Numerical Convergence of the Block-Maxima Approach to the Generalized Extreme Value Distribution

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Abstract In this paper we perform an analytical and numerical study of Extreme Value distributions in discrete dynamical systems. In this setting, recent works have shown how to get a statistics of extremes in agreement with the classical Extreme Value Theory. We pursue these investigations by giving analytical expressions of Extreme Value distribution parameters for maps that have an absolutely continuous invariant measure. We compare these analytical results with numerical experiments in which we study the convergence to limiting distributions using the so called block-maxima approach, pointing out in which cases we obtain robust estimation of parameters. In regular maps for which mixing properties do not hold, we show that the fitting procedure to the classical Extreme Value Distribution fails, as expected. However, we obtain an empirical distribution that can be explained starting from a different observable function for which Nicolis et al. (Phys. Rev. Lett. 97(21): 210602, 2006) have found analytical results.

Keywords Extreme values · Dynamical systems · EVT · GEV · Mixing · Logistic map · Chaos

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1 Introduction

Extreme Value Theory (EVT) was first developed by Fisher and Tippett [21] and formalized by Gnedenko [29] which showed that the distribution of the block-maxima of a sample of independent identically distributed (i.i.d.) variables converges to a member of the so-called Extreme Value (EV) distribution. It arises from the study of stochastical series that is of great interest in different disciplines: it has been applied to extreme floods [30, 50, 56], amounts of large insurance losses [6, 14]; extreme earthquakes [8, 13, 55]; meteorological and climate events [1, 19, 20, 48, 54, 60]. All these events have a relevant impact on socioeconomic activities and it is crucial to find a way to understand and, if possible, forecast them [34, 39].

The attention of the scientific community to the problem of modeling extreme values is growing. An extensive account of recent results and relevant applications is given in Ghil et al. [27]. Such an interest is mainly due to the fact that this theory is also important in defining risk factor in a wide class of applications such as the modeling of financial risk after the significant instabilities in financial markets worldwide [18, 28, 45], the analysis of seismic and hydrological risk [8, 47]. Even if the probability of extreme events decreases with their magnitude, the damage that they may bring increases rapidly with the magnitude as does the cost of protection against them Nicolis et al. [49].

From a theoretical point of view, extreme values represent extreme fluctuations of a system. Very recently, many authors have shown clearly how the statistics of global observables in correlated systems can be related to EV statistics [5, 15]. Clusel and Bertin [9] have shown how to connect fluctuations of global additive quantities, like total energy or magnetization, by statistics of sums of random variables in such a way that it is possible to identify a class of random variables whose sum follows an extreme value distributions.

The so called block-maxima approach is widely used in EVT since it represents a very natural way to look at extremes. It consists of dividing the data series of some observable into bins of equal length and selecting the maximum (or the minimum) value in each of them [11]. When dealing with climatological or financial data, since we usually have limited data-set, the main problem in applying EVT is related to the choice of a sufficiently large statistics of extremes provided that each bin contains a suitable number of observations. Therefore a smart balance between number of maxima and observations per bin is needed [19, 40–42].

Recently a number of alternative approaches have been studied. One consists in looking at exceedance over high thresholds rather than maxima over fixed time periods. While the idea of looking at extreme value problems from this point of view is very old, the development of a modern theory has started with Todorovic and Zelenhasic [57] that have proposed the so called Peaks Over Threshold approach. At the same time there was a mathematical development of procedures based on a certain number of extreme order statistics [36, 52] and the Generalized Pareto distribution for excesses over thresholds [16, 17, 53].

Since dynamical systems theory can be used to understand features of physical systems like climate and forecast financial behaviors, many authors have studied how to extend EVT to these field. When dealing with dynamical systems we have to know what kind of properties (i.e. stability, degree of mixing, correlations decay) are related to Gnedenko’s hypotheses and also which observables we must consider in order to obtain an EV distribution. Furthermore, even if the convergence is achieved, we should evaluate how fast it is depending on all parameters and properties used. Empirical studies show that in some cases a dynamical observable obeys to the extreme value statistics even if the convergence is highly dependent on the kind of observable we choose [58–60]. For example, Balakrishnan et al. [3] and more recently Nicolis et al. [49] and Haiman [33] have shown that for regular orbits of dynamical systems we don’t expect to find convergence to EV distribution.