Neutron star cooling - a challenge to the nuclear mean field

Hoang Sy Than\textsuperscript{1,2}, Dao T. Khoa\textsuperscript{1} and Nguyen Van Giai\textsuperscript{2}

\textsuperscript{1}Institute for Nuclear Science and Technique, VAEC
179 Hoang Quoc Viet Road, Nghia Do, Hanoi, Vietnam.

\textsuperscript{2}Institut de Physique Nucléaire, IN2P3-CNRS/
Université Paris-Sud, 91406 Orsay, France.

(Dated: December 3, 2009)

Abstract

The two recent density-dependent versions of the finite-range M3Y interaction (CDM3Y\textsubscript{n} and M3Y-P\textsubscript{n}) have been probed against the bulk properties of asymmetric nuclear matter (NM) in the nonrelativistic Hartree Fock (HF) formalism. The same HF study has also been done with the famous Skyrme (SLy4) and Gogny (D1S and D1N) interactions which were well tested in the nuclear structure calculations. Our HF results are compared with those given by other many-body calculations like the Dirac-Brueckner Hartree-Fock approach or ab-initio variational calculation using free nucleon-nucleon interaction, and by both the nonrelativistic and relativistic mean-field studies using different model parameters. Although the two considered density-dependent versions of the M3Y interaction were proven to be quite realistic in the nuclear structure or reaction studies, they give two distinct behaviors of the NM symmetry energy at high densities, like the Asy-soft and Asy-stiff scenarios found earlier with other mean-field interactions. As a consequence, we obtain two different behaviors of the proton fraction in the β-equilibrium which in turn can imply two drastically different mechanisms for the neutron star cooling. While some preference of the Asy-stiff scenario was found based on predictions of the latest microscopic many-body calculations or empirical NM pressure and isospin diffusion data deduced from heavy-ion collisions, a consistent mean-field description of nuclear structure database is more often given by some Asy-soft type interaction like the Gogny or M3Y-P\textsubscript{n} ones. Such a dilemma poses an interesting challenge to the modern mean-field approaches.

PACS numbers: 21.30.-x, 21.65.-f, 21.65.Cd, 21.65.Ef, 21.65.Mn

\textsuperscript{*}Electronic address: khoa@vaec.gov.vn
I. INTRODUCTION

The determination of the nuclear equation of state (EOS), which is of vital importance for the nuclear astrophysics, has been a central object of numerous studies of heavy ion (HI) collisions for the last two decades. If the efforts in the early 90’s were concentrated on the determination of the incompressibility $K$ of symmetric nuclear matter (NM), for different types of the EOS are usually distinguished by different $K$ values [1], recent studies of nuclear reactions involving unstable nuclei lying close to the neutron or proton driplines provide us with a unique opportunity to learn about the EOS of asymmetric NM which has a large difference between the neutron and proton densities [2]. The knowledge about the NM symmetry energy (a key ingredient in the EOS of asymmetric NM) is vital not only in studying the dynamics of HI collisions involving radioactive nuclei and/or their structure, but also in studying the neutron star formation or the $r$-process of stellar nucleosynthesis [3, 4, 5]. In particular, the most efficient process of the neutron star cooling, the so-called direct Urca process in which nucleons undergo direct beta (and inverse-beta) decays [6, 7, 8], can take place only if the proton-to-neutron ratio exceeds 1/8 or the proton fraction $x \geq 1/9$ in the $\beta$-equilibrium. The latter is entirely determined from the density dependence of the NM symmetry energy $S(\rho)$ by the following balance equation [5]

$$\hbar c(3\pi^2\rho x)^{1/3} = 4S(\rho)(1 - 2x),$$

(1)

where $\rho$ is the total NM density. It is of fundamental importance whether the direct Urca process is possible or not. If the $x$ value cannot reach the threshold for the direct Urca process, then the neutron star cooling should proceed via the indirect or modified Urca process which has a reaction rate of $10^4 \sim 10^5$ times smaller than that of the direct Urca process and implies, therefore, a much longer duration of the cooling process [6, 7]. Although a recent test [9] of the microscopic EOS against the measured neutron star masses and flow data of HI collisions has shown that the direct Urca process is possible in some cases if the predicted neutron star mass is above a lower limit of $1.35 \sim 1.5$ solar mass ($M_\odot$), the overall cooling time of a neutron star is still unknown as yet [5] due to the uncertainty about the high-density behavior of $S(\rho)$. We also note here a very strong influence by the neutron superfluidity in the inner crust of neutron star [7, 10, 11], where the superfluid effects can reduce the cooling time by a factor of $3 \sim 4$. In the same direction, the cooling time might be further shortened by the possible phase transitions to quark matter or pion condensate
occurring in the core of neutron star and leading, therefore, to a much higher central density and a smaller star radius [4].

Microscopic studies of the EOS of asymmetric NM have been performed in both nonrelativistic and relativistic nuclear many-body theories, using realistic two-body and three-body nucleon-nucleon (NN) forces or interaction Lagrangians (see more details in recent reviews [2, 12]). These many-body studies have shown the important role played by the Pauli blocking as well as higher-order NN correlations in the G-matrix used to generate the NM binding energy at different densities. These medium effects are considered as physics origin of the density dependence introduced into various effective NN interactions used presently in the nonrelativistic mean-field approaches. Among different versions of the effective NN interaction, very popular choice is the so-called M3Y interaction which was originally constructed by the Michigan State University (MSU) group to reproduce the G-matrix elements of the Reid [13] and Paris [14] NN potentials in an oscillator basis. The use of the original density independent M3Y interaction in the Hartree-Fock (HF) calculation of nuclear matter [15] has failed to saturate NM, leading to a collapse at high densities. Since the HF method is the first order of many-body calculation, some realistic density dependences have been introduced into the original M3Y interaction [16] to effectively account for the higher-order NN correlations which cause the NM saturation. During the last decade, different density dependent versions of the M3Y interaction have been used in the HF calculations of symmetric and asymmetric NM [17, 18, 19, 20, 21, 22, 23, 24], in the mean-field studies of nuclear structure [22, 23, 24] as well as in numerous folding model studies of the nucleon-nucleus and nucleus-nucleus scattering [17, 18, 19, 20, 25]. In view of these studies, it is now highly desirable to have a realistic version of the effective NN interaction for consistent use in the mean-field studies of NM and finite nuclei as well as in the nuclear reaction calculations. As an exploratory step, we perform in the present work a systematic HF study of NM using two density-dependent versions of the finite-range M3Y interaction, the CDM3Yn interactions which have been used mainly in the folding model studies of the nucleon-nucleus and nucleus-nucleus scattering [18, 20, 25] and M3Y-Pn interactions which have been carefully parametrized by Nakada [22, 23, 24] for use in the mean-field studies of nuclear structure. For comparison, the same HF study is also performed with some realistic versions of the Gogny [26, 27] and Skyrme [28] interactions.
II. HARTREE FOCK CALCULATIONS OF NUCLEAR MATTER

Like other mean-field approaches, we consider in the present HF study a homogeneous
spin-saturated NM at zero temperature which is characterized by given values of neutron and
proton densities, \(\rho_n\) and \(\rho_p\), or equivalently by its total density \(\rho = \rho_n + \rho_p\) and its neutron-
proton asymmetry \(\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)\). With the direct \((v_D)\) and exchange \((v_{\text{EX}})\) parts
of the interaction determined from the singlet- and triplet-even (and odd) components of
the central NN force, the total NM binding energy is determined within the HF formalism
as
\[
E = E_{\text{kin}} + \frac{1}{2} \sum_{k\sigma \tau \kappa'} \sum_{k'\sigma' \tau'}[<k\sigma\tau, k'\sigma'\tau'|v_D|k\sigma\tau, k'\sigma'\tau'> + <k\sigma\tau, k'\sigma'\tau'|v_{\text{EX}}|k'\sigma\tau, k\sigma'\tau'>],
\]
(2)
where \(|k\sigma\tau>\) are the ordinary plane waves. The nuclear matter EOS is normally classified
by the NM binding energy per particle which can be expressed as
\[
\frac{E}{A}(\rho, \delta) = \frac{E}{A}(\rho, \delta = 0) + S(\rho)\delta^2 + O(\delta^4) + ...
\]
(3)
The NM pressure \(P\) and incompressibility \(K\) are then calculated as
\[
P(\rho, \delta) = \rho^2 \frac{\partial}{\partial \rho} \left[ \frac{E}{A}(\rho, \delta) \right] ; \quad K(\rho, \delta) = 9\rho^2 \frac{\partial^2}{\partial \rho^2} \left[ \frac{E}{A}(\rho, \delta) \right].
\]
(4)
The contribution of \(O(\delta^4)\) and higher-order terms in Eq. (3), i.e., the deviation from the
parabolic law was proven to be negligible \[17, 29\] and the most important physics quantity
is, therefore, the NM symmetry energy \(S(\rho)\) which is the energy required per nucleon to
change the symmetric NM into a pure neutron matter \[2, 9, 17, 29\]. The value of \(S(\rho)\) at
the saturation density of symmetric NM, \(\rho_0 \approx 0.17\, \text{fm}^{-3}\), is also known as the symmetry
energy coefficient \(J = S(\rho_0)\) which has been predicted by numerous many-body calculations
to be around 30 MeV \[17, 29, 30, 31\]. The NM symmetry energy \(S(\rho)\) is often expanded
around \(\rho_0 \) \[33\] as
\[
S(\rho) = J + \frac{L}{3} \left( \frac{\rho - \rho_0}{\rho_0} \right) + \frac{K_{\text{sym}}}{18} \left( \frac{\rho - \rho_0}{\rho_0} \right)^2 + ...
\]
(5)
where \(L\) and \(K_{\text{sym}}\) are the slope and curvature parameters at \(\rho_0\). As discussed above, the
knowledge about the density dependence of \(S(\rho)\) is of vital importance in studying the
neutron star formation and has been, therefore, a longstanding goal of many NM studies.
using either microscopic or phenomenological models. The main method to probe \(S(\rho)\) associated with a given (mean-field) interaction is to test this interaction in the simulation of HI collisions using transport and/or statistical models \([2, 34, 35, 36, 37, 38, 39]\) or in the structure studies of neutron-rich nuclei \([24, 27, 28, 32, 40, 41, 42, 43, 44]\). Based on the constraints set by these studies using the latest experimental data, some extrapolation is then made to draw conclusions on the low- and high-density behavior of \(S(\rho)\). We show below that such conclusions still remain quite divergent in some cases.

In this work we study asymmetric NM at different neutron-proton asymmetries \(\delta\) using two different sets of the density-dependent M3Y interaction named, respectively, as CDM3Y\(n\) \((n = 3, 4, 6)\) \([18]\) and M3Y-P\(n\) \((n = 3, 4, 5)\) \([24]\). Concerning the first set, the *isoscalar* density dependence of the CDM3Y3, CDM3Y4 and CDM3Y6 interactions has been parametrized \([18]\) to properly reproduce the saturation point of symmetric NM and to give \(K = 217, 228\) and 252 MeV, respectively, in the HF approach \(2-5\). These interactions, especially the CDM3Y6 version, have been widely tested in numerous folding model analyses of refractive \(\alpha\)-nucleus and nucleus-nucleus scattering (see a recent review in Ref. \([45]\)). In the present work, the *isovector* density dependence of the CDM3Y\(n\) interactions is parametrized using the procedure developed in Ref. \([20]\), so as to reproduce the Brueckner-Hartree-Fock (BHF) nuclear matter results for the energy- and density-dependent nucleon optical potential (OP) of Jeukenne, Lejeune and Mahaux (JLM) \([46]\). This isovector density dependence is then scaled by a factor \(\sim 1.3\) deduced recently from the folding model analysis \([19, 20]\) of the \((p, n)\) reactions leading to isobaric analog states (IAS) in various targets. In contrast to the CDM3Y\(n\) set, the M3Y-P\(n\) interactions have been carefully parametrized by Nakada \([24]\) in terms of the finite-range M3Y interaction supplemented with a zero-range density-dependent force, to consistently reproduce the NM saturation properties and ground-state (g.s.) bulk properties of double-closed shell nuclei as well as unstable nuclei close to the neutron dripline. These latest versions of the M3Y-P\(n\) interaction have not been used in the HF study of asymmetric NM and it is, therefore, of interest to probe them here. For completeness, the HF calculation \(2-5\) has also been done with two other popular choices of the effective NN interaction: the Gogny (D1S, D1N) \([26, 27]\) and Skyrme (SLy4) \([28]\) forces. Among them, the latest version of the Gogny force D1N has been shown \([27]\) to reproduce the neutron matter EOS better than the older D1S version while still giving a good description of the bulk properties of finite nuclei.
TABLE I: HF results for the energy $E_0 \equiv E/A(\rho_0, \delta = 0)$ and incompressibility $K$ of symmetric NM, symmetry energy coefficient $J$, slope $L$ and curvature $K_{\text{sym}}$ parameters of $S(\rho)$ evaluated at $\rho = \rho_0$ using the CDM3Y$_n$, M3Y-P$_n$, Skyrme (SLy4) and Gogny (D1S, D1N) interactions. Similar results given by the DBHF calculation using the Bonn A interaction [47] and by other mean-field studies [39, 40, 41, 42, 48] are also presented for comparison. $K_\tau = K_{\text{sym}} - 6L$.

| Inter.       | $\rho_0$ | $E_0$  | $K$   | $J$   | $L$   | $K_{\text{sym}}$ | $K_\tau$ | Ref.  |
|--------------|----------|--------|-------|-------|-------|-------------------|-----------|-------|
|              | (fm$^{-3}$) | (MeV)  | (MeV) | (MeV) | (MeV) | (MeV)             | (MeV)     |       |
| CDM3Y6       | 0.17     | -15.9  | 252   | 29.8  | 62.5  | 39.0              | -336      | [18, 20] |
| CDM3Y4       | 0.17     | -15.9  | 228   | 29.0  | 62.9  | 49.8              | -328      | [18]   |
| CDM3Y3       | 0.17     | -15.9  | 217   | 29.0  | 62.5  | 46.2              | -329      | [18]   |
| M3Y-P3       | 0.16     | -16.5  | 245   | 31.0  | 28.3  | -213              | -383      | [24]   |
| M3Y-P4       | 0.16     | -16.1  | 234   | 30.0  | 21.1  | -234              | -361      | [24]   |
| M3Y-P5       | 0.16     | -16.1  | 235   | 30.9  | 27.9  | -217              | -384      | [24]   |
| D1S          | 0.16     | -16.0  | 203   | 31.9  | 23.7  | -248              | -390      | [26]   |
| D1N          | 0.16     | -16.0  | 221   | 30.1  | 32.4  | -182              | -376      | [27]   |
| SLy4         | 0.16     | -16.0  | 230   | 32.1  | 46.0  | -120              | -396      | [28]   |
| DBHF         | 0.18     | -16.1  | 230   | 34.3  | 70.1  | 6.88              | -414      | [47]   |
| $V_{\text{lowk}}$+CT | 0.16     | -16.0  | 258   | 33.4  | 86.8  | -44.6             | -565      | [48]   |
| MDI ($x=-1$) | 0.16     | -16.0  | 211   | 31.6  | 107   | 94.1              | -550      | [39]   |
| MDI ($x=1$)  | 0.16     | -16.0  | 211   | 30.6  | 16.4  | -270              | -369      | [39]   |
| G2           | 0.15     | -16.1  | 215   | 36.4  | 100.7 | -7.5              | -612      | [40]   |
| FSUGold      | 0.15     | -16.3  | 230   | 32.6  | 60.5  | -51.3             | -414      | [41]   |
| Hybrid       | 0.15     | -16.2  | 230   | 37.3  | 119   | 111               | -603      | [42]   |

The properties of symmetric and asymmetric NM described in the HF approximation using CDM3Y$_n$ and M3Y-P$_n$ interactions are summarized in Table I. Since the isovector component of the interaction does not contribute to the total energy of symmetric NM, the interactions having about the same values of nuclear incompressibility $K$ are expected to give similar EOS up to moderate values of the NM density. One can see from the upper panel of Fig. 1 that the HF results for the energy of symmetric NM obtained with the considered interactions, which give $K \approx 200 \sim 250$ MeV (see Table I), are quite close to each other.
FIG. 1: (Color online) Energy per particle $E/A$ of symmetric NM and pure neutron matter calculated in the HF approximation (2)-(3) using the effective NN interactions given in Table I. Circles are microscopic results of the ab-initio variational calculation [49] by Akmal, Pandharipande and Ravenhall (APR).

at densities up to around $2\rho_0$. At densities above $2\rho_0$ the NM energies calculated with the M3Y-P$n$ interactions are significantly larger than those obtained with the CDM3Y$n$ interactions, even though the predicted incompressibilities $K$ are rather close. The main difference here is that the M3Y-P$n$ interactions have been carefully parametrized [24] not only to reproduce the saturation properties of symmetric NM like the parameter choice for the CDM3Y$n$ interactions [18], but also to give good description of the g.s. shell structure of the magic nuclei. For a comparison, we have also plotted in Fig. 1 the microscopic prediction
by Akmal, Pandharipande and Ravenhall (APR) [49] for both symmetric NM and pure neutron matter based on the variational chain summation method, using A18+δv+UIX* version of the Argonne NN interaction.

For the symmetric NM, the HF results given by all considered interactions agree well with the APR predictions at NM densities up to 2ρ₀, but at higher densities the APR results seem to be closer to the CDM3Y6 and D1N curves. For the neutron matter the picture is quite different, with the HF results given by the CDM3Yn and Skyrme (SLy4) interactions agreeing with the APR results at the NM densities up to 4ρ₀, while the neutron matter energies given by the M3Y-Pn and Gogny (D1S, D1N) interactions are by a factor of 2 ~ 3 smaller than the APR results in the same density range (see lower panel of Fig. 1). It is obvious that such a large difference seen in the HF results for the neutron matter energy is due to the difference in the isovector parts of the considered interactions. Since the isovector density dependence of the CDM3Yn interactions has been parametrized [20] to reproduce simultaneously the BHF results for the isospin- and density dependent nucleon OP by the JLM group [46] and charge exchange (p, n) data for the IAS excitations, the high-density behavior of the neutron matter energy given by the CDM3Yn interactions should approximate that given by a BHF calculation of the neutron matter. In this sense, a similarity between the HF results given by the CDM3Yn interactions and microscopic APR results [49] is not unexpected. In contrast to the CDM3Yn interactions, the isovector density dependence of the M3Y-Pn, D1S and D1N interactions were carefully fine tuned against the structure data observed for the neutron (and proton-) dripline nuclei and it is also natural to expect that the EOS of the neutron matter predicted by these interactions should be quite realistic.

The NM pressure [4] is straightforwardly evaluated from the NM energy. The HF results for the NM pressure given by the present mean-field interactions are compared in Fig. 2 with the empirical constraints deduced from the analysis of the collective flow data measured in relativistic HI collisions [34]. For the symmetric NM, all considered interactions give consistently the NM pressure well within the borders of empirical data at densities up to around 4ρ₀ (see lower panel of Fig. 2). For the pure neutron matter, the HF results given by the CDM3Yn and SLy4 interactions agree nicely with the data, while those given by the M3Y-Pn and Gogny interactions are significantly below the data (see upper panel of Fig. 2). Among the two Gogny forces, the D1S version gives a too low pressure in the neutron matter
FIG. 2: (Color online) Pressure of pure neutron matter (upper panel) and symmetric NM (lower panel) calculated in the HF approximation (2)-(4) using the effective NN interactions given in Table I. The shaded areas are the empirical constraints deduced from the HI flow data [34].

which fails badly in the comparison with the data. This result confirms again that the D1S interaction is not suitable for the study of asymmetric NM as found in the previous NM studies [49, 50, 51]. Looking at Fig. 1 of Ref. [27] one might expect that a Gogny-type interaction giving a neutron matter EOS steeper than that given by the D1N interaction and closer to the Friedman-Pandharipande’s curve [50] would improve the HF description of the neutron matter pressure. From Eqs. (3) and (4) one finds easily that the difference observed in the upper part of Fig. 2 is directly related to different density dependences of the NM symmetry energy $S(\rho)$, which in turn is determined by the isovector component of

\[ \rho (\text{fm}^{-3}) \]

\[ P (\text{MeV fm}^{-3}) \]
To show further the difference caused by the isovector component of the interaction, we have plotted in the upper panel of Fig. 3 the HF results for the NM symmetry energy \( S(\rho) \) given by the considered interactions in comparison with both the empirical data at low densities and microscopic APR results. At the saturation density \( \rho_0 \) of symmetric NM all the models predict the symmetry coefficient \( J \approx 29 \pm 3 \) MeV which agrees reasonably well with the empirical values deduced recently from the CC analysis of the charge-exchange \((p,n)\) reaction exciting the IAS states \([19, 20]\) and structure study of the neutron skin in medium and heavy nuclei \([52, 53]\). In the low-density region \((\rho \approx 0.3 \sim 0.6 \rho_0)\) there exist some empirical points extracted from the HI fragmentation data analysis \([35, 37, 38]\) and the \( S(\rho) \) values given by the CDM3Yn interactions are in a very good agreement with these data. The low-density \( S(\rho) \) values given by other effective interactions are slightly larger than the data but rather close to the microscopic APR result \([49]\). Although the studies of HI fragmentation data and/or neutron skin thickness have put some constraints on the NM symmetry energy \( S(\rho) \) at \( \rho \leq \rho_0 \), its behavior at higher NM densities still remains uncertain. In contrast to the CDM3Yn, SLy4 and APR predictions, the NM symmetry energies given by the remaining mean-field interactions reach their maximal values at NM densities around \( 1.5\rho_0 \) and smoothly decrease to negative values at densities approaching \( 4\rho_0 \) (see upper panel of Fig. 3). These two different behaviors have been observed earlier \([2, 37, 54, 55]\) and are often discussed in the literature as the Asy-stiff (with symmetry energy steadily increasing with density) and Asy-soft (with symmetry energy reaching saturation and then decreasing to negative values) behaviors. While SLy4 and other SLyn versions of the Skyrme interaction could also be assigned to be the Asy-stiff type \([2, 28, 55]\), the Gogny and M3Y-Pn interactions belong definitely to the Asy-soft type.

If we compare these two behaviors in terms of the NM pressure shown in Fig. 2 then the Gogny and M3Y-Pn interactions clearly fail to reproduce the empirical pressure \( P(\rho) \) of the pure neutron matter deduced from the HI flow data \([34]\). While this comparison might allow us to conclude that the (Asy-stiff) CDM3Yn interactions have a more appropriate isovector density dependence compared with the (Asy-soft) Gogny and M3Y-Pn interactions, it is not possible to do so based on the nuclear structure results given by the CDM3Yn interactions. Namely, we have performed spherical HF calculations for finite nuclei and found that the CDM3Yn interactions give a much worse description of the g.s. properties of light- and
FIG. 3: (Color online) Upper panel: NM symmetry energies $S(\rho)$ calculated with the interactions of Table I. Empirical data are taken from the studies of neutron skin [53], HI fragmentation [35, 37, 38] and $(p,n)$ excitation of IAS states [19, 20]; Lower panel: Proton fractions (1) corresponding to these interactions, in comparison with the threshold $x_{DU}$ for the direct Urca process [9]. The curves have the same notations as in Figs. 1 and 2.

A widely adopted procedure so far is to assume that the NM properties predicted by an effective NN interaction should be quite realistic if this interaction gives systematically good description of different structure properties of finite nuclei, especially, the unstable nuclei near the neutron dripline. In this sense, the failure of the Gogny and M3Y-P$n$ interactions in reproducing the empirical NM pressure as well as the unsuccessful use of the CDM3Y$n$
interactions in the nuclear structure calculation pose a serious dilemma, which makes it difficult to conclude unambiguously about the high-density behavior of the NM symmetry energy $S(\rho)$ from the present HF results. Although there is no accurate systematics available for all existing effective NN interactions on their consistent use in the mean-field studies of both NM and finite nuclei, an extensive mean-field study of NM by Stone et al. [55] using different Skyrme interactions has shown that out of 87 considered versions of the Skyrme interaction only 27 versions give the Asy-stiff behavior of the NM symmetry, and all the remaining 60 versions give the Asy-soft behavior. It remains, therefore, an interesting question whether the fitting procedure to determine parameters of an effective NN interaction (from an optimal description of the g.s. properties of finite nuclei and saturation properties of symmetric NM), more likely “bends” the NM symmetry energy $S(\rho)$ curve down to some Asy-soft type rather than “raises” it to some Asy-stiff type. We also note in this connection a recent systematics by Klüpfel et al. [56] on different phenomenological choices of parameters in the Skyrme-Hartree-Fock model for a self-consistent description of nuclear structure and NM properties, which shows that the range of NM properties still remains quite broad despite a large sample of the nuclear ground-state properties used in the parameter fit.

It is, therefore, clear that the ability of nuclear structure data to constrain the EOS for asymmetric NM on the mean-field level is still limited.

The difference found between the Asy-soft and Asy-stiff scenarios becomes more drastic in terms of the proton fractions [1] shown in the lower panel of Fig. 3. If the proton fraction $x = \rho_p/(\rho_p + \rho_n)$ exceeds a critical value $x_{DU}$, then the direct Urca (DU) process

$$n \rightarrow p + e^- + \bar{\nu}_e, \quad p + e^- \rightarrow n + \nu_e$$

becomes possible [6, 8]. Since the neutrino and antineutrino momenta are negligible compared with those of protons, neutrons and electrons, the DU threshold $x_{DU}$ can be estimated from the momentum conservation and charge neutrality condition for $p$, $n$ and $e^-$ only [6, 57]. We have plotted in the lower panel of Fig. 3 the averaged DU threshold as a function of the NM density taken from Ref. [9]. At low densities $x_{DU} \approx 1/9$ as found by Lattimer et al. [6] in the muon-free approximation. At densities above $\rho_0$, the charge neutrality is corrected by the muon presence which slightly enhances $x_{DU}$ [9]. From the HF results obtained with the considered effective interactions, only the proton fractions given by the CDM3Yn interactions can reach the DU threshold at moderate densities $\rho \approx 0.6$ fm$^{-3}$. 

12
According to the microscopic APR study [49], such a central density is reachable in a neutron star having mass $M \approx 1.6M_\odot$ which is well above a lower limit of $1.35 \sim 1.5M_\odot$ for the DU process established in Ref. [9]. The NM density $\rho \approx 0.6 \text{ fm}^{-3}$ happens also to be within the range of average central densities of the neutron star estimated from a nuclear EOS with $K \approx 240 \text{ MeV}$ which is quite close to $K$ values given by the CDM3Yn interactions (see Table I). As a result, the direct Urca process (6) is possible if one chooses the CDM3Yn interactions for the in-medium NN interaction in the neutron star matter. Such a scenario for the DU process is also favored by the HF results for the NM pressure shown in Fig. 2 where the CDM3Yn interactions consistently give the best description of empirical data for both the symmetric NM and pure neutron matter. In contrast to the CDM3Yn interactions, the choice of the (Asy-soft) Gogny or M3Y-Pn interactions would definitely exclude the possibility of the DU process because the corresponding proton fractions can never reach the DU threshold as shown in Fig. 3. The microscopic APR results obtained with the A18+$\delta v+\text{UIX}^*$ version of the Argone NN interaction approach the muon-free threshold $x_{\text{DU}} \approx 1/9$ only at $\rho \approx 0.8 \text{ fm}^{-3}$. Such a central density can exist only if the neutron star mass $M \geq 2M_\odot$ (see Fig. 11 of Ref. [49]) and the DU process is, therefore, very unlikely with the EOS given by the APR model. In the case of SLy4 interaction, the numerical integration of the Tolman-Oppenheimer-Volkov equation in Ref. [55] has shown that the central density reached in a neutron star having mass $M \approx 1.4M_\odot$ is only $\rho \approx 0.55 \text{ fm}^{-3}$. Since the proton fraction $x$ given by the Sly4 interaction is reaching the DU threshold at much a too high density of $\rho \approx 1.4 \text{ fm}^{-3}$ [55], the DU process is also not possible with the Sly4 interaction.

The DU process has also been considered in the fully microscopic many-body studies of the EOS using realistic free NN interactions [8, 12], and we found it complementary to compare the present HF results with those of a recent Dirac-Brueckner Hartree-Fock (DBHF) study [47] using an improved treatment of the Bonn-A interaction. It can be seen from the upper panel of Fig. 4 that the NM symmetry energy curve given by this DBHF study is somewhat stiffer than that given by the CDM3Yn interactions. As a result, the proton fraction estimated from the DBHF results is reaching the DU threshold already at NM densities $\rho \approx 0.45 \text{ fm}^{-3}$. Such a critical density for the DU process is higher than that ($\rho \approx 0.37 \text{ fm}^{-3}$) given by the earlier DBHF results (see Fig. 2 of Ref. [9]) and it should correspond to a star mass above the lower limit of $1.35 \sim 1.5M_\odot$ for the DU process. It is interesting to note that the inclusion of three-body forces into the many-body BHF calculations [58] not only
essentially improves the description of the saturation properties of symmetric NM but also gives a much stiffer NM symmetry energy at high densities (see Fig. 4 of Ref. [58]), in the opposite direction from the Asy-soft type interactions. Given highly accurate parametrizations of the bare NN interaction, these microscopic many-body calculations are practically parameter-free and it is natural to assume an Asy-stiff behavior of $S(\rho)$ which allows both the direct and indirect Urca processes to take place during the neutron star cooling. It also is highly desirable that results of such a microscopic many-body study can be accurately reproduced at the mean-field level using some effective (in-medium) NN interaction which is also amenable to the nuclear structure and/or reaction calculations. However, such “microscopic” mean-field interactions remain technically complicated to construct and most of the structure and reaction studies still use different kinds of the effective NN interaction with parameters adjusted to the optimal description of structure and/or reaction data.

An effective NN interaction can be either fully phenomenological like the Skyrme forces or partially based on a microscopic many-body approach like the CDM3Y$n$ interactions considered above. An interesting alternative approach has been suggested recently by the Tübingen group [48] which considers only the low-momentum (below a cut-off $\Lambda = 2 \text{ fm}^{-1}$) part of the bare NN interaction. While this “low $k$” interaction $V_{\text{low}k}$ still describes well the NN scattering data up to the pion threshold, the short-range correlations originated from the high-momentum components are treated phenomenologically at the mean-field level. Namely, the $V_{\text{low}k}$ has been supplemented by an empirical density-dependent contact (CT) interaction adjusted to reproduce the saturation properties of symmetric NM and the empirical symmetry energy $J$ within the HF approximation [48]. This $V_{\text{low}k}+\text{CT}$ interaction was shown to give also a reasonable description of the g.s. properties of some finite nuclei including $^{208}\text{Pb}$. The NM symmetry energy and proton fraction predicted by the $V_{\text{low}k}+\text{CT}$ interaction are shown in Fig. 4 and they are quite close to those predicted by the CDM3Y$n$ interactions shown in Fig. 3. Like the CDM3Y$n$ interactions, the $V_{\text{low}k}+\text{CT}$ interaction should also belong to the Asy-stiff type and allow both the direct and indirect Urca processes during the neutron star cooling. Another famous choice of the effective NN interaction is the Skyrme-type momentum dependent interaction (MDI) which has been first parametrized [39] for the transport model simulation of HI collisions. By varying the $x$ parameter of the MDI interaction, the experimental data from NSCL-MSU on the isospin diffusion have been shown to favor the MDI ($x=-1$) version which gives the NM symmetry energy nearly linear in
FIG. 4: (Color online) Upper panel: NM symmetry energies $S(\rho)$ given by different many-body studies \cite{39, 40, 41, 42, 47, 48, 49}; Lower panel: Proton fraction $X$ given by these symmetry energies in comparison with the threshold $x_{\text{DU}}$ for the direct Urca process \cite{9}. See more details in text.

One can see in Fig. 4 that the NM symmetry energy $S(\rho)$ given by the MDI ($x=-1$) interaction is somewhat stiffer than that predicted by the DBHF calculation using the Bonn A interaction \cite{47}. The proton fraction given by this (Asy-stiff) MDI ($x=-1$) interaction is reaching the DU threshold at NM densities $\rho \approx 0.3 \text{fm}^{-3}$ and is well above $x_{\text{DU}}$ at the typical central density $\rho \approx 0.5 \sim 0.7 \text{fm}^{-3}$ of neutron star. Since the star mass corresponding to such a central NM density should be larger than the lower limit of $1.35 \sim 1.5M_\odot$ for the DU process \cite{9}, the DU process must be possible in
The MDI interaction has also been used to describe neutron skin in finite nuclei in the Skyrme HF model [59], and the MDI interaction with \(x\) between 0 and -1 was shown to reproduce reasonably well the empirical neutron-skin data for \(^{124,132}\)Sn and \(^{208}\)Pb. However, the situation with the MDI interaction becomes somewhat confused after the new FOPI data on the \(\pi^-/\pi^+\) ratio measured in central HI collisions at SIS/GSI energies have been shown to clearly favor the MDI (\(x=1\)) interaction [36]. In terms of symmetry energy, the MDI (\(x=1\)) interaction belongs to the Asy-soft type (see Fig. 4) like the Gogny or M3Y-Pn interactions considered above and it excludes, therefore, the DU process during the neutron star cooling. Given experimental evidences favoring both the Asy-stiff and Asy-soft versions of the MDI interaction, the behavior of the NM symmetry energy at high densities as well as the possibility of the DU process still remain an open question.

The NM symmetry energy has also been the subject of various relativistic mean field (RMF) studies. In the present work, we compare our nonrelativistic HF results with those of some recent RMF studies using carefully chosen parameters for the energy-density functional [40, 41, 42]. The G2 parameter set [40] has been shown to consistently reproduce the g.s. structure of finite nuclei and bulk properties of NM. In particular, the RMF calculations using the G2 parameters reproduce very well the empirical pressure for both the symmetric NM and pure neutron matter [40]. Quite interesting is the FSUgold parameter set developed by Piekarewicz et al. [41] which has been used to study not only the NM properties and g.s. structure of finite nuclei but also the excitation of the nuclear giant monopole resonance (GMR). While the FSUgold parameters give a good description of the GMR in \(^{208}\)Pb, the observed GMR excitation energies in Sn isotopes could not be reproduced using this parameter set. In order to improve the RMF description of the NM saturation properties as well as the monopole strength distribution in Sn isotopes, a hybrid model of the RMF parameters has been developed [42] based on the earlier NL3 model. However, the Hybrid parameters turned out to give a worse description of the GMR in \(^{208}\)Pb compared with that given by the FSUgold model and it remains, therefore, difficult to choose between these two parameter sets. The RMF results using these parameters are shown in Fig. 4 and one can see that the stiffness of the NM symmetry energy is gradually increasing as one goes from FSUgold and G2 to the Hybrid results. It has been found by Steiner [57] that the RMF models typically have a large symmetry energy and a large proton fraction, and the DU process becomes possible at rather low NM densities. This effect can be clearly seen in the Hybrid and G2
results shown in Fig. 4 where the corresponding proton fractions reach DU threshold at the NM densities of $\rho \approx 0.24$ and 0.32 fm$^{-3}$, respectively. We note further that $S(\rho)$ predicted in the Hybrid model is very close to the RMF result by Klähn et al. using the NL$\rho$σ parametrization where the proton fraction is reaching the DU threshold at $\rho \approx 0.28$ fm$^{-3}$.

The behavior of the proton fraction predicted by the FSUgold model is somewhat different from those predicted by the Hybrid and G2 models. Namely, it approaches the muon-free threshold $x$_{DU} $\approx 0.11$ only at $\rho \approx 0.8$ fm$^{-3}$, like the microscopic APR result. With the predicted maximum neutron star mass of $M \approx 1.72 M_\odot$, the FSUgold model has been shown in Ref. [41] to allow partially the DU process in the neutron star cooling. However, if we adopt the averaged DU threshold taken from Ref. [9] which takes into account the muon presence at high densities, then the DU process is unlikely in this case because the proton fraction predicted by the FSUgold model seems to saturate at densities $\rho \geq 0.8$ fm$^{-3}$, around a value of $x \sim 0.11$ (see lower panel of Fig. 4) like the APR results [49] discussed above.

Although the experimental evidences are still divergent with respect to the Asy-stiff and Asy-soft type mean-field interactions, it is of interest to further explore the difference between these two groups in terms of the NM incompressibility. Using the general definition [3]-[4], the NM incompressibility $K(\rho)$ can be written explicitly in terms of the isoscalar ($K_0$) and isovector ($K_1$) parts as

$$K(\rho, \delta) = K_0(\rho) + K_1(\rho)\delta^2 + O(\delta^4) + ...$$  \hspace{1cm} (7)

where

$$K_0(\rho) = 9\rho^2 \frac{\partial^2}{\partial \rho^2} \left[ \frac{E_A(\rho, \delta = 0)}{A} \right]; \quad K_1(\rho) = 9\rho^2 \frac{\partial^2 S(\rho)}{\partial \rho^2}.$$  \hspace{1cm} (8)

It is clear from Eq. [3] that the behavior of the isovector incompressibility $K_1$ should correlate closely with the NM symmetry energy $S(\rho)$. We have plotted in Fig. 5 the density dependence of $K_1$ given by the numerical differentiation of the HF results for $S(\rho)$ and one can see that the $K_1$ value given by the (Asy-stiff) CDM3Y$n$ interactions is positive over the density range $\rho \geq \rho_0$ and gradually increases to 200 $\sim$ 400 MeV at $\rho$ approaching 0.6 fm$^{-3}$ (the DU-onset density found with these interactions). On the contrary, the $K_1$ value given by the (Asy-soft) M3Y-P$n$ and Gogny interactions is negative over the same density range and decreases linearly to about -1000 MeV at $\rho$ approaching 0.6 fm$^{-3}$. Since $K(\rho)$ is actually determined from the first derivative of NM pressure $P(\rho)$ with respect to the density, a strongly negative isovector incompressibility $K_1(\rho)$ should indicate a decrease of $P(\rho)$ in the transition from symmetric NM to the pure neutron matter. Indeed, one can see in Fig. 2...
FIG. 5: (Color online) Isovector part of the NM incompressibility given by HF calculations using different effective interactions. Encircled part shows $K_1$ values near the saturation density $\rho_0$ of symmetric NM, i.e., the $K_{\text{sym}}$ values given in Table I. See more details in text.

that a decrease of $P(\rho)$ in the pure neutron matter was found with the (Asy-soft) M3Y-Pn and Gogny interactions which pulls the calculated $P(\rho)$ out of the empirical boundaries established by the HI collective flow data 34. Based on this discussion, we conclude that the behavior of isovector incompressibility $K_1(\rho)$ given by the (Asy-stiff) CDM3Y$n$ interactions is more consistent with the HI flow data compared with $K_1(\rho)$ given by the (Asy-soft) M3Y-Pn and Gogny interactions.

In the literature, the discussion on the isovector part of the NM incompressibility is very often made based on the $K$ values estimated at the saturation density $\rho_0$ of symmetric NM. It should be noted, however, that the saturation density of asymmetric NM decreases rather quickly with the increasing neutron-proton asymmetry $\delta$, and the pure neutron matter ($\delta = 1$) becomes unbound (see lower panel of Fig. II and also Fig. 2 of Ref. 17). As a result, $\rho_0$ is no more a stable extremum in the NM energy curve and various expansions around it like (5) might not be accurate for large neutron-proton asymmetries $\delta$. For example, the second derivative of the approximated expression (5) for $S(\rho)$ gives a purely parabolic
density dependence of the isovector incompressibility, $K_1(\rho) \approx K_{\text{sym}}(\rho/\rho_0)^2$, which can strongly deviate from the exact HF result at high densities as shown in Fig. 5. In addition, the higher-order $O(\delta^4)$ term in Eq. 5 has been shown by Steiner to be important in determining the critical density for the DU process in neutron stars. In the vicinity of $\rho_0$, the $K_1$ values given by the HF calculation (encircled in Fig. 5) are quite close to the corresponding $K_{\text{sym}}$ coefficients of the expansion, i.e., $K_1(\rho \to \rho_0) \approx K_{\text{sym}}$. In the studies of the HI isospin diffusion or isospin dependence of the GMR excitation, the asymmetry of the NM incompressibility around $\rho_0$ was associated with the quantity $K_\tau = K_{\text{sym}} - 6L$ which has been confined by these data to $K_\tau \approx -550 \pm 100$ MeV. The empirical $K_\tau$ value has been shown by a recent study of the neutron-skin thickness by Centelles et al. to be around $K_\tau \approx -500 \pm 100$ MeV. From Table I one can see that $K_\tau$ values obtained from the (Asy-stiff) DBHF, $V_{\text{lowk}}+\text{CT}$, Hybrid and MDI ($x=-1$) results are in good agreement with the empirical values; the Asy-stiff CDM3Yn interactions give $K_\tau$ values of about -330 MeV which are somewhat above the upper limit of empirical data; the Asy-soft MDI ($x=1$), M3Y-Pn and Gogny interactions give $K_\tau$ values of about -400 MeV, right at the empirical upper limit. However, we note that a very recent mean-field study of asymmetric NM using a large number of Skyrme-type interactions by Chen et al. has found an optimum range $K_\tau \approx -370 \pm 120$ MeV and, thus, cast some doubt on previously adopted empirical $K_\tau$ values. In the context of our paper, this latest analysis seems to give preference to the (Asy-stiff) CDM3Yn interactions. Some further preference for the Asy-stiff interactions can also be made based on the empirical constraints on the $J$ and $L$ parameters established recently by the MSU group in the analysis of the isospin diffusion and ratios of neutron and proton spectra measured in HI collisions. Namely, one can see in Table I that only the $L$ and $J$ values given by the (Asy-stiff) CDM3Yn and SLy4 interactions are lying within the ranges of the double constraint deduced from the isospin diffusion data: $L \approx 40 \sim 70$ MeV and $J \approx 30 \sim 34$ MeV (see Fig. 1 of Ref. 63). The (Asy-soft) Gogny and M3Y-Pn interactions give $L$ values much lower than this limit and this indicates again that the (Asy-stiff) CDM3Yn interactions comply better with the HI data. We note here also a similar empirical range for the slope parameter $L \approx 45 \sim 75$ MeV established recently in a systematic study of the correlation between the neutron-skin thickness and symmetry energy. However, the neutron-skin thickness has been shown by Danielewicz to be mainly sensitive to the surface part of the symmetry energy term in a more elaborate mass
formula for finite nuclei, while the extrapolation to the high-density behavior of the NM symmetry energy is based more on the volume term. Therefore, the empirical ranges for $L$, $J$ and $K_\tau$ values deduced from the neutron-skin studies should be necessary reference points for any mean-field study of asymmetric NM but not sufficient constraints to restrict the behavior of the NM symmetry energy at high densities.

Finally, we note that all the mean-field calculations discussed in the present work do not take into account the hyperon presence in the neutron star. The hyperon population has been estimated to make up about 18% of the neutron star matter and shown to significantly soften the EOS as well as reduce the limiting neutron star mass [63, 64]. In this case, not only the direct Urca process involving hyperons becomes possible [10] but also the proton fraction is significantly enhanced by the hyperon presence. For example, the proton fraction of a hyperon star having mass $M \approx 1.5M_\odot$ is about 50% larger than that of a neutron-proton-lepton star of the same mass (see Fig. 5.28 of Ref. [64]). As a result, if we assume for simplicity a 50% rise in the proton fractions predicted, e.g., by the microscopic APR calculation or FSUgold model at $\rho \approx 0.6 \sim 0.8$ fm$^{-3}$ (see lower panel of Fig. [4]), then the DU process is well allowed within these models. Concerning a typical Asy-soft interaction like the Gogny, M3Y-Pn or MDI (x=1), such a 50% increase of the proton fraction is still not enough to make the DU process possible.

III. SUMMARY

In the framework of the self-consistent HF mean field, we have studied the bulk nuclear matter properties predicted by two different sets (CDM3Yn and M3Y-Pn) of the density-dependent M3Y interaction, SLy4 version of the Skyrme interaction as well as D1S and D1N versions of the Gogny interaction. The HF results for the NM symmetry energy and proton fraction in the $\beta$-equilibrium are also compared with those given by the microscopic many-body studies (DBHF and APR calculations) using the bare NN interaction, and by the RMF studies using different parameter sets.

We have concentrated our discussion on two main aspects: the NM binding energy and pressure in the symmetric NM and pure neutron matter, and the density dependence of the NM symmetry energy $S(\rho)$ and the associated proton fraction [11]. For the symmetric NM, the main conclusion is that all the effective NN interactions used here are more or
less consistent with the microscopic APR prediction and empirical pressure deduced from the HI collective flow data. For the pure neutron matter, the HF predictions for the NM binding energy and pressure show that the considered mean-field interactions are divided into two families which are associated with two different behaviors (Asy-soft and Asy-stiff) of the NM symmetry energy at high densities, where only the Asy-stiff type interactions comply with the empirical NM pressure. These two families were shown to predict two different behaviors of the proton-to-neutron ratio in the β-equilibrium which, in turn, imply two drastically different mechanisms for the neutron star cooling (with or without the direct Urca process).

Although an ambiguity in the high-density behavior of the NM symmetry energy still remains due to the experimental evidences from HI studies favoring both the Asy-soft and Asy-stiff versions of the mean-field interaction, a comparison of the present HF results with the empirical constraints for the symmetry coefficient $J$ and slope parameter $L$, given by the HI isospin diffusion data and ratios of neutron and proton spectra [33] and the systematic study of the neutron-skin thickness [44, 62], seems to provide some evidence favoring the Asy-stiff type interactions. The Asy-stiff behavior is also predicted by the fully microscopic BHF or DBHF calculations [47, 58] which include the higher-order many-body effects and three-body forces, and by the latest RMF studies [40, 41, 42].

A big puzzle remains why on the (nonrelativistic) mean-field level, a wide range of the nuclear structure data can be consistently described only by using some Asy-soft type effective interaction like the famous Gogny forces or M3Y-Pn interaction. In each case, the chosen parameter set for the effective NN interaction depends strongly on the nuclear structure and/or reaction data under consideration [36, 52, 56, 59, 61] and there could be a plethora of systematic uncertainties in different choices for the mean-field interaction which are not under control and can lead, in particular, to the distinct soft and stiff scenarios. In any case, the ability of available structure and/or reaction data to constrain the EOS for high-density neutron rich NM on the mean-field level is still limited and the most interesting challenges are lying ahead.
Acknowledgments

We thank Mario Centelles, Bao-An Li, Jerome Margueron, and Herbert M"uther for their helpful communications and discussions. The present research has been supported by the National Foundation for Scientific and Technological Development (NAFOSTED) under Project Nr. 103.04.07.09. H.S.T. also gratefully acknowledges the financial support from the Asia Link Programme CN/Asia-Link 008 (94791) and the Bourse Eiffel program of the French Ministry of Foreign Affairs during his research stays at IPN Orsay where part of the present nuclear matter study has been performed.

[1] G.F. Bertsch and S. Das Gupta, Phys. Rep. 160, 189 (1988); J. Aichelin, Phys. Rep. 202, 233 (1991).
[2] B.A. Li, L.W. Chen, and C.M. Ko, Phys. Rep. 464, 113 (2008).
[3] H.A. Bethe, Rev. Mod. Phys. 62, 801 (1990).
[4] K. Sumiyoshi and H. Toki, Astro. Phys. J. 422, 700 (1994).
[5] J.M. Lattimer and M. Prakash, Science 304, 536 (2004); J.M. Lattimer and M. Prakash, Phys. Rep. 442, 109 (2007).
[6] J.M. Lattimer, C.J. Pethick, M. Prakash, and P. Haensel, Phys. Rev. Lett. 66, 2701 (1991).
[7] J.M. Lattimer, K.A. Van Riper, M. Prakash, and M. Prakash, Astro. Phys. J. 425, 802 (1994).
[8] D. Page, J.M. Lattimer, M. Prakash, and A.W. Steiner, Astro. Phys. J. Suppl. Series 155, 623 (2004).
[9] T. Kl"ahn et al., Phys. Rev. C 74, 035802 (2006).
[10] D.G. Yakovlev, A.D. Kaminker, O.Y. Gnedin, and P. Haensel, Phys. Rep. 354, 1 (2000).
[11] D.G. Yakovlev and C.J. Pethick, Ann. Rev. Astron. Astrophys. 42, 169 (2004).
[12] M. Baldo and C. Maieron, J. Phys. G 34, R243 (2007).
[13] G. Bertsch, J. Borysowicz, H. McManus, and W.G. Love, Nucl. Phys. A284, 399 (1977).
[14] N. Anantaraman, H. Toki, and G.F. Bertsch, Nucl. Phys. A398, 269 (1983).
[15] D.T. Khoa and W. von Oertzen, Phys. Lett. B304, 8 (1993).
[16] D.T. Khoa and W. von Oertzen, Phys. Lett. B342, 6 (1995).
[17] D.T. Khoa, W. von Oertzen, and A.A. Ogloblin, Nucl. Phys. A602, 98 (1996).
[18] D.T. Khoa, G.R. Satchler, and W. von Oertzen, Phys. Rev. C 56 (1997) 954.
[19] D.T. Khoa and H.S. Than, Phys. Rev. C 71, 044601 (2005).
[20] D.T. Khoa, H.S. Than, and D.C. Cuong, Phys. Rev. C 76, 014603 (2007).
[21] D.N. Basu, P. Roy Chowdhury, C. Samanta, Nucl. Phys. A811, 140 (2008).
[22] H. Nakada and M. Sato, Nucl. Phys. A699, 511 (2002).
[23] H. Nakada, Phys. Rev. C 68, 014316 (2003).
[24] H. Nakada, Phys. Rev. C 78, 054301 (2008).
[25] N.D. Chien and D.T. Khoa, Phys. Rev. C 79, 034314 (2009).
[26] J.F. Berger, M. Girod, and D. Gogny, Comp. Phys. Comm. 63, 365 (1991).
[27] F. Chappert, M. Girod, and S. Hilaire, Phys. Lett. B 668, 420 (2008).
[28] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, and R. Schaeffer, Nucl. Phys. A635, 231 (1998).
[29] W. Zuo, I. Bombaci, and U. Lombardo, Phys. Rev. C 60, 024605 (1999).
[30] M. Brack, C. Guet, and H.B. Håkansson, Phys. Rep. 123, 276 (1985).
[31] J.M. Pearson and R.C. Nayak, Nucl. Phys. A668, 163 (2000).
[32] P. Danielewicz, Nucl. Phys. A727, 233 (2003).
[33] M.B. Tsang, Y. Zhang, P. Danielewicz, M. Famiano, Z. Li, W.G. Lynch, and A.W. Steiner, Phys. Rev. Lett. 102, 122701 (2009).
[34] P. Danielewicz, R. Lacey and W.G. Lynch, Science 298, 1592 (2002).
[35] A. Ono, P. Danielewicz, W.A. Friedman, W.G. Lynch, and M.B. Tsang, Phys. Rev. C 68, 051601(R) (2003).
[36] Z. Xiao, B.A. Li, L.W. Chen, G.C. Yong, and M. Zhang, Phys. Rev. Lett. 102, 062502 (2009).
[37] D.V. Shetty, S.J. Yennello, and G.A. Souliotis, Phys. Rev. C 76, 024606 (2007).
[38] D.V. Shetty, S.J. Yennello, and G.A. Souliotis, Nucl. Inst. and Meth. in Phys. Res. B 261, 990 (2007).
[39] L.W. Chen, C.M. Ko, and B.A. Li, Phys. Rev. Lett. 94, 032701 (2005)
[40] P. Arumugam, B.K. Sharma, P.K. Sahu, S.K. Patra, Tapas Sil, M. Centelles, and X. Viñas, Phys. Lett. B 601, 51 (2004).
[41] B. G. Todd-Rutel and J. Piekarewicz, Phys. Rev. Lett. 95, 122501 (2005).
[42] J. Piekarewicz and M. Centelles, Phys. Rev. C 79, 054311 (2009).
[43] J. Piekarewicz, Phys. Rev. C 76, 064310 (2007).
[44] M. Centelles, X. Roca-Maza, X. Vinas, and M. Warda, Phys. Rev. Lett. \textbf{102}, 122502 (2009).
[45] D.T. Khoa, W. von Oertzen, H.G. Bohlen, and S. Ohkubo, J. Phys. \textbf{G34}, R111 (2007).
[46] J. P. Jeukenne, A. Lejeune and C. Mahaux, Phys. Rev. C \textbf{16}, 80 (1977).
[47] E.N.E. van Dalen, C. Fuchs, and A. Faessler, Eur. Phys. J. A \textbf{31}, 29 (2007).
[48] P. Bozek, D.J. Dean, and H. M"uther, Phys. Rev. C \textbf{74}, 014303 (2006); E.N.E. van Dalen, P. G"ogelein, and H. M"uther, Phys. Rev. C \textbf{80}, 044312 (2009).
[49] A. Akmal, V.R. Pandharipande, and D.G. Ravenhall, Phys. Rev. C \textbf{58}, 1804 (1998).
[50] B. Friedman and V.R. Pandharipande, Nucl. Phys. \textbf{A361}, 502 (1981).
[51] R.B. Wiringa, V. Fiks, and A. Fabrocini, Phys. Rev. C \textbf{38}, 1010 (1988).
[52] B.A. Brown, Phys. Rev. Lett. \textbf{85}, 5296 (2000).
[53] R.J. Furnstahl, Nucl. Phys. \textbf{A706}, 85 (2002).
[54] V. Baran, M. Colonna, V. Greco, and M. Di Toro, Phys. Rep. \textbf{410}, 335 (2005).
[55] J.R. Stone, J.C. Miller, R. Koncewicz, P.D. Stevenson, and M.R. Strayer, Phys. Rev. C \textbf{68}, 034324 (2003).
[56] P. Klüpfel, P.-G. Reinhard, T.J. Bürvenich, and J.A. Maruhn, Phys. Rev. C \textbf{79}, 034310 (2009).
[57] A.W. Steiner, Phys. Rev. C \textbf{74}, 045808 (2006).
[58] Z.H. Li, U. Lombardo, H.J. Schulze, W. Zuo, L.W. Chen, and H.R. Ma, Phys. Rev. C \textbf{74}, 047304 (2006).
[59] L.W. Chen, C.M. Ko, and B.A. Li, Phys. Rev. C \textbf{72}, 064309 (2005).
[60] T. Li \textit{et al.}, Phys. Rev. Lett. \textbf{99}, 162503 (2007).
[61] L.W. Chen, B.J. Cai, C.M. Ko, B.A. Li, C. Shen, and J. Xu, Phys. Rev. C \textbf{80}, 014322 (2009).
[62] M. Warda, X. Vinas, X. Roca-Maza, and M. Centelles, Phys. Rev. C \textbf{80}, 024316 (2009); M. Centelles, private communication (unpublished).
[63] N.K. Glendenning and S.A. Moszkowski, Phys. Rev. Lett. \textbf{67}, 2414 (1991).
[64] N.K. Glendenning, \textit{Compact Stars: Nuclear Physics, Particle Physics and General Relativity} (Springer: Springer-Verlag New York, Inc. 2000).