Holographic description of asymptotically AdS\(_2\) collapse geometries

David A. Lowe and Shubho Roy

Department of Physics, Brown University, Providence, RI 02912, USA

The mapping between bulk supergravity fields in anti-de Sitter space and operators in the dual boundary conformal field theory usually relies heavily on the available global symmetries. In the present work, we study a generalization of this mapping to time dependent situations, for the simple case of collapsing shock waves in two spacetime dimensions. The construction makes use of analyticity of the conformal field theory and the properties of the asymptotic bulk geometry to reconstruct the non-analytic bulk observables. Many of the features of this construction are expected to apply to higher dimensional asymptotically anti-de Sitter spacetimes and their conformal field theory duals.

I. INTRODUCTION

In a series of papers\(^1\),\(^2\),\(^3\) a reformulation of the Lorentzian AdS\(_D\)/CFT\(_{D-1}\) correspondence\(^4\),\(^5\),\(^6\),\(^7\),\(^8\) was worked out in the leading semiclassical \((N \rightarrow \infty)\) approximation. This reformulation was based on mapping normalizable bulk fields, (on the boundary \(\phi(z, x) \sim z^\Delta \phi_0(x)\) as \(z \to 0\)) to local CFT operators \(\mathcal{O}(x)\)\(^9\):

\[
\phi_0(x) \leftrightarrow \mathcal{O}(x)_{\text{CFT}}.
\]

Here \(z \to 0\) on the boundary and \(x\) coordinatizes the boundary. The central aim of this reformulation was to recover approximate locality in the bulk in the most transparent manner - by mapping on-shell bulk insertions to a delocalized boundary (CFT) operator with compact support on the boundary,

\[
\phi(z, x) \leftrightarrow \int dx' K(x'|x, z)\mathcal{O}(x')_{\text{CFT}}.
\]

This was an improvement over earlier attempts\(^10\),\(^11\),\(^12\) which generally involved representation of a local bulk insertion in terms of a nonlocal CFT operator with support over the entire boundary and hence required delicate cancellations to recover bulk locality. This boundary-to-bulk map or the smearing function, \(K(x'|x, z)\) constructed for various coordinate systems was nonvanishing only for points on the boundary spacelike separated from the local bulk insertion. The smearing function immediately reproduces the bulk correlators in terms of the boundary correlators, for example

\[
\langle \phi(x_1, z_1)\phi(x_2, z_2) \rangle = \int dx'_1 dx'_2 K(x'_1|x_1, z_1)K(x'_2|x_2, z_2) \langle \mathcal{O}(x'_1)\mathcal{O}(x'_2) \rangle_{\text{CFT}}.
\]

In the case of accelerating Rindler coordinates in AdS\(_3\), the smearing function could be expressed as a function with support on a disc on the complexified boundary\(^3\). This yielded a more transparent accounting of holographic entropy. The BTZ black hole\(^13\), which can be conveniently obtained as a periodic identification of the AdS\(_3\) Rindler coordinates was then considered. It was shown that for local bulk fields inside the horizon one needed smeared CFT operators on the boundaries of both the left and right Rindler wedges (see also\(^14\)). However, even in these cases, using analytic continuation, one could reduce to CFT operators smeared over a single complexified boundary\(^3\).

In the present paper we generalize these results to an asymptotically AdS\(_2\) spacetime with a null collapsing shockwave. The outline of this paper is as follows. In section II we review the coordinate system(s) employed for this Vaidya spacetime and provide the Penrose diagram. We begin by constructing the boundary to bulk map for points outside the black hole horizon in section III. First, in section III A we consider a massless scalar, where the result is identical to the pure AdS\(_2\) case when written using global null coordinates, despite the time-dependent geometry. In section III B we deal with the slightly more complicated massive case where we need to propagate the field across the shock using appropriate matching conditions. The smearing function again is only nonvanishing in the spacelike separated region on the boundary.

*Electronic address: lowe@brown.edu, sroy@het.brown.edu
The results are extended to points inside the horizon in section IV. It is still possible to express local bulk fields in terms of CFT operators on the single boundary at infinity, provided these are analytically continued to complex values of the boundary coordinates. Thus we accomplish the aim of completely characterizing a local theory propagating in a time-dependent bulk geometry in terms of the analytic boundary theory.

II. THE ADS$_2$ VAIDYA SPACETIME

We consider a 2d black hole which can be obtained by a dimensional reduction of a BTZ formed by the gravitational collapse of a null dust in 3d asymptotically AdS spacetime [13, 16] given by the metric,

$$ds^2 = -f(r)dv^2 + 2dvdr = -f(r)dt^2 + \frac{dr^2}{f(r)}$$ \hspace{1cm} (r > 0),

where

$$dt = dv - \frac{dr}{f(r)},$$

and

$$f(r) = \frac{r^2 - r_0^2}{1 + r^2}, \hspace{1cm} v \geq 0,$$

$$f(r) = \frac{1}{1 + r^2}, \hspace{1cm} v < 0.$$

The two-dimensional action is the Jackiw-Teitelboim gravity theory [17, 18]. Such a black hole is formed by doing cylindrical reduction on a spacetime with a negative cosmological constant $\Lambda$ as a result of the gravitational collapse of a null dust shell. We set $\Lambda = -1$ from now on.

Tortoise coordinates $(t, r_*)$ are defined as,

$$t = v - \int \frac{dr}{f(r)}$$ \hspace{1cm} (2)

and

$$r_* = \int_{\infty}^{r} \frac{dr}{f(r)} = \frac{1}{2r_0} \ln \left| \frac{r - r_0}{r + r_0} \right|, \hspace{1cm} v \geq 0$$

$$\tan^{-1} \frac{r - \pi/2}{v < 0}.$$

Here the constant of integration $-\pi/2$ is chosen to make $r_* \to 0$ as $r \to \infty$ from both sides of the shock. In order to distinguish between the local coordinates defined in the regions $v \geq 0$ and $(v < 0)$, we shall use the superscript $+$ and $-$ respectively. The range of the local coordinates are $-\infty < r_*^+ \leq 0$ for points outside the horizon ($r > 0$) and $-\pi/2 < r_*^- < 0$. Inverting the above relation, we have $r_*^\pm$ in terms of $r$,

$$r = -r_0 \coth (r_*^+ r_0) = -\cot r_*^-.$$ \hspace{1cm} (3)

The Penrose diagram is shown in figure [1].

III. BOUNDARY/BULK DUALITY OUTSIDE THE HORIZON

A. Zero Mass

A free massless scalar minimally coupled to Jackiw-Teitelboim gravity, propagating in such a black hole spacetime satisfies the equation of motion,

$$\frac{1}{\sqrt{-g}} \partial_a (\sqrt{-g} g^{ab} \partial_b \phi) = 0.$$
1. **Smearing function for point having support to either the past or future of the shock**

In tortoise coordinates (2) the wave equation is simply

\[ (-\partial_t^2 + \partial_{r^*}^2)\phi = 0. \]

Now Green’s theorem is,

\[
\phi(x) = \int dS' \sqrt{\eta^a} \left( \partial_a \phi(x')G(x, x') - \phi(x')\partial_a G(x, x') \right)
\]

where \( h \) is the induced metric and \( \eta \) is the normal derivative on the boundary at infinity. We begin by considering points such as S and T in figure 1. If we can find a Green’s function \( G(r^* - r_*, t' - t) \) which has support inside the spacelike cone extending from the bulk point to the boundary, e.g.

\[
G \propto \theta(r^*_s - r_*)\theta(|t - t'|)\theta(r^*_t - r_*)
\]
then we can extract the desired smearing function. We obtain such a Green’s function using contour integration,

\[-\partial_t^2 + \partial_r^2 \] \( G(r'_* - r_*, t' - t) = \delta(r'_* - r_*)\delta(t' - t) \) \ .

(4)

In momentum space,

\[ (\omega^2 - k^2) G(k, \omega) = 1 \] .

So formally,

\[ G(r'_* - r_* , t' - t) = \int \frac{d\omega dk}{(2\pi)^2} \frac{1}{\omega^2 - k^2} e^{i(k \Delta r_* - \omega \Delta t)} \]

where \( \Delta r_* = r'_* - r_* , \Delta t = t' - t \). Now lets do the \( k \) integration first by closing the contour in the upper half-plane,

\[ G(r'_* - r_* , t' - t) = -\int \frac{d\omega}{2\pi} e^{-i\omega \Delta t} \int \frac{dk}{2\pi} \frac{1}{(k - i\epsilon)^2 - \omega^2} e^{ik\Delta r_*} \]

\[ = \frac{1}{2}\theta(\Delta r_*) \int \frac{d\omega}{2\pi} e^{-i\omega \Delta t} \frac{e^{i\omega \Delta r_*} - e^{-i\omega \Delta r_*}}{2\omega} \]

\[ = \frac{1}{4}\theta(\Delta r_*) (\text{sgn}(\Delta r_* + \Delta t) + \text{sgn} (\Delta r_* - \Delta t)) \]

\[ = \frac{1}{2}\theta(\Delta r_*) \theta(\Delta r_* - |\Delta t|) . \]

Now the normalizable modes have the asymptotic fall off,

\[ \phi(r_*, t) \sim r_* \phi_0(t) \]

as \( r_* \to 0 \) since for a massless minimally coupled scalar the scaling dimension is \( \Delta = \frac{1}{2} + \sqrt{\frac{1}{4} + m^2 R^2} = 1 \). So according to to Green’s theorem,

\[ \phi(r_*, t) = \int dt' \phi_0(t') G(0 - r_*, t' - t) = \frac{1}{2} \int_{t+r_*}^{t-r_*} dt' \phi_0(t') \]

(recall that \( r_* < 0 \)) and hence the smearing function is simply the Green’s function with compact support on spacelike separated region on the boundary.

2. Smearing function for point with support to both past and future of shock

In this section we consider points such as \( R \) from figure 2. For convience we will use null coordinates for the calculation. For a null ray

\[ 2dr/f(r) - dv = 0 \]

Introduce new coordinate \( u \)

\[ 2r_* - v = u \]

where \( u \) is a constant over the ray. At the shock \( v = 0 \), we have the relation

\[ u = 2r_* \]
So according to (3) the values of $u$ (on the same null ray) across the shock are related by,

$$r_0 \coth(u^+ r_0/2) = \cot(u^-/2)$$

or,

$$u^- = h(u^+) = 2 \cot^{-1}(r_0 \coth(u^+/2)).$$

So now we can make a **global** null coordinate,

$$u = \begin{cases} 
   u^-, & v < 0 \\
   h(u^+), & v > 0 
\end{cases}$$

In $u,v$ parameters, the metric is,

$$ds^2 = F(u,v)du dv$$
where the conformal factor is,

$$F(u, v) = \begin{cases} 
  f(r(u, v)) = \csc^2 \left( \frac{(u + v)}{2} \right), & v < 0 \\
  f(h^{-1}(u), v) = \frac{2r_0^2 \csc b^2 \left( \frac{1}{(\csc^2 a/2) + rv_0/2} \right)}{(1 + r_0^2) \cos u - (1 - r_0^2)}, & v > 0 .
\end{cases}$$

Spacelike infinity $r_+ = 0$ is where

$$u^\pm + v = \theta(v)h(u) + \theta(-v)u + v = 0 .$$

In the global $u, v$ parameters, the Green’s function equation for a massless minimally coupled scalar is,

$$2\partial_u \partial_v G(u - u', v - v') = \delta(u - u')\delta(v - v')$$

So, we obtain

$$G(u - u', v - v') = \frac{1}{2} \theta(v' - v)\theta(u' - u) .$$

This is a **global** Green’s function since the coordinate $u$ is continuous across the shock. Again this Green’s function is nonvanishing only in the spacelike separated region on the right boundary and hence furnishes a smearing function which has a compact support on the timelike boundary at infinity,

$$\phi(u, v) = \frac{1}{2} \int_{-\infty}^{0} dv' \left( G(u - u', v', v') \partial_{u'} \phi(u', v') \right) |_{u' = -v'} + \frac{1}{2} \int_{0}^{\infty} dv' \left( \frac{dh}{du}(v') \right)^{1/2} (G(u - u', v', v') \partial_{h(u') + v'} \phi(u', v')) |_{h(u') = 0} .$$

$$= \int_{-\infty}^{\infty} dv' \frac{1}{2} \left( \theta(-v') \theta(u + v') \delta(v - v') + \theta(v') \theta(u - h^{-1}(-v')) \delta(v - v') \right) \left( \frac{dh}{du}(v') \right)^{1/2} \phi_0(v') .$$

Recall that normalizable fields have asymptotic behavior $\phi(u, v) \sim (u + v)\phi_0(v)$ where $\lim_{u \rightarrow -v} \phi_0(v) \neq 0$ for $v < 0$ while for $v > 0$, we have $\phi \sim (h(u) + v)\phi_0(v)$ with $\lim_{h(u) \rightarrow -v} \phi_0(v) \neq 0$ again. So the smearing function for massless fields in Kruskal coordinates is,

$$K(v' | u, v) = \frac{1}{2} \left( \theta(-v') \theta(u + v') \delta(v - v') + \theta(v') \theta(u - h^{-1}(-v')) \delta(v - v') \right) .$$

The expression clearly demonstrates the fact that the smearing function cannot be expressed in terms of a AdS2-covariant distance function since the metric is discontinuous (non-analytic) across the shock and cannot be covered by a single analytic coordinate patch although locally it is pure AdS2 everywhere. The $u^\pm$, $v$ coordinates parametrize null geodesics and $u^\pm$ suffers a jump across the shock. As a result, in terms of these coordinates the smearing function has a discontinuity across the shock. Nevertheless the smearing function has support only for points on the right boundary spacelike separated from the bulk point, showing the construction of \[1, 2, 3\] generalizes to time-dependent bulk geometries.

**B. Non-vanishing mass**

The massless minimally coupled field is insensitive to the conformal factor in the metric in the $u, v$ coordinates which leads to a very simple Green’s function in these coordinates. On the other hand, the massive scalar does feel the conformal factor,

$$\frac{1}{\sqrt{-g}} \partial_a (\sqrt{-g} g^{ab} \partial_b G(x, x')) - m^2 G(x, x') = \frac{1}{\sqrt{-g}} \delta(x - x') .$$

Now, lets go back to local parameters $r^\pm_+$, $t^\pm$ since that is more suitable for calculation,
\[ ds^2 = \begin{cases} 
\csc^2 r^- (-dt^2 + dr_-^2), & v < 0 \\
\frac{r_0^2}{csc^2(r_+ r_0)(-dt^2 + dr_+^2)}, & v > 0 
\end{cases} \]

and are related by continuity across the surface of the shock by,

\[
\begin{aligned}
r &= -r_0 \coth(r_+ r_0) = -\cot r^- \\
t^+ &= v - r_+^+ (r^-) = t^- + r_+^+ - r_+^+(r^-). 
\end{aligned}
\] (5)

The Green’s function for the \(v < 0\) region with support over the spacelike separated boundary region is the same as the one for pure AdS\(_2\) with support over spacelike separated region on the right boundary and was worked out in [1] for arbitrary non-vanishing mass,

\[
G^-(r_+^-, r^-_-, t^- - t^-) = \frac{1}{2} P_{\Delta-1}(\sigma) \theta(r_+^--r^-) \theta(r_+^--r_-^- - |t^- - t^-|)
\]

where \(\Delta = \frac{1}{2} + \sqrt{m^2 + \frac{1}{4}}\) and the AdS\(_2\) covariant distance function,

\[
\sigma(x, x') = \frac{\cos(t^- - t^-) - \cos r^- \cos r_-^-}{\sin r^- \sin r^-_-}
\] (6)

and the \(P_{\Delta-1}(\sigma)\) is the Legendre function.

For the \(v > 0\) region, the relevant Green’s function is the one for a point outside the horizon of a BTZ black hole with support on the spacelike region of the right boundary. This is again just the pure AdS\(_2\) global Green’s function transformed to the set of local coordinates [1],

\[
G^+(x, x') = \frac{1}{2} P_{\Delta-1}(\sigma) \theta(\rho' - \rho) \theta(\rho' - \rho - |\tau - \tau'|)
\]

where \(\rho, \tau\) coordinates are defined in terms of the old \(r, t^+\) by,

\[
\begin{aligned}
\frac{\cos \tau}{\sin \rho} &= -\frac{r}{r_0} \\
\frac{\sin \tau}{\cos \rho} &= \tanh t^+ r_0 
\end{aligned}
\] (7)

and the metric is then,

\[ ds^2 = \csc^2 \rho (-d\tau^2 + d\rho^2). \]

In these coordinates we define the boundary field \(\phi_0(\tau)\),

\[
\phi_0(\tau) = \lim_{\rho \to 0} \frac{\phi(\tau, \rho)}{\sin^2 \rho}
\]

and for the \(v < 0\) region,

\[
\phi_0(t^-) = \lim_{r_- \to 0} \frac{\phi(t^-, r^-_0)}{\sin^2 r^-_0}.
\]
1. Matching conditions across the shock

• Continuity of the field

The field is continuous across the shock,

$$\phi(v = 0^+, r) = \phi(v = 0^-, r).$$

• Continuity of the normal derivative of the field

By integrating the d’Alembertian over a Gaussian surface straddling $v = 0$ and noting that there are no sources of the $\phi$-field on $v = 0$,

$$\int d^2x \sqrt{-g} \left( \frac{1}{\sqrt{-g}} \partial_a (\sqrt{-gg^{ab}} \partial_b \phi) - m^2 \phi \right) \eta_a (\sqrt{-gg^{ab}} \partial_b \phi(x)) \bigg|_{v^+}^{-v^-} - \eta_a (\sqrt{-gg^{ab}} \partial_b \phi(x)) \bigg|_{v^+}^{-\epsilon} = 0$$

where $\eta_a$ is the outward unit normal vector to $v = \text{const.}$. In the global coordinates $(r, v)$, the normal derivative is just $\partial_r \phi$ and then the condition is simply,

$$\partial_r \phi(v = 0^+, r) = \partial_r \phi(v = 0^-, r).$$

2. Smearing function for a point with its right spacelike separated regions extending to either sides of the shock

In this case we have a local bulk insertion for a point like $R$ in figure 2. Our strategy is to first use Green’s theorem to express the normalizable bulk field at $x^\tau$ (the superscript indicates it is in the $v < 0$ region) in terms of an integral over spacelike infinity of the $v < 0$ region and the surface of the shock $v = 0$, which bounds the spacelike separated region to the right of $x^\tau$

$$\phi(x^-) = \int dt^- \left( (2\Delta - 1) G^-(x^-, x^-) \sin^{\Delta - 1} r^- \phi_0(t^-) \right) |_{r^- = 0} + \int dr^- \left( \phi(x^-) \left( \frac{\partial}{\partial r^-} - \frac{\partial}{\partial v^-} \right) G^-(x^-, x^-) \right) |_{r^- + v^- = 0}.$$

Then using the matching conditions across the shock we convert the integral over the shock to an integral over the timelike infinity of the $v > 0$ region

$$\int dr^+ \left( \phi(x^+) \left( \frac{\partial}{\partial r^+} - \frac{\partial}{\partial v^+} \right) G^-(x^-, x^-) \right) |_{r^+ + v^+ = 0} = \int dt^+ \phi_0(t^+) \left( \int dr^+ K^+(r^+ | \rho, \tau) \left( \frac{\partial}{\partial r^+} - \frac{\partial}{\partial v^+} \right) G^-(x^-, x^-) \right) |_{\rho + \tau = r^+ + v^+ = 0}$$

where $\rho = \rho(r^-, t^-), \tau = \tau(r^-, t^-)$ and $K^+$ is the smearing function for a field in the $v > 0$ region. So, the full smearing function is,

$$K(x|x^+) = \theta(-t^-) (2\Delta - 1) \left( G^-(x^-, x^-) \sin^{\Delta - 1} r^- \right) |_{r^- = 0} + \theta(t^+) \left( \int dr^- K^+(\tau | \rho, \tau) \left( \frac{\partial}{\partial r^-} - \frac{\partial}{\partial v^-} \right) G^-(x^-, x^-) \right) |_{\rho + \tau = r^+ + v^+ = 0}.$$ (8)
\[ K_1 = \theta(-t^-)(2\Delta - 1) \left( G^-(x^-, x^+) \sin^{\Delta-1} r_*^- \right) \big|_{r_*^- = 0} \]
\[ = \theta(-t^-) \frac{2^{\Delta-1} \Gamma(\Delta + 1/2)}{\sqrt{\pi} \Gamma(\Delta)} \left( \frac{\cos(t^- - t^+) - \cos r_*^-}{\sin r_*^-} \right) \Delta^{-1} \theta(-r_*^- - |t^- - t^|). \quad (9) \]

The second term in (9), the integral along the shock, is a bit more involved. Explicitly (dropping the overall \( \theta(t^+) \)),

\[ K_2 = \int dt_*^- K^+(t^|\rho, \tau)(\partial_{\tau}^- - \partial_\tau)G^-(r_*^+, t^-|r_*^-, t^-) \]
\[ \text{where} \]
\[ K^+(t^|\rho, \tau) = \frac{2^{\Delta-1} \Gamma(\Delta + 1/2)}{\sqrt{\pi} \Gamma(\Delta)} \left( \frac{\cos(t^ - \tau) - \cos \rho}{\sin \rho} \right) \Delta^{-1} \theta(-\rho - |t^ - \tau|) \]

and

\[ G^-(r_*^+, t^- - t^-) = \frac{1}{2} P_{\Delta-1}(\sigma) \theta(r_*^+ - r_-^-) \theta(r_*^- - r_*^+ - |t^- - t^-|) \]

where \( \Delta = \frac{1}{2} + \sqrt{m^2 + \frac{1}{4}} \) and the AdS\(^2\) covariant distance function is given in (6). \( K_2 \) is more conveniently expressed as a sum of two contributions

\[ K_2 = K_{2I} + K_{2II} \]

where

\[ K_{2I} = \theta(t^-) \int dt_*^- K^+(t^|\rho, \tau = -\rho)(\partial_{\tau}^- - \partial_\tau)G^-(r_*^+, t^-|r_*^-, t^-) |_{t^- = -r_*^-} \]
\[ = \left( -2 \sin \frac{\tau^-}{2} \right)^\Delta \left( \frac{2^{\Delta-2} \Gamma(\Delta + 1/2)}{\sqrt{\pi} \Gamma(\Delta - 1)} \right) \]
\[ \times \int dt_*^- \left( \frac{\sin(t^-/2 + \rho)}{\sin \rho} \right)^{\Delta-1} \theta(-\rho - |t^ - \tau|) \left( \frac{\cos r_*^+ - \cos t^-}{\sin r_*^- \sin^2 r_*^-} \right) \frac{\sigma P_{\Delta-1} - P_{\Delta-2} \theta(\Delta r^-_\tau) \theta(\Delta r^-_\rho - |t^- - t^-|)}{\sigma^2 - 1} \]
\[ + \frac{1}{2} \theta(t^-) \theta \left( -\frac{r_*^- + t^-}{2} \right) \left( K^+(t^|\rho, \tau = -\rho) P_{\Delta-1}(\sigma) \right) |_{t^- = -r_*^-} \]

while

\[ K_{2II} = \int dt_*^- \left( G^-(r_*^+, t^-|r_*^-, t^-)(\partial_{\tau}^- - \partial_\tau)K_1(\rho, \tau|t^-) \right) |_{\tau^- = -r_*^-} \]
\[ = \left( -2 \sin \frac{\tau^-}{2} \right)^\Delta \left( \frac{2^{\Delta-3} \Gamma(\Delta + 1/2)}{\sqrt{\pi} \Gamma(\Delta - 1)} \right) \]
\[ \times \int dt_*^- P_{\Delta-1}(\sigma) \theta(\Delta r^-_\tau) \theta(\Delta r^-_\rho - |t^- - t^-|) \frac{\sec^2 r_*^- \sin^{\Delta-2} (t^- + \rho/2)}{1/r_0 + r_0 \tan^2 r_*^- \sin^\Delta r^- \rho} \theta(-\rho - |t^ + \rho|) \]

where on the shock (5) reduces to

\[ \tan \rho = \tan \rho(r_*^-) = r_0 \tan r_*^- \]

So, the full smearing function in this case looks complicated as the bulk geometry changes across the shell from pure AdS\(^2\) to a AdS\(^2\) black hole, but nevertheless has support only on the right boundary region spacelike separated from the bulk point.
IV. POINTS INSIDE THE HORIZON

As has been pointed out in [1] for operator insertions at points inside the future Rindler horizon in pure AdS, the smearing function has support on both boundaries i.e. boundaries of the left and right Rindler wedges. For integer conformal dimension \( \Delta \),

\[
\phi(P) = \int_{-\infty}^{\infty} dt K_{\text{Rindler}}^R(t|P) \phi_0^{\text{Rindler}, R}(t) + (-)^\Delta K_{\text{Rindler}}^L(t|P) \phi_0^{\text{Rindler}, L}(t) \tag{11}
\]

where

\[
K_{\text{Rindler}}^R(t|P) = \frac{2^{\Delta-1} \Gamma(\Delta + 1/2)}{\sqrt{\pi} \Gamma(\Delta)} \lim_{r \to \infty} \left( \frac{\sigma}{r} \right)^{\Delta-1} \theta(\sigma - 1)
\]

\[
K_{\text{Rindler}}^L(t|P) = \frac{2^{\Delta-1} \Gamma(\Delta + 1/2)}{\sqrt{\pi} \Gamma(\Delta)} \lim_{r \to \infty} \left( -\frac{\sigma}{r} \right)^{\Delta-1} \theta(-\sigma - 1)
\]

and \( \sigma = \sigma(t, r|P) \) is the AdS\(2 \) invariant distance. [22]

However the integral over the left Rindler boundary can be mapped back to the right Rindler boundary using the identification,

\[
\phi_0^{\text{Rindler}, L}(t) = \phi_0^{\text{Rindler}, R}(t + i\pi).
\]

This identification was arrived at after noting that a complex change of Rindler coordinates takes one from the left to the right Rindler wedge,

\[
t \to t + i\pi.
\]

So combining these information we have a smearing function which has compact support entirely on the complexified right boundary.

\[
\phi(P) = \phi(P) = \int_{-\infty}^{\infty} dt K_{\text{Rindler}}^R(t|P) \phi_0^{\text{Rindler}, R}(t) + (-)^\Delta K_{\text{Rindler}}^L(t|P) \phi_0^{\text{Rindler}, R}(t + i\pi). \tag{12}
\]

A similar result for the BTZ black hole was also derived in [3].

Now let us extend these results to the shockwave geometry. In this case we no longer have the same global isometries used in the derivation of the expressions (11) and (12). However for bulk points to the future of the shell such as point P in figure 1 by analytic continuation, we can pretend to be in an eternal AdS\(2 \) black hole and the expression (12) carries over [23].

New features appear when we study points to the past of the shock, but inside the horizon, such as Q in figure 1. For this point we first use the spacelike Green’s function to express the bulk scalar in terms of an integral on \( v = 0^- \) and the portion of the boundary \( r^*_+ = 0 \) on the \( v < 0 \) side

\[
\phi(Q) = \int dt^- K_1(Q|t^-) \phi_0(t^-) + \int dt'^- \left( \phi(x'^-) \left( \overrightarrow{\partial_{r^*_+}} - \overrightarrow{\partial_{x'^-}} \right) G^-(Q, x'^-) \right)_{r^*_+ + t'^- = 0}. \tag{13}
\]

We divide the integral on the shock into two parts, one outside and the other inside the horizon \( r = r_0 \) \( (r^*_+ = \tanh^{-1} r_0 - \pi/2) \)

\[
\phi(Q) = \int dt^- K_1(Q|t^-) \phi_0(t^-) + \int dt'^- \left( \phi(x'^-) \left( \overrightarrow{\partial_{r^*_+}} - \overrightarrow{\partial_{x'^-}} \right) G^-(Q, x'^-) \right)_{v=0, r < r_0}
\]

\[
+ \int dt'^- \left( \phi(x'^-) \left( \overrightarrow{\partial_{r^*_+}} - \overrightarrow{\partial_{x'^-}} \right) G^-(Q, x'^-) \right)_{v=0, r > r_0}
\]

where \( K_1 \) is same as in (9). Now we use the matching conditions to express the integral over the shock \textit{outside the horizon} to an integral over the boundary \( r^*_+ = 0 \) just as was done for the point R. Finally for the integral over the
v = 0− surface inside the horizon (r < r0) side we again first match it to quantities just across the shell i.e. v = 0+ and then express it in terms of an integral over analytically continued (complexified) right boundary just as done for the point P. The final expression is,

\[
\phi(Q) = \int dt' K_1(Q|t')\phi_0(t') + \int dr' K_2(Q|\tau')\phi_0(\tau') + \int dt' K_{3I}(Q, \tau')\phi_0^{Rindler, R}(\tau' + i\pi)
\]

where K_2 is given by (10) while K_3’s are

\[
K_{3I}(Q, \tau') = \int dr_+^* \left[ K_{Rindler}^R(x^r|\tau') \left( \frac{\partial}{\partial x_+^r} - \frac{\partial}{\partial x_-^r} \right) G_0^{-}(Q, x^r) \right]_{v=0, r>r_0}
\]

\[
K_{3II}(Q, \tau') = \int dr_+^* \left[ (-)^\Delta K_{Rindler}^L(x^r|\tau') \left( \frac{\partial}{\partial x_+^r} - \frac{\partial}{\partial x_-^r} \right) G_0^{-}(Q, x^r) \right]_{v=0, r>r_0}.
\]

This is a highly nontrivial result. The bulk theory describes propagation in a non-analytic geometry. The boundary CFT nevertheless inherits analytic amplitudes from boundary locality. By exploiting the analyticity of the boundary theory, the expression (13) shows how propagation in the bulk geometry is reconstructed. Bulk non-analyticity shows up only in the smearing function factors.

V. CONCLUSIONS

In this paper we looked at a model time-dependent black hole geometry which has a single asymptotic AdS_2 boundary and constructed a bulk-boundary map which represents local on-shell bulk field operators as non-local boundary operators with compact support but on the boundary. We used analyticity of the boundary CFT as a tool to construct such maps for points inside the black hole horizon. The smearing function is very similar to the one for a pure AdS_2, for points outside the horizon, the function has support over only spacelike separated boundary points. For points inside the horizon the integral is delocalized over the single boundary at infinity, and requires a complex time contour.

One can obtain a more general time-varying geometry by collapsing a series of such null shells. The present construction should generalize straightforwardly to this case, using the general time dependent Vaidya metric [15]. It will also be interesting to generalize this construction to AdS bubble solutions relevant to cosmology, such as those studied in [19, 20, 21].

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Appendix A: COORDINATE SYSTEMS

In this appendix we review the transformations between the different coordinate systems used above. The coordinates (r_+^*, t^+) cover a patch to the future of the shock as described in section III. These are related to the (r, v) coordinates of the Vaidya metric [11] by

\[
r = -r_0 \coth(r_+^* r_0), \quad v = t^+ + r^+.
\]

The coordinates (r_-^*, t^-) cover a patch to the past of the shock where

\[
r = -\cot r_-^*, \quad v = t^- + r^-.
\]

Therefore on the shock v = 0, we have
\[
\begin{align*}
\frac{r_+}{r_0} &= 1 + \cot r_0 \coth^{-1} \left( \frac{\cot r_0}{r_0} \right) \\
t^+ &= -\frac{1}{r_0} \coth^{-1} \left( \frac{\cot r_0}{r_0} \right).
\end{align*}
\]

In section III B we also introduce the coordinates \((\rho, \tau)\) in (7). These are related to \((r_+, t^+)\) by

\[
\rho = -\sin^{-1} \left( \frac{1}{\coth^2 r_0 \cos^2 \tau - \sin^2 r_0 t^+} \right)^{1/2}, \quad \tau = \sin^{-1} \left( \frac{1}{\cosh^2 r_0 \cos^2 \tau - \sin^2 r_0 t^+} \right)^{1/2}.
\]

On the surface of the shock we have

\[
\rho|_{v=0} = -\sin^{-1} \left( \frac{1}{1 + \coth^2 r_0} \right)^{1/2} = -\tau|_{v=0}.
\]

Finally, for completeness, the relation between the \((r_-, t^-)\) and \((\rho, \tau)\) coordinates is

\[
\cos \tau \sin \rho = -\frac{r}{r_0} = \frac{1}{r_0} \cot r_-
\]

\[
\sin(\tau + \rho) = \tanh(r_- + t^-) r_0.
\]

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[22] For non-integer $\Delta$, the result is a bit more complicated and can be found in [1]. The upshot is that a local bulk operator inside the black hole horizon is delocalized over both the left and right boundaries.