Primordial statistical anisotropies: the effective field theory approach

Ali Akbar Abolhasani, Mohammad Akhshik, Razieh Emami and Hassan Firouzjahi

Department of Physics, Sharif University of Technology, P.O. Box 11155-9161, Tehran, Iran
School of Astronomy, Institute for Research in Fundamental Sciences (IPM), P.O. Box 19395-5531, Tehran, Iran
Institute for Advanced Study, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong

E-mail: abolhasani@ipm.ir, m.akhshik@ipm.ir, emami@ipm.ir, firouz@ipm.ir

Received November 15, 2015
Revised February 4, 2016
Accepted February 23, 2016
Published March 8, 2016

Abstract. In this work we present the effective field theory of primordial statistical anisotropies generated during anisotropic inflation involving a background U(1) gauge field. Besides the usual Goldstone boson associated with the breaking of time diffeomorphism we have two additional Goldstone bosons associated with the breaking of spatial diffeomorphisms. We further identify these two new Goldstone bosons with the expected two transverse degrees of the U(1) gauge field fluctuations. Upon defining the appropriate unitary gauge, we present the most general quadratic action which respects the remnant symmetry in the unitary gauge. The interactions between various Goldstone bosons leads to statistical anisotropy in curvature perturbation power spectrum. Calculating the general results for power spectrum anisotropy, we recover the previously known results in specific models of anisotropic inflation. In addition, we present novel results for statistical anisotropy in models with non-trivial sound speed for inflaton fluctuations. Also we identify the interaction which leads to birefringence-like effects in anisotropic power spectrum in which the speed of gauge field fluctuations depends on the direction of the mode propagation and the two polarization of gauge field fluctuations contribute differently in statistical anisotropy. As another interesting application, our EFT approach naturally captures interactions generating parity violating statistical anisotropies.

Keywords: cosmological perturbation theory, inflation

ArXiv ePrint: 1511.03218
1 Introduction

Inflation is widely accepted as the leading paradigm for early universe with its basics predictions being well consistent with cosmological observations [1, 2]. During inflation, the quantum fluctuations of inflaton field(s) and the metric are amplified to cosmological scales which induce nearly scale-invariant, nearly adiabatic and nearly Gaussian perturbations on cosmic microwave background (CMB) maps and large scale structures which are in good agreements with data. The simplest models of inflation are based on a scalar field which is minimally coupled to gravity and rolls slowly over a near flat potential.

Despite all the observational successes of inflation, there is no fundamental understanding of mechanism of inflation. For example, the fundamental questions such as what was the stage of universe prior to inflation or what is the nature of inflaton field are left unanswered within the current working paradigm of inflation. Lacking a fundamental understanding of the mechanism of inflation, there are many phenomenological models of inflation which are consistent with data. Naturally one is lead to ask how far one can capture the most robust predictions of models of inflation without relying on particular realization of inflation model building. Effective Field Theory (EFT) of inflation [3] has been a successful program to answer this question, for a review of general EFT methods see [4, 5]. In the logic of EFT all interactions which are compatible with the underlying symmetries should be considered. Then depending on how one turn on particular interactions governing the dynamics of the light field, different inflationary models are realized. EFT approach was particularly successful in models of single field inflation in classifying their predictions for power spectrum and bispectrum. Similarly, one can extend the method of EFT of inflation to models of multiple fields inflation [6].
Most of models of inflation are based on scalar field dynamics. This is mainly motivated from the fact that the scalar fields are by construction spin-zero fields, naturally apt to generate isotropic cosmological backgrounds, a fundamental requirement of cosmological principle. Having this said, it is natural to examine the role of other type of fields during inflation. In particular, vector fields and gauge fields are ubiquitous in Standard Model of particle physics and in quantum field theory. Therefore, one expects that models of inflation with vector fields can have interesting theoretical motivations which also can be directly confronted with the data. Anisotropic inflation is such a realization based on gauge field dynamics which have captured significant interests in the literature. In most attractive realization of anisotropic inflation, a U(1) gauge field is turned on at the background level with a non-zero electric field energy density. In order to sustain the background electric field energy density and to endow a scale-invariant spectrum for the gauge field perturbations, the gauge field is coupled to the inflaton field. Observationally models of anisotropic inflation predict statistical anisotropy in CMB map which can be tested observationally.

Here our goal is to extend the logic of EFT to the setup of anisotropic inflation which generate statistical anisotropies. We assume the matter sector contains a scalar field $\phi$, playing the role of inflaton, and a U(1) gauge field within the Einstein gravity. With these minimal assumptions, we look for all possible interaction allowed by the underlying symmetries. This generality allows us to go beyond the model-dependent picture of anisotropic inflation and to look for new types of interactions between the inflaton field and the gauge field perturbations. Consequently, we re-derive the previously known results of the power spectrum statistical anisotropies. In addition, we obtain new results for power spectrum statistical anisotropies beyond the known results.

The important starting point to construct the EFT of inflation is to identify the symmetries of the problem at hand. In models of single field inflation, this task is well-understood. To start one chooses a time foliation, known as the unitary gauge, such that the scalar field remains homogeneous. Consequently, all perturbations are transferred into metric sector. In this view, the symmetry of the system contains all coordinate transformation which leaves the time foliation intact. In other words, the general four-dimensional diffeomorphism invariance $x^\mu \rightarrow x^\mu + \xi^\mu$ is reduced to the three-dimensional transformation $x^i \rightarrow x^i + \xi^i (x^\nu)$. The building blocks of this remnant symmetry transformation in unitary gauge are $g^{00}, K_{ij}$ etc in which the latter is the extrinsic curvature of the constant time hypersurface. Equipped with these building blocks one writes down all the possible interactions consistent with the remnant symmetry. Equivalently, one can look at the same problem in an arbitrary coordinate system in which the time coordinate is not fixed. Physically, this corresponds to restoring a scalar field degree of freedom, the so-called Goldstone boson $\pi$, which captures the fluctuations of scalar field perturbations. The advantage of the EFT is when one goes to the decoupling limit in which one can neglect the gravitational back-reaction of $\pi$ with the metric perturbations, corresponding to $M_P \rightarrow \infty$, in which $\pi$ captures the main results of the power spectrum and the bispectrum to leading order in terms of the slow roll parameters.

Now in our setup of anisotropic inflation with an additional gauge field, the role of remnant symmetry and the choice of unitary gauge is somewhat obscure. As in conventional case, we still choose the time foliation such that to keep the scalar field homogeneous, $\delta \phi = 0$. For the gauge field excitations, we can define the unitary gauge to be the gauge in which $\delta A_\mu = 0$. However, this requirement is ambiguous as one has the U(1) gauge symmetry $\delta A_\mu \rightarrow \delta A_\mu + \partial_\mu F$ with $F$ an arbitrary scalar in which the gauge field fluctuations can be turned on again. Therefore, an important task in defining our unitary gauge is to properly
take into account the role of U(1) gauge transformation along with space-time coordinate transformations to correctly identify the remnant symmetry of the setup.

The paper is organized as follows. In section 2 we identify the symmetries and the degrees of freedom and present the invariant action in unitary gauge. In section 3 we restore the Goldstone bosons and re-write the action, including the action of the free fields and the interactions, in terms of the Goldstone bosons. In section 4 we calculate the power spectrum anisotropy generated from various interactions followed by summary and discussions in section 5. We comment that this work is exclusively devoted to power spectrum analysis.

2 Symmetries and degrees of freedom

In this section we briefly review the setup of anisotropic inflation and then identify the physical degrees of freedom and the symmetries of the system to properly identify the starting unitary gauge.

2.1 Anisotropic inflation

As discussed before, our setup contains a scalar field \( \phi \) playing the role of the inflaton field and a U(1) gauge field. The gauge field has a background value which without loss of generality can be taken to be along the x-direction so the gauge field has the form \( \overline{A}^\mu = (0, \overline{A}^1(t), 0, 0) \) in which an overline indicates the background quantity. This also induces a background electric field energy density, breaking the isotropy so the background geometry is in the form of Bianchi type I universe. With this choice of the background gauge field, we still have the rotational symmetry in two-dimensional \( yz \) plane. As mentioned before, in usual models of anisotropic inflation in order for the background electric field to survive the dilution from the exponential expansion, the gauge field is coupled to the inflaton field in the form \( f(\phi)^2 F_{\mu\nu} F^{\mu\nu} \).

The functional form of \( f(\phi) \) is determined by the potential \( V(\phi) \) but in terms of scale factor \( a(t) \), one needs to choose \( f(\phi) \propto a(t)^{-2} \) in order for the background electric field energy density to furnish a nearly constant and sub-leading portion of the total energy density. At the level of perturbations, this specific form of \( f(\phi) \) helps to maintain a scale invariant power spectrum for the gauge field fluctuations. For a review on anisotropic inflation see [7] and for various works related to anisotropic power spectrum and bispectrum see [8–33, 50, 51]. Also see [52–59] for different realizations of statistical anisotropies.

The contribution of the gauge fields into curvature perturbation power spectrum \( P_R \) is in the form of quadrupole anisotropy parametrized as follows [60, 61]

\[
P_R(k) = P_R^{(0)} \left( 1 + g_\ast (\hat{n} \cdot \hat{k})^2 \right),
\]

in which \( P_R^{(0)} \) is the leading isotropic power spectrum, \( k \) is the mode of interest in Fourier space and \( \hat{n} \) represents the direction of anisotropy. The parameter \( g_\ast \) measures the strength of statistical anisotropies. Observational constraints from Planck data require [2, 51] \( |g_\ast| \lesssim 10^{-2} \).

For broad class of potentials, it is shown in [8] that with the appropriate form of the coupling \( f(\phi) \), the system reaches the attractor regime in which the contribution from the electric field energy density reaches a constant and subdominant portion of the total energy density. Denoting the fraction of the electric energy density to total energy density by the parameter \( R_e \), the correction in curvature perturbation power spectrum anisotropy in models with simple chaotic potential is obtained to be

\[
g_\ast = \left( \frac{48 R_e}{e} \right) N^2
\]
in which \( \epsilon \) is the usual slow-roll parameter and \( N \) represents the number of e-folds when the mode of interest \( k \) leaves the horizon till the end of inflation. We mention that the \( N^2 \)-dependence of power anisotropy is a generic feature expected from the accumulative IR effects of the scale-invariant gauge field fluctuations \cite{27}. Imposing the observational constraints on \( g_* \) implies that \( B \epsilon \lesssim 10^{-5} \). In addition, the bispectrum and the trispectrum analysis of the model were performed in \cite{24,28,29} in which the amplitude of local-type non-Gaussianity is obtained to be \( f_{NL} \sim g_* f_{NL} \) with non-trivial anisotropic shape of local-type non-Gaussianity.

We mention that it is shown in \cite{50} that for reasonable values of model parameters consistent with observations it is hard to reach the attractor regime “during inflation”. Nevertheless, one can assume that the duration of inflation somewhat exceeds the minimal 60 e-folds so the gauge field settles down to its attractor solution well before the observable modes exit the horizon.

2.2 Unitary gauge and the general action

After briefly reviewing the models of anisotropic inflation, here we start our study of EFT for these setups.

As in \cite{3} our starting job is to identify the physical degrees of freedom and to determine the proper unitary gauge. Following the logic of EFT of single field inflation \cite{3}, in our setup one can define the unitary gauge as the gauge in which all matter perturbations \( \delta \phi \) and \( \delta A^{\mu} \) are turned off so

\[
A^{\mu} = (0, \overline{A}^1(t), 0, 0), \quad \phi = \overline{\phi}(t) \quad \text{(unitary gauge)}.
\]

Consequently, all perturbations are carried by the metric sector.

The condition \( \delta \phi = 0 \) can be satisfied easily as in \cite{3} by appropriate foliation of spacetime in which the surfaces of constant time coincide with uniform \( \phi \) surfaces. This is motivated from the fact that the inflaton field \( \phi(t) \) can be used as the physical clock.

As for the gauge field the situation is more non-trivial. First note that we work with the contravariant components \( \delta A^{\mu} \) instead of the more natural covariant vector \( \delta A_\mu \). The reason is that in fixing unitary gauge we have to choose our coordinate system such that all perturbations of the field vanish and all degrees of freedom appear in metric. As one may easily check, all covariant components of \( \delta A_\mu \) are transformed by \( \xi^1 \) and therefore the condition \( \delta A_\mu = 0 \) does not involve \( \xi^\mu \) with \( \mu \neq 1 \). However, as we shall see momentarily, \( \delta A^{\mu} \) transformation is controlled by all components of \( \xi^\mu \) and consequently we may easily achieve \( \delta A^{\mu} = 0 \) with the aid of a combination of coordinate transformation and U(1) gauge symmetry.

Second and more importantly, the condition \( \delta A^{\mu} = 0 \) should be taken with care. It is true that part of spatial diffs can be fixed as the condition to put gauge field on its background value. However, things become non-trivial if one keeps in mind that the system should also be invariant under the U(1) gauge transformation

\[
A^{\mu} \rightarrow A^{\mu} + \nabla^{\mu} F = A^{\mu} + g^{\mu\nu} \partial_\nu F
\]

in which \( F(x^\nu) \) is a scalar. Therefore, even if we start with the unitary gauge \( \delta A^{\mu} = 0 \), then the gauge field excitations can be restored by the U(1) transformation eq. (2.4). As a result, in order to read off the correct physical degrees of freedom of the gauge field one has to look into the transformation of the gauge field perturbations both under the U(1) gauge transformation (2.4) and also under the general coordinate transformation

\[
x^\mu \rightarrow x'^\mu = x^\mu + \xi^\mu(x^\nu).
\]
Combining the transformations (2.4) and (2.5) the gauge field perturbations transform effectively as
\[ \delta A^\mu \rightarrow \delta A^\mu + \tilde{A}^1 \partial_1 \xi^\mu + g^{\mu \alpha} \partial_\alpha \mathcal{F}. \tag{2.6} \]
As usual, it is more convenient to decompose the four vector \( \xi^\mu \) into the transverse and the longitudinal parts, \( \xi^\mu_T \) and \( \xi^\mu_L \) respectively as follows
\[ \xi^\mu = \nabla^\mu \xi_L + \xi^\mu_T = g^{\mu \alpha} \partial_\alpha \xi_L + \xi^\mu_T, \tag{2.7} \]
subject to \( \nabla_\mu \xi^\mu_T = 0 \).

Plugging these decompositions in transformation (2.6) yields
\[ \delta A^\mu \rightarrow \delta A^\mu + \tilde{A}^1 \partial_1 \xi^\mu_T + g^{\mu \alpha} \partial_\alpha \mathcal{F} + \tilde{A}^1 \partial_1 (g^{\mu \alpha} \partial_\alpha \xi_L), \]
\[ = \delta A^\mu + \tilde{A}^1 \partial_1 \xi^\mu_T + \tilde{A}^1 (\partial_1 g^{\mu \alpha}) \partial_\alpha \xi_L - \tilde{A}^1 g^{\mu \alpha} \partial_1 \xi_L + g^{\mu \alpha} \partial_\alpha \left( \tilde{A}^1 \partial_1 \xi_L + \mathcal{F} \right). \tag{2.8} \]
The above transformation encodes both the U(1) transformation (2.4) and the coordinate transformation (2.5). Now we are able to see how the unitary gauge defined in eq. (2.3) is feasible. First, we note that by choosing \( \mathcal{F} = -\tilde{A}^1 \partial_1 \xi_L \) we can always cancel the last term above. In other words, with the aid of U(1) symmetry we can partially cancel the variation in \( \delta A^\mu \) which is caused by \( \xi_L \) coordinate transformation. Also, we have to remember that in unitary gauge, we already fixed \( \xi^0 \) to put inflaton on its background value. Now the unitary gauge defined in eq. (2.3), with all matter perturbations turned off, are subject to remnant symmetry \( x^\mu \rightarrow x^\mu + \xi^\mu \) in which
\[ \xi^\mu_L = \xi^\mu_L(t, y, z), \quad (\partial_1 g^{\mu \alpha}) \partial_\alpha \xi_L = -\frac{\tilde{A}^1}{A} g^{\mu \alpha} \partial_1 \xi_L, \quad \text{(remnant symmetry)} \tag{2.9} \]
excluding \( \xi^0 \) component. It is curious that \( \xi^\mu_L \) is independent of the \( x \) coordinate. Note that by remnant symmetry we mean that every term in the EFT action should be invariant under the above symmetries in unitary gauge. The above remnant symmetry in our unitary gauge should be compared with the remnant symmetry in isotropic model containing only a single scalar field \( \phi \) in which \( x^i \rightarrow x^i + \xi(t, x, y, z) \).

Having obtained the remnant symmetry of our system, our next job is to construct all scalars which are invariant under these remnant symmetries. It is important to note that we have fixed the unitary gauge such that there is no matter field perturbations and all perturbations are encoded in metric sector \( \delta g_{\alpha \beta} \). Consequently, all terms in the action in the unitary gauge are constructed from the metric perturbations and their derivatives.

As in the setup of EFT of \( \phi \) involving a single scalar field \( \phi \), \( \delta g^{00} \) is a scalar so we keep \( \delta g^{00} \) as one of our main building block to write down the action in unitary gauge. As for other building blocks constructed from metric sector, we note that neither \( \delta g^{11} \) nor any of other metric component transform as scalars under the remnant symmetry eq. (2.9) so we should look for more non-trivial combinations. However, we see that our symmetry conditions in eq. (2.9) involve \( \partial_1 \) and \( \partial_\alpha \). In particular, we note that \( \partial_1 \xi^\mu_T = 0 \). This suggests that if we works with the metric perturbations with the lower indices, \( \delta g_{\alpha \beta} \), we encounter the objects \( \partial_\alpha \xi_L \) and \( \partial_1 \xi_T \) which help to construct the desired scalars (or tensor). Since \( \partial_1 \xi_T = 0 \), it seems that \( g_{\alpha \beta} \) may be a useful quantity to start with. However, under the general coordinate transformation eq. (2.5) we obtain
\[ g_{\alpha \beta} \rightarrow \Lambda^\alpha_\mu \left( g_{\alpha \beta} + g_{\beta \gamma} \partial_1 g^{\gamma \lambda} \partial_\lambda \xi_L \right) \]

\[ \text{JCAP03(2016)020} \]
\[ L_a' = g_{a1} + \partial_1 \partial_a \xi_L + \frac{\hat{A}^1}{\hat{A}} \delta^0_0 \partial_i \xi_L, \]  

(2.10)

in which we have defined \( \Lambda_a'^\alpha = \frac{\partial \alpha'}{\partial \alpha} \). We note that the presence of last two terms involving \( \xi_L \) tells us that \( g_{1\alpha} \) is not a four-vector with respect to the free index \( \alpha \). Similarly, \( g_{11} \) is not invariant under the remnant symmetry eq. (2.9), so unlike \( g^{00} \), \( g_{11} \) can not be used as a starting building block as expected from the above discussions. This indicates that we have to use a more nontrivial combination of \( g_{1\alpha} \) and its derivatives to construct the proper scalar, four-vector or four-tensor.

Now looking at the derivative of \( g_{a1} \) we obtain

\[ \partial_\beta g_{a1} \to \Lambda_\beta^\gamma \Lambda_a'^\alpha \left[ \partial_\gamma g_{a1} + g_{1\alpha} \xi_{a'\beta'} + \partial_1 \partial_\alpha \partial_\gamma \xi_L + \partial_\gamma \left( \frac{\hat{A}^1}{\hat{A}} \partial_1 \xi_L \right) \delta^0_\alpha \right]. \]  

(2.11)

As before, the presence of \( \xi_L \) and the second term in the bracket, prevent \( \partial_\beta g_{a1} \) to be a four-tensor. However, we note that upon anti-symmetrization with respect to \( \alpha \) and \( \beta \) the second and the third terms in the bracket above cancel and we obtain

\[ \partial_\beta g_{a1} - \partial_\alpha g_{1\beta} \to \Lambda_\beta^\gamma \Lambda_a'^\alpha \left[ \partial_\gamma g_{a1} - \partial_\alpha g_{1\beta} + \partial_\gamma \left( \frac{\hat{A}^1}{\hat{A}} \partial_1 \xi_L \right) \delta^0_\alpha - \partial_\alpha \left( \frac{\hat{A}^1}{\hat{A}} \partial_1 \xi_L \right) \delta^0_\beta \right]. \]  

(2.12)

Only if we can get rid of the term containing \( \xi_L \) above, we can obtain a four-tensor. For this purpose, consider the transformation of the following combination

\[ g_{1\beta} \delta^0_\alpha - g_{a1} \delta^0_\beta \to \Lambda_\beta^\gamma \Lambda_a'^\alpha \left( g_{1\gamma} \delta^0_\alpha - g_{a1} \delta^0_\beta + \partial_1 \partial_\gamma \xi_L \delta^0_\alpha - \partial_1 \partial_\alpha \delta^0_\beta \right). \]  

(2.13)

Combining the above equation with eq. (2.12) we are able to cancel out the undesired term in eq. (2.12) containing \( \xi_L \) with the right combination of the above term. Hence if we define the quantity \( G_{\alpha\beta} \) via

\[ G_{\alpha\beta} = \partial_\alpha g_{1\beta} - \partial_\beta g_{a1} + \frac{\hat{A}^1}{\hat{A}} \left( \delta^0_\alpha g_{1\beta} - \delta^0_\beta g_{a1} \right), \]  

(2.14)

then it not only respects the remnant symmetry (2.9) but also is a four- tensor under the general coordinate transformation in the sense that

\[ G_{\alpha\beta} = \Lambda_\alpha'^\beta \Lambda_\beta'^\alpha G_{\alpha'\beta'}. \]  

(2.15)

Consequently, we can construct proper scalars with the contractions of \( G_{\alpha\beta} \). This is as far as we can go with the metric perturbations and their derivatives.

In addition, the anti-symmetric tensor \( \epsilon^{\alpha\beta\mu\nu} \) can be used to construct the dual of \( G_{\alpha\beta} \) defined via

\[ \tilde{G}^{\mu\nu} \equiv \epsilon^{\alpha\beta\mu\nu} G_{\alpha\beta} \]  

(2.16)

which is a four-tensor too.

In conclusion, our building blocks to construct the action in matter sector in unitary gauge are \( g^{00}, G_{\mu\nu}, \tilde{G}^{\mu\nu} \). The other building blocks like the extrinsic curvature \( K_{ij} \) are geometric in nature and do not come from the matter sector. Since we work in decoupling limit in which the higher derivative terms are neglected, we do not consider the contribution of
geometric building blocks like $K_{ij}$ or their mixings with $g^{(0)}_{\mu\nu}, \tilde{G}^{\mu\nu}$. Below we justify the validity of the decoupling assumption.

The most general action for the matter sector perturbations, up to quadratic order in perturbations, in unitary gauge are

$$S = \int d^4x \sqrt{-g} \left[ \Lambda + \alpha_0 g^{00} + \frac{\alpha_0}{4} (\delta g^{00})^2 - \frac{1}{4} M_1 \delta \left( G^{\alpha\beta} G_{\alpha\beta} \right) - \frac{1}{4} M_2 \delta \left( G^{\alpha\beta} \tilde{G}_{\alpha\beta} \right) \right. $$

$$- \frac{1}{4} M_3 \delta \left( \tilde{G}^{\alpha\beta} \tilde{G}_{\alpha\beta} \right) - \frac{1}{4} M_4 \delta \left( G^{\alpha\beta} \tilde{G}_{\alpha\beta} \right) + \frac{1}{2} \lambda_1 \delta g^{00} \delta \left( G^{\alpha\beta} G_{\alpha\beta} \right) $$

$$\left. + \frac{1}{2} \lambda_2 \delta g^{00} \delta \left( G^{\alpha\beta} \tilde{G}_{\alpha\beta} \right) + \ldots \right] ,$$

(2.17)

in which it is understood that the indices for the four-dimensional tensors are raised and lowered via $g_{\mu\nu}$ and $g^{\mu\nu}$, i.e. $G^{\alpha\beta} = g^{\alpha\mu} g^{\beta\nu} G_{\mu\nu}$ and similarly for $\tilde{G}_{\alpha\beta}$.

The terms $\Lambda$ and $\alpha_0$ are fixed from the tadpole cancelation at the background level. In particular, we note that $\Lambda$ is determined by the value of the potential to support inflation while $\alpha_0 \propto \dot{H}$ in which $H$ is the effective (isotropic) Hubble expansion rate. It worth mentioning that by putting one of $G_{\alpha\beta}$ components on the background the other terms would also make sub-dominant contributions to tadpole terms which can be absorbed by redefinition of $\Lambda$ and $\alpha_0$. Therefore, the symbol $\delta$ behind products of $G_{\alpha\beta}$ etc means that we look at the perturbations of the corresponding quantities, excluding their background values.

The couplings $M_1, M_2, M_3, M_4$ and $\lambda_1, \lambda_2$ are left undetermined in the spirit of EFT. Note that in writing the action we have kept terms to leading orders of derivatives, terms with higher orders of derivatives are suppressed as long as we are working in low energy. However, we note that the terms containing $M_2$ and $M_4$ are higher orders in derivatives respectively compared to $M_1$ and $M_3$ and are non-renormalizable. Therefore, in principle, they can also be ignored to leading order of EFT analysis. However, we keep these two terms which are still leading compared to other higher derivative terms encoded in ... which can have interesting effects for the anisotropy power spectrum.

As usual the unitary action given above represents the action in the matter sector. In addition to this, we also have the usual gravitational action given by the Einstein-Hilbert term. However, we do not elaborate on this part as we will be working on the decoupling limit in which the gravitational back-reactions are suppressed to leading order in slow-roll parameters as we will justify later on.

Before concluding this section, it is instructive to compare our results with the well-studied model of anisotropic inflation [9, 27] based on Maxwell theory with a time-dependent (actually $\phi$-dependent) gauge kinetic coupling:

$$L_{\text{Maxwell}} = - \frac{f(\phi)^2}{4} F_{\mu\nu} F^{\mu\nu} .$$

(2.18)

As mentioned before, in order for the background electric field to contribute a nearly constant energy density to total energy, we require $f(\phi) \propto a(t)^{-2}$, yielding $\tilde{A}^T = H \tilde{A}^T$. Therefore,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu ,$$

$$= \tilde{A}^T \left( \partial_\mu g_{\nu1} - \partial_\nu g_{\mu1} \right) + \tilde{A}^T \left( \delta_\mu^0 g_{\nu1} - \delta_\nu^0 g_{\mu1} \right) ,$$

$$= \tilde{A}^T G_{\mu\nu} ,$$

(2.19)

where we have used the fact that $A_\mu = g_{\mu\nu} A^\nu = g_{\mu1} \tilde{A}^T$. 

- 7 -
Now comparing the Lagrangian eq. (2.18) to our general action eq. (2.17) and using the above relation between $F_{\mu\nu}$ and $G_{\mu\nu}$ yields
\[
M_1 = t^2 \left( \frac{A_1}{\alpha^2} \right)^2 \propto a^{-2}; \quad M_2 = M_3 = M_4 = \lambda_1 = \lambda_2 = c_0 = 0.
\] (2.20)

It is very interesting that the anisotropic inflation based on Maxwell theory is such a simple model compared to general possibilities encoded in eq. (2.17).

## 3 The Goldstone bosons

The action (2.17) are obtained in unitary gauge defined such that $\delta \phi = \delta A^\mu = 0$. As usual in EFT approach, we can leave this gauge to any arbitrary coordinate system in which the full four-dimensional diffeomorphism invariance is explicit. This requires the appearance of Goldstone bosons $\pi^\mu$
\[
x^\mu \rightarrow x'^\mu = x^\mu + \pi^\mu.
\] (3.1)

On the physical grounds, we expect to have more than one Goldstone bosons. The Goldstone boson $\pi^0$ is associated with the breaking of time diffeomorphism which is used to set $\delta \phi = 0$ in unitary gauge. The nature of $\pi^0$ is the same as in [3]: liberating the time coordinate, we introduce the scalar field $\pi^0(x^\mu)$ which encodes the fluctuations of the inflaton in any coordinate system. In addition, restoring the $\delta A^\mu$ fluctuations, we expect to introduce the Goldstone bosons $\pi^i$. This suggests we will have three more Goldstone bosons. However, as we shall see, fixing the U(1) gauge will reduce this to two independent Goldstone bosons, which are the number of transverse polarization degrees of freedom of the gauge field fluctuations.

### 3.1 The quadratic action of Goldstone fields

Now we can restore the Goldstone bosons and perform the so-called Stueckelberg trick. We also work in the decoupling limit in which the metric perturbations are neglected. Also to simplify the notation, we drop the overline over $A^1$, so from now on $A^1$ simply stands for $\overline{A^1}$.

Upon restoring the Goldstone bosons $\pi^\mu$ we have
\[
\delta g^{00} \rightarrow 2\pi^0 + a^{-2}(\pi^0)^2 - (\pi^0)^2, \quad (3.2)
\]
\[
g_{11} \rightarrow g_{11} + 2a_1^2 \pi_1^1 + 2a_1^2 \pi_0^0 \pi_1^1 + 2a_1^2 \pi_1^0 \pi_1^0 - (\pi_0^0)^2 + (\pi_1^1)^2, \quad (3.3)
\]
\[
g_{01} \rightarrow -\pi_0^0 + a_1^2 \pi_1^1 + O(\pi^2), \quad (3.4)
\]
\[
g_{1i} \rightarrow g_{1i} + a_1^2 \pi_1^1 + a_1^2 \pi_1^1 + O(\pi^2). \quad (3.5)
\]

in which the notation “, i ” here and below denotes $\partial_i$, for example $\pi_{ij} = \partial_i \pi^j$ and so on.

Equipped with the above transformation rules of $\delta g_{\alpha\beta}$, we can calculate the quadratic action (2.17) in terms of the Goldstone fields. The key to simplify the analysis is that we should not get any Goldstone field from indices which are contracted in Lorentz invariant manner. For the contraction $G^{\alpha\beta}G_{\alpha\beta}$ we obtain
\[
G^{\alpha\beta}G_{\alpha\beta} \rightarrow -2a^{-2}\left[ \partial_0 \left( \overline{g}_{0\gamma} \Lambda_1^\gamma \right) + \partial_1 \Lambda_1^\gamma + \frac{A_1}{\overline{A}^1} \left( \overline{g}_{0\gamma} \Lambda_1^\gamma \right) \right]^2 + a^{-4} \left[ \partial_i \left( \overline{g}_{ij} \Lambda_1^\gamma \right) - \partial_j \left( \overline{g}_{ij} \Lambda_1^\gamma \right) \right]^2 \]
\[
\rightarrow -2a^{-2}\left( 2H + \frac{A_1}{\overline{A}^1} \right)^2 - 4 \left( 2H + \frac{A_1}{\overline{A}^1} \right) \left( \delta \dot{X}_1 + \pi_{011} + \frac{A_1}{\overline{A}^1} \delta X_1 \right) - 2a^{-2} \left( \delta \dot{X}_i \right)^2
\]
\[-2a^{-2}(\pi_{1i}^0)^2 - 2a^{-2}\left(\frac{\dot{\pi}}{A^1}\right)^2 (\delta X_i)^2 - 4a^{-2}\delta \dot{X}_i \pi_{1i}^0 - 2a^{-2}\frac{\dot{A}^1}{A^1} d\left(\delta X_i\right)^2\]

\[-4a^{-2}\frac{\dot{A}^1}{A^1} \pi_{1i}^0 \delta X_i + 2a^{-4}\left[ (\delta X_{i,j})^2 - (\delta X_{i,i})^2 \right], \quad (3.6)\]

in which we have defined \(\tilde{g}_{\gamma i} \Lambda_i^\gamma \equiv \tilde{g}_{ii} + \delta X_i\) or

\[\delta X_i \equiv a^2 \partial_i \pi^i. \quad (3.7)\]

The above equations indicate that it is \(\delta X_i\) and not \(\pi^i\) itself which is physical. This is a consequence of our remnant symmetry eq. (2.9) which somewhat singles out \(\partial_1\) operation in the sense that \(\partial_1 \xi_T = 0\) while \(\partial_1 \xi_L\) is related to \(\partial_1 \xi_L\).

Similarly, for the contraction \(\tilde{G}^{\alpha\beta} G_{\alpha\beta}\) we obtain

\[\frac{1}{4} \tilde{G}^{\alpha\beta} G_{\alpha\beta} = \frac{1}{4} \epsilon^{\alpha\beta\gamma\delta} G_{\alpha\beta} G_{\gamma\delta} \rightarrow 2\epsilon^{ijk} \left(2H + \frac{\dot{A}^1}{A^1}\right) \delta X_{k,j} + 2\epsilon^{ijk} \delta X_{k,j} \left(\pi_{1i}^0 + \delta \dot{X}_i + \frac{\dot{A}^1}{A^1} \delta X_i\right), \quad (3.8)\]

where we use the convention that \(\epsilon^{0123} = 1\) and define \(\epsilon^{ijk} \equiv \epsilon^{0ijk}\).

The other terms in the action (2.17) can be evaluated in terms of the Goldstone bosons similarly. Combining all terms, the full second order action written in terms of the physical fields \(\pi^0\) and \(\delta X_i\) is obtained to be

\[S = \int d^4x \sqrt{-g} \left\{ \alpha_0 \left[ - (\pi^0)^2 + a^{-2}(\pi_{1i}^0)^2 \right] + c_0 (\pi^0)^2 \right.\]

\[+ \frac{1}{2} M_1 a^{-2} \left[ (\delta \dot{X}_i)^2 + (\pi_{1i}^0)^2 + \left(\frac{\dot{A}^1}{A^1}\right)^2 (\delta X_i)^2 + 2\delta \dot{X}_i \pi_{1i}^0 + \frac{\dot{A}^1}{A^1} d(\delta X_i)^2\right] - \dot{M}_1 \left(2H + \frac{\dot{A}^1}{A^1}\right) \pi^0 (\delta \dot{X}_1 + \pi_{11}^0 + \frac{\dot{A}^1}{A^1} \delta X_1) \right.\]

\[-4M_2 \left(2H + \frac{\dot{A}^1}{A^1}\right)^2 \left[ (\delta \dot{X}_1)^2 + (\pi_{11}^0)^2 + \left(\frac{\dot{A}^1}{A^1}\right)^2 (\delta X_1)^2 + 2\delta \dot{X}_1 \pi_{11}^0\right] + \frac{\dot{A}^1}{A^1} d(\delta X_1)^2 + 2 \frac{\dot{A}^1}{A^1} \pi_{11}^0 \delta X_1 \right.\]

\[+ 2 \epsilon^{ijk} \delta X_{j,k} \left(\pi_{1i}^0 + \delta \dot{X}_i + \frac{\dot{A}^1}{A^1} \delta X_i\right) - M_3 a^{-2} \left(2H + \frac{\dot{A}^1}{A^1}\right)^2 \epsilon^{ijk} \epsilon^{ilm} \delta X_{j,k} \delta X_{l,m} - 4\lambda_1 \pi^0 \left(2H + \frac{\dot{A}^1}{A^1}\right) \left(\delta \dot{X}_1 + \pi_{11}^0 + \frac{\dot{A}^1}{A^1}\right) \right.\]

\[-8\lambda_2 \left(2H + \frac{\dot{A}^1}{A^1}\right) \pi^0 \epsilon^{1ij} \delta X_{i,j} - 2M_3 a^{-2} \epsilon^{1jk} \left(2 + \frac{\dot{A}^1}{A^1} H\right) H \pi^0 \delta X_{j,k}\right\}. \quad (3.9)\]

The above action have many terms. However, in the spirit of EFT we are interested in low energy (comparing to the cut off of EFT) behavior of this system. Therefore, we may adopt the Wilsonian view here and only take into account terms with least number of derivatives so we consistently discard terms with 3 or higher number of derivatives of \(\pi^0\) and \(\delta X_i\) in the following analysis.
3.2 The free fields

Here we read off the action of the free fields from the total action (3.9) and their wave functions.

First we start with $\pi^0$ field which is simpler. After some integrations by parts the action for $\pi^0$ field is obtained to be

$$S^\pi_2 = \int d^4x \sqrt{-g} \left\{ (-\alpha_0) \left[ \left( \dot{\pi}^0 \right)^2 \left( 1 - \frac{c_0}{\alpha_0} \right) - a^{-2} \left( \pi^0 \right)^2 \right] + (2 + n)H \left[ \dot{M}_1 + 2 \left( \dot{\lambda}_1 + 3H\lambda_1 \right) \right] \left( \pi^0 \right)^2 + \ldots \right\},$$

(3.10)
in which $\ldots$ denotes terms with higher number of derivatives. Note that $\alpha_0 \propto \dot{H} < 0$ so the kinetic energy has the proper sign.

The free wave function of $\pi^0$ with the Minkowski initial conditions deep inside the horizon is

$$\pi^0(k) = \frac{H}{2k^{3/2}} \sqrt{\frac{\pi}{2c_s |\alpha_0|}} \left( -kc_s\tau \right)^{3/2} H_{3/2}^{(1)}(-kc_s\tau),$$

(3.11)

where we defined the sound speed of $\pi^0$ fluctuations

$$c_s^{-2} = 1 - \frac{c_0}{\alpha_0}. \quad (3.12)$$

As expected from the discussions of [3] the coefficient $c_0$ controls the sound speed of the $\pi^0$ fluctuations. This can arise for example in the models of k-inflation [62, 63] or DBI inflation [64] as is well-understood in inflation literature. Now the interesting effect is that we can extend the DBI-type model to anisotropic inflation with gauge fields. This may have motivations from string theory in which the world volume of a mobile D3 brane contains the U(1) gauge fields. For a model of anisotropic inflation with DBI type action see [65].

Note that we have discarded the contributions from the terms in second line of eq. (3.10) in the free wave function. In principle we can include the contributions of these terms in curvature perturbations power spectrum via their corrections to $\pi^0$ free wave function. However, their contribution is sub-leading as follows. If we look at the terms containing $M_1$ and $\lambda_1$ we see that these terms come from perturbing the term $G_{\alpha\beta}G^{\alpha\beta}$. Therefore, we will have a contribution from these terms to the energy content during inflation. However, as the dominant source of the background expansion comes from the inflaton sector, the contribution of these terms to $\Lambda$ and the coefficients of tadpole terms should be small. This means that $M_1H^2 \ll |\alpha_0|$ so we can neglect the contributions of the terms in second line of eq. (3.10) to leading order.

Now we calculate the free wave functions of the $\delta X_i$ fields. As usual, we want our canonical fields to be massless. The action of the free $\delta X_i$ fields has the following general form

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left( \dot{N} \phi^2 + m^2 \phi^2 + \ldots \right),$$

(3.13)
in which $N$ is a time-dependent normalization and $\phi$ collectively represents $\delta X_i$ fields. Correspondingly, the canonically normalized field is given by $\phi_c = \sqrt{N}\phi$ and the condition for $\phi_c$ to be massless is

$$-\frac{1}{2} \dot{N}^2 + 3HN\dot{N} + N\ddot{N} + 2Nm^2 = 0. \quad (3.14)$$
Looking at the actions for $\delta X_i$, we see that $\delta X_2$ and $\delta X_3$ have the same coefficients which are different than those of $\delta X_1$. Let us first consider $\delta X_2$ and $\delta X_3$ which are easier. We find that the coefficients $\mathcal{N}$ and $m^2$ for $\delta X_2$ and $\delta X_3$ fields are proportional to the unknown coupling of EFT $M_1$. So far our analysis was generic with no assumptions on the time scaling of EFT coefficients. However, experience from the previous specific models of anisotropic inflation and the structure of our in-in integrals suggest that it is very reasonable to assume a time scaling like

$$M_1 = \overline{M}_1 a^{s_1},$$

(3.15)

with $\overline{M}_1$ and $s_1$ being constant. This is not the most general functional form of $M_1$ but it is generic enough for our purpose which captures all the models of anisotropic inflation studied so far. In addition, as we shall see, this scaling with time is actually what our in-in integrals suggest for interesting physical results. With similar reasoning, we also assume

$$\frac{\dot{A}^1}{A^1} = n H,$$

(3.16)

with $n$ being a constant.

With these scaling ansatz for $M_1$ and $A^1$ we obtain

$$\mathcal{N} = M_1 a^{-2} = \overline{M}_1 a^{-2+s_1}, \quad m^2 = \overline{M}_1 (n^2 - n - ns_1) a^{-2+s_1} H^2.$$  

(3.17)

Plugging these into eq. (3.14) we obtain,

$$s_1^2 + s_1 (2 - 4n) + 4n^2 - 4n - 8 = 0 \Rightarrow s_1 = -1 + 2n \pm 3.$$  

(3.18)

The above equation gives a relation between $n$ and $s_1$, but does not fix them individually. We can fix $s_1$ by checking the contribution of term containing $M_1 G_{\alpha \beta} G^{\alpha \beta}$ into the background inflationary expansion. At the background level we have $G_{0i} = a^2 (2H + \frac{\dot{A}^1}{A^1}) \delta_{i1}$ with the other components being zero. As a result, at the background level we have

$$- \frac{1}{4} M_1 G_{\alpha \beta} G^{\alpha \beta} = \frac{1}{2} M_1 a^2 \left( 2H + \frac{\dot{A}^1}{A^1} \right)^2.$$  

(3.19)

The above term contributes to the background inflation expansion via renormalizing the cosmological constant term. In order to have a long enough period of inflation with small amount of anisotropy, we require that the above term to be nearly time-independent so it only modifies the effective cosmological constant. This requires that $M_1 \propto a^{-2}$. Comparing to ansatz provided in eq. (3.15) this yields $s_1 = -2$. Consequently, from eq. (3.18) we obtain $n = 1$ and $n = -2$. The latter corresponds to $A_1 = $ constant, yielding a zero electric field energy density. Therefore, it is a trivial solution and we conclude that the only allowed value is $n = 1$. Having said that, in order to keep track of the role of parameter $n$ we leave it undetermined, but we will impose the conclusion $n = 1$ in our final results.

Now we look at the free action for $\delta X_1$ field. In addition to common terms similar to the free actions of $\delta X_2$ and $\delta X_3$, we have a new contribution from $M_2$. Motivated from the above discussions, we assume $M_2 = \overline{M}_2 a^{s_2}$ with $\overline{M}_2$ and $s_2$ being constants. In addition, we assume that the time scaling of $M_1 a^{-2}$ is equal to $M_2$ since both of them contribute to the kinetic energy of $\delta X_1$ field and we do not want one of them to dominate over the other during inflation. We will show momentarily that this is indeed a consistent assumption. In conclusion, for $\delta X_1$ field we have,

$$\mathcal{N} = M_1 a^{-2} - 8 M_2 H^2 (2 + n)^2, \quad s_2 = s_1 - 2.$$  

(3.20)
and with \( s_2 = s_1 - 2 \) the mass term \( m \) is given by

\[
m^2 = NH^2(n^2 - n - ns_1).
\] (3.21)

Plugging these in eq. (3.14) we obtain,

\[
- \frac{1}{2}(s_1 - 2)^2 + 3(s_1 - 2) + (s_1 - 2)^2 + 2(n^2 - n - ns_1) = 0.
\] (3.22)

Note that this is exactly the same equation as (3.18) which shows that our assumption on taking \( M_2 \propto M_1 a^{-2} \) was consistent. In conclusion, for the scaling of \( M_2 \) we have \( s_2 = -4 \).

As mentioned before, it seems we will get three independent Goldstone bosons from \( \delta X_1, \delta X_2 \) and \( \delta X_3 \) fields. However, we should recall that these fields are associated with restoring the \( \delta A^\mu \) field after liberating ourselves from the unitary gauge. Therefore, we should be careful of the remnant \( U(1) \) gauge symmetry to be imposed on \( \delta A^\mu \) fluctuations in any coordinate system. To see this more specifically, suppose we move from the unitary gauge to the arbitrary coordinate after restoring the Goldstone bosons \( \pi^i \) as given in eq. (3.1). Then the gauge field perturbations transform as

\[
\delta A^i \rightarrow \delta A^i = \delta A^i + \partial_i \pi^i A^i = \delta A^i + \delta X_i A^i.
\] (3.23)

Already in writing the action in unitary gauge we assumed that the \( U(1) \) gauge is fixed. Here after restoring the coordinate invariance we should check the presumed \( U(1) \) gauge condition.

Now to fix the \( U(1) \) gauge we impose the Coulomb-radiation gauge in which \( A^0 = \partial_i A^i = 0 \). Combining with the above coordinate transformation, this requires

\[
\partial_i \delta X_i = 0.
\] (3.24)

Now decompose \( \delta X_i \) into its longitudinal and transverse parts as follows

\[
\delta X_i = \partial_i \delta X_L + \delta X_{Ti}, \quad \partial_i \delta X_{Ti} = 0.
\] (3.25)

Combining eq. (3.24) with eq. (3.25) we conclude that \( \nabla^2 \delta X_L = 0 \). With the appropriate boundary conditions at infinity, this yield \( \delta X_L = 0 \). Therefore, we come to the important conclusion that the longitudinal part of \( \delta X_i \) perturbations are not physical and only the transverse parts of \( \delta X_i \) are physical. These two physical degrees of freedom are indeed the two transverse polarization of the \( U(1) \) gauge field as anticipated.

Our job now is to find the action of the free fields \( \delta X_T \). Using the relation

\[
e^{ijk}\delta X_{j,k}\delta X_i = \frac{1}{2} \left[ \partial_i \left( e^{ijk}\delta X_{i,l}\delta X_{j,k} \right) - \partial_k \left( e^{ijk}\delta X_{i,l}\delta X_j \right) \right],
\] (3.26)

we obtain the following action for free \( \delta X_{Ti} \) fields

\[
S_{2}^{X_T} = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \left( \delta X_{T_{i}} \right)^2 - \frac{1}{2} a^{-2} \left( \delta X_{T_{i,j}} \right)^2 - a^{-2} \frac{4M_2H^2(2 + n)^2}{M_1 - 8M_2H^2(2 + n)} \left( \delta X_{T_{i,i}} \right)^2 
\right.
\]

\[
+ 2M_3H \left( n - \frac{3M_3}{2HM_3} \right) e^{ijk}\delta X_{T_{i}}\delta X_{T_{j,k}} - M_4 a^{-4}H^2(2 + n)^2 e^{ijkl} \delta X_{T_{i,j}}\delta X_{T_{k,l}} \right\},
\] (3.27)

in which the canonically normalized fields \( \delta X_{T_{i}} \) are defined via

\[
\delta X_{T_{j}} = \sqrt{M_1 a^{-1}}\delta X_{T_{j}}, \quad j = 2, 3
\] (3.28)
\[ \delta X^c_{T1} = \sqrt{M_1 a^{-2} - 8 M_2 H^2 (2 + n)^2} \delta X_{T1}. \]  

(3.29)

Furthermore, it is convenient to decompose \(X^c_{T1}\) in terms of the gauge field polarization base \(\epsilon^s_i(k)\) in Fourier space

\[ \delta X^c_{T1} = \sum_s \delta X^c_{T}^{(s)}(k, t) \epsilon^s_i(k) \]  

(3.30)

where \(\epsilon^s\) denotes the polarization vector and satisfies certain orthogonality relations. We can use either the linear polarization base with \(s = 1, 2\) or the circular (helicity) base with \(s = \pm\) but at this stage we do not fix the base.

Imposing the Minkowski initial conditions deep inside the horizon we obtain

\[ \delta X^c_{T}^{(s)} = -\frac{H \sqrt{\pi}}{2 k^{3/2}} (-k \tau)^{3/2} H^{(1)}_{3/2} (-k \tau), \]  

(3.31)

and finally,

\[ \delta X_{T1} = \frac{a^2}{\sqrt{M_1 - 8 M_2 H^2 (2 + n)^2}} \sum_s \delta X^c_{T}^{(s)} \epsilon^s_1(k) \]  

(3.32)

\[ \delta X_{Tj} = \frac{a^2}{\sqrt{M_1}} \sum_s \delta X^c_{T}^{(s)} \epsilon^s_2(k), \quad j = 2, 3. \]  

(3.33)

In obtaining the above equations we neglect \(M_3\) and \(M_4\) terms which modify different polarization components of gauge field. However, we will take into account their contribution as perturbations to \(\delta X_{T1}\) wave function which can also affect the anisotropic power spectrum.

### 3.3 The interactions

Having calculated the wave functions of the free fields, here we obtain the interaction between the fields which are in the form of exchange vertices. After integration by parts and noting that \(\delta X_{i,i} = 0\), we obtain,

\[
S_{\text{int}}^{(2)} = \int d^4 x \sqrt{-g} \left\{ - \dot{M}_1 (2 + n) H \pi_0^0 \delta \dot{X}_{T1} - 8 M_2 (2 + n)^2 H^2 \pi_{11}^0 \left( \delta \dot{X}_{T1} + n H \delta X_{T1} \right) 
+ 2 M_3 \epsilon^{ijk} \pi_{1i}^0 \delta X_{Tj,k} - 4 \lambda_1 H (2 + n) \pi_0^0 \left( \delta \dot{X}_{T1} + n H \delta X_{T1} \right) 
- 8 \lambda_2 (2 + n) H \epsilon^{ijk} \pi_0^0 \delta X_{Tj,k} - 2 \dot{M}_3 (2 + n) H \epsilon^{ijk} \pi_0^0 \delta X_{Tj,k} 
- \dot{M}_1 (2 + n) n H^2 \pi_0^0 \delta X_{T1} \right\}. 
\]

(3.34)

Fortunately many terms in the interaction Lagrangian above are irrelevant for low energy EFT studies. The terms involving \(M_2, M_3, \dot{M}_3\) and \(\lambda_2\) are suppressed on super-horizon scales due to presence of spatial partial derivatives so they can be discarded in low energy EFT limit. However, one may argue that we do not know the scaling of coefficients \(M_3\) and \(\lambda_2\) so if their time-dependence is singular, i.e. containing positive power of \(a(t)\), then their contributions may not be so obviously suppressed compared to terms containing \(M_1\) and \(\lambda_1\). To answer this concern we estimate the time scaling of these coefficients. First we note that \(M_2 \sim M_1 a^{-2}\) so the term in eq. (3.34) containing \(M_2\) are highly suppressed compared to
terms containing $M_1$ so it can safely be ignored. To obtain the scaling of $\lambda_1$ we note that the term containing $\lambda_1$ comes from perturbing $g^{00}G^2$ which yields

$$\lambda_1 g^{00}GG = -\lambda_1 G^2 - \lambda_1 \delta (G^2) - \lambda_1 g^{00} \delta (G^2),$$

(3.35)
in which $\bar{G}^2$ represents the background value of $G^2$. Hence the above term gives corrections to cosmological constant and also to the coefficient of $\delta (G_{\alpha \beta} G^{\alpha \beta})$ which is $M_1$. Now noting that $\bar{G}^2 \propto a^2$, and in order for the effective cosmological constant to stay nearly constant, we require that $\lambda_1 \propto M_1 \propto a^{-2}$. Therefore, the interaction in eq. (3.34) containing $\lambda_1$ is as relevant as those of $M_1$. As for $\lambda_2$ we see that the term containing $\lambda_2$ originates from perturbing $g^{00}G_{\alpha \beta}G^{\alpha \beta}$,

$$\lambda_2 g^{00}GG = \lambda_2 g^{00}G^2 - \lambda_2 \delta (G^2).$$

(3.36)

Hence, the last term above also contributes to $M_3$ so the scaling of $\lambda_2$ with time must be the same as $M_3$. However, as we will argue in next section, $M_3$ scales like $a^{-5}$ so the interactions containing $\lambda_2$ and $M_3$ are highly suppressed.

After some integration by parts, and going to conformal time $\tau$, the interaction Lagrangian becomes

$$S^\text{int}_2 = \int d\tau d^3x (L_1 + L_2 + L_3),$$

(3.37)
in which,

$$L_1 = a^2 \left[ 2\bar{M}_{1}(n + 2)(n - 1)H^3 \right] \pi^0 \delta X_{T1},$$

(3.38)

$$L_2 = -a \left[ 4\bar{X}_1 H^2 (2 + n) + 2H^2 (2 + n)\bar{M}_{1} \right] \pi^0 \delta X_{T1},$$

(3.39)

$$L_3 = -4\bar{X}_1 H (2 + n) \pi^0 \delta X_{T1},$$

(3.40)

where a $'$ denotes derivative with respect to conformal time and we have defined the scaling of $\lambda_1$ as $\lambda_1 = \bar{X}_1 a^{-2}$ as discussed above.

Before concluding this section, here we discuss the validity of decoupling limit which was used to simplify the analysis significantly and also estimate the UV cutofof the theory due to strong interactions of $\delta X_i$. Let us first start with the justification of our decoupling assumption. The leading term with least number of derivatives which mixes $\pi^i$ field with the metric comes from,

$$M_1 H^2 a^{-2} \delta g_{ij} \delta X_i.$$  

(3.41)

Related to the canonically normalized fields $\delta X_i^c \sim \sqrt{M_1} a^{-1} \delta X_i$ and $\delta g_{ij}^c \sim \delta g_{ij} M_1^{-1}$, this interaction becomes

$$\bar{M}_1^{1/4} M_1^{-1} H^2 \delta g_{ij}^c \delta X_i^c.$$  

(3.42)

Comparing this term with kinetic term $(\delta \dot{X}_i^c)^2$ we are able to estimate the mixing energy,

$$E_{\text{mix}} \sim \frac{\bar{M}_1^{1/4} H}{M_1}.$$  

(3.43)

As we will see in next section, $\bar{M}_1$ controls the fraction of energy density of gauge field (see eq. (4.22)) to the total energy density which is very small. Therefore this mixing energy lies well outside horizon and for energies greater than $E_{\text{mix}}$, we can safely neglect mixing of $\delta X_i$ with gravity.
Now we may estimate the UV cutoff of the theory due to strong interactions of $\delta X_i$. The cutoff of theory due to strong interactions of $\pi^0$ is estimated in [3]. Obviously, the lower cutoff will be the cutoff of our theory at which our effective field theory fails to be weakly interacting. For simplicity, we drop numerical factors and scale factors $a(t)$ in following discussions. From (3.6) and (2.17) it is clear that the first non-trivial interaction between $\delta X_i$ arises from $M_2 \left(G_{\alpha\beta}G^{\alpha\beta}\right)^2$:

$$M_2 H \delta X_i (\partial_t \delta X)^2.$$  \hfill (3.44)

As we shall see in next section, this operator will generate a non-trivial sound speed $c_v$ for gauge field fluctuations, see eq. (4.29). One way to deal with this non-trivial sound speed is to re-scale $x_i \rightarrow \tilde{x}_i = x_i / c_v$. With this rescaling we may define $\tilde{\partial}_\mu = (\partial_0, c_v \partial_i)$ and make our free theory to be explicitly Lorentz-invariant. Note that the Lagrangian changes as $L \rightarrow c_v^3 L$.

Now our operator is a dimension six operator and upon canonical normalization and rescaling sound speed it becomes

$$\frac{M_2^2 M_1^{3/2}}{M_1^{3/2}} H c_v^{-5} \delta X^c (\tilde{\partial}_i \delta X^c)^2.$$  \hfill (3.45)

Hence the cutoff of the theory becomes,

$$E_c^2 \sim \frac{M_1^{3/2} c_v^5}{M_2 H}.$$  \hfill (3.46)

Note that if $M_2 \rightarrow 0$ then $E_c \rightarrow \infty$. This is consistent with our intuition since dropping $M_2 \left(G_{\alpha\beta}G^{\alpha\beta}\right)^2$ there would be no self-interaction for $\delta X$ coming from the term $M_1 \left(G_{\alpha\beta}G^{\alpha\beta}\right)$ and hence the UV cutoff should be as high as $M_P$. However, note that there are self-interactions in other terms of Lagrangian, for example in terms proportional to $M_4$, and dropping $M_2$ term, the UV cutoff of the theory should be determined with this operator. Here we neglected the $M_4$ operator since as it will become clear in next section, unlike $M_2$, it is not relevant for producing observable signatures. As one might expect, lowering the speed of sound tends to make our theory more strongly interacting lowering the value of $E_c$.

It is worth mentioning that we have two different operators for $\delta X^i$. One of them is $M_1 G_{\mu\nu}^2$ which controls the background energy scale of the would be gauge fields in our problem and the other operator is $M_2 G_{\mu\nu}^4$ which controls the UV cut-off of $\delta X^i$ perturbations. As these two operators are totally independent in our EFT, i.e. they are not related by any symmetries, there is no relation between the background energy of would-be gauge fields and the UV cut-off.

4 The anisotropic power spectrum

Having obtained the wave functions of the free theory and the interaction Lagrangians we are able to calculate the anisotropies corrections to the curvature perturbations power spectrum. First, we relate $\pi^0$ to comoving curvature perturbations $\mathcal{R}$ to leading order via

$$\mathcal{R} = -H \pi^0 + O\left(\left(\pi^0\right)^2\right).$$  \hfill (4.1)

Then to calculate the corrections to curvature perturbation power spectrum, we use the standard in-in formalism [66–68] in which

$$\delta P_{ji} = -\int_{-\infty}^{\tau_c} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \left\langle \left[ L_i(\tau_2), \left[ L_j(\tau_1), \pi^{0*}(\tau_c) \pi^{0*}(\tau) \right] \right] \right\rangle,$$  \hfill (4.2)
where $\tau_e$ denotes the time of end of inflation and $L_i$ and $L_j$ stands for either of $L_1, L_2$ and $L_3$ given in eqs. (3.38), (3.39) and (3.40). Note that the relation between $\mathcal{R}$ and $\pi^0$ given in eq. (4.1) has corrections from the direct contributions of gauge field energy density into $\mathcal{R}$. However these corrections are suppressed as we look into leading order curvature perturbation anisotropy.

Below we calculate the anisotropy corrections to power spectrum using eq. (4.2). Before doing that we mention again that the couplings $M_1$ and $\lambda_1$ play differently than the couplings $M_2, M_3$ and $M_4$. The couplings $M_1$ and $\lambda_1$ appear directly in interaction Lagrangians $L_i$ so they plays the role of exchange vertices. The couplings $M_2, M_3, M_4$ do not appear in $L_i$ directly, but they modify the free wave functions of $\delta X_{T1}$ appearing in $L_i$ so they also affect the anisotropic power spectrum. Finally, $\lambda_2$ neither appear in $L_i$ nor modify $\delta X_{T1}$ to leading order so it does not contribute to anisotropic power spectrum.

In order to get better insights about various contributions, it is helpful to look at different limits of parameter space when some couplings are turned off and vice versa.

4.1 The case $M_2 = M_3 = M_4 = 0$

Here we consider the case where $M_2 = M_3 = M_4 = 0$ while $M_1$ and $\lambda_1$ are turned on. Also we allow for $c_0 \neq 0$. As we have seen from eq. (3.12), a non-zero $c_0$ will introduce a non-trivial value of $c_s$ for the sound speed of $\pi^0$ fluctuations. This can arise in models such as k-inflation [62, 63] or DBI inflation [64]. The coupling $M_1$ controls the kinetic energy of $\pi_T$ fluctuations. In simple models of anisotropic inflation based on Maxwell theory with Lagrangian given in eq. (2.18), one has $M_1 = f^2(A^1)^2 = f^2(A^2)^2$. On the other hand, the coupling $\lambda_1$ arises if the gauge kinetic coupling depends on $X \equiv \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi$, such as in theory with

$$L_{\text{int}} = f(X) F_{\mu \nu} F^{\mu \nu}.$$  \hspace{1cm} (4.3)

One can easily check that upon perturbing $X$ we obtain $\delta X \rightarrow \delta g^00 \phi^2$ so $\lambda_1 \propto \phi^2 a^{-2}$ in which the factor $a^{-2}$ is required to obtain the proper time scaling of $\lambda_1$ as discussed around eq. (3.35). As far as we are aware, there is no model of anisotropic inflation in literature which has studied the effects of the coupling $\lambda_1$. This is a manifestation of the power of EFT which allows one to study different types of interactions without relying on particular models in which different possibilities, such as the coupling $\lambda_1$, appear naturally based on symmetry considerations. Having said that, we would like to study in more details the effect of the coupling $\lambda_1$ for anisotropic power spectrum and bispectrum in models such as eq. (4.3) elsewhere.

The structure of in-in integrals is as given in eq. (4.2) in which there are nine possible terms to be calculated in the the form of $\delta P_{ij}$. Here as an example we illustrate how $\delta P_{11}$ is obtained. Using the form of wave functions $\pi^0$ and $\delta X_{T1}$ given in eqs. (3.11), (3.31) and (3.32) we obtain

$$\delta P_{11} = \left[2M_1(n+2)(n-1)H^3\right]^2 \int_{-\infty}^{\tau_e} \frac{d\tau_1}{\tau_1^2 H^2} \text{Im} \left[\pi^0(\tau_1)\pi^{0*}(\tau_e)\right]$$

$$\times \int_{-\infty}^{\tau_1} \frac{d\tau_2}{\tau_2^2 H^2} \text{Im} \left[\pi^0(\tau_2)\pi^{0*}(\tau_e)\delta X_{T1}(\tau_2)\delta X_{T1}^*(\tau_1)\right]$$

$$= \frac{2}{9e_0^2 k^4} (2 + n)^2 (n - 1)^2 c_s(c_s + 1)(c_s^2 - c_s + 1)M_1 H^4 N^2 \sum_s |c_1^s(k)|^2,$$  \hspace{1cm} (4.4)

where $N = -\ln(-k\tau)$ is number of e-folds when the mode $k$ leaves the horizon till the end of inflation.
Similarly, for other contributions we obtain

\[\delta P_{12} = \frac{2}{3a_0^2 k^3}e^4H^4(n-1)(2+n)^2(2n\bar{\lambda}_1 + M_1)N^2 \sum_s |\epsilon_s^+(k)|^2 \quad (4.5)\]

\[\delta P_{21} = \frac{2}{3a_0^2 k^3}c_s(c_s+1)(c_s^2-c_s+1)(n-1)(2+n)^2(2n\bar{\lambda}_1 + M_1)N^2 \sum_s |\epsilon_s^+(k)|^2 \quad (4.6)\]

\[\delta P_{22} = \frac{2}{M_1 a_0^2 k^3}e_s^4H^4(2+n)^2(2n\bar{\lambda}_1 + M_1)^2N^2 \sum_s |\epsilon_s^+(k)|^2 \quad (4.7)\]

\[\delta P_{33} = \frac{32}{M_1 a_0^2 k^3}(2+n)^2e_s^4\lambda_1^2N^2 \sum_s |\epsilon_s^+(k)|^2 \quad (4.8)\]

\[\delta P_{33} = \frac{8}{3a_0^2 k^3}e_s^4H^4(2+n)^2(n-1)\bar{\lambda}_1N^2 \sum_s |\epsilon_s^+(k)|^2 \quad (4.9)\]

\[\delta P_{32} = \frac{8}{M_1 a_0^2 k^3}c_s^2(2+n)^2H^4\lambda_1(2n\bar{\lambda}_1 + M_1)N^2 \sum_s |\epsilon_s^+(k)|^2 \quad (4.10)\]

\[\delta P_{23} = \delta P_{32}. \quad (4.12)\]

Adding all terms together, yields our final result for the anisotropy correction in power spectrum

\[\delta P = \frac{2H^4c_s(2+n)^3N^2}{9k^3M_1 a_0^2}[(c_s^2-1)\bar{\lambda}_1^3 + \frac{1}{2}(c_s^2(c_s+n)-1)(2+n)^2(n-1)\bar{\lambda}_1^3 + \frac{1}{2}(c_s^2(c_s+n)-1)(2+n)^2(n-1)\bar{\lambda}_1^3 \sum_s |\epsilon_s^+(k)|^2. \quad (4.13)\]

In the above expression, we have left the parameter \(n\) undetermined, but as we argued below eq. (3.19), the only allowed value is \(n = 1\) which simplifies the above results to some extent.

To simplify the result further, we use the symmetry in the \(yz\) plane to choose the wave number as

\[k = k(\cos \theta, \sin \theta, 0), \quad (4.14)\]

where \(\theta\) represents the angle between the wave number and the preferred direction \(\mathbf{n}\), i.e. \(\cos \theta = k \cdot \mathbf{n}\) in which in our case \(\mathbf{n}\) is along the \(x\) direction. As for the polarization vectors, we can use either the linear base or the helicity base. For the former, a convenient choice is

\[\epsilon^{(1)} = (\sin \theta, \cos \theta, 0), \quad \epsilon^{(2)} = (0, 0, 1). \quad (4.15)\]

Consequently, the helicity base can be expressed in terms of the linear base as follows

\[\epsilon^{(+)} = \frac{i}{\sqrt{2}}(\epsilon^{(1)} + i\epsilon^{(2)}), \quad \epsilon^{(-)} = \frac{-i}{\sqrt{2}}(\epsilon^{(1)} - i\epsilon^{(2)}). \quad (4.16)\]

Using either base we obtain \(\sum_s |\epsilon_s^+(k)|^2 = \sin^2 \theta\).

Usually, we are interested in fractional change in power spectrum, \(\frac{\delta P}{P_{\pi^0}}\), in which \(P_{\pi^0}\) represents the power spectrum of the \(\pi^0\) field which is

\[P_{\pi^0} = \frac{H^2}{4|\alpha_0|^2 k^3 c_s}, \quad (4.17)\]
in which \( \alpha_0 = -\epsilon M_\parallel^2 H^2 \) from tadpoles cancellation. Using \( \delta P \) obtained in eq. (4.13) we obtain

\[
\frac{\delta P}{\delta P_{\pi^0}} = \frac{8 M_1 c_\parallel^2}{9 \epsilon M_\parallel^2} (n + 2)^3 \left(1 + \frac{6 \lambda_1}{M_1}\right) \left[ c_\parallel^2(n + 2) \left(1 + \frac{6 \lambda_1}{M_1}\right) + (n - 1) \right] N^2 \sin^2 \theta. \tag{4.18}
\]

Comparing the above expression with the amplitude of quadrupole anisotropy defined in eq. (2.1) and taking \( n = 1 \) yields

\[
g_* = \frac{72 M_1 c_\parallel^5}{c M_\parallel^2} \left(1 + \frac{6 \lambda_1}{M_1}\right)^2 N^2. \tag{4.19}
\]

As mentioned before, the observational constraints require that \(|g_*| \lesssim 10^{-2}\). This can be used to fix a combination of the parameters such as \( c_\parallel, M_1, \lambda_1 \). As in simple models of anisotropic inflation, we see again the \( N^2 \) structure of the anisotropic power spectrum. As discussed in [27], this is a consequence of the accumulative contributions of IR modes which have left the horizon and become classical, modifying the background anisotropy.

Now let us apply the result above to the simple model of anisotropic inflation based in Maxwell theory given in eq. (2.18) with \( c_\parallel = 1, \lambda_1 = 0 \) and with potential \( V(\phi) = \frac{m^2}{2} \phi^2 \). As mentioned before, in order for the gauge field furnish a sub-dominant but nearly constant portion of the total energy density, the functional form of \( f(\phi) \) have to be fine-tuned. As shown in [8] if one choses

\[
f(\phi) = \exp \left(\frac{c \phi^2}{2 M_\parallel^2}\right), \tag{4.20}
\]

with \( c > 1 \) being a constant, then the system reaches the attractor solution in which the electric field energy density is a sub-dominant but constant contribution to the total energy density. Denoting the fraction of electric field energy density to total energy density by parameter \( R \), we obtain

\[
R \equiv \frac{\dot{A}_1^2 f(\phi)^2 a^{-2}}{2V} \simeq \frac{I}{2}\epsilon \tag{4.21}
\]

in which \( I \equiv \frac{c-1}{c} \) and \( \epsilon \) is the usual slow-roll parameter \( \epsilon = -\frac{\dot{H}}{H^2} \). Correspondingly, we can relate our \( M_1 \) to \( R \) via

\[
M_1 = a^2 f^2 \left(\dot{A}_1^2\right)^2 = \frac{1}{9 H^2} f^2 a^{-2} \left(\dot{A}_1^2\right)^2 = \frac{2}{3} R M_\parallel^2 = \frac{1}{3} \epsilon IM_\parallel^2. \tag{4.22}
\]

Now plugging these values in our expressions for \( g_* \) in eq. (4.19) yields

\[
g_* = 24 IN^2, \quad \text{(Maxwell theory)} \tag{4.23}
\]

which is in exact agreements with the results obtained in [9, 23–28].

From our analysis we conclude that \( g_* \propto N^2 \). Having this said, the relation \( g_* \propto N^2 \) was revisited in [50] in which the assumption of the attractor regime as employed in [8] was dropped. This corresponds to an intermediate stage in which the system has not reached the attractor regime or the total number of e-folds are limited so the IR modes which have left the horizon did not accumulate enough to modify the background. Compared to our analysis, this corresponds to imposing different time-scaling for \( M_i \) and \( \lambda_i \) than obtained in previous section. For example, as we have seen before, the condition \( M_1 \propto a^{-2} \) was achieved demanding that the anisotropic solution follows the isotropic background so the gauge field’s
contribution to total energy density is sub-leading but nearly constant, i.e. \( R \sim I \epsilon \) as seen above. If we drop this assumption, then \( M_1 \) and other couplings will acquire a different time-dependence than we used above, yielding a more complicated \( N \)-dependence in \( g_s \). As we mentioned before, we are interested in physically well-motivated situations in which the system has reached the attractor regime and our assumptions on the time scaling of various couplings are justified. In this limit, the relation \( g_s \propto N^2 \) is a generic prediction of our analysis.

One interesting conclusion from our result eq. (4.19) is that a small enough value of \( c_s \) may help to relax the observational bound on \( R \). For example, imposing the observational constraint \( |g_s| \lesssim 10^{-2} \), from the conventional formula eq. (4.23) one obtains the tight bound \( R \lesssim 10^{-9} \). However, using our more general result eq. (4.19) this bound relaxes to \( R \sim \frac{M_1}{M^*} \lesssim 10^{-9} c_s^{-5} \). Of course, this is based on the assumption that \( c_s \) does not appear strongly in background parameters such as \( M_1 \). It would be interesting to perform the analysis in a particular model of k-inflation to verify the above conclusion.

### 4.2 The case \( M_2, M_4 \neq 0 \)

Now we extend the previous analysis to case in which \( M_2 \) and \( M_4 \) are non-zero. These are the coefficients of \( \delta (G_{\alpha\beta}G^{\alpha\beta})^2 \) and \( \delta (G_{\alpha\beta}G^{\alpha\beta})^2 \) in our starting unitary gauge action (2.17) which also appear in the quadratic action of transverse modes in eq. (3.27). Compared to Maxwell theory, these are the terms \( \delta (F_{\alpha\beta}F^{\alpha\beta})^2 \) and \( \delta (F_{\alpha\beta}F^{\alpha\beta})^2 \) which are the fourth orders in derivatives and are non-renormalizable. In quantum electrodynamics these interactions represent the photon-photon scattering and is known as the Euler-Heisenberg Lagrangian. In the spirit of EFT these terms are irrelevant in low energy processes compared to terms coming with from \( M_1 \) and \( \lambda_1 \). In this view, \( M_2, M_4 \lesssim M_1 E_c^{-2} \) in which \( E_c \) is the cut-off of the EFT. At an energy scale \( E \ll E_c \) in which EFT is applicable, the contribution of the term containing \( M_2 \) and \( M_4 \) in the action compared to the leading term containing \( M_1 \) is approximately given by \( \frac{M_2 E^4}{M_1 E_c^4} \sim \left( \frac{E}{E_c} \right)^2 \ll 1 \). Therefore, in our analysis below, the effects of \( M_2 \) and \( M_4 \) should be viewed as small sub-leading corrections compared to those of \( M_1 \) and \( \lambda_1 \).

Unlike \( M_1 \) and \( \lambda_1 \) the interactions \( M_2 \) and \( M_4 \) do not show up explicitly in the interaction Lagrangian and in exchange vertices in eqs. (3.38), (3.39) and (3.40). However, as can be seen from the quadratic action eq. (3.27), their presence affects the wave functions of \( \delta X_T \) so their presence are felt via the corrections in \( \delta X_T \) in \( L_i \) in eqs. (3.38), (3.39) and (3.40).

To calculate the corrections from \( M_2 \) and \( M_4 \) in \( \delta X_T \) it is much easier to work in linear polarization bases given in eq. (4.15). Expanding \( \delta X_T \) in linear base as

\[
\delta X_T = \delta X_T^{(1)} \epsilon_i^{(1)} + \delta X_T^{(2)} \epsilon_i^{(2)} \tag{4.24}
\]

yields

\[
\delta X_{T_1} = -\delta X_T^{(1)} \sin \theta, \quad \delta X_{T_2} = \delta X_T^{(1)} \cos \theta, \quad \delta X_{T_3} = \delta X_T^{(2)}. \tag{4.25}
\]

Consequently, for the corresponding terms in eq. (3.27) we easily obtain

\[
(\partial_i \delta X_T)^2 = \left( \partial_i \delta X_T^{(1)} \right)^2 \sin^2 \theta \tag{4.26}
\]

and

\[
\epsilon^{ij} \delta X_{T_1} \delta X_{T_2} = ik \delta X_T^{(2)} \sin \theta \tag{4.27}
\]
The above relation indicates that the mode $δX_T^{(2)}$ does not affect the power spectrum to leading order. This is because $π^0$ couples only to $δX_T^{(1)} = −δX_T^{(1)} sin θ$ in interaction Lagrangians $L_1$, $L_2$ and $L_3$. In addition, from eq. (4.27) we find that the term containing $M_4$ in action (3.27) contains only $δX_T^{(2)}$ which does not couple to $π^0$. Therefore, the effects of $M_4$ to anisotropy power spectrum can be ignored to leading orders.

Now, working only with the relevant component $δX_T^{(1)}$, the action (3.27) yields

$$S_2^{X_T^{(1)}} = \int d^4x \sqrt{-g} \left[ \frac{1}{2} (δX_T^{(1)c})^2 - \frac{1}{2σ^2} \left( 1 + \frac{8M_2H^2(2 + n)^2sin^2θ}{M_1 - 8M_2H^2(2 + n)^2} (δX_T^{(1)c})_j^2 \right) \right]$$

in which the relation between the normalized field $X_T^{(1)c}$ and $δX_T^{(1)}$ is given as in eq. (3.29). The above action suggests that the speed of propagation for $X_T^{(1)c}$ is different than unity, given by (neglecting $O(\frac{M_2}{M_2^2})$)

$$c_v^2 \simeq 1 + \frac{8M_2H^2(2 + n)^2}{M_1} \sin^2θ.$$ (4.29)

There are two interesting conclusions here. First, depending on the sign of $M_2$, the speed of propagation of $X_T^{(1)c}$ can be super-luminal or sub-luminal. Second, this speed also depends on the direction of mode propagation, given by the angle $θ(\hat{k})$. Through $c_v$, these non-trivial effects also show up in the power spectrum anisotropy which may be interpreted as birefringence-like phenomena.

The wave function of the normalized field is

$$δX_T^{(1)c} = \frac{iH}{\sqrt{2(kc_v)^3}}(1 + ikc_vτ)e^{-ikc_vτ}.$$ (4.30)

After taking into account the normalization relation between $δX_T^{(1)c}$ and $δX_T^{(1)}$ given in eq. (3.29), for $δX_T^{(1)}$ which appears in the interaction Lagrangians we obtain

$$δX_T^{(1)} = \frac{-i \sin θ}{Hτ^2 \sqrt{2M_1(kc_v)^3}} \frac{(1 + ikc_vτ)e^{-ikc_vτ}}{\sqrt{1 - \frac{c_v^2 - 1}{\sin^2 θ}}}.$$ (4.31)

The interactions are given as before by eqs. (3.38), (3.39) and (3.40) with $δX_T^{(1)}$ given above.

Performing the in-in integrals as before, the corrections in power spectrum is obtained to be

$$\frac{δP}{δP_{π^0}} = \frac{8H^2M_1c_v^3}{9|α_0|} \frac{(n + 2)^3(1 + \frac{6M_1}{M_1})}{1 - \frac{c_v^2 - 1}{\sin^2 θ}} \left[ c_v^3(n + 2) \left( 1 + \frac{6M_1}{M_1} \right) + (n - 1) \right] N^2 \sin^2 θ$$

$$= \frac{72M_1}{c_v^5(1 - \frac{c_v^2 - 1}{\sin^2 θ})} \left( 1 + \frac{6M_1}{M_1} \right) N^2 \sin^2 θ$$

in which the second line is obtained allowing $n = 1$. Note in particular that when $M_2 = 0$ and $c_v = 1$, the above result reduces to eq. (4.18) as expected.

Now let us apply the above result to the conventional model of anisotropic inflation based on Maxwell theory as summarized below eq. (4.19) in previous sub-section. We obtain

$$\frac{δP}{δP_{π^0}} = 24IN^2 \sin^2 θ \left[ 1 - \frac{36H^2M_2}{M_1}(1 - 3\cos^2 θ) \right].$$ (4.33)
We see that the presence of the non-renormalizable term $M_2$ modifies the shape of anisotropy. We have both $\ell = 2$ and $\ell = 4$ harmonics for power anisotropy. Also note that while both polarization $\delta X_T^{(1)}$ and $\delta X_T^{(2)}$ contribute into leading statistical anisotropy, but it is only $\delta X_T^{(1)}$ which contributes into the sub-leading corrections containing $M_2$.

4.3 The case $M_3 \neq 0$

Now we go back to renormalizable models and assume $M_2 = M_4 = 0$, but allow for a non-zero coupling $M_3$ which is the coupling of the interaction $G_{\alpha\beta}\tilde{G}^{\alpha\beta}$. In terms of Maxwell theory this corresponds to the interaction $F_{\mu\nu}\tilde{F}^{\mu\nu}$. It is well known that this interaction breaks the parity. Usually the coupling to this interaction is controlled by the vev of a pseudo scalar known as the axion. The phenomenology of this interaction has been extensively studied in [69–73]. Our analysis below will be somewhat similar to analysis performed in [71, 73].

We note that, like the situation involving $M_2$ and $M_4$, the coupling $M_3$ affects the free wave function of $\delta X_T$ fields so its presence change the anisotropic power spectrum through the modification in $\delta X_T^{(1)}$ in interaction Lagrangians $L_i$. Therefore, similar to the case with $M_2$ and $M_4$, our job is to calculate the corrections in $\delta X_T^{(1)}$ wave function in the presence of $M_3$.

With $M_2 = M_3 = 0$, the quadratic action (3.27) reduces to

$$S^V = \int d^4x \sqrt{-g} \left[ \frac{1}{2} (\delta \dot{X}_T^i)^2 - \frac{1}{2a^2} (\delta X_{Tij}^{(s)})^2 + \frac{2M_3Ha^4}{M_1} \left( n - \frac{3}{2} \frac{\dot{M}_3}{2M_3H} \right) \epsilon^{ijk} \delta X_T^{(c)} \delta X_T^{(c)} \right] \right]$$

in which $\delta X_T^{(c)} = \sqrt{M_1a^{-1}} \delta X_T^i$.

To proceed further we need to find the time variation of $M_3$. This term does not appear in the background since it gives rise to magnetic field which is zero for our choice of background containing only the electric field. Therefore, the scaling of this term with time is free. However, if it scales differently form spatial gradient part then things become non-trivial from competition of these two terms during inflation. One intuitive argument to set the scaling of $M_3$ with time is to demand that the equation for the free wave function in Fourier space to depend only on the combination $k/a$. This is motivated from the fact that the physical wave number is $k/a$. For example, the usual gradient term in action (4.34) yields $k^2/a^2$. Demanding that only the combination $k/a$ appears for the term containing $M_3$ requires that $M_3a^4 \propto k/a$ so we conclude

$$M_3 = \bar{M}_3a^{-5}.$$  \hspace{1cm} (4.35)

We mention that the above argument may provide a natural expectation for the scaling of $M_3$ as given in eq. (4.35) but it does not seem exhaustive. As a result, in principle, one may allow for different time scaling than used in eq. (4.35).

To solve the free wave function in the presence of $M_3$, this time it is more convenient to switch to the helicity (circular) base given in eq. (4.16) in which

$$\delta X_T^{(c)}(k) = \sum_{s=\pm} \delta X_T^{(s)}(k) \epsilon^s(k).$$  \hspace{1cm} (4.36)

Using the relation

$$\epsilon^{mjl}\delta X_T^{(c)} = -k \sum_{s=\pm} s \delta X_T^{(s)} \epsilon^s(k),$$  \hspace{1cm} (4.37)
the equation of motion for the free wave function is obtained to be
\[
\delta \dddot{X}^{(s)} + 3H\dot{\delta} X^{(s)} + \frac{k^2}{a^2} \delta X^{(s)} + 4(n+1)\frac{k}{M_1} \delta X^{(s)} = 0. \tag{4.38}
\]
As demanded, the coefficients in the above equation depend on the combination \( \frac{k}{a} \). Now going to conformal time and defining \( \delta X^{(s)} = -H\tau \delta V^{(s)} \) we obtain,
\[
\delta V^{(s)\nu} + \left(k^2 + \frac{2sk}{\tau} \xi - \frac{2}{\tau^2}\right) \delta V^{(s)} = 0, \tag{4.39}
\]
where we have defined
\[
\xi = -2(n+1)\frac{M_3}{M_1}. \tag{4.40}
\]
As expected, this equation has the same form in the model studied in \cite{71, 73} so our argument from here will be mostly similar to those of \cite{71, 73}.

If \( |\xi| \ll 1 \) then the effect of term containing \( M_3 \) will be suppressed cosmologically, and so, we may consider opposite limit in which \( |\xi| \gg 1 \). With this assumption the phenomenology originating from equation (4.39) is very interesting. Note that for scales deep inside the horizon only the first term in the bracket in eq. (4.39) is important so both of the polarizations are in Minkowski vacuum as expected. On the other hand, in the regime \( 0 \ll |k\tau| \ll |\xi| \) the second term dominates while its sign depends on polarizations through the pre-factor \( s \) and the sign of \( \xi \). For the moment let us assume that \( \xi > 0 \) so from eq. (4.39) we see that only the positive helicity, \( s = + \), is amplified so at the end of inflation \( \delta X^{(s)} \) is highly polarized with positive helicity. Inversely, if \( \xi < 0 \), then the negative helicity is amplified and \( \delta X^{(s)} \) becomes a pure negative helicity at the end of inflation. However, the overall amplitude of these polarizations are the same on super-horizon scales and as their couplings to \( \pi^0 \) are also the same, the final result will not change. As a result, without loss of generality, we may simply take \( \xi > 0 \).

The general solution of eq. (4.39) is presented in \cite{73} which on super-horizon scales, \( k\tau \to 0 \), simplifies to
\[
\delta V^+ = \frac{e^{\pi\xi}}{\xi^{3/2}} \frac{(-\tau)^{-1}}{2\sqrt{\pi k^3}}, \quad k\tau \to 0, \tag{4.41}
\]
which leads to,
\[
\delta X_{T1} = \frac{-i}{\sqrt{2M_1 k^3 H\tau^2}} \frac{e^{\pi\xi}}{2\sqrt{\pi \xi^3}} \quad k\tau \to 0. \tag{4.42}
\]
The in-in integrals are easy to calculate noting that the main contribution to the in-in integrals comes from super-horizon scales. Now comparing the wave function in (4.42) with the wave function in (4.31) towards the end of inflation, it is easy to see that the only difference here is that \( c_v = 1 \) while the power of \( \delta X_{T1} \) will be amplified with the additional factor \( (\frac{e^{\pi|\xi|}}{2\sqrt{\pi|\xi|}})^2 \). Therefore, we obtain
\[
g_* = \frac{8H^2 M_1 e_v^2}{9|\alpha_0|} \left( \frac{e^{2\pi|\xi|}}{4\pi|\xi|^3} \right) \left[ (n+2)^3 N^2 \left( 1 + \frac{6\lambda_1}{M_1} \right) \sin^2 \theta \right] \left[ c_v^2(n+2) \left( 1 + \frac{6\lambda_1}{M_1} \right) + (n-1) \right]
\]
\[
= \frac{72H^2 M_1 e_v^5}{|\alpha_0|} \left( \frac{e^{2\pi|\xi|}}{4\pi|\xi|^3} \right) \left( 1 + \frac{6\lambda_1}{M_1} \right)^2 N^2 \sin^2 \theta, \tag{4.43}
\]
in which the final result is obtained setting \( n = 1 \).
In particular, for the model studied in [71, 73] with \( c_s = 1, \lambda_1 = 0, M_1 = \frac{f_0}{\sqrt{3}} M_P^2 \) and \( \alpha_0 = -\epsilon H^2 M_P^2 \) we obtain

\[
g_\ast = 24I \left( \frac{e^{2\pi|\xi|}}{4\pi|\xi|^3} \right) N^2 \sin^2 \theta \tag{4.44}
\]

in agreement with the results of [71, 73].

5 Summary and discussions

As argued before, EFT of inflation is a powerful tool to study inflation model-independently. In particular, EFT approach is very helpful to classify different models of inflation based on their predictions for power spectrum and bispectrum. So far most of the EFT studies were based in inflation in FRW setup involving scalar fields. In these setups one chooses a space-time foliation which sets the scalar field fluctuations to zero. Consequently, all perturbations are transferred into metric perturbations. However, the system enjoys the remnant three-dimensional diffeomorphism invariance \( x^i \rightarrow x^i + \xi^i(x^\nu) \). Having presented the most general action in unitary gauge which respects the remnant symmetry, one obtains all interactions after restoring the Goldstone boson \( \pi \) associated with the time diffeomorphism breaking.

Our goal in this study was to extend the EFT approach to the models of anisotropic inflation in which a background gauge field, in the form of an electric field, contributes to the inflationary dynamics, for relevant works but in different setups see [74–76]. The background is intrinsically anisotropic in the form of Bianchi I universe. To simplify the analysis we work in the decoupling limit where the gravitational back-reactions are negligible on dynamics of \( \delta \phi \) and \( \delta A_\mu \) perturbations. In particular, within this assumption, one can approximate the Bianchi I background by the usual FRW metric and take all three scale factors to be the same as far as the gauge field perturbations are concerned. Physically, this means that the leading contributions to statistical anisotropies are sourced by matter perturbations. This was specifically demonstrated in the simple model of anisotropic inflation in [23].

The important task in our analysis was to understand the nature of the underlying symmetry and to read off the physical degrees of freedom. These are the necessary steps to define the unitary gauge and to present the starting general action invariant under the remnant symmetry. As in single field model of inflation, we can still use inflaton as the proper clock to define our time foliation. However, the situations with gauge field is more non-trivial. This is mainly because we have to enforce the U(1) gauge symmetry on gauge field perturbations. Putting specifically, even if we start with \( \delta A^\mu = 0 \), there is always a U(1) gauge transformation which can restore \( \delta A^\mu \). Upon taking care of both coordinate diffeomorphism and the U(1) invariance we have identified the remnant symmetry of the system as given in eq. (2.9). Obviously this symmetry is smaller than the remnant symmetry in single field model with no gauge field. However, thanks to the crucial roles of the U(1) gauge symmetry, this remnant symmetry is still large enough to prevent the appearance of pathologies such as ghost or tachyon. Indeed we have checked that if one does not reinforce the gauge symmetry, i.e. take \( A^\mu \) as a mere 4-vector, the remnant symmetry is smaller than eq. (2.9) and many new terms pop up in the unitary gauge action. The situation may get out of control as some of the new terms may have ghosts and other unwanted pathologies. This seems an interesting question and we would like to come back to this question elsewhere.

Having presented the proper unitary gauge and the corresponding remnant symmetry, we have identified the building blocks to present the invariant action as given in eq. (2.17). As
we have seen the coupling $M_1$ represents the known models of anisotropic inflation based on Maxwell theory. Interestingly, the couplings $\lambda_1, \lambda_2, M_2, M_3$ and $M_4$ represent new types of interaction. Also the parameter $c_0$ measures the sound speed of curvature perturbations. Upon performing the so-called Stueckelberg trick, we restore the Goldstone bosons. In total we have three Goldstone bosons, $\pi^0$ and $\delta X_T = a^2 \partial_1 \pi_T$, in which $\pi^0$ is associated with breaking the time diffeomorphism, representing the inflaton perturbations. The other two Goldstone bosons represent the two transverse polarization degrees of freedom of gauge field fluctuations.

After presenting the wave function of the free theory and the leading interactions, we have calculated the anisotropy corrections to curvature perturbation power spectrum for various couplings. As expected, we have recovered the known results for power anisotropies in known models of anisotropic inflation. In addition, we have shown that the sound speed $c_s$ and the coupling $\lambda_1$ can play non-trivial roles. We have seen that the non-renormalizable term containing $M_2$ introduce a phenomena similar to birefringence in which the speed of gauge field propagation $c_v$ depends on the direction of the propagating mode. In addition, the two polarization of gauge fields contribute asymmetrically in curvature perturbation anisotropies. Finally, we have seen that the coupling $M_3$ captures the parity violating model $F\tilde{F}$ as studied in the past literature.

We comment that here we have assumed that there is only one scalar field degree of freedom. If one considers the cases involving multiple light scalar fields then the number of Goldstone bosons will be different. For example, if one starts with the charged U(1) setup in which the gauge field is charged under a complex scalar field, as studied in [19–23], then one expects to have four Goldstone bosons. Assuming that the potential is a function of the radial part of the complex scalar field, then the additional Goldstone boson represents the axial part of the complex scalar field. Upon Higgs symmetry breaking, the scalar’s axial degree of freedom is eaten by the gauge field, creating its longitudinal degree of freedom. It will be an interesting exercise to present the EFT description of these symmetry breaking scenarios.

There are few directions which we would like to pursue in future works. One natural question is the bispectrum analysis. As is well-known, the EFT approach is specially powerful in non-Gaussianity analysis. Therefore one expects that our EFT approach will be very rich in understanding the generic features of non-Gaussianity in models of anisotropic inflation. The bispectrum and trispectrum for simple models of anisotropic inflation were studied in [24, 27, 28]. We would like to study model-independently the implications of our EFT approach for non-Gaussianity. Another question is the role of gravitational waves. As is well-known, in models of anisotropic inflation there will be mixing between the curvature perturbations and tensor perturbations $h_{ij}$ yielding a non-zero cross-correlation $\langle \zeta h_{ij} \rangle$. This effect was studied in [26, 71]. We expect our general EFT approach to go beyond these analysis and yield more non-trivial results for CMB $TT, TB$ or $EB$ cross-correlations. Also the question of statistical anisotropies beyond the lore of anisotropic inflation in which there is no gauge field and the anisotropies are generated by a generic four vector is another question of interest. In these setups the remnant symmetries are even smaller than the model with U(1) fields and many interactions are allowed. Finding a healthy theory within this setup and looking for their predictions for statistical anisotropies is an interesting question which deserves further investigations.
Acknowledgments

We thank W. Goldberger, J. Gong, E. Komatsu, M. Mirbabayi, L. Senatore, J. Soda, G. Tasinato and M. Yamaguchi for useful discussions and correspondences. H. F. would like to thank Munich Institute for Astro- and Particle Physics (MIAPP) for the hospitality during the workshop “Cosmology after Planck” where this work was in progress. R. E. is supported by Grant HKUST4/CRF/13G issued by the Research Grants Council (RGC) of Hong Kong.

References

[1] PLANCK collaboration, P.A.R. Ade et al., Planck 2013 results. XVI. Cosmological parameters, Astron. Astrophys. 571 (2014) A16 [arXiv:1303.5076] [insPIRE].
[2] PLANCK collaboration, P.A.R. Ade et al., Planck 2013 results. XXII. Constraints on inflation, Astron. Astrophys. 571 (2014) A22 [arXiv:1303.5082] [insPIRE].
[3] C. Cheung, P. Creminelli, A.L. Fitzpatrick, J. Kaplan and L. Senatore, The effective field theory of inflation, JHEP 03 (2008) 014 [arXiv:0709.0293] [insPIRE].
[4] A.V. Manohar, Effective field theories, Lect. Notes Phys. 479 (1997) 311 [hep-ph/9606222] [insPIRE].
[5] C.P. Burgess, Introduction to effective field theory, Ann. Rev. Nucl. Part. Sci. 57 (2007) 329 [hep-th/0701053] [insPIRE].
[6] L. Senatore and M. Zaldarriaga, The effective field theory of multifield inflation, JHEP 04 (2012) 024 [arXiv:1009.2093] [insPIRE].
[7] R. Emami, Anisotropic inflation and cosmological observations, arXiv:1511.01683 [insPIRE].
[8] M.-A. Watanabe, S. Kanno and J. Soda, Inflationary universe with anisotropic hair, Phys. Rev. Lett. 102 (2009) 191302 [arXiv:0902.2833] [insPIRE].
[9] M.-A. Watanabe, S. Kanno and J. Soda, The nature of primordial fluctuations from anisotropic inflation, Prog. Theor. Phys. 123 (2010) 1041 [arXiv:1003.0056] [insPIRE].
[10] J. Ohashi, J. Soda and S. Tsujikawa, Observational signatures of anisotropic inflationary models, JCAP 12 (2013) 009 [arXiv:1308.4488] [insPIRE].
[11] J. Ohashi, J. Soda and S. Tsujikawa, Anisotropic power-law k-inflation, Phys. Rev. D 88 (2013) 103517 [arXiv:1310.3053] [insPIRE].
[12] J. Ohashi, J. Soda and S. Tsujikawa, Anisotropic non-Gaussianity from a two-form field, Phys. Rev. D 87 (2013) 083520 [arXiv:1303.7340] [insPIRE].
[13] S. Kanno, J. Soda and M.-A. Watanabe, Anisotropic power-law inflation, JCAP 12 (2010) 024 [arXiv:1010.5307] [insPIRE].
[14] K. Murata and J. Soda, Anisotropic inflation with non-Abelian gauge kinetic function, JCAP 06 (2011) 037 [arXiv:1103.6164] [insPIRE].
[15] S. Yokoyama and J. Soda, Primordial statistical anisotropy generated at the end of inflation, JCAP 08 (2008) 005 [arXiv:0805.4265] [insPIRE].
[16] M.-A. Watanabe, S. Kanno and J. Soda, Imprints of anisotropic inflation on the cosmic microwave background, Mon. Not. Roy. Astron. Soc. 412 (2011) L83 [arXiv:1011.3604] [insPIRE].
[17] K. Yamamoto, M.-A. Watanabe and J. Soda, Inflation with multi-vector-hair: the fate of anisotropy, Class. Quant. Grav. 29 (2012) 145008 [arXiv:1201.5309] [insPIRE].
[18] A. Ito and J. Soda, Designing anisotropic inflation with form fields, Phys. Rev. D 92 (2015) 123533 [arXiv:1506.02450] [insPIRE].
[19] R. Emami, H. Firouzjahi, S.M. Sadegh Movahed and M. Zarei, Anisotropic inflation from charged scalar fields, JCAP 02 (2011) 005 [arXiv:1010.5495] [nSPIRE].

[20] R. Emami and H. Firouzjahi, Issues on generating primordial anisotropies at the end of inflation, JCAP 01 (2012) 022 [arXiv:1111.1919] [nSPIRE].

[21] S. Baghram, M.H. Namjoo and H. Firouzjahi, Large scale anisotropic bias from primordial non-Gaussianity, JCAP 08 (2013) 048 [arXiv:1303.4368] [nSPIRE].

[22] R. Emami and H. Firouzjahi, Clustering fossil from primordial gravitational waves in anisotropic inflation, JCAP 10 (2015) 043 [arXiv:1506.00958] [nSPIRE].

[23] R. Emami and H. Firouzjahi, Curvature perturbations in anisotropic inflation with symmetry breaking, JCAP 10 (2013) 041 [arXiv:1301.1219] [nSPIRE].

[24] A.A. Abolhasani, R. Emami, J.T. Firouzjaee and H. Firouzjahi, $\delta N$ formalism in anisotropic inflation and large anisotropic bispectrum and trispectrum, JCAP 08 (2013) 016 [arXiv:1302.6986] [nSPIRE].

[25] A.A. Abolhasani, R. Emami and H. Firouzjahi, Primordial anisotropies in gauged hybrid inflation, JCAP 05 (2014) 016 [arXiv:1311.0493] [nSPIRE].

[26] X. Chen, R. Emami, H. Firouzjahi and Y. Wang, The TT, TB, EB and BB correlations in anisotropic inflation, JCAP 08 (2014) 027 [arXiv:1404.4083] [nSPIRE].

[27] N. Bartolo, S. Matarrese, M. Peloso and A. Ricciardone, Anisotropic power spectrum and bispectrum in the $f(\varphi)F^2$ mechanism, Phys. Rev. D 87 (2013) 023504 [arXiv:1210.3257] [nSPIRE].

[28] M. Shiraishi, E. Komatsu, M. Peloso and N. Barnaby, Signatures of anisotropic sources in the squeezed-limit bispectrum of the cosmic microwave background, JCAP 05 (2013) 013 [arXiv:1210.3257] [nSPIRE].

[29] M. Shiraishi, E. Komatsu and M. Peloso, Signatures of anisotropic sources in the trispectrum of the cosmic microwave background, JCAP 04 (2014) 027 [arXiv:1312.5221] [nSPIRE].

[30] K. Dimopoulos, M. Karciauskas, D.H. Lyth and Y. Rodriguez, Statistical anisotropy of the curvature perturbation from vector field perturbations, JCAP 05 (2009) 013 [arXiv:0809.1055] [nSPIRE].

[31] A.E. Gumrukcuoglu, B. Himmetoglu and M. Peloso, Scalar-scalar, scalar-tensor and tensor-tensor correlators from anisotropic inflation, Phys. Rev. D 81 (2010) 063528 [arXiv:1001.4088] [nSPIRE].

[32] T.R. Dulaney and M.I. Gresham, Primordial power spectra from anisotropic inflation, Phys. Rev. D 81 (2010) 103532 [arXiv:1001.2301] [nSPIRE].

[33] K. Yamamoto, Primordial bispectrum from inflation with a triad of background gauge fields, Phys. Rev. D 85 (2012) 123504 [arXiv:1203.1071] [nSPIRE].

[34] H. Funakoshi and K. Yamamoto, Primordial fluctuations from inflation with a triad of background gauge fields, Class. Quant. Grav. 30 (2013) 135002 [arXiv:1212.2615] [nSPIRE].

[35] T. Fujita and S. Yokoyama, Higher order statistics of curvature perturbations in IFF model and its Planck constraints, JCAP 09 (2013) 009 [arXiv:1306.2992] [nSPIRE].

[36] S.R. Ramazanov and G. Rubtsov, Constraining anisotropic models of the early universe with WMAP9 data, Phys. Rev. D 89 (2014) 043517 [arXiv:1311.3272] [nSPIRE].

[37] S. Nurmi and M.S. Sloth, Constraints on gauge field production during inflation, JCAP 07 (2014) 012 [arXiv:1312.4946] [nSPIRE].

[38] R.K. Jain and M.S. Sloth, On the non-Gaussian correlation of the primordial curvature perturbation with vector fields, JCAP 02 (2013) 003 [arXiv:1210.4361] [nSPIRE].
[39] F.R. Urban, *Pseudoscalar N-flation and axial coupling revisited*, *Phys. Rev.* D 88 (2013) 063525 [arXiv:1307.5215] [SPIRE].

[40] M. Thorsrud, D.F. Mota and S. Hervik, *Cosmology of a scalar field coupled to matter and an isotropy-violating Maxwell field*, *JHEP* 10 (2012) 066 [arXiv:1205.6261] [SPIRE].

[41] S. Bhowmick and S. Mukherji, *Anisotropic power law inflation from rolling tachyons*, *Mod. Phys. Lett.* A 27 (2012) 1250009 [arXiv:1105.4455] [SPIRE].

[42] S. Hervik, D.F. Mota and M. Thorsrud, *Inflation with stable anisotropic hair: is it cosmologically viable?*, *JHEP* 11 (2011) 146 [arXiv:1109.3456] [SPIRE].

[43] C.G. Boehmer and D.F. Mota, *CMB anisotropies and inflation from non-standard spinors*, *Phys. Lett.* B 663 (2008) 168 [arXiv:0710.2003] [SPIRE].

[44] T. Koivisto and D.F. Mota, *Vector field models of inflation and dark energy*, *JCAP* 08 (2008) 021 [arXiv:0805.4229] [SPIRE].

[45] J.P. Beltran Almeida, Y. Rodriguez and C.A. Valenzuela-Toledo, *The Suyama-Yamaguchi consistency relation in the presence of vector fields*, *Mod. Phys. Lett.* A 28 (2013) 1350012 [arXiv:1112.6149] [SPIRE].

[46] Y. Rodriguez, J.P. Beltran Almeida and C.A. Valenzuela-Toledo, *The different varieties of the Suyama-Yamaguchi consistency relation and its violation as a signal of statistical inhomogeneity*, *JCAP* 04 (2013) 039 [arXiv:1301.5843] [SPIRE].

[47] D.H. Lyth and M. Karciauskas, *The statistically anisotropic curvature perturbation generated by $f(\phi)^2 F^2$*, *JCAP* 05 (2013) 011 [arXiv:1302.7304] [SPIRE].

[48] T.Q. Do and W.F. Kao, *Anisotropic power-law inflation for the Dirac-Born-Infeld theory*, *Phys. Rev.* D 84 (2011) 123009 [SPIRE].

[49] T.Q. Do, W.F. Kao and I.-C. Lin, *Anisotropic power-law inflation for a two scalar fields model*, *Phys. Rev.* D 83 (2011) 123002 [SPIRE].

[50] A. Naruko, E. Komatsu and M. Yamaguchi, *Anisotropic inflation reexamined: upper bound on broken rotational invariance during inflation*, *JCAP* 04 (2015) 045 [arXiv:1411.5489] [SPIRE].

[51] J. Kim and E. Komatsu, *Limits on anisotropic inflation from the Planck data*, *Phys. Rev.* D 88 (2013) 101301 [arXiv:1310.1605] [SPIRE].

[52] N. Bartolo, S. Matarrese, M. Peloso and A. Ricciardone, *Anisotropy in solid inflation*, *JCAP* 08 (2013) 022 [arXiv:1306.4160] [SPIRE].

[53] N. Bartolo, M. Peloso, A. Ricciardone and C. Unal, *The expected anisotropy in solid inflation*, *JCAP* 11 (2014) 009 [arXiv:1407.8053] [SPIRE].

[54] M. Akhshik, R. Emami, H. Firouzjahi and Y. Wang, *Statistical anisotropies in gravitational waves in solid inflation*, *JCAP* 09 (2014) 012 [arXiv:1405.4179] [SPIRE].

[55] M. Akhshik, *Clustering fossils in solid inflation*, *JCAP* 05 (2015) 043 [arXiv:1409.3004] [SPIRE].

[56] X. Chen, R. Emami, H. Firouzjahi and Y. Wang, *CMB statistical anisotropies of classical and quantum origins*, *JCAP* 04 (2015) 021 [arXiv:1408.2096] [SPIRE].

[57] X. Li, S. Wang and Z. Chang, *Anisotropic inflation in the Finsler spacetime*, *Eur. Phys. J.* C 75 (2015) 260 [arXiv:1502.02256] [SPIRE].

[58] C. Pitrou, T.S. Pereira and J.-P. Uzan, *Predictions from an anisotropic inflationary era*, *JCAP* 04 (2008) 004 [arXiv:0801.3596] [SPIRE].

[59] G. Esposito-Farese, C. Pitrou and J.-P. Uzan, *Vector theories in cosmology*, *Phys. Rev.* D 81 (2010) 063519 [arXiv:0912.0481] [SPIRE].
[60] L. Ackerman, S.M. Carroll and M.B. Wise, *Imprints of a primordial preferred direction on the microwave background*, Phys. Rev. D 75 (2007) 083502 [Erratum ibid. D 80 (2009) 069901] [astro-ph/0701357] [nSPIRE].

[61] A.R. Pullen and M. Kamionkowski, *Cosmic microwave background statistics for a direction-dependent primordial power spectrum*, Phys. Rev. D 76 (2007) 103529 [arXiv:0709.1144] [nSPIRE].

[62] C. Armendariz-Picon, T. Damour and V.F. Mukhanov, *k-inflation*, Phys. Lett. B 458 (1999) 209 [hep-th/9904075] [nSPIRE].

[63] J. Garriga and V.F. Mukhanov, *Perturbations in k-inflation*, Phys. Lett. B 458 (1999) 219 [hep-th/9904176] [nSPIRE].

[64] M. Alishahiha, E. Silverstein and D. Tong, *DBI in the sky*, Phys. Rev. D 70 (2004) 123505 [hep-th/0404084] [nSPIRE].

[65] K. Dimopoulos, D. Wills and I. Zavala, *Statistical anisotropy from vector curvaton in D-brane inflation*, Nucl. Phys. B 868 (2013) 120 [arXiv:1108.4424] [nSPIRE].

[66] S. Weinberg, *Quantum contributions to cosmological correlations*, Phys. Rev. D 72 (2005) 043514 [hep-th/0506236] [nSPIRE].

[67] X. Chen, *Primordial non-Gaussianities from inflation models*, Adv. Astron. 2010 (2010) 638979 [arXiv:1002.1416] [nSPIRE].

[68] Y. Wang, *Inflation, cosmic perturbations and non-Gaussianities*, Commun. Theor. Phys. 62 (2014) 109 [arXiv:1303.1523] [nSPIRE].

[69] K. Dimopoulos and M. Karciauskas, *Parity violating statistical anisotropy*, JHEP 06 (2012) 040 [arXiv:1203.0230] [nSPIRE].

[70] R. Namba, M. Peloso, M. Shiraishi, L. Sorbo and C. Unal, *Scale-dependent gravitational waves from a rolling axion*, JCAP 01 (2016) 041 [arXiv:1509.07521] [nSPIRE].

[71] N. Bartolo, S. Matarrese, M. Peloso and M. Shiraishi, *Parity-violating CMB correlators with non-decaying statistical anisotropy*, JCAP 07 (2015) 039 [arXiv:1505.02193] [nSPIRE].

[72] N. Bartolo, S. Matarrese, M. Peloso and M. Shiraishi, *Parity-violating and anisotropic correlations in pseudoscalar inflation*, JCAP 01 (2015) 027 [arXiv:1411.2521] [nSPIRE].

[73] C. Caprini and L. Sorbo, *Adding helicity to inflationary magnetogenesis*, JCAP 10 (2014) 056 [arXiv:1407.2809] [nSPIRE].

[74] D. Cannone, J.-O. Gong and G. Tasinato, *Breaking discrete symmetries in the effective field theory of inflation*, JCAP 08 (2015) 003 [arXiv:1505.05773] [nSPIRE].

[75] Y. Hidaka, T. Nouni and G. Shiue, *Effective field theory for spacetime symmetry breaking*, Phys. Rev. D 92 (2015) 045020 [arXiv:1412.5601] [nSPIRE].

[76] C. Lin and L.Z. Labun, *Effective field theory of broken spatial diffeomorphisms*, arXiv:1501.07160 [nSPIRE].