Bulk and brane radiative effects in gauge theories on orbifolds *

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Abstract

We have computed one-loop bulk and brane mass renormalization effects in a five-dimensional gauge theory compactified on the $\mathcal{M}_4 \times S^1/\mathbb{Z}_2$ orbifold, where an arbitrary gauge group $\mathcal{G}$ is broken by the orbifold action to its subgroup $\mathcal{H}$. The space-time components of the gauge boson zero modes along the $\mathcal{H}$ generators span the gauge theory on the orbifold fixed point branes while the zero modes of the higher-dimensional components of the gauge bosons along the $\mathcal{G}/\mathcal{H}$ generators play the role of Higgs fields with respect to the gauge group $\mathcal{H}$. No quadratic divergences in the mass renormalization of the gauge and Higgs fields are found either in the bulk or on the branes. All brane effects for the Higgs field masses vanish (only wave function renormalization effects survive) while bulk effects are finite and can trigger, depending on the fermionic content of the theory, spontaneous Hosotani breaking of the brane gauge group $\mathcal{H}$. For the gauge fields we do find logarithmic divergences corresponding to mass renormalization of their heavy Kaluza-Klein modes. Two-loop brane effects for Higgs field masses are expected from wave function renormalization brane effects inserted into finite bulk mass corrections.

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1 INTRODUCTION

Extra dimensions (with respect to the four space-time dimensions) are a common ingredient in all fundamental theories aiming to unify gravity with the rest of the known interactions. However, unlike gravitational interactions that propagate in the bulk of the higher dimensional space (with ten/eleven dimensions in string/M theory), gauge interactions can propagate on a (4+d)-dimensional (d ≥ 1) slice of space-time, e.g. in the worldvolume of a D-brane in type I/I’ strings [1]. Moreover, it has been shown that in these theories the radius R of the compact dimensions where the gauge interactions propagate can be large enough [2] for the corresponding excitations to be at the reach of future accelerators [3][11], while the string (or higher dimensional quantum gravity) scale Ms can be lowered to the TeV range [12] and show up in colliders [13,14] and gravitational experiments [15]. This fact opened up for the first time exciting possible experimental accessibility to fundamental theories and provided new insight into long-standing problems of particle physics such as the hierarchy problem.

The hierarchy problem of the Standard Model i.e. the appearance of quadratic divergences in the quantum corrections to the Higgs mass is one of the most outstanding problems in particle physics. It has motivated, as the prototype perturbative solution, the introduction of supersymmetry – a symmetry responsible for the absence or cancellation of quadratic divergences – that is being looked for extensively in experimental searches. However, in view of the elusiveness of supersymmetry to show up in direct searches and the robustness of the Standard Model predictions that is pushing up the scale of supersymmetry breaking, it is interesting to explore new avenues and possible alternative solutions to the hierarchy problem. The existence of TeV extra dimensions where the Standard Model fields propagate provides new and useful tools for this search.

Large extra dimensions have shown to shed new light on the hierarchy problem. In particular, in higher-dimensional supersymmetric theories it has been proved that one-loop radiative corrections to the Higgs mass are finite (ultraviolet insensitive) and ∼ 1/R [16–25]. Of course, since the theory is non-renormalizable, higher-loop effects introduce through wave function renormalization a certain ultraviolet sensitivity to the Higgs mass that can be absorbed by the renormalization group running of the coupling constants and hence it does not make this sensitivity explicit at low energy [26]. In this sense, the higher dimensionality of the theory allows to improve the solution to the hierarchy problem with respect to four dimensional supersymmetry.

However, in the presence of large compact dimensions supersymmetry is not as necessary an ingredient as it is in four dimensions. In fact, non-supersymmetric solutions to the hierarchy problem based on toroidal compactifications were already explored in the literature [27,28]. In those cases the Standard Model Higgs field should be identified with an extra dimensional component of a gauge field and electroweak symmetry breaking proceeds by the Hosotani mechanism [29,30]. The higher dimensional gauge invariance protects the Higgs mass from quadratic divergences at the quantum level and radiative corrections to the Higgs mass are finite and ∼ 1/R. In short, in these non-supersymmetric models the role of supersymmetry preventing quadratic divergences is played by higher dimensional gauge invariance.

A word of caution should be said here about the proposed (perturbative) solutions
to the hierarchy problem. All of them are based upon introducing a symmetry at an intermediate scale $M_0$ between the electroweak scale $M_{\text{weak}}$ and the Standard Model cutoff (quantum gravity or string scale $M_s$) such that quadratic divergences are canceled at scales $\mu > M_0$. In this way quadratic divergences survive only for scales smaller than $M_0$ and radiative corrections to the Higgs mass are $\sim M_0$. For the case of four dimensional supersymmetry, $M_0 = M_{\text{SUSY}}$, and at scales $\mu > M_{\text{SUSY}}$ the Higgs mass is protected by supersymmetry. For the case of a higher-dimensional non-supersymmetric theory with electroweak Hosotani breaking, $M_0 = 1/R$. For scales $\mu < 1/R$ the theory is four dimensional and it is not protected from quadratic divergences, while for $\mu > 1/R$ the theory is higher-dimensional and the gauge invariance protects the Higgs mass from quadratic divergences. In both cases, for the mechanism to be effective, the scale $M_0$ has to be stabilized and should be not much higher than the electroweak scale. This comment applies to both the scale of supersymmetry breaking $M_{\text{SUSY}}$ and to the compactification scale $1/R$. In particular, fixing the radius $R$ implies considering the gravitational sector of the theory involving the radion field. This problem is outside the scope of the present paper and we will assume that the radius has been fixed and stabilized by some mechanism \[31\].

In the construction of higher dimensional theories the nature of the compact space plays a prominent role in physics. In particular, theories with more than four flat dimensions or higher dimensional theories compactified on tori are non-chiral from the four dimensional point of view. The simplest solution to this problem \[3\] is compactification on tori modded out by a discrete symmetry group acting non-freely (with fixed points) on the compact space, or orbifold compactifications \[33, 34\]. Orbifolds are not smooth manifolds but have singularities at the fixed points which are four-dimensional hypersurfaces or boundaries of the higher-dimensional space. Those boundaries will be (and are) often named “branes” by an abuse of language. At the field theory level brane contributions arise in the higher-dimensional Lagrangian by means of Dirac delta functions. Non-supersymmetric models on orbifold compactifications were already proposed in \[35, 36\].

It has been proved that under radiative corrections a theory with no brane couplings will generally flow to one with non-trivial physics on the brane \[37–40\]. In particular, wave function renormalization effects localized on the brane have been found. We could understand the appearance of those renormalization effects since they are consistent with the four dimensional nature and symmetries of the branes. In the case at hand, we perform electroweak breaking by the Hosotani mechanism in a higher-dimensional gauge theory broken by the orbifold action to the Standard Model gauge theory on the brane. The Higgs is a scalar field from the point of view of the brane and its mass is not a priori protected by the higher-dimensional gauge invariance from acquiring quadratic divergences localized on the brane. Possible mass terms on the branes that are not protected by the residual gauge invariance are not obviously protected from the higher dimensional symmetries either and will be investigated in detail in this paper. Their absence should be essential for any phenomenological applications aiming to solve the hierarchy problem in the absence of supersymmetry.

The plan of this paper is as follows. In section 2 we will consider a five dimensional

\[1\] There are other solutions that have been proposed involving smooth manifolds with non-trivial backgrounds \[32\].
(5D) theory compactified on the orbifold $\mathcal{M}_4 \times S^1/Z_2$ where an arbitrary gauge group $\mathcal{G}$ is broken to the subgroup $\mathcal{H}$ by the orbifold action. We also consider fermions in an arbitrary representation $\mathbf{R}$ of the gauge group and the associated, consistent with the orbifold, $Z_2$ parity action. We fix the gauge consistently with the properties of the 5D theory and introduce the corresponding Faddeev-Popov ghosts with their transformation properties under the orbifold action. The general Feynman rules for fields propagating in the bulk of the orbifold are exhibited explicitly as well as some useful group theoretical formulas that will be used in the rest of the paper. A general discussion of the allowed orbifold gauge breaking patterns and associated consistent fermion representations is postponed to appendix A while the gauge fixing conditions and the unitary gauge are discussed in appendix B. In section 3 we discuss the general structure of corrections generated in the bulk and on the orbifold fixed planes by radiative corrections in the bulk. We will restrict ourselves to radiative corrections to mass terms, i.e. corrections generated by diagrams with vanishing external four-momentum: $p_\mu = 0$. In section 4 the one-loop radiative corrections to all field masses are computed in the theory where an arbitrary gauge group $\mathcal{G}$ is broken to the subgroup $\mathcal{H}$ by the orbifold action with the fermions in an arbitrary representations $\mathbf{R}$ of the gauge group. We have considered separately gauge and fermion sectors and bulk and brane effects. We have found no quadratic divergences either in the bulk or on the branes. While this effect in the bulk is justified from the higher dimensional gauge invariance, its interpretation on the brane for the extra-dimensional components of gauge fields is less clear, although it might be related to the higher-dimensional Lorentz and gauge symmetries. On the other hand, squared mass terms are generated for the extra-dimensional components of the gauge fields opening up the possibility of spontaneous breaking of the residual $\mathcal{H}$ gauge symmetry on the branes. In particular, the contribution from the gauge (fermion) sector to the squared mass terms is positive (negative). In section 5 the conditions for Hosotani breaking are discussed and shown to depend on the group-theoretical invariants of the gauge group and fermion representations. Also the possibility of reducing the rank of $\mathcal{H}$ by the Hosotani breaking is briefly discussed. This possibility is essential in model building where one could identify $\mathcal{H}$ with the Standard Model gauge group. Finally some comments about two-loop corrections and our conclusions are drawn in section 6.

2 Broken gauge symmetry on the $\mathbb{Z}_2$ orbifold

The model we will consider is a gauge theory coupled to matter in five flat space-time dimensions with coordinates $x^M = (x^\mu, x^5)$ and metric signature $(+,−,−,−,−)$. The 5D geometry is $\mathcal{M}_4 \times S^1/Z_2$, i.e. the fifth dimension is compactified on a $Z_2$ orbifold, whereas the remaining part is four dimensional Minkowski space with metric $\eta_{\mu\nu}$. We have neglected gravity so the fixed planes having no tension are rigid geometrical boundaries. Keeping this remark in mind we will refer to these planes as ”branes”. We denote the radius of the compact circle by $R$. The gauge group in the bulk is $\mathcal{G}$ (we denote $\dim(\mathcal{G}) \equiv d_\mathcal{G}$) and it is broken to $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots$ (we denote $\dim(\mathcal{H}) \equiv d_\mathcal{H}$) on the fixed hyperplanes by our choice of the orbifold projection. For matter, we couple Dirac fermions $\Psi_R$ that transform in the representation $\mathbf{R}$ (we denote $\dim(\mathbf{R}) \equiv d_\mathbf{R}$) of $\mathcal{G}$ to the gauge
We will use capital letters from the beginning of the Latin alphabet to denote gauge indices \((A, B, C, \cdots)\), capital letters from the middle of the Latin alphabet to denote five dimensional Lorentz indices \((M, N, R, \cdots)\), small letters from the middle of the Greek alphabet to denote four dimensional Lorentz indices \((\mu, \nu, \rho, \cdots)\), and small letters from the middle of the Latin alphabet to denote the discrete fifth dimensional momentum \((k, l, m, \cdots)\).

Our starting point is the action
\[
S_5 = \int d^5x \text{Tr} \left\{ -\frac{1}{2} F_{MN} F^{MN} + i \bar{\Psi}_R \gamma^M D_M \Psi_R \right\},
\]
where \(F_{MN} = F_{MN}^A T^A_R, F_{MN}^A = \partial_M A_N^A - \partial_N A_M^A + g f^{ABC} A_M^A A_N^B A_C^C\) with the indices \(A, B, C\) running over the adjoint representation of the gauge group and \(f^{ABC}\) the corresponding structure constants. The gauge covariant derivative is \(D_M = \partial_M - igA_M^A T^A_R\), where \(T^A_R\) are matrices corresponding to the representation \(R\) of the gauge group satisfying
\[
[T^A_R, T^B_R] = if^{ABC} T^C_R.
\]

Our parity assignment is defined by
\[
A^A_M(x^\mu, -x^5) = \alpha^M \Lambda^{AB} A^B_M(x^\mu, x^5) \quad \text{(no sum over } M) \tag{2.3}
\]
\[
\Psi_R(x^\mu, -x^5) = \lambda_R \otimes (i\gamma^5) \Psi_R(x^\mu, x^5), \tag{2.4}
\]
where \(\Lambda\) and \(\lambda_R \otimes (i\gamma^5)\) represent the \(\mathbb{Z}_2\) action on the gauge bosons and the fermions respectively \((\lambda_R\) acts on the representation indices), \(\gamma^5 = \text{diag}(-i, i)\) and \(\alpha^\mu = +1, \alpha^5 = -1\). In addition, \(\lambda_R\) is a hermitian matrix that squares to one and therefore unitary. Consistency of the 5D gauge symmetry with the orbifold action requires the condition
\[
f^{ABC} = \Lambda^{AA'} \Lambda^{BB'} \Lambda^{CC'} f^{A'B'C'}, \tag{2.5}
\]
where summation over repeated indices is understood. The above constraint comes from the requirement that under the \(\mathbb{Z}_2\) action \(F_{MN}^A \rightarrow \alpha^M \Lambda^{AB} F_{MN}^B\) (no sum over \(M\)), so that \(F_{MN}^A F^{A'MN}\) is invariant and it is straightforward to check that it is an automorphism of the Lie algebra of \(G\).

On the other hand, the invariance of the fermion kinetic term requires the transformation
\[
\mathcal{P}_\Psi \gamma^M \mathcal{P}_\Psi = \pm \alpha^M \gamma^M, \quad \text{(no sum)}, \tag{2.6}
\]
while the invariance of the fermion-gauge boson coupling implies in addition that
\[
\mathcal{P}_\Psi T^A_R \mathcal{P}_\Psi = \Lambda^{AB} T^B_R. \tag{2.7}
\]
In five dimensions the only solution to these equations is \(\mathcal{P}_\Psi = \lambda_R \otimes (i\gamma_5)\) (with \(\lambda_R\) satisfying Eq. \((2.7)\)), which corresponds to the lower sign in Eq. \((2.6)\). With no loss of

\(^2\text{There should be no confusion between the representation } R \text{ and the radius } R.\)

\(^3\text{When the subscript } R \text{ is omitted it is implied that the matrices are in the fundamental representation.}\)
generality we can diagonalize \( \Lambda^{AA'} = \eta^A \delta^{AA'} \) with \( \eta^A = \pm 1 \), consequently Eq. (2.5) takes the simpler form

\[
 f^{ABC} = \eta^A \eta^B \eta^C f^{ABC}, \quad \text{(no sum).} \tag{2.8}
\]

We will then often express \( \text{(2.5)} \) succinctly as

\[
 A^A_M(x^\mu, -x^5) = \alpha^M \eta^A A^A_M(x^\mu, x^5), \quad \text{(no sum).} \tag{2.9}
\]

A few remarks are in order. From (2.8) we can see that since the orbifold can break \( G \) completely only if \( G = U(1) \), the unbroken subgroup is always (except in the \( U(1) \) case) a non trivial subgroup of \( G \). Under the \( \mathbb{Z}_2 \) action we can naturally split the adjoint index \( A \) into an unbroken part \( a \) and a broken part \( \hat{a} \) so that the generators of the subgroup \( H \) are \( T^a_R \) and the generators of the coset \( K = G/H \) (we denote \( \dim(K) \equiv d_K \) are \( T^a_R \).

Notice that constraint (2.8) simply means that the matrix \( \Lambda \) is a diagonal matrix with \( d_H \) elements equal to +1 and with the rest of the \( d_K \) elements, corresponding to the broken part of \( G \), equal to −1. According then to Eq. (2.9), only \( A^A_\mu \) with \( \eta^A = +1 \) and \( A^A_5 \) with \( \eta^A = -1 \) acquire zero modes and they are non-vanishing at the fixed planes. The former appear as the (massless) four dimensional gauge fields that correspond to \( H \) and the latter as massless scalar fields in four dimensions. Also (2.4) results in a non trivial constraint on the possible bulk fermion representation choices. Let us assume for now that we have made a consistent choice for \( \lambda_R \). Then, the fermions that will appear massless in four dimensions will be chiral because of the \( i\gamma^5 \) appearing in (2.4) and they will transform in some representation \( R = \bigoplus_i r_i \) of \( H = H_1 \otimes H_2 \otimes \ldots \). Another important constraint is that the resulting massless chiral spectrum on the fixed hyperplane should be anomaly free. This puts further restrictions on realistic model building. There are several different ways to arrive at an anomaly free model but this is not the subject of the present work. In appendix A we work out a few simple examples to illustrate issues related to the \( \mathbb{Z}_2 \) action on the fermions.

We will now use the formalism of \cite{37,38}, i.e. we will work with exponential modes for the fields. The modes for any field \( \phi \) are related by

\[
 \phi^{-m} = P_\phi \phi^m, \tag{2.10}
\]

where \( P_\phi \) is the parity operator of the field \( \phi \). For fermions it is \( P_\psi = \lambda_R \otimes (i\gamma^5) \) as in (2.4), whereas for the gauge bosons it is \( P_A = \alpha^M \delta^M_M' \Lambda^{AB} \) with eigenvalues \( \alpha^M \eta^A \) as in (2.9).

Eq. (2.10) is automatically satisfied if the fields are expressed by unconstrained ones:

\[
 \phi^m = \frac{1}{2} (\varphi^m + P_\phi \varphi^{-m}). \tag{2.11}
\]

The gauge propagator can therefore be written as

\[
 \langle A^m A^{m'} \rangle = \frac{1}{2} \left( G^{(A)}(p_\mu, p_5) + P_A G^{(A)}(p_\mu, -p_5) P_A \right) \delta_{m-m'} + \frac{1}{2} \left( P_A G^{(A)}(p_\mu, -p_5) + G^{(A)}(p_\mu, p_5) P_A \right) \delta_{m+m'}. \tag{2.12}
\]

\(^4\)In particular, for models where the bulk gauge symmetry coincides with the gauged superalgebra of a locally supersymmetric theory, this means that an extended bulk supersymmetry (\( N > 1 \)) cannot be completely broken on the fixed planes by the \( \mathbb{Z}_2 \).
In the above, we have denoted by $G^{(A)}(p_\mu, p_5)$ the 5D propagator that corresponds to the compactification on $S^1$. $p_\mu$ is the momentum along $\mathcal{M}_4$, $p_5 = m/R$ is the momentum in the compact direction and we have used the notation $\delta_k \equiv \delta_{k,0}$. Using the covariance of $G^{(A)}(p_\mu, p_5)$ under parity transformations (see (2.19))

$$\mathcal{P}_A G^{(A)}(p_\mu, -p_5) \mathcal{P}_A = G^{(A)}(p_\mu, p_5) \tag{2.13}$$

Eq. (2.12) can be simplified to

$$\langle A^m A'^{m'} \rangle = \frac{1}{2} G^{(A)}(p_\mu, p_5) (\delta_{m-m'} + \mathcal{P}_A \delta_{m+m'}) \tag{2.14}$$

The fermionic propagator can be computed similarly taking into account that $\mathcal{P}_\psi \gamma^M \mathcal{P}_\psi = -\alpha^M \gamma^M$ (no sum), which in turn implies the transformation (see (2.21))

$$\mathcal{P}_\psi G^{(\psi)}(p_\mu, -p_5) \mathcal{P}_\psi = -G^{(\psi)}(p_\mu, p_5) \tag{2.15}$$

Then the simplified fermion propagator reads

$$\langle \Psi^m \bar{\Psi}^{m'} \rangle = \frac{1}{2} G^{(\psi)}(p_\mu, p_5) (\delta_{m-m'} - \mathcal{P}_\psi \delta_{m+m'}) \tag{2.16}$$

The vertices then conserve 5D momentum and are the ones of the unorbifolded 5D theory. All the information about the non-trivial $\mathbb{Z}_2$ action is encoded in the propagators in a particularly simple way.

The next issue is gauge fixing and ghosts. We will work in the 5D covariant gauge $\partial_M A^M = 0$. The modified Lagrangian then including the gauge fixing term and the ghost fields $c^A$ is in the standard way

$$\mathcal{L}_5 \rightarrow \mathcal{L}_5 - \frac{1}{2\xi} \partial^M A^B_M \partial^N A^B_N + \text{Tr} \partial^M \bar{c} D_M c. \tag{2.17}$$

By looking at the ghost-gauge field interaction term above, we can see that the ghost $c^B$ has the same parity as the gauge field with the same index $B$, i.e.

$$c^A(x^\mu, -x^5) = \Lambda^{AB} c^B(x^\mu, x^5) \tag{2.18}$$

The propagators and vertices can then be taken over from any standard textbook with the indices properly generalized to five dimensions. For completeness we give their explicit forms in 5D Minkowski space-time [42]:

$$G^{(A)}(p_\mu, p_5) = \begin{array}{c} B, M \end{array} \begin{array}{c} C, N \end{array} \frac{\delta^{BC}}{p^2 - p_5^2} \left( g_{MN} - (1 - \xi) \frac{p_M p_N}{p^2 - p_5^2} \right) \tag{2.19}$$

$$G^{(c)}(p_\mu, p_5) = \begin{array}{c} B, M \end{array} \begin{array}{c} C, N \end{array} \frac{\delta^{BC}}{p^2 - p_5^2} \tag{2.20}$$
$$G^{(\Psi)}(p_\mu; p_5) = \frac{a}{(p, p_5)} \frac{b}{(p, p_5)} = i \frac{\delta^a_b}{\gamma^\mu p_\mu + \gamma^5 p_5}$$

(2.21)

Finally, in our loop computations, we will make use of the following formulas:

$$f^A_{BC} f^{A'B'C'} = C_2(\mathcal{G}) \delta^{AA'}$$

(2.26)

$$f^A_{BC} f^{A'B'C'} \eta^{C} = \left(C_2(\mathcal{H}_A) - \frac{1}{2} C_2(\mathcal{G})\right) (\eta^A + 1) \delta^{AA'}$$

(2.27)

$$f^A_{BC} f^{A'B'C'} \eta^{B} = C_2(\mathcal{G}) \eta^A \delta^{AA'}$$

(2.28)

$$\text{tr} \left(T^A_R T^{A'}_R\right) = C_R \delta^{AA'}$$

(2.29)

$$\text{tr} \left(T^A_R \lambda_R T^{A'}_R \lambda_R\right) = C_R \eta^A \delta^{AA'}$$

(2.30)

where $C_2(\mathcal{G})$ is the quadratic Casimir of $\mathcal{G}$ and $C_R$ is the Dynkin index of the representation $R$ satisfying $d_R C_2(R) = C_R d_\mathcal{G}$. We normalize the fundamental representation to have its Dynkin index equal to 1/2. In the second identity we have called $\mathcal{H}_A$ the unbroken
subgroup which $T^A_R$ belongs to. Note that this makes sense since the whole expression vanishes if $T^A_R$ is a broken generator $T^A_R = T^\hat{a}_R$, (i.e. when $\eta^A = -1$).

We will find it convenient to work in the Feynman gauge, $\xi = 1$. For any other value of $\xi$, including the 5D Landau gauge $\xi = 0$, there is a tree-level mixing $\langle A^A_5 A^B_5 \rangle$ as can be seen from Eq. (2.19). Furthermore in the class of gauges defined by $\partial_M A^M = 0$ there is no value of $\xi$ for which $\langle A^A_5, n A^B_5 \rangle = 0, n \neq 0$, which would correspond to the unitary gauge since the fields $A^A_5 (n \neq 0)$ are the Goldstone bosons corresponding to the orbifold gauge breaking. To reach the unitary gauge the gauge fixing condition has to be defined in a 5D non-covariant fashion, just consistent with the compactified theory. This issue was discussed in Ref. [43] for an abelian gauge theory. It is discussed in appendix B for a general non-abelian group broken to a subgroup by the orbifold compactification.

3 THE STRUCTURE OF LOOP CORRECTIONS

In this section we will analyze what kind of terms in the effective action could be generated in the bulk and on the orbifold fixed planes by radiative corrections in the bulk. We will focus on the bilinear terms in the effective action and also look only at terms not involving derivatives with respect to infinite directions, $\partial^\mu$. The most general terms are then:

$$\Gamma[A] = \int d^5 x \left\{ A^B_M \Pi^{BC}_{MN} [-\partial^2_5] A^C_N + \frac{1}{2} \left( \delta(x_5) + \delta(x_5 - \pi R) \right) A^B_M \Pi^{BC}_{MN} [i \partial_5, i \partial_5] A^C_N \right\}. \tag{3.1}$$

The arrows on the $\partial_5$ indicate the field whose derivative is to be taken. The bulk and brane terms in Eq. (3.1) can be read off from the generic two-point function

$$\langle A^B_M A^C_N \rangle = \sum_{m,m'} A^B_M A^C_N \Pi^{BC}_{MN} (m^2) \delta_{m-m'} \Pi^{-BC}_{MN} (m^2) \delta_{m+m'}. \tag{3.2}$$

in the following way: The contributions to Eq. (3.2) for which the five momentum in the outgoing line is conserved, $m' = \pm m$, will give rise to bulk terms. We will denote them as

$$\langle A^B_M A^C_N \rangle_{\text{bulk}} = \Pi^{+BC}_{MN} (m^2) \delta_{m-m'} + \Pi^{-BC}_{MN} (m^2) \delta_{m+m'}. \tag{3.3}$$

Contracting this expression with the external fields one finds

$$\sum_{m,m'} A^B_M A^C_N \Pi^{+BC}_{MN} (m^2) \delta_{m-m'} + \Pi^{-BC}_{MN} (m^2) \delta_{m+m'} \tag{3.4}$$

We now take the Fourier transform of Eq. (3.1), sum over $m'$:

$$\int dx_5 A^B_M \Pi^{BC}_{MN} [-\partial^2_5] A^C_N = \sum_{m,m'} A^B_M \Pi^{BC}_{MN} (m^2) \delta_{m-m'} A^C_N \tag{3.5}$$

5Since in the subspace of unbroken generators $\Lambda$ is just the identity one can find a basis in which the groups $\mathcal{H}_i$ are generated by some $\{T^A_i\}$. 

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and read off the correspondence:

\[ \Pi_{MN}^{BC}(m^2) = \Pi_{MN}^{+BC}(m^2) + \alpha^N \eta^C \Pi_{MN}^{-BC}(m^2). \]  

(3.6)

In the second term after summing over \( m' \) there appears a factor of \( A_N^{C,-m} \) which has to be transformed into \( A_N^{C,+m} \) at the cost of an \( \alpha^M \eta^C \), according to (2.10). By doing a Taylor expansion around \( m = 0 \) we obtain the possible bilinear operators in the bulk:

\[ A^m \Pi(m^2) A^m = A^m \Pi(0) A^m + \frac{1}{2} A^m \Pi(2) m^2 A^m + \frac{1}{24} A^m \Pi(4) m^4 A^m + \ldots. \]  

(3.7)

The summation over \( m \) and the gauge and Lorentz indices are left implicit and \( \Pi^{(n)} \) is the \( n \)th derivative with respect to \( m \) evaluated at \( m = 0 \). The bulk mass term is just given by \( \Pi(0) \) (= \( \Pi(0) \)).

The terms in Eq. (3.1) proportional to \( \delta \)-functions are generated by contributions in Eq. (3.2) which do not conserve five-momentum. Instead we will find expressions of the form

\[ \langle A_M^{B,m} A_N^{C,m'} \rangle_{brane} = \sum_l \left\{ \tilde{\Pi}_{MN}^{+BC}(m, l) \delta_m - m' - 2l + \tilde{\Pi}_{MN}^{-BC}(m, l) \delta_m + m' - 2l \right\}. \]  

(3.8)

Note that the sum over \( l \) is now constrained through the Kronecker-\( \delta \), yielding an amplitude which depends on the two independent variables \( m \) and \( m' \). To identify \( \tilde{\Pi}_{MN}^{BC}(m, m') \) we contract this matrix with \( A_M^{B,m} \) and \( A_N^{C,m'} \) and perform the sum over \( l \):

\[ \sum_{m, m'; m \pm m' even} A_M^{B,m} \left\{ \tilde{\Pi}_{MN}^{+BC}(m, \frac{m - m'}{2}) + \tilde{\Pi}_{MN}^{-BC}(m, \frac{m + m'}{2}) \right\} A_N^{C,m'}. \]  

(3.9)

To compare this with Eq. (3.1) one takes the Fourier transform of the latter with respect to the compact dimension:

\[
\int dx_5 \frac{1}{2} (\delta(x_5) + \delta(x_5 - \pi R)) A_M^{B} \tilde{\Pi}_{MN}^{BC} \left[ i \partial_5, i \partial_5 \right] A_N^{C} \\
= \sum_{m, m', k} \int dx_5 \frac{1}{2} e^{i(m+m'+k)x_5} (1 + e^{i k \pi}) A_M^{B,m} \tilde{\Pi}_{MN}^{BC}(m, m') A_N^{C,m'} \\
= \sum_{m, m'; m \pm m' even} A_M^{B,m} \tilde{\Pi}_{MN}^{BC}(m, m') A_N^{C,m'}. 
\]  

(3.10)

We conclude that

\[ \tilde{\Pi}_{MN}^{BC}(m, m') = \tilde{\Pi}_{MN}^{+BC}(m, \frac{m - m'}{2}) + \tilde{\Pi}_{MN}^{-BC}(m, \frac{m + m'}{2}). \]  

(3.11)

To understand the brane terms a bit better, one can make a Taylor expansion of (3.9) around \( m, m' = 0 \). One gets for even \((E)\) and odd \((O)\) fields respectively:

\[ E^m \tilde{\Pi}(m, m') E^{m'} = E^m \tilde{\Pi}^{(0, 0)}(0, 0) E^{m'} + \frac{1}{2} (m^2 E^m) \tilde{\Pi}^{(2, 0)}(m, 0) E^{m'} + \frac{1}{2} E^m \tilde{\Pi}^{(0, 2)}(0, m) E^{m'} + \ldots, \]  

(3.12)

\[ O^m \tilde{\Pi}(m, m') O^{m'} = (m O^m) \tilde{\Pi}^{(1, 1)}(m, m') + \frac{1}{6} (m^3 O^m) \tilde{\Pi}^{(1, 3)}(m, m') + \ldots, \]
where we have suppressed the gauge and Lorentz indices for clarity and the (independent) summations over \( m, m' \) are implicit. In particular it is now clear that the brane mass term for even fields is just \( \tilde{\Pi}^{(0,0)} (= \Pi(0,0)) \), given by setting \( m = m' = 0 \) in the momentum-violating terms of Eq. (3.2). In contrast to Eq. (3.7) the expansion coefficients are now not diagonal but democratic matrices in mode space.

Bulk terms are not expected to generate any divergences that were not present in the original 5D theory. Brane terms, however, could generate new divergences. In particular, we want to investigate the possible appearance of a scalar mass on the brane. Had we introduced fundamental massless scalars in the 5D theory, their masses would pick up corrections proportional to the cutoff of the five dimensional theory and the same would happen with their zero modes on the branes (provided they survive the orbifold projection). In our model there are no scalars in the five dimensional theory. The only bosonic fields are the components of the bulk gauge fields whose masses are zero at tree level and remain zero at all orders because they are protected by gauge invariance. On the branes, however, there are scalars. Some of them are massless at tree level, namely the zero modes of positive \( \mathbb{Z}_2 \) parity fields. The natural question that arises then is whether these masses are protected against radiative corrections. We know that on general grounds this is not the case and in order to avoid the Higgs mass to pick up corrections proportional to the cutoff, it has to be protected by some symmetry, for example supersymmetry. Our model is not supersymmetric and therefore such a mechanism is not possible. On the other hand since the scalars are extra dimensional components of the original gauge field, one would hope that gauge invariance still protects them. The problem is that it is on the branes where the zero modes of positive \( \mathbb{Z}_2 \) parity fields are seen as massless scalar fields and we have also seen that it is on the branes that the only surviving symmetry is \( \mathcal{H} \); there is no apparent symmetry that prohibits dangerous corrections to their masses. In the following we will see in a one-loop calculation that even though there is no such apparent symmetry on the branes, the larger, original bulk gauge symmetry arranges the couplings of the zero modes and the Kaluza-Klein (KK) towers in such a way that the masses of the zero modes remain protected.

4 One Loop Corrections

The one-loop corrections in the effective action coming from the exchange of 5D gauge fields are given in Figs. 1 and 2. We first discuss the general structure of the brane and bulk terms. As a first step, we will not compute the exact values of the diagrams. It is enough to observe their general structure to deduce whether and when there appear bulk
or brane terms. Then, once we have separated bulk from brane terms, we carry out the actual calculation for each case separately and interpret the results.

Since we are mainly interested in mass corrections we will explicitly evaluate the one-loop graphs only for vanishing external four-momentum ($p^\mu = 0$). The amplitudes corresponding to each graph will be denoted as in Eqs. (3.3) and (3.8). In addition, we add a superscript ($i$), i.e.

$$\Pi_{MM'}^{(i)+AA'}(m^2), \quad \tilde{\Pi}_{MM'}^{(i)+AA'}(m, l),$$

where $i = 1$ for the tadpole, $i = 2$ for the gauge loop, $i = 3$ for the ghost loop and $i = 4$ for the fermion loop. The other indices are as described in the beginning of section 2. All computations in this section will be done in Euclidean space [44].

### 4.1 The Gauge Sector

In the sector where 5D gauge bosons are exchanged as internal lines there are the three different Feynman diagrams shown in Fig. 1. All our computations will be carried out in the $\xi = 1$ (Feynman–’t Hooft) gauge and dimensional regularization (with renormalization scale $\mu$) will be used to handle divergent diagrams. Let us examine each case in turn.

#### 4.1.1 The Tadpole

The tadpole is the first graph appearing in Fig. 1. It is proportional to

$$\delta_{m-l'+m'}(\delta_{l-l'} + \alpha^M \eta^B \delta_{l+l'})\delta_{BB'} f^{ABC} f^{A'B'C},$$

which, using the identities $(2.26)$ – $(2.30)$, can be written as

$$C_2(G) \delta_{m-m'} \delta^{AA'} + \alpha^M \left( C_2(H_A) - \frac{1}{2} C_2(G) \right) (\eta^A + 1) \delta_{m-m'} - 2 \delta^{AA'}.$$  

Notice that there is no term proportional to $\delta_{m+m'}$ and so no contributions to the two-point function $\Pi_{MM'}^{(1)+AA'}$ will appear. The first term in the above is five momentum conserving and thus gives only a bulk term. Its dependence on the gauge-index $A$ is trivial and the contribution is the same for all $A_M^A$. The second term is momentum non-conserving and gives rise to a brane term. Since it contains the factor $(\eta^A + 1)$ it is zero if $\eta^A = -1$. Thus there are only brane terms if the external lines correspond to a field $A_M^a$ where $a$ is an index of the unbroken group $H$.

Applying the Feynman rules, in the bulk we obtain for either $\eta^A = \pm 1$

$$\Pi_{MM'}^{(1)+AA'} = -\frac{d}{2} g^2 \delta_{MM'} \delta^{AA'} C_2(G) \mu^{4-d} \int \frac{d^d q}{(2\pi)^d} \sum_{l=-\infty}^{+\infty} \frac{1}{q^2 + \frac{l^2}{R^2}}.$$  

Here we have defined $d = \delta_{\mu\nu}$. This expression is obviously the same along $M_4$ or $S^1/\mathbb{Z}_2$.  

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The contribution of the tadpole to brane localized terms is nonzero only for \( \eta^A = +1 \) and is equal to

\[
\tilde{\Pi}_{MM'}^{(1) + AA'}(m, l) = \left( \frac{d - 1 - \alpha M}{4} \right) g^2 \delta_{MM'} \delta^{AA'} 
\]

\[
(\eta^A + 1) \left( 2C_2(\mathcal{H}_A) - C_2(\mathcal{G}) \right) \mu^{4-d} \int \frac{d^dq}{(2\pi)^d} \frac{1}{q^2 + \frac{l^2}{R^2}}. \tag{4.5}
\]

Observe that there is no sum over the loop fifth momentum.

**4.1.2 The gauge and the ghost loop**

The second diagram in Fig. 1 is proportional to

\[
\delta_{m-l-k} \delta_{l-k'} \delta_{l+l'} \delta_{N N'} \delta_{R R'} \delta_{AA'} \delta_{NN'} \delta_{RR'}(m, l) = - (d-1-\alpha M) \frac{2C_2(\mathcal{H}_A) - C_2(\mathcal{G})}{(2\pi)^d} \delta_{MM'} \delta_{AA'} \delta_{NN'} \delta_{RR'} \tag{4.6}
\]

which gives rise to the four terms

\[
C_2(\mathcal{G}) \delta_{m-m'} \delta^{AA'} \delta_{NN'} \delta_{RR'}, \tag{4.7}
\]

\[
\alpha^N \alpha^R \eta^A C_2(\mathcal{G}) \delta_{m-m'} \delta^{AA'} \delta_{NN'} \delta_{RR'}, \tag{4.8}
\]

\[
\alpha^R (\eta^A + 1) \left( C_2(\mathcal{H}_A) - \frac{1}{2} C_2(\mathcal{G}) \right) \delta_{m-m'} \delta^{AA'} \delta_{NN'} \delta_{RR'}, \tag{4.9}
\]

\[
\alpha^N (\eta^A + 1) \left( C_2(\mathcal{H}_A) - \frac{1}{2} C_2(\mathcal{G}) \right) \delta_{m-m'} \delta^{AA'} \delta_{NN'} \delta_{RR'}. \tag{4.10}
\]

As in the case of the tadpole, there are bulk terms for both \( \eta^A = \pm 1 \). However, once again, one finds brane terms only for \( \eta^A = +1 \).

The structure for the ghost diagram is slightly simpler since the internal propagators do not carry vector indices. The conclusion is unmodified, brane terms are proportional to \( (\eta^A + 1) \).

The diagrams with momentum conserving external lines corresponding to (4.7) and (4.8) and the analogous ghost diagrams give, for \( p_\mu = 0 \),

\[
\Pi_{MM'}^{(i) + AA'}(m^2) = \frac{1}{8} g^2 \delta^{AA'} C_2(\mathcal{G}) \mu^{4-d} \int \frac{d^dq}{(2\pi)^d} \sum_{l=-\infty}^{+\infty} \frac{N_{MM'}^{(i)}}{(q^2 + \frac{l^2}{R^2})(q^2 + (m-l)^2/R^2)}, \tag{4.11}
\]

\[
\Pi_{MM'}^{(i) - AA'}(m^2) = \alpha^{MM'} \eta^A \Pi_{MM'}^{(i) + AA'}, \tag{4.12}
\]

where here of course \( i = 2, 3 \) only. For the gauge loop the numerators are given by

\[
N_{\mu\nu}^{(2)} = \delta_{\mu\nu} \left( \frac{2l^2 + 5m^2 - 2ml}{R^2} + 2(3 - \frac{1}{d})q^2 \right), \tag{4.13}
\]

\[
N_{55}^{(2)} = 2q^2 + \frac{d(m-2l)^2}{R^2}, \tag{4.14}
\]
while the ghost loop gives

\[ N^{(3)}_{MM'} = 2q_M(p - q)_{M'}. \] (4.15)

Evaluated explicitly this becomes

\[ N^{(3)}_{\mu\mu'} = -\frac{2}{d}q^2\delta_{\mu\mu'}, \] (4.16)
\[ N^{(3)}_{55} = \frac{2l(m - l)}{R^2}. \] (4.17)

Obviously all contributions to \( \Pi^{(i)AA'}_{MM'} \) are diagonal, in particular there is no mixing between \( M = \mu \) and \( M = 5 \). In deriving the above one should not forget that the fifth component of the momenta entering in the Feynman rules for the vertices may be flipped due to the \( \delta_{l+l'} \), etc. in the propagators.

Brane terms can be computed as well. We will again perform the calculation for \( p_\mu = 0 \). We find

\[ \tilde{\Pi}^{(i)\pm AA'}_{MM'}(m, l) = \frac{1}{16}g^2\delta^{AA'} \]
\[ (\eta A + 1)(2C_2(H_A) - C_2(\mathcal{G}))\mu^{4-d} \int \frac{d^dq}{(2\pi)^d} \left( \frac{\tilde{N}^{(i)\pm}_{MM'}}{(q^2 + \frac{l^2}{R^2})(q^2 + \frac{(m-l)^2}{R^2})} \right), \] (4.18)

where

\[ \tilde{N}^{(2)\pm}_{\mu\mu'} = -\frac{6l^2 + 3m^2 - 6ml}{R^2} - \frac{2}{d}(5 - 3d)q^2\delta_{\mu\mu'}, \] (4.19)
\[ \tilde{N}^{(2)\pm}_{55} = \pm d\frac{m(m - 2l)}{R^2}, \] (4.20)
\[ \tilde{N}^{(3)\pm}_{\mu\mu'} = -\frac{2}{d}q^2\delta_{\mu\mu'}, \] (4.21)
\[ \tilde{N}^{(3)\pm}_{55} = \pm 2\frac{l(m - l)}{R^2}. \] (4.22)

4.1.3 Bulk effects from the gauge sector

In this section we will compute the bulk effects from the gauge sector obtained in sections 4.1.1 and 4.1.2. It is generally known from finite temperature field theory that by compactifying on a circle no new divergences appear. We can give some reasoning on why this should not happen by looking at a general one-loop amplitude

\[ \int \frac{d^dq}{(2\pi)^d} \frac{1}{R} \sum_{l=-\infty}^{+\infty} g(q_\mu, l/R). \] (4.23)

We can perform a Poisson re-summation

\[ \frac{1}{R} \sum_l g(q_\mu, l/R) = \sum_k \tilde{g}(q_\mu, 2\pi k R), \] (4.24)
where \( \tilde{g} \) is the Fourier transform of \( g \) with respect to \( q_5 = l/R \). This allows us to rewrite the amplitude as
\[
\int \frac{d^d q}{(2\pi)^d} \int \frac{dq_5}{2\pi} g(q_\mu, q_5) + \int \frac{d^d q}{(2\pi)^d} \sum_{k \neq 0} \int \frac{dq_5}{2\pi} e^{i(2\pi k R)q_5} g(q_\mu, q_5) \tag{4.25}
\]

Here we have extracted the term in the sum corresponding to \( k = 0 \) which is just the five dimensional amplitude. The remaining terms summed over give typically an exponentially suppressed function of the four momentum squared. We will see explicit examples below.

To collect the total bulk contribution from the gauge sector, we have to add the different terms according to Eq. (3.6):
\[
\Pi_{MM'}^{AA'}(m^2) = \sum_{i=1}^{3} \left( \Pi_{MM'}^{(i)+AA'}(m^2) + \alpha M' \eta A' \Pi_{MM'}^{(i)-AA'}(m^2) \right) \tag{4.26}
\]

Concentrating first on the scalar sector, we find from Eqs. (4.4), (4.11) and (4.12)
\[
\Pi_{55}^{AA'}(m^2) = \frac{1}{8} g^2 \delta^{AA'} C_2(G) \mu^{d-4} \int \frac{d^d q}{(2\pi)^d} \sum_{l=-\infty}^{+\infty} \left( -\frac{4d}{q^2 + \frac{l^2}{R^2}} + \frac{2q^2 + \frac{d(m-2)^2 + 2(m-l)^2}{R^2}}{(q^2 + \frac{l^2}{R^2})(q^2 + \frac{(m-l)^2}{R^2})} \right) \tag{4.27}
\]

Notice that the signs \( \eta \) and \( \alpha \) in Eq. (4.12) exactly cancel the ones in Eq. (4.26) yielding a global factor of 2 in the second term. Decomposing into partial fractions we obtain
\[
\Pi_{55}^{AA'}(m^2) = \frac{1}{8} g^2 \delta^{AA'} C_2(G) \mu^{d-4} \int \frac{d^d q}{(2\pi)^d} \sum_{l=-\infty}^{+\infty} \left( -\frac{2d}{q^2 + \frac{l^2}{R^2}} + 2d \frac{1}{q^2 + \frac{(m-l)^2}{R^2}} - 4(d-1) \frac{q^2 + \frac{l(m-l)}{R^2}}{(q^2 + \frac{l^2}{R^2})(q^2 + \frac{(m-l)^2}{R^2})} \right) \tag{4.28}
\]

The first observation is that one can shift the summation index \( l \) in the second term by \( m \) so that it cancels against the first term. To interpret the remaining term, let us do the integral first. Naive power counting indicates that a quadratic divergence and a logarithmic divergence will appear in the result. Recall that in dimensional regularization the appearance of quadratic divergences in \( d = 4 \) are signaled by poles in \( d = 2 \) and notice that the usual factor of \( d - 2 \) multiplying the pole that appears in conventional gauge theories is missing. Despite this fact we will now show that all divergences (quadratic and logarithmic) are actually absent. According to Eq. (3.7), we can extract the bulk mass term from Eq. (4.28) by evaluating it at \( m = 0 \):
\[
\Pi_{55}^{AA'}(0) = -\frac{1}{8} g^2 \delta^{AA'} C_2(G) \mu^{d-4} \int \frac{d^d q}{(2\pi)^d} \sum_{l=-\infty}^{+\infty} \left( 4(d-1) \frac{q^2 - \frac{l^2}{R^2}}{(q^2 + \frac{l^2}{R^2})^2} \right) \tag{4.29}
\]

\(^6\)The separation of the amplitude into a five dimensional part and a finite part is similar to the approach of [15].
There seems to be a quadratically divergent piece left over. This is not unexpected at this stage since the four dimensional gauge invariance alone does not protect these scalars from acquiring divergences. However, five-dimensional gauge-invariance does: According to our discussion below Eq. (4.23) one can extract the five dimensional part of the amplitude by substituting the summation by an integration over \( q_5 = l/R \). In this case, however, this integral turns out to be zero and therefore quadratic divergences are absent. The remaining terms of the Poisson re-summation (sum over \( k \neq 0 \)) give \( \sim \sinh^{-2}(\pi R q) \) which is exponentially suppressed for large \( q \) and renders the integration finite. Performing the integration in (4.29) we find

\[
\Pi^{AA'}_{55}(0) = -\frac{9}{32\pi^4 R^2} g^2 \delta^{AA'} C_2(G) \zeta(3). \tag{4.30}
\]

It is a manifestly finite result, as expected, since it is the same result as the one we would have obtained in an \( S^1 \) compactification.

The \( m \)-dependent terms and therefore all terms involving \( \partial_5 \) derivatives vanish, which can be seen by writing Eq. (4.28) for \( m \neq 0 \) as

\[
\frac{q^2 + \frac{l(m-l)}{R^2}}{(q^2 + \frac{l^2}{R^2})(q^2 + \frac{(m-l)^2}{R^2})} = \frac{R}{m} \left( \frac{l}{R^2} - \frac{l-m}{R^2} \right). \tag{4.31}
\]

Shifting the summation index \( l \to l + m \) in the second term cancels the contribution from the first one.

Let us now compute \( \Pi_{\mu\mu'} \). As in the case of \( \Pi_{55} \) the contributions of \( \Pi^{(2,3)_+} \) and \( \Pi^{(2,3)_-} \) effectively add, yielding a total of

\[
\Pi^{AA'}_{\mu\mu'}(m^2) = \frac{1}{2} g^2 \delta^{AA'} C_2(G) \mu^{4-d} \int \frac{d^d q}{(2\pi)^d} \sum_{l=-\infty}^{+\infty} \frac{(1-d) \left( \frac{1}{d}(d-2)q^2 + \frac{l^2}{R^2} \right) + \frac{1}{2} \frac{(5m^2-2ml)}{R^2}}{(q^2 + \frac{l^2}{R^2})(q^2 + \frac{(m-l)^2}{R^2})}. \tag{4.32}
\]

Performing the integral first we expect a quadratic divergence by power counting. However, these divergences again cancel. This can be seen in dimensional regularization by noticing that the coefficient of the pole is proportional to \( d-2 \). Here the factor of \( d-2 \) is necessary because the integral that it multiplies is not zero. The logarithmically divergent part however is now non zero for \( m \neq 0 \). It fact it gives a contribution to the mass renormalization of the heavy modes of the gauge bosons which is expected since the KK modes being massive should not be protected by gauge invariance against such divergences. On the other hand for \( m = 0 \) this would be a contribution (together with (4.4)) to the mass of the massless \( H \) gauge bosons which should be forbidden by the gauge invariance of the zero mode sector. Indeed, for \( m = 0 \) (4.32) reduces to

\[
\Pi^{AA'}_{\mu\mu'}(0) = \frac{1}{2} g^2 \delta^{AA'} C_2(G) \mu^{4-d} \int \frac{d^d q}{(2\pi)^d} \sum_{l=-\infty}^{+\infty} \left( (1-d) \frac{1}{q^2 + \frac{l^2}{R^2}} - \frac{2}{d} \frac{q^2}{(1-d)(q^2 + \frac{l^2}{R^2})^2} \right), \tag{4.33}
\]

and one can check that the whole expression is zero in any \( d \).
Alternatively, we could have re-summed first and separated the five from the four dimensional part from the beginning. The five dimensional part would then have a pole at \( d = 1 \) (corresponding to a cubic divergence in \( d = 5 \)). Then, it is the factor of \( 1 - d \) in (4.32) that would protect against these divergences, while the remaining integration would be found to be logarithmically divergent for \( m \neq 0 \) and zero for \( m = 0 \).

### 4.1.4 Brane effects from the gauge sector

We will here gather brane effects from the brane sector results of sections 4.1.1 and 4.1.2. We have to add the different contributions to \( \tilde{\Pi}_{55} \) according to Eq. (3.11). The contributions are taken from Eqs. (4.5) and (4.18) together with Eqs. (4.20) and (4.22):

\[
\tilde{\Pi}_{AA'}^{AA'}(m, m') = -\frac{d}{8} g^2 \delta^{AA'}(2C_2(H_A) - C_2(G))(\eta^A + 1)\mu^{4-d} \int \frac{d^d q}{(2\pi)^d} \left( \frac{1}{q^2 + m_-^2} + \frac{1}{q^2 + m_+^2} \right),
\]

where we have defined \( m_- = \frac{1}{2R}(m - m') \) and \( m_+ = \frac{1}{2R}(m + m') \). The last expression seems to indicate a quadratic divergence. However, a closer look shows that this is not the case. To see this, observe that \( \tilde{\Pi}_{55}^{AA'}(m, m') \) in (4.34) is an even function of \( m \) (and of \( m' \)) and therefore the sum

\[
\sum_{m, m'} A_{5}^{a,m} \tilde{\Pi}_{55}^{AA'}(m, m')A_{5}^{a,m'} = 0,
\]

since only gauge components with \( \eta^A = +1 \) contribute to brane effects and therefore we are considering only negative parity scalar fields \( A_{5}^{a,m} \). Thus, here we find a different reason for the absence of divergences from the one we found in the bulk. There, the poles canceled between the tadpole, the gauge and the ghost loop, as it happens in a \( d = 4 \) gauge theory; here they are simply zero.

Next let us examine the brane terms for the gauge bosons \( M = \mu \). We find from Eqs. (4.5) and (4.18) together with Eqs. (4.19) and (4.21):

\[
\tilde{\Pi}_{\mu\mu'}^{AA'}(m, m') = -\frac{1}{16} g^2 \delta^{AA'} \delta_{\mu\mu'}(2C_2(H_A) - C_2(G))(\eta^A + 1)\mu^{4-d} \int \frac{d^d q}{(2\pi)^d} \left( \frac{4(d-2)(d-3)q^2 + 3m^2 + m'^2}{(q^2 + m^2)(q^2 + m'^2)} \right),
\]

The first term is quadratically divergent but the pole cancels in \( d = 2 \) as expected. Notice that here the divergence is quadratic (since there is no sum over \( l \)) so the poles should necessarily cancel in \( d = 2 \). The brane mass term according to (3.12) is the zero’th order of the above in an \( m, m' \) expansion evaluated at \( m = m' = 0 \). It is simply

\[
\tilde{\Pi}_{\mu\mu'}^{AA'}(0, 0) = 0\]

as can be checked by doing the integral explicitly in dimensional regularization. The rest of the terms in (4.36) correspond to a logarithmically divergent piece and a finite piece.
Recall that this last computation corresponds to the vacuum polarization of the unbroken \( \mathcal{H} \) gauge bosons, so naively one would have guessed that not only one should find no quadratic divergences but moreover the logarithmically divergent and finite parts should also be absent. What seems even more surprising is that the above amplitude is non zero for say \( m' = 0 \) and \( m \neq 0 \) which implies a mixing between the zero mode and the heavy modes thus apparently breaking gauge invariance. We believe that the resolution to this puzzle is similar to the one in the Standard Model where the mixing between the photon and the \( Z \) gauge boson computed in an \( R_\xi \) gauge is non zero and gauge non-invariant until vertex and box contributions are taken into account. We will not pursue this question any further, since renormalization of these theories is not the main topic of this work.

### 4.2 The Fermion Sector

The diagram contributing from fermions living in the representation \( \mathbf{R} \) is given in Fig. 2 and evaluates to

\[
-(-ig)^2 \delta_{m-l+k} \delta_{l'-k'-m'} \text{tr} \left\{ \frac{-i}{q_\rho \gamma_\rho + \frac{i}{R} \gamma_5} \frac{1}{2} \left( \delta_{l-l'} - \lambda_R \otimes (i \gamma^5) \delta_{l+l'} \right) \gamma_{M'} T_{R}^{A'} \right. \\
\left. \frac{-i}{q_\sigma \gamma_\sigma + \frac{i}{R} \gamma_5} \frac{1}{2} \left( \delta_{l'-k} - \lambda_R \otimes (i \gamma^5) \delta_{l'+k} \right) \gamma_{M} T_{R}^{A} \right\}. \quad (4.38)
\]

Expanding the latter and using the identities \( (2.26) - (2.30) \), we obtain the momentum conserving terms

\[
\Pi^{(4)_{AA'}}_{MM'} = -\frac{g^2}{4} C_R \delta^{AA'} \frac{1}{q^2 + \frac{i}{R} q^2 + \frac{(l-m)^2}{R^2}} \text{tr} \left\{ \gamma_R \gamma_{M'} \gamma_S \gamma_M \right\} q_R r_S ;
\]

\[
\Pi^{(4)_{AA'}}_{MM'} = -\frac{g^2}{4} C_R \delta^{AA'} \frac{1}{q^2 + \frac{i}{R} q^2 + \frac{(l-m)^2}{R^2}} \text{tr} \left\{ \gamma_R (i \gamma_5) \gamma_{M'} \gamma_S (i \gamma_5) \gamma_M \right\} q_R r_S \alpha_S , \quad (4.40)
\]

where \( q_R = (q_\rho, l/R) \), \( r_S = (q_\sigma, k/R) \). Using \( \gamma_5 \gamma_S = -\alpha_S \gamma_5 \gamma_S \) together with Eq. \( (3.6) \) gives

\[
\Pi^{(4)_{MM'}}_{AA'} = -\frac{g^2}{2} C_R \delta^{AA'} \frac{1}{q^2 + \frac{i}{R} q^2 + \frac{(l-m)^2}{R^2}} \text{tr} \left\{ \gamma_R \gamma_{M'} \gamma_S \gamma_M \right\} q_R r_S . \quad (4.41)
\]

The momentum violating terms are

\[
\tilde{\Pi}_{MM'}^{(4)_{AA'}} = \frac{g^2}{4} \text{tr} (\lambda_R T_{R}^{AA'} \frac{1}{q^2 + m_+^2 q^2 + m_-^2} \text{tr} \left\{ \gamma_R (i \gamma_5) \gamma_{M'} \gamma_S \gamma_M \right\} q_R q_+^S \alpha_S , \quad (4.42)
\]

\[
\tilde{\Pi}_{MM'}^{(4)_{AA'}} = \frac{g^2}{4} \text{tr} (\lambda_R T_{R}^{AA'} \frac{1}{q^2 + m_+^2 q^2 + m_-^2} \text{tr} \left\{ \gamma_R \gamma_{M'} \gamma_S (i \gamma_5) \gamma_M \right\} q_+^R q_-^S , \quad (4.43)
\]

where here \( q_R^+ = (q_\rho, m_+) \), \( q_-^S = (q_\sigma, m_-) \). These are potential brane terms for any value of \( \eta^A \). However, one can easily see that they are zero because of their gamma matrix structure:

\[
\tilde{\Pi}_{55}^{(4)_{MM'}} = 0 , \quad \tilde{\Pi}_{\mu\nu}^{(4)_{MM'}} = 0 . \quad (4.44)
\]
The bulk contribution (4.41) will be first evaluated for 

\[
\Pi^{(4)AA'}_{55}(m^2) = \frac{2[\frac{d}{2}]}{2} g^2 C_R \delta^{AA'} \mu^{4-d} \int \frac{d^d q}{(2\pi)^d} \sum_{l=-\infty}^{+\infty} \frac{q^2 + \frac{l(m-l)}{R^2}}{(q^2 + \frac{l^2}{R^2})(q^2 + \frac{(m-l)^2}{R^2})}. \tag{4.45}
\]

where \([x]\) is defined as usual as the integer part of \(x\). In (4.29) we saw that this integral is divergence free and that the only non vanishing contribution comes from \(m = 0\). Thus, the result after summing and integrating, becomes

\[
\Pi^{(4)AA'}_{55}(m^2 \neq 0) = 0 \quad \text{and} \quad \Pi^{(4)AA'}_{55}(0) = \frac{12}{32\pi^4 R^2} g^2 \delta^{AA'} C_R \zeta(3). \tag{4.46}
\]

We stress that here we did not have factors of \(d-2\) \((d-1)\) to protect us against quadratic (cubic) divergences so it is fortunate that this contribution is completely finite.

After a similar calculation, we obtain for the components along \(M_4\)

\[
\Pi^{(4)AA'}_{\mu\mu'}(m^2) = \frac{2[\frac{d}{2}]}{2} g^2 C_R \delta^{AA'} \mu^{4-d} \int \frac{d^d q}{(2\pi)^d} \sum_{l=-\infty}^{+\infty} \frac{-\frac{1}{d}(d-2)q^2 + \frac{l(m-l)}{R^2}}{(q^2 + \frac{l^2}{R^2})(q^2 + \frac{(m-l)^2}{R^2})}. \tag{4.47}
\]

There is again no quadratic divergences since the corresponding pole vanishes in dimensional regularization in \(d = 2\). The logarithmic divergences for \(m \neq 0\) correspond to mass renormalization of the heavy KK gauge bosons just as in the contribution (4.32) from the gauge sector. For \(m = 0\) though these logarithmic divergences are absent, since for \(d = 4\) the integral is proportional to (4.33) which was found to be zero, i.e.

\[
\Pi^{(4)AA'}_{\mu\mu'}(0) = 0. \tag{4.48}
\]

As a final consistency check, let us see how does the cancellation of the pole work if we perform the sum over \(l\) first. As we have said earlier, cubic divergences which should somehow cancel will typically appear. Indeed, we can rewrite (4.47) as

\[
\Pi^{(4)AA'}_{\mu\mu'}(m^2) = \frac{2[\frac{d}{2}]}{2} g^2 \delta_{\mu\mu'} \delta^{AA'} C_R \mu^{4-d} \int \frac{d^d q}{(2\pi)^d} \sum_{l=-\infty}^{+\infty} \frac{\frac{2}{d}(1-d)q^2 + \left(q^2 + \frac{l(m-l)}{R^2}\right)}{(q^2 + \frac{l^2}{R^2})(q^2 + \frac{(m-l)^2}{R^2})}. \tag{4.49}
\]

We have seen in Eq. (4.28) that the term in the bracket corresponds to just a single finite contribution from \(m = 0\). The first term is the one that has the cubic pole but it is multiplied by the factor of \(1-d\) so that the cubic divergence in \(d = 5\) is actually absent.
5 The Hosotani mechanism

In this section we will discuss on the possibility that one of the scalars $A_5^{a,0}$ radiatively acquires a vacuum expectation value (VEV) and breaks the gauge group $\mathcal{H}$ on the brane to a subgroup. This would be relevant in model building for example when $SU(2)_W \times U(1)_Y$ is a subgroup of $\mathcal{H}$, the scalars $A_5^{a,0}$ are interpreted as Higgs fields and the VEV

$$\omega^a \equiv \frac{1}{2} \langle A_5^{a,0} \rangle R \quad (5.1)$$

breaks $\mathcal{H}$ down to $U(1)_Q$. This mechanism is called the Hosotani mechanism [29, 30]. It is not easy to carry out a discussion as general as the one we had up to this point so we will make a few simplifying assumptions. First, we will assume that there is only one type of fermions, transforming either in the adjoint or in the fundamental representation of $G$. We allow though for multiple flavors of fermions and we will call the number of different flavors by $N_f$. Second, we assume that only one of the Higgs fields $A_5^{a,0}$ takes a VEV, we will call this field $h$ and write $\omega = vR/2$, where $v \equiv \langle h \rangle$.

The first step is to look at the squared mass at the origin of the Higgs field. Adding the contribution (4.46) multiplied by the number of fermion flavors to the result (4.30) from the gauge sector and noticing that in Euclidean space we are computing the negative mass squared, we obtain the result

$$m_h^2 = \frac{3}{32\pi^4 R^2} g_5^2 \zeta(3) (3C_2(G) - 4C_R N_f). \quad (5.2)$$

We see that for models satisfying (we use $C_2(G) = C_G$)

$$\frac{C_G}{C_R} < \frac{4}{3} N_f \quad (5.3)$$

this is negative and therefore $\omega$ could indeed break $\mathcal{H}$. Of course, even when $m_h^2 > 0$ the true vacuum can be at some nonzero $\omega$. To be more precise, we have to look at the full effective potential which can be expressed as

$$V = \frac{1}{128\pi^6 R^4} Tr \left( V(r_F) - V(r_B) \right), \quad (5.4)$$

with

$$V(r) = 3(L_5(r) + L_5(r^*)), \quad r_{F,B} = e^{2\pi i q_{F,B}(\omega)} \quad (5.5)$$

and where $q_F$ and $q_B$ are the shifts in the fermion and boson KK masses according to

$$m_n^F = \frac{n + q_F(\omega)}{R}, \quad m_n^B = \frac{n + q_B(\omega)}{R}, \quad n = 0, \pm 1, \pm 2, \cdots . \quad (5.6)$$

The question then is whether this potential has a minimum for some non-zero value of $\omega$, which would then trigger the breaking of $\mathcal{H}$, and if this minimum is a global minimum. It is hard to answer this question in full generality but one can make some progress in special cases. Two simple examples have been provided in Ref. [46]. For $N_f$ fermions in
the fundamental representation of $SU(2)$ it was found that there is a global minimum at $\omega = \frac{1}{2}$ which becomes degenerate with the one at $\omega = 0$ when $N_f \to 0$. Thus, there is a regime $0 < N_f \leq 3$ where the minimum at $\omega = 0$ does not correspond to the true vacuum. However it was also pointed out that the $U(1)$ symmetry generated by $\sigma^3$ which is left over after the orbifolding remains unbroken. This is because the Wilson-line associated to the vacuum $\omega = \frac{1}{2}$ becomes $-1$ which commutes with $\sigma^3$. In the other example, $G = SU(3)$ with fermions in the fundamental, it was shown that there again exists a minimum at $\omega = \frac{1}{2}$, which already becomes the true vacuum when $N_f > \frac{3}{2}$. This is a less stringent bound than Eq. (5.3), $N_f > \frac{9}{2}$, the critical value at which the minimum at $\omega = 0$ turns into a maximum. By computing the Wilson line associated to $\omega = \frac{1}{2}$ it was finally shown that this breaks only the $SU(2)$ subgroup of $\mathcal{H}$ down to $U(1)$. As in the case of $SU(2)$ the rank remains preserved.

If the fermions transform in the adjoint representation of $G$ there is a slight simplification because $q_F = q_B$. The $\omega$ dependence becomes the same for fermionic and bosonic contributions and the symmetry breaking is determined by the global factor $3 - 4N_f$ in front of the potential. For $SU(2)$ one finds two degenerate minima which lie at $\omega = 0, \frac{1}{2}$ for $N_f < \frac{3}{2}$ and at $\omega = \frac{1}{2}, \frac{3}{2}$ for $N_f > \frac{3}{2}$. For $SU(3)$ one has only a minimum at $\omega = 0$ for $N_f < \frac{3}{4}$ and two degenerate minima at $\omega \approx 0.29, 0.71$ for $N_f > \frac{3}{4}$.

Our next observation is the following statement: The Hosotani mechanism does not reduce the rank of $\mathcal{H}$ if the symmetry breaking global minimum is at $\omega = \frac{1}{2}$. To show this statement, we can compute the Wilson line due to the VEV $\frac{1}{R}$ of a scalar along the $T^A$ direction:

$$\langle W \rangle = e^{i\pi T^A}. \quad (5.7)$$

It is straightforward to show that $\exp(i\pi T^A)$ is a diagonal matrix. Thus, we always have that

$$[\langle W \rangle, H_i] = 0, \quad (5.8)$$

where $H_i$ are the generators corresponding to the Cartan subalgebra of $G$, i.e. that the Wilson loop commutes with at least those generators and therefore it leaves at least a $U(1)_1 \times \cdots \times U(1)_{\text{rank}(\mathcal{H})}$ unbroken. On the other hand if $\omega \neq \frac{1}{2}$ then one can reduce the rank of $\mathcal{H}$ by Hosotani breaking. An example of this possibility was provided above where $N_f > \frac{3}{4}$ fermions in the adjoint representation of $SU(N)$ triggered a Hosotani breaking with $\omega \neq \frac{1}{2}$.

One should however be careful with the interpretation of this effect. Let us look at the example of the orbifold breaking $SU(2) \overset{\text{orb}}{\to} U(1)$, where the subsequent Hosotani mechanism according to the above argument apparently leaves the gauge group unbroken when $\omega = 1/2$: $U(1) \overset{\text{Hos}}{\to} U(1)$. Strictly speaking, this is not correct. By looking at the symmetry breaking pattern in detail it can be seen that the gauge boson $A_{3,0}^\mu$ of $\mathcal{H} = U(1)$ becomes massive by the Hosotani vacuum expectation value and therefore $\mathcal{H}$ breaks to nothing on the brane for generic values of $\omega$. However, for special values of $\omega$, such as the value $\omega = 1/2$ corresponding to fermions transforming in the fundamental

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7 Note that the parameter $\alpha$ in Ref. [46] is related to our $\omega$ as $\alpha = 2\omega$.

8 According to Eq. (5.3), $\omega = 0$ is a maximum for $N_f \geq 3$. 

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representation of $SU(2)$, additional gauge bosons can become massless, as can be seen from the mass matrix of the massive KK gauge bosons which has eigenvalues $n^2, (n + 2\omega)^2, (n - 2\omega)^2$. Clearly, since $n > 0$, the gauge boson associated to the $n = 1$ level (some linear combination of $A_{\mu}^{1,1}$ and $A_{\mu}^{3,1}$ in this case) becomes massless, so the correct statement is $U(1) \xrightarrow{HC} U(1)'$. In fact, in general, for special values of $\omega$ there will be an enhanced gauge symmetry at the orbifold fixed points. The interesting feature here is that the special value of $\omega$ that results in gauge symmetry enhancement is not arbitrary, instead it is obtained by minimizing the fixed (once a bulk fermion representation is chosen) effective potential. Recall that $\omega \sim Rv$ and that having decoupled gravity, $R$ is a parameter assumed to be fixed to some reasonable value. Then, $v$ is essentially the (only) classical modulus of the theory. In supersymmetric theories $v$ might remain a modulus even at the quantum level but in non-supersymmetric theories such as the ones we analyze here, it can be fixed through a non-trivial one loop potential as we have seen above.

It would be interesting to see if, by turning on gravity, $R$ could be stabilized in the same manner. Of course, in such a case, another important issue would be if $R$, being the extra component of the metric (i.e. $g_{55}$), is protected by gauge invariance against quadratic divergences like $A_{a}^{5}$, gauge invariance now being general coordinate invariance.

6 CONCLUSIONS AND OUTLOOK

In this paper we have analyzed the one-loop bulk and brane induced radiative effects in a higher dimensional gauge theory compactified on an orbifold that breaks an arbitrary gauge group $G$ into its subgroup $H$. We have restricted our explicit analysis to a five-dimensional theory compactified on $M_4 \times S^1/\mathbb{Z}_2$. The gauge group at the fixed point branes is $H$ and the higher dimensional components of the gauge bosons along $G/H$ are even scalar fields whose (massless) zero modes can become tachyonic by radiative corrections in the bulk and trigger spontaneous Hosotani breaking of $H$, i.e. they play the role of Higgs fields with respect to the gauge group $H$.

Mass renormalization of Higgs fields in the bulk is expected to be protected from quadratic divergences by the higher dimensional gauge invariance $G$ of the theory. However, since the branes localized at the fixed points are four dimensional space-times, mass renormalization of the Higgs fields on the branes is not a priori protected from quadratic divergences by the higher dimensional theory.

We have computed the one-loop mass renormalization of the Higgs (and gauge) fields in the bulk and on the branes and found no quadratic divergences at all for any of them. While this effect in the bulk is justified from the higher dimensional gauge invariance its interpretation on the brane for the Higgs fields is less clear, although we believe it might be related to the higher-dimensional Lorentz and gauge symmetries. In fact, we have used a five dimensional covariant gauge (Feynman-‘t Hooft gauge) and dimensional regularization that are both consistent with the higher dimensional gauge invariance.

In particular, for the Higgs fields all brane effects vanish while bulk effects are finite and can trigger, depending on the fermionic content of the theory, spontaneous Hosotani

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9Gauge symmetry enhancement at orbifold fixed points is a known effect in string theory.
breaking of the brane gauge group $\mathcal{H}$. For the gauge fields we find only logarithmic divergences consistent with the mass renormalization of heavy KK modes.

Our results are also consistent with the Higgs fields acquiring a VEV and thus spontaneously break the gauge symmetry $\mathcal{H}$ on the branes with one-loop insensitivity to the ultraviolet cutoff of the higher dimensional theory. This insensitivity seems to be a remnant of the properties of the higher dimensional theory where the brane is embedded in. Our results prove that the higher dimensional gauge theory provides a one-loop solution to the hierarchy problem, i.e. it replaces the cutoff by the scale $1/R$ above which the effective theory becomes higher dimensional, modulo the stabilization of the compactification radius that should involve the gravitational sector of the theory. In any case, a quadratic divergence on the brane would have recreated the hierarchy problem.

Of course our framework cannot be considered as a full solution to the absence of quadratic divergences until they are proved to vanish at any order of perturbation theory, or the symmetry protecting them is clearly identified. In fact, a naive analysis of two-loop diagrams prove that there should be non-vanishing Higgs mass renormalization brane effects from two-loop diagrams. However, since there are one-loop wave function renormalization effects localized on the branes, they should induce on finite one-loop mass diagrams non-vanishing two-loop effects localized on the branes. This means that two-loop mass renormalization effects on the branes are mandatory. Only a genuine two-loop calculation can disentangle a two-loop effect induced by the wave function renormalization localized on the brane (and thus absorbable by renormalization group running as happens in supersymmetric theories) from a genuine Higgs mass counter-term localized on it. A two-loop calculation is beyond the scope of the present paper but will be the subject of future investigation by the present authors.

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A ORBITAL GAUGE BREAKING PATTERNS

We will not try to derive general rules for the allowed gauge breaking patterns and the associated consistent fermion representations. Most of the general features of orbifold actions on gauge fields and fermions have appeared in one way or another in the early string theory literature. Instead, in this appendix, we work out a few examples demonstrating the simplicity but also the restrictiveness of the $\mathbb{Z}_2$ orbifold action on the fermion representations. We recall that the gauge group in the bulk is $\mathcal{G}$, which breaks by the orbifold action represented by

$$\Lambda = \begin{pmatrix} 1_{d_H} & 0 & 0 \\ 0 & -1_{d_K} \end{pmatrix},$$

such that

$$f^{ABC} = \Lambda^{AA'} \Lambda^{BB'} \Lambda^{CC'} f^{A'B'C'},$$

(A.1)

10A treatment of parity assignments to pure gauge-theories and a list of all possible breaking patterns can be found in Ref. [11,17].
\[ G \rightarrow \mathcal{H}, \quad (A.2) \]

where \( \mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \cdots \) and the broken generators parametrize the coset \( \mathcal{K} = G/\mathcal{H} \). The fermion representation of the bulk then breaks up according to this, as

\[ R = r_1 \oplus r_2 \oplus \cdots, \quad (A.3) \]

where the \( r_i \) have to be determined by the orbifold action on the fermions

\[ \Psi_R(x^\mu, -x^5) = \lambda_R \otimes (i\gamma^5)\Psi_R(x^\mu, x^5) \quad (A.4) \]

and from the requirement that the coupling \( igA_M^A \overline{\Psi}_R \gamma^M T^A_R \Psi_R \) is \( \mathbb{Z}_2 \) invariant. The resulting constraint from the latter is

\[ \lambda_R T^A_R \lambda_R = \eta^A T^A_R; \quad (A.5) \]

which can be simplified to

\[ [\lambda_R, T^a_R] = 0 \quad \{\lambda_R, T^a_R\} = 0. \quad (A.6) \]

To put it in simple words, for a given representation \( R \), \( \lambda_R \) has to be chosen in such a way that it commutes with the unbroken generators \( T^a_R \) and anti-commutes with the broken generators \( T^a_R \).

The first class of models of interest is when \( \Lambda \) is such that \( \text{rank}(G) = \text{rank}(\mathcal{H}) \) (inner automorphism). In particular this means that none of the \( T^a_R \) is diagonal and therefore \( \lambda_R \) can always be chosen to be diagonal of the form

\[ \lambda_R = \begin{pmatrix} 1_{d_1} & 0 \\ 0 & -1_{d_2} \end{pmatrix}, \quad (A.7) \]

where \( d_1 \) and \( d_2 \) are model dependent numbers. This is not the case for the second interesting class of models, the one with \( \Lambda \) chosen such that the rank of \( G \) is reduced (outer automorphism). Reduced rank in particular implies that some of the \( T^a_R \) are diagonal and therefore (for those diagonal \( T^a_R \)) the second equality of \( \{A.6\} \) can never be satisfied if \( \lambda_R \) is diagonal. Thus, for the case of outer automorphism we have to find a non-diagonal (and unitary) \( \lambda_R \) that solves \( \{A.6\} \). The most interesting case will be

\[ \Lambda^{AB} T^B = -(T^A)^T, \quad (A.8) \]

which e.g. breaks \( SU(N) \rightarrow SO(N) \). From Eq. \( (A.5) \) it then follows that possible representations must be real:

\[ -(T^A_R)^T = \lambda_R T^A_R \lambda_R \quad (A.9) \]

(recall that \( \lambda = \lambda^\dagger = \lambda^{-1} \)). For nonreal representations \( \mathcal{R} \) one can always choose \( R = \mathcal{R} \oplus \overline{\mathcal{R}} \) with generators

\[ T^A_R = \begin{pmatrix} T^A_\mathcal{R} & 0 \\ 0 & -(T^A_\mathcal{R})^T \end{pmatrix}. \quad (A.10) \]
Comparing with Eq. (A.9) this fixes $\lambda_R$ to take the block form

$$
\lambda_R = \begin{pmatrix}
0 & 1_R \\
1_R & 0
\end{pmatrix}.
$$

(A.11)

Thus $\lambda_R$ has $d_R$ eigenstates $(\hat{e}_i, \hat{e}_i)$ with positive parity and $d_R$ eigenstates $(\hat{e}_i, -\hat{e}_i)$ with negative parity, where with $\hat{e}_i$ we denote the usual unit vectors. The zero mode spectrum resulting from this action will always be vector-like and therefore anomaly free. Let us now present a few examples for both the rank preserving and the rank breaking orbifold actions.

The first example in the inner automorphism class is an $SU(2)$ gauge group in the bulk with a pair of Dirac fermions transforming as a doublet under the fundamental representation of $SU(2)$. First, we have to choose the action $\Lambda$ on the gauge fields. We have essentially three choices. The first is to take $\Lambda = diag(+1, +1, +1)$. This choice corresponds to an unbroken $SU(2)$ on the fixed planes and therefore it is not an interesting choice from our point of view. The second possibility is to take $\Lambda = diag(-1, -1, -1)$ which corresponds to a completely broken gauge group on the fixed planes. However, we have seen in section 2 that this choice is not compatible with the automorphism constraint on the Lie algebra so this possibility cannot be realized. The third choice is to choose $\Lambda = diag(-1, -1, +1)$, which breaks $SU(2)$ down to $U(1)$ on the brane. The gauge boson of positive parity corresponding to the unbroken $U(1)$ is $A_3^\mu$, whereas the broken coset is spanned by the negative parity $A_1^\mu, A_2^\mu$. Similarly, the zero modes of the positive parity $A_1^\mu, A_2^\mu$ are seen as massless scalars in the four dimensional theory but the negative parity $A_3^\mu$ does not have a zero mode. This is an interesting possibility, so let us look at the action on the fermions in the fundamental representation $R = 2$ of $SU(2)$. One can easily check that $\lambda_2 = diag(+1, -1)$ ($d_1 = d_2 = 1$) commutes with $T_3^R = \frac{1}{2}\sigma^3$ and anti-commutes with $T_1^R = \frac{1}{2}\sigma^1$ and $T_2^R = \frac{1}{2}\sigma^2$. The surviving fermions on the brane are then (say) left handed Weyl fermions with $U(1)$ charge +1 and right handed fermions with $U(1)$ charge −1. Thus, the orbifold action resulted in a broken gauge group and chiral fermions on the brane. The theory on the brane is anomaly free as can be readily checked.

As a second example, let us look at the breaking pattern $SU(3) \rightarrow SU(2) \otimes U(1)$. To achieve this breaking pattern, we take $\Lambda = diag(+1, +1, +1, -1, -1, -1, +1)$, so that the generators corresponding to $H = SU(2) \otimes U(1)$ have positive parity and the generators corresponding to $K = SU(3)/(SU(2) \otimes U(1))$ have negative parity. We also choose the fermions to transform in the fundamental representation of $SU(3)$, i.e. $R = 3$. Then, $\lambda_3 = diag(+1, +1, -1)$ ($d_1 = 2, d_2 = 1$) commutes with the generators of $H$ and anti-commutes with the generators of $K$, so on the brane we will have two massless Weyl fermions transforming as a doublet under $SU(2)$ and a singlet massless Weyl fermion of the opposite chirality. Under $SU(3) \supset SU(2) \otimes U(1)$ we have

$$
3 = 2_{-1} \oplus 1_{+2}.
$$

(A.12)

This model as it stands has a cubic $U(1)$ and an $SU(2)$ anomaly. One has, in principle, to make modifications that render it anomaly free.\footnote{An example can be found e.g. in Ref. 39.}
The third and last example in the inner automorphism class is an $SU(5)$ in the bulk with fermions transforming under some representation of the gauge group. We will not go through all the possibilities, instead we look at the breaking pattern $SU(5) \rightarrow SU(3) \otimes SU(2) \otimes U(1)$. Following the previous examples, we take $\Lambda$ to have $+1$'s along the diagonal corresponding to the 12 generators of $SU(3) \otimes SU(2) \otimes U(1)$ and $-1$'s along the rest of the diagonal elements. Let us assume that the representation is $R = 5$. Next we have to look for a 5 by 5 matrix $\lambda_R$ that commutes with all the generators of $H$ and anti-commutes with the generators of $K$. These constraints then fix $\lambda_5 = diag(+1, +1, +1, -1, -1)$ ($d_1 = 3, d_2 = 2$), since for $SU(5) \supset SU(3) \otimes SU(2) \otimes U(1)$

$$\bar{5} = (\bar{3}, 1)_{+1/3} \oplus (1, 2)_{-1/2}. \quad (A.13)$$

The fermionic zero mode sector consists then of an $SU(3)$ triplet of Weyl fermions of say right handed chirality, and an $SU(2)$ doublet of Weyl fermions of left handed chirality. It is also possible to carry out the same exercise for $R = 10$. Here it is more convenient to express $\lambda_{10}$ in a tensor rather than a matrix form since the ten is the antisymmetrized tensor product of the fundamental with itself. We find

$$\lambda_{10}^{il}_{mn} = \frac{1}{2} (\lambda_5^i_m \lambda_5^l_n - \lambda_5^i_n \lambda_5^l_m), \quad (A.14)$$

which implies that in

$$10 = (3, 2)_{1/6} \oplus (\bar{3}, 1)_{-2/3} \oplus (1, 1)_1 \quad (A.15)$$

the zero mode spectrum consists of a $(3, 2)$ of left handed chirality and a $(\bar{3}, 1) \oplus (1, 1)_1$ of right handed chirality. For a model that has Dirac fermions in the $\bar{5} \oplus 10$ in the bulk, the zero mode spectrum is clearly anomaly free, since the spectrum is that of the Standard Model.

Before turning to examples of outer automorphisms, we would like to make the connection between the general formalism reviewed in [41] and formulas (A.11) and (A.5) through simple examples. We recall that the action of the orbifold group on the fields, when the action is an inner automorphism, is via group elements such that

$$g = e^{-2\pi i V \cdot H}, \quad (A.16)$$

where $H = \{ H_i \}, i = 1, \cdots, \text{rank}(G)$ are the generators of the Cartan subalgebra of $G$ and $V$ is the twist vector specifying the orbifold. For such group elements it is always true that

$$g H_i g^{-1} = H_i \quad \text{and} \quad g E_\alpha g^{-1} = e^{-2\pi i \alpha \cdot V} E_\alpha, \quad (A.17)$$

where $\alpha = \{ \alpha_i \}$ are the roots of $G$ and $E_\alpha$ the corresponding ladder generators in the Cartan-Weyl basis, satisfying

$$[H_i, E_\alpha] = \alpha_i E_\alpha. \quad (A.18)$$

Let us first look at the $SU(2) \rightarrow U(1)$ example. Taking the Cartan generator of $SU(2)$ in the adjoint representation, i.e. $H_1 = T^A_3$ and requiring that $g = \Lambda = diag(+1, +1, -1)$
in (A.16), fixes \( V = -1/2 \). Then, \( \lambda_2 \) is given again by (A.16) with \( H_1 = T_2^3 \). A simple calculation yields \( \lambda_2 = diag(+1, -1) \) (up to a sign) as we had found earlier. The exponent in the second of Eq. (A.17) is non-zero for all non-zero roots of \( SU(2) \) \( (\alpha = \pm 1, \text{so } \exp(-2i\pi \alpha \cdot V) = -1) \) and therefore the only unbroken generator is \( H_1 \). A similar calculation for the \( SU(3) \to SU(2) \otimes U(1) \) example gives \( V = (0, \sqrt{3}) \) and therefore \( \lambda_3 = diag(+1, +1, -1) \) (up to a sign). In this case, however, from (A.17) we can see that in addition to \( H_1 \) and \( H_2 \) there are two more unbroken generators, namely \( E_{\pm 1} \), corresponding to \( \alpha_{\pm 1} = (\pm 1, 0) \), since for those it is \( \alpha_{\pm 1} \cdot V = 0 \). For the rest of the generators \( E_{\pm 2} \) and \( E_{\pm 3} \) we find \( \exp(-2i\pi \alpha \cdot V) = -1 \) as expected.

Finally, we will present two examples with rank breaking actions. The first example is the simplest possible one, namely a \( U(1) \) in the bulk breaking down to nothing on the branes. The choice that performs this breaking is \( \Lambda = -1 \) and is a simple realization of Eq. (A.8). Charged fermions are necessarily accompanied by oppositely charged partners. Thus, the fermionic zero mode spectrum is a vector-like pair of Weyl fermions.

The second simplest example with rank breaking action is \( SU(3) \to SO(3) \). Solving Eq. (A.8) to obtain \( \Lambda \), one has to give positive parities to antisymmetric and negative ones to symmetric generators. This results in the choice

\[ \Lambda = diag(-1, +1, -1, -1, +1, -1, +1, -1). \] (A.19)

The parities for fermions are given by Eq. (A.11). Since the positive generators \( T^2, T^5, T^7 \) form the fundamental representation of \( SO(3) \), we find that matter in \( 3 \oplus \bar{3} \) of \( SU(3) \) will transform in \( 3 \oplus 3 \) of \( SO(3) \). We checked also that after diagonalizing \( \lambda_{3\oplus 3} \) the positive and negative parity eigenstates transform in separate \( 3 \) representations of \( SO(3) \). The fermionic zero mode spectrum is therefore an \( SO(3) \) triplet of Weyl fermions plus their vector-like partners. Finally we mention that other choices of outer automorphisms not obtained from Eq. (A.8) are related to this one by an inner automorphism.

### B The unitary gauge

In this appendix we will study the problem of gauge fixing and the physical (unitary) gauge in the class of 5D models compactified on the orbifold \( S^1/\mathbb{Z}_2 \) considered in this paper, where the gauge group \( G \) is broken by the orbifold action into its subgroup \( H \).

As we have seen in section 5 in the presence of non-vanishing background values for the scalars \( A_{5,0}^A \) in the adjoint representation of \( G \) the subgroup \( H \) can be further broken and the mass pattern induced by the orbifold breaking will be modified. We will first consider, for simplicity the case of zero VEV for \( A_{5,0}^A \). The Hosotani breaking case will be subsequently studied.

We have seen that the choice of the gauge

\[ G^A = \frac{1}{\sqrt{\xi}} \partial^M A_M^A \] (B.1)

does not lead, for any value of the parameter \( \xi \), to the unitary gauge. In the absence of VEV for the fields \( A_{5,0}^A \) this can be achieved for the gauge fixing condition

\[ G^A = \frac{1}{\sqrt{\xi}} \left( \partial^\mu A_\mu^A + \xi \partial^5 A_5^A \right), \] (B.2)
consistent with the orbifold action. Using now the gauge-fixing condition \( (B.2) \) and the infinitesimal transformation of the field \( A_M \) under the gauge transformation \( \alpha(x^\mu, x^5) \),

\[
A_M \rightarrow A_M + \frac{1}{g} D_M \alpha,
\]

standard techniques yield the Faddeev-Popov Lagrangian

\[
\mathcal{L}_{FP} = -\operatorname{Tr} \bar{c} \left( \partial^\mu D_\mu + \xi \partial^5 D_5 \right) c. \tag{B.3}
\]

The propagators can be worked out as in \((2.19)\) and \((2.20)\) and yield

\[
G^{(A_\mu)}(p, p_5) = -i \frac{\delta^{BC}}{p^2 - p_5^2} \left( g_{\mu\nu} - \frac{(1 - \xi)p_\mu p_\nu}{p^2 - \xi p_5^2} \right), \tag{B.4}
\]

\[
G^{(A_5)}(p, p_5) = -i \frac{\delta^{BC}}{p^2 - \xi p_5^2}; \tag{B.5}
\]

\[
G^{(c)}(p, p_5) = -i \frac{\delta^{BC}}{p^2 - \xi p_5^2}. \tag{B.6}
\]

After mode decomposition \( p_5 = n/R \) and so for \( n \neq 0 \) one reaches the unitary gauge in the limit \( \xi \rightarrow \infty \). In this limit the massive modes of \( A_5^A \) and \( c^A \) decouple while the massive modes of \( A_\mu^A \) only propagate their physical degrees of freedom. This corresponds to the gauge fixing condition \( \partial^5 A_5^A = 0 \), that does not fix the gauge in the zero-mode sector. For the zero modes left out by the orbifold breaking the gauge symmetry is unbroken and, in the absence of Hosotani breaking, one cannot define a unitary gauge, as it is obvious from Eqs. \((B.4)\) to \((B.6)\).

Turning now VEVs for fields \( A_5^{A,0} \) some of the massless gauge bosons in \( \mathcal{H} \) acquire a mass and the definition of the unitary gauge can be enlarged to also take into account this effect. The analysis can be readily done in full generality as follows. Let us consider that some fields \( A_5^{A,0} \) will acquire a VEV by quantum corrections. Their tree level potential is flat, since they are part of the gauge bosons in 5D, and we can write the general decomposition for them into a classical part and quantum fluctuations as

\[
A_5^{A,0}(x^\mu) = v^A + \chi^A(x^\mu). \tag{B.7}
\]

If we restrict ourselves to \( x^\mu \)-dependent gauge transformations \( \alpha = \alpha(x^\mu) \) we can move away from \( v^A \) by means of gauge transformations \( \delta A_5^{A,0} = f_A^{BC} A_5^{B,0} \alpha^C \) which shows that field fluctuations along \( f_A^{CB} v^C \) correspond to Goldstone bosons for the zero mode sector. In fact, if we define the mass matrix

\[
M_B^A = g f_A^{CB} v^C, \tag{B.8}
\]

the zero modes of the gauge bosons acquire the squared mass matrix

\[
(M_\lambda^2)^{AB} = (M M^T)^{AB}. \tag{B.9}
\]
In order to incorporate the Hosotani breaking into the 5D formalism we can modify the gauge fixing condition (B.2) and define the $R_\xi$ gauge

$$G^A = \frac{1}{\sqrt{\xi}} \left[ \partial^\mu A^A_\mu + \xi \left( \partial^5 A^A_5 + \mathcal{M}_B^A \chi^B \right) \right]. \quad (B.10)$$

The Goldstone bosons (zero modes) acquire similarly a mass

$$(\mathcal{M}_G^2)_{AB} = \xi (\mathcal{M}^T \mathcal{M})_{AB}. \quad (B.11)$$

Of course, not only the zero modes will acquire a symmetry breaking mass but also all massive modes will get the mass

$$(\mathcal{M}^n)_B^A = m_n \delta_B^A + \mathcal{M}_B^A, \quad (B.12)$$

where $m_n = \pm n/R$ is the compactification mass and $\mathcal{M}_B^A$ is the symmetry breaking mass given in (B.8). Then propagators (B.4), (B.5) and (B.6) become

$$G^{(A_\mu)}(p, n) = \left[ \frac{-i}{p^2 - \mathcal{M}^n (\mathcal{M}^n)^T} \left( g_{\mu\nu} - \frac{(1 - \xi) p_\mu p_\nu}{p^2 - \xi \mathcal{M}^n (\mathcal{M}^n)^T} \right) \right]^{AB}, \quad (B.13)$$

$$G^{(A_5)}(p, n) = -i \left[ \frac{1}{p^2 - \xi (\mathcal{M}^n)^T \mathcal{M}^n} \right]^{AB}, \quad (B.14)$$

$$G^{(c)}(p, n) = -i \left[ \frac{1}{p^2 - \xi \mathcal{M}^n (\mathcal{M}^n)^T} \right]^{AB}. \quad (B.15)$$

The matrix character of the propagators implies that the matrix $\mathcal{M}^n$ should be invertible, i.e. it satisfies the condition

$$\det [\mathcal{M}^n] \neq 0, \quad (B.16)$$

which is the necessary condition for gauging away the corresponding Goldstone boson.

References

[1] J. Polchinski, String theory. Vol. 2: Superstring theory and beyond (Cambridge, UK: Univ. Pr., 1998)

[2] I. Antoniadis, Phys. Lett. B246 (1990) 377

[3] I. Antoniadis, C. Munoz and M. Quirós, Nucl. Phys. B397 (1993) 515, hep-ph/9211309

[4] I. Antoniadis, K. Benakli and M. Quirós, Phys. Lett. B331 (1994) 313, hep-ph/9403290
[5] P. Nath and M. Yamaguchi, Phys. Rev. D60 (1999) 116004, hep-ph/9902323
[6] I. Antoniadis, K. Benakli and M. Quirós, Phys. Lett. B460 (1999) 176, hep-ph/9905311
[7] T.G. Rizzo and J.D. Wells, Phys. Rev. D61 (2000) 016007, hep-ph/9906234
[8] R. Casalbuoni, S. De Curtis, D. Dominici and R. Gatto, Phys. Lett. B462 (1999) 48, hep-ph/9907355
[9] A. Delgado, A. Pomarol and M. Quirós, JHEP 01 (2000) 030, hep-ph/9911252
[10] E. Accomando, I. Antoniadis and K. Benakli, Nucl. Phys. B579 (2000) 3, hep-ph/9912287
[11] I. Antoniadis and K. Benakli, Int. J. Mod. Phys. A15 (2000) 4237, hep-ph/0007226
[12] J.D. Lykken, Phys. Rev. D54 (1996) 3693, hep-th/9603133
[13] G.F. Giudice, R. Rattazzi and J.D. Wells, Nucl. Phys. B544 (1999) 3, hep-ph/9811291
[14] E.A. Mirabelli, M. Perelstein and M.E. Peskin, Phys. Rev. Lett. 82 (1999) 2236, hep-ph/9811337
[15] EOT-WASH Group, E.G. Adelberger, (2002), hep-ex/0202008
[16] A. Pomarol and M. Quirós, Phys. Lett. B438 (1998) 255, hep-ph/9806263
[17] I. Antoniadis, S. Dimopoulos, A. Pomarol and M. Quirós, Nucl. Phys. B544 (1999) 503, hep-ph/9810410
[18] A. Delgado, A. Pomarol and M. Quirós, Phys. Rev. D60 (1999) 095008, hep-ph/9812489
[19] R. Barbieri, L.J. Hall and Y. Nomura, Phys. Rev. D63 (2001) 105007, hep-ph/0011311
[20] N. Arkani-Hamed, L.J. Hall, Y. Nomura, D.R. Smith and N. Weiner, Nucl. Phys. B605 (2001) 81, hep-ph/0102090
[21] A. Delgado and M. Quirós, Nucl. Phys. B607 (2001) 99, hep-ph/0103058
[22] A. Delgado, G. von Gersdorff, P. John and M. Quirós, Phys. Lett. B517 (2001) 445, hep-ph/0104112
[23] R. Contino and L. Pilo, Phys. Lett. B523 (2001) 347, hep-ph/0104130
[24] H.D. Kim, (2001), hep-th/0109101
[25] V. Di Clemente, S.F. King and D.A.J. Rayner, Nucl. Phys. B617 (2001) 71, hep-ph/0107290
[26] A. Delgado, G. von Gersdorff and M. Quirós, Nucl. Phys. B613 (2001) 49, hep-ph/0107233
[27] H. Hatanaka, T. Inami and C.S. Lim, Mod. Phys. Lett. A13 (1998) 2601, hep-th/9805067
[28] H. Hatanaka, Prog. Theor. Phys. 102 (1999) 407, hep-th/9905100
[29] Y. Hosotani, Phys. Lett. B126 (1983) 309
[30] Y. Hosotani, Ann. Phys. 190 (1989) 233
[31] E. Ponton and E. Poppitz, JHEP 06 (2001) 019, hep-ph/0105021
[32] G.R. Dvali, S. Randjbar-Daemi and R. Tabbash, Phys. Rev. D65 (2002) 064021, hep-ph/0102307
[33] L.J. Dixon, J.A. Harvey, C. Vafa and E. Witten, Nucl. Phys. B261 (1985) 678
[34] L.J. Dixon, J.A. Harvey, C. Vafa and E. Witten, Nucl. Phys. B274 (1986) 285
[35] I. Antoniadis, K. Benakli and M. Quirós, Nucl. Phys. B583 (2000) 35, hep-ph/0004091
[36] I. Antoniadis, K. Benakli and M. Quirós, (2001), hep-th/0108005
[37] H. Georgi, A.K. Grant and G. Hailu, Phys. Rev. D63 (2001) 064027, hep-ph/0007350
[38] H. Georgi, A.K. Grant and G. Hailu, Phys. Lett. B506 (2001) 207, hep-ph/0012379
[39] W.D. Goldberger and M.B. Wise, Phys. Rev. D65 (2002) 025011, hep-th/0104170
[40] R. Contino, L. Pilo, R. Rattazzi and E. Trincherini, Nucl. Phys. B622 (2002) 227, hep-ph/0108102
[41] A. Hebecker and J. March-Russell, Nucl. Phys. B625 (2002) 128, hep-ph/0107039
[42] M.E. Peskin and D.V. Schroeder, An Introduction to quantum field theory , Reading, USA: Addison-Wesley (1995) 842 p
[43] J. Papavassiliou and A. Santamaria, Phys. Rev. D63 (2001) 125014, hep-ph/0102019
[44] P. Ramond, Field Theory: A Modern Primer (Benjamin/Cummings, 1989)
[45] S. Groot Nibbelink, Nucl. Phys. B619 (2001) 373, hep-th/0108185
[46] M. Kubo, C.S. Lim and H. Yamashita, (2001), hep-ph/0111327
[47] R. Slansky, Phys. Rept. 79 (1981) 1.