Original Research Article

Impact of different centroid means on the accuracy of orthometric height modelling by geometric geoid method

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ABSTRACT

Background: Orthometric height, as well as geoid modelling using the geometric method, requires centroid computation. And this can be obtained using various models, as well as methods. These methods of centroid mean computation have impacts on the accuracy of the geoid model since the basis of the development of the theory of each centroid mean type is different. This paper presents the impact of different centroid means on the accuracy of orthometric height modelling by geometric geoid method.

Methods: DGPS observation was carried out to obtain the coordinates and ellipsoidal heights of selected points. The centroid means were computed with the coordinates using three different centroid means models (arithmetic mean, root mean square and harmonic mean). The computed centroid means were entered accordingly into a Microsoft Excel program developed using the Multiquadratic surface to obtain the model orthometric heights at various centroid means. The root means square error (RMSE) index was applied to obtain the accuracy of the model using the known and the model orthometric heights obtained at various centroid means.

Results: The computed accuracy shows that the arithmetic mean method is the best among the three centroid means types.

Conclusions: It is concluded that the arithmetic mean method should be adopted for centroid computation, as well as orthometric height modelling using the geometric method.

Keywords: Geoid, Centroid, Arithmetic, Root mean square, Harmonic, Mean, Orthometric height

INTRODUCTION

A centroid is defined as the point whose coordinates are the average values of the coordinates of all points of the figure (polygon).\(^1\) It relates to the concept of central tendency of mean, median and mode.\(^2\) Further, that the mean is a type of average that is a single value used to represent every element in the dataset but can be classified into arithmetic mean, root mean square, harmonic mean and geometric mean.

It is very important to realize that mean has a centring property which implies that when it (mean) is subtracted from each observation in the dataset, the sum of the differences is zero for a normally distributed data. The use of normalized coordinates in the interpolation of geoid undulation and determination of orthometric heights require of the centroid mean techniques along with the observed coordinates in the absolute form. This is required for referencing coordinates for manipulation and computation of geoid and subsequent geoid computation.

Real numbers are governed by the following features of: measurability i.e. has three sides on a number line: infinite negative side; infinite positive side; a zero in between. Value can be attached e.g. 7.345. Manipulated by mathematical operations (subtraction, addition,
division and multiplication) of real numbers to yield another set of real numbers.

e.g., 5 \times 4 = 20, \frac{3}{2} = 1.5

Due to the features of real numbers espoused above, some relationships were derived to enable the computation of different centroid means as listed under the theoretical background.¹ In summary, a commonly accepted meaning of the centroid is equivalent to the ‘centre of mass’.² Centroid determination allows x, y coordinates of an area to locate the points whose orthometric heights have the least root mean square error (RMSE) among arithmetic mean, root mean square mean and harmonic mean. In this, the centroid is computed with the same dataset using the arithmetic mean, root mean square mean and harmonic mean. The differences between the various centroid mean types and the observations (orthometric heights) are evaluated to obtain the residuals. The centroid mean type with the least sum of the square of the residuals has the least root mean square error. Thus, has the highest accuracy.

Practical application area of centroid principle is in the modelling of various aspects of ‘orientation disorder in crystals in electron density distribution for example in distraction experiments’ i.e., in crystallographic problems.³ Other areas are in geoidal undulation determination and terrain correction in gravity studies.

The ellipsoid and the geoid reference surfaces are used to represent the figure of the earth that are involved in geodetic, as well as geophysical applications. Several studies have determined the local geoid models of various areas using the geometric method.⁶ This method requires the computation of the centroid of the study area. The centroid computation can be done with several methods such as geometric mean, arithmetic mean, root mean square and harmonic mean centroids. Each of these methods of centroid computation has an effect on the accuracy of the determined geoid model as the development of each centroid mean type is based on different theories. None of the studies on local geoid model determination using the geometric method has applied the various methods of centroid mean computation and compared their accuracy to determine the best method. So, there is a need to determine the impact, as well as the effect of the various methods of centroid computation on the accuracy of the geometric geoid, as well as orthometric height modelling. The objectives of this study are: to determine and compare the impacts of different centroid means on the accuracy of orthometric height modelling by geometric geoid method and to determine the best method for centroid mean computation.

**Theoretical background**

Mean as a type of average comprises geometric, arithmetic, root mean square and harmonic means.³ The four centroid means and the geometric means are given in equation.

\[
M_p = \left(\frac{x_1^p + x_2^p + \ldots + x_n^p}{n}\right)^{1/p} \tag{1}
\]

Where, \(M_p\) is pth-power mean

\[
x_1, x_2, \ldots, x_n \text{ set of real numbers}
\]

**Geometric mean**

The geometric mean \(G\) is given as

\[
G = (x_1, x_2, \ldots, x_n)^{1/n} \tag{2}
\]

**Arithmetic mean centroid \((M_{\bar{\bar{x}}})\)**

The arithmetic mean centroid is obtained by putting \(p = 1\) into equation (1), the model for the computation of the centroid \((\bar{x}, \bar{y})\) mean is given in equation (3).

\[
\bar{x} = \frac{\sum_{k=1}^{n} x_k}{n} \quad \text{and} \quad \bar{y} = \frac{\sum_{k=1}^{n} y_k}{n} \tag{3}
\]

**Root mean square centroid \((M_{\sqrt{2}})\)**

Substituting \(p = 2\) into equation (1), the root mean square centroid \((\bar{x}, \bar{y})\) can be computed as

\[
\bar{x} = \sqrt{\frac{\sum_{k=1}^{n} x_k^2}{n}} \quad \text{and} \quad \bar{y} = \sqrt{\frac{\sum_{k=1}^{n} y_k^2}{n}} \tag{4}
\]

**Harmonic mean centroid \((M_{-1})\)**

Also, \(p = -1\), the harmonic centroid \((\bar{x}, \bar{y})\) can be computed as

\[
\frac{1}{\bar{x}} = \frac{1}{\sum_{k=1}^{n} x_k} \quad \text{and} \quad \frac{1}{\bar{y}} = \frac{1}{\sum_{k=1}^{n} y_k} \tag{5}
\]

Equations (3) to (5) were used to compute the various centroid values needed for interpolation of geoid and orthometric heights.

The other type of average is the central value termed the median which is not considered in this investigation since
it is based on different assumptions. For a set of \( n \) numbered values arranged from either smallest to largest or vice versa, when \( n \) is odd, then the middle value becomes the median. For \( n \) is even the mean of a pair of middle values is taken as the median.

**Geometric geoid models**

Geometric geoid models are determined by finding the differences between the ellipsoidal and the orthometric heights of selected points to obtain the geoid heights of the points. A geometric geoid surface is then fitted to the computed geoid heights of the points to enable geoid heights of new points within the study area be computed. Polynomial surfaces are used to represent continuous surfaces over study areas. The Multiquadratic surface used for the interpolation of geoid heights is given in equation (6).

\[
N = a_0 + a_1X + a_2Y + a_3X^2 + a_4Y^2 + a_5XY + a_6X^3Y + a_7XY^3 + a_8X^2Y^2
\]

Where,

\[
Y = \text{ABS}(y - y_o) \\
X = \text{ABS}(x - x_o)
\]

\( y \) = Northing coordinate of observed station.
\( x \) = Easting coordinate of observed station.
\( y_o \) = Northing coordinate of the origin (average of the northing coordinates).
\( x_o \) = Easting coordinate of the origin (average of the easting coordinates).
\( N \) is geoidal undulation at the point of interest.
\( a_0, a_1, a_2, \ldots, a_n \) unknown parameters.

The relationship between the orthometric height (\( H \)), ellipsoidal height (\( h \)) and the geoid height (\( N \)) is given in equation (7).

\[
H = h - N
\]

**Computation of root mean square error**

The modelling of orthometric heights using the geometric geoid method requires the computation of the model accuracy with the root mean square error (RMSE) index. To compute the accuracy of the model, the orthometric heights of points from the model that is, the orthometric heights of the points obtained from the differences between the model geoid heights and the ellipsoidal heights of the points are compared with their corresponding existing, as well as spirit levelling orthometric heights to obtain the orthometric height residuals. The orthometric height residuals and the total number of the selected points are used for the computation of the RMSE, as well as the accuracy of the model. The RMSE index for the computation of the model accuracy is given in equation (8).

\[
\text{RMSE} = \pm \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\delta H_{\text{Residual}})^2}
\]

Where,

\[
\delta H_{\text{Residual}} = H_{\text{KNOWN}} - H_{\text{Model}}
\]

\( H_{\text{KNOWN}} \) = Known Orthometric Height

\( H_{\text{Model}} \) = Model Orthometric Height

\( n \) = Number of Points

**Figure 1: Map of the Nigerian states and FCT Abuja**

source: Arcinfo shapefile 2010 (ESRI).

**Dataset available**

In this study, the following datasets were available: list of coordinates obtained from the survey and mapping department, federal capital development authority (FCDA), Abuja. CSRS-PPP post-processed DGPS coordinates and ellipsoidal heights. Nigeria and federal capital territory (FCT) maps (Figures 1 and 2).

**Figure 2: Map of FCT six area councils’ source:**

survey and mapping department, FCDA, Abuja.
METHODS

**DGPS coordinates (observations)**

Hi-target V30 pro geodetic DGPS dual-frequency receivers were used for data capture in the relative mode of GPS observation. During the observation, each point was occupied for about 2 hours. The observations (RINEX data) were post-processed using CSRS-PPP online software. The results are shown in (Table 1).

Using the post-processing results presented in Table 1, the computations of root mean square centroid was done with equation (4), that of arithmetic mean centroid was done using equation (3) and that of the harmonic mean centroid was carried out with equation (5) as respectively shown in (Table 2 and 3).

The summary, as well as the results of the arithmetic mean, root mean square and the harmonic mean centroids are shown in (Table 4).

A microsoft excel program that was developed using the multiquadratic surface (equation (6)) for interpolation of geoid heights, as well as modelling of orthometric heights within the federal capital territory (FCT), Abuja was used in this study. The computed centroid means were entered accordingly one after the other into the developed program to obtain their respective model orthometric heights as given in (Table 5).

The differences in the respective orthometric heights in Table 5, may arise from the different basis of the development of the theory of each centroid mean type.

The accuracy of the model was computed for each of the calculated centroid means.

The computation was done by comparing the obtained orthometric heights for each centroid mean with the known/spirit levelling orthometric heights using equation (8).

### Table 1: Control points coordinates and ellipsoidal heights.

| Control point | Easting (m) | Northing (m) | Existing orthometric height, H (m) | Elip heigth h(m) |
|---------------|-------------|--------------|-----------------------------------|------------------|
| FCC11S        | 331888.114  | 998442.043   | 485.447                           | 509.396          |
| FCT260P       | 255881.175  | 993666.807   | 201.944                           | 224.74           |
| FCT103P       | 340639.766  | 998375.578   | 532.558                           | 556.836          |
| FCT12P        | 333743.992  | 1008308.73   | 735.707                           | 760.192          |
| FCT19P        | 337452.408  | 996344.691   | 635.644                           | 659.824          |
| FCT2168S      | 310554.927  | 1009739.93   | 431.087                           | 455.274          |
| FCT24P        | 322719.776  | 1001884.85   | 453.804                           | 477.987          |
| FCT276P       | 351983.716  | 1025998.314  | 625.572                           | 649.848          |
| FCT4154S      | 329953.882  | 1003831.28   | 476.981                           | 501.232          |
| FCT4159S      | 326124.422  | 1003742.86   | 452.230                           | 476.553          |
| FCT66P        | 299148.035  | 998114.283   | 297.111                           | 321.115          |
| FCT9P         | 329821.512  | 1007612.091  | 497.253                           | 521.693          |
| FCT35P        | 322183.38   | 992926.363   | 427.171                           | 451.299          |
| FCT57P        | 303234.27   | 992916.402   | 323.844                           | 347.795          |
| FCT4028S      | 330164.634  | 1001388.24   | 449.592                           | 473.942          |
| FCT53P        | 308943.361  | 993406.773   | 351.943                           | 375.955          |
| FCT4652S      | 329441.767  | 997474.808   | 462.711                           | 487.113          |
| FCT162P       | 270791.291  | 934625.533   | 189.696                           | 215.091          |
| FCT130P       | 330982.584  | 952889.869   | 695.608                           | 719.383          |
| FCT2327S      | 282526.612  | 973821.47    | 183.287                           | 207.482          |
| FCT2652S      | 271370.273  | 945385.429   | 138.952                           | 163.741          |
| FCT2656S      | 272644.591  | 941062.46    | 204.724                           | 229.237          |
| FCT83P        | 332954.205  | 987231.606   | 568.752                           | 592.801          |
| XP382         | 284074.729  | 983364.863   | 274.586                           | 298.421          |

### Table 2: Root mean square centroid computation.

| Point ID | Easting | SQ Easting | Northing | SQ Northing |
|----------|---------|------------|----------|-------------|
| FCC11S   | 331888.114 | 11014972014.477 | 998442.043 | 996868513230.014 |
| FCT260P  | 255881.175 | 65475175719.381 | 993666.807 | 987373723333.575 |
| FCT103P  | 340639.766 | 116035450180.535 | 998375.578 | 996753794746.834 |

Continued.
### Table 3: Arithmetic and harmonic means centroid computation.

| Control point | Coordinate register value | Existing orthometric height, (m) | CSRS-PPP Elip height (m) |
|---------------|---------------------------|----------------------------------|-------------------------|
| FCC11S        | 331888.114                | 998442.043                       | 485.447                 | 509.396                |
| FCT260P       | 255881.175                | 993666.807                       | 201.944                 | 224.74                 |
| FCT103P       | 340639.766                | 998375.578                       | 532.558                 | 556.836                |
| FCT12P        | 333743.992                | 1008308.73                       | 735.707                 | 760.192                |
| FCT19P        | 337452.408                | 996344.691                       | 635.644                 | 659.824                |
| FCT2168S      | 271370.273                | 73641825068.095                 | 495385.429              | 893753609365.514      |
| FCT266S       | 272644.591                | 74335073001.557                 | 941062.46               | 885598553621.252      |
| FCT24P        | 322719.776                | 1001884.85                       | 453.804                 | 477.987                |
| FCT276P       | 351983.716                | 1025998.314                      | 449.592                 | 473.942                |
| FCT4154S      | 329953.882                | 992926.363                       | 427.171                 | 451.299                |
| FCT4159S      | 322183.38                 | 992916.402                       | 497.253                 | 521.693                |
| FCT4028S      | 302324.27                 | 992916.402                       | 323.844                 | 347.795                |
| FCT4208S      | 330164.634                | 9009865544.354                  | 1001388.24              | 97918036249.808       |
| FCT83P        | 323954.205                | 11085802627.182                 | 987231.606              | 989535.439             |
| XP382         | 284074.729                | 806984156.423                    | 983864.863              | 96700643738.009       |

Total no. of points = 24

| Summation of SQ Easting (A) = | Summation of SQ Northing (B) = |
|-------------------------------|-------------------------------|
| 2365875207883.440             | 23500329269995.400            |

(A)/24 = 98578133661.810
(B)/24 = 97918036249.808

Computed RMS centroid 313971.5491

Continued.
RESULTS

Table 6, presents the accuracy of the model at various centroid means. This was done to show that the various centroid means types used in local geoid, as well as orthometric height modelling, have impacts on the accuracy of the model and to determine which of the centroid mean types is the best for geometric geoid model centroid mean computation.
DISCUSSION

It can be seen from Table 6, that the computed RMSE, as well as the accuracy of the arithmetic mean, root mean square and the harmonic mean centroids are respectively 0.110m, 0.187m and 0.303m. This implies that the centroid mean type used in geometric geoid, as well as orthometric height modelling, has an impact on the accuracy of the model. The variation in the accuracy of the model was as a result of the different theory which each centroid mean type development was based. It can also be seen from Table 6, that the arithmetic mean centroid has the smallest RMS error, as well as the highest accuracy which also shows that the arithmetic mean centroid is the best for geometric geoid, as well as orthometric height modelling. The centroid mean whose accuracy is the highest gives the most accurate interpolated orthometric height. The various centroid mean types have not really been applied and compared in any previous study.6,8

CONCLUSION

In conclusion, the results obtained in this study has shown that the centroid mean type used in local geometric geoid, as well as orthometric height modelling, has an impact on the accuracy of the model. The study has also revealed that the arithmetic mean centroid computation method is the best.

Recommendations

It is recommended that the arithmetic mean method should be adopted for centroid computation, as well as orthometric height modelling using the geometric method.

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