Nonsymmetrized Hyperspherical Harmonics approach to \( A = 6 \) system. *

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The Hyperspherical Harmonics basis, without a previous symmetrization step, is used to calculate binding energies of the nuclear \( A = 6 \) systems using a version of the Volkov potential acting only on \( s \)-wave. The aim of this work is to illustrate the use of the nonsymmetrized basis to deal with permutational-symmetry-breaking term in the Hamiltonian, in the present case the Coulomb interaction.

I. INTRODUCTION

The Hyperspherical Haromtics (HH) basis set has been extensively used to describe bound states and scattering processes in \( A = 3, 4 \) systems [1]. One of the main reasons of using the HH basis resides in its flexibility, however it suffers from a large degeneracy, which, up to now, has prevented a systematic use with realistic potentials for \( A \geq 4 \) systems. The authors have recently proposed an approach [2-4] in which the HH basis is not symmetrized. In particular, in Ref. [4] the nonsymmetrized basis has been used to describe systems up to \( A = 6 \) particles. It was shown that the large degeneracy of HH can be tackled noticing that the Hamiltonian can be expressed as an algebraic combination of sparse matrices, and the diagonalization procedure was implemented by means of an iterative diagonalization, where only the action of the Hamiltonian on a vector is required. In this work we continue the study on the \( A = 6 \) system, interacting via a two-body Volkov potential acting in \( s \)-wave, analyzing how the introduction of Coulomb interaction breaks the original permutation symmetry \( S_6 \), in the case of two or three protons.

II. THE METHOD

Following Ref. [2], we introduce the Jacobi coordinates, \( x_1, \ldots, x_N \), with \( N = A - 1 \), and we “adapt” the coordinates to the particle pair \( (i,j) \) defining \( x_N = r_i - r_j \), where \( r_i \) and \( r_j \) are the Cartesian coordinates of particles \( i \) and \( j \). From the Jacobi coordinates, we introduce the hyperspherical coordinates, that means an hyper-radius \( \rho \) and \( 3N - 1 \) hyperangles \( \Omega_{ij}^N = (\hat{x}_1, \ldots, \hat{x}_N, \phi_2, \ldots, \phi_N) \).

The HH functions with fixed angular momentum \( LM \), \( \gamma_{LM}^N(\Omega_{ij}^N) \), are defined as the eigenfunctions of the grand angular momentum. They are labelled by the grand angular quantum number \( K \) and a set of \( 3N - 1 \) quantum numbers indicated by \( [K] \). In Ref. [4] it was shown that the potential energy, at fixed values of the hyper-radius, can be written as

\[
\sum_{ij} V_{ij}(\rho) = \sum_{ij} [B_{ij}^{LM}]^\dagger V_{12}(\rho) B_{ij}^{LM},
\]

where the potential matrix \([V_{12}(\rho)]_{[K']}_{[K]} = \langle \gamma_{[K']}^{LM}(\Omega_{ij}^N) | V(1,2) | \gamma_{[K]}^{LM}(\Omega_{ij}^N) \rangle\) is a sparse matrix and the matrix \( B_{ij}^{LM} \) is given by a product of sparse matrices corresponding to the transformation of the HH vector defined on \( \Omega_{ij}^N \) to the one defined on \( \Omega_{ij}^N \). The potential energy matrix is obtained after integrating on \( \rho \) using a Laguerre basis. Furthermore, the kinetic energy is diagonal in HH basis, and this allows to write the full Hamiltonian as an algebraic combination of sparse matrices. The diagonalization of the Hamiltonian is obtained via an iterative scheme, where only the action of the Hamiltonian on a vector is required.

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TABLE I: Binding energies calculated with Volkov’s potential in $s$-wave, for $A = 6$ particles and $L = 0$, with and without Coulomb interaction, using 11 Laguerre’s polynomials.

| $K_{\text{max}}$ | $N_{\text{HH}}$ | $E_0$ (MeV) $[4 \, 2]$ | $E_{[6\, \text{He}]}$ (MeV) $[2] \otimes [2^2]$ | $E_{[6\, \text{Li}]}$ (MeV) $[2 \, 1] \otimes [2 \, 1]$ |
|------------------|-----------------|-----------------|-----------------|-----------------|
| 2                | 15              | 24.793          | 24.064          | 22.974          |
| 4                | 120             | 28.791          | 28.016          | 26.988          |
| 6                | 680             | 31.645          | 30.851          | 29.889          |
| 8                | 3045            | 33.723          | 32.935          | 31.947          |
| 10               | 11427           | 35.723          | 34.935          | 33.947          |
| 12               | 37310           | 37.723          | 36.935          | 35.947          |
| 14               | 108810          | 39.723          | 38.935          | 37.947          |
| 16               | 288990          | 41.723          | 40.935          | 39.947          |
| 18               | 709410          | 43.723          | 42.935          | 41.947          |
| 20               | 1628328         | 45.723          | 44.935          | 43.947          |
| 22               | 3527160         | 47.723          | 46.935          | 45.947          |

III. RESULTS

For our numerical application, we chose a Volkov potential $V(r) = V_R e^{-r^2/R_1^2} + V_A e^{-r^2/R_2^2}$, (2)

with $V_R = 144.86$ MeV, $R_1 = 0.82$ fm, $V_A = -83.34$ MeV, and $R_2 = 1.6$ fm, which only acts in $s$-wave, and with the mass such that $\hbar^2/m = 41.47$ MeV fm$^2$. We have calculated the binding energy for a system of $A = 6$ particles, and we repeated our calculations for the same system with Coulomb interaction between two particles, i.e. a model of $^6$He, and between three particles, i.e. $^6$Li.

In Table I we show the results for the $L = 0$ state. Without the Coulomb interaction the symmetry group is $S_6$ and to antisymmetrize the wave function, taking also into account the spin and isospin degree of freedoms, the eigenvalue must belong to the irreducible representation $[4 \, 2]$. We want to stress the fact that, using the nonsymmetrized basis, the eigenvectors belong to all of the irreducible representations of $S_6$, and in particular that of interest.

When we add Coulomb interaction between two particles, the symmetry is broken as $S_6 \rightarrow S_2 \otimes S_4$, and the original level, having degeneracy 9, is split into 4 sub-levels

$$[4 \, 2] \rightarrow [1 \, 2] \otimes [3 \, 1] + [2 \, 2] \otimes [4] + [2 \, 1] \otimes [3 \, 1] + [2] \otimes [2^2],$$ (3)

where only $[2 \otimes [2^2]$ is physical, as is the only one that can be antisymmetrized with respect the four neutrons using the spin degree of freedom, and describes the ground state of $^6$He. When the Coulomb interaction is extended to three particles, the symmetry breaking is $S_6 \rightarrow S_3 \otimes S_3$, and the split reads

$$[4 \, 2] \rightarrow [2 \, 1] \otimes [3] + [2 \, 1] \otimes [2 \, 1] + [3] \otimes [3] + [3] \otimes [2 \, 1],$$ (4)

with the only physical state, describing $^6$Li, being $[2 \, 1] \otimes [2 \, 1]$.

We have shown the power and the flexibility of the nonsymmetrized HH approach. In particular we were able to include basis states up to $K = 22$ for a six-body system (corresponding to a basis set of 38,798,760 elements using 11 Laguerre polynomials). Furthermore, using a symmetry-adapted Lanczos algorithm, we were able to trace the irreducible representation of the eigenvector of interest and to select the corresponding eigenvalue.

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