We consider the multiplet structures, appearing in hadronic physics. The basic types of the multiplets revealed are parity doublets, chiral multiplets, and multispin-parity clusters. We elucidate a new type of isospin supermultiplets in the baryon sector. The creation of parity doublets, chiral multiplets and their interrelation with multispin-parity clusters has been uncovered. The role of chiral symmetry breaking and restoration in the formation of the hadronic spectra has been scrutinized.

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I. INTRODUCTION

Chiral symmetry plays a vital role in strong interaction physics. Since its genesis, the quark model has had to deal with the different facets of the quarks. From the field-theoretical point of view, bare quarks are massless or have very small mass of a few MeV (u,d-quarks). These are also called the ”current quarks”. The current quark masses are determined by the QCD-sumrules. On the other hand there exists a big class of quark models - potential, etc., which deals with so-called ”constituent quarks” with masses of about 1/3 of the nucleon’s mass.

The idea behind chiral models is this lightness of the light quarks compared to typical hadronic scales (take roughly 1GeV or the nucleon mass). One starts with massless quarks and realized that in this so-called chiral limit, the QCD Lagrangian shows an additional symmetry: $SU(2)_L \times SU(2)_R$. This symmetry is broken only by the very small u,d quark masses. However this symmetry is not directly observed in the world - it is hidden, i.e. spontaneously broken symmetry (SBS) [1].

Creation of the universe in cosmology, generation of gauge bosons, Higgs particles, t’Hooft monopoles, instantons and solitons in high energy elementary particle physics, and phenomena of laser, superradiance, superconductivity, superfluidity, and phase transition, are all well understood in terms of SBS. Those phenomena are all known as order-creating phenomena in nature.

Symmetry and order are two mutually complementary concepts; when we have rotational symmetry, there is no particular direction singled out as being different. When we notify a specific direction, there is directional order, and the rotational symmetry should be lost to create such an order. The mechanism of SBS was first demonstrated theoretically in quantum field theory. In the system of infinitely many degrees of freedom described by the Lagrangian manifesting the rotational symmetry, only one state is chosen spontaneously among the infinitely degenerate ground state of the system and the rotational symmetry of the system is broken without recourse to any external environment. Although SBS is a mechanism characteristic to the system of infinitely many degrees of freedom described by quantum field theory,
it can equally be applied to the system of finite but many degrees of freedom. "Spontaneous" means the following fact. The rotational symmetry is broken not by imposing certain external force, but by the fact that the system itself chooses one and only one ground state among the infinitely many possible ground states and the transition of the chosen ground state into other ones cannot be realized. Due to a simple calculation in quantum field theory, the more degrees of freedom of the system becomes, the more stable the spontaneously chosen ground state is. Thus in the case of infinitely many degrees of freedom, the ground state with SBS becomes highly stable but manifests the broken rotational symmetry.

The idea that the fundamental strong interaction theory should possess an approximate $SU(2)_L \times SU(2)_R$ or $(SU(3)_L \times SU(3)_R)$ chiral symmetry dates back to the 1960s. One of the most important insights from this was that this symmetry must be spontaneously broken in the vacuum (i.e. realized in the Nambu-Goldstone mode). The most important early arguments were: i) the absence of parity doublets (PD) in the hadron spectrum (if the chiral symmetry were realized in the Wigner-Weyl mode - i.e. if the vacuum were trivial - then the hadron spectrum would have to reveal the multiplets of the chiral group which are manifested as parity doublets); (ii) the exceptionally low mass of pions, which are taken to be pseudo-Goldstone bosons associated with the spontaneously broken axial symmetry.

During the last few years it has been realized that the spectra of baryon and mesonic resonances exhibit few startling features. First of all it is a clustering, or multiplet structure of the experimental spectra. More precisely, the majority of the hadronic resonances are grouped into some kind of clusters, and we term them as "chiral multiplets" (the meaning of this term will become clear later). Prof. Höehler was probably the first who realized a cluster structure of the baryonic spectra, i.e. baryon resonance should not be treated as an individual states, but rather as a set of clusters, "Höehler clusters" (or a Höehler poles) [2]. Another startling feature is the occurrence of the parity doublets in the hadronic spectra. By this we mean the mass degeneracy among physical resonances with the same total angular momentum $J$, but the opposite $P$- parities [3].

In what follows we will try to uncover the origins of these truly remarkable phenomena. We will also establish their similarities and differences, and what are the real dynamics hidden behind all of this. We will scrutinize what is the role of the chiral symmetry breaking and restoration in the formation of parity doublets and chiral multiplets. These mechanisms and manifestation of chiral symmetry are different for baryons and mesons.

We will analyze the whole baryon spectra and will construct parity doublets and chiral multiplets. In so doing, we analyze a few different models of this phenomena: Klempt, Inopin, Cohen-Glozman, Hatsuda, Kirchbach. We discover duality between the different models and how it sheds light on this problem. New types of symmetries describing this multifaceted phenomena will be uncovered.

II. Chiral Multiplets

We will attack this issue from different directions simultaneously. On one hand, we will consider experimental spectra and look for the emergence of clusters. On the other hand, the block of modern quark models will be presented, where authors show the multiplet structure from the dynamical and symmetry arguments.
a) Cohen-Glozman model

The authors consider only $N, \Delta$ spectra [4]. They suggested that the parity doublet structure seen in the spectrum of highly excited baryons may be due to effective chiral restoration for these states. The authors have argued how the idea of chiral symmetry restoration high in the spectrum is consistent with the concept of quark-hadron duality (QHD). If chiral symmetry is effectively restored for high-lying states, then the baryons should fall into representation of $SU(2)_L \times SU(2)_R$ that are compatible with the given parity of the states-the parity-chiral multiplets. The authors classify all possible parity-chiral multiplets: (i) $(1/2, 0) \oplus (0,1/2)$ that contain parity doublet for the nucleon spectrum, (ii) $(3/2, 0) \oplus (0,3/2)$ consists of the parity doublet for delta spectrum, (iii) $(1/2, 1) \oplus (1,1/2)$ contains one parity doublet in the nucleon spectrum and one parity doublet in the delta spectrum of the same spins that are degenerate in mass. They show that the available spectroscopic data for non-strange baryons in the $\sim 2\text{GeV}$ range are consistent with all possibilities, but the approximate degeneracy of parity doublets in nucleon and delta spectra support the latter possibility with excited baryons approximately falling into $(1/2, 1) \oplus (1,1/2)$ representation of $SU(2)_L \times SU(2)_R$ with approximate degeneracy between positive and negative parity $N$ and $\Delta$ resonances of the same spin [4].

The main conjecture here is that high enough in the spectrum, chiral symmetry is restored and this is precisely the reason of the parity doublet occurrence and chiral multiplets creation. This actually has not been proven. Another moment is the choice of 2 GeV for the role of the high mass region’s threshold. Beside the fact that it is hardly possible to draw such a boundary line between the low- and high-mass region, there are simply too many baryon resonances (BR) above 2 GeV and they stretch up to 4.1 GeV [5]. So, 2 GeV happens to be just in the middle of $N, \Delta$ spectrum. Let’s present the major result of Cohen-Glozman model in a Table 1.

As we can see, some chiral multiplets have a big mass spread and hardly could be considered as a degenerate set of physical states: $J = 1/2, \delta(\text{spread}) = 200 \text{ MeV}; J = 3/2, \delta = 180 \text{ MeV}; J = 5/2, \delta = 295 \text{ MeV};$

**Table 1: Chiral Multiplets in Cohen-Glozman Model**

| $J$   | $N^+$ | $N^-$ | $\Delta^+$ | $\Delta^-$ |
|-------|-------|-------|------------|------------|
| 1/2   | (2100)| (2090)| (1910)     | (1900)     |
| 3/2   | (1900)| (2080)| (1920)     | (1940)     |
| 5/2   | (2000)| (2200)| (1905)     | (1930)     |
| 7/2   | (1990)| (2190)| (1950)     | (2200)     |
| 9/2   | (2220)| (2250)| (2300)     | (2400)     |
| 11/2  | ?     | (2600)| (2420)     | ?          |
| 13/2  | ?     | ?     | ?          | $\Delta^+$ (2750) |
| 15/2  | ?     | ?     | ?          | ?          |

$J = 7/2, \delta = 250 \text{ MeV}; J = 9/2, \delta = 180 \text{ MeV}; J = 11/2, \delta = 180 \text{ MeV (unfilled)}; J = 13/2, \delta = 50 \text{ MeV (unfilled)}; J = 15/2, \text{unfilled}.$

So for the filled quartets, the spread varies in a range $180 \text{ MeV} < \delta < 295 \text{ MeV}$, with $< \delta >= 221 \text{ MeV}$, which is much greater than the pion mass. This could mean possible transitions between the quartet members with pion emission.

Recently, the SAPHIR Collaboration (ELSA) tentatively discovered a few $N^*$, precisely: $N_{1/2}^- (1897), N_{1/2}^+ (1896),$ and $N_{3/2}^- (1895)$ [5]. We can try to insert this triplet into Cohen-Glozman model and see if the chiral multiplets get narrower. The authors [4] also neglect $N, \Delta (\sim 3000 \text{ region})$ data from the PDG2002 which has been well-known for years. Namely, we
are talking about the following class of resonances, obtained by Koch and Hendry:

Table 2: \(N, \Delta \sim 3000\) Region, from PDG2002

| \(J\) | \(\Delta\) | \(N\) |
|---|---|---|
| 11/2 | \(-\) (2850) | \(+\) (3200) |
| 13/2 | \(-\) (3500) | \(+\) (3300) |
| 15/2 | \(-\) (3100) | \(+\) (3800) |
| 17/2 | \(-\) (3750) | \(+\) (3500) |
| 19/2 | \(-\) (3100) | \(+\) (3500) |
| 21/2 | \(-\) (4100) | \(+\) (3700) |

So, we have in total 3+12=15 extra BR’s which we will include in this analysis. But that’s not all. Recently we have developed a potential quark model, which could describe \(N\), \(\Delta\) and strange resonances with both small \(J\), \(M\) and large \(J\), \(M\). In particular this model predicts a whole series of high-lying \(N\), \(\Delta\) resonances, represented in Table 3 \([6],[7]\). After much experimenting with Tables 1, 2 and 3 and the SAPHIR states we can suggest a modified Cohen-Glozman chiral multiplets, presented in Table 4.

One can see that \(J= 1/2\) quartet now has 89 MeV mass spread (it was 180 MeV). This is significant step forward in getting really narrow multiplets! We have marked all of the new entries by an asterisk and in color in Table 4. Another interesting feature of this modification is that some multiplets have gotten filled in more, i.e.: \(J= 11/2\) has 4 states instead of 2; \(J= 13/2\) has 4 states instead of 2; \(J= 15/2\) has 4 quite close states instead of just one. The third startling feature is the emergence of a new sector with \(J= 17/2\), 19/2 and 21/2.

Table 3: High-Lying \(N, \Delta\) states, predicted in our potential model \([6],[7]\)

| \(N\) | \(\Delta\) |
|---|---|
| \(11/2^+\) (2390) | \(13/2^-\) (2900) |
| \(15/2^+\) (3000) | \(19/2^+\) (3450) |
| \(21/2^-\) (3950) | \(17/2^+\) (3490) |
| \(15/2^-\) (3480) | \(19/2^-\) (4000) |

Originally Cohen-Glozman has 25 states, compared to the modified scheme with 42 states, which is about a 40 % difference . Table 4 is filled up to 95%.

Let’s return again to the original Cohen-Glozman scheme (Table 1) and analyze more quantitatively their parity doublet and chiral multiplet structure (see Fig. 1).

\(J=1/2\): \(N^+ (2100) - N^- (2090)\); \(< \PD_N >= 2095 \text{ MeV}, \delta_N = 10 \text{ MeV}\)
\(\Delta^+ (1910) - \Delta^- (1900)\); \(< \PD_{\Delta} >= 1905 \text{ MeV}, \delta_{\Delta} = 10 \text{ MeV}\)
Center of Gravity (c.g.)=2000 MeV; full mass spread = \(\delta = 200 \text{ MeV}\).

As we see, both \(N\) and \(\Delta\) doublets fit excellently the definition of the parity doublets, but \(N\) and \(\Delta\) parity doublets have very small correlation between them.

\(J=3/2\): \(N^+ (1900) - N^- (2080)\); \(< \PD_N >= 1990 \text{ MeV}, \delta_N = 180 \text{ MeV}\)
\(\Delta^+ (1920) - \Delta^- (1940)\); \(< \PD_{\Delta} >= 1930 \text{ MeV}, \delta_{\Delta} = 20 \text{ MeV}\)
c.g.=1960 MeV; \(\delta = 180 \text{ MeV}\).

\(\Delta\) doublet is a good parity doublet, but nucleon’s doublet hardly fits the parity doublet definition. It’s interesting that \(\Delta\) parity doublet is located inside of the nucleon’s parity doublet. Another peculiar feature is that \(J = 3/2\) c.g. is lower than \(J = 1/2\) c.g.
Table 4: Modified Cohen-Glozman-Inopin Multiplets

| J  | N**(1986) | N*-(1897) | Δ+(1910) | Δ-(1900) |
|----|-----------|-----------|----------|----------|
| J= 1/2 |          |          |          |          |
| J= 3/2 | N+(1900) | N*-(1995) | Δ+(1920) | Δ-(1940) |
| J= 5/2 | N+(2000) | N-(2200) | Δ+(1905) | Δ-(1930) |
| J= 7/2 | N+(1990) | N-(2190) | Δ+(1950) | Δ-(2200) |
| J= 9/2 | N+(2220) | N-(2250) | Δ+(2300) | Δ-(2400) |
| J= 11/2 | N+(2390) | N-(2600) | Δ+(2420) | Δ**(2850) |
| J= 13/2 | N+(2700) | N-(2900) | Δ**(3200) | Δ-(2750) |
| J= 15/2 | N+(3000) | N*-(3100) | Δ**(2950) | Δ-(3480) |
| J= 17/2 | N**(3500) | N*-(3450) | Δ**(3490) | Δ**(3300) |
| J= 19/2 | N**(3500) | N**(3750) | Δ**(3500) | Δ**(3500) |
| J= 21/2 | N**(3950) | N*-(3950) | N**(4100) | Δ**(4100) |

\[ J=5/2: \quad N^+(2000) - N^-(2200); <PD_N> = 2100 \text{ MeV}, \delta_N = 200 \text{ MeV} \]
\[ \Delta^+(1905) - \Delta^-(1930); <PD_\Delta> = 1918 \text{ MeV}, \delta_\Delta = 25 \text{ MeV} \]
c.g. = 2009 MeV; \delta = 295 MeV.

\[ J=7/2: \quad N^+(1990) - N^-(2190); <PD_N> = 2090 \text{ MeV}, \delta_N = 200 \text{ MeV} \]
\[ \Delta^+(1950) - \Delta^-(2200); <PD_\Delta> = 2075 \text{ MeV}, \delta_\Delta = 250 \text{ MeV} \]
c.g. = 2083 MeV; \delta = 250 MeV.

\[ J=9/2: \quad N^+(2220) - N^-(2250); <PD_N> = 2235 \text{ MeV}, \delta_N = 30 \text{ MeV} \]
\[ \Delta^+(2300) - \Delta^-(2400); <PD_\Delta> = 2350 \text{ MeV}, \delta_\Delta = 100 \text{ MeV} \]
c.g. = 2293 MeV; \delta = 180 MeV.

\[ J=11/2, 13/2, \text{ and } 15/2: \therefore \text{there are only two, two and one state here correspondingly, so there is no point of discussing chiral multiplets for this sector.} \]

\[ \text{Total average width for nucleons is } <\delta_N> = 124 \text{ MeV.} \]
\[ \text{Total average width for deltas is } <\delta_\Delta> = 81 \text{ MeV.} \]

\[ \text{As we see the deltas are much better PD's than the nucleons.} \]
\[ \text{Grand average width for CG chiral multiplets } <\delta_{\text{GRAND}}> = 221 \text{ MeV.} \]

\[ \text{Original CG has 5 filled quartets. Quartets with } J=1/2, 5/2, \text{ and } 9/2 \text{ have no overlap between } N \text{ and } \Delta \text{ PD's, and only quartets with } J=3/2, 7/2 \text{ could be considered as a chiral multiplets.} \]

\[ \text{Let's return again to the Table 4 and quantify the Cohen-Glozman-Inopin multiplets (see Fig.1, right column).} \]
\[ J=1/2: \quad N^**(1986) - N^*-(1897); <PD_N> = 1942 \text{ MeV}, \delta_N = 89 \text{ MeV} \]
\[ \Delta^+(1910) - \Delta^-(1900); <PD_\Delta> = 1905 \text{ MeV}, \delta_\Delta = 10 \text{ MeV} \]
c.g. = 1923 MeV; \delta = 89 MeV.

\[ \text{Both } N \text{ and } \Delta \text{ doublets qualify as PD's. Delta doublet is inscribed exactly inside the nucleon PD, making together a perfect quartet (see Fig.1).} \]
\( J=3/2 \): \( N^+(1900) - N^-(1895) \); \( PD_N = 1898 \text{ MeV}, \delta_N = 5 \text{ MeV} \)
\( \Delta^+(1920) - \Delta^-(1940) \); \( PD_{\Delta} = 1930 \text{ MeV}, \delta_{\Delta} = 20 \text{ MeV} \)
\( \text{c.g.}=1914 \text{ MeV}; \delta = 45 \text{ MeV} \)

Both \( N \) and \( \Delta \) doublets qualify as a PD’s. \( N \) and \( \Delta \) doublets are clearly separated, but located close to each other, with full width of only 45 MeV. This is the best chiral quartet so far.

\( J=5/2, 7/2 \) and \( 9/2 \): CG quartets remained intact.

\( J=11/2 \): This line was unfilled in CG model, and now we have a full quartet.
\( N^+(2390) - N^-(2600) \); \( PD_N = 2495 \text{ MeV}, \delta_N = 210 \text{ MeV} \)
\( \Delta^+(2420) - \Delta^-(2850) \); \( PD_{\Delta} = 2635 \text{ MeV}, \delta_{\Delta} = 430 \text{ MeV} \)
\( \text{c.g.}=2565 \text{ MeV}; \delta = 460 \text{ MeV} \)

We see that both \( N \) and \( \Delta \) doublets have quite a large mass spread, and hardly could qualify for a PD's. On the other hand \( N \) and \( \Delta \) doublets overlap substantially, showing a clear tendency for the chiral quartet creation. Although we have a big total width for the \( J=11/2 \) quartet, we have to remember, that for such a high-momentum, baryon resonances are becoming ill-defined (masses) and large spacings are natural.

\( J=13/2 \): CG has two states. Now we have:
\( N^+(2700) - N^-(2900) \); \( PD_N = 2800 \text{ MeV}, \delta_N = 200 \text{ MeV} \)
\( \Delta^+(3200) - \Delta^-(2750) \); \( PD_{\Delta} = 2975 \text{ MeV}, \delta_{\Delta} = 450 \text{ MeV} \)
\( \text{c.g.}=2888 \text{ MeV}; \delta = 500 \text{ MeV} \)

As we see, both \( N \) and delta doublets have a very large mass spread, and cannot qualify as a PD. There is clear overlap between \( N \) and \( \Delta \) doublets, which indicates attraction and possible chiral multiplet creation (after the masses will be better measured).

\( J=15/2 \): CG has only one state here. Now we have:
\( N^+(3000) - N^-(3100) \); \( PD_N = 3050 \text{ MeV}, \delta_N = 100 \text{ MeV} \)
\( \Delta^+(2950) - \Delta^-(3480) \); \( PD_{\Delta} = 3215 \text{ MeV}, \delta_{\Delta} = 530 \text{ MeV} \)
\( \text{c.g.}=3133 \text{ MeV}; \delta = 530 \text{ MeV} \)

Nucleon doublet definitely fit as a PD. Delta doublet is no good as a PD. It is wonderful that \( N \) doublet is inscribed exactly inside \( \Delta \) doublet, making this quartet a chiral multiplet (see Fig.1).

\( J=17/2 \): CG has zero states in this line. Now we have:
\( N^{++}(3500) - N^{--}(3450) \); \( PD_N = 3475 \text{ MeV}, \delta_N = 50 \text{ MeV} \)
\( \Delta^+(3490) - \Delta^-(3300) \); \( PD_{\Delta} = 3395 \text{ MeV}, \delta_{\Delta} = 190 \text{ MeV} \)
\( \text{c.g.}=3435 \text{ MeV}; \delta = 200 \text{ MeV} \)

Nucleon doublet happened to be a perfect PD with mass spread of only 50 MeV. Delta doublet is a fair candidate for the PD. Two doublets have almost 100% overlap, making a real chiral multiplet with total spread of only 200 MeV for such a high momentum (see Fig.1).

\( J=19/2 \): CG has zero states in this line. Now we have a quartet. Nucleon doublet is 250 MeV wide, and \( \Delta \) doublet is 500 MeV wide. New data and PWA could considerably shrink this quartet. \( N \) doublet is inscribed exactly inside \( \Delta \) doublet.
\( N^+(3500) - N^{+-}(3750) \); \( PD_N = 3625 \text{ MeV}, \delta_N = 250 \text{ MeV} \)
\( \Delta^+(3500) - \Delta^-(4000) \); \( PD_{\Delta} = 3750 \text{ MeV}, \delta_{\Delta} = 500 \text{ MeV} \)
\( \text{c.g.}=3688 \text{ MeV}; \delta = 500 \text{ MeV} \)

\( J=21/2 \): CG has zero states in this line. So far we’ve got single \( N^- \) and single \( \Delta^- \), lying close to each other. Hopefully soon \( N^+ \) and \( \Delta^+ \) partners will be discovered to form a chiral multiplet.
\( N^-(3950) - \Delta^{*-}(4100); \delta = 150 \text{ MeV} \)
Finally we compare full mass spreads for original CG (Table 1) and modified CGI schemes (Table 4). For the totally filled quartets \( J=1/2, \ldots ,1/9 \) we have:

\[ \langle \delta_{\text{grand}} \rangle_{\text{CG}} = 221 \text{ MeV} ; \quad \langle \delta_{\text{grand}} \rangle_{\text{CGI}} = 172 \text{ MeV} \]

Clearly, in the modified CGI scheme we have much more narrow quartets, making the conjecture of chiral multiplets plausible. If we consider CGI multiplets for \( J=11/2, \ldots ,21/2 \), where CG scheme failed, we get dispersed mass spreads from \( \delta = 150 \) MeV to \( \delta = 530 \) MeV, with \( \langle \delta_{\text{grand}} \rangle = 390 \) MeV.

We have to admit that these results contradict the basic CG idea: chiral symmetry is restored after 2 GeV mass region, and one should see clear chiral multiplets, which has to be more degenerate in high \( J, M \) region. In fact, one can see multiplet’s spread growing with \( J, M \). Klempt came recently to the similar conclusions from a different venue \[8\].

b) Klempt Model

Recently Klempt published a series of papers devoted to baryon spectroscopy \[8\]. Based on analysis of data, Klempt proposed a new baryon mass formula:

\[ M^2 = M^2_{\Delta} + \frac{n_s M^2_s}{3} + a(L + N) - s_i I_{\text{sym}}, \]

where \( M^2_s = (M^2_{\Omega} - M^2_{\Delta}) \), \( s_i = (M^2_{\Delta} - M^2_N) \), \( n_s \) the number of strange quarks in a baryon, \( L \) the intrinsic orbital angular momentum. \( N \) is the principal quantum number. \( I_{\text{sym}} \) is the fraction of the wave function (normalized to the nucleon wave function) which is antisymmetric in spin and flavor.

The author claimed that formula (1) reproduces nearly all known baryon masses. But it is evident that Eq. (1) is based on the presumption that any baryon state clearly can be defined as a pure state in \( L, N, I_{\text{sym}} \) functional space. In other words, there is no admixtures with different “\( L, N \) - orbitals” in a given baryon WF. But we know from our experience and the results of many classical papers, that at least for some of the BR’s this could be 100% wrong. Just take for example \( N_{3/2}^+ (1720) \) B.R. It’s defined as a strong superposition of five \( L \)-orbitals, with its major component having only about 40-50% of all wave function.

We will analyze Klempt’s findings on clustering and chiral properties of baryon sector.

The most interesting are the multiplets with assigned \( S = 3/2 \) intrinsic spin. First he identified the ”stretched” states with \( J = L + S \); \( L = 0, 1, \ldots , 6 \) and \( S = 3/2 \), i.e. resonances with quantum numbers

\[ J^P = 3/2^+, 5/2^-, 7/2^+, 9/2^-, 11/2^+, 13/2^-, 15/2^+ \]

These resonances are shown in Table 5 in the fifth column. Omitted are the decuplet ground states \( (L = 0) \) which also fall into this category. In the nonrelativistic quark model (NRQM), we expect single resonances for \( L = 0 \) (the ground states), triplets for \( L = 1 \), and quartets for higher \( L \). The multiplet structure is clearly visible in Table 5, even though the multiplets are not complete.

As we look more carefully at this scheme in Table 5, we uncover an extremely interesting link between the dynamics and symmetry. When we construct the tower of states, the first floor with \( L = 1 \) is:

\[ L=1: \ N \to \Sigma \to \Lambda \to \Delta \]
The second floor is:  
\[ L=2: \Delta \rightarrow \Sigma \rightarrow \Lambda \rightarrow N \]

The third floor is:  
\[ L=3: N \rightarrow \Delta \]

The fourth floor is:  
\[ L=4 : \Delta, \]

where by \( N, \Delta, \Sigma, \Lambda \) here we understand corresponding multiplets of \( N^*, \Delta^*, \Sigma^*, \Lambda^* \), from the Table 5 (see Fig.2).

There is evidently a new symmetry group that arises, which reflects the total isospin degeneracy for a given floor:

\[ (I = 0) + (I = 1/2) + (I = 1) + (I = 3/2) \rightarrow \text{Supermultiplet} \]

It seems that at low \( L = 1 \) we have a grand \( SU_I(4) \) group, for the \( L = 2 \) it is still \( SU_I(4) \), then for \( L = 3 \) it decays into chain of the subgroups, represented by \( SU_I(2) \), then for \( L = 4 \) \( SU_I(2) \) subsequently decays into \( SU_I(1) \). (We omit here singlets with \( L = 5, 6 \).) We can think of the following chain:

\[ SU_I^1(4) \rightarrow SU_I^2(4) \rightarrow SU_I(2) \rightarrow SU_I(1) \]

Table 5: **Klempt’s S=3/2 Multiplets**

| \( L \) | \( J=L -3/2 \) | \( J=L -1/2 \) | \( J=L +1/2 \) | \( J=L +3/2 \) | Average |
|---|---|---|---|---|---|
| 1 | - | \( N_{1/2} - (1650) \) | \( N_{3/2} - (1700) \) | \( N_{5/2} - (1675) \) | 1675 |
| 1 | - | \( \Sigma_{1/2} - (1750) \) | - | \( \Sigma_{5/2} - (1763) \) | 1763 |
| 1 | - | \( \Lambda_{1/2} - (1800) \) | - | \( \Lambda_{5/2} - (1815) \) | 1815 |
| 1 | - | \( \Delta_{1/2} - (1900) \) | \( \Delta_{3/2} - (1940) \) | \( \Delta_{5/2} - (1923) \) | 1923 |
| 2 | \( \Delta_{1/2}^+ (1910) \) | \( \Delta_{3/2}^+ (1920) \) | \( \Delta_{5/2}^+ (1905) \) | \( \Delta_{7/2}^+ (1950) \) | 1921 |
| 2 | - | \( N_{3/2}^+ (1900) \) | \( N_{5/2}^+ (2000) \) | \( N_{7/2}^+ (1990) \) | 1963 |
| 2 | - | \( \Sigma_{3/2}^+ (2080) \) | \( \Sigma_{5/2}^+ (2070) \) | \( \Sigma_{7/2}^+ (2030) \) | 2060 |
| 2 | - | - | \( \Lambda_{5/2}^+ (2110) \) | \( \Lambda_{7/2}^+ (2020) \) | 2065 |
| 3 | - | \( N_{5/2} - (2200) \) | \( N_{7/2} - (2190) \) | \( N_{9/2} - (2250) \) | 2213 |
| 3 | - | \( \Delta_{5/2} - (2350) \) | - | \( \Delta_{9/2} - (2400) \) | 2375 |
| 4 | - | \( \Delta_{7/2}^+ (2390) \) | \( \Delta_{9/2}^+ (2300) \) | \( \Delta_{11/2}^+ (2420) \) | 2370 |
| 5 | - | - | - | \( \Delta_{13/2}^- (2750) \) | 2750 |
| 6 | - | - | - | \( \Delta_{15/2}^- (2950) \) | 2950 |

As one can see from Fig.2 we significantly generalize Klempt’s model and create a supermultiplets in the ”Isospin Space” instead of Klempt’s ”L-Space” multiplets. New supermultiplets are represented in Fig.2 as a creature with four floors. As we see, the topology realized as plane \((D = 2) \rightarrow \) plane \((D = 2) \rightarrow \) line \((D = 1) \rightarrow \) point \((D = 0)\). Our own analysis of the PDG2002 data lead to the different supermultiplets shown in a Figure 3. We will return to the comparison of different models a bit later.

An interesting feature of the Klempt’s \( S = 3/2 \) multiplets is a ”good definition” of a floor structure, i.e. different floors are really separated in the mass. The ”widths” of the floors with increasing \( L \) are (in MeV): 248\((L = 1) \rightarrow 144(L = 2) \rightarrow 162(L = 3) \rightarrow 0(L = 4)\). So they are smoothly decreasing with \( L \) from 248 MeV to zero at \( L = 4 \) (see Fig. 4).
One can see, that beginning from the $L = 2$ the *quartet structure* is allowed and arise in reality. This is reminiscent of the CG model where the $N, \Delta$-quartets appeared from the chiral symmetry.

Let’s make a detailed analysis of the multiplet’s mass spreads from the Table 5.

$L = 1$: $N(1650 - 1700), \delta = 50\,MeV$

$\Sigma(1750 - 1775), \delta = 25\,MeV$

$\Lambda(1800 - 1830), \delta = 30\,MeV$

$\Delta(1900 - 1940), \delta = 40\,MeV$

For the $L = 1$ floor individual mass spreads vary from 25 to 50 MeV, and $<\delta_{L=1}> = 36$ MeV only. Total width of the L=1 supermultiplet is 290 MeV, with c.g. = 1794 MeV.

$L = 2$: $\Delta(1905 - 1950), \delta = 45\,MeV$

$N(1900 - 2000), \delta = 100\,MeV$

$\Sigma(2030 - 2080), \delta = 50\,MeV$

$\Lambda(2020 - 2110), \delta = 90\,MeV$

For the $L = 2$ floor individual mass spreads vary from 45 to 100 MeV, and $<\delta_{L=2}> = 71$ MeV. Total width of $L = 2$ supermultiplet is 210 MeV, with c.g. = 2002 MeV. One can see that total width of the $L = 2$ floor is diminished, compared to the $L = 1$ floor.

$L = 3$: $N(2190 - 2250), \delta = 60\,MeV$

$\Delta(2350 - 2400), \delta = 50\,MeV$

For the $L = 3$ floor individual mass spreads vary from 50 to 60 MeV, with $<\delta_{L=3}> = 55$ MeV. Total width of the $L = 3$ floor is 210 MeV, the same as $L = 2$, with c.g. = 2294 MeV.

$L = 4$: $\Delta(2300 - 2420), \delta = 120\,MeV$

$L = 4$ supermultiplet shrinks down to the $\Delta$ triplet, but it’s might be just experimental problem which will be resolved in coming years. It’s evident that our supermultiplets are getting narrower with $L$ increasing and the masses increasing, which roughly resembles the chiral symmetry restoration scenario.

In his last paper in the series [8], Klempt discussed the mass spectrum of $N, \Delta$ resonances only. The author compared CG conjecture with the possibility that high-mass states are organized into $(L, S)$-multiplets with defined intrinsic quark spins and orbital angular momenta. Table 6, which is adapted from the CG papers, shows $N^*$ and $\Delta^*$ masses above 1.9 GeV, for states with positive and negative parity. In many cases, the effect of PD is striking; states with identical $J$ but opposite parity often have very similar masses. This does not of course automatically imply that parity doublets are generated by restoration of chiral symmetry. Consider the first six $\Delta$ states in Table 6 with $J = 1/2, 3/2, 5/2$. The masses are clearly degenerate; they form three PD’s.

(Klempt suggested that the states marked with a (*)& in Table 6 should have considerably higher masses than their chiral partners while the other five states in color should be degenerate in mass with corresponding states of opposite parity).
The $\Delta_{7/2^+}(1950)$ and the $\Delta_{7/2^-}(2200)$ should also form a PD but the $\Delta_{7/2^+}(1950)$ has a mass which is very close to the other three positive-parity BR. These four positive-parity $\Delta$'s rather seem to belong to a spin quartet of states with $L = 2$ and $S = 3/2$ coupling to $J = 1/2,...7/2$. The question arises if the PD’s are really due to restoration of chiral symmetry or do the PD’s reflect a symmetry of the underlying quark dynamics?

The CG model requires the existence of a $\Delta_{11/2^-}$ and $N_{11/2^+}$ at about 2500 MeV, of a $\Delta_{13/2^+}$ and a $N_{13/2^+}$ at 2750 MeV, and of three additional states at 2950 MeV. Klempt investigated if the occurrence of PD’s can be understood naturally within SU(6) multiplet structure of BR. In this interpretation PD’s occur naturally, but the prediction for so-far unobserved PD’s differs.

A decision, if $N^*$’s form PD’s, requires a quantitative analysis. First we notice that according to Eq.(1) the mass difference between two resonances with consecutive $L$ and otherwise identical quantum numbers vanishes asymptotically:

$$M_{L+1}^2 - M_L^2 = (M_{L+1} - M_L)(M_{L+1} + M_L) = a,$$

hence

$$M_{L+1} - M_L = \frac{a}{(M_{L+1} + M_L)}.$$

Asymptotically, all mass separations vanish with $1/M$ and chiral symmetry is trivially restored. Klempt finally compared the consistency of the data with the assumption of parity doublets and, alternatively, with their consistency with $(L, S)$ multiplets with vanishing $LS$ coupling. But the author didn’t in fact compare the consistency of the chiral multiplets assumption, which is of paramount importance.

First Klempt computed the mean mass deviation of BR’s when they are interpreted as parity doublets:

$$\sigma_{PD's} = \sqrt{\frac{1}{10} \sum_{i=1,20} (M_i - M_{\pm})^2} = 97 MeV$$

where $M_{\pm}$ are the mean masses of positive- and negative-parity resonances paired to one parity doublet (see Table 6). Now author determined the deviation of baryon masses from the mean value of a $(L, S)$ multiplet:

$$\sigma_{spin\ multiplets} = \sqrt{\frac{1}{13} \sum_{i=1,20} (M_i - M_{cg})^2} = 39 MeV$$

Table 6: CG versus Klempt Models

| $J$ = 1/2 | 1 | $N_{1/2^+}(2100)$ | $N_{1/2^-}(2090)$ | a | $\Delta_{1/2^+}(1910)$ | $\Delta_{1/2^-}(1900)$ |
| $J$ = 3/2 | 2 | $N_{3/2^+}(1900)$ | $N_{3/2^-}(2080)$ | b | $\Delta_{3/2^+}(1920)$ | $\Delta_{3/2^-}(1940)$ |
| $J$ = 5/2 | 3 | $N_{5/2^+}(2000)$ | $N_{5/2^-}(2200)$ | c | $\Delta_{5/2^+}(1905)$ | $\Delta_{5/2^-}(1930)$ |
| $J$ = 7/2 | 4 | $N_{7/2^+}(1990)$ | $N_{7/2^-}(2190)$ | d | $\Delta_{7/2^+}(1950)$ | $\Delta_{7/2^-}(2200)$ |
| $J$ = 9/2 | 5 | $N_{9/2^+}(1990)$ | $N_{9/2^-}(2250)$ | e | $\Delta_{9/2^+}(2300)$ | $\Delta_{9/2^-}(2400)$ |
| $J$ = 11/2 | 6 | $N_{11/2^+}$ | $N_{11/2^-}(2600)$ | f | $\Delta_{11/2^+}(2420)$ | $\Delta_{11/2^-}(\ast)$ |
| $J$ = 13/2 | 7 | $N_{13/2^+}(2700)$ | $N_{13/2^-}$ | g | $\Delta_{13/2^+}$ | $\Delta_{13/2^-}(2750)$ |
| $J$ = 15/2 | 8 | $N_{15/2^+}$ | $N_{15/2^-}$ | h | $\Delta_{15/2^+}(2950)$ | $\Delta_{15/2^-}(\ast)$ |
where the $M_{cg}$ are the mean values (centers of gravity). The comparison of the two hypotheses reveals that evidence for PD’s in the high-mass spectrum is weak, at most. The data is better described in terms of $(L,S)$-multiplets embracing $SU(6)$ multiplets of different $J$ but having the same $L$ and $S$. Klempt claimed that the symmetry leading to PD’s is the vanishing of spin-orbit forces and not a phase transition to chiral dynamics. We have to note that CG a few times stressed that this mechanism is rather a crossover and not a phase transition [4].

As we have seen in the previous section, the failure of the CG scheme on PD’s and chiral multiplet’s widths and spacings with growing $J$ and $M$.

c) **Data Analysis**

$N - \Delta$

Clustering structure of the baryonic spectra was revealed recently by us by examining the PDG2000 issue [9],[10] (there is no changes in baryonic sector in PDG2002). Let us briefly remind the specifics of this analysis. The nonstrange sector is very rich, comprising 23 $N$ and 22 states. We will include in the analysis three new resonances, recently discovered at ELSA, SAPHIR: $N_{3/2}^-(1895), N_{1/2}^-(1897)$, and $N_{1/2}^+(1886)$. We will also include in the analysis the so-called $N(\sim 3000$ Region) and $\Delta(\sim 3000$ Region) (see Table 2). So, altogether we have 31 $N$ and 28 $\Delta$ states. $N$ and $\Delta$ spectra exhibit very interesting clustering properties. In the nucleon sector we see the following four clusters: sextet, $\Delta = 70 MeV$:

$$N_{1/2}^- (1650) - N_{5/2}^- (1675) - N_{5/2}^+ (1680) - N_{3/2}^- (1700) - N_{1/2}^+ (1710) - N_{3/2}^+ (1720)$$

**triplet:**

$$N_{3/2}^- (1895) - N_{1/2}^- (1897) - N_{3/2}^+ (1900), \delta = 5 MeV$$

**triplet:**

$$N_{3/2}^- (2080) - N_{1/2}^- (2090) - N_{1/2}^+ (2100), \delta = 20 MeV$$

**quartet:**

$$N_{7/2}^- (2190) - N_{5/2}^- (2200) - N_{9/2}^+ (2220) - N_{9/2}^- (2250), \delta = 60 MeV$$

So the average width is 39$MeV$ only. First $N$-cluster is split into three PD’s:

$$N_{1/2}^- (1650) - N_{1/2}^+ (1710), \delta = 60 MeV$$

$$N_{3/2}^- (1700) - N_{3/2}^+ (1720), \delta = 20 MeV$$

$$N_{5/2}^- (1675) - N_{5/2}^+ (1680), \delta = 5 MeV$$

Second $N$-cluster has one parity doublet:

$$N_{3/2}^- (1895) - N_{3/2}^+ (1900), \delta = 5 MeV$$

Third $N$-cluster has one parity doublet:

$$N_{1/2}^- (2090) - N_{1/2}^+ (2100), \delta = 10 MeV$$

Fourth $N$-cluster has one parity doublet:

$$N_{9/2}^+ (2220) - N_{9/2}^- (2250), \delta = 30 MeV$$ (see Figs. 5-6)
One can see that our PD’s structure of \(N\)-spectrum *differs drastically* from the CG picture: most of the parity doublets appears in low energy region, and the rest of them are sparsely distributed over the high energy region. Our PD’s also much better fit the definition: mass spreads vary from 5 to 60 MeV, with \(<\delta_N> = 22\) MeV only.

In \(\Delta\) sector we find the following two clusters:

**septet:**

\[
\Delta_{1/2^-}(1900) - \Delta_{5/2^+}(1905) - \Delta_{1/2^+}(1910) - \Delta_{3/2^+}(1920) - \\
\Delta_{5/2^-}(1930) - \Delta_{3/2^-}(1940) - \Delta_{7/2^+}(1950), \Delta = 50\text{MeV}
\]

**triplet:**

\[
\Delta_{7/2^+}(2390) - \Delta_{9/2^-}(2400) - \Delta_{11/2^+}(2420), \delta = 30\text{MeV}
\]

First \(\Delta\)-cluster is split into three PD’s plus one extra state:

\[
\Delta_{1/2^-}(1900) - \Delta_{1/2^+}(1910), \delta = 10\text{MeV}
\]

\[
\Delta_{3/2^+}(1920) - \Delta_{3/2^-}(1940), \delta = 20\text{MeV}
\]

\[
\Delta_{5/2^+}(1905) - \Delta_{5/2^-}(1930), \delta = 25\text{MeV}
\]

Second cluster has no PD’s.

One can see that \(\Delta\)-clusters are shifted upwards as a whole against \(N\) clusters, playing more in accord with CG conjecture. \(N\)-cluster’s center of gravity lying at 1883 MeV, and \(\Delta\)-cluster’s c.g lies at 2067 MeV, resulting in 184 MeV difference (see Figs. 5, 6).

Let’s look at our picture of parity doublets, whether we can create some chiral multiplets. It is quite evident that only sector with \(J^P = 3/2^+\) gives a good sample of chiral multiplet:

\[
J = 3/2 : (N^+(1900), N^-(1895), \Delta^+(1920), \Delta^-(1940)), \delta = 45\text{MeV}
\]

\[
\Lambda - \Sigma
\]

Because \(\Lambda - \Sigma\) BR’s have only one strange quark, their shape is still not so deformed. If we can imagine that we have 3 balls in a bag, and two of them are of almost the same weight, while third a bit heavier than the two, the bag will take the form of a pear. It will be reflectionally asymmetric. Near the rest, the deformation is perhaps still not so dramatic and the lowest excitations are similar to those of the nonstrange baryons. If the heavier ball starts to gain rotational energy, the deformation will increase. The pear shape gets more pronounced and when the pear *oscillates* it gives rise to parity doublets, which we already see.

Full listing [5] give to us 18 \(\Lambda\) and 26 \(\Sigma\) BR’s. Some of the states are lacking the \(J^P\) assignments. State \(\Lambda(2000)\) does not have \(J^P\), but data from Cameron 78 allowed tentatively, the \(J^P = 1/2^-\) assignment. Further evidence came from the recent paper by Iachello [11] and older one by Capstick-Isgur [12]. Therefore we assign \(J^P = 1/2^-\) to the \(\Lambda(2000)\). The \(\Lambda\)-states with highest masses, \(\Lambda(2350)\) and \(\Lambda(2585)\) were not described theoretically and there are no clear claims from the experiments. For this reason we will not include \(\Lambda(2350), \Lambda(2585)\) in our analysis.

The situation with \(\Sigma\) hyperons is even more interesting. Two low-lying \(\Sigma(1480)\) and \(\Sigma(1560)\) do not have any \(J^P\) assignments from the experiment and theory can’t predict them either. We will exclude \(\Sigma(1480), \Sigma(1560)\) from our analysis. The production experiments [5] give strong evidence for \(\Sigma(1620)\), tentatively claiming \(J^P = 1/2^+\). This claim is in accord with Iachello [11]. So with newly defined \(\Sigma^{*1/2^+}(1620)\), we form an *exact* parity doublet \(\Sigma^{*1/2^-}(1620)\) —
\[ \Sigma_{1/2}^+(1620) \]. The production experiments \[5\] give strong evidence for \( \Sigma(1670) \) bumps without \( J^P \) assignments. Using predictions by Iachello and Isgur, we clearly get \( J^P = 1/2^- \) for \( \Sigma(1670) \). It’s interesting that this way we have two resonances with the same mass and different \( J^P \). The state \( \Sigma(1690) \) has most likely claim from the data as \( J^P = 5/2^+ \). We will assign \( J^P = 5/2^+ \) to \( \Sigma(1690) \) in our analysis. Next \( \Sigma \) without \( J^P \) assignment will be \( \Sigma(2250) \). Using results from Iachello and Isgur we assigned \( J^P = 5/2^- \) to \( \Sigma(2250) \). Last few bumps, \( \Sigma(2455) \), \( \Sigma(2620) \), \( \Sigma(3000) \), and \( \Sigma(3170) \) has no experimental claims for \( J^P \), and there are no theoretical predictions so far for such a high masses. For this reason we will not include \( \Sigma(2455) \), \( \Sigma(2620) \), \( \Sigma(3000) \), \( \Sigma(3170) \) in our analysis.

Clustering pattern is very nontrivial in \( \Sigma \) spectrum. We clearly see four clusters here.

**Doublet:** \( \Sigma_{1/2}^-(1620) - \Sigma_{1/2}^+(1620) \)

**Quartet:** \( \Sigma_{1/2}^-(1660) - \Sigma_{3/2}^-(1670) - \Sigma_{1/2}^-(1670) - \Sigma_{5/2}^+(1690), \delta = 30 \text{MeV} \)

**Triplet:** \( \Sigma_{1/2}^-(1750) - \Sigma_{1/2}^+(1770) - \Sigma_{5/2}^-(1775), \delta = 25 \text{MeV} \)

**Triplet:** \( \Sigma_{5/2}^+(2070) - \Sigma_{3/2}^+(2080) - \Sigma_{7/2}^-(2100), \delta = 30 \text{MeV} \)

One can see that our \( \Sigma \)-clusters are mostly grouped in low energy region (9 states), and only one cluster (triplet) is located above 2000 MeV. This result clearly contradicts basic CG conjecture.

**First** cluster is just a parity doublet:

\[ \Sigma_{1/2}^-(1620) - \Sigma_{1/2}^+(1620). \]

**Second** cluster has one parity doublet:

\[ \Sigma_{1/2}^-(1660) - \Sigma_{1/2}^-(1670). \]

**Third** cluster has one parity doublet:

\[ \Sigma_{1/2}^-(1750) - \Sigma_{1/2}^+(1770). \]

**Fourth** cluster has no parity doublets. We still have to understand why \( \Sigma \)-sector produces PD’s with \( J^P = 1/2^+ \) only.

In \( \Lambda \) sector we witness only one cluster; this is **quartet**:

\[ \Lambda_{1/2}^-(1800) - \Lambda_{1/2}^+(1810) - \Lambda_{5/2}^+(1820) - \Lambda_{5/2}^-(1830), \delta = 30 \text{MeV} \]

This quartet is split into two parity doublets:

\[ \Lambda_{1/2}^-(1800) - \Lambda_{1/2}^+(1810), \delta = 10 \text{MeV} \]

\[ \Lambda_{5/2}^+(1820) - \Lambda_{5/2}^-(1830), \delta = 10 \text{MeV} \]

\( \Sigma \)-cluster’s c.g. = 1790 MeV, \( \Lambda \)-cluster’s c.g. = 1815 MeV, so there is a good overlap between the \( \Lambda - \Sigma \) clusters in a global sense (see Figs. 5-7).

From our \( \Lambda - \Sigma \) clusters we can construct only one chiral multiplet:

\[ J = 1/2 : (\Sigma^+(1770), \Sigma^-(1750), \Lambda^+(1810), \Lambda^-(1800)), \delta = 60 \text{MeV} \]

One can see that \( \Lambda - \Sigma \) chiral multiplet is located in a low-energy region, in contrast with CG predictions (see Figure 8 for overall pattern in \( N - \Delta - \Lambda - \Sigma \) sector).

On Figure 9 we have made a comparison between Klempf’s \( S = 3/2 \) multiplets and our clusters. For the most, the two schemes have a very good overlap. In particular our \( 1\Delta(1900-1950) \) split into two Klempf’s multiplet \( 2L = 1(1900-1940) \) and \( 2L = 2(1905-1950) \) making 100% overlap. Klempf’s multiplet \( 1L = 4(2300-2420) \) included his \( 2L = 3(2350-2400) \) and our \( 2\Delta \) multiplet, making a double overlap. Few of our multiplets, do not have corresponding partners: \( 3N, 1\Sigma \) and \( 2\Sigma \).

### d) Kirchbach Model
The quantum numbers of the resonances belonging to a particular cluster fit into Lorentz group fields. To be specific, one finds the plane a well pronounced spin and parity clustering. There, it was further shown, that \( M/J \) been performed by Kirchbach [13], where it was found that Breit-Wigner masses reveal on resonances from the full listing in PDG spread with spin and parity. Such an analysis has dynamics. To uncover it, one has first to analyze isospin by isospin how the masses of the group of baryon spectra appears as a key tool in constructing the underlying strong interaction with a satisfactory description of baryon excitations. The knowledge on the degeneracy spectra. It is natural to expect from the models of hadron structure that they should supply us with a satisfactory description of baryon excitations. The knowledge on the degeneracy group of baryon spectra appears as a key tool in constructing the underlying strong interaction dynamics. To uncover it, one has first to analyze isospin by isospin how the masses of the group of baryon spectra appears as a key tool in constructing the underlying strong interaction

\[ P\sigma_{\eta,Lm} = \eta e^{i\pi \sigma} \sigma_{\eta,L-m}, \quad L = 0^\eta, 1^\eta, \ldots, (\sigma - 1)^\eta, \quad m = -L, \ldots, L \]

In coupling a Dirac spinor, \( \{1/2, 0\} \otimes \{0, 1/2\} \), to the Coulomb multiplets \( \{k/2, k/2\} \) from above, the spin \( (J) \) and parity \( (P) \) quantum numbers of the BR’s are created as

\[ J^P = 1/2^\eta, 1/2^{-\eta}, 3/2^{-\eta}, \ldots, (k + 1/2)^{-\eta}, \quad k = \sigma - 1 \]

The RS fields are finite-dimensional nonunitary representations of the Lorentz group which, in being described by totally symmetric traceless rank-k Lorentz tensors with Dirac spinor components, \( \psi_{\mu_1\mu_2\ldots\mu_k} \), have the appealing property that spinorial and four-vector indices are separated. They satisfy the Dirac equation (DE) according to:

\[ (i\partial \gamma - M)\psi_{\mu_1\mu_2\ldots\mu_k} = 0 \]

The author claimed that, in terms of the notations introduced above, all reported baryons with masses below 2500 MeV, are completely accommodated by the RS fields \( \psi_{\mu} \), \( \psi_{\mu_1\mu_2\mu_3} \), and \( \psi_{\mu_1\mu_2\mu_5} \), having states of the highest spin \(-3/2^-, 7/2^+, \) and \( 11/2^+ \), respectively (see Figs.10-12 for 3D representation). In each one of the \( N, \Delta, \Lambda \) and spectra, the RS cluster of lowest mass is always \( \psi_{\mu} \). For the nonstrange baryons, the \( \psi_{\mu} \) cluster is followed by \( \psi_{\mu_1\mu_2\mu_3} \), and \( \psi_{\mu_1\mu_2\mu_5} \), while for the \( \Lambda \) hyperons a parity doubling of the resonances starts above 1800 MeV. In the following we will extend the notation of the RS clusters to include isospin (I) according to \( \sigma_{2I,\eta} \). For example, the first \( N \) cluster is denoted by \( 2_1^+, \) while \( 2_2^+ \) and \( 2_3^+ \) stand in turn for the corresponding \( \Delta^- \) and \( \Lambda^- \)-hyperon ones. From Eqs[5] and [6] follows that the \( 2_{2I,+} \) clusters, where \( I = 1/2, 3/2, \) and 0, always unite the first spin \(-1/2^+, 1/2^-, \) and \( 3/2^- \) resonances.

Indeed, the relative \( \pi N \) momentum \( L \) takes for \( l = 0^+ \) the value \( L = 1^+ \) and corresponds to the \( P_{2I,1} \) state, while for \( l = 1^- \) it takes the two values \( L = 0^- \) and \( L = 2^- \) describing in turn the \( S_{2I,1} \) and \( D_{2I,3} \) resonances. The natural parity of the first \( O(4) \) harmonics reflects the arbitrary selection of a scalar vacuum [14] through the spontaneous breaking of chiral symmetry.
Therefore, up to three lowest \( N, \Delta, \) and \( \Lambda \) excitations, chiral symmetry is still in the Nambu-Goldstone mode. The Fock space of the \( 2_{2I,+} \) clusters will be denoted in the following by \( F_+ \). Note, that in this context the first \( P_{11} \) and \( S_{11} \) states do not pair, because their internal \( L \) differ by one unit, instead of being equal but of opposite parities. All the remaining \( N, \Delta \) have been shown to belong to either \( 4_{2I,-} \), or \( 6_{2I,-} \). They have been viewed to reside in a different Fock space, which is built on top of a pseudoscalar vacuum [14].

For example, one finds all the seven \( \Delta \)'s \( S_{31}, P_{31}, P_{33}, D_{33}, D_{35}, F_{35}, \) and \( F_{37} \) from the \( 4_{3,-} \) cluster to be squeezed within the narrow mass region from 1900 MeV to 1950 MeV, while the \( I = 1/2 \) resonances paralleling them, of which only the \( F_{17} \) state is still ”missing” from the data, are located around 1700 MeV (compare Figs. 10,11).

In continuing by paralleling baryons from the third \( N \) and \( \Delta \) clusters with \( \sigma = 6 \), one finds in addition the four states \( H_{1,11}, P_{31}, P_{33}, \) and \( D_{33} \) with masses above 2000 MeV to be ”missing” for the completeness of the new classification scheme. The \( H_{1,11} \) state is needed to parallel the well established \( H_{3,11} \), while the \( \Delta^{-} \)-states \( P_{31}, P_{33}, \) and \( D_{33} \) are required as partners to the (less established) \( N_{1/2^+}(2100), N_{3/2^+}(1900) \), and \( N_{3/2^-}(2080) \).

The degeneracy group of the \( N, \Delta \) spectra found in [13] on the grounds of the successful RS classification is

\[
SU(2)I \times SU(3)C \times O(1,3)_{LS}
\] (8)

Traditionally, hadrons are classified in terms of \( SU(6)_{SF} \times O(3)_{L} \) multiplets. This classification is one of the most important paradigms in hadron spectroscopy. According to it, states like \( N_{3/2^+}(1720), N_{5/2^+}(1680), \Delta_{5/2^+}(1905), \) and \( \Delta_{7/2^+}(1950) \), are viewed to belong to a 56\((2^+)\)-plet, the \( N_{1/2^+}(1710) \) excitation is treated as a member of a 70\((0^+)\)-plet, while the negative parity baryons \( N_{1/2^-}(1535), N_{3/2^-}(1520), N_{1/2^-}(1650), N_{3/2^-}(1700), \) and \( N_{5/2^-}(1675) \) are assigned to a 70\((1^-)\)-plet. The above examples clearly illustrate that states from Kirchbach’s RS clusters separated by only few MeV, such as \( N_{5/2^-}(1675), N_{5/2^+}(1680), \) and \( N_{1/2^+}(1710) \) from \( 4_{1^-} \), are distributed over three different \( SU(6)_{SF} \times O(3)_{L} \) representations, while on the other hand, resonances from different RS ”packages” separated by about 200 MeV, such as \( N_{3/2^-}(1520) \) from \( 2_{1^+} \), and \( N_{3/2^-}(1700) \) from \( 4_{1^-} \) are assigned to the same multiplet. This means that \( SU(6)_{SF} \times O(3)_{L} \) cannot be viewed as the degeneracy group of baryon spectra. Further, the \( SU(6)_{SF} \times O(3)_{L} \) multiplets appear approximately only ”half-filled” by the reported resonances. Several dozens states are ”missing” for the completeness of this classification scheme. As long as observed and ”missing” states are part of the same multiplets, they are indistinguishable from the viewpoint of the underlying \( SU(6)_{SF} \times O(3)_{L} \) symmetry and there is no reason not to believe to the observability of all states. On the contrary, within the RS scheme, observed and ”missing” states will fall apart and be attributed to Lorentz multiplets of different space-time properties.

The author also argued that the algebra of the degeneracy group from Eq[8] is also partly the spectrum generating algebra. Indeed, the reported mass averages of the resonances from the RS multiplets with \( L = 1, 3, \) and \( 5 \) are well described by means of the following simple empirical recursive relation:

\[
M_{\sigma'} - M_{\sigma} = m_1 \left( \frac{1}{\sigma^2} - \frac{1}{(\sigma')^2} \right) + m_2 \left( \frac{(\sigma'^2 - 1)}{4} - \frac{(\sigma^2 - 1)}{4} \right),
\] (9)

where, again, \( \sigma = k + 1 \).

The two mass parameters take the values \( m_1 = 600 \text{ MeV} \), and \( m_2 = 70 \text{ MeV} \) (so, \( m_1 \gg m_2 \)), respectively. The first term is the typical difference between the energies of two single particle states of principal quantum numbers \( \sigma \), and \( \sigma' \), respectively, occupied by a particle with mass
m, moving in a Coulomb-like potential of strength $\alpha_C$ with $m_1 = \alpha_C^2 m/2\hbar^2$. The term

$$\frac{\alpha^2 - 1}{2} = k(k/2 + 1), \text{ with } k = \sigma - 1,$$

in Eq.10 is the generalization of the three-dimensional $J(J + 1)$ rule (with $J = k/2$) to four Euclidian dimensions and describes a generalized O(4) rotational band. The parameter $1/m_2 = 2.82$ fm corresponds to the moment of inertia $\tau = 2/5$ MR$^2$ of some “effective” rigid-body resonance with mass $M = 1085$ MeV and a radius $R = 1.13$ fm. Therefore, the energy spectrum in Eq.10 can be considered to emerge from a quark-C-hyperquark model with a Coulomb-like potential ($H_{Coul}$) and a four-dimensional rigid rotator ($T^{(4)}_{rot}$). The corresponding Hamiltonian $H^{QHM}$ that is diagonal in the basis of the O(4) harmonics is given by

$$H^{QHM} = H_{Coul} + T^{(4)}_{rot} = -\alpha_c/r + (1/2\tau)F^2$$

Here, $(1/2\tau)F^2$ denotes a rigid rotator in four Euclidian dimensions as associated with a collective effect there. The author claimed: while the splitting between the Coulomb-like states decreases with increasing $\sigma$, the difference between the energies of the rotational states increases linearly with $\sigma$ so that the net effect is an approximate equidistancy of the baryon cluster levels.

Let’s check this out with our modified CGI multiplets (Table 4). As one can see from modified CGI mass differences, there is no equidistancy in the baryon cluster levels. Instead the separation is basically increasing with $J$ and $M$, while strong fluctuations occurred. An average mass separation is 220 MeV.

So the author showed that $N^*$ and $\Delta^*$, instead of being uniformly distributed in mass, as naively expected on the basis of a 3q-Hilbert space without degeneracy, form well-pronounced spin- and parity-clusters. The masses of the RS-clusters and their spacing were shown to follow O(4) rotational bands slightly modified by a Balmer-like term (see also [15]).

We still have to point to some inconsistencies in the Kirchbach's model. Let’s put $N, \Delta, \Lambda, \Sigma$ spectra on one plot (see Fig.13). One can definitely witness quite strange features of RS clusters:

- First $N^*$-cluster has 95 MeV width, second $N^*$-cluster has 70 MeV width, and third $N^*$-cluster has 350 MeV width. So the last $N^*$-cluster has a huge width, which is incompatible with reasonable definition of the cluster.

- First $\Delta^*$-cluster has 130 MeV width, second $\Delta^*$-cluster has 50 MeV width, and third $\Delta^*$-cluster has 420 MeV width. Again, the last $\Delta^*$-cluster has a huge width, which is incompatible with reasonable cluster definition. $\Lambda$-hyperons have one cluster in this scheme at low masses, but have big 195 MeV width, which hardly fits cluster definition.

- First $\Sigma^*$-cluster has a reasonable 80 MeV width. Second $\Sigma^*$-cluster and third $\Sigma^*$-cluster have 360 and 370 MeV widths correspondingly, which make them bad candidates for the realistic clusters. But that’s not all: second and third $\Sigma^*$-cluster have a strong overlap for a 110 MeV, making a $\Sigma$-sector a real failure of the Kirchbach’s model.

If we will try to merge $N - \Delta - \Lambda - \Sigma$ spectra onto one plot of the concentric rings, like in our model, nothing similar will happen. RS-rings will mostly fuse into a couple of very broad rings, with practically no spacings between them (see Fig.14). It is interesting to find some correspondence between RS-clusters and our clusters (see Fig.13). We see that our 1N(1650 − 1720) completely coincides with 2N*(1650 − 1720) RS from Kirchbach.

Third $N^*$ RS cluster $\psi_{\mu\nu\rho\tau}(1900 − 2250)$ covers completely our 2N, 3N, and 4N clusters. In other words our 2, 3, 4-N-clusters are inserted exactly into $\psi_{\mu\nu\rho\tau}$.

First $\Delta^*$ RS cluster $\psi_{\mu}(1620 − 1750)$ covers totally second $N^*$ RS cluster $\psi_{\mu\nu}(1650 − 1720)$, and our 1N−, and 1Σ− clusters.

Second $\Delta^*$ RS cluster $\psi_{\mu\nu}(1900 − 1950)$ is identical to our 1Δ.

Third $\Delta^*$ RS, $\psi_{\mu\nu\rho\tau}(2000 − 2420)$ cover totally our 3N, 4N, 3Σ, and 2Δ.
Third $N^*$ RS and third $\Sigma^*$ RS are totally corresponding to each other: $3N^*(1900 - 2250)$ is exactly inserted into $3\Sigma^*(1880 - 2250)$.

III. CONCLUSIONS

In this paper we analyze four state-of-the-art models of hadronic structure. Chiral symmetry plays an eminent role in the formation of the spectra. This symmetry is broken only by the very small $u, d$ quark masses. However, this symmetry is not directly observed in the world - it is hidden, i.e. spontaneously broken.

The Cohen-Glozman model with their chiral quartets, is based on the idea of chiral symmetry breaking on a vacuum level and then full restoration of it high in the spectrum, precisely from 2 GeV. The authors claim that upwards from 2 GeV parity doublets and chiral quartets arise - and the precise chiral symmetry restoration is responsible for the genesis of these doublets and quartets.

Our results together with the Klempt model show that parity doublets and chiral multiplets are indeed observed, but:

1) This start to happened far below 2 GeV.
2) The multiplet’s width is growing as we are moving higher and higher in mass and $J$.
3) Standard GC chiral multiplets are not properly filled from $J = 11/2$.
4) The experimental chiral multiplet structure could even better be described in $LS$ multiplet sheme ( or, using $SU(6)_{SF} \times O(3)_L$ group).

So, in fact one can see the sprouts of chiral symmetry restoration above 2 GeV, but there are definitely a few mechanisms, which are responsible for the parity doublets and chiral multiplets formation in the current resonance energy region.

Klempt’s model has a few nice features - formation of the $LS$ multiplets, reminiscent of the experimental clusterings (see Figs.2-4), and introduction of the solitary states and an attraction between them. On the other hand, neglecting of the $LS$ coupling and mixing of the different orbitals in baryon WF sometimes lead to wrong results (see the discussion above on $N_{3/2^+}(1720)$ and other similar states).

Kirkbach aspires to have discovered a degeneracy group of the baryonic spectra: $SU(2)_I \times SU(3)_C \times O(1,3)_{LS}$. This group was inspired by Höehler discovery of the $N, \Delta$ spectra clustering phenomenon, i.e. - Höehler poles, or Höehler clusters. All resonances are classified as Rarita-Schwinger multispinors, and spin-parity multiplets arise. To the advantage of this model we have to attribute the prediction of both existing and missing BR’s, belonging to Lorentz multiplets of different space-time properties. Nevertheless we have to admit that this model, RS-clusters are getting very broad at already low masses. What’s even more - second and third $\Sigma$-clusters have a strong overlap, making RS-scheme non-applicable in $\Sigma$-sector.

We have considered baryonic structure, based on three different groups:

\[
\begin{align*}
&SU(2)_R \times SU(2)_L \quad \text{(and, } SU(3)_R \times SU(3)_L) \\
&O(3)_L \times SU(6)_{SF} \\
&SU(2)_I \times O(1,3)_{LS} \times SU(3)_C
\end{align*}
\]

All of these groups predict clustering patterns in $N, \Delta, \Lambda, \Sigma$ spectra, but these clusters are different. Even the topology of resulting towers is different and reflected in regular and flipped pyramidal structures (see Fig.15).

Klempt’s and Inopin multiplets exhibit ”rings structure”, or ”solar-type structure” with exactly 10 rings - copies of the Solar planets (see Fig.16). The distances between the rings vary - so does this happen in the Solar system as well.

We certainly hardly could give preference to any of the above models - each has its own bold features, which are absent in the others. We have also uncovered a number of interesting
symmetries and multiplet structures in mesonic sector, but this story will be unfolded in a separate paper.

Finally we have to mention the model of Hatsuda et al [16]. This model has the spirit of CG model, with interesting "mirror ansatz". Unfortunately, the authors consider only low-lying $N - \Delta$ quartets, which makes their model of a limited value.

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References

[1] T.P. Cheng, L.F. Li, *Gauge Theory of Elementary Particle Physics*, Clarendon Press, Oxford, 1984.

[2] G. Höehler, In: *Physics with GeV -particle beams*, eds. H. Machner and K. Sistemich (World Scientific, Singapore) 1995, p.198.

[3] F. Iachello, Phys.Rev.Lett., v.62 (1989), 2440.

[4] T.D. Cohen, L.Ya. Glozman, Int.J.Mod.Phys.A, vol.17, (2002), 1327.

[5] Particle Data Group, K. Hagiwara et al., Phys.Rev.D 66, 010001 (2002).

[6] A.E. Inopin, E.V. Inopin, Sov.J.Nucl.Phys. 53(2), (1991), 351.

[7] A.E. Inopin, G.S. Sharov, Phys.Rev.D 63, 054023 (2001).

[8] E. Klempt, *nucl-ex/0203002*, Phys.Rev.C 66, 058201 (2002); [hep-ph/0212241](http://arxiv.org/abs/hep-ph/0212241)

[9] A.E. Inopin, [hep-ph/0012248](http://arxiv.org/abs/hep-ph/0012248)

[10] A.E. Inopin, [hep-ph/0110160](http://arxiv.org/abs/hep-ph/0110160) (review).

[11] F. Iachello et al., Ann.Phys. v.284 (2000), 89.

[12] N. Isgur, S.Capstick, Phys.Rev.D 34 (1986), 2809.

[13] M. Kirchbach, Int.J.Mod.Phys. A15 (2000), 1435.

[14] P.W. Milonni, *The Quantum Vacuum. An Introduction to Quantum Electrodynamics*. Academic Press, 1994, 522p.

[15] M.Kirchbach, M. Moshinsky, Yu.F. Smirnov, Phys. Rev. D64, 114005 (2001).

[16] T. Hatsuda, D. Jido, T. Kunihiro, Phys.Rev.Lett. v.84 (2000), 3252.
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