Motivated by a host of empirical evidences revealing the bursty character of human dynamics, we develop a model of human activity based on successive switching between an hesitation state and a decision-realization state, with residency times in the hesitation state distributed according to a heavy-tailed Pareto distribution. This model is particularly reminiscent of an individual strolling through a randomly distributed human crowd. Using a stochastic model based on the concept of anomalous and non-Markovian Lévy walk, we show exactly that successive decision-making processes drastically slow down the progression of an individual faced with randomly distributed obstacles. Specifically, we prove exactly that the average displacement exhibits a sublinear scaling with time that finds its origins in: (i) the intrinsically non-Markovian character of human activity, and (ii) the power law distribution of hesitation times.

1. Introduction

The influence of individual behaviors on various complex problems of social activity represents a key target of modern social complexity science. A seminal work in this field is due to Barabási [1], however, many aspects of individual human dynamics remain far from being understood [2, 3]. The key pioneering achievement of Ref. [1] is the fact that generally human actions are not governed by the Poisson probability law, as it was conveniently assumed before to quantify various aspects of social human activity. Instead, the real distribution is heavy tailed and close to the
Pareto one \cite{1-3}. Here, we consider the implications of such power-law distributions of human activity and demonstrate that successive decision-making processes can drastically slow down individual movement through a randomly distributed human crowd.

The behavior of an individual in a crowd environment is a part of the more general problem of crowd behavior as a whole \cite{4,5}. Every individual in a crowd will certainly have his own goal and make the corresponding decisions independently resulting in so-called social forces \cite{4}. It has become an important problem for different researchers, from psychologists to architects and urban planners, to be able to predict crowd behaviors as well as its complex dynamics. Mainstream investigations were so far concentrated on the simulation of the crowd behavior as a whole complex system revealing collective effects and self-organizing phenomena \cite{6}. However, the human motion dynamics of a particular individual within a crowd has been relatively overlooked. Kirchner’s field-based model uses a stochastic approach to the problem of crowd dynamics with a focus on individual behaviors \cite{7}. Kirchner’s model takes inspiration from the process of chemotaxis developed by some social microorganisms \cite{8}, and accounts for two individual motives: (i) desire to move toward an initially established target, and (ii) desire to follow the others in a crowd. A similar idea underlies the active-walker models used for the simulation of trail formation in pedestrian dynamics \cite{9}. The above models, however, do not account for true individual decision-making, but rather deal with the common choice for all crowd members.

The ability of cognizant organisms to make decisions yields fundamental differences in their individual and group behaviors as compared to non-cognizant ones. Indeed, using all our sensory modalities, we collect all available external signals, process them and eventually make a decision based on a complex integration process, which is far from being clearly understood \cite{10}. The aim of this Letter is to show that successive decision-making processes can drastically slow down individual movement towards a goal. The concept of anomalous and non-Markovian Lévy walk \cite{11} is applied to the movement of an individual through a crowd—this individual being subjected to the necessity of making decisions at stochastically-distributed points in both time and space. Various diffusion models have been applied in the past to the study of human behavior involving decision-making processes \cite{12}. The most popular model, applied to a variety of two-choice reaction time paradigms, has been developed by Ratcliff \cite{13}, who essentially modified the Wiener diffusion process and applied it to various types of decision-making related phenomena \cite{14,15,16}.

\footnote{Those include lexical decision, short-term and long-term recognition memory tasks, same/different letter-string matching, numerosity judgements, visual-scanning tasks,
The above-mentioned model and its variations only address single-event two-choice decision making processes. Single-event multi-choice decision making processes have primarily been studied in the frame of practical problems such as tourists hesitation in route planning (see Refs. [17, 18] and references therein). In this Letter, we consider the more general case of a multi-event multi-choice continuous decision-making process as epitomized by the simple process of a man strolling through a crowded environment. However, the general formalism is actually applicable to many processes in which multiple decision-making events occur during a certain finite time interval—i.e. when “waiting time” (making the decision) alternates with “jump” (decision realization).

2. Stochastic model of movement through a crowd

Our stochastic model for the individual moving through a crowd is derived from the Lévy walk model [11], such that an individual can be in two distinct states: (i) “straight-motion” (s) state during which he moves towards a chosen target with constant velocity, say in the positive x-axis direction, and (ii) “hesitation” (h) state for which he is immobile while making a decision for further movement. Although pedestrian movements are two dimensional, here we restrict ourselves to a one-dimensional (1D) problem—x is the coordinate along a chosen direction towards a given target—to allow for a complete analytical study. The mechanism of switching between s and h states cannot be described deterministically, hence a stochastic s–h switching is considered. While in the s-state, the individual moves with constant speed v along the chosen direction during a random time $T_s$ before switching to the h-state, in which he stays immobile for a random time $T_h$ (decision time) until another switch to the s-state occurs. The random character of $T_s$ (resp. $T_h$) is fully characterized by its probability density function (PDF) $\psi_s(\tau)$ (resp. $\psi_h(\tau)$). The key concept of “hesitation” requires further elaboration and a univocal definition. Here, we adopt the definition by Cho et al. [19], where hesitation is the preliminary thought process preceding any decision-making process.

Let us now introduce the space-time PDFs for both states $P_s(t, x)$ and $P_h(t, x)$; non-Markovian switching processes [20] are considered given the importance of past actions in human dynamics and mobility [2]. We intend to derive the integro-differential equations for $P_s$ and $P_h$ considering specific residence time PDFs $\psi_s(\tau)$ and $\psi_h(\tau)$. In our non-Markovian

\[ \text{brightness discrimination etc. For a comprehensive review of the Ratcliff model: http://star.psy.ohio-state.edu/coglab/} \]
framework, the balance equations for $P_s$ and $P_h$ are

$$P_s(t, x) = \int_0^t i_h(t - \tau, x - v\tau)\Psi_s(\tau)d\tau + P_s^{(0)}(x - vt),$$

$$P_h(t, x) = \int_0^t i_s(t - \tau, x)\Psi_h(\tau)d\tau + P_h^{(0)}(x)\Psi_h(\tau),$$

where $P_s^{(0)}(x) = P_s|_h(0, x)$ and $\Psi_s|_h(t) = \int_t^\infty \psi_s|_h(\tau)d\tau$ denote the corresponding survival probabilities. The balance equations for the switching rates $i_s(t, x)$ and $i_h(t, x)$, between $s$ and $h$ states respectively, read

$$i_s(t, x) = \int_0^t \psi_s(\tau)\Psi_h(t - \tau, x - v\tau)d\tau + P_s^{(0)}(x - vt)\psi_s(t),$$

$$i_h(t, x) = \int_0^t \psi_h(\tau)i_s(t - \tau, x)d\tau + P_h^{(0)}(x)\psi_h(t).$$

The Master equations for $P_s$ and $P_h$ are obtained by differentiating Eqs. (1) and (2) with respect to time:

$$\frac{\partial P_s}{\partial t} + v\frac{\partial P_s}{\partial x} = -i_s(t, x) + i_h(t, x),$$

$$\frac{\partial P_h}{\partial t} = -i_h(t, x) + i_s(t, x).$$

The switching rates $i_s(t, x)$ and $i_h(t, x)$ can be found by means of the Laplace transform

$$i_s(t, x) = \int_0^t K_s(t - u)P_s(u, x - v(t - u))du,$$

$$i_h(t, x) = \int_0^t K_h(t - u)P_h(u, x)du,$$

where the memory kernels $K_s(t)$ and $K_h(t)$ have the standard representations

$$\tilde{K}_s(s) = \frac{\tilde{\psi}_s(s)}{\tilde{\Psi}_s(s)}, \quad \tilde{K}_h(s) = \frac{\tilde{\psi}_h(s)}{\tilde{\Psi}_h(s)},$$

the Laplace transform being denoted by the tilde superscript.

3. Characterization of the subdiffusive displacement

To quantify the walker’s displacement, we seek its average position $\langle x(t) \rangle$. In line with empirical observations on human dynamics reported in
Refs. [1,2], we take the hesitation time PDF $\psi_h(\tau)$ to be a power-law distribution:

$$\psi_h(\tau) \sim \left( \frac{\tau_h}{\tau} \right)^{1+\mu}, \quad 0 < \mu < 1$$

as $\tau \to \infty$, $\mu$ being the anomalous exponent and $\tau_h$ is a time scale. The Laplace transform $\tilde{\psi}_h(s)$ corresponding to (10) can be approximated by

$$\tilde{\psi}_h(s) \sim 1 - (\tau_h s)^\mu, \quad 0 < \mu < 1$$

for small $s$. As expected, the mean hesitating time $\langle T_h \rangle = \int_0^\infty \tau \psi_h(\tau) d\tau$ is infinite in this case. For the $s$ state, since we consider a randomly-distributed crowd, the process of facing an obstacle or a decision to make can be considered to be Poissonian. Thus the PDF $\psi_s(\tau)$ is assumed to be exponential:

$$\psi_s(\tau) = \nu_s e^{-\nu_s \tau},$$

with a constant and finite switching rate $\nu_s = 1/\langle T_s \rangle$. Its Laplace transform reads

$$\tilde{\psi}_s(s) = \frac{\nu_s}{\nu_s + s}.$$  

Let us show that our model predicts a subballistic behavior $\langle x(t) \rangle \sim t^\mu$ with $0 < \mu < 1$. The Laplace transform of the mean displacement $\langle x(t) \rangle$ is

$$\langle \tilde{x}(s) \rangle = t \frac{dP(s, k)}{dk} |_{k=0},$$

where $P(s, k)$ is the Laplace–Fourier transform of $P(t, x) = P_s(t, x) + P_h(t, x)$ defined by

$$P(s, k) = \int_\mathbb{R} \int_0^\infty e^{-ikx+st} P(t, x) dt \, dx.$$  

From Eqs. (5)–(8), $P(s, k)$ can be explicitly derived as

$$P(s, k) = P_s^{(0)}(k) \frac{\tilde{\Psi}_s(s + ikv) + \tilde{\Psi}_h(s)\tilde{\psi}_s(s + ikv)}{1 - \tilde{\psi}_h(s)\tilde{\psi}_s(s + ikv)} +$$

$$P_h^{(0)}(k) \frac{\tilde{\Psi}_h(s) + \tilde{\psi}_h(s)\tilde{\Psi}_s(s + ikv)}{1 - \tilde{\psi}_h(s)\tilde{\psi}_s(s + ikv)}.$$  

Using Eqs. (11) and (13) inside Eq. (16), together with $\tilde{\Psi}_{s|h}(s) = (1 - \tilde{\psi}_s|_h(s))/s$, we obtain the average position $\langle x(t) \rangle$ in the limit $t \to \infty$:

$$\langle x(t) \rangle \sim \frac{vt^\mu}{\Gamma(1 + \mu)\nu_s \tau_h^\mu}, \quad 0 < \mu < 1,$$
where $\Gamma$ denotes the classical Gamma function. Interestingly, the scaling of $\langle x(t) \rangle$ in (17) is sublinear. This is due to anomalous switching \textsuperscript{[20]} described by heavy-tailed hesitation residence time PDF (10) with infinite mean residence time. It is important to note that if the PDF $\psi_h$ were considered to be a short-tailed Poisson distribution, $\psi_h(\tau) = \nu_h e^{-\nu_h \tau}$, with a finite average hesitation time $\langle T_h \rangle = 1/\nu_h$, then the overall process would be Markovian with a linear ballistic scaling of the mean position $\langle x(t) \rangle$ on time $t$:

$$\langle x(t) \rangle = \frac{\nu_h}{\nu_h + \nu_s} vt.$$  \hspace{1cm} (18)

To gain further insight into the appearance of the nonlinear scaling in time $t^\mu$ in Eq. (17), one can also use the following idea. The average position $\langle x(t) \rangle$ can alternatively be found as the product of the average number of jumps $\langle N(t) \rangle$ from $h$-state to $s$-state, and the distance $v\langle T_s \rangle$ covered in the $s$-state—$\langle T_s \rangle = 1/\nu_s$ is the average time spent in the $s$-state according to the PDF (12) for $\psi_s$. Then

$$\langle x(t) \rangle = \frac{v\langle N(t) \rangle}{\nu_s}.$$  \hspace{1cm} (19)

It is well known from the renewal theory (see, e.g., \textsuperscript{[21,22]}), that the Laplace transform of $P(n,t) = \Pr(N(t) = n)$ is given by

$$\tilde{P}(n,s) = \frac{\tilde{\psi}_h(n)(1 - \tilde{\psi}_h(s))}{s}.$$  \hspace{1cm} (20)

Therefore, the Laplace transform of the average number of jumps from hesitation state $\langle N(t) \rangle$ is

$$\langle \tilde{N}(s) \rangle = \sum_{n=0}^{\infty} n\tilde{P}(n,s) = \frac{\tilde{\psi}_h(s)}{s(1 - \tilde{\psi}_h(s))}.$$  

It follows from (11) that $\langle \tilde{N}(s) \rangle \sim \tau_h^{-\mu} s^{-(1+\mu)}$ as $s \to 0$ and

$$\langle N(t) \rangle \sim \frac{t^\mu}{\Gamma(1 + \mu)\tau_h^\mu}.$$  

This formula together with (19) allows us to recover the sublinear scaling in time of $\langle x(t) \rangle$ previously obtained in (17).

Although we consider a simple model, it enables us to uncover the central fact that this sublinear scaling in time originates from the presence of memory in the agent’s dynamics. Indeed, we found that if past actions do not affect decision making, i.e. in the Markovian case, the average displacement given by Eq. (18) is purely ballistic. The importance of memory in human dynamics has already been highlighted based on a host of empirical evidences, albeit for processes occurring over much longer
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spatial and temporal ranges as compared to the ones in our study \cite{2,3,23}. For instance, the individual-mobility model proposed by Song et al. \cite{2,3} to explain some of these empirical evidences is essentially non-Markovian owing to both the exploration and preferential return mechanisms and was designed to capture the long-term spatial and temporal scaling patterns. In comparison, the present model investigates the short-term scaling in individual mobility that is not captured in Ref. \cite{2,3}.

Our 1D individual-stroller model has three parameters, $0 < \mu < 1$, $\tau_h > 0$ and $\nu_s = 1/\langle T_s \rangle$, the former two associated with the hesitation state while the latter fully characterizes the $s$ state. Values for those parameters could easily be obtained from empirical observations based on trackings of human strolling along a crowded narrow corridor. Other testable experiments highly relevant to the present study abound in the field of sports science, and more specifically with the study of some team sports from the standpoint of complex dynamical systems \cite{24,25}. Indeed, with the aim of improving sports performance, scientists have used video-based and electronic tracking systems to study space-time coordination dynamics during basketball and soccer games among others \cite{24,25}. To the best of our knowledge, the data gathered are mostly analyzed in a compound way, hence delivering team performance indicators. Apparently, without any change in methods, individual players kinematics could be analyzed, thereby providing the PDFs $\psi_s$ and $\psi_h$, along with the associated values for the three parameters of our model. It is worth adding that data from basketball games would provide an excellent match with the details of our model, given that, typically, basketball players switch from periods of straight forward running ($s$ state) with periods during which they are immobile, dribbling, and hesitating ($h$ state) before eventually completing a pass.

The slowed-down dynamics that emerges from successive decision-making processes should obviously not impugn the role and importance of sensory modalities in real-life decision-making situations. Interestingly, our model reveals the counterintuitive fact that making multiple (even seemingly right) decisions over a long period of time contributes to slowing us down in reaching our goal. This important fact applies to an individual making his way through a dense crowd, as well as a basketball player on the court or, more generally, to any agent whose non-Markovian dynamics alternates between two states such that the residency time in one of these two states is heavy tailed.
4. Conclusion

Returning to our initial focus on human traffic flow, our analytical study offers unique insights into the origin of the slowed-down dynamics of individuals moving about a space in the presence of obstacles [26,31]. For instance, in Ref. [30] the authors conducted a series of experiments with a group of pedestrians walking through angled corridors. Specifically, they investigated the dependence of the overall speed of an ‘average’ pedestrian on the corridor angle. It appears that the speed is strongly dependent on the angle value and decreases drastically at some critical value $\theta_c$. The latter varies in a range between $\pi/3$ and $\pi/2$ depending on the pedestrian’s motivation. Obviously, in the particular case of an individual walker meandering through a multi-angled corridor the overall dynamics is slowed down and the speed is highly dependent on the actual sequence of angles. Although this specific experiment does not exactly match our model—one-dimensional curvilinear path with angles analogous to randomly distributed individuals in a given human crowd—the dynamics at play can readily be understood in our framework. Indeed, in our case, the individual’s straightforward motion is disrupted by the presence of fellow crowd members leading to sequences of hesitation periods of varying durations distributed according to a power law. In the experimental framework of Ref. [30], the individual is compelled to execute successive different decision-making processes owing to the different values of the angles imposed. This leads to successive different decision realizations, which is essentially equivalent to different hesitation periods followed by decision realizations as per our model.

Acknowledgments

Present work was financially supported by the SUTD-MIT International Design Centre (IDC). SF gratefully acknowledges the support of the EP-SRC under Grant No. EP/J019526/1.

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