Numerical simulation using the bubble size distribution model of cryogenic cavitation around an axial helical inducer

Y Ito, X Zheng, and T Nagasaki
Department of Mechanical Engineering, Tokyo Institute of Technology, 4259-G3-33-402, Nagatsuta-cho, Midori-ku, Yokohama, Kanagawa 226-8502 Japan
E-mail: itoyu110@00.alumni.u-tokyo.ac.jp

Abstract. A mathematical model using a bubble size distribution was developed by the author for a cryogenic high-speed cavitating flow in turbomachinery. In this model, bubble growth/decay is solved for each class of the bubble mass (as an accurate expression of bubble size) when bubbles with various masses mix in the same spatially discretized calculation region. The bubble growth/decay calculations employ a combination of two rigorous methods: a Rayleigh–Plesset equation for the bubbles’ oscillation, and a heat conduction equation in a thermal boundary layer around a bubble to evaluate the mass rate of evaporation or condensation. This mathematical model was coded using the OpenFOAM C++ toolbox, and a numerical simulation was conducted for a 3D rotational inducer in a cryogenic fluid, the results of which showed that bubbles of a certain specified size were distributed at certain regions on the inducer. This study demonstrated that the mathematical model of cavitation exhibits potential to contribute to fidelity in simulation for turbomachinery considering a fluid with strong thermodynamic effect.

1. Introduction
Axial helical inducers have been used as auxiliary pumps to improve the net positive suction head (NPSH) performance in cryogenic high-flow pumps such as rocket turbopumps [1-3] and submerged liquefied natural gas cargo pumps [4]. The strong thermodynamic effect of cryogens suppresses cavitation growth by reducing the saturation pressure of the surrounding liquid via latent heat removal as cavitation grows, a feature not seen in water cavitation [5]. This effect suppresses cavitation breakdown until a lower NPSH value is reached [6]. More studies are required to develop cryogenic inducers with higher performance and increased stability, at lower cost. To date, only a few visualization experiments have been conducted for cryogenic cavitation [1, 2], including previous works by the present authors [7, 8], because of the level of technical difficulty involved. In contrast, a considerable number of numerical analyses have been conducted for inducers with cryogenic cavitation [9-11]. However, mathematical models more suitable to cryogenic cavitation in turbomachinery are desired as a tool to analyze cryogenic cavitation and to design turbomachinery. Therefore, a mathematical model tuned for cryogenic cavitation in turbomachinery was developed [12, 13]. In this work, this mathematical model was coded using the OpenFOAM C++ toolbox, and a numerical simulation was conducted for a 3D rotational inducer in liquid nitrogen.

2. Features of the Mathematical Model of Cavitation
A mathematical model tuned for cryogenic cavitation in turbomachinery was developed and utilized in this work. It includes the following features:
1. In cryogenic cavitation, a vapor phase is dispersed into a liquid phase. As each part of the phase is sized on the order of micrometers to millimeters, its shape is governed by the surface tension and tends toward the spherical. Therefore, the proposed mathematical model of cavitation assumes that cryogenic cavitation in turbomachinery is a flow of many spherical bubbles in a liquid continuum.

2. In cryogenic cavitation without pressurization by a gas with a lower boiling temperature (commonly helium), there is no non-condensable gas effect. This mathematical model of cavitation thus assumes that each bubble is purely saturated vapor.

3. In turbomachinery, each bubble oscillates as the pressure fluctuates. Furthermore, the strong thermodynamic effect of cryogenic cavitation forms a thermal boundary layer around each bubble that moves with the bubble. This thermal boundary layer determines the rates of evaporation or condensation in each bubble. Therefore, the proposed mathematical model of cavitation applies the unsimplified Rayleigh–Plesset equation alongside the temperature distribution equation of the thermal boundary layer, coupling these for each bubble.

4. In turbomachinery, bubbles with various masses (as an accurate expression of bubble size) mix in the same region. Thus, this mathematical model of cavitation classifies bubbles into several classes by mass. The Rayleigh–Plesset and temperature distribution equations are all applied to the average bubble of each class. Furthermore, when the average bubble of a given class grows, the number of bubbles belonging to this class decreases, whereas the number of bubbles belonging to the next heavier class increases, and vice versa.

5. Cavitation nucleation rate is affected by the density of impurities as well as by local superheating, for which this mathematical model of cavitation applies the heterogeneous nucleation theory.

In addition to the above-mentioned points, the mathematical model of cavitation should basically consider the slip velocity between each bubble and the surrounding liquid. This is because the translational velocity of bubbles is influenced by a steep pressure gradient field with a strong centrifugal force in turbomachinery. However, because this was an introduction study, the slip velocity effect was ignored, and the translational velocity of bubbles was set at that of their surrounding liquid.

3. Mathematical Model of Cavitation Using Bubble Size Distribution

The following equation system for liquid is considered:

$$\frac{\partial Q}{\partial t} + \sum_i \frac{\partial F_{i,1}}{\partial \xi_i} = 0,$$

$$Q_{i} = \frac{1}{J} \left[ \begin{array}{c} \rho _{L} u_{L} \\ \rho _{L} v_{L} \\ \rho _{L} w_{L} \\ \rho _{L} E_{L} + \rho _{L} K_{L} \end{array} \right], \quad F_{i,1} = \frac{1}{J} \left[ \begin{array}{c} \rho _{L} U_{L,1} \\ \rho _{L} U_{L,2} \\ \rho _{L} U_{L,3} \\ \rho _{L} U_{L,4} \end{array} \right],$$

$$U_{L,1} = \xi _{x,1} u_{L} + \xi _{y,1} v_{L} + \xi _{z,1} w_{L},$$

$$U_{L,2} = \xi _{x,2} u_{L} + \xi _{y,2} v_{L} + \xi _{z,2} w_{L},$$

$$U_{L,3} = \xi _{x,3} u_{L} + \xi _{y,3} v_{L} + \xi _{z,3} w_{L},$$

$$U_{L,4} = \xi _{x,4} u_{L} + \xi _{y,4} v_{L} + \xi _{z,4} w_{L}.$$  

(2)

Here, $t$ is the temporal coordinate; $\xi _{i}$ is the spatial coordinate in the considered direction $i$; $Q_{i}$ is the vector of solved variables; $F_{i,1}$ is a flux vector; $\rho _{L}$ is the density; $u_{L}$, $v_{L}$, and $w_{L}$ are the velocities in the $x$, $y$, and $z$ directions, respectively; $E_{L}$ is the static specific energy; $K_{L}$ is the kinetic specific energy; $U_{L,1}$ is the contravariant velocity in the $\xi _{1}$ direction; $p$ is the pressure; $\tau $ is the viscosity tensor; $\psi $ is the energy dissipation vector; $J$ is a Jacobian matrix; and $\xi _{x,i}$, $\xi _{y,i}$, and $\xi _{z,i}$ are the factors for the coordinate transform from the considered direction $i$ to the $x$, $y$, and $z$ directions, respectively.

The equation system for bubbles in vector form according to the above assumptions is

$$\frac{\partial Q}{\partial t} + \sum_i \frac{\partial F_{i,1}}{\partial \xi_i} + \frac{\partial H_{1}}{\partial s} + N_{G} I_{2} = 0.$$  

(3)
Here, \( s \) is the coordinate of the mass of one bubble in kilograms. To formulate assumption no. 4, we introduce the probability function of the bubble number density \( (N_G, 1/\text{m}^3) \). Figure 1(a) shows an example of the distribution of \( N_G \) in the \( s \) direction. The integration of \( N_G \) with respect to \( s \) is the number of bubbles per unit volume \( (n_G, 1/\text{m}^3) \), and the integration of \( s N_G \) with respect to \( s \) is the mass of bubbles per unit volume \( (m_G, \text{kg/m}^3) \). When some bubbles grow, the distribution of \( N_G \) shifts toward a greater \( s \). When some bubbles decay, the distribution of \( N_G \) shifts toward a lower \( s \). The coordinate in the direction of each bubble’s radius in meters is \( r \). To formulate assumption no. 3, we introduce the temperature distribution \( (T_{layer}, K) \) of the thermal boundary layer formed around each bubble that moves with the bubble, as shown in Figure 2(a). Figure 2(b) shows an example of the distribution of \( T_{layer} \) in the \( r \) direction. \( T_{layer} \) is the same as the bubble temperature \( T_G \) at the bubble surface \( r = R \), and \( T_{layer} \) is the same as the liquid temperature \( T_L \) at the outer edge of the thermal boundary layer. The distribution of \( T_{layer} \) approaches linearity as time goes on. Finally, the distribution of \( T_{layer} \) becomes a linear distribution and a steady heat transfer is realized. At that time, the Nusselt number \( Nu = \frac{2hR}{k_l} \), becomes \( 2 \) [14], where \( h \) is the heat transfer coefficient in watts per meter squared kelvin \( (\text{W/m}^2\cdot\text{K}) \) and \( k_l \) is the thermal conductivity in watts per meter kelvin \( (\text{W/m} \cdot \text{K}) \). At that time, heat flux \( q \) through the bubble surface \( r = R \) is expressed in two ways (as follows), where the thermal boundary layer thickness is \( b \), which is the required thickness for the calculation of \( T_{layer} \) in the thermal boundary layer:

\[
q_{ln} = k_l \frac{dT_{layer}}{dr}, \quad \text{and} \quad q_{ln} = h[T_L - T_G], \quad (4)
\]

\[
\frac{hb}{k_l} = 1 = \frac{Nu}{2} = \frac{2hR}{2k_l}, \quad \text{so} \quad b = R. \quad (5)
\]

Namely, the thermal boundary layer thickness \( b \) requires the same value as the bubble radius \( R \), as shown in Figure 2(b). For equation (3), \( Q_2 \) is the vector of solved variables, and \( F_{2,i} \), \( H_2 \), and \( I_2 \) are flux vectors.

\[
Q_2 = \frac{1}{J} \begin{bmatrix}
N_G \\
sN_G \\
R N_G \\
T_{layer,1} N_G \\
\vdots \\
T_{layer, \text{max}} N_G
\end{bmatrix} \quad , \quad F_{2,i} = \frac{1}{J} \begin{bmatrix}
N_G U_{G,i} \\
sN_G U_{G,i} \\
R N_G U_{G,i} \\
T_{layer,1} N_G U_{G,i} \\
\vdots \\
T_{layer, \text{max}} N_G U_{G,i}
\end{bmatrix} \quad , \quad H_2 = \frac{1}{J} \begin{bmatrix}
N_G Y_G - \Pi_G \delta (s = s_0) \\
sN_G Y_G - s_G \Pi_G \delta (s = s_0) \\
R N_G Y_G - R_G \Pi_G \delta (s = s_0) \\
T_{layer,1} N_G Y_G - T_{layer,1} \Pi_G \delta (s = s_0) \\
\vdots \\
T_{layer, \text{max}} N_G Y_G - T_{layer, \text{max}} \Pi_G \delta (s = s_0)
\end{bmatrix},
\]

\[
I_2 = \frac{1}{J} \begin{bmatrix}
0 \\
-\gamma_B \\
\vdots \\
\frac{1}{\rho_R} \left( p_B - p_L - \frac{3}{2} \rho_B R^2 + \frac{2 \sigma_B}{R} - \frac{4 \mu_B}{R} \right) \\
\frac{u_{\text{surface}}}{r^2} \frac{\partial T_{layer,1}}{\partial r} - \frac{a_1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T_{layer,1}}{\partial r} \right) \\
\frac{u_{\text{surface}}}{r^2} \frac{\partial T_{layer, \text{max}}}{\partial r} - \frac{a_1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T_{layer, \text{max}}}{\partial r} \right)
\end{bmatrix}, \quad \text{U}_{G,i} = \xi_{G,i} U_G + \xi_{G,i} v_G + \xi_{G,i} w_G \quad (6)
\]
Figure 1. Bubble size distribution in the $s$ direction. (a) Actual bubble size distribution in a certain region. (b) Classification assumed constant distribution of $N_G$ on $s$. (c) Classification assumed triangle or trapezoid distribution of $N_G$ on $s$.

Figure 2. Temperature distribution in the $r$ direction. (a) Thermal boundary layer formed around each bubble. (b) Temperature distribution in the thermal boundary layer.

\[ \Pi_G = d_{\text{impurity}} \frac{3\sigma_s}{2m_{\text{molecule}}} \exp \left( \frac{-16\pi\sigma_s^3 \phi}{3k_{\text{Boltzmann}}T_l \left( \eta p_G - p_L \right)^2} \right), \quad \phi = \frac{2 + 3\cos \theta - \cos^3 \theta}{4}, \quad \eta = \exp \left( \frac{p_L - p_{\text{sat}}(T_l)}{\rho_L RT_l} \right), \]

where $R$ is the bubble radius; $\dot{R}$ is the time derivative of $R$; $U_{G,i}$ is the bubbles’ contravariant velocity in the considered direction $i$; $\Pi_G$ is the nucleation rate of the bubbles; $d_{\text{impurity}}$ is the number density of the impurities; $\sigma_s$ is the surface tension; $\mu_L$ is the liquid viscosity; $m_{\text{molecule}}$ is the molecular mass; $k_{\text{Boltzmann}}$ is the Boltzmann constant; $T_l$ is the liquid temperature; $p_G$ and $p_L$ are the bubble and liquid pressures, respectively; $\theta$ is the contact angle between the bubble nucleus and the impurity; $R_0$ is the initial radius of the bubble nuclei; $s_0$ is the mass of the bubble nuclei; $\delta(s-s_0)$ is the Dirac’s delta function, which is 1 for $s = s_0$ and 0 for $s \neq s_0$; $\rho_G$ and $\rho_L$ are the bubble and liquid densities, respectively; $\gamma_G$ is the evaporation rate at the surface of each bubble; $L$ is the latent heat; $u_{\text{surface}}$ is the liquid velocity at the bubble surface in the $r$ direction; and $a_L$ is the liquid thermal diffusivity.

4. Numerical Method

The above mathematical model of cavitation using a bubble size distribution was coded using version 3 of the OpenFOAM C++ toolbox. No turbulence model was used because this was an introduction study. A cryogenic inducer model was made using the 3D CAD software Pro/ENGINEER. This inducer model was discretized into an unstructured grid system formed on coordinates $x$, $y$, and $z$, as shown in Figure 3. The number of grids was approximately 300 thousand, and this was not sufficient to ensure fidelity in simulation; however, this was the upper limit for the capacity of the hardware used. The flow came in from the top boundary and went out to the bottom boundary. Only the inducer section rotated, and the sliding mesh method was applied at the connected surfaces. Bubbles located in...
the same spatially discretized volume were discretized into a “bubble mass class” on coordinate $s$, as shown in Figure 1(b) and (c). When the constant distribution of $N_G$ inside a class like that shown in Figure 1(b) is used, the average mass of one bubble cannot be changed inside a class. In this case, bubbles cannot grow or decay inside a class. Instead, when the triangle or trapezoid distribution of $N_G$ inside a class like that shown in Figure 1(c) is used, the average mass of one bubble can be changed inside a class and bubbles can grow or decay inside a class. Thus, the triangle or trapezoid distribution of $N_G$ inside a class like that shown in Figure 1(c) was employed in this study. The temperature distribution $T_{\text{layer}}$ of the thermal boundary layer was discretized on coordinate $r$, as shown in Figure 2(a). Table 1 organizes the solved variables by the arguments $i$, $\ell$, and $j$. Here, $i$ is the index of the spatial coordinates $x$, $y$, and $z$; $\ell$ is the index of the bubble mass coordinate $s$; and $j$ is the index of the radial coordinate $r$ of each bubble.

![Spatial grid system](image1)

**Figure 3.** Spatial grid system.

![Velocity distribution](image2)

**Figure 4.** Velocity distribution in the vertical direction.

| Table 1. Solved variables. |
|----------------------------|
| Variables being a function of spatial coordinate $i$ only | Variables being a function of both spatial coordinate $i$ and bubble mass class $\ell$ | Variable being a function of spatial coordinate $i$, bubble mass class $\ell$, and the radial coordinate of each bubble $j$ |
| $\rho_L$, $\rho_L u_L$, $\rho_L v_L$, $\rho_L w_L$, $\rho_L E_L$, and $\rho_L K_L$ | $N_G$, $s N_G$, $R_G N_G$, and $R N_G$ | $T_{\text{layer}}$ |

-3.9 m/s (down) 2.8 m/s (up)
5. Results

The calculation was conducted under the following conditions. The working fluid was liquid nitrogen. The rotational rate of the inducer was 4480 rpm. At the inlet of the computational domain, the velocity was 1.869 m/s, with no swirl, in the vertical direction, and the temperature was fixed at 78.0 K. At the outlet of the computational domain, the pressure was fixed at 117.0 kPa. The number density of impurities $d_{\text{impurity}}$ was $10^9$ 1/m$^3$, and the contact angle $\theta$ was 178°. These conditions correspond to 0.125 of the flow coefficient, 0.0539 of the head coefficient, and 0.0001 of the cavitation number.

Figure 4 shows the numerical result of the velocity distribution in the vertical direction on the cross section through the inducer axis. The red color indicates an upward velocity, whereas the blue color indicates a downward one. At the clearance region between the inducer blade tip and the casing wall, an upward swirling flow was observed. This flow penetrated close to the inlet boundary and then was pushed back downward by the main flow from the inlet boundary to the inducer blades. The main flow has little swirling velocity. This shear between the backward swirling flow and the main non-swirling flow created vertical vortices on the inducer. These vertical vortices rotated around themselves; furthermore, they revolved around the inducer axis at a slower revolving velocity than the rotating speed of the inducer. The backflow vortex cavitation was numerically observed in this shear region as well as the authors experimentally observed a backflow vortex cavitation in this region [8].

![Figure 4](image)

**Figure 4.** Numerical result of the velocity distribution in the vertical direction on the cross section through the inducer axis.

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Figure 5 shows the numerical result of the cavitation distribution by a gray void (which denotes the region where the cell void fraction is 1% or more), and the pressure distribution on the inducer surface by a colored contour. The red color indicates a high-pressure region and the blue color indicates a low-pressure region. The pressure was low beyond several mm downward of the leading edge because the incline of the suction surface of the inducer blade increases here. In addition, the pressure was low at the tip region. At these low-pressure regions, cavitation formed. In addition, a tornado-like cavitation formed at the clearance region between the inducer blade tip and the casing wall.

![Figure 5](image)

**Figure 5.** Pressure (colored) and cavitation (gray sections denote regions where cell void fraction is 1% or more) distributions.

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Figure 6. Cavitation distribution of each class (colors denote the dominant class).

![Figure 6](image)

**Figure 6.** Cavitation distribution of each class.
formed at the backflow vortex region. The backflow vortex cavitation revolves with a velocity slower than the rotating speed of the inducer.

Figure 6 shows the numerical result of the cavitation distribution of each class. The red and blue colors indicate, respectively, that the cavitation of classes 1 and 2 is most dominant. Small bubbles of class 1 were observed at the tip vortex regions and at the outer boundaries of cavitation regions, whereas large bubbles of class 2 were observed in low-pressure regions on the inducer surfaces and near the vortex cores. Bubbles of class 2 were usually covered by bubbles of class 1; however, bubbles of class 2 sometime appeared at the outer boundaries of cavitation regions and on the casing walls.

6. Conclusion

The mathematical model of cavitation using a bubble size distribution was tuned for cryogenic cavitation in turbomachinery. It includes the following features:

1. In cryogenic cavitation, a vapor phase is dispersed into a liquid phase. As each part of the phase is sized on the order of micrometers to millimeters, its shape is governed by the surface tension and tends toward the spherical. Therefore, the proposed mathematical model of cavitation assumes that cryogenic cavitation in turbomachinery is a flow of many spherical bubbles in a liquid continuum.

2. In cryogenic cavitation without pressurization by a gas with a lower boiling temperature (commonly helium), there is no non-condensable gas effect. This mathematical model of cavitation thus assumes that each bubble is purely saturated vapor.

3. In turbomachinery, each bubble oscillates as the pressure fluctuates. Furthermore, the strong thermodynamic effect of cryogenic cavitation forms a thermal boundary layer around each bubble that moves with the bubble. This thermal boundary layer determines the rates of evaporation or condensation in each bubble. Therefore, the proposed mathematical model of cavitation applies the unsimplified Rayleigh–Plesset equation alongside the temperature distribution equation of the thermal boundary layer, coupling these for each bubble.

4. In turbomachinery, bubbles with various masses (as an accurate expression of bubble size) mix in the same region. Thus, this mathematical model of cavitation classifies bubbles into several classes by mass. The Rayleigh–Plesset and temperature distribution equations are all applied to the average bubble of each class. Furthermore, when the average bubble of a given class grows, the number of bubbles belonging to this class decreases, whereas the number of bubbles belonging to the next heavier class increases, and vice versa.

5. Cavitation nucleation rate is affected by the density of impurities as well as by local superheating, for which this mathematical model of cavitation applies the heterogeneous nucleation theory. This mathematical model was coded using the OpenFOAM C++ toolbox. A model of a 3D rotational inducer was prepared and discretized to an unstructured grid. A numerical simulation was carried out for an inducer rotating in liquid nitrogen. The number of grids was approximately 300 thousand. Although the number of grids was not sufficient for ensuring fidelity in simulation, a backflow vortex cavitation formed on the inducer surfaces. In addition, this numerical simulation using the bubble size distribution model showed that bubbles of a certain specified size were distributed at certain specified regions on the inducer. For example, small bubbles of class 1 were observed at the tip vortex regions and at the outer boundaries of cavitation regions, whereas large bubbles of class 2 were observed in low-pressure regions on the inducer surfaces and near the vortex cores. Bubbles of class 2 were usually covered by bubbles of class 1; however, bubbles of class 2 sometime appeared at the outer boundaries of cavitation regions and on the casing walls. This study demonstrated that the developed mathematical model of cavitation exhibits potential to contribute towards achieving fidelity in simulation for turbomachinery with respect to a fluid having strong thermodynamic effect.

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