Kaon properties in (proto-)neutron star matter

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Abstract

The modification on kaon and antikaon properties of in the interior of (proto-)neutron stars is investigated using a chiral SU(3) model. The parameters of the model are fitted to nuclear matter saturation properties, baryon octet vacuum masses, hyperon optical potentials and low energy a kaon-nucleon scattering lengths. We study the kaon/antikaon medium modification and explore the possibility of antikaon condensation in (proto-)neutron star matter at zero as well as finite temperature/entropy and neutrino content. The effect of hyperons on kaon and antikaon optical potentials is also investigated at different stages of the neutron star evolution.

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I. INTRODUCTION

The study of kaon and antikaon properties is relevant for neutron star phenomenology as well as relativistic heavy-ion collision experiments. It was first suggested by Kaplan and Nelson [1] that the drop in the mass of antikaons in nuclear medium, arising from an attractive antikaon-nucleon interaction, might lead to Bose-Einstein condensation in the interior of neutron stars. Since then, a lot of work has been done in the topic of antikaon condensation in compact stars [2]. The s-wave $K^-$ condensation sets in when the in-medium energy of $K^-$ at zero momentum equals the chemical potential of $K^-$. The effect of the $K^-$ condensate is to replace the electrons in maintaining the charge neutrality condition. The formation of $\bar{K}^0$ condensate in neutron stars in a mean field approach has also been investigated [3]. The condition for the onset of neutral antikaon $\bar{K}^0$ condensation is $\omega_{\bar{K}^0}(k = 0) = 0$.

The gross properties of (proto-)neutron stars depend sensitively on the equation of state of dense, electric charge neutral matter. On the other hand, recent neutron star observations impose constraints on possible nuclear matter equations of state (EOS’s). As a result, nuclear matter EOS’s obtained using effective models should be consistent with astrophysical bounds in order to be used for neutron star matter calculations [4]. Recently, nuclear matter EOS’s have also been investigated consistently with collective flow data from heavy ion collision experiments [5]. The in-medium modification of kaon/antikaon properties have been studied using effective hadronic models based on meson exchange [6], chiral perturbation theory (using data from kaonic atom [7]) as well as using coupled channel approach [8]. These in-medium properties can be observed experimentally in relativistic heavy ion collisions [9] as well as in neutron star phenomenology [1, 2, 3]. Due to charge neutrality, the bulk matter in proto-neutron stars is isospin asymmetric. The isospin effects in hot and dense hadronic matter (already investigated in [10]) are thus important in the context of isospin asymmetric heavy-ion collision experiments [11] as well as for matter in the interior of neutron stars.

In the present investigation we use a chiral SU(3) model [12, 13] for the description of matter inside (proto-)neutron stars. The kaon (antikaon) energies, modified in the medium due to their interaction with nucleons, were also studied within this framework [14, 15] consistently with the low energy KN scattering data [16, 17]. In the present work, we
investigate the kaon and antikaon optical potentials in the asymmetric nucleonic/hyperonic matter. We also study in detail the possibility of kaon condensation on different stages of the neutron star cooling. These stages are defined by finite temperature/entropy and neutrino content.

The outline of the paper is as follows: In section II we briefly review the SU(3) model used in the present investigation. Section III describes the medium modification of the $K(\bar{K})$ mesons in this effective model. In section IV, we discuss the results obtained for the optical potentials of kaons and antikaons in (proto-)neutron stars. Section V summarizes our results and presents the conclusions.

II. THE HADRONIC CHIRAL $SU(3) \times SU(3)$ MODEL

In this section the various terms of the effective hadronic Lagrangian density used

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{vec}} + \mathcal{L}_{\text{scal}} + \mathcal{L}_{\text{SB}},$$

are discussed. This expression derives from a relativistic quantum field theoretical model of baryons and mesons constructed adopting a nonlinear realization of chiral symmetry [18, 19, 20] and broken scale invariance (for details see [12, 13, 21]) to describe strongly interacting nuclear matter. The chiral $SU(3) \times SU(3)$ model was already successfully used to describe nuclear matter, finite nuclei, hypernuclei and neutron stars. The Lagrangian density contains the baryon octet, the spin-0 and spin-1 meson multiplets as elementary degrees of freedom. In Eq. (1) $\mathcal{L}_{\text{kin}}$ is the kinetic energy term and $\mathcal{L}_{\text{int}}$ contains the baryon-meson interactions in which the baryon-spin-0 mesons generate the baryon masses. The term $\mathcal{L}_{\text{vec}}$ describes the dynamical mass generation of the vector mesons and contains additionally quartic self-interactions of the vector fields. The term $\mathcal{L}_{\text{scal}}$ contains the scalar meson self-interaction terms, that induce spontaneous breaking of chiral symmetry, as well as a scale invariance breaking logarithmic potential. Finally, the term $\mathcal{L}_{\text{SB}}$ describes the explicit chiral symmetry breaking.

The baryon-scalar meson interactions generate the baryon masses through the coupling of the baryons to the non-strange $\sigma(\sim \langle \bar{u}u + \bar{d}d \rangle)$ and the strange $\zeta(\sim \langle \bar{s}s \rangle)$ scalar quark condensates. The parameters corresponding to these interactions are adjusted to fix the
baryon masses to their measured vacuum values. It should be emphasized that the nucleon mass also depends on the \textit{strange condensate} \( \zeta \). In analogy to the baryon-scalar meson coupling there exist two independent baryon-vector meson interaction terms corresponding to the antisymmetric and symmetric couplings. Here we use the antisymmetric coupling \[12, 15\] following the universality principle \[22\] and the vector meson dominance model which shows that the symmetric coupling should be small. Additionally we choose parameters so as to decouple the strange vector field \( \phi_\mu \sim \bar{s} \gamma_\mu s \) from the nucleons \[12, 15\], that corresponds to an ideal mixing between \( \omega \) and \( \phi \). A small deviation of the mixing angle from the ideal mixing (used in \[23, 24, 25\]) has not been taken into account in the present investigation.

The Lagrangian density terms corresponding to the self-interaction for the vector mesons \( L_{\text{vec}} \), scalar mesons \( L_{\text{scal}} \) and the one corresponding to the explicit chiral symmetry breaking \( L_{\text{SB}} \) have been described in detail in references \[12, 15\]. For the non linear interaction of vector mesons, we use the invariant which does not generate the \( \rho-\omega \) coupling. That is in general agreement with the observed small mixing between the two mesons \[26\] and allows for more massive neutron stars.

To investigate the hadronic properties in the medium, we write the Lagrangian density within the chiral SU(3) model in the mean field approximation \[27\]. With this we can determine the expectation values of the meson fields by minimizing the thermodynamical potential \[13, 21\].

III. KAON (ANTIKAON) INTERACTIONS IN THE CHIRAL SU(3) MODEL

In this section, we derive the dispersion relations for the \( K(\bar{K}) \) \[28\] and calculate their optical potentials in asymmetric hadronic matter \[15\]. In the present model, the interactions of kaons and antikaons with scalar fields \( \sigma \) (non-strange) and \( \zeta \) (strange) and the scalar-isovector field \( \delta \) as well as a vector interaction with the nucleons (the so-called Weinberg-Tomozawa interaction) modify the energy of K(K) mesons in the medium. It might be noted here that the interaction of pseudoscalar mesons with the vector mesons, in addition to the pseudoscalar meson–nucleon vector interaction, leads to double counting in the linear realization of the chiral effective theory \[29\]. Within the nonlinear realization of chiral effective theories, such an interaction does not arise in the leading or sub-leading order,
but only as a higher order contribution \[29\]. Hence, the vector meson-pseudoscalar meson interaction is not considered within the present investigation.

In the following, we derive the dispersion relations for kaons and antikaons and study the dependence of kaon and antikaon optical potentials on isospin asymmetry. In order to do this, we include effects of isospin asymmetry originating from the scalar-isovector $\delta$ field as well as a vector interaction with nucleons \[15\]. In addition, in the present investigation we consider an isospin dependent range term arising from the interaction with nucleons, which was not taken into account in Ref. \[15\].

The term in the Lagrangian density that represents the interaction between baryons and kaons modifies the energies of the $K(\bar{K})$-mesons, and is given by

\[
\mathcal{L}_{KB} = -\frac{i}{4f_K^2} \left[ 2\bar{p}\gamma^\mu p + \bar{n}\gamma^\mu n - \Sigma^-\gamma^\mu \Sigma^- + \Sigma^+\gamma^\mu \Sigma^+ - 2\Xi^-\gamma^\mu \Xi^- - \Xi^0\gamma^\mu \Xi^0 \right] \\
\times \left( K^- (\partial_\mu K^+) - (\partial_\mu K^-) K^+ \right) \\
+ \left( \bar{p}\gamma^\mu p + \bar{n}\gamma^\mu n + \Sigma^-\gamma^\mu \Sigma^- - \Sigma^+\gamma^\mu \Sigma^+ - \Xi^-\gamma^\mu \Xi^- - 2\Xi^0\gamma^\mu \Xi^0 \right) \\
\times \left( \bar{K}^0 (\partial_\mu K^0) - (\partial_\mu \bar{K}^0) K^0 \right) \\
+ \frac{m_K^2}{2f_K} \left[ (\sigma + \sqrt{2}\zeta + \delta)(K^+ K^-) + (\sigma + \sqrt{2}\zeta - \delta)(K^0 \bar{K}^0) \right] \\
- \frac{1}{f_K} \left[ (\sigma + \sqrt{2}\zeta + \delta)(\partial_\mu K^+)(\partial^\mu K^-) + (\sigma + \sqrt{2}\zeta - \delta)(\partial_\mu K^0)(\partial^\mu \bar{K}^0) \right] \\
+ \frac{d_1}{2f_K^2} (\bar{p}p + \bar{n}n + \bar{K}^0 K^0 + \Sigma^0 \Sigma^0 + \Sigma^\pm \Sigma^\pm + \Xi^- \Xi^- + \Xi^0 \Xi^0)(\partial_\mu K)(\partial^\mu K) \\
+ \frac{d_2}{2f_K^2} \left[ (\bar{p}p + \frac{5}{6} \bar{K}^0 K^0 + \frac{1}{2} \Sigma^0 \Sigma^0 + \Sigma^\pm \Sigma^\pm + \Xi^- \Xi^- + \Xi^0 \Xi^0)(\partial_\mu K^+)(\partial^\mu K^-) \right] \\
+ (\bar{n}n + \frac{5}{6} \bar{K}^0 K^0 + \frac{1}{2} \Sigma^0 \Sigma^0 + \Sigma^\pm \Sigma^\pm + \Xi^- \Xi^- + \Xi^0 \Xi^0)(\partial_\mu K^0)(\partial^\mu \bar{K}^0) \right].
\]

In Eq. 2 the first line stands for the vector interaction term (Weinberg-Tomozawa term) obtained from the kinetic term of the Lagrangian density \[15\]. The second term, which gives an attractive interaction for the $K$-mesons, is obtained from the explicit symmetry breaking term \[14, 15\]. The third term arises within the present chiral model from the kinetic term of the pseudoscalar mesons \[15\]. The fourth and fifth terms in Eq. 2 arise from

\[
\mathcal{L}_{(d_1)}^{BM} = \frac{d_1}{2} Tr(u_\mu u^\mu) Tr(\bar{B}B),
\]

and

\[
\mathcal{L}_{(d_2)}^{BM} = d_2 Tr(\bar{B}u_\mu u^\mu B),
\]
according to Ref. [14, 15]. The last three terms in Eq. (2) represent the range term with the last part being the isospin asymmetric interaction. The Fourier transformation of the equation-of-motion for kaons (antikaons) leads to the dispersion relation

\[-\omega^2 + \vec{k}^2 + m_K^2 - \Pi(\omega, |\vec{k}|, \rho) = 0, \quad \text{(5)}\]

where \(\Pi\) denotes the kaon (antikaon) self energy in the medium.

Explicitly, the self energy \(\Pi(\omega, |\vec{k}|)\) for the kaon doublet \((K^+, K^0)\) arising from the interaction (Eq. (2)) is given by

\[
\Pi(\omega, |\vec{k}|) = -\frac{1}{4f_K^2} \left[ 3(\rho_p + \rho_n) \pm (\rho_p - \rho_n) \pm 2(\rho_{\Sigma^+} - \rho_{\Sigma^-}) - (3(\rho_{\Xi^+} + \rho_{\Xi^0}) \pm (\rho_{\Xi^-} - \rho_{\Xi^0})) \right] \omega
+ m_K^2 \frac{2}{2f_K} (\sigma' + \sqrt{2}\zeta' \pm \delta')
+ \left[ -\frac{1}{f_K} (\sigma' + \sqrt{2}\zeta' \pm \delta') + \frac{d_1}{2f_K^2} (\rho_p^p + \rho_n^n + \rho_A^0 + \rho^{\Sigma^+} + \rho^{\Sigma^-} + \rho^{\Sigma^0} + \rho^{\Xi^+} + \rho^{\Xi^-} + \rho^{\Xi^0})
+ \frac{d_2}{4f_K^2} \left( (\rho_p^p + \rho_n^n) \pm (\rho_p - \rho_n) \pm 2(\rho_{\Sigma^+} \pm \rho_{\Sigma^-} + \rho^{\Sigma^0}) \rho_{\Sigma^+} + (\rho_{\Sigma^+} - \rho_{\Sigma^0}) \right)
+ 2(\rho_{\Xi^+} + \rho_{\Xi^-} + \rho_{\Xi^0}) \right] (\omega^2 - k^2), \quad \text{(6)}
\]

where the \(\pm\) signs refer to \(K^+\) and \(K^0\), respectively. In the equation above \(\sigma'(\equiv \sigma - \sigma_0), \zeta'(\equiv \zeta - \zeta_0)\) and \(\delta'(\equiv \delta - \delta_0)\) are the fluctuations of the scalar-isoscalar fields \(\sigma\) and \(\zeta\), and the third component of the scalar-isovector field \(\delta\) from their vacuum expectation values.

The vacuum expectation value of \(\delta\) is zero \((\delta_0=0)\) because otherwise the vacuum isospin symmetry would be broken. It is important to note that in the present work the small isospin breaking effect coming from the mass and charge difference of the up and down quarks has been neglected. The variables \(\rho_i\) and \(\rho^i\) stand for number density and scalar density of baryon type \(i\), with \(i = p, n, \Lambda, \Sigma^+, \Sigma^0, \Xi^-, \Xi^0\). Similarly, the self-energy for the antikaon doublet \((K^-, K^0)\) is calculated as

\[
\Pi(\omega, |\vec{k}|) = \frac{1}{4f_K^2} \left[ 3(\rho_p + \rho_n) \pm (\rho_p - \rho_n) \pm 2(\rho_{\Sigma^+} - \rho_{\Sigma^-}) - (3(\rho_{\Xi^+} + \rho_{\Xi^0}) \pm (\rho_{\Xi^-} - \rho_{\Xi^0})) \right] \omega
+ m_K^2 \frac{2}{2f_K} (\sigma' + \sqrt{2}\zeta' \pm \delta')
+ \left[ -\frac{1}{f_K} (\sigma' + \sqrt{2}\zeta' \pm \delta') + \frac{d_1}{2f_K^2} (\rho_p^p + \rho_n^n + \rho_A^0 + \rho^{\Sigma^+} + \rho^{\Sigma^-} + \rho^{\Sigma^0} + \rho^{\Xi^+} + \rho^{\Xi^-} + \rho^{\Xi^0})
+ \frac{d_2}{4f_K^2} \left( (\rho_p^p + \rho_n^n) \pm (\rho_p - \rho_n) \pm 2(\rho_{\Sigma^+} \pm \rho_{\Sigma^-} + \rho^{\Sigma^0}) \rho_{\Sigma^+} + (\rho_{\Sigma^+} - \rho_{\Sigma^0}) \right)
+ 2(\rho_{\Xi^+} + \rho_{\Xi^-} + \rho_{\Xi^0}) \right] (\omega^2 - k^2),
\]

where \(\frac{\rho_i}{\rho_n}\) and \(\frac{\rho^i}{\rho^n}\) stand for number density and scalar density of baryon type \(i\), with \(i = p, n, \Lambda, \Sigma^+\) and \(\Xi^+, \Xi^-\), respectively.
\[ + 2\rho_s \Xi - 2\rho_s \Xi_0 \right) \right] (\omega^2 - \vec{k}^2), \] 

(7)

where the ± signs refer to \( K^- \) and \( \bar{K}^0 \), respectively.

The optical potentials are calculated from kaon and antikaon energies using

\[ U(\omega, k) = \omega(k) - \sqrt{k^2 + m_K^2}, \] 

(8)

where \( m_K \) is the kaon (antikaon) vacuum mass. The parameters \( d_1 \) and \( d_2 \) are calculated from empirical values of the KN scattering lengths for \( I=0 \) and \( I=1 \) channels. These are taken to be (as in \([16, 30, 31]\))

\[ a_{KN}(I = 0) \approx -0.09 \text{ fm}, \quad a_{KN}(I = 1) \approx -0.31 \text{ fm}, \] 

(9)

leading to the isospin averaged KN scattering length

\[ \bar{a}_{KN} = \frac{1}{4} a_{KN}(I = 0) + \frac{3}{4} a_{KN}(I = 1) \approx -0.255 \text{ fm}. \] 

(10)

IV. RESULTS AND DISCUSSIONS

The results for the numerical analysis of the presence of kaons and antikaons in (proto-)neutron star matter are presented in this section. Using a chiral SU(3) model, the kaon and antikaon energies are investigated in the electric charge neutral, isospin asymmetric nucleonic matter as well as in hyperonic matter. The effects of finite temperature/entropy as well as neutrino trapping on the in-medium kaon (antikaon) masses and the possibility of antikaon condensation are studied under these circumstances. In the present calculation, the model parameters are determined by fitting the baryon vacuum masses, nuclear matter saturation properties and hyperon optical potentials. The coefficients \( d_1 \) and \( d_2 \), calculated from the empirical values of KN scattering lengths for \( I=0 \) and \( I=1 \) channels (Eq. 9), are 2.5/\( m_K \) and 0.72/\( m_K \) respectively.

In the interior of (proto-)neutron stars, the weak interaction processes \( n \to p + e^- + \bar{\nu}_e \) and \( e^- \to K^- + \nu_e \) can occur. Assuming these processes to be in chemical equilibrium, we have

\[ \mu_n - \mu_p = \mu_e - \mu_{\nu_e} = \mu_{K^-}. \] 

(11)
FIG. 1: Kaon and antikaon energies (at zero momentum) as well as electron chemical potentials plotted for nucleonic matter in (a) and (b) and hyperonic matter in (c) and (d) at T=0 and in beta equilibrium.

For the onset of $K^{-}$ condensation $\mu_{K^{-}} = \omega_{K^{-}}$.

In Fig. 1, we plot the kaon and antikaon energies at zero momentum in (a) and (b) as functions of the baryon density for charge neutral and beta equilibrated nucleonic matter at zero temperature. The modifications to these energies are also shown when hyperons
FIG. 2: Kaon and antikaon energies (at zero momentum) as well as electron chemical potentials plotted for nucleonic matter in (a) and (b) and hyperonic matter in (c) and (d) at $T=0$ and with $Y_{Le}=0.4$. The electron chemical potential plotted as a function of density in (b) and (d) show whether there is a possibility of antikaon condensation in the nucleonic/hyperonic matter. For nucleonic matter, the onset for $K^-\text{condensation}$ takes place at a density of about $5.3 \rho_0$. The net effect of the $K^-$
FIG. 3: Kaon and antikaon energies (at zero momentum) as well as electron chemical potentials plotted for nucleonic matter in (a) and (b) and hyperonic matter in (c) and (d) with $S/B = 2$ and in beta equilibrium.

Condensation is that the respective meson takes the role of electrons maintaining charge neutrality. The inclusion of hyperons shifts the condensation threshold density to $6.1 \rho_0$, a higher value compared to the case that doesn't take hyperons into account (as in $[2, 3]$). This is due to the fact that in the presence of hyperons, the negatively charged baryons
FIG. 4: Kaon and antikaon energies (at zero momentum) as well as electron chemical potentials plotted for nucleonic matter in (a) and (b) and hyperonic matter in (c) and (d) with $S/B = 2$ and $Y_{Le} = 0.4$.

take the role of electrons in maintaining charge neutrality. Consequently the value of the electron chemical potential is reduced, thus shifting the threshold density for onset of $K^-$ condensation to a higher value as compared to the threshold density for nucleonic matter. As a result, in the presence of hyperons the $K^-$ condensation takes place more towards the
Right after the supernova explosion, the proto-neutron star has a high amount of
neutrinos. They are included in the calculation with a chemical potential \( \mu_{\nu_e} \) by fixing the lepton number defined as \( Y_{L_e} = (\rho_e + \rho_{\nu_e})/\rho_B \), where \( \rho_B \) is the total baryonic number density. In this case, there is not only a large number of neutrinos in the star, but also an increased electron density. Therefore, demanding charge neutrality, the proton number density increases and with higher proton density the bulk matter in the star becomes more isospin symmetric, leading to a decrease in the neutron Fermi energy. Thus increasing lepton number softens the equation of state (EOS) and consequently leads to smaller star maximum masses. In Fig. 2 we plot \( K \) and \( \bar{K} \) energies at zero momentum in charge neutral matter with finite lepton number fraction \( Y_{L_e} \approx 0.4 \) and zero temperature. We see that the threshold density for the onset of \( K^- \) condensation is shifted to higher values in the case with neutrino trapping compared to the neutrino free one (in agreement with Ref. \[2, 3\]). For nucleonic matter, our model predicts \( K^- \) condensation at a density of around 9.3 \( \rho_0 \) and in the presence of hyperons, we do not see antikaon condensation even up to a density of around 10 \( \rho_0 \).

Contrary to the case of neutron stars, the temperature in proto-neutron stars cannot be considered to be zero. In this case, we include the heat bath of hadronic and quark quasiparticles within the grand canonical potential of the system. Also, instead of having a constant temperature throughout the star, we fix the entropy per baryon to \( S/B = 2 \) allowing the temperature to increase from zero in the edge to 50 MeV in the center of the star following Ref. \[26\]. This assumption is more realistic and in agreement with dynamical simulations of star evolution and cooling \[32, 33, 34, 35\]. In Fig. 3 we plot the kaon and antikaon energies at zero momentum for the nucleonic and hyperonic matter for the case of finite entropy per baryon. In this case, it is seen that for charge neutral nucleonic matter the onset of \( K^- \) condensation is at a density of around 6 \( \rho_0 \). We might note that in an earlier calculation \[36\], a temperature of around 30 MeV is enough to melt the anti-kaon condensation in nucleonic matter. With the inclusion of hyperons, our calculations show that there is \( K^- \) condensation at a density of around 7.4 \( \rho_0 \) as can be seen from Fig. 3.

Putting together these two features we can simulate the environment existent in proto-neutron stars. In Fig. 4 we plot \( K \) and \( \bar{K} \) energies at zero momentum in neutral matter.
with entropy per baryon $S/B = 2$ and finite lepton number $Y_{Le} = 0.4$. For nucleonic matter, our model predicts $K^-$ condensation at a density of about $10 \rho_0$, whereas the inclusion of hyperons does not give a possibility of $K^-$ condensation even up to a density of $10 \rho_0$. These results show that there is no possibility for antikaon condensation in the earlier moments of the neutron star evolution independent of the composition considered to exist inside the star.

In the present model, the energy of $\bar{K}^0$ at zero momentum is never zero and hence the condition for $\bar{K}^0$ condensation, i.e., $\mu_{\bar{K}^0} = 0$ is never met. In other words, the condensation of $\bar{K}^0$ does not take place at any moment of the neutron star evolution in the our calculations using the SU(3) chiral model.

V. SUMMARY AND CONCLUSIONS

To summarize, within a chiral SU(3) model we have investigated the density dependence of $K$, $\bar{K}$-meson energies in (proto-) neutron star bulk matter. The model parameters are fitted to reproduce hadron masses in vacuum, nuclear matter saturation properties, hyperon optical potentials and low energy KN scattering data. The model takes into account all terms up to the next to leading order arising in the chiral perturbative expansion for the interactions of $K(\bar{K})$-mesons with baryons. The medium modification of antikaons due to isospin asymmetry in dense matter can lead to the onset of antikaon condensation in the bulk charge neutral matter in (proto-) neutron stars.

We concluded that the drop in the antikaon mass in the medium leads to a possibility of antikaon condensation in cold neutron star matter. Our calculations show that in this case the onset of $K^-$ condensation takes place at a density of about $5.3 \rho_0$ when the charge neutral matter comprises of only neutrons, protons, electrons and muons. This threshold density is seen to shift to higher densities of around $6.1 \rho_0$ when hyperons are included. The effect of neutrino trapping as well as finite temperature/entropy is a shift of the threshold density for the antikaon condensation in proto-neutron star matter. In this sense we conclude that there is no possibility for anti-kaon condensation in proto-neutron stars. Only after about a minute from the supernova explosion, when the star cools down via mainly neutrino emission the condensation can take place. Still, even in this case the anti-kaon condensation happens
only in the very center of the star and does not cause considerable changes in macroscopic properties of the star, such as mass and radius.

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