Transverse beam emittance measurement by undulator radiation power noise

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Generally, turn-to-turn power fluctuations of incoherent spontaneous synchrotron radiation in a storage ring depend on a 6D phase-space distribution of the electron bunch. In some cases, if only one parameter of the distribution is unknown, this parameter can be determined from the measured magnitude of these power fluctuations. In this Letter, we report an absolute measurement (no free parameters or calibration) of a small vertical emittance (5–15 nm rms) of a flat beam by this method, under conditions, when it is unresolvable by a conventional synchrotron light beam size monitor.

Most often, noise is encountered in a negative context and is considered something that needs to be minimized. However, there are multiple examples where noise is used as a non-invasive probe into the parameters of a certain system, and even to measure fundamental constants. Examples include the determination of the Boltzmann constant $k_B$ by the thermal noise in an electrical conductor [1] and the measurement of the elementary charge $e$ by the shot noise of the electric current in a vacuum tube [2]. In fact, the latter effect is also relevant to accelerators and storage rings, where it is known as Schottky noise [3] due to the finite number of charge carriers in the beam, as described by Schottky [4]. Many beam parameters, such as the momentum spread, the number of particles and even transverse rms emittances, are imprinted into the power spectrum of Schottky noise. It is often used in beam diagnostics [5–7].

Synchrotron radiation is generated by individual electrons in the beam. Hence, Schottky noise in the beam current must pass on to the synchrotron radiation power in some way. Therefore, one could assume that the synchrotron radiation power noise may carry information about beam parameters as well. This assumption is, in fact, correct. Three decades ago, Ref. [8] reported the results of an experimental study into statistical properties of wiggler radiation in a storage ring. It was noted that the magnitude of turn-to-turn intensity fluctuations depends on the dimensions of the electron bunch. The potential in beam instrumentation was soon realized [9] and a number of papers followed. However, to this day, mostly measurements of a bunch length via these fluctuations were discussed [10–12]. Only Ref. [13] reported an order-of-magnitude measurement of a transverse emittance. In this Letter, we describe a new fluctuations-based technique for an absolute measurement of a transverse emittance. There are no free parameters in our equations, nor is a calibration required. However, the transverse and longitudinal focusing functions of the storage ring are assumed to be known. This technique is tested at the Integrable Optics Test Accelerator (IOTA) storage ring at Fermilab [14]. For a beam with approximately equal and relatively large transverse rms emittances, the results agree with conventional visible synchrotron light monitors (SLMs) [15]. Then, in a different regime, we measure a much smaller vertical emittance of a flat beam, unresolvable by our SLMs. These emittance measurements agree with estimates, based on the beam lifetime. We also discuss possible further improvements.

Let us assume that we have a detector that can measure the number of detected synchrotron radiation photons $N$ at each revolution in a storage ring. Then, according to [8, 16, 17], the variance of this number is

$$\text{var}(N) = \langle (N - \langle N \rangle)^2 \rangle = \langle N \rangle + \frac{1}{M} \langle N \rangle^2, \quad (1)$$

where the linear term represents the photon shot noise, related to the quantum discrete nature of light. This effect would exist even if there was only one electron, circulating in the ring. Indeed, the electron would radiate photons with a Poisson distribution [18–20]. The quadratic term in Eq. (1) corresponds to the interference of fields, radiated by different electrons. Changes in relative electron positions and velocities, inside the bunch, result in fluctuations of the radiation power, and, consequently, of the number of detected photons. In a storage ring, the effect arises from the betatron and synchrotron motion, radiation induced diffusion, etc. The dependence of $\text{var}(N)$ on the 6D phase-space distribution of the electron bunch is introduced through the parameter $M$, which is conventionally called the number of coherent modes [8, 16, 17]. In addition to bunch parameters, $M$ depends
on the specific spectral-angular distribution of the radiation, the angular aperture and the detection sensitivity (as a function of wavelength). Previously, we derived an equation for $M$ \cite{21, Eq. (2)} for a Gaussian transverse beam profile and an arbitrary longitudinal bunch density distribution $\rho(z)$ (normalized), assuming an rms bunch length much longer than the radiation wavelength. In this Letter, $M$ is calculated by this equation numerically, using our computer code \cite{22}, as a function of transverse $\rho$ distribution $\mathcal{M}$, the angular aperture and the detection sensitivity on the specific spectral-angular distribution of the radiation, because the quadratic term in Eq. (1), sensitive to the bunch parameters, is greater for undulators and wigglers than for dipole magnets \cite{16}. The undulator parameter is $K_u = 1.0$ with the number of periods $N_u = 10.5$ and the period length $\lambda_u = 5.5$ cm.

\begin{align*}
d^3N = &\, C \exp \left[ -\frac{(k - k_0)^2}{2\sigma_k^2} - \frac{\theta_x^2}{2\sigma_{\theta_x}^2} - \frac{\theta_y^2}{2\sigma_{\theta_y}^2} \right], \quad (2) \end{align*}

where $k$ is the magnitude of a wave vector, $\theta_x$ and $\theta_y$ represent the horizontal and vertical angles of the direction of the radiation in the paraxial approximation, $C$ is a constant. Then \cite{10, 21}

\begin{align*}
M = &\, \sqrt{1 + 4\sigma_k^2\sigma_x^2} \sqrt{1 + 4\sigma_0^2\sigma_{\theta_x}^2 \sigma_x^2} \sqrt{1 + 4\sigma^2_0 \sigma_{\theta_x}^2 \sigma_y^2}, \quad (3)
\end{align*}

where $\sigma_x$, $\sigma_y$, $\sigma_z$ are the rms sizes (determined by beam emittances) of a Gaussian electron bunch. In addition, it is assumed that the radiation is longitudinally incoherent $k_0\sigma_z \gg 1$, and the radiation bandwidth is very narrow $\sigma_k \ll 1/(\sigma_x\sigma_{\theta_x})$, $\sigma_k \ll 1/(\sigma_y\sigma_{\theta_y})$. In general, the distribution parameters $k_0$, $\sigma_k$, $\sigma_{\theta_x}$, $\sigma_{\theta_y}$ are determined by both the properties of the emitted synchrotron radiation and by the properties of the detecting system, e.g., spectral filters, its angular acceptance, the detector efficiency. In Eq. (3), the beam divergence is neglected and $M$ depends on $\sigma_x$ and $\sigma_y$, as opposed to a more general result \cite{21, Eq. (2)}, where it depends on $\epsilon_x$ and $\epsilon_y$.

A photodetector was installed in a dark box above the M4R dipole magnet, see Figs. 1(a),(b). The light produced in the undulator was directed to the photodetector (3.5 m away) by a system of two mirrors ($\approx 2^\circ$), see Fig. 1(b). Then, it was focused by a lens ($\approx 2^\circ$, focal distance $F = 150$ mm) into a spot, smaller than the sensitive area of the detector ($\approx 1.0$ mm). The measurements were performed in the vicinity of the fundamental harmonic, $\lambda_1 = \lambda_0(1 + K_u^2/2)/(2\gamma^2) = 1.16$ mm, where $\gamma$ is the Lorentz factor.

As a photodetector we used an InGaAs PIN photodiode \cite{23}. It has a high quantum efficiency (about 80\%) around the fundamental. Figure 1(c) illustrates our photodetection circuit. First, the radiation pulse is converted into a photocurrent pulse by the photodiode. Then, the photocurrent pulse is integrated by an op-amp-based RC integrator and converted to a voltage signal $A_i f(t)$, where $A_i$ is the signal amplitude at the $i$th turn and $f(t)$ is the average signal for one turn, normalized so that its maximum value is 1. The number of detected photons (photoelectrons) at the $i$th turn can be obtained as

\begin{align*}
N_i = &\, \chi A_i, \quad (4)
\end{align*}

where $\chi = 2.08 \times 10^7$ photoelectrons/V, with a $\pm 5\%$ uncertainty, as per the characteristics of our integrator and the photodiode, for details see \cite{21}. The op-amp was capable of driving a 50-Ω input load of a fast digitizing scope, located $\approx 100$ m away. In our measurements, the amplitude $A_i$ was in the range 0–1.2 V.

The expected relative fluctuation of $A_i$ was $10^{-4}$–$10^{-3}$ (rms), which is considerably lower than the digitization resolution of our 8-bit broad-band oscilloscope. To overcome this problem, we employed a passive comb (notch) filter \cite{24}, see Fig. 1(c). In this filter, the input signal first passes through a two-way splitter. Further, one arm is delayed relative to the other by exactly one IOTA revolution. Then, a difference and a sum of the two signals are produced in the output $\Delta$- and $\Sigma$-channels, see Fig. 1(c). For an ideal comb filter,

\begin{align*}
\Delta_i(t) = &\, \xi(A_i - A_{i-1}) f(t), \quad (5)
\Sigma_i(t) = &\, \xi(A_i + A_{i-1}) f(t). \quad (6)
\end{align*}
In our filter, $\xi = 0.31$, which was measured by comparing input and output pulses. Now, since the offset was removed (Eq. (5)), we were able to directly observe the sub-mV turn-to-turn fluctuations in the $\Delta$-channel and the oscilloscope operated in the appropriate scale setting with negligible digitization noise.

For each measurement, we recorded 1.5 ms-long waveforms (about 11 250 IOTA revolutions) of $\Delta$- and $\Sigma$-channels with the oscilloscope at 20 GSa/s. The beam current decay is negligible during this 1.5 ms. Then, the photoelectron count variance $\text{var}(N)$ and the photoelectron count mean $\langle N \rangle$ were obtained from the 11 250 collected amplitudes, $\Delta_i(t_{\text{peak}})$ and $\Sigma_i(t_{\text{peak}})$, using the equations

\[
\text{var}(N) = \chi^2 \text{var}(A) = \frac{\chi^2 \text{var}(\Delta(t_{\text{peak}}))}{2\xi^2}, \tag{7}
\]

\[
\langle N \rangle = \chi \langle A \rangle = \frac{\chi \langle \Sigma(t_{\text{peak}}) \rangle}{2\xi}, \tag{8}
\]

which follow from Eqs. (4) to (6), $t_{\text{peak}}$ is the time within each turn, corresponding to the peak of the signal, $f(t_{\text{peak}}) = 1$. It should be noted that our comb filter was not perfect. There was a small cross-talk ($< 1\%$) between the output channels. However, its effect is negligible in Eqs. (7) and (8). Also, there was some instrumental noise contribution to $\text{var}(\Delta(t_{\text{peak}}))$. In terms of $\text{var}(N)$ it was $2.0 \times 10^{-8}$ as per measurements at zero beam current. Primary sources of this noise are the integrator’s op-amp and the oscilloscope’s pre-amp. In [21], we show that this noise level is independent of $\langle N \rangle$ via measurements with an independent test light source. Therefore, it can be simply subtracted every time. Reference [21] also has the details of the photocurrent integrator and the comb filter.

As we discussed before, the number of coherent modes $M$ and, hence, the fluctuations $\text{var}(N)$ depend on the following bunch parameters, $\epsilon_x$, $\epsilon_y$ (or mode emittances $\epsilon_1$, $\epsilon_2$), $\sigma_p$, $\sigma_z^{\text{eff}}$. Therefore, in a situation, when only one of them is unknown and var$(N)$ is known (or measured), we can numerically solve Eq. (1), using our general formula for $M$ [21, Eq. (2)], to find the unknown bunch parameter. We consider two such situations in this Letter.

In the first case, we consider a strongly coupled [25, 26] transverse focusing optics in IOTA, which was specifically designed to keep the two mode emittances equal $\epsilon_1 = \epsilon_2 = \epsilon$. This was empirically confirmed to be true with a few percent precision. We will call this beam round. The longitudinal bunch profile was measured by a high-bandwidth wall-current monitor [27] to determine $\sigma_z^{\text{eff}}$ and estimate $\sigma_p$. The fluctuations, measured using Eq. (7) are shown in Fig. 2(a), where the error bars represent the statistical error of our method, $\pm 2.7 \times 10^8$, estimated with an independent test light source, for details see [21]. Hence, the only unknown parameter in Eq. (1) was $\epsilon$. The numerical solution of Eq. (1) with $M$ from [21, Eq. (2)] was performed on the Midway2 cluster at the University of Chicago Research Computing Center. The results for $\epsilon$ are shown in Fig. 2(c) (red points), the error bars correspond to the error bars of the fluctuations data in Fig. 2(a). Apart from this statistical error there is also a systematic error due to the $\pm 1 \text{ MeV}$ uncertainty of the beam energy. This systematic error varies from $\pm 10 \text{ nm}$ at lower beam currents, to $\pm 14 \text{ nm}$ at higher currents (not shown).

In IOTA, transverse beam sizes are monitored by seven SLMs, at MIL-M4L and at MIR-M3R, see Fig. 1(a). Beam emittances can be determined from the measured beam sizes using the known design Twiss functions. Such measurements for $\epsilon$ of the round beam (blue line in Fig. 2(c)) agree with the fluctuations-based $\epsilon$ within the uncertainties. The smallest reliably resolvable emittance by the SLMs in our experiment configuration was about 20 nm. The measured transverse emittance $\epsilon$ of the round beam is 75–100 nm (rms, unnormalized), primarily due to intrabeam scattering [28, 29]. The expected zero-current value is $\epsilon \approx 12 \text{ nm}$.

In the second case, we consider uncoupled focusing, with the vertical emittance much smaller than the horizontal one. We will call this beam flat. The horizontal emittance $\epsilon_x$ of the flat beam can still be reliably measured by the SLMs; $\sigma_z^{\text{eff}}$ and $\sigma_p$ can still be measured by the wall-current monitor. However, the seven SLMs provided very inconsistent estimates for the much smaller $\epsilon_y$ — the max-to-min variation for different SLMs reached a factor of eight. We believe this happened because the beam images were close to the resolution limit, set by a combination of factors, such as the diffraction limit, the point spread function of the cameras, the chromatic aberration, the effective radiator size of the dipole magnet radiation ($\approx 20 \text{ µm}$), the camera pixel size ($\approx 10 \text{ µm}$ in terms of beam size). Therefore, the monitor-to-monitor emittance variation primarily came from the Twiss beta-function variation ($\sigma_y^{\text{max}} / \sigma_y^{\text{min}} \approx 12$). Although the resolution of the SLMs may be improved in the future [21], at present, $\epsilon_y$ of the flat beam is unresolvable by the SLMs, and, therefore, is truly unknown. However, the measured fluctuations for the flat beam, shown in Fig. 2(b), were of the same order as for the round beam, with the same statistical error. Hence, we were able to reconstruct $\epsilon_y$ in the same way as $\epsilon$ in Fig. 2(c). The results are shown in Fig. 2(d) (red points) along with the SLMs data for $\epsilon_y$ (blue line). In addition to the statistical error of $\epsilon_y$, shown in Fig. 2(d), there was also a systematic error due to the $\pm 1 \text{ MeV}$ uncertainty of the beam energy (from $\pm 2.5 \text{ nm}$ at lower currents, to $\pm 5 \text{ nm}$ at higher currents), and a systematic error due to the $\pm 50 \text{ nm}$ uncertainty of $\epsilon_x$ (from $\pm 1.3 \text{ nm}$ at lower currents, to $\pm 2.4 \text{ nm}$ at higher currents). These systematic errors are relatively high. However, they can be reduced by future improvements of beam characterization in IOTA. The measured vertical emittance is 5–15 nm, most likely due to a nonzero
residual transverse coupling. The expected zero-current flat beam emittances were $\epsilon_x \approx 50 \text{ nm}$, $\epsilon_y \gtrsim 0.33 \text{ pm}$ (set by the quantum excitation in a perfectly uncoupled ring).

In these simulations, we numerically calculated the spectral-angular distribution for the undulator by our computer code [30], based on the equations from [31]. Further, we used the manufacturers’ specifications to account for the spectral properties of the vacuum chamber window at the M4R dipole magnet, the two mirrors, the focusing lens, and the quantum efficiency of the InGaAs photodiode. The resulting spectral width of the radiation was 0.14μm (FWHM), and the angular size was $\approx 2$ mrad, which could be fully transmitted through the $\sqrt{2}$ optical system, see [21] for details. The simulated expected average number of photoelectrons per one electron was $9.1 \times 10^{-3}$ photoelectrons/electron. The empirical value was $8.8 \times 10^{-3}$ photoelectrons/electron. There were no free adjustable parameters in this simulation.

Furthermore, the vertical emittance $\epsilon_y$ of the flat beam in IOTA could also be estimated from the measured beam lifetime $[I/(dI/dt)]$, assuming that it is determined solely by Touschek scattering [32], which is a good approximation at beam currents $I \gtrsim 0.5 \text{ mA}$ [26]. In storage rings, the Touschek lifetime is determined by the effective momentum acceptance $\delta_{\text{acc}}^{(\text{eff})}$, see [33], which is smaller than or equal to the rf bucket half-height, $\delta_{\text{rf}} = 2.8 \times 10^{-3}$ in IOTA. We measured the IOTA beam lifetime (550–1000sec) for both round and flat beams as a function of beam current. Using the known round-beam emittance and the bunch length, we arrived at the following estimate for IOTA, $\delta_{\text{acc}}^{(\text{eff})} = 2.0 \times 10^{-3}$, by comparing the calculated [34, 35] Touschek lifetime and the measured beam lifetime, for details see Appendix D of [21]. The orange line in Fig. 2(c) illustrates the emittance of the round beam $\epsilon$, if it were to be determined from the measured beam lifetime using the Touschek lifetime calculation with $\delta_{\text{acc}}^{(\text{eff})} = 2.0 \times 10^{-3}$. Then, we used this value of $\delta_{\text{acc}}^{(\text{eff})}$ and the values of $\epsilon_x$, measured by the SLMs, to estimate the vertical emittance $\epsilon_y$ of the flat beam via the Touschek lifetime. The results (orange line in Fig. 2(d)) agree rather well with the fluctuations-based measurement.

During our measurements, the rms and the effective bunch lengths $\sigma_z, \sigma_{z}^{\text{eff}}$ were 26–31 cm and 24–30 cm, respectively, primarily due to intrabeam scattering. They were different because the longitudinal bunch shape was not exactly Gaussian due to beam interaction with its environment [36]. The relative rms momentum spread was $\sigma_p \approx 9.1 \times 10^{-6} \times \sigma_z[\text{cm}]$, based on the rf cavity and ring parameters. The expected zero-current values are $\sigma_x = \sigma_{x}^{\text{eff}} = 9 \text{ cm}$, $\sigma_y = 8.3 \times 10^{-5}$. The uncoupled case Twiss beta-functions in the undulator are $\beta_x = 204 \text{ cm}$, $\beta_y = 98 \text{ cm}$, for more details see [21].

Certainly, other emittance monitors (wire scanners, Compton-scattering monitors [37, 38]) could provide better resolution in IOTA. However, in some cases, if a bright synchrotron light source is available, our fluctuations-based monitor may be a better inexpensive non-invasive alternative. It has a unique feature, namely, its sensitivity increases with decreasing beam size. Indeed, smaller $\sigma_x, \sigma_y$ in Eq. (3) lead to greater fluctuations $\text{var}(N)$ in Eq. (1). Therefore, it may be particularly beneficial for existing and next generation low-emittance accelerators. There is a limit, however. If $\sigma_x \ll 1/(2k_0 \sigma_{\theta_z})$, $\sigma_y \ll 1/(2k_0 \sigma_{\theta_y})$, then $M$ and $\text{var}(N)$ become independent of $\sigma_x, \sigma_y$ — the radiation becomes transversely coherent. Let us use for estimation, $\sigma_k \ll k_0/(\hbar N_u)$, $k_0 = 2\pi\hbar/\lambda_1$ and $\sigma_{\theta_z} = \sigma_{\theta_y} = \gamma^{-1} \sqrt{(1 + K_z^2/2)/(\hbar N_u)}$. 

![Figure 2](image_url)
\[ \sigma_x, \sigma_y \ll \frac{\lambda_0}{8\pi\gamma} \sqrt{\frac{N_0}{h}} \left(1 + \frac{K^2}{2}\right). \]  

(9)

In IOTA, this corresponds to \( \sigma_x, \sigma_y \ll 50\mu m \) (in the undulator), or \( \epsilon_x \ll 1.0\,nm, \epsilon_y \ll 2.2\,nm \).

Lastly, the presented technique can be used to measure \( \epsilon_x \) and \( \epsilon_y \) individually by appropriately restricting the angular aperture. To illustrate, consider the model in Eq. (3) with an addition of a vertical slit in front of the detector. The effect of the slit can be approximated as a very small \( \sigma_{\theta_\epsilon} \). If the slit is so narrow that \( \sigma_{\theta_\epsilon} \ll 1/(2k_0\sigma_x) \), then

\[ M = \sqrt{1 + 4\sigma^2_x \sigma_y^2 \sqrt{1 + 4K^2 \sigma^2_\epsilon \sigma_y^2}}. \]  

(10)

i.e., \( \sigma_y \) can be deduced from the fluctuations even if \( \sigma_x \) is unknown. In addition to slits, masks can be applied to analyze fluctuations in various portions of the angular distribution of the radiation.

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