On the ISW-cluster cross-correlation in future surveys

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Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT

We investigate the cosmological information contained in the cross-correlation between the Integrated Sachs-Wolfe (ISW) of the Cosmic Microwave Background (CMB) anisotropy pattern and galaxy clusters from future wide surveys. Future surveys will provide cluster catalogues with a number of objects comparable with galaxy catalogues currently used for the detection of the ISW signal by cross-correlation with the CMB anisotropy pattern. By computing the angular power spectra of clusters and the corresponding cross-correlation with CMB, we perform a signal-to-noise ratio (SNR) analysis for the ISW detection as expected from the eROSITA and the Euclid space missions. We discuss the dependence of the SNR of the ISW-cluster cross-correlation on the specifications of the catalogues and on the reference cosmology. We forecast that the SNRs for ISW-cluster cross-correlation are slightly smaller compared to those which can be obtained from future galaxy surveys but the signal is expected to be detected at high significance, i.e. more than $>3\sigma$. We also forecast the joint constraints on parameters of model extensions of the concordance ΛCDM cosmology by combining CMB and the ISW-cluster cross-correlation.

Key words: galaxies:clusters:general – cosmology:cosmic background radiation – cosmology:cosmological parameters

1 INTRODUCTION

One of the key predictions of the recent accelerated expansion is the late Integrated Sachs-Wolfe (ISW) effect (Sachs & Wolfe 1967) in the Cosmic Microwave Background (CMB) anisotropy pattern. The ISW effect is caused by the time evolution of the gravitational potentials encountered by CMB photons during their journey from the last scattering surface to the observer. The change of the gravitational potentials on large scales is caused by the accelerated expansion of the Universe driven by dark energy (Kofman & Starobinsky 1985) and is therefore correlated with density fluctuations at low redshifts.

The late ISW effect is just a small fraction of the CMB temperature anisotropy signal for a ΛCDM concordance cosmology and is maximum at largest angular scales (Kofman & Starobinsky 1985), where cosmic variance is dominant.

An extraction of the ISW effect was predicted as feasible by cross-correlating CMB with large scale structure (LSS) tracers of the matter distribution (Crittenden & Turok 1996). Several measurements of the ISW-LSS cross-correlation have been reported since the release of the WMAP first-year data (Nolta et al. 2004) by using different dark matter (DM) tracers. These measurements have been performed by using different estimators in different domains (e.g. Fosalba & Gaztanaga (2004); Boughn & Crittenden (2004); Afshordi et al. (2004); Vielva et al. (2006); McEwen et al. (2007); Ho et al. (2008); Giannantonio et al. (2008); Hernandez-Monteagudo (2010); Schiavon et al. (2012); Bianchini et al. (2016); Stölzner et al. (2017)). See Dupe et al. (2011) and references therein for a review of pre-Planck ISW measurements.

A detection at 2.9 σ level of the ISW effect has been obtained by Ade et al. (2016) by cross-correlating the Planck temperature map with a compilation of publicly available galaxy surveys (see Ade et al. (2014) for the results based on the nominal mission). This cross-correlation measurement of
the ISW effect is compatible with the predictions of a concordance ΛCDM model with $\Omega_\Lambda \sim 0.7$ (Ade et al. 2016). Cross-correlation between CMB lensing and matter tracers improve the significance of the ISW detection, see Ferraro et al. (2015); Manzotti & Dodelson (2014); Ade et al. (2016); Shajib & Wright (2016). Note also that the ISW could be targeted by using its cross-correlation with CIB (Ille et al. 2011) or SZ (Taburet et al. 2011).

Future LSS experiments, such as DESI 1 (Levi et al. 2013; Aghamousa et al. 2016a,b), Euclid 2 (Laureijs et al. 2011; Amendola et al. 2013), LSST 3 (Abell et al. 2009), SKA 4 (Maartens et al. 2015) will provide surveys with a larger number of objects and sky fraction probing a larger volume, then allowing a better determination of the ISW signal. The next generation of LSS surveys will not only lead to an improvement in the ISW measurement with matter tracers already employed for this purpose, but could also provide different catalogues of matter tracers suitable for novel detections of the ISW effect (Racanelli et al. 2015, 2016; Poursidou et al. 2017).

Current cluster surveys are not optimal for the detection of the ISW effect since photometric catalogues are limited to small redshift $z \lesssim 0.4$ (Ro Zo et al. 2010) and X-ray catalogues cover small patches of the sky (Mantz et al. 2015). 5

Otherwise, the cluster catalogues from the next generation of LSS surveys are indeed expected to contain a higher number of clusters over a wider redshift range. These future surveys will allow precision cosmology with clusters, as from the cosmological parameter forecasts from the Euclid cluster survey (Sartoris et al. 2016). It is therefore interesting to study the potential for an ISW detection by a cross-correlation with future cluster catalogues as matter tracers.

In this paper, by computing the angular power spectra of cluster counts and of their cross-correlation with CMB, we perform a signal-to-noise ratio (SNR) analysis for the ISW detection with two representative examples for future cluster catalogues, as those expected from the eROSITA (Predehl et al. 2010) and Euclid (Laureijs et al. 2011) space missions and we forecast how ISW-cluster cross-correlation could improve the constraints on cosmological parameters for different extended models. This analysis is complementary to the stacking of CMB data in correspondence of superclusters (Granett et al. 2008; Papai et al. 2011; Ille et al. 2011; Ade et al. 2014, 2016), whose theoretical interpretation is more difficult, although consistent with ΛCDM.

Our paper is organized as follows. In Sec. 2 we review the angular power spectrum of the cross-correlation between CMB and a generic matter tracer, such as clusters or galaxies. In Sec. 3 we review the calculations of the bias and the number density distribution for cluster catalogues. In Sec. 4 we describe the future surveys considered in our work: eROSITA and Euclid for clusters, Euclid photometric and spectroscopic surveys for galaxies. In Sec. 5 we compute the auto- and CMB cross-correlation angular power spectra for these surveys and perform a SNR analysis. In Sec. 6 we present forecasted constraints on cosmological parameters of extensions of the concordance ΛCDM model by combining CMB and the CMB-LSS cross-correlation. We draw our conclusions in Sec. 7.

Throughout this work we adopt a fiducial cosmological model compatible with the most recent Planck data (Aghanim et al. 2016), corresponding to $\omega_\Lambda \equiv \Omega_\Lambda h^2 = 0.02214$, $\omega_c \equiv \Omega_c h^2 = 0.1206$, $H_0 = 68.89$, $\tau = 0.0581$, $n_s = 0.9625$, and log $(10^{10} A_s) = 3.053$.

2 CROSS-CORRELATION OF ISW EFFECT WITH NUMBER COUNTS

We consider the projected density contrast of a tracer of matter X (clusters or galaxies in this paper) in the direction $\hat{n}$ as:

$$\delta_X(\hat{n}) = \int dz \, b_X(z) \frac{dN_X}{dz} \delta_{L}(\hat{n}, z).$$

(1)

In Eq. (1) we denote by $b_X(z)$ the linear bias and by $dN_X/dz$ the redshift distribution of the tracer. The observed tracer density is correlated with the ISW contribution to temperature fluctuations in direction $\hat{m}$:

$$\frac{\delta T_{\text{ISW}}}{T}(\hat{m}, z) = - \int dz \, e^{-\tau(z)} \left( \frac{d\Phi}{dz}(\hat{m}, z) + \frac{d\Psi}{dz}(\hat{m}, z) \right),$$

(2)

where $\Phi$ and $\Psi$ are the gravitational potentials in the longitudinal gauge and $e^{-\tau(z)}$ is the visibility function.

With these assumptions, the angular power spectrum of the tracer $C^{XX}_{\ell}$ and of its cross-correlation with CMB temperature $C^{TX}_{\ell}$ are respectively:

$$C^{XX}_{\ell} = 4 \pi \int \frac{dk}{k} \Delta^2(k) \left[ I_{\ell}^X(k) \right]^2,$$

(3)

$$C^{TX}_{\ell} = 4 \pi \int \frac{dk}{k} \Delta^2(k) I_{\ell}^{\text{ISW}}(k) I_{\ell}^X(k),$$

(4)

where we denote by $\Delta^2(k)$ the scale invariant matter power spectrum $\Delta^2(k) \equiv k^3 P(k)/(4\pi^2)$, and define the kernels of the counts distribution and of the cross-correlation as:

$$I_{\ell}^X(k) = \int dz \, b_X(z) \frac{dN_X}{dz} D(z) j_\ell (k \chi(z)),$$

(5)

$$I_{\ell}^{\text{ISW}}(k) = - \int dz \, e^{-\tau} \left( \frac{d\Phi}{dz} + \frac{d\Psi}{dz} \right) j_\ell (k \chi(z)),$$

(6)

where $D(z)$ is the growth factor and $\chi(z)$ is the conformal distance.

3 CLUSTER NUMBER DENSITY

We now review the computation of the number density for clusters and the corresponding bias that we will use in Eq. 3. The number density of expected clusters within the solid angle $\Delta \Omega$ is:

$$\frac{dN(z)}{dz} = \int_{\Delta \Omega} d\Omega \frac{dV_c}{dz} d\Omega \int_0^\infty dM X(M, z) n_h(M, z),$$

(7)
where $V_c$ is the comoving volume, $n_h(M,z)$ and $X(M,z)$ are the dark matter halo mass function and the survey characteristic function, respectively.

### 3.1 Halo mass function

The distribution $n_h(M_h, z)$ of DM halos in mass, position, and redshift describes the overall abundance and clustering properties of galaxies, and the dependence on the underlying cosmological model. As halo mass function, we use (Tinker et al. 2008):

$$n_h(M, z) = -f(\sigma) \frac{\rho_m}{M} \frac{d \ln \sigma(M, z)}{dM},$$

$$f(\sigma) = C_1(z) \left[ 1 + \left( \frac{\sigma(M, z)}{C_3(z)} \right)^{-C_2(z)} \right] e^{-C_0(\sigma(M, z))^2},$$

where $\rho_m$ is the total physical matter density and $\sigma(M, z)$ is the root mean square density fluctuation within a sphere. The variance of the fractional density fluctuation:

$$\sigma^2(M, z) = \int_0^\infty \frac{k^2 dk}{2\pi^2} P(k, z) W^2(k, M),$$

is calculated by integrating the linear matter power spectrum $P(k, z)$ smoothed by a top-hat window in real space window function $W(k, M)$.

According to Tinker et al. (2008), we parametrize the linear evolution of the parameters of the mass function in Eq. (8) as different power law functions of $(1+z)$:

$$C_1(z) = C_{1,0}(1+z)^{-0.14},$$
$$C_2(z) = C_{2,0}(1+z)^{-0.06},$$
$$C_3(z) = C_{3,0}(1+z)^{-\gamma},$$

$$\log(\gamma) = - \left( \frac{0.75}{\log(\Delta_c/75)} \right)^{1.2},$$

where the subscript 0 denotes the value of the quantity at $z = 0$ and $\Delta_c$ defines the mean overdensity of the spherical halos in terms of the critical density of the universe. The quantities $C_0, C_{1,0}, C_{2,0}, C_{3,0}$ depend on $\Delta_c$ and we refer the reader to Tinker et al. (2008) for their numerical values. We use for our forecasts $\Delta_c = 200$ and $500$ for Euclid and eROSITA respectively.

### 3.2 Halo bias function

We now specify the halo bias function, $b_{\text{halo}}(z)$. The linear bias weighted by the halo mass function is:

$$b_{\text{halo}}(z) = \frac{1}{n_h(z)} \int_0^\infty dM n_h(M, z) b_h(M, z) X(M, z),$$

where the integrated halo redshift distribution is:

$$n_h(z) = \int_0^\infty dM n_h(M, z) X(M, z).$$

According to Tinker et al. (2010), the Eulerian halo bias is:

$$b_h(M_h, z) = 1 - c_1 \frac{\nu^4}{\nu_0^4 + \delta_c^4} + c_2 \nu^5 + c_3 \nu^6,$$

where $\nu = \delta_c / \sigma(M_h, z)$ and $\delta_c$ is the critical density for collapse (in all calculations we use $\delta_c = 1.686$) and the parameters of the halo bias function in Eq. (14) as a function of $\Delta_c$ are:

$$c_1 = 1 + 0.24 \nu e^{-(4/\nu)^4},$$
$$c_2 = 0.183,$$
$$c_3 = 0.019 + 0.107 \nu + 0.19 \nu^{-(4/\nu)^4},$$
$$c_4 = 0.44 \nu - 0.88,$$
$$c_5 = 1.5,$$
$$c_6 = 2.4,$$

where $x \equiv \log \Delta_c$.

### 3.3 Survey characteristic function

We express the survey characteristic completeness function in Eq. (7) as:

$$X(M, z) = \int_{M_{\text{min}}(z)}^\infty dM_{\text{obs}} P(M_{\text{obs}} | M),$$

where $M_{\text{min}}(z)$ represents the minimum value of the observed mass for a cluster to be included in the survey, and it is determined by the survey selection function. The function $P(M_{\text{obs}} | M)$ gives the probability that a cluster of true mass $M$ has a measured mass $M_{\text{obs}}$ and takes into account the uncertainties that a scaling relation introduces in the knowledge of the cluster mass. Following Lima & Hu (2005) and Sartoris et al. (2010), the probability of assigning to a cluster of true mass $M$ an observed mass $M_{\text{obs}}$ can be well approximated with a lognormal density distribution:

$$P(M_{\text{obs}} | M) = \frac{1}{M_{\text{obs}} \sqrt{2\pi \sigma^2_{\text{log-M}}}} \exp \left[ -\frac{(\ln M_{\text{obs}} - B_M - \ln M)^2}{2\sigma^2_{\text{log-M}}} \right],$$

where the parameter $B_M = B_{M,0} + \alpha \log (1+z)$ represents the fractional value of the systematic bias in the mass estimate and $\sigma^2_{\text{log-M}} = \sigma^2_{\text{log-M,0}} - 1 + (1+z)^{2\beta}$ is the intrinsic scattering. We treat $B_{M,0} = 0, \alpha = 0, \sigma_{\text{log-M,0}} = 0.2$, and $\beta = 0.125$ as fiducial as in Sartoris et al. (2016) and we consider them as nuisance parameters in the following analysis.

### 4 FUTURE SURVEYS

In this section we describe two future cluster catalogues suitable for the ISW detection by cross-correlation with CMB, i.e. those expected from eROSITA (Predehl et al. 2010) and Euclid (Laureijs et al. 2011). For comparison, we also present the specifications for the photometric and spectroscopic galaxy surveys expected from Euclid, which are expected to improve on the current statistical significance of the ISW detection with galaxy surveys.
4.1 eROSITA cluster survey

eROSITA\(^6\) (Predehl et al. 2010) is the primary science instrument onboard the Spectrum Roentgen-Gamma (SRG) satellite\(^7\), launched in 2017.

eROSITA is expected to perform an X-ray all-sky survey with a sensitivity \(\sim 30\) times better than ROSAT and to cover a redshift range of \(0 \leq z \leq 1.5\) with a sky coverage of 27,000 deg\(^2\). We consider the eROSITA cluster selection function presented in Pillepich et al. (2012), computed for a mass at \(\Delta_2 = 500\) with an exposure time of \(1.6 \times 10^3\) s and a detection threshold of 50 photons. In addition, we adopt as lower cut for the minimum mass \(5 \times 10^{13} M_\odot/h\).

The resulting specifications of the eROSITA cluster catalogue (hereafter referred eR\(_{cl}\)) are summarized in Tab. 1, leading to a total number of clusters \(\sim 8.1 \times 10^4\). The corresponding redshift density distribution and the linear bias for eROSITA are shown in Fig. 1.

4.2 Euclid cluster survey

The European Space Agency (ESA) Cosmic Vision mission Euclid (Laureijs et al. 2011) is scheduled to be launched in 2021, with the goal of exploring the dark sector of the Universe.

As selection function for the expected Euclid cluster catalogue we consider the one presented in Sartoris et al. (2016) computed for a mass at \(\Delta_2 = 200\) with a threshold for the significance of the clusters detection of 3 in terms of the ratio between the cluster galaxy number counts and the field rms (hereafter referred Eu\(_{cl}\)Opt) with a sky coverage of 15,000 deg\(^2\) over a redshift range of \(0.2 \leq z \leq 2\). We also consider a detection threshold of 5 as a more conservative case (hereafter referred Eu\(_{cl}\)Con). This selection function was obtained following a phenomenological approach. In summary, the number of cluster galaxies is obtained by integration of observed cluster luminosity function down to the H-band magnitude limit of the Euclid survey, and the variance in the field counts takes into account both Poisson noise and cosmic variance (see Sartoris et al. (2016) for more details). Even if the galaxy cluster mass selection function used for the Euclid photometric survey is not constant (see Sartoris et al. (2016)), a mass limit of \(8 \times 10^{13} M_\odot/h\) and \(3 \times 10^{14} M_\odot/h\) approximately mimics the counts of Eu\(_{cl}\)Opt and of Eu\(_{cl}\)Con, respectively.

The resulting specifications of the optimistic and conservative Euclid expected cluster catalogues are summarized in Tab. 1, leading to a total number of clusters \(1.3 \times 10^6\) and \(2.0 \times 10^5\), respectively. See also Fig. 1 for the corresponding redshift density distribution and the linear bias.

4.3 Euclid galaxy surveys

Euclid will measure the galaxy clustering in a spectroscopic survey of tens of millions of \(\text{H}\alpha\) emitting galaxies and the cosmic shear in a photometric survey of billions of galaxies.

For the Euclid wide spectroscopic survey (hereafter re-
ferred Eu_gal_sp), we consider an area of 15,000 deg$^2$. According to the updated predictions obtained by Pozzetti et al. (2016), the Euclid wide single-grism survey will reach a flux limit of $F_{c,\text{lim}} > 2 \times 10^{-16}$ erg cm$^{-2}$ s$^{-1}$ and will cover a redshift range $0.9 \leq z \leq 1.8$.

As specifications for the Euclid photometric survey (hereafter referred Eu_gal_ph), we adopt dN/dz $\propto z^2 \exp[-(z/z_0)^{3/2}]$, with $z_0 = z_{\text{mean}}/1.412$ the peak of the distribution and $z_{\text{mean}}$ the median (Amendola et al. 2013). We choose $z_{\text{mean}} = 0.9$, a surface density $\bar{n}_g = 30$ per arcmin$^2$ and a bias $b_g(z) = \sqrt{1 + 2}$ (Amendola et al. 2013).

The specifications for the Euclid spectroscopic and photometric galaxy surveys are summarized in Tab. 1. In Fig. 2 we plot the normalized redshift density distribution and the effective bias.

### Table 1. Summary of the characteristics of the surveys studied in this paper: fraction of the sky available $\ell_{\text{sky}}$, and surface density of tracer $\bar{n}_X$.

| Survey            | $\ell_{\text{sky}}$ [deg$^2$] | $\bar{n}_X$ [deg$^{-2}$] |
|-------------------|--------------------------------|--------------------------|
| eR_cl             | 27,000                         | 2.9                      |
| Eu_gal_opt        | 15,000                         | 72                       |
| Eu_gal_con        | 15,000                         | 10.8                     |
| Eu_gal_sp         | 15,000                         | 3,960                    |
| Eu_gal_ph         | 15,000                         | 108,000                  |

5 A SIGNAL-TO-NOISE ANALYSIS

We now investigate the detection level of the ISW effect by computing the signal-to-noise ratio (SNR), which we define as (Cooray 2002; Afshordi 2004):

$$\left( \frac{S}{N} \right)_X^2 = \sum_{\ell=2}^{\ell_{\text{max}}} \left( \frac{S}{N} \right)_{X, \ell}^2,$$

and

$$\left( \frac{S}{N} \right)_{T, \ell}^2 = (2\ell + 1) \frac{\ell \bar{f}_{\text{sky}} (C_{T \ell}^{XX})^2}{(C_{T \ell}^{XX})^2 + C_{T \ell}^{TT} C_{T \ell}^{EE}},$$

where $\bar{f}_{\text{sky}}$ is the sky coverage of the survey and an overall bar stands for the sum of the signal and its noise. By $C_{T \ell}^{XX}$ we denote the total CMB temperature angular power spectrum, which includes both the ISW and the Sachs-Wolfe terms, plus the effective noise:

$$\bar{C}_{T \ell}^{XX} = C_{T \ell}^{XX} + \bar{N}_{T \ell}.$$

At low and intermediate multipoles, Planck provides a measurement with a high signal-to-noise of the total anisotropy signal and the effective noise in temperature can therefore be neglected. The noise contribution for the mass tracer associated to a finite number of counts corresponds to a shot-noise as:

$$\bar{N}_{T \ell} = \frac{1}{\bar{n}_X},$$

where $\bar{n}_X$ is the surface density of tracers per steradian.

We show in Fig. 3 and Fig. 4 the angular power spectra and the noises, for cluster and galaxy surveys respectively, calculated with the assumed fiducial cosmology and used in the SNR analysis.

The ISW effect leaves also an imprint on the E-mode polarization that can be quantified in analogy with Eq. (17), by substituting T with E. We have checked that the SNR is at most 0.2 for the surveys considered here. Whereas the cross-correlation between the CMB E-mode polarization and LSS tracers is small as expected (Cooray & Melchiorri 2006), the inclusion of polarization data allows a reduction of the error bars in the overall detection of the ISW effect (Frommert & Enßlin 2009). Indeed, the E-mode polarization is correlated with primary temperature anisotropy (Zaldarriaga 1997) and therefore indirectly helps in disentangling the ISW effect. It is possible to extend the SNR analysis in Eq. (17) to the inclusion of polarization as in Frommert & Enßlin (2009):

$$\left( \frac{S}{N} \right)_{T, E, \ell}^2 = (2\ell + 1) \frac{\ell \bar{f}_{\text{sky}} (C_{T \ell}^{XX})^2}{(C_{T \ell}^{XX})^2 + C_{T \ell}^{TT} (C_{E \ell}^{XX})^2} C_{T \ell}^{TE}.$$

(21)

In the denominator of the latter equation the power spectrum of temperature anisotropies is substituted by the corresponding one of the E-mode uncorrelated temperature map:

$$a_{T \ell} a_{E \ell} \rightarrow a_{T \ell} a_{E \ell} - \frac{C_{E \ell}^{XX} a_{E \ell}}{C_{T \ell}^{EE}} C_{T \ell}^{XX}.$$

(22)
The E-uncorrelated temperature map has been already introduced in the ISW-LSS cross-correlation analysis of the Planck 2015 release (Ade et al. 2016).

We summarize in Tab. 2 the SNR for the cluster and galaxy catalogues introduced in Sec. 2 with the estimators in Eqs. (17,21). Future cluster catalogues from eROSITA and Euclid could lead to a detection of the ISW effect at high significance $>3\sigma$ thanks to the high number of clusters detected, i.e. a small noise level. We find that the SNR for these two cluster surveys will be comparable to the one obtainable from future galaxy surveys. In particular, for a Euclid survey, the SNR from clusters is smaller than the one expected from the photometric survey, but it is still larger than the SNR forecasted for the spectroscopic survey. The reason resides in the high bias expected for clusters which compensate the larger shot noise in the observed cluster maps. The inclusion of CMB polarization information in the analysis increases the SNR by approximately 18% as compared to the temperature only, with a weak dependence of the type of survey considered; this is fully consistent with Fromert & Enßlin (2009); Gianantonio et al. (2012). We find the possibility of detecting the ISW effect at $4\sigma$ both the cluster and the galaxy Euclid photometric surveys by adding the CMB polarization information. A comparison between the two SNR calculated from the two estimators is also shown in Fig. 5.

We now briefly discuss the dependence of the SNR on $M_{\text{min}}$. We vertically shift a redshift independent selection function around $8 \times 10^{13}$ $M_{\odot}$, as representative of the selection function from (Sartoris et al. 2016) for the photometric Euclid cluster survey. The results in Fig. 6 (bottom left panel) show that the expected selection function for Euclid would be optimal in terms of SNR.

We can conclude that for the ISW detection is more important to have wide surveys, covering a large fraction of the sky, than deeper ones, since the surveys analysed are already signal dominated on the scales relevant for the ISW.

We now test the dependence of the SNR obtainable for clusters on some of the specifications of the surveys, namely the smallest multipoles $\ell_{\text{min}}$, the covered sky fraction $f_{\text{sky}}^N$, the maximum redshift $z_{\text{max}}$, and the minimum detectable mass $M_{\text{min}}$. For the sake of brevity, we present this analysis for the optimistic Euclid cluster (Eu$_{\text{cl},\text{opt}}$) survey only and considering the inclusion of polarization in the SNR estimation.

Given the possibility to lose the first multipoles because of some mask effects and possible systematics on the largest scales, we show in Fig. 6 (top left panel) the dependence of SNR on $\ell_{\text{min}}$. The SNR remains approximately constant when removing the first few multipoles, and is decreased approximately by 0.5 $\sigma$ cutting the first ten multipoles.

The SNR is sufficiently robust to a reduction of $z_{\text{max}}$; the forecast SNR remains above a $4\sigma$ detection for $z_{\text{max}} \gtrsim 1.4$ for the optimistic Euclid cluster survey when the CMB polarization is added.

We finally study the dependence of the SNR when changing $M_{\text{min}}$. We vertically shift a redshift independent selection function around $8 \times 10^{13}$ $M_{\odot}$, as representative of the selection function from (Sartoris et al. 2016) for the photometric Euclid cluster survey. The results in Fig. 6 (bottom left panel) show that the expected selection function for Euclid would be optimal in terms of SNR.

We now briefly discuss the dependence of the SNR on the underlying cosmology. The cross-correlation power spectrum is sensitive to the total matter density $\Omega_m$ and to the dark energy equation of state $w_0$. They both change the amplitude of $C^{TX}_{\ell}$ (since for a flat universe an increased amount

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**Figure 4.** Galaxy angular power spectra $C^{gg}_{\ell}$ (top panel) and cross-correlation angular power spectra $C^{TX}_{\ell}$ (bottom panel) for two surveys of galaxies: spectroscopic Euclid (solid orange), and photometric Euclid (solid gray). In the top panel the corresponding shot-noise (dashed lines) is also shown.

**Figure 5.** Comparison of $(S/N)_{\ell,\ell}$ for temperature only (dashed) and including polarization (solid) according to Eq. (17) and Eq. (21) respectively, for the eROSITA and optimistic Euclid survey of clusters, together with the Euclid photometric survey of galaxies.

**5.1 Robustness from the specifications of the survey and cosmology**

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**Table 2.** SNR for the cluster and galaxy surveys considered in Sec. 2 with the estimators in Eqs. (17,21). Future cluster catalogues from eROSITA and Euclid could lead to a detection of the ISW effect at high significance $>3\sigma$ thanks to the high number of clusters detected, i.e. a small noise level. We find that the SNR for these two cluster surveys will be comparable to the one obtainable from future galaxy surveys. In particular, for a Euclid survey, the SNR from clusters is smaller than the one expected from the photometric survey, but it is still larger than the SNR forecasted for the spectroscopic survey. The reason resides in the high bias expected for clusters which compensate the larger shot noise in the observed cluster maps. The inclusion of CMB polarization information in the analysis increases the SNR by approximately 18% as compared to the temperature only, with a weak dependence of the type of survey considered; this is fully consistent with Fromert & Enßlin (2009); Gianantonio et al. (2012). We find the possibility of detecting the ISW effect at $4\sigma$ both the cluster and the galaxy Euclid photometric surveys by adding the CMB polarization information. A comparison between the two SNR calculated from the two estimators is also shown in Fig. 5.
Table 2. SNR for ISW detection with future cluster and galaxy catalogues. The first row refers to the SNR with CMB temperature in Eq. (17); the second one to the estimate taking into account both CMB temperature and E-mode polarization as in Eq. (21).

|          | eR_cl | Eu_cl_opt | Eu_cl_con | Eu_gal_sp | Eu_gal_ph |
|----------|-------|-----------|-----------|-----------|-----------|
| T        | 2.3   | 3.5       | 3.3       | 2.2       | 3.7       |
| T,E      | 2.8   | 4.1       | 3.9       | 2.5       | 4.3       |

![Figure 6](image_url)

Figure 6. We plot the SNR for the optimistic Euclid cluster survey (Eu_cl_opt) as a function of $\ell_{\text{min}}$ (top left), the maximum redshift probed by the survey (top right), the area of the sky survey (bottom left), and the minimum observed mass as over the redshift range $0.2 \leq z \leq 2$ (bottom right). Dashed line represents the expected SNR for the Eu_cl_opt specifications. All the SNR take into account of the contribution from CMB E-mode polarization according to Eq. (21).

5.2 Combining clusters with galaxies

Different LSS surveys can be combined to increase the significance of the ISW detection. The best improvement comes for uncorrelated LSS surveys for which the SNR squared can be simply added. In this case we would obtain for the ISW detection a SNR of 3.2/4.1/4.0 for the combination of Eu_gal_sp and 4.3/5.1/5.0 for the combination of Eu_gal_ph with eR_cl, Eu_cl_opt, Eu_cl_con respectively. However, this ideal case is usually reached for surveys which cover different redshifts or for distributions of tracers which peak at different redshifts.

The Euclid’s cluster catalogue will be extracted from the photometric galaxy survey. Thus, they will trace by construction the same cosmological information mostly, having a similar redshift distribution and covering the same redshift range.

In order to take into account the cross-correlation be-
between different LSS tracers, we extend the expression for the SNR (17) to:

$$\left( \frac{S}{N} \right)^2_{\text{CMB}} = \sum_{\ell, x, y, g} C_{\ell}^{TX} \left[ \text{Cov}_{\ell}^{-1} \right]_{XY} C_{\ell}^{TY}$$

where:

$$[\text{Cov}]_{XY} = C_{\ell}^{TX} C_{\ell}^{TY} + (2\ell + 1) \sqrt{f_{\text{sky}}^{X} f_{\text{sky}}^{Y}}$$

In this case the SNR by adding the information from Eu_gal_ph becomes 3.7 for both Eu_cl_opt and Eu_cl_con. There is no actual improvement compared to using Eu_gal_ph alone but nevertheless it offers the possibility of a more robust ISW detection when cosmological parameters are not kept fixed.

The combination of the two Euclid cluster surveys with Eu_gal_ph brings a little improvement of +0.2 in the SNR.

Finally, we find that for eROSITA the combination of cluster survey with the galaxy survey from Euclid will be more promising in order to raise the SNR thanks to the low-z coverage from eROSITA. We find a SNR of 3.2/4.1 by combining and cross-correlating eR_cl with Eu_gal_sp and Eu_gal_ph.

Also in this case the addition of the CMB polarization information increases the SNR by a further 18% for all the combinations.

6 COSMOLOGICAL FORECASTS WITH CMB-CLUSTER CROSS-CORRELATION

In this section we use the Fisher matrix technique (Tegmark et al. 1997) to compute the joint CMB and cross-correlation forecast constraints on few extensions of the ΛCDM cosmological concordance model.

6.1 Methodology

We use the Fisher matrix derived from the Gaussian approximation of the full likelihood L. By using the Gaussian hypothesis for the data d, the likelihood function of the cosmological parameters θ reads as:

$$\mathcal{L}(d|\theta) \propto \frac{1}{\sqrt{|C(\theta)|}} \exp \left\{ -\frac{1}{2} \left( d^{\text{data}} \right)^{\top} \left[ C(\theta) \right]^{-1} d^{\text{data}} \right\},$$

where:

$$C = \left( \begin{array}{cc} C_{TT} & C_{TE} \\ C_{TE} & C_{EE} \\ C_{EX} & C_{XX} \end{array} \right),$$

is the theoretical covariance matrix of the modeled data d = \{d^T, d^E\} where X = \{c, g\} and |C(\theta)| its determinant. From Eq. (25), we write the Fisher matrix in compact form as:

$$F_{\alpha\beta} = \frac{\partial^2 \log \mathcal{L}}{\partial \theta_\alpha \partial \theta_\beta} = \frac{1}{2} \sum_{\ell} \left( 2\ell + 1 \right) f_{\text{sky}}^{X} \frac{\partial C_{\ell}^{AB}}{\partial \theta_\alpha} \frac{\partial C_{\ell}^{CD}}{\partial \theta_\beta},$$

where $C_{\ell}^{AB} = C_{\ell}^{AB} + \delta_{AB} N_{\ell}^A$ is the sum of the signal and the noise, with $N_{\ell}^{TT}, N_{\ell}^{TN}, N_{\ell}^{EN} = 0$. For the temperature and polarization angular power spectra, here $N_{\ell}^{TT} = \sigma_T^2 b_{\text{FWHM}}^{-2}$ is the isotropic noise deconvolved with the instrument beam, $b_{\text{FWHM}}$ is the beam window function, assumed Gaussian, with $b_{\text{FWHM}} = e^{-\left( \frac{1}{2} \right)^2 \theta_{\text{FWHM}}^2 / \theta_{\text{FWHM}}^2}$ is the full width half maximum (FWHM) of the beam in radians; $\sigma_T$ and $\sigma_E$ are the square of the detector noise level on a steradian patch for temperature and polarization, respectively. See the appendix A for the full structure of the covariance matrix Cov in Eq. (27).

As representative specifications for CMB we consider the Planck 143 GHz channel full mission sensitivity and angular resolution as given in Adam et al. (2016) and used in Ballardini et al. (2016). These correspond to $\theta_{\text{FWHM}} = 7.3$ arcmin, $\sigma_T = 33$ arcmin $\mu K$ and $\sigma_E = 70.2$ arcmin $\mu K$. For cluster and galaxy specifications we use those listed in Tab. 1.

We consider as set of cosmological parameters in our forecast analysis the six standard ΛCDM parameters $\omega_c$, $\omega_b$, $H_0$, $\tau$, $n_s$, and (log $10^{10}$ $A_s$), plus some extra parameters described in the following subsection. We also consider the bias as a nuisance parameter marginalizing over it.

In the case of clusters, in addition to the variation on all the cosmological parameters we consider the dependence on
6.2 Results

In this paper we restrict ourselves to forecast the joint CMB and ISW-cluster cross-correlation constraints on few relevant cosmological models as in Douspis et al. (2008). With this setting the impact of ISW-cluster cross-correlation can be easily evaluated, whereas it would be hardly visible if we were considering the full combination of CMB and cluster cosmological information. As for the SNR in the previous section, we compare these results with those obtained by the joint CMB and ISW-galaxy cross-correlation expected from the Euclid spectroscopic and photometric surveys.

We first forecast the constraints on the redshift-independent parameter of state of dark energy \( w_0 \). Fig. 8 shows how the CMB-cluster cross-correlation improves on the CMB results alone. Our results show that CMB-cluster cross-correlation could lead to constraints similar to the one expected from the CMB-galaxy one, improving the constraints on \( w_0 \) obtained from CMB data alone by around 20-30%.

As second cosmological scenario we show the constraints for a cosmology in which the parameter of state of dark energy is allowed to be redshift-dependent (Chevallier & Polarski 2001; Linder 2003):

\[
\begin{align*}
    w(z) &= w_0 + \frac{z}{1+z} w_a, \\
    w(z) &= \exp(-\lambda w_a) \\
    \frac{\Delta M(z)}{M(z)} &= \frac{\Delta w(z)}{w(z)}.
\end{align*}
\]

where \( w_a \) is its first derivative with respect to the scale parameter \( a(t) \). Also in this case, we see from Fig. 9 that the addition of CMB-cluster cross-correlations leads to a significant improvement for the dark energy parameters, with a similar performance between eROSITA and Euclid. The improvement with respect to CMB-only constraints for the \( w_0 - w_a \) plane is about 10% for the Euclid and 5% for the CMB- \( w_0 \). The Figure of Merit (FoM) for the dark energy parameters (Albrecht et al. 2009) improves by a factor 1.5 when the ISW-cluster cross-correlation is included. In Fig. 9, the difference between the orientation of the contour of eROSITA and the one of Euclid in the \( w_0 - w_a \) parameters space is not only due to the different redshift range probed by the two surveys but it is also due to the larger sky coverage from eROSITA which helps in breaking the degeneracy between the two parameters.

We consider a cosmological model in which there is a suppression on large scales of the primordial curvature power spectrum (Contaldi et al. 2003):

\[
P_\ell(k) = P_{\ell,0}(k) \left( 1 - \exp \left[ - \frac{k}{k_c} \right] \right),
\]

where \( k_c \) is the relevant scale of deviation from a power-law spectrum and \( \lambda \) adjusts the stiffness of the suppression. For this model the addition of the CMB-cluster cross-correlation leads to a hardly visible improvement in the constraints on \( \{\lambda, k_c\} \) with respect to the CMB constraints, since, even if on large scales, the suppression induces a change in the SW contribution rather than on the ISW one. Nevertheless, the ISW-LSS cross-correlation is useful to break degeneracies in the physical effects at play on the largest scales.

As a semi-idealized case, we also consider the constraints on the \( w_0 - w_a \) plane by assuming perfect knowledge of the scaling relation between the true and the observed cluster mass, i.e. by fixing all the corresponding nuisance parameters: this semi-idealized case would cast the cluster case at the same footing of the galaxy one, in which no nuisance parameter is considered. In Fig. 10 we show the comparison of the constraints in the \( w_0 - w_a \) parameters space between the CMB-cluster cross-correlation from Euclid, with and

---

**Figure 8.** In the bottom panel, we show the joint constraints on \((w_0, H_0)\) at the 68% and 95% CL. We show forecasts for the combination of the CMB and the CMB-LSS cross-correlation from eROSITA (green) and from the optimistic Euclid photometric cluster survey (blue). We show also the results for the CMB alone (yellow). In the top panel, we show the corresponding posterior distributions for \( w_0 \).
Figure 9. Joint constraints on \((w_0, w_a)\) at the 68% and 95% CL. We show forecasts for the combination of the CMB and the CMB-LSS cross-correlation from eROSITA (green) and from the optimistic Euclid photometric cluster survey (blue). We show also the results for the CMB alone (yellow).

Figure 10. Joint constraints on \((w_0, w_a)\) at the 68% and 95% CL. We show forecasts for the combination of the CMB and the CMB-LSS cross-correlation from the optimistic Euclid photometric cluster survey with (red) and without (blue) assuming a perfect knowledge of the scaling relation, and from the photometric Euclid galaxy survey (gray). We also show the results for the CMB alone for comparison (yellow).

without perfect knowledge of the scaling relation, and the CMB-galaxy cross-correlation from Euclid. Assuming a perfect knowledge of the nuisance parameters connected to the cluster selection function leads to an improvement in the constraints obtainable with the help of the CMB-cluster cross-correlation. In particular, we found an improvement of 10% for \(w_0\), of 20% and 40% for \(w_0\) and \(w_a\), of 5% for \(k_c\) and \(\lambda_c\).

7 CONCLUSIONS

Cosmology with clusters is a rapidly evolving field, thanks to improvements in observations. Current catalogues are already sufficient to obtain an accurate measurement of the baryon acoustic oscillation with galaxy clusters as from the Sloan Digital Sky Survey leading to a distance-redshift relation in full agreement with Planck observations and with uncertainties similar to the one obtained by galaxy surveys (Veropalumbo et al. 2014, 2016). In the perspective of future surveys in several wavelengths which will provide catalogues with a higher number of clusters over a wider redshift range, we have studied the capabilities to detect the ISW effect by a CMB-cluster cross-correlation. Note that this ISW-cluster cross-correlation is not internal to CMB as the previously studied ISW-tSZ cross-correlation (Taburet et al. 2011). In this perspective, it will be essential to control the SZ cluster residuals in CMB maps that could bias the cross-correlation measurements between clusters and CMB (Chen et al. 2018).

As two representative cases, we have considered the specifications of the cluster catalogues as expected from two coming space missions, i.e. eROSITA (Predehl et al. 2010) and Euclid (Laureijs et al. 2011). We have found that the CMB-cluster cross-correlation could be used for a statistically significant detection of the ISW effect. As two general remarks, we have found that: (1) the signal-to-noise of the CMB-cluster cross-correlation benefits of the large cluster bias which balances the larger shot noise in cluster catalogues compared to galaxy surveys; (2) the dependence of the cluster bias on cosmology needs to be taken into account to extract the full information from the cross-correlation with CMB. Whereas for eROSITA the cross-correlation with CMB would be an added value for the legacy of its cluster catalogue, for Euclid it would constitute a sort of coarse graining of the CMB galaxy cross-correlation and a key verification of the structure formation process. Nevertheless, the CMB-cluster would be another valuable cross-correlation within the Euclid probes and would add useful information on the bias for the cluster catalogue. The cross-correlation studied here would be of interest for other surveys as well, as LSST (Abell et al. 2009) and SKA (Mao et al. 2015).

ACKNOWLEDGEMENTS

We thank Nabila Aghanim and Carlo Baccigalupi for useful comments to the paper. We acknowledge financial contribution from the agreement ASI n.1/023/12/0 “Attività relative alla fase B2/C per la missione Euclid”. The support by the ”ASI/INAF Agreement 2014-024-R.0 for the Planck LFI Activity of Phase E2” is also acknowledged. MB, DP, FF and LM, acknowledge the support from the grant MIUR PRIN 2015 "Cosmology and Fundamental Physics: illuminating the Dark Universe with Euclid”. MB was supported by the South African Radio Astronomy Observatory, which is a facility of the National Research Foundation, an agency of the Department of Science and Technology and the Claude Leon Foundation. BS acknowledges financial support from the University of Trieste through the program "Finanziamento di Ateneo per progetti di ricerca scientifica - FRA 2015", a grant from "Consorzio per la Fisica - Trieste" and from the PRIN 2015W7KAWC project, funded by the Italian Minister for University and Research.

MNRAS 000, 1–12 (2018)
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APPENDIX A: THE COVARIANCE IN THE FISHER APPROACH

Since we are interested in the effect coming from the cross-correlation between the CMB and LSS surveys alone, we do not consider information coming from the LSS tracer angular power spectrum, i.e. we use $C_{\ell}^{XX}$ just for the covariance. The non-vanishing elements of the symmetric angular power spectrum covariance matrix Cov at the $\ell$th multipole are:

$$\text{Cov}(C_{\ell}^{AB}, C_{\ell}^{CD}) = C_{\ell}^{AB} C_{\ell}^{CD} + C_{\ell}^{AD} C_{\ell}^{BC}. \tag{A1}$$

The exact structure of Cov$^{-1}$ is more useful compared to its inverse since it allows to derive all the reduced cases for its inverse when some cross-correlation terms go to zero:

$$\text{Cov}^{-1} = \frac{1}{(|C(\theta)|^2)} \begin{pmatrix}
\theta_{TTTT} & \theta_{TTTE} & \theta_{TTXX} & \theta_{TTEE} & \theta_{TTEE} & \theta_{TXXX} & \theta_{TTEX} & \theta_{TXXE} & \theta_{TXXX} & \theta_{TTEE} \\
\theta_{TTTE} & \theta_{EEEE} & \theta_{ETEE} & \theta_{EEXX} & \theta_{ETEX} & \theta_{EEEX} & \theta_{ETXE} & \theta_{EEXX} & \theta_{EEEX} & \theta_{EEXX} \\
\theta_{TTXX} & \theta_{ETEE} & \theta_{TEEE} & \theta_{ETXX} & \theta_{TEXX} & \theta_{ETXX} & \theta_{TEXX} & \theta_{EEXX} & \theta_{EEXX} & \theta_{EEXX} \\
\theta_{TTEE} & \theta_{ETEE} & \theta_{TEEE} & \theta_{ETXX} & \theta_{TEXX} & \theta_{ETXX} & \theta_{TEXX} & \theta_{EEXX} & \theta_{EEXX} & \theta_{EEXX} \\
\theta_{TTEE} & \theta_{ETEE} & \theta_{TEEE} & \theta_{ETXX} & \theta_{TEXX} & \theta_{ETXX} & \theta_{TEXX} & \theta_{EEXX} & \theta_{EEXX} & \theta_{EEXX} \\
\theta_{TXXX} & \theta_{ETEX} & \theta_{TEEX} & \theta_{ETXX} & \theta_{TEXX} & \theta_{ETXX} & \theta_{TEXX} & \theta_{EEXX} & \theta_{EEXX} & \theta_{EEXX} \\
\theta_{TTEX} & \theta_{ETEX} & \theta_{TEEX} & \theta_{ETXX} & \theta_{TEXX} & \theta_{ETXX} & \theta_{TEXX} & \theta_{EEXX} & \theta_{EEXX} & \theta_{EEXX} \\
\theta_{TXXE} & \theta_{ETXE} & \theta_{TEXE} & \theta_{ETXX} & \theta_{TEXX} & \theta_{ETXX} & \theta_{TEXX} & \theta_{EEXX} & \theta_{EEXX} & \theta_{EEXX} \\
\theta_{TXXX} & \theta_{ETXX} & \theta_{TEXX} & \theta_{ETXX} & \theta_{TEXX} & \theta_{ETXX} & \theta_{TEXX} & \theta_{EEXX} & \theta_{EEXX} & \theta_{EEXX} \\
\theta_{TTEE} & \theta_{ETEE} & \theta_{TEEE} & \theta_{ETXX} & \theta_{TEXX} & \theta_{ETXX} & \theta_{TEXX} & \theta_{EEXX} & \theta_{EEXX} & \theta_{EEXX}
\end{pmatrix}. \tag{A2}$$

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where the auto-correlation are:

\[ \Sigma_{TTTT} = \left[ (C_{TE}^{EX})^2 - C_{TE}^{EE} C_{TE}^{XX} \right]^2, \quad (A3) \]

\[ \Sigma_{EEEE} = \left[ (C_{TE}^{TX})^2 - C_{TE}^{TT} C_{TE}^{XX} \right]^2, \quad (A4) \]

\[ \Sigma_{TETE} = 2 \left\{ 2 \left( C_{TE}^{TX} \right)^2 \left( C_{TE}^{TE} \right)^2 \right. \]
\[ + \left. \left( C_{TE}^{XX} \right)^2 \left[ (C_{TE}^{TX})^2 - C_{TE}^{TT} C_{TE}^{EE} \right] \right. \]
\[ - C_{XX} \left[ 2 C_{TE}^{TX} C_{TE}^{EX} + C_{TE}^{TT} (C_{TE}^{EX})^2 \right] \]
\[ + \left. C_{TE}^{EE} \left( C_{TE}^{TX} \right)^2 \right\}, \quad (A5) \]

\[ \Sigma_{XXXX} = \left[ (C_{TE}^{XX})^2 - C_{TT}^{EE} C_{TE}^{EX} \right]^2, \quad (A6) \]

\[ \Sigma_{TXXT} = 2 \left\{ -2 C_{TE}^{EE} C_{TE}^{TX} C_{TE}^{EX} \right. \]
\[ + \left. \left( C_{TE}^{EX} \right)^2 \left[ (C_{TE}^{TX})^2 - C_{TE}^{TT} C_{TE}^{EE} \right] \right. \]
\[ + C_{TE}^{EE} \left[ C_{TE}^{TX} C_{TE}^{EX} + C_{TE}^{EE} \left( C_{TX}^{EX} \right)^2 \right] \]
\[ - C_{XX} \left[ 2 C_{TE}^{TX} C_{TE}^{EX} + C_{TE}^{EE} C_{TX}^{EX} \right] \left\}, \quad (A7) \right. \]

\[ \Sigma_{EXEX} = 2 \left\{ 2 \left( C_{TE}^{TX} \right)^2 \left( C_{TE}^{XX} \right)^2 \right. \]
\[ + \left( C_{TE}^{TX} \right)^2 \left[ (C_{TE}^{EX})^2 + C_{TE}^{EE} C_{XX} \right] \right. \]
\[ - C_{TX} \left\{ 2 C_{TE}^{TX} C_{TE}^{EX} + C_{TE}^{EE} C_{TX}^{EX} \right\} \}, \quad (A8) \right. \]

and the cross-correlation are:

\[ \Sigma_{TTTE} = \left[ C_{XX} C_{TE}^{TE} - C_{TE}^{EE} C_{TX}^{EX} \right]^2, \quad (A9) \]

\[ \Sigma_{TTEE} = 2 \left\{ \left( C_{TE}^{TX} \right)^2 - C_{TE}^{EE} C_{XX} \right. \left( C_{TE}^{TX} C_{TE}^{EX} \right. \]
\[ - C_{TX} \left) C_{TE}^{XX} - C_{TE}^{EE} C_{XX} \right\} \right\}, \quad (A10) \]

\[ \Sigma_{TTXX} = \left( C_{TE}^{EE} C_{TX}^{EX} \right)^2, \quad (A11) \]

\[ \Sigma_{TTTX} = 2 \left\{ \left( C_{TE}^{TX} \right)^2 - C_{TE}^{EE} C_{XX} \right. \left( C_{TE}^{TX} C_{TE}^{EX} \right. \]
\[ - C_{TX} \left) C_{TE}^{XX} \right\} \right\}, \quad (A12) \]

\[ \Sigma_{TTXX} \]
\[ \Sigma_{TTXX} = 2 \left\{ \left( C_{TE}^{TX} \right)^2 - C_{TE}^{EE} C_{XX} \right. \left( C_{TE}^{TX} C_{TE}^{EX} \right. \]
\[ - C_{TX} \left) C_{TE}^{XX} \right\} \right\}, \quad (A13) \]

\[ \Sigma_{EEET} = 2 \left\{ \left( C_{TE}^{TX} \right)^2 + C_{TT}^{TX} C_{XX} \right. \left( C_{TE}^{TX} C_{TE}^{EX} \right. \]
\[ - C_{TX} \left) C_{TE}^{XX} \right\} \right\}, \quad (A14) \]

\[ \Sigma_{EEXX} = \left( C_{TE}^{TX} C_{XX} - C_{TE}^{TX} C_{EX} \right)^2, \quad (A15) \]

\[ \Sigma_{EETX} = 2 \left\{ \left( C_{TE}^{TX} C_{XX} \right)^2 - C_{TE}^{TX} C_{XX} \right. \left( C_{TE}^{TE} C_{TX} \right. \]
\[ - C_{TX} \left) C_{TE}^{XX} \right\} \right\}, \quad (A16) \]

\[ \Sigma_{EEEX} = 2 \left\{ \left( C_{TE}^{TX} \right)^2 - C_{TE}^{TX} C_{XX} \right. \left( C_{TE}^{TX} C_{TE}^{EX} \right. \]
\[ - C_{TX} \left) C_{TE}^{XX} \right\} \right\}, \quad (A17) \]

\[ \Sigma_{TXXX} = 2 \left( C_{TE}^{EE} C_{XX} C_{TX}^{EX} \right) \left( C_{TE}^{TT} C_{TE}^{EX} - C_{TE}^{TE} C_{TX} \right), \quad (A18) \]

\[ \Sigma_{TXXX} = -2 \left( -2 C_{TE}^{EE} C_{TX}^{EX} C_{TX}^{XX} - C_{TT}^{XX} \left( C_{TE}^{EX} \right)^3 \right) \]
\[ + C_{TE}^{EX} \left[ \left( C_{TE}^{TX} \right)^2 C_{TE}^{EE} \right] \]
\[ + C_{XX} \left[ \left( C_{TE}^{TX} \right)^2 + C_{TE}^{TT} C_{TE}^{EE} \right] \), \quad (A19) \]

\[ \Sigma_{TXXX} = -2 \left( C_{TE}^{EE} \left( C_{TE}^{TX} \right)^3 - C_{TX}^{XX} \left( C_{TE}^{TX} \right)^3 \right) \]
\[ + C_{XX} \left[ \left( C_{TE}^{TX} \right)^2 C_{TX}^{XX} - 2 C_{TE}^{TT} C_{TE}^{EX} \right. \right. \]
\[ + C_{TE}^{EE} \left( C_{TE}^{TX} \right)^3 \}], \quad (A20) \]

\[ \Sigma_{TXXX} = 2 \left( C_{TE}^{TX} \right)^2 \left( C_{TE}^{EX} - C_{TE}^{TX} \right) \left( C_{TE}^{TX} C_{XX} - C_{TE}^{TX} C_{TE} \right), \quad (A21) \]

\[ \Sigma_{TXXX} = 2 \left( C_{TE}^{TX} \right)^2 \left( C_{TE}^{TX} C_{XX} - C_{TE}^{TX} C_{TE} \right), \quad (A22) \]

\[ \Sigma_{TXXX} = -2 \left( \left( C_{TE}^{TX} \right)^2 + C_{TE}^{TX} C_{EX} \right) \]
\[ + C_{TX} \left[ \left( C_{TE}^{TX} \right)^2 \right. \]
\[ + C_{XX} \left[ \left( C_{TE}^{TX} \right)^3 \right] \right\}, \quad (A23) \]

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