Neutrino masses and mixings from an anomaly free $SMG \times U(1)^2$ model.

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Abstract

A natural solution to the fermion mass hierarchy problem suggests the existence of a partially conserved chiral symmetry. We show that this can lead to a reasonably natural solution to the solar and atmospheric neutrino problems without fine-tuning or the addition of new low energy fermions. The atmospheric neutrino atmospheric neutrino anomaly is given by large mixing between $\nu_\mu$ and $\nu_\tau$, with $\Delta m_{atm}^2 \sim 10^{-3}\text{eV}^2$, and the solar neutrino deficit is due to nearly maximal electron neutrino vacuum oscillations. We present an explicit model for the neutrino masses which is an anomaly free Abelian extension of the standard model that also yields a realistic charged fermion spectrum.
1 Introduction

The observed hierarchy of charged fermion masses and quark mixing angles strongly suggests the existence of an approximate chiral flavour symmetry \cite{1} beyond the standard model (SM). In a previous paper \cite{2} we discussed the implications of such a symmetry for neutrino masses and mixings. We showed that the most natural scenario would correspond to nearly maximal mixing between $\nu_e$ and $\nu_\mu$ being responsible for both the solar and atmospheric neutrino problems. However, the recent data on the atmospheric neutrino zenith angle dependence from Super-Kamiokande \cite{3} indicate that this solution no longer gives an acceptable fit to the atmospheric neutrino data. In this paper we show that approximately conserved chiral symmetries can still lead to a reasonably natural solution to the solar and atmospheric neutrino problems, if we relax the assumptions we made in \cite{2}. We shall also present an explicit model for the neutrino masses and mixings, in which the chiral flavour symmetry comes from an Abelian extension of the standard model gauge group.

Previously we made two assumptions for the models with approximately conserved chiral symmetries:

i. The low energy fermion spectrum of the model is the same as in the standard model – in particular we have only three left-handed neutrinos.

ii. The chiral symmetries lead to elements of the effective light neutrino mass matrix $M_\nu$ which are of different orders of magnitude, apart from those elements which are equal due to the symmetry $M_\nu = M_\nu^T$.

As we discussed in our earlier paper \cite{2}, the only natural solution to the solar and atmospheric neutrino problems with these assumptions is if we have nearly maximal $\nu_e - \nu_\mu$ mixing, and small mixing with $\nu_\tau$. This no longer gives a good description of the atmospheric neutrino data. We cannot obtain any other types of solution as a direct consequence of the assumptions (i) and (ii).
Assumption (i) implies that we must have a $3 \times 3$ symmetric effective Majorana-like neutrino mass matrix. With a hierarchy between the elements of a symmetric mass matrix there are essentially two different forms for the matrix depending on whether or not the diagonal elements dominate all of the eigenvalues. The first case leads to small mixing between all three neutrinos, and this is unsuitable for a solution to the atmospheric neutrino problem. The second case gives large mixing between two nearly degenerate neutrinos, and small mixing with the third (non-degenerate) neutrino. Since we have only three neutrinos we have two independent mass-squared differences ($\Delta m_{ij}^2$) for the neutrinos. The smaller of these $\Delta m^2$s determines the wavelength of oscillation for the two largely mixed neutrinos, which we must take to be $\nu_\mu$ and $\nu_\tau$ with $\Delta m_{23}^2 \sim 10^{-3} \text{eV}^2$ (and consequently the other mass-squared differences $\Delta m_{12}^2 \sim \Delta m_{13}^2 > 10^{-3} \text{eV}^2$) if we wish to explain all of the data on the atmospheric neutrino problem. However, we cannot also explain the solar neutrino problem, since the electron neutrino is then only slightly mixed and the small angle MSW solution requires $\Delta m^2 \sim 10^{-5} \text{eV}^2$. Hence we see that it is necessary to relax our assumptions.

The first assumption was made because of the desire for minimality in our theory. We do not wish to introduce extra low energy fermions unless it is absolutely necessary, and consequently we will retain assumption (i) in this paper. The second assumption is often satisfied in models with chiral (gauged) symmetry breaking; however, it is not uncommon to find two order of magnitude equal elements in the mass matrices. Indeed in the explicit model (based on the anti-grand unified model (AGUT), [4, 5]) from our previous paper we found that the $(1, 1)$ and $(2, 2)$ elements of the neutrino mass matrix were approximately equal, although in that case this did not have any effect on the phenomenology. Hence, in this paper we shall relax the second assumption and consider the case where there are two order of magnitude equal elements in our mass matrix (other than those elements which are exactly equal due to the symmetry of the mass matrix). We do not expect these elements to be exactly equal, since that would generally require fine-tuning which we are careful to avoid.

In the next section we discuss the structure of the neutrino mass matrix we would expect to have for natural models of this type, and the phenomenology of the neutrino oscillations. We will show that (with no fine-tuning) we
would typically obtain nearly maximal $\nu_e$ vacuum oscillations (with a linear combination of $\nu_\mu - \nu_\tau$) for the solar neutrinos, and large $\nu_\mu - \nu_\tau$ oscillations for atmospheric neutrinos. We would expect to see nothing at LSND, much of the parameter space for which has already been ruled out by Karmen \[6\], and Bugey \[7\].

Whilst there are numerous examples of models \[4, 5, 8, 9, 10\] which explain the fermion spectrum using global $U(1)$ symmetries, or which cancel gauged $U(1)$ anomalies using the supersymmetric Green-Schwarz mechanism, it seems to have become a common belief \[11, 12\] that it is not possible to construct an anomaly free gauged Abelian extension of the SM which yields a realistic fermion mass spectrum. We present here an explicit anomaly free model with gauge group $SMG \times U(1)^2$ (where $SMG$ is the SM gauge group), which (with a non-minimal Higgs field spectrum) fits the charged fermion mass spectrum and yields solutions to the solar and atmospheric neutrino problems. The charged fermion mass spectrum in this model is identical to that predicted by the AGUT model. However, the neutrino mass spectrum is considerably different from that given by the AGUT, and we show in section 4 that it can yield neutrino masses of the form suggested in section 3. In order to obtain the required neutrino spectrum, it is necessary to introduce an $SU(2)$ triplet Higgs field with a suitable vacuum expectation value. We also discuss some difficulty in naturally obtaining such a vacuum expectation value for this Higgs field from the scalar potential.

## 2 Neutrino Phenomenology

In this section we shall examine the possible structures of the effective $3 \times 3$ light neutrino mass matrix, which can arise in models with approximately conserved chiral symmetries in a reasonably natural way. In the following discussion we shall use the convention that

$$\Delta m^2_{ij} = |m_{\nu_i}^2 - m_{\nu_j}^2|, \quad (1)$$

$$\Delta m^2_{12} < \Delta m^2_{23}, \quad (2)$$

where $\nu_i$ is the $i$th neutrino mass eigenstate. We then require $\Delta m^2_{23} \sim 10^{-3} \text{eV}^2$ and large $\nu_\mu - \nu_\tau$ mixing for the atmospheric neutrinos. We can
have several types of solution to the solar neutrino problem, such as the well
known MSW and ‘just-so’ solutions to the solar neutrino problem with

\[ \Delta m_{solar}^2 \sim 10^{-5}, 10^{-10} \text{eV}^2 \]  

(3)

respectively. There is also some variation in the solar neutrino fluxes pre-
dicted by different solar models and this theoretical uncertainty means that
it is also possible to have an ‘energy-independent’ vacuum oscillation solution
to the solar neutrino problem [13]. By ‘energy-independent’ we mean that
\[ \Delta m_{solar}^2 \] is sufficiently large that many oscillation lengths lie between the sun
and the earth, and what we observe is the averaged flux suppression which
is the same for solar neutrinos of all energies. Hence we can have

\[ 10^{-10} \lesssim \Delta m_{12}^2 = \Delta m_{solar}^2 \lesssim 10^{-4} \text{eV}^2, \]  

(4)

where the upper limit comes from the constraint that electron neutrino mix-
ing does not make a large contribution to the atmospheric neutrinos. This
type of solution does not agree well with the solar neutrino data if we take
both the experimental and theoretical solar neutrino rates at face value.
(The Bahcall-Pinsonneault (BP98) model [14] rules out this possibility at
99\% C.L.) However, we note that there is still some freedom allowed in the
choice of solar model.

The analysis of [13] examines the possibility of having an energy-independent
solution if the true solar model lies somewhere within the range of currently
allowed solar models. Taking the energy-independent flux suppression (\( F \))
as a free parameter they find

\[ F = 0.50 \pm 0.06 \]  

(5)

with a minimum \( \chi^2 \) of 8. If \( F = 0.5 \) is not a free parameter (as in our model
below) then this corresponds to a confidence level of 5\%. Even if the BP98 so-
lar model is correct, the requirement for an energy-dependent solution to the
solar neutrino problem rests essentially on only one experiment (the Chlorine
experiment.) Given the possibility of unknown systematic errors we would
prefer to avoid relying too strongly on the result of any single experiment.
Hence, whilst the MSW and ‘just-so’ solutions to the solar neutrino problem
are empirically favoured we still consider the simpler energy-independent so-
lution (with maximal mixing between two neutrinos) to be a viable solution.
The amount of mixing will be large for the vacuum oscillation solutions, and may be either large or small for the MSW solutions.

As we saw in our previous paper if we have a completely hierarchical mass matrix (with all independent elements of different orders of magnitude), the only solution to the solar and atmospheric neutrino problems is to have nearly maximal $\nu_e - \nu_\mu$ mixing responsible for both, which seems to be no longer compatible with the atmospheric neutrino data. Hence we shall now look at the possible mass matrices with order of magnitude degeneracies between the elements. One possibility would be to have an order of magnitude degeneracy in the charged lepton mass matrix, leading to large mixing coming from the charged sector. It has been shown elsewhere in the literature [15, 16, 17] that this can yield an acceptable phenomenology, and we do not consider it further here. So we now consider order of magnitude equal elements in the neutrino mass matrix. There are essentially three types of matrix which could potentially yield an acceptable phenomenology with a small number of approximately equal elements,

$$
\begin{pmatrix}
A & \times & \times \\
\times & \times & A \\
\times & A & \times
\end{pmatrix}
\quad
\begin{pmatrix}
\times & \times & \times \\
\times & A & B \\
\times & B & C
\end{pmatrix}
\quad
\begin{pmatrix}
\times & A & B \\
A & \times & \times \\
B & \times & \times
\end{pmatrix}
$$

(6)

where $\times$ denotes small elements and in each case $A \sim B \sim C$. We shall call these textures I, II and III respectively.

From the form of texture I we see that this texture would require the imposition of an exact flavour symmetry relating $(M_\nu)_{11}$ to $(M_\nu)_{23}$, for which we have no good reason. Hence we will not use texture I. In order to have a good phenomenology, type II would require $AC \sim B^2$, which is not unlikely to occur by chance. However, it also requires three order of magnitude equal elements in the neutrino mass matrix, which we do not consider likely in most models with approximately conserved chiral symmetries. Nevertheless, it has been obtained in a supersymmetric extension of the standard model with approximately conserved gauged chiral symmetries [10]. Type III has only two approximately equal elements and, as we shall see in section 4, can occur reasonably naturally in a specific model. In fact type III has previously been considered in the literature in [18], where the structure of
The mass matrix is assumed to be due to a global $L_e - L_\mu - L_\tau$ symmetry. The fine-tuned case where $B = A$ corresponds to the popular ‘bi-maximal mixing’ solution to the neutrino problems [19, 20]. All of the textures (I, II, and III) examined here have previously been discussed in [21] by three of the authors of [18]. However, they claim there that flavour symmetries which lead to textures II and III also yield large mixing from the charged lepton mass matrix. We do not find this to be the case here.

The mass matrix texture of type III has the eigenvalues:

$$\pm \sqrt{A^2 + B^2}, 0$$

(7)

and can be diagonalised by the mixing matrix:

$$U_\nu \sim \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \theta & -\sin \theta \\
0 & \sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
0 & 0 & 1
\end{pmatrix}
$$

(8)

$$= \begin{pmatrix}
\frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\
\frac{1}{\sqrt{2}} \cos \theta & \frac{1}{\sqrt{2}} \cos \theta & \sin \theta \\
\frac{1}{\sqrt{2}} \sin \theta & \frac{1}{\sqrt{2}} \sin \theta & \cos \theta
\end{pmatrix}
$$

(9)

where

$$\tan \theta = \frac{B}{A}.$$ 

(10)

From the first row of eq. 9 we can see that $\nu_e$ is maximally mixed between $\nu_1$ and $\nu_2$, so that its mixing does not contribute to the atmospheric neutrino anomaly, and there will be no effect observable at Chooz [22] since we take $\Delta m^2_{12} < 10^{-4}$ eV$^2$. The atmospheric neutrino anomaly will be entirely due to large $\nu_\mu - \nu_\tau$ mixing and, in order that the mixing be large enough, we need $\sin^2 2\theta \gtrsim 0.7$ (95\% C.L) which requires

$$0.56 \lesssim \frac{B}{A} \lesssim 1.8.$$ 

(11)

So although $A$ and $B$ must be of order of magnitude degenerate, it is not necessary to do any fine tuning. The solar neutrino problem is explained by vacuum oscillations, although whether it is an ‘energy-independent’ or a ‘just-so’ solution will depend on the small elements which we have neglected.
It is not entirely clear which of these types of solution will be more likely to occur in models with chiral symmetry breaking. We note however that the elements of $M_\nu$ which contribute to the $\Delta m_{12}^2$ have to be about 8 orders of magnitude smaller than the large elements A and B for the ‘just-so’ solution. The solar neutrino problem cannot be explained in this model by an MSW type solution, since the mixing of the electron neutrino is too large for this type of solution.

### 3 Constructing an anomaly free $SMG \times U(1)^2$ model

We now introduce an anomaly free Abelian extension of the SM which we shall use in the next section to obtain a neutrino mass spectrum of the form we have just discussed. This extension has the gauge group

$$SMG \times U(1)_{f_1} \times U(1)_{f_2}$$

and we have only the standard model fermion spectrum at low energies. We shall break $U(1)_{f_1}$ and $U(1)_{f_2}$ with a non-minimal set of three Higgs fields, which are required to give a realistic charged fermion spectrum and which leave the $SMG$ unbroken. The $SMG$ will be broken down to $SU(3) \times U(1)$ by the usual Weinberg-Salam Higgs field, although this will now also carry charges under $U(1)_{f_1}$ and $U(1)_{f_2}$. We shall also introduce a further Higgs field to generate a realistic spectrum of neutrino masses in the next section.

The fermions will each have different charges under the chiral symmetries $U(1)_{f_1}$ and $U(1)_{f_2}$, which will prevent most of them from acquiring masses by a direct Yukawa coupling with the Weinberg-Salam Higgs field. However, after the spontaneous breaking of $U(1)_{f_1}$ and $U(1)_{f_2}$ at some high mass scale $M_F$, the charged fermions will all acquire effective mass terms in the low-energy effective theory via diagrams such as figure 1. The intermediate states are taken to be vector-like fermions of mass $M = O(M_F)$, and we assume that the fundamental couplings are $O(1)$. Figure 1 then gives an effective mass to the bottom quark.
Figure 1: Feynman diagram for bottom quark mass in the full theory. The crosses indicate the couplings of the Higgs fields to the vacuum.

\[ m_b \sim \langle \phi_{WS} \rangle \langle W \rangle \langle \theta \rangle^2 \frac{M_F}{M_F^2}, \]

where \( \langle W \rangle, \langle \theta \rangle \) are the vacuum expectation values of Higgs fields \( W \) and \( \theta \) used to spontaneously break the \( SMG \times U(1)^2 \) down to the standard model. The other charged fermions acquire their mass via similar diagrams.

As we discussed earlier we do not wish to extend the low-energy fermion spectrum for reasons of minimality, so we have the usual SM fermion spectrum with their usual representations under \( SMG \). The fermion charges under \( U(1)_{f_1} \) and \( U(1)_{f_2} \) are then severely constrained by the requirement that all the anomalies involving them cancel. If we denote the charges of the fermions under \( U(1)_{f_1} \) and \( U(1)_{f_2} \) by \( Q_{fi}(u_L) = u_{Li} \) \((i = 1, 2)\) etc., then the anomaly constraints are given by:

\[
\begin{align*}
\text{Tr}[SU(3)^2U(1)_{f1}] &= 2(u_{Li} + c_{Li} + t_{Li}) \\
&\quad - (u_{Ri} + d_{Ri} + s_{Ri} + c_{Ri} + t_{Ri} + b_{Ri}) = 0, \\
\text{Tr}[SU(2)^2U(1)_{f1}] &= 3(u_{Li} + c_{Li} + t_{Li}) + e_{Li} + \mu_{Li} + \tau_{Li} = 0, \\
\text{Tr}[U(1)_{f1}^2U(1)_{f1}] &= u_{Li} + c_{Li} + t_{Li} - 8(u_{Ri} + c_{Ri} + t_{Ri}) \\
&\quad - 2(d_{Ri} + s_{Ri} + b_{Ri}) + 3(e_{Li} + \mu_{Li} + \tau_{Li}) \\
&\quad - 6(e_{Ri} + \mu_{Ri} + \tau_{Ri}) = 0, \\
\text{Tr}[U(1)_{f1}U(1)_{f1}] &= u_{Li}^2 + c_{Li}^2 + t_{Li}^2 \\
&\quad + d_{Ri}^2 + s_{Ri}^2 + b_{Ri}^2 - 2(u_{Ri}^2 + c_{Ri}^2 + t_{Ri}^2) \\
&\quad + 6(e_{Ri}^2 + \mu_{Ri}^2 + \tau_{Ri}^2) = 0,
\end{align*}
\]
Table 1: An anomaly free choice of Abelian charges for the fermion fields

|       | (1st. gen.) | cL | tL | cR | sR | tR | bR | µL | τL | µR | τR |
|-------|-------------|----|----|----|----|----|----|----|----|----|----|
| Qf1   | 0           | 0  | 1  | 4  | 0  | 0  | -2 | 0  | -3 | 0  | -6 |
| Qf2   | 0           | -1 | 0  | -1 | 1  | -3 | 1  | 3  | 0  | 5  | 1  |

\[ \text{Tr}[U(1)_{f1}U(1)_{f2}U(1)_{f3}] = 6(u_{Li}u_{Lj}u_{Lk} + c_{Li}c_{Lj}c_{Lk} + t_{Li}t_{Lj}t_{Lk}) \\
-3(d_{Ri}d_{Rj}d_{Rk} + s_{Ri}s_{Rj}s_{Rk} + b_{Ri}b_{Rj}b_{Rk}) \\
+u_{Ri}u_{Rj}u_{Rk} + c_{Ri}c_{Rj}c_{Rk} + t_{Ri}t_{Rj}t_{Rk}) \\
+2(e_{Li}e_{Lj}e_{Lk} + \mu_{Li}\mu_{Lj}\mu_{Lk} + \tau_{Li}\tau_{Lj}\tau_{Lk}) \\
-(e_{Ri}e_{Rj}e_{Rk} + \mu_{Ri}\mu_{Rj}\mu_{Rk} + \tau_{Ri}\tau_{Rj}\tau_{Rk}) = 0, \]  

\[ \text{Tr}[(\text{graviton})^2U(1)_{f1}] = 6(u_{Li} + c_{Li} + t_{Li}) - 3(u_{Ri} + d_{Ri} + s_{Ri} + c_{Ri}) \\
+3(t_{Ri} + b_{Ri}) + 2(e_{Li} + \mu_{Li} + \tau_{Li}) \\
-(e_{Ri} + \mu_{Ri} + \tau_{Ri}) = 0. \]  

A possible choice of charges (which is based on the AGUT Abelian charges) satisfying these constraints is given in table 1 and, as we shall see, a realistic charged fermion mass spectrum can be obtained for these charges by making a suitable choice of Higgs fields. The set of charges in table 1 is not the only one which is anomaly free. For example, the AGUT has four \( U(1) \)s, with linearly independent sets of charges which satisfy the anomaly constraints of eq. (14). In the AGUT (14) one of these \( U(1) \)s is broken before the others at the Planck scale, leaving three unbroken \( U(1) \) generators. In this paper we choose the fermion charges to be a linear combination of the charges under these unbroken generators. (Our choice of charges is given by \( Q_Y = y_1 + y_2 + y_3, Q_{f1} = 3y_3 \) and \( Q_{f2} = -3y_2 + Q_f \) where \( y_{1,2,3} \) and \( Q_f \) are the AGUT fermion charges of reference [1]). We could alternatively have chosen to use the charges under the broken \( U(1) \) for \( Q_{f1} \) or \( Q_{f2} \); however, we are unaware of any choice of charges (with only two non-standard model \( U(1) \)s) involving this broken \( U(1) \) which yields a realistic charged fermion spectrum.

The Weinberg-Salam Higgs field, \( \phi_{WS} \), charges are chosen so that the top quark obtains its mass directly from its Yukawa coupling with \( \phi_{WS} \), and \( M_t \)
is thus unsuppressed. The other fermions cannot couple directly to $\phi_{WS}$ since such couplings are protected by the chiral symmetries. Hence we introduce three other Higgs fields $W$, $\xi$ and $\theta$ to break the $U(1)_{f1}$ and $U(1)_{f2}$ with charges and vacuum expectation values chosen to give a realistic fermion spectrum. The charges and vacuum expectation values of the Higgs fields are given in table 2. We take the Higgs fields at the fundamental scale to be singlets under the standard model symmetries. The charged fermion effective SM Yukawa matrices are then given by

$$H_U \sim \begin{pmatrix} 
\langle W \rangle \langle \theta \rangle^4 \langle \xi \rangle^2 & \langle W \rangle^2 \langle \theta \rangle^2 \langle \xi \rangle & \langle W \rangle \langle \theta \rangle^4 \langle \xi \rangle \\
\langle W \rangle \langle \theta \rangle^4 \langle \xi \rangle^3 & \langle W \rangle^2 \langle \theta \rangle^2 & \langle W \rangle \langle \theta \rangle^4 \\
\langle W \rangle \langle \theta \rangle^4 \langle \xi \rangle^3 & \langle W \rangle^2 \langle \theta \rangle^2 & 1
\end{pmatrix},$$

(15)

$$H_D \sim \begin{pmatrix} 
\langle W \rangle \langle \theta \rangle^4 \langle \xi \rangle^2 & \langle W \rangle \langle \theta \rangle^4 \langle \xi \rangle & \langle \theta \rangle^6 \langle \xi \rangle \\
\langle W \rangle \langle \theta \rangle^4 \langle \xi \rangle & \langle W \rangle \langle \theta \rangle^4 & \langle \theta \rangle^6 \\
\langle W \rangle^2 \langle \theta \rangle^8 \langle \xi \rangle & \langle W \rangle^2 \langle \theta \rangle^8 & \langle W \rangle \langle \theta \rangle^2
\end{pmatrix},$$

(16)

$$H_E \sim \begin{pmatrix} 
\langle W \rangle \langle \theta \rangle^4 \langle \xi \rangle^2 & \langle W \rangle \langle \theta \rangle^4 \langle \xi \rangle & \langle W \rangle \langle \theta \rangle^8 \langle \xi \rangle \\
\langle W \rangle \langle \theta \rangle^4 \langle \xi \rangle & \langle W \rangle \langle \theta \rangle^4 \langle \xi \rangle & \langle W \rangle \langle \theta \rangle^8 \langle \xi \rangle \\
\langle W \rangle \langle \theta \rangle^{10} \langle \xi \rangle^3 & \langle W \rangle^2 \langle \theta \rangle^8 & \langle W \rangle \langle \theta \rangle^2
\end{pmatrix},$$

(17)

where the Higgs field vacuum expectation values $\langle W \rangle$, $\langle \xi \rangle$ and $\langle \theta \rangle$ are in units of the fundamental scale, $M_F$. These mass matrices yield exactly the same masses and mixings at the fundamental scale as we obtained in the AGUT model in previous papers [3], as can be seen by substituting the Higgs field combination $\theta^2$ in this paper by the Higgs field $T$ in the AGUT, and relabelling the $c_R$ and $t_R$ fields. This is because (after this trivial relabelling of fermion fields) the charges on the fermion fields are the same as a linear combination of the remaining Abelian fermion charges in the AGUT after one of the AGUT $U(1)$’s is spontaneously broken. The choice of Higgs fields in the $SMG \times U(1)^2$ model is however different and, whilst this leads to the same charged fermion spectrum as in the AGUT (see table 3 for the best fit spectrum from [3]), it does not yield the same neutrino spectrum. The AGUT cannot produce the same neutrino mass matrix structure (without increasing the number of Higgs fields), since it is not possible to choose a consistent set of non-Abelian representations for the Higgs fields. We shall see however that, within the $SMG \times U(1)^2$ model, we can obtain an acceptable neutrino spectrum.
Table 2: Higgs field charges which have been chosen to give a realistic charged fermion spectrum, and the vacuum expectation values for the chiral symmetry breaking Higgs fields in units of the fundamental scale $M_F$.

|       | $y/2$ | $Q_{f1}$ | $Q_{f2}$ | Vacuum expectation value |
|-------|-------|----------|----------|--------------------------|
| $\phi_{WS}$ | $\frac{1}{2}$ | -1 | -3 | $\langle W \rangle$ |
| $W$ | 0 | 3 | $\frac{5}{3}$ | 0.158 |
| $\theta$ | 0 | $\frac{1}{2}$ | $\frac{1}{6}$ | 0.266 |
| $\xi$ | 0 | 0 | 1 | 0.099 |

4 Neutrino masses and mixings from an explicit model

Neutrino masses can be generated in this model by the Weinberg-Salam Higgs field, via a see-saw like mechanism, giving a dominant off-diagonal element in the neutrino mass matrix,

$$M_\nu \sim \phi_{WS}^2 \frac{M_F}{M_F} \begin{pmatrix} \langle W \rangle^2 \langle \theta \rangle^8 \langle \xi \rangle^4 & \langle W \rangle^2 \langle \theta \rangle^8 \langle \xi \rangle & \langle W \rangle^2 \langle \theta \rangle \langle \xi \rangle^3 \\ \langle W \rangle^2 \langle \theta \rangle^8 \langle \xi \rangle & \langle W \rangle \langle \theta \rangle^{10} & \langle W \rangle^2 \langle \theta \rangle \\ \langle W \rangle^2 \langle \theta \rangle \langle \xi \rangle^3 & \langle W \rangle^2 \langle \theta \rangle & \langle W \rangle^2 \langle \theta \rangle^2 \langle \xi \rangle^2 \end{pmatrix}. \quad (18)$$

This yields nearly maximal $\nu_\mu - \nu_\tau$ mixing between a nearly degenerate pair of neutrinos. As we discussed earlier this does not lead to an acceptable phenomenology, and hence we require a different mechanism to generate the dominant contribution to the neutrino masses and mixings. We do this here by introducing an $SU(2)$ triplet Higgs field $\Delta$. The charges on this Higgs field are then chosen so that the $(1, 2)$ and $(1, 3)$ elements of $M_\nu$ are suppressed by equal amounts, giving

$$\left( y, Q_{f1}, Q_{f2} \right) = \left( 1, \frac{3}{2}, -\frac{3}{2} \right). \quad (19)$$
Table 3: Best fit to conventional experimental data. All masses are running masses at 1 GeV except the top quark mass which is the pole mass.

|       | Fitted  | Experimental |
|-------|---------|--------------|
| $m_u$ | 3.6 MeV | 4 MeV        |
| $m_d$ | 7.0 MeV | 9 MeV        |
| $m_e$ | 0.87 MeV| 0.5 MeV      |
| $m_c$ | 1.02 GeV| 1.4 GeV      |
| $m_s$ | 400 MeV | 200 MeV      |
| $m_\mu$| 88 MeV  | 105 MeV      |
| $m_t$ | 192 GeV | 180 GeV      |
| $m_b$ | 8.3 GeV | 6.3 GeV      |
| $m_\tau$| 1.27 GeV| 1.78 GeV     |
| $V_{us}$| 0.18    | 0.22         |
| $V_{cb}$| 0.018   | 0.041        |
| $V_{ub}$| 0.0039  | 0.0035       |

The neutrino mass matrix,

\[
M_\nu \sim \langle \Delta^0 \rangle \langle \theta \rangle^3 \begin{pmatrix}
\langle \xi \rangle^2 & \langle \xi \rangle & \langle \xi \rangle \\
\langle \xi \rangle & \langle \xi \rangle^4 & \langle \xi \rangle^2 \\
\langle \xi \rangle & \langle \xi \rangle^2 & \langle \theta \rangle^6
\end{pmatrix},
\]  

(20)

is then generated by diagrams such as figure 2. We have ignored CP violating phases here, and there are unknown $O(1)$ factors in front of each of the mass matrix elements.

This mass matrix gives

\[
\frac{\Delta m_{12}^2}{\Delta m_{23}^2} \sim \langle \xi \rangle
\]  

(21)

which is not small enough for the ‘just-so’ or MSW solutions to the solar neutrino problem if we take

\[
\Delta m_{23}^2 \sim 10^{-3} \text{eV}^2
\]  

(22)

for the atmospheric neutrino problem. Hence we shall use the ‘energy-independent’ vacuum oscillation solution to the solar neutrino problem. The
mixing from this mass matrix is similar to that given by eq. 3, although the elements of order $\langle \Delta^0 \rangle \theta^3 \xi^2$ in the mass matrix can have some effect on the mixing leading to some small deviations from the form of eq. 3. The electron neutrino mixing remains very close to maximal regardless of the $O(1)$ factors in the mass matrix, and makes almost no contribution to the atmospheric neutrino mixing. Depending on the $O(1)$ factors the muon and tau neutrino mixing can differ slightly from that given by eq. 3, although if eq. 11 is satisfied then the mixing between them remains large enough to solve the atmospheric neutrino problem.

Hence if we take $\langle \Delta \rangle \sim 12 \, \text{eV}$ to give suitable masses for the atmospheric neutrino problem then we have

$$\Delta m_{12}^2 \sim 10^{-4} \, \text{eV}^2, \quad \sin^2 2\theta_{12} \sim 1 \quad (23)$$
$$\Delta m_{23}^2 \sim 10^{-3} \, \text{eV}^2, \quad \sin^2 2\theta_{23} = 0.7 - 1.0 \quad (24)$$

for the solar and atmospheric neutrinos respectively. This means we will have an electron neutrino flux suppression of 1/2 for all of the solar neutrinos, and the atmospheric neutrino problem will be due to large $\nu_e - \nu_\tau$ mixing. The neutrino masses are too small to make a significant contribution to dark matter, or to the anomaly observed at LSND [23]. Hence we predict that the LSND result will prove to be unfounded. The amplitude of neutrinoless double beta decay is proportional to $(M_\nu)_{ee}$, which we predict to be $M_{\nu_{ee}} \sim 2 \times 10^{-3} \, \text{eV}$, which is much less than the current limit of $(M_\nu)_{ee} \leq 0.45 \, \text{eV}$ [24] and the sensitivities of current or planned experiments.

In obtaining the spectrum of neutrino masses we have simply chosen $\langle \Delta^0 \rangle$ to have the required value for the atmospheric neutrinos. However, there is some unnaturalness in obtaining a suitable value for $\langle \Delta^0 \rangle$ from the scalar potential. If we write down the low energy effective scalar potential we have

$$V(\phi_{WS}, \Delta) \sim \lambda\{\phi_{WS}^\dagger \phi_{WS}\}^2 + \lambda'(\Delta^\dagger \Delta)^2 + \lambda'' M_F \phi_{WS}^\dagger \Delta \langle W \rangle \langle \xi \rangle^2 \langle \theta \rangle$$
\[-\eta M_F^2 \Delta^\dagger \Delta - \frac{\mu^2}{\lambda} \phi_W \phi_W \{ \phi \phi \} \]  

(25)

where we would typically expect \( \lambda', \lambda'', \eta = O(1) \). However, this leads to a vacuum expectation value for \( \Delta \) of

\[ \langle \Delta^0 \rangle \sim \frac{\langle \phi_W \phi_W \rangle}{M_F} \langle W \rangle \langle \xi \rangle^2 \langle \theta \rangle. \]  

(26)

Whilst we can choose \( M_F \) to give the required vacuum expectation value for \( \Delta \) we then find that, since \( \langle \Delta \rangle \) is much less than the see-saw scale \( \langle \phi_W \phi_W \rangle^2 / M_F \), the neutrino mass matrix is dominated by the see-saw type diagrams which as we noted earlier, do not yield an acceptable phenomenology. Hence, in order to avoid this problem, we would require a \( \phi_W^2 \Delta \) coupling which is for some unknown reason much larger than expected. Of course the scalar potential is in any case not well understood, since the lightness of the Weinberg-Salam Higgs field is also something of a mystery.

It should be noted that, whilst in this case we have some difficulty in obtaining a suitable vacuum expectation value for the triplet Higgs field, this will not necessarily be the case for other models which use this mechanism for generating the neutrino masses. If the see-saw neutrino masses are sufficiently suppressed by the symmetry breaking parameters, then the masses coming from the triplet Higgs field will dominate and there will be no problem.

5 Conclusions

We have shown that models with only the 3 standard model neutrinos (in the low energy spectrum), and chiral symmetry breaking can explain the solar and atmospheric neutrino problems including the Super-Kamiokande zenith angle distribution. This can occur if the chiral symmetry does not lead to (independent) elements in \( M_\nu \) which are all of different orders of magnitude (as we assumed in a previous paper). The atmospheric neutrino problem is explained by large \( \nu_\mu - \nu_\tau \) mixing, and (for the mass matrix structure we examined) the solar neutrino deficit is due to nearly maximal electron neutrino vacuum oscillations, which can be either ‘just-so’ or ‘energy-independent’. We presented an explicit model, which is an anomaly free
Abelian extension of the SM, yielding this type of phenomenology, although there are unresolved problems in the scalar potential. This model is an extension of a model which gives a realistic 3 parameter fit to the charged fermion masses and mixings. It gives an ‘energy-independent’ solar neutrino suppression of $1/2$, with $\Delta m_{solar}^2 \sim 10^{-4} \text{eV}^2$. We also predict that the signal at LSND will not be confirmed by other experiments, and that the neutrinos will not make a significant contribution to hot dark matter.

The prospects for examining this scenario are good. Experiments such as SNO [25], Borexino [26] and KamLand [27] should provide us with more information on the solar neutrino spectrum. Super-Kamiokande will also provide data on the day-night asymmetry and seasonal variations which will be important in determining the type of solution to the solar neutrino problem. Long baseline experiments such as K2K [28] and MINOS [29] should enable us to confirm the nature of the atmospheric neutrino oscillations with a better understood neutrino source, and should tell us whether the $\nu_\mu$ oscillations are to $\nu_\tau$ or a sterile neutrino. The LSND result will also be further tested by Karmen at 95% C.L., and definitively by MiniBoone; neither of which we would expect to find evidence of oscillations. In conclusion, we predict the atmospheric neutrino problem to be due to large $\nu_\mu - \nu_\tau$ oscillations with $\Delta m^2 \sim 10^{-3} \text{eV}^2$, and the solar neutrino deficit to be due to electron neutrino vacuum oscillations of either the ‘just-so’ or ‘energy-independent’ type. This scenario should be confirmed or denied by a number of experiments in the near future.

Acknowledgements

H.B.N. and C.F. acknowledge funding from INTAS 93-3316-ext, and the EU grant HMC 94-0621. M.G. is grateful for a PPARC studentship. We would also like to thank M. Jezabek for useful discussions.

References

[1] C.D. Froggatt and H.B. Nielsen, Nucl. Phys. B147 (1979) 77.
[2] C.D. Froggatt, M. Gibson, H.B. Nielsen, Phys. Lett. B409 (1997) 305.
[3] M. Takita, Talk given at ICHEP ’98.
[4] C.D. Froggatt, G. Lowe and H.B. Nielsen, Nucl. Phys. B414 (1994) 579.
[5] C.D. Froggatt, H.B. Nielsen and D.J. Smith, Phys.Lett. B385 (1996) 150; C.D. Froggatt, M. Gibson, H.B. Nielsen, and D.J. Smith, Int. J. Mod. Phys. A13 (1998) 5037.
[6] K. Eitel and B. Zeitnitz (KARMEN collab.), talk presented at Neutrino ’98, hep-ex/9809007.
[7] B. Achkar et al., Nucl. Phys. B434 (1995) 503.
[8] M. Leurer, Y. Nir and N. Seiberg, Nucl. Phys. B398 (1993) 319; ibid. B420 (1994) 468.
[9] L.E. Ibanez and G.G. Ross, Phys. Lett. B332 (1994) 100.
[10] N. Irges, S. Lavignac, and P. Ramond, Phys. Rev. D58 (1998) 035003.
[11] J. Bijnens and C. Wetterich, Nucl. Phys. B283 (1987) 237.
[12] P. Binétruy, S. Lavignac, and P. Ramond, Nucl. Phys. B477 (1996) 353.
[13] G. Conforto, C. Grimani, F. Martelli and F. Vetrano, talk presented at Neutrino ’98, Takayama, Japan, hep-ph 9807306.
[14] J.N. Bahcall, P.I. Krastev, and A. Yu. Smirnov, hep-ph/9807210.
[15] Y. Grossman, Y. Nir, and Y. Shadmi, hep-ph/9808353.
[16] G.K. Leontaris, S. Lola and G.G. Ross, Nucl. Phys. B454(1995) 25.
[17] J. Pati, based on talk presented at Neutrino ’98, hep-ph/9807315.
[18] R. Barbieri, L.J. Hall, D. Smith, A. Strumia and N. Weiner, hep-ph/9807235.
[19] V. Barger, S. Pakvasa, T.J. Weiler and K. Whisnant, hep-ph/9806387.
[20] M. Jezabek, and Y. Sumino, hep-ph/9807310.
[21] R. Barbieri, L.J. Hall, and A. Strumia, hep-ph/9808333.

[22] CHOOZ collaboration, M. Apollonio et al., Phys. Lett. B420 (1998) 397.

[23] LSND Collaboration, C. Athanassopoulos et al., Phys. Rev. Lett. 81 (1998) 1774; ibid. 77 (1996) 3082.

[24] H.V. Klapdor-Kleingrothaus, L. Baudis, J. Hellig, M. Hirsch, S. Kolb, H. Päts, and Y. Ramachers, hep-ph/9712381.

[25] SNO Collaboration, A.B. McDonald, Nucl. Phys. B(proc. Suppl.) 48 (1996) 357.

[26] BOREXINO Collaboration, C. Arpesella et al. “INFN Borexino proposal”, Vols. I and II, eds. G. Bellini, R. Ragahaven et al. (Univ. Milan, 1992).

[27] A. Suzuki, Talk presented at Neutrino ’98, Takayama, Japan.

[28] Y. Oyama, hep-ex/9803014.

[29] MINOS Collaboration, “Neutrino Oscillation Physics at Fermilab: The NuMI-MINOS Project”, NuMI-L-375, May 1998.