Random Surfaces that Suppress Single Scattering

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We present a method for generating numerically a one-dimensional random surface, defined by the equation $x_3 = \zeta(x_1)$, that suppresses single-scattering processes in the scattering of light from it within a specified range of scattering angles. Rigorous numerical calculations of the scattering of light from surfaces generated by this approach show that the single-scattering contribution to the mean scattered intensity is indeed suppressed within that range of angles.
In theoretical and experimental studies of multiple-scattering effects in the scattering of light from randomly rough surfaces it is often desirable to be able to suppress the contribution to the mean scattered intensity from single-scattering processes: effects such as enhanced backscattering or the presence of satellite peaks become more readily observable in the absence of the background provided by single-scattering processes.

In theoretical studies it is possible to separate the contribution from single-scattering processes to the mean intensity of the light scattered incoherently from the contribution from multiple-scattering processes. This separation of single- and multiple-scattering contributions is particularly simple in the context of small-amplitude perturbation theory \(^{(1)}\), and not much more difficult in a computer simulation approach \(^{(2)}\). However, it is not so easy to achieve experimentally. In the case of the scattering of light from two-dimensional random surfaces, the in-plane, cross-polarized scattering of p-polarized light suppresses the single-scattering contribution to the mean intensity of the light scattered incoherently. In the case of the scattering of light incident normally on a weakly rough one-dimensional random metal surface, the use of a surface whose roughness is characterized by a power spectrum \(g(|k|)\) that vanishes identically for \(|k| < k_{\text{min}} \leq \omega/c\) eliminates the contribution from single-scattering processes to the mean intensity of the incoherent component of the scattered light for scattering angles smaller in magnitude than \(\sin^{-1}(ck_{\text{min}}/\omega)\). However, such surfaces are difficult to fabricate.

In this paper we explore a different approach to the design of random surfaces that suppress the single-scattering contribution to the incoherent component of the light scattered from them, that is not restricted to weakly rough surfaces, and that appear to be easier to fabricate than surfaces characterized by a West-O’Donnell power spectrum.

To motivate this approach, let us consider the scattering of an s-polarized plane wave of frequency \(\omega\) from a one-dimensional, perfectly conducting, random surface, when the plane of incidence is perpendicular to the generators of the surface. We recall that if in the inhomogeneous Fredholm equation for the normal derivative of the single nonzero component of the electric field in the vacuum, evaluated on the surface, is solved by iteration, the inhomogeneous term yields the Kirchhoff approximation to the mean scattered intensity, a single-scattering approximation, the first iterate yields the pure double-scattering contribution, and so on \(^{(2)}\). Consequently, if a surface can be designed with the property that the Kirchhoff approximation to the mean intensity of the light scattered from it vanishes for the scattering angle \(\theta_s\) in the interval \((-\theta_m, \theta_m)\), all the scattered intensity within this range of scattering angles will be due to multiple-scattering processes. Consequently, our aim is to design a one-dimensional, perfectly conducting random surface for which the Kirchhoff approximation to the mean differential reflection coefficient vanishes identically for \(\theta_s\) in the interval \((-\theta_m, \theta_m)\). The analysis required is simplified significantly by working in the geometrical optics limit of the Kirchhoff approximation. However, the results obtained still display the behavior sought.

Thus, we consider a one-dimensional, randomly rough, perfectly conducting surface defined by the equation \(x_3 = \zeta(x_1)\), that is illuminated by an s-polarized plane wave of frequency \(\omega\). The surface profile function \(\zeta(x_1)\) is written in the form \(^{(4)}\)

\[
\zeta(x_1) = \sum_{\ell=-\infty}^{\infty} c_\ell s(x_1 - \ell 2b), \tag{1}
\]

where the \(\{c_\ell\}\) are independent, positive, random deviates, \(b\) is a characteristic length, and the function \(\zeta(x_1)\) is defined by \(^{(4)}\)

\[
s(x_1) = \begin{cases} 
0, & x_1 < -(m+1)b, \\
-(m+1)bh - hx_1, & -(m+1)b < x_1 < -mb, \\
-b, & -mb < x_1 < mb, \\
-(m+1)bh + hx_1, & mb < x_1 < (m+1)b, \\
0, & (m+1)b < x_1, 
\end{cases} \tag{2}
\]

where \(m\) is a positive integer. Due to the positivity of the coefficient \(c_\ell\), its probability density function (pdf) \(f(\gamma) = \delta(\gamma - c_\ell)\) is nonzero only for \(\gamma > 0\).

It has been shown that for the random surfaces defined by Eqs. (1) and (2) the mean differential reflection coefficient in the geometrical optics limit of the Kirchhoff approximation is given by \(^{(4)}\)

\[
\left\langle \frac{\partial R}{\partial \theta_s} \right\rangle = \frac{1}{2h} \frac{[1 + \cos(\theta_0 + \theta_s)]^2}{\cos(\theta_0)\cos(\theta_0 + \cos \theta_s)} \left[ f \left( \frac{\sin \theta_0 - \sin \theta_s}{h(\sin \theta_0 + \cos \theta_s)} \right) + f \left( \frac{\sin \theta_s - \sin \theta_0}{h(\sin \theta_0 + \cos \theta_s)} \right) \right], \tag{3}
\]

where \(\theta_0\) and \(\theta_s\) are the angles of incidence and scattering, measured counterclockwise and clockwise from the normal to the mean scattering surface, respectively. This result shows that \(\langle \partial R/\partial \theta_s \rangle\) is given in terms of the pdf of the coefficient \(c_\ell\) and is independent of the wavelength of the incident light. It simplifies greatly in the case of normal incidence \((\theta_0 = 0^\circ)\),
\[
\langle \partial R / \partial \theta_s \rangle = \left( 1 + \tan^2 \frac{\theta_s}{2} \right) \frac{f(-\frac{1}{4} \tan \frac{\theta_s}{2}) + f(\frac{1}{4} \tan \frac{\theta_s}{2})}{4h}.
\]

and we will restrict ourselves to this case in what follows. From Eq. (4) we find that if we wish \(\langle \partial R / \partial \theta_s \rangle\) to have the form, say,

\[
\langle \partial R / \partial \theta_s \rangle = \begin{cases} 0, & 0 < |\theta_s| < \theta_m, \\ \frac{\cos \theta_s}{2(1-\sin \theta_m)}, & \theta_m < |\theta_s| < \pi/2, \\ \end{cases}
\]

we must choose for \(f(\gamma)\)

\[
f(\gamma) = \begin{cases} 0, & 0 < \gamma < \gamma_m, \\ \frac{1 + h^2 \gamma_m^2}{2h(1-h \gamma_m)^2}, & \gamma_m < \gamma < \frac{\pi}{4}, \\ \end{cases}
\]

where \(\gamma_m = [\tan(\theta_m/2)]/h\). From this form for \(f(\gamma)\) a long sequence of \(\{c_i\}\) can be generated, e.g. by the rejection method\(^{(5)}\), and the surface profile function generated by the use of Eqs. (1) and (2).

The surface profile functions \(\zeta(x_1)\) generated in this way are not zero-mean Gaussian random processes, and are not stationary. Indeed, the mean square height of the surface, \(\delta^2 = \langle \zeta^2(x_1) \rangle - \langle \zeta(x_1) \rangle^2\), is a periodic function of \(x_1\) with a period \(2b\) and for \(m = 1\) is given by \(\delta^2 = \langle c^2 \rangle - \langle \zeta(x_1) \rangle^2 \rangle h^2 b^2 / 3\) for \(b \leq h \leq b\). The average of this function over a period, \(\delta^2 = \langle \zeta^2(x_1) \rangle - \langle \zeta(x_1) \rangle^2 \rangle h^2 b^2 / 3\), can be used to estimate the rms height of the surface. Similarly, the mean square slope of the surface is given by \(s^2 = \langle \zeta(x_1)^2 \rangle - \langle \zeta(x_1) \rangle^2 \rangle h^2 b^2 / 3\), from which the rms slope can be determined. The averages \(\langle c \rangle\) and \(\langle c^2 \rangle\) appearing in these expressions, the first two moments of \(f(\gamma)\), are given by

\[
\langle c \rangle = \frac{1}{h} \frac{1 + h^2 \gamma_m^2}{(1-h \gamma_m)^2} \left\{ \cos \theta_m + 2 \ln \frac{\cos \frac{\pi}{4}}{\cos \frac{\theta_m}{2}} \right\}
\]

\[
\langle c^2 \rangle = \frac{2}{h^2} \frac{1 + h^2 \gamma_m^2}{(1-h \gamma_m)^2} \left\{ \frac{\pi}{2} - \theta_m + \tan \left( \frac{\theta_m}{2} \right) \right\}
\]

An example of a surface generated in this way is presented in Fig. 1. The pdf \(f(\gamma)\) used in its generation is the one defined by Eq. (6), with \(\theta_m = 40.1^\circ\). The parameters entering the definition of the function \(s(x_1)\) are \(b = 3\lambda\), \(m = 3\), and \(h = 0.2\). For these values of the parameters we find that \(\delta_{av} = 0.9\lambda\), and \(s = 0.57\), so that the surface is moderately rough. It was sampled at the points \(x_p = [(p+\frac{1}{2})b]/N\), where \(p = 0, \pm 1, \pm 2, \ldots\) and \(N = 100\), and is seen to consist of a succession of triangular peaks and valleys.

To show that this random surface suppresses single-scattering processes for \(|\theta_s| < 40.1^\circ\), we have plotted in Fig. 2 the contribution to the mean differential reflection coefficient from the incoherent component of the scattered light, \(\langle \partial R / \partial \theta_s \rangle_{incoh}\), for scattering from this surface, calculated by a computer simulation approach in the Kirchhoff approximation\(^{(2)}\), with and without invoking the geometrical optics limit of the latter. The results for a total of 2000 realizations of the surface were used in carrying out the ensemble average required for obtaining the mean differential reflection coefficient. It is seen that in the geometrical optics limit of the Kirchhoff approximation \(\langle \partial R / \partial \theta_s \rangle_{incoh}\) vanishes for \(\theta_s < 40.1^\circ\). In the Kirchhoff approximation \(\langle \partial R / \partial \theta_s \rangle_{incoh}\) is not identically zero in this region of scattering angles, but is quite small. The difference between these two results shows how well the geometrical optics limit of the Kirchhoff approximation reproduces the result of the Kirchhoff approximation itself. In this figure we have also plotted the total contribution to the mean differential reflection coefficient from the incoherent component of the scattered light, including all multiple-scattering contributions. This result for \(\langle \partial R / \partial \theta_s \rangle_{incoh}\) was calculated exactly by a computer simulation approach\(^{(2)}\), in which the results for 2000 realizations of the surface were averaged. We see from this figure that there is now a low background for \(\theta_s < 40.1^\circ\), due to multiple-scattering, on which is superimposed an enhanced backscattering peak in the retroreflection direction (\(\theta_s = 0^\circ\)), whose height is nearly twice that of the background at its position. The latter is the result expected when the contribution from single-scattering processes has been subtracted\(^{(10)}\).

Although the theory underlying the approach to generating random surfaces that suppress single scattering presented here was based on the assumption that the scattering surface is perfectly conducting, the resulting approach also works very well for finitely conducting surfaces. In Fig. 3 we have plotted a rigorous computer simulation result for \(\langle \partial R / \partial \theta_s \rangle_{incoh}\) in the case that s-polarized light of wavelength \(\lambda = 612.7\) nm is incident normally on a one dimensional random silver surface (\(\epsilon(\omega) = -17.2 + i0.498\)). The surface is characterized by the parameters \(b = 3\lambda\), \(m = 3\), \(h = 0.2\), and \(\theta_m = 40.1^\circ\). Results for 2000 realizations of the surface were averaged in obtaining this figure.
The strong suppression of $\langle \partial R/\partial \theta_s \rangle_{\text{incoh}}$ in the interval $|\theta_s| < 40.1^\circ$ is clearly seen, and an enhanced backscattering peak at $\theta_s = 0^\circ$ rises to about twice the height of the background at its position.

In this letter we have presented a method for generating numerically a one-dimensional random surface profile function $\zeta(x_1)$ that has the property that it suppresses the single scattering of s-polarized light from it, a property that in the case of a perfectly conducting surface is independent of the wavelength of the incident light. The extension of the present approach to the generation of one-dimensional random surfaces that suppress the single scattering of p-polarized light is straightforward. The method described is not restricted to the generation of weakly rough surfaces. Surfaces defined by Eqs. (1) and (2), with a different form of the pdf $f(\gamma)$, have been fabricated successfully in the laboratory(4), and their fabrication appears to be simpler than that of surfaces characterized by a West-O’Donnell(3) power spectrum. The approach to the design of two-dimensional random Dirichlet surfaces that act as band-limited uniform diffusers developed recently(7), can be used to design two-dimensional random Dirichlet surfaces that suppress single-scattering processes. These and other applications of the approach described here will be presented elsewhere.

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FIG. 1. A one-dimensional random surface profile function \( \zeta(x_1) \) obtained from Eq. (1) by the use of the pdf given by Eq. (6) together with a function \( s(x_1) \) defined by Eq. (2) with \( b = 3\lambda, m = 3, h = 0.2, \) and \( \theta_m = 40.1^\circ. \)

FIG. 2. \( \langle \partial R/\partial \theta_s \rangle_{\text{incoh}} \) calculated by a computer simulation approach for the random surface displayed in Fig. 1, when s-polarized light of wavelength \( \lambda \) is incident normally on it. (\cdots\cdots\cdots\cdots) The geometrical optics limit of the Kirchhoff approximation; (\_\_\_\_\_\_\_) the Kirchhoff approximation; (\_\_\_\_\_\_\_) the result with all multiple-scattering contributions included.

FIG. 3. \( \langle \partial R/\partial \theta_s \rangle_{\text{incoh}} \) calculated by a computer simulation approach for the case that s-polarized light of wavelength \( \lambda = 612.7 \) nm is incident normally on a one-dimensional random silver surface. The arrows indicate the positions of the angles \( \pm \theta_m. \)
Figure 1
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Figure 2
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Figure 3
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