On the electron-positron energy asymmetry in

\[ K_L \rightarrow \pi^0 e^+e^- \]

Dao-Neng Gao\(^\dagger\)

*Interdisciplinary Center for Theoretical Study and Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026 China*

**Abstract**

Motivated by some new experimental and theoretical efforts, we update the theoretical analysis of the electron-positron energy asymmetry, which arises, in \( K_L \rightarrow \pi^0 e^+e^- \), from the two-photon intermediate state in the standard model. It is found that the measurement of this asymmetry in future experiments may increase our understanding of the \( K_L, S \rightarrow \pi^0 e^+e^- \) decays, which would thus provide some useful information on quark flavor physics. Meanwhile, in the standard model the electron-positron energy asymmetry in the decay of \( K_S \rightarrow \pi^0 e^+e^- \) is expected to be vanishingly small, therefore the asymmetry in \( K_S \rightarrow \pi^0 e^+e^- \) might be a very interesting quantity to explore new physics scenarios.

\(^\dagger\) E-mail: gaodn@ustc.edu.cn
The flavor-changing neutral-current process $K_L \to \pi^0 e^+ e^-$ has been recognized as one of the most interesting rare kaon decays for a long time \cite{11 2}. It is known that the decay rate of the transition is given by a sum of comparable CP-conserving, direct and indirect CP-violating contributions \cite{3 4}, which in general leads to some difficulties in carrying out an accuracy analysis of the decay. Very recently, the authors of Ref. \cite{5}, based on two new experimental results from NA48 Collaboration: the first observation of $K_S \to \pi^0 e^+ e^-$ \cite{6} and the precise measurement of the $K_L \to \pi^0 \gamma \gamma$ spectrum \cite{7}, have argued that the CP-conserving part is essentially negligible for the rate of $K_L \to \pi^0 e^+ e^-$, and the standard model (SM) contribution to its branching ratio has been predicted as $BR(K_L \to \pi^0 e^+ e^-)_{\text{SM}} \approx 3 \times 10^{-11}$, which is dominated by CP-violating components with about 40% from the direct CP-violating amplitude, through the interference with the indirect CP-violating one. Recalling the new experimental upper bound $BR(K_L \to \pi^0 e^+ e^-) < 2.8 \times 10^{-10}$ (90% C.L.) \cite{8}, observation of the decay rate substantially higher than the SM expectation would therefore signal new physics. On the other hand, in order to deeply understand this decay mode, many interesting observables besides the decay rate of $K_L \to \pi^0 \ell^+ \ell^-$ ($\ell = e$ and $\mu$), such as the muon polarization effects and the lepton energy asymmetries, have been studied in the past literature \cite{9 10 11 12}. Motivated by the new experimental and theoretical efforts mentioned above, in this paper we will update the theoretical analysis of the electron-positron energy asymmetry in the decay of $K_L \to \pi^0 e^+ e^-$. 

The general invariant amplitude for $K_L(p) \to \pi^0(p_\pi)e^+(p_+)e^-(p_-)$ in terms of the scalar, pseudoscalar, vector, and axial-vector form factors, denoted by $F_S, F_P, F_V,$ and $F_A$, can be parameterized as \cite{13 14}

\[ \mathcal{M} = F_S \bar{e}e + i F_P \bar{e} \gamma_5 e + F_V \bar{p}^\mu \gamma_\mu e + F_A \bar{p}^\mu \gamma_\mu \gamma_5 e, \]

where $p, p_\pi, p_\pm$ are the four-momenta of $K_L, \pi^0$, and $e^\pm$, respectively. The differential decay rate in the $K_L$ rest frame is

\[ \frac{d\Gamma}{dE_+ dE_-} = \frac{1}{2^{1+2}} \left[ |F_S|^2 \frac{1}{2} (s - 4m^2_\pi) + |F_P|^2 \frac{1}{2} s + |F_V|^2 m^2_K (2E_+ E_- - s/2) + |F_A|^2 m^2_K (2E_+ E_- - s/2 + 2m^2_\pi) + 2 \text{Re}(F_S^* F_V) m_\pi m_K (E_+ - E_-) + \text{Im}(F_P F_A^*) m_\pi (m^2_\pi - m^2_K - s) \right], \]

where $s = (p_+ + p_-)^2$ and $E_\pm$ denotes the energy of $e^\pm$ in the $K_L$ rest frame. One can find that, the terms which are antisymmetric under the exchange of $E_+ \leftrightarrow E_-$ in Eq. (2) will lead to an asymmetry in the electron-positron energy distribution

\[ A = \frac{N(E_+ < E_-) - N(E_+ > E_-)}{N(E_+ > E_-) + N(E_+ < E_-)}. \]

It is convenient to introduce two dimensionless variables, $z = s/m^2_K$, and $\theta$, the angle between the three-momentum of the kaon and the three-momentum of the $e^-$ in the dilepton rest frame, to rewrite Eq. (2) as \cite{14}

\[ \frac{d\Gamma}{dz d\cos \theta} = \frac{m^6_K \beta \lambda^{1/2} (1, z, \pi^2)}{2^8 \pi^3} \left\{ \left| \frac{F_S}{m_K} \right|^2 z \beta^2 + \left| \frac{F_P}{m_K} \right|^2 z + \frac{|F_V|^2}{4} \frac{1}{4} \lambda (1, z, \pi^2) (1 - \beta^2 \cos^2 \theta) \right\}. \]
\[
F_{V} = -\frac{\alpha G_{F}}{2\pi m_{K}^{2}} \left[ G_{F} m_{K}^{2} (a_{S} + b_{S} z) + W_{S}^{\pi\pi}(z) \right],
\]

where \(a_{S}\) and \(b_{S}\) encode local contributions up to \(O(p^{6})\) in chiral expansion, and the non-analytic function \(W_{S}^{\pi\pi}(z)\), which is generated by the \(\pi\pi\) loop and can be completely determined in terms of the physical \(K \to 3\pi\) amplitude, is known to be very small due to the \(\Delta I = 3/2\) suppression of the \(K_{S} \to \pi^{+}\pi^{-}\pi^{0}\) amplitude \[15\]. Therefore as a good approximation, Eq. (9) can be simplified as

\[
F_{V}^{S} = -\frac{\alpha G_{F}}{2\pi}(a_{S} + b_{S} z).
\]
Motivated by vector meson dominance (VMD), we have [15]

$$b_S = \frac{a_S}{r_V},$$ (11)

where $r_V = m_V/m_K$, and $m_V$ is the vector meson mass. The first experimental evidence of $K_S \rightarrow \pi^0 e^+e^-$ has been reported by NA48 Collaboration [6]:

$$\text{Br}(K_S \rightarrow \pi^0 e^+e^-)_{m_{ee}>165\text{MeV}} = (3.0^{+1.5}_{-1.2} \pm 0.2) \times 10^{-9},$$ (12)

which leads to the determination of $a_S$ [6]

$$|a_S| = 1.06^{+0.26}_{-0.21}.$$ (13)

The direct CP-violating amplitude for $K_L \rightarrow \pi^0 e^+e^-$ is dominated by short-distance dynamics and is calculable with high accuracy in perturbation theory [5, 16, 17]. Thus the CP-violating vector form factor including indirect and direct CP-violating parts can be written as

$$(F_V^L)^{\text{CPV}} = -\frac{\alpha G_F}{2\pi} \left( 1 + \frac{z}{r_V} \right) \left[ a_S|\epsilon|e^{i\phi_\epsilon} - i\frac{4\pi y_{\gamma V}(\mu)}{\sqrt{2}r_1}\text{Im}\lambda_t \right],$$ (14)

where $|\epsilon| = (2.28 \pm 0.02) \times 10^{-3}$ and $\phi_\epsilon = 43.5^\circ$ [18] encode the indirect CP-violating contribution due to $K^0 - \bar{K}^0$ mixing, and the $y_{\gamma V}$ part in Eq. (14) is from the short-distance direct CP violation, $y_{\gamma V} = (0.073 \pm 0.04)\alpha(M_Z)$ with $\alpha(M_Z) = 1/129$ [16], and $\lambda_t = V_{ts}^*V_{td}$ with $\text{Im}\lambda_t = (1.36 \pm 0.12) \times 10^{-4}$ [19]. Note that the VMD relation [Eq. (11)] has been taken into account in deriving Eq. (14).

The CP-conserving amplitude for $K_L \rightarrow \pi^0 e^+e^-$ is generated by the long-distance transition $K_L \rightarrow \pi^0 \gamma^* \gamma^* \rightarrow \pi^0 e^+e^-$, which can contribute to both $F_S$ and $F_V$. The general invariant amplitude for the $K_L(p) \rightarrow \pi^0(p_\pi)\gamma(q_1, \epsilon_1)\gamma(q_2, \epsilon_2)$ decay is given by

$$A(K_L \rightarrow \pi^0 \gamma \gamma) = \frac{G_{S\alpha}}{4\pi} \epsilon_{\mu\nu}(q_1)\epsilon_{2\nu}(q_2)[A(y, z)(q_2^\mu q_1^\nu - q_1 \cdot q_2 g^{\mu\nu})$$

$$+ \frac{2B(y, z)}{m_K^2} (p \cdot q_1 q_2^2 p^\nu + p \cdot q_2 p^\mu q_1^\nu - p \cdot q_1 p \cdot q_2 g^{\mu\nu} - q_1 \cdot q_2 p^\mu p^\nu)],$$ (15)

where $y = p \cdot (q_1 - q_2)/m_K^2$, $z = (q_1 + q_2)^2/m_K^2$, and $|G_{S\alpha}| = 9.2 \times 10^{-6}$ GeV$^{-2}$. Within the framework of chiral perturbation theory, the amplitude $A(y, z)$ will receive non-vanishing contribution at $O(p^4)$ [20]; while the leading order contribution to $B(y, z)$ starts from $O(p^6)$, which has been extensively studied by including both the unitarity corrections and the local terms generated by vector resonance exchange [21, 22, 23, 24, 25, 26]. It is known that, via the transition $K_L \rightarrow \pi^0 \gamma^* \gamma^* \rightarrow \pi^0 e^+e^-$, the leading order $A$ amplitude only contributes to $F_S$, and the $B$ amplitude contributes to both $F_S$ and $F_V$ [31, 44, 10, 27, 28]. On the other hand, chiral symmetry enforces that the scalar form factor $F_S$ be proportional to $m_e$, thus one can find from Eqs. (4) and (6) that the electron-positron energy asymmetry $A$ due to the interference between $F_S$ and $F_V$ is suppressed by $m_e^2$, which is expected to be negligible,
as in the case of $K^+ \to \pi^+ e^+ e^-$ \cite{14, 29}. Therefore in the following we are only concerned about contributions from the $B$ amplitude to the vector form factor $F_V$, which are given by

$$
\mathcal{M}(K_L \to \pi^0 \gamma^* \gamma^* \to \pi^0 e^+ e^-)^B = \frac{2iG_F}{m_K^2} \int \frac{d^4 q}{(2\pi)^4} \frac{B(y,z)\bar{u}(p_-)\gamma_\mu v(p_+)}{q^2(Q-q)^2(p_- - q)^2} \times \left\{ p^\mu \left[ 2p \cdot q - 2p \cdot q - 4p \cdot q \cdot (p_+ - p_-) \right] + q^\mu \left[ 2p \cdot q \cdot (p_+ - p_-) - 2p \cdot q + 2p \cdot q \cdot q \right] \right\},
$$

where $Q = p_+ + p_-$, $s = Q^2$, and $q$ is the loop momentum for the internal photon. The amplitude $B(y,z)$ (which is actually independent of $y$) including unitarity corrections and local contributions at $O(p^6)$ reads \cite{23}

$$
B(y,z) = c_2 \left\{ \frac{4\pi^2}{z} F \left( \frac{z}{r_\pi^2} \right) + \frac{2}{3} \left[ 10 - \frac{z}{r_\pi^2} \right] \left[ \frac{1}{6} + R \left( \frac{z}{r_\pi^2} \right) \right] + \frac{2}{3} \ln \frac{m_\pi^2}{m_\rho^2} \right\} + \beta - 8a_V,
$$

where $c_2 = 1.1$ is the coefficient ruling the strength of the unitarity corrections from $K \to 3\pi$, $\beta = -0.13$, which is originated from the $O(p^6)$ local terms except the vector resonance contribution, and the $a_V$ part characterizes the $O(p^6)$ local contributions generated by the vector resonance exchange with $a_V = -0.46 \pm 0.05$ fixed in the recent NA48 experiment \cite{7}. The functions $F(z/r_\pi^2)$ and $R(z/r_\pi^2)$ are generated by $\pi$-loop diagrams, which can be defined as

$$
F(x) = \begin{cases} 
1 - \frac{4}{x} \arcsin^2 \left( \frac{\sqrt{x}}{2} \right) & x \leq 4, \\
1 + \frac{1}{x} \left( \ln \frac{1 - \sqrt{1 - 4/x}}{1 + \sqrt{1 - 4/x}} + i\pi \right)^2 & x \geq 4,
\end{cases}
$$

and

$$
R(x) = \begin{cases} 
-\frac{1}{6} + \frac{2}{x} - \frac{2}{x} \sqrt{4/x - 1} \arcsin \left( \frac{\sqrt{x}}{2} \right) & x \leq 4, \\
-\frac{1}{6} + \frac{2}{x} + \frac{1}{x} \sqrt{1 - 4/x} \left( \ln \frac{1 - \sqrt{1 - 4/x}}{1 + \sqrt{1 - 4/x}} + i\pi \right) & x \geq 4.
\end{cases}
$$

Since the integral in Eq. \cite{16} is logarithmically divergent, only its absorptive part contribution can be calculated unambiguously. Actually for the off-shell photons, the $B(y,z)$ amplitude corresponding to the on-shell photons, should be replaced by $B[y, z, q^2, (Q - q)^2]$. At present, there is no model-independent way to obtain the off-shell $K_L \to \pi^0 \gamma^* \gamma^*$ form factor. In analogy with the analysis of the $K_L \to \gamma^* \gamma^* \to \mu^+ \mu^-$ \cite{30}, the authors of Ref. \cite{5} proposed the following ansatz

$$
B[y, z, q^2, (Q - q)^2] = B(y,z) \times f[q^2, (Q - q)^2]
$$

\footnote{Actually $O(p^6)$ unitarity corrections to $K_L \to \pi^0 \gamma \gamma$ will induce a dependence on $y^2$ in the $A$ amplitude, which can also contribute to $F_V$ of $K_L \to \pi^0 e^+ e^-$; however, as pointed out in Ref. \cite{5}, this is numerically rather suppressed \cite{22, 23}.}
with the form factor
\[ f[q^2, (Q - q)^2] = 1 + a \left( \frac{q^2}{q^2 - m_V^2} + \frac{(Q - q)^2}{(q^2 - m_V^2)} \right) + b \frac{q^2(Q - q)^2}{(q^2 - m_V^2)[(Q - q)^2 - m_V^2]} \] (21)
to obtain the ultraviolet integral by imposing the condition
\[ 1 + 2a + b = 0. \] (22)

The parameters \( a \) and \( b \) are expected to be \( O(1) \) by naive dimensional chiral power counting, and in a special case for \( a = -b = -1 \), the form factor \( f \) will be identical to the one adopted in Ref. [11] for the \( K_L \to \pi^0 \gamma^* \gamma^* \) transition. Neglecting terms which are suppressed by powers of \( 1/m_V^2 \) and eliminating \( b \) by means of Eq. (22), one can get
\[ \mathcal{M}(K_L \to \pi^0 \gamma^* \gamma^* \to \pi^0 e^+ e^-)^B = F_V^{\gamma \gamma} p \bar{u}(p_-) \gamma_\mu v(p_+), \] (23)
where
\[ F_V^{\gamma \gamma} = \frac{G_F \alpha_s^2 B}{8\pi^2 m_K^2} p \cdot (p_+ - p_-) \left[ \frac{2}{3} \left( \ln \frac{r_\pi^2}{z} + i\pi \right) - \frac{1}{9} + \frac{4}{3}(1 + a) \right]. \] (24)

Since \( F_V^{\gamma \gamma} \) is proportional to \( p \cdot (p_+ - p_-) \), the energy asymmetry \( \mathcal{A} \) can be generated from the interference between \( F_V^{\gamma \gamma} \) and \((F_V^{L})_{CPV}\). Using the relation
\[ p \cdot (p_+ - p_-) = -\frac{m_K^2}{2} \beta \lambda^{1/2} (1, z, r_\pi^2) \cos \theta, \] (25)
on one can get
\[ \mathcal{A}(z) = \frac{m_K^7 \beta^2 \lambda^2 (1, z, r_\pi^2)(1 - \beta^2/2)}{2^{10} \pi^3} \text{Re}[\tilde{f}_V^{\gamma \gamma}(F_V^{L})_{CPV}]/(d\Gamma/dz), \] (26)
where \( \tilde{f}_V = -F_V^{\gamma \gamma}/p \cdot (p_+ - p_-) \), and \( d\Gamma/dz \) is the differential decay rate after integrating the angle \( \theta \) in Eq. (4) (neglecting the contributions from \( F_S, F_P \) and \( F_A \)).

Now the electron-positron energy asymmetry defined in Eq. (23) can be determined up to a free parameter \( a \), which is \( O(1) \) from the naive dimensional chiral power counting, however, cannot be fixed from the present theoretical and experimental studies.\(^2\) For the \( O(1) \) values of \( a \), the differential asymmetry \( \mathcal{A}(z) \) can be \( O(10^{-1}) \), which has been plotted in Fig. 1 with \( a = -1, 0, +1 \) and \( a_S \) and \( a_V \) fixed, respectively. It is found that the order of magnitude of the \( \mathcal{A}(z) \) in the present paper is consistent with the one given in Ref. [4]; however, the distribution of the differential asymmetry against \( z \) is not. We would like to give some remarks on this point here. First, we use \( a_V = -0.46 \) fixed in the new NA48 experiment [7] instead of \( a_V = -0.96 \) in [4]; the vector form factor of \( K_S \to \pi^0 e^+ e^- \) in Eq. (10) has been evaluated up to \( O(p^6) \) in chiral perturbation theory while only the leading

\(^2\)As pointed out in Ref. [5], one can expect to obtain some constraints on this parameter from the observation of \( K_L \to \pi^0 \ell^+ \ell^- \gamma \); however, the present experimental data is not accurate enough to extract any significant constraints [31].
order contribution was considered in [4]. Second, the vector form factor of $K_L \rightarrow \pi^0 e^+ e^-$ via the two-photon intermediate state in Eq. (24), which was obtained in [5], is inconsistent with the one in [4] for the parameter $a = -1$, where the form factor (21) is identical to the one adopted in [4]. As pointed out by the authors of [5], the main difference between their result and the one in [4] is that they did not find any singularity in the limit of $m_e \rightarrow 0$, and the lacking of this kind of singularity has been already noticed in [28].

The relevant contribution from the direct CP-violating component to the decay rate of $K_L \rightarrow \pi^0 e^+ e^-$ is mainly through the interference between it and the indirect CP-violating one, therefore it is easily understood that the asymmetry $A(z)$ does not depend significantly on the direct CP violation. The main uncertainty of the asymmetry, besides the one encoded in the parameter $a$, will come from the indirect CP-violating contribution due to $K^0 - \bar{K}^0$ mixing and the value of $a_V$. By taking into account the present experimental errors, we have shown the sensitivity of $A(z)$ to $a_V$ and $a_S$ in Fig. 2 and Fig. 3, respectively, with the fixed parameter $a = -1$. The similar distributions of the asymmetry as those in Figs. 2 and 3 can be obtained for $a = 0$ and $a = 1$.

Note that in plotting Figs. 1, 2, and 3, we are not concerned about the relative sign of $a_S$ and $G_S$, which will be sensitive to the sign of the asymmetry $A(z)$. The new NA48 experimental study of $K_S \rightarrow \pi^0 e^+ e^-$ [6] has only given the absolute value of $a_S$, the sign
Figure 2: The differential electron-positron energy asymmetry $A(z)$ as a function of $z$ with $|a_S| = 1.06$ and $a = -1$. The full line is for $a_V = -0.46$, the dotted-dashed line for $a_V = -0.51$, and the dashed line for $a_V = -0.41$.

of $a_S$, which however cannot be fixed from this measurement, is important to get the constructive or destructive interference between direct and indirect CP-violating components when computing the SM prediction to the total decay rate of $K_L \rightarrow \pi^0 e^+ e^-$. Theoretically, the authors of Ref. [5], using $\Delta I = 1/2$ isospin relation plus VMD argument, have established the sign of $a_S$ in terms of the sign of $G_8$. Since the indirect CP-violating amplitude is dominant in this transition, it is not surprising from Eq. (26) that the different relative sign of $a_S$ and $G_8$ will lead to the differential asymmetries with different sign, which has been shown in Fig. 4. As an example, we take the parameter $a = -1$ in plotting Fig. 4, and the same conclusions can be reached for $a = 0$ and $a = 1$. Thus the observation of the electron-positron energy asymmetry $A(z)$ in $K_L \rightarrow \pi^0 e^+ e^-$ in future experiments may also be useful to determine the relative sign of $a_S$ and $G_8$.

We have reexamined the SM contribution to the electron-positron energy asymmetry in $K_L \rightarrow \pi^0 e^+ e^-$, induced by the long-distance transition from the two-photon intermediate state, $K_L \rightarrow \pi^0 \gamma^* \gamma^* \rightarrow \pi^0 e^+ e^-$. The present analysis shows that the differential asymmetry $A(z)$ can be $O(10^{-1})$, which implies that more than $10^2$ events would give the signal of this asymmetry. If the experimental challenges posed by the so-called Greenlee background [32] can be overcome in observing this process, one can therefore expect to get some interesting information on quark flavor physics by studying the electron-positron energy asymmetry in
The electron-positron energy asymmetry $A(z)$ as a function of $z$ with $a_V = -0.46$ and $a = -1$. The full line is for $|a_S| = 1.06$, the dotted-dashed line for $|a_S| = 0.85$, and the dashed line for $|a_S| = 1.32$.

Figure 3: The differential electron-positron energy asymmetry $A(z)$ as a function of $z$ with $a_V = -0.46$ and $a = -1$. The full line is for $|a_S| = 1.06$, the dotted-dashed line for $|a_S| = 0.85$, and the dashed line for $|a_S| = 1.32$.

the $K_L \rightarrow \pi^0 e^+ e^-$ decay.

The electron-positron energy asymmetry $A(z)$ in $K_S \rightarrow \pi^0 e^+ e^-$ can be discussed in analogy with the above analysis of the decay $K_L \rightarrow \pi^0 e^+ e^-$. Because the $K_S \rightarrow \pi^0 e^+ e^-$ amplitude is dominated by the long-distance transition via one-photon exchange, and its form factors $F_S$ and $F_V$ receive no contribution from the two-photon intermediate state in the limit of CP invariance, $A(z)$ in this mode is expected to be vanishingly small in the SM. Thus the measurement of the electron-positron energy asymmetry in the $K_S \rightarrow \pi^0 e^+ e^-$ decay might be very interesting to probe new physics scenarios beyond the SM.

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Figure 4: The differential electron-positron asymmetry $A(z)$ as a function of $z$ with $a = -1$, $|a_S| = 1.06$, and $a_V = -0.46$. The full line is for the same sign of $a_S$ and $G_8$, and the dashed line for the different sign of $a_S$ and $G_8$.

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