A numerical benchmark test for continuous casting of steel III

B Šarler1,2 and R Vertnik1,3
1 Laboratory for Simulation of Materials and Processes, Institute of Metals and Technology, Lepi pot 11, SI-1000 Ljubljana, Slovenia
2 Laboratory for Multiphase Processes, University of Nova Gorica, Vipavska 13, SI-5000 Nova Gorica, Slovenia
3 Store Steel d.o.o., Research, Železarska 3, SI-3220 Štore, Slovenia

E-mail: bozidar.sarler@imt.si, robert.vertnik@store-steel.si

Abstract. This paper represents a continuation of numerical results regarding the recently proposed industrial benchmark test [1], obtained by a meshless method. A part of the benchmark test, involving turbulent fluid flow with solidification in two dimensions, was elaborated in [2]. A preliminary macrosegregation upgrade was presented in [3], and in [4], a first three dimensional test was performed. Previous tests were bound to calculations in mold and sub-mold regions only. In the present paper, reference calculations in two dimensions are presented for the entire strand. The physical model is established on a set of macroscopic equations for mass, energy, momentum, species, turbulent kinetic energy, and dissipation rate. The mixture continuum model is used to treat the solidification system. The mushy zone is modeled as a Darcy porous media with Kozeny-Karman permeability relation, where the morphology of the porous media is modeled by a constant value. The incompressible turbulent flow of the molten steel is described by the Low-Reynolds-Number k-ε turbulence model, closed by the Abe-Kondoh-Nagano closure coefficients and damping functions. Lever microsegregation model is used. The numerical method is established on explicit time-stepping, collocation with scaled multiquadrics radial basis functions with adaptive selection of its shape on non-uniform five-nodded influence domains. The velocity–pressure coupling of the incompressible flow is resolved by the explicit Chorin’s fractional step method. The advantages of the method are its simplicity and efficiency, since no polygonisation is involved, easy adaptation of the nodal points in areas with high gradients, almost the same formulation in two and three dimensions, high accuracy and low numerical diffusion.

1. Introduction
Continuous casting [5] is the most common process in the production of steel. The process starts by pouring the molten metal into to the water cooled mold, where cooling intensity is high enough to solidify steel around the inner surface of the mold and generates the solid shell with molten metal in the center of strand. After several minutes, the strand is pulled into the secondary cooling system, which contains water spray systems with much smaller cooling intensity as in the water channels in the mold, and rollers, which support and guide the strand up to the end of the casting machine. The quality of the casted product (round or square billets, blooms or slabs) depends mainly on the process parameters in the mold region, where complex physical phenomena occur. The liquid metal, poured with a high velocity from the submerged entry nozzle (SEM) into the mold, produces turbulent flow with several re-circulating zones. Large heat fluxes, extracted from the mold, are a consequence of very high flow rates of the cooling water in the mold channels. It is impossible to measure temperature and velocity field inside the strand due to the very high temperatures of steel and inaccessibility of the region during the process. The numerical models [6] help to better understand the casting behavior and
to further improve and optimize the process, particularly in such experimentally sophisticated situations. Various numerical methods have already been used to simulate the described spectra of problems. However, until now, there is a lack of benchmark tests that would enable verification of numerical methods for solving transport phenomena in continuous casting of steel in an ordered way, similar as it was done for binary solidification [7]. We recently tried to fill this gap with proposing a first such benchmark [1], elaborated in detail in a journal paper [2]. First related macrosegregation simulations have been displayed in [3]. The simulations from [2] have been in [4] upgraded to three dimensions. In this paper, the two dimensional turbulent heat transfer, fluid flow, and species transfer solidification discussion in [3] is displayed for the whole length of the strand and not only to the mold and near sub-mold regions. The main aim of the test is to get new reference results and to be able to further compare the results of different numerical methods for the same physical model. The presented physical model considers turbulent fluid flow and macrosegregation of carbon steel. Turbulent flow is modeled by the two-equation eddy-viscosity model with the low-Re corrections. The numerical results are shown for the case with simplified lever microsegregation model of 0.8 wt%C steel. The novel meshless Local Radial Basis Function Collocation Method (LRBFCM) is employed for solving the coupled system of six partial differential equations in two dimensions. The objective of the present paper is to produce a first reference numerical solution of the given problem for the strand of the length 12 m, where solidification is completed. LRBFC was first developed in [8] for diffusion problems, to convection diffusion problems in [9], and to driven cavity and natural convection problems in [10, 11]. The turbulence was tackled in [12, 13]. The melting benchmark [14] was successfully solved in [15] and the macrosegregation benchmark [7] in [16, 17]. The magneto-hydrodynamic effects were tackled in [18] for natural convection in a cavity and in [19] for backward facing step situation.

2. Governing equations

The system of equations, describing the coupled transport phenomena in the continuous casting of steel, is derived based on the mixture continuum formulation by Bennon and Incropera [20] and Reynolds time-averaging approach for modelling incompressible turbulent flow [21]. The time-averaged transport equations for mass, energy, momentum and solute conservation are

\[ \nabla \cdot \mathbf{u} = 0, \]

\[ \rho \frac{\partial \mathbf{u}}{\partial t} + \rho \nabla \cdot (\mathbf{uu}) = -\nabla P + \nabla \left[ 2(\mu + \mu_t) \nabla \mathbf{u} \right] - \frac{2}{3} \rho \nabla k - \frac{K_0}{f_L} L (\mathbf{u} - \mathbf{u}_s) + \mathbf{F}_h, \]

\[ \rho \frac{\partial h}{\partial t} + \rho \nabla \cdot (\mathbf{uh}) = \nabla \cdot (\lambda \nabla T) + \rho \nabla \cdot \left[ f_s (h_l - h_s)(\mathbf{u} - \mathbf{u}_s) \right] + \nabla \cdot \left[ \frac{f_L \mu_t}{\sigma_T} \nabla h_l \right], \]

\[ \rho \frac{\partial C}{\partial t} + \rho \nabla \cdot (\mathbf{uC}) = \nabla \cdot \left( f_i D_i \nabla C_i + f_s D_s \nabla C_s \right) + \rho \nabla \cdot \left[ (\mathbf{u} - \mathbf{u}_s) (C_l - C) \right] + \nabla \cdot \left( \frac{f_L \mu_t}{\sigma_c} \nabla C_l \right) \]

with \( \mathbf{u} \), \( P \), \( h \), \( T \) and \( C \) for velocity, pressure, enthalpy, temperature and solute concentration, respectively, and \( \rho \), \( \mu \), \( \mu_t \), \( f \), \( \lambda \), \( D \) are standing for density, molecular dynamic viscosity, turbulent dynamic viscosity, volume phase fraction, thermal conductivity, and diffusivity of the material. In above equations, subscripts \( L \) and \( S \) denote liquid and solid phase. \( \mathbf{S} \) stands for the strain-rate tensor. The fourth term in equation (2) represents the Darcy term, where \( K_0 \) is the morphology constant of the porous medium and \( \mathbf{u}_s \) is velocity of the solid phase. The fifth term in equation (2) represents the buoyancy forces, defined as
\[ F_b = \rho \left[ \beta_T g(T - T_{ref}) + \beta_C g(C_L - C_{ref}) \right] \]  

(5)

where \( \beta_T, \beta_C, T_{ref} \) and \( C_{ref} \) stand for thermal expansion coefficient, solute expansion coefficient, reference temperature and reference concentration, respectively. The turbulent dynamic viscosity is defined as \( \mu = \rho \nu_f f_p k^2 / \varepsilon \), where \( k \) and \( \varepsilon \) are kinetic energy and dissipation rate, respectively. They are calculated by the following transport equations

\[ \rho \frac{\partial k}{\partial t} + \rho \nabla \cdot (uk) = \nabla \cdot \left[ \left( \mu_t + \frac{H_t}{\sigma_k} \right) \nabla k \right] + P_k - \rho \varepsilon + \rho D + \mu_t \frac{K_h(1 - f_k^2)}{f_k^3} k, \]  

(6)

\[ \rho \frac{\partial \varepsilon}{\partial t} + \rho \nabla \cdot (u\varepsilon) = \nabla \cdot \left[ \left( \mu_t + \frac{H_t}{\sigma_\varepsilon} \right) \nabla \varepsilon \right] + \rho \left( c_{\varepsilon t_f} f_i - c_{\varepsilon x f_i} \right) \varepsilon + \rho E + \mu_t \frac{K_h(1 - f_k^2)}{f_k^3} \varepsilon. \]  

(7)

In equations (3,4,6,7) \( c_p, f_p, c_{i_f}, c_{x_f}, f_i, \sigma_f, \sigma_e, \sigma_k \) and \( \sigma_\varepsilon \) are the closure coefficients, while \( D \) and \( E \) are additional source terms of the low-Reynolds turbulent model [21]. The constitutive temperature-enthalpy relationship \( h_s \) and \( h_L \) in equation (3) are defined as

\[ h_s = \int_{T_{ref}}^{T_s} c_{p,s} dT, \quad h_L = \int_{T_{ref}}^{T_L} c_{p,L} dT + \int_{T_L}^{T} c_{p,L} dT + h_m = h_s(T) + \int_{T_s}^{T} (c_{p,L} - c_{p,S}) dT + h_m, \]  

(8,9)

respectively. The liquid fraction \( f_L \), concentration of solute in liquid phase \( C_L \), partition ratio \( k_p \), and liquidus temperature \( T_L \) are determined based on the lever rule microsegregation model, i.e.

\[ f_L = 1 - \frac{1}{1 - k_p} \frac{T - T_L}{T - T_m}, \quad C_L = \frac{C_s}{1 + f_S (k_p - 1), k_p = \frac{C_s}{C_L}, T_L = T_m + (T_e - T_m) \frac{C_e}{C_s}} \]  

(10,11,12,13)

where \( T_m, T_e \) and \( C_e \) are melting temperature, eutectic temperature and solute concentration in eutectic alloy, respectively. They are obtained from the phase diagram for the iron-carbon system.

3. Solution procedure

Our solution procedure is based on the novel meshless technique that follows the following steps. We seek the solution of the equations for the velocity, pressure, temperature, concentration, turbulent kinetic energy, and dissipation at time \( t + t_0 \) by assuming known fields \( u, p, T, C, k, \) and \( \varepsilon \) at time \( t_0 \) and known boundary conditions at time \( t > t_0 \). The coupled set of mass conservation equation (1) and momentum conservation equations (2) are solved by the fractional step method [22], where the continuity of the mass (1) is considered by constructing the pressure Poisson’s equation. The governing equations are discretized by using the explicit time discretization. This leads to the following algorithm, summarized through the following 7 steps: (I) The intermediate velocity components are calculated without the pressure gradient, (II) the pressure Poisson equation is treated, (III) the velocity components are corrected by the pressure gradient, (IV), after the solution of the velocity field, the transport equation (3) for energy is solved, (V) the temperature field is calculated from the enthalpy field, calculated in previous step, using the inverse of the constitutive temperature-enthalpy relationships (8,9), i.e. \( T = T(h) \), (VI) species concentration (4), turbulent kinetic energy (6) and dissipation rate (7) fields are calculated, and finally (VII) the turbulent viscosity, solute concentrations (11,12) and liquidus temperature (13), are updated for each node and the solution is set ready for the next time step. All derivatives of the transport equations are calculated by the LRBFCM [8], where the collocation is made locally on overlapped sub-domains. On each sub-domain, the scalar function \( \Phi \) (standing for temperature, species concentration, velocity components, pressure, turbulent kinetic energy, and turbulent dissipation rate) is represented over a set of (in general) non-equally
spaced nodes $p_k$; $n = 1, 2, \ldots, N$ in the following way $\Phi(p) = \sum_{k=1}^{K} \psi_{\alpha_{kk}}(p) \alpha_{kk} \cdot \psi_{\alpha_{kk}}(p) = \left[ r_{kk}^2 + c_k^2 \right]^{1/2}$, where $\psi_{\alpha_{kk}}$ stands for the multi-quadric radial basis shape functions [23], $\alpha_{kk}$ for the coefficients of the shape functions, $K$ the number of the shape functions, $c_k$ the shape parameter, and $r_{kk}$ the radial distance between two points in the influence domain. Five nodes are used in each of the influence domains. The radial distance is scaled as $r_{kk} = (p_k - p_{sk})^2 / p_{sk}^2 + (p_k - p_{sl})^2 / p_{sl}^2$. The higher the shape parameter is, more accurate are the results. However, higher value of shape parameter induces ill-conditioning of the collocation matrix. The shape parameter of the multiquadrics is respectively adaptively scaled in each influence domain in such a way that the condition number of the local collocation matrix does not exceed $10^{10}$. The detailed procedure of calculating the derivatives with the LRBFCM can be found in [2].

Figure 1. The upper part of geometry with typical dimensions. Figure 2. Considered boundary condition types.

4. Description of the benchmark test

The proposed benchmark represents a simplified 2D model of the continuous casting process. The geometry of the simplified casting machine is shown in Fig. 1. A narrow side cross-section of a slab is assumed. The computational domain represents half of the longitudinal section of the slab with length 12 m, shown in Fig. 2. This simplification is used mainly for straightforward verification of the developed numerical model, and in order to reduce the CPU time of the simulations. The dimensions of the casting geometry of the slab are narrow side width $a_h = 0.18$ m, diameter of the SEN $d_s = 0.035$ m, and mould height $l_m = 0.8$ m.

The material properties of steel are temperature dependent. However, for the benchmark purpose, constant values are used for each phase. They are $\rho = 7200$ kgm$^{-3}$, $c_p = 700$ Jkg$^{-1}$K$^{-1}$, $\lambda = 30$ Wm$^{-1}$K$^{-1}$, $\mu = 0.006$ kgm$^{-1}$s$^{-1}$, $h_m = 250000$ Jkg$^{-1}$, $K_0 = 1.6 \times 10^8$, $D_\delta = 1.6 \times 10^{-11}$ m$^2$s$^{-1}$, $D_\beta = 1.0 \times 10^{-9}$ m$^2$s$^{-1}$, $\beta_\mu = 0.0001$ K$^{-1}$, $\beta_\delta = 0.004$. The inlet carbon concentration is $C_0 = 0.8$ wt%, inlet temperature is $T_0 = 1800$ K, and the slab casting velocity is $u_{cast} = 1.1$ m/min, respectively.
Figure 3: Left - Temperature field. Center - absolute velocity field. Right - carbon segregation field.

The boundary conditions for the velocity, temperature, turbulent kinetic energy, dissipation rate, and species concentration, are set as follows

**SEN outlet:** Constant values are prescribed for all variables $u_{\text{inlet},x} = 0$, $u_{\text{inlet},y} = a_y u_{c,y} / d_z$, $k_{\text{inlet}} = 1.5 (I_e u_{\text{inlet},y})^2$, $\epsilon_{\text{inlet}} = C_{\mu}^{0.75} k_{\text{inlet}}^{1.5} 0.07 d_z$, $T_{\text{inlet}} = T_0$, $C_{\text{inlet}} = C_0$, where $u_{c,y}$ is the casting velocity in the casting direction, and $I_e$ is the turbulent intensity calculated from the following relation

$$I_e = 0.16 \text{Re}^{-0.1}, \text{Re} = \frac{\rho u_{\text{inlet},y} d_z}{\mu}. \quad (14,15)$$

Normal gradient of pressure is set to zero, i.e. $\frac{\partial P}{\partial y} = 0$.

**Slab end:** The pressure outlet is prescribed, where the following boundary conditions are used
\[ P = 0, \quad \frac{\partial u}{\partial y} = 0, \quad \frac{\partial u_y}{\partial y} = 0, \quad \frac{\partial k}{\partial y} = 0, \quad \frac{\partial \varepsilon}{\partial y} = 0, \quad \frac{\partial T}{\partial y} = 0, \quad \frac{\partial C}{\partial y} = 0. \] 

**Top surface (meniscus):** At the meniscus, the symmetry boundary conditions are used (free surface flow). The normal derivative of all variables is set to zero, except the vertical velocity \( u_y \), which is set to zero.

**Moving walls:** The walls with the solidified steel move with casting velocity along the casting direction. At the walls, where the liquid phase exists, the no-slip boundary conditions for the velocity are set. In the mould, the Robin boundary condition is used, with the surface heat transfer coefficient equal to \( 2000 \text{ Wm}^{-2}\text{K}^{-1} \). Below the mould, at the secondary cooling system, the heat transfer coefficient is equal to \( 800 \text{ Wm}^{-2}\text{K}^{-1} \). The normal derivative of the solute concentration is set to zero.

5. **Numerical results**

The low-Re turbulence model by Abe-Kondoh-Nagano (AKN) [23] is selected, where the normal distance in closure coefficients is calculated up to the wall. The simulation was performed on the node arrangement with 84707 nodes. The steady-state solution shown in Figure 3 is approached by a false transient calculation using a fixed time-step of \( 310^{-3} \text{ s} \). The results of the simulation, which runs about one week on 20 contemporary cores, are shown in Fig. 3.

6. **Conclusions**

This paper represents a continuation of numerical benchmarking [1,2,3,4] proposed for the continuous casting of steel. The geometry and material properties of the continuous casting process are simplified in order to enable straightforward performance comparisons with various numerical methods. In the paper, our in-house developed meshless numerical method, based on the local radial basis function collocation, is used to get the steady state solution of the turbulent fluid flow with heat transfer, solidification, and macrosegregation of carbon along the whole longitudinal geometry of the two-dimensional strand. A symmetric half of the narrow side of the slab was considered. The results were obtained without electromagnetic forces [25] in the momentum equation, and with a simplified lever microsegregation model. The benchmark will be further upgraded and adapted, based on the feedback from the colleges in the field of the numerical modelling of solidification processes. A journal paper with tabulated results of the present simulations, similar as [2], is underway. A web page of the benchmark results with systematically tabulated results of different numerical methods will be established, similar to [7].

**Acknowledgement:** The financial support of Slovenian Grant Agency and Štore Steel company under grant L2-6775 is kindly acknowledged.

**References**

[1] Šarler B, Vertnik R and Mramor K 2012 MCWASP XIII, IOP Conf.Series: Mat.Sci.Eng. 33 012012.
[2] Vertnik R and Šarler B 2014 Eng. Anal. Bound. Elem. 45 45-61
[3] Šarler B and Vertnik R 2014 Materials Science Forum 790-91 279-284
[4] Vertnik R and Šarler B 2014 ICASP IV, IOP Conf. Series: Mat.Sci.Eng (to appear)
[5] Irving W R 1993 Continuous Casting of Steel, (London: The Institute of Materials)
[6] Thomas B G 2002 Modeling of the Continuous Casting of Steel - Past, Present Future, www.brimacombecourse.org/pdf/2001_Thomas.pdf
[7] Bellet M, Combeau H, Fautrelle Y, Gobin D, Rady M, Arquis E, Budenkova O, Dussoubs B, Duterrail Y, Kumar A, Gandin C A, Goyeau B, Mosbah S and Založnik M 2009 Int. J. Therm. Sci. 48 2013-16
[8] Šarler B and Vertnik R 2006 Comput. Math. Appl. 51 1269-82
[9] Vertnik R and Šarler B 2006 Int. J. Numer. Method Heat Fluid Flow 16 617-640
[10] Mramor K, Vertnik R and Šarler B 2013 Comput. Mater. Cont. 36 1-21
[11] Kosec G and Šarler B 2009 Int. J. Numer. Method Heat Fluid Flow 18 868-82
[12] Vertnik R and Šarler B 2009 Comput. Model. Engin. Sci. 44 65-95
[13] Vertnik R and Šarler B 2011 Advances in Applied Mathematics and Mechanics 3 259-279
[14] Gobin D and Le Quéré P 2000 Comput. Assist. Mech. Eng. Sci. 7 289-306
[15] Kosec G and Šarler B 2009 Comput. Model. Eng. Sci. 47 191-216
[16] Kosec G Založnik M, Šarler B and Combeau H 2011 Comput. Mater. Cont. 22 169-59
[17] Kosec G and Šarler B 2014 Eng. Anal. Bound. Elem. 45 36-44
[18] Mramor K, Vertnik R and Šarler B 2013 Comput. Model. Eng. Sci. 92 327-352.
[19] Mramor K, Vertnik R and Šarler B 2014 Eng Anal. Bound. Elem. 
doi: 10.1016/j.enganabound.2014.04.013
[20] Bennon W D and Incropera F P 1988 Numer. Heat Transf. A-Appl. 13 277-96
[21] Wilcox, D C 1993 Turbulence modeling for CFD (California: DCW Industries, Inc.)
[22] Chorin A J 1967 J. Comput. Phys. 2 12-26
[23] Buhmann M D 2003 Radial Basis Function: Theory and Implementations (Cambridge: 
Cambridge University Press)
[24] Abe K, Kondoh T and Nagano Y 1994 Int. J. Heat Mass Transf. 37 139-51
[25] Mramor K, Vertnik R and Šarler B 2014 Mat. Technol. 48 281-288.