Air-fluidized balls in a background of smaller beads

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Abstract. We report on quasi-two-dimensional granular systems in which either one or two large balls are fluidized by an upflow of air in the presence of a background of several hundred smaller beads. A single large ball is observed to propel ballistically in nearly circular orbits, in direct contrast to the Brownian behavior of a large ball fluidized in the absence of this background. Further, the large ball motion satisfies a Langevin equation with an additional speed-dependent force acting in the direction of motion. This results in a non-zero average speed of the large ball that is an order of magnitude faster than the root-mean-square speed of the background balls. Two large balls fluidized in the absence of the small-bead background experience a repulsive force depending only on the separation of the two balls. With the background beads present, in contrast, the ball–ball interaction becomes velocity-dependent and attractive. The attraction is long-ranged and inconsistent with a depletion model; instead, it is mediated by local fluctuations in the density of the background beads which depends on the large balls’ motion.

Keywords: granular matter, disordered systems (experiment), stochastic processes (experiment)

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1. Introduction

A major challenge of modern physics is to understand the behavior of non-thermal systems [1, 2], such as granular media [3]–[5]. Just as for their thermal counterparts, these systems can display well-defined statistical distributions that encapsulate their behavior. For thermal systems, the standard theory of statistical mechanics can be used to predict all such distributions from equations of motion based on interaction forces plus stochastic forces set by temperature [6]. For non-thermal systems, in contrast, there is no such general approach. But in some special cases, experiments have found thermal-like behavior in the shape of distributions and in the agreement of distinctly defined effective temperatures [7]–[17]. Unfortunately there is no general framework for determining when such a thermal analogy ought to hold. To make progress, experimentally, it is sensible to compile further examples of different kinds of driven systems where microscopic statistical distributions may be measured.

Here we report on measurements of a system consisting of a monolayer of hundreds of small beads, together with one or two large balls, fluidized in a steady upflow of air. These beads and balls all roll on a horizontal sieve without slipping and experience in-plane forces from collisions with one another and from the air that flows up through them at high Reynolds number. Previous experiments with this apparatus have shown that a single ball [13], as well as dense collections of many beads [16], all behave thermally. However, for two spheres the thermal analogy is progressively broken by increasing the size disparity [15]. Here, for one large ball in a background of small beads, it is therefore unclear in advance whether or not to expect thermal behavior. Furthermore, for two large balls in a background of small beads, it is unclear in advance whether to expect the interaction between two large balls to be repulsive due to turbulence, as in [15], or to be attractive due to a depletion-like entropic force mediated by the thermal background.
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Figure 1. (a) A large sphere, radius \(a = 1.9\) cm and mass 2.7 g, fluidized in the presence of a bidisperse background of smaller spheres—radii 0.477 cm and 0.397 cm and masses 0.21 g and 0.165 g, respectively. The diameter ratio of the large ball to the background beads is 4.4. The radius of the system is \(R = 14.3\) cm. The vectors define position \(\vec{r}\), velocity \(\vec{v}\), as well as the polar angle \(\theta\) as measured from the horizontal axis. (b) A 1 min long trace of the position of the large ball shown in (a). To emphasize the observed circular motion, the trace is colored red for clockwise motion and blue for counterclockwise.

of small beads. The latter possibility of a non-equilibrium depletion force was found in [18] for a system of large balls in a background of small beads, all subject to vertical vibration. Our motivation is both to explore these specific issues, as well as to provide a well-documented experimental system to help motivate and test future statistical theories for non-thermal systems.

Our approach is based on high speed digital video, to track the sphere positions versus time. We begin with the behavior of the one-ball system in section 3. The large ball is observed to propel ballistically through the background medium, creating compressed and rarefied regions in the background beads. We quantify these local fluctuations in the background density in section 3.1. We then characterize the large ball behavior by measuring statistical distributions, calculating the ball dynamics and identifying its equation of motion. We find that the background generates a novel speed-dependent force on the large ball which accelerates it forward, causing it to move much faster than the small beads. In section 4, we analyze the two-ball system and calculate the same time-independent statistical distributions and dynamics for the two large balls. Lastly, in section 5, we deduce the interaction between the two simultaneously fluidized large balls, which we show to be long-ranged and attractive. The magnitude and the range are both larger than for a depletion force.

2. Experimental details

The principal system that we investigate, shown in figure 1(a), involves a monolayer of bidisperse plastic spheres of radii 0.477 cm and 0.397 cm and with masses 0.21 g and 0.165 g, respectively. This monolayer constitutes the background and occupies an area fraction of 55% in the absence of larger fluidized balls. The larger balls fluidized in the presence of this background are ping-pong balls, radius \(a = 1.9\) cm and mass \(m = 2.7\) g.

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that have been spray-painted silver to aid in visualization. Note that the diameter ratio of the large ball to the average diameter of the background beads is 4.4. Other size ratios and airspeeds were also employed, and the phenomena we describe below were found not to be particularly sensitive to these details.

The full system of background beads and one or two large balls is fluidized by an upflow of air. The superficial air speed is $300 \text{ cm s}^{-1}$, made spatially and temporally homogeneous to within $\pm 10 \text{ cm s}^{-1}$ and 0.5 s, as verified by a hot-wire anemometer. The Reynolds number based on ball size is of the order of $10^4$, such that the motion of the balls is driven stochastically by the shedding of turbulent wakes. Note that the airflow is below the terminal free-fall speed of the balls, which ranges from 700–800 cm s$^{-1}$ according to $mg = 0.43\left(\rho_{\text{air}}/2\right)v^2\pi r^2$; therefore, all balls maintain contact with the sieve and move by rolling without slipping. As such, we define an effective mass $m_{\text{eff}} = m + I/r^2$, where $I$ is the moment of inertia.

The apparatus, fluidization method and lighting set-up are described in [19]. The apparatus is a rectangular windbox, $1.5 \times 1.5 \times 4 \text{ ft}^3$, positioned upright, with two nearly cubical chambers separated by a perforated metal sheet. A blower attached to the windbox base by a flexible cloth sleeve provides vertical airflow perpendicular to a circular brass testing sieve with mesh size $150 \mu\text{m}$ and radius $15.3 \text{ cm}$ that rests horizontally on top. To prevent background spheres from getting caught in a small groove at the inner edge of the sieve, we place a $0.953 \text{ cm}$ diameter Norprene tube around its inside edge. Thus, the system has an actual radius, $R = 14.3 \text{ cm}$.

The particles are illuminated from above by six $100 \text{ W}$ incandescent bulbs arranged in a $0.3 \text{ m}$ diameter ring positioned $1 \text{ m}$ above the sieve. A digital CCD camera placed at the center of this ring captures the raw video data, typically for 4 h at a time at 120 frames s$^{-1}$. We threshold these long videos to binary as they save to buffer so that only the highly reflective large ball is seen. Post-processing of the video data is accomplished using LabView, with custom particle-tracking programs. From the center-of-mass position obtained from the video data, we determine velocity and acceleration by fitting the position data to a third-order polynomial over a window of $\pm 4$ frames. The window is Gaussian-weighted, vanishing at the window edge, to ensure the continuity of the derivatives. Scatter around the fits gives us an estimate of the error in the position data of $\pm 18 \mu\text{m}$.

We use the above process to track and characterize the background beads. The 55% density of the background beads is large enough that they were uniformly distributed across the system. It is also small enough that the background bead dynamics are described by only a single ballistic-to-diffusive timescale, with no caging effects. In particular, the relaxation time is $\tau = 1.7 \text{ s}$ for the background beads to achieve an rms displacement comparable to their size. From the short-time mean square displacement, we obtain the rms speed of the background beads as $v_{\text{rms}} = 1.18 \text{ cm s}^{-1}$. This gives a granular kinetic effective temperature of $kT = m_{\text{eff}}\langle v^2 \rangle/2 = 0.16 \text{ erg}$. For these conditions, the equation of state for air-fluidized beads [20] gives the pressure as $P = 0.25 \text{ kg s}^{-2}$.

3. Single-ball behavior

In this section we consider the behavior of a single large ball in a background of smaller beads. A single ping-pong ball fluidized in isolation behaves like a Brownian particle,
obeying a thermal analogy [19]. When fluidized in the presence of a background of smaller beads, the behavior is strikingly different. As soon as the large ball is placed in the background and fluidized, it begins to propel itself ballistically around the system. Typically, it will propel in a straight line until it reaches the boundary or the background balls jam in front of the large ball, at which point it is forced to change direction. When this occurs at the boundary, the particle begins to propel along the boundary edge, resulting in circular motion at some stable orbit position. We emphasize the observed circular behavior in figure 1(b). Here, we show a 1 min time trace of the large-ball center-of-mass position. Whenever the ball is moving clockwise, we color the trace red; counterclockwise motion is blue. Several circular orbits of both types can be seen in the figure.

This ballistic propulsion behavior is general, but does depend on the size ratio of the large ball to the background beads. For small background balls, the particle is observed to almost always orbit in a circle. The background does not have enough ‘stopping power’, so to speak, to change the direction of the large ball. As the diameter of the background beads increases, the background balls are large enough that they are able to ‘kick’ the large ball in other directions. For the case of figure 1(b), the diameter ratio is 4.3 and we see that the orbits are not well-developed, with the large ball traversing the center of the system often. For smaller background balls, the time traces show very circular orbits at a radial position well away from both the center and the outer boundary. For background beads bigger than about half the diameter of the large balls, the self-propelling effect vanishes. The effect is further general in that it is insensitive to airspeed, as long as it is sufficient for fluidization; it is also insensitive to the density of the background beads, but decreases as jamming is approached.

The physical mechanism by which the large ball plows through the background is likely similar to that for a ‘chasing’ phenomenon discussed in [15] for two air-fluidized spheres of different sizes. Ordinarily two balls repel with a force mediated by wake–wake interactions, just as the wake from a single sphere causes repulsion from the boundary [19]. However, if the size disparity is large, and if the two balls happen to come into contact, then the contact persists and the two-ball unit accelerates in a straight line as though the large ball is pushing the small ball; the unit can develop a speed much larger than the rms speed of either ball. This was said to be due to an asymmetry that develops in the shedding of the turbulent wakes due to the asymmetry of the two-ball unit [15]; however, we could not visualize the airflow to test this idea. As quantified by the equal-time velocity cross-correlation [15], the two-ball chasing effect becomes noticeable for size ratios greater than about 1.5, peaks around 3 and falls off only slowly for even larger ratios.

For the case here of a large ball in a background of beads with four times smaller diameter, the chasing effect ought to occur. If the background is uniform and the large ball fluctuates in some direction into contact with a neighboring small bead, then a force will develop to push the large ball in the direction of the original fluctuation. Thus the fluctuation is not healed but rather is amplified. This instability grows until the asymmetric wake-shedding force on the large sphere is matched by its speed times mobility for motion through the finite density of small beads. Also it results naturally in a compression of small beads in the direction of motion and a lacuna behind, as seen in figure 1(a). Besides physically explaining the development of a constant speed relative to compression/rarefaction of the background, this also suggests that the average size
Figure 2. Time-averaged density of the background for three different ranges of the large ball speed, as specified below each image. The large ball (outlined in white), radius 1.9 cm, is always moving horizontally to the right. The density scale linearly interpolates between 0% area fraction (black) and a jammed region at 84% (white).

of the compression/rarefaction depends on the average speed of the large ball and that fluctuations in packing density can cause a stochastic force to alter the direction of the large ball’s velocity. Note also that the pressure drop for the upflow of air is strongly dominated by the sieve, essentially independent of the bead packing fraction; thus, there is no alteration of the local upward flux of air by this phenomenon.

To quantify these observations, we begin by analyzing the effect of the large ball on the distribution of background balls; we then calculate time-independent probability distributions and the dynamics of the large ball.

3.1. Background beads

Rather than track the individual background beads, we quantify the local density fluctuations graphically. We first track the large ball position and then obtain its velocity in each frame. Then, each frame in the video was rotated and had its origin shifted such that the large ball was centered and moving to the right along the horizontal axis. Since we have suggested that the local density is dependent on the large ball speed, we bin these processed images according to the speed of the large ball and then average over all images within each speed bin. The results, for three different ranges of large ball speeds, are shown in figure 2. The images have been color-coded by linearly interpolating the grayscale values between 0% area fraction, shown as black, and jammed particles at 84%, shown as white.

The effect is very dramatic. In the left image, the large ball—outlined by a white circle—is moving very slowly and the compressed and dilute regions are relatively small. As we increase speed from left to right, we observe that both the compressed region in front of the ball and the dilute wake behind the ball become larger. The slight asymmetry in the image is due to the ball circulating in the counterclockwise direction more often than clockwise for the video analyzed.
To quantify the difference in density between the compressed and rarefied regions, we obtain the average packing fraction within hemispherical annular areas both in front of, $\phi_{\text{ahead}}$, and behind, $\phi_{\text{behind}}$, the large ball. We then plot the difference between these packing fractions as a function of the large ball speed, as shown in figure 3. The results show that the extent of the compression and rarefaction, and thus interaction of the large ball with the background, depends on the speed $v$ of the large ball. The effect vanishes in the limit of zero speed and saturates at $\phi_{\text{ahead}} - \phi_{\text{behind}} \approx 0.6$ for $v > 10$ cm s$^{-1}$.

3.2. Statics

Because our system reaches a steady state very quickly, the simplest way in which to characterize it is by compiling statistics of time-independent quantities. Intuitively, we thought that the presence of the turbulent background would serve to overwhelm repulsive interactions of the large ball with the bounding walls. In other words, the large ball would move in a random walk, sampling all of the system space equally. In this case, the radial probability distribution $P(r)/r$, where we divide by $r$ to account for the fact that there are more points in the phase space further from the origin, would be constant from the origin $r = 0$ to the effective reduced radius of the system $R' = R - a = 12.4$ cm, at which point the distribution would discontinuously vanish.

The observed $P(r)/r$ for our system is shown in figure 4(a). In no region is the distribution constant, suggesting that the background may actually enhance the interaction with the boundary. There is a large peak at an intermediate radius as the large ball begins to detect the wall, showing that the large ball is repelled from the wall. This peak radius roughly corresponds to the radial position at which the particle prefers to orbit circularly.

Similarly, for a thermal particle obeying equipartition of energy, we would expect the velocity distribution $P(v)$ to be Gaussian, symmetric about zero, and that each velocity component would be equivalent. The compiled $P(v)$ for our system is shown in figure 4(b). Here, we have decomposed the velocity into radial and polar components to emphasize the circular motion observed for this system. The radial velocity distribution is roughly Gaussian, although having shorter tails. However, the polar velocity exhibits two peaks, near $\pm 7$ cm s$^{-1}$, consistent with approximately constant speed orbiting behavior.
peaks are symmetric about the origin, showing that the particle does not preferentially orbit in any particular direction. According to the physical mechanism discussed above, the characteristic orbiting speed would be set by force balance as the ratio of asymmetric wake propulsion force to large ball mobility.

3.3. Dynamics

We further characterize the large ball behavior by quantifying its dynamics. As discussed above, the large ball propels itself ballistically around the system. This is directly observable in the mean square displacement (MSD) for the large ball, shown in figure 5(a). Here, at short times, we see ballistic motion ($\propto v_{\text{rms}}^2 \tau^2$) characterized by a root-mean-square speed of $v_{\text{rms}} \approx 4 \text{ cm s}^{-1}$. By comparison, recall that tracking of individual background beads yields a root-mean-square speed of $1.18 \text{ cm s}^{-1}$ that is about 3–4 times smaller. The system is too small for us to see any indication of a crossover to diffusive behavior. The MSD saturates within about 2 s at the optimum orbital radius, followed by oscillations about this value.

To get a sense of the characteristic timescales in our system, we consider the velocity and acceleration autocorrelation functions as shown in figures 5(b) and (c). The velocity autocorrelation has a long plateau that rolls off after approximately 1 s and then oscillates as the function decays to zero. This timescale corresponds roughly to the relaxation time of the large ball $a_{\text{rms}}/v_{\text{rms}} \approx 0.95$ s. The strong oscillations are indicative of the circular motion exhibited by the ball. The acceleration autocorrelation decorrelates much more quickly than the velocity does, having one small oscillation at 0.08 s, before decaying to zero at approximately 0.3 s. This separation of timescales is further emphasized by including an overlay of the mean square acceleration change on the mean square displacement, figure 5(a).
Figure 5. (a) Mean square displacement (solid) and mean square acceleration fluctuations (red dashed), (b) velocity autocorrelation and (c) acceleration autocorrelation for a single large sphere fluidized in a background of smaller beads. The effective radius of the system is the difference \( R' = R - a \) between the system and the large ball radii.

3.4. Equation of motion

In this section, we analyze the large ball motion in terms of a model of the forces acting on a single large ball fluidized in the presence of a background of smaller beads. A simple conceivable equation of motion for the large ball is

\[
m_{\text{eff}} \ddot{\mathbf{r}} = C(r) \hat{\mathbf{r}} + D(v) \hat{\mathbf{v}} + \zeta(t),
\]

where the forces \( D(v) \) and \( C(r) \) are functions of the large ball’s speed and radial position, respectively, and \( \zeta(t) \) is a stochastic force satisfying \( \langle \zeta(t) \rangle = 0 \). The position-dependent force \( C(r) \hat{\mathbf{r}} \) must be radially symmetric because of the symmetry of the system. We suppose that, as for the case of a single large ball without the presence of the background beads, the wakes shed by the large ball interact with the boundary and give rise to a speed-independent attraction towards the center [13, 19]. As for the velocity-dependent force, \( D(v) \hat{\mathbf{v}} \), we suppose that there is a drag linearly proportional to speed as well as a term to account for the propulsion effect; both these mechanisms ought to be independent of position. Thus, we write \( D(v) \hat{\mathbf{v}} = -\gamma \hat{\mathbf{v}} + \beta(v) \hat{\mathbf{v}} \), where \( -\gamma \hat{\mathbf{v}} \) is the drag term and \( \beta(v) \hat{\mathbf{v}} \) is the force caused by the background that we assume to be in the direction of the large ball velocity \( \hat{\mathbf{v}} \).

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Figure 6. Forces acting on a large ball in a background of smaller beads: (a) central force $C(r)$ as given by (2) with a fit to $C(r) \propto r^3/(r - R')^{0.65}$ (solid) and the radial force obtained via thermal approximation according to $C_{th}(r) = -(d/dr)kT_{eff} \log(-P(r)/r)$ (red dashed) and (b) speed-dependent force $D(v)$ as given by (3) with an empirical fit to $D(v) = D_o\{2/[1+\exp(-v/v_{\text{switch}})]-1\} - \gamma v$ (solid). The effective system boundary is marked at $r = R - a$ (vertical dotted blue line).

We will now examine each term of the equation using a dynamical approach. We can isolate the central force by taking the cross product of (1) with $\hat{v}$. Rearranging:

$$C(r) - \left[ \frac{\vec{\xi} \times \hat{v}}{\vec{r} \times \hat{v}} \right] = \left( \frac{\vec{a} \times \hat{v}}{\vec{r} \times \hat{v}} \right) m_{eff}. \quad (2)$$

Similarly, we can isolate the speed-dependent force by taking the cross product of (1) with $\hat{r}$, giving

$$D(v) - \left[ \frac{\vec{\zeta} \times \hat{r}}{\vec{v} \times \hat{r}} \right] = \left( \frac{\vec{a} \times \hat{r}}{\vec{v} \times \hat{r}} \right) m_{eff}. \quad (3)$$

If we assume the stochastic force is independent of position and velocity, the time average of the square brackets in both (2) and (3) must be zero. Thus, we can readily obtain $C(r)$, as shown in figure 6(a). We see that there is essentially zero force at small radii. At increasing $r$, as the ball approaches the bounding walls, the force becomes repulsive. The solid line in the figure is a fit to $C(r) \propto r^3/(r - R')^{0.65}$. This is in contrast to the linear dependence of this term observed for a large sphere fluidized in an empty sieve [13, 19]. As a further comparison, the dashed curve is the radial force obtained if we had approximated the system as thermal and calculated the radial force using the data for $P(r)$, according to $C(r) = -(d/dr)kT_{eff} \log(-P(r)/r)$. The discrepancy between the two methods for deducing $C(r)$ shows that the system is not behaving thermally.

We can also readily obtain $D(v)$, as shown in figure 6(b). For speeds between 0 cm s$^{-1}$ and roughly 10 cm s$^{-1}$, the speed-dependent force is positive, causing the large ball to speed up. For speeds larger than about 10 cm s$^{-1}$, the force is negative, thus slowing the particle. This is consistent with our earlier observations of the compression/rarefaction that accompanies the large ball motion. Once the ball receives a kick and begins to move,
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Figure 7. (a) Probability distribution of the stochastic force parallel to (dashed) and perpendicular to (solid) the large ball velocity for \( v = 4.5 \) cm s\(^{-1}\). The dotted curve is a Gaussian fit to \( P(\zeta_{\perp}) \). (b) Standard deviation of the stochastic force distributions as a function of large ball speed.

it compresses a region of the background in front of it. This creates a feedback mechanism that causes the particle to continue gaining speed. However, if the large ball moves too quickly, the compressed region becomes jammed and is able to slow the large ball down. Furthermore the polar velocity distributions in figure 4(b) suggest that there is a stable polar speed of approximately 10 cm s\(^{-1}\) at which the particle orbits the system in circular motion. This is consistent with the speed at which \( D(v) \) crosses the horizontal axis—the stable speed at which there is no speed-dependent force. The solid curve shows a fit to the empirical form
\[
D(v) = D_o \left\{ \frac{2}{1 + \exp \left( -v/v_{\text{switch}} \right)} - 1 \right\} - \gamma v,
\]
where \( v_{\text{switch}} = 0.5 \) cm s\(^{-1}\), \( \gamma = 1.9 \times 10^{-5} \) N s cm\(^{-1}\) and \( D_o = 0.2 \) mN.

Lastly, we examine the stochastic force \( \vec{\xi} \). Our analysis thus far has assumed that the stochastic force is independent of the large ball position and velocity. Since the behavior of the background parallel to the large ball motion is quite different than perpendicular, we will analyze the stochastic force with respect to the direction of the large ball’s motion. To do so, we first subtract the parameterized forces from (1) and take the dot and cross product with \( \hat{v} \) to isolate the stochastic force parallel \( \zeta_{\parallel} \) and perpendicular \( \zeta_{\perp} \) to the large ball velocity:
\[
\zeta_{\parallel}(t) \equiv \vec{\zeta} \cdot \hat{v} = [m_{\text{eff}} \vec{a} - C(r) \vec{r}] \cdot \hat{v} - D(v) \tag{4}
\]
\[
\zeta_{\perp}(t) \equiv \vec{\zeta} \times \hat{v} = [m_{\text{eff}} \vec{a} - C(r) \vec{r}] \times \hat{v}. \tag{5}
\]
Since the speed directly enters these equations, we show the stochastic force probability functions in figure 7(a) at the particular speed \( v = 4.5 \) cm s\(^{-1}\). \( P(\zeta_{\perp}) \), shown as the solid curve, is nearly Gaussian. In contrast, the distribution of the parallel stochastic force is significantly skewed, having much larger probability to provide a kick in the direction of motion. This shows that our assumption that the stochastic force is independent of the speed was incorrect. The shape of these distributions does not significantly change depending on the magnitude of the large ball speed. This is verified in figure 7(b) where we plot the standard deviations \( \sigma \) of the probability distributions \( P(\zeta_{\parallel}(t)) \) and \( P(\zeta_{\perp}(t)) \) as a function of the large ball speed. Both distributions remain roughly the same width until 8 cm s\(^{-1}\), at which point the distributions narrow. Interestingly, this directional...
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Figure 8. Probability distributions for two large balls (solid line, subscript ‘2’), compared with that for one lone large ball (dashed line, subscript ‘1’), each in a background of smaller balls. (a) Radial distribution $P(r)/r$, (b) radial velocity distribution $P(v_r)$ and (c) theta velocity distribution $P(v_\theta)$. The similarity of the distributions with one and two large balls suggests an additional large ball does not drastically change the first ball’s behavior.

dependence of the stochastic force has been seen before in the behavior of fluidized rods [21]. In that experiment, it was observed that the rods self-propel, much like the large ball is observed to do in this experiment.

4. Two-balls behavior

Having documented the behavior of a single large ball in a background of smaller balls, we now add a second large ball to the system. When fluidized in an empty sieve, the force between two balls is characterized by hard-core repulsion and a persistent repulsive force over all separation distances [19]. With the inclusion of background beads, we might expect a short-range depletion interaction that would attract the two balls together as well as longer-range forces mediated by the background. When we place the two large balls in the system, they both propel ballistically and orbit about the system. Often, the two balls will become trapped in one another’s wake and travel as a pair. However, they are rarely observed to come into contact; when cooperatively traveling, the two balls are always some small distance apart.

We begin by characterizing the behavior of the individual balls in the same way that we did for a single fluidized ball. The radial position, radial velocity and polar velocity probability distributions are shown in figures 8(a)–(c). The distributions for the single large ball are shown as dashed curves to highlight differences. The distributions are qualitatively unchanged, suggesting the behavior of one large ball in a background of smaller balls is not greatly affected by another large ball.

Next we compile the probability distribution for separation distance, $P(|\vec{r}_1 - \vec{r}_2|) = P(\rho)$ versus $\rho$. This is shown as the solid line in figure 9. The distribution is zero for separations less than a particle diameter of 3.8 cm, as expected for hard spheres. At small separations, the probability reaches a maximum, showing that the balls strongly prefer to stay near one another. This immediately suggests that the nature of the interaction
between the two balls has dramatically changed and become attractive. However, we note that depletion interactions act on a length scale of the size of the small balls \( \sim 0.87 \text{ cm} \). The broadness of the peak indicates that the interaction is over a length scale of approximately 5 cm and cannot be attributed to depletion interactions. In fact, the absence of a strong peak within one small-ball diameter from the large-ball diameter suggests that depletion interactions play no significant role here.

To stress the role of the background beads, we generate the distribution \( P_{\text{HC}}(\rho) \) expected for hard-core repulsion alone, shown as the dashed curve, using a Monte Carlo simulation in which pairs of positions were generated randomly for two fictitious balls by randomly choosing a phase and radial position weighted by \( P_1(r) \) data for each. The comparison shows an enhancement of close separations at the expense of large separations, which must be due to an effective long-range attraction. This attraction vanishes, however, if we anchor one of the large balls to the center and allow the other to move freely: The resulting separation probability distribution is essentially identical to the one-ball radial probability distribution \( P_1(r) \). Therefore, the attractive force between two free balls depends on their motion through the background. It may be linked to fluctuations in the local density induced by the ‘chasing effect’, or it may be due to correlations between position and velocity inherent in orbital motion. For example, if the two large balls are traveling in the same direction, then the one behind may experience a greater mobility—and hence acquire a greater speed—due to the lacuna of small beads behind the leading large ball.

5. Ball–ball interaction

In this section we seek to quantify the interaction force between two balls in a background of smaller beads. We assume the same equation of motion as for one large ball, but add interaction forces:

\[
\begin{align*}
    m_{\text{eff}} \ddot{r}_1 &= C(r_1) \dot{r}_1 + D(v_1) \dot{v}_1 + \zeta_1(t) + \vec{F}_{12}, \\
    m_{\text{eff}} \ddot{r}_2 &= C(r_2) \dot{r}_2 + D(v_2) \dot{v}_2 + \zeta_2(t) + \vec{F}_{21}.
\end{align*}
\]

That \( C(r) \) and \( D(v) \) should be the same as for one ball is supported to the extent of the agreement of the distributions in figure 8. Assuming \( \vec{F}_{12} \) depends only on separation \( \rho \),

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Figure 10. Interaction potential for two large balls in a background of smaller beads, calculated from the measured separation distribution \( P(\rho) \) and assuming thermal equilibrium. The solid line is a fit to \( V_s^{\text{fit}}(\rho) = \alpha [(\rho - 2a)/a]^\alpha + V_0 \exp[-(\rho - 2a)/\lambda] \). The dashed line shows the depletion potential \( V_{\text{dep}}(\rho) \), multiplied by a factor of 50.

independent of ball velocities, we define an interaction potential \( V_s(\rho) \) according to

\[
\tilde{F}_{12}(\rho) = [-dV_s(\rho)/d\rho] \hat{\rho}_{12} \quad \text{where} \quad \hat{\rho}_{12} = (\vec{r}_1 - \vec{r}_2)/|\vec{r}_1 - \vec{r}_2|.
\]  

As demonstrated below, there is actually a non-negligible velocity dependence, as suggested already by the anchoring experiment above; however, there is greater dependence on separation than on velocity.

To deduce the interaction potential \( V_s(\rho) \) from separation distribution data, \( P(\rho) \), we must assume the system behaves thermally. This of course is contradicted by the non-Maxwellian velocity distributions and hence introduces an unknown systematic error. However, it does allow us to systematically test for velocity dependence of the interaction potential and to compare at least qualitatively with the thermal depletion interaction. Furthermore, it should be cautioned that, since turbulence plays a role, it is unlikely that interactions between more than two large balls would be pairwise additive. Proceeding, with all these provisos in mind, we write

\[
P(\vec{r}_1, \vec{r}_2) \propto \exp\left\{ -\left[ V_c(r_1) + V_c(r_2) + V_s(\rho) \right]/T_{\text{eff}} \right\}
\]  

where the central potential for one large ball is given by \( V_c(r) = -T_{\text{eff}} \ln(P_1(r))/r \). Rearranging (9) and integrating over the remaining variables, \( V_s(\rho) \) can be obtained from the measured \( P(\rho) \):

\[
V_s(\rho) = -T_{\text{eff}} \ln[P(\rho)/g(\rho)] + \text{const.}
\]  

where

\[
g(\rho) = \rho \int_0^{R-a} dr_1 \int_0^{2\pi} d\varphi_1 r_1 \exp\left[ -\frac{(V_c(r_1) + V_c(r_2))}{T_{\text{eff}}} \right].
\]  

The resulting interaction potential is shown as the points in figure 10. As expected due to hard-core repulsion, the potential diverges at one-ball diameter. The data has been fitted to the empirical form \( V_s^{\text{fit}}(\rho) = \alpha [(\rho - 2a)/a]^\alpha + V_0 \exp[-(\rho - 2a)/\lambda] \).

It is straightforward to show that the inferred potential is inconsistent with a depletion attraction. A depletion interaction between large balls is expected when they are sufficiently close that small balls cannot access the region between them. The

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Air-fluidized balls in a background of smaller beads

Figure 11. Interaction potentials for two large balls in a background of smaller beads for different relative motion of the large balls, calculated from the measured separation distribution $P(\rho)$ and assuming thermal equilibrium: moving in the same direction (→→, red squares), moving apart (←→, blue triangles) and moving closer together (→←, green diamonds). The solid curves are best-fit lines to the empirical form $V_\text{fit}(\rho) = V_\alpha [\rho - 2a]^{\alpha} + V_0 \exp[-(\rho - 2a)/\lambda].$

background beads impart a constant pressure $p$ on all accessible surfaces. This pressure $p$ can be estimated by assuming that the background beads obey a reduced-volume ideal gas equation of state $p = N_b T_b / A_{\text{reduced}}$, where $T_b = m_{b,\text{eff}} \langle \nu_b^2 \rangle$ and the reduced area $A_{\text{reduced}}$ is the area available to the balls’ centers, minus the minimum area they could occupy. By integrating over the exposed circumference of one of the large balls, we can derive the depletion force $F_{\text{dep}}(\rho) \hat{\rho}$ on ball 1 due to ball 2. This force will be non-zero and finite only for the region $2a < \rho < 2(a + b)$, where $a$ is the large ball radius and $b$ is the average background bead radius. In this region, the force is given as $F_{\text{dep}}(\rho) = -2ap \sqrt{1 - (\rho / (2a + 2b))^2}$. Then, we are able to obtain the interaction potential by integration:

$$V_{\text{dep}}(\rho) = -2ap \left\{ \frac{\rho}{2} \sqrt{1 - \left[ \frac{\rho}{2(a + b)} \right]^2} + (a + b) \sin^{-1} \left[ \frac{\rho}{2(a + b)} \right] \right\}. \quad (12)$$

This expression has been plotted alongside our data as the dashed curve in figure 10, multiplied by a factor of 50. Both the range and strength of a depletion attraction are much smaller than that obtained from the data. Even though the inferred potential is subject to unknown systematic errors, it still seems safe to rule out depletion.

Now let us reconsider the role of ball speed. Just as the force on a single ball is speed-dependent, it is likely that the ball–ball interaction is also dependent on how the balls are moving relative to one another. To examine this possibility, we first separated the video data into three scenarios of large ball motion:

(i) (→→): moving in the same direction (i.e. $\vec{v}_1 \cdot \vec{v}_2 > 0$)
(ii) (←→): moving apart (i.e. $\vec{v}_1 \cdot \vec{v}_2 < 0$ and $d\rho/dt > 0$)
(iii) (→←): moving closer together (i.e. $\vec{v}_1 \cdot \vec{v}_2 < 0$ and $d\rho/dt < 0$).

We then repeat the above analysis to determine the interaction potential for different types of relative motion. Our results are shown in figure 11. Note that the three potentials differ from each other, such that the strongest attraction occurs when the balls move in the same direction. This is direct evidence that the interaction force depends on the
velocity of the large balls as well as their separation. However, to the extent that each case displays short-range repulsion and longer-range attraction of the same general shape, the interaction depends more on separation than on velocity.

6. Conclusions

We have characterized the behavior of, and forces, acting on a large sphere fluidized in a bidisperse background of smaller beads. The large ball self-propels ballistically through the medium, strongly perturbing the local density of the background. The presence of the background not only modifies the central force felt by a fluidized ball moving in an empty sieve but also mediates a novel speed-dependent force. The speed-dependent force acts to keep the large ball propelling itself at a stable speed of the order of 10 cm s$^{-1}$. Furthermore, the stochastic force is directionally dependent and preferentially kicks the large ball in its direction of motion. When two large balls are fluidized simultaneously, there is a short-range repulsion mediated by the airflow, as in [19] but with a range that is cut off. Also there is a long-range attraction mediated by the background beads and dependent on the relative motion of the two balls. Further, we demonstrated that the attraction is not consistent with the depletion interaction.

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