Heat transfer enhancement of a horizontal microchannel using ferro-nanoparticles injection

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Abstract. A numerical study of Ferro-nanoparticles dispersion has been performed inside a micro-channel by using two phase method. Euler-Lagrange approach was conducted to simulate the presence of Ferro-nanoparticles in the horizontal micro-channel under laminar flow condition. Considering slip velocity and temperature jump due to the effect of decreasing hydrodynamic diameter of channel, temperature and velocity profiles were evaluated and effect of some different parameters on heat transfer has been discussed. Drag, Gravity and Brownian forces were considered using a Lagrangian approach. Furthermore, in order to consider slip effects in micro-channels, Cunningham slip correction factor has been pondered in correcting drag force influence.

1. Introduction

Solid-Gas two-phase flows are encountered frequently in many environmental, industrial and pharmaceutical uses. Furthermore, over the last decade there has been an enormous interest in micro and nano-technology. As systems approach microscopic scales, it has been reported that some phenomena may differ from those in macroscopic counterparts. Therefore, a number of these factors that are dominant in micro scale have been identified through various experimental, analytical and numerical works. Among them, noncontinuum, compressibility, and different surface effects have been under vigorous investigation [1].

Noncontinuum effect: According to different investigation, it would be possible classifying flows into four regimes in order to consider nonequilibrium effects by using a dimensionless number named Kn. Also, Kn number can be defined as the ratio of molecular mean free path to characteristic length of channel. The regime 0≤Kn≤0.1 is referred to as slip flow; no-slip is captured by Kn=0. For Kn≥0.1 the continuum description is expected to fail, and the regime 0.1<Kn≤10 is referred to as the transition regime because the molecular motion undergoes a transition from diffusive (continuum) for Kn≤0.1, to ballistic for Kn≥10 (free molecular flow) [2].

Compressibility effect: In a microchannel, the high Mach number and large pressure drop can be reached even at low Reynolds numbers [3]. Therefore in order to not consider compressibility effect, as we have done in this paper, paying attention to flow Reynolds number would be completely vital.
Roughness effect: At the microscale level, it is impossible to obtain a completely smooth wall surface. According to the traditional knowledge for macrosystems, when the relative roughness is less than 5%, its effect on the friction factor is negligible. But for microscale channels, previously reported experimental and computational results have drawn a conclusion that surface roughness has a significant influence on flow and heat transfer [3, 4].

To analyze microscales gas flows, a number of different analytical and numerical studies have been conducted. Some of these investigations have been done using Direct-Simulation Monte Carlo (DSMC) and the others have been considered Navier-Stokes methods with slip boundary condition. However, the DSMC is still believed to be inefficient tool for low-speed microscale gas flows, due to its huge computational load and large inevitable statistical noise [5]. Therefore, the Navier-Stokes approach including suitable slip boundary condition can analyze the velocity and temperature slip effect in an economical way without losing any accuracy in flow modeling [6]. Inman was the first investigator who studied the constant-wall-temperature Nusselt number in the slip flow regime using continuum theory subject to slip boundary conditions (both velocity and temperature) [7]. Inman's work shows that for fully accommodating walls the Nusselt number reduces due to slip boundary condition. Kavehpour et al. conducted a series of calculations in the slip flow regime examining the relative effects of rarefaction and compressibility [8]. Furthermore, as it has been mentioned before, for microscale channels, previously reported experimental and computational results have shown a considerable influence of roughness on flow and heat transfer. As an example, Kandlikar et al. have conducted an experiment indicating that for a 0.62 mm tube with relative roughness of 0.355%, the effect of roughness on friction factor and heat transfer is significant [4]. Mala and Li studied the effect of roughness in a number of channels with diameters ranging from 50 to 254 mm and relative roughness height 0.7-3.5%. They have stated that in their model, the pressure gradient was higher than that expected through classical theory. In addition, the friction factor increased by increasing Reynolds number and an early transition from laminar to turbulent flow happened at Reynolds number less than 2300. Usami et al. considered gas flow through a two-dimensional channel using a DSMC approach by changing the surface roughness distribution and Kn number. They have indicated the reduction of flow conductivity as a result of surface roughness in transition flow [9]. Karniadakis et al used a more accurate gas flow model and discovered that surface roughness can show a more significant effect with increasing the Kn number [10].

In addition, to analyze the second part of this paper's investigation, injection of Ferro-nanoparticles, it would be necessary to have a look at some invaluable experiments which have done by a number of investigators in the field of two-phase flow in micro-channels. About a century ago, Cunningham [11] derived a correction factor for Stokes drag coefficient to consider slip effects on the drag force acting on a spherical particle. He presented his conclusion in the form of \(1 + AKn\). Another experiment was done by Millikan who identified the linear dependency of Cunningham correction factor on the gas mean free path [12]. Knudsen and Weber investigated more rarefied flow regimes. They expressed the A parameter at Cunningham correction factor as a function of Knudsen number.

\[
C(\text{Kn}) = 1 + AKn \\
A = \alpha + \beta \exp\left(-\frac{\gamma}{Kn}\right) \\
Kn = \frac{\lambda}{L} 
\]

In the above equation, \(\lambda\) is gas mean free path and \(L\) is flow characteristic length. The constants \(\alpha\), \(\beta\) and \(\gamma\) are determined experimentally. Until recently, researchers have tried to propose
slip correction factors based upon above equation. Moshfegh et al brought a total look at different existed conclusions in their paper [13].

All in all, Rapid Progress in the development of micro-fluidic devices has led to a big interest in developing of investigation in micro-channels and totally micro-fluidics. The various relevant applications include devices for flow control, biomedical systems, and chemical reaction systems. These devices, consisting of many micrometer-sized components such as channels, nozzles, and, valves, still need to be improved to enhance performance [6]. Also, the fluid which past these devices can be in two-phase flow in many industrial or environmental applications. Therefore, any kind of progress in the field of micro-channels besides two-phase flow can result in huge advantages in environmental, industrial and pharmaceutical issues.

2. Problem definition and CFD model

2.1. Model development

In this paper we are trying to have a combination of two aforementioned concepts, Gas-Solid two-phase flow and micro-channel. Therefore, previously reported investigation and governing equations which can include the whole geometry would be applied. Here we consider the constant-wall-temperature heat-transfer characteristics of two-dimensional channels that are sufficiently long for flow to be fully developed [2]. Actually, a two-dimensional plane duct geometry has been analyzed in order to simulate Gas-Solid two-phase flow in a microscale channel. To remove compressibility effect, inflow Reynold number respecting the characteristic length of channel has been considered less than 1. In addition, to study roughness effect, Walls have been studied in both smooth and rough situation. A schematic of two described geometries (smooth and rough) has been shown in figure 1. L is the duct length and H defines its height. Also, roughness on channel surfaces is considered as triangular peaks with height r and width w. These elements are uniformly and symmetrically distributed on the top and bottom surfaces. Although this geometry is not exactly the same as actual rough surfaces, it is deliberated as a close approximation to study the roughness effect on flow heat transfer. A parameter called "relative roughness height" is defined as \( e = \frac{r}{H} \). In addition, another parameter called "peak density (Pd)" has been defined for the number of roughness elements in each 0.1 mm of microchannels length. In most micro-fluidic systems, the relative roughness heights are evaluated at 0.1-6% [1]. Therefore, in this study, we consider the mentioned range for analyzing the roughness effect.

![Figure 1. Microchannel with smooth walls](image-url)
2.2. Governing equations

As mentioned before, this work is a combination of two different concepts, Gas-Solid two-phase flow and microchannels. Therefore to have a better look at the whole problem, the governing equations describing the problem geometry will be presented in two separated section as below.

2.2.1. Governing equations describing single-phase micro-channel. As we know, the Navier-Stokes equation is the first order approximation to the Chapman-Enskog solution and can be extended to the slip flow region with the Maxell's slip boundary condition and Smoluchowskyi's temperature jump. In this paper flow has been considered as a steady laminar incompressible one with constant thermophysical properties. Therefore, the needed governing equations are continuity, momentum and energy that must be solved with suitable boundary conditions which will be stated in the next section. Equations (2)-(5):

\[
\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0
\]  
\[
\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \mu \left[ \frac{\partial^2 u}{\partial x^2} + \frac{1}{3} \left( \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x \partial y} \right) \right] + \frac{1}{3} \left( \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y} \right)
\]  
\[
\rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y} + \mu \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{1}{3} \left( \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x \partial y} \right) \right]
\]  
\[
\rho C_p u \frac{\partial T}{\partial x} + \rho C_p v \frac{\partial T}{\partial y} = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + u \frac{\partial P}{\partial x} + v \frac{\partial P}{\partial y}
\]  
\[
\quad + \mu \left[ 2 \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 + 2 \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)^2 \right] - \frac{2}{3} \mu \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)^2
\]
Where \( u, v, T \) and \( P \) are gas streamwise velocity, normal velocity, temperature and pressure, respectively. \( C_p, \mu \) and \( k \) are the specific heat, viscosity, thermal conductivity of the gas that are considered as constant. The above equations can be non-dimensionalized with respect to the channel height \( H \), inlet mean velocity \( u_i \), inlet temperature \( T_i \), inlet mean density \( \rho_i \) and inlet pressure \( P_i \).

\[
\frac{\partial (\rho^* U)}{\partial X} + \frac{\partial (\rho^* V)}{\partial Y} = 0
\]

\[
\text{Re}_i \rho^* (\xi U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y}) = -\frac{\xi \text{Re}_i \rho^* \frac{\partial P^*}{\partial X}}{\gamma Ma_i^2} + \left[ \xi^2 \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + \frac{1}{3} \left( \xi^2 \frac{\partial^2 U}{\partial X^2} + \xi \frac{\partial^2 V}{\partial X \partial Y} \right) \right]
\]

\[
\text{Re}_i \rho^* (\xi U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y}) = -\frac{\text{Re}_i \rho^* \frac{\partial P^*}{\partial Y}}{\gamma Ma_i^2} + \left[ \xi^2 \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} + \frac{1}{3} \left( \xi^2 \frac{\partial^2 V}{\partial X^2} + \xi \frac{\partial^2 U}{\partial X \partial Y} \right) \right]
\]

\[
\xi U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Pe_i \rho^*} \left[ \xi^2 \frac{\partial}{\partial X} \left( \frac{\partial \theta}{\partial X} \right) + \frac{\partial}{\partial Y} \left( \frac{\partial \theta}{\partial Y} \right) \right]
\]

\[
= Ec_i \frac{P_i}{\rho_i u_i^2} \left[ \xi U \frac{\partial P}{\partial X} + V \frac{\partial P}{\partial Y} \right] + Ec_i \left[ 2 \left( \frac{\partial U}{\partial X} \xi \right)^2 + 2 \left( \frac{\partial V}{\partial Y} \xi \right)^2 + \left( \frac{\partial V}{\partial X} + \xi \frac{\partial U}{\partial Y} \right)^2 - \frac{2}{3} \left( \xi \frac{\partial U}{\partial X} + \xi \frac{\partial V}{\partial Y} \right)^2 \right]
\]

Where the parameters for non-dimensionalizing are as belows:

\[
\theta = \frac{T - T_w}{T_i - T_w}, \quad \xi = \frac{H}{L}, \quad X = \frac{x}{L}, \quad Y = \frac{y}{H}, \quad \rho^* = \frac{\rho}{\rho_i}
\]

\[
P^* = \frac{P}{P_i}, \quad U = \frac{u}{u_i}, \quad V = \frac{v}{u_i}, \quad C_p^* = \frac{C_p}{C_p_i}, \quad \alpha_i = \frac{K_i}{\rho_i C_p_i}, \quad K^* = \frac{K}{K_i}
\]

Also during non-dimensionalizing equations some important dimensionless number appear. Respecting above process, the dimensionless numbers will be as follows:

\[
\text{Re}_i = \frac{\rho_i u_i H}{\mu}, \quad \text{Ma}_i = \frac{u_i}{\sqrt{\gamma R T_i}}, \quad \text{Pe}_i = \frac{u_i H}{\alpha_i}, \quad Ec_i = \frac{u_i^2}{C_p (T_i - T_w)}, \quad Kn_i = \sqrt{\frac{\pi}{2}} \frac{Ma_i}{\text{Re}_i}
\]
2.2.2. **Governing equations describing two-phase Euler-Lagrange approach.** In Euler-Lagrange approach, the fluid is considered as a continuous phase while particles are dispersed in it. Therefore, the effect of particles on fluid is encountered as a source term in momentum equation and other conservative equations will be kept the same. As mentioned before, the two following equations can be considered as momentum equations.

\[
\begin{align*}
\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} &= -\frac{\partial P}{\partial x} + \mu \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{1}{3} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} \right) \right] \\
\rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} &= -\frac{\partial P}{\partial y} + \mu \left[ \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial y^2} + \frac{1}{3} \left( \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 u}{\partial x \partial y} \right) \right]
\end{align*}
\]

(12)

(13)

We can write the above momentum equations in the total form of below:

\[
\nabla.(\rho_f V_f V_f) = -\nabla P + \nabla.(\mu_f \nabla V_f)
\]

(14)

Consequently, the momentum equation considering the source-term describing the effect of suspended particles on continues phase is written as below:

\[
\nabla.(\rho_f V_f V_f) = -\nabla P + \nabla.(\mu_f \nabla V_f) + S_p
\]

(15)

Where \( S_p \) in equation (15) is the source term that represents the momentum transfer between the fluid and particles and is found by computing the momentum variation of particles as they pass through the fluid phase control volume, is calculated by:

\[
S_p = \sum m_p F \Delta t
\]

(16)

Where \( m_p \) is the particle mass and \( F \) is the total force per unit particle mass acting on the dispersed phase elements and will be discussed later. Also, \( F \) (total force per unit particle mass) is defined as the rate of momentum change within the control volume.

\[
F = \frac{dV_p}{dt}
\]

(17)

Where \( V_p \) is the particle velocity.

In this paper we have considered gravity, drag and Brownian forces as acting forces on particles. So, we can describe \( F \) as below:

\[
F = F_G + F_D + F_B
\]

(18)

Where \( F_G \) is the gravity force and can be calculated by below formula:

\[
F_G = \frac{(\rho_p - \rho)g}{\rho_p}
\]

(19)

Furthermore, \( F_D \) is the drag force acting on particles and as mentioned before there are several investigations to find a suitable value for Cunningham correction factor that can be used to correct
Stokes drag coefficient for sub-micron particles. Here we will use Davis slip correction factor that can be obtained from below formula:

\[ C_c = 1 + Kn \left[ 1.257 + 0.400 \exp(-1.100/Kn) \right] \]  

(20)

Where \( Kn \) number is one of the most important dimensionless number in microscale channels. It can be defined as below:

\[ Kn = \frac{\lambda}{L} \]  

(21)

\[ \lambda = \frac{kT}{\sqrt{2\pi\sigma^2P}} \]  

(22)

In the equation (21) \( \lambda \) is molecular mean free path and \( L \) can be considered as characteristic length of channel. For two-dimensional plane duct as we have deliberated here, the hydraulic diameter of channel can be considered as channel characteristic length and it is defined as 2H. Also in the definition of molecular mean free path, \( k, T, \sigma \) and \( P \) are Boltzmann Constant, gas temperature, collision diameter of the molecules and gas pressure. Respecting all aforementioned definitions, we will use the following formula in order to calculate drag force acting on particles.

\[ F_D = \frac{18\mu}{d_p^2\rho_pC_c} (V - V_p) \]  

(23)

Where \( d_p, \rho_p, C_c, \mu, V, V_p \) are particle diameter, particle density, Cunningham correction factor, fluid dynamic viscosity, fluid velocity and particle velocity.

Another force that can be considered in our analysis is Brownian force. If the size of suspended particles is almost submicron or less than that, particles motion will be affected by the discrete nature of molecular motion, exhibiting a random motion due to collisions of fluid molecules with the particles. This is called Brownian motion. For sub-micron particles, the effects of Brownian motion can be optionally included in the additional force term. The components of the Brownian force \( F_B \) are modeled as a Gaussian white noise process with spectral intensity \( S_{n,i} \) given by (24):

\[ S_{n,i} = S_0 \delta_{ij} \]  

(24)

Where \( \delta_{ij} \) is the Kronecker delta function, and

\[ S_0 = \frac{216\nu kT}{\pi^2 \rho d_p^2 (\frac{P_p}{\rho})^2 C_c} \]  

(25)

\( T \) is the absolute temperature of the fluid, \( \nu \) is the kinematic viscosity, and \( k \) is the Boltzmann constant. Also amplitudes of the Brownian force components are of the form:

\[ F_{bi} = \zeta_i \sqrt{\frac{2\pi S_0}{d_p}} \]  

(26)

Where \( \zeta_i \) are zero-mean, unit-variance-independent Gaussian random numbers. The amplitudes of the Brownian force components are evaluated at each time step.
2.3. Boundary condition

In this paper slip flow regime \((0.001 \leq Kn \leq 0.1)\) between inlet and outlet of a two-dimensional microscale channel is studied. To analyze this problem slip boundary conditions have to be used besides Navier-Stokes equations. Until now there have been several models in order to examine slip regime. As an example we can state second order slip model [3], Langmuir[6], and a number of other first and second order boundary conditions. Here we will use Maxwell's model for slip velocity and Von Smoluchowski's one for temperature jump. Cartesian forms of these boundary conditions have been shown below:

\[
\begin{align*}
    u_{\text{fluid}} - u_{\text{wall}} &= \frac{2 - \sigma_v}{\sigma_v} \lambda \frac{\partial u}{\partial y} + \frac{3}{4} \frac{\mu}{\rho T} \frac{\partial T}{\partial x} \\
    T_{\text{fluid}} - T_{\text{wall}} &= \frac{2 - \sigma_T}{\sigma_T} \frac{2\gamma}{\gamma + 1} \frac{\lambda}{Pr} \frac{\partial T}{\partial y}
\end{align*}
\]  

(27)  

(28)

Where \(\sigma_v\) is momentum accommodation coefficient and \(\sigma_T\) represents energy accommodation coefficient. Considering \(\sigma_v = 1\) can make a good agreement with a number of experimental works. But all these works have done on smooth surfaces. Therefore here we will choose \(\sigma_v = 0.9\) respecting Ji et al's work [3] in their study on rectangular roughness elements. Also it is necessary to consider some other boundary conditions at inlet and outlet of the channel. We can state these boundary conditions as bellow:

\[
\begin{align*}
    \text{at } X = 0 & \quad U = 1, \quad V = 0, \quad \theta = 1, \\
    \text{at } X = 1 & \quad \frac{\partial U}{\partial X} = 0, \quad \frac{\partial V}{\partial X} = 0, \quad \frac{\partial \theta}{\partial X} = 0, \quad P^{*} = \frac{P_{\text{out}}}{P_j}
\end{align*}
\]  

(29)

In addition, to reduce computational work, only one half of the channel has been considered as a result of symmetrical conditions that exists at \(Y=H/2\).

Equations (27) & (28) for slip and temperature jump apply to flat surfaces in non-rotating domains. As it has been mentioned by other investigators, neglecting the effect of surface curvature or rotating motion on slip behavior leads to false evaluation. Therefore introducing modifications for curved surfaces, following formulas have been stated:

\[
\begin{align*}
    u_{\text{fluid}} - u_{\text{wall}} &= \frac{2 - \sigma_v}{\sigma_v} \lambda \left( \frac{\partial u}{\partial n} + \frac{\partial u}{\partial t} \right) + \frac{3}{4} \frac{\mu}{\rho T} \frac{\partial T}{\partial t} \\
    T_{\text{fluid}} - T_{\text{wall}} &= \frac{2 - \sigma_T}{\sigma_T} \frac{2\gamma}{\gamma + 1} \frac{\lambda}{Pr} \frac{\partial T}{\partial n}
\end{align*}
\]  

(30)  

(31)

Where \(n\) and \(t\) stand for normal and tangential to surface at each point of the wall.
Numerical Simulation Procedure and validation

Our simulation is based on five cases as shown in Table 1. As stated before, rarefaction, compressibility and roughness play important roles in gaseous flows and will result in varied flow characteristics and heat transfer behavior. Here we have removed compressibility effect by choosing inflow Reynolds number for all cases less than 1. We have considered five sub-groups for each of our five cases. On the other hand, we have divided our five cases into five different parts respecting the inflow Reynolds number. Also for each of these cases (except of case 1 and 4) we have examined three various wall roughnesses.

Furthermore, Air has been chosen as the working gas. For all cases the inlet flow temperature is 250 K and the constant-wall-temperature has been considered as 350 K. The injection volume fraction in our different cases varies from 0 to 10% according to studies situation. Where the volume fraction has been considered as the ratio of injected nanoparticles volume to the total volume of duct. Also the influence of these values of Ferro-nanoparticles injection has been studied on heat transfer enhancement. Constant properties of Ferro-nanoparticles have been encountered as 447 /KgK, 80.4 /mKW and 7870 /m3Kg for specific heat, thermal conductivity and density. Nanoparticles diameter have been chosen as 50 nm for all different cases and they are released from inflow in continuously-distributed way.

Aforementioned governing equations with suitable boundary conditions have been solved by employing the SIMPLE algorithm, a finite volume method. The entire microchannel length has been taken as the computational domain to study the effect of roughness and Ferro-nanoparticles injection on heat transfer enhancement. The coupling between fluid and nanoparticles is two-way and discrete phase elements are updated at every flow iteration. Because of triangular shapes of the roughness elements, triangular meshes have been used all over the computational domain. Also the grids are refined near the wall region to obtain highly accurate numerical solutions around the roughness elements. To evaluate the grid size effect on the accuracy of numerical solutions, grid-independence tests have been performed in the way that grid size has been made refined until acceptable differences between the last two grid sizes were found. Four different grid sizes (480×64, 280×48, 160×32 and 128×24) have been examined. For a model with Kn=0.0033 and relative roughness=1.25% the maximum differences in h (heat transfer coefficient) were found to be 0.75%, 1.8% and 4%. By balancing between the computation time and cost besides accuracy, the grid size 280×48 has been selected.

Table 1. Details of five cases using in this study

| Cases | L/H | Re      | Kn  | Relative roughness (%) | Peak density | Injection volume fraction (%) |
|-------|-----|---------|-----|------------------------|-------------|------------------------------|
| 1     | 20  | 0.1, 0.3, 0.5, 0.7, 1 | 0.0033 | Smooth | - | Without Injection, 0.5 |
| 2     | 20  | 0.1, 0.3, 0.5, 0.7, 1 | 0.0033 | Smooth, 1.25, 2.5 | 10, 50 | Without Injection |
| 3     | 20  | 0.1, 0.3, 0.5, 0.7, 1 | 0.0033 | Smooth, 1.25, 2.5 | 10, 50 | Without Injection, 0.5 |
| 4     | 20  | 0.1, 0.3, 0.5, 0.7, 1 | 0.0033 | 1.25 | 10, 50 | Without Injection, 0.5, 3, 10 |
| 5     | 20  | 0.1, 0.3, 0.5, 0.7, 1 | 0.0033, 0.01, 0.033, 0.067 | Smooth, 1.25 | 10 | Without Injection |
To demonstrate the validity and precision of the model and numerical procedure, comparison with the reported numerical and experimental data has been done. It is necessary to mention it that respecting to our studies there is not any previously-reported data which can cover the effect of roughness and nanoparticles injection on heat transfer. Therefore we have used the available data about examining the influence of slip boundary condition on heat transfer of single-phase micro-channel by Hadjiconstantin and Simek [2] to almost validate our numerical results. Figure 3 shows graphical conclusions that are calculated by us in comparison with Hadjiconstantin and Simek’s results. The range of numbers and process for both of them are almost the same.

![Figure 3. Variation of fully-developed Nusselt number with Knudsen number](image)

3. Results and discussion
In this section the effect of wall roughness and Ferro-nanoparticles injection on heat transfer of rarefied, incompressible flows are investigated. In order to have a better look at influences of these parameters, the related results have been presented in four distinct parts.

3.1. Effect of Ferro-nanoparticles injection on smooth rarefied flow
To study heat transfer the most important factor is dimensionless Nusselt number. Here the most conventional definition of Nusselt number is followed as equation below:

$$ Nu = \frac{q'' D_h}{(T_w - T_b)k} $$

(32)

Where $q''$, $D_h$, $T_w$, $T_b$, $k$ are heat flux, hydraulic diameter, wall temperature, gas bulk temperature and thermal conductivity.

Also we can define hydraulic diameter for two-dimensional plane duct as below:

$$ D_h = \frac{4(\text{cross section area})}{(\text{cross section perimeter})} = 2H $$

(33)

In addition, we calculate bulk temperature by using below equation:
\[ T_b = \frac{\int_0^h \rho u T \, dy}{\int_0^h \rho u \, dy} \]  
(34)

Where \( \rho \) is the gas density.

Figure 4 shows the heat transfer enhancement of rarefied micro-channel (Kn=0.0033) by injecting of Ferro-nanoparticles (volume fraction= 0.5%). As we are considering gas-solid two-phase flow, we have to know the equivalent thermal conductivity of this two-phase flow to calculate Nusselt number. According to our studies there is no information about equivalent thermal conductivity of micro-channels. Existed results are related to macro-channels that in this area most of the current investigations have been considering the equivalent thermal conductivity as a combination of two single phase thermal conductivity or finding more accurate results by experimental methods [14, 15]. As we could not find suitable equivalent thermal conductivity we have presented the influence of nanoparticles injection on heat transfer enhancement in the form of \( h \) (convective heat transfer coefficient). It is clear in the figure 4 that we can have improvement in heat transfer by using constant volume fraction as a result of increasing Re number. In addition we should pay attention to this fact that increasing Re number do not affect heat transfer in single-phase significantly.

Figures 5&6 show the internal temperature distribution of previously-described microchannel at Re=1. As it has shown in the figures, We can have a smoother temperature distribution by using nanoparticles injection. Also as a result of heat transfer enhancement concept the gas bulk temperature has been increased at output from 289.69 to 315.064.

Figure 5. Distribution of microchannel interior temperature (K) without injection at Re=1
Figure 6. Distribution of microchannel interior temperature (K) with injection (0.5%) at Re=1

Figs. 7&8 show the internal relative pressure distribution of previously-described microchannel at Re=1. According to the figs, it can be clear that with small amount of injection (as an example volume fraction=0.5%) we will not have a big change in pressure drop. But still we have a bit earlier start in decreasing of the stream-wise pressure due to nano-particles injection.

Figure 7. Distribution of microchannel interior relative pressure (Pa) without injection at Re=1

Figure 8. Distribution of microchannel interior relative pressure (Pa) with injection (0.5%) at Re=1

3.2. Effect of Ferro-nanoparticles injection on rough rarefied flow

Here we want to study wall roughness on heat transfer and then investigating combination of both roughness and Ferro-nanoparticles injection on Heat transfer. First it is better to have a conclusion about wall roughness on heat transfer. As figure 9 Shows, increasing wall roughness can improve heat transfer in an effective way. The only exception is peak density=10 and relative roughness of 1.25%. In this case we can see a reduction in heat transfer due to wall roughness. It seems logical considering the geometry of problem. Since the number of elements are not a lot. Increasing of relative roughness can make a hole-like shape at wall. Therefore this can make a number of preventive circulations inside the hole- shape. As a conclusion these probable circulations can prevent from wall heat flux penetration through flow and clear decrease in heat transfer. Figure 10 represents one of these preventive circulations. But totally we can have a conclusion which increasing wall roughness in an
optimized way can improve heat transfer and an increase in numbers of wall roughness (peak density) will result in a better heat transfer enhancement in comparison with increasing of relative roughness.

Figure 9. Variation of heat transfer coefficient with Re number as a result of roughness increase

Figure 10. A sample of hole-shape roughness elements and preventive circulations at peak density=10 & relative roughness=2.5%

Now we want to examine what we have pondered in the previous section. It means that studying the influence of Ferro-nanoparticles injection on heat transfer. But the difference is studying injection effect on rough-wall cases instead of smooth-wall ones.

Figure 11 presents some information on the discussed issue. As it has been shown, like smooth-wall cases, injection of Ferro-nanoparticles can improve heat transfer. But there is an obvious difference with smooth-wall cases. Rough-wall cases show more resistance to the effect of nanoparticles injection. It means we cannot have heat transfer enhancement related to nanoparticles injection as before. Also another issue which can absorb our attention is comparing two various cases, relative roughness of 1.25% and 2.5% besides peak density of 50. As it has been shown in the figure, for both cases without injection heat transfer coefficient is the same. But since increasing wall roughness can cause more resistance to heat transfer improvement with nanoparticles injection, case with relative roughness of 1.25% and peak density of 50 shows a better improvement as a result of injection in comparison with relative roughness of 2.5%. Therefore, finally we have a better heat transfer coefficient in the mentioned case (relative roughness=1.25% and peak density=50).
The last studied factor in this paper is considering rarefaction effect on heat transfer. As we have stated in the section 2.4., Hadjiconstantin and Simek's results show that with increasing rarefaction effect we will have a reduction in Nusselt number and therefore in \( h \) (heat transfer coefficient). Here we want to consider roughness effect and rarefaction together. According to our studies, roughness in an optimized way can increase heat transfer coefficient although the increasing of rarefaction shows a reverse effect. It means we have an increase as a result of roughness and a decrease due to rarefaction progress. It is clear that the final result must be as a result of superposition of these two effects. According to our investigations, roughness plays a more important role by increasing rarefaction. On the other hand, increasing heat transfer because of a constant roughness will increase by rarefaction progress. Consequently, decreasing heat transfer due to increasing rarefaction will be hidden behind a huge increase in heat transfer coefficient because of roughness effect. Figure 12 shows the discussed issue clearly.

**Figure 11.** Effect of Ferro-nanoparticles injection on smooth and rough-wall microchannels. Density=10 & relative roughness=2.5%

**Figure 12.** Effect of wall roughness and rarefaction together

### 4. Conclusions
The effect of wall roughness and Ferro-nanoparticles injection on heat transfer of rarefied gas flows were investigated and the following conclusions have been reached:

- Nanoparticles injection can cause an absolute increase in the heat transfer coefficient of smooth-wall cases.
Increasing roughness in an optimized way can result in the increase of heat transfer coefficient. Although in some cases as a result of recirculation, roughness will cause a heat transfer reduction, totally it can help heat transfer.

Rough-wall cases in comparison with smooth-wall ones show more resistance to increasing heat transfer coefficient as a result of nanoparticles injection. Therefore, we can have a better enhancement in the less roughness.

Increasing rarefaction will cause a decrease in the heat transfer. Also, roughness will play a more important role with increasing rarefaction in the heat transfer enhancement. Therefore, we will have a huge increase in the heat transfer as a result of increasing rarefaction in a constant roughness.

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