1 Introduction

In 2012 a scalar particle with the mass of 125 GeV has been found at the LHC.\textsuperscript{1,2} In order to confirm that this is the Standard Model Higgs boson, its couplings have to be measured. Among these couplings, one of the most interesting is the triple boson coupling which is, up to a conventional coefficient, equal to the following ratio:

$$g_{hhh} \sim \frac{m_h^2}{v},$$

where $m_h$ is the Higgs boson mass, and $v$ is the vacuum expectation value. We know the vacuum expectation value with good precision from the Fermi coupling in muon decay, and now we have measured the higgs mass. Thus, any difference in the triple coupling constant from the theoretically predicted value would speak of New Physics in the scalar sector.

The triple coupling constant can be measured in the double higgs production process. However, the Standard Model prediction for such a process is very small, just 40 fb at the center-of-mass energy 14 TeV.\textsuperscript{3} Such value can only be measured at the HL-LHC. But if there indeed is New Physics, it might increase the double higgs production cross section, and we may be able to observe it during the Run 2. In this paper three extensions of the scalar sector of the Standard Model that might provide such an increase are considered.

2 Isosinglet

First, let us consider a model with an extra isosinglet.\textsuperscript{4} The extended scalar sector consists of two fields:

$$\Phi = \left(\frac{1}{\sqrt{2}}(v_\Phi + \phi + i\eta)\right),\quad X = v_X + \chi,$$

where $\Phi$ is the same isodoublet as in the SM, and $X$ is the new isosinglet. Both fields have their own vacuum expectation value, $v_\Phi$ and $v_X$, and $\phi$ and $\chi$ are the two neutral scalar particles. Two additional terms appear in the potential:

$$V_1(\Phi, X) = -\frac{1}{2} m_\Phi^2 \Phi^\dagger \Phi + \frac{\lambda}{2} (\Phi^\dagger \Phi)^2 + \frac{1}{2} m_X X^2 + \mu \Phi^\dagger \Phi X.$$  

They describe the bare mass of the isosinglet, and the mixing of the neutral particles. There are more terms allowed by Lorentz invariance, but we assume that they are multiplied by small coupling constants. We introduce the mixing angle $\alpha$,

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \phi \\ \chi \end{pmatrix}. \tag{4}$$


where $h$ and $H$ are the physical eigenstates of the neutral scalar particles.

In total, there are six parameters in the lagrangian: two vacuum expectation values $v_\Phi$ and $v_X$, two bare masses $m_\Phi$ and $m_X$, and constants $\lambda$ and $\mu$. Four of them are fixed: (1, 2) for two fields, we get two equations describing the minimum of the potential; (3) since the isosinglet does not couple to fermions, we get $v_\Phi$ from the muon decay just as in the SM; (4) we assume that it is $h$ that was discovered at the LHC, so we set $m_h = 125$ GeV. We are left with two free parameters, and we choose them to be $\sin \alpha$ and the mass of the second boson, $m_H$.

$H$ decay widths are just like those of the SM higgs, except that they are multiplied by $\sin^2 \alpha$.

In addition, a brand new decay mode appears:

$$\Gamma(H \rightarrow hh) = \frac{(2m_h^2 + m_H^2)^2}{32\pi v_\Phi^2 m_H} \sin^2 \alpha \cos \alpha \sqrt{1 - \frac{(2m_h^2)}{m_H^2}}. \quad (5)$$

$H$ production cross section is the same as the SM higgs production cross section, times $\sin^2 \alpha$.

In order to compute the double $h$ production, we multiply it by the corresponding branching ratio:

$$\sigma(pp \rightarrow H \rightarrow hh) = \sigma(pp \rightarrow h)_{SM} \cdot \sin^2 \alpha \cdot B(H \rightarrow hh). \quad (6)$$

Experimentalists provide us with measurements of the following values:

$$\mu_i = \frac{\sigma(pp \rightarrow h) \cdot B(h \rightarrow f_i)}{(\sigma(pp \rightarrow h) \cdot B(h \rightarrow f_i))_{SM}}, \quad (7)$$

where $h \rightarrow f_i$ describe different decay modes. In the isosinglet model $\mu_i = \cos^2 \alpha$. Experimental values combined into a single quantity are:

$$\mu = 1.36^{+0.18}_{-0.17} \text{ by the ATLAS collaboration}; \quad (8)$$

$$\mu = 1.00^{+0.14}_{-0.13} \text{ by the CMS collaboration.}$$

To take experimental values into account in a more robust way we have calculated a fit of electroweak observables and the measurements of $\mu$. The fit was calculated with the help of the LEPTOP program. Fit results are presented in fig. 1. The minimum of $\chi^2$ is reached at line $\sin \alpha = 0$, with $\chi^2 = 19.6$ for the 13 degrees of freedom listed in our paper. From the fit it follows that $\sin \alpha$ cannot be large, with the maximum value of about 0.35 for confidence probability 95% and $m_H = 300$ GeV.

The golden mode for the search of the new heavy higgs boson is the $H \rightarrow ZZ$ decay mode, just as it was for the Standard Model higgs. The expected signal strength

$$R \equiv \frac{\sigma(pp \rightarrow H)B(H \rightarrow ZZ)}{(\sigma(pp \rightarrow h)B(h \rightarrow ZZ))_{SM}} = \frac{\sin^4 \alpha}{\sin^2 \alpha + \frac{\Gamma(H \rightarrow hh)}{\Gamma_{SM}}}. \quad (9)$$

Note that it does not depend on collision energy. Contour lines of $R(\sin \alpha, m_H)$ are presented in fig. 2. For the allowed region of $\sin \alpha < 0.35$ we get $R < 0.1$. Experimental data has not set bounds on that level, with only some tension being observed in e.g., CMS paper, fig. 5 at $m_H \approx 250$ GeV and $m_H \approx 300$ GeV.

Double higgs production cross section for the center-of-mass energy 14 TeV is shown in fig. 3. In the allowed region it varies from about 0.4 pb at $m_H = 300$ GeV down to the order of several fb as $m_H$ reaches 1 TeV.
3 Isotriplet

In the isotriplet model the extra fields are conventionally represented by a \(2 \times 2\) matrix:

\[
\Delta = \frac{\Delta \bar{\sigma}}{\sqrt{2}} = \begin{bmatrix}
\frac{\delta^+}{\sqrt{2}} & \delta^{++} \\
\frac{1}{\sqrt{2}}(v_\Delta + \delta + i\eta) & -\frac{\delta^+}{\sqrt{2}}
\end{bmatrix},
\]

where \(\delta\) is the new neutral particle. The isotriplet also has its own vacuum expectation value \(v_\Delta\). The other fields \((\eta, \delta^+, \delta^{++})\) are of no interest to us at this moment. We will use the same notation for physical eigenstates as in the case of the isosinglet (4) (with \(\chi\) replaced with \(\delta\)).

In contrast to the isosinglet, the isotriplet couples to gauge bosons. Consequently, its vacuum expectation value produces contributions to masses of gauge bosons:

\[
m_W^2 = \frac{\tilde{g}^2}{4}(v_\Phi^2 + 2v_\Delta^2), \quad m_Z^2 = \frac{\tilde{g}^2}{4}(v_\Phi^2 + 4v_\Delta^2),
\]

where \(g\) is the \(SU(2)_L\) coupling and \(\tilde{g} = g / \cos \theta_W\), \(\theta_W\) is the Weinberg angle. The fact that gauge boson masses are changed nonuniformly breaks custodial symmetry of the model. The breaking is characterized by the quantity

\[
\rho \equiv \frac{m_W}{m_Z \cos \theta_W} \approx \left( \frac{m_W}{m_Z \cos \theta_W} \right)_{SM} \left( 1 - \frac{v_\Delta^2}{v_\Phi^2} \right).
\]

The value of \(\rho\) provided by PDG\(^{10}\) is 1.00040±0.00024. Although it is greater than 1, we can set the bound \(v_\Delta \lesssim 5\) GeV at 3\(\sigma\) level, and this is the strongest bound in the isotriplet model. To estimate the upper bound on the cross section, we will use the value \(v_\Delta = 5\) GeV in following.
From the Fermi coupling constant we get that \( v_\Phi \approx 246 \text{ GeV} \), just like in the SM. It follows that \( \sin \alpha \approx \frac{2v_\Delta}{v_\Phi} \approx 1/25 \), so only one model parameter remains which is \( m_H \). We will consider the case of \( m_H = 300 \text{ GeV} \) so that \( H \) has enough mass to decay to \( hh \), but not to \( t\bar{t} \).

It is a peculiar property of this model that \( H \to WW \) decay is suppressed as \( (m_h/m_H)^4 \), so \( H \to ZZ \) is the “golden mode” for the heavy Higgs boson discovery. As for the \( H \to hh \) decay, its branching ratio approximately equals 0.8 for the chosen case of \( m_H = 300 \text{ GeV} \).

The SM Higgs boson is produced at the LHC through the six main channels: \( gg, WW \) and \( ZZ \) fusions, \( t\bar{t}, W \) and \( Z \) associated productions. Same is true for the heavy Higgs boson of the isotriplet model. Corresponding cross sections can be calculated from cross sections of the SM with \( (m_h)_{SM} = m_H \). Noting that \( gg \) fusion and \( t\bar{t} \) associated production share the same higgs vertex, we get

\[
\frac{\sigma(gg \to H)}{\sigma(gg \to h)_{SM}} = \frac{\sigma(gg \to Htt)}{\sigma(gg \to htt)_{SM}} = 2.4 \cdot 10^{-3},
\]

Similarly,

\[
\frac{\sigma(ZZ \to H)}{\sigma(ZZ \to h)_{SM}} = \frac{\sigma(Z^* \to ZH)}{\sigma(Z^* \to Zh)_{SM}} = 1.0 \cdot 10^{-3},
\]

\[
\frac{\sigma(WW \to H)}{\sigma(WW \to h)_{SM}} = \frac{\sigma(W^* \to WH)}{\sigma(W^* \to Wh)_{SM}} = 7.3 \cdot 10^{-5}.
\]

In the SM the gluon fusion channel dominates, being an order of magnitude greater than the second biggest one, \( WW \) fusion. With these numbers it is clear that gluon fusion dominates even stronger for the heavy Higgs boson of the isotriplet model. For \( \sqrt{s} = 14 \text{ TeV} \) we get \( \sigma(gg \to H) = 25 \text{ fb} \), so the 125 GeV Higgs boson production cross section gets enhanced to

\[
\sigma(pp \to hh) = 40 \text{ fb (SM)} + 25 \text{ fb (H production)} \cdot 0.8 \text{ (branching)} = 60 \text{ fb}.
\]

Custodial symmetry can be saved through introduction of yet another, real isotriplet, with its vacuum expectation value equal to \( v_\Delta \). The corresponding model is referred to as the Georgi-Machacek model. In this case the bound coming from (12) is removed, and signal strength measurements allow \( v_\Delta \) to reach 50 GeV. Double higgs production cross section can then be as high as 2 pb.

An interesting property of the Georgi-Machacek model is that when the mixing angle is small, decays of \( H \) to vector bosons are severely suppressed, with about 98% decays going through the \( H \to hh \) channel for \( m_H \) near 300 GeV. Hence, search in the \( ZZ \) final mode at the LHC will not lead to new limits on model parameters.

\section{Conclusions}

Significant enhancement of the cross section for double production of 125 GeV Higgs bosons can be observed in the isosinglet model. Depending on model parameters, it can be as high as 0.4 pb for the collision energy of 14 TeV, an order of magnitude greater than the SM value of 40 fb. Primary model constraints are set by the signal strength measurements, with experiment data right now becoming sensitive to the \( H \to ZZ \) decay mode.

On the contrary, the isotriplet model is severely constrained by its inherent custodial symmetry breaking. With only 20 extra fb of the cross section for the most favourable value of \( m_H = 300 \text{ GeV} \), we have little hope to test this model through double higgs production before we will reach the level of accuracy that would allow us to test the SM directly.

However, further extension of the isotriplet model to the Georgi-Machacek model changes the picture entirely. In this case the cross section can reach 2 pb, and with the decays of the
second Higgs boson to vector bosons possibly suppressed, double higgs production might be the best mode to test this model at the LHC.

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