Applying a Mathematical Model to the Calculations of Layer Moduli: In Relation to the Falling Weight Deflectomer

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Abstract-- Back calculations of layer stiffness are preceded by initial analyses of the deflection bowl in conjunction with the measured or assumed thickness of the layers. This step is important in understanding the strains, stresses, and moduli in the individual layers. An accurate modelling of the stiffness of the subgrade is observed to be important because a failure to achieve accuracy implies that disproportionately large errors are likely to arise during the back-analysis. Notably, the latter is applicable in for the provision of the upper layer moduli (State Highway Administration, 2016). With the existence of packages that aid in bob-linear subgrade analysis, the use of a system such as the ELMOD package uses deflections for calculating “n” and C. the relationship holds:

$$E = C \left( \frac{\sigma_z}{\sigma'} \right)^n$$

where n and C refer to constants

- $\sigma_z$ = vertical stress
- $\sigma'$ = reference stress
- E = modulus of elasticity

1. Introduction

The approach enables accurate and quick modeling while offering an additional advantage in such a way that it allows for the broad identification of the subgrade soil type. On the other hand, “n” refers to the subgrade modulus' measure of non-linearity (Shirazi, Abdallah and Nazarian, 2009). In situations where “n” is zero, the outcome suggests linear elastic material. An example of such materials lies in hard granular components. On the other hand, markedly non-linear and soft cohesive soils reveal “n” values that lie in the range -1 to -0.3. In turn, an iterative process leads to the determination of the moduli of an intermediate layer and an upper stiff layer (if present) (Choubane & McNamara, 2000). Specifically, the iterative process utilizes the deflection bowl’s shape and the total central deflection under FWD’s loading plates.

Primarily, approaches such as the Odemark method do not consider the moduli directly. Rather, they consider layer stiffness. To determine the isotropic layer moduli, the overall stiffness of layers is determined as $h^3 E/(1-\mu^2)$ where,

- $h$ = Assumed layer thickness
- E = Layer modulus
- $\mu$ = Poisson’s ratio

With an assumed layer thickness (h) and the layer modulus (E), backanalyses imply that small errors in the thickness of the layers are likely to cause significant or large errors in modulus (Varma, Kutay and Chatti, 2013). A similar sensitivity or trend is also observed after adopting and implementing other analysis techniques (such as CIRCLY); with these alternative approaches affirmed to utilize numerical integration. Imperative to highlight is that there is a need to consider general orders of the magnitudes of the moduli of layers because a failure could translate into inaccurate results (Kutay, Chatti and Lei, 2011). However, the trend does not necessarily apply in situations involving subgrade moduli because values in the latter are established explicitly, yielding reliable outcomes. The observation is also informed by the fact that in
situations involving subgrade moduli, the attribute of stiffness is used in the place of layer modulus; translating into minimal effects to the design overlay thickness.  

Whereas load application to subgrades seeks to determine resilient modulus through the imposition of elastic and low strain conditions, the CBR test is adopted in imposing plastic and high strain deformation (Wang and Al-Qadi, 2013). However, modulus-CBR correlations have been documented to only be indicative because of variations of a factor of three or two in the modulus (slope of stress-strain curves) for cohesive soils. These outcomes are illustrated in figure 6.  

From these outcomes, it becomes critical to consider the modulus and resultant degrees of non-linearity while evaluating the rehabilitation design options and pavement distress mechanisms (Varna, 2015). Thus, the CBR parameter plays the important role of informing the nature of the design of new pavements.  

During the application of FWD, unbound granular surfacing on pavements poses the complication of base course modulus non-linearity. The most adopted relationship states $E = K_1θK_2$ where:  

$θ = \text{Total principle stresses}$  

$K_1$ and $K_2 = \text{Material parameters}$  

Notably, the total principle stresses are witnessed when the deviatoric stress is at the maximum (Tarefder & Ahmed, 2014). The following figure (7) highlights the correlation between the degree of stress and compaction state and the modulus of unbound granular mate  

From the outcomes, base course materials attract non-linear elastic models. However, Domitrovic and Rukavina (2013)  

2. Significance of FWD Data Outcomes and Storages  

One of the aspects depicting the critical application of FWD data lies on the issue of residual life. According to Shirazi, Abdallah and Nazarian (2009), residual life refers to the number of equivalent standard axles (ESAs) that a given pavement can accommodate prior to its declaration as one that is no longer serviceable. Through FWD data, the terminal roughness condition can be compared with the existing roughness and establish the correlation between the number of load repetitions and the allowable material strain (Choubane & McNamara, 2000). Figure 8 illustrates this correlation.  

Another merit of FWD data lies in the mechanistic design that informs processes and the nature of establishing rehabilitation treatments. Upon completing the process of deflection bowl analysis and establishing the test point’s layer moduli, operators can evaluate the rehabilitation options. One of the techniques that have been observed to gain application in this procedure is that which involves CIRCLY, a forward-analysis program (Kutay, Chatti and Lei, 2011). In turn, suitable overlay thicknesses can be established before confirming the extent of acceptability in the layer strains. In summary, FWD is an important device because of the significance arising from its data processing, analysis, and storage. The significance is felt in terms of understanding the residual life of a surface by calculating the number of ESAs a given pavement can accommodate before being declared as one that is no longer serviceable. Indeed, the application aids in curbing potential adversities such as accidents and other related environmental concerns that are likely to accrue from the continued use of an unworthy pavement.  

3. Conclusion  

In conclusion, technological advancements in civil engineering have seen the construction industry embraces technology in managing, controlling, and assessing the state of the existing infrastructure. One of the applications that have continued to gain adoption is the Falling Weight Deflectometer (FWD), which refers to a testing device responsible for evaluating physical properties of a pavement. Particularly, FWD yields an understanding of vertical deflection responses upon subjecting surfaces to impulse loads. This paper has assessed the processing and analysis of data received from FWD, upon which implications for practice (upon storing the data) have been highlighted. One of the important attributes of FWD outcomes concerns the basic calculations. The role of these calculations lies in the backcalculation of the state of stiffness, with initial analyses of the deflection bowl occurring in conjunction with the measured or
assumed layer thicknesses. Indeed, backcalculation is critical because it aids in understanding the strains, stresses, and moduli in the individual layers. Another FWD aspect concerns layer thickness sensitivity. One of the methods, Odemark, has been found to play an important role in fostering the determination of isotropic layer moduli, leading to the establishment of the overall stiffness of layers. This study has also established that FWD data has its application lying in the estimation of subgrade CBR. By imposing elastic and low strain conditions, resilient modulus is determined. Coupled with the additional application of analyzing unbound granular materials, FWD data processing, analysis, and storage has been found to be of significance due to its capacity to give insight into rehabilitation design options and informing the nature of designing new pavements.

4. References

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Appendices

The Falling Weight Deflectometer (FWD)

FWD is currently the most practical system for accurate measurement of deflection response
Geophones determine the deflection bowl produced by the impulse of the falling weight.

Figure 1: FWD highlighted
Figure 2: A schematic illustration of fwd
Figure 3: A schematic illustration of the backcalculation procedure