Anomaly-induced charges in nucleons

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Abstract. We suggest a novel charge structure inside nucleons in electromagnetic field due to the chiral anomaly. We use Skyrmions, where nucleons appear as solitons of mesons, to calculate the charge distributions in a single nucleon and find that an additional non-integer charge proportional to the magnetic field would be produced. This might look surprising, but the magnitude of the induced charge is evaluated to be tiny enough to have not been observed yet.

1. Introduction

Recent advance in observations and experiments explores new effects of strong electromagnetic fields on fundamental particles. Since matter consists of baryons, electromagnetic properties of protons and neutrons are of most importance. Under the strong magnetic fields such as in neutron stars, supernovae and heavy ion collisions, tiny quantum effect of quantum chromodynamics may lead to an unveiled and significant consequence.

In this article, we investigate baryons under external electromagnetic fields. For describing the baryons, we use the Skyrme model [2] with Wess-Zumino-Witten (WZW) term [3, 4] including electromagnetism. The consequence is amazing: Nucleons in the external electromagnetic fields have anomalous charge distribution due to the chiral anomaly. Nonzero net charge, which is generally non-integer, is induced even for neutrons. Correspondingly, we will show that the Gell-Mann-Nishijima formula, \( Q = I_3 + N_B/2 \) (\( Q \): electric charge, \( I_3 \): the third component of isospin, \( N_B \): baryon number), has an additional term due to the quantum anomaly.

Figure 1 shows a schematic description of the phenomenon. Due to the anomalous interaction with a quark loop through the WZW term, nucleons (= Skyrmions) have an additional interaction to the electromagnetic field \( A_\mu \). Under the external electromagnetic fields, the anomalous coupling induces the additional electric charge.

2. Skyrmions and anomaly

We adopt the Skyrme model for a concrete illustration in this article. The essential idea of the Skyrme model is to unify baryons and mesons: baryons are described as topological solitons of mesons. This model, known to reproduce experimental data of nucleons within 30% accuracy, is suitable for our purpose. This is because we concentrate on the anomalous contribution to baryons, which is described by the coupling between mesons and photons shown in Fig. 1. Any baryons wearing mesonic clouds will follow our mechanism of anomalous charge generation.

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1 This is study in collaboration with M. Eto (Yamagata Univ.), K. Hashimoto (RIKEN & Osaka Univ.), H. Iida (YITP) and T. Ishii (Seoul National Univ.), and summarized in Ref. [1]
The action of two-flavor Skyrme model \[2, 5, 6\] coupled with electromagnetic field is

\[
S = \int d^4x \left( \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{mass}} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right),
\]

\[
\mathcal{L}_{\text{kin}} = \frac{F_\pi^2}{16} \text{tr}(\hat{R}_\mu \hat{R}^{\mu}) + \frac{1}{32 e_s^2} \text{tr}([\hat{R}_\mu, \hat{R}_\nu][\hat{R}^{\mu}, \hat{R}^{\nu}]),
\]

\[
\mathcal{L}_{\text{mass}} = \frac{F_\pi^2}{16} m_\pi^2 \text{tr}(U + U^\dagger - 2), \quad \hat{R}_\mu = D_\mu U U^\dagger,
\]

where \(m_\pi\) and \(F_\pi\) are the pion mass and the pion decay constant, respectively. \(e_s\) is a dimensionless constant and \(D_\mu U \equiv \partial_\mu U + ie A_\mu [q, U]\) with \(q \equiv \text{diag}(2/3, -1/3)\). The pseudoscalar field \(U\) is an SU(2) matrix which transforms as \(U \to G_L U G_R^{\dagger}\) with \(G_L \in \text{SU}(2)_L\) and \(G_R \in \text{SU}(2)_R\). In the following, we use dimensionless variables: \(r \to r / (e_s F_\pi)\) and \(m_\pi \to (e_s F_\pi) m_\pi\). In the Skyrme model, a general hedgehog-type ansatz in the absence of the electromagnetic background is written as

\[
U = G U_0 G_0^{\dagger} = G \exp(i f(r) \hat{x} \cdot \tau) G_0^{\dagger}, \quad G = a_0 + i a \cdot \tau \in \text{SU}(2)_{L+R}, \quad (a_0^2 + a^2 = 1),
\]

where \(\hat{x} \equiv x / |x|\), \(\tau\) are Pauli matrices, and \((a_0, a)\) are moduli parameters spanning \(\text{SU}(2)_{L+R} \simeq S^3\). We treat electromagnetic effects as a perturbation in terms of \(e\). The equation of motion gives

\[
\left( \frac{r^2}{4} + 2 \sin^2 f \right) f'' + \frac{r}{2} f' + \sin(2f) \left( f'^2 - \frac{1}{4} - \frac{\sin^2 f}{r^2} \right) - \frac{m_\pi^2 r^2}{4} \sin f = 0.
\]

Solving this under the boundary conditions, \(f(0) = \pi\) and \(f(r \to \infty) = 0\), one obtains a solution with baryon number \(B = 1\). The solution is a topological soliton, called Skyrmion.

We focus on the coupling between mesons and photons in the WZW term. In the two-flavor case, this can be given by \[7, 4, 3\]

\[
S_{\text{WZW}} = - \int d^4x \ A_\mu \left( \frac{e N_c}{6} J_\mu^B + \frac{1}{2} J_\mu^{anm} \right),
\]

\[
J_\mu^B = \frac{1}{24\pi^2} e^{\mu\nu\rho\sigma} \text{tr}[R_\nu R_\rho R_\sigma],
\]

\[
J_\mu^{anm} = - \frac{ie^2 N_c}{96\pi^2} e^{\mu\nu\rho\sigma} F_{\nu\rho} \text{tr}[\tau_3(L_\sigma + R_\sigma)],
\]
where $j^\mu_B$ is a baryon current giving an integer baryon number, $L_\mu = U^\dagger \partial_\mu U$, $R_\mu = \partial_\mu U U^\dagger$, and $\varepsilon_{0123} = -1$ in this article.

In the presence of background electromagnetic fields, not only the first term but also the second term in Eq. (3) is important. The electric charge $Q$ with the contribution from anomaly $(N_c = 3)$ is written as

$$Q = I_3 + \frac{N_B}{2} + \frac{Q_{\text{ann}}}{2},$$

where $N_B = \int d^3x J_0^\mu$ and $Q_{\text{ann}} = \int d^3x J_{0 \text{ann}}^\mu$. Thus, the Gell-Mann-Nishijima formula is corrected under background electromagnetic fields. Substituting Eq. (1) into Eq. (5), we obtain

$$\begin{align*}
  j_{\text{ann}}^\mu &= \frac{i e^2 N_c}{96 \pi^2} \epsilon^{\mu
u\rho\sigma} F_{\nu\rho} P_3, \\
  P_\mu &= 4i \left[ (\partial_\mu f) \hat{x}_3^{\text{rot}} + \frac{\sin(2f)}{2} \partial_\mu (\hat{x}_3^{\text{rot}}) \right], \\
  \hat{x}_3^{\text{rot}} &= (a_0^2 + a_1^2 - a_1^2 - a_0^2) \hat{x}_3 + 2(a_1 a_3 + a_0 a_2) \hat{x}_1 + 2(a_2 a_3 - a_0 a_1) \hat{x}_2.
\end{align*}$$

This is a classical anomalous current for the general hedgehog solutions.

### 3. Induced charges from quantized Skyrmion

To obtain physical values of the anomalous charge depending on the baryon states, we need to quantize the Skyrmion. By solving a quantum mechanics of the $S^3$ moduli parameters on the Skyrmion, quantum states of a nucleon with spin quantized along $x^3$ are given by $\psi_{\rho \uparrow} = (a_1 + i a_2)/\pi$, etc.\[5\]. We evaluate matrix elements of the anomalous current,

$$\langle j_{\text{ann}}^\mu \rangle_{I_3S_3} \equiv \int d\Omega_3 \psi_{I_3S_3}^\dagger j_{\text{ann}}^\mu(a_0, a) \psi_{I_3S_3},$$

where $d\Omega_3$ denotes the integration over the $S^3$, and $\psi_{I_3S_3}$ is the baryon states labeled by the third components of isospin and spin. The matrix elements are calculated as

$$\begin{align*}
  \langle j_{\text{ann}}^\mu \rangle_{I_3S_3} &= \frac{i e^2 N_c}{96 \pi^2} \epsilon^{\mu
u\rho\sigma} F_{\nu\rho} \langle P_3 \rangle_{I_3S_3}, \\
  \langle P_0 \rangle_{I_3S_3} &= 0, \\
  \langle P_{0\uparrow} \rangle_{I_3S_3} &= \frac{16i}{3} I_3 S_3 \left( f' - \frac{\sin(2f)}{2r} \right) \hat{x}_a \hat{x}_3, \\
  \langle P_3 \rangle_{I_3S_3} &= \frac{16i}{3} I_3 S_3 \left[ \left( f' - \frac{\sin(2f)}{2r} \right) \hat{x}_3^2 + \frac{\sin(2f)}{2r} \right].
\end{align*}$$

In the following, we concentrate on the case with magnetic-field backgrounds $B_t$. The anomalous charge density is indeed induced in nucleons:

$$\langle j_{\text{ann}}^0 \rangle_{I_3S_3} = \frac{i e^2 N_c}{48 \pi^2} B_t \langle P_t \rangle_{I_3S_3}. \quad (7)$$

Figure 2 shows the baryon number distribution, and the anomalous charge distribution under magnetic field along the 3rd- and the 1st-axes of quantized Skyrmions at $m_\pi = 0$. The configurations of the charge distribution look like wave functions of an electron in a hydrogen atom.
Figure 2. The constant-height surfaces of (a) density distribution of baryon number, (b) electric charge under magnetic field along the 3rd-axis, and (c) electric charge under magnetic field along the 1st-axis.

In contrast, we find that the matrix element of the spatial component of the current density vanishes, \( \langle j_{i}^{anm} \rangle_{I_{3},S_{3}} = 0 \). Thus, the electric current is not induced [1].

Let us calculate the total electric charge from the anomalous effect of \( \langle P_{i} \rangle_{I_{3},S_{3}} \) over the whole space gives

\[
\int d^{3}x \langle P_{i} \rangle_{I_{3},S_{3}} = \begin{cases} 
0 & (i = 1, 2), \\
-16\pi i \left( 4I_{3}S_{3} \right)c_{0} & (i = 3),
\end{cases}
\]

where \( c_{0} = \int dr \{ r^{2}f' + \sin(2f) \} \). Its numerical value is \( c_{0} = (-5.32, -12.3, -10.2, -7.32) \) for pion masses \( m_{\pi} = (0, m_{\pi}^{\text{phys}}/2, m_{\pi}^{\text{phys}}, 2m_{\pi}^{\text{phys}}) \), respectively \( (m_{\pi}^{\text{phys}} \equiv 0.263 \) is the physical value of the pion mass in the unit of \( eF_{\pi} \), determined from the mass splitting between nucleon and \( \Delta \) [5]). We obtain the anomalous charge for nucleons

\[
Q_{anm} = \frac{4e^{2}N_{c}}{27\pi}I_{3}S_{3} \frac{c_{0}B_{3}}{(eF_{\pi})^{2}}.
\]  

Equation (8) shows that an electric charge is actually induced by the anomalous effect even for a neutron. We further find that dipole moment vanishes while quadrupole moment appears as a leading multipole [1].

4. Observation

Let us argue possibilities to observe the anomalous charge. First, we estimate the amount of induced charge in a nucleon. Using Eq. (8), we obtain \( Q_{anm} \sim e \times 10^{-20}I_{3}S_{3}[G^{-1}] \times B_{3}[G] \). Under the terrestrial magnetic field \( B \sim 1[G] \), the induced charge \( Q_{anm} \) is about \( 10^{-20}e \), which would be too small to observe. On the surface of a magnetar, which is a neutron star with very strong magnetic field of order \( 10^{15}[G] \), \( Q_{anm} \) is about \( 10^{-5}e \). In heavy ion collisions, magnetic field of order \( 10^{17}[G] \) would be created. However, \( Q_{anm} \) is about \( 10^{-3}e \) even for such an extremely strong magnetic field. Hence, it is natural that the electric charge of neutrons has never been detected until now.

Next, electric dipole moment (EDM) of nucleons is not induced from the anomaly. This is consistent with the experimental results that there is no evidence for the existence of neutron EDM (see, e.g., [8]), which is performed under a magnetic field. In our study, the leading multipole is a quadrupole, \( Q_{13} = -2Q_{11} = -2Q_{22} \sim e \times 10^{-19}I_{3}S_{3}[\text{fm}^{2}G^{-1}] \times B_{3}[G] \). Its experimental measurement would be interesting.
5. Summary

We found that an additional non-integer charge would be induced on nucleons in external magnetic fields, based on the Skyrme model with Wess-Zumino-Witten term. We have also found nonzero multipoles such as a quadrupole moment. The detailed analysis of the validity of the computation will be given in [1], as the generation of the charge might sound surprising and counter-intuitive. Here we presented the evaluation of the magnitude of the induced charge, and found it was tiny enough to have not been observed yet. To see the universality of the generation of the anomalous charge, confirmation in other approaches is desirable. For instance, lattice QCD simulation with external electromagnetic fields is a reliable approach. Holographic QCD is also helpful for gaining insights. The novel mechanism due to the anomaly is intriguing, regardless of the smallness of the induced charges. Neutrons play an important role on the frontiers of hadron physics, such as neutron stars and heavy-ion collisions, where strong magnetic fields exist. Such neutrons would have anomalous charges which may be physically significant. Our results will bring new aspects of the dynamics of hadrons.

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