Assisted Inflation in Randall-Sundrum Scenario

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Abstract

We extend the Randall-Sundrum(RS) model by adding a fundamental scalar field in the bulk, then study the multi-field assisted inflationary solution on brane. We will show that this model satisfies not only the observation, but also provides a solution to hierarchy problem. Furthermore in comparison with the chaotic inflation model with a single field in four space-time dimension, the parameters in our model required for a successful inflation are natural.

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I. INTRODUCTION

Over the past two years the Randall and Sundrum (RS) model has received much attention. In the RS scenario two 3-branes with opposite tension are taken to sit at the fixed points of an $S^1/Z_2$ orbifold with $AdS_5$ bulk geometry. It is shown that the gravity is localized on the brane of positive tension and the exponential warp factor in the space-time metric generates a scale hierarchy on the brane of negative tension. In this paper we will study the cosmological implications of the RS model. In the literature the related topics about the cosmological solution of the brane-world picture have been discussed intensively in the recent years.

In modern cosmology inflation plays an important role. Recently the observations by Boomerang and MAXIMA on the location of the first peak in the Microwave Background Radiation anisotropy strongly support for a flat universe and favors the predictions of inflation. In spite of these remarkable success of the inflation models, there are some serious difficulties associated with the fine-tuning of the model parameters. To overcome these difficulties Liddle et al. recently have proposed an assisted inflation model in which multi-scalar fields are introduced to drive the inflation cooperatively.

In this paper we propose a specific assisted inflation model by introducing a fundamental bulk scalar field in the RS scenario, and we will show that due to the large number of the Kaluza-Klein (KK) modes of the bulk scalar field, this model in the absence of the fine-tuning of the model parameters satisfies the observation, meanwhile it keeps the nice features of the RS scenario in solving the hierarchy problem. We have noticed that the inflationary solutions in the RS scenario in, however, the fine-tuning of the model parameters is still required in these models. The paper is organized as follow: in section 2 we introduce the model and study the inflationary solution. Section 3 is our conclusion.
II. THE MODEL

Our model is an extension of the RS model [1] by adding a fundamental scalar field $\tilde{\Phi}$ with mass $m$ in the bulk as shown in $S_{\text{bulk scalar}}$ below. The action of the model is $S = S_{\text{bulk}} + S_{\text{brane}} + S_{\text{bulk scalar}}$ where

$$S_{\text{bulk}} = \int d^4x dy \sqrt{\tilde{g}} \left( \frac{1}{2\kappa^2} \tilde{R} - \tilde{\Lambda} \right),$$

$$S_{\text{brane}} = \int d^4x \sqrt{-g}(L_{m,0} - V_0)|_{y=0} + \int d^4x \sqrt{-g}(L_{m,l} - V_l)|_{y=l},$$

$$S_{\text{bulk scalar}} = \int d^4x dy \sqrt{\tilde{g}} \left( \frac{1}{2} \partial A \tilde{\Phi} \partial A \tilde{\Phi} - \frac{1}{2} m^2 \tilde{\Phi}^2 \right).$$

The RS model described by action $S = S_{\text{bulk}} + S_{\text{brane}}$ in Eqs. (1) and (2) consists of two 3-branes in an $AdS_5$ space with a negative cosmological constant $\Lambda$, and two 3-branes are assigned to have opposite brane tensions and located at the orbifold fixed points $y = 0$ and $y = l$. Note that the variables with “$\tilde{\ }$” in Eqs.(1)-(3) represent the five dimensional one and $x^A = (x^\mu, y)$. The distance between the 3-branes is assumed to be stabilized by the mechanisms in [8,9].

In general the five dimensional metric that respects the cosmological principle on the brane can be written as

$$ds^2 = -n^2(t, y)dt^2 + a^2(t, y)dx^2 + b^2(t, y)dy^2.$$  

Following Ref. [3], we consider the energy density of the scalar field in the bulk and the matter in both 3-branes much less than the bulk cosmological constant and the brane tension, so that this metric can be linearized around the RS solution [3]

$$a(y, t) = a(t) \exp \left( -\sigma(y)b(t) \right) (1 + \delta a(y, t))$$

$$n(y, t) = \exp \left( -\sigma(y)b(t) \right) (1 + \delta n(y, t))$$

$$b(y, t) = b(t) (1 + \delta b(y, t)).$$
where $\sigma(y) = ky$ with $k^2 = (-\Lambda/\kappa^2)/6$ the bulk curvature, and $\delta n, \delta a, \delta b \sim O(\rho_0, \rho_l)$ with $\rho_0, \rho_l$ the energy density of matter on the two branes since in the limit of $\rho_0, \rho_l \to 0$ the RS solution should be recovered. Therefore, without loss of generality, $b = 1$ is taken in this paper and the metric in Eq.(4) is reduced to

$$ds^2 = e^{-2\sigma(y)}g_{\mu\nu}dx^\mu dx^\nu - dy^2,$$

where $g_{\mu\nu} = dt^2 - a^2(t)dx^2$ is the flat Friedmann-Robertson-Walker (FRW) metric. And one can see that this metric gives rise to a expansion rate observed by the observer of two 3-branes identically [3,4].

Assisted inflation model requires multi-scalar fields [5,6]. In our model, upon compactification the scalar field $\tilde{\Phi}$ gives rise to a set of effective scalars in the four dimension and the KK modes of the bulk scalar serve as inflaton in the assisted inflation scenario. Now we closely follow the procedure in Ref. [10] and work out the effective potential of the KK scalars.

Starting with the action in Eq.(3), the metric in Eq.(6) and making use of the integration by parts we can rewrite the action of the bulk scalar field as

$$S_{bulk\text{scalar}} = \frac{1}{2} \int d^4x \int dy \left(e^{-2\sigma(y)} \partial_\mu \tilde{\Phi} \partial^\mu \tilde{\Phi} + \tilde{\Phi} \frac{\partial}{\partial y} \left(e^{-4\sigma(y)} \frac{\partial \tilde{\Phi}}{\partial y} \right) - m^2 e^{-4\sigma(y)} \tilde{\Phi}^2 \right).$$

Defining $\Psi_i(y)$ a set of scalar fields in the fifth dimension which satisfies

$$\int dy \Psi_i(y) \Psi_j(y) \exp(-2\sigma) = \delta_{ij}$$

and

$$\frac{\partial}{\partial y} \left(e^{-4\sigma(y)} \frac{\partial \Psi_i}{\partial y} \right) - m^2 e^{-4\sigma(y)} \Psi_i = -m_i^2 e^{-2\sigma(y)} \Psi_i.$$  

We expand $\tilde{\Phi}(x, y)$ as a sum over modes, i.e. $\tilde{\Phi}(x, y) = \sum_i \Phi_i(x) \Psi_i(y)$. Then, after integrating the five dimensional action over the extra dimension $y$, we obtain an effective four dimensional action which describes the evolution of the scale factor $a(t)$ on the two 3-branes

$$S = \int d^4x \sqrt{-g} \frac{1}{2\kappa^2} R + \frac{1}{2} \sum_i \int d^4x \sqrt{-g} \left(\partial_\mu \Phi_i \partial^\mu \Phi_i - m_i^2 \Phi_i^2 \right).$$

In Eq.(9) $\frac{1}{2\kappa^2} = \frac{1}{2k^2}(1 - e^{-2kt})$, from which one can see that for $e^{kt} \gg 1$, $M \sim k \sim m_p$ with $m_p$ the four dimensional Planck scale.
The solution of Eq. (8) is

$$\Psi_i(y) = a_{i\nu} e^{2\sigma(y)} \left( J_\nu \left( \frac{m_i e^{\sigma(y)}}{k} \right) + b_{i\nu} N_\nu \left( \frac{m_i e^{\sigma(y)}}{k} \right) \right),$$

where $a_{i\nu}$ is a normalization factor, $J_\nu$ and $N_\nu$ are the Bessel function. The jump conditions for two 3-branes give rise to two relations which can be used to solve for $m_i$ and $b_{i\nu}$. As usual, the bulk field $\tilde{\Phi}(x, y)$ upon compactifications manifests itself to an observer in the four space-time dimension as an infinite “tower” of scalars with masses $m_i$. And their mass spectrum satisfy equation

$$2 J_\nu \left( \frac{m_i e^{kl}}{k} \right) + m_i e^{kl} J'_\nu \left( \frac{m_i e^{kl}}{k} \right) = 0,$$

with $\nu = \sqrt{4 + \frac{m^2}{k^2}}$.

From Eq. (11), we will have a relation between $m_i$ and $m$. In Fig. 1 we plot the KK scalar mass spectrum as function of the bulk scalar mass $m$. One can see that for $m \leq 0.1k \sim 0.1m_p$, the dependence of the mass spectrum $m_i$ on the mass scale $m$ of the bulk scalar field is very weak. Therefore, we approximate the mass formula as

$$m_i \simeq \xi_1 (1 + (i - 1)\pi)k e^{-kl}. \quad (12)$$

In the usual single-field inflation in four space-time dimension or the assisted inflation in the large extra dimension [11], the scalar mass is determined by the COBE observation, so one has a strong constrain on the mass $m \sim 10^{-6}m_p$. We will show below that this will not be the case in our model.

For the infinite “tower” of the scalar the natural cut off of the mass spectrum is the Plank scale $m_p$. With such a cutoff we will have the total number of the KK fields

$$N_{tot} \approx \frac{\exp (kl)}{\pi \xi_1}. \quad (13)$$

Now we study the inflationary solution of this scenario. Given the action (9) we have the equations of motion

$$H^2 = \frac{4\pi}{3m_p^2} \sum_i (\dot{\Phi}_i^2 + m_i^2 \Phi_i^2), \quad (14)$$
\[ \dot{\Phi}_i + 3H \dot{\Phi}_i + m_i^2 \Phi_i = 0. \quad (15) \]

In the slow-rolling approximation, one drops the second derivative term in Eq. (15), and obtains that

\[ \frac{\Phi_i(t)}{\Phi_i(0)} = \left( \frac{\Phi_1(t)}{\Phi_1(0)} \right)^{m_i^2/m_1^2}, \quad (16) \]

where the Hubble parameter \( H \equiv \dot{a}/a \) represents the expansion rate of two 3-branes and \( \Phi_i(0) \) is the initial values of \( \Phi_i(t) \).

To calculate the amplitude of adiabatic perturbation generated during inflation and the corresponding index of spectrum, we use the usual formula for the perturbations of the multi-field in the four dimensional space-time. Following Refs. [12] and [13], we have for our model the e-folding number

\[ N = \frac{2\pi}{m_p^2 \sum_i \Phi_i^2}, \quad (17) \]

and the amplitude of adiabatic perturbation

\[ \delta_H = \frac{H}{2\pi} \sqrt{\sum_i \left( \frac{\partial N}{\partial \Phi_i} \right)^2}. \quad (18) \]

In general the initial values of the multi scalars in the assisted inflation model could be taken to be different, however for simplicity as what has been done in [11] we assume the same for the initial values of all KK fields. With such an assumption, the discussions will be much simple. After substituting (16) into (17) and (18), we obtain that

\[ N = \frac{2\pi}{m_p^2} \Phi_1^2(0) \sum_i r_i, \quad (19) \]

\[ \delta_H = \frac{\sqrt{4\pi} m_1 \Phi_1^2(0)}{3 m_p^3} \sqrt{\left( \sum_i \mu_i r_i \right) \left( \sum_i r_i \right)}, \quad (20) \]

where \( \mu_i \equiv (m_i/m_1)^2 \) and \( r_i \equiv (\Phi_1(t)/\Phi_1(0))^{2\mu_i} \).

During the inflation the KK fields with mass less than the Hubble parameter drive the inflation, however, those with mass larger are diluted quickly. Quantitatively, for \( m_N^2 \leq H^2 < m_{N+1}^2 \), we have
\[
\mu_N \leq \frac{4\pi \Phi_1^2(0)}{3m_p^2} \sum_{i=1}^{N_1} \mu_i r_i < \mu_{N+1},
\]
where \( N \) denotes the number of KK fields which are in the process of slow-rolling during the inflation. Because that the difference between \( m_N \) and \( m_{N+1} \) is the same order as \( m_N \) (see Eq. (12)), \( \mu_N \sim \mu_{N+1} \), Eq. (21) can be reduced to
\[
\mu_N \simeq \frac{4\pi \Phi_1^2(0)}{3m_p^2} \sum_{i=1}^{N} \mu_i r_i.
\]

Making use of the definitions of \( \mu_i \) and \( r_i \), Eq. (19) and Eq. (22) can be rewritten as
\[
N = \frac{2\pi}{m_p^2} \Phi_1^2(0) \sum_{i=1}^{N} \frac{\Phi_1(t)}{\Phi_1(0)} \left[1 + (i-1)\pi\right]^2 \left[1 + (i-1)\pi\right].
\]

Taking \( N \approx 60 \) as required when the COBE scale exits the Hubble radius, and eliminating \( \Phi_1(t) \) in (23) and (24), we obtain a relation between \( N \) and \( \Phi_1(0) \), which is shown in Fig. 2. One can see that the number of the KK fields participating in the slow-rolling process increases as \( \Phi_1(0) \) decreases.

Combining (19), (20) and (22), we have
\[
\delta_H \simeq \frac{1}{\sqrt{2\pi m_p}} \sqrt{N \mu_N}.
\]

Making use of the COBE observation \( \delta_H \approx 10^{-5} \), and \( m_1 \approx \xi_1 e^{-kl} \), we have
\[
\exp (-kl) \approx \frac{10^{-6}}{N}.
\]

Note that \( e^{kl} \approx 10^{16} \) which is required to solve the hierarchy problem between the Planck scale and the electroweak scale, Eq. (26) shows the number of the KK field which are in slow rolling is large: \( N \sim 10^{10} \). From Fig. 2, we see that in this case \( \Phi_1(0) \ll m_p \).

The corresponding index of spectrum is
\[ n - 1 = \frac{d \ln \delta_H^2}{d \ln k} = - \frac{d \ln \delta_H^2}{dN} \]
\[ \simeq - \frac{1}{N} \frac{2 dN}{NN'}, \] (27)
which is close to zero i.e. a near scale-invariant spectrum.

The effective potential of the total KK fields in our model on the 3-brane can be written as:
\[ V \simeq \frac{1}{2} m_1^2 \phi_1^2(0) \left( \sum_i \mu_i r_i \right). \] (28)
Substituting (22) into (28), and taking into account that \( \mu_N \equiv (\frac{m_N}{m_1})^2 \sim N^2 \) (notice that \( N \sim 10^{10} \gg 1 \)), we obtain
\[ V \sim \frac{3}{8\pi} m_1^2 m_1^4 N^2. \] (29)
Since \( m_1 \simeq \xi_1 k e^{-kl} \) and \( e^{-kl} \sim 10^{-16} \), we have \( V \sim 10^{-12} m_1^4 \). This shows that although the number of KK fields are quite large \( \sim 10^{10} \) the potential energy \( V \) is still far below the Planck scale. Therefore, the assumption that energy density of matter on the 3-brane is much less than both the bulk cosmological constant and the brane tension is reasonable and the quadratic term of energy density is not important in our consideration [2].

III. CONCLUSION

In summary, we have considered a model by including a bulk scalar field in the RS scenario and studied the inflationary solution. We have calculated the amplitude of scalar perturbation generated during inflation and the index of the spectrum. Our results show that the initial value of the inflaton is not required to be higher than the Planck scale and the scalar boson mass \( m \) is not required to be much less than the Plank scale, furthermore this model solves the hierarchy problem. All of these features are attractive compared with the chaotic inflation model of single scalar field in the four dimensional space-time in which the initial value of the scalar field must be higher than the Planck scale to make inflation happen and its mass must be far less than the Planck scale to satisfy the COBE observation.
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FIG. 1. Plot of KK field masses vs the bulk scalar mass. The $x$-axis is $\frac{m}{k}$, and the $y$-axis is $\frac{m_i}{k} e^{-kl}$. The three curves corresponds to the three lightest KK modes.

FIG. 2. Plot of $N$ as function of $\Phi_1(0)$. The $x$-axis is $\Phi_1(0)$ in unit of $m_p$, the $y$-axis is $N$. In the numerical calculation we take $N = 60$. 