Quantum Decoherence and Weak Localization at Low Temperatures

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We develop a theory of a fundamental effect of the interaction-induced decoherence of the electron wave function in a disordered metal. With the aid of the Keldysh technique and the path integral formalism we derive a formally exact equation of motion for the electron density matrix in the presence of interaction. We demonstrate that the effect of interaction of the electron with other electrons and lattice ions in a disordered metal is equivalent to that of an effective dissipative environment. Quantum noise of this environment causes quantum decoherence even at $T = 0$. Our analysis explicitly accounts for the Pauli principle which plays an important role for inelastic scattering processes but turns out not to affect quantum decoherence. Our results seriously challenge the existence of strong localization in low dimensional disordered metals.

I. INTRODUCTION

Recent experiments attracted a great deal of attention to an old but fundamental question: how fast can a quantum particle lose information about its initial state in the presence of interaction? In other words, how fast can interaction destroy the quantum phase coherence? The answer on this question essentially depends on the type of interaction.

It follows from general principles of quantum mechanics that quantum coherence of the wave function cannot be destroyed due to elastic interaction with an external potential. Another physical situation may take place if the quantum particle interacts with other (quantum) degrees of freedom which play the role of an effective environment. In this case quantum dynamics of the particle cannot be described by the wave function but only by the density matrix. Various examples (to be discussed below) show that such interactions may lead to a complete destruction of quantum coherence.

A general approach to the problem was formulated by Feynman and Vernon who demonstrated that the effect of environment can be taken into account by means of averaging over its all possible quantum states. As a result the environment variables are integrated out and quantum dynamics of the particle can be described only in terms of its own degrees of freedom. Within this approach interaction with the external environment is taken into account by means of the so-called influence functional which appears in the (effective) action for the particle as a result of averaging over the bath variables. It is quite clear that specific properties of the environment are not important unless they explicitly enter the expression for the influence functional. In other words, the particle does not “feel” the difference between physically different baths provided they are described by the same influence functional.

These ideas were developed further by Caldeira and Leggett who showed that the above arguments can be used to describe quantum dynamics of dissipative systems and derived the effective action for the case of linear Ohmic dissipation. The same type of analysis was also developed by Schmid who formulated a quasiclassical Langevin equation approach describing real time dynamics of a quantum particle in the presence of dissipation and quantum noise.

Although the above papers are dealing with the model of a bosonic environment it is obvious that the ideas can be applied to a fermionic bath as well. This was done e.g. by Ambegaokar, Eckern and Schön in the case of superconducting tunnel junctions and later by Schön and one of the present authors in a somewhat broader context of a metallic system with dissipation. Although the microscopic Hamiltonian describing electrons in a metal is quite different from one used in the model in the final expressions for the influence functionals for various metallic systems and tunnel junctions obtained in Refs. turn out to be similar or even completely equivalent to those considered in Refs. This equivalence is just an illustration of the property discussed above: the effect of physically different environments is indistinguishable provided they are described by the same influence functional.

In Refs. quantum dynamics of a certain collective variable of interest (the phase) was considered. This variable was extracted “from interaction”, after that electronic degrees of freedom were integrated out and the effective action for the phase was derived. One can also generalize this procedure and describe quantum dynamics of superconductors considering the phase as a quantum field. In all these cases the collective variable is intimately linked to the electronic bath, quantum dynamics of the former does not exist without the latter at all.
In this paper we will analyze a somewhat different situation. Namely, we will study quantum dynamics of an electron propagating in a disordered metal and interacting with other electrons which play the role of effective environment. It is well known that quantum interference of electrons scattered on impurities lead to quantum corrections to the classical Drude conductivity. These so-called weak localization corrections have been extensively discussed in the literature (see e.g. [10] for review). The magnitude of these corrections is known to be determined by the time within which electrons in a metal can be described by a phase coherent wave function. At times exceeding this so-called decoherence time $\tau_{\phi}$ quantum coherence is destroyed, quantum interference is not anymore possible and therefore the classical diffusion picture is restored.

The decoherence time $\tau_{\phi}$ in a disordered metal can be determined by various physical processes, such as electron-electron and electron-phonon interactions, electron scattering on magnetic impurities etc. (see e.g. [12]). It was shown by Altshuler, Aronov and Khmelnitski that at not very low temperatures the effect of electron-electron interaction on the decoherence time is equivalent to that of classical Nyquist noise in a disordered conductor. In this case one finds [12],

$$\tau_{\phi} \propto T^{2/(d-4)},$$

where $d$ is the effective system dimension. This result demonstrates that decoherence effect of electron-electron interaction becomes weaker as the temperature is lowered.

Down to which temperature does the above result remain correct? Or, more generally, does $\tau_{\phi}$ increase with decreasing $T$ at all temperatures thus going to infinity at $T \to 0$? A positive answer on the latter question would mean that at $T = 0$ the electron in a disordered metal can be described by the phase coherent wave function even in the presence of Coulomb interaction with other electrons. This scenario faces several important problems:

(i) Even if one assumes that electrons in a metal are isolated from the external environment it is hard to believe that the $N$-electron wave function $\Psi_N$ ($N = 10^6$ is the total number of electrons in a conductor, $a \gg 1$) can be factorized into the product of single electron wave functions. This would imply that Coulomb interaction is effectively renormalized to zero at $T = 0$. We see no reason to expect that.

(ii) In any real physical situation electrons in a metal are coupled to the external environment, e.g. via external leads. Electrons can enter and leave a piece of metal, interact with electrons in the leads etc. Due to this reason the whole system of $N$ electrons in a metal (and not only one electron!) should be treated as open and thus cannot be described by the wave function $\Psi_N$ even at $T = 0$.

(iii) In the case of Caldeira-Leggett type of models it is well known that after a sufficiently long time even at $T = 0$ quantum coherence is destroyed due to interaction with the bath no matter how weak this interaction is (see e.g. [13]). For strong enough interaction the particle exhibits practically no quantum features and can even get localized in a potential well [14]. Since the Caldeira-Leggett models are qualitatively similar to the problem in question (in both cases one quantum degree of freedom interacts with many others playing the role of a bath) it is quite natural to expect also the decoherence time for interacting electrons in a disordered metal to remain finite even at $T = 0$.

(iv) Saturation of the temperature dependence for $\tau_{\phi}$ at low $T$ was clearly observed in experiments as well as in numerous earlier experimental studies (see e.g. [15]). Even a qualitative interpretation of these results in terms of eq. (1) is not possible. There exist sufficiently strong experimental reasons to believe that this saturation is not due to heating of the sample or the effect of magnetic impurities.

All the above arguments strongly suggest that the decoherence time $\tau_{\phi}$ should remain finite down to $T = 0$. Although the experimental evidence for the low temperature saturation of $\tau_{\phi}$ is available for quite a long time the physical reason for this effect was not convincingly explained in the literature. An earlier attempt [16] to provide this explanation in terms of zero-point fluctuations of impurities has failed. Only very recently it was suggested [17] that quantum decoherence in a disordered conductor is caused by the mechanism qualitatively similar to one existing in the Caldeira-Leggett model, namely the interaction of a particle with an external quantum bath. In the present case an electron propagating in a disordered metal interacts with a fluctuating electric field produced by other electrons which play the role of an effective dissipative environment. Due to randomness of such fluctuations the quantum coherence is lost within a finite time $\tau_{\phi}$.

In fact, this mechanism is not new. Exactly the same effect was considered in Ref. [18] where only the low frequency (classical) fluctuations of the environment $\omega < T$ were taken into account. This approximation is sufficient at high temperatures and yields the result (1). However, as the temperature is lowered there remain less and less classical modes in the fluctuating environment. Therefore it is reasonable to expect that below a certain temperature $T_q$ quantum fluctuations of the environment will take over and determine the decoherence time $\tau_{\phi}$. The generalization of the corresponding analysis is straightforward and allows to evaluate both the classical-to-quantum crossover temperature $T_q$ and the decoherence time $\tau_{\phi}$ at low $T$. The results [19] agree quite well with the experimental findings [20].

Several fundamentally important questions should still be answered, however. Perhaps the main one is about the role of the Pauli principle in our description of the electron-electron interaction. At sufficiently high temperatures
there are always enough empty states into which electrons can get scattered provided the energy transfer due to interaction does not exceed $T$. In this case the Pauli principle does not play a very important role. On the other hand, at $T \to 0$ in equilibrium all quantum states below the Fermi energy are occupied. Thus scattering into any of these states is forbidden due to the Pauli principle and the electron energy cannot decrease. At $T = 0$ it also cannot increase due to interaction with other electrons because they are also in the ground state. Thus at $T = 0$ inelastic electron-electron scattering is forbidden due to the Pauli principle. If so, how can one expect quantum decoherence to occur as a result of the electron-electron interaction at $T = 0$? One might think that due to the Pauli principle the electron is simply insensitive to zero-point fluctuations of other electrons, and quantum decoherence does not take place.

Another though somewhat related question is about the effect of phonons. It is quite obvious that at a finite $T$ decoherence can also be caused by electron-phonon interaction. However at $T \to 0$ there are no real phonons in the system and therefore no quantum decoherence due to phonons should be expected. On the other hand, if we admit that zero-point fluctuations of electrons lead to decoherence, zero point fluctuations of the lattice ions should have qualitatively the same effect. Is it possible to match this conclusion e.g. with the well known fact that electrons do not scatter on zero-point lattice fluctuations, and therefore the phonon contribution to the conductance of a metal vanishes at $T = 0$?

On a more formal level it is sometimes conjectured that the above arguments correspond to the exact cancellation of Keldysh diagrams for $\tau_\varphi$ in the limit $T \to 0$. If this is indeed the case the inverse decoherence time $1/\tau_\varphi$ would be exactly equal to zero at $T = 0$ and (at least qualitatively) would match with the result (1).

The main goal of this paper is to demonstrate that this is not the case. The key point is that all the above arguments apply as far as the electron inelastic relaxation time $\tau_i$ is concerned. The latter can be found by means of a quantum kinetic approach which will be fully reproduced within the framework of our analysis. However, as it has been pointed out in Ref. and discussed later in other works, the decoherence time $\tau_\varphi$ is entirely different from $\tau_i$ and, in contrast to the latter, cannot be determined with the aid of any kind of kinetic analysis. Roughly speaking, the decoherence time controls the decay of off-diagonal elements of the electron density matrix and (even at high temperatures) has little to do with inelastic scattering of electrons. Therefore the quasiclassical kinetic treatment is insufficient and a fully quantum mechanical analysis should be elaborated to determine the time $\tau_\varphi$.

The paper is organized as follows. In Section 2 we make use of the general formalism of the Green-Keldysh functions and derive a formally exact equation of motion for the electron density matrix in the presence of Coulomb interaction. This equation explicitly accounts for the Pauli principle and allows for a clear understanding of its role in the process of electron-electron interaction in a metal. In Section 3 the effective action (or the influence functional) for the fluctuating scalar potential in a metallic conductor is derived. We also demonstrate that in equilibrium this influence functional satisfies the fluctuation-dissipation theorem and establish the relation with the real time effective action derived in the Caldeira-Leggett model. In Section 4 with the aid of these general results we will derive the real time effective action for the electron propagating in a metal and determine the decoherence time $\tau_\varphi$ and the weak localization correction to conductivity at low temperatures. Our formalism naturally includes both electron-electron and electron-phonon interactions and allows to establish the corresponding contributions to $\tau_\varphi$ from each of these processes. In Section 5 we derive the quasiclassical kinetic equation and demonstrate the relation of our analysis to the standard kinetic approach which allows to evaluate the inelastic scattering time $\tau_i$. We also derive the quasiclassical Langevin equation which under certain conditions can be used to describe propagation of electrons in a disordered metal. Discussion of the results is given in Section 6. We discuss the relation of our results with previous works as well as possible consequences of the low temperature saturation of $\tau_\varphi$ for the existing picture of strong localization in low dimensional conductors. We also briefly compare our theoretical predictions with available experimental data. Some details of our calculation are presented in Appendices.

II. DENSITY MATRIX

We will consider a standard Hamiltonian describing electrons in a disordered metal

$$H_{el} = H_0 + H_{int},$$

where

$$H_0 = \int dr \psi_r^+ \left( -\frac{\nabla^2}{2m} - \mu + U(r) \right) \psi_r(r),$$

$$\psi_r(r) = \int J(r) d^3q e^{i\mathbf{q}\cdot\mathbf{r}}.$$
\[ H_{\text{int}} = \frac{1}{2} \int \, dr \int \, dr' \psi_\sigma^+(r) \psi_\sigma^+(r') e^2 v(r - r') \psi_\sigma(r') \psi_\sigma(r). \]  

(4)

Here \( \mu \) is the chemical potential, \( U(r) \) accounts for a random potential due to nonmagnetic impurities, and \( v(r) = 1/|r| \) represents the Coulomb interaction between electrons.

Let us define the generating functional for the electron Green-Keldysh functions in terms of the path integral over the Grassman fields \( \psi \) and \( \bar{\psi} \)

\[
J[\eta, \eta^*] = \frac{1}{\mathcal{Z}} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp \left( iS_{\text{eff}}[\bar{\psi}, \bar{\psi}, V] + i \int dt \left( \bar{\psi}(t, r) \eta(t, r) + \psi(t, r) \eta^*(t, r) \right) \right),
\]

(5)

where \( S_{\text{eff}} \) is the effective action

\[
S_{\text{eff}}[\bar{\psi}, \bar{\psi}, V] = \int dt \left( \int dr \left[ i\bar{\psi}(t, r) \partial_t \psi(t, r) - e\bar{\psi}(t, r) \psi(t, r) V(t, r) \right] - H_0[\bar{\psi}, \psi] \right) + \frac{1}{2} \int dt \int dr \int dr' V(t, r) v^{-1}(r - r') V(t', r'),
\]

where \( v^{-1}(r - r') = -\nabla^2/4\pi \). Integration over time \( t \) in (4) goes along the Keldysh contour \( K \) which runs in the forward and then in the backward time direction. In (4,5) we performed a standard Hubbard-Stratonovich transformation introducing the path integral over a scalar potential field \( V \) in order to decouple the \( \psi^\dagger \)-interaction in (4). The electron Green-Keldysh function \( \hat{G} \) can be determined from (5) by taking the derivatives with respect to the source fields \( \eta \) and \( \eta^* \):

\[
\hat{G}(t, r; t', r') = \left( \frac{\delta}{\delta \eta^*(t, r)} \frac{\delta}{\delta \eta(t', r')} \right) J[\eta, \eta^*]_{\eta=\eta^*=0}.
\]

(7)

Making use of (4,5) and the definition of the Green-Keldysh function for an electron interacting with the field \( V \)

\[
\hat{G}_V(t, r; t', r') = -i \int \mathcal{D}\psi \int \mathcal{D}\bar{\psi} \psi(t, r) \bar{\psi}(t', r') \exp(iS_{\text{eff}}[\bar{\psi}, \bar{\psi}, V])
\]

\[
\int \mathcal{D}\psi \int \mathcal{D}\bar{\psi} \exp(iS_{\text{eff}}[\bar{\psi}, \bar{\psi}, V])
\]

(8)

it is easy to prove the identity

\[
\hat{G} = \int \mathcal{D}V_1 \mathcal{D}V_2 \hat{G}_V e^{iS[V_1, V_2]} \int \mathcal{D}V_1 \mathcal{D}V_2 e^{iS[V_1, V_2]},
\]

where

\[
iS[V_1, V_2] = 2\text{Tr} \ln \hat{G}_V^{-1} + i \int dt \int dr \frac{\nabla V_1)^2 - (\nabla V_2)^2}{8\pi}.
\]

(9)

(10)

The factor 2 in front of the trace comes from the summation over a spin index. In (6,10) we explicitly defined the fields \( V_1(t) \) and \( V_2(t) \) equal to \( V(t) \) respectively on the forward and backward parts on the Keldysh contour \( K \). Analogously \( \hat{G} \) and \( \hat{G}_V \equiv \hat{G}[V_1, V_2] \) are the 2×2 matrices in the Keldysh space:

\[
\hat{G} = \begin{pmatrix}
G_{11} & -G_{12} \\
G_{21} & -G_{22}
\end{pmatrix}.
\]

(11)

The matrix function \( \hat{G}_V \) obeys the equation

\[
\left( i \frac{\partial}{\partial t} - \hat{H}_0(r_1) + e\hat{V}(t_1, r_1) \right) \hat{G}_V = \delta(t_1 - t_2) \delta(r_1 - r_2);
\]

\[
(12)
\]

where

\[
\hat{H}_0 = H_0 \hat{1} = \begin{pmatrix}
-\frac{\nabla^2}{2m} - \mu + U(r) & 0 \\
0 & -\frac{\nabla^2}{2m} - \mu + U(r)
\end{pmatrix} ; \quad \hat{V} = \begin{pmatrix}
V_1(t, r) & 0 \\
0 & V_2(t, r)
\end{pmatrix}.
\]

(13)
Note that the function \( \hat{G}_V \) is to some extent similar to the Green-Keldysh function of an electron in an external field. However there exists an important difference: in our case the electron interacts with a fluctuating (quantum) field \( V \). Formally this implies that the fields \( V(t, r) \) on two parts of the Keldysh contour differ \( V_1(t, r) \neq V_2(t, r) \) while for the external field one always has \( V_1(t, r) = V_2(t, r) \).

The general solution of the equation (12) can be expressed in the form
\[
\hat{G}_V(t_1, t_2) = -i\hat{U}_V(t_1, t_2)[\theta(t_1 - t_2)\hat{a} - \theta(t_2 - t_1)\hat{b} + \hat{f}_V(t_2)],
\]
(14)

Here we defined
\[
\hat{a} = \left( \begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array} \right), \quad \hat{b} = \left( \begin{array}{cc} 0 & 0 \\ 0 & 1 \end{array} \right),
\]
(15)

\( \hat{U}_V(t_1, t_2) \) is the matrix evolution operator
\[
\hat{U}_V(t_1, t_2) = \left( \begin{array}{cc} u_1(t_1, t_2) & 0 \\ 0 & u_2(t_1, t_2) \end{array} \right),
\]
(16)

which consists of the scalar evolution operators
\[
u_{1,2}(t_1, t_2) = T \exp \left[ -i \int_{t_1}^{t_2} dt' \left( H_0 - eV_{1,2}(t') \right) \right]
\]
\[
= \int_{r(t_1)=r_i}^{r(t_2)=r_f} D\mathbf{r}(t') \exp \left[ i \int_{t_1}^{t_2} dt' \left( \frac{m\mathbf{\dot{r}}^2}{2} - U(r') + eV_{1,2}(t', r') \right) \right],
\]
(17)

\( T \) is the time ordering operator. In eq. (14) and below we always imply integration over the internal coordinate variables in the product of operators, whereas integration over time is written explicitly. For the sake of brevity we also do not indicate the coordinate dependence in (14) and many subsequent expressions. This dependence can be trivially restored if needed.

Note that eq. (14) is completely equivalent to the standard representation of the Green-Keldysh matrix which elements can be expressed in terms of retarded, advanced and Keldysh Green functions. The representation (14) defines a general solution of the linear differential equation (12): the term \( \hat{U}_V(t_1, t_2)\hat{f}_V(t_2) \) with an arbitrary matrix operator \( \hat{f}_V(t_2) \) represents a general solution of the homogeneous equation, while the terms with \( \theta \)-functions give a particular solution of the inhomogeneous equation. The operator function \( \hat{f}_V(t_2) \) in (14) is fixed by the Dyson equation
\[
\hat{G}_V(t_1, t_2) = \hat{G}_0(t_1, t_2) - \int_0^t dt' \hat{G}_0(t_1, t')e\hat{V}(t')\hat{G}_V(t', t_2).
\]
(18)

The matrix \( \hat{G}_0 \) is the electron Green-Keldysh function without the field. This function is defined by eqs. (14-17) with \( V_{1,2}(t, r) \equiv 0 \) and \( \hat{f}_0(t_2) \) has the form
\[
\hat{f}_0(t_2) = \left( \begin{array}{cc} -\rho_0(t_2) & \rho_0(t_2) \\ 1 - \rho_0(t_2) & \rho_0(t_2) \end{array} \right),
\]
(19)

where \( \rho_0(t) = e^{-iH_0t}\rho(0)e^{iH_0t} \) is the electron density matrix for \( V_{1,2} = 0 \) at a time \( t \).

The equation (15) can be solved perturbatively in \( e\hat{V} \). Combining this solution with eq. (11) one reproduces the standard Keldysh diagrams. This way of treating the problem is quite complicated in general and becomes particularly nontransparent in the interesting limit of low temperatures.

We will proceed differently.

It is well known that the 1,2-component of the Green-Keldysh matrix \( \hat{G} \) is directly related to the exact electron density matrix
\[
\rho(t; r, r') = -i\hat{G}_{12}(t; r, r'),
\]
(20)
which contains all necessary information about the system dynamics in the presence of interaction. Analogously one can define the “density matrix” \( \rho_V(t) \equiv f_{1,2}(t) \) related to the 1,2-component of the matrix \( G_V \) by the equation equivalent to (20). Our strategy is as follows. First we will derive the exact equation of motion for the density matrix \( \rho_V(t) \) which turns out to have a very simple and transparent form. Already at this stage we will clarify the role of the fluctuating fields \( V_{1,2} \) and the Pauli principle in our problem. Then we will evaluate the influence functional \( S[V_1, V_2] \) (1) and find the density matrix \( \rho \) from the equation

\[
\rho(t; r, r') = \langle \rho_V(t; r, r') \rangle_{V_1, V_2},
\]

where the average over fields \( V_1 \) and \( V_2 \) is defined in (1).

The derivation of the equation for the density matrix \( \rho_V(t) \) is straightforward. Let us perform the time integration in the last term of eq. (18). Integrating by parts and making use of eq. (12) after a simple algebra (see Appendix A) we obtain

\[
\hat{G}_0(t_1, t)\hat{G}(t, t_2) - \hat{G}_0(t_1, 0)\hat{G}(0, t_2) = 0.
\]

Substituting the representation (14) into (22) we arrive at the matrix equation which relates the matrix \( \hat{f}_V(t) \), the evolution operator \( \hat{U}_V(t) \) and the initial density matrix \( \rho_0(0) \) defined for \( V_{1,2} = 0 \). With the aid of this equation one determines the 1,2-component of the matrix \( \hat{f}_V(t) \) and thus the density matrix \( \rho_V(t) \). The details of this calculation are presented in Appendix A. As a result we find

\[
[1 - \rho_0(t)(u_2(t, 0)u_1(0, t) - 1)]\rho_V(t) = \rho_0(t).
\]

One can also rewrite this result in the form of the differential equation describing the time evolution of the density matrix:

\[
\frac{i}{\hbar} \frac{\partial \rho_V}{\partial t} = [H_0, \rho_V] - (1 - \rho_V)eV_1 \rho_V + \rho_V eV_2(1 - \rho_V), \quad \rho_V(0) = \rho_0(0).
\]

The equation (24) is the main result of this Section. We would like to emphasize that our derivation was performed without any approximation, i.e. the result (24) is exact. It contains all information about the system dynamics hidden in the four components of the Green-Keldysh matrix. In the absence of the fluctuating field \( V_{1,2} = 0 \) eq. (24) reduces to the standard equation for the electron density matrix with the Hamiltonian \( H_0 \). In the presence of the field \( V_{1,2} \) the equation (24) exactly accounts for the Pauli principle. This is obvious from our derivation which automatically takes care about the Fermi statistics through the integration over the Grassman fields \( \psi \) and \( \bar{\psi} \). This is also quite clear from the form of the last two terms in the right hand side of eq. (24). In Section 5 we will demonstrate that within the quantum kinetic analysis these terms are responsible for the standard in- and out-scattering terms in the collision integral.

In order to understand the role of the Pauli principle let us rewrite the equation (24) in the form

\[
\frac{i}{\hbar} \frac{\partial \rho_V}{\partial t} = [H_0 - eV^+, \rho_V] - (1 - \rho_V)\frac{eV^-}{2} \rho_V - \rho_V \frac{eV^-}{2} (1 - \rho_V),
\]

where we defined \( V^+ = (V_1 + V_2)/2 \) and \( V^- = V_1 - V_2 \). It is quite obvious from (25) that the field \( V^+(t, r) \) plays the same role as an external field. All electrons move collectively in this field, its presence is equivalent to local fluctuations of the Fermi energy \( \mu \rightarrow \mu + eV^+(t, r) \). The Pauli principle does not play any role here. Below we will demonstrate that quantum fluctuations of the field \( V^+ \) are responsible for the low temperature saturation of the decoherence time \( \tau_\rho \) and the weak localization correction to conductivity in disordered metals.

The field \( V^- \) is, on the contrary, very sensitive to the Pauli principle. It will be shown below that this field is responsible for damping due to radiation of an electron which moves in a metal. Corresponding energy losses can be only due to electron transitions into lower energy states. At \( T = 0 \) in equilibrium all such states are already occupied by other electrons, therefore such processes are forbidden and the electron energy remains unchanged due to the Pauli principle. We will demonstrate, however, that these processes are irrelevant for the decay of the off-diagonal elements of the electron density matrix and therefore the Pauli principle can hardly affect quantum decoherence in a disordered metal even at \( T = 0 \).

III. INFLUENCE FUNCTIONAL FOR THE FIELD

Let us now derive the expression for the influence functional (effective action) \( S[V_1, V_2] \) for the field \( V \). A formally exact action \( S[V_1, V_2] \) obtained by integration over all electron degrees of freedom is given by eq. (10). Let us expand
this expression up to the second order in \( V_{1,2} \). The first order order terms of this expansion vanish because the Green function \( G_0 \) corresponds to a zero current and zero charge density state of the system. In the second order we obtain

\[
i\delta S^{(2)} = -\text{Tr}(G_0 e\dot{V} \hat{G}_0 e\dot{V})
\]

\[
= -\hat{e}^2 \text{tr} \left\{ \begin{array}{c} G_{11}V^+G_{11}V^+ - G_{12}V^+G_{21}V^+ - G_{21}V^+G_{12} + G_{22}V^+G_{22}V^+ \\
G_{11}V^+G_{11}V^- - G_{12}V^+G_{21}V^- + G_{21}V^+G_{12}V^- - G_{22}V^+G_{22}V^- \\
\frac{1}{4}G_{11}V^-G_{11}V^+ - \frac{1}{4}G_{12}V^-G_{21}V^+ - \frac{1}{4}G_{21}V^-G_{12}V^+ - \frac{1}{4}G_{22}V^-G_{22}V^- \end{array} \right\}
\]

(26)

Here the subscript \( 0 \) for the Green functions is omitted for the sake of simplicity, all the Green functions here and below in this Section are defined for \( V_{1,2} = 0 \). The expression (26) can be simplified with the aid of the identities

\[
G_{11} = G_{12} + G^R = G_{21} + G^A, \quad G_{22} = G_{12} - G^A = G_{21} - G^R,
\]

(27)

which allow to exclude the functions \( G_{11} \) and \( G_{22} \) from the action (26). The terms containing \( V^+V^+ \) are reduced to

\[
\text{tr}(G^R V^+G^R V^+ + G^A V^+G^A V^+),
\]

(28)

Making use of the expressions

\[
G^R = -i\theta(t_1 - t_2)u_0(t_1, t_2), \quad G^A = i\theta(t_2 - t_1)u_0(t_1, t_2),
\]

(29)

and writing the traces (29) in the time-space representation, we immediately observe that the product \( \theta(t_1 - t_2)\theta(t_2 - t_1) = 0 \) appears under the integral, and the whole combination (28) vanish. In other words, the terms \( V^+V^+ \) give no contribution to the action (26). The remaining terms in combination with the last two terms in (10) describing the free field action yield

\[
iS[V_1, V_2] = \int_0^t dt' \int_0^t dt \int \frac{(\nabla V^+\nabla V^-)}{4\pi} - \int_0^t dt_1 \int_0^t dt_2 \int d\mathbf{r}_1 d\mathbf{r}_2 \times
\]

\[
\left\{ iV^-(t_1, \mathbf{r}_1, t_2, \mathbf{r}_2)\chi(t_1, t_2, \mathbf{r}_1, \mathbf{r}_2) + V^-(t_1, \mathbf{r}_1)\eta(t_1, t_2, \mathbf{r}_1, \mathbf{r}_2) \right\},
\]

(30)

where

\[
\chi(t_1, t_2, \mathbf{r}_1, \mathbf{r}_2) = -ie^2 \left\{ G^R(t_1, t_2, \mathbf{r}_1, \mathbf{r}_2)(G_{21}(t_2, t_1, \mathbf{r}_2, \mathbf{r}_1) + G_{12}(t_2, t_1, \mathbf{r}_2, \mathbf{r}_1)) + (G_{12}(t_2, t_1, \mathbf{r}_1, \mathbf{r}_2) + G_{21}(t_2, t_1, \mathbf{r}_1, \mathbf{r}_2))G^A(t_2, t_1, \mathbf{r}_1, \mathbf{r}_2) \right\},
\]

(31)

\[
\eta(t_1, t_2, \mathbf{r}_1, \mathbf{r}_2) = \frac{e^2}{2} \left\{ G_{12}(t_1, t_2, \mathbf{r}_1, \mathbf{r}_2)G_{21}(t_2, t_1, \mathbf{r}_2, \mathbf{r}_1) + G_{21}(t_1, t_2, \mathbf{r}_1, \mathbf{r}_2)G_{12}(t_2, t_1, \mathbf{r}_1, \mathbf{r}_2) \right\}
\]

(32)

The expressions (29) and (32) define the influence functional for the field \( V \) in terms of the Green-Keldysh functions for the (in general nonequilibrium) electron subsystem. It is easy to check that the above expressions satisfy the requirement of causality: in the \( V^-(t_1)V^+(t_2) \) terms the time \( t_1 \) is always larger than \( t_2 \) due to analytic properties of retarded and advanced Green functions [29]. It is also straightforward to demonstrate (see Appendix B) that in thermodynamic equilibrium the kernels \( \eta(\omega) \) and \( \text{Im}\chi(\omega) \) (Fourier transformed with respect to the time difference \( t_1 - t_2 \)) satisfy the equation

\[
\eta(\omega, \mathbf{r}_1, \mathbf{r}_2) = -\frac{1}{2} \coth \left( \frac{\omega}{2T} \right) \text{Im}\chi(\omega, \mathbf{r}_1, \mathbf{r}_2).
\]

(33)

The latter equation is just the fluctuation-dissipation theorem [30].

Finally, for a homogeneous system one can also perform the Fourier transformation with respect to \( \mathbf{r}_1 - \mathbf{r}_2 \) and find

\[
iS[V_1, V_2] = i \int \frac{d\omega d^3k}{(2\pi)^4} \left\{ \frac{\epsilon(\omega, k)}{4\pi} V^+(\omega, k) - \frac{1}{2} \frac{\text{Im}\epsilon(\omega, k)}{4\pi} \coth \left( \frac{\omega}{2T} \right) V^-(\omega, k) \right\}.
\]

(34)
where \( \epsilon(\omega, k) \) is the dielectric susceptibility of the system. For a homogeneous electron gas it is given by the standard RPA formula:

\[
\epsilon(\omega, k) = 1 + \frac{e^2}{\pi^2 k^2} \int d^3 p \frac{n_{p+k} - n_p}{\omega - \xi_{p+k} + \xi_p + i0}.
\] (35)

Eq. (35) includes only the electron contribution to the susceptibility. In general the effect of ions should also be accounted for. Here we will describe this effect within a very simple approximation which is, however, sufficient for our analysis. Namely, the ion contribution to the susceptibility will be taken in the form \( \delta \epsilon = -\omega_{pi}^2/\omega^2 \), where \( \omega_{pi} \) is the ion plasma frequency. Then the phonon spectrum is determined by the equation \( \epsilon(\omega, k) = 0 \). In the long wave limit this approximation works sufficiently well for longitudinal phonons which mainly interact with electrons. The effect of transverse phonons cannot be described within this simple model. But such phonons are weakly coupled to the electrons anyway, and therefore their effect can be safely ignored. If needed, further generalizations of this simple model can be also incorporated into our analysis.

In the relevant case of a disordered metal a direct calculation of polarization bubbles (see e.g. 9) yields

\[
\epsilon(\omega, k) = 1 + \frac{4\pi\sigma}{-i\omega + Dk^2} - \frac{\omega_{pi}^2}{\omega^2}.
\] (36)

Here \( \sigma = 2e^2N_0D \) is the classical Drude conductivity, \( N_0 \) is the metallic density of states and \( D = v_F l/3 \) is the diffusion coefficient.

The expression (36) is valid for wave vectors smaller than the inverse elastic mean free path \( k \lesssim 1/l \) and for small frequencies \( \omega \lesssim 1/\tau_e \), where \( \tau_e = l/v_F \) is the elastic mean free time. Note, that if one neglects the effect of phonons and considers only nearly uniform in space \( (k \approx 0) \) fluctuations of the field \( V \), one immediately observes that eqs. (34,36) exactly coincide with the real time version of the Caldeira-Leggett action (see 8) in this limit. For a disordered metal this action was derived by means of the quasiclassical Eilenberger equations in Refs. 7,8. Our analysis (see also 4) reproduces these results and generalizes them to the case of nonuniform fluctuations of the field \( V \).

Taking into account only uniform fluctuations of the electric field one can also derive the Caldeira-Leggett action expressed in terms of the electron coordinate only. In the quasi-one-dimensional situation one should simply write down the electron action on the Keldysh contour, take into account the potential energy \(-Ex\) of the electron with the coordinate \( x(t) \) in the fluctuating electric field \( E(t) \) and add the action for the field \( \phi(t) \) (with the last term in (34) being dropped). After identification \( E(\omega) = -i\omega V(\omega, k) \rightarrow -V(\omega)/L \) (\( L \) is the sample length) and integration over the fluctuating field \( V \) one arrives at the Caldeira-Leggett action for the electron coordinate \( x(t) \). In this case the effective viscosity in the Caldeira-Leggett influence functional is proportional to \( 1/\sigma \) (in contrast to the effective viscosity for the field \( V \) which is proportional to \( \sigma \)). The whole procedure is completely analogous to that discussed in details in Ref. 29 where we considered the real time effective action for a dissipative system characterized by two collective degrees of freedom (the phase and the charge). Integrating over the charge variable one arrives at the Caldeira-Leggett action in the "phase only" representation. The same can be done here if we use a formal analogy of \( x(t) \) with the phase and \( V/\omega \) with the charge (as defined in Ref. 29).

For our present purposes it is not sufficient to restrict ourselves to uniform fluctuations of the collective coordinate \( V \) of the electron environment. We will see that fluctuations with nonzero \( k \) play an important role and should be taken into account in the quantitative analysis. The corresponding effective action will be derived in the next section. However the main message is clear already from the simple example considered above: in a disordered metal the effect of Coulomb interaction of the electron with other electrons is equivalent to that of an effective dissipative environment with the correspondent effective viscosity governed by the Drude conductivity \( \sigma \).

IV. DECOHERENCE TIME AND CONDUCTANCE

A. Conductance and electron effective action

In order to evaluate the system conductance we will determine the single particle density matrix \( \rho \) in the presence of an external electric potential \( V_x(x) \) applied to the metal. Generalization of the results obtained in the previous sections to the case \( V_x \neq 0 \) is straightforward. The density matrix is determined by the equation

\[
\rho(t) = \langle \rho_V(t, V_x) \rangle_{V_1, V_2} = \frac{1}{\int D V_1 D V_2 \rho_V(t, V_x) e^{i S_{V_x}[V_1, V_2]}} \int D V_1 D V_2 e^{i S_{V_x}[V_1, V_2]};
\] (37)

where the effective action \( S_{V_x}[V_1, V_2] \) reads:
\[ iS_{V_z}[V_1, V_2] = 2\text{Tr} \ln \dot{G}_{V_z}^{-1} + i \int_0^t dt' \int dr (\nabla V_1)^2 - (\nabla V_2)^2 \cdot \frac{8\pi}{\delta^2}, \quad (38) \]

where the subscript \( V + V_z \) indicates the shift of the fields \( V_1 \rightarrow V_1 + V_z \). Here we use the formally exact expression for the effective action, the approximation \( (34) \) will be introduced after the expansion in \( V_z \) will be carried out.

The density matrix \( \rho_V(t, V_z) \) obeys the equation \( (37) \) with \( V^+ \rightarrow V^+ + V_z \). Assuming the field \( V_z \) sufficiently small one can linearize the equation for \( \rho_V(t, V_z) = \rho_V(t) + \delta \rho_V(t, V_z) \) and get

\[ i\frac{\partial \delta \rho_V}{\partial t} = H_1 \delta \rho_V - \delta \rho_V H_2 - [eV_x, \rho_V]; \quad (39) \]

where

\[
H_1 = H_0 - eV^+ - \frac{1}{2}(1 - 2\rho_V)eV^-, \\
H_2 = H_0 - eV^+ + \frac{1}{2}eV^-(1 - 2\rho_V). \quad (40)
\]

The formal solution of the equation \( (39) \) can be easily found:

\[
\delta \rho_V(t) = i \int_0^t dt' U_1(t, t')[eV_x, \rho_V(t')]U_2(t', t), \quad (41)
\]

where

\[
U_{1,2}(t_1, t_2) = \text{exp} \left[ -i \int_{t_1}^{t_2} dt' H_{1,2}(t') \right]. \quad (42)
\]

The operators \( H_{1,2} \) \( (40) \) are nonlocal (since they contain the density matrix), therefore the path integral representation for the evolution operators \( (42) \) contains an additional integration over momentum. The operators \( (40) \) can be written in the form:

\[
H_1(p, r) = \frac{p^2}{2m} + U(r) - eV^+(t, r) - \frac{1}{2}[1 - 2n(H_0(\mathbf{p}, r))]eV^-(t, r), \\
H_2(p, r) = \frac{p^2}{2m} + U(r) - eV^+(t, r) + \frac{1}{2}eV^- (t, r)[1 - 2n(H_0(\mathbf{p}, r))]. \quad (43)
\]

where \( n(\xi) = 1/[\exp(\xi/T) + 1] \) is the Fermi function. In deriving \( (43) \) from \( (40) \) we set \( \rho_V \) to be an equilibrium density matrix. We also neglected the effect of Coulomb interaction in the expression for \( \rho_V(t) \). This approximation is justified as long as Coulomb interaction is sufficiently weak. Note that the same approximation for \( \rho_V(t) \) can be used in eq. \( (11) \).

The evolution operators \( (42) \) acquire the form:

\[
U_{1,2}(t_1, t_2; r_f, r_i) = \int_{r(t_1) = r_i}^{r(t_2) = r_f} D\mathbf{r}' \int D\mathbf{p}' \exp \left[ i \int_{t_1}^{t_2} dt' (\mathbf{p}' - H_{1,2}(\mathbf{p}, r)) \right], \quad (44)
\]

For the sake of generality we note that in the presence of interaction there exists an additional (linear in the field) correction to the density matrix. In order to see that let us expand the action \( (38) \) to the first order in \( V_z \):

\[
i\delta S[V_z] = -2\text{et}(G_{11} - G_{22})V_z. \quad (45)
\]

This correction to the action gives an additional contribution to the density matrix \( (37) \). Expressing the functions \( G_{11} \) and \( G_{22} \) in terms of the density matrix \( \rho_V \) and the evolution operators \( u_{1,2} \) and combining this correction to the density matrix with one defined in \( (11) \) we find

\[
\delta \rho(t) = \langle \delta \rho_V \rangle_{V^+, V^-} + \langle \delta \rho_m \rangle_{V^+, V^-}, \quad (46)
\]
where $\delta \rho_V$ is given by \[1\] and $\delta \rho_{\text{int}}$ has the form

$$
\delta \rho_{\text{int}} = -2i \rho_V(t) \int_0^t dt' \text{tr}(u_1(t, t') [eV_s(t'), \rho_V(t')] u_2(t', t)).
$$

(47)

In the limit of weak interaction between electrons the averaging in (46) may be performed with the approximate action $S[V_1, V_2]$ \[14\].

It is easy to observe that the second term in (46) is small in the limit of weak interaction and vanishes completely if interaction is neglected. The weak localization correction is described by the first term in eq. (46) which will be only considered further below.

Making use of a standard definition of the current density $j$:

$$
\vec{j}(t, \vec{r}) = \frac{ie}{m} \left( \nabla \rho(t, \vec{r}_1, \vec{r}_2) - \nabla \rho_r(t, \vec{r}_1, \vec{r}_2) \right)|_{\vec{r}_1 = \vec{r}_2 = \vec{r}},
$$

(48)

combining it with the above equations and assuming the external electric field to be constant in space and time, $V_x = -E r$, we arrive at the expression for the system conductance

$$
\sigma = \frac{e^2}{3m} \int dt \int d\vec{r}_1 d\vec{r}_2 \left( \nabla \rho_{1f} - \nabla \rho_{2f} \right)|_{\vec{r}_1 = \vec{r}_2 = \vec{r}} \vec{J}(t, \vec{r}; \vec{r}_{1f}, \vec{r}_{2f}; \vec{r}_{1i}, \vec{r}_{2i}) (\vec{r}_{1i} - \vec{r}_{2i}) \rho_0 (\vec{r}_{1i}, \vec{r}_{2i}).
$$

(49)

Here we have shifted the initial time to $-\infty$. The function $\vec{J}$ is the kernel of the operator

$$
\vec{J} = \sum_V \vec{U}[V] \langle V | \vec{U}^+ 
$$

where the sum runs over all possible states of the electromagnetic environment. This function can be expressed in terms of the path integral

$$
\int_{r_1(t)=r_{1f}}^{r_2(t)=r_{2f}} D\vec{r}_1 \int_{r_1(t')=r_{1i}}^{r_2(t')=r_{2i}} D\vec{r}_2 \int D\vec{p}_1 D\vec{p}_2 \times
$$

$$
\langle e^{iS_0[\vec{r}_1, \vec{p}_1] - iS_0[\vec{r}_2, \vec{p}_2] + \int_{t'}^t dt'' \int d\vec{r} (f_-^+ f^+ V^-)} \rangle_{V^+, V^-}.
$$

(50)

Here the action $S_0[\vec{r}, \vec{p}]$ has the form

$$
S_0[\vec{r}, \vec{p}] = \int_{t'}^t dt'' \left( \vec{p}\dot{\vec{r}} - \frac{\vec{p}^2}{2m} - \vec{U}(\vec{r}) \right).
$$

(51)

and the “charge densities” $f^-, f^+$ are defined by the equations:

$$
f^-(t, \vec{r}) = e\delta(\vec{r} - \vec{r}_1(t)) - e\delta(\vec{r} - \vec{r}_2(t)),
$$

$$
f^+(t, \vec{r}) = \frac{1}{2} \left( e \left[ 1 - 2n(\vec{p}_1(t), \vec{r}_1(t)) \right] \delta(\vec{r} - \vec{r}_1(t)) + e \left[ 1 - 2n(\vec{p}_2(t), \vec{r}_2(t)) \right] \delta(\vec{r} - \vec{r}_2(t)) \right).
$$

(52)

Averaging over $V^+, V^-$ in eq. (50) amounts to calculating Gaussian path integrals with the action \[14\] and can be easily performed. We obtain

$$
\int_{r_1(t)=r_{1f}}^{r_2(t)=r_{2f}} D\vec{r}_1 \int_{r_1(t')=r_{1i}}^{r_2(t')=r_{2i}} D\vec{r}_2 \int D\vec{p}_1 D\vec{p}_2 \times
$$

$$
\times \exp \{ iS_0[\vec{r}_1, \vec{p}_1] - iS_0[\vec{r}_2, \vec{p}_2] - iS_R[\vec{r}_1, \vec{p}_1, \vec{r}_2, \vec{p}_2] - S_f[\vec{r}_1, \vec{r}_2] \};
$$

(53)

where
The kernel of this operator and that of \( |1\) at the scales exceeding the mean free path. With this in mind we arrive at the standard result (cf. e.g. (51)) and

\[
S_R[r_1, p_1, r_2, p_2] = \frac{e^2}{2} \int \frac{dt}{t'} \int dt' R(t - t', r_1(t_1) - r_1(t_2)) [1 - 2n(p_1(t_2), r_1(t_2))] - R(t_1 - t_2, r_2(t_1) - r_2(t_2)) [1 - 2n(p_2(t_2), r_2(t_2))] + R(t_1 - t_2, r_1(t_1) - r_2(t_2)) [1 - 2n(p_2(t_2), r_2(t_2))] - R(t_1 - t_2, r_2(t_1) - r_1(t_2)) [1 - 2n(p_1(t_2), r_1(t_2))];
\]

(54)

and

\[
S_I[r_1, r_2] = \frac{e^2}{2} \int \frac{dt}{t'} \int dt' \left\{ I(t_1 - t_2, r_1(t_1) - r_1(t_2)) + I(t_1 - t_2, r_2(t_1) - r_2(t_2)) - I(t_1 - t_2, r_1(t_1) - r_2(t_2)) - I(t_1 - t_2, r_2(t_1) - r_1(t_2)) \right\}.
\]

(55)

At the scales \(|r| \gtrsim l\) the functions \(R\) and \(I\) are defined by the equations

\[
R(t, r) = \int \frac{d\omega dk}{(2\pi)^3} \frac{4\pi}{k^2} \epsilon(\omega, k) e^{-i\omega t + ikr}
\]

(56)

\[
I(t, r) = \int \frac{d\omega dk}{(2\pi)^3} \text{Im} \left( \frac{-4\pi}{k^2} \epsilon(\omega, k) \right) \coth \left( \frac{\omega}{2T} \right) e^{-i\omega t + ikr}.
\]

(57)

If necessary, more general expressions for \(R\) and \(I\) for the whole range \(|r| \gtrsim 1/p_F\) can be easily derived. We will avoid doing this here because the approximation \([77]\) is already sufficient for our present purposes.

Note that the expression in the exponent of eq. \((53)\) defines the real time effective action of the electron propagating in a disordered metal and interacting with other electrons. The first two terms represent the electron action \(S_0\) on two branches of the Keldysh contour while the last two terms \(S_R\) and \(S_I\) determine the influence functional which comes from the effective electron (and/or phonon) environment. As can be seen from eqs. \((54, 57)\) this influence functional is in general not identical to one derived in the Caldeira-Leggett model. However on a qualitative level the similarity is obvious: in both models the influence functionals describe the effect of a certain effective dissipative environment.

Let us now analyze the expression \([19]\) and establish connection with other conductivity calculations. It is convenient to introduce the Wigner function \(\rho(p, r) = \int d\xi e^{-i\xi p} \rho(r + \xi r, r - \xi r/2)\) instead of the density matrix. For homogeneous systems it does not depend on \(r\) at the scales exceeding the mean free path. With this in mind we obtain

\[
\sigma = -\frac{2e^2}{3m} \int \frac{dt}{t'} \int \frac{d^3p}{(2\pi)^3} \rho \hat{W}(t, t') \frac{\partial \rho(p)}{\partial t};
\]

(58)

where \(\hat{W}(t, t')\) is the evolution operator for the Wigner function:

\[
n(t, x, p) = \hat{W}(t, t') n(t', x, p).
\]

(59)

The kernel of this operator and that of \(J\) are related by means of the Fourier transformation with respect to \(r_{1i, 1f} - r_{2i, 2f}\). In equilibrium one has \(n(p) = 1/(\exp(\xi/T) + 1)\), and therefore at small \(T\) one finds

\[
\frac{\partial n(p)}{\partial p} \simeq -v_F \delta(\xi)
\]

and arrives at the standard result (cf. e.g. \([3]\))

\[
\sigma = \frac{2e^2N_0}{3} \int \frac{dt}{t} \langle v(t) v(t') \rangle.
\]

(60)
B. Weak localization correction and decoherence time

Let us analyse the structure of the function $J(\mathbf{r})$ in the same spirit as it has been done in Ref. [13]. In the zero order approximation one can neglect the terms $S_0$ and $S_f$ describing the effect of Coulomb interaction. Then in the quasiclassical limit $p_F l \gg 1$ the path integral (53) is dominated by the saddle point trajectories for the action $S_0$ which are just classical paths determined by the Hamilton equations

$$\dot{p} = -\frac{\partial H_0(p,r)}{\partial r}, \quad \dot{r} = \frac{\partial H_0(p,r)}{\partial p} \tag{61}$$

with obvious boundary conditions $r_1(t') = r_{1i}$, $r(t) = r_{1f}$ for the action $S_0[r_1,p_1]$ and $r_2(t') = r_{2i}$, $r_2(t) = r_{2f}$ for the action $S_0[r_2,p_2]$. Substituting these saddle point trajectories into (53) and integrating out small fluctuations around them one finds

$$J(t,t';r_{1f},r_{2f};r_{1i},r_{2i}) = \sum_{r_1} A_{r_1} \sum_{r_2} A_{r_2} \exp \left( iS_0(t,r_{1f},r_{1i}) - iS_0(t,t';r_{2f},r_{2i}) \right) \tag{62}$$

where the actions $S_0(t,t';r_{01},r_{1i})$ are taken on the classical paths $r_{1,2}(t)$ and

$$A_{r_1} = \sqrt{\frac{e^3}{8\pi^3}} \left| \det \left( \frac{\partial^2 S_0(r_{1f},r_{1i})}{\partial r_{1f} \partial r_{1i}} \right) \right| \tag{63}$$

The value $A_{r_2}^*$ is defined analogously.

Since in a random potential $U(r)$ there is in general no correlation between different classical paths $r_1(t)$ and $r_2(t)$ these paths give no contribution to the double sum (62): the difference of two actions in the exponent of (62) may have an arbitrary value and the result averages out after summation. Thus only the paths for which $S_0[r_1,p_1] \approx S_0[r_2,p_2]$ provide a nonvanishing contribution to (62). Two different classes of such paths can be distinguished (see e.g. [13]):

i) The two classical paths are almost the same: $r_1(t'') \approx r_2(t'')$, $r_{1f} \approx r_{2f}$. In other words, in the path integral (53) one integrates only over trajectories with $|r_1(t'') - r_2(t'')| \lesssim 1/p_F$. Physically this corresponds to the picture of electrons propagating as nearly classical particles which can be described by the diagonal elements of the density matrix only. In the diffusive limit these paths give rise to diffusons (see e.g. [12]) and yield the standard Drude conductance.

ii) The pairs of time reversed paths. In this case $r_{1i} \approx r_{2f}$, $r_{1f} \approx r_{2i}$. In the path integral (53) one integrates over trajectories with $|r_1(t'') - r_2(t'')| \lesssim 1/p_F$, however the difference $|r_1(t'') - r_2(t'')|$ may be arbitrarily large in this case. In a disordered metal these paths essentially determine the dynamics of off-diagonal elements of the electron density matrix. They correspond to Cooperons and give rise to the weak localization correction to conductivity. This correction is defined by the following equation [14, 19]

$$\delta \sigma = -\frac{2e^2D}{\pi} \int_{\tau_c}^\infty dt W(t) = -\frac{2e^2D}{\pi} \int_{-\infty}^{t-\tau_c} dt' W(t' - t). \tag{64}$$

Here we changed the parameter of integration in order to make the relation with the equation (13) more transparent. The quantity $W(t)$ represents the effective probability for the diffusive path to return to the same point after the time $t$. Note that $W(t)$ contains the contribution from time reversed paths and therefore differs from a classical probability. However, in the absence of any kind of interaction which breaks the time reversal symmetry this value coincides with the classical return probability and is given by the formula $W_0(t) = (4\pi D t)^{-d/2} a^{-(3-d)}$, where $d$ is the system dimension and $a$ is the transverse sample size (the film thickness for $d = 2$ and the square root of the wire cross section for $d = 1$).

The weak localization correction (64) diverges for $d \leq 2$. This divergence can be cured by introducing the upper limit cutoff for the integral (64) at a certain time $\tau_c$. This time is usually referred to as decoherence time. As we have already discussed, in a disordered metal the time $\tau_c$ is determined by electron-electron, electron-phonon and other types of interaction which may destroy quantum coherence. From (44) one finds [14, 19]

$$\delta \sigma_d = \begin{cases} -\frac{\pi e^2}{2\pi^{d/2} a^{(3-d)}} \tau_c, & d = 3, \\ -\frac{e^2}{2\pi^{d/2} a^{(3-d)}} \ln \frac{\tau_c}{\tau_d}, & d = 2 \\ -\frac{e^2}{2\pi^{d/2}} \sqrt{D \tau_d}, & d = 1. \end{cases} \tag{65}$$
\[ W(t - t') \approx W_0(t - t') \left\langle e^{-iS_R(t', r, p; r_1, p_1) - S_l(t, t', r, r_1)} \right\rangle_r, \]  

where the average is taken over all diffusive paths returning to the initial point. The average in \((66)\) decays exponentially in time, therefore we may define \(\tau_e\) as follows:

\[ e^{-\frac{(t-t')}{\tau_e}} = \left\langle e^{-iS_R(t', r, p; r_1, p_1) - S_l(t, t', r, r_1)} \right\rangle_r \]

\[ \approx e\left\langle -iS_R(t', r, p; r_1, p_1) - S_l(t, t', r, r_1) \right\rangle_r, \quad (t - t') \to +\infty; \]  

where the average is again taken over all classical paths returning to the same point at the time \(t\).

Let us consider the term \(S_R\) \((64)\). Making use of the obvious relations \(R(t, -r) = R(t, r), n(-p, r) = n(p, r)\) after a trivial algebra we get:

\[ S_R[t, t'; r, p; r_1, p_1] = \frac{e^2}{2} \int_{t'}^t dt_1 \int_{t'}^t dt_2 \left\{ 2R(t_1 - t_2, r(t_1) - r(t_2))[n(p(t_1), r(t_1)) - n(p(t_2), r(t_2))] + \\
+ R(t_1 + t_2 - t - t', r(t_1) - r(t_2))[1 - 2n(p(t_2), r(t_2))] - \\
- R(t_1 + t_2 - t - t', r(t + t' - t_1) - r(t + t' - t_2))[1 - 2n(p(t + t' - t_2), r(t + t' - t_2))] \right\} \] (68)

It is clear that the difference of the occupation numbers \(n\) in the first term of eq. \((68)\) vanish: \(n\) depends only on the energy and not on time because the energy is conserved along the classical path \((11)\). Thus the first term in the integral \((68)\) is identically zero already before averaging over diffusive paths. The difference of two other terms vanishes after averaging over such paths. It can be easily seen if we again bear in mind that the occupation numbers do not depend on time and make use of the fact that the average \(\langle R(t_3, r(t_1) - r(t_2))\rangle\), is a function of the time \(t_3\) and the absolute value of the time difference \(|t_1 - t_2|\). This implies that after averaging the two last terms in eq. \((68)\) are equal and cancel each other exactly. As a result the whole functional \(S_R\) does not give any contribution to \(1/\tau_e\).

The same analysis can be applied to \(S_l\). We find:

\[ S_l[t, t'; r; r_1] = \frac{e^2}{2} \int_{t'}^t dt_1 \int_{t'}^t dt_2 \left\{ 2I(t_1 - t_2, r(t_1) - r(t_2)) - \right\]
\[-I(t_1 + t_2 - t - t', r(t_1) - r(t_2)) - I(t_1 + t_2 - t - t', r(t + t' - t_1) - r(t + t' - t_2))\] (69)

Averaging over diffusive paths \(r(t)\) and taking the limit \(t - t' \rightarrow +\infty\), we observe that \(\tau_\varphi\) is determined by the first term under the integral, the other terms give only the time independent (and therefore irrelevant) constant. Thus we get

\[
\frac{1}{\tau_\varphi} = e^2 \int_{-\infty}^{+\infty} dt \langle I(t, r(t) - r(0)) \rangle_r
\] (70)

To find the average over the diffusive paths, we introduce the Fourier transform of the function \(I(t, r)\) and replace \(\langle e^{-ik(r(t) - r(t'))} \rangle_r\) by \(e^{-Dk^2|t-t'|}\). Then we obtain

\[
\frac{1}{a_3-a} \int_{-\infty}^{+\infty} dt \int d\omega d^4k \frac{\omega}{(2\pi)^d+1} \text{Im} \left( \frac{-4\pi}{k^2\epsilon(\omega, k)} \right) \coth \left( \frac{\omega}{2T} \right) e^{-i\omega t - Dk^2|t|} \] (71)

As it was already discussed, eq. (71) includes the effect of both electron-electron and electron-phonon interactions.

To evaluate \(\tau_\varphi\) we use the expression (36) for the dielectric susceptibility. For typical metallic systems one can usually neglect the first term in the expression for \(\epsilon(\omega, k)\) (34). Then we find

\[
\text{Im} \left( \frac{-1}{\epsilon(\omega, k)} \right) = \frac{\omega}{4\pi\sigma} \left[ 1 + \frac{e^4 k^4 - c^4 \omega^2 / D^2}{(\omega^2 - c^2 k^2)^2 + c^4 \omega^2 / D^2} \right].
\] (72)

Here \(c = \omega_{ph}(D/4\pi\sigma)^{1/2}\) is the speed of sound in our system. Possible corrections to (72) may become important only in special cases of 1d and 2d semiconductor systems, where the capacitance may become important. We will come back to this point later.

The expression (72) can be significantly simplified for \(c/D \lesssim k \lesssim 1/l\). In this limit we obtain

\[
\text{Im} \left( \frac{-1}{\epsilon(\omega, k)} \right) \simeq \text{Im} \left( \frac{-1}{\epsilon(\omega, k)} \right)_{ee} + \text{Im} \left( \frac{-1}{\epsilon(\omega, k)} \right)_{eph},
\] (73)

where

\[
\text{Im} \left( \frac{-1}{\epsilon(\omega, k)} \right)_{ee} = \frac{\omega}{4\pi\sigma}\] (74)

and

\[
\text{Im} \left( \frac{-1}{\epsilon(\omega, k)} \right)_{eph} = \frac{Dck^3}{8\sigma} (\delta(\omega - ck) - \delta(\omega + ck)).
\] (75)

 Phonons with small wave vectors \(k \lesssim c/D\) are strongly damped. For such \(k\) we may put \(\text{Im} \left( \frac{-1}{\epsilon(\omega, k)} \right)_{eph} \simeq 0\).

**C. Results**

With the aid of the above results we can now calculate the decoherence time \(\tau_\varphi\). Let us first take into account only electron-electron contribution to \(\epsilon\) and obtain the result for a quasi-one-dimensional system with \(a \lesssim l\). Substituting (74) into (41) and integrating over time and the wave vector we arrive at the integral over \(\omega\) which diverges at both low and high frequencies. The low frequency divergence is cured in a standard manner by neglecting the effect of environmental fluctuations with frequencies below \(1/\tau_\varphi\). At high frequencies the integral should be cut at the scale of the order of the inverse transport time because at higher \(\omega\) the approximation of electron diffusion becomes incorrect. Then we obtain

\[
\frac{1}{\tau_\varphi(T)} = e^2 \frac{\sqrt{2D}}{\sigma_1} \int_{1/\tau_\varphi}^{1/\tau_\varphi} d\omega \frac{\coth(\omega/2T)}{2\pi} \frac{\sqrt{\omega}}{\sqrt{\omega}}.
\] (76)
Eq. \((70)\) yields
\[
\frac{1}{τ_φ} = \frac{e^2}{πσ_1} \sqrt{\frac{2D}{τ_e}} \left[2T \sqrt{τ_φτ_e} + 1\right]. \tag{77}
\]

At sufficiently high temperature the first term dominates and the standard result \(τ_φ \sim (σ_1/e^2D^{1/2}T)^{2/3}\) is recovered. This is a classical contribution to \(τ_φ\). As \(T\) is lowered the number of classical (low frequency) modes decreases and eventually vanishes in the limit \(T → 0\). At \(T \lesssim T_q^{(1)} \sim 1/\sqrt{τ_φτ_e}\) the expression \((77)\) is dominated by the second term and \(τ_φ\) saturates at the value
\[
τ_φ ≈ πσ_1/e^2 v_F. \tag{78}
\]

The estimate for the crossover temperature \(T_q^{(1)}\) is obvious from \((77,78)\):
\[
T_q^{(1)} ≈ e v_F/2 \sqrt{σ_1 l}. \tag{79}
\]

Making use of eqs. \((68,78)\) it is also easy to find the weak localization correction \(δσ_1\) to the Drude conductance in the limit \(T = 0\). For \(T \lesssim T_q^{(1)}\) we obtain
\[
\frac{δσ_1}{σ_1} = -\frac{e^2}{2πσ_1} \sqrt{Dτ_φ} ≈ -\frac{1}{p_F s^{1/2}}, \tag{80}
\]

i.e. \(δσ_1 \approx −σ_1/\sqrt{N_{ch}}\), where \(N_{ch} \sim p_F^2 s\) is the effective number of conducting channels in a 1d mesoscopic system.

For 2d and 3d systems the same analysis yields
\[
\frac{1}{τ_φ} = \frac{e^2}{4πσ_2τ_e} [1 + 2Tτ_e ln(Tτ_φ)], \quad 2d, \tag{81}
\]
\[
\frac{1}{τ_φ} = \frac{e^2}{3π^2σ_1 \sqrt{2Dτ_φ}} [1 + 6(Tτ_e)^{3/2}], \quad 3d, \tag{81}
\]

According to \((81)\) in 2d and 3d systems the decoherence time becomes independent of \(T\) already at relatively high temperatures. In the 3d case such temperatures can be of the order of the inverse transport time, while for 2d systems we have \(T_q^{(2)} \sim (2τ_e ln(p_F^2 a l))^{-1}\).

Note that in the 2d case we again assumed \(a \lesssim l\). Only provided this condition is satisfied the above results for 1d and 2d systems are valid for the whole temperature range. At sufficiently high temperatures this condition can be softened because the cutoff \(τ_e\) is determined by the low frequency fluctuations of the environment. Then the system can be considered as a quasi-1d (quasi-2d) one if its transversal dimension is smaller that the corresponding phase breaking length \(a \ll L_φ \sim \sqrt{Dτ_φ}\). However at low temperatures high frequency modes become important and the situation changes. E.g. in the limit \(a \gg l\) the diffusion process has two (and sometimes even three) stages: at short times (i.e. at frequencies higher than \(D/a^2\)) diffusion is obviously 3d, whereas for longer times it can be 1d or 2d.

For \(a \gg L_φ\) the system is obviously 3d at all \(T\). In the intermediate case \(l \ll a \ll L_φ\) one should use the corresponding 1d or 2d formulas for \(τ_φ\) at high temperatures \(T \gg T_q^{(d)}\) and the 3d result \((81)\) in the low temperature limit \(T \ll T_q^{(d)}\). The crossover temperature \(T_q^{(d)}\) can be determined either directly from the integral \((77)\) or just by comparison of the corresponding expressions for \(τ_φ\). We get \(T_q^{(d)} \approx T_q^{(d)} (a/πl)^{3−d}\). Thus in the limit \(a \gg l\) the low temperature saturation of \(τ_φ\) takes place already at somewhat higher temperatures than in the case of small \(a\). The saturation value of \(τ_φ\) becomes smaller for large \(a\) since it is defined by the 3d result \((81)\). Such details may be quite important for a quantitative comparison with experimental results (see below).

The above discussion applies to metallic systems. In the case of two-dimensional electron gas in semiconductors with \(l \ll a \ll L_φ\) the same arguments hold, but the saturation value \(τ_φ\) at low \(T\) is given by 2d (not 3d) result. The classical-to-quantum crossover takes place at \(T \sim T_q^{(1)} (a/4πl)^{−1}\).

Now let us analyze the effect of the electron-phonon interaction on the decoherence time \(τ_φ\). Substituting \((73)\) into \((74)\) and applying the same cutoff procedure after a simple integration one obtains
\[
\frac{1}{τ_φ^{eph}} = \frac{e^2c}{4π^2σ_1} \ln \left(\frac{v_F}{3c}\right) + \frac{e^2D}{4πσ_1c} T, \quad 1d, \tag{74}
\]
\[
\frac{1}{\tau_{\varphi}^{\text{ph}}} = \frac{e^2 c}{8\pi^2 \sigma_2 v_F \tau_c} + \frac{e^2}{4\pi^2 \sigma_2} T \ln \left( \min \left\{ \frac{v_F}{3c}, \frac{D T}{c^2} \right\} \right), \quad 2d,
\]
\[
\frac{1}{\tau_{\varphi}^{\text{ph}}} = \frac{e^2 c}{16\pi^3 \sigma_2 v_F^2 \tau_c} + \frac{e^2}{4\pi^3 \sigma_2 v_F \tau_c} T \min \left\{ 1, \frac{T}{c} \right\}, \quad 3d.
\]

Comparing these expressions with the above results for \(\tau_{\varphi}\) we find \(\tau_{\varphi}^{\text{em}} / \tau_{\varphi}^{\text{ph}} \sim c/v_F \ll 1\), i.e. at low \(T\) the decoherence effect due to zero-point fluctuations of lattice ions is always much smaller than that of quantum fluctuations in the electron bath. The temperature dependent term in the expression for the inverse dephasing time due to phonons is of the same order as the corresponding electro-electron term in 1d and 2d, and can be even bigger in 3d at relatively high temperatures \(T \gtrsim c/l\). Thus we can conclude that in 1d and 2d systems the decoherence effect due to phonons is never important, while in 3d systems it can modify the temperature dependence of the decoherence time at sufficiently high \(T\).

The above results in 1d and 2d are valid if the number of conducting channels \(N_{ch}\) in the system is sufficiently big. In typical metallic systems this condition is usually well satisfied. However, in semiconductors one can in principle achieve the situation with \(N_{ch} \lesssim 10\). In this case the first term in the expression for \(\epsilon\) cannot be neglected in general. Moreover for 1d and 2d samples with small \(N_{ch}\) the energy of the electromagnetic field outside the sample may also give a substantial contribution. In order to account for this effect we introduce the effective capacitance of the system \(C\). Then the influence functional for the field \(V\) has the form (see above)

\[
iS_C = i \int \frac{d\omega d^d k}{(2\pi)^{d+1}} V^-(\omega, -k) \left[ \frac{C(\omega, k)}{2} + \frac{k^2 (\epsilon(\omega, k) - 1)}{4\pi} \right] V^+(\omega, k) - \frac{1}{2} \int \frac{d\omega d^d k}{(2\pi)^{d}} V^-(\omega, -k) \frac{k^2 \sigma_{v}(\omega, k)}{4\pi} \coth \left( \frac{\omega}{2T} \right) V^-(-\omega, k);
\]

where \(C(\omega, k) \simeq (1 + \epsilon_s(\omega))/4 \ln(1/ka)\) for a 1d wire and \(C(\omega, k) = (1 + \epsilon_s(\omega))(k/8\pi)\) for a 2d film. Here \(\epsilon_s(\omega)\) is the dielectric susceptibility of the substrate. The Fourier transform of the function \(I(t, r)\) takes the form

\[
I_{\omega, k} = \langle |V^+_{k, \omega} |^2 \rangle = \frac{\omega \coth \left( \frac{\omega}{2T} \right)}{\sigma_d k^2 + \frac{\sigma_{v}(\omega, k) D}{\sigma_d}}.
\]

Substituting this expression into (71) we get for 1d wire

\[
\frac{1}{\tau_{\varphi}(T)} = \frac{e^2 \sqrt{2D}}{\sigma_1 \tau_{\varphi}} \int_{1/\tau_{\varphi}}^{1/\tau_c} \frac{d\omega}{2\pi} \coth(\omega/2T) \sqrt{\omega (1 + 2f(\omega))} \left( 1 - \sqrt{\frac{f(\omega)}{1 + f(\omega)}} \right),
\]

where \(f(\omega) = \frac{(1 + \epsilon_s(\omega)) D}{4 \sigma_1 \ln(1/k_{0a})}\) and the value \(k_{0a}\) is roughly of order one. We estimate

\[
f(\omega) \sim \frac{1 + \epsilon_s(\omega)}{\epsilon^2 N_{ch}^2 a} \sim \frac{1 + \epsilon_s(\omega)}{N_{ch} p_{F}r_B},
\]

where \(r_B = 1/me^2 \simeq 0.5\ \text{Å}\) is the Bohr radius. For metallic wires \(p_{F}r_B \sim 1\), \(N_{ch} \gg 1\) and the function \(f(\omega)\) is usually small unless \(\epsilon_s(\omega) \gg 1\) at frequencies of the order of \(1/\tau_{\varphi}\). However for semiconductors \(f(\omega)\) may be large and \(\tau_{\varphi}\) may become significantly longer than one could expect from eq. (71).

The same analysis can be carried out for 2d films. In this case the effect of capacitance is described by the function

\[
f_2(\omega) = \frac{(1 + \epsilon_s(\omega)) \sqrt{D\omega}}{8\pi \sigma_2} \sim \frac{(1 + \epsilon_s(\omega)) \sqrt{\omega \tau_{\varphi} r_B}}{p_{F}l\alpha}.
\]

Again one can conclude that this effect is typically negligible for metallic films. For semiconductors with small \(N_{ch}\) the above effect might cause an increase of \(\tau_{\varphi}\).
V. QUANTUM KINETIC APPROACH AND LANGEVIN EQUATION

Let us now demonstrate how the usual quantum kinetic description can be derived from our analysis. We start from the equation for the density matrix \( \rho_V \) \(^{(23)}\). Rewriting this equation in the “interaction representation”, i.e. substituting \( \rho_V \rightarrow e^{-iH_0t}\rho_V e^{iH_0t} \) we find

\[
i\frac{\partial \rho_V}{\partial t} = -e\tilde{V}^+(t)\rho_V + \rho_V e\tilde{V}^+(t) - \frac{e}{2}((1 - \rho_V)\tilde{V}^-(t)\rho_V + \rho_V \tilde{V}^-(t)(1 - \rho_V)),
\]

where \( \tilde{V}(t) = e^{iH_0t}V(t)e^{-iH_0t} \). Let us integrate this equation over time, then substitute the resulting expression for \( \rho_V \) into the right hand side of eq. \((87)\) and average over \( V^\pm \). If the Coulomb interaction is sufficiently weak one can proceed perturbatively in \( V \) and neglect the dependence of the density matrix \( \rho_V \) on this field in the right hand side of the resulting equation. Then the result of averaging can be expressed in terms of the correlation functions \( \langle VV \rangle \). More precisely, two such functions turn out to be important:

\[
\langle V^+(t_1, r_1)V^+(t_2, r_2) \rangle = I(t_1 - t_2, r_1 - r_2),
\]

\[
\langle V^+(t_1, r_1)V^-(t_2, r_2) \rangle = iR(t_1 - t_2, r_1 - r_2),
\]

The function \( \langle V^+(t_1, r_1)V^-(t_2, r_2) \rangle \) differs from zero only for \( t_1 > t_2 \). The correlation function \( \langle V^+V^- \rangle \) is zero for all times. Taking this into account we obtain

\[
\frac{\partial \rho}{\partial t} = e^2 \int_{-\infty}^{t} dt' \left\{ -\tilde{V}^+(t)\tilde{V}^+(t')\rho(t') + \tilde{V}^+(t)\rho(t')\tilde{V}^+(t') + \tilde{V}^+(t')\rho(t')\tilde{V}^+(t) - \rho(t')\tilde{V}^+(t')\tilde{V}^+(t) - \frac{1}{2}\tilde{V}^+(t)(1 - \rho(t'))\tilde{V}^-(t')\rho(t') - \frac{1}{2}\tilde{V}^+(t')\rho(t')\tilde{V}^-(t')(1 - \rho(t')) + \frac{1}{2}(1 - \rho(t'))\tilde{V}^-(t')\rho(t')\tilde{V}^+(t) + \frac{1}{2}\rho(t')\tilde{V}^-(t')(1 - \rho(t'))\tilde{V}^+(t) \right\}_{V^+, V^-},
\]

where \( \rho(t) = \rho_{V=0}(t) \).

For simplicity let us consider a clean metal. Making use of the momentum conservation one can significantly simplify the equation \((88)\). In this case the density matrix is given by \( \rho(r_1 - r_2) = \int \frac{d^3p}{(2\pi)^3} n_p e^{ip(r_1 - r_2)} \). The operator \( e^{iH_0t} \) reduces to \( e^{-i\omega t} \). Performing the averaging with the aid of eqs. \((88), (50), (77)\) we find

\[
\frac{dn_p}{dt} = e^2 \int d\omega d^3k \text{Im} \left( \frac{-1}{k^2\epsilon(\omega, k)} \right) \delta(\omega + \xi_{p-k} - \xi_p) \left[ \coth \left( \frac{\omega}{2T} \right) (n_{p-k} - n_p) - n_p(1 - n_{p-k}) - n_{p-k}(1 - n_p) \right] .
\]

The right hand side of this equation represents the standard collision integral which vanishes in equilibrium, i.e. for \( n_p = 1/\text{exp}(\xi_p/T) + 1 \).

The equation \((88)\) can be also rewritten in the following form:

\[
\frac{dn_p}{dt} = 2e^2 \int_{0}^{+\infty} d\omega \int d^3k \text{Im} \left( \frac{-1}{k^2\epsilon(\omega, k)} \right) \left\{ \delta(\omega + \xi_{p-k} - \xi_p) N_\omega n_{p-k}(1 - n_p) - \delta(\omega + \xi_{p-k} - \xi_p)(1 + N_\omega)n_p(1 - n_{p-k}) + \delta(\omega + \xi_p - \xi_{p-k})(1 + N_\omega)n_{p-k}(1 - n_p) - \delta(\omega + \xi_p - \xi_{p-k})N_\omega n_p(1 - n_{p-k}) \right\},
\]

where \( N_\omega = 1/\text{exp}(\omega/T) - 1 \) is Bose function. This equation describes the standard photon absorption and emission processes and thus establishes a transparent relation between our approach and one describing the kinetics of an electron interacting with the quantized electromagnetic field. In our case the field \( V \) is due to fluctuations of conducting electrons (or lattice ions – see below). It is quite clear however that the physical nature of this field is not important for the electron dynamics, at least as long as this Bose field remains in equilibrium.

It is important to emphasize that the effect of electron-phonon interaction is also taken into account in eq. \((91)\). The phonon spectrum is determined by the equation \( \epsilon(\omega, k) = 0 \), i.e. the function \( \frac{1}{\epsilon(\omega, k)} \) has a pole at \( \omega = \omega_{ph}(k) - i\eta \). Therefore one can write
where other contributions to $\epsilon^{-1}$ are denoted by dots. The value $A(k)$ determines the strength of electron-phonon interaction. Within the simple model (75) one has $A(k) = Dck^3/8\sigma = ck^3/16e^2N_0$.

Substituting the expression (72) into (71) and integrating over $\omega$ we reproduce the standard electron-phonon collision integral:

$$I_{eph} = \frac{2e^2}{\pi^2} \int d^3k \frac{A(k)}{k^2} \left\{ \delta(\omega_{ph}(k) + \xi_{p-k} - \xi_p)N_{\omega_{ph}(k)}n_{p-k}(1-n_p) - \\
- \delta(\omega_{ph}(k) + \xi_{p-k} - \xi_p)(1+N_{\omega_{ph}(k)})n_p(1-n_{p-k}) + \delta(\omega_{ph}(k) + \xi_p - \xi_{p-k})(1+N_{\omega_{ph}(k)})n_{p-k}(1-n_p) - \\
- \delta(\omega_{ph}(k) + \xi_p - \xi_{p-k})N_{\omega_{ph}(k)}n_p(1-n_{p-k}) \right\},$$

(92)

This result demonstrates that the function $\epsilon(\omega, k)$ correctly describes both electron-phonon and electron-electron interactions. It is not surprising, because this function just accounts for the collective effect of the environment. Electrons propagating in a metal “feel” only the fluctuating electric field produced by the environment, both by electrons and lattice ions. Therefore it is quite natural that both contributions can be successfully treated within the same approach.

The equations (94, 93) are applicable if the distribution functions $n_p$ and $N_\omega$ are close to the equilibrium Fermi and Bose functions. It is not difficult to generalize this approach for stronger deviations from equilibrium. Actually the electron-phonon collision integral (92) remains the same in this case, only the distribution function $N_k$ can deviate far from the Bose function. In order to generalize the electron-electron collision integral we make use of the following nonequilibrium formulas:

$$\text{Im} \left( \frac{-1}{\epsilon(\omega, k)} \right) = -\frac{e^2}{\pi k^2|\epsilon(\omega, k)|^2} \int d^3p \delta(\omega - \xi_{p+k} + \xi_p)(n_{p+k} - n_p),$$

$$\langle V^+V^+ \rangle_{\omega,k} = \frac{4e^2}{k^2|\epsilon(\omega, k)|^2} \int d^3p \left[ n_{p+k}(1-n_p) + n_p(1-n_{p+k}) \right] \delta(\omega - \xi_{p+k} + \xi_p),$$

(94)

which can be easily derived from (85) and (86) respectively. Substituting these expressions into the equation (80) we arrive at the electron-electron collision integral for the degenerate plasma:

$$I_{ee} = \int \frac{d^3k}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \left( \frac{4\pi e^2}{k^2} \right)^2 \frac{8\pi}{|\epsilon(\omega_{p+k} - \xi_{p-k} - \xi_p)|^2} \delta(\xi_{p+k} + \xi_{p-k} - \xi_p) \times \\
\times \left[ n_{p+k}n_{p-k}(1-n_{p+k})(1-n_{p-k})n_p(1-n_{p-k})(1-n_{p-k}) \right]$$

(95)

Thus the kinetic equation can be written in a standard form

$$\frac{dn_p}{dt} = I_{eph} + I_{ee},$$

(96)

where the collision integrals $I_{eph}$ and $I_{ee}$ are defined respectively by eqs. (92) and (95).

In order to estimate the characteristic electron scattering time we have to substitute the function $n_p + \delta n_p$ instead of $n_p$ in the collision integral (90). The inverse inelastic scattering time $1/\tau_i$ is then defined as a coefficient in front of the term $\delta n_p$ describing deviations from equilibrium. Making use of an obvious identity $1 - 2n_{p-k} = \tanh((\xi_p - \omega)/2T)$ we get

$$\frac{1}{\tau_i(p)} = \frac{e^2}{\pi} \int d\omega d^3k \text{Im} \left( \frac{-1}{k^2 \epsilon(\omega, k)} \right) \delta(\omega + \xi_{p-k} - \xi_p) \left( \coth \frac{\omega}{2T} + \tanh \frac{\xi_p - \omega}{2T} \right).$$

(97)

It is clear from this equation that the time $\tau_i$ becomes infinite at zero temperature and at the Fermi energy due to the Pauli principle. The same is true for the inelastic scattering time due to electron-phonon interaction. Thus within the same general formalism we have demonstrated a clear distinction between the decoherence time $\tau_\phi$ and the inelastic scattering time $\tau_i$. The latter becomes infinite at $T = 0$ because the excitation due to noise ($\coth(\omega/2T)$) and the energy losses due to radiation ($\tanh((\xi - \omega)/2T)$, see the discussion below) exactly compensate each other. This is in
the agreement with the Pauli principle: electrons cannot change their energy because at \( T = 0 \) all states below the Fermi level are occupied. At the same time the value \( \tau_\sigma \) remains finite even at \( T = 0 \) because this time is sensitive to random noise of the environment only (be it classical or quantum). The effect of such noise leads to decoherence. We will return to this discussion below.

The above kinetic equations were derived for a simple case of a clean system and do not account for the effect of elastic scattering. In the case of a disordered metal the electron momentum is not conserved and the whole derivation becomes much more complicated. One can demonstrate (see e.g.\([2]\)) that in the diffusive limit the result is roughly equivalent to a substitution

\[
\delta(\omega + \xi_{p-k} - \xi_p) \rightarrow \text{Re} \left[ \frac{1}{i\omega + Dk^2} \right]
\]

in the expression \((57)\). An extended analysis of the inelastic scattering time in various limits is given in Ref. \([28]\).

One can also formulate an alternative approach and derive the quasiclassical Langevin equations describing electron dynamics in a disordered metal. In doing so, we follow the same procedure as one described in Refs. \([12,22]\).

Consider only close electron paths for which the values \( r^- = r_1 - r_2 \) and \( p^- = p_1 - p_2 \) are small. Then we can expand the effective action in the exponent of eq. \((53)\) in powers of \( r^- \) and \( p^- \) keeping only the quadratic terms. The action becomes Gaussian in terms of these variables and the integral \((53)\) is dominated by the saddle point trajectories: \( \delta S / \delta p^- = 0 \) and \( \delta S / \delta r^-= 0 \). The first equation coincides with one without dissipation: \( \dot{r} = p / m \). With the aid of this equation the momentum can be easily excluded and we get:

\[
m\dot{r} + \nabla U(r) + e^2 \left[ 1 - 2n(r, m\dot{r}) \right] \int_{-\infty}^{t} dt' \nabla_r R(t-t', r(t) - r(t')) = -eE(t, r).
\]

(98)

Here \( E(t, r) \) is a fluctuating electric field. Equilibrium fluctuations of this field are described by the correlator

\[
\langle E_i(t_1, r_1) E_j(t_2, r_2) \rangle = 4\pi\delta_{ij} \int \frac{d\omega d^3k}{(2\pi)^4} \text{Im} \left( \frac{-\coth \frac{\omega}{T}}{\epsilon(\omega, k)} \right) e^{-i\omega(t_1-t_2)+ik(r_1-r_2)}
\]

(99)

If needed, the generalization of \((98,99)\) to a strongly nonequilibrium situation can be also provided. Also more general expressions for the kernel \( R(t, r_1, r_2) \) and for the correlation function \((93)\) for the case \( |r_1 - r_2| < l \) can be easily derived. Combining these expressions with \((98)\) one can obtain a quasiclassical description of electron dynamics also at scales \( \ll l \). However such details are not important for us here, eq. \((98)\) is presented merely to illustrate important physical effects.

The equation \((98)\) obviously satisfies the requirement of causality and captures all the essential features of electron dynamics in a metal. E.g. it demonstrates that electrons in a metal cannot infinitely decrease their energy: effective damping due to the presence of the environment (described by the last term in the left hand side of \((98)\)) is zero at the Fermi energy \( (n = 1/2) \) and becomes negative below this energy. Thus electrons with the initial energy above \( \mu \) will loose it before they reach the Fermi level. On the contrary, holes with the initial energy below \( \mu \) will be pushed up to the Fermi surface. This simple example demonstrates again that our analysis accounts explicitly for the Pauli principle. The corresponding information is contained in the influence functional which depends on the occupation numbers.

The damping term in eq. \((98)\) depends on the function \( R(t, r) \) which is determined by the correlation function \( \langle V^+ V^- \rangle \) (see \((98)\)). The physical origin of this damping term is quite transparent: the electron (or the hole) propagating in a metal produces the screened electric potential due to the presence of other electrons and “feels” this potential itself. In this sense eq. \((98)\) is similar to the equation of motion of a high energy particle (e.g. muon) in a metal. The important difference between these two cases, however, lies in the factor \( 1 - 2n \) which is present in our case due to the Pauli principle. Formally this factor enters due to fluctuations of the field \( V^- \) which is “sensitive” to the Pauli principle. The fluctuating electric field \( E \) in the right hand side of eq. \((98)\) is, on the contrary, not affected by the Pauli principle because its correlation function depends only on the field \( V^- \).

With the aid of eq. \((98)\) it is also easy to understand why the real part of the influence functional \( S_R \) \((94)\) does not contribute to the decoherence time. According to \((98)\) the phase difference acquired by the electron propagating along some classical path can be split into two parts: the regular contribution due to damping \( (S_R) \) which depends only on the electron trajectory, and irregular part due to noise \( (S_I) \). Considering now the contribution from a pair of time reversed paths, we observe that the regular contributions are the same and cancel each other because they enter with a different sign. Only irregular contributions due to noise survive and determine \( \tau_\sigma \). For each path the regular contribution may have a different value depending on the path and energy and even vanish (for energies at the Fermi level). However by no means this affects the noise terms and thus \( \tau_\sigma \) which always remains finite.
VI. DISCUSSION

With the aid of a general formalism of Green-Keldysh functions we have analyzed a fundamental effect of quantum decoherence of the electron wave function in a disordered metal due to interaction. Our treatment was carried out with no more assumptions than the usual ones in the weak localization theory: the elastic mean free path was considered large as compared to the Fermi wavelength \( p_F l \gg 1 \) and interactions were assumed to be sufficiently weak. Actually the main effect does not really depend even on these general assumptions, it can be observed already from a formally exact equation for the density matrix (24, 25).

We have demonstrated that the effect of interaction of the electron with other electrons and lattice ions in a disordered metal is equivalent to the effect of a dissipative environment. This effect is similar (although not exactly equivalent) to that of a dissipative Caldeira-Leggett bath. Fluctuations in the dissipative environment play the key role in the effect of quantum decoherence.

Although the environment is dissipative it is not inelastic scattering processes with energy transfer between real quantum states that cause quantum decoherence at low \( T \). Rather this is the effect of quantum noise in a dissipative environment. Due to interaction with quantum fluctuations of this environment the electron not only “goes” through virtual states with different energies but after a finite time \( \tau_\phi \) also loose information about its initial phase. At the same time no real processes with energy transfer exist at \( T = 0 \). It is also interesting to note that – as can be seen e.g. from eq. (84) – voltage fluctuations and therefore the decoherence rate \( 1/\tau_\phi \) vanish in both limits \( \sigma \to 0 \) and \( \sigma \to \infty \).

The presence of Feynman paths which strongly deviate from the classical paths depicted in Fig. 1a is also crucially important for the whole effect. In the case of disordered metals these are time reversed paths (Fig. 1b). In the case of Caldeira-Leggett models other paths (e.g. instantons, see 1817) play a similar role. If the important paths go sufficiently far from each other, electrons gain an additional phase due to the environmental noise. This phase always vanishes for the classical paths of Fig. 1a.

We clarified the role of the Pauli principle in the effect of quantum decoherence. Although the Pauli principle plays a crucial role in the inelastic electron-electron collisions it does not affect quantum decoherence. The physical reason for that is transparent: fluctuations of the collective electric field in a metal are equivalent to local fluctuations of the Fermi energy or the electron density (see Section 2). Such fluctuations are essentially the same for many electron states, and the Pauli principle does not play any role.

On a very simple level the decoherence effect at low \( T \) can be understood as follows. During interaction with other electrons the electron phase changes. These “other” electrons (or some of them) “go out of the game” (the system is open!) and after some time the information about the initial electron phase is lost. The same mechanism works for a particle interacting with the Caldeira-Leggett bath.

We would like to emphasize that no special assumptions about the “open system” were made in our analysis. We just used the standard formalism of Green functions applicable to any grand canonical ensemble in condensed matter physics. Already by fixing the chemical potential \( \mu \) and allowing the total number of particles to vary one defines the system as open. Thus our results are general and in principle can be applied to any disordered system perhaps except for ultrasmall objects with fixed numbers of particles. The latter appear to be irrelevant for weak localization measurements.

Our analysis provides no support for the point of view according to which the effect of high frequency (quantum) fluctuations of the electronic environment on \( \tau_\phi \) should be detemined by inelastic processes with high energy transfers and therefore should be obtained from the kinetic analysis. The kinetic treatment deals with paths of Fig. 1a and allows to calculate only \( \tau_i \) but not \( \tau_\phi \). In contrast, the latter is determined by time reversed paths of Fig. 1c for all relevant frequencies. In this sense there is no qualitative difference between the effect of classical and quantum fluctuations of the environment on \( \tau_\phi \). Both should be treated on equal footing.

There is also no evidence that the Pauli principle “forbids” decoherence at low \( T \). Therefore the suggestion to account for the Pauli principle by introducing an additional term proportional to \( \tanh(\omega/2T) \) into the expression for \( \tau_\phi \) does not appear to be justified. We have demonstrated that the combination “coth – tanh” enters the expression for \( 1/\tau_i \) but not for the inverse decoherence time \( 1/\tau_\phi \). The latter contains only “coth” and does not vanish at \( T \to 0 \).

We believe that the same should hold for a sometimes conjectured cancellation of Keldysh diagrams at \( T = 0 \). To the best of our knowledge a rigorous formulation of the diagramatic representation for \( \tau_\phi \) is still lacking in the literature. An attempt to provide this formulation was made by Fukuyama and Abrahams 34. But it is well understood by now that only diagrams relevant for the inverse inelastic time \( 1/\tau_i \) were taken into account in 34. Cancellation of these diagrams at \( T = 0 \) is quite natural but it does not yet tell anything about \( \tau_\phi \). Since our analysis should in principle include all diagrams we see no reason to expect any cancellation of diagrams for \( \tau_\phi \) even at \( T = 0 \).

The low temperature saturation of \( \tau_\phi \) can cause dramatic consequences for the existing picture of strong
localization in lower dimensions. There seems to be no reason to challenge this picture as long as one neglects Coulomb interaction between electrons. But – as was demonstrated above – it is exactly this interaction that leads to quantum decoherence at \( T = 0 \). If one compares the effective decoherence length \( L_\varphi \) with the localization length \( L_{\text{loc}} \) one immediately sees that for the parameters of a typical metal the former is always smaller than the latter. E.g., in 1d our analysis yields \( L_\varphi \sim \sqrt{D/\tau_\varphi} \sim l\sqrt{N_{\text{ch}}} \). This expression is parametrically smaller than the localization length \( L_{\text{loc}} \sim lN_{\text{ch}} \) for large number of conducting channels. Analogously in 2d at low \( T \) the weak localization correction to conductivity saturates at the level \( \delta\sigma_2 \sim -(e^2/2\pi^2)\ln(p_F a_l) \) and the crossover from weak to strong localization never occurs. In other words, all that implies that (at least for \( p_F l \gg 1 \)) due to electron-electron interaction the electron will loose coherence already before it can get localized, i.e. strong localization does not take place at all and the 1d and 2d metals do not become insulators even at \( T = 0 \).

We can also add that it would be interesting to investigate the effect of electron-electron interaction on more sophisticated weak localization corrections, like ones due to quantum interference of diffusons. Here the point is that (unlike Cooperons) diffusons do not decay at \( T = 0 \) even in the presence of interaction. Nevertheless, we cannot exclude that even at \( T = 0 \) quantum interference of diffusons can be also destroyed due to the same mechanism of quantum noise in a dissipative electronic environment as it was discussed above for Cooperons. This problem will be studied elsewhere.

Irrespective to such details it is quite clear that an adequate description of electron-electron interaction can seriously change the existing picture of strong localization in low dimensional disordered metals. This perspective is very interesting but not very surprising. In fact, the important role of electron-electron interaction was already pointed out long ago by Finkelstein.

One more comment concerns the relation between the low temperature saturation of the decoherence time \( \tau_\varphi \) and the applicability of the Fermi liquid theory. It is sometimes believed that the applicability condition for the latter is \( T\tau_\varphi \gg 1 \). From a practical point of view this condition is not very restrictive: e.g. for typical experimental values of Ref.\( ^{41} \) the value \( \tau_\varphi \) was found to be in the range \( 10^{-1} \div 10 \) ns and the above condition can be violated only at temperatures below \( 1 \div 100 \) mK. However theoretically it is very important to understand if a finite decoherence time at \( T \to 0 \) would imply the breakdown of the Fermi liquid theory at low \( T \). We believe that there is no direct relation between \( \tau_\varphi \) and the lifetime of quasiparticles. The latter should be rather defined by some inelastic time (e.g. by \( \tau_i \)) which is always much longer than \( \tau_\varphi \) at low \( T \). Thus it appears that even for \( T\tau_\varphi \ll 1 \) there is still no evidence for the breakdown of the Fermi liquid theory in metals. Previously an analogous conclusion was reached in Ref.\( ^{39} \).

Finally, we briefly discuss the agreement between our results and the available experimental data. The comparison between theoretical and experimental values for the decoherence length \( L_\varphi = \sqrt{D/\tau_\varphi} \) at zero temperature is given in the Table I. To calculate \( L_\varphi \) we first estimate the decoherence time with the aid of (77). At \( T = 0 \) the time \( \tau_\varphi \) can be conveniently expressed in terms of measurable quantities:

\[
\tau_\varphi = \sqrt{\frac{R_q L}{\text{Ref.}^4}},
\]

where \( R_q = \pi/2e^2 = 6453 \) \( \Omega \) is a quantum resistance, \( L \) is the wire length and \( R \) is the total resistance. The Fermi velocity for gold wires was taken to be\( ^{21} 1.39 \times 10^6 \) m/s. The diffusion coefficient \( D \) was estimated with the aid of the Drude formula \( \sigma = 2e^2N_0D \). The density of states for gold is chosen to be\( ^{44} N_0 = 6 \times 10^{12} \) s/m\(^3\). Note that the numerical values for \( D \) are not identical to those given in the Table I.

**TABLE I.** Theoretical values of the decoherence length \( L_\varphi = \sqrt{D/\tau_\varphi} \) in comparison with the experimental results.

| Sample | \( w, \text{nm} \) | \( t, \text{nm} \) | \( L, \text{m} \) | \( \frac{R}{L}, \frac{\Omega}{\text{m} \mu \text{m}} \) | \( D, \text{m}^2/\text{s} \) | \( L_{\text{exp}}, \mu \text{m} \) | \( L_{\text{theor}}, \mu \text{m} \) |
|--------|----------------|----------------|--------------|--------------------------|----------------|----------------|----------------|
| Au-1   | 60             | 25             | 57.9         | 29.14                    | 7.8            | 5.54           | 1.8            |
| Au-2   | 110            | 60             | 207          | 1.46                     | 35.5           | 16             | 16.5           |
| Au-3   | 100            | 35             | 155          | 9.31                     | 10.5           | 5.2            | 3.6            |
| Au-4   | 60             | 25             | 57.9         | 31.29                    | 7.3            | 3.6            | 1.6            |
| Au-5   | 190            | 40             | 18.9         | 191.7                    | 0.24           | 0.35           | 0.12           |
| Au-6   | 180            | 40             | 155          | 2.91                     | 16.3           | 8              | 8.1            |

The width, the thickness and the length of the wire are denoted respectively by \( w, t \) and \( L; R/L \) is the resistance per unit length and \( L_{\text{exp}}, L_{\text{theor}} \) are experimental and theoretical values of the decoherence length. The agreement between both looks very reasonable for all samples, especially if one takes into account an uncertainty in a numerical prefactor in our formulas due to the cutoff procedure and possible effects of the sample geometry. It is also important to emphasize that our comparison involves no fitting parameters.

The temperature dependence of the inverse decoherence time \( 1/\tau_\varphi \) is plotted in Fig.\( ^2 \). The agreement between theoretical and experimental results is again reasonable. It appears, that within the temperature interval of Fig.\( ^2 \)
temperature dependence of the experimental data is not far from a linear one. Note, that the temperature dependent part of eq. (77) is sensitive to details of the low frequency cutoff. The plotted theoretical dependence corresponds to the lower cutoff frequency equal to $1/\tau_\varphi$ at $T = 0$. With a slight adjustment of this cutoff one can in principle reach a perfect agreement with the experimental data. However, we believe that this adjustment is not needed: within the accuracy of our calculation the agreement is already very good.

![Graph](image)

FIG. 2. The dependence of the decoherence rate $1/\tau_\varphi$ on temperature for one of the thin gold wires. The wire has the following parameters: $w = 35$ nm, $t = 20$ nm, $L = 231$ $\mu$m, $R = 4411$ $\Omega$, $D = 2.56 \times 10^{-2}$ $m^2$/s. Experimental points are shown by squares, the solid curve represents the theoretical dependence (77). The experimental data were kindly provided by P. Mohanty.

Although we do not present the comparison of our theoretical results with the experimental data from Refs. 21, 22 we note that all these data are sufficiently well described by our formulas (77,81). The agreement with the experimental results by Mueller et al. 24 for 2d gold films turns out to be worse: the value $\tau_\varphi$ estimated from eq. (81) is considerably shorter than the experimentally measured value. The reason for this discrepancy is not clear at the moment. We can speculate that it might be due to the high purity of the films which can make the diffusive approximation insufficient. If this is the case the resistance of the films is rather due to electron scattering at the boundaries and our theoretical analysis should be substantially modified. This problem will not be discussed here.

In all the above (and many other) experiments no crossover from weak to strong localization was not detected even at the lowest temperatures $20 \div 40$ mK. Very recently in Ref. 42 a sharp increase of the resistance by several orders of magnitude was observed in quasi-1d GaAs structures with small number of conducting channels $N_{\text{ch}} \sim 10$. This increase of the resistance was interpreted as a crossover from weak to strong localization. Although this could be one possible scenario we believe the interpretation of the above effect is still an open problem. Indeed, if one estimates the crossover temperature from the standard criterion $L_\varphi \sim L_{\text{loc}}$ and makes use of (1) for the parameter $L_{\text{loc}}$ one arrives at the crossover temperature $T \sim 1 \div 10$ mK. However the increase of the resistance was observed at a much higher temperature $T \gtrsim 1$ K. This discrepancy might indicate the important role of Coulomb effects in the system. Analogously Coulomb effects were found to play the key role in many experiments with disordered 2d systems, see e.g. for the corresponding discussion. Further experiments are definitely needed for better understanding of the electron-electron interaction effects in disordered low dimensional systems.

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APPENDIX A: DENSITY MATRIX

With the aid of eq. (1) the time integral in the last term of the Dyson equation (18) can be transformed as follows:

$$\int_{0}^{t} dt' \hat{G}_0(t_1,t_2) e^{i\hat{V}(t')} \hat{G}(t',t_2) = \hat{G}_0(t_1,t_2) - \int_{0}^{t} dt' \hat{G}_0(t_1,t') \left( i \frac{\partial}{\partial t'} - \tilde{H}_0(t') \right) \hat{G}(t',t_2)$$

$$= \hat{G}_0(t_1,t_2) - i \hat{G}_0(t_1,t) \hat{G}(t,t_2) + i \hat{G}_0(t_1,0) \hat{G}(0,t_2) +$$

$$\int_{0}^{t} dt' \left( i \frac{\partial}{\partial t'} + \tilde{H}_0(t') \right) \hat{G}_0(t_1,t') \hat{G}(t',t_2)$$

$$= \hat{G}_0(t_1,t_2) - \hat{G}(t_1,t_2) + i \hat{G}(t_1,t) \hat{G}(t,t_2) + i \hat{G}_0(t_1,0) \hat{G}(0,t_2)$$

(A1)

Here we performed the integration by parts over the time $t'$ and made use of the equation

$$\left( i \frac{\partial}{\partial t} + \tilde{H}_0(x) \right) \hat{G}_0(t_1,t_2) = -\delta(t_1 - t)\delta(x_1 - x),$$

which defines the field-free Green-Keldysh function $\hat{G}_0$. Substituting the result (A1) into eq. (18) we arrive at (22.

Let us substitute the representation (14) into (22). Then we find

$$\hat{U}_0(t_1,t)(\hat{\tilde{b}} + \tilde{f}_0(t))\hat{U}_V(t_2,t)(\hat{\tilde{a}} + \tilde{f}_V(t_2)) - \hat{U}_0(t_1,0)(\hat{\tilde{a}} + \tilde{f}_0(0))\hat{U}_V(0,t_2)(\hat{\tilde{b}} + \tilde{f}_V(t_2)) = 0$$

(A2)

It is now convenient to represent the operator $\tilde{f}_V$ in the form

$$\tilde{f}_V(t_2) = \hat{U}(t_2,0)\tilde{f}(t_2)\hat{U}_V(0,t_2).$$

(A3)

Then for $\tilde{g}$ we get

$$[1 + (\hat{\tilde{b}} - \tilde{f}_0(0))(\hat{U}_0(0,t)\hat{U}_V(t_2,0) - 1)] \tilde{g}(t_2) = \tilde{f}_0(0) - (\hat{\tilde{b}} - \tilde{f}_0(0))(\hat{U}_0(0,t)\hat{U}_V(t,0) - 1)\tilde{a}.$$ (A4)

Let us put $t_2 = t$ and introduce the scattering matrix $S = U_0(0,t)U_V(t,0) = s_1\hat{a} + s_2\hat{b}$. This matrix is diagonal because both $U_0$ and $U_V$ are the diagonal matrices. The matrix elements $s_1$ and $s_2$ are defined as $s_{1,2} = u_0(0,t)u_{1,2}(t,0)$. Making use of the above notations and rewriting the equation (A4) in components we find

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \rho_0 & -\rho_0 \\ -\rho_0 & 1 - \rho_0 \end{pmatrix} \begin{pmatrix} s_1 - 1 & 0 \\ 0 & s_2 - 1 \end{pmatrix} \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} =$$

$$= \begin{pmatrix} -\rho_0 & \rho_0 \\ 1 - \rho_0 & \rho_0 \end{pmatrix} - \begin{pmatrix} \rho_0 & -\rho_0 \\ -\rho_0 & 1 - \rho_0 \end{pmatrix} \begin{pmatrix} s_1 - 1 & 0 \\ 0 & s_2 - 1 \end{pmatrix}.$$

(A5)

Multiplying matrices and keeping only the part of the resulting matrix equation which depends on $g_{12}$ and $g_{22}$ we obtain

$$\begin{pmatrix} 1 + \rho_0(s_1 - 1) & -\rho_0(s_2 - 1) \\ -s_1 + 1 + \rho_0(s_1 - 1) & s_2 - \rho_0(s_2 - 1) \end{pmatrix} \begin{pmatrix} g_{11} \\ g_{22} \end{pmatrix} = \begin{pmatrix} \rho_0 \\ \rho_0 \end{pmatrix}.$$ (A6)

Subtracting the second equation from the first one we get $s_1g_{12} - s_2g_{22} = 0$, or $g_{22} = s_2^{-1}s_1g_{12}$. Substituting this result into the first equation we find

$$[1 + \rho_0(s_2^{-1}s_1 - 1)]g_{12} = \rho_0.$$ (A7)

Note that the S-matrices enter the above equation only in the combination $s_2^{-1}s_1$ which does not contain the evolution operator $u_0$, i.e. we have $s_2^{-1}s_1 = u_2(0,t)u_1(t,0)$ and

$$[1 - \rho_0(1 - u_0(0,t)u_1(t,0))]g_{12}(t) = \rho_0(0).$$ (A8)

Rewriting (A3) for the 1,2-component of the matrix $\tilde{f}_V(t)$

$$f_{12}(t) = u_1(t,0)g_{12}(t)u_2(0,t),$$

and making use of the identity $\rho_V(t) = f_{12}(t)$ we arrive at the result (B7).
APPENDIX B: EFFECTIVE ACTION AND FDT

For $V_{1,2}$ the electron Green functions $G_{12}$ and $G_{21}$ can be expressed in the form

$$G_{12} = i u_0(t_1, 0) \rho_0 u_0(0, t_2), \quad G_{21} = -i u_0(t_1, 0)(1 - \rho_0) u_0(0, t_2).$$

(B1)

In thermodynamic equilibrium we have $u_0(t, 0)\rho_0 u_0(0, t) = \rho_0$ for any time $t$.

Let us introduce the basis of the eigenfunctions for the single electron Hamiltonian, $H_0 \psi_k = \xi_k \psi_k$. Without loss of generality we can choose these eigenfunctions to be real. The initial density matrix $\rho_0$ is assumed to be diagonal in the basis $\psi_k$, namely $\rho_0 = \sum_k n_k |\psi_k\rangle \langle \psi_k|$. This assumption is justified only for weakly interacting particles. Then the functions $G_{12}, G_{21}$ (31), $G_R$ and $G_A$ (29) can be written in the form

$$G_{12}(t_1, t_2, r_1, r_2) = i \sum_k e^{-i \xi_k (t_1 - t_2)} n_k \psi_k(r_1) \psi_k(r_2),$$

$$G_{21}(t_1, t_2, r_1, r_2) = -i \sum_k e^{-i \xi_k (t_1 - t_2)} (1 - n_k) \psi_k(r_1) \psi_k(r_2),$$

$$G_R(t_1, t_2, r_1, r_2) = -i \theta(t_1 - t_2) \sum_k e^{-i \xi_k (t_1 - t_2)} \psi_k(r_1) \psi_k(r_2),$$

$$G_A(t_1, t_2, r_1, r_2) = i \theta(t_2 - t_1) \sum_k e^{-i \xi_k (t_1 - t_2)} \psi_k(r_1) \psi_k(r_2).$$

(B2)

With the aid of these expressions the kernels (31,32) can be represented as follows

$$\chi(t, r_1, r_2) = 2i e^2 \theta(t) \sum_{k,q} e^{-i(\xi_k - \xi_q)t} (n_k - n_q) \psi_k(r_1) \psi_k(r_2) \psi_q(r_2) \psi_q(r_1);$$

(B3)

$$\eta(t, r_1, r_2) = \frac{1}{2} e^2 \sum_{k,q} e^{-i(\xi_k - \xi_q)t} [n_k (1 - n_q) + n_q (1 - n_k)] \psi_k(r_1) \psi_k(r_2) \psi_q(r_2) \psi_q(r_1).$$

(B4)

Performing the Fourier transformation in time:

$$\chi(\omega, r_1, r_2) = -2e^2 \sum_{k,q} \frac{n_k - n_q}{\omega - \xi_k + \xi_q + i0} \psi_k(r_1) \psi_k(r_2) \psi_q(r_2) \psi_q(r_1),$$

(B5)

$$\eta(\omega, r_1, r_2) = \pi e^2 \sum_{k,q} \delta(\omega - \xi_k + \xi_q) [n_k (1 - n_q) + n_q (1 - n_k)] \psi_k(r_1) \psi_k(r_2) \psi_q(r_2) \psi_q(r_1),$$

(B6)

and substituting the equilibrium distribution function $n_k = 1/(e^{\xi_k/T} + 1)$ one immediately arrives at (B3).

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