Gradient-Aware Model-based Policy Search

Pierluca D’Oro∗
Politecnico di Milano, Milan, Italy
pierluca.doro@mail.polimi.it

Alberto Maria Metelli∗
Politecnico di Milano, Milan, Italy
albertomaria.metelli@polimi.it

Andrea Tirinzoni
Politecnico di Milano, Milan, Italy
andrea.tirinzoni@polimi.it

Matteo Papini
Politecnico di Milano, Milan, Italy
matteo.papini@polimi.it

Marcello Restelli
Politecnico di Milano, Milan, Italy
marcello.restelli@polimi.it

Abstract

Traditional model-based reinforcement learning approaches learn a model of the environment dynamics without explicitly considering how it will be used by the agent. In the presence of misspecified model classes, this can lead to poor estimates, as some relevant available information is ignored. In this paper, we introduce a novel model-based policy search approach that exploits the knowledge of the current agent policy to learn an approximate transition model, focusing on the portions of the environment that are most relevant for policy improvement. We leverage a weighting scheme, derived from the minimization of the error on the model-based policy gradient estimator, in order to define a suitable objective function that is optimized for learning the approximate transition model. Then, we integrate this procedure into a batch policy improvement algorithm, named Gradient-Aware Model-based Policy Search (GAMPS), which iteratively learns a transition model and uses it, together with the collected trajectories, to compute the new policy parameters. Finally, we empirically validate GAMPS on benchmark domains analyzing and discussing its properties.

1 Introduction

Model-Based Reinforcement Learning (MBRL) [44, 32] approaches use the interaction data collected in the environment to estimate its dynamics, with the main goal of improving the sample efficiency of Reinforcement Learning (RL) [44] algorithms. However, trying to model the dynamics of the environment in a thorough way can be extremely complex and, thus, require the use of very powerful model classes and considerable amounts of data, betraying the original goal of MBRL. Fortunately, in many interesting application domains (e.g., robotics), perfectly modeling the dynamics across the whole state-action space is not necessary for a model to be effectively used by a learning agent [1, 32, 30]. Indeed, a wiser approach consists in using simpler model classes, whose estimation requires few interactions with the environment, and focus their limited capacity on the most relevant parts of the environment. These parts could present a local dynamics that is inherently simpler than the global one, or at least easier to model using prior knowledge.

The vast majority of MBRL methods employs a maximum likelihood estimation process for learning the model [12]. Nonetheless, the relative importance of the different aspects of the dynamics greatly

∗Equal contribution
depends on the underlying decision problem, on the control approach, and, importantly, on the policy played by the agent. For instance, suppose a learning agent acts deterministically in a certain region of the environment, possibly thanks to some prior knowledge, and has no interest in changing its behavior in that area; or that some regions of the state space are extremely unlikely to be reached by an agent following the current policy. There would be no benefit in approximating the corresponding aspects of the dynamics, since that knowledge cannot contribute to the agent’s learning process. Therefore, with a limited model expressiveness, an approach for model learning that explicitly accounts for the current policy and for how it will be improved can be more powerful than the traditional maximum likelihood estimation.

In this paper, motivated by these observations, we propose a model-based policy search [12, 45] method that leverages awareness of the current agent’s policy in the estimation of a forward model, used to perform policy optimization. Unlike existing approaches, which typically ignore all the knowledge available on the running policy during model estimation, we incorporate it into a weighting scheme for the objective function used in model learning. The contributions of this paper are theoretical, algorithmic and experimental. After having introduced our notation and the required mathematical preliminaries (Section 2), we formalize the concept of Model-Value-based Gradient (MVG), an approximation of the policy gradient that combines real trajectories along with a value function derived from an estimated model (Section 3). MVG allows finding a compromise between the large variance of a Monte Carlo gradient estimate and the bias of a full model-based estimator. Contextually, we present a bound on how the bias of the MVG is related to the choice of an estimated transition model. In Section 4, we derive from this bound an optimization problem to be solved, using samples, to obtain a gradient-aware forward model. Then, we integrate it into a batch policy optimization algorithm, named Gradient-Aware Model-based Policy Search (GAMPS), that iteratively uses samples to learn the approximate forward model and to estimate the gradient, used to perform the policy improvement step. After that, we present a finite-sample analysis for the single step of GAMPS (Section 5), that highlights the advantages of our approach when considering simple model classes. Finally, after reviewing related work in model-based policy search and decision-aware model learning areas (Section 6), we empirically validate GAMPS against common baselines, both model-based and model-free, and discuss its peculiar features in Section 7.

2 Preliminaries

A discrete-time Markov Decision Process (MDP) [40] is described by a tuple $\mathcal{M} = (S, A, r, p, \mu, \gamma)$, where $S$ is the space of possible states, $A$ is the space of possible actions, $r(s, a)$ is the reward received by executing action $a$ in state $s$, $p(\cdot|s, a)$ is the transition model that provides the distribution of the next state when performing action $a$ in state $s$, $\mu$ is the distribution of the initial state and $\gamma \in [0, 1]$ is a discount factor. When needed, we assume that $r$ is known, as common in domains where MBRL is employed (e.g., robotic learning [11]), and that rewards are uniformly bounded by $|r(s, a)| \leq R_{\text{max}} < +\infty$. The behavior of an agent is described by a policy $\pi(\cdot|s)$ that provides the distribution over the action space for every state $s$. Given a state-action pair $(s, a)$ we define the action-value function $Q(s, a)$, or Q-function, as

$$Q^\pi(s, a) = r(s, a) + \gamma \int_S p(s'|s, a) \int_A \pi(a'|s') Q^\pi(s', a') ds' da'$$

and the state-value function, or V-function, as $V^\pi(s) = \mathbb{E}_{a \sim \pi(\cdot|s)} [Q^\pi(s, a)]$, where we made explicit the dependence on the policy $\pi$ and on the transition model $p$. The goal of the agent is to find an optimal policy $\pi^*$, i.e., a policy that maximizes the expected return: $J^\pi = \mathbb{E}_{s_0 \sim \mu} [V^\pi(s_0)]$.

We consider a batch setting [28], in which the learning is performed on a previously collected dataset $D = \{\tau^i\}_{i=1}^N = \{(s_0^i, a_0^i, s_1^i, a_1^i, ..., s_{T_i-1}^i, a_{T_i-1}^i, s_{T_i}^i)\}_{i=1}^N$ of $N$ independent trajectories $\tau^i$, each composed of $T_i$ transitions, and further interactions with the environment are not allowed. The data is generated by an agent that interacts with the environment, following a known behavioral policy $\pi_b$. We are interested in learning a parameterized policy $\pi_\theta$ (for which we usually omit the parameter subscript in the notation) that belongs to a parametric space of stochastic policies $\Pi_\Theta = \{\pi_\theta : \theta \in \Theta \subseteq \mathbb{R}^d\}$. In this case, the gradient of the expected return w.r.t. $\theta$ is provided by the policy gradient theorem (PGT) [45, 44]:

$$\nabla_{\theta} J(\theta) = \frac{1}{1 - \gamma} \int_S \int_A \delta^\pi_{\mu}(s, a) \nabla_{\theta} \log \pi(a|s) Q^\pi(s, a) ds da,$$

where $\delta^\pi_{\mu}(s, a)$ is the $\gamma$-discounted state-action distribution [45], defined as $\delta^\pi_{\mu}(s, a) = (1 - \gamma)^t \sum_{i=0}^{\infty} \gamma^i \Pr(s_t = s, a_t = a|\mathcal{M}, \pi)$. We call $\nabla_{\theta} \log \pi(a|s)$ the score of the policy $\pi$ when ex-
executing action $a$ in state $s$. Furthermore, we denote with $\delta^{\pi, \hat{p}}_{s', a'}(s, a)$ the state-action distribution under policy $\pi$ and model $\hat{p}$ when the environment is deterministically initialized by executing action $a'$ in state $s'$ and with $\zeta^{\pi, \hat{p}}_{\tau}(\tau)$ the probability density function of a trajectory $\tau$. In batch policy optimization, the policy gradient is typically computed for a policy $\pi$ that is different from the policy $\pi_b$ having generated the data (off-policy estimation [39]). To correct for the distribution mismatch, we employ importance sampling [24, 35], re-weighting the trajectories based on the probability of being observed under the policy $\pi$. Namely, we define the importance weight relative to a subtrajectory $\tau_{t': t''}$ of $\tau$, occurring from time $t'$ to $t''$, and to policies $\pi$ and $\pi_b$ as

$$\rho_{\pi/\pi_b}(\tau_{t': t''}) = \frac{\zeta^{\pi, \hat{p}}_{\tau}(\tau_{t', t''})}{\zeta^{\pi_b, \hat{p}}_{\tau}(\tau_{t', t''})} = \prod_{t=t'}^{t''} \frac{\pi(a_t|s_t)}{\pi_b(a_t|s_t)},$$

3 Model-Value-based Gradient

Most model-based policy search approaches employ the learned forward model for generating rollouts, which are used to compute an improvement direction $\nabla_{\theta} J(\theta)$ either via likelihood-ratio methods or by propagating gradients through the model [20, 11]. Differently from these methods, we consider an approximation of the gradient, named Model-Value-based Gradient (MVG), defined as follows.

**Definition 3.1.** Let $p$ be the transition model of a Markov Decision Process $M$, $\Pi_\theta$ a parametric space of stochastic policies, $\mathcal{P}$ a class of transition models. Given $\pi \in \Pi_\theta$ and $\hat{p} \in \mathcal{P}$, the Model-Value-based Gradient (MVG) is defined as:

$$\nabla_{\theta}^{\text{MVG}} J(\theta) = \frac{1}{1-\gamma} \int_{S} \int_{A} \delta^{\pi, \hat{p}}(s, a) \nabla_{\theta} \log \pi(a|s) Q^{\pi, \hat{p}}(s, a) ds \, da. \tag{2}$$

Thus, the MVG employs experience collected in the real environment $\rho$, i.e., sampling from $\delta^{\pi, \hat{p}}(s, a)$, and uses the generative power of the estimated transition kernel $\hat{p}$ in the computation of an approximate state-action value function $Q^{\pi, \hat{p}}$ only. In this way, it is possible to find a compromise between a full model-based estimator, in which the experience is directly generated from $\delta^{\pi, \hat{p}}(s, a)$ [11, 12], and a Monte Carlo estimator (e.g., GPOMDP [4]), in which also the Q-function is computed from experience collected in the real environment. Therefore, the MVG limits the bias effect of $\hat{p}$ to the Q-function approximation $Q^{\pi, \hat{p}}$.

At the same time, it enjoys a smaller variance w.r.t. a Monte Carlo estimator, especially in an off-policy setting, as the Q-function is no longer estimated from samples but just approximated using $\hat{p}$. Existing approaches can be interpreted as MVG. For instance, the ones based on model-based value expansion [17, 8], that use a fixed-horizon unrolling of an estimated forward model for obtaining a better value function in an actor-critic setting.

A central question concerning Definition [3, 1] is how the choice of $\hat{p}$ affects the quality of the gradient approximation, i.e., how much bias an MVG introduces in the gradient approximation. To this end, we bound the approximation error by the expected KL-divergence between $p$ and $\hat{p}$.

**Theorem 3.2.** Let $q \in [1, +\infty]$ and $\hat{p} \in \mathcal{P}$. Then, the $L^q$-norm of the difference between the policy gradient $\nabla_{\theta} J(\theta)$ and the corresponding MVG $\nabla_{\theta}^{\text{MVG}} J(\theta)$ can be upper bounded as:

$$\|\nabla_{\theta} J(\theta) - \nabla_{\theta}^{\text{MVG}} J(\theta)\|_q \leq \frac{\sqrt{2Z R_{\max}}}{(1-\gamma)^2} \sqrt{\mathbb{E}_{s, a \sim \eta^{\pi, \hat{p}}_{\theta}} [D_{KL}(p(\cdot|s, a)||\hat{p}(\cdot|s, a))]},$$

where

$$\eta^{\pi, \hat{p}}_{\theta}(s, a) = \frac{1}{Z} \int_{S} \int_{A} \delta^{\pi, \hat{p}}(s', a') \| \nabla_{\theta} \log \pi_{\theta}(a'|s') \| q \delta^{\pi, \hat{p}}_{\mu}(s, a) ds' \, da'$$

is a probability distribution over $S \times A$ and $Z = \int_{S} \int_{A} \delta^{\pi, \hat{p}}(s', a') \| \nabla_{\theta} \log \pi_{\theta}(a'|s') \| q ds' \, da'$ is a normalization constant, both independent from $\hat{p}$.

Similarly to what was noted for other forms of decision-aware MBRL [16], a looser bound in which the expectation on the KL-divergence is taken under $\delta^{\pi, \hat{p}}_{\mu}$ can be derived (see Appendix [A, 2]). This motivates the common maximum likelihood approach. However, our bound is tighter, and clearly

---

2It is worth noting that when the environment dynamics can be approximated locally with a simple model or some prior knowledge on the environment is available, selecting a suitable approximator $\hat{p}$ for the transition model is easier than choosing an appropriate function approximator for a critic in an actor-critic architecture.

3We need to assume that $Z > 0$ in order for $\eta^{\pi, \hat{p}}_{\theta}$ to be well-defined. This is not a limitation, as if $Z = 0$, then $\nabla_{\theta} J(\theta) = 0$ and there is no need to define $\eta^{\pi, \hat{p}}_{\theta}$ in this case.
shows that not all collected transitions have the same relevance when learning a forward model that is used in estimating the MVG. Overall, the most important \((s, a)\) pairs to be considered are those that are likely to be reached from the policy starting from high gradient-magnitude state-action pairs.

4 Gradient-Aware Model-based Policy Search

Inspired by Theorem 3.2, we propose a policy search algorithm that employs an MVG approximation, combining trajectories generated in the real environment together with a model-based approximation of the Q-function obtained with the estimated transition model \(\hat{p}\). The algorithm, Gradient-Aware Model-based Policy Search (GAMPS), consists of three steps: learning the forward model \(\hat{p}\) (Section 4.1), computing the state-action value function \(Q^{\pi, \hat{p}}\) (Section 4.2) and updating the policy using the estimated gradient \(\hat{\nabla}_\theta J(\theta)\) (Section 4.3).

4.1 Learning the Transition Model

To learn \(\hat{p}\), we aim at minimizing the bound in Theorem 3.2 over a class of transition models \(\mathcal{P}\), using the trajectories \(D\) collected with \(\eta_{\pi, \hat{p}}\). However, to estimate an expected value computed over \(\eta_{\pi, \hat{p}}\), as in Theorem 3.2, we face two problems. First, the policy mismatch between the behavioral policy \(\pi_b\) used to collect \(D\) and the current agent’s policy \(\pi\). This can be easily addressed by using importance sampling. Second, given a policy \(\pi\) we need to be able to compute the expectations over \(\eta_{\pi, \hat{p}}\) using samples from \(\eta_{\pi, \hat{p}}\). In other words, we need to reformulate the expectation over \(\eta_{\pi, \hat{p}}\) in terms of expectation over trajectories. To this end, we provide the following result.

Lemma 4.1. Let \(\pi\) and \(\pi_b\) be two policies such that \(\pi \ll \pi_b\) (\(\pi\) is absolutely continuous w.r.t. to \(\pi_b\)). Let \(f : \mathcal{S} \times \mathcal{A} \to \mathbb{R}^k\) be an arbitrary function defined over the state-action space. Then, it holds that:

\[
\mathbb{E}_{s,a \sim \eta_{\pi, \hat{p}}}[f(s, a)] = \frac{(1 - \gamma)^2}{Z} \mathbb{E}_{\tau \sim \eta_{\pi, \hat{p}}} \left[ \sum_{l=0}^{\infty} \gamma^l \rho_{\pi/\pi_b}(\tau_{0:t}) \sum_{l=0}^{\infty} \| \nabla_\theta \log \pi(a_t|s_t) \|_q f(s_t, a_t) \right].
\]

To specialize Lemma 4.1 for our specific case, we just set \(f(s, a) = D_{KL}(p(\cdot|s, a)||\hat{p}(\cdot|s, a))\). Note that \(Z\) is independent from \(\hat{p}\) and thus it can be ignored in the minimization procedure. Furthermore, minimizing the KL-divergence is equivalent to maximizing the log-likeness of the observed transitions. Putting it all together, we get to the objective:

\[
\hat{p} = \arg \max_{p \in \mathcal{P}} \frac{1}{N} \sum_{i=1}^{N} \sum_{t=0}^{T_i-1} \omega_i^t \log p(s_{i+1}^t|s_i^t, a_i^t), \quad \omega_i^t = \gamma^t \rho_{\pi/\pi_b}(\tau_{0:t}) \sum_{l=0}^{t} \| \nabla_\theta \log \pi(a_t^l|s_t^l) \|_q.
\]

The factors contained in the weight \(\omega_i^t\) accomplish three goals in weighting the transitions for the model. The discount factor \(\gamma^t\) encodes that later transitions are exponentially less important in the gradient computation. The importance weight \(\rho_{\pi/\pi_b}(\tau_{0:t})\) is larger for the transitions that are more likely to be generated by the current policy \(\pi\). This incorporates a key consideration into model learning: since the running policy \(\pi\) can be quite different from the policy that generated the data \(\pi_b\), typically very explorative [12, 42], an accurate approximation of the dynamics for the regions that are rarely reached by the current policy is not useful. Lastly, the factor \(\sum_{t=0}^{T_i-1} \| \nabla_\theta \log \pi(a_t^l|s_t^l) \|_q\) prefers the transitions that occur at the end of a subtrajectory \(\tau_{0:t}\) with a high cumulative score-magnitude. This score accumulation resembles the expression of some model-free gradient estimators, such as G(PO)MDP [4]. Intuitively, the magnitude of the score of a policy is related to its opportunity to be improved, i.e., the possibility to change the probability of actions. Our gradient-aware weighting scheme encourages a better approximation of the dynamics for states and actions found in trajectories that can potentially lead to the most significant improvements to the policy.

4.2 Computing the value function

The estimated transition model \(\hat{p}\) can be used to compute the action-value function \(Q^{\pi, \hat{p}}\) for any policy \(\pi\). This amounts to evaluating the current policy using \(\hat{p}\) instead of the actual transition probability kernel \(p\). In the case of finite MDPs, the evaluation can be performed either in closed form or in an iterative manner via dynamic programming [5, 14]. For continuous MDPs, \(Q^{\pi, \hat{p}}\) cannot, in general, be represented exactly. A first approach consists of employing a parameterized function approximator \(\hat{Q}_\omega \in \mathcal{Q}\) and apply approximate dynamic programming [7]. However, this method requires a proper
Assumption 1. The second moment of \( \tau \) is finite. More precisely, we need the following assumptions.

1. \( \sum \tau \leq K \) for any \( \tau \).
2. \( \sum \tau s \leq K \) for any \( \tau \).
3. \( \sum \tau a \leq K \) for any \( \tau \).
4. \( \sum \tau s^2 \leq K \) for any \( \tau \).
5. \( \sum \tau a^2 \leq K \) for any \( \tau \).

This allows us to define the following functions. Let \( \bar{\tau} \) be a trajectory, \( \bar{\pi} \) be a policy, and \( \bar{\pi} \) be a probability distribution defined on \( \bar{\tau} \). We define the following functions:

\[ Q(s, a) = \sum_{i=1}^{N} \sum_{t=0}^{T-1} \gamma^t \rho(s_t, a_t) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) Q_{\bar{\pi}}(s_i, a_i) \]

\[ \nabla_{\theta} J(\theta) = \sum_{i=1}^{N} \sum_{t=0}^{T-1} \gamma^t \rho(s_t, a_t) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) Q_{\bar{\pi}}(s_i, a_i). \]

### 4.3 Estimating the policy gradient

After computing \( Q_{\bar{\pi}} \) (or some approximation \( \hat{Q} \)), all the gathered information can be used to improve policy \( \pi \). As we are using a model-value-based gradient, the trajectories we will use have been previously collected in the real environment. Furthermore, the data have been generated by a possibly different policy \( \pi_b \), and to account for the difference in the distributions, we need importance sampling again. Therefore, by writing the sample version of Equation (2) we obtain:

\[ \hat{Q}(s, a) = \frac{1}{N} \sum_{i=1}^{N} \sum_{t=0}^{T-1} \gamma^t \rho(s_t, a_t) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) Q_{\bar{\pi}}(s_i, a_i). \]

This approach has the advantage of avoiding the harsh choice of an appropriate model complexity \( M \) and the definition of the regression targets, while providing an unbiased estimate for the quantity of interest. More details on how to derive an approximation of \( Q_{\bar{\pi}} \) are reported in Appendix C.2.

### 5 Theoretical Analysis

In this section, we provide a finite-sample bound for the gradient estimation of Equation (5) assuming to have the exact value of \( Q_{\bar{\pi}} \). This corresponds to the analysis of a single iteration of GAMPS. We first define the following functions. Let \( \tau \) be a trajectory, \( \pi \in \Pi_\Theta \) and \( \bar{\pi} \in \bar{\Pi} \). We define \( l^{\pi, \bar{\pi}}(\tau) = \sum_{i=1}^{T} \omega_i \log \bar{\pi}(s_i, a_i) \) and \( g^{\pi, \bar{\pi}}(\tau) = \sum_{i=1}^{T} \gamma^i \rho(s_t, a_t) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) Q_{\bar{\pi}}(s_i, a_i). \) To obtain our result, we need the following assumptions.

**Assumption 1.** The second moment of \( l^{\pi, \bar{\pi}} \) and \( g^{\pi, \bar{\pi}} \) are uniformly bounded over \( \Pi_\Theta \) and \( \bar{\Pi}_\Theta \). In this case, given a dataset \( D = \{\tau_i\}_{i=1}^{N} \), there exist two constants \( c_1, c_2 \) such that:

\[ \sup_{\pi \in \Pi_\Theta} \sup_{\bar{\pi} \in \bar{\Pi}_\Theta} \max_{\tau_i} \left\{ \frac{1}{N} \sum_{i=1}^{N} l^{\pi, \bar{\pi}}(\tau_i)^2, \frac{1}{N} \sum_{i=1}^{N} g^{\pi, \bar{\pi}}(\tau_i)^2 \right\} \leq c_1^2, \]
Assumption 1 and 2, for any \( \delta \). We now revise prior work in MBRL, focusing on the methods that employ a policy search technique when it achieves a reasonable trade-off between the errors in approximation and estimation.

The pseudo-dimension of the hypothesis spaces \( \{P^p : p \in \mathcal{P}, \pi \in \Pi\} \) and \( \{g^p : \hat{p} \in \mathcal{P}, \pi \in \Pi\} \) are bounded by \( \upsilon < +\infty \).

Assumption 1 is requiring that the overall effect of the importance weight \( \rho_{s,a} \), the score \( \nabla_\theta \log \pi \) and the approximating transition model \( \hat{p} \) preserves the finiteness of the second moment. Clearly, a sufficient (albeit often unrealistic) condition is requiring all these quantities to be uniformly bounded (more details in Appendix A.4). Assumption 2 is necessary to state learning theory guarantees. We are now ready to present the main result, which employs the learning theory tools of [10].

**Theorem 5.1.** Let \( q \in [1, +\infty) \), \( d \) be the dimensionality of \( \Theta \) and \( \bar{p} \in \mathcal{P} \) be the maximizer of the objective function in Equation (3), obtained with \( N > 0 \) independent trajectories \( \{\tau^i\}_{i=1}^N \). Under Assumption 2, for any \( \delta \in (0, 1) \), with probability at least \( 1 - 4\delta \) it holds that:

\[
\| \nabla_\theta J(\theta) - \nabla_\theta J(\bar{\theta}) \|_q \leq 2\gamma \|Z\|_{\max} \inf_{p \in \mathcal{P}} \int_{s,a \sim \pi^p} [D_{KL}(p(\cdot|s,a)||\pi^p(\cdot|s,a))] + 2R_{\max} \left( \frac{\delta}{\epsilon} \epsilon + \frac{\gamma \|Z\|_{\max}}{1 - \gamma} \right),
\]

where \( \epsilon = \sqrt{\frac{\epsilon\log \frac{\epsilon}{\delta} N}{N} + \log \frac{2d(\epsilon+1)}{\delta}} \Gamma \left( \sqrt{\frac{\epsilon\log \frac{\epsilon}{\delta} N}{N} + \log \frac{2d(\epsilon+1)}{\delta}} \right) \) and \( \Gamma(\xi) := \frac{1}{2} + \sqrt{1 + 4 \log \frac{1}{\xi}} \).

The theorem justifies the intuition behind the gradient estimation based on MVG. A model \( \bar{p} \) is good when it achieves a reasonable trade-off between the errors in approximation and estimation. In the case of scarce data (i.e., small \( N \)), it is convenient to choose a low-capacity model class \( \mathcal{P} \) in order to reduce the error-enlarging effect the pseudo-dimension \( \upsilon \). However, this carries the risk of being unable to approximate the original model. Nonetheless, the approximation error depends on an expected value under \( \eta^p, \pi \). Even a model class that would be highly misspecified w.r.t. an expectation computed under the state-action distribution \( \delta^\pi \) can, perhaps surprisingly, lead to an accurate gradient estimation using our approach.

### 6 Related Works

We now revise prior work in MBRL, focusing on the methods that employ a policy search technique and those that include some level of awareness of the underlying control problem into model learning.

**Policy Search with MBRL**. The standard approach consists in using a maximum likelihood estimation of the environment dynamics to perform simulations (or imaginary rollouts) through which a policy can be improved without further or with limited interactions with the environment [12]. This approach has taken different forms, with the use of tabular models [47], least-squares density estimation techniques [46] or, more recently, combinations of variational generative models [26] and recurrent neural networks [21] employed in world models based on mixture density networks [19] or more complex video predictors [24]. Several methods incorporate the model uncertainty into policy updates, by using Gaussian processes and moment matching approximations [11], Bayesian neural networks [13] or ensembles of forward models [27][8][9]. MBRL works that are particularly related to GAMPS are those employing estimated forward models that are accurate only locally [11][33][20], or using a model-value based gradient formulation [17][8][20] as described in Section 6.3.

**Decision-aware MBRL**. The observation that, under misspecified model classes, the dynamics of the environment must be captured foreseeing the final task to be performed led to the development of decision-aware approaches for model learning [17]. While one of the first examples was a financial application [6], the idea was introduced into MBRL [22][8] and the related adaptive optimal control literature [33] by using actual evaluations of a control policy in the environment as a performance index for model learning. More similarly to our approach, but in the context of value-based methods, a theoretical framework called value-aware MBRL [16] was proposed, in which the model is estimated by minimizing the expected error on the Bellman operator, explicitly considering its actual use in the control algorithm. Starting from this, further theoretical considerations and approaches have been proposed, such as an iterative version of the original algorithm [15] and the equivalence, for

\[\text{It is worth noting that the estimation error is } \tilde{O}(N^{-\frac{1}{2}}).\]
Lipschitz MDPs, of value-awareness with the use of the Wasserstein metric as a loss for model learning [2]. Awareness of the final task to be performed has been also incorporated into stochastic dynamic programming [13] and, albeit implicitly, into neural network-based works [34, 43], in which value functions and models consistent with each other are learned.

7 Experiments

We now present an experimental evaluation of GAMPS, whose objective is two-fold: assessing the effect of our weighting scheme for model learning and comparing the performance in batch policy optimization of our algorithm against model-based and model-free policy search baselines.

7.1 Two-areas Gridworld

This experiment is meant to show how decision-awareness can be an effective tool to improve the accuracy of policy gradient estimates when using a forward model. The environment, depicted in Figure 1a, is a 5 × 5 gridworld, divided into two areas (lower and upper) with different dynamics: the effect of a movement action of the agent is reversed in one area w.r.t. the other. Once the agent gets to the lower area, it is not possible for it to go back in the upper one. We collect experience with a linear policy π_\hat{b} that is deterministic on the lower area and randomly initialized in the upper area, which is also used as initial policy for learning.

The first goal of this experiment is to show that, with the use of gradient-awareness, even an extremely simple model class can be sufficiently expressive to provide an accurate estimate of the policy gradient. Hence, we use a forward model which, given the sole action executed by the agent (i.e., without knowledge of the current state), predicts the effect that it will cause on the position of the agent (i.e., up, down, left, right, stay). This model class cannot perfectly represent the whole environment dynamics at the same time, as it changes between the two areas. However, given the nature of policy π, this is not necessary, since only the modeling of the upper area, which is indeed representable with our model, would be enough to perfectly improve the policy. Nonetheless, this useful information has no way of being captured using the usual maximum likelihood procedure, which, during model learning, weights the transitions just upon visitation, regardless the policy. To experimentally assess how our approach addresses this intuitive point, we generate 1000 trajectories running π_\hat{b} in the environment, and we first compare the maximum likelihood and the gradient-aware weighting factors, \( \delta_{\mu}^{\pi,p}(s,a) \) and \( \eta_{\mu}^{\pi,p}(s,a) \). The results (Figure 1b and 1c) show that our method is able, in a totally automatic way, not to assign importance to the transitions in which the policy cannot be improved.

We further investigate the performance of GAMPS compared to batch learning with the maximum likelihood transition model (ML) and two classical model-free learning algorithms REINFORCE [48] and PGT [45]. To adapt the latter two to the batch setting, we employ importance sampling in the same way as described in Equation (5), but estimating the Q-function using the same trajectories (and importance sampling as well). The results obtained by collecting different numbers of trajectories and evaluating on the environment are shown in Figure 2. When the data are too scarce, all compared...
algorithms struggle in converging towards good policies, experiencing high variance. It is worth noting that, with any amount of data we tested, the GAMPS learning curve is consistently above the others, showing superior performance considering the best iteration of any algorithm.

7.2 Minigolf

In the minigolf game, the agent hits a ball using a flat-faced golf club (the putter) with the goal of reaching a hole in the minimum number of strokes. This problem was originally proposed for RL in [29]. Here, we adapt the dynamics to the more realistic model developed by [36]. At the beginning of each trial, the ball is placed at random, between 2 m and 0 m far from the hole. Given only the distance to the hole, the agent chooses, at each step, the angular velocity of the putter which determines the next position of the ball. The episode terminates when the ball enters the hole, with reward 0, or when the agent overshoots, with reward −100. In all other cases, the reward is −1 and the agent can try other hits. We further suppose that the minigolf course is divided into two areas, one twice larger than the other, with different terrains: the first, nearest to the hole and biggest, has the friction of a standard track; the second has a very high friction, comparable to the one of an area with sand. Hence, the effect of each action significantly changes between these regions. We use Gaussian policies that are linear on six radial basis function features. The model predicts the difference from the previous state by sampling from a Gaussian distribution with parameterized mean and standard deviation, using the ball position as the only feature (detail in Appendix B.2).

We evaluate GAMPS against the same baselines employed for the previous experiments. We collect a dataset of 50 trajectories using an explorative policy. The results, shown in Figure 3, show that GAMPS is able to reach a good performance, corresponding to a policy that allows the ball to reach the hole most of the times. The other algorithms, instead, are prone to overfitting after the optimized policy has become too different from the one that generated the data.

8 Discussion and Conclusions

In this paper, we presented GAMPS, a batch gradient-aware model-based policy search algorithm. GAMPS leverages the knowledge about the policy that is being optimized for learning the transition model, by giving more importance to the aspects of the dynamics that are more relevant for improving its performance. We derived GAMPS from the minimization of the bias of the model-value-based

\[ \text{bias} = \mathbb{E}_{s,s'} \left[ (v_{s,a} - v_{s',a})^2 \right] \]

...
gradient, an approximation for the policy gradient that mixes trajectories collected in the real environment together with a value function computed with the estimated model. Our theoretical analysis validates the intuition that, when dealing with shallow models, it is convenient to focus their representation capabilities to the portions of the environment that are most crucial for improving the policy. The empirical validation demonstrates that, even when extremely simple model classes are considered, GAMPS is able to outperform comparable baselines. Future work could focus on adapting GAMPS to the interactive scenario, mixing on-policy and off-policy experience, in order to make GAMPS scale on complex high-dimensional environments.

References

[1] Pieter Abbeel, Morgan Quigley, and Andrew Y Ng. Using inaccurate models in reinforcement learning. In Proceedings of the 23rd international conference on Machine learning, pages 1–8. ACM, 2006.

[2] Kavosh Asadi, Evan Cater, Dipendra Misra, and Michael L Littman. Equivalence between wasserstein and value-aware model-based reinforcement learning. arXiv preprint arXiv:1806.01265, 2018.

[3] Somil Bansal, Roberto Calandra, Ted Xiao, Sergey Levine, and Claire J. Tomlin. Goal-driven dynamics learning via bayesian optimization. 2017 IEEE 56th Annual Conference on Decision and Control (CDC), pages 5168–5173, 2017.

[4] Jonathan Baxter and Peter L Bartlett. Infinite-horizon policy-gradient estimation. Journal of Artificial Intelligence Research, 15:319–350, 2001.

[5] Richard Bellman et al. The theory of dynamic programming. Bulletin of the American Mathematical Society, 60(6):503–515, 1954.

[6] Yoshua Bengio. Using a financial training criterion rather than a prediction criterion. International journal of neural systems, 8 4:433–43, 1997.

[7] Dimitri P Bertsekas, Dimitri P Bertsekas, Dimitri P Bertsekas, and Dimitri P Bertsekas. Dynamic programming and optimal control, volume 1. Athena scientific Belmont, MA, 1995.

[8] Jacob Buckman, Danijar Hafner, George Tucker, Eugene Brevdo, and Honglak Lee. Sample-efficient reinforcement learning with stochastic ensemble value expansion. In Advances in Neural Information Processing Systems, pages 8224–8234, 2018.

[9] Kurtland Chua, Roberto Calandra, Rowan McAllister, and Sergey Levine. Deep reinforcement learning in a handful of trials using probabilistic dynamics models. In Advances in Neural Information Processing Systems, pages 4754–4765, 2018.

[10] Corinna Cortes, Spencer Greenberg, and Mehryar Mohri. Relative deviation learning bounds and generalization with unbounded loss functions. arXiv preprint arXiv:1310.5796, 2013.

[11] Marc Deisenroth and Carl E Rasmussen. Pilco: A model-based and data-efficient approach to policy search. In Proceedings of the 28th International Conference on machine learning (ICML-11), pages 465–472, 2011.

[12] Marc Peter Deisenroth, Gerhard Neumann, Jan Peters, et al. A survey on policy search for robotics. Foundations and Trends® in Robotics, 2(1–2):1–142, 2013.

[13] Priya Donti, Brandon Amos, and J Zico Kolter. Task-based end-to-end model learning in stochastic optimization. In Advances in Neural Information Processing Systems, pages 5484–5494, 2017.

[14] Damien Ernst, Pierre Geurts, and Louis Wehenkel. Tree-based batch mode reinforcement learning. Journal of Machine Learning Research, 6(Apr):503–556, 2005.

[15] Amir-massoud Farahmand. Iterative value-aware model learning. In Advances in Neural Information Processing Systems, pages 9072–9083, 2018.
[16] Amir-massoud Farahmand, Andre Barreto, and Daniel Nikovski. Value-aware loss function for model-based reinforcement learning. In Artificial Intelligence and Statistics, pages 1486–1494, 2017.

[17] Vladimir Feinberg, Alvin Wan, Ion Stoica, Michael I Jordan, Joseph E Gonzalez, and Sergey Levine. Model-based value estimation for efficient model-free reinforcement learning. arXiv preprint arXiv:1803.00101, 2018.

[18] Yarin Gal, Rowan McAllister, and Carl Edward Rasmussen. Improving pilco with bayesian neural network dynamics models. In Data-Efficient Machine Learning workshop, ICML, volume 4, 2016.

[19] David Ha and Jürgen Schmidhuber. Recurrent world models facilitate policy evolution. In Advances in Neural Information Processing Systems, pages 2450–2462, 2018.

[20] Nicolas Heess, Gregory Wayne, David Silver, Timothy Lillicrap, Tom Erez, and Yuval Tassa. Learning continuous control policies by stochastic value gradients. In Advances in Neural Information Processing Systems, pages 2944–2952, 2015.

[21] Sepp Hochreiter and Jürgen Schmidhuber. Long short-term memory. Neural computation, 9(8):1735–1780, 1997.

[22] Joshua Mason Joseph, Alborz Geramifard, John W. Roberts, Jonathan P. How, and Nicholas Roy. Reinforcement learning with misspecified model classes. 2013 IEEE International Conference on Robotics and Automation, pages 939–946, 2013.

[23] Herman Kahn and Andy W Marshall. Methods of reducing sample size in monte carlo computations. Journal of the Operations Research Society of America, 1(5):263–278, 1953.

[24] Lukasz Kaiser, Mohammad Babaeizadeh, Piotr Milos, Blazej Osinski, Roy H Campbell, Konrad Czechowski, Dumitru Erhan, Chelsea Finn, Piotr Kozakowski, Sergey Levine, et al. Model-based reinforcement learning for atari. arXiv preprint arXiv:1903.00374, 2019.

[25] Diederik P. Kingma and Jimmy Ba. Adam: A method for stochastic optimization. In 3rd International Conference on Learning Representations, ICLR 2015, San Diego, CA, USA, May 7-9, 2015, Conference Track Proceedings, 2015.

[26] Diederik P. Kingma and Max Welling. Auto-Encoding Variational Bayes. arXiv:1312.6114 [cs, stat], December 2013.

[27] Thanard Kurutach, Ignasi Clavera, Yan Duan, Aviv Tamar, and Pieter Abbeel. Model-ensemble trust-region policy optimization. In International Conference on Learning Representations, 2018.

[28] Sascha Lange, Thomas Gabel, and Martin Riedmiller. Batch reinforcement learning. In Reinforcement learning, pages 45–73. Springer, 2012.

[29] Alessandro Lazaric, Marcello Restelli, and Andrea Bonarini. Reinforcement learning in continuous action spaces through sequential monte carlo methods. In Advances in neural information processing systems, pages 833–840, 2008.

[30] Sergey Levine and Pieter Abbeel. Learning neural network policies with guided policy search under unknown dynamics. In Advances in Neural Information Processing Systems, pages 1071–1079, 2014.

[31] Alberto Maria Metelli, Mirco Mutti, and Marcello Restelli. Configurable Markov decision processes. In Jennifer Dy and Andreas Krause, editors, Proceedings of the 35th International Conference on Machine Learning, volume 80 of Proceedings of Machine Learning Research, pages 3488–3497, Stockholmsmässan, Stockholm Sweden, 10–15 Jul 2018. PMLR.

[32] Duy Nguyen-Tuong and Jan Peters. Model learning for robot control: a survey. Cognitive processing, 12(4):319–340, 2011.

[33] Duy Nguyen-Tuong, Matthias Seeger, and Jan Peters. Model learning with local gaussian process regression. Advanced Robotics, 23(15):2015–2034, 2009.
[34] Junhyuk Oh, Satinder Singh, and Honglak Lee. Value prediction network. In Advances in Neural Information Processing Systems, pages 6118–6128, 2017.

[35] Art B. Owen. Monte Carlo theory, methods and examples. 2013.

[36] AR Penner. The physics of putting. Canadian Journal of Physics, 80(2):83–96, 2002.

[37] Jan Peters and Stefan Schaal. Reinforcement learning by reward-weighted regression for operational space control. In Proceedings of the 24th international conference on Machine learning, pages 745–750. ACM, 2007.

[38] L. Piroddi and W. Spinelli. An identification algorithm for polynomial narx models based on simulation error minimization. International Journal of Control, 76(17):1767–1781, 2003.

[39] Doina Precup, Richard S. Sutton, and Satinder P. Singh. Eligibility traces for off-policy policy evaluation. In Pat Langley, editor, Proceedings of the Seventeenth International Conference on Machine Learning (ICML 2000), Stanford University, Stanford, CA, USA, June 29 - July 2, 2000, pages 759–766. Morgan Kaufmann, 2000.

[40] Martin L. Puterman. Markov decision processes: discrete stochastic dynamic programming. John Wiley & Sons, 2014.

[41] Martin Riedmiller. Neural fitted q iteration—first experiences with a data efficient neural reinforcement learning method. In European Conference on Machine Learning, pages 317–328. Springer, 2005.

[42] Stéphane Ross and J Andrew Bagnell. Agnostic system identification for model-based reinforcement learning. In Proceedings of the 29th International Conference on International Conference on Machine Learning, pages 1905–1912. Omnipress, 2012.

[43] David Silver, Hado van Hasselt, Matteo Hessel, Tom Schaul, Arthur Guez, Tim Harley, Gabriel Dulac-Arnold, David Reichert, Neil Rabinowitz, Andre Barreto, et al. The predictron: End-to-end learning and planning. In Proceedings of the 34th International Conference on Machine Learning-Volume 70, pages 3191–3199. JMLR. org, 2017.

[44] Richard S Sutton and Andrew G Barto. Reinforcement learning: An introduction. 2018.

[45] Richard S Sutton, David A McAllester, Satinder P Singh, and Yishay Mansour. Policy gradient methods for reinforcement learning with function approximation. In Advances in Neural Information Processing Systems, pages 1057–1063, 2000.

[46] Voot Tangkaratt, Syogo Mori, Tingting Zhao, Jun Morimoto, and Masashi Sugiyama. Model-based policy gradients with parameter-based exploration by least-squares conditional density estimation. Neural networks, 57:128–140, 2014.

[47] Xin Wang and Thomas G Dietterich. Model-based policy gradient reinforcement learning. In Proceedings of the 20th International Conference on Machine Learning (ICML-03), pages 776–783, 2003.

[48] Ronald J Williams. Simple statistical gradient-following algorithms for connectionist reinforcement learning. Machine learning, 8(3-4):229–256, 1992.
A Proofs and Derivations

In this appendix, we report the proofs of the results presented in the main paper, together with some additional results and extended discussion.

A.1 Proofs of Section 3

The following lemma is used in proving Theorem 3.2.

**Lemma A.1.** Considering the state-action distributions \( \delta^{\pi,p}_{\mu} \) and \( \delta^{\pi,\hat{\pi}}_{\mu} \) under policy \( \pi \) and models \( p \) and \( \hat{\pi} \), the following upper bound holds:

\[
\| \delta^{\pi,p}_{\mu} - \delta^{\pi,\hat{\pi}}_{\mu} \|_1 \leq \frac{\gamma}{1 - \gamma} \mathbb{E}_{s,a \sim \delta^{\pi,p}_{\mu}} \left[ \| p(\cdot|s,a) - \hat{\pi}(\cdot|s,a) \|_1 \right].
\]

**Proof.** Recalling that \( \delta^{\pi,p}_{\mu}(s,a) = \pi(a|s)d^{\pi,p}_{\mu}(s) \) we can write:

\[
\begin{align*}
\| \delta^{\pi,p}_{\mu} - \delta^{\pi,\hat{\pi}}_{\mu} \|_1 &= \int_S \int_A \left| \delta^{\pi,\hat{\pi}}_{\mu}(s,a) - \delta^{\pi,p}_{\mu}(s,a) \right| \, ds \, da \\
&= \int_S \int_A \pi(a|s) \left| d^{\pi,\hat{\pi}}_{\mu}(s) - d^{\pi,p}_{\mu}(s) \right| \, ds \, da \\
&= \int_S \left( d^{\pi,\hat{\pi}}_{\mu}(s) - d^{\pi,p}_{\mu}(s) \right) \int_A \pi(a|s) \, da \\
&= \int_S \left( d^{\pi,\hat{\pi}}_{\mu}(s) - d^{\pi,p}_{\mu}(s) \right) \, ds \leq \left\| d^{\pi,\hat{\pi}}_{\mu} - d^{\pi,p}_{\mu} \right\|_1,
\end{align*}
\]

where \( d^{\pi,\hat{\pi}}_{\mu}(s) = (1 - \gamma) \sum_{t=0}^{\infty} \gamma^t \Pr(s_t = s|\mathcal{M}, \pi). \) In order to bound \( \| d^{\pi,\hat{\pi}}_{\mu} - d^{\pi,p}_{\mu} \|_1 \), we can use Corollary 3.1 from [31]:

\[
\left\| d^{\pi,\hat{\pi}}_{\mu} - d^{\pi,p}_{\mu} \right\|_1 \leq \frac{\gamma}{1 - \gamma} \mathbb{E}_{s,a \sim \delta^{\pi,p}_{\mu}} \left[ \| p(\cdot|s,a) - \hat{\pi}(\cdot|s,a) \|_1 \right].
\]

Now, we can prove Theorem 3.2.

**Theorem 3.2.** Let \( q \in [1, +\infty) \) and \( \hat{\pi} \in \mathcal{P}. \) Then, the \( L^q \)-norm of the difference between the policy gradient \( \nabla_\theta J(\theta) \) and the corresponding MVG \( \nabla^\text{MVG}_\theta J(\theta) \) can be upper bounded as:

\[
\| \nabla_\theta J(\theta) - \nabla^\text{MVG}_\theta J(\theta) \|_q \leq \frac{\gamma \sqrt{2} R_{\max}}{(1 - \gamma)^2} \sqrt{\mathbb{E}_{s,a \sim \delta^{\pi,p}_{\mu}} \left[ D_{KL}(p(\cdot|s,a)||\hat{\pi}(\cdot|s,a)) \right]},
\]

where

\[
\eta^{\pi,p}_{\mu}(s,a) = \frac{1}{Z} \int_S \int_A \delta^{\pi,\hat{\pi}}(s',a') \| \nabla_\theta \log \pi_{\theta}(a'|s') \|_q \delta^{\pi,p}_{\mu}(s,a) \, ds' \, da'
\]

is a probability distribution over \( S \times A \) and \( Z = \int_S \int_A \delta^{\pi,\hat{\pi}}(s',a') \| \nabla_\theta \log \pi_{\theta}(a'|s') \|_q \, ds' \, da' \) is a normalization constant, both independent from \( \hat{\pi}. \)

**Proof.**

\[
\begin{align*}
\| \nabla_\theta J(\theta) - \nabla^\text{MVG}_\theta J(\theta) \|_q &= \left\| \frac{1}{1 - \gamma} \int_S \int_A \delta^{\pi,p}_{\mu}(s,a)(Q^{\pi,p}(s,a) - Q^{\pi,\hat{\pi}}(s,a)) \nabla_\theta \log \pi(\cdot|s,a) \, ds \, da \right\|_q \\
&\leq \frac{1}{1 - \gamma} \int_S \int_A \delta^{\pi,p}_{\mu}(s,a) \left| Q^{\pi,p}(s,a) - Q^{\pi,\hat{\pi}}(s,a) \right| \| \nabla_\theta \log \pi(\cdot|s,a) \|_q \, ds \, da \\
&= \frac{Z}{1 - \gamma} \int_S \int_A \delta^{\pi,p}_{\mu}(s,a) \left| Q^{\pi,p}(s,a) - Q^{\pi,\hat{\pi}}(s,a) \right| \, ds \, da \\
&= \frac{Z}{1 - \gamma} \int_S \int_A \delta^{\pi,p}_{\mu}(s,a) \left| \int_S \int_A r(s',a')(\delta^{\pi,p}_{\mu}(s',a') - \delta^{\pi,\hat{\pi}}_{\mu}(s',a')) \, ds' \, da' \right| \, ds \, da \\
&\leq \frac{Z R_{\max}}{1 - \gamma} \int_S \int_A \delta^{\pi,p}_{\mu}(s,a) \left| \int_S \int_A \delta^{\pi,\hat{\pi}}_{\mu}(s',a') - \delta^{\pi,p}_{\mu}(s',a') \right| \, ds' \, da' \, ds \, da \\
&\leq \frac{Z R_{\max}}{1 - \gamma} \int_S \int_A \delta^{\pi,p}_{\mu}(s,a) \left| \delta^{\pi,\hat{\pi}}_{\mu} - \delta^{\pi,p}_{\mu} \right|_1 \, ds \, da.
\end{align*}
\]
We now show that maximum-likelihood model estimation is a sound way of estimating the policy
where in Equation (7), we define a new probability distribution

\[ \eta^\pi_{\mu^p}(s, a) = \frac{1}{Z} \delta^\pi_{\mu^p}(s, a) \parallel \nabla_\theta \log \pi(a|s) \parallel_q \]

by means of an appropriate normalization constant \( Z \), assumed \( Z > 0 \). In Equation (8), we use the definition of Q-function as \( Q^\pi_{\mu^p}(s, a) = \int_S \delta^\pi_{\mu^p}(s', a') r(s', a') ds' da' \). After bounding the reward in Equation (9), in Equation (10) we apply Lemma A.1. Then we obtain Equation (11) by employing Pinsker’s inequality, defining

\[ J_\theta = \int_A \int_S \eta^\pi_{\mu^p}(s, a) \sqrt{2D_{KL}(p(\cdot|s,a)||\tilde{p}(\cdot|s,a))} dsda \]

where in Equation (11), we define a new probability distribution \( \eta^\pi_{\mu^p}(s, a) = \frac{1}{Z} \delta^\pi_{\mu^p}(s, a) \parallel \nabla_\theta \log \pi(a|s) \parallel_q \)

In order to understand how the weighting distribution \( \eta^\pi_{\mu^p}(s, a) \) enlarges the relative importance of some transitions with respect to others, we can use the auxiliary distribution \( \eta^\pi_{\mu^p}(s', a') = \frac{1}{Z} \parallel \nabla_\theta \log \pi(a'|s') \parallel_q \delta^\pi_{\mu^p}(s', a') \)
and rewrite the weighting distribution as:

\[ \eta^\pi_{\mu^p}(s, a) = \int_A \int_S \eta^\pi_{\mu^p}(s', a') \delta^\pi_{\mu^p}(s', a') \parallel \nabla_\theta \log \pi(a'|s') \parallel_q ds'da'. \]

Intuitively, \( \eta^\pi_{\mu^p}(s', a') \) is high for states and actions that are both likely to be visited executing \( \pi \) and corresponding to high norm of its score; \( \delta^\pi_{\mu^p}(s, a) \) is the state-action distribution of \( (s, a) \) after executing action \( a' \) in state \( s' \). Each state-action couple \( (s', a') \) with high score magnitude that precedes \( (s, a) \) brings a contribution to the final weighting factor for \( (s, a) \).

### A.2 Gradient-Unaware Model Learning

We now show that maximum-likelihood model estimation is a sound way of estimating the policy

\[ \parallel \nabla_\theta J(\theta) - \nabla_\theta \text{MVG} J(\theta) \parallel_q \leq K \]

where in Equation (11), we define a new probability distribution \( \eta^\pi_{\mu^p}(s, a) = \frac{1}{Z} \delta^\pi_{\mu^p}(s, a) \parallel \nabla_\theta \log \pi(a|s) \parallel_q \)

Then we obtain Equation (12) by employing Pinsker’s inequality, defining

\[ J_\theta = \int_A \int_S \eta^\pi_{\mu^p}(s, a) \sqrt{2D_{KL}(p(\cdot|s,a)||\tilde{p}(\cdot|s,a))} dsda \]

where in Equation (11), we define a new probability distribution \( \eta^\pi_{\mu^p}(s, a) = \frac{1}{Z} \delta^\pi_{\mu^p}(s, a) \parallel \nabla_\theta \log \pi(a|s) \parallel_q \)

Intuitively, \( \eta^\pi_{\mu^p}(s', a') \) is high for states and actions that are both likely to be visited executing \( \pi \) and corresponding to high norm of its score; \( \delta^\pi_{\mu^p}(s, a) \) is the state-action distribution of \( (s, a) \) after executing action \( a' \) in state \( s' \). Each state-action couple \( (s', a') \) with high score magnitude that precedes \( (s, a) \) brings a contribution to the final weighting factor for \( (s, a) \).

### A.2 Gradient-Unaware Model Learning

We now show that maximum-likelihood model estimation is a sound way of estimating the policy

\[ \parallel \nabla_\theta J(\theta) - \nabla_\theta \text{MVG} J(\theta) \parallel_q \leq K \]

where in Equation (11), we define a new probability distribution \( \eta^\pi_{\mu^p}(s, a) = \frac{1}{Z} \delta^\pi_{\mu^p}(s, a) \parallel \nabla_\theta \log \pi(a|s) \parallel_q \)

Intuitively, \( \eta^\pi_{\mu^p}(s', a') \) is high for states and actions that are both likely to be visited executing \( \pi \) and corresponding to high norm of its score; \( \delta^\pi_{\mu^p}(s, a) \) is the state-action distribution of \( (s, a) \) after executing action \( a' \) in state \( s' \). Each state-action couple \( (s', a') \) with high score magnitude that precedes \( (s, a) \) brings a contribution to the final weighting factor for \( (s, a) \).

### A.2 Gradient-Unaware Model Learning

We now show that maximum-likelihood model estimation is a sound way of estimating the policy

\[ \parallel \nabla_\theta J(\theta) - \nabla_\theta \text{MVG} J(\theta) \parallel_q \leq K \]

where in Equation (11), we define a new probability distribution \( \eta^\pi_{\mu^p}(s, a) = \frac{1}{Z} \delta^\pi_{\mu^p}(s, a) \parallel \nabla_\theta \log \pi(a|s) \parallel_q \)

Intuitively, \( \eta^\pi_{\mu^p}(s', a') \) is high for states and actions that are both likely to be visited executing \( \pi \) and corresponding to high norm of its score; \( \delta^\pi_{\mu^p}(s, a) \) is the state-action distribution of \( (s, a) \) after executing action \( a' \) in state \( s' \). Each state-action couple \( (s', a') \) with high score magnitude that precedes \( (s, a) \) brings a contribution to the final weighting factor for \( (s, a) \).

### A.2 Gradient-Unaware Model Learning

We now show that maximum-likelihood model estimation is a sound way of estimating the policy

\[ \parallel \nabla_\theta J(\theta) - \nabla_\theta \text{MVG} J(\theta) \parallel_q \leq K \]

where in Equation (11), we define a new probability distribution \( \eta^\pi_{\mu^p}(s, a) = \frac{1}{Z} \delta^\pi_{\mu^p}(s, a) \parallel \nabla_\theta \log \pi(a|s) \parallel_q \)

Intuitively, \( \eta^\pi_{\mu^p}(s', a') \) is high for states and actions that are both likely to be visited executing \( \pi \) and corresponding to high norm of its score; \( \delta^\pi_{\mu^p}(s, a) \) is the state-action distribution of \( (s, a) \) after executing action \( a' \) in state \( s' \). Each state-action couple \( (s', a') \) with high score magnitude that precedes \( (s, a) \) brings a contribution to the final weighting factor for \( (s, a) \).
where we started from Theorem 3.2. Equation (15) follows from the fact that $\int \delta_{\mu, p}(s', a') \delta_{\mu, p}(s, a) ds' da' = \delta_{\mu, p}(s, a)$, as we are actually recomposing the state-action distribution that was split at $(s', a')$ and Equation (16) is obtained by observing that $Z \leq K$.

Therefore, the bound is less tight than the one presented in Theorem 3.2. We can in fact observe that

$$
\|\nabla \theta J(\theta) - \nabla \theta^{TVG} J(\theta)\|_q \leq \frac{\gamma \sqrt{2Z R_{\text{max}}}}{(1 - \gamma)^2} \frac{\mathbb{E}_{s, a \sim \eta_{\mu, p}} [D_{KL}(p(\cdot | s, a) || \tilde{p}(\cdot | s, a))]}{\mathbb{E}_{s, a \sim \delta_{\mu, p}} [D_{KL}(p(\cdot | s, a) || \tilde{p}(\cdot | s, a))]}. 
$$

This reflects the fact that the standard approach for model learning in MBRL does not make use of all the available information, in this case related to the gradient of the current agent policy.

### A.3 Proofs of Section 4

We start introducing the following lemma that states that taking expectations w.r.t. $\delta_{\mu, p}$ is equivalent to taking proper expectations w.r.t. $\zeta_{\mu, p}$.

**Lemma A.2.** Let $f : S \times A \to \mathbb{R}^k$ an arbitrary function defined over the state-action space. Then, it holds that:

$$
\mathbb{E}_{s, a \sim \delta_{\mu, p}} [f(s, a)] = (1 - \gamma) \sum_{t=0}^{+\infty} \gamma^t \mathbb{E}_{\tau_0 \sim \tau_{||} \sim \zeta_{\mu, p}} [f(s_t, a_t)] = (1 - \gamma) \sum_{t=0}^{+\infty} \gamma^t \mathbb{E}_{\tau \sim \zeta_{\mu, p}} [f(s_t, a_t)].
$$

**Proof.** We denote with $T$ the set of all possible trajectories. We just apply the definition of $\delta_{\mu, p}$ [45]:

$$
\mathbb{E}_{s, a \sim \delta_{\mu, p}} [f(s, a)] = \int_s \int_a \delta_{\mu, p}(s, a) f(s, a) ds da
$$

$$
= (1 - \gamma) \sum_{t=0}^{+\infty} \gamma^t \int_s \int_a \mathbb{P}(s_t = s, a_t = a | \mathcal{M}, \pi) f(s, a) ds da
$$

$$
= (1 - \gamma) \sum_{t=0}^{+\infty} \gamma^t \int_T \int_s \int_a \mathbb{Z}_{\mu, p}(\tau_0, t) \mathbb{1}(s_t = s, a_t = a) f(s, a) ds da d\tau_0 t
$$

$$
= (1 - \gamma) \sum_{t=0}^{+\infty} \gamma^t \int_T \int_s \mathbb{Z}_{\mu, p}(\tau_0, t) d\tau_0 t
$$

$$
= (1 - \gamma) \sum_{t=0}^{+\infty} \gamma^t f(s_t, a_t) d\tau,
$$

where we exploited the fact that the probability $\mathbb{P}(s_t = s, a_t = a | \mathcal{M}, \pi)$ is equal to the probability that a prefix of trajectory $\tau_{t-1}$ terminates in $(s_t, a_t)$, i.e., $\int_T \mathbb{Z}_{\mu, p}(\tau_0, t) d\tau_0 t$. The last passage follows from the fact that $f(s_t, a_t)$ depends on random variables realized at time $t$ we can take the expectation over the whole trajectory.

We can apply this result to rephrase the expectation w.r.t. $\eta_{\mu, p}$ as an expectation w.r.t. $\zeta_{\mu, p}$.

**Lemma A.3.** Let $f : S \times A \to \mathbb{R}^k$ an arbitrary function defined over the state-action space. Then, it holds that:

$$
\mathbb{E}_{s, a \sim \eta_{\mu, p}} [f(s, a)] = \left(1 - \gamma \right)^2 \mathbb{E}_{\tau \sim \zeta_{\mu, p}} \left[ \sum_{t=0}^{+\infty} \gamma^t \sum_{l=0}^{+\infty} \|\nabla \theta \log \pi(a_t | s_t)\|_q f(s_t, a_t) \right].
$$

**Proof.** We just need to apply Lemma A.2 twice and exploit the definition of $\eta_{\mu, p}$:

$$
\mathbb{E}_{s, a \sim \eta_{\mu, p}} [f(s, a)] = \int_s \int_a \eta_{\mu, p}(s, a) f(s, a) ds da
$$
We are now ready to prove Lemma 4.1.

Let us first focus on the expectation w.r.t. \( \delta_{s,t}^{\pi_p}(s', a') \). By applying Lemma A.2 with \( f(s', a') = \| \nabla_\theta \log \pi(a'|s') \|_q \delta_{s', a'}^{\pi_p}(s, a) ds'da' \), we have:

\[
\int_S \int_A \int_A \delta_{s,t}^{\pi_p}(s', a') \| \nabla_\theta \log \pi(a'|s') \|_q \delta_{s', a'}^{\pi_p}(s, a) ds'da' = (1 - \gamma) \sum_{t=0}^{+\infty} \gamma^t \int_T \zeta_{s,t}^{\pi_p}(\tau_{0:t}) \| \nabla_\theta \log \pi(a_t|s_t) \|_q \delta_{s_t, a_t}^{\pi_p}(s, a) d\tau_{0:t}.
\]

Now, let us consider \( \delta_{s,t}^{\pi_p}(s, a) \). We instantiate again Lemma A.2

\[
\mathbb{E}_{s,a \sim \eta_{\pi_p}^{\pi}} [f(s, a)] = \frac{(1 - \gamma)}{Z} \int_S \int_A \int_T \sum_{t=0}^{+\infty} \gamma^t \int_T \zeta_{s,t}^{\pi_p}(\tau_{0:t}) \| \nabla_\theta \log \pi(a_t|s_t) \|_q \delta_{s_t, a_t}^{\pi_p}(s, a) f(s, a) d\tau_{0:t} ds d\tau.
\]

where the last passage derives from observing that, for each \( t \) and \( l \) we are computing an integral over the trajectory prefixes of length \( h := t + l \) and observing that \( (s_l, a_l) \) can be seen as the \( h \)-th state-action pair of a trajectory \( \tau \sim \eta_{\pi_p}^{\pi} \). We now rearrange the summations:

\[
\sum_{t=0}^{+\infty} \| \nabla_\theta \log \pi(a_t|s_t) \|_q \sum_{h=0}^{+\infty} \gamma^h f(s_h, a_h) = \sum_{h=0}^{+\infty} \gamma^h f(s_h, a_h) \sum_{t=0}^{+\infty} \| \nabla_\theta \log \pi(a_t|s_t) \|_q.
\]

By changing the names of the indexes of the summations, we get the result.

We are now ready to prove Lemma 4.1

**Lemma 4.1.** Let \( \pi \) and \( \pi_b \) be two policies such that \( \pi \ll \pi_b \) (\( \pi \) is absolutely continuous w.r.t. to \( \pi_b \)). Let \( f : S \times A \rightarrow \mathbb{R}^b \) be an arbitrary function defined over the state-action space. Then, it holds that:

\[
\mathbb{E}_{s,a \sim \eta_{\pi}^{\pi}} [f(s, a)] = \frac{(1 - \gamma)}{Z} \mathbb{E}_{\tau \sim \eta_{\pi}^{\pi}} \left[ \sum_{t=0}^{+\infty} \gamma^t \rho_{\pi \ll \pi_b}(\tau_{0:t}) \frac{1}{t} \sum_{l=0}^{t} \| \nabla_\theta \log \pi(a_t|s_t) \|_q f(s_t, a_t) \right].
\]

**Proof.** What changes w.r.t. Lemma A.2 is that we are now interested in computing the expectation w.r.t. to a target policy \( \pi \) while trajectories are collected with a behavioral policy \( \pi_b \), fulfilling the hypothesis stated in the lemma. We start from Lemma 4.1 and we just need to apply importance weighting [15].

\[
\mathbb{E}_{\tau \sim \eta_{\pi_b}^{\pi}} \left[ \sum_{t=0}^{+\infty} \gamma^t \frac{1}{t} \sum_{l=0}^{t} \| \nabla_\theta \log \pi(a_t|s_t) \|_q f(s_t, a_t) \right] = \sum_{t=0}^{+\infty} \gamma^t \mathbb{E}_{\tau \sim \eta_{\pi_b}^{\pi}} \left[ \sum_{l=0}^{t} \| \nabla_\theta \log \pi(a_t|s_t) \|_q f(s_t, a_t) \right] = \sum_{t=0}^{+\infty} \gamma^t \mathbb{E}_{\tau \sim \eta_{\pi_b}^{\pi}} \left[ \rho_{\pi \ll \pi_b}(\tau_{0:t}) \frac{1}{t} \sum_{l=0}^{t} \| \nabla_\theta \log \pi(a_t|s_t) \|_q f(s_t, a_t) \right] = \mathbb{E}_{\tau \sim \eta_{\pi_b}^{\pi}} \left[ \sum_{t=0}^{+\infty} \gamma^t \rho_{\pi \ll \pi_b}(\tau_{0:t}) \sum_{l=0}^{t} \| \nabla_\theta \log \pi(a_t|s_t) \|_q f(s_t, a_t) \right].
\]

\[\square\]
A.4 Details about Assumption 1

Assumption 1 is equivalent to require that there exists two finite constants $c_1 < +\infty$ and $c_2 < +\infty$ such that:

\[
\mathbb{E}_{\tau \sim \gamma^{b,p}} \left[ \sum_{t=0}^{+\infty} \gamma^t \rho_{\pi/\pi_b}(\tau_{0:t}) \sum_{l=0}^{t} \left\| \nabla_{\theta} \log \pi(a_l|s_l) \right\|_q \log p(s_{t+1}|s_t, a_t) \right]^2 \leq c_1^2, \tag{19}
\]

\[
\mathbb{E}_{\tau \sim \gamma^{b,p}} \left[ \sum_{t=0}^{+\infty} \gamma^t \rho_{\pi/\pi_b}(\tau_{0:t}) \nabla_{\theta} \log \pi(a_t|s_t) Q^\tau P(s_t, a_t) \right]^2 \leq R_{\max}^2 c_2^2, \quad j = 1, \ldots, d. \tag{20}
\]

We now state the following result that allows decoupling Assumption 1 into two separate conditions for the policies $\pi$ and $\pi_b$ and the transition models $p$ (the real one) and $\hat{p}$ (the approximating one).

**Corollary A.3.1.** Assumption 1 is satisfied if there exist three constants $\chi_1$, $\chi_2$ and $\chi_3$, with $\chi_1 < \frac{1}{\gamma}$.

\[
\sup \sup \mathbb{E}_{\pi \in \Pi_a, s \in S} \left[ \frac{\left( \frac{\pi(a|s)}{\pi_b(a|s)} \right)}{\left( \frac{\pi_b(a|s)}{\pi_b(a|s)} \right)} \right]^2 \leq \chi_1,
\]

\[
\sup \sup \mathbb{E}_{\pi \in \Pi_a, s \in S} \left[ \frac{\left( \frac{\pi(a|s)}{\pi_b(a|s)} \right)}{\left( \frac{\pi_b(a|s)}{\pi_b(a|s)} \right)} \left\| \nabla_{\theta} \log \pi(a|s) \right\|_q \right]^2 \leq \chi_2,
\]

\[
\sup \sup \mathbb{E}_{\pi \in \Pi_a, s \in S} \left[ \left( \log \hat{p}(s'|s, a) \right)^2 \right] \leq \chi_3.
\]

In such case, Equation (19) and Equation (20) are satisfied with constants:

\[
c_1^2 = \frac{\chi_3 \chi_2 (1 + \gamma \chi_1)}{(1 - \gamma)(1 - \gamma \chi_1)^2}, \quad c_2^2 = \frac{\chi_3 \chi_2}{(1 - \gamma)^3 (1 - \gamma \chi_1)}.
\]

**Proof.** Let us start with Equation (19). We first apply Cauchy Swartz inequality to bring the expectation inside the summation:

\[
\mathbb{E}_{\tau \sim \gamma^{b,p}} \left[ \sum_{t=0}^{+\infty} \gamma^t \rho_{\pi/\pi_b}(\tau_{0:t}) \sum_{l=0}^{t} \left\| \nabla_{\theta} \log \pi(a_l|s_l) \right\|_q \log p(s_{t+1}|s_t, a_t) \right]^2
\]

\[
\leq \sum_{t=1}^{+\infty} \gamma^t \mathbb{E}_{\tau \sim \gamma^{b,p}} \left[ \sum_{l=0}^{t} \rho_{\pi/\pi_b}(\tau_{0:t}) \left\| \nabla_{\theta} \log \pi(a_l|s_l) \right\|_q \log p(s_{t+1}|s_t, a_t) \right]^2
\]

\[
\leq \frac{1}{1 - \gamma} \sum_{t=0}^{+\infty} \gamma^t \mathbb{E}_{\tau \sim \gamma^{b,p}} \left[ \rho_{\pi/\pi_b}(\tau_{0:t}) \sum_{l=0}^{t} \left\| \nabla_{\theta} \log \pi(a_l|s_l) \right\|_q \log p(s_{t+1}|s_t, a_t) \right]^2.
\]

Let us fix a timestep $t$. We derive the following bound:

\[
\mathbb{E}_{\tau \sim \gamma^{b,p}} \left[ \rho_{\pi/\pi_b}(\tau_{0:t}) \sum_{l=0}^{t} \left\| \nabla_{\theta} \log \pi(a_l|s_l) \right\|_q \log p(s_{t+1}|s_t, a_t) \right]^2
\]

\[
= \mathbb{E}_{\tau \sim \gamma^{b,p}} \left[ \sum_{l=0}^{t} \rho_{\pi/\pi_b}(\tau_{0:t}) \left\| \nabla_{\theta} \log \pi(a_l|s_l) \right\|_q \log p(s_{t+1}|s_t, a_t) \right]^2
\]

\[
\leq \mathbb{E}_{\tau \sim \gamma^{b,p}} \left[ (t + 1) \sum_{l=0}^{t} \rho_{\pi/\pi_b}(\tau_{0:t}) \left\| \nabla_{\theta} \log \pi(a_l|s_l) \right\|_q \log p(s_{t+1}|s_t, a_t) \right]^2,
\]

where we applied Cauchy-Swartz inequality to bound the square of the summation. We now rewrite the expectation in a convenient form to highlight the different components.

\[
\mathbb{E}_{\tau \sim \gamma^{b,p}} \left[ (t + 1) \sum_{l=0}^{t} \rho_{\pi/\pi_b}(\tau_{0:t}) \left\| \nabla_{\theta} \log \pi(a_l|s_l) \right\|_q \log p(s_{t+1}|s_t, a_t) \right]^2
\]

\[
= (t + 1) \sum_{l=0}^{t} \mathbb{E}_{\tau \sim \gamma^{b,p}} \left[ \rho_{\pi/\pi_b}(\tau_{0:t}) \left\| \nabla_{\theta} \log \pi(a_l|s_l) \right\|_q \log p(s_{t+1}|s_t, a_t) \right]^2 \mathbb{E}_{s_{t+1} \sim \hat{p}(s_{t+1}, a_t)} \left[ \log p(s_{t+1}|s_t, a_t) \right]^2
\]

16
\[(t + 1)\chi_3 \sum_{i=0}^{\infty} \mathbb{E}_{\tau_0 \sim \zeta_\mu^p} \left[ \left( \rho_{\pi/s_0}(\tau_{0:t}) \| \nabla \theta \log \pi(a_i|s_i) \|_q \right)^2 \right].\]

Let us fix \(l\) and bound the expectation inside the summation, by unrolling the trajectory and recalling the definition of \(\rho_{\pi/s_0}(\tau_{0:t})\):

\[
\mathbb{E}_{\tau_0 \sim \zeta_\mu^p} \left[ \left( \rho_{\pi/s_0}(\tau_{0:t}) \| \nabla \theta \log \pi(a_i|s_i) \|_q \right)^2 \right] \\
= \mathbb{E}_{a_0 \sim \pi(\cdot | s_0)} \left[ \left( \frac{\pi(a_0|s_0)}{\pi_\theta(a_0|s_0)} \right)^2 \right] \times \mathbb{E}_{a_1 \sim \pi(\cdot | s_1)} \left[ \left( \frac{\pi(a_1|s_1)}{\pi_\theta(a_1|s_1)} \| \nabla \theta \log \pi(a_1|s_1) \|_q \right)^2 \right] \\
\leq \chi_2 \chi_1^2.
\]

Plugging this result in the summation we get the result, recalling that \(\gamma \chi_1 < 1\) and using the properties of the geometric series, we obtain:

\[
\frac{1}{1 - \gamma} \sum_{t=0}^{\infty} (t + 1)^2 \gamma^t \chi_2 \chi_3 = \frac{\chi_3\chi_2(1 + \gamma \chi_1)}{(1 - \gamma)(1 - \gamma \chi_1)^2}.
\]

We now consider Equation (20) and we apply Cauchy Swartz as well:

\[
\mathbb{E}_{\tau \sim \zeta_\mu^p} \left[ \left( \sum_{t=0}^{\infty} \gamma^t \rho_{\pi/s_0}(\tau_{0:t}) \nabla \theta \log \pi(a_t|s_t) Q^\phi,\pi(s_t, a_t) \right)^2 \right] \\
\leq \frac{1}{1 - \gamma} \sum_{t=0}^{\infty} \gamma^t \mathbb{E}_{\tau \sim \zeta_\mu^p} \left[ \left( \rho_{\pi/s_0}(\tau_{0:t}) \nabla \theta \log \pi(a_t|s_t) Q^\phi,\pi(s_t, a_t) \right)^2 \right].
\]

By observing that \(\| Q^\phi,\pi(s_t, a_t) \| \leq \frac{R^2_{\text{max}}}{(1 - \gamma)^2} \) we can use an argument similar to the one used to bound Equation (19) to get:

\[
\mathbb{E}_{\tau \sim \zeta_\mu^p} \left[ \left( \rho_{\pi/s_0}(\tau_{0:t}) \nabla \theta \log \pi(a_t|s_t) Q^\phi,\pi(s_t, a_t) \right)^2 \right] \leq \frac{R^2_{\text{max}}}{(1 - \gamma)^2} \chi_2 \chi_1^2.
\]

Plugging this result into the summation, we have:

\[
\frac{1}{1 - \gamma} \sum_{t=0}^{\infty} \gamma^t \chi_1 \chi_2 \chi_3 \frac{R^2_{\text{max}}}{(1 - \gamma)^2} = \frac{\chi_3\chi_2 R^2_{\text{max}}}{(1 - \gamma)^2(1 - \gamma \chi_1)}.
\]

A.5 Proofs of Section 5

Under Assumption 1 we prove the following intermediate result about the objective function in Equation (5).

Lemma A.4. Let \(\hat{\mu} \in \mathcal{P}\) be the maximizer of the objective function in Equation (3), obtained with \(N > 0\) independent trajectories \(\{\tau^i\}_{i=1}^N\). Under Assumption 1 and 2 for any \(\delta \in (0, 1)\), with probability at least \(1 - 2\delta\) it holds that:

\[
\mathbb{E}_{\tau \sim \zeta_\mu^p} \left[ \frac{f_{\pi,\hat{\mu}}(\tau)}{f_{\pi,\mu}(\tau)} \right] \geq \sup_{\hat{\mu} \in \mathcal{P}} \mathbb{E}_{\tau \sim \zeta_\mu^p} \left[ \frac{f_{\pi,\hat{\mu}}(\tau)}{f_{\pi,\mu}(\tau)} \right] - 4c_1 \epsilon,
\]

where \(\epsilon = \sqrt{\frac{\log 2\delta N + \log \frac{4}{\epsilon}}{N}} \left( \sqrt{\frac{\log 2\delta N + \log \frac{4}{\epsilon}}{N}} \right) \) and \(\Gamma(\xi) := \frac{1}{2} + \sqrt{1 + \frac{1}{4} \log \frac{1}{\xi}} = \tilde{O}(1)\).

Proof. We use a very common argument of empirical risk minimization. Let us denote with \(\bar{\pi} \in \arg \max_{\pi \in \mathcal{P}} \mathbb{E}_{\tau \sim \zeta_\mu^p} f_{\pi,\theta}(\tau)\) and \(\hat{\mu}_n = \frac{1}{N} \sum_{i=1}^N f_{\pi,\hat{\mu}}(\tau^i)\):

\[
\mathbb{E}_{\tau \sim \zeta_\mu^p} \left[ f_{\pi,\hat{\mu}}(\tau) \right] \leq \mathbb{E}_{\tau \sim \zeta_\mu^p} \left[ f_{\bar{\pi},\hat{\mu}}(\tau) \right] \leq \mathbb{E}_{\tau \sim \zeta_\mu^p} \left[ f_{\pi,\theta}(\tau) \right] - \mathbb{E}_{\tau \sim \zeta_\mu^p} \left[ f_{\pi,\theta}(\tau) \right] = \mathbb{E}_{\tau \sim \zeta_\mu^p} \left[ f_{\pi,\theta}(\tau) \right] - \mathbb{E}_{\tau \sim \zeta_\mu^p} \left[ f_{\pi,\hat{\mu}}(\tau) \right] + \mathbb{E}_{\tau \sim \zeta_\mu^p} \left[ f_{\pi,\hat{\mu}}(\tau) \right] = \mathbb{E}_{\tau \sim \zeta_\mu^p} \left[ f_{\pi,\theta}(\tau) \right] - \mathbb{E}_{\tau \sim \zeta_\mu^p} \left[ f_{\pi,\hat{\mu}}(\tau) \right] + \mathbb{E}_{\tau \sim \zeta_\mu^p} \left[ f_{\pi,\hat{\mu}}(\tau) \right].
\]
We are now ready to prove the main result. We can derive a concentration result for the gradient estimation (Equation (5)), recalling the fact that $g^{\pi,p}$ is a vectorial function.

**Lemma A.5.** Let $q \in [1, +\infty]$, $d$ be the dimensionality of $\Theta$ and $\tilde{p} \in \mathcal{P}$ be the maximizer of the objective function in Equation (3), obtained with $N > 0$ independent trajectories $\{\tau^i\}_{i=1}^N$. Under Assumption 1 and 2, for any $\delta \in (0, 1)$, with probability at least $1 - 2d\delta$, simultaneously for all $\tilde{p} \in \mathcal{P}$, it holds that:

$$\left\| \nabla_{\theta} J(\theta) - \nabla_{\theta}^{\text{MVG}} J(\theta) \right\|_q \leq 2d^{\frac{1}{2}} R_{\text{max}} c_2 \epsilon,$$

where $\epsilon = \sqrt{\frac{2\log 2N + \log 2(1 + \delta)}{N}}\Gamma\left(\sqrt{\frac{2\log 2N + \log 2(1 + \delta)}{N}}\right)$ and $\Gamma(\xi) := \frac{1}{2} + \sqrt{1 + \frac{1}{2} \log \frac{1}{\xi}} = O(1)$.

**Proof.** We observe that $\tilde{\nabla}_{\theta} J(\theta)$ is the sample version of $\nabla_{\theta}^{\text{MVG}} J(\theta)$. Under Assumption 1 and 2, and using Corollary 14 in [10], as in Lemma A.4, we can write for any $j = 1, \ldots, d$ the following bound that holds with probability at least $1 - 2d\delta$ simultaneously for all $\tilde{p} \in \mathcal{P}$:

$$\left\| \tilde{\nabla}_{\theta_j} J(\theta) - \nabla_{\theta_j}^{\text{MVG}} J(\theta) \right\| \leq 2R_{\text{max}} c_2 \epsilon.$$

Considering the $L^q$-norm, and plugging the previous equation, we have that with probability at least $1 - 2d\delta$ it holds that, simultaneously for all $\tilde{p} \in \mathcal{P}$:

$$\left\| \tilde{\nabla}_{\theta} J(\theta) - \nabla_{\theta}^{\text{MVG}} J(\theta) \right\|_q \leq \left( \sum_{j=1}^d \left\| \nabla_{\theta_j}^{\text{MVG}} J(\theta) - \tilde{\nabla}_{\theta_j} J(\theta) \right\|_q^2 \right)^{\frac{1}{2}} \leq 2d^{\frac{1}{2}} R_{\text{max}} c_2 \epsilon,$$

having exploited a union bound over the dimensions $d$.

We are now ready to prove the main result.

**Theorem 5.1.** Let $q \in [1, +\infty]$, $d$ be the dimensionality of $\Theta$ and $\tilde{p} \in \mathcal{P}$ be the maximizer of the objective function in Equation (3), obtained with $N > 0$ independent trajectories $\{\tau^i\}_{i=1}^N$. Under Assumption 1 and 2, for any $\delta \in (0, 1)$, with probability at least $1 - 4d\delta$ it holds that:

$$\left\| \tilde{\nabla}_{\theta} J(\theta) - \nabla_{\theta} J(\theta) \right\|_q \leq \frac{\gamma \sqrt{Z} R_{\text{max}}}{(1 - \gamma)^2} \inf_{\pi \in \mathcal{P}} \left\{ \frac{\mathbb{E}_{s,a \sim \eta_{\hat{n}}^p}}{\mathbb{E}_{s,a \sim \eta_{\hat{n}}^p}} \left[ D_{KL}(p(\cdot|s,a)||\hat{p}(\cdot|s,a)) \right] + 2R_{\text{max}} \left( d^{\frac{1}{2}} \epsilon + \frac{\gamma \sqrt{Z} c_2 \epsilon}{1 - \gamma} \right) \right\},$$

where $\epsilon = \sqrt{\frac{2\log 2N + \log 2(d+1)}{N}}\Gamma\left(\sqrt{\frac{2\log 2N + \log 2(d+1)}{N}}\right)$ and $\Gamma(\xi) := \frac{1}{2} + \sqrt{1 + \frac{1}{2} \log \frac{1}{\xi}} = O(1)$.

**Proof.** Let us first consider the decomposition, that follows from triangular inequality:

$$\left\| \tilde{\nabla}_{\theta} J(\theta) - \nabla_{\theta} J(\theta) \right\|_q \leq \left\| \tilde{\nabla}_{\theta} J(\theta) - \nabla_{\theta}^{\text{MVG}} J(\theta) \right\|_q + \left\| \nabla_{\theta}^{\text{MVG}} J(\theta) - \nabla_{\theta} J(\theta) \right\|_q.$$

We now bound each term of the right hand side. (i) is bounded in Lemma A.5. Let us now consider (ii). We just need to apply Theorem 3.2 and Lemma A.4, recalling the properties of the KL-divergence. From Theorem 3.2

$$\left\| \nabla_{\theta}^{\text{MVG}} J(\theta) - \nabla_{\theta} J(\theta) \right\|_q \leq \frac{\gamma \sqrt{Z} R_{\text{max}}}{(1 - \gamma)^2} \sqrt{\frac{\mathbb{E}_{s,a \sim \eta_{\hat{n}}^p}}{\mathbb{E}_{s,a \sim \eta_{\hat{n}}^p}} \left[ \int_S p(s'|s,a) \log p(s'|s,a) ds' - \int_S p(s'|s,a) \log \hat{p}(s'|s,a) ds' \right]}$$

(24)
\[ 1 \] − where Equation (24) and Equation (29) follow from the definition of KL-divergence and Lemma 4.1. Equation (25) is derived from Lemma 4.1 where \( E \tau \sim \zeta_{\mu, b}, p \sum_{t=0}^{\infty} \omega_t \log p(s_{t+1} | s_t, a_t) \) and bound (ii) w.p. 1 \( - 2 \delta \) and bound (ii) w.p. 1 \( - 2 \delta \). By rescaling \( \delta \) we get the result. \( \square \)

B Experimental details

B.1 Two-areas Gridworld

The gridworld we use in our experiments features two subspaces of the state space \( S \), to which we refer to as \( S_1 \) (lower) and \( S_2 \) (upper). The agent can choose among four different actions: in the lower part, a sticky area, each action corresponds to an attempt to go up, right, down or left, and has a 0.9 probability of success and a 0.1 probability of causing the agent to remain in the same state; in the upper part, the four actions have deterministic movement effects, all different from the ones they have in the other area (rotated of 90 degrees). Representing as \( (p_1, p_2, p_3, p_4, p_5) \) the probabilities \( p_1, p_2, p_3, p_4 \) and \( p_5 \) of, respectively, going up, right, down, left and remaining in the same state, the transition model of the environment is defined as follows:

\[
\begin{array}{c}
\text{s} \in S_1 : p(\cdot | s, a) = \\
(0, \frac{0.9}{a = 0}, 0, 0, \frac{0.1}{a = 0}), & \text{if } a = 0 \\
(0, \frac{0.9}{a = 1}, 0, 0, \frac{0.1}{a = 0}), & \text{if } a = 1 \\
(0, \frac{0.9}{a = 0}, 0, 0, \frac{0.1}{a = 0}), & \text{if } a = 2 \\
(0, \frac{0.9}{a = 0}, 0, 0, \frac{0.1}{a = 0}), & \text{if } a = 3 \\
\end{array}
\]
We learn both the policy and the models by minimizing the corresponding loss function via gradient descent. We use the Adam optimizer \cite{Kingma2014} with a learning rate of 0.2 for the former and of 0.01 for the latter, together with $\beta_1 = 0.9$ and $\beta_2 = 0.999$. These hypeparameters were chosen by trial and error from a range of $(0.001, 0.9)$. In the computation of our gradient-aware weights, we use $\| \nabla \log \pi(a|s) \|_q$ with $q = 2$.

In order to understand the properties of our method for model learning, we compare the maximum likelihood model (ML) and the one obtained with GAMPS, in terms of accuracy in next state prediction and MSE with the real Q-function w.r.t. to the one derived by dynamic programming; lastly, we use the computed action-value functions to provide two approximations to the sample version of Equation 2. The intuitive rationale behind decision-aware model learning is that the raw quality of the estimate of the forward model itself or any intermediate quantity is pointless: the accuracy on estimating the quantity of interest for improving the policy, in our case its gradient, is the only relevant metric. The results, shown in Table 1, illustrate exactly this point, showing that, although our method offers worse performance in model and Q-function estimation, it is able to perfectly estimate the correct direction of the policy gradient. The definitions of the metrics used for making the comparison, computed over an hold-out set of 1000 validation trajectories, are now presented. The model accuracy for an estimated model $\hat{p}$ is defined as $\text{acc}(\hat{p}) = \frac{1}{|\mathcal{D}|} \sum_{(s,a,s') \in \mathcal{D}} \mathbb{1}(s' = \arg \max_{a'} \hat{\pi}(s'|s,a))$. The MSE for measuring the error in estimating the tabular Q-function is computed by averaging the error obtained for every state and action. Lastly, the cosine similarity between the real gradient $\nabla_{\theta} J(\theta)$ and the estimated gradient $\hat{\nabla}_{\theta} J(\theta)$ is defined as $\text{sim}(\nabla_{\theta} J(\theta), \hat{\nabla}_{\theta} J(\theta)) = \frac{\nabla_{\theta} J(\theta) \cdot \hat{\nabla}_{\theta} J(\theta)}{\max(\|\nabla_{\theta} J(\theta)\|_2, \|\hat{\nabla}_{\theta} J(\theta)\|_2, \epsilon)}$, where $\epsilon$ is set to $10^{-8}$.

| Approach | $\hat{p}$ accuracy | $\hat{Q}$ MSE | $\hat{\nabla}_{\theta} J$ cosine similarity |
|----------|---------------------|---------------|------------------------------------------|
| ML       | 0.765 ± 0.001       | 11.803 ± 0.158| 0.449 ± 0.041                             |
| GAMPS    | 0.357 ± 0.004       | 633.835 ± 12.697| 1.000 ± 0.000                             |

Table 1: Estimation performance on the gridworld environment comparing Maximum Likelihood estimation (ML) and our approach (GAMPS). 1000 training and 1000 validation trajectories per run. Average results on 10 runs with a 95% confidence interval.
B.2 Minigolf

In the minigolf game, the agent has to shoot a ball with radius $r$ inside a hole of diameter $D$ with the minimum number of strokes. We assume that the ball moves along a level surface with a constant deceleration $d = \frac{5}{2} \rho g$, where $\rho$ is the dynamic friction coefficient between the ball and the ground and $g$ is the gravitational acceleration. Given the distance $x_0$ of the ball from the hole, the agent must determine the angular velocity $\omega$ of the putter that determines the initial velocity $v_0 = \omega l$ (where $l$ is the length of the putter) to put the ball in the hole in one strike. For each distance $x_0$, the ball falls in the hole if its initial velocity $v_0$ ranges from $v_{\text{min}} = \sqrt{\frac{2}{\rho} x_0}$ to $v_{\text{max}} = \sqrt{(2D - r)^2 \frac{\rho}{2r} + v_{\text{min}}^2}$. $v_{\text{max}}$ is the maximum allowed speed of the edge of the hole to let the ball enter the hole and not to overcome it. At the beginning of each trial the ball is placed at random, between 2000cm and 0cm far from the hole. At each step, the agent chooses an action that determines the initial velocity $v_0$ of the ball. When the ball enters the hole the episode ends with reward 0. If $v_0 > v_{\text{max}}$ the ball is lost and the episode ends with reward $-100$. Finally, if $v_0 < v_{\text{min}}$ the episode goes on and the agent can try another hit with reward $-1$ from position $x = x_0 - \frac{v_0^2}{2\rho}$. The angular speed of the putter is determined by the action $a$ selected by the agent as follows: $\omega = a(1 + \epsilon)$, where $\epsilon \sim \mathcal{N}(0, 0.3)$. This implies that the stronger the action chosen the more uncertain its outcome will be. As a result, the agent is disencumbered by trying to make a hole in one shot when it is away from the hole and will prefer to perform a sequence of approach shots. The state space is divided into two parts: the first one, bigger twice the other, is the nearest to the hole and features $\rho_1 = 0.131$; the second one is smaller and has an higher friction with $\rho_1 = 0.19$.

We use a linear-Gaussian policy that is linear on six equally-spaced radial basis function features. Four of the basis functions are therefore in the first area, while two are in the other one. The parameters of the policy are initialized equal to one for the mean and equal to zero for the standard deviation.

As a model class, we use parameterized linear-Gaussian models that predict the next state by difference of the forward model (order of $MNH$) transitions, using the trajectories imagined by the model for obtaining the value function (order of $NH$). For all the learning algorithms, we employ a constant learning rate of $0.08$ for the Adam optimizer, with $\beta_1 = 0$ and $\beta_2 = 0.999$. For training the model used by GAMPS and GAMPS-ML, we minimize the MSE weighted through our weighting scheme, again using Adam with learning rate $0.02$ and default betas. For the estimation of the Q-function, we use the on-the-fly procedure outlined in Section 4.2 with an horizon of 20 and averaging over 10 rollouts. Also in this experiment we set $q = 2$ for the q-norm $\|\nabla_{\theta} \log \pi(a|s)\|_q$ of the score. We use $\gamma = 0.99$.

C Algorithm

C.1 Time complexity of Algorithm

Let us consider that the algorithm is run for $K$ iterations on a dataset of $N$ trajectories. Suppose a parametric model class for which at most $E$ epochs are necessary for estimation. We define $H$ as the maximum length of a trajectory (or horizon) and use an estimate of the Q-function derived by sampling $M$ trajectories from the estimated model, as described in Section 4.2. For every iteration, we first compute the weights for every transition in every trajectory $\mathcal{O}(NH)$ and then estimate the corresponding forward model (order of $NH$). Then, we estimate the gradient given all the transitions, using the trajectories imagined by the model for obtaining the value function (order of $NMH^2$). The overall time complexity of the algorithm is therefore $\mathcal{O}(KNHE + KNMH^2)$.

C.2 Approximation of the value function

We now briefly review in a formal way how $Q_{\pi, \hat{\theta}}$ can be estimated. For the discrete case, the standard solution is to find the fixed point of the Bellman equation:

$$Q^*(s, a) = r(s, a) + \gamma \mathbb{E}_{s', a' \sim p(\cdot|s, a)} \left[ Q^*(s', a') \right],$$

(30)
that can be found either in exact form using matrix inversion or by applying Dynamic Programming. In the continuous case, one can use approximate dynamic programming. For instance, with one step of model unrolling, the state-action value function could be found by iteratively solving the following optimization problem:

\[
\hat{Q} = \arg\min_{Q} \sum_{t} \left( r(s_t, a_t) + \gamma \mathbb{E}_{s_{t+1} \sim \hat{p} \mid s_t, a_t \sim \pi} [Q(s_{t+1}, a_{t+1})] \right)^2.
\] (31)

The expected value in Equation (31) can be approximated by sampling from the estimated model \(\hat{p}\) and the policy \(\pi\). In practice, a further parameterized state-value function \(\hat{V}(s) \approx \mathbb{E}_{a \sim \pi(s)} \left[ \hat{Q}(s, a) \right]\) can be learned jointly with the action-value function.

The third approach, that is the one employed in GAMPS, is to directly use the estimated model for computing the expected cumulative return starting from \((s, a)\). We can therefore use an ephemeral Q-function, that is obtained by unrolling the estimated model and computing the reward using the known reward function.

### D A connection with reward-weighted regression

Interestingly, our gradient-aware procedure for model learning has some connections with the reward-weighted regression (RWR) techniques, that solve reinforcement learning problems by optimizing a supervised loss. To see this, we shall totally revert our perspective on a non-Markovian decision process. First, we interpret a model \(\hat{p}\) parameterized by \(\phi\) as a policy, whose action is to pick a new state after observing a previous state-action combination. Then, we see the policy \(\pi\) as the model, that samples the transition to the next state given the output of \(\hat{p}\). Finally, the cumulative absolute score at time \(t\) is the (non-markovian) reward. To strengthen the parallel, let us consider an appropriate transformation \(u_c\) on the weights \(\omega_t\).

We can now give an expectation-maximization formulation for our model learning problem as reward-weighted regression in this newly defined decision process:

**E-step:**

\[
q_{k+1}(t) = \frac{p_{\phi_k}(s_{t+1} \mid s_t, a_t) u_{c_k}(\omega_t)}{\sum_{t'} p_{\phi_k}(s_{t'+1} \mid s_{t'}, a_{t'}) u_{c_k}(\omega_{t'})}
\] (32)

**M-step for model parameters:**

\[
\phi_{k+1} = \arg\max_{\phi} \sum_t q_{k+1}(t) \log p_{\phi}(s_{t+1} \mid s_t, a_t)
\] (33)

**M-step for transformation coefficient:**

\[
\tau_{k+1} = \arg\max_c \sum_t q_{k+1}(t) u_c(\omega_t)
\] (34)

Assuming a Gaussian-linear model \(\hat{p} = \mathcal{N}(s_{t+1} \mid \mu(s_t, a_t), \sigma^2 I)\) and a transformation \(u_c(x) = c \exp(-cx)\), the update for the model parameters and the transformation parameter is given by:

\[
\phi_{k+1} = (\Phi^T W \Phi)^{-1} \Phi^T W Y
\] (35)

\[
\sigma^2_{k+1} = \| Y - \Phi_{k+1}^T \|_W^2
\] (36)

\[
c_{k+1} = \frac{\sum_t u_c(\omega_t)}{\sum_{t'} u_c(\omega_{t'})}
\] (37)

where \(\Phi, Y\) and \(W\) are the matrices containing, respectively, state-action features, successor state features and cumulative score weights on the diagonal.

As in the case of the original RWR, this learned exponentiation of the weights could in practice improve the performance of our algorithm. We leave this direction to future work.