Some Consequences of the Law of Local Energy Conservation in the Gravitational Field

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Abstract

At gravitational interactions of bodies and particles there appears the defect of masses, i.e. the energy yields since the bodies (or particles) are attracted. It is shown that this changing of the effective mass of the body (or the particle) in the external gravitational field leads to changes the measurement units: velocity and length (relative to the standard measurement units). The expression describing the advance of the perihelion of the planet (the Mercury) has been obtained. This expression is mathematically identical to Einstein’s equation for the advance of the perihelion of the Mercury.

1 Introduction

At gravitational interactions there appears a defect masses [1], i.e. some an energy yields since the bodies (or particles) are attracted. In the previous work [1] it was shown, that the radiation spectrum (or energy levels) of atoms (or nuclei) in the gravitational field has a red shift since the effective mass of radiating electrons (or nucleons) changes in this field. This red shift is equal to the red shift of the radiation spectrum in the gravitational field measured in existing experiments. The same shift must arise when the photon (or \( \gamma \) quantum) is passing through the gravitational field if it participates in gravitational interactions. The absence of the double effect in the experiments means that photons (or \( \gamma \) quanta) are passing through the gravitational field without interactions.

This work is devoted to search for the influence of gravitational interaction on the body (or particle) characteristics.
2 Some Consequences of the Law of Local Energy Conservation in Gravitational Field

a). We will consider the influence of the external gravitational field \( \varphi = -G \frac{M}{r} \) (\( M \) is a mass of the external body, \( r \) is a distance and \( G \) is a gravitational constant) on characteristics of a body (or particle) having a small velocity. The law of local energy conservation in the classical case is presented in the following form:

\[
E = \frac{mv_1^2}{2} + m\varphi_1 = \frac{mv_2^2}{2} + m\varphi_2
\]  \( (1) \)

or

\[
\frac{m(v_1^2 - v_2^2)}{2} = m(\varphi_2 - \varphi_1)
\]  \( (2) \)

The Eqs. (1) and (2) characterize the balance between kinetic and potential energies (the smaller one energy, the bigger another energy and back). It is necessary to take into account the fact that in equations (1), (2) the mass \( m \) is included as a factor. Formally we can delete this factor and then we came to the following senseless equation:

\[
\frac{(v_1^2 - v_2^2)}{2} = (\varphi_2 - \varphi_1)
\]  \( (2) \)

In a more strict form the law of local energy conservation can be rewritten in the form:

\[
E = mc^2 + \frac{mv_1^2}{2} + m\varphi_1 = mc^2 + \frac{mv_2^2}{2} + m\varphi_2.
\]  \( (3) \)

Then the Eq. (3) can be rewritten in the following form:

\[
E = mc^2(1 + \frac{\varphi_1}{c^2}) + \frac{mv_1^2}{2} = mc^2(1 + \frac{\varphi_2}{c^2}) + \frac{mv_2^2}{2}.
\]  \( (4) \)

After introduction the new masses:

\[
m' = m(1 + \frac{\varphi_1}{c^2}) \quad m'' = m(1 + \frac{\varphi_2}{c^2}),
\]  \( (5) \)

and new velocities:

\[
v_1'^2 = \frac{v_1^2}{(1 + \frac{\varphi_1}{c^2})} \quad v_2'^2 = \frac{v_2^2}{(1 + \frac{\varphi_2}{c^2})}.
\]  \( (5') \)
Eq. (4) acquires the following form:

\[ E = m'c^2 + \frac{m'v^2}{2} = m''c^2 + \frac{m''v'^2}{2}. \]  

(6)

The eq. (6) means that in an external gravitational field the effective mass body (or particle) changes. For clarification of this question, let us consider a body (or particle) with mass \(m\) in the external gravitational field \(\varphi\) in point \(r\) and write the law of local energy conservation for this system

\[ mc^2 = E = mc^2 + \frac{mv^2}{2} + m\varphi m'c^2 + \frac{m'v^2}{2}, \]  

(7)

where

\[ m' = m(1 + \frac{\varphi}{c^2}) \quad \Delta m = m - m' = m\frac{\varphi}{c^2}, \quad \text{or} \quad \frac{\Delta m}{m} = \frac{\varphi}{c^2}, \]

and

\[ \Delta v^2 = v^2 - v'^2 = -v^2(\frac{\varphi}{c^2}) \quad \text{or} \quad \frac{\Delta v^2}{v^2} = \frac{\varphi}{c^2}. \]  

(8)

Eqs. (7), (8) mean that changing of mass (\(\Delta mc^2\)) of the body (or particle) in the external gravitational field goes on kinetic energy of this body (or particle).

And what is the result we have come to? In contrast to classical physics, the velocity of the body (or particle) is \(v'\) and

\[ \Delta v^2 = v^2 - v'^2 = -v^2\frac{\varphi}{c^2}, \quad \Delta v = -v\frac{\varphi}{2c^2}. \]  

(9)

The real velocity of the body (or particle) differs on \(\Delta v\) from the classical one. This difference can be measured by using Doppler and Mösbauer effects at moving the radiation atoms in the gravitational field of the Earth. We also can measure this effect at rotating planet around the star (the Sun). The real velocity of the rotating planet will differ from the classical one of the rotating planet.

b) Since the difference between the real and classical velocities \(\Delta v\) is very small, then this displace is very small. So, we can see this effect at multiple rotation of the planet around the star (the Sun). From equation (8) we see that this difference is bigger if the value \(|\frac{\varphi}{c^2}|\) bigger is. For the Sun system the biggest effect will be obtained for the Mercury since
its distance from the Sun is the smallest (this effect is known as advance of perihelia of the Mercury).

Let come to the computation of this effect.

The rotation period of a planet around a star (the Sun) on a nearly circle orbit is given by the following expression [2]:

\[ P = 2\pi \sqrt{\frac{r^3}{GM}}, \]  

(10)

where \( G \) is a gravitational constant and \( M \) is the Sun mass.

If the circular velocity of the planet (the Mercury) is \( v \) and it does one circle for period \( P \), then perimeter \( D \) is

\[ D = P \cdot v. \]

(11)

If the real velocity of the Mercury is \( v' \) and \( p' \) is the real period of its rotation, then the real distance \( D' \) is

\[ D' = P' \cdot v' \]

(11)

Let us introduce the following definition :

\[ \epsilon = \frac{GM}{c^2r} = -\frac{\varphi}{c^2} \]

(12)

The centrifugal force is equal to the attractive force on the absolute value at the rotating planet around the star (the Sun) and therefore

\[ \frac{mv^2}{r} = G\frac{mM}{r^2}. \]

(13)

From Eq. (13) we obtain the following correlation:

\[ v = \sqrt{\frac{GM}{r}}, \]

(14)

then the velocity variation \( \Delta v \) is

\[ \Delta v = -\frac{1}{2} \sqrt{\frac{GM}{r^{1.5}}} \Delta r, \]

then using eq. (14) we obtain the relation

\[ \frac{\Delta v}{v} = -\frac{1}{2} \frac{\Delta r}{r} \]

(15)
Since the time measurement unit is fulfilled by the atomic clock, then the decrease of mass results in the increase of length (Bohr orbit [1, 3]), then the decrease of the planet mass results also in increasing the distances (or standard measure unit).

\[ r \sim \frac{1}{m}, \quad m \to m - \Delta m (\Delta m > 0) \]

\[ r' = r + \Delta r = r (1 + \frac{\Delta r}{r}) \]

\[ \frac{1}{m'} = \frac{1}{m - \Delta m} \simeq \frac{1}{m} (1 + \frac{\Delta m}{m}) = \frac{1}{m} (1 + \epsilon) \quad (16) \]

So, we obtain

\[ \frac{\Delta r}{r} = \epsilon \quad (17) \]

Besides we must take into account the relativistic factor \( \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \), since the planet is moving around the star (the Sun) with velocity \( v \)

\[ \frac{1}{\sqrt{1 - v^2/c^2}} \simeq 1 + \frac{v^2}{2c^2} = 1 + \delta. \quad (18) \]

Putting the eq. (14) in eq. (18) we get

\[ \delta = \frac{v^2}{2c^2} = \frac{\epsilon}{2}. \quad (19) \]

Collecting eqs (17), (19) and using eq. (12) we obtain the following equation:

\[ \frac{\Delta r}{r} = \frac{\epsilon}{2} + \epsilon = 1.5 \epsilon, \quad (20) \]

or

\[ r' = r (1 + 1.5 \epsilon). \]

Putting eq. (20) in eq. (10), we come to the following expression for the period of the planet:

\[ P' = P (1 + 2.25 \epsilon). \quad (21) \]

Using eqs (15) and (20), we obtain the following expression for the velocity displace (variation):

\[ \frac{\Delta v}{v} = -\frac{1}{2} \frac{\Delta r}{r} = -\frac{1.5}{2} \epsilon. \quad (22) \]
The Kepler’s law states that the smaller the distance from the Sun, the more the planet velocity is, i.e.
\[
\frac{\Delta v}{v} = \frac{v - v'}{v} < 0.
\]
It takes place since the length measure unit grosses up and, correspondingly, the velocity must gross on the absolute value. If to take into account the Kepler’s law, then
\[
\frac{\Delta v}{v} = \frac{1}{2} \frac{\Delta r}{r} = \frac{1.5\epsilon}{2} = 0.75\epsilon
\]
(23)
or
\[
v' = v(1 + 0.75\epsilon)
\]
(23’)
Putting equations (21) and (23’) in eq. (11), we obtain
\[
D' = P'v' = Pv(1 + 2.25\epsilon)(1 + 0.75\epsilon) \simeq Pv(1 + 3\epsilon) = D(1 + 3\epsilon)
\]
(24)
or
\[
D' = D(1 + \frac{3GM}{c^2r})
\]
(24’)
Equation (24’) as function of angle \(\varphi\) has the following form:
\[
\varphi' = \varphi + \Delta\varphi, \quad \Delta\varphi = 2\pi 3\epsilon = 6\pi \frac{GM}{c^2r},
\]
(25)
for elliptic orbit \(r \rightarrow r(1 - e^2)\) \((e\) is a larger eccentricity and \(\Delta\varphi\) is
\[
\Delta\varphi = \frac{6\pi GM}{c^2r(1 - e^2)}
\]
(26)
Eqs. (25), (26) are mathematically identical to Einstein’s equation [4]. These equations are obtained in the flat space by taking into account changing of the effective mass of the planet in the external gravitational field, i.e. by using the law of local energy conservation. A more detailed consideration of this problem is presented in work [3].

3 Conclusion

So, the effective mass \(m'\) of the planet rotation around the star (the Sun) with mass \(M\) on distance \(r\) is
\[
m' = m\left(1 - \frac{\varphi}{c^2}\right), \quad \varphi = \frac{GM}{r},
\]
and the corresponding connection energy is

\[ \Delta E = m | \varphi | = \Delta mc^2. \]

Half of this energy is the energy of the planet rotation around the star (the Sun)

\[ m'v^2 = G \frac{m'M}{r} = \frac{m'}{2} | \varphi |, \quad v^2 = \varphi, \]

\[ E_{\text{kin}} = \frac{m'v^2}{2} = \frac{m'\varphi}{2}, \]

and another half-

\[ E_{\text{con}} = \frac{m'\varphi}{2} \]

is the energy lost of planet rotating around the star (the Sun).

At gravitational interactions of bodies and particles there appears the defect of masses, i.e. the energy yields since the bodies (or particles) are attracted. It was shown that this changing of the effective mass of the body (or the particle) in the external gravitational field leads to changes the measurement units: velocity and length (relative to the standard measurement units). The expression describing the advance of the perihelion of the planet (the Mercury) has been obtained. This expression is mathematically identical to Einstein’s equation for the advance of the perihelion of the Mercury.

References
1. Kh.M. Beshtoev, JINR Commun. P4-2000-45, Dubna, 2000; quanta-ph/0004074.
2. P.I. Bakulin, E.V. Kononovich, V.I. Moroz, Course of General Astronomy, M., Nauka, 1977.
3. P. Marmet, Physics Essays, 1999, v.3, p.468.
4. A. Einstein, Ann. Phys., 1911, v.35, p.898; M. Born, Einstein’s Theory of Relativity, Dover, N.Y., 1962; C.M. Will, Intern. Journal of Modern Phys., 1992, v.1, p.13.