NONLINEAR STRESS ANALYSIS
FOR CFRP-SHEET-BONDED STEEL PLATES
UNDER UNIAXIAL TENSILE LOADING

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To clarify the mechanical behavior, peeling mechanism, and peeling strength of the adhesion layer inserted between a steel member and a carbon fiber-reinforced plastic (CFRP) sheet on reinforced steel structures, this study established a nonlinear theoretical analysis method with a CFRP-sheet-bonded steel plate under uniaxial tensile loading. The proposed analysis method considered the nonlinear material properties of all members of the analytical model. The material model of the adhesion layer was constructed based on the relationship between shear stress and relative displacement. To verify the accuracy of the proposed method, 2D geometric nonlinear finite element method (FEM) analyses were implemented. Comparison of the results of the theoretical and FEM analyses confirmed that accurate evaluation of the peeling strength of the analytical model and the mechanical behavior of all members is possible.

Key Words : nonlinear theoretical analysis, peeling mechanism, peeling strength, adhesion layer, CFRP

1. INTRODUCTION

The applicability of carbon fiber-reinforced plastic (CFRP) as a material for repairing and strengthening aging or damaged structures has been intensively investigated worldwide, especially in the USA since the 1970s1), and in Japan and China since the 1980s2). CFRPs in sheet form are usually used to repair and strengthen steel structures with reduced load-carrying capacity due to natural hazards, human errors, corrosion damage, fatigue damage, and the changes in the purpose of the structure utility.

However, the application of CFRP sheets to steel structures for corrosion and fatigue damage is the most common one. To maximize the effectiveness of repair and reinforcement methods using CFRP sheets and to prevent the peeling failure between the CFRP layers and the steel member under large deformations such as buckling; polyurea putty3) with a low elastic modulus (55 MPa–75 MPa) and high elongation (300%–500%) is usually inserted between the steel member and the CFRP sheet. Further, in many cases, peeling failure, specifically adhesive failure, occurs on the interface between the adhesion layer and steel members, although these steel members are still within the elastic range. Therefore, the maximum strength of repaired or reinforced steel members is often determined by the peeling failure of this adhesion layer. For this reason, to perform the most effective repair and reinforcement, it is necessary to understand the mechanical behavior, actual peeling mechanism, and peeling strength of the adhesion layer inserted between the steel member and CFRP.

In previous studies, Hart-Smith (1973)4), 5) developed a theoretical analysis of the adhesive-bonded double-lap joint using the elastic-plastic analytical model of the adhesion layer from the classical elastic analysis of Volkersen (1938)6). In these studies, the material model of the adhesion layer was constructed using the stress–strain relation. Further, Kamiharako et al. (1999)7) proposed a theoretical analysis to simulate bonding and the peeling behavior of the adhesion layer under the double-strap joint of concrete bonding the continuous fiber (CF) sheet with the material constitutive rule of the adhesion layer as the bilinear model between shear stress and relative
of the proposed method, a two-dimensional geomet-

tions (the bond relation between shear stress and relative displace-

members of the analytical model (Fig. 1), whereby the primary Young’s modulus is $E_{s1}$ and the secondary modulus after yield is $E_{s2}$. The employed CFRP in sheet form is light and has a large tensile strength. The material model is shown in Fig. 3(b). The CFRP sheet has an elastic modulus that is 1.2–3.2 times higher than that of steel. In addition, the elastic modulus of the adhesion layer was smaller by two orders of magnitude than that of steel and the CFRP sheet; hence, under uniaxial tensile loading, the adhesion layer was considered only shear stress, and the steel plate and CFRP sheet was considered only tensile stress. As a bonding constitution rule, the bond–slip model of the adhesion layer was constructed based on the shear stress–relative displacement relationship. From the data directly tested in the laboratory, numerous existing research studies identified a simple bilinear bond–slip relationship of the adhesion layer as shown in Fig. 3(c). The bilinear bond–slip relationship was defined by three key points: the initial point (0, 0), the peak shear stress point, and the ultimate point.

In Fig. 2, $t_s$, $t_{cf}$, and $h$ are the thicknesses of the steel plate, CFRP sheet, and the adhesion layer, respectively; $L$ is the length of the original area bonding the CFRP sheet; $b$ is the width of the steel plate and CFRP sheet; $\sigma_{st}$ is the stress at the top of the steel plate, in which the CFRP sheet is not bonded; $\sigma_{s}$, $\sigma_{sY}$, and $\tau$ are the axial stress values of the steel plate in the areas bonding the CFRP sheet, the axial stress of the CFRP sheet, and the shear stress of the adhesion layer, respectively; and $\delta$ is the relative displacement between the steel plate and the CFRP sheet in the infinitesimal region $dx$.

In Fig. 3, $\sigma_{s}$, $\epsilon_{s}$, $E_{s1}$, and $E_{s2}$ are the yield strength, yield strain, primary Young’s modulus, and the secondary modulus after the yield of steel, in that order; $\sigma_{sY}$, $\epsilon_{sY}$, and $E_{c}$ are the tensile strength, strain at the tensile strength, and the primary Young’s modulus of the CFRP sheet, respectively; and $\tau_{s}$, $\delta_{s}$, and $\delta_{n}$ are the shear strength, relative displacement at the shear strength, and the relative displacement at the peeling stage of the adhesion layer, respectively.

2. THEORETICAL ANALYSIS

(1) Differential equations

a) Analytical object

As the first premise of the nonlinear theoretical analysis method, a steel plate, which bonded only one layer of the CFRP sheet under uniaxial tensile loading (Fig. 1), was used to determine the mechanical behavior, peeling mechanism, and peeling strength of the adhesion layer inserted between the steel plate and the CFRP sheet. In this analytical object, the width of the CFRP sheet was the same as that of the steel plate. In addition, the principal direction of the bonded CFRP sheet was matched with that of the applied load.

b) Modeling

The calculation model was described as a quarter model of the CFRP-sheet-bonded steel plate (Fig. 2) and considered as the nonlinear material model of all members of the analytical model (Fig. 3). As a base metal, a type of carbon steel was used. Its stress–strain curve relationship was bilinear (Fig. 3(a)), whereby the primary Young’s modulus is $E_{s1}$ and the secondary modulus after yield is $E_{s2}$. The employed CFRP in sheet form is light and has a large tensile strength. The material model is shown in Fig. 3(b). The CFRP sheet has an elastic modulus that is 1.2–3.2 times higher than that of steel. In addition, the elastic modulus of the adhesion layer was smaller by two orders of magnitude than that of steel and the CFRP sheet; hence, under uniaxial tensile loading, the adhesion layer was considered only shear stress, and the steel plate and CFRP sheet was considered only tensile stress. As a bonding constitution rule, the bond–slip model of the adhesion layer was constructed based on the shear stress–relative displacement relationship. From the data directly tested in the laboratory, numerous existing research studies identified a simple bilinear bond–slip relationship of the adhesion layer as shown in Fig. 3(c). The bilinear bond–slip relationship was defined by three key points: the initial point (0, 0), the peak shear stress point, and the ultimate point.

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Constitutive models of materials.

Boundary between elastic and plastic stage of steel plate

Boundary between linear and softening stage of adhesion layer

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### Table 1: Cases implemented in the calculation program.

| Steel  | Adhesion | Elastic | Elastic + Plastic | Completely plastic |
|--------|----------|---------|-------------------|--------------------|
| Linear | Case 1   | Case 2  | Case 3            |
| Softening | Case 4 | Case 5 and Case 6 | Case 7 |

In Case 5, $l_e$ was smaller than $l_s$; and in Case 6, $l_e$ was larger than $l_s$ (Fig. 4).

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Fig. 4: Peeling mechanism of the adhesion layer (1/4 model).

Outline of CAL

1. Set the peeling area $lp = 0$

   - **Case 1:** Completely linear of Adhesion & Completely plastic of Steel
     - (The length of softening area of adhesion = 0)
     - (The length of plastic area of steel = 0)
     - (The differential equation (6a))

   - **Case 2:** Completely linear of Adhesion & (Elastic + Plastic) of Steel
     - (The length of softening area of adhesion = 0)
     - (The length of plastic area of steel = 0)
     - (The differential equations (6a) & (6b))

   - **Case 3:** Completely linear of Adhesion & Completely elastic of Steel
     - (The length of softening area of adhesion = 0)
     - (The length of plastic area of steel = 0)
     - (The differential equation (6a))

   - **Case 4:** Softening stage of Adhesion
     - (Elastic + Plastic) of Steel
     - (The length of softening area of adhesion = 0)
     - (The length of plastic area of steel = 0)
     - (The differential equations (6a) & (6b))

   - **Case 5:** Softening stage of Adhesion & Completely elastic of Steel
     - (The length of softening area of adhesion = 0)
     - (The length of plastic area of steel = 0)
     - (The differential equations (6a) & (7a))

   - **Case 6:** Softening stage of Adhesion & Completely plastic of Steel
     - (The length of softening area of adhesion = 0)
     - (The length of plastic area of steel = 0)
     - (The differential equations (6a) & (7b))

   - **Case 7:** Softening stage of Adhesion & Completely plastic of Steel
     - (The length of softening area of adhesion = 0)
     - (The length of plastic area of steel = 0)
     - (The differential equations (6a) & (7b))

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Fig. 5: Flowchart of the calculation process.

- **Step 1:** Completely linear stage
- **Step 2:** Softening stage
- **Step 3:** Peeling stage
- **Step 4:** Developing peeling damage

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**Fig. 3:**

(a) Steel ($\sigma$-$\varepsilon$)

(b) CFRP sheet ($\sigma$-$\varepsilon$)

(c) Adhesion layer ($\tau$-$\delta$)
**Table 2** Boundary conditions and converged-calculated conditions.

| No. | Calculated cases | Continuous and boundary conditions | Conditions of convergence calculation |
|-----|------------------|-----------------------------------|--------------------------------------|
| 1   | Case 1           | Equation (6a): \( \partial(0) = 0, \partial(L) = \delta, \) and \( e_s(L) = 0 \) | \( e_s(L) \times 10^6 \approx e_s(L) \times 10^6 \) |
| 2   | Case 2           | Equation (6a): \( \partial(0) = 0, \partial(L) = \delta, \) and \( e_s(L) = 0 \) | \( e_s(L) \times 10^6 \approx e_s(L) \times 10^6 \) |
| 3   | Case 3           | Equation (6b): \( \partial(0) = 0, \partial(L) = \delta, \) and \( e_s(L) = 0 \) | \( e_s(L) \times 10^6 \approx e_s(L) \times 10^6 \) |
| 4   | Case 4           | Equation (6a): \( \partial(0) = 0, \partial(L) = \delta, \) and \( e_s(L) = 0 \) | \( e_s(L) \times 10^6 \approx e_s(L) \times 10^6 \) |
| 5   | Case 5           | Equation (6a): \( \partial(0) = 0, \partial(L) = \delta, \) and \( e_s(L) = 0 \) | \( e_s(L) \times 10^6 \approx e_s(L) \times 10^6 \) |
| 6   | Case 6           | Equation (6b): \( \partial(0) = 0, \partial(L) = \delta, \) and \( e_s(L) = 0 \) | \( e_s(L) \times 10^6 \approx e_s(L) \times 10^6 \) |
| 7   | Case 7           | Equation (6b): \( \partial(0) = 0, \partial(L) = \delta, \) and \( e_s(L) = 0 \) | \( e_s(L) \times 10^6 \approx e_s(L) \times 10^6 \) |

**c) Relationship between stress and strain**

When considering a location in the direction along the length of the steel plate with the coordinate \( x \), the relationship between stress and strain on the steel plate and the CFRP sheet, and the relationship between shear stress and relative displacement on the adhesion layer are described as follows:

\[
\sigma_x = \begin{cases} 
E_{sl} \frac{du}{dx} & (\varepsilon_x \leq \varepsilon_y) \\
(E_{sl} - E_{sf}) \varepsilon_y + E_{sf} \frac{du}{dx} & (\varepsilon_x > \varepsilon_y)
\end{cases} \tag{1}
\]

\[
\sigma_{xy} = \begin{cases} 
\frac{\tau_{xy}}{\delta_y} & (0 \leq \delta \leq \delta_y) \\
0 & (\delta_y < \delta \leq \delta_y)
\end{cases} \tag{2}
\]

\[
\tau = \begin{cases} 
\frac{\tau_{xy}}{\delta_y - \delta} & (\delta_y < \delta \leq \delta_y)
\end{cases} \tag{3}
\]

where, \( x \) is the coordinate in the direction along the length of the steel plate; \( u_x, \varepsilon_x \) and \( u_y, \varepsilon_y \) are the displacement and the strain of the steel plate and the CFRP sheet, respectively on the \( x \) coordinate; and \( \delta \) is the relative displacement between the steel plate and the CFRP sheet in the infinitesimal region \( dx \).

From **Fig. 2**, when considering the balance forces in the infinitesimal region \( dx \), balance equations were obtained as follows:

\[
t \frac{d\sigma_x}{dx} - \tau = 0 \tag{4}
\]

\[
t \frac{d\sigma_{xy}}{dx} + \tau = 0 \tag{5}
\]

**d) Differential equations**

By implementing some basic transformations on Equations (4) and (5) together with Equations (1) to (3) of the material models, the differential equations that described the relative displacement of the adhesion layer were obtained as Equations (6) and (7). Moreover, a general solution for the relative displacement of the adhesion layer was obtained at each loaded step by solving Equations (6) and (7). Unknown coefficients in this general solution were determined from continuous and boundary conditions for stress, strain, and relative displacement in each member of the analytical model.

In the case of \( 0 \leq \delta \leq \delta_y \):

\[
d^2 \delta(x) - \lambda_1 \delta(x) = 0 \quad \text{if} \quad (\varepsilon_x \leq \varepsilon_y) \tag{6a}
\]

\[
d^2 \delta(x) - \lambda_2 \delta(x) = 0 \quad \text{if} \quad (\varepsilon_x > \varepsilon_y) \tag{6b}
\]

In the case of \( \delta_y < \delta \leq \delta_y \):

\[
d^2 \delta(x) - \lambda_1' \delta(x) = 0 \quad \text{if} \quad (\varepsilon_x \leq \varepsilon_y) \tag{7a}
\]

\[
d^2 \delta(x) - \lambda_2' \delta(x) = 0 \quad \text{if} \quad (\varepsilon_x > \varepsilon_y) \tag{7b}
\]

where, \( \lambda_1 = \left( \frac{2}{E_{sl} t_s} + \frac{1}{E_{sf} t_{sf}} \right) \frac{\tau_y}{\delta_y} \tag{8a} \)

\( \lambda_2 = \left( \frac{2}{E_{sf} t_s} + \frac{1}{E_{sf} t_{sf}} \right) \frac{\tau_y}{\delta_y} \tag{8b} \)

\( \lambda_1' = \left( \frac{2}{E_{sl} t_s} + \frac{1}{E_{sf} t_{sf}} \right) \frac{\tau_y}{\delta_y} \tag{8c} \)
\[
\lambda'' = \left( \frac{2}{E_s t_s} + \frac{1}{E_f t_f} \right) \frac{\tau_y}{\delta_y - \delta_t}
\] (8d)

(2) Calculation process

This study has referenced the calculation process of Kamiharako et al.\(^7\), which proposed a theoretical analysis to simulate bonding and the peeling behavior of the continuous fiber (CF) sheet with a model of the concrete bonding a layer of the CF sheet. Kamiharako et al.\(^7\) assumed that the concrete member was the complete rigid body, and the CF sheet worked only at the linear phase of the material. In addition, the material constitutive model of the adhesion layer was considered as the relationship between shear stress and relative displacement. This study developed the calculation process of Kamiharako et al.\(^7\) by considering the nonlinear material properties of all members of the analytical object: the steel plate, CFRP sheet, and the adhesion layer.

The peeling mechanism of the adhesion layer on the analytical object is shown in Fig. 4. In the analytical object, peeling damage occurred when the maximum value of the relative displacement on the adhesion layer exceeded the value of \(\delta_t\) of the adhesion material. Further, this maximum value was reached at the top of the fixing section of the bonded CFRP sheet. Therefore, to simplify the calculation process, the control type of the applied load was relative displacement. This means that the calculation process in the proposed theoretical analysis method was controlled by gradually increasing the relative displacement of \(\delta_t\) at the top of the fixing section of the CFRP sheet (Fig. 4).

In Fig. 4, \(l_f\), \(l_e\), and \(l_p\) are the length of the original area bonded with the CFRP sheet, the length of the fixing area of the CFRP sheet, and the length of the peeled area of the CFRP sheet, respectively; \(l_f\) and \(l_e\) are the length of the elastic area on the steel plate and the length of the linear area on the adhesion layer, respectively; \(\delta_t\) is the relative displacement applied to the top of the fixing section of the CFRP sheet in the calculation process; and \(P\) is the applied load in the analytical object.

Based on the level of the applied relative displacement and the material model of the adhesion layer, the calculation was classified into four stages: completely linear stage, softening stage, peeling stage, and development stage of peeling damage of the adhesion layer. Table 1 lists all seven potential cases implemented in the calculation program, and Fig. 5 shows the flowchart of this program. The stress and strain values on the steel plate, CFRP layer, and the adhesion layer were obtained by solving the differential conditions (6) and (7) at each applied load step. Unknown coefficients in the general solutions were determined from the continuous and boundary conditions for stress, strain, and relative displacement in each member of the analytical model, as listed in Table 2. Furthermore, in cases 2, 4, 5, 6, and 7 listed in Table 1, \(l_f\) of the length of the elastic area on the steel plate, and \(l_e\) of the length of the linear area on the adhesion layer were determined from the conditions of the convergence calculation (Table 2) using the Newton-Raphson method with a convergence tolerance of \(10^{-3}\). At each step of the applied relative displacement load, the calculation program is stopped if the stress on the CFRP layer is greater than the tensile strength of the CFRP sheet. The stages of the program are described in detail in the following paragraphs.

Step 1 (\(0 \leq \delta < \delta_t\)): The mechanical behavior of the adhesion layer along the length of the CFRP sheet was completely linear. The mechanical behavior on the steel plate was classified into three potential cases: fully elastic (Case 1), partly elastic and partly plastic (Case 2), and completely plastic (Case 3) (Table 1 and Fig. 5).

Step 2 (\(\delta_t < \delta < \delta_t\)): A part of the adhesion layer closest to the top of the fixing location of the CFRP sheet reached the softening condition. The mechanical behavior on the steel plate was classified into three potential cases: fully elastic (Case 4), partly elastic and partly plastic (Case 5 and Case 6), and completely plastic (Case 7) (Table 1 and Fig. 5).

Step 3 (\(\delta = \delta_t\)): The mechanical properties of the adhesion layer fully satisfied the peeling condition and began to develop the peeling damage at the top of the fixing location of the CFRP sheet. The mechanical behavior on the steel plate was classified into three potential cases: fully elastic (Case 4), partly elastic and partly plastic (Case 5 and Case 6), and completely plastic (Case 7) (Table 1 and Fig. 5).

Step 4 (\(\delta = \delta_t\)): To express the progress of the peeling damage on the adhesion layer, after the peeling condition was fully satisfied in step 3, the lengths of the peeling and fixing areas were newly set, and the same calculation procedure as in step 3 was repeated. The calculation was ended when the length of the set fixing area of the CFRP sheet became very short, or when it was impossible to solve the differential equations (6) and (7).

In each step of the procedure, the equations used to determine the mechanical behavior of all members in the seven potential cases are summarized in the Appendix.
3. EXAMPLE AND DISCUSSION

(1) Calculation model

In the example, the calculation model and the material properties of all members are shown in Fig. 6 and Table 3, respectively. The length and the thickness of the steel plate were 400 mm and 9 mm, respectively; and those of the CFRP sheet were 400 mm and 0.143 mm, respectively. In this model, the width of the CFRP sheet was the same as that of the steel plate with 60 mm. The CFRP sheet was bonded to the steel plate using an adhesion layer with a thickness of 0.517 mm. Carbon steel was used as the base metal. The stress–strain relationship of the carbon steel is bilinear; the primary Young’s modulus is \( E_{s1} = 2 \times 10^5 \) MPa, and secondary modulus after yield is \( E_{s2} = E_{s1}/100 = 2000 \) MPa. The Poisson ratio is 0.3. The employed FTS-C8-30 CFRP, in the sheet form, is lightweight (2.1 g/cm³), has a large tensile strength (2430 MPa), and is durable in harsh environments. The FTS-C8-30 CFRP sheet has an elastic modulus that is 3.2 times higher than that of the steel with the elastic modulus of 6.4 x 10⁵ MPa. The material properties of the adhesion layer used the bilinear model tested by Zhang et al. This material model was formulated from the strain distribution measured on the 25 mm x 25 mm x 600 mm steel-plate-specimens, bonding the CFRP sheet under the uniaxial tensile tests. Its shear strength \( \sigma_y \), relative displacement \( G_y \) at the shear strength, and relative displacement \( G_u \) at the peeling stage were 17 MP, 0.11 mm, and 0.25 mm, respectively.
To show the results of all seven potential cases listed in Table 1, three groups of material (material 1, 2, and 3) were considered as shown in Table 3. The model with the group of material 1 showed the results of potential cases 1, 2, 3, and 7; the model with the group of material 2 showed the results of cases 1, 2, and 6; and the model with the group of material 3 showed the results of cases 1, 4, and 5. In the groups of material 1 and 2, all the material properties of the CFRP sheet and the adhesion layer were fixed; and only the yield stress of the steel plate was changed to the value of 250 MPa and 417 MPa. However, in the group of material 3, if only the yield stress of the steel plate was changed, the required yield stress value of the group of material 3, if only the yield stress of the steel would be very large. Therefore, the shear plate was changed, and the required yield stress value of the CFRP sheet and the adhesion layer were fixed; and only the yield stress of the steel plate was changed to the value of 250 MPa and 417 MPa. However, in the group of material 3, if only the yield stress of the steel plate was changed, the required yield stress value of steel would be very large. Therefore, the shear strength of the adhesion layer in this group was set as 5 MPa, and then the yield stress of the steel plate was set as 720 MPa (Table 3). This means that the nominal values were the yield stress of steel plate in three groups of material, and the shear strength of the adhesion layer in the group of material 3. The actual values were shown for the other ones.

(2) Finite element analysis

To confirm the accuracy of the proposed nonlinear theoretical method, a two-dimensional geometric nonlinear FEM analysis was implemented with a quarter model of the CFRP-sheet-bonded steel plate, using a distribution load as shown in Fig. 7. The analysis software used in this study was DIANA 9.6. The steel plate and the CFRP sheet were constructed using the plane stress element (eight-node CQ16M). The adhesion layer was simulated using the interface element (three-node CL12I element), because this element has material models that are based on the relationship between stress and relative displacement. The boundary conditions were considered on two symmetrical sides, with the fixed perpendicular direction, and free in the other direction, as shown in Fig. 7.

The resolution of the finite element mesh was 2 mm for the direction along the length of the model. The division of the thickness of the steel plate and the CFRP sheet was 10 divisions. As the default, the division of the interface element, which simulated the adhesion layer, was one division. With the sizes of the finite element mesh mentioned above, it was confirmed that the FEM analytical results were almost unchanged when these sizes were constructed smaller. All the material properties applied to the FEM analytical model were selected from the values listed in Table 3. Particularly, the elastic modulus of the adhesion layer was smaller by two orders of magnitude than that of steel and the CFRP sheet. Therefore, in the FEM analytical model, the material model of the adhesion layer in the shear direction was considered as the bilinear model defined in Fig. 3(c). Its material model in the normal direction was assumed to be the linear model with a stiffness gradient of 105 N/mm3. The von Mises yield condition was applied to simulate the steel material, and geometric nonlinearity was considered.

(3) Results of the proposed method

a) Model with the group of material 1

This section presents the results calculated using the model with the group of material 1. The relationship between load and relative displacement at the top of the fixing location of the CFRP sheet is shown in Fig. 8 and is used to compare the FEM analytical (FEA) and the proposed theoretical analytical (CAL) results. The results of the CAL indicate that the load-relative displacement curve began to change at the load of 135 kN (initial plastic load) because the stress on a part of the steel plate reached the plastic condition. Meanwhile, at this load level, the CFRP sheet and the adhesion layer still worked at the linear stage. This trend was consistent with the behavior of the adhesion layer in the shear direction, as shown in Fig. 7. Figure 9 describes the stress distribution of the steel plate, CFRP sheet, and the adhesion layer obtained from the results of CAL and FEA at the loads of 100 kN, 140 kN, 150 kN, 156 kN, 158.25 kN, and 158.25 kN after the peeling damage. The stress of the CFRP sheet was much smaller than its tensile strength during the loading process. Moreover, Fig. 9 also indicates the agreement between the CAL and FEA results. Under the load value of 100 kN, the mechanical behavior of all members of the calculated model was linear (Fig. 9(a)). At 140 kN, a part of the steel plate on the calculated model reached the plastic condition, while the stress of the adhesion layer was still smaller than its shear strength (Fig. 9(b)). At 150 kN, the steel plate fully reached the plastic condition, and the stress of the adhesion layer at the top of the fixing section reached its shear strength (Fig. 9(c)).
At 156 kN, a part of the adhesion layer reached the softening condition (Fig. 9(d)). Then, when the applied relative displacement value increased and attained to the value of $\delta_0 = 0.25$ mm, the peeling damage of the adhesion layer began to appear at the load of 158.25 kN (Fig. 9(e)). This also means that the peeling strength of the adhesion layer was determined when the shear stress of the adhesion layer at the top of the fixing location of the CFRP sheet reached the value of zero (Fig. 9(e)). Therefore, the peeling load of the example model with the group of material 1 was 158.25 kN. After the peeling condition was fully satisfied at the load of 158.25 kN, the lengths of the peeling and fixing areas were newly set. The stress distribution of all members after peeling damage with the length of the peeled area of 30 mm are shown in Fig. 9(f). The behavior in Fig. 9(f) was almost the same as that of the first stage of the adhesion layer, which satisfied the peeling condition (Fig. 9(e)), and the stress of the CFRP sheet and adhesion layer on the peeling area was zero.

As a result, the mechanical behavior of the steel plate, CFRP sheet, and the adhesion layer could be evaluated accurately by using the proposed theoretical analysis method. Additionally, the peeling strength of the calculated model in the proposed method was determined when the shear stress of the adhesion layer at the top of the fixing location of the CFRP sheet reached the value of zero.

b) Model with the group of material 2

This section presents the results calculated in the model with the group of material 2. In this model, the stress on the steel plate also reached the yield stress before the shear stress on the adhesion layer entered the softening stage. However, the stress on the steel plate did not achieve the complete plastic condition during the loading process. The load–relative displacement relationship and the stress distribution of the calculated model shown in Figs. 10 and 11 indicate the consistency between the results of the FEA and CAL. The mechanical behavior of the steel plate, CFRP sheet, and the adhesion layer could be evaluated accurately using the proposed theoretical analysis method. The peeling load of the calculated model was 247.52 kN (Fig. 11(d)).

c) Model with the group of material 3

This section shows the results calculated in the model with the group of material 3. This model was unlike other models: the shear stress on the adhesion layer entered the softening stage before the stress on the steel plate reached the yield stress. Further, in this model, the stress on the steel plate did not achieve the complete plastic condition during the loading process. The load–relative displacement relationship and the stress distribution of the calculated model shown in Figs. 12 and 13 indicate the consistency between the results of the FEA and CAL. The mechanical behavior of the steel plate, CFRP sheet, and adhesion
Steel plate                                    CFRP sheet                                    Adhesi
(b) At the load of 140 kN (Case 2 listed in Table 1)

Steel plate                                    CFRP sheet                                    Adhesi
(c) At the load of 150 kN (Case 3 listed in Table 1)

Steel plate                                    CFRP sheet                                    Adhesi
(d) At the load of 156 kN (Case 7 listed in Table 1)

Steel plate                                    CFRP sheet                                    Adhesi
(e) At the load of 158.25 kN (Case 7 listed in Table 1)

Steel plate                                    CFRP sheet                                    Adhesi
(f) At the load of 158.25 kN, after the peeling damage with the length of a peeled area of 30 mm

Fig. 9 Comparison of stress distribution between CAL and FEA in the model with the group of material 1.
**Fig. 10** Load-relative displacement relationship in the model with the group of material 2.

**Fig. 11** Comparison of stress distribution between CAL and FEA in the model with the group of material 2.
(e) At the load of 247.52 kN, after the peeling damage with the length of a peeled area of 30 mm

Fig.11 Comparison of stress distribution between CAL and FEA in the model with the group of material 2.

Fig.12 Load-relative displacement relationship in the model with the group of material 3.

(a) At the load of 100 kN (Case 1 listed in Table 1)

(b) At the load of 367 kN (Case 4 listed in Table 1)

(c) At the load of 393.22 kN (Case 5 listed in Table 1)

Fig.13 Comparison of stress distribution between CAL and FEA in the model with the group of material 3.
layer could be evaluated accurately using the proposed theoretical analysis method. The peeling load of the calculated model was 393.22 kN (Fig. 13(c)).

For the steel members, which bonded a layer of the CFRP sheet under uniaxial-tensile-stress conditions, it was possible to clarify the actual peeling mechanism of the adhesion layer through the calculation procedure of the proposed nonlinear theoretical analysis method. Moreover, the peeling strength of the analytical model and the mechanical behavior of all members: the steel plate, CFRP sheet, and the adhesion layer, were also accurately evaluated.

4. CONCLUSION

By using a steel plate, which bonded a layer of the CFRP sheet under uniaxial tensile loading, a nonlinear theoretical analysis method was established considering the actual peeling condition of the adhesion layer, and the nonlinear material condition of all members of the analytical model. The comparison of the results with the FEM analysis indicated that it was possible to accurately evaluate the peeling strength of the model and the mechanical behavior of the steel plate, CFRP sheet, and the adhesion layer. Moreover, this study also clarified the peeling mechanism of the adhesion layer through the calculation procedure of the proposed nonlinear theoretical analysis method. From this peeling mechanism of the adhesion layer, to reproduce the peeling failure of the adhesion layers on the repaired or reinforced members under the environment of FEM analysis, the use of the interface element is strongly preferred for the simulation of the adhesion layer.

In future study, the implementation of loading tests with the analytical model at the laboratory will be required to allow for comparison with the results obtained from the proposed method. Moreover, a nonlinear analysis method for the model of the steel plate with multilayered CFRP sheets under uniaxial loading and bending, and a model that is able to fully consider the material properties (shear direction and normal direction) of the adhesion layer, will be developed.

APPENDIX: EQUATIONS USED IN SEVEN POTENTIAL CASES

At each step of the calculating procedure, the equations used to determine the mechanical behavior of all the members in seven potential cases are summarized as follows:

(1) Case 1

\[
\delta(x) = C_1 \sinh(\sqrt{\lambda_1}x) \\
\tau(x) = \frac{\tau_y}{\delta_y} C_1 \sinh(\sqrt{\lambda_1}x) \\
e_{sf}(x) = C_{0sf} \left( \cosh(\sqrt{\lambda_1}x) - \cosh(\sqrt{\lambda_1}L) \right) \\
\sigma_{sf}(x) = E_{sf} C_{0sf} \left( \cosh(\sqrt{\lambda_1}x) - \cosh(\sqrt{\lambda_1}L) \right) \\
\varepsilon_s(x) = E_{sf} C_{0sf} \cosh(\sqrt{\lambda_1}x) - E_{sx} C_{0sf} \cosh(\sqrt{\lambda_1}L) \\
P = t_1 \sigma_s(L)b \\
\]

where, \( C_1 = \frac{\delta_y}{\sinh(\sqrt{\lambda_1}L)} \) (9h)

\[
C_{0sf} = \left( \frac{\tau_y}{\delta_y E_{sf} \sqrt{\lambda_1}} \right) \frac{\delta_y}{\sinh(\sqrt{\lambda_1}L)} \\
C_{0s} = \left( \sqrt{\lambda_1^2 - \frac{\tau_y}{\delta_y E_{sf} \sqrt{\lambda_1}}} \right) \frac{\delta_y}{\sinh(\sqrt{\lambda_1}L)} \\
\]

(2) Case 2

a) Plastic area of steel plate \((l_s \leq x \leq L)\)

\[
\delta(x) = C_1 e^{\sqrt{\lambda_1}x} + C_2 e^{-\sqrt{\lambda_1}x} \\
\tau(x) = \frac{\tau_y}{\delta_y} \left( C_1 e^{\sqrt{\lambda_1}x} + C_2 e^{-\sqrt{\lambda_1}x} \right) \\
\varepsilon_{sf}(x) = C_{0sf} \left( C_1 e^{\sqrt{\lambda_1}x} - C_2 e^{-\sqrt{\lambda_1}x} \right) + k_1 \\
\]

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\[ \sigma_{cf} = E_c C_{o_{cf}2} \left( C_1 e^{\sqrt{\lambda_1} x} - C_2 e^{-\sqrt{\lambda_1} x} \right) + E_c k_1 \]  
\[ \varepsilon_{cf} = \left( E_{s_1} - E_{s_2} \right) \varepsilon_y + E_{s_2} \left( C_{o_{cf}2} + \sqrt{\lambda_2} \right) \left( C_1 e^{\sqrt{\lambda_1} x} - C_2 e^{-\sqrt{\lambda_1} x} \right) + E_{s_2} k_1 \]
\[ P = t \sigma_1 (L) b \]

\( b) \text{ Elastic area of steel plate (} 0 \leq x \leq L \) \]
\[ \delta(x) = C_1 \left( e^{\sqrt{\lambda_1} x} - e^{-\sqrt{\lambda_1} x} \right) \]  
\[ \tau(x) = \frac{\tau_y}{\delta_y} C_1 \left( e^{\sqrt{\lambda_1} x} - e^{-\sqrt{\lambda_1} x} \right) \]
\[ \varepsilon_{cf} = C_{o_{cf}1} C_1 \left( e^{\sqrt{\lambda_1} x} + e^{-\sqrt{\lambda_1} x} \right) + k_2 \]
\[ \sigma_{cf} = E_c C_{o_{cf}1} \left( e^{\sqrt{\lambda_1} x} + e^{-\sqrt{\lambda_1} x} \right) + E_c k_2 \]
\[ \varepsilon_x = C_1 \left( C_{o_{cf}1} + \sqrt{\lambda_1} \right) \left( e^{\sqrt{\lambda_1} x} + e^{-\sqrt{\lambda_1} x} \right) + E_s k_2 \]
\[ \sigma_x = E_s C_1 \left( C_{o_{cf}1} + \sqrt{\lambda_1} \right) \left( e^{\sqrt{\lambda_1} x} + e^{-\sqrt{\lambda_1} x} \right) + E_s k_2 \]

where, \( C_{o_{cf}1} = \left( -\frac{\tau_y}{\delta_y E_{cf} \sqrt{\lambda_2}} \right) \)
\[ C_{o_{cf}2} = \left( -\frac{\tau_y}{\delta_y E_{cf} \sqrt{\lambda_2}} \right) \]
\[ D = -2 e^{\sqrt{\lambda_1} (L-L)} \left[ \sinh \left( \sqrt{\lambda_1} l_1 - \frac{\sqrt{\lambda_1}}{\sqrt{\lambda_2}} \cosh \left( \sqrt{\lambda_2} l_1 \right) \right) \right] 
- 2 e^{\sqrt{\lambda_1} (L-L)} \left[ \sinh \left( \sqrt{\lambda_1} l_1 + \frac{\sqrt{\lambda_1}}{\sqrt{\lambda_2}} \cosh \left( \sqrt{\lambda_2} l_1 \right) \right) \right] \]
\[ D_1 = -2 e^{\sqrt{\lambda_1} l_1} \left[ \sinh \left( \sqrt{\lambda_1} l_1 + \frac{\sqrt{\lambda_1}}{\sqrt{\lambda_2}} \cosh \left( \sqrt{\lambda_2} l_1 \right) \right) \right] \]
\[ D_2 = -2 e^{\sqrt{\lambda_1} l_1} \left[ \sinh \left( \sqrt{\lambda_1} l_1 - \frac{\sqrt{\lambda_1}}{\sqrt{\lambda_2}} \cosh \left( \sqrt{\lambda_2} l_1 \right) \right) \right] \]
\[ D_3 = -2 \delta \]
\[ C_1 = \frac{D_1}{D} \]
\[ C_2 = \frac{D_2}{D} \]
\[ \delta \]
\[ \delta \]

\( C_3 = \frac{D_3}{D} \)
\[ k_1 = -C_{o_{cf}2} \left[ C_1 e^{\sqrt{\lambda_1} t} - C_2 e^{-\sqrt{\lambda_1} t} \right] \]
\[ k_2 = -C_{o_{cf}2} \left[ C_1 e^{\sqrt{\lambda_1} t} - C_2 e^{-\sqrt{\lambda_1} t} \right] \]
\[ + k_1 - 2 C_c C_{o_{cf}1} \cosh \left( \sqrt{\lambda_1} t \right) \]

\( (3) \text{ Case 3} \)
\[ \delta(x) = C_1 \sinh \left( \sqrt{\lambda_2} x \right) \]
\[ \tau(x) = \frac{\tau_y}{\delta_y} C_1 \sinh \left( \sqrt{\lambda_2} x \right) \]
\[ \varepsilon_{cf} = C_{o_{cf}} \left[ \cosh \left( \sqrt{\lambda_2} x \right) - \cosh \left( \sqrt{\lambda_2} L \right) \right] \]
\[ \sigma_{cf} = E_c C_{o_{cf}} \left[ \cosh \left( \sqrt{\lambda_2} x \right) - \cosh \left( \sqrt{\lambda_2} L \right) \right] \]
\[ \varepsilon_x = C_{o_{cf}} \cosh \left( \sqrt{\lambda_2} x \right) - C_{o_{cf}} \cosh \left( \sqrt{\lambda_2} L \right) \]
\[ \sigma_x = \left( E_{s_1} - E_{s_2} \right) \varepsilon_y + E_{s_2} C_{o_{cf}} \cosh \left( \sqrt{\lambda_2} x \right) \]
\[ - E_{s_2} C_{o_{cf}} \cosh \left( \sqrt{\lambda_2} L \right) \]
\[ P = t \sigma_1 (L) b \]

where, \( C_1 = \frac{\delta_i}{\sinh \left( \sqrt{\lambda_2} L \right)} \)
\[ C_{o_{cf}} = -\frac{\tau_y}{\delta_y E_{cf} \sqrt{\lambda_2} \sinh \left( \sqrt{\lambda_2} L \right)} \]
\[ C_{o_{cf}} = \frac{\sqrt{\lambda_2} - \frac{\tau_y}{\delta_y E_{cf} \sqrt{\lambda_2} \sinh \left( \sqrt{\lambda_2} L \right)}}{\sinh \left( \sqrt{\lambda_2} L \right)} \]

\( (4) \text{ Case 4} \)
\[ \delta(x) = \delta_a - \left( C_1 \cos \left( \sqrt{\lambda_1} x \right) + C_2 \sin \left( \sqrt{\lambda_1} x \right) \right) \]
\[ \tau(x) = \frac{\tau_y}{\delta_a - \delta_y} \left( C_1 \cos \left( \sqrt{\lambda_1} x \right) + C_2 \sin \left( \sqrt{\lambda_1} x \right) \right) \]
\[ \varepsilon_{cf} = C_{o_{cf}'} \left( C_1 \sin \sqrt{\lambda_1} x - C_2 \cos \sqrt{\lambda_1} x \right) + k_1 \]
\[ \sigma_{cf} = E_c C_{o_{cf}'} \left( C_1 \sin \sqrt{\lambda_1} x - C_2 \cos \sqrt{\lambda_1} x \right) + E_c k_1 \]
\[ \varepsilon_x = \left( C_{o_{cf}'} + \sqrt{\lambda_1} \right) \left( C_1 \sin \sqrt{\lambda_1} x - C_2 \cos \sqrt{\lambda_1} x \right) + k_1 \]
\[ \sigma_s = E_{st} \left( C_{s\text{of}1} + \sqrt{L_1} \right) \left( C_1 \sin \sqrt{L_1} x - C_2 \cos \sqrt{L_1} x \right) + E_{st} k_1 \]
\[ P = t_s \sigma_s (L) + (12f) \]

b) Linear area of adhesion layer \((0 \leq x \leq l_s)\)

\[ \delta(x) = C_3 \left( e^{\sqrt{L_1}x} - e^{-\sqrt{L_1}x} \right) \]
\[ \tau(x) = \frac{\tau_s}{\delta_y} C_3 \left( e^{\sqrt{L_1}x} - e^{-\sqrt{L_1}x} \right) \]
\[ \varepsilon_{cf}(x) = E_{cf} C_{cf1} \left( e^{\sqrt{L_1}x} + e^{-\sqrt{L_1}x} \right) + k_2 \]
\[ \sigma_{cf}(x) = E_{cf} C_{cf1} \left( \sqrt{L_1} \right) \left( e^{\sqrt{L_1}x} + e^{-\sqrt{L_1}x} \right) + E_{cf} k_2 \]
\[ (12g) \]

where, \( C_{cf1} = \left( \frac{\tau_s}{\delta_y E_{cf} \sqrt{L_1}} \right) \)
\[ C'_{cf1} = \left( \frac{\tau_s}{(\delta_y - \delta_y) E_{cf} \sqrt{L_1}} \right) \]
\[ D = -2 \sinh \left( \sqrt{L_1} L \right) \sin \left( \sqrt{L_1} \left( L - l_s \right) \right) \]
\[ D_1 = 2 \left( \delta_y - \delta_y \right) \left( \sqrt{L_1} L \right) \sin \left( \sqrt{L_1} \left( L - l_s \right) \right) \]
\[ -2 \left( \delta_y - \delta_y \right) \left( \sqrt{L_1} \left( L + l_s \right) \right) \sin \left( \sqrt{L_1} \left( L - l_s \right) \right) \]
\[ D_2 = 2 \left( \delta_y - \delta_y \right) \left( \sqrt{L_1} \left( L + l_s \right) \right) \cos \left( \sqrt{L_1} \left( L - l_s \right) \right) \]
\[ -2 \left( \delta_y - \delta_y \right) \left( \sqrt{L_1} \left( L - l_s \right) \right) \cos \left( \sqrt{L_1} \left( L + l_s \right) \right) \]
\[ D_3 = -\delta_y \sin \left( \sqrt{L_1} \left( L - l_s \right) \right) \]
\[ C_1 = \frac{D_1}{D} \]
\[ C_2 = \frac{D_2}{D} \]
\[ C_3 = \frac{D_3}{D} \]
\[ k_1 = -C'_{cf1} \left[ \sin \left( \sqrt{L_1} L \right) - C_2 \cos \left( \sqrt{L_1} L \right) \right] \]
\[ k_2 \left( C_{cf1} + \sqrt{L_1} \right) \left[ C_1 \sin \left( \sqrt{L_1} L \right) - C_2 \cos \left( \sqrt{L_1} L \right) \right] \]
\[ + k_1 - 2C_3 \left( C_{cf1} + \sqrt{L_1} \right) \cosh \left( \sqrt{L_1} l_s \right) \]
\[ (12k) \]

(5) Case 5

a) Plastic area of steel plate and softening area of adhesion layer \((l_s \leq x \leq L)\)

\[ \delta(x) = \delta_u - \left( C_3 \cos \sqrt{L_1} x + C_4 \sin \sqrt{L_1} x \right) \]
\[ \tau(x) = \frac{\tau_s}{\delta_u - \delta_y} \left( C_3 \cos \sqrt{L_1} x + C_4 \sin \sqrt{L_1} x \right) \]
\[ \varepsilon_{cf}(x) = C'_{cf1} \left( C_3 \sin \sqrt{L_1} x - C_4 \cos \sqrt{L_1} x \right) \]
\[ \sigma_{cf}(x) = E_{cf} C'_{cf1} \left( C_3 \sin \sqrt{L_1} x - C_4 \cos \sqrt{L_1} x \right) \]
\[ + E_{cf} k_2 \]
\[ (13a) \]

where, \( \delta_u \) is the initial plastic deformation

b) Elastic area of steel plate and softening area of adhesion layer \((l_s \leq x \leq L)\)

\[ \delta(x) = \delta_u - \left( C_3 \cos \sqrt{L_1} x + C_4 \sin \sqrt{L_1} x \right) \]
\[ \tau(x) = \frac{\tau_s}{\delta_u - \delta_y} \left( C_3 \cos \sqrt{L_1} x + C_4 \sin \sqrt{L_1} x \right) \]
\[ \varepsilon_{cf}(x) = C'_{cf1} \left( C_3 \sin \sqrt{L_1} x - C_4 \cos \sqrt{L_1} x \right) \]
\[ \sigma_{cf}(x) = E_{cf} C'_{cf1} \left( C_3 \sin \sqrt{L_1} x - C_4 \cos \sqrt{L_1} x \right) \]
\[ + E_{cf} k_2 \]
\[ (13b) \]

\[ (13c) \]

\[ (13d) \]

\[ (13e) \]

\[ (13f) \]

\[ (13g) \]

\[ (13h) \]

\[ (13i) \]

\[ (13j) \]

\[ (13k) \]

\[ (13l) \]

\[ (13m) \]
c) Elastic area of steel plate and linear area of adhesion layer \( (0 \leq x \leq l_e) \)

\[
\delta(x) = C_s \left( e^{\sqrt{\delta_x} x} - e^{-\sqrt{\delta_x} x} \right) \quad (13n)
\]

\[
\tau(x) = \frac{\tau_y}{\delta_y} C_s \left( e^{\sqrt{\delta_x} x} - e^{-\sqrt{\delta_x} x} \right) \quad (13o)
\]

\[
\varepsilon_{ef}(x) = C_{oef1} C_s \left( e^{\sqrt{\delta_x} x} + e^{-\sqrt{\delta_x} x} \right) + k_s \quad (13p)
\]

\[
\sigma_{ef}(x) = E_{ef} C_{oef1} C_s \left( e^{\sqrt{\delta_x} x} + e^{-\sqrt{\delta_x} x} \right) + E_{ef} k_s \quad (13q)
\]

\[
\varepsilon_r(x) = \left( C_{oef1} + \sqrt{\delta_1} \right) C_s \left( e^{\sqrt{\delta_x} x} + e^{-\sqrt{\delta_x} x} \right) + k_3 \quad (13r)
\]

\[
\sigma_r(x) = E_{sl} \left( C_{oef1} + \sqrt{\delta_1} \right) C_s \left( e^{\sqrt{\delta_x} x} + e^{-\sqrt{\delta_x} x} \right) + E_{sl} k_3
\]

where, \( C_{oef1} = -\frac{\tau_y}{\delta_y E_{ef} l_f \sqrt{\delta_1}} \quad (14a) \)

\[
C_{oef1}^t = -\frac{\tau_y}{(\delta_u - \delta_y) E_{ef} l_f \sqrt{\delta_1}} \quad (14b)
\]

\[
C_{oef2} = -\frac{\tau_y}{(\delta_u - \delta_y) E_{ef} l_f \sqrt{\delta_2}} \quad (14c)
\]

\[
q_1 = \sin\left(\sqrt{\delta_1} l_i\right) \cot\left(\sqrt{\delta_1} l_i\right) - \cos\left(\sqrt{\delta_1} l_i\right) \quad (14d)
\]

\[
q_2 = \frac{\sin\left(\sqrt{\delta_1} l_i\right)}{\sin\left(\sqrt{\delta_2} l_i\right)} \left( \delta_u - \delta_y \right) \quad (14e)
\]

\[
q_3 = -\frac{\sqrt{\delta_1}}{\sqrt{\delta_2}} \sin\left(\sqrt{\delta_2} l_i\right) + \cos\left(\sqrt{\delta_2} l_i\right) \cot\left(\sqrt{\delta_2} l_i\right) \quad (14f)
\]

\[
q_4 = -\sqrt{\frac{\delta_1}{\delta_2}} \cos\left(\sqrt{\delta_2} l_i\right) \left( \delta_u - \delta_y \right) \quad (14g)
\]

\[
D = q_1 \cos\left[\sqrt{\delta_1} (L - l_i)\right] - q_3 \sin\left[\sqrt{\delta_2} (L - l_i)\right] \quad (14h)
\]

\[
D_1 = (\delta_u - \delta_y) \left[ q_1 \cos\left(\sqrt{\delta_2} l_i\right) + q_3 \sin\left(\sqrt{\delta_2} l_i\right) \right] - \sin\left(\sqrt{\delta_2} L\right) q_2 q_3 - q_1 q_4 \quad (14i)
\]

\[
D_2 = \cos\left(\sqrt{\delta_2} L\right) q_2 q_3 - q_1 q_4 - (\delta_u - \delta_y) \left[ q_1 \cos\left(\sqrt{\delta_2} l_i\right) - q_3 \sin\left(\sqrt{\delta_2} l_i\right) \right] \quad (14j)
\]

\[
D_3 = -q_1 \cos\left(\sqrt{\delta_2} (L - l_i)\right) - q_3 \sin\left(\sqrt{\delta_2} (L - l_i)\right) + q_4 \quad (14k)
\]

\[
C_1 = \frac{D_1}{D} \quad (14l)
\]

\[
C_2 = \frac{D_2}{D} \quad (14m)
\]

\[
C_3 = \frac{D_3}{D} \quad (14n)
\]

\[
C_4 = \frac{1}{\sin\left(\sqrt{\delta_2} l_i\right)} \left[ \left( \delta_u - \delta_y \right) - C_3 \cos\left(\sqrt{\delta_2} l_i\right) \right] \quad (14o)
\]

\[
k_1 = -C_{oef2} \left[ C_1 \sin\left(\sqrt{\delta_2} l\right) - C_2 \cos\left(\sqrt{\delta_2} l\right) \right] \quad (14p)
\]

\[
k_2 = C_{oef2} \left[ C_1 \sin\left(\sqrt{\delta_2} l\right) - C_2 \cos\left(\sqrt{\delta_2} l\right) \right] + k_1 - C_{oef1} \left[ C_3 \sin\left(\sqrt{\delta_2} l\right) - C_4 \cos\left(\sqrt{\delta_2} l\right) \right] \quad (14q)
\]

\[
k_3 = \left( C_{oef1} + \sqrt{\delta_1} \right) \left[ C_1 \sin\left(\sqrt{\delta_2} l\right) - C_2 \cos\left(\sqrt{\delta_2} l\right) \right] \quad (14r)
\]

\[
k_4 = C_{oef1} \left( C_1 \sin\left(\sqrt{\delta_2} l\right) - C_2 \cos\left(\sqrt{\delta_2} l\right) \right) \quad (14s)
\]

(6) Case 6
a) Plastic area of steel plate and softening area of adhesion layer \( (l_e \leq x \leq L) \)

\[
\delta(x) = \delta_u - \left( C_1 \cos\left(\sqrt{\delta_2} x\right) + C_2 \sin\left(\sqrt{\delta_2} x\right) \right) \quad (15a)
\]

\[
\tau(x) = \frac{\tau_y}{\delta_y} \left( C_1 \cos\left(\sqrt{\delta_2} x\right) + C_2 \sin\left(\sqrt{\delta_2} x\right) \right) \quad (15b)
\]

\[
\varepsilon_{ef}(x) = C_{oef2} \left( C_1 \sin\left(\sqrt{\delta_2} x\right) - C_2 \cos\left(\sqrt{\delta_2} x\right) \right) \quad (15c)
\]

\[
\sigma_{ef}(x) = E_{ef} C_{oef2} \left( C_1 \sin\left(\sqrt{\delta_2} x\right) - C_2 \cos\left(\sqrt{\delta_2} x\right) \right) \quad (15d)
\]

\[
\varepsilon_r(x) = k_1 \quad (15e)
\]

\[
\sigma_r(x) = \left( E_{sl} - E_{ef}\right) \varepsilon_r \quad (15f)
\]

\[
P = t_1 \sigma_r(L) b \quad (15g)
\]
b) Plastic area of steel plate and linear area of adhesion layer \((l_i \leq x \leq l_e)\)

\[
\begin{align*}
\delta(x) &= C_3 e^{v_{1i}^2 x} + C_4 e^{-v_{1i}^2 x} \\
\tau(x) &= \frac{\tau_y}{\delta_x} \left( C_3 e^{v_{1i}^2 x} + C_4 e^{-v_{1i}^2 x} \right) \\
\varepsilon_\sigma(x) &= C_{\sigma/(2)} \left( C_3 e^{v_{1i}^2 x} - C_4 e^{-v_{1i}^2 x} \right) + E_{\sigma} k_2 \\
\varepsilon_\varepsilon(x) &= \left( C_{\varepsilon/(2)} + \sqrt{\lambda_2} \right) \left( C_3 e^{v_{1i}^2 x} - C_4 e^{-v_{1i}^2 x} \right) + E_{\varepsilon} k_2 \\
\sigma_\sigma(x) &= \left( E_{\sigma} - E_{\varepsilon} \right) \varepsilon_\varepsilon + E_{\varepsilon} k_2 \\
\sigma_\varepsilon(x) &= \varepsilon_{\varepsilon} \left( C_{\sigma/(1)} + \sqrt{\lambda_2} \right) C_3 \left( e^{v_{1i}^2 x} + e^{-v_{1i}^2 x} \right) + E_{\varepsilon} k_3 \\
\end{align*}
\]

where, \(C_{\sigma/(1)} = -\frac{\tau_y}{\delta_x E_{\varepsilon} \sqrt{\lambda_2}}\) \hspace{1cm} (16a)

\[
\begin{align*}
C_{\varepsilon/(2)} &= -\frac{\tau_y}{\delta_x E_{\sigma} \sqrt{\lambda_2}} \\
C_{\sigma/(2)} &= -\frac{\tau_y}{\delta_x E_{\sigma} \sqrt{\lambda_2}} \left( \delta_\sigma - \delta_\varepsilon \right) \left( E_{\sigma} - E_{\varepsilon} \right) \\
\delta_1 &= \frac{\sin(\sqrt{\lambda_2^2 L})}{\cos(\sqrt{\lambda_2^2 L})} \\
C_1 &= \frac{\sin(\sqrt{\lambda_2^2 L})}{\cos(\sqrt{\lambda_2^2 L})} \\
C_2 &= \tan(\sqrt{\lambda_2^2 L}) \\
\end{align*}
\]

\[D = -2e^{-\sqrt{\lambda_2^2 l_i}} \left[ \sinh\left( \sqrt{\lambda_2 l_i} \right) - \frac{\lambda_2}{\sqrt{\lambda_2^2}} \cosh \left( \sqrt{\lambda_2 l_i} \right) \right] - 2e^{\sqrt{\lambda_2^2 l_i}} \left[ \sinh\left( \sqrt{\lambda_2 l_i} \right) + \frac{\lambda_2}{\sqrt{\lambda_2^2}} \cosh \left( \sqrt{\lambda_2 l_i} \right) \right] \] \hspace{1cm} (16f)

\[D_2 = -2\delta_\varepsilon e^{\sqrt{\lambda_2^2 l_i}} \left[ \sinh\left( \sqrt{\lambda_2 l_i} \right) - \frac{\lambda_2}{\sqrt{\lambda_2^2}} \cosh \left( \sqrt{\lambda_2 l_i} \right) \right] \] \hspace{1cm} (16g)

\[D_3 = -2\delta_\varepsilon e^{\sqrt{\lambda_2^2 l_i}} \left[ \sinh\left( \sqrt{\lambda_2 l_i} \right) + \frac{\lambda_2}{\sqrt{\lambda_2^2}} \cosh \left( \sqrt{\lambda_2 l_i} \right) \right] \] \hspace{1cm} (16h)

\[C_3 = \frac{D}{D} \] \hspace{1cm} (16i)

\[C_4 = \frac{D_k}{D} \] \hspace{1cm} (16j)

\[k_i = -C_{\sigma/(2)} \left[ C_1 \sin(\sqrt{\lambda_2^2 L}) - C_2 \cos(\sqrt{\lambda_2^2 L}) \right] \] \hspace{1cm} (16m)

\[k_2 = \left( C_{\sigma/(2)} + \sqrt{\lambda_2^2} \right) \left[ C_1 \sin(\sqrt{\lambda_2^2 l_i}) - C_2 \cos(\sqrt{\lambda_2^2 l_i}) \right] + k_1 \left( C_{\sigma/(2)} + \sqrt{\lambda_2^2} \right) \left[ C_1 e^{\sqrt{\lambda_2^2 l_i}} - C_2 e^{\sqrt{\lambda_2^2 l_i}} \right] \] \hspace{1cm} (16n)

\[k_3 = \left( C_{\sigma/(2)} + \sqrt{\lambda_2^2} \right) \left[ C_1 e^{\sqrt{\lambda_2^2 l_i}} - C_2 e^{\sqrt{\lambda_2^2 l_i}} \right] + k_2 - 2C_3 \left( C_{\sigma/(2)} + \sqrt{\lambda_2^2} \right) \cos(\sqrt{\lambda_2 l_i}) \] \hspace{1cm} (16o)

(7) Case 7

a) Plastic area of steel plate and softening area of adhesion layer \((l_i \leq x \leq L)\)

\[
\begin{align*}
\delta(x) &= \delta_\sigma - \left( C_1 \cos(\sqrt{\lambda_2^2 x}) + C_2 \sin(\sqrt{\lambda_2^2 x}) \right) \\
\tau(x) &= \frac{\tau_y}{\delta_x - \delta_\varepsilon} \left( C_1 \cos(\sqrt{\lambda_2^2 x}) + C_2 \sin(\sqrt{\lambda_2^2 x}) \right) + \delta_\varepsilon \varepsilon_{\varepsilon} + \delta_\sigma \varepsilon_{\sigma} \\
\varepsilon_\sigma(x) &= C_{\sigma/(2)} \left( C_1 \sin(\sqrt{\lambda_2^2 x}) - C_2 \cos(\sqrt{\lambda_2^2 x}) \right) + k_1 \\
\varepsilon_\varepsilon(x) &= C_{\varepsilon/(2)} \left( C_1 \sin(\sqrt{\lambda_2^2 x}) - C_2 \cos(\sqrt{\lambda_2^2 x}) \right) + k_2 \\
\sigma_\sigma(x) &= E_{\sigma} C_{\sigma/(2)} \left( C_1 \sin(\sqrt{\lambda_2^2 x}) - C_2 \cos(\sqrt{\lambda_2^2 x}) \right) + E_{\varepsilon} k_1 \\
\sigma_\varepsilon(x) &= E_{\sigma} C_{\varepsilon/(2)} \left( C_1 \sin(\sqrt{\lambda_2^2 x}) - C_2 \cos(\sqrt{\lambda_2^2 x}) \right) + E_{\varepsilon} k_2 \\
\end{align*}
\]
\[ \varepsilon_s(x) = k_1 \]
\[ + \left( C_{02f} + \sqrt{\lambda_2} \right) \left( C_1 \sin(\sqrt{\lambda_2} x) - C_2 \cos(\sqrt{\lambda_2} x) \right) \]
\[ \sigma_s(x) = (E_{s1} - E_{s2}) \varepsilon_s \]
\[ + E_{s2} \left( C_{02f} + \sqrt{\lambda_2} \right) \left( C_1 \sin(\sqrt{\lambda_2} x) - C_2 \cos(\sqrt{\lambda_2} x) \right) \]
\[ + E_{s2} k_1 \]
\[ P = t \sigma_s(L)b \]

b) Plastic area of steel plate and linear area of adhesion layer \((0 \leq x \leq l_e)\)

\[ \delta(x) = C_{02f} \left( C_1 e^{\sqrt{\lambda_2} x} - C_4 e^{-\sqrt{\lambda_2} x} \right) + k_2 \]

\[ \tau(x) = \frac{\tau_y}{\delta_x} \left( C_1 e^{\sqrt{\lambda_2} x} - C_4 e^{-\sqrt{\lambda_2} x} \right) \]

\[ \varepsilon_{cf}(x) = C_{02f} \left( C_1 e^{\sqrt{\lambda_2} x} - C_4 e^{-\sqrt{\lambda_2} x} \right) + k_2 \]

\[ \sigma_{cf}(x) = E_{cf} \left( C_0 \right) \left( C_1 e^{\sqrt{\lambda_2} x} - C_4 e^{-\sqrt{\lambda_2} x} \right) + k_2 \]

\[ \varepsilon, (x) = \left( E_{s1} - E_{s2} \right) \varepsilon_s \]

where, \( C_{02f} = \left( -\frac{\tau_y}{\delta_x E_{cf} t_f \sqrt{\lambda_2}} \right) \)

\[ C_{02f} = \left( \frac{\tau_y}{\delta_x E_{cf} t_f \sqrt{\lambda_2}} \right) \]

\[ \delta_l = \frac{\sin(\sqrt{\lambda_2} L)}{\sin(\sqrt{\lambda_2} t_e)} \]

\[ \cos(\sqrt{\lambda_2} L) - \cos(\sqrt{\lambda_2} t_e) \]

\[ C_1 = \frac{\sin(\sqrt{\lambda_2} L)}{\cos(\sqrt{\lambda_2} L) - \tan(\sqrt{\lambda_2} L) \tan(\sqrt{\lambda_2} t_e)} \]

\[ k_1 = -C_{02f} \left[ C_1 \sin(\sqrt{\lambda_2} L) - C_2 \cos(\sqrt{\lambda_2} L) \right] \]

\[ k_2 = C_{02f} + \sqrt{\lambda_2} \left[ C_1 \sin(\sqrt{\lambda_2} t_e) - C_2 \cos(\sqrt{\lambda_2} t_e) \right] \]

\[ + k_1 - 2C_1 \left( C_{02f} + \sqrt{\lambda_2} \right) \cosh(\sqrt{\lambda_2} t_e) \]

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