Dynamical Quark Effects in QCD

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We discuss latest results of lattice QCD simulations with dynamical fermions. Special emphasis is paid to the subjects of the static quark potential, the light hadron spectrum, Υ spectrum, and the pion-nucleon-sigma term.

1. INTRODUCTION

The investigation of the influence of sea quarks on the physics of strong interaction has been a field of vital interest ever since the formulation of lattice quantum chromodynamics. However, the computer simulation of full QCD processes is a very challenging task, and it is only in these days, that computers and algorithms might become powerful enough to produce results which can be connected reliably to continuum physics.

Several large scale lattice simulations of full QCD have been started recently, unfortunately most of them being still in the phase of lattice calibration or program development. The CP-PACS, UKQCD and QCDSP projects belong to this category. Status reports and first (preliminary) results can be found in [1,2], [3,4], [5]. The bulk of – at least semi-final – results achieved this year comes from the SESAM and the TχL collaborations. Therefore, these results will serve as a guideline for this talk. In the first section I will discuss the status of static potential measurements. The second section deals with slowly moving quarks, i.e. the NRQCD results come into focus. The third section summarizes the SESAM/TχL investigations on the light spectrum, and finally the lattice measurement of the pion-nucleon-sigma term σN is reviewed in section four.

2. STATIC POTENTIAL

One of the most exciting features that we expect to be a property of full QCD, but not of quenched QCD, is that the energy string between a quark and its anti-quark should break once the distance between these particles becomes large enough to produce a quark anti-quark pair out of the vacuum. Clearly, one would like to verify this important quality in full QCD lattice simulations. One prominent quantity, which should exhibit a clear sign of string breaking, is the potential between static quarks. The signature that we expect is a flattening out of the potential at a given distance rbreak. Unfortunately, this has not been observed yet in lattice QCD simulations.
up to distances of \( r \approx 1.5 \text{fm} \) and quark masses down to approximately the strange quark mass. The SESAM/T\( \chi \)L collaboration has performed a high precision measurement of the static potential \(^{12}\) with \( n_f = 2 \) dynamical Wilson fermions at \( \beta = 5.6 \). SESAM has used a lattice size \( 16^3 \times 32 \) at four different values of the sea quark mass, \( \kappa_{\text{sea}} = 0.1560, 0.1565, 0.1570, 0.1575 \), corresponding to \( m_\pi/m_\rho = 0.839(4), 0.807(7),0.755(7),0.69(1) \), with 200 statistically independent gauge configurations. The T\( \chi \)L collaboration has used lattice size \( 24^3 \times 40 \), two sea quark masses, \( \kappa_{\text{sea}} = 0.1390, 0.1395 \), corresponding to \( m_\pi/m_\rho \approx 0.69,0.55 \), and a statistics of presently \( \sim 80 \) configurations.

The result – at two different values of the quark mass – is shown in fig.1. Obviously, there is still no clear sign of string breaking in this range of quark masses and up to a physical distance of \( r \approx 2.0 \text{fm} \).

The result of SESAM/T\( \chi \)L is supported by the (preliminary) findings of the UKQCD collaboration\(^2\). Their result, obtained with an improved clover action\(^{13}\) at \( n_f = 2, \beta = 5.2 \) and a lattice volume of \( 12^3 \times 24 \) is displayed in fig.2. In order to guess, at which energy \( V(r_{\text{break}}) \) the string should break, UKQCD has also calculated the mass of the static light meson \( (M_{SL}) \) at \( \kappa_{\text{light}} = 0.1390 \). Naively, one expects \( V(r_{\text{break}}) \approx 2 \times M_{SL} \). The data in fig.2, however, doesn’t seem to obey this naive expectation, although there is still a small chance that they may do so (at \( \kappa_{\text{sea}} = 0.1390 \)). This uncertainty can be removed soon, as UKQCD’s simulation proceeds.

So, finally one has to tackle the question why string breaking could not be observed yet in full QCD determinations of the static potential. One explanation could be that currently available sea quarks are still too heavy. The energy necessary to produce a quark anti-quark pair out of the vacuum would then be available only at larger distances. This is however in sharp contrast to the naive expectation and would therefore require further investigation.

Another explanation, which appears much more likely to the author, is that the Wilson loop operator, which is used to calculate the potential, does not have sufficient overlap with the state in which the string is broken. In order to corroborate this scenario one would have to compare the potential energy with the ground state energy of an operator, which is designed to describe the physical situation of a broken string. Such an operator could be that of a system of two static-light mesons. Its ground state would have to be evaluated with respect to the spatial distance of the mesons. The crossover point between \( V(r) \) and the ground state energy of this operator would then determine \( r_{\text{break}} \).

Before we turn to the discussion of NRQCD results, we would like to emphasize that the influence of dynamical fermions can be seen clearly in the short distance part of the static potential, i.e. in its coulombic contribution. Fig. 3 compares the values of the coulomb coefficient \( e \) at several sea quark masses and in the quenched approximation. The difference between quenched and full QCD results is statistically significant and reveals that the coulombic part becomes stronger in full QCD.

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\(^{12}\) Configuration with this sea quark mass are currently analyzed.

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**Figure 2.** UKQCD collaboration (preliminary). The rescaled static potential in full and in quenched (\( \beta = 5.7 \)) QCD, \( r/r_0 = 3 \) corresponds roughly to \( r \approx 1.5 \text{fm} \). The naive expectation of string breaking is displayed at the right edge. For details see \(^3\).
3. Υ Spectrum

The determination of heavy particle properties with lattice methods is a notoriously difficult task, as large cutoff effects may contaminate the results. A very promising attempt to get rid of these contaminations is the lattice formulation of NRQCD, which, in principle, is able to control its systematic errors. As the computational effort needed to exploit NRQCD is small compared to standard lattice simulations, it appears to be an ideal laboratory to study the influence of sea quarks on the spectrum of heavy mesons with high statistical precision.

It has been reported last year [8] that the Υ spectrum energy levels are shifted towards their experimental values once sea quarks are switched on. This behaviour had been seen in the spin independent as well as in the spin dependent parts of the spectrum.

Encouraged by this result the SESAM collaboration has now performed an NRQCD analysis of the Υ spectrum with $n_f = 2$ Wilson fermions[14]. With this data in hand, which we display in fig. 4, the situation now looks much less definite. Although SESAM consolidates the last years picture with respect to the spin independent part of the spectrum, there is no significant effect of sea quarks in the spin dependent part.

In order to explain the difference between the results of the NRQCD collaboration[9] and the SESAM results, one has to remember that the definition of the NRQCD Hamiltonian is by far not unique. The NRQCD collaboration has included only terms up to $O(Mv^4)$ and used the plaquette definition of the average link for tadpole improvement. SESAM went up to $O(Mv^6)$ and used the Landau gauge definition of the average link for tadpole improvement. The effects of such changes have been studied in detail by [14,15] within the quenched approximation. For the spin dependent splittings of the Υ system they...
find them to be of the order 10% – 20%. We conclude that, using state of the art NRQCD, unquenching effects in the spin dependent part may be hidden by systematic uncertainties.

4. LIGHT SPECTRUM

The accurate calculation of the light hadron spectrum, including the strange quark sector, is still an open problem for full lattice QCD. Apart from the lack of statistics, finite size and finite cutoff effects, one reason is that one cannot include light and strange sea quarks at the same time yet.

Thus, the usual way of analyzing spectrum data has been to look at light hadrons in a sea of light quarks only, whereas the properties of hadrons containing strange quarks have been approximated by use of a sea of strange quarks.

The SESAM collaboration has now developed a more general type of analysis, which goes beyond the restriction of equal values of sea and valence quark mass. With that method one exploits a given full QCD gauge configuration – with a definite value of the sea quark mass \( m_{\text{sea}} \) – with respect to several valence quark masses \( m_{\text{val}} \). Guided by chiral perturbation theory one can then parameterize hadron mass results with respect to both \( m_{\text{sea}} \) and \( m_{\text{val}} \). A simultaneous fit to all hadron data finally allows to access arbitrary points in the \((m_{\text{sea}}, m_{\text{val}})\) plane. Of course, the validity of this ansatz has to be checked carefully with the data.

SESAM has applied this method to the set of 3 (sea quarks) \&times; 5 (valence quarks) combinations, with a high statistics of 200 independent gauge configurations per sea quark, using a linear ansatz both in \( m_{\text{val}} \) and \( m_{\text{sea}} \). The valence quark masses are located in the same range as the sea quark masses (see above). Fig. 3 illustrates the quality of the approach for the case of pseudo-scalar and vector mesons. Although the data is reasonably well described with a linear ansatz, one cannot rule out quadratic terms in \( m_q \) at this stage of the analysis. In order to check on this SESAM is currently collecting data at one additional value of the sea quark mass.

With the current setup – 3 values of dynamical quarks and linear parameterization – SESAM finds no significant unquenching effect in the light hadron spectrum. This is similar to the findings of the MILC collaboration [10], which uses staggered sea quarks.

Finite size effects are a potentially dangerous source of contamination, especially if one tries to push lattice masses towards the chiral point. As almost nothing is known about the size of these effects in the case of dynamical Wilson fermions [18], the T\(\chi\)L collaboration has extended the SESAM lattice to size \( 24^3 \times 40 \). The simulation is now being performed at the smallest SESAM sea quark mass, \( \kappa = 0.1575 \), and at an even smaller mass, \( \kappa = 0.1580 \). With the current statistics – 80(200) T\(\chi\)L(SESAM) configurations – a direct comparison reveals finite size effects of \( O(5\%) \), but at this stage of the simulation it is clearly too early to draw definite conclusions.

5. PION-NUCLEON-SIGMA TERM

The determination of the pion-nucleon-sigma term

\[
\sigma_N = m_l \langle P | \bar{u} u + \bar{d} d | P \rangle , \quad m_l = \frac{1}{2} (m_u + m_d) \tag{1}
\]

has a long and eventful history, reaching back into the late seventies. Although experimentally as well as theoretically difficult to determine, it is an important quantity, measuring chiral symmetry breaking in QCD. It is also a fascinating quantity, as it is expected that pure vacuum fluctuations make up at least half of its size. Since we are looking for sea quark effects, it appears mandatory to study \( \sigma_N \) in full lattice QCD.

The cheap way to calculate \( \sigma_N \) in full lattice QCD is simply to determine the proton mass at several quark masses and to apply the Feynman-Hellman theorem

\[
m_l \langle P | \bar{u} u + \bar{d} d | P \rangle = m_l \frac{\partial m_P}{\partial m_l} . \tag{2}
\]

However, in a statistical simulation, the outcome of this method may be rather unprecise: suppose one would be able to determine \( m_P(m_q) \) to a precision which allows to resolve the quadratic term
of this function. Then, because of eq. 2, the quark mass dependence of $\sigma_N$ would be known only to the order linear in $m_q$. Thus, in order to extract maximal information from a given set of configurations, it seems advantageous to choose the hard way, i.e. to calculate the connected and the disconnected parts of eq. 1.

The SESAM collaboration has followed this way of analysis. The connected part has been determined with the standard procedures. The disconnected contribution to $\sigma_N$ is given by the correlation between the proton propagator and the quark loops. The latter were calculated by application of the stochastic estimator method with complex $Z_2$ noise. The statistical quality of the correlation signals could be increased substantially by using a plateau-like sampling of quark loops instead of the usual summation over all quark loops in space and time.

Fig. 5 displays the SESAM result for the (unrenormalized) amplitude $\langle P | \bar{u} u + \bar{d} d | P \rangle$. Both contributions, connected and disconnected, increase as the quark mass decreases. A linear extrapolation to the light quark mass yields

$$\sigma_N = 19.65(4.39) \text{MeV}.$$  \hfill (3)

Here we have multiplied with the light quark mass, $m_l = 0.5(1/\kappa_l - 1/\kappa_c)$, $\kappa_l = 0.158415$, $\kappa_c = 0.158458$, and with the inverse lattice spacing, $a^{-1} = 2.33 \text{GeV}$. This value (eq. 3) is almost a factor two smaller than the results obtained from phenomenology \cite{19} and from quenched QCD \cite{20, 21}. We emphasize that no renormalization factor is involved here, as $\sigma_N$ is a renormalization group invariant quantity.

In order to find out where this large deviation comes from, we compare in fig. 6 the SESAM results with previous quenched results. Obviously all amplitudes are in reasonable agreement. Thus we conclude that the large deviation in $\sigma_N$ is caused by the large difference between quenched and unquenched quark masses found in \cite{21, 22, 23}. It is an open issue, whether present lattice techniques are sufficient to determine $m_l$ reliably in full QCD. But lattice calculations of $\sigma_N$ appear

\footnote{This has been anticipated by M. Okawa.}
to exclude already that uncertainties due to mass renormalization factors could be responsible.

6. CONCLUSIONS

Sea quarks are shy objects. The only places where we could at least see the footprints of Wilson fermions are the short distance part of the static potential and the spin independent part of the $\Upsilon$ spectrum. Obviously very accurate simulations, very refined analysis methods and a clear understanding of the underlying physics are needed to observe sea quarks at work. The first promising steps in this direction have been done recently, and we are looking forward to the next generation of full QCD simulations, namely UKQCD, CP-PACS and QCDSP.

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