Thermodynamic analysis of changes the internal energy of an ideal gas under adiabatic throttling

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Abstract. The analysis of the irreversible process of adiabatic throttling of an incompressible liquid using the construction of physical and mathematical models of the process is carried out. The nature of irreversible dissipation of mechanical energy in the fluid flow is revealed. An analytical expression is obtained for the always positive so-called specific heat of friction, the value of which depends on the nature of the flow. This value can be equal to zero only in the case of a vortex-free or helical movement. Therefore, only with such movements, the mechanical energy of the flow does not dissipate.

1. Introduction

The stage of pre-project development of new technical solutions in the field of forest engineering often includes the creation of physical models of processes implemented in various installations. In such models, it is often necessary to analyze the energy exchange during the flow of the gas phase through channels with friction. Improving process models allows, ultimately, to improve the quality of manufactured equipment.

The throttling process has an analogy with the flow of a flow through a channel with friction. The flow with friction can be considered as «stretched» along the length of the channel throttling. On the other hand, a throttle device, for example, made in the form of a washer that partially blocks the passage section of the channel, can be considered as a significant local roughness of the channel, compared to which the effect on the flow of other small roughnesses can be ignored. Based on the latter, further analysis of the interaction of the flow with the channel surface will not consider [1].

The relevance of practical application and research related to the throttling process is reflected in a number of works [2-9].

2. Methods and Materials

The aim of this work is to comprehensively analyze the factors that affect the change in the internal energy of an ideal gas during its adiabatic throttling. The analysis method is based on the construction of a physical model of the process and its thermodynamic description.

The qualitative difference between the concepts of kinetic energy and internal energy of the body is as follows. Kinetic energy is that part of the total energy that depends on the choice of reference system and therefore is not a physical characteristic of the body as such. Internal energy is that part of the total energy of the body that does not depend on the choice of reference system, is related to the
body itself and therefore is a physical characteristic of the body, for example, the characteristic of the allocated volume of gas [10].

It should be noted that the internal energy is an indifferent scalar, that is, it does not depend on the choice of the reference system, despite the fact that one of its essential components is the kinetic energy of all gas molecules located inside the selected volume in chaotic motion. The marked property of internal energy, resulting from its definition, indicates the following. The velocity of gas molecules, which determines the value of internal energy, must always be measured in a frame of reference in which the entire volume of gas released is in thermodynamic equilibrium and, as a whole, at rest. As such a reference system, the $S_{CM}$ coordinate system can be chosen with the $x'$, $y'$, $z'$ axes and the reference point $O'$, which coincides with the point of the center of mass (CM) of the gas volume and is rigidly connected to it. Thus, in the coordinate system of the $S_{CM}$, the allocated volume of gas is always in thermodynamic equilibrium, and the speed and acceleration of its CM are equal to zero.

To analyze changes in internal energy in the process of adiabatic gas throttling, the following model is considered, schematically shown in figure 1. A stationary gas flow with subsonic speed moves along a channel of constant cross-section. A throttle device is installed inside the channel, for example, in the form of a washer that partially covers the passage section of the channel. When the flow part of the throttle device passes through, the parameters of the elementary particles that make up the gas flow change. These parameters include, in particular, the absolute pressure $p$, Pa; absolute temperature $T$, K; specific internal energy $u$, J/kg; speed of CM, the isolated elementary particle, or the isolated final volume $\bar{c}$, m/s. The area of flow inside the channel where its parameters change significantly is the area of heterogeneity. We will assume that the area of heterogeneity occupies a relatively small area, compared to the volume of the entire channel. Outside the area of heterogeneity, the flow parameters are almost unchanged. Before the throttle device, their values are $p_1$, $u_1$, $T_1$, $u_1$, $c_1$, and after $p_2$, $u_2$, $T_2$, $u_2$, $c_2$.

![Figure 1. Installation diagram for implementing adiabatic gas throttling in the slab coordinate system $S_{lab}$: where 1 – the channel; 2 – the throttle device made in the form of a washer; 3 – the support to which the channel is attached; 4 and 5 – fixed pistons.](image)

Two cross sections of the channel will allocate some volume in the gas flow in front of the throttle device. One section is located on the surface of the washer. The second is at some distance from it. If the selected volume is large enough, the area of heterogeneity can be ignored and it can be assumed that the entire selected volume is characterized by the parameters $p_1$, $u_1$, $T_1$, $u_1$, $c_1$.

Using the diagram in figure 1, we will trace the throttling process in the reference frame associated with the earth’s surface, that is, in the inertial laboratory coordinate system $S_{lab}$ with the $x$, $y$, $z$ axes and the reference point $O$ (figure 1), which is rigidly connected to the channel mounting support.

After all the elementary particles of the volume under consideration pass through the flow part of the throttle device, in accordance with the above, the parameters that characterize it will have the
values \( p_2, v_2, T_2, u_2, c_2 \). In this case, we can say unequivocally that \( p_1 > p_2, v_1 < v_2, c_1 < c_2 \). The diagram in figure 1 contains the following elements. Channel of constant cross-section \( I \). Rigidly fixed in the channel 1 washer 2. Support 3, fixed in the laboratory coordinate system \( S_{\text{lab}} \), to which the channel 1 is rigidly attached. Weightless pistons 4 and 5, limiting the allocated volume of gas in the channel. The CM point indicates the center of mass of the selected volume. At the initial moment of time, the entire allocated volume of gas is located directly in front of the washer 2. Accordingly, the piston 4 is located at some distance to the left of the washer 2, and the piston 5 – to the right of the washer 2, directly adjacent to it.

It is obvious that in the laboratory coordinate system of the \( S_{\text{lab}} \), the channel 1 and the washer 2 remain stationary during the entire process of passing the allocated volume through the flow part of the throttle device. During the process, the piston 4 moves to the right at a constant speed \( c_1 \). At the end of the process, the piston 4 is adjacent to the washer 2 on the left. The piston 5 moves to the right at a constant speed \( c_2 \).

The speed \( c_2 \) is higher than \( c_1 \) due to the fact that the specific volume of gas increases when throttling, that is, the gas expands, and the channel section remains unchanged. Thus, during the entire process, the CM point of the volume under consideration moves with acceleration. As a result of this acceleration, the speed of the CM point increases from \( c_1 \) to \( c_2 \).

Taking into account the properties of internal energy, it is convenient to analyze the process of energy exchange during throttling by observing it from the \( S_{\text{CM}} \) reference system, which is rigidly connected to the center of mass of the gas volume allocated in the flow. In the \( S_{\text{CM}} \) system, the throttling process looks different. The scheme of realization of such a process is shown in figure 2. The origin of the system CM point \( O' \) in figure 2 is at point CM the total mass of gas stored in the selected volume. The numbers in figure 2 correspond to figure 1.

\[ L_w = \frac{V_1}{S} + \frac{(V_2 - V_1)}{2S} = \frac{(V_1 + V_2)}{2S} \]

\[ (1) \]
where, \( L_w \) is the path traversed by the process inlet washer, m; \( V_1 \) and \( V_2 \) are the volumes of the released gas mass before and after passing the throttle device, m\(^3\); \( S \) is the surface area of the pistons equal to the area of the passage channel, m\(^2\).

Due to the positive pressure difference \( (p_1 - p_2) \) from the flow side, a force \( F \), H, directed in the opposite direction of the washer movement will act on the washer during its movement (figure 2). Accordingly, this force will perform negative work. That is, as a result of the throttling process, energy equal to the absolute value of this work will be brought to the allocated volume of gas from the outside.

Thus, when considering the throttling process from the \( S_{CM} \) coordinate system, energy is supplied to the flow from outside due to the movement of the throttle device.

This energy transfer mechanism is most clearly seen for the flow of an incompressible fluid. In this case, the flow does not expand, the pistons remain stationary, and the washer 2 together with the support 3 move at a constant speed. In practice, this process can be carried out if the support 3 in figure 2 is sufficiently massive. With a sufficiently large mass of support, its speed will not change significantly during the process, despite the effect of the braking force \( F_{br} \). However, due to the action of the braking force \( F_{br} \), the kinetic energy of the support will necessarily decrease by a finite amount. It is this component of the kinetic energy lost by the support 3 during the throttling process that is transferred to the allocated volume of liquid as a result of the movement of the throttle device in the \( S_{CM} \) coordinate system.

For the process of adiabatic gas throttling, the scheme of which is shown in figure 2, the first law of thermodynamics for the allocated volume can be formulated as follows. The change in the internal energy of the gas as a result of the adiabatic throttling process is equal to the total amount of energy supplied to it or withdrawn from it in the form of mechanical work. Energy in the form of mechanical work is supplied to the gas molecules or diverted from them when these molecules collide with moving surfaces. In the process under consideration, such collisions occur as a result of the movement of three objects: two pistons and a washer. In accordance with the above, the first law of thermodynamics is written as

\[
U_2 - U_1 = \epsilon_{int}^{CM} - A^{CM}.
\]  

where \( U_1 \) and \( U_2 \) are the internal energy of the gas mass located in the allocated volume at the beginning and end of the adiabatic throttling process, respectively, J; \( \epsilon_{int}^{CM} \) – the energy transmitted to the gas from outside as a result of the movement of the washer, J; \( A^{CM} \) – the energy withdrawn from the gas as a result of its expansion caused by the movement of two pistons, J.

In physical terms, the \( A^{CM} \) value is the work performed when the gas expands in a non-equilibrium adiabatic throttling process.

After dividing (2) by the mass \( M, \) kg, of the gas in the selected volume, the expression for specific values is obtained:

\[
u_2 - \nu_1 = \epsilon_{int}^{CM} - \dot{I}^{CM},
\]  

where \( \nu_1 \) and \( \nu_2 \) are the specific internal energy of the gas before and after the throttle device, J/kg; \( \epsilon_{int}^{CM} \) – the specific energy supplied to the gas from the outside, when considering the process from the coordinate system \( S_{CM}, \) J/kg; \( \dot{I}^{CM} \) – the specific expansion work, J/kg.

The \( \dot{I}^{CM} \) value is calculated using the formula (3):

\[
\dot{I}^{CM} = \frac{(p_1 + p_2) \cdot (\nu_2 - \nu_1)}{2}.
\]  

Equality (4) shows that the expansion work in a non-equilibrium adiabatic gas throttling process is equal to the expansion work in an equilibrium isobaric process, provided that the specific volume
changes within the same range from $v_1$ to $v_2$, and the constant pressure is equal to the arithmetic mean between the pressures in the flow before and after the throttle device $(p_1 + p_2)/2$.

From (3), taking into account (4), we get:

$$u_2 - u_1 = e_{\text{int}}^{CM} + \frac{(p_1 + p_2)(v_2 - v_1)}{2}.$$  \hspace{1cm} (5)

Equality analysis (5) shows the following. In the right part (5), another purely positive value $e_{\text{int}}^{CM}$ is subtracted from one purely positive value. Accordingly, the difference in (5) on the right can be a positive or negative value, depending on the ratio between them.

The change in the internal energy of an ideal gas is uniquely related to the change in its temperature. Therefore, the temperature of the ideal gas, after it passes through the throttle device, can increase, decrease, or remain unchanged. The specific result is determined by the ratio of values $e_{\text{int}}^{CM}$ and $k^{CM}$ in each specific case.

Rewrite (5) as:

$$e_{\text{int}}^{CM} = (u_2 - u_1) + \frac{(p_1 + p_2)(v_2 - v_1)}{2}.$$  \hspace{1cm} (6)

In the left part of the equation (6) there is a positive value, the value of which is equal to the specific energy supplied to the gas from outside during the adiabatic throttling process.

Equality (6) allows us to reformulate the first law of thermodynamics for the case in question as follows. The algebraic sum of the change in the specific internal energy and the specific expansion work is equal to the specific energy supplied to the gas during adiabatic throttling from the outside.

In the right part of equality (6) there are terms, each of which does not depend on the choice of the reference system in which the process is considered, and in the left part there is a value that depends on the reference system. For example, in the laboratory reference system of the $S_{\text{lab}}$, the value $e_{\text{int}}^{CM}$ will be zero, since in the $S_{\text{lab}}$ the washer remains stationary and, accordingly, the force $F_{fr}$ does not perform any work. The latter does not mean that in this case the energy from outside will not be supplied to the gas. In the $S_{\text{lab}}$ reference system, the energy to the elementary particles that make up the gas flow will be supplied by the work of volumetric and surface forces acting in a continuous medium. It should be noted that as a result of this work, in addition to the above consequences, in the form of changes in internal energy and the completion of the expansion work, the kinetic energy of the gas volume in question will change. Accordingly, for this case, the first law of thermodynamics can be formulated as follows. The algebraic sum of the change in specific internal energy, specific kinetic energy, and specific expansion work is equal to the specific energy supplied to the gas during adiabatic throttling. At the same time, energy is supplied to the gas from outside due to the presence of density distributions of volume and surface forces.

Writing the first law of thermodynamics in the form of expression (5) allows us to interpret a purely positive value $e_{\text{int}}^{CM}$ as the specific heat of friction $q_{fr}$, J/kg, which is transmitted to the elementary particles of the flow when passing the flow part of the throttle device.

This is particularly evident in the case of incompressible liquid throttling, in which the difference $(u_2 - u_1)$ is equal to $q_{fr}$. This indicates that when an incompressible liquid is throttled, its internal energy always increases, which means that the temperature of the flow behind the throttle device also increases.

3. Results and Discussion

It is found that the change in internal energy when throttling a certain volume of gas is convenient to analyze in a reference system that is rigidly connected to its center of mass. In this system, gas expansion is performed, resulting in energy being diverted to the environment. At the same time,
energy is transferred from the environment to the gas by the movement of the throttle device. The sign of the increment of internal energy of the considered volume of ideal gas in the non-equilibrium adiabatic throttling process is determined by the ratio between the two named energy exchange processes. The set ratio affects how the gas temperature changes during adiabatic throttling.

4. Conclusion
The differences that arise when analyzing the throttling process in different reference systems are shown. The conditions for changing the temperature of an ideal gas under adiabatic throttling are revealed. The analysis of energy exchange processes in adiabatic throttling of an ideal gas is carried out, taking into account the properties of internal energy as an indifferent scalar.

Clarification of ideas about energy exchange during throttling may allow, in the end, to increase the effectiveness of the design of various devices used in forest engineering.

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