RELATIVISTIC EFFECTS IN TWO-BODY SYSTEMS:

$\pi$- AND $K$- MESONS AND DEUTERON

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Abstract

The electromagnetic form factors of $\pi$ and $K$ mesons and deuteron are calculated in modified impulse approximation using instant form of relativistic Hamiltonian dynamics. The different model wave functions are used. Meson wave function parameters are fixed by fitting the mean square radius of meson. The internal quark structure is taken into account through electromagnetic quark form factor and quark anomalous magnetic moments. Results of our calculations of electroweak structure of pion and kaon and electromagnetic deuteron properties agree well with the available experimental data. The meson form factors asymptotics at large momentum transfer is the same as in perturbative QCD. Some predictions about CEBAF experiments are given.

In recent years the interest has been renewed to the study of electromagnetic form factors of composite systems: of pseudoscalar mesons and, particularly, $\pi$- and $K$- mesons, and of the deuteron. This fact is due, first of all, to the experiments at CEBAF concerning the measurement of pion, kaon and deuteron form factors. It is possible that these experiments will enable to choose between different theoretical models whose predictions differ rather strongly.

Such a difference of theoretical results seems to be quite natural, because one encounters a lot of difficulties while calculating the structure of composite particles. The main difficulties are the following two. First, the importance of relativistic effects in the whole range of momentum transfer. Second, the problem of calculation of the "soft" structure which cannot be obtained from perturbative QCD. The soft part, which describes the structure at long and intermediate distances, needs nonperturbative approaches. (A controversy still exists concerning a scale of momentum transfers characteristic for the boundary between the nonperturbative to perturbative regime.)

The relativistic Hamiltonian dynamics (RHD) (for a review see [1]) is one of such nonperturbative ways. RHD unifies potential approach to composite systems and the condition of Poincaré-invariance. This method is based on the direct realization of the Poincaré group algebra or, in other words, of the relativistic invariance condition in the

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few body Hilbert space. RHD can be formulated in different ways (different relativistic dynamics). At present the light front dynamics is the most popular one.

Here we present a relativistic treatment of the problem of soft electromagnetic structure in the framework of alternative form of relativistic dynamics, the so called instant form (IF) of RHD. Our approach, which describes well the available data for the elastic form factors for the charged $\pi$ (see figs.1 – 3) and $K$ (see fig.4) mesons and deuteron (see figs.5 – 7), will be briefly outlined here. The details are partially given in [2]. The IF form of relativistic dynamics, although not widely used, has some advantages. The calculations can be performed in a natural straightforward way without special coordinates.

IF is particularly convenient to discuss the nonrelativistic limit of relativistic results. The space reflection and time reversal operators do not depend on the interaction (contrary to the case of light front dynamics) [3]. This approach is obviously rotationally invariant so that IF is the most suitable for spin and polarization problems. Our method gives the same form factors asymptotics as in QCD. It is worth to notice that the relativization is performed in the same way for quark–antiquark systems as for nucleon–nucleon composite systems. At last, for the systems with spin 1 (e.g., deuteron and $\rho$ – meson) the solution for the formfactors is unique (contrary to the case of light front dynamics).

In our approach it is possible to construct the current operator of composite system which satisfies the conservation law and the relativistic covariance condition. The calculation is based on the representation of two-body current matrix element in terms of the free two-particle form factors $g(s, Q^2, s')$ where $s, s'$ are invariant masses of free two-particle system and $Q^2$ is transfer momentum square. $g(s, Q^2, s')$ is a matrix in orbital moments $l, l'$ in the case of deuteron. The final expression for electromagnetic form factor has the following form:

$$F(Q^2) = \int d\sqrt{s} \sqrt{s'} \varphi(s) g(s, Q^2, s') \varphi(s')$$

Here $\varphi(s)$ is wave function which is calculated in RHD. Results of our calculations of electroweak structure of pion and kaon and electromagnetic deuteron properties agree well with the available experimental data.

The following meson wave functions were utilized:

1. A gaussian or harmonic oscillator (HO) wave function

$$u(k) = N_{HO} \exp \left( -k^2/2b^2 \right).$$

(curve 1 on figs.1,4; for comparison the nonrelativistic limit of (1) is given by the curve 6 on figs.1,4).

2. A power-law (PL) wave function (see e.g. [4], [5])

$$u(k) = N_{PL} (k^2/b^2 + 1)^{-n}, \quad n = 2, 3.$$  \hspace{1cm} (3)

(curves 2,3 on figs.1,4).

3. The wave function with linear confinement from Ref. [6]:

$$u(r) = N_T \exp(-\alpha r^{3/2} - \beta r), \quad \alpha = \frac{2}{3} \sqrt{2M a}, \quad \beta = M b.$$  \hspace{1cm} (4)
$a, b$ – parameters of linear and Coulomb parts of potential, respectively. (curve 4 on figs.1,4).

The values of the parameters are choosen so as to describe experimental mean square radius and lepton decay constant of mesons.

The internal quark structure can be described by the introduction of electromagnetic quark form factor and anomalous quark magnetic moment. Electromagnetic quark structure was chosen in the form:

$$f_q(Q^2) = \frac{1}{1 + \ln(1 + <r^2_q > / Q^2 / 6)}.$$ (5)

$<r^2_q >$ – is the mean square radius of the quark. The choice of form (5) does not violate the asymptotics which takes place in our approach at $Q^2 \to \infty$, $M_q \to 0$, which in our approach is:

$$F_c(Q^2) \sim Q^{-2}.$$ (6)

The model dependence of electromagnetic pion form factor in region of CEBAF experiments occurs to be comparatively weak. On the other hand, pion electromagnetic form factor depends strongly on the anomalous quark magnetic moments. Analogous results take place for kaon.

Our results lead to the following conclusions.

1. Most likely the CEBAF experiments on the measurements of $\pi^-$ and $K^-$ mesons form factors at $Q^2 \leq 3$ (GeV/c)$^2$ will not allow for the possibility to extract the best model of quark interaction in view of weak dependence on the choice of wave function.

2. It seems reasonable to say that CEBAF experiments will give the possibility to estimate the manifestation of internal quark structure.

3. In the IF of RHD the meson form factors asymptotics at large momentum transfer is the same as in perturbative QCD. The asymptotics is now determined by relativistic kinematics only, specifically by the relativistic effect of spin rotation, and does not depend on the choice of the quark wave function, that is of the quark interaction model.

4. IF of RHD describes well the experimental data on elastic $ed$ – scattering.

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Figure 1: Electromagnetic pion form factor, $Q^2 F_\pi(Q^2)$, at high momentum transfer. 

$M_u = M_d = 0.25 \, GeV/c$. 1 – harmonic oscillator wave function Eq.(2), $b = 0.207 \, GeV$; 
2 – power-law wave function Eq.(3), $n = 2$, $b = 0.274 \, GeV$; 3 – power-law wave function Eq.(3), $n = 3$, $b = 0.388 \, GeV$; 4 – wave function Eq.(4) from model with linear confinement [6], $a = 0.0183 \, GeV^2$, $b = 0.7867$. 5 – harmonic oscillator wave function Eq.(2), $b = 0.207 \, GeV$; without spin rotation. 6 – harmonic oscillator wave function Eq.(2), $b = 0.207 \, GeV$; nonrelativistic limit of Eq.(1). Experimental data are taken from [7],[8].
Figure 2: $\pi$-meson form factor for different values of quark anomalous magnetic moments and different wave functions. Two sets of curves: the upper set is for the sum of quarks anomalous magnetic moments ($-0.1$), the lower for ($+0.1$). If the quark structure is taken into account the following effects are found: 1. Weak dependence on the wave function choice. 2. Strong dependence on the value of quark anomalous magnetic moments.
Figure 3: The relative contribution of the effect of Wigner spin rotation to $\pi$-meson form factor, $W(Q^2) = \frac{F_{R+SR}(Q^2) - F_R(Q^2)}{F_{R+SR}(Q^2)}$. Here $F_{R+SR}(Q^2)$ – pion form factor, spin rotation included, $F_R(Q^2)$ – without spin rotation. $M_u = M_d = 0.25$ GeV. $<r_q^2> \simeq \frac{0.3}{M_q} = 0.187$ Fm$^2$ (see e.g. [4]). The same line code as in Fig.1 is used. 1 – HO, $b = 0.350$; 2 – PL $n = 2$, $b = 0.396$; 3 – PL $n = 3$, $b = 0.571$; 4 – model with linear confinement $a = 0.083$, $b = 0.7867$. The curves 3 and 4 practically coincide.
Figure 4: Electromagnetic kaon form factor at high momentum transfer. The same line code as in Fig.1 is used. $M_s = 0.35$ GeV, $1 - b = 0.255$ GeV, $2 - b = 0.339$ GeV, $3 - b = 0.480$ GeV, $4 - a = 0.0318$ GeV$^2$, $b = 0.7867$. Experimental data are taken from [9].
Figure 5: Deuteron structure function $A(Q^2)$ for Paris wave functions from [10] and nucleon form factors from [11]. Solid line is our relativistic result, dashed–dot line is relativistic result from [12], dashed line is nonrelativistic calculation. Data are from [13].
Figure 6: The deuteron charge form factor $G_c(Q^2)$ near the point of zero value. Solid line – Paris wave functions from [10], short–dashed line – Bonn (R) [14], long–dashed line – dispersion wave functions from [15]. For nucleon form factor the dipole fit is used.
Figure 7: Deuteron tensor polarization $T_{20}(Q^2)$. $\vartheta = 70^\circ$. Solid line – Paris wave functions from [10], short–dashed line – Bonn (R) [14], long–dashed line – dispersion wave functions from [15], dashed–dot line is calculation in the Paris model from [12]. Data are from [16].