A large amount of new results have been presented at TAU2006. The highlights of the workshop, the present status of a few selected topics on lepton physics (universality, QCD tests, $V_{us}$ determination from $\tau$ decay, $g - 2$, $\nu$ oscillations, lepton-flavour violation) and the prospects for future improvements are briefly summarized.

1. INTRODUCTION

The known leptons provide clean probes to perform very precise tests of the Standard Model and search for signals of new dynamics. The electroweak gauge structure has been successfully tested at the 0.1% to 1% level, confirming the Standard Model framework \cite{1}. Moreover, the hadronic $\tau$ decays turn out to be a beautiful laboratory for studying strong interaction effects at low energies \cite{2,3,4}. Accurate determinations of the QCD coupling, $|V_{us}|$ and the strange quark mass have been obtained with $\tau$ decay data.

The first hints of new physics beyond the Standard Model have also emerged from the lepton sector. Convincing evidence of neutrino oscillations has been obtained by SNO \cite{5} and SuperKamiokande \cite{6,7}. Combined with data from other neutrino experiments \cite{8,9,10}, it shows that $\nu_e \rightarrow \nu_\mu$ and $\nu_\mu \rightarrow \nu_\tau$ transitions do occur.

The huge statistics accumulated at the B Factories allow to explore lepton-flavour-violating $\tau$ decay modes with increased sensitivities beyond $10^{-7}$, which could be further pushed down to few $10^{-9}$ at future facilities. Moreover, BESIII will soon start taking data at the new Beijing Tau-Charm Factory. With the excellent experimental conditions of the threshold region, complementary information on the $\tau$ should be obtained, such as an improved mass measurement.

The large amount of new results presented at this workshop shows that $\tau$ physics is entering a new era, full of interesting possibilities and with a high potential for new discoveries.

| Table 1 | Present constraints on $|g_l/g_e|$ \cite{3,11,12,13}. |
|---------|----------------------------------------------------------|
| $B_{\tau\rightarrow\mu}/B_{\tau\rightarrow e}$ | 1.0000 ± 0.0020 |
| $B_{\pi\rightarrow\mu}/B_{\pi\rightarrow e}$ | 1.0017 ± 0.0015 |
| $B_{K\rightarrow\mu}/B_{K\rightarrow e}$ | 1.012 ± 0.009 |
| $B_{W\rightarrow\mu}/B_{W\rightarrow e}$ | 0.997 ± 0.010 |

| $g_{\tau}/g_{\mu}$ |
|---------------------|
| $B_{\tau\rightarrow\mu\tau}/\tau$ | 1.0004 ± 0.0022 |
| $\Gamma_{\tau\rightarrow\gamma}/\Gamma_{\tau\rightarrow \mu}$ | 0.996 ± 0.005 |
| $\Gamma_{\tau\rightarrow K}/\Gamma_{K\rightarrow \mu}$ | 0.979 ± 0.017 |
| $B_{W\rightarrow\tau}/B_{W\rightarrow \mu}$ | 1.039 ± 0.013 |

| $|g_{\tau}/g_e|$ |
|-----------------|
| $B_{\tau\rightarrow\mu\tau}/\tau$ | 1.0004 ± 0.0023 |
| $B_{W\rightarrow\tau}/B_{W\rightarrow \mu}$ | 1.036 ± 0.014 |

2. LEPTON UNIVERSALITY

In the Standard Model all lepton doublets have identical couplings to the $W$ boson. Comparing the measured decay widths of leptonic or semileptonic decays which only differ in the lepton flavour, one can test experimentally that the $W$ interaction is indeed the same, i.e. that $g_e = g_\mu = g_\tau \equiv g$. As shown in Table 1 the present data verify the universality of the leptonic charged-current couplings to the 0.2% level\footnote{Br($W \rightarrow \nu_e \tau$) is $2.1 \sigma/2.7 \sigma$ larger than Br($W \rightarrow \nu_\mu/\nu_\mu$). The stringent limits on $|g_{\tau}/g_{\mu}|$ from $W$-mediated decays make unlikely that this is a real effect.}.
The $\tau$ leptonic branching fractions and the $\tau$ lifetime are known with a precision of 0.3%. A slightly improved lifetime measurement could be expected from BABAR and BELLE [14]. For comparison, the $\mu$ lifetime is known with an accuracy of 10^{-9}, which should be further improved to 10^{-6} by the MuLan experiment at PSI [15].

The universality tests require also a good determination of $m_\tau^2$, which is only known to the 0.08% level. Two new preliminary measurements of the $\tau$ mass have been presented at this workshop:

$$m_\tau = \begin{cases} 1776.71 \pm 0.13 \pm 0.35 \text{ MeV} & \text{[BELLE]}, \\ 1776.80^{+0.25}_{-0.23} \pm 0.15 \text{ MeV} & \text{[KEDR]}. \end{cases}$$

BELLE [16] has made a pseudomass analysis of $\tau \to \nu_\tau 3\pi$ decays, while KEDR [17] measures the $\tau^+\tau^-$ threshold production, taking advantage of a precise energy calibration through the resonance depolarization method. In both cases the achieved precision is getting close to the present BES-dominated value, $m_\tau = 1776.99^{+0.29}_{-0.26}$ [11]. KEDR aims to obtain a final accuracy of 0.15 MeV. A precision better than 0.1 MeV should be easily achieved at BESIII [18], through a detailed analysis of $\sigma(e^+e^- \to \tau^+\tau^-)$ at threshold [19][20].

3. HADRONIC TAU DECAYS

The $\tau$ is the only known lepton massive enough to decay into hadrons. Its semileptonic decays are then ideally suited for studying the hadronic weak currents in very clean conditions. The decay $\tau^- \to \nu_\tau H^-$ probes the matrix element of the left-handed charged current between the vacuum and the final hadronic state $H^-$. For the decay modes with lowest multiplicity, $\tau^- \to \nu_\tau \pi^-$ and $\tau^- \to \nu_\tau K^-$, the relevant matrix elements are already known from the measured decays $\pi^- \to \mu^- \bar{\nu}_\mu$ and $K^- \to \mu^- \bar{\nu}_\mu$. The corresponding $\tau$ decay widths can then be accurately predicted. As shown in Table [1], the predictions are in good agreement with the measured values and provide a test of lepton universality. Assuming universality in the quark couplings, these decay modes determine the ratio [11][21]

$$\frac{|V_{us}|}{|V_{ud}|} f_\pi = \begin{cases} 0.27618 \pm 0.00048 & \frac{\Gamma_{K/\pi \to \nu_\mu \nu}}{\Gamma_{\tau \to \nu_\tau K/\pi}}, \\ 0.267 \pm 0.005 & \frac{\sigma_{\tau \to \nu_\tau K/\pi}}{\sigma_{\tau \to \nu_\tau}}. \end{cases}$$

The very different accuracies reflect the present poor precision on $\Gamma(\tau^- \to \nu_\tau K^-)$.

For the two–pion final state, the hadronic matrix element is parameterized in terms of the so-called pion form factor $[s \equiv (p_{\pi^-} + p_{\pi^0})^2]$

$$\langle \pi^- \pi^0 | d\gamma^\mu u(0) \rangle \equiv \sqrt{2} F_\pi(s) (p_{\pi^-} - p_{\pi^0})^\mu .$$

A dynamical understanding of the pion form factor can be achieved [23][24][25][26], using analyticity, unitarity and some general properties of QCD, such as chiral symmetry [27] and the short-distance asymptotic behavior [28][29]. Putting all these fundamental ingredients together, one gets

$$F_\pi(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left\{ -\frac{s \Re A(s)}{96\pi^2 f_\pi^2} \right\},$$

where

$$A(s) \equiv \log \left( \frac{m_\rho^2}{M_\rho^2} \right) + 8 \frac{m_\rho^2}{s} - \frac{5}{3} + \sigma_\pi^2 \log \left( \frac{\sigma_\pi + 1}{\sigma_\pi - 1} \right)$$

contains the one-loop chiral logarithms, $\sigma_\pi \equiv \sqrt{1 - 4m_\pi^2/s}$ and the off-shell $\rho$ width [24][23] is given by $\Gamma_\rho(s) = \theta(s - 4m_\pi^2) \sigma_\pi^2 M_\rho s/(96\pi f_\pi^2)$. This prediction, which only depends on $M_\rho$, $m_\rho$ and the pion decay constant $f_\pi$, is compared with the data in Fig. [1]. The agreement is rather impressive and extends to negative $s$ values, where the $e^-\pi$ elastic data (not shown in the figure) sits.
Figure 2. Preliminary BELLE measurement of the pion form factor from $\tau^\rightarrow \nu_\tau \pi^- \pi^0$ [30].

The small effect of heavier $\rho$ resonance contributions and additional next-to-leading in $1/N_C$ corrections can be easily included, at the price of having some free parameters which decrease the predictive power [23,26]. This gives a better description of the $\rho'$ shoulder around 1.2 GeV (continuous line in Fig. 1). A clear signal for the $\rho''(1700)$ resonance in $\tau^\rightarrow \nu_\tau \pi^- \pi^0$ events has been reported by BELLE, with a data sample 20 times larger than in previous experiments [30].

The decay $\tau \rightarrow \nu_\tau K\pi$ is characterized by two form factors with $J^P = 1^-$ and $0^+$. The vector form factor $F_{K\pi}^{V}(s)$ can be described in an analogous way to $F_\pi(s)$. The scalar component $F_{K\pi}^{S}(s)$ has been recently studied [31], taking into account additional information from $K\pi$ scattering data through dispersion relations [21,32]. The decay width is dominated by the $K^*(892)$ contribution, with a predicted branching ratio $\text{Br}[\tau \rightarrow \nu_\tau K^*] = (1.253 \pm 0.078)\%$, while the scalar component is found to be $\text{Br}[\tau \rightarrow \nu_\tau (K\pi)_{S\text{-wave}}] = (3.88 \pm 0.19) \cdot 10^{-4}$. The preliminary measurements of the $\tau^\rightarrow \nu_\tau K^0 S\pi^-$ (BELLE [39]) and $\tau^\rightarrow \nu_\tau K^- \pi^0$ (BABAR [34]) distributions, presented at this workshop, show a clear evidence for the scalar contribution at low invariant mass and a $K^*(1410)$ vector component at large $s$.

Figure 3. Predicted $\tau \rightarrow \nu_\tau K\pi$ distribution, together with the separate contributions from the $K^*(892)$ and $K^*(1410)$ vector mesons as well as the scalar component residing in $F_{K\pi}^{S}(s)$ [31].

The dynamical structure of other hadronic final states can be investigated in a similar way. The $\tau \rightarrow \nu_\tau 3\pi$ decay mode was studied in Ref. [35], where a theoretical description of the measured structure functions [36,37,38] was provided. A detailed analysis of other $\tau$ decay modes into three final pseudoscalar mesons is in progress [39]. The more involved $\tau \rightarrow \nu_\tau 4\pi$ and $e^+e^- \rightarrow 4\pi$ transitions have also been studied [40].

BABAR has presented preliminary measurements of $\tau^\rightarrow \nu_\tau \pi^+ \pi^- \pi^0$ and $\tau^\rightarrow \nu_\tau \pi^+2\pi^-\eta$ decays. The $4\pi$ distribution is found to have
a large $\omega\pi^-$ contribution, while the $f_1(1285)\pi^-$ component is seen to be the primary source of $\tau^-\rightarrow\nu\pi^+\pi^-\pi^0$ events [1]. The large statistics collected by BABAR ($2 \times 10^8 \tau^+\tau^-$ pairs) also allow to put limits on decay modes with 7 charged pions at the level of few $10^{-7}$ (90\% CL) [42].

CLEO has investigated rare $\tau$ decay modes with kaons in the final state, obtaining the results $\text{Br}(\tau^-\rightarrow\nu\tau^-\pi^-\pi\pi^0) = (7.4\pm0.8\pm1.1) \times 10^{-4}$ ($K^0$ excluded) and $\text{Br}(\tau^-\rightarrow\nu\tau^-K^+\pi^-\pi^0) = (5.5 \pm 1.4 \pm 1.2) \times 10^{-5}$ [43].

### 4. THE TAU HADRONIC WIDTH

The inclusive character of the total $\tau$ hadronic width renders possible an accurate calculation of the ratio $\frac{R_{\tau}}{R_{\tau,V}} = \frac{\Gamma(\tau^-\rightarrow\nu\tau^-\text{hadrons}(\gamma))}{\Gamma(\tau^-\rightarrow\nu\tau^-e^-\nu_e(\gamma))}$, (2)

using analyticity constraints and the Operator Product Expansion. One can separately compute the contributions from specific quark currents:

$$R_{\tau} = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}.$$  (3)

$R_{\tau,V}$ and $R_{\tau,A}$ correspond to the Cabibbo-allowed decays through the vector and axial-vector currents, while $R_{\tau,S}$ contains the remaining Cabibbo-suppressed contributions.

The theoretical prediction for $R_{\tau,V+A}$ can be expressed as [46]

$$R_{\tau,V+A} = N_C |V_{ud}|^2 S_{\text{EW}} \left\{ 1 + \delta_P + \delta_{NP} \right\},$$  (4)

where $N_C = 3$ and $S_{\text{EW}} = 1.0201 \pm 0.0003$ contains the electroweak radiative corrections [49].

The dominant correction ($\sim 20\%$) is the perturbative QCD contribution $\delta_P$, which is fully known to $O(\alpha_s^3)$ [46] and includes a resummation of the most important higher-order effects [47].

Non-perturbative contributions are suppressed by six powers of the $\tau$ mass [49] and, therefore, are very small. Their numerical size has been determined from the invariant-mass distribution of the final hadrons in $\tau$ decay, through the study of weighted integrals [50],

$$R_{\tau}^{kl} = \int_0^{m_{\tau}^2} ds \left( 1 - \frac{s}{m_{\tau}^2} \right)^k \left( \frac{s}{m_{\tau}^2} \right)^l \frac{dR_{\tau}}{ds},$$  (5)

Figure 5. Measured values of $\alpha_s$ at different scales. The curves show the energy dependence predicted by QCD, using $\alpha_s(m_{Z}^2)$ as input. The corresponding extrapolated $\alpha_s(M_{Z}^2)$ values are shown at the bottom, where the shaded band displays the $\tau$ decay result within errors [51].

which can be calculated theoretically in the same way as $R_{\tau}$. The predicted suppression of the non-perturbative corrections has been confirmed by ALEPH [38], CLEO [52] and OPAL [53]. The most recent analysis [55] gives

$$\delta_{NP} = -0.0043 \pm 0.0019.$$  (6)

The QCD prediction for $R_{\tau,V+A}$ is then completely dominated by the perturbative contribution; non-perturbative effects being smaller than the perturbative uncertainties from uncalculated higher-order corrections. The result turns out to be very sensitive to the value of $\alpha_s(m_{\tau}^2)$, allowing for an accurate determination of the fundamental QCD coupling [45-49]. The experimental measurement $R_{\tau,V+A} = 3.471 \pm 0.011$ implies [51]

$$\alpha_s(m_{\tau}^2) = 0.345 \pm 0.004_{\text{exp}} \pm 0.009_{\text{th}}.$$  (7)

The strong coupling measured at the $\tau$ mass scale is significantly larger than the values ob-
tained at higher energies. From the hadronic decays of the Z, one gets \( \alpha_s(M_Z^2) = 0.1186 \pm 0.0027 \) [12], which differs from the \( \tau \) decay measurement by more than twenty standard deviations. After evolution up to the scale \( M_Z \) [51], the strong coupling constant in (7) decreases to \( \delta R \) [56,57,58,59,60,61]. Since the longitudinal contribution to \( \Delta_{kl}(\alpha_s) \) does not converge well, the \( \bar{V}_{us} \) data has the potential to provide the best determination of \( m_s \), which is much more precise because it is dominated by far by the well-known kaon pole. The small non-perturbative contribution, \( \delta O_4 \equiv \langle 0 | m_s \bar{s}s - m_d \bar{d}d | 0 \rangle = (-1.5 \pm 0.4) \times 10^{-3} \text{GeV}^4 \), has been estimated with Chiral Perturbation Theory techniques [5].

From the measured moments \( \delta R^{(k)}_\tau \) \( (k = 0, 1, 2, 3, 4) \) [62,63], it is possible to determine the strange quark mass; however, the extracted value depends sensitively on the modulus of the Cabibbo–Kobayashi–Maskawa matrix element \( |V_{us}| \). It appears then natural to turn things around and, with an input for \( m_s \) obtained from other sources, to actually determine \( |V_{us}| \) [60]. The most sensitive moment is \( \delta R^{(0)}_{\tau,V} \).

\[
|V_{us}|^2 = \frac{R^{(0)}_{\tau,S}}{R^{(0)}_{\tau,V} + A} - \delta R^{(0)}_{\tau,\text{th}}. \tag{10}
\]

Using \( m_s(2 \text{ GeV}) = (94 \pm 6) \text{ MeV} \), which includes the most recent determinations of \( m_s \) from lattice and QCD Sum Rules [21], one obtains \( \delta R^{(0)}_{\tau,\text{th}} = 0.240 \pm 0.032 \) [60]. This prediction is much smaller than \( R^{(0)}_{\tau,V} |V_{us}|^2 \), making the theoretical uncertainty in (10) negligible in comparison with the experimental inputs \( R^{(0)}_{\tau,V} = 3.471 \pm 0.011 \) and \( R^{(0)}_{\tau,S} = 0.1686 \pm 0.0047 \) [51]. Taking \( |V_{us}| = 0.97377 \pm 0.00027 \) [11], one gets [60]

\[
|V_{us}| = 0.2220 \pm 0.0031 \text{ exp} \pm 0.0011 \text{ th}. \tag{11}
\]

This result is competitive with the standard \( K_{e3} \) determination, \( |V_{us}| = 0.2236 \pm 0.0029 \) [21]. The precision should be considerably improved in the near future because the error is dominated by the experimental uncertainty, which can be reduced with the much better data samples from BABAR [51], BELLE [38,63], and the forthcoming BESIII detector [18]. Therefore, the \( \tau \) data has the potential to provide the best determination of \( |V_{us}| \).

With future high-precision \( \tau \) data, a simultaneous fit of \( m_s \) and \( |V_{us}| \) should also become possible. A better understanding of the perturbative QCD corrections \( \Delta_{kl}(\alpha_s) \) would be very helpful to improve the resulting \( m_s \) accuracy [59,60].

![Figure 6. OPAL measurement of the spectral function distribution in \( |\Delta S| = 1 \) \( \tau \) decays [33].](image)
6. LEPTON MAGNETIC MOMENTS

The most stringent QED test comes from the high-precision measurements of the $e$ and $\mu$ anomalous magnetic moments $a_l \equiv (g_l - 2)/2$ \cite{65,66,67}. A recent measurement of $a_e$, using a one-electron quantum cyclotron, has reduced the experimental uncertainty by a factor of six \cite{68}:

$$a_e = (1.159\,652\,180.85 \pm 0.76) \cdot 10^{-12}. \quad (12)$$

To a measurable level, $a_e$ arises entirely from virtual electrons and photons; these contributions are known to $O(\alpha^4)$ \cite{65,66,67,69}. The theoretical error is dominated by the uncertainty in the input value of $\alpha$. Turning things around, the measured value of $a_e$ provides the most precise determination of the fine structure constant \cite{70}:

$$\alpha^{-1} = 137.035\,999\,710 \pm 0.000\,000\,096. \quad (13)$$

This number agrees with other precise determinations of $\alpha$, but it has an uncertainty (0.70 ppb) 10 times smaller than any other method.

The BNL-E821 experiment has recently published its final value for $a_\mu$ \cite{71}:

$$a_\mu = (11.659\,208.0 \pm 6.3) \cdot 10^{-10}. \quad (14)$$

The anomalous magnetic moment of the muon is sensitive to small corrections from virtual heavier states; compared to $a_e$, they scale as $m_\pi^2/m_\mu^2$. The Standard Model prediction can be decomposed in five types of contributions:

$$10^{10} \cdot a_\mu^{th} = 11\,658\,471.81 \pm 0.02 \text{ QED} \quad [67,69]$$

$$+\ 15.4 \pm 0.2 \quad \text{EW} \quad [72]$$

$$+\ 698.9 \pm 9.6 \quad \text{had}^{\text{LO}} \quad [51,73]$$

$$-\ 9.8 \pm 0.1 \quad \text{had}^{\text{NLO}} \quad [74]$$

$$+\ 12.0 \pm 3.5 \quad \text{lbl} \quad [66,75]$$

$$= 11\,659\,188.3 \pm 10.2.$$  

This result differs by 1.6 $\sigma$ from the experimental value \cite{14}.

The main theoretical uncertainty on $a_\mu$ has a QCD origin. Since quarks have electric charge, virtual quark-antiquark pairs induce hadronic vacuum polarization corrections to the photon propagator (Fig. 7c). Owing to the non-perturbative character of QCD at low energies, the light-quark contribution cannot be reliably calculated at present. Fortunately, this effect can be extracted from the measurement of the cross-section $\sigma(e^+e^- \to \text{hadrons})$ and from the invariant-mass distribution of the final hadrons in $\tau$ decays. The largest contribution comes from the 2$\pi$ final state. The $\tau$ decay determination includes a careful investigation of isospin breaking effects \cite{76}, using the pion form factor expression of ref. \cite{24}, which amount to an overall $-(2.2 \pm 0.5)\%$ correction \cite{73}.

At present, there is a discrepancy between the 2$\pi$ contributions extracted from $e^+e^-$ and $\tau$ data, which translates into slightly different values (2.9 $\sigma$) for $a_\mu^{\text{had},\text{LO}}$ \cite{73}:

$$a_\mu^{\text{had},\text{LO}} = \begin{cases} 
(690.8 \pm 4.4) \cdot 10^{-10} \quad (e^+e^-), \\
(710.3 \pm 5.2) \cdot 10^{-10} \quad (\tau) . 
\end{cases} \quad (15)$$

Therefore, from $e^+e^-$ data one gets the prediction $a_\mu^{th} = (11\,659\,180.2 \pm 5.6) \cdot 10^{-10}$, which disagrees with the measured value by 3.3 $\sigma$, while the $\tau$ data gives $a_\mu^{th} = (11\,659\,199.7 \pm 6.3) \cdot 10^{-10}$, in much better agreement (0.9 $\sigma$) with the BNL measurement of $a_\mu$ \cite{71}. In order to quote a reference number for $a_\mu^{th}$, I have used a weighted average of these two determinations, increasing the error with the appropriate scale factor \cite{11}.

New precise $\mu$ and $\tau$ data sets are needed to settle the true value of $a_\mu^{\text{had},\text{LO}}$. The present experimental situation is very unsatisfactory, showing internal inconsistencies among different $e^+e^-$ and $\tau$ measurements. The KLOE $e^+e^-$ invariant-mass distribution \cite{77} does not agree with CMD2 and SND, while the most recent BELLE measurement of the $\tau$ decay spectrum \cite{30} slightly disagrees with ALEPH and CLEO \cite{73}. Using CVC, one predicts from $e^+e^-$ data $\text{Br}(\tau \to \nu_\tau 2\pi) =$

![Figure 7. Feynman diagrams contributing to $a_l$.](image)
(24.48 ± 0.18)% which is 4.5σ smaller than the direct τ measurement (25.40 ± 0.10)% [73]. It is difficult to explain such a large disagreement as an isospin-breaking effect [78]. Since the e+e− prediction involves a delicate integration over all the spectrum, while the τ number is a more robust branching ratio measurement (on which all τ experiments agree), the e+e− analysis appears to me more suspect as a source of underestimated systematic uncertainties. The radiative return method [79], already used by KLOE [77], will allow the B Factories to provide some light on this issue. Preliminary analyses from BABAR have been already presented at this workshop [80].

Additional QCD uncertainties stem from the smaller light-by-light scattering contributions (Fig. 4). The most recent evaluations of these corrections [75], have uncovered a sign mistake in previous calculations, improving the agreement with the experimental measurement.

If funded, the Brookhaven E969 proposal could reduce the aμ experimental uncertainty by a factor of two or more [81]. A meaningful test of the electroweak contributions at this level of precision requires a better control of the QCD corrections. A factor of three improvement also seems possible in the aee measurement [68]. On the QED side a formidable effort to perform the fifth-order calculation has already started [69].

7. NEUTRINO OSCILLATIONS

The flux of solar νe neutrinos reaching the earth has been measured by several experiments [52] to be significantly below the standard solar model prediction [53]. The Sudbury Neutrino Observatory has provided strong evidence that neutrinos do change flavour as they propagate from the core of the Sun [5], independently of solar model flux predictions. SNO is able to detect neutrinos through three different reactions: the charged-current process νe, d → e−pp which is only sensitive to νe, the neutral current transition νe, d → νe, pn which has equal probability for all active neutrino flavours, and the elastic scattering νe, e− → νe, e− which is also sensitive to νμ and ντ, although the corresponding cross section is a factor 6.48 smaller than the νe one. The measured neutrino fluxes, shown in Fig. 8 demonstrate the existence of a non-νe component in the solar neutrino flux at the 5.3σ level. These results have been further reinforced with the KamLAND data, showing that νe from nuclear reactors disappear over distances of about 180 Km [5].

Another evidence of oscillations has been obtained from atmospheric neutrinos. The known discrepancy between the experimental observations and the predicted ratio of muon to electron neutrinos has become much stronger with the high precision and large statistics of Super-Kamiokande [78]. The atmospheric anomaly appears to originate in a reduction of the νμ flux, and the data strongly favours the νμ → ντ hypothesis. This result has been confirmed by K2K [9] and MINOS [10], observing the disappearance of accelerator ντ’s at distances of 250 and 735 Km, respectively. Super-Kamiokande has recently reported statistically significant evidence of ντ appearance at the 3.2σ level [7]. The direct detection of the produced ντ is the main goal of the ongoing CERN to Gran Sasso neutrino program [83].

Thus, we have now clear experimental evidence that neutrinos are massive particles and there is mixing in the lepton sector. The present solar, atmospheric, accelerator and reactor neutrino data, leads to the following preferred ranges for the os-
oscillation parameters [11]:
\[
\Delta m^2_{21} = (8.0^{+0.4}_{-0.3}) \cdot 10^{-5} \text{eV}^2 ,
1.9 \cdot 10^{-3} < |\Delta m^2_{32}| / \text{eV}^2 < 3.0 \cdot 10^{-3} ,
\sin^2(2\theta_{12}) = 0.86^{+0.03}_{-0.04} ,
\sin^2(2\theta_{23}) > 0.92 ,
\sin^2(2\theta_{13}) < 0.19 ,
\]
where \(\Delta m^2_{ij} \equiv m_i^2 - m_j^2\) are the mass squared differences between the neutrino mass eigenstates \(\nu_{i,j}\) and \(\theta_{ij}\) the corresponding mixing angles in the standard three-flavour parametrization. The ranges indicate 90\% CL. In the limit \(\theta_{13} = 0\), solar and atmospheric neutrino oscillations decouple because \(\Delta m^2_{32} \ll \Delta m^2_{2}\). Thus, \(\Delta m^2_{21}, \theta_{12}\) and \(\theta_{13}\) are constrained by solar data, while atmospheric experiments constrain \(\Delta m^2_{32}\), \(\theta_{23}\) and \(\theta_{13}\). The angle \(\theta_{13}\) is strongly constrained by the CHOOZ reactor experiment [80].
New planned reactor experiments [82], T2K and NOvA [84] are expected to achieve sensitivities around \(\sin^2(2\theta_{13}) \sim 0.01\).

8. NEW PHYSICS

Non-zero neutrino masses constitute a clear indication of new physics beyond the Standard Model. Right-handed neutrinos are an obvious possibility to incorporate Dirac neutrino masses. However, the \(\nu_{1R}\) fields would be \(SU(3)_C \otimes SU(2)_L \otimes U(1)_Y\) singlets, without any Standard Model interaction. Moreover, the gauge symmetry would allow for a right-handed Majorana neutrino mass term of arbitrary size, not related to the ordinary Higgs mechanism.

Adopting a more general effective field theory language, without any assumption about the existence of right-handed neutrinos or any other new particles, one can write the most general \(SU(3)_C \otimes SU(2)_L \otimes U(1)_Y\) invariant lagrangian, in terms of the known low-energy fields (\(\nu_{1L}\) only).

The Standard Model is the unique answer with \(d = 4\). The first contributions from new physics appear at \(d = 5\) and have also a unique form [87], which violates lepton number by two units:
\[
\Delta \mathcal{L} = - \frac{\epsilon_{ij}}{\Lambda} \bar{L}_i \phi \phi^c L_j + \text{h.c.} ,
\]
where \(\phi\) and \(L_i\) are the scalar and \(i\)-flavoured lepton \(SU(2)_L\) doublets, \(\phi \equiv i \tau_2 \phi^c\) and \(L_i^c \equiv CL_i^c\). Similar operators with quark fields are forbidden, due to their different hypercharges, while higher-dimension operators would be suppressed by higher powers of the new-physics scale \(\Lambda\).
Table 2
Best published limits (90% CL) on lepton-flavour-violating decays \[11,88,89\].

| Decay                                      | Limit                  |
|--------------------------------------------|------------------------|
| $\text{Br}(\mu^- \to e^- \gamma)$         | $< 1.2 \times 10^{-11}$ |
| $\text{Br}(\tau^- \to \mu^- \gamma)$     | $< 6.8 \times 10^{-8}$  |
| $\text{Br}(\tau^- \to e^- K_S)$           | $< 5.6 \times 10^{-8}$  |
| $\text{Br}(\tau^- \to \Lambda \pi^-)$    | $< 7.2 \times 10^{-8}$  |
| $\text{Br}(\tau^- \to \mu^- \pi^0)$      | $< 1.1 \times 10^{-7}$  |
| $\text{Br}(\mu^- \to e^- 2\gamma)$       | $< 7.2 \times 10^{-11}$ |
| $\text{Br}(\tau^- \to e^- 2\gamma)$      | $< 1.1 \times 10^{-7}$  |
| $\text{Br}(\tau^- \to \mu^- K_S)$         | $< 4.9 \times 10^{-8}$  |
| $\text{Br}(\tau^- \to \mu^- e^+ e^-)$     | $< 1.3 \times 10^{-7}$  |
| $\text{Br}(\tau^- \to e^- e^+ e^-)$       | $< 1.0 \times 10^{-12}$ |
| $\text{Br}(\tau^- \to e^- e^+ \pi^-)$     | $< 0.7 \times 10^{-7}$  |
| $\text{Br}(\tau^- \to e^- \pi^+ \pi^-)$   | $< 1.2 \times 10^{-7}$  |
| $\text{Br}(\tau^- \to e^- \pi^0)$         | $< 1.4 \times 10^{-7}$  |

After spontaneous symmetry breaking, $\langle \phi^{(0)} \rangle = v/\sqrt{2}$, $\Delta \mathcal{L}$ generates a Majorana mass term:

$$
\mathcal{L}_M = -\frac{1}{2} \bar{\nu}_L M_{ij} \nu^c_J + \text{h.c.}, \quad M_{ij} = \frac{c_{ij} v^2}{\Lambda}.
$$

(18)

Thus, Majorana neutrino masses should be expected on general symmetry grounds. The relation \[15\] generalizes the well-known seesaw mechanism ($m_{\nu L} \sim m^2/\Lambda$) \[90\]. Taking $m_{\nu} \gtrsim 0.05$ eV, as suggested by atmospheric neutrino data, one gets $\Lambda/c_{ij} \lesssim 10^{15}$ GeV, amazingly close to the expected scale of Grand Unification.

With non-zero neutrino masses, the leptonic charged-current interactions involve a flavour mixing matrix $U$. The present data on neutrino oscillations imply that all elements of $U$ are large, except for $U_{e3} < 0.18$ \[91\]. Therefore, the mixing among leptons appears to be very different from the one in the quark sector.

The smallness of neutrino masses implies a strong suppression of neutrinoless lepton-flavour-violating processes, which can be avoided in models with other sources of lepton flavour violation, not related to $m_{\nu L}$ \[92\]. The B Factories are pushing the experimental limits on neutrinoless $\tau$ decays beyond the $10^{-7}$ level \[88,89\], increasing in a drastic way the sensitivity to new physics scales. Future experiments could push further some limits to the $10^{-9}$ level \[88,91\], allowing to explore interesting and totally unknown phenomena. Complementary information will be provided by the MEG experiment, which will search for $\mu^+ \rightarrow e^+ \gamma$ events with a sensitivity of $10^{-13}$ \[95\]. There are also ongoing projects at J-PARC aiming to study $\mu \rightarrow e$ conversions in muonic atoms, at the $10^{-18}$ level \[96\].

An important question to be addressed in the future concerns the possibility of leptonic CP violation and its relevance for explaining the baryon asymmetry of our universe through leptogenesis.

9. OUTLOOK

Our knowledge of the lepton properties has been considerably improved during the last few years. Lepton universality has been tested to a rather good accuracy, both in the charged and neutral current sectors. The Lorentz structure of the leptonic $l \rightarrow \nu l^\prime \bar{\nu}_l$ decays has been determined with good precision in the $\mu$ decay and relevant constraints have been obtained for the $\tau$ \[11\]. An upper limit of 3.2% (90% CL) has been already set on the probability of having a (wrong) decay from a right-handed $\tau$ \[23,34\].

The quality of the hadronic $\tau$ decay data has made possible to perform quantitative QCD tests and determine the strong coupling constant very accurately, providing a nice experimental verification of asymptotic freedom. Information on the strange quark mass has also been obtained from Cabibbo-suppressed hadronic $\tau$ decays; these decay modes are expected to provide soon the most precise determination of $|V_{us}|$.

The recent measurement of the electron anomalous magnetic moment has substantially improved the determination of $\alpha$, while the BNL investigation of $a_\mu$ has reached the needed sensitivity to explore higher-order electroweak corrections. Further experimental progress could be possible. To perform a meaningful precision test of the electroweak theory, it is necessary to control better the QCD contributions. The present $e^+e^-$ versus $\tau$ experimental controversy on the photon vacuum polarization should be resolved, and a more accurate determination of the light-by-light scattering contribution is needed.
The first hints of new physics beyond the Standard Model have emerged recently, with convincing evidence of neutrino oscillations from solar, atmospheric, accelerator and reactor neutrino experiments. The existence of lepton flavour violation opens a very interesting window to unknown phenomena, which we are just starting to explore. It seems possible to push the present limits on neutrino-less $\tau$ decays beyond the $10^{-8}$ or even $10^{-9}$ level, probing the underlying lepton flavour dynamics to a much deeper level of sensitivity. At the same time, new neutrino oscillation experiments will measure the small mixing angle $\theta_{13}$ and will investigate whether CP violating phases are also present in the lepton mixing matrix.

The huge $\tau$ data sample accumulated at the B Factories will soon be complemented with the BESIII $\tau^+\tau^-$ pairs, collected at threshold. Moreover, a possible Super-B Factory is already under study [93], and further ideas towards a low-energy Tau-Charm Factory with luminosities beyond $10^{35}$ cm$^{-2}$s$^{-1}$, using a high-intensity $e^+e^-$ linear collider, have been presented at this workshop [94]. Therefore, $\tau$ physics will continue being a very active field of research in the next years. Among the new topics which could be investigated in the future, it is worth mentioning the search for CP violating signals in $\tau$ decays. Decays of heavier particles into $\tau$ leptons are another interesting field of research, as exemplified by the recent determination of $f_B|V_{ub}|$ from the $B \to \tau\nu$ branching ratio [97]. The Tevatron has also shown the advantages of the $\tau$ lepton as a tool for new physics searches [98]. The $\tau$ provides a clean signature and the possibility to perform polarization analyses, which makes $\tau$ identification a key ingredient for new discoveries. Strategies to exploit the full $\tau$ potential at LHC are being actively investigated at present [99].

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