Resolved photon processes

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Abstract. We review high-energy scattering processes that are sensitive to the hadronic structure of the photon, describing theoretical predictions as well as recent experimental results. These processes include deep-inelastic electron–photon scattering at $\text{e}^+\text{e}^-$ colliders; and the production of jets, heavy quarks and isolated photons in the collision of real photons at $\text{e}^+\text{e}^-$ colliders, as well as in photon–photon collisions at $\text{e}^+\text{e}^-$ colliders. We also comment on minijet based calculations of total $\gamma p$ and $\gamma \gamma$ cross-sections, and discuss the possibility that future linear $\text{e}^+\text{e}^-$ colliders might produce very large photon fluxes due to the beamstrahlung phenomenon; in the most extreme cases, we predict more than one hadronic $\gamma \gamma$ event to occur at every bunch crossing.

Keywords. Photon structure functions; minijets; $\text{e}^+\text{e}^-$ colliders; linear $\text{e}^+\text{e}^-$ colliders.

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1. Introduction

Among the quarks, leptons and gauge bosons now thought to be truly elementary particles, the photon occupies a special place. Together with the electron it was the first elementary particle correctly identified as such; the understanding of reactions involving these two particles spawned the theory of gauge interactions, now thought to describe all observed (electroweak, strong and gravitational) interactions. In view of this long, distinguished history, it may come as a surprise that there is one large class of photonic interactions about which only relatively little is known: The interaction of real (on-shell) photons with hadrons (or other real photons).

Such reactions can proceed in two quite different ways: The photon can couple directly to a quark or gluon in the struck hadron; in this case the whole energy of the photon goes into the hard (partonic) scattering process. Alternatively, the photon can undergo a transition into a (virtual) hadronic state before encountering the target hadron. In this case a quark or gluon “in” the photon can react, via strong interactions, with partons in the struck hadron. Notice that now only a fraction of the photon’s energy goes into the partonic scattering; the rest is carried away by a “spectator jet” produced by the break-up of the photon. (In both cases the break-up of the struck hadron will also produce such a spectator or remnant jet, at least in the photon–hadron centre-of-mass frame.) In this article we generally refer to the first kind of reaction as direct processes, while those of the second kind are termed resolved photon processes. Cross sections for direct processes are computable from perturbative QCD (assuming the reaction is “hard” enough, i.e. involves a sufficiently large momentum exchange) in terms of parton densities inside the hadron. Similarly, the cross sections for resolved photon processes depend on the parton densities “inside” the photon.

Since the pioneering SLAC measurement [1] of deep-inelastic electron–nucleon scattering, a large body of data on the parton distributions inside nucleons has been accumulated [2]. Further constraints on nucleonic parton densities are imposed by several sum rules, which can be derived directly from QCD. In comparison the picture looks much more sketchy where the parton content of the photon is concerned. Until very recently, the only relevant data were measurements of the electromagnetic structure function $F_2$ in deep-inelastic $e\gamma$ scattering; as we will see in more detail in §2, these measurements suffer from large theoretical and/or experimental uncertainties. Moreover, they only cover a limited kinematical range. Finally, they are not sensitive to the gluon content of the photon, which plays an important role in many resolved photon processes. During the last year experimental analyses of jet production in $\gamma\gamma$ collisions in terms of resolved photon processes have started to appear [3, 4]. Even more recently, the $ep$ collider HERA has started operations; photoproduction processes, including resolved photon reactions, play an important role in its physics programme [5, 6]. It seems therefore timely to review what is known about resolved photon processes and the parton content of the photon, and what we can hope to learn in the near future. We will see in §5 that this information might be crucial for assessing the physics potential of future linear $e^+e^-$ supercolliders.

This article is organized as follows. In §2 we give a short introduction to the photon structure function, describing the theoretical understanding of as well as experimental data on $F_2$. We also briefly compare existing parametrizations of photonic parton densities. In §3 we discuss hard $\gamma\gamma$ processes at existing and planned $e^+e^-$ colliders, while §4 is devoted to hard $\gamma p$ reactions. We will see in both cases that the relative importance of resolved photon processes steadily increases with the
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available centre-of-mass energy. As a result, at existing \( e^+ e^- \) colliders one can probably only study processes with large partonic cross sections, i.e. jet and heavy quark production; in contrast, at HERA and certain \( e^+ e^- \) supercolliders a large variety of final states that receive contributions from resolved photon processes can be produced with detectable rates. In \( \S \) 5 we discuss to what extent "minijet" calculations allow us to predict total \( \gamma p \) and \( \gamma \gamma \) cross sections. We will show that even if the total \( pp \) cross section and the parton densities in the photon were known, sizable theoretical uncertainties would remain due to our insufficient understanding of multiple parton scattering. Finally, \( \S \) 6 contains a summary of our results.

2. The photon structure function \( F_2^\gamma \).

As we know, information about the proton structure is obtained by studying deep inelastic scattering (DIS) of high energy leptons of energy \( E \) off proton targets.

\[
e^- + p \rightarrow e^- + X
\]  

shown in figure 1a. The structure of the proton as revealed to a photon probe of invariant mass \( -Q^2 \) depends on the value of \( Q^2 \). In the DIS regime the process of (2.1) is characterised by two independent kinematic variables, \( y = v/E \) where \( v \) is the energy carried by the probing photon in the laboratory frame, and \( x = Q^2/(2Mv) \) where \( M \) is the proton mass. We also know that the double differential cross-section for this process factorises in the quark parton model (QPM) as,

\[
\frac{d^2 \sigma_{ep}\rightarrow x}{dx dy} = \frac{2\pi\alpha^2_s}{Q^4} \times [(1 + (1 - y)^2)F_2^p(x) - y^2 F_L^p(x)],
\]

where

\[
F_2^p(x) = \sum_q e_q^2 xq^p(x);
\]

\[
F_L^p(x) = F_2^p(x) - 2xF_1^p(x)
\]

are the two electromagnetic structure functions of the proton. The longitudinal structure function \( F_L^p(x) \) is zero in QPM, \( q^p(x) \) the probability for quark \( q \) to carry a momentum fraction \( x \) of the proton and \( e_q \) denotes the electromagnetic charge of quark \( q \) in units of the proton charge. This factorisation of the \( x \) and \( Q^2 \) dependence in (2.2) is of course only approximate. In general \( F_2^p \) depends on \( x \) as well as \( Q^2 \) and \( F_L^p(x) \) is nonzero. QCD predicts the \( Q^2 \) dependence of the structure functions, given by the Gribov–Lipatov–Altarelli–Parisi (GLAP) [7] equations but does not say anything about the shape of hadronic structure functions. According to QCD

\[
\begin{align*}
\text{(a)} \quad & e^+ e^- (\text{Probe}) \\
\text{(Target)} & \rightarrow \gamma^* \rightarrow \text{hadrons} \\
\text{(b)} \quad & e^+ e^- (\text{Probe}) \\
\text{(Target)} & \rightarrow \gamma \rightarrow \text{hadrons}
\end{align*}
\]

Figure 1. Deep inelastic scattering for the proton and photon.
predictions all the hadronic structure functions shrink to lower values of $x$ as $Q^2$ increases.

The idea that photons behave like hadrons when interacting with other hadrons dates back to the early days of strong interaction physics and is known to us under the name of the Vector Meson Dominance (VMD) picture. This essentially means that at low 4-momentum transfer, the interaction of a photon with hadrons is dominated by the exchange of vector mesons which have the same quantum numbers as the photon. While this picture works reasonably well for "soft" processes (i.e., reactions characterized by small 4-momentum transfer), it is not at all clear that it should describe the whole story of interactions of photons with hadrons at high energies as well.

Since a photon is known to behave like a hadron, it seems reasonable that it should be possible to probe its structure also in a DIS experiment. Such an experimental situation is provided at $e^+e^-$ colliders in $e^+e^- \rightarrow \gamma^*$ reactions as shown in figure 1b. Here the virtual photon with invariant mass square $-Q^2$ probes the structure of the real photon. If the VMD picture were the whole story then one would expect that such an experiment will find

$$F_2^\gamma \simeq F_2^{\gamma,\text{VMD}} \propto F_2^{p^0} \simeq F_2^{e^0}.$$  \hspace{1cm} (2.4)

Then with increasing $Q^2$, the structure function $F_2^\gamma$ will behave just like a hadronic proton structure function. However, there is a very important difference in case of photons, i.e., photons possess pointlike couplings to quarks. This has interesting implications for $\gamma^*\gamma$ interactions as first noted in the framework of the QPM by Walsh \cite{8}. It essentially means that $\gamma^*\gamma$ scattering in figure 1 contains two contributions as shown in figure 2. The contribution of figure 2a can be estimated by (2.4), whereas that of figure 2b was calculated in the QPM \cite{8}. This is done by considering the cross-section for the reaction

$$\gamma + \gamma^* \rightarrow q + \bar{q}.$$  

Due to $t$ and $u$ channel poles this can be calculated only when one considers quarks with finite masses. The result can be recast in a form equivalent to (2.2):

$$\frac{d^2 \sigma^{\gamma\gamma \rightarrow X}}{dx dy} = \frac{2\pi x^2 s_{e^+e^-} \times 3x}{Q^4} \sum_q e_q^4 \left\{ (1 + (1 - y)^2) \times [x(x^2 + (1 - x)^2) \times \ln \frac{W^2}{m_q^2} ight.$$  

$$+ 8x^2(1 - x) - x] - y^2[4x^2(1 - x)] \right\}, \hspace{1cm} (2.5)$$

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure2.png}
\caption{Two contributions to $F_2^\gamma$.}
\end{figure}
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where \( W^2 = Q^2(1-x)/x \). On comparing (2.2) and (2.5) we see that the factors in square brackets in the above equation have the natural interpretation as photon structure functions \( F_2^\gamma \) and \( F_L^\gamma \) and one has

\[
F_2^\gamma,\text{pointlike}(x, Q^2) = \frac{3}{\pi} \sum_q e_q^2 \left[ x(x^2 + (1-x)^2) \times \ln \frac{W^2}{m_q^2} + 8x^2(1-x) - x \right] \\
= \sum_q e_q^2 xq^\gamma,\text{pointlike}(x, Q^2).
\]

(2.6)

Two points are worth noting: the function \( F_2^\gamma,\text{pointlike}(x, Q^2) \) can be completely calculated in QED and secondly this contribution to \( F_2^\gamma \) increases logarithmically with \( Q^2 \). So in this simple “VMD + QPM” picture, \( F_2^\gamma \) consists of two parts, \( F_2^\gamma,\text{pointlike} \) and \( F_2^\gamma,\text{VMD} \), with distinctly different \( Q^2 \) behaviour and with the distinction that for one part both the \( x \) and the \( Q^2 \) dependence can be calculated completely from first principles.

This QPM prediction received further support when it was shown by Witten [9] that at large \( Q^2 \) and at large \( x \), both the \( x \) and \( Q^2 \) dependence of the quark and gluon densities in the photon can be predicted completely even after QCD radiation is included. An alternative way of understanding this result is to consider the evolution equations [10] for the quark and gluon densities inside the photon. These contain an inhomogeneous term on the r.h.s. proportional to \( \alpha \), which describes \( \gamma \rightarrow q\bar{q} \) splitting, i.e. the pointlike coupling of photons to quarks. In the ‘asymptotic’ limit of large \( Q^2 \) and large \( x \), the \( q_i^\gamma(x, Q^2) \) have the form

\[
q_i^\gamma,\text{asymp}(x, Q^2) \propto Q \times \ln \left( \frac{Q^2}{\Lambda_{\text{QCD}}^2} \right) F_i(x) \\
\approx \frac{\alpha}{\alpha_s} F_i(x),
\]

(2.7)

where \( \Lambda_{\text{QCD}} \) is the usual QCD scale parameter and the \( x \) dependence of the \( F_i(x) \) is completely calculable. Comparing (2.6) and (2.7) we see that the essential change, apart from the different \( x \) dependence, between the leading order (LO) QCD and QPM predictions is the replacement of \( m_q^2 \rightarrow \Lambda^2 \). We then have

\[
F_2^\gamma,\text{asymp} = \sum_q e_q^2 xq^\gamma,\text{asymp}.
\]

(2.8)

The asymptotic form of \( G^\gamma(x, Q^2) \) can also be uniquely calculated. These asymptotic predictions, however, show unphysical divergences as \( x \rightarrow 0 \) indicating thereby that at small \( x \) the “hadronic” part of \( F_2^\gamma \) can not be neglected, and is not adequately described by a regular, VMD-inspired ansatz. The initial enthusiasm that this completely calculable prediction can be used as a test of QCD and also for a high precision measurement of \( \alpha_s \), suffered a big set back by the observation [11] that the degree of the \( x \rightarrow 0 \) pole becomes larger in higher orders of perturbation theory; we therefore have to conclude that the separation of \( F_2^\gamma \) in two parts \( F_2^\gamma,\text{VMD} \) and \( F_2^\gamma,\text{asymp} \) is not physical. One way out of this is to add a hadronic contribution to \( F_2^\gamma,\text{asymp} \) with similar divergence so as to get a finite result at small \( x \) [12]. This involves arbitrary parameters, but keeps the hope of being able to use the calculable pointlike contribution for QCD tests. An alternative suggestion is to use the experimental data.
on $F_2$ to fit a finite input distribution for $q^r$ at a scale $Q_0^2 \approx 1 \text{GeV}^2$, thus retaining the predictability only of the $Q^2$ dependence but gaining the ability to calculate physically meaningful quantities for all combinations of $x$ and $Q^2 \geq Q_0^2$ [13]. The theoretical debate on the subject is still not completely closed [14].

By now several parametrizations of the parton densities inside the photon $q^r(x, Q^2) \equiv (q^r_i, G^r)(x, Q^2)$ exist. The oldest parametrizations [15, 16] are based on the “asymptotic” LO predictions. However, these parametrizations are even more singular as $x \to 0$ than the exact “asymptotic” prediction. Care must therefore be exercised if these parametrizations are to be used at small $x$.

The first parametrization that followed the suggestion of ref. [13] to fit input distributions at some low scale $Q_0^2$ such that data at higher $Q^2$ are reproduced is the “DG” parametrization of ref. [17]. In 1984, when this parametrization was constructed, only a single set of data on $F_2$ existed [18]. Since then, more data have become available [19, 4]. Most of these newer data were taken into account in the “LAC” fits of ref. [20]. Indeed, taken at face value, low-$Q^2$ data from the TPC/27 collaboration [21] at PEP disfavour the older DG parametrization compared to the LAC fits. However, the interpretation of these data is not entirely straightforward. A general problem of the measurement of the $x$ dependence of $F_2$ is that, unlike in deep-inelastic $ep$ scattering, $x$ has to be determined from the hadronic final state, since the energy of the target photon is not known [22]. Unfortunately some of the final state particles are usually lost in the beam pipe. One therefore needs fairly sophisticated “unfolding” techniques in order to determine the true value of $x$ from the observed final state. Notice that some model of $F_2$ has to serve as an input for this unfolding procedure. Usually a simple “QPM + VMD” description is used for this purpose, even though we have seen above that this picture is not very meaningful within QCD.

Low-$Q^2$ data suffer from two additional problems. First of all, higher twist contributions might be quite important. Often only the region $W > 2 \text{GeV}$ is used for QCD comparisons, i.e. the region of large $x$ is discarded; it is not clear, however, whether this is sufficient to really make higher twist contributions negligibly small. Secondly, at low $x$ and low $Q^2$ one actually has $W^2 > Q^2$; in this case it is not clear that $Q^2$ is indeed the relevant scale in the process. Ideally one would like a unified treatment of real $\gamma\gamma$ (no-tag) and $\gamma^*\gamma$ (single-tag) data, allowing for a smooth transition from one to the other. Work along these lines is in progress [23]. At present we do not think it advisable to base the exclusion of a parametrization of $q^r(x, Q^2)$ on these low-$Q^2$ data alone.

The parametrizations of refs. [17, 20] are the only ones that allow an arbitrary form at least of the quark densities at scale $Q_0^2$; this leads to a fairly large number of free parameters which, in view of the paucity of good data, cannot be determined very precisely. The authors of ref. [24] have therefore used theoretical considerations (or prejudices) to fix the shape of the input distributions, reducing the number of free fit parameters to one. They assume that (2.4) is valid at the rather low input scale $Q_0^2 = 0.25 \text{GeV}^2$; this scale has been taken from the “dynamical” fits to proton structure functions by the same authors [25]. Similarly, their pionic structure functions [26] were used to fix the shape of the input distributions. The only free parameter is then the overall size of the input distributions; the idea is that this parameter describes the contribution of vector mesons beyond the $\rho$.

Finally, for the “GS” parametrization of ref. [27] an intermediate approach has been taken. In order to avoid the ambiguities of low-$Q^2$ data, rather high input scale
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\( Q_0^2 = 5.3 \text{ GeV}^2 \) has been chosen. At this scale, \( F_2^e \) is assumed to be described by the "QPM + VMD" model; however, the gluon and sea-quark distributions in the pion, the overall size of the VMD (pionic) contribution, as well as the quark masses in eq. (2.6) are all allowed to vary within reasonable limits.

In figure 3a we compare various parametrizations for \( F_2^e(x, Q^2 = 5.3 \text{ GeV}^2) \) with each other and with experimental data from the PLUTO collaboration [18]. Since only the contributions from \( u, d \) and \( s \) quarks have been included in the calculation, charm-subtracted data have been used. We observe that all shown parametrizations describe the data reasonably well, although none of the fits is perfect. The differences between the various parametrizations only amount to typically 20% in the region \( x > 0.05 \). Due to the inhomogeneous term in the evolution equation, these differences tend to be even smaller at higher \( Q^2 \); statistical errors are also larger for these theoretically cleaner high-\( Q^2 \) data. The best possibility to discriminate between different parametrizations using data on \( F_2^e \) therefore seems to lie in high-statistics measurements at intermediate values of \( Q^2 \) and small \( x \).

In figure 3b we compare the parametrizations of the gluon density at the same value of \( Q^2 \). Obviously the various parametrizations differ much more strongly here than for the quark densities that determine \( F_2^e \). The extreme behaviour of the LAC parametrizations is especially noticeable; this is because in these parametrizations no theoretical assumptions about \( G^g(x, Q^2) \) were made. The very hard gluon density of LAC3 is necessary to explain the rapid increase of \( F_2^e(x \simeq 0.1, Q^2 < 4 \text{ GeV}^2) \) with \( Q^2 \) seen in the data [21]; at \( Q^2 > Q^2_0 \), some of the gluons at large \( x \) are converted into \( q\bar{q} \) pairs, leading to the maximum of \( F_2^e(x \simeq 0.1) \) shown in figure 3a for this parametrization. These low-\( Q^2 \) data were ignored for the LAC2 fit, which uses \( Q^2_0 = 4 \text{ GeV}^2 \); now a very soft gluon density is favoured. It should be noted, however, that the LAC3 parametrization also describes the high-\( Q^2 \) data adequately; this shows that \( F_2^e \) is not very sensitive to \( G^g \).

For the other three parametrizations the gluon input is essentially fixed from the quark input. We already saw that the GRV fit only contains a single free parameter, which is fixed from data on \( F_2^e \). In the DG parametrization it is assumed that gluons are only produced radiatively from quarks, i.e. there is no truly intrinsic (VMD-like or otherwise) gluon content of the photon; as a result, the DG gluon density falls below that of the other parametrizations, except for very small \( x \) where it resembles the GRV and LAC3 gluons. Note, however, that the DG parametrization has been obtained using a rather large value for the QCD scale parameter \( \Lambda = 400 \text{ MeV} \), while all other parametrizations assume \( \Lambda = 200 \text{ MeV} \). The same value of \( \Lambda \) should also be used in the factors of \( a_q \) that occur in resolved photon cross-sections. As a result, the DG parametrization sometimes leads to larger predictions for such cross-sections, in spite of its smaller gluon content. Finally, in the GS2 parametrization, \( G^g(x, Q^2_0) \) receives contributions both from a VMD-like (pionic) term, and from radiation off the (QPM) quarks [28].

There are also versions of the GRV and GS parametrizations that include higher order effects, i.e. where the 2-loop evolution equations [11] have been used. However, in view of the large uncertainties at least in the gluon densities, this seems at present an unnecessary refinement. Moreover, full higher order calculations do not yet exist for most resolved photon processes; we will come back to this point later.

This concludes our discussion of deep-inelastic \( e\gamma \) scattering. While theoretically relatively clean, since one computes a fully inclusive cross section, we have seen that this is probably not the best way to give us detailed information about the partonic...
structure of the photon, as witnessed by the large differences between parametrizations of the gluon density. As we will see in the following sections, this information might be provided by resolved photon processes involving only real photons.
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3. Real $\gamma\gamma$ scattering at $e^+ e^-$ colliders

Effects of the photon structure and especially the gluon component of the photon are best studied in processes involving real photons. In this section we will discuss only the possibilities of probing the photon structure at $e^+ e^-$ colliders. Possible effects of the hadronic component of the photon at $ep$ colliders and at fixed target photoproduction experiments will be discussed in the next section. At $e^+ e^-$ colliders processes other than DIS which could yield information about the photon structure function are jet production, heavy flavour production and prompt production initiated by partons in the photon. Compared to DIS (single-tag) processes, real $\gamma\gamma$ reactions have cross-sections which are enhanced by a factor $O(s/(4m_Q^2))^2 \approx 20$ at existing colliders. The scale at which the parton densities in the photon $q(x, Q^2)$, will be probed is decided by the $p_T^2$ of the jets, which is comparable to the higher end of $Q^2$ values accessible to DIS experiments at present colliders.

3.1 $e^+ e^- \rightarrow jets + X$

Jet production in $\gamma\gamma$ collisions can receive contributions from three different types of diagrams [29] as shown in figure 4. The ‘direct process’ of figure 4a is due to $\gamma\gamma \rightarrow q\bar{q}$ production, present already in the naive quark–parton model. Figure 4b depicts the case where only one photon is resolved into its partonic components, which then interact with the other photon; we call these the ‘once-resolved’ processes (‘1-res’ for short). Finally, figure 4c shows the situation where both photons are resolved, so that the hard scattering is a pure QCD $2 \rightarrow 2$ process; we call these the ‘twice-resolved’ contributions (‘2-res’ for short). It is very important to note here that every resolved photon will produce a spectator jet of hadrons with small transverse momentum relative to the initial photon direction, which for (quasi-) real photons coincides with
the beam direction. The resolved contributions of figure 4b and c can therefore be separated if one can tag on these spectator jets.

The cross-section for jet production in $\gamma\gamma$ collisions for the '2-res' processes can be written schematically as [30, 31]

$$d\sigma = f_{\gamma/e}(x_1)q^q(x_2, Q^2)f_{\gamma/e}(x_3)q^q(x_4, Q^2)d\hat{s},$$

where the $\delta$ are the cross-sections for the hard $2 \rightarrow 2$ subprocesses [32, 15], $q^q(x, Q^2)$, $f_{\gamma/e}$ denote parton densities inside the photon and photon fluxes inside the electron respectively; we include non-leading contributions to $f_{\gamma/e}$, following [33]. For the '1-res' (direct) processes, one (both) of the parton density functions $q^q(x, Q^2)$ have to be replaced by $\delta(1-x)$, and the proper hard sub-process cross-sections have to be inserted [15]. Recall (2.7) for $q^q$. This makes it clear that all three classes of diagrams are of the same order in $\alpha$ and $\alpha_s$.

While it is clear that our present knowledge of $q^q$ is not precise enough to make absolute predictions, one can check how sensitively the predicted cross-sections depend on the choice of $q^q$. The DG parametrization will usually give us the most conservative prediction of the available parametrizations, as can be seen from figure 3.

In figure 5 we show the energy dependence of the cross-section for the production of two jets with $p_T = 3$ GeV, as predicted [31] by the DG parametrization, in the range covered by the PETRA and TRISTAN colliders. For this choice of $p_T$ the cross-section is quite sizable; recall that the total luminosity collected at PETRA amounts to several hundred pb$^{-1}$ per experiment, while as of this writing, TRISTAN has collected about 100 pb$^{-1}$. The cross-section is also well above the background from annihilation events with hard initial state radiation (dotted curve). We also note that the twice-resolved contribution grows faster than $\sqrt{s}$ with increasing machine energy and, for this choice of $p_T$, begins to dominate the cross-sections in the energy range of TRISTAN. Notice that “jets” with smaller transverse momentum can even originate from the poorly understood soft (VMD) contribution to the $\gamma\gamma$ cross-section; this contribution is essentially negligible if $p_T \geq 2$ to 3 GeV. Thus TRISTAN is in a unique position to probe the structure of the photon through jet production. Our detailed studies [31] do indeed indicate that studying the production of high $p_T$ jets and heavy flavour (charm) at TRISTAN should be able to probe the hadronic content of photon in some detail.

![Graph](image_url)

**Figure 5.** $d\sigma/dp_T$ at $p_T = 3$ GeV as a function of $\sqrt{s}$ [31].
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This is demonstrated in figure 6, where we compare the $p_T$ spectrum of jets produced in $\gamma\gamma$ collisions at $\sqrt{s(e^+e^-)} = 60$ GeV, as predicted by the DG (a) and LAC3 (b) parametrizations. We already saw in the previous figure that the DG parametrization predicts the three classes of processes to contribute roughly equally at $p_T = 3$ GeV. At larger values of $p_T$ the direct process starts to dominate. The reason is simply

![Figure 6](image)

**Figure 6.** The transverse momentum spectrum of jets produced in real $\gamma\gamma$ scattering at $\sqrt{s} = 60$ GeV, for (a) DG and (b) LAC3 parametrizations.
that a resolved photon has to split its energy between the parton participating in the hard scattering on the one hand, and a spectator jet on the other; therefore the cross-section for resolved photon processes will depend more strongly on the available phase space, i.e. will have a steeper $p_T$ spectrum. Nevertheless, even for the DG parametrization the sum of once and twice resolved contributions exceeds the direct contribution out to $p_T \approx 4.5$ GeV.

The extremely hard gluon density of the LAC3 parametrization (see figure 3b) greatly enhances the cross-section for twice resolved processes compared to the predictions of the DG parametrization. Note that out of the eight $2 \rightarrow 2$ QCD scattering matrix elements, that for $gg \rightarrow gg$ scattering is the largest, followed by the one for $qg \rightarrow qg$ [32]. As a result, LAC3 predicts the 2-res contribution to be almost an order of magnitude larger than DG; it also predicts an approximately two times larger 1-res contribution. The LAC3 parametrization therefore predicts the high-$p_T$ jet cross-section to be dominated by resolved photon contributions up to $p_T \approx 9.5$ GeV.

What is the experimental situation? Note that even at the lower end of the curves in figure 5, at PEP/PETRA energies, one expects a sizable contribution to the jet cross-section from resolved processes in addition to the 'direct' process. In this context it is interesting to note that almost all the groups at PEP/PETRA observed [34] such an excess of jet events. These experiments compared data with a two component model where they added to the direct process a soft component (with exponential $p_T$ spectrum) expected from the VMD picture and always failed to reproduce the data. The data always had yet another 'third component' with a $p_T$ spectrum softer than QPM but broader than VMD and thrust distribution broader than the QPM prediction. Both features are expected for resolved contributions.

The first experimental analysis including resolved photon contributions has recently been performed by the AMY collaboration at TRISTAN [3]. They modelled the three classes of hard contributions to high-$p_T$ jet production using eq. (3.1), but included the full machinery of initial and final state parton showers predicted by QCD, as well as parton → hadron fragmentation. They conclude that inclusion of resolved photon processes as predicted using the DG parametrization greatly improves the agreement between Monte Carlo predictions and data.

An example is shown in figure 7 [35], which shows the $p_T$ spectrum of their data

![Figure 7](image_url)

**Figure 7.** Data on $\gamma \gamma \rightarrow$ jets and predictions with and without 'res' contributions [35].
Resolved photon processes

Sample. It should be noted that, like previous analyses of $\gamma\gamma$ scattering [34], AMY does not use a jet finding algorithm. Rather, the entire event is divided into two hemispheres, perpendicular to the thrust axis; the $p_T$ shown in the figure is then simply the sum over the transverse momenta of all particles in one hemisphere. Notice also that the AMY trigger requires the event to contain at least one charged particle with $p_T \geq 1.0$ GeV. This suppresses events with very small $p_T$ per hemisphere, and further complicates the relation between the partonic and hemispheric transverse momentum.

The agreement between QCD MC predictions (solid histogram) and data (points) shown in figure 7 is indeed quite impressive, in particular when compared to the prediction of the traditional “QPM + VMD” model (dashed histogram). It should be noted that the AMY Monte Carlo contains a number of free parameters beyond those determining the $q^2(x, Q^2)$. The most important one is the cut-off $p_{T,\text{min}}$, which is the smallest partonic transverse momentum allowed in the hard scattering diagrams of figure 4. Of course, these diagrams diverge badly as $p_T \to 0$; QCD does not tell us, however, just how large the partonic $p_T$ has to be for its predictions to become trustworthy. AMY thus simply fits $p_{T,\text{min}}$ from their data with $p_{T,\text{min}} > 1.5$ GeV; they find $p_{T,\text{min}} = 1.6$ GeV for the DG parametrization, if only the three light flavours of quarks are assumed to be present in the photon. The same value of $p_{T,\text{min}}$ also leads to a good description of the $p_T^{\text{thrust}}$ spectrum in the theoretically cleaner region beyond 3 GeV. Other free parameters include the amount of intrinsic $p_T$ allowed for the partons inside the photon, and parameters describing the hadronization process. The experimental observables that have been studied so far, in particular the thrust distribution, do not seem to depend much on the former, but are quite sensitive to the latter. More detailed analyses of higher statistics data taken with an upgraded AMY detector are now being carried out. However, the existing data are already good enough to rule out the LAC3 parametrization; the data are clearly incompatible [35] with the huge rate of resolved photon events predicted by this parametrization, see figure 6. In contrast, the LAC1 or LAC2 parametrizations, with $p_{T,\text{min}} = 2.0$ GeV, seem to describe the recent AMY data slightly better than the DG parametrization does [36]. On the other hand, a toy-model with zero gluon content of the photon falls short of the data, even if $p_{T,\text{min}}$ is allowed to be as small as 1.0 GeV [3]. Finally, by comparing their own data with data taken at the PETRA collider, AMY could show that the importance of the resolved photon processes increases with energy [3], as expected from figure 5.

More recently, other experimental groups have also entered the fray. In particular, the TOPAZ collaboration at TRISTAN has presented a preliminary analysis [37] of their data. Compared to the AMY detector, TOPAZ has the advantage of a lower trigger threshold, which merely requires the presence of 2 charged tracks with $p_T \geq 0.3$ GeV in the event; for a given luminosity, this leads to an approximately two times larger $\gamma\gamma$ data sample than at AMY. This should allow for a more detailed study of the region with low and intermediate $p_T$, which is however difficult to interpret theoretically. The $p_T$ distribution presented by TOPAZ is in good agreement with the AMY result, while the thrust distributions seem to differ somewhat.

Very recently TOPAZ has also presented [38] preliminary results of an analysis which, for the first time in $\gamma\gamma$ physics, actually requires jets to be reconstructed, using an algorithm familiar from hadron collider studies of jets. They used an unfolding procedure to extract the partonic cross-section from the measured jet rates. The results
for the cross-section integrated over $2.5 \text{ GeV} \leq p_T (\text{parton}) \leq 8 \text{ GeV}$ are:

$$\sigma(|y_1|, |y_2| \leq 0.7) = 23.4 \pm 2.7 \pm 1.7 \text{ pb}$$

$$\sigma(|y_1| \leq 0.7, y_2 \text{ anywhere}) = 96.7 \pm 3.7 \pm 8.5 \text{ pb.} \quad (3.2)$$

The first result corresponds to the situation where both high-$p_T$ jets are reconstructed, while the second result includes events where only one high-$p_T$ jet is seen. Notice that these are $e^+e^-$, not $\gamma\gamma$, cross-sections. The systematic errors include an estimate of the effect of varying $p_{T,\text{min}}$ between 1.6 and 2 GeV. An additional 7% uncertainty is included in the single jet cross-section; this is the estimate for the contribution from soft processes. (Their contribution to the di-jet cross-section is negligible.) These numbers are preliminary; in particular, the error caused by fragmentation uncertainties has not yet been included. These numbers are reproduced equally well by the DG and LAC1, 2 parametrizations; e.g., DG predicts 21.4 and 90.9 pb for the first and second cross-section in (3.2), respectively. The predictions of the LAC3 parametrization are almost three times too large, so that this parametrization is clearly excluded; the DO + VMD parametrization is also disfavoured. The extraction of a partonic cross-section is an important step, since this allows to directly compare the predictions of theoretical models of $q^r(x, Q^2)$ with their data; it also simplifies the comparison of experimental data from different groups.

Finally, it should be mentioned that two LEP groups, ALEPH and DELPHI, have also presented first preliminary results on $\gamma\gamma$ scattering [39, 40]. The size of their data samples is only about 20% of those of the TRISTAN groups; in addition, the very large annihilation cross-section at $\sqrt{s} \approx m_Z$ poses special background problems [41]. Nevertheless, both groups confirm that traditional "QPM + VMD" models are in conflict with their data, while the inclusion of resolved photon contributions, as predicted from the DG parametrization, leads to satisfactory agreement between MC results and data. Notice that the good angular coverage of LEP detectors can give them an advantage when trying to find direct evidence for spectator jets, which so far have only been observed in $\gamma\gamma$ collisions via their contribution to the thrust distribution. Good angular coverage is also necessary for the study of events where one or both high-$p_T$ jets emerge at small angles, which would allow to extend the range of $x$ values probed by a given experiment.

We have seen that the jet cross-sections and especially the resolved photon contributions fall off sharply with increasing $p_T$; e.g. in figure 6a the '2-res' processes dominate only up to $p_T \approx 3 \text{ GeV}$. However, if we consider $d\sigma(jj)/dm_{jj}$ where $m_{jj}$ is the invariant mass of the two high-$p_T$ jets, then the resolved processes dominate to higher values. This can be understood from the fact that only 2-res processes receive contributions from diagrams where a gluon is exchanged in the $t$- or $u$- channel, leading to a more singular dependence of the hard subprocess cross-section on the square of momentum transfer $\hat{t}$ for these contributions. Thus the study of invariant mass distributions, as well as rapidity distributions, can help us to get more detailed information on $q^r$ [31]. If one can measure the spectator jet energy instead of only using the spectator jet activity to 'tag' the 'res' contributions, it may be possible to separate gluon initiated events from quark initiated ones. Since $G^r(x, Q^2)$ is peaked at smaller values of $x$ as compared to $q^r(x, Q^2)$, the spectator jet energies will be higher for events initiated by gluons in the photon.
3.2 Heavy quark production

As already stressed at the end of §2, at present we have much less information about the gluon content of the photon than about its quark content. Unfortunately, the production of central jets with $p_T \geq 3 \text{GeV}$ is sensitive mainly to the region $x \geq 0.15$ at TRISTAN and LEP1 energies [42]. As a result, the study of high-$p_T$ jet production at these colliders is unlikely to discriminate between different ansätze for $G^\gamma$, provided only that it is “soft” (this rules out LAC3, as we have seen). The production of open or hidden charm might offer better opportunities to get information about the gluon content of the photon. First of all, there is no need of a $p_T$ cut to get rid of a “soft” component, since there is none. The invariant energy going into the hard scattering can therefore be almost a factor of 2 smaller than for the clean sample of high-$p_T$ jets, and correspondingly smaller values of $x$ can be probed. Secondly, even if it turns out that some $p_T$ cut has to be imposed to allow for the identification of charm events, the sensitivity to $G^\gamma$ is still greater than for inclusive jet production, because there is less background from resolved photon events initiated by quarks: The only 1-res contribution comes from $\gamma g$ fusion, and the 2-res contribution from $q\bar{q}$ annihilation is predicted to be very small. Any nonzero signal for $c\bar{c}$ production via resolved photon processes would therefore allow a direct measurement of $G^\gamma$.

In figure 8 we compare predictions [31] for the total $c\bar{c}$ cross-section as calculated from the DG and LAC1 parametrizations, in the energy range covered by TRISTAN and LEP. For this and the following figures of this section, we have modified the estimate of the flux of resolved bremsstrahlung photons. The standard expression of ref. [33] has been derived by integrating the photon propagator $\propto 1/P^2$ over the full kinematically allowed range of the photon virtuality $-P^2$. However, if $P$ is larger than the scale $Q$ characterizing the hard scattering process, the picture of real (on-shell) partons residing “in” the photon is no longer valid. Therefore the upper limit of the

![Figure 8](image-url)  

Figure 8. The total cross-section for $e^+e^-\rightarrow e^+e^- c\bar{c}X$ as a function of $\sqrt{s}$.  

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$P^2$ integration should be of order $Q^2$. Moreover, it has been known for some time [43] that the parton content of virtual photons with $\Lambda_{\text{QCD}}^2 < P^2 < Q^2$ is suppressed compared to the parton content of on-shell photons. We attempt a crude estimate of this effect by introducing a further suppression factor of 0.85 for the bremsstrahlung flux of resolved photons; this number has been estimated from numerical results of Rossi [43]. Altogether we thus have:

$$f_{g/e}^{\text{brems}}(x) = 0.85 \frac{\alpha}{2\pi} [1 + (1 - x)^2] \ln \frac{Q^2}{m_e^2}. \quad (3.3)$$

The effect of this refinement will obviously be larger for larger ratio $s/Q^2$; this is why we did not introduce it in our predictions for jet production at TRISTAN energies and below. Of course, the formula of [33] is still applicable for the flux of bremsstrahlung photons interacting directly. It may be noted here that as has been pointed out [44] recently the abovementioned suppression ought to be different for quarks and gluons. Since in perturbation theory gluons can only be radiated off quarks, which are themselves off-shell if $P^2 \neq 0$, their density in the photon drops faster with increasing virtuality of the photon than the quark content does.

We see that, with the exception of the immediate vicinity of the Z pole, the two-photon cross section is larger than the one for the corresponding annihilation process $e^+e^- \rightarrow c\bar{c}$, by a factor of at least 8 (200) at $\sqrt{s} = 60 (200) \text{GeV}$. Secondly, just as in case of jet production, the contributions from resolved photon processes grow substantially faster with energy than that of the direct $\gamma\gamma \rightarrow c\bar{c}$ process. However, at least for the more conservative DG parametrization, the direct contribution still dominates the total cross-section even at $\sqrt{s} = 200 \text{GeV}$. Cuts on the transverse momentum or angle of the produced charm quarks will further reduce the importance of the resolved photon contribution, since it has a softer $p_T$ spectrum and a more asymmetric angular distribution than the direct contribution. If the actual gluon content is indeed described by the DG parametrization, $c\bar{c}$ production can therefore only be used to measure $G^\gamma$ if the direct contribution can be suppressed by detecting the spectator jet from the resolved photon, either directly or via the total thrust distribution. The GRV parametrization even predicts slightly (by 5 to 10%) smaller resolved photon contributions, due to the smaller value of $\alpha_s$ that has to be used with this parametrization; note that at the rather low energy scales characteristic for charm production, $\alpha_s$ depends quite sensitively on $\Lambda_{\text{QCD}}$.

On the other hand, the LAC1 parametrization predicts the resolved photon contribution to dominate already for $\sqrt{s} \geq 55 \text{GeV}$; at $\sqrt{s} = 200 \text{GeV}$ it predicts an almost 3 times larger total $c\bar{c}$ cross-section than the DG parametrization does. (Similar results also hold for the LAC2 parametrization.) If the LAC1 predictions turn out to be close to the truth, isolation of the resolved photon contribution to $c\bar{c}$ production should not be very difficult. However, even in this extreme case the 2-res contribution only amounts to 1-5% of the total at the highest LEP energy; any resolved photon signal in $c\bar{c}$ production can therefore safely be identified as stemming from 1-res photon-gluon fusion process.

### 3.3 $J/\Psi$ Production

Another interesting process at existing $e^+e^-$ colliders is the production of $J/\Psi$ mesons
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in the reaction

\[ e^+ + e^- \rightarrow \gamma + \gamma \rightarrow J/\Psi + X. \quad (3.4) \]

In this case the relevant hard scattering process is \( \gamma + g \rightarrow J/\Psi + g \), which can be estimated from the colour singlet model \([45]\). Notice that this requires one of the photons to be resolved; in leading order in \( \alpha \) and \( \alpha_s \), there is no direct contribution. Moreover, as in case of open charm production, the 2-res contribution is negligible. As a result the process is an extremely clean probe of \( G^\prime(x, Q^2) \) for \( x \sim 0.01-0.05 \). The main problem in this case is the rather small total cross section; the DG parametrization predicts the cross-section to grow from 0.5 pb at \( \sqrt{s} = 60 \text{ GeV} \) to 4.0 pb at \( \sqrt{s} = 200 \text{ GeV} \) \([31]\). The LAC1, 2 parametrizations again lead to more than 3 times larger cross-sections. However, almost certainly only the 12% of all \( J/\psi \) mesons that decay into \( e^+ e^- \) or \( \mu^+ \mu^- \) pairs will be detectable. Moreover, there are indications from the photoproduction of \( J/\psi \), which proceeds via the same hard scattering process, that the LO prediction of the colour singlet model might be too low by as much as a factor of 5 \([46]\). While this is good news as far as the observability of the \( J/\psi \) signal at \( e^+ e^- \) colliders is concerned, this large theoretical uncertainty means that at present this process cannot be used for a reliable measurement of the absolute size of the gluon component of the photon \([47]\).

3.4 Direct photon production

Given a sufficiently large data sample, one might even attempt to look for \( \gamma \gamma \) processes that occur only in higher orders in \( \alpha \). An example is the study of the production of prompt photons in \( \gamma \gamma \) collisions via the processes,

\[ q^\gamma + \gamma \rightarrow q + q \]
\[ q^\gamma + \bar{q}^\gamma \rightarrow \gamma + g \]
\[ g^\gamma + q^\gamma \rightarrow q + q \quad (3.5) \]

These processes, though suppressed by a factor of \( \alpha \) compared to the case of jet production, have the advantage of having a cleaner final state. In the PEP to TRISTAN energy range the 1-res contribution clearly dominates \([48]\); requiring \( p_T^\gamma (= p_T^\text{jet} \text{ in leading order}) \) to be larger than 1.5 GeV (a value very close to the value of \( p_{T,\text{min}} \) as determined from the AMY jet analysis \([3]\) described above), the DG parametrization predicts a total \( \gamma + \text{jet} \) cross-section of about 1 pb at \( \sqrt{s} = 60 \text{ GeV} \). At LEP energies the 2-res contributions become more significant, and might allow to extract additional nontrivial information about the parton content of the photon \([48]\).

3.5 Beamstrahlung and hard \( \gamma \gamma \) processes at linear colliders

We have seen repeatedly that, for fixed transverse momenta of the particles produced in the hard scattering process or fixed invariant mass of the system produced in that scattering, the importance of resolved photon events increases quite rapidly with increasing beam energy; under the same circumstances, the ratio of \( \gamma \gamma \) to annihilation events also grows rapidly with \( \sqrt{s} \), as we have seen for the example of the total charm production cross-section. At future \( e^+ e^- \) colliders it might therefore no longer be possible to consider \( \gamma \gamma \) events as a background that can be suppressed easily.
This is demonstrated in figure 9, where we show the energy dependence of the total cross-section for the production of a pair of central jets (with rapidity $|\eta_{1,2}| \leq 2$) with $p_T \geq 5$ GeV in $\gamma\gamma$ collisions at high energy $e^+e^-$ colliders. Here we have conservatively ignored all effects of beamstrahlung (see below), and have used (3.3) for the flux of resolved photons. Nevertheless, even the DG parametrization predicts resolved photon contributions to be dominant already at $\sqrt{s} = 200$ GeV; at $\sqrt{s} = 500$ GeV, which is now foreseen as the likely operating energy of the next (linear) $e^+e^-$ collider, resolved photon processes are predicted to dominate the direct one by a ratio of 6:1. Once again the LAC2 parametrization predicts both a more rapid increase with energy, and a considerably larger absolute value, of the $\gamma\gamma\rightarrow$jets cross-section. Notice finally that the annihilation cross-section at $\sqrt{s} = 500$ GeV only amounts to 0.4 pb for $\mu^+\mu^-$ pairs, and 8 pb for $W^+W^-$ pairs; this is to be compared to a $\gamma\gamma$ cross-section of at least 150 pb at the same energy, even using the relatively strong cuts of figure 9; this cross-section is as large as 500 pb if the photon structure is better described by the LAC1 parametrization.

In figure 9 we have only included the bremsstrahlung contribution to the photon spectrum. However, it is well known that synchrotron radiation makes the construction of $e^+e^-$ storage rings with $\sqrt{s}$ significantly beyond the reach of LEP2 prohibitively expensive; one will therefore have to use linear colliders (linacs) if higher energies are to be reached in $e^+e^-$ collisions. In such linacs, each electron or positron bunch has only a single chance to produce a reaction; at the same time, the total luminosity of the machine has to grow $\propto s$ if a useful rate of annihilation events is to be maintained. These two constraints imply that the luminosity per bunch crossing has to be much larger than at present storage rings; this in turn necessitates the use of very dense bunches, with transverse dimensions of the order of a few dozen nm. This leads to a large charge density, which produces very strong electromagnetic
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fields. Immediately before and during bunch collisions the particles in one bunch feel the field produced by the other bunch, and are accelerated. The radiation produced by this acceleration is known as beamstrahlung [49].

This qualitative discussion shows that machines with large luminosity per bunch crossing generally produce more beamstrahlung [50]. The necessary luminosity per bunch crossing is obviously inversely proportional to the number of bunch collisions per second; this number in turn depends on the design of the accelerating structures.

Some typical examples of beamstrahlung spectra for $e^+e^-$ colliders operating at $\sqrt{s} = 500$ GeV are shown in figure 10 [52]; these curves have been computed using approximate analytical expressions given in ref [53]. The acronyms P-G, P-F, D-D and T stand for the Palmer-G, Palmer-F, DESY-Darmstadt and TESLA designs, respectively, while wbb (nbb) denotes the wide (narrow) band beam option of the D-D design. We see that machines that utilize accelerating RF fields with wavelength in the “X band” region (P-G, P-F as well as the Japanese Linear Collider JLC, whose first stage is somewhat similar to the Palmer-F design) have harder beamstrahlung spectra than designs using the longer wavelengths of the “S band” (D-D), or the TESLA design, which is based on superconducting cavities. For comparison we also show the Weizsacker–Williams (WW) bremsstrahlung spectrum (dotted curve). For large fractional photon momentum $x$, the beamstrahlung contribution is exponentially suppressed; this end of the spectrum is therefore still dominated by the bremsstrahlung contribution. However, at smaller values of $x$, beamstrahlung photons are more abundant; the cross-over point between the regions dominated by beam- and bremsstrahlung depends sensitively on the machine parameters.

Finally, it has been pointed out [54] that an $e^+e^-$ collider can be converted into a $\gamma\gamma$ collider by shining very intense laser light on the particle beams; some laser photons then undergo Compton backscattering. The dash-dotted curve in figure 10 shows the spectrum that results if the laser energy is chosen such that the invariant mass of a laser and $\gamma$ backscattered photon is just below $2m_e$, and both laser and electron beam are unpolarized. Notice that these backscattered photons, as well as beamstrahlung photons, are truly on-shell, unlike bremsstrahlung photons.

Obviously beamstrahlung can greatly enhance rates of two-photon events. For

![Figure 10](image_url)

Figure 10. The beamstrahlung photon spectrum of 4 typical designs of 500 GeV $e^+e^-$ linacs, as well as of bremsstrahlung photons (dotted) and of backscattered laser photons (dot-dashed). From [52].
example, had we included the beamstrahlung contribution to \( f_{\gamma/e} \) in figure 9, the cross-section would have grown [52] to between 180 pb (for TESLA) and 4.5 nb (for Palmer-G). Since the luminosity of those designs is \( 2 \times 10^{33} \text{ cm}^{-2} \cdot \text{s}^{-1} \), this corresponds to approximately 4 (250) million events with total hard \( E_T > 10 \text{ GeV} \) per year for the TESLA (Palmer-G) collider! Of course, in principle one can get rid of most of these events by setting a rather high trigger threshold on the transverse momentum of the jets, or the total \( E_T \) in the event [55]. However, then one risks to lose interesting annihilation events containing massive stable neutral particles, as predicted e.g. by supersymmetric theories. Moreover, the \( \gamma\gamma \) events are interesting in their own right. In our view it is therefore preferable to use a low trigger threshold, even if this means that the amount of data to be handled is rather large for \( e^+e^- \) colliders; it is still small compared to the amount of information that has to be manipulated at typical LHC or SSC detectors.

Beamstrahlung also changes the electron spectrum [53]; obviously an electron will lose some of its energy when emitting a hard photon. This effect has to be added to the smearing of the beam energy due to the machine parameter independent initial state radiation. For designs with hard beamstrahlung spectrum (e.g., Palmer-G), the \( e^+e^- \) luminosity spectrum is distorted by beamstrahlung even for energies far below the nominal \( \sqrt{s} \) of the collider. At small invariant masses one thus generally has a competition between \( \gamma\gamma \) and \( e^+e^- \) events. This is demonstrated in figure 11, where we show the invariant mass spectrum of events with two central jets with \( p_T \geq 20 \text{ GeV} \) [52]. The annihilation contribution exhibits a prominent peak at \( \sqrt{s} = M_Z \); by comparing events in that peak with events with \( M_{jj} \approx \sqrt{s} \), one can hope to study the QCD evolution of the hadronic system with increasing invariant mass in a single detector, thereby reducing experimental (systematic) errors. However, this figure shows that at this collider it would be very difficult to extract a clean sample of annihilation events with \( M_{jj} \approx M_Z \); in spite of the rather severe cut on \( p_T \) which reduces the \( \gamma\gamma \) contribution considerably, the \( Z \) peak will hardly stand out in the total sample of di-jet events once detector resolution effects are included. In colliders with soft
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Table 1. Total $c\bar{c}$ cross-sections from two-photon processes at the 4 $e^+e^-$ colliders of figure 10, as well as for a $\gamma\gamma$ collider made from an $e^+e^-$ collider with $\sqrt{s} = 500$ GeV. We have used the DG parametrization to estimate the resolved photon contributions. $\sigma(q\bar{q})$ and $\sigma(gg)$ stand for the 2-res $q\bar{q}$ annihilation and gluon fusion cross-sections, $\sigma(\gamma g)$ for the 1-res photon gluon fusion contribution, and $\sigma(\gamma\gamma)$ for the direct contribution; $\sigma(J/\psi)$ is the 1-res $\gamma + g \rightarrow J/\psi + g$ cross-section in the color singlet model. All cross-sections are in nb.

| Collider  | $\sigma(q\bar{q})$ | $\sigma(gg)$ | $\sigma(\gamma g)$ | $\sigma(\gamma\gamma)$ | $\sigma(\text{tot})$ | $\sigma(J/\psi)$ |
|-----------|--------------------|--------------|--------------------|------------------------|---------------------|-----------------|
| T         | 0.010              | 0.038        | 1.8                | 2.2                    | 4.0                 | 0.014           |
| D-D (wbb) | 0.041              | 0.11         | 7.0                | 6.4                    | 13.5                | 0.053           |
| P-F       | 0.017              | 0.08         | 4.0                | 2.4                    | 6.4                 | 0.030           |
| P-G       | 0.14               | 1.1          | 38                 | 9.9                    | 49                  | 0.28            |
| $\gamma(500)$ | 0.13               | 7.6          | 130                | 0.14                   | 140                 | 0.89            |

beamstrahlung spectrum (TESLA or the nbb option D-D) the annihilation cross-section at $M_{jj} = M_Z$ is reduced by a factor of 3, but the $\gamma\gamma$ contribution is almost 30 times smaller than at Palmer-G, enabling one to isolate a rather clean sample of annihilation events from the Z peak.

As a final example of the importance of beamstrahlung we list in table 1 estimates of total $c\bar{c}$ and $J/\psi$ production cross-sections, using the DG parametrization. Since these processes are sensitive to the region of small $x$, even the soft beamstrahlung spectrum of the TESLA collider leads to a sizable enhancement of the rate. This is especially true for the direct contribution, whose cross-section decreases with increasing $W_{\gamma\gamma}$, unlike those for the resolved photon contributions; without beamstrahlung the direct and 1-res total $c\bar{c}$ cross-sections would only amount to 0.6 and 0.85 nb, respectively. Due to the different dependence on $W_{\gamma\gamma}$ resolved photon events are more important at colliders with harder beamstrahlung spectrum. However, even for the Palmer-G design we find [52] that the cross-section for the production of central $c\bar{c}$ pairs with $p_T > 5$ GeV is dominated by the direct contribution. This is because the 1-res contribution has a softer $p_T$ spectrum and, due to the asymmetric initial state, is peaked at small angles. If, on the other hand, the $e^+e^-$ collider is converted into a $\gamma\gamma$ collider, even the 2-res contribution will be larger than the direct one; notice that in this case the 2-photon luminosity actually falls at small $W_{\gamma\gamma}$.

Qualitatively similar results hold for total $b\bar{b}$ production, except that the cross-sections are smaller by a factor between 100 and 200. It has been claimed [56] that at future linacs total $t\bar{t}$ production might also be dominated by the $\gamma\gamma$ contribution. We find [52], however, that even at the Palmer-G collider the $\gamma\gamma$ contribution amounts to at most 5% of the total; for the other designs of 500 GeV linacs this number is closer to 1%. Even for the third stage of the JLC, which operates at $\sqrt{s} = 1.5$ TeV and also has a rather hard beamstrahlung spectrum, the annihilation contribution is still dominant if $m_t > 130$ GeV. At such very high energy colliders, beamstrahlung and initial state radiation also increase the annihilation contribution by as much as 60%, due to the reduction of the average centre-of-mass energy of $e^+e^-$ pairs. In principle one could increase beamstrahlung even further, e.g. by using round beams. However, we will argue in § 5 that in this case one will have to deal with qualitatively new beamstrahlung induced backgrounds, including the existence of an “underlying
event which will make experiments at such $e^+e^-$ linacs similar to those at hadron colliders, so that the detailed study of $t\bar{t}$ events will become very difficult. In contrast, most of the $e^+e^-$ colliders discussed here could quite easily accumulate a clean sample of $t\bar{t}$ events from $e^+e^-$ annihilation. We therefore see no advantage of operating future linacs in the domain of high beamstrahlung.

Of course, at present predictions for total $c\bar{c}$ and $b\bar{b}$ production cross-sections at high energy linacs suffer from large uncertainties, since, one is probing the parton content of the photon at values of $x$ as small as $10^{-3}$, where no experimental information exists so far. With the advent of the $ep$ collider HERA this is expected to change soon, however, as we discuss in the next section.

4. Resolved photon reactions in $\gamma p$ scattering

The discussion in earlier sections indicates that we need to probe the parton content of the photon, especially the gluon content, at small values of $x$ and large $Q^2$. For reasons discussed earlier, DIS experiments are limited by statistics in the region of large $Q^2$ and probe $G^p$ only indirectly. Jet production in $\gamma\gamma$ collisions at TRISTAN and LEP will certainly provide useful information. But the only other possibility to go to higher values of $\sqrt{s}$ for photon interactions and hence increase the range of $x$ and $Q^2$ values at which the photon can be probed, is at present the high energy $ep$ collider HERA.

The suggestion to use $\gamma p$ collisions to study $q^\gamma$ is not new [57]. Theoretically the situation is actually somewhat simpler than for real $\gamma\gamma$ scattering, since we only have to deal with two classes of contributions: Direct ones, where the photon directly interacts with the partons in the photon; and resolved photon reactions, where the partons in the photon scatter off partons in the proton. The low energy of photon beams available at fixed target experiments reduces the contribution of 'res' processes in current experiments but it still plays an important role [57, 15, 58]. At the high energy HERA collider with an $ep$ centre-of-mass energy $\sim 300$ GeV, the situation is quite different. In a large number of QCD processes such as high $p_T$ jet production [59, 60, 61], heavy flavour production [59, 62], direct photon production [63] and Drell Yan lepton pair production [64] the hadronic structure of the photon not only plays an extremely important role but even dominates in some cases. The cross-section for the various QCD processes is given by expressions very similar to (3.1), where one replaces one photon by the proton and includes the partonic subprocesses corresponding to the QCD process under consideration. We now describe some reactions in more detail.

4.1 $ep \rightarrow jets + X$

We start with a discussion of inclusive jet pair production, which offers the highest cross-section of all hard scattering processes at HERA. Here the direct processes are the same as the 1-res processes of $\gamma\gamma$ scattering, and the resolved photon reactions correspond to the 2-res contributions to $\gamma\gamma$ collisions.

In figure 12 we show the ratio $R_\gamma$ of the cross-sections of the resolved and direct processes as a function of the transverse momentum $p_T$ of the jets [59]. We see that even the DG parametrization predicts the resolved photon contribution to be larger than the direct one out to $p_T \approx 35$ GeV. In $\gamma p$ collisions we have to pay the price (in terms of reduced phase space) of producing an additional spectator jet only once
Resolved photon processes

Figure 12. Ratio of resolved and direct contributions for \( \frac{d\sigma(ep \rightarrow \text{jets})}{dp_T} \) as a function of \( p_T \) [59].

when we want to gain access to the QCD \( 2 \rightarrow 2 \) scattering processes, whose matrix elements are enhanced by gluon exchange in the \( t- \) or \( u- \) channel, as discussed earlier; in \( \gamma \gamma \) collisions this price has to be paid twice, since both photons have to be resolved. Moreover, the proton has a relatively larger gluonic component than the photon, at least if the DG parametrization is close to the truth. This further favours resolved photon processes over direct ones, since the QCD \( 2 \rightarrow 2 \) matrix elements containing gluons in the initial and final state are enhanced by colour factors, while the direct \( \gamma g \) fusion process is actually colour-suppressed (by a factor of 3/8) compared to \( \gamma q \) scattering. These two observations explain why at HERA resolved photon processes dominate jet production out to larger values of \( x_T = 2p_T/\sqrt{s} \) than at \( e^+ e^- \) colliders: see figure 6. Figure 12 also illustrates that existing parametrizations of proton structure functions (DO2 [65] vs. GHR [66]) differ much less than those for the photon, at least in the region \( x > 0.01 \) relevant for the production of high-\( p_T \) jets.

In figure 13a, b we show the \( p_T \) spectrum in absolute units. In these figures, we also show the contributions from different final states separately; for most events the parton composition of the initial and final states are identical. We see that, unlike at hadron colliders, the \( gg \) final state dominates only at very low values of \( p_T \), below 3 GeV for the DG parametrization. This is because the difference in shape between quark and gluon distribution functions is larger for photons than for nucleons. In the latter case all parton densities fall with increasing \( x \), while \( x \cdot q' \) has a maximum at \( x \approx 0.9 \), as shown in figure 3.

For most of the \( p_T \) range where resolved photon contributions dominate, the largest contribution comes from the mixed \( qg \) final state. The DG parametrization predicts that in most cases the quark comes from the photon and the gluon from the proton; this is again a result of the relative softness of \( G' \) and the large gluon content of nucleons. Finally, figure 13b shows that the direct contribution is only dominated by photon-gluon fusion for values of \( p_T \) where the total di-jet cross-section is dominated by res contributions; this might make it difficult to extract the gluon density of the nucleon from measurements of jet production at HERA.

Of course, in principle direct and res events can be distinguished by the presence of the spectator jet from the photon going in the electron beam direction, which is the hallmark signature of resolved photons. However, while there are arguments suggesting [59] that this jet should be rather broad and hence easily detectable in
most cases, the exact value of the efficiency for tagging on this jet clearly depends on the details of the jet formation model (i.e., it is not an “infrared safe quantity”), as well as on the detector acceptances. It is therefore tempting to try and find differences between direct and resolved contributions in the distributions of the high-$p_T$ jets themselves, which can be predicted directly from perturbative QCD.

One possibility [61] is to look at the cross-section as a function of the centre-of-mass scattering angle. Due to the presence of diagrams with gluon exchange in the $t$- or $u$-channel the resolved photon contribution will be more strongly peaked at small angles than the direct contribution.

Another possibility [59] is to study the triple-differential cross-section $\frac{d\sigma}{dp_Tdy_1dy_2}$, where the $y_i$ are the rapidities of the two high-$p_T$ jets. The results of figure 13 show that the cross-section should be large enough to allow such detailed studies even with less than the full HERA design luminosity of about 100 pb$^{-1}$/yr. In this case we can make use of purely kinematical considerations to separate the two classes of contributions. Obviously a parton “in” a photon will have less energy than the photon itself. For a given invariant mass of the produced partonic system, the parton from the proton will therefore have to supply more energy in resolved photon events than in direct ones; this results in a boost of the partonic system in the direction of the proton beam. Since, as we have emphasized repeatedly, $G^\gamma$ is expected to be much softer than the $q^\gamma$ [67], this boost will be stronger if the parton that is “pulled out” of the photon is a gluon. Parametrizations with larger $G^\gamma$ will thus tend to predict a rapidity distribution that is more strongly peaked at larger rapidities.

This is demonstrated in figure 14, which shows the shape of the rapidity distributions (for $y_1 = y_2 = y$) at $p_T = 10$ GeV. In order to avoid “$k$-factor” uncertainties, all curves have been normalized to give the same single-differential cross-section $\frac{d\sigma}{dp_T} = 8.8$ nb/GeV at $p_T = 10$ GeV. The DO + VMD distribution with its rather large gluon content leads to a much more pronounced peak at $y \approx 2.2$ than the DG parametrization does, although the position of the peak is not shifted very much; note that $y \rightarrow y_{\text{max}}$ corresponds to $x_p \rightarrow 1$, where $q^p$ vanishes. On the other hand, a toy model with zero gluon content predicts the peak in the rapidity distribution to be shifted towards smaller $y$ by about 1.5 units. We have already seen that AMY data require [3] a
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\[ \frac{d^2 \sigma}{dy_1 dy_2} \]

\[ y_1 = y_2 \]

\[ p_T = 10 \text{ GeV} \]

\[ \text{(DO+VMD)} \times 0.62 \]

\[ \text{(DG, } G^2 = 0) \times 1.5 \]

\[ \text{DG} \]

Figure 14. Comparison [59] of the shape of the rapidity distribution of jets produced at HERA. Note that the dotted and dashed curves have been normalized, as described in the text.

We thus see that the presence of resolved photon contributions leads to 3 qualitative predictions [59] that ought to be testable quite easily: Large jet production cross sections; spectator jets from the photon; and a rapidity distribution that is peaked at positive \( y \) (corresponding to the proton beam direction). Even though HERA experiments so far have only taken a few nb\(^{-1}\) of data, they are already starting to confirm these predictions.

In figure 15 we show the transverse energy spectrum of the photoproduction events identified by the ZEUS collaboration [5] in the first (pilot) run of HERA. For \( E_T > 10 \text{ GeV} \) the soft (VMD) contribution is found to be negligible; the events in this region therefore have to be explained by hard scattering processes. The dashed curve shows the prediction of the HERWIG generator [68] using the DG parametrization with \( p_{T,\text{min}} = 1.5 \text{ GeV} \); we see that it describes the data quite well. Recall that almost the same value of \( p_{T,\text{min}} \) has been found to describe the TRISTAN data. From their data sample ZEUS extracts a total cross-section for the production of events with \( E_T > 10 \text{ GeV} \) of \( 2.4 \pm 0.1 \pm 0.7 \mu b \). A glance at figure 13 shows immediately that a cross-section of this size cannot be explained from direct processes alone; indeed, the detailed MC study of the ZEUS group shows that without the resolved photon contributions theory falls short of the data by at least one order of magnitude. Similar results have been reported by the H1 collaboration [6].

Both groups also find evidence for the spectator jet from the photon in their data. As an example we show in figure 16 the energy flow measured [6] in the H1 calorimeter as a function of the angle (\( \theta = 0 \) is the direction of the proton beam); only events where both high-\( p_T \) jets emerge at \( \theta \leq 100^\circ \) have been included. At small angles a
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Figure 15. The total $ep$ cross-section measured [5] for transverse energies larger than $E_T^0$. The curve is the HERWIG prediction, using the DG parametrization with $p_{T,\text{min}} = 1.5 \text{ GeV}$.

Figure 16. Histogram of energy flow per event versus polar angle. The open points represent the data, while the full and dotted lines give the MC prediction with and without resolved photon contributions. From ref. [6].

large amount of energy is deposited by the proton remnants. At intermediate angles both soft and hard processes contribute. However, direct events are unable to populate the region around the electron beam direction, in conflict with the data, which show a constant or even slowly rising energy deposition at $\theta \approx 180^\circ$. This is well described by the MC generator once resolved photon contributions are included. ZEUS also finds [5] evidence for the spectator jet in their sample of reconstructed jet events: Some events have sizable energy deposit around the electron beam direction even though all high-$p_T$ jets are 2 or more units of rapidity away. Their data indicate that the efficiency for tagging on this spectator jet should be around 40%, in qualitative agreement with earlier MC studies [69].

Finally, H1 also finds [6] that their jets populate a quite different angular (or rapidity) region than what one would expect from direct events. In particular, they find that a large fraction of their events have one or both jets relatively close to the proton beam direction; we have seen in figure 14 that direct $\gamma p$ scattering produces jets preferably at large angles. Our three main predictions are therefore all borne out
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at least qualitatively by the data; we are looking forward to more detailed analyses of higher statistics data samples.

Before closing this subsection we should point out that recently first results of next-to-leading order calculations of the photoproduction of jets at HERA energies have become available. NLO corrections to the direct processes have been computed already some time ago by Aurenche et al and independently by Baer et al [58]; they have recently been re-done and applied to HERA in ref. [70]. The production of three jets in direct and resolved photon interactions has been studied in ref. [61]. Full NLO corrections to the resolved photon contribution have been computed in ref. [71]. Finally, so far the only paper that includes a full next-to-leading order calculation of all contributions to jet production is ref. [72]. Such a comprehensive analysis is necessary since some divergent corrections to the direct process have to be absorbed in the parton distribution functions of the photon, thereby blurring the distinction between direct and resolved photon contributions. These studies indicate that inclusion of NLO corrections reduces the artificial dependence of the cross-section on factorization and renormalization scales. However, if these scales are chosen to be equal to the transverse momentum of the jets and a cone size $\Delta R = 0.7$ is chosen in the jet definitions, NLO corrections appear to be quite modest. The results of this subsection, which have been obtained from leading order calculations, should therefore retain their validity also in NLO.

4.2 Heavy quark production

We have seen in the last subsection that inclusive jet production at HERA will probably only allow a rather indirect determination of $G^7$ due to the large background from quark-initiated resolved photon events. Just as in case of $\gamma\gamma$ scattering, one can enhance the importance of gluon-initiated processes by studying specific final states. Among those, the production of a pair of heavy quarks offers the largest cross-section. We focus here on $b$-quarks, which should be easier to identify than $c$ quarks, and where fragmentation effects should be smaller.

The total $b\bar{b}$ cross-section at HERA as predicted from the DG parametrization is [59, 62] about 1 nb, which corresponds to 100,000 $b\bar{b}$ pairs per year. Unfortunately the resolved photon contribution only amounts to about 20% of the total; a separation of the two classes of contributions thus becomes mandatory if $b\bar{b}$ production is to be used to determine $G^7$. Fortunately we have seen at the end of the previous subsection that it seems to be possible to tag spectator jets from the photon with reasonable efficiency; this should allow to accumulate a rather clean sample of resolved $b\bar{b}$ events.

In figure 17 we show the $p_T$ spectrum of the $b$ (or $\bar{b}$) quark for the symmetric configuration $y_1 = y_2 = y$; the res contribution has again been estimated from the DG parametrization. We see that the slope of the spectrum at high $p_T$ depends quite sensitively on $y$; recall that large $y$ correspond to large $x_p$, where the parton densities in the nucleon decrease rapidly. One can also conclude that a detailed study of the resolved photon contribution will only be possible if $b$ quarks with $p_T$ below 10 GeV can be identified efficiently and reliably; otherwise the rate will be too small.

Figure 17 also shows that resolved photon contributions are much more important at large $y$. After the discussion of the rapidity distribution of jets in the previous subsection this should not be surprising; since the res contribution to $b\bar{b}$ pair production is dominated by $gg$ fusion, res $b\bar{b}$ events will usually undergo a strong
boost in the proton beam direction. This is further illustrated in figure 18, where we show the $b\bar{b}$ cross-section as a function of $y_2$ for fixed $y_1$, at $p_T = 5$ GeV. Notice that in this figure the res contribution has been computed from the DO + VMD parametrization, which predicts a ratio of direct to resolved contributions of about 2:1, as opposed to 4:1 for the DG parametrization. We have seen above that the efficiency for tagging the photonic spectator jet might be around 40 to 50% for generic high-$p_T$ jet events; it might be somewhat smaller for the more spherical $b\bar{b}$ events. Simply requiring the absence of such a tag would thus leave a $b\bar{b}$ sample that still contains 20 to 30% resolved photon events, if $G'$ is similar to the DO + VMD parametrization; this could complicate the extraction of the gluon content of the nucleon from studies of $b\bar{b}$ pair production at HERA. Fortunately figure 18 shows that even the large res contribution predicted by the DO + VMD parametrization can be suppressed to an insignificant level by requiring either the $b$ or the $\bar{b}$ to emerge at small (negative) rapidity; this restriction still allows to probe $G'(x_p)$ for $x_p$ between $2 \cdot 10^{-3}$ and 1 by varying $y_2$ within its kinematically allowed limits. The same conclusion holds for all parametrizations of $qV(x, Q^2)$ that predict $G'(x)$ to be soft, as seems to be required by TRISTAN data. By separating the total $b\bar{b}$ sample into events with and without
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a photonic spectator jet, and studying the rapidity distribution in each sample, it should therefore be possible to extract important information about both $G^p$ and $G^\gamma$ from $b\bar{b}$ pair production at HERA.

Finally, we remark that so far only partial NLO calculations for the photoproduction of heavy quark pairs exist [62, 73, 74]. In these papers the corrections to the direct process are included, but the resolved photon contribution, which occurs at the same order in $\alpha_s$, has only been included at tree level.

4.3 Direct photon production

Another process that can be studied at HERA is the production of hard direct photons in $ep \rightarrow e\gamma X$ [75]. Of course, the cross-section is now $\mathcal{O}(\alpha^3)$ and thus approximately two orders of magnitude smaller than the jet cross-section. On the other hand, the direction and energy of a hard photon can be determined much more precisely than those of a jet; this should help in the reconstruction of the Bjorken-$x$ variables of the partons participating in the hard scattering.

In [63] a fairly comprehensive study of this reaction has been presented for HERA energies. NLO corrections to the direct process $\gamma q \rightarrow \gamma q$ [76] are included, but the resolved photon contributions ($gq \rightarrow \gamma q$ and $q\bar{q} \rightarrow \gamma g$) are treated at the Born level. If a hard photon within a jet can be detected, one can also study the fragmentation of a parton into a photon, which is the inverse of $\gamma \rightarrow$ parton splitting described by the parton densities in the photon. Even if these contributions are included, resolved photon processes dominate the total cross-section only for $p_T^2 \leq 15$ GeV, according to the DG parametrization. The reason is that again one has to produce two additional jets (the spectator jet from the photon, and the remnants of parton $\rightarrow$ photon fragmentation) before the QCD $2 \rightarrow 2$ processes become accessible.

Nevertheless the study of photons with $p_T \simeq 5$ GeV or so should yield information about $q^\gamma$, especially $G^\gamma$. Kinematics again implies that events of the type $q^p g^\gamma \rightarrow q\gamma$ should be strongly boosted in the proton direction; in addition, the hard matrix

![Figure 19. Energy spectrum of photons produced in $\gamma p$ scattering at $s = 30,000$ GeV$^2$, as predicted [77] from the DG parametrization.](image-url)
element favours the photon to emerge close to the direction of the incident quark. The combination of these two effects implies [77] that res contributions dominate at small angles relative to the proton beam direction, as shown in figure 19. Here a fixed energy of the incoming photon has been assumed, $E_\gamma = 9$ GeV; experimentally this means that the ongoing electron has to be tagged in a forward spectrometer, which is also used for luminosity measurements. Since the transverse momentum of the outgoing photon has been fixed to 5 GeV, there is a one-to-one relation between the energy and the angle of the photon, with small angles corresponding to large energies. The coverage of the electromagnetic calorimeter of HERA experiments starts approximately 4 degrees from the proton beam pipe [77]; this means that photons with energy as large as 110 GeV should be detected with sizable rates. Notice also that at this angle even the DG parametrization predicts the resolved photon contribution to be a full order of magnitude above the direct one. Even if the outgoing electron is not tagged, i.e. after integration over the Weizsäcker–Williams spectrum, at this angle res contributions are at least two times bigger than direct ones.

The proximity of the spectator jet from the proton should not compromise the observability of this signal, since this jet is not expected to contain photons of this very high energy. Finally, due to the softness of $G'$, almost the whole energy of the incoming photon will go into the spectator jet. If the energy of the photon is known (by measuring the energy of the outgoing electron), this information can be used to study a sample of photonic spectator jets with known energy, which might provide valuable information for the study of other resolved photon processes.

4.4 $J/\psi$ Production

Just like the production of heavy quark pairs, the process $ep \to eJ/\psi X$ has originally been proposed [78] as a way to determine the gluon density of the proton; results from such analyses have been reported from fixed-target photoproduction experiments [46]. It was realized later [64], however, that at the much higher energies which can be achieved at HERA this final state also receives sizable contributions from resolved photon processes. $J/\psi$ mesons can be produced from $\gamma g \to J/\psi g$ (direct process) as well as $gg \to J/\psi g$ (res process); in addition, they can be produced in the decay of $\chi$ mesons or $b$ quarks. Indeed, this latter process dominates [79] for $p_T > 5$ GeV; moreover, the cross-section becomes quite small in this region. Most studies of $J/\psi$ production at HERA therefore focus on the region $1 \text{ GeV} < p_T < 5 \text{ GeV}$.

Although the total resolved photon contribution estimated from the DG parametrization amounts to about 0.5 nb, extraction of the signal may not be trivial. First of all, only the leptonic decays will be detectable at HERA, which reduces the signal by a factor of 7. Since most $J/\psi$'s produced via res processes emerge at large rapidity, i.e. small angle to the proton beam, requiring both leptons to be detected further degrades the signal; in this case there is no rapidity region left where the res contribution clearly dominates [80]. Of course, requiring the spectator jet from the photon to be detected will suppress the direct contribution, while $J/\psi$'s produced with negative rapidity will overwhelmingly come from direct processes, as in case of open heavy flavour production.

Another possibility is to tag the outgoing electron [81]. This selects events with incident photon energy $E_\gamma > 7$ GeV, since otherwise the outgoing electron is too energetic to be bent out of the beam, which is necessary for its detection in the forward spectrometer. As shown in figure 20, this is sufficient to suppress the direct contribution
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![Graph showing rapidity distribution of J/ψ mesons](image)

Figure 20. Rapidity distribution [81] of J/ψ mesons produced at HERA in events where the outgoing electron is tagged, as estimated using the D02 and DG parametrizations for the proton and photon, respectively.

at positive rapidities to an insignificant level. The price is the reduction of the overall signal by a factor of 10 or so; on the other hand, the analysis is no longer sensitive to the details of the spectator jet formation.

Finally, it has been pointed out in ref. [80] that there is no direct contribution if the J/ψ is produced in association with a hard photon; this process has subsequently been studied in ref. [82]. The main problem is again the small event rate; after mild acceptance cuts, the DG parametrization predicts an observable cross-section of only 0.08 pb. However, as already mentioned in connection with J/ψ production at e⁺e⁻ colliders, the "colour singlet" model [45], which has been used in all cross-section calculations, might underestimate the rate by as much as a factor of 5 [46]. While this would make the detection of the signal easier, the presence of a k-factor of this magnitude casts doubt on the leading order analyses presented here. Nevertheless, J/ψ production at HERA has the potential to probe gluon densities down to very small values of x, of order 10⁻³ or less.

4.5 Other processes

We close this section with a brief survey of other processes that receive contributions from resolved photon reactions, although limitations of space do not allow to discuss them in detail.

Of great theoretical interest is the production of W and Z bosons at HERA. In leading order only the resolved photon (Drell–Yan) process qq̅ → W contributes [83]; note that the corresponding cross-section is O(1/α_s), since q² ∝ 1/α_s. On the other hand, the direct process γq → Wq', while formally of higher order in α_s (∼ α²), is sensitive to the γWW coupling [84]. In order to study this dependence the p_T → 0 divergent pieces of the direct contribution, which are already included in the res part, have to be subtracted to avoid double-counting. Several subtraction procedures have been suggested recently [85]. The resulting cross-section in the standard model is about 0.5 pb for W bosons, but probably only the leptonic decay mode can be identified; the cross-section for Z bosons is even smaller.

In § 4.3 we have discussed the production of real (on-shell) photons at HERA. The same processes can also give rise to off-shell photons, and hence to lepton pairs; this
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has been studied in refs [64, 74]. As expected from our previous discussions, resolved photon contributions are quite important at small transverse momentum and/or small invariant mass of the dilepton system. However, compared to direct photon production the cross-section is down by another factor of $\alpha$. This process can therefore only yield useful information about parton densities after a large amount of data has been accumulated.

It has also been suggested [86] to study the production of a photon whose transverse momentum is balanced by a charm quark as a means to constrain the heavy flavour content of the proton as well as the photon. The charm quark is detected via its decay into a muon. The total cross-section after acceptance cuts is expected to be a few pb; the exact number depends on the way mass effects are included in heavy flavour density distributions.

As a last process we mention the production of two hard photons at HERA [87]. The cross-section is rather small, being of order $\alpha^4$, so that only a limited range of transverse momenta can be studied experimentally. On the other hand, this process receives important contributions from $gg$ fusion, via a box diagram which (up to trivial coupling and colour factors) is equivalent to the famous light-by-light scattering diagram; although first studied more than 50 years ago [88], the effect of this diagram has still not been detected experimentally.

5. Minijets and total cross-sections

So far we have only discussed “hard” processes, where the applicability of perturbative QCD is not in doubt. However, we have already seen in § 3 that one has to introduce at least one parameter that cannot be predicted from perturbative QCD if one wants to describe existing $\gamma\gamma$ data in the intermediate region where both soft and hard processes contribute; this parameter is the cut-off $p_{T,\text{min}}$, which parametrizes the applicability of perturbative QCD. TRISTAN data indicate that this parameter has to be chosen around 1.5 (2.0)GeV if data are to be described by the DG (LAC1) parametrization. However, with such values of $p_{T,\text{min}}$, the total jet pair cross-section grows very rapidly with energy, and eventually even exceeds the value of the total cross-section measured at lower energies.

The rapid growth of the inclusive jet cross-section due to the copious production of “minijets” with $p_T \approx p_{T,\text{min}}$ via resolved photon processes has first been pointed out in ref. [89] for the case of $\gamma p$ scattering; an example is shown in figure 21, for $p_{T,\text{min}} = 2$ GeV and various parametrizations of $q'$. It was conjectured in that paper that this increase of the cross-section might help to explain the mysteriously large number of muons observed [90] in photon-induced cosmic air showers. Later detailed Monte Carlo calculations [91] showed that, while resolved photon processes might boost the muon yield by a factor of 2–3, they are not sufficient to explain the data by themselves.

Of course, the total cross-section cannot grow indefinitely at the rate shown in figure 21; some mechanism will have to unitarize it. This problem is well known for hadronic ($pp$ or $p\bar{p}$) collisions; indeed, it was suggested almost 20 years ago [92] that minijet production might contribute to the growth of total hadronic cross-sections. In this case unitarization is usually achieved by eikonalization. The crucial observation here is that LO QCD predictions for cross-sections, like those shown in figure 21, refer to inclusive jet cross-sections; in other words, they differ from the jet production contribution to the total cross-section by a factor of the average jet pair multiplicity.
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\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure21.png}
\caption{Predictions [89] of the increase of the inclusive (mini) jet cross-section in $\gamma p$ collisions with $\sqrt{s}$, for $p_{T\text{, min}} = 2$ GeV and various parametrizations for $q^2$.}
\end{figure}

Figure 21. Predictions [89] of the increase of the inclusive (mini) jet cross-section in $\gamma p$ collisions with $\sqrt{s}$, for $p_{T\text{, min}} = 2$ GeV and various parametrizations for $q^2$.

\[ \langle n_{\text{jet}} \rangle. \] Formally one writes [93]

\[ \sigma_{pp}^{\text{inel}} = \int d^2 b \{ 1 - \exp\left[ - \left( \sigma_{pp}^{\text{hard}}(s) + \chi_{pp}^{\text{soft}} \right) A(b) \right] \}, \quad (5.1) \]

Here $b$ is the two-component impact parameter, $A(b)$ described the transverse distribution of partons in nucleons, $\sigma_{pp}^{\text{hard}}$ is the perturbative QCD prediction for the minijet cross-section (obtained by integrating $d\sigma/dp_T$ in the region $p_T > p_{T\text{, min}}$), and $\chi_{pp}^{\text{soft}}$ is the non-perturbative (soft) contribution to the eikonal, which is fitted from low-energy data. In (5.1) it has been assumed that the transverse distribution is independent of $x$ and $Q^2$, and that different partonic scatterings are uncorrelated, i.e. obey Poisson statistics. Eikonalized minijet models with $p_{T\text{, min}}$ around 1.5 to 2 GeV and standard parametrizations for $q^2$ not only reproduce the rise of the total and inelastic $pp$ and $p\bar{p}$ cross-sections [93], but also correctly describe many details of “minimum-bias” events as well as events containing hard jets [94].

However, as pointed out by Collins and Ladinsky [95], (5.1) will have to be modified before it can be applied to photonic cross-sections. This can easily be seen [96] by expanding the exponential; one finds that the cross-section for the production of 2 jet pairs is proportional to the square of the hard QCD cross-section. In case of $\gamma p$ scattering this hard cross-section is of $\mathcal{O}(a_s)$, so that (5.1) would predict $\sigma(2$ pairs$) \sim \mathcal{O}(a^2 a_s^2)$. On the other hand, once the photon has undergone its transition into a (virtual) hadronic state, no additional factor of $a$ is necessary to produce additional jet pairs; rather, one would expect $\sigma(2$ pairs$) \sim \mathcal{O}(a a_s^2)$. Similar arguments hold for even larger number of jet pairs. This can be achieved by introducing a parameter $P_{\text{had}}$ describing the probability that the photon goes into a hadronic state; clearly $P_{\text{had}} \sim \mathcal{O}(a)$. Equation (5.1) then becomes [95]

\[ \sigma_{\gamma p}^{\text{inel}} = \int d^2 b P_{\text{had}} \{ 1 - \exp\left[ - \left( \sigma_{\gamma p}^{\text{hard}}(s) + \chi_{\gamma p}^{\text{soft}} \right) A(b)/P_{\text{had}} \right] \}, \quad (5.2) \]

A similar expression can be derived for $\gamma\gamma$ collisions, but here $P_{\text{had}}$ has to be replaced by $P_{\text{had}}^2$ [97].
Unfortunately there are many unknown quantities in (5.2). First of all, we cannot predict the hard scattering cross-section, since we do not (yet) know the parton densities in the photon at sufficiently small values of \( x \). TRISTAN data give some indication what \( p_{T,\text{min}} \) should be, but it is not clear that the same value should be used in \( \gamma p \) scattering, or that it should be independent of energy (although first HERA data do seem to point in that direction). Finally, it is not clear how \( P_{\text{had}} \) and \( A(b) \) are to be determined. In most papers [95, 97, 98] VMD ideas are used to estimate these quantities. In particular, \( P_{\text{had}} \) is taken to be the \( \gamma \rightarrow \rho \) transition probability \( \approx 1/300 \), and \( A(b) \) is computed from the Fourier transform of some pionic form factor. However, it should be stressed that these are assumptions which are not inherent to perturbative QCD or even to the idea that minijets drive the increase of hadronic cross-sections. Recall, for example, that in the GRV parametrization the "naive" VMD estimate of \( q^2(x, Q^2) \) had to be doubled [24] in order to describe data at higher \( Q^2 \). Finally, if one estimates [96] \( P_{\text{had}} \) as \( \int_0^1 dx x q^2(x, p_{T,\text{min}}^2) \), one finds a value around \( 1/150 \) even for the DG parametrization. We therefore have to conclude that theoretical considerations at present only allow to estimate \( P_{\text{had}} \) up to a factor of 2 or so. The uncertainty in \( A(b) \) has so far not been discussed in the literature, but might be of similar magnitude.

In view of these ambiguities it is not surprising that predictions [95, 96, 97, 99, 100] for the total \( \gamma p \) cross-section at HERA energies differed quite widely prior to its measurement. Some examples are shown in figure 22, together with low-energy data and the recent ZEUS measurement [101]; a very similar value has been reported by the H1 collaboration [102]. The two solid curves show fits to low-energy data based on Pomeron phenomenology. The two dot-dashed curves show minijet predictions [100] using the DG parametrization with \( p_{T,\text{min}} = 1.4 \) (upper) and 2.0 (lower curve) GeV, while the dotted and dashed curves have been obtained from the LAC1 parametrization using the same values of \( p_{T,\text{min}} \). The LAC parametrization seems disfavoured, but in view of the theoretical and experimental uncertainties it might be premature to exclude it altogether. The DG minijet prediction with \( p_{T,\text{min}} = 2 \) GeV is certainly in agreement with the data. Notice, however, that all minijet calculations

![Figure 22](image-url)
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Figure 23. Integrated two jet cross-section for $p_T \geq p_{T,min}$ as a function of $p_{T,min}$ for the photon spectra of figure 10, as predicted [52] from the DG parametrization.

predict a substantially larger slope of the cross-section than the Pomeron-based fits do; a measurement of the energy-dependence of the total $\gamma p$ cross-section at HERA might therefore help to distinguish these models [103]. Finally, we have already seen that, in the case of hadronic collisions, minijet models also reproduce details of event shapes, e.g. multiplicity fluctuations and various correlations [94]. The measurement of similar quantities at HERA should help to distinguish between models.

Minijets are also expected to play an important role in $\gamma \gamma$ collisions at $e^+ e^-$ colliders [104, 52]. Indeed, the minijet cross-section at $e^+ e^-$ colliders rises even faster than at hadron colliders, since not only the $\gamma \gamma$ cross-section but also the $\gamma$ flux increases with energy, especially once beamstrahlung becomes important. Some examples of the resulting minijet cross-sections are shown in figure 23 [52], for the same photon spectra introduced in figure 10. We see that the DG parametrization with $p_{T,min} \approx 1.6$ GeV predicts a cross-section between about 20 and 500 nb, depending on the machine parameters; at a $\gamma \gamma$ collider this cross-section would be as large as 2 $\mu$b.

For the Palmer-G (Palmer-F) design this corresponds to about 25(0.5) minijet pairs per bunch train collision; for the wbb option of the D-D and TESLA designs one expects 0.02 and 0.004 minijet pairs, respectively, in a 100 nanosecond interval. Of course, the minijet cross-section is sensitive to the parton content at small $x$ values, where so far no experimental data exist. On the other hand, "shadowing" effects, which can be important for $x < 10^{-3}$, are not expected to be relevant for colliders with $\sqrt{s} \leq$ 1 TeV.

We have just seen that (inclusive) jet cross-sections can be larger than the total cross-section. However, with the possible exception of the Palmer-G design whose hard beamstrahlung spectrum implies that the average $W_{\gamma \gamma}$ and hence $\sigma(\gamma \gamma \rightarrow \text{jets})$ averaged over the photon spectrum is quite large, eikonalization effects are not expected to change the number of hadronic (minijet) events at 500 GeV $e^+ e^-$ colliders significantly even if a conservative, VMD-based eikonalization scheme is used. This can be seen from the fact [52] that the minijet cross-section (as predicted from the DG parametrization) is smaller than or of the same order of magnitude as the total $e^+ e^- \rightarrow e^+ e^- + \text{hadrons}$ cross-sections estimated using a constant $\gamma \gamma \rightarrow \text{hadrons}$ cross-section of 250 nb for $W_{\gamma \gamma} > 5$ GeV. We thus have to face the unpleasant fact
that some designs for $e^+e^-$ colliders predict several hadronic events to occur at each bunch train collision already at $\sqrt{s} = 500$ GeV. It is usually accepted that a 500 GeV collider should be designed such that it can be upgraded to $\sqrt{s} \geq 1$ TeV; of course, beamstrahlung and hadronic 2-photon backgrounds become worse at higher energies.

This problem might be alleviated somewhat if detectors achieve a very good time resolution. E.g., at Palmer-F or -G, a bunch train consists of 10 bunches in time intervals of 1.4 ns. A time resolution of about 2 ns seems achievable at least for the tracking system, so that this part of the detector would “see” at most two superimposed bunch crossings; this would obviously reduce the number of minijets in the smallest time unit measurable by the detector by a factor of 5. On the other hand, it seems unlikely that similarly fast calorimeters can be built. Notice that about 35 to 40% of the energy of a hadronic jet is carried by neutral particles, which are only detectable in calorimeters.

What are the consequences of “always” having >1 minijet event present in the detector? Basically it means that one now has an “underlying event”; i.e. every annihilation event (and every hard $\gamma\gamma$ event) will be accompanied by several minijet events. Every event will thus have some hadronic activity. This situation is of course well known from hadron colliders, but the absence of an underlying event, i.e. the “cleanliness” of the experimental environment, is usually considered to be one of the main advantages of $e^+e^-$ colliders. The presence of a few (or even a few dozen) soft hadrons does not usually affect the possibility to detect “new physics” signals very much, although some care has to be taken when defining what is meant by an isolated lepton or photon, or by a hadronically quiet event; and it has to be kept in mind that fluctuations in the underlying event might fake elements of a signal, e.g. missing $p_T$. However, the ability of future linacs to study new particles in detail might be compromised severely by the presence of a large underlying event. First of all, the beam energy constraint would no longer be applicable, since the visible energy can be larger than $\sqrt{s}$. This already excludes the possibility of precision measurements of the mass of a hadronically decaying particle at energies far above threshold. An underlying event would also make it more difficult to discriminate between hadronically decaying $W$ and $Z$ bosons. Moreover, a large multiplicity of soft particles might make it impossible to operate a microvertex detector, which is deemed necessary for efficient $b$ and $c$ quark tagging. We estimate that one minijet event will deposit between 6 and 10 GeV of transverse energy in the detector (from both the minijets itself and the outer fringes of the spectator jets), corresponding to a charged multiplicity of about 8. Finally, an underlying event would also complicate the study of hard $\gamma\gamma$ events, since there would always be some spectator jet activity in the forward and backward directions, making it much more difficult to distinguish between hard, direct and resolved photon events.

It therefore seems much preferable to us to construct future $e^+e^-$ linacs and their detectors such that an underlying event can be avoided. This ought to be relatively easy at $\sqrt{s} = 500$ GeV, but might prove challenging [52] for colliders operating at $\sqrt{s} \geq 1$ TeV.

6. Summary and conclusions

The measurement of $F_2'$ in deep-inelastic $e\gamma$ scattering at present $e^+e^-$ colliders does not yield sufficient information for decisive tests of QCD, nor for a discrimination
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of different ansätze for the parton content of real photons (§ 2). This is partly due to rather poor statistics (which is 3 or 4 orders of magnitude worse than for typical fixed-target deep-inelastic lepton-nucleon scattering experiments), partly due to kinematical constraints (which do not allow measurements at small Bjorken-x), and partly because $F_2^\gamma$ is not very sensitive to $G_\rho$. The situation might improve at future colliders, where smaller values of $x$ become accessible in DIS; in this "sea" region, gluons do contribute to $F_2^\gamma$. The ideal experiment of this type could be performed [105] if an $e^+e^-$ linac can be converted into an $e^+\gamma$ collider by backscattering laser photons.

In the last year the existence of resolved photon contributions had evolved from a theoretical prediction into an experimental fact. Their presence has first been demonstrated by the AMY group at TRISTAN, and has been confirmed by TOPAZ at TRISTAN and by the LEP experiments ALEPH and DELPHI (§ 3). Very recently the HERA experiments H1 and ZEUS have also reported that their data from the first (pilot) run show clear evidence of resolved photon events. The three main theoretical predictions [59]—large jet cross-sections at small and moderate transverse momentum; jet rapidity distribution peaked at large values; and the presence of a photonic spectator jet—have already been confirmed experimentally.

This first analyses of resolved photon events have already contributed to our understanding of the hadronic structure of the photon. TRISTAN data clearly exclude one parametrization of photonic parton densities (LAC3); the measurements of the total $\gamma p$ cross-section at HERA are in conflict with predictions from the more extreme variety of minijet models. TRISTAN and LEP data will improve due to increased statistics, improved angular coverage of the detectors (at TRISTAN) and increased beam energy with less annihilation backgrounds (at LEP). The next year should see the HERA data sample grow by at least 3 orders of magnitude. The number of resolved photon events detected at HERA will then greatly exceed that of all $e^+e^-$ colliders combined, allowing for detailed studies of jet production as well as searches for many other final states (§ 4). Nevertheless, $e^+e^-$ data will continue to play an important role. On the one hand, these lower energy (in the $\gamma\gamma$ or $\gamma p$ centre-of-mass system) machines can probe the parton densities in the photon at large $x$ but moderate $Q^2$, while at higher energies large $x$ usually imply large $Q^2$. Recall that all models converge towards the asymptotic prediction if both $x$ and $Q^2$ are large, while there are sizable differences at large $x$ and moderate $Q^2$. Moreover, $e^+e^-$ colliders also allow to study events with rather small invariant mass, which are usually boosted out of the detector at HERA; this should help us in understanding the transition between soft and hard interactions.

Soft and semihard (minijet) $\gamma\gamma$ events can lead to an "underlying event" at future $e^+e^-$ supercolliders, spoiling the traditional cleanlines of $e^+e^-$ colliders (§ 5). The main question here is whether beamstrahlung can be kept under control. Existing designs indicate that this should be fairly easy at centre-of-mass energies up to 500 GeV, but can become increasingly difficult at higher energies.

Our general conclusion is that the importance of resolved photon contributions increases with beam energy, and thus with time. We therefore expect great progress to be made in this field over the next few years. This is the heroic age of resolved photons!
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