Gravimagnetic shock waves in the anisotropic plasma

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Abstract

The relativistic magnetohydrodynamic equations for the anisotropic magnetoactive plasma are obtained and accurately integrated in the plane gravitational wave metrics. The dependence of the induction mechanism of the gravimagnetic shock waves on the degree of the magnetoactive plasma anisotropy is analyzed.

1 Introduction

In [1] on the basis of an exact solution of the relativistic magnetohydrodynamic (RMHD) equations on the background of the plane gravitational wave (PGW) metrics a new class of relativistic essentially non-linear phenomena was found, which arise in a highly magnetized plasma under the PGW influence, was named as gravimagnetic shock waves (GMSW). The essence of the GMSW phenomenon is that the highly magnetized plasma

$$\alpha^2 = \frac{H_\perp^2}{4\pi(\varepsilon_0 + p_0)} \gg 1,$$

(1)

(where $H_\perp$ is the magnetic field intensities component perpendicular to the PGW propagation direction, $\varepsilon_0$, $p_0$ are the imperturbed energy density and plasma pressure without the magnetic field) very actively reacts even on a weak PGW by rather large values of the second parameter of GMSW:

$$\Upsilon \equiv 2\beta_0\alpha^2 > 1,$$

(2)

where $\beta_0$ is the maximum PGW amplitude.

In [2] on the basis of the energy balance model of the plasma and PGW it was shown, that under the condition (2) the PGW energy is practically fully transformed into the magnetoactive plasma acceleration (par excellence in the PGW propagation direction) and into creating a shock wave with high densities of the plasma energy and the magnetic field. In this case the sublight velocity of the plasma motion in pointed out direction is achieved.

The essential change of the electromagnetic radiation characteristics of the plasma in GMSW was used in [2] in order to creat a new experimental test for the detection of the gravitational pulsars radiation. In particular, in [3] it was shown that, the so-called giant impulses in the radio radiation of the pulsar NP 0532 may be explained by the GMSW mechanism.

In the mentioned above papers the locally isotropic plasma was studied ($p_\perp = p_\parallel = p$); the anisotropy was created by the exclusively magnetic field.
In the strong magnetic fields due to the magnetobremssstrahlung the local thermodynamic equilibrium (LTE) of the plasma is destroyed. Therefore generally writing:

\[ p_\perp \neq p_\parallel . \]  

(3)

The present paper is devoted to the investigation of the degree influence of the plasma anisotropy on the forming of the electro-magnetic reaction of the plasma to the PGW. The relativistic magnetohydrodynamic equations are placed into the base of the theoretical model of the magnetooactive plasma description in the PGW field [1]. These equations, as it follows from [4], can be also obtained from the kinetic equations to the collisionless plasma in a drift approximation:

\[ \omega \ll \omega_B , \]  

(4)

where \( \omega \) is the PGW frequency, \( \omega_B = eH/m_e \) is the Larmor’s frequency. Throughout the paper the metrics signature (−1, −1, −1, +1) is used and the fundamental system of units: \( G = c = \hbar = 1 \) are used.

2 RMHD equations

2.1 RMHD equations for an arbitrary structure of the plasma MET

Generally the RMHD equations are obtained from the conservation law of the whole energy-momentum tensor (MET) of the plasma and electromagnetic field:

\[ \left( T^{ij} + f^{ij} \right) ,j = 0 \]  

(5)

and from the first group of the Maxwell equations:

\[ \ast F_{ij} ,j = 0 \]  

(6)

if the Maxwell tensor satisfies the following requirements [1]:

\[ F^{ij} \ast F_{ij} = 0 ; \]  

(7)

\[ F^{ij} F_{ij} = 2 H^2 > 0 . \]  

(8)

Then it turns out that the timelike eigenvectors of the plasma MET and of the electromagnetic field MET coincide, and moreover this vector of the plasma dynamic velocity, \( v^i \), is the eigenvector of the Maxwell tensor simultaneously:

\[ F_{ij} v^j = 0 . \]  

(9)

\[ ^4 \text{In [1] this condition was called “condition of the magnetic field embedding in the plasma”}. \]
Analogical to (9) contraction of the velocity vector of the dual Maxwell tensor is the magnetic field intensity vector:

\[ H_i = v^k \tilde{F}_{ki}^*, \quad (10) \]

This vector is spacelike:

\[ (H, H) = -H^2 \quad (11) \]

and orthogonal to the plasma velocity vector:

\[ (v, H) = 0. \quad (12) \]

Besides, Eq. (11) follows from Eqs. (5) - (8) (1) the second group of the Maxwell equations is:

\[ F_{ij}^{*} = -4\pi J_i^d \quad (13) \]

with the spacelike drift current:

\[ J_i^d = \frac{2F_{ik}^p T_{k,l}^{i,l}}{F_{jm}F_{jm}}; \quad (14) \]

and the differential relations:

\[ v^i T^k_{i,k} = 0, \quad (15) \]

\[ H^i T^k_{i,k} = 0. \quad (16) \]

Let us also note useful differential identities following from Eqs. (6) - (10) (1):\n
\[ v^i H^k_{,k} + v^i v^k H^k - v^k u^k H^i = 0; \quad (17) \]

\[ - v_{i,k} H^i v^k = H_{i,k} v^i v^k = H^k_{,k}; \quad (18) \]

\[ H_{i,k} v^i H^k = -v_{i,k} H^i H^k = H(H v^k)_{,k}. \quad (19) \]

The pointed above relations represent a complete set of the algebraic and differential consequences (5) - (8), and at the plasma ETM fixing by the set they are a set of the plasma relativistic magnetohydrodynamic equations in the gravitational field.
2.2 RMHD equations for the anisotropic plasma

Let introduce a single spacelike vector \( h^i \) \(^1\):

\[
h^i = \frac{H^i}{H}; \quad (h, h) = -1.
\]  

(20)

Two independent vectors \( v^i \) and \( h^i \) and the metrics tensor \( g^{ij} \) define the following algebraic structure of the anisotropic plasma MET:

\[
T^{ij} = (\epsilon + p_{\perp}) v^i v^j - p_{\perp} g^{ij} + (p_{\parallel} - p_{\perp}) h^i h^j,
\]  

(21)

and the timelike vector of velocity satisfies normalization relation:

\[
(v, v) = 1.
\]  

(22)

The MET trace (21) is equal to:

\[
\dot{\epsilon} = \epsilon - p_{\parallel} - 2p_{\perp},
\]  

(23)

therefore, in the consequence of the known theorem of the virial (see, for instance \([5]\)) the following condition must take place:

\[
p_{\parallel} + 2p_{\perp} \leq \epsilon.
\]  

(24)

In this case the electromagnetic field MET is equal to (1) :

\[
\dot{f} \epsilon_{ij} = \frac{1}{8\pi} \left( 2H^2 v_i v_j - 2H_i H_j - g_{ij} H^2 \right).
\]  

(25)

Substituting the plasma ETM written in the form (21) in (15) and (16) and using the identities (17) - (19) we obtained equations:

\[
\epsilon, i v^i + (\epsilon + p_{\parallel}) v^i, i + (p_{\parallel} - p_{\perp}) v^i (\ln H), i = 0;
\]  

(26)

\[
(\epsilon + p_{\parallel}) H^i, i - (p_{\parallel} - p_{\perp}) H^i (\ln H), i + (p_{\parallel}), i H^i = 0.
\]  

(27)

3 The RMHD equations solution in PGW metrics

3.1 PGW metrics and the initial conditions

The PGW metrics has the form \([6]\):

\[
ds^2 = 2dudv - L^2 e^{2\beta} (dx^2)^2 + e^{-2\beta} (dx^3)^2,
\]  

(28)

where \( \beta(u) \) is the amplitude, and \( L(u) \) is the PGW background factor; \( u = \frac{1}{\sqrt{2}} (t - x^1) \) is the retarded time, \( v = \frac{1}{\sqrt{2}} (t + x^1) \) is the advanced. The metrics (28) admits the group of motions \( G_5 \), associated with three linearly independent Killing vector:

\[
\xi^i = \delta^i_v; \quad \xi^i = \delta^i_2; \quad \xi^i = \delta^i_3.
\]  

(29)
Let in the PGW absence \((u \leq 0)\):

\[
\beta(u_{|u\leq0} = 0; \quad L(u)_{|u\leq0} = 1, \tag{30}
\]

the plasma is homogeneous and at rest:

\[
v_{|u\leq0}^v = v_{|u\leq0}^u = \frac{1}{\sqrt{2}}; \quad v^2 = v^3 = 0;
\]

\[
\varepsilon_{|u\leq0} = \varepsilon_0;
\]

\[
(p_{||})_{|u\leq0} = p_{||}; \quad (p_{\perp})_{|u\leq0} = p_{\perp}, \tag{31}
\]

and the homogeneous magnetic field is directed in the plane \(\{x^1, x^2\}\):

\[
H_{1|u\leq0} = H_0 \cos \Omega; \quad H_{2|u\leq0} = H_0 \sin \Omega;
\]

\[
H_{3|u\leq0} = 0; \quad E_{|u\leq0} = 0, \tag{32}
\]

where \(\Omega\) is the angle between the axis \(0x^1\) (the PGW propagation direction) and the magnetic field direction \(\mathbf{H}\). The conditions (32) correspond to the vector potential:

\[
A_v = A_u = A_2 = 0;
\]

\[
A_3 = H_0(x^1 \sin \Omega - x^2 \cos \Omega); \quad (u \leq 0). \tag{33}
\]

In [1] it is shown that in consequence of Eqs. (13) and (14) in the PGW component presence \(A_3\) of the vector potential takes the form:

\[
A_3 = -H_0x^2 \cos \Omega + \frac{1}{\sqrt{2}}H_0[v - \psi(u)] \sin \Omega, \tag{34}
\]

where \(\psi(u)\) is an arbitrary differentiated function satisfying the initial condition:

\[
\psi_{|u\leq0} = u. \tag{35}
\]

The components of the magnetic field intensity vector corresponding to the vector potential (34) are equal to:

\[
H_v = -H_0L^{-2}(v_v \cos \Omega + \frac{1}{\sqrt{2}}v_2 \sin \Omega);
\]

\[
H_u = H_0L^{-2}(v_u \cos \Omega - \frac{1}{\sqrt{2}}\psi' v_2 \sin \Omega);
\]

\[
H_2 = -\frac{1}{\sqrt{2}}H_0e^{2\beta} \sin \Omega(v_u + v_v \psi'); \quad H_3 = 0, \tag{36}
\]

and the embedding conditions (10) are reduced to one equation [1]:

\[
\frac{1}{\sqrt{2}}(v_v \psi' - v_u) \sin \Omega + v^2 \cos \Omega = 0. \tag{37}
\]
In this case the scalar \( H_2 \) is equal to:

\[
H^2 = H_0^2 \left( \frac{\cos^2 \Omega}{L^4} + \frac{\sin^2 \Omega}{L^2} e^{2\beta} \right).
\]

(38)

In \cite{1} it is shown that the relation of the velocity vector normalization \( (22) \) by means of Eqs. \((36)\) and \((38)\) may be represented in the equivalent form:

\[
(v_v \cos \Omega + \frac{1}{\sqrt{2}} v_2 \sin \Omega)^2 = \frac{H^2}{H_0^2} v_v^2 L^4 - \frac{\sin^2 \Omega}{2} L^2 e^{2\beta}.
\]

(39)

3.2 Integrals of the motion

In the consequence of the existence of the motions \( (29) \) the conservation laws of the whole plasma MET in the PGW field have the following integrals \cite{1}:

\[
L^2 \xi^i (v_v) T_{vi} = C_a = \text{Const}; \quad (a = 1, 3),
\]

(40)

of which only the first two are non-trivial. In \cite{1} it was shown that the consequence of these laws is equivalent to the single non-trivial Maxwell equation \((13)\). Substituting in the integrals \((42)\) the expressions to the MET of the plasma and electromagnetic field \((21)\) and \((25)\), and using the relations \((36) - (41)\) and also the initial conditions \((30)\) and \((31)\) we reduce non-trivial integrals to form:

\[
2L^2 v_v^2 (\varepsilon + p) - e^{2\beta} \left( \frac{\sigma_{\perp}}{H^2} \right)^2 (p\| - p\perp) =
\]

\[
= (\varepsilon + p) \Delta(u);
\]

(41)

\[
L^2 (\varepsilon + p\|) v_v v_2 + \frac{1}{\sqrt{2}} e^{2\beta} \cos \Omega \sin \Omega \frac{H_0^2}{H^2} (p\| - p\perp) =
\]

\[
= (e^{2\beta} - 1) \frac{H_0^2 \cos \Omega \sin \Omega}{4\sqrt{2}\pi} + \frac{1}{\sqrt{2}} \cos \Omega \sin \Omega (p\| - p\perp),
\]

(42)

where:

\[
\sigma^0 = \cos^2 \Omega p\| + \sin^2 \Omega p\perp;
\]

(43)

\[
(\sigma_{\perp})^2 = H_0^2 \sin^2 \Omega
\]

(44)

and the so-called GMSW governing function is introduced:

\[
\Delta(u) = \left[ 1 - \alpha^2 (e^{2\beta} - 1) \right],
\]

(45)

it differs from analogous function to the isotropic plasma \cite{1}, \cite{7} by the dimensionless parameter \( \alpha^2 \) determination:

\[
\alpha^2 = \frac{(\sigma_{\perp})^2}{4\pi(\varepsilon + p)}.
\]

(46)
Solving Eqs. (41) and (42) relatively \( v_2 \) and \( v_v \) we get the expressions for the coordinates of the velocity vector via scalar: \( \varepsilon, p_\parallel, p_\perp, \psi \) and the explicit functions of retarded time:

\[
v_v^2 = \frac{\varepsilon + p_\parallel}{2L^2(\varepsilon + p_\parallel)^2} \Delta(u) + e^{2\beta} \frac{L^2}{2L^2(\varepsilon + p_\parallel)} (H_\perp)^2; \quad (47)
\]

\[
\frac{v_2}{v_v} = \frac{1}{\sqrt{2}} \sin \Omega \cos \Omega 	imes \left( (e^{2\beta} - 1) \frac{H_\perp^2}{4\pi} - \left[ e^{2\beta} \frac{H_\perp^2}{H^2} (p_\parallel - p_\perp) (p_\parallel - p_\perp) \right] \right), \quad (48)
\]

By means of (47) and (48) from the relation of the velocity vector normalization (22) we obtain the coordinate \( v_u \) of this vector:

\[
\frac{v_u}{v_v} = \frac{1}{2} \left[ e^{-2\beta} \left( \frac{v_2}{v_v} \right)^2 + \frac{1}{v_v^2} \right], \quad (49)
\]

and from the embedding condition (??) let us find the value of the potential derivative \( \psi' \):

\[
\psi' = \frac{v_u}{v_v} + \frac{e^{-2\beta}}{L^2} \cotg \Omega \frac{v_2}{v_v}, \quad (50)
\]

due to it the scalar \( H^2 \) is determined from the relation (58).

Using the integrals (41) and (42) it can be shown that from the two differential equations (26) and (27) only one is independent. Let Eq. (26) be an equation of this kind, which in the PGW metrics takes the form:

\[
L^2 \varepsilon v_v + (\varepsilon + p_\parallel)(L^2 v_v)' + \frac{1}{2} L^2 (p_\parallel - p_\perp) v_v (\ln H^2)' = 0. \quad (51)
\]

Thus, in the final analysis the rest equation (51) represents itself a differential equation for the three unknown scalar functions: \( \varepsilon, p_\parallel \) and \( p_\perp \). Such sub-diffiniteness is the known consequence of the incompleteness of the hydrodynamic plasma description. For the solution to this equation it is necessary to impose two supplementary relations between the functions \( \varepsilon, p_\parallel \) and \( p_\perp \), i.e., to introduce state equation of the form:

\[
p_\parallel = f(\varepsilon); \quad p_\perp = g(\varepsilon). \quad (52)
\]

### 4 Barothropic state equation

#### 4.1 General expressions

Let us study the barothropic behaviour of the anisotropic plasma, when the relations (52) are linear:

\[
p_\parallel = k_\parallel \varepsilon; \quad p_\perp = k_\perp \varepsilon, \quad (53)
\]
moreover due to Eq. (24) the constant coefficients \( k_\parallel \) and \( k_\perp \) satisfy the inequality:

\[
k_\parallel + 2k_\perp \leq 1,
\]

in the consequence of which it is always:

\[
p_\parallel \leq 1; \quad p_\perp \leq \frac{1}{2}.
\]

The equation (51) at the relations (53) is easily integrated, and we get one more integral:

\[
\varepsilon (\sqrt{2}L^2 v_v) (1 + k_\parallel) H^{(k_\parallel k_\perp)} = 0 \quad H^{(k_1 - k_\perp)}.
\]

Thus, formally the problem is solved, as it was reduced to solving algebraic equations set which, however, is still very complicated for its solving and analyzing. The solution of the problem is essentially defined by the three non-dimensional parameters: \( \cos \Omega \), \( k_\perp \) and \( k_\parallel \). Below we shall investigate the private values of these parameters.

4.2 Transverse propagation of the PGW

In this case \( \cos \Omega = 0 \), and from (48) it follows immediately:

\[
v_2 = 0.
\]

Then (22), (37) and (38) give:

\[
v_u = \frac{1}{2v_v}; \quad \psi' = \frac{1}{2v_v}.
\]

\[
H = \frac{H_0e^\beta}{\sqrt{2}Lv_v}.
\]

and the substitution of Eq. (59) in Eq. (47) leads to the result:

\[
v_v^2 = \frac{\varepsilon}{2L^2 \varepsilon} \Delta (u).
\]

Substituting (59) and (60) in (63), we obtain a closed equation for the variable \( \varepsilon \), solving it we finally get:

\[
\varepsilon = 0 \quad \left[ \Delta^{1+k_\perp}L^{2(1+k_\parallel)} e^{2\beta(k_\parallel - k_\perp)} \right]^{-\frac{1}{2}};
\]

\[
v_v = \frac{1}{\sqrt{2}} \left[ \Delta L^{k_\parallel+k_\perp} e^{\beta(k_\parallel - k_\perp)} \right]^{\frac{1}{2}};
\]

\[
H = H_0 \left[ \Delta L^{(1+k_1)} e^{-\beta(k_1)} \right]^{-\frac{1}{2}};
\]
where
\[ g_\perp = \frac{1}{1 - k_\perp} \in [1, 2]. \] (64)

In particular, for the ultrarelativistic plasma with the zero longitudinal pressure:
\[ k_\parallel \to 0; \quad k_\perp \to \frac{1}{2} \] (65)

we get from Eqs. (61) - (64):
\[ v_v = \frac{1}{\sqrt{2}} L\Delta^2 e^{-\beta}; \quad (66) \]
\[ \varepsilon = \varepsilon \Delta L^{-4} e^{-2\beta}; \quad H = H_0 L^{-2} \Delta^{-2} e^{2\beta}. \] (67)

### 4.3 Ultrarelativistic plasma with the outbedding transverse impulse

In this case:
\[ k_\perp = 0; \quad k_\parallel = 1, \] (68)

and we obtain:
\[ v_2 = \frac{1}{\sqrt{2}} L e^\beta ctg\Omega [1 - \Delta]; \quad (69) \]
\[ v_v = \frac{1}{\sqrt{2}} L e^\beta; \quad (70) \]
\[ \varepsilon = \varepsilon \Delta^{-1} L^{-4} e^{-2\beta}; \quad (71) \]
\[ H = H_0 L^{-2} \Delta^{-1}. \] (72)

### 5 The plasma anisotropy influence on the GMSW effectiveness

In [1] it is pointed out that the singular state arising in the magnetoactive plasma by fulfilling the condition (2) on the hyperspace
\[ \Delta(u) = 0, \] (73)

is destroyed by the back action of the magnetoactive plasma on the PGW, that leads to an effective absorbing the PGW energy by the plasma and to the restriction on the PGW amplitude. In [2] a simple model of energoballance allowing to describe this process is bilt. The gravitational wave described by
the metrics \( \mathcal{g} \) corresponds to the "effective tensor of the energy impulse" has alone non-zero component \( 0 \) with the single non-zero component:

\[
\mathcal{T}_{uu} = \frac{1}{4\pi}(\beta')^2.
\] (74)

Let \( \beta_*(u) \) be the PGW vacuum amplitude and \( \beta(u) \) be the PGW amplitude with the energy absorption in the plasma. Then the energy balance equation in a short-wave approximation takes the form:

\[
(\beta_*)^2 = (\beta')^2 + 4\pi(T_{uu} - \mathcal{T}_{uu}),
\] (75)

where \( T_{ik} \) is whole MET of the plasma. At the condition \( \|1\| \) in case of the transverse PGW propagation direction, when the GMSW effect is maximum, the equation (75) can be written in the form (see also \( \|2\| \)):

\[
\dot{q}_*^2 = \dot{q}^2 + \xi^2 V(q),
\] (76)

where \( q = \beta/\beta_0 \) the point means derivation by the variable \( \sqrt{2}\omega u \), \( \omega \) is the PGW frequency, \( V(q) \) is the potential function which in a weak PGW takes the form:

\[
V(q) = \left[ \Delta(q) \right]^{-4g_\perp} - 1,
\] (77)

and \( \xi^2 \) is the so-called first GMSW parameter \( \|2\| \):

\[
\xi^2 = \frac{H_0^2}{4\beta_0^2\omega^2}.
\] (78)

From (76) we obtain the minimally possible value of the governing function:

\[
\Delta_{min} = \left( \frac{1}{\xi^2} + 1 \right)^{-\gamma_\perp},
\] (79)

where:

\[
\gamma_\perp = \frac{1}{4g_\perp} = \frac{1 - k_\perp}{4} \leq \frac{1}{4}.
\] (80)

In this case the maximally achievable density of the magnetic field energy is equal to:

\[
\left( \frac{H^2}{8\pi} \right)_{max} = \frac{H_0^2}{8\pi} \sqrt{1 + \frac{1}{\xi^2}}
\] (81)

and generally does not depend on the state equations of the plasma \( \|52\| \). The plasma velocity in the GMSW is also turns out to be independent on the equation of state. Maximum density of the plasma energy without the magnetic field turns out to be dependent on the degree of the plasma anisotropy:

\[
\varepsilon_{max} = \varepsilon_0 (1 + \frac{1}{\xi^2})^{\frac{1}{4}(1+k_\perp)}
\] (82)
and is maximum for the ultrarelativistic plasma in the case when longitudinal pressure is equal to zero.

Thus, the maximum value of the magnitude of the local reaction of the highly magnetized plasma with the linear equation of state \( \text{(52)} \) does not depend on the plasma anisotropy degree and its equation of state. Obtained in \([2]\) the dependence of the maximal value of local response from the equation of state of plasma, just like the dependence of pulse duration GMSW from the second its argument, are the consequences of computed errors arising at the solution of the equation \( \text{(78)} \) alongside of maximum of function \( q(u) \). We shall come back to this question in the following article.

Global magnetobremsstrahlung GMSW-response of the plasma is dependent on the extents of its anisotropy and equation of state, since a magnetobremsstrahlung intensity radiation of one electron is proportional to the product of the square of magnetic strength on the square of lateral momentum \( P_\perp^2 \) connected with matching component of the pressures, \( p_\perp \). For the detection of this connection and determining real equation of state stinstead \( \text{(52)} \) it is necessary the building of kinetic analog GMSW in anisotropic plasma.

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