The question of selective absorption of light in space viewed from the viewpoint of the dynamics of the universe

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Abstract. The selective light absorption in space has been raised in astronomical literature. The substance producing the absorption must have some mass; thus the question is how large it is. We develop a dynamical model of the Milky Way system, assuming that it can be represented by a flattened ellipsoid of rotation. We use the spatial distribution of δ-Cephei and Algol type variable stars, and mean velocities of stars according to Campbell to calculate the dynamical density of the Milky Way near the Sun, $0.100 \, M_\odot/pc^3$. We find that the dynamical density is equal to the mean density of stars in the vicinity of the Sun. Our conclusion is that the intrinsic gravity of stars fully explains their motion, and the existence of any other matter in any significant quantity seems unlikely. Therefore, the existence of noticeable selective absorption seems to be absolutely improbable, unless one admits the existence in the space of particles much smaller than atoms of elements known to us. Normal absorption may exist if the particle diameter is of the order of a millimetre or less, and their mass is comparatively small. This absorption has not yet been reliably detected; the fact that the number of stars increases with stellar magnitude more slowly than theory requires in case of uniform distribution of stars in space, can be equally explained by both light absorption and decrease in number of stars with distance.

Keywords: methods: analytical – Galaxy: fundamental parameters – Galaxy: kinematics and dynamics – Galaxy: solar neighborhood – galaxies: dark matter

Recently, in astronomical literature, the question of light absorption in space has been raised with increasing frequency, mostly selective absorption, i.e. absorption that is stronger the smaller the wavelength; a review of work on determining the magnitude of selective absorption was recently given by Kapteyn (1914). The existence of absorption has not yet been proved with certainty, and how shaky may be the grounds for its determination, is shown by the work of Adams & Kohlschütter (1914), which demonstrates that the well-known fact of a decrease of the violet end of the spectrum with the decrease of the proper motion, i.e. with the increase of the star’s distance, is not confirmed yet. Namely, with equal apparent brightness the more distant stars are absolutely brighter and

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2 Abstract and Keywords added by editors of the English translation Jaan Einasto and Peeter Tenjes.
bigger in their sizes and, in all probability, are surrounded by accordingly thicker atmosphere.

I would like to draw attention to one side of the question, which, as far as I know, has not been covered so far. The substance producing the absorption must have some mass; the question is how large it is. As it is known, selective absorption is created by particles small in size in relation to the wavelength, such as gas molecules; the amount of the absorption depends only on the number of these molecules per unit volume. Let $\mu$ be the average molecular weight (relative to air) of the substance filling the space, and $a$ – the visual absorption coefficient expressed in magnitudes per light-year, $\rho$ – the density of the absorbing matter (relative to the Sun). Based on Abbot & Fowle (1911) measurements at Mount Whitney at pressure $p = 440$ mm the coefficient of transparency for visual rays ($\lambda = 550$ nm) is equal to 0.918, which corresponds to the absorption of 0.070 magnitudes. The air mass at Mt. Whitney is equal to 6000 kg per square meter; in such a case the mass of interstellar matter inside a parallelepiped, whose base is 1 square metre and whose length is 1 light year, is $6000a\mu/0.07$ kg; because 1 light year $= 9.45 \times 10^{15}$ meters and the density of the Sun is $1410$ kg/m$^3$, it is not difficult to calculate that the density of the interstellar substance is equal to

$$\rho = 6.4 \times 10^{-15}a\mu \ (\odot = 1).$$

Let $\delta$ be the “density of stellar matter in space”, i.e. the density of some imaginary matter, which, being evenly distributed in space, will have the mass equal to the mass of encapsulated in this space stars. It is not difficult to determine the order of magnitudes of $\delta$. According to Kaptein’s studies, within a sphere of $r = 500$ light years there are about $1.5 \times 10^6$ stars; let their average mass ($\odot = 1$) be $\overline{m}$, then (as the radius of the Sun is $0.72 \times 10^{-7}$ light years)

$$\delta = \frac{1.5 \times 10^6 (0.72 \times 10^{-7})^3}{500^3} \overline{m} = 0.44 \times 10^{-23} \overline{m}.$$

Comparing $\rho$ and $\delta$, we see that even at the lowest value of $a$ the former significantly exceeds the latter. Indeed, taking $a = 0.0001$ (most investigators give larger values), $\mu = 1/14$ (molecular weight of hydrogen), we obtain

$$\rho = 4.7 \times 10^{-18};$$

the quantity $\overline{m}$ is close to unity, and

$$\delta = 0, 44 \times 10^{-23}.$$

Thus, at absorption of 1 magnitude per 10,000 light-years the density, and consequently, also the mass of the absorbing substance, must be at least 100 000 times greater than the mass of all stars! Obviously, in such a distribution of absorbing substance in space the movements of the stars should depend almost entirely not
on their mutual gravity, but on the attraction of this absorbing matter scattered in space. However, such a large value of the mass of the interstellar matter seems very unlikely – the velocities of stars would accordingly be much larger than the observed values, and would be measured in thousands, not tens, of kilometres per second. Therefore we have to choose between the following suppositions (or to admit them both): 1) the value of selective absorption is much lower than is commonly assumed; 2) the molecular weight of the substance causing selective absorption is much lower than the molecular weight of hydrogen. This strongly suggests the idea of electrons scattered in space.

Since even the most negligible selective absorption must be dominated by interstellar matter, it is of some interest to find answer to the question about the laws of motions inside a star system, enveloped by evenly dispersed nebular matter of considerable mass. (The question on dynamics of a whole stellar system has already been developed by Eddington [1913] for a spherical system, consisting of only stars). Our main task will be to determine the order of matter density, filling the Milky Way system. Although, as it will turn out later, our assumptions will not correspond to the truth precisely, however, they will not influence the order of magnitude of the sought density.

Let us make the following assumptions: the interstellar matter is distributed as an ellipsoid of rotation of indefinite dimensions, uniform density \( \rho \), and with the axial ratio \( q \). Inside the ellipsoid there are stars scattered within it, the stellar density \( \delta \) (it is proportional to the number of stars per unit volume) reaches its maximum value at the ellipsoid centre, and the surfaces of equal density \( \delta \) (the surface level) are ellipsoids, similar and concentric to the above mentioned one. If the equation of such a surface is

\[
q^2x^2 + y^2 + z^2 = u^2
\]

(the minor axis of the ellipsoid lies on the \( x \)-axis), then the stellar density is

\[
\delta = f(u^2).
\]  

(3)

At last, we assume that \( \delta \) is significantly smaller than \( \rho \), so the mass of the stars can be neglected.

The gravitational acceleration at a given point inside the ellipsoid of rotation of uniform density \( \rho \) is expressed by the following formulae:

\[
\begin{align*}
\frac{d^2x}{dt^2} &= -cx \\
\frac{d^2y}{dt^2} &= -c_1y \\
\frac{d^2z}{dt^2} &= -c_1z
\end{align*}
\]

(4)
where
\[ c = 4\pi n \rho \frac{q^2}{(q^2-1)^{3/2}} \left( \sqrt{q^2-1} - \arctan \sqrt{q^2-1} \right) \]
\[ c_1 = 2\pi n \rho \frac{q^2}{(q^2-1)^{3/2}} \left( \arctan \sqrt{q^2-1} - \frac{\sqrt{q^2-1}}{q^2} \right) \]  

(5)

here \( q \) is the axis ratio of the ellipsoid, and \( n \) is the gravitational constant.

It is clear from this that the motion projections on coordinate axes are harmonic oscillations, and the oscillation term for \( x \)-axis is different than for \( y \)-axis and \( z \)-axis. Having solved these equations, they can be reduced to the following form:

\[
\begin{align*}
    x &= H \cos(t\sqrt{c}) \\
    y &= K \cos(t\sqrt{c_1} - \eta) \\
    z &= L \cos(t\sqrt{c_1} - \zeta) \\
\end{align*}
\]

(6)

where
\[
H, K \text{ and } L \text{ are obviously the values of the greatest distance of a given point mass along the coordinate axes, } \eta \text{ and } \zeta \text{ are the phase differences of the oscillations. Let us call } H, K, \text{ and } L \text{ the amplitudes along the coordinate axes.}
\]

Let us denote by \( u, v \) and \( w \) the velocity components along the coordinate axes; then it is easy to obtain the following relations from equations (6):

\[
\begin{align*}
    u &= \pm \sqrt{c} \sqrt{H^2 - x^2} \\
    v &= \pm \sqrt{c_1} \sqrt{K^2 - y^2} \\
    w &= \pm \sqrt{c_1} \sqrt{L^2 - z^2} \\
\end{align*}
\]

(7)

Let the number of stars whose amplitudes along the \( x \)-axis are enclosed between \( H \) and \( H + dH \) be

\[ dn_H = \varphi(H)dH, \]

(8)

and similarly for other axes

\[ dn_K = \psi(K)dK, \quad dn_L = \psi(L)dL. \]

(9)

The functions \( \varphi \) and \( \psi \) give the distribution functions of amplitudes along the axes; for the \( y \) and \( z \)-axis, identical function is assumed due to symmetry. Let us determine what form these functions should take in order for the system to be in equilibrium at a given \( \delta = f(u^2) \). For this purpose we define the number of stars whose abscissae are between \( x \) and \( x + dx \), \( y + dy \) and \( z + dz \). On the one hand this number is obviously

\[
\begin{align*}
    dn_x &= dx \iint \delta \, dy \, dz \\
    dn_y &= dy \iint \delta \, dx \, dz \\
    dn_z &= dz \iint \delta \, dx \, dy \\
\end{align*}
\]

(11)
The integration limits for an unbounded stellar system are constant; the lower limits are 0, the upper limits – infinity.

On the other hand, \( \frac{dn_x}{dx} \) depends on the form of the function \( \psi(H) \). We note that for the same star its \( x \) coordinate varies in time from 0 to \( H \) (pay attention only to the absolute value of \( x \)). Therefore at a given distance \( x \) there can be only these stars for which \( H \geq x \); the time at which a given star changes \( x \) from \( x \) to \( x + dx \) will be equal to

\[
\frac{dt}{\tau_x} = \frac{dx}{|u|},
\]

where \( \tau_x = \frac{2\pi}{\sqrt{c}} \) is the period of oscillation, and \( |u| = \sqrt{c(\sqrt{H^2 - x^2}} \). Substituting these variables we obtain that the number of stars with a given \( H \), enclosed between \( x \) and \( x + dx \), is \( dx/\sqrt{H^2 - x^2} \); but the number of stars with the given \( H \) is \( \varphi(H) dH \), therefore we obtain

\[
\frac{dn_x}{dx} = \frac{dH}{\sqrt{H^2 - x^2}} \int_{H=x}^{H=\infty} \varphi(H) dH.
\]  

Comparing formulae (11) and (12) we obtain

\[
\int \int \delta dy \ dz = \int \frac{\varphi(H) dH}{\sqrt{H^2 - x^2}}.
\]  

This equation can be called an equation of state of a given system; for a given \( \delta \) we can determine \( \varphi \) and vice versa.

Let the stellar density law be similar to the normal distribution \( i.e. \) let

\[
\delta_0 = \delta e^{-k^2(q^2x^2+y^2+z^2)}.
\]  

This law seems rather probable for the Milky Way stellar system; moreover, we do not care about the exact form of the function, but it is only important that it satisfies some general conditions – that it decreases with distance, the level surfaces are ellipsoids, and the system is unbounded.

Substituting (14) into (13) and integrating, we obtain

\[
\int_x^{\infty} \frac{\varphi(H) dH}{\sqrt{H^2 - x^2}} = \frac{A}{k^2 e^{-k^2 q^2 x^2}},
\]  

where

\[
A = \frac{\pi \delta_0}{4}.
\]
The first part of this equation can be represented in the form:

\[
\int_{x}^{\infty} \frac{\varphi(H)}{\sqrt{H^2 - x^2}} dH = \left[ \frac{\varphi(H)\sqrt{H^2 - x^2}}{H} \right]_{H=x}^{H=\infty} - \int_{0}^{\infty} \sqrt{H^2 - x^2} \left[ \frac{H \varphi'(H) - \varphi(H)}{H^2} \right] dH;
\]

suppose that when \( H = \infty \) \( \varphi(H) = 0 \);

then, as it is easy to see, the first term in the right side of the equality will be 0, and we will have (taking into account equality (15))

\[
- \int_{x}^{\infty} \sqrt{H^2 - x^2} \left[ \frac{H \varphi'(H) - \varphi(H)}{H^2} \right] dH = \frac{A}{k^2} e^{-k^2 q^2 x^2}.
\]

Differentiate this equation by \( x \), applying the formula

\[
\left( \frac{d}{da} \int_{a}^{b} f(x, a) da \right) = -f(a, a) + \int_{a}^{b} f'(x, a) da,
\]

then we will have after a reduction by \( x \):

\[
\int \frac{[H \varphi'(H) - \varphi(H)] dH}{H^2 \sqrt{H^2 - x^2}} = -2A q^2 e^{-k^2 q^2 x^2},
\]

or by excluding \( e^{-k^2 q^2 x^2} \) with formula (15) and transferring everything to the left-hand side, we will have

\[
\int_{x}^{\infty} \frac{1}{\sqrt{H^2 - x^2}} \left[ 2k^2 q^2 \varphi(H) \right. \left. - \frac{\varphi(H)}{H^2} + \varphi'(H) \right] \frac{dH}{H} = 0.
\]

This identity is correct when

\[
\left( 2k^2 q^2 - \frac{1}{H^2} \right) \varphi(H) + \frac{\varphi'(H)}{H} = 0,
\]

or

\[
\frac{\varphi'(H)}{\varphi(H)} = -2k^2 q^2 H + \frac{1}{H};
\]

hence by a simple integration we find

\[
\begin{align*}
\varphi(H) &= C H e^{-k^2 q^2 H^2} \\
\psi(K) &= C \frac{K}{q} e^{-k^2 K^2} \\
\psi(L) &= C \frac{L}{q} e^{-k^2 L^2}
\end{align*}
\]

(16)
These formulae give the amplitude distribution function. It is not difficult to determine the average value of the velocity components of the stars along the axes for any point. Let’s find the average absolute value of the velocity component parallel to the $x$-axis for a certain $x$-value; it will obviously be equal to

$$
\overline{u}_x = \frac{\int_x^\infty u \varphi(H) \sqrt{H^2 - x^2} \, dH}{\int_x^\infty \varphi(H) \sqrt{H^2 - x^2} \, dH};
$$

substituting

$$u = \sqrt{c} \sqrt{H^2 - x^2},$$

we find after the integration

$$\overline{u}_x = \frac{\sqrt{c}}{kq\sqrt{\pi}}.$$  \hspace{1cm} (17)

We see that the average value of the velocity component of stars under the chosen density function is a constant for all points of the system. For the average value of the velocity components on the other axes we will get the same expression:

$$\overline{u}_y = \overline{u}_z = \frac{\sqrt{c_1}}{k\sqrt{\pi}}.$$  \hspace{1cm} (18)

The average absolute value of the velocity can be assumed to be (approximately)

$$V = \sqrt{\overline{u}_x^2 + \overline{u}_y^2 + \overline{u}_z^2} = \frac{1}{k\sqrt{\pi}} \sqrt{\frac{c}{q^2} + 2c_1}.$$  \hspace{1cm} (19)

The constants $c$ and $c_1$, as seen from formulae (5), depend on the density and shape of the system; $k$ determines the rate at which the number of stars decreases with distance from the centre. For a spherical body, at $q = 1$, the following expression is obtained for $c$ instead of (5):

$$c = \frac{4}{3} \pi n \rho.$$  

Let us choose the constant $n$ so that, if the unit of length is kilometre and the unit of time is second, then the unit of the density would be the solar density; we have

$$\frac{d^2x}{dt^2} = -\frac{4}{3} \pi n \rho x,$$

and for the Sun

$$\rho = 1, \quad x = 695000, \quad \frac{d^2x}{dt^2} = 0.274;$$

from here

$$4\pi n = 1.2 \times 10^{-6}.$$
Let us now try to apply the results of our reasoning to the Milky Way. According to the currently known data, it is a flat disc or a flattened ellipsoid of rotation with a ratio of axes approximately equal to 10. For determination of the approximate value of the constant \( k \) we use the results by Hertzsprung (1913) and Russell & Shapley (1914), obtained from the examination of the spatial distribution of variable stars of \( \delta \)-Cephei and Algol type. It turns out, that these stars are heavily concentrated to the plane of the Milky Way, with average distance from the plane for type \( \delta \)-Cephei at 260 light years, for Algol at 440 light years; the difference between these two numbers could be real, but could be also due to some systematic reasons, depending on some accepted assumptions. It is possible to accept an average value of \( \bar{x} = 350 \) light years or \( 3.3 \times 10^{15} \) km. On the other hand, it follows from formula (14) that with fixed \( y \) and \( z \) values the stellar density (number of stars) is expressed by the formula

\[ \delta = Ce^{-k^2q^2x^2}; \]

their average distance from the plane of the Milky Way will be

\[ \bar{x} = \frac{\int_{x}^{\infty} xe^{-k^2q^2x^2} \, dx}{\int_{x}^{\infty} e^{-k^2q^2x^2} \, dx} = \frac{1}{kq\sqrt{\pi}}. \]

Accepting

\[ q = 10, \quad \bar{x} = 3.3 \times 10^{15}, \]

we found

\[ k = 1.72 \times 10^{-17}. \]

Then at \( q = 10, 4\pi n = 1.2 \times 10^{-6} \) our constants \( c \) and \( c_1 \), will be \( 10^{-6}\rho \) and \( 0.08 \times 10^{-6}\rho \), respectively.

Substituting these values into formula (19), and taking according to Campbell (1915) \( V = 30 \) km/s, we find

\[ \rho = 0.48 \times 10^{-23} \]

in units of solar density\(^3\).

Consequently, the effective density for the hypothetical homogeneous medium turns out to be exactly equal to the “stellar density”, which was calculated in the beginning of this paper. From this it is clear that the intrinsic gravity of stars fully explains their motion and the existence of any other matter in any significant quantity seems unlikely. Therefore, the existence of noticeable selective absorption seems to be absolutely improbable, unless one admits the

\(^3\)Since solar density is 1410 kg/m\(^3\) and \( 1 \, \text{M}_\odot/\text{pc}^3 = 6.770 \times 10^{-20} \, \text{kg/m}^3 \), we get the density in solar masses per cubic parsec: \( \rho = 0.100 \, \text{M}_\odot/\text{pc}^3 \) (Note added by editors of the English translation Jaan Einasto and Peeter Tenjes).
existence in the space of particles much smaller than atoms of elements known to us.

In addition to selective absorption, normal absorption equal to all wavelengths can exist, produced by particles with a diameter larger than the wavelengths. If these particles are completely opaque, \(d\) their thickness in millimetres, then, taking their density equal to the density of water, it is easy to obtain the following expression for the effective density of the space filled with them:

\[
\rho = 0.75 \, a \, d \times 10^{-19},
\]

where \(a\) is, as before, the absorption coefficient in magnitudes per 1 light year. At \(a = 0.0001\) we have

\[
\rho = 0.75 \, d \times 10^{-23}.
\]

Consequently, normal absorption may well exist if the particle diameter is of the order of a millimetre or less, and their mass is comparatively small. However, it must be remembered that this absorption has not yet been reliably detected; the fact that the number of stars increases with stellar magnitude more slowly than theory requires in case of uniform distribution of stars in space, can be equally explained by both light absorption and decrease in number of stars with distance.

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Comment added by editors Jaan Einasto and Peeter Tenjes.

The paper by Öpik (1915) is the first one where the problem of invisible (dark) matter in solar neighbourhood of Galaxy was discussed. His results were confirmed by Kapteyn (1922), who introduced the term “dark matter” to denote the possible invisible matter in the solar neighbourhood. This problem was studied by Jeans (1922), Oort (1932) and in Tartu Observatory by Kuzmin (1952, 1955), Eelsalu (1958) and Jõeveer (1972, 1974, 1975).

The dynamical density \(\rho\) can be calculated from the gradients of the gravitational potential \(\Phi\) via the Poisson equation, which has in cylindrical coordinates the form

\[
4\pi G \rho = -\frac{\partial^2 \Phi}{\partial z^2} - \frac{\partial^2 \Phi}{\partial R^2} - \frac{1}{R} \frac{\partial \Phi}{\partial R},
\]

(1)

where \(G\) is the gravitational constant. To determine the dynamical density Kuzmin (1952) made several innovations compared with earlier studies. First, he noticed that planar subsystems rotate practically with a circular velocity. In this case we can express the sum of second and third terms in Eq. (1) near the Galactic plane through Oort galactic rotation parameters \(A\) and \(B\):

\[
\frac{\partial^2 \Phi}{\partial R^2} + \frac{1}{R} \frac{\partial \Phi}{\partial R} = -2 \frac{dV_c}{dR} \frac{V_c}{R} = 2(A^2 - B^2),
\]

(2)
where $V_c$ is the circular velocity. Kuzmin introduced for the first term of Eq. (1) the designation

$$C^2 = -(\partial^2 \Phi / \partial z^2)_{z=0}.$$  

The parameter $C$ has the same dimension as $A$ and $B$, and is a necessary complement to the rotational Oort parameters. Assuming that velocities $v_z$ of a stellar population have normal distribution Kuzmin showed that near the Galactic plane also their $z$ coordinates have normal distribution, and the parameter $C$ can be calculated from the ratio of the dispersions of vertical velocities $\sigma_z$ and spatial $z$-coordinates $\zeta$:

$$C = \frac{\sigma_z}{\zeta}.$$  

(3)

In other words, to derive the dynamical density, it is not needed to calculate the gravitation attraction, $K_z = -\partial \Phi / \partial z$, over a large $z$-interval, as done by Kapteyn (1922), Jeans (1922) and Oort (1932), but only its gradient near the Galactic plane. Since the gradient changes near the Galactic plane rather quickly, only flat stellar populations are suitable to find dispersions $\sigma_z$ and $\zeta$.

The second innovation by Kuzmin (1952) was to use for determination of dispersions of spatial position and velocities of identical samples of A and gK stars near the galactic plane within galactic latitudes $\pm 3$ degrees. To find the velocity dispersion $\sigma_z$ he used vertical components of proper motions, to calculate spatial dispersion $\zeta$ he used parallaxes. This eliminates possible sampling errors and errors in parallaxes. His final result (Kuzmin 1955) was

$$C = 68 \pm 3 \text{ km/sec/kpc}.$$  

and for the density:

$$\rho = (5.2 \pm 0.5) \times 10^{-24} \text{g/cm}^3 = 0.077 \pm 0.008 \text{ M}_\odot / \text{pc}^3.$$  

This value confirmed Öpik (1915) and Kapteyn (1922) conclusion that the gravity of known stars fully explains their motion, and there is no need for local dark matter. Further determinations of $C$ were made in Tartu by Eelsalu (1958), and Jõeveer (1972, 1974, 1975). These studies by Kuzmin, Eelsalu and Jõeveer formed their PhD theses (candidate theses according to Soviet rules). The final result of the Eelsalu analysis is: $C = 67 \pm 3 \text{ km/sec/kpc}$. The mean value of the Jõeveer analysis is $C = 70 \text{ km/sec/kpc}$, and $\rho_{dyn} = 0.09 \text{ M}_\odot / \text{pc}^3$.

Different results were obtained by Oort (1960) and Bahcall (1984), Bahcall et al. (1992). Their analyses suggested the presence of local dark matter approximately in the same quantity as the known matter. This discrepancy led to a discussion, for overviews see Gilmore et al. (1989), Read (2014) and McKee et al. (2015). Kuijken & Gilmore (1989) made a careful analysis of the Bahcall (1984) study and found flaws in it. Kuijken & Gilmore (1989) made a reanalysis of F dwarf and gK giant stars and found no evidence for any missing matter near the Sun.
Flat rotation curves of galaxies (Rubin et al. 1978, Bosma 1978) suggest the presence in galaxies almost spherical extended coronas of dark matter. Models of the Galaxy including coronas were developed by Einasto et al. (1976), Bienayme et al. (1987) and Haud & Einasto (1989). These models predict the presence of local dark matter in solar vicinity with density of the order $\rho_{DM} \simeq 0.01$ M$_{\odot}$/pc$^3$.

Recent determination of the density of matter in solar vicinity have led to values $0.097 \pm 0.013$ M$_{\odot}$/pc$^3$, the estimated sum of the stellar and gas mass densities $\rho_* = 0.084 \pm 0.012$ M$_{\odot}$/pc$^3$, which yield for the density of dark matter $\rho_{DM} = 0.013 \pm 0.003$ M$_{\odot}$/pc$^3$ (Bienayme et al. (1987); Bienaymé et al. (2006), Creze et al. (1998), Holmberg & Flynn (2000), McKee et al. (2015)). Local matter density determinations based on the Gaia satellite data fully confirm these results (Kipper et al. (2018), Buch et al. (2019), Guo et al. (2020), Salomon et al. (2020)).
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