Efficient Document Indexing Using Pivot Tree

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Abstract. We present a novel method for efficiently searching top-\(k\) neighbors for documents represented in high dimensional space of terms based on the cosine similarity. Mostly, documents are stored as bag-of-words tf-idf representation. One of the most used ways of computing similarity between a pair of documents is cosine similarity between the vector representations, but cosine similarity is not a metric distance measure as it doesn’t follow triangle inequality, therefore most metric searching methods can not be applied directly. We propose an efficient method for indexing documents using a pivot tree that leads to efficient retrieval. We also study the relation between precision and efficiency for the proposed method and compare it with a state of the art in the area of document searching based on inner product.

1 Introduction and Related Work

There are two main areas of research in information retrieval 1.) Search in metric spaces 2.) Search in non-metric spaces. A metric space basically refers to a similarity measure which follows all the metric properties like reflexivity, symmetry, non-negativity and triangle-inequality. All other properties can be achieved by trivial transformations, but triangle inequality is considered most important out of all others, since it is difficult to achieve it using trivial transformations and it can be effectively used in pruning elements. On the other hand, a non-metric space basically refers to a similarity measure that doesn’t follow triangle inequality. In such spaces, metric access methods using triangle inequality can not be applied directly. The retrieval approaches in non-metric spaces can be broadly categorized as \textit{embedding} and \textit{classification}.

A metric space is described by a similarity measure that follows the properties of reflexivity, nonnegativity, symmetry and most importantly triangle inequality. A number of methods have been developed in the past to search metric spaces\cite{4, 11, 14}, most metric spaces can be search efficiently using the triangle inequality. \cite{12, 8} discuss the use of range queries and kNN in metric space. A lot of research has been done in the field of information retrieval from nonmetric spaces, but a lot of similarity measures do not follow triangle inequality. In such non metric spaces, there are broadly two approaches followed, embedding and classification.

Embedding basically refers to the conversion of nonmetric space into metric spaces. There are certain embedding methods which perform exact conversion\cite{5}
to metric space whereas others do approximate conversion to metric space [2].
A number of approximate embedding methods have been developed such as [2]
that converts data objects into vector space. They introduce a query sensitive
distance function together with the embedding method, in order to give different
importance to different embedding dimensions for each query object. TriGen
is another method developed to convert nonmetric spaces into metric spaces
by using metric preserving and similarity invariant modifiers. But the author
himself acknowledges, not all nonmetric measures are suitable to be converted
by these methods. In the case of exact embedding methods, the most prominent
is LCE [5], which tried to divide objects into groups and then adds a small local
constant to all pairwise distances within a group to make them follow triangle
inequality. This method although exact, becomes completely unscalable for large
datasets, since it requires the computation of all possible triplets of objects
within a cluster, which can be a huge computational cost. A number of other
approximate embedding techniques like Fastmap [6], Metric Map [13], and Sparse
Map [7] exist, but the only exact method with no false dismissals are LCE and
CSE [10]

[3] presented a non-metric clustering method based on distances to the so-
called fiduciary templates (some selected random objects from the set). The
distances to these fiduciary templates form a vector, which is used to decide
in which cluster a new object belongs. [1] proposed a k-median clustering al-
gorithm for nonmetric functions (specifically, the Kullback-Leibler divergence)
that computes $a(1 + \epsilon) -$ approximation of the k-median problem.

Recently [9] published maximum inner product based approach for querying
documents. Their approach is based on creating tighter bounds as the query
object traverses down the tree because of reduced number of documents at each
new level and therefore a reduced radius. In the proposed method, we project
query object on a set of orthogonal pivots as we descend down the pivot tree. We
use the previous pivots to construct an orthogonal pivot to all other pivots in the
descend path of the query object. We avoid any euclidean addition/subtraction
operations that are expensive in high dimensional spaces. The proposed method
is based on maximizing the projection for group of documents on a set of
orthogonal projectors.

2 Proposed Method

We observe experimentally that the following relation holds for any given query
$q \in \mathbb{R}^v$, where $v$ is the vocabulary size, projector $S \in \mathbb{R}^{v \times v}$, document $d \in \mathbb{R}^v$
and orthogonal projector $S^\perp \in \mathbb{R}^{v \times v}$

$$q^T d \leq \|Sq\|\|Sd\| + \|S^\perp q\|\|S^\perp d\|$$
$$\leq 1 + 2\|Sq\|\|Sd\| - \|Sq\| - \|Sd\|$$

We can bound the distance between a given document $d$ and query $q$ using the
above inequality. We use the above inequality to bound $\|q^T d\|$ for all documents
contained in the subtree rooted at node $N_p$, we select a random pivot $p_{n+1}$ from all such documents.

### 2.1 Updating the Projector

We construct basis $B_n$ for the subspace spanned by the vectors (pivots) $p_1, ..., p_n$ in the descend path to the node $N_p$ from the root of the tree.

$$B_n = P_n A_n \text{ with } P_n = (p_1 \ldots p_n)$$

Let $p_{n+1}$ be the new vector to be added to the subspace then, we have the new basis $B_{n+1}$:

$$B_{n+1} = \left( B_n x \right)$$

such that:

$$x = \frac{y}{\|y\|} \text{ with } y = (\text{Id} - B_n B_n^\dag) p_{n+1}$$

We can get a projection vector ($y$) orthogonal to $B_n$ using the relation:

$$\|y\|^2 = \|p_{n+1}\|^2 - \|B_n B_n^\dag p_{n+1}\|^2 = \|p_{n+1}\|^2 - \|B_n^\dag p_{n+1}\|^2$$

Then, denoting $\alpha = \|y\|^{-1}$,

$$B_{n+1} = \left( P_n A_n \alpha \left( \text{Id} - B_n B_n^\dag \right) p_{n+1} \right)$$

$$= \left( P_n p_{n+1} \right) \begin{pmatrix} A_n - \alpha A_n A_n^\dag p_{n+1} \\ 0 \end{pmatrix}$$

### 2.2 Updating the Similarity

We compute the value of $\|B_{n+1}^\dag D\|$ from $\|B_n^\dag D\|$ for all the documents ($D$) contained in the subtree rooted at node $N_p$. Each node of pivot tree contains $\max(\|B_{n+1}^\dag D\|^2)$ and $\min(\|B_{n+1}^\dag D\|^2)$ $\forall D \in D_p$ where $D_p$ is the set of all documents contained in the subtree rooted at node $N_p$.

$$\|D^\dag B_{n+1}\|^2 = \left\| (D^\dag P_n D^\dag p_{n+1}) \begin{pmatrix} A_n - \alpha A_n A_n^\dag p_{n+1} \\ 0 \end{pmatrix} \right\|^2$$

$$= \left\| (D^\dag P_n \alpha D^\dag p_{n+1} - \alpha D^\dag P_n A_n A_n^\dag p_{n+1}) \right\|^2$$

$$= \|D^\dag B_n\|^2 + \|\alpha D^\dag p_{n+1} - \alpha D^\dag P_n A_n A_n^\dag p_{n+1}\|^2$$
2.3 Algorithm

In this section we describe the algorithm we use to construct the pivot tree. We then describe an algorithm to search the pivot tree using a given query.

1. Algorithm SelectPivot(Data S)

```
SelectPivot(Data S)
    Pick some random pivots P ∈ S
    Choose a random pivot p ∈ P s.t. \( \arg \max_p (\sum \|p^T pS_i\|^2) \) ∀\( S_i \in S \)
    return (p)
```

2. Algorithm MakeSplit(Data S, Pivot p)

```
MakeSplit(Data S, Pivot p)
    A ← \{ s ∈ S: \|D^T p_{n+1}\|^2 > c \}
    B ← S/A
    return (A,B)
```

3. Algorithm UpdateProjections(Data \( D_l \), Pivot p, A)

```
UpdateProjections((Data \( D_l \), Pivot p, A))
    \( D_l \).Projections ← update\( (D_l,p,A) \) ∀\( D_i \in D_l \); # Using eqn. 5
```

4. Algorithm BuildTree(Data S)

```
BuildTree(Data S)
    Input ← S
    Output ← Tree T
    T.S ← S
    T.min ← min(S.Projections)
    T.max ← max(S.Projections)
    if (|S| ≤ N_0)
        return T
    T.p ← SelectPivot(Data S)
    \( D_l,D_r \) ← MakeSplit(T.S, T.p)
    # Using eqn. 4
    T.A ← UpdateA(pivot p)
    # Using eqn. 4
    T.P ← UpdateP(pivot p)
    # Using eqn 7.
    \( D_l \).Projections ← UpdateProjections(Data \( D_l \), Pivot p, T.A)
    T.left ← BuildTree(Data \( D_l \))
    T.right ← BuildTree(Data \( D_r \))
    return T
```

5. Algorithm SearchTree(Query S, Tree T)
SearchTree(Query S, Tree T)
Input ← Query S, Tree T
Output ← Document Set D

\[ B_l ← \text{ComputeBound(Tree T.left, Query q)} \] # using eqn 2
\[ B_r ← \text{ComputeBound(Tree T.right, Query q)} \] # using eqn 2

#getLast: Returns the element with least similarity with query
if \( B_l \geq \text{getLast(queue)} \)
    searchL=True
if \( B_r \geq \text{getLast(queue)} \)
    searchR=True

if(searchL and searchR)
    if \( B_l > B_r \)
        queue ←SearchTree(Query S, Tree T.left)
    else
        queue ←SearchTree(Query S, Tree T.right)
else if (searchL and !searchR)
    queue ←SearchTree(Query S, Tree T.left)
else if (!searchL and searchR)
    queue ←SearchTree(Query S, Tree T.right)
else
    return (queue)

3 Experimentation and Results

We present in this section experimental results for the proposed method based on the MTA (Maximized Trace Approach) against state of the art method MIP (Maximum Inner Product) approach. The precision versus prunes is drawn for both approaches by reducing the bound artificially, reduction in bound leads to more prunes, but reduced precision. We can see in Figure 1 that MTA outperforms MIP\cite{9} in terms of both ranking (as measured by spearman distance) and precision for different values of prunes.

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Fig. 1: The figure in the left presents the prunes against precision. The figure on the right presents ranking performance of the two methods for different number of prunes.

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