Abstract. The dipole strength function and the photoabsorption cross section for some He even-even isotopes close to the neutron drip-line region are systematically studied within a selfconsistent random phase approximation method in the continuum. In the systematic analysis several parametrizations of the Skyrme interaction with density and momentum dependent terms are used as the p–h residual interaction. Proton and neutron partial contributions to the total dipole strength and cross section are determined and the ratio \( R = \frac{\sigma(\gamma, p)}{\sigma(\gamma, p)} \) is calculated and compared to the measured data. On the same footing, calculations of the mean square radius and interaction cross section are carried out for the secondary beams of \(^4\)He, \(^6\)He and \(^8\)He produced through the fragmentation of \(^{11}\)Be primary beam with targets \(^9\)Be, \(^{12}\)C and \(^{27}\)Al at 800 MeV/nucleon. It is determined that some of the Skyrme force parametrizations give better results than others when compared to the experimental data. Finally, a determination of nuclear skins for the He isotopes is given.

1. Introduction

A great number of studies have been performed on photonuclear reactions in many nuclei throughout the periodic table. These studies are well justified by the well known fact that the electromagnetic radiation interaction with matter is the best understood example in nuclear physics. In particular, the dipole resonance has been of central interest both experimentally and theoretically speaking since it corresponds to the absorption of the electric dipole radiation component of the electromagnetic field by the nucleus. The most simple picture for this mode
of nuclear excitation is represented by a proton oscillation produced by the electric field of certain frequency $\omega$ and as to keep the centre of mass fixed, a subsequent neutron movement in the opposite direction [1]. In single-particle shell-model calculations, the nuclear resonance states are viewed as highly correlated collective states of a superposition of $1p$–$1h$, $2p$–$2h$, $3p$–$3h$, etc, states. Possibly the most important effect giving place to resonance states is the interaction of the excited particles (particle-states) with the holes left below the Fermi surface (hole-states). This p–h interaction causes a mixing of the possible shell model configurations with the result that some of the final states obtained in this mixing have energies appreciably larger than those of simple particle–hole shell model transitions. Since the transition matrix elements between these final resonant states and the closed shell ground state must be large, it then becomes clear that the residual p–h interaction used in the calculations is of vital importance [2]. It is also well known that since these resonant collective states lie well beyond the energy threshold for particle emission where the density level is high, they must decay either by particle emission or by damping (spreading) into more complex compound nuclear states.

Over the course of many years, the collective nature of these excited resonant states has been well established by numerous studies. These studies range from conventional shell model diagonalization techniques to extensions of the shell model to the continuum [2, 3]. In this work, we present a random phase approximation (RPA) approach in the continuum applied to the photoexcitation of $^4$He and to the halo nuclei $^6$He and $^8$He. In our method, the single-particle continuum is treated exactly while the bound single-particle energies and wavefunctions are constructed selfconsistently. The p–h residual interaction is considered to be the Skyrme force with velocity and density dependent terms. Several parametrizations of such an interaction are considered and their effect on the dipole strength energy distribution as well as on the energy weighted sum rule (EWSR) are systematically studied. The inhomogeneous coupled-channel (CC) equations that appear in the RPA formalism are solved by the use of the Lanczos method as was done in a previous work [4]. The usefulness and efficiency of this way of solving the CC-equations has been proved by the calculations performed on the $2^+$-strength response function on $^{16}$O, $^{40}$Ca.

The nucleus $^4$He represents an ideal example to test the usefulness of any theory in the continuum since all its excited states are already scattering states. There is also an interesting controversy generated by different theoretical calculations and experimental measurements regarding the relative values of the photo-emission cross sections for protons $\sigma(\gamma, p)$ and neutrons $\sigma(\gamma, n)$. Some years ago, Berman et al [5] reported that the $(\gamma, p)$ cross section resulted almost twice the corresponding $(\gamma, n)$ cross section over most of the resonance energy and gave place to calculations in which some charge symmetry breaking components were introduced to account for the difference. However, later measurements have shown [6], in disagreement with the data of [5], that the $(\gamma, p)$ and $(\gamma, n)$ cross section ratio was almost unity over a wide range of energies and therefore showed no need for any charge symmetry breaking factors. In this work, we intend to explicitly calculate the partial contributions $(\gamma, p)$ and $(\gamma, n)$ of the total cross section for $^4$He and for the isotopes $^6$He and $^8$He and compare the resultant ratio $R = \sigma(\gamma, p)/\sigma(\gamma, n)$ to the data of [6].

It has been argued that thick neutron skins must appear in the nuclei $^6$He and $^8$He [7], this conclusion has been drawn from direct experimental measurements of the interaction and reaction cross sections for reactions of He-isotopes with some stable nuclei, such as $^{12}$C, $^9$Be and $^{27}$Al at 800 MeV/nucleon [8]–[11]. The experimental results suggest that the calculation
of the root mean square radius $R_{\text{rms}}$ for $^6$He and $^8$He might indicate the existence of thick nuclear skins in these nuclei. We intend to find out if such nuclear skins can be determined within our RPA model. A comparison of our theoretical results for the $R_{\text{rms}}$ radius and for the interaction cross sections will be made with those of [7] for the same nuclear reactions. The remainder this paper is organized as follows: in section 2 a brief description of the RPA theoretical formalism is presented and in section 3 the calculations and a discussion of the results are given.

2. Brief theoretical description

2.1. The response function

The response of a nuclear system to an external field $\hat{F}$ is generally given by the transition strength defined by

$$S(E) = \sum_\nu |\langle \nu | \hat{F} | 0 \rangle|^2 \delta (E_\nu - E).$$

(1)

This strength function $S(E)$ has poles for discrete states $\nu$ and a resonant behaviour for continuum states. The external force $\hat{F}$ acting on the nuclear system is a one-body operator which in a second quantization representation is described by

$$\hat{F} = \int dr \hat{\Psi}^\dagger (r) F(r) \hat{\Psi}(r),$$

(2)

where the creation and annihilation particle operators at point $r$ are $\hat{\Psi}^\dagger (r) = \sum_i \phi_i^\dagger (r) \hat{a}_i^\dagger$ and $\hat{\Psi}(r) = \sum_j \phi_j (r) \hat{a}_j$ respectively. It is easily found that the transition strength can be expressed by

$$S(E) = -\frac{1}{\pi} \text{Im} \int dr \, dr' F^*(r) R(E, r, r') F(r'),$$

(3)

where $R(E, r, r')$ is known as the response function and is given by

$$R(E, r, r') = \langle \phi_0 | \hat{\Psi}^\dagger (r) \hat{\Psi}(r) \hat{G}(E, r, r') \hat{\Psi}^\dagger (r') \hat{\Psi}(r') | \phi_0 \rangle + \langle \phi_0 | \hat{\Psi}^\dagger (r) \hat{\Psi}(r) \hat{G}(-E, r, r') \hat{\Psi}^\dagger (r') \hat{\Psi}(r') | \phi_0 \rangle,$$

(4)

where $\phi_0$ is the Hartree–Fock nuclear ground state and $\hat{G}(E) = 1/(E - H + i\omega)$ is the Green function that correlates a p–h excitation from one point to another. The total Hamiltonian of a nuclear system is defined by

$$H = H_0 + V,$$

(5)

where $H_0 = T + U$ is the Hartree–Fock Hamiltonian and $V$ the residual interaction. If the residual interaction $V$ is assumed to be local and its exchange part of a $\delta$-form, then by the use of $\hat{G} = \hat{G}_0 + \hat{G}_0 V \hat{G}$, the response function $R(E, r, r')$ can be written as

$$R(E, r, r') = R_0(E, r, r') + \int dr_1 \, dr_2 \, R_0(E, r_1, r_2) V(r_1, r_2) R(E, r_2, r'),$$

(6)

where $R_0(E, r, r')$ is the free response defined by

$$R_0(E, r, r') = \langle \phi_0 | \hat{\Psi}^\dagger (r) \hat{\Psi}(r) \hat{G}_0(E, r, r') \hat{\Psi}^\dagger (r') \hat{\Psi}(r') | \phi_0 \rangle + \langle \phi_0 | \hat{\Psi}^\dagger (r) \hat{\Psi}(r) \hat{G}_0(-E, r, r') \hat{\Psi}^\dagger (r') \hat{\Psi}(r') | \phi_0 \rangle,$$

(7)
where as before \( \hat{G}_0 = 1/(E - H_0 + i\omega) \) is the free Green function. In a 1p–1h approximation, the expression for \( R_0(E, r, r') \) can equivalently be written in a more computable form, i.e.,

\[
R_0(E, r, r') = \sum_{m_i} \left[ \phi_i^{(0)*}(r) \phi_m^{(0)}(r) \left( \frac{1}{E - \epsilon_m + i\omega} + \frac{1}{E - \epsilon_m - i\omega} \right) \phi_{m_i}^{(0)*}(r') \phi_i^{(0)}(r') \right] \tag{8}
\]

where \( \phi_i^{(0)}, \phi_m^{(0)}, \epsilon_m \) and \( \epsilon_i \) are selfconsistent Hartree–Fock single-particle and hole wavefunctions and corresponding energies which are solutions of \( H_0 \). As usual, the indices \( m, n, l, \ldots \) mean single-particle levels above the Fermi surface (particle-states) and \( i, j, k, \ldots \) are levels below (hole-states). The energy difference \( \epsilon_{mi} \) of equation (8) is the p–h transition energy defined by \( \epsilon_{mi} = \epsilon_m - \epsilon_i \). In principle, the strength response function \( S(E) \) can be obtained by introducing equation (8) into (6) and in turn into equation (3). The result is then solved iteratively until convergence is achieved. In order to simplify the calculations, it is convenient to define the correlated source function \( \lambda(r) \) by the identity

\[
\int \, dr' \, R(r, r', E) F(r') = \int \, dr' \, R_0(r, r', E) \lambda(r'). \tag{9}
\]

From equation (6), it is clear that the equation that \( \lambda \) must satisfy is

\[
\lambda(r) = F(r) + \int \, dr' \, dr'' \, V(r, r') R_0(r', r'', E) \lambda(r''). \tag{10}
\]

Integrating this equation over the spin-angle variables, the forward and backward RPA amplitudes \( \lambda_{ph}(r) \) and \( \tilde{\lambda}_{hp}(r) \) can be obtained, that is

\[
\lambda_{ph}(r) = F_{ph}(r) + \sum_{p' h'} \int \, dr' \, dr'' \, V_{ph, p'h'}(r, r') g_{p'h'}^{(+)}(r', r'') \lambda_{p'h'}(r'') + \sum_{h' p'} \int \, dr' \, dr'' \, V_{ph, h'p'}(r, r') g_{h'p'}^{(+)}(r', r'') \tilde{\lambda}_{h'p'}(r''), \tag{11}
\]

and

\[
\tilde{\lambda}_{hp}(r) = \tilde{F}_{hp}(r) + \sum_{p' h'} \int \, dr' \, dr'' \, V_{hp, p'h'}(r, r') g_{p'h'}^{(+)}(r', r'') \lambda_{p'h'}(r'') + \sum_{h' p'} \int \, dr' \, dr'' \, V_{hp, h'p'}(r, r') g_{h'p'}^{(+)}(r', r'') \tilde{\lambda}_{h'p'}(r''). \tag{12}
\]

In this equation, \( g^{(+)}(r) \) is the radial Green function for the excited particle states. The particle–hole matrix elements \( V_{ph, p'h'}(r, r') \) are given by

\[
V_{ph, p'h'}(r, r') = \sum \langle j_p m_p j_h m_h | JM \rangle \langle j_{p'} m_{p'} j_{h'} m_{h'} | JM \rangle rr'(u_p \phi_i^{(0)} | V | u_{p'} \phi_{h'}^{(0)}), \tag{13}
\]

where \( () \) denotes integration over spin-angle variables and \( u_p \) is the radial part of the particle state \( p \). The radial matrix elements \( V_{ph, p'h'}(r, r') \) depend exclusively on the radial dependence of the residual interaction and clearly the other combinations are obtained by simple interchange of indices. Regarding the external force \( F(r) \) of equation (2), it is well known that for dipole excitations this field corresponds to the dipole component of the electric potential that is

\[
F(r) = \sum_{\mu} e_{p, n} r Y_{1\mu}(\hat{\Omega}), \tag{14}
\]

where \( e_{p, n} \) are the proton and neutron effective charges that are given by

\[
e_p = \frac{Z}{A} e, \quad e_n = -\frac{N}{A} e, \tag{15}
\]

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N, Z and A denote the neutron, proton and mass numbers respectively. As for the effective residual p–h interaction $V$ of equation (5), we propose to use the Skyrme interaction [12] and in particular those parametrizations of it with velocity and nuclear density terms. The two-body Skyrme interaction used in successful selfconsistent calculations is of the form [13]

$$
V(r_1, r_1) = v_0(1 + x_0 P_\sigma) \delta(r_{12}) + \frac{1}{3} v_1(1 + x_1 P_\sigma)(k' \cdot \delta(r_{12}) + \delta(r_{12}) k^2) +
$$

$$
t_2(1 + x_2 P_\sigma) k' \cdot \delta(r_{12}) k + \frac{1}{6} t_3(1 + x_3 P_\sigma) P(r_{c.m.}) \delta(r_{12})
$$

$$
+ i W_0 \sigma \cdot k' \times \delta(r_{12}) k,
$$

where $k'$ is the adjoint of $k'$, $P_\sigma$ is the spin-exchange operator and $\rho$ the nuclear density. The RPA calculations of the nuclear strength will be systematically executed for several parametrizations of these interactions as will be described in what follows.

3. Numerical details and calculations

Feldman et al [6] have published data for the radiative capture on $^3$He and photoabsorption on $^4$He in the photon energy range from 21.3 to 31.1 MeV. They found that the $\sigma(p, \gamma)$ cross section resulted about 35% lower than previous measurements [5]. In fact, the measured ratio $\sigma(\gamma, p)/\sigma(\gamma, n)$ resulted about 1.09±0.17 in the range of energies from 24 to 31 MeV. Concerning the nuclear interaction, this means that there is no need for any charge symmetry breaking factor to be introduced into the theoretical calculations. In this work, we will attempt to correctly calculate and fit the experimental energy distribution of the $(\gamma, p)$ cross section for $^4$He. Clearly the $\sigma(\gamma, n)$ can also be determined and the ratio $R = \sigma(\gamma, p)/\sigma(\gamma, n)$ can be compared to the experimental measurements. The total photoabsorption cross section $\sigma(\gamma, E)$ can be calculated directly from the total strength function $S(E)$ by

$$
\sigma(\gamma, E) = \frac{8\pi^3(\lambda + 1)}{\lambda((2\lambda + 1)!)^2} k^{2\lambda-1} S(E),
$$

where $\lambda = 1$ for dipole excitations.

It is true that the Skyrme nuclear interaction has been very useful in the description of the nuclear mean field, therefore we shall adopt this interaction as the residual force that correlates the particles and holes in the excited resonance state of He isotopes. The parametrizations of the Skyrme interaction that will be considered in the calculations are presented in table 1. We consider linear ($\alpha = 1$) density dependent interactions such as the SI, SIII and SIV used by Vautherin et al [14] and Bartel et al [15] in selfconsistent calculations of ground-state properties of spherical nuclei [16]. The nonlinear Ska and Skp interactions used in studies of nuclear surface properties, mass formula and stiffness parameters [17, 18], the SGI and SGII parametrizations adopted by Van Giai and Sagawa [19] in their RPA calculations of dipole strength distributions and finally the SkM* interaction [15] used for static nuclear properties of spherical and deformed nuclei. As seen in table 1, in all the parametrizations of the interaction, the parameter $t_3$ takes values in a wide range. This fact is important since the nuclear density term will be the most crucial in the fitting of the dipole strength resonance. In figures 1(a) and (b), we show the calculated total strength function for $^4$He with the Skyrme interactions SI, SIII, SGI and SGII (figure 1(a)) and with SIV, Ska, Skp and SkM* (figure 1(b)). It can be seen in both figures that the dipole resonance is located at slightly different energies around 22.5 and 24 MeV. The location depends basically on the repulsive character of the density dependent term of

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Table 1. Parameters of the Skyrme interaction, $t_0$ (MeV fm$^3$), $t_1$ (MeV fm$^5$), $t_2$ (MeV fm$^5$), $t_3$ (MeV fm$^3$).

|   | $t_0$  | $t_1$  | $t_2$  | $t_3$  | $x_0$ | $x_3$ | $\alpha$ |
|---|-------|-------|-------|-------|-------|-------|----------|
| SI | -1057.3 | 239.5 | -100.0 | 14463.5 | 0.56  | 1.0   | 1.0      |
| SIII | -1128.75 | 395.0 | -92.00 | 14000.0 | 0.45  | 1.0   | 1.0      |
| SIV | -1205.6 | 765.0 | 35.01  | 5000.0  | 0.05  | 1.0   | 1.0      |
| Ska | -1602.8 | 570.88| -67.7  | 8000.0  | -0.02 | -0.286| 1/3      |
| SkM* | -2645.0 | 410.0 | -135.0 | 15595.0 | 0.09  | 0.06  | 1/6      |
| SGI | -1603.0 | 515.9 | 84.5   | 8000.0  | -0.02 | 0.1381| 1/3      |
| SGII | -2645.0 | 340.0 | -41.9  | 15595.0 | 0.09  | 0.06  | 1/6      |
| Skp | -2931.7 | 320.62| 337.41 | 18708.9 | 0.292 | 0.181 | 1/6      |

The corresponding interaction. For instance, the SI, SIII and SGII interactions (figure 1(a)) are repulsive enough so as to wash off the peak at 20.7 MeV corresponding to direct particle emission from the $s_{1/2}$-level.

The partial emission contributions to the total strength from proton and neutron transitions can be easily obtained from equation (17), the photoabsorption cross section $\sigma(\gamma, p)$ is

$$\sigma(\gamma, p)(E) = \frac{16\pi^3}{9} \left(\frac{E}{\hbar c}\right)S_p(E),$$

where $S_p$ is the dipole strength contribution from protons. The corresponding part for neutrons $\sigma(\gamma, n)$ has a similar expression. The calculated $\sigma(\gamma, p)$ cross section corresponding to the interactions used in figures 1(a) and (b) are shown in figures 2(a) and (b) respectively. As seen in these figures the interactions SI, SIII of figures 2(a) and Skp of figure 2(b) fit well the data [6] for energies less than 26 MeV. In any case the calculated resonance width is not as big as expected. The reason for this underestimation may lie in the fact that no absorption potential is explicitly considered in the RPA calculations for the excited particle states and that only $1p$–$1h$ excitations are allowed. However, as the proton dipole resonance is basically fitted by our calculation, it may be said that the major contribution to the resonance is given by direct emission to the continuum. It is interesting to note that the position of the resonance depends basically on the density dependent term of the interaction, giving the closer fits for those interactions with higher $t_3$ values. Thus, the density dependence of the Skyrme interaction plays a fundamental role to push the dipole resonance up to the required energies. Regarding the ratio $R = \sigma(\gamma, p)/\sigma(\gamma, n)$, we have calculated it with the interaction SIII that fits well the total and proton dipole strengths. This is shown in figure 3 together with the data of [6]. We see that our result agrees very well with the experimental data for a wide range of energies. Also shown in figure 3 is the interesting fact that the calculation of the ratio $R$ with the momentum dependent part of the Skyrme interaction gives slightly closer values to unity in the low-energy region (dashed-curve) than the calculation without velocity dependent terms (full-curve). This feature was also found in the calculations with the other parametrizations of the interaction. As seen also in figure 3, the ratio $R$ apparently goes to infinity for energies less than 21 MeV. The reason for this is that the proton and neutron binding energies of the $s_{1/2}$-level of $^4$He are 19.7 and 20.8 MeV respectively, thus there can be some proton emission and zero neutron emission for energies between the binding energies.
Figure 1. (a) Dipole strength energy distribution for $^4$He calculated with the Skyrme interactions; SI, SIII, SGII and SGI. (b) Dipole strength energy distribution for $^4$He calculated with the Skyrme interactions; SkM*, Ska, SIV and Skp.
Figure 2. (a) Proton contribution to the total dipole cross section for the interactions, SGI, SGII, SIII and SI. The data correspond to [6]. (b) Proton contribution to the total dipole cross section for the interactions, SIV, Ska, SkM* and Skp.
A very different situation is found for the isotopes $^6\text{He}$ and $^8\text{He}$. In figure 4(a) the total dipole strength function as well as its proton and neutron contributions are shown for $^6\text{He}$ with the SGI Skyrme interaction while in figure 4(b) the same calculations are presented for $^8\text{He}$ with the SkM* Skyrme interaction. As seen in these figures, there is a strong difference between the proton and neutron emission strength. It is clear in both cases that the main contribution to the dipole resonance comes from neutron emission (dotted-curves) up to 20.5 MeV, for higher energies the proton contribution takes over. This is due to the fact that there are neutron levels with very low energy in the p–h shell configuration of He isotopes. That is, in the HF calculation for $^6\text{He}$ the halo neutrons are bound to the $1p_3/2$-orbit with a binding energy of 0.96 MeV while the other two neutrons and two protons are in the $1s_1/2$-orbit with energies of 20.8 and 19.7 MeV. As for the $^8\text{He}$ isotope, the neutron shell configuration used is two valence neutrons in the $1p_1/2$-level with 2.13 MeV, two neutrons in the $1p_3/2$-level with 2.52 MeV while the other two neutrons and two protons are kept in the $1s_1/2$-level with energies very close to the $^4\text{He}$ core.

It is also evident that for both isotopes $^6\text{He}$ and $^8\text{He}$ a great amount of the energy integrated dipole strength comes from the low-energy region. From general principles the dipole strength function of equation (1) with the external field of equations (2) and (14) satisfies an EWSR given by [20]

$$ S = \sum_\nu |\langle \nu | F | 0 \rangle |^2 (E_\nu - E_0) = \frac{\hbar^2}{2m} \frac{3}{4\pi} \frac{NZ}{A} (1 + \alpha), $$

(19)
Figure 4. (a) Total dipole strength for $^6$He with proton (dashed curve) and neutron (dotted curve) contributions for a calculation with the SGI Skyrme interaction. (b) Total dipole strength for $^8$He with proton (dashed curve) and neutron (dotted curve) contributions for a calculation with the SkM* Skyrme interaction.
Figure 5. Energy integrated strength function for $^4$He (full curve), $^6$He (dashed curve) and $^8$He (dotted curve).

where

$$\alpha = \frac{1}{4} \left( t_1 + t_2 \right) \left\langle \sum_{ij} \delta (\vec{r}_i - \vec{r}_j) \right\rangle \left( \frac{\hbar^2}{2m} \right) A,$$

(20)

therefore, the upper limit that the EWSR can get depends on the parameters $t_1$ and $t_2$ of the parametrization used. The factor $\left\langle \sum_{ij} \delta (\vec{r}_i - \vec{r}_j) \right\rangle$ can be easily found once the level wavefunctions are found within a HF-selfconsistent calculation. We have calculated the values of $S$ for all the parametrizations of the effective Skyrme interaction given in table 1. By integrating of the calculated strength function up to 100 MeV, we found that the highest EWSR correspond to $5.03e^2$ MeV fm$^2$ for $^4$He with the Skp interaction, $6.91e^2$ MeV fm$^2$ for $^6$He with the SI interaction and $7.24e^2$ MeV fm$^2$ for $^8$He with the SIV parametrization. It is interesting to notice that for a momentum independent interaction ($t_1, t_2 = 0$), the EWSR calculated with the right-hand side of equation (19) does not depend on the parametrization used and are respectively $4.95e^2$ fm$^2$ MeV, $6.64e^2$ fm$^2$ MeV and $7.43e^2$ fm$^2$ MeV for $^4$He, $^6$He and $^8$He. Evidently only the $^8$He calculation is still below the momentum independent limit. On the other hand, the limit values calculated with equation (19) using (20) are $5.34e^2$ fm$^2$ MeV for $^4$He, $7.01e^2$ fm$^2$ MeV for $^6$He and $7.58e^2$ fm$^2$ MeV for $^8$He with the Skp, SI and SIV parametrizations respectively. Therefore, our calculations show that up to 100 MeV, 94, 98 and 95% of the EWSR have been exhausted.

From some time it has been known that nuclear size poses a very important problem in nuclear physics, for this reason several experimental techniques have been developed to determine it with more accuracy. Similarly, measurements of the proton and neutron mass distributions have been done and compared to the predictions of some theoretical nuclear structure calculations [21].

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Table 2. The mean square radii (fm) for nuclear matter, protons and neutrons. Also shown are the calculated and experimental interaction cross sections (mb). The experimental data are shown in parentheses [7, 8, 11].

| Material | $R_{\text{rms}}$ | $R_{\text{rms,p}}$ | $R_{\text{rms,n}}$ | $R_{\text{rms,np}}$ | $\sigma_I(^{12}\text{C})$ | $\sigma_I(^{9}\text{Be})$ | $\sigma_I(^{27}\text{Al})$ |
|----------|----------------|-----------------|-----------------|-----------------|----------------|----------------|----------------|
| $^4\text{He}$ | 1.78 | 1.79 | 1.77 | -0.02 | 565.3(503 ± 5) | 504.6(485 ± 4) | 797.1(780 ± 13) |
| SkM* | 1.80 | 1.79 | 1.79 | 0.0 | 570.4 | 509.5 | 803.1 |
| SIII | 2.43 | 1.78 | 2.79 | 1.01 | 704.5(722 ± 6) | 636.9(672 ± 7) | 961.5(1063 ± 8) |
| $^6\text{He}$ | 2.47 | 1.80 | 2.76 | 0.96 | 715.7 | 647.2 | 973.9 |
| SkM* | 2.55 | 1.83 | 2.84 | 1.01 | 707.7(817 ± 6) | 639.6(757 ± 4) | 964.5(1197 ± 9) |
| SIII | 2.59 | 1.83 | 2.86 | 1.03 | 731.2 | 661.9 | 991.2 |
| $^8\text{He}$ | 2.55 | 1.83 | 2.84 | 1.01 | 707.7(817 ± 6) | 639.6(757 ± 4) | 964.5(1197 ± 9) |

The experimental determination of the interaction cross section $\sigma_I$ through the production of exotic-isotope beams from the projectile fragmentation process in heavy-ion reactions has been achieved recently. In particular, the nuclear matter, proton and neutron mass distribution radii of $^4\text{He}$, $^6\text{He}$ and $^8\text{He}$ isotopes have been determined from the collision of the secondary beam projectile (He-isotopes) with targets $^{9}\text{Be}$, $^{12}\text{C}$ and $^{27}\text{Al}$ at 800 MeV/nucleon [8, 9]. From these observations the nuclear size has been calculated using

$$\sigma_I = \pi [R_p + R_t]^2,$$

where $R_p$ and $R_t$ are the projectile and target interaction radii.

As a last calculation, a Hartree–Fock selfconsistent determination of the mean square radius $R_{\text{rms}}$ for nuclear, proton and neutron mass distributions will be performed. Also, the predictions of equation (21) are found for the interaction cross section of nuclear reactions involving He isotopes with $^{9}\text{Be}$, $^{12}\text{C}$ and $^{27}\text{Al}$ at 800 MeV/nucleon. Finally, we will look into the plausibility of the so-called thick nuclear skins for He-isotopes as predicted by Tanihata et al [7]. According to their definition, nuclear skins can be identified by the values of the radial root mean square difference between neutrons and protons; $R_{\text{rms, np}} = R_{\text{rms,n}} - R_{\text{rms,p}}$ and by the corresponding Fermi energy difference $\Delta E_F = E_{F,n} - E_{F,p}$. In table 2, the $R_{\text{rms, np}}$ values are listed for the SkM* and SIII interactions. For $^4\text{He}$, the Fermi energy difference for these interactions are 18.73 and 19.3 MeV respectively. As for $^8\text{He}$, we have $R_{\text{rms, np}} = 1.01$ fm for SkM* and 1.03 fm for the SIII with an energy difference $\Delta E_F = 17.57$ and 18.45 MeV. These
\(R_{\text{rms, np}}\) and \(\Delta E_F\) values agree perfectly well with those of [7]. Therefore our results also show that He isotopes have indeed thick neutron skins and even more, they show that the neutron mass distribution extends farther than that of protons as found in previous calculations.

In summary, we have shown that the RPA method can give reasonable good predictions for the energy distribution of the dipole strength function and of the partial contributions from protons and neutrons for some He isotopes. In the calculations, different parametrizations of the Skyrme residual interaction were used and their effect on the dipole strength, photonuclear cross section and on the EWSRs was also investigated. On the same footing, quadratic mean square radii were calculated as well as interaction cross sections of nuclear reactions of He isotopes with the stable nuclei \(^{12}\text{C}\), \(^{9}\text{Be}\) and \(^{27}\text{Al}\) at 800 MeV/nucleon. In general our results for \(R_{\text{rms}}\) for nuclear, proton and neutron mass distributions and of interaction cross sections \(\sigma_I\) are in good agreement with the experimental values. Finally our results for \(R_{\text{rms}}\) show that nuclear skins appear for the \(^6\text{He}\) and \(^8\text{He}\) isotopes.

References

[1] Goldhaber M and Teller E 1948 Phys. Rev. 74 1046
[2] Goeko K and Speth J 1982 Ann. Rev. Nucl. Part. Sci. 32 65
  Van der Woude A 1987 Prog. Nucl. Part. Nucl. Phys. 18 217
[3] Bloch C and Gillet V 1965 Phys. Lett. 16 62
  Buck B and Hill A 1967 Nucl. Phys. A 95 271
[4] Gómez A and Udagawa T 1992 Rev. Mex. Phys. 38 43
[5] Berman B L, Fultz F C and Kelly M A 1971 Phys. Rev. C 4 723
  Berman B L, Firk F W and Wu C P 1972 Nucl. Phys. A 179 791
[6] Feldman G, Balbes M J, Kramer L H, Williams J Z and Weller H R 1990 Phys. Rev. C 42 R1167
[7] Tanihata I, Hirata D, Kobayashi T, Shimoura S, Sugimoto K and Toki H 1992 Phys. Lett. B 289 261
[8] Tanihata I 1985 Phys. Lett. B 160 380
[9] Tanihata I, Hamagaki H, Hashimoto O, Shida Y, Yoshikawa N, Sugimoto K, Yamakawa O, Kobayashi T and Takahashi N 1985 Phys. Rev. Lett. 55 2676
[10] Tanihata I, Hamagaki H, Hashimoto O, Nagamiya S, Shida Y, Yoshikawa N, Yamakawa O, Sugimoto K, Kobayashi T, Greiner D E, Takahashi N and Nojiri Y 1985 Phys. Lett. B 160 380
[11] Kobayashi T and Yamakawa O 1988 Phys. Rev. Lett. 60 2599
[12] Skyrme T 1959 Nucl. Phys. 9 615
[13] Gómez J M G, Pérez J C and Prieto C 1993 Nucl. Phys. A 551 451
[14] Vautherin D and Brink D M 1972 Phys. Rev. C 5 626
  Negele W and Vautherin D 1972 Phys. Rev. C 5 1472
[15] Bartel J, Quentin P, Brack M, Guet C and Hakansson H B 1982 Nucl. Phys. A 386 79
[16] Beiner M, Flocard H, Nguyen Van Giai and Quentin P 1975 Nucl. Phys. A 238 29
[17] Kohler H S 1976 Nucl. Phys. A 258 301
[18] Dobaczewski J, Flocard H and Treiner J 1984 Nucl. Phys. A 422 103
[19] Van Giai N and Sagawa H 1981 Nucl. Phys. A 371 1
[20] Bertsch G F and Tsai S F 1975 Phys. Rep. C 18 140
[21] Sato H and Okuhara Y 1985 Phys. Lett. B 162 217