Research Article

Fractal Statistical Study on the Strength of Jointed Rock Mass

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The failure probability and strength of jointed and fractured rock mass under water pressure action place important constraints on the safety and stability of rock mass engineering. In this study, two strength calculation models for rock mass joint rupture failure and shear slip failure are established according to different relative sizes and working conditions, and the effects of fractal dimension, water pressure, and number of cracks on the strength and failure probability of rock mass are analyzed. The most unfavorable dip angle under shear-slip conditions is calculated. The proposed models accurately reflect the failure probability and strength of jointed rock mass, and different strength calculation methods should be selected according to the specific working conditions. The effect of water pressure on the strength of jointed rock mass is sensitive to the failure mode. Reasonable drainage should be carried out according to the different failure modes when constructing tunnels with abundant groundwater, and certain drainage or water plugging construction schemes are not necessarily applicable to all cases. The dip angle at which joint fissures are most prone to shear slip is given as $\pi/4 + \phi/2$, where $\phi$ is the jointed rock mass strength parameter. An increase in the fractal dimensions, external stress, and number of fractures will enhance the rock mass failure probability regardless of failure mode. Controlled blasting and advanced grouting reinforcement measures are recommended when a tunnel traverses through a joint fissure zone to reduce the number of cracks generated in the surrounding rock and thus improve its strength and stability.

1. Introduction

Weak structural planes (e.g., bedding, faults, joints, and fissures) in rock mass control the mechanical properties and stability of rock mass engineering. Rock mass can be destroyed along structural planes under certain conditions, which cannot be mitigated using classical elastic-plastic mechanics [1–3]. A comprehensive understanding of the mechanical properties of rock mass with structural planes according to actual engineering needs is therefore urgent.

The construction of dams, tunnels, and slopes in jointed, water-bearing rock causes complex interactions between the joint deformation and effective stress [4]. Accurate strength estimations of weak rock mass have posed a long-lasting challenge in geotechnical engineering due to its complex nature and limited definition [5]. Several recent studies have found appropriate methods to evaluate the failure behavior of rock joints by means of laboratory experiments, geological surveys, theoretical calculations, and numerical simulations. The morphological features of rock joints directly affect the rock mass peak shear strength, and displacement has been verified by tests and numerical analyses. The mechanical properties, geometric properties, and spatial distribution of joint surfaces are the main factors that affect the mechanical properties and hydraulic behavior of the overall rock mass [6]. The joint roughness coefficient (JRC) and 10 standard JRC contour curves have been proposed to illustrate the joint surface roughness (JSR) and have subsequently gained acceptance and been widely used [7]. Numerous studies have described joint surfaces using their undulating height, mean dip, surface anisotropy, mean joint gradient, and surface distortion features to better describe the JSR and depict the 3D morphology characteristics of the joint surfaces [8]. The established empirical model is related to shear displacement because the parameters are simply derived from the joint surface geometry, whereas current roughness indices are relatively insufficient to describe the shear mechanical properties of joints [9].
Rock mass classification is a complex engineering problem that involves analyzing fracture distribution and joints, factors that will critically impact the mechanical behavior and engineering properties of rocks [10]. Fractal geometry provides a new approach to investigate fractured materials, and several studies have shown that the fractal dimension is a geometric parameter related to material microstructures, the evolution of organisms, and deformation and fracture properties [11–13]. Thus, alternative models that are based on the theory of fractal geometry have been developed in an effort toward simplifying the rock mass classification problem [14]. Scholars extended the research by establishing empirical formulas that describe the relationship between JRC (joint roughness coefficient) and fractal dimension (description of the length of fracture trace, amputate of surface wave, and distribution of joint density) [15–17]. Moreover, some researchers have analyzed the fractal characteristics of rock joints by using the Variogram method, which demonstrated that the application of fractal dimension alone cannot sufficiently capture the effects of fracture roughness on shear strength [18, 19]. In a word, fractal geometry theory and statistical theory have been used to describe the geometrical characteristics of joints and fractures. The size distribution of joints in rock mass is described in a power-law function, regardless of they are small (a few mm) or large (thousands of m), and thus, the shear strength of rock joints is commonly estimated using a statistical approach [20, 21].

In addition to the description of joints, substantial research attention has also focused on the shear strength of rock mass. To study the interaction of influencing joint parameters, the shear behavior of rock joints has been modeled with regard to the individual impacts of the specific influencing parameters and their interactions [22, 23]. Moreover, the peak shear strength and shear behavior of rock joints have been modeled considering their geometrical distribution, dilation angle evolution, and filled versus unfilled nature [24–27]. The relationship among peak shear strength, normal stress, residual strength, expansion, and roughness with normal stress is generally obtained by direct shear tests. A new peak shear strength criterion was expressed as follows [8, 9]:

\[
\begin{align*}
N &= CL^{-D}, \\
f(L) &= \frac{D}{L_0} \left( \frac{L}{L_0} \right)^{-(1+D)},
\end{align*}
\]

where \(L\) is the joint size, \(N\) is the number of joints larger than \(L\), \(f(L)\) is the probability density distribution function of the joint size, \(L_0\) is the characteristic size of the joint in the rock mass, \(C\) is the proportional constant, and \(D\) is the fractal dimension of the joint size distribution.

2. Description of Rock Mass Joint Distribution

2.1. Joint Fissure Distribution. The failure and instability of rock mass are the result of the continuous initiation, fracture, and expansion of its internal joints and fissures. The basis of fractal theory promotes a thorough understanding of the mechanical properties of jointed and fractured rock masses. A large number of studies have shown that the distribution of joints and fissures in rock mass has a significant self-similarity and that the size distribution follows a power law. The joint distribution in the rock mass can be expressed as follows [8, 9]:

\[
\begin{align*}
N &= CL^{-D}, \\
f(L) &= \frac{D}{L_0} \left( \frac{L}{L_0} \right)^{-(1+D)},
\end{align*}
\]

where \(L\) is the joint size, \(N\) is the number of joints larger than \(L\), \(f(L)\) is the probability density distribution function of the joint size, \(L_0\) is the characteristic size of the joint in the rock mass, \(C\) is the proportional constant, and \(D\) is the fractal dimension of the joint size distribution.

2.2. Strength Parameters of Jointed and Fractured Rock Mass. The connectivity rate of jointed rock mass affects its strength and failure. The methods for calculating the rock mass connectivity rate mainly include the path search method and bandwidth projection method. The bandwidth projection
method is widely used to study jointed rock mass due to its simple and understandable calculation. The bandwidth projection method refers to the proportion of all structural planes with an included angle in the shear failure direction smaller than the allowable value within a certain bandwidth range. The connectivity rate calculated by the bandwidth projection method is as follows:

\[ k = \frac{\sum TL}{L}, \]  

(2)

where \( k \) is the connectivity rate of the rock mass, \( TL \) is the projection length of the structural plane in the shear direction, and \( L \) is the total length in the shear direction. The specific calculation method is shown in Figure 1, where the length of the jointed rock mass in the shearing direction is \( L \) and the width is \( B \). Joints 1–4 are all within the range of bandwidth of \( B \) and thus meet the requirement of having an angular difference with the shearing direction, which can be calculated by projection. Joints 6 and 7 are not within the range, and joint 5 is not eligible for projection calculation because it does not satisfy the angular difference with the shear direction. The calculated connectivity rate in Figure 1 is, thus, as follows [32]:

\[ k = \frac{\sum TL}{L} = \frac{(TL_1 + TL_2 + TL_3 + TL_4)}{L}. \]  

(3)

where \( TL_1, TL_2, TL_3, \) and \( TL_4 \) represent the effective projection length of joints 1, 2, 3, and 4 in the shear direction, respectively.

After obtaining the rock mass connectivity rate, the strength parameters can be calculated using the respective proportions of the rock bridge and structural plane in the shear direction. The strength parameters of the rock mass are calculated as follows:

\[
\begin{align*}
    f &= f_r \times m\% + f_j \times n\%, \\
    c &= c_r \times m\% + c_j \times n\%,
\end{align*}
\]

(4)

where \( f \) and \( c \) represent the friction coefficient and cohesion, respectively; \( f_r \) and \( f_j \) represent the friction coefficients of the rock bridge and structural plane, respectively; \( c_r \) and \( c_j \) represent the cohesion of the rock bridge and structural plane, respectively; and \( m\% \) and \( n\% \) represent the proportions of the rock bridge and structure plane in their respective shear directions.

3. Modes and Conditions of Instability and Failure of Rock Mass Joints

3.1. Calculation Model for Water-Bearing Jointed Rock Mass

Figure 2 shows the calculation model established for water-bearing jointed and fractured rock mass. The water pressure in this paper refers to the water pressure of the aquifer, which mainly acts on rock fractures. The included angle between the joint and maximum principal stress \( \sigma_1 \) is \( \beta \), and the dip angle is \( \theta \), where \( \beta + \theta = \pi/2 \), the joint length is \( 2L \), the water pressure is \( p_w \), and the third principal stress is \( \sigma_3 \). According to the calculation model for water-bearing jointed rock mass, the normal stress \( \sigma_n \) and shear stress \( \tau \) on the joint surface under the action of water pressure can be calculated as follows [33]:

\[
\begin{align*}
    \sigma_n &= \sigma_1 \sin^2 \beta + \sigma_3 \cos^2 \beta - p_w \\
    &= \frac{\sigma_1 + \sigma_3}{2} - \frac{\sigma_1 - \sigma_3}{2} \cos 2\beta - p_w, \\
    \tau &= \frac{1}{2} \sin 2\beta (\sigma_1 - \sigma_3).
\end{align*}
\]

(5)

The failure mode and instability of jointed rock mass are associated with factors such as joint size, dip angle, and connectivity rate. Previous studies and a large number of field observations have shown that there are two classical modes of failure and instability in jointed rock mass [34]: rupture failure and shear-slip failure. There are three typical failure modes in fracture mechanics (Figure 3). For the rupture failure of rock mass joints, joint crack propagation failure can be divided into type I-II tension-shear propagation failure and type II compression-shear propagation failure according to whether the normal direction of the joint fissure experiences tensile stress or compressive stress.

3.2. Rupture Failure of Rock Mass Joints. Rock mass is generally hard and brittle. When the joint size is considerably smaller than the rock mass size, high local stress develops at
and for I-II type tension-shear propagation failure as follows:

\[
K_{lc} = K_1 + K_{IIc},
\]

\[
K_{lc} = \left(\frac{\sigma_1 + \sigma_3 - \sigma_1 - \sigma_3}{2} \cos 2\beta - p_w + \frac{\sigma_1 - \sigma_3}{2} \sin 2\beta\right)\sqrt{\pi L},
\]

(7)

where \(K_{lc}\) is the type I-II tension-shear propagation stress intensity factor, \(K_1\) is the type I fracture stress intensity factor, and \(K_{IIc}\) is the type II fracture stress intensity factor. When the water pressure is small and the normal stress of the joint surface is compressive, the joint will produce type II compression-shear propagation failure, and its type II compression-shear propagation stress intensity factor \(K_{IIc}\) is as follows:

\[
K_{IIc} = \frac{\sigma_1 - \sigma_3}{2} \sin 2\beta \sqrt{\pi L}.
\]

(8)

This approach assumes that at critical failure, the length of the joint is \(L_c\) and \(\sigma_1 = \sigma\). To facilitate the subsequent calculations, let \(\sigma_3 = \sigma\) to more concisely express the joint length \(L_c\) at critical failure. The critical joint length \(L_c\) for type I-II tension-shear propagation failure and type II compression-shear propagation failure is calculated as follows:

\[
K_{lc} = \frac{4}{\pi} \left[\frac{\sigma_1}{(1 + \epsilon)\sigma - (1 - \epsilon)\sigma \cos 2\beta - 2p_w + (1 - \epsilon)\sigma \sin 2\beta}\right]^2
\]

(tension – shear propagation failure),

(9)

\[
K_{IIc} = \frac{4}{\pi} \left[\frac{\sigma_1}{(1 - \epsilon)\sigma \sin 2\beta}\right]^2
\]

(compression – shear propagation failure).

3.3. Shear-Slip Failure of Rock Mass Joints. When the joint length is large, its length cannot be ignored compared with the size of the rock mass, and the jointed rock mass is more prone to shear slip failure along the joint plane. For example, a slope with joint fissures is susceptible to shear slip instability. The shear slip failure of jointed rock mass can be calculated according to the classic Mohr–Coulomb strength criterion. By referring to existing research for nonpenetrating joints within rock mass [32], the Mohr–Coulomb strength criterion can be amended as follows:

\[
\tau = \sigma_n \tan \varphi + (1 - \rho) c,
\]

(10)

where \(\rho = L/M\) and \(M\) is the size of the rock mass. The strength parameters \(f\) and \(c\) of the rock mass are obtained according to the connectivity rate of the rock mass joints. According to the relationship between the friction coefficient \(f\) and internal friction angle \(\varphi (\tan(\varphi) = f)\), Equation (10) can be expressed as follows:

\[
\tau = \sigma_n f + \left(1 - \frac{L}{M}\right) c.
\]

(11)

By substituting equation (5) into equation (11), we obtain

\[
\sin 2\beta (\sigma_1 - \sigma_3) = f [\sigma_1 + \sigma_3 - (\sigma_1 - \sigma_3)\cos 2\beta - 2p_w] + 2\left(1 - \frac{L}{M}\right) c.
\]

(12)

When rock mass joint fissures undergo slip failure instability, the joint length \(L_c\) in the critical state can be calculated from equation (12) as follows:
\[ L_c = M \left\{ 1 - \sin \frac{2\beta (1 - \epsilon) \sigma - f[(1 + \epsilon)\sigma - (1 - \epsilon)\sigma \cos 2\beta - 2p_w]}{2\epsilon} \right\}. \] (13)

4. Statistical Strength of Jointed Rock Mass

4.1. Failure Probability and Statistical Strength of Jointed Rock Mass. Rock mass strength is affected by the size, length, dip angle, and number of joints. A complex combination of rock blocks and structural planes leads to significant anisotropy and random distribution characteristics of the rock strength. Most scholars describe the dip angles of joints based on statistical theory. The two most common statistical description methods for dip angles of joints are given for uniform and normal distributions, and the probability density can be expressed as follows:

\[
\begin{cases}
  g(\theta) = \frac{1}{\pi}, \\
  g(\theta) = \frac{1}{\sqrt{2\pi} \delta} \exp \left[ -\frac{(\theta - \bar{\theta})^2}{2\delta^2} \right],
\end{cases}
\] (14)

where \( g(\theta) \) is the probability density of the joint dip angle \( \theta \), \( \bar{\theta} \) is the mean value of the joint dip angle, and \( \delta^2 \) is the variance of the joint dip angle. Assuming that the dip angle distribution and size distributions of the joint fissures are independent of each other, the probability density of the size distribution is \( f(L) \), and the probability density function of the dip angle is \( g(\theta) \). The probability that a joint is located between \( L \leq L + dL \) and \( \theta \leq \theta + d\theta \) is, thus, as follows [32]:

\[
P[L \leq x \leq L + dL, \theta \leq \theta + d\theta] = f(L) f(\theta) dL d\theta.
\] (15)

When a joint fissure fails and destabilizes, its size and length will be greater than or equal to its critical value. The failure probability a joint fissure can be calculated by integrating equation (15) according to the joint failure conditions. When the stress is \( \sigma \), the probability for a joint to fail is given as follows:

\[
F(\sigma) = \int_{-\pi/2}^{\pi/2} f(\theta) d\theta \int_{L_c}^{\infty} f(L) dL.
\] (16)

When the rock mass contains \( N \) joints and fissures, the strength of the rock mass should be assigned the average value of the critical failure stress when the rock mass fails and destabilizes according to the weakest-link principle of statistical fracture theory. The failure and instability probability of a rock mass with \( N \) joint fissures and corresponding strength are as follows [12]:

\[
\begin{align*}
  P_f &= 1 - \exp\left[ -NF(\sigma) \right] = 1 - \exp\left[ -N \int_{-\pi/2}^{\pi/2} f(\theta) d\theta \int_{L_c}^{\infty} f(L) dL \right], \\
  \bar{\sigma} &= \int_{0}^{\infty} \sigma \ dP_f = \int_{0}^{\infty} (1 - P_f) d\sigma = \int_{0}^{\infty} \exp\left[ -N \int_{-\pi/2}^{\pi/2} f(\theta) d\theta \int_{L_c}^{\infty} f(L) dL \right] d\sigma.
\end{align*}
\] (17)

When the dip angle of a joint fissure follows a uniform distribution and the joint size follows a power function distribution, Equation (17) can be expressed as follows:

\[
\begin{align*}
  P_f &= 1 - \exp\left[ \frac{N}{\pi} \int_{-\pi/2}^{\pi/2} \left( \frac{L_0}{L_c} \right)^{D} d\theta \right], \\
  \bar{\sigma} &= \int_{0}^{\infty} \exp\left[ \frac{N}{\pi} \int_{-\pi/2}^{\pi/2} \left( \frac{L_0}{L_c} \right)^{D} d\theta \right] d\sigma.
\end{align*}
\] (18)

4.2. Probability and Statistical Strength of Rupture Failure of Rock Mass Joints. Based the above analysis, the rupture propagation failure of joint fissure can be divided into type I tension-shear propagation failure and type II compression-shear propagation failure. For ease of calculation, assuming that the dip angle of the joint fissure follows a uniform distribution and the joint size follows a power distribution, the critical dimension \( L_c \) calculated by equation (9) can be substituted into equation (18) to calculate the probability of two fracture propagation failures and the rock mass strength of the joint fissure, respectively. Before calculation, a trigonometric function transformation is performed following the relationship between \( \beta \) and \( \theta \) of \( \beta + \theta = \pi/2 \). We thus obtain

\[
\begin{align*}
  \sin 2\beta &= \sin 2\theta, \\
  \cos 2\beta &= -\cos 2\theta.
\end{align*}
\] (19)

The critical dimension \( L_c \) can be expressed by a function containing the dip angle \( \theta \), and by substituting equation (19) into equation (9) to obtain the following equation:

\[
\begin{align*}
  L_c &= \frac{4}{\pi} \left[ \frac{K_{lc}}{(1 + \epsilon)\sigma + (1 - \epsilon)\sigma \cos 2\theta - 2p_w + (1 - \epsilon)\sigma \sin 2\theta} \right]^2, \\
  L_c &= \frac{4}{\pi} \left[ \frac{K_{lc}}{(1 - \epsilon)\sigma \sin 2\theta} \right]^2.
\end{align*}
\] (20)
When the rock mass undergoes type I-II tension-shear propagation failure, Equation (20) can be substituted into equation (18) to calculate its failure probability and strength according to

\[
P_f = 1 - \exp \left\{ - \frac{N}{\pi} \int_{-\pi/2}^{\pi/2} \left( \frac{\pi L_0}{4^D} \right)^D \left[ \frac{(1 + \epsilon) \sigma + (1 - \epsilon) \cos 2\theta - 2p_w + (1 - \epsilon) \sin 2\theta}{K_{lc}} \right]^{2D} d\theta \right\},
\]

\[
\bar{\sigma} = \int_0^\infty \exp \left\{ - \frac{N}{\pi} \int_{-\pi/2}^{\pi/2} \left( \frac{\pi L_0}{4^D} \right)^D \left[ \frac{(1 + \epsilon) \sigma + (1 - \epsilon) \cos 2\theta - 2p_w + (1 - \epsilon) \sin 2\theta}{K_{lc}} \right]^{2D} d\theta \right\} d\sigma.
\]

Similarly, when the rock mass undergoes type II compression-shear propagation failure, its failure probability and strength are as follows:

\[
P_f = 1 - \exp \left\{ - \frac{N}{\pi} \int_{-\pi/2}^{\pi/2} \left( \frac{\pi L_0}{4^D} \right)^D \left[ \frac{(1 - \epsilon) \sin 2\theta}{K_{lc}} \right]^{2D} d\theta \right\},
\]

\[
\bar{\sigma} = \int_0^\infty \exp \left\{ - \frac{N}{\pi} \int_{-\pi/2}^{\pi/2} \left( \frac{\pi L_0}{4^D} \right)^D \left[ \frac{(1 - \epsilon) \sin 2\theta}{K_{lc}} \right]^{2D} d\theta \right\} d\sigma.
\]

4.3. Probability and Statistical Strength of Shear-Slip Failure of Rock Mass Joints. The critical size \( L_c \) when slip failure occurs in a joint fissure of rock mass can be calculated as described above. To calculate the probability and strength of the shear-slip failure of rock joints, equation (19) is first substituted into equation (13) to calculate the critical length \( L_c \) of the joint with a dip angle \( \theta \), where \( L_c \) is calculated as follows:

\[
L_c = M \left\{ 1 - \frac{2c}{\pi} \sin 2\theta (1 - \epsilon) \sigma - f (1 + \epsilon) \sigma + (1 - \epsilon) \cos 2\theta - 2p_w \right\}. \tag{23}
\]

By substituting the obtained critical length \( L_c \) of the joint into equation (18), we obtain the probability and strength of the shear-slip failure of the rock mass joint, given as follows:

\[
P_f = 1 - \exp \left\{ - \frac{N}{\pi} \int_{-\pi/2}^{\pi/2} \left( \frac{L_0}{M} \right)^D \left[ \frac{2c}{2c - \sin 2\theta (1 - \epsilon) \sigma + f (1 + \epsilon) \sigma + (1 - \epsilon) \cos 2\theta - 2p_w} \right]^{D} d\theta \right\},
\]

\[
\bar{\sigma} = \int_0^\infty \exp \left\{ - \frac{N}{\pi} \int_{-\pi/2}^{\pi/2} \left( \frac{L_0}{M} \right)^D \left[ \frac{2c}{2c - \sin 2\theta (1 - \epsilon) \sigma + f (1 + \epsilon) \sigma + (1 - \epsilon) \cos 2\theta - 2p_w} \right]^{D} d\theta \right\} d\sigma. \tag{24}
\]

5. Discussion

5.1. Relationship between Rock Mass Strength and Fractal Dimension. The fractal dimension of jointed rock mass is related to the joint distribution density and can be used to determine the rock mass strength to a certain extent. Rock integrity is associated with the number of joint fissures in the rock mass. Equation (18) presents the failure probability and strength calculation formula for jointed rock mass. To analyze the effects of the fractal dimension on the rock mass strength, equation (18) is used to obtain the derivative of the fractal dimension \( D \), yielding

\[
\frac{dP_f}{dD} = \frac{N}{\pi} \exp \left\{ - \frac{N}{\pi} \int_{-\pi/2}^{\pi/2} \left( \frac{L_0}{L_c} \right)^D d\theta \right\} \left[ \int_{-\pi/2}^{\pi/2} \left( \frac{L_0}{L_c} \right)^D \ln \left( \frac{L_0}{L_c} \right) d\theta \right] > 0
\]

\[
\frac{d\bar{\sigma}}{dD} = \frac{N}{\pi} \int_0^\infty \exp \left\{ - \frac{N}{\pi} \int_{-\pi/2}^{\pi/2} \left( \frac{L_0}{L_c} \right)^D d\theta \right\} \left[ \int_{-\pi/2}^{\pi/2} \left( \frac{L_0}{L_c} \right)^D \ln \left( \frac{L_0}{L_c} \right) d\theta \right] d\sigma < 0. \tag{25}
\]
5.2. Effects of Water Pressure on the Strength of Jointed Rock Mass. According to porous media fluid dynamics and groundwater dynamics, the force applied on a confined aquifer is borne by the rock skeleton and water. A schematic diagram of the force is shown in Figure 4. According to the flow dynamics of porous media and groundwater dynamics, water pressure can also share a part of the load on the rock, thereby increasing the rock strength to a certain extent. IX_his is because when the water-bearing rock stratum can be expressed as follows [35]:

\[ \sigma = \sigma_s + p_w, \]  

(26)

Equation (25) shows that when the failure probability is used to obtain the derivative of the fractal dimension, its value is greater than 0, whereas when the rock mass strength is used to obtain the derivative of the fractal dimension, its value is less than 0. This indicates that when the fractal dimension increases, the failure probability will also increase and the rock mass strength will decrease. Similarly, the derivative of the number of fissures \( N \) can be obtained, and the result is the same. Therefore, more joints and fissures will facilitate rock mass fracturing. Mountains with developed joints and fissures should therefore be avoided as much as possible when selecting sites for actual projects (e.g., slope projects and tunnel projects).

\[
P_f = 1 - \exp \left\{ -\frac{N}{\pi} \frac{(\pi L_0)^D}{4^D} \int_{-\pi/2}^{\pi/2} F(\epsilon, \sigma, \theta, p_w, K_L) \right\}^{2D} d\theta, 
\]

\[
\bar{\sigma} = \int_0^\infty \exp \left\{ -\frac{N}{\pi} \frac{(\pi L_0)^D}{4^D} \int_{-\pi/2}^{\pi/2} F(\epsilon, \sigma, \theta, p_w, K_L) \right\}^{2D} d\sigma, 
\]

\[
F(\epsilon, \sigma, \theta, p_w, K_L) = \left( 1 + \epsilon \right) \sigma + \left( 1 - \epsilon \right) \sigma \cos 2\theta - 2p_w + (1 - \epsilon) \sigma \sin 2\theta. 
\]

(27)

Equation (27) is then used to obtain the derivative of the water pressure \( P_w \), yielding

\[
\frac{dP_f}{dp_w} = \frac{4D}{4^DK_L\pi} \left\{ \int_{-\pi/2}^{\pi/2} F(\epsilon, \sigma, \theta, p_w, K_L) \right\}^{2D-1} d\theta \exp \left\{ -\frac{N}{\pi} \frac{(\pi L_0)^D}{4^D} \int_{-\pi/2}^{\pi/2} F(\epsilon, \sigma, \theta, p_w, K_L) \right\}^{2D} d\theta, 
\]

(28)

\[
\frac{d\sigma}{dp_w} = \frac{4D}{4^DK_L\pi} \int_0^\infty \left\{ \int_{-\pi/2}^{\pi/2} F(\epsilon, \sigma, \theta, p_w, K_L) \right\}^{2D-1} d\theta \exp \left\{ -\frac{N}{\pi} \frac{(\pi L_0)^D}{4^D} \int_{-\pi/2}^{\pi/2} F(\epsilon, \sigma, \theta, p_w, K_L) \right\}^{2D} d\sigma. 
\]

When rock mass undergoes type I-II tension-shear propagation failure, the derivative of the water pressure obtained through the failure probability is negative, and the derivative of the water pressure obtained through the rock mass strength is positive. This implies that when the water pressure increases, the failure probability decreases and the rock mass strength increases. This is because when the size of joint fissures is relatively small, the unpenetrated joint fissures...
are filled with water, which bears a part of the load borne by the rock mass and thus increases the rock mass strength to a certain extent. This is consistent with the conclusions in porous media fluid dynamics and groundwater dynamics. According to the study of dynamics of fluids in porous media, the force applied on a confined aquifer is borne by the rock skeleton and water, and when the water pressure increases, the pressure on the rock skeleton will decrease [33]. Therefore, when rock mass undergoes type I-II tension-shear propagation failure, the increase of water pressure can improve the strength of jointed rock mass to a certain extent.

The failure probability and strength when rock mass joints undergo shear-slip failure can be calculated from equation (24), which can be further simplified to

\[
P_f = 1 - \exp\left\{ \frac{N}{\pi} \int_{-\pi/2}^{\pi/2} \left[ F(L_0, M, \theta, \phi, \sigma, \epsilon) \right]^D d\theta \right\},
\]

\[
\sigma = \int_0^{\infty} \exp\left\{ \frac{N}{\pi} \int_{-\pi/2}^{\pi/2} \left[ F(L_0, M, \theta, p_w, \epsilon, \sigma) \right]^D d\theta \right\} d\sigma,
\]

\[
F(L_0, M, f, \theta, p_w, \epsilon, \sigma) = \frac{L_0}{M} 2c - \sin 2\theta(1 - \epsilon)\sigma + f [1 + (1 + \epsilon)\sigma + (1 - \epsilon)\sigma \cos 2\theta - 2p_w].
\]

Equation (29) is used to obtain the derivative of the water pressure \( p_w \), yielding

\[
\frac{dp_f}{dp_w} = \frac{2DMN}{\pi} \int_{-\pi/2}^{\pi/2} \left[ F(L_0, M, \theta, \phi, \sigma, \epsilon) \right]^{D+1} d\theta \exp\left\{ -\frac{N}{\pi} \int_{-\pi/2}^{\pi/2} \left[ F(L_0, M, \theta, p_w, \epsilon, \sigma) \right]^D d\theta \right\},
\]

\[
\frac{d\sigma}{dp_w} = \frac{2DMN}{\pi} \int_0^{\infty} \int_{-\pi/2}^{\pi/2} \left[ F(L_0, M, \theta, p_w, \epsilon, \sigma) \right]^{D+1} d\theta \exp\left\{ -\frac{N}{\pi} \int_{-\pi/2}^{\pi/2} \left[ F(L_0, M, \theta, p_w, \epsilon, \sigma) \right]^D d\theta \right\} d\sigma.
\]

Equation (30) indicates that when rock mass joints undergo shear-slip failure, the derivative of the water pressure obtained through the failure probability is positive, and the derivative of the water pressure obtained through the rock mass strength is negative. This implies that when the water pressure increases, the failure probability will increase and the rock mass strength will decrease. This is because when the joint size is considerably larger than the rock mass size, the joint surface will be filled with water and penetrated when the water pressure increases, which makes it easier for the rock mass to slip along the structural surface. The effects of water pressure on the rock mass strength should be avoided in actual engineering when the joint size is relatively large. For example, for a slope with penetrated joints, when the water pressure inside the joint increases, the slope is more likely to slip along the joint surface and destabilize. According to the above discussion, the effects of water pressure tend to differ depending on the jointed rock mass failure mode.

5.3. Analysis of the Dip Angle of Shear-Slip Failure of Jointed Rock Mass. The overall slip instability failure of rock mass results in serious hazards, such as landslide disasters and tunnel failure caused by fault slip. It is therefore necessary during engineering surveying and design to analyze the shear-slip failure dip angle of jointed rock mass. When rock mass undergoes shear slip failure, let its critical joint size be \( L_\alpha \), and substitute \( L_\alpha \) into equation (12) to obtain the following equation:

\[
\sin 2\beta(\sigma_1 - \sigma_3) = \tan \phi
\]

\[
\left[ \sigma_1 + \sigma_3 - (\sigma_1 - \sigma_3)\cos 2\beta - 2p_w \right] + 2\left( 1 - \frac{L_\alpha}{M} \right)c.
\]

To analyze the joint dip angle when a rock mass undergoes shear slip failure according to equations (19) and (31), a function \( F(\theta) \) is established with respect to \( \theta \), which is given as follows:

\[
F(\theta) = \sin 2\theta(\sigma_1 - \sigma_3) - \tan \phi \left[ \sigma_1 + \sigma_3 + (\sigma_1 - \sigma_3)\cos 2\theta - 2p_w \right] - 2\left( 1 - \frac{L_\alpha}{M} \right)c.
\]
The function $F(\theta)$ is used to obtain the derivative of the angle $\theta$. When the result of the derivation is equal to 0, the function $F(\theta)$ takes the critical value and the corresponding angle $\theta$ is the dip angle when the rock mass joint undergoes shear failure. According to the derivation result, the calculated $\theta$ value that is most likely to undergo shear failure is as follows:

$$\theta = \frac{\pi}{4} + \frac{\varphi}{2} = \frac{\pi}{4} + \arctan(f)$$

(33)

6. Application of Failure Probability and Strength Models in a Case Study

Tunnel drilling-blasting not only creates new fissures in surrounding rock but also aggravates the propagation of existing fissures. The stability of jointed surrounding rock mass will directly affect the safety of tunnel construction and operation under water pressure action, especially when building tunnels in karst areas. The Miaoziping Tunnel on the Emei-Hanyuan Expressway is located in Leshan City, Sichuan Province, China. This tunnel has rich groundwater with well-developed joints and fissures in the surrounding rock (Figure 5). The safety of the Miaoziping Tunnel surrounding rock is therefore evaluated using the proposed calculation formula for the failure probability of the jointed rock mass. Design and construction suggestions are made on the basis of the actual on-site working conditions combined with the evaluation results.

A detailed geological survey was conducted on the jointed section of the surrounding rock at the exit of the Miaoziping Tunnel. The area of joint development is dominated by karst fissure water with low water pressure, and minor water pressure changes occur during the rainy season. The joints are not completely connected and relatively small in size with a low possibility of shear slip failure and type I-II tension-shear propagation. The jointed rock mass is therefore dominated by type II compression-shear propagation failure. The probability of rock mass failure is calculated by equation (22), and the jointed rock mass of the tunnel is evaluated with the related rock mass parameters, as shown in Table 1.

By substituting the fractal dimension $D$ of the surrounding rock in Table 1 into a simplified form of equation (22), we obtain the following equation:

$$P_f = 1 - \exp\left\{ -\frac{N}{\pi} \left( \frac{\pi L_0}{4} \right)^2 \left( \frac{1 - \varepsilon}{K_{\text{IIc}}} \right)^4 \times \frac{3\pi}{8} \right\}$$

(34)

The other surrounding rock parameters are then substituted into equation (34) to calculate the failure probability of the tunnel surrounding rock for different numbers of joint fissures $N$ and stress conditions $\sigma$. The calculation results are shown in Figure 6. When $N$ or $\sigma$ increase, the failure probability of the surrounding rock will increase, resulting in surrounding rock instability and large convergence deformation of the initial support. When the tunnel traverses through the joint fissure zone, controlled blasting and advanced grouting reinforcement measures are recommended to reduce the number of cracks generated in the surrounding rock and thus improve the surrounding rock strength and stability. On-site monitoring and measurements of the tunnel indicate that the deformation of the initial support is more likely to stabilize when the failure probability is below 0.3. It can be assessed in advance whether to adjust the tunnel blasting parameters based on the failure probabilities for different $N$ and $\sigma$ values, thereby reducing the construction safety risks.

![Figure 5: (a) Miaoziping Tunnel entrance. (b) Joints and fractures of the tunnel surrounding rock.](image)

Table 1: Jointed rock mass parameters.

| $N$ | $L_0/m$ | $\varepsilon$ | $K_{\text{IIc}}$/MPa | $D$ | $\sigma$/MPa |
|-----|---------|---------------|----------------------|-----|--------------|
| 1~8 | 10      | 0.3           | 120                  | 2   | 50~80        |

Shock and Vibration 9
7. Conclusions

Strength models for rock mass joint rupture failure and shear-slip failure are established based on different sizes and failure modes of rock mass joints. The effects of fractal dimension, water pressure, and the number of cracks on the strength and failure probability of the rock mass are analyzed. The most unfavorable dip angle when shear slip occurs in rock mass is calculated. The conclusions are summarized as follows:

(1) Two strength calculation formulas for rupture failure and shear slip failure of rock mass joints are obtained according to the relative joint sizes and groundwater pressure conditions. The proposed rock failure probability and strength calculation formulas appropriately reflect the failure probability and strength of jointed rock mass. When calculating and evaluating the strength of jointed rock mass, different calculation methods should be selected according to the specific working conditions.

(2) The effect of water pressure on the strength of jointed rock mass is sensitive to the failure mode. For jointed rock mass that undergoes rupture failure, the water and rock mass skeleton jointly bear the external load of the rock mass, and an increase in water pressure leads to an increase in the rock mass strength to a certain extent. For jointed rock mass that undergoes shear slip failure, the rock mass strength will decrease when the water pressure increases and shear slip failure is more likely to occur. In the construction of tunnels with abundant groundwater, reasonable drainage should be carried out according to the different failure modes that may occur in jointed rock mass, and drainage or water plugging construction schemes are not always applicable to all cases.

(3) The dip angle at which joint fissures are most prone to shear slip is $\pi/4 + \phi/2$, where $\phi$ is the strength parameter of jointed rock mass. When the fractal dimensions, external stress, and number of fissures increase, the failure probability of the rock mass will increase regardless of the failure mode. When a tunnel traverses through a joint fissure zone, controlled blasting and advanced grouting reinforcement measures are recommended to reduce the number of fissures generated in the surrounding rock and thus improve its strength and stability.

Data Availability

The data generated during the study appear in the submitted article and that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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