Measuring the length of a spiral when evaluating the plastic processability by injection molding

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Abstract. The evaluation of the plastic processability by injection molding could be achieved by measuring the length of a spiral part obtained by pressing the plastic found in a melted state in an adequate cavity of the mold. During the injection molding process materialized on a simple injection molding equipment, when using different types of materials and, respectively, heating temperatures and mold temperatures, it was seen that the spiral channel is differently filled under the action of a known pressure and in certain experimental conditions. The aim of this paper was to show that the length of the spiral channel filled with solidified plastic can be calculated considering more types of spirals and using certain design software. A theoretical analysis allowed the elaboration of some considerations concerning the ways of determining the length of the spiral part obtained by injection of the initially melted plastic into a spiral cavity found in the mold.

1. Introduction

Plastics are based on polymers, compounds derived from macromolecules based on chemical polyreactions that determine certain structures specific to each material. The two major categories of plastics that exists are: thermoplastics, which have the linear structure and are soften upon heating, while the three-dimensional, structures make the material heat-resistant, being called thermosets. Additives are added to these polymers according to the needs of the future plastics [1].

Due to the low density, the resistance to corrosive factors, the optical properties, the resistance to chemical and mechanical stresses, the plastics are used in medicine, construction, aviation, automotive, electrotechnics and others [2].

The current requirements for the development of technologies and the improvement of plastic materials determine continuous research in this direction. Processability of the plastics is an important aspect in the process of obtaining finished plastics products. On the one hand, this property shows how easily a product can be obtained from a certain plastic material and at the same time how the finished product is in relation to the base material and whether the product fulfills the requirements to meet the needs of the human user.

Being a complex factor, many types of researches aimed at observing the processability of certain materials and how their use can be improved considering their properties. The same problem was previously valid in the case of metallic materials. For these metallic materials, there is a method that uses a spiral channel so that depending on how much the melted material channel is filled, the metallic material processability by casting could be evaluated. Such an effect leads to the idea that the method could also be used in the case of plastics. For this purpose, it is necessary to have equipment that makes it possible to measure the length of the channel filled with solidified plastic [3].
Among the most widely used processes used for obtaining products from plastics, there is the plastics injection, a complex process to large series or mass production. By means of the injection and a spiral channel mold, it is possible to materialize the premises of some measurements based on which it is possible to investigate the processability of the plastic materials, considering a direct proportionality between the length of the plastic filled channel and the degree of processability of the plastic material.

Once designing the necessary equipment and making the plastic injection, these measurements may raise certain difficulties, including the possibilities of measuring the length of the plastic filled canals. Measuring the length of spirals, which could have with various shapes and equations, has been the subject of some research. There are analytical methods or even software dedicated to the solving of this problem. For example, the Stack Exchange forum provides clues with reference to the mathematical equations corresponding to the Archimedean spiral, whose length can be calculated [4]. Also, on another website, there is another method by which the Archimedean spiral can be measured; the method is based on the mathematical equations that characterize the Archimedean spiral. Interestingly, this source provides indications that can be used for other types of spirals such as golden spiral, using some examples from nature, recalling that many discoveries in science are based on elements of animal or vegetal nature [5]. In the same way, another source provides suggestive illustrations that can facilitate the calculations of arcs length of certain shapes of curves or spirals in plan or space through animations when spirals defined by mathematical equations [6].

In the Iasi Machine Manufacturing Technology department, from the Technical University of Iasi, a plastics injection equipment was developed, with a spiral channel mold for the research of the processability of the plastics [7]. Once the material has been injected, it has filled the channel over a certain length, and this size will highlight the processability by injections of the plastics. The purpose of this paper is to use CAD software to measure the length of the plastic filled channel, these measurements being made for several types of spirals so that a more general image can be offered on the possibilities of this type of measurement.

2. Theoretical considerations

A spiral is an open flat curve that rotates, wraps around a fixed point, more and more departs from it, observing a certain rule of wrapping often characterized by a mathematical formula.

The spiral has been studied in detail by the Archimedes who defines it as being a segment with one of the extremity fixed, rotated at a uniform angular velocity in a plane until it returns to the same initial angular position, at the same time as the rotation of the line, a point moves at uniform speed along the straight line, starting from the fixed end.

There are several types of spirals, each of which is distinctly characterized by a mathematical view. Generally, the spiral is the curve corresponding to the mathematical equation:

$$r = f(\theta),$$  \hspace{1cm} (1)

where \(r\) is the distance from the origin, \(\theta\) – the angle from the horizontal axis, \(\theta > 0\)

Thus, there are known the Archimedean spiral, the golden spiral, the logarithmic spiral, the parabolic spiral or Fermat spiral, the lituus spiral, the hyperbolic spiral, the spherical spiral. The parametric combination of coefficients can generate other spirals similar to those above-mentioned. Among these spirals, they seem to be of interest and significant for the possibilities of using as a cavity for the mold the following three alternatives: the Archimedean spiral, the logarithmic spiral, and the golden spiral. In this regard, the following spirals were analyzed using Autocad 2017 as well as using Matlab R2018b.
What was achieved was the parametric definition of spirals, as well as their 2D drawing. The problem of designing spirals is to determine the length of spirals arcs so that one can determine the length of the filled plastic channel in the mold cavity.

As mentioned above, a method of evaluating the processability of plastics takes into consideration the injection. This complex process consists of several stages that are cyclically repeated. The injection equipment was made piece by piece and then assembled. At the base of the design, there were used methods of axiomatic design which led to the optimization of the design process. An overview of the equipment is shown in figure 1. The main parts of the equipment are the cylinder inside of which the piston moves. It is operated by a rotary handle that can be operated manually or can be operated freely, directly proportional to the weight placed on a plate.

This type of actuation provides variable piston pressure on the material and thus multiple filling possibilities of the mold. The mold is designed with a spiral channel in order to meet the above-mentioned requirements. The equipment also has a controller and a sensor connected by which the melting temperature of the plastic can be set and at the same time it can be measured and controlled.

3. Specific elements of the experimental test
In the measurements made on the equipment, it was noted that it is important to set the exact optimal temperature for melting the plastic material to flow properly into the mold cavity, but also do not alter the structure of the material by a too high temperature. At the same time, the mold tightness is an important aspect so that material flow is only in the specially designed channel. Finally, a predetermined length of the mold channel is ensured by the chosen spiral, which must be long enough so that the molten material will fit into the mold cavity.

The first case is the Archimedean spiral, or the so-called arithmetic spiral characterized by a constant pitch at each turn. Archimedes was the first person who discovered and studied it. Its representation can be seen on vinyl discs, in helical springs, mechanisms of clocks and not only. The characteristic equation:

$$ r = a + b\theta, $$

(2)

where $a$ and $b$ are constants, $\theta$- the angle between the line from the center and the tangent to the spiral,

The representation can be done by parametrically decomposing the axes $x$ and $y$ as follows:

$$ x = \theta \ast \cos(\theta) $$

(3)

$$ y = \theta \ast \sin(\theta), $$

(4)

where $\theta \in [0; 2 \ast \pi \ast k], \ k \in \mathbb{N}$.

Graphic representation in Matlab can be achieved by:

$$ t = 0: 0.01: 12 \ast p; \ x = t \ast \cos(t); \ y = t \ast \sin(t); \ \text{plot} \ (x, y), $$

(5)

where $\theta \leftrightarrow t, k = 6$.

The case of the logarithmic spiral is also found in nature; the spiral has an exponential increase in radius from the center. Logarithmic spiral is also called equiangular spiral due to the fact that the angle formed between the line that starts from the center of the spiral and the tangent line to the spiral is

![Figure 2](image-url)

**Figure 2.** Different types of spirals: a) Archimedean spiral b) logarithmic spiral c) golden spiral
constant in increasing it. Those who studied it were Descartes and Bernoulli. The characteristic equation is:

\[ r = a \cdot e^{b\theta} \]  \hspace{1cm} (6)

where \( a \) is a constant, \( \theta > 0 \).

The parametric equations consist of:

\[ x = e^{b\theta} \cdot \cos(\theta) \]  \hspace{1cm} (7)

\[ y = e^{b\theta} \cdot \sin(\theta) \]  \hspace{1cm} (8)

where \( e \) is the base of natural logarithms.

The graphical representation in Matlab can be realized using the equations:

\[ t = 0:0.01:12 \cdot \pi; \hspace{0.5cm} x = (\exp(0.5 \cdot t)) \cdot \cos(t); \hspace{0.5cm} y = (\exp(0.5 \cdot t)) \cdot \sin(t); \hspace{0.5cm} plot(x, y) \]  \hspace{1cm} (9)

where \( \theta \leftrightarrow t \), \( k = 6 \), \( b = 0.5 \).

Speaking about the golden spiral, it can be mentioned that it is found in the structure of many elements of nature, and its base is Fibonacci's string. It is a special case of the logarithmic spiral. The difference is the growth factor. If on the previous spiral the growth factor was constituted by \( e \), in this case, the growth factor is \( \phi \), called the golden ratio whose value is approximated by \( \phi \approx 1.6180339887 \).

This spiral can be found in the arrangement of the galaxies, in the arrangement of phyllotaxis leaves.

The characteristic equation is:

\[ r = \phi^{\frac{\theta}{\pi}} \]  \hspace{1cm} (10)

where \( \phi \) is constant, \( \theta > 0 \).

The parametrically equations are:

\[ x = \phi^{\frac{\theta}{\pi}} \cdot \cos(\theta) \]  \hspace{1cm} (11)

\[ y = \phi^{\frac{\theta}{\pi}} \cdot \sin(\theta) \]  \hspace{1cm} (12)

Graphical representation in Matlab can be realized by:

\[ \phi = 1.61803398875; \hspace{0.5cm} t = 0:0.01:12 \cdot \pi; \hspace{0.5cm} x = (f. \hat{\cdot}(t * 2/\pi)) \cdot \cos(t); \hspace{0.5cm} y = (f. \hat{\cdot}(t * 2/\pi)) \cdot \sin(t); \hspace{0.5cm} plot(x, y) \]  \hspace{1cm} (13)

where \( \phi \rightarrow f \), \( \theta \rightarrow t \), \( k=6 \).

After the spirals were made, the next step was to find a way to measure the length of the spiral arc. For this, the 3 spirals were represented in Autocad. The idea of this measurement is to decide at the end which of the 3 is most suitable to be used as a mold cavity so that the filling of the filled channel is an important index of the processability of the plastic used. For this, the research started from the premise that the equipment mold will have limited dimensions, being for laboratory research, so that the 3 spirals were framed in a pre-defined geometrical figure. A circle with a diameter of 75 mm was used as the base, inside of which there were drawn the Archimedean spiral, the logarithmic spiral, and the golden spiral. Their representations can be seen in figure 2. After the representation, the DimCurve application (an application that can be attached to the Autocad base program) was used for the measurement. This allows the measurement of any curve, entirely, on sections or between points. The measurement results
are exemplified in figure 3. As it can be seen, the length of the Archimedean spiral arc is the greatest, ensuring the optimization of the use of space available inside the basic circle.

The method may be useful when someone wants to inject more material, but also not to protrude out of the channel due to lack of space or to the small size of the cavity. Therefore, it has been decided to use as the cavity for the mold an Archimedean spiral channel within plastic material will be injected.

4. Use of a 3D CAD software for determining the length of an arc of the spiral

After determining the required spiral elements, the channel function was tested by an injection sample. The mold was made of aluminum in the form of a square with the side of 50 mm, and the spiral groove was machined on a CNC by milling and the milling cutter had a diameter of 1 mm. Small dimensions are due to the purpose of experimental research on minimal dimensional channels.

According to figure 4, which exemplifies the result of the injection process, for the material used, an EVA type copolymer (based on vinyl acetate/ethylene), characterized by a melting temperature of 160-180°C, the filling of the cavity was made on the colored length in the shade of black. The material is transparent, but for greater visibility, it was colored after injection. The problem is to measure this length so that the result is conclusive on the processability of the tested material.

There have been several attempts to measure the channel filled with material, and the most convenient method was using the CatiaV5 program. A photo was achieved as one could see in figure 4. The photo had to be resized so there was a proportion between the virtual dimensions and the actual dimensions of the spiral. It required the use of the GIMP 2.10.8 program that allowed size adjustment and clarity as needed. The next step was to make a sketch in Catia, which was the basis for inserting the picture of the mold. The proper insertion came from a combination of Wireframe design, Apply Material properties with its correspondent commands. After insertion and scaling to 1:1 so that the picture corresponds to the initial dimensions of the projected mold, Generative Shape Design was used in order to insert a spiral corresponding to the initial spiral of the filled channel, imitating the plastic material path that filled the mold channel. Finally, by means of the Measure function determined the length of the spiral arc was determined (figure 5).

After finding the result, the problem was whether the length of the channel arc could be calculated in an analytical way by calling the mathematical equations and corresponding functions.

Thus, returning to the initial equation of the Archimedean spiral:

\[
r = a + b\theta
\]

the following formula for spiral length was reached:

\[
L = \int_{\alpha_1}^{\alpha_2} \left( r^2 + \left( \frac{dr}{d\theta} \right)^2 \right)^{1/2} \, d\theta
\]

The next step was the matrix analysis to determine the numerical values of the coefficients required in the calculation.
Thus, the inner radius of the spiral is 1mm so that a=1.

Also, for the limits, the starting point is $a_2 = 0$.

The number of the turns of the spiral is 7.5, so the end point of the spiral is $a_2 = 7.5 \cdot 2 \cdot \pi = 15 \cdot \pi$.

Measuring on the channel of the mold, one can see that in order to determine the constant $b$, it has to measure the distance between 2 consecutive coils, so that $2\pi b = 1.9$ and $b = 0.3023943$.

The formula of the spiral became:

$$ r = 1 + 0.3023943 \cdot \theta $$

In order to solve the integral, one could see that:

$$ \frac{dr}{d\theta} = 0.3023943 $$

Thus, the integral can be written as:

$$ \int_{0}^{15\pi} \sqrt{(1 + 0.3023943 \cdot \theta)^2 + (0.3023943)^2} d\theta = 383.292. $$

One could notice that close values were obtained by using 2 distinct calculation methods. The channel measurement was performed using both empirical method by means of CAD software and analytical method by solving the integral corresponding to channel arc length filled with material. It can be seen that the difference between the two results of the two applied methods is only 1.6 mm, which is a good approximation of the results. Thus, it is possible to consider that any of the methods is good to apply, depending on the user's preferred method and the technical and software availabilities. Also, the idea is that having some of this kind of numerical calculations, the results can be compared. So, it was seen that depending on the specific properties, some materials flow faster so the processability is better than others, this being the reason for their applications.

5. Conclusions

The existence of equipment for the plastics injection process can be a good element in carrying out research on the processability of plastics. A method by which the processability of plastic materials can be analyzed is to perform injection tests using a mold cavity in the form of a spiral groove.

There are several types of spirals, and the most convenient in terms of calculations and spatial layout so that it can be conveniently and optimally designed for the mold channel is the Archimedean spiral.

A problem whose solving was the purpose of the work is represented by the determination of the length of the spiral channel projected as a cavity of the mold so that the measurements can be centralized and constitute a factor in determining the processability of the plastics.

In the future, further research is needed so that more materials could be tested and ranked according to its processability by injection.

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