [5] et al. is

$$M_{ij} = \sum D_i(j, J_n) D_j(j, J_n) U(1jJ_f J_n; j J_i)$$

(4)

We put the results of the calculated matrix elements in Table 4.1.

Table 4.1

| $^{7/2-7/2}$ | E(0) | MBZE | E(7) | Q.Q |
|-------------|------|------|------|-----|
| $^{43}$Sc   | 0.3849 | -0.2088 | -0.10160 | 0.1207 |
| $^{45}$Sc   | 0.2666 | 0.0927 | -0.0027 | 0.0255 |
| 7/2-5/2(43) | zero  | 0.2020 | 0.2902 | 0.2763 |
| 7/2-5/2(45) | zero  | 0.0459 | -0.0022 | 0.000792 |
| 7/2-9/2(43) | zero  | -0.0818 | 0.0168 | 0.008380 |
| 7/2-9/2(45) | zero  | 0.0008 | -0.0028 | -0.02399 |

The results for the $7/2^+$ to $7/2^-$ transitions are shown in the first 2 rows above. We see that $J=0$ T=1 pairing gives the largest matrix element, MBZE is in the middle and E(J_{max}) the smallest. Thus we have the systematic that deviations for $J=0$ T=1 pairing lead to reduced Gamow-Teller matrix elements. It is not surprising that the realistic case, MBZE, is in the middle because the two-body interaction used in that calculation has both an a low lying $J=0$ part but also a low lying $J=7$ part. Of perhaps greatest interest is the fact that the matrix elements of GT for the E(0) interaction vanish when $J_f$ is different than $J_i$. We have here considered the cases $J_i=(7/2)_1$ and $J_f=5/2$ or 9/2, both for $^{43}$Sc and $^{45}$Sc. There is considerable discussion of the pairing interaction in the 1993 book by Talmi[8]. He has a discussion of odd tensor operators in space and spin. It is there shown that these operators conserve seniority. In this work on GT we have a product of an odd tensor operator in spin and an odd tensor operator in isospin. The general selection rules for overall isospin are that $T_f$ can be equal to $T_i$, $T_i + 1$ or $T_i - 1$; although in the cases considered here, the latter does not apply. We will soon see that in general the GT operator does not conserve seniority. For the $J=0$ T=1 pairing interaction the lowest state in $^{43}$Sc with $J_i=j=7/2$ has seniority $v=1$. All other states for this and all other angular momenta have seniority 3 except the T=3/2, J=j state which also has $v=1$. In the $f_{7/2}$ shell the latter state is unique. We see from Table 4.1 that if our initial state is a $v=1$ state with $J=j$ ($7/2$ in this case) and isospin T=1/2 there is a non vanishing matrix element to a $v=1$ T=3/2 state and $J_{f}=j$. However with a $J=0$ T=1 pairing interaction the matrix element from the $v=1$ state to $v=3$ states with $J=j+1$ or $J=j-1$ vanish. It should be noted that although one
constructs a $J = j$, $v=1$ state in $^{43}$Sc by first adding 2 neutrons coupled to $J_n=0$ to the single proton that is not the end of the story. One must introduce isospin wave functions and antisymmetrize. The values of $D(J_n)$ for the $v=1$ $J = j$ $T=1/2$ state for $J_n=0, 2, 4$ and 6 are respectively

$$(0.8660, 0.2152, -0.0887, 0.3469).$$

Consider the matrix element

$$M' = N \left( \psi_{J_f} \sum \sigma t_+ (1 - P_{12} - P_{13}) \left[ j(1) \left[ j(2)j(3) \right]^0 \right]^2 p(1)n(2)n(3) \right)$$

(5)

where $t_z = -1/2$ for a proton and $+1/2$ for a neutron. We can replace $\sum \sigma t_+$ by $3\sigma(1)t_+$. Since $t_{z=0}$ we see that the $(-P_{12} - P_{13})$ terms will not contribute. We are left with $3N(j^2) \left( \psi_{J_f} \left[ j(1)j(2)j(3) \right]^0 \right)$. We can write $\psi_{J_f} = \sum D_{J_f}(J_n v) \left[ j_p, J_n \right]^{J_f}$. Hence the last factor is simply $D_{J_f}(0)$. However for a seniority $v=3$ final state $D_{J_f}(0)$ is equal to zero. As mentioned before the only $T=3/2$ state with seniority $v=1$ is the one with $J_f = j$. The $J=5/2$ and $9/2$ states all have seniority 3 and hence the matrix element $M'$ vanishes for those cases, but there is a problem. The state on the right is a mixture of $J=7/2 \ v=1\ T=1/2$ and $J=7/2 \ v=1\ T=3/2$. We next show that the $T=3/2$ part also vanishes and this will imply that the $T=1/2$ part will also vanish. That is, we consider a transition from $J=7/2 \ v=1\ T=3/2$ in $^{43}$Sc to $J=5/2$ or $9/2$ \ $v=3$ in $^{44}$Ca. There is a close relation between Gamow-Teller transitions and isovector magnetic dipole (M1) transitions. If one removes the orbital part of the M1, keeping only the spin there is an isospin relation between the two transitions. We can transform the GT problem to one of M1 transitions in $^{43}$Ca. But it is well known that for a single $j$ shell of particles of one kind, i.e. only neutrons, all M1 transitions vanish. We had previously displayed a formula for single $j$ shell M1 transitions from an $I=0^+$ ground state to an $I=1^+$ state of an even-even nucleus, $[9]$. This can be generalized to an expression given in the appendix (11). Note that the term with $J_p=0$ does not contribute. From this and the previous discussion on GT we see that this will also vanish for $J=0 \ v=0$ to $J=1 \ v=4$ $[15]$. (Note that this expression implies that isoscalar transitions vanish in the single $j$ shell limit i.e. $g_p-g_n=0$). In ref $[2]$ the energy shifts and $B(GT)$’s starting from the initial $J=0 \ v=0 \ T=0$ state in $^{44}$Ti were given, although no proof of the selection rule was given.

### BM1 Values for $I=1$ to $I=0$ Pairing Interaction

The values from the $i$th state of $I=1$ into the $j$th state of $I=0$

| I   | 0₁    | 0₂    | 0₃    | 0₄    |
|-----|-------|-------|-------|-------|
| ₁   | 2.69963 | 8.0995 | 1.92994 | 0.898554 |
| ₂   | 0      | 0     | 0.11174 | 7.6793  |
| ₃   | 0      | 0     | 2.89221 | 1.91866 |

### BM1 Values for $I=1$ to $I=2$ Pairing Interaction

The values from the $i$th state of $I=1$ into the $j$th state of $I=2$

| I    | 2₁    | 2₂    | 2₃    | 2₄    | 2₅    | 2₆    | 2₇    | 2₈    | 2₉    |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| ₁    | 1.02858 | 17.5613 | 0.0475777 | 2.29634 | 0     | 0     | 5.14334 | 0     |
| ₂    | 0.181872 | 1.45084 | 0.0330685 | 1.8904 | 0     | 0     | 0.909075 | 8.2364 |
| ₃    | 0.525607 | 1.4562 | 2.07128 | 3.32567 | 0     | 0     | 2.62751 | 0.465319 |
BM1 Values for I=1 to I=0 Q.Q Interaction

The values from the ith state of I=1 into the jth state of I=0

| I  | 01   | 02   | 03   | 04   |
|----|------|------|------|------|
| 1  | 1.31736 | 1.80213 | 0.183327 | 0.0413715 |
| 2  | 0.00146367 | 6.14543 | 9.041392 | 0.057738 |
| 3  | 0.000661312 | 0.153487 | 0.953011 | 0.205204 |

BM1 Values for I=1 to I=2 Q.Q Interaction

The values from the ith state of I=1 into the jth state of I=2

| I  | 21   | 22   | 23   | 24   | 25   | 26   | 27   | 28   | 29   |
|----|------|------|------|------|------|------|------|------|------|
| 1  | 0.885292 | 5.0018 | 0.0301273 | 0.0533339 | 0 | 0.0781581 | 3.36014 | 0 |
| 2  | 0.0126882 | 3.30166 | 18.1444 | 8.08602 | 0 | 0.0339801 | 0.347936 | 0 |
| 3  | 0.0000924735 | 0.180103 | 0.27692 | 0.534714 | 0 | 5.13135 | 8.26883 | 0 |

We see that with the J=0 pairing interaction there is a nonzero transition is from a J=0 v=0 state to a J=1 v=2 state i.e. the M1(or GT) operator does not conserve seniority. We can, in analogy with what we did for Sc, form a $^{44}$Ti state $[jj]^0[jj]^0$ and antisymmetrize. But this will be an admixture of J=0 v=0 T=0 and J=0 v=0 T=2. We now have to show that the T=2 part vanishes when we overlap with a J=1 v=4 T=1 state and this will lead to the desired result that the T=0 part vanishes. It is easier to use an isospin transformation and consider the transition between a unique J=0 v=0 T=2 state in $^{44}$Ca to a v=4 T=1 state in $^{44}$Sc. The T=2 state can be obtained by forming the 4 neutron state $[jj]^0[jj]^0$ and antisymmetrizing. However, as shown before, we do not have to antisymmetrize in the matrix element. And clearly; the v=4 T=1 J=1$^+$ state will, even after antisymmetrization not have any component $[jj]^1[jj]^0$. Thus, the T=2 part vanishes and so will the T=0 part.

We briefly compare the results of J=0 T=1 pairing and Q.Q for 2 protons and 2 neutrons in the $g_{9/2}$ shell.

We consider M1 transitions from J=0 T=0 to J=1 T=1. With J=0$^+$ T=1 pairing the value of MM$^2$ is 12.0254 and all the strength is to one state with v=2. With Q.Q the summed strength to all states is 4.5345 with most of the strength (4.4833) going to the lowest J=1$^+$ T=1 state.

Closing Remarks

It should be noted that the relation between B(M1) and B(GT) has been previously discussed by L. Zamick and D.C.Zheng[17].

With regards to selection rules for the M1 (or GT) operator we find with the J=0 T=1 interaction that for the odd- even Sc isotopes e.g $^{43}$Sc and $^{45}$Sc, seniority is conserved, i.e. $\Delta v=0$. For $^{44}$Ti seniority is not conserved. We can and do have $\Delta v=2$transitions. However $\Delta v=4$ transitions are not allowed. This is shown by the zeros for the transitions 12 to 01 and 04 as well as 13 to 01 and 04. We further note that a change of reduced isospin by 2 units is forbidden. This is shown by the zero for 11 to 29 (t=0 to t=2). These are the main results concerning selection rules.

Some well known rules also come into play. From I=1 T=1 we get vanishing matrix elements to I=2 T=1.

This has been discussed by both Lawson [7] and Talmi [8]. This can be connected to the fact that the Clebsh-Gordan coefficient (1100—10) vanishes.

Equally important in this work is the comparison of B(M1)(GT) strengths with various interactions. In the odd-even Sc isotopes and for I=1 to I=0 transitions and in $^{44}$Ti the pairing interaction gives larger strengths than does Q.Q. The reverse transition from 01 to 11 is often called the “spin-scissors transition”.

There is even a much stronger transition from I=1 to I=2 state 04 with the pairing interaction as compared with Q.Q. In our opinion the fact that these transitions are larger with the pairing interaction than with Q.Q is not well known. We also note very strong transitions from I=1 to I=2 with the Q.Q interaction e.g. 18.1444 $\mu_N$ from 12 to 24. Can such large transitions be found experimentally?

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5 Appendix

Formulas For $B(GT)$

\[ X_1 = \sum_{J_pJ_n} D^f(J_pJ_n) D^i(J_pJ_n) U(1J_pJ_n; J_pJ_n) \sqrt{J_p(J_p + 1)} \] (6)

\[ X_2 = \sum_{J_pJ_n} D^f(J_pJ_n) D^i(J_pJ_n) U(1J_pJ_n; J_pJ_n) \sqrt{J_n(J_n + 1)} \] (7)

\[ B(GT) = 0.5 \frac{2I_f + 1}{2I_i + 1} f(j)^2 \left[ \frac{\langle 1T_i1M_T_i|T_jM_T_j \rangle}{\langle 1T_i0M_T_i|T_jM_T_j \rangle} \right]^2 (X_1 - (-1)^{I_f-I_i} X_2)^2 \] (8)

Where $f(j) = \begin{cases} \frac{1}{2l+1} & \text{if } j = l + 1/2 \text{ e.g. } f_{7/2} \\ \frac{1}{2l} & \text{if } j = l - 1/2 \text{ e.g. } f_{5/2} \end{cases}$ (9)

\[ ft = \frac{2177}{B(F) + 1.583B(GT)} \] (10)

Formulas For $B(M1)$

\[ B(M1) = 3 \frac{2I_f + 1}{4\pi 2I_i + 1} \left[ g_{jp} X_1 + (-1)^{I_f-I_i} g_{jn} X_2 \right]^2 \] (11)

Here $g_j = g_i \pm \left( \frac{g_s - g_l}{2l + 1} \right)$ (12)

\[ g_{sp} = 5.586 \quad g_{ip} = 1 \] (13)

\[ g_{sn} = -1.913 \quad g_{in} = 0 \] (14)

For the case $T_f$ is not equal to $T_i$ we find:

\[ X_1 = (-1)^{I_f-I_i} X_2 \] (15)

\[ B(M1) = 3 \frac{2I_f + 1}{4\pi 2I_i + 1} (g_{jp} - g_{jn})^2 X_i^2 \] (16)

With this simplification we see that $B(GT)$ is proportional to $B(M1)$.

Using bare values we find $B(GT)/B(M1) = 0.1411$ for $j = 7/2$.

The magnetic moment is:

\[ \mu \frac{T}{I} = \frac{g_{jp} + g_{jn}}{2} + \frac{g_{jp} - g_{jn}}{2(I+1)} \left[ \sum_{J_pJ_n} |D(J_pJ_n)|^2 [J_p(J_p + 1) - J_n(J_n + 1)] \right] \] (17)

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