The Effect of Form Factor Shape on the Nuclear Stopping Power of Calcium, Iron, Zirconium, and Lead Elements

Shafik S. Shafik¹,², Muna A. Saeed², and Rana M. Yas²
¹University of Alayen, Thi-Qar, Iraq
²Department of Physics, College of Science, University of Baghdad, Baghdad, Iraq
¹Corresponding author: shafeqsh76@gmail.com ; shafeq_sh@scbaghdad.edu.iq

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Abstract
This paper examined the effect of the form factor of the nucleus and its type on the nuclear stopping power of Ca – 40, Fe – 56, Zr – 90, and Pb – 206 isotopes, respectively. The collision, radiative, and total electronic stopping power of these elements was also calculated. The results of the electronic stopping power showed that the collision part exceeds the radiative up to a specific energy that depends on the element's atomic number, after which the electronic stopping power will prevail. As for the nuclear stopping power of electrons, the results indicated its great dependence on the form factor of the nucleus, but this dependence begins at a specific energy for each studied isotope and this energy changes with the isotope change. Also, the results did not show distinct significant differences between the different types of form factors, which are exponential, geometric, and uniform. On the whole, the results of the nuclear stopping power containing the exponential form factor are decreased by increasing the mass number of the isotopes. Finally, the behavior of the electronic and nuclear stopping powers as a function of the incident electron energy of the studied elements and isotopes behaved like what is expected and known.

1. Introduction
The stopping power is one of the most important parameters in the field of radiation physics. It is involved in many applications, including medical, industrial, and even nuclear reactors [1-3]. It is usual when calculating the stopping power to neglect the contribution of the atomic nucleus and relying only on collision between the incident particle and the atomic levels [4, 5]. This reaction can be divided into two important classes if the projectile particle is the electron: the first is the collision reaction with the target atom's electrons, and the second is the radiative that the projectile loses its energy when it passes through a charged medium (the bremsstrahlung radiation). But
as it is known, the nucleus of an atom possesses the positively charged due to protons, which can act as a Coulomb barrier, which contributes to increasing the stopping power [4]. On the other hand, the shape of the target nucleus changes with the change in the number of nucleons within it, as well as the effects of its shell (if the nucleus is magic, then the general shape has a spherical shape) [6]. The form factor (FF) of the nucleus plays the main role in determining the shape of the nucleus. There are several mathematical formulae for FF, including Exponential, Gaussian, and Uniform, whose mathematical details will be shown in the next section. In this paper, the effect of FF shape (Exponential, Gaussian, and Uniform) on the nuclear stopping power of electrons incident on Ca – 40, Fe – 56, Zr – 90, and Pb – 206 isotopes will be studied. Furthermore, the electronic stopping power both collision and radiative will be calculated for these isotopes. To achieve this goal, the online codes of National Institute of Standards and Technology (NIST) [7] and Screened Relativistic (SR) Nuclear Stopping Power Calculator (version 6.9.0) [8] have been used, respectively.

2. The Electronic Stopping Power

The electronic stopping power can be defined as the process of losing energy to the particles and ions charged during the path unit and that the process of calculating it relied on the basic factors of the incident particle and the target material through speed, charge and mass in relation to the incident particle as well as on the characteristic of that target (i.e. the target material) so therefore, the process differs energy loss through the nature and type of incident particle. However, using the first Born approximation, Bethe (in 1930) depends on quantum mechanics approach [9] to calculate the electronic stopping power (the electronic energy loss). Though, the cross section of stooping in c.g.s. units, is given by [10]:

\[
S_e = \frac{4\pi Z_1^2 e^4}{mv^2} Z_2 \ln \left( \frac{2mv^2}{I} \right),
\]

where \( v \) is the projectile’s speed, \( Z_{1e} \) is the nuclear charge of the projectile, \( Z_{2e} \) is the nuclear charge of the target, \( m \) and \( e \) are mass and charge of the electron, and \( I \) is the mean excitation energy of the target.

By adopting the relativistic effects, equation (1) becomes:

\[
S_e = \frac{4\pi Z_1^2 e^4}{mv^2} Z_2 \left[ \ln \left( \frac{2mv^2}{I} \right) - \ln(1 - \beta) - \frac{\beta^2}{1} \right],
\]

\[
\beta = \frac{v}{c},
\]

where \( c \) is the speed of light.

The linear electronic stopping power is related to the electronic stopping cross section as follows:

\[
S_{lin} = -\frac{dE}{dx} = nS_e
\]

where \( n \) is the no. of target atoms. The mass stopping power can be given by:

\[
S_{lin/\rho} = \frac{4\pi Z_1^2 mc^2}{\beta^2} \frac{1}{A_2} Z_2 \left[ \ln \left( \frac{2mv^2}{I} \right) - \ln(1 - \beta) - \frac{\beta^2}{1} \right]
\]
\[ S_{\text{lin}/p} = (0.307075 \text{ MeV} \text{cm}^2/\text{g}) \frac{Z_2^2}{A_2} Z_1^2 \left[ \ln \left( \frac{2mu^2}{I} \right) - \ln(1 - \beta) - \beta^2 \right] \ldots \quad (6) \]

where \( r_e^2 = \frac{e^2}{m_c^2} \) is the classical electron radius, \( u \) is the atomic mass unit, \( A_2 \) is the relative isotopic mass of the target atom, and standard values of the various atomic constants have been used.

3. The Nuclear Stopping Power

Whereas, the nuclear stopping power can be defined as the average rate of energy loss per unit path length due to the transfer of energy to recoiling atoms in elastic collisions, which is contribute in the total stopping power for charged particles (especially heavy projectile). When the projectile energy becomes high, nuclear stopping is not important, and can be neglected in the calculations. For practical calculations, nuclear stopping is given by [11]

\[ S_n(E) = \frac{8.46210^{-15} Z_1 Z_2 M_1 S_n(\varepsilon)}{(M_1 + M_2)(Z_1^{0.23} + Z_2^{0.23})} \text{eV/atom/cm}^2 \ldots \quad (7) \]

where \( \varepsilon \) is the reduced energy and calculated from

\[ \varepsilon = \frac{32.53 M_2 E}{Z_1 Z_2 (M_1 + M_2)(Z_1^{0.23} + Z_2^{0.23})} \ldots \quad (8) \]

where \( E \) is the energy in keV, and \( M_1, M_2 \) are the masses of projectile and target atom, respectively. The reduced nuclear stopping is then given by

\[ S_n(\varepsilon) = \frac{\ln(1+1.1383 \varepsilon)}{2 \varepsilon} \quad \text{for} \quad \varepsilon \leq 30 \ldots \quad (9) \]

\[ S_n(\varepsilon) = \frac{\ln(\varepsilon)}{2 \varepsilon} \quad \text{for} \quad \varepsilon > 30 \ldots \quad (10) \]

To this extent, the mathematical treatment of the nuclear stopping power is constructed with the assumption that the atomic nucleus can be regarded as point-like shape [12]. But in fact, the nucleus has its own structure and shape that depends on several parameters. The mathematical treatment of this situation has been done depending on the work of M.J. Boschini and et.al. in their wonderful paper [4] and on the references therein. As example for this treatment, they used the following formula for exponential charge distribution \( FF \) (\( FF_{\text{exp}} \)) [13]:

\[ FF_{\text{exp}} = \left[ 1 + \frac{1}{12} \left( \frac{q r_n}{h} \right)^2 \right]^{-2} \ldots \quad (11) \]

where \( q \) is the momentum transfer, \( r_n \) is the nuclear radius, and \( h \) is the reduced Plank constant. The nuclear radius \( r_n \) can be given by [13]:

\[ r_n = 1.27 A^{0.27} \text{fm} \ldots \quad (12) \]

where \( A \) is the atomic mass number.
For Gaussian charge distribution $FF (F_{gau})$ and uniform–uniform folded charge distribution $FF (F_{u})$, the formulae suggested by reference [13] and references therein and by references [14] and [15] and references therein were depended in this work.

4. Results and Discussions

Before we present the results of the nuclear stopping power and the effect of changing the shape of the nucleus by studying the form factor, it is appropriate and necessary to display the electronic stopping power of the elements that were studied, namely calcium, iron, zirconium and lead. Figure (1) shows the results of the stopping power of the studied elements. For each element, the collision stopping power, radiative stopping power and the total stopping power are drawn. In general, the radioactive stopping power contributes effectively to high energies that exceed 10 MeV, whereas the collision stopping power is predominant from the start of the reaction to the energy of 10 MeV and then its effect is weakened. As for the total electronic stopping power, its behavior is almost constant for all elements, regardless of the values, which increase with increasing the atomic number of the element, as is evident from Figure (2).

Figure (3) shows the nuclear stopping power in the absence and presence of FF with its three variations that have been adopted in this paper, which are: exponential, geometric, and uniform for all the studied isotopes; Ca – 40, Fe – 56, Zr – 90, and Pb – 206. There are three important observations that can be inferred from this figure. The first one is that the nuclear stopping power is not dependent on FF of the nucleus till to a specific energy for each nucleus at which the apparent influence of FF on the total value of the nuclear stopping begins. The values of this energy are approximately 40, 30, 20 and 15 MeV for the isotopes of Ca – 40, Fe – 56, Zr – 90, and Pb – 206, respectively. The difference in this energy is due to important things related to the quantum state of the nucleus, such as its total angular momentum and other parameters, which could be future research projects. The second important observation is that the nuclear stopping power is not affected by the type of FF used to some extent, and this gives freedom to use any FF, especially if the goal is to calculate the total electron stopping power. The third observation is that the nuclear stopping power dependent on the exponential modulus coefficient is more valuable than others. On this basis, figure (4) shows that the nuclear stopping power dependent on the exponential FF decreases with increasing mass number and this is a logical result due to the effects of the electronic increase in these isotopes.
Figure (1): The collision, radiative, and total stopping powers of electrons incident on Calcium, Iron, Zirconium, and Lead elements, respectively, as a function of energy.

Figure (2): The total stopping powers of electrons incident Calcium, Iron, Zirconium, and Lead elements, respectively as a function of energy.
Figure (3); The nuclear stopping powers of electrons incident Ca – 40, Fe – 56, Zr – 90, and Pb – 206 isotopes, respectively, as a function of energy. The effect of the shape of FF is shown for these isotopes.

Figure (4); The exponential FF nuclear stopping powers of electrons incident Ca – 40, Fe – 56, Zr – 90, and Pb – 206 isotopes, respectively, as a function of energy. The effect of mass number is shown for these isotopes.
5. Conclusions

The results of this study showed important conclusions that can be summarized as follows:

i. The electronic stopping power appeared as expected in terms of values and general behavior, and that the radiative stopping power starts its effect at a threshold energy that changes with the change of the atomic number of the element.

ii. The electronic stopping power increases with increasing atomic number, as is well known.

iii. The nuclear stopping power is largely dependent on the nucleus form factor and this dependence begins at threshold energy that change with the change of the studied isotope, and this threshold energy can depend on other parameters that can be studied in the future.

iv. In general, the shape of the nucleus form factor does not significantly affect the results.

v. The nuclear stopping power is reduced by increasing the mass number of the isotopes.

vi. The behavior of the electronic and nuclear stopping powers as a function of the incident electron energy of the studied elements and isotopes behaved like what is expected and known.

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