Measurement of the total and leptonic decay widths of the $J/\psi$ resonance with an energy scan method at BESIII
39 Nankai University, Tianjin 300071, People's Republic of China
40 National Centre for Nuclear Research, Warsaw 02-093, Poland
41 North China Electric Power University, Beijing 102206, People's Republic of China
42 Peking University, Beijing 100871, People's Republic of China
43 Qufu Normal University, Qufu 273165, People's Republic of China
44 Shandong Normal University, Jinan 250014, People's Republic of China
45 Shandong University, Jinan 250100, People's Republic of China
46 Shanghai Jiao Tong University, Shanghai 200240, People's Republic of China
47 Shaxi Normal University, Linfen 041004, People's Republic of China
48 Shandong University, Jinan 250006, People's Republic of China
49 Sichuan University, Chengdu 610064, People's Republic of China
50 Soochow University, Suzhou 215006, People's Republic of China
51 South China Normal University, Guangzhou 510006, People's Republic of China
52 Southeast University, Nanjing 211100, People's Republic of China
53 State Key Laboratory of Particle Detection and Electronics, Beijing 100049, Hefei 230026, People's Republic of China
54 Sun Yat-Sen University, Guangzhou 510275, People's Republic of China
55 Suranaree University of Technology, University Avenue 111, Nakon Ratchasima 30000, Thailand
56 Tsinghua University, Beijing 100084, People's Republic of China
57 Turkish Accelerator Center Particle Factory Group, (A)Istinye University, 34010, Istanbul, Turkey; (B)Near East University, Nicosia, North Cyprus, Mersin 10, Turkey
58 University of Chinese Academy of Sciences, Beijing 100049, People's Republic of China
59 University of Groningen, NL-9747 AA Groningen, The Netherlands
60 University of Hawaii, Honolulu, Hawaii 96822, USA
61 University of Jinan, Jinan 250022, People's Republic of China
62 University of Manchester, Oxford Road, Manchester, M13 9PL, United Kingdom
63 University of Muenster, Wilhelm-Klemm-Strasse 9, 48149 Muenster, Germany
64 University of Oxford, Keble Road, Oxford OX13RH, United Kingdom
65 University of Science and Technology Liaoning, Anshan 114051, People's Republic of China
66 University of Science and Technology of China, Hefei 230026, People's Republic of China
67 University of South China, Hengyang 421001, People's Republic of China
68 University of the Punjab, Lahore-54590, Pakistan
69 University of Turin and INFN, (A)University of Turin, I-10125, Turin, Italy; (B)University of Eastern Piedmont, I-15121, Alessandria, Italy; (C)INFN, I-10125, Turin, Italy
70 Uppsala University, Box 516, SE-75120 Uppsala, Sweden
71 Wuhan University, Wuhan 430072, People's Republic of China
72 Xiangyang Normal University, Xiangyang 441000, People's Republic of China
73 Yunnan University, Kunming 650500, People's Republic of China
74 Zhejiang University, Hangzhou 310027, People's Republic of China
75 Zhengzhou University, Zhengzhou 450001, People's Republic of China
76 Also at the Moscow Institute of Physics and Technology, Moscow 141700, Russia
77 Also at the Novosibirsk State University, Novosibirsk, 630090, Russia
78 Also at the NRC "Kurchatov Institute", PNPI, 188300, Gatchina, Russia
79 Also at Goethe University Frankfurt, 60323 Frankfurt am Main, Germany
80 Also at Key Laboratory for Particle Physics, Astrophysics and Cosmology, Ministry of Education; Shanghai Key Laboratory for Particle Physics and Cosmology; Institute of Nuclear and Particle Physics, Shanghai 200240, People's Republic of China
81 Also at Key Laboratory of Nuclear Physics and Ion-beam Application (MOE) and Institute of Modern Physics, Fudan University, Shanghai 200443, People's Republic of China
82 Also at State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, People's Republic of China
83 Also at School of Physics and Electronics, Hunan University, Changsha 410082, China
84 Also at Guangdong Provincial Key Laboratory of Nuclear Science, Institute of Quantum Matter, South China Normal University, Guangzhou 510006, China
85 Also at Frontiers Science Center for Rare Isotopes, Lanzhou University, Lanzhou 730000, People's Republic of China
86 Also at Lanzhou Center for Theoretical Physics, Lanzhou University, Lanzhou 730000, People's Republic of China
87 Also at the Department of Mathematical Sciences, IBA, Karachi, Pakistan
Abstract

Using $e^+ e^-$ annihilation data sets collected with the BESIII detector, we measure the cross sections of the processes $e^+ e^- \rightarrow e^+ e^-$ and $e^+ e^- \rightarrow \mu^+ \mu^-$ at fifteen center-of-mass energy points in the vicinity of the $J/\psi$ resonance. By a simultaneous fit to the measured, center-of-mass energy dependent cross sections of the two processes, the combined quantities $\Gamma_{ee}\Gamma_{ee}/\Gamma_{tot}$ and $\Gamma_{ee}\Gamma_{\mu\mu}/\Gamma_{tot}$ are determined to be $(0.346 \pm 0.009)$ and $(0.335 \pm 0.006)$ keV, respectively, where $\Gamma_{ee}$, $\Gamma_{\mu\mu}$, and $\Gamma_{tot}$ are the electronic, muonic, and total decay widths of the $J/\psi$ resonance, respectively. Using the resultant $\Gamma_{ee}\Gamma_{ee}/\Gamma_{tot}$ and $\Gamma_{ee}\Gamma_{\mu\mu}/\Gamma_{tot}$, the ratio $\Gamma_{ee}/\Gamma_{\mu\mu}$ is calculated to be $1.031 \pm 0.015$, which is consistent with the expectation of lepton universality within about two standard deviations. Assuming lepton universality and using the branching fraction of the $J/\psi$ leptonic decay measured by BESIII in 2013, $\Gamma_{tot}$ and $\Gamma_{ll}$ are determined to be $(93.0 \pm 2.1)$ and $(5.56 \pm 0.11)$ keV, respectively, where $\Gamma_{ll}$ is the average leptonic decay width of the $J/\psi$ resonance.

Keywords: $J/\psi$, decay width, lepton universality, energy scan, BESIII

1. Introduction

The total and electronic decay widths $\Gamma_{tot}$ and $\Gamma_{ee}$ of the $J/\psi$ resonance, present in the Breit-Wigner formulae for all the decay modes of $J/\psi$ produced in $e^+ e^-$ collisions [1], are among its most important parameters. Theoretically, these decay widths, reflecting $J/\psi$ internal interactions, are predicted by various potential models [2–6] and lattice quantum chromodynamics [7]. In 2014, BESIII measured the $\Gamma_{ee}/\Gamma_{\mu\mu}$ ratio [9] and used it to test the lepton universality assumption [8]. Based on the assumption, the ratio is derived to be [9]

$$\Gamma_{ee}/\Gamma_{\mu\mu} = \frac{\beta_e (3 - \beta_e^2)}{\beta_\mu (3 - \beta_\mu^2)},$$  \hspace{1cm} (1)

where $m_e$, $m_\mu$, and $M$ are the masses of electron, muon, and the $J/\psi$ resonance, respectively. The values of $m_e$, $m_\mu$, and $M$ from the Particle Data Group (PDG) [1], $\Gamma_{ee}/\Gamma_{\mu\mu}$ is calculated to be $1.00000814211(6)$, which has a deviation from 1 that is far less than the experimental precision at present. Thus, any observed, significant deviation of $\Gamma_{ee}/\Gamma_{\mu\mu}$ from 1 will be a hint of physics beyond the Standard Model [10].

Since the discovery of the $J/\psi$ resonance in 1974 [11, 12], its decay widths have been measured by many experiments [13–17]. The precision of the measurements has been improved significantly in the past two decades. In 2004 and 2006, the $J/\psi$ decay widths have been measured by studying the $J/\psi$ samples produced in the initial state radiation (ISR) return process $e^+ e^- \rightarrow \gamma^{ISR}\mu^+ \mu^-$ collected at the $\Upsilon(4S)$ and $\psi(3770)$ peaks by BaBar [18] and CLEO [19], respectively. In 2010, KEDR improved the measurement precision by performing an energy scan (ES) around the $J/\psi$ peak and studying the $J/\psi$ production in the processes $e^+ e^- \rightarrow e^+ e^-$ and $e^+ e^- \rightarrow \mu^+ \mu^-$ [20]. In 2018 and 2020, KEDR presented new results with the $J/\psi$ production in the processes $e^+ e^- \rightarrow e^+ e^-$ and $e^+ e^- \rightarrow$ inclusive hadrons [21].

Operating in the $\tau$-charm energy region, the high luminosity of the BEPCII collider [22] and the excellent performance of the BESIII detector [23] offer us a good opportunity for the precision measurements of the $J/\psi$ decay widths. In 2016, BESIII measured the $J/\psi$ decay widths by applying the ISR return technique to the data sample collected at the $\psi(3770)$ peak, and obtained a result with improved precision [24]. In this Letter, we report a new precision measurement of the $J/\psi$ decay widths with the ES method, confirming and complementing the above measurement.

Since the $J/\psi$ resonance contributes to the vacuum polarization in the time-like region, the cross sections of the processes $e^+ e^- \rightarrow e^+ e^-$ and $e^+ e^- \rightarrow \mu^+ \mu^-$ are functions of the $J/\psi$ decay widths [25, 26]. Specifically, the cross section ($\sigma_0$) of each process with respect to the center-of-mass (CM) energy ($W_0$) can be written as

$$\sigma_0(W_0) = \sigma_0^C(W_0) + \sigma_0^R(W_0) + \sigma_0^I(W_0),$$  \hspace{1cm} (3)

where $\sigma_0^C$, $\sigma_0^R$, and $\sigma_0^I$ are the continuum, resonance and interference terms, respectively. The formula still
holds after considering the ISR effect, and we take $\sigma_0^C$, $\sigma_0^R$, $\sigma_0^R$ and $\sigma_0^C$ here as the quantities with ISR considered. Unlike the term $\sigma_0^C$, the terms $\sigma_0^R$ and $\sigma_0^C$ depend on the $J/\psi$ decay widths, and their analytic forms are derived in Ref. [27] using the structure function method [28, 29].

In Eq. (3), $\sigma_0^R$ is the primary term related to the $J/\psi$ decay widths, and its major subterm is proportional to $\Gamma_{ee} \Gamma_{ee}/\Gamma_{tot}$ and $\Gamma_{ee} \Gamma_{\mu\mu}/\Gamma_{tot}$ for the processes $e^+e^- \rightarrow e^+e^-$ and $e^+e^- \rightarrow \mu^+\mu^-$, respectively [27]. Therefore, we can determine $\Gamma_{ee} \Gamma_{ee}/\Gamma_{tot}$ and $\Gamma_{ee} \Gamma_{\mu\mu}/\Gamma_{tot}$ by fitting to the measured, CM energy dependent cross sections of the two processes. Then, $\Gamma_{ee} \Gamma_{ee}/\Gamma_{tot}$ can be evaluated as the ratio of $\Gamma_{ee} \Gamma_{ee}/\Gamma_{tot}$ to $\Gamma_{ee} \Gamma_{\mu\mu}/\Gamma_{tot}$ by fitting to the measured, CM energy dependent cross sections of the two processes. Combined with the branching fraction of the $J/\psi$ leptonic decay measured by BESIII in 2013 [30], the total and leptonic $J/\psi$ decay widths can be obtained from $\Gamma_{ee} \Gamma_{ee}/\Gamma_{tot}$ and $\Gamma_{ee} \Gamma_{\mu\mu}/\Gamma_{tot}$ as well.

2. Experimental facilities and data sets

The data used in this work were collected with the BESIII detector [23], which operates at the south crossing point of the BEPCII collider [22]. BEPCII is a superconducting solenoid magnet with a nominal magnetic field of 1 T (0.9 T in 2012) parallel to the beam direction; (5) a muon chamber (MDC) measuring the momenta of charged particles; (4) a superconducting solenoid magnet providing a nominal magnetic field of 1 T (0.9 T in 2012) parallel to the beam direction; (5) a muon chamber system made of resistive plate chambers with position resolution about 2 cm.

In addition, a beam energy measurement system (BEMS), located at the north crossing point of the BEPCII storage rings, is used to determine the BEPCII beam energies by measuring the energies of Compton back-scattered photons [31].

In 2012, an ES experiment was performed at fifteen CM energy points in the vicinity of the $J/\psi$ resonance. The measured CM energies and integrated luminosities are summarized in Table 1. The CM energies are

| Prop. $\sqrt{s}$ (MeV) | Calib. BEMS $\sqrt{s}$ (MeV) | Int. $L$ (pb$^{-1}$) |
|------------------------|-------------------------------|---------------------|
| 3050.0                 | 3049.642±0.026±0.033          | 14.919±0.029±0.158  |
| 3060.0                 | 3058.693±0.028±0.033          | 15.060±0.029±0.158  |
| 3083.0                 | 3082.496±0.023±0.033          | 4.769±0.017±0.052   |
| 3090.0                 | 3088.854±0.022±0.033          | 15.558±0.030±0.162  |
| 3093.0                 | 3091.760±0.025±0.033          | 14.910±0.030±0.157  |
| 3094.3                 | 3094.697±0.084±0.033          | 2.143±0.011±0.023   |
| 3095.2                 | 3095.430±0.081±0.033          | 1.816±0.010±0.019   |
| 3095.8                 | 3095.826±0.075±0.033          | 2.135±0.011±0.023   |
| 3096.9                 | 3097.213±0.076±0.033          | 2.069±0.011±0.024   |
| 3098.2                 | 3098.340±0.075±0.033          | 2.203±0.011±0.023   |
| 3099.0                 | 3099.042±0.093±0.033          | 0.756±0.007±0.008   |
| 3101.5                 | 3101.359±0.106±0.033          | 1.612±0.010±0.018   |
| 3105.5                 | 3105.580±0.090±0.033          | 2.106±0.011±0.022   |
| 3112.0                 | 3112.051±0.093±0.033          | 1.720±0.010±0.019   |
| 3120.0                 | 3119.878±0.115±0.033          | 1.264±0.009±0.013   |
measured by the BEMS and calibrated according to the 
$J/\psi$ mass value given by the PDG [1]. As a conse-
quence, the $J/\psi$ mass can not be determined in this
work. The CM energy calibration process fits to the
$J/\psi$ lineshapes in the $e^+e^-$ and $\mu^+\mu^-$ final states si-
multaneously to their preliminary measured, CM en-
dependent cross sections. Adding the uncertainties of the $J/\psi$ masses from the fit and from the PDG in qua-
drature gives a total calibration uncertainty of 0.033
MeV, which is comparable with the systematic uncer-
tainty (0.043 MeV) of the calibration (via the inclu-
sive hadronic decay mode) to the small $J/\psi$ scan data
used for the $\tau$ mass measurement [32]. The correspond-
ing integrated luminosities are measured offline with $e^+e^- \rightarrow \gamma\gamma$ events [33].

To determine the signal detection efficiencies, Monte Carlo (MC) simulated events of the processes
$e^+e^- \rightarrow e^+e^-$ and $e^+e^- \rightarrow \mu^+\mu^-$ in the polar
angle ranges of $34^\circ$-$146^\circ$ and $0^\circ$-$180^\circ$, respectively, in-
corporating the ISR and final state radiation (FSR)
effects, are simulated with a revised version of the
BABAYAGA-3.5 [34] generator, which is modified by
the authors to explicitly involve the $J/\psi$ resonance
in the vacuum polarization. In addition, MC events of
the processes $e^+e^- \rightarrow \text{inclusive hadrons and}$
$\gamma^*\gamma^* \rightarrow X$ ($e^+e^-e^+e^-, e^+e^−\mu^+\mu^-$, $\ldots$) are gen-
erated for background studies with the CONEXC [35]
and BESTWOGAM [36] generators, respectively. When
generating these events, the calibrated BEMS CM energies
are used.

A GEANT4 [37] based MC simulation program in-
cluding the geometric description and response of the
detector is used to simulate the interaction of final state
particles in the detector. Both the experimetal data
and simulated MC are reconstructed and analysed under the
GAUDI [38] based offline software system.

3. Event selection

The signal candidates of $e^+e^- \rightarrow e^+e^-$ and $e^+e^- \rightarrow \mu^+\mu^-$ events are required to have two op-
positely charged tracks in the MDC. Each charged track
has to fulfill the following requirements: it must origi-
nate from the interaction region of $|V_x| < 1 \text{ cm}$ and
$|V_z| < 10 \text{ cm}$, where $|V_x|$ and $|V_z|$ are its closest
approach relative to the collision point in the x-y plane
and along the z axis (taken as the axis of the MDC), re-
spectively; it must hit the detector in the barrel region of
$|\cos \theta| < 0.8$, where $\theta$ is the polar angle of the recon-
structed momentum vector with respect to the z axis.

For $e^+e^- \rightarrow e^+e^-$ candidates, the next two criteria
are applied further to each of the selected tracks: its mo-
tum ($P$) is required to be larger than 0.7 times the
beam energy ($E_{\text{beam}}$), and its energy deposited in the
EMC ($E$) has to be larger than 0.6 times the momen-
uum.

For $e^+e^- \rightarrow \mu^+\mu^-$ candidates, the following con-
ditions are required in addition: for each of the selected
tracks, $P$ must be larger than 0.8$E_{\text{beam}}$, $E$ has to be
larger than 25 MeV and less than 0.25$P$, and valid tim-
ing information is required to be left in the TOF; at
the event level, no neutral showers with a deposited en-
ergy above 25 MeV are allowed in the EMC, and the
difference of the flight time of the two charged tracks ($\Delta t_{\text{TOF}}$) obtained with the TOF has to be less than 1.5
ns to suppress cosmic rays.

Figure 3 shows the comparison between data and
MC simulation of variables used in the event selection.
The data shown in the figure are those of surviving can-
didate events subtracting the residual background esti-
many of the processes $e^+e^- \rightarrow \mu^+\mu^-$, the effi-
ciences obtained from the signal MC samples are about
70% (80%), and the background levels estimated with
the MC simulation are less than 0.05% (0.5%). Closer
examination with a generic event type analysis tool,
TOPOANA [39], shows that the backgrounds mainly
arise from events with $\pi^+\pi^-$, $K^+K^-$ or $e^+e^-e^+e^−$,$(e^+e^-\mu^+\mu^-)$ final states.

4. Cross section measurement

4.1. Nominal results with statistical uncertainties

Usually, the cross section $\sigma$ is determined from

$$\sigma = \frac{N_{\text{sig}} - N_{\text{bkgs}}}{L \cdot \epsilon_{\text{trg}} \cdot \epsilon_{\text{recsel}}} \cdot f,$$  \hspace{1cm} (4)

where $N_{\text{sig}}$ is the number of signal events selected from
data, $N_{\text{bkgs}}$ is the number of residual background
events, $L$ is the integrated luminosity, $\epsilon_{\text{trg}}$ is the trig-
ger efficiency, $\epsilon_{\text{recsel}}$ is the reconstruction-selection ef-
ciciency and $f$ is a reconstruction efficiency correction
factor.

The trigger efficiency is taken as 100% in this
work [40]. The correction factor $f$ is related to the
imperfection of the detector simulation. In practice, the
reconstruction efficiencies (including the tracking effi-
cency in the MDC and the cluster reconstruction effi-
cency in the EMC) from MC simulation deviate those
from data. We study the corresponding reconstruction
Fig. 1: Comparison between data and MC simulation of the distributions of the variables used in the selection criteria for the processes $e^+e^- \rightarrow e^+e^-$ and $e^+e^- \rightarrow \mu^+\mu^-$ at $\sqrt{s} = 3096.9$ MeV. For all the plots, dots with error bars show the background-subtracted data (the background level is evaluated with the MC simulation) and the histograms denote the signal MC. The small discrepancy in the last plot is due to the imperfection of the MC simulation, and it has a negligible effect on the cross section measurement of $e^+e^- \rightarrow \mu^+\mu^-$. 

The measured cross sections and related input quantities at all individual CM energy points of the processes $e^+e^- \rightarrow e^+e^-$ and $e^+e^- \rightarrow \mu^+\mu^-$ are summarized in Tables 2 and 3, respectively.

4.2. Systematic uncertainties

The systematic uncertainties of the measured cross sections arise mainly from the integrated luminosities, trigger efficiencies, CM energies, reconstruction and selection efficiencies, efficiency correction factors and residual backgrounds.

The uncertainties due to the integrated luminosities are estimated to be less than 1.40% (1.26% at $\sqrt{s} = 3096.9$ MeV) [33], while those resulting from trigger efficiencies are evaluated as 0.10% [40].

efficiencies for leptons in different $\cos \theta$ bins. To compensate the deviation of the reconstruction-selection efficiency, the correction factor $f$ is introduced as

$$f = \frac{N_{\text{MC}}}{N_{\text{obs}}} \left( \sum_n \sum_m N_{\text{obs}}^{MC} (m,n) \cdot \frac{\epsilon_{\text{data}} (m)}{\epsilon_{\text{MC}} (m)} \cdot \frac{\epsilon_{\text{MC}} (n)}{\epsilon_{\text{MC}} (n)} \right).$$

Here, $N_{\text{MC}}$ stands for the number of surviving events of signal MC samples, $m$ ($n$) for the $m^{th}$ ($n^{th}$) $\cos \theta$ bin of positively (negatively) charged leptons, $\epsilon_{\text{data}}$ and $\epsilon_{\text{MC}}$ ($\epsilon_{\text{MC}}$ and $\epsilon_{\text{MC}}$) for the MDC tracking efficiency (EMC cluster reconstruction efficiency) of leptons from data and MC simulation, respectively.
The largest changes of the efficiencies with respect to the similar method. Specifically, for the selection of additional sets of MC samples are generated by increasing or decreasing the CM energies by one standard deviation with respect to their nominal values. The largest changes of the efficiencies with respect to their nominal values are taken as the uncertainties. The uncertainties associated with the momentum requirement for the process \( e^+e^- \rightarrow e^+e^- \) are estimated by changing the selection criteria from \( P > 0.7E_{\text{beam}} \) to \( P > 0.6E_{\text{beam}} \). The resultant changes in the calculated cross sections are taken as the uncertainties. The uncertainties related to other requirements are estimated with the similar method. Specifically, for the selection of \( e^+e^- \rightarrow \mu^+\mu^- \) events, the analysis is carried out with the alternative criteria of \( |\cos \theta| < 0.7 \) and \( E/P < 0.7 \), individually, while for \( e^+e^- \rightarrow \mu^+\mu^- \), the analysis is repeated with the alternative criteria of \( P > 0.9E_{\text{beam}} \), \( 0.7 \), \( E/P < 0.35 \) and \( \Delta t^\mu_{\text{TOF}} < 2.5 \) ns.

### Table 3: Measured cross sections and related input quantities of the process \( e^+e^- \rightarrow e^+e^- \) at all individual CM energy points. The uncertainties of the input quantities are statistical, while the first and second uncertainties of cross sections are statistical and systematic, respectively. Here, the cross sections are defined in the polar angle range of \( 0^\circ - 180^\circ \).

| Prop. \( \sqrt{s} \) (MeV) | \( N_{\text{sig}} \) | \( N_{\text{bkg}} \) | \( \epsilon_{\text{recsel}} \) | \( f \) | \( \sigma \) (nb) |
|-----------------------------|----------------|----------------|-----------------|---------|------------|
| 3.0500                     | 2274639 ± 1508 | 164 ± 13      | 0.6909 ± 0.0002 | 0.9989 ± 0.0002 | 220.4 ± 0.2 ± 2.5 |
| 3.0600                     | 2286953 ± 1512 | 145 ± 12      | 0.6910 ± 0.0002 | 0.9989 ± 0.0002 | 219.5 ± 0.2 ± 2.5 |
| 3.0830                     | 710011 ± 843   | 57 ± 8        | 0.6913 ± 0.0002 | 0.9989 ± 0.0002 | 215.1 ± 0.3 ± 2.6 |
| 3.0990                     | 2341309 ± 1530 | 182 ± 13      | 0.6918 ± 0.0002 | 0.9989 ± 0.0002 | 217.3 ± 0.2 ± 2.4 |
| 3.0930                     | 2240003 ± 1497 | 181 ± 13      | 0.6918 ± 0.0002 | 0.9989 ± 0.0002 | 216.9 ± 0.2 ± 2.5 |
| 3.0943                     | 345449 ± 588   | 34 ± 6        | 0.6989 ± 0.0002 | 0.9990 ± 0.0002 | 230.0 ± 0.4 ± 2.9 |
| 3.0952                     | 335948 ± 580   | 52 ± 7        | 0.7180 ± 0.0002 | 0.9991 ± 0.0002 | 257.3 ± 0.4 ± 3.4 |
| 3.0958                     | 487256 ± 698   | 61 ± 8        | 0.7329 ± 0.0002 | 0.9992 ± 0.0002 | 311.2 ± 0.4 ± 6.9 |
| 3.0969                     | 577995 ± 760   | 174 ± 13      | 0.7580 ± 0.0002 | 0.9994 ± 0.0002 | 368.2 ± 0.5 ± 5.2 |
| 3.0982                     | 443694 ± 666   | 104 ± 10      | 0.7286 ± 0.0002 | 0.9992 ± 0.0002 | 276.2 ± 0.4 ± 4.2 |
| 3.0990                     | 127832 ± 358   | 19 ± 4        | 0.7082 ± 0.0002 | 0.9991 ± 0.0002 | 238.6 ± 0.7 ± 4.3 |
| 3.1015                     | 242275 ± 492   | 26 ± 5        | 0.6958 ± 0.0002 | 0.9990 ± 0.0002 | 215.7 ± 0.4 ± 2.8 |
| 3.1055                     | 313080 ± 560   | 32 ± 6        | 0.6929 ± 0.0002 | 0.9990 ± 0.0002 | 214.3 ± 0.4 ± 2.7 |
| 3.1120                     | 251731 ± 502   | 21 ± 5        | 0.6919 ± 0.0002 | 0.9990 ± 0.0002 | 211.3 ± 0.4 ± 2.8 |
| 3.1200                     | 185572 ± 431   | 13 ± 4        | 0.6913 ± 0.0002 | 0.9990 ± 0.0002 | 212.1 ± 0.5 ± 2.8 |
individually.

As shown in Tables 2 and 3, the statistical uncertainties of the efficiency correction factors are 0.02% and 0.03% for the processes \(e^+e^-\rightarrow e^+e^-\) and \(e^+e^-\rightarrow \mu^+\mu^-\), respectively, which are determined from the statistics of the samples used to study the reconstruction efficiencies. On the other hand, detailed studies show that the purities of the control samples for the electron tracking, electron clustering, muon tracking, and muon clustering efficiencies in data are about 99.99%, 99.81%, 98.45%, and 99.52%, respectively. Considering other factors, such as the background contaminations, the uncertainties resulting from the efficiency correction factors can be roughly and conservatively estimated to be 0.10% for both the \(e^+e^-\rightarrow e^+e^-\) and \(e^+e^-\rightarrow \mu^+\mu^-\) processes.

The numbers of residual background events, estimated with the MC simulation, are subtracted from the numbers of surviving events in the calculation of the cross sections, and hence the uncertainties of background levels need be taken into account. Since the uncertainties of the cross sections for some dominant background channels (for example, \(e^+e^-\rightarrow K^+K^-\)) set in the generator are as large as 100%, we therefore take the background levels themselves as the related uncertainties. As a result, the uncertainties for the processes \(e^+e^-\rightarrow e^+e^-\) and \(e^+e^-\rightarrow \mu^+\mu^-\) at \(\sqrt{s} = 3096.9\) MeV are 0.03% and 0.25%, respectively.

Table 4 shows a summary of the systematic uncertainties of the measured cross sections of the processes \(e^+e^-\rightarrow e^+e^-\) and \(e^+e^-\rightarrow \mu^+\mu^-\) at \(\sqrt{s} = 3096.9\) MeV. The total systematic uncertainties, 1.40% and 1.29% for the two processes individually, are the square root of the quadratic sum of the individual uncertainties and dominated by those associated with the integrated luminosities. The systematic uncertainties of the measured cross sections at other CM energy points are estimated with the same method, and they are summarized in Tables 2 and 3 together with the statistical uncertainties.

### 4.3. Correlation analysis

To consider the correlations between the measured cross sections of the same process at different CM energy points, the corresponding covariance matrices are estimated. To estimate such a covariance matrix, contributions from all related uncertainty sources are analysed and estimated according to their nature and the method of uncertainty propagation. To get an impression of the strength of these correlations, the correlation coefficient matrices of the measured cross sections of the processes \(e^+e^-\rightarrow e^+e^-\) and \(e^+e^-\rightarrow \mu^+\mu^-\) are shown in Fig. 2. We find that the correlations are strong and can not be neglected.

In the covariance matrix analysis above, the corresponding covariance matrix of the measured luminosities at different CM energy points is estimated in advance with the similar method. This matrix is required when estimating the covariance matrices of the measured cross sections and constructing the global \(\chi^2\) function for the simultaneous fit of the processes \(e^+e^-\rightarrow e^+e^-\) and \(e^+e^-\rightarrow \mu^+\mu^-\) (see Section 5.2).
5. Decay width determination

5.1. Energy spread and final state radiation

To determine the $J/\psi$ decay widths, a simultaneous fit to the measured, CM energy dependent cross sections of the processes $e^+ e^- \to e^+ e^-$ and $e^+ e^- \to \mu^+ \mu^-$ is required. In the theoretical formulae used in the fit, the effects of the beam energy spread and FSR are taken into account as well.

By assuming the CM energy spread follows a Gaussian distribution, the theoretical cross section is

$$\sigma(W) = \int \sigma_0(W_0) \left( \frac{1}{\sqrt{2\pi}S_W} e^{-\frac{(W_0-W)^2}{2S_W^2}} \right) dW_0. \tag{6}$$

Here, $W (= \sqrt{s})$ and $S_W$ are the mean and standard deviation of the CM energy distribution, respectively. According to the formula and the expression of $\sigma_0$ in Eq. (3), $\sigma$ can also be divided into three terms: the continuum term ($\sigma^C$), the resonance term ($\sigma^R$) and interference term ($\sigma^I$). In practice, $\sigma^C$ is evaluated with the BABAYAGA-3.5 generator [34] with the effects of $J/\psi$ and FSR switched off, while $\sigma^R + \sigma^I$ is calculated using the analytic formulae for $\sigma_0^R + \sigma_0^I$ in Ref. [27].

In Eq. (3), only the ISR effect is involved in $\sigma_0$. To take into account the FSR effect, we introduce a correction factor $R_{\text{FSR}}$ into the theoretical cross section:

$$\sigma_{\text{theor}}(W) = \sigma(W) \cdot R_{\text{FSR}}(W). \tag{7}$$

In practice, $R_{\text{FSR}}$ is obtained with the BABAYAGA-3.5 generator [34] as the ratio of the calculated cross sections with and without the FSR effect. For example, at $\sqrt{s} = 3096.9$ MeV, $R_{\text{FSR}}$ is 0.980 and 0.998 for the processes $e^+ e^- \to e^+ e^-$ and $e^+ e^- \to \mu^+ \mu^-$, respectively.

Due to the high-order corrections related to the FSR effect, the cross sections of the processes $e^+ e^- \to e^+ e^-$ and $e^+ e^- \to \mu^+ \mu^-$ are calculated by the BABAYAGA-3.5 generator with the uncertainties of 0.5% and 1.0%, respectively [34]. Thus, systematic deviations of $R_{\text{FSR}}(W)$ from their truth values probably appear in the vicinity of the $J/\psi$ resonance. To take the possible deviations into consideration, we implement one free scaling parameter in the theoretical cross section formula of each process for the simultaneous fit,

$$\sigma_{\text{theor}}(W) = \sigma_{\text{theor}}^0(W) \cdot F. \tag{8}$$

Specifically, the free scaling parameters for the processes $e^+ e^- \to e^+ e^-$ and $e^+ e^- \to \mu^+ \mu^-$ are referred to as $F_{ee}$ and $F_{\mu\mu}$, respectively.

5.2. Global $\chi^2$ function

To take into account the correlations between the measured cross sections of different processes and/or at different CM energy points, as well as the uncertainties of the CM energies, we construct a global $\chi^2$ function for the simultaneous fit according to the standard covariance matrix method as

$$\chi^2 = \Delta\sigma^T \cdot V^{-1} \cdot \Delta\sigma, \tag{9}$$

where

$$\Delta\sigma(i) = \begin{cases} \sigma_{\text{exper}}(i) - \sigma_{\text{theor}}(i) & i = 1, 2, \ldots, 14, 15 \\ \sigma_{\text{exper}}(i-15) - \sigma_{\text{theor}}(i-15) & i = 16, 17, \ldots, 29, 30 \end{cases}$$

Fig. 2: Correlation coefficient matrices of the measured cross sections of the processes $e^+ e^- \to e^+ e^-$ and $e^+ e^- \to \mu^+ \mu^-$. 

\[\begin{array}{c|c|c|c} \hline \sigma & e^+ e^- & e^+ e^- \\ \hline e^+ e^- & 0.980 & 0.998 \\ \hline e^+ e^- & 0.980 & 0.998 \\ \hline \end{array}\]
and

\[
V(i, j) = \begin{cases} 
V_{ee}(i, j) + \delta(i, j) \left( \frac{\partial \sigma_{ee}^{\text{theor}}(i)}{\partial W} (\Delta W(i))^2 \right) & (1) \\
\frac{\sigma_{ee}^{\text{theor}}(i) \sigma_{ee}^{\text{theor}}(j - 15)}{L(i) L(j - 15)} V_{L}(i, j - 15) + \delta(i, j - 15) \frac{\partial \sigma_{ee}^{\text{theor}}(i)}{\partial W} (\Delta W(i))^2 & (2) \\
\frac{\sigma_{ee}^{\text{theor}}(j) \sigma_{ee}^{\text{theor}}(i - 15)}{L(i - 15) L(j)} V_{L}(i - 15, j) + \delta(i - 15, j) \frac{\partial \sigma_{ee}^{\text{theor}}(j)}{\partial W} (\Delta W(j))^2 & (3) \\
V_{\mu\mu}(i - 15, j - 15) + \delta(i - 15, j - 15) \left( \frac{\partial \sigma_{\mu\mu}^{\text{theor}}}{\partial W} (i - 15) (\Delta W(i - 15)) \right)^2 & (4)
\end{cases}
\]

\[
\sigma_{ee}^{\text{theor}} = \begin{cases} 
\sigma_{ee}^{\text{theor}} & (1) \\
\sigma_{ee}^{\text{theor}} & (2) \\
\sigma_{ee}^{\text{theor}} & (3) \\
\sigma_{ee}^{\text{theor}} & (4)
\end{cases}
\]

Here, \( \sigma_{ee}^{\text{theor}} \) and \( \sigma_{ee}^{\text{theor}} \) (\( \sigma_{ee}^{\text{theor}} \) and \( \sigma_{ee}^{\text{theor}} \)) are the experimental measured and theoretical predicted cross sections of the processes \( e^+e^- \rightarrow e^+e^- (\mu^+\mu^-) \), \( V_{ee} \) (\( V_{ee} \)) is the covariance matrix of the measured cross section of \( e^+e^- \rightarrow e^+e^- (\mu^+\mu^-) \), \( V_L \) is the covariance matrix of the measured luminosities, \( i \) and \( j \) are the horizontal and vertical indices of the 30 \( \times \) 30 covariance matrix \( V \), \( \delta \) is the Kronecker delta function, and \( \Delta W \) is the statistical uncertainty of the CM energy as listed in Table 1, whose systematic uncertainty, 0.033 MeV, will be taken into account by examining the changes of the fit result due to the changes of the CM energies by 0.033 MeV.

5.3. Simultaneous fit and parameter transformation

By minimizing the global \( \chi^2 \) function, the simultaneous fit to the measured, CM energy dependent cross sections of the processes \( e^+e^- \rightarrow e^+e^- \) and \( e^+e^- \rightarrow \mu^+\mu^- \) is carried out. In the fit, the following six parameters \( M, \Gamma_{ee}, \Gamma_{ee}/\Gamma_{\text{tot}}, \Gamma_{\mu\mu}/\Gamma_{\text{tot}}, S_W, F_{ee} \) and \( F_{\mu\mu} \) are float, while \( \sigma_C(W) \) and \( R_{\text{FSR}}(W) \) are expressed as piecewise linear interpolation functions based on hundreds of pairs of \( (W, \sigma_C^2) \) and \( (W, R_{\text{FSR}}^2) \) values obtained with the BABAYAGA-3.5 generator. The resultant fit curves are shown in Fig. 3, and the corresponding fit quality is \( \chi^2_{\text{min}}/\text{ndf} \approx 23.0/24 \approx 1.0 \), where \( \chi^2_{\text{min}} \) and \( \text{ndf} \) are the minimized global chi-squared and the number of degrees of freedom, respectively.

![Fig. 3: Simultaneous fit to the measured, CM energy dependent cross sections of the processes $e^+e^- \rightarrow e^+e^-$ and $e^+e^- \rightarrow \mu^+\mu^-$. For clear display of the interference effect [denoted by $\sigma_0^2$ in Eq. (3) and mainly illustrated by the small dip in front of the peak], the plot for the process $e^+e^- \rightarrow \mu^+\mu^-$ is drawn with a logarithmic vertical axis, while the plot for $e^+e^- \rightarrow e^+e^-$ is drawn with a linear vertical axis, because in this process the interference effect is less noticeable due to the existence of the scattering channel. In the plots, the red points with error bars and blue curves are from data and fitting, respectively.](image-url)
The fit result of $\Gamma_{ee}/\Gamma_{tot}$ and $\Gamma_{ee}/\Gamma_{tot}$ are $(0.346 \pm 0.009)$ and $(0.335 \pm 0.006)$ keV with the covariance and correlation coefficient between them as $0.000046 \text{ keV}^2$ and 0.83, respectively. Taking into account the correlation term, we evaluate $\Gamma_{ee}/\Gamma_{tot}$ to be $1.031 \pm 0.015$, which is consistent with the expectation of lepton universality within about 2σ. Besides, $S_W$ is fitted to be $(0.916 \pm 0.018)$ MeV, which is consistent with the designed energy spread of the BEPCII collider, and $F_{ee}$ and $F_{\mu\mu}$ are fitted to be $0.995 \pm 0.009$ and $1.015 \pm 0.011$, respectively, which are compatible with the precision levels of the BABAYAGA-3.5 generator within uncertainties.

Assuming lepton universality, $\Gamma_{ee} = \Gamma_{\mu\mu}$, and referring to $\Gamma_{ee}$ and $\Gamma_{\mu\mu}$ as $\Gamma_{ll}$, $\Gamma_{ee}/\Gamma_{tot}$ and $\Gamma_{ee}/\Gamma_{tot}$ are combined and referred to as $\Gamma_{ll}/\Gamma_{tot}$, the resultant $\Gamma_{ll}/\Gamma_{tot}$ is $(0.332 \pm 0.006)$ keV, which is smaller than the individual values of $\Gamma_{ee}/\Gamma_{tot}$ and $\Gamma_{ee}/\Gamma_{tot}$ because of the correlation between the latter two.

Combining the resultant $\Gamma_{ll}/\Gamma_{tot}$ with the branching ratio of the $J/\psi$ leptonic decay measured by BESIII in 2013, $B(J/\psi \rightarrow l^+l^-) = \Gamma_{ll}/\Gamma_{tot} = (5.978 \pm 0.040) \%$ [30], $\Gamma_{tot}$ and $\Gamma_{ll}$ are determined to be $(93.0 \pm 2.1)$ and $(5.56 \pm 0.11)$ keV, respectively.

As mentioned previously, the impact of the systematic uncertainty (0.033 MeV) of the CM energies requires additional consideration. By increasing and decreasing the CM energies by 0.033 MeV, we repeat the entire simultaneous fit process twice, the relative changes of the results are less than 0.1%, and are neglected.

### 5.4. Result and comparison

The results obtained in this work are summarized as follows:

| Collab. | Method | Year | $\Gamma_{tot}$ (keV) | $\Gamma_{ll}$ (keV) | Ref. |
|--------|--------|------|----------------------|---------------------|-----|
| BaBar  | ISR return | 2004 | $94.7 \pm 4.4$ | $5.61 \pm 0.21$ | [18] |
| CLEO   | ISR return | 2006 | $96.1 \pm 3.2$ | $5.71 \pm 0.16$ | [19] |
| KEDR   | ES     | 2010 | $94.1 \pm 2.7$ | $5.59 \pm 0.12$ | [20] |
| BESIII | ISR return | 2016 | — | $5.58 \pm 0.09$ | [24] |
| KEDR   | ES     | 2018 | $92.5 \pm 2.0$ | $5.55 \pm 0.11$ | [21] |
| PDG    | —      | 2020 | $92.9 \pm 2.8$ | $5.53 \pm 0.10$ | [1] |
| This work | ES   | 2021 | $93.0 \pm 2.1$ | $5.56 \pm 0.11$ | — |

The uncertainties quoted here are total uncertainties, which are obtained with all the statistical and systematic uncertainties of the input quantities taken into consideration.

The result of $\Gamma_{ee}/\Gamma_{tot}$ is consistent with and more precise than the result $(1.002 \pm 0.025)$ given by KEDR with the same method [20]. It is also in agreement with but less precise than the previous BESIII result $(1.0017 \pm 0.0037)$ obtained with a different approach [30]. Table 5 shows a comparison of the $\Gamma_{tot}$ and $\Gamma_{ll}$ obtained in this work with those from other works and the PDG. The results given by this work agree with all other results; they come up to a new precision level, together with previous results obtained with the ISR return technique at BESIII and ES method at KEDR.

### 6. Summary

Based on the data samples collected with the BESIII detector at fifteen CM energy points in the vicinity of the $J/\psi$ resonance, the cross sections of the processes $e^+e^- \rightarrow e^+e^-$ and $e^+e^- \rightarrow \mu^+\mu^-$ are measured and summarized in Tables 2 and 3, respectively. By performing a simultaneous fit of the cross sections of the two processes as functions of the center-of-mass energy, $\Gamma_{ee}/\Gamma_{tot}$ and $\Gamma_{ee}/\Gamma_{tot}$ of the $J/\psi$ resonance are determined to be $(0.346 \pm 0.009)$ and $(0.335 \pm 0.006)$ keV, respectively.

Using the obtained $\Gamma_{ee}/\Gamma_{tot}$ and $\Gamma_{ee}/\Gamma_{tot}$, the ratio of the $J/\psi$ leptonic decay widths, $\Gamma_{ee}/\Gamma_{\mu\mu}$,
is evaluated to be $1.031 \pm 0.015$, which agrees with the expectation of lepton universality within about two standard deviations. Under the assumption of lepton universality and combining with $B(J/\psi \rightarrow l^+l^-)$ measured by BESIII [30], the total and leptonic decay widths of the $J/\psi$ resonance, $\Gamma_{tot}$ and $\Gamma_{ll}$, are determined to be $(93.0 \pm 2.1)$ and $(5.56 \pm 0.11)$ keV, respectively. These results are consistent with the previous results and reach the world leading level.

7. Acknowledgments

The BESIII collaboration thanks the staff of BEPCII and the IHEP computing center for their strong support. This work is supported in part by National Key R&D Program of China under Contracts Nos. 2020YFA0406400, 2020YFA0406300; National Natural Science Foundation of China (NSFC) under Contracts Nos. 11275211, 11475090, 11335008, 11635010, 11735014, 11935015, 11935016, 11935018, 1196114010, 12022510, 12025502, 12035009, 12035013, 12122509, 12192260, 12192261, 12192262, 12192263, 12192264, 12192265; the Chinese Academy of Sciences (CAS) Large-Scale Scientific Facility Program; Joint Large-Scale Scientific Facility Funds of the NSFC and CAS under Contract No. U1832207; 100 Talents Program of CAS; The Institute of Nuclear and Particle Physics (INPAC) and Shanghai Key Laboratory for Particle Physics and Cosmology; ERC under Contract No. 758462; European Union’s Horizon 2020 research and innovation programme under Marie Skłodowska-Curie grant agreement under contract No. 894790; German Research Foundation DFG under Contracts Nos. 443159800, Collaborative Research Center CRC 1044, FOR 2359, GRK 2149; Istituto Nazionale di Fisica Nucleare, Italy; Ministry of Development of Turkey under Contract No. DPT2006K-120470; National Science and Technology fund; National Science Research and Innovation Fund (NSRF) via the Program Management Unit for Human Resources & Institutional Development, Research and Innovation under Contract No. B16F640076; STFC (United Kingdom); Suranaree University of Technology (SUT), Thailand Science Research and Innovation (TSRI), and National Science Research and Innovation Fund (NSRF) under Contract No. 160355; The Royal Society, UK under Contracts Nos. DH140054, DH160214; The Swedish Research Council; U. S. Department of Energy under Contract No. DE-FG02-05ER41374

8. References

[1] P.A. Zyla et al., Particle Data Group, Prog. Theor. Exp. Phys. 2020 (2020) 083C01.
[2] N. Brambilla et al., CERN Yellow Report, CERN-2005-005 (2005) 487; N. Brambilla et al., Eur. Phys. J. C 71 (2011) 1534.
[3] A.M. Badalian, I.V. Danilkin, Phys. Atom. Nucl. 72 (2009) 1206.
[4] O. Lakhina, E.S. Swanson, Phys. Rev. D 74 (2006) 014012.
[5] E. Eichten, K. Gottfried, T. Kinoshita, K.D. Lane and T.M. Yan, Phys. Rev. D 17 (1978) 3090; Phys. Rev. D 21 (1980) 313; Phys. Rev. D 21 (1980) 203.
[6] J.L. Richardson, Phys. Lett. B 82 (1979) 272.
[7] J.J. Dudek, R.G. Edwards, D.G. Richards, Phys. Rev. D 73 (2006) 074507.
[8] A. Pich, NATO Sci. Ser. B 363 (1997) 173.
[9] R. Van Royen, V.F. Weisskopf, Nuovo Cim. A 50 (1967) 617.
[10] R.H. Bernstein, P.S. Cooper, Phys. Rept. 533 (2012) 27.
[11] J.J. Aubert et al., Phys. Rev. Lett. 33 (1974) 1404.
[12] I.E. Augustin et al., Phys. Rev. Lett. 33 (1974) 1406.
[13] M. Boyarski et al., Phys. Rev. Lett. 34 (1975) 1357.
[14] R. Baldini-Celio et al., Phys. Lett. B 58 (1975) 471.
[15] B. Esposito et al., Lett. Nuovo Cim. 14 (1975) 73.
[16] R. Brandelik et al., Z. Phys. C 1 (1979) 233.
[17] J.Z. Bai et al., BES Collaboration, Phys. Lett. B 355 (1995) 374.
[18] B. Aubert et al., Bubar Collaboration, Phys. Rev. D 69 (2004) 011103.
[19] G.S. Adams et al., CLEO Collaboration, Phys. Rev. D 73 (2006) 051103.
[20] V.V. Anashin et al., KEDR Collaboration, Phys. Lett. B 685 (2010) 134.
[21] V.V. Anashin et al., KEDR Collaboration, J. High Energ. Phys. 05 (2018) 119 [Addendum: J. High Energ. Phys. 07 (2020) 112].
[22] C. Zhang, Sci. China Phys. Mech. Astron. 53 (2010) 2084.
[23] M. Ablikim et al., BESIII Collaboration, Nucl. Instrum. Methods A 614 (2010) 345.
[24] M. Ablikim et al., BESIII Collaboration, Phys. Lett. B 761 (2016) 98.
[25] R.P. Feynman, Photon-Hadron Interactions, Boulder, Colorado: Westview Press, 1998.
[26] V.V. Anashin et al., KEDR Collaboration, Phys. Lett. B 711 (2012) 280.
[27] X.Y. Zhou, Y.D. Wang, L.G. Xia, Chin. Phys. C 41 (2017) 083001.
[28] E.A. Kuraev, V.S. Fadin, Sov. J. Nucl. Phys., 41 (1985) 466.
[29] F.Z. Chen, P. Wang, J.M. Wu, et al., HEP & NP 14 (7) (1990) 1206.
[30] M. Ablikim et al., BESIII Collaboration, Phys. Rev. D 90 (2014) 012001.