Nuclear Production of Charmonium at ELFE Energies

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NUCLEAR PRODUCTION OF CHARMONIUM AT ELFE ENERGIES

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ABSTRACT

I discuss the diffractive production of charmonium from nuclei and the anomalous nuclear attenuation — termed “color transparency” — attributed to such processes, focusing on the few tens of GeV energy regime germane to the ELFE project. A critical review of color transparency calculations is presented, and the major sources of uncertainty are emphasized.

1 Introduction — why charmonium?

In diffractive charmonium production, the produced charmonium states emerge with nearly all the momentum kinematically accessible to them. In a semi-inclusive photoproduction experiment, in which only charmonium is detected, the constraint of diffractive production requires that the transverse component of the charmonium momentum be small. Diffractive production from a nuclear target is of interest, as in this kinematic regime the nuclear attenuation can vary markedly from standard estimates \[1\]. That is, the longitudinal momentum transfer to convert one charmonium state to another under interactions with the nuclear medium vanishes as the charmonium momentum increases \[2\], so that the amplitudes of the individual charmonium states (e.g., $J/\psi, \psi'$) are scrambled in their exit through the nucleus. In this manner the nuclear attenuation can suffer large excursions from that of a Glauber model estimate, in which the charmonium’s internal structure is neglected. Such anomalous nuclear attenuation effects
have been discussed in the context of processes such as the \((e,e'p)\) reaction at large momentum transfer and small missing energy and termed “color transparency” \cite{3}, as produced states of small transverse size are expected to interact with the nuclear medium in an anomalously weak manner, due to the above interference effects \cite{4}. One typically posits that any departure from the Glauber estimate is a signal of color transparency, though other corrections, such as the inclusion of nuclear correlations \cite{5}, as well as any change in the assumed reaction mechanism with energy \cite{6,7}, can modify the nuclear dependence as well and mimic the excursions arising from the nucleon’s internal structure. Thus, an accurate “baseline” calculation in the absence of color transparency effects is essential to detecting the onset of the phenomenon.

I focus on charmonium production, yet the issues here — that is, the existence and magnitude of any anomalous nuclear attenuation — are generic to the diffractive photoproduction of all vector mesons. The nucleon’s internal structure is modelled in a nonrelativistic constituent quark model, and the computation performed in the charmonium rest frame, for tractability. For sufficiently large quark mass \(m_q\), the energy splitting of the \(1S\) and \(2S\) quarkonium states will be small relative to the ground state mass, thus justifying the nonrelativistic treatment of charmonium’s structure. Thus, the consideration of large quark masses is convenient theoretically, though this begs the question of whether a completely consistent calculation can exist in the large \(m_q\) limit, as the longitudinal momentum transfer to produce other charmonium states must be able to vanish as the photon energy increases. These limits are compatible, as the production and propagation of charmonium in a nucleus is a two-scale problem. Crudely, the length scale for charmonium production is inversely proportional to the off-shellness of the real photon if it were to fluctuate to a \(J/\psi\), so that the “creation length” \(l_c\) is

\[
l_c \sim \frac{1}{\sqrt{E_\gamma^2 + m_{J/\psi}^2 - E_\gamma}},
\]

whereas the “formation length” \(l_f\), or the inverse of the longitudinal momentum transfer to produce the next low-lying charmonium state, is given by

\[
l_f \sim \frac{2p_{J/\psi}}{(m_{\psi'}^2 - m_{J/\psi}^2)}.
\]

The denominator in the above saturates as \(m_q\) grows large — note that the splitting of the lowest lying \(1^{--}\) states is 660 MeV, 588 MeV, and 563 MeV in the \(s\bar{s}\), \(c\bar{c}\), and \(b\bar{b}\) sectors — so that \(l_f\) can exceed \(l_c\) in the large \(m_q\) limit.

Here I delineate an archetypal color transparency calculation \cite{8}, and discuss the various model estimates and their deficiencies, considering the special needs of a near-threshold treatment. Certain constraints on the diffractive charmonium-nucleus interaction exist as \(s \to \infty\), and I consider how these constraints may relax at modest \(s\) and what experiments are germane to elucidating these departures.

2 An archetypal color transparency calculation

The \(A\)-dependence of the nuclear transparency for the production of charmonium state \(j\), defined as

\[
T_A^j(E_{LAB}) = \frac{\int d^3r \rho_A(\vec{r})\langle \psi_j | \int_z^\infty dz' U(z', \vec{b}) | \psi_i \rangle^2}{A \langle \psi_j | \psi_i \rangle^2},
\]

should weaken as the photon energy increases, if color transparency exists. This suggestion itself assumes the production mechanism is not intrinsically \(A\)-dependent, or, less stringently, that its
$A$-dependence is not energy-dependent \cite{9}. The above formula is appropriate to charmonium production which is incoherent from the nucleus, yet coherent from the nucleon, so that the transverse momentum component of the produced charmonium must be nonzero, yet less than the pion mass. The sums over the possible intermediate nuclear states which exist for every application of $\hat{U}$ are suppressed — the momentum of every produced charmonium state is presumed much larger than the variation in energy of the nuclear intermediate states, so that the closure approximation can be employed. Note that $\rho_A(\vec{r})$ denotes the matter density of the nucleus $A$, $|\psi_i\rangle$ denotes the initial charmonium state, $|\psi_f\rangle$ denotes the final charmonium state detected, and $\hat{U}$ denotes the evolution operator. Note that the calculation is realized in the charmonium rest frame.

Certain assumptions underlying Eq.(3) are crudely violated. Noting Eq.(1), we see that the reaction mechanism certainly changes with photon energy; the impact of this change on $T_A$ has been estimated in Ref. \cite{6} and studied in detail in Ref. \cite{7}. The above formula also models the nucleus as a continuous distribution of nuclear matter; the inclusion of nuclear correlations can modify the energy dependence of $T_A$ substantially \cite{5}. How are the pieces in $T_A$ modelled?

### 2.1 $|\psi_f\rangle$: the charmonium state detected.

The nonrelativistic quark model is assumed. Harmonic oscillator potentials have been used exclusively \cite{10,6,11}: the fits to the charmonium spectrum are not quantitative. Consider the $s$-wave charmonium states in the nonrelativistic quark model. In MeV, we have

\begin{align*}
\text{EXPT} & \quad \text{HO}_1 & \quad \text{HO}_2 & \quad \text{Richardson} \\
1S & 3097 & 3097 & 3097 & 3095 \\
2S & 3686 & 3686 & 3569 & 3684 \\
3S & 4040 & 4275 & 4040 & 4096 \\
4S & 4415 & 4864 & 4512 & 4440
\end{align*}

where $\text{HO}_1$ denotes a harmonic oscillator fit in which $2\hbar\omega = m_{\psi(3097)} - m_{\psi(3685)}$ \cite{10}, $\text{HO}_2$ denotes the same sort of fit with $2\hbar\omega = m_{\psi(4040)} - m_{\psi(3097)}$ \cite{6}, and “Richardson” denotes a fit with the Richardson potential \cite{12}. The $\text{HO}_2$ fit is a better caricature of the higher-lying portion of the charmonium spectrum, though a realistic potential, such as the Richardson potential, is preferred. The impact of this simplification could be numerically large.

### 2.2 $|\psi_i\rangle$: the “initial state”.

The “initial state” is the charmonium production amplitude. Two different sorts of estimates exist. One estimate uses the amplitude for the photon to fluctuate to a free $c\bar{c}$ pair in the infinite momentum frame times the cross section associated with the diffractive charmonium-nucleon interaction \cite{13}. This procedure neglects mutual $c\bar{c}$ interactions, which are equally important. If they were included in any realistic model, the dependence on the longitudinal and transverse components of the $c\bar{c}$ separation would not factorize, so that the transformation to the rest frame, required for the subsequent $T_A$ calculation, would be unclear. Moreover, the above estimate is only appropriate in the large photon energy limit. Other work simply makes an Ansatz for the initial amplitude \cite{10,6} in the near threshold regime; this is unsatisfying.

Perhaps a better estimate of the production amplitude can be constructed. Borrowing from threshold $e^+e^- \rightarrow t\bar{t}$ studies \cite{14}, the non-relativistic three-point function for charmonium photoproduction is given by the Green function

$$\left[-\Delta + V(\vec{r}_{c\bar{c}}) - E\right]G(\vec{r}_{c\bar{c}}; E) = \delta(\vec{r}_{c\bar{c}})$$  \hspace{1cm} (5)
where the source is localized at \( \vec{r}_{c\bar{c}}' = 0 \) and \( V(\vec{r}_{c\bar{c}}) \) describes the mutual \( c\bar{c} \) interactions, as well as the soft interactions with the medium. In the context of top quark production, this is really a perturbative QCD estimate, as the enormous decay width associated with the heavy top quark effectively screens the confining portion of the potential \[14\]; here it is a model calculation.

2.3 \( \hat{U} \): the evolution operator.

The quantum mechanical evolution operator is associated with

\[
H(t) = H_{c\bar{c}} + H_{\text{soft}},
\]

where the time \( t \) in the \( c\bar{c} \) rest frame is connected to the lab coordinate \( z \) via \( z = v\gamma t \), noting \( v \) is the charmonium velocity and \( \gamma \) is the Lorentz factor. \( H_{\text{soft}} \) is typically modelled in a two-gluon exchange picture, so that the cross section for the \( c\bar{c} \)-nucleon interaction is \[15, 10\]

\[
\sigma(\rho) = \frac{16\alpha_s^2}{3} \int d^2k \frac{[1 - \exp(i\vec{k} \cdot \vec{\rho})][1 - F(\vec{k})]}{(k^2 + m_g^2)^2},
\]

where \( m_g \) is the effective gluon mass, required to regulate \( \sigma(\rho) \) for arbitrary \( \rho \), and \( F(\vec{k}) = \langle N|\exp(i\vec{k} \cdot (\vec{r}_1 - \vec{r}_2))|N\rangle \), where \( |N\rangle \) is the nucleon wave function. Finally, \( H_{\text{soft}} \) in the charmonium rest frame is chosen to be \[10\]

\[
H_{\text{soft}} = -\frac{i}{2}\sigma(\rho)\rho_A(\vec{r}(t)),
\]

as the two-gluon exchange amplitude is purely imaginary in the \( s \to \infty, t \) fixed limit. Eq.(8) is not derivable from Eq.(7); one assumes as per Ref. \[10\] that the bulk of diffractive charmonium interactions are with the lowest momentum partons in the nucleon — the “wees” — which are uncontracted even though the nucleon momentum is large in the charmonium rest frame. Practically, \( H_{\text{soft}} \sim C\rho^2 \) is chosen, though corrections have been estimated perturbatively \[11\]. There is no reason, however, why the construction of the soft interaction should stop at two-gluon exchange. Consider three-gluon exchange, for example. These exchanges are \( C \)-odd, as well as \( C \)-even, in the \( t \)-channel. The empirical support for the Pomeranchuk theorem, that is, \( \sigma_{\text{hp}}^{\text{tot}} \sim \sigma_{\bar{h}p}^{\text{tot}} \) as \( s \to \infty \), can be interpreted to mean that \( C \)-odd exchanges are suppressed in the large \( s \) limit. Yet the merely asymptotic approach in \( s \) to the Pomeranchuk prediction indicates itself that the predictions of the infinite \( s \) limit are modified at finite energies. This is important to the ELFE energy regime! The consequence is that one has diffractive production of both natural \( (J/\psi, \psi') \) and unnatural \( (\chi) \) parity charmonium states in the nucleus, as the final-state nucleons are undetected. Ref. \[6\] estimates the numerical impact of the inclusion of both \( C \)-odd and \( C \)-even exchanges in the near threshold regime in an Abelian string model, though the model is schematic. In the absence of data, it is difficult to constrain the inclusion of three-gluon exchange effects.

The \( T_A \) calculation can now be effected with the above model ingredients. The hard part is retaining all the intermediate charmonium states throughout the calculation. For harmonic oscillator potentials, one can use the exact path integral result for the evolution operator, and the problem becomes greatly simplified \[10\]. For other potentials, one must construct the evolution operator numerically and sum over all the states \[3\]. Arbitrary two-channel truncations have also been performed, though they are of unclear reliability.

The model calculations all show an increase in the nuclear transparency with increasing photon energy, though a complete calculation in the near threshold regime has not been performed. On the theoretical side, a calculation with a quantitative \( H_{c\bar{c}} \), a production amplitude
estimate as per $G(\vec{r}_c; E)$, and a diffractive interaction which includes two- and three-gluon exchange is required for an improved estimate of the nuclear dependence of diffractive charmonium production in the ELFE energy regime. On the experimental side, one would like to study the breaking of the Gribov-Morrison rule, which presumes all diffractively produced states to have the quantum numbers of the photon, in the nucleus at moderate to large $E_\gamma$.

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