Bethe-Salpeter equations for the collective-mode spectrum of a superfluid Fermi gas in a moving optical lattice

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We derive the Bethe-Salpeter (BS) equations for the collective-mode spectrum of superfluid Fermi gases of equal mixture of atomic Fermi gas of two hyperfine states loaded into a moving optical lattice. In a moving lattice the superfluid state is unstable due to spontaneous emission of the short-wavelength rotonlike excitations which appear in the spectrum of the collective modes. It is shown that the spectrum obtained by the BS equations is in an excellent agreement with the collective-mode dispersion calculated by the perturbation approach, while there are some differences between the results obtained by density response function method and by the BS approach. The difference increases with increasing the lattice velocity, which can be seen in the analytical approximations for the dispersion relation in the long-wavelength limit in a weak-coupling regime.

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I. INTRODUCTION

The experimental ability to create a superfluid alkali atom Fermi gas in optical lattices\textsuperscript{23–25} opens a new opportunity to study strongly correlated quantum many-particle systems and to emulate high-temperature superconductors. Theoretical description of ultracold atomic Bose and Fermi gases in optical lattices has attracted much attention\textsuperscript{23–25} Near the Feshbach resonance the atom-atom interaction can be manipulated in a controllable way because the scattering length $a_{\text{s}}$ can be changed from the BCS side (negative values) to the BEC side (positive values) reaching very large values close to resonance. On BEC side of the resonance the spin-up and spin-down atoms can form diatomic molecules, and these bosonic molecules can undergo a BEC at low enough temperature.\textsuperscript{3–5} In what follow we focus our attention on the BCS side (negative scattering length) where BCS superfluidity is expected analogous to superconductivity. In particular, we consider an equal mixture of atomic Fermi gas of two hyperfine states with contact interaction loaded into an optical lattice. The two hyperfine states are described by pseudospins $\sigma = \uparrow, \downarrow$. We also assume that the number of atoms in each hyperfine state per site (the filling factor) is smaller than unity, and that the lattice potential is sufficiently deep such that the tight-binding approximation is valid. The system in this case is well described by the single-band attractive Hubbard model:

$$
H = -J \sum_{<i,j>,\sigma} \psi_{i,\sigma}^\dagger \psi_{j,\sigma}^\dagger - \mu \sum_{i,\sigma} \hat{n}_{i,\sigma} - U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}. \quad (1)
$$

Here, the Fermi operator $\psi_{i,\sigma}^\dagger (\psi_{i,\sigma})$ creates (destroys) a fermion on the lattice site $i$ with pseudospin $\sigma = \uparrow, \downarrow$ and $\hat{n}_{i,\sigma} = \psi_{i,\sigma}^\dagger \psi_{i,\sigma}$ is the density operator on site $i$ with a position vector $\mathbf{r}_i$. $\mu$ is the chemical potential, and the symbol $\sum_{<i,j>}$ means sum over nearest-neighbor sites. $J$ is the tunneling strength of the atoms between nearest-neighbor sites, and $U$ is the on-site interaction. On the BCS side the interaction parameter $U$ is positive (the atomic interaction is attractive). We assume $\hbar = 1$ and the lattice constant $a = 1$.

In the case when the periodic array of microtraps is generated by counter propagating laser beams with different frequencies, the lattice is not stationary anymore. It is expected that the formation of a BCS superfluidity is possible, but due to the presence of supercurrent (or quasimomentum $\mathbf{q} = m \mathbf{v}$, where $m$ is the mass of the trapped atoms and $\mathbf{v}$ is the lattice velocity) the superflow can break down. Recently, the stability of superfluid Fermi gases loaded into an optical lattice in the presence of supercurrent has been studied using the second-order time-dependent perturbation theory\textsuperscript{22} (this method is a generalization of the earlier approach by Belkhir and Randeria\textsuperscript{23}) and the Green’s function formalism.\textsuperscript{24,25} In what follows, we derive the Bethe-Salpeter (BS) equations for the spectrum of the collective modes in the case of a moving optical lattice, and compare the BS formalism with the above-mentioned two methods.

This paper is organized as follows. In Sec. II, we derive the BS equations for the dispersion of the collective excitations. In Sec. III, the BS approach is compared with the results obtained by the perturbation theory, and by the Green’s function formalism.

II. BETHE-SALPETER EQUATIONS IN THE PRESENCE OF SUPERCURRENT

We consider an imbalance mixture of $^6$Li atomic Fermi gas of two hyperfine states $|F = 1/2, m_f = \pm 1/2 > \Rightarrow |F = 1/2, m_f = \uparrow \downarrow >$ with contact interaction loaded into a moving optical lattice. The total number of $^6$Li atoms is $M$, and they are distributed along $N$ sites. The tight-binding form of the mean-field electron energy is $\xi(\mathbf{k}) = 2J (1 - \sum_{\nu} \cos k_{\nu} d) - \mu$. In the presence of supercurrent with velocity $\mathbf{v}$, the Cooper’s pair has quasimomentum $2m\mathbf{v}$.

The Fourier transform of the single-particle Green’s
function is a $2 \times 2$ matrix $\hat{G} = \begin{pmatrix} G_{\uparrow \uparrow} & G_{\uparrow \downarrow} \\ G_{\downarrow \uparrow} & G_{\downarrow \downarrow} \end{pmatrix}$. In the mean-field approximation the corresponding matrix elements are as follows:

$$G_{q}^{\uparrow \uparrow}(k, \omega_{m}) = \frac{u_{q}^{2}(k)}{\omega_{m} - \omega_{+}(k, q)} + \frac{v_{q}^{2}(k)}{\omega_{m} + \omega_{+}(k, q)}$$

$$G_{q}^{\uparrow \downarrow}(k, \omega_{m}) = \frac{u_{q}^{2}(k)}{\omega_{m} - \omega_{+}(k, q)} + \frac{v_{q}^{2}(k)}{\omega_{m} + \omega_{+}(k, q)}$$

$$G_{q}^{\downarrow \uparrow}(k, \omega_{m}) = G_{q}^{\uparrow \downarrow}(k, \omega_{m}) = \frac{1}{\omega_{m} - \omega_{+}(k, q)} - \frac{1}{\omega_{m} + \omega_{+}(k, q)}.$$

The symbol $\omega_{m}$ denotes $\omega_{m} = (2\pi/\beta)(m + 1/2); m = 0, \pm 1, \pm 2, ...$, $\beta = (k_{B}T)^{-1}$, $k_{B}$ is the Boltzmann constant, $T$ is the temperature. As can be seen, the one-particle excitations in mean-field approximation are coherent combinations of electronlike $\omega_{+}(k, q) = E(k, q) + \eta(k, q)$ and holelike $\omega_{-}(k, q) = E(k, q) - \eta(k, q)$ excitations. The coherent factors $u_{q}(k)$ and $v_{q}(k)$ give the probability amplitudes of these states in the actual mixture. Here, $E(k, q) = \sqrt{\chi^{2}(k, q) + \Delta^{2}}$, $u_{q}(k) = \sqrt{\frac{1}{2} \left[ 1 + \frac{\chi(k, q)}{E(k, q)} \right]}$, $v_{q}(k) = \sqrt{\frac{1}{2} \left[ 1 - \frac{\chi(k, q)}{E(k, q)} \right]}$, and we have used the following notations $\eta(k, q) = \frac{1}{2}[\xi(k + q) - \xi(q - k)]$, $\chi(k, q) = \frac{1}{2}[\xi(q + k) + \xi(q - k)]$.

The number and the gap equations in this case are as follows ($f = M/N$ is the filling factor):

$$1 - f = \frac{1}{N} \sum_{k} \frac{\chi(k, q)}{E(k, q)}, \quad 1 = \frac{U}{N} \sum_{k} \frac{1}{2E(k, q)}.$$

The BS equations for the collective mode $\omega = \omega_{q}(Q)$ and corresponding BS amplitudes

$$\tilde{\Psi}_{q}(k, Q) = \begin{pmatrix} \psi_{q}^{\uparrow \downarrow}(k, Q) \\ \psi_{q}^{\downarrow \uparrow}(k, Q) \\ \psi_{q}^{\uparrow \downarrow}(k, Q) \\ \psi_{q}^{\downarrow \uparrow}(k, Q) \end{pmatrix}$$

in the general random phase approximation (GRPA) are as follows:

$$\tilde{\Psi}_{q}(k, Q) = -\frac{U}{2} \hat{D} \sum_{p} \tilde{\Psi}_{p}(p, Q) + \frac{U}{2} \hat{M} \sum_{p} \tilde{\Psi}_{p}(p, Q),$$

The first and second terms in the last equation represent the direct and exchange interactions:

$$\hat{D} = \begin{pmatrix} K_{q}^{(\downarrow \downarrow \uparrow \uparrow)}(k, Q, \omega_{p}, \omega_{q}) & K_{q}^{(\downarrow \downarrow \uparrow \downarrow)}(k, Q, \omega_{p}, \omega_{q}) & 0 & 0 \\ K_{q}^{(\uparrow \downarrow \uparrow \downarrow)}(k, Q, \omega_{p}, \omega_{q}) & K_{q}^{(\uparrow \downarrow \downarrow \uparrow)}(k, Q, \omega_{p}, \omega_{q}) & 0 & 0 \\ K_{q}^{(\uparrow \uparrow \downarrow \uparrow)}(k, Q, \omega_{p}, \omega_{q}) & K_{q}^{(\uparrow \uparrow \uparrow \downarrow)}(k, Q, \omega_{p}, \omega_{q}) & 0 & 0 \\ K_{q}^{(\downarrow \uparrow \downarrow \uparrow)}(k, Q, \omega_{p}, \omega_{q}) & K_{q}^{(\downarrow \uparrow \uparrow \downarrow)}(k, Q, \omega_{p}, \omega_{q}) & 0 & 0 \end{pmatrix}, \quad \hat{M} = \begin{pmatrix} 0 & 0 & K_{q}^{(\downarrow \downarrow \uparrow \downarrow)}(k, Q, \omega_{p}, \omega_{q}) & K_{q}^{(\uparrow \downarrow \downarrow \uparrow)}(k, Q, \omega_{p}, \omega_{q}) \\ 0 & 0 & K_{q}^{(\uparrow \uparrow \downarrow \uparrow)}(k, Q, \omega_{p}, \omega_{q}) & K_{q}^{(\uparrow \uparrow \uparrow \downarrow)}(k, Q, \omega_{p}, \omega_{q}) \\ 0 & 0 & K_{q}^{(\uparrow \downarrow \uparrow \uparrow)}(k, Q, \omega_{p}, \omega_{q}) & K_{q}^{(\uparrow \downarrow \downarrow \uparrow)}(k, Q, \omega_{p}, \omega_{q}) \\ 0 & 0 & K_{q}^{(\downarrow \uparrow \down \uparrow)}(k, Q, \omega_{p}, \omega_{q}) & K_{q}^{(\downarrow \up \up \down)}(k, Q, \omega_{p}, \omega_{q}) \end{pmatrix}.$$

Here, $\omega_{p} = (2\pi/\beta)p; p = 0, \pm 1, \pm 2, ...$ is a Bose frequency, and we have introduced the two-particle propagator $K_{q}^{(i,j,k,l)}(i, j, k, l = \{\uparrow, \downarrow\})$:

$$K_{q}^{(i,j,k,l)}(k, Q, \omega_{l}) = \sum_{\omega_{m}} G_{q}^{i,j}(k + Q, \omega_{p} + \omega_{m}) G_{q}^{k,l}(k, \omega_{m}).$$

As in the case of a stationary lattice, the BS equations can be reduced to a set of two equations. At zero temperature, the corresponding equations are as follows:

$$[\omega + \Omega_{q}(k, Q) - \varepsilon_{q}(k, Q)] \hat{G}_{q}^{\uparrow}(k, Q) = \frac{U}{2N} \sum_{p} \left[ \gamma_{k, Q}^{q} \gamma_{p}^{q} + \gamma_{k, Q}^{q} \gamma_{p}^{q} \right] \hat{G}_{q}^{\uparrow}(p, Q)$$

$$- \frac{U}{2N} \sum_{p} \left[ \gamma_{k, Q}^{q} \gamma_{p}^{q} - \gamma_{k, Q}^{q} \gamma_{p}^{q} \right] \hat{G}_{q}^{\uparrow}(p, Q) - \frac{U}{2N} \sum_{p} \left[ \gamma_{k, Q}^{q} \gamma_{p}^{q} \right] \left( \hat{G}_{q}^{\uparrow}(p, Q) - \hat{G}_{q}^{\downarrow}(p, Q) \right)$$

$$+ \frac{U}{2N} \sum_{p} \left[ m_{k, Q}^{q} m_{p, Q}^{q} \hat{G}_{q}^{\uparrow}(p, Q) + \hat{G}_{q}^{\downarrow}(p, Q) \right],$$
Here

\[ \varepsilon_q(k, Q) = E(k + Q, q) + E(k, q), \]

\[ \Omega_q(k, Q) = 2J \sum \nu [\sin(k_\nu d) - \sin(Q_{\nu d} + k_\nu d)] \sin(q_\nu d), \]

and the form factors are defined as follows:

\[ \gamma_{k, Q}^q = u_{k, k+Q}^q + v_{k, k+Q}^q \gamma_{i, k}^q = u_{k, k+Q}^q - v_{k, k+Q}^q, \]

\[ \tilde{\gamma}_{k, Q}^q = u_{k, k+Q}^q - u_{k+Q, k}^q \gamma_{i, k}^q = u_{k, k+Q}^q + u_{k+Q, k}^q \gamma_{i, k}^q. \]

The BS equations (3)- (4) lead to a set of four coupled linear homogeneous equations. The existence of a non-trivial solution requires that the secular determinant

\[
\begin{vmatrix}
U^{-1} + J_{\gamma, \gamma}^q & f_{\gamma, \gamma}^q & f_{\gamma, m}^q & f_{m, m}^q \\
i_{\gamma, \gamma}^q & U^{-1} + f_{\gamma, \gamma}^q & f_{\gamma, m}^q & f_{m, m}^q \\
i_{\gamma, m}^q & f_{\gamma, m}^q & U^{-1} + f_{\gamma, m}^q & f_{m, m}^q \\
J_{\gamma, m}^q & f_{\gamma, m}^q & f_{m, m}^q & U^{-1} + J_{m, m}^q
\end{vmatrix}
\]

is equal to zero. Here we have introduced symbols \( f_{a, b} \) and \( J_{a, b} \):

\[
f_{a, b}^q = \frac{1}{N} \sum_k \frac{\varepsilon_q^k \gamma_{k, Q}^q \varepsilon_q^k (k, Q)}{\[\omega + \Omega_q(k, Q)\]^2 - \varepsilon_q^2(k, Q)}, \]

\[
J_{a, b}^q = \frac{1}{N} \sum_k \frac{a_{k, Q} \gamma_{k, Q}^q [\omega + \Omega_q(k, Q)]}{[\omega + \Omega_q(k, Q)]^2 - \varepsilon_q^2(k, Q)}. \]

The quantities \( a_{k, Q}^q \) and \( b_{k, Q}^q \) could be one of the four form factors: \( \gamma_{k, Q}^q, m_{k, Q}^q, \tilde{\gamma}_{k, Q}^q \) or \( \tilde{\gamma}_{k, Q}^q \).

The secular determinant (3) is the main result in our BS approach. This determinant provides the spectrum of the collective excitations (in the particle and spin channels) \( \omega(Q) \) in a uniform manner.

III. COMPARISON WITH OTHER APPROACHES

There exist another two methods that can be used to calculate the spectrum of the collective excitations of Hamiltonian (1). Decades ago, Belkhir and Randeria calculated the collective modes by linearizing the Anderson- Rickayzen equations. In the GRPA the Anderson- Rickayzen equations are reduced to a set of three coupled equations and the collective-mode spectrum is obtained by solving the secular equation \( \text{Det}[\Delta] = 0 \), where

\[
\Delta = \begin{pmatrix}
U^{-1} + I_{\gamma, \gamma} & J_{\gamma, \gamma} & I_{\gamma, m} \\
I_{\gamma, \gamma} & U^{-1} + I_{\gamma, \gamma} & J_{\gamma, m} \\
I_{\gamma, m} & J_{\gamma, m} & U^{-1} + I_{m, m}
\end{pmatrix},
\]

The quantities \( I \) and \( J \) are defined by Eq. (6) when \( q = 0 \). The perturbation approach by Ganesh et al. provides the following 3x3 secular determinant (see Eqs. (B8) in Ref. 22, where there is a negative sign in front of the sum in the definition of \( \chi_0 \)).
Since $\text{Det}|D| = \text{det}|A|$, where

\[
\text{Det}|A| = \begin{vmatrix} U^{-1} + I_{\gamma,m} & J_{\gamma,\gamma}^q & J_{\gamma,m}^q \\ J_{\gamma,\gamma}^q & U^{-1} + I_{\gamma,\gamma}^q & J_{\gamma,m}^q \\ J_{\gamma,m}^q & J_{\gamma,m}^q & U^{-1} + I_{\gamma,m}^q \end{vmatrix},
\]

one can say that the perturbation method is a generalization of the Belkhir and Randeria method to the case of moving optical lattice.

The secular determinant can be rearranged as $|A B^T| = 0$, where $B^T$ means the transpose matrix, and $B$ and $C$ are $1 \times 3$ and $1 \times 1$ blocks, respectively:

\[
B = |I_{\gamma,\gamma} \ J_{I_{\gamma,\gamma}} |, \quad C = -U^{-1} + I_{\gamma,\gamma}.
\]

Thus, the BS secular equation assumes the form $\text{Det}|A - B^T C^{-1} B| = 0$. It should be mentioned that numerical calculations show that the contributions to $A$ due to the term $B^T C^{-1} B$ are negligible, and therefore, there is an excellent agreement between the dispersions obtained by Ganesh et al. in Ref. [22] and by the BS secular equation not only in the case of a stationary lattice (see the conclusion section in Ref. [21]), but in the case of a moving lattice as well.

The method used by Yunomae et al., known as the Green’s function approach, is a generalization of the Côté and Griffin approach to the collective excitations in $s$-wave layered superconductors. According to this approach the spectrum of the collective excitations of an interacting electron gas (the model includes a Coulomb interaction $v(r) = e^2/|r|$ and a short-range attractive interaction $g(r)$ between electrons) is obtained from the poles of the density and spin response functions in the GRPA. The response functions are obtained by using the Kadanoff and Baym formalism. The Kadanoff and Baym formalism is equivalent to the BS formalism. The BS equation for the two-particle Green’s function is $K^{-1} = K^{(0)-1} - I$, where $K^{(0)}$ is the free two-particle propagator, and the kernel $I = I_d + I_{\text{ex}}$ is a sum of two parts. $I_d$ takes into account the direct interaction between the electrons, while $I_{\text{ex}}$ describes their exchange interaction. Côté and Griffin have ignored the long-range Coulomb interaction keeping in $I_d$ only the ladder diagrams involving the short-range interaction. The exchange interaction involves bubble diagrams with respect to both unscreened Coulomb interaction $v(r)$ and short-range interaction attractive $g(r)$. Thus, Eq. (2.33) in Ref. [24] is equivalent to the BS equation $K = K^{(0)} + K^{(0)} I_d K$.

and Eq. (2.32) in Ref. [26] corresponds to the BS equation $K = K + K I_{\text{ex}} K$. In other words, the BS equation $K^{-1} = K^{(0)-1} - I_d - I_{\text{ex}}$ is equivalent to both, Eqs. (2.32) and (2.33) in the paper by Côté and Griffin. The kernel of the BS equations in the case when $v(r) = 0$ and $g(r)$ replaced by a contact interaction does take into account the direct interaction (terms which depend on $\gamma$ and $l$) and exchange interaction (terms which depend on $\bar{\gamma}$ and $m$), and therefore, the Côté and Griffin results in

![FIG. 2: Collective mode energy in a 2D optical lattice at coupling $U = 4.5J$ and filling factor $f = 0.8$ The superfluid flows along $(\pi, \pi)$ direction, and $Q_x = Q_y = Q$. (a) Stationary lattice (the gap and the chemical potential are $\Delta = 1.60J$ and $\mu = 3.32J$). (b) Moving lattice $q = 0.260$ (the gap and the chemical potential are $\Delta = 1.60J$ and $\mu = 3.32J$). Solid lines are obtained by the Bethe-Salpeter method while the dotted lines represent the spectrum obtained by the Green’s function method (see FIG. 11 in Ref. [24]).]
the case of an attractive Hubbard model should be similar to the results obtained by the BS approach. This statement is supported by FIG. 1 and FIG. 2. The two approaches provide very similar results, but the difference between them increases with increasing the lattice velocity. To illustrate this statement, we compare the two analytical results for the speed of sound in a weak-coupling regime in a moving 1D optical lattice. By following the calculations that have already been given by Belkhir and Randeria\textsuperscript{23} we reduce our secular determinant \((\%)\) to the following \(2 \times 2\) determinant:

\[
\begin{vmatrix}
-\frac{Q^2 \alpha (\mu^2 - 4J\mu + \Delta^2)}{4\Delta^2 \cos^2(q)} - \frac{\omega^2 \alpha}{4\Delta^2} & \frac{2J}{2\Delta} - \frac{\omega \alpha}{2\Delta} \\
\frac{2J}{2\Delta} - \frac{\omega \alpha}{2\Delta} & 1 - \frac{\alpha}{\alpha} \\
\end{vmatrix} = 0, \quad (11)
\]

\(A_{11} = -\frac{Q^2 \alpha (\mu^2 - 4J\mu + \Delta^2)}{4\Delta^2 \cos^2(q)} - \frac{\omega^2 \alpha}{4\Delta^2} + \frac{(2J - \mu)Q \omega \tan(q)}{2\Delta^2} - \frac{1}{4}Q^2 \tan^2(q) - \frac{[2J^2 - \Delta + 4J^2 \cos(2q)]Q^2 \alpha \tan^2(q)}{2\Delta^2} \quad (13)
\]

The last determinant provides for the phononlike dispersion in the long-wavelength limit:

\[
\omega(Q) = (2J - \mu) \tan(q) Qa + Qh \sqrt{1 - \frac{\alpha}{\alpha}} \sqrt{V_F^2 + 4J^2 [2 - 3 \cos^2(q)] \tan^2(q) - \Delta^2 \left[1 + \frac{1 - \alpha}{\alpha} \tan^2(q)\right]} \quad (14)
\]

Here, \(V_F = (a/h) \sqrt{4J^2 \cos^2(q) - (2J - \mu)^2}\) is the Fermi velocity in the presence of a supercurrent. The speed of sound obtained by the Green’s function method is\textsuperscript{25}:

\[
\omega(Q) = (2J - \mu) \tan(q) Qa + Qh \sqrt{1 - \frac{\alpha}{\alpha}} \sqrt{V_F^2 - \Delta^2 \left[1 + \frac{1 - \alpha}{\alpha^2} \tan^2(q)\right]} \quad (15)
\]

It should be mentioned that in the case when \(\Delta/J < 1\), the term \(4J^2 [2 - 3 \cos^2(q)] \tan^2(q)\) is more important than the term \(\Delta^2(1 - \alpha) \tan^2(q)/\alpha^2\) in (14) and the term \(\Delta^2(1 - \alpha) \tan^2(q)/\alpha^2\) in (15).

**IV. CONCLUSION**

We have derived the BS equations for the spectrum of the collective modes in the case when an equal mixture of atomic Fermi gas of two hyperfine states with contact interaction is loaded into a moving optical lattice. It is worth mentioning that there are two advantages of the BS formalism over the Green’s function approach. First, within the BS approach we obtain the poles of density and spin response functions in a uniform manner, i.e. one secular equation provides not only the poles of the density response function, but the poles of the spin response function as well. Second, to obtain the poles of the density (spin) response function within the Green’s function method, one has first to take into account the ladder diagrams with respect to both, the short- and long-range interactions.

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