Modelling and experimental characterisation of the rate dependent fracture properties of gelatine gels

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1. Introduction

Gelatine gels are frequently used in the food industry to thicken and stabilise various products such as desserts, yogurts, candies and jellies. The mechanical behaviour of gels is therefore very important as it affects the texture of the end product, which in turn will influence consumer acceptance and preference. Moreover, hydrogels, which in contrast to gelatine gels contain both physical and chemical crosslinks (Hellio & Djabourov, 2006) are popular materials for biological and biomedical applications since they exhibit properties similar to natural soft tissues. Due to their high water content and the resulting biocompatibility, they have been successfully used for soft contact lenses, wound dressings, super-absorbents, and drug-delivery systems (Balakrishnan, Mohanty, Umashankar, & Jayakrishnan, 2005; López-Alemany, Compañ, & Refojo, 2002; Parida & Mishra, 2012; Saltzman & Radomsky, 1991; Zhao, Su, Fang, & Tan, 2005). Recently, hydrogels have also been employed in studies concerning cell-based therapeutics and soft tissue engineering for example as scaffolds in tissue regeneration such as cartilage replacement (Baumberger & Ronsin, 2009) or as simulants of soft tissue in robotic surgery studies (Oldfield, Dini, Giordano, & Rodríguez y Baena, 2013).

All gels are defined as solid like systems formed by a solid substance finely dispersed or dissolved in a liquid phase (Djabourov, 1991). The two phases are interconnected by a network which extends continuously throughout the whole system. The cross-linking in this network can be either chemical or physical giving rise to the so-called ‘chemical’ and ‘physical’ gel terms respectively. The structure of gelatine gel, which is the material under study in this work, has been thoroughly investigated in the literature (Djabourov, 1991; Hellio & Djabourov, 2006; Mc Evoy, Ross-Murphy, & Clark, 1985 and citations therein). Gelatine is a protein derived from the collagen inside animals’ skin and bones. It is now generally accepted that during cooling, gelation occurs via the molecular chains rearranging into an ordered triple helix arrangement which is typical of collagen. With time, the triple helix regions propagate, aggregate and are separated at junctions by disordered regions. This process effectively provides physical crosslinks in a gelatine gel such as the one studied here which are thermoreversible.

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Due to their numerous applications, the number of research studies on the mechanical behaviour of hydrogels has increased dramatically in the last decade (e.g. Chester, 2012; Wang & Hong, 2012; Zhang, An, Yazzie, Chawla, & Jiang, 2012). A significant part of the published literature is concerned with the behaviour of gels at small deformations. Large deformations alter the structure of the gel considerably, meaning that theories describing the relationship between small deformation properties and structure of the gel are no longer applicable (van Vliet & Walstra, 1995). As most of the applications mentioned above involve large deformations, fracture or separation phenomena, there is a need to develop predictive models for the behaviour of the gels in this large deformation regime as well as establish failure criteria to predict fracture in these complex materials. Experimental investigations show that for most gels, the fracture behaviour depends noticeably on the strain rate (Ross-Murphy & Todd, 1983; van Vliet, Luyten, & Walstra, 1991, 1993; Zoon, van Vliet, & Walstra, 1989) as opposed to the deformation properties such as elastic modulus which are largely independent of strain rate (Gamonpilas, Charalambides, & Williams, 2009). This is a rather odd behaviour as usually strain rate dependent materials show rate dependencies in both their deformation (i.e. stress-strain curve before fracture point, modulus) and fracture properties (i.e. energy release rate, fracture stress and strain). A number of studies have been reported with the aim of explaining the rate dependent fracture of gels (Chester, 2012; Wang & Hong, 2012; Zhang et al., 2012), Bonn, Kellay, Prochnow, Ben-Djemia, and Meunier (1998) investigated the fracture of agarose polymer gels and reported that in three point bending experiments under constant load, cracks did not occur simultaneously but happened after a time delay. This delay correlated with the magnitude of the applied constant force, even though no visco-elastic relaxations were observed in the storage modulus versus frequency plot. In addition, all the gels studied had the same moduli, in spite of the observed large difference in their fracture behaviour. This phenomenon was attributed to the fact that the time delay was affected by the activation energy for crack nucleation; i.e. high activation energy resulted in a longer time delay. It was argued that differences in the time delay — applied force relationships for agarose gels were due to the different fractal dimensions of the gels that were investigated.

van Vliet and Walstra (1995) suggested that in polymer gels which are physically cross-linked, large deformations of the gel leads to ‘unzipping’ of the junctions. This process takes a certain time and as a result the fracture parameters become dependent on the deformation rate. Thermally activated ‘unzipping’ of the non-covalent cross linked network and the associated viscous solvent drag during chain pull-out was also suggested as a fracture mechanism by Baumberger and Ronsin for alginate and gelatine hydrogels (Baumberger & Ronsin, 2009). A study on starch gels suggested that the rate dependent fracture behaviour might arise from energy dissipation due to frictional processes occurring between the granules sliding against each other at large deformation (Luyten & van Vliet, 1995). For this reason, fracture — deformation rate relationships are not simple and depend on the structure of the gel (Gamonpilas et al., 2009; van Vliet & Walstra, 1995).

Another important aspect linked to the mechanical behaviour of gelatine is the role of the solvent diffusing through the network of the porous matrix in the presence of pressure gradients, which has been the focus of many recent studies. In particular, applications using hydrogels as potential articular cartilage replacements have placed much emphasis on the fluid pressurisation induced by deformation. This stems from early studies showing that if cartilage tissues were considered as simply viscoelastic, it was not possible to explain the role of this interstitial fluid flow on the globally observed mechanical behaviour (Hayes, Keer, Herrman, & Mockros, 1972; Hayes & Mockros, 1971; Parsons & Black, 1977). Although Higginson, Litchfield, and Snaith (1976) and Johnson, Dowson, and Wright (1977) observed that under uniaxial cyclic compressive loads the short-term creep deformation response was elastic and was unaffected by interstitial fluid flow, Elmore, Sokoloff, Norris, and Carmeci (1963) had earlier shown that the creep response in indentation was mainly due to the exudation of the interstitial fluid. Subsequently, Mow, Kuei, Lai, and Armstrong (1980) proposed a biphasic theory to explain deformational behaviour for materials made up of two immiscible phases: a fluid phase and a solid phase. In particular, in articular cartilage, the fluid phase is the interstitial fluid, which is incompressible and inviscid, while the solid phase is the extra-cellular matrix which is made of collagen fibres, proteoglycans and other components and is porous, permeable and compressible. This model forms the basis of many of the models developed since then to understand the role of flow-dependent and flow-independent viscoelastic mechanisms in the response of biphasic polymeric media, including biological tissues and hydrogels. For instance, Kalyanam, Yapp, and Insana (2009) have studied the poro-viscoelastic behaviour of gelatine hydrogels under compression to separate the flow-dependent and flow-independent contributions to the relaxation response of gelatine. In their studies, they have also shown that creep shear strain experiments using a rheometer provide information about the viscoelastic response of the solid matrix. This is because no fluid flow takes place under infinitesimal dynamic shear loading which is a constant volume process.

In this work, the mechanical behaviour of gelatine gels is investigated; specifically their large deformation and fracture properties. The global compressive stress—strain properties are reported considering effects of strain rate and gelatine concentration. For fracture properties, fracture mechanics based methodologies are used in the form of wire cutting analysis (Gamonpilas et al., 2009). A simplified analytical model is used to predict the dependence of fracture on test rate. In addition, the gel is simulated as a poroelastic material with a porous solid matrix fully saturated with liquid, using Finite Element Analysis. It is shown that a simple critical stress failure criterion in the solid matrix suffices to explain the observed experimental trends.

2. Experiments: materials and methods

Gelatine gels were produced by mixing distilled water and beef gelatine in powder form (SuperCook LLP, UK). Throughout the sample preparation process, the temperature was monitored using a thermocouple. Distilled water was heated to 80 °C and maintained at this temperature using a hot plate. The powdered gelatine was stirred in the water using a magnetic stirrer for 5 min. A timer was used to assure a repeatable process. The solution was left in the beaker to cool down to 60 °C at room temperature before it was poured into cylindrical moulds made of Polytetrafluoroethylene (PTFE). Paraffin oil was applied on the surface of the moulds as a lubricant in order to facilitate the subsequent removal of the fragile samples. The moulds were then covered with cling film to prevent drying and left in a domestic refrigerator kept at 4 °C overnight; samples were tested on the following day. The cooling storage time was kept as constant as possible (18 h and 30 ± 5 min) throughout the study to ensure comparable gel properties and reproducible data. For each series of tests, a dummy sample with a thermocouple inserted in the centre, was used to monitor the specimen temperature. The preparation of all specimens and the testing were performed at controlled room conditions, 18.5 ± 5 °C with 50% relative humidity.

Uniaxial compression and wire cutting tests were performed. Monotonic as well as cyclic loading experiments were undertaken.
in the uniaxial mode in order to further investigate the rheological behaviour of the gel. Gelatine concentrations of 5% w/w, 7.5% w/w, 10% w/w, 12.5% w/w and 15% w/w were used for the compression tests, whereas the cutting and cyclic loading tests were only performed with a gelatine concentration of 10% w/w.

The uniaxial compression test is suitable for gel specimens because it is simple and eliminates the problem of premature failure arising from clamping as in tensile tests. An Instron 5534 with a 100 N load cell was used to perform these tests. Cylindrical samples with a diameter and height of 20 mm, were tested at three different constant true strain rates of 0.25, 2.5 and 25 mm⁻¹. In order to achieve these constant strain rates, the crosshead speed decreased exponentially during the test. Silicon oil with a kinematic viscosity of $5 \times 10^{-4}$ m²/s was applied at the interface between the sample and the compression platens in order to minimise friction effects (Charalambides, Goh, Lim, & Williams, 2001; Charalambides, Goh, Wanigasooriya, Williams, & Xiao, 2005). At least six specimens were tested at each loading rate. Since the tests involved large deformations, the true stress, $\sigma$, and true strain or Hencky strain, $\epsilon$, were calculated assuming a constant volume deformation:

$$\sigma = \frac{F}{\pi R^2 h}$$

$$\epsilon = -\ln \frac{h}{h_0}$$

where $F$, $R$, $H$ and $h$ are the applied load, original sample radius, original sample height and current sample height respectively.

Loading and unloading tests in compression mode were conducted using a Zwick testing machine with a 100 N load cell. The cyclic tests were performed at constant true strain rates of 0.25, 2.5 and 25 mm⁻¹. The test consisted of two steps: the sample is initially subjected to an increasing strain up to a maximum value (loading), followed by the strain being gradually removed (unloading) until the stress value reaches a quasi-null value (~0.01 N). The loading and unloading strain rates for each cyclic test were kept equal.

Wire cutting tests (Gamonpilas et al., 2009) were performed at the five wire cutting speeds, $v$, of 1, 10, 100, 500 and 1000 mm/min. A schematic of the tests is shown in the inset of Fig. 6a. The specimens were rectangular blocks of length 30 mm, width 20 mm and height 20 mm. Steel wires of diameter, $\phi$, of 0.05, 0.1, 0.125, 0.25 and 0.5 mm were used. An Instron 5534 machine was employed with a 10 N load cell since the cutting load was small. For each value of $\phi$ and $v$, at least three specimens were tested (apart from the case of $v = 0.1$ mm/min and $\phi = 0.05$ mm where only one sample was successfully tested).

During the wire cutting process, fracture, large deformation and surface friction occur simultaneously (Goh, Charalambides, & Williams, 2005; Kamyab, Chakrabarti, & Williams, 1998; Luyten & van Vliet, 1995). The force $F$ required for cutting is proportional to the wire diameter and there is a constant component arising from the energy release rate, $G_c$, also referred to as fracture toughness:

$$F = G_c + \sigma_c (1 + \mu_k) \phi$$

where $F$ is the steady state cutting force, $b$ is the width of the sample being cut, $\sigma_c$ is a characteristic stress of the material and $\mu_k$ is the kinetic coefficient of friction. By plotting $F/b$ against $\phi$ and linearly extrapolating to zero wire diameter, $G_c$ can be determined. The slope of such linear fits to experimental data will be approximately equal to $\sigma_c$ for small values of $\mu_k$.

### 3. Experimental results

#### 3.1. Uniaxial compression

Fig. 1 shows the compression behaviour of gelatine at 5% w/w and 10% w/w concentrations at three different strain rates. The results from all replicate tests are shown to indicate the level of scatter in the measurements; this is acceptable considering the biological nature of the material. During the tests, it was observed that samples did not show signs of frictional barrelling due to the use of the silicon oil. Both gels are seen to be almost rate independent at small deformations. However, at large deformations, the fracture response seems to be strongly dependent on the strain rate, with the gels reaching higher stresses and strains before failure. This finding is in agreement with literature (Gamonpilas et al., 2009; Gamonpilas, Charalambides, Williams, Dooling, & Gibbon, 2010; Ikeda, Sangu, & Nishinari, 2003; Luyten & van Vliet, 1995).

The average compression response of gelatine at various concentrations and a rate of 25 mm⁻¹ is shown in Fig. 2. As expected, the higher concentrations led to stiffer and stronger gels, due to the denser network that is formed (Djabourov, 1991). The exception is the 10% and 12.5% data, where the 10% gel displays higher stress values at the larger strains. The reason for this discrepancy is uncertain.

Using averages of the data in Fig. 1, plots of the fracture stress and fracture strain versus strain rate for the 5% and 10% gelatine concentrations are provided in Fig. 3a and b respectively. Once again, it is apparent that higher rates enable the material to ‘travel’ higher up on the stress—strain curve before fracture. Fig. 4 illustrates the effect of gelatine concentration, using data collected at the strain rate of 25 min⁻¹. While the fracture stress is found to increase with increasing concentration, the fracture strain stays almost constant. The reason behind this observation is unclear at present.

Fig. 5 shows the loading-unloading compression test results at a rate of 2.5 min⁻¹. The two loops shown correspond to two consecutive loading-unloading cycles. The unloading and loading paths seem to be almost overlapping. Even though some hysteresis is observed at larger deformations, it remains very small. Similar observations were obtained for the rest of the test rates (results not shown). Therefore, the gels behave in a non-linear elastic manner, at least on the global scale.
Young's moduli of the gels as a function of gelatine concentration were determined from the initial gradient of the stress–strain curves in Fig. 2 by using linear regression to approximate the curves up to a strain level of 0.4. The results are shown in Table 1.

### 3.2. Wire-cutting

Force–displacement curves from the wire cutting tests on gelatine (10% gelatine concentration) for wire diameters varying from 0.05 mm to 0.5 mm and cutting speeds of 1–1000 mm/min were recorded and one example is shown in Fig. 6a for a wire diameter of 0.05 mm. The load first rises steeply. This corresponds to the phase where the wire is simply indenting into the gel, without any cutting or fracture occurring. This indentation part is found to be similar for all rates in agreement with the behaviour shown in Fig. 1. When the peak load point is reached, the gel starts to cut and new surfaces are formed. Thereafter, the load drops to a constant value corresponding to the steady state phase of the cutting process. There are some differences in the peak values and this is due to the fact the fracture initiates at flaws. Such flaws are randomly occurring from sample to sample, hence the variation seen. In addition in Fig. 6a, a stick-slip crack growth behaviour is
observed for the 1 mm/min cutting speed. This was not a systematic occurrence as it was not observed for the lower speed tests performed with the other wire diameters. In all the other tests, constant steady state force values were observed which increased markedly with cutting speed for all wire diameters. This agrees with the increasing trend in failure properties with rate as observed in the uniaxial compression experiments. In Fig. 6b, the averaged values of the steady state cutting force divided by sample width ($F/b$) are plotted against the wire diameter. It is observed that the steady state cutting force increases almost linearly with the wire diameter ($R^2 > 0.94$), in agreement with equation (3). By linearly extrapolating the data to a theoretical zero wire diameter, the energy release rate, $G_c$, can be determined as a function of the cutting rate. The values from Fig. 6b, as expected for this kind of materials, are relatively small and are found to be between 1 and 12 J/m².

The $G_c$ versus cutting rate data are plotted in Fig. 7 on a log-log scale. The observed behaviour can be divided into two regimes; the first one up to 10 mm/min ($\log \dot{x} = -3.78$ with $x$ measured in m/s) is characterised by a very low propagation speed with a negligible influence on $G_c$ giving a constant value of approximately 1.1 J/m². The second region, from 10 mm/min to 1000 mm/min, points to a large influence of the speed on $G_c$. The data in this region can be well approximated with a line of slope equal to 0.50. An analytical model describing the behaviour shown in Fig. 7 is presented in the following section.

4. Analytical model for the rate dependent fracture

The rather surprising observation that the fracture of gels is rate dependent whilst the deformation is not, is probably due to the pressure-driven flow of water within the porous structure. Similar behaviour has been observed in environmental stress crack growth in polymers (Williams & Marshall, 1975). A simple model of this mechanism is provided by the analysis of laminar flow in a circular pipe of diameter $d$, giving a mean velocity, $\pi$, as (Massey, 1983, chap. 6):

$$\pi = \frac{d^2}{32\mu} \left( \frac{dp}{dx} \right)$$  \hspace{1cm} (4)

where $\mu$ is the fluid viscosity, $p$ is pressure and $x$ is the distance along the length of the pipe.

Now for a fracture process to occur, the transverse pressure gradient is defined by the fracture cohesive stress, $\sigma_c$, across the length of the process zone, $r_c$, given by (Broek, 1986):

$$r_c = \frac{1}{6\pi} \frac{EG_c}{\sigma_c^2}$$  \hspace{1cm} (5)

where $E$ is the elastic modulus and $G_c$ is the fracture toughness, assuming plane strain conditions at the crack tip. Thus under steady state conditions:

$$\frac{dp}{dx} = \frac{\sigma_c}{r_c}$$  \hspace{1cm} (6)

In addition, under steady state conditions, the crack or cutting rate, $\dot{x}$, and the fluid flow mean velocity, $\pi$, appearing in equation

| C (% w/w) | $E$ (kPa) |
|-----------|-----------|
| 5         | 3.1       |
| 7.5       | 7.1       |
| 10        | 14.3      |
| 12.5      | 15.1      |
| 15        | 24.6      |

Table 1

Modulus from uniaxial compression at a strain rate of 25 min⁻¹ as a function of gelatine concentration.
(4), have to be equal. Therefore substituting equations (6) and (5) into 4, we obtain:

\[ \Pi = \frac{3\pi \sigma_c^2 d^2}{16E_G \mu} \]  

(7)

It is also common that many polymeric systems have a relationship between \( G_c \) and \( \sigma_c \) such that the crack opening displacement, \( \delta_c \), is independent of rate (Williams, 1984, chap. 7):

\[ G_c = \delta_c \sigma_c \]  

(8)

where \( \delta_c \) is constant.

Substituting equation (8) in 7 gives the result:

\[ \dot{\sigma}_c = \frac{4}{d} \left( \frac{E\delta_c \mu}{3\pi} \right)^{0.5} \dot{x}^{0.5} \]  

(9)

and

\[ \sigma_c = \frac{4}{d} \left( \frac{E\delta_c \mu}{3\pi} \right)^{0.5} \dot{x}^{0.5} \]  

(10)

The theoretical model described above predicts a power law rate dependence of \( G_c \) with the value of the power being 0.5. This agrees with the value of the exponent 0.50 derived from the experimental data shown in Fig. 7.

In addition to \( G_c \), the linear approximations shown in Fig. 6b can be used to determine the values of the characteristic stress, \( \sigma_c \). From equation (3), the slopes of the fitted lines are equal to \( \sigma_c \) for small coefficients of friction. The assumption is justified as the water solvent is thought to help lubricate the wire as it cuts through the gel. Therefore \( \sigma_c \) values are derived as a function of rate. Fig. 8 shows \( G_c \) plotted versus the corresponding \( \sigma_c \) values which confirm that \( G_c \) is proportional to \( \sigma_c \) as suggested by equation (8), with a \( \delta_c \) of 0.12 mm.

In addition, Fig. 9 shows log \( \sigma_c \) plotted versus log \( \dot{x} \) and for \( \dot{x} > 10 \) mm/min, a line with a slope of 0.5 fits the data reasonably well. Below this speed, \( \sigma_c \) remains constant at about 7.4 kPa and \( G_c \) is approximately 1.1 J/m\(^2\) (see Fig. 7).

![Graph showing \( G_c \) versus cohesive stress, \( \sigma_c \).](image)

**Fig. 8.** \( G_c \) versus cohesive stress, \( \sigma_c \).

Using a value of \( \mu = 10^{-3} \) Pa s for the viscosity of water, a modulus of \( E = 14.3 \) kPa for the 10% w/w concentration (Table 1) and the previously calculated \( \delta_c = 0.12 \) mm, approximate values of \( d \) can be derived from equations (9) and (10). Values of \( d \) equal to 68 nm and 88 nm are derived from the constants 1.98 (Fig. 7) and 5.79 (Fig. 9) respectively. These magnitudes are reasonable and are close to values stated by Djabourov (1991) where characteristic lengths of 20–100 nm are quoted for a 2% w/w gelatine gel and 10–30 nm for a 10% w/w gelatine gel.

The physical picture emerging from these results, is that of an elastic, porous gel with small pores filled with water. At low fracture speeds, i.e. lower than 1–10 mm/min, \( G_c \) is the material’s true fracture toughness which is rate independent as expected for an elastic material. Superimposed on this behaviour is an additional process in which work is done forcing water from the pores with an enhanced apparent toughness. Note that an earlier paper (Gamonpilas et al., 2009) reported similar fracture rate effects for starch gels. The range of wire cutting rates was smaller and the changes in \( \sigma_c \) and \( G_c \) were much less that those measured here for the gelatine gels. There was no clear evidence of transition in behaviour as reported here.

5. Numerical modelling of the rate dependent fracture

In order to further explain the rate-dependent fracture behaviour of gelatine, a numerical simulation of the uniaxial compression of the 10% w/w gel was performed using a poroelastic material model. The commercial software package Abaqus (Abaqus software manual, 2013) was used for all analyses. In a standard poroelastic model, the porous solid matrix is modelled as an elastic material. Furthermore the model allows the material behaviour to be divided into a rate-independent and a rate-dependent response. These arise from the deformation of the elastic solid network and the flow of the liquid through this porous medium respectively. Assuming the medium is fully saturated, the total stress at a point, \( \sigma \), is given by:

\[ \sigma = \Pi - pl \]

where \( \Pi \) is the effective stress in the porous material skeleton and \( p \) is the pressure stress in the wetting liquid. Based on previous studies where the Ogden hyperelastic model was successfully
employed to characterise the global behaviour of the gel (Gamonpilas et al., 2010; Gamonpilas et al., 2009), the rate-independent response of the solid network which defines $\mathbf{\sigma}$ was assumed to follow the Ogden hyperelastic model. This model has a strain energy potential $U$ defined as:

$$U = \frac{2\nu}{\alpha^2} \left( \frac{\psi}{\alpha} + \frac{\psi^2}{2} + \frac{\psi^3}{3} \right) + \frac{1}{D^2} \left( 1 - \frac{\psi}{\alpha} \right)^2$$  \hspace{1cm} (11)

where $\psi_i$ are the deviatoric principal stretches and they are equal to $\psi_i = J^{-1/2} \lambda_i$, $\lambda_i$ are the principal stretches, $\psi$, $\alpha$, and $D$ are material parameters and $J$ is the volume strain (equal to $\lambda_1 \lambda_2 \lambda_3$). Note that $\psi$ is the initial shear modulus whereas the bulk modulus $K$ is related to $D$ and Poisson’s ratio, $\nu$, through:

$$D = \frac{2}{K} = \frac{3(1 - 2\nu)}{1 - 2\nu}$$  \hspace{1cm} (12)

The stresses are then given by partial differentiation of equation (11), i.e. $\mathbf{\sigma}$ = $\mathbf{dU}/d\lambda$.

Furthermore, the gel is assumed to be a fully saturated, porous medium with the water flow being governed by Darcy’s law:

$$n \mathbf{v} = -\frac{k}{\gamma_w} (\nabla p - \rho_w g)$$  \hspace{1cm} (13)

where $\mathbf{v}$ is the fluid flow velocity vector, $n$ is the porosity of the medium, $\gamma_w$ is the specific weight of the fluid (here water hence equal to 9.879 kN/m$^3$), $\nabla p$ is the pressure gradient vector, $k$ is the hydraulic conductivity of the medium, $\rho_w$ is the fluid density (998.2 kg/m$^3$) and $g$ is defined as the gravitational acceleration vector:

$$g = -g \nabla z$$

where $g$ is the gravitational constant (9.812 m/s$^2$) and $z$ is the elevation above some datum. Note that the hydraulic conductivity, $k$ (units m/s), is related to conventional permeability, $\Pi$ (units m/s), through:

$$\Pi = k \rho_w g$$

As already mentioned, the gelatine specimen is assumed to be fully saturated with water, i.e. all voids in the material are filled up with the wetting liquid, water. In addition, the void ratio, $e$, is defined as the ratio of volume of wetting liquid, $V_w$, to the sum of the volumes of the solid $V_s$ and trapped liquid $V_t$:

$$e = \frac{V_w}{V_s + V_t}$$

Therefore the porosity, $n$, in equation (13) is related to void ratio, $e$, through:

$$n = \frac{e}{1 + e}$$

The analysis requires the value of the initial void ratio; this was taken from Yakimets et al. and Kalyanam et al. (Kalyanam et al., 2009; Yakimets, Wellner, Smith, Wilson, Farhat, & Mitchell, 2005) to be equal to 9. Therefore 90% of the volume is free water able to exude from the solid matrix. A hydraulic conductivity value of $1.25 \times 10^{-6}$ m/s was assumed as measured by Dreesmann, Hajosch, Ahlers, Vaz Nuernberger, and Schlosshauer (2008) from experiments on gelatine. This completes the material definition for the model.

The uniaxial compression tests were simulated using an axisymmetric geometry as shown in Fig. 10 with a radius of 10 mm and height 20 mm. Mesh convergence tests were performed to validate the final mesh density which was chosen when results deviated by only 0.12%. A 4-node, axisymmetric quadrilateral, reduced integration, hourglass control hybrid element, which includes pore pressure was employed. The bottom surface was constrained in the direction of compression while it was free to move and expand horizontally. An axisymmetric boundary condition was applied to the rotational axis on the left. A free draining condition was assigned to the outside surface to allow the fluid to drain out of the specimen. The top surface was displaced in the vertical direction such that the three different true strain rates used in the experiments were simulated, i.e. 0.25 min$^{-1}$, 2.5 min$^{-1}$ and 25 min$^{-1}$.

The three parameters of the Ogden model (equation (11)) were calibrated in order to obtain the best fit between the experimental stress–strain results and those derived from the model. Table 2 summarises the final coefficients that were chosen, whilst Fig. 11 shows the comparison between the model results and the experimental data. In order to obtain the model data shown in this figure, the nodal forces at the top surface were summed to derive the total

![Fig. 10. Axisymmetric uniaxial compression model with boundary conditions. The top surface is displaced such that the true strain rate, $\dot{\epsilon}$, is constant.](image)
force \( F \) and the true stress and true strain were then calculated using equations (1) and (2). Note that the value of \( D \) shown in Table 2 implies a Poisson's ratio of 0.4 and that the value of \( j \) is close to the value of 4.8 kPa derived by dividing by three the modulus of the 10% w/w gel shown in Table 1. It is worth emphasising that this Poisson's ratio of 0.4 in the numerical model is to be considered only for the solid matrix, while the fluid in Abaqus is considered incompressible by default. The apparent Poisson's ratio of the poroelastic gel will depend on the applied strain rate as well in literature (Brinker & Scherer, 1990).

Next, in order to model the rate-dependent fracture behaviour of gelatine, the solid and fluid contributions were split and visualised separately. The vertical solid stress and the pore pressure contours are shown in Fig. 12 for the case of 25 min\(^{-1}\) strain rate and a strain of 0.7. Since the vertical stresses in the solid matrix and the fluid do not vary with depth, stress values were calculated at the top surface as highlighted in Fig. 12. As shown in this figure the pore pressure and solid stress do not vary in the axial direction. Therefore a 1D model could have been sufficient in order to simulate the material behaviour. Unfortunately this approach was not attempted because of the lack of 1D porous elements in the software.

These results were then normalised and plotted along the top surface for the three different strain rates as shown in Fig. 13. Note that the pressure and stress values were normalised by dividing by their corresponding maximum values attained along the top surface. For the pore pressure the maximum was at the centre of the specimen whereas for the vertical stress, the maximum occurred at the free edge of the sample. It is observed that the two contributions to the global stress are entirely complementary, showing how the response of the gel is effectively a sum of the solid and fluid reactions to the applied load. As the rate increases, the pore pressure rises. As a result the stress taken by the solid decreases. Fig. 14 shows the values of the solid stress and the pore pressure averaged along the top surface versus the applied strain, for the three different strain rates. In order to be able to predict the strain rate dependent fracture behaviour, the assumption is made that the gel fractures when the solid stress reaches a critical value. The contribution of the fluid phase is negligible at the lowest strain rate of 0.25 min\(^{-1}\) (see Fig. 14). Therefore we suggest that the breaking stress detected for the lowest strain rate is only related to the solid matrix hence is a true material property. Further, at this low strain rate of 0.25 min\(^{-1}\), the fracture strain is 0.9\( \pm 1.0 \) (see Fig. 3b). In Fig. 14, the strain of 0.9\( \pm 1.0 \) corresponds to a solid stress of approximately 16–19 kPa at 0.25 min\(^{-1}\), which now defines the critical stress range. This range is illustrated by the horizontal

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**Table 2**

| Parameter | Value |
|-----------|-------|
| \( \psi \) (kPa) | 6.21 |
| \( \alpha \) | 2.64 |
| \( D \) (1/Pa) | 69 \times 10^{-6} |

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**Fig. 11.** Comparison between experimental and numerical compression stress–strain curves as a function of strain rate. All data are for 10% w/w gelatine gel.

**Fig. 12.** Pore pressure (MPa), \( p \), and vertical solid stress, \( \sigma^* \), (MPa) contours. Top surface where forces were summed is highlighted. The strain rate is 25 min\(^{-1}\) and the true strain is 0.7.

**Fig. 13.** Normalised pore pressure, \( p \), and solid stress, \( \sigma^* \), variation along the radius of the sample.
shaded region in Fig. 14. Using this critical stress range and looking at the solid stress curves, the fracture strains of 1.0 e 1.1 and 1.2 e 1.3 are obtained for the rates of 2.5 min~1 and 25 min~1 respectively. These are to be compared with the breaking strains measured experimentally in Fig. 3b which were 1.1–1.2 for 2.5 min~1 and around 1.6 for 25 min~1. For 2.5 min~1, the agreement is very good whereas for the 25 min~1 rate, the predicted values are somewhat lower than the experimental. The correct trends are however shown, i.e. the fracture strain rises with applied strain rate.

Figs. 13 and 14 both show that the solid matrix stresses, which are argued here to be responsible for specimen fracture, decrease as the strain rate increases. This is as expected as the fluid contribution is almost null at quasi-static procedures (i.e. unconfined compression at very low strain rates) and the liquid can easily move within the solid matrix network. However, as the rate increases, the liquid becomes ‘trapped’ in the solid network and in so doing, it gradually influences the material response by sharing the applied load. This effect is illustrated clearly in Fig. 15, where the average value of the void ratio, e, along the top surface of the sample is plotted versus strain for the three strain rates. In fact, for the 25 min~1 curve, e remains almost constant and close to the initial value of 9.0, since the liquid does not have sufficient time to flow out of the sample.

The results shown in Fig. 16 highlight that the incompressibility assumption made in equation (1) might lead to errors in the stress calculations. Therefore the correct contact area from Fig. 16 was used to recalculate the true compressive stress when using both

Fig. 14. Pore pressure, p, and solid stress, s*, versus strain as a function of strain rate. Highlighted vertical regions indicate the fracture range for each strain rate.

Fig. 15. Void ratio, e, versus strain as a function of strain rate.

Fig. 16. Specimen contact area versus strain as function of strain rate. The results are compared with the constant volume assumption data.

Fig. 17. Corrected experimental and numerical compression stress–strain curves as a function of strain rate. These are the initial data from Fig. 11 which were corrected by taking into account the numerically calculated contact area of the sample shown in Fig. 16.
experimental and numerical force – displacement data. The results are shown in Fig. 17 and these can be compared to the curves shown in Fig. 11. As expected, the correction was negligible for the case of 25 min⁻¹. For the lowest rate of 0.25 min⁻¹, the stress increased by approximately 20% at the fracture point, i.e. at the largest strain. This brought the average experimental stress–strain curves at different strain rates closer to each other as well as closer to the model results. As already stated above, from the numerical simulation of the uniaxial compression, the critical stress range at the lowest strain rate is approximately 16–19 kPa. The fracture stress measured at a rate 0.25 min⁻¹ as shown in Fig. 3a is 11.5–13.9 kPa. However, this stress range changes to 15.3–18.2 kPa after the contact area correction described above. Therefore the fracture stress values from experiment (15.3–18.2 kPa) and numerical model (16–19 kPa) are now very close which gives further confidence to the model’s relevance. Nevertheless, in the earlier analysis of the wire cutting, the characteristic, cohesive stress, σc, was found to be about 7.4 kPa. This was the stress obtained at low cutting speeds, when the effect of water flow through the network pores on the cutting process was argued to be insignificant. It is not clear at this stage whether the σc values quoted here are close enough for an experimental study of this nature or whether the uniaxial compression values should indeed be higher than the ones calculated from wire cutting. The reason for the latter could be that in wire cutting, the stress state under the wire is hydrostatic compression. In the uniaxial compression test, the hydrostatic stress component is 1/3 of the uniaxial, applied stress. Therefore, the value of σc derived from the wire cutting analysis could well be 1/3 of the critical stress determined from the uniaxial compression test. Further studies are needed to be able to determine this, employing further experiments and possibly numerical simulations of the fracture test.

6. Conclusions

It was shown that the stress–strain characteristics of gelatin gels are independent of rate but the fracture point varied significantly with rate; fracture stress and strain increased as the rate increased. Similarly, the energy release rate, Gc, measured from the cutting experiments also increased with rate except at the lowest crack speeds where Gc was almost constant. This effect was explained quantitatively through modelling the gel as a poroelastic material with fluid (water) flow through the porous solid polymer network. An analytical model was derived which accurately captured the power law dependence of Gc and characteristic stress σc on rate with a power index of 0.5. At low rates, fluid flow occurs readily and does not contribute greatly to the load carrying and fracture resistance of the gel, therefore Gc and σc remain constant, as would be expected of an elastic solid. At higher rates, the contribution to the fracture resistance from the fluid flow is significant and therefore Gc (and σc) both rise. This same rate effect was highlighted through a numerical simulation of the compression test using a poroelastic material model. The simulation enabled the solid stress and pore pressure contributions to the overall stress to be quantified. In addition, a critical maximum solid stress fracture criterion was able to show that it is possible for a gel to exhibit a rate independent deformation (and hence approximately constant initial moduli) but a strong rate dependent fracture. It is found that a simple maximum solid stress criterion captures the dependence of fracture strain on strain rate.

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