Bianchi Type VI Cosmological Model with Dynamical Cosmological Parameters $G$ and $\Lambda$ in the presence of Bulk Viscous Stress

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ABSTRACT

In the present study, we have obtained Bianchi type VI anisotropic model of the universe filled with a bulk viscous stress in the presence of variable gravitational and cosmological constants. Here we have assumed the cosmological term in the form $H \propto \Omega$ to discuss the effect of cosmological variables. It is found that the bulk viscosity coefficient is a decreasing function of time. The expression for proper distance, luminosity distance, angular diameter distance, look back time and distance modulus curve have been analyzed and also the distance modulus curve of derived model nearly matches with Supernova Ia (SN Ia) observations.

Keywords: Bianchi Type VI, Bulk Viscous Stress, Gravitational and Cosmological Parameters

I. INTRODUCTION

Cosmology is one of the great intellectual achievements of all time beginning from its origin. It is that branch of astronomy, which deals with the large scale structure of the universe. Using astronomical data, our resulting cosmological model of universe compare with the present day universe. Cosmic fluid is considered as perfect fluid in most treatments of cosmology. However, bulk viscosity is expected to play an important role at certain stages of an expanding universe. Nowadays viscosity mechanism has attracted the attention of many researchers. At early stages of evolution of the universe, when radiation is in the form of photons as well as neutrino decoupled, the matter behaved like a viscous fluid.

In recent years there has been considerable interest in the cosmological models with variable gravitational constant $G$ and the cosmological constant $\Lambda$. Variation of gravitational constant was first suggested by Dirac [1] in an attempt to understand the appearance of certain very large numbers, when atomic and cosmic world are compared. He postulates that the gravitational constant $G$ decreases inversely with cosmic time. Beesham [2] has studied the creation with variable $G$ and pointed out the variation of the form $G \sim t^{-1}$ originally proposed by Dirac [1].

On the other hand, in 1917 Einstein introduced the cosmological constant $\Lambda$ to account for a stable static universe as appeared to him at that time. It is believed that the cosmological constant was very large in the early universe, relaxed to its present small value in the course of expansion of the universe by creating massive or massless particles. Cosmological constant problem arises because of large discrepancy between the observational value of $\Lambda$ and the theoretical value. Many solutions convening to this problem were
proposed by considering dynamical $\Lambda$ [3]-[10]. It is found that the presence of a non-zero cosmological constant is one of the most important reason for the acceleration [11, 12]. Discussion on cosmology with a time varying cosmological constant is presented by several authors. Samdurkar and Sen [13] investigated the effect of bulk viscosity on Bianchi Type V cosmological models with varying $\Lambda$ in general relativity. Dwivedi analyzed that Bulk viscous Bianchi type -V cosmological models with stiff fluid and time dependent cosmological term [14]. Some Bianchi type VI viscous fluid cosmological models were investigated by Patel and Kopper [15]. Dunn and Tupper [16] studied a class of Bianchi type VI cosmological models with electromagnetic field. Arbab [17] investigated the cosmological models with variable cosmological and gravitational constants and bulk viscous fluid. A new class of LRS Bianchi type VI0 universe with free gravitational field and Decaying vacuum energy density were proposed by Pradhan et.al. [18]. A number of authors [19]-[30] have studied the variation of $G$ and $\Lambda$ within the framework of general relativity in different context.

Eckart [31] has made the first attempt to develop the relativistic theory of nonequilibrium thermodynamics to study the effect of bulk viscosity. But Eckart theory has several drawbacks including violation of causality and stability. Readers interested in the general theory of causal thermodynamics are urged to consult the excellent survey report of Maartens [32] and Zimdahl [33] and references cited therein.

With the above motivations, in this paper we have studied Bianchi type VI cosmological model filled variable $G$ and $\Lambda$ in the presence of bulk viscous stress in the framework of general relativity. We have also obtained bulk viscosity coefficient in Eckart, Truncated and FIS causal theory. The expression for proper distance, luminosity distance, angular diameter distance, look back time and distance modulus curve have been analyzed and also the distance modulus curve of derived model nearly matches with Supernova Ia (SN Ia) observations.

**II. METRIC AND FIELD EQUATIONS**

It is well-known that the energy-momentum tensor $T_{ij}$ in the form of bulk viscous stress may be considered as

$$T_{ij} = (\rho + p + \Pi) u_i u_j - (p + \Pi) g_{ij}$$

(1)

Here $\rho$, $p$ and $\Pi$ represents energy density, perfect fluid pressure and bulk viscous stress respectively and $u$ is the four velocity vector.

Here we consider the spatially homogeneous and anisotropic Bianchi type VI space time in the form

$$ds^2 = -dt^2 + A_1^2 dx^2 + A_2^2 dy^2 + A_3^2 dz^2$$

(2)

where $A_1$, $A_2$ and $A_3$ are functions of the cosmic time $t$ and $\alpha$ is a constant.

The Einstein field equations are

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi G (\rho + p + \Pi) + \Lambda g_{ij}$$

(3)

Here $R_{ij}$ is the Ricci tensor, $R$ is the Ricci scalar curvature, $T_{ij}$ is the energy--momentum tensor of matter.

In co-moving coordinate system $u_0= (0, 0, 0, 1)$, the Einstein’s field equation together with (1) for the metric (2) yields

$$\frac{\dot{A}_1}{A_1} + \frac{\dot{A}_2}{A_2} + \frac{\dot{A}_3}{A_3} + \frac{\alpha^2}{A_1^2} = -8\pi G (\rho + p + \Pi) + \Lambda$$

(4)

$$\frac{\dot{A}_1}{A_1} + \frac{\dot{A}_2}{A_2} + \frac{\dot{A}_3}{A_3} - \frac{\alpha^2}{A_2^2} = -8\pi G (\rho + p + \Pi) + \Lambda$$

(5)

$$\frac{\dot{A}_1}{A_1} + \frac{\dot{A}_2}{A_2} + \frac{\dot{A}_3}{A_3} - \frac{\alpha^2}{A_3^2} = -8\pi G (\rho + p + \Pi) + \Lambda$$

(6)

$$\frac{\dot{A}_1}{A_1} + \frac{\dot{A}_2}{A_2} + \frac{\dot{A}_3}{A_3} - \frac{\alpha^2}{A_1^2} = -8\pi G (\rho + p + \Pi) + \Lambda$$

(7)

$$\frac{\dot{A}_1}{A_1} = 0$$

(8)
where a dot denotes ordinary differentiation with respect to t.

The average scale factor \( R \) and volume \( V \) are given by
\[
R^3 = V = A_i A_j A_k
\]
(9)

The energy conservation equation is
\[
\dot{\rho} + 3(\rho + p + \Pi) = 0
\]
(10)

which splits into
\[
\dot{\rho} + 3(\rho + p + \Pi) H = 0
\]
(11)

and
\[
8\pi G \dot{p} + \dot{k} = 0
\]
(12)

The generalized mean Hubble parameter \( H \) is defined by
\[
H = \frac{k^2}{R} - \frac{1}{3}(H_i + H_j + H_k)
\]
(13)

where the directional Hubble parameters \( H_i, H_j \) and \( H_k \) are given by
\[
H_i = \frac{k}{A_i} \quad H_j = \frac{k}{A_j} \quad H_k = \frac{k}{A_k}
\]
(14)

The scalar expansion and shear scalar are given by
\[
\theta = \frac{\dot{k}}{k} + \frac{1}{A} + \frac{1}{A}
\]
(15)

\[
2\sigma^2 = \sum_{i=1}^{n} H_i - \frac{\theta}{3}
\]
(16)

The deceleration parameter \( q \) is defined by
\[
q = \frac{\dot{k}\ddot{k}}{k^2}
\]
(17)

The sign of \( q \) indicates whether the model inflates or not. A positive sign of \( q \) corresponds to the standard decelerating model whereas the negative sign of \( q \) indicates inflation. The recent observations of SNIa (Reiss et.al. [34], Perlmutter et.al.[35]) reveal that the present universe is accelerating and the value of DP lies somewhere in the range -1<q<0.

**III. SOLUTION OF FIELD EQUATIONS**

From Eqn. (8), we get
\[
A_2 = A_3
\]
(18)

Using (18), Eqn. (4)-(7) reduces to

\[
\frac{2\dot{k}}{k} + \frac{\dot{k}}{A_i} + \frac{\dot{k}}{A_j} = -8\pi G (p + \Pi) + \Lambda
\]
(19)

\[
\dot{k} + \frac{\dot{k}}{A_i} + \frac{\dot{k}}{A_j} = -8\pi G (p + \Pi) + \Lambda
\]
(20)

\[
\frac{2\dot{k}}{k} + \frac{\dot{k}}{A_i} + \frac{\dot{k}}{A_j} + \frac{\dot{k}}{A_k} = 8\pi G (p + \Pi) + \Lambda
\]
(21)

Here we assume the cosmological term as a function of Hubble parameter \( H \) in the following form (Arbab [36])
\[
\Lambda = 3\beta H^2
\]
(22)

In order to solve the above Eqns., we use the physical condition that expansion scalar is proportional to shear scalar, which leads to
\[
A_i = A_2^n \quad \text{where} \quad n > 1
\]
(23)

Combining Eqn. (9), (18) and (23), we have
\[
\frac{n}{V^3} = A_2^n
\]
(24)

and
\[
\frac{n}{V^3} = A_2^n
\]
(25)

Equating Eqn. (19) and (20),
\[
\frac{2\dot{k}}{k} + \frac{\dot{k}}{A_i} + \frac{\dot{k}}{A_j} + \frac{2\dot{k}}{A_k} + 2\sigma^2 = 0
\]
(26)

Substituting Eqn. (24) and (25) in (26), we get
\[
\frac{\dot{k}}{V} = \frac{2\sigma^2(n+2)}{(n-1) V^3}
\]
(27)

after the first integral of Eqn. (27), we obtain
\[
\int \frac{dV}{V^{3/2} + K_0} = \frac{\sigma (n+2)}{\sqrt{n-1}}
\]
(28)

where \( K_0 \) is an arbitrary constant.

It is clear that Eqn. (28) is not possible to solve in general, so in order to solve the problem completely we have to choose \( K_0 = 0 \) so that (28) be integrable.

Hence the solution of Eqn. (28) is
\[
V = \left( \frac{na}{\sqrt{n-1}} \right)^{n} (t + m)^{\frac{n+2}{n}}
\]
(29)

Hence we have
The energy density should be positive for the condition \( \beta < \frac{3}{n+2} \). Also it can be easily seen that the energy density is always a decreasing function during evolution of the universe.

By assuming EOS \( p = \gamma \rho \), we write

\[
p = \frac{\gamma (n+2) \{3-\beta (n+2)\}}{24\pi n^2 M_0} \left[ \frac{1}{(t+m)^{\frac{6}{3-\beta(n+2)}}} \right]
\]  

Hence from (19), we have

\[
\Pi = \frac{\beta (n+2)^2 + 3(n-1) - \gamma(n+2)3\beta(n+2)}{24\pi n^2 M_0} \left[ \frac{1}{(t+m)^{\frac{6}{3-\beta(n+2)}}} \right]
\]  

In (Extended irreversible thermodynamics (EIT), the bulk viscous stress \( \Pi \) satisfies a transport equation given by

\[
\Pi + \tau \Pi_4 = -3\xi H - \frac{\varepsilon}{2} \tau \Pi \left[ 3H + \frac{\tau_4 - \xi_4}{\tau} - \frac{T_4}{T} \right]
\]  

where \( \tau \) relaxation time and \( T \) is temperature. The parameter \( \varepsilon \) takes the value 0 or 1. Here \( \varepsilon = 0 \) represents truncated Israel-Stewart theory, \( \varepsilon = 1 \) represents full Israel-Stewart causal theory and \( \tau = 0 \) recovers the non-causal Eckart theory.

The Gibb’s integrability condition [Maarten (1995)] is defined as

\[
T = \exp \left( \int \frac{dp(\rho)}{\rho + p(\rho)} \right)
\]  

Using EOS, (45) reduces to

\[
T = T_0 \rho^{\frac{\tau_4}{\tau}}
\]  

where \( T_0 \) is a constant.

Using (41) in (45), the temperature \( (T) \) is as follows

\[
T = T_0 \left( \frac{(n+2)\{3-\beta(n+2)\}}{24\pi n^2 M_0} \right)^{\frac{\tau}{1+\gamma}} \left[ \frac{1}{(t+m)^{\frac{6\gamma}{(1+\gamma)(3-\beta(n+2))}}} \right]
\]
It is been observed that the density \( \rho \), the cosmological constant \( \Lambda \) and the temperature \( T \) are decreasing function of time which can be seen in Fig. 1, Fig. 2 and Fig. 3 whereas the gravitational constant \( G \) is increasing function of time which is shown in Fig. 4.

**Bulk Viscosity in Eckart’s Theory:**

The evolution Eqn. (44) for bulk viscosity in non-causal Eckart’s theory reduces to

\[
\Pi = -3\xi H
\]

with the help of (33) and (43), we get bulk viscosity coefficient as

\[
\xi = -\frac{\Pi}{3H} = \frac{n\gamma n\beta(n+2) - \beta(n+2)^2 - 3(n-2)}{24\pi n^3 M_0} \frac{1}{(t+m)^{3/2}}
\]

**Bulk Viscosity in Truncated Theory:**

In truncated theory \( \varepsilon = 0 \), the evolution Eqn. (44) for bulk viscosity reduces to

\[
\Pi + \tau \Pi = -3\xi H
\]
The relation between \( \tau \) and coefficient of bulk viscosity \( \xi \) is given by

\[
\tau = \frac{\xi}{\rho}
\]

(51)

Thus the Eqn. (50) reduces to

\[
\Pi + \frac{\xi}{\rho} \Pi_4 = -3\xi H
\]

(52)

Using Eqns. (33), (41) and (43), we obtain

\[
\xi = \frac{(n + 2)(3 - \beta(n + 2))^2 \beta_0}{24\pi n M_0 \left(6n\beta_0 - (n + 2)^2(3 - \beta(n + 2))^2\right)} \left(\frac{1}{(t + m)}\right) (t + m)^{3 - \beta(n + 2)}
\]

(53)

Bulk Viscosity in FIS Causal Theory:

In FIS theory (i.e. \( \epsilon = 1 \)), the evolution Eqn. (44) for bulk viscosity reduces to

\[
\Pi + \tau \Pi_4 = -3\xi H \left(\frac{1}{2}\right) \tau \Pi \left[3H + \frac{\tau_4}{\tau} - \frac{\xi_4}{\xi} - \frac{T_4}{T}\right]
\]

Thus we obtain

\[
\xi = \frac{(n + 2)(3 - \beta(n + 2))^2(1 + \gamma)\beta_0}{24\pi n M_0 \left(3n\beta_0 - (n + 2)(1 + \gamma)(3 - \beta(n + 2))(\beta_0 + n + 2)\right)} \frac{1}{(t + m)} (t + m)^{2 - \beta(n + 2)}
\]

(54)

To investigate the consistency of the model (32), we measure the physical parameters such as proper distance, luminosity distance, angular diameter etc.

Proper Distance

The proper distance \( d(z) \) is defined as the distance between a cosmic source emitting light at any instant \( t = t_1 \) located at \( r = r_1 \) with redshift \( z \) and an observer at \( r = 0 \) and \( t = t_0 \) receiving the light from the source emitted i.e.

\[
d(z) = R_0 r_1
\]

(55)

where \( r_1 = \int_{t_1}^{t_0} \frac{dt}{R(t)} \)

Hence

\[
d(z) = \frac{(n + 2)}{2(n - 1)} H_0 (1 + z)^{\frac{2n-2}{n^2}} - 1
\]

(56)

where \( 1 + z = \frac{R_0}{R} \) is redshift and \( R_0 \) is the present scale factor of the universe.

Luminosity Distance

Luminosity distance is the important concept of theoretical cosmology of a light source. The luminosity distance is a way of expanding the amount of light received from a distant object. It is defined in such a way as generalizes the inverse-square law of the brightness in the static Euclidean space to an expanding curved space.

The luminosity distance of a light source is defined as

\[
d_L = \frac{L}{4\pi l^2}
\]

(57)

where \( L \) is the absolute luminosity and \( l \) is the apparent luminosity of source. Therefore one can write

\[
d_L = (1 + z)d(z)
\]

(58)

Using (56), Eqn. (58) reduces to

\[
H_0 d_L = \frac{(n + 2)}{2(n - 1)} \frac{(1 + z)^{\frac{2n-2}{n^2}} - 1}{(1 + z)^{\frac{3n}{n^2}}}
\]

(59)

Angular Diameter Distance

The angular diameter distance is a measure of how large objects appear to be. As with the luminosity distance, it is defined as the distance that an object of known physical extent appears to be at, under the assumption of the Euclidean geometry.

The angular diameter \( d_A \) of a light source of proper distance is given by

\[
d_A = (1 + z)^{-2} d_L
\]

(60)

Using (59), we get

\[
d_A = H_0^{-1} \frac{(n + 2)}{2(n - 1)} \frac{(1 + z)^{\frac{2n-2}{n^2}} - 1}{(1 + z)^{\frac{3n}{n^2}}}
\]

(61)

Look back Time
The look back time is defined as the elapsed time between the present age of universe $t_0$ and the time $t$ when the light from a cosmic source at a particular redshift $z$ was emitted.

In the context of our model it is given by

$$ t_0 - t = \int_0^R \frac{\kappa}{R} dR $$  \hspace{2cm} (62) 

which on simplification gives

$$ H_0 (t_0 - t) = \frac{n + 2}{3n} \left( 1 - (1 + z)^{\frac{3n}{n+2}} \right) $$  \hspace{2cm} (63) 

**Distance Modulus Curve**

The distance modulus is given by

$$ \mu = 5 \log d_L + 25 $$  \hspace{2cm} (64) 

Using (59), we obtain the expression for distance modulus ($\mu$) in terms of red shift parameter ($z$) as

$$ \mu = 5 \log \left( \frac{(n+2)}{2(n-1)H_0} \left( 1 + \frac{2n-4}{n-1} (1 + z)^{\frac{2n-4}{n-1}} - 1 \right) \frac{1}{(1 + z)^{2 n/2}} \right) + 25 $$  \hspace{2cm} (65) 

The observed value of distance modulus at different redshift parameters are employed to draw the curve corresponding to the calculate value of $\mu(z)$. The plot of observed $\mu(z)$ (dotted line) and calculated $\mu(z)$ (solid line) versus redshift parameter ($z$) showed in Fig. 5.

| Redshift ($z$) | Supernovae Ia ($\mu$) | Our model ($\mu$) |
|---------------|-----------------------|-------------------|
| 0.014         | 33.73                 | 32.72             |
| 0.026         | 35.62                 | 34.07             |
| 0.036         | 36.39                 | 34.79             |
| 0.040         | 36.38                 | 35.02             |
| 0.050         | 37.08                 | 35.51             |
| 0.063         | 37.67                 | 36.01             |
| 0.079         | 37.94                 | 36.51             |
| 0.088         | 38.07                 | 36.75             |
| 0.101         | 38.73                 | 37.06             |
| 0.160         | 39.08                 | 38.08             |
| 0.240         | 40.68                 | 39.00             |
| 0.380         | 42.02                 | 40.05             |

**IV. CONCLUSION**

In this paper, we have investigated exact solution of Einstein’s field equations for a Bianchi type VI cosmological model filled variable $G$ and $\Lambda$ in the presence of bulk viscous stress under the assumption that the cosmological term is a function of Hubble parameter ($H$) and expansion scalar is proportional to shear scalar. In this paper, the form of cosmological term $\Lambda$ is physically reasonable as observations suggest that $\Lambda$ is very small in the present universe. It can also be observed that scalar expansion ($\theta$), Hubble parameter ($H$) and shear scalar ($\sigma$) decreases with
increase of time. The bulk viscosity coefficient \( \xi \) decrease as time increases. From (41) and (42), we can see that energy density and pressure will vanish with the increase of cosmic time. Hence they represent vacuum cosmological models in general relativity for large values of \( t \) also the bulk viscosity coefficient \( \xi \) decrease as time increases. We find \( \frac{\sigma}{\partial} = \text{constant} \), which shows that the anisotropy in the universe is maintained throughout. However it becomes isotropic for \( n = 1 \). We have also taken an account of the consistency of our model with observational parameters such as proper distance, luminosity distance, angular diameter distance, look back time. Also we compared the observe value of distance modulus with the calculated value of derived model (Fig.5 and Table).

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