Large-scale outflows in galaxies

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ABSTRACT

We discuss massive outflows in galaxy bulges, particularly the ones driven by accretion episodes where the central supermassive black hole reaches the Eddington limit. We show that the quasar radiation field Compton-cools the wind shock until this reaches distances $\sim 1$ kpc from the black hole, but becomes too dilute to do this at larger radii. Radiative processes cannot cool the shocked gas within the flow time at any radius. Outflows are therefore momentum driven at small radii (as required to explain the $M-\sigma$ relation). At large radii, they are energy driven, contrary to recent claims.

We solve analytically the motion of an energy-driven shell after the central source has turned off. This shows that the thermal energy in the shocked wind can drive further expansion for a time $\sim 10$ times longer than the active time of the central source. Outflows observed at large radii with no active central source probably result from an earlier short (few Myr) active phase of this source.

Key words: accretion, accretion discs – black hole physics – galaxies: evolution – quasars: general.

1 INTRODUCTION

The large-scale structure of galaxies often has surprisingly close connections to properties of their nuclei. The $M-\sigma$ relation between the supermassive black hole (SMBH) mass $M$ and the bulge velocity dispersion $\sigma$ is the most striking of these. Similar relations hold between black hole and galaxy stellar bulge mass $M_b$, and between the mass of nuclear star clusters and $\sigma$ in galaxies where there is no strong evidence for the presence of an SMBH ($\sigma \lesssim 150$ km s$^{-1}$).

Massive gas outflows (Pounds et al. 2003) driven by the central object offer a way of connecting these apparently disparate scales. A fast wind from the nucleus collides with the host galaxy’s interstellar medium (ISM), driving a reverse shock into the wind, and a forward shock into the ISM. This shock pattern moves outwards at a speed mainly determined by whether or not the reverse shock cools on a shorter time-scale than the outflow time-scale ($R/R_\ast$) (cf. Dyson & Williams 1997; Lamers & Cassinelli 1997). In the first case (efficient cooling), only the ram pressure of the original outflow is communicated to the ambient medium. This is a momentum-driven flow. In the second case (inefficient cooling), the full energy of the fast wind is communicated to the ambient medium through its thermal expansion after the shock. This is an energy-driven flow, which expands at higher speed and so can have a much larger effect on the bulge of the host galaxy.

Both types of outflow are important in galaxy formation. This Letter is mainly concerned with the large-scale effects of energy-driven flows. The existence of flows of this type has recently been questioned, so we first set the problem in context. We show that energy-driven outflows do occur, and are ubiquitous on large scales. Solving the outflow equations analytically, we give a simple relation between the time that the outflow is driven by the central source and the time over which it can be observed as coasting after this source turns off. This relation means that observed outflows can be used to constrain the past activity of a source. In this Letter, we deal with the case where this source is a quasar, which we model as an Eddington-accreting SMBH. Similar considerations apply in cases where the driving source is a nuclear star cluster.

2 MOMENTUM OR ENERGY DRIVING

The first proposal that outflows might relate SMBH and galaxy properties was by Silk & Rees (1998, hereafter SR98), who considered the effect of an Eddington wind from the black hole colliding with the host ISM. Requiring the shock pattern to move with the escape velocity, and so presumably cutting off accretion to the black hole, they found $M \propto \sigma^4$, with an undetermined coefficient of proportionality. Later, King (2003, 2005) pointed out that SR98 implicitly assumed an energy-driven outflow, whereas Compton cooling in the radiation field of the active nucleus was likely to produce a momentum-driven flow. The condition that this flow should be able to escape the immediate vicinity of the black hole, and so cut-off accretion, predicts a black hole mass

$$M = \frac{f_k}{\pi G^2} \sigma^4 \simeq 2 \times 10^8 M_\odot \sigma_b^4,$$

(1)

in good agreement with the observed relation (Ferrarese & Merritt 2000; Gebhardt et al. 2000) (which is itself probably an upper limit...
to the SMBH mass; cf. Batcheldor 2010; King 2010b). Here \( f_g \approx 0.16 \) is the gas fraction, \( \kappa \) is the electron scattering opacity and \( \sigma_{200} \) is the velocity dispersion in units of 200 km s\(^{-1}\). By contrast, an energy-driven outflow as in SR98 would have produced a mass smaller than equation (1) by a factor \( \sim \sigma/c \sim 10^{-3} \) (e.g. King 2010a).

A later application of similar ideas (McLaughlin, King & Nayakshin 2006; Nayakshin, Wilkinson & King 2009) to outflows driven by nuclear star clusters shows that these produce an offset \( M-\sigma \) relation between the total cluster mass and the velocity dispersion, the offset resulting from the fact that star clusters produce roughly 20 times less outflow momentum per unit mass compared with an accreting black hole.

There have also been attempts to explain the relation between black hole and bulge stellar mass in terms of Eddington outflows from accreting SMBH. The observed relation \( M \sim 10^{-5} M_8 \) (cf. Haring & Rix 2004) means that this is an inherently more complex problem than \( M-\sigma \), since \( M_8 \) is apparently the small part that remains after some process has almost swept the bulge clear of its original baryon content. Two recent papers discuss this problem.

Power et al. (2011) suggest that star formation in a galaxy bulge is self-limiting, and this limit largely determines the bulge stellar mass \( M_8 \). They further suggest that an energy-driven outflow from the central black hole clears away the remaining gas. This process cannot be totally effective: King (2010b) shows that energy-driven outflows are Rayleigh–Taylor unstable since the rapid expansion of the shocked wind leads to a large density contrast with the ambient medium. Thus a fraction of the gas can still remain even after the outflow passes.

In contrast, Silk & Nusser (2010) assert that energy-driven outflows do not occur at all in galaxy bulges. It is easy to show that momentum-driven outflows cannot clear the remaining gas from the bulge (Silk & Nusser 2010; Power et al. 2011, appendix). Accordingly, Silk & Nusser (2010) suggest that star formation must be able to remove it.

### 3 SHOCK COOLING

To decide whether energy-driven outflows exist or not, we consider an Eddington wind \( (M_{\text{out}} \approx M_{\text{Edd}}) \) from an SMBH propagating in an approximately isothermal galaxy bulge, with gas density

\[
\rho = \frac{f_g \sigma^2}{2 \pi G R^2}.
\]

The gas mass inside radius \( R \) is

\[
M(R) = 4 \pi \int_0^R \rho r^2 dr = \frac{2 f_g \sigma^2 R}{G}.
\]

As we have seen, the important question for the gas motions is whether the reverse shock cools. The pre-shock wind has a velocity \( v \approx n_c \approx 0.1 c \) (King & Pounds 2003; King 2010a), which implies a (reverse) shock temperature

\[
T_s \approx \frac{3 m_{\text{H}}}{{16} k} \frac{\sigma^2}{v^2} \approx 1.6 \times 10^{10} \ \text{K}.
\]

This gas is so hot it must be fully ionized, so the only losses cooling it are Compton and free–free. The mass conservation equation for the Eddington outflow gives a post-shock number density

\[
N = 4 \times \frac{M_{\text{out}}}{4 \pi R^2 \mu_{\text{H}} v} \approx 1 \times 10^{-3}(M_{\text{out}}/M_{\text{Edd}}) \frac{M_8}{R_{\text{kpc}}^2} \ \text{cm}^{-3},
\]

where \( M_8 \) is the SMBH mass in units of \( 10^8 \ M_\odot \) and \( R_{\text{kpc}} \) is the radial distance in kpc. This gives a radiative (free–free) cooling time for the shocked gas of

\[
t_{\text{rad}} \approx 2 \times 10^{11} M_8^{-1} R_{\text{kpc}}^2 \ \text{yr}.
\]

King (2003, equation 8) shows that the Compton cooling time of this gas in the quasar radiation field is

\[
t_c = \frac{2 c R^2}{3 GM} \left( \frac{m_e}{m_p} \right)^2 \left( \frac{c}{v} \right)^2 b \approx 10^7 R_{\text{kpc}}^2 b M_8^{-1} \ \text{yr},
\]

where \( b \approx 1 \) is the fractional solid angle of the outflow and \( m_e \) and \( m_p \) are the electron and proton masses, and we have set \( v = 0.1 c \) in the original equation.

To decide if cooling is effective, we compare these time-scales with the flow time-scale for a momentum-driven outflow, which is

\[
t_{\text{flow}} = \frac{R}{\dot{M} c_s} = 5 \times 10^8 R_{\text{kpc}} M_8^{-1/2} \ \text{yr},
\]

(cf. King 2003, equations 9 and 14). We find directly

\[
\frac{t_c}{t_{\text{flow}}} = 1.8 R_{\text{kpc}} M_8^{-1} M_8^{-1/2}.
\]

We see that Compton cooling is effective only out to about \( R = 1 \ kpc \) (cf. Ciotti & Ostriker 1997), while the radiative (free–free) cooling is always far longer than the flow time. Silk & Nusser (2010) claim the opposite, but appear to have considered the cooling of the ambient gas rather than the shocked wind which contains all the energy. Their adopted cooling function (Sutherland & Dopita 1993) only goes to temperatures of \( 10^{10} - 10^9 \ K \), far below the shock temperature \( T_s \approx 10^{10} \ K \). We recover the result (King 2003, 2005) that in a galaxy bulge, an Eddington outflow is momentum driven when very close to the SMBH, but becomes energy driven outside a typical radius \( \sim 1 \ kpc \).

Many galaxies show evidence for massive high-speed \( (v \sim 1000 \ \text{km s}^{-1}) \) gas outflows on large scales \( (\sim 20 \ kpc) \); e.g. Tremonti, Moustakas & Diamond-Stanic 2007; Holt, Tadhunter & Morganti 2008). By the arguments of this section, these must be energy driven. Their ultimate cause may be starbursts or active galactic nucleus (AGN) activity by the central SMBH. However, these nuclear phenomena are often absent or weak when the outflows are observed. So to understand the connection between the observed outflow and its original cause, we need to know how the outflow coasts and ultimately stalls in the absence of driving.

### 4 ENERGY-DRIVEN OUTFLOWS

The equation governing the movement of the shock pattern in an energy-driven outflow in an isothermal potential is (King 2005)

\[
\frac{n_f}{n_{\text{Edd}}} L_{\text{Edd}} = \frac{2 f_g \sigma^2}{G} \left( \frac{1}{2} R^2 R + 3 R R + R^3 \right) + 10 f_b \frac{\sigma^4}{G} R,
\]

where \( n \approx 0.1 \) is the accretion efficiency, \( L_{\text{Edd}} \) is the Eddington luminosity of the central black hole, \( \sigma \) is the velocity dispersion of the ambient medium and \( f_b \) is the gas fraction relative to all matter in this medium. The latter quantity may be depleted relative to its value \( f_c \) prevailing when the earlier momentum-driven outflow establishes the \( M-\sigma \) relation (1). Using the expression \( M = \frac{L_{\text{Edd}}}{\pi \sigma^2 n^4} \) in \( L_{\text{Edd}} \) gives

\[
\eta \approx \frac{\sigma^2 f_g}{f_b} = \left( \frac{1}{2} R^2 R + 3 R R + \frac{1}{2} R^3 \right) + 5 \sigma^2 R.
\]

This equation has a solution of the form \( R = v_c t \), with

\[
\frac{n f_c}{f_b} = 3 \frac{v_c^4}{\sigma^2} + 10 v_c.
\]
The assumption \( v_e \ll \sigma \) leads to a contradiction \( (v_e \simeq 0.02c[f_e/f_g] \gg \sigma) \), so the equation has the approximate solution

\[
v_e \simeq \left( \frac{2nf_e}{3f_g} \right)^{1/3} \simeq 92.5\sigma_{200}^{2/3}(f_e/f_g)^{1/3} \text{ km s}^{-1}.
\] (13)

This solution is an attractor. At radii \( R \) large enough that Compton cooling becomes ineffective, the extra gas pressure makes the previously momentum-driven shock pattern accelerate to this value.

At still larger radii, it may happen that the quasar supplying the driving term on the left-hand side of equation (11) switches off. Evidently, the shock pattern will continue to propagate outwards for a time, because of the residual gas pressure in the shocked wind. Its equation of motion now becomes

\[
\frac{1}{2} R^2 \ddot{R} + 3 \dot{R} \dot{R} + \frac{3}{2} \dot{R}^2 + 5 \sigma^2 \dot{R} = 0.
\] (14)

As the independent variable \( t \) does not appear in this equation, we let \( \dot{R} = p \), which implies that \( \ddot{R} = pp' \), \( \dot{R} = p'p^2 + pp^2 \), where the primes denote differentiation with respect to \( R \). After a little algebra, the equation takes the form

\[
\frac{R^2}{2} \frac{d}{dR} (pp') + 3Rpp' + \frac{3}{2} p^2 + 5 \sigma^2 = 0
\] (15)

or

\[
\frac{1}{4} R^2 y'' + \frac{3}{2} R y' + \frac{3}{2} y + 5 \sigma^2 = 0,
\] (16)

where \( y = p^2 \). Now we write \( y = y_1 - 10 \sigma^2/3 \) to reduce the equation to the algebraically homogeneous form

\[
R^2 y_1' + 6R y_1 + 6y_1 = 0,
\] (17)

which has linearly independent solutions \( y_1 \propto R^{-2}, R^{-3} \). Reversing the earlier substitutions, we have

\[
p^2 = \dot{R} = \frac{A_2}{R^2} + \frac{A_3}{R^3} - \frac{10 \sigma^2}{3}.
\] (18)

We now choose the constants \( A_2, A_3 \) to fulfil the boundary conditions \( \dot{R} = 0, \quad R = v_e \) at the shock position \( R = R_0 \), where the quasar turns off. This gives finally

\[
R^2 = 3 \left( v_e^2 + \frac{10}{3} \sigma^2 \right)^{1/2} \left( \frac{1}{x^2} - \frac{2}{3x^3} \right) - \frac{10}{3} \sigma^2,
\] (19)

where \( x = RR_0 \geq 1 \). Fig. 1 shows numerical solutions of the full equation of motion. With an arbitrary initial condition at small \( R \), the shock pattern rapidly adopts the constant velocity \( v_e \). Once the quasar switches off, the velocity decays as predicted by the exact solution (19).

Equation (19) gives the velocity of the shock pattern after the quasar switches off. This pattern stalls (i.e. \( \dot{R} = 0 \)) when

\[
\frac{1}{x^2} - \frac{2}{3x^3} = \frac{10 \sigma^2}{9(v_e^2 + 10 \sigma^2/3)}.
\] (20)

Since \( v_e \gg \sigma \), we must have \( x \gg 1 \), so we can neglect the \( 1/x^3 \) term on the right-hand side of (20) to get

\[
x_{\text{stall}} \simeq \frac{9}{10} \left( \frac{v_e^2}{\sigma^2} + \frac{10}{3} \right) \simeq \frac{9v_e^2}{100 \sigma^2},
\] (21)

where we have used \( v_e \gg \sigma \) at the last step. So finally

\[
R_{\text{stall}} \simeq 0.95 \frac{v_e}{\sigma} R_0 \simeq 0.95 \frac{2nf_e}{3f_g} R_0.
\] (22)

We can find a good approximation for the delay between quasar turn-off and the shock stalling by integrating equation (19). Again neglecting the \( 1/x^3 \) term, this reduces to a quadrature of the form

\[
t = \int_{R_0}^{(C/D)^{3/2}} \frac{RAR}{(C - DR^2)^{3/2}}.
\] (23)
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where \( t_{\text{acc}} \) is the time-scale over which the central black hole accretes at the Eddington rate. Using (22), we can rewrite this as

\[
R_{\text{stall}} \approx \frac{v_e^2}{\sigma} t_{\text{acc}},
\]

which of course implies

\[
t_{\text{stall}} \approx \left( \frac{v_e}{\sigma} \right)^2 t_{\text{acc}}. \]

This last relation is interesting, because it shows that outflows persist for quite a long time after the quasar switches off. Using (13), we find

\[
t_{\text{stall}} \approx 10 t_{\text{acc}} \sigma_{200}^{-2/3} (f_c / f_e)^{2/3}.
\]

Hence, outflows can in principle persist for an order of magnitude longer than the driving phases giving rise to them.

5 ESCAPE

We can use the results of the last section to find the conditions for SMBH growth to remove gas from the host galaxy bulge. Attempts to explain the relation between SMBH and bulge mass (e.g. King 2003, 2005; Silk & Nusser 2010) often invoke this kind of process. A complication so far not treated is that energy-driven outflows are Rayleigh–Taylor unstable, and the bulge mass remaining may depend on the non-linear growth of these instabilities. Nevertheless, it seems probable that significant mass removal requires much of the shocked gas to escape the galaxy.

This happens if the shock pattern reaches the galaxy’s virial radius,

\[
R_V \approx \frac{\sigma}{7H} = \frac{\sigma h}{7h(z)},
\]

before stalling. Here \( H = H_0 h(z) \), with \( H_0 \) the Hubble constant, and \( h(z) \) gives the redshift dependence. Requiring \( R_{\text{stall}} > R_V \) and using (27) gives

\[
t_{\text{stall}} > 1 \times 10^6 \left( \frac{\eta_{0.1} f_c}{f_e} \right)^{2/3} \sigma_{200}^{2/3} \text{yr},
\]

where \( \eta_{0.1} = \eta/0.1 \). This is about twice the Salpeter time-scale for the mass growth of the SMBH, almost independently of other parameters. Apparently, the black hole must grow significantly in order to remove a significant amount of bulge mass.

We may compare this accretion time-scale with the time required for the SMBH luminosity to unbind the gas in the galaxy. Using equation (3) with \( R = R_V \) from equation (30), assuming that the gas binding energy is \( E_b \sim M \sigma^2 \) and the SMBH energy input is \( E_{\text{BH}} = 0.05 \xi \sigma L_{\text{Edd}} t_{\text{wind}} \) (typical for an energy-driven outflow), gives

\[
t_{\text{wind}} \sim \frac{f_c M_e}{f_e M} R_V \approx 1.5 \times 10^7 \frac{f_c}{f_e} \xi^{-1} \text{yr}.
\]

We see that the time it takes for an Eddington-limited accreting SMBH to inject enough energy into the gas to unbind it is, in principle, shorter than the Salpeter time. However, crucially, this luminosity has to be communicated to the gas in the host galaxy. Communication via an energy-driven wind therefore requires an accretion time-scale \( t_{\text{acc}} \) due to the wind outflow having \( v_e \ll c \).

If the galaxy is inside a cluster, the outflow may reheat the cluster gas (King 2009). In this case, the virial radius of a galaxy is not well defined, but we may consider how long it takes for an outflow from the central cluster galaxy to reach the typical cluster cooling core radius \( R_{\text{cool}} \sim 150 \sigma_{100}^{1/2} \text{kpc} \), where \( \sigma_{100} \) is the cluster velocity

\[
R_0 \approx v_e t_{\text{acc}},
\]

with

\[
C = 3 \left( v_e^2 + \frac{10}{3} \sigma^2 \right) R_0^2 \quad \text{and} \quad D = \frac{10}{3} \sigma^2.
\]

We find

\[
t \approx \frac{(C - DR_0^2)^{1/2}}{D} \approx \frac{R_0 v_e}{2 \sigma^2} \approx \frac{R_{\text{stall}}}{2 \sigma}.
\]

The shock pattern moves at the speed \( v_e \) for almost all the time that the quasar is on, so we can write

\[
R_0 \approx v_e t_{\text{acc}}.
\]

Figure 2. Same as Fig. 1, but with \( R_0 = 50 \text{pc} \) and \( v_e = 200 \text{km s}^{-1} \) and varying mean gas fractions: black solid – \( f_g = 3 \times 10^{-3} \); blue dashed curve – \( f_g = 10^{-2} \); red dot-dashed curve – \( f_g = 3 \times 10^{-2} \) and green triple-dot-dashed curve – \( f_g = 10^{-1} \).
dispersion in units of 1000 km s\(^{-1}\). If the galaxy has \(\sigma = 200\) km s\(^{-1}\) and \(f_k \sim f_c\), then the outflow cannot propagate into the intracluster medium, as \(v_s \simeq \sigma_c\). However, if we take the velocity dispersion of the surrounding material to be similar to that in a galaxy, then the accretion duration is

\[ t_{\text{acc}} \simeq 3.4 \times 10^7 \sigma_{200}^{-1/3} \left( \frac{f_c}{f_k} \right)^{-2/3} \text{yr}, \quad (33) \]

and the stalling time is \(t_{\text{stall}} \simeq 3.7 \times 10^8 \sigma_{200}^{-1} \text{yr}\). This is the timescale on which the intracluster medium is replenished by the outflow from the central galaxy, provided that the outflow occurs. As long as the AGN duty cycle of the SMBH at the centre of that galaxy is greater than \(t_{\text{stall}}/\tau_{\text{Edd}} \simeq 2.7\) per cent, the intracluster medium is continuously replenished and reheated, as the temperature of the gas in the snowplough phase (the outer shock) of the outflow is \(T_{\text{out}} \sim 10^8\) K, similar to the virial temperature of the cluster gas.

6 VISIBILITY

The most favourable case for viewing outflows is when each quasar phase is sufficiently short that the associated outflow has not left the visible galaxy by the time it stalls. If for example we take the visible galaxy to have a size \(\sim 20\) kpc, we want \(R_{\text{stall}} \lesssim 20\) kpc, which by (27) requires

\[ t_{\text{acc}} \lesssim 1.7 \times 10^6 \sigma_{200}^{-1/3} \left( \frac{f_c}{f_k} \right)^{-2/3} \text{yr}. \quad (34) \]

Thus short growth episodes like these are most favourable for seeing outflows. The fraction of galaxies actually showing outflows then depends on the growth time of their black holes. The frequency of detectable outflows in principle offers a way of constraining the growth history of the SMBHs.

7 DISCUSSION

This Letter has discussed massive outflows in galaxy bulges, chiefly those driven by accretion episodes where the central SMBH reaches the Eddington limit. We have shown that these outflows are momentum driven at sizes \(R \lesssim 1\) kpc, as required to explain the \(M-\sigma\) relation, but become energy driven at larger radii because the quasar radiation field becomes too diluted to cool the wind shock within the flow time. Radiative cooling is incapable of doing this in any regime, contrary to recent claims.

We derive an analytic solution of the equation governing the motion of an energy-driven shell after the central source has turned off. We show that the thermal energy in the shocked wind is able to drive further expansion for a time typically 10 times longer than the original driving time. Outflows observed at large radii with no active central source probably result from an earlier short (few Myr) active phase of this source.

Energy-driven outflows from longer lasting accretion episodes escape the galaxy, and may well be responsible for removing ambient gas from the bulge, as required in some pictures of the black hole–bulge stellar mass relation. We stress, however, that since these outflows are Rayleigh–Taylor unstable, some gas may leak through the shocks and not be swept out. This problem is impossible to handle analytically and is currently numerically intractable. The inherent difficulty is that the instability sets in at very short wavelengths, placing great demands on spatial resolution.

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