Viscosity in spherically symmetric accretion

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1 November 2018

ABSTRACT

The influence of viscosity on the flow behaviour in spherically symmetric accretion, has been studied here. The governing equation chosen has been the Navier-Stokes equation. It has been found that at least for the transonic solution, viscosity acts as a mechanism that detracts from the effectiveness of gravity. This has been conjectured to set up a limiting scale of length for gravity to bring about accretion, and the physical interpretation of such a length-scale has been compared with the conventional understanding of the so-called “accretion radius” for spherically symmetric accretion. For a perturbative presence of viscosity, it has also been pointed out that the critical points for inflows and outflows are not identical, which is a consequence of the fact that under the Navier-Stokes prescription, there is a breakdown of the invariance of the stationary inflow and outflow solutions – an invariance that holds good under inviscid conditions. For inflows, the critical point gets shifted deeper within the gravitational potential well. Finally, a linear stability analysis of the stationary inflow solutions, under the influence of a perturbation that is in the nature of a standing wave, has indicated that the presence of viscosity induces greater stability in the system, than has been seen for the case of inviscid spherically symmetric inflows.

Key words: accretion, accretion discs – hydrodynamics – methods: analytical

1 INTRODUCTION

The influence of viscosity, in a spherically symmetric accreting system, at least in qualitative terms, has been the object of the study that has been undertaken here. Accretion flows are governed by the equations of fluid dynamics. This in itself makes a convincing case for viscosity to be accorded some importance, since viscosity is an intrinsic property of fluids. Indeed, in accretion processes, viscosity does have a prominent role to play – as has been established particularly for the axially symmetric case of thin accretion discs, where viscosity, being purportedly a mechanism for the outward transport of angular momentum, effectively causes the infall of matter.

However, the spherically symmetric accreting system has been studied very widely, using only inviscid hydrodynamical equations [Bondi 1952; Frank et al. 1992]. In this study, instead of the inviscid Euler equation, the Navier-Stokes equation has been considered as one of the governing equations of the flow. For a compressible viscous flow with constant coefficients of viscosity, the Navier-Stokes equation can be rendered in the form [Landau & Lifshitz 1987],

\[
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} + \frac{\nabla P}{\rho} + \frac{GM}{r^2} \hat{\mathbf{r}} = \frac{1}{\rho} \left[ \eta \nabla^2 \mathbf{v} + \left( \frac{\eta}{3} + \zeta \right) \nabla (\nabla \cdot \mathbf{v}) \right]
\]  

(1)

For molecular viscosity, for which the actual magnitude may be quite small, the coefficients of viscosity \( \eta \) and \( \zeta \), as has been mentioned before, have been taken to be positive constants [Landau & Lifshitz 1987]. This greatly simplifies the mathematics, without largely compromising the underlying physics. In so far as viscosity is dependent on temperature (which can be connected to a dependence on the density of the accreting matter), its spatial variation is much less compared with that of the velocity of the flow, and in an actual accreting system where radiative processes facilitate efficient cooling and keep the system somewhat close to being isothermal, it is an expectation that the assumption of constancy of \( \eta \) and \( \zeta \) largely holds true [Balbus & Hawley 1998].

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For the present purpose, $\eta$ and $\zeta$ have been taken to be the conventional first and second coefficients of viscosity. They are expected to be of the same order of magnitude (Landau & Lifshitz 1987). Even as molecular viscosity is understood to be of not much significance in quantitative terms, its introduction in one of the governing equations should at least reveal in what qualitative manner would it influence the flow dynamics – especially the behaviour of the critical points of the flows and the stability of the steady inflow solutions under the effect of a linearized perturbation.

Considering now the vector identity,
\[ \nabla \times (\nabla \times \mathbf{v}) = \nabla (\nabla \cdot \mathbf{v}) - \nabla^2 \mathbf{v} \] (2)
it is seen that for an irrotational flow, such as in a spherically symmetric accreting system, the left-hand side of (2) vanishes and leaves behind the condition,
\[ \nabla (\nabla \cdot \mathbf{v}) = \nabla^2 \mathbf{v} \] (3)
a result whose use could be made in (1).

It is to be noted here that for a compressible fluid, the equation of continuity
\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \] (4)
which is the other governing equation of the flow, does not give the result $\nabla \cdot \mathbf{v} = 0$, as a steady-state solution. Comparing this fact with the steady solution of (1), and by using the identity given in (3), it can be seen that the compressibility of accreting matter allows for viscosity to have a role to play, in a spherically symmetric flow under steady-state conditions.

2 THE EQUATIONS OF THE FLOW

For a spherically symmetric system, in which $\mathbf{v} \equiv \mathbf{v}(r,t)$ and $\rho \equiv \rho(r,t)$, the equations given by (1) and (4) are rendered respectively as,
\[ \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \frac{\partial \mathbf{v}}{\partial r} + \frac{1}{\rho} \frac{\partial P}{\partial r} = \frac{1}{\rho} \left( \frac{4}{3} \eta + \zeta \right) \frac{\partial}{\partial r} \left( \frac{1}{r^2} \frac{\partial}{\partial r} (vr^2) \right) \] (5)
and
\[ \frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (\rho vr^2) = 0 \] (6)

Use has also been made of a general polytropic equation of state
\[ P = k \rho^\gamma \] (7)
where $\gamma$ is the polytropic exponent with an admissible range given by $1 < \gamma < 5/3$, the restrictions being imposed by the isothermal limit and the adiabatic limit, respectively.

The speed of sound $c_s$, with which the velocity of the flow is suitably scaled, is given by
\[ c_s^2 = \gamma k \rho^{\gamma-1} \] (8)

The set of equations (5)–(8), completely describes the problem. It can be seen that in (5), the two coefficients of viscosity add up like simple scalars, which makes it possible to define a total viscosity $\eta_{\text{tot}} = (4/3)\eta + \zeta$.

In the steady state, explicit time dependence vanishes, i.e. $(\partial/\partial t) \equiv 0$, and from (5) and (6) will then follow the results
\[ \frac{1}{\rho} \frac{d}{dr} \left( \frac{v^2}{r^2} \right) = \frac{\eta_{\text{tot}}}{\rho} \frac{d}{dr} \left[ \frac{1}{r^2} \frac{d}{dr} (vr^2) \right] \] (9)
and
\[ \frac{d}{dr} (\rho vr^2) = 0 \] (10)
respectively. From (10) it further follows that
\[ \frac{1}{r^2} \frac{d}{dr} (vr^2) = -\frac{v}{\rho} \frac{dp}{dr} \] (11)

Using (7), (8) and (11), yields from (9) the expression,
\[ \frac{1}{2} \frac{d}{dr} \left( \frac{v^2}{2} \right) + \frac{1}{\gamma - 1} \frac{1}{r^2} \frac{d}{dr} \left( c_s^2 \right) + f_{\text{eff}} = 0 \] (12)
where the effective force $f_{\text{eff}}$ is given by,
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\[ f_{\text{eff}} = \frac{GM}{r^2} - \frac{2}{\gamma - 1} \frac{\eta_{\text{tot}}}{\rho} \frac{d}{dr} \left( -\frac{v}{c_s} \frac{dc_s}{dr} \right) \] (13)

The second expression on the right-hand side of (13) is now to be studied closely. For accretion, it is a convention that \( v < 0 \). When \( r \) decreases, \( dc_s/dr \) increases with a negative sign and for the transonic solution \((-v/c_s)\) increases with a positive sign. Hence, at least for the transonic solution, the product \((-v/c_s) (dc_s/dr)\) increases with a negative sign for decreasing \( r \), i.e.

\[ \frac{d}{dr} \left( -\frac{v}{c_s} \frac{dc_s}{dr} \right) > 0 \] (14)

On using this result in (13), it may be concluded that \( f_{\text{eff}} < f_{\text{grav}} \equiv GM/r^2 \). This implies that the presence of viscosity detracts from the effectiveness of gravity in spherically symmetric accretion. Since it is gravity that drives the process of accretion in spherical symmetry, viscosity has an inhibiting effect on it, which is quite contrary to what is seen in axially symmetric accretion, where viscosity, in effecting the outward transport of angular momentum, actually aids the process of accretion (Pringle 1981; Frank et al. 1992).

It can also be seen that with the introduction of viscosity in a governing hydrodynamical equation, it would be possible to obtain a limiting scale of length for the effectiveness of gravity to bring about the infall of matter. Viscosity is a microscopic phenomenon. Very far away from the accretor, the bulk influence of gravity, having been greatly weakened, becomes comparable with the inhibiting influence of viscosity at the microscopic level, and at such a distance, a scale of length may be obtained which, in a manner of speaking, may be understood to be a “viscous shielding radius”, \( r_{\text{vis}} \). By dimensional arguments it is also possible to have some quantitative understanding of \( r_{\text{vis}} \). The physical quantity \( \dot{m}/\eta_{\text{tot}} \) has the dimension of length and this quantity may be identified with \( r_{\text{vis}} \). Here \( \dot{m} \) is the mass accretion rate, which is dependent on the mass of the accretor and the “ambient conditions” (Frank et al. 1992), although for the viscous system being discussed here, this dependence – as opposed to that of the inviscid case, where the dependence is exactly known (Frank et al. 1992) – may be known only in an order-of-magnitude sense. With the identification that \( r_{\text{vis}} \equiv \dot{m}/\eta_{\text{tot}} \), the “viscous shielding radius” would then conform to what can be intuited concerning the role of viscosity, that for a non-zero magnitude of \( \eta_{\text{tot}} \), the length \( r_{\text{vis}} \) would set a finite spatial limit to the accretion process. This understanding of the physical nature of \( r_{\text{vis}} \) can be compared with the somewhat loosely defined concept of what has conventionally been known to be the “accretion radius” (Frank et al. 1992), given by \( r_{\text{acc}} \equiv GM/c_s^2 (\infty) \). In all realistic cases with molecular viscosity, the condition \( r_{\text{vis}} \gg r_{\text{acc}} \) prevails. The accreting system would then very likely be bound by the scale of length \( r_{\text{acc}} \). In such cases, the mass accretion rate \( \dot{m} \), would be very close to the value that would be obtained for the inviscid case.

Viscosity is also expected to influence the critical points of the flow solutions in the \( v^2 - r \) space. Critical points of the flow are given by the condition that in such a space, for a first-order differential equation in \( v \) and \( r \), both the numerator and the denominator of \( d (v^2) / dr \) vanish simultaneously (Chakrabarti 1990). The flow equation (10), however, is a second-order differential equation. So to get the critical point condition, ideally a first integral of (10) is to be obtained, which, unfortunately, cannot be done for compressible flows. Hence an approximation is resorted to, that viscosity has a very small perturbative influence on the conditions governed by the inviscid equations. For molecular viscosity the validity of such an approximation is quite evident. Moreover, for large distances, the second derivative of \( v \) is comparatively small. The expectation here is that the critical point would be at a distance where this condition would hold. A cautionary note to be made here is that for the branch of the transonic solution very close to the accretor, the second derivative of \( v \) would be large.

Using (7), (8) and (10), would then deliver from (9), the result

\[ \left( v - \frac{c_s^2}{v} - 2 \frac{\eta_{\text{tot}}}{\rho r} \right) \frac{dv}{dr} = \frac{2c_s^2}{r} - \frac{GM}{r^2} + \frac{\eta_{\text{tot}}}{\rho} \left( \frac{d^2v}{dr^2} - \frac{2v}{r^2} \right) \] (15)

The relation \( \rho \equiv \rho_{\infty} \left[ c_s/c_{\text{sc}} (\infty) \right]^{2n} \) is now to be used, where \( n = (\gamma - 1)^{-1} \). It is then possible to obtain from the right hand side of (15), the condition

\[ \frac{2c_{\text{sc}}^2}{r_c} - \frac{GM}{r_c^2} + \frac{\eta_{\text{tot}}}{\rho_{\infty}} \left[ \frac{c_s (\infty)}{c_{\text{sc}}} \right]^{2n} \left( \frac{d^2v}{dr^2} - \frac{2v}{vc_s^2} \right) = 0 \] (16)

where the subscript label \( c \) indicates critical point values. From (16), it is possible to have some idea of the influence of viscosity on the critical point of the transonic inflow solution.

A rearrangement of terms in (16) gives,

\[ r_c = \frac{GM}{2c_{\text{sc}}^2} + \frac{\eta_{\text{tot}}}{\rho_{\infty}} \left[ \frac{c_s (\infty)}{c_{\text{sc}}} \right]^{2n} \frac{vc_s}{c_{\text{sc}}^2} \left( 1 - \frac{1}{2} \frac{r_c^2}{v_c^2} \frac{d^2v}{dr^2} \right) \] (17)

For the transonic solution it may be said that a power law of the form \( v \sim r^{-\delta} \) is followed. For free fall, \( \delta = 1/2 \). For the hydrodynamical case being discussed here, it may be supposed that in the vicinity of the critical point it will be \( 1/2 < \delta < 1 \). That will make the second term in parentheses in (17), a dimensionless positive number less than unity. Since for inflows,
convention has it that \( v_c < 0 \), it can then be said that the presence of viscosity causes the critical point to be shifted inwards along the \( r \)-axis, as compared with the inviscid case, where \( r_c = GM/2c_{\text{sc}}^2 \) [Frank et al. 1992]. This, in itself, is consistent with the fact that viscosity has the effect of weakening the influence of gravity. At the critical point, the pressure term balances the gravity term. With gravity having been weakened, the pressure term is capable of balancing the gravity term deeper within the potential well.

The use of the Navier-Stokes equation gives rise to another feature of the flow. A look at (14) or its spherically symmetric adaptation (15), makes it evident that unlike in the inviscid Euler equation, \( v \) makes an appearance here in the first power (in the viscous term). A direct consequence of this fact is that in the steady state, the Navier-Stokes equation (15), governing the spherically symmetric flow, is not invariant under the transformation \( v \rightarrow -v \). The occurrence of \( v \) in the first power in (14), causes the invariance to break down. The important conclusion to be drawn here is that unlike in the case of inviscid flows, the critical points of the transonic inflow and outflow solutions, are not identical in the steady state solutions of (5) and (6) are

\[
\frac{\partial \rho'}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \rho' v' \right) = 0
\]

With the use of (19), successive differentiation of (18) with respect to time yields, first

\[
\frac{\partial v'}{\partial t} = \frac{1}{\rho_0 r^2} \left( \frac{\partial f'}{\partial t} + v_0 \frac{\partial f'}{\partial r} \right)
\]

and then

\[
\frac{\partial^2 v'}{\partial t^2} = \frac{1}{\rho_0 r^2} \left[ \frac{\partial^2 f'}{\partial t^2} + v_0 \frac{\partial}{\partial r} \left( \frac{\partial f'}{\partial r} \right) \right]
\]

Now linearizing in terms of the perturbation variables, gives from (15) the result

\[
\frac{\partial v'}{\partial t} + \frac{\partial}{\partial r} \left( v_0 v' + c_{\text{sc}}^2 \frac{\rho'}{\rho_0} \right) = \frac{\eta_{\text{vis}}}{\rho_0} \left\{ \frac{\partial}{\partial r} \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 v' \right) \right] - \frac{\rho'}{\rho_0} \frac{\partial}{\partial r} \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 v_0 \right) \right] \right\}
\]

where \( c_{\text{sc}} \) is the speed of sound in the steady state.

Partially differentiating (22) with respect to time, and using the conditions given by (10), (20) and (21) will finally deliver the perturbation equation as

\[
\frac{\partial^2 f'}{\partial t^2} + 2 \frac{\partial}{\partial r} \left( v_0 \frac{\partial f'}{\partial t} \right) + \frac{1}{v_0} \frac{\partial}{\partial t} \left[ v_0 \left( v_0^2 - c_{\text{sc}}^2 \right) \frac{\partial f'}{\partial r} \right] = \eta_{\text{vis}} r^2 \left\{ \frac{\partial}{\partial r} \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial f'}{\partial r} \right) + v_0 \frac{\partial f'}{\partial r} \right] \right\} + \frac{1}{\rho_0} \frac{\partial f'}{\partial r} \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 v_0 \right) \right]
\]

It is quite evident that the related perturbation equation for the inviscid case (Theuns & David 1992), will be readily obtained for \( \eta_{\text{vis}} = 0 \).

For (23), a solution of the form \( f' = g(r)e^{\Omega t} \) is chosen, in which \( g(r) \) is the spatial part of the perturbation and \( \Omega \) in general is complex. This leads to a quadratic equation in \( \Omega \), which is

\[
A\Omega^2 + B\Omega + C = 0
\]

where
\[ A = g \]

\[ B = 2 \frac{d}{dr} (v_0 g) - \eta_\text{tot} r^2 \frac{d}{dr} \left[ \frac{1}{r^2} \frac{d}{dr} \left( \frac{g}{\rho_0} \right) \right] \]

\[ C = \frac{1}{v_0} \frac{d}{dr} \left[ v_0 \left( v_0^2 - c_{\infty}^2 \right) \frac{dg}{dr} \right] - \eta_\text{tot} \left\{ r^2 \frac{d}{dr} \left[ \frac{1}{r^2} \frac{d}{dr} \left( \frac{v_0}{\rho_0} \frac{dg}{dr} \right) \right] + 1 \frac{dg}{dr} \frac{d}{dr} \left[ \frac{1}{r^2} \left( r^2 v_0 \right) \right] \right\} \]

(25)

Stability of the subsonic inflows has been studied here with a particular emphasis. In such flows, it is readily understandable that for a perturbation which is in the form of a standing wave, there are two spatial points, one very close to the accretor and one very far away from it, where it is possible to have boundary conditions which would constrain the perturbation to die out (Petterson et al. 1980). Multiplying (24) by \( v_0 g \) and integrating over the range of \( r \), that is bounded by the two points where the perturbation dies out, will give,

\[ A \Omega^2 + B \Omega + C = 0 \]

(26)

where

\[ A = \int v_0 g^2 dr \]

\[ B = \frac{4 \pi \eta_\text{tot}}{(-\dot{m})} \int r^{-2} \left( \frac{d}{dr} (v_0 g^2) \right)^2 dr \]

\[ C = -\int v_0 \left( v_0^2 - c_{\infty}^2 \right) \frac{dq}{dr} \frac{d}{dr} \frac{g}{\rho_0} \frac{dg}{dr} \frac{d}{dr} \left( -\frac{v_0}{\rho_0} \frac{dg}{dr} \right) dr \]

(27)

The results given by (26) and (27) have been arrived at by introducing the simplification of partial integrations, and by requiring that all terms at the two boundaries vanish (Petterson et al. 1980). Use has also been made of an integrated form of (10), which is \( \rho_0 v_0 r^2 = (-\dot{m})/4 \pi \), for spherically symmetric inflows (Frank et al. 1992). The integration constant \( \dot{m} \) is physically identified with the mass infall rate and therefore \( \dot{m} > 0 \).

The solution of (26) is given by,

\[ \Omega = -\frac{B}{2A} \pm \sqrt{\frac{B^2}{4A^2} - \frac{C}{A}} \]

(28)

The result in (28) indicates that \( \Omega \) has a real part and to ensure stability it is necessary that \( \Re(\Omega) < 0 \). An examination of \( A \) and \( B \) will give,

\[ \frac{B}{A} = \frac{4 \pi \eta_\text{tot}}{(-\dot{m})} \left( \int v_0 g^2 dr \right)^{-1} \int r^{-2} \left( \frac{d}{dr} (v_0 g^2) \right)^2 dr \]

(29)

For inflows, the condition is \( (-v_0) > 0 \), which, for a real \( g \), would then imply that \( (B/A) > 0 \). Instabilities could still arise if \( (-C/A) > 0 \), because \( \Omega \) would then have one positive root. It is seen that

\[ -\frac{C}{A} = \left[ \int v_0 \left( v_0^2 - c_{\infty}^2 \right) \frac{dq}{dr} dr \right] + \eta_\text{tot} \left( \frac{v_0 g}{\rho_0} \right) \frac{d}{dr} \left( -\frac{v_0}{\rho_0} \frac{dg}{dr} \right) dr \]

\[ + \eta_\text{tot} \int r^{-2} \left( \frac{d}{dr} \left( v_0 g^2 \right) \right) \frac{d}{dr} \left( -\frac{v_0}{\rho_0} \frac{dg}{dr} \right) dr \]

(30)

In the inviscid term of (30), for subsonic inflows, the governing condition is \( v_0^2 < c_{\infty}^2 \). This would make the term always negative for any real \( g \). For the viscous terms, the asymptotic properties of the integrands near the outer boundary, would have to be studied. For \( r \to \infty \), the boundary conditions are \( \rho_0 \to \rho_\infty, v_0 \to 0 \) and \( g \to 0 \). In that case, for the first viscous term, asymptotically at least, the products of the derivatives within the integrand, would be negative. For the second viscous term, the integrand in the numerator would be positive asymptotically, but having a negative denominator (since \( \eta_\text{tot} \)), the term would be negative overall. It is to be expected that the conclusion drawn on the basis of the asymptotic argument, could be logically extended over the entire range of the integration. This would then lead to the conclusion that \( (-C/A) < 0 \) for all situations of physical interest, and thus makes it possible to avoid having instabilities to develop in the system.

It is to be noted here that for inviscid subsonic inflows, a similar analysis would give the result that \( \Omega \) would be purely imaginary (Petterson et al. 1980), i.e. the perturbation would be oscillatory with a constant amplitude. The stability analysis for the viscous flow solutions developed above, indicates that even if the perturbation were to be oscillatory, its amplitude would decrease. Viscosity is fluid friction in its essence and as such it damps out the amplitude of the perturbation. This
would therefore establish that the presence of viscosity would induce greater stability in the system. This is a result that is somewhat surprising, in view of the fact that viscosity is a dissipative mechanism that releases heat into the system. To understand this seeming anomaly, it would be worthwhile to go back to (9), in which, the viscous term gives a mode for the loss of mechanical energy from the system. However, in the choice of equations here, no account has been taken of an equation for the energy balance, that, for viscous dissipation of the mechanical energy, would monitor the concomitant rise in the internal energy of the system. This loss of mechanical energy manifests itself as greater stability for the system, compared with the inviscid case.

For the stability analysis of the transonic solution, a different line of reasoning would have to be adopted, because there is no physically feasible inner boundary condition for the inflow ($\eta_{\text{tot}}^2 < c_{\text{s,c,0}}^2$), and neither is the condition $v_0^2 < c_{\text{s,c,0}}^2$ satisfied within the sonic radius. Themis & David (1992) have shown that in the case of inviscid flows, for the subsonic branch of the transonic solution, the flow stability is dependent on conditions prevailing at the outer boundary. For the supersonic branch, recourse is to be had to the argument of Garlick (1977) that for a finite supersonic region, a disturbance would be carried away in a finite time, and in this manner would ensure the stability of the solution. These arguments, it is to be expected, would hold good for the transonic solution for viscous flows. The expectation is based on the analogous reasoning that for the inviscid flow solutions, the conclusions drawn about the stability of the subsonic inflows are largely and qualitatively the same as those about the stability of the transonic inflow. More so, when, for viscous flow solutions, it has been argued that the system shows itself to have greater stability for the subsonic stationary inflow solutions under the influence of a linearized perturbation. Hence, it may be safely established that a linear stability analysis indicates that in the absence of any energy balance equation, viscosity, as incorporated in the Navier-Stokes equation, would apparently induce greater stability in the stationary solutions in a spherically symmetric accreting system.

4 CONCLUDING REMARKS

It has been seen that the presence of viscosity in a spherically symmetric accreting system, adversely affects the effectiveness of gravitation in bringing about the infall of matter – at least for the transonic solution. To the extent that the transonic solution is the one that is most likely to be realizable (Bond 1952; Garlick 1973; Ray & Bhattacharjee 2002), the effect of viscosity on such flows can have important implications – especially so, when a significantly large and scale-dependent effective “turbulent viscosity” is taken into consideration. Such a prescription is not entirely without its context or relevance, because turbulent viscosity prescribed for the Navier-Stokes equation. The change could be even more significant, considering that such an effective turbulent viscosity, being scale-dependent, would have a spatial variation (Mészáros & Silk 1977) – not to mention factoring in the accompanying rise in the internal energy of the system.

ACKNOWLEDGMENTS

This research has made use of NASA’s Astrophysics Data System. The financial assistance provided by the Council of Scientific and Industrial Research, Government of India, is being gratefully acknowledged here. Gratitude is also to be expressed to
Prof. J.K. Bhattacharjee for his very helpful advice and comments. The suggestion of an anonymous referee that the term “viscous shielding radius” should be introduced is also being acknowledged.

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