Modelling and analysis of surface topography generated in end milling process

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Abstract

The surface topography of workpiece plays an important role in the performance and service life of workpiece. Complex surface parts are widely used in shipbuilding, aerospace and other industries. At present, the study of milling surface topography is mainly on 3-axis milling. A prediction model of milling surface topography is proposed, which can obtain the machined workpiece surface topography and roughness directly from cutting parameters, cutter location file and workpiece surface geometry. The effects of cutting parameters on surface roughness is discussed. Different milling experimental conditions are set up to validate the proposed model. This method can be used to analyze the surface topography of milling, and further to optimize the cutting parameters to improve the surface quality.

Keywords: End milling; Surface topography; Surface roughness

1. Introduction

The surface topography has an important impact on the performance and fatigue of the workpiece [1,2]. The prediction of milling surface topography is necessary, which is the theoretical basis of cutting parameter optimization. With the development of intelligent manufacturing, data processing method is used to analyze cutting characteristics, such as cutting force, cutting temperature and surface topography. Khorasani et al.[3] used artificial neural networks method to analysis the surface roughness, and the data is obtained by designed experiments. Liu et al.[4] used multi-sensor information to reconstruction of 3D surface topography modeling. Ngermontg et al.[5] used Taguchi method and experimental results to analysis and modeling the machined surface roughness. Chen et al.[6] designed orthogonal regression experiment method to predict the micro-milling surface roughness. In previous works[3-6], the surface topography and surface roughness prediction model can be obtained by analyzing and processing the experimental data.
This method is used in specific material and cutting parameters, which is impossible to analyze the formation process of milling surface topography theoretically.

By analyzing the motion characteristics of milling process, the surface residual height after machining forms surface topography. The geometry of complex surface is difficult to be expressed analytically, and the discrete method is usually used, such as z-map method, z-buffer method. Z-map method is used to calculation the tool-workpiece engagement in ball end milling [7]. Gao et al. [8] used z-map method to simulate the machined surface topography and roughness in ball end milling processes. Similarly, Zhang et al. [9] developed a improved z-map method to modeling the surface topography in ball end milling. Based on the proposed model, the effect of cutting parameters on the surface topography is discussed. B. Denkena et al.[10] simulate the surface topography in five-axis milling and the tool’s dynamic excitation is considered. Peng et al.[11] developed the surface topography based on the cutting edge motion trajectory model, and the effect of inclination angle is discussed. Liu et al.[12] developed the prediction model of the surface topography in ball screw whirling milling. Zhang et al.[13] proposed a new surface topography model which considered the cutter deflection in micro-side milling. Kasim et al.[14] used experimental results to analysis the surface topography in ball nose end milling process, the difficult-to cut materials and tool wear are considered in the model. Arizmendi et al.[15] proposed the model of face milling surface topography based on the cutting edge geometry and trajectory. Wang et al.[16] developed 3D surface topography of ruled surface milling, and the complex workpiece surface is described by point cloud. Xu et al.[17] present a surface topography model for a ball-end cutter, and the feed rate is considered in high speed milling. Torta M et al.[18] developed a framework model for estimating the surface texture in complex surface milling process.

The above studies is mainly on ball-end mill and 3-axis milling. The study of surface topography model with end mill is universal and necessary. The modeling of milling surface topography is introduced in Section 2. In order to analysis the effect of cutting parameters on surface topography and roughness, and validate the proposed model, experiment and results are discussed in Section 3. In section 4, the conclusion of this study is summarized.

2. Theoretic modeling of surface topography

In the milling process, the residual height in the feed direction and intermittent feed direction after milling will have an impact on the machined surface topography. Therefore, it is necessary to establish the point cloud swept by the cutting edge in the milling process of end milling according to the geometric model of milling cutter, milling process kinematics and tool path file in the milling process. The point cloud formed in the process updates the discrete points of the
workpiece surface, and finally obtains the machined surface topography.

2.1 Geometry model of the torus end mill

The geometric model of torus end mill is shown in Fig.1. \( D \) is diameter, \( r \) is the bottom radius, \( r(z) \) is the filleted radius, and point \( O_T \) is the tool center. As shown in Fig. 1c, the effective cutting radius of point \( P \) can be calculated based on the axial position:

\[
r(z) = D - r + \sqrt{r^2 - (r - z)^2}
\]

(1)

![Fig.1. Geometric model of the torus end mill [19]](image)

For circular inserted cutter, the helix angle of the cutting edge of the arc segment is not a fixed value. The helix angle of element \( P \) can be expressed as the following equations:

\[
i(z) = \cot \left( \tan^{-1} \left( \frac{r(z) - (D/2 - r) \tan i_0}{r} \right) \right)
\]

(2)

where \( i_0 \) is maximum helix angle.

The radial lag angle \( \psi(z) \) and axial immersion angle \( \kappa(z) \) as follows:

\[
\psi(z) = \frac{z}{r} \tan i(z), \quad 0 \leq z < r
\]

(3)

\[
\kappa(z) = \arccos \left( \frac{r - z}{r} \right), \quad 0 \leq z < r
\]

(4)

The radial immersion angle of point \( P \) at level \( z \) can be calculated:
\[ \phi_j(z, \theta) = \theta + (j-1) \frac{2 \pi}{N} - \psi(z) \]  

(5)

where \( \theta \) is tool rotation angle and \( N \) is cutter edge number.

2.2 The movement track of discrete points of cutting edge

By analyzing the influence of tool size, tool movement relative to workpiece, tool inclination angle and different cutting edges on the trajectory of discrete points, the trajectory equation of each discrete point in the workpiece coordinate system in the machining process is established. The movement track of cutting edge of torus-end milling on the \( m \)-th cutting path is shown in Fig.2. Fig. 2a shows the projection of the motion track of the point on the cutting edge on the second cutting path on the plane. Because the points on the cutting edge rotate around the cutter axis and move along the feed direction in the milling process, the trajectory of the points on the cutting edge is asymmetric. When the torus end mill moves along the feed direction of the first cutting path, the points on the cutting edges 1 and 2 will sweep away the material on the workpiece. Suppose that in the process of cutting, the cutting edge 1 cuts the workpiece before the cutting edge 2, and the residual material after cutting the cutting edge 1 will be cut again by the cutting edge 2 (as shown in Fig.2b). After cutting in the first cutting path, the residual height on the workpiece surface will be cut off by the cutting edge in the next cutting path. Therefore, residual height in the feed direction is formed by the sweeping tracks of four cutting edges( \( L_1^n, L_2^n, L_1^{n+1}, L_2^{n+1} \)), which are on two cutting paths. As shown in Fig.3, the residual height in the feed direction is formed by the sweeping cutting on two cutting paths. The position with filling color is the position of feed residual height. It can be clearly seen from Fig.3 that the residual height is formed by cutting four cutting edges on two adjacent cutting paths.
2.3 Simulation of surface topography

In the process of milling surface topography simulation, milling tool workpiece contact judgment is an important process. The determination of the contact area is a dynamic process that needs to be updated with the cutting tool and workpiece surface. As shown in Fig.4, the projection point of the cutting edge looks for the point closest to the point on the grid to update the height Z value. Therefore, if the size of the discrete points of the cutting edge is inconsistent with the size of the grid points, the Z vector of some grid points cannot be updated.
In the simulation algorithm, Z-map method is mainly used to judge the cutting edge of the milling tool and update the workpiece geometry during the milling process. As shown in Fig.5, The meshing point \((x_i, y_j)\) is determined by:

\[
\begin{align*}
X_i &= X_0 + i\Delta X \\
Y_j &= Y_0 + j\Delta Y
\end{align*}
\]

(6)

The \(Z_g\) coordinate of each meshing point represents the height of the workpiece:

\[
Z_g = f(X_i, Y_j)
\]

(7)

A set of coordinate systems are established, as shown in Fig. 5. \(O_w - X_w Y_w Z_w\) represents a workpiece coordinate system, \(O - X_f Y_f Z_f\) represents local tool coordinate system, \(OZ_f\) is along the main shaft, \(OX_f\) is the direction of two cutting edges tangent direction at point \(O_f\), \(OY_f\) determined by \(OX_f\) and \(OZ_f\).

In the simulation, the cutter was discretized into \(k\) disks, the \((x_c, y_c, z_c)\) at \(k\) disks at local coordinate system \(O - X_f Y_f Z_f\) can expressed by the following:

\[
\begin{align*}
x_c &= r \cos(\varphi) \\
y_c &= r \sin(\varphi) \\
z_c &= (k - 1)dz
\end{align*}
\]

(8)

where \(\varphi = \omega t + (j - 1)\times\phi_p - \frac{z_c}{R} \tan(i_p)\), \(\omega = 2\pi n / 60\).

In the proposed method, the tool orientation and position are obtained from the CL file. The cutter tip orientation and position in the tool path files is represent as \((x, y, z, \alpha, \beta, \gamma)\) in the \(O - XYZ\) system, the \((x, y, z)\) represent the position and the \((\alpha, \beta, \gamma)\) represent the orientation. The tool axis vector \((x_t, y_t, z_t)\) is:
\begin{align*}
zt &= \mathbf{n}_2 \\
yt &= (\mathbf{n}_1 \times \mathbf{n}_2) / \| \mathbf{n}_1 \times \mathbf{n}_2 \| \\
xt &= (yt \times zt) / \| (yt \times zt) \| 
\end{align*}
\tag{9}

where \( \mathbf{n}_1 = [0, 0, 1]^T \), \( \mathbf{n}_2 = [\alpha, \beta, \gamma]^T \).

The rotation matrix \( \mathbf{R}_t \) and transform matrix \( \mathbf{T}_t \) is following:
\[ \mathbf{R}_t = [xt, yt, zt] \]
\[ \mathbf{T}_t = [x, y, z]^T \]
\tag{10}

Consequently, the overall transformation matrix \( \mathbf{M} \) from \( OXyZ \) to \( wXyZ \) is written as:
\[ \mathbf{M} = \begin{bmatrix} \mathbf{R}_t & \mathbf{T}_t \\ 0 & 1 \end{bmatrix} \]
\tag{11}

The general form of the trajectory equation of point \( P \) in \( OXyZ \) is:
\[ P_w = \mathbf{M} \times [x, y, z, 1]^T \]
\tag{12}

Assuming that the height of the point on workpiece surface corresponding to \( (X_i, Y_i) \) is \( Z_{ij} \), the scallop height of \( z \) after material removal is following:
\[ \text{If } P_w(z) < Z_{ij}, Z = P_w(z); \text{ If } P_w(z) > Z_{ij}, Z = Z_{ij} \]
\tag{13}

Surface roughness is the arithmetic mean deviation of contour (within the sampling length, the average of the absolute
value of the distance from each point of on the actual contour to the contour center line). Through the height z-value of the workpiece surface, the z-value can extract within a sampling length, and the roughness can be calculated.

The z-value height on the machined workpiece surface is \( i, (i = 1, ..., n) \), the average value is:

\[
\bar{t} = \frac{1}{n} \sum_{i=1}^{n} t_i
\]  

(14)

The roughness is the arithmetic mean deviation of contour \( R_a \)

\[
R_a = \frac{1}{n} \sum_{i=1}^{n} (t_i - \bar{t})
\]  

(15)

As shown in Fig. 6, in the whole milling profile simulation system, tool path file (including tool position and posture), spindle speed and feed speed, geometric parameters of tool and workpiece are included. In the preparation stage of simulation, the coordinates of discrete points are used to represent the workpiece surface. The next step is to use these input parameters for simulation. In the simulation process, the main task is to check the tool workpiece contact, that is, to determine whether the cutting edge of the tool has cut to the workpiece surface. If the workpiece has been cut to the cutting edge, the Z value of the grid point needs to be updated to the surface height value after cutting. Repeat the above process until the whole workpiece has been cut, and the final calculation will be made. The specific calculation steps are as follows:

1. Input the tool path, cutter location file, the tool geometry parameters, workpiece surface geometry.
2. Divide the workpiece in plane, and discrete point file stored in the initialization matrix.
3. Calculated the discrete points on the cutting edge in the workpiece coordinate system.
4. Calculate and update the cutter scallop height for the next cutting edge.
5. Repeat (3) and (4) steps for each cutting edge.
6. Repeat (3), (4), and (5) steps for each cutting point.
7. Output the surface topography.
3. Simulation and experiment results

3.1 Experiment set up

The dry milling cutting test is shown in Fig. 7. The workpiece is nickel-aluminium bronze[21]. The carbide cutter is Sandvik Coromant 1B230-1200-XA 1630, 2 flutes, 6mm radius, normal rake angle is 5°, 30° helical angle. Cutting forces were measured using a Kistler Dynamometer 9257B. The machine tool is 5-axis vertical machine.

To validate the surface topography model, the cutting parameters is set to obtain large roughness for observation obvious. It is carried out on a block of 100 mm length, 10mm width and 10 mm height, which is divided into seven areas in the length direction and processed with different processing parameters. After all milling is completed, the three-dimensional surface topography is measured by the keinsys vh-m100 micro measurement system. The keinsys ultra depth of field micro measurement system can reconstruct, display and measure the undulation of the surface. Therefore,
the images before and after the experimental processing can be easily compared, and the effectiveness of the simulation software can be tested. Surface roughness Ra is measured by surface roughness test device (Mitutoyo SJ-210).

![Milling center](Milling center.png)![Workpiece](Workpiece.png)![Dynamometer](Dynamometer.png)

(a) Milling Experiment setup (b) SJ210 Roughness measuring instrument

Fig. 7 Milling experiment setup and Roughness measuring instrument

3.2 The effect of cutting parameters

Table 1 is the different spindle speed in milling. Ra is the roughness of the surface which is measured by instrument. In Table 1, the surface roughness of the workpiece after cutting under different spindle speeds parameters is shown. Fig. 8 is the feed direction roughness with different spindle speed. The feed direction roughness is decrease with the spindle speed increase. Since the spindle speed increase, intermittent cutting time between the two cutting edges decrease, the more conducive to the formation of relatively small surface residual height.

| No. | spindle speed (r/min) | Feed speed (mm/min) | Cutting depth (mm) | Lead angle (°) | Tool radius (mm) | Residual height (mm) | Ra in feed direction |
|-----|-----------------------|---------------------|--------------------|---------------|------------------|----------------------|---------------------|
| 1   | 500                   | 800                 | 1                  | 10            | 6                | 0.05                 | 1.983               |
| 2   | 600                   | 800                 | 1                  | 10            | 6                | 0.05                 | 1.901               |
| 3   | 700                   | 800                 | 1                  | 10            | 6                | 0.05                 | 1.356               |
| 4   | 800                   | 800                 | 1                  | 10            | 6                | 0.05                 | 1.331               |
Table 2 is the different feed speed in milling. Fig. 9 is the feed direction roughness with different feed speed. As shown in Fig. 9, the roughness value in the milling feed direction is increasing with the increase of milling feed speed. But in the actual production, the lower the feed rate is not the better, because in the actual production, not only the surface quality of the workpiece after processing is concerned, but also the processing efficiency is concerned. Choosing a lower milling feed rate is conducive to improving the surface quality, but it will reduce the production efficiency.

Table 2  the different feed speeds in milling

| No. | spindle speed (r/min) | Feed speed (mm/min) | Cutting depth (mm) | Lead angle (°) | Tool radius (mm) | Residual height (mm) | Ra in feed direction |
|-----|-----------------------|---------------------|-------------------|----------------|-----------------|---------------------|---------------------|
| 1   | 800                   | 200                 | 1                 | 10             | 6               | 0.05                | 0.528               |
| 2   | 800                   | 300                 | 1                 | 10             | 6               | 0.05                | 0.651               |
| 3   | 800                   | 400                 | 1                 | 10             | 6               | 0.05                | 0.710               |
Table 3 is the different lead angle milling tests and the roughness in feed direction. Fig. 10 is the surface roughness results after milling with the different incline angle. When the tool inclination angle is 0 degree, the cutter tip velocity is zero, which lead to the poor surface quality. A suitable tool inclination can improved the tool life, surface quality significantly. The roughness decreases rapidly with tool inclination angle increase. And when the tool inclination angle increased to 15 degree, roughness changed slowly. When the inclination angle is increase to 20deg, the roughness increase. In the milling process, the tool elastic deformation is increase with the tool-surface inclination increase. An inclined angle of 10 degree to 15 degree is suitable to obtain a good surface quality after milling.

Table 3 The inclination angle effect on surface topography and roughness test

| No. | spindle speed (r/min) | Feed speed (mm/min) | Cutting depth (mm) | Lead angle (°) | Tool radius (mm) | Residual height (mm) | Ra in feed direction |
|-----|-----------------------|---------------------|-------------------|---------------|-----------------|----------------------|----------------------|
| 1   | 700                   | 800                 | 1                 | 0             | 6               | 0.05                 | 5.602                |
|   |  |  |  |   |  |  |  |
|---|---|---|---|---|---|---|---|
| 2 | 700 | 800 | 1 | 5 | 6 | 0.05 | 1.706 |
| 3 | 700 | 800 | 1 | 7.5 | 6 | 0.05 | 1.430 |
| 4 | 700 | 800 | 1 | 10 | 6 | 0.05 | 1.387 |
| 5 | 700 | 800 | 1 | 12.5 | 6 | 0.05 | 1.374 |
| 6 | 700 | 800 | 1 | 15 | 6 | 0.05 | 1.376 |
| 7 | 700 | 800 | 1 | 20 | 6 | 0.05 | 1.379 |
| 8 | 700 | 800 | 1 | 25 | 6 | 0.05 | 1.531 |

Fig. 10 The surface topography after milling with the different incline angle

Fig. 11 shows the simulated surface topographies with different incline angle. The roughness decrease and the workpiece surface is more smooth when the incline angle increase. Since inclination angle can improve the surface quality, five-axis is used widely in precision milling of free surface.
Fig. 11 Simulated results in the ball end milling process with different inclination angle
3.3 Surface topography and scallop height validation

For comparison, the measured surface topography measured by the optical microscope is shown in Fig.12. From Fig.12, it can be seen that on the surface of the workpiece after milling, the residual height of the workpiece in two directions is left after milling by ball end milling cutter. From the simulation results in Fig.12 b,c, it can be seen that the simulation program can clearly show the residual height of the workpiece in two directions after milling and the residual height of the workpiece surface after milling.

(a) Measurement of workpiece surface profile
(b) Simulation of workpiece surface profile
(c) Single tool mark measurement results
(d) Single tool mark simulation results

Fig.12 Measurement and simulation results of surface topography

The distribution characteristics of the three-dimensional surface topography of the surface are analyzed. Because the use of Keith vhx-1000 ultra depth of field three-dimensional microscope can only observe the surface topography profile, does not have the measurement function, can not well explain the degree of agreement between the simulation results and the experimental results. So in order to further verify the correctness of the simulation program, the roughness meter is used to measure the waviness curve of the surface topography in the two-dimensional direction and compare with the simulation results. Fig.13 is the scallop height result of simulated and measured at different cutting parameters: No.4 in table 3,No.8 in table 3, No.8 in table 1. The extracted surface waviness is compared, which can effectively verify the
correctness of the simulation program. The results are consistent with the experimental values, although there are some errors between them, and the experimental values will be slightly larger than the simulation values. Therefore, the method of 3D surface milling simulation based on discrete cutting edge proposed in this paper can well simulate the workpiece surface topography formed after milling.

| Sample length (mm) | Simulated | Measured |
|-------------------|-----------|----------|
| 0                 | 0.5       | 1        |
| 1.5               | 2         | 2.5      |
| 2                 | 3         | 3.5      |
| 3                 | 4         | 4.5      |

Fig.13 The scallop height in feed direction

### 4. Conclusions

An analytical model is proposed for the prediction surface topography in end milling. The surface topography calculated using z-map method directly from CAM data. The tool runout, tool deformation, tool wear and vibration are not considered in this study. The following conclusions can be summarized:

1. The proposed model is used to predict surface topography directly from the CAM data and cutting parameters.

2. The effect of cutting parameters (spindle speed, feed speed, inclination angle) on surface topography is
discussed.

(3) The comprehensive experiments are used to validate the model, and the results of surface roughness is consistent and have certain errors between measured and simulation.

Declarations

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Authors' contributions

Ruihu Zhou designed and performed the manuscript, analyzed the data, and drafted the manuscript. Qilin Chen designed and carried out the experiments. All authors have read and agreed to the published version of the manuscript.

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