Corrigendum: The quantum phase slip phenomenon in superconducting nanowires with a low-Ohmic environment

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On the second page, after equation (1), the statement ‘…where \(R_N\) is the normal state resistance, \(R_Q = h/(2e) = 6.45\) k\(\Omega\) is the superconducting quantum resistance, …’ contains a misprint. The correct expression is \(R_Q = h/(2e)^2 = 6.45\) k\(\Omega\).
The quantum phase slip phenomenon in superconducting nanowires with a low-Ohmic environment

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Abstract

In a number of recent experiments it has been demonstrated that in ultra-narrow superconducting channels quantum fluctuations of the order parameter, alternatively called quantum phase slips, are responsible for the finite resistance well below the critical temperature. Acceptable agreement between those experiments and the models describing quantum fluctuations in quasi-one-dimensional superconductors has been established. However, the very concept of phase slip is justified when these fluctuations are relatively rare events, meaning that the effective resistance of the system should be much smaller than the normal state equivalent. In this paper we study the limit of the strong quantum fluctuations where the existing models are not applicable. In the particular case of ultra-thin titanium nanowires, it is demonstrated that below the expected critical temperature the resistance does not demonstrate any trend towards the conventional for a superconductor zero-resistivity state even at negligibly small measuring currents. The application of a small magnetic field leads to an unusual negative magnetoresistance, which becomes more pronounced at lower temperatures. The origin of the negative magnetoresistance effect is not clear.

(Some figures may appear in colour only in the online journal)

1. Introduction

Since the early years of superconductivity studies it has been noticed that any superconducting transition $R(T)$ always has a finite width. Very often sample inhomogeneity is the dominating factor. However, with refinement of fabrication methodology it became clear that even in the most homogeneous samples the $R(T)$ transition is not infinitely narrow. The broadening of the $R(T)$ dependence is most pronounced in systems with reduced dimensions where the thermodynamic fluctuations have a larger effect. In the particular case of quasi-one-dimensional (1D) channels, fluctuation-driven phase slips, the concept first introduced by Little in 1967 [1], are responsible for the finite resistivity in a narrow region below the critical temperature [2, 3] $R(T) \sim \exp(-F_0/k_BT)$, where $F_0$ is the condensation energy of the smallest statistically independent volume of the wire. Soon after formulation of the model [2, 3], experiments [4, 5] confirmed the validity of the concept of the thermally activated phase slips (TAPS).

At lower temperatures the number of thermally activated phase slips is exponentially suppressed and no measurable resistance should be expected well below the critical temperature. However, later experiments [6–11] in extremely narrow superconducting nanowires have demonstrated the finite resistance even at temperatures $T \ll T_c$. The effect has been associated with quantum fluctuations of the order parameter, alternatively called quantum phase slips (QPS) [12, 13]. Yet another confirmation of the QPS reality came from the experiments studying the persistent current in nanorings [14] resulting in building a quantum two level system: qubit [15].

Though the quantum fluctuation phenomenon has already received experimental confirmation from several independent
sources, the physics behind it is still under debate. Of special interest are the recent theoretical developments predicting that a QPS junction is dual to a Josephson junction [16–18]. Contrary to conventional transport measurements with low-Ohmic contacts [6–11], if a nanowire governed by QPSs (‘QPS junction’) is truly current biased, then one should expect the development of the insulating state, the Coulomb blockade [19], in full accordance with a Josephson junction in the regime of Bloch oscillations [20, 21, 16].

The mandatory prerequisite of such a non-trivial experimental configuration is the high enough rate of QPSs being of the same order as the experimentally observed Coulomb gap. Contrary to such a sample current biased through high-Ohmic electrodes, a similar superconducting nanowire of the same diameter, but with low-Ohmic probes (e.g. superconducting), should not demonstrate the Coulomb gap, but its $R(T)$ and $I–V$ dependences should show no traces of ‘conventional’ superconductivity either. The objective of this paper is to study exactly the limit of extremely narrow nanowires where the QPS rate is high enough to significantly affect the transport properties.

2. Theory background

The impact of the quantum fluctuations on the shape of the superconducting transition is qualitatively described by the expression, similar to the TAPS expression [13],

$$R(T) \sim \exp(-F_0/h\Gamma_{\text{QPS}}),$$

where the rate of QPS is

$$\Gamma_{\text{QPS}} \approx \frac{R_Q \Delta(T)}{R_N} \frac{L}{\hbar} \left(\frac{\xi(T)}{\xi}\right)^2 \exp(-S_{\text{QPS}}),$$

where $R_N$ is the normal state resistance, $R_Q = h/2e = 6.45 \text{k}\Omega$ is the superconducting quantum resistance, $L$ is the length of the wire, and $\Delta(T)$ and $\xi(T)$ are the temperature-dependent superconducting energy gap and coherence length. $S_{\text{QPS}} = A(R_Q/R_N)L/(\xi(T))$ is the QPS action, where constant $A \approx \infty$ and cannot be more precisely defined from the theory [12]. It is the only true fitting parameter of the model; other parameters can be derived from the experimental data. For our dirty limit samples $\xi \ll l$ the superconducting coherence length can be estimated as $\xi \approx \sqrt{\xi_0(T)}$, where $\xi_0 \approx h\nu_F/\Delta(T)$ is the coherence length in the clean limit $\xi_0 \gg l$ and $\nu_F$ is the Fermi velocity. Utilizing the text-book expression for the normal state resistance of a wire with cross-section $\sigma$, length $L$ and resistivity $\rho_N$, one comes to a conclusion that the effective resistance of a QPS-governed system exponentially depends on the sample parameters $R(T) \sim \exp(-\sigma\sqrt{\tau_c}/\rho_N)$. Hence, thin nanowires of low-$\tau_c$ superconductors with high resistivity (in normal state) are of advantage for observation of a pronounced contribution of the QPS effect.

It should be noted that the model [12] is based on the assumption that quantum fluctuations are relatively rare events. Or in other terms, the corresponding QPS-related effective resistance of the nanowire should be much smaller than the normal state resistance $R(T \ll \tau_c) \ll R_N$. As it comes from the formulated above objective of the paper, the limit of strong fluctuations violates this requirement. Unfortunately, to

our best knowledge, the limit of strong quantum fluctuations has not yet been properly treated theoretically. Hence, expression (1) can be considered only as a certain guideline. As will be shown below, indeed when the contribution of the QPS effect on the shape of the $R(T)$ transition is already noticeable, the further reduction of the nanowire cross-section $\sigma$ leads to complete flattening of the $R(T)$ dependence not described by the model [12].

3. Results and discussion

The research related to mesoscopic superconductivity is limited by the fabrication technology. In the particular case of the QPS effect, to reach the regime of interest $h\Gamma_{\text{QPS}} \lesssim \Delta$ the nanowire should have a very small cross-section. For example, for aluminum, a cross-section of below 10 nm (still maintaining the high level of uniformity!) is mandatory [9]. Following equation (1), for materials like Nb, Sn or Pb with a relatively high critical temperature one gets even more pessimistic estimations approaching the 1 nm scale. This note explains why in the particular case of niobium no traces of the QPS effect have been observed down to 7 nm scales [22]. With proper material selection the extremely tough fabrication requirements can be somehow relaxed. From our previous studies we have already learnt that in titanium the QPS effect is observable at sub-40 nm scales [11, 14], which is achievable with the standard e-beam lithography technique. In addition, titanium is an easy to work material and the extended microscopic and elemental analysis reveals no severe structural defects or/and impurities [11]. For the purpose of the present work we prepared several titanium nanowires with the length $L = 10 \mu m$ and the effective diameter $\sqrt{\sigma}$ between 27 and 48 nm. The structures were fabricated with standard lift-off e-beam lithography, and the titanium was deposited in a UHV e-beam evaporator at a residual pressure of $\sim5 \times 10^{-9} \text{mbar}$. The substrate covered with an exposed PMMA/MAA mask was cleaned with low-energy $O_2$ plasma immediately before the metal evaporation. Based on the measured resistivity $\rho_N = 0.5 \times 10^{-6} \Omega \text{m}$, which is comparable with the tabulated value for the clean bulk titanium, one may conclude that the quality of the metal thin film is acceptable. Previous TEM and TOF-ERDA analyses of titanium nanostructures fabricated using identical conditions and parameters confirmed the high material quality; the highest concentration of foreign inclusions inside the titanium matrix corresponds to $\sim0.4 \text{at.\%}$ of oxygen [11]. This low concentration of impurities, combined with TEM analysis, disables any speculations about the presence of sample inhomogeneities which can significantly broaden the $R(T)$ dependence. The SPM analysis of the studied samples (figure 1) confirmed the ‘standard’ sample quality with the surface roughness of about several nanometres [11, 14, 23, 24]. The mean free path $l$ for the wire can be evaluated from the measured normal state resistance $R_N$ and the known material constant $l_{\text{PNI}} \approx 10 \times 10^{-16} \Omega \text{m}^2$ varying slightly in different literature sources. For our samples, estimations give the mean free path $l \approx 1–2 \text{nm}$. The result agrees well with the SPM and TEM measured grain size of roughly 2–3 nm.
In the thinnest samples the zero-biased differential resistance some residual ‘critical current’ peculiarities can be traced. 

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R(T) dependences (figure 3) for the thickest samples with pronounced QPS contribution (figure 4). The double shape of the transition Tc is not within the scope of the present paper. Yet one more interesting observation is the negative magnetoresistance (nMR) which is observed in the thinnest samples at 20 mK [25]. For sufficiently thick nanowires with an effective diameter σ ≳ 40 nm the shape of the R(T) dependences follows our earlier findings: pronounced superconducting transition with the low temperature data R(T) < R0/10 which can be described by the QPS model [12] with the realistic set of fitting parameters [11]. However, the thinner the sample, the less the resistance R(T) drops below the certain ‘critical temperature’ (figure 2). It should be noted that for titanium the critical temperature Tc decreases with the decrease of the cross-section σ. The size dependence of the critical temperature for low dimensional superconductors is a well-known effect, though a commonly accepted explanation of the phenomenon Tc, necessary for the theory fitting (equation (1)), is rather problematic due to the strongly broadened R(T) transition. Nevertheless, for the thinnest samples σ ≳ 35 nm no set of realistic QPS model parameters can fit the experimental data (figure 2). For these ultra-thin samples, the experimental R(T) dependency is so weak that the model [12] applicability criterion R(T) ≪ R0 is not satisfied down to the lowest experimentally obtainable temperatures. 

At temperatures well below the critical temperature T ≪ Tc the dV/dI(T) dependences (figure 3) for the thickest studied samples demonstrate the conventional destruction of the zero-resistance state by current with the well-pronounced critical current IC. The double shape of the transition (e.g. ∼18 nA and ∼27 nA for the 48 nm sample) presumably originates from the slightly different effective cross-section of the samples at the node regions. With the decrease of the nanowire cross-section, as expected from the R(T) dependences, the zero-resistance state disappears and only some residual ‘critical current’ peculiarities can be traced. In the thinnest samples the zero-biased differential resistance dV/dI (I → 0) very slightly increases, indicating the presence of a weak Coulomb blockade. Observation of a pronounced Coulomb effects requires the true current biasing (including the high frequencies!) [19] and this experimental realization is not within the scope of the present paper.

Yet one more interesting observation is the negative magnetoresistance (nMR) which is observed in the thinnest samples with pronounced QPS contribution (figure 4). The effect increases with lowering the temperature. A similar phenomenon has been reported in aluminium [10], niobium and MoGe [28] and lead [29] nanowires. The origin of the nMR in these quasi-1D channels is still under debate. In the case of niobium and MoGe it was conjectured that...
some rogue magnetic moments might be present, and their pair breaking contribution, active at lower fields, is suppressed by higher fields leading to the observed nMR [28]. However, this mechanism needs an independent proof of the existence of these magnetic moments, and cannot explain the nMR in such material as aluminum [10], where it is a well-known fact that the majority of magnetic impurities can obtain a non-negligible magnetic moment only at relatively high concentrations [30]. Another explanation of the nMR phenomenon deals with the magnetic field suppression of the charge imbalance accompanying each phase slip event [31]. A somewhat related mechanism, capable of providing nMR, deals with more effective suppression of superconductivity in (wider) electrodes affecting the phase slip formation in the (thinner) nanowire [32].

4. Conclusions

$R(T, B)$ and $V$–$I$ characteristics of titanium nanowires with sub-50 nm diameters were measured. The shape of the $R(T)$ dependence for the ‘not-too-narrow’ samples with effective diameters $\sqrt{\sigma} \gtrsim 40$ nm confirms the earlier findings: a pronounced superconducting transition with the low temperature data $R(T) < R_N/10$ which can be described by the quantum phase slip model [12]. In thinner samples, the $R(T)$ transitions dramatically flatten disabling any comparison with the existing fluctuation models, which assume that the phase slips are still rare events and hence the effective resistance should be much smaller than the normal state resistance. The $dV/dI(I)$ dependences confirm the $R(T)$ data conclusion about the absence of the truly zero-resistance state in the thinnest samples. The negative magnetoresistance is observed, while the origin of the effect is not clear.

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