Massive torsion modes from
Adler-Bell-Jackiw and scaling anomalies

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Regularization of quantum field theories introduces a mass scale which breaks axial rotational and scaling invariances. We demonstrate from first principles that axial torsion and torsion trace modes have non-transverse vacuum polarization tensors, and become massive as a result. The underlying reasons are similar to those responsible for the Adler-Bell-Jackiw (ABJ) and scaling anomalies. Since these are the only torsion components that can couple minimally to spin 1/2 particles, the anomalous generation of masses for these modes, naturally of the order of the regulator scale, may help to explain why torsion and its associated effects, including CPT violation in chiral gravity, have so far escaped detection. As a simpler manifestation of the reasons underpinning the ABJ anomaly than triangle diagrams, the vacuum polarization demonstration is also pedagogically useful.

hep-th/9905001; PACS numbers: 11.15.-q, 11.40.Ha, 04.62.+v

I. INTRODUCTION AND OVERVIEW

Torsion arises naturally in Riemann-Cartan spacetimes when the vierbein, $e_{\mu A}$, and spin connection, $A_{\mu AB}$, are assumed to be independent. There is no compelling physical reason which forces torsion to vanish identically. However not all components of torsion interact naturally with matter. In minimal coupling schemes only spinors couple to torsion, and even then only the axial and trace modes of torsion couple to spin 1/2 particles. Actually, in a hermitian theory, only the axial torsion mode, $\tilde{A}_{\mu}$, interacts minimally with spin 1/2 matter.

Investigations have nevertheless uncovered that torsion can have quite interesting quantum field theoretic effects. Recent works have revealed that even the well-studied Adler-Bell-Jackiw (ABJ) anomaly [1] receives further contributions from torsional invariants [2–4]. Moreover, because of the axial vector coupling, vacuum polarization diagrams with two external axial torsion vertices are not transverse, with the divergence being controlled by the Nieh-Yan term [5]. This breakdown in transversality occurs in addition to that manifested by the ABJ triangle diagrams that give rise to a term quadratic in the curvatures.

A striking consequence of non-transversality in the polarization tensor is the generation of mass. We may think of the axial torsion mode as an “axial torsion photon” [4], coupled to a current whose conservation has been compromised because of anomaly considerations. The associated “gauge” invariance, the $\gamma^5$ rotational symmetry, is therefore broken, and a mass results. In this paper, we describe explicitly how this phenomenon takes place.

It should be noted that the breakdown in current conservation poses no consistency problems. The reason is that the axial torsion modes are not gauge field modes, and are not responsible for any local symmetries. As shown in Appendix A, the axial torsion field is itself a combination of vierbein and spin connections and transforms covariantly under Lorentz and diffeomorphism transformations. The presence of a mass term therefore does not jeopardize consistency of the quantum field theory. The computations to be described below are carried out using a regularization that preserves local Lorentz and diffeomorphism symmetries, and the internal symmetries of the standard model, so all these invariances and their associated Green functions satisfy the usual Ward identities.

This paper will also examine one characteristic aspect of the standard model and its implications in gravity. The standard model incorporates maximal parity and charge conjugation non-conservation by assigning left- and right-handed fermions to different representations of the internal gauge group. In 4D, it is always possible to re-write a

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1 In Appendix A, notations and definitions are clarified, and a few relevant identities are briefly introduced.
right-handed fermion as a left-handed field with conjugate properties. In this respect, therefore, the standard model can be defined using only fermion fields of one chirality.

In the presence of gravity, fields of one chirality are coupled to one set of spin connections, and those of the opposite chirality are coupled to their conjugates. In Appendix A, we show that the result in using only left-handed fermion fields is a minimal coupling to the axial torsion and torsion trace fields via the combination $J^\mu C_\mu \equiv J^\mu (iB_\mu + A_\mu)$. Here $J_\mu$ is the total singlet current over all left-handed fermion fields, $\hat{A}_\mu = 4eA_\mu$, the axial torsion field, while $B_\mu$ is the trace of the torsion field. The first term in $C_\mu$ is anti-hermitian relative to the second, and hermitizing the action would eliminate it completely. But doing so requires bringing in right-handed conjugate Weyl fields, which in turn are coupled to the self-dual or right-handed spin connection fields. These fields are distinct components of the spin connection, and it has already been shown in Ref. [6] that general relativity field equations can be reproduced without reference to them. They are therefore irrelevant in determining the equations of motion. The system can be defined by an action that is dependent upon only fields of one chirality, thereby extending this characteristic feature of the standard model to cover gravity interactions as well. In this scheme of things, the appearance of $iB_\mu$ is entirely natural.

Such an action preserves holomorphy in the chiral fields. In the regularization scheme adopted in this paper [7], this property is maintained, and both torsional modes must therefore have the same mass. The generation of mass for the $B$ field is not self-evident, since it has a vector coupling to the fermion fields rather than an axial vector coupling. Based upon our experience with QED, we might have argued that the associated quanta remain massless. The difference arises because of the anti-hermitian coupling. The operator appearing in the regularized integrals involves the positive definite form in Euclidean space $i\mathcal{D}(i\mathcal{D})$. Now $(i\mathcal{D})$ differs from $i\mathcal{D}$, because of the anti-hermitian nature of the coupling. For conventional internal gauge fields, the two forms of the covariant derivatives are the same, and lead to hermitian Dirac operators. We detail in Section II how this difference causes the vacuum polarization tensor for $B_\mu$ to be different from the usual result for gauge theories and how $B_\mu$ acquires an anomalously generated mass. The coefficient is equal to a similar piece involving the axial torsion field.

We begin our discussion by showing explicitly below how the axial torsion field can acquire a mass as a result of the considerations outlined above, utilizing two methods of computation. Next, in Section II, we generalize the discussion to cover the completely chiral action in a general curved spacetime manifold with torsion. We show how both the axial and trace torsion components must acquire masses at the same time through the covariant combination $C_\mu = iB_\mu + A_\mu$. We end in Section III with further remarks on the distinction between gauged and ungauged symmetries, Lorentz invariance and Abelian couplings, and also CPT violation in chiral gravity. Appendix A contains a summary of the notations used in the paper. Appendix B contains details on the computation of the polarization tensor for internal gauge fields within the completely chiral context employed in the paper. The results are not new [10,11]; without spontaneous symmetry breaking, the tensors are all purely transverse. We have included the details for a number of reasons. First, this invariant regularization differs from conventional schemes in utilizing an infinite tower of massive Pauli-Villars regulators. This is necessary for Weyl fermions when the gauge coupling representation $T^a$ is complex, as it is in the standard model. Second, the doubling in the regulators typical of this method is done in internal $T^a$ gauge space rather than by including both left- and right-handed chiral fermions. This difference is not really significant for the vacuum polarization of internal gauge fields in flat spacetime, but for gravitational fields doubling in the internal space allows us to avoid the introduction of right-handed fermions and right-handed spin connections and thereby preserve the chirality of the gravitational interaction even at the level of regularization. Thus even in teleparallel spacetimes with flat vierbein, the chirality of the torsion coupling of Weyl fermions to $C_\mu = iB_\mu + A_\mu$ will be ensured at the quantum field theoretic level. Appendix B also supplies the check that computations with doubling in internal gauge space can be carried out consistently. Finally, and more importantly, the explicit intermediate steps allow us to compare and contrast with the results of the polarization of the torsion modes in Section II.

A. Vacuum polarization and the ABJ Current

Consider the action of a bispinor theory in teleparallel spacetimes with flat vierbein $e^{A}_\mu = \delta^A_\mu$ but with nontrivial axial torsion coupling

$$S = -\frac{1}{2} \int d^4x e^{\nabla^\mu (i\partial_\mu + \frac{1}{4e}\hat{A}_\mu \gamma^5)\Psi + H.c.}$$

(1.1)

The coupling is through the axial ABJ current $J^{5\mu} = e^{\nabla^\mu \gamma^5}\Psi$ which has expectation value

$$\langle J^{5\mu}(x) \rangle = -\langle \frac{\delta S}{\delta A_\mu(x)} \rangle \approx -\lim_{x \rightarrow y} \text{Tr} \{ \gamma^\mu \gamma^5 \frac{1}{(i\theta + \hat{A}\gamma^5)}\delta(x - y) \},$$

(1.2)
where we have defined $A_{\mu} \equiv \frac{1}{2} \tilde{A}_{\mu}$ for convenience. The corresponding vacuum polarization tensor, $\Pi^{\mu\nu}$, is defined by the Fourier transform

$$\frac{\delta (J^5_{\mu}(x))}{\delta A_{\nu}(y)} \Big|_{A_{\nu}=0} = \int \frac{d^4k}{(2\pi)^4} \Pi^{\mu\nu}(k) e^{ik(x-y)},$$

(1.3)

from which

$$\Pi^{\mu\nu}(k) \propto \int d^4p \text{Tr}\{\gamma^{\mu}\gamma^5 \frac{1}{\not{p} + k} \gamma^{\nu}\frac{1}{\not{p}}\}.$$  

(1.4)

Had this expression been well defined, it would have been no more than

$$\Pi^{\mu\nu}(k) \propto \int d^4p \text{Tr}\{\gamma^{\mu}\frac{1}{\not{p} + k} \gamma^{\nu}\frac{1}{\not{p}}\}$$

(1.5)

since the two $\gamma^5$'s cancel out in the trace, and we would have obtained the usual vacuum polarization amplitude for which we do not expect a longitudinal component. But it is not, for the integration over fermion loop momentum diverges. We will need a regularization scheme, Pauli-Villars for instance, to tame this divergence before performing any Dirac algebra. Any gauge-invariant scheme however will compromise symmetry generated by the axial current; this is the essence of the ABJ anomaly. For example, in the scheme to be used in this paper, the regulator fields carry any Dirac algebra. Any gauge-invariant scheme however will compromise symmetry generated by the axial current; this is the essence of the ABJ anomaly. For example, in the scheme to be used in this paper, the regulator fields carry masses $\{m_n\}$. Summing over the propagators for all the fields, including the massive regulators, results in

$$\Pi^{\mu\nu}(k) \propto \sum_n C_n \int d^4p \text{Tr}\{\gamma^{\mu}\gamma^5 \frac{1}{(\not{p} + k) + im_n} \gamma^{\nu}\frac{1}{\not{p} + im_n}\},$$

(1.6)

with $C_n = \pm 1$, depending on whether the regulators are anticommuting or commuting. (For the original fermion multiplet, $C_0 = 1$ and $m_0 = 0$. We assume analytic continuation to Euclidean Green functions.) By moving the second $\gamma^5$ to the left to cancel out the first, and bearing in mind that $\gamma^5$ anticommutes with the Dirac matrices, we observe that $m_n$ changes its relative sign with respect to $(\not{p} + k)$ in the denominator. Consequently,

$$\Pi^{\mu\nu}(k) \propto \sum_n C_n \int d^4p \text{Tr}\{\gamma^{\mu}\frac{1}{(\not{p} + k) - im_n} \gamma^{\nu}\frac{1}{\not{p} + im_n}\}.$$  

(1.7)

The integrals over the loop momentum will be well defined for a suitable set $\{C_n, m_n\}$ which satisfies the Pauli-Villars conditions. In Section II an explicit set of $\{C_n, m_n\}$ shall be presented for the Weyl theory. It is also applicable to the bispinor theory here, consistently yielding a polarization magnitude which is twice that of the single Weyl fermion. The important point is that if we had started with a vector (instead of the axial vector) coupling, the result for Eq. (1.7) would have been

$$\Pi^{\mu\nu}(k) \propto \sum_n C_n \int d^4p \text{Tr}\{\gamma^{\mu}\frac{1}{(\not{p} + k) + im_n} \gamma^{\nu}\frac{1}{\not{p} + im_n}\}$$

(1.8)

instead. This integral would have produced a transverse polarization tensor. As it is, by rewriting one of the propagators in Eq. (1.7) as

$$\frac{1}{(\not{p} + k) - im_n} = \frac{1}{(\not{p} + k) + im_n} - \frac{2im_n}{(p + k)^2 + m_n^2},$$

(1.9)

we obtain two terms, the first of which is identical to the integral in Eq. (1.8). However, there is now an additional anomalous term which is given by

$$\sum_n C_n \int d^4p \text{Tr}\{\gamma^{\mu}\frac{2im_n}{(p + k)^2 + m_n^2} \gamma^{\nu}(\not{p} - im_n)\} = 8g^{\mu\nu} \sum_n C_n \int d^4p \frac{m_n^2}{[(p + k)^2 + m_n^2][p^2 + m_n^2]}.$$  

(1.10)

This non-transverse part of the vacuum polarization tensor, above and beyond the usual transverse $(k^\mu k^\nu - \not{p}^\mu \not{p}^\nu)\Pi(k^2)$ term from Eq. (1.8), is clearly generated anomalously through massive regulators which break the $\gamma^5$ symmetry of the classical action. Both the axial vector coupling and the presence of nontrivial $m_n$'s from the regularization are required for the argument to go through. From
\[ \Pi^{\mu\nu} = (k^\mu k^\nu - g^{\mu\nu} k^2) \Pi + g^{\mu\nu} \Pi' \]  

and the divergence of Eq. (1.3), we deduce that
\[ k_\mu \Pi^{\mu\nu} \propto k^\nu \quad \text{and} \quad \langle \partial_\mu J^{5\mu} \rangle_{\text{Reg}} \propto \partial_\mu \hat{A}^\mu \neq 0 \]  

at the level of vacuum polarization diagrams. The anomalous \( g^{\mu\nu} \Pi' \) contribution in the vacuum polarization tensor also implies that besides the usual \( F_{\mu\nu} F^{\mu\nu} \) piece required for the transverse polarization, a mass counter term of the form \( A_\mu A^\mu \) is also needed for the non-zero longitudinal component in the effective action. \( A_\mu \) becomes massive as a result.

In vector QED, curbing divergences by naive momentum truncation also results in a non-transverse photon polarization tensor \[ \frac{1}{2} \delta_{\mu\nu} e^{A_\mu} e^{A_\nu} \]. But this apparent breakdown of gauge invariance is an artifact of symmetry breaking “regularization” which can be removed altogether by proper gauge-invariant regularization schemes. One may therefore suspect that, in the axial-vector coupling of torsion to fermions, the non-transverse polarization exhibited here is likewise the consequence of a “fake anomaly” resulting from “improper regularization” which breaks the symmetry of axial rotations. That this is not the case is guaranteed by the ABJ anomaly. The anomaly assures us that there are no regularization schemes that preserve singlet axial rotations, and at the same time respect all of the local symmetries, such as Lorentz and other gauge symmetries, which are present. The non-transverse polarization for axial torsion can thus be regarded as another manifestation of this phenomenon. As discussed in Appendix A, \( A_\mu \) transforms covariantly under diffeomorphisms, and is Lorentz invariant. The counter term necessary to compensate for a non-transverse polarization tensor, \( A_\mu A^\mu \), is therefore completely consistent with local Lorentz and diffeomorphism invariance. A similar term in QED would have violated local gauge invariance.

Before leaving this topic, it is instructive to exhibit and confirm the same effect using an alternative regularization method. For bispinors, we may write the effective action in curved space as
\[ \Gamma_{\text{eff.}} = -i \text{Tr} \ln[e^{\frac{1}{2} \hat{\Delta} e^{-\frac{1}{2}}}], \]
\[ \hat{\Delta} = \gamma^\mu (i \partial_\mu + \frac{i}{2} \sigma_{\mu AB} \sigma^{AB} + A_\mu \gamma^5). \]  

With heat kernel regularization and the Schwinger-DeWitt expansion \[ \frac{1}{2} \delta_{\mu\nu} e^{A_\mu} e^{A_\nu} \], the ABJ anomaly with Euclidean signature has been demonstrated to take the form \[ \frac{1}{2} \delta_{\mu\nu} e^{A_\mu} e^{A_\nu} \]
\[ \langle \partial_\mu J^{5\mu} \rangle_{\text{Reg}} = 2i \lim_{t \to 0} \lim_{x \to x'} \frac{1}{(4\pi t)^2} \text{Tr} \langle x' | \gamma^5 \exp[-t(\hat{\Delta})^2] | x \rangle \]
\[ = 2i \lim_{t \to 0} \frac{1}{(4\pi t)^2} \text{Tr} \left[ \sum_{n=0}^{\infty} e^{\gamma^5 a_n t^n} \right]. \]  

Furthermore, it is known that the traces of the coefficients \( a_0, a_1 \) and \( a_2 \) contribute to the divergent part of the effective action. For instance, \( a_0 = I \) leads to the renormalization of the cosmological constant. We focus on the relevant coefficient \( a_1 \) which, in our notation, is \( a_1 = \frac{1}{4} R - 2 A_\mu A^\mu - \gamma^5 e^{-1} \partial_\mu (e A^\mu) \). Clearly \( \text{Tr}(e a_1) \) (and thus the effective action) contains both the familiar Einstein-Hilbert term, \( e R \), as well as the \( e A_\mu A^\mu \) axial torsion contribution which we also found to be required by the vacuum polarization computations above. Moreover, it follows from Eq.(1.13) that the ABJ anomaly is
\[ \langle \partial_\mu J^{5\mu} \rangle_{\text{Reg}} = -\frac{2i}{(4\pi t)^2} \partial_\mu \hat{A}^\mu + \frac{2i}{(4\pi t)^2} \text{Tr}(e \gamma^5 a_2). \]  

\( \text{Tr}(e \gamma^5 a_2) \) is the more familiar regulator scale independent part of the ABJ anomaly. But there is also the \( t \)-dependent first term \( t \) has the physical dimension of inverse regulator mass squared). This is precisely the Nieh-Yan contribution to the ABJ anomaly. The linear dependence on \( A_\mu \) shows that the vacuum polarization is indeed the correct Feynman diagram process to consider for this purpose \[ \frac{1}{2} \delta_{\mu\nu} e^{A_\mu} e^{A_\nu} \]. Therefore the non-transversality of the polarization tensor is not only genuine, but is in fact necessary to understand the origin of the Nieh-Yan contribution to the ABJ anomaly in perturbation theory.

**II. WEYL FERMIONS AND VACUUM POLARIZATION TENSOR OF TORSION FIELDS**

The classical Weyl action for a left-handed fermion multiplet in a general curved spacetime is
\[ S_L = - \int d^4 x \bar{\Psi}_L i \overleftrightarrow{D}_L \Psi_L, \] (2.1)

with \( iD = \gamma^\mu (i\partial_\mu + \frac{i}{2} A_{\mu AB} \sigma^{AB} + W_{\mu a} T^a ) \). It is known that if the representation of the internal gauge field \( T^a \) is perturbatively anomaly-free \( \text{Tr}(T^a) = \text{Tr}(T^a \{ T^b, T^c \}) = 0 \), then all fermionic loops in background gauge and gravitational fields of the theory can be regularized in an explicitly gauge, Lorentz and diffeomorphism invariant manner through an infinite tower of Pauli-Villars regulators which are doubled in the internal space (see Ref. [9] for further details.) This generalizes the invariant scheme first introduced by Frolov and Slavnov [14]. Specifically, to form general invariant masses, the internal space is doubled from \( T^a \) to

\[ \mathcal{T}^a = \begin{pmatrix} (T^a)^* & 0 \\ 0 & T^a \end{pmatrix}, \] (2.2)

and the original fermion multiplet is projected as \( \Psi_L = \frac{1}{2} (1 - \sigma^3) \Psi_L \), where

\[ \sigma^3 = \begin{pmatrix} 1_d & 0 \\ 0 & -1_d \end{pmatrix}, \] (2.3)

and \( d \) is the number of Weyl fermions in the \( \Psi_L \) multiplet. Written in full, the regularized action is

\[ S_{L_{\text{reg}}} = -\int d^4 x e \left\{ \bar{\Psi}_L i \overleftrightarrow{D}_L \Psi_L + \frac{1}{2} \sum_n m_n (\bar{\Psi}_{L_r} \sigma^1 C_4 \Psi_{L_r} + \bar{\Psi}_{L_r} \sigma^1 C_4 \Psi_{L_r} + \bar{\Psi}_{L_r} \sigma^1 C_4 \Psi_{L_r}) \right\} \]

\[ -\sum_{s=1,3} \bar{\Psi}_{L_s} \sigma^3 i \overleftrightarrow{D}_L \Psi_{L_s} + \frac{1}{2} \sum_n m_n (\bar{\Psi}_{L_r} \sigma^1 \sigma^3 C_4 \Psi_{L_r} + \bar{\Psi}_{L_r} \sigma^1 \sigma^3 C_4 \Psi_{L_r}). \] (2.4)

The sums are over all even natural numbers for the anticommuting and over all odd natural numbers for the commuting fields. With the exception of the original undoubled and massless \( (m_0 = 0) \) multiplet (written as \( \Psi_{L_0} \equiv \Psi_L \)), all other anticommuting \( \Psi_{L_r} \) and commuting \( \Psi_{L_s} \) multiplets are generalized Pauli-Villars regulator fields, doubled in internal space, and endowed with Majorana masses, which we take for definiteness to satisfy \( m_n = n \Lambda \). \( C_4 \) is the charge conjugation matrix in four dimensions, and in the covariant derivative all fields are now coupled to \( W_{\mu a} T^a \). We emphasize that because all the multiplets are left-handed, there are no couplings to the right-handed spin connection which neither needs nor should be introduced for a truly Weyl theory. The regularization therefore preserves the chirality of the theory with respect to the gravitational interaction. It can be shown [9] that the net effect of the regularization is to replace the \( \frac{1}{2} (1 - \sigma^3) \) projection of the bare currents by \( \frac{1}{2} (f (\overleftrightarrow{D}^\dagger) - \sigma^3) \) where \( f \) is the regulator function,

\[ f(\overleftrightarrow{D}^\dagger / \Lambda^2) = \sum_n C_n \frac{i \overleftrightarrow{D}(i \overleftrightarrow{D})^\dagger}{|i \overleftrightarrow{D}(i \overleftrightarrow{D})^\dagger + m_n^2|} = \sum_{n=-\infty}^{\infty} \frac{(-1)^n i \overleftrightarrow{D}(i \overleftrightarrow{D})^\dagger}{|i \overleftrightarrow{D}(i \overleftrightarrow{D})^\dagger + n^2 \Lambda^2|}. \] (2.5)

To concentrate on the vacuum polarization of the torsion fields, we specialize to \( W_{\mu a} = 0 \), and flat vierbein \( e_{\mu A} = \eta_{\mu A} \) but retain nontrivial torsion couplings. To wit, the Weyl action reduces to

\[ S_L = \int d^4 x \bar{\Psi}_L e^{-\overleftrightarrow{D}_L \gamma^\mu i \partial_\mu \Psi_L + C_\mu \bar{\Psi}_L e \gamma^\mu \Psi_L}, \] (2.6)

with \( C_\mu = i B_\mu + \frac{1}{4 \pi} A_\mu \). Note that the torsion interaction appears exclusively in this specific combination. Discussion in Appendix A shows that the vector \( C_\mu \) is covariant relative to diffeomorphisms, but is otherwise invariant under local Lorentz and internal symmetry transformations. The bare current

\[ \langle J_{L}(x) \rangle_{\text{bare}} = \lim_{x \to y} \text{Tr} \{ \gamma^\mu \frac{1}{2} (1 - \gamma^5) \frac{1}{i \overleftrightarrow{D}} - \frac{1}{2} \sigma^3 \delta(x - y) \} \] (2.7)

is modified by the Pauli-Villars regulators to become

\[ \langle J_{L}(x) \rangle_{\text{Reg}} = \lim_{x \to y} \text{Tr} \{ \gamma^\mu \frac{1}{2} (1 - \gamma^5) \frac{1}{i \overleftrightarrow{D}} - \frac{1}{2} \sigma^3 \delta(x - y) \} \]

\[ = \lim_{x \to y} \text{Tr} \{ \gamma^\mu \frac{1}{2} (1 - \gamma^5) \{ \frac{1}{2} \sum_n \frac{(-1)^n i \overleftrightarrow{D}}{|i \overleftrightarrow{D}^\dagger + m_n^2|} \} - \frac{1}{2 i \overleftrightarrow{D}} \sigma^3 \delta(x - y) \}. \] (2.8)
As demonstrated in Ref. [3], the $\sigma^5$ part vanishes automatically for fermion loops with four or less external vertices, and hence does not contribute to vacuum polarization diagrams.

The full curved space Dirac operator satisfies

$$(i\slashed{D})^\dagger = i\slashed{D} + 2iB.$$  \hspace{1cm} (2.9)

with respect to the Euclidean inner product $\langle X|Y \rangle = \int d^4x e X^\dagger Y$. Therefore

$$i\slashed{D} = i\partial - \frac{1}{4e}\tilde{\alpha}\gamma^5,$$

$$(i\slashed{D})^\dagger = i\partial + \frac{1}{4e}\tilde{\alpha}\gamma^5,$$  \hspace{1cm} (2.10)

and the positive-definite operator which appears in the regulator function $f$ is

$$i\slashed{D}(i\slashed{D})^\dagger = -\Box + i\partial (i\slashed{D} + \frac{1}{4e}\tilde{\alpha}\gamma^5) - (i\slashed{D} + \frac{1}{4e}\tilde{\alpha}\gamma^5) i\partial - (i\slashed{D} + \frac{1}{4e}\tilde{\alpha}\gamma^5)(i\slashed{D} + \frac{1}{4e}\tilde{\alpha}\gamma^5)$$  \hspace{1cm} (2.11)

where $\Box = \partial_\mu \partial^\mu$. In computing the vacuum polarization $\Pi^{\mu\nu}$ defined as

$$\int \frac{d^4k}{(2\pi)^4} \Pi^{\mu\nu} e^{ik(x-y)} \equiv \frac{\delta\langle J^\mu_L(x) \rangle}{\delta C_\nu(y)} |_{C_\nu = 0},$$  \hspace{1cm} (2.12)

we need retain only terms linear in $\tilde{\alpha}_\mu$ and $B_\mu$ in the regularized current. Displaying only the relevant terms,

$$\langle J^\mu_L(x) \rangle_{\text{Reg}} = \lim_{x \rightarrow y} \text{Tr} \left[ \frac{1}{2} \gamma^\mu \frac{1}{2} (1 - \gamma^5) \sum_n (-1)^n \left[ ... + \frac{m_n}{(-\Box + m_n^2)} (i\slashed{D} + \frac{1}{4e}\tilde{\alpha}\gamma^5) \frac{m_n}{(-\Box + m_n^2)} \right] \right.$$

$$\left. + \frac{i\partial}{(-\Box + m_n^2)} (i\slashed{D} + \frac{1}{4e}\tilde{\alpha}\gamma^5) \frac{i\partial}{(-\Box + m_n^2)} + ... \right] \delta(x-y).$$  \hspace{1cm} (2.13)

After moving the $\gamma^5$ associated with $\tilde{\alpha}_\mu$ to the left, this works out to be

$$\langle J^\mu_L(x) \rangle_{\text{Reg}} = \lim_{x \rightarrow y} \text{Tr} \left[ \frac{1}{2} \gamma^\mu \frac{1}{2} (1 - \gamma^5) \sum_n (-1)^n \left[ ... + \frac{m_n}{(-\Box + m_n^2)} \right] \right.$$

$$\left. + \frac{i\partial}{(-\Box + m_n^2)} \frac{i\partial}{(-\Box + m_n^2)} + ... \right] \delta(x-y);$$  \hspace{1cm} (2.14)

Again $C_\mu$ is the only torsion combination that appears in the final result. In the above expansion of the current it is crucial to note the two displayed terms within square brackets has the same sign. This is to be contrasted with the result for internal gauge current in Eq. (B4) of Appendix B in which a sign difference occurs. Thus this origin of non-transverse torsion polarization for Weyl theory is essentially the same as that for the bispinor theory of Section IA. By comparing the intermediate steps of Eqs.(2.8)-(2.14) with those in Appendix B, these differences in the signs with Eq. (B4) can be traced precisely to the $\gamma^5$ which comes with $\tilde{\alpha}_\mu$ and the non-hermitian $iB_\mu$ interaction in $i\slashed{D}$.

The relevant Feynman diagram processes for vacuum polarization can also be read off from the above expression of the current. The first term arises from nontrivial $\overline{\Psi}_L \overline{\Psi}_L$ and $\Psi_L \Psi_L$ propagators due to regulator Majorana masses, while the second term is associated with $\overline{\Psi}_L \overline{\Psi}_L$ propagators. Proceeding as in Appendix B, we arrive at

$$\Pi^{\mu\nu} = d \sum_n (-1)^n \int \frac{d^4p}{(2\pi)^4} (-I^{\mu\nu}_1 + I^{\mu\nu}_2),$$  \hspace{1cm} (2.15)

where $I_{1,2}$ are as defined in Eqs.(B9) and (B10). Again the crucial difference with Eq. (B11) for internal gauge fields is the negative sign multiplying $I^{\mu\nu}_1$. We decompose the expression

$$\Pi^{\mu\nu} = (A^{\mu\nu}_L + A^{\mu\nu}_R),$$  \hspace{1cm} (2.16)

2\footnote{This can be shown with the identity $\partial_\mu \gamma^\mu + (\partial_\mu \ln e) \gamma^\mu + \frac{1}{4} A_\mu A^\mu |\sigma^{AB}, \gamma^\mu| = 2B$, which follows from $\gamma^\mu = E^\mu_A \gamma^A$ and themetricity $(\nabla_\mu E^{\nu A} = \nabla_\nu E^{\mu A}) = 0$ of the GL(4,R) connection $\Gamma$ introduced in $\nabla = d + \Gamma + A$.}

3\footnote{In continuing from Lorentzian $(-,+,+,+)$ to Euclidean signature $(+,+,+,+)$ our Dirac matrices satisfy $\gamma^5 = \gamma^\mu$.}
into transverse and longitudinal contributions $A_T^{\mu\nu}$ and $A_L^{\mu\nu}$. The transverse piece

$$A_T^{\mu\nu} = \frac{1}{24\pi^2}(k^\mu k^\nu - g^{\mu\nu}k^2)[\ln\left(\frac{k^2}{\Lambda^2}\right) - \frac{5}{3} + 2\ln\left(\frac{\pi}{2}\right)]$$  \hspace{1cm} (2.17)

is identical to the result obtained in Eq. (B19); while the nontrivial anomalous longitudinal component is given by

$$A_L^{\mu\nu} = -2\sum_{n=-\infty}^{\infty} (1)^n\int \frac{d^4p}{(2\pi)^4} I^{\mu\nu n}$$

$$= 4g^{\mu\nu}\int_0^1 dz \int \frac{d^4p}{(2\pi)^4} \sum_{n=-\infty}^{\infty} (-1)^n \frac{n^2\Lambda^2}{[p^2 + k^2z(1-z) + n^2\Lambda^2]^2}$$

$$= g^{\mu\nu}\Lambda^2 \int_0^1 dz \int_0^\infty (dp^2)p^2 \frac{d}{dp^2}[p^2 + s(f(p^2 + s))]$$

$$= g^{\mu\nu}\Lambda^2 \int_0^1 dz \int_0^\infty \frac{\pi p^2}{\sinh(\pi p)} dp. \hspace{1cm} (2.18)$$

$f$ and the properties associated with it are as in Eqs. (B12)-(B14), and $s = \frac{k^2z(1-z)}{\Lambda^2}$. Although the result for the longitudinal component can be written as integrals of polylogarithmic functions, for finite $\Lambda^2 \gg k^2$, it is more enlightening to express it as a convergent power series in terms of Bernoulli numbers and gamma functions as

$$A_L^{\mu\nu} = -g^{\mu\nu}\left[\frac{7\zeta(3)}{4\pi^4}\Lambda^2 + k^2\sum_{n=0}^{\infty} \frac{(2n-1)!(2n-2)\sqrt{\pi}}{2^{2n-4}(2n)!\Gamma(n+\frac{3}{2})}\left(\frac{k^2}{\Lambda^2}\right)^n\right]. \hspace{1cm} (2.19)$$

The longitudinal part of the polarization therefore diverges as the square of the regulator mass scale.

In the effective propagator with vacuum polarization insertions, it is known that a non-trivial longitudinal polarization causes a shift to a physically massive pole, even if the bare propagator is massless in the beginning (see, for instance, Ref. [2]). Since $\Pi^{\mu\nu}$ is the Fourier transform of $\left[\frac{\delta^4\Gamma}{\delta C_L^{\mu}\delta C_P^{\nu}}\right]_{C=0}$, this implies that in addition to the more familiar curvature squared counter term $g^{\mu\nu}g^{\rho\sigma}(\partial_\mu C_\nu - \partial_\nu C_\mu)(\partial_\rho C_\sigma - \partial_\sigma C_\rho)$ required by the logarithmic divergence of the transverse part $A_T^{\mu\nu}$ of $\Pi^{\mu\nu}$, a counter term proportional to $g^{\mu\nu}C_\mu C_\nu$ for the longitudinal component $A_L^{\mu\nu}$ of $\Pi^{\mu\nu}$ is also needed in the Lagrangian. The presence of these terms in the effective action implies that as a result $C_\mu$ becomes massive and obeys the Proca equation. We discuss next what implications this mass will have on the local invariances of the action.

### III. FURTHER REMARKS

The Weyl action of Eq. (2.6) would be gauge invariant under local $\gamma^5$ and scaling transformations

$$\Psi_L \rightarrow \exp(i\alpha(x)\gamma^5 - \frac{3}{2}\beta(x))\Psi_L = T\Psi_L, \quad e\overline{\Psi}_L\gamma^\mu \rightarrow e\overline{\Psi}_L\gamma^\mu T^{-1},$$

with $T(x) = \exp[-(i\alpha + \frac{3}{2}\beta)]$ if we pretend that $C_\mu = (iB_\mu + A_\mu)$ is a complex Abelian gauge connection which transforms as

$$C_\mu \rightarrow TC_\mu T^{-1} - T\frac{\partial}{\partial x}\gamma^\mu T^{-1} \hspace{1cm} (3.1)$$

i.e.

$$A_\mu \rightarrow A_\mu + \partial_\mu \alpha, \quad B_\mu \rightarrow B_\mu - \frac{3}{2}\partial_\mu \beta. \hspace{1cm} (3.2)$$

Note that $B_\mu$ comes with an $i$ in the complex combination $C_\mu$ because, unlike $\gamma^5$ rotations, the group parametrized by $\exp(-\frac{3}{2}\beta)$ is noncompact rather than $U(1)$. However the massive regulators of Eq. (2.4) break both of these symmetries, and the current $J_\mu^L$ coupled to $C_\mu$ is not conserved. As shown, $k_\mu \Pi^{\mu\nu} \propto k^\nu$ at the vacuum polarization level.
full Weyl theory exhibits no inconsistencies because these invariances are really not gauged as local symmetries. The theory is on the other hand diffeomorphism, and local internal gauge and Lorentz invariant, with internal symmetries gauged by $W_{\mu}$ and local Lorentz invariance by the full spin connection $A_{\mu AB}$. In fact, $C_\mu$ transforms covariantly under general coordinate transformations, and is invariant under Lorentz and gauge transformations. Under global $\gamma^5$ transformations, $\epsilon_{\mu A}$ and $A_{\mu AB}$ are inert; while $\epsilon_{\mu A} \rightarrow \exp(\beta)\epsilon_{\mu A}$ but $A_{\mu AB}$ remains unchanged under global scaling. Hence $C_\mu$ is invariant under both. These global transformations are however symmetries of the bare Weyl action. As a result $\partial_\mu J_\mu^\mu = 0$ and the vanishing of the trace of the energy momentum tensor, $\epsilon T_\mu^\mu = 0$, hold at the classical level, but these equations are nevertheless anomalous at the quantum level. For the Weyl theory, there is actually an interesting relation

$$\langle \epsilon T_\mu^\mu \rangle_{\text{Reg}} = \lim_{x \rightarrow y} \{ \frac{1}{2}(1 - \gamma^5)\frac{1}{2}(f - \sigma^3) \delta(x - y) \} + 2i \langle \partial_\mu J_\mu^\mu \rangle_{\text{Reg}}$$

(3.4)

connecting the ABJ and conformal anomalies.

The upshot is that the coupling of $C_\mu$ to an anomalous current poses no consistency problems in the quantum theory. In particular, the presence of the mass term $C_\mu C^\mu$ is compatible with all local invariances in the action. Its Green functions can and does satisfy modified Ward identities with additional terms that imply non-transversality. With regard to the Abelian part of gauged internal symmetries, we still require $\text{Tr}(\bar{T}_\nu)$ to vanish for the regularization to work in curved space.

The appearance of a mass term for $C_\mu$ does have one important consequence. The combination $C_\mu C^\mu$ is complex, and the counterterms of the Lorentzian signature Lagrangian required by the vacuum polarization diagrams are of the form

$$eg^{\mu \nu}g^{\nu \beta}(\partial_\mu C_\nu - \partial_\nu C_\mu)(\partial_\alpha C_\beta - \partial_\beta C_\alpha) = eg^{\mu \alpha}g^{\nu \beta}[-(\partial_\mu B_\nu - \partial_\nu B_\mu)(\partial_\alpha B_\beta - \partial_\beta B_\alpha)]$$

$$+ 2i(\partial_\mu B_\nu - \partial_\nu B_\mu)(\partial_\alpha A_\beta - \partial_\beta A_\alpha)$$

$$+ (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial_\alpha A_\beta - \partial_\beta A_\alpha)],$$

(3.5)

and

$$eg^{\mu \nu}C_\mu C_\nu = eg^{\mu \nu}(-B_\mu B_\nu + 2iB_\mu A_\nu + A_\mu A_\nu).$$

(3.6)

Therefore the anti-hermitian cross terms are Lorentz invariant, but CPT-odd. In a related work, it was pointed out that a truly Weyl theory which preserves the chirality of the gravitational interaction should violate CPT through $B_\mu$ effects because of the non-hermitian coupling; and that these signatures of chiral gravity will already be manifest at the level of quantum field theory in curved spaces in Weyl fermion loops processes with external chiral gravitational fields. An example is precisely the vacuum polarization diagram with two complex left-handed spin connection vertices. The flat vierbein limit with nontrivial $C_\mu$ evaluated here indeed confirms the presence of these CPT-violating terms in the effective action.

In general, we may decompose the torsion as $T_{\mu \nu} = \frac{2}{3}(B_\mu e_\nu A - B_\nu e_\mu A) + \frac{1}{3}\epsilon_{\mu \nu \alpha}A^\alpha + Q_{\mu \nu A}$, where $Q_{\mu \nu A}$ is constrained by $E^\nu A Q_{\mu \nu A} = \epsilon^{\nu \alpha}A^\alpha Q_{\mu \nu A} = 0$. Since $Q_{\mu \nu A}$ does not couple to fermions, the spin connection in the fermion coupling can be restricted to

$$A_{\mu AB} = \omega_{\mu AB} + \frac{2}{3}(B_\nu E_A^\nu e_{\mu B} - B_\nu E_B^\nu e_{\mu A}) - \frac{1}{6}\epsilon_{AB\mu \alpha}A^\alpha$$

(3.7)

As a result, the Samuel-Jacobson-Smolin action (which is the (anti)-self-dual part of the Einstein-Hilbert-Palatini action), is equivalent to $\frac{1}{192\pi G} \int d^4x(eR - 4i\partial_\mu(eC^\mu)) + \frac{3}{8}C_\mu C^\mu$. Modulo the total divergence term which is not reproduced by perturbation theory, the $C_\mu C^\mu$ counter term required by vacuum polarization computations is of the same form as the Samuel-Jacobson-Smolin action in the teleparallel limit.

Phenomenological consequences of massive axial torsion modes have been discussed before. In our present context, we wish to pursue the extent to which chirality can be used as a defining characteristic of particle interactions, including gravity. That context required us to use Weyl spinors, and also the Ashtekar formulation of gravity. A net consequence is that both axial torsion as well as vector torsion trace are needed, but with a relative phase which ruins CPT invariance because of the chiral nature of the fields. Massive modes appear as consequences of the anomalous non-conservation of the current to which these torsion modes are coupled. Our results to some degree extend those of Ref. to cover the case of torsion trace as well. At low energies compared to the torsion mass, the fermion-torsion interaction produces a four-fermion coupling. Present high energy experimental data on four-fermion vertices sets the lower bound for torsion mass at above roughly 200GeV.
The question of mass generation via anomalies has had a storied past. In the Schwinger model in 2D, the physical 

degree of freedom of the photon is equivalent to a free massive boson \[16,17\] since the interaction term can be 

transformed away by an axial rotation. The mass of the boson stems from the ABJ anomaly, which gives rise to an 

infra-red pole in the polarization tensor. The value of the mass is uniquely determined when the vacuum polarization 

tensor is regularized to satisfy vector gauge invariance at the expense of axial vector current conservation. On the 

other hand, the chiral Schwinger model in 2D is anomalous albeit exactly solvable. The resultant photon mass, while 

still finite, carries an ambiguity as the previous condition of gauge invariance is now absent. It can be made to 

vanish, while preserving the (V-A) form for the coupling, but we then lose unitarity \[13\]. This paper discusses the 

corresponding chiral situation in 4D, but without loss of any local gauge invariance. By retaining explicitly all local 
gauge symmetries and the holomorphic dependence on the left-handed spin connection in the regularization, we end up 

with a vacuum polarization tensor that is non-transverse, and gives a mass to \( C_\mu = i B_\mu + A_\mu \). These torsion modes 

are massive because of ABJ and scaling anomalies, with generated masses naturally of the order of the regulator scale. 

Since these are the only modes that can couple to spin 1 fermions, large regulator masses, or high cut-off scales in the 

context of effective field theories, naturally explain why torsion and its associated effects, including CPT violations 

from \( B_\mu \) couplings, have so far escaped detection.

ACKNOWLEDGMENTS

The research for this work has been supported in part by funds from the U.S. Department of Energy under Grant 

No. DE-FG05-92ER40709, the National Center for Theoretical Sciences, Taiwan, and the National Science Council 

of Taiwan under Grant No. NSC 89-2112-M-006-050.

APPENDIX A: NOTATIONS AND SOME RELEVANT IDENTITIES

Lorentz indices are denoted by uppercase Latin letters while Greek symbols are spacetime indices. The flat 

Lorentzian metric is \( \eta_{\alpha\beta} = \text{diag}(-1,1,1,1) \), and \( g_{\mu\nu} = \eta_{\alpha\beta} e^A_{\mu} e^B_\nu \).

Let us recall from the definition of torsion

\[
T_A = \frac{1}{2} T_{A\mu
\nu} dx^\mu \wedge dx^\nu = de_A + A_{AB} \wedge e^B, \tag{A1}
\]

that the general solution for invertible vierbein is

\[
A_{\mu AB} = \omega_{\mu AB} - \frac{1}{2} \left[ T_{A\sigma \mu} e^\sigma_B - T_{B\sigma \mu} e^\sigma_A - T_{C\rho \sigma} e^\rho_C e^\sigma_B e^\sigma_A \right], \tag{A2}
\]

with \( E^{\mu A} \) being the inverse of the vierbein \( e_{\mu A} \); and \( \omega_{\mu AB} \) is the torsionless spin connection \((de_A + \omega_{\mu AB} \wedge e^B = 0)\) which can be solved as

\[
\omega_{\mu AB} = \frac{1}{2} \left[ E^A_B \left( \partial_\mu e^\nu_B - \partial_\nu e^\nu_B \right) - E^A_B \left( \partial_\mu e^\rho_A - \partial_\rho e^\rho_A \right) - E^A_B \left( \partial_\rho e^\sigma_C - \partial_\sigma e^\sigma_C \right) e^\rho_B \right]. \tag{A3}
\]

Spin \( \frac{1}{2} \) fermions couple minimally to torsion through the spin connection \( \frac{i}{2} A_{\mu AB} \sigma^{AB} \), \((\sigma^{AB} = \frac{1}{2} [\gamma^A, \gamma^B])\), in the Dirac operator \( i D = \gamma^\mu (i \partial_\mu + \frac{i}{2} A_{\mu AB} \sigma^{AB} + W_{\mu a} T^a) \). Here \( W_{\mu a} \) denotes the generic internal gauge connection in the \( T^a \) representation. By substituting for \( A_{\mu AB} \) the interaction reduces to

\[
e^{i} A_{\mu AB} \sigma^{\nu A} \Psi = \frac{1}{2} (-i \omega_{\mu AB} J^\nu - \frac{1}{2} \epsilon_{AB} e^\mu_{\nu B} \omega_{CD} J^\nu e^A_{\nu} - (i B_{\mu} J^\nu - \frac{1}{4} \tilde{A}_\mu J^\nu) e^{A\nu} \Psi + \frac{1}{4} \tilde{A}_\mu J^\nu \tag{A4}
\]

where \( B_{\mu} = \frac{1}{2} T_{\mu
\nu} E^{\nu A} \) and \( \tilde{A}_\mu = \frac{1}{g_{\mu A}} \varepsilon^{\alpha\beta A} e_{\nu A} T_{\alpha\beta} \) are precisely the trace and axial parts of the torsion, while \( J^\mu = \overline{\Psi} e_{\gamma^\mu} \Psi \) and \( J^\mu \) are the respective vector and axial-vector currents. Note that only the axial torsion and torsion trace interactions are present. In particular, for chiral fermions,

\[
e^{i} A_{\mu AB} \sigma^{\nu A} \Psi_{L,R} = \frac{1}{2} (-i A_{\mu AB} \pm \frac{1}{2} \epsilon_{AB} A_{\mu CD} e^{A\nu} J^\nu_{L,R} \\
= \frac{1}{2} (-i \omega_{\mu AB} \pm \frac{1}{2} \epsilon_{AB} e^\mu_{\nu B} \omega_{CD} e^A_{\nu} J^\nu_{L,R} - (i B_{\mu} \pm \frac{1}{4} \tilde{A}_\mu) J^\nu_{L,R} \tag{A5}
\]

with \( J^\mu_{L,R} = \overline{\Psi}_{L,R} e_{\gamma^\mu} \Psi_{L,R} = \mp J^\mu_{L,R} \). This shows that left(right)-handed chiral fermions couple to the left(right)-handed (or anti-self-dual(self-dual)) projection of the spin connection, and chiral fermions interact only with the

\( (i B_{\mu} \pm \frac{1}{4} \tilde{A}) \) components.
Appendix B: Vacuum Polarization Tensor of Internal Gauge Fields

We begin by observing that the gauge current $J^\mu_a = -\overline{\Psi}L e^\mu T^a \Psi_L$ is regularized as

$$
\langle J^\mu_a (x) \rangle_{\text{Reg}} = \frac{1}{2} (1 - \gamma^5) - \lim_{x \to y} \text{Tr} \{ \gamma^\mu T^a \frac{1}{2} (1 - \gamma^5) \} \frac{1}{i\partial} \sum_n C_n \left[ \frac{i\partial}{[i\partial + m_n^2]} - \sigma^3 \delta(x - y) \right].
$$

(B1)

For pure internal gauge fields in flat spacetime we set $\hat{A}_\mu = B_\mu = 0$ in the Dirac operator to yield

$$
i\partial = (i\partial)^\dagger = i\partial + W_a T^a.
$$

(B2)

The positive-definite operator which appears in the regulator function is

$$i\partial(i\partial)^\dagger = -\Box + i\partial W_a T^a + W_a T^a i\partial + W_a T^a W_b T^b$$

(B3)

where $\Box = \partial_\mu \partial^\mu$. Thus the current has the expansion

$$
\langle J^\mu_a (x) \rangle_{\text{Reg}} = \lim_{x \to y} \frac{1}{2} \text{Tr} \{ T^a T^b \} \text{Tr} \{ \gamma^\mu \frac{1}{2} (1 - \gamma^5) \} \sum_n C_n \left[ -\frac{m_n^2}{(\Box + m_n^2)} W_b \left( \frac{m_n}{\Box + m_n^2} \right) + \frac{im_n}{(\Box + m_n^2)} \right] \delta(x - y).
$$

(B4)

In the above, we display only terms which will contribute to the vacuum polarization tensor i.e. terms which are linear in $W_{\mu a}$ since the vacuum polarization $\Pi^{\mu a k}$ is defined through the Fourier transform

$$
\frac{1}{(2\pi)^4} \int d^4k \Pi^{\mu ab} e^{ik(x-y)} = \frac{\delta \langle J^\mu_a (x) \rangle}{\delta W_{\nu b}(y)} \big|_{W=0} = \frac{1}{(2\pi)^4} \int d^4p e^{ip(x-z)} \left[ \frac{1}{(\Box + m_n^2)} \gamma^\nu \delta(x-y) \right],
$$

(B5)

Denoting the Dirac delta functions by

$$
\delta(x - y) = \frac{1}{(2\pi)^4} \int d^4k e^{ik(x-y)}, \quad \delta(x - z) = \frac{1}{(2\pi)^4} \int d^4p e^{ip(x-z)},
$$

(B6)

and bearing in mind that $\frac{1}{2} \text{Tr} \{ T^a T^b \} = \text{Tr} \{ T^a T^b \}$, the vacuum polarization tensor is therefore

$$
\Pi^{\mu ab} = \text{Tr} \{ T^a T^b \} \sum_n C_n \int \frac{d^4p}{(2\pi)^4} \text{Tr} \{ \gamma^\mu \frac{1}{2} (1 - \gamma^5) [ -\frac{m_n}{(p + k)^2 + m_n^2} \gamma^\nu \frac{m_n}{(p^2 + m_n^2)} + \frac{m_n}{(p + k)^2 + m_n^2} \gamma^\nu \frac{p}{(p^2 + m_n^2)} ] \}.
$$

(B7)

By using the Feynman parametrization

$$
\frac{1}{(p + k)^2 + m_n^2} \frac{m_n}{(p^2 + m_n^2)} = \frac{1}{(p' + k)^2 + m_n^2} = \frac{1}{(p' + k)^2 + m_n^2} \frac{m_n}{(p^2 + m_n^2)} = \frac{1}{(p + k)^2 + m_n^2} \frac{m_n}{(p^2 + m_n^2)} \frac{m_n}{(p^2 + m_n^2)}
$$

(B8)

with $p' \equiv p + k (1 - z)$, the first term of Eq. (B7) can be rewritten as

$$
\int \frac{d^4p}{(2\pi)^4} H^{\mu ab} = -\int \frac{d^4p}{(2\pi)^4} \text{Tr} \{ \gamma^\mu \frac{1}{2} (1 - \gamma^5) [ -\frac{m_n}{(p + k)^2 + m_n^2} \gamma^\nu \frac{m_n}{(p^2 + m_n^2)} + \frac{m_n}{(p + k)^2 + m_n^2} \gamma^\nu \frac{p}{(p^2 + m_n^2)} ] \}.
$$

(B9)
while the final contribution to $\Pi^{\mu\nu ab}$ of Eq. (B7) can be reexpressed as

$$\int \frac{d^4p}{(2\pi)^4} f^\mu_{2\nu n} = \int \frac{d^4p}{(2\pi)^4} \text{Tr}\left\{ \gamma^\mu \frac{1}{2} (1 - \gamma^5) \frac{(p + k)}{[(p + k)^2 + m_n^2]} \gamma^\nu \frac{f}{(p^2 + m_n^2)} \right\}$$

$$= 2 \int_0^1 dz \int \frac{d^4p}{(2\pi)^4} \frac{-2z(1-z)(k^\mu k^\nu - k^2 g^{\mu\nu}) - g^{\mu\nu} [(p^2/2) + k^2 z(1-z)]}{[p^2 + k^2 z(1-z) + m_n^2]^2}.$$  \hspace{1cm} (B10)

In total,

$$\Pi^{\mu\nu ab} = \text{Tr}\{ T^a T^b \} \sum_n C_n \int \frac{d^4p}{(2\pi)^4} \left( I^{\mu\nu mn}_1 + I^{\mu\nu mn}_2 \right),$$  \hspace{1cm} (B11)

with $C_n = (-1)^n$ and $m_n = n\Lambda$. For the explicit computation of the polarization tensor, a few identities related to our regulator function $f(p^2)$ are required. We have

$$\tilde{f}(p^2) = \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{(p^2 + n^2)^2} = \frac{\pi}{\sqrt{p^2} \sinh(\pi \sqrt{p^2})} = \frac{f(p^2)}{p^2},$$  \hspace{1cm} (B12)

$$\frac{d\tilde{f}}{dp^2} = - \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{(p^2 + n^2)^2},$$  \hspace{1cm} (B13)

and

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n n^2}{(p^2 + n^2)^2} = \tilde{f}(p^2) + p^2 \frac{d\tilde{f}}{dp^2} = \frac{d}{dp^2} [p^2 \tilde{f}(p^2)].$$  \hspace{1cm} (B14)

By writing $\Pi^{\mu\nu ab}$ as a sum of transverse and non-transverse parts,

$$\Pi^{\mu\nu ab} = \text{Tr}\{ T^a T^b \} (A^{\mu\nu}_{NT} + A^{\mu\nu}_{T}),$$  \hspace{1cm} (B15)

we observe that the non-transverse contribution vanishes identically since

$$A^{\mu\nu}_{NT} = -2g^{\mu\nu} \int_0^1 dz \int \frac{d^4p}{(2\pi)^4} \sum_n C_n \frac{k^2 z(1-z) + (p^2/2) + m_n^2}{[p^2 + k^2 z(1-z) + m_n^2]^2}$$

$$= 2g^{\mu\nu} \int_0^1 dz \int \frac{d^4p}{(2\pi)^4} \sum_n (-1)^n \frac{1}{[p^2 + k^2 z(1-z) + n^2\Lambda^2] + 2[p^2 + k^2 z(1-z) + n^2\Lambda^2]^2}$$

$$= -g^{\mu\nu} \Lambda^2 \int_0^1 dz \int \frac{1}{(16\pi^2)} \int_0^\infty dp^2 \frac{\partial}{\partial p^2} [p^4 \tilde{f}(p^2 + s)] = 0.$$  \hspace{1cm} (B16)

In the above, we have labeled $s = \frac{k^2 z(1-z)}{\Lambda^2}$ for convenience. The transverse piece is

$$A^{\mu\nu}_T = 2 \int_0^1 dz \int \frac{d^4p}{(2\pi)^4} \sum_n (-1)^n \frac{-2z(1-z)(k^\mu k^\nu - k^2 g^{\mu\nu})}{\Lambda^4 [p^2/\Lambda^2 + k^2 z(1-z)/\Lambda^2 + n^2]^2}$$

$$= \frac{1}{(4\pi^2)} (k^\mu k^\nu - g^{\mu\nu} k^2) \int_0^1 dz z(1-z) \int_0^\infty dp^2 p^2 \frac{\partial}{\partial p^2} \tilde{f}(p^2 + s)$$

$$= \frac{1}{(4\pi^2)} (k^\mu k^\nu - g^{\mu\nu} k^2) \int_0^1 dz 2z(1-z) \ln \tanh \left( \frac{\pi}{2} \sqrt{s} \right),$$  \hspace{1cm} (B17)

on rescaling $p/\Lambda \to p$ in the intermediate step, and noting that

$$\int_0^\infty dp^2 p^2 \frac{\partial}{\partial p^2} \tilde{f}(p^2 + s) = p^2 \tilde{f}(p^2 + s) \bigg|_{p^2=0} - \int_0^\infty dp^2 \tilde{f}(p^2 + s)$$
\[
= - \int_s^\infty \frac{\pi d(p^2 + s)}{\sqrt{p^2 + s \sinh(\pi \sqrt{p^2 + s})}} \\
= -2 \ln \tanh \left( \frac{\pi x}{2} \right) \bigg|_{x = \sqrt{s}}^{\infty}.
\]

We therefore arrive at

\[
\Pi^{\mu \nu a b} = \text{Tr} \left\{ T^a T^b \right\} A_\mu^{\nu \nu} \\
= \frac{\text{Tr} \left\{ T^a T^b \right\}}{24\pi^2} (k^\mu k^\nu - g^\mu \nu k^2) [\ln \left( \frac{k^2}{\Lambda^2} \right) - \frac{5}{3} + 2 \ln \left( \frac{\pi}{2} \right)]
\]

after integration over \( z \). This agrees with result obtained previously in Refs. [10] and [11] using the generalized Pauli-Villars scheme with doubling in external left-right fermionic space (rather than the method here of doubling in the internal \( T^a \) space).

The vacuum polarization tensor for internal gauge fields is therefore still transverse, and internal gauge fields remain massless. This is expected since the regularization explicitly respects the local internal symmetries gauged by \( W_\mu a \).

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