The Theoretical Model And Numerical Solution Of Heat Flow Coupling Considering Fractal Characteristics Of Rock Fractures

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Abstract: In order to analyze the temperature distribution law in rough fissured rock mass more truly, the fractal dimension of fractal theory is used to make statistics on the roughness of fissured surface, and a heat flow coupling theoretical model considering the fractal characteristics of fissured surface of rock mass is established. By building a geometric model containing rough fissured surface and numerical simulation by software, the more real temperature distribution law in rough fissured rock mass is analyzed. The simulation results show that the temperature distribution in rough fissured rock mass appears channeling phenomenon, and convection plays a leading role in the temperature distribution, and the existence of fissured seepage leads to a relatively low temperature zone in the rock mass.

1. Introduction
With the massive construction of the reservoir dam, the phenomenon of dam failure caused by the infiltration and destruction of underground and deep rock mass fissures has attracted extensive attention, such as the great landslide of bedrock on the left bank abutment of vajon arch dam in Italy in 1963 and the shear dislocation of the dam foundation on the right bank of Meishan multi-arch dam in China in 1962. Under complex hydrogeological conditions such as water pressure, in-situ stress and hydrogeochemistry, fissured rock mass tends to form concentrated leakage channels on large rock mass fissure structural surfaces [1]. Further erosion of rock mass and soil mass under the action of seepage leads to further expansion of seepage channels, which eventually leads to the occurrence of safety accidents such as slope rock mass collapse, dam foundation instability and dam break.

Concentrated leakage will cause abnormal changes in the temperature field of fractured rock mass [2], so whether concentrated leakage occurs in fractured rock mass can be judged by observing the changes in the temperature field of fractured rock mass [3–5]. Therefore, studying temperature distribution law of a fractured rock mass is of great engineering significance for dam safety operation management and maintenance.

In recent years, many scholars have done a lot of research on the temperature distribution law of rock mass, and established corresponding fluid-solid-thermal coupling models [6–8], and simplified them to get the analytical solution of the model. However, most of the fluid-solid-thermal coupling
models established are continuous media models, which are only suitable for porous continuous media, and not suitable for rock masses with large fracture structural planes. Other scholars took flat single-fracture rock mass \[9\sim11\] as the research object to conduct research, established a fluid-solid-thermal coupling theoretical model, and simplified the analytical solution of temperature distribution of single-fracture rock mass, but assumed that the premise was smooth fracture surface, which was inconsistent with the actual rough fracture surface.

In this paper, fractal dimension of fractal theory is used to statistics the roughness of fracture surface, and a heat flow coupling theoretical model considering the fractal characteristics of rock mass fracture is established, and the more real temperature distribution law in rough fracture rock mass is analyzed.

2. The theoretical model

2.1 The seepage control equation
In fractured rock mass, because the permeability of the rock is relatively small, it is assumed that the fracture is the main channel of seepage and the rock is impermeable. According to the assumption that the fluid is incompressible, viscous, there is no water flow exchange on the fracture wall and the water flow is laminar flow, the cubic law of seepage in smooth fracture rock mass can be obtained \[13\], as shown in the following formula:

\[
q = \frac{b^3 \gamma}{12 \mu} J \tag{1}
\]

Where: \(q\) is the single-width flow in the rock mass fissure, \(j\) is the hydraulic gradient, which is the specific gravity of groundwater, \(b\) is the fissure opening, and \(\mu\) is the dynamic viscosity coefficient.

Based on the cubic law of fissure water flow, it can be seen that the single width flow on the fissure cross section is directly proportional to the cubic square of the fissure opening, thus the average flow velocity through the single fissure is obtained as follows:

\[
V = \frac{b^2 \gamma}{12 \mu} J = K J \tag{2}
\]

Where \(k\) is the permeability coefficient, \(K = \frac{b^2 r}{12 \mu}\).

In ideal cracks, the permeability coefficient of cracks can be described by cubic law. In the actual crack, the crack has a certain area contact ratio, which is between two and three dimensions. The fracture is divided into many units. Assuming that the water flow in each unit conforms to the cubic law of the fracture, regardless of the influence of the rough fracture surface, the permeability coefficient of each unit can be expressed by the following formula:

\[
k_i = \frac{\gamma}{12 \mu} b(x_i, y_i)^2 \tag{3}
\]

Where \(k_i\) is the permeability coefficient; \(b (x_i, y_i)\) is the opening degree of the crack.

According to Darcy’s law and the continuity equation of seepage, the seepage control equation in rock mass fissures can be obtained:

\[
\frac{\partial}{\partial x} (k_i b_i \frac{\partial H}{\partial x}) + \frac{\partial}{\partial y} (k_i b_i \frac{\partial H}{\partial y}) = 0 \tag{4}
\]

2.2 The temperature control equation
The General differential equations describing energy convection conduction are:

\[
\frac{\partial T}{\partial t} + V \cdot \frac{\partial T}{\partial x} + V \cdot \frac{\partial T}{\partial y} + V \cdot \frac{\partial T}{\partial z} = \frac{\lambda}{c \rho} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + Q \tag{5}
\]
Where $T$ is the temperature, $V_x$, $V_y$ and $V_z$ respectively represent the seepage velocity in $x$, $y$ and $z$ directions, and $\lambda$ is the coefficient of thermal conductivity, $\rho$ is density, $c$ is the specific heat and $Q$ is the heat source term.

Assuming that the seepage flow in the fracture is a stable two-dimensional laminar flow, the scale in $z$ direction is much smaller than that in other two directions, and $V_z = 0$. $Q = 0$ without considering phase change and heat source. In a stable temperature field, the water flow temperature does not change with time, therefore $\frac{\partial T}{\partial t} = 0$. The temperature conduction equation of rock mass fracture can be simplified as follows:

$$V_x \frac{\partial T}{\partial x} + V_y \frac{\partial T}{\partial y} = \frac{\lambda}{\rho c} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right)$$  \hspace{1cm} (6)

In the rock mass, $V_x = 0$, $V_y = 0$, $V_z = 0$, because it is assumed that the fracture of the rock mass is a seepage channel and the rock itself is impermeable to water. Because it is a stable temperature field and has no heat source, therefore $\frac{\partial T}{\partial t} = 0$, $Q = 0$, the temperature control equation in the rock mass can be reduced to:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$  \hspace{1cm} (7)

Simultaneous equations (4), (6) and (7) can be used to obtain the heat flow model governing equations of fractured rock masses, as shown in equation (8). The seepage field and the temperature field influence each other, causing the temperature field of fractured rock mass to be redistributed under the action of thermal convection. The change of temperature will affect the dynamic viscosity coefficient and density of the fluid, thus affecting the flow of water. Therefore, the seepage field and the temperature field interact and influence each other. In the model established in this paper, because the temperature change of rock mass is small, the change of fluid characteristics is also small. The effect of both is mainly reflected in the convective heat transfer effect of seepage on temperature, ignoring the effect of temperature on fluid.

$$\begin{cases}
V_x \frac{\partial T}{\partial x} + V_y \frac{\partial T}{\partial y} + V_z \left( k_b \frac{\partial H}{\partial x} + k_b \frac{\partial H}{\partial y} \right) = 0 & \Omega \in \text{fracture} \\
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 & \Omega \in \text{rock mass}
\end{cases}$$  \hspace{1cm} (8)

3. The geometric model

3.1 The crack opening simulation
The two-dimensional fractional Brownian motion function can construct various random surfaces, which have statistical self-affine characteristics. Therefore, according to the Brownian motion function, a group of fracture surfaces can be generated. The general method for generating fracture opening is to consider the fracture as [10] formed by superposition of two fractal fracture surfaces. Given a dislocation value, the fracture opening is generated. As shown in fig. 1, a group of fracture surfaces are generated within the range of $10m \times 20m$. A group of rough fracture surfaces [13] can be obtained by setting the fracture surface fractal dimension. The fracture surface fractal dimension is 2.3, the parameter $\lambda = 1.25$, the dislocation value (0.0002, 0.0002) is taken, the area contact ratio of the fracture surface is 7.3 %, and the average fracture width is 0.8 mm.
3.2 The geometric model and boundary conditions
This paper establishes a geometric model with a length of 20 m, a width of 10 m, and a height of 20 m. A fissure surface is set in the middle. As shown in fig. 2, water flows in from a, and the boundary conditions are shown in the following formula.

\[
\begin{align*}
H_{FA} &= 1 \text{ m} \\
H_{FB} &= 0 \text{ m} \\
T_{FA} &= 283 \text{ K} \\
T_{FB} &= 293 \text{ K} \\
T_{FE} &= 293 \text{ K}
\end{align*}
\] (9)

In addition, the boundaries of B, C, and D are free exit boundaries of heat flow, and the others are insulated boundaries.

3.3 The model parameters
The model parameters are shown in table 1.

| Density \( \rho \) (kg/m\(^3\)) | Coefficient of heat conduction \( \lambda \) (W/m·K) | Specific heat \( c \) (J/kg·K) | The Dynamic viscosity coefficient \( \mu \) (kg/m·s) |
|---|---|---|---|
| 1000 | 0.5 | 4200 | 0.001 |

4. The simulation results and analysis
The above geometric model was simulated by software. The simulation results are shown in figs. 3 - 6. Figure 3 shows the distribution diagram of water head in cracks. From the diagram, it can be seen that the distribution of water head is obviously uneven, which is significantly different from that in parallel cracks of equal width. Combined with figure 4, the distribution of flow velocity in cracks is also significantly different, which shows obvious channel flow phenomenon. This is mainly due to the uneven distribution of opening in cracks and the flow of water along places where the opening is larger and the cracks are relatively connected, thus forming a relatively dominant seepage channel, and finally causing this channel flow phenomenon.
Fig. 3 Head pressure distribution within the fractured

Fig. 4 The velocity distribution within the fractured

Fig. 5 shows the temperature distribution diagram in the fracture when the average hydraulic gradient is 0.05. From the diagram, it can be seen that the temperature distribution in the fracture also shows non-uniform characteristics, which shows that the existence of seepage obviously changes the temperature distribution law in the fracture. Comparing fig. 4, it can be seen that the temperature is lower where the flow velocity is larger and the flow velocity is smoother, while the temperature is higher where the flow velocity is smaller and the fracture is relatively closed. This is mainly due to the re-distribution of temperature due to the convection effect of water flow, showing non-uniform characteristics, while the heat conduction effect is relatively small, maintaining this non-uniform characteristics, which shows that the convection effect in the fracture is at temperature. The temperature at the entrance of the crack is the lowest, and the temperature goes further into the crack. With the heat convection and conduction, the temperature gradually increases. It can be predicted that the water flow in the crack will reach equilibrium under the heat exchange, and eventually the original temperature of the rock will be reached without heat exchange.

Fig. 6 shows the temperature distribution diagram in rock mass. Generally speaking, the lower the temperature is closer to the fissure, and the closer the temperature is to the original temperature of the rock away from the fissure. Due to the large temperature difference at the entrance of the crack, the heat exchange is also strongest. With the occurrence of heat exchange, the water flow temperature gradually increases, the rock temperature gradually decreases, and finally reaches a state of equilibrium. The existence of cracks leads to a relatively low temperature zone in rock mass, which can be revealed by drilling and temperature monitoring, so the seepage situation in rock mass cracks can be analyzed by temperature.
5. Conclusions
In this paper, fractal dimension of fractal theory is used to make statistics on the roughness of fracture surface, and a heat flow coupling theoretical model considering the fractal characteristics of rock mass fracture is established. By constructing a geometric model containing rough fracture surface and using software to carry out numerical simulation, the temperature distribution law in more real rough fracture rock mass is analyzed. The main conclusions are as follows:

(1) the groove flow phenomenon in the rough fissure rock mass also causes the groove phenomenon to appear in the temperature distribution, which is obviously different from the temperature distribution law in the smooth fissure; The temperature is lower where the flow velocity is larger and the seepage flow is smoother in the rough fissured rock mass, while the temperature is higher where the flow velocity is smaller and the fissured rock mass is relatively closed, which plays a leading role in the convection in the temperature distribution.

(2) the temperature difference at the crack entrance is large and the heat exchange is intense. With the increase of distance, the heat exchange tends to decay, and the fluid in the crack finally reaches the original temperature of the rock.

(3) the existence of cracks creates a relatively low temperature region in the rock mass. This low temperature region can be revealed through drilling holes and temperature monitoring. Therefore, the seepage situation in the rock mass cracks can be analyzed through temperature.

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