THE KECK APERTURE-MASKING EXPERIMENT: NEAR-INFRARED SIZES OF DUSTY WOLF-RAYET STARS

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ABSTRACT

We report the results of a high angular resolution near-infrared survey of dusty Wolf-Rayet stars using the Keck I Telescope, including new multwavelength images of the pinwheel nebulae WR 98a, WR 104, and WR 112. Angular sizes were measured for an additional eight dusty Wolf-Rayet stars using aperture-masking interferometry, allowing us to probe characteristic sizes down to $\sim$20 mas ($\sim$40 AU for typical sources). With angular sizes and specific fluxes, we can directly measure the wavelength-dependent surface brightness and size relations for our sample. We discovered tight correlations of these properties within our sample that could not be explained by simple spherically symmetric dust shells or even the more realistic “pinwheel nebula” (three-dimensional) radiative transfer model, when using Zubko’s optical constants. While the tightly correlated surface brightness relations we uncovered offer compelling indirect evidence of a shared and distinctive dust shell geometry among our sample, long-baseline interferometers should target the marginally resolved objects in our sample in order to conclusively establish the presence or absence of the putative underlying colliding-wind binaries thought to produce the dust shells around WC Wolf-Rayet stars.

Subject headings: binaries: general — circumstellar matter — instrumentation: interferometers — radiative transfer — stars: individual (WR 11, $\gamma$ Vel, WR 48a, WR 76, WR 95, WR 98a, WR 104, WR 106, WR 112, WR 113, WR 118, WR 140, CV Ser) — stars: winds, outflows — stars: Wolf-Rayet

Online material: color figures

1. INTRODUCTION

The existence of dust shells around Wolf-Rayet (W-R) stars has long posed a mystery: how can dust nucleate and survive near these hot stars in the presence of harsh ultraviolet radiation and gas densities lower than those found in other dust-forming environments (e.g., winds of asymptotic giant branch stars)? While novel chemical pathways for producing special carbonaceous grains have been proposed (Zubko 1998; Cherchneff et al. 2000), recent theoretical and observational results point strongly to a very different solution.

Inspired by the periodic dust formation episodes around the colliding-wind binary WR 140 (Moffat et al. 1987; Williams et al. 1990), Usov (1991) suggested that dust formation might be catalyzed in the compressed layer of gas at the interface of colliding winds in W-R+OB binary systems. This hypothesis was confirmed by the surprising imaging of a “pinwheel” nebula dust spiral around WR 104 (Tuthill et al. 1999). Indeed, Monnier et al. (1999a) went so far as to suggest that perhaps all dusty W-R systems are hiding colliding-wind binary systems (see also Dougherty & Williams 2000), and this was further supported by the discovery of nonthermal radio emission around WR 104, WR 98a, and WR 112 (Monnier et al. 2002b).

The previously cited imaging work was carried out using the diffraction-limited capabilities of the world’s largest optical telescope at the Keck Observatory. High-resolution imaging was achieved using aperture-masking interferometry, whereby the Keck I primary mirror is converted to a VLA-style interferometric array (Tuthill et al. 2000b) of many subapertures. This technique has been shown to be superior to current adaptive optics systems for high-fidelity imaging and size estimations for marginally resolved objects (Rajagopal et al. 2004).

Here we report an extension to the initial imaging work done on WR 104 and WR 98a. We observed all W-R stars (accessible from Mauna Kea) brighter than $m_K = 6.5$ in order to investigate the binary hypothesis as to the origin of dust production. Although imaging was only possible for one additional source (WR 112), we report the characteristic size measurements for a total of 11 sources and discuss the significance of these findings.

2. OBSERVATIONS

Our group has been carrying out aperture-masking interferometry at the Keck I telescope since 1996. We have published images and size measurements with unprecedented angular resolution on topics ranging from young stellar objects to carbon stars, red supergiants, and photospheric diameters of Mira variables (e.g., Monnier et al. 1999b; Tuthill et al. 2000a, 2000b; Danchi et al. 2001). A full description of this experiment can be found in Tuthill et al. (2000b), with further discussion of systematic errors in Monnier et al. (2004).

The NIRC camera with image magnifier (Matthews et al. 1996) was used in conjunction with the aperture-masking hardware. For this work, we used an aperture mask with an annulus that is 8 m in diameter as projected onto the Keck primary. This mask gives us complete UV coverage and sensitivity to targets brighter than $m_K \sim 6$ with a modest loss in calibration precision due to additional redundancy noise (for further discussion, see Tuthill et al. 2000b). The data frames were taken in speckle mode ($T_{int} = 0.14$ s) to freeze the atmosphere. A variety of filters were used, and the characteristic center wavelengths and widths can be found...
in Table 1 (spectral scans can be found in Harrison & Goodrich 1999).

Here we report the full body of Wolf-Rayet size measurements collected during the entire period of the Keck aperture-masking experiment. A target list and observing log can be found in Tables 2 and 3. For V^2 and closure phase data are available from the authors; all data products are stored in the FITS-based, optical interferometry data exchange format (OI-FITS) recently described in Pauls et al. (2005).

3. DETERMINING CHARACTERISTIC SIZES

3.1. Methodology

While the basic data reduction has been described in previous papers (Tuthill et al. 2000b), this paper is the first Keck aperture-masking paper to explicitly deal with a sample of partially resolved objects. This is important here, since we intend to measure the two-dimensional size and shape of Wolf-Rayet dust shells in order to search for asymmetries. If we measure an ellipticity in our data, what confidence do we have that an elongation is not due to poor calibration of the optical transfer function? To answer this question we have undertaken a systematic investigation of our calibration errors.

The errors for this experiment are dominated by statistical and seeing calibration errors. The statistical error arises from the contribution of photon noise and read noise to our measurement process. Calibration error arises because the optical transfer function varies with time and telescope pointing. In order to correct for this latter effect, we always observe a point-source reference star nearby in time and angle from our target. In the process of calibration, raw visibilities from the target are divided by those from nearby in time and angle from our target. In the process of calibrating the optical transfer function, raw visibilities from the target are divided by those from the calibrator. Hence, calibration error is multiplicative and affected high-visibility data the most in absolute terms (ΔV^2). For well-resolved objects, these two types of error are comparable. For small objects (≤25 mas), calibration error is dominant and limits our ability to say whether an object is resolved.

3.1.1. Functional Form and Statistical Errors

As part of our analysis, the calibrated visibility data are fitted to the following generic function (based on a two-dimensional Gaussian):

\[ V^2 = \left( V_0 e^{-(\alpha u^2 + \beta v^2 + \gamma uv)} + V_p \right)^2, \]

where \( V_p \) is constant and represents a point-source contribution (which was set to zero for this study). The parameters \( \alpha, \beta, \) and \( \gamma \) are not constrained to be positive, thus allowing the function to curve upward in cases where calibration errors are extreme.

Under a rotation through angle \( \phi \), this function can be projected onto the major and minor axes (\( u', v' \)), written in the form

\[ V^2 = \left( V_0 e^{-4 \ln^2 \left[ \left( u'/u'_\text{FWHM} \right)^2 + \left( v'/v'_\text{FWHM} \right)^2 \right] + V_p \right)^2, \]

with the parameters of this fit being given by

\[ \phi = \frac{1}{2} \arctan \left( \frac{\gamma}{\alpha - \beta} \right), \]

\[ (u'_{\text{FWHM}})^2 = \frac{4 \ln 2 \cos 2\phi}{\alpha - (\alpha + \beta) \sin^2 \phi}, \]

\[ (v'_{\text{FWHM}})^2 = \left[ \frac{\alpha + \beta}{4 \ln 2} - \frac{1}{(u'_{\text{FWHM}})^2} \right]^{-1}. \]

These expressions can be converted to the angular FWHM along the major/minor axis using the formula FWHM\_major = (4 ln 2)/(πu'\_FWHM) and FWHM\_minor = (4 ln 2)/(πv'\_FWHM).

Statistical errors are dealt with by using bootstrap sampling (Efron & Tibshirani 1993) with the function given in equation (1). The fitting routine MPFIT in C. Markwardt’s MINPACK⁶ suite of IDL programs is used as the basis for the bootstrap.

3.1.2. Monte Carlo Method for Determining Calibration Error

Quantifying the magnitude of the calibration errors in this experiment has been difficult. Due to the very limited time available on a Keck telescope, the number of stars used as calibrators for a given filter is usually small, ranging from less than five up to perhaps a dozen for the most commonly used bands. While precise systematic errors are difficult to determine due to small-number statistics, it is possible to estimate the magnitude of the calibration accuracy.

The first step in the analysis of calibration errors is to cross-calibrate all of the calibration stars of a given night (with the same bandpass filter and aperture mask) against each other and fit the resulting visibility curves with the quadratic function introduced in equation (2). At this point, the calibrator-calibrator pairs are weighted according to their proximity to each other in time and elevation according to exp[-(Δt/Δt_forms)^2 - (Δθ_el/Δθ_forms)^2], with Δt_forms ~ 2000 s and Δθ_forms ~ 20°.

In order to propagate calibration errors into the errors on the Gaussian FWHM sizes that we wish to measure, we start with a visibility curve with a known FWHM (using the same baselines as the Keck data). Then we multiply this perfect simulated data by various calibrator-calibrator pair data and then recalculate a

⁶ See http://cow.physics.wisc.edu/~craigm/idl/.
| Source Names | R.A. (J2000.0) | Decl. (J2000.0) | V<sup>a</sup> (mag) | J<sup>a</sup> (mag) | H<sup>a</sup> (mag) | K<sup>a</sup> (mag) | Spectral Type | Refs. | Distance (kpc) | Refs. | Luminosity<sup>b</sup> <sup>log (L/L<sub>☉</sub>)</sup> | Refs. | Photometry Refs. |
|--------------|--------------|----------------|-----------------|-----------------|-----------------|-----------------|--------------|------|----------------|------|-----------------|------|-----------------|
| γ<sup>2</sup> Vel, WR 11....... | 08 09 31.96 | −47 20 11.8 | 1.81 | 2.15 | 2.25 | 2.10 | WC8 + O7.5 III–V | 1 | 0.26 | 2 | 5.5<sup>c</sup> | 3, 4 | 5 |
| WR 48a............. | 13 12 39.65 | −62 42 55.8 | ... | 8.74 | 6.80 | 5.09 | WC8ed + ? | 1 | 1.2 | 1 | ... | 5, 6 |
| WR 76............. | 16 40 05.3 | −45 41 10 | 15.36 | 8.46 | 6.51 | 4.88 | WC9d | 1 | 5.35 | 1 | ... | 5, 6 |
| WR 95............. | 17 36 19.76 | −33 26 10.9 | 14.00 | 8.29 | 6.67 | 5.27 | WC9d | 1 | 2.09 | 1 | ... | 5, 6 |
| WR 98a............. | 17 41 13.051 | −30 32 30.34 | ... | 9.14 | 6.51 | 4.33 | WC8-9vd + OB? | 1 | 1.9 | 7 | ... | 6 |
| WR 104............. | 18 02 04.123 | −23 37 42.24 | 13.54 | 6.67 | 4.34 | 2.42 | WC9d + B0.5 V | 1 | 2.3 | 8 | 5.4<sup>d</sup> | 9 | 5 |
| WR 106............. | 18 04 43.66 | −21 09 30.7 | 11.93 | 7.94 | 6.28 | 4.82 | WC9d | 1 | 2.3 | 1 | ... | 5, 6 |
| WR 112............. | 18 16 33.489 | −18 58 42.47 | 17.7 | 8.68 | 6.26 | 4.26 | WC9d + OB? | 1 | 4.15 | 1 | ... | 5, 6 |
| WR 113............. | 18 19 07.36 | −11 37 59.2 | 9.43 | 7.02 | 6.28 | 5.49 | WC8d + O8–9 | 1 | 1.79 | 1 | ... | 5, 6 |
| WR 118............. | 18 31 42.3 | −09 59 15 | 22 | 8.10 | 5.41 | 3.65 | WC9 | 1 | 3.13 | 1 | ... | 5, 6, 10 |
| WR 140............. | 20 20 27.98 | +43 51 16.3 | 6.9 | 5.55 | 5.43 | 5.04 | WC7pd + O4–5 | 1 | 1.85 | 11 | 6.1<sup>c</sup> | 12 |

**Note.**—Units of right ascension are hours, minutes, and seconds, and units of declination are degrees, arcminutes, and arcseconds.

<sup>a</sup> These magnitudes (V band from SIMBAD and JHK bands from 2MASS) are merely representative, since some of the targets are variable.

<sup>b</sup> Luminosity here has been corrected for the adopted distance used in this paper; not all objects have well-established luminosities, due to high extinction.

<sup>c</sup> Binary system total luminosity.

**References.**—(1) Van der Hucht 2001; (2) Perryman et al. 1997; (3) De Marco et al. 2000; (4) De Marco & Schmutz 1999; (5) Williams et al. 1987; (6) Cutri et al. 2003; (7) Monnier et al. 1999a; (8) Tuthill et al. 1999; (9) Harries et al. 2004; (10) SIMBAD; (11) Dougherty et al. 2005; (12) Williams et al. 1990.
| TARGET  | DATE (UT) | FILTER | $\lambda_0$ (\(\mu\)m) | APERTURE MASK | CALIBRATORS  |
|---------|-----------|--------|-----------------|-------------|--------------|
| $\gamma^2$ Vel       | 1999 Feb 5 | CH4    | 2.269            | Golay-21    | HD 68553 K3 4.39 ± 0.44 |
| WR 48a           | 2000 Jan 26 | K      | 2.2135           | Annulus-36  | HD 115399 K5 1.2 ± 0.7 |
| WR 76           | 1998 Jun 5 | H      | 1.6575           | Annulus-36  | HD 151834 G3 1.2 ± 0.4 |
|                | 1999 Apr 26 | K      | 2.2135           |             | HD 153258 K4 2.4 ± 0.9 |
| WR 95           | 2000 Jun 24 | H      | 1.6575           | Annulus-36  | HD 158774 M0 2 ± 2 |
| WR 98a          | 2000 Jun 24 | K      | 2.2135           | Annulus-36  | HD 158774 |
|                |             | CH4    | 2.269            |             | HD 163428 K5 4.1 ± 1.0 |
| WR 104          | 2000 Jun 24 | H      | 1.6575           | Annulus-36  | HD 165813 M0 1.7 ± 0.9 |
| WR 106          | 1998 Jun 5 | CH4    | 2.269            | Annulus-36  | HD 167036 M0 3.0 ± 0.6 |
|                | 1999 Jul 30 | K      | 2.2135           |             | HD 167036 |
| WR 112          | 2000 Jun 24 | H      | 1.6575           | Annulus-36  | HD 165813 |
| WR 113          | 2000 Jun 24 | K      | 2.2135           | Annulus-36  | HD 168366 K2 1.03 ± 0.12 |
| WR 118          | 1998 Jun 5 | CH4    | 2.269            | Annulus-36  | HD 168000 K3 1.42 ± 0.8 |
|                | 1998 Jun 6 | H      | 1.6575           | Annulus-36  | HD 170474 K0 3.5 ± 1.1 |
|                | 1999 Apr 26 | CH4    | 2.269            | Annulus-36  | HD 175775 K0 1.3 ± 0.3 |
| WR 140          | 1999 Jul 30 | FeII   | 1.6471           | FFA         | HD 193631 K0 0.5 ± 0.1 |
|                | 2001 Jul 30 | Kcont  | 2.25965          | FFA         | HD 192867 M1 1.6 ± 0.6 |

Calibrator size estimates made using getCal, which is maintained and distributed by the Michelson Science Center (http://msc.caltech.edu).
size using the quadratic model developed above. This simulates what the measured visibility would look like for a star of a known size using different calibrators. By Monte Carlo sampling over many calibrator-calibrator pairs according to the weights described above, we build up an estimate of the likely error in the final size measurement.

The results are summarized in Figure 1 for the 2000 June epoch in the CH4 filter using the annulus mask. The most important feature of these plots is the determination of the effective angular resolution limit of this experiment. We see that simulations of an unresolved point source, when passed through the calibration study described above, can yield apparent sizes as large as 12 mas (Gaussian FWHM). From this, we conclude that our resolution limit corresponds to Gaussian FWHMs of 12–15 mas (λ ~ 2.2 µm), approximately 4 times better than the formal Keck diffraction limit of 1.22λ/D ~ 55 mas. For nights of poor seeing, our size upper limit on FWHM increases to 15–20 mas.

As stated before, we would like to use the Keck masking data to measure modest asymmetries in the dust shells of Wolf-Rayet stars, since this can tell us about the dust production mechanism (single-star vs. binary interaction). In order to test our sensitivity to asymmetries, we fitted two-dimensional Gaussians to our calibrator study data, and these results are also shown in Figure 1. Our study shows that if we impose typical calibration errors on a circularly symmetric object with a FWHM of 20 mas, we will regularly find asymmetries with measured ratio FWHMminor/FWHMmajor = 0.8. Thus, except for extremely asymmetric objects, any study on asymmetries from these data will have to restrict itself to objects that are larger than about 25–30 mas.

We note that the calibration errors determined by this method are overestimates, since typical calibrator-calibrator pairs suffer from greater time differences and angular distances than the target-calibrator pairs used for actual science measurements (this is confirmed for a few objects where we have multiple independent data sets). Thus, we can assume that the errors calculated in this section are conservative and that the true calibration errors should be up to a factor of 1.5–2 smaller for nights of comparable seeing.

The greatest uncertainty in this analysis is that it uses a very limited number of calibrator stars spread out over a night. Our understanding of the errors would improve greatly if a series of identical exposures of a calibrator star were to be taken sequentially. This would allow us to see the short-timescale variations in a calibrator’s visibility curve, and thus how much variance there is in the calibration process. If very large variations in the quadratic fits remain on timescales as short as this, the calibration process may need further refinement.

The same study has been done for the H filter (λ = 1.65 µm) and for the PAHcs filter (λ = 3.08 µm), and these results are also included (in the same format as Fig. 1) in Figures 2 and 3.

4. RESULTS

4.1. Multiwavelength Characteristic Sizes

Circularly symmetric Gaussian emission profiles were fitted to the azimuthally averaged visibility data for all targets, and the results can be found in Table 4. The uncertainty estimates were based on the combined error from statistical variations (using bootstrap) and from the calibration errors discussed in the previous section. Calibration error is typically the dominant error for these data.

7 The Keck Time Allocation Committee is unlikely to give time to such a proposal.
Fig. 2.—Same as Fig. 1, but for the results for the fitting of recalibrated simulated data using calibrators from the 2000 June epoch for the H filter using the annulus mask.

Fig. 3.—Same as Fig. 1, but for the results for the fitting of recalibrated simulated data using calibrators from the 2000 June epoch for the PAHcs filter using the annulus mask.
The largest dust shells (WR 48a, WR 98a, WR 104, WR 112, and WR 140) show strong deviations from the Gaussian at the longer Keck baselines. This is not at all surprising since WR 98a, WR 104, WR 112, and WR 140 are all known to have spiral and/or asymmetric dust shells (Tuthill et al. 1999; Monnier et al. 1999a; Ragland & Richichi 1999; Monnier & Tuthill 2002c; Marchenko et al. 2002). It is likely that the other smaller dust shells in our sample are also not simple Gaussians, although we were unable to detect deviations due to insufficient angular resolution (all marginally resolved objects look Gaussian-like to an interferometer).

In order to calculate a consistent “characteristic size” for all targets, we only fit to short-baseline data where the visibility is greater than 0.5. By fitting only to short-baseline, high-visibility data, the reported characteristic sizes represent the overall scale of emission and are not strongly affected by the spiral and/or small-scale structures of the nebulae, if present. Our results are graphically shown for 2.2, 1.65, and 3.08 μm data in Figures 4–6, respectively.

Subject to the (large) uncertainties in ellipticity discussed in § 3.1.2, we did not find evidence for elongation in the emission except in the cases of the largest dust shells, WR 98a, WR 104, and WR 112. For these, we have measured Gaussian FWHM and the ratio of the minor axis to the major axis. Just as for the characteristic sizes discussed above, this ellipticity parameter only refers to the “large-scale” emission component of the dust shell and not the smaller scale details, such as the inner spiral windings of WR 104.

### 4.2. Correction for Stellar Contribution

While for most stars the near-infrared emission is completely dominated by dust emission, some targets in our sample have significant stellar contributions. By carrying out simple spectral energy density (SED) fitting, we can estimate the fraction of light coming from the star (compared to dust) and apply a correction factor. Essentially, the visible photometry is fitted with a Kurucz model allowing the stellar size and reddening to vary; the stellar IR flux can then be estimated by extrapolation. With this knowledge, we can more accurately estimate the true size of the dust shell (not the star+dust emission). This is a common procedure in other areas of astronomy, such as estimating the sizes of disks around young stellar objects, and we follow procedures documented elsewhere (e.g., Millan-Gabet et al. 2001; Monnier et al. 2005).

This correction is strongest for γ Vel (WR 11), which does not show much infrared excess, but since the observed characteristic size was consistent with a point source, we do not apply the correction. For cases in which the correction is more than 5% (WR 48a, WR 95, WR 98a, WR 106, and WR 113), we have included the new size estimates (and the estimates for the dust fraction) in Table 5. We estimate that the correction factor is only known to 50% due to difficulty in uniquely fitting the broadband SEDs, and this error has been included in the corrected sizes in Table 5.

Note that the postoutburst WR 140 dust shell is so large that it was obviously necessary to correct for the stellar contribution (see top curves of Fig. 4), and this was done for Table 4. The WR 140 dust shell was considered in detail in Monnier et al. (2002c), and we refer the reader to this paper for further discussion on this object.

### 4.3. Aperture Synthesis Imaging

Three systems in our sample are sufficiently resolved that they can be imaged using aperture synthesis techniques. The maximum entropy method (MEM; Skilling & Bryan 1984; Gull & Skilling 1984) has been used to create diffraction-limited images from the interferometric data, as implemented in the VLBMEM package by Sivia (1987).

Figures 7–9 show new multiwavelength images of WR 98a, WR 104, and WR 112, respectively, at wavelengths of 1.65, 2.2, and 3.08 μm. The resolution degrades with increasing wavelength, and we adopt an effective angular resolution of 21, 28, and...
Wolf-Rayet Results: 2.2 μm FWHMs

Wolf-Rayet Results: 1.65 μm FWHMs

Fig. 4.—Azimuthally averaged visibility curves at 2.2 μm for Wolf-Rayet stars in our sample, along with best-fit Gaussian curves. Each visibility curve is offset by 0.25 from the next. For the most resolved dust shells, the Gaussian curves were only fitted to the “large-scale” component of the visibility data ($V > 0.5$). See Table 4 for detailed fitting results.

Fig. 5.—Same as Fig. 4, but for the azimuthally averaged visibility curves at 1.65 μm.

39 mas for 1.65, 2.2, and 3.08 μm, respectively, corresponding to $\Delta \Theta = \lambda / 2 \beta_{\text{max}}$. While we have presented images of WR 98a and WR 104 in the past, the images given here are at a new epoch and we have included additional wavelengths.

Figure 9 contains the first published resolved images of the near-infrared dust shell around WR 112 (preliminary results were shown at the “Interacting Winds from Massive Stars” workshop in 2000; Monnier et al. 2002a). The asymmetric nature of this dust shell had already been discovered using lunar occultations (Ragland & Richichi 1999). Recently, Marchenko et al. (2002) reported a spiral-like dust plume around WR 112 at mid-infrared wavelengths. Interestingly, we do not see an obvious spiral structure in these new near-infrared images; rather, we see only evidence for filaments/arcs and one-sided nebulosity.

5. DISCUSSION

This paper significantly expands the number of W-R stars with known angular sizes, and we wish to use this information to probe the nature of the dusty outflows that are so common in WC systems. The Wolf-Rayet systems with the largest dust shells, WR 48a, WR 98a, WR 104, WR 112, and WR 140, are all in confirmed colliding-wind systems (from detection of nonthermal radio emission). Unfortunately, we do not have enough angular resolution to definitely resolve filamentary or spiral structure in the other objects in our survey.

5.1. Surface Brightness Relations

Secure identification of binarity is critical. Monnier et al. (1999a) emphasize that if all dusty W-R stars are in binaries, this has profound implications on massive stellar evolution and the nature of carbon-rich (WC) W-R stars in particular. One method to achieve this end is to see if all the objects in our survey follow similar surface brightness relations. In order to calculate a specific surface brightness $S_\lambda$ we applied the following formula:

$$S_\lambda = \frac{F_\lambda \times 10^{-(\text{mag}/2.5)}}{0.68 \pi \Theta_{\text{FWHM}}},$$

where $F_\lambda$ represents the flux density for a zero-magnitude star (here using units of $W \text{ m}^{-2} \text{ μm}^{-1}$), “mag” is the magnitude of the W-R dust shell at the given wavelength (correcting for stellar emission when appropriate; see § 4.2), and $\Theta_{\text{FWHM}}$ is the FWHM of the Gaussian fit. Note that the 0.68 in the formula is a correction factor that converts a Gaussian FWHM size estimate to the equivalent uniform disk for a better definition of “surface brightness.” The final units of $S_\lambda$ are $W \text{ m}^{-2} \text{ μm}^{-1} \text{ sr}^{-1}$.
Figure 10 shows the observed surface brightness relations. The objects in the survey all appear to follow the same surface brightness relations, indicating that all the systems share a similar near-infrared emission mechanism. Assuming the emission comes from optically thin carbon dust (described by the optical constants of Zubko 1998), we can estimate the color temperature, and we find \( T_{\text{color}} \approx 1000 \text{ K} \) (considering the relation between 1.65 and 2.2 \( \mu \text{m} \)) and \( T_{\text{color}} \approx 650 \text{ K} \) (considering 2.2 and 3.08 \( \mu \text{m} \)). The fact that the surface brightness relations cannot be described by a single temperature reflects the fact that the dust from a wide range of temperatures contributes to the near-IR emission.

We compared our observed surface brightness relations with the predicted relations from two models. First, we used the radiative transfer code DUSTY (Ivezić et al. 1999) to fit spherically symmetric dust shell models (assuming uniform outflow and the dust constants of Zubko 1998) to the SED. We could vary the amount of dust optical depth and calculate the model surface brightness subject to an overall fit to the SED. We present the calculated surface brightness relations in Figure 10, spanning \( r_{2.2 \mu m} = 0.01 - 0.77 \). We note that the SED fits were not perfect (usually underpredicting 3.1 \( \mu \text{m} \) fluxes), and thus we take these relations as representative but not fully optimized.

Table 5

| TARGET     | DUST FRACTION (%) | GAUSSIAN FWHM\(^*\) (mas) |
|------------|-------------------|---------------------------|
| WR 48a...... | ...               | 85                        |
| WR 95....... | 73                | 89                        |
| WR 98a...... | 90                | ...                       |
| WR 106...... | 78                | 93                        |
| WR 113...... | 38                | 65                        |

\* These characteristic sizes are corrected for the presence of the star (which contributes significant near-IR flux in some cases; see § 4.2). Refer to Table 4 for size measurements for the rest of the sample, which do not require corrections.

Harries et al. (2004) recently calculated the emergent flux from a series of spiral dust shell models using the three-dimensional (Monte Carlo) radiative transfer code TORUS (Harries 2000). We refer to this as the “WR 104 reference model” and used this sophisticated physical model to calculate the surface brightness relations as a function of inclination angle (the model only reasonably fits the WR 104 SED for low inclinations). These results can also be seen in Figure 10.

Comparing the model calculations to the data, we find that neither of these models are a good fit to all the 1.65, 2.2, and 3.1 \( \mu \text{m} \) surface brightness relations, although the models can reproduce some of the observations. More generically, this figure shows us that any particular dust shell geometry yields a family of surface brightness relations. The tightly correlated surface brightness relations measured for our sample are suggestive of a shared and distinctive dust shell geometry, although neither the spherically symmetric model nor the three-dimensional WR 104 reference model can fully explain our results. One could improve the fits of a given model to the data by tuning the adopted optical constants (which are not well known), although the differences between models would persist. Before discussing the implications of this, we look for more information in the “size ratio” relations.

5.2. Size Ratio Relations

Another possibly distinctive (and distance-independent) signature to differentiate spherically symmetric dust shells from colliding-wind systems is the ratio of dust shell characteristic sizes at different wavelengths. We have collected all of the size ratios and present these (with errors) in Figure 11. Assuming that all the objects are drawn from the same class, we find the following mean size relations:

\[
R_{1.65 \mu m/2.2 \mu m} = \frac{\text{FWHM at } 1.65 \mu m}{\text{FWHM at } 2.2 \mu m} = 0.73 \pm 0.05, \quad (7)
\]

\[
R_{3.08 \mu m/2.2 \mu m} = \frac{\text{FWHM at } 3.08 \mu m}{\text{FWHM at } 2.2 \mu m} = 1.36 \pm 0.07. \quad (8)
\]

This compares favorably with calculations for an optically thin, spherically symmetric dust shell (\( R_{1.65 \mu m/2.2 \mu m} = 0.78, R_{3.08 \mu m/2.2 \mu m} = 1.22 \)), as performed using DUSTY (Ivezić et al. 1999) and tuned to fit the SED. We have also calculated the size ratios predicted by the WR 104 reference model discussed in the previous section. As shown in Harries et al. (2004), this model does a remarkable job in fitting the morphology, SED, and overall size. The series of dust shell models used to calculate the size relations (as well as surface brightness relations) can be found.
Fig. 7.—Multiwavelength aperture synthesis images of WR 98a on UT 2000 June 24, at 1.65, 2.2, and 3.08 \( \mu m \). Note that the resolution is lower at the longer wavelengths, as indicated by the “beam” spot located in the bottom left corner of each panel [representing the best achievable angular resolution, \( \Delta \theta = \lambda / (2B_{\text{max}}) \)]. [See the electronic edition of the Journal for a color version of this figure.]

Fig. 8.—Same as Fig. 7, but for WR 104. [See the electronic edition of the Journal for a color version of this figure.]

Fig. 9.—Same as Fig. 7, but for WR 112. [See the electronic edition of the Journal for a color version of this figure.]
Figure 10.—Surface brightness relations for dusty Wolf-Rayet systems. We plot the best-fit linear relation corresponding to $T_{\text{color}} \sim 1000$ K (left) and $T_{\text{color}} \sim 650$ K (right), assuming optical constants for Zubko (1998) dust. The open circles show the surface brightness relations for a series of spherically symmetric dust shell models with $r_{2.2 \mu m} = 0.01, 0.1, 0.3$, and $0.67$ (higher $r$ yields higher $S_i$). The filled circles show the same relations for the WR 104 reference model of Harries et al. (2004) for inclination angles of $i = 0^\circ$, $30^\circ$, $60^\circ$, and $90^\circ$ (higher $i$ yields higher $S_i$). The arrow shows how the surface brightness changes for an interstellar reddening of $A_V = 5$, assuming standard interstellar medium dust.

Figure 11.—Near-infrared characteristic size ratios vs. K-band magnitude. The left panel shows the results for the ratio of the FWHM at 1.65 $\mu m$ to the FWHM at 2.2 $\mu m$, while the right panel shows the results for the ratio of the FWHM at 3.08 $\mu m$ to the FWHM at 2.2 $\mu m$. We plot the mean size ratio (dashed line) and also mark with arrows the expected size ratios for (1) an optically thin, spherically symmetric dust shell and (2) the WR 104 reference model of Harries et al. (2004), viewed from a $30^\circ$ inclination angle.

in Figure 12. Analysis of these synthetic images yields the following mean size relations, where $i$ is the inclination:

\[
\begin{align*}
  &i = 0^\circ: \quad R_{1.65 \mu m}/2.2 \mu m = 0.92, \quad R_{3.08 \mu m}/2.2 \mu m = 1.04; \\
  &i = 30^\circ: \quad R_{1.65 \mu m}/2.2 \mu m = 0.91, \quad R_{3.08 \mu m}/2.2 \mu m = 1.07; \\
  &i = 60^\circ: \quad R_{1.65 \mu m}/2.2 \mu m = 0.95, \quad R_{3.08 \mu m}/2.2 \mu m = 1.14; \\
  &i = 90^\circ: \quad R_{1.65 \mu m}/2.2 \mu m = 0.98, \quad R_{3.08 \mu m}/2.2 \mu m = 1.28.
\end{align*}
\]

Paradoxically, the simple spherically symmetric model fits the WR 104 size relations better than the model made specifically for WR 104. This can be explained if the optical depth of the shell in the reference model of Harries et al. (2004) is somewhat too high. A high value of $\tau$ in the inner windings of the spiral casts a strong shadow across the nebula that causes the temperature profile to be very steep (analogous to the temperature profile for an optically thick dust shell). Most likely, the observed size relations can be accommodated by the pinwheel model by adjusting the distribution of dust in the innermost part of the nebulae.

5.3 Comments on Individual Objects

WR 112.—We know that WR 112 is in a colliding-wind binary system (like WR 98a and WR 104) from its clear nonthermal radio emission (Chapman et al. 1999; Monnier et al. 2002b). Furthermore, we believe that we are viewing the binary near the orbital plane (not face-on) because the nonthermal radio emission is highly variable, suggesting that our line of sight periodically passes through the optically thin O-star wind (as is well documented for WR 140; White & Becker 1995). In this context, it is not surprising that the colliding-wind dust spiral takes on a more complicated shape due to projection effects (see discussion of this in Monnier et al. 2002c; Harries et al. 2004). Indeed, the $i = 60^\circ$ model for WR 104 (Fig. 12) bears an obvious similarity to the "horseshoe" structure that we observed for WR 112 (middle panel of Fig. 9).

This suggestion of an edge-on viewing angle contradicts the apparent (near) face-on dust spiral geometry seen in the mid-IR (Marchenko et al. 2002). We speculate that projection effects of
the outer spiral windings (seen at high inclination) could also explain the mid-IR data, yielding bright arcs and filaments that may resemble a face-on spiral outflow (e.g., Monnier et al. 2002b). Alternatively, if the underlying colliding-wind system is indeed viewed from a face-on position as implied by the mid-IR images, then the variable radio emission might be caused by an as yet undetected third stellar component to the system. Further analysis of this system will be included in a future paper, where we will present multipoch near- and mid-IR images, as well as VLA monitoring photometry.

WR 113, CV Ser.—Van der Huch (2001) lists WR 113 as a nearly edge-on binary system (i ~ 70°) with colliding winds. We expect such edge-on systems to show significant dust shell elongations in the near-IR (see Fig. 12), although our data indicate that the dust emission is symmetric (the ratio r = FWHM_{minor}/FWHM_{major} ≃ 0.8). This higher level of symmetry could result if the cone opening angle of the colliding-wind interface was larger than that found for WR 104 or if there was significant dust entrained in the W-R outflow in a more spherical pattern. Perhaps the hint of a third component in the system (Niemela et al. 1996) could also explain this surprising lack of dust shell asymmetry in this object.

WR 118.—Our K-band size estimate is compatible with the earlier speckle measurement of Yudin et al. (2001).

Quintuplet Stars.—Tuthill et al. (2006) recently reported resolving two bright Quintuplet WC stars into pinwheel nebulae. These objects show a size ratio R_{3.08 μm}/2.2 μm ~ 2, larger than that than for the stars reported here, along with a correspondingly low 3.1 μm surface brightness. We have not included these stars in this paper given the difficulty in correcting for extinction and the possible importance of local heating in this unusually dense and active cluster.

6. CONCLUSIONS AND FUTURE WORK

We have presented a multiwavelength survey of near-infrared angular sizes of dusty Wolf-Rayet systems from the Keck aperture-masking program. Aperture synthesis images were presented for WR 112 for the first time, and we found strong evidence for interacting/colliding winds. In addition, we presented new epochs of WR 104 and WR 98a images, as well as the first results at 3.1 μm, confirming the spiral nature of the dust shells for WR 104 and WR 98.

Using these data, we discovered tightly correlated surface brightness relations and also common size ratios between different near-infrared bands. The observed relations could not be reproduced in detail using either a spherically symmetric dust shell model or the three-dimensional radiative transfer model of WR 104 (Harries et al. 2004). The high-quality data presented here will act as an observational foundation for a new generation of modeling efforts.

We find these results to be compelling indirect evidence that all these dusty W-R stars here share a common emission geometry, presumably related to the obvious spiral distribution of WR 104 and WR 98a (and confirmed recently for two Quintuplet WC stars; Tuthill et al. 2006). Thus, while not conclusive, this study can be viewed as further evidence that WC Wolf-Rayet stars are associated with binarity (Monnier et al. 1999a; Williams & van der Hucht 2000).

In order to make further progress on identifying binarity, we must pursue multiple approaches. While traditional direct spectroscopic identifications of binarity have proved difficult (e.g., Williams & van der Hucht 2000), other promising methods have emerged for unambiguously determining whether dusty W-R systems are binaries. We recommend that the following strategies be pursued:

Higher resolution infrared data.—Baselines longer by a factor of 3 will allow unambiguous detection of asymmetry and nonzero closure phases, if spirals and/or filaments exist in our sample. This corresponds to about 30 m baselines, easily achievable with the current interferometers (the Very Large Telescope is particularly well suited to this). The main challenge will be the low visible flux, which makes tip-tilt tracking difficult; infrared star trackers would be helpful in this regard.

Visible or infrared photometry.—Photometric fluctuations have been reported for WR 98a (Monnier et al. 1999a) and WR 104 (Kato et al. 2002). The variations seem to correlate to the known orbital periods, and we strongly encourage observers to monitor all dusty WC stars to establish the periods of the putative underlying binaries. This method can work for objects at large distances (e.g., Galactic center, Local Group); however, one has to contend with intrinsic variability and must have the determination to monitor sources over many years (or decades; Williams et al. 1990).

Nonthermal radio emission.—For systems with periods ≳ 1 yr, we expect detectable nonthermal radio emission from the colliding winds (Dougherty & Williams 2000; Monnier et al. 2002b). This requires sensitive measurements at multiple frequencies but should be possible and would yield unambiguous results. Recent successful attempts to use this method include those of Leitherer et al. (1997), Chapman et al. (1999), Monnier et al. (2002b), and Cappa et al. (2004).

While skeptics will not be convinced yet that the WC phenomena has a necessary connection to binarity, the evidence in favor of this scenario continues to accumulate. The proposed observational efforts to conclusively establish binarity complement ongoing theoretical studies to explore the far-reaching consequences of the role of binarity in the theory of massive star evolution. Indeed, a complete understanding of gamma-ray burst progenitors, the Galactic black hole population, and the high-mass end of the initial mass function hinges on an accurate picture of massive binary stellar evolution.

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