Double Compactification

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Abstract
A cosmological scenario according to which our universe experienced space-time compactifications twice in its early development is investigated through toy models. In this scenario gauge configurations on an extra space play essential roles to bring about a change of the dimensionality of the compactified space. Simple models are offered and their behaviour at finite temperature is examined. A possibility of causing inflation and problems on our scenario is argued briefly.

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1 Introduction
During the last unified theories of forces, including gravity, in higher-dimensional space-time have gained much interest. Promising unification schemes such as supergravity [1] and superstring theories [2] seem to make clear sense in higher dimensions. Usually the extra dimensions, except for the ordinary space-time of four dimensions, are assumed to be curled up in a tiny compact space [3].

In addition, the theories which contain fundamental gauge fields in higher dimensions have been studied extensively [3]. The theory of this type may be deduced from superstring theories. In such theories, the gauge field configuration on the extra spaces can play important roles in compactification scheme. The energy and pressure of the gauge field strengths on the compact space and the cosmological constant can lead to stable compactification with a suitable balance.

In this case, the cosmological constant must be tuned by hand in order that our universe in large dimensions results in a flat space-time. The same attitude is taken in the usual inflationary universe scenario [4].
When we imagine the early era of the evolution of the universe, we can presume the process of dynamical compactification. The dynamical compactification has received much attention regarding its cosmological aspects [5]. There are various types of cosmological scenarios with dynamical compactification which bring about inflationary expansion [6, 7].

In this paper, we show a new possible mechanism to serve the condition for inflation in compactified theory with nontrivial configuration of gauge field in extra dimensions. We think ‘phase transition’ of the gauge configuration. The phase transition is accompanied with the change of the background geometry; even the dimensionality of compact subspace changes. We wish to call the scenario ‘double compactification’. We believe it worthwhile to analyse simple models on the basis of this attractive possibility. It is shown that there appears a large cosmological constant during the phase transition. The details and subtleties are discussed later.

2 Nontrivial configuration of gauge fields on an extra space

We take the Einstein-Yang-Mills Lagrangian

$$\frac{1}{\sqrt{-g}} L = -\frac{1}{2\kappa^2} R + \frac{1}{4e^2} \text{tr}(F_{MN}F^{MN}) + \Lambda. \quad (1)$$

Here $\kappa^2 = 8\pi G$; $G$ is the Newton constant; $e$ is a gauge coupling constant; $\Lambda$ is a cosmological constant. The scalar curvature of $S^N$ with unit radius is defined as $R = +N(N-1)$.

We consider the $SU(2)$ gauge theory here. This may be regarded as a subgroup of a large unified symmetry group.

Let us suppose the gauge fields on $S^3$ of unit radius with metric

$$d\Omega^2(S^3) = \tilde{g}_{mn}dy^m dy^n = d\psi^2 + \sin^2\psi(d\theta^2 + \sin^2\theta d\phi^2), \quad (2)$$

where $0 \leq \psi, \theta < \pi$ and $0 \leq \phi < 2\pi$.

We adopt the following ansatz in the matrix form of the gauge field configuration:

$$A_{\psi} = f \times \begin{pmatrix} \cos \theta & \sin \theta \exp[-i\phi] \\ \sin \theta \exp[i\phi] & -\cos \theta \end{pmatrix},$$

$$A_{\theta} = f \times \sin \psi \cos \psi \begin{pmatrix} -\sin \theta & \cos \theta \exp[-i\phi] \\ \cos \theta \exp[i\phi] & \sin \theta \end{pmatrix} + f \times \sin^2 \psi \begin{pmatrix} 0 & -i \exp[-i\phi] \\ i \exp[i\phi] & 0 \end{pmatrix},$$

$$A_{\phi} = f \times \sin^2 \psi \sin \theta \begin{pmatrix} \sin \theta & -\cos \theta \exp[-i\phi] \\ -\cos \theta \exp[i\phi] & -\sin \theta \end{pmatrix} + f \times \sin \psi \cos \psi \sin \theta \begin{pmatrix} 0 & -i \exp[-i\phi] \\ i \exp[i\phi] & 0 \end{pmatrix}. \quad (3)$$
These configurations have been used in ref. [8] and in ref. [9]. For the moment, we regard $f$ as a constant.

The energy density of the gauge field strength is

$$\frac{1}{4e^2}\text{tr}(F_{MN}F^{MN}) = \frac{12}{e^2}(f - f^2)^2 \equiv V(f) \quad (f \text{ is a constant}),$$

(4)

where the field strength is defined by

$$F_{mn} = \partial_m A_n - \partial_n A_m + i[A_m, A_n].$$

(5)

$A$ being a constant, the values of $f$ satisfying the field equation

$$D_m F^{mn} = \nabla_m F^{mn} + i[A_m, F^{mn}] = 0,$$

(6)

with $\nabla_m$ representing the covariant derivative on $S^N$, are $f = 0$, $1/2$ and $1$ [9]. These values precisely correspond to the stationary point of the potential. The gauge fields of $f = 0$ and $f = 1$ trivially satisfy the equation of motion (6) because they lead vanishing field strengths. The gauge configuration (3) with $f = 1$ turns out to be locally gauge-equivalent to the trivial configuration $f = 0$ [8]. Later on, we will pay attention to the configuration with $f = 1/2$.

Incidentally, this configuration may remind us of the physics of “sphaleron” [10]. But the configuration considered here exists in pure Yang-Mills theory and can have finite energy only in a compact space. Now let us get back to our subject. We shall work in the $(d + 1 + 3)$-dimensional space-time with metric

$$ds^2 = ds^2(d + 1) + b^2 d\Omega^2(S^3),$$

(7)

and we assume the $(d + 1)$-dimensional space-time admits the metric of an Einstein space. First we suppose the previous gauge field (3) with $f = \text{const}$ on the extra space.

The substitution of the above configuration into the Einstein equations following from (1) leads to

$$R_{\mu\nu}^{(d+1)} = \frac{\kappa^2}{d + 2} \left(2\Lambda - \frac{24(f - f^2)^2}{e^2 b^4}\right) g_{\mu\nu}^{(d+1)},$$

(8)

$$R_{mn} = \frac{\kappa^2}{d + 2} \left(2\Lambda + 8(2d + 1)(f - f^2)^2\right) g_{mn},$$

(9)

where $R_{\mu\nu}^{(d+1)}$ is a Ricci tensor derived from the metric $g_{\mu\nu}^{(d+1)}$ of the $(d + 1)$-dimensional Einstein space, while $R_{mn}$ is a Ricci tensor for the compactified space and $g_{mn} = b^2 \tilde{g}_{mn}$.

In the cases with $f = 0$ and $f = 1$, eqs. (8, 9) have a static solution in which the extra-space is expressed as a static sphere with radius $\{(d + 1)\kappa^2\Lambda\}^{1/2}$ and the $(d + 1)$-dimensional space-time looks like the de Sitter space. However, this solution is unstable; the extra-space will collapse ($b \rightarrow 0$) or will become large indefinitely ($b \rightarrow \infty$).
In this case with \( f = 1/2 \), (9) has a static and stable solution for appropriate values of \( \Lambda, \kappa^2 \) and \( \epsilon^2 \) and the \((d+1)\)-dimensional space-time becomes de Sitter space.

The preferred value of the cosmological constant will be decided in the next section. We suspend an analysis of the solution of (7) until the examination of the classically stable structure of the gauge field.

3 Compactification caused by a domain wall

In this section we consider \( f \) in (3) as a function of one of the space coordinates, say \( z \). In a flat-space background, the existence of a solitonlike structure of the gauge field has been found [8]. Here, we take the curved background geometry and Einstein equations into account. To this end, we consider the following metric in \((d + 1 + 3)\)-dimensional space-time:

\[
\text{ds}^2 = \text{ds}^2(d) + a(z)^2 \text{dz}^2 + b(z)^2 \text{d}Ω^2(S^3),
\]

with \( d \)-dimensional space-time of the Einstein metric and \( a(z) \) and \( b(z) \) are functions of \( z \). The field equation for the Yang-Mills field (6) is reduced to

\[
(a^{-1}bf')' = 4ab^{-1}(f - f^2)(1 - 2f),
\]

where \( ' \) denotes the derivative with respect to \( z \). To solve this equation, we take an ansatz

\[
a(z) \propto b(z).
\]

Then eq. (11) becomes

\[
f'' = 4(a/b)^2(f - f^2)(1 - 2f),
\]

where \( (a/b) \) is a constant. This equation is the same as that in flat space, up to inclusion of an undetermined constant \( (a/b) \). For this case, we have the kink solution, which exists in the \( \phi^4 \)-theory [8, 11]:

\[
f(z) = \frac{1}{2} \left( 1 + \tanh \left( \frac{a}{b} z \right) \right).
\]

We need the Einstein equations to determine the function \( a(z) \) or \( b(z) \). The equations are reduced to

\[
R^{(d)}_{\mu\nu} = \frac{\kappa^2}{d+2} \left\{ 2\Lambda - \frac{1}{\epsilon^2} \left( \frac{6(f')^2}{a^2b^2} + \frac{24(f - f^2)^2}{b^4} \right) \right\} g^{(d)}_{\mu\nu},
\]

\[
R_{zz} = \left( -3 \frac{b'}{ab} \right)^2 + \frac{b'}{ab} \right\} g_{zz}
\]

\[
= \frac{\kappa^2}{d+2} \left\{ 2\Lambda + \frac{1}{\epsilon^2} \left( \frac{6(d+1)(f')^2}{a^2b^2} - \frac{24(f - f^2)^2}{b^4} \right) \right\} g_{zz},
\]

4
\[
R_{mn} = \left( -\frac{1}{a} \left( \frac{b'}{ab} \right)' - 3 \left( \frac{b'}{ab} \right)^2 + \frac{N-1}{b^2} \right) g_{mn}
\]

\[
= \frac{\kappa^2}{d+2} \left\{ 2\Lambda + \frac{1}{e^2} \left( \frac{2(d-1)(f')^2}{a^2 b^2} + \frac{8(2d+1)(f-f^2)}{b^4} \right) \right\} g_{mn}
\]

where \( R^{(d)}_{\mu\nu} \) is a Ricci tensor derived from the metric \( g^{(d)}_{\mu\nu} \) of \( d \)-dimensional space-time and \( R_{zz} \) is a component of the Ricci tensor and \( g_{zz} = a^2(z) \). In addition, we shall look for solutions such that

\[
R^{(d)}_{\mu\nu} = 0, \tag{18}
\]

i.e. we demand flat large dimensions. By substituting the kink solution of \( f(z) \) and flat \( d \)-dimensional metric, we find the following solution:

\[
a(z) = A \{ \cosh(A/B)z \}^{-1}, \tag{19}
\]

\[
b(z) = B \{ \cosh(A/B)z \}^{-1} \tag{20}
\]

with

\[
B^2 = \frac{\kappa^2}{2e^2}. \tag{21}
\]

(Here \( A \) is an arbitrary constant which defines the unit of length scale in \( z \)-direction) and algebraic relations between \( \kappa^2, e^2 \) and \( \Lambda \):

\[
\Lambda = \frac{6e^2}{\kappa^4}. \tag{22}
\]

(Note that eqs. (19, 20, 21) and (22) are independent of the dimensionality \( d \).) Does this space-time realize the ‘universe in a domain-wall’ suggested by Rubakov and Shaposhnikov [12]? Unlike their consideration, our solution is coupled with gravity. However, a close examination reveals another aspect of compactification.

Let us try to rewrite the solution using a new coordinate as

\[
r = C \exp[(A/B)z], \tag{23}
\]

where \( C \) is a constant.

Performing this substitution in the solution above, we get the metric

\[
ds^2 = ds^2(d) + \frac{4C^2 B^2}{(C^2 + r^2)^2} \{ dr^2 + r^2 d\Omega^2(S^3) \} . \tag{24}
\]

Thus this turns out to be a direct-product space \( M_d \times S^4 \) (\( d \)-dimensional Minkowski space-time \( \times \) four-dimensional sphere). (If one considers another variable \( y \) defined as \( \sin y = \tanh(A/B)z \), then one obtains

\[
ds^2 = ds^2(d) + B^2 \{ dy^2 + \cos^2 y d\Omega^2(S^3) \}. \]

If we transform the coordinates into the solution of gauge structure, we find the coincidence with the solutions considered in ref. [13], dubbed as ‘instanton-induced compactification’. Therefore, fortunately, the stability of the solution
has been guaranteed. Moreover, it is known that there can be massless fermion field coupled to the gauge field and the model can be taken as a quasi-realistic model.

Substituting the algebraic relation back into the eqs. (8), (9), we find the \((d+1)\)-dimensional space-time is a de Sitter space as long as eq. (9) has static solutions. This point is discussed again later.

4 Cosmological scenarios

Now we are ready to state a new cosmological scenario. Suppose that, once upon a time, we lived in \((d+1)\)-dimensional space-time with the compact space \(S^3\) and the gauge configuration (3) with \(f = 1/2\) is realized by some reason (see later). During or after the de Sitter expansion, the universe was divided into domains in which the gauge configuration on \(S^3\) is expressed as the form of (2) with \(f = 0\) or \(f = 1\), i.e. the potential minima. Then the domain wall between the domains of \(f = 0\) and of \(f = 1\) appeared. Once the domain wall was made, the compactification of the perpendicular direction to the wall occurred. Then, if we were in the domain wall, we lived in flat \(d\)-dimensional space-time with the compact space \(S^4\), provided the algebraic relation between the coupling constants holds. Note also that we can consider the gauge symmetry breaking caused by the nontrivial gauge configuration.

Here we must suppose the size of \(S^3\) is contracting \((b \to 0)\) during the phase transition in the region of \(f = 0\) and \(f = 1\), because of the form of the solution (19, 20, 21). The domain-wall structure between the regions in which the size of the extra-space expands indefinitely is presently unknown. This will be clarified in future work.

The universe filled with infinite domains was considered by Linde [14]. He considered that even the dimensionality of space-time was different from domain to domain [15]. But in our scenario, we should live in one domain-wall, not in a domain.

Now, we must consider the realization of the initial configuration of gauge fields. We have two ideas at present. One of the ideas is that we consider a radiation-dominated era before the ‘inflation’ as in the ‘old and new’ inflationary cosmology scenario [4]. To explain this feasibility, we propose a simpler model for our “double compactification” scenario.

Let us consider a two-dimensional sphere with standard polar coordinates. We consider an \(SU(2)\) gauge structure of the following type:

\[
A_\theta = f \times \begin{pmatrix} 0 & -i \exp[-i\phi] \\ i \exp[i\phi] & 0 \end{pmatrix},
\]

\[
A_\phi = f \times \sin \theta \begin{pmatrix} \sin \theta & -\cos \theta \exp[-i\phi] \\ -\cos \theta \exp[i\phi] & -\sin \theta \end{pmatrix}.
\]
a solitonic solution or a domain-wall structure in this simple model, though “topological” stability is not guaranteed unlike the previous $S^3$ case (see ref. [8]). The configuration satisfies the general spherical symmetric ansatz which yields spherically symmetric distributions to the energy density, like the previous case.

When $f = 1/2$, the configuration is gauge-equivalent to the monopole configuration [16, 17]. To see this, we take the following gauge transformation:

\[ A_m \rightarrow A'_m = \Omega A_m \Omega^\dagger - i \Omega \partial_m \Omega^\dagger, \]

with

\[ \Omega = \begin{pmatrix} \cos(\theta/2) & \sin(\theta/2) \exp[-i\phi] \\ -\sin(\theta/2) \exp[i\phi] & \cos(\theta/2) \end{pmatrix}. \]

Then we get, provided $f = 1/2$,

\[ A'_\phi = \frac{1}{2} \begin{pmatrix} 1 - \cos \theta & 0 \\ 0 & - (1 - \cos \theta) \end{pmatrix}, \]

\[ A'_\theta = 0. \]

The classical instability at this configuration was discussed in ref. [17], and the quantum correction to vacuum energy was investigated by Hosotani in ref. [16]; Note that gauge symmetry is broken at this point, $f = 1/2$ [16].

We examine the free energy at finite temperature in addition to the potential energy at the typical points with $f = 0$ and $f = 1/2$. We adopt $SU(2)$-doublet fermions as matter fields. The one-loop free energy of the massless fermion gas at temperature $T$ in the background space-time $M_{d+1} \times S^2$ and no gauge-background are formally given by

\[ F(0) = \frac{4N_F V_d \cdot 2^{(d+1)/2}}{\beta(4\pi)^{d/2}} \sum_{n=-\infty}^{\infty} \sum_{l=0}^{\infty} (l+1) \left( \frac{(l+1)^2}{b^2} + \left( \frac{2\pi}{\beta} \right)^2 (n + 1/2)^2 \right)^{d/2}, \]

where $\beta = T^{-1}$ and $N_F$ is the number of fermion species. $V_d$ is the $d$-dimensional volume of the system.

Here we consider the large dimensions as a flat space, because before the “phase transition” the universe is assumed to be dominated by radiation and appears to be a flat space approximately as in ordinary inflationary scenarios. The free energy of the fermions coupled to the gauge configuration (25, 26) with $f = 1/2$ is expressed as

\[ F(1/2) = \frac{N_F V_d \cdot 2^{(d+1)/2}}{\beta(4\pi)^{d/2}} \sum_{n=-\infty}^{\infty} \sum_{l=0}^{\infty} d(l) \left[ \frac{l(l+1)}{b^2} + \left( \frac{2\pi}{\beta} \right)^2 (n + 1/2)^2 \right]^{d/2}, \]

where $d(l) = 2(2l + 1)$ when $l > 0$ and $d(0) = 1$.

At high temperature ($T \gg 1/b$), the free energy in each case is approximately given by

\[ F(0) \sim -\frac{4N_F (4\pi b^2)^2 V_d \cdot 2^{(d+1)/2}}{(4\pi)^{(d+3)/2}} \Gamma \left( \frac{d + 3}{2} \right). \]
\[
\times \left(1 - \frac{1}{2d+2}\right) \zeta(d+3) \frac{2^{d+3}}{\beta^{d+3}} - \frac{1}{3(d+1)} \left(1 - \frac{1}{2d}\right) \zeta(d+1) \frac{2^{d+1}}{b^2 \beta^{d+1}}\right] (33)
\]

\[
F(1/2) \sim - \frac{4N_F(4\pi b^2) V_d \cdot 2^{(d+1)/2}}{(4\pi)^{(d+3)/2}} \Gamma\left(\frac{d+3}{2}\right)
\times \left[1 - \frac{1}{2d+2}\right] \zeta(d+3) \frac{2^{d+3}}{\beta^{d+3}} + \frac{2}{3(d+1)} \left(1 - \frac{1}{2d}\right) \zeta(d+1) \frac{2^{d+1}}{b^2 \beta^{d+1}}\right] (34)
\]

Thus if \(F(0) - F(1/2) > 1/4e^2b^4 \times V_d \times (4\pi b^2)\), the difference of one-loop free energy overcomes the difference of the tree level potential energy. In other words, at sufficiently large \(T (> N_F(e^2b^2)^{-1/(d+1)})\), the vacuum expectation value of \(f\) is 1/2 in the presence of fermion doublets If a sufficiently large number of fermions exist, their contribution overwhelm the radiation of gauge particles and other matter contents in the system under consideration.

This result supports the following speculation: in a very early universe, it is very hot and filled with radiations of matter fields. In a later era in the history of the universe, the vacuum value of \(\langle f \rangle\) cannot tolerate its initial value \(f = 1/2\) as temperature decreases. Then there appear many domains and domain walls; “our universe” should be contained in one of the domain wall. During this transition inflation may take place.

The previous model with extra-space of \(S^3\) is expected to have a similar property at finite temperature; phase transition of gauge configuration could take place.

The point \(f = 1/2\) might not be global minimum but local minimum of the potential. In any case, the domain wall is, at least, unstable. The precise shape of the effective potential is crucial for cosmological evolution as in the inflationary universe scenario.

Another attractive scenario works in a cold universe. At first our universe has \((d+1)\) large dimensions and \(S^3\) (or \(S^2\)) as a compact space. Suppose that the matter field contribution to energy density is negligible. In the early stage of cosmological evolution, the \((d+1)\)-dimensional space can be de Sitter or de Sitter-like space regardless of the value of \(f\). In other words, we assume the extra-sphere had not yet collapsed nor been decompactified. On the other hand, the effective action of \(f\) is precisely the same as that of a scalar field with self-couplings. In the de Sitter space, it is known that the quantum fluctuation drives the vacuum value of scalar fields \([18]\). This dynamics has been studied by many authors through a Fokker-Planck-type equation, and often called stochastic dynamics \([19]\). Thus the vacuum value of \(f\) can cross over the potential barrier between \(f = 0\) and \(f = 1\); stochastic processes could account for the completion of such a transition. Consequently it is possible to make many domains in which the gauge configuration takes \(f = 0\) or \(f = 1\).

This is a “cold” universe scenario. It seems very interesting to investigate the development of \(f\) in this scenario using computer simulations.
5 Comments

Now several comments are in order. First, in this paper, we tacitly assume the static extra-space in its size before the “phase transition” as in the arguments of Wilson-loop symmetry breaking [20]. At the classical level, i.e. as a solution of (9), we can obtain static extra-spheres if \( d \geq 9 \) in our former model with \( S^3 \) and \( d \geq 10 \) in our latter model with \( S^2 \). (In each case, \((d + 1)\)-dimensional space-time becomes de Sitter space of which the distance of horizon is of order \( \sim (e^2/\kappa^2)^{-1/2} \sim (\kappa^2 \Lambda)^{-1/2} \)). Otherwise, if we want a static compact space before the phase transition when \( d < 9 \), we obtain anti-de Sitter space after the compactification of another dimension. If this is true, we must consider this model as an intermediate stage of the history of the universe, i.e. successive compactifications occur until the large dimensions left become four.

However, we must take quantum corrections from matter and gravitational fields into account. Furthermore, even at the classical level, higher-order terms in the curvatures in the action can save the stability of the extra space. These possibilities will be examined in our subsequent work [21].

Secondly, we looked for de Sitter solutions of gravitational equations of motion to test the possibility of the inflationary scenario. There is a subtlety, however, in the occurrence of inflation, because of the simultaneous change in spatial dimensionality. Moreover, the model with \( d < 9 \) has a possibility in serving another dynamical evolution including inflating large spaces. Note that an inflationary universe scenario does not require an “exact” de Sitter space. The study of dynamical evolution of the model along with the cosmic time is necessary.

Several initial-condition problems are known in inflation in Kaluza-Klein theories. In one of the scenarios, a selected initial condition for scale factors or background geometry is crucial [6]. Another scenario makes use of the dynamical evolution of scale factors of the compact space as a source of the rapid expansion in large dimensions [7]. But even in this type of scenario, the compatibility with the stable compactification after inflation requires a restriction to the initial condition in the scale factors of large and small dimensions [22].

Though our scenario of “double compactification” is very different from the above mechanisms, the initial condition of our scenario for inflation may be restricted within some range of variable quantities. We must carefully investigate dynamical evolution of double compactification. We would like to suggest that a numerical simulation like in ref. [23] may be effective, for example, in our “hot” universe scenario.

Thirdly, if we consider the “hot” scenario at finite temperature, we should consider finite-temperature instability for compactification [24]. The quantum effects in curved space and self-consistent background in terms of Einstein equations have to be considered [25]. Moreover, if we wish to investigate our models closely, we must consider dynamical quantum effects [26] too. Even thermal effects in a nonequilibrium system ought to be studied. Together with the previous problem of spatial dimensionality admitting stable compactification, the aspects of quantum and thermal effects will be studied extensively [27].
There are many other topics; the effect of $F^4$ term or higher-order terms in field strengths in the action suggested in the string theory; a search for nontrivial gauge configurations in other gauge groups on complicated compact spaces; the nontrivial structure of gauge fields on a flat compact space, such as a torus, which may have relation with the string theory; the reheating mechanism after the phase transition; the universe before the double compactification or “triple” compactification which explains the origin of gauge field and compact spaces. Each topic is an interesting subject of much worth to study.

Finally we mention the cosmological constant problem [28]. Coleman suggested a possible solution to the cosmological constant in four-dimensions in the framework of quantum gravity [29]. In his scenario even the fundamental couplings behave like dynamical variables through quantum-gravity effects. If we believe this scenario, we must re-examine our model; the algebraic relation to admit a flat-space solution might become an “equation of motion” of coupling “constants”, $\Lambda$, $\kappa^2$ and $e^2$.

Several authors made efforts to describe other theories of a Kaluza-Klein type in the context of quantum gravity [30]. We will report on this topic in relation to our model.

In this paper, we outlined the double-compactification scenario which provides the feasibility of causing inflationary expansion in our large dimensions. Many subjects to study further are left for future work. We must start from the investigation of this new scenario from various points of view.

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