On the experimental feasibility of continuous-variable optical entanglement distillation

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Entanglement distillation aims at preparing highly entangled states out of a supply of weakly entangled pairs, using local devices and classical communication only. In this note we discuss the experimentally feasible schemes for optical continuous-variable entanglement distillation that have been presented in [D.E. Browne, J. Eisert, S. Scheel, and M.B. Plenio, Phys. Rev. A 67, 062320 (2003)] and [J. Eisert, D.E. Browne, S. Scheel, and M.B. Plenio, Annals of Physics (NY) 311, 431 (2004)]. We emphasize their versatility in particular with regards to the detection process and discuss the merits of the two proposed detection schemes, namely photodetection and homodyne detection, in the light of experimental realizations of this idea becoming more and more feasible.

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The ability to distribute entanglement over large distances is one of the key pre-requisites for many practical implementations of quantum communication schemes. Quite spectacular experimental progress has indeed been made in recent years towards reaching this aim. Several functioning medium-distance quantum key distribution schemes have been reported and successful tests indicating a violation of Bell’s inequalities have been carried out.

Needless to say, any mechanism leading to losses and decoherence will eventually deteriorate entangled states into merely classically correlated quantum states. Such states may then, for example, no longer be useful in the sense that the generation of a secure classical key cannot be guaranteed. To regain the ability to distribute entanglement in the presence of noise, some instance of an entanglement distillation scheme or quantum repeaters is required. In such entanglement distillation schemes 1, highly entangled states are extracted from a situation where entanglement is present in only a dilute form. In practical optical schemes such methods form one of the building blocks towards making long-distance quantum communication possible.

A number of distillation schemes have been devised for discrete degrees of freedom of light (in particular polarization degrees of freedom), and in some instances even been experimentally realized 2,3,4. Considering the photon-number or continuous-variable degree of freedom, in turn, offers an interesting alternative to the former setting, allowing also in principle the realization of event-ready entanglement distillation without the need of destructive post-selection or photon counters. In this context Gaussian states and operations are of particular interest as they are experimentally relatively easily accessible. It came as a surprise, yet, that with Gaussian operations alone, continuous-variable entanglement can not be distilled 5,6,7. This refers to Gaussian input states, and manipulation with passive and active optical elements, homodyne detection, and vacuum projections. Fortunately, it was then demonstrated in Refs. 8,9 that one only needs to leave the Gaussian setting in a single step to break this no-go theorem. From there on entanglement distillation is indeed possible, making use of Gaussian operations, in fact using passive linear optics and vacuum projections or homodyne measurements only.

As the realizations of the experimentally feasible schemes of Ref. 8,9 is coming closer in view of the techniques that have been developed in recent years, it seems worth discussing and emphasizing the versatility of this approach without presenting mathematical details. This is the aim of this brief note. As has already been discussed in Ref. 8, photon detectors (with relatively small detection efficiency) are suitable in the iteration of the scheme, as well as homodyning techniques. Under no circumstances, highly efficient photon counters with photon number resolution are required.

FIG. 1: One step of the procedure. The dotted rectangle represents the local detection.

The iteration. – In the following we discuss the basic iterative scheme for entanglement distillation of general non-Gaussian states developed in Refs. 8,9 (Non-Gaussian here means that the states are non-Gaussian in the photon number degree of freedom; hence, the Wigner function of the states is non-Gaussian 10,11). Two copies of a non-Gaussian two-mode state $\rho$ serve as inputs to the procedure 10. These inputs are mixed locally at a 50/50 beam splitter each, represented by $U_{BS}$, leading to

$$\rho' = (U_{BS} \otimes U_{BS}) (\rho \otimes \rho) (U_{BS} \otimes U_{BS})^\dagger. \quad (1)$$

Here, the symbol $\otimes$ denotes a tensor product with respect to the two identical two-mode states. In turn, $\otimes$ is meant as the tensor product between the two remote parties (see Fig. 1). Then, one of the outputs of each of the beam splitters undergoes a Gaussian measurement. The outputs of the other arms
of the beam splitters are retained, in case of a successful measurement event, and are then used as the input of the next step of the procedure. This can be done in an iteration, and each instance will from now on be referred to as one step of the procedure, or a “Gaussification step”. One such step is presented in Fig. 1. In practice, already a single step alone can significantly increase the degrees of entanglement, and this is the setting that seems indeed realistic to reach with present technology. In Refs. [6, 7], the action of such an iteration on all non-Gaussian two-mode states has been studied in detail. In particular, (i) a proof of convergence to Gaussian states has been delivered for all pure or mixed initial states (compare also Fig. 5), (ii) the increase of the degree of entanglement has been studied, (iii) the increase in the degree of squeezing, and (iv) necessary and sufficient criteria have been presented to decide when an exact purification of the input state is possible. In the single mode case, enhancement of squeezing and the loss of non-classical features has been discussed. The action of the scheme under one or a few steps has also been studied in detail.

The non-Gaussian states can be thought of as resulting from a number of possibilities; in Refs. [6, 2], in particular, an idea has been explored that employs the use of photon subtraction. Very much related steps for the creation of a non-Gaussian state have already been demonstrated experimentally [13, 14, 15, 16].

FIG. 2: Photon detection variant of one of the two necessary measurements.

Photon detection variants. – To realize the feasible distillation scheme of Refs. [6, 3] measurements have to be carried out, retaining the outcomes corresponding to a Gaussian projection. For this key element of the procedure a number of approaches are possible. In one variant, the Gaussian projection is obtained by projecting onto the vacuum, making use of photon detectors which can discriminate between the presence or absence of photons. Such a device – which we assume to be perfectly functioning for simplicity, an assumption that can easily be relaxed – can be theoretically described as implementing a measurement with Kraus operators $|0\rangle\langle0|$ (“no click”) and $I - |0\rangle\langle0| = \sum_{n=1}^\infty |n\rangle\langle n|$ (“click”), see Fig. 4. One step of the procedure, in a successful event of a vacuum projection, hence amounts to the transformation of two two-mode input states $\rho$ into

$$\rho'' = (|0\rangle\langle0| \otimes U_{\text{BS}} \otimes U_{\text{BS}}) (\rho \otimes \rho) (U_{\text{BS}} \otimes U_{\text{BS}})^\dagger |0\rangle\langle0| / N$$

(2)

where $N$ is an appropriate normalization.

This state will have a higher degree of entanglement, and may or may not be used as the input of the next step of the procedure. Note that this scheme does not require the ability to count photons, but merely requires photon detectors discriminating between presence or absence of photons. This is the variant in terms of which most of the results in Refs. [6, 3] have been stated. In Ref. [3], the implications of detection inefficiencies are discussed in great detail. Quite surprisingly, the scheme is robust with respect to low detection efficiencies, lower than the ones that are already available with present technology. The degree of entanglement, measured in terms of the logarithmic negativity $E_N$ as a measure of entanglement [17], after a number of steps of the procedure, as a function of the detection efficiency $\eta$ ($\eta = 1$ corresponds to perfect detectors) is depicted in Fig. 3. The initial two-mode state $\rho$ is taken in this plot to be $\rho = |\psi\rangle\langle\psi|$ with

$$|\psi\rangle = (|0\rangle, 0 + \varepsilon|1\rangle, 1) / (1 + \varepsilon^2),$$

(3)

for $\varepsilon = 0.95$. Other examples are discussed in Ref. [3].

FIG. 3: The logarithmic negativity as a function of the detector efficiency $\eta$ after one (dashed line) and after 10 steps (dotted line) of the procedure, for the initial state given in eq. (3). The solid line represents the logarithmic negativity of the initial state prior to the implementation of the procedure.

Homodyne detection variants. – As has been discussed in Ref. [3], homodyne detection is a feasible alternative to photon-detection in the implementation of the basic protocol detailed above. The advantage of such an approach is that the detection efficiencies are higher for such measurement schemes than for photon detection.

Of particular interest here is balanced homodyne detection, see Fig. 4. In homodyne detection, the signal field is combined via a beam splitter with a reference field, referred to as local oscillator. In balanced homodyne detection with a sufficiently
strong local oscillator field, field quadratures can be measured. In the scheme depicted in Fig. 4 having an additional input, followed by passive linear optics, and homodyne detection, the above measurement can indeed be realized. This setup is in fact the familiar setup to measure to $Q$-function of a single mode. Concerning the choice of the passive optics, see also the appendix. This has been discussed in the context of entanglement distillation in Ref. [9], but is as such a well-known observation on general measurement schemes [18, 19, 20]. Refs. [6, 7] discuss the possibility of using homodyne detection in order to realize projections onto Gaussian states in a deterministic manner in the language of covariance matrices, i.e., moments of quadrature operators. Note that other amplification schemes, even phase insensitive amplification, would in principle also be suitable.

Balanced homodyne detection leads to the implementation of a projection on a coherent states $|\alpha\rangle$ or, in other words, a measurement with POVM elements

$$\{|\alpha\rangle\langle \alpha|/\pi : \alpha \in \mathbb{C}\}.$$  \(4\)

As a consequence, such a balanced homodyne detection leads us to replace the expression eq. 2 by

$$\rho'' = (\alpha \otimes \beta)(U_{\text{BS}} \otimes U_{\text{BS}})(\rho \otimes \rho)(U_{\text{BS}} \otimes U_{\text{BS}})^\dagger |\alpha\rangle \otimes |\beta\rangle /N$$  \(5\)

for any complex $\alpha, \beta$ and, again, for appropriate normalization $N$. In the language of moments, each of these projections refers to one onto a single-mode Gaussian state with second moments given by $\gamma = I$, and first moments $d = (\text{Re}(\alpha), \text{Im}(\alpha))$. If one now accepts only those measurements that correspond to complex $\alpha$ and $\beta$ close to the origin – effectively introducing a cut-off – it is effectively as if the original vacuum projection has been implemented. That is, one would for a given $x > 0$ only accept outcomes for which in phase space $|\alpha| < x$. For small $\alpha$, we can approximate $|\alpha\rangle = |0\rangle + \alpha|1\rangle + O(\alpha^2)$, so higher order contributions would be orthogonal (in Hilbert space) to an arbitrarily good approximation, depending on $x$. See also Ref. [21] for an effective realization of vacuum projections with homodyne detection. So for the purposes of entanglement distillation, these approaches are equivalent. With no exception, all results concerning the convergence in the iteration towards Gaussian states, the increase in the degrees of entanglement and squeezing are just as applicable as in the previous variant, without modification [8, 9], as the postselected state is identical to arbitrary approximation under appropriate filtering.

There is a trade off between the achieved rate when filtering successful outcomes and the quality of the distilled state: a too weak filtering has the same effect as having non-unit detection efficiencies in the previous variant, see above. The interesting feature of this variant is as has been pointed out in Ref. [8] – that the high detection efficiencies of homodyning techniques can be made use of. The disadvantage may be a lower rate in the full scheme due to filtering.

Remarks on other homodyning variants. – Finally, it is worth mentioning that a direct homodyne detection of one of the output modes of the two parties will again lead to the same predictions under appropriate filtering. The action of a homodyning measurement is up to displacements the same as a local squeezing, followed by a projection onto the vacuum state, in the idealized limit of infinite squeezing. For any real non-zero squeezing parameter $s$, we have that

$$[U_{\text{BS}}, S(s) \otimes S(s)] = 0,$$  \(6\)

and hence,

$$\langle 0|(S(s) \otimes I)U_{\text{BS}} = \langle 0|(I \otimes S(s))U_{\text{BS}}(S(s) \otimes S(s)).$$  \(7\)

This is, for any finite squeezing parameter $s$, the situation including squeezing is the same as if the input state had been appropriately squeezed, followed by an inverse squeezing of the final outputs. Again, if one does a homodyne detection and filters with respect to outcomes close to the origin, the results stated in Ref. [8, 9] concerning a general convergence to Gaussian states can be applied. Also, all statements in an increase of entanglement are valid in the same manner.

![FIG. 5: An instructive plot of the scheme used as a single-mode procedure: this plot depicts the Wigner function of the single-mode state after zero, one, and two steps of the procedure. The initial state is taken to be $|0\rangle + \varepsilon|1\rangle)/(1 + \varepsilon^2)$, where again $\varepsilon = 0.95$ (taken from Ref. [4]).](image)

Non-Gaussian steps. – The above described procedure will take non-Gaussian input and yield an output that is a state that is closer to being Gaussian. In the experimental preparation of the non-Gaussian states, there is a large variety of possible approaches. The scheme is sufficiently versatile to be applicable as such to all non-Gaussian states. If one encounters a Gaussian input from a lossy Gaussian channel, modeling photon loss and thermal noise [22], then photon subtraction-type [13, 23] is one possible reasonable step [8, 9], making
use of photon detectors and using the outcome associated with \( I - |0\rangle\langle 0| = \sum_{n=1}^{\infty} |n\rangle\langle n| \) but any operation yielding a non-Gaussian state is acceptable. A fair figure of merit for an assessment of the quality of the procedure would than be the increase of the degrees of entanglement or purity of the channel output compared to the final output of the scheme. In special instances, one may also suffer mere classical ignorance, due to classical displacements in phase space due to phase noise. This kind of phase noise is related to the so-called classical noise channel \([22]\). This also leads to mixed non-Gaussian states, in case the classical weight of the mixing is non-Gaussian. Again, the above procedure would be applicable in cases where the classical information about the random displacements could in principle be retrieved, however, it may be advantageous to directly correct for such errors compared to resorting to entanglement distillation. In any case, the entanglement after Gaussification will typically be higher than the one of the mixed state before Gaussification (but generally smaller compared to the state before mixing).

Notably, there are still many challenges to be overcome in a full experimental realization of such an idea: mode matching at the two beam splitters is definitely an issue, which points towards the requirement of realizing the scheme entirely within fibers, to avoid coupling losses \([24]\). The scheme is obviously also demanding in that a number of squeezers with the capability to create large degrees of squeezing is required. Dark counts and inefficient detectors can to some extent be harmful, the former of which may be significantly diminished by appropriate temporal gating. All these obstacles constitute a challenge to the experimental realization, but do not render it prohibitively difficult. After all, all ingredients of this scheme have already been experimentally realized.

To summarize, in this note we have emphasized the experimental feasibility of the scheme presented in Refs. \([3,5,8,11]\). It is notably sufficiently versatile to allow for homodyne detection techniques as measurement schemes, instead of photon detection. This may be advantageous when it comes to exploiting higher detection efficiencies, together with appropriate filtering. This alternative was discussed already in Ref. \([5]\), and holds as well for the results of the later Ref. \([6]\). Yet, with experimental implementations becoming increasingly feasible, this point seems worth emphasizing. It is the hope that this note fosters further experimental work in this direction.

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Appendix. – In this appendix, we represent the arguments of Refs. \([3,4,7]\) for realizing the above POVM elements. In one case, we consider a bi-partite system, consisting of parts 1 and 2. Part 2 embodies exactly one mode, part 1 may consist of a number of \( n \) modes. In this case, we consider a projection onto the vacuum \( |0\rangle\langle 0| \) in system 2. In the other case, we take a tri-partite system. Here, 3 is initially prepared in \( |0\rangle\langle 0| \), corresponding to an empty port. Now 2 and 3, each consisting of one mode, undergo a certain beam splitter transformation. Then they are measured with homodyne detection.

It is not difficult to see that the resulting transformation of system 1 is identical, up to displacements in phase space. The beam splitter transformation for this to be true is specified by a matrix \( S \) acting on the quadratures \((x_1, p_3, x_2, p_2, x_{1,1}, p_{1,1}, ... , x_{1,n}, p_{1,n})\) of the tri-partite system. This matrix is found to be

\[
S = \begin{bmatrix}
a & \ldots & a \\
\ldots & a & \ldots & a \\
-a & a & \ldots & a \\
\ldots & \ldots & \ldots & a
\end{bmatrix},
\]

(8)

where \( a = 1/\sqrt{2} \). When acting on Gaussian inputs, for example, both schemes will result in a transformation of the covariance matrix

\[
\gamma_{1,2} = \begin{bmatrix}
A_2 & C_{1,2} \\
C_{1,2}^T & B_1
\end{bmatrix}
\]

(9)

of the system 1 and 2, according to \([5]\)

\[
\gamma_1' = B_1 - C_{1,2}^T(A_2 + \mathbb{1}_2)^{-1}C_{1,2}.
\]

(10)

The covariance matrix, in turn, is defined for a state \( \rho \) centered at the origin as the matrix with entries

\[
(\gamma_{1,2})_{j,k} = 2\text{Re} \text{tr} \rho O_j O_k,
\]

(11)

where \( O = (x_2, p_2, x_{1,1}, p_{1,1}, ... , x_{1,n}, p_{1,n}) \). For a more detailed analysis, see Refs. \([5,7,8]\).

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