I. INTRODUCTION

The Standard Model (SM) of particle physics has proved to be a very successful theory, but still has problems. Among these, one that is considered fundamental is the Hierarchy Problem, which might be solved by considering extra dimensions. In higher dimensional spacetimes the gravitational theory of Einstein can be generalized in several ways. The simplest of these generalizations is due to Cartan.

Cartan’s generalization of general relativity, which introduces torsion to the picture, can be coupled to fermionic matter in a natural way. Since the equation of motion for the spin connection is a constraint, related to the contortion, it can be used to get rid of the torsion in the original action. The new action contains standard general relativity and matter fields with an additional contact four fermion interaction [1].

The effective four fermion interaction term has a coupling constant proportional to Newton’s gravitational constant, $G_N \sim M_{pl}^{-2}$. Therefore, at first glimpse this interaction is highly suppressed. Nevertheless, in the last twenty years diverse scenarios have proposed that the existence of extra dimensions could explain the hierarchy problem, and thus the (higher dimensional) fundamental gravity scale might be roughly $M_\ast \sim \mathcal{O}(1) \text{ TeV}[2-5]$. Recently, limits to the size of the extra dimensions have been set up by direct searches of quantum black holes [6] and the influence of the exchange of virtual gravitons on dilepton events [7].

On the other hand, the ATLAS collaboration has presented experimental limits for the coupling constant of four fermion contact interaction [7-9]. This result is particularly important for imposing bounds on the value of the fundamental gravity scale, $M_\ast$, and by extension in order to find limits on the dimensionality or size of the space-time.

The aim of this work is to find natural bounds and limits on the typical size of the extra dimensions, as well as on the gravitational scale, $M_\ast$. In Sec. II, for the sake of completeness, a brief presentation of Cartan’s generalisation of gravity coupled with fermions is presented. In the next sections constraints are found for different higher dimensional scenarios, such as those proposed byArkani-Hamed, Dimopoulos and Dvali [2, 3] in Sec. III A, and by Randall and Sundrum [4, 5] in Sec. III B. In Sec. IV a summary of results and conclusions are presented.

II. CARTÁN-EINSTEIN GRAVITY WITH FERMIONS

It is a well-known fact that Einstein’s gravity is a field theory for the metric, and the space-time connection is required to be torsion-free.

However, when the first order formalism of pure gravity is considered, whose fields are the vielbeins and spin connection, the torsion-free imposition is nothing but the equation of motion for the spin connection.

Moreover, for gravity coupled with matter (specially fermionic matter), this condition changes and introduces a four-fermion contact interaction. The modification of the fermionic Lagrangian due to the presence of torsion is presented below.

The Cartan-Einstein action in $D$-dimensions is,

$$S_{gr} = \frac{1}{2\kappa^2} \int \frac{\epsilon_{a_1\cdots a_D}}{(D-2)!} \hat{R}^{a_1a_2} \wedge \hat{e}^{a_3} \wedge \cdots \wedge \hat{e}^{a_D},$$  

(1)

where $\hat{e}^a$ and $\hat{\omega}^a_{\dot{b}}$ are the vielbein and spin connection 1-forms respectively, and $\hat{T}^a$ and $\hat{R}^{a}_{\dot{b}}$ are the torsion and curvature 2-forms satisfy the structure equations

$$d\hat{e}^a + \hat{\omega}^a_{\dot{b}} \wedge \hat{e}^\dot{b} = \hat{T}^a,$$

(2)

$$d\hat{\omega}^a_{\dot{b}} + \hat{\omega}^c_{\dot{d}} \wedge \hat{\omega}^d_{\dot{e}} \wedge \hat{\omega}^a_{\dot{c}} = \hat{R}^{a}_{\dot{b}}.$$

(3)

Additionally, the Dirac action in arbitrary dimension is

$$S_\Psi = -\int \frac{\epsilon_{a_1\cdots a_D}}{(D-1)!} \bar{\Psi} \hat{e}^{a_1} \wedge \cdots \wedge \hat{e}^{a_{D-1}} \gamma^{a_D} \hat{D} \Psi$$

$$- m \int \frac{\epsilon_{a_1\cdots a_D}}{D!} \bar{\Psi} \hat{e}^{a_1} \wedge \cdots \wedge \hat{e}^{a_D} \Psi,$$

(4)

where $\epsilon_{a_1\cdots a_D}$ is the totally anti-symmetric $D$-index tensor.
with $\hat{\mathcal{D}}$ the exterior derivative twisted by the spin connection.

Since the total action is

$$S = S_{\text{gr}} + S_{\Psi},$$

the equations of motion for the whole system are,

$$\hat{\mathcal{K}}^{\hat{a}}_{\hat{a}} - \frac{1}{2} \hat{\mathcal{R}}^{\hat{a}}_{\hat{a}} = \kappa^2 \bar{\Psi} \{ \gamma^{\hat{a}} \hat{\mathcal{D}} \bar{\Psi} - (\hat{\mathcal{D}} + m) \} \Psi,$$

$$\hat{\mathcal{K}}_{\hat{a}\hat{b}\hat{c}} = - \frac{\kappa^2}{4} \bar{\Psi} \gamma_{\hat{a}\hat{b}\hat{c}} \Psi.$$  \hspace{1cm} (7)

Since Eq. (7) is a constraint, it can be substituted into the action, using that

$$\hat{\omega}^{\hat{a}}_{\hat{b}} \rightarrow \hat{\omega}^{\hat{a}}_{\hat{b}} + \hat{\mathcal{K}}^{\hat{a}}_{\hat{b}},$$

where the former is a general spin connection, while the later is a torsion-free one.

The new action is

$$S = \int d^Dx \, |\epsilon| \left[ \frac{1}{2k^2} \hat{\mathcal{R}} - \bar{\Psi} \left( \hat{\mathcal{D}} + m \right) \Psi + \frac{\kappa^2}{32} \bar{\Psi} \gamma_{\hat{a}\hat{b}\hat{c}} \Psi \bar{\Psi} \gamma^{\hat{a}\hat{b}\hat{c}} \Psi \right],$$

which is a torsion-free theory of gravity coupled to a fermion with a four-fermion contact interaction.

In the next sections the gravitational aspect of this model is not considered, based in the fact that the universe is essentially flat, and gravitational forces are extremely weak in comparison to other known interactions. Nonetheless, the existence of torsion still leaves behind a four-fermion interaction. Hereon, our objective will be to compare the four-fermion interaction coming from this gravity model with the four fermion interaction limits found in ATLAS experiment.

### III. BOUNDS ON FOUR-FERMI-ON INTERACTION

Early proposals of contact four-fermion interaction signals in colliders are found in [10–12]. These works inspired searches at the Large Hadron Collider (LHC) experiments. In particular, the ATLAS collaboration has found limits on the scale of four-fermion contact interaction, by analyzing the invariant mass and angular distribution of dijets [7–9]. The strongest constraint comes from an interaction of chiral fermions in the form

$$\mathcal{L}_{\text{qqqq}} = \pm \frac{g^2}{2\Lambda^2} \bar{\psi}_q \gamma^a \psi_q \bar{\psi}_L \gamma_\alpha \psi_L,$$

which is experimentally excluded for $\Lambda < 9.5$ TeV, assuming that the coupling constant $g \sim O(1)$.

The aim of this section is to compare the above result with the four-fermion interaction coming from the Cartan-Einstein gravity action [13],

$$\mathcal{L}_{\Psi} = \frac{\kappa^2}{32} \bar{\Psi} \gamma_{\hat{a}\hat{b}\hat{c}} \Psi \bar{\Psi} \gamma^{\hat{a}\hat{b}\hat{c}} \Psi.$$  \hspace{1cm} (10)

It becomes clear that if one starts in four-dimensions, where $\kappa^2 \sim M_{pl}^{-2}$, $\gamma_{abc} \sim \gamma^{d\gamma^a}\gamma^b$ and $\Psi = \bar{\psi}_L + \psi_R$, the comparison implies $\Lambda \sim M_{pl} \gg 10$ TeV, which is tautological.

Nonetheless, there exist models where the fundamental scale of gravity is not $M_{pl} \sim 10^{18}$ GeV, but rather a much lower one, $M_*$, which could be of order of the electroweak scale, i.e. $M_* \sim M_{EW}$, giving a natural solution to the Hierarchy Problem. These models, however, require extra dimensions. Therefore, in the following the space-time will be considered to be a $(4+n)$-dimensional, where the $n$ extra dimensions are either compact or extended, and differentiation between models is made according to this characteristic.

In order to achieve the goal of comparison, it is necessary to reduce the dimension of the space-time from $D$ down to four. Although the dimensional reduction could be a complex procedure [14], we will sketch how a term like Eq. (10) appears.

First, in the contraction $\gamma_{\hat{a}\hat{b}\hat{c}} \gamma^{\hat{a}\hat{b}\hat{c}}$ one considers only the four-dimensional part, i.e., $\gamma_{abc} \gamma^{abc}$. Second, the antisymmetric product of three elements of the four-dimensional Clifford algebra is equal to the product of the missing element of the Clifford algebra times the chiral element, i.e., $\gamma_{abc} \sim \gamma^d \gamma^e$. Next, the higher dimensional spinor $\Psi$ can be decomposed as the product of the four-dimensional times $n$-dimensional spinors,

$$\Psi(x, \xi) = \sum_i \left( \psi_L^{(i)}(x) + \psi_R^{(i)}(x) \right) \otimes \lambda_i(\xi).$$  \hspace{1cm} (11)

Therefore, after integration of the extra dimensions, the effective four-dimensional theory would have a term of the desired form

$$\mathcal{L}_{\text{eff}} = \frac{\kappa^2}{32} \bar{\psi}_q \gamma^a \psi_q \bar{\psi}_L \gamma_a \psi_L.$$  \hspace{1cm} (12)

Dimensional analysis gives two possibilities. Either the effective coupling constant is directly related to the fundamental gravitational scale, $\kappa_{\text{eff}} \sim M_*^{-1}$, or is related to the effective four-dimensional one $\kappa_{\text{eff}} \sim \left( M_{pl}' \right)^{-1}$, where $M_{pl}'$ is the redefined Planck mass after the dimensional reduction.

#### A. ADD Models

ADD models [2, 3] consist of a four-dimensional space-time with a set of $n$ compact extra dimensions, with typical length $R$. Matter is confined to the four-dimensional space-time for energies below $\Lambda \sim \frac{1}{R}$, while gravity propagates through the whole space-time. This configuration allows to solve the hierarchy problem, because the natural scale for gravity is not the effective four-dimensional one, but rather the $(4+n)$-dimensional.

The relation between the fundamental gravitational scale, $M_*$, and the four-dimensional effective one, $M_{pl}$
the relation between the four-dimensional Planck mass, $M_{pl}$, and the fundamental (five-dimensional) Planck scale, $M_*$, is given by

$$M_{pl}^2 = \frac{M_*^3}{k^2}. \quad (17)$$

Since a well-known modulus stabilization method would ensure that the product $kr_c \sim 10^{[15]}$, the relation between the gravitational scales and the length of the extra dimension is found to be

$$M_{pl}^2 \sim \frac{M_*^3 r_c}{10}. \quad (18)$$

Analyzing both limits as in the previous section, the limit on the extra dimension size is

$$10^{-13} \text{ m} < r_c < 10^{19} \text{ m}. \quad (19)$$

The range is particularly wide because there is a single extra dimension. Although brane-worlds of codimension higher than one have been considered[16–20], without a carefully thought-out moduli stabilization process the bounds on the extra dimensions sizes are equal to those found in Sec. III A (shown in Table I).

IV. CONCLUDING REMARKS

In this work we have used the limits on four-fermion chiral contact interactions obtained by the ATLAS collaboration in order to constrain the typical size of eventual extra dimensions in models where gravity admits torsion in the bulk. For a codimension 2 we have found an upper bound for the radius of the extra dimensions of the order of $10^{-6} \text{ m}$, which is comparable to the limits obtained from direct search of the Kaluza-Klein excitations of the graviton [21]. Nevertheless, for higher numbers of extra dimensions we find that the constrains are much more stringent. This is due to the fact that the fundamental gravitational scale, $M_*$, is related with the effective one, say $M'_{pl}$, through higher powers, reducing the dependence of the model on the size of the extra dimensions. This result provides an example of the possibility of testing non-trivial extensions of General Relativity using collider data.

ACKNOWLEDGEMENTS

We would like to thank to C. Dib, N. Neills, J.C. Helo and A. Carcamo for fruitful discussions and encourage through the realisation of this work.

This work was supported in part by Fondecyt Project No. 11000287, Fondecyt Project No. 1120346.

[1] Daniel Z. Freedman and Antoine van Proyen. Supergravity. Cambridge University Press, 2012.

[2] Nima Arkani-Hamed, Savas Dimopoulos, and G. R.
Dvali. The hierarchy problem and new dimensions at a millimeter. *Phys. Lett.*, B429:263–272, 1998.

[3] Nima Arkani-Hamed, Savas Dimopoulos, and G. R. Dvali. Phenomenology, astrophysics, and cosmology of theories with sub-millimeter dimensions and TeV scale quantum gravity. *Phys. Rev.*, D59:086004, 1999.

[4] Lisa Randall and Raman Sundrum. A large mass hierarchy from a small extra dimension. *Phys. Rev. Lett.*, 83:3370–3373, 1999.

[5] Lisa Randall and Raman Sundrum. An alternative to compactification. *Phys. Rev. Lett.*, 83:4690–4693, 1999.

[6] Douglas M. Gingrich and Krishan Saraswat. Model uncertainties on limits for quantum black hole production in dijet events from ATLAS. 2012.

[7] Georges Aad et al. Search for contact interactions and large extra dimensions in dilepton events from pp collisions at $\sqrt{s} = 7$ TeV with the ATLAS detector. 2012.

[8] Georges Aad et al. Search for New Physics in Dijet Mass and Angular Distributions in pp Collisions at $\sqrt{s} = 7$ TeV Measured with the ATLAS Detector. *New J.Phys.*, 13:053044, 2011.

[9] Georges Aad et al. ATLAS search for new phenomena in dijet mass and angular distributions using pp collisions at $\sqrt{s} = 7$ TeV. 2012.

[10] E. Eichten, I. Hinchliffe, K. Lane, and C. Quigg. Supercollider physics. *Rev. Mod. Phys.*, 56:579–707, Oct 1984.

[11] E. Eichten, I. Hinchliffe, K. Lane, and C. Quigg. Erratum: Supercollider physics. *Rev. Mod. Phys.*, 58:1065–1073, Oct 1986.

[12] P. Chiappetta and M. Perrottet. Possible bounds on compositeness from inclusive one jet production in large hadron colliders. *Phys.Lett.*, B253:489–493, 1991.

[13] Note that four-dimensional fermions are denoted with lower case symbol $\psi$, whilst the fermion in arbitrary dimension is denoted with the capitalized one, $\Psi$.

[14] The difficulty for finding the effective theory comes from the fact that in higher dimensional space-times, the spinorial representation of the Lorentz group could have dimension different than four. Therefore, one should be aware of the decomposition of the spinors (including the profiles through the extra dimensions), as well as the Clifford algebra elements, prior to the integration of the extra dimensions.

[15] Walter D. Goldberger and Mark B. Wise. Modulus stabilization with bulk fields. *Phys.Rev.Lett.*, 83:4922–4925, 1999.

[16] Sean M. Carroll and Monica M. Guica. Sidestepping the cosmological constant with football shaped extra dimensions. 2003.

[17] S.L. Parameswaran, S. Randjbar-Daemi, and A. Salvio. Gauge Fields, Fermions and Mass Gaps in 6D Brane Worlds. *Nucl.Phys.*, B767:54–81, 2007.

[18] C.P. Burgess, Claudia de Rham, and Leo van Nierop. The Hierarchy Problem and the Self-Localized Higgs. *JHEP*, 0808:061, 2008.

[19] Allan Bayntun, C.P. Burgess, and Leo van Nierop. Codimension-2 Brane-Bulk Matching: Examples from Six and Ten Dimensions. *New J.Phys.*, 12:075015, 2010.

[20] Leo van Nierop. *Phenomenology of codimension-2 brane worlds: the importance of back-reaction*. PhD thesis, McMaster University, 2011.

[21] J. Beringer et al. (Particle Data Group). Review of Particle Physics. *Phys. Rev.*, D86:010001, 2012.