Wideband Corrugated Feedhorns, for Radar, Communications, Radiometry and Quasi-Optics

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ABSTRACT A wide variety of desirable antenna beam patterns can be synthesized by optimal excitation and phasing of the HE\textsubscript{11} and HE\textsubscript{12} modes in scalar corrugated feedhorns. However, the bandwidth of such two-mode horns is often limited by modal dispersion. In this paper we introduce a class of low dispersion, two-mode feedhorns that can operate, in some cases, over operating bandwidths of 40-50%. We provide example designs that include horns with high coupling efficiency to: 1) a pure HE\textsubscript{11} mode for single-mode excitation of corrugated pipe transmission lines; 2) a LG\textsubscript{00} and LG\textsubscript{02} combination for radiometry, with narrow beams; 3) a pure Laguerre Gaussian LG\textsubscript{00} mode for quasi-optical instrumentation with constant phase centers; 4) a constant gain antenna for uniform illumination with frequency; 5) Airy patterns or “top hat” patterns for radar or communications applications, designed to maximize aperture efficiencies when used with larger reflect or lens antennas. More generally, we show methods to generate and phase multiple HE\textsubscript{1n} modes, to synthesize symmetric output beams at any desired frequency or gain.

INDEX TERMS Aperture antennas, antenna feeds, corrugated horns, horn antennas, quasi-optics, ultra-wideband antennas.

I. INTRODUCTION

Corrugated feedhorns have long been the gold standard for feed antennas that require the highest levels of beam quality combined with the lowest levels of cross-polarization. They find numerous applications in quasi-optical instrumentation, in communication systems, astronomical telescopes, imaging radar and radiometry for remote sensing.

Corrugated feedhorns are essentially a series of mode transformers. The standard design usually involves transforming from an input rectangular TE\textsubscript{10} waveguide mode to a circular TE\textsubscript{11} mode and then to a dominant HE\textsubscript{11} mode at the horn throat. The horn profile then determines the amplitude of higher order modes excited within the horn and their relative phasing at the aperture that in turn determine the final radiation beam pattern.

Today, beam patterns from corrugated feedhorns can be modelled with extraordinary accuracy using mode matching software and there is now a significant literature on their design and optimization including a number of books and review articles [1]–[4]. Often, initial designs are based on general geometric arguments and improved by computational optimization programs [5]. This has encouraged a view that corrugated feedhorn design is largely a solved problem. However, practical experience has shown this is simply not true, especially over wide bandwidths. Due to the very large parameter space it is nearly always necessary to start from a near optimal solution, strongly informed by previous designs, and guided by the intuition of the design engineer [5]. There are always trade-offs in performance, and it is rarely clear from first principles if published designs are close to optimal.
This paper aims to establish optimal mode sets for common applications, and provide example designs that excite these mode sets, with low dispersion, over wide bandwidths.

The waveguide modes in corrugated feedhorns are most conveniently expressed as a set of $HE_{1n}$ and $EH_{1n}$ modes but have often been (less helpfully) expressed as set of $TE_{1n}$ and $TM_{1n}$ modes in the literature, as it is usually these modes that are directly calculated in mode matching software.

All advanced corrugated feedhorn designs involve excitation of higher order modes within the feedhorn and a number of wideband designs have been described in the literature. Teniente has described a wideband design based on two stacked Gaussian profiles with $< -30$ dB sidelobes over $40\%$ bandwidths [6]. Dual profile designs covering $20\%$ and $30\%$ bandwidths have been described for the LFI [7] and HFI [8] of the Planck telescope. These horns are part of a wider family of Gaussian profiled horns whose bandwidth is discussed in [9]. Other wideband designs covering $50\%$ bandwidths, mainly targeting aperture efficiency, have also been described for the ALMA telescope [10].

However, virtually no papers tackle the design problem from the point of view of first identifying the optimal $HE_{1n}$ mode set optimal for a given application. We are also not aware of any that then explicitly try to minimize modal dispersion to maximize bandwidth for a specific desired excitation. This is the design approach we have taken here, where we believe we demonstrate significantly improved wideband performance relative to previous designs in the literature.

In most practical cases, the mode conversion process in a corrugated horn is dominated by the $HE_{11}$ mode converting to higher order $HE_{1n}$ or $EH_{1n}$ modes. The coupling $\Lambda_{1n}$ of the $HE_{11}$ mode to a different $HE_{1n}$ mode, at any point in the horn, is proportional to the slope of the profile $dr/dL$ divided by the profile cross-section radius $r$ where $L$ is the longitudinal distance along the horn axis [11], given by:

$$\Lambda_{1n} \propto \frac{dr/dL}{r}$$  \hspace{1cm} (1)

with different constants of proportionality for different modes (lower for higher order modes). Thus, higher order modes will always be excited within a corrugated horn for any non-trivial profile. In this paper the normalized amplitude of a $HE_{1n}$ mode, at a given radius $r$, is specified as $a_{1n}$ where $\Sigma a_{1n}^2 \sim 1$ (neglecting the usually very small contribution from $EH$ modes). The relative phase of the $HE_{1n}$ mode with respect to the $HE_{11}$ mode is specified as $\theta_{1n}$. At any given point an excited $HE_{1n}$ mode can interfere constructively or destructively with an existing propagating $HE_{1n}$ mode, depending on their relative phases. Thus, it is possible to design profiles where certain modes interfere destructively and cancel at the aperture, while favorable modes are enhanced. It is also possible to bring any two given modes into phase at the aperture by simply adding a straight parallel length of corrugated guide, where no extra higher order mode excitation takes place.

In a traditional long conical corrugated horn with a constant slope profile, a small amount of $HE_{12}$ mode will always be excited and end up approximately $90^\circ$ out of phase with the $HE_{11}$ mode at the aperture. It is the relative phasing of this extra mode that accounts for both the curved phase front at the aperture and the position of the phase center inside the horn. For compact horns with wide flare angles, this phasing can also lead to a degradation in the output beam profile with respect to a pure $HE_{11}$ mode design. For this particular case, methods to reduce $HE_{12}$ excitation can improve beam quality, as previously described by Clarricoats et al. [2] and Olver and Xiang [12].

As an alternative, it is possible to add a parallel section to a conical horn (or other profiles) that adds another $270^\circ$ of relative phase shift to bring the two modes back into phase at the aperture. If the amplitude of the $HE_{12}$ mode is chosen correctly this can significantly improve the quality of the beam. This is the design principle behind the $\sin^2$-parallel horn we have described before [13], which is now used in many quasi-optical systems. This horn is relatively easy to design, and typically produces highly symmetric beams with $-37$ dB to $-40$ dB sidelobes at the center frequency (limited by unwanted excitation of the $HE_{13}$ mode). This is, in effect, a variation on the well-known smooth walled Potter horn design, and works well over $20\%$ bandwidths, limited by modal dispersion.

We have also described a more compact version of this design with a tanh-linear profile that optimized the mode amplitudes for both the $HE_{12}$ and $HE_{13}$ modes ($0.31$ and $0.03$ respectively) [14]. We tested a $20$ dBi horn using this design, which had $>99.95\%$ coupling to a fundamental Gaussian beam with sidelobes at the $-60$ dB level at $94$ GHz, and sidelobes close to $-40$ dB over a $16\%$ bandwidth [15]. Now although the bandwidth and performance of this type of horn is significantly better than a Potter horn, the effective bandwidth is nevertheless still limited by modal dispersion, which causes beam distortion and movement of phase centers at the band edges.

The design challenge for wideband horns is thus to excite an $HE_{12}$ mode with a target amplitude $a_{12}$ and relatively constant phase $\theta_{12}$ (usually $|\theta_{12}| < 35^\circ$) over a wide frequency range, whilst reducing excitation of all other unwanted higher modes, usually the $EH_{11}$ and $HE_{13}$ modes.

In this paper, we propose a set of dual-profiled horn designs that have very low modal dispersion and hence can operate over $40-50\%$ fractional bandwidths.

In many of these two-section horns the first section excites a pure $HE_{11}$ mode over a wide frequency range at the approximate desired aperture radius $r_0$. The second section of the horn then excites the required higher order modes and brings them into phase (or out of phase) with the $HE_{11}$ mode at the aperture.

Section II first identifies the optimal $HE_{1n}$ mode excitations for common applications and shows that it is often sufficient just to excite and correctly phase the $HE_{12}$ mode with respect to the $HE_{11}$ mode. Section III examines how the
HE\textsubscript{12} mode content defines the effective phase center and beam waist, and thus the gain of the horn, as well as discussing other important metrics of horn performance such as compactness, return loss and cross-polar performance. Section IV then provides a number of illustrative wideband designs for a variety of applications.

All data shown in this paper are based on horn simulations using the mode matching software CORRUG (S.M.T. Consultancies Ltd.),\textsuperscript{1} which derives the full scattering matrix at each corrugation. Simulations of co-polar and cross-polar beam patterns from this package have been experimentally validated many times by multiple international groups over a wide range of frequencies. For example, projects which feature CORRUG designed feedhorns include: the HFI Planck space telescope, which operates in 8 bands from 100 GHz to 800 GHz [8]; a limb sounder at 600 GHz for the International Space Station [16]; and more recently in the design of the quad-band TROPICS pathfinder cubesat system [17]. We have previously used CORRUG to design feedhorns with sidelobes as low as $-60$ dB [15]. In each case excellent agreement was found with measured antenna patterns. It should be noted that full wave simulation packages such as HFSS or CST Microwave Studio are less suited to optimizing corrugated feed designs, as they take considerably longer to simulate beam patterns accurately, especially for some of the relatively high gain ($\sim 25$ dBi) horns used to illustrate performance in this paper.

II. OPTIMAL HE\textsubscript{1N} MODE SETS

For many wideband applications it is sufficient to consider excitation of just the HE\textsubscript{11} and HE\textsubscript{12} modes, and only exceptionally the HE\textsubscript{13} mode. In all applications considered here, the HE\textsubscript{12} mode should ideally be in phase ($\theta_{12} = 0^\circ$), or out of phase ($\theta_{12} = 180^\circ$), at the aperture. The amplitudes $a_{12}$ of the HE\textsubscript{12} excitation is chosen dependent on the application. Fig. 1 shows the effect on the aperture mode pattern of exciting just the HE\textsubscript{11} mode and HE\textsubscript{12} modes for four different HE\textsubscript{12} amplitudes: $a_{12} = 0$, $a_{12} = +0.31$, $a_{12} = -0.37$ and $a_{12} = +0.81$ where a plus sign indicates the two modes are in phase and a minus sign indicates they are $180^\circ$ out of phase at the aperture.

Fig. 2 then shows the resulting expected far field beam patterns. These respectively produce, the typical beam pattern obtained from a HE\textsubscript{11} mode, a good approximation to a fundamental Gaussian mode, a “top hat” pattern and an Airy pattern. Note the Airy and “top hat” patterns are effectively Fourier transform pairs. These beam patterns are of interest when the aperture efficiency of a system is the primary design goal.

A. PURE HE\textsubscript{11} HORN

Excitation of a pure HE\textsubscript{11} mode (i.e., $a_{11} = 1$, $a_{1n} = 0$ for $n > 1$) is useful whenever the design goal is to transmit power over relatively long distances, with very low loss, using a corrugated pipe. Corrugated pipe transmission lines are commonly used in fusion plasma heating and diagnostics, high power mm-wave radar systems, high frequency electron magnetic resonance and dynamic nuclear polarization experiments. If other higher order HE\textsubscript{1n} modes are excited within the pipe these modes will continuously move in and out of phase with the HE\textsubscript{11} mode, making the output beam pattern a function of frequency and pipe length. Pure HE\textsubscript{11} mode horns and pipes are also potentially useful in horn-to-horn, quasi-optical, materials characterization measurements using VNAs. In these systems it is important to avoid higher order modes being trapped resonantly inside the system and to have a constant phase center at the horn aperture. A feed of radius $r_0$ that produces a pure HE\textsubscript{11} mode will couple with 98% efficiency to a fundamental Laguerre Gaussian LG\textsubscript{00} mode with a beam waist radius of 0.6435...
mode, for a VOLUME 3, 2022 973
LG
00
Fig. 3 indicates the coupling efficiency to the LG
00
horns of length
mode, over extended bandwidths. In Section IV we describe
extra LG
00
vantageous as the subtle change of beam profile caused by
mode with 1.6% efficiency. This is not necessarily disad-
r
0
FIGURE 3. Maximal coupling efficiency, as a function of HE12 mode amplitude: to a
LG
00
mode (solid line), to a LG
00
and LG
02
mode (dashed line), and LG
00
, LG
01
and
LG
02
modes (dotted line). It can be seen that for
−0.05 < a12 < +0.65 the output can be approximated as coupling to a fundamental LG
00
mode with a small LG
02
content. Outside of that range from
−0.25 < a12 < +0.8, the LG
01
mode makes a significant
contribution.

r0
positioned exactly at the aperture, where
r0
is the aper-
ture radius. This creates an output beam with sidelobes at
the −28 dB level. This beam will also couple to the LG
02
mode with 1.6% efficiency. This is not necessarily disad-
vantangeous as the subtle change of beam profile caused by
extra LG
02
Gaussian mode content can lead to improved
antenna efficiency with reflection optics, relative to a pure
Gaussian beam. Equation (1) shows that in general any gra-
dient within a horn profile will excite higher order modes.
Thus, a specific profile is required to excite a pure HE11
mode, over extended bandwidths. In Section IV we describe
horns of length \( \sim 3r_0^2/\lambda_c \) with HE11 coupling efficiencies
above 99.75% across full waveguide bands, where
r0
is the
horn aperture radius and \( \lambda_c \) is the center wavelength. It is
also possible to use shorter horns with lower coupling effi-
ciency or longer horns which can have coupling efficiencies
>99.9% across full waveguide bands. These pure HE11 horns
form the starting point for most other wideband designs.

B. LAGUERRE GAUSSIAN LG
00
HORNS

Fig. 3 indicates the coupling efficiency to the LG
00
, LG
01
and
LG
02
modes, as a function of HE12 amplitude, where it is
assumed no higher order HE1n modes are excited. In this
graph the chosen Gaussian mode set, at each point, is the
one that maximizes power in the LG
00
mode. It can be seen
that the highest coupling to a fundamental LG
00
Laguerre
Gaussian beam (or ‘Gaussicity’) is achieved at \( a_{12} = +0.31 \),
where 99.87% of power is in the fundamental LG
00
beam.

It can also be seen that to a very good approximation,
for most HE12 excitations (0.05 < a12 < 0.6), corrugated
feedhorns primarily couple to just the LG
00
and LG
02
beams
(>99.5% power coupling, with 99.95% power coupling to
just these two modes for \( a_{12} = +0.34 \)). For some applica-
tions it can be advantageous to excite both modes, when it is
then also important to take the LG02 mode into account
when calculating beam profiles and specifying the size of

FIGURE 4. First sidelobe level as a function of HE12 amplitude assuming no other
higher order modes are present and the HE11 and HE12 modes are in phase at
the aperture. The lowest sidelobes are achieved when the Gaussicity is largest at
\( a_{12} = +0.31 \).

C. HORNS FOR RADIOMETRY

Radiometers are used to measure accurately the brightness
temperature of extended thermal sources and are commonly
employed in satellite systems to aid weather forecasting.
Such systems often need to operate over extended band-
widths and horn specifications are driven by the need to
reduce unwanted extraneous contributions to the measured
temperature via sidelobe or spill-over loss. Thus, primary
reflectors are often specified with relatively modest edge
tapers and far-field beam patterns are primarily determined
by the design of the feedhorn. Here, an important characteris-
tic is the fractional power contained in the main beam. This is
often specified relative to a multiple of the 3 dB beamwidth.
For this application, a pure LG
00
Gaussian mode is not nec-
essarily the optimum choice. Fig. 5 shows the fractional
power within 2.5 times the 3 dB beamwidth as a function of
HE12 excitation. This has a local maximum around \( a_{12} = +0.14 \),
which is equivalent to adding a small amount of
LG02 mode in phase with the LG00 mode. The main design

optics. It should be noted that for beams containing only
even LG02 modes the profile at the aperture will be the
same as the far field.

Higher order mode content is a critical determinant of
sidelobes, which can be important in many remote sensing
and radar applications. Fig. 4 shows the sidelobe levels as a
function of HE12 amplitude. These can be as low as
−60 dB
at \( a_{12} = +0.31 \) for the ideal case, although in practice they
can be limited by unwanted low-level excitation of the HE13
or higher order modes.

It should be noted that sidelobe levels by themselves are
not necessarily good indicators of beam quality. It is entirely
possible to construct horns that excite a combination of
higher order HE1n and Gaussian modes (not necessarily in
phase) that produce apparently low sidelobe levels in the far
field, but also yield broad beams. These are rarely optimal
for any application.

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challenge is keeping the HE$_{11}$ and HE$_{12}$ modes in phase at the aperture over wide bandwidths. In Section IV we provide illustrative designs where $a_{12}$ can be chosen to approximate $+0.14$ and where the fractional power within 2.5 times the 3 dB beamwidth can exceed 99.5% over a full waveguide band.

It should be noted that the maxima for $a_{12} > +0.40$ correspond to flatter output beams in the far field where the 3 dB points are artificially broad (see Section II-F). Whilst their suitability needs to be evaluated for any given application, this type of excitation can still offer useful solutions where trade-offs are needed between maximizing aperture efficiency, reducing sidelobes and increasing spill-over efficiency.

**D. HORNS FOR QUASI-OPTICS**

Excitation of a nearly pure Laguerre Gaussian LG$_{00}$ mode is often desirable in generic quasi-optical instrumentation to maximize transmission from feedhorn to feedhorn. In quasi-optical systems using frequency independent optics [18], [19] it is important that the size and position of the beam waist in the horn remains constant as a function of frequency to maintain optimum coupling between horns, as indicated in Fig. 6. In Section III it is shown that for the beam waist to be exactly at the aperture the HE$_{12}$ mode should be kept in phase with the HE$_{11}$ mode at the aperture, across the desired bandwidth.

**E. CONSTANT GAIN HORNS**

Most LG$_{00}$ feed designs have approximately constant beam sizes at the aperture with respect to wavelength. This means the gain of the horn will vary with frequency, typically by around 4 dB across a waveguide band. Fig. 6 shows the intermediate focus where the beam waist size will now be proportional to the wavelength. It can sometimes be desirable to produce a horn that can couple to this intermediate focus and in effect produce a constant gain horn. Constant gain horns have previously been outlined by Clarricoats et al. [2], who pointed out that wide angle linear tapered horns had approximately constant gain for certain taper lengths over limited bandwidths, albeit with rather poor broad beam patterns. A highly compact design with a constant gain operating over 20% bandwidths has also been described for a body scanner application, with low sidelobes and cross-polar $<-30$ dB and return loss $<-22$ dB [4]. It can be shown that such a constant gain horn requires precise dispersive properties between modes and in Section IV we demonstrate a design that can operate over full waveguide bands with reasonable beam quality.

**F. HORNS TO MAXIMISE APERTURE EFFICIENCY**

In a radar, telescope or communications system an important design specification is often the system aperture efficiency over the desired bandwidth. For example, the feedhorns for bands 2 and 3 combined, for the ALMA Cassegrain telescope are specified to provide system aperture efficiencies >80% at the secondary (also limited by a focusing lens), with sidelobe levels lower than $-20$ dB and cross-polar better than $-25$ dB, over 50% bandwidths [10].

For many applications involving a telescope or radar system, the feedhorn should ideally couple with maximum efficiency to the parallel beam received by the primary antenna. A point source in the far field imaged via a circular focusing element will produce an Airy pattern at its focus, as indicated schematically in Fig. 7a. If this Airy-pattern is then imaged using another lens or reflector a “top hat” image of the circular focusing element will form, as indicated schematically in Fig. 7b. The aperture efficiency of an ideal imaging system can be maximized by matching the feed pattern either to the Airy pattern at the first focus (Fig. 7a) or to the top hat pattern at the second focus (Fig. 7b).

The “Airicity” of a beam was a concept introduced by Clarricoats and Poulton [20] and refers to the coupling efficiency at the feedhorn aperture to an Airy pattern given by $(J_1(u))/u$, where $J_1(u)$ is the Bessel function of the first kind. In most dish antenna systems, the feeds are designed simply to match to the central lobe of this Airy pattern at their aperture. This limits the maximum possible aperture efficiency to around 84%. A fundamental LG$_{00}$ Gaussian beam matched to this central lobe can provide up to 81.5%
efficiency. However, it is possible also to match to the next outer lobe in which case the theoretical aperture efficiency can be increased to approximately 91%.

The idea of creating a feed whose output also matches the outer lobes of an Airy pattern is an old one [21], [22], but it had previously been assumed that the bandwidth of such a horn would necessarily be extremely narrow due to the need to excite and precisely phase a number of higher order modes [20]. Such an Airy pattern, at the horn aperture, can be obtained by exciting the HE_{12} mode with an amplitude $a_{12}$ of approximately +0.81, and in this paper we show that good efficiencies can be obtained over bandwidths of order 7%. In practical systems, bandwidth may also be partially limited as the width of the main lobe of the Airy pattern also scales with $1/\lambda$.

An alternative to matching to the Airy pattern at the horn aperture is to match to the “top hat” image of the primary produced at the horn aperture, as indicated in Fig. 7b. The optics in the ALMA telescope are designed to achieve this [23]. The width of this pattern will now remain constant as a function of frequency, and the horn will produce an approximation to an Airy beam in the far field. In Section IV-B, we show the design of a horn where $a_{12} \sim -0.15$ ($\theta_{12} = 180^\circ$) over a full waveguide band for which aperture efficiencies at reflector antennas are predicted to be above 90%. Higher efficiencies can be achieved over rather narrower bandwidths by choosing $a_{12} \sim -0.33$ and adding correctly phased higher order modes to match to the higher lobes of the Airy beam in the far field (optics permitting).

Fig. 8 shows the predicted aperture efficiency for these two schemes. The solid line shows theoretical coupling efficiency to an Airy beam at the aperture of the horn and the dotted line shows coupling efficiency to a “top hat” beam. Both schemes offer aperture efficiencies above 90%. However, because of the reduced excitation, the top hat horn can be more compact and operates over larger bandwidths, at the expense of slightly more complicated optics in a telescope.

G. HORMS FOR IMAGING ARRAYS

For some imaging array applications, where a flat response across the aperture is desirable, one can simply maximize the aperture efficiency of individual horns [24]. This is the same as maximizing the coupling efficiency to a “top hat” beam whose width is defined as being equal to the aperture radius of the horn. A corrugated horn with good Gaussian coupling efficiency typically has only 45% aperture efficiency, whereas an HE_{11} horn has almost 70%. The aperture efficiency of an individual horn can increase to more than 80% by adding an HE_{12} component of $a_{12} = -0.40$. To improve on this, further higher order modes must be included at the cost of bandwidth. A design with $a_{12} = -0.37$ and $a_{13} = +0.24$ and $a_{14} = -0.17$ would theoretically provide horn aperture efficiencies above 95%. Simulations show this can be achieved over a narrow bandwidth (few %) and we indicate in Section IV how excitation of these mode-sets can be achieved in practice.

III. HORN CHARACTERISTICS

A. COUPLING TO A GAUSSIAN BEAM

For many corrugated feedhorn applications the object is to maximize coupling to a fundamental LG_{00} Gaussian beam. By taking a coupling integral between the HE_{1n} modes and a fundamental Gaussian mode, it can be shown that the coupling is maximized ($> 99.9\%$) when the HE_{11} and HE_{12} modes are in phase at the aperture and $a_{12} \sim +0.31$. For this case the beam waist $\omega_0$ is approximately $0.50r_0$, where $r_0$ is the aperture radius, and where we assume no other modes are excited. However, from Fig. 3 it can be seen that Gaussian coupling remains very high ($> 99.9\%$) for a wide range of HE_{12} excitations corresponding to an $a_{12}$ of between +0.15 to beyond +0.40, giving beam waist radii of between 0.60$r_0$ and 0.45$r_0$ respectively [14]. In practice, coupling efficiency is limited by excitation of higher order modes.

As noted in Section I, we have previously constructed compact horns with $a_{12} \sim +0.31$ (and $a_{13} \sim +0.03$), which
had very high Gaussicity and −60 dB sidelobes at the center frequency [15]. However, for designs that will operate over a full waveguide band, we show in Section IV (B-C) that it is easier to reduce modal dispersion and unwanted excitation of other modes when \( a_{12} \sim +0.15 \) to +0.20. This leads to beams with narrower central lobes than a pure LG00 Gaussian mode, but where the Gaussian coupling efficiency still remains relatively high. (See Section II-C).

**B. HORN GAIN AND PHASE CENTER**

Because coupling to the fundamental Gaussian mode is so high for a wide range of HE12 excitations, it is often useful to be able to work out the gain of the horn and the position of its phase center as a function of the HE12 mode content. Kowalski et al. [25] has derived analytical expressions that allow the beam radius \( \omega \) and the radius of curvature \( R \) of the dominant fundamental Gaussian beam to be related to the amplitude and relative phase of the HE12 mode at the horn aperture:

\[
\frac{\omega^2}{\lambda} = 0.436a_{11}^2 + 0.622a_{12}^2 - 0.696a_{11}a_{12}\cos(\theta_{12}) \tag{2}
\]

\[
\frac{R}{\lambda} = \frac{0.436a_{11}^2 + 0.622a_{12}^2 - 0.696a_{11}a_{12}\cos(\theta_{12})}{4.302a_{11}a_{12}\sin(\theta_{12})} \tag{3}
\]

where \( r_0 \) is the radius of the aperture of the horn, \( k = 2\pi/\lambda \), \( a_{11} \) and \( a_{12} \) are the normalized amplitudes of the HE11 and HE12 modes and \( \theta_{12} \) represents the relative phase between the HE11 and HE12 modes at the aperture. It is also understood that these formulae assume the use of the Gaussian mode set that maximizes the power in the fundamental mode. These quantities can then be easily related to the effective beam waist radius \( \omega_0 \) of a fundamental Gaussian beam and its position \( z \) with respect to the aperture using well known formulae [19]:

\[
\omega_0 = \frac{\omega}{\sqrt{1 + \left(\frac{\pi \omega^2}{\lambda r_0}\right)^2}} \tag{4}
\]

\[
z = \frac{R}{1 + \left(\frac{R}{\pi \omega_0^2}\right)^2}. \tag{5}
\]

This analysis was originally applied to excitation of corrugated pipes, but equally applies to corrugated horns. Inspection of these formulae shows that the position of the beam waist (and to a good approximation the phase center) is largely determined by the relative phasing of the HE12 mode with respect to the HE11 mode. If the HE12 mode is in phase at the aperture, the beam waist will be at the aperture and will couple to a fundamental Gaussian beam with a relatively small beam waist and sidelobe levels will be low. If the HE12 mode is in antiphase, the beam waist will also be positioned at the aperture. However, it will now couple to a fundamental Gaussian beam with a large beam waist relative to the aperture. This leads to higher sidelobes, but the horn aperture efficiency will be higher. In the general case where the phase \( \theta_{12} \) is intermediate, the effective beam waist for the fundamental mode will be positioned either behind or in front of the horn aperture and coupling to the fundamental Gaussian mode is reduced.

For many feedhorns, we can use (2) – (5) to identify the Gaussian mode set (given by specifying \( \omega_0 \) and \( z \)) that maximizes coupling to the fundamental Gaussian beam given the amplitude \( a_{12} \) and relative phase \( \theta_{12} \) of the HE12 mode. We then use a coupling integral at the aperture to calculate the coupling efficiency to the desired output beam. This will be a good approximation as long as HE12 is the only other dominant mode and \( a_{12} \) lies between +0.05 and +0.60. It becomes less exact as \( \omega_0 \) approaches \( \lambda \), when higher order modes are required to describe the beam.

If \( a_{12} \) remains constant and \( \theta_{12} \) remains close to 0° (or 180°) then to a good approximation the beam waist size \( \omega_0 \) and its position will also remain constant as a function of frequency.

**C. HORN COMPACTNESS**

The total length of the feedhorn is an important specification for many applications. This is particularly true at low microwave frequencies where the length and weight of the horns can easily determine the dimensions of an entire optical sub-assembly, e.g., in a multi-channel radiometer.

Most horns can be scaled for any desired center frequency, and to a reasonable approximation to higher gains (with some caveats related to excitation of higher order modes). The length \( L \) of the horn scales with \( G \), \( \lambda_c \), or equivalently \( r_0^2/\lambda_c \), where \( G \) is the linear horn gain, \( r_0 \) is the horn aperture radius, and \( \lambda_c \) is the center wavelength [11]. To allow comparisons of different horn designs we also find it useful to define a dimensionless figure of merit, \( N_{19dB} \), which is the length \( L_{19dB} \) of a horn with a gain of \( 8\pi^2 \) (19 dBi) measured in wavelengths, at the center wavelength:

\[
N_{19dB} = \frac{L_{19dB}}{\lambda_c}. \tag{6}
\]

A horn with a gain \( G \) of \( 8\pi^2 \) is chosen as the reference, since the gain \( G \) of a fundamental Gaussian beam is given by \( 8\pi^2(\omega_0^2/\lambda^2) \) where \( \omega_0 \) is the effective beam waist radius, [10]. For horns that produce optical outputs that approximate fundamental Gaussian beams, this allows their approximate length \( L \) to be conveniently calculated from:

\[
L = L_{19dB}\left(\frac{\omega_0}{\lambda_c}\right)^2 = (N_{19dB}\lambda_c)\left(\frac{\omega_0}{\lambda_c}\right)^2 \tag{7}
\]

where the beam waist \( \omega_0 \) is chosen to maximize the power in the fundamental mode and measured at the center frequency. It should be noted the ratio \( \omega_0/r_0 \) will in general be different for each design and is a function of \( a_{12} \) and \( \theta_{12} \) as shown by (2).

In general, most of the wideband horns described here have \( N_{19dB} > 7 \) wavelengths, but we note it is possible to design extremely compact feeds with very good performance.
over more limited bandwidths. We have previously described a modified tanh-linear design with $N_{19\text{dBi}} \sim 3.8$ wavelengths, which has more than 99% coupling efficiency to a fundamental Gaussian beam over 10% bandwidths and $-40$ dB sidelobe levels at its center frequency [15]. This particular design was recently adapted to provide two highly compact antennas for the quad-band TROPICS cubesat system [17]. In general, there is a trade-off between compactness, bandwidth and performance, particularly at higher gains.

D. RETURN LOSS AND CROSS POLARIZATION

Although this paper is mainly concerned with mode synthesis, the bandwidth of practical feedhorns is also determined by their return loss and cross-polar properties. For the corrugated horns described here, the return loss is largely determined by the matching section at the throat of the horn. The cross-polar component is mainly determined by the level of EH$_{11}$ mode present at the aperture. This is often determined by the throat design, however, the EH$_{11}$ mode can also be excited if the slope of the feed profile becomes too large.

Nevertheless, the design of the matching section at the throat is well understood and has been the subject of several analyses, most notably by Clarricoats et al. [2], Dragone [26] and Zhang [27]. For all the horns described, we have adapted the design approach of Zhang. With the exception of the Airy horn (Section IV-F) this throat design leads to simulated return loss and cross-polar excitation better than $-30$ dB across a full waveguide band (neglecting reflections from the rectangular to circular waveguide transition) as shown in Fig. 9. It is possible to extend the bandwidth to 50% (cross-polar and return loss better than $-30$ dB) at the cost of a small loss of performance at the center of the band.

IV. EXAMPLE BROADBAND HORNS

In the following section a number of example horn profiles are given that illustrate wideband performance for the applications described in Section II.

Unless otherwise stated, the following horns all have the same quarter wavelength corrugation depth and the same throat designs leading to very similar predicted return loss and cross-polarization properties as illustrated in Fig. 9. All simulations were carried out using the mode-matching program CORRUG. All horns also give highly symmetric patterns with only slight distortions at the band edges. For reasons of clarity, we have chosen to show only the co-polar diagonal (D-plane) antenna pattern, which in almost all cases is a very good approximation to both the E and H patterns for the main lobe.

A. PURE HE$_{11}$ MODE

We have examined a number of profiles to achieve excitation of a pure HE$_{11}$ mode (i.e., $a_{11} = 1$, and $a_{1n} = 0$ for $n > 1$) over large bandwidths in reasonably compact designs. One practical dual-profile design is to have an initial profile of length $L$ and radius $r(z)$, given by:

$$r(z) = r_i + r_f \sin\left(\frac{\pi z}{2L}\right)^{0.85} \tag{8}$$

where $r_i$ and $r_f$ are the initial and final radii. This is then followed by a second section consisting of a short shallow linear taper that further improves the cancellation of the HE$_{12}$ mode (and other higher order modes). This particular horn does not have a fixed length and in general higher purity HE$_{11}$ modes can be obtained with longer horns, although performance starts to degrade for shorter horns than described here. We obtained this profile through an iterative design optimization process.

The horn has some similarities to a previous design of Granet et al. [28], who used a horn with a $\sin(\pi z/2L)^{0.8}$ profile as a feed for Earth observation studies, and to the corrugated Winston Cone described by Maffei et al. [29], although neither was designed with pure HE$_{11}$ mode excitation in mind.

An example of a 25 dBi HE$_{11}$ mode horn is shown in Fig. 10a, with a center frequency of 94 GHz, where the bowl-like profile is 80 mm long, the shallow linear taper is 35 mm long and the horn aperture is 22 mm diameter. The length $L$ of this horn is $L \sim 3r_0^2/\lambda_c$, which corresponds to $N_{19\text{dBi}} = 7.3$ wavelengths.

Fig. 10b shows the far field beam pattern, with sidelobes around $-28$ dB (as expected for a pure HE$_{11}$ mode) and beamwidths consistent with a constant beam waist at the aperture, independent of frequency. The illustrative design shown here provides $\geq 99.9\%$ power coupling efficiency to the HE$_{11}$ mode at the center of the band and $\geq 99.75\%$ power coupling efficiency to the HE$_{11}$ mode over a 50% bandwidth with the HE$_{12}$ mode largely making up the residual ($\langle a_{12} \rangle < 0.04$). It is also possible to increase coupling efficiency further by using the same generic design but lengthening the horn.
A pure HE$_{11}$ mode will produce beams with sidelobes at $-28$ dB.

**B. GAUSSIAN EXCITATION VIA A SHORT LINEAR TAPER**

It is then possible to improve the Gaussicity of the output beam by adding another steeper linear taper to the pure HE$_{11}$ horn to deliberately excite the HE$_{12}$ mode, as shown in Fig. 11a. The length of this taper should be small compared to the relevant phasing wavelength $\lambda_{12}(r) = (1/\lambda_{HE_{11}}(r) - 1/\lambda_{HE_{12}}(r))^{-1}$. In this case, the HE$_{12}$ excitation will be almost constant with frequency, and there will be a small constant phase difference between the modes, leading to a fixed phase center. There is a trade-off regarding the length and slope of this taper. If the taper is too long then there is too much dephasing of the HE$_{12}$ mode, introducing beam distortions (over extended bandwidths). However, if it is too short there will either be too little HE$_{12}$ excitation, or if too steep, too much excitation of the HE$_{13}$ and EH$_{11}$ modes, which, respectively, increases sidelobe content and cross-polar content. Typically choosing a value of $a_{12}$ between $+0.12$ and $+0.20$ and a phase difference $\theta_{12}$ not much more than $35^\circ$ leads to a beam waist with an approximately constant phase center positioned behind the aperture. This provides relatively high quality beam patterns over 50% bandwidths as shown in Fig. 11b.

Sidelobes are between $-30$ dB and $-37$ dB and are limited largely by unwanted excitation of the HE$_{13}$ mode that becomes more significant at higher frequencies. This type of horn is particularly suitable for wideband coupling between two feedhorns in quasi-optical systems. This design appears similar to a wideband horn recently developed using numerical optimization techniques [30] and is of the same design family as the horns used for the Planck HFI telescope [8]. For the particular design shown, $N_{19dBi} \sim 7.8$ wavelengths, or $L \sim 2.25r_0^2/\lambda_c$ where $r_0$ is the aperture radius. Note that the expression for $L$ appears smaller than for the previous pure HE$_{11}$ horn design, because the aperture radius $r_0$ is larger (for comparable gain).

Numerous papers have also suggested using a Gaussian flare that mimics the shape of the Gaussian beam (following excitation of a mode that approximates an HE$_{11}$ mode). The general claim is that high purity Gaussian beams are naturally produced, as long as the flare is a reasonable fraction of a Raleigh range. Whilst this can be true over a range of frequencies, especially at lower gains [9], at this aperture we did not find this scheme worked as well over extended bandwidths, compared to the approaches outlined above, and in the next section.

**C. GAUSSIAN BEAM WITH HALF SINUSOID PROFILE**

In the previous short linear taper design, there will always be a constant phase shift between the two main modes. As an alternative that produces higher quality beams over a marginally reduced bandwidth, it is also possible to add a half sinusoidal section with a radial profile $r(z)$ given by:

$$r(z) = r_0 + A_2 \sin\left(\frac{2\pi z}{\lambda_{12}(r) + \phi_2}\right) - A_2 \sin(\phi_2)$$

where $A_2$ is the radial modulation amplitude and $\phi_2$ represents the starting phase of the radial modulation for the HE$_{12}$ mode. The phasing wavelength $\lambda_{12}(r) =$
Fig. 11 due to lower HE_{13} excitation.

(b) The resulting far-field pattern. Sidelobes are lower compared to the horn in Fig. 11 due to lower HE_{13} excitation.

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where λ is the center wavelength and λ_L is the wavelength at the low frequency end of the band. This corresponds to N_{19dBi} \sim 15.5 wavelengths. Far-field patterns across a waveguide band are shown in Fig. 12b.

This type of horn is suited both for wideband quasi-optics, and radiometric (and radar) applications, where high-quality narrow beams with constant phase centers are important.

D. “TOP HAT” BEAM WITH HALF SINUSOID PROFILE

A variation of the previous horn design where now \phi_2 = 0^\circ, is shown in Fig 13a. This design puts the HE_{12} mode almost exactly 180° out of phase with the HE_{11} mode and now produces an approximation to a “top hat” profile at the aperture, (and an Airy-pattern in the far-field), across a full waveguide band, as indicated in Fig. 13b. The length of this horn, compared to the previous example, can also be calculated from L \sim 3r_0^2/\lambda_c + 1.6r_0^2/\lambda_L but N_{19dBi} now approximates 9.4 wavelengths, due to the higher horn aperture efficiency.

This type of horn has a horn aperture efficiency of 75% and can couple to a “top hat” beam with \sim 90% efficiency, both across a full waveguide band. This efficiency can be improved by exciting higher order modes as further discussed in Section IV-G.

E. GAUSSIAN HORNS WITH MULTIPLE SINUSOIDS

In general, for a single half sinusoid there is a limit to the radial modulation amplitude A_2 before excitation of higher order modes (HE_{13} and EH_{11}) limits performance. Thus, for larger excitations (e.g., a_{12} > +0.2) or more complex excitations, more radial oscillations are required, and the horn becomes longer and bandwidth decreases. This bandwidth can nevertheless be substantially greater than methods that excite the higher order mode(s) near the throat of the horn. They are also easier to scale for any desired gain.
F. AIRY BEAM HORN

As another example of a horn that uses multiple sinusoidal cycles, Fig. 15a shows a horn that produces an Airy-pattern at its aperture. The resulting “top hat” far-field beam patterns are shown in Fig. 15b. The horn is comprised of a pure HE\textsubscript{11} horn to which 5 sinusoidal half-cycles have been added. This horn now becomes relatively long ($L \sim 3r_0^2/\lambda_c + 8r_0^2/\lambda_L$) as several modulation periods are needed to excite the required amplitude $a_{12} \sim +0.81$, without excessively exciting the HE\textsubscript{13} or EH\textsubscript{11} modes. This horn can allow aperture efficiencies > 90% to be obtained over ~7% bandwidths, limited by modal dispersion. For this specific horn, the cross-polar level is degraded with respect to the performance shown in Fig. 9, with levels more in the range of -20 dB to -30 dB.

G. COMPLEX MODE EXCITATIONS

Synthesis of more complex output beams requires excitation and phasing of multiple higher order Gaussian beams, requiring excitation and phasing of multiple HE\textsubscript{1n} modes.

In general, at a given frequency, this can be achieved by sinusoidally modulating the horn radius $r(z)$ over a length $z$ at a number of different radial frequencies, with different radial amplitudes $A_n$ and different starting phases $\phi_n$ according to the desired excitation. The profile is then given by:

$$r(z) = r_0 + \sum\left\{A_n \sin\left(\frac{2\pi z}{\lambda_{1n}(r_0)} + \phi_n\right) - A_n \sin(\phi_n)\right\} \quad (10)$$

where the phasing wavelength $\lambda_{1n}(r)$ at radius $r$ is given by:

$$\lambda_{1n}(r) = \left(\frac{1}{\lambda_{HE11}(r)} - \frac{1}{\lambda_{HE1n}(r)}\right)^{-1} \quad (11)$$

where $\lambda_{HE1n}(r)$ is the wavelength of the HE\textsubscript{1n} mode at radius $r$, $A_n$ is the radial modulation amplitude, $\phi_n$ is a starting phase (often 0° or 180° for HE\textsubscript{12}) and the summation is just over the first few HE\textsubscript{1n} modes. In practice, optimization algorithms need to be run as the mode excitations are not completely independent. However, optimization is now relatively quick due to the limited number of input variables. Bandwidth decreases as higher order modes are added.

For example, it is possible to increase the “top hat” and horn aperture efficiency of the feedhorn by adding higher order modes within the horn, to create a better approximation to a “top hat” excitation at the aperture e.g., $a_{12} = -0.37$, $a_{13} = +0.23$, $a_{14} = -0.17$.

Figure 16a shows a design where the first two modes are excited using (10), and Fig. 16b shows the resulting aperture pattern. This horn couples to a “top hat” beam with efficiencies of 88% across the band with 95% at band center and has a horn aperture efficiency of 80% across a waveguide band with 87% at band center.

H. CONSTANT GAIN FEEDHORNS

In all the horns described above, the effective beam waist $\omega_0$ stays roughly constant, and so horn gain $G$ will vary with frequency, typically by 4 dB over a waveguide band.
This can be seen from the expression for gain $G$ of a fundamental Gaussian beam, $G = 8\pi^2(\omega_0^2/\lambda^2)$. However, it is also possible to design feedhorns with constant gain, where $\omega_0$ must now be made to scale with $\lambda$. Inspection of (2) – (5) shows that this requires the feedhorn to be strongly dispersive (both $a_{12}$ and $\theta_{12}$ varying with frequency).

One wideband solution is to utilize the modal phase dispersion naturally present in the $\sin^2$-parallel horn [13]. It is found for full band horns that the necessary dispersion can be achieved if the length of the initial section is chosen to excite a HE$_{12}$ mode amplitude $a_{12} = +0.35$ at the center frequency. The phasing length is then chosen so that the HE$_{12}$ mode is in phase with the HE$_{11}$ mode at the highest frequency over the desired bandwidth. In other words, the phase center is positioned at the aperture at the highest frequency and its position moves further inside the horn as frequency is lowered. In general, there is a trade-off between beam quality and required bandwidth, and sidelobe performance can be improved considerably over narrower bandwidths.

This design is illustrated schematically above in Fig. 17a. Fig. 17b shows a plot of simulated far-field antenna patterns at 5 GHz intervals over a full waveguide band (75-110 GHz). Beam patterns are effectively constant out to a 15 dB edge taper. The phase center now moves over $\pm 0.2$ of a Raleigh range $z_R = \pi \omega_0^2/\lambda c$, which can be acceptable, but the effect on phase efficiency would need to be checked for a given design requirement. The example presented is for a horn with a center frequency of 94 GHz and a gain of 25.5 dBi. This corresponds to a horn $L \sim 4.3 r_0^2/\lambda_c$ or $N_{Sidelobe} \sim 17.3$ wavelengths. This horn is illustrative of one design approach. Much higher performance is possible at reduced bandwidths.

V. DISCUSSION AND SUMMARY
We have shown that many wideband high-performance feed designs can simply depend on the correct excitation ($a_{12}$) and relative phasing ($\theta_{12}$) of the HE$_{12}$ mode. A simple conical corrugated horn can have acceptable performance, especially compared to a smooth horn, but is rarely optimal as it excites the HE$_{12}$ mode (where $a_{12}$ is length dependent from (2)) approximately 90° out of phase with the HE$_{11}$ mode.

An alternative approach (at least for horns designed to excite a Gaussian LG$_{00}$ beam) is the generic Gaussian flared horn approach [9], [31]. These designs can provide good beam patterns in rather compact horns, over reasonable bandwidths, especially at low gains [9]. This style of horn usually has a beam waist whose position is both a function of frequency and relatively far inside the horn which will lead to a complex mode distribution at the aperture. This makes them less amenable to more complex mode synthesis, at least over extended bandwidths.

In this paper, we have indicated optimal excitations for different applications, and have shown how low dispersion structures can lead to wideband performance, relative to other published wideband designs [9], [31], [32]. Many of the designs described have near constant phase centers and produce optical beams with very similar mode content and modal phasing across their specified bandwidth. The different designs discussed in this paper are summarized in Table 1, along with other lower bandwidth “dispersive” horns also mentioned in this paper. Table 1 gives sidelobe level (SL) at the center frequency, approximate (conservative) percentage bandwidth (BW), and two measures of relative compactness for different designs.

The HE$_{11}$ horn (Fig. 10), with $a_{12} \sim 0$, provides excellent coupling to a corrugated pipe transmission line.
TABLE 1. Table indicating some of the key characteristics of the horns illustrated in this paper. SL is the sidelobe level in dB at the center frequency. BW gives an indication of the approximate percentage bandwidth. \(a_{12}\) and \(d_{12}\) give the target excitations of the HE_{12} mode where applicable. \(N_{\text{beam}}\) is the effective length of a 19 dB horn at the center wavelength and length gives the approximate length of the horn in units of \(\sqrt{\lambda/2}\) where \(r_0\) is the aperture radius and \(\lambda\) is the center wavelength.

| Horn Type (advantage)          | SL (dB) | BW (%) | \(a_{12}\) at \(\lambda_0\) | \(d_{12}\) (deg) | \(N_{\text{beam}}\) (\(\lambda_0\)) | Length (\(r_0^2/2\lambda\)) |
|--------------------------------|---------|--------|-----------------------------|------------------|-----------------------------------|-----------------------------|
| Conical (simple)               | -28     | var    | var                         | \(-90\)          | var                               | var                         |
| Sin-par\parallel [15] (Easy to design) | -37     | 20     | 0.2                         | \(\pm 35\)       | 15.5                              | 4.4                         |
| Tumb-LinI [15] (very compact)  | -40     | 20     | 0.2                         | \(\pm 35\)       | 3.8                               | 0.94                        |
| Tumb-LinI2 [15] (high Gauscnity) | -60    | 20     | 0.31                        | \(\pm 35\)       | 12.4                              | 3.0                         |
| Low Dispersion Feeds (this paper) |         |        |                              |                  |                                   |                             |
| HE_{11} (Fig. 10) coupling to corrugated pipe | -28 | 50     | 0                            | NA               | 7.3                               | 3.0                         |
| HE_{11} + Flare (Fig. 11) opt. 2.5 x 3dB | -34 | 50     | 0.15                        | 30               | 7.8                               | 2.25                        |
| HE_{11} + 1/2 x Sine (Fig. 12) opt. 2.5 x 3dB | -37 | 40     | 0.15                        | 0                | 15.5                              | 4.3                         |
| HE_{11} + 2 x Sine (Fig. 13) high Gauscnity | -50 | 30     | 0.25                        | 0                | 20.8                              | 6                           |
| HE_{11} + 5 x Sine (Fig. 15) Airy Horn | NA | 7      | 0.81                        | 0                | NA                                | 9.6                         |
| HE_{11} – 1/2 x Sine (Fig 14) Top Hat Horn | -22 | 40     | 0.2                         | 180              | 9.4                               | 4.8                         |
| Max App Eff (Fig. 16) Horn array | -18 | 10     | 0.37                        | 180              | 10.6                              | 6.5                         |
| Constant Gain (Fig. 17)        | -18     | 40     | var                         | var              | 17.3                              | 4.3                         |

For radiometry, \(a_{12} \sim +0.14\) (with \(\theta_{12} = 0\)) excites a LG_{00} and LG_{02} combination that provides relatively low sidelobes and narrow beams that maximizes power within 2.5 of the 3 dB beam angle. This can be achieved by a simple taper (Fig. 11) or a single half-sinusoid with slightly better performance (Fig. 12). These types of horns are also suitable for wideband quasi-optical instrumentation.

Gaussicity (pure LG_{00} excitation) improves as \(a_{12}\) increases further, which can be achieved by adding extra half sinusoids at the cost of a slight reduction in bandwidth (Fig. 13, with \(a_{12} \sim +0.25\)). For \(a_{12} > +0.32\) the beam starts to distort, and at \(a_{12} \sim +0.81\) the beam resembles an Airy pattern at the aperture (Fig. 15).

High system aperture efficiency is typically desired for radar and communication systems, and both “Airy” and “top hat” design horns can, in principle, be used to improve system aperture efficiency to around 90% with ideal dish antennas. “Top hat” designs are more compact and operate over wider bandwidths than “Airy” designs, but require an additional focusing element if used in a telescope (see Fig. 7b).

“Top hat” beam patterns at the aperture can be obtained by making \(\theta_{12} = 180^\circ\) and Fig. 14 illustrates a wideband design where \(a_{12} = -0.2\). Higher coupling efficiency to “top hat” beams is also desirable for horn arrays and can be further improved by exciting and phasing other higher order modes. Fig. 16 shows a design where \(a_{12} \sim -0.37\), and \(a_{13} \sim -0.2\), where 80% horn aperture efficiencies across a full waveguide band can be achieved, with higher efficiencies at the center frequency.

Finally, Fig. 17 shows a design that is deliberately dispersive to provide a feedhorn that has constant gain over a full waveguide band.

All feedhorns described in this paper can be scaled both for center frequency and for gain, although, it should be noted that better performance can be expected for smaller apertures (and lower gain) where higher order modes can be below or near to cut-off. Simulations also show it is possible to further improve performance for a given design if the total bandwidth is reduced, even moderately.

We do not claim to have fully explored the parameter space, or fully optimized the horns given, and there is always scope to trade-off different performance parameters within a given design, particularly when highly compact horns are required. However, we believe these designs give new starting points, which allow efficient optimization algorithms to be run, with a restricted set of input parameters for a given target specification. They also give system designers an indication of what can be realistically achieved for a given target mode excitation. In many cases we believe the wideband characteristics of many of these feeds already represent state-of-the-art performance.

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