Distributions of the charged particles in the electromagnetic showers initiated by 5-1000 GeV electrons in Fe, W and Pb

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Abstract. The results of calculations of the charged particles multiplicity, energy, radial and time distributions in the electromagnetic showers initiated by electrons with energies from 5 to 1000 GeV in Fe, W and Pb are presented. It is shown that the multiplicity distributions are well fitted by the inverse sum of two exponents in a wide range of energy and lead depth. The shapes of energy and radial distributions at the shower maximum weakly depend on the electron energy. Radial distributions for all materials studied are similar if radius is expressed in g/cm$^2$. The time spread of the charged particles at the shower maximum turned out to be in the picosecond range.

1. Introduction
In 1970 A.A.Tyapkin proposed to use a simple detector consisting of a lead convertor and a scintillation or Cherenkov counter behind it for $e, \gamma$ energy measurements [1]. This idea was based on the Rossi approximation of the electromagnetic shower development [2]. As an optimal convertor thickness A.A.Tyapkin suggested to use the value $t_{\text{max}}$ corresponding to the maximum flux $N_{\text{max}}$ of the charged shower particles since $t_{\text{max}}$ weakly depends on the energy $E_0$ of the initial $e, \gamma$ and $N_{\text{max}}$ is almost proportional to $E_0$. At present this type of detector is often referred to as shower maximum detector. Since 1970 a number of studies (see, for example, [3-14]) were performed to investigate shower maximum detectors characteristics like energy and space resolutions and $e$/hadron and $\gamma$/\ipa separations which strongly depend on $N_{\text{max}}$ fluctuations and space and energy distributions of the charged shower particles.

In this report the results of calculations of the charged particles energy, radial and time distributions at $t_{\text{max}}$ in Pb, W and Fe are presented and multiplicity distributions in Pb in a wide $t$-range are considered. The calculations are based on GEANT4 10.01.p02 (Physical list FTFP_BERT) [15] with 700 micron range cut for all materials. Corresponding energy thresholds for $e^+$ and $e^-$ are close to 1 MeV for Pb and Fe and 1.6 MeV for W. Increasing or decreasing the range cut by a factor of two does not affect $N_{\text{max}}$ value within 0.5% statistical uncertainty [16] since the energy thresholds are much less than the average particle energy of $\sim$50 MeV (see below). Note that the charged particles flux at $t_{\text{max}}$ consists mainly of $e^+$, $e^-$. For example the admixture of other particles at $E_0=200$ GeV is 0.02% only[16]. Diameter of all converters studied is 70 cm.
2. Charge particle multiplicity distributions
In our previous studies [16] it was shown that multiplicity distributions at $t_{\text{max}}$ are strongly asymmetric and have long tails at low multiplicities due to late developments of some showers. They are well described by the inverse sum of two exponents:

$$\frac{dP}{dN} = \frac{p_0}{e^{p_1(N-p_2)} + e^{p_3(N-p_1)}}, \quad p_0 = \frac{p_1 - p_2}{\pi} \sin \frac{\pi p_1}{p_1 - p_2},$$

where $p_0$ is a normalization factor and $p_1$, $p_2$, $p_3$ are free parameters. This function is defined from $-\infty$ to $+\infty$ and because of $N>0$ it can be used only if multiplicity $N>>1$ and $dP/dN(0)$ is close to 0.

![Graphs showing charged particle multiplicity distributions in lead for 200 GeV electrons at different times.](image)

**Figure 1.** Charged particle multiplicity distributions in lead for 200 GeV electrons at $t$ of 5 (a), 7 (b), 9 (c), 12 (d), 14 (e) and 16 (f) radiation lengths.
The most probable and average \( N \) values and RMS of this distribution can be presented by the following formulas:

\[
N_{mp} = p_3 - \frac{\ln p_1 - \ln(-p_2)}{p_1 - p_2}, \quad <N> = p_3 + \mu \cdot \text{ctg} \mu p_1, \quad \sigma = \mu \cdot \sin^{-1} \mu p_1 = \frac{1}{p_0}, \quad \mu = \frac{\pi}{p_1 - p_2}.
\]

The main purpose of this report is to check how well the formula (1) works at the depths \( t \) other than \( t_{\text{max}} \) and to define the evolution of the \( dP/dN \) shape as a function of \( t \). \( dP/dN \) distributions obtained for different \( E_0 \) values at \( E_0=200 \text{ GeV} \) are presented in figure 1 (see [17] for other energies). They are reasonably well described by formula (1). The same conclusion is valid for other \( E_0 \) and \( t \) values investigated [17].

Figure 2 demonstrates the \( t \)-dependencies of \( p_1 \) and \( p_2 \) parameters obtained from the fit to formula (1) for \( E_0=10, 40, 200 \) and \( 1000 \text{ GeV} \) (see [17] for other energies). From figure 2 it follows that at some depths \( t_1 \) below and \( t_2 \) above \( t_{\text{max}} \) which are equal to 6, 7, 9 and 11 r.l. for \( E_0 \) of 10, 40, 200 and 1000 GeV correspondingly \( p_1 \) and \( p_2 \) parameters become equal (two \( p_1, p_2 \) intersections below \( t_{\text{max}} \) at 1000 GeV are not discussed in this paper). This means that at these \( t \)-values \( dP/dN \) distributions become symmetric (see figure 1 and [17]) and as follows from equation (1) can be described by the following function

\[
\frac{dP}{dN} = \left( \frac{p}{\pi} \right) \cdot \text{ch}^{-1} \left( p \left[ N - p_3 \right] \right) \quad \text{with} \quad <N>=p_3 \quad \text{and} \quad \sigma = \frac{\pi}{2p},
\]

where \( p=p_1=-p_2 \). This function reaches maximum of \( P/\pi \) at \( N=p_3 \). Asymmetric \( dP/dN \) distributions between \( t_1 \) and \( t_2 \) have tails on the left side of the \( dP/dN \) peaks while in the region of \( t>t_2 \) the tail appears on the right side as shown in figure 1 and Ref. [17].

![Figure 2](image_url)

**Figure 2.** \( t \)-dependencies of the \( p_1 \) (black circles) and \( -p_2 \) (open circles) parameters for different \( E_0 \) energies.

### 3. RMS/<\( N \)> vs \( E_0 \) and \( t \)

An important parameter of the particle multiplicity distributions is RMS/<\( N \>\. \( t \)-dependencies of RMS/<\( N \> values for different \( E_0 \) energies are shown in figure 3. From figure 3 it follows that the positions \( t_0 \) of minimal RMS/<\( N \> values do not coincide with \( t_{\text{max}} \) where particle flux reaches maximum value, but instead close to \( t_2 \). The \( t_0 \) and \( t_{\text{max}} \) energy dependencies are presented in figure 4. Both follow logarithmic dependence:
$t_{\text{max}} = (3.06 \pm 0.08) + (1.129 \pm 0.016) \ln E_0,$ 
$t_0 = (4.31 \pm 0.06) + (1.177 \pm 0.012) \ln E_0$

with similar slopes while the $t_0$ constant term is 1.25 r. l. above the constant term for $t_{\text{max}}$. The energy dependence of RMS/$\langle N \rangle$ minimal values is shown in figure 5. It is fitted by the formula 

$$(\text{RMS}/\langle N \rangle)_{\text{min}} = (0.46 \pm 0.03)/E^{0.49 \pm 0.04} + (0.074 \pm 0.005).$$

(2)

The first term in (2) can be interpreted as a stochastic term independent of the place in the converter where the EM shower started to develop. The constant term is mainly due to fluctuations of the shower starting point. Formula (2) is an estimate of the best energy resolution of a simple $e, \gamma$ detector, consisting of a high $Z$ converter and a counter of $e^+, e^-$ behind it.

Figure 3. $t$-dependence of RMS/$\langle N \rangle$ for the EM showers initiated by 10 to 1000 GeV electrons in lead. Triangles show the depths $t_{\text{max}}$ of the maximum particle flux.
4. Energy distributions

Integrated energy distributions of the charged particles at \( t_{\text{max}} \) for W are presented in table 1 (tables for Fe and Pb and differential distributions can be found in Ref. [18]). They weakly depend on the electron energy \( E_0 \). For example the fraction of particles with energy below 50 MeV is varied from 0.742 to 0.752 (Fe), from 0.792 to 0.806 (W) and from 0.799 to 0.810 (Pb) when \( E_0 \) is changed from 5 to 1000 GeV.

| \( E_0 \), GeV | 5    | 10   | 20   | 30   | 50   | 100  | 200  | 300  | 500  | 1000 |
|----------------|------|------|------|------|------|------|------|------|------|------|
| 5              | 0.210| 0.364| 0.561| 0.673| 0.792| 0.901| 0.959| 0.978| 0.991| 0.998|
| 10             | 0.214| 0.371| 0.568| 0.679| 0.794| 0.900| 0.957| 0.976| 0.989| 0.997|
| 20             | 0.222| 0.381| 0.577| 0.688| 0.801| 0.903| 0.958| 0.976| 0.988| 0.996|
| 30             | 0.222| 0.381| 0.576| 0.686| 0.799| 0.901| 0.956| 0.974| 0.987| 0.995|
| 40             | 0.224| 0.384| 0.580| 0.689| 0.801| 0.902| 0.956| 0.974| 0.987| 0.995|
| 80             | 0.229| 0.388| 0.584| 0.691| 0.802| 0.901| 0.955| 0.973| 0.986| 0.995|
| 120            | 0.230| 0.390| 0.586| 0.694| 0.804| 0.902| 0.955| 0.972| 0.986| 0.994|
| 160            | 0.231| 0.391| 0.586| 0.693| 0.803| 0.901| 0.954| 0.972| 0.985| 0.994|
| 200            | 0.231| 0.391| 0.586| 0.693| 0.803| 0.901| 0.954| 0.972| 0.985| 0.994|
| 300            | 0.234| 0.395| 0.590| 0.697| 0.805| 0.902| 0.955| 0.972| 0.985| 0.994|
| 500            | 0.236| 0.397| 0.592| 0.698| 0.806| 0.902| 0.954| 0.972| 0.985| 0.994|
| 1000           | 0.234| 0.395| 0.588| 0.694| 0.802| 0.899| 0.952| 0.970| 0.983| 0.993|

Figure 6 demonstrates the dependencies of average particle energy \( \langle E \rangle \) vs \( E_0 \). They are fitted to the formula

\[
\langle E \rangle = f_1 \cdot \ln E_0 + f_2
\]

where \( f_1 \) and \( f_2 \) are free parameters (see table 2).

| Material | Fe       | W       | Pb       |
|----------|----------|---------|----------|
| \( f_1 \), GeV | 4.55±0.13 | 2.71±0.15 | 2.39±0.10 |
| \( f_2 \), GeV | 51.92±0.61 | 40.19±0.68 | 40.52±0.47 |
Figure 6. Average particle energy at $t_{\text{max}}$ for Fe (▲), W (○) and Pb (●). The solid and dash-dotted line present the fit to equation (3) for W and Fe with parameters shown in table 2. The results for W and Pb almost coincide with each other.

5. Radial distributions

Integrated radial distributions of the charged particles at $t_{\text{max}}$ are shown in figure 7 for Fe, W and Pb and table 3 for W (tables for Fe and Pb and differential distributions can be found in Ref.[18]). From figure 7 it follows that the distributions for different materials become close to each other if radius is expressed in g/cm$^2$. They are reasonably well fitted to the function

$$f(r) = 1 - f_0 \cdot e^{-sr} - (1 - f_0) \cdot e^{-tr},$$

(4)

where $f_0$, $s$, and $t$ are free parameters (see table 4). It is worth to mention that the particles are strongly concentrated near the shower axis: the ring with 1 mm radius contains from 66% to 59% (W) and from 48% to 44% (Pb) of particles in the energy range from 5 to 1000 GeV.

Figure 7. Fraction of particles inside the ring of radius $r$ for $E_0=200$ GeV. The curves present the fit to equation (4) for W and Pb (top) and Fe (bottom) with parameters shown in table 4.
Table 3. Fraction of particles inside the circle of radius \( r \) at \( t_{\text{max}} \) in W.

| \( E_0', \text{ GeV} \) | \( r, \text{ mm} \) | 0.5 | 1   | 2   | 3   | 5   | 10  | 20  | 30  | 50  | 100 |
|-------------------------|-----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 5                       |                 | 0.417 | 0.655 | 0.851 | 0.919 | 0.961 | 0.982 | 0.990 | 0.992 | 0.995 | 0.998 |
| 10                      |                 | 0.409 | 0.641 | 0.841 | 0.912 | 0.957 | 0.981 | 0.989 | 0.992 | 0.994 | 0.997 |
| 20                      |                 | 0.392 | 0.620 | 0.823 | 0.899 | 0.951 | 0.978 | 0.988 | 0.991 | 0.994 | 0.997 |
| 30                      |                 | 0.395 | 0.620 | 0.822 | 0.898 | 0.950 | 0.978 | 0.988 | 0.991 | 0.993 | 0.997 |
| 40                      |                 | 0.390 | 0.614 | 0.817 | 0.894 | 0.948 | 0.977 | 0.988 | 0.991 | 0.993 | 0.997 |
| 80                      |                 | 0.385 | 0.606 | 0.809 | 0.888 | 0.944 | 0.975 | 0.987 | 0.990 | 0.993 | 0.997 |
| 120                     |                 | 0.382 | 0.601 | 0.804 | 0.885 | 0.942 | 0.974 | 0.986 | 0.990 | 0.993 | 0.996 |
| 160                     |                 | 0.383 | 0.601 | 0.803 | 0.883 | 0.941 | 0.974 | 0.986 | 0.989 | 0.992 | 0.996 |
| 200                     |                 | 0.382 | 0.600 | 0.802 | 0.882 | 0.940 | 0.974 | 0.986 | 0.989 | 0.992 | 0.996 |
| 300                     |                 | 0.377 | 0.593 | 0.796 | 0.878 | 0.937 | 0.972 | 0.986 | 0.989 | 0.992 | 0.996 |
| 500                     |                 | 0.375 | 0.589 | 0.792 | 0.874 | 0.935 | 0.972 | 0.985 | 0.989 | 0.992 | 0.996 |
| 1000                    |                 | 0.375 | 0.594 | 0.794 | 0.875 | 0.935 | 0.971 | 0.985 | 0.989 | 0.992 | 0.996 |

Table 4. Parameters values in formula (4), \( r \) is in g/cm\(^2\).

| Convertor | \( f_0 \) | \( s \) | \( t \) |
|-----------|--------|--------|--------|
| Fe        | 0.13   | 0.085  | 0.47   |
| Pb, W     | 0.14   | 0.086  | 0.59   |

6. Time distributions

Time distributions of the charged particles for 200 GeV showers in W are shown in figure 8. They are extremely narrow: 90% of particles are in the time window of 4 ps. For the 90% of particles inside a circle with \( r=1 \text{ mm} \) the time spread is equal to 0.8 ps.

![Figure 8](image-url)

Figure 8. Time distributions for all charged particles (b) and those inside 1 mm circle (a) at \( t_{\text{max}} \) for W converter \((E_0=200 \text{ GeV})\).

7. Conclusions

Calculations of the EM showers initiated by 10 to 1000 GeV electrons in lead are performed using GEANT4 to investigate fluctuations of the charged particles fluxes. It is shown that for all studied electron energies and lead depths probability distributions \( dP/dN \) of the charged particles multiplicities are well described by the inverse sum of two exponents with three free parameters. At the certain depths \( t_1<t_{\text{max}} \) and \( t_2>t_{\text{max}} \) \( dP/dN \) distributions are symmetric. In the interval from \( t_1 \) to \( t_2 \) \( dP/dN \) distributions are asymmetric with the tail on the left side of the peak and in the region \( t>t_2 \) the tail appears on the right side of the peak. Such evolution of \( dP/dN \) shape can be explained by the late development of some EM showers. Minimal RMS/\( \langle N \rangle \) values of the multiplicity distributions are
achieved at the depths close to \( t_2 \). As follows from formula (2) their \( E_0 \)-dependence obeys a power law. This formula can be considered as an estimate of the best energy resolution of a simple \( e, \gamma \) detector consisting of a high Z converter and a detector of secondaries \( e^+, e^- \) behind it.

Studies of energy, radial and time distributions of charged particles at the maximum of electromagnetic showers initiated by 5 to 1000 GeV electrons in Fe, W, and Pb are also performed using GEANT4. It is shown that the shapes of energy distributions and the average particles energy weakly depend on the incident particle energy and radial distributions for different materials become close to each other if radius is expressed in g/cm\(^2\). The radial and time distributions are narrow. For example 200 GeV incident electron produces at \( t_{\text{max}} \) in W about \( 5 \times 10^2 \) particles within the radius \( r<1 \) mm. Their time spread is 0.8 ps and average energy is 82 MeV. Therefore a high Z material with thickness of \( t_{\text{max}} \) placed in a high energy electron beam can be used as a source of short and intense bunches of ultrarelativistic positrons and electrons with subpicosecond time spread.

Acknowledgments

We gratefully acknowledge the help of D.S. Denisov, T.Z. Gurova, A.V.Kozelov and D.A. Stoyanova in preparation of this manuscript. This work was supported in part by the Russian Foundation for Basic Research under grant #17-02-00120.

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