What rare K decays can tell about the MSSM

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Abstract. Supersymmetric contributions to the theoretically clean $K^+ \to \pi^+ \nu\bar{\nu}$, $K_L \to \pi^0 \nu\bar{\nu}$, $K_L \to \pi^0 e^+ e^-$ and $K_L \to \pi^0 \mu^+ \mu^-$ decays are briefly reviewed. Particular emphasis is laid on the information one could get on the MSSM flavor sector from a combined study of the four modes.

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1 Introduction

The FCNC-induced decays, $K^+ \to \pi^+ \nu\bar{\nu}$, $K_L \to \pi^0 \nu\bar{\nu}$, $K_L \to \pi^0 e^+ e^-$ and $K_L \to \pi^0 \mu^+ \mu^-$, are very suppressed in the Standard Model (SM), where they can be predicted very accurately. Therefore, these modes are ideal for probing possible New Physics effects[1]. In the present talk, the signatures of supersymmetry, in its simplest realization as the MSSM, are reviewed.

2 Rare K decays in the Standard Model

In the SM, the electroweak processes driving the rare $K$ decays are the $W$ box, $Z$ and $\gamma$ penguins[2], see Fig.1a. In this section, the excellent theoretical control reached on these contributions is summarized briefly.

The $K \to \pi \nu\bar{\nu}$ decays in the SM: The $t$-quark contribution to the Wilson coefficient of the dimension-six FCNC operator $(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$ is known at NLO[2], while the $c$-quark one has recently been obtained at NNLO[3]. The matrix-elements for this operator can be extracted from $K_{\ell}\pi$ decays, including NLO isospin corrections[4]. For $K^+ \to \pi^+ \nu\bar{\nu}$, residual $c$-quark effects from dimension-8 operators, along with long distance $u$-quark contributions, have also been computed[5]. For $K_L \to \pi^0 \nu\bar{\nu}$, the indirect CP-violating contribution (ICPV), $K_L \to K_1 \to \pi^0 \nu\bar{\nu}$, is of about 1%[6], and the CP-conserving one is less than 0.01%[7]. Altogether, the SM predictions are

$$B(K_L \to \pi^0 \nu\bar{\nu})_{SM} = (2.49 \pm 0.39) \cdot 10^{-11},$$

$$B(K^+ \to \pi^+ \nu\bar{\nu})_{SM} = (7.83 \pm 0.82) \cdot 10^{-11}.$$ 

The error on $K_L \to \pi^0 \nu\bar{\nu}$ is mainly parametric, i.e. dominated by $\text{Im} \lambda_1$, $\lambda_1 \equiv V_{td}^* V_{ut}$. For $K^+ \to \pi^+ \nu\bar{\nu}$, which receives a significant $c$-quark contribution, the total error could be reduced with a better knowledge of $m_c$ and through a lattice study of higher-dimensional operators[8].

The $K_L \to \pi^0 \ell^+ \ell^-$ decays in the SM: The situation is more involved because there are a priori three competing processes.

First, the $t$ and $c$-quark contributions, known at NLO[2], generate both the dimension-six vector and axial-vector operators:

$$H_{\ell\ell} = y_{\ell\ellV} (\bar{s}d)_{V} (\bar{\ell}\ell)_{V} + y_{\ell\ellA} (\bar{s}d)_{V} (\bar{\ell}\ell)_{A}.$$ 

The former produces the $\ell^+ \ell^-$ pair in a $1^-_L$ state, the latter in both $1^{+-}$ and $0^{-+}$ states.

Secondly, the ICPV contribution is related to $K_S \to \pi^0 \ell^+ \ell^-$, which is dominated by the Chiral Perturbation Theory (ChPT) counterterm $\alpha_S[9]$. NA48 measurements give $|\alpha_S| = 1.2 \pm 0.2[10]$. Producing $\ell^+ \ell^-$ in a $1^-_L$ state, it interferes with the $(\bar{s}d)_{V}(\bar{\ell}\ell)_{V}$ contribution, arguably constructively[11,12]. This sign could also be fixed experimentally from $A_{FB}^{\ell\ell}$, the integrated forward-backward, or muon-energy asymmetry[13].

The final piece is the CP-conserving two-photon contribution, which produces the lepton pair in either a helicity-suppressed $0^{+-}$ or phase-space suppressed $2^{++}$ state. The LO corresponds to the finite two-loop process $K_L \to \pi^0 P^+ P^- \to \pi^0 \gamma \gamma \to \pi^0 \ell^+ \ell^-$, $P = \pi, K$, exactly predicted by ChPT, and produces only $0^{++}$ states. Higher order corrections are estimated using experimental data on $K_L \to \pi^0 \gamma \gamma$ for both the $0^{++}$ and $2^{++}$ contributions[13,14].

Altogether, the predicted rates are

$$B(K_L \to \pi^0 e^+ e^-)_{SM} = 3.54^{+0.98}_{-0.85} (1.56^{+0.62}_{-0.49}) \cdot 10^{-11},$$

$$B(K_L \to \pi^0 \mu^+ \mu^-)_{SM} = 1.41^{+0.28}_{-0.26} (0.95^{+0.22}_{-0.21}) \cdot 10^{-11},$$

for constructive (destructive) interference. The errors are detailed in [11,13,14], and are currently dominated by the one on the $K_S \to \pi^0 \ell^+ \ell^-$ rate measurements.

3 Rare K decays and supersymmetry

Even though the minimal supersymmetrization of the SM requires one super-partner for each SM particle
(and two Higgs doublets), it is very constrained and involves only a few free parameters. However, SUSY must be broken, and the precise mechanism still eludes us. Therefore, in practice, an effective description is adopted, introducing all possible explicit soft-breaking terms allowed by the gauge symmetries. In the squark sector, there are \( LL \) and \( RR \) mass terms and trilinear couplings giving rise to \( LR \) mass-terms after the Higgses acquire their VEV’s, \( \langle H_u,d \rangle = v_{u,d} \).

\[
\begin{align*}
\mathcal{L}_{soft}^{LL,RR} &= -\tilde{Q} \tilde{m}_Q^2 \tilde{Q} - \tilde{U} \tilde{m}_U^2 \tilde{U}^\dagger - \tilde{D} \tilde{m}_D^2 \tilde{D}^\dagger, \\
\mathcal{L}_{soft}^{LR} &= -\tilde{U} \tilde{A}^U \tilde{Q} \tilde{H}_u + \tilde{D} \tilde{A}^D \tilde{Q} \tilde{H}_d,
\end{align*}
\]

with \( \tilde{Q} = (\tilde{u}_L, \tilde{d}_L)^T \), \( \tilde{U} = \tilde{u}_R^\dagger \), \( \tilde{D} = \tilde{d}_R^\dagger \). Obviously, \( \tilde{m}_Q^2, \tilde{A}^U \) and \( \tilde{A}^D \), which are \( 3 \times 3 \) matrices in flavor space, generate a very rich flavor-breaking sector as squark mass eigenstates can differ substantially from their gauge eigenstates.

**What to expect from SUSY in rare \( K \) decays**: In the SM, the \( Z \)-penguin is the dominant contribution, and is tuned by \( \lambda_i \) (Fig.1a). The four MSSM corrections depicted in Figs.1b – e (together with box diagrams), represent the dominant corrections, and are thus the only MSSM effects for which rare \( K \) decays can be sensitive probes. Let us briefly describe each of them. First, there is the charged Higgs contribution to the \( Z \)-penguin (Fig.1b), which, is at moderate \( \tan \beta = v_u/v_d \), aligned with the SM one (\( \sim \lambda_1 \)). Then, there is the supersymmetrized version of Figs.1a – b, with charginos – up-squarks in place of \( W^\pm, H^\pm \) – up-squarks in the loop (Fig.1c), and which is sensitive to the mixings among the six up-squarks (\( Z^U \)), a priori not aligned with the CKM mixings. Another purely supersymmetric contribution, relevant only for charged lepton modes, is the gluino electromagnetic penguin (Fig.1d), sensitive to down-squark mixings (\( Z^D \)). The last class of effects consists of neutral Higgs FCNC (Fig.1e), and arises at large \( \tan \beta \approx 50 \). Indeed, the 2HDM-II structure of the Higgs couplings to quarks, required by SUSY, is not preserved beyond leading order due to \( \mathcal{L}_{soft} \), and the “wrong Higgs”, \( H_u \), gets coupled to down-type quarks, \( \mathcal{L}_{eff} \supset \tilde{d}_R \tilde{Y}_d^* (\tilde{H}_d^0 + \tilde{Y}_u^T \tilde{Y}_d^0) \tilde{s}_R \). Clearly, once the Higgses acquire their VEV’s, there is a mismatch between quark mass eigenstates and Higgs couplings; both are no longer diagonalized simultaneously and Higgs FCNC are generated [10].

**Bottom-up approach and Minimal Flavor Violation**: There are too many parameters in \( \mathcal{L}_{soft} \) to have any hope to fix them all from rare \( K \) decays. At the same time, however, the observed suppression of FCNC transitions and CP-violating phenomena seem to indicate that only small departures with respect to the SM are possible. Therefore, one starts from a lowest-order basis in which the flavor-breakings due to \( \tilde{m}_Q, \tilde{A}^U \) are minimal. This can take the form of \( mSUGRA \), alignment of squarks with quarks or the Minimal Flavor Violation hypothesis (MFV). In a second stage, one probes the possible signatures of departures from this minimal setting. The goal being, ultimately, to constrain SUSY-breaking models, which imply specific soft-breaking structures. At that stage, information from rare \( K \) decays, colliders and B-physics must of course be combined.

Here we adopt MFV as the lowest order basis, i.e. we impose that the SM Yukawas \( Y_{u,d} \) are the only sources of flavor-breaking [17]. In practice, this means that \( \mathcal{L}_{soft} \) terms can be expanded as \( (u_i, b_i \sim O(1), \text{ and } A_0, m_0 \text{ set the supersymmetry-breaking scale}) \)

\[
\begin{align*}
\tilde{m}_Q^2 &= m_Q^2(a_1 + b_1 Y_u^* Y_u + b_2 Y_d^* Y_d) \\
& + b_3(Y_d^* Y_u Y_u^* Y_u + Y_u^* Y_u Y_d Y_d), \\
\tilde{m}_U^2 &= m_U^2(a_2 + b_1 Y_u^* Y_u), \\
\tilde{A}^U &= A_0 Y_u(a_1 + b_2 Y_d Y_d),
\end{align*}
\]

and similarly for \( \tilde{m}_D^2 \) and \( \tilde{A}^D \), such that all FCNC’s and CP-violation are still essentially tuned by the CKM matrix. For example, the dominant contributions to the \( Z \)-penguin are those breaking the \( SU(2)_L \) gauge-symmetry [15,19]. In the SM, this breaking is achieved through a double top-quark mass insertion (Fig.1f). Similarly, in the MSSM, it is the double \( t_L - t_R \) mixing via the \( \tilde{A}^U \) trilinear terms which plays the dominant role (Fig.1g in the sCKM basis) [20]. Within MFV, this gives a factor \( m_t^2 \lambda_i |a_4 - c \beta \mu_*^2| \) [21], still enhanced by \( m_t^2 \) and tuned by \( \lambda_i \).
4 Supersymmetric effects in $K \to \pi \nu \bar{\nu}$

SUSY effects in the (axial)-vector operators, $(\bar{s}d)_{V+A}(\nu \bar{\nu})_{V-A}$, cannot be distinguished since only $(\bar{s}d)_{V}(\nu \bar{\nu})_{V-A}$ contributes to the $K \to \pi \nu \bar{\nu}$ matrix-element. All MSSM effects are thus encoded into a single complex number, $X^\nu \equiv y^\nu_L + y^\nu_R$ [19].

$$H_{eff} = y^\nu_L (\bar{s}d)_{V-A}(\nu \bar{\nu})_{V-A} + y^\nu_R (\bar{s}d)_{V+A}(\nu \bar{\nu})_{V-A}$$

At moderate $\tan \beta$, chargino penguins are the dominant MSSM contributions because of their quadratic sensitivity to up-squark mass-insertions (Figs.1c, 1g). Within MFV, this means, given the $m_t$ enhancement present in the $\delta_{LR}^f$ sector, that $K \to \pi \nu \bar{\nu}$ are particularly sensitive. Still, a significant enhancement would require a very light stop and chargino [21], mostly because of the constraint from the $\Delta \rho$ [22]. Any enhancement $\gtrsim 5\%$ would thus falsify MFV if sparticles are found above $\sim 200$ GeV, and if $\tan \beta \gtrsim 5$ (to get rid of the $H^\pm$ contribution). Turning on generic $A^U$ terms, the largest deviations arise in $K \to \pi \nu \bar{\nu}$, see Fig.2a [21]. Further, the decoupling is slower than for observables sensitive to chargino boxes like $\varepsilon_K$. All in all, given that the effect could be for $K_L \to \pi^0 \nu \bar{\nu}$? By an extensive, adaptive scanning over the MSSM parameter space, Ref.[23] has shown that the GN model-independent bound [21] can be saturated, which represents a factor $\sim 30$ enhancement of $B(K_L \to \pi^0 \nu \bar{\nu})$ over the SM.

At large $\tan \beta$, the chargino contributions may no longer represent the dominant effect. While the Higgs FCNC obviously does not contribute (Fig.1e), higher order effects in the $H^\pm$ contribution to the $Z$-penguin (Fig.1h), sensitive to $\delta_{RR}^B$, can become sizeable beyond MFV [25]. Further, this contribution is slowly decoupling as $M_t$ increases compared to tree-level neutral Higgs exchanges, as for example in $B_{s,d} \to \mu^+ \mu^-$. SUSY effects in other dimension-six operators, $(\bar{s}d)(\bar{\nu} (1, \gamma_5) \nu)$ and $(\bar{s}d_{\mu \nu})(\bar{\nu} \gamma^\mu (1, \gamma_5) \nu)$, require active right-handed neutrinos and will not be discussed here [20]. Another possible class of operators, since the neutrino flavors are not detected, are $(\bar{s}\Gamma_i^A d)(\bar{(\nu} \Gamma^B \nu))$ with $i \neq j$ and $\Gamma^{A,B}$ some Dirac structures. In the MSSM, such lepton-flavor violating operators arise only from suppressed box diagrams, and cannot lead to significant effects [27]. However, they could be sizeable in the presence of R-parity violating terms [27,28].

5 Supersymmetric effects in $K_L \to \pi^0 \ell^+ \ell^-$

Though the SM predictions for these modes are less accurate than for $K \to \pi \nu \bar{\nu}$, they are sensitive to more types of New Physics operators [13]. Indeed, the final-state leptons are now charged and massive. Therefore, besides electromagnetic effects, common to both the muon and electron modes, the relatively large muon mass opens the possibility to probe a whole class of helicity-suppressed effects.

SUSY effects in the QCD operators, i.e. in the chromomagnetic $s_\mu d G_{\mu \nu}$ or four-quark operators, have no direct impact on $K_L \to \pi^0 \ell^+ \ell^-$. Indeed, as said in Sect. 2, the two-photon CPC piece is fixed entirely in terms of the measured $K \to \pi \pi \pi$, $\pi \gamma$ modes [11,14], while the ICPV contribution is fixed from the measured $\varepsilon_K$ and $K_S \to \pi^0 \ell^+ \ell^-$ rate [9]. At the low scale $\mu \lesssim m_{s},$ new physics can thus explicitly enter through semi-leptonic FCNC operators only.

SUSY effects in the SM operators, which are the vector and axial-vector operators, can in principle be disentangled thanks to the different sensitivities of the two modes to the axial-vector current (as discussed in Sec. 2, it also produces $\ell^+ \ell^-$ in a helicity-suppressed $0^+ \to 0^+$ state). Various MSSM contributions can enter in $y_{7A}$ and $y_{7V}$. First, chargino contributions to the Z-penguin (Fig.1c) enter as $y_{7A}, y_{7V} \sim (\delta_{RL}^i)^2 (\delta_{RL}^j)$, and are thus directly correlated to the corresponding contribution to $K \to \pi \nu \bar{\nu}$ [21,22]. Within MFV, the maximal effect for $K_L \to \pi^0 \ell^+ \ell^-$ is about one third of the one for $K_L \to \pi^0 \nu \bar{\nu}$, hence may be inaccessible due to theoretical uncertainties. Secondly, gluino contributions to the electromagnetic operator $s_\mu d F_{\mu \nu}$ (Fig.1d) can be absorbed into $y_{7V} \sim (\delta_{RL}^i)$ [21]. Even if directly correlated with $\varepsilon'/\varepsilon$, sizeable effects in $K_L \to \pi^0 \ell^+ \ell^-$ are

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**Image 83x567 to 268x697**

**Fig. 2.** a) Sensitivity of $K^+ \to \pi^+ \nu \bar{\nu}$ to $A^U$ terms, compared to $B$-physics observables. b) Schematic representation of the neutral Higgs FCNC beyond MFV, at large $\tan \beta$. c) Impacts of dim-6 FCNC operators in the $B(K_L \to \pi^0 \mu^+ \mu^-)$ vs. $B(K_L \to \pi^0 e^+ e^-)$ plane.
\[ \pi^0 \ell^+ \ell^- \] are still possible[30]. Finally, \( H^\pm \) contributions arise at large tan \( \beta \) (Fig.1b), with \( y_{T_A} y_{T^V} \sim (\delta^D_{LR})_{12} \), and are directly correlated with those for \( K \to \pi \nu \bar{\nu} \)[25].

**SUSY effects in the (pseudo)-scalar operators**, which can be helicity-suppressed or not:

\[ \mathcal{H}_{eff} = y_S (\bar{\sigma} d) (\bar{\ell} \ell) + y_P (\bar{\sigma} d) (\bar{\ell} \gamma_5 \ell) + y_S (\bar{\sigma} \gamma_5 d) (\bar{\ell} \ell) + y_P (\bar{\sigma} \gamma_5 d) (\bar{\ell} \gamma_5 \ell) . \]

The first (last) two operators contribute to \( K_L \to \pi^0 \ell^+ \ell^- (K_L \to \ell^+ \ell^-) \). In the MSSM at large tan \( \beta \), they arise from Higgs FCNC[31], and are thus helicity-suppressed (Fig.2b). Sizeable effects for the muon mode are possible beyond MFV, where they are sensitive to \( (\delta^D_{LR,LL})_{12} \) and \( (\delta^D_{RR,LL})_{23} \) mass-insertions. Also, even if this contribution is correlated to the one for \( K_L \to \mu^+ \mu^- \), given the large theoretical uncertainties for this mode, a factor \( \sim 4 \) enhancement is still allowed (Fig.2c)[13]. On the other hand, helicity-allowed contributions to these operators do not arise in the MSSM. They could appear in the presence of R-parity violating couplings, but, being fine-tuning, their effects must be small to avoid overproducing \( K_L \to e^+e^- \)[13]. Further, they cannot arise from R-parity violating couplings.

**SUSY effects in the (pseudo)-tensor operators**, \( (\bar{\sigma}_\mu \nu d)(\bar{e} \rho^\mu \nu (1, \gamma_5 \ell)) \), the last possible dimension six semi-leptonic FCNC operators, are helicity-suppressed in the MSSM[32], and, being also phase-space suppressed, do not lead to any significant effect[13]. Further, they cannot arise from R-parity violating couplings.

### 6 Conclusion

The \( K^+ \to \pi^+ \nu \bar{\nu}, K_L \to \pi^0 \nu \bar{\nu}, K_L \to \pi^0 e^+ e^- \) and \( K_L \to \pi^0 \mu^+ \mu^- \) decay modes are the only theoretically clean windows into the \( \Delta S = 1 \) sector. If SUSY is discovered, the pattern of deviations they could exhibit with respect to the SM (see Table 1) will be essential to constrain the MSSM parameter-space, and hopefully unveil the nature of the SUSY-breaking mechanism.

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### Table 1

| MSSM scenario | \( K \to \pi \nu \bar{\nu} \) | \( K_L \to \pi^0 e^+ e^- \) |
|---------------|-------------------------------|-------------------------------|
| MFV, \( \tan \beta \approx 2 \) | Best sensitivity, but maximal enhancement < 20-25% | Less sensitive, but precisely correlated with \( K \to \pi \nu \bar{\nu} \) |
| MFV, \( \tan \beta \approx 50 \) | Negligible effects? | \( \delta^L_{LR} \) correlated with \( K \to \pi \nu \bar{\nu} \) |
| General, \( \tan \beta \approx 2 \) | Best probes of \( \delta^L_{LR} \) (quadratic dependence in \( \delta^U_{LR} \)) | \( \delta^L_{LR} \) correlated with \( \epsilon' \epsilon \) (but cleaner) |
| General, \( \tan \beta \approx 50 \) | Good probes of \( \delta^R_{RR} \) (slow decoupling as \( M_H \to \infty \)) | Good probes of \( \delta^R_{RR,LL} \) (cleanly) |

\[ \Delta S \]