Deorbiting of space debris by using the Lorentz force along the spiral of Archimedes

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Abstract. The proposals had been considered for development an analytical engineering methodology for the preliminary assessment of the impact of the space debris (SD) value object charge on the dynamics of its descent from orbit. The Earth's magnetic field is taken as a dipole, and the charge of SD is taken as a point charge. The orbital motion of the SD is approximated by the Archimedes spiral, the choice of its parameter (spiral step) provides a close approximation to the orbital motion of the SD. The use of the proposed analytical engineering methodology allows to make reasonable design decisions on the control system and charge formation on the SD at the early stages of designing systems propellantless deorbiting. The results have a methodological nature and are intended to provide preliminary estimates.

1. Introduction
The increase of the number and total mass of space debris (SD) leads to the fact that the orbiting functioning spacecraft have to be protected by different means [1].

There are concepts for the active deorbiting of SD through the use of maneuvering space vehicle which requires a number of tasks: guidance, docking, towing SD into the orbit of disposal [2]. In addition to these tasks, significant fuel reserves are required onboard the space vehicle. Existing other methods of removal SD from working orbits to the disposal orbit based on the installation of electrodynamic tethers solar sails, systems of interaction with the Earth's magnetic field and much more can be attributed to propellantless methods [3]. In [4] the use of Lorentz force is considered to decrease the relative speed of approach of SD to the "invader" on which the jet deorbiting system is installed.

In conditions of low Earth orbit (with a height of up to 2000 km) it is promising to use a system for providing an electric charge, on which the Lorentz force acts in the Earth's magnetic field.

The gravitational and magnetic fields of the Earth together have influenced the motion of SD. Therefore, when compiling the equation of motion of the charged SD in general, in addition to the gravitational force of gravity, we will take into account the Lorentz force. Note also that in the calculation of circular orbits to determine the position of SD it is more convenient to use a spherical coordinate system.

From the equations of motion of SD it is possible to express the specific charge, which can be numerically calculated by setting the trajectory of motion, thereby solving the inverse problem of dynamics. With great accuracy, the orbital motion could be replaced to the motion of the spiral...
Archimedes. At the moment, aerodynamic brakes are already used when braking SD about the upper atmosphere during the lowering from orbit [5], which facilitates the task of cleaning the low Earth orbit, it is only enough to take it to an orbit 80-100 km high from the Earth's surface.

The aim of the study is to develop an engineering methodology for the project assessment of the magnitude of the change in the required charge over time for its descent from orbit.

2. Methodology of estimation of motion parameters of charged SD under the action of Lorentz force

Methodology includes drawing up the equations of motion of the charged SD in the Earth's magnetic field in a general form in spherical coordinates and determines the specific charge of SD moving in the Equatorial plane along the spiral of Archimedes.

2.1. Assumptions

1) The whole mass of SD is concentrated in the center of mass and it is constant in time;
2) The SD has point charge variable with on-board system forming charge;
3) The Electrostatic charge is changing by the on-board charge formation system in the shortest period of time, which can be neglected and considered that the value of charge can be changed instantly;
4) On SD act only the forces of gravity and Lorentz, the influence of disturbing forces, such as the pressure of sunlight, the force of the Coulomb and others can be neglected;
5) The Magnetic field of the Earth has been replaced the ideal magnetic field of the dipole, located in the center of the planet. The axis of the dipole combined with the axis of rotation of the Earth. The angle of the dipole is equal to the angle of the Earth's axis. The dipole rotates synchronously with the Earth.

2.2. Equation of motion of SD under the action of Lorentz force

Figure 1 shows a spherical coordinate system with unit vectors describing the position of SD relative to the origin point, which combined with the center of the Earth's magnetic dipole.

![Figure 1. Position of SD in a spherical coordinate system [6]](image)

In a spherical coordinate system to determine the position of SD it is necessary to know three parameters:

1) $r$ – distance from the center of the Earth's dipole to SD;
2) $\theta$ – latitude angle;
3) $\phi$ – longitude angle.
As mentioned earlier, only the Lorentz forces and the Earth’s gravity act on the SD, which are potential (conservative), then according to the principle of least action, the Lagrange equation of the 2nd kind is applicable for the conservative system:

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{r}} \right) - \frac{\partial L}{\partial r} = 0
\] (1)

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\phi}} \right) - \frac{\partial L}{\partial \phi} = 0
\] (2)

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = 0,
\] (3)

where the Lagrange function is equal to the difference between the kinetic \( T \) and the potential energy \( \Pi \) SD:

\[
L = T - \Pi
\] (4)

The kinetic energy of SD in a spherical coordinate system is equal [6]:

\[
T = \frac{m}{2} \left( \dot{r}^2 + r^2 \cdot \dot{\theta}^2 + r^2 \cdot \dot{\phi}^2 \cdot \sin^2 \theta \right).
\] (5)

Potential energy folding from the potential energy of the Earth’s gravitational force and the potential energy of The Earth’s magnetic field. The Earth’s gravity vector:

\[
\vec{F}_g = -\frac{m \cdot \mu \cdot \vec{r}}{r^3}
\] (6)

And its potential energy gravitational field:

\[
\Pi_g = -\frac{\mu \cdot m}{r}.
\] (7)

The Lorentz force vector [7]:

\[
\vec{F}_L = q \cdot (\vec{E} + \vec{V}_{rel} \times \vec{B})
\] (8)

The electric field of the Earth \( E \) in the first approximation can be neglected \( E = 0 \) therefore equation (8) takes the form:

\[
\vec{F}_L = q \cdot (\vec{V}_{rel} \times \vec{B}),
\] (9)

where is the relative speed SD \( V_{rel} \) is equal to the difference between the absolute speed of SD and the portable speed of the Earth's magnetic field.

\[
\vec{V}_{rel} = \vec{V} - \vec{\omega} \times \vec{r}
\] (10)

The module of the relative velocity is equal to:

\[
V_{rel} = |\vec{V} - \vec{\omega} \times \vec{r}| = \dot{\phi} \cdot r \cdot \sin \theta - \omega \cdot r \cdot \sin \theta = r \cdot \sin \theta \cdot (\dot{\phi} - \omega)
\] (11)

Potential energy of magnetic field [8]:

\[
\Pi_L = q \cdot \vec{V}_{rel} \cdot \vec{A},
\] (12)
where the vector potential of the magnetic field [8] is equal:

\[ \vec{A} = \left( \frac{B_0 \cdot \sin \theta}{r^2} \right) \cdot \hat{\phi}. \]

(13)

The vectors of the relative velocity and the vector potential of the magnetic field are collinear and oppositely directed, the cosine of the angle between them is equal to \(-1\). The charge \(q\) presented as a product of the specific charge \(\lambda\) weight \(SD\) \(m\): \(q = m\lambda\).

\[ \Pi_L = q \cdot \vec{V}_{rel} \cdot \vec{A} = \lambda \cdot m \cdot r \cdot \sin \theta \cdot (\dot{\phi} - \omega) \cdot \frac{B_0 \cdot \sin \theta}{r^2} \cdot \cos(180^\circ) = -\lambda \cdot m \cdot \sin^2 \theta \cdot \frac{B_0}{r} \cdot (\dot{\phi} - \omega) \]

(14)

Substituting the resulting expression of the potential energy of the gravitational and magnetic fields in the in equation (4), we obtain the Lagrange function:

\[ L = T - \Pi_L = \frac{m}{2} \left( \dot{\vec{r}}^2 + r^2 \cdot \dot{\theta}^2 + r^2 \cdot \dot{\phi}^2 \cdot \sin^2 \theta \right) - \left( -\frac{\mu \cdot m}{r} - \lambda \cdot m \cdot \sin^2 \theta \cdot \frac{B_0}{r} \cdot (\dot{\phi} - \omega) \right) = \]

\[ = \frac{m}{2} \left( \dot{r}^2 + r^2 \cdot \dot{\theta}^2 + r^2 \cdot \dot{\phi}^2 \cdot \sin^2 \theta \right) + \frac{\mu \cdot m}{r} + \lambda \cdot m \cdot \sin^2 \theta \cdot \frac{B_0}{r} \cdot (\dot{\phi} - \omega) \]

(15)

Substituting the Lagrange function in equations (1), (2) and (3), we obtain the equations of motion:

\[ m \left[ \dot{\vec{r}} - r \cdot \left( \ddot{\theta} + \dot{\phi}^2 \cdot \sin^2 \theta \right) \cdot \frac{\mu}{r^2} + \lambda \cdot \sin^2 \theta \cdot \frac{B_0}{r^2} \cdot (\dot{\phi} - \omega) \right] = 0 \]

(16)

\[ m \left[ r^2 \cdot \ddot{\theta} + 2 \cdot \dot{r} \cdot r \cdot \dot{\theta} \cdot \sin \theta \cdot \cos \theta \cdot \left( r^2 \cdot \dot{\phi}^2 + 2 \lambda \cdot \frac{B_0}{r} \cdot (\dot{\phi} - \omega) \right) \right] = 0 \]

(17)

\[ m \left[ \ddot{\phi} \cdot r^2 \cdot \sin \theta + 2 \dot{\phi} \cdot r \cdot \sin \theta \cdot \left( \sin \theta \cdot \dot{r} + r \cdot \cos \theta \cdot \dot{\theta} \right) + \lambda \cdot \frac{B_0}{r} \cdot \sin \theta \cdot \left( 2 \cdot \cos \theta \cdot \dot{\phi} - \sin \theta \cdot \dot{r} \right) \right] = 0 \]

(18)

3. Approximation of the orbital motion SD in the form of a spiral of Archimedes

For the preliminary estimate, let us assume that SD moves in the plane of the equator, the Lorentz force in this case is maximal, because the angle between the magnetic induction vectors and the velocity of SD is direct.

Provided that the orbit is Equatorial, therefore:

\[ \theta = 90^\circ, \ \dot{\theta} = 0, \ \ddot{\theta} = 0. \]

(19)

A new system of equations:

\[ m \left[ \dot{\vec{r}} - r \cdot \dot{\phi}^2 + \frac{\mu}{r^2} + \lambda \cdot \frac{B_0}{r^2} \cdot (\dot{\phi} - \omega) \right] = 0 \]

(20)

\[ m \left[ \ddot{\phi} \cdot r^2 + 2 \dot{\phi} \cdot \dot{r} \cdot r - \lambda \cdot \frac{B_0}{r^2} \dot{r} \right] = 0 \]

(21)
And specific charge:

\[ \lambda = \frac{r^3 \cdot \dot{\varphi}^2 - \mu - r^2 \cdot \dot{r}}{B_0 \cdot (\dot{\varphi} - \omega_r)} \]  

(22)

The orbital motion of the SD is approximated by the Archimedes spiral, the choice of its parameter (spiral step) provides a close approximation to the orbital motion of the SD.

The equation of Archimedean spiral in polar coordinates:

\[ r = r_0 - \frac{a}{2\pi} \varphi , \]  

(23)

The longitude angle:

\[ \varphi = \dot{\varphi} \cdot t , \]  

(24)

Taking into account the constancy of the angular velocity SD \( \dot{\varphi} = \text{const} \):

\[ r = r_0 - \frac{a}{2\pi} \dot{\varphi} \cdot t \]  

(25)

The dependence of the specific charge on the flight time \( t \):

\[ \lambda = \frac{\left( r_0 - \frac{a}{2\pi} \dot{\varphi} \cdot t \right)^3 \cdot \dot{\varphi}^2 - \mu}{B_0 \cdot (\dot{\varphi} - \omega_r)} \]  

(26)

4. Example of methodology implementation

According to the source data (table 1) the specific charge was determined and graph was built depending on the time (Figure 2) [9]. For example was chosen as the second stage rocket Kosmos – 3M, located at an altitude of 700 km. The calculated altitude has been defined at which it will be after 24 years. The spiral step is taken \( a = 5 \) m. Calculations are carried out using Mathcad 14.
After 24 years, the SD will be at an altitude of 60 km from the Earth's surface.

Comparing the formula (26) and the graph (Figure 2) at first sight you might think that they do not correspond to each other: graph (Figure 2) linear, but the formula has cubic dependence. This paradox is due to the fact that the calculated coefficient $k$ before time $t$ in the formula (26) is equal to:

$$k = \frac{a}{2\pi} \phi = 8.45 \cdot 10^{-7} \text{km/s}.$$ 

Opening the cube of difference from (26) we have:

$$(r_0 - k \cdot t)^3 = r_0^3 - 3 \cdot r_0^2 \cdot k \cdot t + 3 \cdot r_0 \cdot (k \cdot t)^2 - (k \cdot t)^3,$$

where the last two members are very small and can be neglected.

$$(r_0 - k \cdot t)^3 = r_0^3 - 3 \cdot r_0^2 \cdot k \cdot t.$$ (28)

We obtain a linear dependence on the flight time (29), which is confirmed by its graph (Figure 3):

$$\lambda = \frac{(r_0 - 3 \cdot k \cdot t) \cdot r_0^2 \cdot \phi^2 - \mu}{B_0 \cdot (\phi - \omega_f)}$$ (29)

![Figure 3](image)

**Figure 3.** The linear graph The dependence of the value of the specific charge SD time of flight along the spiral of Archimedes

The relative error of the linear approximation at the end of the deorbiting trajectory:

$$\Delta = \left| \frac{12.469 - 13.668}{12.469} \right| = 9.615\%$$ (30)

This error is acceptable for the first approximation, so the calculation of the required specific charge will be based on the linear formula (29). These results will be used in further design of the charge formation system. Examples of such a system are already exist in [10].
5. Conclusions
The proposals had been considered for development an analytical engineering methodology for the preliminary assessment of the impact of the space debris (SD) value object charge on the dynamics of its descent from orbit. Analysis of existing propellantless methods of descent SD has been performed. The fundamental interaction based on Lorentz forces is chosen as the basic method.

The orbital motion of the SD is approximated by the Archimedes spiral, the choice of its parameter (spiral step) provides a close approximation to the orbital motion of the SD. Acceptable engineering accuracy is achieved by choosing the step of the Archimedes spiral, which is necessary to estimate the value of the required charge SD.

The use of the proposed analytical engineering methodology allows to make reasonable design decisions on the control system and charge formation on the SD at the early stages of designing systems propellantless deorbiting. The results have a methodological nature and are intended to provide preliminary estimates. The problem has been solved when the orbit parameters of the approximated Archimedes spiral reach the value of about 100 km.

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