Can Non-unitary Effect be Prominent in Neutrino Oscillation Measurements?

Lei Lü, Wenyu Wang, Zhaohua Xiong
Institute of Theoretical Physics, College of Applied Science,
Beijing University of Technology, Beijing 100124, China

Subject to the neutrino experiments, the mixing matrix of ordinary neutrinos can still have small violation from unitarity. We introduce a quasi-unitary matrix to interpret this violation and propose a natural scheme to parameterize it. A quasi-unitary factor $\Delta_{QF}$ is defined to be measured in neutrino oscillation experiments and our numerical results show that improvement of experimental precision may help us figure out the secret of neutrino mixing.

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1. Introduction The definite evidence of neutrino oscillation has confirmed that neutrinos are massive and they exhibit non-trivial mixing, which strengthened our belief in new physics beyond Standard Model (SM). The compelling experiments verifying neutrino masses and mixing angles are from the neutrino oscillation measurements including solar (SNO, KamLAND) [1, 2], atmosphere (SK) [3] and reactor (CHOOZ) [4] neutrino experiments. These experiments can give the squared-mass splitting and trigonometric function of mixing angles. Up to now, all the other experiments to measure the absolute values of neutrino masses can only give upper limits and relatively rough mixing angles.

To explain neutrino masses, adding the neutrino Yukawa coupling (which gives the Dirac mass of neutrino) to the SM lagrangian is straightforward and causes no anomaly, but the real problem is the huge hierarchy between the up component ($\nu_e$, $\nu_\mu$, $\nu_\tau$) and the down component ($e$, $\mu$, $\tau$) of lepton isospin doublet which is very different from that in quark sector. Among the mechanisms proposed to generate very light neutrino masses, the most popular one is the seesaw mechanism [5-7]. In seesaw models heavy right-handed Majorana neutrinos are introduced. The Dirac masses of neutrinos at electroweak scale are suppressed by Majorana mass terms to be ultralight Majorana neutrinos which are (primarily) left-handed. This mechanism embodies not only the mixing of ordinary light neutrino flavors which is similar to the quark mixing, but also the mixing between ordinary and additional heavy neutrinos which is different from charged fermions.

Note current experimenters and data analyzers work in the framework of three-flavor neutrino, i.e., as the CKM matrix in quark sector, the mixing angles in parameterization of the mixing matrix extracted from the experiments just are based on a assumption that the mixing matrix related the neutrino mass eigenstates to the interaction ones is a $3 \times 3$ unitary matrix, namely PMNS matrix [3]. However, if there do exist more than three neutrino species, the matrix that transforms the ordinary neutrino mass eigenstate to the flavor eigenstate should not be inherently unitary [8, 9] as the additional heavy neutrinos can mix with the three ordinary ones and make the unitarity of the PMNS matrix deviated.

The non-unitary effects of ordinary neutrino mixing has been extensively studied in literature, giving possible correction to unitarity and further with new parameterizations for the neutrino mixing matrix [10, 11, 12, 13, 14]. While models like these emphasized on obtaining a more physical interpretation of neutrino mixing, there is still one pressing question needed to be answered: Can the non-unitary effects stand out within current or forthcoming measurement precision? That is to say whether the violation from unitarity tests in experiments will be more of experimental error or theoretical non-unitary effects itself. The answer of the question is more fundamental in understanding of neutrino mixing.

In this Letter we will start from a relatively simpler but reasonably parameterization for the ordinary neutrino mixing matrix. A quasi-unitary matrix is proposed to interpret the violation from unitary. For testing the unitary of the neutrino mixing matrix, we will introduce a quasi-unitary factor $\Delta_{QF}$, for demonstrate we also will give some numerical results and show in which situation the non-unitary effects can be directly measured in neutrino oscillation experiments. Finally we will present some discussions and summary.

2. Quasi-unitary mixing and the parameterization To test the unitarity of neutrino mixing is important because the unitary mixing implies only three Dirac neutrinos with masses coming form the Yukawa coupling while the quasi-unitary mixing will imply the existence of more than three neutrino species and new mechanism for the generation of neutrino mass. Unfortunately, up to now, by test of $\mathbf{NN}^\dagger$ ($\mathbf{N}$ is neutrino mixing matrix measured from experiments) [9] we can not determine the error is whether from experiment measurements or the ordinary neutrino mixing matrix is non-unitary theoretically. Subject to constraints from weak decays, the mixing matrix can be violated from unitarity at the order of $1\%$ level [9]. If this mixing matrix is inherently quasi-unitary with the definition:

$$A = (I + X)U$$

where $X$ is a small matrix at subleading order (1st order), and $U$ is a unitary mixing matrix:

$$UU^\dagger = I.$$
If an ideal condition $X + X^\dagger = 0$ is employed, then

$$AA^\dagger = I + XX^\dagger, \quad (3)$$

thus the violation from unitarity slightly occurs at the second order. In this case, for three generation ordinary neutrinos, just three parameters are introduced to parameterize the anti-symmetric matrix $X$, the unitarity-deviation of $A$ is at a lower order than the deviation from $I$ of $AA^\dagger$.

As mentioned earlier, in our letter we try to use a simple parameterization for neutrino mixing. At first feeling, it seems that an anti-symmetric matrix $X$ to interpret the violation is a ideal choice. However, this is not reliable in many neutrino mixing models such as seesaw models. For example, in seesaw mechanism the mixing matrix for the diagonalization of left- and right-handed neutrinos should be an overall unitary matrix $O$ instead of the $3 \times 3$ PMNS matrix for the left-handed neutrinos which is only part of $O$. To be more clear, we denote the mixing matrix $O$ as

$$O_{2n \times 2n} = \begin{pmatrix} A_{n \times n} & C_{n \times n} \\ D_{n \times n} & B_{n \times n} \end{pmatrix}, \quad (4)$$

where $A$ is the matrix that transforms mass eigenstates of three left-handed neutrinos to the flavor eigenstates with $n$ as the flavor number. The unitarity of $O$ requires that:

$$AA^\dagger + CC^\dagger = I. \quad (5)$$

We also denote $A = (I + X)U$ as the quasi-unitary definition in Eq. (1), where $X$ is a small arbitrary matrix at subleading order. Thus we have

$$AA^\dagger = I + X + X^\dagger + XX^\dagger = I - CC^\dagger \quad (6)$$

In fact, the matrix $O$ diagonalizes the neutrino mass matrix in two steps. The first step is a block diagonalization which reduces the problem to a two by two problem and the rotation is essentially a generalized $2 \times 2$ Euler rotation. The second step is the diagonalization of light neutrino mass matrix by a unitary matrix $U_1$ and that of the heavy one by $U_2$. Then the overall matrix is given by

$$O_{2n \times 2n} = \begin{pmatrix} \cos \Theta U_1 & \sin \Theta U_1 \\ -\sin \Theta U_2 & \cos \Theta U_2 \end{pmatrix}, \quad (7)$$

where the $\sin \Theta$ and $\cos \Theta$ are to be interpreted as series expansions of a small matrix $\Theta = m_D/m_M^R$. From Eq. (1)

$$A = \cos \Theta U_1 = \left( I - \frac{\Theta^2}{2!} + \frac{\Theta^4}{4!} - \cdots \right) U_1, \quad (8)$$

$$C = \sin \Theta U_1 = \left( \Theta - \frac{\Theta^3}{3!} + \frac{\Theta^5}{5!} - \cdots \right) U_1. \quad (9)$$

we can see that $X + X^\dagger$ is at the same order of $CC^\dagger$ thus $A$ is violated from unitarity at the first order. Since we know nothing about $m_D$ and $m_M^R$, in order to simplify the problem, we assume that

$$C \sim U/\eta, \quad (10)$$

where $1/\eta$ is a small number. Then we have

$$AA^\dagger \sim I + X + X^\dagger \sim I - \frac{1}{\eta^2}. \quad (11)$$

Although the choice of the anti-symmetric $X$ is not true in many models, for simplification, we still can treat $X$ as a real matrix. (Strictly speaking, the matrix $X$ should be hermitian in seesaw mechanism, the deviation will occur at 1st order. Here we just use a approximate matrix to substitute the transformation matrix $O$, the diagonalization is not consistent with the seesaw mechanism, but it can simplify the parameterization for the quasi-unitary matrix and give us a way to test order of unitarity violation.) thus $X$ can be parameterized as

$$X = \begin{pmatrix} -\epsilon & x & y \\ -x & 1-\epsilon & z \\ -y & -z & 1-\epsilon \end{pmatrix}, \quad (12)$$

for two-flavor case and

$$X_{2 \times 2} = \begin{pmatrix} 1-\epsilon & x \\ -x & 1-\epsilon \end{pmatrix} \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \quad (13)$$

$$X_{3 \times 3} = \begin{pmatrix} 1-\epsilon & x & y \\ -x & 1-\epsilon & z \\ -y & -z & 1-\epsilon \end{pmatrix} \times \begin{pmatrix} c_{13} c_{12} & s_{12} & s_{13} \\ -c_{23} s_{12} - s_{23} s_{13} c_{12} & c_{23} c_{12} - s_{23} s_{13} s_{12} & s_{23} c_{13} \\ s_{23} s_{12} - c_{23} s_{13} c_{12} & -s_{23} c_{12} & c_{23} c_{13} \end{pmatrix} \quad (14)$$

for three-flavor case. Here $\theta$ ($\theta_{ij}$) is the rotation angle used to parameterize the unitary matrix $U_{2 \times 2}$ ($U_{3 \times 3}$) and $s_{ij}$ ($c_{ij}$) means $\sin \theta_{ij}$($\cos \theta_{ij}$). We do not consider the CP phase $\delta$ in three-flavor case. In addition to the unitary parameterization, there are two more parameters for the $2 \times 2$ quasi-unitary matrix and four more for the $3 \times 3$ one. We should note that

1. Here we just treat $\epsilon$ as a parameter which can be set as any small real number even zero to make $A$ a quasi-unitary matrix, thus our parameterization is not totally based on seesaw mechanism but on a much simpler and more generalized way to parameterize the quasi-unitary matrix;

2. Unlike the previous works [11, 12, 13, 14], in our parameterization $AA^\dagger$ is deviated from $I$ only in the diagonal elements at the first order when $\epsilon \neq 0$. This is consistent with the result of [9] in which $NN^\dagger$ is deviated from $I$ apparently in the diagonal elements;
3. In seesaw interpretation, parameters $\epsilon$, $x$, $y$, $z$ are to be too small (typically $\mathcal{O}(10^{-26})$) to give any implication to the neutrino measurements, thus the neutrino mixing matrix $A$ can be safely parameterized as a unitary matrix. But in general, $\epsilon$, $x$, $y$, $z$ can be not so small, they may make unitary parameterization deviated with observable errors, which should be theoretical errors.

3. Implication on neutrino oscillation Now let us focus on the implication of such parameterization in neutrino oscillation experiments. Ever since the issue of neutrino mixing was on the table, it has always been assumed that the neutrino flavor eigenstates $|\nu_{\alpha}\rangle$ ($\alpha = e, \mu, \tau$) were linear superposition of the neutrino mass eigenstates $|\nu_i\rangle$ ($i = 1, 2, 3$) through a unitary leptonic mixing matrix: $U^*_{\alpha i}$ and $|\nu_{\alpha}\rangle = \sum U^*_{\alpha i}|\nu_i\rangle$. By oscillating when propagating in vacuum, the oscillation probabilities can be expressed as (two flavors case $n = 2$)\[15\):

$$P(\nu_{\alpha} \rightarrow \nu_{\beta}) \simeq S_{\alpha \beta} \sin^2 \left[ 1.27 \Delta m^2 (L/E) \right]$$

(15)

for $\alpha \neq \beta$ and

$$P(\nu_{\alpha} \rightarrow \nu_{\beta}) \simeq 1 - 4T_\alpha (1 - T_\alpha) \sin^2 \left[ 1.27 \Delta m^2 (L/E) \right]$$

(16)

for $\alpha = \beta$, where $E$ is the energy of neutrinos, $L$ is the length they travel, $\Delta m^2$ is the neutrino mass squared splitting and

$$S_{\alpha \beta} \equiv 4 \sum_{i \in U_p} |U_{\alpha i}|^2, \quad T_\alpha \equiv \sum_{i \in U_p} |U_{\alpha i}|^2.$$  

(17)

Here ‘i up’ denotes a sum over only those neutrino mass eigenstates that lie above $\Delta m^2$ or, alternatively, only those that lie below it. The unitarity of $U$ guarantees that $S_{\alpha \beta}$ and $4T_\alpha (1 - T_\alpha)$ get exactly the same results in Eq.(15) and Eq.(16), which yield to:

$$4T_\alpha (1 - T_\alpha) = S_{\alpha \beta} = \sin^2 2\theta.$$  

(18)

This is the two-flavor case and the same result occurs for the three-flavor case.

Inspired by Eq.(18), we define a ‘Quasi-unitary factor’ to test the unitarity of neutrino mixing:

$$\Delta_{QF} = 4T_\alpha (1 - T_\alpha) - S_{\alpha \beta}.$$  

(19)

For the unitary mixing $\Delta_{QF} = 0$ theoretically, so if we extract this factor from the experimental data, a non-zero result must have come from the experimental errors. But in the quasi-unitary case, a non-zero result should be partially from the quasi-unitarity. If $\epsilon = 0$, $\Delta_{QF}$ still equates to zero at the leading order since

$$4T_\alpha (1 - T_\alpha) \simeq S_{\alpha \beta} \simeq \sin^2 2\theta + 2x \sin 4\theta,$$

(20)

which is similar to Eq.(18), but the situation changes when $\epsilon \neq 0$, we have

$$4T_\alpha (1 - T_\alpha) \simeq \sin^2 2\theta + 2x \sin 4\theta - 4\epsilon \sin^2 2\theta + 8\epsilon \sin^2 \theta,$$

(21)

$$S_{\alpha \beta} \simeq \sin^2 2\theta + 2x \sin 4\theta - 4\epsilon \sin^2 2\theta,$$

(22)

to zero:

$$\Delta_{QF} \simeq 8\epsilon \sin^2 \theta + \mathcal{O}(x^2, \epsilon^2, \ldots).$$

(23)

Thus, by comparing the neutrino oscillation data between $P(\nu_{\alpha} \rightarrow \nu_{\beta})$ and $P(\nu_{\alpha} \rightarrow \nu_{\beta})$, we can test the unitarity of neutrino mixing. One should realize the mixing angle $\theta$ appeared in Eq.(18) and Eq.(22) has a different meaning from the ordinary one measured under the unitary parameterization, which is essentially an effective mixing angle, denoted as $\theta^2$ in our subsequent discussions. Meanwhile, as indicated by Eq.(23), it is the diagonal parameter $\epsilon$ in $X$ that dominates $\Delta_{QF}$ while off-diagonal effect by $x$, $y$, $z$ appears at subleading order thus can be absorbed into the effective unitary parameterization of the mixing matrix. But from Eq.(19), we know that $x$, $y$, $z$ together with $\epsilon$ can make $\theta^2$ (maybe greatly) deviated from $\theta$ and make $\Delta_{QF}$ big enough to be measured in the neutrino oscillation experiments.

Now we inspect the actual three-neutrino mixing situation and try to give the ‘quasi-unitary correction’ to certain explicit measurements. As mentioned above, in order to test the unitarity we need to measure $\Delta_{QF}$ dependent on a same $\theta^2_{ij}$, which needs data from different oscillation patterns. In case of $n = 3$, however, such checks are very difficult, all the current accelerator, reactor, solar and atmospheric neutrino data are described within the framework of the 3 x 3 PMNS matrix. And different experiments measure different $\theta_{ij}$ and $\Delta m^2$ without considering the non-unitarity of neutrino mixing. We find that short distance $\nu_e$ oscillation measurements may give us some hints. For short distance (L<5 km) it is a good approximation to express the $\nu_e$ oscillation probabilities in the unitary scheme as\[15\]:

$$P(\bar{\nu}_e \rightarrow \nu_e) \simeq 1 - \sin^2 (2\theta_{13}) \sin^2 (\Delta m^2_{13} L/4E)$$

$$P(\nu_e \rightarrow \nu_\mu) \simeq \sin^2 (2\theta_{13}) \sin^2 (\Delta m^2_{12} L/4E)$$

$$P(\nu_e \rightarrow \nu_\tau) \simeq \sin^2 (2\theta_{13}) \cos^2 (\Delta m^2_{13} L/4E).$$

(24)

This takes the similar two-neutrino form with $\theta_{13}$ and $\Delta m^2_{12}$. If the neutrino mixing is quasi-unitary, $\Delta_{QF}$ can be defined as:

$$\Delta_{QF} = 4T_\epsilon (1 - T_\epsilon) - S_{e\mu} - S_{e\tau}$$

$$\simeq 8\epsilon \sin^2 \theta^2_{13} + \mathcal{O}(x^2, \epsilon^2, \ldots),$$

(25)

which has the similar form to Eq.(23). The measurements of $P(\bar{\nu}_e \rightarrow \nu_e)$ have already been taken in the reactor neutrino experiments, however, $P(\nu_e \rightarrow \nu_\mu)$ and $P(\nu_e \rightarrow \nu_\tau)$.
TABLE I: Comparison of mixing angle ranges under different schemes.

| Unitary scheme [16] | Quasi-unitary scheme |
|---------------------|----------------------|
| Range               | Range               |
| $\theta_{12}^Q$    | $31.8^\circ - 36.4^\circ$ | $24.4^\circ - 43.1^\circ$ |
| $\theta_{13}^Q$    | $37.2^\circ - 50.9^\circ$ | $31.9^\circ - 56.6^\circ$ |
| $\theta_{13}^Q$    | $< 10.9^\circ$       | $< 18.4^\circ$                      |

$P(\nu_e \to \nu_{\tau})$ are difficult to measure in accelerator neutrino experiments. Thus we need more precise experiments to measure $\Delta_{QF}$ for the test of unitarity. Though there is no data for such test, the current experiments data can give us some implications about the quasi-unitary mixing.

4. Testing unitarity by experiments With the expression for the quasi-unitary factor prepared, we can present some numerical results and discuss the constraints on parameter space from current experiments and furthermore, the probability of test the unitarity by current or near future neutrino experiments. As known, Solar, Atmospheric and reactor neutrino experiments are sensitive to different neutrino oscillations, which give us the best fit mixing angles and the errors in unitary scheme. In the numerical calculation, our general method was to require the new matrix elements $A_{\alpha\beta}$ in Eq.(14) to satisfy corresponding constraints derived from the latest data [16].

1). Compare the effective mixing angles extracted in the unitary scheme and the quasi-unitary scheme. We scatter $\theta_{ij}^Q$ from $0^\circ$ to $90^\circ$, and scan the parameters $x, y, z$ randomly between $-0.1 \sim 0.1$ and $\epsilon$ randomly between $0 \sim 0.1$, keeping all experimentally allowed points. The results are listed in TABLE I in comparison with unitary scheme data derived from Ref.[16].

As seen in TABLE I with $x, y, z, \epsilon \neq 0$ and the same required constraints for neutrino mixing, the allowed ranges for neutrino mixing angles are expanded. If the parameters $x, y, z, \epsilon$ are comparable to $10^{-1}$, there will be highly deviated mixing angles. Though such mixing angles have no direct physical meaning (the physical value is $A_{\alpha\beta}$ in Eq.[14]), we can see from Eq.[20] that the measurement of trigonometric function has been changed, which means the angles we measured under unitary scheme are just effective values.

2). Evaluate the quasi-unitary factor in case of $x, y, z \neq 0$. Since the quasi-unitary factor prepared, we can reflect the discrepancies of different measurements of $\theta_{ij}^Q$, we can also calculate the quasi-unitary factor from the simulation above. The numerical result is shown as the green points (gray $\times$) in FIG.1. As the figure indicated, under our quasi-unitary parameterization, nowadays experimental data cannot allow $\epsilon$ bigger than 0.05 and $\Delta_{QF}$ can reach up to $2.6 \times 10^{-2}$ at most, corresponding to deviated $\theta_{13}^Q \lesssim 18.35^\circ$.

If $\epsilon$ and $x, y, z$ in $A$ are big enough, we can measure a sizable $\Delta_{QF}$ in the quasi-unitary scheme, however, the error ranges in Ref.[16] cannot be totally from the non-unitary effect because the experimental errors also take a share. Unlike the general seesaw-predicted non-unitary effect which is too small in comparison to the experimental error, the quasi-unitary factor parameterized in this work with $x, y, z \neq 0$, $\Delta_{QF}^T$ may remain sizable in the measurements. It is expected the precision improvement of experiments will eliminate the experimental background as much as possible, leaving a relatively pure constitution of $\Delta_{QF}$ so that the effect may stand out.

FIG. 1: Magnitude of quasi-unitary factor versus $\epsilon$. The green points (gray $\times$) are obtained by scattering $x, y, z$ randomly between $-0.1 \sim 0.1$ and $\epsilon$ randomly between $0 \sim 0.1$, corresponding to the deviation ($\Delta_{QF}^E$) from unitarity theoretically. The blue points (dark $o$) are obtained by only scanning $\epsilon$ ($x, y, z = 0$), corresponding to the deviation ($\Delta_{QF}^E$) from unitary experimentally.

3).Evaluate the quasi-unitary in case of $x = y = z = 0$. Since $NN^\dagger$ is violated from the unit matrix with definite corrections ($\sim 1\%$) on the diagonal elements while only upper limits for the corrections on the off-diagonal elements [9], which suggests that the diagonal corrections dominate the unitarity violation, we also consider the case with $x, y, z = 0$ in our parameterization and regard the non-zero parameter $\epsilon$ as being originated from the experimental error. This consideration is quite naive, though, it can help us distinguish the differences between theoretical unitary violation and the experimental one. The simulation result ($\Delta_{QF}^E$) is also displayed in FIG.1 by the blue points (dark $o$) present, for sake of comparing it with $\Delta_{QF}^E$.

It is clear that in this case, $\Delta_{QF}^E$ can reach $1.0 \times 10^{-2}$ at most, corresponding to deviated $\theta_{13}^Q \lesssim 11.20^\circ$. Generally speaking, if $\epsilon$ in non-unitary matrix theoretically is the same order as the experimental error in the diagonal matrix elements, $\Delta_{QF}^T$ will be larger than $\Delta_{QF}^E$ for the participation of $x, y, z$. This can be seen in FIG.1.
where $\Delta T_{QF}$ can be two times larger than $\Delta E_{QF}$. Meanwhile, with the precision improvement, the tendencies of these two factors are to be entirely different, that $\Delta T_{QF}$ will remain non-zero while $\Delta E_{QF}$ will become smaller and smaller and eventually vanish.

5. Conclusion As the mixing matrix of ordinary neutrinos may be unitary or quasi-unitary, we parameterize the mixing as a quasi-unitary matrix with four additional parameters $\epsilon$, $x$, $y$, $z$ and define a quasi-unitary factor $\Delta_{QF}$ for the test of unitarity. Our numerical analysis gives the possible deviations in mixing angles without violating any current experimental constraints, as well as the magnitude of the $\Delta T_{QF}$ and $\Delta E_{QF}$ which separately describes the theoretical and experimental sensibility of discrepancies in future $\theta_{13}$ measurements. The improvement of experimental precision may help us figure out the secret of unitarity of neutrino mixing and tell us whether the seesaw framework is a reasonable model for neutrino masses.

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