Analytical calculation of deformations of a truss for a long span covering

Mikhail N. Kirsanov
National Research University “Moscow Power Engineering Institute” (MPEI);
Moscow, Russian Federation

ABSTRACT

Introduction. The method of induction based on the number of panels is employed to derive formulas designated for deflection of a square in plan hinged rod structure, which has supports on its sides and which consists of individual pyramidal rod elements. The truss is statically determinable and symmetrical. Some features of the solution are identified on the curves constructed according to derived formulas.

Materials and methods. The analysis of forces arising in the rods of the covering is performed symbolically using the method of joint isolation and operators of the Maple symbolic math engine. The deflection in the centre of the covering is found by the Maxwell–Mohr’s formula. The rigidity of truss rods is assumed to be the same. The analysis of a sequence of analytical calculations of trusses, having different numbers of panels, is employed to identify coefficients, designated for deflection and reaction at the supports, in the final design formula. The induction method is employed for this purpose. Common members of sequences of coefficients are derived from the solution of linear recurrence equations made using Maple operators.

Results. Solutions, obtained for three types of loads, are polynomial in terms of the number of panels. To illustrate the solutions and their qualitative analysis, curves describing the dependence of deflection on the number of panels are made. The author identified the quadratic asymptotics of the solution with respect to the number of panels. The quadratic asymptotics is linear in height.

Conclusions. Formulas are obtained for calculating deflection and reactions of covering supports having an arbitrary number of panels and dimensions if exposed to three types of loads. The presence of extremum points on solution curves is shown. The dependences, identified by the author, are designated both for evaluating the accuracy of numerical solutions and for solving problems of structural optimization in terms of rigidity.

KEYWORDS: Space truss, deflection, induction, Maple, analytical solution

FOR CITATION: Kirsanov M.N. Analytical calculation of deformations of a truss for a long span covering. Vestnik MGSU [Monthly Journal on Construction and Architecture]. 2020; 15(10):1399-1406. DOI: 10.22227/1997-0935.2020.10.1399-1406 (rus.).

Аналитический расчет деформаций фермы пространственного покрытия

М.Н. Кирсанов
Национальный исследовательский университет «МЭИ» (НИУ «МЭИ»); г. Москва, Россия

АННОТАЦИЯ

Введение. Методом индукции по числу панелей дается вывод формул для прогиба квадратной в плане шарнирной стержневой конструкции, имеющей опоры по сторонам и состоящей из отдельных пирамидальных стержневых элементов. Ферма статически определимая, симметричная. На кривых, построенных по выведенным формулам, отмечаются некоторые особенности решения.

Материалы и методы: Расчет усилий в стержнях покрытия выполняется в символной форме методом вырезания узлов с использованием операторов системы символьной математики Maple. Прогиб середины покрытия находится по формуле Максвелла–Мора. Жесткость стержней фермы принимается одинаковой. Из анализа последовательности аналогичных расчетов ферм различим числом панелей методом индукции выводятся коэффициенты в итоговой расчетной формуле для прогиба и реакций опор. Общие члены последовательностей коэффициентов находятся из решения линейных рекуррентных уравнений, составленных с помощью операторов Maple.

Результаты: Решения, полученные для трех видов нагрузки, имеют полиномиальную по числу панелей форму. Для иллюстрации полученных решений и их качественному анализу построены кривые зависимости прогиба от числа панелей. Обнаружена квадратическая асимптотика решения по числу панелей и линейная по высоте.

Выводы. Получены формулы для вычисления прогиба и реакций опор покрытия с произвольным числом панелей, размерами под действием трех типов нагрузок. Показано наличие точек экстремума на кривых решения. Найденные зависимости предназначены как для оценки точности численных решений, так и для решения задач оптимизации конструкции по жесткости.

КЛЮЧЕВЫЕ СЛОВА: пространственная ферма, прогиб, индукция, Maple, аналитическое решение

ДЛЯ ЦИТИРОВАНИЯ: Кирсанов М.Н. Аналитический расчет деформаций фермы для покрытия в пространстве // Вестник МГСУ. 2020. Т. 15. Вып. 10. С. 1399–1406. DOI: 10.22227/1997-0935.2020.10.1399-1406
INTRODUCTION

The calculation of complex building structures, in particular, trusses, is traditionally performed in specialized packages, which, as a rule, are based on the finite element method [1–4]. This allows taking into account all the features of the problem, the type of loads, the property of materials, the nature of supports. In some cases, calculations enable the use of simple though sufficiently accurate models that correspond to analytical methods that are not associated with the problem of choosing the calculation accuracy typical for numerical solutions. If independent parameters of an analytical solution, written in the form of a finite formula of foreseeable complexity, have a number that determines the degree of complexity of the structure, for example, the number of panels in a truss, then such a solution is of great importance. The problem of identification and analysis of statically definable regular trusses was first raised by Hutchinson R. G., Fleck N.A. [5, 6]. Some theoretical problems of periodic rod structures are described in [7]. The induction method was applied to a number of regular planar trusses having a lattice structure by using computer mathematics systems; simple formulas describing dependence of deflection on the number of panels were obtained [8–14]. The calculation of arch trusses in the analytical form is given in [15–18]. A more difficult task is the problem of deformation of spatial structures [19]. A survey of papers using the induction method to derive formulas for regular structures is available in [20].

In this paper, we consider a truss (Fig. 1) consisting of \( n^2 \) identical rod pyramids with a square base having height \( h \) (Fig. 2). The length of rods at the edges of the pyramid is equal to \( c = \sqrt{2a^2 + h^2} \). All connections in the truss are hinged, and the sides of the truss are supported by vertical posts. In angle \( A \) there is a spherical joint corresponding to three rods, in angle \( B \) there is a cylindrical joint modeled by two rods. All support rods are assumed to be non-deformable. Together with the support rods, the truss contains \( n_s = 6n^2 + 6n + 3 \) rods. The number of support rods, including those that simulate angular spherical and cylindrical joints, is \( 4n + 3 \). The truss is regular, and an inductive method for deriving formulas for components of the stress-strain state as functions of the number of panels is applicable to it. Consider the case of an even number of panels on the sides of the covering: \( n = 2k, k = 1, 2, \ldots \).

MATERIALS AND METHODS

To determine deflection in the analytical form (vertical displacement of node \( C \) in the centre of the covering) according to the Maxwell – Mohr’s formula, it is necessary to calculate the forces arising in the rods in the analytical form. Any modern computer mathematical system such as Maple, MathCad, Mathematica, Maxima, Derive, Reduce, etc. can handle this task. However, inductive generalization of solutions still requires a system that has operators for determining the General terms of sequences, the choice is limited to the two systems that are current leaders: Maple and Mathematica. Here the choice was made in favour of the Maple system [21], which has a more intuitive interface. In the language of this system, there is a software for calculating forces in the truss rods [22]. We use this software to calculate the proposed coverage. The program enters coordinates of the hinges. The coordinates of the corner points of the pyramid bases are:

---

**Fig. 1.** The truss, \( k = 2 \)

**Fig. 2.** Pyramid element of covering
The coordinates of the pyramid vertices are:
\[ x_k = 2a(i - 1), \quad y_k = 2a(j - 1), \quad z_k = f_k, \]
\[ k = i + (j - 1)(n + 1), \quad i, j = 1, ..., n + 1. \]

The functions \( f_1(i, j), f_2(i, j) \) define the shape of the covering surface. In this case, these are constants \( f_1 = 0, f_2 = h \). By changing these functions, you can get different types of surfaces of statically defined structures (Fig. 3, 4).

The reference book [23] considers 500 different types of surfaces for coverings and structures of building structures. Ellipsoidal surfaces of constructions of domes, shells and high-pressure vessels are considered in the review [24].

![Fig. 3. Two versions of the surface: I — \( f(i, j) = hi^2 \); II — \( f(i, j) = -h(j - 1)(j - n - 2) + h_2 \).](image)

The lattice structure is organized on the basis of special ordered lists consisting of the numbers of ends of rods, just as graphs are attributed in discrete mathematics. Using data on the structure and coordinates of nodes, guiding cosines of forces that make up the matrix of the system of equilibrium equations for all nodes are calculated. This system also includes support reactions as unknown values [22].

Using the values of identified forces, deflection is calculated using the Maxwell – Mohr’s formula in the form
\[
\Delta = P \sum \frac{s_j f_j}{EF},
\]
where \( f_j \) and \( s_j \) are the length and force in the \( j \)-th rod caused by the load, \( s_j \) is the force arising from the unit vertical force applied to the central node \( C \), \( E \) is the elastic modulus of the rods, \( F \) is the cross section area. Summation is performed only for deformable rods. All support rods are assumed to be rigid. In regular systems, the type of solution does not depend on its order, in this case the number of panels matter
\[
\Delta = P \left( C_1 a^3 + C_2 c^3 \right) / \left( h^2 EF \right).
\]

Only \( C_1 = C_1(k), \quad C_2 = C_2(k) \) coefficients depend on the number of panels \( k \). Determination of these dependencies for different types of loads is one of the tasks of this work.

**RESULTS**

Consider the load uniformly distributed over the covering nodes. Let’s imagine it as the sum of two loads — the load on the nodes at the base of the pyramids (Fig. 4) and on their vertices (Fig. 5). By calculating the deflection using the Maxwell – Mohr’s formula (1) of a truss with a consistently increasing number of panels, we get a sequence of coefficients.

In the cycle for the number of panels \( k \), the program outputs a sequence of coefficients before cubes \( a^3, c^3 \). The following numbers are obtained for coefficient

![Fig. 4. Uniform loading of the pyramid bases, \( n = 4 \).](image)
By using these data, the `rgf_findrecur` operator generates a sixth order recurrent equation with binomial coefficients

$$C_{k,k} = 6C_{k,k-1} - 15C_{k,k-2} + 20C_{k,k-3} -$$

$$- 15C_{k,k-4} + 6C_{k,k-5} - C_{k,k-6}.$$  

The solution for this equation using initial data

$$C_{1,1} = 3, C_{1,2} = 120, ..., C_{1,6} = 27768$$

represents the sequence: –3, 13, 45, 93, 157, ... . Recurrent equation

$$Z_k = 3Z_{k-1} - 3Z_{k-2} + 3Z_{k-3}$$

is obtained using Maple system operator `rgf_findrecur`. The solution of the equation has the form

$$Z_k = P(8k^2 - 8k - 3)/4.$$  

It should be noted that only when \( k = 1 \), the reaction of the support is less than zero, i.e. the support rod is compressed and the support supports the covering, in all other cases the support acts as a holding bond. The reactions of the supports located on the sides of the covering are equal, and the formula depending on the number of panels is simple:

$$Z_n = -P(1 + 4k)/2.$$  

The distribution of forces in the rods (Fig. 6), if caused by the action of the uniformly distributed load over all nodes, shows that all the rods along the contour of the covering are compressed (highlighted in blue), some of the rods of pyramid ribs are compressed, and some are stretched (highlighted in red). The thickness of rod segments is proportional to the moduli of the forces. The numbers show the values of the forces referred to the \( P \) value.

If all nodes of the covering are loaded, the formula deflection has the form (2), where \( C_2 = k^4 \), and \( C_1 \) coefficient is the sum of solutions (3) and (4):  

$$C_1 = k^2(2k + 1)(5k^2 - 2)/3.$$  

The split of the distributed load into two is made in order to more accurately account for the real situation, when, for example, during precipitation, the major portion of the load falls on recessed parts of the roof, and the smaller part — on its tops.  

For an action of a single concentrated force in central node \( C \), coefficients in (2) represent

$$C_1 = k(4k^2 - 1), \quad C_2 = k^2.$$  

A linear combination of solutions for three types of loads, taking into account the arbitrariness of the number of panels, allows us to apply the obtained formulas for a fairly wide class of problems.  

Resulting formulas are checked by comparing them with the numerical solution.  

In the process of deriving formulas for deflection, forces in all rods were obtained in the analytical form. Here are the values of the support reactions.  

- For all surface nodes (Fig. 4, 5) are exposed to the combined action of loads, reactions of angular supports represent common members of the following sequence: –3, 13, 45, 93, 157, ... .  

- For a fairly wide class of problems, the growth of deflection accompanied by an increase in the number of panels depends on truss height \( h \). For small \( h \), the curve has a minimum, which means that in this case it is possible to optimize the structure in terms of rigidity by choosing the optimal number of panels. Here, for \( h = 2 \) m, the optimal number of panels is \( n = 2k = 10 \), which corresponds to size \( a = 2.5 \) m.

The analytical form of the solution allows using Maple to find the quadratic asymptotics of the curves:  

$$\lim_{k \to \infty} \Delta^2/k^2 = \frac{h}{(4L)}.$$
The dependence of the solution on the height is nonlinear (Fig. 8). With an increase in the height of the truss with a small number of panels, its deflection naturally decreases, however, for \( k > 2 \), after a certain value of the height, the deflection begins to grow. The asymptotics is linear here. The angle of inclination of the asymptotes is

\[
\lim_{h \to \infty} \frac{\Delta'}{h} = \frac{2k^4}{(8k^2 + 4k + 1)L}.
\]

Let us also focus on a characteristic of the analytical calculation of 3D structures. The Maple system, like other systems of symbolic mathematics, has an extremely low pace of transformations, which is invisible when calculating planar trusses containing a small number of rods [8–18]. As for 3D problems, the increase in the number of structure elements accompanied by an increase in the number of panels is quadratic if this number of panels is calculated on both sides of the structure. That is why, here, during the in-
duction process, the computation time increased so that if the first calculations of \( k = 1, 2, 3 \) took seconds and minutes, then for \( k > 5 \), they take hours and even days to complete the computation and obtain the formula (2). The use of the latest versions of Maple, theoretically having an algorithm for parallelizing the counting process by individual cores, does not help here either. It was possible to calculate the coefficients only due to the simple intuitively clear form of coefficient \( C_1 \). Coefficient \( C_1 \) was calculated in the numeric form, in which the system works quickly at \( \alpha = 1.0 \text{ m} \) as the difference between (2) and the value of \( C_1\varepsilon^3 \). The corresponding operator of the Maple program has the form

\[
C_1 [k] = \text{round} (\text{del} \times h^2 – k^4 \times c^3).
\]

**DISCUSSION AND CONCLUSIONS**

The proposed 3D truss design is distinguished by the presence of many supports that do not contradict its static determinability. The mathematical model of the structure made it possible, using the inductive method in the system of computer mathematics, to obtain relatively simple formulas to identify dependence of deflection on the number of panels, allowing asymptotic analysis. Unfortunately, the extreme points found in solution points cannot be obtained in the analytical form. However, the very fact of their existence gives optimism for obtaining optimal solutions when choosing the size of a structure based on an increase in its rigidity. It is also noticed that corner supports exposed to the vertical load on the truss do not have a supporting role, but a holding one, since these support rods have positive (tensile) forces. This makes designers to design retaining ties at the corner points, for example, in the form of cables with anchors at the base. It was also found that for a uniformly distributed load, responses of all supports (except for angular ones) are equal. This fact can greatly simplify the design, since all support elements can have the same section and height.

**REFERENCES**

1. Villegas L., Moran R., Garcia J. J. Combined culm-slat Guadua bamboo trusses. *Engineering Structures*. 2019; 184:495-504. DOI: 10.1016/j.estruct.2019.01.114

2. Dong L. Mechanical responses of snap-fit Ti-6Al-4V warren-truss lattice structures. *International Journal of Mechanical Sciences*. 2020; 173:105460. DOI: 10.1016/j.ijmecsci.2020.105460

3. Mathieson C., Roy K., Clifton G., Ahmadi A., Lim J.B.P. Failure mechanism and bearing capacity of cold-formed steel trusses with HRC connectors. *Engineering Structures*. 2019; 201:109741. DOI: 10.1016/j.estruct.2019.109741

4. Vatin N.I., Havula J., Martikainen L., Sinelnikov A.S., Orlova A.V., Salamakhin S.V. Thin-walled cross-sections and their joints: tests and fem-modelling. *Advanced Materials Research*. 2014; 945-949:1211-1215. DOI: 10.4028/www.scientific.net/AMR.945-949.1211

5. Hutchinson R.G., Fleck N.A. Microarchitecture of cellular solids – the hunt for statically determinate periodic trusses. *ZAMM*. 2005; 85(9):607-617. DOI: 10.1002/zamm.200410208

6. Hutchinson R.G., Fleck N.A. The structural performance of the periodic truss. *Journal of the Mechanics and Physics of Solids*. 2006; 54(4):756-782. DOI: 10.1016/j.jmps.2005.10.008

7. Zok F.W., Latture R.M., Begley M.R. Periodic truss structures. *Journal of the Mechanics and Physics of Solids*. 2016; 96:184-203. DOI: 10.1016/j.jmps.2016.07.007

8. Sud I.B. Derivation of formulas for deflection of the girder truss with an arbitrary number of panels in the maple system. *Structural Mechanics and Structures*. 2020; 2(25): 25–32. (rus.)

9. Terze S.V. Analytical calculation of the dependence of cantilever rack deformations on the number of panels in the maple system. *Structural Mechanics and Structures*. 2020; 2(25):16-24. (rus.)

10. Vorobiev O.V. About methods of obtaining analytical solution for eigenfrequencies problem of trusses. *Structural Mechanics and Structures*. 2020; 1(24):25-38. (rus.)

11. Belyankin N.A., Boyko A.Yu. Formulas for the deflection of a beam girder with an arbitrary number of panels under uniform loading. *Structural Mechanics and Structures*. 2019; 1(20): 21-29. (rus.)

12. Tkachuk G.N. The formula for the dependence of the deflection of an asymmetrically loaded planar truss with reinforced braces on the number of panels. *Structural mechanics and structures*. 2019; 2(21):32-39. (rus.)

13. Rakhmatulina A.R., Smirnova A.A. Analytical calculation and analysis of planar springel truss. *Structural mechanics and structures*. 2018; 2(17):72-79.

14. Kirsanov M.N. *Planar Trusses: Schemes and Formulas*. Cambridge Scholars Publishing Lady Stephenson Library, Newcastle upon Tyne, NE6 2PA, UK. 2019; 206.

15. Rakhmatulina A.R., Smirnova A.A. The dependence of the deflection of the arched truss loaded on the upper belt, on the number of panels. *Science Almanac*. 2017; 2-3(28):268-271. DOI: 10.17117/na.2017.02.03.268 (rus.)

16. Kazmiruk I.Yu. On the arch truss deformation under the action of lateral load. *Science Almanac*. 2016; 3-3(17):75-79. DOI: 10.17117/na.2016.03.03.075 (rus.)
17. Bolotina T.D. The deflection of the flat arch truss with a triangular lattice depending on the number of panels. Bulletin of Scientific Conferences. 2016; 4-3(8):7-8.

18. Voropai R.A., Kazmiruk I.Yu. Analytical study of the horizontal stiffness of the flat statically determinate arch truss. Bulletin of Scientific Conferences. 2016; 2-1(6):10-12.

19. Kirsanov M.N. Analytical study on the rigidity of statically determinate spatial truss. Vestnik MGSU [Proceedings of Moscow State University of Civil Engineering]. 2017; 12(2):165-171. DOI: 10.22227/1997-0935.2017.2.165-171 (rus.).

20. Tinkov D.V. Comparative analysis of analytical solutions to the problem of truss structure deflection. Magazine of Civil Engineering. 2015; 75(7):66-73. DOI: 10.5862/MCE.57.6 (rus.).

Received April 20, 2020.
Adopted in a revised form on June 17, 2020.
Approved for publication July 31, 2020.

**Bionotes:** Mikhail N. Kirsanov — Doctor of Physics and Mathematics, Professor, Department of robotics, mechatronics, dynamics and strength of machines; National Research University “Moscow Power Engineering Institute” (MPEI); 14 Krasnokazarmennaya st., Moscow, 111250, Russian Federation; ID RISC: 118571; C216@ya.ru.

**Literature**

1. Villegas L., Moran R., Garcia J.J. Combined culm-slat Guadua bamboo trusses // Engineering Structures. 2019. Vol. 184. Pp. 495–504. DOI: 10.1016/j. enstructure.2019.01.114

2. Dong L. Mechanical responses of snap-fit Ti-6Al-4V Warren-truss lattice structures // International Journal of Mechanical Sciences. 2020. Vol. 173. P. 105460. DOI: 10.1016/j.ijmecsci.2020.105460

3. Mathieson C., Roy K., Clifton G., Ahmadi A., Lim J.B.P. Failure mechanism and bearing capacity of cold-formed steel trusses with HRC connectors // Engineering Structures. 2019. Vol. 201. P. 109741. DOI: 10.1016/j. engstruct.2019.109741

4. Vatin N.I., Havula J., Martikainen L., Sinelnikov A.S., Orlova A.V., Salamakhin S.V. Thin-walled cross-sections and their joints: tests and fem-modeling // Advanced Materials Research. 2014. Vol. 945–949. Pp. 1211–1215. DOI: 10.4028/www.scientific.net/ AMR.945.1211

5. Hutchinson R.G., Fleck N.A. Microarchitected cellular solids – the hunt for statically determinate periodic trusses // ZAMM. 2005. Vol. 85. Issue 9. Pp. 607–617. DOI: 10.1002/zamm.200410208

6. Hutchinson R.G., Fleck N.A. The structural performance of the periodic truss // Journal of the Mechanics and Physics of Solids. 2006. Vol. 54. Issue 4. Pp. 756–782. DOI: 10.1016/j.jmps.2005.10.008

7. Zok F.W., Lattice R.M., Begley M.R. Periodic truss structures // Journal of the Mechanics and Physics of Solids. 2016. Vol. 96. Pp. 184–203. DOI: 10.1016/j. jmps.2016.07.007

8. Суд И.Б. Вывод формул для прогиба шпренгельной балочной фермы с произвольным числом панелей в системе maple // Строительная механика и конструкции. 2020. Т. 2. № 25. С. 25–32.

9. Терзег С.В. Аналитический расчет зависимост изо деформаций консольной стойки от числа панелей в системе maple // Строительная механика и конструкции. 2020. Т. 2. № 25. С. 16–24.

10. Воробьев О.В. О методах получения аналитического решения для проблемы собственных частот шарнирных конструкций // Строительная механика и конструкции. 2020. Т. 1. № 24. С. 25–38.

11. Белянкин Н.А., Бойко А.Ю. Формулы для прогиба балочной фермы с произвольным числом панелей при равномерном загружении // Строительная механика и конструкции. 2019. Т. 1. № 20. С. 21–29.

12. Ткачук Г.Н. Формула зависимости прогиба несимметрично нагруженной плоской фермы с усиленными раскосами от числа панелей // Строительная механика и конструкции. 2019. Т. 2. № 21. С. 32–39.

13. Rakhmatulina A.R., Smirnova A.A. Analytical calculation and analysis of planar springel truss // Structural mechanics and structures. 2018. Vol. 2. № 17. Pp. 72–79.
14. Kirsanov M.N. Planar trusses: schemes and formulas. Cambridge Scholars Publishing Lady Stephenson Library, Newcastle upon Tyne, NE6 2PA, UK. 2019. 206 p.

15. Рахматулина А.Р., Смирнова А.А. О зависимости прогиба арочной фермы, загруженной по верхнему поясу, от числа панелей // Научный альманах. 2017. № 2–3 (28). C. 268–271. DOI: 10.17117/na.2017.02.03.268

16. Казьмирук И.Ю. О деформации арочной фермы под действием боковой нагрузки // Научный альманах. 2016. № 3–3 (17). C. 75–79. DOI: 10.17117/na.2016.03.03.075

17. Bolotina T.D. The deflection of the flat arch truss with a triangular lattice depending on the number of panels // Bulletin of Scientific Conferences. 2016. № 4–3(8). C. 7–8.

18. Voropai R.A., Kazmiruk I.Yu. Analytical study of the horizontal stiffness of the flat statically determinate arch truss // Bulletin of Scientific Conferences. 2016. 2–1(6). C. 10–12.

19. Кирсанов М.Н. Аналитическое исследование жесткости пространственной статически определимой фермы // Вестник МГСУ. 2017. Т. 12. № 2 (101). C. 165–171. DOI: 10.22227/1997-0935.2017.2.165-171

20. Тиньков Д.В. Сравнительный анализ аналитических решений задачи о прогибе ферменных конструкций // Инженерно-строительный журнал. 2015. № 5 (57). C. 66–73. DOI: 10.5862/MCE.57.6

21. Greene R.L. Classical Mechanics with Maple. Springer-Verlag New York, 1995. 174 p. DOI: 10.1007/978-1-4612-4236-9

22. Бука-Вайваде К., Кирсанов М.Н., Сердюк Д.О. Calculation of deformations of a cantileverframe planar truss model with an arbitrary number of panels // Вестник МГСУ. 2020. Т. 15. Вып. 4. C. 510–517. DOI: 10.22227/1997-0935.2020.4.510-517

23. Krivoshapko S.N., Ivanov V.N. Encyclopedia of Analytic Surfaces. Librocom, 2019. 560 p.

24. Krivoshapko S.N. Research on general and axisymmetric ellipsoidal shells used as domes, pressure vessels, and tanks // Applied Mechanics Reviews. 2007. Vol. 60. Issue 6. Pp. 336–355. DOI: 10.1115/1.2806278

Поступила в редакцию 11 августа 2020 г.
Приема в доработанном виде 14 октября 2020 г.
Одобрена для публикации 30 октября 2020 г.

ОБ АВТОРЕ: Михаил Николаевич Кирсанов — доктор физико-математических наук, профессор кафедры робототехники, межатроники, динамики и прочности машин; Национального исследовательского университета «МЭИ» (НИУ «МЭИ»); 111250, г. Москва, ул. Красноказарменная, д. 14; РИНЦ ID: 118571; c216@ya.ru.