Visualizing Volume to Help Students Understand the Disk Method on Calculus Integral Course

F Tasman, and D Ahmad
Mathematics Department of Universitas Negeri Padang, Jl. Prof. Hamka, Padang, Sumatera Barat, Indonesia 25131
fridgo_tasman@fmipa.unp.ac.id, defriahmad88@gmail.com

Abstract. Many research shown that students have difficulty in understanding the concepts of integral calculus. Therefore this research is interested in designing a classroom activity integrated with design research method to assist students in understanding the integrals concept especially in calculating the volume of rotary objects using disc method. In order to support student development in understanding integral concepts, this research tries to use realistic mathematical approach by integrating geogebra software. First year university student who takes a calculus course (approximately 30 people) was chosen to implement the classroom activity that has been designed. The results of retrospective analysis show that visualizing volume of rotary objects using geogebra software can assist the student in understanding the disc method as one way of calculating the volume of a rotary object.

1. Introduction
Calculus is the heart of mathematics curriculum and one of the most important subjects because it is the foundation for the next course especially in the natural science, engineering, and mathematics classes[1]. This means that if students does not understand the calculus well, they will get difficulties for the next course. Because of the importance of this course, calculus becomes a compulsory course for university students in Indonesia. In Universitas Negeri Padang (UNP), this course is given to first year mathematics and science students.

One of the two important topics of calculus is integral[2]. Where on the definite integral of a function topic becomes important part. However, many studies suggest that students have difficulty in understanding the concept of integral calculus[3]. This also happened to the first year students in mathematics and science faculty of Universitas Negeri Padang. Based on the data about 30 percent of the students who took the calculus courses did not pass the course and some of them pass with unsatisfactory grade that impacted their success for next course. One of the causes that make they fail is lack of understanding of integrals concept. This happen because most of students tend to focus on integrating procedures without strengthening their understanding on integral concept itself. Tasman argues that educators should not rush to explain the procedure without a good understanding of the concept [4]. Therefore this topic needs a suitable approach to teach to the students in order to strengthen their understanding.

When students are learning about Integral concepts, they are expected to understand the rieman sum, limit, area and other concepts [5]. To understand the integral concept students must be able to connect those concepts well by building their knowledge. Integral is one part that cannot be separated
from calculus. In the calculus textbook[6], integral is introduced as an inverse of a derivative and continues to application of integral. One of its application is counting solids volume where one of the methods to count solids volume is disk method.

Students answers sheet on final test on the course of calculus tell us that most of students know the formula to count solids volume. However, most of them are wrong in determining the bound of the integral of the problem. it is happens because lack of their understanding on the concept. Based on this situation, the researcher is interested to design a series of integrated classroom activities in a design research method by using geogebra software to assist students understanding the integrals concept. In this article one of the activities discussed is counting the solids volume of rotary object by using disk methods. In order to support students development in understanding the disk method researcher try to apply realistic mathematical approach.

Tasman stated five tenets of realistic mathematics approach that adapted into this research [7]. These tenets are (1) Constructions stimulated by concreteness. The research does not start with formal level but starts with a situation that is experimentally real for students by seeing the volume of solid of rotary objects using geogebra software. (2) Developing mathematical tools to move from concreteness to abstraction. This tenets bridging from concreteness to a more formal level by using models and symbols. (3) Stimulating free production and reflection. Students own construction or production assumed will be meaningful for them. (4) Stimulating the social activity of learning by interaction. the social university classroom made the students have interaction between each other that provoke them to share idea and have a good discussion to solve the problems. (5) Intertwining learning strands in order to get mathematical idea structured. The topic, application of integral, have a strong relation to the topic of integral and the derivatives.

2. Method and Design

The research was conducted under design research methodology. Edelson stated the reason to chose this methodology [8]. First, the design research provide a productive perspective for theory development. Second, the design research has typical usefulness of its results and third, design research directly involved the researcher in the improvement in mathematics education.

Bakker identified five tenets that apply to different types of design research [9]. First is that its purpose to develop theories about learning and the means that are designed to support learning. Instructional theory for university students are designed which means to support students in learning the application of integral especially in counting volume of an object using the disk methods. The second feature is the interventionist nature. This methodology makes the researcher not constrained to improve the design after an experiment phase that has been carry out. The third, cross-cutting feature is that design research has a prospective and reflective component that cannot be separated by an experiment. The researcher confronts conjecture in a prospective with actual learning that he observe in a reflective part. The fourth feature is the cyclic character of design research where the invention and the revision occur as iterative process. Conjecture of learning are sometime refuted and alternative conjectures can be generated and tested. The fifth crosscutting feature of design research is that theory is relatively humble in the sense that it is developed for a specific domain that is the application integral especially counting the volume of solids using disk method therefore it must be general enough to be applicable in different contexts such as different classroom in other countries.

The main objectives of design research is to develop theories together with instructional material. This design research consist of cycle of three phases. They are:

2.1. A preparation and design phase

The first phase start with clarifications of mathematical learning goals, combined with anticipatory thought how to reach the learning goal in classroom application. The result is our conjectures of instructional activities which consist of three components, they are (1) Learning goal for students, (2) Planned instructional activities and the tools that will be used, (3) Conjectured of learning processes in
which one anticipates how students' thinking and understanding could evolve when instructional activities used in classroom [10]

2.2. A teaching experiment

Instructional activities are tried in this phase to collect data. The collected data will explain how a certain set of instructional activities could work. After that the instructional activities are revised on daily basis with the purpose to develop a well considered and empirically grounded local instructional theory.

2.3. A retrospective analysis

All collected data are analyzed in this phase. Our hypothetical learning trajectory (HLT) is compared with students actual learning. The exploration is refined to form a new cycle in the emergence of local instructional theory.

3. Findings and Discussion

A learning trajectory is defined as a description of the path of learning activities that the students can follow to construct their understanding of application of integral especially counting volume of solid by disk method, where in that path considers the learning goal, the learning activities and the conjecture of learning process. The learning trajectory is hypothetical because Students' interpretations, ideas and strategy never be sure until the students really work on that problems. The general overview of the Hypothetical Learning Trajectory (HLT) of application of integral in counting solid volume using disk method showed in Table 1.

| Name of Activity       | Students Activity                                      | Learning Goal                             | Mathematical Idea                                                                 | Strategy                                           |
|------------------------|--------------------------------------------------------|-------------------------------------------|----------------------------------------------------------------------------------|---------------------------------------------------|
| Lets count the volume   | Counting the volume of rotary object using disk method | Students are able to count the volume of  |
|                        |                                                        | rotary object using disk method.          | ▪ Partition the surface of the object which give us the area of circle as a result of |
|                        |                                                        |                                            | ▪ add up the area of the circle which means the integration from a (lower bound) to b (upper bound) |
|                        |                                                        |                                            | Giving the illustration of the volume by using geogebra software.                |

In, the design phase, students worksheet is designed. In the worksheet, students were asked to draw area that represented the \( \int_0^4 \sqrt{x} \, dx \), where the cartesius diagram and the function already on the answer box. After that the students were asked to draw the result of rotation of the picture on x axis 360 degrees. After that the students were asked the steps to count the volume.

Our conjecture in this activity are (1) Students will have difficulties in drawing the volume. If this happens the volume will be showed by using geogebra software. (2) Some students will have idea to count the volume, if this happen they will be asked to share their idea to others. If all students do not have idea, the lecture will ask the students to see the figure on geogebra and guide them to found the volume.

In the teaching experiment, students were asked to answer problems on their answers sheet by group. Most of students can draw the area of \( \int_0^4 \sqrt{x} \, dx \). However, most of them cannot draw the volume. Therefore visualization of the volume showed to the students using geogebra software. The visualization showed in figure 1.
After seeing the animation on the geogebra, the students are able to draw the volume of the object. Figure 2 shows some students’ representation on the object. When they complete to draw the representation of the object, counting the volume became another issue to discuss in classroom discussion.

In counting the volume of the object, students have to understand that it can be done by slicing the volume vertically (in this case), so they get many circle with different radius. They have to sum up all of those circle. However, the radius of the circle is not a constant. In order to know the radius of the circle, the prior knowledge of the function plays an important rule. Students have to know that the radius was implied in the function. After that shifting it into integral and doing some calculation on it is the last mathematical idea that the students have to be mastered.

The classroom discussion reveals that most of students do not have idea how to count the volume of the object. However, giving the idea of the volume of rectangular prism that the base times the height can provoke the students to have idea to slice the object and get the circle. Their knowledge about the area of the circle lead and their understanding of integral lead them to know the volume of the object.

Most of the students are able to count the volume after see the visualization of the object by geogebra software. However, some of them still have mistake for its results. Figure 3 showed students
error in counting volume of the object. In that figure, students already connect his knowledge about circle and their understanding about integral. However, he forgot to square the radius that implied in the function which lead them to the error. Therefore we suggest the lecture and the teacher have to ask the students to check their answers.

![Mathematical expression and diagram]

**Figure 3. Student' Steps to Count the Volume**

The research confirm the finding of Milanovic that the importance of using multimedia in the math classes [11]. By seeing the volume of object using geogebra software, students can see the rotation of the curve which produce a circle when it rotated in 360 degrees. This can make the students sure that they can use the formula $\pi r^2$ to count the volume and see that the radius implied in the function. The understanding of the radius implied in the function can build by seeing the visualization in the geogebra software. Students will see that the radius always change over the x axis.

### 4. Conclusion

The research found that the use of geogebra in visualizing the object in building students understanding on disk method in order to count volume of solid is important. The students need to see or imagine the object before they have an idea to count the volume of objects in disk method and build their understanding on the concept. Therefore we suggest the lecture/the teacher, in teaching the integral especially disk method topic, showed the volume first and let the students mathematize with it.

### Acknowledgment

The research that we discuss in this article is a part of a research entitle Design Research in Calculus Integral which funded by KEMENRISTEK DIKTI Indonesia.

### References

[1]. Bressoud, D. M. Why Do We Teach Calculus? Author ( s ): David M . Bressoud Source : The American Mathematical Monthly , Vol . 99 , No . 7 ( Aug . - Sep ., 1992 ), pp . 615-617 Published by : Mathematical Association of America Stable URL: http://www.jstor.org/stable/23. 99, 615–617 (2016).

[2]. Zakaria, Effandi & Salleh, T. S. Using Technology in Learning Integral Calculus . Mediterr. J. Soc. Sci. 6, 144–148 (2016).

[3]. Mahir, N. Conceptual and procedural performance of undergraduate students in integration. Int. J. Math. Educ. Sci. Technol.40, 201–211 (2009).
[4]. Tasman, F., den Hertog, J. & Hartono, Y. Helping students acquainted with multiplication in rectangular model. *IndoMS J.M.E*, 185–198 (2011).

[5]. Serhan, D. Students’ Understanding of the Definite Integral Concept. *Int. J. Res. Educ. Sci.*, 1, 84–88 (2015).

[6]. Dale Varberg, Edwin J. Purcell, S. E. R. *Calculus (9rd Edition)* - Dale Varberg, Edwin Purcell and Steve Rigdon.pdf. (Erlangga, 2010).

[7]. Tasman, F. *Supporting Second Graders’ on Learning Multiplication*. (Lambert Academic Publishing, 2017).

[8]. Edelson, D. C. Design research: What we learn when we engage in design. *J. Learn. Sci.*, 11, 105–121 (2002).

[9]. Bakker, A. Design research in statistics education: On symbolizing and computer tools. (2004).

[10]. Gravemeijer, K. Local instruction theories as means of support for teachers in reform mathematics education. *Math. Think. Learn.*, 6, 105–128 (2004).

[11]. Milovanović, M., Takači, & Milajić, A. Multimedia approach in teaching mathematics - example of lesson about the definite integral application for determining an area. *Int. J. Math. Educ. Sci. Technol.*, 42, 175–187 (2011).