Signatures of Supersymmetry in $B$ Decays - A Theoretical Perspective

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Signatures of Supersymmetry in $B$ Decays - A Theoretical Perspective

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We discuss precision tests of the standard model in radiative and semileptonic rare $B$-decays and CP-violating asymmetries, and possible signatures of supersymmetry in these processes. Motivated by current data, and with an eye on the forthcoming measurements, we restrict ourselves to the processes in which the benchmarks set by the standard model have either been met by ongoing experiments or are expected to be met shortly. This includes the mixing-induced CP asymmetry, measured through $\sin^2 \beta$, and branching ratios for the radiative and semileptonic rare decays $B \to X_s \gamma$, $B \to K^* \gamma$, $B \to X_s \ell^+ \ell^-$ and $B \to (K, K^*) \ell^+ \ell^-$, where $\ell^\pm = e^\pm, \mu^\pm$.

1 Introduction

Experimental $B$ physics is making big strides, thanks largely to the superb performance of the KEK and SLAC $B$ factories and the BELLE and BABAR detectors, but also due to the solid foundation provided by CLEO, the LEP and SLC experiments, and by the CDF collaboration. The rich harvest of new experimental results includes the first measurements of CP violation in the $B$-meson system $B^0_d$ and $B^0_{d2}$, involving statistically significant results by the BABAR and BELLE collaborations. The CP asymmetry in question, $a_{J/\psi K_S}$, is measured through the time-dependent difference in the decay rates for $B^0_d \to J/\psi K_S$ and its CP-conjugate process $\bar{B}^0_{d2} \to J/\psi K_S$ (as well as a number of other related final states such as $\psi(2S)K_S$, $J/\psi K_L$ etc.). Data on this asymmetry, allowing a clean determination of $\sin 2\beta$ (where $\beta$ is one of the angles of the unitarity triangle UT), is now quantitative and can be meaningfully compared with the standard model (SM) predictions of the same. Likewise, it can be used to put constraints on additional CP-violating phases in beyond-the-SM (BSM) scenarios and we will illustrate this in terms of an additional phase, called $\theta_d$, in the context of a supersymmetric theory.

Equally interesting from the point of view of precision tests of the SM and searches of BSM physics are a number of flavour-changing-neutral-current (FCNC) processes, of which the decay $b \to s \gamma$ is so far the most significant. Here also, SM has not only survived a rather crucial experimental test involving quantum (loop) effects in the FCNC sector, it has done so with a comfortable ease.
to cross (or, have probably already crossed) the next milestone in rare $B$-decays, involving semileptonic FCNC decays. We have in mind here the decays $B \to (X_s, K^*, K) \ell^+ \ell^-$, where the current experimental limits are fast approaching the SM-sensitivity. In fact in the decays $B \to K \ell^+ \ell^-$, $\ell = e, \mu$, first measurements by the BELLE collaboration are at hand, though the BABAR collaboration still does not see any signal in this channel. So, more data is needed, which luckily is on its way. The next experimental milestone in this field will be reached with the measurement of the inclusive decays $B \to X_s \ell^+ \ell^-$, since the SM-theory is now quantitative, in particular in the low dilepton invariant mass region. Data can be used to extract effective Wilson coefficients in a general theoretical scenario which we shall discuss, drawing heavily from a recent analysis.

2 Constraints on $\sin 2\beta$ and an additional CP-violating phase $\theta_d$

As is by now folk-lore, the measurement of $a_{J/\psi K}$ yields $\sin 2\beta$ in the SM. The current experimental value of this quantity (including a scale factor in conformity with the Particle Data Group prescription) is $\sin 2\beta = 0.79 \pm 0.12$, and is now dominated by the BABAR measurement ($\sin 2\beta = 0.59 \pm 0.14^{\text{stat}} \pm 0.05^{\text{syst}}$), and the BELLE measurement ($\sin 2\beta = 0.99 \pm 0.14^{\text{stat}} \pm 0.06^{\text{syst}}$) measurements. SM-predictions of $\sin 2\beta$ based on indirect measurements of the sides of the unitarity triangle lie in the range $\sin 2\beta = 0.6 - 0.8$. From this, we tentatively conclude that the current experiments and SM are in reasonable agreement with each other in $\sin 2\beta$. However, this rapport will be tested very precisely in future and it is sensible to estimate the magnitude of a BSM-phase allowed by current data.

In popular extensions of the SM, such as the minimal supersymmetric standard model (MSSM), one anticipates supersymmetric contributions to FCNC processes, in particular $\Delta M_{B_d}, \Delta M_{B_s}$ (the mass differences in the $B_d^0 - \overline{B_d^0}$ and $B_s^0 - \overline{B_s^0}$ systems), and $\epsilon_K$, characterizing the mixing-induced CP-asymmetry $A_{\text{CP}}^{\text{mix}}$ in the $K^0 - \overline{K^0}$ system. However, if the Cabibbo-Kobayashi-Maskawa (CKM) matrix remains effectively the only flavour changing (FC) structure, which is the case if the quark and squark mass matrices can be simultaneously diagonalized, and all other FC interactions are associated with rather high scales, then all hadronic flavour transitions can be interpreted in terms of the same unitarity triangles which one encounters in the SM. In particular, in these theories $a_{J/\psi K}$ measures the same quantity $\sin 2\beta$ as in the SM. These models, usually called the minimal flavour violating (MFV) models, are structured so that the SUSY contributions to $\Delta M_{B_d}, \Delta M_{B_s},$ and $\epsilon_K$ have the same CKM-dependence as the SM top quark contributions in the box diagrams.
(denoted below by \( C_1^{Wt} \)). Hence, supersymmetric effects for the UT-analysis can be effectively incorporated in terms of a single common parameter \( f \) by the following replacement:

\[
\epsilon_K, \; \Delta M_{B_s}, \; \Delta M_{B_d}, \; a_{\psi K_s} : C_1^{Wt} \rightarrow C_1^{Wt}(1 + f).
\]

The parameter \( f \) is positive definite and real, implying that there are no new phases in any of the quantities specified above. The size of \( f \) depends on the parameters of the supersymmetric models. Given a value of \( f \), the CKM unitarity fits can be performed in these scenarios much the same way as they are done for the SM. Qualitatively, the CKM-fits in MFV models yield the following pattern for the three inner angles of the UT:

\[
\beta^{MFV} \approx \beta^{SM}; \quad \gamma^{MFV} < \gamma^{SM}; \quad \alpha^{MFV} > \alpha^{SM},
\]

and a recent CKM-fit along these lines yields the following central values for the three angles:

\[
f = 0 \text{ (SM)} : \; (\alpha, \beta, \gamma)_{central} = (95^\circ, 22^\circ, 63^\circ),
\]

\[
f = 0.4 \text{ (MFV)} : \; (\alpha, \beta, \gamma)_{central} = (112^\circ, 20^\circ, 48^\circ).
\]

leading to \( \sin 2\beta^{SM}_{central} \approx 0.70 \) and \( \sin 2\beta^{MFV}_{central} \approx 0.64 \). Thus, what concerns \( \sin 2\beta \), the SM and the MFV models give similar results from the UT-fits, unless much larger values for the parameter \( f \) are admitted which are now unlikely due to the existing constraints on the MFV-SUSY parameters.

However, in a general extension of the SM, one expects that all the quantities appearing on the l.h.s. in Eq. (1) will receive independent additional contributions. In this case, the magnitude and the phase of the off-diagonal elements in the \( B_0^d - \overline{B}_0^d \) and \( B_0^s - \overline{B}_0^s \) mass matrices can be parametrized as follows:

\[
M_{12}(B_d) = \frac{(B_d | H_{eff}^{B_d=2} | B_d)}{2M_{B_d}} = r_d^2 e^{2i\theta_d} M_{12}^{SM}(B_d),
\]

\[
M_{12}(B_s) = \frac{(B_s | H_{eff}^{B_s=2} | B_s)}{2M_{B_s}} = r_s^2 e^{2i\theta_s} M_{12}^{SM}(B_s).
\]

where \( r_d \) (\( r_s \)) and \( \theta_d \) (\( \theta_s \)) characterize, respectively, the magnitude and the phase of the new physics contribution to the mass difference \( \Delta M_{B_d} (\Delta M_{B_s}) \). It follows that a measurement of \( a_{\psi K_s} \) would not measure \( \sin 2\beta \), but rather a combination \( \sin 2(\beta + \theta_d) \). In this scenario, one also expects new contributions in \( M_{12}(K^0) \), bringing in their wake additional parameters (\( r_s, \theta_s \)). They will alter the profile of CP-violation in the decays of the neutral kaons.
It is obvious that in such a general theoretical scenario, which introduces six a priori independent parameters, the predictive power vested in the CKM-UT analysis is lost. If the idea is to retain this predictivity, at least partially, then one has to work within a more limited framework. A model along these lines was introduced using the language of minimal insertion approximation (MIA) in a supersymmetric context. In this model, all FC transitions which are not generated by the CKM mixing matrix are proportional to the properly normalized off–diagonal elements of the squark mass matrices:

\[ (\delta_{ij})_{AB} = \frac{(M^2_{ij})_{AB}}{M_{\tilde{q}_i}M_{\tilde{q}_j}} \]  

(5)

where \( i,j = 1,2,3 \) and \( A,B = L,R \). The dominant effect of the non-CKM structure contained in the MIA-parameters is to influence mainly the \( b \to d \) and \( s \to d \) transitions while the \( b \to s \) transition is governed by the MFV-SUSY and the SM contributions alone. For what concerns the quantities entering in the UT analysis, the following pattern for the supersymmetric contributions emerges in this model:

\[ \Delta M_{B_s} : C_1^{Wtt} \to C_1^{Wtt}(1 + f) \]
\[ \epsilon_K, \Delta M_{B_d}, a_{\psi K_S} : C_1^{Wtt} \to C_1^{Wtt}(1 + f) + C_1^{M I} \equiv C_1^{Wtt}(1 + f + g) \]  

(6)

(7)

where the parameters \( f \) and \( g = g_R + ig_I \) represent normalized (w.r.t. the SM top quark \( Wtt \)) contributions from the MFV and MIA sectors, respectively. Hence, in the UT-analysis the contribution from the supersymmetric sector can be parametrized by two real parameters \( f \) and \( g_R \) and a parameter \( g_I \), generating a phase \( \theta_d \), which is in general non-zero due to the complex nature of the appropriate mass insertion parameter. A precise measurement of \( a_{\psi K_S} \) would fix this argument (= \( \theta_d \)).

The impact of the Extended-MFV model on the profile of the unitarity triangle in the \((\rho, \eta)\) plane is shown in Fig. 1, which also shows the corresponding profiles in the SM and MFV models: the solid contour corresponds to the SM 95% C.L., the dashed one to a typical MFV case \( (f = 0.4, g = 0) \) and the dotted–dashed one to an allowed point in the Extended-MFV model \( (f = 0, g_R = -0.2 \text{ and } g_I = 0.2) \). All three models give comparable fits. In Fig. 2 the \( CP \) asymmetry \( a_{\psi K_S} \) is plotted as a function of arg \( \delta_{\tilde{u}_L \tilde{t}_2} \) (expressed in degrees). Here, \( \delta_{\tilde{u}_L \tilde{t}_2} \) is a linear combination of \( (\delta_{13})_{LR} \) and \( (\delta_{13})_{LL} \). The light and dark shaded bands correspond, respectively, to the SM and the experimental 1 \( \sigma \) allowed regions. The solid line is drawn for \( f = 0 \) and \( |g| = 0.28 \). The experimental band flavours arg \( \delta_{\tilde{u}_L \tilde{t}_2} \) in the range \([0^\circ, 100^\circ]\). Employing
the explicit dependence

$$\theta_d = \frac{1}{2} \text{arg}(1 + f + |g|e^{2i\text{arg}\delta_{LL}^s}) \pmod{\pi},$$

(8)

the above phase interval is translated into

$$-3^\circ < \theta_d < 8^\circ,$$

(9)

for the assumed values of $|g|$ and $f$, which is a typical range for $\theta_d$ for the small angle solution with the current values of $a_{\psi K}\bar{S}$.

Such a non-zero angle would have measurable consequences in $b \to d$ transitions, such as the isospin-violating ratio $\Delta^{\pm 0} = \frac{\Gamma(B^{\mp \to \rho^\pm \gamma})}{2\Gamma(B^{\mp \to \rho^{\mp} \gamma})} - 1$, and in direct CP-violating asymmetries $A_{\text{CP}}(\rho^{\pm} \gamma) = \frac{\mathcal{B}(B^{\mp \to \rho^\pm \gamma}) - \mathcal{B}(B^{+ \to \rho^- \gamma})}{\mathcal{B}(B^{+ \to \rho^+ \gamma}) + \mathcal{B}(B^{- \to \rho^- \gamma})}$, which may deviate measurably from their SM ranges. Recently, these quantities have been calculated in the SM by taking into account explicit $O(\alpha_s)$ corrections, using the so-called Large-Energy-Effective-Theory (LEET) approach. While there is considerable parametric uncertainty in the determination of $A_{\text{CP}}(\rho^{\pm} \gamma)$ (and its analogue $A_{\text{CP}}(\rho^{0} \gamma)$), the estimates of $\Delta^{\pm 0}$ are quantitative. Testing these predictions should be feasible at the B factories in the next several years.

### 3 Inclusive Decay rate for $B \to X_s \gamma$ in the SM and SUSY

The effective Hamiltonian in the SM inducing the $b \to s\gamma$ transitions, obtained by integrating out the heavier degrees of freedom, can be expressed as follows:

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) O_i(\mu),$$

(10)

where $G_F$ is the Fermi coupling constant and the CKM dependence has been made explicit; $O_i(\mu)$ are dimension-six operators at the scale $\mu$, and $C_i(\mu)$ are the corresponding Wilson coefficients. Of these, the dominant operators are $O_1 \sim (\bar{s}_L\gamma_\mu T^a q_L)(\bar{q}_L\gamma^\mu T^a b_L)$, $O_2 \sim (\bar{s}_L\gamma_\mu q_L)(\bar{q}_L\gamma^\mu b_L)$, and the magnetic moment operators $O_7 \sim (\bar{s}_L\sigma_{\mu\nu} b_R) F^{\mu\nu}$ and $O_8 \sim (\bar{s}_L\sigma_{\mu\nu} b_R) T^a G^{a,\mu\nu}$. Here, $F^{\mu\nu}(G^{a,\mu\nu})$ is the electromagnetic (chromomagnetic) field strength tensor, with $T^a (a = 1, \ldots, 8)$ being the SU(3) group generators. Current theoretical precision of the $b \to s\gamma$ decay rate is limited to $O(\alpha_s)$, consisting of the anomalous dimension matrix in the next-to-leading order (NLO), the commensurate matching conditions, and the virtual and bremsstrahlung contributions. Also,
the leading power corrections in $1/m_b$ and $1/m_c$ have been calculated. The present experimental average of the branching ratio

$$B(B \to X_s \gamma) = (3.22 \pm 0.40) \times 10^{-4},$$

(11)
is in good agreement with the next-to-leading order prediction of the same in the SM, estimated as

$$B(B \to X_s \gamma)_{\text{SM}} = (3.35 \pm 0.30) \times 10^{-4}$$

for the pole quark mass ratio $m_c/m_b = 0.29 \pm 0.02$, rising to

$$B(B \to X_s \gamma)_{\text{MS}} = (3.73 \pm 0.30) \times 10^{-4},$$

if one uses instead the input value $m_c^{\text{MS}}(\mu)/m_b^{\text{pole}} = 0.22 \pm 0.04$, where $m_c^{\text{MS}}(\mu)$ is the charm quark mass in the MS-scheme, evaluated at a scale $\mu$ in the range $m_c < \mu < m_b$. The inherent uncertainty reflects the present accuracy of the theoretical branching ratio and the imprecise knowledge of the quark masses, in particular $m_c$ and $m_b$. Precise measurements of the photon energy spectrum in $B \to X_s \gamma$ decays may help in decreasing some of these uncertainties.

The agreement between experiment and the SM for the $B \to X_s \gamma$ decay rate is quite impressive and this has been used to put non-trivial constraints on the BSM-physics, in particular supersymmetry. A recent analysis along these lines is discussed here for illustration. Following earlier works, the integrated $B \to X_s \gamma$ branching ratio can be solved as a function of the quantities $R_{7,8}(\mu_W) \equiv C_{7,8}^{\text{SM}}(\mu_W)/C_{7,8}^{\text{MS}}(\mu_W)$, where $R_{7,8}^{\text{tot}} = R_{7,8}^{\text{NP}} + R_{7,8}^{\text{SM}}$. Taking the scale $\mu_W = M_W$ for the purpose of the renormalization group evolution (RGE) also for the supersymmetric contributions, and imposing the experimental bound $B(B \to X_s \gamma) = (3.22 \pm 0.40) \times 10^{-4}$, the corresponding allowed regions in the $[R_7(\mu_W), R_8(\mu_W)]$ plane are worked out. Evolving the allowed regions to the scale $\mu_b = 2.5 \text{ GeV}$ and assuming that new physics only enters in the effective Wilson coefficients $C_{7,8}$, the corresponding low–scale bounds in the plane $[R_7(2.5 \text{ GeV}), R_8(2.5 \text{ GeV})]$ are obtained, yielding

$$0.785 \leq R_7(2.5 \text{ GeV}) \leq 1.255 \Rightarrow -0.414 \leq C_7^{\text{tot}, <0}(2.5 \text{ GeV}) \leq -0.259,$$

$$-1.555 \leq R_7(2.5 \text{ GeV}) \leq -1.200 \Rightarrow 0.396 \leq C_7^{\text{tot}, >0}(2.5 \text{ GeV}) \leq 0.513. \quad (12)$$

Depending on the sign of $C_7^{\text{eff}}$, there are two allowed solutions - called $C_7^{\text{tot}^+}$–positive and $C_7^{\text{tot}^-}$–negative solutions. SM corresponds to the point $(R_7(\mu), R_8(\mu)) = (1, 1)$. Flavour-blind supersymmetric theories, such as SUGRA, allow points in the vicinity of the SM, though in a more general supersymmetric scenario, both $C_7^{\text{tot}^+}$–positive and $C_7^{\text{tot}^-}$–negative solutions are allowed. These two scenarios can be distinguished, in principle, by measurements of the decays $B \to (X_s, K, K^*)\ell^+\ell^-$, which we discuss next.
4 Inclusive Decays $B \to X_s \ell^+\ell^-$ in the SM and SUSY

The inclusive decays $B \to X_s \ell^+\ell^-$ and the corresponding exclusive decays such as $B \to (K,K^*)\ell^+\ell^-$ allow to get more detailed information on the flavour structure of the SM, and hence offer new search strategies for BSM physics. The effective Hamiltonian governing these decays in the SM is obtained by enlarging the sum given in Eq. (10) by the addition of two more terms involving the four-Fermi operators, denoted by $O_9$ and $O_{10}$:

$$
O_9 \sim (\bar{s}_L \gamma_\mu b_L) \sum_\ell (\bar{\ell}\gamma^\mu \ell),
$$

$$
O_{10} \sim (\bar{s}_L \gamma_\mu b_L) \sum_\ell (\bar{\ell}\gamma^\mu \gamma_5 \ell),
$$

weighted, respectively, by the corresponding Wilson coefficients $C_9(\mu)$ and $C_{10}$.

In most supersymmetric theories, the SM basis is sufficient to describe the generic transitions $b \to s\gamma$ and $b \to s\ell^+\ell^-$, and we shall confine ourselves to discussing some possible BSM effects in this context.

Experimentally, the goal is to precisely measure a number of differential distributions, of which the dilepton invariant mass spectrum and the forward-backward asymmetry of the charged leptons in the dilepton rest frame are the best studied theoretically. BSM physics could manifest itself through additional contributions in the Wilson coefficients, shifting their values from the ones in the SM. This will lead to possible distortions in the two mentioned decay distributions, as well as some others not discussed for lack of space.

From a theoretical point of view, inclusive decays $B \to X_s \ell^+\ell^-$ are more robust, in particular in the dilepton mass region below the $J/\psi$ resonance, as the explicit $\mathcal{O}(\alpha_s)$ improvements in the dilepton invariant mass distributions are now available in this region. Furthermore, the long-distance contributions, implemented through the matrix elements of the operators $O_1$ and $O_2$, can be brought under control by a judicious choice of the experimental cuts, or estimated theoretically.

The dilepton invariant mass distribution for the inclusive decay $B \to X_s \ell^+\ell^-$ can be written as

$$
\frac{d\Gamma(b \to X_s \ell^+\ell^-)}{d\hat{s}} = \left( \frac{\alpha_{em}}{4\pi} \right)^2 \frac{G_F^2 m_b^5}{48\pi^3} |V_{ts}^* V_{tb}|^2 (1 - \hat{s})^2 \times
$$

$$
\left( 1 + 2\hat{s} \right) \left( \left| \tilde{C}_{9}^{\text{eff}} \right|^2 + \left| \tilde{C}_{10}^{\text{eff}} \right|^2 \right) + 4 \left( 1 + 2/\hat{s} \right) \left| \tilde{C}_{7}^{\text{eff}} \right|^2 + 12 \text{Re} \left( \tilde{C}_{7}^{\text{eff}} \tilde{C}_{9}^{\text{eff}} \right). \tag{14}
$$

The effective Wilson coefficients $\tilde{C}_{7}^{\text{eff}}$, $\tilde{C}_{9}^{\text{eff}}$ and $\tilde{C}_{10}^{\text{eff}}$, including explicit $\mathcal{O}(\alpha_s)$ corrections, have been calculated.
The dilepton invariant mass distribution for the process $B \rightarrow X_s e^+ e^-$ calculated in NNLO is shown in Fig. 3 for the three choices of the scale $\mu = 2.5$ GeV, $\mu = 5$ GeV and $\mu = 10$ GeV (solid curves). In this figure, the left-hand plot shows the distribution in the very low invariant mass region ($\hat{s} \in [0, 0.05]$, with 0 to be understood as the kinematic threshold $s = 4m_e^2 \approx 10^{-6}$ GeV$^2$, yielding $\hat{s} = 3.7 \times 10^{-8}$), and the right-hand plot shows the dilepton spectrum in the region beyond $\hat{s} > 0.05$, and hence this also holds for the decay $B \rightarrow X_s \mu^+ \mu^-$. It should be stressed that a genuine NNLO calculation only exists for values of $\hat{s}$ below 0.25, which is indicated in the right-hand plot by the vertical dotted line. For higher values of $\hat{s}$, an estimate of the NNLO result is obtained by an extrapolation procedure discussed in detail elsewhere. The so-called partial NNLO dilepton spectrum is also shown in each of these cases for the same three choices of the scale $\mu$ (dashed curves). Note that the NNLO dilepton invariant mass spectrum in the right-hand plot ($\hat{s} > 0.05$) lies below its partial NNLO counterpart, and hence the partial branching ratios for both the $B \rightarrow X_s e^+ e^-$ and $B \rightarrow X_s \mu^+ \mu^-$ decays are reduced in the full NNLO accuracy.

This framework has recently been used to calculate the branching ratio for $B \rightarrow X_s \ell^+ \ell^-$ in the SM, yielding

$$B(B \rightarrow X_s e^+ e^-) = (6.9 \pm 1.0) \times 10^{-6} \quad (\delta B_{X_s ee} = \pm 15\%) ,$$

$$B(B \rightarrow X_s \mu^+ \mu^-) = (4.2 \pm 0.7) \times 10^{-6} \quad (\delta B_{X_s \mu\mu} = \pm 17\%) .$$

The theoretical errors shown are obtained by estimating the errors from the individual input parameters, and the details can be seen in the original work. Current experimental data sees no signal for these decays, yielding the following upper limits

$$B(B \rightarrow X_s \mu^+ \mu^-) \leq 19.1 \times 10^{-6} \text{ at } 90\% \text{ C.L.},$$

$$B(B \rightarrow X_s e^+ e^-) \leq 10.1 \times 10^{-6} \text{ at } 90\% \text{ C.L.} .$$

Thus, the current experimental sensitivity is typically a factor 3 away from the SM-estimates.

We now turn to the modifications of the effective Wilson coefficients $\tilde{C}_7^{\text{eff}}$, $\tilde{C}_8^{\text{eff}}$ and $\tilde{C}_{10}^{\text{eff}}$ in the presence of new physics which modifies the Wilson coefficients $C_7$, $C_8$, $C_9$ and $C_{10}$ at the matching scale $\mu_W$. For lack of complete NLO calculations, we assume that only the lowest non-trivial order of these Wilson coefficients get modified by new physics, which means that $C_7^{(1)}(\mu_W)$, $C_8^{(1)}(\mu_W)$, $C_9^{(1)}(\mu_W)$, $C_{10}^{(1)}(\mu_W)$ get only indirectly modified. The shifts of the Wilson coefficients at the scale $\mu_W$ can be written as ($i = 7, \ldots, 10$):

$$C_i(\mu_W) \rightarrow C_i(\mu_W) + \frac{\alpha_s}{4\pi} C_i^{NP}(\mu_W) .$$
These shifts at the matching scale are translated through the RGE step into modifications of the coefficients $C_i(\mu_b)$ at the low scale $\mu_b$. The bounds implied by the experimental results given in Eq. (18) (being the more stringent of the two limits) have been computed in the $[C_{NP}^{9}(\mu_W), C_{NP}^{10}]$ plane. We shall show the cumulative bounds resulting from the combined analysis of all the inclusive and exclusive semileptonic decay in the next section.

5 Exclusive Decays $B \to K^*\gamma$ and $B \to K^*\ell^+\ell^-$ in the SM and Supersymmetry

Concentrating first on the transitions $B \to K^*\gamma^{(*)}$, with a real or virtual photon, the general decomposition of the matrix elements on all possible Lorentz structures present in the effective Hamiltonian admits seven form factors, which for the dilepton final state are functions of the momentum squared $q^2$ transferred from the heavy meson to the light one. When the energy of the final light meson $E$ is large (the large recoil limit), one can expand the interaction of the energetic quark in the meson with the soft gluons in terms of $\Lambda_{QCD}/E$. Using HQET for the interaction of the heavy $b$-quark with the gluons, one can derive non-trivial relations between the soft contributions to the form factors. The resulting theory (LEET) reduces the number of independent form factors from seven in the $B \to K^*\gamma$ transitions to two in this limit. The relations among the form factors in the symmetry limit are broken by perturbative QCD radiative corrections arising from the vertex renormalization and the hard spectator interactions. To incorporate both types of QCD corrections, a factorization formula for the heavy-light form factors at large recoil is useful:

$$f_k(q^2) = C_{\perp}k\xi_{\perp} + C_{\parallel}k\xi_{\parallel} + \Phi_B \otimes T_k \otimes \Phi_\rho,$$

where $f_k(q^2)$ is any of the seven independent form factors in the $B \to K^*$ transitions at hand; $\xi_{\perp}$ and $\xi_{\parallel}$ are the two independent form factors remaining in the LEET-symmetry limit; $T_k$ is a hard-scattering kernel calculated in $O(\alpha_s)$; $\Phi_B$ and $\Phi_\rho$ are the light-cone distribution amplitudes of the $B$- and $\rho$-meson convoluted with $T_k$; $C_k = 1 + O(\alpha_s)$ are the hard vertex renormalization coefficients. An $O(\alpha_s)$ proof of the validity of Eq. (20) for radiative decays has, in the meanwhile, been provided by several groups, yielding

$$B_{th}(B \to K^*\gamma) = \tau_B \Gamma_{th}(B \to K^*\gamma)$$

$$= \tau_B \frac{G_F^2|V_{tb}V_{ts}^*|^2}{32\pi^4} \frac{m_b}{m_b,pole} M^3 \left[\xi_{\perp}(K^*)\right]^2 \left(1 - \frac{m_{K^*}^2}{M^2}\right)^3$$

$$\times \left|C_{7}^{(0)\text{eff}} + A^{(1)}(\mu)\right|^2,$$
where $\alpha = \alpha(0) = 1/137$ is the fine-structure constant, $M$ and $m_{K^*}$ are the $B$- and $K^*$-meson masses, and $\tau_B$ is the lifetime of the $B^0$- or $B^+$-meson. The function $A^{(1)}$ in Eq. (21) lumps all three explicit $O(\alpha_s)$ contributions from the Wilson coefficient $C^H_7$, $b \to s \gamma$ vertex, and the hard-spectator corrections to the $B \to K^* \ell \nu$ amplitude. NLO corrections yield a typical "K-factor" of 1.6, yielding

$$B_{\text{th}}(B \to K^* \ell \nu) \simeq (7.2 \pm 1.1) \times 10^{-5} \left( \frac{\tau_B}{1.6 \text{ ps}} \right) \left( \frac{m_{b, \text{pole}}}{4.65 \text{ GeV}} \right)^2 \left( \frac{\xi^{(K^*)}(0)}{0.35} \right)^2$$

$$= (7.2 \pm 2.7) \times 10^{-5},$$

(22)

where the enlarged error in the second equation reflects the assumed error in the nonperturbative quantity, $\xi^{(K^*)}(0) = 0.35 \pm 0.07$. The estimates presented in Refs. 34, 27 are similar. The LEET-based estimates are larger than the experimental branching ratio for $B \to K^* \ell \nu$:

$$B(B^\pm \to K^{*\pm} \ell \nu) = (3.82 \pm 0.47) \times 10^{-5},$$

$$B(B^0(\bar{B}^0) \to K^{*0}(\bar{K}^{*0}) \ell \nu) = (4.44 \pm 0.35) \times 10^{-5},$$

(23)

though the attendant theoretical error, estimated as $\pm 40\%$, does not allow to draw a completely quantitative conclusion.

The price of agreement between the LEET approach and data can be specified in terms of the LEET-form factor $\xi^{(K^*)}(0)$. To that end, it is advantageous to calculate the following ratio of the exclusive to inclusive branching ratios:

$$R_{\text{exp}}(K^* \ell \nu/X s \ell \nu) \equiv \frac{B_{\text{exp}}(B \to K^* \ell \nu)}{B_{\text{exp}}(B \to X s \ell \nu)} = 0.13 \pm 0.02,$$

(24)

where the current experimental value of this ratio is also given, averaging over the charged and neutral $B$-decays. At NLO, the ratio $R(K^* \ell \nu/X s \ell \nu)$ yields

$$\xi^{(K^*)}(0) = 0.25 \pm 0.04, \quad \left[ T_1^{(K^*)}(0, \bar{m}_b) = 0.27 \pm 0.04 \right],$$

(25)

where $T_1^{(K^*)}(0, \bar{m}_b)$ is the form factor in full QCD, determined using the quark masses in the $\overline{MS}$-scheme. This values is significantly smaller than the corresponding predictions from the QCD sum rules analysis, $T_1^{(K^*)}(0) = 0.38 \pm 0.06$, and from the lattice simulations, $T_1^{(K^*)}(0) = 0.32^{+0.04}_{-0.02}$. The reason for this mismatch is not obvious and this point deserves further theoretical study.
In view of this, one can not insist that the absolute rates in exclusive decays can be calculated reliably in the LEET-approach. It should, however, be emphasized that any measurable CP asymmetry in the exclusive \((B \rightarrow K^{*}\gamma)\) or inclusive \((B \rightarrow X_{s}\gamma)\) decay will be a sure sign of BSM physics, as the SM CP asymmetry in either of these modes is not expected to exceed \(\frac{1}{2} \%\). Present experimental bounds on the CP asymmetry are\[^{37}\] \(A_{\text{CP}}(B \rightarrow X_{s}\gamma) = (-0.079 \pm 0.018 \pm 0.022)\), and \[^{38}\] \(A_{\text{CP}}(B \rightarrow K^{*0}\gamma) = (-0.035 \pm 0.094 \pm 0.012)\), obtained in the \(K^{+}\pi^{-}\) mode. The first error in both cases is statistical and the second systematic. They still allow a lot of room for the BSM physics; however, not in the MFV and Extended-MFV models, discussed earlier.

\[5.1 \quad B \rightarrow K^{*}\ell^{+}\ell^{-} \text{ Decays}\]

The NNLO corrections for \(B \rightarrow X_{s}\ell^{+}\ell^{-}\) calculated by Bobeth et al.\[^{16}\] and by Asatrian et al.\[^{17}\] for the short-distance contribution have been recently harnessed\[^{15}\] to study the exclusive decays \(B \rightarrow K^{(*)}\ell^{+}\ell^{-}\). This input is then combined with the form factors calculated with the help of the QCD sum rules\[^{14}\], ignoring the so-called hard spectator corrections, calculated in the decays \(B \rightarrow K^{*}\ell^{+}\ell^{-}\).\[^{34}\] The rationale of this is the following: Beneke et al.\[^{34}\] have shown that the dilepton invariant mass distribution in the low invariant mass region is rather stable against the explicit \(O(\alpha_{s})\) corrections, and the theoretical uncertainties are dominated by the form factors and other non-perturbative parameters specific to the large-energy factorization approach. As already discussed, current data on \(B \rightarrow K^{*}\gamma\) decay yields typically a range \(T_{1}(0) = 0.27 \pm 0.04\). To accommodate this, a value \(T_{1}(0) = 0.33 \pm 0.05\), corresponding to the lower set of values in the QCD sum rules have been used in the NNLO analysis,\[^{15}\] yielding

\[
\mathcal{B}(B \rightarrow K^{*}\ell^{+}\ell^{-}) = (0.35 \pm 0.12) \times 10^{-6} \quad (\delta\mathcal{B}_{K\ell\ell} = \pm 34%) , \quad (26)
\]

\[
\mathcal{B}(B \rightarrow K^{*}\ell^{+}\ell^{-}) = (1.58 \pm 0.49) \times 10^{-6} \quad (\delta\mathcal{B}_{K^{*}\ell\ell} = \pm 31%) , \quad (27)
\]

\[
\mathcal{B}(B \rightarrow K^{*}\mu^{+}\mu^{-}) = (1.19 \pm 0.39) \times 10^{-6} \quad (\delta\mathcal{B}_{K^{*}\mu\mu} = \pm 33%) . \quad (28)
\]

These estimates are lower than the NLO estimates due to two reasons: the explicit \(O(\alpha_{s})\) corrections lower the decay rates and the central values of the input form factors are also reduced so as to accommodate the \(B \rightarrow K^{*}\gamma\) branching ratios in the same accuracy in \(O(\alpha_{s})\). They have to be confronted with the BELLE data summarized below:

\[
\mathcal{B}(B \rightarrow K^{*}\ell^{+}\ell^{-}) = (0.75^{+0.25}_{-0.21} \pm 0.09) \times 10^{-6} , \quad (29)
\]

\[
\mathcal{B}(B \rightarrow K^{*}\mu^{+}\mu^{-}) \leq 3.0 \times 10^{-6} \text{ at } 90\% \text{ C.L.} , \quad (30)
\]

\[
\mathcal{B}(B \rightarrow K^{*}\ell^{+}\ell^{-}) \leq 5.1 \times 10^{-6} \text{ at } 90\% \text{ C.L.} . \quad (31)
\]
Very recently, upper limits on these decays have been set by the BABAR collaboration, which at 90% C.L. are posted as:

\[
\mathcal{B}(B \to K^{ \ast \pm} \ell^{-}) \leq 0.50 \times 10^{-6},
\]

\[
\mathcal{B}(B \to K^{\ast \pm} \ell^{-}) \leq 2.9 \times 10^{-6},
\]

where a ratio \( \mathcal{B}(B \to K^{\ast e^+ e^-})/\mathcal{B}(B \to K^{\ast \mu^+ \mu^-}) = 1.2 \), following from the NLO QCD-SR estimate \(^{14}\), has been used to combine the \( K^{\ast e^+ e^-} \) and \( K^{\ast \mu^+ \mu^-} \) modes. As opposed to the BELLE collaboration, reporting a statistically significant signal in the \( B \to K\ell^{ \pm} \ell^{-} \) modes, BABAR data has no signal in this mode. However, the BABAR upper limit is not inconsistent with the BELLE measurement fluctuated down by slightly over a standard deviation.

To quantify the agreement between the SM and current data, we show the bounds implied by the experimental results given above in the \([C^{NP}_9 (\mu_W), C^{NP}_{10}]\) plane. In Fig. 4 the bounds from the inclusive radiative decays and inclusive and exclusive semileptonic decays have been combined in a single plot. Note that the overall allowed region is driven by the constraints emanating from the decays \( B \to X_s e^+ e^- \) and \( B \to K\ell^{ \pm} \ell^{-} \). In showing the constraints from \( B \to K\ell^{ \pm} \ell^{-} \), we have used the BELLE measurement to get the following bounds:

\[
0.37 \times 10^{-6} \leq \mathcal{B}(B \to K\ell^{ \pm} \ell^{-}) \leq 1.2 \times 10^{-6} \text{ at 90\% C.L.},
\]

resulting in carving out an inner region in the \([C^{NP}_9 (\mu_W), C^{NP}_{10}]\) plane. The two plots shown in these figures correspond respectively to the \( C^{tot}_7 \)-negative and \( C^{tot}_7 \)-positive solutions discussed earlier. In Fig. 5 four regions are identified which are allowed by the constraints on the branching ratios that present very different forward–backward asymmetries. In Fig. 6 we show the shape of the FB asymmetry spectrum for the SM and other three sample points. The distinctive features are the presence or not of a zero and global sign of the asymmetry. A rough indication of the FB asymmetry behavior is thus enough to rule out a large part of the parameter space that the current branching ratios can not explore.

In order to explore the region in the \([C^{NP}_9, C^{NP}_{10}]\) plane (where \( C^{NP}_{9,10} \) are the sum of MFV and MI contributions) that is accessible to these models, a high statistic scanning over the EMFV parameter space has been recently performed \(^{15}\) requiring each point to survive the constraints coming from the sparticle masses lower bounds and \( b \to s\gamma \). The surviving points are shown in Fig. 8 together with the model independent constraints. Note that the region spanned by these points has been drastically reduced by the presence of the \( b \to s\gamma \) constraint.
6 Summary

We summarize the main points of this talk.

- SM is in comfortable agreement with the measurements of the CP-asymmetry $a_{J/ψK_s}$, yielding $\sin 2\beta$. However, current data also allows a BSM phase, with a typical range $-3^\circ < \theta_d < 8^\circ$.

- SM is also in comfortable agreement with data on $B \rightarrow X_s\gamma$. Due to the inherent two-fold ambiguity on the sign of the effective Wilson coefficient, both $C_7^{\text{tot}} > 0$ and $C_7^{\text{tot}} < 0$ solutions are allowed in supersymmetry involving different regions of the parameter space.

- Theoretical precision in exclusive decays is at present compromised by the imprecise knowledge of form factors and some other non-perturbative quantities. Ratios of the branching ratios, and some asymmetries (due to isospin-violations or CP-violation in $B \rightarrow \rho\gamma$ and $B \rightarrow K^*\gamma$) are more reliably calculable in this framework, and can be used to search for BSM physics. A quantitative test of the SM in these decays will be undertaken in inclusive decays $B \rightarrow X_s\ell^+\ell^-$.

- Despite theoretical uncertainties, the experimental sensitivity on rare semileptonic $B$ decays is already strong enough to provide non-trivial bounds on the SUSY parameter space. Indeed, for the $C_7^{\text{tot}} > 0$ case, the larger portion of the SUSY allowed points is already ruled out.

- SUSY models can account only for a small part of the region allowed by the model independent analysis of current data. In the numerical analysis discussed here, integrated branching ratios have been used to put constraints on the effective coefficients. They allow a multitude of solutions in the effective Wilson parameter space and can be disentangled from each other only with the help of both the dilepton mass spectrum and the forward-backward asymmetry in semileptonic rare $B$ decays. Only such measurements would allow us to determine the exact values and signs of the Wilson coefficients $C_7$, $C_9$ and $C_{10}$, also limiting $C_8$ and decipher the physics behind flavour transitions.

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Figure 1: Allowed 95 % C.L. contours in the $(\rho, \eta)$ plane. The solid contour corresponds to the SM case, the dashed contour to the Minimal Flavour Violation case with $(f = 0.4, g = 0)$ and the dashed-dotted contour to the Extend-MFV model discussed in the text $(f = 0, g_R = -0.2, g_I = 0.2)$. (From Ref. 25.)

Figure 2: The $CP$ asymmetry $a_{\psi K_S}$ as a function of $\arg \delta_{u_L t_2}$ expressed in degrees. The solid curve corresponds to the Extended-MFV model $(f = 0, |g| = 0.28)$. The light and dark shaded bands correspond, respectively, to the allowed 1 $\sigma$ region in the SM $(0.58 \leq a_{\psi K_S} \leq 0.82)$ and the current 1 $\sigma$ experimental band $(0.67 \leq a_{\psi K_S} \leq 0.91)$. The plot on the right shows the correlation between $\arg \delta_{u_L t_2}$ and the angle $\theta_d$: $\theta_d = \frac{1}{4} \arg(1 + f + |g|^2 \arg \delta_{u_L t_2})$, $(\mod \pi)$. The experimentally allowed region flavours $0^\circ < \arg \delta_{u_L t_2} < 100^\circ$ that translates into $-3^\circ < \theta_d < 8^\circ$. (From Ref. 25.)
Figure 3: Partial (dashed lines) vs full (solid lines) NNLO computation of the branching ratio $B \to X_s e^+ e^-$. In the left plot ($\hat{s} \in [0,0.05]$) the lowest curves are for $\mu = 10$ GeV and the uppermost ones for $\mu = 2.5$ GeV. In the right plot the $\mu$ dependence is reversed: the uppermost curves correspond to $\mu = 10$ GeV and the lowest ones to $\mu = 2.5$ GeV. The right-hand plot also holds for the decay $B \to X_s \mu^+ \mu^-$. (From Ref. 15.)

Figure 4: NNLO Case. Superposition of all the constraints. The plots correspond to the $C_7^{\text{tot}} (2.5 \text{ GeV}) < 0$ and $C_7^{\text{tot}} (2.5 \text{ GeV}) > 0$ case, respectively. The points are obtained by means of a scanning over the EMFV parameter space and requiring the experimental bound from $B \to X_s \gamma$ to be satisfied. (From Ref. 15.)
Figure 5: Differential Forward-Backward asymmetry for the decay $B \rightarrow X_s \ell^+ \ell^-$. The four curves correspond to the points indicated in Fig. 4. (From Ref. 15.)