How much is enough?:
Data requirements for statistical NLP

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Abstract
Feeding training data to statistical representations of language has become a popular pastime for computational linguists, but our understanding of what constitutes a sufficient volume of data remains shadowy. For example, Brown et al. (1992) used over 500 million words of text to train their language model. Is this enough? Could devouring even more data further improve the accuracy of the model learnt? In this paper I explore a number of issues in the analysis of data requirements for statistical NLP systems. A framework for viewing such systems is proposed and a sample of existing works are compared within this framework. Finally, the first steps toward a theory of data requirements are made by establishing an upper bound on the expected error rate of a class of statistical language learners as a function of the volume of training data.

1 Introduction
Statistical approaches to natural language processing are becoming increasingly popular, being applied to a wide variety of tasks. For example, Weischedel et al. (1993) explores part-of-speech tagging, parsing and acquisition of lexical frames. Nonetheless, all these tasks share some important characteristics, not the least of which is the requirement for a sizable corpus of training data. One question which has largely been ignored is how much data is enough? For example, given a limited body of training data, it is essential to know which statistical NLP methods are likely to be accurate before pursuing any one. Also, given a particular method, when will acquiring further training data cease to improve the system accuracy? Currently, the field is conspicuously lacking a general theory of data requirements for statistical NLP.

In this paper, I present the first steps towards the development of such a theory. I begin by formulating a framework for statistical NLP systems designed to capture some of the elements crucial to data requirements analysis. I will then review a sample of existing approaches, showing how they fit into the framework, and where they vary from it. Even though several of these introduce complexities which are not captured by the framework, reasoning in the framework still supports some important insights into these systems. Finally, I present some preliminary work on establishing a closed form upper bound on data requirements for a class of statistical NLP systems. This latter work owes much to Mark Johnson, Brown University, who is responsible for several key mathematical ideas in Section 4.

Statistical NLP systems are designed to make choices; hopefully in an informed manner. To do this they use indicators, upon which their choices are conditioned. The purpose of computing statistics is to inductively establish the relationship between the indicators and the choices to be made. Consider for example a next word predictor which attempts to predict the next word on the basis of the preceding word. To do this it must have an understanding of the relationship between the indicator (the preceding word) and the choice...
(the next word). It is possible to acquire this understanding by computing statistics over a large corpus, a process called training. Once trained, the system may then be applied to a new text and its accuracy evaluated. This paper is concerned with the dependence of a system’s accuracy on the size of the training corpus. In the following section, the notions of indicators, choices and training data will be made more formal.

2 Statistical Processors

2.1 A Framework

A statistical NLP system deals with a certain linguistic universe. Formally, there is a set of linguistic events $\Omega$ from which every training example and every test instance will be drawn. In the next word predictor, this need only be the set of all pairs of words which may be adjacent in text.

Let $V$ be a finite set of values that we would like to assign to a given linguistic input. This defines the range of possible answers that the analyser can make. In the predictor, this is the set of words plus an end of sentence symbol. Let $J : \Omega \rightarrow V$ be the random variable describing the distribution of values that linguistic events take on. We also require a set of indicators, $B$, to use in selecting a value for a given linguistic event. I will refer to each element of $B$ as a bin. In the predictor, the set of bins is the set of words plus a start of sentence symbol. Let $I : \Omega \rightarrow B$ be the random variable describing the distribution of bins into which linguistic events fall.

The task of the analyser is to choose which value is most likely given only the indicator. Therefore, it is a function $A : B \rightarrow V$. The task of the learning algorithm is to acquire this function by computing statistics on the training set.

Putting these components together, we can define a Statistical Processor, $S$ as a tuple $\langle \Omega, B, V, A \rangle$, where:

- $\Omega$ is the set of all possible linguistic events
- $B$ and $V$ are finite sets, the bins and values respectively
- $A$ is the trained analysis function

Amongst all such statistical processors, there is a special class in which we are interested. Define a probabilistic analyser to be a statistical processor which computes a function $p : B \times V \rightarrow [0, 1]$ such that $\sum_{v \in V} p(b, v) = 1$ for all $b \in B$ and then computes $A$ as:

$$A(b) = \arg\max_{v \in V} p(b, v)$$

The problem of acquiring $A$ is thus transformed into one of estimating the function $p$ using the training corpus. Generally, $p(b, v)$ is viewed as an estimate of the probability $\text{Pr}(J = v | I = b)$.

2.2 Training Data

Formally, a training corpus, $c$, of $m$ instances, is an element from $(B \times V)^m$ where each pair $(b, v)$ is sampled according to the random variables $I$ and $J$ from $\Omega$. For probabilistic analysers, there are a variety of methods by which an appropriate function $p$ can be estimated from a corpus; one simple example being the Maximum Likelihood Estimate. Regardless of the learning algorithm used, each possible training corpus, $c$, results in the acquisition of some function, $P_c$. Our aim is to explore the dependence of the expected accuracy of $P_c$ on the magnitude of $m$.

Surprisingly, it is not always obvious how many training instances have been used to train a statistical method. It is not generally sufficient to report the size of the corpus in words. A system which collects word associations using a window of cooccurrence 10 words wide will find 819 instances in a 100 word corpus, while one collecting the objects of the preposition on from the same corpus, would most likely find only a few instances. Therefore, before any conclusions can be drawn about data requirements, the training corpus must be measured in terms of instances.

Each of these instance falls into a particular bin by virtue of its associated indicator. In choosing the indicators, we have implicitly defined equivalence classes for instances. The statistical processor will treat every instance in a bin identically. Further, once the bins are chosen, the greater the number of training instances that fall into a bin, the greater our confidence in the statistical inference made by
the processor for test cases in that bin. For instance, the next word predictor is more likely to be correct when the preceding word is common than when it is a rare word.

It is not always obvious how many bins a given statistical method employs. Often multiple indicators are used. For instance, a trigram tagger uses the tags of the two preceding words and the current word to choose a new tag. In this case, \( B = T \times T \times W \) where \( T \) is the tagset and \( W \) is the vocabulary.

This example demonstrates an important point. By choosing to take into account the tags of two preceding words, the trigram tagger requires \(|T| \) times as many bins as a bigram tagger (where \( B = T \times W \)). With more bins, the trigram tagger is sensitive to a broader range of context and thus can in principle achieve a greater accuracy. However, because there are more bins, there are fewer training instances in each bin. Thus, statistical estimation will be less accurate. In practice, high accuracy requires at least a few training instances per bin. Thus increasing the number of indicators may actually decrease the overall accuracy.

For probabilistic analysers it is useful to define the number of slots, \( L \), to be \(|B|(|V| - 1)|\), which is the number of independent parameters needed to define the function \( p \).

### 2.3 Error Rates and Optimality

For any non-trivial general statistical processor the indicators used cannot perfectly represent the entire linguistic event space. Thus, in general there exist values \( v_1, v_2 \in V \), for which both \( \text{Pr}(J = v_1 | I = b) > 0 \) and \( \text{Pr}(J = v_2 | I = b) > 0 \) for some \( b \in B \). Suppose without loss of generality that \( A(b) = v_1 \). The analyser will be in error with probability at least \( \text{Pr}(J = v_2, I = b) \). This is the root of a rather difficult problem in statistical NLP because no matter how inaccurate a trained statistical processor is, the inaccuracy may be due to the imperfect representation of \( \Omega \) by \( B \).

Probabilistic analysers always select just one value for each bin, the one which maximises \( p \). Let \( v_{\text{mode}}(p, b) = \arg \max_{v \in V} p(b, v) \). This leads to a definition for the expected error rate of a function \( p \), \( R(p) \):

\[
R(p) = \sum_{b \in B} \text{Pr}(I = b) \left( \sum_{v \in V \setminus \{v_{\text{mode}}(p, b)\}} \text{Pr}(J = v | I = b) \right)
\]

This is the probability of the analyser being in error on a randomly selected element of \( \Omega \). Let \( p_{\text{opt}} \) be any function which minimises the expected error rate and \( r_{\text{opt}} = R(p_{\text{opt}}) \). Given \( B \) and \( V \), \( r_{\text{opt}} \) is the smallest possible expected error rate. Any probabilistic analyser which achieves an accuracy close to this is unlikely to benefit from further training data.

Unless large volumes of manually annotated data exist, measuring the size of \( r_{\text{opt}} \) in any given statistical processor presents a difficult challenge. Hindle and Rooth (1993) have attempted a similar task using human subjects on the problem of prepositional phrase attachment. Subjects were given only the preposition and the preceding verb and noun and then were asked to select the attachment. This was precisely the task facing their statistical processor. The subjects could only perform the attachment correctly in around 86% of cases. If we assume that the subjects incorrectly analysed the remaining 14% of cases because these cases depended on knowledge of the wider context, then any statistical learning algorithm based only on these indicators cannot do better than 86%. Of course, if there is insufficient training data the system may do considerably worse.

Assuming that human performance on the task accurately reflects the value of \( r_{\text{opt}} \) is the only means known at present to estimate the value of \( r_{\text{opt}} \). Unfortunately, this approach is expensive to apply and makes a number of questionable psychological assumptions. For example, it assumes that humans can accurately reproduce parts of their language analysis behaviour on command. It may also suffer when representational aspects of the analysis task cannot be explained easily to experimental subjects. A worthwhile goal for future research is to establish a statistical method for estimating or bounding \( r_{\text{opt}} \) using language data.

### 3 Statistical Learning

\footnote{Unless a more accurate statistical processor based on the same indicators already exists.}
3.1 Existing Methods

In this section, I show how a number of existing statistical NLP works fit into the framework, including a tagger, a sense disambiguator and three syntactic analysers. For each, I consider how the various elements of the general statistical processor are instantiated.

Weischedel et al. (1993) uses (among other experiments) a trigram hidden Markov model to tag text for part of speech. The training data is four million words of the University of Pennsylvania Treebank, tagged with a set of 47 different tags. I shall regard $B$ as consisting of the two previous tags ($T \times T$), while $V$ is simply the tagset. The system also takes into account lexical tag frequencies (that is, $B = T \times T \times W$). I will assume however that data sparseness does not affect the lexical tag frequency estimates. Since the trigram estimates and the lexical tag frequencies are combined as independent factors, ignoring the lexical component does not seem unreasonable. The situation is further complicated because probability is maximised over a sequence of words, rather than for a single word. The framework needs to be extended to capture these mechanisms, but for the moment the approximations I have made may be useful. Since every word in the corpus (bar the first two) is used for training, we have $m = 4$ million and $L = 47 \times 47 \times 46$. The accuracy is reported to be around 97%, which is approximately the accuracy of human taggers using the whole context.

Yarowsky (1992) describes a sense disambiguation system which uses a 100 word window of cooccurrences. He uses a mutual information-like measure which combines the cooccurrence statistics for all words in each category of Roget’s thesaurus. The result is a profile of contexts for a category which can be used to estimate how likely each category is within a certain context. Comparing the different possible categories for the word provides a broad sense discrimination. The training corpus is Grolier’s encyclopedia which contains on the order of 10 million words. Each of these provides 100 training instances (every other word in the window), so $m \approx 1$ billion. Since the evidence from each word in the context is combined independently, it is reasonable to regard $B$ as simply the set of distinct words in Grolier’s. Again, further work is needed to make this approximation unnecessary. $V$ is the set of Roget’s categories ($|V| = 1042$), so assuming the vocabulary is around 100,000, $L \approx 100$ million. The average accuracy reported is 92%.

Hindle and Rooth (1993) propose a system to syntactically disambiguate prepositional phrase attachments. Unambiguous examples of attachments are used to find lexical associations (a likelihood ratio) between prepositions and the nouns or verbs they attach to. They cyclically apply this technique, adding disambiguated attachments into the training set, until all the training data (ambiguous or not) has been used. This approach can be approximated by a probabilistic analyser. Each association value is ascribed to a pair $(w, p)$ where $w$ is a verb or noun and $p$ is a preposition. Thus $B$ is a product of two indicator spaces: the set of verbs and nouns and the set of prepositions. Assuming they used 10,000 nouns and verbs (5,000 of each) and 100 prepositions, $|B| = 1$ million. The analyser computes a probability for each of two possible attachments, nominal and verbal, so $V$ is binary. The training set consists of 754,000 noun attachments and 468 thousand verb ones giving $m = 1.22$ million. The accuracy reported is close to 80%, while human subjects given the same indications could achieve 85–88% accuracy. If the latter figure reflects the optimal error rate, it appears there is still room for improvement by adding training data or changing the statistical measures.

Lauer (1994) describes a system for syntactically analysing compound nouns. Two-word compounds extracted from Grolier’s encyclopedia were used to measure mutual information between every pair of thesaurus categories (using Roget’s thesaurus) and the results used to select a bracketing for three-word compounds. Since an association value is computed for every pair of thesaurus categories, $|B|$ is equal to $1043 \times 1043$. There are only two possible bracketings to choose from, so again $V$ is binary. The training corpus consists of about 35,000 two-word compounds, giving $m = 35,000$ and $L \approx 1$ million. The accuracy reported is 75%.

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3 Some stemming is performed, so it is the number of stems in the vocabulary that we want.

4 I have allowed all the training examples as instances, even though some are acquired by cyclic refinement.
Resnik and Hearst (1993) aim to enhance Hindle and Rooth’s (1993) work by incorporating information about the head noun of the prepositional phrase in question. Thus $B$ is now a product of three spaces: the set of nouns and verbs, the set of prepositions and the set of nouns. To reduce the data requirement, a freely available on-line thesaurus, called WordNet is used (Beckwith et al., 1991). WordNet groups words into synsets, categories of synonymous words. These synsets are arranged in a taxonomy, so that every word is also provided with a list of hypernyms. The system then adds together the frequency counts for nouns within a synset, providing more data about each. This reduces the number of bins, since it is the synsets which are taken as indicators rather than individual words. If we assume roughly 1000 synsets, 1000 verbs and 100 prepositions, then $|B| = 200$ million. $V$ is still binary. Their training corpus is an “order of magnitude smaller than” Hindle and Rooth’s, so $m$ is around 100,000. Unlike Hindle and Rooth’s, their corpus is parsed, which should give better results. Interestingly, they combine evidence from large groups of synsets within WordNet’s hypernym hierarchy using a t-test. This causes the effective number of synsets for nouns to be reduced, perhaps by as much as a factor of 10 (thus $|B| \approx 20$ million). I will therefore assume that $L \approx 20$ million. Even given the additional information about the head noun of the prepositional phrase, the accuracy reported fails to improve on that of Hindle and Rooth, being 78%. It is possible that insufficient training data is the cause of this shortfall.

Table 1 shows a summary of the above systems, ordered on the ratio $m : L$. A strong correlation is evident between the value of this ratio and the success rate. This suggests that the success of a statistically based system is strongly dependent on the confidence permitted by the training set size as measured by this ratio.

3.2 An Important Trade-off

The model formulated above and the empirical data presented support a number of qualitative inferences about the potential of systems given a fixed training set size. Because training data will always be limited, such reasoning is an important part of system design. Therefore before turning to some quantitative analyses, I will examine a few such inferences.

The most important of these is in regard to linguistic sophistication, that is the degree to which the system uses knowledge of the patterns of language. This kind of knowledge is extremely important, since it often allows just the right distinctions to be made. More simplistic systems will inevitably assign one choice to two different inputs because their linguistic knowledge fails to support a distinction. Therefore, it seems desirable to incorporate as much linguistic sophistication as possible. While this is a tempting direction to take for improving system performance, there is a barrier.

Consider, for instance, the effect on data requirements of incorporating new indicators. Each indicator increases the number of distinctions which the system can make. For example, Resnik and Hearst (1993) take into account the object of the preposition. In doing so, they distinguish cases which Hindle and Rooth (1993) did not. As a result, the number of cases their system considers is substantially larger than those considered by Hindle and Rooth’s. In terms of the framework, Resnik and Hearst have many more bins than Hindle and Rooth.

It is easy to see that incorporating a new indicator increases the number of bins combinatorially. The size of $B$ is multiplied by the range of the new indicator. This results in the ratio $m : L$ falling by the same factor, which, as I have argued above, can be detrimental to the overall accuracy.

The situation is worse still if the training set is not hand annotated. In this case, introducing the new indicator creates additional ambiguity in the training set since the value of the new indicator must be determined for each training example. This effectively decreases the number of training instances resulting in a further decrease in $m : L$.

Thus, linguistic sophistication presents a trade-off between accuracy and data sparseness. It is a balance between poor modeling of the language and insufficient data for accurate statistics. If we are to strike a satisfactory compromise, we need a strong theory.

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5 Brackets indicate measured on different data and/or under different conditions.
6 Reported in Resnik (1993).
7 Reported in Dras and Lauer (1993).
of data requirements and ways to make more economic use of data.

One such method is termed conceptual association, as defined in Resnik and Hearst (1993). By collecting statistics based on concepts rather than individual words, the number of bins is usually reduced. The idea is to generalise findings about words to cover other words which have the same meaning. The advantages of this approach are extensively argued in Resnik (1993) and the method is used in Lauer (1994). While concepts can help, the ambiguity introduced (namely what concept does a given word belong to) may undermine the increased accuracy. Further work is needed to establish the effects on data requirements of employing this strategy.

A novel extension to this approach that has not yet been employed, would be to collect statistics at various levels of granularity. Statistics computed on counts of individual words would provide fine sensitivity, while statistics computed on counts of a small set of semantic primitives (such as ANIMATE, ABSTRACT, etc.) would provide the coarsest evidence. As many levels as desired between these two extremes could be employed in this way. The level used to make each choice could then be selected according to the degree of confidence available at each level. If insufficient data has been seen to allow a confident selection at one level, a coarser grained level would be tried. Resnik and Hearst (1993) seem to be simulating this when they perform a t-test across all levels of the WordNet hierarchy.

4 First Steps Towards a Theory

4.1 A Simple Learning Scheme

In this section I shall establish some lower bounds on the accuracy of a simple training scheme within the framework developed. The mathematics presented in Sections 4.2 through 4.3 was for the most part developed by Mark Johnson of Brown University and completed by the author. I wish to thank him for his many communications in this regard.

Let \( t(b, c) = \{(b, v)|v \in V, (b, v) \in c\} \), the training instances in a corpus \( c \) that fall into bin \( b \).

Let \( f(v, t) = |\{(b, v)|b \in B, (b, v) \in t\}| \), the frequency of the value \( v \) in the set of training instances, \( t \).

Let \( \text{mode}(b, c) = \arg\max_{v \in V} f(v, t(b, c)) \), the most common value for instances from a corpus \( c \) in a bin \( b \). Where several values have equal frequencies, one should be chosen at random.

Define the learning algorithm such that:

\[
P_c(b, v) = \begin{cases} 
1 & \text{if } v = \text{mode}(b, c) \\
0 & \text{otherwise}
\end{cases}
\]

Since each bin has only one value with non-zero probability, \( V \) is effectively a binary set (either the instance has the non-zero value or it does not). Thus, \( L = |B| \). Notice also that the value assigned highest probability by \( P_c \) is the one most frequently falling into the bin. That is, \( v_{\text{mode}}(P_c, b) = \text{mode}(b, c) \).

Two possible cases arise when the analyser is faced with making a decision on the basis of some indication, \( b \). Either the corpus contains no occurrences of \((b, v)\) for any value \( v \in V \) (Case A) or there is some training data which falls into the bin (Case B).

4.2 Empty Bins

Case A arises when none of the training instances fall into the bin. Let \( p_b \) denote

| System                | Training Source          | \( m \) | \( L \) | \( m : L \) | Accuracy | Humans     |
|-----------------------|--------------------------|---------|-------|-----------|----------|------------|
| Weischedel et al.     | Manual Supervision       | 4M      | 100k  | 40        | 97%      | \( \leq 97\% \) 5 |
| Yarowsky              | Unsupervised             | 1G      | 100M  | 10        | 92%      | -          |
| Hindle & Rooth        | Automatic Supervision    | 1.2M    | ~1M   | ~1        | 80%      | 85–88%     |
| Lauer                 | Automatic Supervision    | 35k     | 1M    | 0.035     | 75%      | \( \leq 80\% \) 6 |
| Resnik & Hearst       | Manual Supervision       | 100k    | \( \geq 20M \)  \( \leq 0.005 \) | 78%      | \( \leq 92\% \) 7 |

Table 1: Summary of a sample of statistical NLP systems
Pr(I = b). The probability of bin b being empty after training on a randomly selected corpus is \((1 - p_b)^m\). Thus the probability over all test inputs of there being no training data in the bin for that input is:

\[
w = \sum_{b \in B} p_b (1 - p_b)^m
\]

Since we know that \(\sum_{b \in B} p_b = 1\), it is possible to show that the maximum for \(w\) occurs when \(\forall b \in B\) \(p_b = \frac{1}{|B|}\). Therefore:

\[
w \leq (1 - \frac{1}{|B|})^m \leq e^{-m/|B|}
\]

So for even quite small values of \(m/|B|\), the probability that any given test sample falls into a bin for which we received no training samples is very low. For example, when \(m/|B| \geq 3\), they occur in less than 5% of inputs.

### 4.3 Non-empty Bins

In Case B, we have at least one instance in the corpus for the given bin. Let \(n = |t(b, c)| \geq 1\). An optimal function, \(v_{opt}\), will be one which chooses for bin \(b\) the value \(v\) that maximises \(Pr(J = v|I = b)\). Let \(v_{opt}(b) = \arg\max_{v \in V} Pr(J = v|I = b)\), the most likely value in bin \(b\). Let \(q(b) = Pr(J = v_{opt}(b)|I = b)\), the probability of this value given an instance in bin \(b\). Notice from equation (2) that the expected error rate is minimised when \(\forall b \in B\) \(v_{mode}(p, b) = v_{opt}(b)\). Therefore:

\[
r_{opt} = \sum_{b \in B} Pr(I = b)(\sum_{v \in V\{v_{opt}(b)\}} Pr(J = v|I = b))
\]

\[
= \sum_{b \in B} Pr(I = b)(Pr(J \neq v_{opt}(b)|I = b))
\]

\[
= \sum_{b \in B} Pr(I = b)(1 - q(b))
\]

Since \(r_{opt}\) is the best possible error rate, it follows that \(q(b)\) must be high for most bins if the system is to work at all. Therefore, \(v_{opt}(b)\) should be a frequent value in each bin. Now if more than half of the instances in a bin have the value \(v_{opt}\), then this must be the most common value in the bin. Thus, if \(f(v_{opt}(b), t(b, c)) > \frac{n}{2}\), then \(mode(b, c) = v_{opt}(b)\). So by computing the probability of \(f(v_{opt}(b), t(b, c)) > \frac{n}{2}\), we can obtain a lower bound for the accuracy on bin \(b\).

\[
Pr(v_{opt}(b) = v_{mode}(p, b)|I = b)
\]

\[
= Pr(mode(b, c) = v_{opt}(b)|I = b)
\]

\[
\geq \sum_{i = \frac{n+1}{2}}^{n} Pr(f(v_{opt}(b), t(b, c)) = i|I = b)
\]

\[
= \sum_{i = \frac{n+1}{2}}^{n} \binom{n}{i} (1 - q(b))^{n-i} q(b)^i
\]

Thus:

\[
\sum_{v \in V\{v_{mode}(p, b)\}} Pr(J = v|I = b)
\]

\[
= 1 - Pr(J = v_{mode}(p, b)|I = b)
\]

\[
\leq 1 - q(b) \sum_{i = \frac{n+1}{2}}^{n} \binom{n}{i} (1 - q(b))^{n-i} q(b)^i
\]

since

\[
Pr(J = v_{mode}(p, b)|I = b)
\]

\[
\geq Pr(J = v_{opt}(b)|I = b)
\]

\[
Pr(v_{opt}(b) = v_{mode}(p, b)|I = b)
\]

Let \(U_n(b) = 1 - q(b) \sum_{i = \frac{n+1}{2}}^{n} \binom{n}{i} (1 - q(b))^{n-i} q(b)^i\)

This is an upper bound on the expected error rate for bin \(b\). So:

\[
R(P_c) \leq \sum_{b \in B} Pr(I = b)U_n(b)
\]

As noted above \(r_{opt} = \sum_{b \in B} Pr(I = b)(1 - q(b))\). A comparison between the upper bound \(U_n(b)\) and the optimal error rate \(1 - q(b)\) shows that for reasonably high values of \(q(b)\) that \(U_n(b)\) is close to \(1 - q(b)\). For example, when \(q(b) \geq 0.9\), \(U_5(b) \leq 1.26(1 - q(b))\) and \(U_5(b) \leq 1.08(1 - q(b))\).

\(^*\)The argument shown holds for all odd \(n\). A variation of the argument that bounds the expected accuracy for all even \(n\) is simple to construct.

\(^\dagger\)For odd \(n\). When \(n\) is even it can be shown that is \(U_{n-1}(b)\) is an upper bound.
In fact, we can derive a bound for any \( n \) as follows:

\[
U_n(b) \leq U_1(b) = 1 - q(b) \Rightarrow (1 + q(b))(1 - q(b)) \leq 2(1 - q(b)) = 2 \Pr(J \neq v_{opt}(b)|I = b)
\]

Thus in all bins which have training instances in the corpus, the expected error rate for the bin never exceeds twice the optimal error rate for that bin. It is interesting to note that Hindle and Rooth’s (1993) system has roughly one instance per bin and an optimal error rate of 12% (assuming the human accuracy of 88% is optimal), so that equation (3) predicts a lower bound of 77% accuracy. [3]

When 5 instances from the training corpus fall into the bin, the expected error rate approaches the optimal error rate closely and when there are an average of 3 instances per bin, very few bins do not have instances from the training corpus. So, in general it appears that 3–5 instances per bin will be sufficient.

4.4 Skewed Bins

An obvious question is why systems such as Lauer (1994) and Resnik and Hearst (1993) work at all given that far less than one instance is expected for each slot. One possible answer is that different bins have widely differing frequencies. The system quickly learns about the most frequent cases at the expense of less frequent ones.

This can be modeled by considering the different distributions, \( p_b \) defined for Case A above. In that analysis, the probability of encountering an empty bin was maximised over all possible distributions. However, if something is known about the distribution, in principle a tighter bound is possible. For example, suppose some fraction of the bins have very low probability. That is, \( \exists B' \subseteq B \) such that \( \sum_{b \in B'} p_b = c \) for some small \( c \). Let \( B'' = B \setminus B' \) and \( \beta = \frac{|B''|}{|B|} \). Now:

\[
w = \sum_{b \in B} p_b(1 - p_b)^m \leq \sum_{b \in B''} p_b(1 - p_b)^m \leq c + \sum_{b \in B'} p_b(1 - p_b)^m \leq c + \sum_{b \in B''} p_b(1 - p_b)^m \leq c + \sum_{b \in B'} p_b(1 - p_b)^m
\]

\[
= \sum_{b \in B''} p_b(1 - p_b)^m + \sum_{b \in B'} p_b(1 - p_b)^m
\]

Now the second term is maximised when \( \forall b \in B'' p_b = \frac{1}{|B''|} = \frac{1}{|B'|} \). So letting \( \beta_c = \frac{1}{\beta} \),

\[
w \leq c + (1 - \frac{c}{\beta_c|B'|})^m \leq c + (1 - \frac{1}{\beta_c|B'|})^m \leq c + e^{-m/\beta_c|B'|}
\]

So knowing a pair of values \( c \) and \( \beta \) is a useful, if primitive, means of lowering the upper bound on data requirements. Since the distribution of bins does not depend on the values we are seeking to learn, it should be possible to develop simple techniques for estimating values of \( c \) and \( \beta \).

4.5 Future Work

A great deal of work remains to be done. I will mention only a few directions where the work begs to be extended. First, the mathematical model doesn’t capture several aspects of existing models, such as maximising probabilities over sequences of words and combining evidence from multiple sources. Second, the simple learning algorithm presented differs from those used in practice in several ways. It would be useful to explore the relationship between the algorithm I have proposed and others in existing statistical methods (for example, the backoff method in Katz, 1987). Third, smoothing is frequently used to alleviate data sparseness (see Dagan et al., 1993), but the model does not include any means to represent the process of smoothing. Finally, almost all statistical NLP systems deal with some noise in the training data. This is especially important in systems like Yarowsky (1992) where training is unsupervised. The mathematical results need to be extended to reflect noisy training data and to support reasoning about the sensitivity of data requirements to noise.

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\[\text{I wish to thank Eugene Charniak for pointing out this fact.}\]
5 Conclusion

In this paper I have indicated the lack of a general theory of data requirements within the field of statistical NLP. As a first step in the development of such a theory I have presented a framework for statistical NLP systems. I have shown how several prominent works in the field fit this model and demonstrated a number of mathematical results which support inferences about data requirements. I believe this represents a significant first step along the road to a better understanding of when and how statistical NLP methods can be applied.

6 Acknowledgments

Without Mark Johnson’s interest and collaboration, this paper would not exist. Many of the key ideas originate with him and I am indebted to him for his patience and attention. The work has also benefited from assistance from Richard Buckland, Robert Dale, Mark Dras, Mike Johnson and Steven Sommer. Financial support is gratefully acknowledged from the Australian Government and the Microsoft Institute.

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