Sustainable Multi-Product Seafood Production Planning Under Uncertainty

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Abstract. A multi-product fish production planning produces simultaneously multi fish products from several classes of raw resources. The goal in sustainable production planning is to meet customer demand over a fixed time horizon divided into planning periods by optimizing the trade-off between economic objectives such as production cost, waste processed cost, and customer satisfaction level. The major decisions are production and inventory levels for each product and the number of workforce in each planning period. In this paper we consider the management of small scale traditional business at North Sumatera Province which performs processing fish into several local seafood products. The inherent uncertainty of data (e.g. demand, fish availability), together with the sequential evolution of data over time leads the sustainable production planning problem to a nonlinear mixed-integer stochastic programming model. We use scenario generation based approach and feasible neighborhood search for solving the model.

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1. Introduction

Generally, the production planning problem aims to fulfill production and sourcing decisions so as to meet customer demand subject to production capacity, workforce availability and inventory restrictions and is inherently an optimization problem. The objective of the problem is to minimize the total cost or to maximize profit.

In production planning problems, mathematical models can be broadly classified into two classes: deterministic models and stochastic models. Deterministic models assume that the data are known and typically model the uncertainty using “best guesses” of uncertain values. Although various human judgments based and quantitative models have been developed to forecast these variables with uncertainty such as demand, these deterministic models typically end up solving “mean-value” or “worst-case” problems. The solution to such “worst-case” or “mean-value” problems are often inadequate–large error bound arise when one solves “mean-value” problems and “worst-case” formulations that can produce very conservative and expensive solutions ([2]). Without considering uncertainty, the deterministic production planning models, though widely studied in the literature, are less acceptable and deployed in practice.

In this paper we consider production planning problem which arises in marine fisheries industry in Indonesia. Marine fisheries play an important role in the economic development of Indonesia. This industry could also provide employment to people who live at coastal areas, to increase the financial gain of local government, and to conserve sustainability. Fisheries industrial sector can be classified into three different parts, i.e., open sea fishing, fish cultivation and processed fish. This paper is focusing on the latter sector.
Generally the processed fish industry in Indonesia can be found at the coastal area. There are a lot of variety of fish processed can be produced, such as smoked fish, salted fish, crunchy bashed of fish, fish bowl, terrain (preserved fish), etc. The management of processed fish industry is still dominated by the small local traditional business, using conventional management strategy. Consequently, they do not have enough information regarding the market demand and price. In this production planning situation, the current information may be certain, but future events are inevitably stochastic.

The proposed model explicitly permits the incorporation of uncertain parameters. Most of the references concerning optimization problems in the presence of the uncertainty appear under the heading of stochastic programming. See ([1], [5], [8]), for a good presentation of the basics. Two-stage stochastic programs with recourse typify a particularly important class of models. In such models, the objective function commonly corresponds to the minimization of expected costs or to the maximization of expected benefits (linear or nonlinear), although it can also refer to the expected value of the absolute or quadratic deviations of certain specific goals or the variance of the second-stage recourse function. Two kinds of decision variables exist. Those determined before the random variables have been revealed are called first-stage or here-and-now decision variables and represent proactive decisions; in this paper, they correspond to the production cost and workforce of the first period. Those determined after the realization of the random variables are called second-stage or recourse decision variables and represent reactive decisions made in recourse or response to the uncertainty factor. See also ([6], [7], [9]) for more information.

A review of some of the existing literature of production planning under uncertainty is provided in [15]. Stochastic programming ([2], [3], [7]) and robust optimization [16] have made several successful applications in production planning. In [4] a multi-stage stochastic programming approach was used for addressing a multi-product production planning model with random demand. Bakir and Byrune [1] developed a stochastic LP model based on the two-stage deterministic equivalent problem to incorporate demand uncertainty in a multi-period multi-product production planning model. Huang [5] proposed multi-stage stochastic programming models for production and capacity planning under uncertainty. Kazemi et al. ([8], [10]) proposed a two-stage stochastic model for addressing multi-product production planning with uncertain yield. [12] proposed a robust optimization model for stochastic aggregate production planning. In [11] a robust optimization model was developed to address a multi-site aggregate production planning problem in an uncertain environment. Wu [22] applied the robust optimization approach to uncertain production loading problems with import quota limits under the global supply chain management environment. [9] proposed two robust optimization models with different recourse cost variability measures to address multi-product production planning with uncertain yield.

In the case of discrete random variables, the resulting two-stage recourse models are usually large and complex, and thus must be solved numerically using suitable algorithmic strategies. Most of these algorithms apply decomposition strategies that break the model down by scenario or stage in an iterative scheme, allowing the resolution of smaller models (smaller in comparison to the deterministic equivalent model in its extensive form which gave rise to the original two-stage model). In this paper, a feasible neighborhood search method is proposed to solve an extended deterministic equivalent model, in which each first-stage
variable is replicated for each scenario, with an imposed equality for the new variable values (non-anticipatively constraints).

This article is organized as follows: section two briefly reviews the two-stage recourse model. Section three presents the problem background. The stochastic programming model of the problem can be found in section four. In section five we present the solution basic approach. Computational results and conclusions, respectively, in section six and seven.

2. Framework of Two-Stage Recourse Model

In the following, the framework of two-stage stochastic integer programming model is described briefly. For detail, the reader is referred to van der Vlerk and Haneveld (1999). The stochastic linear programming model is expressed as follows:

\[
\begin{align*}
\text{min} & \quad c^T x + \sum_{s=1}^{S} p_s (q^T y^s) \\
\text{s.t.} & \quad Ax = b \\
& \quad T^s x + W y^s = h^s \quad s = 1, \ldots, S \\
& \quad x, y^s \geq 0 \quad s = 1, \ldots, S
\end{align*}
\]

Equations (2) represent the first-stage model and (3) represent the second-stage model. \(x\) is the vector of first-stage decision variables which is scenario-independent. The optimal value of \(x\) is not conditional on the realization of the uncertain parameters. \(c\) is the vector of cost coefficient at the first-stage. \(A\) is the first-stage coefficient matrix and \(b\) is the corresponding right-hand-side vector. \(y\) is the vector of second-stage (recourse) decision variables. \(q\) is the vector of cost (recourse) coefficient matrix and \(h\) is the corresponding right-hand-side vector and \(T\) is the matrix that ties the two stages together where \(s \in \Omega\) represents scenarios in future and \(p_s\) is the probability that scenario \(s\) occurs. In the second-stage model, the random constraint defined in (3), \(h^s - T x\), is the goal constraint: violations of this are allowed, but the associated penalty cost, \(q^T y\), will influence the choice of \(x\). \(q^T y\) is the recourse penalty cost or second-stage value function and \(\sum_{s=1}^{S} p_s (q^T y^s)\) denotes the expected value of recourse penalty cost (second-stage value function).

3. Problem Background

Fish and its processed products are the most affordable source of animal protein in the diet of most people. In Indonesia, most of the processed fish industries are found at the coastal area. In these industries fish are processed traditionally. There are eight kinds of fish product to be produced by the community, namely, dried fish, salted fish, BBQ fish, pindang fish, smoked fish, preserved fish, pressed fish, and fish bowl.

Model parameter and decision variables used throughout this paper are defined as follows.

Sets
- \(T\) = number of periods
- \(N\) = set of products
- \(M\) = set of resources
- \(S\) = set of scenarios
Variables

- $X_{jt}$: Quantity of product $j \in N$ in period $t \in T$ (ton)
- $J = 1$ for dried fish
- $J = 2$ for salted fish
- $J = 3$ for BBQ fish
- $J = 4$ for pindang fish
- $J = 5$ for smoked fish
- $J = 6$ for preserved fish
- $J = 7$ for pressed fish
- $J = 8$ for fish bowl
- $u_{it}$: Additional amount of resource $i \in M$ to purchase in $t \in T$ (unit)
- $k_t$: Number of workers required in period $t \in T$ (man-period)
- $k_t^-$: Number of workers laid-off in period $t \in T$ (man-period)
- $k_t^+$: Number of additional workers in period $t \in T$ (man-period)
- $I_{jt}$: Quantity of product $j \in N$ to be stored in period $t \in T$ (units)
- $B_{jt}$: Under-fulfillment of product $j \in N$ in period $t \in T$ (units)

Parameters

- $\alpha, \beta, \gamma, \delta, \mu, \lambda, \eta$ are all costs
- $D_{jt}$: Demand for product $j \in N$ in period $t \in T$ (units)
- $U_{jt}$: Upper bound on $u_{it}$
- $r_{ij}$: Amount of resource $i \in M$ needed to produce one unit of product $j \in N$
- $f_{it}$: Amount of resource $i \in M$ available at time $t \in T$ (units)
- $a_j$: Number of worker needed to produce one unit of product $j \in N$
- $w_{jt}$: Waste of fish product $j \in N$ in period $t \in T$ (units)

4. The Stochastic Programming Model

Minimize

$$
\sum_{j \in N} \sum_{t \in T} \alpha_j x_{jt} + \sum_{i \in M} \sum_{t \in T} \beta_i u_{it} + \sum_{i \in M} \mu_i k_t^- + \sum_{i \in M} \gamma_i k_t^+ + \sum_{i \in M} \delta_i k_t^+ + \sum_{s \in S} \sum_{j \in N} \sum_{t \in T} \eta_{js} w^s_{jt}$$

$$+ \sum_{i \in M} \sum_{t \in T} \rho_{ij} f_{it} + \sum_{s \in S} p_s \sum_{j \in N} \sum_{t \in T} \gamma_j x_{jt} + \sum_{s \in S} p_s \sum_{j \in N} \sum_{t \in T} \lambda_j x_{jt}$$

$$+ \sum_{s \in S} p_s \left( \sum_{j \in N} \sum_{t \in T} \left( \rho_{ij} f_{it} \right)^2 + \sum_{t \in T} \left( \sum_{j \in N} \sum_{s \in S} \left( \gamma_j x_{jt} - \sum_{s \in S} \lambda_j x_{jt} \right)^2 \right) \right)$$

Subject to

$$\sum_{j \in N} r_j x_{jt} \leq f_{it} + u_{it} \quad \forall i \in M, \forall t \in T$$

$$u_{it} \leq U_{it} \quad \forall i \in M, \forall t \in T$$

$$\sum_{j \in N} a_j x_{jt} \leq k_t^- \quad \forall t \in T$$
In this production planning problem we will decide:

- The quantity of each processed fish product to be produced in each period
- The additional resource to be used
- The number of regular additional and laying-off workers in each period
- To minimize the cost of processing fish waste, and
- To minimize the variability

The demand for each period is not known with certainty. Under the random demand in each period we should decide the number of each product to be stored in inventory or to fulfill the under fulfillment for each period.

All of these decisions are formulated in expression (5) of the model as an objective function. Constraint (6) expresses that the amount of resource \( i \in M \) needed to produce product \( j \in N \) at least should have the same amount of resources available at time \( t \in T \) together with the additional resource needed. However, the additional resource needs to have an upper bound (expression (7)). In constraint (8), we have the number of workers needed to produce one unit product \( j \in N \). The amount of fish waste where it should be between 10% - 20% can be found in (9). Constraint (10) expresses that the process of fish waste should be within the capacity \( C^\rho \). Ranges for the variability can be found in constraints (11) and (12). Constraint (13) ensures that the available workers in any period equal the number of worker from the previous period plus any change in the number of worker level during the current period. The change in the number of worker level may be due to either adding extra workers or laying-off redundant workers. Constraint (14) determines either the quantity of product to be stored in inventory or to purchase from outside to fulfill the shortfall in meeting market demand.

Model formulated in expression (5) through to (15) is in deterministic equivalent form, due to the fact that, the random form has been represented by scenario and in the objective function of these random terms have been premultiplied by the corresponding probabilities \( p_s \). The method for transforming a stochastic programming model to its deterministic equivalent model was addressed in Mawengkang et al. (2006) and Irvan and Mawengkang (2007).
5. The Algorithm

Cycle 1.

After solving the relaxed problem, the procedure for searching a suboptimal but integer-feasible solution from an optimal continuous solution can be described as follows.

Let $x = [x] + f$, $0 \leq f \leq 1$
be the (continuous) solution of the relaxed problem, $[x]$ is the integer component of non-integer variable $x$ and $f$ is the fractional component.

Step 1. Get row $i^*$ the smallest integer infeasibility, such that
$$
\delta_{i^*} = \min(f_i, 1 - f_i)
$$

Step 2. Calculate
$$
v_{i^*}^T = c_{i^*}B^{-1}
$$
this is a pricing operation

Step 3. Calculate $\sigma_{ij} = v_{i^*}^T a_j$

With $j$ corresponds to $\min_j \left| \frac{d_j}{\sigma_{ij}} \right|

I. For nonbasic $j$ at lower bound

If $\sigma_{ij} < 0$ and $\delta_{i^*} = f_i$ calculate $\Delta = \frac{(1 - \delta_{i^*})}{-\sigma_{ij}}$

If $\sigma_{ij} > 0$ and $\delta_{i^*} = 1 - f_i$ calculate $\Delta = \frac{(1 - \delta_{i^*})}{\sigma_{ij}}$

If $\sigma_{ij} < 0$ and $\delta_{i^*} = 1 - f_i$ calculate $\Delta = \frac{\delta_{i^*}}{-\sigma_{ij}}$

If $\sigma_{ij} > 0$ and $\delta_{i^*} = f_i$ calculate $\Delta = \frac{\delta_{i^*}}{\sigma_{ij}}$

II. For nonbasic $j$ at upper bound

If $\sigma_{ij} < 0$ and $\delta_{i^*} = 1 - f_i$ calculate $\Delta = \frac{(1 - \delta_{i^*})}{-\sigma_{ij}}$

If $\sigma_{ij} > 0$ and $\delta_{i^*} = f_i$ calculate $\Delta = \frac{(1 - \delta_{i^*})}{\sigma_{ij}}$

If $\sigma_{ij} > 0$ and $\delta_{i^*} = 1 - f_i$ calculate $\Delta = \frac{\delta_{i^*}}{\sigma_{ij}}$

If $\sigma_{ij} < 0$ and $\delta_{i^*} = f_i$ calculate $\Delta = \frac{\delta_{i^*}}{-\sigma_{ij}}$
Otherwise go to next non-integer nonbasic or superbasic \( j \) (if available). Eventually the column \( j^* \) is to be increased form LB or decreased from UB. If none, go to next \( i^* \).

**Step 4.** Calculate

\[ \alpha_{j^*} = B^{-1} \alpha_{j^*} \]

i.e. solve \( B \alpha_{j^*} = \alpha_{j^*} \) for \( \alpha_{j^*} \)

**Step 5.** Ratio test; there would be three possibilities for the basic variables in order to stay feasible due to the releasing of nonbasic \( j^* \) from its bounds.

If \( j^* \) lower bound

Let

\[
A = \min_{i' \neq i, j^* \neq 0} \left\{ \frac{x_{B_{i'}} - l_i}{a_{ij^*}} \right\}
\]

\[
B = \min_{i' \neq i, j^* \neq 0} \left\{ \frac{u_{i'} - x_{B_{i'}}}{a_{ij^*}} \right\}
\]

\[ C = \Delta \]

the maximum movement of \( j^* \) depends on:

\[ \theta^* = \min(A, B, C) \]

If \( j^* \) upper bound

Let

\[
A' = \min_{i' \neq i, j^* < 0} \left\{ \frac{x_{B_{i'}} - l_i}{a_{ij^*}} \right\}
\]

\[
B' = \min_{i' \neq i, j^* > 0} \left\{ \frac{u_{i'} - x_{B_{i'}}}{a_{ij^*}} \right\}
\]

\[ C' = \Delta \]

the maximum movement of \( j^* \) depends on:

\[ \theta^* = \min(A', B', C') \]

**Step 6.** Exchanging basis for the three possibilities

1. If \( A \) or \( A' \)
   - \( x_{B_{i'}} \) becomes nonbasic at lower bound \( l_i \)
   - \( x_{j^*} \) becomes basic (replaces \( x_{B_{i'}} \))
   - \( x_{i^*} \) stays basic (non-integer)

2. If \( B \) or \( B' \)
   - \( x_{B_{i'}} \) becomes nonbasic at upper bound \( u_i \)
   - \( x_{j^*} \) becomes basic (replaces \( x_{B_{i'}} \))
   - \( x_{i^*} \) stays basic (non-integer)

3. If \( C \) or \( C' \)
   - \( x_{j^*} \) becomes basic (replaces \( x_{i^*} \))
• \( x_i \) becomes superbasic at integer-valued
   repeat from step 1.

Cycle2, pass1: adjust integer-infeasible superbasics by fractional steps to reach complete
integer-feasibility.

Cycle2, pass2: adjust integer feasible superbasics.

This phase aims to conduct a highly-localized neighbourhood search.

6. Computational Result

The processed fish industry under investigation is located at the eastern coastal area of North
Sumatra province of Indonesia. The industry run by the community of that area has to make a
production plan for these eight processed fish products to fulfill market demand over each
period of time \( t \), \( t = 1, \ldots, T \). In this case each period equals to three months. Therefore there
will be four periods in a year, i.e. \( T = \{1, 2, 3, 4\} \).

After we had a survey to the location, then we found out that the market situation for the
eight processed fish products could come out within three possible situation i.e., good, fair
and poor, with associated probabilities of 0.30, 0.50 and 0.20 respectively. Nevertheless, the
method addressed in Mawengkang and Suherman (2007) could be used in order to get an
efficient number of scenarios. The data of the problem can be found in Mawengkang [13].

Table 1. The Number of Each Product to be Produced (ton)

| Product | Period |
|---------|--------|
|         | 1      | 2      | 3      | 4      |
| 1       | 250    | 250    | 250    | 300    |
| 2       | 900    | 900    | 900    | 950    |
| 3       | 200    | 200    | 200    | 300    |
| 4       | 200    | 200    | 200    | 300    |
| 5       | 200    | 200    | 200    | 300    |
| 6       | 200    | 200    | 200    | 300    |
| 7       | 200    | 200    | 200    | 300    |
| 8       | 200    | 200    | 200    | 300    |

Table 2. Additional Resources to be used (ton)

| Resources   | Period |
|-------------|--------|
|             | 1      | 2      | 3      | 4      |
| Machine 1   | 12.20  | 12.20  | 12.20  | 16.95  |
| Machine 2   | 9.80   | 9.80   | 9.70   | 13.80  |
| Machine 3   | 8.65   | 8.75   | 8.65   | 12.55  |

Table 3. Workforce Plan

| Policy     | Period |
|------------|--------|
|            | 1      | 2      | 3      | 4      |
| Reg. workforce | 38     | 35     | 35     | 47     |
Table 1 reports the optimal production plan for each product in each period. It can be seen that the number for all product in period 4 increase. Therefore, correspondingly there is an increase in additional resources to be used and the need of workforce in period 4, as shown in Table 2 dan Table 3, respectively.

7. Conclusions

In this paper, we develop a two-stage recourse model for production planning problem of a processed fish industry at coastal area of North Sumatra province of Indonesia with stochastic demand. The model is adequate for solving the planning problem faced by the management of the industry. The model includes the computation of worker which is very useful for the industry in order they will be able to schedule a number of local people, and to conserve sustainability. We also propose an algorithm for solving the mixed integer stochastic programming problem. It is observed that the proposed model would be able to provide a credible and effective methodology for real-world sustainable production planning problems under uncertainty. However, there is still room for further investigation, if scheduling situation is imposed in the model. In this case the proposed algorithm needs to explore particularly in CYCLE 2 of the algorithm.

References

[1] M. A. Bakir, and M. D. Byrune, “Stochastic linear optimization of an MPMP production planning model”, International Journal of Production Economics 1998; 55:87-96
[2] J. R. Birge, and F. V. Louveaux, “Introduction to stochastic programming”. New York: Springer; 1997.
[3] G. B. Dantzig, “Linear programming under uncertainty”, Management Science 1955; 1: 197-206
[4] L. F. Escudero, P. V. Kamesam, A. J. King, and R...J-B. Wets, “Production planning via scenarios”, Annals of Operations Research 1993; 34:311-335
[5] K. Huang, “Multi-stage stochastic programming models for production planning”. Thesis(PhD), School of Industrial and Systems Engineering,Georgia Institute of Technology;2005
[6] Irvan, H. Mawengkang, “Characteristics of deterministic equivalent model for multi-stage integer stochastic programs”, Mathematics Journal, Special Edition Part II, Universiti Teknologi Malaysia; 2008
[7] P. Kall, and S. Wallace, Stochastic programming. Chichester: Wiley; 1994.
[8] M. K. Zanjani, M. Nourelfath, and D. Ait-Kadi, “A stochastic programming approach for production planning in a manufacturing environment with random yield”, CIRRELT; 2007(58), working document, Quebec (Canada)
[9] M. K. Zanjani, D. Ait-Kadi, M. Nourelfath, “Robust production planning in a manufacturing environment with random yield: a case in Sawmill production planning”, CIRRELT; 2008(52), working document, Quebec (Canada).
[10] M. K. Zanjani, M. Nourelfath, and D. Ait-Kadi, “A multi-stage stochastic programming approach for production planning with uncertainty in the quality of raw materials and demand”, CIRRELT;2009(09), working document, Quebec (Canada).
[11] S. C. H. Leung, S. O. S Tsang, Ng W.L, and Y. Wu, “A robust optimization model for multi-site production planning problem in an uncertain environment”, European Journal of Operational Research 2007;181(1):224-238.
[12] S. C. H Leung, and Y. Wu, “A robust optimization model for stochastic aggregate production planning”, Production Planning & Control 2004; 15(5):502-514.

[13] H. Mawengkang (2010). Production Planning of Fish Processed Product under Uncertainty, Anziam J. Vol. 51, pp C715 – C733.

[14] H. Mawengkang, S. Suwilo, O. S. Sitompul, “Revision modeling of two-stage stochastic programming problem”, Journal of Industrial System 2006;7(4):6-10.

[15] H. Mawengkang, Suherman, “A heuristic method of scenario generation in multi-stage decision problem under uncertainty”. Journal of Industrial System 2007; 8(2): 98-105.

[16] J. Mula, R. Poler, J. P. Garcia-Sabater, and F. C. Lario, “Models for production planning under uncertainty: a review”, International Journal of Production Economics 2006; 103:271-285.

[17] J. M. Mulvey, R. Van derbei, and S. Zenios, “Robust optimization of large scale systems”, Operations research 1995; 43 (2): 264-281.

[18] Rico-V. Ramirez, “Two-stage stochastic linear programming”: a tutorial. SIAG/OPT Views-and-News 2002; 13 (1): 8-14.

[19] A. Ruszczynski, A. Shapiro, editors, “Stochastic programming”. Handbooks in operations research and management science, vol. 10. New York; North-Holland; 2003.

[20] S. Sen, J. l. Higle, “Introductory tutorial on stochastic linear programming models”, Interfaces 1999; 29 (2): 33-61.

[21] M. H. Van der Vlerk, and W. K. Haneveld, “Stochastic integer programming: General models and algorithms”, Annals of Operations Research, 85:39-57, 1999.

[22] Wu Yue. “Robust optimization applied to uncertain production loading problems with import quota limits under global supply chain management environment”. International Journal of Production Research 2006; 44(5):849-882.