MEASURING STRUCTURAL PARAMETERS THROUGH STACKING GALAXY IMAGES

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ABSTRACT

It remains challenging to detect the low surface brightness structures of faint high-\textit{z} galaxies, which are key to understanding the structural evolution of galaxies. The technique of image stacking allows us to measure the averaged light profile beneath the detection limit and probe the extended structure of a group of galaxies. We carry out simulations to examine the recovery of the averaged surface brightness profile through stacking model \textit{Hubble Space Telescope}/Advanced Camera for Surveys imaging of a set of galaxies as functions of the Sérsic index ($n$), effective radius ($R_e$) and axis ratio (AR). The Sérsic profile best fitting the radial profile of the stacked image is taken as the recovered profile, in comparison with the intrinsic mean profile of the model galaxies. Our results show that, in general, the structural parameters of the mean profile can be properly determined through stacking, though systematic biases need to be corrected when spreads of $R_e$ and AR are counted. We find that the Sérsic index is slightly overestimated and $R_e$ is underestimated at AR < 0.5 because the stacked image appears to be more compact due to the presence of inclined galaxies; the spread of $R_e$ biases the stacked profile to have a higher Sérsic index. We stress that the measurements of structural parameters through stacking should take these biases into account. We estimate the biases in the recovered structural parameters from stacks of galaxies when the samples have distributions of $R_e$, AR and $n$ seen in local galaxies.

\textit{Key words:} galaxies: photometry – Galaxy: evolution – Galaxy: structure

1. INTRODUCTION

The evolution of galaxies is found to both theoretically and observationally correlate with stellar mass (e.g., Kauffmann et al. 2003; Bundy et al. 2006; Dekel & Birnboim 2006; Guo & White 2008; Peng et al. 2010), yielding fundamental relationships between stellar mass and color (Baldry et al. 2004), size (Shen et al. 2003), metallicity (Tremonti et al. 2004), and star formation rate (Brinchmann et al. 2004) among local galaxies (see Blanton & Moustakas 2009 for a review). These relationships evolve significantly out to high redshifts (e.g., Erb et al. 2006; Zheng et al. 2007; Wuyts et al. 2010, 2011; Brammer et al. 2011; Shapley 2011, and references therein). Much effort has been made to characterize the structural properties of galaxies at different cosmic epochs in order to dissect different physical processes regulating galaxy evolution (Conselice 2014). The size of massive galaxies has been found to increase on average by a factor of approximately three to five since $z \sim 2$ (e.g., Trujillo et al. 2006, 2007; Toft et al. 2007; Zirm et al. 2007; van der Wel et al. 2008; Mancini et al. 2010; Damjanov et al. 2011; Newman et al. 2012; Krogager et al. 2013; Belli et al. 2014; van der Wel et al. 2014). van Dokkum et al. (2010) found that an extended stellar halo around massive galaxies was gradually built up over cosmic time, suggesting that accretion of satellite galaxies plays a key role in governing the size growth of the massive galaxies (Naab et al. 2009; Oser et al. 2010). While physical interpretations of the dramatic size evolution are still under debate (e.g., Hopkins et al. 2010), the spatially resolved brightness profile as a function of redshift turns out to be crucial to unveiling the assembly histories of galaxies (e.g., Trujillo et al. 2011; Hilz et al. 2013). In particular, the brightness profiles of low-mass galaxies at high-\textit{z} are poorly explored.

It is technically challenging to measure the brightness profiles toward large radius for typical ($L^*$) and low-mass galaxies at high redshifts, even with deep imaging of the \textit{Hubble Space Telescope} (\textit{HST}; Szomoru et al. 2012). The size of galaxies may be underestimated if the extended structure of low surface brightness is not detected (e.g., Ferguson & Bingelli 1994; Naab et al. 2007, 2009; Bezanson et al. 2009; Mancini et al. 2010). Stacking is a powerful tool to suppress background noise and detect fluxes beneath the detection limit for individual objects. It has been applied successfully in studies with optical (e.g., Zibetti et al. 2004; van der Wel et al. 2008; van Dokkum et al. 2010), infrared (e.g., Zheng et al. 2006; Lee et al. 2010; Bourne et al. 2012; Guo et al. 2013), and radio (e.g., White et al. 2007; Garn & Alexander 2009; Hancock et al. 2011) imaging data. van Dokkum et al. (2010) examined the systematical effects in parameterizing the mean structure of massive galaxies via stacking ground-based images, finding that the averaged size and Sérsic index can be recovered when each of the stacked galaxies is characterized by a single Sérsic profile. In practice, galaxies tend to have multiple components (e.g., bulge+disk) with different surface brightness profiles; galaxies of similar stellar masses have an effective radius and/or axis ratio (AR; or inclination angle) spanning over a range (Shen et al. 2003; Hao et al. 2006; Padilla & Strauss 2008). Further investigation is demanded to address how the scatter in effective radius, AR, and Sérsic index affects the recovered structural parameters from the stacked images and to what extent the results of stacking are accurate and robust.

In this paper, we present the results of our simulation to characterize the dependences of the averaged structural parameters of faint galaxies derived from stacking on effective radius ($R_e$), AR, index of Sérsic profile ($n$), and the
distributions of these parameters. We describe our methodology in Section 2. In Section 3, we present the simulation results. We discuss our results and summarize them in Section 4. We assume a cosmology with $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}, \Omega_M = 0.3$, and $\Omega_{\Lambda} = 0.7$ throughout this paper.

2. METHODOLOGY

2.1. Galaxy Models

The existing deep optical and near-infrared imaging data from large surveys with $HST$, including GEMS (Rix et al. 2004), COSMOS (Scoville et al. 2007), and CANDELS (Grogin et al. 2011; Koekemoer et al. 2011), provide the basis for a stacking analysis of faint high-$z$ galaxies. In particular, the $HST$/Advanced Camera for Surveys (ACS) imaging of COSMOS over 1.48 deg$^2$ (Koekemoer et al. 2007) through the F814W ($i$) filter allows for morphological examination for large samples of galaxies. In our simulations, we adopt a pixel size of 0.05 (same as $HST$/ACS pixel size) and ACS point-spread function (PSF) in the $i$ band to generate galaxy model images. A physical scale of 100 kpc then corresponds to [280, 250, 239] pixels at $z = [0.7, 1, 2]$. A size of 351 × 351 pixels is chosen for the model images to have the radial surface brightness profile extended to $R = 50$ kpc and have background estimation out to $R = 70$ kpc for galaxies at $z > 0.7$.

Three structural parameters are used to characterize the two-dimensional model image of a galaxy: index of Sersic profile ($n$), effective radius ($R_e$) and AR. Position angle is randomly chosen between 0$^\circ$ and 180$^\circ$ for AR $< 1$. The Sersic profile is described by

$$I(r) = I_0 \exp \left(-b_n \left(\left(r/r_e\right)^n - 1\right)\right).$$

(1)

The AR, defined as the ratio of minor axis $b$ over major axis $a$, measures the elongation in morphology for early-type galaxies or the inclination for late-type galaxies. The model galaxy is centered at the centroid of the image, which is then convolved with the empirical PSF derived from $HST$/ACS $i$-band images using a number of isolated bright stars. The full width at half maximum of the PSF is $R_{PSF} = 0.011$ or 2.2 pixels. The total brightness of the model image is scaled to match the total flux in analog-digital units for galaxy ranges from 24 to 24.75 mag (i.e., a factor of three spread in flux) in the $HST$ images from COSMOS. We adopt a faint-end slope of $-0.47$ for early-type galaxies and $-1.37$ for late-type galaxies to determine the distribution of magnitude of the galaxies (Tomczak et al. 2014), and we add photon noise and background noise to match the noise level in the $HST$ images of COSMOS in order to address the effect of noise. An IDL code SIMULATE_GALAXY. PRO$^5$ (see Häussler et al. 2007, for more technical details) is used to create a galaxy model image at given $n$, $R_e$, and AR. We also randomly locate the center of a model galaxy within a pixel to match observations. A set of galaxy model images is shifted and aligned to the same center before stacking. Due to the noises, the measured center of a model galaxy slightly differs from the actual center. We will discuss the effects of this issue on the final conclusions in Section 3.7.

In practice, the structural parameters are barely known for individual faint high-$z$ galaxies; the stacked image is often obtained by directly co-adding the aligned galaxy images without corrections for inclination, orientation, and size (but see, e.g., Zibetti et al. 2005); the stacked surface brightness profile is usually derived from the stacked image using circular apertures. We generate galaxy models with structural parameters spanning over sufficiently wide ranges: $1 \leq n \leq 6$, 0.05 $\leq R_e \leq 0.75$, and 0.1 $\leq$ AR $\leq 1.0$. First, we examine the measurement of surface brightness profiles derived using circular apertures from a single model image in relations to each of structural parameters $n$, $R_e$, and AR. Second, we test the recovery of the mean surface brightness profile when a set of stacked galaxies has two parameters fixed and the third parameter following a certain distribution. Each set contains 700 model images, which is representative of practical cases for stacking. We divide galaxy models into late-type ($1 \leq n \leq 2.5$) and early-type ($2.5 < n \leq 6$) because the two populations are distinct in structure (Blanton & Moustakas 2009). We adopt the log-normal distributions of $R_e$ for the two populations from Shen et al. (2003). The log-normal distribution of $R_e$ is described by the scatter $\sigma_{\ln(R_e)}$ and the median $\langle R_e \rangle$. More

\footnote{http://www.mpia.de/GEMS/fitting_utilities/simulate_galaxy.pro}
massive galaxies have larger \( R_e \). The AR distribution of late-type galaxies comes from Padilla & Strauss (2008) and that of early-type galaxies from Hao et al. (2006) is adopted. At a given AR distribution, the other two parameters \( n \) and \( R_e \) vary across the corresponding parameter space. When \( n \) falls into \( 1 \leq n \leq 2.5 \) and \( 2.5 < n \leq 6 \), the AR distribution of the late-type and of the early-type galaxies are used, respectively. Furthermore, \( R_e \) ranges from \( 0''05 \) to \( 0''75 \) (1–15 pixels), corresponding to a physical scale of 0.5–6.0 kpc at \( z = 1 \). Similarly, we generate each set of model images for stacking at a fixed \( n \) and \( AR = 1.0 \) with \( R_e \) following the given distribution. Third, we let AR and \( R_e \) follow the corresponding distributions and generate models as functions of \( n \) and \( R_e \) to see the integrated effect of the spreads of these parameters. Finally, we let AR and \( R_e \) follow the distributions as described above, and Sérsic index \( n \) follows a uniform distribution between one and two for late-type galaxies, and three and four for early-type galaxies. This simulates the case that faint galaxies are usually selected by color (or type) and mass (or luminosity), and their AR, \( R_e \), and \( n \) often spread over a certain range. Table 1 lists the structural parameters of galaxy models adopted in our simulations. Figure 1 demonstrates the surface brightness profiles of these single-profile models with \( AR = 1 \) for comparison.

### 2.2. Stacking Galaxy Images

A set of model images for stacking have the same size of \( 351 \times 351 \) pixels, with photon noise and background noise counted. Before stacking a set of model galaxy images, we first measure the positions of the galaxies using sextractor (Bertin & Arnouts 1996) and shift them to the same position in all images. We note that the measurement errors of positions and shifting of images introduce uncertainties into the stacked profile. We also stack model images free from noise and offsets in position to quantify the corresponding uncertainties due to the noises and errors in aligning the galaxies. Each set contains 700 galaxy model images and is combined using the averaging algorithm. Because the position angle of model galaxies is randomly distributed, the averaged profile is rotationally symmetric. The radial surface brightness profile is thus sufficient to characterize the averaged profile. The radial profile is derived from the stacked image using \( 21 \) circular apertures with radii from 0.5 to 140 pixels evenly split in logarithm. The software tool aper.pro from the IDL Astronomy User’s Library\[6\] is used to perform aperture photometry. We use an annulus of \( r = 6''24 \) to \( r = 8''74 \) (about 50–70 kpc at \( z \sim 1 \)) for sky background estimation.

### 3. ANALYSIS AND RESULTS

#### 3.1. The Elongation/Inclination Effects

The elongation (or inclination for disk galaxies) of individual stacked galaxies is usually not corrected in stacking. Here we examine how the elongation/inclination affects the recovery of the structural parameters of a Sérsic profile. We extract the radial surface brightness profiles using circular apertures from the single-profile galaxy model images listed in Table 1. We note that these model images are convolved with ACS PSF. Figure 2 shows the single Sérsic profiles as functions of \( n \), \( R_e \), and AR. The PSF profile is also presented with an arbitrary normalization for clarity. Red and blue lines show AR = 0.34 and 0.68 in each panel. We can see that the surface brightness profile of a galaxy appears to be more compact at edge-on than at face-on when circular apertures are adopted. The bias becomes larger for late-type galaxies with larger \( R_e \).

We measure the structural parameters from a radial surface brightness profile using the method described in Section 2.3. Figure 3 shows the effects of the elongation/inclination on the

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\[6\]  http://idlastro.gsfc.nasa.gov/
recovery of the intrinsic structural parameters of these galaxy models. When AR = 1, the recovered Sérsic index $n$ and effective radius $R_e$ perfectly match the input values when the input Sérsic index is low, but the recovered Sérsic index is lower than the input value when the input Sérsic index is high. For model galaxies with high Sérsic index and large effective radius, the effective radius of the stacked profile tends to be underestimated. This is because such galaxies exhibit profiles with extended wings out to large radii, leading to an oversubtraction of the background and thus underestimate in both Sérsic index and effective radius. We plot the fraction of light outside $R = 125$ pixels (about $6''25$ and $\sim 50$ kpc at $z \sim 1$) as a function of Sérsic index $n$ and effective radius $R_e$ in Figure 4. We can see that for late-type galaxies, the light beyond $R = 6''25$ is negligible. For early-type galaxies, however, the light out of $R = 6''25$ dramatically increases with the effective radius, and reaches up to 10% for galaxies with $n = 6$ and $R_e = 15$ pixels. This indeed leads to the oversubtraction of background in our measurements, and consequently to the underestimate of both Sérsic index $n$ and effective radius $R_e$ for early-type galaxies. We note that the underestimate of Sérsic index due to background oversubtraction can be corrected once a larger annulus is adopted for the background estimate. For a model galaxy with $n = 6$ and $R_e = 15$ pixels, the background estimate from outer regions of $R > 465$ pixels (186 kpc at $z = 1$) may suppress light contamination from the galaxy to <1%; and the Sérsic index can be recovered as well as for galaxies with $n = 1$.

Figure 3 shows that the Sérsic index $n$ is increasingly overestimated for galaxies with lower AR (i.e., more inclined). The overestimate is significant only at AR < 0.5, and becomes larger for larger $R_e$ at small $n(n < 2.5)$, from $\Delta n \leq 1$ when $R_e = 15$ pixels (6 kpc at $z = 1$), to $\Delta n \leq 0.5$ when $R_e = 1$ pixel. Meanwhile, $R_e$ is increasingly underestimated at
and effective radius $R_e$ as a function of axis ratio AR. Black squares (barely seen behind blue circles for late-type galaxies) mark the starting points at given $(n, R_e)$. Color-coded circles represent the recovered $(n, R_e)$ at corresponding AR from the starting points (black squares).

Figure 4. Fraction of light out of $R = 125$ pixels as a function of Sérsic index $n$ and effective radius $R_e$. We can see that the light in the outer regions is marginal for late-type galaxies, but increases as the $R_e$ increases for early-type galaxies.

Figure 3. Recovery of structural parameters Sérsic index $n$ and effective radius $R_e$ as a function of axis ratio AR. The recovered $R_e$ deviates from the intrinsic value by up to 50% at the minimal AR; and the deviation does not strongly depend on $R_e$ and $n$.

We conclude that elongation/inclination (AR) influences the estimate of structural parameters $n$ and $R_e$ when the radial surface brightness profile is extracted from a galaxy image using circular apertures. The galaxy would appear to be more compact, i.e., with larger $n$ and smaller $R_e$, at decreasing AR, if the elongation/inclination is ignored. We point out that the bias in recovering structural parameters through stacking from elongation effects is negligible for early-type galaxies ($n > 2.5$) because the early-type galaxies usually have AR $> 0.5$ (at least in the local universe, Hao et al. 2006), and the main source of bias is from background estimation. For late-type galaxies ($n \leq 2.5$), the inclination leads to the over-estimate of $n$ and the underestimate of $R_e$.

The next step is to test the measurement of the mean surface brightness profile from the stacked image of individual galaxy models with two parameters fixed and the third parameter following a given distribution.

### 3.2. Effects of AR Spread

For each pair of structural parameters $(n, R_e)$ listed in Table 1, a set of 700 galaxy model images is generated to have a fixed $R_e$ and $n$ but AR spreading within a distribution. We adopt the AR distribution of late-type galaxies ($n \leq 2.5$) from Padilla & Strauss (2008) and that of early-type galaxies ($n > 2.5$) from Hao et al. (2006). These images are stacked together and a radial surface brightness profile is then obtained. The intrinsic parameter of the mean profile is given by $(n, R_e)$.

Figure 5 shows the results of our simulation when AR is spread according to a realistic distribution rather than fixed. The two empirical AR distributions are shown in the inner panels. The left plot gives the difference between recovered $R_e$ and input $R_e$ as a function of the input $n$ and the right plot presents the deviation of recovered $n$ from input $n$ as a function of the input $n$. The recovered $R_e$ is systematically smaller by 12%–27% over $1 \leq R_e \leq 15$ pixels. For late-type galaxies, the degree of the underestimate in $R_e$ does not depend on $R_e$ itself, suggesting that the AR distribution is responsible for the underestimate. However, for early-type galaxies, the degree of underestimate in $R_e$ increases as the $R_e$ increases. Given that the oversubtraction of background becomes more serious for early-type galaxies of larger size, it is reasonable to attribute the increase of the degree of underestimate in $R_e$ to the oversubtraction of background. Apparently, the estimate of $R_e$ is more biased for late-type galaxies due to the highly inclined ones (AR $< 0.5$), which are absent in the early-type galaxies when the effective radius is not very large. Similarly, $n$ is overestimated by up to 20% for late-type galaxies because of the inclination effect due to those with AR $< 0.5$. Again, the overestimate of $n$ does not rely on $R_e$ significantly. For early-type galaxies, $n$ can be underestimated by up to 15%, and the underestimate of $n$ increases as $n$ increases. As most of them are with AR $> 0.5$ in terms of the AR distribution from Hao et al. (2006), the overestimate of $n$ can be ignored for early-type galaxies, thus the oversubtraction of background dominates the estimation of $n$ and makes $n$ underestimated, especially for galaxies with larger $n$, which have a more extended halo in the outskirts. As shown in Figure 3, the recovery of $n$ is marginally biased by inclination/elongation effects at AR $> 0.5$. These results denote that the function of the AR distribution regulates the deviation of the recovered $R_e$ and $n$ from the original values for late-type galaxies, and the estimation of background affects the estimation of structural parameters for early-type galaxies.

Differing from the claim in van Dokkum et al. (2010) that the recovery of structural parameters is not sensitive to the distribution of AR, we show that the AR distribution may significantly bias the recovery of $R_e$ and $n$, dependent on the fraction of highly inclined or elongated ones; the stacked profile appears to be more compact (i.e., with smaller $R_e$ and higher $n$ at the same time) if the elongation/inclination effect is not corrected.

### 3.3. Effects of $R_e$ Spread

Figure 6 shows our simulation results of stacking model galaxies with a fixed $n$ but $R_e$ following a log-normal distribution. Here AR $= 1$ is adopted to get rid of the AR...
effect. The log-normal distribution is described by the median $R_{e,0}$ and scatter $\sigma_{\ln(R_e)}$. We adopt $\sigma_{\ln(R_e)} = 0.3$ dex for early-type galaxies and $\sigma_{\ln(R_e)} = 0.5$ dex for late-type galaxies from Shen et al. (2003). As shown in the left panel of Figure 6, the input median effective radius $R_{e,0}$ is well recovered through stacking for late-type galaxies and early-type galaxies with small effective radius, but for early-type galaxies with large radius, the underestimation of $R_e$ can be up to 15% due to the oversubtraction of background. However, $n$ is increasingly overestimated by up to 60% at decreasing $n$. This tendency has no dependence on the median of the log-normal distribution of $R_e$. For early-type galaxies, the $n$ can still be underestimated due to the oversubtraction of background.

One can infer from Figure 6 that the median of a log-normal distribution of $R_e$ can be properly measured from the stacked profile of galaxies; but the spread of $R_e$ leads to an overestimate of $n$ for late-type galaxies ($n \leq 2.5$).

### 3.4. Effects of AR and $R_e$ Spreads

We have shown that the spread of AR biases the estimates of structural parameters of the mean profile through stacking, leading the stacked profile to be more compact, say with smaller $R_e$ and higher $n$; and the spread of $R_e$ does not influence the estimate of median $R_e$ but deviates $n$ to be higher. The effects of the two spreads are significant only for the stacking of late-type galaxies ($n \leq 2.5$). Now we include both the two spreads in stacking and examine their effects on the recovery of the structural parameters. Again, AR and $R_e$ follow distinct distributions for early- and late-type galaxies as mentioned before. We note that the effects of the two spreads are not correlated with the median $R_{e,0}$.

Figure 7 shows the results of stacking with both the AR and $R_e$ spreads involved. We also overplot the results from Figures 5 and 6 for comparison. We can see that the mixture of AR and $R_e$ spreads biases the estimates of median $R_{e,0}$ and $n$ in the same way as the effects of the two spreads combine linearly together. The median $R_{e,0}$ is underestimated by 20%–27% and 10%–20% for late-type and early-type galaxies, depending on the $R_e$ of the galaxies, respectively, caused by the AR spread and the estimation of background; and $n$ is increasingly overestimated by up to 70% at decreasing $n$, equal to a linear combination of the deviations caused by each of the two spreads.

In summary, our simulations manifest that the measured size ($R_e$) and Sérsic index ($n$) of the averaged profile by stacking a set of galaxies deviate from the input values when the stacked galaxies disperse in AR and/or the half-light radius ($R_e$). The deviations depend on the type of stacked galaxies ($n$) and distribution functions of AR and $R_e$. The stacked profile tends to be more compact for late-type galaxies, and the over-subtraction of background dominates the estimation of structural parameters for early-type galaxies and cause the
profile to be smaller in both $R_e$ and $n$. With given distributions of AR and/or $R_e$, the deviations in estimates of $R_e$ and $n$ can be quantitatively determined and thus corrected accordingly.

It is worth noting that the effects caused by the spread of $n$ depend strongly on the distribution function of $n$. The averaged profile of a set of Sérsic profiles with fixed $R_e$ and $AR = 1$ is much closer to the median one. The derived $n$ from stacking are rather reliable with uncertainties of $<0.5$ for both late-type and early-type galaxies (see the Appendix of van Dokkum et al. 2010, for more details). We will also discuss this issue in Section 3.5. However, it remains to be explored when two distinct types of profiles are stacked together.

### 3.5. Effects of $n$ Spread

Generally speaking, a set of galaxies of similar properties (e.g., stellar mass or color) is often selected for stacking to derive their mean profile. For instance, galaxies are often divided into two populations: star-forming and quiescent. The star-forming galaxies tend to have a Sérsic index between 1 and 2 and the quiescent galaxies have a Sérsic index that is systematically higher. In order to test how the spread of $n$ affects the stacked results, we adopt a uniform distribution of $n$ between 1 to 2 for late-type galaxies and 3 to 4 for early-type galaxies, with fixed $R_e$ and $AR = 1.0$. Figure 8 shows that the $R_e$ is almost identical to the input values for LTGs and small ETGs, and for large ETGs, the $R_e$ can be slightly underestimated due to the oversubtraction of background, which can be seen from Figure 3. The median Sérsic index can be recovered well for both late-type galaxies and early-type galaxies, though the recovered Sérsic index is slightly lower than the input median Sérsic index for the early-type galaxies with large effective radius.

#### 3.6. Effects of Spreads in AR, $R_e$, and $n$

We account for the spreads in AR, $R_e$, and $n$ together in our stacking exercises, and examine whether the averaged structural parameters can be well recovered through stacking. Our results shown in Figure 9 denote that the $R_e$ may be underestimated by up to 20%–26% for late-type galaxies but only 10%–15% for early-type galaxies. The presence of highly inclined late-type galaxies is believed to cause the additional bias to the underestimate of the averaged effective radius. The median $n$ can be well recovered for early-type galaxies, but overestimated by $bn \sim 0.7$ for late-type galaxies. This is due to late-type galaxies having larger dispersion in $R_e$ and highly inclined ones. The early-type galaxies have smaller dispersion in $R_e$, leading to a smaller overestimate of $n$, which can be diluted by the underestimation of $n$ caused by the oversubtraction of background.
3.7. Uncertainties in Aligning Images

For faint galaxies, the measured position is affected by noise and thus is offset slightly from the true position. In our stacking analysis, we measure the positions of model galaxies using SExtractor and shift the galaxies to the same position in all images before stacking. Figure 10 shows that the 68 percentile of the measured positions deviates by less than 1 pixel from the true positions. At fixed magnitudes, galaxies with more extended structure (i.e., larger effective radius) exhibit larger deviation in measuring their positions. In order to quantitatively estimate the uncertainties caused by the errors in aligning...
images, we repeat the stacking exercises presented in Figure 9 but with model galaxies generated to exactly center at the same position in all images. No image aligning is employed here. Figure 11 shows the corresponding results. We can see that the recovered parameters of the stacked profiles at all $R_e$ and $n$ from Figure 11 are nearly identical to those given in Figure 9, indicating that the error in aligning galaxy images is not a source to bias the recovery of the mean structural parameters through stacking, at least for galaxies with 24–24.75 mag and brighter.

3.8. Stacking Bulge+disk Galaxies

The composite-type galaxies are commonly composed of two distinct Sérsic components, i.e., bulge+disk. We generate bulge+disk galaxy models to test the recovery of structural parameters through stacking such objects. We assume that bulges are classical (i.e., de Vaucouleurs type with $n = 4$) and disks are exponential ($n = 1$), as often used in morphological studies of high-$z$ galaxies (e.g., Bruce et al. 2012; Lang et al. 2014). The effective radius of the bulges $R_{e,B}$ is set not to be larger than the effective radius of the disks $R_{e,D}$. Here we adopt $R_{e,D} = [5, 10, 15]$ pixels and $R_{e,B}$ is chosen accordingly, as listed in Table 2. A new parameter, bulge-to-total-light ratio $(B/T)$, is used to measure the fraction between the two components in a galaxy. The $B/T$ is set to be $0.1, 0.2, 0.25, 0.375, 0.5, 0.625, 0.75, 0.8, and 0.9$.

We first fit the 1D profiles of individual noise-free bulge+disk models with single Sérsic profiles, selecting the recovered profiles of best-fit parameters $n_T$ and $R_{e,T}$. Meanwhile, we also obtain the corresponding single Sérsic profiles best fitting the 1D profiles of the bulge+disk models without PSF convolution using the method presented in Section 2.3. The best-fit profiles to the PSF-free models are taken as the reference profiles. Here AR = 1 is adopted for all galaxy models. We do not include noises in these bulge+disk decomposition exercises in order to test how well the actual parameters of bulge+disk models can be derived from the 1D profile fitting. Second, we examine the estimate of structural parameters through stacking bulge+disk models with spreads in AR and $R_e$ counted only for the disk component, aiming at addressing how the recovered structural parameters depend on the input model parameters. We stack images of bulge+disk models of given ($R_{e,B}$, $R_{e,D}$, $B/T$) with spreads in AR$_B$ and $R_{e,D}$. The AR$_D$ spread in Section 3.2 and the spread of $R_e$ for late-type galaxies in Section 3.3 are adopted. Third, the spread of $R_{e,B}$ for the bulge component and the AR$_D$ and $R_{e,D}$ spreads for the disk component are taken into account in stacking bulge+disk models of given ($R_{e,B}$, $R_{e,D}$, $B/T$). Similarly, the best-fit Sérsic profile to the 1D profile of the stacked image is taken as the recovered profile and the reference profile is obtained from the best-fit profile with removal of noise and PSF effect. Table 2 presents the recovered structural parameters in the above three cases for three representative values $B/T = 0.1, 0.5$ and 0.9. These stacking processes deal with the model galaxy images with photon noise and background included to match the actual observations.
Figure 12 presents the measured global structural parameters $n_T$ and $R_{e,T}$ as functions of $B/T$ and the ratio of $R_{e,B}$ to $R_{e,D}$ for individual bulge+disk models (left), stacked profiles of bulge +disk models with spreads in $A R_D$ and $R_{e,D}$ (middle) counted, and stacked profiles of bulge+disk models with spreads in $A R_D$ and $R_{e,D}$ and $R_{e,B}$ included (right). By default, $n = 4$ is adopted for the bulge and $n = 1$ for the disk. Color codes $R_{e,B}/R_{e,D} = 0.25$ (blue), 0.5 (cyan), and 1 (red) with $R_{e,D} = 10$ pixels. Circles represent the recovered profiles from PSF-convolved models, while triangles in all panels mark the reference profiles from the individual PSF-free models.

3.9. Dual-Sérsic Profile Fitting

Resolving high-z galaxies into bulge and disk components even in a statistical sense is key to drawing an empirical picture for bulge growth. Here we test the recovery of the structural parameters of the two components through fitting the global 1D profile of a stacked image with bulge+disk composite profiles. We test this method and see how the measurements rely on the parameters of galaxy models for stacking. In our two-component fitting, we fix the Sérsic index for the bulge ($n = 4$) and the disk ($n = 1$). Again, the models presented in Table 2 are used in our simulations. The best fitting is selected using the least squares method across a parameter space of $R_{e,D}$, $R_{e,B}$, and $B/T$. The best-fit dual-Sérsic profile provides the recovered structural parameters $R_{e,D}$, $R_{e,B}$, and $B/T$ as the bulge and disk component of a target. The results of the dual-Sérsic profile fitting are shown in Table 2, in comparison with the input model parameters when $B/T = 0.1, 0.5$ and 0.9.

The left panel of Figure 13 shows the results of dual-Sérsic fitting to the 1D profile of individual bulge+disk models, with $R_{e,D} = 10$ pixels and $R_{e,B} = [10, 5, 2.5]$ pixels. Note that the results are not dependent on $R_{e,D}$. One can see that the three key structural parameters $R_{e,D}$, $R_{e,B}$, and $B/T$ can be properly recovered using the method of dual-Sérsic profile fitting.

Accounting for the spreads of AR and $R_e$ for the disk component of bulge+disk models, we derive the structural parameters of the bulge and disk component of the stacked image of the models through dual-Sérsic profile fitting and show the results in the middle panel of Figure 13. Clearly, the
AR and $R_e$ spreads affect the recovery of the structural parameters of the two components. While the average size of the disk component ($R_{e,D}$) is systematically underestimated by about 20%, independent from $B/T$, the average size of the bulge component ($R_{e,B}$) is increasingly overestimated at decreasing $B/T$, particularly for lower $R_{e,B}/R_{e,D}$. The $B/T$ is increasingly overestimated by decreasing $B/T$. These biases in recovering the structural parameters are obviously caused by the inclined disks in stacking. The degree of bias in $B/T$ is strongly dependent on the intrinsic $B/T$, but free from $R_{e,B}/R_{e,D}$.

Along with the spreads of AR and $R_e$ added to disks, the spread in bulge $R_e$ is also included in the stacking of bulge+disk models. The right panel of Figure 13 presents the recovered structural parameters using the method of dual-Sérsic fitting to the global 1D profile of the stacked model image. Again, the recovered structural parameters deviate from the input ones. The deviations of $(R_{e,D})$, $(R_{e,B})$, and $(B/T)$ are similar to what is given in the middle panel of Figure 13, suggesting that the bulge $R_e$ spread has little influence on the estimate of the averaged structural parameters of the stacked bulge+disk models. Furthermore, the biases in $(R_{e,B})$ and $(B/T)$ are mainly caused by the AR and $R_e$ spreads of the disk component.

### 3.10. Measurements of Pseudo Bulges

Not all bulges are classical types with a Sérsic index of $n = 4$. In fact, many bulges with smaller Sérsic indices in the local universe are recognized as pseudo bulges and thought to be built up through secular evolution (e.g., Graham 2001; Balcells et al. 2003; Kormendy & Kennicutt 2004, and references therein). For high-$z$ galaxies, the bulges formed through inward migration of disk clumps are expected to differ from the classical ones, which are usually formed via mergers (e.g., Bournaud et al. 2007). Since the pseudo bulges are less distinct from disks compared to the classical bulges, it is important to examine how the global properties depend on the model parameters and to what degree the pseudo bulge component can be resolved through a dual-Sérsic profile fitting to a 1D surface brightness profile. We therefore repeat the simulations in Section 3.8, but set $n = 2.5$ for the bulge component.

Again, we derive global structural parameters $h_T$ and $R_e,T$ by fitting single Sérsic profiles to the simulated profiles in three cases: individual pseudo bulge+disk models, stacking of pseudo bulge+disk models with AR and $R_e$ spreads added to the disk component, and stacking of such models with $R_e$ spread added to the bulge component and AR and $R_e$ spreads added to the disk component. Table 3 lists the results only for $B/T = [0.1, 0.5, 0.9]$. Figure 14 shows recovery results of the global structural parameters. Similar to those present in Figure 12: the recovered global $h_T$ increases from 1 to 2.5 as $B/T$ increases from 0 to 1; when $B/T$ is fixed, a lower ratio $R_{e,B}/R_{e,D}$ generally leads $h_T$ to be slightly higher; the global size can be approximately seen as the light-weighted combination of $R_{e,B}$ and $R_{e,D}$; the recovered parameters agree well with the reference parameters (triangles) derived from the PSF-free models; the AR and $R_e$ spreads of the disk component leave the global Sérsic index increasingly overestimated and the global size increasingly underestimated at decreasing $B/T$; the $R_e$ spread of the pseudo bulge component has an insignificant influence on the global structural parameters.

We point out that fitting the stacked profiles of pseudo bulge+disk models with dual-Sérsic profile results in similar results for classical bulge+disk models presented in Figure 13 if the two Sérsic profiles have $n = 1$ for one (disk) and $n = 2.5$ for the other (pseudo bulge). Moreover, it is interesting to see how the recovery of structural parameters is affected when $n = 4$ is adopted as the bulge component in the dual-Sérsic profile fitting. Accordingly, the recovered $R_{e,B}, R_{e,D}$, and $B/T$ as a
Table 3
Recovery of Structural Parameters Through Stacking Pseudo Bulge \((n = 2.5)\)+disk Galaxy Models

| Input Parameters | Recovered Parameters |
|------------------|----------------------|
| \(R_{e,D}\) | \(R_{e,B}\) | \(B/T\) | \(\eta_T\) | \(R_{e,T}\) | \(R_{e,D}\) | \(R_{e,B}\) | \(B/T\) | \(\eta_T\) | \(R_{e,T}\) | \(R_{e,D}\) | \(R_{e,B}\) | \(B/T\) | \(\eta_T\) |
| 15.0 | 15.0 | 0.1 | 14.9 | 20.0 | 0.07 | 1.1 | 14.8 | 10.9 | 14.3 | 0.44 | 1.8 | 11.3 | 10.9 | 13.8 | 0.44 | 1.8 | 11.2 |
| 15.0 | 10.0 | 0.1 | 14.7 | 14.5 | 0.09 | 1.2 | 14.3 | 10.5 | 14.3 | 0.45 | 1.8 | 11.0 | 10.7 | 13.3 | 0.45 | 1.8 | 10.9 |
| 15.0 | 5.0 | 0.1 | 14.6 | 7.9 | 0.12 | 1.4 | 13.3 | 10.0 | 13.1 | 0.50 | 1.9 | 10.4 | 10.1 | 13.0 | 0.51 | 1.9 | 10.4 |
| 10.0 | 10.0 | 0.1 | 10.0 | 11.7 | 0.07 | 1.1 | 9.9 | 7.6 | 8.5 | 0.43 | 1.7 | 7.5 | 7.6 | 8.4 | 0.43 | 1.7 | 7.5 |
| 10.0 | 5.0 | 0.1 | 9.8 | 6.1 | 0.09 | 1.2 | 9.3 | 7.4 | 7.4 | 0.43 | 1.7 | 7.1 | 7.4 | 7.5 | 0.44 | 1.8 | 7.1 |
| 10.0 | 2.5 | 0.1 | 9.8 | 3.6 | 0.12 | 1.3 | 8.6 | 7.3 | 6.5 | 0.48 | 1.9 | 6.8 | 7.3 | 6.6 | 0.49 | 1.9 | 6.8 |
| 5.0 | 5.0 | 0.1 | 5.1 | 4.1 | 0.07 | 1.1 | 5.0 | 4.2 | 3.8 | 0.48 | 1.8 | 3.9 | 4.2 | 3.8 | 0.49 | 1.8 | 3.9 |
| 5.0 | 2.5 | 0.1 | 5.0 | 1.7 | 0.07 | 1.1 | 5.0 | 4.0 | 3.5 | 0.48 | 1.8 | 3.7 | 4.1 | 3.3 | 0.49 | 1.8 | 3.7 |
| 5.0 | 1.0 | 0.1 | 5.0 | 0.9 | 0.10 | 1.2 | 5.0 | 4.0 | 3.0 | 0.55 | 2.0 | 3.4 | 4.0 | 3.0 | 0.54 | 2.0 | 3.4 |

Figure 14. Measured global structural parameters \(\eta_T\) and \(R_{e,T}\) as functions of \(B/T\) and the ratio of \(R_e,B\) to \(R_e,D\) for individual pseudo bulge+disk models (left), the stacked profiles of pseudo bulge+disk models with AR and \(R_e\) spreads added to the disk component (middle), and the stacked profiles of pseudo bulge+disk models with AR and \(R_e\) spreads for the disk component and \(R_e\) spread for the bulge component (right). Here \(n = 2.5\) is adopted for pseudo bulges and \(n = 1\) for disks. Color codes \(R_e,B/R_e,D = 0.25\) (blue), 0.5 (cyan), and 1 (red) at \(R_e,D = 10\) pixels. Circles represent the recovered profiles from PSF-convolved models, while triangles in all panels mark the reference profiles from individual PSF-free models.
function of input $B/T$ are presented in Figure 15, for individual pseudo bulge+disk models, stacking of these models with the spreads of AR and $R_e$ included into the disk component, and stacking of these models with AR spread counted for the disk component and $R_e$ spread for both components, respectively.

When the pseudo bulge in the pseudo bulge+disk models is mistaken as a classical one, the bulge+disk decomposition ends up with an increasing underestimate of $B/T$ (also $R_{e,B}$ and $R_{e,D}$) with increasing input $B/T$. This is understandable in that the pseudo bulge component is divided into a classical bulge and an additional disk. At $R_{e,B}/R_{e,D} < 1$, the additional disk biases the recovered $R_{e,D}$ smaller. Taking the spreads of AR and $R_e$ into account, the recovery of the averaged structural parameters of ($R_{e,B}$), ($R_{e,D}$), and ($B/T$) from the stacked profile of pseudo bulge+disk models is further affected mainly by the AR and $R_e$ spreads of the disk, as described in Section 3.9.

4. SUMMARY

The stacking technique is a powerful tool to probe signals under the detection limit of individual images for faint galaxies and enable us to obtain the averaged surface brightness profile toward larger radii. We carried out simulations to test the stacking of galaxy models with spreads in AR (elongation or inclination) and effective radius ($R_e$) counted, and to explore how the recovered structural parameters of the averaged surface brightness profile depend on the effective radius ($R_e$), AR, index of Sérsic profile ($n$) of the models and their spreads. We also addressed the recovery of structural parameters through stacking bulge+disk models in order to simulate the real observations.

In our simulations, we use circular apertures to derive the surface brightness profile of a galaxy image. This is the usual way for stacking of faint high-$z$ galaxies, whose structural parameters are barely known so that corrections for inconsistence in inclination, orientation, and size are often ignored. Following Szomoru et al. (2012), we fit the 1D radial surface brightness profile of a stacked galaxy image with a library of 1D Sérsic profiles to obtain the best-fit profile using the method of least squares. The best-fit profile is taken as the intrinsic profile for the stacked image. Galaxy models used in our simulations have structural parameters spanning sufficiently wide ranges: $1 \leq n \leq 6$, $0^\circ.05 \leq R_e \leq 0^\circ.75$ and $0.1 \leq AR \leq 1.0$.

We examined (1) the dependence of the measured surface brightness profile solely on each parameter of $n$, $R_e$ and AR; (2) the recovery of structural parameters of the mean surface brightness profile when a set of galaxy models for stacking have two parameters fixed and the third parameter spreading within a certain distribution; (3) the recovery of structural parameters of the mean surface brightness profile of galaxy models with spreads counted in both AR and $R_e$ as functions of $n$ and mean effective radius ($R_e$); (4) the recovery of structural parameters of the mean surface brightness profile of galaxy models with spreads added to all parameters, including AR, $R_e$, and $n$, for late-type and early-type galaxies; (5) the fitting of the stacked profile of bulge+disk models with both single Sérsic profiles and dual-Sérsic profiles to see to which extent the stacking can recover the averaged structural parameters of a galaxy population.

The striking results from our simulations are that the structural parameters $n$ and $R_e$ of the mean-stacked image of a group of galaxies may be biased up to 70% by spreads in AR and $R_e$, depending on the distribution functions of the spreads; the bias can be quantitatively corrected once the spread functions are known. We summarize our results as follows.

1. The spread in AR leads the mean-stacked image of a group galaxies to appear more compact, i.e., with a lower mean effective radius ($\langle R_e \rangle$) and a higher $n$. The inclusion
of highly inclined (or elongated) galaxies with low AR in stacks biases the recovered $n$ and $\langle R_e \rangle$.

2. For early-type galaxies with large Sérsic index $n$, which have extended halos in the outskirts, the oversubtraction of background biases the estimate of structural parameters. The mean Sérsic index $n$ may be underestimated, and the effective radius $R_e$ may be underestimated for early-type galaxies with large sizes. This indicates that the estimation of background is very important for the estimation of structural parameters, especially for early-type galaxies which have extended halos in the outskirts.

3. Accounting for the AR spread of local disk galaxies from Padilla & Strauss (2008) in stacking galaxies with $n \leq 2.5$, the median effective radius $\langle R_e \rangle$ of the stacked galaxies is underestimated by 23% and the mean Sérsic index $n$ is overestimated by up to 20%. Similarly, the AR spread of local early-type galaxies from Hao et al. (2006) in stacking galaxies with $n > 2.5$ leads to an underestimate of 12% in $\langle R_e \rangle$ and little influence on the estimate of $n$.

4. The spread in $R_e$ plays a different role from that of the spread in AR, which leads the mean-stacked profile of galaxies to be more concentrated. Taking the log-normal distribution of $R_e$ from Shen et al. (2003) into account, $\langle R_e \rangle$ is not biased for either early-type or late-type galaxies, and Sérsic index $n$ is overestimated by 50% for the late-type galaxies but not significantly biased for the early-type galaxies. When $R_e$ scatters within a spread, the stacked galaxies with a size smaller than the median contribute more to the central part of the integrated light, and those with a size larger than the median contribute more to the extended wing, resulting in a mean-stacked profile with a higher Sérsic index and an effective radius equal to the median of the $R_e$ spread.

5. The effects of the spreads in AR and $R_e$ are linearly co-added on the estimate of structural parameters of a stacked image. The actual corrections for these effects rely on the spread functions.

6. Accounting for the spread of $n$ with a uniform distribution, we find that the recovered structural parameters remain unchanged compared to those derived with no spread in $n$ counted, suggesting that the spread of $n$ does not significantly affect the stacked results.

7. In the stacking analysis, the galaxies are often classified by the stellar mass and type, and the AR, $R_e$ and $n$ all have spreads. In this case, we find that the effective radius $R_e$ can be underestimated by 20% to 27% for late-type galaxies, and only 10% to 15% for early-type galaxies, due to the AR distribution of galaxies. The Sérsic index $n$ can be well recovered for early-type galaxies, but can be overestimated for late-type galaxies due to both the AR and $R_e$ distribution.

8. For faint galaxies, the center we find will have an offset to the real center of galaxies due to the noise. We also test this effect by stacking galaxies, using the real center instead of the center found by SExtractor. We find that for galaxies in the range of 24–24.75 mag, the 68% percentile of the centering offset is not more than 1 pixel and does not influence the results.

9. Recovery of structural parameters of bulge+disk galaxies are regulated by the bulge-to-total ratio $B/T$, the $R_e$ ratio of the two components and the Sérsic index of the bulge if the distortion is exponential ($n = 1$). The global Sérsic index $n_T$ mainly depends on the Sérsic index of the bulge and $B/T$. The global effective radius $R_{e,T}$ can be approximately seen as the light-weighted combination of bulge size $R_{e,B}$ and disk size $R_{e,D}$. Furthermore, the ratio $R_{e,B}/R_{e,D}$ also has influence on $n_T$ at a second-order level. The AR and $R_e$ spreads of the disks and the $R_e$ spread of the bulges lead $n_T$ to be increasingly overestimated and $R_{e,T}$ to be increasingly underestimated at decreasing $B/T$.

10. By dual-Sérsic profile fitting to the stacked profile of bulge+disk galaxies, the averaged structural parameters of the bulge and disk components can be determined, though the uncertainties are large. The AR and $R_e$ spreads of disks are the main causes of the increasing overestimate of $B/T$ (also $R_{e,B}$) at decreasing intrinsic $B/T$. The issue of the bulges being pseudo or classical dramatically changes the results of the dual-Sérsic profile fitting for a composite profile. The caveat that the measurement of bulge growth in a statistical sense is significantly affected by the AR and $R_e$ spreads of the disks and Sérsic index of the bulges in the bulge+disk decomposition through stacking. We thus stress that the interpretation of stacking results should take these biases into account and the corrections for the biases are strongly dependent on the structural parameters of stacked galaxies and their spread functions.

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