Bottomonium-like states: physics case for energy scan above the $B\bar{B}$ threshold at Belle-II

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Abstract

The Belle-II experiment is expected to collect large data samples at the $\Upsilon(4S)$ and $\Upsilon(5S)$ resonances to study primarily $B$ and $B_s$ mesons. We discuss what other data above the $B\bar{B}$ threshold are of interest. We propose to perform a high-statistics energy scan from the $B\bar{B}$ threshold up to the highest possible energy, and to collect data at the $\Upsilon(6S)$ and at higher mass states if they are found in the scan. We emphasize the interest in increasing the maximal energy from 11.24 GeV to 11.5 – 12 GeV in the future. These data are needed for investigation of bottomonium and bottomonium-like states.

1 Introduction

Conventional bottomonium is an approximately non-relativistic system. Out of 34 expected $b\bar{b}$ energy levels below the $B\bar{B}$ threshold [1] 15 have been observed [3]. The masses and decays of these states are well described by potential models [4].

There are five states with $b\bar{b}$ pairs above the $B\bar{B}$ threshold: three isospin-zero vector states $\Upsilon(10580), \Upsilon(10860)$ and $\Upsilon(11020)$ [or $\Upsilon(4S), \Upsilon(5S)$ and $\Upsilon(6S)$ according to the potential model assignment], and two isospin-one axial states $Z_b(10610)$ and $Z_b(10650)$. The isospin-one states are obviously exotic with minimal quark content $[b\bar{b}ud]$. But the isospin-zero states also have properties unexpected for a pure $b\bar{b}$ pair. The mass splitting between the $\Upsilon(4S)$ and $\Upsilon(5S)$ is larger by $73 \pm 11$ MeV/$c^2$ than that between the $\Upsilon(3S)$ and $\Upsilon(4S)$, while for a pure $b\bar{b}$ system it is expected to be smaller by about 40 MeV/$c^2$ [4]. The rates of $\Upsilon(5S) \to \Upsilon(nS)\pi^+\pi^-$ and $\Upsilon(6S) \to \Upsilon(nS)\pi^+\pi^-$ transitions are two orders of magnitude higher than expected for a pure bottomonium [5,6]. The $\eta$ transitions, that in a pure bottomonium involve spin of heavy quark and are suppressed by three orders of magnitude relative to $\pi^+\pi^-$ transitions, are not strongly suppressed in case of the $\Upsilon(5S)$ [7] and are even enhanced in case of the $\Upsilon(4S)$ [8]. More puzzling results on hadronic transitions are discussed below. Thus, there is a strong evidence that the structure of the vector states above the $B\bar{B}$ threshold is more complicated than the pure $b\bar{b}$, i.e. it is exotic.

In this paper we briefly review the experimental status of the “bottomonium-like” states, their interpretation and prospects for future studies. Our main goal is to discuss what we can learn about them based on the increased $\Upsilon(4S)$ and $\Upsilon(5S)$ data samples at Belle-II, and what dedicated data taking is needed. We propose to perform a high-statistics energy scan from the $B\bar{B}$ threshold up to the highest possible energy to measure exclusive open and hidden flavor cross sections. We also propose to collect data at the peak of $\Upsilon(6S)$ and at higher mass states – if they are found in the energy scan – to study their properties and to search for missing bottomonia, $P$-wave $B_s$ mesons or exotic states – partners of the $Z_b(10610)$ and $Z_b(10650)$.

Favored interpretation of the bottomonium-like states is related to the presence of heavy mesons in their wave functions. The $Z_b(10610)$ and $Z_b(10650)$ states have purely molecular structures, $B\bar{B}^*+B^*\bar{B}$, respectively, while the $\Upsilon(4S), \Upsilon(5S)$ and $\Upsilon(6S)$ states are mixtures of the $b\bar{b}$ and $B_s^{(*)}\bar{B}_s^{(*)}$ pairs [with $B_s^{(*)}$ we denote $P$-wave excitations of $B$ or $B_s$ mesons]. We discuss measurements that are needed to establish this interpretation and point out strategy to search for other types of exotic hadrons – compact tetraquarks [9] or hadroquarkonia [10].

The paper is organized as follows. We first discuss the results obtained using on-resonance data, since they constitute the bulk of our current knowledge about the bottomonium-like states, and then move to the discussion of the energy scans. We conclude with a summary of the physics program for the proposed dedicated data taking.

2 On-resonance data

The results on bottomonium-like states presented in this section are based on large data samples collected by the Belle and BaBar experiments. These are 711 fb$^{-1}$ and 433 fb$^{-1}$ collected by Belle and BaBar, respectively, at the
the QCD multipole expansion \[21, 22\]. The size of the quarkonium is small compared to the gluon wavelength \( s \), the transitions can be described using hadrons (see Fig. 1 (left)). Such transitions are OZI suppressed, accordingly, their partial widths are small. If \( \Gamma \) is the width of the \( \Upsilon(5S) \) and \( \Upsilon(6S) \) partial

2.1 Mechanism of hadronic transitions and structure of vector bottomonium-like states

Known hadronic transitions from vector bottomonium and bottomonium-like states and corresponding partial widths are presented in Table 1. There are difficulties with the determination of the \( \Upsilon(5S) \) and \( \Upsilon(6S) \) partial widths, since, in particular, the corresponding \( e^+e^- \) widths are not yet known. Thus the partial widths are defined as \( \sigma_{\text{vis}}/\sigma_{b\bar{b}} \times \Gamma \), where \( \sigma_{\text{vis}} \) is the visible cross section of the considered process, \( \sigma_{b\bar{b}} \) is the total \( b\bar{b} \) cross section and \( \Gamma \) is the width of the \( \Upsilon(5S) \) or \( \Upsilon(6S) \). This definition likely underestimates the partial widths \[20\].

In a pure \( b\bar{b} \) bottomonium the hadronic transitions proceed via emission of gluons, which convert into light hadrons (see Fig. 1 (left)). Such transitions are OZI suppressed, accordingly, their partial widths are small. If the size of the quarkonium is small compared to the gluon wavelength, the transitions can be described using the QCD multipole expansion \[21\] \[22\]. The \( \pi^+\pi^- \) transitions between the \( \Upsilon(nS) \) occur via emission of two \( E1E1 \) gluons, while \( \eta \) transitions require emission of \( E1M2 \) gluons. Such magnetic transitions involve the spin of the \( b\bar{b} \) bottomonium the hadronic transitions proceed via emission of gluons, which convert into light hadrons.

![Click here to view the image](image.png)

| Transition         | Partial width (keV) | Reference |
|--------------------|---------------------|-----------|
| \( \Upsilon(2S) \rightarrow \) |                     |           |
| \( \Upsilon(1S) \pi^+\pi^- \) | 5.7 ± 0.5           | \[3\]     |
| \( \Upsilon(1S) \eta \) | (9.3 ± 1.5) \times 10^{-3} | \[3\]     |
| \( \Upsilon(3S) \rightarrow \) |                     |           |
| \( \Upsilon(1S) \pi^+\pi^- \) | 0.89 ± 0.08         | \[3\]     |
| \( \Upsilon(1S) \eta \) | < 2 \times 10^{-3}  | \[3\]     |
| \( \Upsilon(2S) \pi^+\pi^- \) | 0.57 ± 0.06         | \[3\]     |
| \( \Upsilon(4S) \rightarrow \) |                     |           |
| \( \Upsilon(1S) \pi^+\pi^- \) | 1.7 ± 0.2           | \[3\] \[13\] |
| \( \Upsilon(1S) \eta \) | 4.0 ± 0.8           | \[8\]     |
| \( \Upsilon(2S) \pi^+\pi^- \) | 1.8 ± 0.3           | \[8\]     |
| \( h_b(1P) \eta \) | 45 ± 7              | \[14\]    |
| \( \Upsilon(5S) \rightarrow \) |                     |           |
| \( \Upsilon(1S) \pi^+\pi^- \) | 238 ± 41            | \[15\]    |
| \( \Upsilon(1S) \eta \) | 39 ± 11             | \[7\]     |
| \( \Upsilon(1S) K^+K^- \) | 33 ± 11             | \[5\]     |
| \( \Upsilon(2S) \pi^+\pi^- \) | 428 ± 83            | \[13\]    |
| \( \Upsilon(2S) \eta \) | 204 ± 44            | \[7\]     |
| \( \Upsilon(3S) \pi^+\pi^- \) | 153 ± 31            | \[14\]    |
| \( \chi_b(1P) \omega \) | 84 ± 20             | \[10\]    |
| \( \chi_b(1P) (\pi^+\pi^-\pi^0)_{\text{non-}\omega} \) | 28 ± 11             | \[10\]    |
| \( \chi_b(2P) \omega \) | 32 ± 15             | \[10\]    |
| \( \chi_b(1P) (\pi^+\pi^-\pi^0)_{\text{non-}\omega} \) | 33 ± 20             | \[10\]    |
| \( \Upsilon_J(1D) \pi^+\pi^- \) | \(~ 60\)            | \[17\]    |
| \( \Upsilon_J(1D) \eta \) | 150 ± 48            | \[18\]    |
| \( Z_b(10610)^{+}\pi^\mp \) | 2070 ± 440          | \[19\]    |
| \( Z_b(10650)^{+}\pi^\mp \) | 1200 ± 300          | \[19\]    |
| \( \Upsilon(6S) \rightarrow \) |                     |           |
| \( \Upsilon(1S) \pi^+\pi^- \) | 137 ± 32            | \[6\]     |
| \( \Upsilon(2S) \pi^+\pi^- \) | 183 ± 43            | \[6\]     |
| \( \Upsilon(3S) \pi^+\pi^- \) | 77 ± 28             | \[6\]     |
| \( Z_b(10610,10650)^{+}\pi^\mp \) | 1300 − 6600         | \[12\]    |

\footnote{We do not list transitions to the \( h_b(nP)^{+}\pi^- \) final states, since they proceed entirely via the intermediate \( Z_b \) states \[11\] \[12\]. We do not subtract the \( Z_b \) contributions from the \( \Upsilon(nS)^{+}\pi^- \) final states; they should be relatively small in all transitions except \( \Upsilon(6S) \rightarrow \Upsilon(3S)^{+}\pi^- \).}
heavy $b$ quark and are strongly suppressed, which is a manifestation of the Heavy Quark Spin Symmetry (HQSS). These expectations are well fulfilled for the $\Upsilon(2S)$ and $\Upsilon(3S)$ states, which are below the $B\bar{B}$ threshold.

For the states above the $B\bar{B}$ threshold the pattern of transitions changes. In case of $\Upsilon(4S)$ the $\eta$ transition to $\Upsilon(1S)$ is not suppressed, but rather enhanced by a factor of $2.4 \pm 0.4$ relative to the $\pi^+\pi^-$ transition, which corresponds to a very strong violation of HQSS. Similarly, the $\Upsilon(4S) \rightarrow h_b(1P)\eta$ transition in the multipole expansion corresponds to emission of HQSS-breaking $E1M1$ gluons, but it is found to be even further enhanced. In case of the $\Upsilon(5S)$ and $\Upsilon(6S)$, the partial widths are at the level of hundred keV, which is two orders of magnitude higher than expected for a pure bottomonium, and is a clear violation of the OZI rule. The $\eta$ transitions have comparable rates to the $\pi^+\pi^-$ transitions, which also corresponds to a significant violation of HQSS. In the case of the $\Upsilon(5S) \rightarrow \chi_{bJ}(1P)$ transitions the HQSS prediction for the relative yield of the $\chi_{bJ}(1P)$ states with different values of $J$ also appears to be strongly violated [23 24].

The fact that the states $\Upsilon(4S)$, $\Upsilon(5S)$ and $\Upsilon(6S)$ contain not only $b\bar{b}$ pairs, but also a “molecular” admixture of heavy mesons, $B_s^{(*)}B_s^{(*)}$, is crucial for understanding their puzzling properties. It was realized at the time of observation of the $\Upsilon(4S)$, $\Upsilon(5S)$ and $\Upsilon(6S)$ in 1980s [25 26] that too high splitting between the $\Upsilon(5S)$ and $\Upsilon(4S)$ states is due to the contribution of hadron loops [27], which is another language to discuss the molecular admixture. Enhanced transitions into hidden flavor final states are due to rescattering of the on-shell heavy mesons (see Fig. 1 (right)) [28 29]. Finally, the molecular admixture is also responsible for the violation of HQSS. Indeed, an admixture of a specific $B\bar{B}$, $B\bar{B}^*$ or $B^*\bar{B}^*$ meson pair is not an eigenstate of the $b\bar{b}$ total spin. Table 2 presents the decomposition of the $P$-wave $B_s^{(*)}B_s^{(*)}$ pairs with $J^{PC}=1^{--}$ into the $b\bar{b}$ spin eigenstates $\psi_{ij}$, where $i$ is the total spin of the $b\bar{b}$ pair and $j$ is the total angular momentum contributed by all other degrees of freedom, including both the spin of light quarks and the orbital angular momentum $L = 1$ [30 31]. Various $\psi_{ij}$ components give rise to transitions that are forbidden by HQSS for pure $b\bar{b}$ states. Experimental signatures for $\psi_{ij}$ components are presented in Table 3.

Table 2: The decomposition of the $P$-wave $B_s^{(*)}B_s^{(*)}$ pairs with $J^{PC}=1^{--}$ into the $b\bar{b}$ spin eigenstates.

| State | Decomposition into $b\bar{b}$ spin eigenstates |
|-------|-----------------------------------------------|
| $B\bar{B}$ | $\frac{1}{\sqrt{3}}\psi_{10} + \frac{1}{\sqrt{6}}\psi_{11} + \frac{2}{\sqrt{3}}\psi_{12} + \frac{1}{\sqrt{2}}\psi_{01}$ |
| $B\bar{B}^*$ | $\frac{1}{\sqrt{3}}\psi_{10} + \frac{1}{\sqrt{6}}\psi_{11} - \frac{\sqrt{2}}{\sqrt{3}}\psi_{12}$ |
| $(B^*\bar{B})_{S=0}$ | $-\frac{1}{2}\psi_{10} - \frac{1}{2\sqrt{3}}\psi_{11} - \frac{\sqrt{2}}{2}\psi_{12} + \frac{\sqrt{2}}{2}\psi_{01}$ |
| $(B^*\bar{B})_{S=2}$ | $\frac{\sqrt{2}}{2}\psi_{10} - \frac{\sqrt{2}}{2\sqrt{3}}\psi_{11} + \frac{1}{\sqrt{2}}\psi_{12}$ |

Table 3: Experimental signatures for the $b\bar{b}$ spin eigenstates $\psi_{ij}$.

| Spin eigenstate | Expected decays |
|----------------|----------------|
| $\psi_{10}$ | $\Upsilon(nS)\pi^+\pi^-$, $\Upsilon(nS) K^+K^-$ in $S$ wave |
| $\psi_{11}$ | $\Upsilon(nS)\eta$, $\Upsilon(nS)\eta'$ |
| $\psi_{11}, \psi_{12}$ | $\Upsilon(nS)\pi^+\pi^-$, $\Upsilon(nS) K^+K^-$ in $D$ wave |
| $\psi_{01}$ | $\eta_b(nS)\omega$, $\eta_b(nS)\phi$, $h_{b}(nP)\eta$, $h_{b}(nP)\eta'$ |

One can expect that the closer is the physical state to the threshold, the larger is the admixture of the corresponding meson pairs. Given that the $\Upsilon(4S)$ is only 20 MeV above the $B\bar{B}$ threshold and 25 MeV below the
threshold, the \(B\bar{B}\) should be the dominant admixture in the \(\Upsilon(4S)\) wave function \([31]\). The \(\psi_{10}\) component has the same quantum numbers as a pure \(b\bar{b}\) pair. It can contribute to the \(\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^-\) transitions, but since the phase space for the \(B\bar{B}\) pair is still small, the corresponding partial width is not enhanced. The \(\Upsilon(4S) \rightarrow \Upsilon(1S)\eta\) transition could be due to the \(\psi_{11}\) component, thus the HQSS breaking finds a very natural explanation. The \(\psi_{11}\) and \(\psi_{12}\) components should lead to a \(D\)-wave contribution in the \(\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^-\) decay, which was not yet studied experimentally. Finally, the \(\psi_{01}\) component could contribute to the \(\Upsilon(4S) \rightarrow h_0(1P)\eta\) transitions. The same component could result in the \(\eta_b(1S)\omega\) final state, which has not yet been studied experimentally.

In case of the \(\Upsilon(5S)\) the dominant HQSS-violating contribution could be the \(B^*_s\bar{B}^*_s\) component, since the corresponding threshold is only 40 MeV below the \(\Upsilon(5S)\) peak \([31]\). In this case one can expect that the \(D\)-wave component in the \(\Upsilon(5S) \rightarrow \Upsilon(1S)K^+K^-\) final state and the spin-singlet channels \(h_0(nP)\eta, h_0(1P)\eta'\) and \(\eta_b(1S)\phi\) are enhanced. Only results on \(\eta\) transitions were reported with relatively weak upper limits \([38]\). The distance to the \(B^{(*)}\) thresholds of about 260 MeV/\(c^2\) is relatively big compared to the splitting between the thresholds of 45 MeV/\(c^2\), which possibly suppresses the contribution of the non-strange \(B\) meson pairs to the HQSS violation. Indeed, there is no non-resonant contribution in the \(\Upsilon(5S) \rightarrow h_0(nP)\pi^+\pi^-\) transitions. The light quark content of the heavy meson admixture can be tested via the ratio of the \(\Upsilon(1S)\eta'\) vs. \(\Upsilon(1S)\eta\) and \(h_0(1P)\eta'\) vs. \(h_0(1P)\eta\) decay rates \([31]\).

The \(\Upsilon(6S)\) is situated near the \(B_1(5721)\bar{B}\) threshold, where \(B_1(5721)\) is a narrow \(P\)-wave excitation with the spin-parity of the light degrees of freedom \(j^p = 3/2^+\). A contribution of the \(B_1(5721)\bar{B}\) pairs to the \(\Upsilon(6S)\) decays has a very clear experimental signature: the \(Z_b(10610)\pi\) final state should be produced, while the \(Z_b(10650)\pi\) should not \([32]\). This prediction is distinct from the observations at \(\Upsilon(5S)\) (see discussion in Ref. \([12]\)), despite the two states being relatively close in energy. The detailed comparison of the two states is of interest and requires an increase of statistics at the \(\Upsilon(6S)\), which can be realized at Belle-II.

Energy dependence of the cross sections was measured only for the \(\Upsilon(nS)\pi^+\pi^-\) \((n = 1, 2, 3)\) and \(h_0(mP)\pi^+\pi^-\) \((m = 1, 2)\) final states \([6, 12]\). The cross sections exhibit no continuum contribution, and the parameters of the \(\Upsilon(5S)\) and \(\Upsilon(6S)\) peaks agree well among different channels (it is interesting to confirm experimentally that the same is true for other hidden flavor channels). Thus for a measurement of the relative rates of hadronic transitions the on-resonance data are sufficient. The situation with open flavor decays (like \(BB, BB^{*\ldots}\)) is completely different, since a significant continuum contribution is seen \([33, 35]\) and quite different line-shapes are expected in different channels \([34]\). Therefore, studies of open flavor channels require an energy scan; we discuss this in Section \([3]\) below.

In the next two sections we consider the possibility of using the hadronic transitions to search for missing conventional bottomonia and for molecular states – expected partners of the \(Z_b(10610)\) and \(Z_b(10650)\).

### 2.2 Search for missing bottomonia below the \(B\bar{B}\) threshold

The 121 \(fb^{-1}\) data sample at the \(\Upsilon(5S)\) was highly instrumental in finding missing bottomonium levels. Belle observed the \(h_0(1P)\) and \(h_0(2P)\) states using \(\Upsilon(5S) \rightarrow h_0(nP)\pi^+\pi^-\) transitions \([11]\). (We do not list these transitions in Table \([1]\) since they proceed entirely via the intermediate \(Z_b(10610)\) and \(Z_b(10650)\) states \([35]\).) Belle also found first evidence for the \(\eta_b(2S)\) state, precisely measured the \(\eta_b(1S)\) mass and for the first time measured its width using prominent radiative transitions from \(h_0(nP)\) \([36]\). In addition, Belle observed the \(\Upsilon(5S) \rightarrow \Upsilon_J(1D)\pi^+\pi^-\) and \(\Upsilon(5S) \rightarrow \Upsilon_J(1D)\eta\) transitions; the accuracy in the \(\Upsilon_J(1D)\) mass is competitive with measurements that use \(\Upsilon(3S) \rightarrow \Upsilon_J(1D)\gamma\) transitions \([3]\).

Bottomonium levels below the \(B\bar{B}\) threshold that are still missing are shown in Table \([1]\). The spin-singlet members of the \(3S, 3P\) and \(1D\) multiplets are not known, as well as the complete \(2D, 1F\) and \(1G\) multiplets. They can be searched for using transitions listed in Table \([1]\). Most of the thresholds for final state particles are above \(\Upsilon(5S)\) and \(\Upsilon(6S)\), thus one needs first to find a new vector bottomonium-like state with sufficiently high mass. However, quite a few final states are accessible already at the \(\Upsilon(5S)\) or \(\Upsilon(6S)\).

No result was reported yet for the \(\Upsilon(5S) \rightarrow \Upsilon_J(2D)\pi^+\pi^-\) transition. The chiral soft pion theorems ensure that the amplitude of a \(\pi^+\pi^-\) transition is bilinear in the momenta of the pions, independently of the structure of the vector bottomonium states \(\Upsilon(5S), \Upsilon(6S)\) such as presence of a molecular component, \(S - D\) mixing, etc. For this reason the transitions \(\Upsilon(5S) \rightarrow \Upsilon_J(2D)\pi^+\pi^-\) are strongly kinematically suppressed in comparison with \(\Upsilon(5S) \rightarrow \Upsilon_J(1D)\pi^+\pi^-\). Using the predictions of potential models for the mass of the \(2^3D_2\) state about 10450 MeV (see, e.g., Ref. \([37]\)), one readily estimates that the rate of the \(\Upsilon(5S) \rightarrow \Upsilon_J(2D)\pi^+\pi^-\) carries a
kinematic suppression factor of approximately $4 \times 10^{-3}$ as compared to $\Upsilon(5S) \to \Upsilon_J(1D)\pi^+\pi^-$. The relative kinematical suppression factor is not as dramatic at the higher energy of the $\Upsilon(6S)$ peak: approximately $2.5 \times 10^{-2}$, but still is quite strong. For the case of $\eta$ transitions to the $\Upsilon_J(2D)$ bottomonium, only the $\Upsilon(6S)$ peak is at or slightly above the kinematical threshold. Given that the $\eta$ transition is a $P$-wave process, a very strong kinematical suppression of the process $\Upsilon(6S) \to \Upsilon_J(2D)\eta$ is to be expected. It is certainly possible that dynamical factors in the transition amplitudes do compensate, partially or fully, the kinematical suppression. However, realistically a search for the $\Upsilon_J(2D)$ bottomonium states would likely require exploring the $e^+e^-$ annihilation at energies above the $\Upsilon(6S)$ peak.

The production in $e^+e^-$ annihilation of the final states with the spin-singlet $h_{23}(1F)$ and $\eta_{23}(1G)$ bottomonium requires breaking of HQSS and is expected to be suppressed, unless such breaking is enhanced by molecular resonance effects, e.g. similar to the production of $h_{03}(mP)\pi\pi$ within the $\Upsilon(5S)$ and $\Upsilon(6S)$ resonances due to intermediate exotic $Z_b$ molecular states. Furthermore, the light mesons produced in association with either spin-singlet or spin-triplet 1F or 1G bottomonium should carry a large angular momentum, resulting in a kinematical suppression of the yield near the thresholds. A possible exception from this threshold behavior may be applicable to the production of the final state $\chi_{22}(1F)\omega$ [or $\chi_{22}(1F)\phi$], where an $S$-wave process is kinematically allowed. However, a presence of an $S$-wave requires breaking of HQSS, so that an observation of such process would definitely provide an insight into the mechanisms for violation of HQSS.

The transitions listed in Table \ref{table:4} can be reconstructed inclusively using missing mass of the emitted light hadrons. In case of the spin-triplet levels there are also final states convenient for exclusive reconstruction. The dominant transitions between the bottomonia below the $B\bar{B}$ threshold are radiative $E1$ transitions that change orbital angular momentum of the $b\bar{b}$ pair by one unit and conserve the $b\bar{b}$ spin. Thus the chain $\Upsilon_J(1G) \to \gamma\chi_{23}J(1F) \to \gamma\gamma\Upsilon_J(1D) \to \gamma\gamma\chi_{23}J(1P) \to \gamma\gamma\gamma\gamma\Upsilon(1S)$ corresponds to dominant transitions and can be used for exclusive reconstruction with $\Upsilon(1S) \to e^+e^-$ or $\mu^+\mu^-$. More details on the bottomonium decays can be found in e.g. Ref. \cite{1}.

### 2.3 Search for molecular states – partners of the $Z_b(10610)$ and $Z_b(10650)$

In the single pion transitions from the $\Upsilon(5S)$ and $\Upsilon(6S)$ Belle observed isovector bottomonium-like states $Z_b(10610)$ and $Z_b(10650)$ \cite{33}. The Breit-Wigner masses of the $Z_b(10610)$ and $Z_b(10650)$ are located within the experimental uncertainty of about 2 MeV/$c^2$ at the $BB^*$ and $B^*B^*$ thresholds, respectively. These states are known to decay to $\Upsilon(nS)\pi$ ($n = 1, 2, 3$) and $h_0(mP)\pi$ ($n = 1, 2$) channels; however, the dominant decays with about 80% branching fractions are $Z_b(10610) \to BB^*$ and $Z_b(10650) \to B^*B^*$ \cite{19}. Such a decay pattern is a “smoking gun” of the molecular structure, $BB^*$ for the $Z_b(10610)$ and $B^*B^*$ for the $Z_b(10650)$ \cite{33}. Measured spin and parity of $J^P = 1^+$ for both states \cite{15} correspond to $BB^*$ and $B^*B^*$ in the $S$-wave, which supports the molecular interpretation. Recent combined analysis of the $B^{(*)}B^*$ and $h_0(mP)\pi$ channels using phenomenologically motivated expressions for amplitudes indicates that $Z_b(10610)$ and $Z_b(10650)$ may in fact be virtual molecular states with poles within 2 MeV from the corresponding thresholds \cite{59, 40}.

In the limit of exact HQSS the mechanism that binds $B$ mesons to form the $Z_b$ states is determined by light degrees of freedom, while the total spin of the $b\bar{b}$ pair plays a classification role only: rotating this spin one can find other molecular states that are partners of the $Z_b$'s \cite{11}. Table \ref{table:5} gives the predicted states with their composition and quantum numbers. All the expected, but not yet observed, isovector states have negative $G$-parity, which is opposite to that of the $Z_b$ resonances. For this reason they can not be produced in $e^+e^-$ annihilation with the emission of a single pion. The most natural channel for their production is the emission of $\rho(770)$ meson. For the transitions from $\Upsilon(5S)$ the maximal $\pi^+\pi^-$ invariant mass is only 300 MeV/$c^2$ which makes such a production,
The total hadronic cross section is measured up to 11.2 GeV, while the exclusive cross sections are measured similar to that at Belle and thus is close to 5 MeV, no narrow peak will be missed if the step of the scan is 10 MeV. Due to the low-mass ‘tail’ of the $\rho$ resonance, strongly suppressed. At the $\Upsilon(6S)$ the maximal mass is 440 MeV/c², thus there are better chances to observe the $W_{60}$. An alternative way to produce the electrically neutral isotopic components of these molecular states at the $\Upsilon(5S)$ is provided by emission of a photon. Naturally, the rates of such radiative processes carry the suppression factor of $\alpha$.

No isosinglet states were seen yet in the bottomonium sector, and at present they are purely hypothetical. It has to be mentioned that only the isovector states in the Table 5 are related to the observed $Z_0^\ast$ resonances. For the isosinglet ones the interaction of the light components is different, and it is not known whether this interaction results in near-threshold singularities for the heavy meson-antimeson pairs. Furthermore, the isosinglet states are affected by mixing with a pure $b\bar{b}$ bottomonium, which mixing can result in yet not fully predictable modification of their properties. In particular, the admixture of a compact $b\bar{b}$ component should result in that the isoscalar states, unlike those with $I = 1$, can be produced in hard processes, e.g., at LHC. An example of such a behavior in the charmonium sector is the $X(3872)$ resonance. Being essentially a threshold singularity in the $D^0\bar{D}^{*0} + D^0\bar{D}^{0}$ channel, it apparently contains a short-distance $c\bar{c}$ charmonium core through which it is produced in hard processes: the $B$ meson decays and at high-energy proton-(anti)proton colliders (see, e.g., in the review [42]).

There is also a good reason to avoid deducing the existence and the properties of the hypothetical $J^{PC} = 1^{++}$ isoscalar bottomonium-like resonance $X_0$ from those of the $X(3872)$. Namely, in the latter the isotopic symmetry is badly broken by the isotopic mass differences of the $D$ mesons, so that the $X(3872)$ is a strong mixture of $I = 0$ and $I = 1$ states. For the $B$ mesons the isotopic mass differences are very small, and the separation between isoscalar and isovector states should be very well preserved.

The isosinglet states can be produced in $\eta$ or $\omega$ transitions (depending on the $C$-parity), however, in this case the $\Upsilon(6S)$ energy is insufficient, and one has to find a higher vector state with the mass above 11.43 GeV/c².

Details of the interaction resulting in the existence of the $Z_0^\ast$ peaks are not yet understood. One might speculate at this point that if the dominant one is the interaction between the gluonic degrees of freedom in the $B^{(0)}$ mesons, some form of flavor SU(3) symmetry may be present between the molecular states giving rise to existence of strange analogs of the $Z_0$ resonances. Such bottomonium-like strange resonances, $Z_{0s}$, related to the combinations of the channels $B_s^\ast\bar{B}$ and $B_s^\ast\bar{B}^\ast$ and also to $B_s^\ast\bar{B}^\ast$, should have masses, respectively, near 10695 MeV and 10740 MeV, and can be produced in $e^+e^-$ annihilation in association with a kaon: $e^+e^- \to Z_{0s}K$, at energies above 11.20 GeV. These resonances would decay into the states of bottomonium plus a kaon, and also to heavy meson pairs with one $B$ meson being either $B_s$ or $B_s^\ast$. By any measure, an observation of such bottomonium-like molecules with strangeness would be of paramount importance for studies of hadronic dynamics.

### 3 Energy scan of the cross sections

Measurement of the total hadronic cross section does not require high statistics. In recent energy scans BaBar and Belle used 25 pb⁻¹ and 50 pb⁻¹ per point, respectively [33, 6]. Much more data are necessary to measure exclusive cross sections. Belle performed a high-statistics energy scan with roughly 1 fb⁻¹ per point and measured the $\Upsilon(nS)\pi^+\pi^-$ ($n = 1, 2, 3$), $h_0(nP)\pi^+\pi^-$ ($m = 1, 2$) and $B^{(*)}\bar{B}^{(*)}$ cross sections [6] [12] [43]. Results for $B\bar{B}$, $BB^*$, $B^*\bar{B}^*$ and $B^{(*)}\bar{B}^{(*)}\pi$ are still expected [44]. The statistical uncertainty of the measurements is rather large, thus at Belle-II it is useful to collect about 10 fb⁻¹ per scan point. Given that expected energy smearing at Belle-II is similar to that at Belle and thus is close to 5 MeV, no narrow peak will be missed if the step of the scan is 10 MeV. The total hadronic cross section is measured up to 11.2 GeV, while the exclusive cross sections are measured up
to 11.02 GeV. The maximal energy of the SuperKEKB collider, limited by the injection system, is 11.24 GeV.

3.1 Cross sections for open flavor channels

The measured total hadronic cross section, usually presented as \( R_b = \sigma(e^+e^- \rightarrow bb)/\sigma_{\mu\mu}^0 \), where \( \sigma_{\mu\mu}^0 \) is the Born \( e^+e^- \rightarrow \mu^+\mu^- \) cross section, has several features (see Fig. 2 (left)): these are peaks of the \( \Upsilon(4S) \), \( \Upsilon(5S) \) and \( \Upsilon(6S) \), and dips near the \( B\bar{B}^* \), \( B^*\bar{B}^* \) and \( B_s^*\bar{B}_s^* \) thresholds [6, 33]. The channels with the bottomonium in the final state contribute only a few percent to the total \( bb \) cross section; the rest is due to the open flavor channels, such as \( B\bar{B} \), \( BB^* \), \( B^*\bar{B}^* \), \( B\bar{B}\pi \), \( BB^*\pi \), \( B^*\bar{B}\pi \), \( B_s\bar{B}_s \), \( B_s\bar{B}_s^* \), \( B_s^*\bar{B}_s \) etc. The corresponding exclusive cross sections are expected to have significantly more features than their sum. As an example, Fig. 2 (right) shows the predictions of the Unitarized Quark Model [34]. The oscillatory behavior of exclusive cross sections in this model is due to the nodes of the \( \Upsilon(5S) \) and \( \Upsilon(6S) \) wave functions. The individual cross sections contain considerably more information than their sum. Their measurements are extremely important for understanding this energy region. In particular, they will allow to determine the \( e^+e^- \) widths of \( \Upsilon(5S) \) and \( \Upsilon(6S) \), that are needed to determine their branching fractions. Moreover, a combined analysis of the energy dependence of the open-flavor and hidden-flavor cross sections in the framework of the coupled-channel models is crucial for determining their most essential basic properties, such as the positions of the poles and inter-channel couplings. Thus, the studies along these lines could establish the picture of the vector bottomonium-like states as superpositions of a pure \( bb \) bottomonium and a molecular \( B_{(s)}^*\bar{B}_{(s)}^* \) component.

3.2 Cross sections for hidden-flavor channels

Hidden-flavor cross sections provide input for the coupled-channel analysis and play important role in searching for new states.

One alternative to the molecular picture, currently discussed in the literature, is the diquark-antidiquark model [9], where the quarks and antiquarks making up a four-quark state are the same as in a molecule, however they are arranged (or clustered) differently: the two quarks form a diquark, while the two antiquarks form an antidiquark. Both diquark and antidiquark are colored objects, therefore such a system is very compact contrary to meson-antimeson molecule. The (anti-)quarks within the (anti-)diquark are more tightly bound than the diquark and anti-diquark. This model can also explain the OZI rule violation and the HQSS breaking, however, the \( B_s \) final state contribute only a few percent to the total hadronic cross section; the rest is due to the open flavor channels, such as \( B\bar{B} \), \( BB^* \), \( B^*\bar{B}^* \), \( B\bar{B}\pi \), \( BB^*\pi \), \( B^*\bar{B}\pi \), \( B_s\bar{B}_s \), \( B_s\bar{B}_s^* \), \( B_s^*\bar{B}_s \) etc. The corresponding exclusive cross sections are expected to have significantly more features than their sum. As an example, Fig. 2 (right) shows the predictions of the Unitarized Quark Model [34]. The oscillatory behavior of exclusive cross sections in this model is due to the nodes of the \( \Upsilon(5S) \) and \( \Upsilon(6S) \) wave functions. The individual cross sections contain considerably more information than their sum. Their measurements are extremely important for understanding this energy region. In particular, they will allow to determine the \( e^+e^- \) widths of \( \Upsilon(5S) \) and \( \Upsilon(6S) \), that are needed to determine their branching fractions. Moreover, a combined analysis of the energy dependence of the open-flavor and hidden-flavor cross sections in the framework of the coupled-channel models is crucial for determining their most essential basic properties, such as the positions of the poles and inter-channel couplings. Thus, the studies along these lines could establish the picture of the vector bottomonium-like states as superpositions of a pure \( bb \) bottomonium and a molecular \( B_{(s)}^*\bar{B}_{(s)}^* \) component.

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In the charmonium sector there are states that exhibit themselves as peaks in hidden-flavor channels (e.g., \( J/\psi\pi^+\pi^- \), \( \psi(2S)\pi^+\pi^- \), \( h_c\pi^+\pi^- \)), but are not seen in the total hadronic cross section, or even in exclusive open-
charm final states \cite{3}. The nature of these charmonium-like states is not yet understood; given that each peak is dominantly seen in only one final state, the notion of hadrocharmonium has been proposed \cite{10}. This “selectivity” of final states makes hadrocharmonium distinct from a compact tetraquark. There could exist a $b\bar{b}$ partner of the hadrocharmonium, the hadrobottomonium.

The states with molecular admixture should be seen in the open-flavor channels or in the total hadronic cross section, as is the case for the $\Upsilon(4S)$, $\Upsilon(5S)$ and $\Upsilon(6S)$, since dominance of the open-flavor channels is a “smoking gun” of the molecular structure. Nevertheless, hidden-flavor channels could have better sensitivity because of higher reconstruction efficiency. Moreover, in open-flavor channels there is always a continuum contribution, which is zero in hidden-flavor channels in all known so far cases.

Thus, the hidden-flavor channels could have better sensitivity than the open-flavor ones in searches for states with molecular admixture; they are unique for exotic states with different than molecular structures, such as compact tetraquarks or hadrobottomonia. The final states to be investigated are the same as already found at the $\Upsilon(5S)$ or proposed to be searched for (see Tables 1 and 3).

3.3 Promising $e^+e^-$ energy regions

Molecular states are naturally located (and produce the largest effects) near the corresponding threshold. The positions of the thresholds are listed in Table 6 where we consider only narrow $S$- and $P$-wave mesons and baryons. The regions of $B^{(*)}\bar{B}^{**}$ and $B^{(*)}_{s}\bar{B}^{**}_{s}$ are basically within the current reach of the SuperKEKB collider of 11.24 GeV. An increase of the energy by at least 100 MeV will give a possibility to explore the $\Lambda_b\bar{\Lambda}_b$ threshold region and to search for baryon-antibaryon molecular states. Such states are almost certainly there, as can be judged from the charmonium sector.

It is also quite important to find a high-mass state to serve as a source of missing bottomonia below the $B\bar{B}$ threshold and of molecular states – partners of the $Z_b(10610)$ and $Z_b(10650)$, as discussed in Sections 2.3 and 2.2. The thresholds for various transitions are listed in Tables 1 and 5. The maximal threshold is at 11.55 GeV. In order to cover the molecular states at the $B^{(*)}_{s}\bar{B}^{**}_{s}$ thresholds the peak should be above 11.61 GeV. This is the region of $B^{(*)}\bar{B}^{**}$, $B^{(*)}_{s}\bar{B}^{**}_{s}$, $\Lambda_b\bar{\Lambda}_b$ and $\Sigma^{(*)}_{b}\bar{\Sigma}^{(*)}_{b}$ thresholds. Finally, investigation of the complete region of resonances would require to increase the energy up to 12 GeV.

In the next section we discuss the $B^{(*)}_{s}\bar{B}^{**}_{s}$ threshold, which is within the current reach of SuperKEKB.

3.3.1 Search for narrow excited $0^+$ and $1^+$ strange $B_s$ mesons

Based on the heavy quark symmetry one should expect existence in the $b$-flavored sector of analogs of the charmed strange $D_{s0}(2317)$ and $D_{s1}(2460)$ mesons: $B_{s0}$ and $B_{s1}$. Their expected masses can be estimated within the heavy quark expansion as 5715 MeV and 5763 MeV, respectively. These mesons can be produced in $e^+e^-$ annihilation in pairs with the ground-state strange $B_s$ and $B^{*}_{s}$ mesons. Moreover, an $S$-wave production is allowed for the pairs $B_{s0}B^{*}_{s} + c.c.$, $B_{s1}B_{s} + c.c.$ and $B_{s1}B^{*}_{s} + c.c.$. The thresholds for the former two channels practically coincide at the c.m. energy of 11.13 GeV, while for the latter the threshold is expected to be at approximately 11.18 GeV, and one can expect, due to a presence of the $S$-wave, a measurable production cross section at energies just above the threshold. The final states in these processes necessarily contain $B_s$ and $\bar{B}_s$ mesons, and the excited $B_{s0}$ and $B_{s1}$ strange bottom mesons can be sought for by studying the spectrum of the invariant mass recoiling against a reconstructed $B_s$ meson.

Another interesting possibility may potentially arise if there is a molecular $S$-wave resonance, $\Upsilon_{ss}$, near the threshold of the $B_{s0}B^{*}_{s} + c.c.$ and $B_{s1}B_{s} + c.c.$ pairs, i.e. near 11.13 GeV. An existence of such a resonance (or a double resonance) can be affected by the coincidence of the thresholds for the two types of pairs (which is not the case for the $B_{s1}B^{*}_{s} + c.c.$). One can expect that such a resonance, in addition to decay channels
with open \(b\) flavor, would have a small, but possibly measurable, branching fraction for decay into bottomonium and light mesons: e.g. \(Y_{ss} \rightarrow Y(nS)K\bar{K}\) (with \(n = 1, 2\)), or \(Y_{ss} \rightarrow \chi_{bJ}(1P)\phi\). [The former decay is somewhat similar to the part of decays \(Y(5S) \rightarrow Y(nS)\pi\pi\) not associated with the \(Z_b\) resonances. The branching fraction for this part is in the ballpark of (a few) \(10^{-3}\).] Thus a scan of these channels with hidden strangeness near the c.m. energy of 11.13 GeV can reveal existence of molecular resonances with the excited strange bottom meson. It can be also mentioned that \(S\)-wave decays into \(h_b(mP)\eta\) are also allowed for \(Y_{ss}\), in analogy with the decay \(Y(4S) \rightarrow h_b(1P)\eta\), although it is not clear at present, how strong the breaking of HQSS, that is required for such decays, is. However, if these transitions from \(Y_{ss}\) proceed at a measurable rate, they can include the decay \(Y_{ss} \rightarrow h_b(3P)\eta\) and thus provide a unique gateway to studies of the \(h_b(3P)\) state of bottomonium.

4 Conclusions

All hidden-beauty hadrons above the \(B\bar{B}\) threshold have properties inconsistent with their interpretation as pure \(b\bar{b}\) bottomonia. The vector bottomonium-like states \(Y(10580), Y(10860)\) and \(Y(11020)\) [or the \(Y(4S), Y(5S)\) and \(Y(6S)\)] are likely mixtures of conventional \(b\bar{b}\) bottomonia and pairs of \(B\bar{B}\) or \(B_s\bar{B}_s\) mesons in ground or excited states. The isospin-one axial states \(Z_b(10610)\) and \(Z_b(10650)\) likely have purely molecular structures of \(B\bar{B}\) and \(B_s\bar{B}_s^*\), respectively. We propose a dedicated data taking program that will establish the above interpretation, check its predictions and search for new bottomonium and bottomonium-like states.

We propose to perform a high-statistics energy scan from the \(B\bar{B}\) threshold up to the highest possible energy, with 10 fb\(^{-1}\) per point and 10 MeV step. These data will allow to measure cross sections of \(e^+e^-\) annihilation into exclusive open-flavor and hidden-flavor final states, such as \(B\bar{B}, B\bar{B}^*, B^*\bar{B}, BB\pi, BB^*\pi, B^*B^*\pi, B_s\bar{B}_s, B_s\bar{B}_s^*, Y(nS)\pi^+\pi^-, Y(nS)\eta, Z\pi\) and others. Combined analysis of these cross sections using coupled-channel framework will determine all basic properties of the vector bottomonium-like states, such as their pole positions and interchannel couplings. These results are crucial for establishing the nature of the vector bottomonium-like states. The hidden-flavor channels will be used to search for states that do not exhibit themselves in the total cross section or in open-flavor channels; among such states could be compact tetraquarks and hadroquarkonia.

The states with molecular admixture are naturally located near the corresponding threshold. The present energy limit of the SuperKEKB accelerator of 11.24 GeV will allow to investigate the \(B_s^{(*)}\bar{B}_s^{(*)}\) and \(B_s^{(*)}\bar{B}_s^*\) threshold regions. Increase of maximal energy by at least 100 MeV will allow to explore the \(\Lambda_b\bar{\Lambda}_b\) threshold region and search for baryon-antibaryon molecular states, which should be there, as can be judged from the charmonium sector. The region of promising thresholds extends up to 12 GeV.

Observation of new vector states is of utmost importance for studies of molecular states – partners of the \(Z_b(10610)\) and \(Z_b(10650)\). Transitions from high mass bottomonium-like states provide a unique way to produce purely molecular states. Energy above 11.5 GeV is of special interest, as most of the relevant hadronic transitions become kinematically allowed. Thus searches for the molecular states provide additional motivation for increase of the SuperKEKB energy up to at least 11.5 – 11.6 GeV.

At each new vector state we propose to collect 500 fb\(^{-1}\) to perform a detailed study of corresponding transitions, since they provide information on the structure of the state, to search for missing conventional bottomonia and to search for molecular states. A region near the \(B_s^{(*)}\bar{B}_s^{(*)}\) threshold is also of interest to search for missing \(P\)-wave \(B_s\) mesons.

Available data at \(Y(6S)\) are limited to 5 fb\(^{-1}\) that were taken at five scan points, with the effective luminosity of 3 fb\(^{-1}\). Only the most prominent transitions \(Y(6S) \rightarrow Y(nS)\pi^+\pi^-\) and \(Y(6S) \rightarrow Z_b\pi\) are known. An observed pattern of transitions is somewhat different from that at the \(Y(5S)\), despite that the two states are close in energy. More detailed comparison requires an increase of statistics at the \(Y(6S)\) and could provide interesting information on the structure of both states. Given that many transitions were observed at the \(Y(5S)\) and there are still channels to be investigated, the list of measurements to be performed at the \(Y(6S)\) is quite long. Even non-observation of some channels will be of interest for comparison of the \(Y(5S)\) and \(Y(6S)\). The \(Y(6S)\) data will provide a rich physics output even for relatively small ammount of a few tens of fb\(^{-1}\). This makes \(Y(6S)\) an attractive option for the Belle-II initial data taking.

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\(^2\)By applying the same picture in the charm sector, one can expect a similar phenomenon due to the \(S\)-wave production in \(e^+e^-\) annihilation of the pairs \(D_{s0}(2317)D_{s*}\) c.c. and \(D_{s1}(2460)D_{s}\) c.c. at and/or above their common threshold at 4.43 GeV. The available data of a study using initial state radiation at Belle [33] indicate a certain activity in the final state \(J/\psi KK\) above 4.4 GeV with possible structures. However more detailed data on the yield of strange \(D_s\) mesons and/or of \(J/\psi KK\) in that energy range are needed for further conclusions.
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