Bettin, S.; Drappeau, S.
Modularity and value distribution of quantum invariants of hyperbolic knots. (English)
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Summary: We obtain an exact modularity relation for the \( q \)-Pochhammer symbol. Using this formula, we show that Zagier’s modularity conjecture for a knot \( K \) essentially reduces to the arithmeticity conjecture for \( K \). In particular, we show that Zagier’s conjecture holds for hyperbolic knots \( K \neq T_2 \) with at most seven crossings. For \( K = 4_1 \), we also prove a complementary reciprocity formula which allows us to prove a law of large numbers for the values of the colored Jones polynomials at roots of unity. We conjecture a similar formula holds for all knots and we show that this is the case if one assumes a suitable version of Zagier’s conjecture.

MSC:

57K16 Finite-type and quantum invariants, topological quantum field theories (TQFT)
11B65 Binomial coefficients; factorials; \( q \)-identities
11F03 Modular and automorphic functions
11F23 Relations with algebraic geometry and topology
60F05 Central limit and other weak theorems

Full Text: DOI arXiv

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