More on boundary conditions for warped AdS$_3$ in GMG

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Abstract In this paper, we study the Aggarwal, Ciambelli, Detournay, and Somerhausen (ACDS) boundary conditions (Aggarwal et al. in JHEP 22:013, 2020) for Warped AdS$_3$ (WAdS$_3$) in the framework of General Massive Gravity (GMG) in the quadratic ensemble. We construct the phase space, the asymptotic structure, and the asymptotic symmetry algebra. We show that the global surface charges are finite, but not integrable, and also we find the conditions to make them integrable. In addition, to confirm that the phase space has the same symmetries as that of a Warped Conformal Field Theory (WCFT), we compare the bulk entropy of Warped BTZ (WBTZ) black holes with the number of states belonging to a WCFT.

1 Introduction

One of the interesting achievements of string theory in the last two decades is the Anti de-Sitter/Conformal Field Theory (AdS/CFT) correspondence. This correspondence has opened a new approach to studying two different areas of physics, i.e. quantum field theory and gravity theory. After the introduction of the AdS/CFT correspondence in [1–4], or more generally gauge/gravity duality, many different questions on the field theory side have been investigated utilizing the gravity side [5–9] (for more details see [10] and references therein). This duality proposes a correspondence between a quantum field theory in d-dimensional space-time and gravity theory in (d+1)-dimensional space-time. Fields, parameters, and quantities on the gauge theory side are translated to equivalent quantities on the gravity side. For instance, the vacuum state and thermal state on the field theory side correspond to the pure-AdS and black hole on the gravity theory, respectively. In addition, an extension of AdS/CFT correspondence to non-AdS geometries is Flat/Bondi–Metzner–Sachs invariant field theories (Flat/BMSFT) correspondence. According to this duality, asymptotically flat spacetimes in (d+1) dimensions are dual to d-dimensional BMSFTs [11–22].

As we know, the study of asymptotic symmetries in gravity theories is an old topic that has recently received attention. In the context of the AdS/CFT correspondence, the asymptotic symmetries of the gravity theory in the bulk spacetime correspond to the global symmetries of the dual quantum field theory in the boundary through the holographic dictionary. Therefore, with strong control of asymptotic symmetries, new holographic dualities can be investigated. The asymptotic symmetries are bulk residual transformations that preserve the boundary conditions but change the asymptotic field space (that is, they have non-vanishing surface charges). The asymptotic symmetry group is the group of residual gauge diffeomorphisms preserving the boundary conditions with associated non-vanishing charges. The boundary conditions determine the structure of the asymptotic symmetry group. Brown and Henneaux studied asymptotic symmetries of three-dimensional AdS space (AdS$_3$) and found that the symmetry algebra forms two copies of Virasoro algebra with a non-vanishing central charge ($c = 3l/2G$) [23]. This implies that bulk theories with these boundary conditions are dual to CFTs with this central charge. Strominger and collaborators have extended these results to extremal Kerr black holes in what is known as the Kerr/CFT correspondence. Compere, Song, and Strominger (CSS) [24] have demonstrated a family of specific alternative boundary conditions in which the asymptotic symmetry algebra of a 3D theory turns out to consist of a semi-direct product of a Virasoro and $u(1)$ Kac–Moody algebras which are symmetries of the 2-dimensional WCFT’s (that is invariant under chiral scaling and translations but not rotations). In [25,26], Topologically Massive Gravity (TMG) and General Minimal Massive Grav-
ity (GMMG) with the CSS boundary conditions are studied. We refer interested reader to [27–37] for more details.

In [38], a new set of boundary conditions in three-dimensional TMG has been introduced so that the dual field theory is a WCFT in the quadratic ensemble. The boundary dimensional TMG has been introduced so that the dual field. We refer interested reader to [27–37] for more details.

In Sect. 4, we compute the bulk thermodynamic entropy and of the solution space one can obtain the integrable charges. and their corresponding surface charges in GMG. We find match once the vacuum is correctly identified. Finally, we provide some conclusions in Sect. 5.

2 GMG under ACDS boundary conditions

The generalized massive gravity theory is realized by adding both the Chern–Simons (CS) deformation term and the higher derivative deformation term to pure Einstein gravity with a negative cosmological constant. This theory has two mass parameters and TMG and New Massive Gravity (NMG) are just two different limits of this generalized theory [39–43].

The action for the generalized massive gravity theory can be written as [44,45]

\[ S_{\text{GMG}} = \frac{1}{8\pi G} \int d^3x \sqrt{-g} \left[ sR - 2\lambda + \frac{1}{\mu} L_{\text{CS}} + \frac{1}{\xi^2} L_{\text{NMG}} \right], \]

where

\[ L_{\text{CS}} = \frac{1}{2} \epsilon^{\lambda \mu \nu} \left( \Gamma^\sigma_{\lambda \rho} \partial_\mu \Gamma^\rho_{\sigma \nu} + \frac{2}{3} \Gamma^\rho_{\lambda \sigma} \Gamma^\sigma_{\mu \tau} \Gamma^\tau_{\rho \nu} \right), \]

\[ L_{\text{NMG}} = R_{\mu \nu} R_{\mu \nu} - \frac{3}{8} R^2, \]

and \( \mu \) and \( \xi \) are the mass parameters of TMG and NMG, respectively. \( \lambda \) is a cosmological parameter with the dimension of mass squared, and \( s \) is a conventional sign. Varying the action with respect to the metric, one gets the following equations of motion

\[ \mathcal{E}_{\mu \nu} = \delta g_{\mu \nu} + \lambda g_{\mu \nu} + \frac{1}{\mu} C_{\mu \nu} + \frac{1}{2\xi^2} K_{\mu \nu}, \]  

where \( G_{\mu \nu} \) is the Einstein tensor, \( C_{\mu \nu} \) the Cotton tensor, and \( K_{\mu \nu} \) is given by

\[ K_{\mu \nu} = -\frac{1}{2} \nabla^2 R g_{\mu \nu} - \frac{1}{2} \nabla_\mu \nabla_\nu R + 2\nabla^2 R_{\mu \nu} + 4 R_{\mu \nu} \nabla \bar{R}^{ab} g_{\mu \nu} - \frac{3}{2} R R_{\mu \nu} - \nabla_a \nabla_\mu \nabla_\nu \nabla^a g_{\mu \nu} + \frac{3}{8} R^2 g_{\mu \nu}. \]

The parameters \( \delta \) and \( \lambda \) are parameters defined in terms of other parameters like \( s \), \( \mu \) and \( \xi \). The Fefferman–Graham gauge in three spacetime dimensions in coordinates \( x^\mu = (r, x^+, x^-) \) with three gauge-fixing conditions is

\[ g_{rr} = \frac{L^2}{r^2}, \quad g_{ra} = 0, \]

where \( x^\pm = t/L \pm \phi \). The line element takes the form

\[ ds^2 = \frac{L^2}{r^2} dr^2 + \eta_{ab}(r, x) dx^a dx^b. \]

We consider the following fall-offs of the metric [38]

\[ g_{rr} = \frac{L^2}{r^2} + O(r^{-4}), \quad g_{++} = O(r^4), \]

\[ g_{+-} = O(r^2), \quad g_{--} = O(1). \]

Therefore, the field equations (3) give us

\[ g_{++} = j_{++} r^4 + h(x^+) r^2 + f_{++}(x^+) + \frac{(1 - A^2)(A^2 - 1) h(x^+) + 4 f_{++}(x^+)}{8 j_{++} r^2 (1 + A^2)} h(x^+) + \frac{(A^2 - 1)^2 (4 j_{++} f_{++}(x^+) + (A^2 - 1) h(x^+))^2}{64 (1 + A^2)^2 f_{++} r^4} \]

\[ + \frac{(A^2 - 1)(4 f_{++} f_{++}(x^+) + (A^2 - 1) h(x^+))}{8 j_{++} r^2} \]

\[ + \frac{(1 - A^2)(A^2 - 1) h(x^+) + 4 f_{++}(x^+)}{8 f_{++} r^2 (1 + A^2)^2} \]

\[ \gamma_{++} = \frac{\xi_{++} r^2}{j_{++}} \]

\[ \gamma_{--} = \frac{(1 - A^2)\xi_{--}^2}{j_{++}}, \]

with

\[ \lambda = \frac{1}{3087 \mu^4 L^4 \xi^2} [2 L \xi^2 (3 L^2 \xi^4 - 145 L^2 \mu^2 \xi^2 - 56 \mu^2 \sqrt{84 \mu^2 + \xi^2 L^2 (9 \xi^2 - 42 \zeta)} - 18 L^4 \xi^8 \]

\[ \zeta = 3087 \mu L \xi^2 \]
\( R = 2(1 - 4A^2) / A^2L^2 = 6\lambda, \) \hspace{1cm} (12)

where

\[ A = \text{RootOf} \left\{ (16\mu + 4\lambda L^4\mu^2 \zeta^2)_Z^4 + 63\mu - 16\lambda \zeta^2 \right\}_Z^2 \]

\[ + (-80\mu + 4\mu L^2\zeta^2 \bar{s})_Z^2 + 16\zeta^2 L \] \hspace{1cm} (13)

Therefore, it is negative as long as \( \bar{\lambda} \) is. As it can be shown, the solution space is characterized by four quantities: two constants \( j_{++} \) and \( \zeta_{++} \) and two functions \( h(x_+) \) and \( f_{++}(x_+) \). By writing the WBTZ black holes (93) in the Fefferman–Graham gauge with the boundary conditions (7), one gets

\[ h = 0, \] \hspace{1cm} (14)

\[ j_{++} = -\frac{119\mu^2 - 6\zeta^4 L^2 + 2\zeta^2 L \sqrt{84\mu^2 + \zeta^2 L(9\zeta^2 - 42\mu^2 \bar{s})} + 14\mu^2 \zeta^2 L^2}{1176GL(LM - J)\mu^2}, \] \hspace{1cm} (15)

In the case of \( A = 1 \) and arbitrary \( j_{++} \), the metric becomes

\[ ds^2 = \frac{L^2}{r^2} dr^2 + (j_{++}r^4 + h(x^+)) r^2 + f_{++}(x^+) dx^2 + \zeta_{++} r^2 dx^+ dx^-, \] \hspace{1cm} (19)

and

\[ \bar{\lambda} = -\frac{12L\zeta^2 - 35\mu}{4\mu L^4 \zeta^2}, \bar{s} = \frac{6\zeta^2 L - 17\mu}{2\mu L^2 \zeta^2}. \] \hspace{1cm} (20)

This metric is not a solution for the Einstein equation, because the Cotton tensor and NMG part have non-vanishing components \( (C_t^+ = \frac{12\zeta^2 r_{j_{++}}^2}{\zeta^2 - L^2}, \xi_t^+ = -\frac{136\zeta^2 r_{j_{++}}^2}{\zeta^2 - L^2}) \). In the case of \( A = 1, \mu = \frac{6\zeta^2 L}{1 + 2\alpha \zeta^2 L^2} \) and \( j_{++} \rightarrow 0 \) but keeping the ratio \( \Delta = A^2/\zeta_{++} \) constant, the line element becomes

\[ ds^2 = \frac{L^2}{r^2} dr^2 + [h(x^+) r^2 + f_{++}(x^+)] dx^2 + \Delta_{++} dx^+ dx^-, \] \hspace{1cm} (21)

\[ h(x^+) \Delta [4f_{++}(x^+) + \Delta h^2(x^+)] 16r^2 dx^2 + \Delta \zeta_{++} r^2 dx^+ dx^- \]

This metric is the CSS metric [24].

3 Symmetries and charges

The residual gauge diffeomorphisms are generated by the vector \( \xi \) satisfying

\[ \xi \bar{g}_{rr} = 0, \xi \bar{g}_{ra} = 0, \] \hspace{1cm} (22)

where \( \bar{\xi} \) denotes the Lie derivative. The solutions to these equations are

\[ \xi = \xi^\mu \partial_\mu = \xi^r \partial_r + \xi^+ \partial_+ + \xi^- \partial_-, \] \hspace{1cm} (23)

\(^2\) RootOf is a command used as a placeholder for roots of equations in Maple [46].
Therefore, the transformation of 

\[ \xi^f = r \eta(x^+) , \]

\[ \xi^+ = \epsilon + \frac{2L^2 j_{++} \eta'(A^4 - 1)}{A^2((A^2 - 1)^2 h^2 + 4 j_{++} f_{++} (A^2 - 1) + 8r^4 j_{++}^2 (A^2 + 1))} , \]

\[ \xi^- = \sigma + \frac{j_{++} L^2 \eta'(A^2 + 1)(A^2 - 1)h - 4j_{++} r^2}{2A^2 \zeta_{++} - [(A^2 - 1)^2 h^2 + 4j_{++} f_{++} (A^2 - 1) + Aj_{++}^2 r^4 (A^2 + 1)]} , \]  

(24)

(25)

In these expressions, \( \sigma(x^+) \) and \( \epsilon(x^+) \) are field-independent arbitrary functions. Varying the metric (6) along \( \xi \), we find the variation of solution space as follows

\[ \delta_{\xi} j_{++} = 0 , \]

\[ \delta_{\xi} h = (h \epsilon)' + 2 \zeta_{+-} \sigma' , \]

\[ \delta_{\xi} f_{++} = \epsilon f_{++}' + 2 f_{++} \epsilon' - \frac{L^2 \epsilon'' (A^2 + 1)}{4A^2} + \frac{h \sigma' \zeta_{+-} (1 - A^2)}{f_{++}} , \]

(26)

(27)

The general symmetry generators, using (31), are as follows

\[ \delta_{\xi} j_{++} = 2j_{++}(\epsilon' + 2 \eta) , \]

\[ \delta_{\xi} h = 2h \eta + 2 \zeta_{+-} \sigma' + h \epsilon' + 2h \epsilon' , \]

\[ \delta_{\xi} f_{++} = \epsilon f_{++}' + 2f_{++} \epsilon' + \frac{L^2 \eta'' (A^2 + 1)}{2A^2} + \frac{h \sigma' \zeta_{+-} (1 - A^2)}{f_{++}} , \]

(28)

(29)

and

\[ \delta_{\xi} \gamma_{+-} = \mathcal{L}_\xi \gamma_{+-} \quad \rightarrow \quad \delta_{\xi} \zeta_{+-} = \zeta_{+-}(2 \eta + \epsilon') . \]

(30)

By requiring \( j_{++} \) to be constant, from (27) we get

\[ \eta = -\frac{1}{2} \epsilon' + \eta_0 . \]

(31)

Therefore, the transformation of \( j_{++} \) becomes

\[ \delta_{\xi} j_{++} = 4j_{++} \eta_0 . \]

(32)

If we assume \( \eta_0 = 0 \), then we obtain

\[ \delta_{\xi} j_{++} = \delta_{\xi} \zeta_{+-} = 0 . \]

(33)

This means that \( j_{++} \) and \( \zeta_{+-} \) are fixed along the residual orbits. Finally, we find the full residual variation of solution space as

\[ \delta_{\xi} j_{++} = 2j_{++}(\epsilon' + 2 \eta) , \]

\[ \delta_{\xi} h = 2h \eta + 2 \zeta_{+-} \sigma' + h \epsilon' + 2h \epsilon' , \]

\[ \delta_{\xi} f_{++} = \epsilon f_{++}' + 2f_{++} \epsilon' + \frac{L^2 \eta'' (A^2 + 1)}{2A^2} + \frac{h \sigma' \zeta_{+-} (1 - A^2)}{f_{++}} , \]

(34)

(35)

(36)

(37)

\[ \mathcal{L}_\xi Q^a = \int_{\Sigma} dS_i F^{ai}(g, h) , \]

(38)

where \( A \) is defined in (13). The residual symmetries (37) depend on two arbitrary chiral functions \( \epsilon(x^+) \) (generating the usual Witt algebra) and \( \sigma(x^+) \) (generating an abelian algebra). Therefore, the total asymptotic symmetry algebra is a direct sum of a Witt and a \( u(1) \) algebra.

3.1 Charges and algebra

The surface charges are computed using [47–49] as follows

\[ F_E^{ai}(\xi) = \xi_b \tilde{\nabla}^a h^{ib} - \tilde{\xi}_b \tilde{\nabla}^i h^{ab} + \tilde{\xi}^a \tilde{\nabla}^i h - \tilde{\xi}_b \tilde{\nabla}^a \tilde{h}_b - h^{ib} \tilde{\nabla}^a \tilde{\xi}_b + \tilde{\xi}^i \tilde{\nabla} h^{ab} - \tilde{\xi}^a \tilde{\nabla} h^{ib} + h \tilde{\nabla}^a \tilde{\xi}_i , \]

(39)
\[
\begin{align*}
&+ \frac{1}{\sqrt{8}} \xi_{\lambda} \left( e^{a\lambda} \delta G_{\rho} - \frac{1}{2} e^{a\lambda} \delta G \right) \\
&+ \frac{1}{2\sqrt{8}} e^{a\rho} \left[ \xi_{\rho} h^{b} G_{\lambda} + \frac{1}{2} \left( \xi_{\sigma} G_{\rho} + \frac{1}{2} \xi_{\rho} R \right) \right],
\end{align*}
\] (40)

\[
F_{R_{1}}^{ab}(\xi) = 2R_{E}^{ab}(\xi) + 4\xi^{[a} \nabla^{b]} \delta R + 2\xi^{[a} h_{b]} \nabla_{a} R, \quad (41)
\]

\[
F_{R_{2}}^{ab}(\xi) = \nabla^{2} F_{E}^{ab} + \frac{1}{2} F_{R_{2}}^{ab} - 2 F_{E}^{[a} R_{b]} - 2\xi^{[a} h_{b]} \nabla_{a} R,
\]

where \( \delta R = -R_{E}^{ab} h_{ab} + \nabla^{a} \nabla^{b} h_{ab} - \nabla^{2} h, \) \( \eta^{\mu} = \epsilon^{\nu} \delta_{\nu} \hat{\nabla} \rho \hat{\nabla} \delta h, \) and \( h = \delta_{g_{\mu\nu}}(\delta \alpha, \alpha) = \delta_{g_{\mu\nu}}(\delta \alpha) \). For the \((1, 1)\) sector and the Killing vector \( \sigma = (x^\nu) \partial_{\nu}, \) the surface charge becomes

\[
\begin{align*}
\delta Q_{z} &= \left( A^{2} - 1 \right) \frac{4A^{5} \mu L^{2} \xi^{2} j_{++}^{2}}{2\pi} d \Phi(x^{+})[2(-8\mu A^{4} + 4\xi^{2} L A^{3} + 2A^{2}(\delta L^{2} \xi^{2} + 34) - 2\xi \xi (LA - 63 \mu) \xi_{+}-] j_{++} \delta h - j_{++} \delta \xi_{+} - (4(-14 - 3\xi L^{2} \xi^{2}) \mu A^{4} + 14 A^{3} \xi^{2} + 2A^{2}(4\xi L^{3} \xi^{2} + 7) - 8\xi \xi (LA - 63 \mu) h + (2\mu A^{2}(10 + \tilde{L}^{2} \xi^{2} + 3) - 3\xi \xi L^{3} A^{3} - (5\xi L^{2} \xi^{2} + 83) \mu A^{2} + 2\xi \xi (LA + 63 \mu) \xi_{+-} + (A^{2} - 1) \xi_{++} \times j_{++}(\xi_{+-} - h)(\mu A^{2}(10 + 5\tilde{L}^{2} \xi^{2} - L A \xi^{2} - 63)].
\end{align*}
\] (43)

where we have used \( \xi = \xi, \) with \( \xi^{r}, \xi^{+}, \) and \( \xi^{-} \) provided in (37) and \( \sigma = 0. \) To obtain the above surface charges we evaluated equations (38)-(42) first at \((r, x^{+})\) fixed, and second at \((r, x^{-})\) fixed, then added them together, and finally sent \( r \rightarrow \infty. \) These charges are finite but are not integrable. Non-integrability of charges implies that the finite charge expressions rely on the particular path that one chooses to integrate on the solution space, which is a common feature of a dissipative system. If \( \delta j_{++} = \delta \xi_{+} = 0 \) the charges become integrable. Also, one can find a combination of vectors such that these charges become integrable even when \( \delta j_{++} \neq 0, \) \( \delta \xi_{+} = 0. \) Utilizing the integrable charges, the charge algebra is obtained. Therefore, in the case \( \delta j_{++} = \delta \xi_{+} = 0, \) the charges read as

\[
\begin{align*}
\delta Q_{z} &= \left( A^{2} - 1 \right) \frac{(A^{2} - 1) (-8\mu A^{4} + 4\xi^{2} L A^{3} + 2A^{2}(\delta L^{2} \xi^{2} + 34) - 2\xi \xi (LA - 63 \mu) \xi_{+}-)}{4A^{5} \mu L^{3} \xi^{2} j_{++}} \int_{0}^{2\pi} d \Phi(x^{+}) \delta h,
\end{align*}
\] (45)

The Virasoro charges are

\[
\begin{align*}
\delta Q_{z} &= \frac{1}{16\mu L^{3} \xi^{2} A^{2}(A^{2} + 1) j_{++}^{2} \xi_{+-}} \\
&- \int_{0}^{2\pi} d \phi L^{2} j_{++}(A^{2} + 1) e^{\phi} \\
&\times ((A^{2} - 1)(8\mu A^{2} - 2\mu A^{2}(-14 + \tilde{L}^{2} \xi^{2})
\end{align*}
\]

\[
\begin{align*}
&\times 9A^{2} L^{2} \xi^{2} + 21\mu)) \xi_{+-} - A j_{++} - A \delta \xi_{+-} + (10A^{4} L^{2} \xi^{2} + 3A^{3}(40 \mu + 4\mu A \tilde{L}^{2} \xi^{2}) - 25A^{2} L^{2} \xi^{2} - 42 \mu A \\
&+ (11L^{2} \xi^{2}) - A^{2} e(A^{2} - 1) \xi_{+-} j_{++} \delta h(-12A^{4} L^{2} \xi^{2} - 32A^{3} \mu(4 + \tilde{L}^{2} \xi^{2}) + 10A^{2} L^{2} \xi^{2} + L^{2} \xi^{2} + 144 \mu) A \\
&\times A + \xi_{+} j_{++} \delta h_{+} (24L^{2} A^{5} \xi^{2} - 24 \mu A^{4}(4 + 2L^{2} \xi^{2}) + 44A^{3} L^{2} \xi^{2}) + 8 \mu A^{2}(35 + 2L^{2} \xi^{2}) - 20A^{2} L^{2} \xi^{2} - 168 \mu) - (A^{2} - 1) \xi_{+-} j_{++}(-4\mu A^{2} L^{2} \xi^{2} + 16A^{3} \mu A^{2} L^{2} \xi^{2}) - 72 \mu A - 2L^{2} \xi^{2}) + (A^{2} + 1) \xi_{+-} h(16A^{3} L^{2} \xi^{2} - 4A L^{2} \xi^{2} - 84 \mu + 8 \mu A^{2}(10 + \tilde{L}^{2} \xi^{2}) - 20(2A^{2} + 1) A L f_{++} \xi^{2} j_{++} \\
&\times (A^{4} - 1) \xi_{+-} h (16 \mu A^{2}(10 + \tilde{L}^{2} \xi^{2}) - 16A L^{2} \xi^{2} - 168 \mu) + 8A L \\
&\times \xi^{2} f_{++} j_{++}(10A^{4} - 5A^{2} - 3)))
\end{align*}
\] (44)

The Virasoro charges are

\[
\begin{align*}
\delta Q_{z} &= \frac{1}{8j_{++} A^{3} L^{3} \xi^{2}(A^{2} + 1)} \\
&\times \int_{0}^{2\pi} d \phi \left( h \right) (A^{2} - 1) \delta h (6A^{4} L^{2} \xi^{2} - 16A^{3} \mu(4 + \tilde{L}^{2} \xi^{2}) \\
&+ 5A^{2} L^{2} \xi^{2} + 72 \mu A + L^{2} \xi^{2}) + (16A^{2} - 14) \xi_{+-} L^{2} \xi^{2}(A^{2} + 1))
\end{align*}
\]
Using the mode decomposition representation $\epsilon_1 = e^{i m x^+}$, $\epsilon_2 = e^{i n x^+}$, and calling $Q_{\bar{z}} = P_m$, $Q_z = P_n$, it is easy to obtain

$$ i \{P_m, P_n\} = \frac{k}{2} \delta_{m+n,0}, \quad (51) $$

where

$$ k = \frac{2\pi (A^2 - 1) (-8\mu A^4 + 4\xi^2 L A^3 + 2\mu A^2 (\tilde{s} L^2 \xi^2 + 34) - 2\xi^2 L A - 63\mu) \xi^2_{+}}{A^5 \mu L^3 \xi^2 j_{++}}. \quad (52) $$
where

\[
c = \frac{3\pi(12L\zeta^2A^5 - 24\mu A^4(2 + \tilde{s}L^2\zeta^2) + 22L^3A^3\zeta^2 + 4\mu A^2(35 + 2\tilde{s}^2L^2) - 10AL\zeta^2 - 84\mu)}{4\mu LA^3\zeta^2}.
\]  

(55)

In summary, the algebra is

\[
i \{L_m, L_n\} = (m - n)L_{m+n} + \frac{c}{12}m^3\delta_{m+n,0}, \tag{56}
i \{L_m, P_n\} = -nP_{m+n} \tag{57}
i \{P_m, P_n\} = m\frac{k}{2}\delta_{m+n,0} \tag{58}
\]

with central extensions

\[
c = \frac{3\pi(12L\zeta^2A^5 - 24\mu A^4(2 + \tilde{s}L^2\zeta^2) + 22L^3A^3\zeta^2 + 4\mu A^2(35 + 2\tilde{s}^2L^2) - 10AL\zeta^2 - 84\mu)}{4\mu LA^3\zeta^2},
\]

\[
k = \frac{2\pi(A^2 - 1)(-8\mu A^4 + 4\zeta^2LA^3 + 2\mu A^2(\tilde{s}L^2\zeta^2 + 34) - 2\zeta^2LA - 63\mu)\zeta_+^2}{A^5\mu L^3\zeta^2j_{++}}. \tag{59}
\]

Therefore, from (56)–(58) with the associated central charges (59), the bulk solution space has a symmetry algebra identified with that of a WCFT in the quadratic ensemble. In the limit \(\zeta \to \infty\), and \(\mu > 0, L > 0\) one obtains [38]

\[
c = \frac{\mu^2L^2\tilde{s}^2 + 9}{3\mu}, \quad k = -\frac{\zeta_+^2 - (\mu^2L^2\tilde{s}^2 - 9)}{\mu^2L^2j_{++}}, \tag{60}
\]

while in the case of \(\mu \to \infty\), we have [50]

\[
c = \frac{16\pi}{7}\sqrt{\frac{2}{21}}\frac{(\zeta^2L^2 + 2)^{\frac{3}{2}}}{\zeta^2L^2}, \quad k = -\frac{8\pi\sqrt{42}(2\zeta^2L^2 - 17)\zeta_+^2}{21\zeta^2L^3\sqrt{\zeta^2L^2 + 2}j_{++}}. \tag{61}
\]

This algebra is one of the centrally extended group

\[
Vir \otimes U(1). \tag{62}
\]

Now, we study the null warped limit in the case \(A = 1\). In this case, the \(u(1)\) level and charges vanish identically \((k = 0)\), we are left with a Virasoro symmetry algebra with central extension

\[
c = \frac{18\pi(6\mu - L\zeta^2)}{\mu L\zeta^2}, \quad A \to 1. \tag{63}
\]

As we know, the CSS limit can be achieved setting \(A = 1\) and \(j_{++} = 0\) while keeping \(\Delta = \frac{d-1}{2j_{++}}\) constant. The charges read

\[
Q_\sigma = \frac{\Delta - (2\zeta^2L - 5\mu)}{\mu L\zeta^2} \int_0^{2\pi} d\phi \sigma(x^+)(h + h_0), \quad Q_\zeta = -\frac{1}{4\mu L^3\zeta^2} \int_0^{2\pi} d\phi \epsilon(x^+)\left[6f_{++}(6\mu - \zeta^2L) + \Delta(6h^2(3\mu - \zeta^2L) + \zeta_+L\zeta^2h)\right], \tag{64}
\]

while the central extensions become

\[
c = \frac{18\pi(6\mu - L\zeta^2)}{\mu L\zeta^2}, \quad k = \frac{8\pi\Delta - (2\zeta^2L - 5\mu)}{\mu L^3\zeta^2}. \tag{66}
\]

This limit coincides with our results in [26]. We now turn our attention to the solution space of WBTZ black holes in (14)–(18). Therefore, their charges in the quadratic ensemble take the form

\[
P_m = \frac{2\pi h_0(A^2 - 1)(LM - J)(2H^2 - 1)}{A^5\mu L^2\zeta^2H^2} \times [-8\mu A^4 + 4\zeta^2LA^3 + 2\mu A^2(\tilde{s}L^2\zeta^2 + 34) - 2\zeta^2LA - 63\mu]\delta_{m,0}, \tag{67}
\]

\[
L_m = \frac{\pi G(J + ML)(H^2 - 1)}{2\mu A^3L^2\zeta^2(A^2 + 1)} \times [12L\zeta^2A^5 - 24\mu A^4(2 + \tilde{s}L^2\zeta^2)] + 22L^3A^3\zeta^2 + 4\mu A^2(35 + 2\tilde{s}^2L^2) \times L^2 - 10AL\zeta^2 - 84\mu]\delta_{m,0}. \tag{68}
\]

where \(M\) and \(J\) are Einstein charges. The GMG mass and angular momentum of these solutions are defined as

\[
\mathcal{M} = Q_{\partial_+} + \frac{1}{L}(Q_{\partial_+} + Q_{\partial_-}), \quad \mathcal{J} = Q_{\partial_+} - Q_{\partial_-}. \tag{69}
\]
and we also have

\[ P_0 = 0, \quad Q_0 = L_0. \]  

\[ \text{Then, the relation between the GMG mass, angular momentum, and the zero modes of the charges can be obtained as follows} \]

\[ \mathcal{M} = \frac{1}{L}(P_0 + L_0) \]

\[ = \frac{\pi}{2\mu L^3 \zeta^2 A^5 H^2(A^2 + 1)} \times \left[ 4h_0(A^4 - 1)(2H^2 - 1)(LM - J) \right. \]

\[ \times (8\mu A^4 + 4\zeta^2 L A^3 + 2\mu A^2) \]

\[ \times (5L^2 \zeta^2 + 34) - 2 \zeta^2 L A - 63\mu \]

\[ + (H^2 - 1)A^2 H^2(J + ML)(12L^2 A^5 - 24\mu A^4) \]

\[ \times (2 + 5\zeta^2 L^2 - 10AL^2 - 84\mu)) \],

\[ \mathcal{J} = L_0 - P_0 \]

\[ = \frac{\pi}{2\mu L^2 \zeta^2 A^5 H^2(A^2 + 1)} \times \left[ (H^2 - 1)A^2 H^2(J + ML) \right. \]

\[ \times (12L^2 A^5 - 24\mu A^4 (2 + 5\zeta^2 L^2) \]

\[ + 22LA^3 \zeta^2 + 4\mu A^2 (35 + 25\zeta^2 L^2) \]

\[ - 10AL^2 \zeta^2 - 84\mu) \]

\[ \times (LM - J)(-8\mu A^4 + 4\zeta^2 L A^3 \]

\[ + 2\mu A^2 (5L^2 \zeta^2 + 34) - 2\zeta^2 L A - 63\mu), \]

where \( \mathcal{M} \) and \( \mathcal{J} \) are the mass and angular momentum of WBTZ black holes.

4 Entropy matching

The WBTZ black hole solution in ADM form is given as [38,51]

\[ ds^2 = -N(r)^2 dt^2 + \frac{dr^2}{f(r)^2} + R(r)^2 (\varphi^2 + d\varphi^2), \]

where

\[ N^\varphi(r) = \frac{H^2 r^4 - 8MH^2 L r^2 - 16JL^2 (J (H^2 - 1) + LM (1 - 2H^2))}{L (H^2 r^4 + 16H^2 J^2 L^2 - 4L r^2 (ML + J (2H^2 - 1)))}. \]

Taking \( \xi = \partial_t + \frac{r^2}{L} \partial_\varphi \) and given (73), the entropy of black hole in GMG is obtained as [52,53]

\[ S^\text{GMG} = \frac{\pi^2}{4 \mu L^2 \zeta^2 A^5 H^2(A^2 + 1) \sqrt{M^2 L^2 - J^2}} \]

\[ \times [4h_0(A^4 - 1)(2H^2 - 1)(LM - J)(-8\mu A^4 \]

\[ + 4\zeta^2 L A^3 + 2\mu A^2 (5L^2 \zeta^2 + 34) - 2\zeta^2 L A - 63\mu) \]

\[ \times (H^2 - 1)A^2 H^2(J + ML)(12L^2 A^5 - 24\mu A^4) \]

\[ \times (2 + 5\zeta^2 L^2 - 10AL^2 - 84\mu) \]

\[ \times \sqrt{ML^2 + L \sqrt{M^2 L^2 - J^2}} + ((H^2 - 1)A^2 H^2) \]

\[ \times (J + ML)(12L^2 A^5 - 24\mu A^4 (2 + 5\zeta^2 L^2 + 22LA^3 \zeta^2 + 4\mu A^2 \]

\[ \times (35 + 25\zeta^2 L^2) - 10AL^2 - 84\mu) \]

\[ - 2\zeta^2 L A - 63\mu)]) \sqrt{ML^2 - L \sqrt{M^2 L^2 - J^2}}, \]

where \( r_\pm \) are the horizons of black holes (solutions of the equation \( f(r) = 0 \)) and are given by

\[ r_\pm = 2\sqrt{G L} \sqrt{LM \pm \sqrt{M^2 L^2 - J^2}}. \]

The Hawking temperature and angular velocity of the black hole are given as

\[ T = \frac{r_+^2 - r_-^2}{2 \pi r_+ L^2} = \frac{2 \sqrt{L^2 M^2 - J^2}}{\pi L^2 \sqrt{ML + \sqrt{M^2 L^2 - J^2}}}, \]

and

\[ \Omega = \frac{r_-}{L r_+} \frac{L \sqrt{ML - \sqrt{L^2 M^2 - J^2}}}{L \sqrt{ML + \sqrt{M^2 L^2 - J^2}}}. \]

As expected, the above thermodynamic quantities (71)–(81) satisfy the first law

\[ d\mathcal{M} = T dS + \Omega d\mathcal{J}. \]
symmetries to global symmetries which imposes $2\pi$ periodicity in $\phi$ (for more details see [38]). Therefore, a particular vacuum solution is obtained by setting $J = 0, M = -1/8G$ as [38]

$$d_{vac}^2 = (L^2 + r^2)(2H^2r^2 + L^2(2H^2 - 1)) \frac{dt^2}{L^4} + \frac{L^2dr^2}{L^2 + r^2} + 4H^2r^2(L^2 + r^2) \frac{d\phi}{L^2} + \left(r^2 + \frac{2H^2r^4}{L^2}\right) d\phi^2.$$  

(83)

We see that in the case of $H = 0$, the metric becomes global $AdS_3$. We have two values of GMG charges that the mass charge is

$$M = \frac{\pi}{8L^2\mu(A^2 + 1)A^7\zeta L^2(-16\mu h_0 A^8 - 16L\zeta^2 A^7 H^2 h_0 + 12\mu A^6\zeta L^2 H^4 - 12\mu A^6\zeta L^2 H^4 \times H^2 + 8\mu A^6\zeta L^2 H^2 h_0 + 4\mu A^6\zeta L^2 H^2 h_0 - 272\mu A^6 H^2 h_0 + 11L\zeta^2 A^5 H^4 - 44L^2 A^5 h_0 - 70\mu A^5 H^4 + 70\mu A^5 H^2 - 110\mu A^5 h_0 + 5L\zeta^2 A^3 H^4 - 5L\zeta^2 A^3 H^2 - 8L\zeta^2 A^3 h_0 + 42) \times A^3 H^4 - 42A^3 H^2 - 136A^3 H^2 h_0 - 252\mu h_0 H^2}{8L^2\mu(A^2 + 1)A^7\zeta L^2}{(-16\mu h_0 A^8 - 16L\zeta^2 A^7 H^2 h_0 + 12\mu A^6\zeta L^2 H^4 - 12\mu A^6\zeta L^2 H^4 \times H^2 + 8\mu A^6\zeta L^2 H^2 h_0 + 4\mu A^6\zeta L^2 H^2 h_0 - 272\mu A^6 H^2 h_0 + 11L\zeta^2 A^5 H^4 - 44L^2 A^5 h_0 - 70\mu A^5 H^4 + 70\mu A^5 H^2 - 110\mu A^5 h_0 + 5L\zeta^2 A^3 H^4 - 5L\zeta^2 A^3 H^2 - 8L\zeta^2 A^3 h_0 + 42) \times A^3 H^4 - 42A^3 H^2 - 136A^3 H^2 h_0 - 252\mu h_0 H^2}}.$$  

(84)

As can be seen from (85), the GMG angular momentum of vacuum solution does not equal to zero. This interesting result has been observed from other three dimensional gravitational theories containing parity-odd terms [54,55]. In the quadratic ensemble, the warped Cardy formula takes the form

$$S_{WCF} = 4\pi \sqrt{-P_{vac}^0 P_0} + 4\pi \sqrt{-L_{vac}^0 L_0},$$  

(86)

where the zero modes for vacuum metric become

$$P_{vac}^0 = \frac{\pi h_0(-1 + 2H^2)(A^2 - 1)(8\mu A^4 - 4\zeta^2 LA^3 - 2A^2\mu \zeta L^2 \zeta^2 - 68A^2\mu + 2\zeta^2 LA + 63\mu)}{4L^2 A^5\mu \zeta^2}. \quad \text{(87)}$$

$$L_{vac}^0 = -\frac{\pi(H^2 - 1)}{8L^2 A^3\mu(A^2 + 1)\zeta^2}(6A^5 L\zeta^2 - 24A^4 - 12A^4 \mu \zeta L^2 \zeta^2 + 11\zeta^2 LA^3 + 4A^2\mu \zeta L^2 \zeta^2 + 70A^2\mu - 5\zeta^2 LA - 42\mu). \quad \text{(88)}$$

Inserting this in (86), one finds

$$S_{WCF} = \frac{\sqrt{2}\pi^2}{H^2 A^5\mu \zeta^2(A^2 + 1)}(2h_0(A^4 - 1)(2H^2 - 1) \times \sqrt{4}(8\mu A^4 - 4\zeta^2 A^3 - 4\mu A^2(34 + \zeta^2) + 4\zeta^2 A + 63\mu) + H^2(H^2 - 1)A^2 \times \sqrt{X}(6\zeta^2 A^3 - 12\mu A^4(2 + \zeta^2) + 11\zeta^2 A^3) + 2uA^2(2\zeta^2 + 35) - 5A^2 - 42\mu), \quad \text{(89)}$$

where $X = ML + J, Y = ML - J$. After some manipulations, this expression matches the bulk thermodynamic WBTZ entropy (78) provided that

$$h_0 = \frac{H^2 A^2(H^2 - 1)(r_+ + r_-)(2X + \sqrt{2X}(r_+ - r_-)C)}{2(2H^2 - 1)(A^4 - 1)(r_+ - r_-)D}, \quad \text{(90)}$$

where $C$ and $D$ are constants.
In this section, we obtained the entropy of WBTZ via thermodynamical approach and the entropy of WCFT via Cardy formula. Finally, we showed that $S_{WCFT} = S_{WBTZ}$ if $h_0$ satisfied (90).
\[ \tilde{\lambda} = \frac{1}{5087\mu^4L^4\zeta^2} [12L\zeta^2(3L^2\zeta^4 - 145L^2\mu^2\zeta^2 - 56\mu^2) \times \sqrt{84L^2 + \zeta^2L^2(9\zeta^2 - 425)} - 18L^4\mu^8 + 126\delta \mu^2\zeta^6L^4 + (252 - 147\tilde{s}^2\mu^2L^2)\mu L^2\zeta^4 - 4704\delta \mu^2L^2 - 2352\mu^4]. \]

For \( \tilde{s} = (6\delta^2L - 17\mu)/(2\mu L^2\zeta^2) \), \( H \) becomes zero and \( \tilde{\lambda} = -(12\delta^2L - 35\mu)/4\mu^2L^4 \) and the WBTZ metric becomes the BTZ metric. For \( \zeta \to \infty \), and assuming \( L > 0, \mu > 0 \) we have
\[ \tilde{\lambda} = -\frac{36\delta + \tilde{s}^2 L^2}{27L^2}, \quad H^2 = \frac{17}{42} + \frac{\tilde{s}^2 L^2}{21}, \]
which are the same as \([38]\) for TMG. In the case of \( \mu \to \infty \)
\[ \tilde{\lambda} = -\frac{\tilde{s}^2 L^4 + 32\tilde{s}^2 L^2 + 16}{21\zeta^2 L^4}, \quad H^2 = \frac{17}{42} \frac{\tilde{s}^2 L^2}{21}, \]
which are the same as \([50]\) for NMG.

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