Dark energy model with higher derivative of Hubble parameter

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Abstract

In this letter we consider a dark energy model in which the energy density is a function of the Hubble parameter $H$ and its derivative with respect to time $\rho_{de} = 3\alpha \ddot{H}H^{-1} + 3\beta \dot{H} + 3\gamma H^2$. The behavior of the dark energy and the expansion history of the Universe depend heavily on the parameters of the model $\alpha$, $\beta$, and $\gamma$. It is very interesting that the age problem of the well-known three old objects can be alleviated in this models.

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It has been strongly confirmed that our Universe is undergoing an accelerated expansion by many observations including the Type Ia supernova (SN) \[^1\], cosmic microwave background (CMB) \[^2\] and large scale structure (LSS) \[^3\], and so on. The late time cosmic acceleration challenges our understanding of the standard models of gravity and particle physics. Within the framework of Einstein’s general relativity, the Universe is supposed to be filled with dark energy to explain this observed phenomena. The dark energy is an exotic energy component with negative pressure and the experiments have indicated that today it constitutes about 70% of present total cosmic energy. However, so far, the nature of dark energy is still unclear.

It seems that the preferred candidate for dark energy is the cosmological constant \[^4\], which is a term that can be added to Einstein’s equations. This term acts like a perfect fluid with an equation of state \(\omega = -1\), and the energy density is associated with quantum vacuum. Recent investigations indicate that the cosmological constant is consistent with observational data. However, it is very difficult to understand in the modern field theory since the vacuum energy density is far below the value predicted by any sensible quantum field theory. Moreover, it has also been plagued by the so-called coincidence problem, namely, “why are the vacuum and matter energy densities of precisely the same order today?” In order to eliminate these problems, a lot of the dynamical scalar fields, such as quintessence \[^5\], k-essence \[^6\], phantom \[^7\] and quintom field \[^8, 9\], have been put forth as an alternative of dark energy. The another possible way of explaining the cosmic accelerated expansion is that Einstein’s theory should be modified, for example the \(f(R)\) theory \[^10\] and the DGP model \[^11\].

It is well known that the holographic principle \[^12\] plays an important role in the black hole and string theory, which is based on the fact in quantum gravity, the entropy of a system scales not with its volume, but with its surface area \(L^2\). Inspired by the holographic principle, A. Cohen et al \[^13\] suggested that the vacuum energy density is proportional to the Hubble scale \(l_H \sim H^{-1}\). In this model, both the fine-tuning and coincidence problems can be alleviated, but it can not explain the cosmic accelerated expansion because that the effective equation of state for such vacuum energy is zero. Recently, M. Li \[^14\] proposed that the future event horizon of the Universe to be used as the characteristic length \(l\). This holographic dark energy model not only presents a reasonable value for dark energy density, but also leads to an acceleration solution for the cosmic expansion. In fact, the choice of the characteristic length \(l\) is not the unique for the holographic dark energy model. Gao et al \[^15\] assumed that the length \(l\) is giving by the the inverse of Ricci scalar curvature, i.e., \(|R|^{-1/2}\), which is the so-called holographic Ricci dark energy model. It is argued that this model can
solve the coincidence problem entirely. Thus, the properties of such holographic Ricci dark energy have been investigated widely recently. L. N. Granda et al. proposed a modified Ricci dark energy model in which the density of dark energy is a function of the Hubble parameter $H$ and its derivative with respect to time $\dot{H}$. However, all of these models have been plagued with the age problem of the well-known three old objects, LBDS 53W091 ($z = 1.55, t = 3.5Gyr$), LBDS 53W069 ($z = 1.43, t = 4.0Gyr$), and APM 08279+5255 ($z = 3.91, t = 2.1Gyr$). In this letter, we present a generalized dark energy model in which density of dark energy contains the second order derivative with respect to time $\ddot{H}$ and find that the age problem of the old objects above can be alleviated.

It is well known that in the Einstein general relativity theory, a flat universe is described by the standard Friedmann equation

$$H^2 = \frac{1}{3} \rho.$$  \(\text{(1)}\)

This means that the total density $\rho$ of the Universe is a function of the Hubble parameter $H$. Since the dark energy occupies almost 70% of the content of the universe today, it is rational to assume that the density of dark energy is a function of the Hubble parameter $H$ and its derivatives with respect to time. For the mathematical simplicity, we here assume that the density of dark energy contains the Hubble parameter $H$, the first order and the second order derivatives (i.e, $\dot{H}$ and $\ddot{H}$). The concrete expression of the density of dark energy is given by

$$\rho_{de} = 3\alpha \dot{H} H^{-1} + 3\beta \ddot{H} + 3\gamma H^2,$$  \(\text{(2)}\)

where $\alpha, \beta$ and $\gamma$ are three arbitrary dimensionless parameters. The main reason we choose such a form for the density of dark energy is that the Friedman equation \(\text{(1)}\) can be rewritten as a second order differential equation with constant coefficients (see Eq.\(\text{(3)}\) below) and we can obtain an analytical general solution for it, which is very convenient for us to study the properties of the model in the following calculations. Moreover, we introduce the inverse of Hubble parameter (i.e, $H^{-1}$) in the first term in the density of dark energy so that the dimensions of the terms in the equation \(\text{(2)}\) are identical. When $\alpha = 0$, it can be reduce to the modified Ricci dark energy model. Since possessing an extra free parameter $\alpha$, the model \(\text{(2)}\) is more general than the modified Ricci dark energy. The similar dark energy models have also been studied in \[31, 32\].

Setting the variable $x = \ln a$ and substituting the density of dark energy \(\text{(2)}\) into Eq.\(\text{(1)}\), the Friedman equation can be written as

$$\frac{\alpha}{2} \frac{d^2 h^2}{dx^2} + \frac{\beta}{2} \frac{dh^2}{dx} + (\gamma - 1)h^2 + \Omega_m a^3 e^{-3x} = 0,$$  \(\text{(3)}\)
where \( h = H / H_0 \), \( \Omega_{m0} = \rho_{m0} / (3H_0^2) \) and \( H_0 \) is the Hubble constant. Neglecting the contribution from the radiation, the general solution of the differential equation (3) can be expressed by

\[
h^2 = \Omega_{m0} e^{-3x} + f_0 e^{-\frac{\beta - \sqrt{\beta^2 - 8\alpha(\gamma - 1)}}{2\alpha} x} + f_1 e^{\frac{\beta + \sqrt{\beta^2 - 8\alpha(\gamma - 1)}}{2\alpha} x} + \frac{9\alpha - 3\beta + 2\gamma}{2 - 9\alpha + 3\beta - 2\gamma} \Omega_{m0} e^{-3x},
\]

(4)

where \( f_0 \) and \( f_1 \) are integration constants. From the initial condition we find that \( f_0 \) and \( f_1 \) satisfy

\[
f_0 + f_1 + \frac{2}{2 - 9\alpha + 3\beta - 2\gamma} \Omega_{m0} = 1.
\]

(5)

From this equation we obtain that the integration constants \( f_0 \) and \( f_1 \) can not be decided exactly. In other word, there exists a free integration constant between \( f_0 \) and \( f_1 \). Here we only consider the cases \( f_1 = 0 \), \( f_0 = 0 \) and \( f_1 = f_0 \) for simplicity, and then we study the properties of the dark energy model (2) and check the age problem of three old objects.

Let us first consider the case \( f_1 = 0 \). It is very easy to obtain that the density and pressure of dark energy can be expressed by

\[
\rho_{de} = H_0^2 \left[ f_0 e^{-\frac{\beta - \sqrt{\beta^2 - 8\alpha(\gamma - 1)}}{2\alpha} x} + \frac{9\alpha - 3\beta + 2\gamma}{2 - 9\alpha + 3\beta - 2\gamma} \Omega_{m0} e^{-3x} \right],
\]

(6)

and

\[
p_{de} = -\rho_{de} - \frac{1}{3} \frac{d\rho_{de}}{dx} = -\frac{6\alpha - \beta + \sqrt{\beta^2 - 8\alpha(\gamma - 1)}}{6\alpha} f_0 H_0^2 e^{-\frac{\beta - \sqrt{\beta^2 - 8\alpha(\gamma - 1)}}{2\alpha} x},
\]

(7)

respectively. Here

\[
f_0 = \frac{2 - 9\alpha + 3\beta - 2\gamma - 2\Omega_{m0}}{2 - 9\alpha + 3\beta - 2\gamma}.
\]

(8)

Thus, the equation of state is

\[
\omega = \frac{p_{de}}{\rho_{de}} = -\frac{(6\alpha - \beta + \sqrt{\beta^2 - 8\alpha(\gamma - 1)})(2 - 9\alpha + 3\beta - 2\gamma - 2\Omega_{m0})e^{\frac{6\alpha - \beta + \sqrt{\beta^2 - 8\alpha(\gamma - 1)}}{2\alpha} x}}{6\alpha[(9\alpha - 3\beta + 2\gamma)\Omega_{m0} + (2 - 9\alpha + 3\beta - 2\gamma - 2\Omega_{m0})e^{\frac{6\alpha - \beta + \sqrt{\beta^2 - 8\alpha(\gamma - 1)}}{2\alpha} x}]}.
\]

(9)

Obviously, the equation of state of dark energy depends on the parameters \( \alpha, \beta, \gamma, \Omega_{m0} \) and the variable \( x \). From Eq.(9), it is easy to find that when \( 9\alpha - 3\beta + 2\gamma = 0 \) the equation of state of dark energy is a constant \( \omega = -1 + \frac{\beta - \sqrt{\beta^2 - 8\alpha(\gamma - 1)}}{6\alpha} \). Especially, when \( \gamma = 1 \) and, the model (2) can recover the cosmological constant [4]. As \( \alpha(\gamma - 1) > 0 \) (\( \alpha(\gamma - 1) < 0 \)), it is corresponded to the constant \( \omega \) quintessence [5] (phantom [7]) model.

For the case \( 9\alpha - 3\beta + 2\gamma \neq 0 \), it describes the model in which the equation of state of dark energy is variable with the time. The behavior of dark energy (2) depends on the values of \( \alpha, \beta, \gamma \) and \( \Omega_{m0} \). From the Fig. (1), we find that for chosen \( \alpha, \beta \) and \( \gamma \) the dark energy (2) has the behaviors of “quintom-like” field (such as
FIG. 1: The change of the equation of state $\omega$ with the redshift $z$ for different $\alpha$ and $\gamma$. Here we set $\beta = 1$ and $\Omega_{m0} = 0.27$.

FIG. 2: The change of the age of Universe with the redshift $z$. The solid, dotted and dashed lines denote the cases $(\alpha = 0.2, \beta = 2.4, \gamma = 0.8)$, $(\alpha = 0.1, \beta = 2.2, \gamma = 0.9)$ and $(\alpha = 0.3, \beta = 2.6, \gamma = 1)$, respectively. The large points denote the ages of the three old objects LBDS 53W091 ($z = 1.55, t = 3.5$ Gyr), LBDS 53W069 ($z = 1.43, t = 4.0$ Gyr) and APM 08279+5255 ($z = 3.91, t = 2.1$ Gyr). Here we set $\Omega_{m0} = 0.27$ and $h_0 = 0.78$.

$\alpha = 0.2, \beta = 1, \gamma = 0.8$ \cite{8,9} and of “quintessence-like” matter (such as $\alpha = 0.1, \beta = 1, \gamma = 1.2$) \cite{5}, which are similar to that of the modified holographic Ricci dark energy \cite{15,26,27}.

The evolution of age of the Universe is described as

$$t = \frac{1}{H_0} \int_{-\infty}^{-\ln(1+z)} \frac{dx}{h},$$

where $x = \ln a$ and $h = H/H_0$. From the observations of the Hubble Space Telescope Key project, the present
Hubble parameter is constrained to be $H_0 = 9.776 h_0^{-1}$, where $0.64 < h_0 < 0.80$. In Fig.(2) we plotted the curves of age of the Universe for different values of $\alpha$, $\omega$ and $\Omega_{m0}$. We also check the ages problem of several old objects, LBDS 53W091 ($z = 1.55, t = 3.5 Gyr$) [28], LBDS 53W069 ($z = 1.43, t = 4.0 Gyr$) [29] and APM 08279+5255 ($z = 3.91, t = 2.1 Gyr$) [30] and find that for chosen parameter the ages problem can be alleviated in this case.

Now let us consider the second case $f_0 = 0$. Similarly, the density and pressure of dark energy can be written as

$$\rho_{de} = H_0^2 \left[ f_1 e^{-\frac{\beta+\sqrt{\beta^2-8\alpha(\gamma-1)}}{2\alpha} x} + \frac{9\alpha - 3\beta + 2\gamma}{2 - 9\alpha + 3\beta - 2\gamma} \Omega_{m0} e^{-3x} \right],$$

and

$$p_{de} = -\rho_{de} - \frac{1}{3} \frac{d\rho_{de}}{dx} = -\frac{6\alpha - \beta - \sqrt{\beta^2 - 8\alpha(\gamma - 1)}}{6\alpha} f_1 H_0^2 e^{-\frac{\beta+\sqrt{\beta^2-8\alpha(\gamma-1)}}{2\alpha} x},$$

respectively. The integration constants $f_1$

$$f_1 = \frac{2 - 9\alpha + 3\beta - 2\gamma - 2\Omega_{m0}}{2 - 9\alpha + 3\beta - 2\gamma}.$$  

Thus, the equation of state is

$$\omega = \frac{p_{de}}{\rho_{de}} = -\frac{(6\alpha - \beta - \sqrt{\beta^2 - 8\alpha(\gamma - 1)})(2 - 9\alpha + 3\beta - 2\gamma - 2\Omega_{m0})e^{3x}}{6\alpha[(9\alpha - 3\beta + 2\gamma)\Omega_{m0} e^{\frac{\beta+\sqrt{\beta^2-8\alpha(\gamma-1)}}{2\alpha} x} + (2 - 9\alpha + 3\beta - 2\gamma - 2\Omega_{m0})e^{3x}]}.$$

Similarly, as $9\alpha - 3\beta + 2\gamma = 0$ the equation of state of dark energy is a constant $\omega = -1 + \frac{\beta+\sqrt{\beta^2-8\alpha(\gamma-1)}}{6\alpha}$. When $\alpha < -2/9$ and $\beta = (9\alpha + 2)/3$, we find that the equation of state $\omega = -1$. It is shown in Fig. (3) that

![Fig. 3: The change of the equation of state $\omega$ with the redshift $z$ for different $\alpha$ and $\gamma$. Here we set $\beta = -1$ and $\Omega_{m0} = 0.27.$](image-url)
the dark energy (2) has the behaviors of “quintom-like” field and of “quintessence-like” matter for different values of $\alpha$, $\beta$ and $\gamma$ in this case. Moreover, as in the case I, the age problem of old high redshift objects can be alleviated in the model (2).

![Figure 4: The change of the age of Universe with the redshift $z$. The solid, dotted and dashed lines denote the cases $(\alpha = -0.9, \beta = -1, \gamma = 0.9)$, $(\alpha = -1, \beta = -1, \gamma = 0.92)$ and $(\alpha = -0.9, \beta = -1, \gamma = 1)$, respectively. The large points denote the ages of the three old objects LBDS 53W091 ($z = 1.55, t = 3.5Gyr$), LBDS 53W069 ($z = 1.43, t = 4.0Gyr$) and APM 08279+5255 ($z = 3.91, t = 2.1Gyr$). Here we set $\Omega_{m0} = 0.27$ and $h_0 = 0.78$.]

Now we consider the case III: $f_0 = f_1$. From the initial condition (5), we obtain that

$$f_0 = f_1 = \frac{2 - 9\alpha + 3\beta - 2\gamma - 2\Omega_{m0}}{2(2 - 9\alpha + 3\beta - 2\gamma)}. \quad (15)$$

Repeating the previous operation, we find that the density, pressure and the equation of state of dark energy are

$$\rho_{de} = H_0^2 f_1 \left[ e^{-\frac{\beta + \sqrt{\beta^2 - 8\alpha(\gamma - 1)}}{2\alpha} x} + e^{-\frac{\beta - \sqrt{\beta^2 - 8\alpha(\gamma - 1)}}{2\alpha} x} \right] + \frac{9\alpha - 3\beta + 2\gamma}{2 - 9\alpha + 3\beta - 2\gamma} \Omega_{m0} e^{-3x}, \quad (16)$$

$$p_{de} = -\rho_{de} - \frac{1}{3} \frac{d\rho_{de}}{dx} = f_1 H_0^2 \left[ \left( \beta + \sqrt{\beta^2 - 8\alpha(\gamma - 1)} - 6\alpha \right) e^{-\frac{\beta + \sqrt{\beta^2 - 8\alpha(\gamma - 1)}}{2\alpha} x} \right. \right.$$

$$+ \left( \beta - \sqrt{\beta^2 - 8\alpha(\gamma - 1)} - 6\alpha \right) e^{-\frac{\beta - \sqrt{\beta^2 - 8\alpha(\gamma - 1)}}{2\alpha} x}] \left. \right], \quad (17)$$

and

$$\omega = \frac{p_{de}}{\rho_{de}} = \frac{f_1 \left[ \left( \beta + \sqrt{\beta^2 - 8\alpha(\gamma - 1)} - 6\alpha \right) e^{-\frac{\beta + \sqrt{\beta^2 - 8\alpha(\gamma - 1)}}{2\alpha} x} + \left( \beta - \sqrt{\beta^2 - 8\alpha(\gamma - 1)} - 6\alpha \right) e^{-\frac{\beta - \sqrt{\beta^2 - 8\alpha(\gamma - 1)}}{2\alpha} x} \right]}{6\alpha f_1 \left( e^{-\frac{\beta + \sqrt{\beta^2 - 8\alpha(\gamma - 1)}}{2\alpha} x} + e^{-\frac{\beta - \sqrt{\beta^2 - 8\alpha(\gamma - 1)}}{2\alpha} x} \right) + \frac{9\alpha - 3\beta + 2\gamma}{2 - 9\alpha + 3\beta - 2\gamma} \Omega_{m0} e^{-3x}}. \quad (18)$$
respectively. Similarly, as $2 - 9\alpha + 3\beta - 2\gamma = 0$ and $\beta^2 - 8\alpha(\gamma - 1) = 0$, the equation of state of dark energy becomes a constant $\omega = -1 + \frac{\beta}{6\alpha}$. It is easy to obtain that as $\gamma = 1, \beta = 0$ it reduces to cosmological constant. In the general case, the equation of state of dark energy is variable with the time which is shown in Fig.(5). We also examine the age problem of old high redshift objects and find it can be alleviated in this case.

In summary, we studied a dark energy model with higher derivative of Hubble parameter. This model can
be reduced to the dark energy with the constant $\omega$, Ricci-like dark energy models, and so on. The behavior of the dark energy and the expansion history of the Universe depend heavily on the parameters of the model $\alpha$, $\beta$ and $\gamma$. We check the age problem of the three old objects for three special cases in the model and find it can be alleviated for the chosen parameters. It implies that this kind of studies can help us to understand more about dark energy.

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