Heat transfer enhancement in nanofluids. A numerical approach.

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Abstract.
The aim of the reported investigation is to assess the effect of brownian and thermophoretic diffusion in nanofluids convective heat transfer. In order to capture these effects, a new equation for particles distribution had to be considered. Momentum and energy equations have been reformulated in order to include brownian and thermophoretic diffusion. These modes of diffusion have been suggested extensively in the literature but their effect on momentum and energy transport has not yet been numerically analyzed. In order to obtain a solution for the modified set of governing equations, a new CFD solver had to be devised. The new solver has been applied to a case study involving hydrodynamic and thermally developing laminar flow regime in a pipe. Pure base fluid solutions have been used to assess the accuracy of the model. Numerical nanofluid solutions compare reasonably well with both experimental results obtained elsewhere and the Churchill and Ozoe correlation. The observed heat transfer enhancement by the nanofluid has been attributed to its transport properties rather than to another transport mechanism.

1. Introduction
Heat transfer enhancement is not a closed topic in the field of nanofluids momentum and energy transport investigation, though several arguments have been used to explain it [1], [2], [3], [4], [5], [6], [7] and [9]. Nanofluids related heat transfer enhancement has been observed mostly by experimental studies involving both internal laminar and turbulent flow [2] to, [6] and [7]. In a recently published study, Buongiorno [8] performed an order of magnitude analysis of the different mechanisms that plausibly intervene in the momentum and heat transfer of a flowing nanofluid. In his analysis, Buongiorno introduced, among other mechanisms, the Brownian and Thermophoretic nano-particles diffusion. One of the most important contributions by this study has been the recognition and modelling of the effect of the transport properties in the region close to the heating wall under turbulent flow conditions.

The Boussinesq assumption, though considering temperature effects on the density, does not contemplate continuity conditions, and, as a result, it affects significantly the flow pattern. Given that the density is affected by both the temperature and the nanoparticles concentration, assumptions such as constant density or Boussinesq approximation are inadequate. In addition, to the best knowledge of the authors, up until now no study has approached the problem by including Browninan and thermophoretic diffusion effects along with the nanoparticles concentration equation. In the numerical investigation reported herein these effects have been included aiming at determining how heat transfer enhancement is affected by variable transport
properties, which depend both on the temperature and nanoparticles concentration, along with nanoparticles diffusion.

The present numerical approach has been applied to hydrodynamic and thermally developing laminar flow of a nanofluid. It has been determined that momentum and energy transport are affected by the nanoparticles concentration, which is also affected by counteracting Brownian and thermophoresis diffusion effects in the region close to the wall. Whereas higher nanoparticles concentration in the central region of the tube tends to promote migration of nanoparticles towards the wall, thermophoresis tends to move the nanoparticles away from the heated wall where higher temperature gradients occur. The simultaneous action of both opposing diffusion effects promotes the development of a concentration boundary layer along the tube wall which affects the hydrodynamic and thermal field in the region close to the tube wall and promotes the development of unusual velocity and temperature patterns.

2. Governing equations
The study reported herein deals with the flow of a nanofluid in the entrance region of a tube under laminar flow conditions. The flow is assumed hydrodynamic and thermally undeveloped. The entrance conditions are assumed to be the conventional ones, that is, uniform velocity, temperature and nanoparticle concentration at the entrance test section. The nanofluid is considered as a homogeneous mixture, designated by the subscript nf, of a base fluid, designated by the subscript bf, and nanoparticles designated by the subscript p. The following are the working governing equations:

- Mixture continuity:
  \[ \frac{\partial \rho_{nf}}{\partial t} + \nabla \cdot \left( \rho_{nf} \vec{U} \right) = 0 \]  

- Momentum:
  \[ \frac{\partial \left( \rho_{nf} \vec{U} \right)}{\partial t} + \nabla \cdot \left( \rho_{nf} \vec{U} \vec{U} \right) = -\nabla P + \nabla \cdot \left[ \mu_{nf} \left( \nabla \vec{U} + \nabla \vec{U}^T \right) \right] \]

- Energy:
  \[ \frac{\partial (\rho_{nf} C_{nf} T)}{\partial t} + \nabla \cdot \left( \rho_{nf} C_{nf} \vec{U} T \right) = -\nabla \cdot \left( \rho_{nf} C_{nf} \vec{U} \vec{U} \right) + \rho_p C_p \left( D_B \vec{\nabla} \alpha \cdot \vec{\nabla} T + D_T \frac{\vec{\nabla} T \cdot \vec{\nabla} T}{T} \right) \]

- Particles distribution:
  \[ \frac{\partial \alpha}{\partial t} + \nabla \cdot \left( \vec{U} \alpha \right) = \nabla \cdot \left( D_B \vec{\nabla} \alpha + D_T \frac{\vec{\nabla} T}{T} \right) \]

Though the expressions of the set of governing equations are the conventional ones, some peculiarities in the present form, particularly those related to the nanoparticles concentration equation, require further explanation. The following are some explanatory notes related to this set of equations:

- Brownian and thermophoretic diffusion effects are included in the Mixture Energy equation, the second right hand side term of Eq. (3), and the nanoparticle Mass Conservation equation, right hand side term of Eq. (4).

- The following is the assumed expression for the Brownian diffusion coefficient, \( D_B \), [8]:
  \[ D_B = \frac{k_B T}{3 \pi \mu_{bf} d_p} \]  

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The assumed expression for the thermophoretic diffusion coefficient, $D_T$, is the following:

$$D_T = S_{coef} \frac{\mu_{bf}}{\rho_{bf}} \alpha = 0.26 \frac{\kappa_{bf}}{2\kappa_{bf} + \kappa_p} \frac{\mu_{bf}}{\rho_{bf}} \alpha$$

As previously mentioned, the nanofluid transport properties variation with temperature and nanoparticles concentration are considered in the present analysis along with the dependence of the base fluid on the temperature. Table I present the assumed expressions for the mixture transport properties. It must be stressed that the transport properties are coupled with the governing equations since they depend on the temperature and nanoparticles concentration.

| Property       | Equation                                      |
|----------------|-----------------------------------------------|
| Density        | $\rho_{nf} = \rho_{bf} (1 - \alpha) + \rho_p \alpha$ (7) |
| Viscosity      | $\mu_{nf} = \mu_{bf} (1 + A_\mu \alpha + B_\mu \alpha^2)$ (8) |
| Thermal conductivity | $\kappa_{nf} = \kappa_{bf} (1 + A_\kappa \alpha + B_\kappa \alpha^2)$ (9) |
| Specific heat  | $\rho_{nf} C_{nf} = \rho_{bf} C_{bf} (1 - \alpha) + \rho_p C_p \alpha$ (10) |

There is another important issue related to the governing equations, namely the boundary conditions. In the case of Eqs. (1), (2) and (3), the boundaries are treated as usual, and are generally available in all CFD software packages. In addition, they do not need a special treatment to use and adapt them to the present case of nanofluids. The last equation, Eq. (4), requires unusual boundary conditions. There are no special comments regarding symmetry, periodic, fixed value, etc. However, since the tube wall is impermeable, the nanoparticles mass flow rate there must be zero through the wall. Thus, considering the fact that there is no particle transport by convection at the wall and the flow is in steady state, the nanoparticles diffusion by thermophoresis is equal and opposite to the Brownian diffusion, that is,

$$\vec{\nabla} \alpha|_w = -\frac{D_T}{D_B} \frac{\vec{\nabla} T|_w}{T}$$

Through a dimensional analysis, Buongiorno obtained the following dimensionless relation between Brownian and Thermophoresis diffusion that he designated by $N_{BT}$:

$$N_{BT} = \frac{\alpha_b D_B T_b}{\Delta T D_T}$$

where the subscript b designates bulk nanofluid properties. This dimensionless parameter controls the nanoparticles concentration in the region close to the wall. In the case of heating walls, the temperature gradient tends to diffuse nanoparticles away from the wall by thermophoresis, whereas Brownian diffusion, as a result of the concentration gradient, tends to move them closer to the wall. Thus, the lower the value of $N_{BT}$, the lower must be the average nanoparticles concentration in the region close to the wall.
3. Numeric algorithm

The open source code OpenFoam \cite{12} has been adjusted to comply with the following requirements related to solution procedure of the set of governing equations:

- The solver must work with the present form of Eq. (1) since the Boussinesq approximation is not adequate. It has been determined that for a velocity divergence free flow, the mass flow rate is not conserved. The use of this assumption has led to errors of up to about 20% for a nanofluid flowing in a pipe at Reynolds number of 750. Errors of this order in the velocity field result in low accuracy Nusselt numbers.
- The nanoparticles distribution equation, Eq.(4), had to be overrelaxed to prevent oscillations that could lead to the final divergence of the whole process.
- A fast convergence should not be expected since the boundary conditions like Eq. (11) depend themselves on the solution.
- Special care must be taken in keeping all diffusion coefficients higher than zero every time and over the whole control volume. In the design of this solver any kind of summation to the diffusion coefficients has been avoided. Usually most solvers overcome this problem introducing artificial diffusion.
- The thermophoresis diffusion coefficient, \( \frac{D_T}{m^2 s^{-1} K^{-1}} \), has achieved values varying from \( 5.25 \times 10^{-14} \) up to \( 2.74 \times 10^{-13} \). The brownian diffusion coefficient, \( D_B \ [m^2 s^{-1}] \), assumes values from \( 6.79 \times 10^{-12} \) up to \( 2.10 \times 10^{-11} \).
- In order to avoid possible spurious values when calculating both the fluid properties and the diffusion coefficients, values of the nanoparticles volumetric concentration must be positive and lower that twice the bulk concentration during the numerical procedure. However, values of particles concentration have not reached these limits in the solutions shown in the present study.

4. Numerical model. Case set up.

4.1. Nanofluid selection.

The nanofluid selected for this study is made up of water, as a base fluid, and alumina (\( \text{Al}_2\text{O}_3 \)) nanoparticles. Transport properties of the alumina and the nanoparticles assumed diameter are shown in Table 2. Water properties have been assumed to be temperature dependent and their variation with temperature has been determined by curve fitting data from the 1995 Formulation for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific Use, issued by The International Association for the Properties of Water and Steam (IAPWS) \cite{13}.

| Concept               | Value                      |
|-----------------------|----------------------------|
| Particles diameter \( d_p \) | 45 nm.                     |
| Particles density \( \rho_p \) | \( 3.8 \times 10^3 kg/m^3 \). |
| Particles conductivity \( \kappa_p \) | 30.0 W/m-K.                |
| Specific heat \( C \)    | 0.875 kJ/kg-K.             |
4.2. Geometry selection.
As mentioned in the introduction, as a case study a developing laminar flow regime has been considered in a pipe 970 mm long and 4.5 mm diameter. This particular pipe geometry has been chosen to allow a comparison of obtained numerical results with the experimental results reported by Wen et al.\(^2\), who used the same pipe geometry and nanofluid in their experiments. Wen et al. worked with nanoparticles volumetric concentrations varying in the range from 0 to 1.6%, whereas the Reynolds number varied from 750 to 1750. It must be stressed that the aim of the present study is to numerically determine the variation of the local (cross section) Nusselt number. For that purpose the following boundary conditions have been used in order to comply with the experimental conditions reported by Wen et al.\(^2\):

- **Pipe entrance section:**
  - Fixed and uniform velocity depending on the chosen Reynolds number. This is not a straightforward procedure since the average properties of the Reynolds number are not known a priori.
  - As usual in this kind of models, the pressure gradient is assumed equal to zero.
  - A fixed and uniform temperature, which in the present case was set equal to 283 K for all the cases.
  - A fixed and uniform nanoparticles volumetric concentration. In the present paper just pure water and 1.6% nanoparticles concentration have been considered.

- **Solid wall:**
  - No slip for the velocity
  - As usual in this kind of models, the pressure gradient is assumed equal to zero.
  - A fixed and uniform value of the heat flux along the tube wall, \(q''\), which is not equivalent of assuming a uniform temperature gradient since the thermal conductivity of the nanofluid varies along the tube. This boundary condition is written according to the following expression:
    \[
    \nabla T|_w = \frac{q''}{\kappa_{nf}}
    \]

  - The heat flux has been assumed as the one corresponding to the maximum test section power reported by Wen et al.\(^2\).
  - Equation (11) has been used for particles concentration.

- **Pipe exit section:** The gradient of all the significant physical parameters (velocity, temperature, concentration) have been assumed to be equal to zero.

4.3. Numerical mesh.
5. Results
The case study to be presented is based on tests conditions by Wen et al.\(^2\). Accordingly, as previously mentioned, a water/alumina nanofluid flowing at a Reynolds number of 750 will be considered. The maximum test section power reported by Wen et al.\(^2\) will be assumed (300 W) along with the nanofluid test section entrance temperature, 283 K. In order to assess the nanofluid thermal performance, obtained results will be compared with those of the base fluid (water) flowing in the same pipe at the same Reynolds number. Though the water solution could be used to check for the accuracy of the numerical model, no analytical solution has been found in the literature for the simultaneous hydrodynamic and thermal development of laminar flow in a pipe. Some of the available solutions assume Langhaar velocity profiles as starting point\(^14\), \(^15\). Others are the result of either numerical or curve fitting procedures\(^16\), \(^17\), \(^18\). These developing laminar flow solutions assume constant properties, an aspect not contemplated in the present paper, which includes the temperature effect over the transport properties. Thus,
the comparison of numerical results from the present investigation with available correlations lacks of similarity of conditions, though they are very close to those from the Churchill and Ozoe correlation [18]. It must be added at this point that due to the variation of the viscosity from the center of the pipe to the wall, the velocity will tend to be higher than that for a constant viscosity in the region closer to the wall, with the opposite occurring at the center of the pipe. Thus the velocity profile resulting from the present numerical solution must be slightly deformed with respect to both the developing and the parabolic developed velocity profile. This trend must lead to a slight heat transfer enhancement with respect to the constant property solutions over the whole length of the pipe even under developing flow conditions.

The heat transfer coefficient has been determined as the ratio between the heat flux and the wall and average temperature difference. The average temperature in each section has been determined according to the conventional way; however, in the present case, effects of temperature and particle concentration on the density and the specific heat have been taken into account for water (pure fluid) and nanofluid in each cross section.

![Figure 1](image1.png)  
**Figure 1.** Particles profiles for sections in [2] at $R_n=750$.

![Figure 2](image2.png)  
**Figure 2.** Local heat convection transport coefficient for sections in [2] for the base and nanofluid at $R_n=750$. Full symbols correspond to nanofluid.

The plot of Fig. 1 illustrates the development of the concentration boundary layer. The nanoparticles volumetric concentration with respect to the bulk one at the entrance (1.6%) is plotted against the dimensionless distance from the wall at cross sections located at several distances from the entrance of the pipe. It can be noted that the initial uniform concentration at the entrance section, corresponding to 1, dimensionless, progressively is deformed, with nanoparticles being displaced from the wall region towards the center. A concentration boundary layer can clearly be noted at each section, with volumetric concentration diminishing at each layer with the distance from the wall. This is an expected result if thermophoretic effects are considered. In that respect, the $N_{BT}$ dimensionless parameter in the concentration boundary layer for the conditions of Fig. 1 is of the order of 0.05.

Heat transfer results are displayed in the plot of Fig. 2, where the local heat transfer coefficient is plotted against the dimensionless distance from the entrance section for water and nanofluid along with experimental data from Wen et al [2]. Only two data points are available from the Wen et al experiments for a Reynolds number of 750. It can be noted that numerical data are very close to experimental data for both pure water and nanofluid, a result that confirms the adequacy of the present numerical procedure. The nanofluid heat transfer coefficient clearly is higher than the one for the base fluid (water). However, it must be emphasized that this
comparison is based on the same Reynolds number, which requires higher nanofluid average velocity than that of water. In fact, the velocity of the nanofluid at the test section entrance is equal to 0.307 m/s whereas that of water is 0.149 m/s. This could explain the observed heat transfer enhancement with the nanofluid also emphasized by Wen et al. A more reasonable heat transfer enhancement evaluation would be a comparison in terms of the mass flow rate. The result of such a comparison is shown in the plot of Fig. 3 which is based on a Reynolds number of 750 for water, corresponding to a mass flow rate of \(2.365 \times 10^{-3}\) kg/s. and entrance velocity of 0.149 m/s. For the same mass flow rate, the corresponding nanofluid Reynolds number and entrance velocity are equal to 426 and 0.143 m/s. It can be noted that even for the same mass flow rate, the nanofluid clearly enhances the rate of heat transfer. The enhancement varies in the range of 11.99\% at the outlet section and 7.40\% at the first section. It must be noted at this point that the observed enhancement by the nanofluid results from the value of its transport properties, other physical mechanism could not be devised under the present investigation. The latter statement is confirmed by expressing the numerical data in terms of dimensionless parameters. Laminar developing flow heat transfer is generally expressed in terms of two dimensionless parameters: the Nusselt and the Graetz number \([14], [15], [16], [17], [18]\), given by the following expressions:

\[
Nu_x = \frac{hD}{\kappa} \quad \text{and} \quad Gz = \frac{RnPr}{xD}.
\]

Numerical data for Reynolds number of 750 have been reduced to these dimensionless parameters for both the base fluid (water) and the nanofluid and plotted as shown in Fig. 4. Numerical Nusselt and Graetz are based on a cross section average transport properties in contrast with experimental data by Wen et al which are based on the test section average values. Their experimental data have been overlaid in the same plot along with the Churchill and Ozoe correlation \([18]\), which is the same either for water or the nanofluid as should be expected. Equal mass flow rate conditions would result in similar trends, not shown in this paper. The plot of Fig. 4 displays results that deserve being analyzed in detail as in the following comments:

- Water and nanofluid numerical data collapse into a single line when plotted in terms of the governing dimensionless parameters.
- Wen et al experimental data follow a very close trend to the numerical data, though, in the present case study, only two data points are available for water and nanofluid.

Figure 3. Base / nanofluid local heat convection transport coefficient for sections in [2]. (Water Rn=750 - Same mass flow rate for both Nanofluid and Water). Filled points correspond to nanofluid.

Figure 4. Base / nanofluid local Nusselt number comparison for sections in [2] both for Rn=750. Filled points correspond to nanofluid.
The Churchill and Ozoe equation [18] correlates closely the numerical and experimental data both for water and nanofluid.

The plot of Fig. 4 clearly indicates that laminar developing flow is adequately correlated by the Nusselt and Graetz numbers both for a pure fluid and a nanofluid, which apparently behaves as a pseudo pure fluid.

Nanofluid heat transfer enhancement results from the numerical investigation reported herein have not revealed effects other than those related to the transport properties of the nanofluid, as the trends of the plot of Fig. 4 in terms of dimensionless parameters, clearly display.

6. Conclusions
The numerical investigation reported in the present paper focused on a case study involving a nanofluid made up of water and alumina flowing in a pipe under developing laminar conditions for a Reynolds number of 750. A single volumetric concentration of alumina has been considered (1.6%) at the pipe entrance section in a trial to reproduce experimental results obtained elsewhere [2]. It has been found that the literature regarding laminar flow simultaneous hydrodynamic and thermal development is scarce with the equation proposed by Churchill and Ozoe [18] being the one that correlates best the present numerical and experimental results. The following are further conclusions from the present investigation:

- A concentration boundary layer development has been observed which affects the velocity and temperature ones with respect to the pure fluid boundary layers.
- Heat transfer enhancement of nanofluid with respect to the base fluid is not significant at least for the conditions reported in the present paper. Comparison has been made with both equal Reynolds number and equal mass flow rate for the base and nanofluid.
- Results do confirm that the nanofluid behaves as a pure fluid with combined transport properties, with the local Nusselt number closely being correlated by the local Graetz number.
- Further research is needed involving higher heat fluxes and cooling of the fluid. The former is related to higher temperature gradients at the wall that might enhance thermophoresis diffusion with respect to the Browninan one, inducing a stronger removal of nanoparticles from the wall. Cooling instead of heating tests could allow a better assessment of thermophoresis diffusion and nanoparticles concentration effects on heat transfer since in that case both diffusion effects would tend to move the nanoparticle toward the wall, as suggested by Buongiorno [8].

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