Effect of the variable gradient of sound speed on pulsating combustion of the solid fuel in Helmholtz resonator-type setup

V M Larionov¹, O V Iovleva¹, A O Malahov¹, I V Larionova²

¹Kazan Federal University, 18 Kremlyovskaya str., Kazan 420008, Russian Federation
²Kazan National Research Technical University named after A. N. Tupolev, 10 K. Marx St., Kazan 420111, Russian Federation

E-mail: ovi_b@rambler.ru

Abstract. In previous works, a theoretical study of pulsating combustion of solid fuel in Helmholtz-resonator-type setup was carried out under the condition that the distribution of the sound speed in the combustion products is constant, i.e. independent of the amplitude of gas oscillations. In this work, it is assumed that the gradient of sound speed linearly depends on the amplitude of pressure fluctuations in the combustion chamber. It is shown that the frequency of oscillations calculated for variable gradient of the sound speed differs significantly from the frequency that corresponds to a constant gradient.

1. Introduction

It is known that the energy approach [1-3] is an effective tool for studying the pulsating combustion of solid fuel in setup such as Helmholtz resonator. The results of the available work were obtained under the condition that the gradient of sound velocity in the combustion products is a constant value, independent of the amplitude of gas oscillations. Actually, in the regime of pulsating combustion, the heat transfer to the chamber walls accelerates. Therefore, the longitudinal distribution of gas temperature and sound velocity will depend on the amplitude of the gas oscillations. The aim of this work is to assess the effect of a variable gradient of sound velocity on the frequency and amplitude of gas oscillations during pulsed combustion of solid fuel in a Helmholtz resonator-type setup.

2. Mathematical model

The self-excitation of gas oscillations in Helmholtz resonator-type setup (Figure 1) for burning of solid fuel was studied in [4].
The linear distribution of the sound speed in the combustion products was used:

\[ c_2(x) = c^* - b_0 x, \quad b_0 = \frac{(c^* - c_{L,0})}{l_2}, \]  

(1)

where \( c^* = c_2(0) \) is the sound speed at the inlet of the resonance tube of length \( l_2 \), \( b_0 \) is the gradient of the sound speed, \( c_{L,0} \) is the sound speed at the output of the resonance tube in the absence of oscillations. Oscillations of gas accelerate heat transfer to the walls; therefore, the gas temperature and the sound speed at the output of the resonance tube decrease, i.e. \( c_i < c_{L,0} \). Suppose that the sound speed gradient depends linearly on the amplitude of pressure fluctuations \( p_c \) in the combustion chamber, i.e. \( b = b_0 + b_1 p_c \). Then the distribution of the sound speed in the resonance tube has the form:

\[ c_2(x) = c^* - (b_0 + b_1 p_c) x. \]  

(2)

If we know the speed of sound \( c_i \) for oscillations with a certain amplitude \( p_c^* \), the coefficient \( b_1 \) can be found from relation (2). Substitution of expression (1) for coefficient \( b_0 \) into relation (2) in which \( x = l_2, \ p_c = p_c^* \), \( c_2(l) = c_i \) gives:

\[ b_1 = \frac{c_{L,0} - c_i}{l_2 p_c^*}. \]  

(3)

The frequency equation, which corresponds to distribution (1), is known [6] and has the form:

\[ \frac{b_0}{\omega} + \beta_0 \tan \varphi_2 + F^{-1} = 0, \quad \beta_0 = \sqrt{1 - \left(\frac{b_0}{\omega l_2^2}\right)^2}, \]  

(4)

where \( \varphi_2 = -\omega \frac{l_2^2}{c^*}, \ l_2^2 = l_2^2 + 0.61R_2, \ \omega \) - cyclic frequency, \( R_2 \) - radius of the resonance tube. The function \( F \) depends on the volume of the combustion chamber, the radius and length of the air supply tube and was determined in [4–6].

The amplitude of the steady-state pressure fluctuations in the combustion chamber is represented by relation:

\[ c_2(x) = c^* - (b_0 + b_1 p_c) x. \]  

(2)
\[ p_c = \frac{a_{cL} - a_{w1} - a_{w2} - a_{L1} - a_{L2}}{a_{cN} + a_{1,N} + a_{2,N}}. \]  

(5)

Functions with the first index “c” characterize the generation of acoustic energy in the combustion zone, functions with the first index “w” characterize the loss of acoustic energy on the tube walls, functions with the first index “1” or “2” characterize the loss of acoustic energy caused by sound emission from open ends of tubes, index “L” corresponds to a linear approximation, index “N” corresponds to a nonlinear approximation. All functions were defined and presented in [4-7].

3. Results of work

In this work, the following experimental setup was considered: the volume of the combustion chamber is 1,1l, the length of the resonance tube is \( l_2 = 1,06 \, \text{m} \), the radius is 15mm, and the radius of the air supply tube is 20mm. We use the experimental data obtained for the combustion of wood particles: \( c^* = 601 \, \text{m}/\text{s}, \, c_{l0} = 503 \, \text{m}/\text{s}, \, p_{c}^* = 200 \, \text{Pa}, \, c_l = 362 \, \text{m}/\text{s} \). Substituting these data in relations (1) - (3) allows us to find \( b_0, \, b_1 \) and the distribution of the sound speed (2).

The length of the air tube \( l_1 \) varied from 0 to 0,25m. The iteration method was used. For a fixed value of \( l_1 \) and distribution (1) from equation (4) we find the initial oscillation frequency. From relation (5), we determine the initial amplitude of pressure fluctuations in the combustion chamber and the corresponding distribution of sound velocity (2). For this distribution, we find a new frequency and amplitude of pressure oscillations. The calculations end when the frequency change becomes less than 1Hz, and the change in the amplitude of the pressure fluctuations corresponds to 1dB.

The calculation results are presented in (Figure 2, 3), where curves (a) and (b) are obtained for distributions (1) and (2), respectively. For a fixed length of the air supply pipe, the vibration frequency calculated for the distribution of the sound speed in the resonance tube in the absence of oscillations is higher than the frequency found for the distribution, which depends on the amplitude of the gas oscillations. The higher the amplitude of the oscillations, the greater this difference. However, the amplitude of the pressure fluctuations and the corresponding sound pressure level \( I_c \) in the combustion chamber is almost independent of the type of sound velocity distribution in the resonance tube.

![Figure 2. Dependence of the frequency on the length of the air supply tube](image-url)
4. Conclusion
In previous works, the influence of the amplitude of gas oscillations on the gradient of the sound speed in a resonance tube was not considered. In this paper, it is assumed that this gradient increases in direct proportion to the amplitude of the pressure fluctuations in the combustion chamber. It is shown that the frequency of gas oscillations is significantly lower compared with the frequencies obtained for a constant distribution of the sound speed. At the same time, the inconstancy of the sound speed gradient has almost no effect on the amplitude of gas oscillations. This fact requires further study.

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