Dilaton Stabilisation in $D$-term Inflation

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Abstract

Dilaton stabilisation is usually considered to pose a serious obstacle to successful $D$-term inflation in superstring theories. We argue that the physics of gaugino condensation is likely to be modified during the inflationary phase in such a way as to enhance the gaugino condensation scale. This enables dilaton stabilisation during inflation with the $D$-term still dominating the vacuum energy at the stable minimum.
Due to its intrinsic elegance, inflation has become the almost universally accepted dogma for accounting for the flatness and homogeneity of the universe. There are various classes of inflation that have been proposed, but possibly the most successful, and certainly one of the most popular versions these days is hybrid inflation. In hybrid inflation, there are (at least) two fields at work: the slowly rolling inflaton field $I$, and a second field which we shall call $\psi$ whose value is held at zero during inflation and whose role is to end inflation by developing a non-zero vacuum expectation value (VEV) when $I$ passes a certain critical value during its slow roll. With $\psi = 0$, the potential along the $I$ direction is approximately flat, with the flatness lifted by a $I$ mass which must be small enough to satisfy the slow-roll conditions for inflation.

The natural framework for hybrid inflation is supersymmetry (SUSY). Supersymmetry can naturally provide flat directions along which the inflaton can roll, and additionally ensures that the scalar inflaton mass does not have quadratic divergences. The natural size of the inflaton mass in SUSY is of order the SUSY breaking scale, and the slow roll conditions and COBE constraints then determine the height of the potential during inflation $V_0^{1/4}$ to be some intermediate scale below the grand unification (GUT) scale. To be precise, during inflation we may set the field $\psi = 0$ so that, assuming a flat direction, the potential simplifies to:

$$V = V_0 + \frac{1}{2} m_I^2 I^2.$$  

The slow roll conditions are given by:

$$\epsilon_N = \frac{1}{2} M_{Pl} \left( \frac{V''}{V} \right)^2 = \frac{1}{2} \frac{M_{Pl}^2 m_I^4 I_N^2}{V_0^2} \ll 1,$$  

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\[ |\eta_N| = M_{Pl}^2 \left( \frac{V''}{V} \right) = M_{Pl}^2 \frac{|m^2_I|}{V_0} \ll 1. \]  

The subscripts \( N \) means that \( I \) and \( \epsilon \) have to be evaluated \( N \sim 50 \) e-folds before the end of inflation, when the largest scale of cosmological interest crosses the horizon.

Here \( M_{Pl} \equiv \frac{M_P}{\sqrt{8\pi}} = 2.4 \times 10^{14} \) GeV is the reduced Planck mass. The spectrum of the density perturbations is given by the quantity \[ \delta^2_H = \frac{32}{75} \frac{V_0}{M_{Pl}^2} \frac{1}{\epsilon_N}, \]  

with the COBE value, \( \delta_H = 1.95 \times 10^{-5} \) \cite{4}, expressed in terms of the (unreduced) Planck mass.

If one accepts the above framework, one is driven almost inevitably to supergravity (SUGRA). The origin of the vacuum energy \( V_0 \) which drives inflation can only be properly understood within a framework which allows the possibility for the potential energy to settle to zero at the global minimum, and hence lead to an acceptable cosmological constant, and this implies SUGRA. In this way we are led to consider SUGRA models in which the fields are displaced from their global minimum values during inflation. The potential in SUGRA is given by:

\[ V = e^G \left[ G_i (G_j)^{-1} G^{ij} - 3 \right] + V_D \]  

in the usual notation, i.e. natural units where \( M_{Pl} \) is set equal to unity, with subscripts \( i \) (\( \bar{i} \)) referring to partial differentiation with respect to the generic field \( \phi_i \) (\( \phi_i^\ast \)). We have written the \( D \)-terms very schematically as \( V_D \). The Kähler function \( G \) is:

\[ G = K + \ln |W|^2, \]  

where the Kähler potential \( K \) is a real function of generic fields \( \phi, \phi^\ast \) and the superpotential \( W \) is an analytic (holomorphic) function of \( \phi \) only. The Kähler metric is \( G_i^{\bar{j}} \)
and its inverse satisfies \((G^i_j)^{-1}G^j_k = \delta^i_k\). The SUGRA \(F\)-terms are

\[
F^i = e^{G/2}(G^i_j)^{-1}G^j_i,
\]

whose non-zero value signals SUSY breaking, with a gravitino mass

\[
m_{3/2} = e^{G/2}.
\]

Using the preceding results we can schematically write the potential in SUGRA as

\[
V = |F|^2 + V_D - 3m_{3/2}^2M_{Pl}^2,
\]

where we have put back the reduced Planck mass. Eq. (7) shows that there are two possible sources for the positive vacuum energy \(V_0\) which drives inflation: the \(F\)-term or the \(D\)-term. The negative term also allows for eventual cancellation of the potential energy, and is the main motivation for considering SUGRA. If the \(D\)-terms are zero and the \(F\)-terms of the same order of magnitude during inflation as they are at the end of inflation (in order to cancel the vacuum energy and lead to an acceptable cosmological constant) we have

\[
V_0 = |F|^2 - 3m_{3/2}^2M_{Pl}^2 \sim m_{3/2}^2M_{Pl}^2 \sim (10^{11}\text{ GeV})^4.
\]

Here the gravitino mass is assumed to be given by,

\[
m_{3/2}^2 = \langle e^G \rangle = e^K|W|^2 \sim (1\text{ TeV})^2,
\]

Now if we make the further assumption that the inflaton \(\phi\) mass is of order \(m_{3/2}\), this line of reasoning leads to a violation of the slow roll condition in Eq. (3); in fact from Eq. (10) we predict \(|\eta_N| \sim 1\). This is the so-called \(\eta\) or “slow roll” problem.
An attractive solution to the $\eta$ problem is to suppose that the energy which drives inflation originates from the $D$-term \[3\]. Then $V_D$ is allowed to take a higher value than that in Eq. (10), providing it cancels to zero at the end of inflation, thereby solving the $\eta$ problem. In a string-inspired toy model one introduces an anomalous $U(1)_X$ gauge symmetry, with the fields $\phi^\pm$ having charge $\pm1$ while the inflation $I$ carries no anomalous charge, and couples with a superpotential,

$$W = \lambda I \phi^+ \phi^-.$$ (12)

The $D$-term in such a theory is given by

$$V_D = \frac{g_X^2}{2} (|\phi^+|^2 - |\phi^-|^2 + \xi)^2,$$ (13)

where $\xi$ is the Fayet-Iliopoulos $D$-term related to the Green-Schwarz mechanism of anomaly cancellation in string theories \[4\]

$$\xi = \frac{g_X^2}{4} \delta_{GS} M_{Pl}^2$$ (14)

and

$$\delta_{GS} = \frac{1}{192\pi^2} \text{Tr} Q_X.$$ (15)

The idea is that during inflation the inflaton exceeds some critical value which results in $\phi^+ = \phi^- = 0$ and $V_D \neq 0$, and inflation is ended when the critical value of $I$ is reached and allows $\phi^-$ to develop a VEV which cancels the $D$-term. Assuming that, during inflation, the $D$-term energy dominates over the $F$-term, the required COBE normalisation may be estimated to be \[7\]:

$$\sqrt{\xi_{\text{COBE}}} = 6.6 \times 10^{15} \text{ GeV}.$$ (16)
From the point of view of conventional string theory this scale looks rather low, however it may be possible to introduce additional $U(1)$’s to ameliorate this problem \[8\]. Some level of flexibility may be allowed in the case in which, in the strong coupling limit, the ten-dimensional $E_8 \otimes E_8$ heterotic string can be described as the compactification of an eleven-dimensional theory known as M-theory \[9\]. When the ten-dimensional heterotic coupling is large, the fundamental eleven-dimensional mass parameter $M_{11}$ becomes of the order of the unification scale and it might be that the value of the Fayet-Illiopoulos $D$-term may be reduced to a value close to the one required by COBE. However, to our knowledge no explicit example has been constructed so far.

In the context of string theories there is a further problem with $D$-term inflation which forms the focus of this paper, namely the problem of dilaton stabilisation. In weakly-coupled string theory the gauge coupling of $U(1)_X$ is given by

$$g^2_{X} = \frac{2}{S + S^*},$$

where $S$ is the dilaton field whose VEV is supposed to determine the gauge couplings. Thus in terms of the dilaton field the potential during inflation is given by:

$$V_D = \frac{\delta^2_{GS} M_{Pl}^4}{4(S + S^*)^3}.$$ (18)

The problem is that, assuming that $V_D$ dominates over $V_F$, the potential during inflation appears to prefer $s \equiv S + S^* \to \infty$ and $V_D \to 0$, where we have defined $s$ to be twice the real part of $S$. This is the $D$-term inflation equivalent of the dilaton runaway problem that appears in string theories \[3\].

\[3\]Other moduli fields, like the moduli $T$, may have the same problem, but the $T$ run-away problem may disappear in some particular models if the one-loop thresholds corrections are taken into account \[10\]. In such a case the moduli $T$ is stabilized around the string scale.
In the context of gaugino condensation the dilaton runaway problem may find, however, a solution \cite{11}. In $E_8 \otimes E_8$ superstring theory one of the gauge factors is supposed to provide a hidden sector providing a natural mechanism for breaking SUSY via gaugino condensation. In the effective four-dimensional theory one may have in natural units a Kähler potential

$$K = -\ln(S + S^*) - 3 \ln(T + T^* - \phi^* \phi),$$

(19)

where $T$ is a modulus field $\phi$ is some generic matter field, and an effective superpotential of the form

$$W = e^{-3S/2b_0} - c,$$

(20)

where $b_0 = \frac{3N_c - N_f}{16\pi^2}$ is the $\beta$-function of the hidden sector gauge group $G_h$ defined by the renormalization group equation

$$\mu \frac{\partial g_h}{\partial \mu} = -\frac{3}{2} b_0 g_h^3,$$

(21)

being $g_h$ the gauge coupling constant of $G_h$ and $\mu$ the renormalization scale.

The origin of this effective superpotential for the dilaton field is the following. In $E_8 \otimes E_8$ superstring theories, a strongly interacting gauge theory undergoes gaugino condensation,

$$\langle {\bar{\chi}} \chi \rangle \equiv \Lambda_c^3 = M_{Pl}^3 e^{-3S/2b_0}.$$

(22)

Notice that the dynamical scale $\Lambda_c$ carries an anomalous $U(1)_X$ charge since under a $U(1)_X$ gauge transformation with parameter $\alpha$ the dilaton transforms as $S \rightarrow S + \frac{i}{2} \delta_{GS} \alpha$.

If one takes into account the coupling to the field strength $F_{\mu \nu \rho}$ of the ten-dimensional second-rank tensor field $a_{\mu \nu}$, the terms in the lagrangian that depend
upon $F_{\mu\nu\rho}$ and $\text{Tr}[\chi\Gamma_{\mu\nu\rho}\chi]$ form a perfect square. This means that the contribution of gaugino condensation to the vacuum energy is canceled by $\langle F_{\mu\nu\rho} \rangle \sim \langle F_{ijk} \rangle \sim c\epsilon_{ijk}$. The resulting potential may be reproduced in four-dimensions by the effective superpotential of the form (20). The constant $c$, since it appears as an additive term in $W$, it is not renormalized.

Now, the $F$-term part of the potential which results from Eqs. (5), (6), (19) and (20) is

$$V_F = \frac{1}{(S + S^*)(T + T^* - \phi^*\phi)^3} \left| (S + S^*) \frac{\partial W}{\partial S} - W \right|^2 - 3e^G = \frac{1}{(S + S^*)(T + T^* - \phi^*\phi)^3} \left| e^{-3S/2b_0} \left[ 1 + \frac{3(S + S^*)}{2b_0} \right] - c^2 \right|. \quad (23)$$

The no-scale structure means that the part $-3e^G$ of the potential has cancelled. In the limit $c \to 0$ (or, equivalently, in the limit in which the gauginos do not condense) the potential is minimised at zero by the infinite runaway dilaton solution. However with a finite positive $c$ a second minimum develops also with zero energy at finite $S$, thereby stabilising the dilaton potential in this framework.

Physically we might wish to stabilise the dilaton such that $\alpha_{GUT} = 1/24$ which would require $s = S + S^* = 4$. If the dilaton provides the dominant source of SUSY breaking via the gravitino mass $m_{3/2} \sim 1$ TeV then we might also wish to arrange that the gaugino condensate scale is given by $\Lambda_c = e^{-S/2b_0} = e^{-s/4b_0} \sim 10^{-6}$ which would imply $b_0 \sim 0.07$, where we have assumed that the imaginary part of $S$ is zero. One also finds that $c \sim e^{-3S/2b_0} \left( 1 + \frac{3(S + S^*)}{2b_0} \right) \sim 10^{-16}$ in natural units. Thus in the region of the minimum the typical scale of the potential is of order $V_F \sim c^2 \sim 10^{-32}$, although exactly at the minimum point we have $V_F = 0$. By comparison the COBE normalised $D$-term from an anomalous $U(1)_X$ as in the $D$-term inflation scenario
would be $V_D^{\text{COBE}} = 1.4 \times 10^{-11}$ in natural units (corresponding to a height of $4.6 \times 10^{15}$ GeV) which is 21 orders of magnitude larger! Thus at first sight it would appear that, even if one accepts the above mechanism for dilaton stabilisation, it is inadequate to resolve the dilaton problem during $D$-term inflation.

This conclusion, however, is based on the assumption that during the inflationary phase the values of $b_0$ and $c$ which characterise the hidden sector are the same as those which characterise the hidden sector in the present non-inflationary epoch. The main observation of the present paper is that, clearly, they need not be, i.e the value of the $\beta$-function during inflation $b_0^{\text{inf}}$ may be much different from its present day value $b_0^{\text{tod}}$. For example the inflaton $I$ may couple to some hidden sector matter superfields thereby giving them a large effective mass during inflation, and effectively decoupling them from the $\beta$-function. By reducing $N_f$ the value of $b_0$ can be increased, $b_0^{\text{inf}} \gg b_0^{\text{tod}}$ and hence the gaugino condensate $\Lambda_c$ and $c$ can both be increased by orders of magnitude, thereby allowing the dilaton to be stabilised during $D$-term inflation. A typical example may be provided by SUSY-QCD based on the gauge group $SU(N_c)$ in the hidden sector with $N_f \leq N_c$ flavors of “quarks” $Q^i$ in the fundamental representation and “antiquarks” $\bar{Q}_i$ in the antifundamental representation of $SU(N_c)$. If the inflaton field $I$ gets very large values, of the order of $M_{\text{Pl}}$, and it couples to (some of) these matter superfields in the superpotential via $IQ^i\bar{Q}_i$, the corresponding value of $b_0$ will be naturally changed. This means that during inflation the values of the gaugino condensation scale $\Lambda_c$ and the constant $c$ are different from the values they acquire in the present true vacuum. Indeed, during inflation, the gauge group may undergo gaugino condensation at a scale larger than the traditional $10^{13}$ GeV and this will balance the $D$-term driving inflation and stabilize the dilaton $S$. 

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For definiteness let us consider that during inflation the total potential is given by

\[ V_T = V_D + V_F \]  

(24)

where \( V_D \) is given in Eq. (18) and assume for definiteness that \( T + T^* = 1 \). We assume that the \( D \)-term is COBE normalized, i.e. \( V_D \) is of order \( 10^{-11} \) in natural units. We also assume that \( s = 4 \) both during inflation and after. As discussed we shall suppose that the value of \( b_0^{\text{inf}} \) during inflation can be increased sufficiently that the \( F \)-term dilaton stabilisation will stabilise the total potential \( V = V_F + V_D \). In fact the precise value of \( b_0^{\text{inf}} \) is not crucial, providing it is not so small that the potential is not stabilised, or so large that \( V_F \) dominates. Suitable typical values are \((0.2 - 0.25)\).

For example \( b_0^{\text{inf}} = 0.2 \) corresponds to a gaugino scale \( \Lambda_c \sim 7 \times 10^{-3} \) with the potential stabilised at \( s = 4 \) by a constant term \( c \sim 10^{-5} \). Since \( V_F \) is very close to zero at the minimum, it is obvious that the value of \( V_D \) will dominate at this point as is required in \( D \)-term inflation. We find that at the minimum, \( V_D \approx 10^{-11} \) in natural units with \( V_F \approx 7 \times 10^{-14} \) so that \( V_D \) dominates during inflation as desired. The behavior of the three potentials \( V_F, V_D, V_T \) in the inflationary phase with the above choice of parameters is given in Fig.1.

The above values correspond to the case in which the dilaton is stabilised during inflation at its correct present physical value \( s = 4 \). Of course, one might envisage situations where, during inflation, it is not sitting at its present value, but our assumption represents the most economical and surely the most harmless one. If for example the dilaton were stabilized at large values during inflation by gaugino condensation, \( s \gg 1 \), it would run away to infinity after inflation when \( b_0 \) assumes its present value \( b_0^{\text{inf}} \), with the dilaton separated from the true physical value \( s = 4 \) at
Figure 1: The behaviour of the potential $V_T$ (dark) and its component potentials $V_F$ (medium) and $V_D$ (light) as a function of $s$ for $b_0^{\text{inf}} = 0.2$ with the potential stabilised at $s = 4$ by a constant term $c \sim 10^{-5}$. 
the minimum of the potential by an insurmountable barrier. Our assumption clearly avoids this.

One may wonder how the perfect square approach to dilaton stabilisation survives in strongly coupled theories such as M-theory. This question has recently been addressed by a number of authors \[13\]. The answer seems to be inconclusive. What happens is that the effective superpotential receives additional corrections, and in particular develops a $T$ dependence with $S$ replaced by $S + \alpha T$. The presence of the $T$ dependence violates the no-scale structure of the potential which is no longer a perfect square. However it is possible that dilaton stabilisation survives even though the potential may no longer be zero at the minimum. Another effect of M-theory is to split the gauge couplings in the hidden sector $g_h$ from the observable sector $g_o$ with:

$$g_o^2 = \frac{2}{s + \alpha t}, \quad g_h^2 = \frac{2}{s - \alpha t}. \quad (25)$$

Thus the hidden sector gauge coupling $g_h$ may be naturally larger than in weakly coupled heterotic string theories. This feature means that the values of both $b_{0}^{\text{rad}}$ and $b_{0}^{\text{inf}}$ will be smaller in these theories than the values we estimated earlier. In particular the value of $b_{0}^{\text{inf}} \sim 0.2$ which may seem rather large may be reduced to a more comfortable value in M-theory.

To summarise, our mechanism for dilaton stabilisation in $D$-term inflation is based on a particular mechanism for $F$-term dilaton stabilisation in the weakly coupled heterotic string, namely to appeal to hidden sector gaugino condensation in the presence of the field strength $F_{\mu\nu\rho}$ which results in the perfect square form of the potential, and the cancellation of the vacuum energy for a particular value of the dilaton field. We have pointed out that this same mechanism to stabilise the total potential can be
used during inflation, since the scale of gaugino condensation during the inflationary phase may be much higher than in the physical vacuum so that $V_F$ may be increased to of order $V_D$ in the vicinity of the minimum. This allows usual $D$-term inflation to proceed, and answers the question of dilaton stabilisation both during and after inflation. Finally we point out that a similar idea may be used for any situation in which the dilaton is stabilised in the $F$-term of the potential. For example there may be non-perturbative corrections to the Kähler potential of the dilaton which serve to stabilise the dilaton potential (for a particular parametrisation see ref. [12]). Clearly the strength of such non-perturbative corrections may differ between the inflationary phase and the present vacuum, and so one could envisage an analogous scenario to that presented here in which the total potential is stabilised during inflation by such non-perturbative effects.

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