Photon Damping in a Strongly Magnetized Plasma

A. A. Yarkov* a, D. A. Rumyantsev, and M. V. Chistyakov a

a Demidov Yaroslavl State University, Yaroslavl, 150000 Russia
b Yaroslavl Higher Military School of Air Defense, Yaroslavl, 150001 Russia
* e-mail: a12l@mail.ru

Received December 30, 2021; revised December 30, 2021; accepted January 12, 2022

Abstract—The process of electromagnetic wave propagation in a strongly magnetized (magnetic fields exceeding 10^13 G) charge-symmetric plasma is investigated. Taking the change in the dispersion properties of a photon in a magnetic field and plasma into account, it is found that, as in the case of a pure magnetic field, the process of photon damping in a magnetized plasma has a nonexponential character. It is shown that the effective photon absorption width is significantly smaller in comparison with the results in the literature.

Keywords: strong magnetic field, plasma, photon splitting

DOI: 10.1134/S1063778822100647

1. INTRODUCTION

When considering some phenomena in different astrophysical objects, the problem arises of describing the propagation of electromagnetic fields in an active medium. Of special interest are objects with fields of the so-called critical value scale \( B_c = m^2/e \approx 4.41 \times 10^{13} \) G (we use the natural system of units where \( c = \hbar = \mu_0 = 1 \), \( m \) is the electron mass, and \( e > 0 \) is the elementary charge). Recent observations allow one, in particular, to identify some astrophysical objects, such as soft gamma repeaters (SGRs) and anomalous X-ray pulsars (AXPs), with magnetars [1].

According to the currently best-known model (see, for example, [2]), a strong magnetic field reaching a value of \( 10^{15} - 10^{16} \) G can exist in the vicinity of such objects. In addition, analysis of the emission spectrum of some of these objects indicates the presence of a relatively hot and dense electron-positron plasma [3] with temperature of \( T \sim 1 \) MeV in their vicinity.

It is under such conditions that it is of interest to consider the process of photon damping due to reactions of photon absorption by an electron (positron), \( \gamma \bar{e}^\mp \rightarrow e^\pm \), and creation of \( e^+e^- \) -pairs, \( \gamma \rightarrow e^+e^- \), which are kinematically forbidden in a vacuum but become possible in the presence of an external magnetic field and/or plasma and are important in astrophysics of magnetized neutron stars [4, 5]. It should be noted that the expression for the decay width in the limit of a strongly magnetized plasma contains singularities of the root type at points of cyclotron resonances. As emphasized in [6], this points to the impossibility of interpreting the given decay width calculated by the perturbation theory near cyclotron resonances as the damping coefficient. In this case, the primary issue for determining the damping coefficient is the time dependence of the photon wave function in the presence of a magnetic field and plasma. In [6], as a method for determining this dependence, it was proposed to solve the dispersion equation with allowance for vacuum polarization in the magnetic field and plasma with complex values of photon energy. However, in our opinion, this method has some disadvantages. First, it is well known (although rarely mentioned) that solutions of the dispersion equation with complex energies on a physical sheet do not exist. The solutions are found on nonphysical Riemann sheets (the domain of analyticity of the polarization operator) the number of which is, generally speaking, infinite. This, in turn, leads to an infinite number of solutions of the dispersion equation both with positive and with negative values of the imaginary part of the energy. Second, this approach does not make it possible to correctly describe the character of damping near cyclotron resonances; in this case, the character significantly differs from exponential. Thus, the method of describing damping electromagnetic waves in magnetized plasma by solving the dispersion equation is not self-sufficient.

In this work, photon decay is considered as a result of the \( \gamma \bar{e}^\pm \rightarrow e^\mp \) and \( \gamma \rightarrow e^+e^- \) processes in a strongly magnetized plasma, \( eB \gg T^2 \) at a temperature \( T \sim 1 \) MeV and chemical potential \( \mu = 0 \). We use the method applied in the field theory at finite temperatures and in plasma physics [7]. It consists in find-
ing a retarded solution of the electromagnetic field equation in the presence of an external source with allowance for vacuum polarization in magnetized plasma.

### 2. PHOTON PROPAGATION IN A MAGNETIZED MEDIUM

To describe the evolution of an electromagnetic wave $\mathcal{A}_\alpha(x)$, $x_\mu = (t, x)$, in time, we use the technique that was expounded in detail in [8]. Let us consider the linear response of the system ($\mathcal{A}_\alpha(x)$ and vacuum polarized in the magnetic field) to an external source which is switched on adiabatically at $t = -\infty$ and switched off at the time instant $t = 0$. At $t > 0$, the electromagnetic wave will develop independently. Thus, the source is necessary for creating the initial state. For this purpose, the source function should be chosen in the form

$$\mathcal{J}_\alpha(x) = j_\alpha e^{ikx}e^{i\omega(t)}; \quad \varepsilon \to 0^+.$$  

(1)

Here, $j_\alpha = (0, j)$, $j \cdot k = 0$, is the law of current conservation. As well, for simplicity, we consider the evolution of a monochromatic wave.

The dependence of $\mathcal{A}_\alpha(x)$ on time is determined by the equation

$$\left[ \partial_t^2 - \partial_x^2 \right] \mathcal{A}_\beta(x) + \int d^4x' \mathcal{P}_{\alpha\beta}(x-x')\mathcal{A}_\beta(x') = \mathcal{J}_\alpha(x),$$  

(2)

where $\mathcal{P}_{\alpha\beta}(x-x')$ is the polarization operator of the photon in the magnetic field and plasma. $q^\mu = (q_0, k)$ is the 4-vector of the photon pulse.

In a magnetized plasma, in the general case, the photon is ellipsically polarized and has three polarization states. However, in the limit of $B \gg B_c$ and charge-symmetric plasma ($\mu = 0$), the polarization vectors are the same as in a pure magnetic field up to $O(1/eB)$ and $O(\alpha^2)$ [9]:

$$\varepsilon^{(1)}_\alpha(q) = \frac{(q_0)p_\alpha}{\sqrt{q_0^2 + q_\perp^2}}, \quad \varepsilon^{(2)}_\alpha(q) = \frac{(q_0)\bar{p}_\alpha}{\sqrt{q_0^2 + q_\perp^2}}.$$  

(3)

Hereinafter, four-dimensional vectors with subscripts $\perp$ and $\parallel$ are related to the Euclidean and Minkowski subspaces $\{1,2\}$ and $\{0,3\}$, respectively, in the frame of reference where the magnetic field is directed along the third axis; $(ab)_\perp = (a\varphi_\beta b) = a_\alpha\varphi_\alpha\varphi_\beta b_\beta$ and $(ab)_\parallel = (a\varphi_\beta b) = a_\alpha\varphi_\alpha b_\beta \bar{b}_\beta$. $\varphi_{\alpha\beta} = F_{\alpha\beta}/B$ and $\varphi_{\alpha\beta} = \frac{1}{2}e_{\alpha\beta\mu\nu}\varphi_{\mu\nu}$ are the dimensionless tensor of the electromagnetic field and the dual tensor, respectively.

The solution of Eq. (2) for photons of modes $\lambda = 1, 2$ can be represented in the form

$$\mathcal{A}_\lambda(x) = V_\lambda^{(2)}(0, x) \Re F^{(\lambda)}(t),$$  

(4)

where

$$V_\lambda^{(2)}(0, x) = 2e^{ikx}e^{i(\lambda_0^1(x) + \lambda_0^2)},$$  

(5)

The function $F^{(\lambda)}(t)$ can be represented in the form of two summands:

$$F^{(\lambda)}(t) = F_{\text{pole}}^{(\lambda)}(t) + F_{\text{cut}}^{(\lambda)}(t),$$  

(6)

the first of which is determined by the residue at the point $q_0 = 0$ which is a solution of the dispersion equation, $q^2 - \mathcal{P}^{(\lambda)}(q) = 0$, in the kinematic region where the eigenvalue of the photon polarization operator, $\mathcal{P}^{(\lambda)}(q)$, is real. The second summand determines the dependence of the electromagnetic field on time in the region between cyclotron resonances and has the form of a Fourier integral:

$$F_{\text{cut}}^{(\lambda)}(t) = \int_{-\infty}^{\infty} \frac{dq_0}{2\pi} F_{\text{cut}}^{(\lambda)}(q_0) e^{-iq_0t},$$  

(7)

$$F_{\text{cut}}^{(\lambda)}(q_0) = \frac{2|\mathcal{P}^{(\lambda)}(q)|}{q_0(|q_0^2 - k^2 - R^{(\lambda)}|^2 + |I^{(\lambda)}|^2)},$$  

(8)

where $R^{(\lambda)} \equiv \Re \mathcal{P}^{(\lambda)}(q_0)$ is the real part of the photon polarization operator in magnetized plasma and $I^{(\lambda)} \equiv -\Im \mathcal{P}^{(\lambda)}(q_0 + i\epsilon)$ is its imaginary part. The imaginary part can be obtained from the photon absorption coefficient

$$W_{\text{abs}}^{(\lambda)}(\omega) = W_{\text{pol}}^{(\lambda)}e^{-\omega} + W_{\text{cut}}^{(\lambda)}e^{-\omega}.$$  

(9)

With allowance for processes of photon emission, (9) can be represented in the following form (see, e.g., [6, 10, 11]):

$$\Im \mathcal{P}^{(\lambda)} = 2q_{0\lambda}1 - \exp(-q_{0\lambda}/T)W_{\text{abs}}^{(\lambda)}.$$  

(10)

The expressions for $W_{\text{cut}}^{(\lambda)}e^{-\omega}$ for $\lambda = 1, 2$ can be obtained from [8] and represented in the following form:

$$W_{\text{cut}}^{(\lambda)}(\omega) = \frac{\alpha eB}{2q_{0\lambda}}$$  

$$\times \sum_{l = 0}^{\infty} \sum_{n = -\infty}^{\infty} \sum_{\epsilon = \pm 1} f_{L^l}^{(\lambda)}(1 - f_{L^l + L_{\epsilon}}^{(\lambda)})$$  

$$\times \{(L_{\epsilon})^2 - (L_{\epsilon}^2 + L_{\epsilon}^2 - q_0^2) = 2q_{0\lambda}^2M_l^2$$  

$$- 8eB\sqrt{nI_{L_{\epsilon} - L_{\epsilon}}I_{L_{\epsilon} - L_{\epsilon}}},$$  

$$W_{\text{cut}}^{(\lambda)}(\omega) = \frac{\alpha eB}{2q_{0\lambda}}$$  

$$\times \sum_{l = 0}^{\infty} \sum_{n = -\infty}^{\infty} \sum_{\epsilon = \pm 1} f_{L^l}^{(\lambda)}(1 - f_{L^l + L_{\epsilon}}^{(\lambda)})$$  

$$\times \{(L_{\epsilon})^2 - (L_{\epsilon}^2 + L_{\epsilon}^2 - q_0^2) = 2q_{0\lambda}^2M_l^2$$
\[
\times \left\{ \frac{(2eB(n - \ell))^2}{q^2} - 2eB(n + \ell) - 4m^2 \right\} \quad (12)
\]
\[
\times \left( I_{n,1}^2 + I_{n-1,1}^2 - 8eB\sqrt{nI_{n,1}I_{n-1,1}} \right)
\]
\[
E_\ell = \frac{1}{2q^2} \left[ q_0(M_n^2 - M_\ell^2 - q_0^2) \right]
\]
\[
+ \epsilon k_x \sqrt{(M_n^2 - M_\ell^2 - q_0^2)^2 - 4q_0^2M_\ell^2},
\]

where \( M_\ell = \sqrt{m^2 + 2eB\ell} \), \( f_\ell = \{\exp(E_\ell/T) + 1\}^{-1} \),

\[
I_{n,\ell}(x) = \frac{I_{\ell-1}(x)}{n!} e^{-x/2} x^{(n-\ell)/2} \,,
\]
\[
I_{\ell,n}(x) = (-1)^{n+\ell} I_{n,\ell}(x), \quad n \geq \ell,
\]

where \( L_n^\ell(x) \) are generalized Laguerre polynomials,

\[
n_0 = \ell + \left[ \frac{q_0^2 + 2M_\ell \sqrt{q_0^2}}{2eB} \right] \quad (14)
\]

\([x]\) is the integral part of \( x \).

Values of \( W^{(1)}_{\gamma e} \) can be obtained from (11) and (12) with the use of crossing symmetry.

The real part of the polarization operator can be reconstructed by its imaginary part using the dispersion relationship with a single subtraction:

\[
\Theta^{(1)}(t) = \int_0^\infty \text{Im}(\tilde{\Theta}(t'))dt' - \Theta^{(1)}(0), \quad t = q_0^2.
\]

Expressions (7)–(9) with allowance for (15) solve the problem about finding the time dependence of the photon wave function in the presence of a strongly magnetized plasma.

Strictly speaking, due to the threshold behavior of the Fourier image \( F_{c\ell}(q_0) \), the character of the time decay of the function \( F_{c\ell}(t) \) and, therefore, the wave function \( \mathcal{A}_{\ell,\mu}(t) \), differs from exponential. However, during a certain characteristic time interval \( (\sim [W^{(1)}_{\gamma e}]^{-1}) \), the dependence of the wave function can be approximately described as exponentially damping harmonic oscillations:

\[
\mathcal{A}_{\ell,\mu}(t) \sim e^{-i\omega t + \phi_0} \cos(\omega t + \phi_0).
\]

Here, \( \omega \) and \( \gamma(\lambda) \) are the effective frequency and mode \( \lambda \) photon absorption coefficient, respectively. They are to be found using (7)–(9) for each value of the pulse \( k \), which determines the effective law of photon dispersion in the region of its instability.

3. NUMERICAL ANALYSIS

The quantity \( \gamma(\lambda) \) that determines the intensity of \( \gamma \)-quantum absorption in the magnetic field due to the \( \gamma \rightarrow e^+e^- \) and \( \gamma e^\pm \rightarrow e^\pm \) processes plays an important part in astrophysical applications. The expression for the absorption coefficient usually used in astrophysics contains root singularities (see, e.g., \([12, 5]\)). As mentioned in \([6]\), this leads to overstating the intensity of creation of \( e^+e^- \)-pairs. Our analysis shows that calculation of the absorption coefficient (the decay width) using the complex solution on the second Riemann sheet \([6]\) also leads to a significantly overestimated result in the vicinity of cyclotron resonances \( \omega = 2m \) and \( \omega = \sqrt{m^2 + 2eB - m} \), as it is seen in Figs. 1 and 2. Our analysis demonstrates (see Figs. 1 and 2) that calculation of the absorption coefficient with allowance for the nonexponential character of the damping leads to a finite expression for the photon absorption coefficient in the vicinity of the resonances \( \omega = 2m \) and \( \omega = \sqrt{m^2 + 2eB - m} \).

As seen from Fig. 1 and formula (11) in a strongly magnetized plasma \( (B = 200B_c) \), the mode 1 photon will damp in the region \( 0 \leq \omega/2m \leq (1/2)(\sqrt{1 + 2B/B_c} - 1) \approx 9.5 \) and especially near its upper boundary, which kinematically corresponds to the process of photon absorption by an electron at the zero Landau level with electron creation at the first Landau level, \( \gamma e_0 \rightarrow e_1 \). In the region \( \omega/2m \geq (1/2)(\sqrt{1 + 2B/B_c} - 1) \), the mode 1 photon will remain quasi-stable (in our conditions, population of Landau levels with \( n > 1 \) is low).
As follows from Fig. 2 and formula (12), the mode 2 photon in the regions $0 \leq \omega/2m < 1$ and $\omega/2m \geq (1/2)(\sqrt{1 + 2B/B_e} - 1)$ will be quasi-stable, while in the region $1 \leq \omega/2m \leq 9.5$ and especially near its boundaries it will effectively damp, which kinematically corresponds to the process of creation of the $e^+e^-$-pair at the ground Landau level and absorption reaction $\gamma e_0 \to e_1$.

**4. CONCLUSIONS**

The process of electromagnetic wave propagation in a strongly magnetized charge-symmetric plasma has been studied. Taking into account the change in dispersion properties of photons in the magnetic field and plasma, it has been found that, similarly to the case of a pure magnetic field, the process of photon damping in magnetized plasma has a nonexponential character.

It has been shown that the obtained values for the photon absorption coefficient are significantly modified in the vicinity of the resonances $\omega = 2m$ and $\omega = \sqrt{m^2 + 2eB} - m$ compared to results known in the literature.

**FUNDING**

This work was supported by the Russian Foundation for Basic Research, project no. 20-32-90068, and within the framework of the “Comprehensive studies of complex physical systems” project no. AAAA-A16-116070610023-3.

**CONFLICT OF INTEREST**

The authors declare that they have no conflicts of interest.

**REFERENCES**

1. S. A. Olausen and V. M. Kaspi, Astrophys. J. Suppl. 212, 6 (2014).
2. C. Thompson, M. Lyutikov, and S. R. Kulkarni, Astrophys. J. 574, 332 (2002).
3. R. C. Duncan and C. Thompson, Astrophys. J. 392, L9 (1992).
4. A. Kostenko and C. Thompson, Astrophys. J. 869, 44 (2018).
5. A. Philippov, A. Timokhin, and A. Spitkovsky, Phys. Rev. Lett. 124, 245101 (2020).
6. A. E. Shabad, Tr. Fiz. Inst. Akad. Nauk SSSR 192, 5 (1988).
7. D. Boyanovsky, H. de Vega, D. Lee, et al., Phys. Rev. D: Part. Fields 59, 105001 (1999).
8. N. V. Mikheev and N. V. Chistyakov, JETP Lett. 73, 642 (2001).
9. M. V. Chistyakov and D. A. Rumyantsev, Int. J. Mod. Phys. A 24, 3995 (2009).
10. D. A. Rumyantsev, D. Shlenev, and A. Yarkov, J. Exp. Theor. Phys. 125, 410 (2017).
11. H. A. Weldon, Phys. Rev. D 28, 2007 (1983).
12. A. C. Harding, M. G. Baring, and P. L. Gonthier, Astrophys. J. 476, 246 (1997).

*Translated by A. Nikol’skii*