Study of diffusion influence on plasma channel while transporting a low-energy high intensity electron beam in the low pressure gas

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Abstract. This work studies the mathematical model of plasma channel. We consider the beam current in the range of 100-400 A and the external magnetic field in the range of 100-1500 G. It is shown that plasma channel expands under the influence of diffusion. The channel expansion is inversely proportional to the external magnetic field magnitude.

1. Basic equations
Gas ionization in the drift space is caused by the gas atom ionization by fast beam electrons as well as by fields on the front edge of the beam. The change in plasma channel parameters such as an electron density, plasma temperature and plasma conductivity will depend on the gas pressure, geometry of the drift tube and beam parameters. This will lead to the plasma current appearance, the current neutralization of the beam and conditions of the beam transport change. The plasma current changes the magnetic field of the beam, and this current must be taken into account while calculating the vector potential.

We consider a model that describes the beam transport in gas at pressure of $p < 0.1$ Torr under the full charge neutralization of the beam. An additional model of initial conditions was used to obtain the full charge neutralization.

Let us assume that an electron beam with radius $r_0$ is injected into a plane $z = 0$ along the axis $z$ of the drift tube with radius $R_c$. The basic heterogeneity is associated with beam and plasma density distribution along the radius because the beam transit time through the drift tube is much smaller than the beam rise time. The beam and plasma fields in the cylindrical coordinate system are described by a nonlinear equation:

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial A_z}{\partial r} \right) = -\frac{4\pi}{c} \left( j_{be} + j_{pc} \right),$$  \hspace{1cm} (1)

where $A_z$ – vector potential, which satisfies the boundary conditions $A_z(r=R_c) = \partial A_z/\partial r |_{r=0} = 0$ and determines an electric field $E_z = -1/c(\partial A_z/\partial t)$ and a magnetic field $B_0 = -\partial A_z/\partial r$; $j_{be} = e\nu_b n_b$ – beam current density; $j_{pc}$ – plasma current density; $c$ – speed of light; $e$ – elementary electric charge; $\nu_b$ – velocity of beam electrons; $n_b$ – beam density. Let us suppose the drift tube has an ideal wall conductivity. The drift tube has ground potentials on its ends. The plasma current is equal to zero at the reference time. The initial condition for the vector potential is derived from the equation (1).

The interaction among the plasma current density, the electric field and the plasma conductivity is determined by the equation:
\[
\frac{1}{\nu_e} \frac{\partial j_{pe}}{\partial t} = \sigma E_z - j_{pe},
\] (2)

where \(\sigma = e^2 n_e / (m_e \nu_{e0})\) – plasma conductivity; \(n_e\) – electron mass; \(n_e\) – plasma electron density; \(\nu_{e0} = \nu_{e0(1)} + \nu_{e0(2)}\) – effective collision frequency of plasma electrons and heavy particles; \(\nu_{e0}\) – collision frequency of plasma electrons and gas atoms; \(\nu_{e0} = 1.45 \times 10^{-6} n_i T_e^{-3/2} \ln(2.4 \times 10^{20} T_e^{1/2} n_i^{(3)})\) – collision frequency of plasma electrons and ions; \(n_i^{(3)}\) – the atomic \((l=1)\) and molecular \((l=2)\) ion density; \(T_e\) – electron plasma temperature in eV [1]; \(\nu_{oa} = 3.7 \times 10^{-8} n_e T_e\) for argon, \(\nu_{oa} = 4.4 \times 10^{-8} n_e T_e^{0.5}\) for helium [2]; \(n_g\) – gas density.

The balance of particles is described by the following equations:

\[
\frac{\partial n_{i}\left(t\right)}{\partial t} = \left(\sigma_{ib}\nu_b\right) n_p n_g + \left(\sigma_{ie}\nu_e\right) n_p n_g - K_i n_i^{(1)} n_e^{(2)} - \alpha_r n_i^{(2)} n_e^{(2)} - \ldots
\] (3)

\[
\ldots - \alpha_{re} n_i^{(1)} n_e^{(2)} - \alpha_{rd} n_i^{(2)} n_e^{(2)} + \text{div} \left(D_{\perp} \text{grad} n_i^{(2)}\right),
\]

\[
\frac{\partial n_{e}}{\partial t} = K_i n_i^{(1)} n_e^{(2)} - \alpha_{re} n_i^{(2)} n_e^{(2)} - \ldots
\] (4)

\[
\ldots + \alpha_{re} n_i^{(1)} n_e^{(2)} + \alpha_{rd} n_i^{(2)} n_e^{(2)} + \alpha_{rd} n_i^{(1)} n_e^{(2)} + 2\alpha_{rd} n_i^{(2)} n_e^{(2)},
\]

\[
n_e = n_i^{(1)} + n_i^{(2)} - n_b,
\] (5)

where \(\sigma_{ib}, \sigma_{ie}\) – ionization cross-sections by beam and plasma electrons [3]; \(\nu_e\) – velocity of plasma electrons; \(\alpha_{1}, \alpha_{2}, \alpha_{3}, \alpha_{rd}\) – recombination coefficients: impact, neutral third particle, radiation and dissociative; \(K_i\) – conversion coefficient of atomic ions into molecular ones; \(D_{\perp}\) – ambipolar diffusion coefficient. Boundary conditions for the plasma ion density and the ambipolar diffusion coefficient are similar to the vector potential boundary conditions. The recombination coefficients for argon atoms are determined by formulas:

\[
a_{1} = 4.7 \times 10^{-5} T_e^{-6/2} \text{ cm}^3/\text{s}, \quad a_{2} = 1.09 \times 10^{-3} T_e^{-5/2} \text{ cm}^3/\text{s},
\]

\[
a_{3} = 2.7 \times 10^{-13} T_e^{-3/4} \text{ cm}^3/\text{s}, \quad a_{rd} = 9.1 \times 10^{-7} (0.026/T_e^{0.01}) \text{ cm}^3/\text{s}.
\]

The ambipolar diffusion coefficient is calculated by:

\[
D_{\perp} = \frac{c^2}{\nu_e \frac{\Omega_e^2 + \nu_{ie}^2}{m_e} + \frac{4 \Omega_i^2 + \nu_{ie}^2}{m_i} \nu_{ia}},
\] (6)

where \(T_i\) – plasma ion temperature in eV; \(\Omega_e, \Omega_i\) – cyclotron frequencies of the plasma electrons and ions; \(m_i\) – ion mass; \(\nu_{ia} = 4.6 \times 10^{10} n_i T_i^{0.5}\) – collision frequency of plasma ions and gas atoms [1].

The connection of the plasma electron temperature and the electric field that includes both elastic and non-elastic collisions of plasma electrons with heavy particles, is determined by [7]:

\[
\frac{\partial T_e}{\partial t} = \frac{2}{3} \frac{E_0 r_0}{m_e \nu_{e0}} \left(\frac{E_e^2}{p^2} \left(\frac{T_e}{k}\right)^k\right),
\] (7)

where \(k = 11.65\) for argon and \(k = 18\) for helium [2]; \(r_0\) – classical electron radius; \(E_0\) – electron mass energy equivalent in eV.

As the aforementioned model describes the beam transport under the full charge neutralization, it is necessary to determine the initial conditions for the plasma density at various beam injection modes.
2. Initial conditions
When the electron beam is injected into a neutral gas, the basic processes of the plasma channel are the beam passing current ionization and the plasma electron ionization in the space-charge field of the beam:

\[
\frac{dn_i}{dt} = \sigma_{ib} n_g \frac{j_q}{e} + \sigma_{ie} n_g v_e n_e,
\]

\[
\frac{dn_e}{dt} = \frac{dn_i}{dt} - n_e / \tau_e,
\]

\[
n_g = 3.5 \cdot 10^6 p - n_i, \tag{11}
\]

where \(n_i\) – plasma ion density; \(j_q\) – passing current density; \(\tau_e = L / v_e\) – mean time of plasma electron liberation at the drift tube; \(L\) – length of the drift tube.

When the beam is injected in the neutral gas, there is no space-charge neutralization of the beam. When an external magnetic field \(B_z\) is strong, the beam is kept from the transverse separation. The losses of the transported beam current are associated with the virtual cathode when

\[
I_b \geq I_{pr}, \tag{12}
\]

where \(I_b\) – injected beam current; \(I_{pr}\) – passing current. The virtual cathode (VC) arises at a distance of \(d = 1.5 \cdot 10^{-3} U^{0.4} j_{ib}^{0.5}\) from the injection plane, where \(j_{ib}\) – injected beam current density; \(U\) – VC voltage. Only one part of the injected beam current passes through the drift tube. It does not exceed the limiting current, which equals

\[
I_{pr} = I_{pr}^{(0)} / (1 - f), \tag{13}
\]

\[
I_{pr}^{(0)} = \frac{2}{9} I_{pr}^{(0)} \beta \left(1 + \frac{4}{3} \ln \left(\frac{R}{r_b}\right)\right)^{-3/2}, \tag{14}
\]

where \(I_{pr}^{(0)}\) – limiting vacuum current [8], \(f = n_i/n_b\) – charge neutralization of the beam; \(I_{pr}^{(0)}\) – Alfven critical current; \(\beta\) – relative electron velocity. Formula (14) gives satisfactory results when \(R_0/r_b\) changes from 1 to 10. VC disappears when the inequality (12) is false, and the beam current starts to pass to a full degree.

The passing beam electrons ionize the gas. As a result, the degree of the beam charge neutralization increases with time. This leads to an increase of the passing current. The full charge neutralization is achieved at the moment, when \(f\) is equal to one. The values of the beam and plasma parameters at this moment are taken as initial conditions.

In this work, we describe the initial conditions when there is a strong external magnetic field. The given initial conditions model is a separate part of the total ionization and plasma channel program. This model takes into account the transient mode. A similar model for the high-current beams in the absence of an external magnetic field is presented in work [9].

3. Simulated results
We consider a uniform beam injection into the drift tube filled with the inert gas. The beam parameters correspond to experiment [10]: \(r_b = 2\) cm, \(R_e = 4.1\) cm, \(L = 20\) cm. The beam profile [10] (current \(\sim 300\) A, pulse duration \(\sim 145\) \(\mu\)s, leading edge pulse time \(\sim 20\) \(\mu\)s) is shown in figure 1.

The slow change of the beam current is the cause of the plasma current smallness and the electric field strength of the total current (fig. 1, 2). At the edge of the injected current pulse, the plasma current is nearly non-existent. This occurs as the pulse duration is big enough and the fall in the plasma current is quite fast. At the falling edge, the plasma current changes its direction and increases the total current. The result does not depend on the external magnetic field.

The electric and the magnetic fields become lower at smaller values of the beam current maximum (fig. 2). The magnetic field has a common radius distribution throughout the entire pulse – it is equal to zero on the axis and has the maximum value on the beam boundary.
The figure 3 shows leading edge erosion of the beam due to the formation of VC. At a higher pressure, the virtual cathode disappears or is not formed. It is proved that the virtual cathode is not formed in helium when the pressure is $3 \times 10^{-3}$ Torr and it is formed when pressure is $10^{-3}$ Torr. Other parameters influence the virtual cathode too.

**Figure 1.** The time dependences of the plasma current (dashed line), the beam current (dotted line) and the total current (dash-and-dot line). $p = 10^{-3}$ Torr, $e_b = 30$ keV, $I_{b0} = -300$ A, $B_z = 100, 300, 1500$ G, Ar.

**Figure 2.** (a) The time dependences of the electric field strength of the total current on the axis; (b) The radial dependences of the magnetic field of the total current at the pulse peak. $p = 10^{-3}$ Torr, $e_b = 30$ keV, $B_z = 300$ G, Ar. The solid line $- I_{b0} = -100$ A, the dashed line $- I_{b0} = -300$ A, the dotted line $- I_{b0} = -400$ A.
The argon ionization cross-sections are higher than helium. As a result, the argon is ionized faster than helium. In argon, the full gas ionization occurs quickly. The figure 4 shows the dependence of diffusion on the external magnetic field. It is seen that there is no diffusion in argon, and helium, when the external magnetic field equals 1.5 kG. The plasma channel expands rapidly when the external magnetic field equals 100 G.

**Figure 3.** The leading edge erosion of the beam. He, $\varepsilon_b = 30$ keV, $I_{b0} = -300$ A, $B_z = 1500$ G. The solid line $- p = 5 \times 10^{-4}$ Torr, the dashed line $- p = 10^{-3}$ Torr, the dotted line $- p = 3 \times 10^{-3}$ Torr, the injected beam current.

**Figure 4.** The radial dependences of the plasma ion density at the pulse peak. $I_{b0} = -100$ A, $p = 10^{-3}$ Torr, $\varepsilon_b = 30$ keV. The solid line $- B_z = 100$ G, the dashed line $- B_z = 300$ G, the dotted line $- B_z = 1500$ G. (a) Ar, (b) He.

4. Conclusion
The developed model allows calculating the plasma current, the electric and magnetic fields, the diffusion of the plasma channel, the plasma electron temperature, the gas ionization and recombination, the leading edge erosion of the beam, etc. The model is suitable for a wide range of parameters: the gas pressure is $10^{-4}$-$10^2$ Torr, the beam energy is 10-30 keV, the beam current is 100 A-15 kA and the microsecond impulses.

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