Impurity in a Fermi gas under non-Hermitian spin–orbit coupling

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Abstract. We study the fate of an impurity in a two-component, non-interacting Fermi gas under a non-Hermitian spin–orbit coupling (SOC) which is generated by dissipative Raman lasers. While SOC mixes the two spin species in the Fermi gas thus modifies the single-particle dispersions, we consider the case where the impurity only interacts with one of the spin species. As a result, spectral properties of the impurity constitute an ideal probe to the dissipative Fermi gas in the background. In particular, we show that dissipation destabilizes polarons in favor of molecular formation, consistent with previous few-body studies. The dissipative nature of the Fermi gas further leads to broadened peaks in the inverse radio-frequency spectra for both the attractive and repulsive polaron branches, which could serve as signals for experimental observation. Our results provide an exemplary scenario where the interplay of non-Hermiticity and interaction can be probed.

1 Introduction

Non-Hermitian systems have attracted much research interest lately due to their unique properties. In general, the dynamics of a system can effectively be driven by a non-Hermitian Hamiltonian when it is coupled to an environment [1]. While some non-Hermitian systems still acquire purely real eigenspectra thanks to the parity-time symmetry [2, 3], some may possess properties such as skin effects and non-Bloch bulk-boundary correspondence [4–12], with deep topological origins that are non-existent in their Hermitian counterparts [13, 14]. A key issue in the study of non-Hermitian systems is the interplay of interaction and non-Hermiticity in a many-body setting [15–32]. It has been shown that such an interplay gives rise to significantly modified dynamics in bosons [15–17], as well as unconventional pairing superfluid in fermions [19, 20]. In a very recent study, a non-Hermitian version of the spin–orbit coupling (SOC) has been proposed [32], under which the SOC-dressed and dissipative single-particle dispersion results in a dissipation-facilitated molecular state. The impact of such a non-Hermitian synthetic gauge field in a genuine many-body system is still largely unexplored.

In this work, we bridge this knowledge gap by investigating an intermediate scenario interpolating the few- and many-body physics under a non-Hermitian SOC. Specifically, we study an impurity immersed in a two-component Fermi gas under the dissipative SOC.
light of recent experimental progress in dissipative cold atomic gases [33–39], our results would be helpful for future experimental studies.

Our work is organized as follows. In Sect. 2, we present our model and the single-particle dispersion. In Sect. 3, we show the complex energy spectra of attractive and repulsive polarons, as well as those of molecular states. We then calculate the inverse r.f. spectroscopy in Sect. 4. A summary is given in Sect. 4.

2 Model and single-particle dispersion

Following the derivation in Ref. [32], a non-interacting, two-component Fermi gas under a non-Hermitian SOC is governed by the non-Hermitian Hamiltonian

$$H_f = \sum_k \Psi^\dagger_k h_f \Psi_k,$$

where

$$h_f = \begin{bmatrix} \frac{\hbar^2}{2m} \left( k + k_0 e_x \right)^2 - i \Gamma_x & \Omega - i \Gamma_x \\ \Omega - i \Gamma_x & \frac{\hbar^2}{2m} \left( k - k_0 e_x \right)^2 - i \Gamma_x \end{bmatrix},$$

and \( \Psi_k = [a_{k\uparrow}, a_{k\downarrow}]^T \), with \( a_{k\sigma} \) the annihilation operator of fermions with momentum \( k \) and spin \( \sigma \) (\( \sigma = \uparrow, \downarrow \)). Here, \( \Omega - i \Gamma_x \) (\( \Gamma_x > 0 \)) is the SOC amplitude, \( 2\hbar k_0 \) the momentum transfer in the Raman process generating the SOC, \( m \) is the atomic mass.

The single-particle dispersion of fermions \( \xi_{k,\pm} \) can be derived by diagonalizing \( h_f \), with

$$\xi_{k,\pm} = \frac{\hbar^2 (k^2 + k_0^2)}{2m} - \Omega \pm \sqrt{\left( \frac{\hbar^2 k_0 k_x}{m} \right)^2 + (\Omega - i \Gamma_x)^2},$$

where \( \pm \) labels the two helicity branches. The corresponding creation operators \( a_{k,\pm}^\dagger \) for the right eigenstates of the helicity branches are given by

$$a_{k,\pm}^\dagger = c_1 + a_{k,\uparrow}^\dagger + c_2 + a_{k,\downarrow}^\dagger,$$

$$a_{k,\pm} = c_1 - a_{k,\uparrow} + c_2 - a_{k,\downarrow},$$

where

$$c_{1\pm} = \frac{\sqrt{(\Omega^2 + \Gamma_x^2)^2 - \epsilon_k^2}}{\sqrt{\rho} [2\epsilon_k \rho \cos \frac{\theta}{2} \pm (\Omega^2 + \Gamma_x^2 - \epsilon_k^2 - \rho)]},$$

$$c_{2\pm} = -\epsilon_k^\prime \pm \sqrt{\rho e^{i\theta/2}} \times c_{1\pm}.$$

Here, \( \epsilon_k^\prime = \hbar^2 k_0 k_x / m, \rho = (\epsilon_k^2 + \Omega^2 - \Gamma_x^2)^2 + 4\Omega^2 \Gamma_x^2 \), and \( \tan \theta = -2\Omega \Gamma_x / (\epsilon_k^2 + \Omega^2 - \Gamma_x^2) \). In Fig. 1, we show the numerically evaluated single-particle dispersion. For numerical calculations, we take \( E_F = \hbar^2 k_F^2 / 2m \) as the unit of energy, where \( k_F = (3\pi^2 n)^{1/3} \) and \( n \) is the density of the Fermi sea. Throughout the work, we will focus on the parameter regime where a double-well structure exists in the lower helicity branch. Other configurations of the single-particle dispersion exist, which could considerably complicate the calculation. We therefore leave these cases to future studies.

Under the non-Hermitian Hamiltonian \( H_f \), right single-particle eigenstates \( |k, \pm\rangle_R \) are different from the left ones \( |k, \pm\rangle_L \), where \( H_f |k, \pm\rangle_R = \xi_{k,\pm} |k, \pm\rangle_R \) and \( H_f^\dagger |k, \pm\rangle_L = \xi_{k,\pm}^\ast |k, \pm\rangle_L \). However, right/left eigenstates are not orthonormal nor complete by themselves; rather, we have \( \langle L|k_1, \lambda_1|k_2, \lambda_2 \rangle_R = \delta_{k_1,k_2}\delta_{\lambda_1,\lambda_2} \) and \( \sum_{k,\lambda} |k, \lambda\rangle_R \langle k, \lambda | = 1 \), where \( 1 \) is the identity operator. It is therefore convenient to adopt a biorthogonal description, with the additional field operators

$$a_{k,\pm}^{L/R} = \frac{c_2 - c_1 + c_2^{\pm} - c_1 - c_2^{\pm}}{c_1 - c_1 + c_2^{\pm} - c_1 - c_2^{\pm}} a_{k,\pm},$$

where \( |k, \pm\rangle_R = a_{k,\pm}^{R1}\psi_{vac} \) and \( |k, \pm\rangle_L = a_{k,\pm}^{L1}\psi_{vac} \), such that \( \{ a_{k,\lambda}^{R1}, a_{k,\lambda}^{R1}\} = \delta_{k,k'}\delta_{\lambda,\lambda'} \) [19,30,32].

For the many-body setting, we assume that the Fermi energy \( E_h \) is the real part of the total energy of a Fermi sea filling the complex single-particle spectrum, where single-particle states with \( \text{Re}(\xi_{k,\pm}) < E_h \) are occupied. We focus on the case where \( E_h \) is larger than the peak of the central barrier of the lower helicity branch, but smaller than the lowest energy of the higher helicity branch in Fig. 1a, so that only a single Fermi surface exists in the system. The actual Fermi energy \( E_h \) is then a function of total density, as well as the SOC parameters [40]. An impurity interacts with the spin-up component of the Fermi sea, with the Hamiltonian

$$H_{\text{imp}} = \sum_k \epsilon_k b_k^\dagger b_k + \frac{U}{V} \sum_{k,k',\lambda} a_{k,\lambda}^{\dagger} a_{\frac{k',\lambda}{2}} b_{\frac{k,\lambda}{2}} b_{-k',\lambda} a_{\frac{k,\lambda}{2}} a_{\frac{k,\lambda}{2}}.$$

where \( b_k \) (\( b_k^\dagger \)) the annihilation (creation) operator of the impurity atom, \( V \) is the quantization volume, and \( \epsilon_k = \hbar^2 k^2 / 2m \) is the kinetic energy of the impurity atom, with the assumption that the impurity has the same mass as that of a fermion. The bare interaction strength \( U \) is related to the s-wave scattering length \( a_s \) between the impurity and the spin-up state through the standard renormalization relation in three dimensions

$$1/U = \frac{m}{4\pi \hbar^2 a_s} - 1/V \sum_k 1/(2c_k).$$

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3 Polarons and molecules

Following the Chevy’s ansatz [41,42], we write the polaron and molecular states as

$$|P\rangle = (\phi_0 b_+^\dagger + \sum_{\lambda_1, \lambda_2, \lambda} \phi_{k, q}^{\lambda_1, \lambda_2} b_{-k, \lambda_1}^\dagger a_{q, \lambda_2}^R |FS\rangle)_N,$$

$$|M\rangle = \sum_{\lambda, k} \phi_{k, \lambda} b_{-k, \lambda}^\dagger a_{k, \lambda}^R |FS\rangle_{N-1},$$

where $\phi_0$, $\phi_{k, q}^{\lambda_1, \lambda_2}$, and $\phi_{k, \lambda}$ are the corresponding wave functions, $|FS\rangle_N$ represents a Fermi sea with $N$ fermions. We note that it is not a priori clear whether the Chevy’s ansatz should provide a good description for the non-Hermitian system discussed here. Nevertheless, in a Hermitian setting, it is known that Chevy’s ansatz can provide a reasonably good description for repulsive polarons, which acquire a finite width (imaginary self-energy) due to their decay to low-lying states [41,42]. While a key difference here is that the dissipative nature of the polaron comes from the single-particle dispersions of the environment, we still expect the Chevy’s ansatz to provide a reasonable approximation, as long as the polaron can still be viewed as a quasi-particle, i.e., an impurity dressed by the surrounding Fermi gas. Such a picture should be valid on a self-consistent level, when the calculated imaginary component of the polaron energy is small compared to the real component. We also note that the introduction of left field operators $a_{k, \lambda}^L$ in the ansatz above is consistent with the understanding that Fermi sea consists of right eigenstates of the non-Hermitian single-particle Hamiltonian $H_t$, with $|FS\rangle_N = \sum_{E < E_h} a_{k, \lambda}^R |\text{vac}\rangle$.

From the Schrödinger’s equations ($H_t + H_{\text{imp}})|P(M)\rangle = E_{P(M)}|P(M)\rangle$, we have the closed equations

$$E_P = \sum_{\lambda_2 = \pm, q} \frac{\beta_{q, \lambda_2}^2}{U - \sum_{\lambda_1 = \pm, k} E_{P} - \epsilon_{q, -k, \lambda_1, \lambda_2}^L - \xi_{\lambda_1, \lambda_2, q}},$$

$$1/U = \sum_{\lambda = \pm, k} \frac{c_{1\lambda} \beta_{k, \lambda}^2}{E_{M} - \xi_{k, \lambda} - \epsilon_k}(-1)^\lambda,$$

where $\beta_{q, \lambda_2}^2 = (c_{1+2} - c_{1+2}^- - c_{1-2}^+ - c_{1-2}^-)/2(c_{1+2}^- + c_{1-2}^+)$, $\beta_{k, \lambda}^2 = -(c_{1-2}^- - c_{1-2}^+)/2(c_{1+2}^- + c_{1-2}^+)$. While both $E_P$ and $E_M$ are complex, the ground state of the system, which is essentially a quasi-steady state in the short time scale, can be determined by comparing Re($E_P$) and Re($E_M$) - $E_h$. 

Fig. 1 Real component of the single-particle dispersion under a non-Hermitian SOC for $\Omega = 0.2$, $\Gamma_e = 0.3$, and with a fixed $k_0 = k_0/k_F = 0.5$. Color code indicates the spin-up fraction. See main text for the definition of $\theta$. While the single-particle dispersion can take different shapes by tuning $\Omega$ and $\Gamma_e$, we focus on the case where the lower helicity branch has a double-well structure, and the Fermi surface (dashed line) lies within the gap of the lower and upper helicity branches. The dimensionless parameters are defined as $\tilde{\Omega} = \Omega/E_P$, $\tilde{\Gamma_e} = \Gamma_e/E_P$, $\tilde{k}_0 = k_0/k_F$. The unit of energy $E_P$ and wave vector $k_F$ are defined in the main text.

Fig. 2 a Real components of the attractive polaron Re($E_P$) (blue) and molecule Re($E_M$) - $E_h$ (red) energies with varying interaction strength. b Imaginary components of the attractive polaron Im($E_P$) (blue) and molecule Im($E_M$) (red) energies. c Real components of the attractive polaron Re($E_P$) (blue) and molecule Re($E_M$) - $E_h$ (red) energies with increasing $\Gamma_e$. d Imaginary components of the attractive polaron Im($E_P$) (blue) and molecule Im($E_M$) (red) energies. e Ratio between the imaginary and real components of the attractive polaron (blue) and molecule (red) energies, with varying interaction strength. f Ratio between the imaginary and real components of the attractive polaron (blue) and molecule (red) energies, with varying dissipation rate. We set $\tilde{\Omega} = 1$ and $\tilde{\Gamma_e} = 0.5$ in (a)-(b); $\tilde{\Omega} = 1$ and $1/(k_F a_s) = 1$ in c, d. We fix $k_0 = 0.5$, and the dimensionless parameters are defined in the same way as those in Fig. 1, with Re($E_P$) = Re($E$)/$E_P$ for instance.
In Fig. 2, we show the real and imaginary components of the polaron (blue) and molecular (red) energies, as functions of the interaction strength [(a)(b)] and dissipation [(c)(d)]. Conventionally, a polaron or molecular excitation in a many-body setting is considered ill-defined when the imaginary component of its self-energy is large compared to the real component [41,42], particularly as their dispersions cannot be arranged in a quadratic form with an interaction-induced effective mass. For our case here, we find that both the polaron and molecule remain well-defined either deep in the BEC regime where interaction effects dominate over dissipation, or close to resonance but with small dissipation $\Gamma_x$ [see Fig. 2e, f]. Further, a polaron–molecule transition can still be observed in Fig. 2a. This is illustrated in detail in Fig. 3, where the polaron–molecule transition is found to shift toward the Feshbach resonance with increasing dissipation. This result is consistent with the previous finding that dissipation can facilitate molecular formation in vacuum.

Equation (13) also allows us to solve for polarons of the repulsive branch. This is shown in Fig. 4, where we demonstrate how the real and imaginary components of the repulsive polaron energy vary with interaction [(a)(b)] and dissipation [(c)(d)]. Under a fixed dissipation strength, the imaginary components of the repulsive polaron energy decrease rapidly toward the BEC regime. This originates from the fact that the attractive polaron branch becomes much lower in energy, and that dissipation effect from the non-Hermitian SOC becomes suppressed in the strong-interaction regime. Therefore, despite a decreasing tendency of the polaron energy in the BEC regime, the repulsive branch remains well-defined. On the other hand, when dissipation becomes sufficiently large, the width of the repulsive branch should become appreciable, and the polaron description would fail.

4 Detecting polarons: r.f. spectroscopy

While the impurity serves as a probe for the non-Hermitian Fermi sea, it can leave key signatures in the r.f. spectroscopy. We focus on the inverse r.f. spectroscopy here, where we prepare the impurity atom in a bystander state that does not interact with the Fermi sea, and then couple the bystander state to the actual impurity state that interacts with the spin-up fermions. We note that such a scheme does not require a detailed protocol on preparing the quasi-steady polaron state: it can be resolved so long as it is a well-defined quasiparticle with the imaginary component of its energy much smaller than the real component.

Following the linear-response approach outlined in Ref. [32], the population transfer or the signal of the inverse r.f. spectroscopy can be written as:

$$R_i(\omega) = -\text{Im} \sum_k \sum_{i\omega_n} G_{\text{imp}}(k, i\omega_n) G_{11}(k, i\omega_n - \omega)$$

where $\omega$ and $\omega_n$ are both Fermionic Matsubara frequencies, $G_{\text{imp}}(k, i\omega_n)$ is the Green’s function for the impurity state, with

$$G_{\text{imp}}(k, i\omega_n) = \frac{\langle \text{FS} | b_k | \Phi_F^R \rangle \langle \Phi_F^L | b_k^\dagger | \text{FS} \rangle}{i\omega_n - E_j}.$$  

(16)
Fig. 5 a Inverse r.f. spectra $R(\omega)$ for $\tilde{\Gamma}_x = 0.1$ (black dashed), $\tilde{\Gamma}_x = 0.5$ (blue solid), and $\tilde{\Gamma}_x = 1$ (red dash-dotted). We take $\tilde{\Omega} = 1$, $1/(k_Fa_s) = 1$ for the calculation. b Comparison of the real component of polaron energies from the variational calculations Eq. (13) (blue) and the peaks in the r.f. spectrum (red dashed). We take $\tilde{\Omega} = 1$, $\tilde{\Gamma}_x = 0.5$, and $\tilde{k}_0 = 0.5$ here.

Here, $|\Phi_{R(L)}^j\rangle$ is the right (left) polaron state with energy $E_j^\pm (E_j^*)$. The subscript $j$ runs over both the attractive and repulsive polaron branches, following the spirit of pole expansion of quasi-particles. $G_{11}(k,i\omega_n)$ is the Green’s function for the non-interacting bystander state, with

$$G_{11}(k,i\omega_n) = \frac{1}{i\omega_n - \epsilon_k}, \quad (17)$$

Performing the Matsubara sums and taking the zero-temperature limit, we have

$$R_i(\omega) = -\text{Im} \frac{1}{\omega - \Sigma(0,\omega)}, \quad (18)$$

where the self-energy is given by

$$\Sigma(0,\omega) = \sum_{\lambda_2=\pm q} U^{-1} - \sum_{\lambda_1,k} \frac{\beta_{\lambda_2}^\lambda_2}{\omega - \epsilon_{q-k}^\lambda_1 - \xi_{\lambda_1,k} + \xi_{\lambda_2,q}^\lambda_2}. \quad (19)$$

In Fig. 5a, we show the calculated inverse r.f. spectra with different dissipation strength $\Gamma_x$. Both attractive and repulsive polaron branches are clearly visible as peaks in the spectrum, where the widths of peaks increase for larger $\Gamma_x$. Note that compared with Eq. (13), $\omega$ on the right-hand side of Eq. (19) is real. Therefore, under the non-Hermitian SOC, peaks in the inverse r.f. spectrum do not exactly correspond to $\text{Re}(E_P)$ calculated from Eq. (13). This is in sharp contrast to the Hermitian case. However, as we illustrate in Fig. 5b, positions of the spectral peaks do not deviate much from polaron energies calculated using the variational approach. Hence, the dissipative polarons can be fully resolved by the inverse r.f. spectroscopy.

5 Summary

In this work, we study the quasi-particles induced by an impurity in a two-component Fermi gas under a non-Hermitian SOC. We show dissipation affects the polaron–molecule transition in the system, stabilizing the molecular state. We further demonstrate that both attractive and repulsive polarons can be resolved by the inverse r.f. spectroscopy, thus providing an ideal experimental probe to the system. Our configuration has highly tunable parameters: the dissipative SOC is directly tunable through laser parameters of the Raman process; the interaction is tunable through the Feshbach resonance; the Fermi surface can be adjusted by controlling the fermion densities as well as the SOC parameters. Further, our detection scheme using the inverse r.f. spectroscopy does not require a polaron state to exist in the dissipative system prior to detection, thus circumventing a key difficulty in non-Hermitian many-body systems where stringent requirements on the hierarchy of time scales may hinder experimental preparation and detection. The remaining key time scale is that the inverse r.f. spectroscopy be probed within $t \ll \hbar/\Gamma_x$, where $H_I$ is still valid. Our work therefore provides an ideal scenario for the investigation of non-Hermitian many-body systems.

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Data availability This manuscript has no associated data or the data will not be deposited. [Authors’ comment: This is a theoretical study and no experimental data has been listed.]

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