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Successive Interference Cancellation Decoding With Adaptable Decision Regions for NOMA Schemes

RAFAEL CAMPELLO, GABRIEL CARLINI, CARLOS E. C. SOUZA, CECILIO PIMENTEL, (Senior Member, IEEE), AND DANIEL P. B. CHAVES, (Member, IEEE)
Department of Electronics and Systems, Federal University of Pernambuco, Recife 50670-901, Brazil
Corresponding author: Cecilio Pimentel (cecilio.pimentel@ufpe.br)

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ABSTRACT In this work, we propose adaptable decision regions in the successive interference cancellation (SIC) decoder for the three-user uplink/downlink non-orthogonal multiple access (NOMA) scheme by exploiting the knowledge of the channel gains. We present scenarios in which the modified SIC decoder outperforms the traditional one and is able to replicate the performance achieved by a deep learning-based decoder. We present analytical symbol error rate (SER) expressions for the three users, compare them to simulation results, and discuss possible consequences of our analysis, such as the optimal power allocation. We also employ heatmaps to analyze the joint influence on the SER of some system parameters, such as signal-to-noise ratio and channel gains.

INDEX TERMS Non-orthogonal multiple access, successive interference cancellation, symbol error rate, decision regions.

I. INTRODUCTION

Channel resource sharing techniques have been gaining importance due to the necessity of achieving higher spectral efficiency in the next generations of wireless communications systems [1]. Among them, power domain non-orthogonal multiple access (NOMA) is an important technique in accomplishing this task [2], which is also frequently linked to 5G technologies [3]. In this scheme, signals from multiple users with different power levels share the same channel resources, and the successive interference cancellation (SIC) is the usual decoding method, in which the decoding at each user is performed sequentially according to the ordering of their power levels.

Multiple analytical expressions for NOMA system performance have been derived under different assumptions and scenarios [4]–[20]. A two-user uplink scheme with QPSK is analyzed in [4] and with BPSK/QPSK in [5]. In [6]–[9], two downlink users with arbitrary modulation order are considered. In [10], the same scenario is analyzed under imperfect channel state information assumption. The three-user scenario is considered for downlink with QPSK [11], and space-shift keying [12]. In [13], theoretical analysis and experimental validation are done for multi-user downlink. Arbitrary users analysis is considered in [14]–[16], [21]. Imperfect SIC modeling is done in [15], [17], [22], and [10] by considering different approaches for the error propagation phenomenon. In [15], the interference caused by SIC decoding is modeled by a zero-mean Gaussian random variable. A linear model to represent imperfect SIC is employed in [22]. In [17], a precise analysis of imperfect SIC is considered for the two-user downlink case. To the best of our knowledge, sufficient power spacing between users is generally assumed to avoid unnecessary interference. Traditionally, the usage of SIC in scenarios where interference is significant results in error floor [15], [20], [21], in which inadequate power allocation between users can increase the error propagation in SIC decoding [14]. In particular, results from [21] suggest that the usage a neural network-based decoding is capable of decisively outperforming the traditional SIC decoding. Therefore, we also consider in this work the scenario where arbitrary interference is present and a
SIC modification is employed to handle decoding in high interference imperfect SIC cases by exploiting channel gain knowledge.

We consider an uplink/downlink scenario where three users using QPSK modulation communicate with a base station. There is no restriction under the channel gain parameters (apart from the ordering restriction). We propose a decision region modification for the first step in SIC decoding that is able to justify and replicate the improved performance in deep learning-based methods [21] in situations where channel gain parameters are assumed to be known at the decoder.

The proposed method enables classical techniques to be used in the analysis of the system performance and decoder implementation, which contributes to the interpretability of the results and possible extensions to more generalized scenarios. We also present an analytical expression for the symbol error rate (SER) of the first decoding step where the modified decision region is used and approximate formulas for the SER of the other users. Comparisons between simulated and analytical results are conducted and we employ our results to justify a power allocation analysis where users are constrained to transmit limited power. We also present a joint parameter analysis using heatmaps which enables a more general overview and intuition about the system performance. Although the analysis is initially done focusing on the uplink scenario, we utilize a theoretical adaptation that enables the extension of the uplink scenario to the downlink one.

The rest of this paper is structured as follows. We define the uplink communication system and present the proposed decoder in Section II. In Section III, we present an analytical SER analysis for the proposed decoder. In Section IV, we present various SER results for the considered scenarios. We extend previous results to the downlink scenario in Section V and also present simulation results. Finally, in Section VI we discuss the final considerations.

II. THE COMMUNICATION SYSTEM

We consider a power-domain uplink NOMA system consisting of a single-antenna base station and three single-antenna users. We consider that the complex channel coefficients between the user and the base station, denoted by \( h_i \), \( i \in \{1, 2, 3\} \), are perfectly known at the base station [23]. For coherent detection, given that the phase of \( h_i \) is compensated perfectly, the received signal \( y = (y_1, y_Q) \) is given by

\[
y = \sum_{i=1}^{3} |h_i|\sqrt{P_i}x_i + n
\]  

where \( P_i \) is the transmitted power of the user \( i \), \( x_i \) is a unit energy QPSK symbol transmitted by user \( i \), \( x_i \in \mathcal{A} = \{\pm B, \pm B\} \), \( B = \sqrt{\frac{2}{2}} \), and \( n = (n_Q, n_Q) \) is the AWGN noise modeled as a zero-mean Gaussian random variable with variance \( N_0/2 \) per dimension. We define \( h_i = |h_i|\sqrt{P_i} \) as the multiplicative term of the symbol \( x_i \) and consider \( h_1 \) fixed during transmission. Users are labeled according to \( h_1 > h_2 > h_3 \).

The received signal is decoded at the base station using the SIC technique, following the order indicated by the multiplicative term \( h_i \). In the considered scenario, user 1 is decoded first, followed by user 2 and user 3. The estimated symbols for each user \( \hat{x}_i, i \in \{1, 2, 3\} \), are obtained as follows. The user 1 symbol is directly decoded by treating the symbols of the remaining users as interference

\[
\hat{x}_1 = \arg \min_j |y - h_1 x_j|^2
\]

where \( x_j \in \mathcal{A} \), and the arg min \( j \) notation indicates that we take the corresponding \( x_j \) which minimizes the evaluated expression. In the sequel, the interference from user 1 is subtracted from the received vector yielding \( y_2 \triangleq y - h_1 \hat{x}_1 \). Thus

\[
\hat{x}_2 = \arg \min_j |y_2 - h_2 x_j|^2.
\]

Finally, the estimation of \( x_3 \) is achieved from \( y_3 \triangleq y_2 - h_2 \hat{x}_2 \) as

\[
\hat{x}_3 = \arg \min_j |y_3 - h_3 x_j|^2.
\]

A. PROPOSED SIC DECODER

We consider the 64 possible signals \( y \) in absence of AWGN noise in (1) for different configurations of the parameters \( h_i \) (the combination of three QPSK users creates a 64-NOMA constellation). In the three-user case, the inequality \( D : h_1 > h_2 + h_3 \) determines the distribution of the signals in the I/Q plane. If \( D \) is satisfied, all transmitted symbols from user 1 remain in their respective quadrant for any combination of the transmitted symbols from users 2 and 3. If \( D \) is not satisfied, denoted by \( \overline{D} \), the interference from users 2 and 3 is able to modify the quadrant of the symbol transmitted from user 1.

Fig. 1 shows the received constellations (in the absence of noise) for the cases \( D \) (Fig. 1a) and \( \overline{D} \) (Fig. 1b). For both figures, the colors of each signal indicate the transmitted symbol from user 1. The background colors indicate the decision regions for the cases \( D \) and \( \overline{D} \) (using the minimum Euclidean distance criterion). Fig. 1b provides an intuitive explanation to the error floor phenomenon for the case \( \overline{D} \) when the traditional SIC is used. In this scenario, 28 out of 64 symbols are moved outside of their respective quadrants, which results in decoding errors in the traditional SIC algorithm for user 1 (even in the noiseless scenario), and additionally causes error propagation at the decoding of users 2 and 3.

Therefore, the first step of the proposed decoder is the evaluation of the inequality \( D \). If it is satisfied, we employ the traditional SIC decoding for user 1. Otherwise, we employ the customised decision regions in the SIC decoding of this user, following Fig. 1b, and the traditional SIC algorithm for the remaining users, since the correct estimation of \( \hat{x}_1 \) is sufficient to remove the error floor phenomenon in the decoding of
users 2 and 3 (since $h_2 > h_3$). We do not consider the situation $h_1 = h_2 + h_3$, since this equality means that there exist different combinations of transmitted symbols that are mapped into the same received constellation symbol.

It is necessary to determine in Fig. 1b the threshold that separates decision regions with different background colors. For instance, we consider two symbols indicated by red circles in Fig. 1b, which are associated with different transmitted symbols from user 1. The in-phase (I) components of these symbols are $Bh_1 - Bh_2 + Bh_3$ and $-Bh_1 + Bh_2 + Bh_3$, which indicates that the in-phase threshold $L = Bh_3$ separates the corresponding regions. Due to the constellation symmetry, analogous thresholds separate the other regions at $\pm Bh_3$ at both in-phase and quadrature (Q) components, beyond the trivial threshold 0 which separates the quadrants. The thresholds at $I = L$ and $Q = L$ are shown in black dashed lines in Fig. 1b.

The procedure Modified-SIC is given in Algorithm 1. The inputs are the parameters $h_1, h_2, h_3$, and $y = (y_I, y_Q)$, and the outputs are the decoded symbols $\hat{x}_1, \hat{x}_2, \hat{x}_3$. At the given pseudocode, line 1 evaluates the inequality $D$ and decides which decision regions are used by user 1. The loop in line 4 seeks to obtain $\hat{x}_1$ by determining in which of the 16 disjoint decision regions in Fig. 1b $y$ lies. For such, the if in line 7 performs reflections in I/Q components enabling the evaluation of Boolean variables $reg_1, reg_2, reg_3, reg_4$ in lines 14, 15, 16 and 17. The variable $reg_k$ indicates that $y$ belongs to the $k$-th region of the same background color in Fig. 1b (there are four disjoint regions of the same background color). The rest of the procedure from line 20 is identical to the traditional SIC.

Since Algorithm 1 fixes the number of users as three, the application of the proposed technique in scenarios with more users may be achieved by employing auxiliary techniques such as pairing algorithms [24]. In a typical scenario, $N$ users are split into clusters, each one with a subset of users. The proposed method allows the formation of clusters of three users with high interference.

III. SER ANALYSIS
In this section, we present a closed-form SER expression for user 1, and SER approximations for users 2 and 3 (following
some simplifications presented in [10]). We consider different expressions conditioned to the veracity of the inequality $\mathcal{D}$. Let $P(c_1)$ be the probability of decoding a correct symbol for user 1. We first compute the probability of a correct decoding for user 1, for the case $\mathcal{D}$, by initially fixing the transmitted symbol, $P(c_1 | \mathcal{D}, x_1 = (B, B))$. Then, we divide the 16 symbols of the received constellation (considering $x_1 = (B, B)$) in ten cases, as indicated in Fig. 2. We assume that all symbols have equal probability of being transmitted. By symmetry, $P(c_1 | \mathcal{D}) = P(c_1 | \mathcal{D}, x_1 = (B, B))$. Then

$$P(c_1 | \mathcal{D}) = \frac{1}{16} \sum_{x_a \in A} \sum_{x_b \in A} P(c_1 | \mathcal{D}, x_1 = (B, B), x_2 = x_a, x_3 = x_b)$$

(5)

Expressions for each probability in (6) are shown in the appendix with respect to $h_1, h_2, h_3$ and the signal-to-noise ratio (SNR), defined as $\text{SNR} \triangleq \rho \triangleq 1/N_0$. For each of the ten cases, we disregard the probability of the AWGN noise moving a symbol between disjoint regions with the same background color. This is adequate consideration for practical SNR situations, as will be seen in Section IV, in which we compare analytical and simulation results. Substituting (14)-(23) (shown in the appendix) into (6) yields

$$P(c_1 | \mathcal{D}) = \frac{1}{16} \left[ 2Q(-B\sqrt{2}\rho(h_1 - h_2)) + Q(-B\sqrt{2}\rho(h_1 - h_2 - h_3)) + Q(-B\sqrt{2}\rho(h_1 + h_2 - h_3)) + Q(-B\sqrt{2}\rho(h_1 - h_2 - h_3)) \right]^2.$$  

(7)

An analogous procedure is applied to obtain $P(c_1 | \mathcal{D})$, where the probabilities in (6) are derived from Fig. 1a. This calculation is performed in [16, Eq. (58)] yielding

$$P(c_1 | \mathcal{D}) = \frac{1}{16} \left[ Q(-B\sqrt{2}\rho(h_1 + h_2 + h_3)) + Q(-B\sqrt{2}\rho(h_1 - h_2 + h_3)) + Q(-B\sqrt{2}\rho(h_1 + h_2 - h_3)) + Q(-B\sqrt{2}\rho(h_1 - h_2 - h_3)) \right]^2.$$  

(8)

Let $P(e_1)$ be the probability of decoding an erroneous symbol for user 1. The conditional SER for user 1 is $P(e_1 | X) = 1 - P(c_1 | X)$, for $X \in \{\mathcal{D}, \mathcal{D}^c\}$. To derive an approximate expression for the conditional probability of a correct decoding for user 2, we consider as a necessary condition the correct decoding for user 1 [10]. Therefore

$$P(c_2 | X) \approx P(c_2 | c_1)P(c_1 | X), \quad X \in \{\mathcal{D}, \mathcal{D}^c\}$$  

(9)

where $P(c_1 | X)$ is given either by (7) or (8), and $P(c_2 | c_1)$ refers to the conditional probability of a correct decoding for user 2, given a correct decoding for user 1. This probability corresponds to the same uplink NOMA scenario, considering only two users. This is calculated in [16, Eq. (14)]

$$P(c_2 | c_1) = 4\left[ Q(-B\sqrt{2}\rho(h_2 - h_3)) + Q(-B\sqrt{2}\rho(h_2 + h_3)) \right]^2.\quad (10)$$

For user 3, we apply again the consideration of the correct decoding for users 1 and 2. Therefore

$$P(c_3 | X) \approx P(c_3 | c_2 \cap c_1)P(c_2 \cap c_1 | X), \quad X \in \{\mathcal{D}, \mathcal{D}^c\}.$$  

(11)

We consider $P(c_2 \cap c_1 | X) = P(c_2 | X)$ and $P(c_3 | c_2 \cap c_1) = Q^2(-Bh_3\sqrt{2}\rho)$ is the SER of the QPSK constellation with symbols at $(\pm Bh_3, \pm Bh_3)$.  

IV. RESULTS FOR THE UPLINK SCENARIO

In the analyses conducted in this section we consider $h_3 = 1$ and varying parameters $h_1, h_2$, and SNR. Different values for $h_3$ yield similar results. We utilize the convention to define the parameters $h_i$ by the relation $h_i^2 = h_i^2 + \xi_i$ (dB), $i = 1, 2$, and $h_3^2 = 0$ (dB). Therefore, we consider the SER system performance with respect to $\xi_1, \xi_2$, and SNR, and fix one or two variables resulting in, respectively, a two-dimensional or a one-dimensional plot.

We consider some scenarios to compare the simulated and analytical SER for each user. In the first one, shown in Fig. 3, we fix $\xi_1 = \xi_2 = 3$ dB (case $\mathcal{D}^c$) and vary the SNR. In another one, the SNR is fixed at 13 dB and 18 dB, and we consider the restriction $\xi = \xi_1 = \xi_2$ and vary $\xi$, as shown in Figs. 4 and 5. These figures show a good agreement between analytical and simulation results for users 1 and 2 and some disagreement for user 3 for some choices of $\xi$ in the region of high SER. As already discussed in Section III, the derived analytical SER considers the correct decoding of the previous users, and therefore, $P(e_1) \leq P(e_2) \leq P(e_3)$, which is not correct for all values of $\xi_1$ and $\xi_2$, as shown by the simulated results.
Comparison between the simulated and analytical SER for each user considering $\xi = \xi_1 = \xi_2$ in Figs. 4 and 5. The solid lines represent the analytical solution, whereas the circled markers indicate the simulation results. The threshold for the inequality $D$ (indicated in the figure by $\xi_D$) corresponds to $\xi_D = 20\log_{10}\left(\frac{1+\sqrt{3}}{2}\right) \approx 4.18$ dB, which is obtained by combining the inequality $D$ with the $\xi_i$ definition, under the restriction $\xi = \xi_1 = \xi_2$. The SER for the case $D$ (region to the right of $\xi_D$) decreases monotonically with increasing $\xi$. For the case $\overline{D}$ (region to the left of $\xi_D$), the SER dependence on $\xi$ is not monotonic, with an optimal spacing, denoted by $\xi^*$, for each user. It is also important to note that the optimal spacing $\xi^*$ for user 1 does not always correspond to optimal $\xi^*$ for other users. A meaningful result obtained by comparing Figs. 4 and 5 is the dependence of the optimal spacing with the SNR. For user 1 and SNR $= 13$ dB (Fig. 4), $\xi^* = 2.83$ dB, while for SNR $= 18$ dB (Fig. 5), $\xi^* = 2.75$ dB, both values agree with the simulations and can be obtained numerically from the provided SER expression and employing the restriction $\xi = \xi_1 = \xi_2$. We also show in Figs. 4 and 5 a special value $\xi^* = 20\log_{10}\left(\frac{1+\sqrt{3}}{2}\right) \approx 2.71$ dB, which refers to a choice of $h_i$ such that the points indicated by #1, #2, #3 and #4 in Fig. 2 are symmetrically positioned in their respective regions, under the restriction $\xi = \xi_1 = \xi_2$. In other words, if we choose $\xi$ such that the point indicated by #1 in Fig. 2 is at the coordinate $\left(\frac{1}{2}, \frac{1}{2}\right)$, the center of its respective square, we obtain a suboptimal SER. The disjoint decision regions also add complexity to the optimal constellation design.

It should be noted that under case $\overline{D}$, we have a non-monotonic behaviour. This can be explained by the atypical disjoint decision regions. Fig. 6 shows three receiving constellations at the base station, similar to those shown in Fig. 1, for extreme $\xi_i$ values. In Fig. 6a, $\xi = 1$ dB, results in different user 1 symbols mapped to overlapping positions under the specified SNR, at regions near $|L| = L$ or $|Q| = L$, resulting in poor performance. In Fig. 6b, $\xi = 2.7$ dB, which is close to the optimal $\xi^*$ value, resulting in well-spaced constellation points and good performance. In Fig. 6c, we consider a value of $\xi$ close to the threshold $\xi_D$ ($\xi = 4.1$ dB), resulting in poor performance due to different symbols being mapped to overlapping positions near $L = 0$ or $Q = 0$.

We now consider a scenario in which the SER of user 3 is significantly lower than that of users 1 and 2, which occurs when $h_1 \approx h_2$ and $h_2$ is sufficiently larger than $h_3$. Fig. 7 shows the SER versus SNR of each user for $\xi_1 = 1.5$ dB and $\xi_2 = 5$ dB (case $\overline{D}$). A possible explanation for this phenomenon is that a decoding error at user 1 corresponds to a vector addition of $-h_1\hat{x}_1, \hat{x}_1 \neq x_1$, which can induce an error at user 2, $\hat{x}_2 \neq x_2$, and a subsequent addition of $-h_3\hat{x}_2$. It is also important to note that the errors of users 1 and 2 are correlated due to the nature of the sequential SIC decoding, which can result in a cancellation of the added error vectors, which in practice means that the symbol of user 3 is corrupted by an error propagation factor proportional to $B(h_1 - h_2)$, which is not necessarily sufficient to cause an error at user 3. The exact scenarios where the analytical expression for user 3
is inadequate are discussed later in this section. Recall that the discrepancy between analytical and simulations results for user 3 is due to the assumption of correct decoding of the previous users.

We show next the SER comparisons for three decoder strategies: Traditional-SIC, Modified-SIC, and a neural network (NN) similar to the presented in [21]. Fig. 8 shows the SER versus SNR for the cases $D$ and $\overline{D}$. For the case $D$, shown in Fig. 8a for $\xi_1 = \xi_2 = 5$ dB, all decoders have very similar performance. Fig. 8b uses $\xi_1 = \xi_2 = 3$ dB and shows that the Modified-SIC decisively outperforms the Traditional-SIC for the case $\overline{D}$, which confirms that the proposed customized decision regions enable better performance in some cases. The NN model performs almost identically to the Modified-SIC decoder, which suggests that the NN boost in performance over the Traditional-SIC reported in [21] occurs due to the learning of decision regions similar to those shown in Fig. 1b.

Since the model in [21] deals with NOMA decoding in scenarios in which $\xi_i$ is low, we compare the proposed decoder with its NN and argue that the proposed decoder is an alternative to the NN approach for low modulation orders $M$, in the three-user scenario. According to [21], the NN scheme has a computational complexity which is empirically determined, given as $O(60 \times 80)$, and is essentially fixed for the investigated scenario. The proposed decoder has a complexity $O(M^2)$, valid for the three-user scenario, since for user 1 decoding we have to loop over its $M$ possible transmission hypotheses (line 4 of Algorithm 1) and perform $M$ operations to check if $(y'_1, y'_Q)$ belongs to each one of the possible $M$ regions (lines 14-17), in the worst case. Our decoder also provides theoretically guaranteed performance and a better understanding of the performed decoding steps. As a consequence of the similar performance in the proposed scenario, it could also be argued that such decoder can be viewed as a theoretical estimate or reference to the expected performance of the NN model.

A. HEATMAP ANALYSIS
The heatmaps for the analytical SER expressions for users 1, 2, and 3 are shown in Figs. 9a, 9b, and 9c, respectively. The heatmaps aim to show the overall performance behavior by considering joint changes in the parameters $\xi_1$ and $\xi_2$, given a fixed SNR = 18 dB. It is also indicated (in a dashed line) the threshold for the inequality $D$. For all users, these figures indicate that the performance is poor for $h_i$ configurations close to this threshold ($h_1 \approx h_2 + h_3$) and it is possible to obtain better results by moving away from the threshold. It can be seen that low $\xi_1$ values correspond to poor performance, for any $\xi_2$, since if $h_1 \approx h_2$ the interference in the decoding of user 1 is high, and errors in first step of SIC decoding affect the remaining users (users 2 and 3). In Figs. 9b and 9c, a cut-off value for performance of users 2 and 3 is seen at around $\xi_2 \approx 3$ dB for any $\xi_1$ used, which can be explained by considering that if correct user
1 decoding occurs, the $\xi_2$ parameter controls the interference between the remaining users. Because the employed derivation hypothesis $P(e1) \leq P(e2) \leq P(e3)$, the regions where good performance for users 2 and 3 occur are necessarily a subset of the regions where good performance occurred for user 1. This behavior can be seen in the theoretical heatmaps.

For validation purposes, we also present simulated heatmaps, which confirm the accuracy of the SER expressions for users 1 and 2 (Figs. 10a and 10b), and outline where disagreements occur for user 3 (Fig. 10c). The simulations are performed for a low number of bits ($10^5$) per cell, on a $50 \times 50$ grid due to hardware limitations. As discussed before,
user 3 may have lower SER (for a given SNR) than users 1 and 2, which is not considered in the analytical expressions. This is shown in the simulated heatmap for user 3 (Fig. 9c) for $\xi_1 < 2$ dB and $\xi_2 > 3$ dB. For these values of $\xi_1$ and $\xi_2$, the proposed analytical expression for user 3 is invalid.

The average SER heatmaps ($\frac{1}{3} \sum_{i=1}^{3} P(e_i | X), X \in \{D, \overline{D}\}$), are shown in Fig. 11 for SNR = 18 dB. Fig. 11a presents the analytical result and Fig. 11b the simulated one. It is seen that both versions agree for all considered $\xi_i$. Thus, the average SER can be expressed with the provided analytical expressions. Similar experiments, not shown here, were also conducted for different SNRs to confirm the result.

The analytical expressions can be employed by the users to adjust their own transmitted power $P_i$ aiming to minimize the SER, given that the channel gain of each user is known. This situation is plausible in a scenario where the base station communicates with the users informing the current channel gain obtained from pilot symbol transmissions. In the following, we present an analysis targeting the SER of user 1, which is chosen due to the strong influence on the overall system performance, although any user could be chosen. Fig. 12 shows the heatmap for user 1 with respect to $\xi_1$ and SNR, in which $\xi_2$ is fixed at 3 dB. This scenario seeks to reflect the situation where $h_2, h_3$ are known by user 1 and this user seeks to adjust $P_1$ and consequently $\xi_1$ to obtain optimal SER for a given SNR (under the assumption that $h'_1$ is known). By inspecting Fig. 12 we conclude that different scenarios may be employed according to the SNR. By considering SNR = 25 dB, increasing or decreasing $P_1$ around a reference value, $\xi_1 \approx 4.65$ dB (dashed line) reduce the SER. The reduction of $P_1$ under SNR = 9 dB does not provide the same gains.

We now consider a restriction on the total transmitted power by considering $\sum_{i=1}^{3} h_i^2 \leq K$, under fixed SNR assumption. We consider a scenario where we seek to optimize the SER of user 1 under the constraint that users 2 and 3 must have SER $\leq 10^{-3}$ in addition to the transmitted power restriction. The provided theoretical formula can be used to quickly find the optimal points under the constraints. We delimit four regions in the SER heatmap of user 1, for
$K \in \{4, 7, 10, 14\}$, indicated by black dashed lines in Fig. 13, meaning that the power constraint limits the analyzed region to the upper side of the black dashed lines. The green stars in Fig. 13 indicate the optimal SER values for each considered restriction. The yellow dashed line indicates the $D$ inequality, in which the right side represents case $D$, and the left side, $\overline{D}$. For $K = 14$, the optimal values are $\xi_1 \approx 7.5$ dB and $\xi_2 \approx 2.9$ dB, which indicates case $D$. For $K = 10$, the optimal SER for user 1 is achieved for $\xi_1 \approx 2.1$ dB and $\xi_2 \approx 5.3$ dB, which indicates $\overline{D}$. In this case, it is not possible to achieve better performance using the right side of the yellow line, since the performance for users 2 and 3 does not satisfy the constraints. For $K = 7$, optimal SER for user 1 is achieved for $\xi_1 \approx 2.5$ dB and $\xi_2 \approx 3.3$ dB, which
indicates $\overline{D}$. For $K = 4$, it is impossible to satisfy the SER restriction for users 2 and 3 in the considered scenario.

V. DOWNLINK

We now consider the 3-user single-antenna NOMA downlink scenario. The received signal for user $u, u \in \{1, 2, 3\}$, is written as [10]

$$y_u = \sqrt{P_u} h_u' \sum_{i=1}^{3} (\sqrt{\alpha_i} x_i) + n_u$$

(12)

where $P_u$ is the transmit power of the base station, $h_u'$ is the channel gain for user $u$, which is perfectly phase compensated, $\alpha_i$ is the power allocation coefficient for the transmitted signal corresponding to user $i$, which satisfies $\sum_{i=1}^{3} \alpha_i = 1$, $x_i$ is the unit energy QPSK symbol transmitted to user $i$ and $n_u = (n_1, n_Q)$ is the AWGN noise. We assume that $\alpha_1 > \alpha_2 > \alpha_3$, and the SIC decoding follows this ordering. We define $h_u \triangleq |h_u'| \sqrt{P_u}$ as the multiplicative term of user $u$.

By analogy to the uplink scenario, we interpret (12) as distinct realizations of (1) for each user. Therefore, we set

$$h_u \triangleq h_u \sqrt{\alpha_i}$$

(13)

and employ the SER analysis for the uplink scenario shown in Section III using the resulting nine $h_u'$. In the downlink scenario, we employ for user 1 the uplink result $P(e_1|X)$ (7-8), $X \in \{D, \overline{D}\}$ by setting $h_1 = h_1'$, for $i = \{1, 2, 3\}$. For user 2 we employ (9) by setting $h_1 = h_2'$, and for user 3 we employ (10) by setting $h_1 = h_3'$.

We verify the adequacy of the conversion given in (13) by comparing the analytical and simulation results under two $\alpha_i$ configurations. We utilize the convention to define the parameters $h_u$ by the relation $h_u^\alpha = h_u'^{\alpha_i} + \gamma_u$ (dB), $u = \{1, 2\}$, and $h_3 = 0$ (dB). The power allocation coefficients $\alpha_i$ determine if the corresponding scenario corresponds to $D$ or $\overline{D}$ according to inequality $E : \sqrt{\alpha_1} + \sqrt{\alpha_2} + \sqrt{\alpha_3}$, which follows immediately from inequality $D$ and (13). Fig. 14 shows the comparison results for two scenarios $E$ and $\overline{E}$ and confirms that the analysis from Section III can be employed by converting downlink-uplink gain parameters according to (13). We use in this figure $\gamma_1 = \gamma_2 = 3$ dB.

We next present heatmaps for the downlink scenario. We fix $\gamma_u$ and present the result in terms of $\alpha_i$. We set $\gamma_1 = \gamma_2 = 3$ dB and consider the previously presented restrictions for $\alpha_i$: $\sum_{i=1}^{3} \alpha_i = 1$ and $\alpha_1 > \alpha_2 > \alpha_3$. The analytical and simulated heatmaps are presented in Figs. 15 and 16, respectively, where invalid choices of $\alpha_i$ are shown in green. It is shown the inequality $E$ and the corresponding usage of the Modified-SIC decoder. Similar to the uplink scenario, regions closer to the inequality threshold result in poor performances. These figures show adequate agreement between analytical expressions and simulated results for users 1 and 2.
User 3 continues to have a significant mismatch between analytical and simulated results for a restricted region in the Modified-SIC side of the inequality $\mathcal{E}$.

VI. CONCLUSION

In this paper, we proposed a modified SIC decoding for the three-user NOMA scenario by assuming channel gain knowledge. We investigated the SER performance for different decoders under both scenarios $\mathcal{D}$ and $\overline{\mathcal{D}}$ (or $\mathcal{E}$ and $\overline{\mathcal{E}}$), in which results holds for both uplink and downlink scenarios, with a minor necessary adaptation. It was shown that the proposed decoder can be seen as an alternative way to obtain previously encountered results in the literature using deep learning techniques [21]. Furthermore, the joint bidimensional analysis enables a power allocation approach and shows that the proposed decoder can outperform the traditional one under specific power allocation and channel gain constraints.

Future work can investigate generalization to a higher number of users and the decision regions for each user. A possible approach to achieve this generalization is to employ a decision tree (DT) or support vector machine (SVM) model as part of the decoding algorithm. DT could be used alternatively to NN since the decision regions obtained after training can be extracted directly from the tree. This model typically provides a better understanding about the task than NN since the tree can be effectively visualized, and any model prediction can be explained using Boolean logic. We also argue that the DT, although less powerful, is enough to tackle the low number of users scenario while using a white box model, in contrast to the NN’s black box. Investigation of imperfect knowledge of channel gains can also be considered for a better understanding of the proposed method’s practical usefulness, since the decision regions depend on these variables. Another line of investigation is the analysis of related techniques for $N$ users generalization, such as the influence of the proposed algorithm in pairing algorithm schemes or beamforming.

APPENDIX

Expressions for probabilities $P(c_1|\#i)$, $i = 1 \ldots 10$ are shown in (14)–(23), as shown at the bottom of the previous page, for case $\mathcal{D}$. We use $Q(z) = \frac{1}{\sqrt{2\pi}} \int_{z}^{\infty} e^{-t^2/2} dt$.

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GABRIEL CARLINI was born in Recife, Brazil, in 1998. He is currently pursuing the B.Sc. degree in electronics engineering from the Federal University of Pernambuco, Recife. His research interests include digital communications and machine learning.

CARLOS E. C. SOUZA received the Ph.D. degree in electrical engineering from the Federal University of Pernambuco, Recife, Brazil, in 2018. He has been a Postdoctoral Researcher with the Federal University of Pernambuco, since 2018. His current research interests include chaos communication, information theory, quantum chaos, and quantum transport in nanostructures.

CECILIO PIMENTEL (Senior Member, IEEE) was born in Recife, Brazil, in 1966. He received the B.Sc. degree in electrical engineering from the Federal University of Pernambuco, Recife, in 1987, the M.Sc. degree in electrical engineering from the Catholic University of Rio de Janeiro, Rio de Janeiro, Brazil, in 1990, and the Ph.D. degree in electrical engineering from the University of Waterloo, ON, Canada, in 1996. From 2007 to 2008, he was a Visiting Research Scholar with the Department of Mathematics and Statistics, Queen’s University, Kingston, Canada. Since 1996, he has been with the Department of Electronics and Systems, Federal University of Pernambuco, where he is currently a Professor. His research interests include digital communications, information theory, chaos communication, and error correcting coding. He is a Senior Member of the Brazilian Telecommunications Society.

DANIEL P. B. CHAVES (Member, IEEE) received the B.Sc. degree in electronics engineering and the M.Sc. degree in electrical engineering from the Federal University of Pernambuco, Recife, Brazil, in 2004 and 2006, respectively, and the Ph.D. degree in electrical engineering from the State University of Campinas, São Paulo, Brazil, in 2011. Since 2012, he has been with the Department of Electronics and Systems, Federal University of Pernambuco, where he is currently an Associate Professor. His current research interests include information theory, coding theory, symbolic dynamics, system modeling, chaos communication, chaotic circuits, and chaos-based random number generators.