Gauge Hierarchy, Planck Scale Corrections
And The Origin of GUT Scale In
Supersymmetric (SU(3))^3

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Abstract

Within a supersymmetric unified framework we explore the
resolution of the gauge hierarchy problem taking account of the non-
renormalizable terms in the superpotential. For [SU(3)]^3 supple-
mented by a discrete R parity, we find the remarkable property that
the vacuum configuration corresponding to the correct gauge symme-
try breaking remains flat (in the supersymmetric limit) to all orders
in M^{-1}_{Planck}. The grand unification scale arises from an interplay of
the Planck and supersymmetry breaking scales. An ‘internal’ Z_3 ⊗ Z_4
symmetry protects a pair of electroweak doublets from becoming su-
perheavy, yielding at the same time the supersymmetric ‘µ term’ with

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the right order of magnitude. The $Z_4$ symmetry acts as matter parity and eliminates the dangerous baryon number violating couplings.
1 Introduction

In a recent letter, hereafter referred to as [I], an attempt was undertaken to resolve the gauge hierarchy problem within a specific grand unified theory, to wit, supersymmetric $G \equiv SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$ ($SU(3)^3$ for short). It was found that a combination of discrete symmetries suffices to ensure that a pair of electroweak higgs doublets remains ‘light’. More importantly perhaps, it was noted that a combination of suitable discrete symmetries may provide an ‘all order’ resolution of the gauge hierarchy problem in $G$. The main purpose of this paper is to address this and the related problem of the origin of the GUT scale $M_G$ within the framework of $G$. It turns out that the $SU(3)^3$ approach does have some unique features which we will highlight in this work.

The resolution of the hierarchy problem in supersymmetric grand unified theories (SUSY GUTS) would answer the following question: How can a theory with superheavy scales $M_G$ and $M_{\text{Planck}}$ ($= M_P$ for short) arrange itself in such a way that a pair of light (mass $\sim M_W$) electroweak doublets survive? There exist several mechanisms (missing partner [2], pseudogolstone [3], custodial symmetry [4]), which (more or less naturally\footnote{Criteria of “naturalness” vary in these approaches}) allow one to obtain light doublets at the level of a renormalizable potential. However, in SUSY GUTs (especially those based on superstrings or supergravity) there is no obvious reason why one should restrict attention only to the renormalizable couplings in the superpotential (and/or in the Kahler potential). But then we are immediately confronted by the problem of stability of the tree
level hierarchy in the presence of the higher order terms. Even if kept ‘light’
(by some mechanism) at the tree level, the doublet mass can (and in general
will) be disturbed by the Planck scale operators (which, for the SUSY GUT
vacuum expectation values (vevs) cannot necessarily be regarded as small
corrections).

Let us suppose that we want to solve the hierarchy problem in all orders
in a theory in which the only allowed input mass scales in the superpotential
are $M_P$ and $M_G$. The scale $M_G$ either can be included in the superpotential
from the very beginning as an explicit input mass parameter, or it can be
dynamically or radiatively generated after SUSY breaking. In the latter case,
in the unbroken SUSY limit, the scale $M_G$ is represented by the undetermined
(sliding) vev $< S >$ along the flat vacuum direction, which gets fixed at
$< S > = M_G$ only after SUSY breaking.

A natural solution of the hierarchy problem in such a model would imply
that due to some symmetry, a pair of electroweak Higgs doublets $H^{(1)}, H^{(2)}$
has no mass term and/or renormalizable couplings with the superlarge vev
$< S >$ in the superpotential. However, an absolutely decoupled doublet can
be problematic, since in the effective low energy theory we do need a ‘small’
($\sim M_W$) supersymmetric mass term $\mu H^{(1)} H^{(2)}$ for the doublets. Thus, the
doublets better couple to the vev $< S >$ at some order in $M_P^{-1}$. Consider
therefore the following term in the superpotential:

$$\frac{S^{n+1}}{M_P^n} H^{(1)} H^{(2)}, n \geq 1$$

(1)

After GUT symmetry breaking this operator will produce an effective $\mu$-term
of order $(M_G/M_P)^n M_G$. With $M_G/M_P \sim 10^{-2} - 10^{-3}$ we will obtain the right
order of magnitude for $n = 5 - 6$.

Naively one might expect that this program is readily realized by imposing some appropriate discrete or continuous symmetries (which of course should be respected by the Planck scale physics). However, there are two major obstacles to be overcome. First of all, in ordinary GUTs (e.g. SU(5) SO(10),...), the Higgs doublet is not an independent field but a member of some irreducible representation of the GUT symmetry. This representation usually includes a colored triplet component which can mediate rapid proton decay unless it is superheavy\footnote{However, as was shown in [5], this is not a necessary condition; in some cases the theory can be arranged in such a way that the colored triplet is light but does not mediate proton decay.}. Thus, we typically do need to couple this multiplet to the large vevs, and the challenge is to keep the doublet(s) light. This is the famous doublet-triplet splitting problem in GUTs which often gets confused with the hierarchy problem.

It is difficult, however, to achieve this splitting to all orders, since the symmetries which allow the renormalizable couplings needed for the correct doublet-triplet splitting also permit nonrenormalizable terms with rather low dimensionality that can give large masses to the doublets. Perhaps the best motivated candidates for an ‘all order’ resolution of the hierarchy problem are theories in which the weak doublets and the colored triplets do not reside in the same representation. An explicit example of such a theory is provided by $G \equiv SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$.

Another potential problem is that the higher dimensional operators in the superpotential cannot necessarily be regarded as small corrections since they often provide a mechanism for the generation of the vevs, and can strongly
alter both the magnitude of the GUT scale as well as the symmetry breaking pattern. The reason is that to suppress the dangerous low dimensional operators as in (1), one often needs to assign an additional (discrete or continuous) quantum number(s) to the superfield $S$ whose vev breaks the GUT symmetry. However, this symmetry may also forbid the lowest (renormalizable) selfcouplings of this superfield, allowing only the higher dimensional invariants in the superpotential.

To conclude, we find that the simplest candidate theories for an ‘all order’ solution of the hierarchy problem are the ones which satisfy the following two conditions:

1. Electroweak higgs doublets do not reside in the same representation as the colored triplets;

2. All higher order invariants in the superpotential respect both the correct pattern of the symmetry breaking, as well as the magnitude of the GUT scale. Indeed, we are particularly interested in models in which the GUT scale appears after SUSY breaking, with the higher dimensional operators playing an important role.

In addition, we give preference to theories that do not require the introduction of additional ‘auxiliary’ multiplets which are not otherwise necessary for the symmetry breaking. In other words, the $SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$ scheme automatically fits in this class of desired theories. The minimal Higgs representation that can induce the correct symmetry breaking and also contain the electroweak Higgs doublets are two pairs of $(1.3.3) + (1.3.\bar{3})$ representations. Condition (1) above is automatically satisfied since there is no colored component in this representation and thus no doublet-triplet split-
ting problem. As we have indicated in [1], by supplementing \([SU(3)]^3\) with an appropriate discrete symmetry \(Z_3 \otimes Z_4 \otimes R\)-parity, condition (2) can also be satisfied, thereby solving the hierarchy problem in all orders! More than that, it was argued that the desired symmetry breaking pattern corresponds to a vacuum direction that is unique and respected by all possible operators to all orders! The SUSY GUT scale arises from an interplay of the SUSY breaking parameters and higher dimensional operators.

2 Gauge Hierarchy to All Orders

Let us now discuss this question in more detail. There are two pairs of the Higgs superfields \(\lambda^A_\alpha, \bar{\lambda}^a_A\) (and \(\lambda'^A_\alpha, \bar{\lambda}'^a_A\)) transforming as \((1, \bar{3}, \bar{3}) + (1, 3, 3)\) respectively. Here and below we shall denote by the Latin \((A, B, C...) = 1, 2, 3...\) and Greek \((\alpha, \beta, \gamma = 1, 2, 3)\) symbols the \(SU(3)_L\) and \(SU(3)_R\) indices respectively. For the correct gauge symmetry breaking \([SU(3)]^3 \rightarrow SU(3)_C \otimes SU(2)_L \otimes U(1)\), the vevs should be oriented along the directions

\[
|\lambda| = |\bar{\lambda}^*| = \begin{bmatrix} 0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & N \end{bmatrix}, \quad |\lambda'| = |\bar{\lambda}'^*| = \begin{bmatrix} 0 & 0 & 0 \\
0 & 0 & 0 \\
0 & \nu^{\epsilon} & 0 \end{bmatrix}
\]

Let us find the minimal discrete symmetry which will guarantee that the vacuum given by (2) is respected to all orders. Surprisingly enough, it turns out that all one needs is a discrete \(R\)-parity under which the chiral superfields as well as the superpotential change sign. This simply means that the superpotential will only include odd power invariants in the superfields. It is easily shown that ANY odd power invariant as well as its first derivatives (with respect to the superfields) automatically vanish for the configuration of the
vevs given by (2). Thus, all derivatives of the superpotential are identically zero and the desired vacuum configuration is respected to all orders!

To prove this let us consider an arbitrary $SU(3)_L \otimes SU(3)_R$ invariant product of the n-superfields transforming as $(1, \bar{3}, 3)$ (such as $\lambda, \lambda'$) and m-superfields transforming as $(1, 3, \bar{3})$ (such as $\bar{\lambda}, \bar{\lambda}'$). Due to R-parity the total number $(m + n)$ of superfields participating in this invariant should be odd and thus $m \neq n$. Any such invariant is obtained from the direct product of the superfields

$$\lambda^{A_1}_{\alpha_1} \otimes \cdots \lambda^{A_n}_{\alpha_n} \otimes \bar{\lambda}^{\beta_1}_{B_1} \otimes \cdots \bar{\lambda}^{\beta_m}_{B_m}$$

(3)

with all possible $SU(3)_L \otimes SU(3)_R$-invariant contractions of the indices. For simplicity we have dropped the prime symbol in (3). The number of primed and unprimed fields is not important since they transform identically under the gauge group. To form the invariants, each lower $\alpha, B$ (upper $\beta, A$) index should be contracted either with some other upper $\beta, A$ (lower $\alpha, B$) index or with the totally antisymmetric epsilon tensor. But we know that $n \neq m$ and the number of the upper and lower indices (for each $SU(3)_{L,R}$) are different. Thus, not all of them can be paired with each other. Therefore, there should be at least one antisymmetric epsilon contraction among the indices of each $SU(3)_{L,R}$.

In other words, any appropriate invariant can be written in the general form

$$\epsilon_{\alpha\beta\gamma} \epsilon_{ABC} f_{\alpha\beta\gamma;ABC}$$

(4)

where $f_{\alpha\beta\gamma;ABC}$ is some odd product of the fields $\lambda, \bar{\lambda}, \lambda', \bar{\lambda}'$ with all other indices contracted, and $\alpha, \beta, \gamma$ and $ABC$ are upper (or lower) indices belonging
to the components of $\lambda, \bar{\lambda}, \lambda', \bar{\lambda}'$. Of course, inside $f$ there can be some other epsilon contractions too, but for us it is sufficient that there exists at least two epsilon contractions which are explicitly indicated in (4). Any such invariant as well as its derivatives with respect to the fields $\lambda, \bar{\lambda}, \lambda', \bar{\lambda}'$, will automatically vanish along the configuration (2). This is clear since along this direction, the possible nonzero components are $|\lambda_3^3| = |\bar{\lambda}_3^3| \neq 0, |\lambda_3^2| = |\bar{\lambda}_2^3| \neq 0$. But, $\lambda_3^3, \lambda_3^2$ and $\bar{\lambda}_3^3, \bar{\lambda}_2^3$ share one index, and therefore any epsilon contraction as well as its derivative will automatically vanish along this direction. Thus, the configuration (2) corresponding to the right gauge symmetry breaking pattern automatically ensures a vanishing first derivative of any existing invariant in the superpotential, thereby satisfying to all orders and for any value of the parameters the minimum (F-flatness condition):

$$\frac{\partial W}{\partial \lambda} = \frac{\partial W}{\partial \bar{\lambda}} = \frac{\partial W}{\partial \lambda'} = \frac{\partial W}{\partial \bar{\lambda}'} = 0$$

(5)

Next let us ask the following question: What is the minimal ‘internal’ discrete symmetry that can satisfy the following requirements:

(1) Protect a pair of the electroweak doublets $H^{(1)}, H^{(2)}$ from getting large masses;

(2) At the same time guarantee that there will appear a small supersymmetric mass term $\mu H^{(1)} H^{(2)}$ with $\mu \sim M_W$.

(3) Allow large $(\sim M_G)$ masses for the other $SU(3)_C \otimes SU(2)_L \otimes U(1)$-nonsinglet states, except, of course, the quark and lepton superfields (and possibly some other states that may form complete $SU(5)$ multiplets so as not to disturb unification of the gauge couplings).
(4) Eliminate the dangerous baryon number violating operators.

It is remarkable that all of these requirements are met by an appropriate $Z_4 \otimes Z_3$ symmetry. Indeed we will now show that the $Z_4 \otimes Z_3$ symmetry is the unique ‘minimal’ choice satisfying all of the requirements (1) – (4) above. First, we need to identify those states that may play the role of the electroweak Higgs doublets after $[SU(3)]^3$ breaks to the standard model gauge group. Inside the Higgs supermultiplets $\lambda, \bar{\lambda}, \lambda', \bar{\lambda}'$ there are 12 electroweak doublet states. Half of them ($\bar{H}^{(1)}, H^{(2)}, L, \bar{H}^{(1)'}, H^{(2)'}, L'$) carry quantum numbers of the “down”-type Higgs doublets, while the other half $H^{(1)}, \bar{H}^{(2)}, L, H^{(1)'}, H^{(2)'}, \bar{L}'$ carry quantum numbers of the “up-”type Higgs doublet. However, due to the $[SU(3)]^3$ symmetry, only the $H^{(1)}, H^{(2)}$ pair (from $\lambda$ or $\lambda'$) has the correct Yukawa couplings with the quarks and leptons. The three generations of matter fermions (leptons, quarks, antiquarks) transform as $(1, \bar{3}, \bar{3}), (3, 3, 1), (\bar{3}, 1, \bar{3})$. We shall denote them as $\lambda_i, Q_i, Q_c^i$, where $i = 1, 2, 3$ is the family index. It is obvious that the matter superfields do not have cubic invariant couplings with $\bar{\lambda}$ or $\bar{\lambda}'$ fields which transform as $(1,3,\bar{3})$. The doublets $\bar{H}^{(1)}_1, H^{(2)}_1, \bar{L}, H^{(1)'}_1, H^{(2)'}_1, L'$ belonging to $\bar{\lambda}'$, $\bar{\lambda}$ do not (at least at the renormalizable tree level) couple with the matter fields.

The trilinear “Yukawa” couplings are of the form:

$$\lambda \lambda_i \lambda_j + \lambda Q_i Q^c_j + \lambda' \lambda_i \lambda_j + \lambda' Q_i Q^c_j$$

Decomposing this into $SU(3)_3 \otimes SU(2)_L \otimes U(1)$-invariant pieces, we can easily find that $H^{(2)}, H^{(2)'}$ and $H^{(1)}, H^{(1)'}$ doublets have the correct down-type and up-type coupling with the quarks and leptons, just as in the minimal SUSY standard model. However, to minimize the number of ‘light’ doublets, we
make the choice that only the pair \( H^{(1)}, H^{(2)} \) be protected by a discrete symmetry. This is achieved by ensuring that the cubic invariant \( \lambda^3 \) (which contains the term \((H^{(1)}H^{(2)}N)\) and is allowed by R-parity) is forbidden. The minimal symmetry which accomplishes this is \( Z_2 \), under which \( \lambda \rightarrow -\lambda \). Since the Yukawa couplings of \( \lambda \) with the matter superfields must be allowed, \( \lambda_i \) should transform under \( Z_2 \) in such a way that \((\lambda_i\lambda_j) \rightarrow - (\lambda_i\lambda_j)\). Here we assume that all the families \((i, j = 1, 2, 3)\) transform in the same way under \( Z_2 \). This automatically fixes the transformation properties of the leptons, \( \lambda_i \rightarrow i\lambda_i \) \((i = 1, 2, 3)\). It is natural to assume that the quarks transform in the same way, namely, \((Q_i, Q^c_i) \rightarrow i(Q_i, Q^c_i)\). Thus, it turns out that the symmetry which acts as a \( Z_2 \) on \( \lambda \), acts as a \( Z_4 \) on the matter multiplets. Note that this \( Z_4 \) symmetry also forbids the dangerous baryon number violating operators which are trilinear in the matter fields. The proton is essentially stable in this theory.

What are the transformation properties of the remaining Higgs superfields \( (\bar{\lambda}, \lambda', \bar{\lambda}') \) under \( Z_4 \)? In order to prevent a proliferation of light doublets we should allow the trilinear invariants

\[
\bar{\lambda}^3, \lambda'^3, \bar{\lambda}'^3 \tag{7}
\]

in the superpotential. Thus \( \bar{\lambda}, \lambda', \bar{\lambda}' \) should not transform under \( Z_4 \). But this is not the end of the story. The superfields \( \bar{\lambda}, \lambda', \bar{\lambda}' \) must transform under some other discrete symmetry, otherwise invariants of the form

\[
\lambda\lambda' \bar{\lambda} + \frac{(\lambda \bar{\lambda})\lambda^3}{M_P^2} + ... \tag{8}
\]

will give very large masses to the doublets \( H^{(1)}, H^{(2)} \). The only possible symmetry that allows trilinear invariants \((7)\) is a \( Z_3 \) symmetry, and it is quite
remarkable that this symmetry automatically forbids (8)! We will assume that the $\bar{\lambda}, \lambda', \bar{\lambda}'$ fields transform under $Z_3$ as:

$$(\bar{\lambda}, \bar{\lambda}') \rightarrow e^{i\alpha}(\bar{\lambda}, \bar{\lambda}'), \quad \text{and} \quad \lambda' \rightarrow e^{i2\alpha}\lambda', \alpha = \frac{2\pi}{3} \quad (9)$$

We now reach the important conclusion that the lowest dimensional coupling which gives rise to the ‘$\mu$-term’ is given by

$$\frac{\lambda^3(\lambda\bar{\lambda})^3}{M_P^6} \quad (10)$$

which, for $|\lambda| = |\bar{\lambda}| \sim M_G$, gives $\mu \sim \left(\frac{M_G}{M_P}\right)^6 M_G$, as desired!

In conclusion, starting from some fairly general considerations, we are inevitably led to the unique discrete symmetry $Z_3 \times Z_4$ for satisfying the four requirements listed below equation (5).

### 3 Supersymmetry Breaking and the Generation of GUT Scale

We have shown in the previous section that in the supersymmetric limit the configuration in (2) describes a valid vacuum in all orders (including all possible renormalizable and nonrenormalizable terms), for arbitrary values of the parameters compatible with R-parity. The allowed invariants (as well as their derivatives) identically vanish for this configuration of the vevs, so that the magnitude of the vevs are not fixed, and our “correct” vacuum corresponds to a flat direction. The existence of “accidental” flat directions in the vacuum is characteristic for supersymmetric theories. However, the present case is a very unusual (and as far as we know the first realistic) example of
a flat direction that can survive to all orders in $M_P^{-1}$. Consequently, the GUT scale in our theory can never be fixed (in any order in $M_P^{-1}$) in the unbroken SUSY limit. It is determined only after supersymmetry breaking has occurred, offering as a consequence the exciting possibility of explaining why $M_G$ happens to be close to $M_p \sim 10^{18} GeV$. [There are previous examples (e.g. see [6], [7], [4]) in which a sliding scale in the unbroken SUSY limit is fixed at $M_G$ after supersymmetry breaking.]

In conventional supergravity scenarios it is usually assumed that SUSY breaking takes place in some “hidden” sector of the theory which communicates with the “observable” fields ($\lambda, \bar{\lambda}, \lambda', \bar{\lambda}', Q_i, Q'_c, \lambda_i$) via some nonrenormalizable Planck scale operators. In the present work we do not wish to advocate any particular mechanism which leads to SUSY breaking in the hidden sector. This breaking can be induced, for example, through the vev of some fundamental scalar field (as in the simplest Polony potential [8]), or dynamically due to some strongly coupled metacolor force (like in models with gaugino condensation [9]). For us, the important point about this breaking is that it induces soft SUSY violating terms in the effective potential of the observable fields, which in the minimal case has a well known form [10]

$$V = \sum_{Z_i} \left| \frac{\partial W}{\partial Z_i} \right|^2 + m^{*2}_3 A_n W^{(n)} + h.c. + m^2_3 \sum_{Z_i} | Z_i |^2 + |D - terms|$$  \hspace{1cm} (11)

Here $| m^{*2}_3 |$ is a gravitino mass which happens to be the messenger of SUSY breaking in the visible sector, and $A_n$ are $(n = 1,2,...)$ numbers, typically of order unity. The $W^{(n)}$ denote the n-linear pieces of the superpotential and the sum over $Z_i$ refers to all the scalar components of the observable sector.
As we have shown in section 1, all \( \frac{\partial W}{\partial Z_i} \) and all \( W^{(n)} \) (for \( n = 1, \ldots, \infty \)), as well as D-terms, identically vanish for the configuration of the vevs given by (2), corresponding to the correct pattern of symmetry breaking. Therefore, the only term that can destabilize the vevs is \( m^2_3 |N|^2 + |\bar{N}|^2 + |\nu^{\prime\prime}|^2 + |\bar{\nu}^{\prime\prime}|^2 \). At the tree level (at scale \( \sim M_P \)), the quantity \( m^2_3 \) is positive and thus the vevs are confined to the origin. However, it is well known that the sign of \( m^2_3 \) can change due to renormalization effects, since “Yukawa” interactions (as well as cubic couplings \( \bar{\lambda}^3, \lambda^3, \bar{\lambda}^a \)) can drive them negative. If this happens, more precisely if \( M^2_N + M^2_{\bar{N}} < 0, M^2_{\nu^{\prime\prime}} + M^2_{\bar{\nu}^{\prime\prime}} < 0 \), then the potential (11) formally becomes unbounded from below and the minimum is established at \( < N > = \infty, < \nu^{\prime\prime} > = \infty \). Fortunately this is not the case, since for large enough vevs, one must take into account the back reaction of the observable vevs on the hidden sector [the limit \( M_P \to \infty \) with \( m^2_3 = \text{fixed} \), under which the effective potential (11) was obtained becomes invalid.]

One possible mechanism which seems quite attractive assumes that the spontaneous breaking of the R-parity (as well as SUSY) in the hidden sector induces some R-parity noninvariant terms in the observable sector which will help stabilize the GUT scale vevs. In general this will happen if the two sectors communicate through the gravitational (\( M^{-1}_P \) suppressed) couplings in the superpotential (or at least in the Kahler potential). This communication can be established, for instance, via some additional gauge singlet fields which play the role of ‘connectors’: they pick up a nonzero (\( R \)-noninvariant) VEV from tree level interactions with the hidden sector fields, and transfer this breaking to the observable sector through the Planck scale couplings.
Of course, it is very difficult to answer (without better knowledge of the Planck scale physics) precisely what stabilizes the vevs at the high scales. It is intriguing, however, to see how far we can go in understanding the appearance of the GUT scale with our present knowledge of (super) gravity. We will see that under plausible assumptions, the vevs tend to be stabilized at high scales which are somewhat below $M_P$. A key point is that R-parity will inevitably be broken in the hidden sector by the nonzero vev of the hidden sector superpotential $\langle h \rangle$ which changes sign under the R-parity transformation. Note that $\langle h \rangle$ cannot vanish in the nonsupersymmetric minimum with a zero cosmological constant.

As we have mentioned before, the spontaneous breaking of the R-parity in the hidden sector can (and in general will) induce effective R-parity non-invariant terms in the observable sector through the general (super) gravity couplings. On dimensional grounds it is clear that such terms are suppressed by powers of $M_R/M_P$ (where $M_R$ is a $R$-noninvariant VEV communicating with the observable sector). Being noninvariant under R-parity, they sooner or later lift the flat direction (2), and thereby stabilize the vevs at some high scale.

To estimate the order of magnitude of these vevs, we need to know the leading gravitational couplings between the hidden and observable sectors that involve the R-parity breaking vevs. Let us assume that the messenger of the R-parity breaking is a gauge singlet $R$ which picks up the VEV through a tree level interaction with the hidden sector fields. This VEV, in general, can be arbitrary. However, if we assume that these are the only two (input)
scales $M_P$ and $M_S \sim \sqrt{M_P m_{3/2}}$ in the hidden sector, then it is natural to relate $< R >$ with one of them. We will assume that $< R > \sim M_S$.

The lowest dimensional couplings in the superpotential compatible with $Z_4 \otimes Z_3 \otimes$ R-parity are $\frac{R(\lambda' \lambda')^2}{M_P^2}$ and $\frac{R(\lambda \lambda)^6}{M_P^6}$ for the prime and nonprime sectors respectively. After the R-parity breaking these will induce effective terms in the potential of the form:

$$m_{\frac{1}{2}} \frac{|\lambda'|^6}{M_P^2} \quad \text{and} \quad m_{\frac{1}{2}} \frac{|\lambda|^22}{M_P^{19}}$$

which will stabilize the vevs at scales $< \nu'^* > \sim M_P \left(\frac{m_{\frac{1}{2}}}{M_P}\right)^{\frac{1}{2}}$ and $< N > \sim M_P \left(\frac{m_{\frac{1}{2}}}{M_P}\right)^{\frac{1}{2}}$ respectively. Admittedly, the $< N >$ vev is somewhat higher (and $\nu'^*$ somewhat lower?) than what we would prefer ($\sim 10^{16}$ GeV), but the important message here is that the origin of SUSY GUT scale possibly can be understood within the framework of supersymmetric ($SU(3))^3$ models.

4 Conclusion

The gauge symmetry $G \equiv SU(3)_C \times SU(3)_L \times SU(3)_R$ provides an attractive alternative to ‘standard’ SUSY GUTs such as $SU(5)$ or $SO(10)$. In contrast to the latter (however see [11]), $G$ it seems can arise from superstring theories. From a practical viewpoint, it allows for an elegant (‘all order’) resolution of the hierarchy problem with a minimal set of higgs supermultiplets. Furthermore, the SUSY GUT scale arises from an interplay of the Planck and SUSY breaking scales, with the higher dimensional operators playing an essential role. [Note that if the GUT scale is put in by hand, a single R-symmetry is
enough to solve the hierarchy problem with \( G \equiv SU(3)_C \times SU(3)_L \times SU(3)_R \).

See [1] for details.] Both the proton and the lightest supersymmetric particle are stable in the approach we have described. \( \alpha \varepsilon \)6

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