Conical metal-rubber support with quasi-zero stiffness effect

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Abstract. The study relates to the important branch of applied mechanics – the theory of vibration isolation of vibroactive objects such as generators, motors, pumps, compressors, fans, pipelines, etc. The design and mathematic model of a prospective support structure using quasi-zero stiffness effect were considered. The support consists of an elastomeric shock reducer as a shell in cross section having a truncated cone shape with inserted cylindrical elastomer spring, specially selected thanks to its elastic characteristics and size, to obtain characteristics with quasi-zero stiffness near support equilibrium. The mathematical model of the support, allowing to select options to reduce the factor of the force transmission to the base in a certain frequency range was obtained. The calculations for the model and charts show the efficiency of vibration isolation for the proposed support.

1. Introduction

Vibration protection has been the recent problem in modern machine building, as functioning reliability of processing equipment and operator's safety depend on the efficiency of vibration protection systems. The constant increase of motion speed and the power of propulsion plants and processing equipment results in growing levels of vibrations affecting the design. This raises the necessity in improving vibration protection systems and implementing new technical solutions. Vibration isolation supports with the quasi-zero stiffness of different designs are offered. To protect operational equipment from harmful vibrations it is useful to apply passive systems to be the simplest and most cost-effective. One of the main characteristics of the vibration isolator is a frequency of its free oscillations. The lower it is, the wider the frequency range of driving force, in which operation of the vibration isolator is effective. For obtaining vibration isolation systems with low oscillation frequency it is possible to use systems with quasi-zero stiffness [1], [2]. They are different because in the working range of frequencies these systems have a flat sloping area of the power characteristics, i.e. low stiffness, but they have a high load-carrying capacity in equilibrium position. This makes them promising for use as means of vibration control of objects. They consist of carrying and corrector springs, the stiffness of which is subtracted from the support spring stiffness to obtain the area with quasi-zero stiffness. Corrector springs are performed on the base of helical, leaf or air springs [3], [4], [5], [6], [7], [8], [9]. Despite the undoubted effectiveness of systems with quasi-zero stiffness, their implementation is hampered by a small number of designs simple to manufacture, with small weight and dimensions.

2. Description of the study subject and preparation of its mathematical model

For vibration isolation of vibroactive objects of different type we offer the support consisting of an elastomer shock reducer as a shell in cross section having a truncated cone shape [10] with inserted cylindrical elastomer spring, specially selected thanks to its elastic characteristics and size, to obtain characteristics with quasi-zero stiffness near support equilibrium (Figure 1). The shock reducer I has a conical side elastomer surface connected with the upper and lower metal supporting bottoms. Inside it
a specially selected spring 2 is mounted and bolted 3. In the unloaded state between the low bottom of
the shock reducer and the lower spring base there is a calculated gap.
Without internal cylindrical spring the shock buckles and, therefore, has a force load characteristic
with the negative stiffness area (curve 1 in figure 2 [10]). Also in this figure the curve 2 shows the
characteristics of a specially chosen cylindrical elastomeric spring, and the curve 3 - the total load
characteristic of the support with the required small (quasi-zero) stiffness area. In its middle,
corresponding to the displacement \( x = 0.02 \text{m} \) there is the initial operating point, where the load is
equal to the gravitation force per support. The support operates with small oscillation amplitudes no
more than ± 5 mm near the operating point on the load area with quasi-zero stiffness.

1 – elastomeric shock reducer with conical surface; 2 – a cylindrical elastomer spring; 3 – lower
support bottom bolts

Figure 1. Design of vibration isolation support with quasi-zero stiffness: a) the support without load,
b) the support under load.

To assess the vibration isolation efficiency of the proposed support it is required at various
frequencies to find the coefficient of force transmission to the foundation \( K_R \), as the ratio of the
foundation reaction amplitudes and vibrating force. The foundation reaction was calculated according
to the principle of motion of center of mass

\[
R(t) = P_0 \sin \omega t - m \ddot{x},
\]

(1)

where \( P_0, \omega \) is the amplitude and the frequency of vibrating force; \( m \) is the mass of the vibration-
isolated object per one support; \( \dddot{x} \) is the acceleration of the mass \( m \) defined by nonlinear equation of motion

\[
m \dddot{x} + b \dot{x} + P_{ymp} (x) = P_0 \sin \omega t,
\]

(2)

where \( P_{ymp} (x) \) is the elastic force created by the support and shown on curve 3 in figure 2.
The calculations were performed by computer programme Simulink (Matlab). The mass per one support is \( m = 264 \, kg \), the amplitude of vibrating force is \( P_0 = 30 \, H \). The material and sizes of the cylindrical elastomer spring as well as resistance coefficient were selected on the base of references given in [11], so that its stiffness compensated the negative one of the cone shock reducer. Thereby the area with quasi-zero stiffness appeared near support equilibrium. The material of the cylindrical spring is a rubber 194 with dynamic elastic modulus \( E = 4.6 \, MPa \), taken for the average of frequencies from the studied frequency range. For this rubber grade the coefficient of resistance for this frequency was taken \( b = 700 \, Nsec/m \). The stiffness of the cylindrical spring in the operating range of elastic deformation amounted \( C_0 = 140000 \, N/m \). The elastic force created by the cone elastomer shock reducer was taken from the experimental graph given in [10] and approximated by the polynomial of seventh degree. Then the resulting elastic force of the support within the operating range of displacements is

\[
P_{upp}(x) = C_0 x + 3.78 \times 10^{15} x^7 - 5.61 \times 10^{14} x^6 + 3.22 \times 10^{13} x^5 - 8.82 \times 10^{11} x^4 + 1.17 \times 10^{10} x^3 - 7.42 \times 10^{7} x^2 + 3.66 \times 10^5 x - 3.42
\]  

(3)

On the basis of the differential equation (2) with (3) the parameters of the vibration-isolated object motion were determined by computer. Their substitution in the equation (1) at every integration step and stating \( R(t) \) divided by \( P_0 \) defined the value \( K_p \) with the envelope frequency amplitude of dimensionless reaction in the function of vibroexcitation frequency. The frequency was varied in the range from 2 to 12 Hz with low velocity 0.01 Hz/sec.

3. Results of numerical experiments

The graphs in figure 3 show the dependence of the amplitude of the object vibration and coefficient of the force transmission to the base on the frequency for the given support (curve 1). To compare two parallel vibration isolation supports AKCC – 120/100 with similar static 240 kg load and the same vibroexcitation, the calculations were made and the graph of the force transmission coefficient to the base was shown (curve 2). Their total stiffness is equal to \( C = 4 \times 10^5 \, N/m \).
1 – for vibration isolation support with quasi-zero stiffness; 2 for two parallel vibration isolation supports AKCC–120/100 with a similar static 240 kg load and the same vibroexcitation.

Figure 3. Graphs of dependences of the object vibration amplitude and coefficient of the power transmission to the base on the frequency.

4. Conclusion
From the changes graph of force transmission to the base in figure 3 it can be seen that the proposed support, having the area with quasi-zero stiffness on the load characteristics, can significantly improve equipment vibration isolation in comparison with installing on the standard AKCC supports with a similar static load and the same vibroexcitation. As it can be seen in figure 3, the amplitude of vibration-isolated object oscillation throughout the frequency range from 3 Hz and above does not exceed the area with quasi-zero stiffness ± 5 mm. In addition, the support structure is simple.

In this paper, the example of the conical shock reducer given in [10] was studied and designed for heavy loads. However, such shock reducers with area of negative stiffness in the load characteristic can be made for smaller loads. The appropriate springs can be selected, to be installed inside the shock reducer and obtain the effect of quasi-zero stiffness for improving the vibration isolation of various technological objects.

5. References
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