The Cosmological Dynamics of Interacting Logarithmic Entropy Corrected Holographic Dark Energy Model

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We investigate the cosmological dynamics of interacting Logarithmic Entropy Corrected Holographic Dark Energy model with Cold Dark Matter. Fixed points are determined and their corresponding cosmological models are presented. Moreover, the dynamical properties of these fixed points are derived.

Keywords: Logarithmic Entropy Correction, Holographic Dark Energy, Fixed points.
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I. INTRODUCTION

Recent cosmological and astrophysical data from the observations of type Ia supernovae, Cosmic Microwave Background radiation (CMB) and Large Scale Structure (SSL) have proved that the universe experiences an accelerated expansion phase [1]. It is commonly believed that the accelerated expansion have been driven by an enigmatic energy component with negative pressure so called dark energy (DE). The cosmological constant is the simplest candidate for dark energy. However, this candidate suffers from two major problems, namely the fine-tuning and the cosmic coincidence problems [2,3]. The nature of dark energy is unknown, thus there are several suggested models for DE, the well-known of them are Thachyon, K-essence, Phantom, Quintom, chaplygin gas, Quintessence and modified gravity [2,4].

Recently, a new model of DE within the framework of quantum gravity, so called Holographic Dark Energy model (HDE), was suggested [5,6], and its energy density, based on the holographic principle, was introduced as [7]

$$\rho_\Lambda = 3c^2 M_p^2 L^{-2},$$

(1)

where $c$ is a numerical constant, $M_p$ is the reduced Planck mass and $L$ is the cut-off length.

In the HDE model, the Bekenstein-Hawking entropy ($S_{BH} = A/G$) plays an essential role and is satisfied on the horizon [8] (where $A \sim L^2$ is the area of horizon). Since this model is connected to the area of entropy, any correction in the entropy formula will affect the energy density of HDE model. Corrections may arise because of the quantum field theory, thermal and quantum fluctuations in Loop Quantum Gravity (LQG) and string theory [8,11]. One correction to the entropy is the logarithmic correction [12]

$$S_{BH} = \frac{A}{4G} + \hat{\alpha} \ln\left(\frac{A}{4G}\right) + \hat{\beta}.$$

(2)

Here $\hat{\alpha}$ and $\hat{\beta}$ are two dimensionless constants. The energy density of the Logarithmic Entropy-Corrected Holographic Dark Energy (LECHDE) can be obtained as [13]

$$\rho_\Lambda = 3c^2 M_p^2 L^{-2} + \alpha L^{-4} \ln(M_p^2 L^2) + \beta L^{-4},$$

(3)
where $\alpha$ and $\beta$ are dimensionless constants, $c$ is a positive constant and the IR cut-off parameter $L$, selected to be the radius of the event horizon as measured on the sphere of the horizon, is described as

$$ L = ar(t), $$

(4)

$$ \int_0^{r(t)} \frac{dr}{\sqrt{1 - kr^2}} = \frac{R_h}{a}, $$

(5)

which yields

$$ r(t) = \frac{1}{\sqrt{k}} \sin y, $$

(6)

where $y = \frac{\sqrt{\pi} \rho_h}{a}$ and $R_h$ is the radius of the event horizon scaled along the $r$ direction

$$ R_h = a \int_t^\infty \frac{dt}{a}. $$

(7)

We can rewrite Eq. (3)

$$ \rho_\Lambda = 3c^2M_p^2L^{-2}\gamma_\alpha, $$

(8)

where

$$ \gamma_\alpha = 1 + \frac{1}{3c^2M_p^2L^2} \left( \alpha \ln(M_p^2L^2) + \beta \right). $$

(9)

Eq. (8) is reduced to (1), if $\alpha = \beta = 0$.

In this paper, we will generalize the LECHDE model interacting with Cold Dark Matter (CDM), studied in [14]. We will determine the system of first-order differential equations and obtain the corresponding fixed points, the attractors, repellers and saddle points.

II. STABILITY OF INTERACTING LOGARITHMIC ENTROPY CORRECTED HOLOGRAPHIC DARK ENERGY MODEL SOLUTIONS

We suppose that there is an interaction between Logarithmic Entropy Corrected Holographic Dark Energy (LECHDE) model and dark matter. The continuity equations yield

$$ \dot{\rho}_\Lambda + 3H\rho_\Lambda(1 + \omega_\Lambda) = -Q, $$

(10)

$$ \dot{\rho}_m + 3H\rho_m = Q, $$

(11)

where $Q = \Gamma\rho_\Lambda$ is an interaction term whose form is not unique. Taking a ratio of two energy densities as $r = \frac{\rho_m}{\rho_\Lambda}$ and using Eqs. (10) and (11) we obtain

$$ \dot{r} = 3Hr \left[ \omega_\Lambda + \frac{\Gamma}{3H} \frac{(1 + r)}{r} \right]. $$

(12)

1 The second and third terms in Eq. (3) are comparable to the first term only when $L$ takes a very small value, because the corrections determined by these terms are important at early universe. When the universe becomes large, corrections are ignorable and the Logarithmic entropy-corrected holographic dark energy reduces to the ordinary holographic dark energy. Therefore, estimation of the parameters in the present model of early universe is not an easy task and needs accurate data from early universe which are not so available and reliable. Of course, for the ordinary holographic dark energy model, the confrontation with present observations have been studied in [20, 21].
Assuming that there is a transfer from the dark energy component to the matter component, we have $\Gamma > 0$. Also, it is clear from Eq. (12) that any restriction on the parameter $\omega_\Lambda$ will set constraint on the quantity $\Gamma$. Now, we define

\[
\omega_\Lambda^{\text{eff}} = \omega_\Lambda + \frac{\Gamma}{3H},
\]

\[
\omega_m^{\text{eff}} = -\frac{\Gamma}{3Hr}.
\]

Using Eqs. (13) and (14), the continuity equations can be written as

\[
\dot{\rho}_\Lambda + 3H\rho_\Lambda (1 + \omega_\Lambda^{\text{eff}}) = 0,
\]

\[
\dot{\rho}_m + 3H\rho_m (1 + \omega_m^{\text{eff}}) = 0.
\]

We assume the non-flat FRW universe

\[
ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right).\]

Here, $k$ denotes the curvature of space with $k = 0, 1, -1$ for flat, closed and open universe, respectively. The first Friedmann equation is given by

\[
H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2} \left[ \rho_\Lambda + \rho_m + \rho_r + \rho_b \right],
\]

where $H$ is the Hubble parameter and $\rho_\Lambda, \rho_m, \rho_r$ and $\rho_b$ correspond to the dark energy, CDM, radiation and baryons densities, respectively. Also, we assume that baryons and radiation have no interaction with dark energy, so they obey the continuity equations

\[
\dot{\rho}_b + 3H\rho_b = 0,
\]

\[
\dot{\rho}_r + 4H\rho_r = 0.
\]

We define the dimensionless fractional contributions of baryons, radiation, CDM, ECHDE and curvature as follows

\[
\Omega_b = \frac{\rho_b}{3M_p^2 H^2},
\]

\[
\Omega_r = \frac{\rho_r}{3M_p^2 H^2},
\]

\[
\Omega_m = \frac{\rho_m}{3M_p^2 H^2},
\]

\[
\Omega_\Lambda = \frac{\rho_\Lambda}{3M_p^2 H^2},
\]

\[
\Omega_k = \frac{k}{a^2 H^2}.
\]
and consider the decay rate to be

\[ \Gamma = 3b^2(1 + r)H, \]

where \( b^2 \) is a coupling constant. Using Eqs. 13, 14 and 26, we obtain

\[ \omega^e_{\Lambda} = \omega_{\Lambda} + b^2 \left( \frac{\Omega_{\Lambda} + \Omega_m}{\Omega_{\Lambda}} \right), \]

\[ \omega^e_{\Lambda} = -b^2 \left( \frac{\Omega_{\Lambda} + \Omega_m}{\Omega_m} \right). \]

Taking the time derivative of Eq. (3) and using Eqs. 4, 7, 10 and 26, we obtain

\[ \omega^e = -1 - \frac{2}{3} \left[ 1 - \frac{\sqrt{\Omega_{\Lambda} \cos y}}{\gamma_{\alpha}} \right] \left[ 1 + \frac{\gamma_{\alpha} - 1}{\gamma_{\alpha}} - \frac{\alpha H^2 \Omega_{\Lambda}}{3c^4 M_p^2 \gamma_{\alpha}^2} \right], \]

which for the non-interacting case \( b^2 = 0 \) reduces to

\[ \omega^e_{\Lambda} = -1 + \frac{2}{3} \left[ 1 - \frac{\sqrt{\Omega_{\Lambda} \cos y}}{\gamma_{\alpha}} \right] \left[ 1 + \frac{\gamma_{\alpha} - 1}{\gamma_{\alpha}} - \frac{\alpha H^2 \Omega_{\Lambda}}{3c^4 M_p^2 \gamma_{\alpha}^2} \right]. \]

Now, inserting Eq. 29 in 27, we obtain

\[ \omega^e_{\Lambda} = -1 + \frac{2}{3} \left[ 1 - \frac{\sqrt{\Omega_{\Lambda} \cos y}}{\gamma_{\alpha}} \right] \left[ 1 + \frac{\gamma_{\alpha} - 1}{\gamma_{\alpha}} - \frac{\alpha H^2 \Omega_{\Lambda}}{3c^4 M_p^2 \gamma_{\alpha}^2} \right], \]

where the \( b^2 \) term is canceled out. Taking \( \alpha = \beta = 0 \) in Eq. 30, we recover Eq. (31) in paper [17]. Also, using Eqs. 8 and 24 we obtain

\[ \Omega_{\Lambda} = \frac{c^2 \gamma_{\alpha}}{H^2 L^2}. \]

Now, we obtain the solutions of the model under investigation and determine their stability. Following 14, we use the quantity

\[ D = \sqrt{H^2 + \frac{k}{a^2}}, \]

and define the dimensionless variables

\[ Z = \frac{H}{D}, \]

\[ \hat{\Omega}_{\Lambda} = \frac{\rho_{\Lambda}}{3M_p^2 D^2}, \]

\[ \hat{\Omega}_m = \frac{\rho_m}{3M_p^2 D^2}, \]

\[ \hat{\Omega}_r = \frac{\rho_r}{3M_p^2 D^2}, \]

\[ \hat{\Omega}_b = \frac{\rho_b}{3M_p^2 D^2}. \]
Then, the Friedmann equation takes the following form
\[ \hat{\Omega}_\Lambda + \hat{\Omega}_m + \hat{\Omega}_r + \hat{\Omega}_b = 1. \] (39)

Taking time derivative of Eq. (18) and using (15), (16), (19), (20), (33) and (34), we obtain the deceleration parameter
\[ q = -\frac{\dot{H}}{H^2} - 1 = \frac{3\left[\hat{\Omega}_{\Lambda}\omega_{\Lambda}^{eff} + \hat{\Omega}_m\omega_{m}^{eff}\right]}{2Z^2} + \hat{\Omega}_r + 1. \] (40)

Taking time derivative of Eqs. (33), (34), (35), (36), (37), (38) and using (15), (16), (19), (20), (34), (35), (37), (38), (11), (10), (11), we obtain
\[ \dot{D} = -Z^2D(q + \frac{1}{Z^2}), \] (41)
\[ \dot{Z} = Z^2\left[1 - q + Z^2(q + \frac{1}{Z^2})\right], \] (42)
\[ \hat{\Omega}_\Lambda = \hat{\Omega}_\Lambda Z\left[-3(1 + \omega_{\Lambda}^{eff}) + 2Z^2(q + \frac{1}{Z^2})\right], \] (43)
\[ \hat{\Omega}_m = \hat{\Omega}_m Z\left[-3(1 + \omega_{m}^{eff}) + 2Z^2(q + \frac{1}{Z^2})\right], \] (44)
\[ \hat{\Omega}_r = \hat{\Omega}_r Z\left[-4 + 2Z^2(q + \frac{1}{Z^2})\right], \] (45)
\[ \hat{\Omega}_b = \hat{\Omega}_b Z\left[-3 + 2Z^2(q + \frac{1}{Z^2})\right], \] (46)
respectively, where \( \dot{\cdot} = \frac{\partial \cdot}{\partial t} \). Here \(-1 \leq Z \leq 1\) and the Hubble parameter can be positive for an expanding cosmological model or negative for a contracting cosmological model. We study the dynamical system for the variables \( \hat{\Omega} = (Z, \hat{\Omega}_\Lambda, \hat{\Omega}_m, \hat{\Omega}_b, \hat{\Omega}_r) \), defined by the Eqs. (42), (43), (44), (45), (46). The dynamical character of this system of equations with their fixed points is determined by the corresponding matrix of linearization. The real parts of its eigenvalues will tell us that the cosmological solutions are repeller, attractor or saddle points [19]. For \( Q \neq 0 \) and \( Q = 0 \), the eigenvalues of dynamical system are given in table 1 and table 4, respectively.

The cosmological models denoted by \( DE_+ \) and \( DE_- \) are the dark energy dominated expanding \( (H > 0) \) and contracting \( (H < 0) \) models, respectively. The cosmological models denoted by \( DM_+ \) and \( DM_- \) are the matter dominated expanding and contracting models, respectively. The cosmological models denoted by \( R_+ \) and \( R_- \) are the expanding and contracting radiation dominated models, respectively. The cosmological models denoted by \( B_+ \) and \( B_- \) are the baryon dominated expanding and contracting models, respectively. The cosmological model denoted by \( E \) is the Einstein universe \( (H = 0) \) and \( M_+ \) and \( M_- \) are the expanding and contracting matter-baryon dominated models, respectively.

The dynamical property of the fixed point is defined by the sign of the real part of the eigenvalues. If all of the eigenvalues are positive, the point is said to be a repeller; if all of the eigenvalues are negative, the point is said to be an attractor; otherwise the fixed point is called a saddle point. For \( Q \neq 0 \), in table 2 and table 3, we determine the attractor, repeller and saddle point characters for the fixed points given in table 1; and for \( Q = 0 \), in table 5 and table 6, we determine the attractor, repeller and saddle point characters for the fixed points given in table 4.
Table 1. Fixed points and eigenvalues for $Q \neq 0$.

| Model | Coordinates | Eigenvalues |
|-------|-------------|-------------|
| $(DE_+)$ | $(1, 1, 0, 0)$ | $1 + 3\omega_A^{eff}, 3\omega_A^{eff}$, $3(\omega_A^{eff} - \omega_m^{eff}), -1 + 3\omega_m^{eff}$ |
| $(DM_+)$ | $(1, 0, 1, 0)$ | $1 + 3\omega_m^{eff}, 3\omega_m^{eff}$, $3(\omega_m^{eff} - \omega_A^{eff}), -1 + 3\omega_A^{eff}$ |
| $(R_+)$ | $(1, 0, 0, 1)$ | $1, 1 - 3\omega_A^{eff}, 1 - 3\omega_m^{eff}$ |
| $(B_+)$ | $(1, 0, 0, 1)$ | $-1, 0, 1 - 3\omega_A^{eff}, -3\omega_m^{eff}$ |
| $(E)$ | $(0, -\delta\Omega_m + \delta\Omega_r + 3\omega_A^{eff} + 3\Omega_m\omega_m^{eff} + \Omega_r^2 + 1, \Omega_m, \Omega_r, -\delta\Omega_m\omega_m^{eff} + \Omega_r^2 + 1)$ | $0$ |

Table 2. Repeller, Attractor and Saddle points for $Q \neq 0$.

| Model | Repeller | Attractor | Saddle point |
|-------|----------|-----------|--------------|
| $(DE_+)$ | $\omega_A^{eff} < \frac{1}{2}, \omega_m^{eff} < \omega_A^{eff}$ | $\omega_A^{eff} < \omega_m^{eff}$ | $\omega_A^{eff} < \omega_m^{eff}$ |
| $(DM_+)$ | $\omega_m^{eff} < \frac{1}{2}, \omega_m^{eff} < \frac{1}{2}$ | $\omega_m^{eff} < \omega_m^{eff}$ | $\omega_m^{eff} < \omega_m^{eff}$ |
| $(R_+)$ | $\omega_A^{eff} < \frac{1}{2}, \omega_m^{eff} < \omega_A^{eff}$ | $\omega_A^{eff} < \omega_m^{eff}$ | $\omega_A^{eff} < \omega_m^{eff}$ |
| $(B_+)$ | $\omega_A^{eff} < \omega_A^{eff}$ | $\omega_A^{eff} < \omega_A^{eff}$ | Saddle point |
| $(E)$ | $\omega_m^{eff} < \frac{1}{2}, \omega_m^{eff} < \omega_m^{eff}$ | $\omega_m^{eff} < \omega_m^{eff}$ | Saddle point |

Table 3. Repeller, Attractor and Saddle points for $Q \neq 0$.

| Model | Repeller | Attractor | Saddle point |
|-------|----------|-----------|--------------|
| $(DE_+)$ | $\omega_A^{eff} > \frac{1}{2}, \omega_A^{eff} > \omega_m^{eff}$ | $\omega_A^{eff} > \omega_m^{eff}$ | $\omega_A^{eff} > \omega_m^{eff}$ |
| $(DM_+)$ | $\omega_m^{eff} > \frac{1}{2}, \omega_m^{eff} > \omega_m^{eff}$ | $\omega_m^{eff} > \omega_m^{eff}$ | $\omega_m^{eff} > \omega_m^{eff}$ |
| $(R_+)$ | $\omega_A^{eff} > \omega_A^{eff}$ | $\omega_A^{eff} > \omega_m^{eff}$ | $\omega_A^{eff} > \omega_m^{eff}$ |
| $(B_+)$ | $\omega_m^{eff} > \omega_m^{eff}$ | $\omega_m^{eff} > \omega_m^{eff}$ | Saddle point |

Table 4. Fixed points and eigenvalues for Q=0.

| Model | Coordinates | Eigenvalues |
|-------|-------------|-------------|
| $(DE_+)$ | $(1, 1, 0, 0)$ | $1 + 3\omega_A^{non-int}, 3\omega_A^{non-int}, -1 + 3\omega_A^{non-int}$ |
| $(M_+)$ | $(1, 0, 1 - \hat{\Omega}_b, \hat{\Omega}_b, 0)$ | $1, 0, -1, -3\omega_A^{non-int}$ |
| $(R_+)$ | $(1, 0, 0, 1)$ | $1, 2, 1 - 3\omega_A^{non-int}$ |
| $(E)$ | $(0, \frac{-\hat{\Omega}_b + 1}{3\omega_A^{non-int}}, \frac{-3(\hat{\Omega}_b + \hat{\Omega}_r - 1)\omega_A^{non-int} + \hat{\Omega}_b^2 + 1}{3\omega_A^{non-int}}, \hat{\Omega}_b, \hat{\Omega}_r)$ | $0$ |
| $(DE_+)$ | $(-1, 1, 0, 0)$ | $-1 + 3\omega_A^{non-int}, 3\omega_A^{non-int}, 1 - 3\omega_A^{non-int}$ |
| $(M_-)$ | $(-1, 0, 1 - \hat{\Omega}_b, \hat{\Omega}_b, 0)$ | $-1, 0, 1, 3\omega_A^{non-int}$ |
| $(R_-)$ | $(-1, 0, 0, 1)$ | $-1, -2, 1 + 3\omega_A^{non-int}$ |
| Model | Repeller | Attractor | Saddle point |
|-------|----------|-----------|--------------|
| $DE_+$ | $\omega_A^{\text{non-int}} > -\frac{1}{3}$ | ----- | ----- |
| $M_+$ | ----- | ----- | Saddle point |
| $R_+$ | $\omega_A^{\text{non-int}} < -\frac{1}{3}$ | ----- | ----- |
| $(E)$ | ----- | ----- | Saddle point |
| $DE_-$ | ----- | $\omega_A^{\text{non-int}} > -\frac{1}{3}$ | ----- |
| $M_-$ | ----- | ----- | Saddle point |
| $R_-$ | ----- | $\omega_A^{\text{non-int}} < -\frac{1}{3}$ | ----- |

III. CONCLUDING REMARKS

In this work, we have discussed the cosmological dynamics of interacting Logarithmic Entropy Corrected Holographic Dark Energy model. We have determined the system of first-order differential equations that explains the evolution of the five dimensionless quantities. In addition, for $Q \neq 0$, the nine fixed points of the mentioned cosmological model are obtained and the dynamical properties of these fixed points are presented. Also, for $Q = 0$, the seven fixed points of the mentioned cosmological model are obtained and the dynamical properties of these fixed points are presented.

In particular, for $Q \neq 0$, it is shown that the Dark Energy dominated models ($DE_+$ and $DE_-$) have a set of attractor and repeller points. Considering the conditions $\omega_{DE}^{\text{eff}} < -\frac{1}{3}$ and $\omega_{DE}^{\text{eff}} < \omega_m^{\text{eff}}$, we have shown that the expanding Dark Energy dominated model ($DE_+$) and the contracting Dark Energy dominated model ($DE_-$) are attractor and repeller, respectively. Similarly, considering the conditions $\omega_m^{\text{eff}} < -\frac{1}{3}$ and $\omega_m^{\text{eff}} < \omega_{DE}^{\text{eff}}$, we have shown that the expanding Dark Matter dominated model ($DM_+$) and the contracting Dark Matter dominated model ($DM_-$) are attractor and repeller, respectively. Finally, considering the conditions $\omega_{DE}^{\text{eff}} < -\frac{1}{3}$ and $\omega_m^{\text{eff}} < \omega_{DE}^{\text{eff}}$, we have shown that the expanding Radiation dominated model ($R_+$) and the contracting Radiation dominated model ($R_-$) are repeller and attractor, respectively. We have shown that the expanding early universe model ($B_+$), the contracting early universe model ($B_-$) and the Einstein universe model ($E$) are saddle points.

Also, for $Q = 0$, it is shown that the Dark Energy dominated models ($DE_+$ and $DE_-$) have a set of attractor and repeller points. Considering the conditions $\omega_A^{\text{non-int}} < -\frac{1}{3}$, we have shown that the expanding Dark Energy dominated model ($DE_+$) and the contracting Dark Energy dominated model ($DE_-$) are attractor and repeller, respectively. Similarly, considering the conditions $\omega_A^{\text{non-int}} < -\frac{1}{3}$, we have shown that the expanding radiation dominated model ($R_+$) and the contracting radiation dominated model ($R_-$) are repeller and attractor, respectively. Finally, we have shown that the expanding matter-baryon universe model ($M_+$), the contracting matter-baryon universe model ($M_-$) and the Einstein universe model ($E$) are saddle points.

[1] A. G. Riess, et al., Astron. J. 116, 1009 (1998);
S. Perlmutter, et al., Astrophys. J. 517, 565 (1999);
References:

P. de Bernardis, et al., Nature 404, 955 (2000);
S. Perlmutter, et al., Astrophys. J. 598, 102 (2003);
U. Seljak, et al., Phys. Rev. D 71, 103515 (2005).

[2] E. J. Copeland, M. Sami, S. Tsujikawa, International Journal of modern Physics D, 15, 1753 (2006).

[3] S. Weinberg, Reviews of Modern Physics, 61, 1 (1989).

[4] T. Padmanabhan, Phys. Rept. 380, 235 (2003).

[5] A. G. Cohen, D. B. Kaplan, A. E. Nelson, Phys. Rev. Lett. 82, 4971 (1999).

[6] M. Li, Phys. Lett. B 603, 1 (2004).

[7] L. Susskind, J. Math. Phys. 36, 6377 (1995).

[8] R. M. Wald, Phys. Rev. D 48, 3427 (1993).

[9] N. Radicella, D. Pav’on, Phys. Lett. B 691, 121 (2010).

[10] A. Ashtekar, J. Baez, A. Corichi, K. Krasnov, Phys. Rev. Lett 80, 904 (1998).

[11] R. M. Wald, Chicago, University of Chicago Press, 504 (1984).

[12] R. Banerjee, B. R. Majhi, Phys. Lett. B 662, 62 (2008);
R. Banerjee, B. R. Majhi, JHEP 06, 095 (2008).

[13] H. Wei, Commun. Theor. Phys. 52, 743, (2009).

[14] M. R. Setare, E. C. Vagenas, Int. J. Mod. Phys. D 18, 147 (2009).

[15] Yu. L. Bolotin, A. Kostenko, O. A. Lemets, D. A. Yerokhin, Int. J. Mod. Phys. D 24, 1530007 (2015).

[16] D. Pavon, W. Zimdahl, AIP Conf. Proc. 841, 356 (2006).

[17] M. R. Setare, Phys. Lett. B 642, 1 (2006).

[18] H. Kim, H. W. Lee, Y. S. Myung, Phys. Lett. B 632, 605 (2006).

[19] D. Iakubovskyi, Y. Shtanov, Class. Quant. Grav. 22, 2415 (2005).

[20] M. R. Setare, J. Zhang and X. Zhang, JCAP, 0703, 007 (2007).

[21] M. Li, X. Li, S. Wang and X. Zhang, JCAP, 0906, 036 (2009).