The \( H_0 \) Tension in Non-flat QCDM Cosmology

Haitao Miao and Zhiqi Huang

School of Physics and Astronomy, Sun Yat-sen University, 2 Daxue Road, Tangjia, Zuhai, People’s Republic of China

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Abstract

The recent local measurement of the Hubble constant leads to a more than 3σ tension with Planck + ΛCDM. In this article we study the \( H_0 \) tension in non-flat QCDM cosmology, where Q stands for a minimally coupled and slowly or moderately rolling quintessence field \( \phi \) with a smooth potential \( V(\phi) \), and CDM refers to cold dark matter. By generalizing the QCDM one-parameter and three-parameter parameterizations in Huang et al. to a non-flat universe and using the latest cosmological data, we find that the \( H_0 \) tension remains above the 3.2σ level for this class of model.

Key words: cosmic background radiation – cosmological parameters – dark energy – large-scale structure of universe

1. Introduction

Confirmed late-time acceleration of the universe by observational data including Type Ia supernovae (SNe; Riess et al. 1998; Perlmutter et al. 1999; Betoule et al. 2014), cosmic microwave background (CMB) radiation (Planck Collaboration et al. 2016a, 2016b, 2016c, 2016d), and baryon acoustic oscillations (BAO; Beutler et al. 2011; Ross et al. 2015; Gil-Marín et al. 2016a, 2016b) indicates that about 70% of the energy density of the universe today consists of dark energy, which is a supposed exotic component of the universe that induces a negative pressure on large scales.

In the standard ΛCDM cosmology, the late-time cosmic acceleration is explained by Einstein’s cosmological constant \( \Lambda \), whose microscopic nature is interpreted as vacuum energy. There is, however, a serious fine-tuning problem with this interpretation: the measured energy scale of \( \Lambda \) is \( \sim 10^{120} \) times smaller than a naive dimension analysis. This discrepancy motivated theorists to construct alternative models, among which the first suggestion is quintessence, namely, a minimally coupled canonical scalar field with a potential \( V(\phi) \) (Ratra & Peebles 1988; Wetterich 1988; Caldwell et al. 1998; Zlatev et al. 1999). A slowly rolling quintessence field can provide a negative pressure that drives the cosmic acceleration. Physicists have also proposed many other more exotic models, such as k-essence (Armendariz-Picon et al. 2000, 2001), f(R) gravity (Capozziello et al. 2003; Carroll et al. 2004; Nojiri & Odintsov 2006; Hu & Sawicki 2007), and the Dvali-Gabadadze-Porrati (DGP) model (Dvali et al. 2000). For a comprehensive list of dark energy models, the reader is referred to Copeland et al. (2006), Yoo & Watanabe (2012), and Arun et al. (2017).

Over the last two decades, the ΛCDM model continued to be the simplest model to explain the observational data, and other candidates seemed to be disfavored by Occam’s razor. Recently, the improved local measurement of the Hubble constant \( H_0 \) starts to challenge this picture: the locally measured \( H_0 = 73.48 \pm 1.66 \) km s\(^{-1}\) Mpc\(^{-1}\) (Riess et al. 2018b) is about 3σ (or even more, depending on which combination of CMB data sets is used) higher than the CMB + ΛCDM favored value \( H_0 = 67.8 \pm 0.9 \) km s\(^{-1}\) Mpc\(^{-1}\) (Planck Collaboration et al. 2016c).

In minimally coupled or weakly coupled dark energy models the dark energy component can often be approximated as a perfect fluid. The equation of state (EOS) of the fluid, defined as the ratio of the pressure to the energy density, depending on the underlying model, can be approximately constant or strongly time-dependent. For the ΛCDM model, the EOS is equal to −1. Whereas for a quintessence field, the EOS is characterized by a time-dependent function \( w(a) \geq -1 \), with \( a \) being the scale factor in the Friedmann–Robertson–Walker (FRW) metric. To make definitive predictions that can be compared with the data, observers need to specify a function from \( w(a) \). The Chevallier–Polarski–Linder (CPL; Chevallier & Polarski 2001; Linder 2003) parameterization, \( w = w_0 + (1 - a)w_a \), is the most popular one in the literature. However, such a parameterization is not based on any physical model. This arbitrariness in CPL parameterization leads people to investigate more theoretically motivated dark energy trajectories described by a few physical parameters. For quintessence models, an analytic approximation of \( w(a) \) has been derived in Huang et al. (2011; HBK). HBK’s three-parameter approximation \( w(a; \varepsilon, \varepsilon_0, \varepsilon_\infty) \) fits well in the ensemble of trajectories for a wide class of potentials \( V(\phi) \). The slope parameter \( \varepsilon_0 \) characterizes the slope of the potential. They found that a reasonable pivot to measure the slope of the potential is at \( a = a_{eq} \), where the energy densities of dark energy and matter are equal, and so \( \varepsilon_0 \) is defined. The tracking parameter \( \varepsilon_\infty \) and the running parameter \( \varepsilon_0 \) induce necessary corrections if the quintessence has early-time dynamics.

HBK’s \( w \) formula was based on flat-space assumption, a well-established observational fact in ΛCDM cosmology. In other words, HBK implicitly assumed that the constraint on the spatial curvature is not sensitive to the choice of the dark energy model. In this paper we re-examine this assumption by generalizing HBK parameterization to the FRW metric with nonvanishing spatial curvature.

This article is organized as follows. Section 2 generalizes the HBK \( w \) parameterization to the non-flat FRW metric and Section 3 describes the observational data sets. In Section 4 we constrain the generalized HBK \( w \) parameterization and study the \( H_0 \) tension in this class of model and Section 5 concludes.

Throughout the paper we work with natural units with \( c = \hbar = 1 \). The reduced Planck mass is defined as \( M_p \equiv \frac{1}{\sqrt{8\pi G_N}} \), where \( G_N \) is Newton’s gravitational constant. We assume three species of light neutrinos with a default sum of mass of \( \sum m_i = 0.06 \) eV.
2. Generalizing the HBK Parameterization

We begin with FRW metric
\[ ds^2 = dt^2 - a^2(t)\left[\frac{1}{1 - kr^2}dr^2 + r^2(d\theta^2 + \sin \theta d\phi^2)\right]. \]
(1)
The scale factor \( a \) is normalized to unity today and can be related to cosmological redshift \( z \) via \( a = \frac{1}{1 + z} \). Cosmological expansion is characterized by the Hubble parameter
\[ H = \frac{\dot{a}}{a}, \]
(2)
where \( \dot{a} \) denotes time derivative \( d/dt \).

The Hubble parameter today, namely the Hubble constant, is denoted as \( H_0 \). The combination of \( H_0 \) and the parameter \( k \) in the FRW metric yields a dimensionless parameter
\[ \Omega_k = - \frac{k}{H_0^2}, \]
(3)
which characterizes the spatial curvature of the current universe.

At the background level, the Klein–Gordon equation of the quintessence field is
\[ \ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \]
(4)
where \( V'(\phi) = dV/d\phi \).

The pressure of the quintessence field is the difference between the kinetic energy and the potential energy,
\[ p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \]
(5)
The energy density is the sum of
\[ \rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi). \]
(6)
Because a slowly rolling quintessence field has a subdominant kinetic energy of \( \frac{1}{2}\dot{\phi}^2 \ll V(\phi) \), its EOS, \( w = p_\phi/\rho_\phi \), is close to \(-1\). To the lowest order approximation, dark energy behaves like a cosmological constant: \( 1 + w \approx 0 \) and \( \rho_\phi \approx \) const. Our goal is to compute the next order correction \( 1 + w \), or equivalently, the time evolution of \( p_\phi \).

It is useful to introduce a dimensionless parameter,
\[ \theta = \arcsin \frac{\dot{\phi}}{\sqrt{2p_\phi}}. \]
(7)
Using equation (4) and after a few lines of algebra, we achieve
\[ \frac{d\theta}{d \ln a} = \sqrt{\epsilon_V} \frac{\sqrt{\rho_\phi}}{HM_p} \cos \theta - \frac{3}{2} \sin 2\theta, \]
(8)
where
\[ \epsilon_V \equiv \frac{M_p^2}{2} \left(\frac{V'}{V}\right)^2 \]
(9)
describes the slope of the (logarithm) potential.

At low redshift where dark energy is relevant, we can ignore the radiation and light neutrinos and compute the Hubble parameter via
\[ H = H_0 \sqrt{\Omega_m a^{-3} + \Omega_k a^{-2} + \frac{\rho_\phi}{3H_0^2M_p^2}}, \]
(10)
where \( \Omega_m \) is the ratio of today’s matter density to the critical density \( \rho_m = 3H_0^2M_p^2 \). Similarly we denote today’s value of \( \rho_\phi \) as \( \Omega_\phi \).

We proceed with a slow-roll approximation \( \theta \ll 1 \). Following Huang et al. (2011), we bootstrap from the zeroth order approximations \( \rho_\phi \approx 3H_0^2M_p^2\Omega_\phi \) and \( \epsilon_V \approx \epsilon_s \). The evolution equation of \( \theta \) approximated to the lowest order reads as
\[ \frac{d\theta}{d \ln a} \approx \sqrt{\epsilon_s} \frac{\sqrt{\rho_\phi}}{HM_p} \left(\frac{\Omega_m}{\Omega_\phi}\right) \approx \frac{\sqrt{\epsilon_s}}{\sqrt{\Omega_\phi}} - 3\theta. \]
(11)
For the moment we assume no early-time dynamics of the quintessence field, i.e.,
\[ \theta_{t=0} = 0. \]
(12)
The solution for (11) and (12) is
\[ \theta \approx \frac{\sqrt{\epsilon_s}}{3} \int_0^x \left(\frac{\Omega_m}{\Omega_\phi}\right)^{1/2} \left(\frac{\Omega_m}{\Omega_\phi}\right)^{1/2} dt. \]
(13)
where
\[ F(\lambda, x) = \int_0^x \sqrt{\frac{t^3}{1 + \lambda t + t^3}} dt. \]
(14)

In HBK’s parameterization, \( \lambda = 0 \) and
\[ F(0, x) = \frac{1}{x^{3/2}} - \frac{\ln [x^{3/2} + \sqrt{1 + x^3}]}{x^{3/2}} \]
(15)
are used instead.

The EOS is then
\[ w \approx -1 + 2\theta^2 \approx -1 + \frac{2\epsilon_s}{3} \left(\frac{\Omega_k}{\Omega_m}\right)^{1/2} \left(\frac{\Omega_m}{\Omega_\phi}\right)^{1/2}. \]
(16)
A seemingly trivial but key observation is that the above formula can be achieved by replacing \( F(0, x) \) in HBK’s one-parameter approximation with the full \( F(\lambda, x) \).

Equation (16) as a generalization of HBK’s one-parameter parametrization is valid for flat potential models where the scalar field is frozen (\( \phi \to 0 \) and \( w \to -1 \)) by a large Hubble friction in the early universe. To cover another popular class of models where the scalar field has a tracking behavior \( (w \sim \text{constant} > -1) \) in the early universe, HBK proposed a three-parameter approximation, \( w(a; \epsilon_s, \epsilon_\phi, \lambda_s) \). The slope parameter \( \epsilon_s \) is defined as \( \epsilon_V \) at matter-dark-energy density equality \( a = a_{eq} \). The running parameter \( \epsilon_\phi \) is defined as \( \epsilon_V \phi_{eq} \) at \( a = a_{eq} \). (For tracking models this quantity is approximately a constant.) The running parameter is defined
in a more sophisticated way as
\[
\zeta_f \equiv \left. \frac{dq}{dy} \right|_{\Omega = \Omega_{eq}} - \left. \frac{dq}{dy} \right|_{\Omega = 0^+},
\]
where
\[
q = \sqrt{f W P_\phi} \frac{H}{\dot{H}}, \quad y = \sqrt{\frac{a}{a_{eq}}}.
\]

HBK showed that \( \zeta_f \) is related to the second \( \phi \)-derivative of \( \ln V \), explicitly so in the \( \epsilon_{\phi, \infty} \to 0 \) limit.

In Friedmann equations, the matter density term is proportional to \( 1 + z^3 \) and the spatial curvature term to \( (1 + z)^2 \). Therefore, a small \( [\Omega_k] \lesssim \Omega_k \) has negligible impact at high redshift. We thus expect most of the \( \Omega_k \) effect to be captured by replacing \( F(0, x) \) in HBK's three-parameter parameterization with \( F(\lambda, x) \). On the other hand, in the limit where \( \frac{\epsilon_{\phi, \infty}}{M^2 H^2} \) is exactly a constant, the slow-roll terms, which alter the tracking solution at low redshift, should vanish. This requirement suggests a replacement of
\[
\sqrt{\epsilon_{\phi}} - \sqrt{2 \epsilon_{\phi, \infty}} \to \sqrt{\epsilon_{\phi}} - \frac{2 \epsilon_{\phi, \infty}}{1 - \Omega_k}.
\]
Finally we arrive at a generalized three-parameter parameterization,
\[
w = -1 + \frac{2}{3} \left( \sqrt{\epsilon_{\phi, \infty}} + \left( \sqrt{\epsilon_{\phi}} - \sqrt{\frac{2 \epsilon_{\phi, \infty}}{1 - \Omega_k}} \right) \right) \times \left[ F \left( \frac{\Omega_k a_{eq}}{\Omega_m}, \frac{a}{a_{eq}} \right) + \zeta F_2 \left( \frac{a}{a_{eq}} \right) \right]^2,
\]
where \( a_{eq} \) can be approximated by
\[
a_{eq} = \left( \frac{\Omega_m}{\Omega_\phi} \right)^{1/7},
\]
and
\[
\delta \equiv \left\{ \sqrt{\epsilon_{\phi, \infty}} + \left[ 0.91 - \frac{0.78 \Omega_m}{1 - \Omega_k} + (0.24 - \frac{0.76 \Omega_m}{1 - \Omega_k}) \zeta_f \right] \right. \times \left( \sqrt{\epsilon_{\phi}} - \sqrt{\frac{2 \epsilon_{\phi, \infty}}{1 - \Omega_k}} \right) \right\}^2 + \left[ \sqrt{\epsilon_{\phi, \infty}} + (0.53 - 0.09 \zeta_f) \left( \sqrt{\epsilon_{\phi}} - \sqrt{\frac{2 \epsilon_{\phi, \infty}}{1 - \Omega_k}} \right) \right]^2
\]
is a fitting formula describing the variation of dark energy density in the slow-roll regime \( a \geq a_{eq} \). The shape correction function
\[
F_2(x) = \sqrt{2 - \ln \left( 1 + \frac{x^3}{3} \right)} - \sqrt{1 + \frac{x^3}{3}} + \ln \left( 1 + \frac{x^3}{3} \right) + \sqrt{1 + \frac{x^3}{3}}
\]
formulates the impact of the running parameter \( \zeta_f \).

A few concrete examples are given in Figure 1 to illustrate the accuracy of the Huang–Bond–Kofman–Miao (HBKM) parameterization, i.e., the generalized HBK parameterization in Equation (20).

3. Data Sets

To compare parameterizations (16) and (20) with the observations, we use the publicly available software CosmoMC (Lewis & Bridle 2002) and replace its default CPL parameterization with Equations (16) and (20). Our parameterization include \( \epsilon_{\phi, \infty}, \zeta_f, \Omega_k \), and the standard six parameters: the baryon density \( \Omega_b h^2 \), the cold dark matter density \( \Omega_c h^2 \), the angular extension of sound horizon on the last scattering surface \( \theta_{\text{MC}} \), the CMB optical depth \( \tau \), the primordial scalar metric fluctuation amplitude \( A_s \), and its spectral index \( n_s \). The Hubble constant \( H_0 \) and present matter density fraction \( \Omega_m \) can be derived from these parameters. Flat priors \( 0 \leq \epsilon_{\phi, \infty} \leq 1.5 \), \( 0 \leq \zeta_f \leq 1 \), and \( -1 \leq \zeta_f \leq 1 \) are used.

The following data sets are used for the Monte Carlo Markov Chain (MCMC) calculations.

3.1. SNe

SNe are known as standard candles at cosmological distance. The joint light curve analysis samples, used in Planck Collaboration et al. (2016c) and this paper, are a joined data set of the Supernova Legacy Survey data (Conley et al. 2011), Sloan Digital Sky Survey (SDSS) SNe data (Kessler et al. 2009), and some low-redshift SNe samples.

3.2. CMB

The CMB is a powerful tool to measure the primordial fluctuations, the matter content, and the geometry of the universe. We use the full Planck 2015 release (TT,TE, EE + lowP + lensing). The details of the Planck mission and its data description can be find in Planck Collaboration et al. (2016a, 2016b, 2016c), and references therein.

3.3. BAO

BAO are standard rulers that measure the geometry of the late-time universe. They are based on known physics and very few assumptions about the universe, and are considered to be very reliable and almost free of systematics. Here we use the recent SDSS data release 12 (Gil-Marín et al. 2016a, 2016b) together with some low-redshift data sets (Beutler et al. 2011; Ross et al. 2015). The full data set covers redshift up to \( z \sim 0.6 \).

4. Results

To demonstrate the impact of different data sets, we do an MCMC calculation for the CMB only, SNe + BAO, and all three together, respectively, for the one-parameter parameterization Equation (16). With CMB only or SNe + BAO, the
Figure 1. Examples of $w(a)$ trajectories for the exponential potential (left panel), power-law potential (middle panel), and negative power-law potential (right panel). For the flat potential models (left and middle panels) the initial condition is given by $\dot{a}_{\text{ini}} = 0$, whereas for the tracking models (right panel) the initial condition is given by its tracking solution. Solid gray lines are the exact $w(a)$ solutions. Dotted red lines represent the HBKM parameterization, i.e., Equation (20).

Table 1

| Parameter Description | Three-parameter CMB + SNe + BAO | One-parameter CMB + weak $H_0$ prior | SNe + BAO + weak $H_0$ prior |
|-----------------------|---------------------------------|-------------------------------------|-------------------------------|
| $\Omega_b h^2$        | 0.02223$^{+0.00017}_{-0.00016}$ | 0.02224$^{+0.00016}_{-0.00016}$    | unconstrained                 |
| $\Omega_c h^2$        | 0.1194$^{+0.016}_{-0.015}$      | 0.1195$^{+0.016}_{-0.015}$         | unconstrained                 |
| 1000$\theta_{\text{MC}}$ | 1.04082$^{+0.00033}_{-0.00034}$ | 1.04082$^{+0.00035}_{-0.00034}$   | 1.25$^{+0.10}_{-0.12}$        |
| $\tau$               | 0.0751$^{+0.014}_{-0.013}$      | 0.067$^{+0.013}_{-0.013}$          | 0.069$^{+0.016}_{-0.016}$     |
| $\Omega_k$            | 0.0035$^{+0.0028}_{-0.0029}$    | 0.0022$^{+0.0022}_{-0.0021}$       | 0.003$^{+0.0053}_{-0.0053}$   |
| $\epsilon_r$         | 0.00$^{+0.16}_{-0.36}$          | 0.00$^{+0.29}_{-0.56}$             | 0.00$^{+0.55}_{-1.12}$        |
| $\epsilon_{\phi,\text{MC}}$ | 0.00$^{+0.28}_{-0.67}$        | 0.00$^{+0.28}_{-0.67}$             | 0.00$^{+0.37}_{-0.63}$        |
| $\zeta'$              | unconstrained                   | unconstrained                      | unconstrained                 |
| ln$(10^{10}A_s)$      | 3.079$^{+0.027}_{-0.025}$       | 3.069$^{+0.023}_{-0.024}$          | 3.07$^{+0.03}_{-0.03}$        |
| $n_s$                 | 0.965$^{+0.005}_{-0.005}$       | 0.965$^{+0.005}_{-0.005}$          | 0.965$^{+0.005}_{-0.005}$     |
| $H_0$                 | 67.3$^{+0.9}_{-0.8}$            | 67.3$^{+0.8}_{-0.9}$               | 66.7$^{+0.7}_{-0.7}$          |
| $\Omega_m$            | 0.314$^{+0.009}_{-0.008}$       | 0.314$^{+0.009}_{-0.008}$          | 0.321$^{+0.025}_{-0.024}$     |

Note. For $\epsilon_r$ and $\epsilon_{\phi,\text{MC}}$ that are bound from below by the theory, a 68.3% confidence level upper limit and a 95.4% confidence level upper limit are shown. A dash indicates an unused parameter, while unconstrained means that the parameter is not constrained by the data (posterior $\approx$ prior).

The degeneracy between geometrical parameters is very strong. We use a very weak Gaussian prior $H_0 = 70.6 \pm 3.3$ (see Efstathiou 2014) to avoid the MCMC chains exploring nonphysical regions. With all three data sets together (CMB + SNe + BAO), the degeneracy is not strong and we do not use the $H_0$ prior.

In Table 1 we show the results of the abovementioned three runs for the one-parameter parameterization and of a CMB + SNe + BAO run for the three-parameter parameterization Equation (20). The combination of SNe + BAO + weak $H_0$ prior only constrains background geometrics at low redshift. Thus, in this case $\Omega_b h^2$ is perfectly degenerate with $\Omega_c h^2$ and only their sum, $\Omega_m h^2$, can be constrained.

In the three-parameter parameterization case with CMB + SNe + BAO, the posterior of $H_0$ is close to a Gaussian distribution of $H_0 = 67.25 \pm 0.84 \text{ km s}^{-1} \text{ Mpc}^{-1}$, which is in 3.3$\sigma$ tension with $H_0 = 73.48 \pm 1.66 \text{ km s}^{-1} \text{ Mpc}^{-1}$ from Riess et al. (2018b). In the one-parameter case, the deviation of the $H_0$ posterior from a Gaussian distribution ($67.30 \pm 0.86$ constructed from mean and standard deviation) is more significant. While using the full posterior distribution constructed from the MCMC calculation, we find the tension slightly drops to 3.2$\sigma$, which is not a significant change of the story. In the left panel of Figure 2, we visualize the $H_0$ tension by plotting the CMB + SNe + BAO posterior, for both one-parameter and three-parameter parameterizations, against the Riess et al. (2018b) result.

In the three-parameter parameterization $\epsilon_r$ is anomalously constrained better than that in the one-parameter case. Similar
results have been shown in Huang et al. (2011) and Planck Collaboration et al. (2016d), and were understood as an effect due to the anticorrelation between $\varepsilon_s$ and $\varepsilon_{\phi,0}$ shown in the left panel of Figure 3.

In the right panel of Figure 3 we show the impact of different data sets on the constraints on $\varepsilon_s$ and $H_0$ for the one-parameter parametrization. We find $\varepsilon_s$ and $H_0$ to be slightly anticorrelated. Thus, compared to the $\Lambda$CDM $\varepsilon_s = 0$ case, the relaxation of $\varepsilon_s$ may even worsen the $H_0$ tension.

Although the combined low-redshift data SNe $+$ BAO $+$ weak $H_0$ prior slightly prefers a negative $\Omega_k$ ($\sim 1.7\sigma$ level), the CMB data drives $\Omega_k$ back toward zero and gives much tighter bounds. We thus have justified the flatness assumption used in Huang et al. (2011) with a fully self-consistent calculation.

5. Conclusion and Discussion

The $H_0$ tension between Planck Collaboration et al. (2016c; assuming $\Lambda$CDM) and Riess et al. (2018b), if taken at face value, suggests evidence for new physics beyond $\Lambda$CDM at more than a $99\%$ confidence level. We explored one of the simplest alternatives to $\Lambda$CDM: a slowly or moderately rolling quintessence field with a smooth potential in a non-flat FRW universe. We generalized the model-independent HKB parameterization to a non-flat FRW metric. Using the latest CMB, SNe, and BAO data, we find that the four additional degrees of freedom ($\varepsilon_s$, $\varepsilon_{\phi,0}$, $\zeta$, and the spatial curvature $\Omega_k$) do not ease, if not worsen, the tension between local and high-redshift measurements of the Hubble constant.

There are many other interesting attempts to explore the $H_0$ tension with alternative cosmologies. Among those some are phenomenological models, such as free lensing amplitude (Grandis et al. 2016), late-time spatial curvature (Bolejko 2018), the $\Omega\Lambda$CDM model (Khosravi et al. 2017), and XCDM cosmology (Ooba et al. 2018), and the others are self-consistent models, such as Galileon gravity (Renk et al. 2017) and hot axions (D’Eramo et al. 2018). Some phenomenological models, despite their lack of physical consistency, can mitigate the $H_0$ tension.

The $H_0$ tension may also be subject to some unknown observational biases and systematics. Efstathiou (2014) argued that there might be uncounted systematic effects in Hubble Space Telescope (HST) Cepheid calibration. Thanks to the recent Gaia data release 2 (Gaia Collaboration et al. 2018), Riess et al. (2018a) was able to do a more accurate calibration and found no significant migration of Cepheid standards. Meanwhile, many other efforts have been made to obtain an unbiased $H_0$ from existing $H_0$ and $H(z)$ measurements. Chen & Ratra (2011) applied median statistics, which are supposed to be less sensitive to outliers with unknown systematics, on 553 $H_0$ measurements and obtained $68 \pm 5.5\, \text{km\,s^{-1}\,Mpc^{-1}}$. More recently, Yu et al. (2018) used a Gaussian process method to determine a continuous $H(z)$ function, and found a similar result of $H_0 = 67 \pm 4\, \text{km\,s^{-1}\,Mpc^{-1}}$. This result was soon updated to $67.06 \pm 1.68\, \text{km\,s^{-1}\,Mpc^{-1}}$ by Gómez-Valent & Amendola (2018), who added SNe into their analysis. These measurements, independent of HST and Planck constraints, seem to favor a lower $H_0$ value that is more consistent with the Planck result. This conclusion was further embraced by Zhang et al. (2018), who combined BAO measurements with a tomography Alcock–Paczynski method, and Addison et al. (2018), who did a more detailed study by combining galaxy BAO data with a variety of data that are independent of HST and Planck measurements.

Orcid iDs

Zhiqi Huang @ https://orcid.org/0000-0002-1506-1063

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