Developing mathematical model for calculating forces affecting to Ship motions

Abstract

Mathematical model of ship motion is considered as an artificial brain deciding the capability of a bridge simulation system and ensuring the reality of ship maneuvering. Due to the huge development of shipbuilding industry ships have been being equipped with a variety of non-conventional propellers and rudders the existing mathematical models need to be improved accordingly. Moreover, training simulators nowadays require more external and environmental forces to be integrated into the system for more reality. This study aims to review previous studies on mathematical models of ship motions and proposes a development to the synthetic model of forces for systematically and simply simulating ship motion in six degrees of freedom in real-time mode.

Keywords: mathematical modeling, hydrodynamics forces, ship hydrodynamics, ship simulation

Introduction

The mathematical model of ship motions is considered as an artificial brain deciding the processing capability of a bridge simulation system and ensuring the reality of ship maneuvering. Due to the quick development of shipping industry, ships, nowadays, have been being equipped with advanced propellers such as nozzled, azimuth, surface-piercing propellers, contra-rotating propellers etc. The variety in types and numbers of propellers and rudders fixed on the ship causing a challenge for calculating and computerizing all of the forces impacting to ship hulls on simulation systems.

By establishing a mathematical model based on Newton’s equation, hydrostatic, hydrodynamic, aerodynamic theories with empirical data the status equations of ship motions are set up. A simple mathematical model including one equation was introduced by NOMOTO K (1957). Davidson and Schiff (1946) described yawing and drifting in 2 DOF. Norrbin (1971), Inoue (1981), Ankudinov (1993) and other researchers developed in 3 DOF model including surging, swaying, yawing. Eda (1980), Hirano (1980) and Oltmann (1993) described 4 DOF model by adding rolling motion. By adding heaving and pitching motions Ankudinov (1983) and Hooft & Pieffers (1988) did establish 6DOF model.1,2 Thor I Fossen1 systemized 6 DOF model with status equations in which coefficients were described in the form of a matrix:

\[
M \cdot \ddot{X} + C(X, \dot{X}) \cdot \dot{X} + g(X) = f
\]  

\[
M_\alpha \cdot \ddot{X} + C_\alpha(X, \dot{X}) \cdot \dot{X} + D(X, \dot{X}, \ddot{X}) = T
\]  

Where \( M \) is generalized mass matrix of the ship and added mass, \( C \), \( C_\alpha \) and \( D \) are Coriolis and centripetal matrixes of the ship and added mass due to motion or rotation about the initial frame, \( D(X, \dot{X}, \ddot{X}) \) is damping matrix; \( f = \{u, v, w, p, q, r\}^T \) is velocity matrix, \( X = \{u, v, w, p, q, r\}^T \) is acceleration matrix. \( f \) is generalized gravitational/buoyancy forces and moments. \( T = \{X, Y, Z, K, M, N\}^T \) is matrix of external forces and moments effecting to the ship (Table 1).

Table 1 Parameters defined in the body-fixed reference frame

| DOF | Description | Velocities | Forces |
|-----|-------------|------------|--------|
| 1   | surge - motion in x direction | \( u \) | \( X \) |
| 2   | sway - motion in y direction  | \( v \) | \( Y \) |
| 3   | heave - motion in z direction | \( w \) | \( Z \) |
| 4   | roll – rotation about x axis | \( p \) | \( K \) |
| 5   | pitch - rotation about y axis | \( q \) | \( M \) |
| 6   | yaw - rotation about z axis  | \( r \) | \( N \) |

The principal for calculating of added mass is based on work of Ursell1 and Frank1 for an arbitrary symmetric cross section. Then Keil1 introduced a method for any arbitrary water depth based on a variation of the method of Ursell1 with Lewis conformal mapping. Frank1 described the pulsating source method for deep water.

Nils Salvesen et al.1 introduced new method to predict heave, pitch, sway and yaw motions as well as wave-induced vertical and horizontal shear forces, bending moments, and torsional moments for a ship advancing at a constant speed with arbitrary heading in regular waves.

For calculating damping coefficients a simple set of equations is presented by Society of Naval Architects and Marine Engineers (SNAME) in 3 DOFs including surge, sway and yaw.3 Fedyaevsky et al. introduced equations to calculate cross-flow Drag in sway and yaw. Nils Salvesen et al.1 suggested a method to calculate damping components in “Ship Motions and Sealoads”.

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Recent studies on the calculation of ship resistance have tended to improve the accuracy of previous methods or apply computational fluid dynamics (CFD).

K Zelazny\textsuperscript{10} introduced a method to improve the accuracy of ship resistance at preliminary stages of design. Mucha et al.\textsuperscript{11} had a validation study on numerical prediction of resistance in shallow water based on the solution of the Reynolds-averaged Navier-Stokes (RANS) equations, a Rankine Panel method and a method based on slender-body. The application of CFD can be typically referred to the study of Yasser M Ahmed et al.\textsuperscript{12} For roll damping coefficients, it can be referred to study of Frederick Jaouen et al.\textsuperscript{13} and the calculation of Yang Bo et al.\textsuperscript{14} by using numerical simulation based on CFD. Burak Yildiz et al.\textsuperscript{15} introduced a URANS prediction of roll damping due to the effects of viscosity based on CFD while Min Gu et al.\textsuperscript{16} presented a roll damping calculation based on numerical simulation on the RANS model in calm water. In 2017, D Sathyaseelan et al.\textsuperscript{17} introduced an efficient Legendre wavelet spectral method (LWSM) to ship roll motion model for investigating the nonlinear damping coefficients.

The forces \( f=\{X,Y,Z,K,M,N\} \) includes ship propulsion forces and external forces caused by environmental effects such as current, wind, wave, squat, bank effect, ship interaction, anchor, mooring line, towing, grounding, collision. The calculations of these forces were solved and published separately by various researchers.

The propulsion forces are created by the ship’s propellers and rudders. Since there is a wide range of various types of propellers and rudders, the methods to calculate forces are diversified and have been solved by separate studies which can be referred to as Habil Nikolai Kornev,\textsuperscript{18} John P. Breslin et al.,\textsuperscript{19} Öyvind Notland Smogeli,\textsuperscript{20} Tran Cong Nghĩ,\textsuperscript{21} Edward M Lewandowski,\textsuperscript{8} and other various researchers.

The ship to ship interaction can be based on the paper of Wang S\textsuperscript{22}and Cummins WE.\textsuperscript{23} The estimation of squat can be referred to the formula suggested by Gourlay et al.\textsuperscript{24} based on slender-body theory while the bank effect can be determined by application of the description of Evert Lataire et al.\textsuperscript{25}

The external forces caused by the anchor, mooring line, towing, grounding, collision can be described in the same approach. Each certain force can be considered as single component affecting the ship as a force vector at given position. Therefore, these forces can be described according to the principle of basic physics.

As the above mentioned, the methods to calculate every single force components are available in previous studies. However, number and properties of these forces vary continuously during the real-time ship motion. The problem, therefore, is how to develop an equation to describe all unlimited force components that have not been posted in previous researches yet.

This paper is aimed to focus on the systematization of all force components and suggest a total force model \( \sum f \) for the purpose of ship simulation in real-time motion in 6 DOFs.

**Establishing total force model**

Basically, forces effecting on the ship hull consist of:

a) Hydrodynamic forces: include Coriolis forces and damping forces.

These forces are considered calculated in the hydrodynamic coefficients \( C(v), D(v) \).

b) Hydrostatic forces: include buoyancy forces and restoring moments.

c) Propulsion forces: are created by propellers and rudders.

d) External forces: consist of forces caused by current, wind, wave, squat, bank suction, interaction of ships, mooring line, towing, tug support, anchor, collision, and grounding (Figure 1).

![Figure 1 Describing force components effecting on ship's hull.](image)

In practice, the force components are very complex depending strictly on the ship propulsion system and loading condition and environmental condition. They change time to time at every real-time condition. The total forces can be described:

\[
f = f_c + f_p + f_{cu} + f_{si} + f_{sq} + f_{sk} + f_{sr} + f_{sm} + f_{sp} + f_{pl} + f_{cl}
\]

(3)

Where:

- \( f_c \): matrix of restoring forces and moments
- Propulsion force group:
  - \( f_p \): matrix of forces and moments caused by rudder system
  - \( f_{cu} \): matrix of forces and moments caused by propeller system
- External force group:
  - \( f_{cu} \): matrix of forces and moments created by current
  - \( f_{sm} \): matrix of forces and moments created by wind
  - \( f_{sq} \): matrix of forces and moments created by wave
  - \( f_{sk} \): matrix of forces and moments created by squat
  - \( f_{sr} \): matrix of forces and moments created by bank effect
  - \( f_{sp} \): matrix of forces and moments created by ship to ship interaction
  - \( f_{pl} \): matrix of forces and moments created by ship anchors
  - \( f_{cl} \): matrix of forces and moments created by towing lines
  - \( f_{pl} \): matrix of forces and moments created by tug pushing
  - \( f_{cl} \): matrix of forces and moments created by grounding

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For simpler expression, the external forces can be grouped as the force matrix $f_e$:

$$f_e = f_{ex} + f_{ey} + f_{ez} + f_{ex} + f_{ey} + f_{ez} + f_{px} + f_{py} + f_{pz} + f_{cx} + f_{cy} + f_{cz}$$

Hereafter, $F_i$ is defined as the $i^{th}$ force (Figure 2), $\sigma_i$ as azimuth angle and $\gamma_i$ as declination angle of the forces vector in oyz frame at a position $O_i$:

$$f_i = [X_i, Y_i, Z_i, K_i, M_i, N_i]^T$$

$$F_i = \sqrt{X_i^2 + Y_i^2 + Z_i^2}$$

Based on basic physics and mathematics, the mathematical model of total forces and moments of surfing, swaying and heaving motions are expressed:

$$X_i = F_i \cos(\sigma_i) \sin(\gamma_i)$$

$$Y_i = \sin(\sigma_i) \sin(\gamma_i)$$

$$Z_i = \cos(\gamma_i)$$

Thus, the moment in rolling, pitching and yawing rotations are obtained:

$$K_i = F_i \cdot z_i \cdot (\sin(\sigma_i) \sin(\gamma_i))$$

$$M_i = F_i \cdot x_i \cos(\gamma_i)$$

$$N_i = F_i \cdot y_i \left[ \cos(\sigma_i) \sin(\gamma_i) + x_i \sin(\sigma_i) \sin(\gamma_i) \right]$$

Where $x_i, y_i, z_i$ are lever arm of the force $F_i$ over axis OX, OY, OZ:

$$x_i = OX_i ; y_i = OY_i ; z_i = OZ_i$$

Thus, the matrix of total forces and moments:

$$f = \sum_{i=1}^{n} F_i$$

$$f = \begin{bmatrix}
X_1 & Y_1 & Z_1 & K_1 & M_1 & N_1 \\
X_2 & Y_2 & Z_2 & K_2 & M_2 & N_2 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
X_n & Y_n & Z_n & K_n & M_n & N_n
\end{bmatrix}$$

$$f = \begin{bmatrix}
X_{p1} & Y_{p1} & Z_{p1} & K_{p1} & M_{p1} & N_{p1} \\
X_{p2} & Y_{p2} & Z_{p2} & K_{p2} & M_{p2} & N_{p2} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
X_{pj} & Y_{pj} & Z_{pj} & K_{pj} & M_{pj} & N_{pj}
\end{bmatrix}$$

$$f = \begin{bmatrix}
X_{ek} & Y_{ek} & Z_{ek} & K_{ek} & M_{ek} & N_{ek}
\end{bmatrix}$$

$$f = \sum_{i=1}^{n} f_i$$

$$f = \sum_{i=1}^{n} X_i + \sum_{j=1}^{n} Y_j + \sum_{k=1}^{n} Z_k + \sum_{l=1}^{n} K_l + \sum_{m=1}^{n} M_m + \sum_{n=1}^{n} N_n$$

Where,

$$f_\eta = \begin{bmatrix} X_\eta & Y_\eta & Z_\eta & K_\eta & M_\eta & N_\eta \end{bmatrix}$$

is restoring force matrix;

$$f_{ri} = \begin{bmatrix} X_{ri} & Y_{ri} & Z_{ri} & K_{ri} & M_{ri} & N_{ri} \end{bmatrix}$$

is force matrix of the $i^{th}$ rudder;

$$f_{pj} = \begin{bmatrix} X_{pj} & Y_{pj} & Z_{pj} & K_{pj} & M_{pj} & N_{pj} \end{bmatrix}$$

is force matrix of the $j^{th}$ propeller;

$$f_{ek} = \begin{bmatrix} X_{ek} & Y_{ek} & Z_{ek} & K_{ek} & M_{ek} & N_{ek} \end{bmatrix}$$

is force matrix of the $k^{th}$ external forces.

With such the calculation, all the forces can be considered as separate components $i^{th}$, $j^{th}$, $k^{th}$. This enables to calculate and add single force into the general equations (2) in real-time simulation.

Figure 2 Describing components of the $i^{th}$ force.
Algorithm and simulation

The computerizing algorithm (Figure 3) for simulating ship motions is set up based on the general equation (2). General diagram for calculating all forces are described:

For this study, the ship model Container 18000 TEU triple E and Genting Dream are used (Table 2).

| Ship Model | Triple E | Genting Dream |
|------------|----------|---------------|
| Type       | Container| Cruise        |
| L (m)      | 399      | 335.3         |
| B (m)      | 59       | 39.7          |
| T (m)      | 16       | 8.3           |
| Displ. (MT)| 257,343  | 70,330        |
| Propulsion | 2 conventional rudders/propellers 2 bow thrusters | 2 azipod propellers/3 bow thrusters |

For assessing the propriety of the established formula, the algorithm is transferred into Matlab in the full equation (2). The hydrodynamics coefficients C(v), D(v) are referred to previous studies. For simplifying the assessment, it is assumed that the ship is moving in calm water without external forces. In this case, the propulsion forces, hydrodynamic forces, and hydrostatic forces are applied. The other external forces can be added separately without influencing to final results of the method Figures 5–8.

The plotting curves in MATLAB of the sample models show that the forces and moments caused by the propulsion system are reasonably and logically.

Due to the limitation of time and work for studying, only propulsion forces in combination with hydrodynamic forces and hydrostatic forces are simulated and tested. However, the other external forces can be added into the equation (2) separately in other steps.

The above results are extracted from the Genting Dream model:

![Figure 3 Diagram of total forces affecting to ship motions.](image)

![Figure 4 Control units made in Matlab for different types of propellers/rudders.](image)

![Figure 5 Genting Dream model.](image)

![Figure 6 Wet area of the model created by Matlab.](image)

![Figure 7 Turning circle of the model when applying rudder turning 15 degrees on portside.](image)

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Figure 8 Turning circle of the model when applying rudder turning 20 degrees on starboardside.

Figure 9 Turning parameters of the model when applying rudder turning 20 degree on starboardside.

Conclusion

The suggested mathematical model of total forces affecting ship motions can be used for calculating and computerizing forces effecting to ship’s hull to simulate ship motions in 6DOF in real-time mode.

The details of each propulsion forces and external forces can be referred to available studies.

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Conflict of interest

Authors declare there is no conflict of interest in publishing the article.

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