Formation of new phase inclusions in the system of quasi-equilibrium magnons of high density

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The paper studies the spatial variation of the magnetization in a nonconducting magnetic sample with an excess number of magnons in comparison to the equilibrium. The phenomenon is considered using the Landau-Lifshits equation with additional terms describing the longitudinal relaxation of the magnetization, the magnon diffusion and the magnon creation by external pumping. The free energy of the system is presented in the mean field approximation. It is shown that, if the pumping exceeds some critical value, regions of the new phase arise where the magnetic moments are oriented opposite to the magnetization of the magnetic sample. The phenomenon is similar to the appearance of droplets of the condensed phase in the supersaturated vapor. The appearance of the new phase either in the form of a single domain or a periodical lattice is demonstrated. The studied process is a competitor to the process of the Bose-Einstein condensation of magnons.

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I. INTRODUCTION

Spatial structures in non-equilibrium nonlinear systems have been studied widely and successfully during the last half century. The processes of the formation of such structures are referred to as the non-equilibrium phase transitions. Non-equilibrium phase transitions are various and in their majority differ from the equilibrium phase transitions. There is a class of specific non-equilibrium phase transitions in systems of particles (or quasiparticles in crystals), which have finite values of the lifetime. If combining particles produces an energy gain and the lifetime of particles is much larger than the time of inter-particle collisions, the particles may form a condensed phase. The formation of drops of the electron-hole condensed phase coexist, the spatial distribution of the same parameters for the infinite value of the lifetime. But for the range of parameters, where the gas and the lifetime of the lifetime particles has particular features which will be discussed later.

Magnons in magnetic materials are the classical example of particles with the finite lifetime. The chemical potential of magnons is equal to zero in the equilibrium state because their number is not conserved due to the magnon-magnon and magnon-phonon interactions. In presence of an external pumping, additional magnons appear besides the equilibrium magnons. As the result, the chemical potential is not equal to zero. Magnons are Bose particles and when their concentration exceeds some threshold value, the appearance of the Bose-Einstein condensation (BEC) could be expected together with its interesting manifestations: accumulation of particles at a certain level, superfluidity and so on. An interesting effect was observed in the works in which the authors investigated magnons in the yttrium-iron-garnet (YIG) films. The magnons were excited by the parametric longitudinal pumping. The analysis of the magnon spectra was carried out using Brillouin light scattering (BLS) spectroscopy. The authors showed from the analysis of experiments the manifestation of the Bose-Einstein condensation of magnons. In particular, an increase of the magnon concentration in the state with the wave vector which corresponded to the minimum of the magnon band was observed. The appearance of the spontaneous coherence in BLS by magnons was observed if the pumping exceeded a critical value. The emergence of the periodical variation of the magnon density at a high level of the magnon excitation was also demonstrated. The latter phenomenon was explained by the presence of two minima in the magnon dispersion law leading to the formation of two condensates and to the interaction between the condensates.

Since then the investigation of the magnon condensation in YIG obtained further development in numerous works which further advanced the explanation of the phenomenon observed and suggested different other effects related to Bose-Einstein condensation. Thus, the stability of BEC in the high density magnon system in YIG was analyzed in the paper. The microwave emission from the uniform mode generated by BEC studied in. The spatial structure of interacting bosons with two minima in the dispersion law was investigated in the papers. The dramatic peak in the density of the proposed condensed magnons after switching off the pumping was observed in the work. It is interesting that the time by the peak increased coincided with the time of the magnon decay and the time by the peak decreased was much larger than the magnon decay time. The problem of the spin current in the system of an isolator and a conductor was theoretically studied under the
condition of BEC of magnons in the isolator in\cite{14}. In the papers\cite{15,16}, the Josephson oscillations in the magnon density between two spatially separated magnon clouds were calculated and also the methods of the magnon current measurement are analyzed. In the paper\cite{17} the temporal decrease of the magnon condensate density in YIG at the gradient of the temperature, created by a laser, after the pumping shutdown was observed. The authors explained this effect by the appearance of a supercurrent in the condition of BEC. But there are works in which the doubt is expressed about the correctness of the interpretation of the results observed in\cite{14}. The authors of the work\cite{18} showed that the reason of an accumulation of particles created by pumping at the lowest state may be caused by the peculiarities of the Bose-Einstein condensation of quasi-particles. In the paper\cite{19} the authors described the time evolution of the magnon condensate under pumping by the classical stochastic Landau-Lifshitz-Gilbert equation including magnon-phonon hybridization and came to the conclusion that the phenomenon observed in\cite{14} has a purely classical nature.

In the current paper, we present another version of the processes in a ferromagnet with the magnon density exceeding the equilibrium value. We show that, if there are additional magnons, created by an external pumping, the evolution of the system may choose the scenario alternative to the Bose-Einstein condensation. This new scenario is the formation of regions in the ferromagnetic material where the magnetic moments are oriented opposite to the orientation of the magnetic moment of the sample. Similar to the Bose-Einstein condensation, this phenomenon appears in crystals with the magnon density higher than the equilibrium value.

The system of magnons is in a way equivalent to a gas of particles. If the concentration of the particles exceeds the equilibrium value, the gas is referred to as "over-saturated" or "supersaturated". The processes of the precipitation of the regions of the new phase are known to occur in the over-saturated gas. Similar systems arise also in mixtures of liquids or solids after rapid cooling. There are two popular models that describe processes of the unmixing of mixtures from one thermodynamic phase to form two coexisting phases: the model of the spinodal decomposition\cite{20,21} and the model of the nucleation and growth\cite{22}. The subject of our interest, the system with the magnon pumping, is "over-saturated" with magnons. So, during the relaxation the individual magnons would cluster forming inclusions of the new phase. Within these inclusions, the orientation of the magnetic moments would be opposite to the magnetic moment of the crystal. A qualitative picture for the dynamics of the magnon system is shown in Fig. 1.

Let us assume that the magnon state of Fig. 1 presents the uniform quasi-equilibrium magnon distribution which arises due to both the thermal excitation and the external pumping. There are two scenarios for the further development of the uniform magnon distribution. According to the first scenario, the Bose-Einstein condensation shall occur if the magnon concentration exceeds the critical value. Such process is investigated in the cited papers\cite{4,5}. But, the second scenario, according to which the formation of the new phase occurs in the over-saturated magnon system, is also realistic. There is a strong short-range interaction between magnons. When magnons are collected in a cluster, the energy per magnon decreases by the value of order of 0.1 eV\cite{24}, which significantly exceeds the thermal energy at the room temperature. The equilibrium state of the system is determined by the minimum of the free energy and not by the minimum of the energy. But, if the magnon concentration exceeds the equilibrium one, the magnon clusters have to form with the opposite orientation of their magnetic moments to the magnetic moment of the other part of the crystal. The magnon clusters are inclusions of regions of the magnon condensed phase. But it is not the Bose-Einstein condensation, it is the conventional condensation in the coordinate space due to the interaction between particles. The clusters of the new condensed phase may be shaped variously. A structure in the form of a single domain is drawn in Fig. 1. The presented paper investigates the processes of the formation of the regions of the new phase in the supersaturated magnon gas.

Because the magnon lifetime is finite, the arising structures may exist only during the continuous magnon pumping. Similar studies of the phase transitions in the systems of particles with the finite lifetimes have been carried out for different types of quasi-particles: for the radiation defects\cite{24,27}, for the excitons\cite{28,29}. The the-
ories of these works have modified and generalized the stochastic model of the nucleation and growth (Lifshiz-Slyosov\textsuperscript{33}) and the model of the spinodal decomposition (Cahn-Hilliard\textsuperscript{20,21}) to make them applicable to systems of particles with a finite lifetime. The theories have been successful in explaining the unconventional experimental results obtained by different authors (mainly by the Timofeev’s\textsuperscript{34} and Butov’s\textsuperscript{35} groups) during investigation of the light emission by excitons from the double quantum well heterostructures in semiconductors at low temperature. Excitons were created by lasers and, at high density, formed islands of the excitonic condensed phase. Sometimes the islands were localized periodically in the space. The references to the applications of the theory of phase transitions in systems of unstable particles to the explanation of experiments with excitons are given in\textsuperscript{31,32,35}. The formation of the new phase in a system of unstable particles has distinct features compared to the phase transition in the system of stable particles. The distinctions include: 1) the size of the regions of the new phase is restricted, 2) there is a correlation between the regions of the new phase, which may cause the appearance of periodical structures, 3) the regions of the new phase exist only at the external pumping. The structures are the results of self-organization processes in non-equilibrium systems.

In the presented paper, we apply to the many-magnon system the approach developed in the above papers (the paper\textsuperscript{35} and the references therein) devoted to the study of the phase transitions in systems of unstable particles. We shall show the possibility of an appearance of the new phase inclusion in the magnetic with supersaturated magnon gas. The qualitative analysis, given in the last section, argues that effects caused by the new inclusions may be similar to the effects, which were observed in\textsuperscript{31,32} and explained by the manifestation of BEC.

II. TAKING INTO ACCOUNT MAGNON DIFFUSION AND MAGNON PUMPING INTO EQUATION FOR MAGNETIZATION

We shall consider a nonconducting magnetic crystal in the magnetic field oriented along the crystalline axis $\hat{Z}$. We assume that the non-equilibrium magnons are excited in the system by the two-magnon longitudinal pumping and the number of magnons is larger than the equilibrium one. Due to the strong magnon-magnon and magnon-phonon interactions, the magnons are in the quasi-equilibrium state. Our aim consists in the determination of the spatial variation of the magnetization $\mathbf{M}$. To this end, we shall study the clustering of magnons into a new phase with the creation of regions that have the magnetic moment oriented opposite to the orientation of the main magnetization. After the creation of the regions of the new inverse phase, the magnetization is non-uniform, though the initial system and external fields (the static magnetic field, the pumping) are assumed to be uniform. The processes of the self-organization in the system spontaneously break the symmetry. Describing the non-uniform system we assume that the principle of the local equilibrium holds. In this case, in a small vicinity of some spatial point, the thermodynamic functions are the same functions of the local microscopic variables (magnon density, temperature) as in the equilibrium system. This assumption allows introduction of the free energy in non-equilibrium system. The local free energy depends on the magnon density and the magnon density depends on the spatial coordinates. The principle of the local equilibrium is conventionally used in the majority cases when considering the self-organization problems in non-equilibrium systems\textsuperscript{36}.

We shall use the phenomenological approach for the solution of the problem. Let us analyze the phenomenological equation for the magnetization $\mathbf{M}$, solution of which will be investigated in the paper. The equation for the change of the magnetization in the unit time contains the dynamic and the relaxation parts. The relaxation terms of the Landau-Lifshits (LL) and the Landau-Lifshits-Gilbert (LLG) equations cannot be used in our paper because they require the conservation of the magnetization. The pumping creates magnons and decreases the absolute value of the magnetization. The LL and LLG equations do not describe the equilibration of the magnetization after the pumping is switched off. Taking into account the processes of the establishment of the equilibrium state is important in a description of the non-equilibrium system. The equation for the evolution of the magnetization which do not require the conservation of the absolute value of the magnetization is given in the monograph of Akhiezer, Baryakhtar and Peletminskii\textsuperscript{37}. But neither the equation in\textsuperscript{37} nor the LL and LLG equations take into account the diffusion of magnons. The diffusion processes induce spatial redistribution of magnons due to the interaction between them, which may be a reason for the formation of the new phase. The diffusion is important in the formation of the new phase at spinodal decomposition processes\textsuperscript{20,22}. So, we presented the main equation for the magnetization in the form given in\textsuperscript{36} and added to its right hand side the terms describing the magnon diffusion $(\partial \mathbf{M}/\partial t)_D$ and the pumping $\mathbf{P}$.

$$\left( \frac{\partial \mathbf{M}}{\partial t} \right)_D = -\gamma [\mathbf{M}, \mathbf{H}_{\text{eff}}] + \mathbf{R} + \mathbf{P} \quad \text{(1)}$$

where $\gamma$ is the gyromagnetic ratio, $\mathbf{H}_{\text{eff}}$ is the effective magnetic field determined as the variational derivative of the free energy with respect to the magnetization

$$\mathbf{H}_{\text{eff}} = -\frac{\delta F}{\delta \mathbf{M}}. \quad \text{(2)}$$

$\mathbf{R}$ is the relaxation term

$$\mathbf{R} = -\gamma_{R1} [\mathbf{n}_M, [\mathbf{n}_M, \mathbf{H}_{\text{eff}}]] + \gamma_R \mathbf{H}_{\text{eff}} + \left( \frac{\partial \mathbf{M}}{\partial t} \right)_D, \quad \text{(3)}$$

$\gamma_R$ and $\gamma_{R1}$ are the relaxation rates, $\mathbf{n}_M = \mathbf{M}/M$. 

$$\gamma_R = \frac{1}{\tau}, \quad \gamma_{R1} = \frac{1}{\tau_{R1}}$$

$$\tau, \quad \tau_{R1}$$
The two first terms in the right hand side of Eq. (3) determine the relaxation in the book. The first term in Eq. (3) is equal to the relaxation term of the LL equation. For a such relaxation, the conservation of the absolute value of the magnetization holds. The LLG equation for the magnetization is equivalent to the LL equation: the LLG equation may be obtained from the LL equation by redefining parameters. Therefore, the first term cannot describe the magnetization in the condition of the external magnetic field. The second and the third terms in Eq. (3) are important for the description of the development of the non-uniform variation of the magnetization.

To describe the contribution of the exchange interaction into the free energy, we use the method of the self-consistent field and present the free energy in the form

$$F(M, \nabla M) = \frac{aM^2}{2} + \frac{bM^4}{4} + \frac{K}{2}(|\nabla M|^2) - MH \cos \theta - \frac{1}{2}M \cdot H^{(m)} + K_{an} \sin^2 \theta,$$

(4)

where $K_{an}$ is the anisotropy constant, $\theta$ is the angle between the magnetic field and the crystal axis, $H^{(m)}$ is the magnetic field created by the magnetic moment $M$, $a$, $b$ and $K$ are parameters depending on the temperature and not depending on the spatial coordinates. The first three terms describe the exchange interaction. The free energy may be given in the form of Eq. (4) at the temperature close to the critical temperature of the phase transition. Such case will be studied in this paper.

As mentioned, the regions of the new phase, appearing due to the pumping, may have different shapes. We shall consider the formation of a magnetic domain with the orientation of the magnetic moments opposite to the orientation of the magnetic moment of the crystal and parallel to its easy-axis. Domains of such type are simple and widespread defects in magnetics.

In the transition area, where one orientation of the magnetic moments change to another, the exchange energy increases. The transition may occur either by a rotation of the magnetic moment which preserves its absolute value or by changing the value of the moment. Usually the first way dominates due to the high magnitude of the exchange interaction. But in the vicinity to the phase transition, the exchange interaction decreases and the second way is plausible (see the textbook problem in). Since we study the processes nearby the critical temperature, the transition between the domain and the matrix is considered by changing the value of the magnetic moment without its rotation.

In the framework of the chosen approach, when the normal to the domain plane is perpendicular to the magnetic field $H$, the orientation of the magnetic moments has the form presented schematically in Fig. 1. The magnetic moment inside the domain has the single component $M_z$ and depends on the single variable $y$. Therefore, $M \rightarrow M_z(y)$ and $H^{(m)} = 0$. For a such orientation of the magnetic moment, the angle $\theta$ in Eq. (4) is zero and both the first term in the right hand side of Eq. (4) and the first term in the right hand side of Eq. (3) disappear.

Let us apply the phenomenological approach to determine how the diffusion contributes to the time evolution of the magnetic moment. The density of the magnon current in the non-homogeneous system at the uniform distribution of the temperature may be expressed by the gradient of the chemical potential $\mu$

$$j = -K_M \nabla \mu,$$

(5)

where the coefficient $K_M$ (mobility) depends on the temperature. $K_M$ may be evaluated from the Boltzmann equation for the magnon distribution function taking into account the magnon scattering on magnons, phonons and impurities.

Let us introduce the tensor of magnetization current $\Pi_{Mik}$, that describes the density current of the $i$-th component of the magnetic moment when the magnons are moving along the axis $k$. It is equal to the product of the $i$-th component of the magnetic moment of a single magnon and $k$-th component of the magnon current density

$$\Pi_{Mik} = m_{ik}.\theta.$$

(6)

The single magnon has the following magnetic moment

$$m = -gB nM,$$

(7)

The vector $n_M$ is oriented along the axis $z$. It may assume two values $\pm 1$. The sign "+" takes place in the matrix where the moment is oriented along the magnetic field. The sign "−" is realized in the domain. The rate of the change of the magnetization is equal to

$$\left(\frac{\partial M}{\partial t}\right)_D = \frac{\partial \Pi_{Mik}}{\partial x_k}.$$

(8)

The magnon current creates the magnetic moment current

$$\Pi_{Mik} = -gB n_{Mik}.$$

(9)

In the considered case, the tensor of the magnetization current has a single nonzero component $\Pi_{Mzy}$. The contribution of the magnetic moment current into the rate of the magnetization change is equal to

$$\left(\frac{\partial M_z}{\partial t}\right)_D = \frac{\partial \Pi_{Mzy}}{\partial y} = gB \frac{\partial}{\partial y} \left( n_{Mz} K_M \frac{\partial \mu}{\partial y} \right).$$

(10)

The chemical potential may be determined via the free energy by the formula $\mu = \delta F/\delta n$ at the constant temperature, where $n$ is the magnon density. The magnon density may be related to the magnetization by the expression

$$M = n_M (M_z - gB n).$$

(11)
where the saturation magnetization $M_s = N \mu_B$, $N$ is the number of the crystal cells in a unit volume. Eq. (11) does not take into account Walker’s modes. But since we consider the high temperature case, the main contribution into the decrease of the magnetization comes from the magnons with the high value of the wave vectors. The authors of the paper\textsuperscript{39} showed that notion of the magnons as quasiparticles may hold almost up to the temperature of the phase transition.

Using the free energy of Eq. (2) and Eqs. (11) and (14) we obtain the chemical potential

$$\mu = \frac{\delta F}{\delta M} \frac{\partial M}{\partial t} = -H_{\text{eff}}(-g \mu_B M) = -g \mu_B(aM + bM^3 - n_{Mz}H - K_{M} n_{M} \Delta M).$$

As seen from Eq. (12), the chemical potential may be presented as the interaction energy of the effective magnetic field with the magnetic moment of a magnon ($-m_B g n_{M} n_{M}$). Without the pumping ($P = 0$), Eq. (12) has the solution $H_{\text{eff}} = 0$ and, therefore, $\mu = 0$, as follows from Eq. (12).

Let us consider now the contribution of the pumping into Eq. (11). The rate of the magnon injection into the sample depends on the method of the injection and numerous other conditions: the frequency and amplitude of the external field, the state of the magnetization of the sample, the magnon spectrum and so on\textsuperscript{23,40}. We shall not consider the processes of the magnon creation by the external microwave field. We assume only that the value of $P$, which determines the change of the magnetization in the unit time and the unit volume due to the pumping, does not depend on time or spatial position. The magnons created by the pumping come very quickly to the quasi-equilibrium state due to the magnon-magnon and the magnon-phonon interactions and the dynamics of system is determined by the total number of magnons. It is the major assumption of the BEC studies as well. The pumping causes the decrease of the magnetization. So, the value of $P$ should be negative with respect to the crystal magnetization and positive in the domain, where the magnetization changes sign, and should be equal to zero at the point where $M_s = 0$. We shall present the pumping in a simplified form. Its value will be approximated by the formula

$$P = -q M_z.$$

where $q$ is a positive phenomenological parameter. Its value is deduced from the experimental decrease of the magnetization due to the pumping.

Using Eqs. (10), (12) and (13) in Eq. (11) and taking into account that the dynamic term in the considered case is zero, we obtain the equation for the magnetization

$$\frac{\partial M_z}{\partial t} = R_z + P,$$

where

$$R_z = \gamma_R H_{\text{eff}} - g \mu_B K_M \frac{\partial^2 H_{\text{eff}}}{\partial y^2}$$

$$H_{\text{eff}} = H - aM_z - bM_z^3 + K \frac{\partial^2 M_z}{\partial y^2},$$

Since the dynamic term in the Eq. (14) for the magnetization is zero in the considered system, its right hand side consists of the relaxation term given by Eq. (13) and the pumping. It is seen that dissipative term in Eq. (15) contains the component with the second derivative of the effective magnetic field. The form of the dissipative term $R_z$ coincides with the form of the dissipative term $R_{\text{Bar}}$ obtained for the magnetization in the Landau-Lifshitz-Baryakhtar equation\textsuperscript{31-43}

$$\left( \frac{\partial M}{\partial t} \right) = -\gamma(M, H_{\text{eff}}) + R_{\text{Bar}},$$

where

$$R_{\text{Bar}} = \lambda_1 H_{\text{eff}} - \lambda_2 \Delta H_{\text{eff}}$$

where $\lambda_1$ and $\lambda_2$ are tensor coefficients in the general case.

Comparing the relaxation term of Eq. (18) with the Baryakhtar’s expression of Eq. (15), we obtain the values of the coefficients: $\lambda_1 = \gamma_R$, $\lambda_2 = g \mu_B K_M$ for our particular case of the magnon diffusion.

Baryakhtar built the equation for the magnetization using the Onsager kinetic equations and the crystal symmetry. As shown in a number of works, the terms additional to the LL and the LLG equations are important for the explanation of many processes in ferromagnets: the dynamics of domain walls\textsuperscript{42,44}, the dynamics of solitons\textsuperscript{45}, the damping of the spin waves with the high values of the wave vectors\textsuperscript{44,47}, the anisotropic damping in the ferromagnetics\textsuperscript{48}, the temporal spin evolution in the magnetic heterostructures disturbed by femtosecond laser pulses\textsuperscript{49}. The general theory\textsuperscript{48} does not specify the numerical value of the coefficients. Their values are determined for particular systems. For example, the inclusion of the conductivity electrons in the magnetization dynamics of conducting ferromagnets\textsuperscript{44,47} give the additional terms that depend on the conductivity. The value of the coefficient we obtained is determined by the diffusion of magnons.

Let us introduce the dimensionless variables $\tilde{y} = y/l_0$, $l_0 = (K/(-a))^{1/2}$, $M = M/M_{00}$, $H = H/H_{00}$, $t = Kt/(a^2 g \mu_B K_M)$, $\tilde{q} = qK/(a^2 g \mu_B K_M)$, $\gamma_R = \gamma_R K/((-a)q^2 \mu_B^2 K_M)$, where $M_{00} = ((-a)/b)^{1/2}$ and $H_{00} = (-a)((-a)/b)^{1/2}$ are the value of the magnetization and the effective magnetic field, respectively, in the absence of the magnetic field and pumping.

Eq. (13) for the magnetization in the dimensionless variables takes the form (the diacritic “\text{"\textasciitilde}” above the notations for $M_z$, $H$, $q$, $t$ and $y$ will be omitted from now on)

$$\frac{\partial M_z}{\partial t} = \frac{\partial^2}{\partial y^2} \left( -M_z + M_z^3 - H - \frac{\partial^2 M_z}{\partial y^2} \right)$$

$$-\gamma_R \left( -M_z + M_z^3 - H - \frac{\partial^2 M_z}{\partial y^2} \right) - q M_z.$$
Eq. (19) determines the variation of the magnetization in the region of the phase transition in the presence of the magnon pumping. The first and the second terms in its right hand side describe dissipative processes. The first term originates from the processes of the magnon diffusion. The second term describes the relaxation of the magnetization to the equilibrium value.

Let us consider expressions which may be used for the estimation of the numerical values of parameters. The mobility $K_M$ may be related to the magnon diffusion coefficient by the Einstein’s relation $K_M = Dn/kT$, where $T$ is the temperature. The diffusion coefficient may be obtained from the solution of the Boltzmann equation for magnons. According to Eq. (11), $n = (M_s - M)/g\mu_B$. In the vicinity of the phase transition, where $M \ll M_s$, we have $n \sim M_s/g\mu_B$. In the mean field approximation, the parameters of the free energy are

$$a = -\Delta t c\frac{Tc}{M\mu_B}, \quad b \sim \frac{\kappa Tc}{3 M^3\mu_B}, \quad K \sim \frac{\kappa Tc}{M\mu_B} d^2,$$

where $T_c$ is the phase transition temperature, $\Delta t = (T_c - T)/T_c$, $d$ is the period of the crystal lattice. In the approach of Eq. (20) in dimensionless variables, the length unit is equal to $l_o = d/(\Delta t)^{1/2}$, the magnetization unit is $M_{bo} = (\Delta t)^{1/2}\sqrt{3} M_s$, the magnetic field unit is equal to $H_{bo} = M_s (\Delta t)^{3/2}\sqrt{3} (\kappa T_c)/(\mu_B M_s)$. Let us do the estimations of the values of the parameters in the dimensionless units. We consider the parameters of the yttrium-iron-garnet crystal, in which $T_c = 560$ K, $M_s = 140$ Gs. We assume that $D = 100$ cm$^2$/s. Since the dimensionless magnetization is the ratio of the magnetization to the value of the magnetization of the sample without the magnetic field and pumping, its magnitude is of order of unity or smaller ($M \ll 1$). The magnetic fields which varies in the experiments are from 0 to and to 1000 Oe is described in the dimensionless units by the values that are less than the unity ($H \ll 1$). For example, the magnetic field of 1000 Oe is equal in the dimensionless units to 0.00077 and 0.00222 at the temperatures $\Delta t = 0.2$ and $\Delta t = 0.1$, correspondingly. We shall choose the parameter of the pumping $q$ in Eq. (19) in the way that ensures a small decrease of the magnetization due to the pumping (less than several percent of the magnetization value). The dimensionless coefficient $\gamma_R$ is found to be very small. For $\Delta t = 0.1$, varying the parameter $\gamma_R$ from $10^{-6}$ s$^{-1}$ to $10^{-3}$ s$^{-1}$ leads to the decrease of the dimensionless coefficient $\gamma_R$ from $10^{-6}$ to $10^{-9}$. Therefore, in the dimensionless units, the new magnon diffusion related term in the LL equation has the coefficient which is large compared to the typically used relaxation term. But, because this term contains derivatives of the higher (second) order than other terms, its effect becomes important only in the case of the strongly non-uniform states. For example, for Walker’s modes in the sample with the size of order of 1 mm, the presence of the second derivative in the first term in the left part of Eq. (19) decreases this term, presented in the dimensionless units, by 13 orders of magnitude and it becomes negligible.

Eq. (19) describes domains in the ZOX plane and the magnetization oriented along the $z$ axis and depending on $y$ ($M \equiv M_z(y)$). Let us consider another orientation of the domain plane. Using the free energy of Eq. (19), we may describe the domain in the XOY plane with the magnetization depending on $z$ ($M \equiv M_z(z)$) with an equation, which may be obtained from Eqs. (14,15,16) by the transformations $M_z(y) = M_z(z)$, $H = H - 4\pi M_z(z)$. The new term $4\pi M_z(z)$ may be combined with the term $aM_z$ from the free energy. As the condition $a \gg 2\pi$ holds, the equation for the magnetization for the alternative XOY orientation of the domain plane will assume the form of Eq. (15). Naturally, the resulting solutions will be similar. Therefore, we shall study only Eq. (19) (Eq. (19) in dimensionless units).

### III. INVESTIGATION OF STABILITY OF UNIFORM MAGNETIZATION

At the uniform steady pumping, Eq. (19) has a uniform steady state solution which determines the stationary magnetization $M_0$ that satisfies the following equation

$$M_0^3 - M_0(1 + q/\gamma_R) - H = 0. \quad (21)$$

The solution of this equation in the absence of the pumping ($q = 0$) determines the equilibrium magnetization $M_{eq}$.

In order to investigate the stability of the uniform solution, let us put $M = M_0 + \delta M \exp(\lambda(k)t + iky)$, where $\delta M \ll M_0$. The decrement of the damping obtained from Eq. (19) is equal to

$$\lambda(k) = (1 - 3M_0^2 - k^2)(k^2 + \gamma_R) - q. \quad (22)$$

Eq. (22) implies, that the decrement $\lambda(k)$ is negative in the absence of the pumping for every value of $k$, meaning that the uniform state is stable. The dependence of the damping decrement on $k$ is presented in Fig. 2 for the different values of the pumping $q$. As seen from Fig. 2, the damping decrement $\lambda(k)$ becomes positive with increasing the pumping at a certain wave number $k = k_c$. At $q < 3.3424 \cdot 10^{-8}$ the uniform state is stable. At the increased pumping $q > 3.34261 \cdot 10^{-8}$ the uniform solution becomes unstable with respect to the creation of the periodical magnon density variation with the wave number $k_c$. The instability occurs in the region of the spinodal decomposition, where the second derivative of the free energy with respect to the order parameter ($M_z$ in the considered case) changes sign ($\partial^2 F/\partial M_z^2 = 0$). But the numerical value of the drop of the magnetization $M_z$ caused by the pumping needed for reaching the instability is very large. For the example considered in Fig. 2, the magnetization $M_z$ changes from 1.0005 (in the dimensionless units) at $q = 0$ to 0.577244 at $q = q_c$. Such a decrease of the magnetization requires an extremely strong pumping, that is likely to change the temperature.
state of the crystal. We shall not consider the region in the vicinity to the spinodal decomposition and such strong pumping.

So, at \( \lambda(k) < q_* \) the uniform state is stable even in the presence of the pumping and for the finite lifetime of the non-equilibrium state. But non-uniform stable states may coexist with the uniform state even at \( \lambda(k) < q_* \) in a certain range of the pumping intensity. These states arise when the parameters of the system belong to the region between the binodal and the spinodal. For a magnetic system, the magnetization in this region is restricted by the conditions imposed on \( \partial F / \partial M = 0 \) and \( \partial^2 F / \partial M^2 = 0 \) (this region is slightly affected by the pumping and the finite value of the particle’s lifetime). The condition \( \partial F / \partial M = -H_{eff} = 0 \) is realized in the equilibrium state and determines the equilibrium magnetization. The region between \( \partial F / \partial M = 0 \) and \( \partial^2 F / \partial M^2 = 0 \) arises at the magnetization smaller than the equilibrium value, that can be caused by the pumping, which creates the magnons and decreases the magnetization. The total number of magnons in this region exceeds the equilibrium value. The system is supersaturated with magnons. The gas of the particles in the state between the binodal and the spinodal may remain in the uniform supersaturated state or could transfer due to fluctuations to the state with nuclei of the new condensed phase. Subsequently, these nuclei grow with time. In the case of stable particles, the growth of the condensed phase nuclei slows down with time because their number in the matrix is limited. To the contrary, in a system of constantly created particles with the finite lifetime, the spatially localized regions of the new phase may be stabilized by the interplay of the steady generation and decay. The stationary localized islands of the condensed phase of indirect excitons created by light in double quantum well heterostructures were studied in \( 49,50 \) in the parameter ranges at which the uniform and the non-uniform solutions are stable simultaneously. The localized solutions are called either the autosolitons (static solitons) according to the classification by the paper \( 51 \) or the breathers according to \( 52 \). In the next section, we shall use the Eq. \( 19 \) to study the non-uniform non-equilibrium stationary states in the magnetic sample at the steady pumping.

IV. FORMATION OF A SINGLE DOMAIN

We shall consider the steady state solutions of the Eq. \( 19 \) for the magnetization. The non-uniform states studied in the paper are domains with the magnetic moment oriented opposite to the magnetization of the matrix. As have been mentioned, we shall consider the solutions in the region on the magnetization diagram where the uniform solution is stable, but the non-uniform solutions appear as well due to the magnon created by pumping. In the search of the non-homogeneous solutions, we follow the procedure applied in \( 49,50 \). Two approaches are used. In the first approach, we solve Eq. \( 19 \) for a certain value of the pumping choosing the initial magnetization in the form of the non-homogeneous function \( M_i(y,t=0) = M_{in}(y) \) depending on some parameters, the value of \( q \) is given. In the second method, the external pumping is presented in the form of \( q \rightarrow q + Q_i(y,t) \), where \( Q_i(y,t) \) is a function containing some parameters and tending to zero at \( t \rightarrow \infty \), the initial magnetization being uniform. Thus, the solution of the evolution equation converges with time to the solution of Eq. \( 19 \) with the given value of \( q \). Varying the functions \( M_{in}(y) \) and \( Q_i(y,t) \), different non-uniform stationary solutions for the magnetization may be obtained. These solutions give \( M(y,t) \rightarrow M_{eq} \) at \( q \rightarrow 0 \). The solutions are stable because they do not change at \( t \rightarrow \infty \). They do not change with the variation of the functions of \( M_{in}(y) \) and \( Q_i(y,t) \) in some limited region of parameters of the functions. In other words, for every solution there is a region of parameters of the functions \( M_{in}(y) \) and \( Q_i(y,t) \) in which the solution is the same. This region determines “the attraction basin” for the given solution.

In the absence of pumping (\( q = 0 \)), the solution of Eq. \( 19 \) is uniform. Non-uniform solutions are possible at \( q \neq 0 \). Firstly, let us find one such non-uniform solution localized in the center of the sample. It may be obtained choosing the initial magnetization in the form

\[
M_{in}(y) = M_{in0} \exp[-(y-L/2)^2/s^2],
\]

where \( L \) is the length of the sample, \( M_{in0} \) and \( s \) are parameters.

If the function given by Eq. \( 23 \) with certain values of parameters belongs to the attraction basin of some solution of Eq. \( 19 \), it converges to this solution with time.
We consider such a solution in the time limit \( t \to \infty \) as one of the desired solutions. The example of the magnetization variation obtained in such way is presented in Fig. 3. In the center of the sample, the region is seen where the orientation of the magnetic moment is opposite to the orientation of the magnetization of the remaining part of the sample. The change of the parameters \( M_{\text{init}} \) and \( s \) of the trial function Eq. (23) within the some limits gives the same solution of Eq. (19), presented in Fig. 3. This confirms that the applied method for the solution Eq. (19) is correct. Fig. 3 is the manifestation of the scenario shown in Fig. 1. The inverted spins (magnons) created by the pumping are clustered into a domain. The applied method allows determination of the possible states in the system. The realization of certain state depends on the boundary initial condition and on fluctuations. The calculations showed that the time of an establishment of the steady size of the inclusions is very long (much longer than magnon lifetime). This fact should be taken into account performing an investigation of the magnon distribution at a pulse excitation.

The stationary state of the domain may exist if there is an additional inflow of magnons from outside. There are minimal values of the domain thickness and the sample thickness \( L \) at which the domain could develop. The domain thickness grows if the value of \( L \) rises, since the region from which the domain harvests magnons increases. But, if \( L \) becomes greater than the diffusion length, the size of the single domain reaches the limit for the fixed value of the pumping rate. This occurs at \( L > (1/\gamma_R)^{1/2} \). In this limit, the results would not depend on the boundary conditions. We studied the magnetization for two types of the boundary conditions: for the periodical conditions and for the fixed magnetization at the boundary.

The domain size grows with increasing the pumping. Fig. 4 presents the domain thickness as a function of the pumping rate. It is seen from Fig. 4 that there is a threshold of the pumping rate for the domain creation. The pumping in Fig. 4 leads to the decrease of the magnetization. The pumping is such, that the relative change of the uniform magnetization is equal to \( 5 \cdot 10^{-3} \) and \( 3 \cdot 10^{-2} \) at \( q = 10^{-8} \) and \( q = 6 \cdot 10^{-8} \), correspondingly. As seen from Fig. 4, the threshold \( q_c \) for pumping is \( 0.8 \cdot 10^{-8} \) for the given value of the magnetic field. Only the uniform solution exists at \( q < q_c \). The threshold grows with increasing the magnetic field. In the dimensional units, at \( T_c/(T_c - T) = 10 \), \( q = 5 \cdot 10^{-8} \), \( D = 100 \text{ cm}^2/\text{s} \), the thickness of the domain reaches 1.2 \( \mu \text{m} \). The size of the domains increases with decreasing the parameter \( \gamma_R \), id est, with increasing the magnon lifetime.

If the decrease of the magnetization is such that the state of the system is between the binodal and the spinodal, the nuclei of a new phase (for example, a domain) are created due to fluctuations. Only those nuclei survive which overcome a barrier. Therefore, in order to describe this process of the domain formation mathematically, we solved the main Eq. (23) with the given initial trial non-homogeneous magnetization. If the width of the initial non-homogeneous magnetization increases (the value of \( s \) in Eq. (23) decreases), the solution with two parallel domain arises. The two domains move apart. The repulsion may be explained in the following way. Since the magnons in domains relax, the domains exist due to the magnon inflow from outside. The region between the domains is common for both domains and, as long as the number of the magnons is restricted, their density is not sufficient to support the stationary state of two close domains. As a result, the domains tend to be situated on a certain distance from one another.

V. SUPERLATTICE OF DOMAINS

Solutions of Eq. (19) with the periodical variation of the magnetization are also possible. Some of them are presented in Figs. 5 and 6. In Figs. 5 and 6, the enlarged regions of one of the domains are given separately.
FIG. 5: Spatial dependence of the magnetization at the pumping parameter $q = 8 \cdot 10^{-9}$, the magnetic field $H = 0.01$, the relaxation rate $\tilde{\gamma}_R = 10^{-5}$ (a). A single domain is given separately in (b).

Comparing Fig. 5, Fig. 6 and Fig. 7, one can see that with increasing the pumping rate the domains widen.

The appearance of a periodical structure of the magnetization was observed in YIG films at the external pumping.

Some comments about the effect of fluctuations are possible. The problem of the fluctuations of the density of unstable particles were investigated in the papers \cite{28-30} by solving the Fokker-Planck functional equation for the free energy in the Landau-Ginzburg form. Both the intrinsic fluctuations and the fluctuations of the pumping were taken into account. The studies show the appearance of a second maximum in the one-point distribution function if the pumping rate exceeded a certain value. This maximum corresponded to the development of the second phase. The separation of two phases may occur if the particle lifetime is greater than a certain threshold value. The two-point correlation function was obtained. The Fourier transform of two-point correlation function calculated in \cite{28} has a sharp maximum at some value of the wave vector that corresponds to the oscillations of the correlation function as a function of coordinates. So, periodical structures appear in the system. The problem has solved in \cite{28} differs from the problem in the current work by the presentation of the lifetime of particles: the lifetime in \cite{28} is constant, it is the function of the spatial coordinate (the second term in right hand side of Eq. \cite{19}). But the first term in the right hand side of Eq. \cite{19} plays the main role in the formation of non-uniform structure. Using these results, we may argue that the fluctuations do not destroy the periodical structures that arise in the considered ferromagnetic system, if the magnon lifetime is much greater than the times of magnon-magnon and magnon-phonon collisions and the quasi-equilibrium state is formed in the system.
VI. DISCUSSION AND CONCLUSION

We have shown that, in the system with the high density of quasi-equilibrium magnons, non-uniform structures may arise similar to the formation of inclusions of the condensed phase in a supersaturated gas or to the separation of the new phase in the crystal supersaturated with impurities. Performing the calculation, we used the Landau presentation of the free energy of magnons, which is valid in the vicinity to the phase transition temperature. The work explored the simplest model, namely, we considered the appearance of simple defects in the form of domains. But, in our opinion, some of the obtained results give insight into the general behavior in more complicated cases. Besides the considered simplest one-dimensional structures, the system may develop other types of non-uniformities having different shapes and being restricted in all directions (disks, balls, ellipsoids, and so on). What new features may be expected when more complex regions of the new phase are formed? Let us make some qualitative analysis of the possible manifestations of the development of non-uniform structures. The following list names the most important of them.

1. The appearance of the regions of the non-uniform magnetization causes additional scattering of the electromagnetic waves. The intensity of the light scattering by clusters of magnons at some frequencies is greater than by the same number of the individually independent particles.

2. Since the non-uniform inclusions are small, the magnon levels in them will be quantized. The lowest states vary slowly in space and manifest themselves intensively in the scattering and absorption of the electromagnetic waves. The levels with small quantum numbers will display themselves most strongly. This may cause an increase in the lowest part of the scattering spectra similar to the effect observed at the Bose-Einstein condensation. It was shown that stationary state becomes established if the time of the pumping pulse duration is much longer that the magnon lifetime. The sizes of inclusions will grow with the increasing duration of the pulse. It will cause to the step-down of the quantized levels of magnons in the inclusions and to the shift of the scattering spectra of electromagnetic waves to the lower frequencies.

3. The spin orientation in an inclusion of the condensed phase is determined by the strong exchange interaction. In order to change the orientation of its spin to the orientation of the matrix spins, a magnon should jump out of the inclusion, where it is bound by the strong exchange interaction. There is another mechanism of the spin relaxation in the inclusion. Firstly, magnons (light arrows in dark regions of Fig. 1) are exited in the domain and then leave the inclusion. This is a two-stage process and therefore it has small probability. As the result the inclusions should live long.

4. The stationary distributions of the number of inclusions and their sizes arise under stationary pumping. In this case the number of magnons captured by the inclusions coincides with the number of magnons, which leaves the inclusions and decay in inclusions. If the magnon pumping goes off, the sample cooling occurs more quickly than the decrease the magnon number. As the result, a some excess of magnons will exist some time in comparison with equilibrium value. Because the number of magnons leaving inclusions will decrease due to the temperature decrease, the number of magnons in the inclusion will rise. Later the excess of magnons drops and inclusions disappear. So, the peak of magnon number in inclusions may arise after switching off the pumping. Such peak was observed during investigation of Bose-Einstein condensation of magnons.

5. The regions of the condensed phase in systems supersaturated with magnons may arrange themselves into structures. These structures would form a periodical distribution of domains with parallel planes. The normal to the domain plane may be oriented either along the external magnetic field or perpendicular to the field. Such picture was observed in Ref.

All these effects, which may be expected due to the inclusions of the new phase, were observed in experiments, and explained as the manifestation of the Bose-Einstein condensation. We cannot object to that attribution because our study is carried out in somewhat different conditions. The majority of experiments were carried out using the magnon excitation by pulses. Our calculations studied the steady states. Yet we would like to underline that, besides the Bose-Einstein condensation, another scenario should be considered when studying the many-magnon systems. A search for the inclusions of the new phase and evaluation of their role in the physical processes should be performed.

Thus, the presented paper studied a magnetic sample with a large magnon concentration created by the external pumping. Due to the long lifetimes, magnons are in the quasi-equilibrium state. It is shown that besides the Bose condensation, the formation of the inclusions of new condensed magnon phase may occur similarly to the development of the condensed phase in a supersaturated gas. The exchange interaction promotes the combining the separate magnons into inclusions. The orientation of magnetic moments in each inclusion is opposite to the magnetic moment of the magnetic matrix. Therefore, the inclusions may exist only in condition of the external pumping. Summarizing, inclusions are dissipative structures that arise as the result of self-organization processes in non-equilibrium conditions. The presented
study considered the inclusions of the condensed phase shaped as individual domains or periodical superlattices of domains.

The possibility of the formation of the inclusions of the new phase should be taken into account side by side with the process of Bose-Einstein condensation when analyzing experiments.

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