$B_c$ Mesons Production at Hadron Colliders by Heavy Quark Fragmentation

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ABSTRACT

We present a reliable estimate on the production rate of $B_c$ mesons in $1S$ and $2S$ states in the large transverse momentum region at hadronic colliders using heavy quark fragmentation functions derived within the framework of perturbative QCD. We also present the transverse momentum distribution for the $B_c$ mesons. The production rate is large enough for the $B_c$ mesons to be identified at the Tevatron. At the SSC/LHC the rate is so large that their properties can be studied in details.

I. INTRODUCTION

Heavy flavor production and decays are very useful for measuring CKM matrix elements of the standard model and for testing bound-state models for mesons and baryons. Ever since the first $B$ meson was discovered, a lot of data have been coming out from, e.g., CLEO, ARGUS, LEP/SLC, FNAL, on the $B_d$ and $B_u$ meson families. From the recent run
at Tevatron the masses and other properties of the $B_s$ have been measured and confirmed \[1\]. The next family of $B$ mesons will be the $B_c$ mesons made up of $\bar{b}c$. The $B_c$-meson family differs from the $J/\psi$ and $\Upsilon$ families and from other $B$ mesons because it is made up of a pair of heavy quark and antiquark of different flavors and masses. The $J/\psi$ and $\Upsilon$ families have played important roles in developing heavy quark bound-state models inspired by QCD. Being quarkonium systems of different flavors and masses $\bar{b}c$ bound-states provide unique opportunities to test different bound-state models of QCD. The decays of $B_c$ mesons also provide rich sources to test the standard model, e.g. the measurement of $|V_{bc}|$, and enables us to see the interplay between strong and weak interactions.

Since the physics of the $B_c$ mesons is so interesting, one would like to know how many can be produced in the present colliders (e.g. Tevatron) and in the future hadronic supercolliders (SSC/LHC). It is the purpose of this letter to present reliable estimate in the high $p_T$ region, by using the heavy quark fragmentation functions $D_{\bar{b}\rightarrow B_c}(z) \ [2]$ which are based on perturbative QCD. We will summarize some features of these heavy quark fragmentations in Sec. II, and present the production rates and the $p_T$ distributions for the $B_c$ mesons in Sec. III. Finally we will conclude in Sec.IV.

II. HEAVY QUARK FRAGMENTATION

Previous estimates of $B_c$ meson production have based on perturbative QCD calculations for $e^+e^-$ colliders \[3\] and monte carlo studies for both $e^+e^-$ and hadronic colliders \[4\]. The monte carlo estimates of the ratio $\sigma(B_c^\pm)/\sigma(\bar{b}\bar{b}X)$ are all of the order $10^{-3}$ for LEP, Tevatron, SSC/LHC, and HERA. This fact leads us to think that the production mechanisms are all of the same nature. In the region of large $p_T$, the major mechanism for producing $B_c$ or any other mesons is heavy quark fragmentation \[4][43\], in which a $\bar{b}$ antiquark is produced at large $p_T$ by a hard-scattering process and it subsequently fragments into the meson. The differential cross section for direct production of the $B_c$ meson at high $p_T$ can be factorized at leading order in $\alpha_s$ as
\[ d\sigma(B_c(p)) = \int_0^1 dz \, d\tilde{\sigma}(\tilde{b}(p/z, \mu))D_{\tilde{b}\rightarrow B_c}(z, \mu), \] (1)

where \( z \) is the longitudinal momentum fraction carried by the \( B_c \), and \( \mu \) is a factorization scale. The physical interpretation is as follow: a heavy \( \tilde{b} \) antiquark is produced in a hard process with four-momentum \( p/z \), and then it fragments into the \( B_c \) meson with a longitudinal momentum fraction \( z \). The fragmentation function \( D_{\tilde{b}\rightarrow B_c}(z) \) satisfies the Altarelli-Parisi evolution equation

\[ \mu \frac{\partial}{\partial \mu} D_{\tilde{b}\rightarrow B_c}(z) = \int_z^1 \frac{dy}{y} P_{\tilde{b}\rightarrow \tilde{b}}(z/y, \mu)D_{\tilde{b}\rightarrow B_c}(y, \mu), \] (2)

at leading order in \( \alpha_s \). The factorization for \( B_u, B_d \), and \( B_s \) productions can be described in the same way as Eqs. (1) and (2), with the corresponding fragmentation functions. These fragmentation functions should be independent of the hard process by which the \( \tilde{b} \) is produced.

The fragmentation of \( \tilde{b} \) into \( B_u, B_d \), and \( B_s \) is a soft process, and can only be described by a phenomenological function \[ \text{[5]} \]. However, \( \tilde{b} \rightarrow B_c \) requires the production of a \( c\bar{c} \) pair and it is therefore a hard process which can be calculated using perturbative QCD \[ \text{[2,6]} \]. This perturbative QCD approach has been shown valid in calculating the fragmentation functions for heavy quarkonium productions \[ \text{[3]} \], including the splitting of gluons and charm quarks into \( S \)-wave charmonium. The fragmentation functions \( D_{\tilde{b}\rightarrow B_c}(z) \) derived in Ref. \[ \text{[2]} \] need only the input parameters of \( \alpha_s, m_b, m_c \), and the radial wave function \( R(0) \) of the bound state at the origin so that it has more predictive power. The initial fragmentation functions are given by

\[ D_{\tilde{b}\rightarrow B_c}(z, \mu_0) = \frac{2\alpha_s(2m_c)^2|R(0)|^2}{81\pi m_c^3} \frac{rz(1-z)^6}{(1-(1-r)z)^6} \times \left[ 6 - 18(1-2r)z + (21 - 74r + 68r^2)z^2 
\quad - 2(1-r)(6 - 19r + 18r^2)z^3 + 3(1-r)^2(1 - 2r + 2r^2)z^4 \right], \] (3)

for the \( ^1S_0 \) \( B_c \) state, and

\[ D_{\tilde{b}\rightarrow B_c^*}(z, \mu_0) = \frac{2\alpha_s(2m_c)^2|R(0)|^2}{27\pi m_c^3} \frac{rz(1-z)^6}{(1-(1-r)z)^6}, \]
\begin{align}
&\times \left[ 2 - 2(3 - 2r)z + 3(3 - 2r + 4r^2)z^2 \right. \\
&\left. - 2(1 - r)(4 - r + 2r^2)z^3 + (1 - r)^2(3 - 2r + 2r^2)z^4 \right],
\end{align}

for the first excited state $B^*_c$ which has the spin-orbital quantum number $^3S_1$, and $r = m_c/(m_b + m_c)$. Note that the scale in $\alpha_s$ is set to be $2m_c$. The perturbative QCD calculation gives directly the fragmentation function at the scale $\mu_0 = 2m_c$, which is the minimum virtuality of the gluon splitting into $c\bar{c}$. Using the Heavy Quark Effective Theory methods \cite{7}, it can be shown that the evolution up to the scale $m_b$ is trivial, which means that higher order radiative corrections will not introduce any logarithm of $m_b/m_c$. A convenient choice for the initial scale is $\mu_0 = m_b + 2m_c$ \cite{2}, because the fragmentation functions for $c \rightarrow B_c$, $B^*_c$ can then be obtained from Eqs. (3) and (4) simply by interchanging $m_b$ and $m_c$.

In the factorization scheme of Eq. (4), all the dependence on the momentum $p$ is in the hard process $\hat{\sigma}$. Large logarithm of $p/\mu$ can be avoided by choosing the factorization scale $\mu$ to be of order $p$. The induced large logarithm of order $\mu/m_b$ in $D(z)$ can be solved by evolving the Eq. (2). To leading order in $\alpha_s$ only the $\bar{b} \rightarrow \bar{b}$ contributes to the evolution. The fragmentation functions $D_{\bar{b}\rightarrow B_c}(z)$ and $D_{\bar{b}\rightarrow B^*_c}(z)$ at the initial scale $\mu_0$ and higher scale have been shown in Fig. 1 of Ref. \cite{2}.

### III. RESULTS & DISCUSSIONS

In this section, we use Eq. (1) to compute the direct production rates of $B_c$ and $B^*_c$ in hadronic collisions. Our calculation includes

\begin{align}
gg &\rightarrow \bb, \quad gb \rightarrow g\bar{b}, \quad \text{and} \quad q\bar{q} \rightarrow b\bar{b},
\end{align}

as the hard subprocesses for the inclusive production of the $\bar{b}$. We choose the scale $\mu$ for the parton distribution functions and for $\alpha_s$ to be the transverse mass of the $\bar{b}$, $\sqrt{p_{T_b}^2 + m_b^2}$. We use the parameterization of HMRS (set b) \cite{8} for parton distribution functions. The running coupling constant $\alpha_s(Q)$ is evaluated at 1-loop by evolving from the experimental value $\alpha_s(m_Z) = 0.12$ \cite{9}, and given by
\[ \alpha_s(Q) = \frac{\alpha_s(m_Z)}{1 + 8\pi b_0 \alpha_s(m_Z) \log(Q/m_Z)}, \]

where \( b_0 = (33 - 2n_f)/48\pi^2 \), and \( n_f \) is the number of active flavors below the scale \( Q \).

The subprocess cross sections are convoluted with \( D(z, \mu) \), as in Eq. 1. The functions \( D(z, \mu_0) \) at the initial scale \( \mu_0 \) are given in Eqs. (3) and (4), and are evolved to the scale \( \mu \) using Eq. (2). For the initial fragmentation functions \( D(z, \mu_0) \) we use the input parameters of \( m_b = 4.9 \) GeV, \( m_c = 1.5 \) GeV, and \( |R(0)|^2 = (1.18 \) GeV\(^3 \).

The \( p_T \) spectrum for the \( B_c \) meson at Tevatron energy is shown in Fig. 1, with the acceptance cuts

\[ p_T(B_c) > 10 \text{ GeV} \quad \text{and} \quad |y(B_c)| < 1. \]

The corresponding spectrum for the \( B_c^* \) is also shown on the same figure. The shape of the two spectra is very similar, because \( D_{b \to B_c}(z) \) and \( D_{b \to B_c^*}(z) \) have similar shapes and differ primarily by an overall normalization difference of about 50%. The corresponding spectra at \( \text{SSC}(\sqrt{s} = 40 \) TeV) and \( \text{LHC}(\sqrt{s} = 14 \) TeV) energies are shown in Fig. 2, but under slightly different acceptance requirements

\[ p_T(B_c) > 20 \text{ GeV} \quad \text{and} \quad |y(B_c)| < 2.5. \]

The integrated cross sections versus \( p_T^{\text{fin}}(B_c) \) are also shown in Fig. 3. The cross sections at the SSC are about three times as large as those at the LHC, and about two order of magnitudes larger than those at the Tevatron.

So far we have only estimated the \( B_c \) meson productions in \( 1^1S_0 \) and \( 1^3S_1 \) states. Since the annihilation channel for the decay of the excited \( B_c \)-meson states is suppressed relative to the electromagnetic and hadronic transitions, all the excited states (\( 1^3S_1, 2S, 1P, 2P, 1D \)) below the \( BD \) threshold will decay to the ground state \( 1^1S_0 \) by emitting photons or pions. They therefore all contribute to the inclusive production of \( B_c \) mesons. A simple modification can be made to estimate the productions in \( 2^1S_0 \) and \( 2^3S_1 \) states, by multiplying with the factor \( |R_{2S}(0)/R_{1S}(0)|^2 \simeq 0.6 \). Therefore, the curves in Figs. 1, 2, and 3 can be
multiplied by 0.6 to get the productions for 2S states. To get the total inclusive production rate, however, we need to include $P$-wave and possibly $D$-wave contributions, which have not been calculated. Therefore, the cross sections presented here are rather conservative to the actual production rates. Table I shows the number of $B_c$ mesons that can be produced inclusively at Tevatron, SSC and LHC, including the contributions from 1S and 2S states, with integrated luminosities of 0.025, 10, and 100 fb$^{-1}$ respectively. It is also informative to give the ratio $\sigma(B_c)/\sigma(\bar{b}X)$, which is simply the fragmentation probability $\int_0^1 dz D_{\bar{b} \to B_c}(z)$. Adding the contributions from 1S and 2S states, the ratio $\sigma(B_c)/\sigma(\bar{b}X)$ is about $1.5 \times 10^{-3}$, which is consistent with the monte carlo studies [4].

The factorization in Eq. (1) is correct up to the order $(m_b/p_T)^2$, which explains why we impose a rather high $p_T$ cut on the $B_c$ mesons. In the low $p_T$ region, mechanisms other than heavy quark fragmentation have to be taken into account. For example, production of pairs of $\bar{b}b$ and $c\bar{c}$ followed by recombination of $\bar{b}$ and $c$ to form a $B_c$ meson contributes at low $p_T$ region; whereas heavy quark fragmentation dominates at high $p_T$. Similar conclusion can be found in the production of $J/\psi$ by heavy quark fragmentation [11], which is dominant over the process $gg \to \psi g$ in the large $p_T$ region. There are other uncertainties arising from higher order QCD corrections, relativistic corrections of the bound state model, and the values of $\alpha_s$ and $R(0)$ used. But the largest source of uncertainties comes from the values of $m_b$ and $m_c$ employed because of the factor $1/m_c^2$ in the initial fragmentation functions as in Eqs. (3) and (4).

IV. CONCLUSIONS

We have used heavy quark fragmentation functions derived from perturbative QCD to calculate the production rates and the $p_T$ distributions for $S$-wave $B_c$ mesons at Tevatron, SSC, and LHC energies. Imposing cuts of $p_T(B_c) > 10$ GeV and $|y(B_c)| < 1$ at the Tevatron, and including the contributions from 1S and 2S states, about 16000 $B^+_c$ and 16000 $B^-_c$ mesons should be produced for 25 pb$^{-1}$ integrated luminosities. The corresponding numbers for SSC
and LHC with $p_T(B_c) > 20$ GeV and $|y(B_c)| < 2.5$ are $7.1 \times 10^7$ and $2.3 \times 10^8$ with integrated luminosities of 10 and 100 fb$^{-1}$ respectively. These $B_c$ mesons can be detected via the decays of the form $J/\psi + X$, and in particular, via

$$B_c^\pm \rightarrow \psi \ell^\pm \nu_\ell \quad \text{and} \quad B_c^\pm \rightarrow \psi \pi^\pm$$

(9)

with $\psi \rightarrow \ell^+ \ell^-$. The first one has a distinct signature of three charged leptons coming off from the same secondary vertex and has a combined branching ratio of about 1%. The second decay channel has a smaller branching ratio of order 0.2-0.4%, but it has the advantage that the $B_c$ can be fully reconstructed. The production rate of $B_c$ mesons given above is large enough that it should be possible to use these decay modes to identify the $B_c$ mesons at the Tevatron, and to study its properties in details at the SSC and LHC.

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TABLES

**TABLE I.** Table showing the number of $B_c$ mesons that can be produced at Tevatron, SSC, and LHC, including the contributions from 1$S$ and 2$S$ states, and under different $p_T^\text{min}(B_c)$ cuts. Another acceptance cut is $|y(B_c)| < 1 (2.5)$ at Tevatron (SSC/LHC). The integrated luminosities are 25 pb$^{-1}$, 10 fb$^{-1}$ and 100 fb$^{-1}$ for the Tevatron, SSC, and LHC respectively.

| $p_T^\text{min}(B_c)$ (GeV) | Tevatron | SSC | LHC |
|-----------------------------|----------|-----|-----|
| 10                          | 16000    | –   | –   |
| 15                          | 4400     | –   | –   |
| 20                          | 1600     | 7.1×10^7 | 2.3×10^8 |
| 30                          | 300      | 2.3×10^7 | 6.6×10^7 |
| 40                          | 82       | 9.8×10^6 | 2.5×10^7 |
| 50                          | 28       | 4.9×10^6 | 1.2×10^7 |
FIGURES

FIG. 1. The dependence of the differential cross section $d\sigma/dp_T(B_c)$ on the transverse momentum $p_T(B_c)$ for the ground state $B_c(1^1S_0)$ and the first excited state $B_c^*(1^3S_1)$ at the Tevatron.

FIG. 2. The dependence of the differential cross section $d\sigma/dp_T(B_c)$ on the transverse momentum $p_T(B_c)$ for the ground state $B_c(1^1S_0)$ and the first excited state $B_c^*(1^3S_1)$ at the SSC and LHC.

FIG. 3. The integrated cross sections $\sigma(p_T(B_c) > p_T^{\text{min}}(B_c))$ for the ground state $B_c(1^1S_0)$ (solid) and the first excited state $B_c^*(1^3S_1)$ (dash) versus $p_T^{\text{min}}(B_c)$ at the Tevatron, SSC, and LHC.