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What role do futures markets play in Bitcoin pricing? Causality, cointegration and price discovery from a time-varying perspective?☆

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ABSTRACT

Recent papers that have explored spot and futures markets for Bitcoin have concluded that price discovery takes place either in the spot, or the futures market. Here, we consider the robustness of previous price discovery conclusions by investigating causal relationships, cointegration and price discovery between spot and futures markets for Bitcoin, using appropriate daily data and time-varying mechanisms. We apply the time-varying Granger causality test of Shi, Phillips, and Hurn [2018]; time-varying cointegration tests of Park and Hahn [1999], and time-varying information share methodologies, concluding that futures prices Granger cause spot prices and that futures prices dominate the price discovery process.

1. Introduction

Bitcoin was the first digital asset established in 2008 and since then, it has increased from less than US$1 in 2010 to reach a peak of approximately US$19,000 in December 2017. During its peak, the Chicago Board Options Exchange (CBOE) and the Chicago Mercantile Exchange future markets (CME) introduced futures contracts for Bitcoin on 10 December 2017 and 18 December 2017, respectively. CBOE and CME are regulated exchanges and both futures are cash-settled in US dollars.1 On March 2019, the CBOE decided not to list additional Bitcoin futures contracts for trading and the last futures contracts expired on 19 June 2019. As a result, the CME remains the only currently traded and regulated exchange. Some comparisons of the contract specifications of the CBOE and CME markets are shown as Table 1, below.

Bitcoin futures contracts have attracted some attention from academics where recently, several studies have attempted to explore price discovery in the spot and futures markets for Bitcoin. Corbet, Lucey, Peat, and Vigne [2018] apply four measures of price discovery including the information share methodology of Hasbrouck [1995], the component share approach of Gonzalo and Granger [1995], the information leadership measure of Yan and Zivot [2010] and the information leadership share measure of Putniņš [2013], between the CBOE and CME futures and spot prices using data, sampled at a one-minute frequency. They typically find that price discovery is focused on the spot market. However, Corbet et al. [2018] use the same Bitcoin spot prices for the CBOE and CME futures in analyzing price discovery. Kapar and Olmo [2019] conclude that the CME futures market dominates the price discovery process at the daily frequency using the price discovery measures of Gonzalo and Granger [1995] and Hasbrouck [1995] to explore the contribution of each market to the price discovery process. Kapar and Olmo [2019] use the Coindesk Bitcoin USD Price Index as the spot price. Akyildirim, Corbet, Katsiampa, Kellard, and Sensoy [2019] find that Bitcoin futures dominate price discovery relative to spot markets using the information share methodology of Hasbrouck [1995], Gonzalo and Granger [1995], Yan and Zivot [2010] and Hauptfleisch, Putniņš, and Lucey [2016]. Baur and Dimpfl [2019] also investigate the price discovery of Bitcoin using the information share methodology of Hasbrouck [1995] and Gonzalo and Granger...
and Baur and Dimpfl [2019] use the same price discovery measures of Corbet et al. [2018], Kapar and Olmo [2019], Akyildirim et al. [2019] and Baur and Dimpfl [2019] use the same price discovery measures of Hasbrouck [1995] and Gonzalo and Granger [1995], they produce different results. Not only are the future contracts of the CBOE and CME different, but the underlying spot prices used are also different, see Table 1. However, this important point seems to have been ignored until very recently, see Alexander and Dakos [2020]. It is well known from the general time series econometrics literature, that some possible cointegrating relationships may be missed if the underlying model formulation is constrained to be time invariant. We, therefore, investigate here the existence of possible cointegrating relationship between spot and futures prices under the assumption of a time-varying cointegrating coefficient based on the Park and Hahn [1999] test. We also add via our analysis, the new date stamping approaches of Shi, Phillips, and Hurn [2018] which allow us to provide more insights into potential time-varying Granger causality relationships, which is also new in this area of research.

In particular our paper contributes to the current and fast-growing literature on Bitcoin in five key ways. First, we apply a newly developed time-varying Granger causality approach of Shi et al. [2018] to explore the causal relationship between Bitcoin spot and futures markets for the first time. There are many advantages of this new approach, in particular, it allows for unknown change points in the causal relationships and also accommodates potential heteroskedasticity, which is typically ignored in the existing literature. This new testing procedure also does not require detrending or differencing of the data. Of particular interest here is that the approach allows practitioners to identify the origination and termination dates for any episodes of Granger causality and hence, the lead-lag relationship between spot and futures markets.

Second, the paper explores, for the first time, the time-varying Granger-causal relationship between spot and futures Bitcoin prices. We also suggest that a time-varying approach is better suited to investigate the causal relationship in Bitcoin markets despite existing work choosing to adopt a time-invariant version Granger causality tests. As shown in Shi, Hurn, & Phillips [2020], there are many reasons to expect the existence of a time-varying causal relationship between variables of interest (e.g., changes in economic policy, regulatory structure, governing institutions, or operating environments). Of particular interest in this paper, we would expect the existence of a time-varying causal relationship for several reasons listed below.

- Bitcoin is well-known for its highly volatile nature, for example, Bitcoin’s dramatic rise and fall in 2017–18 has lead many to see it as a speculation-drive bubble asset.

- Bitcoin isn’t tied to any financial assets and its price is extremely sensitive to financial markets.
- Bitcoin faces regulatory challenges, for example, governments can restrict or ban Bitcoin trading.
- Mining difficulty is adjusted periodically over time (approximately 14 days) as a function of how much hashing power has been deployed by the network of miners.
- Bitcoin mining experiences substantial technological changes in computing power by improving the processing power (hashrate). According to Taylor [2017], the hardware for Bitcoin mining has evolved rapidly since the invention of Bitcoin.
- The power used for Bitcoin mining has changed over time. Power may be generated from fossil, natural-gas, nuclear, solar, wind or hydropower.
- The places used for mining have also changed over time. Bitcoin miners move to places with low electricity costs.

For these reasons, we believe a time-varying approach is best-suited to explore causal relationships in Bitcoin markets.

Third, the paper tests for potential cointegration under a time-varying cointegrating coefficient assumption, using the Park and Hahn [1999] test. Previous literature on this topic takes no account of the possibility of a time-varying cointegrating coefficient. The reason why the cointegration coefficient is considered to be time-varying is consistent with the argument presented by Park and Hahn [1999] who state that “since cointegration reveals the long-run relationship between variables, it can not be ruled out that such a relationship does not hold still throughout a long sample path given the possibility of the changing conditions of both macro and micro fundamental drivers pertaining to market operations and regulatory circumstances”. The general reasons argued by Park and Hahn [1999] for a time-varying cointegrating coefficient can also apply to the long-run relationship between the Bitcoin spot and futures prices. Some features of Bitcoin markets, as summarized above, suggest that market operations and regulatory circumstances that govern markets operations, including the determination of fundamental values, are relevant to both spot and futures prices and would not remain unchanged over time, further suggesting that there might exist dynamic, predictable power from the futures price to the spot, equivalent to a time-varying cointegrating relation. For example, the sample period of this study covers a dramatic and ultimately volatile booming phase for Bitcoin that was followed by subsequent drops in its value. During this period, the regulatory environments facing the Bitcoin markets saw a series of changes as authorities’ regulatory policies on trading Bitcoin underwent a number of adjustments. Mining activities relating to how Bitcoin market operates are also changing over time, contributing to a time variant relationship between Bitcoin spot and futures prices. Examples here include, mining difficulty that adjusts periodically over time, technological changes in computing power to support mining processes, changing energy power for mining, and changing locations for mining activities. Although time variations limiting arbitrage in the Bitcoin spot and futures markets may be transient, variables of regulatory circumstances and market operations that change over time do not rule out a possibility of a time varying spot-futures cointegrating relation. This paper is devoted to exploring evidence for such relationships.

Fourth, the paper also fills a gap by investigating price discovery in Bitcoin spot and futures markets using a time-varying perspective. The existing literature has found that the ability to assimilate new information in stock indices and foreign currency futures markets in the long run varies over time. This might be due to the fact that the activities of informed traders in the market may appear as differing patterns over time, due to market volatility (Chakravarty, Gulen, & Mayhew, 2004; Chen & Gau, 2009; Chen & Gau, 2010); the number of transactions in given time intervals (Ates & Wang, 2005); and bid-ask spreads (Ates & Wang, 2005; Chen & Gau, 2010), all of which appear to significantly drive the evolution of information shares in futures markets. These variables are regarded as proxies reflecting non-constant

| Contract specifications | CBOE futures | CME futures |
|-------------------------|--------------|-------------|
| First trading date      | 10 December 2017 | 18 December 2017 |
| Symbol                  | XBT          | BTC         |
| Contract unit           | 1 Bitcoin    | 5 Bitcoin   |
| Tick size               | $10 per contract | $25 per contract |
| Underlying spot price   | The Gemini auction price | The Bitcoin Reference from the Gemini exchange. |

Note: Table 1 Some key comparisons of the CBOE and CME futures markets.
trading over time. In this study we apply concepts from traditional financial assets to the Bitcoin spot and futures markets, while allowing information shares in the Bitcoin spot and futures markets to be time-varying rather than static. As discussed previously, most of current empirical studies investigate price discovery in the Bitcoin futures markets using the static (time-invariant) information share methodology. In this paper, we follow Avino, Lazar, and Varotto [2015] and explore a time-varying information share by considering a bivariate conditional variance-covariance matrix for Bitcoin spot and futures returns. A bivariate Dynamic-Conditional-Correlation (DCC) GARCH model is employed to model the conditional variance-covariance matrix. According to Engle [2002], the DCC model can provide a better approximation for the second moment of a multivariate return distribution than other GARCH specifications. Furthermore, we take into account non-zero skewness and excess kurtosis of return distributions in the Bitcoin spot and futures markets for the time-varying price discovery measurements. The findings shed more light on the efficiency of Bitcoin futures markets when market data exhibits asymmetry and fat tails.

Fifth, this paper enriches the literature on the empirical analysis of Bitcoin futures markets by correctly specifying the underlying spot prices for the CBOE and CME futures markets. We suggest that future research uses the correct pair of spot-futures prices for subsequent analysis from a reliable and trusted data source for example, Thomson Reuters Datastream. More specifically, this paper contributes to the literature by highlighting, and demonstrating the effects, of using the correct spot Bitcoin products that underlie the assets specified in the futures contracts traded in the CBOE and CME. We also emphasize the reliability of databases that contain low-frequency data of the spot assets such as daily data and weekly one. Such information will prove very useful for future researcher on Bitcoin spot and futures prices.

With the above contributions, we are able to address the following three important questions from a time-varying perspective:

- Do futures prices Granger cause spot prices, or vice versa?
- Do futures prices cointegrate with spot prices? If they do, is cointegration static or time varying?
- Do futures prices lead spot prices in the price discovery process, or vice versa?

The remaining parts of the paper are organized as follows. Section 2 describes the existing literature to which we contribute and the particular financial asset issues related to Bitcoin and the testing of causal relationships in this market based upon spot and futures prices. Section 3 discusses the data and presents the econometric methods. Section 4 presents the empirical findings and Section 5 concludes.

2. Existing literature and issues

Price discovery is generally considered to be an important indicator of the functionality that futures contracts provide towards the underlying spot assets’ transactions, and reflects one of the major contributions of futures markets to the organization of economic activity [Silber, 1981]. A number of empirical studies support the hypothesis that futures prices absorb new information first, which is then transmitted to the underlying spot market via cross-border transactions etc. Futures markets, therefore, are generally regarded for most traditional financial assets to lead the underlying spot price in the long run. Much of the theory supporting Granger causality, in this case, is based upon, for example, the assumption that asset prices are driven by the discounted present value of the long-term earnings of shares, or the price dynamics of a bond. The case for a spot-futures relationship can be illustrated via a number of approaches including a cost-of-carry model which shows that the intrinsic value of the futures price is the continuously compounded value of spot prices within the time to maturity (Chan, 1992; Garbade & Silber, 1983; Stoll & Whaley, 1990; Wahab & Lasbargi, 1993). The cost-of-carry model identifies a no-arbitrage condition for futures prices and how a fair price for futures is determined. It allows us to propose that the spot price should not drift away from the futures price for sufficiently long periods given that the no-arbitrage condition ensures that market efficiency is maintained. The cost-of-carry approach translates into a situation where the long-run equilibrium ties the spot price to the relevant futures price. This theoretical relationship between spot and futures prices translates empirically to a case where estimation and testing can be undertaken within a cointegrating regression framework where the spot and futures prices should be cointegrated as long as the no-arbitrage condition holds. Supporting evidence for such implications have been found widely in the commodity markets and stock index markets (see, e.g., Garbade & Silber, 1983; Stoll & Whaley, 1990, Chan, 1992, Wahab & Lasbargi, 1993, Ghosh, 1993, Koutmos & Tucker, 1996, Pizzi, Economopoulos, & O’Neill, 1998, Yang, Bessler, & Leatham, 2001, Kavussanos, Visvikis, & Alexakis, 2008; Rosenberg & Traub, 2009, Cabrera, Wang, & Yang, 2009, Bohl, Salm, & Schuppli, 2011, Hauptfleisch et al., 2016; among others).

The identification of the price discovery mechanism can be extended to be time variant, since the literature has found that the assimilation of information to reflect intrinsic values can evolve over time. One of the main consequences is that a time-varying variance and covariance of price innovations occurs, which reflects the evolving process of information generation and assimilation. Attempts to capture the time-varying information share have been recently described in the literature, where there appear to be predominantly three methods established to capture the time-varying nature of price discovery. First, the time-varying information share is calculated based upon the time-varying error correction coefficients that can be derived via a rolling-window estimation on the vector error correction model [Bell, Brooks, & Taylor, 2016] or a series of scaling factors imposed on the original adjustment coefficients [Taylor, 2011]. The second involves calculating the information share that varies at low-frequency intervals, by using high-frequency tick data [Ates & Wang, 2005; Chen & Gau, 2009, 2010; Xu & Wan, 2015]. Finally, Avino et al. [2015] propose to generate the time-varying information share by extending the innovation covariance matrix to be conditional on past information, where the multivariate BEKK-GARCH model is employed to estimate the conditional covariance matrix.

The informational role of a futures market has been extensively studied by investigating possible lead-lag relationships between spot and futures markets. Granger causality is widely used to formally test for lead-lag relationships (temporal ordering) to determine which market (the spot or futures prices) leads the other. Care must be exercised here, especially the need for robustness of the results, as it is well-documented that Granger causality tests can be very sensitive to the time period of estimation or to assumptions that causal
relationships do not change (time invariant) over time. The procedure of Shi et al. [2018] allows practitioners to examine whether the causal relationship varies over the time, as may be expected for many economic and financial variables, including Bitcoin.

In addition, the nature of the cointegrating relationship between spot and futures prices has important implications. For futures and spot markets there exist a priori expectations that there is a strong (cointegrating) relationship between the two markets. If spot and futures prices are cointegrated, spot-futures parity exists, indicating that no arbitrage opportunities arise. Moreover, the presence of cointegration also shows that futures markets are efficient. However, when testing for such conditions, conventional cointegration tests assume a static (time-invariant) framework. If this assumption is invalid, cointegration may be falsely rejected as well the implications listed above, hence, it is important to allow for a time-varying cointegration framework (where time invariance is a special case) which enriches the potential interactions between variables when they are driven by the same information set.

The discussion and cited papers above are taken from financial futures markets where the assets/commodities fit within traditional price discovery financial environments/markets where theory might naturally drive us to think Granger causality should run from the long run to the short run with spot and futures markets being naturally cointegrated. However, when the spot/futures markets are cryptocurrency-based, it is not clear what the pricing model(s) should be (the fundamentals) and similarly how the price discovery dynamics should be modeled and whether the relationships are stable. The advent of cryptocurrency trading has led to considerable discussion about what determines their prices and in particular the links between spot and futures prices. As discussed above, grounding cryptocurrency pricing within a cost-of-carry framework embeds the no-arbitrage condition into the pricing process, which empirically translates into the existence of a cointegrating relationship framework. Booth, So, and Tse [1999], Jong and Donders [1998], Fleming, Ostdiek, and Whaley [1996] and Hsieh, Lee, and Yuan [2008] argue that, in a trading cost world, futures trades have a lower trading cost and faster information absorption. The theoretical rationale of futures' superiority in information content has also been well discussed and explained by Fleming et al. [1996], where the theoretical rationale of futures' superiority in information content has also been well discussed and explained by Fleming et al. [1996].

Cryptocurrencies are so different to traditional markets that using the risk-free rate as a key compounding factor may not be appropriate and the key concept of risk neutrality has yet to be fully investigated and tested in this type of market. As such, the question as to whether the cost-of-carry approach is an appropriate one for pricing cryptocurrency-based futures has not been established, but can be addressed via various tests including cointegration, lead-lag causality and price discovery stability. Given some of the unique features of the cryptocurrency spot and futures prices summarized in the Introduction, one might expect some differences in the cryptocurrency futures pricing model compared to other asset markets. Place this in a world of disruption coming from e.g., COVID-19, where for example Bitcoin, may have acted as a safe haven (Corbet, Hou, Hu, Larkin, & Olexyn, 2020) and there is an urgent need to test, using advanced, appropriate and flexible econometric methods, for directions of causality between spot and futures markets and, in an ever changing world, whether such relationships are stable over time.

Hence, in this paper we seek to identify the causal directions, lead-lag dynamics and potential regime-switching that may occur for Bitcoin spot/futures prices using the most flexible and general model as yet applied to these data and this literature. The nature of the data used and econometric flexibilities employed are discussed in Section 3 below.

3. Data and method

3.1. Data

The CBOE and CME were the first two regulated exchanges that have provided futures contracts on Bitcoin. We use the daily settlement price of the CBOE Bitcoin futures from 18 December 2017 to 16 June 2019, and the final settlement value (defined as XBTUS) based on the Gemini auction price at 4 pm Eastern time as the spot prices for the CBOE market. For CME, we use the daily settlement price of the CME Bitcoin futures from 25 December 2017 to 29 July 2019, where the CME Bitcoin future prices are based on the CME Bitcoin Reference Rate (BRR), which is used as a spot price for the CME market. BRR refers to the daily reference rate of the U.S. dollar price of one Bitcoin as developed by the CME. The BRR aggregates the trade flows from the major Bitcoin spot exchanges, for example, Bitstamp, Coinbase, itBit and Kraken at 4 pm London time to ensure transparency and replicability in the underlying spot markets. To construct the CBOE and CME futures price series, we only use daily price observations of the most nearby futures contracts to ensure their liquidity, where the most nearby contracts are the ones that are closest to the expiration dates at each calendar month. We switch the most nearby contract to the second most nearby one if the trading volume of the former is exceeded by the latter at the former's contract month.

The Gemini auction price and the BRR are used to represent the spot markets under consideration in this paper. To align with the futures prices of Bitcoin, we then obtain the daily Gemini auction price and the BRR for the same period with the CBOE and CME futures. After matching the spot and futures data series, we are left with 416 observations for the CME sample and 393 observations for the CBOE sample. Note that our sample excludes data on weekends due to unavailability. Both the spot and future prices are downloaded from Thomson Reuters Datasstream for our empirical analysis. It should be noted that the futures and spot prices used by the CBOE and CME are different, hence the empirical analysis is based on the counterpart spot markets.

All spot and futures prices are transformed to natural logarithms and are presented as Fig. 1. As can be seen from the figure, there is a decreasing trend for both spot and futures prices from the beginning of the sample period until the early of February 2019, which might represent a bear market in both the spot and futures markets. From early February 2019, both prices follow an upward trend until the end of sample period, which suggests a bull market. Second, the patterns of both spot and futures prices look similar. It is possible that there exists a long run relationship between spot and futures prices. This will be further examined formally via tests for cointegration.

We also provide descriptive statistics on Bitcoin spot and futures daily returns in Table 2. As can be seen from the table, the means of spot and futures returns are negative. The volatilities of the four markets are similar. Second, returns of the four markets do not follow a normal distribution as indicated by a Jarque-Bera test. This might be due to non-zero skewness and excess kurtosis, which will be further examined via a semi-nonparametric (SNP) approach. Finally, heteroscedasticity may exist in the spot and futures returns given the existence of significant Ljung-Box Q statistics. This issue will be addressed by using a DCC-GARCH modeling approach.

6 More details related to the CBOE Bitcoin futures can be assessed by the website cache at https://cfe.cboe.com/cfe-products/xbt-cboe-bitcoin-futures/contract-specifications.
7 Gemini is a digital currency exchange.
8 More details about the CME Bitcoin futures can be found from the CME website at https://www.cmegroup.com/education/bitcoin/cme-bitcoin-futures-frequently-asked-questions.html.
9 For a discussion of the choice of spot and futures data, see Baur and Dimpfl [2019].
The volume of CBOE and CME Bitcoin futures contracts are shown in Fig. 2. As can be seen, trading volumes for both the CBOE and CME futures contracts are quite high, suggesting that the Bitcoin traders are active in Bitcoin transactions. The volume of CBOE Bitcoin futures dominates in the early markets from December 2017 to the middle of 2018 after which the CME's product starts to dominate in the market and this phenomenon becomes more evident when the CBOE decided to stop listing its product in March 2019. A comparison of the trading volume for both the CBOE and CME Bitcoin futures is presented in Table 3. Thus it is interesting to investigate the pricing behaviour of futures contracts in these two markets resulting from those transactions.

Daily returns are calculated as the first order differences of log daily prices. Std. Dev., standard deviation. JB, the Jarque-Bera test statistic for normality. LB(12) denote the Ljung-Box Q test statistic for return squares up to lag order 12. *** denotes significance at the 1% level.

3.2. Methods

3.2.1. Time-varying Granger causality tests

The following section is taken from Shi et al. [2018]. We can write an unrestricted VAR(p) in multi-variate regression format simply as:

\[ y_t = \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \ldots + \Phi_p y_{t-p} + x_t + \epsilon_t, \quad t = 1, \ldots, T \]

where \( y_t = (y_{1t}, y_{2t})' \), \( x_t = (1, y_{1t-1}, y_{2t-1}, \ldots, y_{1t-p}, y_{2t-p})' \), and \( \Gamma_{1:2(2p+1)} = [\Phi_1, \Phi_2, \ldots, \Phi_p] \). Let \( \bar{\Pi} \) be the ordinary least squares estimator of \( \Pi \), \( \bar{\Pi} = \hat{\Pi} = T^{-1}Y'Y' \) with \( \hat{\epsilon}_t = y_t - \hat{\Pi}x_t \) and \( X' = [x_1, \ldots, x_T] \) be the observation matrix of the regressors in Eq. 1. In order to test the null hypothesis that \( y_{2t} \) does not Granger cause \( y_{1t} \), the Wald test for such restrictions can be denoted as:

\[ W = [R'\hat{\Omega}^{-1}R][R'\hat{\Omega}^{-1}(XX')^{-1}R']^{-1}[R'\hat{\Omega}^{-1}] \]

where vec(\( \prod_t \) ) denotes the (row vectorized) \( 2(2p+1) \times 1 \) coefficients of \( \hat{\Pi} \) and \( R \) is the \( p \times 2(2p+1) \) matrix. Each row picks one of the coefficients to set to zero under the non-causal null hypothesis. There are \( p \) coefficients on the lagged values of \( y_{2t} \) in Eq. 1.

10 The trading volume of CBOE and CME Bitcoin futures are available from Datastream.

Table 2

Descriptive statistics of the daily returns for spot and futures markets.

| Statistic | Gemini auction price | CBOE futures | BRR | CME futures |
|-----------|----------------------|--------------|-----|-------------|
| Mean      | -0.0018              | -0.0019      | -0.0009 | -0.0009 |
| Median    | 0.0000               | 0.0000       | -0.0004 | 0.0000 |
| Std. Dev. | 0.0470               | 0.0479       | 0.0476 | 0.0514 |
| Skewness  | -0.1906              | -0.1493      | -0.0764 | -0.3198 |
| Kurtosis  | 6.4944               | 6.2973       | 7.1780 | 7.0927 |
| JB        | 201.8147***          | 179.0373***  | 302.2371*** | 296.7135*** |
| LB(12)    | 19.833***            | 20.562*      | 51.658*** | 51.658*** |

Fig. 1. Time series plot of the CBOE futures prices, Gemini auction price, the CME futures prices and the CME Bitcoin Reference Rate prices in natural logarithmic scale.
Following the recent bubble detection tests of Phillips, Shi, and Yu [2015], Shi et al. [2018] develop three tests based on the supremum norm (sup) of a series of recursively evolving Wald statistics for detecting changes in causal relationships using a forward recursive, a rolling window and a recursive evolving algorithm. If the Wald statistic obtained for each subsample regression over a rolling window and a recursive evolving procedure. If the Wald statistic exceeds (goes below) its corresponding critical value, a significant change in causality is detected. The origination (termination) date of a change in causality is identified as the first observation whose test statistic respectively exceeds or falls below the critical value. The dating rules of the rolling and recursive evolving procedures are much higher than that of the forward recursive testing procedure, and the recursive evolving algorithm offers the best finite sample performance. Hence, we investigate the potential causal relationship using these two procedures in this paper.

In estimating the bivariate VAR and implementing tests of Granger causality, the lag order is selected using the Bayesian Information Criterion (BIC) with the maximum lag length 12. The minimum window size $f_0$ is set to 0.2. The critical values are obtained from a bootstrapping procedure with 499 replications. The empirical size is 5% and is controlled over a three-month period.

### 3.2.2. Measurement of price discovery

Let $Y_t$ be an $n \times 1$ vector of $I(1)$ series and assume that there exist $n - 1$ cointegrating vectors; that is, $Y_t$ contains a single common stochastic trend [Stock & Watson, 1988]. Then $Y_t$ can be specified in the following vector error correction model (VECM) [Engle & Granger, 1987]:

$$\Delta Y_t = \pi Y_{t-1} + \sum_{i=1}^{\lambda} A_i \Delta Y_{t-1} + \varepsilon_t,$$  

where $\pi = \alpha \beta'$, $\alpha$ and $\beta$ are $n \times (n - 1)$ matrices with $n - 1$ non-zero eigenvalues. $\beta$ contains ($n - 1$) cointegrating vectors such that $\beta Y_{t-1}$ consists of ($n - 1$) cointegrating equations. Each column of $\alpha$ is comprised of adjustment coefficients. The covariance matrix of the error term is given by $\Omega = E(\varepsilon_t \varepsilon_t')$, where $E[.]$ is the expectation operator. Following Stock and Watson [1988] and Hasbrouck [1995], Eq. 6 can be expressed as:

$$\Delta Y_t = \pi Y_{t-1} + \sum_{i=1}^{\lambda} A_i \Delta Y_{t-1} + \varepsilon_t,$$  

where $\Omega = E(\varepsilon_t \varepsilon_t')$, where $E[.]$ is the expectation operator. Following Stock and Watson [1988] and Hasbrouck [1995], Eq. 6 can be expressed as:

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where $\Omega = E(\varepsilon_t \varepsilon_t')$, where $E[.]$ is the expectation operator. Following Stock and Watson [1988] and Hasbrouck [1995], Eq. 6 can be expressed as:
be transformed into the following vector moving average (VMA) model:
\[ \Delta Y_t = \Psi(L)\varepsilon_t, \]
(7)
\[ Y_t = Y_0 + \Psi(1) \sum_{i=1}^t \varepsilon_i + \Psi^*(L)\varepsilon_t, \]
(8)

According to the Engle-Granger representation theorem Engle and Granger [1987], \( \Psi(1) \) has the following important properties due to the cointegrated unit-root series [De Jong, 2002; Lehmann, 2002]:
\[ \beta_t \Psi(1) = 0 \quad \text{and} \quad \Psi(1)\varepsilon_t = 0. \]

More importantly, \( \Psi(1)\varepsilon_t \) in Eq. 8 represents the long run impact of innovations on the unit-root series [Hasbrouck, 1995]. This term is the major focus of different information share measures.

### 3.2.3. Hasbrouck Information (IS)

In Hasbrouck [1995], all the prices are equal in equilibrium because these series correspond to the prices of the same security being traded in multiple markets. This would impose special restrictions on cointegrating matrix \( \beta \). That is, each of the pairwise cointegrating vector in \( \beta \) is \((1, -1)\). Thus, Eq. 9 implies that the rows of \( \Psi(1) \) are identical. Let \( \psi = (\psi_1, \psi_2, \ldots, \psi_n) \) be the identical row of \( \Psi(1) \). Then \( \psi \varepsilon_t \) represents the long-run impact innovations on each of the price series. Assuming that the covariance matrix \( \Omega \) is diagonal (i.e., the innovations are independent), the IS of market \( j \) is defined as:
\[ S_j = \frac{\psi_j^T \Omega_j^{-1} \psi_j}{\psi_j^T \Omega_j \psi_j}, \]
(10)
where \( \psi_j \) is the \( j \)th element of the row vector \( \psi \). \( \psi_j \Omega_j^{-1} \psi_j \) represents the variance of \( \psi_j \). It can be decomposed as:
\[ \psi_j \Omega_j^{-1} \psi_j = \sum_{i=1}^n \psi_{ij}^2 \Omega_{ii}, \]
(11)

Note that since \( \psi \varepsilon_t \) represents the long-run impact of innovations on unit-root series, the IS of market \( j \) is the proportion of the long-run impact of innovations that is attributable to innovations of market \( j \) [Baillie, Booth, Tse, & Zabotina, 2002]. In other words, the IS of market \( j \) is the contribution of market \( j \) to the total variance of the common efficient price or permanent impact [Lien & Shrestha, 2014]. We can observe that \( \psi \Omega \psi' \) consists of \( n \) terms in Eq. 11. The first (last) represents the contribution to the common factor innovation from the first (last) market [Baillie et al., 2002].

When the covariance matrix is not diagonal, that is, the innovations are not independent, the IS of market \( j \) is given by Hasbrouck [1995],
\[ S_j = \frac{1}{n} \left( \psi_j \Omega_j^{-1} \psi_j \right), \]
(12)
where \( F \) is the Cholesky factorization of \( \Omega \) and is the lower triangular matrix such that \( \Omega = FF' \). \( \psi F \) is the \( j \)th element of the row vector \( \psi F \). Due to the use of Cholesky factorization, Hasbrouck [1995] considers the upper (lower) bound of series \( j \)'s information share when series \( j \) is the first (last) variable in the factorization. That is, the upper (lower) bound of series \( j \)'s information share appears when series \( j \) is the first (last) series in \( Y_t \). This is known as the ordering problem where the calculation of IS using Eq. 12 depends on the particular ordering of the series. Thus the IS measure of any market is not unique.

### 3.2.4. Generalised Information Share (GIS)

Lien and Shrestha [2014] propose a new measure of information share to resolve the ordering problem of the Hasbrouck information share. The new measurement is called generalised modified information share (GIS). GIS utilises a different factor structure that is based upon the correlation matrix of innovations instead of the covariance matrix. The IS measures illustrated thus far depend on the special restrictions imposed on the cointegrating matrix \( \beta \). That is, each of the pairwise cointegrating vectors in \( \beta \) is \((1, -1)\), which results in the rows of \( \varphi(1) \) to be identical. However, this assumption is restrictive since the one-to-one cointegrating relationship does not necessarily hold in the real world. Lien and Shrestha [2014] propose a new IS measure that does not require the cointegrating vector of each pair of series to be \((1, -1)\). Therefore, such new measure can apply to series that do not have the one-to-one cointegrating relationships between them.

When the innovations are independent, the variance of long-run impact on the \( i \)th series is:
\[ \text{Var}(\varphi_i(t)) = \varphi_i^2 \Omega_i \text{-} \frac{1}{n} \sum_{j=1}^n \varphi_{ij}^2 \Omega_{jj}, \]
(13)
where \( \varphi_j \) is the \( j \)th element of the row vector \( \varphi \) and \( \varphi_{ij} \) is the \( j \)th element of the row vector \( \varphi_i \). The contribution of the innovation of series \( j \) to the total variance of the common factor of series \( i \) is then represented by:
\[ S_{ij} = \frac{\varphi_{ij}^2 \Omega_{ij}}{\varphi_i^2 \Omega_i}, \]
(14)
\[ S_j = \frac{\varphi_j^2 \Omega_j}{n}, \]
(15)
where \( \varphi_j = \varphi_i \Omega_i, F_i = F = [G \Gamma^{1/2} G' \Gamma^{-1/2}]^{-1}, \) and \( \varphi_j \) is the \( j \)th element of \( \varphi_j \). It should be noted that the GIS measure uses the factor structure same as the MIS; thus it would also be independent of the ordering problem.

We can compute the time-varying IS and GIS measures which are conditioned on past information by replacing the time-invariant covariance matrix \( \Omega \) of innovations used for calculating IS and GIS measures with its conditional counterpart obtained with Eq. 24. We assume that error correction coefficients in Eq. 6 are constant over the sample period in the calculation.

### 3.2.5. A time-varying cointegration test

Let \( S_i \) and \( F_i \) be the natural logarithms of daily prices of the spot and futures contacts, respectively. If the two series are integrated at the same order, a potential cointegration relationship where the cointegrating coefficient is time variant rather than static, is represented as:
\[ S_i = \beta_0 + \beta_1 \left( \frac{\Delta u_i}{n} \right) + u_i, \]
(16)
where \( \beta_0 \) is a constant mean of the equation and \( u_i \) denotes the error correction term. \( \beta(\frac{t}{n}) \) is the time-varying cointegrating coefficient associated with \( \left( \frac{t}{n} \right) \) where \( t \) is the order of observation in the sample and \( n \) denotes the sample size. We have \( \beta(\frac{t}{n}) = \beta(\lambda) \) such that \( \lambda \in (0, 1] \). Hence \( \beta(\lambda) \) is a smooth function defined on \( [0, 1] \). According to Park and Hahn [1999], the time-series parameters, \( \beta(\lambda) \), are approximated by the Fourier Flexible Form (FFF) functions,
\[ \beta(\lambda) = \alpha_0 + \alpha_1 \lambda + \sum_{i=1}^k \left( \alpha_{2i-1} \cos(2\pi i \lambda) + \alpha_{2i} \sin(2\pi i \lambda) \right), \]
(17)
where \( \alpha_{2i} \in \mathbb{R}^2 \) for \( j = 1, 2, \ldots, 2(k + 1), k \) is a positive integer. Let \( \varphi_{ij}(\lambda) = (\cos 2\pi \lambda, \sin 2\pi \lambda). \)

We also assume that:

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11 According to Park and Hahn (1999), the FFF function used for defining \( \beta(\lambda) \) is superior to alternative specifications, provided that the former yields robust and stable inference for asymptotic distribution of test statistic as well as corresponding critical values.
\begin{align}
\beta_k(\lambda) &= f_k^{\ast}(\lambda)\alpha_k, \\
\text{where } f_k(\lambda) &= (1, \lambda; \varphi_1(\lambda), \ldots, \varphi_2(\lambda))' \\
\text{and } \alpha_k &= (\alpha_{k,1}, \alpha_{k,2}, \ldots, \alpha_{k,2k+1})'.
\end{align}

Therefore, Eq. 17 can be written as:

\begin{equation}
S_t = \beta_0 + \alpha_0 F_{it} + u_{it},
\end{equation}

where \( F_{it} = f_i(\lambda)\), and \( u_{it} = u_t + [\beta(\lambda) - \beta(\lambda)]F_{it} \).

Park and Hahn [1999] employ the superfluous regression approach to test the null hypothesis of the time-varying coefficient cointegration against the alternative of the spurious regression with non-stationary innovations. The corresponding test statistic is defined as:

\begin{equation}
\eta = \frac{RSS_{TVC} - RSS_{TVC}^2}{\alpha_i^2},
\end{equation}

where \( RSS_{TVC} \) and \( RSS_{TVC}^2 \) are the sum of squared residuals from the canonical cointegration regression (CCR) for Eq. 20 and the same regression augmented by \( s \) additional superfluous regressors, respectively. \( \alpha_i^2 \) is the long-run variance of CCR estimation. Under the null hypothesis of a time-varying cointegration model, the limit distribution of \( \eta \) is a Chi-square distribution with \( s \) degree of freedom.

Alternatively, the null hypothesis of the validity of the time-invariant cointegration model, that is, \( \beta(\lambda) = 0 \) in Eq. 17 is static over time, is tested by the test statistic as below:

\begin{equation}
\tau = \frac{RSS_{TVC} - RSS_{TVC}^2}{\alpha_i^2},
\end{equation}

where \( RSS_{TVC} \) and \( RSS_{TVC}^2 \) are the sum of squared residuals from the CCR estimation of Eq. 17 with the time-invariant cointegrating coefficient and the same regression augmented by \( s \) additional superfluous regressors, respectively. The limit distribution of \( \tau \) is the Chi-square distribution with \( s \) degree of freedom under the null. Moreover, the null hypothesis of the time-varying cointegration model against the alternative of the time-varying cointegration model is tested with the null hypothesis \( H_0 : \alpha_{k,2} = \alpha_{k,3} = \ldots = \alpha_{k,2k+1} = 0 \) in Eq. 20 for a specific \( k \). In this paper, following the literature, we choose \( s = 4 \) in addition. We choose \( k \) from a range between 1 and 5. The optimal \( k \) is picked based on the adjusted R-square of CCR. The test statistic follows a Chi-square distribution with \( s \) degree of freedom equal to the number of restrictions.\(^{12}\)

### 3.2.6. DCC-GARCH

To accommodate the time-varying second moments of return distribution, we use the Dynamic-Conditional-Correlation (DCC) BGARCH model proposed by Engle [2002] to model the conditional variance-covariance matrix of innovations of Eq. 6. It should be noted that the conditional variance-covariance matrix underlies the calculation of the conditional variance-covariance matrix of innovations of model proposed by Engle [2002] to model the conditional variance-covariance matrix of innovations of Eq. 6.

The log-likelihood of the multivariate SNP density that each observation at time \( t \) contributes to, without unnecessary constant components, is shown as:

\begin{equation}
\log(\text{SNP}) = -\frac{1}{2}\log(\mathbf{R}_t) - \frac{1}{2}u_t'\mathbf{R}_t^{-1}u_t + \log \left( \sum_{i=1}^{2} \alpha_i u_i^2 \right);
\end{equation}

\begin{equation}
\psi_i(x_{it}) = 1 + \delta_i (x_{it}^3 - 3x_{it}) + k_i (x_{it}^4 - 6x_{it}^2 + 3);
\end{equation}

\begin{equation}
\alpha_i = 1 + x_i^2 + k_i^2;
\end{equation}

\begin{equation}
x_t = (x_{1t}, x_{2t})' = R_{t}^{-1/2}u_t,
\end{equation}

where \( u_t = (u_{1t}, u_{2t})' \) is a \( 2 \times 1 \) vector of standardised residuals denoted by \( u_t = \frac{u_t}{\sqrt{h_{it}(t = 1, 2)}} \). \( h_{it} \) is a standard individual GARCH process. \( Q_t = (1 - \delta_1 - \delta_2)Q_t + \delta_1 u_{it-1}u_{it-1}' + \delta_2 Q_{it-1} \)

where \( u_t = (u_{1t}, u_{2t})' \) is a \( 2 \times 1 \) vector of standardised residuals denoted by \( u_t = \frac{u_t}{\sqrt{h_{it}(t = 1, 2)}} \). \( h_{it} \) is a standard individual GARCH process. \( Q_t = (1 - \delta_1 - \delta_2)Q_t + \delta_1 u_{it-1}u_{it-1}' + \delta_2 Q_{it-1} \)

where \( u_t = (u_{1t}, u_{2t})' \) is a \( 2 \times 1 \) vector of standardised residuals denoted by \( u_t = \frac{u_t}{\sqrt{h_{it}(t = 1, 2)}} \). \( h_{it} \) is a standard individual GARCH process. \( Q_t = (1 - \delta_1 - \delta_2)Q_t + \delta_1 u_{it-1}u_{it-1}' + \delta_2 Q_{it-1} \)

where \( u_t = (u_{1t}, u_{2t})' \) is a \( 2 \times 1 \) vector of standardised residuals denoted by \( u_t = \frac{u_t}{\sqrt{h_{it}(t = 1, 2)}} \). \( h_{it} \) is a standard individual GARCH process. \( Q_t = (1 - \delta_1 - \delta_2)Q_t + \delta_1 u_{it-1}u_{it-1}' + \delta_2 Q_{it-1} \)

where \( u_t = (u_{1t}, u_{2t})' \) is a \( 2 \times 1 \) vector of standardised residuals denoted by \( u_t = \frac{u_t}{\sqrt{h_{it}(t = 1, 2)}} \). \( h_{it} \) is a standard individual GARCH process. \( Q_t = (1 - \delta_1 - \delta_2)Q_t + \delta_1 u_{it-1}u_{it-1}' + \delta_2 Q_{it-1} \)

where \( u_t = (u_{1t}, u_{2t})' \) is a \( 2 \times 1 \) vector of standardised residuals denoted by \( u_t = \frac{u_t}{\sqrt{h_{it}(t = 1, 2)}} \). \( h_{it} \) is a standard individual GARCH process. \( Q_t = (1 - \delta_1 - \delta_2)Q_t + \delta_1 u_{it-1}u_{it-1}' + \delta_2 Q_{it-1} \)

where \( u_t = (u_{1t}, u_{2t})' \) is a \( 2 \times 1 \) vector of standardised residuals denoted by \( u_t = \frac{u_t}{\sqrt{h_{it}(t = 1, 2)}} \). \( h_{it} \) is a standard individual GARCH process. \( Q_t = (1 - \delta_1 - \delta_2)Q_t + \delta_1 u_{it-1}u_{it-1}' + \delta_2 Q_{it-1} \)

where \( u_t = (u_{1t}, u_{2t})' \) is a \( 2 \times 1 \) vector of standardised residuals denoted by \( u_t = \frac{u_t}{\sqrt{h_{it}(t = 1, 2)}} \). \( h_{it} \) is a standard individual GARCH process. \( Q_t = (1 - \delta_1 - \delta_2)Q_t + \delta_1 u_{it-1}u_{it-1}' + \delta_2 Q_{it-1} \)
4. Results

4.1. Time-varying Granger causality

The procedure does not require pre-filtering the data but it require the maximum order of integration for the VAR. Using several unit root tests to check the stationarity of log futures and spot prices, we conclude that all variables are I(1).\(^{13}\) As suggested in Shi et al. [2020], we next conduct the time-varying Granger causality analysis based upon a VAR model that allows for possibly integrated data, and set the lag addition parameter \(d\) to unity.\(^{14}\)

4.1.1. CBOE

Initially we concentrate on the CBOE market and test for any causal effects from the Gemini auction price (spot prices) to the CBOE futures prices. The time-varying Wald test statistics for causal effects from the CBOE futures prices to the CBOE spot prices are shown in Fig. 3. The two rows illustrate the sequences of test statistics obtained from the rolling window and recursive evolving procedures respectively, while the columns of the figure refer to the two different assumptions for the residual error term (homo-skedasticity and heteroskedasticity) for the VAR. Sequences of the test statistics start from April 2018. Under different model and error assumptions presented in Fig. 3, the test statistics of the predictive power of spot prices on the CBOE futures prices are always below their bootstrapped critical values, suggesting there is no evidence to reject the null hypothesis of no Granger causality in all cases. As a result, date-stamping results from Figs. 3a to 3d suggest that spot prices cannot predict the CBOE futures prices, i.e., there is no causal relationship running from spot prices to the CBOE futures prices in all cases. The result suggests that the CBOE spot market may not be able to lead the futures market since the former responds to new information more slowly than the latter.

When we consider the causal effects from the CBOE futures prices to spot prices as shown in Fig. 4. We can see that, first, there is little evidence of Granger causality episodes based on the rolling window procedure as presented in Fig. 4a and b. Second, the recursive evolving approach offers some different results. As shown in Fig. 4c, we find significant evidence of Granger causality episodes running from the CBOE futures prices to spot prices from August 2018 to June 2019 as the test statistic exceeds the critical value sequences in August 2018 until the closure of the CBOE Bitcoin futures market in June 2019. As a result, the null hypothesis of no Granger causality can be rejected. Similarly, under the error assumption of heteroskedasticity, Fig. 4d also presents significant evidence to support the conclusion that the CBOE futures prices do Granger cause spot prices from November 2018 to June 2019. As noted in Shi et al. [2018], the recursive evolving window algorithm provides the most reliable results. Hence, we are confident in concluding that the CBOE futures prices Granger cause spot prices from August 2018 to November 2018 to the end of the CBOE futures market in June 2019. The results indicate that the CBOE Bitcoin futures market leads the spot market since the former responds to new information faster than the latter from August 2018/November 2019 to June 2019. Moreover, the test statistics produce their largest values for November/December 2018. This can be explained as being due to the volume of CBOE Bitcoin futures reaching a peak in November 2018 as shown in Fig. 2.

\(^{13}\) Unit root results are not shown here to save space. They are available upon request.

\(^{14}\) We also carry out additional analysis by setting the lag addition parameter \(d\) to zero for the CBOE and CME markets and the corresponding results in Section 4.1 remain unchanged.

4.1.2. CME

As stated above, the spot and futures prices of the CME are different from those used for the CBOE. The conclusions drawn for the CBOE market do not necessarily hold for those of the CME market. Next, therefore, we undertake our analysis using the CME futures prices and CME BRR to explore the causal relationship between futures and spot markets with the results presented as in Fig. 5 and Fig. 6. Fig. 5 considers tests of Granger causality running from spot prices to the CME futures prices. As shown in Fig. 5, we find quite similar results based on different testing procedures and error assumptions. For example, when we look at the date-stamping outcomes in Fig. 5a and Fig. 5b, the rolling window approach identifies an episode of Granger causality between March 2019 and June 2019 under two different error assumptions. When the recursive evolving procedure is applied as in Fig. 5c and Fig. 5d, we also identify an episode of Granger causality from March 2019 to July 2019. Based on the above results, we can conclude that spot prices Granger cause the CME futures prices from March 2019 to June/July 2019.

Finally, we conduct an analysis of Granger causality running from the CME futures to spot prices. Interestingly, we obtain significant evidence to reject the null of no Granger causality from the CME futures to spot prices as presented in Fig. 6. The rolling window approach finds an episode of Granger causality between April 2018 and March 2019 in Fig. 6a and Fig. 6b with some small breaks. What is even more interesting is that the recursive evolving approach identifies an episode of Granger causality for the whole period between April 2018 and July 2019 as shown in Fig. 6c and Fig. 6d. It is clear that our results are robust to different error assumptions. As the recursive evolving approach has higher power over the rolling window approach, we prefer the results obtained from the recursive evolving approach. Our results, therefore, suggest that the CME futures prices lead spot prices in the short term within the context of time-varying Granger causality. Our result suggests there exists bi-directional Granger causality between the CME Bitcoin spot and futures prices. More importantly, given the duration of the Granger-causal episodes and the magnitude of the test statistics in Fig. 5 and Fig. 6, it was found that the strength of Granger causality from the futures prices to spot prices is stronger than vice-versa. From this we conclude that Granger causality runs from the futures market to the spot market. This result further suggests that the CME Bitcoin futures market leads the spot since the former embeds the new information faster than the latter.

The results from time-varying Granger causality tests present some very important findings. The key results for the CBOE and CME markets are summarized as follows:

- **CBOE market**
  1. There are no Granger causality episodes running from the Gemini auction price (spot prices) to the CBOE futures prices;
  2. The recursive evolving approach detects an episode (August/November 2018–June 2019) running from the CBOE futures prices to spot prices;
  3. Overall, the CBOE futures market dominates the spot market in terms of Granger causality from August/November 2018 to June 2019.

- **CME market**
  1. There is a Granger causality episode running from the BRR (spot prices) to the CME futures prices (March 2019–June/July 2019);
  2. The rolling window approach detects an episode (April 2018–March 2019) and recursive evolving approach detects an episode (April 2018–July 2019) running from the CME futures prices to spot prices;
  3. There is bi-directional causal relationship between spot price and the CME futures prices;
  4. Compared with duration of causal episodes and the magnitude of...
the test statistics in Fig. 5 and Fig. 6, the CME futures market dominates the underlying spot market in terms of Granger causality.

Overall, our result show that there is unidirectional Granger causality running from the CBOE Bitcoin futures to spot markets, whereas there is bidirectional Granger causality between the CME Bitcoin futures and spot prices. It should be noted that even though Granger bidirectionality cannot be rejected, the Granger causality from the futures to spot is stronger than the other way around which suggests that the Bitcoin futures market dominates the spot in terms of strength of the lead-lag responses. Our results enrich the literature by identifying the evolving Granger causality relations between two major Bitcoin futures markets and their spot assets where both futures markets play a leading role in the dynamic Granger causality processes. The result is in line with prior studies on traditional financial futures markets that indicate futures market lead the spot in the between-market interactive processes (see, e.g. Chan, 1992; Wahab & Lashgari, 1993; Koutmos & Tucker, 1996; Yang et al., 2001; Kavussanos et al., 2008; Bohl et al., 2011; among others).

4.2. Time-varying cointegration

Next we use Park and Hann's [1999] procedure to test for the existence of cointegration which allows for the possibility of estimating a time-varying cointegrating coefficient. Via the Engle-Granger Theorem we know that cointegration implies Granger causality in at least one direction such that non-rejection of cointegration strengthens any causality results represented above, although the theorem does not itself identify the direction of Granger causality. In addition, cointegration may vary across time given a time-driven cointegrating coefficient [Park & Hahn, 1999], which has an important implication for the long-run lead-lag relationship. Upon the non-rejection of non-static cointegration, that is, a time varying equilibrium when cointegrated time series hold, a dynamic error correction process between them is identified. This further reinforces the accuracy and robustness of the results of price discovery given the more accurate estimation of error correction coefficients. We also present the movements of the time-varying cointegration coefficients between the futures and spot markets in Fig. 7. The time-varying cointegration coefficients of $\beta_{CBOE}$ and $\beta_{CME}$ for the two futures markets are shown as Fig. 7a and Fig. 7b, respectively.
which suggest that the patterns of $\beta_{\text{CBOE}}$ and $\beta_{\text{CME}}$ are both non-linear and time-varying during the entire sample period.\footnote{We test whether the cointegrating vector is $(1, -1)'$ using a Wald test on the coefficients in the CCR assuming as a null hypothesis, a static cointegrating coefficient for each pair of spot and futures markets. For each paired sample, the Wald test statistic strongly rejects the null hypothesis that the cointegrating vector is $(1, -1)'$. It should be noted that the cointegrating vector of $(1, -1)'$ is a prior condition for the IS measure. To test for this condition holding we use the GIS measure which allows the cointegrating vector to be other than $(1, -1)'$. Table 4 shows that the cointegration coefficient is time-varying. Cointegration between the Bitcoin spot and futures prices does exist, however, provided we allow for time variation. Therefore, the assumption (null hypothesis) that the cointegrating vector is $(1, -1)'$ indeed is not applicable to this study. This also necessitates the use of the GIS measure, since GIS is able to estimate the information share under conditions where a time-varying cointegrating coefficient exists.}

We next present cointegration test results for the CBOE market based on Park and Hahn [1999] test in Table 4. As shown in Panel A of Table 4, we cannot reject the null hypothesis of a time-varying cointegration model as the $p$-value of $\tau_1$ statistic is 0.1091, indicating a significant cointegrating relationship with time-varying coefficients between the spot and CBOE futures markets. However, the null hypothesis of the validity of the time-invariant coefficient cointegration model can be rejected as the $p$-value of $\tau_2$ statistic is 0. Results from $\tau_2$ provide further support for a time-varying cointegration model. Alternatively, the null hypothesis of the time-invariant cointegration model against the alternative of the time-varying model is also tested with the null hypothesis $H_0: \alpha_{k, 2} = \alpha_{k, 3} = \ldots = \alpha_{k, 2(k+1)} = 0$ when $k=8$.\footnote{$k$ is chosen to be 8 for the Park and Hahn test since it generates the highest adjusted R-square of the CCR under the null hypothesis. We also test the null when $k$ is 1, 2, 3, ..., 8 and find the results are qualitatively similar. We also try alternative choices for the equality test.} The result from Panel A provides significant evidence to support a time-varying model as the $p$-value of a Chi-square statistic $\chi^2(2k + 1)$ is 0. We, therefore, prefer the time-varying cointegration model rather than the time-invariant cointegration model.

We also obtain the similar result from Panel B of Table 4 for the CME markets. The null hypothesis of a time-varying cointegration model is not rejected as the $p$-value of $\tau_1$ statistic is 0.3515, suggesting a cointegrating relationship with time-varying coefficients between the spot and CME futures markets. On the other hand, the $p$-value of $\tau_2$ statistic is 0, rejecting the null hypothesis of the time-invariant coefficient cointegration model. The results from $\tau_2$ are in line with $\tau_1$. The validity of the time-invariant coefficient cointegration model can be rejected as the $p$-value of $\tau_2$ statistic is 0. Results from $\tau_2$ provide further support for a time-varying cointegration model. Alternatively, the null hypothesis of the time-invariant cointegration model against the alternative of the time-varying model is also tested with the null hypothesis $H_0: \alpha_{k, 2} = \alpha_{k, 3} = \ldots = \alpha_{k, 2(k+1)} = 0$ when $k=8$.\footnote{$k$ is chosen to be 8 for the Park and Hahn test since it generates the highest adjusted R-square of the CCR under the null hypothesis. We also test the null when $k$ is 1, 2, 3, ..., 8 and find the results are qualitatively similar. We also try alternative choices for the equality test.} The result from Panel A provides significant evidence to support a time-varying model as the $p$-value of a Chi-square statistic $\chi^2(2k + 1)$ is 0. We, therefore, prefer the time-varying cointegration model rather than the time-invariant cointegration model.
A time-varying model is preferred over the time-invariant cointegration model for the CME futures markets as the Chi-square statistic $\chi^2(2k+1)$ is significant at the 1% level. Overall, we prefer to apply a time-varying cointegration model between Bitcoin spot and futures markets due to significant evidence as suggested by Table 4.

Based on Table 4, we further discuss why time variation of the cointegration coefficients $\beta_{\text{CBOE}}$ and $\beta_{\text{CME}}$ for the CBOE and CME Bitcoin futures markets are essential in this study. The Park and Hahn [1999] test results presented in Table 4 clearly show i) that the null hypothesis that the cointegration specification, assuming a static cointegration relationship is statistically rejected, whereas ii) the null hypothesis that the cointegration specification with a time-varying cointegration coefficient, governed by a FFF function of time, is not rejected at any conventional level. The results hold for both futures markets. Our results therefore suggest that time variations in the cointegration coefficients exists, which has implications for the long-run equilibrium between spot and futures prices. Moreover, in Table 4 the null hypothesis that all the coefficients in the FFF time function are jointly zero is rejected, again for both of the futures markets, again supporting the results in terms of time variations of the cointegration coefficients.

Finally, we test three null hypotheses based on values of the variances of the cointegration coefficients. For the CBOE Bitcoin spot and futures markets, we test the null hypotheses that the variance equals 3.00e-07, 3.30e-07 and 4.00e-07, respectively. Note that the sample variance equals 3.30e-07. The test result shows that the first null is rejected, whereas the null hypothesis that variance equals 3.30e-07 is not. These results suggest that the variance of the cointegration coefficient for the CBOE Bitcoin spot and futures is not zero, supporting time variability of that coefficient. For the CME Bitcoin spot and futures markets, we test the null hypotheses that the variances equal 2.00e-06, 2.48e-06 and 3.00e-06, respectively, where 2.48e-06 is the sample variance. Test results show that the nulls that the variance equals 2.00e-06 and 3.00e-06 is rejected, whereas the null that the variance equals 2.48e-07 is not rejected. These results suggest that the variance of the cointegration coefficient for the CME Bitcoin spot and futures is not zero, again supporting time variability of the coefficient. Overall, time variations in the cointegration coefficients are observed, substantiating a dynamic cointegrating model for the long-run relationship between Bitcoin spot and futures markets.

This table presents results of Park and Hahn [1999] test. $r_1$ and $r_2$ are calculated by Eq. 21 and Eq. 22, respectively. $k$ in Panel A refers to the number of pairs of trigonometric polynomial functions in Eq. 18. *** denotes significance at the 1% level.

17 In the test for the CME futures, the optimal value of $k$ is chosen to be 1.
Fig. 6. Tests for Granger causality running from the CME futures to the BRR (spot prices) ($d=1$).

(a) Rolling Window-Homoskedasticity

(b) Rolling Window-Heteroskedasticity

(c) Recursive Evolving-Homoskedasticity

(d) Recursive Evolving-Heteroskedasticity

Fig. 7. Time-varying cointegration coefficient $\beta$ between spot and futures markets (CBOE and CME).

(a) Time-varying cointegration coefficient $\beta_{CBOE}$

(b) Time-varying cointegration coefficient $\beta_{CME}$
Note that when calculating static and time-varying information share measures, we incorporate the cointegration equations with time varying cointegrating coefficients in the computation procedure. This is necessary because Table 4 suggests cointegration exists only when cointegrating coefficients are dynamic.

### Table 4
Cointegration test under time-invariant and time-varying coefficients.

| Panel A: CBOE futures | p-value |
|------------------------|---------|
| Time-varying coefficient | \( P_{t1} : 0.1091 \) |
| Time-invariant coefficient | \( P_{\tau 2} : 0.0000^{***} \) |
| \( H_0 : a_{k, 2} = a_{k, 3} = \ldots = a_{k, 2 + 1} \) \( \chi^2(2k + 1) \) | \( P_{\chi^2(2k + 1)} : 0.0000^{***} \) |

### Table 5
The static (time-invariant) estimates of price discovery measure for Bitcoin spot and two futures markets.

| Panel A | IS measure | GIS measure |
|---------|------------|-------------|
|         | Upper Bound | Lower Bound | Mid-point |
| CBOE futures | 0.9880 | 0.0077 | 0.5029 | 0.5216 |
| Spot | 0.9924 | 0.0020 | 0.4972 | 0.4784 |

| Panel B | IS Measure | GIS Measure |
|---------|------------|-------------|
|         | Upper Bound | Lower Bound | Mid-point |
| CME futures | 0.9540 | 0.0926 | 0.5233 | 0.5464 |
| Spot | 0.9074 | 0.0460 | 0.4767 | 0.4536 |

### 4.3. Time-varying price discovery

Results for the static (time-invariant) price discovery measures are summarized in Table 5.\(^{18}\) With respect to price discovery of the CBOE futures and spot markets in Panel A, the IS (upper bound, lower bound and mid-point) and GIS measures of the CBOE futures are higher than those of spot markets. Hence, price discovery takes place in the CBOE futures market rather than Bitcoin spot market. It should be pointed out that the CBOE futures market dominates the price discovery process. We next consider the results for the CME futures market in Panel B. It is clear that the IS (upper bound, lower bound and mid-point) and GIS measures of the CME futures are higher than those of spot markets, indicating that the CME futures market outperforms in terms of static information shares price discovery. In general, Table 5 suggests that both the CBOE and CME futures markets lead the Bitcoin spot market. This finding is consistent with the results of the time-varying Granger causality approaches reported in Section 4.1.

Results of the time-varying price discovery measures are summarized in Table 6. As can be seen from Panel A, the mean, maximum and minimum estimates of upper bound, lower bound and mid-point of the IS measures for the CME futures are higher than those of spot markets. Similarly, the CBOE futures market outperforms the spot market in terms of conditional information shares as the mean, maximum and minimum estimates of upper bound, lower bound and mid-point of the IS measures for the CBOE futures market are also higher than those of the spot market. Hence, the conditional IS measures suggest that price discovery mainly takes place in the Bitcoin futures markets, rather than spot counterpart. The results are consistent with static measures. In addition, standard deviations of the IS measures for the spot and futures markets are small, indicating that those measures are stable over the entire sample period.\(^{19}\) Results for the conditional GIS of Bitcoin spot and futures are shown in the Panel B of Table 6. The GIS results are similar to the IS measures of Panel A. Moreover, the GIS measures are stable given their small standard deviations. In addition, we test the equality of means of the conditional mid-point IS and that of conditional GIS between Bitcoin spot and futures markets. The \( t \)-statistics are all significant for the IS and GIS measures, for both the CBOE and CME cases. The results indicate that the differences in dynamic price discovery performance between Bitcoin futures and spot markets are

\(^{18}\) Note that when calculating static and time-varying information share measures, we incorporate the cointegration equations with time varying cointegrating coefficients in the computation procedure. This is necessary because Table 4 suggests cointegration exists only when cointegrating coefficients are dynamic.

\(^{19}\) We test a range of null hypotheses relating to the variances of the lower bounds, upper bounds and mid-points of the IS measures of spot and futures markets being equal to non-zero values of the sample variance. We also test the null hypotheses that the variances of the GIS measures of the spot and futures markets are equal to non-zero values of the sample variance. The test results show that the nulls are not rejected at any conventional level, suggesting that the time variations of information share measures in Table 6 are statistically identified.
substantial, which confirms the leading role the futures markets play in the long-run information channels. Overall, both the CBOE and CME futures markets have higher means of the GIS than the spot counterpart, indicating that the Bitcoin futures markets dominate in the dynamic price discovery process.

Furthermore, the time-varying IS and GIS measures of the Bitcoin spot and two futures markets are depicted in Fig. 8 and Fig. 9, respectively. As shown in both figures, the values of the conditional series of the IS and GIS measures for futures markets are higher than those of the spot markets across time, which is consistent with the results of Table 7. Moreover, Figs. 8 and 9 show that during the full sample period, the mid-points of the IS and GIS of Bitcoin futures in the CBOE and CME are consistently above those of the associated Bitcoin spot assets. This shows that the Bitcoin futures markets possess a larger time varying information share than the Bitcoin spot market without evidence of any episode where the futures markets are shown to be inferior in terms of price discovery. There exist no episodes where the Bitcoin spot markets dominates the price discovery processes with regard to Bitcoin futures. This points to a conclusion that the price formation originates solely in the Bitcoin futures market. We can, therefore, conclude that the Bitcoin futures markets dominate the dynamic price discovery process based upon time-varying information share measures. Overall, price discovery seems to occur in the Bitcoin futures markets rather than the underlying spot market based upon a time-varying perspective, which is consistent with results obtained from the static (time-invariant) information share measures in Table 5.

Our results provide a number of evidence-based implications for both market traders and regulators in the Bitcoin markets. For investors, it is suggestive that futures contracts for Bitcoin in the CBOE and CME are an efficient tool, that delivers expected price discovery functionality, which aligns with the theoretical rationale for futures’ functions proposed by Fleming et al. [1996] and that futures markets should be able to guide spot markets by behaving as a focal point of information due to known superiorities in market’s structures, including lower cost and easier access. Based on this superior price discovery performance, both CME and CBOE futures are expected to provide traditional risk–transferring functions such as hedging and arbitrage. For market regulators that pay attention to the Bitcoin market operations, the establishment of a futures markets for Bitcoin has been a success due to the superior price discovery performance of the futures markets which appear to operate well over time. Therefore, it seems that the futures markets uncover new information that is embedded into prices and lead the way for adjustments to innovations in the fundamental values in the spot markets. Hence, the futures markets are capable of delivering a stabilizing effect on spot markets, which is one of the major purposes for launching futures contracts for Bitcoin. Such a result is of considerable importance to regulators and monetary authorities who have shown misgivings about the growth of the cryptocurrency markets.

The estimation results for the DCC-GARCH model with an SNP approach is shown in Table 7. The model estimates are used to predict the conditional variances and covariances of Bitcoin spot and futures markets which determine the conditional IS and GIS measures. As can be seen from Panel A of the table, volatility clustering exists in all the
markets, where the individual variances are driven by the arrival of new shocks. In addition, persistency of variances is significant for all four markets.

Panel B of Table 7 suggests that the correlation between the Bitcoin spot and futures markets is conditioned on past shocks as well as their own lagged values, given the significant estimates of $\delta_1$ and $\delta_2$. Moreover, the SNP approach significantly captures the excess kurtosis of return distribution. Significant estimates of the marginal kurtosis parameters from the SNP distribution respond to large values of sample kurtosis reported in Table 2 and subsequently contribute to the rejection of normality of the return series. The result means that the SNP distribution used for estimating the DCC-GARCH model succeeds in taking into account this non-normality feature of the return distributions, particularly in relation to fat-tail. Furthermore, our results align with the prior literature that stresses the importance of considering non-normality of financial time series in the MLE of the GARCH model and that any loss of estimation efficiency can be mitigated when excess kurtosis and asymmetry of the return distribution are addressed (Engle & Gonzalez-Rivera, 1991; Park & Jei, 2010). Using a log-likelihood ratio test to examine whether the DCC-GARCH model with the SNP distribution for estimating the DCC-GARCH model succeeds in taking into account this non-normality feature of the return distributions, particularly in relation to fat-tail. Furthermore, our results align with the prior literature that stresses the importance of considering non-normality of financial time series in the MLE of the GARCH model and that any loss of estimation efficiency can be mitigated when excess kurtosis and asymmetry of the return distribution are addressed (Engle & Gonzalez-Rivera, 1991; Park & Jei, 2010). Using a log-likelihood ratio test to examine whether the DCC-GARCH model with the SNP distribution is superior to the DCC-GARCH model with the traditional normal distribution we find that the null hypothesis of the DCC-GARCH model with the traditional normal distribution is statistically rejected, indicating that the DCC-GARCH model with the SNP distribution provides a better fit for data and yields more reliable, efficiently estimated, results. It further suggests that the time varying information share measures obtained from the DCC-GARCH model with the SNP distribution, are more robust than those estimated under the assumption of normality. Skewness parameters are estimated, but none of them are significant. It suggests asymmetry of the distribution might not be a significant obstacle for model estimation. Finally, Panel D of Table 7 shows that there is no heteroscedasticity remaining in the standardised innovations, suggesting the model is now well specified.

5. Conclusion

This paper investigates, for the first time in the literature, the existence of causal relationships, cointegration and price discovery between Bitcoin spot and futures in the CBOE and CME markets from December 2017 to June/July 2019 from a time-varying perspective.

Of particular importance from the results of this paper is that we offer more robust evidence to support our key findings. This paper presents three important findings as follows. First, the results from a recently proposed time-varying Granger causality test of Shi et al. [2020, 2018] suggest that the CBOE and CME futures prices Granger cause the underlying spot markets. For the CBOE market, the CBOE futures prices Granger cause spot prices between August/November 2018 and June 2019 based on the recursive evolving testing procedure. However, there are no Granger causality episodes running from spot prices to the CBOE futures prices. For the CME market, there is a Granger causality episode running from spot prices to the CME futures prices (March 2019-June/July 2019) using both rolling window and recursive
other words, the time-varying cointegration model is better suited to

Akyildirim, E., Corbet, S., Katiampa, P., Kellard, N., & Sensoy, A. (2019). The develop-

Notes: This table reports the estimation results of the bivariate DCC-GARCH-SNP model based upon Eq. 24 to Eq. 29. Coefs. denotes coefficients. i = 1 refers to the conditional variance equation of Bitcoin spot markets (Gemini auction price or BRR) while i = 2 refers to the conditional variance equation of Bitcoin futures markets (CBOE or CME). LB^2(12) is the Ljung-Box Q statistics of squared standardised residuals up to lag order 12. ARCH(12) denotes the test statistic for testing the ARCH effect up to lag order 12. Figures in the parentheses are p-values. ***, **, and * denote significance at the 1%, 5% and 10% levels, respectively.

Table 7
The DCC-GARCH-SNP model.

|                  | CBOE          | CME           |
|------------------|---------------|---------------|
| **Panel A:** Conditional variances |              |               |
| \(\omega_i\)                  | \(7.74 \times 10^{-5}\) | \(4.69 \times 10^{-5}\) |
| \(\omega_1\)                  | \(0.01222\) | \(0.01511\) |
| \(\omega_2\)                  | \(0.0291**\) | \(0.0158*\) |
| \(\omega_i'\)                | \(0.0453\) | \(0.0637\) |
| \(\omega_{i'}\)              | \(0.9777***\) | \(0.8765**\) |
| **Panel B:** Conditional correlation |              |               |
| \(\delta_1\)                  | \(0.1474***\) | \(0.0170**\) |
| \(\delta_2\)                  | \(0.6779***\) | \(0.8380***\) |
| **Panel C:** SNP distribution |              |               |
| \(i_1\)                       | \(-0.4167\) | \(0.1794\) |
| \(k_1\)                       | \(-0.10964\) | \(0.1495\) |
| \(k_2\)                       | \(4.9499\) | \(5.6004*\) |
| \(\lambda_1\)                 | \(0.1318\) | \(0.0807\) |
| \(\lambda_2\)                 | \(5.8634\) | \(4.6873*\) |
| **Panel D:** Residual diagnosis |              |               |
| LB(12)                        | \(2.5025\) | \(2.0816\) |
| ARCH(12)                      | \(2.4730\) | \(2.0478\) |

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