Theoretical bases of combined nonlinear optical demodulator-amplifier for broadband microwave photonics system

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Abstract. Our demodulator is based on the use of a strong light pump wave along with a signal modulated light wave. The demodulator is linear in terms of the modulation signal, which makes it possible to obtain a demodulated signal without harmonic distortion. The modulator conversion efficiency and signal gain increase with increasing microwave baseband.

1. Introduction
Microwave photonics is based on the use of laser radiation, modulated by a microwave signal, in the tasks of receiving, transmitting and processing information [1, 2]. The transition to microwave photonics provides noise immunity, information security, broadband, reduced noise distortions during signal processing, increased frequency selectivity, compact hardware implementation. This allows the creation of devices with parameters unattainable for traditional microwave electronics. For ultrafast microwave photonic systems, high-speed light modulators and demodulators are critical. The efficiency of such existing devices drops sharply after exceeding their critical microwave frequency, which for the fastest of them is about 100 GHz [3, 4, 5].

We have proposed a nonlinear optical demodulator, the efficiency of which increases with an increase in the signal modulation frequency.

2. Proposed Nonlinear Optical Demodulation Scheme
2.1. Desired properties of a nonlinear optical demodulator
We would the nonlinear optical demodulator have the following properties:
• it would be linear to the information signal,
• the hardware components of the modulation path could be used in its design,
• it would also be an amplifier at the same time.

2.2. Technological implementation: nonlinear optical planar waveguide inside the strip.
To implement such an optical demodulator, we propose to use the same design as for a nonlinear optical phase light modulator: a plane nonlinear optical waveguide inside a stripline.
Figure 1: Planar nonlinear optical waveguide with microstrip line.

Structurally, such a demodulator is a planar nonlinear single-mode optical waveguide inside a microstrip line. (see Figure 1).

Both a modulated signal light wave and a much more intense light pump wave, which frequency coincides with the carrier frequency of the signal wave, are fed to the input of the waveguide in the direction of axis $z$. A microwave wave arises as a result of the nonlinear optical interaction of these waves. It propagates along the stripline in the same direction. Because the intensity of the pump wave is much higher than the intensity of the signal wave (so the amplitude of the pump wave can be considered constant), the amplitude of the microwave wave turns out to be linearly related to the amplitude of the modulated light wave. Therefore there are no nonlinear distortions in such a demodulation scheme.

3. Theoretical description of nonlinear demodulator

3.1. Equations for dimensionless wave amplitudes

Due to the smallness of nonlinearity of the planar optical waveguide, wave amplitudes vary slightly at distances of the order of their wavelengths. Therefore, the process of nonlinear optical demodulation may describe in the slowly varied amplitudes approximation.

We assume that all waves have linear polarization along the $x$-axis perpendicular to the planar waveguide. In this case, the nonlinear optical conversion in the optical waveguide core depends on the largest component $\chi^{(2)}_{xx} \equiv \chi^{(2)}$ of the second-order nonlinear susceptibility tensor. For the case of harmonic amplitude modulation of a signal wave, the following differential couple-waves equations describe the demodulation process:

$$\frac{\partial \bar{a}}{\partial \tau} + iq\bar{a} = i [C_+ a^* a_+ + C_- a a_+]$$

$$\frac{\partial a_+}{\partial \tau} = ic_+ a\bar{a},$$

$$\frac{\partial a_-^*}{\partial \tau} = -ic_-^* a^*\bar{a},$$

where $a$, $a_-$ ($a_+$), and $\bar{a}$ are dimensionless amplitudes of the pump optical wave, the Stokes (anti-Stokes) spectral component of the modulated optical wave, and the microwave wave, respectively;

$$C_\pm = \sqrt{\frac{(\omega \pm \Omega)}{\omega} \frac{n(\omega)}{n(\omega \pm \Omega)} \chi^{(2)}(\Omega; \mp \omega, \Omega \pm \omega) \chi^{(2)}(0; \omega, -\omega)},$$

$$c_\pm = \sqrt{\frac{(\omega \pm \Omega)}{\omega} \frac{n(\omega)}{n(\omega \pm \Omega)} \chi^{(2)}(\omega \pm \Omega; \omega, \pm \Omega) \chi^{(2)}(0; \omega, -\omega)},$$

(1) (2) (3) (4) (5)
\( q = \left( \frac{V_p - v_g}{v_g} \right) \times L_{NL}, \)  \( \tag{6} \)

\[
L_{NL}^{-1} = 2\pi|\chi^{(2)}(0; \omega, -\omega)| \frac{|E|}{|a|} \sqrt{\frac{k}{n(\omega) n(\Omega)}}, \]  \( \tag{7} \)

\( L_{NL} \) is non-linear interaction length, \( \tau = z/L_{NL}; \) \( \omega, k \) and \( n(\omega) \) are circular frequency, wave number in vacuum and refractive index of the pump wave correspondingly, and \( \Omega, \kappa \) and \( n(\Omega) \) are corresponding quantities for the microwave wave; \( E \) is the electric field amplitude of the pump wave; \( V_p \) and \( v_g \) are phase velocity of the microwave wave and group velocity of the light one, correspondingly. The squares of the moduli of the dimensionless amplitudes of the strong reference \((a)\), modulated \((a_\pm)\) and microwave \((\bar{a})\) waves are equal to the flux fractions of the corresponding quanta from the total quanta flux \( F_0 \):

\[
|a|^2 = \frac{c n(\omega)}{8\pi\hbar\omega} \frac{|E|^2}{F_0}, \]  \( \tag{8} \)

\[
|a_\pm|^2 = \frac{c n(\omega \pm \Omega)}{8\pi\hbar(\omega \pm \Omega)} \frac{|E_\pm|^2}{F_0}, \]  \( \tag{9} \)

\[
|\bar{a}|^2 = \frac{c n(\Omega)}{8\pi\hbar\Omega} \frac{|E|^2}{F_0}. \]  \( \tag{10} \)

Moduli of dimensionless amplitudes are satisfy the follows relations:

\[
|a_\pm|^2, |\bar{a}|^2 \ll |a|^2 \approx 1. \]  \( \tag{11} \)

According to the first equation (1) of the system (1) – (3), the change in the amplitude of the microwave wave is determined by two processes described by two terms in square brackets on the right side of this equation. The first term describes annihilation of the anti-Stokes light quantum with the frequency of \( \omega + \Omega \) and simultaneous creation of the light quantum with the frequency of \( \omega \) and the microwave quantum with the frequency of \( \Omega \). The second term describes annihilation of the light quantum with the frequency of \( \omega \) and the simultaneous creation of the anti-Stokes light quantum with the frequency \( \omega - \Omega \) and the microwave quantum with the frequency \( \Omega \) (see Figure 2).

From the system (1) – (3), we obtain a closed equation for the amplitude of the microwave wave by eliminating the amplitudes of the Stokes and anti-Stokes optical waves from the first equation (1) of the system by means of the of the second (2) and third (3) equations:

\[
\frac{\partial^2 \bar{a}(\tau)}{\partial \tau^2} + i q \frac{\partial \bar{a}(\tau)}{\partial \tau} + g \bar{a}(\tau) = 0, \]  \( \tag{12} \)

where

\[
g \equiv (C_+c_+ - C_-c_-)|a|^2. \]  \( \tag{13} \)

The boundary conditions for the equation (12) are

\[
\bar{a}(0) = 0, \]

\[
\frac{\partial \bar{a}(\tau)}{\partial \tau} \bigg|_{\tau=0} = i \left[ C_+a^*a_+(0) + C_-a^*a_- (0) \right]. \]

There are possible three qualitatively different operation modes of the proposed nonlinear optical demodulator. Just the sign of the combination \((g + \frac{q^2}{4})\) defines the mode.
Figure 2: Energy diagrams and spectral energy transfers illustrating nonlinear optical

demodulation. (a) A microwave quantum is created both during the decay of an anti-Stokes

g + \frac{q^2}{4} > 0

\text{Under condition } g + \frac{q^2}{4} > 0, \text{ the solution to equation (12) can be represented as}

\bar{a}(\tau) = iB \frac{\sin \left( \sqrt{\frac{g + \frac{q^2}{4}}{\tau}} \right)}{\sqrt{\frac{g + \frac{q^2}{4}}{\tau}}} \exp \left( -\frac{i q}{2} \right), \quad (14)

\text{where}

B \equiv \left[ C_+ a^* a_+ (0) + C_- a a^* (0) \right]. \quad (15)

\text{It is convenient to introduce the dimensionless nonlinear coherence length } \tau_c:

\tau_c \equiv \frac{\pi}{2} \frac{1}{\sqrt{|g + \frac{q^2}{4}|}}.

\text{In terms of the nonlinear coherence length } \tau_c, \text{ expression (14) for } \bar{a}(\tau) \text{ takes the form}

\bar{a}(\tau) = \frac{2}{\pi} B \tau_c \sin \left( \frac{\pi \tau}{2 \tau_c} \right) \exp \left( -\frac{i q}{2} \right).

\text{The microwave amplitude modulus } |\bar{a}(\tau)| \text{ is maximal at } \tau = \tau_c \text{ and is equal to } 2B \tau_c / \pi. \text{ In the case of } \tau < \tau_c, \text{ we have } |\bar{a}(\tau)| \approx |B| \tau.

\text{The quantum efficiency } \eta \text{ of the nonlinear optical modulator is the ratio of the flux of quanta of the microwave wave at the output of the modulator to the sum of quanta fluxes of the Stokes and anti-Stokes components of the modulated wave at the input of the modulator:}

\eta = \frac{|\bar{a}(\tau)|^2}{|a_+(0)|^2 + |a_-(0)|^2} = \frac{|C_+ a^* a_+ (0) + C_- a a^* (0)|^2 \sin^2 \left( \sqrt{\frac{g + \frac{q^2}{4}}{\tau}} \right)}{\left( g + \frac{q^2}{4} \right)} \frac{\left( g + \frac{q^2}{4} \right)}{\left( g + \frac{q^2}{4} \right)}. \quad (16)
In the case of $\tau \ll \tau_c$, it is approximately equal to

$$\eta \approx \frac{|C_+ a^* a_+ (0) + C_- a a_-^* (0)|^2}{|a_+ (0)|^2 + |a_- (0)|^2} \tau^2.$$  \hspace{1cm} (17)

### 3.3. Parametric gain mode ($g + \frac{q^2}{4} < 0$)

In this case, the expression for the dimensionless amplitude of the microwave wave takes the form

$$\bar{a} (\tau) = iB \sinh \left( \frac{\sqrt{|g + \frac{q^2}{4}| \tau}}{|g + \frac{q^2}{4}|} \right) \exp \left( -i \frac{q}{2} \tau \right).$$

One can see that the amplitude of the microwave wave increases exponentially at large $\sqrt{|g + \frac{q^2}{4}| \tau}$ in increasing the modulator length $\tau$.

The quantum efficiency $\eta$ of the nonlinear optical modulator for this case is as follows:

$$\eta = \frac{|C_+ a^* a_+ (0) + C_- a a_-^* (0)|^2 \sinh^2 \left( \frac{\sqrt{|g + \frac{q^2}{4}| \tau}}{|g + \frac{q^2}{4}|} \right)}{|g + \frac{q^2}{4}|}. $$

It also exponentially increases at large $\sqrt{|g + \frac{q^2}{4}| \tau}$ in increasing the modulator length $\tau$. In the opposite case, when $\sqrt{|g + \frac{q^2}{4}| \tau} \ll 1$, it takes the already known expression (17).

### 3.4. Linear gain mode ($g + \frac{q^2}{4} = 0$)

The mode is intermediate between the two modes discussed above. In this case, $\tau_c = \infty$, and the amplitude of the microwave wave increases linearly in increasing the length $\tau$ of the modulator:

$$\bar{a} (\tau) = iB \tau \exp \left( -i \frac{q}{2} \tau \right).$$

The quantum efficiency $\eta$, on the other hand, grows quadratically in increasing the length $\tau$ of the modulator:

$$\eta = \frac{|C_+ a^* a_+ (0) + C_- a a_-^* (0)|^2}{|a_+ (0)|^2 + |a_- (0)|^2} \tau^2.$$  

Both regimes considered above tend to this regime in the limit $\sqrt{|g + \frac{q^2}{4}| \tau} \to 0$.

### 4. Estimations and discussion

So, we have shown that the dynamics of the nonlinear optical demodulator and its mode are determined by the sign and value of $g$. In the absence of frequency dispersion $g = 0$, as follows from the definition of $g$ (13) and expressions (4) and (5). Let us take into account the frequency dispersion in the first approximation in $\Omega/\omega \ll 1$:
Let us present our estimates of the demodulator parameters for a nonlinear optical planar waveguide with the section of 2 \( \mu \)m by 1 \( \mu \)m, \( n(\omega) = 1.6 \), and the length \( L=2 \) cm [6]. Let us take the refractive index of the microwave wave equal to 1.5, the power of the pump wave with \( \lambda = 1.3 \mu \) equal to 2 mW, the microwave frequency equal to 100 GHz, and the nonlinear susceptibility equal to \( 2 \times 10^2 \text{pm/V} \) [7, 8]. Let us also assume that the wave velocities are equal (\( V_p = v_g \)). For these conditions, the nonlinear length \( L_{NL} \) and coherent length \( L_c = \tau_c L_{NL} \) are equal to 9.9 cm and 3.1 \( \times 10^2 \) cm, respectively. Since \( L/L_c \ll 1 \), we estimate the quantum efficiency by expression (17). The fraction in this expression is approximately equal to 1, since \( |a| \approx 1 \) according to (11) and \( C_{\pm} \approx 1 \) in the case of the weak dispersion. Therefore, the quantum efficiency is equal to \( \eta \approx (L/L_{NL})^2 \approx 1/25 \). The power conversion factor differs from this expression by the factor \( \Omega/\omega \approx 4.3 \times 10^{-4} \).

The above estimate of the derivative \( \partial \chi^{(2)} / \partial \omega \) is valid for the lack of spectral features in the vicinity of frequency \( \omega \). Otherwise, this estimate may differ from the real derivative by several times and even significantly more.

The attractive parametric amplification mode (see subsection 3.3) is possible only with a negative \( g \). In turn, this is possible only with negative dispersion of the nonlinear susceptibility, according to expressions (18), (19).

Note that even in the case \( L/L_c \ll 1 \), corresponding to the linear regime, the quantum efficiency can be greater than 1 when \( L/L_{NL} > 1 \) (see (17)). The reason for this is as follows. First, both the anti-Stokes quanta and quanta of pumping create the microwave photons (see Figure 2). Second, the number of quanta of pumping is much more than the number of the anti-Stokes quanta.

Acknowledgments
The study was supported by the Ministry of Education and Science of the Russian Federation (project ‘II.10.2.1. Photonics of micro- and nanostructured media’ state registration No. AAAA-A17-117060810014-9).

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ISSN 8756-6990