Photo-Absorption Sum Rules $\sigma_{-1}$ in Different Environments 
(Atoms, Nuclei, Nucleons)$^1$

S. B. Gerasimov
Bogoliubov Laboratory of Theoretical Physics, JINR, Dubna 
e-mail: gerasb@theor.jinr.ru

Abstract—Combining the spin-dependent dispersion GDH-sum rule, the isotopic-spin-dependent Cabibbo-Radicati sum rule, and the relativistic dipole-moment-fluctuation (i.e. generalized Gottfried) sum rule with the three valence quark configuration of nucleons taken into account for the composition of the ground and the excited states of the nucleon, the relevant moments of the distribution and correlation functions of the quark electric dipole moment operators in the nucleon ground state are expressed via the experimentally measurable nucleon resonance photo-excitation amplitudes. These functions are of interest for checking detailed quark-configuration structure of the nucleon state vector. Within the non-relativistic approach to photo-absorption sum rules for the 3N-nuclei a new $\sigma_{-1}$ sum rule proposed which is based on general charge-symmetry (CS) consequences for the “CS-conjugated” triton and $^3$He.

DOI: 10.1134/S1063779617010087

1. INTRODUCTION: 
THE PAST EXPERIENCE

The observed effects of the structure-dependent asymmetries in the static electromagnetic characteristics of hadrons, such as parameters of form-factors or coefficients of the electromagnetic polarizability, or in characteristics of processes induced by lepton- or photon-hadron interactions, e.g., total or partial cross sections, are known as very useful means to study and to understand the underlying dynamics.

The sum rules, in particular, have long served as a reliable constraints on, or relations between measured quantities, depending only on most general principles and statements of the theory.

In this work we shall concentrate mainly on two topics: the sum rules for static electromagnetic characteristics of hadrons and the role of the dynamical short-range correlations of nuclear constituents in description of the behavior of the total and polarized photonuclear or photon-nucleon cross section.

As is known, the non-relativistic dipole sum rules continue to be a useful tool in the theory of the atomic and nuclear photo-effect.

$$\sigma_{n}(E1) = \int_{\text{thr}}^{\infty} d\omega \omega^{n} \sigma_{E1}(\omega).$$

Examples: $n = -2 \rightarrow$ Kramers–Heisenberg sum rule (SR) for static electric-dipole polarizability of a given quantum system;

$n = -1 \rightarrow$ the bremsstrahlung-weighted SR, dependent of charged-“parton” correlation in a given system;

$n = 0 \rightarrow$ the famous Thomas–Reiche–Kuhn SR, known as a precursor of the Quantum Mechanics. As far as the nucleon is the relativistic 3q system, the relativistic generalization of sum rules is needed, including the spin degrees of freedom and spin-dependent interactions and correlations of partons.

2. QED AND ATOMS

In what follows we consider relativistic dipole moment fluctuation sum rules in the “valence-parton” approximation, that is neglecting virtual particle-antiparticle configurations in the ground state of the considered systems or diffractively produced in the final states of photo-absorption reactions. The anomalous magnetic moment sum rules express a model-independent correspondence between static properties of a particle (or bound system of particles) and integrals over the photo-absorption spectrum. For particles with the spin $S = 1/2$ the sum rule for the anomalous magnetic moment $\kappa$ reads [1–3]

$$\frac{2\pi^{2}\alpha\kappa^{2}}{m^{2}} = \int_{\text{thr}}^{\infty} dv E(v)(\sigma(p) - \sigma(n)).$$ (1)
In QED, the validity of the sum rule for free electron was checked in two lowest orders of the perturbation theory [4, 5]. Later on, for the physical reasons, we shall replace $\kappa^2$ entering different sum rules just by its integral expression in the GDH sum rule. In particular, for the relativistic dipole moment fluctuation sum rule

$$4\pi^2\alpha\left[\frac{1}{3}\langle D^2 \rangle - \frac{\kappa^2}{4m^2}\right] = \int \frac{d\nu}{\nu} \sigma_{tot}(\nu)$$

we get another form to be used later

$$4\pi^2\alpha\frac{1}{3}\langle D^2 \rangle = \int \frac{d\nu}{\nu} \sigma_{tot}(\nu).$$

We apply first derived sum rule to the system of the highly ionized atom Pb$^{81+}$, initiated about half-century ago by J.S. Levinger and co-workers [6]. Using the form of the sum rule with our included term $\kappa_{atom}$ we reduced deviation between left- and right-hand sides of the sum rule, discovered in earlier works, to one-half percent. Numerically:

$$4\pi^2\alpha\frac{1}{3}\langle D^2 \rangle [937.26b] - 4\pi^2\alpha\left(\frac{\kappa}{2M}\right)^2 [67.9b]$$

$$= \int \frac{d\nu}{\nu} \sigma_{tot}(\nu) [874b],$$

where for the anomalous magnetic moment of the atom we used the total magnetic moment of electron bound in the lowest S-wave orbit [7].

3. THE NON-RELATIVISTIC $\sigma_{-1}$ SUM RULES FOR FEW-BODY NUCLEI

Besides sum rules including the measurable physical quantities and obtained on the base of the current algebras and the $p_z \to \infty$ techniques, the similar sum rules (but without the explicitly spin-dependent terms) were obtained in the non-relativistic (NR) nuclear physics context. In particular, the NR sum rules $\sigma_{-1}$ for the s-shell nuclei ($A = Z + N \leq 4$) were obtained from the assumption that the ground-state wave function is symmetric in the space coordinates of nucleons [8, 9] for each nucleus of the isotopic doublet $^3H$ and $^3He$ and for $^4He$

$$4\pi^2\alpha \frac{NZ}{A - 1} \left\langle \frac{1}{3} \langle r_{ch}^2 \rangle \right\rangle_{NR} = \int \frac{d\nu}{\nu} \sigma_{el}(\nu).$$

The present-day situation is that the accuracy of the theoretical results [10] is higher than the experimental uncertainties in the important energy range up to the pion production threshold [11] suggesting that new experimental data would be timely. The main objectives of this Section is a comparison of the Charge Independence (CI) and Charge Symmetry (CS) initial assumptions taken for the derivation of the $\sigma_{-1}$ sum rules for the “mirror” $^3He$–$^3H$ and the “charge-self-conjugate” $^4He$ nuclei. The validity of the Foldy [8] and Khokhlov [9] sum rules assumes validity of the (nucleon) charge independence (CI) symmetry. Our objective in this Section is to apply more strong charge symmetry (CS) requirements in derivation of the F-Kh-type sum rules. In accord with the CS-provisions for the mean values of the nucleon radii correlations in the mirror pair $^3H$–$^3He$ nuclei and the “self-conjugate” $^4He$ nucleus, we have get that the values of $\langle D^2 \rangle_{^3He}$, $\langle D^2 \rangle_{^3H}$, and $\langle r_{ch}^2 \rangle_{^3H}$ can be expressed via one and the same linear combination of the introduced and undetermined parameters which is different from the analogues expression for $\langle r_{ch}^2 \rangle_{^3He}$. Taken at face value this means that with our more strict assumption on the underlying symmetry i.e. the CS instead of CI-option, we get only one “mixed” sum rule instead of three (for $^3H$, $^3He$ and $^4He$) earlier mentioned Foldy–Khokhlov sum rules:

$$\sigma_{-1}(^3H) = \frac{4\pi^2\alpha}{3} \left\langle \langle r_{pp/\ell}^2 \rangle_{^3H} \right\rangle = \sigma_{-1}(^3He).$$

$$2.45mb \quad 2.42mb = .242f^2 \quad 2.54 \pm .09mb$$

The nuclei charge radii in the above relations and in what follows are understood as corresponding to the case of “point-charge-protons” ones which reached by subtracting from “physical” values standard corrections, e.g. [12]. The problem of the radii difference in the charge-symmetry-conjugated 3N-nuclei is of similar degree of complexity as the long-discussed binding energy difference of the $^3He$ and $^3H$:

$$E_b(\tilde{^3He}) = 8.52 MeV \quad \text{and} \quad E_b(^3H) = 7.76 MeV,$$

e.g. [13, 14]. For a qualitative understanding of the intended estimation, we resort to dimensional arguments and factors as follows:

$$\frac{E_b(\tilde{^3He})}{E_b(^3H)} \equiv \frac{\langle r_{ch}^2 \rangle_{^3He}^{-1/2}}{\langle r_{ch}^2 \rangle_{^3H}^{-1/2}} = 0.911 \quad 0.899$$

The unexpected numerical closeness of two ratios written down on exclusively dimensional ground signals nevertheless that for its full dynamical explanation one should include into consideration not only the Coulomb interaction effects but all collection of the CSB – explications as in Refs. [13, 14], that represent on the nuclei and hadronic scales the fundamental current u- and d-quark mass difference.
4. RELATIVISTIC CONSTITUENT QUARK MODEL AND NUCLEONS

Following formally to the $p_z \rightarrow \infty$ techniques derivation of the Cabibbo–Radicati [15] or GDH sum rule [16], we can obtain the relation

$$4\pi^2\alpha\left(\frac{1}{3}\langle \mathbf{D}^2 \rangle - \left(\frac{\mathbf{k}_N}{2m_N}\right)^2\right) \int \frac{d\nu}{\nu} \sigma_{\nu\alpha}(\nu).$$

(8)

We use the definitions

$$\mathbf{D} = \int \hat{x}\hat{p}(\mathbf{x})d^3x \sum_{j=1}^{3} Q_j(j)\hat{d}_j,$$

(9)

$$r_j^2 = \int \hat{x}\hat{p}(\mathbf{x})d^3x = \sum_{j=1}^{3} Q_j(j)d_j^2.$$  
(10)

The defined operators $Q_j(j)$ and $d_j$ are the electric charges and configuration and spin variables of point-like interacting quarks in the infinite-momentum frame of the bound system.

Finally, we relate the electric dipole moment correlators consecutively for the proton, the neutron, the pure “isovector-nucleon” and the iso-vector part of the mean-squared radii operators, which all are sandwiched by the nucleon state vectors in the “infinite—momentum frame”, with experimentally measurable data on the resonance parts of the photoabsorption cross sections on the proton and neutron presently known below ~2 GeV [17]. The listed operator mean values are parametrized as follows

$$R_V = \frac{1}{2}(\langle r_1^2 \rangle_p - \langle r_1^2 \rangle_N) = \alpha - \frac{1}{2}\beta,$$

(11)

$$J_p = \frac{1}{3}\langle \mathbf{D}^2 \rangle_p = \frac{8}{27}\alpha + \frac{1}{2}\beta + \frac{8}{27}\gamma - \frac{8}{27}\delta,$$

(12)

$$J_N = -\frac{1}{3}\langle \mathbf{D}^2 \rangle_N = \frac{2}{27}\alpha + \frac{1}{2}\beta + \frac{2}{27}\gamma - \frac{8}{27}\delta,$$

(13)

$$J_V = \frac{1}{3}\langle \mathbf{D}^2 \rangle_V = \frac{2}{3}\alpha + \frac{1}{3}\beta + \frac{2}{3}\gamma - \frac{4}{3}\delta,$$

(14)

where $\langle \hat{d}_1^2 \rangle = \langle \hat{d}_2^2 \rangle = \alpha$, $\langle \hat{d}_3^2 \rangle = \beta$, $\langle \hat{d}_1 \cdot \hat{d}_2 \rangle = \gamma$, $\langle \hat{d}_1 \cdot \hat{d}_3 \rangle = \langle \hat{d}_2 \cdot \hat{d}_3 \rangle = \delta$ indices “1” and “2” refer to the like quarks (i.e. to the $u(d)$—quarks inside the proton (neutron)), and “3” to the odd quark. Evaluation of the relativistic electric dipole moment fluctuation and the isovector charge radius sum rules for the nucleon was carried out with the available compilation [17] of the resonance pion-photoproduction data on the proton and neutron with the helicity $\lambda = 1/2(3/2)$ $A_{1/2}^{(N)}$ and $A_{3/2}^{(N)}$ and all integrals over photoexcited nucleon resonances were taken in the narrow resonance approximation:

$$J_{p(\alpha)} = \frac{4\pi m_n}{m_{res} - m_n^2} A_{3(2)}^{res}$$

(15)

where $m_{n(res)}$ is the nucleon (or resonance) mass. Solving the system of the linear equations and evaluating the $R_V, J_{p,N,V}$ with the help of experimentally known partial amplitudes of main photo-excited resonances, we find our final results for the numerical values $\alpha, \beta$ and the opening angle $\theta_{12}$ and $\theta_{13}$ between vectors $\hat{d}_1$ and $\hat{d}_2$ and vectors $\hat{d}_{(3)}$ and $\hat{d}_3$:

$$\alpha^{1/2} = 0.75 \pm 0.06 \text{f},$$

(16)

$$\beta^{1/2} = 0.77 \pm 0.12 \text{f},$$

(17)

$$\theta_{12} \approx 120^0,$$

(18)

$$\theta_{13} = \theta_{23} = 120^0,$$

(19)

$$\langle r_1^2 \rangle_V = 0.25 \pm 0.02 \text{f}^2 (\exp : 0.29 f^2).$$

(20)

Having got the values of the “global” parameters $\alpha$, $\beta$, etc. defined in the infinite momentum frame, we put all three vectors with the pertinent numerical parameters into the plane transverse to $p_z \rightarrow \infty$ momentum and define parameters of the closed contour circumscribing their ends. The resulting curve is either ellipse with very small excentricitity or the circle. This fact is, in our opinion, at variance with rather popular “quark-diquark” models of the nucleon with, presumably, more noticeable asymmetry of the spatial correlations of the $uu$- and $ud$-quark pairs.

5. CONCLUDING REMARKS

(1) The properness was demonstrated of using the current-algebra and dispersion relation based relativistic sum rules for establishing new relations between different characteristics of the electron bound in the hydrogen-like, highly-charged ions.

(2) With the accuracy about 15%, the isovector charge radius of the nucleon was calculated via the Cabibbo–Radicati sum rule through the integral of total resonance photo-production of hadrons on the nucleons. The mentioned deficit should be referred to the “pion-sea” presence in the nucleon state vector.

(3) According to our suggested Ansatz for the including of the interference of photo-absorption amplitudes on different quarks, their effect is very significant and it strongly diminishes the difference of $J_p - J_N$ calculated for the real photo-absorption. For the virtual photon absorption in the reactions of electro-production hadrons on nucleons, the more strong suppression of the interference amplitudes in the “deep-inelastic” region will result in the enhancement
of mentioned difference, approaching the result to the deep-inelastic version of the generalized Gottfried sum rule [18, 19].

6. ACKNOWLEDGMENTS

The author would like to thank Dr. O.V. Selyugin and Dr. A.I. Titov for fruitful discussions and help in preparation of the paper.

REFERENCES

1. S. B. Gerasimov, “A sum rule for magnetic moments and the damping of the nucleon magnetic moment in nuclei”, Sov. J. Nucl. Phys. 2, 430–433 (1966).
2. S. B. Gerasimov, “Dispersion sum rule for magnetic moments and spin dependence of photoabsorption cross-section”, Sov. J. Nucl. Phys. 5, 902 (1967).
3. S. D. Drell and A. C. Hearn, “Exact sum rule for nucleon magnetic moments”, Phys. Rev. Lett. 16, 908 (1966).
4. S. B. Gerasimov and J. Moulin, “Tests of sum rules for photon total cross sections in quantum electrodynamics and mesodynamics”, Nucl. Phys. B 98, 349 (1975).
5. D. A. Dicus and R. Vega, “The Drell-Hearn sum rule at order α^3”, Phys. Lett. B 501, 44 (2001).
6. W. B. Payne and J. S. Levinger, “Relativistic radiative transitions”, Phys. Rev. 101, 1020 (1956).
7. S. B. Gerasimov, “Quark-hadron duality sum rules for spin-dependent photon-nucleon and photon-photon resonance cross-section”, Czech. J. Phys. 55, A-2009 (2005).
8. L. L. Foldy, “Photodisintegration of the lightest nuclei”, Phys. Rev. 107, 1303 (1957).
9. Yu. K. Khokhlov, “Some sum rules for the cross sections of electric quadrupole transitions in the nuclear photoeffect”, ZhETF 32, 124 (1957).
10. J. Golak, R. Skibinski, W. Glöckle, H. Kamada, A. Nogga, H. Witala, V. D. Efros, W. Leidemann, G. Orlandini, “Benchmark calculation of the three-nucleon photodisintegration”, Nucl. Phys. A 707, 365 (2002).
11. V. N. Fetisov, A. N. Gorbunov, and A. T. Varfolomeev, “Nuclear photoeffect on three-particle nuclei”, Nucl. Phys. 71, 305 (1965).
12. A. Ekström, G. R. Jansen, K. A. Wendt, G. Hagen, T. Papenbrock, B. D. Carlsson, C. Forssén, M. Hjorth-Jensen, P. Navrátil, and W. Nazarewicz, “Accurate nuclear radii and binding energies from a chiral interaction”, Phys. Rev. C 91, 00301(R) (2015).
13. R. A. Brandenburg, S. A. Coon, and P. U. Sauer, “Nuclear charge asymmetry in the A = 3 nuclei”, Nucl. Phys. A 294, 305 (1978).
14. Y. Wu, S. Ishikawa, and T. Sasakawa, “Nuclear charge asymmetry and charge dependence and the 3H–3He binding-energy difference”, Phys. Rev. Lett. 64, 1875 (1990).
15. N. Cabibbo and L. A. Radicati, “Sum rule for the isovector magnetic moment of the nucleon”, Phys. Lett. 19, 687 (1966).
16. M. Hosoda and K. Yamamoto, “Sum rule for the magnetic moment of the Dirac particle”, Prog. Theor. Phys. 36, 425 (1966).
17. K. A. Olive et al. (Particle Data Group), Chin. Phys. C 38, 090001 (2014).
18. K. Gottfried, “Sum rule for high-energy electron-proton scattering”, Phys. Rev. Lett. 18, 1174 (1967).
19. A. L. Kataev, The Gottfried Sum Rule: Theory vs Experiment, arXiv: 0311091 [hep-ph].