Reconstructing interacting entropy-corrected holographic scalar field models of dark energy in the non-flat universe

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Abstract
Here we consider the entropy-corrected version of the holographic dark energy (DE) model in the non-flat universe. We obtain the equation of state parameter in the presence of interaction between DE and dark matter. Moreover, we reconstruct the potential and the dynamics of the quintessence, tachyon, K-essence and dilaton scalar field models according to the evolutionary behavior of the interacting entropy-corrected holographic DE model.

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1. Introduction
The present acceleration of the universe expansion has been well established through numerous and complementary cosmological observations [1]. A component that is responsible for this accelerated expansion is usually dubbed ‘dark energy’ (DE). However, the nature of DE is still unknown, and people have proposed some candidates to describe it (for good reviews see [2, 3] and references therein).

The holographic DE (HDE) is one of the interesting DE candidates that were proposed based on the holographic principle [4]. According to the holographic principle, the number of degrees of freedom of a physical system should scale with its bounding area rather than with its volume [5] and it should be constrained by an infrared cut-off [6]. By applying the holographic principle to cosmology, one can obtain the upper bound of the entropy contained in the universe [7]. Following this line, Li [8] suggested \( \rho_s \leq 3c^2M_p^2L^{-2} \) as a constraint on its energy density, the equality sign holding only when the holographic bound is saturated. In this expression, \( c \) is a numerical constant, \( L \) denotes the IR cut-off radius and \( M_p = (8\pi G)^{-1/2} \) is the reduced Planck mass. The HDE models have been studied widely in the literature [9–12]. Obviously, in the derivation of HDE, the black hole entropy \( S_{BH} \) plays an important role. As is well known, usually, \( S_{BH} = A/(4G) \), where \( A \sim L^2 \) is the area of the horizon. However, in the literature, this entropy–area relation can be modified to [13]

\[
S_{BH} = \frac{A}{4G} + \tilde{\alpha} \ln \frac{A}{4G} + \tilde{\beta},
\]

where \( \tilde{\alpha} \) and \( \tilde{\beta} \) are dimensionless constants of the order of unity. These corrections can appear in the black hole entropy in loop quantum gravity (LQG) [14]. They can also be due to thermal equilibrium fluctuation, quantum fluctuation or mass and charge fluctuations (for a review see [14] and references therein). Using the corrected entropy–area relation (1), the energy density of the entropy-corrected HDE (ECHDE) can be obtained as [14]

\[
\rho_A = 3c^2M_p^2L^{-2} + \alpha L^{-4} \ln(M_p^2L^2) + \beta L^{-4},
\]

where \( \alpha \) and \( \beta \) are dimensionless constants of the order of unity. In the special case \( \alpha = \beta = 0 \), the above equation yields the well-known HDE density. Since the last two terms in equation (2) can be comparable to the first term only when \( L \) is very small, the corrections make sense only at the early stage of the universe.
of the universe. When the universe becomes large, ECHDE reduces to the ordinary HDE [14].

Reconstructing the holographic and agegraphic scalar field models of DE is one of interesting issues that have been investigated in the literature [15–17]. The holographic and the agegraphic DE models originated from some considerations of the features of the quantum theory of gravity. On the other hand, the scalar field models (such as quintessence, tachyon, K-essence and dilaton) are often regarded as an effective description of an underlying theory of DE [17]. The scalar field models can mimic the cosmological constant at the present epoch and can give rise to other observed values of the equation of state (EoS) parameter \( \omega \) (recent data indicate that \( \omega = \omega_\Lambda = 1 \) and is consistent with being below this value) [18]. They can also alleviate the fine-tuning and coincidence problems [18]. Therefore, it becomes meaningful to reconstruct the scalar field models from some DE models possessing some significant features of the LQG theory, such as ECHDE and entropy-corrected agegraphic DE (ECADE) models.

An interesting feature of entropy-corrected DE is that it permits successive acceleration–deceleration phase transitions. Moreover, the cosmic coincidence problem is resolved and the universe eventually tends to de Sitter expansion [19]. Here our aim is to investigate the correspondence between the entropy-corrected version of the interacting HDE model with the quintessence, tachyon, K-essence and dilaton scalar field models in the non-flat universe. These correspondences are essential to understand the connection of various scalar field models of DE with the ECHDE.

This paper is organized as follows. In section 2, we obtain the EoS parameter for the interacting ECADE model in a non-flat universe. In sections 3–6, we suggest a correspondence between the interacting ECHDE and the quintessence, tachyon, K-essence and dilaton scalar field models in the presence of a spatial curvature. We reconstruct the potentials and the dynamics for these scalar field models, which describe accelerated expansion of the universe. Section 7 is devoted to the conclusions.

2. Interacting entropy-corrected holographic dark energy (ECHDE) and dark matter (DM) in the non-flat universe

Within the framework of the standard Friedmann–Robertson–Walker (FRW) cosmology,

\[
\text{d}s^2 = -\text{d}t^2 + a^2(t) \left( \frac{\text{d}r^2}{1 - kr^2} + r^2 \text{d}\Omega^2 \right),
\]

for the non-flat FRW universe containing the ECHDE and DM, the first Friedmann equation takes the form

\[
H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2} (\rho_\Lambda + \rho_m),
\]

where \( k = 0, 1, -1 \) represent a flat, closed and open FRW universe, respectively. Also \( \rho_\Lambda \) and \( \rho_m \) are the energy density of ECHDE and DM, respectively. Observational evidence has implied that our universe is not a perfectly flat universe and that it possesses a small positive curvature [20]. In addition, as usually believed, an early inflation era leads to a flat universe. This is not a necessary consequence if the number of e-foldings is not very large [21]. Additionally the parameter \( \Omega_k \) (discussed below) represents the contribution to the total energy density from the spatial curvature and is constrained as \(-0.0175 < \Omega_k < 0.0085\) with 95% confidence level by current observations [22]. It has been shown that a non-zero positive curvature parameter \( k \) allows for a bounce, thereby preventing the cosmic singularities without violating the null energy condition \( \rho + p \geq 0 \) [23].

From equation (4), we can write

\[
\Omega_m + \Omega_\Lambda = 1 + \Omega_k,
\]

where we have used the following definitions:

\[
\Omega_m = \frac{\rho_m}{3M_p^2H^2}, \quad \Omega_\Lambda = \frac{\rho_\Lambda}{3M_p^2H^2}, \quad \Omega_k = \frac{k}{a^2H^2}.
\]

Recent observational evidence provided by the galaxy cluster Abell A586 supports the interaction between DE and DM [24]. This motivates us to consider the interaction between ECHDE and DM. Hence \( \rho_\Lambda \) and \( \rho_m \) are not conserved separately and the energy conservation equations for ECHDE and DM are

\[
\rho_\Lambda + 3H(1 + \omega_\Lambda)\rho_\Lambda = -Q, \quad \rho_m + 3H\rho_m = Q,
\]

where, following [25], we choose \( Q = \Gamma\rho_\Lambda \) as an interaction term and \( \Gamma = 3b^2H(\frac{\rho_m}{\rho_\Lambda}) \) is the decay rate of the ECHDE component into DM with a coupling constant \( b^2 \). Although this expression for the interaction term may look purely phenomenological, different Lagrangians have been proposed in support of it [26].

Note that in equation (2), taking \( L \) as the size of the current universe, for instance the Hubble scale, the resulting energy density is comparable to the present-day DE. However, as found by Hsu [27], in that case, the evolution of the DE is the same as that of DM, and therefore it cannot drive the universe to accelerated expansion. The same appears if one chooses the particle horizon of the universe as the length scale \( L \). To obtain an accelerating universe, Li [8] proposed that for a flat universe, \( L \) should be the future event horizon \( R_h \), and Huang and Li [21] argued that for the non-flat case, the IR cut-off \( L \) should be defined as

\[
L = a \frac{\sin n(\sqrt{|k|}y)}{\sqrt{|k|}},
\]

where

\[
y = \frac{R_h}{a} = \int_r^{\infty} \frac{dt}{a} = \int_0^{r} \frac{dr}{\sqrt{1 - kr^2}}.
\]

Here \( R_h \) is the radial size of the event horizon measured in the \( r \) direction and \( L \) is the radius of the event horizon measured on the sphere of the horizon [21]. For a flat universe, \( L = R_h \). The last integral in equation (10) has the explicit form

\[
\int_0^{r} \frac{dr}{\sqrt{1 - kr^2}} = \frac{1}{\sqrt{|k|}} \sin^{-1}(\sqrt{|k|}r) = \begin{cases} \sin^{-1}r, & k = 1, \\ r, & k = 0, \\ \sinh^{-1}r, & k = -1. \end{cases}
\]
From the definition $\rho_\Lambda = 3M_p^2 H^2 \Omega_\Lambda$ and using equation (2), we obtain
\[ L = \frac{c}{H} \left( \frac{\gamma_c}{\Omega_\Lambda} \right)^{1/2}, \tag{12} \]
where
\[ \gamma_c = 1 + \frac{1}{3c^2 M_p^2 L^2} \left[ \alpha \ln (M_p^2 L^2) + \beta \right]. \tag{13} \]
Taking the time derivative of equation (9) and using (12) yields
\[ \dot{L} = c \left( \frac{\gamma_c}{\Omega_\Lambda} \right)^{1/2} - \cos n(\sqrt{|k|}y), \tag{14} \]
where
\[ \cos n(\sqrt{|k|}y) = \begin{cases} \cos y, & k = 1, \\ 1, & k = 0, \\ \cosh y, & k = -1. \end{cases} \tag{15} \]
Taking the time derivative of equation (2) and using (12) and (14), one can obtain
\[ \rho_\Lambda = \left( \frac{2H^2 \rho_\Lambda}{\gamma_c} \right) \left[ 1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M_p^2} \left( \frac{\Omega_\Lambda}{c^2 \gamma_c} \right) \right] \times \left[ 1 - \left( \frac{\Omega_\Lambda}{c^2 \gamma_c} \right)^{1/2} \cos n(\sqrt{|k|}y) \right]. \tag{16} \]
Substituting equation (16) into (7) gives the EoS parameter of the interacting ECHDE as
\[ \omega_\Lambda = -1 - b^2 \left( \frac{1 + \Omega_\Lambda}{\Omega_\Lambda} \right) - \frac{2}{3\gamma_c} \left[ 1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M_p^2} \left( \frac{\Omega_\Lambda}{c^2 \gamma_c} \right) \right] \times \left[ 1 - \left( \frac{\Omega_\Lambda}{c^2 \gamma_c} \right)^{1/2} \cos n(\sqrt{|k|}y) \right]. \tag{17} \]
Note that as we already mentioned, at the very early stage when the universe undergoes an inflation phase, the correction terms in the ECHDE density (2) become important. After the end of the inflationary phase, the universe subsequently enters the radiation- and then matter-dominated eras. In these two epochs, since the universe is much larger, the entropy-corrected terms to ECHDE, namely the last two terms in equation (2), can be safely ignored. Therefore, if we set $\alpha = \beta = 0$, then from equation (13) $\gamma_c = 1$ and equation (17) recovers the EoS parameter of the ordinary HDE [28]:
\[ \omega_\Lambda = -\frac{1}{3} - b^2 \left( \frac{1 + \Omega_\Lambda}{\Omega_\Lambda} \right) - \frac{2\sqrt{\Omega_\Lambda}}{3c} \cos n(\sqrt{|k|}y). \tag{18} \]
In the next sections, we suggest a correspondence between the interacting ECHDE model with the quintessence, tachyon, K-essence and dilaton scalar field models in the non-flat universe.

### 3. The entropy-corrected holographic quintessence model

Quintessence is described by an ordinary time-dependent and homogeneous scalar field $\phi$ which is minimally coupled to gravity, but with a particular potential $V(\phi)$ that leads to the accelerating universe. The action for quintessence is given by [3]
\[ S = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]. \tag{19} \]
The energy momentum tensor of the field is derived by varying the action (19) with respect to $g^{\mu\nu}$:
\[ T_{\mu\nu} = - \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}}. \tag{20} \]
which gives
\[ T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left[ \frac{1}{2} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + V(\phi) \right]. \tag{21} \]
The energy density and pressure of the quintessence scalar field $\phi$ are as follows:
\[ \rho_Q = -T^{00} = \frac{1}{2} \dot{\phi}^2 + V(\phi), \tag{22} \]
\[ p_Q = T^i_i = \frac{1}{2} \dot{\phi}^2 - V(\phi). \tag{23} \]
The EoS parameter for the quintessence scalar field is given by
\[ \omega_Q = \frac{\rho_Q}{p_Q} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}. \tag{24} \]
From (24) for $\omega_Q < -1/3$, we find that the universe accelerates when $\dot{\phi}^2 < 2V(\phi)$.

Here we establish the correspondence between the interacting ECHDE scenario and the quintessence DE model; then equating equation (24) with the EoS parameter of interacting ECHDE (17), $\omega_Q = \omega_\Lambda$, and also equating equation (22) with (2), $\rho_Q = \rho_\Lambda$, we have
\[ \dot{\phi}^2 = (1 + \omega_\Lambda) \rho_\Lambda, \tag{25} \]
\[ V(\phi) = \frac{1}{2} (1 - \omega_\Lambda) \rho_\Lambda. \tag{26} \]
Substituting equations (2) and (17) into equations (25) and (26), one can obtain the kinetic energy term and the quintessence potential energy as follows:
\[ \dot{\phi}^2 = 3M_p^2 H^2 \Omega_\Lambda \left( b^2 \left( \frac{1 + \Omega_\Lambda}{\Omega_\Lambda} \right) + \frac{2}{3\gamma_c} \left[ 1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M_p^2} \left( \frac{\Omega_\Lambda}{c^2 \gamma_c} \right) \right] \times \left[ 1 - \left( \frac{\Omega_\Lambda}{c^2 \gamma_c} \right)^{1/2} \cos n(\sqrt{|k|}y) \right] \right). \tag{27} \]
\[ V(\phi) = 3M_p^2 H^2 \Omega_\Lambda \left( 1 + \frac{b^2}{2} \left( \frac{1 + \Omega_\Lambda}{\Omega_\Lambda} \right) + \frac{1}{3\gamma_c} \left[ 1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M_p^2} \left( \frac{\Omega_\Lambda}{c^2 \gamma_c} \right) \right] \times \left[ 1 - \left( \frac{\Omega_\Lambda}{c^2 \gamma_c} \right)^{1/2} \cos n(\sqrt{|k|}y) \right] \right). \tag{28} \]
From equations (27), one can obtain the evolutionary form of the quintessence scalar field as
\[
\phi(a) - \phi(a_0) = M_P \int_{a_0}^{a} (3b^2(1 + \Omega_k) + \frac{2\Omega_L}{\gamma_c} - 2\sqrt{\frac{\alpha H^2}{3c^2 M_P^2}} \frac{\Omega_L}{c^2 \gamma_c}) \left(1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M_P^2} \frac{\Omega_L}{c^2 \gamma_c}\right) \left[1 - \left(\frac{\Omega_L}{c^2 \gamma_c}\right)^{1/2} \cos n(\sqrt{|k|} y)\right]^{1/2} \frac{da}{a},
\]
(29)
where \(a_0\) is the scale factor at the present time.

The above integral cannot be taken analytically. But during the early inflation era, when the correction terms make sense in the ECHDE density (2), the Hubble parameter \(H\) is constant and \(a = a_0 e^{Ht}\). Hence, the Hubble horizon \(H^{-1}\) and the future event horizon \(R_0 = a \int_{t_0}^t dt/a\) will coincide, i.e. \(R_0 = H^{-1} = \text{const}\). On the other hand, since an early inflation era leads to a flat universe, we have \(L = R_0 = H^{-1} = \text{const}\). Also from equations (12) and (15), we have \(\Omega_L = \Omega_0 = 1\) and \(\cos n(\sqrt{|k|} y) = 1\). Therefore, during the early inflation era, equation (29) reduces to
\[
\phi(a) = \phi(a_0) + \sqrt{3} b M_P \ln \left(\frac{a}{a_0}\right).
\]
(30)
For the late-time universe, i.e. \(\Omega_L = 1\) and \(\Omega_k = 0\), the universe becomes large and ECHDE reduces to the ordinary HDE. In this case \(L = R_0 = H^{-1}\) and \(H \neq \text{const}\). Now by setting \(\gamma_c = 1(\alpha = \beta = 0)\) and \(\cos n(\sqrt{|k|} y) = 1\), the Hubble parameter from equations (12) and (14) can be obtained as
\[
H = \frac{H_0}{1 + \left(\frac{c}{\gamma_c}\right) H_0(t - t_0)},
\]
(31)
where \(H_0\) is the Hubble parameter at the present time. After integration of equation (31) with respect to \(t\), the scale factor can be obtained as
\[
a = a_0 \left[1 + \left(\frac{c}{\gamma_c}\right) H_0(t - t_0)\right]^{\gamma_c/(\gamma_c - 1)}.
\]
(32)
Using the above relation, one can rewrite equation (31) as
\[
H = H_0 \left(\frac{a}{a_0}\right)^{(1 - \gamma)/\gamma_c}.
\]
(33)
Finally, for the late-time universe, equation (29) yields
\[
\phi(a) = \phi(a_0) + M_P \left[3b^2 - 2 \left(1 - \frac{1}{\gamma_c}\right)\right]^{1/2} \ln \left(\frac{a}{a_0}\right).
\]
(34)

4. Entropy-corrected holographic tachyon model

In recent years, a great deal of interest has been devoted to studying the inflationary model with the help of the tachyon field. The tachyon field associated with unstable D-branes might be responsible for cosmological inflation in the early evolution of the universe, due to tachyon condensation near the top of the effective scalar potential [29]. Also the tachyonic matter could suggest some new form of DM at late epoch [30]. The tachyon field has emerged as a possible source of the DE. A rolling tachyon has an interesting EoS whose parameter smoothly interpolates between −1 and 0 [31]. This discovery motivated us to take DE as the dynamical quantity, i.e. a variable cosmological constant and model inflation using tachyons. The effective Lagrangian density of tachyon matter is given as [32]
\[
\mathcal{L} = -V(\phi) \sqrt{1 + \partial_\mu \phi \partial^\mu \phi}.
\]
(35)
The energy density and pressure for the tachyon field are as follows [32]:
\[
\rho_T = \frac{V(\phi)}{\sqrt{1 + \partial_\mu \phi \partial^\mu \phi}},
\]
\[
p_T = -V(\phi) \sqrt{1 + \partial_\mu \phi \partial^\mu \phi},
\]
where \(V(\phi)\) is the tachyon potential. The EoS parameter for the tachyon scalar field is obtained as
\[
\omega_T = \frac{p_T}{\rho_T} = \phi^2 - 1.
\]
(38)
If we establish the correspondence between the ECHDE and tachyon DE, then equating equation (38) with the EoS parameter of interacting ECHDE (17), \(\omega_T = \omega_{\Lambda}\), and also equating equation (36) with (2), \(\rho_T = \rho_{\Lambda}\), we obtain
\[
\phi^2 = b^2 \left(\frac{1 + \Omega_k}{\Omega_\Lambda}\right) + \frac{2}{3\gamma_c} \left[1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M_P^2} \frac{\Omega_L}{c^2 \gamma_c}\right]\frac{\Omega_L}{c^2 \gamma_c} \left[1 - \left(\frac{\Omega_L}{c^2 \gamma_c}\right)^{1/2} \cos n(\sqrt{|k|} y)\right]^{1/2},
\]
(39)
\[
V(\phi) = 3M_P^2 H^2 \Omega_\Lambda \left[1 + b^2 \left(\frac{1 + \Omega_k}{\Omega_\Lambda}\right) + \frac{2}{3\gamma_c} \left[1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M_P^2} \frac{\Omega_L}{c^2 \gamma_c}\right]\right]\frac{\Omega_L}{c^2 \gamma_c} \left[1 - \left(\frac{\Omega_L}{c^2 \gamma_c}\right)^{1/2} \cos n(\sqrt{|k|} y)\right]^{1/2}.
\]
(40)
From equation (39), one can obtain the evolutionary form of the tachyon scalar field as
\[
\phi(a) - \phi(a_0) = \int_{a_0}^{a} \frac{da}{H} \left(b^2 \frac{1 + \Omega_k}{\Omega_\Lambda}\right) + \frac{2}{3\gamma_c} \left[1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M_P^2} \frac{\Omega_L}{c^2 \gamma_c}\right]\frac{\Omega_L}{c^2 \gamma_c} \left[1 - \left(\frac{\Omega_L}{c^2 \gamma_c}\right)^{1/2} \cos n(\sqrt{|k|} y)\right]^{1/2}.
\]
(41)
During the early inflation era \((L = R_0 = H^{-1} = \text{const})\), equation (41) yields
\[
\phi(a) = \phi(a_0) + b \frac{H}{\sqrt{\Omega_\Lambda}} \ln \left(\frac{a}{a_0}\right),
\]
(42)
where
\[
\Omega_\Lambda = c^2 + \frac{H^2}{3M_P^2} [\alpha \ln(M_P^2 H^2) + \beta].
\]
(43)
For the late-time universe, i.e. \(\Omega_\Lambda = 1\), \(\Omega_k = 0\) and \(\gamma_c = 1(\alpha = \beta = 0)\), using equation (33), one can take the integral (41) as
\[
\phi(a) = \phi(a_0) + \frac{\left[b^2 - \frac{2}{3} (1 - \frac{1}{\gamma})\right]^{1/2}}{H_0 (1 - \frac{1}{\gamma})^{1 - (1/c)}} \left[\frac{a}{a_0}\right]^{1 - (1/c)} - 1.
\]
(44)
where $p(\phi, \chi) = f(\phi)(-\chi + \chi^2)$

and the energy density of the field is

$$\rho(\phi, \chi) = f(\phi)(-\chi + 3\chi^2).$$

The EoS parameter for the K-essence scalar field is obtained as

$$\omega_K = \frac{p(\phi, \chi)}{\rho(\phi, \chi)} = \frac{\chi - 1}{3\chi - 1}.$$  \hfill (48)

Equating equation (48) with the EoS parameter (17), $\omega_K = \omega_\lambda$, we find the solution for $\chi$

$$\chi = \frac{2b^2 \left(\frac{1}{3} \frac{\Omega_{\phi}}{\Omega_{\chi}} + \frac{1}{16} \left[1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M^2} \left(\frac{\Omega_{\phi}}{\Omega_{\chi}}\right)^2\right]\right)}{4 + 3b^2 \left(\frac{1}{3} \frac{\Omega_{\phi}}{\Omega_{\chi}} + \frac{1}{16} \left[1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M^2} \left(\frac{\Omega_{\phi}}{\Omega_{\chi}}\right)^2\right]\right)}$$

Using $\phi = 2\chi$ and (49), we obtain the evolutionary form of the K-essence scalar field as

$$\phi(a) = \phi(a_0) + \frac{\phi'}{2} \left[\frac{1}{\Omega_{\phi}} \left(\frac{1}{3} \frac{\Omega_{\phi}}{\Omega_{\chi}} + \frac{1}{16} \left[1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M^2} \left(\frac{\Omega_{\phi}}{\Omega_{\chi}}\right)^2\right]\right)^{1/2}\right.$$  \hfill (50)

$$\times \left[1 - \left(\frac{\Omega_{\phi}}{\Omega_{\chi}}\right)^{1/2} \cos n(\sqrt{\chi}y)\right]$$

$$+ \frac{3b^2}{4} \left[\frac{1}{3} \frac{\Omega_{\phi}}{\Omega_{\chi}} + \frac{1}{16} \left[1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M^2} \left(\frac{\Omega_{\phi}}{\Omega_{\chi}}\right)^2\right]\right)^{1/2}\right.$$  \hfill (50)

$$\times \left[1 - \left(\frac{\Omega_{\phi}}{\Omega_{\chi}}\right)^{1/2} \cos n(\sqrt{\chi}y)\right]$$

During the early inflation era, equation (50) reduces to

$$\phi(a) = \phi(a_0) + \frac{1}{H} \left(\frac{4\Omega_\lambda + 2b^2}{4\Omega_\lambda + 3b^2}\right)^{1/2} \ln \left(\frac{a}{a_0}\right),$$  \hfill (51)

where $\Omega_\lambda$ is given by equation (43).}

For the late-time universe, i.e., $L = R_0 \neq H^{-1}$ and $H \neq \text{const}$, using equation (33) one can take integral (50) as

$$\phi(a) = \phi(a_0) + \frac{1 + \frac{c^2}{2} - \frac{1}{2} (1 - \frac{1}{2})}{1 + \frac{3c^2}{2} - \frac{1}{2} (1 - \frac{1}{2})} 1^{1/2} \left[\frac{\Omega_\alpha}{\Omega_0} 1^{1/2} (1 - \frac{1}{2})\right].$$  \hfill (52)

### 6. Entropy-corrected holographic dilaton model

The process of compactification of the string theory from higher to four dimensions introduces the scalar dilaton field, which is coupled to curvature invariants. The coefficient of the kinematic term of the dilaton can be negative in the Einstein frame, which means that the dilaton behaves as a phantom-type scalar field. The pressure (Lagrangian) density and the energy density of the dilaton DE model are given as

$$p_D = -\chi + c' e^{\phi} \chi^2,$$  \hfill (53)

$$\rho_D = -\chi + \gamma + 3c' e^{\phi} \chi^2,$$  \hfill (54)

where $c'$ and $\gamma$ are positive constants and $\chi = \phi^2/2$. The EoS parameter for the dilaton scalar field is given by

$$\omega_D = \frac{p_D}{\rho_D} = \frac{-1 + c' e^{\phi} \chi}{1 + 3c' e^{\phi} \chi}.$$  \hfill (55)

Equating equation (55) with the EoS parameter (17), $\omega_D = \omega_\lambda$, we find the following solution:

$$2b^2 \left(\frac{1}{3} \frac{\Omega_{\phi}}{\Omega_{\chi}} + \frac{1}{16} \left[1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M^2} \left(\frac{\Omega_{\phi}}{\Omega_{\chi}}\right)^2\right]\right)$$

$$= \frac{c' e^{\phi} \chi}{4 + 3b^2 \left(\frac{1}{3} \frac{\Omega_{\phi}}{\Omega_{\chi}} + \frac{1}{16} \left[1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M^2} \left(\frac{\Omega_{\phi}}{\Omega_{\chi}}\right)^2\right]\right)}.$$  \hfill (56)

then using $\phi = 2\chi$, we obtain

$$e^{\phi/2} \frac{\phi'}{2} = \frac{4 + 2b^2 \left(\frac{1}{3} \frac{\Omega_{\phi}}{\Omega_{\chi}} + \frac{1}{16} \left[1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M^2} \left(\frac{\Omega_{\phi}}{\Omega_{\chi}}\right)^2\right]\right)}{4 + 3b^2 \left(\frac{1}{3} \frac{\Omega_{\phi}}{\Omega_{\chi}} + \frac{1}{16} \left[1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M^2} \left(\frac{\Omega_{\phi}}{\Omega_{\chi}}\right)^2\right]\right)}$$

Integrating with respect to $a$, we obtain

$$e^{(\phi/a)} = e^{(\phi/a_0)/2} + \frac{\lambda}{2 \sqrt{\epsilon}}$$

$$\times \left[1 - \left(\frac{\Omega_{\phi}}{\Omega_{\chi}}\right)^{1/2} \cos n(\sqrt{\chi}y)\right]$$

$$\left[4 + 2b^2 \left(\frac{1}{3} \frac{\Omega_{\phi}}{\Omega_{\chi}} + \frac{1}{16} \left[1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M^2} \left(\frac{\Omega_{\phi}}{\Omega_{\chi}}\right)^2\right]\right)\right]^{1/2}\right.$$  \hfill (58)
Therefore, the evolutionary form of the dilaton scalar field is obtained as

\[ \phi(a) = \frac{2}{\lambda} \ln \left[ \frac{\lambda}{\sqrt{2c^2}} \int_0^a \frac{da}{Ha} \left( 2 + b^2 \left( \frac{1 + a_0}{a_0} \right) + \frac{2}{3\gamma} \left[ 1 - 2y + \frac{a H^2}{\lambda^2 M^2} \left( \frac{a}{a_0} \right)^{2\gamma} \right] \right) \right] \]

During the early inflation era, equation (59) yields

\[ \phi(a) = \frac{2}{\lambda} \ln \left[ \frac{\lambda}{\sqrt{2c^2}} \int_0^a \frac{da}{Ha} \left( 2 + b^2 \left( \frac{1 + a_0}{a_0} \right) + \frac{2}{3\gamma} \left[ 1 - 2y + \frac{a H^2}{\lambda^2 M^2} \left( \frac{a}{a_0} \right)^{2\gamma} \right] \right) \right] \]

where \( \Omega_\lambda \) is given by equation (43).

For the late-time universe, using equation (33) one can take the integral (59) as

\[ \phi(a) = \frac{2}{\lambda} \ln \left[ \frac{\lambda}{\sqrt{2c^2}} \int_0^a \frac{da}{Ha} \left( 2 + b^2 \left( \frac{1 + a_0}{a_0} \right) + \frac{2}{3\gamma} \left[ 1 - 2y + \frac{a H^2}{\lambda^2 M^2} \left( \frac{a}{a_0} \right)^{2\gamma} \right] \right) \right] \]

7. Conclusions

Here, we considered the entropy-corrected version of the HDE model that is in interaction with DM in the non-flat FRW universe. The HDE model is an attempt at probing the nature of DE within the framework of quantum gravity [11]. We considered the logarithmic correction term to the energy density of the HDE model. The addition of correction terms to the energy density of HDE is motivated by the LQG, which is one of the promising theories of quantum gravity. Using this modified energy density, we obtained the EoS parameter for the interacting ECHDE. We established a correspondence between the interacting ECHDE model with the quintessence, tachyon, K-essence and dilaton scalar field models in the non-flat FRW universe. These correspondences are important to understand how various candidates of DE are mutually related to each other.

In the present case, the correspondence is established between ECHDE and various scalar field models of DE. We adopted the viewpoint that these scalar field models of DE are effective theories of an underlying theory of DE. Thus, we should be capable of using these scalar field models to mimic the evolving behavior of the interacting ECHDE and reconstruct the scalar field models according to the evolutionary behavior of the interacting ECHDE. We reconstructed the potentials and the dynamics of these scalar field models, which describe accelerated expansion of the universe, according to the evolutionary behavior of the interacting ECHDE model. We also obtained the explicit evolutionary forms of the corresponding scalar fields for both early inflation (\( L = R_h = H^{-1} \), const) and late-time acceleration (\( L = R_h \neq H^{-1} \), and \( H \neq \) const) phases.

late-time acceleration, \( L \) is dynamical and \( \dot{L} \) will contribute to the above expressions and will yield the scalar potentials for the DE. This suggests that the vacuum energy that produced inflation at the early cosmic epoch and the one driving late-time cosmic acceleration are fundamentally different. Hence the same scalar field moves in different potentials at different times.

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