Coupling between time series: A network view

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Abstract – Recently, the visibility graph has been introduced as a novel method for analyzing time series, which maps a time series to a complex network. In this paper we introduce a new algorithm of visibility, “cross-visibility”, which reveals the conjugation of two coupled time series. The correspondence between the two time series is mapped to a network, “the cross-visibility graph”, to demonstrate the correlation between them. We have applied the algorithm to several correlated and uncorrelated time series, generated by the linear stationary ARFIMA process, in order to better understand the results of the cross-visibility of empirical series. The comparison between the degree distribution of coupled and uncoupled (shuffled) series’ networks demonstrates the emergence of super nodes (extremely high-degree nodes) in the uncoupled ones. Furthermore, we have applied the algorithm to real-world data from the financial trades of two companies and oil, and observed significant small-scale coupling in their dynamics.

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Introduction. – Coupled series, and series of long-range correlations could not be thoroughly understood unless a global view of them is presented [1]. Complex networks [2–5], on the other hand, have provided a global understanding of multi-component systems. Recently, the idea of complex networks has been implemented for single variable systems [6,7], and mapping between time series and networks is proposed. For instance, nodes in such networks can be extracted by binning the time axis or the level axis [8].

While some features of time series are not simply observed with standard methods, they can be easily studied when converted to a network [9]. For example, the dynamics of financial trades seem to be similar to a white noise, while they represent memory [10–12] and high clustering when converted to complex networks. Moreover, investigating a map between time series and networks is a method of constructing several prototypes of complex networks [13].

According to a recent work [14], the visibility algorithm has been introduced as a fast computational algorithm of mapping a time series to a network. Based on this algorithm, each point of the time series plays the role of a node in a network. Each two points in the time series would reveal connected nodes in the graph if by some criterion they are visible to each other, and they are disconnected otherwise. This algorithm corresponds a time series to a graph. Several properties of the time series serve in the resulting network’s structure. For example, periodic series result in a regular lattice, fractal series result in a scale-free network and random series are in correspondence with a random graph. Though, the visibility graph is not in any respect more informative than the primary time series, it is capable of highlighting several properties which are not easily captured in the time series.

Visibility is the representation of the question as to how much local information is attained from a global structure. Based on this approach, new global properties are introduced for time series. For example, along with scale-free properties, small-world characteristics [15] are demonstrated for the fractional Brownian motion time series [14]. Moreover, in order to distinguish randomness in time series, a modified horizontal visibility algorithm [16] is proposed. The visibility algorithm is found to be an appropriate approach in different interdisciplinary areas, such as social science [17]. In addition, mathematical structures such as fractal and complex chaotic time series have been studied using the visibility algorithm [14,18].
In this work, we introduce a new visibility algorithm based on the mutual information of two time series, the so-called cross-visibility. According to this method, the visibility criterion is represented for two time series, \( i \), \( j \), a time series looks at its components through the perspective of the other time series. The algorithm provides a framework to understand how complex time series lead each other, and determines the direction of information flow between them. In addition, coupling in different scales is demonstrated. Investigating the cross-visibility between well-known correlated and uncorrelated time series could be a criterion for characterizing the cross-visibility. For this purpose, we have studied the ARFIMA time series.

Previously, it has been demonstrated that the Hurst exponent for the fractional Brownian motion series is distinguished by the structure of the degree distribution in their visibility graph [19]. We have extended this analysis for correlated ARFIMA time series, and characterized the power-law exponent in the resulting cross-visibility networks’ degree distribution, based on their Hurst exponent. Moreover, in order to demonstrate the implementation of the approach to real-world data, we have used the data from recent financial trades for two companies. We have shown the information flow in several scales between pertinent data.

The cross-visibility is the result of several local operations on a time series. Therefore, the local properties serve in the resulting network. Since the horizon of each point in the time series is limited, the resulting network is sparse. Hence, regarding the complexity of the computation it does not make much difference to study short or long time series. There are several relevant approaches like Granger causality [20,21], mutual information and transfer entropy, which investigate the cross-correlation between time series. There are two significant differences between our model and those techniques. Firstly, those techniques are based on the conditional probability which was obtained by averaging over the time series. Therefore, the local features are not captured. While in our proposed method local properties serve at graph edges. Moreover, these approaches are based on the assumption that the time series are Markovian, which is not included in our approach.

**The cross-visibility algorithm.** According to the standard visibility algorithm [14], every component in the time series \( \{y_i\} \), with \( i = 1, \ldots, N \), is mapped to a node of a graph. Node \( i \) and node \( j \) (\( i < j \)) are connected, if they are directly visible to each other, i.e. node \( i \) on the time series can be connected to node \( j \) on the time series without crossing it:

\[
i < \forall k < j; \quad y_k < y_i + \frac{y_j - y_i}{j - i}(k - i). \quad (1)
\]

If node \( i \) is visible to node \( j \), node \( j \) will also be visible to node \( i \), and the corresponding network is called non-directional. Elements in the time series which are located higher above the others have higher degrees, and constitutes the singular nodes of the network.

The cross-visibility algorithm maps two time series, \( \{x_i\} \) and \( \{y_i\} \), into two different networks, the so-called cross-visibility networks. The algorithm consists of two major steps:

a) In order to make the time series comparable, they are normalized to reveal dimensionless variables. If the sequences are stationary, they are normalized to their mean and standard deviations. Therefore, new sequences are generated as \( \{\hat{x}_i\} = \{\frac{x_i - \bar{x}}{\sigma_x}\} \) and \( \{\hat{y}_i\} = \{\frac{y_i - \bar{y}}{\sigma_y}\} \), where \( \bar{x} \) and \( \bar{y} \) are the mean values, \( \sigma_x \) and \( \sigma_y \) are the corresponding standard deviations of the series \( \{x_i\} \) and \( \{y_i\} \), respectively. If the series are non-stationary, they are normalized to their corresponding maximum value.

b) Every component in the time series \( \{\hat{x}_i\} \) is mapped to a node of a graph. Node \( i \) is connected to node \( j \), if:

\[
i < \forall k < j; \quad \hat{y}_k \leq \hat{y}_i + \frac{\hat{x}_j - \hat{x}_i}{j - i}(k - i), \quad (2)
\]

or

\[
i < \forall k < j; \quad \hat{y}_k \geq \hat{y}_i + \frac{\hat{x}_j - \hat{x}_i}{j - i}(k - i). \quad (3)
\]

This rule could be interpreted as node \( i \) looking at the components of \( \{\hat{x}_i\} \) time series, through the obstacles of the shifted time series \( \{\hat{y}_i\} = \{\hat{y}_i - \hat{y}_i + \hat{x}_i\} \). The structure of this network demonstrates how \( \{x_i\} \) leads the time series \( \{y_i\} \). Equation (2) refers to the visibility from the top view, and eq. (3) demonstrates the visibility from the beneath view. This graph is depicted by \( G^{yx} \). However, another network, \( G^{yx'} \), could be generated by changing the role of \( \{x_i\} \) and \( \{y_i\} \).

The cross-visibility can be interpreted as a directional measure of mutual information between the time series.

If the time series \( \{y_i\} \) is permuted randomly to construct a time series \( \{y_p\} \) uncorrelated to \( \{x_i\} \), the structure of the cross-visibility network \( G^{yx'} \) deviates from the original one.

**Characterizing cross-visibility.** In order to interpret the results from the cross-visibility algorithm, it is advantageous to analyze the time series that are confirmed to be coupled or uncoupled. We have utilized the time series generated by the linear stationary ARFIMA [22–24] process. These time series are completely well known, and results from this analysis would be a standard model for investigating any other relevant models.

Through a stochastic recursive rule, the ARFIMA process generates a power-law autocorrelated series \( \{y_k\} \):

\[
y_i = \sum_{j=1}^{\infty} a_j(\rho)y_{i-j} + \eta_i, \quad (4)
\]
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Fig. 1: (Colour on-line) Degree distribution for several cross-visibility networks between ARFIMA series with different Hurst exponents. (Blue) squares demonstrate the result for the coupled series, (red) stars are for the uncoupled series, and (black) triangles are for the permuted series. The coupling between several series with $H \in \{0.6, 0.7, 0.8, 0.9, 1\}$ is investigated. $0.6-0.7$ and $0.7-0.6$ are included in a) and b), $0.6-0.8$ and $0.8-0.6$ are included in c) and d), $0.6-0.9$ and $0.9-0.6$ are included in e) and f), and $0.6-1$ and $1-0.6$ are included in g) and h), respectively. Both axes are sketched in logarithmic scale.

Fig. 2: (Colour on-line) The colour plot demonstrates the exponent of the power-law degree distribution associated with the cross-visibility networks of coupled ARFIMA series with $H \in [0,5,1)$. The result is sketched for the cross-visibility networks $G^{H_1,H_2}$, where $H_1$ and $H_2$ are the corresponding Hurst exponents for the first and the second time series, respectively.

where $a_j(\rho)$ is a weight as a function of an independent real variable $0 < \rho < \frac{1}{2}$ and is defined by $a_j(\rho) = \frac{\Gamma(1-\rho)\Gamma(1+j)}{\Gamma(1-j)\Gamma(1+\rho)}$ and $\eta_i$, the error term, is a member of an i.i.d. sequence, which consists of random normal distribution samples. The Hurst exponent, $H$, is related to $\rho$ as $H = 0.5 + \rho$ [23]. These time series show power-law autocorrelation [25]. Using exactly the same error sequences to generate two ARFIMA series leads to two correlated time series. In contrast, $\eta_i$ independent error terms reveal uncoupled time series [26].

We applied the cross-visibility algorithm to coupled time series, with different independent terms $\rho \in [0,0.5)$, which are in correspondence with Hurst exponents $H \in [0.5,1)$. Figure 1 depicts the degree distribution associated with each network. The cross-visibility network for the uncoupled time series (stars and triangles in fig. 1) have a different structure in contrast to the corresponding network of the coupled series (squares in fig. 1), which is due to the emergence of “super nodes” (nodes with very high degree) [27]. Moreover, the average degree of the distributions is higher for the uncoupled series.

Based on the plots in fig. 1, degree distributions have a power-law–like behavior in tails. To demonstrate the difference between various coupling in ARFIMAs, we depict the power-law exponents of ARFIMAs with Hurst exponents $H \in [0.5,1)$ ($\gamma_{xy}$ for several series which is
The power-law exponent for the degree distribution of cross-visibility graphs associated with coupled ARFIMA series is shown as a function of the difference between the corresponding Hurst exponents. The result is depicted for different Hurst exponents, $H_1$, for the cross-visibility network $G^{H_1,H_2}$. The power-law exponent decays linearly with $H_2 - H_1$.

Fig. 4: (Colour on-line) The closing price of simultaneously recorded daily data for Microsoft and IBM (upper panels) and oil and IBM (lower panels) are analyzed by the cross-visibility algorithm. These data are collected from 1987 until 2012. The degree distribution associated with the original data is depicted by (blue) squares. (Red) dots depict the degree distribution for the cross-visibility network of the permuted time series, $G^{xy_p}$. Panels (a) and (c) refer to IBM-MICROSOFT and IBM-OIL cross-visibility networks, and panels (b) and (d) are for MICROSOFT-IBM and OIL-IBM. The deviation is mostly related with the small scales, and also the peak of each distribution is shifted for uncoupled time series. Both axes are sketched in logarithmic scale.

**Application in empirical data.** For real-world data, it is important to confirm whether the time series are coupled and to what extent they follow each other. In order to determine the directional coupling between the two time series, $\{x_i\}$ and $\{y_i\}$, we compare the associated degree distribution of the cross-visibility network with the corresponding network of the permuted time series.

We have analyzed the cross-visibility between the closing price of two companies, Microsoft (MSFT) and IBM. The cross-visibility method is applied to the normalized log-return of the daily prices of the companies in 1987–2012. For each company, return prices are stationary and are normalized to their average and standard deviations. The degree distribution of the cross-visibility graphs is depicted as a colour plot in fig. 2). In order to determine the exponent of the degree distribution we have used the MLE technique. In fig. 3, $\gamma_{H_1,H_2}$ is plotted as a functional $H_2 - H_1$, which clearly demonstrates the linear dependence of $\gamma_{H_1,H_2}$ on $H_2 - H_1$. This result demonstrates that the Hurst exponents could be distinguished by evaluating the power-law exponents.
It could be useful to compare the results with transfer entropy [28]. Transfer entropy is a measurement which determines the information transferred between two time series by preserving its directionality,

\[ T_{J \rightarrow I} = \sum_{t} p(i_{t+1}, i_{t}^{k}, j_{t}^{l}) \log \left( \frac{p(i_{t+1}|i_{t}^{k}, j_{t}^{l})}{p(i_{t+1}|i_{t}^{k})} \right), \]

where \( T_{J \rightarrow I} \) is the transfer entropy of time series \( J \) to \( I \).

Also, \( l \) and \( k \) are the block sizes of all previous states (i.e., \( i_{t}, \ldots, i_{t-k} \)). First of all, we measured the block sizes \( k \) and \( l \) in time series of return IBM, MSFT, oil, by mutual information in each series [29]. In all these time series the block size is 3. Then we measured the difference of \( T_{J \rightarrow I} \) and \( T_{J, \text{shuffled} \rightarrow I} \) (the second time series is shuffled) which is the effective transfer entropy [29]. The values of the effective transfer entropy between IBM and MSFT, also between oil and IBM are as follows: \( T_{\text{effective}}^{\text{IBM} \rightarrow \text{MSFT}} = 0.0073 \), \( T_{\text{effective}}^{\text{IBM} \rightarrow \text{oil}} = 0.0032 \), \( T_{\text{effective}}^{\text{MSFT} \rightarrow \text{IBM}} = 0.0115 \), \( T_{\text{effective}}^{\text{IBM} \rightarrow \text{MSFT}} = 0.0116 \). The higher value of the effective transfer entropies between IBM and MSFT indicates the higher directional coupling between them (in accordance with the cross-visibility results). A crucial point is that transfer entropy summarizes the coupling into the single value. On the contrary, since each point of the time series is represented as a node in the cross-visibility network, local information is preserved in the network. This means that the network and its corresponding degree distribution are larger than a single value; this depicts a more detailed picture of these couplings, corresponding to the scale of fluctuations. In the degree distribution, larger deviation between original and shuffled series indicates stronger coupling. Hence, in fig. 4, the strongest coupling between IBM and MSFT is in the low-degree region (which corresponds to the low-amplitude fluctuations in the time series). In other words, IBM and MSFT follow each other in low-amplitude fluctuations. But they have an independent behavior with respect to high-amplitude fluctuations.

It is also worthwhile to compare our results with the detrended cross-correlation analysis (DCA) [26,30,31]. Figure 5 shows the results for DCA, and indicates how Microsoft leads the coupling to higher moments (i.e. the coupled generalized Hurst exponent merges with Microsoft ones), and IBM leads the coupling to lower moments (i.e. the coupled generalized Hurst exponent merges with IBM ones). One important difference is that DCA is a symmetric method to analyze the cross-correlation, while the cross-visibility introduces two networks of coupling between the two time series, therefore it is asymmetrical. In addition, the local information will remain in the networks, in contrast with the DCA technique where averaging will eliminate the local information. Indeed, network’s nodes reserve this locality.

**Conclusions.**

- Understanding the coupling between two time series and how information transfers between them is a crucial

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**Fig. 5:** (Colour on-line) General Hurst exponents of IBM, Microsoft and their coupled ones estimated using the multi-fractal detrended cross-correlation analysis.
concept in various areas of complex systems. In this paper we have shown the coupling between two time series as a network map.

- Given two time series, we assume that the information flow between them is directional. In order to determine to what extent each time series follows the other series, the cross-visibility algorithm has been introduced in which the directional coupling of the time series is represented by two networks.

- Our results demonstrate that in average, the mean degree corresponding to the coupled series is less than that of the uncoupled data. There is a shift in the degree distribution to higher degrees in uncoupled time series, which could be a criterion to measure the value of the coupling.

- The results show that increasing the difference between the Hurst exponent of two ARFIMA time series, the corresponding degree distribution decays with a slower slope in higher degrees.

- Cross-visibility network is appropriate for investigating the coupling for different empirical data. This approach shows how the companies in the same category are highly coupled with each other.

- Furthermore, this approach clarifies that the coupling between two time series is not uniform and identical everywhere. There are some situations in which series forget each other. In our results, we find that there is strong coupling in low-degree nodes, but uncoupling appears in higher-degree regions. Studying a single time series when it is coupled to the other ones, does not provide us with a comprehensive information. But we should know when and where this coupling occurs.

- Generally, the coupling networks are not symmetrical, that is, the amount of dependence of one company on the other is not the same as the amount of dependence of the latter on the former. For instance, we have observed how IBM is more coupled to oil price than oil price to IBM.

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