The Cosmic Ray Background as a Tool for Relative Calibration of Atmospheric Cherenkov Telescopes

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Abstract

The atmosphere is an intrinsic part of any ground based Cherenkov $\gamma$-ray telescope, and the telescope response is therefore sensitive to unpredictable changes in the atmospheric transparency which are difficult to measure and interpret in the absence of a calibrated beam of high energy $\gamma$-rays. In this paper, we use the detector response to Cherenkov emission from cosmic ray initiated air showers to obtain a relative calibration for data obtained under different instrumental and atmospheric conditions as well as over a range of source angles to the Zenith. We show that such a relative calibration is useful and efficient for data selection, for correcting the measured $\gamma$-ray rate and for inter-calibration between the elements of an array of Cherenkov telescopes.

1 Introduction

Atmospheric Cherenkov detectors cannot be calibrated using a test beam and the estimation of their sensitivity strongly depends on Monte Carlo simulation programs in which are modeled the atmosphere and the various elements of the detectors. Simulations usually assume a set of fixed conditions while the overall efficiency of the experiment can vary in time due to a number of factors. The most important cause of these variations is the atmosphere itself, and measuring and modeling changes in the $\gamma$-ray detection efficiency due to changing atmospheric conditions is complex. The slow degradation of optical elements until their recoating or replacement or the occasional readjustment
of photo-detector gains also affects the sensitivity of the experiment. When measuring the \( \gamma \)-ray flux from a source, one must correct for these effects.

In this paper we present a method used for the Whipple Atmospheric Cherenkov Imaging Telescope to estimate an overall relative efficiency factor. We also validate the method using observations of the Crab Nebula and present some applications. In its basic form the method is based on the analysis of data taken toward the Zenith and this is presented first. We have realized that the method can be generalized in a way which incorporates the effects of the Zenith angle at which observations are made. While detailed simulations will always be necessary in order to understand variations in telescope sensitivity, a simple correction such as that presented here is a useful tool which may be particularly important when studying the time variability of \( \gamma \)-ray sources. An example of this is the case of flaring active galactic nuclei (AGN) which may be observed over a long period of time and a wide range of Zenith angles and atmospheric conditions. We also present the application of a similar method to CELESTE, a Cherenkov wavefront sampling experiment, which illustrates the utility of the technique as a way of obtaining a relative calibration between individual elements of a detector array.

2 Relative calibration at fixed Zenith angle

2.1 The method

Each recorded Cherenkov event can be characterized by its luminosity, \( Q \), the definition of which may depend on the specific experiment. A relative throughput factor, \( F \), between two observation times can then be defined as the ratio between the luminosity produced by the same atmospheric shower observed at the same Zenith angle but under the two different conditions. For data obtained with the Whipple 10 m telescope, we define the luminosity of an event as the sum of the signals in all the photomultiplier tubes (PMTs) that gave a significant contribution to the image. In order to effectively estimate the throughput factor we use the fact that the cosmic ray spectrum is constant at the energies we observe, and therefore differences in the distribution of \( Q \) obtained at the same Zenith angle with the same detector should only reflect variations in light collection efficiency and gain of the experiment. Practically, in the Whipple data analysis, we construct the histogram of \( Q \) obtained from the Zenith observations during a specific night. This is then used as a reference for the other nights to be calibrated. For each of the other nights we construct the histogram of \( F \times Q \), with \( F \) being a test value for the relative gain between the night to be calibrated and the reference night. We then adjust \( F \) until the distribution best fits the reference
Fig. 1. The event luminosity distributions obtained with and without light collecting cones are shown at the top. The $\chi^2$ is calculated by comparing the distribution obtained without the cones with the distribution obtained with the cones (above a value of 1800 ADC counts) and rescaled by a test value for the throughput factor $F$. The minimum occurs for $F = 0.73 \pm 0.03$ indicating a 27% contribution by the cones to the light collection efficiency.

one. When applying this process one must take care to avoid the regions of the $Q$ distribution where the effects of threshold and saturation of the experiment become important.
2.2 An Application: Calibration of the focal plane light collecting cones

The Whipple telescope focal plane detector is equipped with light collecting cones which reduce the dead space between the photomultiplier tube pixels. The actual light collection improvement due to the cones is given by a combination of their optical properties and the shape and size of the main optics as seen from the focal plane. This factor is usually estimated by simulations based on measurements performed in the lab under simplified conditions. Here we try to quantify this factor using the method described in the previous section by taking some data toward the Zenith with and without the cones installed. The comparison of the two sets of data is presented in figure 1. The data with the cones are used as a reference and a relative throughput factor is calculated for the data obtained without the cones. From this we estimate that the cones are responsible for 27% of the Cherenkov light collection efficiency of the telescope. This result matches very well with the previous estimates (F. Krennrich, private communication).

2.3 Correlation with the sky quality

Every night, the Whipple telescope operator records his estimate of the sky quality on a qualitative scale ranging from $C^-$ to $A$. Data that were obtained under sky qualities less than $B$ are often rejected. In figure 2 we show the average throughput factor as a function of the observer’s estimate of sky quality, with the vertical error bars indicating the standard deviation. It appears that the throughput factor increases as the sky quality improves but the plot also shows that some of the data obtained under a $C$ sky could very well be used. Cirrus cloud occurs typically at an altitude of $\sim 10$ km, with night to night variations of $\pm 3$ km (3). The majority of the Cherenkov light produced by air showers is emitted near to the shower maximum (the altitude at which the number of particles in the shower is greatest) which is $\sim 10$ km for showers initiated by a 500 GeV $\gamma$-ray. It is possible, therefore, that the observer can be influenced in his judgment of the weather conditions by some high altitude cirrus clouds which could be responsible for an increase in the sky background brightness without affecting the quantity of Cherenkov light collected by the telescope. There will also be occasions when the telescope was aimed between the clouds during the observation. Because of this, the idea of calculating the throughput factor using a single test run with the telescope pointing to the Zenith is invalid, as the results cannot be applied to other observations on the same night. This is one of the motivations for generalizing the method to any Zenith angle.
Fig. 2. The relative throughput factor, $F$, measured at the Zenith as a function of the observer’s estimate of sky quality.

3 Relative calibration at any Zenith angle

3.1 Generalization

The relative throughput calibration method as described above already allows us to correct for changes in the telescope which affect the light collection efficiency, as well as helping with data selection. Nevertheless, it is based on data obtained at a fixed Zenith angle, which cannot be strictly contemporaneous with the astronomical observations of interest. Therefore it can not be used with confidence to make corrections to the data. It is, in principle, possible to apply the same method to compare data obtained at different Zenith an-
gles. The value of $F$ then results from differences in atmospheric transparency as well as differences in the detection geometry which affect both the energy threshold and the effective $\gamma$-ray collection area. When observations are made at lower Zenith angles the atmospheric showers produce a Cherenkov light pool which extends over a larger area ([7]).

Figure 3 shows how the throughput factor $F$ corresponds to a combination of two effects. Factor $T$ measures the horizontal shift in the distribution caused by changes in the light collection efficiency which result from changes in the instrument and atmospheric conditions as well as from differences in Zenith angle. Factor $A$ measures the vertical shift (a change in the number of showers observed with a given luminosity) caused by changes in the effective $\gamma$-ray collection area due to different source Zenith angles. Only the factor $F$ can be directly measured from the event luminosity distributions. As the radius of the Cherenkov light pool is defined only by the atmospheric density profile and source Zenith angle, factors $T$ and $A$ should show the same Zenith angle dependence for both $\gamma$-ray and cosmic-ray initiated showers as long as most of the Cherenkov light is emitted from the core of the shower. This allows us to use a generalized throughput factor in our analysis of $\gamma$-ray signals.

3.2 Test and application of the method

In figure 4 the throughput factor is shown as a function of $\theta_z$, the distance from the Zenith. The reference data were taken at $\theta_z \sim 30^\circ$ from the Zenith and so $F$ is close to one at this point. It can be shown that if the atmospheric density profile is assumed isothermal, the area of the Cherenkov light pool is proportional to $\frac{1}{\cos \theta_z}$ (see appendix). Using this, for a luminosity distribution of differential power law index $-\Gamma$, the throughput factor is expected to vary as

$$F \propto (\cos \theta_z)^{2(\Gamma + 1)} \times e^{-\frac{K}{\cos \theta_z}} \quad (1)$$

where the exponential term is used to describe the atmospheric attenuation of Cherenkov light. For our observed $\Gamma = 2.3$ we have

$$F \propto (\cos \theta_z)^{1.13} \times e^{-\frac{K}{\cos \theta_z}} \quad (2)$$

This function gives the curves shown on figure 4 for three different empirically derived values of $K$. Points falling near the upper curve would correspond to data obtained under the best atmospheric conditions while points on the lower curve correspond to data obtained under poorer conditions. We can see on this figure that variations of $\pm 20\%$ arise in the event luminosity even when
Fig. 3. The general principle behind the throughput factor (both axes are in log scale). See text for details.

the observer estimated the sky quality to be good (more than 90% of these observations were graded as A or B weather by the observer). Variations of this magnitude must be corrected for in order to establish accurate γ-ray fluxes, particularly in the case of sources with steep spectra.

In order to use the throughput value to correct the measured γ-ray rate, we must verify that the γ-ray showers are affected by changes in Zenith angle, instrument efficiency and atmospheric transparency in approximately the same way as the background cosmic ray showers which are used to derive the throughput factor. We do this by looking at the γ-ray rate observed in the direction of the Crab Nebula: the Crab is the standard candle of TeV γ-ray astronomy and dedicated studies (using selected data taken close to Zenith and under good weather conditions) have shown its emission at these energies to be constant over a timescale of years (8). If the throughput factor is applied correctly, the Crab Nebula γ-ray rate after correction should remain stable within statistical errors over all elevations and weather conditions.
Fig. 4. The throughput factor as a function of the distance from the Zenith. Each point represents a 28 minute observation (with statistical errors). The curves correspond to a simple isothermal model for the atmosphere with three different values for atmospheric attenuation.

There are different ways in which one could apply the correction. One possible method is to apply the throughput correction directly to the measured Cherenkov photon yield in each PMT, prior to parameterization of the images. The drawback here is that the signal to noise ratio will not remain constant over the range of throughput factors and Zenith angles, and so the efficiency of the γ-ray selection cuts will also change. In addition, for observations where the correction factor is large, the hardware trigger threshold will bias the number of images which pass the selection cuts. Because of this, we prefer to apply the correction directly to the measured rate. This is only strictly accurate if the spectrum of γ-rays from the source follows a simple power law of known
We try here to correct the $\gamma$-ray rate separately for the Zenith angle and atmospheric transparency effects. The Zenith angle dependence of the $\gamma$-ray rate is ideally calculated using Monte Carlo simulations; here we use a simple analytical model which provides a good approximation. The effective collection area, $A$, and threshold energy, $E_{th}$, are both proportional to $\frac{1}{\cos^2 \theta_z}$ and so the $\gamma$-ray rate $\Phi \propto (\cos \theta_z)^{2(\alpha - 1)}$ where $\alpha$ is the integral $\gamma$-ray power law spectral index. For the Crab Nebula, $\alpha = 1.5$ (3) and so $\Phi \propto \cos \theta_z$. This can be used to correct the measured $\gamma$-ray rate to the rate expected at a fixed Zenith angle; we choose to calculate the corrected rate for a Zenith angle of 30°, $\Phi_{30}$.

To apply the throughput correction we first calculate the expected throughput factor, $F_{exp}$, normalized to a Zenith angle of 30° (because the measured throughput factor $F_{meas}$ has been calculated with reference to an observation taken at a Zenith angle of 30°) such that:

$$F_{exp} = \left(\frac{\cos \theta_z}{\cos 30^\circ}\right)^{1.13}$$

(3)

This is equivalent to equation 2 but without atmospheric attenuation. The effects of atmospheric attenuation are automatically incorporated in the throughput correction, which we use to calculate the corrected rate as follows:

$$\Phi_{corr} = \frac{\Phi_{30}}{(F_{meas}/F_{exp})^\alpha}$$

(4)

Figure 5 shows $\Phi_{30}$ as a function of Zenith angle and of $F_{meas}/F_{exp}$. The $\gamma$-ray rate is constant with Zenith angle after the Zenith angle correction, while there is clearly still a correlation with the throughput correction which is well fit by a power law of index $\alpha = 1.5$, as expected for the Crab.

Figure 6 shows the reduction in the width of the rate distributions at each stage of the correction. We note, however, that the width of this distribution is not the best measure of the effectiveness of the correction. Observations at large Zenith angles often result in a measurement which is not statistically significant and so the width of the distribution for these runs will be dominated by statistical fluctuations. Most of these Crab data were taken under good weather conditions; the throughput correction will play an even stronger role when trying to analyse data taken under poorer conditions.
Fig. 5. The averaged Crab nebula γ-ray rate after correction for the Zenith angle as a function of Zenith angle (left) and $F_{\text{meas}}/F_{\text{exp}}$ (right).

3.3 Application to Mrk421 Variability Studies

Markarian 421 (Mrk421) is a bright TeV γ-ray source which has been well studied and is known to be extremely variable [10, 11, 12]. During the 2000/2001 observing season this source was in the most active state yet observed [13], with an average TeV γ-ray flux of 1.5 times the steady flux from the Crab Nebula. Measurements in the 2 – 12 keV X-ray region were also made by the All Sky Monitor (ASM) on board the Rossi X-Ray Timing Explorer (RXTE). Multiwavelength studies of the variable emission from AGN are extremely important to our understanding of the nature of the particles and acceleration mechanisms in jets. In the 2000/2001 Whipple observations of Mrk421, data
Fig. 6. The effect of the elevation and throughput corrections to the Crab Nebula $\gamma$-ray rate. The upper plots show the rate as a function of the Zenith angle. Each point represents a 28 minute observation (with statistical errors) showing the uncorrected rate ($\Phi$), the rate corrected to a fixed Zenith angle ($\Phi_{30}$) and the rate corrected for Zenith angle and throughput ($\Phi_{corr}$). The lower plots are histograms showing the distribution of the three rates.

were taken whenever possible, including at low Zenith angles and during poor weather, so as to provide the best possible sampling of the $\gamma$-ray light curve. The throughput factor provides us with a method to treat these data in a consistent fashion.

Observations of Mrk421 for the night of March 27th 2001 are shown in figure 7. The source was in a very high state and so we observed continuously from a source Zenith angle of 12° down to 60°. The difference between the results before and after throughput factor correction illustrates the importance of
Fig. 7. The light curve of Mkn421 for the observations of March 27\textsuperscript{th} 2001 with and without the correction for elevation and throughput factor being applied.

this correction when studying the detailed structure of flares.

Figure 8 shows the correlation between X-ray and γ-ray measurements both with and without a throughput correction applied to the γ-ray data. The correlation, as defined by the linear correlation coefficient, $r$, improves slightly when the throughput correction is applied. The correlation is not perfect; the remaining scatter may be due to the fact that the measurements are daily average fluxes, where the X-ray and γ-ray observations which have been used to calculate the averages are not exactly contemporaneous. Alternatively, it may be due to the details of the γ-ray production mechanism in the source. Clearly though, the magnitude of the effect of the throughput correction on the measured γ-ray flux shows the importance of the correction in this type
Fig. 8. Correlation between daily averaged ASM quicklook flux and $\gamma$-ray flux for Mrk421. In the right-hand plot the throughput correction has been applied, in the left-hand plot it has not.

of study.

4 Relative Calibration of a Telescope Array: CELESTE

In this section we consider a different use of the cosmic ray background. Rather than attempting to correct for temporal changes in the efficiency of a single telescope we are interested in calculating a relative calibration between the different elements of an array.

The CELESTE experiment (14; 15), situated in the French Pyrenees, uses forty movable 54 m$^2$ mirrors (known as heliostats) of a former solar electrical plant to reflect Cherenkov light from air showers to a detector package at the top of a 100 m tall tower. The detector package consists of a single photomultiplier tube (PMT) equipped with a fast analog-to-digital converter (FADC) for each heliostat, providing a measure of the arrival time and photon density of the Cherenkov light at ground level. This type of Cherenkov wavefront sampling experiment provides a massive total mirror area which allows us to reach an energy threshold of 60 GeV (16). The fundamental problem still exists however; there is no test beam with which to calibrate the experiment. Furthermore, in the case of a heliostat array it is necessary to calculate a relative calibration of the different heliostats which, for a solar plant experiment like CELESTE, will vary as the source position is tracked across the sky and the mirrors present a changing area of reflecting surface to the detector.
For each event which triggers the experiment, the luminosity $Q$ for each heliostat is simply given by the charge measured by the single PMT. The histograms of $Q$ can then be used to calculate a throughput factor for each heliostat in a similar fashion to that described above for the Whipple telescope. The difference in this case is that the throughput factor describes the relative gains of the forty heliostats. In fact, the throughput calculation has been made in a simplified way for CELESTE by simply integrating the $Q$ histograms and measuring the values of $Q$ at two constant fraction levels (5% and 30%) of the total number of events. These fractions were chosen so as to be distant from the regions of the histograms affected by saturation of the ADCs at the higher end and by the trigger conditions at the lower. We then normalize the difference in $Q$ at these two levels to provide the throughput measurement for each heliostat. If the distribution of $Q$ is a perfect power law this method should produce identical results to the $\chi^2$ minimization used for the Whipple telescope data. We have compared the two methods using Whipple data taken over a range of elevations and find the results to be consistent to within $\sim 10\%$. The results of this section illustrate relative changes in the heliostat response and so are not strongly affected by this discrepancy.

An example of the use of this measurement is shown in figure 9. The upper plot shows the layout of the CELESTE heliostat array and the tower which houses the detector package. Also shown are the five groups of heliostats whose signals are summed and used to trigger the experiment and the position of the Cherenkov imaging experiment, CAT. The three central plots show the throughput measurements for three heliostats at different positions in the heliostat array as a function of the Azimuth angle of the source in the sky. As the source moves across the sky, the heliostat efficiency changes, and this change will be greatest for heliostats which reflect the Cherenkov light through the largest angle to the detector in the tower. The three plots correspond to heliostats located on the far left (west), centrally and on the far right (east) of the array. The slope of the measured change in throughput with Azimuth angle reflects these positions. This is illustrated further in the lower plot which shows the slope of the throughput change with Azimuth angle as a function of the heliostat’s angular position in the array defined by the angle heliostat - tower - north. A heliostat due north of the tower has an angular position of 0°, while those to the east and west of the tower have positive and negative angular positions, respectively.

The measured change in throughput with source position has been used to verify the simulation of the rather complex telescope optics with good results (17). Also, the heliostat throughput factors, calculated with the heliostats observing a point due south of the experiment, have been used to calculate an adjustment to the high voltage supply of each PMT such that the relative gain after the adjustment is approximately the same for each heliostat/PMT pair. This correction smooths out the largest differences between the different
groups of heliostats which are used to trigger the experiment and prevents the experiment energy threshold being defined by a few very sensitive detectors. The throughput factors have also been used to identify heliostats with erratic behaviour; for example, heliostats with large tracking errors, which can then be repaired.

5 Conclusion

We have shown that cosmic ray background events observed at fixed Zenith angle can be used to establish a relative calibration for a single atmospheric Cherenkov imaging telescope in order to account for the many unavoidable temporal changes in light collection efficiency, gain and, most importantly, atmospheric conditions. Generalizing the method, we have shown that it can be used for the relative calibration of data obtained at different Zenith angles, taking into account both the geometrical effects due to Zenith angle and the variations in atmospheric conditions.

This calibration method can be used to introduce corrections at various levels. At the most basic level, it can be used to select which data were taken under good conditions. We have also shown that it can be used to rescale the measured $\gamma$-ray fluxes in order to make observations taken under different conditions more comparable. One method of estimating the background due to cosmic rays for $\gamma$-ray observations taken without dedicated background control observations is to choose archival background observations taken under conditions as similar as possible to the source observation being considered. The throughput factor can be used as one of the criteria to judge which background runs are most suitable (18).

The application of the throughput calibration to CELESTE observations shows how useful it may be when considering telescope arrays. The next generation of Cherenkov imaging telescopes are currently being developed; the VERITAS (19), HESS (20) and CANGAROO III (21) projects all involve using multiple telescopes on the same site. Inter-calibration of these telescopes will be difficult without a dedicated test beam. For VERITAS, simulations indicate that an energy resolution of 15% should be possible; in practice, this will require a relative calibration accurate to $<15\%$. The throughput method described here may well prove to be the best solution.
Fig. 9. The throughput variation with heliostat Azimuth angle as a function of the heliostat position. See text for details.
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A Appendix

Figure A.1 shows schematically the emission of Cherenkov light in the atmosphere. We can see that \( R = \frac{H - H_{tel}}{\cos \theta} \tan \psi \). The Cherenkov angle \( \psi \) is given by \( \cos \psi = \frac{1}{n} \) where \( n \) is the atmospheric refraction index at altitude \( H \). In a standard isothermal atmosphere \( n = 1 + 273 \times 10^{-6} e^{-H/8.5} \) where \( H \) is expressed in km. Using the first order small angles approximation for \( \psi \) one finds \( R = \frac{H - H_{tel}}{\cos \theta} \sqrt{\left(546 \times 10^{-6} e^{-H/8.5}\right)} \).

For small values of \( H \), \( R \) increases with \( H \) while for large values of \( H \), \( R \) decreases with \( H \). Therefore \( R \) must take a maximum of value \( R_{\text{max}} \) which corresponds to the rim of the Cherenkov light pool. By solving \( \frac{dR}{dH} = 0 \) for \( R \) one finds: \( R_{\text{max}} = \frac{17}{\cos \theta} \sqrt{546 \times 10^{-6} e^{-\left(17 + H_{tel}\right)/8.5}} \) which, for \( H_{tel} = 2 \text{ km} \), gives \( R_{\text{max}} \sim 130 \text{ m} \cos \theta \). From this it results that the effective collection area should scale as \( \frac{1}{\cos^2 \theta} \).

When we consider cosmic rays which are incident at an angle to the vertical the air shower develops in less dense atmosphere, where the Cherenkov emission per unit of track length is lower. The total track length is longer by an amount which compensates for this and consequently the total quantity of Cherenkov light produced by the shower does not depend on the Zenith angle \( \theta \). However, as the Cherenkov light pool on the ground extends over a larger radius, the light is more diluted and \( Q \), the measured luminosity of a shower, scales as \( \cos^2 \theta \).

In the generalized throughput calculation we compare luminosity distributions obtained under different elevations. The content of a specific luminosity bin will be affected by a factor \( \frac{1}{\cos^2 \theta} \), corresponding to the change in collection area, and by a factor \( \cos^2 \Gamma \theta \), corresponding to the event luminosity scaled for a power law luminosity distribution of spectral index \( \Gamma = 2.3 \) as measured in the case of the Whipple telescope. By combining those two factors raised to the \( 1/\Gamma \), one expects the throughput factor to scale as \( F \propto (\cos \theta) 2^{(1/\Gamma - 1)} \).
Fig. A.1. $\psi$ is the Cherenkov angle. $\theta$ is the angle between the particle arrival direction and the vertical. $R$ is the radius of the circle drawn by the Cherenkov light emitted at $P$ on the plane perpendicular to the arrival direction at $T$, the position of the telescope. $H_{tel}$ is the telescope altitude above sea level.