Electrical Noise under the Fluctuation-Dissipation framework

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Abstract

Although the Fluctuation-Dissipation (F-D) framework for noise processes was published in 1951 by Callen & Welton, today’s model for electrical noise does not fit in it. This model overseeing fluctuations of electrical energy in resistors becomes a limited interpretation of the work published in 1928 by Nyquist. This is why we propose a new, Quantum-compliant model for electrical noise that not only fits in the F-D framework, but also shows that the resistance noise known as 1/f excess noise in Solid-State devices and the phase noise found in electronic oscillators are some of its effects.
1. Introduction

In 1928, H. Nyquist published a work on the electromotive force due to thermal agitation of electric charge in conductors [1] to explain the electrical noise found by J. B. Johnson [2]. These works form the basis of today’s model for the electrical noise in resistors, hereafter referred to as Traditional Model (TM). This TM uses the circuit of Figure 1 to handle the noise of a resistor of resistance $R$ at temperature $T$. In this circuit, a resistance $R$ or conductance $G = 1/R$ Ω$^{-1}$ is shunted by a generator of spectral density $S_I = 4kT G$ $A^2$/Hz (Nyquist noise) or equivalently, a generator of density $S_V = 4kT R$ $V^2$/Hz (Johnson noise) appears in series with $R$. The values of $R$ and $G$ are the Resistance and Conductance measured at low-frequencies in the two-terminal device called resistor.

In this TM resistors are considered as pure resistances generating electrical noise whereas capacitors and inductors, which can accumulate electrical noise generated by other sources [3] do not generate it. The linear dependence of $S_I$ on $G$ and $S_V$ on $R$ likely suggests that $R$ (or $G$) is the source of electrical noise. However, readers used to the generalized Ohm’s Law should realize that this suspicious or striking dependence of the noise generator on the $R$ it has to drive is to sustain in time an average fluctuation of $kT/2$ J in the electrical energy of the Susceptance $B(2\pi f)$ of the resistor, which is the Imaginary part of its Admittance $Y(j2\pi f)$ whose Real part is Conductance $G(2\pi f)$.

2. A Fluctuation-Dissipation model for electrical noise

The last sentence of previous Section led us to consider the electrical noise of a resistor as the measurable effect of thermal Fluctuations of the electrical energy it stores in its capacitance $C$ between terminals [4]. Hence, thermal Equipartition can be applied to this $C$ or to the degree of freedom linked with the energy of the electric field between terminals of the resistor. This way, our model incorporates fluctuations of energy that allow it to fit in the Fluctuation-Dissipation (F-D) framework proposed by H. B. Callen.
and T. A. Welton in their work entitled *Irreversibility and generalized noise* [5]. In our model, electrical noise comes from discrete fluctuations of electrical energy taking place in the capacitance \( C \) of resistors and capacitors. Each of these fluctuations that we called Thermal Actions (TA) initiates a subsequent Device Reaction (DR) to *dissipate* or to remove the energy unbalance set by the preceding TA [4].

Let us note the *Cause-Effect order* underlying our model (e.g. TA first, DR next) that tracks the (Fluctuation first-Dissipation next) dynamics of [5]. This ordering linked with the “*Irreversibility*” of the title of [5] has to do with the circuit of Figure 2 that our model uses in Thermal Equilibrium (TE). As this circuit shows, the Johnson noise of a resistor is born in its capacitance \( C \) between terminals and its \( R \) determines the spectral density of this noise in order to keep Equipartition in \( C \).

Note the parallelism between the *generalized* Ohm’s Law needed to handle the Admittance of Figure 2 and the *generalized* noise handled in [5]. We mean the *need for a Complex function* to properly handle pairs of orthogonal entities like the Displacement and Conduction currents in the Admittance of Figure 2 or like those Fluctuations and Dissipations of energy studied in [5], whose authors had to use a complex Impedance to particularize their model to electrical noise. Therefore, TM is a limited interpretation of a phenomenon that has a two-dimensional nature.

To describe the physical behaviour of a resistor we need a complex admittance because whenever a sinusoidal voltage \( v(t) \) exists between its two terminals, it *converts electrical energy* into heat while its *electrical energy in \( C \) fluctuates* with time. These two processes of energy fluctuation and energy conversion are linked with orthogonal currents in Figure 2, where \( R \) and \( C \) share the \( v(t) \) [4]. Fluctuations of energy linked with the voltage \( v(t) \) between terminals in a resistor mean that this device holds an energy linked with this voltage, thus bearing some *Susceptance* between them. In dual form: to
have Dissipation of electrical energy in a resistor, it must bear a non-null Conductance \( G = 1/R \) between terminals to convert this type of energy into another form like heat as in this case or like photons by the radiation resistance of a circuit. Appendix II of [4] shows how active and reactive powers are linked with dissipation of energy in \( R \) and fluctuation of energy in \( C \) in the circuit of Figure 2.

Although our two-dimensional (2-D) model for electrical noise hardly compares with the TM that only is a 1-D model, it is worth noting that both TM and ours give similar numbers, but not similar meanings in Thermal Equilibrium (TE), which is the case studied in [1]. Let us study the noise of a capacitor of capacitance \( C \) shunted by a resistance \( R \). Following [3] the noise generated by \( R \) at temperature \( T \) accumulates in \( C \) a mean square voltage \( \langle (v_n)^2 \rangle = kT/C \ V^2 \) that is the well-known \( kT/C \) noise of Charge Coupled Devices for example. Accordingly to [4], TE at some temperature \( T \) means that \( C \) and \( R \) that can exchange electrical energy through its parallel connection, also are in thermal contact with its environment to receive a series of TA whose exact nature will be shown in Section 3. This series of TA is an electrical power entering the resistor that is returned to this environment as heat power. This leads to an average fluctuation of \( kT/2 \ J \) in \( C \) (Equipartition) no matter the \( R \) shunting \( C \). This is the starting point of our new model (hereafter NM). Therefore:

\[
\frac{1}{2} C \times \langle v_n^2 \rangle = \frac{1}{2} kT \Rightarrow \langle v_n^2 \rangle = \frac{kT}{C} v^2
\]  

Equation (1) shows that electrical noise in resistors and capacitors has the same origin. The NM uses Equation (1) in TE and out of TE, it uses the shot noise created in \( C \) by TA. For resistors in TE this shot noise simply is their Nyquist noise [4]. When the system is out of TE the TM uses to fail whereas the NM still can give results due to its 2-D nature. To reinforce the role of \( C \), let us say that a voltage noise \( v(t) \) appearing on a circuit element like \( R \) that always absorbs electrical power, calls for a transducer in
parallel to generate and sustain in time this $v(t)$ feeding $R$ the active power it absorbs whenever an electrical voltage exists between its terminals. This transducer is no other than $C$, which replaces the transmission line used in [1] for example.

3. The Quantum nature of the new model for electrical noise

The first step towards our Quantum-compliant model was to link Susceptance with fluctuations of electrical energy in resistors. The energy states used in the Quantum treatment of noisy systems done in [5] will exist in the resistor if a non-null Susceptance is found between its terminals. A resistor will store electrical energy linked with its voltage between terminals if, and only if, it bears a reactive behaviour. We focus on electrical energy linked with the voltage $v(t)$ between terminals because electrical noise is measured by means of this $v(t)$ or as we will see in next Section: by the difference of electrical potentials that appear simultaneously at these two terminals.

The second step was to consider that the charge involved in the Displacement currents through $C$ is discrete and the packet of electrical charge that can be displaced between the plates of $C$ in a resistor is one electron of charge $-q$. This leads to define properly the Thermal Action (TA) in a resistor as: a displacement current carrying one electron between its terminals. This discrete current defines properly each passage of one electron between its terminals proposed in [4]. At the time of writing [6] (where we used a first version of this new model by assuming these displacements were very fast) these TA were hard to accept due to a prejudice we had about conduction current in Solid Matter as due to corpuscles of negative charge drifting under the action of an electric field as the drift model (DM) for electrical conduction in Solids considers.

If the DM was true, each electron involved in a TA would have to cross several mm of Solid Matter without the collisions this DM predicts. To solve this puzzling question we had to replace this corpuscular model by the one sketched in Section 3 of
[7] that leads to a true new view on Conduction current and Joule effect in Solids [8]. Although deeply connected with electrical noise, this view falls out of the scope of this paper and we will solve here this puzzling question by showing that $C$ is an easy path for the passage of single electrons between terminals of devices. This passage of an electron between plates of $C$ is undistinguishable from a fluctuation of the electric field between terminals of the resistor. This displacement of a charge $-q$ between terminals requires (and sets in $C$) this Fluctuation of electrical energy:

$$\Delta U = \frac{q^2}{2C} \text{ J}$$

(2)

For $C=1$ pF we have $\Delta U \approx 1.3 \times 10^{-26}$ Joules. The thermal energy at $T=300K$ is $kT \approx 4 \times 10^{-21}$ Joules. From $kT \approx 3 \times 10^5 \Delta U$ we conclude that the passage of a single electron between terminals of resistors separated several mm is a very likely event at room $T$. This justifies the random passages of single electrons between terminals we proposed to generate in $C$ the electrical noise of resistors [4]. The thermal contact of the resistor with its environment we met in Section 2 would be the set of electromagnetic interactions giving rise to this random series of TA in $C$ (50% in each sense on average) which sustains in time the noise we observe in devices like resistors and capacitors that share the admittance of Figure 2 [6, 9].

4. Especial Relativity and the new model for electrical noise

Let us note that each TA, no matter its sign, involves the passage of only one electron between terminals because two electrons passing “side by side” would lead to an unobserved packet of charge with twice the electronic charge. Thus, each TA will set in $C$ a voltage step of $\pm q/C$ volts as we considered in [4], but for this to be true the transit time of the electron in a TA should be very short (null if possible). Thus, the key question is: What is the transit time $\tau_c$ for the electron involved in a TA? This was an
unsolved question at the time of writing [6] that we will answer right now. This $\tau_c$ gives
the risetime of each noise pulse shown in Figure 3 whose exponential decay with
lifetime $\tau=RC$ in the resistor is the Device Reaction (DR). From [6] we knew that this $\tau_c$
*had to be very short* to allow the sum in power (e.g. incoherently) of all these DR to
obtain the right noise power dissipated in the resistor.

Since the passage of an electron between terminals is a small fluctuation of
electric field between these terminals and given the link between time-varying electric
fields and magnetic ones, each TA is a fluctuation of electromagnetic field between two
points of the resistor (its terminals) separated by a distance $d$. As Einstein showed in
1905, electromagnetic signals must be used to define *simultaneity* at points separated in
space. Thus, a TA synchronizes the clocks of two observers measuring electrical voltage
at the two terminals of the resistor separated by the distance $d$. Therefore, a Thermal
Action becomes an *instantaneous event* for two observers measuring voltages appearing
*simultaneously* on the terminals of a 2TD to obtain their difference, as we do in noise
measurements. This means a *null risetime* $\tau_c=0$ in the noise pulses of Figure 3.

This null $\tau_c$ allows the passage of single electrons independently of one another
whereas a simultaneous passage of two of them is unlike as we have shown. This means
that TA and thus DR *are uncorrelated* and must be added in power to consider properly
the active power in $R$ as we did in [4, 6, 7]. Once a TA is born (instantaneous *Cause*,
Fluctuation of energy linked with *reactive power in C*) the device can respond and a DR
is initiated (slower *Effect*, Dissipation linked with *active power in R*). This justifies our
NM where electrical noise comes from a huge rate $\lambda$ of pulses like those of Figure 3. For
a device of resistance $R$ between terminals this rate is [4]:

$$\lambda = \frac{2kT}{q^2R} \text{ s}^{-1} \quad (3)$$
This $\lambda$ means that the resistor receives from its environment an average reactive power $P_C$ through its $C$ acting as a receiving antenna for TA. In TE, a similar amount of power $P_C$ on average is converted by $R$ into heat (active power) and leaves the device towards its surrounding universe. Under this new model it is not difficult to show that $1/f$ excess noise [6], flicker noise [9] and phase noise [7, 10], are effects of this electrical noise in two-terminal devices like resistors, capacitors and resonators.

We want to say that ref. [3] only has been taken as a typical example of these ideas underlying TM that we ourselves acquired years ago. Although we could give lot of references like this one, we will not do it. This would make our paper similar to those valuable papers plenty of references called Review papers and ours is not this type of review. After reading carefully [1, 2, 5], our paper only offers a different interpretation of electrical noise for which we have not found related references.

Finally, let us say that in the circuit of Figure 2 it is quite clear that the mean square fluctuating current driving the circuit must be proportional to $G=1/R$ to observe a mean energy fluctuation of $kT/2$ Joules in $C$. Note that the product voltage$\times$current is a power similar to the product force$\times$velocity in mechanical ensembles used to study the Brownian motion of a big particle of mass $M$ under the random collisions of a rain of small particles of mass $m$. Replacing random current by random force, random voltage by random velocity, electrical $C$ by inertial $M$ and conductance $G$ by friction factor, Figure 2 offers an electrical ensemble of the F-D Theorem “relating the mean square fluctuating force to the friction factor” [11]. This is no other than the shot noise due to TA or Nyquist noise $4kTG A^2/Hz$ being proportional to the “friction factor” $G$ in TE.

5. Conclusion

What we considered a suspicious dependence of the Nyquist noise generator on the conductance $G$ it had to drive, only is the expression of the Fluctuation-Dissipation
Theorem in electrical form. Unknowing this fact, but looking for the reason of this suspicious dependence we found a new model for electrical noise based on the concept of Admittance excelling the traditional model based on Conductance. Whereas the later only handles Dissipations of electrical energy in Resistances, our new model handles together Fluctuations and Dissipations of electrical energy taking place in Admittances. Adding to this features the discrete nature of electric charge involved in displacement currents, a Quantum-compliant model for electrical noise results that fits well into the Fluctuation-Dissipation framework proposed by Callen&Welton in 1951.

Acknowledgments

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**Figure 1.** Traditional model for electrical noise in resistors.

**Figure 2.** Complex Admittance of a noisy resistor in TE [4] where its Nyquist noise is viewed as the power of charge noise in its capacitance between terminals.

**Figure 3.** Electrical noise viewed as a random series of TA-DR pairs that follows the Cause-Effect dynamics implicit in the Fluctuation-Dissipation framework of [5].