Quantum field-theoretical description of neutrino and neutral kaon oscillations

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Abstract

It is shown that the neutrino and neutral kaon oscillation processes can be consistently described in quantum field theory using only the mass eigenstates of neutrinos and neutral kaons. The distance-dependent and time-dependent parts of the amplitudes of these processes are calculated and the results turn out to be in accord with those of the standard quantum mechanical description of these processes based on the notion of neutrino flavor states and neutral kaon states with definite strangeness. However, the physical picture of the phenomena changes radically: now there are no oscillations of flavor or definite strangeness states, but, instead of it, there is an interference of amplitudes due to different virtual mass eigenstates.

1 Introduction

Neutral kaon and neutrino oscillations are discussed in theoretical physics for more than half a century already. The standard theoretical approach to describing these phenomena is based on considering the states with definite strangeness or lepton number ($K^0$, $\bar{K}^0$ and the neutrino flavor states) that are treated as superpositions of the corresponding mass eigenstates. It is assumed that it is the former states that are produced in the strong and weak interactions. Considering their time evolution in accordance with the Schrödinger equation, one gets the oscillating probabilities of processes. This was first demonstrated for neutral kaons in paper [1] and later for neutrinos in papers [2, 3]. The present day status of neutrino oscillations is discussed in detail in textbook [4] and in review article [5].

However, although this approach to describing neutral kaon and neutrino oscillations is physically transparent and simple, it is not rigorous and in many papers it was argued that it should be considered as a phenomenological one (see, e.g. [6, 7, 8, 9, 10]). Its most noticeable flaw is the violation of the energy-momentum conservation in the processes, in which $K^0$, $\bar{K}^0$ or a neutrino flavor state are produced, because in local quantum field theory, where the four-momentum is conserved in any interaction vertex, different mass-eigenstate components of these states must have different momenta as well as different energies. Thus, if the incoming particles have definite four-momenta, then the produced particles must also have definite four-momenta, which implies that neutral kaons and neutrinos can be produced only in the mass eigenstates.
Apparently, the spread of the momenta of the incoming particles (the use of wave packets) does not solve the problem, if the interaction remains local.

A simple physical idea for solving the problem with the violation of energy-momentum conservation is to go off the mass shell. However, a specific realization of this idea encountered serious difficulties. Its first implementation in QFT was given in paper [6], where it was suggested that the produced neutrino mass eigenstates are virtual and their motion to the detection point should be described by the Feynman propagators. The authors managed to get through rather complicated calculations and reproduced the results of the standard quantum mechanical approach. Later this idea was developed in paper [7]. The authors of this paper proved an important theorem that, for large macroscopic distances between the production and the detection points, the virtual neutrinos are almost on the mass shell, and thus reduced the description in terms of the virtual neutrinos to the standard one. Recently, the approach with virtual neutrinos was also discussed in paper [9], where the authors used a formalism of relativistic wave packets to facilitate the calculations. Nevertheless, the calculations in all these papers are too bulky and the final results are not quite convincing. This is due to the standard S-matrix formalism of QFT used in these papers, which is not appropriate for describing processes at finite distances and finite time intervals.

In the present paper we will modify the standard perturbative S-matrix formalism so as to be able to calculate the amplitudes of the processes at finite distances and finite time intervals and apply it to describing neutrino and neutral kaon oscillations. We start with discussing neutrino oscillation processes, which is simpler because in these processes the massive neutrinos can be treated as stable particles [11].

## 2 Neutrino oscillations

We consider the minimal extension of the Standard Model (SM) by the right neutrino singlets. After the diagonalization of the terms sesquilinear in the neutrino fields, the charged current interaction Lagrangian of leptons takes the form

$$L_{cc} = -\frac{g}{2\sqrt{2}} \left( \sum_{i,k=1}^{3} \bar{l}_i \gamma^\mu (1 - \gamma^5) U_{ik} \nu_k W^-_{\mu} + h.c. \right),$$

where $l_i$ denotes the field of the charged lepton of the i-th generation, $\nu_i$ denotes the field of the neutrino mass eigenstate most strongly coupled to $l_i$ and $U_{ik}$ stands for the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix.

We consider the case, where the neutrinos are produced and detected in the charged current interaction with electrons. Due to the structure of the interaction Lagrangian any process involving the production of a neutrino at one point and its detection at another point, when treated perturbatively, includes a subprocess described by the following diagram
for all the three neutrino mass eigenstates. Depending on the neutrino production process, the external fermion line at the point $x$ can either correspond to an incoming electron or an outgoing positron. An important point is that there is no need to add the diagram with interchanged W-bosons, because they are associated with different spatially separated processes and therefore should not be considered as identical particles. Without loss of generality we can assume that the incoming particles have definite momenta. Therefore all the three virtual neutrino eigenstates and the outgoing particles also have definite momenta.

The amplitude in the coordinate representation corresponding to diagram (2) can be easily written out using the Feynman rules formulated in §23 of textbook [12]. In the present paper we will not calculate the complete amplitude, we will rather calculate only its oscillating part that depends on the distance between the points $x$ and $y$. This means that we can drop the fermion wave functions that depend only on the particle momenta, the polarization vectors of the W-bosons, as well as the coupling constants and the $\gamma$-matrix structures in the vertices. What remains of the amplitude looks as follows:

$$|U_{1i}|^2 e^{-ipx} S^c_i(y - x) e^{iqy},$$

where $S^c_i(y - x)$ is the Feynman propagator of the neutrino mass eigenstate $\nu_i$, $p$ is the sum of the momenta of the external lines at $x$ and $q$ is the sum of the momenta of the external lines at $y$.

According to the prescriptions of the standard perturbative S-matrix theory ([12], §24), next we would have to integrate with respect to $x$ and $y$ over the Minkowski space, which would give us the part of the scattering amplitude corresponding to the inner fermion line of diagram (2). However, in this case we would get the amplitude of the process lasting an infinite amount of time and loose the information about the distance between the production and detection points. In order to retain this information, we have to integrate with respect to $x$ and $y$ in such a way that the distance between these points along the direction of the neutrino propagation remains fixed. Of course, this is at variance with the standard S-matrix formalism. However, we recall that the diagram technique in the coordinate representation was developed by R. Feynman [13] without reference to S-matrix theory. Thus, the Feynman diagrams in the coordinate representation make sense beyond this theory, and for this reason we may integrate with respect to $x$ and $y$ in any way depending on the physical problem at hand. In particular, in the case under consideration we have to integrate in such a way that the distance between the points $x$ and $y$ along the direction of the neutrino propagation equals to $L$. This can be achieved by introducing the delta function $\delta(\vec{p}(\vec{y} - \vec{x}) - |\vec{p}|L)$ into the integral, which gives the amplitude

$$|U_{1i}|^2 \int dx dy e^{-ipx} S^c_i(y - x) e^{iqy} \delta(\vec{p}(\vec{y} - \vec{x}) - |\vec{p}|L).$$

It is convenient to make the change of variables

$$x = u - \frac{z}{2}, \quad y = u + \frac{z}{2}, \quad \frac{D(x, y)}{D(u, z)} = 1$$

(5)
and to integrate with respect to $u$ in the integral in formula (4). This transformation shows that the amplitude is proportional to the delta function of energy-momentum conservation $(2\pi)^4 \delta(q - p)$, which is unimportant for our further considerations and will be dropped, and the distance-dependent propagator of the neutrino mass eigenstate $\nu_i$ in the momentum representation, which will be denoted by $S^c_i(p, L)$ and is defined by the remaining integral:

$$S^c_i(p, L) = \int dz \, e^{ipz} S^c_i(z) \delta(\vec{p}z - |\vec{p}|L).$$

(6)

Obviously, the integration of this distance-dependent fermion propagator with respect to $|\vec{p}|L$ from minus infinity to infinity gives the standard Feynman fermion propagator in the momentum representation. Thus, the distance-dependent fermion propagator can be considered as the distribution of the Feynman propagator with respect to $L$ along the direction of momentum $\vec{p}$.

The integral in formula (6) is evaluated exactly in Appendix 1, and for $\vec{p}^2 > m_i^2 - \vec{p}^2$ the result is given by

$$S^c_i(p, L) = \frac{\hat{p} + m_i}{2|\vec{p}| \sqrt{\vec{p}^2 + \vec{p}^2 - m_i^2}} e^{-i(\vec{p} - \sqrt{\vec{p}^2 + \vec{p}^2 - m_i^2})L}. $$

(7)

We emphasize that this distance-dependent fermion propagator makes sense only for macroscopic distances. As we have already mentioned in the Introduction, the results of paper [7] imply that the virtual particles propagating at macroscopic distances are almost on the mass shell. This means that $|\vec{p}^2 - m_i^2|/\vec{p}^2 \ll 1$ and we can expand the square roots to the first order in $(\vec{p}^2 - m_i^2)/\vec{p}^2$. It is clear that this term can be dropped everywhere, except in the exponential, where it is multiplied by a large macroscopic distance $L$, which results in

$$S^c_i(p, L) = i \frac{\hat{p} + m_i}{2\vec{p}^2} e^{-i\frac{m_i^2 - \vec{p}^2}{2|\vec{p}|}L}. $$

(8)

It is worth noting that this distance-dependent fermion propagator taken on the mass shell has no pole and does not depend on the distance, which is also true for the exact propagator in formula (7). The integration of this propagator with respect to $|\vec{p}|L$ from zero to infinity gives one half of the Feynman fermion propagator in the momentum representation (the other half can be obtained by the integration of $S^c_i(p, -L)$ defined in accordance with formula (6)).

Since the neutrinos produced in the standard processes with electrons or positrons are ultrarelativistic, the terms $m_i/|\vec{p}|$ are negligibly small and we can also drop them. In this approximation distance-dependent propagator (8) takes the simple form

$$S^c_i(p, L) = i \frac{\hat{p}}{2\vec{p}^2} e^{-i\frac{m_i^2 - \vec{p}^2}{2|\vec{p}|}L}. $$

(9)

Accordingly the amplitude of any process, where the neutrinos are produced and detected in the charged current interaction with electrons, is the sum of the amplitudes with all the three virtual neutrino mass eigenstates and contains, due to (4), (6), (9), the scalar factor

$$\sum_{i=1}^3 |U_{1i}|^2 e^{-i\frac{m_i^2 - \vec{p}^2}{2|\vec{p}|}L}. $$

(10)
Therefore the probability of the process is proportional to

\[
\left| \sum_{i=1}^{3} |U_{1i}|^2 e^{-\frac{m_i^2-p^2}{4|\vec{p}|} L} \right|^2 = 1 - 4 \sum_{i,j=1, i < j}^{3} |U_{1i}|^2 |U_{1j}|^2 \sin^2 \left( \frac{m_j^2 - m_i^2}{4|\vec{p}|} L \right).
\]  

(11)

In this way we have obtained the survival probability of the electron, which coincides with the survival probability of the electron neutrino, and reproduced the result of the standard quantum mechanical approach based on the notion of the neutrino flavor states [14], although these states have not been used in the derivation of formula (11).

Similarly, we can find the transition probability from the charged lepton of the k-th generation to the charged lepton of the l-th generation. The corresponding diagram looks like diagram (2) with other incoming and outgoing fermions. If the momentum of the neutrino line satisfies the conditions \( m_i/|\vec{p}| \ll 1 \), the amplitude of this process is proportional to

\[
\sum_{i=1}^{3} \bar{U}_{ki} U_{li} e^{-\frac{m_i^2-p^2}{4|\vec{p}|} L},
\]

(12)

and one can easily obtain the general formulas for the oscillating survival probabilities of charged leptons and transition probabilities from any charged lepton to another charged lepton, which again coincide with the corresponding standard formulas for neutrino flavor states [14]; we will not write them out here.

Several remarks are in order. First, in the SM only the sum of all three diagrams (2) with all the neutrino mass eigenstates is gauge invariant, and therefore it is impossible to derive by this method the formula for the oscillation of only two neutrino flavors as it is usually done in the standard approach. Second, if the neutrino production and detection processes are specified, one can calculate the amplitude of the complete process by drawing the corresponding Feynman diagram, constructing the amplitude in the momentum space in accordance with the standard Feynman rules and replacing the standard fermion propagator in the latter by the distance-dependent propagator in formula (8). By this method, one can also calculate the loop corrections to this amplitude that will determine the actual accuracy of formula (11). Third, to calculate the survival and transition probabilities of charged leptons in realistic experiments one has to generalize the approach by considering the amplitudes of processes, where the distance between the production and detection points lies between \( L_1 \) and \( L_2 \). Such an amplitude can be obtained by integrating the amplitude in formula (1) with respect to \(|\vec{p}|L\) from \(|\vec{p}|L_1\) to \(|\vec{p}|L_2\). However, in the case \( L_2 - L_1 \ll L_1 \), which is specific for experiments with neutrinos, the result is trivial: the amplitude corresponding to the distance \( L_1 \) is just multiplied by \(|\vec{p}|(L_2 - L_1)\). We note once again that the integration of the amplitude in formula (1) with respect to \(|\vec{p}|L\) from minus infinity to infinity gives the standard amplitude with the Feynman propagators. Thus, the amplitude integrated from \(|\vec{p}|L_1\) to \(|\vec{p}|L_2\) can be considered as a portion of the standard Feynman amplitude. Fourth, formula (11) is formally valid for any value of the macroscopic distance \( L \). The momentum spread of the produced neutrinos gives rise to the standard coherence length, which limits its range of applicability. In the framework of the present approach there may exist one more coherence length arising due to a different reason. As we have already noted, the virtual particles propagating at macroscopic distances are almost on the mass shell. However, the theorem proved in paper [7] does not define explicitly the...
admissible deviation of the virtual particle momentum squared from the mass shell depending on the distance traveled. If such an admissible deviation exists, one can imagine the situation, where, for certain large distances, the admissible deviation from the mass shell is less than the minimal absolute value of the differences of the neutrino masses squared. In this case one should expect that either only two of the three diagrams contribute to the amplitude of the process or only one of the three diagrams contributes to the amplitude of the process, which means that there would be either a change of the interference pattern at a certain large distance or no interference at all between the contributions of different neutrino mass eigenstates and, therefore, no oscillation of the process probability depending on the distance. For this reason it would be very interesting to improve the theorem of paper [7] so as to estimate the admissible deviation of virtual particles from the mass shell depending on the distance.

3 Neutral kaon oscillations

Now we turn to discussing neutral kaon oscillations. The production of neutral kaons by 24 GeV protons hitting a platinum target and their subsequent decay to $\pi^+\pi^-$ pairs were studied in paper [15]. It was shown in this paper that the probability of detecting a $\pi^+\pi^-$ pair includes an oscillating component.

The standard theoretical explanation of this phenomenon is based on the assumption that the state with strangeness -1 denoted $K^0$ is predominantly produced in the original interaction of protons with platinum nuclei. This state has no definite mass and can be treated as a superposition of states with definite masses. The time evolution of this superposition of the mass eigenstates gives rise to the oscillating decay probability to $\pi^+\pi^-$ pairs [1, 16].

However, as we have already mentioned in the previous section, in local QFT the production of a state without definite mass violates the energy-momentum conservation. Below we will show, how this problem can be solved by considering distance-dependent and time-dependent amplitudes of processes mediated by the particles $K^0_S$ and $K^0_L$. The situation here is more complicated than in the case of neutrinos for two reason: neutral kaons are states of quark-antiquark pairs bound by the strong interaction and they are unstable.

Let us denote the mass and the width of $K^0_S$ by $m_S, \Gamma_S$ and those of $K^0_L$ by $m_L, \Gamma_L$. Since these particles are produced in the strong interaction processes, we cannot isolate the subprocesses of $K^0_S, K^0_L$ production as definitely, as in the case of only weakly interacting neutrinos. However, we can assume that either a virtual $K^0_S$ or a virtual $K^0_L$ is produced at a point $x$ and decays to $\pi^+\pi^-$ pair at a point $y$, which can be represented by the following diagram:

$$ x \xrightarrow{K^0_S(K^0_L)} y $$

The amplitudes of the $\pi^+\pi^-$ pair production processes mediated by $K^0_S$ and $K^0_L$ differ in the production amplitudes, the propagators and the decay amplitudes to $\pi^+\pi^-$. Denoting by $P_S$ the production amplitude of $K^0_S$ and by $A_{+-,S}$ its decay amplitude to $\pi^+\pi^-$, as well as by $P_L$ the production amplitude of $K^0_L$ and by $A_{+-,L}$ its decay amplitude to $\pi^+\pi^-$, we can write down
The same calculations can be easily repeated for the distance-dependent propagator of
from (14), (15), (18) one gets that the total amplitude is proportional to

\[ P_S e^{-i\pi x} D_S(x-y) e^{i\pi y} A_{+-,S}, \quad P_L e^{-i\pi x} D_L(x-y) e^{i\pi y} A_{+-,L}, \]

where \( D_S(x-y) \) and \( D_L(x-y) \) stand for the propagators of \( K_S^0 \) and \( K_L^0 \), \( p \) is the total momentum entering the vertex at the point \( x \) and \( q \) is the sum of the momenta of \( \pi^+\pi^- \) pair.

To find the distance-dependent amplitude of the process under consideration, we again have to integrate these amplitudes with respect to \( x \) in such a way that the distance between the points \( x \) and \( y \) along the direction of the neutral kaon propagation equals to \( L \). The amplitudes due to virtual \( K_S^0 \) and \( K_L^0 \) having absolutely the same form, we will do it in detail only for the amplitude due to \( K_S^0 \) that looks like

\[ P_S A_{+-,S} \int dx dy e^{-i\pi x} D_S(x-y) e^{i\pi y} \delta(p\vec{y} - \vec{x}) - |\vec{p}|L. \]

Again making the change of variables in accordance with (5), we obtain that the amplitude is proportional to the delta function of energy-momentum conservation \((2\pi)^4 \delta(q - p)\), which is dropped, and the distance-dependent propagator of \( K_S^0 \) in the momentum representation that will be denoted by \( D_S^c(p, L) \) and is defined by the integral

\[ D_S^c(p, L) = \int dz e^{i\pi z} D_S(z) \delta(p\vec{z} - |\vec{p}|L). \]

This integral is evaluated exactly in Appendix 2, the result for \( p^2 > m_S^2 - p^2 \) being given by

\[ D_S^c(p, L) = \frac{i}{2|\vec{p}| \sqrt{p^2 + p^2 - m_S^2 + im_S \Gamma_S}} e^{-i\left(|\vec{p}| - \sqrt{p^2 + p^2 - m_S^2 + im_S \Gamma_S}\right) L}. \]

We recall once more that the virtual particles propagating at macroscopic distances are almost on the mass shell [7], and, therefore, \( |p^2 - m_S^2|/p^2 \ll 1 \). The ratio \( m_S \Gamma_S/|\vec{p}|^2 \) being also very small, we actually have \( |p^2 - m_S^2 + im_S \Gamma_S|/|\vec{p}|^2 \ll 1 \) and can drop the term \( p^2 - m_S^2 + im_S \Gamma_S \) in the square root in the denominator and expand the square root in the exponential to the first order in this term. In this approximation formula (17) for the distance-dependent propagator of \( K_S^0 \) can be written as

\[ D_S^c(p, L) = \frac{i}{2|\vec{p}|} e^{-m_S \Gamma_S/2|\vec{p}|} e^{-i\left(p^2 - m_S^2 - im_S \Gamma_S \right) L}. \]

The same calculations can be easily repeated for the distance-dependent propagator of \( K_L^0 \), and from (14), (15), (18) one gets that the total amplitude is proportional to

\[ e^{-m_S \Gamma_S/2|\vec{p}|} e^{-i\left(p^2 - m_S^2 \right) L} + \frac{P_L \eta_{+-}}{P_S} e^{-i\left(p^2 - m_S^2 \right) L}, \]

where \( \eta_{+-} = A_{+-,L}/A_{+-,S} \) is the standard parameter for describing the decay of neutral kaons to \( \pi^+\pi^- \) pairs [16]. It is convenient to isolate the phases of \( \eta_{+-} \) and \( P_L/P_S \) in accordance with the parameterizations

\[ \eta_{+-} = |\eta_{+-}| e^{i\phi_{+-}}, \quad \frac{P_L}{P_S} = \left| \frac{P_L}{P_S} \right| e^{i\phi_P}. \]
The specificity of the experiments with neutral kaons suggests that their results should be represented not in terms of the distance $L$, but rather in terms of the time $T = Lp^0/|\vec{p}|$. Substituting $L$ in terms of $T$ into formula (19), we get

$$e^{-m_S^2 T} e^{-\frac{S^2 - p^2}{2p^0} T} + \frac{P_L}{P_S} \eta_+ e^{-\frac{S^2 - p^2}{2p^0} T} e^{-i \frac{S^2 - p^2}{2p^0} T}. \tag{21}$$

However, it is clear that time-dependent amplitudes can be obtained directly from amplitudes (14) by multiplying them by the delta function $\delta(\beta(y^0 - x^0 - T))$ and then integrating with respect to $x$ and $y$. Here the parameter $\beta$ has dimension of mass and will be defined later. Similar to the case of the distance-dependent amplitude, the integral entering the expression for the time-dependent amplitude due to virtual $K_S^0$ can be represented as the product of the delta function of energy-momentum conservation and the time-dependent propagator of $K_S^0$ in the momentum representation $D_S(p, T)$ defined by the formula

$$D_S(p, T) = \int dz e^{ipz} D_S^c(z) \delta(\beta(z^0 - T)). \tag{22}$$

The integral in this formula is evaluated exactly in Appendix 3, and the resulting time-dependent propagator looks like

$$D_S(p, T) = \frac{e^{i\beta}}{2\beta\sqrt{(p^0)^2 + m_S^2 - p^2 - im_S\Gamma_S}} e^{i(p^0 - \sqrt{(p^0)^2 + m_S^2 - p^2 - im_S\Gamma_S}) T}. \tag{23}$$

Again we have $|m_S^2 - p^2 - im_S\Gamma_S|/(p^0)^2 \ll 1$ and for this reason we can drop the term $m_S^2 - p^2 - im_S\Gamma_S$ in the square root in the denominator and expand the square root in the exponential to the first order in this term. In this approximation formula (23) takes the form

$$D_S^c(p, T) = \frac{e^{i\beta}}{2\beta p^0} e^{-\frac{m_S^2 - p^2 - im_S\Gamma_S}{2p^0} T}, \tag{24}$$

which coincides with (18), if we choose $\beta = \vec{p}^2/p^0$ and substitute $T = Lp^0/|\vec{p}|$. Thus, if one uses the matched delta functions in the propagator definitions, the distance-dependent propagator and the time-dependent propagator at macroscopic distances and time intervals are, in fact, the same and differ only in the parametrization. It is worth noting that this is also true for the case of neutrinos, where formula (24) can be obtained by substituting $\beta = \vec{p}^2/p^0$, $T = Lp^0/|\vec{p}|$ into the corresponding time-dependent propagator. Moreover, in the case of ultrarelativistic neutrinos one can put $\beta = p^0$ and get the time-dependent propagator with the same accuracy.

Summing the time-dependent amplitudes due to $K_S^0$ and $K_L^0$, one again gets that the total amplitude is proportional to the expression in formula (21). It is customary to write this formula in terms of the proper time, which, in the case under consideration, should be defined as follows:

$$t_p = \frac{2p^0 T}{m_S + m_L}. \tag{25}$$

Formula (21) rewritten in terms of the proper time looks like

$$e^{-m_S^2 T} e^{-\frac{S^2 - p^2}{m_S + m_L} T} + \frac{P_L}{P_S} |\eta_+| e^{i(\phi_p + \phi_+)} e^{-\frac{S^2 - p^2}{m_S + m_L} T} e^{-i \frac{S^2 - p^2}{m_S + m_L} T} \tag{26}.$$
where the phases $\phi_P$ and $\phi_{\pm}$ have been defined in (20). Therefore the probability of the $\pi^+\pi^-$ production process is proportional to

$$
e^{-\frac{2m_S\Gamma_S}{m_S+m_L} tp} + \left| \frac{P_L}{P_S} \right|^2 |\eta_{\pm}|^2 e^{-\frac{2m_L\Gamma_L}{m_S+m_L} tp} +$$

$$+ 2 \left| \frac{P_L}{P_S} \right| |\eta_{\pm}| e^{-\frac{m_S\Gamma_S}{m_S+m_L} tp} \cos (\Delta m_{L,S} tp - \phi_P - \phi_{\pm}),$$

(27)

where $\Delta m_{L,S} = m_L - m_S$. For $P_L = P_S$ and $P_L = -P_S$ this expression coincides exactly with those presented in [16], §4.3, if one neglects the difference between $m_L$ and $m_S$ in all the exponentials. This formula can also be rewritten in the form

$$
e^{-\frac{m_S\Gamma_S}{m_S+m_L} tp} \left( e^{-\frac{m_L\Gamma_L}{m_S+m_L} tp} + \left| \frac{P_L}{P_S} \right|^2 e^{-\frac{m_L\Gamma_L}{m_S+m_L} tp} \right)^2 -$$

$$- 4 \left| \frac{P_L}{P_S} \right| |\eta_{\pm}| e^{-\frac{m_S\Gamma_S}{m_S+m_L} tp} \sin^2 \left( \frac{\Delta m_{L,S} tp}{2} - \frac{\phi_P + \phi_{\pm}}{2} \right),$$

(28)

which is most similar to formula (11) for neutrino oscillations. Thus, we have again reproduced the results of the standard approach without reference to the states with definite strangeness $K^0, \bar{K}^0$.

4 Conclusion

In the present paper we have shown that it is possible to give a consistent quantum field-theoretical description of neutrino oscillations in the SM minimally extended by the right neutrino singlets. To this end we had just to adapt the standard perturbative S-matrix formalism for calculating the amplitudes of the processes passing at finite distances and finite time intervals. The developed approach is physically transparent and, unlike the standard one, has the advantage of not violating the energy-momentum conservation. In its framework, the calculations of amplitudes are much simpler than in papers [6, 9], where a similar approach based on the standard S-matrix description of the propagation of virtual neutrinos is used.

The application of this modified formalism to describing the neutrino and neutral kaon oscillation processes showed that the standard results can be easily and consistently obtained using only the mass eigenstates of these particles. Therefore, the neutrino flavor states and the neutral kaon states with definite strangeness are redundant in the theory and should be amputated by Occam’s razor.

Although the results obtained within the developed approach coincide with the standard results of the neutrino and neutral kaon oscillation theory, the physical picture of the phenomena changes radically. If there are no neutrino flavor states, there is no neutrino oscillation, and the term can be retained only as a historical one. What remains of this theory is the observable oscillating survival probabilities of charged leptons and the observable oscillating transition probabilities from one charged lepton to another charged lepton arising due to the interference of the amplitudes of the processes mediated by different neutrino mass eigenstates. Thus, it is the mass eigenstates $\nu_i, i = 1, 2, 3$, that are the only observable neutrino states in the SM. These states can be consistently treated as particles, and it is natural to call them after the
charged leptons, to which they are most strongly coupled, the electron neutrino, the muon neutrino and the tau neutrino respectively, which restores the quark-lepton symmetry in the SM.

In the present paper we have considered only the case of Dirac neutrinos. However, all the results can be transferred, *mutatis mutandis*, to the case of Majorana neutrinos, because their propagators have essentially the same structure.

A similar situation takes place in the case of neutral kaons, where only the states $K^0_S$ and $K^0_L$ can be considered to be particles. The states $K^0$ and $\bar{K}^0$ are artificial abstract entities that are unnecessary in QFT and cannot exist or even be produced in Nature, where, unlike in the theory, the weak interaction cannot be switched off. Therefore, we have to admit that there is no neutral kaon oscillations, but there is an interference of the amplitudes with virtual $K^0_S$ and $K^0_L$ instead.

Finally, we note that one can construct the amplitudes and find the cross sections of processes with any particles, whose production and detection sites are separated by a macroscopic distance, by drawing the corresponding Feynman diagrams, constructing the amplitudes in the momentum space in accordance with the standard Feynman rules and replacing the standard propagators in the latter by the corresponding distance-dependent or time-dependent propagators introduced in the present paper. Such calculations may be useful for analyzing events in the experiments, where detectors are situated at macroscopic distances from the interaction points.

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**Appendix 1**

Substituting the standard integral representations for the fermion propagator and the delta function into the integral defining the distance-dependent propagator in formula (4), we get

$$S^c_i(p, L) = \frac{1}{(2\pi)^5} \int dzdkdt e^{ipz}e^{-ikt}e^{i(\vec{p}-\vec{n}\omega)t} \frac{\hat{k} + m_i}{m_i^2 - k^2 - i\epsilon},$$  \hspace{1cm} (A1)

where $m_i$ denotes the mass of the neutrino mass eigenstate $\nu_i$.

Next we make the change of variable $t = \omega/|\vec{p}|$ and integrate with respect to $z$, which results in

$$S^c_i(p, L) = \frac{1}{2\pi|\vec{p}|} \int d\omega dk \delta(p^0 - k^0)\delta(\vec{k} - \vec{p} + \vec{n}\omega)e^{-i\omega L} \frac{\hat{k} + m_i}{m_i^2 - k^2 - i\epsilon},$$  \hspace{1cm} (A2)

where $\vec{n} = \vec{p}/|\vec{p}|$.

The integration with respect to $k$ is trivial due to the $\delta$-functions:

$$S^c_i(p, L) = \frac{1}{2\pi|\vec{p}|} \int d\omega e^{-i\omega L} \frac{\hat{p} + \vec{\gamma}n\omega + m_i}{m_i^2 - \vec{p}^2 - 2|\vec{p}|\omega + \omega^2 - i\epsilon}.$$  \hspace{1cm} (A3)
The last integral can be evaluated for $\vec{p}^2 > m^2 - p^2$ by closing the integration contour in the lower complex half-plane and calculating the residue at the simple pole, which gives:

$$S_c^i(p, L) = i \frac{\hat{p} + \gamma \vec{p}}{2|\vec{p}| \sqrt{\vec{p}^2 + p^2 - m_i^2}} \left( 1 - \frac{1}{1 + \frac{p^2 - m_i^2}{\vec{p}^2}} \right) + m_i \frac{1}{m_i^2 - k^2 - im_i \Gamma_i} e^{-i(|\vec{p}| - \sqrt{\vec{p}^2 + p^2 - m_i^2})L}. \quad (A4)$$

**Appendix 2**

Again substituting the standard integral representations for the scalar field propagator and the delta function into the integral in formula (16), we get

$$D_c^S(p, L) = \frac{1}{(2\pi)^5} \int dzdkdt e^{ipz} e^{ikz} e^{i(\vec{p} - |\vec{p}|)t} \frac{1}{m_S^2 - k^2 - im_S \Gamma_S}. \quad (A5)$$

Then we make the change of variable $t = \omega/|\vec{p}|$ and integrate with respect to $z$ and $k$ similar to (A2), (A3), which gives

$$D_c^S(p, L) = \frac{1}{2\pi|\vec{p}|} \int d\omega e^{-i\omega L} \frac{1}{m_S^2 - p^2 - 2|\vec{p}|\omega + \omega^2 - im_S \Gamma_S}. \quad (A6)$$

For $\vec{p}^2 > m_S^2 - p^2$, the residue integration method gives:

$$D_c^S(p, L) = \frac{i}{2|\vec{p}| \sqrt{\vec{p}^2 + p^2 - m_S^2 + im_S \Gamma_S}} e^{-i(|\vec{p}| - \sqrt{\vec{p}^2 + p^2 - m_S^2 + im_S \Gamma_S})L}. \quad (A7)$$

**Appendix 3**

Once more substituting the standard integral representations for the scalar field propagator and the delta function into the integral in formula (22), we get

$$D_c^S(p, T) = \frac{1}{(2\pi)^5} \int dzdkdt e^{ipz} e^{ikz} e^{i(\vec{p} - |\vec{p}|)t} \frac{1}{m_S^2 - k^2 - im_S \Gamma_S}. \quad (A8)$$

To evaluate this integral we make the change of variable $t = \omega/\beta$, then integrate with respect to $z$ and $k$ similar to (A2), (A3) and arrive at the result

$$D_c^S(p, T) = \frac{1}{2\pi\beta} \int d\omega e^{-i\omega T} \frac{1}{m_S^2 - p^2 - 2p^n \omega - \omega^2 - im_S \Gamma_S}. \quad (A9)$$

The residue integration method gives:

$$D_c^S(p, T) = \frac{i}{2\beta \sqrt{(p^n)^2 + m_S^2 - p^2 - im_S \Gamma_S}} e^{i(p^n - \sqrt{(p^n)^2 + m_S^2 - p^2 - im_S \Gamma_S})T}. \quad (A10)$$
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