LEP1 vs. Future Colliders:
Effective Operators And Extended Gauge Group

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ABSTRACT

In an effective Lagrangian approach to physics beyond the Standard Model, it has been argued that imposing $SU(2) \times U(1)$ invariance severely restricts the discovery potential of future colliders. We exhibit a possible way out in an extended gauge group context.

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1. INTRODUCTION

In studying departures from the Standard Model (SM), effective Lagrangians\cite{1,2} prove a convenient tool, since they unify the various manifestations of a given contribution. Usually an operator expansion is used and the various possible operators are ranked by increasing dimensionality. To any given order, only a finite number of operators need be considered, and this is true irrespective of the nature of the underlying SM extension (which only determines the coefficients).

Gauge invariance imposed on an effective Lagrangian and precise measurements at LEP1 have recently been shown\cite{3,4,5} to severely restrict the possible impact of new physics on observations at forthcoming colliders (LEP2, ...). The argument, with which we are in complete agreement, proceeds in two steps:

- In the framework of gauge theories, extra terms induced by new physics in an effective Lagrangian must at least obey the $SU(2)_L \times U(1)$ invariance. In particular, they must be expressible (above the scale of symmetry breakdown) in terms of $[SU(2)_L \times U(1)]$-invariant products of fields, including the SM scalar doublet(s).

- These operators can be classified by increasing dimensionality. If the new physics scale is high enough, only the lowest-dimensional operators will contribute.

Gauge invariance thus strongly limits the number of operators available for a given dimensionality; consequently, the typical effects expected at future colliders, such as new 3-vector vertices, are intimately related to modifications of typical LEP1 observables (widths, asymmetries, ...). When the analysis is limited to dimension 6 operators, and apart from a few blind directions, the LEP1 measurements\cite{6} tend to give stronger bounds on the coefficients of these operators than those expected from the direct observation of the 3-vector vertices.

We found it interesting to explore some limitations of this approach. Our goal was to see how the above predictions would be affected when the implicit
assumption of a very high mass scale for any new physics was relaxed.

We have chosen to study a specific example, built upon the extended group $SU(2) \times U(1) \times U(1)'$\textsuperscript{[7]}. This introduces essentially two new fields, namely an extra gauge boson ($B'_\mu$) and an extra scalar singlet ($\chi$) (an extra $\nu_R$ per generation is also needed, so as to avoid anomalies, but it plays no role in the present discussion). The mass of the extra gauge boson will characterize the scale of this part of the new physics. If this scale lies far beyond the one accessible at future colliders, the extra degrees of freedom may be integrated out in an effective lagrangian approach, which leads, for dimension-6 operators, to the same analysis as Ref.[4].

We have instead assumed the mass of the new gauge boson to be relatively close to the LEP2 scale.

One systematic way to treat this model would be to consider the higher orders in a dimensional expansion. This, however, quickly becomes intractable—in view of the number of new operators involved—and unreliable, since doubts arise about the convergence of the expansion (specially close to a resonance); it is then safer to bring in the full gauge group structure $SU(2) \times U(1) \times U(1)'$.

In the spirit of Ref.[4], we continue to use an effective Lagrangian approach (limited to dimension-6 operators) to parametrize any new physics beyond the $Z'$ (i.e. the heavy-mass eigenstate). The only difference lies in the fact that the effective operators are now to be classified following their invariance under the gauge group $SU(2) \times U(1) \times U(1)'$ rather than $SU(2) \times U(1)$.

At first sight, one might expect that extending the requested symmetry group would further restrict the choice of operators. It turns out that this is more than compensated for by the presence of the new degrees of freedom $Z'$ and $\chi$.

A typical example involves couplings of $W$ bosons. In the approach of Ref.[4], these can only arise from two dimension-6 operators, namely:

$$O_{WB} = \phi^+ \sigma^i \phi W^i_{\mu\nu} B^{\mu\nu}$$ (1)
\[ O_W = \frac{1}{3!} \epsilon_{ijk} W^i_\nu W^j_\lambda W^k_\mu. \] (2)

In the present case, we can obviously add to this list the operator:

\[ O_{WB'} = \phi^+ \sigma^i \phi W^i_\mu B'^{\mu \nu}. \] (3)

Such an operator is absent from the expansion of Ref.[4]. Nevertheless, ((3)) will generate anomalous couplings between the physical \( Z \) (now a mixture of \( W^3, B, B' \)) and \( W \)'s.

One needs to formally integrate out \( B' \) (and \( \chi \)) to compare the two approaches. The situation is most easily exemplified in terms of graphs. In particular, for \( ZW^+W^- \) coupling, we get contributions from:

Upon integrating out \( B' \) and \( \chi \) (the presence of the \( \chi \) field is implicit in the \( B' \) mass term), we get:

containing e.g. the new operator (of dimension 8 in the fields)

\[ \frac{1}{M_{B'}^2} \phi^+ \phi B_{\mu \nu} \phi^+ \tau^j \phi W^{j \mu \nu}. \] (4)

On top of other dimension-8 operators, higher dimensionalities will of course appear.
through the low-momentum expansion of the $B'$ propagator.

Let us now turn back to the full $SU(2) \times U(1) \times U(1)'$ and review briefly the parameters involved (we use $g_2, g_1, g'_1$ for the respective gauge couplings and $Y, Y'$ for the hypercharges $v = \sqrt{2} |\langle \phi \rangle|$, $V = |\langle \chi \rangle|$). The field content and quantum number assignments are detailed in Table 1.

We now include the additional operator (3) in the Lagrangian density:

$$\mathcal{L}_{WB'} = \frac{\varepsilon}{v^2} O_{WB'}.$$  \hfill (5)

We have followed Ref.[4] in the definition of $\varepsilon$.

In addition to 3-gauge boson couplings of interest for future colliders, this implies (after symmetry breaking) corrections to the kinetic energy of neutral gauge bosons. The $Z$-propagator is thus modified, with potential effects at LEP1. It is however easy to check that the propagator of the light $Z$ physical state is not directly affected by the $\varepsilon$ correction, but only by the combination $\varepsilon \theta_3$, where $\theta_3$ is the standard $Z-Z'$ mixing angle. As a result, the departure from a pure $SU(2) \times U(1) \times U(1)'$ is kept minimal once $\theta_3$ remains within its usual bounds$^5$.

The same cannot be said of the $e^+e^- \rightarrow W^+W^-$ cross section. The effective operator (3) manifests itself in two ways in this process:

- The currents are modified by the redefinition of the boson fields (e.g. the $B'$ content of $Z$)

- Direct contributions to the three boson vertices appear. In particular, the $Z'-W^+W^- (O(\theta_3))$ vertex already present in the extended $SU(2) \times U(1) \times U(1)'$ now receives an $O(\varepsilon)$ contribution. That coupling is not suppressed by $\theta_3$ and thus becomes competitive with the other contributions, with which it interferes. Its angular dependence makes it quite conspicuous.
2. OVERVIEW OF THE CALCULATIONS

The differential cross-section for the process $e^+e^- \rightarrow W^+W^-$ (see Ref.[9]) is

$$\frac{d\sigma(e^+e^- \rightarrow W^+W^-)}{d\cos \theta} = s^{1/2}(s/4 - M_W^2)^{1/2}|A|^2,$$

(6)

where $A$ takes into account the contributions from the four usual diagrams corresponding to the t-channel $\nu$ exchange and the s-channel $\gamma, Z, Z'$ exchanges

$$|A|^2 = \frac{1}{8\pi} \sum_{\alpha,\beta} \left( a^V_{\alpha} Spin_{\alpha,\beta} a^V_{\beta} + a^A_{\alpha} Spin_{\alpha,\beta} a^A_{\beta} \right),$$

(7)

with

$$a^V_{\alpha=\nu,\gamma,Z,Z'} = \left( \frac{g_2^2}{4t}, \frac{e^{\gamma} g^\gamma_{V(A)\gamma}}{s - M_Z^2}, \frac{e^{Z g^Z_{V(A)Z}}}{s - M_{Z'}^2} \right),$$

(8)

and

$$Spin_{\nu,\nu} = \left[ \frac{ut}{M_W^4} - 1 \right] \left[ \frac{t^2}{4s^2} + \frac{M_W^4}{s^2} \right] + \frac{t^2}{sM_W^2},$$

(9a)

$$Spin_{\nu,i} = \left[ \frac{ut}{M_W^4} - 1 \right] \left[ \frac{\kappa_i t}{4s} - \frac{M_W^2 t}{2s^2} - \frac{M_W^4}{s^2} \right] + \left(1 + \kappa_i\right) \left[ \frac{t}{2M_W^2} - \frac{t}{s} + \frac{M_W^2}{s} \right],$$

(9b)

$$Spin_{i,j} = \left[ \frac{ut}{M_W^4} - 1 \right] \left[ \frac{\kappa_i \kappa_j}{4} - \frac{M_W^2}{s} \left(1 + \kappa_i \kappa_j\right) \right] + \frac{3M_W^4}{s^2}$$

$$+ \left(1 + \kappa_i\right)\left(1 + \kappa_j\right) \left[ \frac{s}{4M_W^2} - 1 \right]; \quad i = \gamma, Z, Z'$$

(9c)

describing the angular dependence. The different couplings $e^i_{V(A)}$, $g^i$, $\kappa^i$ are defined through the matrix $S$ relating the physical neutral fields to the original gauge fields*:

$$\begin{pmatrix} W_3^\mu \\ B^\mu \\ B'^\mu \end{pmatrix} = S \begin{pmatrix} A^\mu \\ Z^\mu \\ Z'^\mu \end{pmatrix}$$

(10)

* $S$ is not unitary, as $\varepsilon_{W'B'}$ induces a non-canonical kinetic term requiring a rescaling of the fields.
by

\[ (g^\gamma, g^Z, g^{Z'}) = (g_2, 0, 0), S \]
\[ (g^\gamma(1 + \kappa^\gamma), g^Z(1 + \kappa^Z), g^{Z'}(1 + \kappa^{Z'})) = (g_2, 0, \varepsilon), S \]
\[ (e^\gamma_{L(R)}, e^Z_{L(R)}, e^{Z'}_{L(R)}) = (g_2 T_{e_{L(R)}}^3, g_1 Y_{e_{L(R)}}^B, g'_1 Y_{e_{L(R)}}^{B'}), S \]
\[ e_{V(A)} = \frac{1}{2}(e_R + (-)e_L). \]

3. DISCUSSION OF THE NUMERICAL RESULTS

We have not pursued a systematic search of the parameter space, but present here a simple example. While the parameters are obviously chosen to make our point clear, we have not attempted to maximize the effects to the extreme limits allowed by current data. We have taken a smallish value for the new coupling strength \( g'_1 (\lambda = g'_1/g_2 = 0.1) \), which allows for a relatively small \( Z' \)-mass (\( M_{Z'} = 300 \text{ GeV} \)) without dangerous direct contributions. In the same spirit, the \( Z-Z' \) mixing is kept small (\( \theta_3 \sim 0.008 \)) to control the indirect ones \(^8\).

As we mentioned above, all the effects of the additional operator \( O_{WB'} \) at LEP1 are suppressed by a factor \( \theta_3 \varepsilon \). We do not present here complete fits to the LEP1 data, but use instead the evolution of \( \sin^2 \theta_W |_{\text{eff}} \) as the dominant constraint \(^\dagger\). Taking values of \( \varepsilon \) between \(-0.2 \) and \( 0.2 \), we find \( \Delta \sin^2 \theta_W |_{\text{eff}} \leq 0.001 \sim 1\sigma \[^6\].\) The fact that the values for \( \varepsilon \) are larger than those found for the similar parameter \( \varepsilon_{WB}[^4] \) is just an illustration of the screening effect due to the small mixing angle \( \theta_3 \).

We have plotted the differential cross-section at 200 GeV, which corresponds approximately to the maximum of the \( WW \) cross-section. We also give a plot for 260 GeV, to show how the effect of \( \varepsilon \) is boosted by nudging the energy only a little closer to the pole of the \( Z' \).

\(^\dagger\) One can check that the same naïve approach applied to the parameter \( \varepsilon_{WB} \) of Ref.\[^4\] essentially reproduces their global fit.
We first make out some qualitative remarks. With $\varepsilon$ set to 0, the cross-section is indistinguishable from the SM one. It is an interesting question to know whether other channels might reveal the presence of the extended gauge group independently of gauge boson couplings. We have only checked the most obvious channel—$\mu$-pair production—and found that while the relative difference in cross-section was indeed sizeable, the overall value of this cross-section was very small, which might cause a problem with statistics.

The presence of the anomalous gauge boson couplings controlled by $\varepsilon$ reflects in a modification of the cross-section. This is not uniform, and a decrease in the backward part is compensated for by an increase in the forward one. A detailed study of the best observables to detect the effect depends obviously on the detailed properties of the detectors (angular resolution, ...) and falls beyond the scope of this paper. If one wants to use a simple number to quantify the departures from the SM, inspection of the curves suggests resorting to $A_{135} = \frac{\sigma_{\theta>135} - \sigma_{\theta<135}}{\sigma_{\text{total}}}$ (the angle 135 is close to optimal at 200 GeV, and should be adapted as a function of energy) or to the deviation at the maximum of the cross-section.

The numerical results are displayed in figs. 1 and 2 and gathered in table 2 for two values of the centre-of-mass energy (e.g. 200 and 260 GeV). As can be expected, the effects become more important when approaching the $Z'$ pole. We have not taken into account the radiative corrections for $WW$ production (but included them for LEP1) as we do not expect them to qualitatively alter our conclusions. Indeed they were shown in the Standard Model to be almost $\theta$-independent, depleting at most the irrelevant small $\theta$ region by 15%. Although the presence of the $Z'$ will slightly change their behaviour, this can be considered as a second-order correction for our discussion.

As we mentioned above, we have not done an exhaustive exploration of the parameter space, since our purpose was rather to illustrate our proposition than to give a full account of the effects of this kind of operators in the context of extended gauge groups.
4. CONCLUSIONS

The model we have examined here shows one possible way to evade the limits of Ref.[4]. It also gives some idea of the price to pay to achieve this goal. It requires both a relatively light $Z'$ and anomalous couplings of that $Z'$ (themselves attributed to unspecified new physics) which are quite sizeable.

What can we conclude from the above approach?

- Despite strong constraints arising from the (high-luminosity) precise measurements at LEP1 and lower energies, the introduction of a larger gauge group, broken at a scale higher but still comparable with the SM, considerably increases the allowed freedom.

- In particular, the anomalous couplings of an extra $Z$ boson are not considerably restricted at the LEP1 level, and may lead to important departures from SM expectations at energies reachable in the near future.

The present observations do not detract from the importance of Ref.[4], but stress the point that $e^+e^-$ colliders should be designed with enough flexibility to be operated as discovery machines.

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TABLE CAPTIONS

1: Matter content and charges assignment of the extended model $SU(2) \times U(1) \times U(1)'$.

2: $A_{135}$-values for the SM and for $SU(2) \times U(1) \times U(1)'$ with $\varepsilon = 0, \pm 0.2$. $A_{135}$ is defined as follows:

$$A_{135} = \frac{\sigma_{\theta>135} - \sigma_{\theta<135}}{\sigma_{\text{total}}}.$$
| Hypercharge | $\begin{pmatrix} u \\ d \end{pmatrix}_L$ | $u^c_L$ | $d^c_L$ | $\begin{pmatrix} \nu \\ e \end{pmatrix}_L$ | $\nu^c_L$ | $\phi^c_1$ | $\phi^c_2$ | $\chi$ |
|------------|-----------------|--------|--------|-----------------|--------|--------|--------|------|
| $Y$        | $\frac{1}{6}$   | $-\frac{2}{3}$ | $\frac{1}{3}$ | $-\frac{1}{2}$ | 1      | 0      | $\frac{1}{2}$ | 0    |
| $Y'$       | $\frac{1}{5}$   | $\frac{1}{5}$  | $-\frac{3}{5}$ | $-\frac{3}{5}$ | $\frac{1}{5}$ | 1      | $-\frac{2}{5}$ | 1    |

Table 1

| Model  | $\sqrt{s} = 200$ GeV | $\sqrt{s} = 260$ GeV |
|--------|---------------------|---------------------|
| SM     | -0.064              | 0.297               |
| $\epsilon = 0$ | -0.066              | 0.292               |
| $\epsilon = +0.2$ | -0.061              | 0.290               |
| $\epsilon = -0.2$ | -0.089              | 0.180               |

Table 2
FIGURE CAPTIONS

1) Detail of the unpolarized differential cross-section for the process $e^+e^- \rightarrow W^+W^-$ at $\sqrt{s} = 200$ GeV: the solid line shows the SM, the dashed one is for $\varepsilon = 0.2$ and the dash-dotted one for $\varepsilon = -0.2$; $\theta$ is the angle between $e^-$ and $W^+$. 

2) Same as fig.1 at $\sqrt{s} = 260$ GeV.
Figure 1

Figure 2