Model-Free Reinforcement Learning for Symbolic Automata-encoded Objectives

Anand Balakrishnan*, Stefan Jakšić†, Edgar A. Aguilar†, Dejan Ničković†, and Jyotirmoy V. Deshmukh*
*University of Southern California, Los Angeles, California, USA
Email: {anandbal, jdeshmuk}@usc.edu
†AIT Austrian Institute of Technology GmbH, Vienna, Austria
Email: {stefan.jaksic, edgar.aguilar, dejan.nickovic}@ait.ac.at

Abstract—Reinforcement learning (RL) is a popular approach for robotic path planning in uncertain environments. However, the control policies trained for an RL agent crucially depend on user-defined, state-based reward functions. Poorly designed rewards can lead to policies that do get maximal rewards but fail to satisfy desired task objectives or are unsafe. There are several examples of the use of formal language such as temporal logics and automata to specify high-level task specifications for robots (in lieu of Markovian rewards). Recent efforts have focused on inferring state-based rewards from formal specifications; here, the goal is to provide (probabilistic) guarantees that the policy learned using RL (with the inferred rewards) satisfies the high-level formal specification. A key drawback of several of these techniques is that the rewards that they infer are sparse: an agent receives positive rewards only upon completion of the task and no rewards otherwise. This naturally leads to poor convergence properties and high variance during RL.

In this work we propose using formal specifications in the form of symbolic automata: these serve as a generalization of both bounded-time temporal logic based specifications as well as automata. Furthermore our use of symbolic automata allows us to define non-sparse potential-based rewards which empirically shape the reward surface, leading to better convergence during RL. We also show that our potential-based rewarding strategy still allows us to obtain the policy that maximizes the satisfaction of the given specification.

I. INTRODUCTION

Reinforcement learning (RL) is a paradigm for automatically synthesizing controllers that perform a certain task through repeated interaction with the environment [6] [34]. In the standard RL setting, instead of explicitly programming an agent to perform the task – which is often infeasible due to task complexity – a reward function is specified for the actions an agent might take. Thus, by taking actions in an environment and collecting rewards, an agent learns which actions maximize the overall expected reward.

With the advent of deep neural networks in the last decade, associated to an unprecedented increase in computational power, RL was able to solve incredibly complex tasks using simple reward functions [27] [28] [33] [32]. This major accomplishment of deep reinforcement learning was demonstrated in various environments, from video games to robot locomotion. A major ingredient in the success of RL is a well-designed reward function that encodes the task correctly. Engineering a good reward function is still considered to be an art.

A good reward function must ensure that the agent will be able to learn how to reach desired objectives according to designer intentions. A poorly designed reward function can result in a situation where an agent learns to maximize total rewards without actually satisfying the high-level objective intended by the designer. This situation is known as reward hacking [3]. Reward hacking can lead to undesirable behaviors, which can be of particular importance in the context of safety-critical systems, where failures are not acceptable. To avoid reward hacking, there is a need for more principled approaches to reward engineering.

Traditional reward functions are Markovian, i.e. they do not depend on history to complete a given task. This makes them inappropriate for history-aware objectives, such as sequential tasks, where individual objectives must be completed in order, and security surveillance, where an area needs to be patrolled within a time bound.

Techniques from formal methods can be effectively used to address the problem of principled history-aware reward engineering. For instance, automata-based approaches [31] [12] [17] [25] have been used to solve the issue of non-Markovian objectives for omega-regular objectives for infinite behavior. However, these methods are susceptible to the definition of sparse reward functions. Sparse reward functions suffer from long convergence because the agent has to perform a long sequence of actions before seeing any concrete reward. Finally, techniques that use quantitative semantics of temporal logics have shown to be highly effective in finding optimal policies up to some finite control horizon [11] [5]. Unfortunately, such techniques require that some history is stored to remove the non-Markovian nature of the objective. These approaches are also limited by the “horizon” of the tasks at hand, and scale poorly to tasks that require a large history.

A. Main Contributions

In this paper, we propose a novel approach to encode a finite sequence of tasks using symbolic automata [12]. Symbolic automata enable the encoding of history-dependent goals, while providing rich quantitative information about the means to achieve the overall objective. We propose a reward function that uses both the automaton structure and its symbolic information to accelerate the discovery of the optimal policy. Our reward function consists of two components: (1) a sparse base reward
that is given only when the agent reaches the overall objective, and (2) an additive potential function that shapes the base reward to ensure progress towards intermediate goals and the final objective. To the best of our knowledge, we present the first method of using symbolic automata as task specifications for reinforcement learning.

We prove the soundness of our reward function on a product transition system in a model-based RL setting, to show that it solves the problem of maximizing the probability of satisfying SA specification in an MDP. Then, we empirically evaluate it in a model-free RL environment, where we use the Q-learning algorithm and our reward to tackle three different problems: (1) goal reachability, (2) recurrence, and (3) sequential tasks. We demonstrate that our reward function indeed optimizes the learning process towards the goals specified by the symbolic automaton specification. This is done by showing a significant increase in the convergence rate when compared to baseline rewards.

B. Organization

The rest of the paper is organized as follows: Section II provides the theoretical prerequisites of our work. In Section III, we formalize the problem of policy synthesis for a symbolic automaton encoded task in a given MDP, for which we propose a RL rewarding strategy in Section IV. We empirically evaluate our reward strategy in three distinct scenarios in Section V. Section VI and conclude the paper in Section VII.

II. PRELIMINARIES

A. Symbolic Automata

Let \( X = \{x_1, \ldots, x_n\} \) be a set of variables, where each \( x_i \) takes values in some compact set \( D \subseteq \mathbb{R} \). Let \( v: X \rightarrow D \) be a valuation function (or just valuation) that maps a variable \( x \in X \) to the value of \( x \). For example if \( X = \{x, y\} \), then the function \( v : x \mapsto 3, y \mapsto 5 \) defines the values of \( x \) and \( y \) to 3 and 5 respectively. Given the set of variables \( X \), we can abuse notation and use \( v(X) \) to denote \((v(x_1), \ldots, v(x_n))\). In other words \( v(X) \) is some value in \( D^n \). We can define a metric over this space, for example, consider the Manhattan distance between two valuations defined as follows:

\[
d_{\text{man}}(v, v') = \sum_{i=1}^{n} |v(x_i) - v'(x_i)|.
\]

If \( v(X) = (3, 5) \) and \( v'(X) = (2, 1) \), then \( d_{\text{man}}(v, v') = 1 + 4 = 5 \).

Definition 1 (Predicate). A predicate \( \psi \) over \( X \) is defined with the following recursive grammar:

\[
\psi ::= \top | \bot | x \sim k | \neg \psi | \psi \land \psi
\]

where \( x \in X \), \( k \in D \), and \( \sim \in \{<, \leq, >, \geq, =\} \). We denote by \( \Psi(X) \) the set of all predicates over \( X \).

Given a valuation function \( v : X \rightarrow D \), we define the semantics of \( \psi \) in terms of a satisfaction relation \( v \models \psi \) as follows:

\[
v \models \top \iff \top
\]

\[
v \models \bot \iff \bot
\]

\[
v \models x \sim k \iff v(x) \sim k
\]

\[
v \models \neg \psi \iff \psi \not\models \psi
\]

\[
v \models \psi_1 \land \psi_2 \iff (v \models \psi_1) \land (v \models \psi_2)
\]

In the following definition, we assume that there is an appropriately defined distance metric \( d \) between two valuations \( v \) and \( v' \) (for example Manhattan distance as defined in Eq. 1).

Definition 2 (Value-Predicate Distance [20]). Given a predicate \( \psi \in \Psi(X) \) and a valuation \( v \), we define the value-predicate distance as the distance between \( v \) and the set of valuations that satisfy \( \psi \) as follows:

\[
\text{vpd}(v, \psi) = \min_{v' \models \psi} d(v, v'),
\]

where \( d : D^n \times D^n \rightarrow \mathbb{R}_{\geq 0} \) is a distance metric on the space of valuations.

Definition 3 (Symbolic Automaton [12]). A symbolic automaton is a tuple \( A = (X, Q, q_{\text{init}}, F, \Delta, G) \), where \( X \) is a finite set of variables, where each variable takes values in \( D \); \( Q \) is a finite set of locations with initial location \( q_{\text{init}} \); \( F \subseteq Q \) is a set of accepting locations; \( \Delta \subseteq Q \times Q \) is a nonempty set of transitions; and \( G : Q \times Q \rightarrow \Psi(X) \) is the guard labeling the transition.

A run of the symbolic automaton is defined as a sequence of states and valuations for variables in \( X \) as follows:

\[
q_0 \xrightarrow{v_1} q_1 \rightarrow \ldots \rightarrow q_{n-1} \xrightarrow{v_n} q_n.
\]

Here, \( q_0 = q_{\text{init}} \), and for all \( i \in [0, n - 1] \); \((q_i, q_{i+1}) \in \Delta \), and \( v_{i+1} \models G(q_i, q_{i+1}) \). We say that the run is accepting if for some \( n, q_n \in F \). We say that a symbolic automaton \( A \) is terminal accepting if for every accepting state in \( F \), all outgoing transitions are to some state in \( F \). Such an automaton allows us to replace all accepting states by a single “sink” accepting state \( q_F \), s.t. \( \forall(q_F, q) \in \Delta, q = q_F \). In this paper, we restrict our attention to such terminal accepting symbolic automata.

B. Reinforcement Learning

Reinforcement learning (RL) is a technique for an autonomous agent to learn the policy that maximizes some notion of a cumulative expected reward provided to it by a stochastic environment. Typically, the interaction between the RL agent and its environment is modeled as Markov Decision Process (MDP).

Definition 4 (Markov Decision Process (MDP) [34]). An MDP is a tuple \( \mathcal{M} = (S, s_{\text{init}}, A, P, R) \), where \( S \) is a finite set of states with initial state \( s_{\text{init}} \); \( A \) is a finite set of possible actions; \( P : S \times A \times S \rightarrow [0, 1] \) is a (partial) probabilistic transition function, where \( P(s, a, s') = \Pr(s' \mid s, a) \) defines the probability of arriving in state \( s' \) after taking action \( a \) from state \( s \); and \( R : S \times S \rightarrow \mathbb{R} \) is a reward function defined on \( \mathcal{M} \), where \( R(s, s') \) denotes the immediate reward received by transitioning from \( s \) to \( s' \).
An episode $\xi = (s_0, \ldots, s_N)$ is a trace of length $N$ in the MDP $\mathcal{M}$ such that $s_0 = s_{\text{init}}$ and for all $t \in [0, N-1]$, $P(s_t, a_t, s_{t+1}) > 0$ for some $a_t \in A$, and $N$ is the maximum episode length.

Given a set $Y$, we let $\mathcal{D}(Y)$ denote the set of all probability distributions over $Y$.

**Definition 5 (Policy of an MDP).** A policy $\pi : S \to \mathcal{D}(A)$ is a function that maps a state $s \in S$ to a probability distribution over the set of actions $\mathcal{D}(A)$.

Fixing a policy $\pi$ in $\mathcal{M}$ induces a probability space of episodic trajectories characterized by the distribution $\mathcal{M}^\pi$ such that the probability of generating a trajectory $\xi$ in $\mathcal{M}$ under the policy $\pi$ (denoted $\xi \sim \mathcal{M}^\pi$):

$$
\Pr (\xi \sim \mathcal{M}^\pi) = \Pr \left( (s_0, \ldots, s_N) \mid s_0 = s_{\text{init}} \right),
$$

where each action $a_t$ is sampled from the distribution $\pi(s_t)$, and $P(s_t, a_t, s_{t+1}) > 0$.

Let $R_i$ denote the immediate reward given to the agent in time instance $t$ when the MDP transitions from state $s_{t-1} = s$ to $s_t = s'$: $R_t = R(s_{t-1}, s_t) = R(s, s')$.

**Definition 6 (Policy Value functions [34]).** Under a policy $\pi : S \to \mathcal{D}(A)$, the state-value function $V^\pi : S \to \mathbb{R}$ of some state $s \in S$ in time instance $1 \leq t \leq N$ is the expected total reward induced in $\mathcal{M}^\pi$ starting from state $s$:

$$
V^\pi(s) = \mathbb{E}_\pi \left[ \sum_{i=1}^N R_i \mid s_i = s \right] \tag{3}
$$

Moreover, the action-value function under a policy $\pi$, $Q^\pi : S \times A \to \mathbb{R}$ is the expected total reward for taking an action $a \in A$ at some state $s \in S$ at time-step $t$:

$$
Q^\pi(s, a) = \mathbb{E}_\pi \left[ \sum_{i=1}^N R_i \mid s_i = s, a_i = a \right] \tag{4}
$$

The optimal state-value function $V^*(s)$ is defined as

$$
V^*(s) = \max_\pi V^\pi(s), \ \forall s \in S,
$$

and the optimal action-value function $Q^*(s, a)$ is

$$
Q^*(s, a) = \max_\pi Q^\pi(s, a), \ \forall s \in S, \forall a \in A.
$$

Note that the optimal policy value functions have recursive definitions:

$$
V^*(s) = \max_{a \in A} Q^*(s, a) = \max_{a \in A} \mathbb{E} P(s, a, s') > 0 [R(s, s') + V^*(s')]
$$

The goal in reinforcement learning is to find a policy $\pi^*$ on $\mathcal{M}$ such that $V^\pi^*(s_{\text{init}}) = V^*(s_{\text{init}})$, or

$$
\pi^* = \arg \max_{\pi} V^\pi(s_{\text{init}}) \tag{5}
$$

Remark. Usually, the policy synthesis problem is posed for discounted, infinite runs [34]. In this paper, we only consider the episodic reinforcement learning setting [41], where the goal is to maximize expected total returns from trajectories with a finite time bound $N$ and initial state $s_{\text{init}}$.

### III. Problem Statement

Given a Markov Decision Process $\mathcal{M} = (S, s_{\text{init}}, A, P)$, and a simply accepting symbolic automaton specification $A = (X, Q, q_{\text{init}}, F, \Delta, G)$ that represents a task that needs to be performed in $\mathcal{M}$, our goal is to synthesize a policy that maximizes the probability of satisfying the task $A$. To do this, we first define a product transition system which is an MDP that composes $\mathcal{M}$ with $A$.

**Definition 7 (Product MDP with accepting states).** Given an MDP $\mathcal{M} = (S, s_{\text{init}}, A, P)$ with $S \subseteq \mathbb{R}^n$ and a symbolic automaton $A = (X, Q, q_{\text{init}}, F, \Delta, G)$ with a valuation function $v : S \times X \to D$ (where $D \subseteq \mathbb{R}$), we can construct a product MDP (with additional annotation of accepting states) $\mathcal{P} = \mathcal{M} \otimes A$ as a tuple $(S_{\otimes}, s_{\text{init}_{\otimes}}, A_{\otimes}, P_{\otimes}, Acc)$, where:

- $S_{\otimes} = S \times Q$,
- $s_{\text{init}_{\otimes}} = (s_{\text{init}}, q_{\text{init}})$,
- $P_{\otimes} : S_{\otimes} \times A_{\otimes} \times S_{\otimes} \to [0, 1]$ is defined as:

$$
P_{\otimes}((s, q), a, (s', q')) = \begin{cases} P(s, a, s') & \text{if } (q, q') \in \Delta, s \models G(q, q'), \\ 0 & \text{otherwise.} \end{cases}
$$

- $Acc = \{(s, q) \mid q \in F\}$.

An episode $\xi = ((s_0, q_0), \ldots, (s_N, q_N))$ in $\mathcal{P}$ (with $(s_0, q_0) = (s_{\text{init}}, q_{\text{init}})$) is considered accepting if and only if $(s_N, q_N) \in Acc$. We use $\xi \models A$ to denote that the episode $\xi$ is accepted by the specification automaton $A$.

Given a policy $\pi : S \times Q \to \mathcal{D}(A)$, we let $\Pr (\pi \models A)$ denote the probability that an episode sampled from $\mathcal{P}^\pi$ is accepted by $A$ (also called the probability of $\pi$ being accepting):

$$
\Pr (\pi \models A) = \Pr_{\xi \sim \mathcal{P}^\pi} [\xi \models A] = \mathbb{E}_{\xi \sim \mathcal{P}^\pi} \left[ 1(\xi \models A) \right], \tag{6}
$$

where $1(\cdot)$ is the indicator function such that

$$
1(f) = \begin{cases} 1, & \text{if } f \text{ evaluates to } \top \\ 0, & \text{otherwise.} \end{cases}
$$

**Problem 1.** Given an MDP $\mathcal{M} = (S, s_{\text{init}}, A, P, R)$ and a terminally accepting specification automaton $A$, let $\mathcal{P} = \mathcal{M} \otimes A$. Synthesize a policy $\pi^* : S \times Q \to \mathcal{D}(A)$ that maximizes the probability of acceptance.

$$
\pi^* = \arg \max_\pi \Pr (\pi \models A)
$$
IV. REWARDING STRATEGY FOR SYMBOLIC AUTOMATA

GOALS

In the following sections, we will describe a rewarding strategy for the product transition system that solves Problem 1 for the episodic reinforcement learning setting.

For an MDP $\mathcal{M} = (S, S_\text{init}, A, P, R)$, and a given specification automaton $A = (X, Q, q_\text{init}, F, \Delta, G)$, let $\mathcal{P} = \mathcal{M} \otimes A = (S_\otimes, s_\text{init}_\otimes, A, P_{\otimes}, \text{Acc})$ be the product MDP. Then, we define $R : S_\otimes \times S_\otimes \rightarrow \mathbb{R}$ be a reward function on $\mathcal{P}$ such that:

$$R((s, q), (s', q')) = \begin{cases} d_{\text{max}}, & \text{if } (s', q') \in \text{Acc} \\
 & \text{and } (s, q) \notin \text{Acc} \\
0, & \text{otherwise} \end{cases} \quad (7)$$

Given the reward function $R$, we claim that any policy $\pi$ that maximizes the expected total rewards using $R$ will also maximize the probability of satisfying the specification automaton $A$. Formally:

**Theorem 1.** Let $\pi_1$ and $\pi_2$ be some policies on $\mathcal{P}$ such that $V_{\pi_1}(s_\text{init}, q_\text{init}) > V_{\pi_2}(s_\text{init}, q_\text{init})$. Then, $\Pr(\pi_1 \models \mathcal{A}) > \Pr(\pi_2 \models \mathcal{A})$.

**Proof:** For some trajectory $\xi = ((s_0, q_0), \ldots, (s_N, q_N))$ in $\mathcal{P}^\pi$, let the total return for the trajectory be

$$G(\xi) = \sum_{t=0}^{N-1} R((s_t, q_t), (s_{t+1}, q_{t+1}))$$

Since $\mathcal{A}$ is terminally accepting, from $\text{Eq. } 7$ we can see that the set of all trajectories in $\mathcal{P}$ can be partitioned into two sets:

$$\{ \xi \mid G(\xi) = d_{\text{max}} \} \quad \text{and} \quad \{ \xi \mid G(\xi) = 0 \}$$

For notational convenience, we will use $V_{\pi_1}^\text{init}$ (and $V_{\pi_1}^*$) denote the state-value function of policy $\pi$ (and the optimal state-value function) for the initial state $(s_\text{init}, q_\text{init})$ in $\mathcal{P}$.

For some policy $\pi$, we know that

$$V_{\pi_1}^\text{init} = \mathbb{E}_{\xi \sim P_{\pi}} G(\xi) = \mathbb{E}_{\xi \sim P_{\pi}} \left[d_{\text{max}} \mathbb{1}(\xi \models \mathcal{A}) + \mathbb{E}_{\xi \sim P_{\pi}}[0 \mid \xi \models \mathcal{A}] \right] = d_{\text{max}} \mathbb{E}_{\xi \sim P_{\pi}}[1 \mathbb{1}(\xi \models \mathcal{A})] = d_{\text{max}} \Pr(\pi \models \mathcal{A})$$

Thus, if $V_{\pi_1}^\text{init} > V_{\pi_2}^\text{init}, \Pr(\pi_1 \models \mathcal{A}) > \Pr(\pi_2 \models \mathcal{A})$.

**Corollary 1.** Let $p^* = \max_\pi \Pr(\pi \models \mathcal{A})$ be the maximum probability of acceptance of a policy $\pi$ in $\mathcal{P}$, and let $\pi^* = \arg\max_\pi V^*(s_\text{init}, q_\text{init})$ be an optimal policy with respect to the reward function $R : S_\otimes \times S_\otimes$. Then, $\Pr(\pi^* \models \mathcal{A}) = p^*$.

While the reward in $\text{Eq. } 7$ provides theoretical guarantees for an optimal accepting policy, this reward is sparse, i.e., for large episode lengths or task horizons, the agent may not see any rewards from accepting runs in the early stages of its training. This can cause considerable slowdown in the training process, and can potentially make it unfeasible to use RL to synthesize such a controller.

To mitigate this, in the subsequent section will present a reward shaping $[13]$ technique that supplements spatial information to $R((s, q), (s', q'))$ and makes it dense.

A. Potential-based Reward Shaping

To speed up the training process, we need to make the reward function $R$ defined in $\text{Eq. } 7$ more dense, i.e., each transition in $\mathcal{P}$ needs to receive a reward such that:

1) The reward is positive only if the agent moves closer to the goal;
2) The shaped reward function does not alter the set of optimal policies.

To this end, we define a potential-based reward shaping method $[15]$ which uses a symbolic potential function that heuristically takes into account the shortest possible accepting trajectory from the current state in $\mathcal{P}$, solely by looking at the symbolic constraints in $\mathcal{A}$.

We will show that this shaped reward function follows some basic requirements for potential-based reward shaping such that any policy that optimizes this new reward remains optimal under the reward defined in $\text{Eq. } 7$.

**Definition 8** (Task Progress Level, $\eta$). Given a terminally accepting automaton $A = (X, Q, q_0, F, \Delta, \gamma)$, the task progress level is a mapping $\eta : Q \rightarrow \mathbb{N} \cup \{\infty\}$ such that $\eta(q)$ is the length of the shortest simple path from the state $q \in Q$ to the state $q_F \in F$. If there is no such path from $q$ to $q_F$, then $\eta(q) = \infty$.

Let $\{\psi(q, q')\} = \{s \mid s \in S \land s \models \psi(q, q')\}$ be the set of all $s \in S$ that satisfy the predicate $\psi(q, q')$, and let $d_H(\psi_1, \psi_2)$ be the Hausdorff distance between $[\psi_1]$ and $[\psi_2]$ using some distance measure $d$ in $S$.

**Definition 9** (Symbolic Subtask Progress, $\Phi_{\text{sym}}$). The quantity $\Phi_{\text{sym}} : \Delta \rightarrow \mathbb{R}_{\geq 0}$ is a heuristic approximation of distance between a sub-goal set and the final goal set such that, for a transition $(q, q') \in \Delta$,

$$\Phi_{\text{sym}}(q, q') = \begin{cases} 0 & \text{if } q' \in F \\
\min_{q'' \in \Delta} d_H([\psi(q, q')], [\psi(q', q'')]) + \Phi_{\text{sym}}(q', q'') & \text{otherwise} \end{cases} \quad (8)$$

**Definition 10** (Symbolic Potential Function, $\Phi$). Thus, given an MDP $\mathcal{M} = (S, s_0, A, P)$ and a symbolic automaton task specification $A$, we define the function $\Phi : S \times Q \rightarrow \mathbb{R}_{\geq 0}$ in the product MDP $\mathcal{P} = \mathcal{M} \otimes A$ as:

$$\Phi(s, q) = \begin{cases} 0 & \text{if } (s, q) \in \text{Acc} \\
\min_{q' \in \Delta, \eta(q') = \eta(q) + 1} \psi_{\text{d}}(s, \psi(q, q')) + \Phi_{\text{sym}}(q', q') & \text{otherwise} \end{cases} \quad (9)$$

To gain some intuition behind how this potential function works, we refer to $\text{Fig. } 1$. The quantity $\eta(\cdot)$ allows us to check if we are making progress in the specification automaton and disregard the transitions that don’t contribute to the symbolic progress measure $\Phi_{\text{sym}}(\cdot, \cdot)$. $\Phi_{\text{sym}}(\cdot, \cdot)$ computes a symbolic
Theorem 2 (Policy Invariance under Shaping [29]). Let \( p^* \) be the maximum probability of acceptance. Let \( \pi \) be a policy that maximizes expected total rewards with respect to the sparse reward function \( R \), and \( \hat{\pi} \) be one that does so with the potential-based reward shaping \( \hat{R} \). Then,\[
\Pr(\hat{\pi} \models \mathcal{A}) = \Pr(\pi \models \mathcal{A}) = p^*.
\]

Proof: From [29], we know that any reward-optimal policy remains consistent with respect to the value function \( V^*((s_{init}, q_{init})) \) in \( \mathcal{P} \). Since any policy remains consistent under reward-shaping, from Theorem 1 we can see that the policy will remain optimal with respect to the probability of acceptance.

In the subsequent section, we will empirically show the performance improvement gained by using the proposed potential-based reward shaping method.

V. Experiments

In the following case studies, we consider an agent moving through a discrete, grid environment, where the agent can use the actions \( A = \{\uparrow, \downarrow, \leftarrow, \rightarrow, \land, \lor, 0\} \) which allow also for diagonal movements and no movement (0). We model the probabilistic transition function \( P \) in the grid such that if the controller decides to move along a direction, it will move to the next state (if there is no wall) with probability \( 1 - p_{\text{slip}} \), or move along an adjacent direction with probability \( 0.5p_{\text{slip}} \). Here, \( p_{\text{slip}} \) is the probability of the agent “slipping”, and is set to 0.1.

We will compare the performance of our proposed symbolic potential-based reward (Eq. 10) against the sparse rewarding baseline (Eq. 7) and the potential-based reward presented in [25]. We will also compare this with the performance of the purely quantitative approach presented in [1] to contrast some situations where a quantitative method may outperform automata-based methods.

To do so, we will evaluate the training process for the tasks described below, and plot the probability of acceptance for the learned policy at different points in the training process. At fixed intervals, we evaluate the policy 100 times, and aggregate the results across 5 training runs with different random seeds. The probability of satisfaction is computed as the distribution of a binomial distribution with 95% confidence interval.

a) Reachability: Here, the task of the RL agent is to start at some initial location \((s_0, q_0)\) and reach some goal set \(A\) (represented in the automaton as a predicate). Specifically, we are interested in synthesizing a controller in a \(6 \times 6\) grid environment, where the agent starts at state \((0, 0)\) and needs to reach the states satisfying \(x \geq 4 \land y \geq 4\) with a hard deadline of 15 time steps. The efficacy of our approach can be seen in Fig. 3.

From [Fig. 3], we can see that the our proposed method for reward shaping is faster at finding a policy with acceptance probability equal to 1.0 than other methods. We see that the purely quantitative approach proposed in [1] is the next best solution, but suffers from poor stability in its results. Moreover, in this scenario, the sparse rewarding strategy is exactly as performant as the automaton potential-based reward shaping proposed in [25]. This is due to the fact that while a transition hasn’t been taken in the automaton, the potential function in [25] provides no extra information.

b) Recurrence: Based on an environment presented in [1], the goal of the agent in this task is to repeatedly visit two

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**Fig. 1.** Left: The symbolic automaton for a sequential specification for the agent, where the goal is to visit regions \(A, B,\) and \(C\) in order. Middle: The approximate “shortest path” for the agent to satisfy the specification from \(*\). Right: The evaluation of \(\Phi(s, q)\) from Eq. 9 such that the top image is for \(\Phi(s, q_0)\), the middle for \(\Phi(s, q_1)\), and the bottom for \(\Phi(s, q_2)\), for any \(s\).

**Fig. 2.** UP (\(\uparrow\)) transition for \(p_{\text{slip}} = 0.1\)
regions in the map as often as possible within a certain time limit. Here, the goal is to visit two regions in a $4 \times 4$ grid, labeled $x = 2 \land y = 2$ and $x = 1 \land y = 3$.

In this environment we can see that the approach presented in [25] performs poorly. This is due to the fact that in such specifications, using $\eta(\cdot)$ to compute the potential may mislead the agent into taking choices that are local maxima in the rewards. This is similar to the issue present in the “Branching Paths” task presented later.

On the other hand, increasing the task horizon (as defined in [11]) by a few time steps causes the $\tau$-MDP method to perform poorly due to a state-space explosion.

c) Sequential: This task requires an agent to visit regions in a strict sequence. In this example, the agent is placed in a $25 \times 25$ grid environment, with 3 labeled regions:

- $A = \{(x, y) \mid (x \geq 22) \land (y \geq 22)\}$
- $B = \{(x, y) \mid (x \geq 22) \land (y \leq 3)\}$
- $C = \{(x, y) \mid (x \leq 3) \land (y \geq 22)\}$

The goal of the agent is to learn a controller that visits the region A, then the region B, and finally the region C in sequence.

Note. Since the environment and the task horizon are considerably large (even for artificially bounded task specifications), the state space for the $\tau$-MDP construct presented in [11] explodes greatly. This made the experiments unfeasible to run with this method, and thus the results for this method are omitted from [Fig. 5].

d) Branching Paths: In this specification, the agent operates on a $16 \times 16$ grid with a few obstacles, as seen in [Fig. 6]. The goal of the agent is to visit regions in either of the following orders: $A \rightarrow B \rightarrow D$ or $C \rightarrow D$. From the figure, we can see that one of the above orders is significantly longer than the other, but since the agent does not have any prior knowledge of the environment (except for the locations of these regions), it cannot rule out either branch.

Note. For the same reasons as in the “Sequential” task, we do not evaluate the performance of the $\tau$-MDP approach presented in [11].

We can see from [Fig. 6] that our proposed method significantly outperforms the approach presented in [25]. (Note that due to size restrictions, we do not see that there is a small error band around the graph for [25] for when they do find some accepting trajectories.) Surprisingly, we see that the sparse rewarding case also performs well. In the following section, we will analyze this result in detail and show that in this case, the automaton structure can be misleading and lead the agent to local maximums in the optimization problem.

A. Case Study: Bounded Reach

First, we will look at the simple “reach” task, where the goal of the agent is to reach a region $A$ within some bounded time. We will look at a simplified version of the automaton in [Fig. 7]. Here, we can see that for the states $\{q_0, q_1, q_2, q_3\}$, the value for $\eta(\cdot)$ is all equal, i.e., $\eta(q) = 1$ for $q \in \{q_0, q_1, q_2, q_3\}$.

Paraphrasing the potential-based reward function proposed in [Eq. 9]:

In this task, we notice that the sparse reward baseline, along with the potential-based reward shaping presented in [25] do not learn any good information for 50000 training epochs. This is due to the incredibly sparse rewards provided by both the methods. Similar to the results in the “Bounded Reach” task presented earlier, the method in [25] does not provide any information to the agent until it enters the region corresponding to the next task.

On the other hand, our proposed method learns to find satisfying traces relatively quickly due to the information from the potential function in [Eq. 9].
B. Case Study: Branching Paths

Here, we will look at the “branching paths” reachability task presented earlier, represented by the symbolic automaton in Fig. 8.

Notice that for \( q \in \{ q_0, q_1 \} \), \( \eta(q) = 2 \), and \( \eta(q_2) = 1 \). Thus, under the reward strategy presented by [25] (see Eq. 11), the agent will get a positive reward only if it tries to take either the \( (q_1, q_2) \) transition into \( B \) or the \( (q_0, q_2) \) transition into \( C \). This means that the agent will favor to go to \( C \) and then \( D \) due to the automaton structure. But, we can see from Fig. 6 that picking the path to \( C \) will cause the agent to take a much longer path than necessary.

We notice that this problem is mitigated when using our proposed potential function, where both the paths in the automaton are equally favored (without the knowledge of the obstacles in the environment). This, in turn, prevents the agent from getting stuck in a local maximum. Similarly, in the sparse rewarding case, since all non-accepting paths are equally weighted, this local maximum is still mitigated, compared to the approach in [25].

VI. RELATED WORK

Reward engineering in RL based on formal specifications is a well-established research topic [1, 18, 8, 14, 35, 22, 25]. In [1] the authors define an effective approach for learning robust controllers using Q-learning. The history-based dependency of formula satisfaction is resolved by encoding n-step history in every state. The authors use bounded horizon robustness as a reward, which requires transforming MDP by enhancing it with n-step MDP history. A robustness-based approach to reward function was also taken in [5]. In [25], the authors propose the use of a terminal deterministic finite automaton (DFA) to encode task specifications over discrete labeled inputs, and define a state-based potential function on the automaton. Similarly, the authors of [22] propose the use of a custom specification language [21] to generate a similar DFA, but rather than learning a single controller for the entire specification, they propose to learn a controller for each “subtask” encoded on an edge in the form of some guard. These multiple controllers are then scheduled using the automaton structure as a guide. Here we describe a more general method based on task progress, independent on the specification language. Our choice of automata as our specification language was inspired by a general framework for calculating robustness w.r.t. a
Temporal Logic Policy Search was introduced in [26]. Finally, the existence of direct translation from several specification formalisms to automata [13, 20, 9, 10, 21] further supports our decision.

The problem of enabling Non-Markovian Rewards (NMRs) in RL can be understood as a problem of supporting history-based behavior in an inherently memory-less MDP. One of the approaches to supporting NMRs in RL is by introducing the Non-Markovian Reward Decision Process (NMRDP), which allows reward functions to span over a history of states [4]. Such NMRDPs can be transformed into a MDP which incorporates NMRs, previously translated into automata [11]. In [13], we see that obtaining the NMR function by using automata learning techniques comes at a cost of producing sufficient number of relevant traces to feed the learning algorithm.

A model-based technique for synthesizing MDP policies for LTL specifications was shown in [23], where the authors effectively reduce synthesis problem to mixed integer linear programming. A notion of task progress was used in [24] to generate policies for Co-Safe LTL specifications. A recent approach for LTL controller synthesis, which is also able to handle situations when a task cannot be satisfied is presented in [16]. A robustness-based method for model-free RL, called Temporal Logic Policy Search was introduced in [26]. Finally, in [2] the authors assign a “shield” (based on a safety automaton derived from LTL specifications) which helps guide the RL agent towards safe (and more efficient) learning.

VII. CONCLUSION

In this paper, we present a novel approach to using symbolic automata as task specifications to encode complex tasks. We show that, compared to other automata-based solutions, the reward function obtained from this symbolic task specification can encode rich, quantitative information about the environment. We present theoretical guarantees for the correctness of the constructed reward, and empirically compare the approach against related works.

In future work, we hope to 1) extend these results in episodic reinforcement learning to infinite horizon tasks; 2) provide guarantees for this approach in continuous space and continuous time settings; and 3) study how to construct robust plans for multi-agent systems with global and local tasks.

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