Phenomenology of an extended IDM with loop-generated fermion mass hierarchies

A. E. Cárcamo Hernández\textsuperscript{a,*} Sergey Kovalenko\textsuperscript{a} Roman Pasechnik\textsuperscript{b,c,d,‡} and Ivan Schmidt\textsuperscript{a,§}

\textsuperscript{a} Universidad Técnica Federico Santa María and Centro Científico-Tecnológico de Valparaíso, Casilla 110-V, Valparaíso, Chile
\textsuperscript{b} Department of Astronomy and Theoretical Physics, Lund University, Sövegatan 14A, SE-223 62 Lund, Sweden
\textsuperscript{c} Nuclear Physics Institute ASCR, 25068 Řež, Czech Republic
\textsuperscript{d} Departamento de Física, CFM, Universidade Federal de Santa Catarina, C.P. 476, CEP 88.040-900, Florianópolis, SC, Brazil

We perform a comprehensive analysis of the most distinctive and important phenomenological implications of the recently proposed mechanism of sequential loop generation of strong hierarchies in the Standard Model (SM) fermion mass spectra. This mechanism is consistently realized at the level of renormalizable interactions in an extended variant of the Inert Higgs Doublet model, possessing the additional $Z_2 \times Z_4$ discrete and $U_1$ gauge family symmetries, while the matter sectors of the SM are extended by means of $SU_{2L}$-singlet scalars, heavy vector-like leptons and quarks, as well as right-handed neutrinos. We thoroughly analyze the most stringent constraints on the model parameter space, coming from the $Z'$ collider searches, related to the anomaly in lepton universality, and the muon anomalous magnetic moment, as well as provide benchmark points for further tests of the model and discuss possible “standard candle” signatures relevant for future explorations.

I. INTRODUCTION

The hypothetical extensions of the Standard Model (SM) that accommodate a dynamical explanation of the mass and mixing hierarchies in the quark, lepton and neutrino sectors, are typically expected to contain many new interactions and states at high scales of the theory. In particular, additional scalar fields are required to break the high-scale (e.g. discrete or continuous family) symmetries, causing the formation of specific patterns in the fermion mass spectra across generations. The additional inert sectors, such as heavy right-handed neutrinos, are mandatory for see-saw type mechanisms of neutrino mass generation, and play a supplemental but important cosmological role in leptogenesis and also as candidates for DM. In practice, there are no strong constraints on how many additional heavy scalar singlet and vector-like fermion states could be added to the SM at the fundamental level, as they typically produce vanishing direct signatures in collider measurements, but may have indirect (e.g. via radiative corrections) signatures imprinted into the patterns of SM couplings and mass parameters.

In general, additional states are required to explain specific patterns in the SM fermion spectra. For example, to address only the quark sector and to explain the Cabbibo-like structure of the quark mixing simultaneously with the hierarchies in the quark mass spectrum, the addition of a gauged $U_{1X}$ or discrete family symmetry and few extra scalar fields seems to be enough (see Refs. \textsuperscript{a,1-10}). Such models, although not necessarily excluded, may generically suffer from large Flavor-Changing Neutral Currents (FCNCs) and from non-observability of Higgs partners in the few-hundreds GeV mass range. In order to explain the lepton mass hierarchy together with the highly decoupled neutrino mass spectrum, even more additional inputs are required on top of the SM. Due to a large number of states, such theories quickly become cumbersome to deal with and to verify phenomenologically. Therefore, the search for a particular model capable of explaining all the fermion mass and mixing hierarchies in a dynamical and fully renormalizable way, while still having it simple enough for a straightforward phenomenological verification, becomes a challenging and demanding, but very important task for the model-building community.

In addition, models having an extended scalar and (or) fermion sector are motivated by the search of a theoretical explanation for the Lepton Universality Violation (LUV) recently observed by the LHCb experiments. A concise

\textsuperscript{*}Electronic address: antonio.carcamo@usm.cl
\textsuperscript{†}Electronic address: sergey.kovalenko@usm.cl
\textsuperscript{‡}Electronic address: Roman.Pasechnik@thep.lu.se
\textsuperscript{§}Electronic address: ivan.schmidt@usm.cl
review of New Physics models aimed at explaining the LUV and their possible connection to DM is provided in Ref. [11]. Some theoretical explanations for the LUV are discussed in Refs. [12–38]. In Ref. [10] we have proposed such a possible candidate theory, capable of generating the SM fermion mass and mixing hierarchies via a sequential loop suppression mechanism, in terms of model parameters with no intrinsically imposed hierarchies between them. In this framework the only fermion that acquires its mass at tree level is the heavy top quark. Moderate and light quark masses are generated essentially at one- or two-loop level, respectively, while light active neutrinos become massive only via three-loop radiative seesaw mechanisms triggered after the electroweak symmetry breaking. We have found specific conditions on the minimal symmetry and particle content for a theory where this mechanism can be realized without adding the non-renormalizable (higher-dimensional) Yukawa operators or soft family-breaking mass terms. While such a construction is supposedly not unique, its minimality is manifest as every field plays a relevant role for producing the observed patterns in quark, lepton and neutrino sectors of the SM, with a required degree of suppression between the corresponding SM parameters.

II. REVIEW OF THE EXTENDED IDM MODEL

With the aim of generating the hierarchy of SM charged fermion masses via the sequential loop suppression mechanism, proposed for the first time in Ref. [7], we consider an extension of the inert two-Higgs doublet model (ITHDM), where the SM gauge symmetry is supplemented by an exactly preserved $Z_4$ and spontaneously broken $Z_2$ discrete groups, and by an $U_{1X}$ gauge symmetry. The scalar sector of the ITHDM is extended to include nine electrically neutral fields, i.e., $\sigma_j$ ($j = 1, 2, 3$), $\rho_k$ ($k = 1, 2, 3, 4, 5$), $\eta$ and two electrically charged $\varphi_k^\pm$ ($k = 1, 2$) $SU_{2L}$ scalar singlets. The fermion sector of the SM includes additionally six SM gauge-singlet charged leptons $E_{jL}$ and $E_{jR}$ ($j = 1, 2, 3$), four right handed neutrinos $\nu_{jR}$ ($j = 1, 2, 3$), $\Omega_R$ and ten $SU_{2L}$ singlet heavy quarks $T_L$, $T_R$, $\tilde{T}_L$, $\tilde{T}_R$, $B_{KL}$, $B_{KR}$ ($k = 1, 2$). It is assumed that the heavy exotic $T$, $\tilde{T}_k$ and $B_k$ quarks have electric charges equal to $\frac{2}{3}$ and $-\frac{1}{3}$, respectively. The scalar, quark and lepton assignments under the $SU_{3c} \times SU_{2L} \times U_{1Y} \times U_{1X} \times Z_2 \times Z_4$ symmetry are shown in Tables I, II and III, respectively. Let us note that the SM Higgs doublet, i.e., $\varphi_1$, as well as the SM scalar singlets $\sigma_1$ and $\rho_5$ are the only scalar fields neutral under the preserved $Z_4$ discrete symmetry. Since the $Z_4$ symmetry remains unbroken, the SM Higgs doublet $\varphi_1$ and the SM scalar singlets $\sigma_1$ and $\rho_5$ are the only scalar fields which acquire nonvanishing vacuum expectation values. The SM scalar singlet $\sigma_1$ is required to spontaneously break the $U_{1X}$ local symmetry, whereas the scalar singlet $\rho_5$ spontaneously breaks the $Z_2$ discrete symmetry, due to its nontrivial $Z_2$ charge. A detailed explanation for the choice of the aforementioned particle content and symmetries is provided in our previous work in Ref. [10].

With the above specified particle content, the following Yukawa interactions and exotic fermion mass terms are present.
at renormalizable level, invariant under the $SU_{3c} \times SU_{2L} \times U_{1Y} \times U_{1X} \times Z_2 \times Z_4$ symmetry:

\[
\mathcal{L}_Y = y_{33}^{(u)} \bar{q}_{3L} \phi_1 u_{3R} + \sum_{n=1}^{2} x_n^{(u)} \bar{q}_{nL} \phi_2 T_R + \sum_{n=1}^{2} w_j^{(u)} \bar{T}_L \sigma_2 u_{nR} + \sum_{n=1}^{2} z_n^{(u)} \bar{T}_{1L} \rho_1 u_{nR} + y_T \bar{T}_L \sigma_1 T_R
\]

\[
+ \sum_{n=1}^{2} \sum_{m=1}^{2} y_{mn}^{(B)} \bar{T}_n \phi_1 \sigma_1 B_{mR} + \sum_{n=1}^{2} w_j^{(d)} \bar{T}_L \phi_1 d_{nR} + \sum_{n=1}^{2} z_j^{(d)} \bar{T}_L \phi_1 \sigma_1 d_{nR} + \sum_{j=1}^{3} x_j^{(d)} \bar{l}_{1L} \phi_2 l_{2R} + \sum_{j=1}^{3} y_j^{(d)} \bar{l}_{2L} \phi_2 l_{3R} + \sum_{j=1}^{3} z_j^{(d)} \bar{l}_{3L} \phi_2 l_{1R}
\]

\[
+ \sum_{k=1,3}^{(E)} \bar{l}_{3L} \rho_2 E_{1R} + \sum_{k=1,3}^{(E)} \bar{l}_{2L} \rho_1 E_{1R} + \sum_{i=1}^{3} y_i^{(d)} \bar{t}_{iL} \sigma_1^i E_{iR} + \sum_{i=1}^{3} z_i^{(d)} \bar{t}_{2L} \phi_2 \rho_2
\]

\[
+ \sum_{k=1,3}^{(E)} \bar{l}_{3L} \rho_2 E_{1R} + \sum_{k=1,3}^{(E)} \bar{l}_{2L} \rho_1 E_{1R} + \sum_{i=1}^{3} y_i^{(d)} \bar{t}_{iL} \sigma_1^i E_{iR} + \sum_{i=1}^{3} z_i^{(d)} \bar{t}_{2L} \phi_2 \rho_2
\]

\[
+ \sum_{k=1,3}^{(E)} \bar{l}_{3L} \rho_2 E_{1R} + \sum_{k=1,3}^{(E)} \bar{l}_{2L} \rho_1 E_{1R} + \sum_{i=1}^{3} y_i^{(d)} \bar{t}_{iL} \sigma_1^i E_{iR} + \sum_{i=1}^{3} z_i^{(d)} \bar{t}_{2L} \phi_2 \rho_2
\]

\[
+ \sum_{k=1,3}^{(E)} \bar{l}_{3L} \rho_2 E_{1R} + \sum_{k=1,3}^{(E)} \bar{l}_{2L} \rho_1 E_{1R} + \sum_{i=1}^{3} y_i^{(d)} \bar{t}_{iL} \sigma_1^i E_{iR} + \sum_{i=1}^{3} z_i^{(d)} \bar{t}_{2L} \phi_2 \rho_2
\]

\[
+ \sum_{k=1,3}^{(E)} \bar{l}_{3L} \rho_2 E_{1R} + \sum_{k=1,3}^{(E)} \bar{l}_{2L} \rho_1 E_{1R} + \sum_{i=1}^{3} y_i^{(d)} \bar{t}_{iL} \sigma_1^i E_{iR} + \sum_{i=1}^{3} z_i^{(d)} \bar{t}_{2L} \phi_2 \rho_2
\]

where the dimensionless couplings are $O(1)$ parameters. From the quark Yukawa terms it follows that the top quark mass only arises from the interaction with the SM Higgs doublet $\phi_1$. After the spontaneous breaking of the SM electroweak symmetry, the observed hierarchy of SM fermion masses arises by a sequential loop suppression, such that we have: tree-level top quark mass; one-loop bottom, strange, charm, tau and muon masses; two-loop masses for the up, down quarks as well as for the electron. Furthermore, light active neutrinos get their masses from a three-loop level radiative seesaw mechanism. Some of the one-, two- and three-loop Feynman diagrams contributing to the entries of the SM fermion mass matrices are shown in Figure 1. More details are given in our previous work.

### III. CONSTRAINTS ON THE $Z'$ MASS, COUPLINGS AND PRODUCTION AT THE LHC

In this section, we discuss the constraints on the $Z'$ mass and couplings in our model that emerge due to the 2.6σ lepton universality anomaly expressed as the ratio $R_K = \frac{Br(B\to K^{\mu+\mu^-})}{Br(B\to K^{e+e^-})}$ measured by the LHCb collaboration. In addition, we will determine the LEP constraint on the $M_{Z'}/g_X$ ratio. As we will show below, in our model the lepton universality violation is a consequence of the non-universal $U_{1X}$ charge assignments of the fermionic fields. From the $U_{1X}$ assignments for fermions, we find the following $Z'$ interactions with the SM fermions:

\[
\mathcal{L}_{Z'} = gx \bar{q}_{3L} \gamma^\mu q_{3L} Z'_\mu + 2gx \sum_{j=1}^{3} \bar{u}_{jL} \gamma^\mu u_{jR} Z'_\mu - 3g_X \sum_{j=1}^{3} \bar{l}_{jL} \gamma^\mu l_{jR} Z'_\mu
\]

\[
-3g_X \bar{l}_{1L} \gamma^\mu l_{1L} Z'_\mu - 6g_X \bar{l}_{2L} \gamma^\mu l_{2L} Z'_\mu - 3g_X \sum_{k=1,3} \bar{l}_{kL} \gamma^\mu l_{kR} Z'_\mu.
\]
Figure 1: Some of the one-, two- and three-loop Feynman diagrams contributing to the entries of the SM fermion mass matrices. Here, $n, m = 1, 2, l, n = 1, 3$.

Then the non-universal $Z'$ interactions with the SM fermions given above lead to the following effective Hamiltonian, where the fermionic fields are given in the physical basis:

$$\Delta H_{eff} = -\frac{g_X^2}{M_{Z'}^2} (V_{DL})_{32} (V_{DL})_{33} x_{q_3L} \sum_{j=1}^3 \left[ x_{jL} (\bar{\psi} \gamma^\mu P_L b) (\bar{l}_j \gamma^\mu l_j^L) + x_{jL} (\bar{\psi} \gamma^\mu P_L b) (\bar{l}_j R \gamma^\mu l_j^R) \right]$$

$$- \frac{g_X^2}{M_{Z'}^2} (V_{DL})_{32} (V_{DL})_{33} \left[ -3 (\bar{\psi} \gamma^\mu P_L b) (\bar{l}_2 \gamma^\mu l_2^L) - 6 (\bar{\psi} \gamma^\mu P_L b) (\bar{l}_2 R \gamma^\mu l_2^R) \right]$$

$$- 3 (\bar{\psi} \gamma^\mu P_L b) (\bar{l}_3 R \gamma^\mu l_3^R) - \frac{9g_X^2}{2M_{Z'}^2} (V_{DL})_{32} (V_{DL})_{33} (\bar{\psi} \gamma^\mu P_L b) (\bar{\mu} \gamma^\mu \mu) ,$$

(3)
where the following relations have been taken into account:

\[
\tilde{M}_f = (M_f)_{\text{diag}} = V^\dagger_{fL} M_f V_{fR}, \quad f(L,R) = V_{f(R)L} \tilde{f}_{(L,R)},
\]

\[
\tilde{f}_{kL} (M_f)_{ij} f_{jR} = \tilde{f}_{kL} \left( V^\dagger_{jL} (M_f)_{ij} V_{jR} \right)_{kl} \tilde{f}_{kR} = \tilde{f}_{kL} \left( \tilde{M}_f \right)_{kl} \tilde{f}_{kR} = m_{f_k} \tilde{f}_{kL} \tilde{f}_{kR},
\]

(4)

Here, \( f_{kL} \) and \( f_{k(R)R} \) \((k = 1, 2, 3)\) are the SM fermionic fields in the mass and interaction bases, respectively.

Let us note that the \( R_K \) anomaly results from a shift in the Wilson coefficient \( C_9^{\mu\mu} \) appearing in the following \( \Delta B = 1 \) effective Hamiltonian:

\[
\Delta H_{\text{eff}} = -\frac{G_F \alpha_{\text{em}} V_{tb} V^*_{ts}}{\sqrt{2} \pi} \sum_{\hat{l}_R = e, \mu, \tau} C^\mu_9 \left( \hat{\pi}^\mu \gamma^\mu P_L b \right) \left( \tilde{\gamma}^\mu \hat{\pi} \right).
\]

(5)

Then, our model predicts the following correction to the \( C_9^{\mu\mu} \) coefficient relative to its SM value:

\[
\Delta C_9^{\mu\mu} = -\frac{9 g_X^2}{2 M_Z^2} \left( V^\dagger_{DL} \right)_{32} \left( V_{DL} \right)_{33} \frac{\sqrt{2} \pi}{G_F \alpha_{\text{em}} V_{tb} V^*_{ts}} \simeq \frac{9 g_X^2}{2 M_Z^2} \frac{\sqrt{2} \pi}{G_F \alpha_{\text{em}}}.
\]

(6)

On the other hand, the LHCb data provide the constraints on the \( C_9^{\mu\mu} \) coefficient given in Table IV. Requiring for the correction to the \( C_9^{\mu\mu} \) coefficient predicted by our model to be inside the 1σ and 2σ experimentally allowed ranges, we find the constraints for the \( M_{Z'}/g_X \) ratio:

\[
14 \text{ TeV} < \frac{M_{Z'}}{g_X} < 20 \text{ TeV} \text{ at 1σ, 13 TeV} < \frac{M_{Z'}}{g_X} < 26 \text{ TeV} \text{ at 2σ}.
\]

(7)

With respect to the LEP bounds on the \( M_{Z'}/g_X \) ratio, it is worth mentioning that the tightest constraint arises from the \( e^+e^- \to \mu^+\mu^- \) measurement at LEP. Using the effective leptonic interactions

\[
\mathcal{L}_{\text{eff}} = -\frac{g_X^2}{M_Z^2} \sum_{j=1}^{3} \left[ x_{i_{1L}i_{1L}} \left( \tilde{l}_j \gamma^\mu P_L l_1 \right) \left( \tilde{l}_j \gamma^\mu l_{1L} \right) + x_{i_{1L}i_{1R}} \left( \tilde{l}_j \gamma^\mu P_L l_1 \right) \left( \tilde{l}_j \gamma^\mu l_{1R} \right) \right] + x_{i_{1R}i_{1L}} \left( \tilde{l}_j \gamma^\mu P_R l_1 \right) \left( \tilde{l}_j \gamma^\mu l_{1L} \right) + x_{i_{1R}i_{1R}} \left( \tilde{l}_j \gamma^\mu P_R l_1 \right) \left( \tilde{l}_j \gamma^\mu l_{1R} \right),
\]

(8)

we find that the \( e^+e^- \to \mu^+\mu^- \) measurement at LEP imposes the following limit [10]:

\[
2 M_{Z'} \frac{g_X}{\sqrt{g_X} \sqrt{x_{i_{1L}i_{1L}} + x_{i_{1R}i_{1L}} + x_{i_{1R}i_{1R}} + x_{i_{1L}i_{1R}}}} > 4.6 \text{ TeV},
\]

(9)

which for the leptonic charge assignments of our model takes the form:

\[
2 M_{Z'} \frac{g_X}{\sqrt{3} g_X} \simeq 0.38 \frac{M_{Z'}}{g_X} > 4.6 \text{ TeV}.
\]

(10)

The latter yields the following lower bound on the \( M_{Z'}/g_X \) ratio:

\[
\frac{M_{Z'}}{g_X} > 12 \text{ TeV}.
\]

(11)
In what follows, we proceed with computing the total cross section for production of a heavy $Z'$ gauge boson at the LHC via a Drell-Yan (DY) mechanism. In this computation, we consider the dominant contribution due to the parton distribution functions of the light up, down and strange quarks, so that the total $Z'$ production cross section via quark-antiquark annihilation in proton-proton collisions with center-of-mass energy $\sqrt{S}$ reads:

$$
\sigma_{pp \rightarrow Z'}^{DY} (S) = \frac{g^2 \pi}{6 c^2 W_S} \left\{ 4 g_X^2 \int_{-\ln \frac{M_{Z'}^2}{S}}^{-\ln \frac{M_Z^2}{S}} f_{p/u} \left( \frac{M_{Z'}^2}{S} e^y, \mu^2 \right) f_{p/\pi} \left( \frac{M_{Z'}^2}{S} e^{-y}, \mu^2 \right) dy \\
+ g_X^2 \int_{-\ln \frac{M_{Z'}^2}{S}}^{-\ln \frac{M_Z^2}{S}} f_{p/d} \left( \frac{M_{Z'}^2}{S} e^y, \mu^2 \right) f_{p/\pi} \left( \frac{M_{Z'}^2}{S} e^{-y}, \mu^2 \right) dy \\
+ g_X^2 \int_{-\ln \frac{M_{Z'}^2}{S}}^{-\ln \frac{M_Z^2}{S}} f_{p/s} \left( \frac{M_{Z'}^2}{S} e^y, \mu^2 \right) f_{p/\pi} \left( \frac{M_{Z'}^2}{S} e^{-y}, \mu^2 \right) dy \right\},
$$

where $f_{p/u} (x_1, \mu^2)$, $f_{p/d} (x_1, \mu^2)$, and $f_{p/s} (x_1, \mu^2)$ are the distributions of the light up, down and strange quarks (antiquarks) in the proton, respectively, which carry momentum fractions $x_1$ of the proton. Here, $\mu = m_{Z'}$ is the corresponding factorization scale.

Figure 2: The total $Z'$ production cross section via the DY mechanism at the LHC for $\sqrt{S} = 13$ TeV and $g_X = 0.1$ as a function of the $Z'$ mass.

Figure 2 displays the total $Z'$ production cross section via the DY mechanism at the LHC for $\sqrt{S} = 13$ TeV and $g_X = 0.1$ as a function of the $Z'$ mass. The latter is varied from 1.4 TeV up to 2 TeV to satisfy the LEP constraint as well as the constraints imposed by the 2.6σ anomaly in lepton universality. For such a region of $Z'$ masses, we find that the total production cross section is found to be $0.2 - 1$ pb. On the other hand, at a future 100 TeV proton-proton collider this cross section gets significantly enhanced reaching values of $9 - 29$ pb in the same mass interval, as indicated in Figure 3.

IV. MUON ANOMALOUS MAGNETIC MOMENT

In this section, we will determine the constraints on the parameter space of our model imposed by the experimental measurements of the muon anomalous magnetic moment. The latter receives one-loop contributions from vertex diagrams involving the $Z'$ exchange as well as the exchanges of the heavy $Z_1$ charged neutral scalars $\text{Re} (\rho_2)$, $\text{Re} (\phi^0_2)$,
\[ \sigma(pp \rightarrow Z') \text{[pb]} \]

$M_{Z'}[\text{GeV}]$

| $M_{Z'}[\text{GeV}]$ |
|-------------------|
| 1400              |
| 1500              |
| 1600              |
| 1700              |
| 1800              |
| 1900              |
| 2000              |

\[ \Delta a_\mu = y^2 \frac{m_\mu^2}{8\pi^2} (I_S (m_{E_2}, m_{H_2}) - I_P (m_{E_2}, m_{A_2})) \sin \theta + \frac{m_\mu^2}{8\pi^2 M_{Z'}^2} I_V (M_{Z'}) \]

Figure 3: The total $Z'$ production cross section via the DY mechanism at a future $pp$ collider for $\sqrt{S} = 100$ TeV and $g_X = 0.1$ as a function of the $Z'$ mass.

\[\text{Im} (\phi_0^L), \text{Im} (\rho_2), \text{Im} (\rho_2')\]

that couple to the charged exotic lepton $E_2$. The scalar contributions to the muon anomalous magnetic moment include the Yukawa interactions $E_{2L} \rho_2^L \phi_2^R$ and $E_{2L} \phi_2^L \phi_2^R$ as well as the trilinear scalar interactions such as $\rho_2 \left( \phi_1 \cdot \phi_2^L \right) \sigma_1^L$ giving rise to the $\phi_2^L - \rho_2$ mixing, which is crucial to generate those contributions.

In view of a huge amount of free parameters in the scalar potential of our model (which is shown explicitly in our previous work in Ref. [10]), for the sake of simplicity, here we work with a simplified benchmark scenario where $\text{Re} (\rho_2)$ ($\text{Im} (\rho_2)$) and $\text{Re} (\phi_2^L)$ ($\text{Im} (\phi_2^L)$) mix between themselves only and do not mix with other scalar fields. In this scenario, we have the following relations:

\[ \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_S & \sin \theta_S \\ -\sin \theta_S & \cos \theta_S \end{pmatrix} \begin{pmatrix} \text{Re} (\rho_2) \\ \text{Re} (\phi_2^L) \end{pmatrix}, \quad \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_P & \sin \theta_P \\ -\sin \theta_P & \cos \theta_P \end{pmatrix} \begin{pmatrix} \text{Im} (\rho_2) \\ \text{Im} (\phi_2^L) \end{pmatrix} \]

where $H_1$, $H_2$ are the physical CP-even scalars whereas $A_1$ and $A_2$ are the CP-odd scalars in the physical basis. In addition, without any loss of generality we set $\theta_S = \theta_P = \theta$ and $y_{22}^{(i)} = x_{22}^{(i)} = y$. Then, the muon anomalous magnetic moment in this scenario reads:

\[ \Delta a_\mu = y^2 \frac{m_\mu^2}{8\pi^2} (I_S (m_{E_2}, m_{H_2}) - I_S (m_{E_2}, m_{H_2}) + I_P (m_{E_2}, m_{A_2}) - I_P (m_{E_2}, m_{A_2})) \sin \theta + \frac{m_\mu^2}{8\pi^2 M_{Z'}^2} I_V (M_{Z'}) , \]

where the loop integrals are given by [11] [12]:

\[ I_{S(P)} (m, m) = \int_0^1 \frac{x^2 \left( 1 - x \pm \frac{m_\mu}{m_\mu} \right)}{m_\mu^2 x^2 + (m_E^2 - m_P^2) x + m^2 (1 - x)} dx, \]

\[ I_V (M_{Z'}) = \int_0^1 \frac{g_V^2 P_V (x) + g_A^2 P_A (x)}{(1 - x) \left( 1 - \frac{m_{Z'}^2}{M_{Z'}^2} x \right) + \frac{m_{Z'}^2}{M_{Z'}^2} x} dx, \]

\[ P_V (x) = 2x^2 (1 - x), \quad P_A (x) = 2x^2 (1 - x) (x - 4) - 4 \frac{m_\mu^2}{M_{Z'}^2} x^3 \]

\[ g_L = -3g_X, \quad g_L = -6g_X, \quad g_{V,A} = g_R \pm g_L. \]
In our numerical analysis we have fixed \( \tan \theta = \frac{v}{\sqrt{\sigma}} \), \( M_{Z'} = 1.5 \text{ TeV} \) and \( g_X = 0.1 \), in consistency with the 2.6\( \sigma \) \( R_K \) anomaly. Considering that the muon anomalous magnetic moment is constrained to be in the range \( [43 – 45] \), we can write \( \sum a_{\mu} \exp = (26.1 \pm 8) \times 10^{-10} \), \( (16) \),

we plot in Figure 5 the allowed parameter space for \( M_S - M_E \) (left panel) and \( M_A - M_E \) (right panel) planes with different values for \( \Delta a_{\mu} \). Here, we have set \( M_S = \min (m_{H_1}, m_{H_2}) \) and \( M_A = \min (m_{A_1}, m_{A_2}) \) and \( M_E = m_{E_2} \). We found that our model can accommodate the experimental values of \( \Delta a_{\mu} \) for a large region of parameter space.

V. DM PARTICLE CANDIDATES

Note that due to the exact \( Z_4 \) discrete symmetry, our model has several stable scalar DM (DM) candidates, which can be the neutral components of the inert \( SU_{2L} \) scalar doublet \( \phi_2 \) as well as the real and imaginary parts of the SM scalar singlets \( \sigma_2, \sigma_3, \eta \) and \( \rho_j \) \( (j = 1, 2, 3, 4) \). Furthermore, the model can have a fermionic DM candidate, which is the only SM-singlet Majorana neutrino \( \Omega_{1R} \) with a non-trivial \( Z_4 \) charge.

Considering a scenario with a scalar DM candidate, one has to ensure its stability. This can be done by assuming that it is the lightest among the inert scalar particles and is lighter than the exotic fermions. That scalar DM candidate annihilates mainly into \( WW, ZZ, t\bar{t}, b\bar{b} \) and \( hh \) via a Higgs portal scalar interaction \( \left( \Phi_1 \Phi_1 \right)^{\Phi_{DM}^\dagger \Phi_{DM}} \),
where $\phi_1$ is the SM Higgs doublet and $\Phi_{DM}$ the scalar DM candidate in our model. These annihilation channels will contribute to the DM relic density, which can be accommodated for appropriate values of the scalar DM mass and of the quartic scalar coupling of the corresponding Higgs portal scalar interaction $\left(\phi_1^\dagger \phi_1\right) \Phi_{DM}^4 \Phi_{DM}$, similarly as in Refs. [6, 7, 46, 47]. Thus, for DM direct detection prospects, the scalar DM candidate would scatter off a nuclear target in a detector via Higgs boson exchange in the $t$-channel, giving rising to a constraint on the coupling of the $\left(\phi_1^\dagger \phi_1\right) \Phi_{DM}^4 \Phi_{DM}$ interaction. Given the large number of parameters in the scalar potential of our model (which is discussed in detail in our previous work [10]), there is a lot of parametric freedom that allows us to reproduce the observed value of the DM relic density and the parameter space of our model consistent with DM constraints will be similar to the one in Refs. [6, 7, 46, 47].

For instance, in the case, where $m_{\Phi_{DM}}^2 >> v^2$, with $v = 246$ GeV, and neglecting the annihilation channel of the scalar DM candidate into neutrino-antineutrino pairs as in Ref. [6], the freeze-out of heavy scalar DM particle will be largely dominated by the annihilations into Higgs bosons and the corresponding thermally averaged cross section can be estimated as

$$<\sigma v> \simeq \frac{\gamma^2}{128\pi m_{\Phi_{DM}}^4}, \quad (17)$$

which results in a DM relic abundance

$$\frac{\Omega_{DM} h^2}{0.12} = \frac{0.1 pb}{0.12 <\sigma v>} \simeq \left( \frac{1}{\gamma} \right)^2 \left( \frac{m_{\Phi_{DM}}}{1.1 TeV} \right)^2, \quad (18)$$

with $\gamma$ being the quartic scalar coupling of the Higgs portal scalar interaction $\left(\phi_1^\dagger \phi_1\right) \Phi_{DM}^4 \Phi_{DM}$. Consequently, our model naturally reproduces the observed value [18]

$$\Omega_{DM} h^2 = 0.1198 \quad (19)$$

for the DM relic density.

In the scenario with a fermionic DM candidate, it follows from the Yukawa interactions $\Omega_{1R} \eta_1 \nu_{kR}$ and $\Omega_{1R} \eta_{1I}^* \Psi_R$ that the DM candidate $\Omega_{1R}$ can annihilate into a pair of the right-handed Majorana neutrinos $\nu_{kR}$ $(k = 1, 3)$ and $\Psi_R$, via $t$-channel exchange of the real and imaginary parts of the gauge singlet scalar $\eta$. Additionally, the fermionic DM candidate $\Omega_{1R}$ can also annihilate into $\eta_{1R} \eta_{1R}^*$ and $\eta_{1I} \eta_{1I}^*$ via the the $t$-channel exchange of the right handed Majorana neutrinos $\nu_{kR}$ $(k = 1, 3)$ and $\Psi_R$. Thus, the corresponding relic density will depend on the neutrino Yukawa coupling of the aforementioned Yukawa interactions, on the fermionic DM candidate mass $m_{\Omega_{1R}}$, on the masses of the the right-handed Majorana neutrinos $\nu_{kR}$ $(k = 1, 3)$, $\Psi_R$, as well as on the masses of the real and imaginary parts of the gauge singlet scalar $\eta$.

Considering a scenario where $m_{\Omega_{1R}}^2 << m_{\eta_{1R}}^2 \sim m_{\eta_{1I}}^2 \sim m_{\eta}^2$, and the annihilation channel $\Omega_{1R} \Omega_{1R} \rightarrow \nu_{kR} \nu_{kR}$ $(k = 1, 3)$, following Ref. [6] one can estimate the corresponding thermally averaged cross section as

$$<\sigma v> \simeq \frac{9 y_{\eta}^4 m_{\Omega_{1R}}^2}{16\pi m_{\eta}^4}. \quad (20)$$

Then, the DM relic abundance is

$$\frac{\Omega_{DM} h^2}{0.12} = \frac{0.1 pb}{0.12 <\sigma v>} \simeq \left( \frac{1}{y_{\eta}} \right)^4 \left( \frac{400 GeV}{m_{\Omega_1}} \right)^2 \left( \frac{m_{\eta}}{1.9 TeV} \right)^4, \quad (21)$$

showing that in the case of fermionic DM candidate our model also naturally reproduces the observed value [19].

VI. CONCLUSIONS

We have studied some phenomenological aspects of the extended Inert Higgs Doublet model, which incorporates the mechanism of sequential loop-generation of the SM fermion masses, explaining the observed strong hierarchies between them as well as the corresponding mixing parameters. A particular emphasis has been made on analyzing the constraints on the $Z'$ mass and couplings of our model, imposed by the $2.6\sigma$ anomaly in lepton universality, the
LEP constraint on the $M_{Z'}/g_X$ ratio and the constraints arising from the experimental measurements of the muon anomalous magnetic moment. Furthermore, we have studied production of the heavy $Z'$ gauge boson in proton-proton collisions via the Drell-Yan mechanism. We found that the corresponding total cross section at the LHC is equal to $29 \text{ pb}$. Additionally, we have found that the $2.6\sigma$ anomaly in lepton universality yields a tighter constraint than the one obtained from the $e^+e^- \rightarrow \mu^+\mu^- \rightarrow gX$ measurement at LEPI and implies a lower bound of $\approx 13 \text{ TeV}$ on the $M_{Z'}/g_X$ ratio. We have found that our model successfully accommodates the experimental values of the nuon magnetic moment for a large region of parameter space. Finally, we have examined the possible fermion and scalar DM particle candidates of the model and showed that in both cases our predictions are compatible with the observed DM relic density abundance.

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