Econophysics of a religious cult: the Antoinists in Belgium [1920-2000]

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Abstract

In the framework of applying econophysics ideas in religious topics, the finances of the Antoinist religious movement organized in Belgium between 1920 and 2000 are studied. The interest of investigating financial aspects of such a, sometimes called, sect stems in finding characteristics of conditions and mechanisms under which definitely growth AND decay features of communities can be understood. The legally reported yearly income and expenses between 1920 and 2000 are studied. A three wave asymmetric regime is observed over a trend among marked fluctuations at time of crises. The data analysis leads to propose a general mechanistic model taking into account an average GDP growth, an oscillatory monetary inflation and a logistic population drift.

Keywords: econophysics; Antoinism; GDP; logistic; grow; decay
1 Introduction

In modern statistical physics nowadays [1], much work is pertaining to econophysics [2, 3, 4] and/or sociophysics [4, 5, 6]. Among such topics, religious communities have attracted some interest [7, 8, 9, 10, 11, 12, 13]. However the data is often rare, irregular and subject to caution. Moreover, data acquisition is often relying upon opinion pools or indirect statistical survey means. Their validity limit is usually far away from any accepted one in laboratory studies. Two recent examples can be used for illustrating the point, i.e.

\[ \text{Figure 1: Yearly income and expenses of the belgian Antoinist community as reported in the Moniteur Belge} \]
commenting upon (i) the evolution of the partial distribution function or (ii) the evolution of the number of adepts, - either of several religious denominations, on the one hand, or of a given religious movement, on the other hand:

- (i) the International Data Base (IDB) [15], see table 58, gives information on the population of 103 nations worldwide, between 1960 and 1992, specifying the number of adherents for 150 religions. However this is for about 2 billions people, i.e. only 1/3 of the present world population;

(ii) from the World Christian Encyclopedia [16, 17], an enormous compilation, one can obtain the number of adherents as a function of time, but only for 56 religions, from 1900 till 2000, - the data being averaged over a regularly distributed 5 years span.

- for a specific religious movement, i.e. Jehovah’s Witnesses, (i) the evolution in the number of ”publishers”, can be examined [18], but within 5 years averaging windows between 1935 and 1995;

(ii) the precise and reliable high frequency data used by Picoli and Mendes [13] is for 47 years, i.e. between 1959 and 2005.

On the other hand, from a theoretical point of view, there are plenty of studies about the growth or the decay of various ”populations”. Often the approach is based on prey-predator aspects [19], as in language [20, 21] and more generally ”opinion” competitions [22, 23]. This necessarily involves two differential equations. In contrast, the single, concise Verhulst [24] first order differential equation, was proposed to describe the growth of a community, with a finite life time, and can be adapted to describe a population decay. However such a first order differential equation to describe growth and decay on the same footing seems to be missing [25]. To my knowledge, only Bass model [26] describes a growth-death process along a first order differential equation. It is popular in marketing, but rarely used, if at all, in describing the evolution of socio-economic and scientific activities, though apparently governed by similar growth mechanisms, as often demonstrated. It is therefore of interest to find examples of complex systems or processes showing both growth and decay features, and to treat them with in mind a simple

found to obey Benford law.
first order differential equation as a support. Religions and sects are such organised systems. Thus, the possibility is hereby explored that concepts and methods from statistical physics may lead to a better understanding of the mechanisms governing the growth and decay of systems, e.g. of "churches", "sects", and "religious movements".

In order to do so, it is necessary to obtain enough reliable data points, over a sufficiently long time span. However, religious movements ("organized churches") have a long and often extraordinarily complicated life due to various events, whereas "sects" have a very short life and are often somewhat hiding; thus, in both cases, the data is intrinsically rare and spare, though for different reasons. In fact, religious movements and sects have very often no binding to publish data on their activities.

A religious movement community, the Antoinist cult [27, 28, 29], in Belgium, called a sect in France [30], is known to have had a rather huge growth since 1880 or so, but is much decaying nowadays. Moreover, it is of common knowledge that the evolution in the number of adherents of the Antoinist cult is not due to a prey-predator aspect, but rather stems from socio-economic condition improvements. Thus, it can be expected that some interpretation of some reliable, even indirect measure, data might lead to a description of causes ("forces") inducing the overall evolution and mutatis mutandis to suggest ingredients of a model for most, even more complicated, "religious" cases, whatever their size.

The "exact" number of adherents to the Antoinist cult is not known. The more so since the Antoinist cult [27, 28, 29] has neither proselytism nor wealth accumulation as active principles, but rather discretion and anonymous concerns. This goes against the strong recruitment policy as in all expanding cults, sects and religious movements [31]. Again, this indicates that some non-trivial evolution of the Antoinist cult should be expected [32].

Fortunately, yearly legal reports about the budget (expenses and income) of the cult community are known. The data, outlined in Sect. 2, extends between 1922 and 2002, i.e. grossly speaking over the 20-th century. In Sect. 3 it is found that several growth-decay regimes are in fact observable. Following an empirical analysis, a concise mechanistic-like model is proposed. It appears that the data can be understood taking into account various factor

\footnote{Interesting growth-decay cases, on a rather short life time, could be the communist party memberships in various countries, but the data on adherents would be highly unreliable, scribit A. Pękalski}
responses. A conclusion is given in Sect. 4.

2 The data set

The Antoinist cult [27, 28, 29], - see also culteantoiniste.com, developed during the life time of "Father" (Louis) Antoine (1846-1912), mainly in Belgium and France [30]. Father Antoine, after some catholic education, and some spiritism activity, turned toward predication and ritual healing. His "philosophy", based on ten "principles" [27, 28, 29], mainly appealed to steel and coal mine workers, first near Liège, Belgium. Then, the religious movement steadily expanded, as can be observed from the evolution in the number of "temples", - approximately 32 in each country and "reading rooms" [29] at the end of the 30's.

The community structure is legally recognised as an Etablissement d'utilité publique (Organism of Public Utility) in Belgium, since 1922. Thus, it must legally report regularly, every year, financial data, approved by the Cult Administration Board and its General Assembly. Such data must be published in the Belgian yearly official journal (Moniteur Belge), under the supervision of the Minister of Justice, in charge of "churches" and "religious movements" activities in Belgium.

The here below analysed financial data set was obtained through the Moniteur Belge issues, when available in the archives of the Antoinism Library in Jemeppe-sur-Meuse, Belgium. There are a few missing few data points. However, they are without much loss of content for the present discussion and conclusion, as it will be apparently seen below. Moreover due to a change in regulation, since 2002, the data has not to be published anymore in the Moniteur Belge, but deposited at some court office in Liège. In order to obtain them, one needs specific approbation from the Antoinist cult hierarchy, approbation which was presently refused. Thus only data up to 2000 are examined below.

The income is almost originating in (anonymous) gifts. The sales of books is included in such yearly accounts, but they turn out to be a weak contribution to the total sum, - after examining detailed data, not shown, before 1940. Almost all incomes are turned toward expenses, which are mainly for the construction and maintenance of the temples. It is important to stress that the so called "temple desservants" and "lecture room readers"

\footnote{A few temples are also found in Congo (DRC) and Brazil.}
are not paid. The difference between income and expenses goes into savings accounts, - savings which can be later on used. As seen on Fig.1 sometimes the expenses can turn out to be larger than the income, but the yearly budget has always been balanced.

3 Data analysis

3.1 Data visual inspection

The yearly income and expenses raw data, Fig.1, appear as pretty scattered. Notice that, yearly offerings can become very large: already a million of BEF in 1960. Yet, after visual inspection, it appears that, in both income and
Figure 3: Income (logarithmic scale) of the Antoinist community in Belgium suggesting three growth regimes; they are recognised through fits with a logistic function on selected time intervals.

\[ y = \frac{y_0}{1 + \exp(-b(t-t_0))} \]
expenses cases, three growth-decay regimes can be imagined, over weakly overlapping time interval boundaries: [1920-1939], [1939-1967], and [1967-2004]. It seems that one should emphasize that the time interval widths, in which the regimes appear, follow a simple but remarkable progression, i.e. 19, 28 and 37 years, apparently \((10 + 9 \times i)\text{years}\), where \(i\) indicates the order of the regimes over the 80 years or so for which the financial data can be examined.

For example, the presence of a maximum is seen near 1929 followed by a short decay till roughly 1940. A second growth regime is followed by a small decay at the end of the golden sixties. The next regime has a sharp rise which leads to a bump with a maximum near 1985 and seems to end at the beginning of the 21-st century. The latter regime is better seen than the previous ones because of the \(y-\) axis scale. In this regime, the evolution in the income and expenses is more stochastic, with two ”special” years (1976 and 1977) before and two ”very special” years after some sort of maximum. These incomes are attributed to huge and unusual gifts of deceased persons.

Such a historical evolution can be first qualitatively understood according to the social situation in Belgium during the 20-th century. Indeed, recall that Father Antoine, among one of his ”principles”, argued that sickness and low mood are merely questions of faith; since there is no evil, it is thus simply by faith that one can be healed: ... faith in oneself and in the Father, who by imposing hands could pass his fluid to those who could believe to get it and be healed. Such a theory was accepted by many workers, - see on ritual healing [33, 34] as a source of religion. It is easily understood that such a convincing appeal had to be reduced when the health care and other social conditions of workers improved. Whence, one expects some growth-decay regime(s) in the number of adepts, which might be reflected in the financial aspects of the community. Several temples are in fact closing nowadays.

Thus, e.g., from Fig.2 and Fig.3 it appears in semi-log plots, that three regimes can be conceived in both expenses and income evolutions. A test can be made with the growing logistic function, i.e. \(y/y_M = [1 + \exp(-b(t-t_0))]^{-1}\), with parameters appropriate in each regime. On the one hand, one observes that the growth always occurs on a longer interval of time than the decay. Also, the income growth rate is, at first sight, comparing Fig.2 and Fig.3, slightly larger than the expenses growth rate, except in the 3rd regime, - itself occurring after the golden sixties, when the absolute difference between income and expenses is much larger than in the previous regimes,- see Fig.1.
One could also expect that there should be, if the community is well managed, some time delay between the incoming money and the expenses. Time-delay correlations were looked for but no well marked conclusion could be derived, suggesting a short term, management of the finances of the community. Thus one concludes on an immediate reinvestment of the income, with some yearly savings, the evolution of these being not discussed further here. Nevertheless, the year boundary between the various income regimes seems to occur in slightly different years, (+/-1), that the one for the expenses regimes.

3.2 Model

It is obvious that the income and expenses span quite different BEF ranges depending on the time interval. One can note a sort of exponential growth over the century. A search for the trend indicates such an exponential growth indeed. It is therefore reminiscent of the average monetary inflation and GDP growth during the 20-th century:

$$y \simeq A e^{\alpha t} \quad \alpha > 0.$$ (1)

The Belgian GDP growth case is shown in Fig. 4, from estimated 35, after being reconstructed 36, 37, 1835-1990 data. It is noticed that $\alpha \simeq 0.078$. On such a figure, some sort of wave, with some fine correspondence to the Antoinist community financial data, seems also observable, over the exponential trend. Thus, the yearly incomes and expenses being necessarily positive numbers, one can conceive, for such oscillations, the most simple sort of periodic wave

$$y_i \simeq \sin\left(\frac{2\pi t_i}{2T_i}\right)$$ (2)

in each $i$ regime, with $i = 1, 2, 3$, $T_i$ being the corresponding semi-period.

A second aspect which might be taken into account pertains to demography, the possible change in the specific population size of the areas in which the religious movement was expanding. However this is not available. Some information is known concerning the whole country population; see e.g. [http://en.wikipedia.org/wiki/Demographics_of_Belgium]. However, the areas of expansion of the religious movement community were rather limited from a geographical point of view, occurring mainly in the industrial
Figure 4: Belgium GDP (PPP) suggesting a 3 wave pattern, beside an exponential growth trend; relevant dates for the 20-th century are indicated; time $t_0 = 0$ starts in 1920
Figure 5: Yearly expenses of the belgian Antoinist community as reported in the *Moniteur Belge*, with 4 different fits (see text), indicating a three wave structure.

areas. However, this possible population effect suggests to introduce a two-parameter logistic law

\[ \frac{y}{y_M} = \frac{1}{1 + e^{-rt}} \equiv \frac{e^{rt}}{1 + e^{rt}}, \]  

(3)

with usual meanings for \( y_M \) and \( r \), in order to better extract some specificity of the religious movement evolution. Moreover, the introduction of such a Verhulst-like law interestingly allows to reproduce some possible (though limited) proselytism effect, due to the so called ”country capacity”, in a demographic sense, of the areas.

Whence, one can introduce a term like

\[ \sim \frac{B_1 e^{\beta t}}{1 + B_2 e^{\beta t}} \sim \frac{B_3}{1 + B_4 e^{-\beta t}} \sim (1 + bt) \]  

(4)

on which the oscillating evolution is superposed.
In so doing, one may claim to have ingredients depending both on the adept offerings and indirectly their number. They support assumptions in adepts behavior, likely based on imitative behavior \[26, 32, 38\], and economic constraints, for modelling the community finances.

Therefore, both income and expenses data are finally proposed to behave along a concise expression

\[
y_i = A_i e^\alpha t_i \sin \left( \frac{2\pi t_i}{2T_i} + \phi_i \right) + (B_i + b_i t_i),
\]

with appropriate parameters, - to be determined in each regime. Notice that the \(\phi\) and \(B\) values can be taken equal to 0, if the time origin is chosen at some origin of the time interval \(i\) considered for the fit with the first term and if a single time origin is always chosen for the drift term whatever the time interval \(i = 1, 2, 3\). In practice for the drift term, the time origin has been imposed to be 1920, in order to have simple values of the \(x\)-axis ticks in

Figure 6: Yearly income of the belgian Antoinist community as reported in the Moniteur Belge, with best fit (see text), indicating a three wave structure
Table 1: Parameter values of the overall best fit, i.e. over the whole data range, see Fig.5, according to the model equation, Eq.(5), see text, for the reported yearly expenses of the belgian Antoinist community, assuming 3 different regimes, having each a different $T_i$, i.e., 38, 54 and 68 years respectively.

- The displays. Another condition on the fits has been to use integer numbers, both for years and financial numbers, these measured in BEF.

- Note that such nonlinear fits lead, as usual, to several solutions, with equivalent distributions of residuals, depending on the initial conditions imposed at the start of the trial and error process. Such a situation is illustrated on Fig.5, for the yearly expenses, where four different fits, with parameters given in Table 1, over appropriate regime existence intervals, have been found with rather equivalent residual distributions. The order of magnitude of the parameters remain quasi the same.

- The semi-periods $T_i$ of the sine wave have been imposed to be the same one in order to compare each fit meaningfully in each regime, but yet depending on the regime. Notice that the sum of $T_i$’s for the expense regimes, 38+54+68=160, i.e. correspond exactly with the time interval investigated (2000-1920=160/2). Notice also, from the graphs, a computational detail: in order to minimise the residual spread, the period $T_i$ does not correspond always exactly to half the time interval which is investigated, but can differ by a few years. This might be due to having imposed that $T_i$ be an integer. In the four evolution laws for the expenses, the drift term and the exponential growth term can be slightly different from each other, however without much significant difference.

- The income case is discussed from Fig.6. A fit following the model equa-

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4One could more exactly choose 1922, but this ”origin of time” approximation is not likely to be a drastic one, as one can expect.
Table 2: Parameter values of the best fits, see Fig.6, according to the model equation, Eq.(5), see text, for the reported yearly income of the belgian Antoinist community, assuming 3 different regimes.

| $i$ | $[t_0; t_1]$ | $A_i$ | $\alpha_i$ | $2T_i[y]$ | $B_i$ | $b_i$ |
|-----|--------------|-------|------------|----------|-------|-------|
| 1   | [1922;1940]  | $0.11 \times 10^6$ | 0.059      | 38       | 0     | $0.265 \times 10^4$ |
| 2   | [1941;1965]  | $0.11 \times 10^6$ | 0.059      | 56       | 0     | $0.615 \times 10^4$ |
| 3   | [1966;2000]  | $0.09 \times 10^6$ | 0.059      | 74       | 0     | $1.0725 \times 10^4$ |

The data analysis following the suggested model of financial evolution of the community, based on three effects from likely three different causes, leads to reasonable parameters: one fonds (i) coherence between the time intervals, (ii) the drift terms are characterised by a coefficient $b_i$ leading to a value similar to what is expected in demography, $\simeq 0.025$, concerning the global birth-death rate for the evolution of a population, like in Belgium during most of the 20-th century, and (iii) the exponent $\alpha$ is quite similar to that found for the average evolution of the GDP, e.g. of Belgium, during the 20-th century, Fig.4. Note that the T-fit in Fig. 5 is the best of all four: in fact it has an $\alpha$ exponent value, the largest $\simeq 0.062$, and the closest to the one found in Fig. 4 for the Belgium GDP, i.e. $\simeq 0.078$.

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\[ \text{Table 2: Parameter values of the best fits, see Fig.6, according to the model equation, Eq.(5), see text, for the reported yearly income of the belgian Antoinist community, assuming 3 different regimes.} \]

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\[ \text{Observe that the growth-decay curves appear asymmetric.} \]

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\[ \text{Such asymmetry between growth and decay also characterises economic cycles.} \]
4 Conclusion

In summary, the objectives of the study were to find (i) a community for which regular and trusted data exist, using such financial data to compensate the lack of knowledge on the number of adherents, (ii) expectedly behaving with a growth and decay history, (iii) such that one can go beyond a Verhulst population description, but without using a Lotka-Volterra (prey-predator) approach, (iv) finding qualitatively reasonable causes forcing the behaviour in order to describe the evolution along quantitative modelling aspects, and (v) hopefully suggesting a non controversial benchmark case, before studies on more complex community systems.

The income and expenses of the Antoinist cult community in Belgium have been found to fulfill such objectives, and much more. In the main text, a good explanation why the experimental curves have such a theoretical form seems to have been reached, with very reasonable values of the parameters. The ”model” indicates that such religious communities are markedly influenced by external considerations (”external fields”), besides their intrinsic ”religious” goals. Practically, in the present case, as illustrated, the crash of 1929 induces a drop in income, but the second world war increases the community strength. The golden sixties ”reduce” the income: the adepts well being increased, but the adherents reduced their offering, becoming in some sense more egoistical. Therefore, one can deduce that there are two different causes for the drop in income: either a lack of money of the adepts, or in contrast, paradoxically, ”too much” wealth. Similarly, the increase in the religious movement income, at its legal beginning, may result from the enthusiastic thanking for healing the suffering, both of the soul and the body, - but also occurs due to the income explosion until 1985. The variation in expenses are immediately related with such income considerations.

Therefore such a ”model”, - still needing an interpretation of the so called non-universal ”amplitudes”, contains apparently rather ”universal”-like laws, with power exponents, and is expected to be applicable to other societies, - not only religious ones. It seems plausible, that many more societies will be found to have the same behavior and for similar reasons.

In conclusion, it is of interest to study the growth and decay of religious movements, churches, cults from an anthropology [38 10 41 42 43 44], see also [45], or historian [46], but also from a sociology [47] or economist [48] point of view, but combining these also in studies having a socio-physics one [19]. It has to be emphasized that the Antoinist cult was appealing be-
cause of the suffering of people, working under very hard conditions in the Liège area, when Père Antoine started to preach and to give psychological remedies, ”principles”, for accepting one’s life, and demonstrated his healing power. One crucial aspect of the religious movement concerns its survival under much improved economic, social, and health conditions of workers to whom the Père Antoine’s philosophy appealed. Indeed a marked decay in income occurs at the end of the 20-th century, inciting to conclude to a doomed situation, according to the theory in [32], - in contrast to the Jehovahs Witnesses [18]. Indeed, ideologies (whether religious or secular) seem to lack coherence and potency unless they are developed and promulgated by vigorous formal organisations and social movements [50]. Due to the present economic and financial crisis, a phoenix effect [23, 51] might nevertheless take place again.

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