Exploration of The Duality Between Generalized Geometry and Extraordinary Magnetoresistance

Leo Rodriguez, 1, * Shanshan Rodriguez, 1, † Sathwik Bharadwaj, 2, ‡ and L. R. Ram-Mohan 2, §

1 Department of Physics, Grinnell College, Grinnell, IA 50112
2 Department of Physics, Worcester Polytechnic Institute, Worcester, MA 01609

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We outline the duality between the extraordinary magnetoresistance (EMR), observed in semiconductor-metal hybrids, and non-symmetric gravity coupled to a diffusive U(1) gauge field. The corresponding gravity theory may be interpreted as the generalized complex geometry of the semi-direct product of the symmetric metric and the antisymmetric Kalb-Ramond field: \( (g_{\mu\nu} + \beta_{\mu\nu}) \). We construct the four dimensional covariant field theory and compute the resulting equations of motion. The equations encode the most general form of EMR within a well defined variational principle, for specific lower dimensional embedded geometric scenarios. Our formalism also reveals the emergence of additional diffusive pseudo currents for a completely dynamic field theory of EMR. The proposed equations of motion now include terms that induce geometrical deformations in the device geometry in order to optimize the EMR. This bottom-up dual description between EMR and generalized geometry/physics lends itself to a deeper insight into the EMR effect with the promise of potentially new physical phenomena and properties.

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I. INTRODUCTION

Since the advent of the Anti-deSitter/Conformal Field Theory (AdS/CFT) correspondence of string theory, the implementation of gravity duality theories has seen a plethora of use for applications ranging from black hole thermodynamics in quantum gravity to phase transitions and their critical temperatures in condensed matter. The general idea of mapping strong-coupling problems to a weakly coupled gravity theory, where perturbative methods and/or analytic approaches are feasible, has spawned a relatively new paradigm in condensed matter physics.

The critical phenomena related to phase changes in strong coupling theory map back to black hole thermodynamic properties of specific solution spaces of the dual gravity theory. This permits the application of known perturbative techniques of a higher dimensional gravity theory in situations where field-theoretic or numerical methods fail for specific time-domains and/or critical temperatures. Conversely, certain classes of black hole solutions exhibit asymptotic two dimensional chiral conformal symmetries which generate their near-horizon structures. These symmetries allow for the computation of certain quantum gravitational properties of the respective black hole within known renormalizable CFT techniques, thus circumventing the ultraviolet behavior of four dimensional general relativity.

The discovery of the extraordinary magnetoresistance (EMR) effect in hybrid semiconductor-metal structures when cast in the framework of action integral formulation provides another venue towards the realization of the gravity-condensed matter interrelation. Though we should be careful with use of the word gravity, as our construction is not a traditional CFT/geometry correspondence; however, the interrelation described in this work is more reminiscent of an analogue gravity system.

We also note that our analogue model is rooted in a generalized geometric construction, which naturally encodes diffeomorphisms in conjunction with a stringy Kalb-Ramond two-form symmetry, and resulting conserved two-form currents relating to internal dynamical magnetic fields. Similar discoveries have been noted in the (differing but seemingly closely related) application of generalized global one-form symmetry and resulting conserved two-form currents in order to understand dissipative magnetohydrodynamics, holographic duals of specific strongly interacting plasmas, generalized elasticity theory and (more recently) holographic descriptions of the stable quantum matter phases (fracton states) via spin two U(1) gauge field coupled to emergent massless spin two states (gravity).

The EMR phenomenon is very sensitively dependent upon the position and width of the voltage and current ports, and more importantly, the device geometry. The EMR can be optimized by inducing geometrical deformations in the device geometry. The main goal of our duality construction is to provide a mathematical framework to address geometric/shape optimization of the device geometry.

As mentioned above, our construction is not strictly tied to low energy string theory and originates from the semiconductor side of the duality and draws upon specific semiconductor experimental constructs and results. The geometric nature of the correspondence is realized from the previous theoretical developments. In this setting, the conductivity tensor is seen to behave similar to a formulation by Ref. 9 and Ref. 10 on generalized geometry, we are able to construct a fully dynamical theory of all constituent fields of the
EMR effect. This bottom-up dual description, between EMR and generalized geometry/gravity, lends itself to a deeper insight into the EMR effect with potentially new physical phenomena and properties yet to be discovered.

In Sec. II, we briefly consider the phenomenon of EMR and show the van der Pauw configuration for measuring the EMR. The variational integral representation of the governing equation of motion provides an insight into the linking of EMR with gravity. This leads to the geometric sector in which the generalization to 4D considerations with gravity redefines the action to include gravity, the electromagnetic and Ramond-Nuevo-Schwarz \( U(1) \) symmetry. The new Maxwell equation implies a conserved current that is analogous to other examples of tensor currents discovered in recent works on combining gravity with magnetohydrodynamics, etc. Additionally, Sec. II contains all of our main results, as listed below:

- In Sec II C, the construction of a four dimensional covariant field theory of EMR and computation of all respective Euler-Lagrange equations, for a complete dynamical field content including geometry is done. Thus provides a computational avenue for shape optimization in EMR.
- In Sec II D, the discovery of emerging additional diffusive pseudo currents, complementing the fully dynamical field theory of EMR is given.
- The summary of our constructed duality, is detailed in Table I.
- In Sec II E, we break general diffeomorphism symmetry and compute the \((2 + 1)\) dimensional action and field equations for general axial symmetry.

The concluding remarks are delivered in Sec. III and we summarize results and definitions used from the generalized geometry formalism in Appendix A.

II. GENERALIZED GEOMETRIC EMR DUALITY

It has been shown experimentally\(^{20}\) that semiconductor thin films with metallic inclusions display EMR at room temperature, with remarkable enhancements as high as 100–750000 % at magnetic fields ranging from 0.05 to 4T. In the four-probe Hall measurements, the current through the device is held fixed while the voltage difference is measured across ports 3 and 4, as in Fig. 1. The magnetoresistance (MR) is defined as MR\( = \frac{R(H) - R(0)}{R(0)} \), where \( R(H) \) is the resistance at finite field magnetic field \( H \). With a steady current through the metal-semiconductor hybrid structure, using Ohm’s law we obtain \( [V(3) - V(4)]_{\text{in}} = IR(H) \). Thus the MR is determined by measuring the voltage difference across the device. The experiments were initially performed on a composite van der Pauw disk of a semiconductor matrix with an embedded metallic circular inhomogeneity that was concentric with the semiconductor disk. A similar enhancement has been reported\(^{21}\) for a rectangular semiconductor wafer with a metallic shunt on one side. The rectangular geometry with four contacts can be shown to be derivable from the circular geometry by a conformal mapping\(^{22}\). Magnetic materials and artificially layered metals exhibit the so-called giant magnetoresistance (GMR), and manganite perovskites show colossal magnetoresistance (CMR). However, patterned nonmagnetic InSb shows a much larger geometrically enhanced extraordinary MR even at room temperature. This has significant advantages in device design.

FIG. 1: A semiconductor wafer with a concentric metallic disk at the center is shown in Fig. 1a. In Fig. 1b The 4-probe van der Pauw arrangement is displayed with current \( I \) entering through port 1 and exiting through port 2. The contacts at ports 3 and 4 are used to measure the voltage drop across the device. (After Ref. 17).
A. Condensed Matter Sector

The MR can be calculated based on a diffusive current-field relation,

\[ \mathbf{J} = \mathbf{\dot{\sigma}} \cdot \mathbf{E}, \]  

where \( \mathbf{J}, \mathbf{E} \) and \( \mathbf{\dot{\sigma}} \) are the usual current density, electric field and conductivity-(tensor). In the presence of an external (constant) magnetic field \( \mathbf{H} = \beta / \mu \) (where \( \beta \) is a unit-less magnetic field and \( \mu \) the carrier mobility) the (magneto-)conductivity tensor in three dimensional Cartesian coordinates takes the form:

\[
\mathbf{\dot{\sigma}} = \frac{\sigma_0}{1 + |\beta|^2} \times
\begin{pmatrix}
(1 + \beta_x^2) & (-\beta_y + \beta_x \beta_z) & (\beta_y + \beta_x \beta_z) \\
(\beta_x + \beta_y \beta_z) & (1 + \beta_y^2) & (-\beta_x + \beta_y \beta_z) \\
(-\beta_y + \beta_x \beta_z) & (\beta_x + \beta_y \beta_z) & (1 + \beta_z^2)
\end{pmatrix},
\]  

where \( \sigma_0 \) is the intrinsic conductivity in the presence of zero external magnetic field. The carrier mobility is given by \( \mu = \frac{e\tau}{m^*} \), where \( e \) is the electron charge, \( \tau \) is the momentum relaxation time and \( m^* \) is the effective (electron) mass. Thus, the unit-less magnetic field \( \beta \), may equivalently be interpreted as the product; \( \beta = \omega_c \tau \), where \( \omega_c \) is the effective cyclotron frequency of carriers with mass \( m^* \). In 2D, with \( \mathbf{H} = \hat{x} H \) and \( \beta_z = \mu H \), we can reduce Eq. (2) to

\[
\mathbf{\dot{\sigma}} = \frac{\sigma_0}{1 + \beta_z^2} \begin{pmatrix} 1 & -\beta_z \\ \beta_z & 1 \end{pmatrix},
\]  

where only the 2D \((x, y)\) components are present.

A fast, robust and convergent variational approach for calculating the EMR for specific geometric cases in 2D and 3D structures has been developed through the variation of the action integral within the framework of finite element analysis\(^{16–18} \). Using the current continuity condition

\[ \nabla \cdot \mathbf{J} = 0, \]  

we obtain the scalar field equation

\[ -\nabla \cdot (\mathbf{\dot{\sigma}} \cdot \nabla \varphi) = 0, \]  

where \( \mathbf{E} = -\nabla \varphi \) and \( \varphi \) is the electric scalar potential. The action integral for which Eq. (5) is the corresponding Euler-Lagrange equation is given by:

\[
\mathcal{A}_0 = \frac{1}{2} \int d^4x \left( \nabla \varphi \cdot \mathbf{\dot{\sigma}} \cdot \left( \nabla \varphi \right) - \sqrt{-g} \right) = \frac{1}{2} \int d^4x \left\{ \sum_{i=1}^{3} \sum_{j=1}^{3} \sigma^{ij} \partial_i \varphi \partial_j \varphi \right\} = \frac{1}{2} \int d^4x \left( \partial_i \varphi \right) \sigma^{ij} \left( \partial_j \varphi \right),
\]  

where we have introduced index notation, and Einstein summation convention is used throughout this paper for repeated indices omitting the summation symbol. Latin indices will be reserved for spatial components and run from 1, 2, 3, while greek indices will run from 0, 1, 2, 3; where 0 denotes the temporal component. In the case of a four-probe Hall measurement, to measure MR in a composite structure, two additional surface terms are added to the action functional to specify the incoming and outgoing currents.

Looking at Eq. (6), we see that by recasting our initial diffusive equation into a variational form, the conductivity tensor now plays the role of a matrix encoding geometry, commonly referred to as the inverse metric of the gravitational field\(^{24, 25} \). We can now proceed with a functional variational approach to geometric optimization by constructing an appropriate action involving the conductivity tensor given in Eq. (2).

The resistivity, denoted as \( \tilde{\mathcal{G}}_{ij} \), is given by the inverse of Eq. (2). This yields:

\[
\tilde{\mathcal{G}}_{ij} = \frac{\sigma_0}{\sigma_0} \left[ \mathcal{G}_{ij} + \beta_{ij} \right],
\]  

where \( \mathcal{G}_{ij} \) is a 3D metric tensor equal to a diagonal matrix, with not necessarily equal diagonal components, depending on choice of coordinates and

\[
\beta_{ij} = \begin{pmatrix} 0 & \beta_x & -\beta_y \\ -\beta_x & 0 & \beta_z \\ \beta_y & -\beta_z & 0 \end{pmatrix}.
\]  

Further, we wish to recast Eq. (6) into a covariant form (invariant under general coordinate transformations). First, we introduce the electromagnetic field curvature tensor, which in Cartesian coordinates is given by

\[
F_{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y \\ E_x & 0 & -B_z \\ E_y & B_z & 0 \end{pmatrix},
\]  

and relates to the gauge-field four vector potential \( A_\mu = (-\varphi, A) \), which is a combination of both the electric scalar \( \varphi \) and magnetic vector \( \mathbf{A} \) potentials in a four vector formalism, via

\[
F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.
\]  

Clearly,

\[
F_{\mu\nu} = -F_{\nu\mu},
\]

\[
F_{\mu i} = (\partial_i A + \nabla \varphi)_i = -E_i,
\]

\[
F_{ij} = \partial_i A_j - \partial_j A_i = \epsilon_{ij}^k B_k,
\]

in units where the speed of light is given by \( c = 1 \) and where \( \epsilon_{ij}^k \) is the Levi-Civita symbol, or the totally antisymmetric permutation symbol:

\[
\epsilon_{ij}^k = \begin{cases} +1 & (i, j, k) = (1, 2, 3) \text{even permutations} \\ -1 & (i, j, k) = (1, 2, 3) \text{odd permutations} \\ 0 & \text{any } i = j, i = k, j = k \end{cases}
\]
Next, we introduce the four dimensional resistivity $\tilde{\sigma}_{\mu\nu} = (1/\sigma_0) [g_{\mu\nu} + \beta_{\mu\nu}]$, where $\tilde{G}_{ij}$ is as defined in Eq. (7), and the added temporal components are

$$\tilde{G}_{00} = -\frac{1}{\sigma_0}, \quad \tilde{G}_{i0} = \sigma_0 = 0 \quad (13)$$

and the inverse condition reads:

$$\tilde{G}_{\mu\nu} \sigma^{\mu\nu} = \delta_{\mu\nu}. \quad (14)$$

Hence, following from Eq. (2), (7), (14) we have

$$\tilde{G}_{\mu\nu} = \frac{1}{\sigma_0} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & \beta_z & -\beta_y \\ 0 & -\beta_z & 1 & \beta_x \\ \beta_y & -\beta_x & 1 & 0 \end{pmatrix} \quad (15)$$

and

$$\sigma^{\mu\nu} = \frac{\sigma_0}{1 + |\beta|^2} \begin{pmatrix} -1 + |\beta|^2 & 0 & 0 & 0 \\ 0 & (1 + \beta_z^2) & (\beta_y + \beta_z \beta_x) & (\beta_y + \beta_z \beta_x) \\ 0 & (\beta_z + \beta_y \beta_x) & (1 + \beta_y^2) & (-\beta_x + \beta_y \beta_z) \\ 0 & (\beta_y + \beta_z \beta_x) & (-\beta_y + \beta_z \beta_x) & (1 + \beta_x^2) \end{pmatrix}. \quad (16)$$

With these definitions in hand we may recast Eq. (6) into a covariant form via the transformations $d^4x \rightarrow d^4x \sqrt{-g}$, $E$ and $B \rightarrow F_{\mu\nu}$ yielding the action

$$S_{A_0} = -\frac{\lambda}{4} \int d^4x \sqrt{-g} \sigma^{\alpha\beta} F_{\alpha\beta}, \quad (17)$$

where $\lambda$ is an arbitrary coupling to be determined by requiring Eq. (17) to reduce to Eq. (6) in the steady state and $g = \det (g_{\mu\nu})$. To see this, we may expand Eq. (17) in steps (\mu and \nu first), yielding

$$S_{A_0} = -\frac{\lambda}{4} \int d^4x \sqrt{-g} \left\{ \sigma^{00} \sigma^{\alpha\beta} F_{0\alpha} F_{0\beta} + 2 \sigma^{0i} \sigma^{\alpha\beta} F_{0\alpha} F_{i\beta} + \sigma^{ij} \sigma^{\alpha\beta} F_{i\alpha} F_{j\beta} \right\}. \quad (18)$$

The middle term vanishes due to the definitions in Eq. (11) and Eq. (13) and the above equation reduces to

$$S_{A_0} = -\frac{\lambda}{4} \int d^4x \sqrt{-g} \left\{ \sigma^{00} \sigma^{\alpha\beta} F_{0\alpha} F_{0\beta} + \sigma^{ij} \sigma^{\alpha\beta} F_{i\alpha} F_{j\beta} \right\}. \quad (19)$$

Next, expanding in $\alpha$ and $\beta$ we obtain

$$S_{A_0} = -\frac{\lambda}{4} \int d^4x \sqrt{-g} \left\{ \sigma^{00} \sigma^{00} F_{00} F_{00} + 2 \sigma^{00} \sigma^{0i} F_{00} F_{0i} + \sigma^{00} \sigma^{ij} F_{0i} F_{0j} + \sigma^{ij} \sigma^{i0} F_{i0} F_{j0} + \sigma^{ij} \sigma^{jr} F_{ir} F_{js} \right\}. \quad (20)$$

B. Geometric Sector

The overarching goal is to optimize shape or geometry of the metallic insulator/semiconductor such that we obtain desired enhancements to the EMR. This can be best accomplished within an action principle, where the functional variation of the action yields the Euler-Lagrange equations whose solutions are stationary points. The task now is to construct an additional terms to the action integral that determines the geometric shape. We interpret $\tilde{G}_{\mu\nu}$ defined in Eq.(15) as a metric deformation of a generalized geometric structure (algebroid)$^9$. In other words, $\tilde{G}_{\mu\nu}$ is the weighted sum of two fields, $g_{\mu\nu}$ which is symmetric and defines geometry, and $\beta_{\mu\nu}$ which is antisymmetric and encodes the components of the external
magnetic field. The remaining task now is to construct an action for $\tilde{G}_{\mu\nu}$ by looking to string theory as a guide, and implementing techniques of generalized geometry as reviewed in Appendix A.

The metric deformation:

$$\hat{G}_{\mu\nu} = e^{2\psi} (g_{\mu\nu} + \beta_{\mu\nu}) ; \quad (23)$$

includes three fields; $g_{\mu\nu}$ a symmetric Riemannian metric, $\beta_{\mu\nu}$ akin to the antisymmetric Kalb-Ramond field and the dilation $\psi$.\textsuperscript{26} By Riemannian metric, we imply a metric $g_{\mu\nu}$ that is covariantly constant:

$$\nabla_\alpha g_{\mu\nu} = 0, \quad (24)$$

with respect to the covariant derivative $\nabla_\alpha$, such that:

$$(\nabla_\alpha - \partial_\alpha) A_\beta = \Gamma_\alpha^\beta_{\mu\nu} A^\mu. \quad (25)$$

In other words, for auto parallel transport of some vector $A^\mu$ across a smooth manifold, the respective change in $A^\mu$ is measured by the Levi-Civita connection (unique, symmetric and metric compatible\textsuperscript{27}) coefficients $\Gamma_\alpha^\beta_{\mu\nu}$, determined by its Christoffel variant:

$$\Gamma_\alpha^\beta_{\mu\nu} = \frac{1}{2} g^{\gamma\gamma} (\partial_\mu g_{\nu\gamma} + \partial_\nu g_{\mu\gamma} - \partial_\gamma g_{\mu\nu}). \quad (26)$$

Thus from Eq. (25), we see that the covariant derivative $\nabla_\alpha A^\beta = \partial_\alpha A^\beta + \Gamma_\alpha^\beta_{\mu\nu} A^\mu$ is symmetric with respect to general diffeomorphisms.\textsuperscript{28} The Kalb-Ramond field can be naturally interpreted as a rank two antisymmetric field strength given by its exterior (completely antisymmetric) derivative:

$$d\beta = H, \Rightarrow H_{\mu\nu\alpha} = \partial_\mu \beta_{\nu\alpha} + \partial_\nu \beta_{\mu\alpha} + \partial_\alpha \beta_{\mu\nu}. \quad (27)$$

This is analogous to the electromagnetic field strength tensor (see Eq. (11)) $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, coming from the exterior derivative of the rank one gauge-field potential $A_\mu$.

One possible route, in constructing a geometric sector action, may be taken by reinterpreting the above three fields as background fields interacting with a closed bosonic string. This scenario is commonly known as a 2 dimensional non-linear sigma model with a world sheet action $S = S_1 + S_2 + S_3$ comprised of the standard three parts:

$$\begin{align*}
S_1 &= -\frac{1}{4\pi\alpha'} \int d^2\xi \sqrt{h} h^{ab} \partial_a X^\mu \partial_b X^\nu g_{\mu\nu}(X) \\
S_2 &= -\frac{1}{4\pi\alpha'} \int d^2\xi \epsilon^{ab} \partial_a X^\mu \partial_b X^\nu \beta_{\mu\nu}(X) \quad , \quad (28) \\
S_3 &= \frac{1}{4\pi} \int d^2\xi \sqrt{h} R^{(2)} \psi(X)
\end{align*}$$

where $\alpha'$ is the string coupling, $\xi^a = \{\tau, \sigma\}$ are the world sheet coordinates (parameters), $h_{ab}$ is the world sheet metric, $X^\mu$ are the target spacetime coordinates, $g_{\mu\nu}(X)$ is the target spacetime metric as a function of $X^\mu$, $\beta_{\mu\nu}(X)$ is the Kalb-Ramond field as a function of $X^\mu$, $R^{(2)}$ is the induced Ricci scalar curvature of the string world sheet, and $\psi(X)$ is the dilation field as a function of $X^\mu$. We should note that $S_3$ above breaks the desired classical conformal invariance of the respective 2 dimensional non-linear sigma model. However, a standard renormalization group flow analysis\textsuperscript{29} restores conformal symmetry at the quantum field theoretic level by interpreting the resulting one-loop beta functions as field equations, i.e.

$$\begin{align*}
B^g_{\mu\nu} &= R_{\mu\nu} + \frac{1}{4} H_{\mu}^{\lambda\rho} H_{\nu}^{\rho\lambda} - 2 \partial_\mu \partial_\nu \psi = 0, \\
B^\beta_{\mu\nu} &= \nabla_\lambda H^\lambda_{\mu\nu} - 2 \nabla_\lambda \psi H^\lambda_{\mu\nu} = 0, \\
B^\psi &\equiv \frac{1}{\alpha'} (\nabla \psi)^2 - 4 \nabla_\mu \psi \nabla^\mu \psi + R + \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} = 0, \quad (29)
\end{align*}$$

where $R$ is the target spacetime Ricci scalar curvature of $g_{\mu\nu}$. The above field equations may be derived, up to total derivatives, via field variations with respect to $g^{\mu\nu}$, $\beta_{\mu\nu}$ and $\psi$ from the spacetime closed-string effective action:

$$S_{\text{eff}} = -\frac{1}{2\alpha'} \int d^2x \sqrt{-g} e^{-2\psi} \left\{ R - 4 (\nabla \psi)^2 + \frac{1}{12} H^2 \right\}, \quad (30)$$

where $H^2 = H_{\mu\nu\rho} H^{\mu\nu\rho}$. Note that the above action requires a spacetime dimension of 26, in order for the conformal anomaly of $S_3$ to vanish in $B^\psi$.\textsuperscript{29}

Alternatively for a more symmetric and geometric approach, and one that is not necessarily rooted/dependent on the 2 dimensional non-linear sigma model paradigm, we chose to implement the generalized geometric approach of Refs. [30] and [31] (see Appendix A) in order to construct our action principle. The generalized geometric approach has also been shown to reproduce the same effective action of Eq. (30) by interpreting the generalized Ricci tensor as a field equation, however for pure symmetry reasons, we are motivated in interpreting an Einstein-Hilbert action of the generalized metric as our final geometric sector action.

For simplicity and continuity with Appendix A, we will begin with a general metric-deformation given by

$$G_{\mu\nu} = g_{\mu\nu} + \beta_{\mu\nu} \quad (31)$$

and after constructing the action for $G_{\mu\nu}$ it will be a simple exercise to determine the action for $\hat{G}_{\mu\nu}$ by conformally transforming the action for $G_{\mu\nu}$. Looking to diffeomorphism symmetry and gravity theory/string theory as a guide, we imagine the Euler-Lagrange equation of motion for a generalized geometric metric to be given by an Einstein type field equation which is stationary for the respective generalized Einstein-Hilbert action\textsuperscript{32};

$$S_{\tilde{G}} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} R, \quad (32)$$
where $R$ is the Ricci scalar curvature of the general metric connection $\nabla\psi$ and $1/(2\kappa^2)$ is an arbitrary coupling. The computation of $R$ in terms of $\nabla\psi$ (here $\nabla$ without a vector symbol denotes the covariant derivative connection) is detailed in Appendix A and reads

$$R = R^{LC} - \frac{1}{4} \mathcal{H}_{\mu\rho\alpha\beta} \mathcal{H}^{\mu\rho\alpha\beta},$$

and $\mathcal{H}$ is determined in Eq. (27) in terms of the Kalb-Ramond field $\beta_{\mu\nu}$.

Looking back at Eq. (7) we see that the case for the diffusive current field relation is actually a conformally scaled version of $G$, i.e.,

$$G_{\mu\nu} = e^{2\psi} G_{\mu\nu} - \frac{4}{\sigma_0} \left( \nabla \mu \psi \nabla \nu \psi \right),$$

where $\psi$ is an arbitrary function, $e^{2\psi}$ is a general conformal factor and in our case $e^{2\psi} = 1/\sigma_0$. Under the above generalized conformal transformation we have

$$\sqrt{-g} = e^{4\psi} \sqrt{-\tilde{g}}$$

and thus Eq. (32) becomes

$$S_{\tilde{\phi}} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} e^{2\psi} \left\{ R^{LC} - 18 \nabla \mu \psi \nabla \nu \psi + \frac{1}{4} \left[ \mathcal{H}^{2} - 12 \mathcal{H}_{\mu\rho\alpha\beta} \mathcal{H}^{\mu\rho\alpha\beta} + \right. \right. \left. \left. 12 \left( \beta^{2} \nabla \mu \psi \nabla \nu \psi + 2 \beta_{\mu\nu} \nabla \alpha \psi \nabla \nu \psi \right) \right] \right\},$$

and thus Eq. (37) becomes

$$S_{\tilde{\phi}} = \frac{e^{2\psi_0}}{2\kappa^2} \int d^4x \sqrt{-g} \left\{ R^{LC} - \frac{1}{4} \mathcal{H}^{2} \right\},$$

up to total derivative terms. Again, from Eq. (7) the above action simplifies drastically for $\psi \rightarrow \psi_0$ and Eq. (37) becomes

$$S_{\tilde{\phi}} = \frac{e^{2\psi_0}}{2\kappa^2} \int d^4x \sqrt{-g} \left\{ R^{LC} - \frac{1}{4} \mathcal{H}^{2} \right\}.$$

C. The Total Action

Collecting results Eq. (17) and Eq. (38) we have the total action

$$S_{\text{total}} = S_{\tilde{\phi}} + S_{\mathcal{A}_{\mu}} + \frac{e^{2\psi_0}}{2\kappa^2} \int d^4x \sqrt{-g} \left\{ R^{LC} - \frac{1}{4} \mathcal{H}^{2} \right\} - \frac{\lambda}{4} \int d^4x \sqrt{-g} \sigma_{\mu\nu} \sigma_{\sigma\beta} F_{\mu\alpha} F_{\nu\beta}.$$

There are three equations of motion obtained by performing functional variations with respect to $g^{\mu\nu}$, $A_{\mu}$ and $\beta_{\mu\nu}$. Implementing the functional variational relationships

$$\delta \sigma^{\beta\nu} = \frac{1}{\sigma_0} \sigma_0 g_{\alpha\lambda} g_{\mu\rho} \sigma^{\mu\nu} \delta g^{\lambda\rho},$$

$$\delta \sigma^{\beta\nu} = - \frac{1}{\sigma_0} \sigma_0 g^{\nu\beta} \sigma^{\alpha\beta} \delta \beta^{\alpha\nu},$$

we obtain the following field equations:

$$\frac{\delta S_{\text{total}}}{\delta g^{\mu\nu}} = 0 \Rightarrow \frac{e^{2\psi_0}}{2\kappa^2} \left\{ \left( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) + \frac{3}{8} g_{\mu\nu} \mathcal{H}^{2} - \frac{3}{4} \mathcal{H}_{\mu\alpha\beta} \mathcal{H}^{\mu\alpha\beta} \right\} + \frac{\lambda}{4} \left( \frac{1}{2} g_{\mu\nu} F_{\sigma}^{2} + \frac{2}{\sigma_0} \sigma^{\alpha\beta} g_{\alpha\lambda} g_{\mu\rho} \sigma^{\nu\beta} \sigma_{\gamma\rho} F_{\lambda\gamma} F_{\rho\sigma} \right) = 0,$$

$$\frac{\delta S_{\text{total}}}{\delta A_{\alpha}} = 0 \Rightarrow \nabla_{\mu} \left( \sigma^{\alpha\beta} \sigma^{\nu\mu} F_{\beta\nu} \right) = 0,$$

$$\frac{\delta S_{\text{total}}}{\delta \beta_{\mu\nu}} = 0 \Rightarrow \frac{3 e^{2\psi_0}}{2\kappa^2} \nabla_{\mu} \mathcal{H}_{\mu\nu\alpha} + \frac{\lambda}{\sigma_0} \sigma^{\alpha\rho} \sigma^{\mu\nu} \sigma^{\beta\gamma} F_{\rho\gamma} F_{\nu\beta} = 0,$$

where $F_{\sigma}^{2} = \sigma^{\mu\nu} \sigma^{\alpha\beta} F_{\mu\alpha} F_{\nu\beta}$. Equation (42) is identical to (or encodes) Eq. (4) and Eq. (5) for $B_i = 0$, however now includes additional dynamics for the possibility of a non-zero internal magnetic field. In effect, we have now enhanced the EMR geometric optimization problem to four field equations above. They all are coupled partial differential equations containing geometry $g_{\mu\nu}$, electromagnetism $F_{\mu\nu}$ and external magnetic field $\beta_{\mu\nu}$. In the limiting case for constant $g_{\mu\nu}$ and $\beta_{\mu\nu}$ all the curvature terms involving $R_{\mu\nu}$ and $\mathcal{H}_{\mu\nu\alpha}$ are identically zero and Eq. (41) and Eq. (43) reduce to the on-shell condition of Eq. (6). This completes the construction of our generalized-geometry/EMR duality, which provides a fully dynamical theory of constituent fields influencing the EMR phenomena. We should note that the dilation field $\psi$, normally a dynamical field on the string-theory
side, is non-dynamical in our specific duality. This is due to the material properties of the semiconductor under the influence of zero external magnetic fields.

Now, it may be that for a given experiment/design the conductivity tensor and external magnetic field will be “non-dynamical”. However, and as stated earlier, the conductivity tensor (and thus the EMR) depends on the semiconductor shape. Additionally, using time dependent, sinusoidally and radially varying external magnetic fields have become of interest in how they may enhance EMR. The dynamical nature of these fields within our construction allows for the determination, via a variational principle, and selection of the optimum combinations of each field and thus provides parameters for an optimum design for enhancing EMR. Finally, for this section, we summarize our duality construction in Table I.

D. Current Density Tensor

The Maxwell equation, Eq. (42), implies a conserved quantity:

\[ \nabla_\mu (\sigma^{\alpha\beta} \sigma^{\mu\nu} F_{\beta\nu}) = \nabla_\mu (J^{\alpha\mu}) = 0, \]

and thus

\[ J^{\alpha\mu} = \sigma^{\alpha\beta} \sigma^{\mu\nu} F_{\beta\nu}. \]

The above tensor current is antisymmetric \((J^{\mu\nu} = -J^{\nu\mu})\) and in the constant \(g_{\mu\nu}\) and \(\beta_{\mu\nu}\) case:

\[ J^{0i} = \sigma_0 J^i, \]

where \(J^i\) is the electric current in Eq. (1) and

\[ J^{ij} = \sigma_0 \ell^{ij} k J^B, \]

where \(J^B\) is a pseudo-current and is a new dynamical feature of the EMR/Geometry duality, but is identically zero for zero internal magnetic field \(\vec{B}\).

The concept of second rank antisymmetric current densities induced by time independent internal magnetic fields was first considered and discussed in\(^{38}\). A more recent exposition\(^{34}\) shows that these antisymmetric current densities are fundamental features of materials with internal magnetic fields such as nuclear dipole moments, and in conjunction with magnetizability, nuclear magnetic shielding and spin-spin coupling fully characterize magnetic perturbation responses. A similar analogy can be drawn for our consideration as well, since the pseudo-current here originates from the internal magnetic field. We present a fundamental origin of these currents, within a well defined symmetry principles and the action integral framework.

An additional interesting analogue including positive and negative magnetoelastic phenomena has been presented in Ref. [30] and [31]. In these studies electrons, in clean metals and under certain conditions, are seen to behave collectively as a fluid with magnetohydrodynamic properties, including viscosity, heat transport and entropy. Each of these included properties are accompanied by specific conserved currents which contain pseudo contributions. A comprehensive study of the multi-form symmetries of these currents should yield some interesting theoretical frameworks closely related to the one presented here and the overarching fluid-MHD/gravity correspondence\(^{11,12,35}\).

E. \((2 + 1)\)-Action Ansatz and Equations of Motion

In this section we will break diffeomorphism symmetry of Eq. (39) in order to construct a model applicable to device designs exhibiting a constant axially symmetric constant external magnetic field. Typically, experiments for measuring EMR are performed on a semiconductor wafer having a two-dimensional metal semiconductor hybrid structure. Hence, we present the equations of motion specific to a 2D configuration space. We are interested in applying our new EMR formalism to a similar scenario as analyzed in\(^{17,18}\). We will need to perform a dimensional reduction to the total action of Eq. (39) to \(2 + 1\) dimensions, in conjunction with a cylindrical coordinate choice as in\(^{17,18}\). We begin with our symmetric Riemannian metric ansatz, given by:

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -f(r) dt^2 + f^{-1}(r) dr^2 + \ell^2 e^{-2\psi(r)} d\varphi^2 + dz^2, \]

where \(\ell\) is an arbitrary length scale. The generalized metric takes the form:

\[ \tilde{g}_{\mu\nu} = \frac{1}{\sigma_0} \begin{pmatrix} -f(r) & 0 & 0 & 0 \\ 0 & f(r) & r\beta_\zeta & 0 \\ 0 & -r\beta_\zeta & \ell^2 e^{-2\psi(r)} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \]

where we have performed a cylindrical coordinate transformation to Eq. (8), while setting \(\beta_\zeta = \text{constant}\) and \(\beta_\psi = 0\), to coincide with the scenario of\(^{17,18}\). For these above choices, we have \(H = 0\) identically, and the conductivity tensor takes the form:


We make the final ansatz for the $U(1)$ gauge field to be given by:

$$A_\mu = (-\varphi(r), 0, 0, 0).$$  

Next, we will perform a dimensional reduction of the action Eq. (39) from (3+1) dimensions to (2+1), by integrating out $z$ from zero to some arbitrary length (height) scale $L$. This is a convenient choice, since none of the fields in our theory have explicit $z$ dependence and thus all curvature invariants in Eq. (39) are identical in terms of our ansätze $f(r)$, $\psi(r)$ and $\varphi(r)$. Thus the dimensionally reduced action is identical in form to Eq. (39) modulo an overall factor of $L$:

$$S_{\text{total}}^{(2+1)} = S_{\tilde{G}}^{(2+1)} + S_{A_0}^{(2+1)}$$

$$= \frac{L e^{2\psi_0}}{2\kappa^2} \int d^3x \sqrt{-g} \mathcal{R}^{(3)}_{(3)}$$

$$= \frac{L \lambda}{4} \int d^3x \sqrt{-g} \mathcal{R}^{(3)} \mathcal{R}^{(3)}_{(3)}$$

$$= \frac{L e^{2\psi_0}}{2\kappa^2} \int d^3x \mathcal{R}^{(3)}_{(3)} \left( 2f'(r)\psi'(r) - 2f(r)\psi'(r)^2 - f''(r) + 2f(r)\psi''(r) \right)$$

$$A_{(3)} = (-\varphi(r), 0, 0)$$

and in terms of the $(2+1) = (3)$ dimensional invariants generated by the effective fields:

$$ds^2 = g^{(3)}_{\mu\nu} dx^\mu dx^\nu$$

$$= -f(r)dt^2 + f^{-1}(r)dr^2 + \ell^2 e^{-2\varphi(r)} d\varphi^2,$$  

$$A^{(3)}_\mu = (-\varphi(r), 0, 0)$$

and
\[
\sigma^{\mu\nu}_{(3)} = \sigma_0 \begin{pmatrix}
-\frac{1}{f(r)} & 0 & 0 \\
0 & \ell^2 f(r) & e^{2\psi(r)} r^2 \beta_z f(r) \\
0 & \ell^2 + e^{2\psi(r)} r^2 \beta_z f(r) & \ell^2 + e^{2\psi(r)} r^2 \beta_z f(r)
\end{pmatrix}
\]

(54)

The dimensionally reduced action has two general equations of motion; the Einstein field equations, which come from variation with respect to \(g^{\mu\nu}_{(3)}\), and the Maxwell equations, which come from variations with respect to \(A^\mu_{(3)}\):

\[
\frac{\delta S_{(2+1)}^{total}}{\delta g^{\mu\nu}_{(3)}} = 0 \Rightarrow \begin{align*}
\delta S_{(2+1)}^{total} &= \\
\frac{\lambda \sigma_0^2 \ell^2}{\kappa^2} \left( \beta_z^2 e^{2\psi(r)} r^2 f(r) - \ell^2 \right) \psi'(r)^2
+ \frac{\lambda \sigma_0^2 \ell^2}{\kappa^2} \left( \beta_z^2 e^{2\psi(r)} r^2 f(r) + \ell^2 \right) \psi'(r)^2
- \frac{e^{2\psi_0} f'(r) \psi'(r)}{\kappa^2}
= 0
\end{align*}
\]

(55)

and

\[
\frac{\delta S_{(2+1)}^{total}}{\delta A^\mu_{(3)}} = 0 \Rightarrow \beta_z^2 e^{2\psi(r)} r^2 f'(r) \psi'(r) + \ell^2 \left( \psi'(r) \psi''(r) - \psi''(r) + \beta_z^2 e^{2\psi(r)} r f'(r) (\psi'(r) (2 + 3 r \psi'(r)) - r \psi''(r)) \right) = 0.
\]

(56)

The above equations are completely new results in the effort to address the effects of dynamical geometrical deformations in optimizing the EMR, and are not obtainable within the diffusive current-field relation, Eq. (1), alone. Despite specifying a constant external magnetic field, the metric is still dynamical for the initially specified symmetries. A general solution to the above dimensionally reduced action and resulting equations of motion is currently in progress using numerical techniques within the finite element analysis approach\(^{18}\), with results to follow in a forthcoming manuscript\(^{36}\).

As a proof-of-concept we consider a pure simplistic geometry mimicking the device geometry depicted in Fig. 1, for which

\[
f(r) = 1,
\psi(r) = \frac{1}{2} \ln \frac{\ell^2}{r^2},
\]

(57)

and solve the dimensionally reduced action integral with the above assumptions within the framework of finite element analysis\(^{18}\). The device geometry is discretized into a refined finite element mesh. Within each element, we express the potential function as a linear combination of Hermite interpolation polynomials multiplied by as-yet undetermined coefficients. Using the principle of stationary action, variation of the action integral with respect to these undetermined coefficients results in a set of linear equations, which are then solved using sparse matrix solvers.

In Fig. 2a and 2b, we plot the potential function obtained for the two cases with a constant applied magnetic field \(H = 0\) T, and \(H = 1\) T, respectively. We note that for a non-zero external magnetic field, the potential gradient (current density) clearly shows that the current is expelled\(^{16}\) from the metallic region into the semiconductor region, as shown in Figs. 3a and 3b and in accordance with Ref. 18. This is because, while \(E\) is always perpendicular to the metal surface, at \(H \neq 0\), \(J\) deflects away from \(E\) by the amount of the Hall angle. Hence, the magnetoresistance increases significantly with the applied external magnetic field. With three orders of magnitude
difference between the conductivities of semiconductor and the metallic inclusion. The EMR will be very sensitive to the position and width of the voltage and current ports, and more importantly to the device geometry.

III. CONCLUDING REMARKS AND OUTLOOK

Here, we have recognized the existence of a metric deformation of a generalized geometric structure for the magnetoconductivity tensor, that arises in the diffusive model for the current-electric field relation. We have derived a new action integral and the corresponding field equations which encode contributions from the condensed matter and the geometrical sector. This is a bottom-up dual description which provides a fundamental alternative to the inter-relation between condensed matter and CFT/gravity. Though we have motivated this approach through the EMR effect, our approach is very general since we begin with the basic current continuity condition. This should be of interest for a variety of geometrical and material optimization problems with semiconductor-metal hybrid structures in inhomogenous magnetic fields for wider device applications such as magneto-sensors, readheads, and the like.

Finally, some immediate questions/comments for fu-
ture work have arisen:

- The resulting field equations of Section II E are currently being analyzed via the finite element analysis paradigm for a specific choice of geometric optimizations, with final results to be presented in a forthcoming publication\textsuperscript{36}.

- In our construction, we did not address gauge invariance of the Kalb-Ramond field in Eq. (39) ($\beta_{\mu\nu}$). Requiring this symmetry should induce an additional conserved two-form current ($J_2^{\mu\nu}$) as a response from the condensed matter sector.

- The study of the above gauge symmetry might give a relationship to global one-form (or even higher-form) symmetry and their resulting conserved currents, a potentially very interesting relationship that needs to be explored, and thus relating to Ref. \textsuperscript{[11–13]}.  

- Now that we have a full field theory of EMR, the relatively unexplored realm of quantum enhancements of EMR can be explored via path integral tree level perturbation theory from the perspective of EMR. 

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### Appendix A: Generalized Geometry Redux

This section is not intended to provide a full pedagogical introduction to the generalized geometry formalism, but is included for completeness and to demonstrate where the geometric sector action functional Eq. (38) originates from. For a comprehensive educational introduction we refer to the dissertation works of Gualtieri\textsuperscript{19} and Vysoky.\textsuperscript{9}

As mentioned before, the inverse of $\hat{\sigma}$ in Eq. (7) may be interpreted as the metric deformation of a generalized geometric structure (algebroid).\textsuperscript{9} In this setting and following the notation of Ref. [9] and Ref. [10] we consider a general bundle $E = T(M) \oplus T^*(M)$ of the manifold $M$, such that

- $T(M)$ is the tangent bundle, and
- $T^*(M)$ is the co-tangent bundle.

Thus, a smooth section $e \in \Gamma(E)$ is the direct sum

\[ e = X + \xi, \]  

where $X = X^\mu \partial_\mu$ is a vector and $\xi = \xi_\mu dx^\mu$ is a 1-form. A natural pairing invariant under $O(d,d)$ rotations, where $d$ is the dimension of $M$, is given by

\[ \langle e_1, e_2 \rangle = \langle X + \xi, Y + \eta \rangle = i_\gamma \xi + i_\chi \eta \]

\[ = Y^\nu \xi_\mu + X^\mu \eta_\mu, \]  

where $i$ denotes the interior product. A bracket structure (similar to a Lie bracket, but not identical) is given by the Dorfman bracket

\[ [e_1, e_2]_D = [X,Y]_{Lie} + L_X \eta - i_y d\xi, \]  

where $[..]_{Lie}$ is the standard Lie bracket between vectors, $L$ is the Lie derivative and $d$ is the exterior derivative, i.e.,

\[ ([X,Y]_{Lie})^\nu = X^\mu \partial_\mu Y^\nu - Y^\mu \partial_\mu X^\nu \]

\[ (L_X \eta)^\nu = X^\mu \partial_\mu \eta_\nu + \eta_\mu \partial_\mu X^\nu \]

\[ (i_y d\xi)^\nu = Y^\mu \xi_\mu + X^\mu \eta_\mu. \]  

Next, we define the anchor map $a : E \to T(M)$, as the projection

\[ a(e) = a(X + \xi) = X. \]  

Now, for calculational purposes, the collection $\{ E, \langle \cdot, \cdot \rangle; [\cdot, \cdot]_D; a \}$ forms a Courant algebroid such that the following is specifically satisfied

- **Leibniz Rule**
  
  For all $f \in C^\infty(M)$ the Dorfman bracket satisfies
  
  \[ [e_1, f e_2]_D = f [e_1, e_2]_D + (a(e_1) f) e_2; \]  

- **Jacobi Identity**
  
  \[ [e_1, [e_2, e_3]]_D + [e_2, [e_3, e_1]]_D + [e_3, [e_1, e_2]]_D = 0; \]

- **Homomorphism and Leibniz of $a$**
  
  \[ a([e_1, e_2]_D) = [a(e_1), a(e_2)]_D \]  

\[ a(e_1) \langle e_2, e_3 \rangle = \langle [e_1, e_2]_D, e_3 \rangle + \langle e_2, [e_1, e_3]_D \rangle \]

\[ a^1 d \langle e_1, e_2 \rangle = [e_1, e_2]_D + [e_2, e_1]_D, \]

where $a^1 : T^*(M) \to E^* \cong E$.

The $O(d,d)$ symmetries are given by

\[ T = \begin{pmatrix} N & \beta^* \\ \beta & -N^* \end{pmatrix}, \]  

where

\[ N : X^\mu \to N^\mu, X^\nu \]  

Diffeomorphism

\[ \beta : X^\mu \to \beta_{\mu\nu} X^\nu \]  

Kalb – Ramond field

\[ \beta^* : \xi_\mu \to \beta^{\mu\nu} \xi_\nu \]  

Bivector

\[ N^* : \xi_\mu \to N^*_\mu \xi_\nu \]  

Diffeomorphism.  

Given the above, a metric deformation may be introduced
\[
\langle e_1, e_2 \rangle \rightarrow \langle e_1, e_2 \rangle^G
\]
\[
= \langle e^G (e_1), e^G (e_2) \rangle ,
\]
(A13)
and
\[
[e_1, e_2]_D \rightarrow [e_1, e_2]_D^G
\]
\[
e^{-g} [e^G (e_1), e^G (e_2)]_D ,
\]
(A15)
where \( g_{\mu
u} = g_{\mu
u} + \beta_{\mu
u} \) which maps from \( T(M) \rightarrow T^*(M) \) and \( e^G : E \rightarrow E \). Specifically we have
\[
e^G (e) = e + G (a(e), -),
\]
\[
(\text{A17})
\]
i.e., for \( e = X + \xi \) the above reads
\[
e^G (X + \xi) = X + \xi + (g_{\mu
u} + \beta_{\mu
u}) X^\nu dx^\mu.
\]
(A18)
The above definitions imply
\[
\langle e_1, e_2 \rangle^G = \langle e_1, e_2 \rangle + 2g(X, Y),
\]
(A19)
and
\[
[e_1, e_2]_D^G = [e_1, e_2]_D + 2g(\nabla X, Y),
\]
(A20)
where \( \nabla \) is the generalized connection of the non-symmetric metric \( G \) and is defined in terms of the general Koszul formula
\[
2g(\nabla_Z X, Y) = X G(Y, Z) - Y G(X, Z) + Z G(X, Y)
\]
\[
- G (Y, [X, Z]_{Lie}) - G ([X, Y]_{Lie}, Z)
\]
\[
+ G (X, [Y, Z]_{Lie}) .
\]
(A21)

Working out the computational details of the above we see a splitting in the generalized connection such that
\[
g(\nabla X, Y) = g (\nabla_{LC} X, Y) + \frac{1}{2} \mathcal{H}(X, Y, Z),
\]
(A22)
i.e., the contortion of \( \nabla \) is given by the Ramond-Neveu-Schwarz three form \( \mathcal{H} = d \beta \) and \( \nabla_{LC} \) is the Levi-Civita connection obtained from just \( g \). As a result, we obtain the generalized Ricci tensor
\[
R_{\mu\nu} = R_{\mu\nu}^{LC} - \frac{1}{2} \nabla_{\alpha} \mathcal{H}^{\mu\alpha} \mathcal{H}_{\alpha\beta} - \frac{1}{4} \mathcal{H}_{\mu\beta} \mathcal{H}^{\alpha\beta} ,
\]
(A23)
and the Ricci scalar
\[
R = R^{LC} - \frac{1}{4} \mathcal{H}_{\mu\nu\alpha} \mathcal{H}^{\mu\nu\alpha} .
\]
(A24)

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* rodriguezl@grinnell.edu
† rodriguezs@grinnell.edu
‡ sathwik@wpi.edu
§ lrram@wpi.edu

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