Internal radiation losses in semiconductor lasers with stressed InGaAsP/InP quantum wells

N V Pavlov, G G Zegrya
Ioffe Institute, 194021, Russian Federation, St Petersburg, Politekhchnicheskaya st., 26
E-mail: pavlovnv@mail.ru

Abstract. A microscopic analysis of the mechanism of intraband radiation absorption by holes with their transition to a spin-split band for quantum wells based on stressed InGaAsP/InP quantum wells is performed within the framework of the four-band Kane model. The Kane’s equations are written and solved with taking into account nonsphericity of the $kP$ Hamiltonian. It is shown that this process can be a significant mechanism of internal radiation losses for quantum well lasers. It is shown that taking the nonsphericity and the elastic stresses into account leads to the absorption coefficient value decreasing. It is also shown that the maximum of the absorption coefficient is observed at values of the QW width from 40 to 60 Å.

1. Introduction
At present, semiconductor lasers with quantum wells (QWs) InGaAsP/InP quantum wells (QWs) with a generation wavelength of 1.3 and 1.55 μm have been widely used in optoelectronics [1-3]. However, these structures are characterized by strong intraband absorption of radiation, which leads to a disruption of generation [4,5]. The intraband absorption coefficient increases with decreasing thickness of the active region and can reach several tens of reciprocal centimeters. The mechanisms of intraband absorption of radiation in QW lasers have been studied both theoretically and experimentally for many years [6]. Experimental results [7,8] show that the coefficient of intraband absorption of laser radiation is substantially higher than that predicted by the theory [9]. One of the candidates for explaining these results is the process of intraband radiation absorption by holes with the transition to the spin-split (so) zone. In this paper, we use a modification of the four-band Kane model [10] proposed by Polkovnikov and Zegrya [11,12], which is based on the use of the $8 \times 8 kP$ Hamiltonian. It allows us to obtain explicit analytical expressions for energy spectra and wave functions of charge carriers, as well as matrix elements of transitions [13,14]. The authors proposed a modification of this method, allowing to take into account the elastic stresses arising in mismatched heterostructures and the $kP$ Hamiltonian nonsphericity [15]. The aim of this work is to calculate the coefficient of intraband radiation absorption by holes with their transition to the so-zone for QWs based on $\text{Al}_x\text{In}_{1-x}$ semiconductors with taking into account both the elastic stresses and the nonsphericity. The calculations are made for the InGaAsP/InP heterostructure, which is widely used in fabricating semiconductor lasers with a radiation wavelength of 1.55 μm. The structure parameters are taken from [16].
2. Basic Equations

We used $8 \times 8$ $kP$ Hamiltonian [17] that takes interaction with the higher bands into account up to quadratic terms by the wave vector but neglects the relativistic linear terms and the term with heavy electron mass. The Kane equations near the $\Gamma$-point are given by following expressions:

\[
(E_c - E - A_c) u = -i P k \cdot v ,
\]

\[
(E_{\nu} - \delta - E - \frac{\hbar^2}{2m_0} (\gamma_1 - 2\gamma_2) - P_{\nu} - Q_{\nu} - i\delta[\sigma \times v] - \frac{3\hbar^2}{m_0} \gamma_3 k(k \cdot v))
\]

\[
+ \frac{3\hbar^2}{m_0} (\gamma_3 - \gamma_2) \left( \begin{array}{c} k_x v_x \\ k_y v_y \\ k_z v_z \end{array} \right) = -i P k u
\]

Here $E_c$ and $E_{\nu}$ are the energies of conduction and valence band edges, $P$ is the Kane matrix element, $\delta=\Delta_{so}/3$ is the spin-orbit splitting constant, $m_0$ is the free electron mass, $\gamma_1$, $\gamma_2$ and $\gamma_3$ are the generalized Luttinger parameters, $\sigma$ is the Pauli matrices, $u$ and $v$ are the spinor components of wave function with s- and p-symmetry, respectively, $k$ is the wave vector and $E$ is the energy, $P_{\nu} = a_\gamma (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})$, $Q_{\nu} = b (\varepsilon_{yy} + \varepsilon_{zz} - 2\varepsilon_{xx})$, $a_\nu = a_\gamma (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})$, $a_\gamma$, $a_\nu$, and $b$ are the Bir-Pikus deformation potentials. For a strained-layer semiconductor pseudomorphically grown on a (100)-oriented (x axis) substrate, the strain tensor $\varepsilon_{ij}$ components are: $\varepsilon_{yy} = \varepsilon_{zz} = \frac{a_0 - a}{a}$, $\varepsilon_{xx} = -\frac{2C_{12}}{C_{11}} \varepsilon_{yy}$, where $a_0$ and $a$ are the lattice constants of the substrate and the layer material, and $C_{11}$ and $C_{12}$ are the elastic stiffness constants.

The energy spectrum of heavy holes, calculated from (1), is:

\[
E_1^3 - C_h k^2 E_1^2 + \left( C_h^2 k_x^2 + k_y^2 k_3^2 + k_z^2 k_3^2 \right) - 3\delta^2 \right) E_1 - C_h^2 k_x^2 k_y^2 k_z^2 + 2C_h k^2 \delta + 2\delta^3 = 0
\]

(2)

If $k_x = q$ and $k_z = 0$, the wave function of heavy hole is:

\[
v = H_1 \begin{pmatrix}
q \cos k_x x \xi \\
-ik_y \sin k_y x \xi \\
\end{pmatrix}
\]

\[
+ H_2 \begin{pmatrix}
q \sin k_x x \eta \\
-ik_y \cos k_y x \eta \\
\end{pmatrix}.
\]

(3)

Where

\[
E_1 = E + \frac{\hbar^2 k_x^2}{2m_h} + P_{\nu} + Q_{\nu} + \delta - E_{\nu}, \quad m_h = \frac{m_0}{\gamma_1 - 2\gamma_2}, \quad \xi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \eta = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix},
\]

\[
C_h = \frac{3(\gamma_3 - \gamma_2)\hbar^2}{m_0}, \quad H_1 \text{ and } H_2 \text{ are the normalization coefficients. We note that wave function (3) differs from similar one in spherical approximation because } E_1^{-1} \delta \text{ is not unity. In (1) and (2) we considered that } E_{\nu} = \delta.
\]

Energy spectra and wave function of electrons, light and spin-orbit holes are more complex. If $k_z = 0$ they are given by following equations:

\[
E - E_g - \delta = -P f_j (1 + f_j^2) k^2 + 2C k^2 q^2
\]

(4)
$$u = A_1 \cos kx \eta + A_2 \sin kx \xi$$

$$v = A_1 f_3 \begin{cases} 
-qg_x \cos k_x x + g_x k_x \sin k_x \eta \\
-ig \gamma 3_k \cos k_x \eta + i k_x g_x \sin k_x x \\
+f_2 \left\{ k_x (g_x - g_y) \sin k_x x + q(g_x - g_y) \cos k_x \eta \right\} 
\end{cases}$$

$$\begin{align*}
&+ A_2 f_3 \begin{cases} 
-q g_x \sin \gamma 3_k \xi - k_x g_x \cos \gamma 3_k \xi \\
-ig \gamma 3_k \sin \gamma 3_k \eta - k_x g_x \cos \gamma 3_k \eta \\
+f_2 \left\{ k_x (g_x - g_y) \cos \gamma 3_k \xi + q(g_x - g_y) \sin \gamma 3_k \xi \right\} 
\end{cases},
\end{align*}$$

where $g_x = 1 - f_2^2 + C k_x^2$, $g_y = 1 - f_2^2 + C q^2$, $g_i = - f_2^2 - f_2$, $f_3 = \frac{f_1}{g_x g_y - g_i}$, $f_2 = -E_1^\delta$, $\gamma 3 = E_i - A_1$, $A_1$ and $A_2$ are the normalization coefficients.

We can derive the boundary contitions by integrating equations (1) near the interface. Thus we obtain:

$$v_0^+ = 0, \left( \frac{\partial v_0}{\partial x} \right)^+ = 0, \left( \frac{\hbar^2}{2m_0} \frac{\partial v_0}{\partial x} \right)^+ = \left( \frac{i q(\hbar^2 / 2m_0 + C) v_0}{\gamma 3} \right)_-, \left( \frac{\hbar^2}{2m_0} (\gamma 1 + 4 \gamma 2 - 6 \gamma 3) \frac{\partial v_0}{\partial x} \right)_- = \left( -P + \frac{3 \hbar^2}{m_0} \gamma 3 \frac{E - E_c - A_x}{P} u \right)_-$$

$$\text{(6)}$$

Where $m_0 = \frac{2 p^2}{\hbar^2 (E_c - E - A_x)} + m_0 (\gamma 1 + 4 \gamma 2)$

However, we can use more simple equations with practically no loss of accuracy:

$$\left( v_x \right)^-_0 = 0, \left( v_y \right)^-_0 = 0, \left( \frac{\partial v_x}{\partial x} \right)_- = 0, \left( \frac{\hbar^2}{2m_0} (\gamma 1 + 4 \gamma 2 - 6 \gamma 3) \frac{\partial v_x}{\partial x} \right)_- = (-P u)_-$$

$$\text{(7)}$$

3. Results

Following [18] we obtain: $A_x = 65 meV$, $P_x = -13 meV$, $Q_x = 42 meV$. The shifts of the valence subbands are: $\Delta E_h = 26 meV$, $\Delta E_i = -44 meV$, $\Delta E_{io} = -24 meV$. The positive value corresponds to shift up in the energy scale.

Figures 1 and 2 show the frequency dependences of absorption coefficients $\alpha_{par}^\delta$ and $\alpha_{par}^\epsilon$ for the optical transitions from heavy hole subbands to the discrete and continuous spectra of so-holes in the InGaAsP/InP heterostructure at $a = 80 A$ and a holes concentration $p = 10^{12} \text{ cm}^{-2}$. Index par denotes that the wave vector of incident radiation is in the QW plane. The calculation scheme is described in [13]. In the calculations we considered that temperature $T = 300 K$.

We can see that taking the nonsphericity and the elastic stresses into account leads to the graph shifting towards higher energies, and the value of the absorption coefficient decreasing. We can also see that the peak corresponding to transition from the first excited heavy holes subband to the first excited so-holes subband becomes practically indistinguishable.

The decrease of the absorption coefficient value is because if we use wave functions (5) for so-holes, then the matrix element of the transition is proportional to the normalization coefficients $A_1$ and $A_2$, and they are proportional to $f_3^{-1}$. As $f_3 \sim (1 + C k_x^2)^{-1}$ and $C < 0$ then $1 + C k_x^2 < 1$ and normalization coefficients $A_1$ and $A_2$ decrease with taking nonsphericity into account.
Figure 1. Dependence of the absorption coefficient $\alpha_{\text{par}}$ for the discrete spectrum of so-holes on the incident radiation frequency in the InGaAsP/InP heterostructure with the QW width $a=80\,\text{A}$ and the holes concentration $p=10^{12}\,\text{cm}^{-2}$. The solid curve shows the calculation result with taking into account the nonsphericity and the elastic stresses, and the dashed curve shows the result for the four-band Kane model [14].

Figure 2. Dependence of the absorption coefficient $\alpha_{\text{par}}$ for the continuous spectrum of so-holes on the incident radiation frequency in the InGaAsP/InP heterostructure with the QW width $a=80\,\text{A}$ and the holes concentration $p=10^{12}\,\text{cm}^{-2}$. The solid curve shows the calculation result with taking into account the nonsphericity and the elastic stresses, and the dashed curve shows the result for the four-band Kane model [14].

Figure 3. Dependence of the absorption coefficient to the continuous spectrum of spin-split holes on the generation wavelenght on the width of a quantum well in the InGaAsP/InP heterostructure at the holes concentration $10^{12}\,\text{cm}^{-2}$. The solid curve shows the calculation result with taking into account the nonsphericity and the elastic stresses, and the dashed curve shows the result for the four-band Kane model [14].
In Figure 3 the dependence of the absorption coefficient of spin-split holes on the generation wavelength on the width of a quantum well is shown. It can be seen that taking the nonsphericity and the elastic stresses into account leads to the absorption coefficient decreasing. We can also see that the maximum of the absorption coefficient is observed at values of the QW width from 40 to 60 Å, and the absorption is much weaker for values less than 30 Å and greater than 80 Å as well as for the calculations in four-band Kane model.

4. Conclusions
The mechanism of intraband radiation absorption by holes with a transition to the so-band for QWs based on $A_3B_5$ semiconductors is analyzed microscopically with taking into account the nonsphericity of the $kP$ Hamiltonian and the elastic stresses. The analysis is performed for two cases: transitions to a discrete spectrum and to a continuous spectrum of so-holes. It is shown that the intraband absorption can be significant mechanism of internal optical losses for semiconductor lasers on QWs.

It is shown that taking the nonsphericity and the elastic stresses into account leads to the absorption coefficient value decreasing. It is also shown that the maximum of the absorption coefficient is observed at values of the QW width from 40 to 60 Å.

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