Is Dark Energy the Only Solution to the Apparent Acceleration of the Present Universe?

Hideo Iguchi\textsuperscript{1}, Takashi Nakamura\textsuperscript{2} and Ken-ichi Nakao\textsuperscript{3}

\textsuperscript{1}Department of Physics, Tokyo Institute of Technology, Tokyo 152-8550, Japan
\textsuperscript{2}Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan
\textsuperscript{3}Department of Physics, Osaka City University, Osaka 558-8585, Japan

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Even for the observed luminosity distance $D_L(z)$, which suggests the existence of dark energy, we show that an inhomogeneous dust universe solution without dark energy is possible in general. Future observation of $D_L(z)$ for $1 \lesssim z < 1.7$ may confirm or refute this possibility.

\section{Introduction}

Recent measurements of the luminosity distance $D_L(z)$ using Type Ia supernovae\textsuperscript{1)-3} suggest that an accurate value of $D_L(z)$ may be obtained in the near future. In particular, SNAP\textsuperscript{4} should provide us the luminosity distances of $\sim 2000$ Type Ia supernovae with an accuracy of a few percent up to $z \sim 1.7$ every year. Also, from observation of the first Doppler peak of the anisotropy of the CMB, it is suggested that the universe is flat,\textsuperscript{5,6} and this may be proved in the future from observations by MAP and Planck. Under the assumption of the homogeneity and isotropy of our universe, these observations suggest that dark energy is dominant at present.

In an attempt to determine the nature of dark energy, many arguments have been given.\textsuperscript{7} Recently, some mechanisms to account for the observed tiny but finite dark energy are proposed\textsuperscript{8),9} However, at present we do not have a firm and reliable theoretical basis to investigate such a small energy scale compared with the Planck scale. In short, the nature of dark energy under the assumption of the homogeneity and isotropy of our universe is still a great mystery.

From the observed isotropy of the CMB, assuming that we are not in a special part of our universe, the universe should be homogeneous. However, if our position in the universe is special, the universe might be inhomogeneous, although the CMB is isotropic. Such cosmological models have been constructed using spherically symmetric models in which we are near the symmetric center. Some authors have considered such models to interpret the SNIa data for small $z$,\textsuperscript{10} as well as large $z$ assuming a void structure\textsuperscript{11)-13} to avoid dark energy. Such possibilities may be regarded as absurd. However, our point of view in this paper is to construct a possible inhomogeneous dust universe derived from the observed $D_L(z)$. If such a model is consistent with present observational results, the inhomogeneous universe should be examined more seriously, because the dark energy solution is also absurd in the
sense that it is \( \sim 120 \) orders of magnitude smaller than the Planck scale. In short, we suppose that the question we need to answer to roughly reduces to the following: Which is more absurd, dark energy or an inhomogeneous universe? In the former case, there is no reliable theory to examine the problem at present, while the latter case can be studied in the frame-work of known theories. We would like to point out that it is not taste but, rather, future observations that will confirm either dark energy or an inhomogeneous universe.

The analysis of high redshift supernovae gives us the luminosity distance-redshift relation \( D_L(z) \) along the observational past null cone up to \( z \sim 1 \). The data fit well with \( D_L(z) \) in the homogeneous and isotropic universe with \( \Omega_m = 0.3 \) and \( \Omega_A = 0.7 \) given by

\[
D_L(z) = \frac{1}{H_0}(1 + z) \int_0^z \frac{dz'}{\sqrt{\Omega_m(1 + z')^3 + \Omega_A}}. \tag{1.1}
\]

In this paper we assume that \( D_L(z) \) is given by Eq. (1.1) with \( \Omega_m = 0.3 \) and \( \Omega_A = 0.7 \) for \( z \lesssim 1 \). This is done for the sake of simplicity to make the arguments clearer. In particular, we do not wish to claim that \( D_L(z) \) with \( \Omega_m = 0.3 \) and \( \Omega_A = 0.7 \) has been confirmed. While \( D_L(z) \) for \( 1 \lesssim z < 1.7 \) is not certain even at present and will be obtained in the future, for example, by SNAP. Since the scale factor \( a \) obeys

\[
\frac{\ddot{a}}{a} = - \frac{4\pi}{3}(\rho + 3p), \tag{1.2}
\]

\( D_L(z) \) with \( \Omega_m = 0.3 \) and \( \Omega_A = 0.7 \) implies that the present universe is accelerating, while for the dust universe (\( p = 0 \)), \( a \) should be decelerating. Therefore it may be concluded that observations is inconsistent with the inhomogeneous dust model. However, the point is that to determine \( D_L(z) \), we are observing Type Ia supernova events that occurred at past times in spatial positions separated from us. In the inhomogeneous universe model, the time dependence of \( a \) at a point separated from us differs from that of our position, so that we may obtain an apparent accelerating universe even though the dust universe is decelerating locally.

Before ending this introduction, we comment on some other works relevant to this paper. The inhomogeneous scenario is not the only alternative to dark energy. Giving up the assumption that the cosmic substratum is composed of perfect fluids, bulk pressures that differ from the kinetic pressure can be allowed. The assumption of an effective anti-friction force leads to a model that has only one dark component (CDM) and is consistent with the CMB and SNIa data. Also, there is an approach somewhat related to that presented in this paper (though with different motivation) that has been used.

\section{Formulation}

The line element of a spherically symmetric dust universe is given by

\[
ds^2 = -dt^2 + \frac{(R'(t, r))^2}{1 + 2E(r)r^2}dr^2 + R^2(t, r)d\Omega^2, \tag{2.1}
\]
where the prime indicates differentiation with respect to $r$. The solution to the Einstein equations is known as the Lemaitre-Tolman-Bondi (LTB) spacetime, given by

$$
\dot{R} = \sqrt{\frac{2M(r)}{R} + 2E(r)r^2},
$$

$$
4\pi \rho(t, r) = \frac{M'(r)}{R^2 \dot{R}'},
$$

where the dot indicates differentiation with respect to $t$. The solution of Eq. (2.2) is given by

$$
R(t, r) = \frac{M}{\epsilon(r)r^2} \phi(t, r), \quad \dot{t} - t_B(r) = \xi(t, r) \frac{M}{(\epsilon(r)r^2)^{3/2}},
$$

where

$$
\epsilon(r)r^2 = \begin{cases} 
2E(r)r^2, & (E(r) > 0) \\
1, & (E(r) = 0) \\
-2E(r)r^2, & (E(r) < 0)
\end{cases}
$$

and

$$
\phi = \begin{cases} 
\cosh \eta - 1, & (E(r) > 0) \\
\frac{\eta^2}{2}, & (E(r) = 0) \\
1 - \cos \eta, & (E(r) < 0)
\end{cases}
$$

$$
\xi = \begin{cases} 
\sinh \eta - \eta, & (E(r) > 0) \\
\frac{\eta^3}{6}, & (E(r) = 0) \\
\eta - \sin \eta, & (E(r) < 0)
\end{cases}
$$

In the general solutions of the LTB models, there are three arbitrary functions $M(r), E(r)$ and $t_B(r)$. $M(r)$ is regarded as the gravitational mass function, and we can set $M(r) = M_0 r^3$, redefining $r$. $t_B(r)$ corresponds to the local BigBang time. $E(r)$ determines the local curvature radius or the local specific energy. The functions $t_B(r)$ and $E(r)$ should be chosen to reproduce the observed $D_L(z)$. This means that we have only one constraint for two arbitrary functions.

The observational past null cone is specified in the form, $t = \hat{t}(r)$. We denote the areal radius $R$ on $t = \hat{t}(r)$ by $R$. Then, by Eq. (2.2), we can regard $\dot{R}$ on $t = \hat{t}(r)$ as a function of $R$, $E$ and $r$:

$$
\dot{R}(\hat{t}(r), r) = \mathcal{R}_1(R, E, r) \equiv \sqrt{\frac{2M_0 r^3}{R}} + 2E r^2.
$$

By differentiating Eqs. (2.2) and (2.4), $R'$ and $\dot{R}'$ on $t = \hat{t}(r)$ can be expressed as functions of $R$, $\dot{t}$, $E$, $E'$, $t_B$, $t_B'$ and $r$:\(^{(*)}\)

$$
R'(\hat{t}(r), r) = \mathcal{R}_2(R, \dot{t}, E, E', t_B, t_B') \\
\equiv - \left( \mathcal{R} - \frac{3}{2} [\dot{t} - t_B] \mathcal{R}_1 \right) \frac{E'}{E} - \mathcal{R}_1 t_B' \\
+ \frac{R}{r},
$$

\(^{(*)}\) If $E(r) = 0$, we should omit the terms proportional to $E'$ in Eqs. (2.8) and (2.9).
and

\[ \dot{R}'(\dot{t}(r), r) = R_3 \left( R, \dot{t}, E, E', t_B, t'_B, r \right) \]
\[ \equiv \frac{1}{2} \left( R_1 - 3 \frac{M_0 r^3}{R^2} [t - t_B] \right) \frac{E'}{E} \]
\[ + \frac{M_0 r^3}{R^2} t'_B + \frac{R_1}{r}. \]  \quad (2.9)

The observational past null cone \( t = \dot{t}(r) \) satisfies

\[ \frac{d\dot{t}}{dr} = -\frac{R_2(R, \dot{t}, E, E', t_B, t'_B, r)}{\sqrt{1 + 2Er^2}}. \] \quad (2.10)

The redshift \( z(r) \) along the past null cone is given by

\[ \frac{dz}{dr} = \frac{1 + z}{\sqrt{1 + 2Er^2}} R_3(R, \dot{t}, E, E', t_B, t'_B, r). \] \quad (2.11)

The total derivative of \( R \) on the past null cone is written

\[ \frac{dR}{dr} = \left( 1 - \frac{R_1(R, E, r)}{\sqrt{1 + 2Er^2}} \right) \frac{R_2(R, \dot{t}, E, E', t_B, t'_B, r)}{R_3(R, \dot{t}, E, E', t_B, t'_B, r)}. \] \quad (2.12)

Our basic equations are Eqs. (2.10)–(2.12). These three equations can be regarded as a system of first-order ordinary differential equations for three of the five functions \( R(r), \dot{t}(r), E(r), t_B(r) \) and \( z(r) \). In order to integrate these equations, we must specify two conditions on these five functions. The luminosity distance \( D_L(z) \) is related to \( R \) \(^{21}\) as

\[ R = \frac{D_L(z)}{(1 + z)^2}. \] \quad (2.13)

As mentioned above, we assume that \( D_L(z) \) is given by Eq. (1.1). We will specify one further condition on \( E, t_B \) or a combination of them.

\section*{3. Results}

\subsection*{3.1. Results of the BigBang time inhomogeneity}

We first consider a pure BigBang time inhomogeneity. In this case, the curvature function \( E(r) \) is set to a constant value. From Eqs. (2.11) and (2.12), we have equations for \( z(r) \) and the Big-Bang time function \( t_B(r) \). The model is specified by \( \Omega_0 \equiv 2M_0/H_0^2 \), which is the present central density, \( 3M_0/4\pi \), divided by the present central critical density, \( \rho_{\text{crit}} = 3H_0^2/8\pi \), where \( H_0 \) is the present central Hubble parameter, and we set it to unity. We numerically integrated these two differential equations from \( r = 0 \) for ten \( \Omega_0 \) from 0.1 to 1.0. The initial conditions are given by \( z = 0 \) and \( t_B = 0 \).

From Eq. (2.3), \( R' > 0 \) for positive density, while from Eq. (2.11), \( \dot{R}' > 0 \) for monotonically increasing \( z(r) \), so that the integration is terminated when either of the inequalities

\[ R' > 0 \quad \text{or} \quad \dot{R}' > 0 \] \quad (3.1)
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is violated. In Fig. 1, we display the relation between the parameter $\Omega_0$ and the redshift at the time that the integration is terminated. For small value of $\Omega_0$, $\Omega_0 = 0.1 – 0.4$ (open triangles), shell-crossing singularities appear when $dR/dz = 0$. For large value of $\Omega_0$, $\Omega_0 = 0.5 – 1.0$ (open square), the second condition in Eq. (3.1) is violated first. This occurs when $R = 2M$.

![Fig. 1. Plots of the maximum redshift of the time that either of the inequalities in Eq. (3.1) is violated as a function of the present density parameter. The open triangles and the open squares correspond to the Big Bang time inhomogeneity. The crosses correspond to the curvature inhomogeneity.](image1)

Figure 2 plots the Big Bang time functions $t_B$ for each $\Omega_0$. For all $\Omega_0$, the Big Bang time functions $t_B$ decrease as $z$ increases up to $z \sim 0.5$. This result is related to the fact that the expansion of our universe appears to be accelerating up to $z \sim 0.5$. In inhomogeneous models, an apparent acceleration is realized if the recession velocity of mass shells does not increase rapidly along the observational past null cone as in the case of a homogeneous and isotropic universe filled with dust. To construct such a situation in our model, we need to prepare an older shell,
i.e., one that is more decelerated by gravity, for more distant shells on the past null cone. This is the reason that the function $t_B$ decreases.

In Fig. 3 we plot the redshift space density,

$$\hat{\rho}(z) = \rho \frac{4\pi R^2 R'dr}{4\pi z^2 dz} = \Omega_0 \frac{r^2}{z^2} \frac{dr}{dz} \rho_{\text{crit}}, \quad (3.2)$$

along the past null cone. Observations of the mass distribution along the past null cone would give us this density profile.

![Fig. 3. Plots of the redshift space density $\hat{\rho}$ divided by the central critical density. The dotted curve represents the $\Omega_m = 0$, $\Omega_\Lambda = 0$. 3 homogeneous model.](image)

3.2. Results of curvature inhomogeneity

Next, we consider the pure curvature inhomogeneity. In this case the BigBang time function $t_B(r)$ is set to zero. From Eqs. (2.10) – (2.12) we obtain three differential equations for the three variables $z(r)$, $E(r)$ and $\hat{t}(r)$. We numerically integrated these three differential equations from $r = 0$. The initial conditions are given by $z = 0$, $E = (1 - \Omega_0)/2$ and

$$\hat{t}(0) = \frac{\Omega_0}{2} \frac{(\sinh \eta_0 - \eta_0)}{(1 - \Omega_0)^2}, \quad (3.3)$$

where

$$\eta_0 = \ln \left( \frac{2 - \Omega_0}{\Omega_0} + \sqrt{\left( \frac{2 - \Omega_0}{\Omega_0} \right)^2 - 1} \right). \quad (3.4)$$

The present central cosmological time $\hat{t}(0)$ and $\eta_0$ are obtained from Eq. (2.4).

In Fig. 1, we show the relation between the parameter $\Omega_0$ and the redshift at the time that the integration is terminated (cross marks). For the case of a curvature inhomogeneity, it is found that the second condition in Eq. (3.1) is violated first.\(^{(22)}\)

Figure 4 displays the curvature functions $E$ for variations values of $\Omega_0$. We can see $E$ decreases as $z$ increases, except in the $\Omega_0 = 1.0$ case.
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The result that $E$ decreases is consistent with the apparent acceleration. The value of $E$ determines the specific energy of the dust elements, so that the "initial" velocity is smaller for more distant shells. This causes apparent acceleration, because the velocity at $r = 0$ can be largest.

Figure 4. Plots of the curvature functions.

Figure 5 displays the redshift space density $\hat{\rho}$ along the past null cone as a function of $z$.

Fig. 5. Plots of the redshift space density divided by the present central critical density. The dotted curve represents the $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ homogeneous model.

§4. Summary and discussion

In this paper we have constructed inhomogeneous dust models without dark energy. We find that these models are consistent with the observed $D_L(z)$ up to $z = 1$, as from Fig. 1 no difficulties are encountered up to $z \sim 1$ for any set of parameter values in both the BigBang time inhomogeneity and curvature inhomogeneity cases. For $z > 1$, we have difficulties in our inhomogeneous dust models. Recently, the SNIa
at a redshift of $\sim 1.7$ was found with rather large uncertainties. However, only a single SNIa at a redshift of $\sim 1.7$ is not enough to construct an accurate $D_L(z)$, although that result seems to rule out the 'grey-dust' hypothesis. In addition, the results of a recent investigation of the effect of gravitational lensing on this SNIa suggests that the grey-dust model may be consistent with the observational data. If future observations confirm $D_L(z)$ up to $z \sim 2$ with $\Omega_m \sim 0.3$ and $\Omega_L \sim 0.7$, it can be concluded that our inhomogeneous dust models are incompatible with the observations and that some form of dark energy is likely to exist. However, if future observations confirm that $D_L(z)$ for $z > 1$ is not consistent with Eq. (1-1), the plausibility of our inhomogeneous dust models should be studied more extensively. In such a case, the first Doppler peak as well as the higher ones will give us another constraints on the inhomogeneous universe models.

It may believed that the existing observations for $0 < z < 1$, such as (i) evolution of cluster abundance, (ii) lensing rate, and (iii) ages of stellar populations, already rule out the inhomogeneous models.

Using the cluster temperature evolution data for $0.3 < z < 0.8$, it was found that the best-fit value for $\Omega_m$ is $\Omega_m = 0.45 \pm 0.1$ for open universe and $\Omega_m = 0.3 \pm 0.1$ for flat universe. However, recent analysis shows that the systematic error is comparable to the statistical error. Therefore, we may say that $0.1 < \Omega_m < 0.5$ for $0.3 < z < 0.8$ data. It is not clear whether the Press-Schechter formalism can be applied to our inhomogeneous models. One possible estimate of cluster abundance could be based on the locally homogeneous approximation. As we know, massive cluster evolution is very sensitive to matter density. It seems that a model whose local density parameter $\Omega_m$ differs greatly from the best-fit value would not be able to explain the observed cluster evolution. The pure curvature inhomogeneity case with $\Omega_0 \sim 0.2$ may not survive, because it is approximated by a flat universe at large $z$. Also the BigBang time inhomogeneity case with $\Omega_0 \sim 1.0$ cannot survive. However, it can be expected that the pure BigBang time inhomogeneity with $\Omega_0 \sim 0.5$ and the pure curvature inhomogeneity with $\Omega_0 \sim 0.1$ will predict the observed cluster abundances.

The estimate of the lensing rate and the distribution of the separation of the images depend on the model used for the mass distribution of the lensing object and the luminosity function of the source objects as well as the cosmological parameters. However, it has been shown that the dependence on the lens model and parameters is much stronger than that on the cosmological parameters. In addition, the mass distribution of the lensing objects should depend strongly on the baryon density $\Omega_b$. Therefore, we conclude that the estimate of the cosmological parameters from the lensing rate and the distribution of the separation of the images is difficult at present, and for this reason, we cannot rule out the inhomogeneous model.

As shown in Fig. 6, the look back times along the past null cone differ little between the inhomogeneous model and the corresponding homogeneous model with cosmological constant for $z < 0.5$. For $z \sim 1$, a difference appears, but some of the inhomogeneous models do not differ greatly from the homogeneous model even in that case. The ages of a stellar population could not be used to distinguish the inhomogeneous model from the homogeneous one.
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Fig. 6. Plots of the look back time along the past null cone. The solid curve represents the homogeneous $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ case. The broken and dotted lines denote the pure BigBang time and the pure curvature inhomogeneity cases with $\Omega_0 = 0.1, 0.3, 0.7$ in descending order, respectively.

We have found that the model dependence, including various undetermined parameters, and the observational uncertainty are much larger than the dependence on the cosmological parameters. Therefore we believe that these observations cannot easily rule out the inhomogeneous model.

Before finishing, we give a brief comment on how we can be positioned away from the center of the symmetry. A displacement from the center would correspond to a dipole mode of CMB. Therefore we can be positioned $\sim 50$ Mpc away from the center.

In conclusion, dark energy is not the only possible solution of the apparent acceleration of the present universe, as inhomogeneous dust models can also account for current observations.

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