Testing the distance duality relation using Type Ia supernovae and ultracompact radio sources

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ABSTRACT
We test the possible deviation of the cosmic distance duality relation \( D_A(z)(1 + z)^2/D_L(z) \equiv 1 \) using the standard candles/rulers in a fully model-independent manner. Type Ia supernovae are used as the standard candles to derive the luminosity distance \( D_L(z) \), and ultracompact radio sources are used as the standard rulers to obtain the angular diameter distance \( D_A(z) \). We write the deviation of distance duality relation as \( D_A(z)(1 + z)^2/D_L(z) = \eta(z) \). Specifically, we use two parametrizations of \( \eta(z) \), i.e. \( \eta_1(z) = 1 + \eta_0 z \) and \( \eta_2(z) = 1 + \eta_0 z/(1 + z) \). The parameter \( \eta_0 \) is obtained using the Markov chain Monte Carlo methods by comparing \( D_L(z) \) and \( D_A(z) \) at the same redshift. The best-fitting results are \( \eta_0 = -0.06 \pm 0.05 \) and \( -0.18 \pm 0.16 \) for the first and second parametrizations, respectively. Our results depend on neither the cosmological models nor the matter contents or the curvature of the Universe.

Key words: supernovae: general – cosmological parameters – distance scale.

1 INTRODUCTION
In the standard cosmological model, there is a strict correlation between the luminosity distance \( D_L(z) \) and the angular diameter distance \( D_A(z) \) at the same redshift, i.e. \( D_A(z)(1 + z)^2/D_L(z) = 1 \) (Etherington 1933, 2007). This is the so-called distance duality relation (DDR). The DDR holds true in any metric theory of gravity such as general relativity, as long as the photons travel along null geodesics and the photon number is conserved. Any deviation of DDR implies that there are new physics beyond the standard cosmological model. Therefore, testing the validity of DDR arouses great interests in recent years.

Many works have been devoted to testing the validity of DDR (Bassett & Kunz 2004; Uzan, Aghanim & Mellier 2004; Bernardis, Giusarma & Melchiorri 2006; Holanda, Lima & Ribeiro 2010; Fu et al. 2011; Piorkowska et al. 2011; Yang et al. 2013; Santos-da-Costa, Busti & Holanda 2015; Chen et al. 2016; Holanda et al. 2016; Holanda, Busti & Alcaniz 2016; Liao et al. 2016; Lv & Xia 2016; Ma & Corasaniti 2016). All the methods require the measurement of both luminosity distance and angular diameter distance at the same redshift. The luminosity distance \( D_L \) can be obtained from Type Ia supernovae (SNe Ia) with high precision. SNe Ia have an approximately consistent absolute luminosity after correcting for stretch and colour, and therefore are widely regarded as the standard candles in cosmology (Riess et al. 1998; Perlmutter et al. 1999). However, the measurement of angular diameter distance \( D_A \) is not as straightforward as that of \( D_L \). One way to determine \( D_A \) is using the Sunyaev–Zeldovich effect combined with the X-ray data from galaxy clusters (De Filippis et al. 2005; Bonamente et al. 2006). The obtained \( D_A \) in this way depends on the mass model of galaxy clusters, which arouses large uncertainties. Many works have used the \( D_L \) data from SNe Ia and \( D_A \) data from galaxy clusters to test the DDR (see e.g. Yang et al. 2013, and the references therein). Due to the large uncertainty, most work found no evidence for the violation of DDR. A more accurate measurement of \( D_A \) can be obtained from the baryon acoustic oscillations (BAO) (Beutler et al. 2011; Anderson et al. 2013; Kazin et al. 2014; Delubac et al. 2015). However, the measurement of BAO requires the statistics of a large number of galaxies, and the number of measured BAO data points so far is very limited. Ma & Corasaniti (2016) used the \( D_L \) from SNe Ia and \( D_A \) from BAO to test DDR and found 5 per cent constraints in favour of its validity.

Recently, Liao et al. (2016) proposed to test the DDR using the angular diameter distance from strong gravitational lensing systems, in combination with the luminosity distance from SNe Ia. The angular diameter distance can be deduced from the Einstein radius, and the redshifts of lens and source. However, the gravitational lensing systems could only provide the information of distance ratio between lens and source, and between observer and source, i.e. \( R_A \equiv D_{A,l}/D_{A,s} \). To obtain the distance from observer to lens, a flat FLRW cosmology was assumed, which makes the test of DDR not completely model-independent. In addition, the Einstein radius depends on the mass profile of lens, which causes some uncertainty. The luminosity distance is from SNe Ia, which is independent of the gravitational lensing systems. The problem is that the SNe and gravitational lensing systems are usually located at different redshifts, making the direct comparison between \( D_L \) and \( D_A \) impossible. To solve this problem, the authors adopted a matching criterion that if the redshift difference between the SNe and lens or source is no
more than 0.005, then they may be regarded as located at the same redshift. With such a criterion, there are only approximately 60 lensing systems that have matched SNe, although the total number of lensing systems is more than 100. The redshift of SNe Ia is limited to be \( z \leq 1.4 \), much smaller than the redshift of strong gravitation lensing systems. Holanda et al. (2016) combined the SNe, strong gravitational lensing systems and gamma-ray bursts to test the DDR at high redshift, and found that the DDR validity is verified within 1.5\( \sigma \).

The milliarcsecond ultracompact radio sources (RSs) provide a unique tool to measure the angular diameter distance. Kellermann (1993) studied the angular size-redshift relation (\( \theta \sim z \) relation) of 79 ultracompact RSs associated with active galaxies and quasars observed by the Very Long Baseline Interferometry (VLBI), and showed that the \( \theta \sim z \) relation can be naturally explained by geometrical effect (i.e. the further object has the smaller angular size) in the FLRW cosmology. This implies that the linear size of ultracompact RSs, \( d_\theta \), is approximately constant, free of evolution effects. From then on, much efforts have been done to standardize the ultracompact RSs as the cosmological rulers (Gurvits 1994; Jackson & Dodson 1996, 1997; Gurvits, Kellermann & Frey 1999; Jackson 2004; Jackson & Jennett 2006; Jackson, 2008, 2012). The possible cosmological evolution of \( d_\theta \) with luminosity \( L \), redshift \( z \) and spectra index \( \alpha \) has been investigated (Gurvits 1994; Gurvits, Kellermann & Frey 1999; Jackson 2004). With appropriate cuts on \( L \), \( z \) and \( \alpha \), the cosmological evolution of \( d_\theta \) can be neglected (Gurvits 1994; Gurvits, Kellermann & Frey 1999). Assuming that there is no cosmological evolution of \( d_\theta \), Jackson & Jennett (2006) used a large sample of RS data to give a very strict constraint on cosmological parameters.

In this paper, we use the angular diameter distance from ultracompact RSs, and combine with the luminosity distance from SNe Ia, to test the validity of DDR. The main advantage of this method is that it is completely cosmological model-independent. The rest of the paper is organized as follows: In Section 2, we introduce the methodology of testing DDR using SNe Ia and ultracompact RS data in a model-independent way. In Section 3, we introduce the observational data samples that are used in the test of DDR and give the results. Finally, discussions and conclusions are given in Section 4.

## 2 METHODOLOGY

In this section, we illustrate how to test the DDR using SNe Ia and ultracompact RSs. Following Holanda, Lima & Ribeiro (2010), we write the possible violation of the standard DDR as

\[
\frac{D_A(z)(1+z)^2}{D_L(z)} = \eta(z).
\]

We use two different parametrizations of \( \eta(z) \),

\[
\eta(z) = 1 + \eta_0 z, \quad \eta(z) = 1 + \eta_0 \frac{z}{1 + z},
\]

where \( \eta_0 \) is a free parameter representing the amplitude of DDR violation. There is no violation of DDR if \( \eta_0 = 0 \). By comparing \( D_A(z) \) and \( D_L(z) \) at the same redshift \( z \), we can constrain the parameter \( \eta_0 \).

The luminosity distance can be derived from SNe Ia. SNe Ia are widely used as the standard candles in cosmology due to their approximately consistent absolute luminosity (Riess et al. 1998; Perlmutter et al. 1999). The distance modulus of SNe can be extracted from the light curves using the following empirical relation (Tripp 1998; Guy et al. 2005, 2007; Suzuki et al. 2012; Betoule et al. 2014)

\[
\mu_{\text{sn}} = m_B - M_B + \alpha X_1 - \beta C,
\]

where \( m_B \) is the apparent magnitude, \( M_B \) is the absolute magnitude, \( X_1 \) and \( C \) are the stretch factor and colour parameter, respectively. The two parameters \( \alpha \) and \( \beta \) are universal constants and can be fitted simultaneously with cosmological parameters, or more generally, can be marginalized over. The uncertainty of \( \mu_{\text{sn}} \) is propagated from the uncertainties of \( m_B \), \( X_1 \) and \( C \) using the standard error propagation formula,

\[
\sigma_{\mu_{\text{sn}}} = \sqrt{\sigma_{m_B}^2 + \alpha^2 \sigma_{X_1}^2 + \beta^2 \sigma_C^2}.
\]

The angular diameter distance is derived from the ultracompact RSs. Due to the approximately constant linear size of RSs, the angular diameter distance can be easily obtained if the angular size \( \theta \) is observed,

\[
D_A,rs = \frac{d_\theta}{\theta}.
\]

where the linear size \( d_\theta \) is not known a priori and is regarded as a free parameter. Combining equations (1) and (5), the luminosity distance of RSs can be written as

\[
D_L,rs = \frac{d_\theta (1+z)^2}{\eta(z)}.
\]

The distance modulus of RSs is given by

\[
\mu_{\text{rs}} = 5 \log_{10} \frac{D_L,rs}{\text{Mpc}} + 25.
\]

The uncertainty of \( \mu_{\text{rs}} \) is propagated from that of \( \theta \),

\[
\sigma_{\mu_{\text{rs}}} = \frac{5 \sigma_\theta}{\ln 10 \theta},
\]

where ‘ln’ is the natural logarithm.

Difficulties arise when we try to directly compare \( \mu_{\text{sn}} \) with \( \mu_{\text{rs}} \). This is because SNe and RSs are usually located at different redshifts. For a specific SN, there is, in general, no RS located at the same redshift, and vice versa. To solve this problem, we first reconstruct the \( \theta \sim z \) relation using the Gaussian processes (Seikel, Clarkson & Smith 2012). Then, the \( \mu_{\text{rs}}(z) \) function can be reconstructed using equations (6)-(8). Given \( \mu_{\text{rs}}(z) \) function, we can calculate the distance modulus of RSs at any desired redshift. We use the reconstructed \( \mu_{\text{rs}}(z) \) function to calculate the distance modulus of RS at the redshift of each SNe. Therefore, the comparison between \( \mu_{\text{rs}} \) and \( \mu_{\text{sn}} \) at the same redshift is possible.

We use the Markov chain Monte Carlo methods (Foreman-Mackey et al. 2013) to calculate the posterior probability distribution functions of free parameters. The likelihood is given by

\[
\mathcal{L}(p) = \prod \frac{1}{\sqrt{2\pi}\sigma_\mu} \exp \left[ -\frac{1}{2} \left( \frac{\mu_{\text{sn}} - \mu_{\text{rs}}}{\sigma_\mu} \right)^2 \right],
\]

where

\[
\sigma_\mu = (\sigma_{\mu_{\text{sn}}}^2 + \sigma_{\mu_{\text{rs}}}^2 + \alpha^2 \sigma_{\text{int}}^2)^{1/2}
\]

is the uncertainty of distance modulus of SNe and RSs, \( p = (d_\theta, \eta_0, \alpha, \beta, M_B, \sigma_{\text{int}}) \) is the set of free parameters and the product runs over all the SN–RS pairs. We have added the intrinsic scatter term in equation (10) to account for any other uncertainties. Note that \( d_\theta \) is degenerated with \( M_B \), so they cannot be constrained simultaneously. One parameter should be fixed in order to constrain the other. We leave \( d_\theta \) free and fix \( M_B = -19.32 \) (Suzuki et al. 2012).
The SNe sample is taken from Suzuki et al. (2012), i.e. the Union2.1 sample. Union 2.1 consists of 580 SNe Ia with high light curve quality in the redshift range [0.015, 1.414]. Each SN has well-measured redshift \( z \), apparent magnitude \( m^g_\alpha \), stretch factor \( X_1 \) and colour factor \( C \). Note that, Suzuki et al. (2012) also published the distance modulus of each SNe, which is calibrated in the wcdm dark matter (CDM) model. We use the original light-curve parameters rather than the published distance moduli in order to avoid the model dependence. The distance moduli are calculated using equation (3), with \( \alpha \) and \( \beta \) as free parameters.

The RS sample is taken from Jackson & Jannetta (2006). The sample is selected from a compilation of VLBI survey of ultracompact RSs at 2.29 GHz released by Preston et al. (1985). The redshift and radio flux are updated according to the recent observations. The sample consists of 613 RSs in the redshift range \([0.0035, 3.787]\). Among the sample, there are 468 RSs that have redshift larger than 0.5. Data with \( z < 0.5 \) are not appropriate to use as standard rulers because they are more affected by cosmological evolutions (Gurvits 1994; Jackson (2004); Jackson & Jannetta (2006); Jackson & Dodgson (1996, 1997); Gurvits, Kellermann (1994)).

From Fig. 1, we can see that the original data points show large scatter around the \( \theta - z \) theoretical curve. Following Kellermann (1993); Jackson & Dodgson (1996, 1997); Gurvits, Kellermann & Frey (1999); Gurvits (1994); Jackson (2004) and Jackson & Jannetta (2006), we bin the raw data into 18 bins, with 26 data points in each bin. The central value and 1\( \sigma \) uncertainty are taken to be the mean and standard deviation of data points in each bin, respectively. The binned data are plotted in Fig. 2. Based on the binned data, we use the publicly available python package EMCEE (Foreman-Mackey et al. 2013) to do the Markov chain Monte Carlo analysis. The likelihood function is given by equation (9). We use a flat prior on the parameters: \( P(d_\theta) = U[1, 20] \text{pc}, P(\eta_0) = U[-1, 1], P(\alpha) = U[0, 1], P(\beta) = U[0, 10] \) and \( P(\sigma_{\text{int}}) = U[0, 1] \). The absolute magnitude \( M_\alpha \) is fixed to \(-19.32 \) due to the degeneracy between \( d_\theta \) and \( M_\alpha \). The marginalized-likelihood distributions and the two-dimensional confidence regions for the parameters are plotted in Figs 3 and 4 for the first and second parametrizations, respectively. The best-fitting (mean) values and 1\( \sigma \) errors (standard deviations) of the parameters are listed in Table 1. Except for the intrinsic scatter \( \sigma_{\text{int}} \), all the other parameters can be well constrained. The 1\( \sigma \) upper limit of \( \sigma_{\text{int}} \) is 0.05, which is much smaller than the errors of SNe and RSs, and therefore can be neglected. In fact, if we fix \( \sigma_{\text{int}} = 0 \), all the other parameters are almost unaffected. The best-fitting linear size is \( d_\theta = 11.86 \pm 0.54 \) pc in the first parametrization and \( d_\theta = 11.46 \pm 0.92 \) pc in the second parametrization. These are well consistent with the result of Jackson (2012) obtained in \( \Lambda \)CDM cosmology, i.e. \( 7.76 h_0^{-1} \) pc (assuming \( h_0 = 0.67 \) (Ade et al. 2014, 2016)). The best-fitting \( \eta_0 \) is \(-0.06 \pm 0.05 \) in the first (second) parametrization. In both parametrizations, there is no strong evidence for the violation of DDR. Note that we make no assumption on the matter contents or the curvature of the Universe, and therefore our results are completely model-independent.

4 DISCUSSIONS AND CONCLUSIONS

The DDR, which relates the luminosity distance \( D_L \) to the angular diameter distance \( D_\theta \) at the same redshift \( z \), plays an important role in modern cosmology and astronomy. It is a natural inference of
The marginalized-likelihood distributions and two-dimensional confidence regions for the parameters $\eta_1(z) = 1 + \eta_0 z$. The blue lines represent the mean values.

Figure 4. The marginalized-likelihood distributions and two-dimensional confidence regions for the parameters $d_0$, $\eta_0$, $\alpha$, $\beta$ and $\sigma_{\text{int}}$ in the second parametrization $\eta_2(z) = 1 + \eta_0 z/(1 + z)$. The blue lines represent the mean values.

Table 1. The best-fitting parameters and their 1σ uncertainties in two different parametrizations of $\eta(z)$. For the intrinsic scatter, only the 1σ upper limit can be obtained.

| $\eta_1(z)$ | $\eta_2(z)$ |
|------------|------------|
| $d_0$ (pc) | $\eta_0$   | $\alpha$ | $\beta$ | $\sigma_{\text{int}}$ |
| 11.86 ± 0.54 | $-\alpha > 0.06 \pm 0.05$ | 0.07 ± 0.02 | 0.50 ± 0.20 | <0.05 |
| 11.46 ± 0.92 | $-\alpha > 0.18 \pm 0.16$ | 0.07 ± 0.02 | 0.50 ± 0.20 | <0.05 |
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of this work. The different nuisance parameters only cause the discrepancy of SNe distance by 0.1 mag, which is much smaller than the uncertainty of distance of RSs. The main conclusions of our manuscript, therefore, are not affected by the nuisance parameters.

The main shortcoming is that the low redshift \((z < 0.5)\) RSs could not be used as the standard rulers, whereas most SNe have redshift \(z < 0.5\). In the redshift overlapping range \((0.5 < z < 1.414)\), there are only 167 SNe, much smaller than the full Union2.1 sample. Nevertheless, this subsample of SNe is already much larger than the available strong gravitational lensing systems or the galaxy clusters.

We may enlarge the sample by adding some high-redshift data such as gamma-ray bursts to SNe sample, or adding some low-redshift galaxy cluster to the RSs sample. The price, however, is that the uncertainty is also enlarged.

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REFERENCES

Ade P. A. R. et al., 2014, A&A, 571, A16
Ade P. A. R. et al., 2016, A&A, 594, A13
Anderson L. et al., 2014, MNRAS, 441, 24
Bassett B. A., Kunz M., 2004, Phys. Rev. D, 69, 101305
Betoule M. et al., 2014, A&A, 568, A22
Beutler F. et al., 2011, MNRAS, 416, 3017
Bonamente M., Joy M. K., LaRoque S. J., Carlstrom J. E., Reese E. D., Dawson K. S., 2006, ApJ, 647, 25
Chen Z. X., Zhou B., Fu X., 2016, Int. J. Theor. Phys., 55, 1229
de Bernardis F., Giusarma E., Melchiorri A., 2006, Int. J. Mod. Phys. D, 15, 759
De Filippis E., Sereno M., Bautz M. W., Longo G., 2005, ApJ, 625, 108
Delubac T. et al., 2015, A&A, 574, A59
Etherington I. M. H., 1933, Phil. Mag., 15, 761
Etherington I. M. H., 2007, Gen. Relativ. Gravit., 39, 1055
Foreman-Mackey D., Hogg D. W., Lang D., Goodman J., 2013, PASA, 125, 306
Fu X. Y., Wu P.-X., Yu H.-W., Li Z.-X., 2011, Res. Astron. Astrophys. 11, 895
Gurvits L. I., 1999, ApJ, 425, 442
Gurvits L. I., Kellermann K. I., Frey S., 1999, A&A, 342, 378
Guy J., Astier P., Nobili S., Regnault N., Pain R., 2005, A&A 443, 781
Guy J. et al., 2007, A&A, 466, 11
Holanda R. F. L., Lima J. A. S., Ribeiro M. B., 2010, ApJ, 722, L233
Holanda R. F. L., Busi V. C., Alcaniz J. S., 2016, J. Cosmol. Astropart. Phys., 1602, 054
Holanda R. F. L., Busi V. C., Lima F. S., Alcaniz J. S., 2017, J. Cosmol. Astropart. Phys., 1709, 039
Jackson J. C., 2004, J. Cosmol. Astropart. Phys., 0411, 007
Jackson J. C., 2008, MNRAS, 390, L1
Jackson J. C., 2012, MNRAS, 426, 779
Jackson J. C., Dodgson M., 1996, MNRAS, 278, 603
Jackson J. C., Dodgson M., 1997, MNRAS, 285 806
Jackson J. C., Jannetta A. L., 2006, J. Cosmol. Astropart. Phys., 0611, 002
Kazin E. A. et al., 2014, MNRAS, 441, 3524
Kellermann K. I., 1993, Nature, 361, 134
Liao K., Li Z., Cao S., Biesiada M., Zheng X., Zhu Z. H., 2016, ApJ, 822, 74
Lv M. Z., Xia J. Q., 2016, Phys. Dark Univ., 13, 139
Ma C., Corasaniti P. S., preprint (arXiv:1604.04631)
Perlmutter S. et al., 1999, ApJ, 517, 565
Piorikowska A. et al., 2011, Acta Phys. Pol. B, 42, 2297
Preston R. A., Morabito D. D., Williams J. G., Faulkner J., Jauncey D. L., Nicolson G., 1985, AJ, 90, 1599
Riess A. G. et al., 1998, ApJ, 116, 1009
Santos-da-Costa S., Busi V. C., Holanda R. F. L., 2015, J. Cosmol. Astropart. Phys., 1510, 061
Seikel M., Clarkson C., Smith M., 2012, JCAP, 1206, 036
Suzuki N. et al., 2012, ApJ, 746, 85
Tripp R., 1998, A&A, 331, 815
Uzan J. P., Aghanim N., Mellier Y., 2004, Phys. Rev. D, 70, 083533
Yang X., Yu H. R., Zhang Z. S., Zhang T. J., 2013, ApJ, 777, L24

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