The formation of errors caused by temperature processes in mechanisms built on the principles of parallel kinematics

Yu I Brovkina\textsuperscript{1}, A N Sobolev\textsuperscript{2} and A Ya Nekrasov\textsuperscript{2,*}

\textsuperscript{1}Moscow Polytechnic University, 38, Bolshaya Semenovskaya street, Moscow, 107023, Russia
\textsuperscript{2}MSUT “STANKIN”, 1, Vadkovsky per., Moscow, 127994, Russia

*t-64@mail.ru

Abstract. The article discusses issues related to the process of formation of the total error affecting the position error of the output link of mechanisms with a parallel structure caused by thermal deformations of the leading links. Movable rod mechanisms allow you to create lightweight technological machines for complex technological operations: finishing turbine blades, the production of art products, complex configurations, as well as medical devices and navigation equipment. The causes causing a change in the temperature state of the mechanisms under consideration are considered. A technique for determining the position of the output coordinate as a result of deviation from a given trajectory is presented.

1. Introduction
Mechanisms with a parallel structure have found application in various fields. These mechanisms make it possible to create a lightweight portable technological equipment for processing both large-sized and small-sized, but complex shapes. This design has advantages over traditional layouts, and in some cases, only it is possible to carry out certain technological operations. The mechanisms of this group are highly accurate.

Mechanisms with a parallel structure are more often a closed loop consisting of units of variable length connected by rotational or spherical joints [1, 2]. Moreover, depending on the number of kinematic pairs and the order of connection of the links, the output links of such mechanisms can realize three or more movements in space. The overall dimensions of the mechanisms also depend on the purpose and application: for example, a machine for processing parts with a complex curved surface or a test bench for a pilot built on the principle of a parallel structure will be heavier and more massive than equipment for performing medical operations.

One of the key tasks in the design of such mechanisms is to identify the causes that affect the accuracy of positioning. The multivariance of the areas of application of these mechanisms outlines a wide range of reasons for the deviation of the trajectory of the output link. One of the applications of mechanisms with parallel kinematics is the creation of navigation equipment for use in outer space, where the temperature regime changes from low subzero temperatures in the shade to high in open space. Such a temperature regime undoubtedly affects the functioning of the mechanism. But even when working on the ground, the mechanisms are subject to temperature effects that cause deformation of the links, which are unsteady in nature and during the operation of the mechanism
change both in magnitude and direction, causing a constant effect on the formation of quality indicators of the performance of specified operations.

2. Factors affecting the temperature field of the mechanism
The occurrence of temperature deformations in mechanisms with parallel kinematics is due to the same factors as in conventional machines. The main sources of heat can be divided into internal and external. Friction units, lubrication system, electric motors, air temperature fluctuations in the workshop, air flows from neighboring equipment, heating from lighting, etc. affect the links of the mechanism, changing its temperature state (Figure 1).

![Figure 1. Factors affecting the temperature state of the mechanism](image)

The air temperature in the production room is directly related to the technological processes occurring there and changes according to a law close to sinusoidal [3, 4, 5], moreover, the period of temperature fluctuation is approximately 24 hours a week. In workshops that are not equipped with devices for stabilizing the temperature, fluctuations of several degrees are observed. In addition, the air temperature is not the same for the height of the workshop. The heating time of machine parts to ambient temperature depends on the material of the part and the thickness of its walls. It is possible to eliminate the influence of external sources by placing a high-precision machine in a thermoconstant workshop or in a thermoconstant chamber with an air conditioning system with a circulating air flow. But this technique does not apply to all operations performed.

Internal heat sources are associated with the operation of the machine itself, it is usually more difficult to eliminate them, and in the vast majority of cases they are decisive in the formation of the temperature field of the machine and its deformations. Friction pairs, continuous operation of engines and other elements of the machine changes its thermal field. And sometimes, a seemingly insignificant heating of the links, significantly affects the final result of the manufactured products.

3. The formation of positioning errors
Moving the output coordinate in mechanisms of a parallel structure, such as hexapods (Figure 2a), is a function that depends on six independent generalized coordinates, each of which is associated with the i-th moving link. Such mechanisms are several kinematic chains containing one or more drive pairs. Leashes imposing one condition of communication are made in the form of rods with two spherical pairs of the third class, and the base link can have six degrees of freedom. Such mechanisms are controlled by changing the lengths of the movable guides [6, 7, 8, 9]. The peculiarity of using rod elements is that the size of the link in the linear direction exceeds several times the dimensions of the
link in the transverse direction. Under the influence of various heat sources, the core elements experience thermal expansion, as a result of which the accuracy of such a mechanism decreases. Since the controlled coordinates are associated with a change in the length of the rods, even a slight change in the linear direction of such a link in a mechanism with a closed structure leads to a shift in the output coordinate in space in all six directions at once: linear and angular.

![Figure 2. General view of the hexapod (a) and the scheme of formation of the error (b)](image)

Output positioning error $\Delta_{pl}$ is a function dependent on linear strains $\Delta l(t)$ of movable guides - rods (Figure 2b) and can be represented as the sum of the errors along the three coordinate axes in space:

$$\Delta_{pl} = \Delta x_i + \Delta y_i + \Delta z_i + \Delta \phi_i + \Delta \theta_i + \Delta \psi_i$$

(1)

$$\Delta x_i = x' - x = x(l_i + \Delta l_i) - x(l_i);$$

$$\Delta y_i = y' - y = y(l_i + \Delta l_i) - y(l_i);$$

$$\Delta z_i = z' - z = z(l_i + \Delta l_i) - z(l_i);$$

(2)

where $\Delta x_i, \Delta y_i, \Delta z_i$ - describe linear coordinate changes, $\Delta \phi_i, \Delta \theta_i, \Delta \psi_i$ - angular coordinate changes, $x(l), y(l), z(l)$ - ideal coordinates of the center of the platform (functions of changing the lengths of the executive links in time); and $x(l_i + \Delta l_i), y(l_i + \Delta l_i), z(l_i + \Delta l_i)$ - actual coordinates as a function of not only time but also temperature.

The angular displacements of the platform are found through the guide cosines of the coordinate system $X'Y'Z'$ relative to the fixed coordinate system $XYZ$.

Coordinates of hinge centers $A_i$ on a fixed base relative to the base coordinate system $X_0Y_0Z_0$, and coordinates of points $B_i$ belonging to the platform (in the moving coordinate system $X_1Y_1Z_1$ of the platform), respectively equal:

$$A_i(x_{A_i}, y_{A_i}, z_{A_i})$$

$$B_i(x_{B_i}, y_{B_i}, z_{B_i})$$

(3)

The transition between spatial coordinate systems is carried out using the coordinate transformation matrix $\Pi$:
where \( \cos \alpha, \cos \beta, \cos \gamma, \sin \alpha, \sin \beta, \sin \gamma \) – cosines and sines of the platform rotation angles specified by the program around the corresponding coordinate axes, and \( \Delta x, \Delta y, \Delta z \) – moving the center of the platform in three directions. This rotation matrix takes into account six movements, which corresponds to six degrees of mobility of such mechanisms.

In order to determine the coordinates of the hinges on the platform in a fixed (base) coordinate system, you need to multiply the coordinates of the hinges in the moving coordinate system by the transformation matrix:

\[
\Pi \times \begin{pmatrix} x_{Bi} \\ y_{Bi} \\ z_{Bi} \end{pmatrix} = \begin{pmatrix} x'_{Bi} \\ y'_{Bi} \\ z'_{Bi} \end{pmatrix}
\]  

Then the length of the \( i \)-link:

\[
l_i = \left( (x'_{Bi} - x_{Bi})^2 + (y'_{Bi} - y_{Bi})^2 + (z'_{Bi} - z_{Bi})^2 \right)^{1/2}
\]  

The heat transfer problem can be represented in the form of a model that includes the geometric parameters of the mechanism, physical characteristics of the material, and boundary conditions. Physical parameters – thermophysical properties of construction materials and the environment. In most cases, for machine components, the environment is air. The boundary conditions contain the values of the desired variables at all points of the studied region at the initial time and during the entire process at the boundaries of this region.

Representing the links in the form of rods of a given length, we can write the heat distribution equation [10]:

\[
K_{xx} \frac{\partial^2 T}{\partial x^2} = 0
\]

with boundary conditions:

\[
K_{xx} \frac{\partial T}{\partial x} + q = 0 \text{ at } x=0
\]

\[
K_{xx} \frac{\partial T}{\partial x} + h \cdot (T - T_\infty) = 0 \text{ at } x=L
\]

\( K_{xx} \) – thermal conductivity coefficient of the rod material;

\( q \) – heat flux (positive if removed from the body);

\( h \) – heat transfer coefficient;

\( T_\infty \) – ambient temperature.

Along with thermophysical indicators and boundary conditions, one should determine the temporal conditions. You must specify the period of time during which the process is simulated. Usually this is the period of thermal stabilization of the machine. If the period is unknown (for example, from experimental studies), then the process time is set in excess to determine the stabilization period of the thermal state of the machine during the calculation process. To calculate the temperature field of the mechanism of a parallel structure, taking into account heat transfer with the environment, it is necessary to specify the specific heat and density of air, as well as the temperature regime of the room (workshop) in which the mechanism operates. It is necessary to assess the effect of each source of thermal energy on the overall thermal field of the mechanism and identify the hazardous areas that
have the greatest impact. Here we have to consider each specific case, because the mechanisms have a wide range of applications and can work in completely different conditions.

Figure 3. Change in the temperature field of the mechanism from one heat source.

If we talk about the use of hexapod robots as mechanisms for machining parts, then the obvious internal factors affecting its thermal field are engines and friction pairs. As can be seen from the models in Figure 3, a change in the thermal field of one element (in this case, a change in the temperature regime in kinematic pairs is shown) does not affect other control links, but affects the output platform.

4. Conclusions
Thus, it was found that heating of the main elements of the system leads to a change in the relative position of the output link. Moreover, a feature of mechanisms with a parallel structure is that changing even one coordinate, for example, lengthening one link, changes the position of the platform in all six coordinates. Given that the platform links are made in the form of rods and have dimensions in the transverse direction many times larger than in the radial direction, knowing the reason for the heating and the law of temperature distribution, it is possible to calculate the temperature deformations of the system links. But then the task arises of determining the new coordinates of the platform, with the changed lengths of the links. The operating conditions of the mechanism should also be taken into account in order to identify the most significant sources of heat.

References
[1] Glazunov V A, Rashoyan G V, Aleshin A K, Shalyukhin K A and Skvortsov S A 2020 Structural synthesis of spatial l-coordinate mechanisms with additional links for technological robots Advances in Intelligent Systems and Computing 902 683-691
[2] Chekanin V A and Chekanin A V 2016 Implementation of packing methods for the orthogonal packing problems Journal of Theoretical and Applied Information Technology 88 421-430
[3] Glazunov V A 1991 Kinematic analysis of manipulators of parallel structure taking into account special provisions Solid mechanics 4 54-61
[4] Rivkin A V, Nekrasov A Ya and Sobolev A N 2019 The analytical model for calculating the contact stiffness of the double-based toolholder Materials Today: Proceedings. International Conference on Modern Trends in Manufacturing Technologies and Equipment 19 1982-1984
[5] Sheth P N and Uicker J J 1971 IMP (Integrated Mechanisms Program), A Computer-Aided Design Analysis System for Mechanisms and Linkage ASME 193-202
[6] Alvan H M and Sloush A B 2003 On the motion control of a spatial platform with several
degrees of mobility Theory of mechanisms and machines 1 63-69

[7] Segida A P 1962 Calculation of stationary temperature fields of metal cutting machines Vestnik of mechanical engineering 9 37-41

[8] Brovkina Yu I, Sobolev A N and Nekrasov A Ya 2017 The analytical method for calculating the positioning error of the links of the mechanisms of machines with parallel kinematics Vestnik MSUT Stankin 40 52-56

[9] Brovkina Yu I, Sobolev A N and Nekrasov A Ya Research of Characteristics and Parameters of Cycloidal Gear 2019 Lecture Notes in Mechanical Engineering 1169-1179

[10] Sokolov Y N 2003 Thermal deformation of machine tools STIN 10 18-20