Evidence for Ring Caustics in the Milky Way

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(May 7, 2003)

Abstract

The late infall of cold dark matter onto our galaxy produces discrete flows and caustics in its halo. The recently discovered ring of stars near galactocentric distance 20 kpc and a series of sharp rises in the Milky Way rotation curve are interpreted as due to the presence of caustic rings of dark matter in the galactic plane. Their locations are consistent at the 3% level with the predictions of the self-similar infall model for the caustic ring radii. Also, a triangular feature in the IRAS map of the galactic plane is consistent with the imprint of a caustic ring of dark matter upon the baryonic matter. These observations imply that the dark matter in our neighborhood is dominated by a single flow whose density and velocity vector are estimated.
There are compelling reasons to believe that the dark matter of the universe is constituted in large part of non-baryonic collisionless particles with very small primordial velocity dispersion, such as axions and/or weakly interacting massive particles (WIMPs). Generically, such particles are called cold dark matter (CDM). Knowledge of the distribution of CDM in galactic halos, and in our own halo in particular, is of paramount importance to understanding galactic structure and predicting signals in experimental searches for dark matter.

A model of the structure of the halos of isolated galaxies has been developed [1–3] based on the observation that the dark matter particles must lie on a 3-dimensional sheet in phase-space, that this sheet cannot break, and hence that its evolution is constrained by topology. The thickness of the sheet is the velocity dispersion. The primordial velocity dispersion of the leading cold dark matter candidates is extremely small, of order \(10^{-12}c\) for WIMPs and \(3 \cdot 10^{-17}c\) (at most) for axions. To a coarse-grained observer, however, the sheet may have additional velocity dispersion because it is wrapped up on scales which are small compared to the galaxy as a whole. This effective velocity dispersion is associated with the clumpiness of the dark matter before it falls onto the galaxy. The effective velocity dispersion of the infalling dark matter must be much less than the rotation velocity of the galaxy for the model to have validity, say less than 30 km/s for our galaxy. On the other hand, by comparing the model with observations, an upper bound of order 50 m/s will be obtained below.

Where a galaxy forms, the sheet wraps up in phase-space, turning clockwise in any two dimensional cut \((x, \dot{x})\) of that space. \(x\) is the physical space coordinate in an arbitrary direction and \(\dot{x}\) its associated velocity. The outcome of this process is a discrete set of flows at any physical point in a galactic halo [1]. Two flows are associated with particles falling through the galaxy for the first time \((n = 1)\), two other flows are associated with particles falling through the galaxy for the second time \((n = 2)\), and so on. Scattering in the gravitational wells of inhomogeneities in the galaxy (e.g. molecular clouds and globular clusters) are ineffective in thermalizing the flows with low values of \(n\). Recently, Stiff and Widrow [4] have put these discrete flows, also called 'velocity peaks', into evidence in \(N\)-body simulations using a new technique which increases the resolution of the simulations in the relevant regions of phase-space.

Caustics appear wherever the projection of the phase-space sheet onto physical space has a fold [3,5–7]. Generically, caustics are surfaces in physical space. On one side of the caustic surface there are two more flows than on the other. At the surface, the dark matter density is very large. It diverges there in the limit of zero velocity dispersion. There are two types of caustics in the halos of galaxies, inner and outer. The outer caustics are topological spheres surrounding the galaxy. They are located near where a given outflow reaches its furthest distance from the galactic center before falling back in. The inner caustics are rings [3]. They are located near where the particles with the most angular momentum in a given inflow reach their distance of closest approach to the galactic center before going back out. A caustic ring is a closed tube whose cross-section is a \(D_{-4}\) (also called elliptic umbilic) catastrophe [5]. The existence of these caustics and their topological properties are independent of any assumptions of symmetry.

Primordial peculiar velocities are expected to be the same for baryonic and dark matter particles because they are caused by gravitational forces. Later the velocities of baryons and CDM differ because baryons collide with each other whereas CDM is collisionless. However,
because angular momentum is conserved, the net angular momenta of the dark matter and baryonic components of a galaxy are aligned. Since the caustic rings are located near where the particles with the most angular momentum in a given infall are at their closest approach to the galactic center, they lie close to the galactic plane.

A specific proposal has been made for the radii \( a_n \) of caustic rings [3]:

\[
\{ a_n : n = 1, 2, ... \} \simeq (39, 19.5, 13, 10, 8, ...) \text{kpc} \times \left( \frac{j_{\text{max}}}{0.25} \right) \left( \frac{0.7}{h} \right) \left( \frac{v_{\text{rot}}}{220 \text{ km s}^{-1}} \right)
\]

where \( h \) is the present Hubble constant in units of 100 km/(s Mpc), \( v_{\text{rot}} \) is the rotation velocity of the galaxy and \( j_{\text{max}} \) is a parameter with a specific value for each halo. For large \( n \), \( a_n \propto 1/n \). Eq. 1 is predicted by the self-similar infall model [8,2] of galactic halo formation. \( j_{\text{max}} \) is then the maximum of the dimensionless angular momentum \( j \)-distribution [2]. The self-similar model depends upon a parameter \( \epsilon \) [8]. In CDM theories of large scale structure formation, \( \epsilon \) is expected to be in the range 0.2 to 0.35 [2]. Eq. 1 is for \( \epsilon = 0.3 \). However, in the range \( 0.2 < \epsilon < 0.35 \), the ratios \( a_n/a_1 \) are almost independent of \( \epsilon \). When \( j_{\text{max}} \) values are quoted below, \( \epsilon = 0.3 \) and \( h = 0.7 \) will be assumed.

It was pointed out in ref. [2] that including angular momentum in the self-similar infall model results in a depletion of the inner halo and hence an effective core radius. The average amount of angular momentum of the Milky Way halo was estimated [2] by requiring that approximately half of the rotation velocity squared at our location is due to dark matter, the other half being due to ordinary matter. This yields \( \bar{j} \sim 0.2 \) where \( \bar{j} \) is the average of the \( j \)-distribution for our halo. \( \bar{j} \) and \( j_{\text{max}} \) are related if some assumption is made about the shape of the \( j \)-distribution. For example, if the \( j \)-distribution is taken to be that of a rigidly rotating sphere, one has \( j_{\text{max}} = \frac{4}{3} \pi \bar{j} \). Hence \( j_{\text{max}} \sim 0.25 \) for our halo.

Since caustic rings lie close to the galactic plane, they cause bumps in the rotation curve, at the locations of the rings. In ref. [9] a set of 32 extended well-measured rotation curves was analyzed and statistical evidence was found for the \( n = 1 \) and \( n = 2 \) caustic rings, distributed according to Eq. 1, at the 2.6\( \sigma \) and 3\( \sigma \) significance level respectively. That study implies that the \( j_{\text{max}} \) distribution is peaked near 0.27. The rotation curve of NGC3198, one of the best measured, by itself shows three faint bumps which are consistent with Eq. 1 and \( j_{\text{max}} = 0.28 \) [3]. Also the above estimate of \( j_{\text{max}} \) for our own halo is close to the peak value of 0.27.

Eq. 1 with \( j_{\text{max}} = 0.25 \) implies that our halo has caustic rings with radii near \( 40 \text{ kpc}/n \), where \( n \) is an integer. The purpose of this paper is to point out evidence in support of this extraordinary claim.

Recently Yanny et al. [10] and Ibata et al. [11] discovered what appears to be a ring of stars, with radius of order 20 kpc, circling the Galaxy. The origin of this star ring is puzzling. In ref. [10], the ring is interpreted as the tidal stream from an accreted satellite galaxy. However, in this interpretation it is hard to account for the fact that the star population is confined to a nearly circular region [11]. Also, it would have to be an accident that the tidal stream lies in the Galactic plane. Ibata et al. [11] propose instead that the ring of stars is a perturbation of the Galactic disk population. What causes the perturbation is not clear however. Since the ring of stars is near the predicted \( n = 2 \) caustic ring of dark matter, the perturbation may be the attractive gravitational field of the \( n = 2 \) caustic ring.
This interpretation accounts for the 20 kpc radius of the ring, as well as for the fact that the ring lies in the galactic plane.

The spatial coincidence of the 20 kpc ring of stars with the predicted $n = 2$ caustic ring may, of course, be fortuitous. So it is natural to ask whether there is similar evidence for any of the other caustic rings. The answer is yes for the $n = 3$ ring. Binney and Dehnen studied [12] the outer rotation curve of the Milky Way and concluded that its anomalous behavior can be explained if most of the tracers of the rotation are concentrated in a ring of radius $1.6 r_\odot$ where $r_\odot$ is our distance to the galactic center. Throughout this paper we use the standard value $r_\odot = 8.5$ kpc. That value is assumed in Eq. 1, and also in refs. [10,11]. The Binney and Dehnen ring is therefore at 13.6 kpc, which is within 3% of the predicted radius of the $n = 3$ caustic ring. Moreover, there is independent evidence for the existence of the Binney and Dehnen ring.

Olling and Merrifield have recently published [13] a rotation curve for the Milky Way. It is reproduced in Fig. 1. It shows a significant rise between 12.7 and 13.7 kpc. The increase in rotation velocity is 27%, from 220 to 280 km/s. A ring of matter in the Galactic plane produces a rise the rotation curve. The rise expected from the $n = 3$ caustic ring of dark matter, by itself, is only of order 3%. However, the effect of a caustic ring of dark matter is amplified by the ordinary matter (stars, gas, dust ..) which it attracts gravitationally. The amplification would have to be by a factor of order nine in the case of the $n = 3$ ring. One may think, at first, that such a large amplification is implausible because the back reaction of the ring of ordinary matter upon the caustic ring of dark matter would determine the position of the latter, instead of the latter determining the position of the former. However this is not so because the dark matter caustic is not an overdensity of particles which are at rest with respect to the caustic ring. The particles which at a given time make up the caustic ring are moving with great speed, of order $360$ km/s for $n = 3$, and are continually replaced by new particles. As a result, the position of the caustic ring is insensitive to the gravitational field of the matter it attracts.

The existence of rings of ordinary matter precisely where the $n = 2$ and $n = 3$ rings of dark matter are predicted may yet be fortuitous. Fig. 1 does not show a significant rise near the predicted location (10 kpc) of the $n = 4$ caustic ring. Note however that the error bars in Fig. 1 are very large for $r > r_\odot$. The rise near 10 kpc, if indeed there is one, may be too small to show up in the data. On the other hand, the inner ($r < r_\odot$) part of the rotation curve is far better measured, and we may go look for rises there.

Galactic rotation curves are obtained from HI and CO surveys of the Galactic plane. A list of surveys performed to date is given in ref. [14]. Everything else being equal, CO surveys have far better angular resolution than HI surveys because their wavelength is nearly two orders of magnitude smaller (0.26 cm vs. 21 cm). The most detailed inner Galactic rotation curve appears to be that obtained [15] from the Massachusetts-Stony Brook North Galactic Plane CO survey [16]. It is reproduced in Fig. 2. It shows highly significant rises between 3 and 8.5 kpc. Eq. 1 predicts ten caustic rings between 3 and 8.5 kpc. Allowing for ambiguities in identifying rises, the number of rises in the rotation curve between 3 and 8.5 kpc is in fact approximately ten. Below 3 kpc the predicted rises are so closely spaced that they are unlikely to be resolved in the data.

Table I lists ten rises, identified by the radius $r_1$ where they start, the radius $r_2$ where they end, and the increase $\Delta v$ in rotation velocity. The rises are marked as slanted line
segments in Fig. 2. The evidence for the rise between 7.30 and 7.42 kpc is relatively weak, so the corresponding numbers are in parenthesis in the Table.

The effect of a caustic ring in the plane of a galaxy upon its rotation curve was analyzed in ref. [5]. The caustic ring produces a rise in the rotation curve which starts at \( r_1 = a_n \), where \( a_n \) is the caustic ring radius, and which ends a \( r_2 = a_n + p_n \), where \( p_n \) is the caustic ring width. The ring widths depend in a complicated way on the velocity distribution of the infalling dark matter at last turnaround [5] and are not predicted by the model. They also need not be constant along the ring.

In the past, rises (or bumps) in galactic rotation curves have been interpreted as due to the presence of spiral arms [17]. Spiral arms may in fact cause some of the rises in rotation curves. This does not, however, exclude the possibility of other valid explanations. Two properties of the high resolution rotation curve of Fig. 2 favor the interpretation that its rises are caused by caustics rings of dark matter. First, there are of order ten rises in the range of radii covered (3 to 8.5 kpc). This agrees qualitatively with the predicted number of caustic rings, whereas only three spiral arms are known in that range: Scutum, Sagittarius and Local. Second, the rises are sharp transitions in the rotation curve, both where they start \( (r_1) \) and where they end \( (r_2) \). Sharp transitions are consistent with caustic rings because the latter have divergent density at \( r_1 = a \) and \( r_2 = a + p \) in the limit of vanishing velocity dispersion. Finally, there are bumps and rises in rotation curves measured at galactocentric distances much larger than the disk radius, where no spiral arms are seen. In particular, the features found in the composite rotation curve constructed in ref. [9] are at distances 20 kpc and 40 kpc when scaled to our own galaxy.

The self-similar infall model prediction for the caustic ring radii, Eq. 1, was fitted to the eight rises between 3 and 7 kpc by minimizing \( \text{rmsd} \equiv \left[ \frac{1}{8} \sum_{n=7}^{14} (1 - \frac{a_n}{r_{1n}})^2 \right]^\frac{1}{2} \) with respect to \( j_{\text{max}} \), for \( \epsilon = 0.30 \). The fit yields \( j_{\text{max}} = 0.263 \) and \( \text{rmsd} = 3.1\% \). The fourth column of Table I shows the corresponding caustic ring radii \( a_n \). In Fig. 2 the latter are indicated by short vertical line segments.

The velocity increase due to a caustic ring is given by

\[
\Delta v_n = v_{\text{rot}} f_n \frac{\Delta I(\zeta_n)}{\cos \delta_n(0) + \phi_n'(0) \sin \delta_n(0)}.
\]

The \( f_n \), defined in ref. [3], are predicted by the self-similar infall model, but \( \Delta I(\zeta_n), \delta_n(0) \) and \( \phi_n'(0) \), defined in ref. [5], are not. Like the \( p_n \), the latter parameters depend in a complicated way on the velocity distribution of the dark matter at last turnaround. On the basis of the discussion in ref. [5], the ratio on the RHS of Eq. 2 is expected to be of order one, but to vary from one caustic ring to the next. The size of these fluctuations is easily a factor two, up or down. The fifth column of Table I shows \( \Delta v_n \) with the fluctuating ratio set equal to one, i.e. \( \bar{\Delta} v_n \equiv f_n v_{\text{rot}} \).

For the reasons just stated, the fact that the observed \( \Delta v \) fluctuate by a factor of order 2 from one rise to the next is consistent with the interpretation that the rises are due to caustic rings. However the observed \( \Delta v \) (column 3) are typically a factor 5 larger than the velocity increases expected from the caustic rings acting alone (column 5). This is similar to what we found for the \( n = 3 \) caustic ring, and suggests that the effects of the \( n = 5 \ldots 14 \) caustic rings are also amplified by baryonic matter they have accreted. I’ll argue that the
gas in the disk has sufficiently high density and low velocity dispersion to produce such large amplification factors. I’ll also give observational evidence in support of the hypothesis.

The equilibrium distribution of gas is:

$$d_{\text{gas}}(\vec{r}) = d_{\text{gas}}(\vec{r}_0) \exp\left[-\frac{3}{<v_{\text{gas}}^2>}(\phi(\vec{r}) - \phi(\vec{r}_0))\right],$$

(3)

where $d$ is density and $\phi$ gravitational potential. In the solar neighborhood, $d_{\text{gas}} \simeq 3 \times 10^{-24} \text{gr/ cm}^3$ [18], which is comparable to the density of dark matter inside the tubes of caustic rings near us. From the scale height of the gas [18] and the assumption that it is in equilibrium with itself and the other disk components, I estimate $<v_{\text{gas}}^2> \simeq 8 \text{ km/s}$. The variation in the gravitational potential due to a caustic ring over the size of the tube is of order $\Delta \phi_{\text{CR}} \simeq 2 f v_{\text{rot}}^2 p / a \simeq (5 \text{ km/s})^2$. Because $\frac{3}{<v_{\text{gas}}^2>}\Delta \phi_{\text{CR}}$ is of order one, the caustic rings have a large effect on the distribution of gas in the disk. The accreted gas amplifies and can dominate the effect of the caustic rings on the rotation curve. To check whether this hypothesis is consistent with the shape of the rises would require detailed modeling, as well as detailed knowledge on how the rotation curve is measured. However, there is observational evidence in support of the hypothesis.

The accreted gas may reveal the location of caustic rings in maps of the sky. Looking tangentially to a ring caustic from a vantage point in the plane of the ring, one may recognize the tricusp [5] shape of the $D_{-4}$ catastrophe. I searched for such features. The IRAS map of the galactic disk in the direction of galactic coordinates $(l, b) = (80^\circ, 0^\circ)$ shows a triangular shape which is strikingly reminiscent of the cross-section of a ring caustic. The relevant IRAS maps are posted at http://www.phys.ufl.edu/~sikivie/triangle/. They were downloaded from the Skyview Virtual Observatory (http://skyview.gsfc.nasa.gov/). The vertices of the triangle are at $(l, b) = (83.5^\circ, 0.4^\circ), (77.8^\circ, 3.4^\circ)$ and $(77.8^\circ, -2.6^\circ)$. The shape is correctly oriented with respect to the galactic plane and the galactic center. To an extraordinary degree of accuracy it is an equilateral triangle with axis of symmetry parallel to the galactic plane, as is expected for a caustic ring whose transverse dimensions are small compared to its radius. Moreover its position is consistent with the position of a rise in the rotation curve, the one between 8.28 and 8.43 kpc ($n = 5$). The caustic ring radius implied by the image is 8.31 kpc and its dimensions are $p \sim 130$ pc and $q \sim 200$ pc, in the directions parallel and perpendicular to the galactic plane respectively. It therefore predicts a rise which starts at 8.31 kpc and ends at 8.44 kpc, just where a rise is observed. The probability that the coincidence in position of the triangular shape with a rise in the rotation curve is fortuitous is less than $10^{-3}$.

In principle, the feature at $(80^\circ, 0^\circ)$ should be matched by another in the opposite tangent direction to the nearby ring caustic, at approximately $(-80^\circ, 0^\circ)$. Although there is a plausible feature there, it is much less compelling than the one in the $(+80^\circ, 0^\circ)$ direction. There are several reasons why it may not appear as strongly. One is that the $(+80^\circ, 0^\circ)$ feature is in the middle of the Local spiral arm, whose stellar activity enhances the local gas and dust emissivity, whereas the $(-80^\circ, 0^\circ)$ feature is not so favorably located. Another is that the ring caustic in the $(+80^\circ, 0^\circ)$ direction has unusually small dimensions. This may make it more visible by increasing its contrast with the background. In the $(-80^\circ, 0^\circ)$ direction, the nearby ring caustic may have larger transverse dimensions.

Our proximity to a caustic ring means that the corresponding flows, i.e. the flows in
which the caustic occurs, contribute very importantly to the local dark matter density. Using the results of refs. [3,5,2], we can estimate their densities and velocity vectors. Let us assume, for illustrative purposes, that we are in the plane of the nearby caustic and that its outward cusp is 55 pc away from us, i.e. \( a_5 + p_5 = 8.445 \) kpc. The densities and velocity vectors on Earth of the \( n = 5 \) flows are then:

\[
d^+ = 1.7 \times 10^{-24} \frac{\text{gr}}{\text{cm}^3}, \quad d^- = 1.5 \times 10^{-25} \frac{\text{gr}}{\text{cm}^3}, \quad \mathbf{v}^\pm = (470 \hat{\phi} \pm 100 \hat{r}) \frac{\text{km}}{\text{s}},
\]

where \( \hat{r}, \hat{\phi} \) and \( \hat{z} \) are the local unit vectors in galactocentric cylindrical coordinates. \( \hat{\phi} \) is in the direction of galactic rotation. The velocities are given in the (non-rotating) rest frame of the Galaxy. Because of an ambiguity, it is not presently possible to say whether \( d^\pm \) are the densities of the flows with velocity \( \mathbf{v}^\pm \) or \( \mathbf{v}^\mp \). The large size of \( d^+ \) is due to our proximity to the outward cusp of the nearby caustic. Its exact value is sensitive to our distance to the cusp. We do not know that distance well enough to estimate \( d^+ \) with accuracy. However we can say that \( d^+ \) is very large, of order the value given in Eq. 4, perhaps even larger. If we are inside the tube of the nearby caustic, there are two additional flows on Earth, aside from those given in Eq. 4. A list of local densities and velocity vectors for the \( n \neq 5 \) flows can be found in ref. [19]. An updated list is in preparation.

Eq. 4 has dramatic implications for dark matter searches. Previous estimates of the local dark matter density, based on isothermal halo profiles, range from 5 to 7.5 \( 10^{-25} \frac{\text{gr}}{\text{cm}^3} \). The present analysis implies that a single flow (\( d^+ \)) has of order three times (or more) that much local density.

The sharpness of the rises in the rotation curve and of the triangular feature in the IRAS map implies an upper limit on the velocity dispersion \( \delta v_{\text{DM}} \) of the infalling dark matter. Caustic ring singularities are spread over a distance of order \( \delta a \simeq \frac{R \delta v_{\text{DM}}}{v} \) where \( v \) is the velocity of the particles in the caustic, \( \delta v_{\text{DM}} \) is their velocity dispersion, and \( R \) is their last turnaround radius. The sharpness of the IRAS feature implies that its edges are spread over \( \delta a \lesssim 20 \) pc. Assuming that the feature is due to the \( n = 5 \) ring caustic, \( R \simeq 180 \) kpc and \( v \simeq 480 \) km/s. Therefore \( \delta v_{\text{DM}} \lesssim 53 \) m/s. This bound is strongly at odds with the often quoted claim that structure formation on small scales causes all, or nearly all, late infalling dark matter to be in dwarf galaxy type clumps, with velocity dispersion of order 10 km/s.

The caustic ring model may explain the puzzling persistence of galactic disk warps [20]. These may be due to outer caustic rings lying somewhat outside the galactic plane and attracting visible matter. The resulting disk warps would not damp away, as is the case in more conventional explanations of the origin of the warps, but would persist on cosmological time scales.

The caustic ring model, and more specifically the prediction Eq. 4 of the locally dominant flow associated with the nearby ring, has important consequences for axion dark matter searches [21], the annual modulation [19,22,23] and signal anisotropy [24,23] in WIMP searches, the search for \( \gamma \)-rays from dark matter annihilation [25], and the search for gravitational lensing by dark matter caustics [6,26]. The model makes predictions for each of these approaches to the dark matter problem.

The use of NASA’s Skyview facility is gratefully acknowledged. I would like to thank Fred Hamann, Achim Kempf, Will Kinney, Heidi Newberg, Stuart Wick and the members of the Axion Dark Matter Search collaboration for valuable comments. Finally I thank the
Aspen Center for Physics for its hospitality while this paper was being written. This work is supported in part by U.S. DOE grant DEFG05-86ER-40272.
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TABLE I. Radii at which rises in the Milky Way rotation curve of Fig. 2 start ($r_1$) and end ($r_2$), the corresponding increases in velocity $\Delta v$, the caustic ring radii $a_n$ of the self-similar infall model for the fit described in the text, and typical velocity increases $\bar{\Delta}v_n$ predicted by the model without amplification due to accretion of ordinary matter onto the caustic rings.

| $r_1$ (kpc) | $r_2$ (kpc) | $\Delta v$ (km/s) | $a_n$ (kpc) | $\bar{\Delta}v_n$ (km/s) |
|-------------|-------------|-------------------|-------------|-------------------------|
| 41.2        | 26.5        | 12                | 10.5        | 5.0                     |
| 20.5        | 10.6        | 13.9              | 6.8         |                         |
| 10.5        | 5.0         |                   |             |                         |
| 8.28        | 8.43        | 12                | 8.50        | 3.9                     |
| (7.30)      | (7.42)      | (8)               | 7.14        | 3.2                     |
| 6.24        | 6.84        | 23                | 6.15        | 2.6                     |
| 5.78        | 6.01        | 9                 | 5.41        | 2.3                     |
| 4.91        | 5.32        | 15                | 4.83        | 2.0                     |
| 4.18        | 4.43        | 8                 | 4.36        | 1.7                     |
| 3.89        | 4.08        | 8                 | 3.98        | 1.5                     |
| 3.58        | 3.75        | 6                 | 3.66        | 1.3                     |
| 3.38        | 3.49        | 14                | 3.38        | 1.2                     |
| 3.16        | 3.25        | 8                 | 3.15        | 1.1                     |
FIG. 1. Milky Way rotation curve from ref. [13]. The different lines represent the contributions from the bulge (dotted), the stellar disk (filled circles), the HI layer (crosses), the H$_2$ layer (circles), and from a smooth dark halo (dashed). The full line represents the sum of the contributions. Reprinted by permission of the authors and Blackwell Publishing Ltd.
FIG. 2. North Galactic rotation curve from ref. [15]. The locations of the rises listed in the first three columns of Table I are indicated by line segments parallel to the rises but shifted downwards. The caustic ring radii for the fit described in the text are shown as vertical line segments. The position of the triangular feature in the IRAS map of the galactic plane near 80° longitude is shown by the short horizontal line segment. It coincides with a rise in the rotation curve.