Overall stability analysis of combined boom with telescopic main boom and fixed flying jib

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Abstract. On the purpose of accurately analyze the overall stability of the combined boom, and it also provides calculation basis for the structural design of the crane combined boom and the revision of the Chinese Design Rules for Cranes GB3811-2008. In order to analyse the overall stability of the combined boom accurately, the deflection differential equations of combined boom is established based on the vertical and horizontal bending theory. What’s more, combining the constraint and displacement boundary condition, the precise recursive expression of the buckling characteristic equations of the combined boom is derived. Taking the combined boom of 6-section telescopic main boom and fixed flying jib as an example, the proposed method and the ANSYS simulation method are used to calculate the critical bucking force of the combined boom, and the comparison results show that: the deduced instability characteristic equation of the combined boom is correct and accurate, which is necessary to the designing analysis in engineering.

1. Introduction
On the purpose of further improving the lifting height of the truck crane to meet the needs of high-rise and super high-rise building construction, the combined boom with telescopic main boom and fixed flying jib is used widely, as shown in figure 1. In order to effectively reduce the weight of the boom, its fixed flying jib is a variable cross-section lattice beam welded by many members. As the main load-bearing structure of the crane, stability analysis of the jib is one of the key issues in design calculation, and many researchers have conducted in-depth research [1].

In the past, the approximation method represented by the energy method is usually used to analyse the stability of the telescopic boom [4], and its accuracy is affected by the assumed deformation curve. The precise stability analysis of the telescopic boom mainly applies the differential equation method [5, 6], the instability characteristic equation established by this method can calculate the instability critical load of the structure accurately. The theoretical analysis methods for the stability of lattice members principally include equivalent length method, equivalent inertia moment method, differential equation method and finite element method, etc. The equivalent length method [8] and the equivalent moment of inertia method [9, 10] are the methods for the stability analysis of lattice members, both of these methods are equivalent lattice members into solid web members, and the analysis results have certain errors. Standard for Design of Steel Structures [11] and Design Rules for Cranes [12] in China, lattice members are equivalent to solid-web members. In [13], a tapered cantilever is equivalent to solid-web member of inertia moment vary quadratic, then the differential equation method is used to
analyse the stability of the variable cross-section lattice structure. At present, the finite element is mainly used to calculate the stability of the complex combined boom structure in actual engineering, and the designer directly uses finite element software for modelling and analysis [14, 15]. However, the use of finite element software to build models is complicated, and each analysis case has little significance for the design and optimization of this type of structure, and lacks of in-depth theoretical analysis. In previous studies, researchers have conducted in-depth studies on the stability of telescopic booms or lattice booms, however, for the boom structure composed of telescopic boom and lattice boom that its precise recursive mathematical model for stability analysis has not yet been formed.

![Figure 1. Truck crane.](image)

In this paper, the combined boom with a telescopic main boom and a variable cross-section lattice fixed flying jib is used as the research object. The differential equation method is applied to establish deflection differential equations of the combined boom, and the precise recursive expression of the instability characteristic equation is derived. The theoretical derivation and quantitative description of the overall stability analysis of the combined boom structure are given, and providing calculation basis for structural design of truck crane combined boom.

2. Instability characteristic equation of combined boom

In the analysis of overall stability of the combined boom with telescopic main boom and fixed flying jib, the telescopic main boom is equivalent to a multi-stepped column, a variable cross-section lattice fixed flying jib is equivalent to a solid-web member with inertia moment vary quadratic [13], the equivalent mechanical model of the combined boom is shown in figure 2.

![Figure 2. Schematic diagram of combined boom.](image)
In Figure 2, the total length of the telescopic boom is \( l_n \), the length of the fixed jib is \( b \), the total length of the combined boom is \( L = l_n + b \). Suppose the moment of inertia of each telescopic boom is \( I_i (i = 1, 2, \ldots n) \), the elastic modulus is \( E \), for variable cross-section lattice fixed jib, the moment of inertia of the large end section is \( I_{1g} \), the moment of inertia of the small end section is \( I_{2g} \), and then the sectional moment of inertia \( I_g(x) \) at any section of the fixed jib can be expressed as:

\[
I_g(x) = I_{g2}(\frac{L - x}{a})^2
\]

where, \( \bar{L} = L + a \).

Based on the vertical and horizontal bending theory, the deflection differential equation of combined boom can be established as:

\[
\begin{cases}
EI_1 y_1'' = P(\delta - y_1) & (0 \leq x \leq l_1) \\
EI_n y_n'' = P(\delta - y_n) & (l_{n-1} \leq x \leq l_n) \\
EI_g(x) y_{g+1}' = P(\delta - y_{g+1}) & (l_n \leq x \leq L)
\end{cases}
\]

Let \( k_i = \frac{P}{\sqrt{EI_i}} \), the general solution of equation (2) is written as:

\[
\begin{aligned}
y_i(x) &= A_i \sin(k_ix) + B_i \cos(k_ix) + \delta & (i = 1, 2, \ldots n) \\
y_{g+1}(x) &= \left(\frac{L - x}{a}\right) A_{g+1} \sin\left(\varepsilon \ln\left(\frac{L - x}{a}\right)\right) + B_{g+1} \cos\left(\varepsilon \ln\left(\frac{L - x}{a}\right)\right) + \delta
\end{aligned}
\]

In equation (3), \( \varepsilon \) is the axial force coefficient excluding the effect of shear deformation,

\[
\varepsilon = \sqrt{\frac{Pa^2}{EI_g} - \frac{1}{4}}
\]

Applying boundary conditions at \( x = 0, y = 0, y' = 0 \), the integral constant is got as:

\[
\begin{cases}
A_1 = 0 \\
B_1 = -\delta
\end{cases}
\]

According to the displacement boundary conditions of each boom section of the telescopic boom at \( x = l_i, y_i = y_{i+1}, y_i' = y_{i+1}' \), \( i = 1, 2, \ldots n-1 \), the relationship between the integral constants can be obtained.

\[
\begin{aligned}
A_1 \sin(k_1l_1) + B_1 \cos(k_1l_1) &= A_{i+1} \sin(k_{i+1}l_i) + B_{i+1} \cos(k_{i+1}l_i) \\
A_i k_i \cos(k_1l_1) - B_i \sin(k_1l_1) &= A_{i+1} k_i \cos(k_{i+1}l_i) - B_{i+1} \sin(k_{i+1}l_i)
\end{aligned}
\]

Equation (6) is expressed as:

\[
\begin{bmatrix}
A_{i+1} \\
B_{i+1}
\end{bmatrix} = Q_i^{-1} U_i \begin{bmatrix}
A_i \\
B_i
\end{bmatrix}
\]

where

\[
U_i = \begin{bmatrix}
\sin(k_1l_i) & \cos(k_1l_i) \\
k_i \cos(k_1l_i) & -k_i \sin(k_1l_i)
\end{bmatrix}
\]
\[ Q_i = \begin{bmatrix} \sin(k_{i1}l_i) & \cos(k_{i1}l_i) \\ k_{i2} \cos(k_{i1}l_i) & -k_{i2} \sin(k_{i1}l_i) \end{bmatrix} \] (9)

Let
\[ T_i = Q_i^{-1} U_i \] (10)

The recursive expressions of coefficients \( A_n \) and \( B_n \) can be derived.
\[
\begin{cases} A_n \\ B_n \end{cases} = T_{n-1} T_{n-2} \cdots T_1 \begin{cases} A_1 \\ B_1 \end{cases} = \prod_{i=1}^{n} T_i \begin{cases} A_i \\ B_i \end{cases} \] (11)

According to the deformation compatibility condition between the telescopic main boom and the fixed jib, when \( x = l_n \), \( y_n = y_{n+1}' \), then
\[
\begin{align*}
A_n \sin(k_{n1}l_n) + B_n \cos(k_{n1}l_n) &= \sqrt{\lambda} \left( A_{n+1} s + B_{n+1} c \right) \\
A_n k_n \cos(k_{n1}l_n) - B_n k_n \sin(k_{n1}l_n) &= -\frac{A_{n+1}}{a\sqrt{\lambda}} \left( s + \epsilon c \right) - \frac{B_{n+1}}{a\sqrt{\lambda}} \left( \frac{c}{2} - \epsilon s \right)
\end{align*}
\] (12)

In equation (12),
\[
\begin{cases} s = \sin(\epsilon \ln \lambda) \\ c = \cos(\epsilon \ln \lambda) \end{cases}
\] (13)

Where \( \lambda \) is the taper coefficient of the variable cross-section beam, \( \lambda = (a+b)/a = \sqrt{I_{x1}/I_{x2}} \).

Expressing the equation (12) in matrix form, then
\[
\begin{cases} A_{n+1} \\ B_{n+1} \end{cases} = D^{-1} U_n \begin{cases} A_n \\ B_n \end{cases} \] (14)

where
\[
U_n = \begin{bmatrix} \sin(k_{n1}l_n) & \cos(k_{n1}l_n) \\ k_n \cos(k_{n1}l_n) & -k_n \sin(k_{n1}l_n) \end{bmatrix} \] (15)

\[
D = \begin{bmatrix} \sqrt{\lambda} s & \sqrt{\lambda} c \\ -1/a\sqrt{\lambda} \left( s + \epsilon c \right) & -1/a\sqrt{\lambda} \left( \frac{c}{2} - \epsilon s \right) \end{bmatrix} \] (16)

Substituting (11) into (14) gives
\[
\begin{cases} A_{n+1} \\ B_{n+1} \end{cases} = D^{-1} U_n \prod_{i=1}^{n} T_i \begin{cases} A_i \\ B_i \end{cases} \] (17)

By boundary conditions at the top of the combined: \( x = L \), \( y_{n+1} = \delta' \), the following equation is gotten:
\[
\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{cases} A_{n+1} \\ B_{n+1} \end{cases} = 0 \] (18)

By substituting (17) into (18), the recursive expression of instability characteristic equation of combined boom is obtained:
\[
B_{n+1} = \begin{bmatrix} 0 & 1 \end{bmatrix} D^{-1} U_n \prod_{i=1}^{n} T_i \begin{cases} A_i \\ B_i \end{cases} = 0 \] (19)
Solving equation (19), the critical bucking force $P_c$ is obtained when the combined boom structure is instability. In Design Rules for Cranes [12], the cross-section inertia moment of the first boom is usually used to represent the critical instability of the boom, so

$$P_c = \frac{\pi^2 EI}{(\mu L)^2}$$

(20)

To facilitate analysis and comparison, the dimensionless effective length coefficient $\mu$ is often used to express the Euler's critical force of the structure. From equation (20), the effective length coefficient of the combined boom structure can be expressed as:

$$\mu = \frac{\pi}{L} \sqrt{\frac{EI}{P_c}}$$

(21)

3. Numerical examples

Taking the combined boom of 6-section telescopic boom and fixed flying jib as an example, as shown in figure 3. Suppose the elastic modulus of the combined boom is $E = 2.05 \times 10^{11}$, the moment of inertia of the telescopic boom’s first section is $I_1 = 43.26 \times 10^{-3} \text{m}^4$, the relationship of moment of inertia of each section boom are $\beta_2 = I_2 / I_1$, $\beta_3 = I_3 / I_1$, $\beta_4 = I_4 / I_1$, $\beta_5 = I_5 / I_1$, $\beta_6 = I_6 / I_1$, $\beta_7 = I_7 / I_1$, $\beta_8 = I_8 / I_1$, $\beta_9 = I_9 / I_1$, $\beta_{10} = I_{10} / I_1$, $\beta_{11} = I_{11} / I_1$, $\beta_{12} = I_{12} / I_1$, $\beta_{13} = I_{13} / I_1$, $\beta_{14} = I_{14} / I_1$, $\beta_{15} = I_{15} / I_1$, $\beta_{16} = I_{16} / I_1$, $\beta_{17} = I_{17} / I_1$, $\beta_{18} = I_{18} / I_1$, $\beta_{19} = I_{19} / I_1$, $\beta_{20} = I_{20} / I_1$, the total length of the combined boom is $L = 66 \text{m}$, the total length of telescopic boom is $L_0 = 60 \text{m}$, the length of each section boom is $l_1 = 0.2l_0$, $l_2 = 0.4l_0$, $l_3 = 0.6l_0$, $l_4 = 0.8l_0$, $l_5 = 0.8l_0$, $l_6 = 0.8l_0$, $l_7 = 0.8l_0$, $l_8 = 0.8l_0$, $l_9 = 0.8l_0$, $l_{10} = 0.8l_0$, $l_{11} = 0.8l_0$, $l_{12} = 0.8l_0$, $l_{13} = 0.8l_0$, $l_{14} = 0.8l_0$, $l_{15} = 0.8l_0$, $l_{16} = 0.8l_0$, $l_{17} = 0.8l_0$, $l_{18} = 0.8l_0$, $l_{19} = 0.8l_0$, $l_{20} = 0.8l_0$, the length of the fixed flying jib is $b = 0.1l_0$, and the moment of inertia of large end section is $I_{g1} = I_{g6}$, the moment of inertia of small end section is $I_{g2} = I_{g1} / \lambda^2$, the moment of inertia of fixed jib at any cross section is $I_g(x) = I_{g2}\left(\frac{L-x}{a}\right)^2$.

![Figure 3. 6-section telescopic boom and fixed flying jib.](image)

To verify the correctness of the expression results of the instability characteristic equation deduced in this paper, the simulation calculation results of the finite element software ANSYS are compared with the calculation results of equation (19). In ANSYS simulation, the combined boom is equivalent to a cantilever beam model, the telescopic main boom is equivalent to a multi-stage ladder column, the variable section lattice fixed flying jib is equivalent to a solid web structure with the moment of inertia changing twice along the cross-section, then the Beam44 beam element is used to simulate each boom section of the telescopic main boom and the fixed flying jib, where the section moment of inertia of the fixed flying jib changes twice along the section, but the section area remains unchanged, as shown in figure 4. In ANSYS, to ensure the accuracy of analysis, each boom section of the telescopic boom is divided into 10 units, and the fixed jib is divided into 20 units.
In table 1, the instability critical force $P_c$ of the combined boom is calculated under different structural parameters $\beta_i$ and $\lambda$, where $\beta_i = I_{i-1} / I_i$ is the ratio of the moment of inertia of the adjacent boom section of the telescopic main boom, and $\lambda$ is the taper coefficient of the fixed flying jib; in order to facilitate the comparison of results, the dimensionless calculation length coefficient $\mu$ in equation (21) is used to express equation (19) and ANSYS simulation calculation results.

**Table 1.** Effective length coefficients under different sectional inertia moments ratio.

| $\beta$ / $\beta$ / $\beta$ / $\beta$ / $\beta$ | $\lambda$ | Simulation($\mu$) | Equation($\mu$) | Error% |
|-----|-----|-----------------|-----------------|--------|
| 1.3/1.3/1.3/1.3 | 2 | 2.4098 | 2.4100 | 0.008 |
| | 4 | 2.4117 | 2.4119 | 0.008 |
| | 6 | 2.4128 | 2.4130 | 0.008 |
| | 8 | 2.4135 | 2.4137 | 0.008 |
| 1.6/1.6/1.6/1.6 | 2 | 2.9510 | 2.9512 | 0.007 |
| | 4 | 2.9569 | 2.9571 | 0.007 |
| | 6 | 2.9603 | 2.9605 | 0.007 |
| | 8 | 2.9626 | 2.9628 | 0.007 |
| 1.9/1.9/1.9/1.9 | 2 | 3.6525 | 3.6528 | 0.008 |
| | 4 | 3.6672 | 3.6674 | 0.005 |
| | 6 | 3.6758 | 3.6760 | 0.005 |
| | 8 | 3.6818 | 3.6819 | 0.003 |
| 2.5/2.5/2.5/2.5 | 2 | 5.6325 | 5.6328 | 0.005 |
| | 4 | 5.6871 | 5.6873 | 0.004 |
| | 6 | 5.7202 | 5.7202 | 0.000 |
| | 8 | 5.7437 | 5.7434 | -0.005 |
According to the analysis results of the length coefficients of combined boom in table 1, the maximum error between the solution in this paper and the simulation solution of ANSYS is 0.008%, which verifies the correctness and accuracy of the instability characteristic equation derived in this paper.

4. Conclusion
In this paper, the differential equation method is applied to study the overall stability of the combined boom with telescopic main boom and lattice fixed jib, the recursive expression of the instability characteristic equation of the combined boom is established, furthermore, the numerical solution of the recursive expression is compared with the ANSYS simulation solution. The results show that, the maximum error between the calculation results of the instability characteristic equation of the combined boom derived and the ANSYS simulation calculation results is 0.008%, it is proved that it is accurate and reasonable to calculate the instability critical force of the combined boom by applying the derived theoretical formula. It provides a new solution for the design calculation of the combined boom with a telescopic main boom and a lattice fixed jib, and provides theoretical data support for the Chinese crane design code GB/T3811, which has certain practical application value.

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