Optimal decision for two dimensional maintenance service contract involving three parties

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Abstract. A two-dimensional (i.e. time and use) maintenance service contract involving three parties i.e. manufacturer, agent and consumer are considered in this paper. The contract period by the manufacturer/agent begins since the warranty expires \(W\) until the lifetime of the equipment expires \(L\). The use of equipment by consumer is modeled with accelerated failure time (AFT) model. AFT model specifies the limit value of permitted equipment usage \(U_l\). The contract time is limited to \(t = L\), but equipment usage may exceed \(U_l\). High use of equipment will increase consumer revenue. However, the consumer will incur additional costs paid to the manufacturer/agent if the use of equipment exceeds \(U_l\). Therefore, consumers will seek the optimum value of use \((U^*)\) that maximizes the profits. As with consumer, manufacturer and agent also want to maximize their profits. Decision problems in maintenance service contracts involving the three parties will be resolved consider the interaction of the manufacturer and the agent in carrying out maintenance action. One level optimization used if the manufacturer and agent compete with each other. However, if the manufacturer and agent interact in a cooperation, then optimization of the decision model becomes more difficult and should be done with bilevel optimization.

1. Introduction
The company needs equipment to support its business processes. If the equipment used in operating conditions is running well (functioning), then the business is expected to going well. For example, the coal mining business process depends on the conditions of heavy equipment. This is because heavy equipment such as dump trucks, excavators, shovels etc. used by companies to carry out major activities in the business i.e. loading and hauling.

In general, equipment will deteriorate along with age and/or usage. Deterioration can cause failure if it has exceeded a certain threshold. Failure that occurs will reduce the performance of companies that use equipment (consumer). An effective way to slow down the deterioration of equipment so that failure can be minimized and can then keep its availability high is maintenance (Boland, 1982). Because failure is more often characterized by use e.g. in the case of heavy equipment (and other transportation equipment businesses), this study considering two-dimensional maintenance i.e. time and use.

In coal mining companies, taking into account that the company's main business is not a heavy equipment business, usually the company does not carry out maintenance in-house (by company itself) but subcontracts to the manufacturer or original equipment manufacturer (OEM) and/or agent. In this
research, consumer tend to choose two-dimensional maintenance service contracts (MSC) to fulfill the responsibility.

MSC research involving two parties has been carried out a lot by previous researchs. Among them is a MSC that involves the OEM with the consumer or the agent with the consumer. Some of these studies were carried out by Asgharizadeh and Murthy (2000), Kim et al. (2003), Iskandar et al. (2005), Jackson and Pascual (2008), Iskandar and Jack (2011) and Husniah et al. (2014). However, MSC research involving three parties was carried out only by Gamchi et al. (2013) and Esmaili et al. (2014).

In the Gamchi et al. (2013) and Esmaili et al. (2014) researchs, the OEM and agent only offered corrective maintenance (CM) for maintenance action for non repairable equipment. Considering the equipment that supports the company's business performance is usually repairable, it is important to develop models on repairable equipment. This research will develop Esmaili et al. (2014) models on repairable equipment. Modeling failure on repairable equipment is more complex than modeling of non repairable equipment, because there is also considering effect of PM action besides CM action.

Maintenance will be carried out based on performance. Generally performance that used by consumers is availability. If the equipment is always available, the company's production will continue to run, so the company's revenue continues to increase. This study will consider availability as a development of the research of Yeh and Chang (2007). The OEM and agent will offer several maintenance options in the form of MSC to be chosen by the consumer. In this research, optimal decisions of three parties (i.e. OEM, agent and consumer) will be determined, that will provide the best expected profit for each party.

Gamchi et al. (2013) and Esmaili et al. (2014) research succeeded in developing this decision problem between three parties in MSC in an independent model. Decision problem models are made separately, namely the agent's and consumer's decision models and the OEM's and the agent's decision model. The agent will first take the optimal decision on the consumer. Next, the OEM will take optimal decisions based on the steps taken by the agent.

In fact, the maintenance action can be carried out jointly by the OEM and the agent. This is done on the condition of the OEM who does not have sufficient technology and resources. Compared to independent model, the idea integration of each party expected to describe the relationship between OEM and agent. This is done by carried out a Stackelberg game theory to formulate the decision problem and use bilevel optimization to optimize the decision. In the first level, the OEM (as a leader)'s profit maximization model is done by constraining the agent (as the follower) profit model. At the second level, the maximization of the agent (as a leader) profit model is done by making the consumer (as a follower) profit model as a constraint. Solution search for bilevel optimization is carried out by heuristic optimization method (genetic algorithm) to determine the optimal price structure that will maximize the profit of each party.

This paper is organized as follows. The problem formulation is described in Section 2. In Section 3 we formulate the model that including an explanation of warranty situation, maintenance service contracts situation, equipment failure and repairs, and expected profit for each party involved. Section 4 explains about analysis the optimal strategies and solution procedures. In section 5, given numerical examples and discussion of research results. The conclusion is placed in the last section.

2. Problem Formulation
A consumer buys new equipment with warranty. The warranty offered by the OEM is two-dimensional warranty. In general, two-dimensional warranty is offered when product’s failure is influenced by age and usage. In companies that use transportation equipment such as coal companies, the dimensions of use more important to be considered because equipment failure is more caused by the dimension of use.

Throughout the warranty period, OEM responsible for all maintenance actions (PM and CM actions). After warranty ends, the consumer has the responsibilities for it. In this study, the consumer fulfilling the responsibilities by subcontract the maintenance actions to the agent or OEM under two-dimensional MSC. MSC offered generally by previous research has not ensure the equipment performance. In this study, we consider the performance that is in demand by costumer, i.e. availability.
If the equipment continues to be maintained, it is expected that equipment will always be available whenever needed. This research will ensure the minimum availability (e.g. availability = 90%) throughout the lifetime of the equipment. The ensuring of the value of availability at a certain value will have a positive impact on the production carried out by the company. Furthermore, the company will get a lot of income because the equipment is always in good condition.

3. Model Formulation
This section will explain the warranty situation, MSC situation, warranty and MSC options, failure modelling, and PM impact modelling. After that, expected profit will be expressed for the three parties involved (the OEM, agent and consumer) which are the objective functions used in modeling.

3.1. Warranty Situation
A new equipment (e.g. dump truck) is sold by the OEM to the consumer with a warranty. The warranty provided in the form of a two-dimensional warranty. This warranty is characterized by a rectangular shape $\Omega_w = [0, W) \times [0, \infty)$, with W is the time limit (e.g. 1 year) and no limit of usage (see Figure 1).

3.2. Maintenance Service Contract (MSC) Situation
After the warranty expires, the maintenance responsibility is transferred to the consumer. In the case of a consumer is a coal production company, for example, the consumer carries out this responsibility by conducting MSC. MSC is carried out by the OEM and/or the agent. This is done by considering two reasons: (1) the main business of consumers is not a business using dump trucks (loading and hauling processes) (2) consumers do not have human resources or technology to carry out maintenance (expensive).

The OEM and the agent offer an MSC for L (for example 3 year) with a maximum $U_l$ usage (e.g. 90,000 km mileage) at a certain price. The scope of the contract with 2D warranty forms the area $\Omega_s = [W, W + L) \times [U_l, \infty)$ (see Figure 2). However, when usage exceeds certain $U_l$ (e.g. 10,000 km mileage), consumers will be charged additional cost to the OEM (Husniah et. al., 2017). The amount of additional cost is proportional to $\Delta = U_y - U_l$ given by $C_{ac} (U_y - U_l)$.

3.3. Warranty and MSC Options
During maintenance contracts, equipment will deteriorate with time (age) and usage. The OEM and Agent provide comprehensive coverage of maintenance, including PM and CM actions. Three MSC options offered by the OEM are:

Option O1: The OEM provide PM only during $(W, L)$ at $P_1$. 

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**Figure 1.** Warranty region $\Omega_w$.

**Figure 2.** Warranty region $\Omega_w$ and MSC region $\Omega_s$.
Option O2: The OEM provide CM only during \((W, L)\) at \(P_2\).

Option O3: The OEM provide CM only during \((W, L)\) at \(P_3\) per each failure.

Three MSC options offered by the agent are:

Option A1: The agent provide PM only during \((W, L)\) at \(P_4\).

Option A2: The agent provide CM only during \((W, L)\) at \(P_5\).

Option A3: The agent provide CM only during \((W, L)\) at \(P_6\) per each failure.

The combination of options in MSC that can be offered by OEMs and agents can be selected in Table 1 below.

| Consumer’s Option Choices \((C_i)\) | Option Offer by OEM \((O_i)\) | Option Offer by Agent \((A_i)\) | PM | CM |
|-----------------------------------|-------------------------------|---------------------------------|-----|-----|
| 1                                 | 1                             | 2                               | O   | A   |
| 2                                 | 2                             | 1                               | A   | O   |
| 3                                 | 3                             | 1                               | O   | A/failure |
| 4                                 | 3                             | 1                               | A   | O/failure |

As result, there are four options available for consumer to be choses:

Option 1: OEM offers MSC which includes the PM action only for \(P_1\) during \((W, L)\). The agent offers MSC that includes only CM actions to improve maintenance actions for \(P_5\) during \((W, L)\).

Option 2: OEM offers MSC that includes only CM actions for \(P_2\) during \((W, L)\). The agent offers MSC which includes PM actions only to improve maintenance actions for \(P_4\) during \((W, L)\).

Option 3: OEM offers MSC which includes the PM action only for \(P_1\) during \((W, L)\). The agent offers MSC that includes only CM actions to improve maintenance actions for \(P_6\) per each failure during \((W, L)\).

Option 4: OEM offers MSC that includes only CM actions for \(P_3\) per each failure during \((W, L)\). With the offer, the agent also offers MSC which includes PM actions only to improve maintenance actions for \(P_4\) during \((W, L)\).

In each option (option 1-4) that can be chosen by consumer, when usage exceeds certain \(U_l\) limits, consumers will be charged additional cost given by \(C_{ac}(U_y - U_l)\).

### 3.4. Failure Modelling

There are three types of failure modeling that can be used for two-dimensional fields (Jack and Murthy, 2014). One of them is the one-dimensional point process approach used in this study. The first failure of the equipment (dump truck) is expressed as a random point in a two-dimensional (i.e. time and use).

Suppose \(U\) is a constant rate of use for a given dump truck. For \(U = u\), the conditional failure rate function (hazard rate) for the first failure time is given by \(r_u(t)\). Taking into account the degradation of the dump truck caused (the biggest) by the rate of use, the Accelerated Failure Time (AFT) model is used. In the AFT model, the distribution function for \(T_u\) is given by \(F(t, \alpha_u)\), with the scale parameter given by \(\alpha_u = \left(\frac{u}{\lambda}\right)^{\rho}, \rho \geq 1\), where \(\rho\) is the parameter for dumptruck operating condition. Failure rate function and cumulative failure rate corresponding to \(F(t, \alpha_u)\) is given by \(r_u(t) = \frac{f(t, \alpha_u)}{1-F(t, \alpha_u)}\) and \(R_u(t) = \int_0^t r_u(x) dx\), with \(f(t, \alpha_u)\) is density function.

Any failure that occurs throughout the contract period (after the warranty expires) will be maintained with minimal repair (minimum repair) so the reliability of the equipment not affected by the corrective actions taken. As a result, the rate of failure after making repairs is considered the same as before corrective actions (Barlow and Hunter, 1961).

### 3.5. PM Impact Modelling and The Ensure of Availability
PM is done imperfectly by the OEM/agent. Performing imperfect PM actions according to Jiang and Murthy (2008) can result in reduction of (a) virtual age or (b) in failure intensity function. This study uses imperfect PM modeling which results in a reduction in the failure intensity function.

Every time a PM is performed ($T_i$), the failure intensity function will decrease by $\delta$ which is fixed. The amount of reduction ($\delta$) of the failure rate for each PM by OEM and agent in this study is assumed to be same. The intensity function after PM action becomes $r_{i+1}(t) = r_i(t) - \sum_{i=1}^{n} \delta$.

Because the failure between PM actions is done with minimal repair, then the expected numbers of failures at intervals $[T_i, T_{i+1}]$, 0 ≤ $i$ ≤ $n$ given by

$$E[N(T_i, T_{i+1})] = \int_{T_i}^{T_{i+1}} r_{i+1}(t) dt$$

(1)

Availability in the interval $(T_i, T_{i+1})$ is given by

$$A(T_i, T_{i+1}) = \frac{(T_{i+1} - T_i)}{T_{i+1} - T_i + t_{pm} + t_{cm} E[N(T_i, T_{i+1})]}$$

(2)

Based on formula (1) and (2) above, there are two ways to ensure the minimum availability for each interval of PM action. Either by keeping the expected failure same for each interval (see Figure 3) or by ensure the availability throughout the life time of the equipment itself. By ensure the same expected failure, it means that we ensure the reliability throughout the lifetime of the equipment, i.e.

$$E[N(T_1, T_2)] = E[N(T_2, T_3)] = \ldots = E[N(T_{n-1}, T_n)]$$

$$e^{-E[N(T_1, T_2)]} = e^{-E[N(T_2, T_3)]} = \ldots = e^{-E[N(T_{n-1}, T_n)]}$$

![Figure 3. The interval to perform PM actions is obtained by ensure the expected failure.](image)

Whilst by ensure the availability throughout the life time of the equipment i.e.

$$A(T_1, T_2) = A(T_2, T_3) = \ldots = A(T_{n-1}, T_n)$$

3.6. Expected Profit

Maintenance service contract studied involve three parties i.e. the OEM, agent and consumer. The performance measure of interest to each party is the expected profit over the interval $(W, L)$. Before formulating expected profit, the PM cost, CM cost and the expected cost of the penalty will be involved as follows.

**PM Cost:** Suppose $C_p(\delta)$ is PM i-th cost. If $C_p(\delta) = C_0 + C_1 (1-\delta)$, then the total expectation of PM cost from all contract periods is given by $E[N_{PM}] = \sum_{i=0}^{N} C_p(\delta) = NC_0 + NC_1 (1 - \delta)$.

**CM cost:** Suppose that $C_r$ is the cost needed by the OEM or the agent, every time a minimum corrective action is taken, the expectation of minimal repair costs (CM) is given by $E[N_{CM}] = C_r \left( \sum_{i=0}^{n-1} \int_{T_i}^{T_{i+1}} r_{i+1}(t) dt \right)$. We obtain the expected profit for each party involved as follows.

The OEM

Expected profit for OEM Option 1 = MSC price ($P_1$) - (total cost PM (Sigma $C_p$))
\[
\rho_{O1}(P_1) = P_1 - \sum_{i=0}^{n} C_p
\]

Expected profit for OEM Option 2 = MSC price \( P_2 \) – (cost CM \( Cr \)) x Expected failures throughout the maintenance contract period \((W,L)\)

\[
\rho_{O2}(P_2) = P_2 - (Cr) \int_{W}^{L} \left( \frac{\beta}{\alpha} \left( \frac{t}{\alpha} \right)^{\alpha-1} - \sum_{i=0}^{\delta} \right) dt
\]

Expected profit for OEM Option 3 = (MSC price \( P_3 \) – cost CM \( Cr \)) x Expected failures throughout the maintenance contract period \((W,L)\)

\[
\rho_{O3}(P_3) = (P_3 - Cr) \int_{W}^{L} \left( \frac{\beta}{\alpha} \left( \frac{t}{\alpha} \right)^{\alpha-1} - \sum_{i=0}^{\delta} \right) dt
\]

**The Agent**

Expected profit for agent Option 1 = MSC price \( P_4 \) - (total cost PM \( Sigma Cp \))

\[
\rho_{A1}(P_4) = P_4 - \sum_{i=0}^{n} C_p
\]

Expected profit for agent Option 2 = MSC price \( P_5 \) – (cost CM \( Cr \)) x Expected failures throughout the maintenance contract period \((W,L)\)

\[
\rho_{A2}(P_5) = P_5 - (Cr) \int_{W}^{L} \left( \frac{\beta}{\alpha} \left( \frac{t}{\alpha} \right)^{\alpha-1} - \sum_{i=0}^{\delta} \right) dt
\]

Expected profit for agent Option 3 = MSC price \( P_6 \) x Expected failures throughout the maintenance contract period \((W,L)\)

\[
\rho_{A3}(P_6) = (P_6 - Cr) \int_{W}^{L} \left( \frac{\beta}{\alpha} \left( \frac{t}{\alpha} \right)^{\alpha-1} - \sum_{i=0}^{\delta} \right) dt
\]

**The Consumer**

Expected profit for consumer Option 1 = Revenue \( R \) --- MSC price for OEM \( P_1 \) – MSC price for agent \( P_5 \) – Additional fee

\[
\Pi_1 = R - P_1 - P_5 - CaC(U_y - U_l), R = U_y * r/usage
\]

Expected profit for consumer Option 2 = Revenue \( R \) --- MSC price for OEM \( P_2 \) – MSC price for agent \( P_4 \) – Additional fee

\[
\Pi_2 = R - P_2 - P_4 - CaC(U_y - U_l)
\]

Expected profit for consumer Option 3 = Revenue \( R \) --- MSC price for OEM \( P_1 \) – MSC price for agent/failure \( P_6 \) – Additional fee

\[
\Pi_3 = R - P_1 - (P_6) \int_{W}^{L} \left( \frac{\beta}{\alpha} \left( \frac{t}{\alpha} \right)^{\alpha-1} - \sum_{i=0}^{\delta} \right) dt - CaC(U_y - U_l)
\]

Expected profit for consumer Option 4 = Revenue \( R \) --- MSC price for OEM/failure \( P_3 \) – MSC price for agent \( P_4 \) – Additional fee

\[
\Pi_4 = R - (P_3) \int_{W}^{L} \left( \frac{\beta}{\alpha} \left( \frac{t}{\alpha} \right)^{\alpha-1} - \sum_{i=0}^{\delta} \right) dt - P_4 - CaC(U_y - U_l)
\]

4. **Analysis**

This section will analysis the optimal PM intervals that ensure the value of availability and the optimal strategy for each party. After that, a procedure for finding a solution explained.
4.1. Optimal Interval PM for Ensure Availability at $\hat{A}$

The optimal interval of performing PM actions is obtained by maintaining the availability value. Using simple numerical calculations, the values of $T_i$ and $T_{i+1}$ will be obtained by keeping the value of availability (set value, example: $A(T_i, T_{i+1}) = 0.9000$, or $e^{-E[N(T_i, T_{i+1})]} = 0.8500$). The optimal $T_i$ value will be searched with the following algorithm:

1. Set the values of $\delta, \epsilon_{cm}, \epsilon_{pm}$ and $A(T_i, T_{i+1})$ or $e^{-E[N(T_i, T_{i+1})]}$
2. Set $T_0 = 0$, then we will compute $T_i, i = 1$.
3. Input the failure intensity function and the formula of expected failure along $(0, T_1)$.
4. Input the reliability function formula along $(0, T_i)$.
5. Input the formula of availability, by using a solver, the $T_i$ value is obtained.
6. Set $i = i + 1$, repeat steps 3-5 to get $T_i$.
7. Repeat step 6 to the condition that $T_i \geq L$.

4.2. Optimal Strategy for Each Party

The optimal decisions of a maintenance contract from the agent and the OEM can be done directly with a one-level optimization model as Gamchi et. al (2013) and Esmaili et. al. (2014). In other words, the OEM and agent can offer a contract directly to the consumer. However, with this model, the relationship between the OEM and the agent cannot be described interdependently.

In fact, there is a relationship between the OEM and the agent in taking maintenance action of equipment used by the consumer. The relationship between the OEM and the agent can be cooperative, for example by cooperating in providing spare parts and/or taking maintenance action. Each options offered by OEM and agent show the cooperative relationship between the OEM and agent that can be chosen by consumers. Another type of relationship that can occur between the OEM and the agent is the competition that occurs when the two parties carry out their respective maintenance actions without cooperating, this study does not discuss this.

**Upper-level problem: OEM-Stackelberg Model**

$$
\max \rho_0 (P_1, P_2, P_3) = y_1 \{P_1 - \sum_{i=0}^{n} C_p\} + y_2 \{P_2 - (C_r) \int^L_W \left( \frac{\beta}{\alpha} \right)^{\beta-1} \, dt \} + y_3 \{(P_3 - C_r) \int^L_W \left( \frac{\beta}{\alpha} \right)^{\beta-1} - \sum_{i=0}^{n} \delta \, dt \}
$$

s.t.

$$
P'_5 = y_1 \left( R - P_4 - C_{ac} (U_y - U_i) - \Pi_1 \right)
$$

$$
P'_4 = y_2 \left( R - P_2 - C_{ac} (U_y - U_i) - \Pi_2 \right)
$$

$$
P'_6 = \frac{\int^L_W \left( \frac{\beta}{\alpha} \right)^{\beta-1} - \sum_{i=0}^{n} \delta \, dt }{\int^L_W \left( \frac{\beta}{\alpha} \right)^{\beta-1} \, dt } \left( R - P_1 - C_{ac} (U_y - U_i) - \Pi_3 \right)
$$

$$
P'_4 = y_3 \left( R - \frac{\int^L_W \left( \frac{\beta}{\alpha} \right)^{\beta-1} - \sum_{i=0}^{n} \delta \, dt }{\int^L_W \left( \frac{\beta}{\alpha} \right)^{\beta-1} \, dt } - C_{ac} (U_y - U_i) - \Pi_4 \right)
$$

$$
y_1 + y_2 + y_3 = 1
$$

**Lower-level problem: Agent-Stackelberg Model**

$$
\max \rho_A (P_4, P_5, P_6) = z_1 \{P_4 - \sum_{i=0}^{n} C_p\} + z_2 \{P_5 - (C_r) \int^L_W \left( \frac{\beta}{\alpha} \right)^{\beta-1} - \sum_{i=0}^{n} \delta \, dt \} + z_3 \{(P_6 - C_r) \int^L_W \left( \frac{\beta}{\alpha} \right)^{\beta-1} - \sum_{i=0}^{n} \delta \, dt \}
$$

s.t.

$$
z_1 \Pi_1 = R - P_1 - P_5 - C_{ac} (U_y - U_i)
$$
\[
\begin{align*}
    z_2 \Pi_2 &= R - P_2 - P_4 - C_{ac}(U_y - U_l) \\
    z_3 \Pi_3 &= R - P_1 - (P_6) \int_w^t \left( \frac{\beta (t)}{\alpha} \right)^{\beta-1} \left( -\sum_{j=0}^i \delta \right) \ dt - C_{ac}(U_y - U_l) \\
    z_4 \Pi_4 &= R - (P_3) \int_w^t \left( \frac{\beta (t)}{\alpha} \right)^{\beta-1} \left( -\sum_{j=0}^i \delta \right) \ dt - P_4 - C_{ac}(U_y - U_l) \\
    z_1 + z_2 + z_3 &= 1
\end{align*}
\]

4.3. Solution Procedure

To solve a bilevel optimization, the lower level problem will be transformed to the condition of Karush-Kuhn-Tucker (KKT). Thus, bilevel optimization is transformed into a single level problem by changing the lower level model into the KKT condition as follows:

Stationarity
\[
A = \{P_4, P_5, P_6\} ; \nabla_A L(A, \mu, \lambda) = 0
\]
\[
z_1 \left\{ P_4 - (C_P) \right\} + z_2 \left\{ P_5 - (C_r) \right\} \int_w^t \left( \frac{\beta (t)}{\alpha} \right)^{\beta-1} \left( -\sum_{j=0}^i \delta \right) \ dt + z_3 \left\{ (P_6 - C_r) \right\} \int_w^t \left( \frac{\beta (t)}{\alpha} \right)^{\beta-1} - \\
\sum_{i=0}^\delta \ dt \right\} + \mu^t (z_1 + z_2 + z_3 - 1) - \lambda^t \left\{ -z_1 \Pi_1 + R - P_1 - P_5 - C_{ac}(U_y - U_l) - z_2 \Pi_2 + R - \\
P_2 - P_4 - C_{ac}(U_y - U_l) - z_3 \Pi_3 + R - P_1 - (P_6) \int_w^t \left( \frac{\beta (t)}{\alpha} \right)^{\beta-1} \left( -\sum_{j=0}^i \delta \right) \ dt - C_{ac}(U_y - U_l) - \\
z_4 \Pi_4 + R - (P_3) \int_w^t \left( \frac{\beta (t)}{\alpha} \right)^{\beta-1} \left( -\sum_{j=0}^i \delta \right) \ dt - P_4 - C_{ac}(U_y - U_l) \right\}
\]

Inequality constraints
\[
\nabla_{\mu} L(A, \mu, \lambda) = 0
\]

\[
\lambda_j \geq 0 \ for \ j = 1, \ldots, 4
\]
\[
\lambda_j z_j \Pi_j = 0
\]

All the conditions of the KKT above will be an additional constraints for the objective function of the manufacturer in the upper level problem. After being transformed into single level optimization, genetic algorithms (GA) will be used to carry out the search process for solutions. The GA procedures involve three iterative steps as stated in Figure 4.

In more detail, the six-step genetic algorithm are as follows.

**Step 0 (Initialization).** The population size chosen was 50 (De Jongis (1975); Grefenstette (1986); Schaffer, Caruana, Eshelman and Das (1989)). Each decision variable will be represented by binary encodings with a length of 5 bits. Because the decision variable in the second level model becomes a constraint for the upper level model, an additional decision variable is needed, namely \( \mu \) and \( \lambda \).

**Step 1 (Generating initial population).** The initial population is randomly generated as an intermediate number (0.1) as a sized matrix (population size x total bits on the chromosome), i.e. (50x480). Round up so that binary values are formed.
Step 2 (Calculating the initial fitness population). After the binary population is formed, a conversion is made to convert the value of bits into a continuous number to calculate the fitness value, with the formula: for encoding; \( p_{norm} = \frac{p_{hi} - p_{lo}}{2^m} \), \( gene[m] = \text{round} (p_{norm} - 2^{-m} - \sum_{p=1}^{m-1} gene[p]2^{-p}) \) and for decoding; \( p_{quan} = \sum_{m=1}^{N} gene[m]2^{-m} + 2^{-m+1}) = q_m = p_{quan}(p_{hi} - p_{lo}) + p_{lo} \).

Step 3 (Selection). This study selected the population by selecting half of the population that has the best value (maximum).

Step 4 (Crossovers). After selecting half the population (\( M \)) based on the best value, a crossover process is carried out by selecting one crossover point from the random value generation with the formula \( \text{rand}(1,M) \times (\text{total bits} - 1) \).

Step 5 (Mutation). The mutation process is done by selecting a random mutation point such as selecting the step 4 crossover point. Previously determined the number of mutations that will be done by the formula \( \text{population size} - 1 \times \text{number of bits} \times \text{rate mutation} \). The mutation rate is set at 0.15. Next generate data to select column and row addresses (addresses of random variables and generations that state bits) that will be mutated.

Step 6 (Repeat step 2) to the desired maximum value or until the generation runs out.

5. Numerical Example

Assume given usage rate \( u \). The time to the first failure follows the Weibull distribution with \( F_u = 1 - \exp(-\frac{t}{\alpha_u})^\beta \). Failure rate function is \( r_u(t) = \beta \frac{t^{\beta-1}}{\alpha_u} \). The other parameter values are: \( \alpha_0 = 3, \beta = 1.5, t_{pm} = 0.03, t_{cm} = 0.015, L = 10 \) (months) and \( \delta = 0.001 \).

Table 2 shows optimal solutions for PM actions interval \( (T_i, T_{i+1}) \). By performing performance based maintenance, the desired level of availability and reliability is obtained. Based on numerical results, the approach that ensures the availability is better (results in higher availability) than that of the reliability.
Table 2. Optimal PM interval by ensuring reliability and availability.

| i  | $T_i$ (months) | $\bar{A}$ | $T_i$ (months) | $R$ Before | $A$ After | $\Delta$ |
|----|----------------|---------|----------------|------------|------------|---------|
| 1  | 0.9926         | 0.9683  | 0.2731         | 0.9810     | 0.9960     | 0.0150  |
| 2  | 1.5953         | 0.9489  | 0.5480         | 0.9650     | 0.9950     | 0.0300  |
| 3  | 2.1108         | 0.9234  | 0.9431         | 0.9430     | 0.9940     | 0.0610  |
| ...|                |         |                |            |            |         |
| 25 | 9.8324         | 0.8782  | 9.4207         | 0.8370     | 0.9850     | 0.1480  |
| 26 | 9.8016         |         | 0.8350         |            | 0.9850     | 0.1500  |

Table 3. Value of decision variable in optimal decision MSC and expected profit of each party.

| No. | Decision Variable | Iteration | 1   | 2   | 3   | 4   |
|-----|-------------------|------------|-----|-----|-----|-----|
| 1   | $y_2$             | 1          | 1   | 1   | 1   |     |
| 2   | $P_2$             | 1000.0000  | 1000.0000 | 1000.0000 | 967.7419 |
| 3   | $z_1$             | 1          | 1   | 1   | 1   |     |
| 4   | $P_4$             | 580.6452   | 774.1935 | 451.6129 | 516.1290 |
| 5   | $C_{ac}$          | 0.0000     | 0.0000 | 0.0000 | 0.0000 |     |

Expected Profit

|                 | OEM       | Agent     | Customer  |
|-----------------|-----------|-----------|-----------|
| Expected        | 800.0000  | 800.0000  | 800.0000  |
| Profit          | 800.0000  | 800.0000  | 800.0000  |
|                | 767.7419  | 366.1290  | 516.1291  |

The GA result (see Table 3) shows that the OEM, as the leader of the agent will choose option O2 i.e. the OEM provide CM only during ($W, L$) at $P_2$. The agent as the follower of the manufacturer and the leader of the customer, will choose option A1 i.e. the agent provide PM only during ($W, L$) at $P_4$. And the customer has to choose option 2 which optimizes its profit. However, the choice of contract value for the agent ($P_4$) will influence the expected profit obtained by the agent – which is greater than the customer’s expected profits. However, in iterations 3 and 4, the optimal value of $P_4$ obtained will cause a greater expected profit of the consumer.

6. Conclusion

This paper studied performance based two-dimensional (i.e. time and usage) MSC for dump truck after the expired two-dimensional warranty. Availability, as the customer’s interest performance is considered in this paper. By ensuring availability ($\bar{A} = 0.9000$), the reliability after doing imperfect PM is always above 0.8350. Meanwhile, doing imperfect PM by ensuring reliability (the expected failure) $(e^{-E[N(T_i, T_{i+1})]} = 0.8500)$, the availability is always above 0.8782.

The decision problems that involving three parties (manufacturer, agent and consumer) for MSC options are formulated using the Stackelberg game theory formulation and the optimal decision is obtained using a bilevel optimization. The result shows that the SPE obtained, i.e. the OEM choose option O2, the agent choose option A1 and the consumer choose option 2 as the optimal decision.

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References

[1] Ashgarizadeh E and Murthy D N P 2000 Mathematical and Computer Modelling Service contracts 31 11-20
[2] Bard J F 1998 Practical Bilevel Optimization Kluwer Academic Publishers
[3] Boland Philip J 1982 Naval Research Logistics Quarterly Periodic replacement when minimal repair costs vary with time 29 (4) 541-546

[4] Esmaili M, Gamchi N S, and Ashgarizadeh E 2014 European Journal of Operational Research Three-level warranty service contract among OEM, agent and customer: a game-theoretical approach 239 177-186

[5] Gamchi N S, Esmaili M, and Monfared M S 2012 Proceeding of the 2012 International Conference of Industrial Engineering and Operation Management Three-level service contract between OEM, agent and customer (game theory approach July 3-6 Turkey

[6] Iskandar B P, Murthy D N P, and Jack N 2005 Comp Oper Res A new repair-replace strategy based on usage rate for items sold with a two-dimensional warranty 32 669-682

[7] Iskandar B P, Pasaribu U S, and Husniah H 2013 CIE44 Performance based maintenance contract for equipment sold with two-dimensional warranties, Hongkong

[8] Jackson C, and Pascual R 2008 European Journal of Operational Researchs Optimal service contract negotiation with aging equipment 189 387-398

[9] Sadigh A N, Mozafar M, and Karimi B 2012 Advances in engineering Software OEM-retailer supply chain coordination: A bi-level programming approach 45 144-152

[10] Sinha P Malo and K Deb 2017 IEEE Transactions on Evolutionary Computation A review on bilevel optimization: from classical to evolutionary approaches and applications 22 (2) 276-295

[11] Yeh R Y and Chang W L 2007 Mathematical and Computer Modelling Optimal threshold value of failure-rate for leased products with preventive maintenance actions 46 730-737