The deconfining phase transition in SU($N_c$) gauge theories

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We report on our ongoing investigation of the deconfining phase transition in SU(4) and SU(6) gauge theories. We calculate the critical couplings while taking care to avoid the influence of a nearby bulk phase transition. We determine the latent heat of the phase transition and investigate the order and the strength of the transition at large $N_c$. We also report on our determination of the critical temperature expressed in units of the string tension in the large $N_c$ limit.

1. INTRODUCTION

The large-$N_c$ physics of QCD and SU($N_c$) gauge theories is of great theoretical and phenomenological interest. Lattice calculations of string tensions, glueball masses etc.\cite{1} have confirmed that a smooth large-$N_c$ limit is achieved by keeping fixed the 't Hooft coupling $g^2 N_c$ and that, as expected, the leading corrections are $O(1/N_c^2)$.

An interesting question concerns the order of the deconfining phase transition at $N_c \to \infty$. It has been argued\cite{2} that the transition in SU(3) may be accidently first order, due to a cubic term in the effective potential, but in general is second order in SU($N_c$). Our ongoing lattice study\cite{3}, on which we report here, is addressing this question and thereby improving on earlier results\cite{4,5}.

We use the standard plaquette action on $L^3 \times L_t$ lattices with periodic boundary conditions. The deconfining phase transition is studied by keeping $L_t$ fixed and varying $\beta = 2N_c/g^2$ so as to pass the temperature $T = 1/a(\beta)L_t$ through the transition.

In order to distinguish the confining and the deconfining phases we define the spatial average $\bar{l}_p$ of the Polyakov loop and correspondingly the spatial average $\bar{u}_p$ of the plaquette variable $\text{Tr} U_p$.

For a first order transition, e.g., we should find tunneling between the phases near the critical temperature $T_c$ and subsequently a double peak structure in the probability distributions of $\bar{l}_p$ and $\bar{u}_p$.

In order to determine the critical couplings $\beta_c(V)$ at finite volume $V = L^3$ we define a normalised Polyakov loop susceptibility

$$\chi_l/V = (|\bar{l}_p|^2) - (|\bar{l}_p|)^2$$

and, for each volume $V$, obtain $\chi_l$ as a continuous function of $\beta$ using standard reweighting techniques\cite{6}. Then we determine $\beta_c$ from the location at which the susceptibility has its maximum.

In complete analogy we can define the specific heat $C(\beta)$ from the average plaquette $\bar{u}_p$,

$$\frac{1}{\beta^2} C(\beta) = \frac{\partial}{\partial \beta} \langle \bar{u}_p \rangle = N_p \langle \bar{u}_p^2 \rangle - N_p \langle \bar{u}_p \rangle^2,$$

where $N_p = 6L^3 L_t$ is the total number of plaquettes. The value of $\beta$ where $C$ has its maximum provides a different definition for a critical coupling which however should agree with the one from $\chi_l$ in the thermodynamic limit. Moreover we can use the finite size scaling behaviour of the peak value $C(\beta_c, V)$ as a criterion for the determination of the order of the transition. To be more
is first order, as ours clearly is, then the leading
to infinite volume. We note that if the transition
temperature
inverse spatial volume expressed in units of the

\[ \beta \]

for a first order transition.

\[ \beta \]

hibit clear double peaking which is characteristic
probability distributions of these quantities ex-

\[ \beta \]

2.

An important point we need to address is the
influence of a bulk phase transition which can be
close to the physical deconfining transition. Our
choices of \( L_t \) for the different gauge groups ensure that the two transitions are well separated and
can unambiguously be distinguished.

\[ \beta \]

finite-V correction should be

\[ \beta_c(V) = \beta_c(\infty) - \frac{h}{V^{1/3}}, \]

where \( h \) is independent of the lattice action. We find that all our values of \( \beta_c \) are consistent with eq. (1) and we obtain \( \beta_c(\infty) = 10.63709(72) \) in the \( V \to \infty \) limit. Moreover the value of \( h = 0.09 \pm 0.02 \) we thus extract is close to the SU(3) value, \( h \simeq 0.1 \) [7]. This suggests that \( h \) depends only weakly on \( N_c \) and we will therefore use this value to extrapolate to \( V = \infty \) in the SU(6) case where we do not perform an explicit finite volume study.

From \( \beta_c(\infty) \) we obtain the critical temperature \( T_c \) in terms of the string tension \( \sigma \) [8],

\[ T_c \sqrt{\sigma} = 0.6024 \pm 0.0045 \quad \text{at } a = 1/5T_c. \] (2)

Our preliminary \( L_t = 6 \) calculations give a value \( \beta_c(\infty) = 10.780(10) \) which translates to \( T_c/\sqrt{\sigma} = 0.597(5) \). This, taken together with the value in eq. (2), yields the extrapolated continuum value

\[ \lim_{a \to 0} \frac{T_c}{\sqrt{\sigma}} = 0.584 \pm 0.030 \quad \text{in SU(4)}. \] (3)

In fig. 2 we plot \( C(\beta_c)/\sqrt{\sigma} \) against \( 1/\sigma \) and note that the scaling is certainly consistent with a first order one. The intercept at \( 1/\sigma = 0 \) provides a measure of the latent heat, \( \Delta c \), since

\[ \Delta c = \left( \bar{u}_{p,c} - \langle \bar{u}_{p,d} \rangle \right)^2 = \Delta c^2, \]

where \( \bar{u}_{p,c} \) and \( \bar{u}_{p,d} \) are the average plaquette
values at \( \beta_c \) in the confined and deconfined phases, respectively. We obtain \( \Delta c(N_c = 4) = 0.0197(5) \) and observe that if we take the \( L_t = 4 \) SU(3) latent heat in [8] and naively scale it to \( L_t = 5 \), we obtain \( \Delta c(N_c = 3) \simeq 0.0013 \) which is substantially smaller than our above SU(4) value. This shows explicitly that the SU(4) transition is more strongly first order than the SU(3) one.

3. \( T_c \) IN SU(6)

In SU(6) we perform a study on a \( 16^3 \times 6 \) lattice at values of \( \beta \) close to \( \beta = 24.845 \). In physical
units this is like a $14^3$ lattice with $L_t = 5$ and thus we are confident from our SU(4) calculations to be close to the thermodynamic limit.

As before we observe well defined tunneling between confined and deconfined phases, characteristic of a first order phase transition. Using the reweighting technique we extract a critical value $\beta_c = 24.855 \pm 0.003$ which translates into

$$\frac{T_c}{\sqrt{\sigma}} = 0.582(15) + 0.43(13) N_c^2.$$  \hspace{1cm} (6)

Finally, we conclude that the SU($N_c$) transition at $N_c = \infty$ is first order and not particularly weak.

4. CONCLUSIONS

We find that SU(4) gauge theory at $L_t = 5$ reveals a first order deconfining phase transition with a latent heat that is not particularly small. We also find evidence that the SU(6) transition at $L_t = 6$ is first order as well and certainly not weaker than the one in SU(4). We come to that conclusion by comparing the appropriately rescaled latent heat expressed in terms of $T_c$ for the different gauge groups. In doing so we take care to avoid confusion with a nearby bulk phase transition which, on our lattices, can unambiguously be identified.

The $N_c$-dependence of the critical temperature when expressed in units of the string tension, $T_c/\sqrt{\sigma}$, appears to be weak. From a fit we obtain

$$\frac{T_c}{\sqrt{\sigma}} = 0.582(15) + 0.43(13) N_c^2.$$  \hspace{1cm} (6)

REFERENCES

1. B. Lucini and M. Teper, JHEP 0106 (2001) 050 [hep-lat/0103027].
2. R. Pisarski and M. Tytgat, hep-ph/9702340.
3. B. Lucini, M. Teper and U. Wenger, hep-lat/0206023.
4. M. Wingate and S.Ohta, Phys. Rev D63 (2001) 094502 [hep-lat/0006016].
5. R. Gavai, Nucl. Phys. B633 (2002) 127 [hep-lat/0203015].
6. A. M. Ferrenberg and R. H. Swendsen, Phys. Rev. Lett. 63, 1995 (1989); \textit{ibid}. 61, 2635 (1988).
7. B. Beinlich, F. Karsch, E. Laermann and A. Peikert, Eur. Phys. J. C6 (1999) 133 [hep-lat/9707023].
8. N. Alves, B. Berg and S. Sanielevici, Nucl. Phys. B376 (1992) 218 [hep-lat/9107002].
9. L. Del Debbio, H. Panagopoulos, P. Rossi and E. Vicari, JHEP 0201 (2002) 009 [hep-th/0111090]; Phys.Rev. D65 (2002) 021501 [hep-th/0106185].