The leading non-zero electric moment of the nucleon strange-quark vector current is the mean square strangeness radius, \( \langle r_s^2 \rangle \). We evaluate the lightest OZI-allowed contribution to \( \langle r_s^2 \rangle \), arising from the kaon cloud, using dispersion relations. Drawing upon unitarity constraints as well as \( K^+N \) scattering and \( e^+e^- \rightarrow K\bar{K} \) cross section data, we find the structure of this contribution differs significantly from that suggested by a variety of QCD-inspired model calculations. In particular, we find evidence for a strong \( \phi \)-meson resonance which may enhance the scale of kaon cloud contribution to an observable level.

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The reasons for the success of the non-relativistic quark model (NRQM) in accounting for the static properties of low-lying hadrons remains one of the unsolved mysteries of non-perturbative QCD. It has been postulated that – as far as low-energy observables are concerned – constituent quarks of the NRQM effectively account for the QCD degrees of freedom inside hadrons. This suggestion may be tested, in part, by measuring observables which depend only on the sea quarks and gluons, such as the matrix element \( \langle N|\bar{s}\gamma_\mu s|N \rangle \). By now there exists a well-defined program of parity-violating (PV) electron-nucleus scattering experiments dedicated to the determination of this matrix element. The theoretical understanding of this matrix element is much less clear. To date, only two results from the lattice have been reported and they appear to conflict with the recent results for the strangeness magnetic form factor reported by the SAMPLE collaboration. The use of hadronic effective theory, in the guise of chiral perturbation theory, is also limited, as chiral symmetry does not afford an independent determination of the relevant low-energy constants. A third alternative – the use of QCD-inspired models – is equally problematic, and model predictions for the mean square strangeness radius and magnetic moment \( \langle \mu_s \rangle \) vary considerably in magnitude and sign (for a recent review of model calculations, see Ref.).

In this Letter, we analyze the strangeness radius using a fourth approach, namely, dispersion relations (DR’s). Our objective is to identify the hadronic mechanisms which govern the leading strangeness moments without relying on QCD-inspired nucleon models. To that end, we focus on the lightest OZI-allowed contribution, which arises from the so-called “kaon cloud” or \( K\bar{K} \) intermediate state. A variety of model calculations reported to date have assumed that (a) OZI-allowed processes, in the guise of virtual strange intermediate states, give the most important contributions to the leading strangeness moments; and (b) these processes are adequately described by truncating at second order in the strong hadronic couplings, \( g \). In the case of \( \langle r_s^2 \rangle \), the resultant predictions are generally smaller than would be observable in the parity-violating (PV) electron scattering experiments. Moreover, kaon cloud predictions are typically an order of magnitude smaller than those obtained with DR’s under the assumption of of vector meson dominance (VMD), which relies on the extraction of a large \( \phi NN \) coupling from analyses of the isoscalar EM form factors. Assumption (a) has been analyzed elsewhere and shown to be questionable. Regarding assumption (b), we find that the structure of the strangeness form factors differs significantly from the assumptions underlying kaon cloud models, particularly the validity of the second-order approximation. Using the kaon cloud as an illustrative case study, we show that the scale of OZI-allowed contributions depends critically on effects going beyond \( O(g^2) \), and that a proper inclusion of such effects may enhance the scale of OZI-allowed contributions to an experimentally detectable level. We also demonstrate the relationship between resonance and kaon cloud contributions, resolving a long-standing issue in this field.
Of the form factors which parameterize $\langle N|\bar{s}\gamma_{\mu}s|N\rangle$, we focus on the strangeness electric form factor $G_E^{(s)}$, for which we obtain our most reliable results. Since $G_E^{(s)}$ vanishes at $q^2 = 0$, we write a subtracted dispersion relation for this form factor and its leading, non-vanishing moment:

$$G_E^{(s)}(t) = \frac{t}{\pi} \int_{t_0}^{\infty} dt' \frac{\text{Im} \ G_E^{(s)}(t')}{t'(t' - t)} , \quad (1)$$

$$\langle r_s^2 \rangle = \frac{6}{\pi} \frac{dG_E^{(s)}}{dt} \bigg|_{t = 0} = \frac{6}{\pi} \int_{t_0}^{\infty} dt' \frac{\text{Im} G_E^{(s)}(t')}{t'^2} , \quad (2)$$

where $t = q^2$ and $t_0$ begins a cut along the real $t$-axis associated with a given physical intermediate state. The state having the lowest threshold $t_0$ is the $3\pi$ state, whose contribution is nominally OZI-violating. The lightest OZI-allowed contribution arises from the $KK$ state, for which $t_0 = 4m_K^2$. Following the analysis of Ref. [3], we express the $KK$ contribution to the absorptive part of the electric form factor as a product of the appropriate $KK \rightarrow N\bar{N}$ partial wave and the kaon strangeness vector current form factor:

$$\text{Im} \ G_E^{(s)}(t) = \text{Re} \left\{ \left( \frac{Q}{4m_N} \right) b_1^{1/2} \frac{1}{2}(t) F_{KK}^{(s)}(t)^* \right\} . \quad (3)$$

Here, $Q = \sqrt{t/4 - m_K^2}$, $F_{KK}^{(s)}$ parameterizes the matrix element $\langle 0|\bar{s}\gamma_{\mu}s|KK(\bar{K}k_1)\rangle$, and $b_1^{\lambda,\lambda'}$ is the $J = 1$ partial wave for $KK$ to scatter to the state $|N(\lambda)\bar{N}(\lambda')\rangle$ with $\lambda, \lambda'$ denoting the corresponding helicities.

The problem now is to determine $b_1^{1/2, 1/2}$ and $F_{KK}^{(s)}$ as reliably as possible. For $t \geq 4m_K^2$, the physical $NN$ production threshold, one may in principle use $KK \rightarrow NN$ data to determine the scattering amplitude. Alternatively, we note that in this kinematic region, the unitarity of the S-matrix implies that $|b_1^{\lambda,\lambda'}| \leq 1 \ intr. 3\] Given the present quality of $KK \rightarrow NN$ scattering data, it turns out to be more effective to insert the unitarity bound on $b_1^{1/2, 1/2}$ into Eq. (3). The corresponding bound on the contribution from $t \geq 4m_K^2$ to the dispersion integral of Eq. (3) is negligible (see Table 1).

To evaluate $b_1^{1/2, 1/2}$ in the unphysical region $4m_K^2 \leq t \leq 4m_K^2$, one must rely on some method of analytic continuation. The $O(q^2)$ (one-loop) model calculations of $G_E^{(s)}$ implicitly rely on an analytic expression to effect this continuation. The one kaon-loop approximation is equivalent to using a DR in which the $b_1^{\lambda,\lambda'}$ are computed in the Born Approximation (BA) and the kaon strangeness form factor taken to be point-like $[F_{KK}^{(s)}(t) = -1]$ [4]. The BA calculation yields an analytic expression for $b_1^{1/2, 1/2}$, which may be analytically continued to unphysical values of $t$. When computed in this approximation, however, the $b_1^{\lambda,\lambda'}$ violate the unitarity bound by a factor of four or more for $t \geq 4m_K^2$. This violation reflects the omission of important kaon rescattering and other short-distance hadronic mechanisms which render the scattering amplitude consistent with the requirements of unitarity. Given the magnitude of the unitarity violation at the physical threshold, one would infer that the BA represents a rather drastic approximation in the unphysical region and that truncation at $O(q^2)$ is questionable.

An alternate strategy, which we follow here, is to perform a fit to experimental $K^+N$ scattering amplitudes and analytically continue the results for the fit into the unphysical region. The success of this approach depends on the quality of the data, the kinematic range over which it exists, and the stability of the fit. It is advantageous to consider $K^+N$ (s-channel) amplitudes, since the analytic continuation may be performed without encountering problematic singularities occurring in the t-channel reaction $K^-N$. As experimental input, we use the recent phase shift analysis of the VPI-group [5]. The requisite $K^+N$ amplitudes of sufficient quality exists over the range $-8m_K^2 \leq t < 0$. We correspondingly expect our continuation to yield a credible estimate of the scattering amplitude for $0 \leq t \lesssim 8m_K^2$. Although this range does not include the entire unphysical region over which one must compute the dispersion integral, it is sufficiently broad for our present purposes. Indeed, the structure we find in the continued amplitude in this region appreciably affects the kaon cloud contribution to $\langle r_s^2 \rangle$.

We carry out the continuation by using backward dispersion relations. This method relies on the coincidence of s- and t-channel amplitudes in the kinematic domain: $\cos \theta_s = \cos \theta_t = -1$ for a given value of $t$, where $\theta_s$ and $\theta_t$ are the c.m. scattering angles. Thus, we may continue the s-channel amplitude to the unphysical region in $t$, and evaluate this amplitude with the corresponding t-channel amplitude: $A(\cos \theta_s = -1, t_{\text{phys}}) \rightarrow A(\cos \theta_t = -1, t_{\text{unphys}}) = A(\cos \theta_t = -1, t_{\text{unphys}})$. In performing the continuation of the backward s-channel amplitude, we follow conventional procedures [6-9] and work with the discrepancy function, $\Delta A$:

$$\Delta A(t) = A(t) - A^{\text{pole}}(t) - \frac{p}{\pi} \int_{t_0}^{t_p} dt' \text{Im} A^{\exp}(t') \left( t' - t \right) \quad (4)$$

$$\int_{t_0}^{-\infty} dt' \text{Im} A^{\text{model}}(t') \left( t' - t \right) ,$$

where data exist in the range $t_0 \leq t \leq 0$, $A^{\exp}(t)$ gives the experimental amplitude in this range, $A^{\text{pole}}(t)$ gives a model for the high-energy part of the amplitude, and $A^{\text{model}}(t)$ denotes the $\Lambda, \Sigma$ pole terms, which are known exactly (the dependence on $\cos \theta_t$ is implicit). The function $\Delta A$ is analytic for real values of $t$ in the range $t_0 \leq t \leq 0$, whereas the full amplitude has a cut in this region. For $A^{\text{model}}$, we use a Regge exchange model fit [6]. The inferred high-$t$ behavior is $A^{\text{model}}(t) \propto |t|^\alpha$, with $\alpha = -1.2$, and the constant factor is determined from the highest-energy experimental point.

The continuation proceeds by replacing $A(t) \rightarrow \text{Re} A^{\exp}(t)$ in Eq. (3) and performing a fit to the experimental values for $\Delta A$ in the range $t_0 \leq t \leq 0$. This fit is
most easily accomplished by first carrying out a conformal map \( t \rightarrow z(t) \), transforming the domain of analyticity for \( \Delta A \) into a circle of unit radius \([3]\). A convergent expansion of the amplitude is then made using Legendre polynomials, \( P_n(z) \), and the coefficients determined from a fit to the experimental values for \( \Delta A \). In practice, we fit \( g(z(t)) = \Delta A/|t|^n \), which divides out the high energy behavior of \( \Delta A \) and improves the stability of the continuation. The discrepancy function \( \delta \) and, via Eq. (6), the full backward amplitude \( \delta \) is then obtained in the unphysical region by evaluating the Legendre expansion for \( z = z(t_{\text{unphys}}) \).

The final step in obtaining \( b_1^{1/2}, 1/2 \) is to expand \( A(\cos \theta_1 = -1, t_{\text{unphys}}) \) in partial waves, using the helicity basis as in Ref. [9]. Since the amplitude is known only for one value of \( \theta_1 \), a separation of the \( b_1^{J,\lambda} \) from the \( b_1^{J,\lambda} \) relies on several additional observations: (a) the only significant enhancements of the \( J \geq 2 \) partial waves occur via resonances; (b) the lightest \( J \geq 1 \) resonance having a non-negligible branching ratio to the \( KK \) state is the \( f_2(1270) \), whose mass lies near the upper end of the range in \( \sqrt{t} \) for which we expect our continuation to be valid; and (c) the use of a realistic parameterization for \( F_K^{(s)} \) in Eq. (3) dramatically suppresses contributions for \( \sqrt{t} \gtrsim 1.4 \) GeV/\( c \). We thus expect little contamination from the \( f_2(1270) \) and higher-lying resonances in the region near the \( KK \) threshold, which gives the dominant contribution to \( \langle r_2^2 \rangle \), and we correspondingly omit the \( J \geq 2 \) partial waves.

The remaining \( J = 0 \) and \( J = 1 \) partial waves may be separated by drawing on the work of Refs. [10,11]. In those analyses, the strength and approximate peak position of the \( J = 0 \) partial wave were determined from \( NN \) phase shift analyses using backward dispersion relations and its generalization.

As a check on our general procedure, we reproduce the results of Ref. [11,12] for the \( \pi N \) case. Furthermore, we have tested the sensitivity of our results for \( \langle r_2^2 \rangle \) to changes of the high-energy parameter \( \alpha \) and the truncation point of the fitted Legendre series, \( n \). We find that the structure of the continued amplitudes and resultant value of \( \langle r_2^2 \rangle \) change by \( \lesssim 15\% \) as \( \alpha \) and \( n \) are varied over reasonable ranges. We obtain the results shown in Fig. 1 and Table I using \( n = 6 \) and the value for \( \alpha \) taken from the Regge model fit \([3,13]\), \( \alpha = -1.2 \).

The results of this analysis are displayed in Fig. 1, where we plot the kaon cloud contribution to the spectral function for \( \langle r_2^2 \rangle \) as a function of \( t \). We give the spectral function computed using a point-like form factor (PFF) for \( F_K^{(s)} \) and two scenarios for \( b_1^{1/2}, 1/2 \): (I) the BA, and (II) analytic continuation (AC) of \( K^+N \) scattering amplitudes. We also show the upper limit on the spectral function generated by the unitarity bound on \( b_1^{1/2}, 1/2 \) for \( t \geq 4m_K^2 \). The curve obtained in scenario (II) contains a peak in the vicinity of the \( \phi(1020) \) meson, presumably reflecting the presence of a \( KK \leftrightarrow \phi \) resonance in the scattering amplitude. This structure enhances the spectral function over the result obtained at \( \mathcal{O}(g^2) \) near the beginning of the \( K \bar{K} \) cut. As \( t \) increases from \( 4m_K^2 \), the spectral function obtained in scenario (II) falls below that of scenario (I), ostensibly due to \( K \bar{K} \) rescattering which must eventually bring the spectral function below the unitarity bound for \( t \gtrsim 4m_K^2 \). As observed previously in Ref. [14], and as illustrated in Fig. 1b, the \( \mathcal{O}(g^2) \) approximation omits these rescattering corrections and consequently violates the unitarity bound by a factor of four or more even at threshold.

Turning to the kaon strangeness form factor, we note that the assumption of point-like behavior \( [F_K^{(s)}(t) \equiv -1] \) is poorly justified on phenomenological grounds. Data for \( e^+e^- \rightarrow KK \) indicate that the kaon EM form factor is strongly peaked for \( t \approx m_{\phi}^2 \) and falls off sharply from unity for \( t \gtrsim 2 \) (GeV/\( c \))^2. Although there exist no data for \( F_K^{(s)}(t) \), one expects it to behave in an analogous fashion to \( F_K^{EM}(t) \). We therefore follow Ref. [14] and employ a Gounaris-Sakurai (GS) parameterization for \( F_K^{(s)} \), which produces the correct normalization at \( t = 0 \) and a peak in the vicinity of the \( \phi \) meson. The corresponding spectral function is shown in Fig. 1b, where comparison is made with the spectral function computed using a point-like \( F_K^{(s)} \). The use of the GS parameterization significantly enhances the spectral function near the beginning of the \( K \bar{K} \) cut as compared with the point-like case, while it suppresses the spectral function for \( t \gtrsim 2 \) (GeV/\( c \))^2. We note that other parameterizations for \( F_K^{(s)} \), such as a simple \( \phi \)-dominance form, yield similar results for the spectral function.

The numerical consequences of using experimental \( K^+N \) amplitudes to determine \( b_1^{1/2, 1/2} \) and of employing a realistic kaon strangeness form factor are indicated in Table I. The first line gives the \( \mathcal{O}(g^2) \) kaon cloud prediction for \( \langle r_2^2 \rangle \), using a point-like \( F_K^{(s)} \) and \( b_1^{1/2, 1/2} \) computed in the BA. The second line gives results when the GS form factor and analytically continued fit to \( K^+N \) amplitudes for \( b_1^{1/2, 1/2} \) are used (AC/GS). In both cases the unitarity bound on \( b_1^{1/2, 1/2} \) is imposed for \( t \geq 4m_K^2 \). The AC/GS results were obtained by extending \( b_1^{1/2, 1/2} \) to \( 4m_K^2 \). Although we believe the continuation to be trustworthy only for \( t \lesssim 8m_K^2 \), and although the continued \( b_1^{1/2, 1/2} \) amplitude exceeds the unitarity bound for \( t \rightarrow 4m_K^2 \), the GS form factor suppresses contributions for \( t \gtrsim 8m_K^2 \), rendering the overall contribution from \( 8m_K^2 \leq t \leq 4m_K^2 \) negligible. Since we are presently unable to determine the relative phases of \( F_K^{(s)} \) and \( b_1^{1/2, 1/2} \) as a function of \( t \), the AC/GS results represent an upper bound on \( \langle r_2^2 \rangle \) and carry an uncertain overall sign [14]. Hence, we give only the magnitude in Table I.

With these caveats in mind, we observe that the use of a realistic, non-perturbative \( K \bar{K} \) spectral function increases the kaon cloud contribution to \( \langle r_2^2 \rangle \) by roughly a factor of three as compared to the \( \mathcal{O}(g^2) \) calculation.
Moreover, the non-perturbative result approaches the scale at which the PV electron scattering experiments are sensitive, whereas the $\mathcal{O}(g^2)$ prediction is too small to be seen. The enhanced scale of the kaon cloud depends critically on the presence of the resonance structure near $t = m_K^2$ in both $b_1^{1/2, 1/2}$ and $F_K^{(s)}$ (Fig. 1). This structure – obtained without relying on the a priori assumption of pole dominance in $G_K^{(s)}$ or $G_K^{(o)}$ – demonstrates how the resonance contribution arises out of the $K\bar{K}$ continuum and yields a kaon cloud result similar in magnitude to pure VMD predictions.

Although the kaon cloud is only one of the virtual hadronic states through which $s\bar{s}$ pairs may contribute to the strangeness form factors, our result illustrates the importance of including effects to all orders in $g$ when computing any of these hadronic contributions. In particular, we speculate that the results of the NRQM calculation reported in Ref. [1], which includes contributions from a tower of OZI-allowed intermediate states, will be significantly modified when effects beyond $\mathcal{O}(g^2)$ are included. Our analysis suggests that these effects, in the guise of resonant and non-resonant meson rescattering, determine the structure and scale of OZI-allowed contributions. The presently available scattering data is not likely to afford a model-independent, all-orders treatment of higher-mass contributions. Nevertheless, any model estimate of the higher-mass content of $s\bar{s}$ spectral functions must be evaluated by comparing its kaon cloud prediction with the results of the analysis reported here.

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| Moment | Scenario | $4m_K^2 \leq t \leq 4m_N^2$ | $4m_N^2 \leq t \leq Total$ |
|--------|----------|---------------------------|-----------------------------|
| $\langle r_2^2 \rangle [\text{fm}^2]$ | $\mathcal{O}(g^2)$ | $-0.017$ | $-0.007$ | $-0.024$ |
| $\langle |r_2^2| \rangle [\text{fm}^2]$ | AC/GS | $0.065$ | $0.001$ | $0.066$ |

TABLE I. Kaon cloud prediction for $\langle r_2^2 \rangle$ for the two scenarios discussed in the text. Second and third columns give contributions to the dispersion integral of Eq. (2) from integration regions corresponding to unphysical (second col.) and physical (third col.) t-channel scattering amplitudes. In both scenarios, the unitarity bound on $b_1^{1/2, 1/2}$ is imposed for $t \geq 4m_N^2$.

FIG. 1. $K\bar{K}$ contribution to the spectral function for $\langle r_2^2 \rangle$ in units of $(\text{GeV}/c)^{-4}$. Solid curve [(a) and (b)] gives AC/PFF results, while long-dashed curves show unitarity bound for $t \geq 4m_N^2$ using PFF (a) and GS form factor (b). Dash-dotted curve gives AC/GS spectral function (b). Dotted vertical line indicates physical $N\bar{N}$ production threshold.