Tray forming operation of paperboard: A case study using implicit finite element analysis

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Abstract
The possibility to perform advanced forming operations of initially plane paperboard paves the way to making products like food trays, plates, cups and other containers as a part of shifting towards a circular bioeconomy. As a part of the ongoing efforts of expanding the product range using paperboard, we performed analyses of the forming operation using simulations. An implicit non-linear finite element model is built to more accurately than previous studies simulate the tray forming process of paperboard. Two different commercial paperboards are investigated. The use of an implicit solver enabled the inclusion of the creasing pattern into the geometry of the paperboard blank resolving the formation of wrinkles during forming. The material data is extracted from tensile test curves of the investigated paperboards and was fitted accurately using Hill’s plasticity with difference in tension and compression accounted for with subsequent failure evaluation. The results showed that the inclusion of the creases in the geometry is vital for getting a correct shape of the formed tray and important for decreasing the risk of failure. The results also showed that friction has a big impact on the formed shape and hence on the stress levels, and therefore supports the means of lowering friction between the blank holder and the blank during the tray forming operation. A stochastic approach is proposed to determine the probability of failure for the boards. The performed failure evaluation is consistent with the field observations. The developed approach enables more precise simulations of paperboard tray forming.

KEYWORDS
creases, Hill’s plasticity, non-linear finite elements, paperboard, tray forming

1 | INTRODUCTION

Today, the possibility to perform advanced forming operations of paperboard has a big interest in the industry to create products like food trays, plates, cups and other containers. However, compared to plastics and sheet metal, paperboard has lower formability. Deep drawing of sheet metal is widely used in many industries, and plastics are used to form a variety of trays and containers. At the same time, paperboard is an important part of shifting towards a circular bioeconomy and the effort to increase formability and the processes enabling the forming operation is ongoing.

There are several types of forming operations for paperboard, such as hydroforming, press (tray) forming, deep drawing and molding. The first three of them are all variants of pressing down a sheet of paper or paperboard, called the blank, into a die. Hydroforming presses the blank downwards utilizing air pressure. The press forming uses...
a punch, also called male die, to press the blank into the bottom of a die, while deep drawing uses the punch to press the blank into a bottomless die. The methods are described in e.g. Hagman et al. 3 Östlund and Söderberg 9 and Lowe et al. 10 Complex products such as egg packages are molded. 11

As a part of the product development process using paperboard, simulations of the forming operations are used. The behaviour of the paperboard can then be studied, and it can be evaluated if and how material properties should be altered to minimize the risk of failure of the paperboard during the forming operation. The present study focuses on the tray forming operation of paperboard. A numerical approach is presented where the simulation of the operation is further developed compared to what has been done previously.

Deep drawing and tray forming of paperboard has been simulated in earlier studies, 12–14 with different numerical approaches and objectives. In Wallmeier et al. 12 the effects of varying the blank holder force, the die temperature, and the thickness of the paperboard were investigated. They showed, amongst other things, that the friction of the blank holder and die have significant effects on the stress in the blank, implying that low-friction dies and blank holders can considerably reduce the failure probability. In Awais et al. 13 the effect of the number of creases on the strain levels was numerically investigated for paperboard. The creases were modelled with hinge connector elements, and not explicitly included in the geometry. It was seen that the number of creases aided in lowering the strain levels. It could also be seen that this effect was greater for the first paperboard having a higher Young’s modulus compared to the second investigated paperboard, which had a larger strain to failure. The approach with hinge connector elements for the creases is investigated more in detail in Livill et al., 14 where the approach allows for spontaneous wrinkling, that is, the number of creases or the position of them do not need to be known by forehand. The approach is included in deep drawing simulations and shows that the number of creases is about the same as in experiments. The disadvantage of these approaches is the inability to assess the effect of creases on the frictional interaction between the paperboard and the forming unit as the wrinkles were not resolved physically. The inclusion of the creases in a numerical model is important not only to simulate the correct shape, friction interaction and strain levels, but also to investigate the possibilities to add a lid onto the tray to seal it. For the sake of successful sealing operations, the upper edge of the formed tray should be as smooth as possible.

The advances in the current study are as follows:

1. The use of an orthotropic material model with isotropic hardening according to Hill plasticity, fitted to actual tensile tests and compared with two sets of data from papers with different failure properties. The fitting accuracy was improved, which allowed a reliable comparison of different materials.
2. Accounting for the difference in tension and compression for yielding and failure.
3. The detailed resolution of the creases which are introduced through geometrical features rather than through the ad-hoc methods, and the considerably higher number of elements compared to earlier studies, allowing for more exact results. This enables to study the effect of creases on the frictional interaction.
4. The use of implicit time integration during the simulations, which has a direct impact on the accuracy of the test.
5. The analysis of the frictional conditions required to achieve the desired shape.

With all the aforementioned parts included at once in the model, the possibility to simulate advanced forming operations is enhanced. The objective is to increase the precision of the results coming from a simulation model of tray forming, giving more exact results for stresses, strains and the risk of failure compared to previous models. The verification is based on achieving the correct shape of the tray, something not fully attained in earlier studies of tray forming where the flange of the corners has been larger than what is seen in production, even for failed trays. The risk of failure is also determined and compared to reality.

The paperboard is commercially produced under the name Inverform by Iggesund paperboard, which is a part of the Holmen Group. The paperboard is produced on two board machines, but differences in converting performance have been observed and to understand this difference the present investigation was performed. The paperboard coming from the first board machine is here called Board A, and the paperboard coming from the second machine is here called Board B.

In Figure 1, the failure mode of the studied tray using Board A is shown, where the material fails in the corner under the creases. With the new simulation approach, the tray forming operation is simulated using the different paperboards, and the difference in results is evaluated and compared to reality.

## 2 MATERIALS AND METHOD

### 2.1 The tray

Paper and paperboard are anisotropic, heterogeneous, and hygroscopic materials. They are built up by a network of cellulose fibers from softwood or hardwood, or from a combination of the two. The fibers are randomly distributed over the sheet with, partly, random orientations. The fibers are connected via fibre bonds which form spontaneously when water disappears from the web in the papermaking process. The anisotropic material behaviour must be considered in numerical simulations of paperboard. A common way is to model the paperboard as orthotropic, 2,12–16 where material properties are specified in the paperboard machine-direction (MD), cross-direction (CD), and in the Z-direction (ZD), that is, through the thickness of the paperboard.

In Figure 2 the paperboard blank is shown as it is prepared by the tray manufacturer for the forming operation. The blank is laminated with a polymer that is extruded over the blank since the tray must withstand moisture during usage. The creasing pattern with 30 creases in each corner has been pressed into the paperboard so that it folds
FIGURE 1  The failure mode occurring for the studied tray using Board A. In (A) seen from the inside the tray, and in (B) seen from the outside.

FIGURE 2  The paperboard blank before forming operation. (A) The full blank with dimensions and (B) close-up of the creases.

FIGURE 3  The studied tray after a successful forming operation. In (A) seen slightly from above and in (B) seen from the side.
and shapes correctly. The grammage is 330 g/m² and the thickness of the paperboards, including the thin (30 μm) and compliant PET coating, is 0.5 mm.

Figure 3 shows the studied tray after a successful forming operation, that is, without detectable failure. The linear dimensions of the formed tray are 185 × 125 × 25 mm.

2.2 | Material data

Paperboard is an anisotropic material, which may be approximated as an orthotropic material. The material shows different responses in tension and compression which may not always be captured by the materials models available in the standard libraries in the commercial finite element tools. The source for the input data was the physical tensile tests of the two paperboard types considered in this study. The tensile tests are performed under ISO standard conditions in three in-plane directions, MD, CD and 45°, and are shown in Figure 4.

As observed, the biggest difference between the boards is the tensile properties in the 45°-direction. The tensile strain in the MD is about 1.9% for Board A, and 2.2% for Board B. The tensile stress in the MD is about 60 MPa for Board A and 70 MPa for Board B. For the CD, Board A has a tensile strain of about 5.0% at 30 MPa, while for Board B the tensile strain is 6.4% at 32 MPa. The distribution of the tensile test results is discussed later in the paper.

Paperboard exhibit a reduction in yield limit and strength in compression compared to the corresponding values in tension. This is taken into account by assuming the yield stress in compression being 70% of that in tension. For the failure evaluation, the compressive strength is reduced by 50% from the tensile value. The chosen values for yield and failure stress levels in compression are based on the reported values in Xia et al.

2.3 | Elastic material properties

In the following theory, the principal material directions MD, CD and ZD are described with indices 1, 2 and 3, respectively.

The elastic part of the paperboard is described using Hooke’s law $\varepsilon = C \sigma$. For an orthotropic material the full expression reads

$$
\begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
\varepsilon_{23} \\
\varepsilon_{31} \\
\varepsilon_{12}
\end{bmatrix} =
\begin{bmatrix}
1/E_x & -\nu_{12}/E_y & 0 & 0 & 0 & 0 \\
-\nu_{12}/E_y & 1/E_y & 0 & 0 & 0 & 0 \\
0 & 0 & 1/2G_{12} & 0 & 0 & 0 \\
0 & 0 & 0 & 1/2G_{12} & 0 & 0 \\
0 & 0 & 0 & 0 & 1/2G_{12} & 0 \\
0 & 0 & 0 & 0 & 0 & 1/2G_{12}
\end{bmatrix}
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{23} \\
\sigma_{31} \\
\sigma_{12}
\end{bmatrix}
\tag{1}
$$

The paperboards are modelled with 3D shells with plane stress assumption. In the case of plane stress, the expression in Equation 1 reduces to

$$
\begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{12}
\end{bmatrix} =
\begin{bmatrix}
1/E_x & -\nu_{12}/E_y & 0 \\
-\nu_{12}/E_y & 1/E_y & 0 \\
0 & 0 & 1/2G_{12}
\end{bmatrix}
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{12}
\end{bmatrix}
\tag{2}
$$

The two in-plane values for the Young’s modulus $E_i$ in Equation 2 are determined by the fitting procedure, and the in-plane shear modulus $G_{12}$ and Poisson’s ratio $\nu_{12}$ are then calculated using the two separate relations for commercially produced papers.
Table 1: Elastic material parameters

|                | Young’s modulus $E_1$, $E_2$ [MPa] | Shear modulus $G_{12}$ [MPa] | Poisson’s ratio $\nu_{12}$ [-] |
|----------------|----------------------------------|-----------------------------|-----------------------------|
| Board A        | 7143, 3078                       | 2258                        | 0.446                       |
| Board B        | 7501, 2948                       | 1511                        | 0.467                       |

$\nu_{12}$ is determined from the above parameters due to the symmetry of the stiffness matrix.

\[
G_{12} = \frac{\sqrt{E_1 E_2}}{2(1 + \nu_{12})}, \quad \text{and} \quad \sqrt{\nu_{12} E_{21}} = 0.293. \tag{3}
\]

Equation 3 along with the symmetry condition of the compliance matrix, giving $\nu_{21}/E_2 = \nu_{12}/E_1$, give the in-plane elastic material parameters which are listed in Table 1.

The out-of-plane strain $e_{33}$ can be derived as $e_{33} = -\frac{1}{2}\epsilon_{11} - \nu_{21}\epsilon_{22}$ as given by Equation 1 with $\sigma_{22} = 0$, but requires estimations of the Poisson’s ratios $\nu_{21}$ and $\nu_{23}$.

2.4 | Hardening model

Plasticity in paperboard has been modelled in many different studies. In Harrysson and Ristinmaa\textsuperscript{20} a large strain orthotropic elasto-plastic model was developed with a yielding surface based on the Tsai–Wu failure surface,\textsuperscript{21} which made it possible to directly introduce the difference in tensional and compressive yield behaviour for paperboard. Several models based on the complex anisotropic yield surface introduced by Xia et al.\textsuperscript{18} have been developed, such as the in-plane paperboard models established in Li et al.\textsuperscript{22} and Tjahjanto et al.\textsuperscript{23} The latter model is a viscoelastic-viscoplastic small strain approach developed to capture creep and relaxation for transient uniaxial loading. One of the latest publications on the subject is the one by Robertson et al.\textsuperscript{24} where the continuum model is based on previous models\textsuperscript{15,22} using numerous sub-surfaces. In Robertson et al.\textsuperscript{24} results from simulations using solid continuum elements and shell elements are compared for some forming operations. They showed, amongst other things, that the shell elements had a better performance compared to the continuum elements. For the example simulating an actual forming operation from the industry, frictionless contacts, an explicit solver scheme and ideal plasticity were used.

The evolution of the plastic strains in the current study is described using Hill’s plasticity,\textsuperscript{26} which is suitable for composites and a common way to model plasticity for orthotropic composite such as paperboard. Hill’s plasticity is defined as

\[
f(\sigma, \sigma) = F(\sigma_{22} - \sigma_{33})^2 + G(\sigma_{33} - \sigma_{11})^2 + H(\sigma_{11} - \sigma_{22})^2 + 2L\sigma_{23}^2 + 2M\sigma_{31}^2 + 2N\sigma_{12}^2 - \sigma_0^2 = 0 \tag{4}
\]

where $F$, $G$, $H$, $L$, $M$ and $N$ are defined as

and determine the shape of the yield surface, which initial size is determined by the initial yield stresses $\sigma_{0i}$ and the isotropic yield stress $\sigma_0$. The material parameters in Equation 6 are found with the previously mentioned fitting procedure. The stress–strain curve measured in the CD is used as a master curve for the multilinear hardening and the rest of the parameters are fitted in Matlab using the `fmincon` function to minimize the error between the measured and the calculated tensile test curves. The quality of the fit is shown in Figure 5. The curves on the compressive side are only from the numerical tests since no data is given from experiments. In compression, the two paperboards have a 30% reduction of the yield stress and a 50% reduction of the ultimate stress compared to the tensional side, which renders the curves in the compressive side in Figure 5.

The shape of the Hill yield surfaces for the two boards is shown in Figure 6, plotted for zero shear stress, $\tau_{12} = 0$. As seen in Figure 6, the modelled difference in tension and compression for the paperboards render in two yield surfaces per board, one for compression and one for tension. Which surface that applies for the current point is determined by the sign of the hydrostatic stress. With a positive sign, the hardening in the current point is evaluated towards the surface for compression, and a negative sign evaluates the evolution of the plastic strains towards the surface for compression. If a sign change would occur during the analysis, the point remains on the initially assigned surface which is shown with dotted lines in Figure 6. This approach of using two surfaces has an advantage in avoiding the difficulties in simultaneous fitting compressive and tensile behaviour with non-symmetric surfaces. The disadvantage is an abrupt change in the second and fourth quadrants. As the largest part of the paperboard appears in the first and third quadrant and the surface is not swapped during the simulations, this does not present a problem with convergence or thermodynamic inconsistency.

The fitting procedure resulted in different yield surfaces for the two paperboards (see Figure 6) Board B has a yield surface close to circular and an earlier yield point, compared to Board A. In Table 2, the complete set of parameters for the Hill’s plasticity used in this
study is presented. Note that the fit of $R_{33}$ is important even though plane stress is approximated, since $R_{33}$ influences the shape of the yield surface in the MD-CD plane.

2.5 | Failure evaluation

Failure is not included in the numerical model but will be evaluated as a part of the post-processing of the final results using the Tsai–Wu stress failure criterion.\(^{21}\) For plane stress, it reads\(^{27,28}\)

$$\sigma_{TW} = F_1 \sigma_{11} + F_2 \sigma_{22} + F_{12} \sigma_{12}^2 + 2F_{12} \sigma_{11} \sigma_{22} + F_{66} \sigma_{12}^2 < 1, \quad (7)$$

where $F_i$ and $F_{ij}$ are defined as

$$F_1 = \frac{1}{\sigma_{11}'}, \quad F_2 = \frac{1}{\sigma_{22}'}, \quad F_{11} = \frac{1}{\sigma_{11}' \sigma_{11}'},$$

$$F_{22} = \frac{1}{\sigma_{22}' \sigma_{22}'}, \quad F_{66} = \frac{1}{(\sigma_{12}')^2}, \quad F_{12} = k \sqrt{F_{11} F_{22}}, \quad (8)$$

and the indices ‘$t$’ and ‘$c$’ are for ultimate tensile stress and ultimate compressive stress respectively. In Equation 8, $F_{12}$ and the in-plane ultimate shear stress $\sigma_{12}'$ require some extra attention. These cannot be directly determined from tensile and compressive tests and require shear testing where the failure envelope is studied. For the current study, no such data is given for the two paperboards. Some estimations from the literature are required. For $F_{12}$ the constant $k$ is chosen as $k = -0.5$, which is suitable for most composites.\(^{27,29}\) In Li et al.,\(^{27}\)
for the interested reader, $F_{12}$ is analysed not only for closed failure surfaces but also for open surfaces. The ultimate shear stress $\sigma_{12}^*$ in Equation 8 may be estimated by using the geometrical mean of the tensile strength values in MD and CD, as done by Fellers et al.$^{30}$ for evaluation of the compressive modes. This study utilizes the geometrical mean for the tensile modes as

$$\sigma_{12}^* = \sqrt[4]{\sigma_{11}^t \sigma_{22}^t}. \quad (9)$$

In Figure 7 the failure surface is shown for the two boards for zero shear stress. The shape of the two surfaces is very similar, but Board A fails earlier compared to Board B. It is obvious that the combined stress state allows for considerably higher stress levels than can be concluded if only the uni-axial tensile tests are studied.

Strain failure is also evaluated, here using maximum strain theory according to

$$\varepsilon_F = \max \left[ \varepsilon_{11}^t, \varepsilon_{22}^t, \quad \text{abs} \left( \varepsilon_{12}^t \right) \right] < 1. \quad (10)$$

For zero shear strain, the failure envelop is a rectangle limited by the uni-axial tensile and compressive strains.

In Equation 10 the tensile shear strain must be estimated and is in this project estimated in the same way as the tensile shear stress as

$$\varepsilon_{12}^t = \sqrt[4]{\varepsilon_{11}^t \varepsilon_{22}^t}. \quad (11)$$

In Table 3 the material parameters for the failure evaluation are listed, which are given by the expression in Equation 8 along with the end value of the MD and CD curves in Figure 5.

### 2.6 | Finite element model

The simulations are performed with the finite element solver Ansys 2019R1 in a quasi-static regime using an implicit time-integration method. It is common to use explicit solver schemes for models exhibiting large non-linearities to avoid convergence problems. The use of explicit solvers introduces a limitation of the time step, which is limited by the size of the smallest element in the model and often requires the use of increase rate, mass-scaling, and the use of reduced-integration elements to make the solution solvable. In addition, to achieve the accuracy for the spring-back, damping or implicit solver should be used. The motivation for choosing the implicit solver was to avoid these limitations.

The model consists of the paperboard blank, the blank holder, the punch and the die. The blank is modelled with shell element 181, here fully integrated and with five integration points through the thickness. The tools are modelled as rigid bodies. Due to the symmetry, a quarter model is simulated, as seen in Figure 8. Initially, the blank is located so that the MD is parallel with the global x-axis and CD with the global y-axis, compare Figure 2A.

The blank is meshed with 0.5-mm quad elements over the area with the creases, and then up to 1 mm towards the centre of the model, as seen in Figure 9. In total, the model consists of about 50 300 elements. This is the finest mesh among those used to address similar problems. The creases are included in the geometry, in total 30 with a depth of 0.05 mm. The depth is based on the average depth measured on the actual blank. The depth is considerably lower than for creases used for folding corners of, for instance, boxes but it sufficient for this kind of converting operation. The thickness of the blank in the model is 0.5 mm.

The blank holder is force controlled, with a force of 700N acting on it (a quarter of 2800 N due to the use of quarter symmetry).
It corresponds to the actual force applied on the tray with the dead weight during the tray forming operation. The influence of the blank holder force has been investigated in previous studies. In Wallmeier et al.\textsuperscript{31} it was seen that the number of formed wrinkles for blank paperboard depend highly on the blank holder force, but also on the height of the formed product. In Awais et al.\textsuperscript{13} an optimum blank holder force was found to avoid failure during the forming process, and in Tanninen et al.\textsuperscript{32} the use of advanced force control was studied. The current study focuses on the difference between the two paperboards, and so the blank holder force will not be further evaluated here. The punch is displacement controlled, pressing down the blank to form the final tray.

During the real tray forming operation the temperature of the die is in the range of 140–180°C, and the punch 30–55°C. Paperboard is affected by temperature and highly affected by moisture change. In this case, the operation takes about 1 s, and the temperature and moisture change of the paperboard blank during this time is not fully known. Hence the paperboard material properties are based on the tensile tests performed under ISO standard conditions. The temperature of the forming tools also has a great effect on the paperboard-metal friction coefficient. In the current finite element model, all contacts are modelled as frictional contacts. In Huttel et al.\textsuperscript{34} the friction coefficient was determined to be in the range of 0.2–0.3 for tools holding room temperature. In Lenske et al.\textsuperscript{35} the friction coefficient for tools with room temperature was higher, from 0.3 and upwards. However, they also showed that for tools holding 60°C the friction coefficient was about 0.2, and for 120°C 0.1. Similar findings were seen in Wallmeier et al.\textsuperscript{12} but with even lower values: 0.1 and 0.075 respectively. Further, in the manufacturing process, it is possible to apply wax over the corner area to lower the friction when problems with failure are seen. With this background, the friction coefficient is chosen to 0.3 for all contacts, but the critical area where the creases are located, see Figure 10, are modelled with a 0.3, 0.1 and 0 coefficient of friction to study the influence of the friction in this area.

### RESULTS AND DISCUSSION

In the following, the trays are evaluated for stresses, strains, failure and the effect of creases and friction. All results are displayed at the end of the punch stroke step, that is, when the punch is pressing the blank towards the very bottom of the die. The results are displayed for the case of zero friction in the area shown in Figure 10 since this gave the best shape of the tray. The effect of altering the friction coefficient in the corner is shown later as a separate section.

In Figure 11 the shape of the simulated tray is shown and compared to the real tray. The shape of the simulated tray is to a high

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**TABLE 3** Tsai–Wu parameters

|        | Board A |          |          | Board B |          |          |
|--------|---------|----------|----------|---------|----------|----------|
| $F_1$  | $-1.6e-2$ | $-3.3e-2$ | $5.5e-4$ | $-1.4e-2$ | $-3.1e-2$ | $4.1e-4$ |
| $F_2$  | $2.2e-3$ | $5.5e-4$ | $-5.5e-4$ | $1.9e-3$ | $4.4e-4$ | $-4.4e-4$ |
extent agreeing with the shape of the real tray. At position ‘A’ in Figure 11, the material is buckling slightly inwards, which is seen in both the real and simulated tray. The same thing is seen at position ‘C’. At position ‘D’, a small outgoing ‘buckle’ is seen in both the real and simulated tray. At position ‘B’, some difference is seen. Here the material in the simulated tray has not folded enough to form the fully correct corner, but it is only a small difference. Further, some more wrinkles on the edges can be seen in the simulated tray. The appearance of the wrinkles along the edge is undesired as it hinders sealing the tray. The quantification of the wrinkles may be yet another verification possibility.31

3.1 | Normal stresses

In Figure 12 the normal stresses in MD and CD are displayed. The stresses are sampled from the mid-plane of the shell elements. For both boards, the highest tensile and compressive stress levels, in both MD and CD, arise between the creases. Although high stresses can occur over these areas, the numerical model used in this study possibly overestimates the stresses in that region. The reason is that the model uses homogenous material properties, whereas in reality, the paperboard consists of several plies through the thickness with various stiffness and the possibility to delaminate in the folding process. The interplay delamination and subsequent relaxation of stresses are not resolved with the current model. However, for these specific trays, the failure is seen in the lower corner and the current model outside the creased area, and therefore the limitation of not modelling delamination should affect the stresses in this area.

In the MD, the maximum stress outside the region of the creases occurs in the lower corner, see Figure 12A.B. For Board A, the MD stress here is 85 MPa, and for Board B about 65 MPa. For CD, the stress in the corner becomes about 50 MPa for Board A and 35 for Board B, see Figure 12C.D. The maximum stress in CD and MD does not occur in the exact same location.

3.2 | Total normal strains

The total normal strains in the MD and CD, sampled from the mid-plane, are shown in Figure 13. At the observed failure location, that is, the lower corner, the strain in MD is about 2% and 1.3% for Board A
and Board B respectively. For the CD, the strain for Board A is high, almost 12% close the lower part of the creases, and 8% in the same location for Board B. The strain levels may seem high but, as shown in for instance Hagman et al., can be considerably higher under complex loading states than seen in a standard tensile test.

### 3.3 Failure results

The stress and strain levels for the analyses are used to study failure of the trays. In Figure 14 the failure evaluation using Maximum Strain Theory is shown. The expression is shown in Equation 10 and, as seen there, only one component at a time of the in-plane strain components is evaluated. In other words, the contribution from the total strain state is not considered which in most locations (but not all) would allow for higher strain levels before failure. Hence the Maximum Strain Theory is a conservative approach, something seen in the results in Figure 14. In both models, there are many locations where the strain state is outside of the failure surface, that is, \( \varepsilon_F > 1 \), also in the lower corner for Board B where failure is not seen in production.

In Figure 15 the results using the Tsai–Wu Theory for failure evaluation are shown, see Equation 7. Here it can be seen that Board A experiences a stress state outside of the failure surface \( \sigma_{TW} > 1 \) in the lower corner, and Board B stays below \( \sigma_{TW} = 1 \) in the lower corner, which is in line with what has been observed in production. For both Board A and Board B, the area over the creases has several locations with high values of \( \sigma_{TW} \). Although failure can occur over these areas as well, the use of homogenous material properties and ignoring delamination leads to an overestimation of the stresses in these regions.
Failure evaluation

The Maximum Strain Theory was shown to be too conservative for this study. The Tsai–Wu stress $\sigma_{TW}$ for Board A implies that the risk of failure in the lower corner is very high, as seen in Figure 15A, where $\sigma_{TW}$ reaches 1.7 [-] which is far beyond the failure limit. Board B, however, has an area where $\sigma_{TW}$ reaches 0.8 [-] in the lower corner. In the following, these values are further analysed to show the probability of failure in these locations. The analysis is performed as follows:

1. Run FE analysis.
2. Identify the distributions for the tensile and compressive strengths given by tensile tests.
3. Use the modes, that is, the most frequent value, of the tensile and compressive strength distributions to post-process the FE-model for the Tsai–Wu stress $\sigma_{TW}$.
4. Identify critical locations in the model.
5. With the stress state ($\sigma_{11}, \sigma_{22}, \sigma_{12}$) in the critical location, run Monte Carlo simulations of the $\sigma_{TW}$ (Equation 7) where random numbers of the tensile and the compressive stresses are picked from their respective distribution.
6. Analyse the results to see the probability for failure, that is, probability to pass $\sigma_{TW} = 1$, in the critical location.

So far, point 1 to 4 has been performed. The tensile tests from point 2 are shown in Figure 4 and consist of 10 curves in each direction (MD, CD and 45). The number of points is too low to make a reliable estimate of how the tensile strengths spread. However, it has been seen in previous studies\cite{lin97} that the strength parameters for paper can be well described with the Weibull distribution. Hence, the Weibull probability density function, PDF, is fitted to the tensile strengths from the tensile tests performed for this study. The fit is performed utilizing the Matlab wblfit-function. The two-parameter Weibull PDF reads

$$f(x) = \frac{\beta}{\eta} \left(\frac{x}{\eta}\right)^{\beta-1} e^{\left(\frac{x}{\eta}\right)^\beta}$$

(12)
where the scale $\eta$ and shape $\beta$ must be determined. The results of the fit of Equation 12 to the tensile tests in Figure 4 are shown in Table 4.

Since the Weibull distribution returns 0 for a negative variable, the distributions for the compressive stresses were created by dividing the tensile test data by two (since the failure in compression is 50% of that in tension) for the MD and the CD respectively, which were then used for fitting of the Weibull function.

The analysis continues with Monte Carlo simulations of the $\sigma_{TW}$ in the studied locations. As seen in Equation 7, the current stress state $(\sigma_{11},\sigma_{22},\sigma_{12})$ in the studied location is a part of the $\sigma_{TW}$ expression. In Figure 16, a close-up of the corners of the trays is shown. For Board A, the $\sigma_{TW}$ is about 1.7 [-] in the most critical point, and for Board B the $\sigma_{TW}$ is about 0.8 [-]. Note that the locations are not the same. The stress state in the studied location is $\sigma_{11} = 75$ MPa, $\sigma_{22} = 50$ MPa and $\sigma_{12} = 1.5$ MPa for Board A, and for Board B the stress state is $\sigma_{11} = 9.5$ MPa, $\sigma_{22} = 34$ MPa and $\sigma_{12} = 2.7$ MPa.

Now Monte Carlo simulations are performed of Equation 7 where the $F_i$ and $F_y$ coefficients are re-calculated each time with values randomly picked from the Weibull tensile strength distributions. The values picked from the distributions for the compressive stresses are multiply by $-1$ to make the stresses negative. The distribution of the $\sigma_{TW}$ Monte Carlo simulations is expected to have a skewness, partly due to the Weibull distributed input strength parameters, and partly due to the non-linearity of the Tsai–Wu stress equation itself and its parameters, see Equations 7 and 8. The distribution with the best fit to the Monte Carlo simulations was the Generalized Extreme Values (GEV) distribution. The GEV PDF reads

$$f(x) = \frac{1}{\eta} t(x)^{\beta + 1} e^{-t(x)}$$

$$t(x) = \left(1 + \beta \left(\frac{x - \mu}{\eta}\right)\right)^{-\frac{1}{\beta}} \text{ if } \beta \neq 0$$

Equation 13 is fitted to the Monte Carlo simulations for the two boards using the Matlab `gevfit`-function, and the results are shown in Figure 17. The fitted GEV distribution parameters and the modes are shown in Table 5.

With the cumulative distribution function, CDF, for the fitted GEV functions, the probability of staying under the failure value $\sigma_{TW} = 1$ can now be derived. The CDF for the GEV function reads

$$F(x) = e^{-t(x)}$$

and derives the probability of staying below $x$. With Equation 14 the probability of passing $x = 1$, that is, $\sigma_{TW} = 1$, becomes for Board A in practice 100%. For Board B, this probability is about 1%, or 1 out of 100 trays can be expected to fail in this location.

The parameters in Table 5 only apply for the specific stress state $(\sigma_{11},\sigma_{22},\sigma_{12})$ since these are included in the expression for the Tsai–Wu stress, and hence affect the results of the Monte Carlo simulations. Hence, both point 5 and 6 must be performed each time a new location in the model is evaluated. That is, new Monte Carlo simulations must be performed for the studied stress state and then the risk of getting failure must be evaluated again. Other examples of how

| Paperboard | Strength   | Scale $\eta$ [MPa] | Shape $\beta$ [MPa] | Mode [MPa] |
|------------|------------|--------------------|---------------------|------------|
| Board A    | MD tension | 60.0               | 39.6                | 60.0       |
|            | MD compression | 30.0               | 39.6                | (-) 30.0*  |
|            | CD tension  | 29.4               | 45.3                | 29.4       |
|            | CD compression | 14.7               | 45.3                | (-) 14.7*  |
| Board B    | MD tension | 70.6               | 57.0                | 70.6       |
|            | MD compression | 35.3               | 57.0                | (-) 35.3*  |
|            | CD tension  | 32.3               | 60.3                | 32.3       |
|            | CD compression | 16.2               | 60.3                | (-) 16.1*  |

*The negative sign must be added after the fitting procedure.
the Tsai–Wu stress deviates depending on the stress state can be seen in, for instance, Mukherjee et al.\textsuperscript{38} It should be emphasized that at $\sigma_{TW} = 1$ failure is initiated, and is not necessarily the same thing as visible failure. The number of trays with visible failure can be expected to be fewer than what has been calculated in this study. Further, the stochastic approach can be extended to include a variation of the stress state. This means having a variation of the load or the material parameters in the constitutive equations. The assumption of zero variation of the stress state is of course non-conservative. The pressing of the punch can be considered to be quite a controlled process, but friction possibly adds a small variation of the stress levels in the paperboard blank. The non-uniformities of paperboard result in a variation of constitutive material

![Figure 16](image1.png)

**FIGURE 16** Close-up of critical points in the lower corner using the two different boards. In (A) Board A, and in (B) Board B

![Figure 17](image2.png)

**FIGURE 17** Histograms of the Monte Carlo simulations of the Tsai–Wu stress and fitted GEV PDF. (A) Plot of results for Board A, and in (B) for Board B

| Paperboard | Scale [$\eta$] | Shape [$\beta$] | Mode [$\mu$] |
|------------|---------------|----------------|-------------|
| Board A    | 0.154         | -0.067         | 1.71        |
| Board B    | 0.053         | 0.003          | 0.78        |

**TABLE 5** GEV distribution parameters for the fitted PDF seen in Figure 17
parameters, and the drying constraints during the manufacturing of paperboard will further contribute to a variation, as numerically proved in Alzweighi et al.\textsuperscript{39} To include a variation of constitutive parameters requires further testing but, if acquired, is then straightforward to include in the analysis.

The approach may also be more exact if testing of the materials size dependency is performed. As mentioned earlier, the failure stresses and strains for paperboard have been shown\textsuperscript{36} to have a size dependency, that is, locally allowing for higher stresses and strains than seen in standard tensile tests. Data for material size dependency was not accessible for this study and is hence not included, and in that sense the approach is conservative.

The effect of the polymer extruded on the paperboard blank before the tray forming operation could also be investigated. This coating may affect the upper ply allowing for higher failure strain.\textsuperscript{40} Finally, the use of shell elements leaves the contribution of the out-of-plane (ZD) stress outside of the analysis. The ZD stress will influence the stress-based failure surfaces, locally allowing for both lower and higher stress levels.

3.5 | Effect of friction and creases

In the following the numerical model is used for investigating the influence of friction and the importance of including the creases in the geometry. We used only Board A for this evaluation as the results are similar to Board B.

The effect on the shape of changing the friction coefficient in the corner (the area shown in Figure 10) is shown in Figure 18, where the friction coefficient in (a) is 0.3, in (b) 0.1 and in (c) 0. It is obvious that the friction makes the forming operation harder, and that it is an advantage of keeping the friction low in the corner. The shape closest to reality is given by the simulation using zero friction in the corner, shown in Figure 18C, which is also the model used for the above presented results.

In Figure 19, the effect of the friction coefficient in the corner on the Tsai–Wu stress $\sigma_{TW}$ is shown. Here it is seen that the area where $\sigma_{TW} > 1$ is very large for the cases with a friction coefficient of 0.3 and 0.1. In Figure 19C the friction coefficient is zero, and this is the same results as shown in Figure 15A. The results strongly support means of lowering the friction, such as the use of lubricants or increased die tool temperature, in the forming operation to aid problems with failure and bad shapes.

In Figure 20 the influence of the creases is shown. The model has zero friction in the corner area and can be compared with the results

![FIGURE 20](image-url) The shape of the formed tray without the use of creases

![FIGURE 18](image-url) Effect of friction coefficient in the corner on the shape. (A) Friction coefficient 0.3, (B) 0.1 and (C) 0

![FIGURE 19](image-url) Effect of friction coefficient in the corner on the Tsai–Wu stress. (A) Friction coefficient 0.3, (B) 0.1 and (C) 0
in Figure 18C. The model with no creases renders in a shape very different to that observed in the physical samples even for low friction and shows how important the creases are to avoid unwanted shapes. Without the creases, the material is having a hard time to fold leading to a large amount of material left on the upper edge at the end of the forming process.

4 | CONCLUSIONS

An implicit non-linear finite element model with full-integrated shell elements is built to simulate the tray forming process of paperboard. Two different boards with different failure propensities are investigated. The creases are included in the geometry of the paperboard blank which is a new approach compared to earlier studies. The material data are extracted from tensile test curves of the investigated paperboards, and the used material model includes different behaviour in tension and compression. The numerical tensile test curves have a very good fit with the experimental tensile test curves.

The results from the tray forming simulation show a very good agreement of the shape between the real tray and the simulated trays, which is a clear improvement compared to earlier studies of the paperboard tray forming process. The results show that including the creases in the geometry is very important to acquire the correct shape observed in reality. The incorrect shape is associated with an increased risk of failure of the tray during the converting operation. Further, the model shows that the friction between the paperboard blank and the die and blank holder has a great effect on the shape of the tray. High friction leads to incorrect shape and an increased risk of failure. The model supports the measures used in the industry to lower the friction during the converting operation. The area with the creases has several locations with large stresses. Although failure can occur over these areas as well, the use of homogenous material properties and ignoring delamination leads to an overestimation of the stresses in these regions. The failure evaluation using Maximum Strain Theory shows small differences between the two paperboards but is deemed too conservative. The failure evaluation using Tsai–Wu theory shows that Board A has a very high risk of failure, and that the risk of failure using Board B is lower, something that is in full agreement with what has been seen in production. The results from the stochastic failure evaluation based on the numerical model more precisely suggest that the risk of failure in the lower corner using Board A is close to 100%, and for Board B the risk of failure is about 1%. No known problems with Board B are reported from production and the here calculated failure risk is probably conservative.

The model may be further used to estimate the most critical material properties for the tray forming such as elongation, strength, and friction. Among these, in this study, we only probed the impact of the friction in the corner. A stochastic approach is used as a part of the post processing to study the impact of strength. However, the stochastic nature of paper makes it difficult to include the elongation and strength directly in the model as these parameters can vary locally and therefore require characterization and modelling which includes statistical distributions of the paper properties on the relevant scales. The latter is outside the scope of this work.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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