Solutio Theorematum
by Adam Adamandy Kochoński – Latin text with annotated
English translation
translated by Henryk Fukš
Department of Mathematics, Brock University, St. Catharines, ON, Canada
e-mail hfuks@brocku.ca

Translator’s note: The Latin text of Solutio theorematum presented here closely follows the original
text published in Acta Eruditorum [1]. Punctuation, capitalization, and mathematical notation
have been preserved. Several misprints which appeared in the original are also reproduced un-
changed, but with a footnote indicating correction. Every effort has been made to preserve the
layout of original tables. The translation is as faithful as possible, often literal, and it is mainly
intended to be of help to those who wish to study the original Latin text. In the appendix, all
propositions from Euclid’s Elements mentioned in the text are listed in both Latin and English
version.

SOLUTION OF THE THEOREM
proposed by an illustrious man in
this year’s January issue of Acta
on p. 28, given by Adam
Adamandy Kochoński SJ, once
mathematician of Prague.

Fr. Sigismundus Hartman from Soc. of Je-
sus proposed in a public program the problem of
DUALPLICATION of an equilateral triangle with-
out the use of proportions, and conveyed this
question to me from Bohemia to Poland, on the
account of old friendship. I returned to the kind-
est author perhaps twenty solutions, by him thus
examined, so that one of these, together with
others, send to him from elsewhere, was to be
put to light, if only Fate had spared the man so
great.

When, however, those days Acta Eruditorum
of Leipzig came into my hands, and in them so-
lution of this problem of Hartman, given by a
certain illustrious man, directed to Fr. Copil-
lus, successor of the deceased in the mathemat-

\[ \text{1} \] Sigismundus Ferdinands Hartmann SJ (1632–1681) – Bohemian Jesuit and mathematician, professsor of the University of Prague.
\[ \text{2} \] Matthaeus Coppilius SJ (1642–1682) – Bohemian Jesuit and mathematician, author of books on mechanics.
tioni posthumae operum Hartmanni, maximeque *Protei Geometrici*, ab eo nuper promissi incumberet; cum tamen ab obitu Authoris elaboratum nihil, sed prima solum Operis lineamenta reperta fuisses mihi ab Amicis nunciatum fuerit; Eam ob rem non aegre latum spero P. Coppilium, si dum illum alio in opere fructuose versarsi intelligo, hic ejus partes occupare ausus fuero: non enim it temere, aut praesidenter egisse videbor, sed veluti quodam antiquitatis; quod videlicet ante illum in eodem Matheseos Pragensis pulvere quemam Professor fuerim versatus; adeoque prior tempore, licet non eruditione.

Dilatis autem in aliius tempus meis illis *Duplicaionum Particularium, & Universaliun Solutionibus*, una cum *Pythagoricae nova*, ac multiplici Demonstratione, ceterisque meis considerationibus *Geometricis*, usque dum ultroneam Typographi, vel Bibliopolae cujuspiam humanitatem invenerint; suffecerit hoc loco strictim ea persequi, quae pertinent ad geminum illud Theorema Geometricum, quod Anonymus ille Problematis Hartmanniani δείχτης *loco sup. memorato* proposuit.

**ARTICULUS I.**

Circa primum illorum Theorematum considerationa veniunt sequentia. Inprimis dissentire me in eo ab *Illustri Viro*, quod est existimet Theorema Pythagoreum continuari in Sphaera, Pyramidem Rectangulam circumscribente; cum nec Pythagoras suum illud Orthogonium, tanquam circulo inscriptum consideraverit, nec, si universaliter agamus de potentissvis Trianguli, haec considerationi proprius ad Circulum pertinere videatur, sed positus ad Parallelogrammum universim; quando videlicet istud Diametro sua sectum concipitur in duo Triangula aequalia, eaque Orthogonia, Amblygonia, vel Oxygonia, pro diversitate parallelogramm; Harum enim Diame-

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3Kochański stayed in Prague from 1670 to 1672, lecturing in mathematics and (probably) moral philosophy.
trorum Potentiae cum suis lateribus comparentur 47. *Primi*, nec non 13, & 13. *Secundi Elem*. Universalius autem hoc ipsum consideratur 31 *Sexti* saltem quoad orthogonium Triangulum; nam quoad reliqua, videri possunt ea, quae demonstrat Clavius e Pappo, *in Scholio ad* 47. *Primi, ad finem*.

Quamobrem non ineleganter fluente Analogia, nobis dicendum videtur, Pyramides Triangulares Orthogonias, Amblygonias, & Oxygonias, cum suis laterum, ac diametrorum Potentiis, immediate quidem reduci oportere ad Prismata sua, bases triangulares habentis, quorum Pyramides illae sunt partes tertiae per Prop. 7. *Duodecimi Elem*. Haec ipsa vero Prismata, tanquam partes revocantur ad totum Parallelepipedum, cujus sunt medietates. Istas porro relationes partium ad sua Tota intelligi volumus de ordine *Doctrinae* potius, quam *Naturae*, constat enim, Triangulum esse prius, ac simplicius Parallelogrammo, non minus ac Pyramidem Tetrahedram Prisme, vel Parallelepipedo:

Hinc δογµατιχως sperando, & ipsae Linearum Potentiae, non Triangulis aequilateris, sed Quadratis, omnium consensu taxantur, licet illa sint ipsis priora, magisque simplicia. Quamvis autem Pyramidum Polyhedralum aliae inscribi possint Sphaeris, aliae vero Sphaeroidibus, ac tum Potentiae laterum conferri cum Diametro corporis circumscribentis: eadem tamen Pyramides adhuc secari poterunt in Tetrahedras, atque ita Prismatibus suis, ac tum Parallelepipedis veluti postliminio quodam restitui. Si quis nihilominus omnia Trilatera a Quadrilateris, omnia Tetrahedra a Pentahedris & Hexahedris emancipare contenderit cum eo nequaquam cruento Marte dimicabimus.

Powers [squares] of these diameters [diagonals] are compared with powers of their parallelograms; as long as the right triangle is considered. On the other hand, cases which remains can be seen as those demonstrated by Clavius following Pappus *at the end of commentary to prop. 47*. It seems we need to state an elegantly flowing analogy, that it is right to put triangular prisms, right-angled as well as obtuse-angled and acute-angled, with powers [squares] of their sides and diameters, into their respective prisms, having triangular bases, of which these prisms constitute third parts [by volume], by prop. 7 of the *twelfth book*. Such prism are in truth recalled just as if they were parts of a complete parallelepiped, of which they are halves. Hereafter we want these relations of parts to their wholes to be understood from *Principles* rather than from *Nature*, as it is evident that the triangle is prior to and simpler than a parallelogram, not less as the tetrahedral pyramid is prior to and simpler than a prism, or parallelepiped: from there observing the thing dogmatically, powers of own lines taken together are valued not with equilateral triangles, but squares, although those [triangles] are prior to these [squares], and much simpler.

Although some polyhedral pyramids could be inscribed in spheres, and others in spheroids [i.e., ellipsoids of revolution], and then powers of their sides could be matched against diameters of circumscribed bodies: besides, the same pyramids could be divided into tetrahedrons, and therefore also brought into their own prisms, and then parallelepipeds, as if by the right to return home. None the less, if somebody wanted to alienate all trilaterals from quadrilaterals, all tetrahedra from pentahedra and hexahedra, we will by no means spill blood in a fight with him.

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4Obviously, this should be “prop. 12. and prop. 13”.
5Pappus builds parallelograms on sides of an arbitrary triangle, cf. p. 366 of [2].
PROPOSITIO I. Theorema.
In omni Pyramide rectangula, tria Quadrata laterum, Angulum rectum in vertice comprehendentium, aequalia sunt Quadrato Diametri totius Parallelepiedi aeque alti, Pyramidem illam completentis.

Sit Pyramis ABC rectangula ad verticem D. sive jam sit Aequilatera, prout in Cubo Figura I. sive Isosceles, ut est in Fig. II Parallelepipedi, supra basin quadratam ADCF assurgentis; sive demum Scalena, qualem exhibet in Fig. III. solidum rectangulum, supra basin ADCF altera parte longiorem, erectum.

Dico, tria Quadrata DA, DB, DC aequari Quadrato Diametri AE, per oppositos solidi angulos incedentis. Nam primam in Triangulo ADB angulus D rectus est, ex hypoth.

Igitur Qu. lateris AB aequatur (47. I. Elem.) Quadratis AD, DB duorum laterum datae Pyramidis. Deinde Triangulum pariter ABE rectangulum est ad B. (id ostendi potest per 4. Undecimi) Quocirca Quadratum AE aequabitur Quadrato AB, hoc est duobus DA, DB, & insuper Quadrato BE, hoc est ipsi aequali DC, quod est tertium latus datae Pyramidis ABCD.

Tria igitur omnis Pyramidis rectangulae latera, potentia aequantur Diametro Parallelepipedi Pyramidem continentis, qu. e. d.

Corollarium I.
Colligitur hinc, in omni Prismate rectangulo ACDBGA, cui basis est Orthogonium Triangulum ADC cujus Parallelogrammi rectanguli AGEC, quod angulos D. & B. rectos subtendit, Diametrum AE. Potentia aequari iidem tribus lateribus Pyramidis rectangulae AB-

PROPOSITION I. Theorem.
In every right-angled pyramid, sum of squares of three sides coming from the vertex embracing the right angle is equal to the square of the diameter of the complete parallelepiped of equal height, encompassing this pyramid.

Let the pyramid ABC be right-angled at the vertex D, whether equilateral, as in the cube of Fig. I, or isosceles, as in the parallelepiped of Fig. II, rising over the square basis ADCF, or finally scalene, as in the rectangular solid erected over the basin ADCF elongated in the other direction, as displayed in Fig. III.

I say that three squares of DA, DB, and DC are equal in sum to the square of the diameter AE, stretched between opposite angles of the solid. For, of first, in the triangle ADB, the angle D is right, by hypothesis. Therefore the sum of squares of the three sides of the right-angled pyramid is equal to the square of AB, that is sum of squares of DA and DB plus square of BE, which itself is equal to DC, the third side of the given pyramid.

Therefore three sides of any rectangular pyramid are equal in power to the diameter of the parallelepiped enclosing the pyramid, Q.E.D.

Corollary I.
One obtains from there that in all rectangular prisms ACDBGA, whose base is a right triangle ADC, square of the diameter AE of the right-angled parallelogram AGEC, which extends below right angles D and B, is equal to sum of squares of three sides of the right-angled pyramid.

\[ AB^2 = AD^2 + DB^2, \quad AE^2 = AB^2 + BE^2 = AD^2 + DB^2 + BE^2 = AD^2 + DB^2 + DC^2. \]

\[ i.e., \text{sum of their square is equal to the square of...} \]

\[ Having \text{three equal sides.} \]

\[ AB^2 = AD^2 + DB^2, \quad AE^2 = AB^2 + BE^2 = AD^2 + DB^2 + BE^2 = AD^2 + DB^2 + DC^2. \]
CD: eo quod ipsa DE sit eadem omnino cum Diametro totius Solidi GC.

**Corollarium II.**

Hinc quoque manifestum est, Diametrum Sphaerae, quae Pyramidi rectanguli ABCD circumscripta est, eaequari potentia tribus lateribus ejusdem Pyramidis, rectos angulos constituentibus in vertice D. Nam cum omni solido rectangulo Sphaera circumscripsi possit, non minus ac Circulus ejus basi rectangulae, erit Diameter solidi eadem omnino quae circumscriptae Sphaerae, per ea, quae Pappus demonstrat Lemmate 4. apud Clavium ad calcem Lib. 16. Elem.

Si cui placuerit in simili materia ingenium exercere circa Pyramides Triangulares, tam Acutangulas, quam Obtusangulas, itemque mixtis in vertex angulis contentas in illis Potentias lateribus ejusdem Pyramidis comparando; id vero non difficulter poterit expedire ope duarum Propositionum, videlicet penultimae, & antepenultimae Lib. Secundi Elem.

**ARTICULUS II.**

Circa alterum Theorema percutatur Illustris Vir AN sicut in Circulo unica Media Proportionalis, ita etiam insistendo Analogiae, in Sphaera duae Mediae inveniri possint ? Ad hanc quaestionem Respondeo I. Nec Antecedens Analogiae huimus tam ratum esse, ut absolute loquendo, Circulus duas Medias excludere pronunciari possit, aut debeat: Fieri namque potest, ut in Semicirculo ACD (inspice Fig. 4.) continuae sint AB, BC, BD, DA. cujus Problematis Geometricam constructionem Mathematam peritis proponeo, interim vero Arithmeticum sequentibus numeris expono. Ponatur enim

ABCD: consequently DE\(^9\) itself would be entirely the same as the diameter GC of the whole solid.

**Corollary II.**

From this it is evident that the square of the diameter of the sphere circumscribed around a right-angled pyramid ABCD, is equal to the sum of squares of three sides of this pyramid forming the right angle at the vertex D. For while every rectangular solid can be circumscribed by a sphere, as much as its rectangular base [can be circumscribed] by a circle, the diameter of this solid will be entirely the same as the diameter of the circumscribing sphere, as Pappus demonstrated in *Lemma 4 in Clavius’ commentary at the end of book 16\(^{10}\) of Elements.*

If somebody would please to exercise his talent in similar matters regarding triangular pyramids, either acute-angled or obtuse-angled, and likewise stretching over mixed angles, comparing sums of squares of the sides with the diameter of the whole solid; this will certainly not be difficult to obtain by the power of two propositions, namely the second last and the third from the end proposition of the second book of *Elements.*

\(^9\)Clearly a misprint. Should be AE.

\(^{10}\)Book XVI was a medieval addendum to *Elements.*
Diameter AD. - - 2 00000 00000
erit AB. - - 63534 43923. +- 
BC. - - 93114 24637. +- 
BD. - - 1 36465 56077. +- 

Unde per 19. Septimi Elem. erunt aequa-
li Rectangula

From there by Prop. 19 of the seventh book of
Elements [the following] rectangles will be equal

DAB. - - 1 17068 87846 00000 00000.
CBD. - - 1 17068 55798 69049.

Gg.

Et per 20. ejusdem, Quadrata mediarum
aequabuntur Rectanguli sub earundem extreme-

And by Prop. 20 of the same, squares of
means will be equal to [areas of] rectangles under
their outer segments.12

□BC - - - 86702 62877 05305 81769.
ABD - - - 86702 62877 72943 70071.
□BD - - - 1 86288 49276 27056 29929.
ADBC - - - 1 86228 49274 00000 00000.

Respondeo II. De Analoga illa nihil certi
statui posse videtur: Unius enim Mediae in-
ventio, quae Circulo tribuitur, etiam Sphaeræ
congruit; & inventio duarum Mediarum, hic a
nobis demonstranda, aeque ad circulum, sicut
&Sphaeram aptari poterit, quenammodum
ex dicendis constabit.

Secondly, I respond than nothing certain
seems to possible to state about this analogy.
Invention of one mean, assigned to a circle,
suits to sphere too. Invention of two means,
demonstrated here by us, can be adapted as
well to a circle as to a sphere, in what way it
will agree with what has been said.

PROPOSITIO II. Theorema.
In Circulo ADC a Diametro AC descripto du-
cantur utcunque ad peripheriam duae rectae
AD DC: tum ex D cadat ad AC perpendicularis D; similiterque ex E sit ad AD normalis
EF.

Dico in Circulo ADC, haberi Quatuor con-
tinue Proportionales, AF. AE. AD. AC. De-
scrivatur enim Diametro AD Circulus AGDE,
quem EF productae secet in G, & connectantur
G A. GD.

PROPOSITION II. Theorem.
In a circle ADC traced out from a diameter AC
two straight lines are drawn as far as to the
perimeter: then from D a line13 perpendicular
to AC line is led, and similarly from E a line EF
normal to AD.

I say that in the circle ACD four consecutive
proportionals exist, AF, AE, AD, AC. Indeed,
let a circle AGDE with diameter AD be drawn,
which intersects with extension of the line EF at
G, and let G connecting lines GA and GD be

11Prop. 20 is often omitted in modern editions of Elements, as it is considered a later addition, and a direct
consequence of proposition 19.
12“Outer” means neighbours in the sequence. Kochanski considers sequence of linear segments AB, BC, BD, DA,
where both BC and BD are geometric means of their nearest neighbours in the sequence.
13A clear misprint: this should be DE.
Demonstr. Recta AD subtendit Angulum rectum DEA. Igitur Circulus AGDE Diametro AD descripsit transit per verticem Anguli recti DEA. juxta Schol. Clavii ad 31. 3 Elem. Est autem recta EFG perpendicularis ad Diametrum AD. Ergo per 3. 3. Elem. tota EG bifariam secatur in F. Triangula igitur AFG, AFE orthogonia in F, sunt per 4.1. Elem. in-vicem aequalia : Eademque de causa aequan-tur Triangula DFG. DFE, ac proinde & totum DGA toti DEA aequale. Jam sic. In Orthogo-nio AED (par ratio de aequali AGD) ab angu-lo recto E cadit perpendicularis EF in basin AD: Ergo per Coroll. 8.6. Elem. Proportio-nales sunt tres AF.AE.AD. Sed eadem de causa in Orthogonio ADC duabus postremis e praeecedenti serie, videlicet AE. AD. propor-tionalis est tertia AC. Igitur omnes quatuor AF.AE.AD.AC sunt in continua Propotio-ne intra Circulum ADC, q.e.d.

Corollarium.
Non difficulter hinc elicitur, easdem qua-tuor continuas in Spaerae quoque concipi pos-se, non modo praedicta, sed & alia ratione: Fingamus enim Sphaera ADC auferri Segmen-tum AHDA, cujus basis erit Circulus a Dia-metro AD, cujus meditas esto AGDA. Ductis autem Orthogonalius DE. EF. in plano Cir-culi Sphaeræ maximæ ADC, nec non Orthog-onali FG, in altero plano Semicirculi AGD, hoc est basi Segmati AHDA, jungatur AG: erunt enim ut antea, quatuor AF. AD.AC. con-tinue proportionales, id patet e praeecedenti discursu, qui non difficulter hue applicari po-terit, licet plana Circulorum ADG. AGD. sint diversa, & ad rectos invicem collocate.

Notandum vero est, Theorema praece-

14spherical cup

Proof. Line AD extends beneath the right angle DEA. Therefore the circle AGDE determined by the diameter AD passes through the vertex of the right angle DEA, according to commentary of Clavius to prop. 31 of book 3 of Elements. The straight line EFG is in fact perpendicular to the diameter AD. Therefore by prop. 3, book 3 of Elements, the entire EG is divided into two equal parts at F. The triangles AFG, AFE having straight angle at F, are by Prop. 4.1 Elem. equal to each other. For the same reason triangles DFG, DFE are equal, and hence the whole [triangle] DGA is equal to the whole [triangle] DEA. Now in the right-angled triangle AED (similarly reasoning applies to AGD) from the right angle E a perpendicular line EF falls onto the base AD: therefore, by Coroll. 8. 6. Elem., there are three proportionals AF, AE, AD. But for the same reason, in the right-angled triangle ADC, AC is proportional to the last lines from the aforementioned sequence [of proportionals], namely AE and AD. Therefore all four AF, AE, AD, AC are proportional in succession within the circle ADC, Q.E.D.

Corollary.
Form there it is not difficult to elicit that the same four successive proportionals can be devised in a sphere, not by the preceding method, but by a different reasoning: let us namely imagine that a segmentAHDA is taken from a sphere ADC, whose basis is a circle with diameter AD, and whose one half is AGDA. Drawing perpendicular lines DE, EF in the plane of the great circle ADC, and perpendicular line GF, in another planar semicircle AGD, that is, in the base of the segmentAHDA, let they be joined by AG: there will be, as before, four successive proportionals AF, AG, AD, AC. It stands clear from the previous discourse, which could be applied here without difficulty, that it is permitted that planes of circles ADG and
Inter duas datas, duas medias in continua ratione, duobus tantum digitis reperire.

Celeberrimum illud Problema Deliacum quot & quanta totius Orbis eruditi exercuerit ingenia, Geometris est notissimum, ut & variae illius absolvendi Praxes Organicae, a compluribus excogitatae, quorum aliae alii sunt operosiores: Nostra haec videri poterit nonnihil Paradoxa, quod duobus tantum digitis unius manus, absolvat, cum nonnullae requirant, & occupent utramque. Datae sint, in Figura VI. duae AC. AB. quas inter duae mediae quaeruntur. In commun utriusque termino A figatur Regula AZ, instructa Cursore FY, qui semper insistat ad rectos ipsi regulae AZ, idque firmiter, ubicunque collocetur. In illa fumatur AF, qualis datae AB, minori altera AC. Descripto autem super tota AC Semicirculo ADC in plano quopiam verticali, hoc est ad Horizontem recto, cui aquidistet Diameter AC; applicetur ad Peripheriam ADC Stylus quidam gracilis DS, e quo deorsum propendeat filum subtile cum appenso Pondere X, vel certe hujus loco regula quaedam sub gravis, accurate tamen aequilibra: Nam si Stylus DS duobus digitis apprehensus pedenttim promoveatur per Circumferentiam AD, usque dum Perpendicularum DE cum Cursore FE sese mutuo interscent alicubi in recta AC, velut in Puncto E: istud probe notatum offeret quatuor Proportionales, quorum duae AE. AD. inter datas AGD are different, and placed at right angles to each other.

One must observe, however, that the previous theorem, speaking more precisely, is not as tied to a semicircle as to an arbitrary triangle subdivided into similar ones: yet because its proof lends itself to the following problems, certain leniency toward the circle appeared in this place, without which they could to be brought out.

**PROPOSITIO III. Problema.**

Proposition III. Problem.

Between two given [quantities], find two means in successive proportions, with only two fingers.

It is very well known to Geometers how many and how great learned [men] exercised [their] talents on this most famous Delian problem, and how various practical methods of its solution utilizing mechanical instruments, devised by many, have advantage one over another. With our method one could these as paradoxes, because it utilizes only two fingers of one hand, while some other [methods] require and occupy both [hands].

Given are, as in Figure VI, two [quantities] AC and AB, between which two means are sought. At their common end A a ruler AZ is fixed, equipped with cursor FY, which always firmly stands at the right angle to the ruler AZ, wherever placed. With the ruler the distance AF is taken, equal to the given AB, smaller that AC. Somewhere in a vertical plane, that is, perpendicular to the horizontal line, let a semicircle ADC be drawn over the entire AC, whose diameter is equal to AC. Let at the circumference of ADC a thin stylus DS be placed, from which a fine string is hanging down with a weight X attached, and at which finally the ruler is placed in equilibrium under gravity. For if the stylus DS is carefully held with two fingers and moved through the circumference AD, all the way until the perpendicular DE intersects with he cursor FE somewhere on the straight line AC, for instance at point E,
extremas AF hoc est AB nec non AC interpo-
entur.

Demonstratio Problematis hujus, quoad inter-
rem, eadem est; cum adducto praecedenti
Theoremate.

PROPOSITIO IV. Problema.

Id ipsum aliter, una Circini
apertura.

Quoniam praecedens praxis ob situm pla-
ni verticalem, & usum Perpendiculi, nonmi-
hil impedita cuiiam videri possit, quin & a
Geometriae moribus aliena, dabimus alteram,
predictis incommodis haud obnoxiam.

Positis iisdem, in locum perpendiculi DEX
Figurae praecedentis, subrogetur in hac Figu-
ra VII. Parallelogrammum materiale KLMN,
cujus unum Latus KL. fixum sit in plano, al-
terum vero MN mobile, semper tamen ad rec-
tos ipsi AC. Nam si tota AC divisa bifariam
in O Circini pes unus figatur in O, alter au-
tem intervallo OC diductus, tandem in Ar-
cu CD provehatur, impellatque binas Regulas
AZ. & MN in puncto intersectionis D, usque
dum regula MN Cursorem FY secet in puncto
E, posito in recta AC, obtineuntur eadem
mediae AE. AD. longe commodiori ratione,
quam fuerit praecedens; quae tamen ipsa,
si Figura magnae molis fuerit, in vasto quopiam
pariate usu esse poterit Architectis.

Non est cur hoc loco moneam de circi-
no, ejusque Cruribus in regula quapiam mo-
bilibus, nec de acie pedis alterius, quae re-
gulas in puncto D subtiliter impellere debet;
nec denique de nisu quodam regularum AZ,
MN contra Circinum ; hunc enim vel ipsa-
rum regularum pondere, vel Elatare quopiam,
aut unus digiti impulsu consequetur ingenio-
sus quivis, ac istam nostris longe meliora
excogitabit.

this, as it rightly to be noted, produces four
proportionals, of which two, AE and AD, are
interposed between two given outward [lines]
AF (that is AB) and AC.

Demonstration of [correctness of solution of]
this problem follows from the preceding proved
theorem.

PROPOSITION VI. Problem.

The same thing differently, with
one aperture of the compass.

Because the previous method seems to be
hindered by the positioning of the vertical plane
and by the use of the plumbline, which is alien
to customs of geometers, we will give another
[method], not burdened with the aforementioned
inconveniences.

Setting up things as before, in place of the
perpendicular DEX of the preceding figure, let
in Figure VII a material parallelogram KLMN
be substituted, whose one side KL is fixed in
the plane, and another [side] MN is mobile, still
always staying perpendicular to AC. For if the
whole AC is divided into two [equal] parts at
O, and one foot of the compass is placed at O,
and another draws a line with [the aperture of]
the interval AO, as long as it moves along the
arc CD, let two rules AZ and MN, intersecting
at the point D, be pushed until the ruler MN
intersects the cursor FY at point E, positioned
on the line AC. By this the same means AE and AD
will be obtained, with a much more convenient
method than the preceding one. This method, if
the Figure was much larger, [placed] somewhere
on a huge wall, could be of use to architects.

It is not a place for me to give advise about
the compass and its legs moveable is a certain
ruler, or about the sharpness of the other leg,
which should delicately push rules at point D, or
finally about the pressure of rules AZ and MN
against the compass. This indeed someone in-
genious will attempt to achieve by the weight of
rulers, or by some spring, or by the push of one
finger, or will devise something much better than that.

Figures 1 – 7.
Appendix - list of propositions from *Elements* mentioned in the text of *Solutio theorematum*

*The first number indicates proposition number, the second one is the book number. Latin text from [3], English translation of propositions from [2]. Pappus generalization of 47.1 and Clavius’ scholium for Prop. 31.3 translated by H.F.*

**Prop. 4.1**

If two triangles have the two sides equal to two sides respectively, and have the angles contained by the equal straight lines equal, they will also have the base equal to the base, the triangle will be equal to the triangle, and the remaining angles will be equal to the remaining angles respectively, namely those which the equal sides subtend.

**Prop. 47.1**

In right-angled triangles the square on the side subtending the right angle is equal to the squares on the sides containing the right angle.

**Prop. 12.2**

In obtuse-angled triangles the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle by twice the rectangle contained by one of the sides about the obtuse angle, namely that on which the perpendicular falls, and the straight line cut off outside by the perpendicular towards the obtuse angle.

**Prop. 13.2**

In acute-angled triangles the square on the side subtending the acute angle is less than the squares on the sides containing the acute angle by twice the rectangle contained by one of the sides about the acute angle, namely that on which the perpendicular falls, and the straight line cut off within by the perpendicular towards the acute angle.

**Prop. 3.3**

If in a circle a straight line through the centre bisects a straight line not through the centre, it also cuts it at right angles; and if it cut at right angles, it also bisects it.
**Prop. 8.6**
Si in triangulo rectangulo, ab angulo recto ad basim perpendicularis ducatur; quae ad perpendiculararem sunt triangula, et toti, et inter se sunt similia.

**Prop. 31.6**
In triangulis rectangulis figura rectilinea quae sit a latere rectum angulum subtendere, ae- quale est eis quae a lateribus rectum angulum continentibus sunt, similibus et similiter descriptis.

**Prop. 19.7**
Si quatuor numeri proportionales fuerint, qui ex primo, & quarto fit, numerus, aequalis erit ei, qui ex secundo, & tertio fit, numero. Et si, qui ex primo, & quarto fit, numerus, aequalis fuerit ei, qui ex secundo, & tertio fit, numero; ipsi quatuor numeri proportionales erunt.

**Prop. 20.7**
Si tres numeri proportionales fuerint; qui sub extremis continetur, aequalis est ei, qui a medio efficitur: Et si, qui sub extremis continetur, aequalis fuerit ei, qui a medio describitur; ipsi tres numeri proportionales erunt.

**Prop. 4.11**
Si recta linea duabus rectis lineis se invicem secantibus in communi sectione ad rectos angulos insistat, etiam ducto per ipsas plano ad rectos angulos erit.

**Prop. 7.12**
Omne prisma triangulem habens basim, dividitur in tres pyramides aequales inter se, quae triangulares bases habent.

**Pappus generalization of 47.1, as given by Clavius**
In omni triangulo, parallelogramma quaeque super duoibus lateribus descripta, aequalia sunt parallelogrammo super reliquo late- re constituto, cuius alterum latus aequalis sit, & parallellum rectae ductae ab angulo, quae duo illa latera comprehendunt, ad punctum, in quo conveniunt latera parallelogrammorum lateribus trianguli opposita, si ad partes an- guli dicti producantur.

**Prop. 8.6**
If in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar both to the whole and to one another.

**Prop. 31.6**
In right-angled triangles the figure of the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle.

**Prop. 19.7**
If four numbers be proportional, the number produced from the first and fourth will be equal to the number produced from the second and the third; and, if the number produced from the first and fourth be equal to that produced from the second and third, the four numbers will be proportional.

**Prop. 20.7** (stated in the commentary to prop. 19.7 in [2])
If three numbers be proportional, the product of the extremes is equal to the square of the mean, and conversely.

**Prop. 4.11**
If a straight line be set up at right angles to two straight lines which cut one another, at their common point of section, it will also be at right angles to the plane through them.

**Prop. 7.12**
Any prism which has a triangular base is divided into three pyramids equal to one another which have triangular bases.

**Pappus generalization of 47.1, as given by Clavius**
In every triangle, any parallelograms built on two sides are equal to the parallelogram built on the remaining side, whose other side is equal and parallel to the straight line drawn from the angle made by the two sides of the triangle to the point of intersection of extensions of the sides of parallelograms opposite to the sides of the triangle.
Scholium Clavii ad 31.3
Manifestum quoque est conversum huius theorematis. Hoc est, segmentum circuli, in quo angulus constitutus est rectus, semicirculus est. Nam si esset maius, angulus in eo foret acutus; si minus, obtusus.

Corollarium ad Prop. 8.6
Ex hoc manifestum est, perpendiculararem quae in rectangulo triangulo ab angulo recto in basin demittitur, esse mediam proportionalem inter duo basis segmenta.

Clavius’ scholium for Prop. 31.3
It is also clear that the converse of this theorem is true. That is, a segment of a circle, in which a right angle is constituted, is a semicircle. For is it was greater, the angle in it would be acute, if lesser, it would be obtuse.

Corollary to Prop. 8.6
From this is clear that, if a right angled triangle a perpendicular be drawn from the right angle to the base, the straight line so drawn is a mean proportional between the segments of the base.

References

[1] Adam Adamandy Kochański. “Solutio Theorematum Ab illustri Viro in Actis hujus Anni Mense Januario, pag. 28. propositorum”, Acta Eruditorum, Lipsiae 1682, pp. 230–236.

[2] Euclid. The thirteen books of the Elements, translated with introduction and commentary by Sir Thomas L. Heath, Dover, New York, 1956.

[3] Euclides, Christophorus Clavius. Euclidis Elementorum libri XV, Romae apud Vincentium Accoltum, 1574.