Supplementary Materials for

Giant nonreciprocity of surface acoustic waves enabled by the magnetoelastic interaction

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Spin wave spectra in bilayer heterostructures

In this section we propose a dynamic theory that is applicable for any mono-domain magnetic ground state of a magnetic bilayer. We are interested in a case when the ferromagnetic layers are oriented in an antiparallel fashion and the uniaxial magnetic anisotropy axes are collinear in both layers. In the absence of an external magnetic field the magnetization vectors are naturally oriented in opposite directions, while an external magnetic field tilts the magnetization vectors creating a canted ground state. Calculations of the static magnetic states in such a system is quite cumbersome, but well known in literature (20).

A general theory of spin wave propagation in antiparallel magnetic layers has been developed in (11). Here we build on that derivation by developing the theory of magnetoelastic interactions. We consider two parallel ferromagnetic film planes. For simplicity we assume the films are composed of identical magnetic material and have identical thickness $L$. The layers are separated by a nonmagnetic spacer with thickness $d_s$. Also we assume that the layers are sufficiently thin ($L \ll \lambda$) with respect to spin wave wavelength $\lambda$.

Magnetic dynamics in each layer are governed by Landau-Lifshitz equations:

$$\frac{dM_i(t)}{dt} = \gamma B_{i}^{\text{eff}}(t, M_1, M_2) \times M_i(t),$$

(S1)
where \( i = 1,2 \) is the layer index, \( \gamma/(2\pi) \approx 28 \text{ GHz/T} \) is the gyromagnetic ratio, \( M_i \) is the magnetization in each layer and \( B_i^{\text{eff}} \) is the effective magnetic field acting on the \( i \)-th layer. For the sake of simplicity in the following derivation we assume that the magnetization is uniform across a layer, but can be different in each layer. The full derivation is discussed at length in (11).

Here we are interested in the small amplitude dynamics and we can decompose the magnetization into static and dynamic parts as \( M_i(t) = M_s(\mu_i + s_i e^{-i\omega t}) \), where \(|\mu| = 1\) is the vector pointing in the direction of the equilibrium magnetization, \( s_i \) is the spin-excitation vector, and \( \omega \) is the angular frequency of the spin-wave. By definition these vectors are orthogonal to each other: \( \mu_i \cdot s_i = 0 \). This expansion allows us to linearize Eq. S1 using the standard procedure discussed in (23, 24).

Assuming the lateral dimensions of the films are much larger than the distance between them, we can neglect the static part of the interaction between layers. Under this approximation we can find the equilibrium magnetization and the internal magnetic field \( B_i \) as:

\[
B_i \mu_i = B^{\text{ext}} - \mu_0 M_s (\vec{N}_0 + \vec{R}) \cdot \mu_i \tag{S2}
\]

where \( \vec{N}_0 \) and \( \vec{R} \) are the tensors of static demagnetization and uniaxial anisotropy.

The dynamic part of Eq. S1 can be written as:

\[
-i \omega_k s_i = \mu_i \times [\vec{\Omega}_k \cdot s_i + \omega_M \hat{R}^{i,j}_{k} \cdot s_j] \tag{S3}
\]

where \( \vec{\Omega}_k = [\gamma B_i \hat{I} + \omega_M (\vec{N}_k + \vec{R} + \lambda^{2}_{\text{ex}} k^2 \hat{I})] \) is the dynamic self-interaction tensor (24, 25), \( \vec{N}_k \) is the dynamic self-demagnetization tensor (23), \( \hat{I} \) is an identity matrix, \( \hat{R}^{i,j}_{k} \) is the mutual cross-demagnetization tensor which defines the dynamic interaction between the films, \( \lambda_{\text{ex}} \) is the inhomogeneous exchange length, \( j \neq i \), \( \omega_M = \gamma \mu_0 M_s \), and \( k \) is the wave vector of the spin wave. Wave vector dependence is implied for \( s_i \), although the index is dropped.

Equation S3 can be rewritten in a more compact form of a standard eigenvalue problem (24):

\[
-i \omega_k \hat{s} \omega_k \begin{pmatrix} j_1 \\ 0 \end{pmatrix}, \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \hat{\varphi} \cdot \begin{pmatrix} \vec{\Omega}_1 \\ \omega_M \hat{R}^{1,2}_{k} \\ \Omega_2 \end{pmatrix} \cdot \hat{\varphi} \cdot \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}, \tag{S4}
\]

where \( \omega_M = \gamma \mu_0 M_s, \hat{I}_i = \hat{e} \cdot \mu_i, \hat{e} \) is the Levi-Civita tensor, and

\[
\hat{\varphi} = - \begin{pmatrix} j_1 \cdot \hat{j}_1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ j_2 \cdot \hat{j}_2 \end{pmatrix}.
\]

The solution to Eq. S4 is obtained with standard numerical methods obtaining values of \( \omega \) and \( s_i \) as a function of \( k \).
To obtain the explicit form of tensors $\mathbf{N}_k$ and $\mathbf{R}_{k}^{ij}$ we fix the coordinate system as $k = k\mathbf{x}$ and $y$ is the normal to the film surface. In this coordinate system the self-demagnetization tensor (23) is $\mathbf{N}_k = p\mathbf{x} \otimes \mathbf{x} + (1 - p)\mathbf{y} \otimes \mathbf{y}$ with $p = (-1 + |k|L + e^{-|k|L})/|k|L$.

The mutual demagnetization tensor is $\mathbf{R}_{k}^{1,2} = g(\mathbf{x} \otimes \mathbf{x} - \mathbf{y} \otimes \mathbf{y}) + ig\text{sign}(k)(\mathbf{y} \otimes \mathbf{x} + \mathbf{x} \otimes \mathbf{y})$, where

$$g = \int_0^{L} \int_{L + d_x}^{2L + d_y} \frac{1}{2L} e^{-|k|(|y - y'|)} dy dy' = \frac{e^{-|k|L}(L + d_x)(-1 + e^{-|k|L})^2}{2|k|L}. \quad (S5)$$

Note that $\mathbf{R}_{k}^{1,2} = (\mathbf{R}_{k}^{2,1})^\dagger \neq (\mathbf{R}_{-k}^{1,2})$, which constitutes the necessary condition for spin wave nonreciprocity (3, 25-28).

**Magnetoelastic coupling**

The magnetoelastic interaction couples spin waves in the magnetic film and SAWs in the substrate. This coupling leads to a modification in the dispersion characteristics of the SAWs, ultimately changing the propagation behavior. Here we are interested in the modification of losses incurred by SAWs traveling in opposite directions.

A general theory of SAW/spin wave interactions has been developed by Verba et al. (2, 4). Here we employ several approximations to make the analytical calculations manageable. First, we consider the magnetic layer acoustically identical to the substrate material, i.e. we do not take into account the mass loading effect. In general, mass loading is important for SAW IDT matching, however, as the mass loading is a purely mechanical effect it does not contribute to nonreciprocity. Second, we assume that the magnetoelastic coupling energy is much smaller than other interaction energies in the system, which is practically always true for ferromagnets with strong magnetostriction.

In magnetostrictive materials, acoustic and magnetic systems are coupled via magnetoelastic interaction with characteristic energy density:

$$W^{me}(r) = \frac{b_{ijkl}u_{ij}(r)M_i(r)M_j(r)}{M_s^2}, \quad (S6)$$

where $b_{ijkl}$ is the magnetoelastic tensor and $\mathbf{u}$ is the mechanical strain-tensor of the SAWs (29). Here we assume the magnetoelastic coupling to be isotropic and uniform across the ferromagnetic sample $b_{ijkl} = b\delta_{ij}\delta_{kl} + \text{sym}$. Note that the energy density itself does depend on the direction and position in the sample.

The magnetoelastic interaction entangles SAW and spin wave modes. In the weakly coupled oscillator model, the spectrum of magnetoelastic waves can be found as described in (21) as Eq. 1 in the main text. The coupling coefficient can be found as an overlap integral of the SAW and spin wave mode

$$\kappa = \frac{2b}{\sqrt{\lambda_0}} \int_{-\infty}^{0} \mu(y) \cdot \hat{u}^\dagger(y) \cdot s(y) dy \quad (S7)$$
where $\dagger$ stands for Hermitian conjugation and the coefficients $\sqrt{A}$ and $\sqrt{Q}$ are the normalizing constants calculated below (4). In general, it is difficult to obtain useful explicit expressions for the coupling coefficient for an arbitrary configuration of the external magnetic field, magnetic anisotropy, and SAW propagation direction. Some particular cases for a bilayer magnetic film have been considered in (2). Here we use the closed form Eq. S7 and evaluate the integral numerically. The mechanical strain-tensor for a SAW propagating in the z-direction can be obtained as (29):

$$u_{zz} = e^{ik_R z}k_R^{-1}(k_R e^{\kappa i y} + a\kappa_t e^{\kappa_t i y})$$  \hspace{1cm} (S8)

$$u_{yy} = -i e^{ik_R z}k_R^{-2} (\kappa_t^2 e^{\kappa_t i y} + a k_R \kappa_t e^{\kappa_t i y})$$  \hspace{1cm} (S9)

$$u_{yz} = u_{zy} = \frac{1}{2} e^{ik_R z}k_R^{-2}(2k_R \kappa_t e^{\kappa_t i y} + a(\kappa_t^2 + k_R^2)e^{\kappa_t i y})$$  \hspace{1cm} (S10)

$$Q = |k_R^5|\rho c_t \left( 4ak_R + \frac{(k_R^2 + \kappa_t^2)}{2\kappa_t} + \frac{a^2(2\kappa_t^2 + k_R^2)}{\kappa_t^2} \right),$$  \hspace{1cm} (S11)

where $\zeta$ is the Rayleigh coefficient, $c_t$ and $c_l$ are transverse and longitudinal velocities, $k_R = \zeta \omega/c_t$ is the Rayleigh wavenumber, $\rho$ is the density, $a = 1/2k_R/(2 - \zeta^2)/\sqrt{1 - \zeta^2}$, $\kappa_t = |k_R|\sqrt{1 - \zeta^2}$ and $\kappa_l = |k_R|\sqrt{1 - c_t/c_l^2}$.

The magnetic part of Eq. S7 can be obtained in the approximation of a uniform magnetization distribution across the film thickness. The static part is obtained by solving Eq. S2 and using the following function in Eq. S7:

$$\mu(y) = \mu_1 \Pi_1(y) + \mu_2 \Pi_2(y),$$  \hspace{1cm} (S12)

where $\Pi_1 = \theta(y + L) - \theta(y)$, $\Pi_2 = \theta(y + 2L + d) - \theta(y + L + d)$, and $\theta(y)$ is the Heaviside step function. The dynamic part can be constructed analogously by solving Eq. S4 and using the following function: $s(y) = s_1 \Pi_1(y) + s_2 \Pi_2(y)$. The magnetic normalizing constant $A$ can be found from the expression (4, 25).

$$A = i \frac{2LM_x}{\gamma} \sum_{j=1,2} \mu_i \cdot (s_j^+ \times s_j).$$  \hspace{1cm} (S13)

Note that $A$ and $Q$ have the same dimensionality of the action linear density.

**Damping of magnetoelastic waves**

The operating principle of the magnetic SAW isolator is the direction selectivity of the SAW damping. This damping is due to the loss of energy in the magnetic system which is usually several order of magnitude larger than intrinsic SAW losses.

To take into account magnetic losses we substitute the magnetic eigenfrequency as $\omega_k \rightarrow \omega_k + i\Gamma_k$ where the decay rate is calculated as

$$\Gamma = \alpha_G \omega_k \frac{2LM_x}{\gamma A} \sum_{l=1,2} s_l^+ \cdot s^+.$$

\hspace{1cm} (S14)
Where $\alpha_G$ is the Gilbert damping constant. The linear magnetic losses of the SAWs can be approximately calculated from (7) as Eq. 2 in the main text.

Collectively based on the above calculation, we can calculate the damping caused by magnetoelastic coupling in SAWs. For our calculations we take the following parameters for the magnetic material: $\mu_0 M_s = 1.3$ T, $\gamma = 28$ GHz/T, $\alpha_G = 2 \times 10^{-2}$, $b = 9.38$ MJ/m$^3$, $L = 20$ nm, $d_s = 5$ nm, $b_a = 1.8$ mT, $\lambda_{ex} = 4.7$ nm. The elastic properties for LiNbO$_3$ are: $c_i = 2.8$ km/s, $c_t = 3.85$ km/s, $\rho = 4650$ kg/m$^3$.

Supplemental figures

Fig. S1. Nonreciprocal SAW transmission in FeGaB/Al$_2$O$_3$/FeGaB multilayer stack with growth field at 60°. (A) ADFMR plot with 863-MHz SAWs traveling in +z-direction (forward). (B) ADFMR plot with 863-MHz SAWs traveling in -z-direction (reverse).
Fig. S2. Schematic diagram of the sample holder for sputter deposition with in-situ magnetic field. Inset shows the angle relationship between the applied growth fields with respect to the SAW $k$ vector direction. Samples are physically placed in between the permanent bar magnets and measured field along the center line is about 200 Oe.
Fig. S3. Ferromagnetic resonance at 2 GHz. (A) Angle-dependence of FMR showing extreme anisotropy defined by the growth field at 60°. (B) Line cuts at angles near the optimal isolator conditions. Multiple resonances occur near 5 and 45 Oe, similar to that seen in ADFMR. In FMR the uniform mode is excited, a notable difference from ADFMR in which the spin waves are traveling.
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