Influences of geological parameters to probabilistic assessment of slope stability of embankment

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Abstract. This article considers influences of geological parameters to slope stability of the embankment in probabilistic analysis using SLOPE/W computational system. Stability of a simple slope is evaluated with and without pore–water pressure on the basis of variation of soil properties. Normal distributions of unit weight, cohesion and internal friction angle are assumed. Monte Carlo simulation technique is employed to perform analysis of critical slip surface. Sensitivity analysis is performed to observe the variation of the geological parameters and their effects on safety factors of the slope stability.

1. Introduction

Evaluation of the slope stability of the embankment through the analysis of critical slip surface is a useful tool to determine safety factors of embankment structures such as roads, dams, dykes, etc. The evaluation depends on the geological conditions and failure profiles of embankment which may occur. There are many random factors impacting on stability of a slope: scatter of geological profiles, variation of soil properties, natural conditions and stability assessment methods. These random factors then create the randomness of the slope stability evaluation problem. Because randomness is taken into account rarely, the result of analysis is single value or safe/unsafe verdict. Slope stability, therefore, should be analyzed under the scheme of probability in order to have a whole spectrum of possible results. This robust approach also allows to optimize design problems.

According to Vietnamese design standard 22TCN 262–2000 [1], stability analysis of slope can be carried out with two classic approaches on which input parameters are constant: fragmentation approach (Fellenius approach) and simplified approach (Bishop approach). The slope is considered stable when the calculated safety factor (K) is greater than the allowable safety factor ($K_{\text{min}}$). Values of $K_{\text{min}}$ are different in the two mentioned methods: 1.2 in Fellenius method and 1.4 in Bishop one. In practice, mechanical properties of the soil layers such as unit weight (specific gravity) $\gamma$, cohesion $c$ and internal friction angle $\phi$ always show high scatter. Thus analysis results from the two methods do not present exact performance of structures. As a result, many roads in operation are frequently encountered with slippage due to their instability. Figure 1 shows an incident of slope slip of a new–built road due to the instability of its slope [2].
Figure 1. A 140–meter road segment slipping due to instability in Hung Chau commune, Hung Nguyen district, Nghe An province, Vietnam (Photo taken on October 6th 2017, adopted from [2]).

To account for the randomness of the input parameters in problems of embankment’s slope, probabilistic stability analysis method has widely been used. It was first introduced in the 1970s, for example, Alonso 1976 [3], Tang et al. 1976 [4] and Harr M E 1977 [5]. Since then, the concepts and principles of slope stability analysis using probabilistic theory have been continuously developed and improved. In 2002 and 2003, El–Ramly and his colleagues made probabilistic stability analyses for slope and a tailings dyke [6], [7].

Through using probabilistic method, risk level and hence reliability of safety factor of a particular structure can be quantified even in the inadequacy of representative data. For instance, an uncertainty analysis for a slope based on imprecise information was carried out in [8] and a reliability–based design procedure when the statistics of the random variables are incomplete was introduced in [9]. In addition, Finite element methods (FEM) in combination with probabilistic simulation techniques have become useful tools in solving probabilistic problems. FEM was used in probabilistic stability analysis of embankments in [10] and Monte Carlo simulation method was applied to reliability analysis of slope stability [11]. Another example is that Recursive finite element method (RFEM) was used in probabilistic analysis of reinforced slopes with respect to spatial variability of soil properties [12].

Study on critical slip surface for earth slopes was done in [13] based on First Order Reliability Method. Critical slip surface problems can be solved by the two mentioned classic approaches with certain limitations. Nowadays, a probabilistic analysis of stability of a slope can be conveniently done in SLOPE/W [14]. It is a powerful tool developed by Geo–Slope International Limited (Canada) on which Fellenius approach and simplified approach Bishop approach was integrated.

In this paper, influences of geological parameters to slope stability of an embankment are considered through probabilistic analysis using SLOPE/W. Stability of a simple slope is evaluated in two scenarios: existence of pore–water pressure (PWP) and absence of pore–water pressure (non–PWP). Monte Carlo simulation technique was used to take into account of the variability of soil properties. Probability of failure and sensitivity of slope stability are evaluated with assumption of normal distribution of input data. Sensitivity of the analysis to the scatter of input parameters is also studied with three cases. Coefficient of variation of evaluated random input parameters is assumed to be 25%, 50% and 75% for cohesion; 17%, 33% and 50% for internal friction angle and 5%, 10% and 20% for unit weight in three cases, respectively.
2. Analysis of circular slip surface by classic approaches

Classic methods have been used for ages to analyze critical slip surface problems. In these methods, a slip surface is assumed as an arc and equilibrium of sliding mass is considered. Sketch of a typical problem are illustrated in figure 2.

![Sketch of a typical circular slip surface analyzed by classic approaches](image)

According to [16], safety factor of slope stability is defined as:

\[
FS = \frac{T_f}{T}
\]

where:

- \(T_f\): shear stress resistance (kPa);
- \(T\): calculated shear stress (kPa).

In practical calculations, sliding block is divided (by vertical surfaces, parallel to each other) into pieces and then its equilibrium is evaluated. The problem would have many parameters if interactive forces among pieces and deformation of the sliding block were taken into account. So, assumptions were added to simplify the problem [16], as follows:

- Fellenius method assumes that the sum of interactive forces equals to zero if projected to the axis perpendicular to the radius.
- Bishop method assumes that the sum of interactive forces equals to zero if projected to a horizontal axis.

Both these two approaches are simple quantitative methods on which the embankment’s slope is evaluated as stable if the safety factor is greater than 1. The selected safety factor depends on the experience of the design engineers, design standards and other conditions. In addition, stochastic factors of input parameters are not taken into account in the problem. Consequently, the results do not allow accurate estimation of slope stability.

3. Stochastic factors in computation of slope stability of embankment

In evaluation of slope stability, stochastic factors include but not limited to the followings:

- Factors associated with the accident of nature;
- Randomness of input data, including errors in measurements, surveys, experiments, inconsistent data and errors in data processing;
- Randomness of calculation model and calculation method as well as empirical formulas describing the physical characteristics of the system;
- Randomness of the outputs due to imprecise input parameters.

In evaluation of sliding stability, there are three main random factors: mechanical properties of soil, randomness of calculation model as well as the variation of pore–water pressure.

3.1. Randomness of mechanical properties of soil

Random factors in the mechanical properties of soil may be arised by the spatial random distribution of different soil types or by the errors of the experimental process. Spatial variation is the change in
the mechanical characteristics of the same soil layer between the boreholes. Random errors occur in the processes of measurement and testator's carelessness as well as laboratory equipment need to be eliminated before the calculation.

According to Lumb (1966) and Christian (1994), key random factor in circular sliding stability problem is the variation of mechanical properties of soil on which the following parameters are the main ones [17], [18]:

- Specific weight (unit weight), $\gamma$
- Cohesion, $c$
- Friction angle, $\varphi$

3.2. Randomness of calculation model

The difference between applied hypothesis to the calculation and reality generated random factors in the calculation model. These factors include the description of geological layers (different thickness of the layers, presence of weak geological sites) and water pressure (PWP and non–PWP).

3.3. Slope stability probabilistic analysis based on Monte Carlo simulation technique

Distribution functions describing the transformation of random parameters can be normal distribution functions, normal logarithm functions, uniform distribution functions, triangular distribution functions, and curved distribution functions. In this study, normal distribution functions are used to describe the transformation of three soil mechanical parameters ($\gamma$, $\varphi$ and $c$). The normal distribution function is often referred to as the Gaussian distribution function, which describes the variation of inputs in probability analysis, which is expressed by the following expression:

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where:
- $f(x)$: the probability density functions of the normal distribution;
- $\mu$: the mean or expectation of the distribution;
- $\sigma$: the standard deviation.

Variation of soil mechanical properties can be represented by coefficient of variation (COV). It represents the degree of randomness of the soil properties and is defined as:

$$\text{COV} = \frac{\sigma}{\mu}$$

(3)

Reliability function $Z$ is described as:

$$Z = FS$$

(4)

Thus, probability of failure is defined as the probability at which $FS$ is less than 1.0:

$$P_{\text{failure}} = P(Z < 1)$$

(5)

The reliability index ($\beta$) presents the slope stability through the number of times of standard deviations ($\sigma$) deviated from the mean safety factor ($\mu$):

$$\beta = \frac{(\mu-1)}{\sigma}$$

(6)

Monte Carlo is a simulation technique based on a process of repeated random sampling to obtain numerical results. Its methodology includes: creating the sample, running the model and analyzing the data [19]. This method is adopted here to analyze slope stability of the embankment. The number of trials depends on the level of required reliability as well as the number of considered input parameters. To ensure accuracy, therefore, the more input parameters involve, the more Monte Carlo trials are needed. As stated in [20], number of trials is determined as:

$$n_{MC} = \left[ \frac{\sigma^2}{4(1-\beta)^2} \right]^m$$

(7)
where:
\( n_{MC} \) : number of trials;
\( \beta \) : required reliability index (0% ÷ 100%);
\( m \) : number of input parameters.

4. Sample example of probabilistic evaluation of stability of circular sliding surface
Stability of an 7–meter high embankment’s slope of 1H:1V as shown in figure 3 is analyzed in cases of PWP and non–PWP using SLOPE/W.

Monte Carlo simulation technique is used to cope with variability of random parameters. Critical slip surface is firstly determined by Bishop approach integrated in SLOPE/W software. Input parameters are mean values of soil mechanical properties. These parameters are then proceeded with variation of input parameters. The variability of the input parameters is assumed to follow the normal distribution function. Three cases of standard deviation of cohesion (c), internal friction angle (\( \phi \)) and unit weight (\( \gamma \)) were considered as illustrated in table 1. Relationship between \( c \) and \( \phi \) were evaluated with as well as with correlation factor of \(-0.5\). Groundwater table (if any) fluctuates within 5 meters from the ground. From equation (7), necessary number of Monte Carlo trials is 15,625 in case of unit standard deviation and required reliability index of 90%.

Table 1. Input parameters used in the example.

| Parameters               | Mean value, \( \mu \) | Standard deviation, \( \sigma \) |
|-------------------------|------------------------|----------------------------------|
|                         |                        | Case 1  | Case 2  | Case 3  |
| Cohesion, c (kPa)       | 20                     | 5       | 10      | 15      |
| Internal friction angle, \( \phi \) \(^\circ\) | 30                     | 5       | 10      | 15      |
| Unit weight, \( \gamma \) (kN/m³) | 20                     | 1       | 2       | 4       |

5. Results

5.1. When \( c, \phi \) and \( \gamma \) are constant
Table 2 presents safety factors computed by deterministic method i.e. \( c, \phi \) and \( \gamma \) are constant parameters with PWP and non–PWP.
Table 2. FS calculated with Bishop approach and with constant parameters.

| SF (non-PWP) | SF (PWP) |
|--------------|----------|
| 1.647        | 1.282    |

5.2. When \( c, \varphi \) and \( \gamma \) are inconstant and \( c-\varphi \) uncorrelation

It is noticeable that values of FS analyzed by probability method (table 3) are larger than those computed by deterministic method (table 2). In individual case, furthermore, FS in case of PWP is smaller than that of non-PWP. The probability of failure increases as the mechanical properties of the soil increase in values.

Table 3. Results from probabilistic method without \( c-\varphi \) correlation.

| Parameters                  | Case 1 | Case 2 | Case 3 |
|-----------------------------|--------|--------|--------|
| Mean of FS                  | 1.654  | 1.285  | 1.707  |
| Reliability index, \( \beta \) | 3.088  | 2.152  | 1.581  |
| Standard deviation of FS    | 0.212  | 0.132  | 0.447  |
| Minimum of FS               | 0.785  | 0.733  | 0.359  |
| Maximum of FS               | 2.963  | 1.998  | 8.757  |
| Failure probability, P (%)  | 0.064  | 1.473  | 3.835  |
| Number of trials            | 15,625 | 15,625 | 15,625 |

5.3. When \( c, \varphi \) and \( \gamma \) are inconstant and \( c-\varphi \) correlation

According to [22], \( c \) and \( \varphi \) are often negatively correlated with correlation coefficient ranges from \(-0.72 \) to \( 0.35 \). These resulted from laboratory tests on a wide variety of soils. In this example, a \( c-\varphi \) correlation coefficient of \(-0.5 \) is assumed. Results computed by probabilistic method are described in table 4.

Table 4. Results from probabilistic method with \( c-\varphi \) correlation.

| Parameters                  | Case 1 | Case 2 | Case 3 |
|-----------------------------|--------|--------|--------|
| Mean of FS                  | 1.655  | 1.286  | 1.710  |
| Reliability index, \( \beta \) | 5.312  | 5.212  | 2.442  |
| Standard deviation of FS    | 0.123  | 0.063  | 0.291  |
| Minimum of FS               | 0.076  | 0.947  | 1.009  |
| Maximum of FS               | 2.765  | 1.856  | 8.364  |
| Failure probability, P (%)  | 0.018  | 0.038  | 0.798  |
| Number of trials            | 15,625 | 15,625 | 15,625 |

5.4. Sensitivity of safety factor to \( c, \varphi \) and \( \gamma \)

The purpose of sensitivity analysis is to observe the variability of the input parameters and to evaluate their effects on the outputs. Results on this analysis with three parameters \( c, \varphi \) and \( \gamma \) of embankment soil layers are shown in figure 4 and figure 5.

As depicted in figures 4 and 5, the sensitivity of computed factor of safety versus the range of the parameters used in the computation was analyzed with 11 points in the range. It can be seen from the two figures that the rise in unit weight leads to a decrease in the safety factor (inverse relation). Meanwhile, when the cohesion or internal friction angle increases, the safety factor also increases (proportional relation).
For the absence of pore–water pressure (figure 4), the sensitivity of the internal friction angle to the safety factor is greatest, followed by the cohesion and finally the unit weight. With the presence of pore–water pressure (figure 5), however, the sensitivity of cohesion to the safety factor is greatest, followed by the internal friction angle and finally the specific weight.

6. Conclusions
Influences of geological parameters to slope stability of an embankment were studied through an example of probabilistic analysis using SLOPE/W. Results from the analysis revealed that the presence of pore–water pressure increased the sliding ability of the embankment. It is also worth noticing that probability of failure (slide) decreases if correlation between cohesion and internal friction angle are considered. In addition, the higher the standard deviation of the mechanical properties of the soil is, the greater the sliding probability of the embankment results.

The influence of the input parameters on the safety factor depends on the parameter properties itself. In this research, relationship between the input parameters and the safety factor may be an inversion relation (safety factor versus unit weight) or a proportional relation (safety factor versus cohesion/internal friction angle).

The sensitivity of the three considered parameters to the safety factor is at different levels in the two scenarios (non–PWP and PWP). Therefore, it is impossible to say that the nature of the parameter determines its overall sensitivity to the factor of safety.

It can be inferred from the study that embankment with high average safety factor does not mean that it is safe because there is possibility of existence of large failure probability. This means that there is no direct relationship between probability of failure and safety factor. This also reveals that solving
the problem of circular sliding using probabilistic methods allows a better comprehensive view of the stability of the embankment.

Results from sensitivity analysis of the mechanical properties of the soil revealed that the cohesion had the largest and direct impact on the failure probability of the embankment. Nevertheless, this parameter did not affect mean value the safety factor.

This sample example demonstrates the effectiveness of the probabilistic method in solving the sliding stability problem in comparison with the classic deterministic approach. Embankment’s slope stability analysis based on the Monte Carlo simulation technique allows the determination of the probability of failure, wide range of safety factor and its reliability index. This helps not only to minimize random errors in the calculation process but also to predict the fluctuation of safety factor in real conditions. As a result, safety factor and probability of failure could be selected based on nature of the work, specific design requirements to serve as the basis for design as well as evaluation of the embankment structures.

This study only investigated the most disadvantageous position of sliding arcs. In addition, external load, horizontal seismic force and variation of pore–water pressure have not been studied yet. In the future, therefore, this research should be developed with taking into account of external loads including horizontal seismic force and change of pore–water pressure.

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