Syntax and analytic semantics of LISA

Jade Alglave
Microsoft Research Cambridge
University College London
jalglav@microsoft.com, jalglave@ucl.ac.uk

Patrick Cousot
New York University
em. École Normale Supérieure, PSL Research University
pcousot@cims.nyu.edu, cousot@ens.fr

24th August 2016

Abstract

We provide the syntax and semantics of the LISA (for “Litmus Instruction Set Architecture”) language. The parallel assembly language LISA is implemented in the herd7 tool [Alglave and Maranget, 2015] for simulating weak consistency models.

1 Introduction

LISA (which stands for “Litmus Instruction Set Architecture”) has the vocation of being a fairly minimal assembly language, with read and write memory accesses, branches and fences to design consistency models for weakly consistent systems without having to concern oneself with the syntax of the programming language (such as ARM, IBM, Intel x86, Nvidia multiprocessor chips, or languages like C++ or OpenCL), which has proved quite useful at times where said syntax was still in flux.

The weakly consistent semantics of a LISA is analytic in that it is the intersection of an anarchic semantics (without any restriction on communications) and a communication semantics (specified by a cat specification [Alglave, Cousot, and Maranget, 2015c] restricting the allowed communications).

The herd7 tool is a weakly consistent system simulator, which takes as input a cat specification [Alglave, Cousot, and Maranget, 2015c] and a litmus test preferably in LISA, and determines whether the candidate executions of this test are allowed or not under the cat specification and under which conditions on communication events. The semantics of cat and LISA has been implemented in the herd7 tool. The documentation of the tool is available online, at diy.inria.fr/tst7/doc/herd.html. The sources of the tool are available at diy.inria.fr. A web interface of herd7 is available at virginia.cs.ucl.ac.uk/herd.
We define the anarchic true parallel semantics with separate communications where anarchic means that no restriction is made on possible communications. We also formally define the abstraction into candidates executions which are the inputs for the semantics of the cat language placing restriction on the communication events, which defines a weak consistency model.

2 An overview of analytic semantics

We introduce anarchic semantics with true parallelism and unrestricted separate communications in Section 2. Then in Section 2.4 we show how to abstract anarchic execution to candidate executions that a cat specification will allow or forbid based on hypotheses on relations between communication events.

2.1 Executions

The anarchic semantics of a parallel program is a set of executions; an execution has the form $\pi = \varsigma \times rf \in \Pi$, where $\varsigma$ is the computation part and $rf$ is the communication part.

Communications are sets $rf$, which gather read-from relations. A read-from relation $rf[w, r]$ links a (possibly initial) write event $w$ and a read event $r$ relative to the same shared variable $x$ with the same value. Communications are anarchic: we place no restriction on which write a read can read from; restrictions can be made in a cat specification however.

Computations have the form $\varsigma = \{\text{start}\} \times \prod_{p \in P} \tau_p$, where $\{\text{start}\}$ is an execution trace of the prelude process, and $\tau_p$ are execution traces of the processes $p \in P$. A finite (resp. infinite) non-empty trace $\tau_p$, $p \in P \cup \{\text{start}\}$ is a finite (resp. infinite) sequence

$$\tau_p = (\overset{\rightarrow}{p_k} \rightarrow \tau_{p_k} | k \in [0, 1 + m]) \in \mathcal{T}$$

of computation steps $\overset{\rightarrow}{p_k} \rightarrow \tau_{p_k}$ (with $\overset{\rightarrow}{p_k}$ an event and $\tau_{p_k}$ the next state—see below for the definitions of event and state) such that $\tau_{p_0} = \epsilon_{\text{start}}$ is the start event, $|\tau_p| \triangleq m \in \mathbb{N}$ for finite traces and $|\tau_p| \triangleq m = \omega = 1 + \omega$ for infinite traces where $\omega$ is the first infinite limit ordinal so that $[0, 1 + \omega] = [0, \omega] = \mathbb{N}$. This is a true parallelism formalisation since there is a notion of local time in each trace $\tau_p$, $p \in P \cup \{\text{start}\}$ of an execution $\pi = \{\text{start}\} \times \prod_{p \in P} \tau_p \times rf$ but no global time, since it is impossible to state that an event of a process happens before or after an event of another process or when communications do happen.

Events indicate several things:
- their nature, e.g. read ($r$), write ($w$), branch ($b$), fence ($f$), etc.;
- the identifier $p$ of the process that they come from;
- the control label $\ell$ of the instruction that they come from;
- the instruction that they come from—which gives the shared variables and local registers affected by the event, if any, e.g. $x$ and $R$ in the case of a read $r[t] R x$;
their stamp $\theta \in \mathcal{P}(p)$; they ensure that events in a trace are unique. In our examples, stamps gather the control label and iteration counters of all surrounding loops, but this is not mandatory: all we need is for events to be uniquely stamped. Different processes have non-comparable stamps. Stamps are totally ordered per process by $\prec_p$ (which is irreflexive and transitive, while events on different processes are different and incomparable). The successor function $\text{succ}_p$ is s.t. $\text{succ}_p(\theta)$ (but not necessarily the immediate successor); $\text{inf}_p$ is a minimal stamp for process $p$. We consider executions up to the isomorphic order-preserving renaming $\cong$ of stamps;

• their value $v \in \mathcal{D}$, whether ground or symbolic. To name the values that are communicated in invariants, we use pythia variables $\mathcal{P}(p) \triangleq \{x_\theta \mid x \in \mathbb{X} \land \theta \in \mathcal{P}(p)\}$ (note that the uniqueness of stamps on traces ensures the uniqueness of pythia variables). More precisely, traditional methods such as Lamport’s and Owicki-Gries’ name $x$ the value of the shared variable $x$, but we cannot use the same idea in the context of weak consistency models. Instead we name $x_\theta$ the value of shared variable $x$ read at local time $\theta$.

The events $\tau_p$ on a trace $\tau_p$ of process $p$ are as follows:

• register events: $a((p, \ell, \text{mov} \, R, \mathbf{operation} \, \theta), v)$;
• read events: $r((p, \ell, r[ts] \, R, x, \theta, x_\theta))$;
• write events: $w((p, \ell, w[ts] \, x, \mathbf{r-value} \, \theta), v)$;
• branch events are of two kinds:
  - $t((p, \ell, b[ts] \, \mathbf{operation} \, l, \theta))$ for the true branch;
  - $t((p, \ell, b[ts] \, \mathbf{operation} \, l, \theta))$ for the false branch;
• fence events: $m((p, \ell, f[ts] \, \{\{l_1 \ldots l_n\} \, \{l_1 \ldots l_k\}\}, \theta))$;
• RMW events are of two kinds:
  - begin event: $m((p, \ell, \text{beginrmw}[ts] \, x, \theta))$;
  - end event: $m((p, \ell, \text{endrmw}[ts] \, x, \theta))$

**States** $\sigma = s(\ell, \theta, \rho, \nu)$ of a process $p$ mention:

• $\ell$, the current control label of process $p$ (we have $\text{done}[P](p)\ell$ which is true if and only if $\ell$ is the last label of process $p$ which is reached if and only if process $p$ does terminate);
• $\theta$ is the stamp of the state in process $p$;
• $\rho$ is an environment mapping the local registers $R$ of process $p$ to their ground or symbolic value $\rho(R)$;
• $\nu$ is a valuation mapping the pythia variables $x_\theta \in \mathcal{P}(p)$ of a process $p$ to their ground or symbolic value $\nu(x_\theta)$. This is a partial map since the pythia variables (i.e. the domain $\text{dom}(\nu)$ of the valuation $\nu$) augment as communications unravel. Values can be ground, or symbolic expressions over pythia variables.

The prelude process has no state (represented by $\bullet$).

### 2.2 Well-formedness conditions

We specify our anarchic semantics by the means of well-formedness conditions over the computation traces $\zeta = \{\text{start}\} \times \prod_{p \in \Pi} \tau_p$, and the communications $\mathfrak{r}$ of an execution $\pi = \zeta \times \mathfrak{r}$.

**Conditions over computations** $\{\text{start}\} \times \prod_{p \in \Pi} \tau_p$ are as follows:

**Start:** traces $\tau$ must all start with a unique fake start event $\text{start}_0 = \text{start}_0 \land \forall i \in [0, 1 + |\tau|], \tau_i \neq \text{start}_0$.
• **Uniqueness**: the stamps of events must be unique:
  \[ \forall p, q \in \mathcal{P} \cup \{\text{start}\}, \forall i \in [0, 1 + |\tau_p|], \forall j \in [0, 1 + |\tau_q|], (p = q \implies i \neq j) \implies \text{stamp}^p_{\tau_p, i} \neq \text{stamp}^q_{\tau_q, j}. \]

It immediately follows that events of a trace are unique, and the pythia variable \( x_\theta \) in any read \( r((p, \ell, x := x, \theta), x_\theta) \) is unique.

• **Initialisation**: all shared variables \( x \) are initialised once and only once to a value \( v_x \) in the prelude (or to \( v_x = 0 \) by default).

\[ \exists \tau_0, \tau_1 \cdot (\tau_0 = \tau_1 = \tau_0(w((\text{start}, \ell_\text{start}, x := e, \theta), v_x)) \wedge \forall \tau_0, \tau_1, \tau_2 \cdot (\tau_1 \implies \tau_1 \wedge \tau_2 \implies (\text{start} = \tau_0 \implies \sigma \tau_1 \wedge \tau_2)) \implies \tau_1 \wedge (e \in \text{wall} \implies y \neq x). \]

• **Maximality**: a finite trace \( \tau_p \) of a process \( p \) must be maximal \( \text{i.e.} \) must describe a process whose execution is finished. Note that infinite traces are maximal by definition, hence need not be included in the following maximality condition:

\[ \exists \theta, \rho, \nu \cdot \tau_{\rho, \nu} = (\text{start}, \ell, \theta, \rho, \nu) \cap \text{done}[p](p) \ell \]

\( \text{i.e.} \) the control state of the last state of the trace is at the end of the process, as indicated by \( \text{done}[p](p) \ell \).

**Conditions over the communications \( rf \) are as follows:**

• **Satisfaction**: a read event has at least one corresponding communication in \( rf \):

\[ \forall i \in [0, 1 + |\tau_p|], \forall r . (\text{wall}^p_i = r) \implies (\exists w . r(f[w, r] \in rf)). \]

• **Singleness**: a read event in the trace \( \tau_p \) must have at most one corresponding communication in \( rf \):

\[ \forall r, w, w' . (r(f[w, r] \in rf) \wedge r(f[w', r] \in rf)) \implies (w = w'). \]

Note however that a read instruction can be repeated in a program loop and may give rise to several executions of this instruction, each recorded by a unique read event.

• **Match**: if a read reads from a write, then the variables read and written must be the same:

\[ (r(f[w((p, \ell, x := r, \theta), v), r((p', \ell', x' := x', \theta'), x_\theta)) \in rf) \implies (x = x'). \]

• **Inception**: no communication is possible without the occurrence of both the read and (maybe initial) write it involves:

\[ \forall r . (r(f[w, r] \in rf)) \implies (\exists p \in \mathcal{P}, q \in \mathcal{P} \cup \{\text{start}\}, \exists j \in [0, 1 + |\tau_p|], k \in [0, 1 + |\tau_q|], \{\tau_p, \tau_q\} \cdot \text{wall}^p_j = r \land \text{wall}^q_k = w). \]

Note that this does not prevent a read to read from a future write.

**Language-dependent conditions for LISA are as follows:**

• **Start**: the initial state of a trace \( \tau_p \) should be of the form:

\[ \tau_{\text{start}}^p = \text{start}^p \cdot \text{init}^p \cdot \lambda \text{R} \cdot 0. \emptyset. \]

where \( \text{init}^p \) is the entry label of process \( p \) and \( \text{init}^p \) is a minimal stamp of \( p \).
• **Next state**: if at point \(k\) of a trace \(\tau_p\) of process \(p\) of an execution \(\pi = [\text{start}] \times \prod_{p \in \mathcal{P}} \tau_p \times \mathcal{R}\) the computation is in state \(\tau_{p_k-1} = \exists! (\ell, \theta, \rho, \nu)\) then:
  – the next event must be generated by the instruction \(\text{instr} \triangleq \text{instr}[P] \times \ell\) at label \(\ell\) of process \(p\)
  – the next event has the form \(\tau_{p_k} = \epsilon(\langle p, \ell, \text{instr}, \theta \rangle, x_\theta, v)\)
  – the next state \(\tau_{p_k} = \langle \kappa', \theta', \rho', \nu' \rangle\) has \(\kappa' = \ell'\) which is the label after the instruction \(\text{instr}\)
  – the stamp \(\theta' = \text{succ}_p(\theta)\) is larger, and
  – the value \(v\) as well as the new environment \(\rho'\) and valuation \(\nu'\) are computed as a function of the previous environment \(\rho\), the valuation \(\nu\), and the execution \(\pi\).

Formally: \(\forall k \in [0, 1 + |\tau_p|]. \forall \kappa' \rho' \nu' \theta'\).

\[
\begin{align*}
\tau_{p_{k-1}} = & \exists! (\ell, \theta, \rho, \nu) \land \tau_{p_k} = \epsilon(\langle p, \ell, \text{instr}, \theta \rangle, x_\theta, v) \land \\
\tau_{p_k} = & \epsilon(\kappa', \theta', \rho', \nu')) \implies (\kappa' = \ell' \land \theta' = \text{succ}_p(\theta) \land v = \nu(\rho) \land \rho' = \nu(\rho, \rho) \land \nu(\nu, \nu, \nu, \nu')).
\end{align*}
\]

We give the form of the next event \(\tau_{p_k}\) for each LISA instruction:

• **Fence** \(\text{instr} = \ell : \mathcal{F}\{\text{ts}\} [\{\ldots\} \{\ldots\}]; \ell': \ldots)\):
  \[
  \tau_{p_k} = m(\langle p, \ell, \mathcal{F}\{\text{ts}\} [\{\ldots\} \{\ldots\}], \theta \rangle)
  (\rho' = \rho \land \nu' = \nu).
  \]

• **Register instruction** \(\text{instr} = \ell : \text{mov} \ R_i \ \text{operation}\ ell': \ldots)\):
  \[
  \tau_{p_k} = a(\langle p, \ell, \text{mov} \ R_i \ \text{operation}, \theta \rangle, v)
  (v = E[\text{operation}[\rho, \nu] \land \rho' = \nu(\rho, \rho) \land \nu' = \nu).
  \]

  where \(E[e](\rho, \nu)\) is the evaluation of the expression \(e\) in the environment \(\rho\) and valuation \(\nu\).

• **Write** \(\text{instr} = \ell : \text{w}[\text{ts}] \times r\text{-value}\ ell': \ldots)\):
  \[
  \tau_{p_k} = w(\langle p, \ell, \text{w}[\text{ts}] \times r\text{-value}, \theta \rangle, v)
  (v = E[r\text{-value}[\rho, \nu] \land \rho = \rho' \land \nu' = \nu).
  \]

• **Read** \(\text{instr} = \ell : \text{r}[\text{ts}] \ R_i \ x; \ell': \ldots)\):
  \[
  \tau_{p_k} = r(\langle p, \ell, \text{r}[\text{ts}] \ R_i \ x, \theta \rangle, x_\theta)
  (\rho' = \rho(\text{R}_i := x_\theta) \land \exists q \in \mathcal{P} \cup \text{start}. \exists j \in [1, 1 + |\tau_q|]. \exists \ell'', \theta'', v. \langle \tau_{q_j} = w(\langle q, \ell'', \text{w}[\text{ts}] \times r\text{-value}, \theta'' \rangle, v \land \\
  \forall j(\tau_{q_j}, \tau_{p_k}) \in \mathcal{R} \land v' = \nu(\nu(\nu(\nu))), v)\rangle).
  \]

• **RMW** \(\text{instr} = \text{rmw}[\text{ts}] \ \text{r} \ \text{reg-instrs} \ x\): for the begin \(\text{instr} = \text{beginrmw}[\text{ts}] \ x\) and end event \(\text{instr} = \text{endrmw}[\text{ts}] \ x\):
  \[
  \tau_{p_k} = m(\langle p, \ell, \text{instr}, \theta \rangle)
  (\rho' = \rho \land \nu' = \nu).
  \]

• **Test** \(\text{instr} = \ell : \text{b}[\text{ts}] \ \text{operation}\ [\ldots]; \ell': \ldots)\):
  – on the true branch:
    \[
    \tau_{p_k} = t(\langle p, \ell, \text{b}[\text{ts}] \ \text{operation}[\ldots], \theta \rangle)
    (\text{sat}(E[\text{operation}[\rho, \nu] \neq 0) \land \kappa' = \ell' \land \rho' = \rho \land \nu' = \nu).
    \]
  – on the false branch:
\[
\tau_{pk} = t((p, \ell, b[ts] \mathord{\text{operation}}[\ell], \theta)) \quad \text{Wf}_{\text{TS}}(\pi)
\]
\[
(\text{sat}(E \mathord{\text{operation}}[\rho, \nu] = 0) \land \kappa' = \ell' \land \rho = \rho' \land \nu = \nu)
\]

2.3 Anarchic semantics

The anarchic semantics of a program \( P \) is
\[
S_a[\llbracket P \rrbracket] \triangleq \{ \pi \in \Pi \mid \text{Wf}_1(\pi) \land \cdots \land \text{Wf}_{16}(\pi) \}.
\]

2.4 cat specification of a weakly consistent semantics

The candidate execution abstraction \( \alpha_\Xi(\pi) \) abstracts the execution \( \pi = \varsigma \times \mathit{rf} \) into a candidate execution \( \alpha_\Xi(\pi) = (e, \\mathit{po}, \mathit{rf}, iw, fw) \) where
- \( e \) is the set of events (partitionned into fence, read, write, ... events),
- \( \mathit{po} \) is the program order (transitively relating successive events on a trace of each process),
- \( r = \mathit{rf} \) is the set of communications, and
- \( iw \) is the set of initial write events.

Then we define
\[
\alpha_\Xi(\Pi) \triangleq \{ \pi \mid \langle \pi, \alpha_\Xi(\pi) \rangle \in C \land \langle \mathit{allowed}, \Gamma \rangle \in \llbracket Hcm \rrbracket(\Xi) \}
\]
for a cat consistency model \( Hcm \) defined in (Alglave, Cousot, and Maranget, 2015c) and returns communication relations \( \Gamma \) specifying communication constraints on communication events. The analytic semantics of a program \( P \) for a cat specification \( Hcm \) is then
\[
S[\llbracket P \rrbracket] \triangleq \alpha_\Xi(\Pi) \circ \alpha_\Xi(\llbracket S_a[\llbracket P \rrbracket] \rrbracket).
\]

3 An overview of LISA

3.1 Example

To illustrate LISA we use Peterson’s algorithm, given in Figure 1.

The algorithm uses three shared variables \( F1, F2 \) and \( T \):
- two shared flags, \( F1 \) for the first process \( P0 \) (resp. \( F2 \) for the second process \( P1 \)), indicating that the process \( P0 \) (resp. \( P1 \)) wants to enter its critical section, and
- a turn \( T \) to grant priority to the other process: when \( T \) is set to 1 (resp. 2), the priority is given to \( P0 \) (resp. \( P1 \)).

Let’s look at the process \( P0 \): \( P0 \) busy-waits before entering its critical section (see the do instruction at line 3:) until (see the while clause at line 6:) the process \( P1 \) does not want to enter its critical section (viz., when \( F2 = \text{false} \), which in turn means \( \neg R1 = \text{true} \) thanks to the read at line 4:) or if \( P1 \) has given priority to \( P0 \) by setting turn \( T \) to 1, which in turn means that \( R2 = 1 \) thanks to the read at line 5:.

LISA code

- a prelude at line 0:, between curly brackets, which initialises the variables \( F1 \) and \( F2 \) to \text{false} and the variable \( T \) to 0. By default initialisation is to 0 (false);
- two processes, each depicted as a column; let’s detail the first process, on the left-hand side: at line L1: we write 1 (true) to the shared variable \( F1 \)—the LISA syntax for writes is “w[] x e” where \( x \) is a variable and \( e \) an expression over registers, whose value is written to \( x \). At
3.2 Syntax

**LISA programs** $P = \{P_{\text{start}}\} P_0 \ldots P_{n-1}$ on shared variables $x \in \text{loc}[P]$ contain:

- a prelude $P_{\text{start}}$ assigning initial values to shared variables. In the case of Peterson algorithm in Figure 1, the prelude at line 0: assigns the value $\text{false}$ to both variables $F1$ and $F2$, and the value 0 to $T$. This initialization to 0 ($\text{false}$) is implicit in the LISA translation;
- processes $P_0 \ldots P_{n-1}$ in parallel; each process:
  - has an identifier $p \in \mathbb{P}[P] \triangleq [0, n[$; in the case of Peterson we have used $P0$ for the first process (on the left) and $P1$ for the second process (on the right);
  - has local registers ($e.g.$ $R0$, $R1$); registers are assumed to be different from one process to the next; if not we make them different by affixing the process identifier like so: $(p : R)$;
  - is a sequence of instructions.

**Instructions** can be:

- **register instructions** $\text{mov } R1 \text{operation } R2$ where the operation has the shape $\text{op } R2 \text{ r-value}$
  - the operator $\text{op}$ is arithmetic ($e.g.$ $\text{add}$, $\text{sub}$, $\text{mult}$) or boolean ($e.g.$ $\text{eq}$, $\text{neq}$, $\text{gt}$, $\text{ge}$);
  - $R1$ and $R2$ are local registers;
  - $r$-value is either a local register or a constant;
- **read instructions** $r[ts] R x$ initiate the reading of the value of the shared variable $x$ and write it into the local register $R$;
- **write instructions** $w[ts] x e$ initiate the writing of the value of the register expression $e$ into the shared variable $x$;
• **branch instructions** `b[ts] operation l` branch to label `l` if the `operation` has value `true` and go on in sequence otherwise;

• **fence instructions** `f[ts]` `{l₀, ..., lₘ}` `{l₀', ..., lₙ'}`. The optional sets of labels `{l₀, ..., lₘ}` and `{l₀', ..., lₙ'}` indicate that the fence `f[]` only applies between instructions in the first set and instructions in the second set. The semantics of the fence `f[]` as applied to the instructions at `l₀, ..., lₘ` and `l₀', ..., lₙ'`, is to be defined in a cat specification;

• **read-modify-write instructions** (RMW) `rmw[ts] reg-instrs x` are translated into a sequence of instructions delimited by markers `beginrmw[ts] x` and `endrmw[ts] x` as shown opposite. `reg-instrs` is a sequence of register instructions, which computes in `R` the new value assigned to the shared variable `x`.

Any semantics requirement on RMWs, such as the fact that there can be no intervening write to `x` between the read and the write of the RMW, has to be ensured by a cat specification.

Instructions can be labelled (i.e. be preceded by a control label `ℓ`) to be referred to in branches or fences for example. Labels are unique; if not we make them different by affixing the process identifier like so: `p[ℓ] instr[P]_p ₗ` is the instruction at label `ℓ` ∈ `L(p)` of process `p` of program `P`. Moreover, instructions can bear tags `ts` (to model for example C++ release and acquire annotations). Scopes are special tags (e.g. to model e.g. Nvidia PTX [Alglave, Batty, Donaldson, Gopalakrishnan, Ketema, Poetzl, Sorensen, and Wickerson 2015a] and HSA [HSA Foundation, 2015]), whose semantics must be defined in a cat specification. Scopes can be organized in the scope tree. The events created by a process will be automatically tagged by the scope of that process. For example the LISA program `MP-scoped` in Figure 2 is augmented by a scope tree. The scope tree `scopes:` (system (wi P0) (wi P1)) in Figure 2 specifies that the threads P0 and P1 reside in two different scope instances of level wi. By contrast, still as specified by the scope tree, there is one scope instance of level system and both threads reside in this common instance.

### 4 The anarchic true parallel formal semantics with separated communications of LISA

We now instantiate the general definition of an anarchic semantics of a parallel program of Section 2 to the case of the LISA language.

We introduce the anarchic true parallel semantics $S^a$ with separated unconstrained communications in Section 4.1 and provide ground value and symbolic instances of the anarchic semantics for the little language LISA, in Section 4.2. The abstraction of the executions to candidate executions is specified in Section 5.1. This is used in Section 5.2 to specify the semantics of a program with a cat weak consistency model $M$ constraining communications. The analytic semantics is the anarchic semantics with separated communications $S^a$ constrained by a cat the weak consistency model. It is analytic in that it separates the definition of the computational semantics $S^a$ from the communication semantics specified by a cat specification.
The semantics is in two parts. The first part in Section 4.1 is language independent. The second part in Section 4.2 is language dependent for LISA.

4.1 The anarchic true parallel symbolic and ground valued semantics

The anarchic true parallel semantics avoids interleaving thanks to a concurrent representation of execution traces of processes with separate communications. The anarchic semantics can be ground when taking values in a ground set $D_{\ast}$ (for LISA). The anarchic semantics can also be symbolic when values are symbolic expressions in the symbolic variables denoting communicated values (called pythia variables). This generalises symbolic execution (King, 1976) to finite and infinite executions of parallel programs with weak consistency models. Since in a true parallel semantics there is no notion of global time, there is also no notion of instantaneous value of shared variables. The only knowledge on the value of shared variables is local, when a process has read a shared variable. We use a pythia variable to denote the value which is read (at some local time symbolically denoted by a stamp). The usage of pythia variable is not strictly necessary in the ground semantics. It is useful in the symbolic semantics to denote values symbolically. It is indispensable in invariance proof methods to give names to values as required in formal logics.

4.1.1 Semantics

The semantics $S[P]$ of a concurrent program $P = [P_0||\ldots||P_{n-1}] \in \mathcal{P}_g$ with $n \geq 1$ processes $P_0,\ldots,P_{n-1}$ identified by their pids $p \in \mathcal{P}[P] \triangleq [0,n]$ is a set of executions $S[P] \in \phi([[]P])$. We omit $[P]$ when it is understood from the context.

4.1.2 Computations and executions

A computation $\zeta \in \Sigma \triangleq \Sigma_\text{start} \times \prod_{p \in \mathcal{P}[P]} \Sigma_\text{p} \times \Sigma_\text{finish}$ has the form $\zeta = [\text{start}] \times \prod_{p \in \mathcal{P}[P]} \tau_p \times [\text{finish}]$ where $[\text{start}] \notin \mathcal{P}[P] \cup [\text{finish}]$ is the initialization/prelude process, $[\text{finish}] \notin \mathcal{P}[P] \cup [\text{start}]$ is the finalization/postlude process, $[\text{start}] \in \Sigma_\text{start}$ is an execution trace of the initialization process, $\tau_p \in \Sigma_\text{p}$ are execution traces of the processes $p \in \mathcal{P}[P]$, and $[\text{finish}] \in \Sigma_\text{finish}$ is an execution trace of the finalization process.

An execution $\pi \in \Pi \triangleq \Sigma \times \phi([R])$ has the form $\zeta \times \mathcal{E} = [\text{start}] \times \prod_{p \in \mathcal{P}[P]} \tau_p \times [\text{finish}] \times \mathcal{E}$ where $\mathcal{E} \in \phi([R])$ is the communication relation (read-from). $R$ will be defined in Sect. 4.1.1.2 as the set of read-write event pairs on the same shared variable with the same value.

4.1.3 Traces

A finite non-empty trace $\tau_p \in \Sigma_\text{start} \times \mathcal{P}[P] \times \Sigma_\text{finish}$ is a finite sequence of computation steps $\langle \epsilon, \sigma \rangle$, represented as $\epsilon \rightarrow \sigma$ (with event $\epsilon \in \mathcal{E}(p)$ and next state $\sigma \in \mathcal{S}(p)$), of the form

$$
\tau_p = \langle \epsilon_1 \rightarrow \sigma_1, \epsilon_2 \rightarrow \sigma_2, \epsilon_3 \rightarrow \sigma_3, \ldots , \epsilon_{n-1} \rightarrow \sigma_{n-1}, \epsilon_m \rightarrow \sigma_m \rangle
$$

$$
\epsilon_k \rightarrow \sigma_k \mid k \in [1, 1 + m]
$$

such that $\epsilon_1 = [\text{start}]$ is the start event. A computation step $\epsilon \rightarrow \sigma$ represents the atomic execution of an action that (1) creates event $\epsilon$ and (2) changes the state to $\sigma$. The trace $\tau_p$
We assume that stamps are totally ordered per process.

4.1.4 Stamps

\[ \tau \equiv \sigma_1 \xrightarrow{\epsilon_1} \sigma_2 \xrightarrow{\epsilon_2} \sigma_3 \ldots \sigma_{m-1} \xrightarrow{\epsilon_{m-1}} \sigma_m \] since the first event \( \epsilon_1 \) is always the start event \( \epsilon_1 = \varepsilon_{\text{start}} \).

We say that event \( \epsilon_k, \sigma_k, \) and step \( \xrightarrow{\epsilon_k} \sigma_k \) are “at point \( k \in [1, 1 + |\tau|] \) of \( \tau \).

An infinite trace \( \tau = \sigma_1 \xrightarrow{\epsilon_1} \sigma_2 \xrightarrow{\epsilon_2} \sigma_3 \ldots \sigma_{n-1} \xrightarrow{\epsilon_{n-1}} \sigma_n \) has length \( |\tau| = m. \) For brevity, we use the more traditional form \( \tau = \sigma_1 \xrightarrow{\epsilon_1} \sigma_2 \xrightarrow{\epsilon_2} \sigma_3 \ldots \sigma_{n-1} \xrightarrow{\epsilon_{n-1}} \sigma_n \ldots \)

\[ \tau = (\xrightarrow{\epsilon_k} \sigma_k | k \in \mathbb{N}_\omega) \]

and has length \( |\tau| = \infty \) such that \( 1 + \infty = \infty \). It is traditionally written \( \tau = \sigma_1 \xrightarrow{\epsilon_2} \sigma_2 \xrightarrow{\epsilon_3} \sigma_3 \ldots \sigma_{n-1} \xrightarrow{\epsilon_{n-1}} \sigma_n \ldots \)

We let \( \mathcal{T} = \{ \mathcal{T}_k \} \) be the set of (finite or infinite) traces.

We define the sequence of states of a trace \( \tau \) as

\[ \tau \triangleq \sigma_1 \sigma_2 \sigma_3 \ldots \sigma_{n-1} \sigma_n[\ldots] \]

and its sequence of events of a trace \( \tau \) as

\[ \tau \triangleq \epsilon_1 \epsilon_2 \epsilon_3 \ldots \epsilon_{n-1} \epsilon_n[\ldots]. \]

(We write \([\ldots]\) when the elements \( \ldots \) are optional e.g. to have a single notation for both finite and infinite traces.) So a trace has the form \( \tau = (\xrightarrow{\epsilon_k} \sigma_k | k \in [1, 1 + |\tau|]) \).

4.1.4 Stamps

Stamps are used to ensure that events in a trace of a process are unique.

\[ \theta \in \Psi(p) \] the stamps (or postmarks) of process \( p \in \mathbb{P} \cup \{ \text{start}, \text{finish} \} \) uniquely identify events when executing process \( p. \) Different processes have different stamps so for all \( p, q \in \mathbb{P} \cup \{ \text{start}, \text{finish} \} \) if \( p \neq q \) then \( \Psi(p) \cap \Psi(q) = \emptyset; \)

\[ \theta \in \Psi \triangleq \bigcup_{p \in \mathbb{P} \cup \{\text{start}, \text{finish}\}} \Psi(p) \]

We assume that stamps are totally ordered per process.

\[ \leq_p \in \Psi(p) \times \Psi(p) \] totally orders \( \Psi(p) \) (\( <_p \) is the strict version);

\[ \text{succ}_p : \Psi(p) \rightarrow \Psi(p) \] is the successor function (s.t. \( \theta < \text{succ}_p(\theta) \)), not necessarily the immediate successor;

\[ \text{inf}_p \in \Psi(p) \] is the smallest stamp for process \( p \) (s.t. \( \forall \theta \in \Psi(p) \), \( \text{inf}_p \leq_p \theta \)).

\[ \text{With this convention, the domain of a finite or infinite trace is } [1, 1 + |\tau|] \text{ (where } \infty \text{ is the first infinite ordinal). If } |\tau| \text{ is finite then } [1, 1 + |\tau|] = [1, |\tau|]. \text{ Else } |\tau| \text{ is infinite so } 1 + |\tau| = 1 + \infty = \infty \text{ in which case } [1, 1 + |\tau|] = [1, \infty]. \]
These hypotheses allow us to generate a sequence of unique stamps when building traces. Starting from any stamp (e.g. \( m^f_p \)), a trace where events are successors by \( \text{succ}_p \) is guaranteed to have unique stamps (since moreover different processes have different stamps). Stamps are otherwise unspecified and can be defined freely. For example we can use process labels and/or loop counters.

4.1.5 Equivalence of executions and semantics up to stamp renaming

We consider executions \( \Pi \models \cdot \equiv \cdot \) up to the isomorphic renaming \( \equiv \) of stamps. The renaming is an equivalence relation.

\[
\begin{align*}
\text{start} \times \prod_{p \in \Pi} \tau_p \times \text{finish} & \equiv \text{start} \times \prod_{p \in \Pi} \tau'_p \times \text{finish}.
\end{align*}
\]

where \( \text{start} \) and \( \text{finish} \) are the start and finish events of the trace, \( \equiv \) denotes an isomorphism and \( \phi \) is homomorphically extended from stamps to events, states, traces, and communications containing these stamps.

We also consider semantics \( S \models \cdot \equiv \cdot \) up to the isomorphic renaming \( \equiv \) of stamps by defining semantics to be equivalence when identical up to isomorphic stamp renaming of their traces.

\[
\begin{align*}
S \equiv S' \equiv \{ \pi \models \cdot \pi \in S \} = \{ \pi' \models \cdot \pi' \in S' \}.
\end{align*}
\]

4.1.6 Shared variables, registers, denotations, and data

- \( x \in \text{loc}[P] \subseteq X \): shared variables of program \( P \);
- \( r \in R(p) \): local registers of process \( p \in \Pi \cup \{ \text{finish} \} \);
- \( d \in D \): set of all data/value denotations;
- \( 0 \in D \): we assume that registers and, in absence of prelude, shared variables are implicitly initialized by a distinguished initialisation value denoted \( 0 \);
- \( S \equiv S' \equiv \cdot \{ \pi \models \cdot \pi \in S \} = \{ \pi' \models \cdot \pi' \in S' \} \).

4.1.7 Pythia variables

- Pythia variables \( x_\theta \) are used to “store/record” the value of a shared variable \( x \) when accessing this variable. Pythia variables \( x_\theta \) can be thought of as addresses, locations, L-values in buffers, or channels, communication lines to indirectly designate the R-value of a shared variable observed by a specific \text{read} event stamped \( \theta \). The R-value designated by an L-value \( x_\theta \) may be unknown. For example, the actual value may be assigned in the future e.g. in the thin-air case.\(^2\)

\(^2\) Similarly to single assignment languages but not SSA which is a static abstraction.

\(^3\) Again this situation may have to be rejected by a \text{cat} specification.
4.1.8 Expressions

In the symbolic semantics, symbolic values will be expressions on pythia variables \(\mathbb{X}_v\) (and possibly other symbolic variables), in which case \(\mathbb{D} = \mathbb{X}_v\) and the interpretation \(I\) is the identity. In the ground semantics, values belong to a ground set \(\mathbb{D}\) (\(Z\) for LISA).

- \(e, e_1, e_2, \ldots \in E[V]\): set of all mathematical expressions over variables \(v \in V\), \(e := d | v | e_1 \oplus e_2\) (where \(\oplus\) is a mathematical (e.g. arithmetical) operator);
- \(b, b_1, b_2, \ldots \in B[V]\): set of all boolean expressions over variables \(v \in V\), \(b := e_1 \odot e_2 | 1 \odot b\) (where \(\odot\) is a comparison, \(\odot\) is a boolean operator, and \(\odot\) is negation).
- The evaluation \([e]\rho, \nu\) of a mathematical expression \(e \in E[V]\) over variables \(v \in V\) in an environment \(\rho \in V \rightarrow \mathbb{D}\) mapping variables \(v \in V\) to their value \(\rho(v) \in \mathbb{D}\) or to a pythia variable and a valuation \(\nu \in \mathbb{X}_v \rightarrow \mathbb{D}\) mapping pythia variables \(x_0 \in \mathbb{X}_v\) to their value \(\nu(x_0) \in \mathbb{D}\) is defined by structural induction on \(e\) as

\[
\begin{align*}
[E[d]\rho, \nu] & \triangleq I[d] \quad \text{(Def. 16)} \\
[E[v]\rho, \nu] & \triangleq \rho(v) \\
& \triangleq E[\rho(v)]\rho, \nu \quad \text{symbolic semantics or ground value semantics when } \rho(v) \in \mathbb{D} \\
& \triangleq \nu(x_0) \quad \text{ground value semantics when } \rho(v) \notin \mathbb{X}_v \\
[E[e_1 \oplus e_2]\rho, \nu] & \triangleq E[e_1]\rho, \nu I[\oplus] E[e_2]\rho, \nu
\end{align*}
\]

where \(I[d]\) is the interpretation of the constant \(d\) and \(I[\oplus]\) is the interpretation of the mathematical operator \(\oplus\).

- The evaluation \([b]\rho, \nu\) of a boolean expression \(b \in B[V]\) in environment \(\rho \in V \rightarrow \mathbb{D}\) is defined as

\[
[b]\rho, \nu \triangleq \{\text{true}, \text{false}\}
\]
\[ B[e_1 \land e_2](\rho, \nu) \equiv B[e_1](\rho, \nu) I[\land] B[e_2](\rho, \nu) \]  
\[ B[b_1 \land b_2](\rho, \nu) \equiv B[b_1](\rho, \nu) I[\land] B[b_2](\rho, \nu) \]  
\[ B[\neg b](\rho, \nu) \equiv I[\neg] B[b](\rho, \nu). \]  

where \( I[\land] \) is the interpretation of the comparison operator \( \land \), \( I[\lor] \) is the interpretation of the boolean operator \( \lor \), and \( I[\neg] \) is the interpretation of the negation \( \neg \).

4.1.9 Events

Start event.

- \( \epsilon_{\text{start}} \) is the start event at the beginning of traces.

\[ \mathfrak{B} \equiv \{ \epsilon_{\text{start}} \}. \]

The \( \epsilon_{\text{start}} \) event is not indispensable. It is used to represent uniformly traces as sequences of computation steps \( \text{i.e.} \) pairs of an event and a state. Otherwise a trace would be a sequence of states separated by events. This conventional representation is dissymmetric which is why we choose to have a \( \epsilon_{\text{start}} \) event.

Computation events.

- \( c(p) \in C(p) \) is the set of computation events of process \( p \in \mathcal{P} \). In the case of \textsc{lisa}, the computation events \( C[\text{proc}] \) of a process \( \text{proc} \) are defined in Fig. 5. They can be marker events (defined for fences in Fig. 9 and for read-modify-write instructions in Fig. 14), register assignment events (defined in Fig. 10), and test events (defined in Fig. 15).

Read events.

- \( r(p, x, v) \in R[p, x, v] \) is the set of read events of process \( p \in \mathcal{P} \) reading value \( v \in D \) from shared variable \( x \in \text{loc}[\mathcal{P}] \).
- \( r[\text{finish}, x, v] \in R[\text{finish}, x, v] \) is the set of final read events reading value \( v \in D \) from shared variable \( x \in \text{loc}[\mathcal{P}] \).
- \( r(p) \in \mathcal{R}(p) \equiv \bigcup_{x \in \text{loc}[\mathcal{P}] \atop v \in D} R[p, x, v] \) is the set of read events of process \( p \in \mathcal{P} \cup \{ \text{finish} \} \).

Read events for \textsc{lisa} are defined in Fig. 12. Each read event stamped \( \theta \) uses a python variable \( x_\theta \) to store the value read/to be read by the read event. The python variable \( x_\theta \) is always unique in a trace \( \tau \) since the stamp \( \theta \) of the event is assumed (by the forthcoming condition \( Wf(\tau) \)) to be unique on that trace \( \tau \).
Write events.

- \( w(p, x, v) \subseteq W(p, x, v) \) is the set of write events of process \( p \in \Pi \cup \{\text{start}\} \) writing value \( v \in D \) into shared variable \( x \in \text{loc}[P] \);
- \( w(\text{start}, x, v) \subseteq W(\text{start}, x, v) \) is the set of initial write events of value \( v \in D \) into shared variable \( x \in \text{loc}[P] \);
- \( w(p) \subseteq W(p) \triangleq \bigcup_{x \in X} \bigcup_{v \in D} W(p, x, v) \) is the set of write events of process \( P_p, p \in \Pi \cup \{\text{start}\} \);

For the LISA language, write events are defined in Fig. 11.

Events.

- \( \epsilon, \epsilon(p) \subseteq E(p) \triangleq \mathcal{C}(p) \cup \mathcal{R}(p) \cup W(p) \) is the of computation events of process \( p \in \Pi \cup \{\text{start, finish}\} \);
- \( E[P] \triangleq \bigcup_{p \in \Pi \cup \{\text{start, finish}\}} E(p) \) is the set of events of program \( P = [P_1|\ldots|P_n] \in \mathcal{P}_g \);

For the LISA programs, it is defined in Fig. 4.

4.1.10 States

States are the control state and the memory state mapping variables to their value. We use an environment to record the value of local registers, as is classical. However, the instantaneous values of shared variables are unknown to processes. We use instead pythia variables to record the values read by processes. The valuation maps these pythia variables to the value of the corresponding shared variable at the time when it was read (i.e. which in general is not the instantaneous value, otherwise unknown).

We would like all events to be distinct, ordered per process, but not interprocesses. Therefore we had stamps to states (knowing by the hypotheses of Section 4.1.4 that there are ordered per process according to the program order of that process).

- The states of the start and finish processes are meaning less and so are \( \bullet \) where \( S[\text{start}] = S[\text{finish}] = \{\bullet\} \).
- The states \( \sigma \in E[P] \) of process \( p \in \Pi \) have the form \( \sigma = s(\kappa, \theta, \rho, \nu) \) where
  - \( \kappa \in L(p) \) is the current control label/program point of process \( p \) (we have done\( \Pi[p](p)\kappa \) which is true if and only if \( \kappa \) is the last label of process \( p \) which is reached if and only if process \( p \) does terminate);
  - \( \theta \in \mathcal{P}(p) \) is the stamp of the state in process \( p \);
  - \( \rho \in \mathcal{E}(p) \triangleq \mathcal{R}(p) \cup \mathcal{X}[P] \) is an environment mapping local registers \( r \in \mathcal{R}(p) \) of process \( p \) to their ground/symbolic value. This is a map since the process registers are known statically;
4.1.11 Well-formed traces

The execution \( \pi = \tau_{\text{start}} \times \prod_{p \in \mathcal{P}} \tau_p \times \tau_{\text{finish}} \) is well-formed under the following well-formedness conditions. The finalisation \( \tau_{\text{finish}} \) it is necessary to specify the outcome on the program computations (in conjunction with a cat specification of how the final communications should be performed).

- **Start:** traces \( \tau \in \{ \tau_p \mid p \in \mathbb{P} \} \cup \{ \tau_{\text{start}}, \tau_{\text{finish}} \} \in \nu(\mathcal{T}) \) must all start with a unique start event on the trace.

  \[
  \forall i \in [1, 1 + |\tau|] . \tau_i \neq \tau_{\text{start}} . \quad \text{Wf}_1(\pi)
  \]

- **Uniqueness:** the stamps of events in a trace \( \tau \in \{ \tau_p \mid p \in \mathbb{P} \} \cup \{ \tau_{\text{start}}, \tau_{\text{finish}} \} \in \nu(\mathcal{T}) \) must be unique on the trace (the initial write events as well as the final read and communication events are unique per shared variable and so do not need stamps to be distinguished).

  \[
  \forall i, j \in [1, 1 + |\tau|] . (i \neq j \land \tau_i \neq \tau_{\text{finish}} \cup \tau_{\text{start}})) \rightarrow \text{stamp}(\tau_i) \neq \text{stamp}(\tau_j) .
  \]

  (It immediately follows that all events occurring on a trace are unique. Moreover, the symbolic variable \( x_\emptyset \) in any read event \( \tau_k = r((p, \ell : x = x, \theta), x_\emptyset) \) of a trace \( \tau \) is unique on that trace.)

- **Initialisation:** all shared variables \( x \in \mathbb{T} \) are assumed to be initialized once and only once to a value \( v_x \in \mathcal{D} \) in the sequential prelude (or to \( v_x = l[0] \) by default).

  \[
  \forall x \in \mathbb{T} . (\exists \tau_1, \tau_2 . \begin{array}{c}
  \text{start} = \tau_1 \nu(\text{start} x = e, \theta, v_x) \end{array} \land \\
  \begin{array}{c}
  \forall \tau_2, \tau_3, \tau_4 . \begin{array}{c}
  \text{start} = \tau_1 \epsilon \tau_2 \nu(\text{start} \ell, x = e, \theta, v_x) \tau_3 \end{array} \\
  \Rightarrow (e \in \nu(\text{start} y) \land y \neq x)) .
  \end{array}
  \]

- **Finalisation:** all shared variables \( x \in \mathbb{T} \) are finally read once and only once in the postlude and their final value is stored in a fresh register \( r_x \).

  \[
  \forall x \in \mathbb{T} . (\exists \tau_1, \tau_2 . \tau_{\text{finish}} = \tau_1 r((\text{finish}, r_{\text{finish}} x = x, \theta), x_\emptyset) \tau_2) \land \\
  \begin{array}{c}
  \forall \tau_1, \tau_2, \tau_3 . \tau = \tau_1 r((\text{finish}, \ell, r_x = x, \theta), x_\emptyset) \tau_2 \tau_3 \end{array} \Rightarrow \\
  ((e' \in \nu(\text{finish} y) \land y \neq x)) .
  \]

- **Maximality:** finite trace \( \tau_p \) of a process \( p \in \mathbb{P} \) must be maximal i.e. must describe a process which execution is finished. Note that infinite traces are maximal by definition, hence we do not need to include them in the present maximality condition.

- \( \nu \in \nu(\mathcal{T}) \) is a valuation mapping pythia variables of process \( p \) to their ground/symbolic value. This is a partial map since the pythia variables are known dynamically. We write \( \text{dom}(\nu) \) for the domain of definition of \( \nu \) (initially \( \emptyset \) at execution start).
\( \forall p \in \mathcal{P}. (\tau_p \in \mathfrak{T}^+) \implies (\exists \ell \in \mathcal{L}(p) . \exists \theta \in \mathcal{P}(p). \exists p \in \mathcal{L}(p) . \exists v \in \mathcal{V}(p) . \text{Wf}_5(\pi) \wedge \tau_{|\tau_p|} = [\mathcal{L}(\ell, \theta, p, v) \wedge \text{done}[\mathcal{P}(p)\ell]) . \)

(i.e. the control state of the last state of the trace is at the end of the process, as indicated by \( \text{done}[\mathcal{P}(p)\ell] \), which is language dependent, and defined for \text{LISA} in Sect. 4.2.)

### 4.1.12 Well-formed communications

**Communications.**

- \( c(p, w) \in \mathcal{R}(p, w) \) is the set of communications of process \( p \in \mathcal{P} \cup \{\text{finish}\} \) reading from the write \( w \).

\[
c(p, w(q, x, v)) \triangleq \text{tf}[w(q, x, v), r(p, x, v')]
\]

**read event** \( r(p, x, v') \) of process \( p \in \mathcal{P} \) or final read (\( p = \text{finish} \)) reads the value \( v' \) of \( x \) from write event \( w(q, x, v) \) of the same or another process \( q \in \mathcal{P} \) or from initial write (\( q = \text{start} \)) where \( \text{sat}(v = v') \).

- \( c(p) \in \mathcal{R}_p(p) \triangleq \bigcup_{w \in \mathcal{P}(p)} \mathcal{R}(p, w) \) is the set of communications for process \( p \in \mathcal{P} \cup \{\text{finish}\} \);

- \( c(w) \in \mathcal{B}_w(w) \triangleq \bigcup_{p \in \mathcal{P} \cup \{\text{finish}\}} \mathcal{R}(p, w) \) is the set of communications satisfied by write event \( w \);

- \( c \in \mathcal{R}[\mathcal{P}] \triangleq \bigcup_{p \in \mathcal{P} \cup \{\text{finish}\}} \mathcal{R}(p, w) \) is the set of communications of program \( P = [P_1 \parallel \ldots \parallel P_n] \in \mathcal{P} \);

- \( \mathcal{R} \triangleq \bigcup_{P \in \mathcal{P} \cup \{\text{finish}\}} \mathcal{R}[P] \) is the set of all communications.

Notice that communications are not stamped (precisely because we do not want to impose any notion of time between communications). If a stamp is needed to uniquely identify a communication, we can use the one of the read event involved in the communication since it is unique by \( \text{Wf}_5(\pi) \) and \( \text{Wf}_7(\pi) \).

**Well-formed communications.** To be well-formed, an execution \( \pi = (\text{start} \times \prod_{p \in \mathcal{P} \cup \{\text{finish}\}} \tau_p \times \text{finish} \times \tau_f \in \mathcal{I} \) must have communications \( \tau_f \) satisfying the following conditions.

- **Satisfaction:** a read event in the trace \( \tau_p \in \mathfrak{T}^+ \) of a process \( p \in \mathcal{P} \cup \{\text{finish}\} \) must have at least one corresponding communication in \( \tau_f \) (since writes are fair i.e. become ultimately readable and there is always an initial readable write to initialize variables). We impose this condition on reads to avoid the case where a read never reads anything, which would block the execution.

\[
\forall i \in [1, 1 + |\tau_p|] . \forall r \in \mathcal{R}(p) . (\tau_{|\tau_p|} = r) \implies (\exists w \in \mathcal{R}(p). \text{tf}[w, r] \in \tau_f).
\]

- **Singleness:** a read event in the trace \( \tau_p \in \mathfrak{T}^+ \) of a process \( p \in \mathcal{P} \cup \{\text{finish}\} \) must have at most one corresponding communication in \( \tau_f \).
\( \forall r \in R(p) \). \( \forall w, w' \in \mathcal{W} \). \( (r[f][w, r] \in rf \land r[f][w', r] \in rf) \implies (w = w') \). \( \text{Wf}_7(\pi) \)

(Note however that a single read action (i.e. atomic instruction in LISA), can be repeated in a program loop and may give rise to several executions of this action, each recorded by unique read events (each one reading from one, possibly different, past or future write event.)

- **Match**: if a read event in the trace \( \tau \in \mathcal{T} \) reads from a write event, then the variables read and written must be the same.

\[
(r[f][w](⟨p, ℓ, x := r, \theta⟩, v), r(⟨p', ℓ', x' := x', \theta'⟩, x')) \in rf \implies (x = x') . \quad \text{Wf}_8(\pi)
\]

- **Inception**: no communication is possible without the occurrence of both the read and write events it involves (the write may be an initial one).

\[
\forall r \in R(p) . \forall w \in \mathcal{W} . \quad (r[f][w, r] \in rf) \implies (\exists p \in \mathcal{P} \cup \{\text{start}\}, q \in \mathcal{P} \cup \{\text{finish}\}. \exists j \in [1, 1 + |\tau_p|], k \in [1, 1 + |\tau_q|]. \tau_{pj} = r \land \tau_{qk} = w) . \quad \text{Wf}_9(\pi)
\]

(Note that this does not prevent a read to read from a future write.)

### 4.1.13 Well-formed execution

An execution \( \pi = \text{start} \times \prod_{p \in \mathcal{P}} \tau_p \times \text{finish} \times rf \in \mathcal{P} \) is well-formed if and only if it satisfies all conditions \( \text{Wf}_2(\pi) \) to \( \text{Wf}_9(\pi) \). This leads to the definition of the semantic domain.

\[
\mathcal{D} \triangleq \{ S \in \wp(\mathcal{T}_\pi) | \forall \pi \in S. \text{Wf}_2(\pi) \land \ldots \land \text{Wf}_9(\pi) \} \quad (10)
\]

Moreover, for a particular programming language, computation events on process traces must be generated in program order. This well-formedness condition has to be specified for each programming language e.g. \( \text{Wf}_10(\pi) \) to \( \text{Wf}_16(\pi) \) below for LISA.

### 4.1.14 Anarchic semantics

Observe that if \( \forall i \in \Delta. S_i \in \mathcal{D} \) then \( \bigcup_{i \in \Delta} S_i \in \mathcal{D} \) and \( \bigcap_{i \in \Delta} S_i \in \mathcal{D} \) proving that \( \mathcal{D} \subseteq \emptyset, S_a, \cup, \cap \) is a complete sublattice of \( \wp(\mathcal{T}_\pi), \subseteq, \emptyset, \mathcal{T}_\pi, \cup, \cap \). So \( \mathcal{D} \) has an infimum \( \emptyset \) and a supremum \( S_a \triangleq \left( \bigcup_{S \in \mathcal{D}} S \right) \in \mathcal{D} \) called the anarchic semantics.

### 4.2 Litmus Instruction Set Architecture (LISA)

In this section we present a little language that we call LISA, for Litmus Instruction Set Architecture. We provide the syntax, symbolic and ground semantics of LISA.
4.2.1 Programs

The semantics of programs is defined by an attribute grammar (Knuth, 1990) (see Paakki, 1995 for an introduction) given in Figure 3.

The program may have a sequential prelude, viz., a set of initial assignments of values $v_x$ to shared variables $x$, a case covered by $Wf(p)$ and by $p = \text{start}$ in Fig. 11. We omit the specification of the syntax and semantics of the prelude. In absence of prelude, for an empty prelude $\{\}$, or in absence of a specific initialization of some shared variables, the shared variables are assumed to be implicitly initialized to $I[0]$.

Local registers of processes are assumed to be implicitly initialized to $I[0]$, as shown in Fig. 5.

The LISA version used by the herd7 tool also offers the possibility to initialize registers $r$ in the prelude. The register is designated by $(p:r)$ where $p \in P$ is their process identifier. This is equivalent to moving these initializations at the beginning of the corresponding process $p$ since they will be executed after the default initialization of registers to 0.

The LISA version used by the herd7 tool also offers the possibility to define a postlude to check whether there exists a finite execution $\tau$ or for all finite executions $\tau$ a condition on the trace $\tau$ does hold. This boolean condition may involve the final value of registers $(p:r)$ evaluated by $E_p(\tau, |\tau|)$ where $p$ is a process identifier. This boolean condition may also involve the final value of shared variables $x$ as stored in register $r_x$ in Fig. 12 where $p = \text{finish}$.

One can optionally specify a scope tree to be used in the cat communication specification.

The attributes of a program $\text{prog}$ are its number $\text{pid}(\text{prog})$ of processes, the events $E(\text{prog})$ that can be generated by the instructions of the program, and its semantics $S(\text{prog})$.

For clarity, some attributes are left implicit such as the local labels $L(p)$, the local registers $R(p)$ of process $p \in [0, \text{pid}(\text{prog})]$. These attribute definitions including restrictions such as uniqueness can be easily added to the attribute grammar.

The anarchic semantics $S^a(\text{body})$ is the set of all executions satisfying conditions $w(\text{body})$ expressing that the execution $\pi$ must correspond to an execution of the $\text{body}$. Moreover all executions $\pi$ in this anarchic semantics $S(\text{prog})$ must satisfy well-formedness conditions $Wf(\pi)$ . . . , $Wf(\pi)$ as expected by the cat semantics.
Programs —

\[
\begin{align*}
\text{prog} & \in \text{Proc} \\
\text{prog} & ::= \text{body} \mid \{\text{body}\} \mid \text{prelude body} \\
& \quad \mid \text{prelude body postlude} \\
& \quad \mid \text{prelude body scopes: scope-tree} \\
& \quad \mid \text{prelude body scopes: scope-tree postlude}
\end{align*}
\]

In all cases, let \( n = \text{pid(body)} \) in

- \( \text{pid(body)} \triangleq n + 1 \); (last process identifier in \( \text{body} \))
- \( \Pi[0, n] \); (program process identifiers)
- \( \text{done(body)} \triangleq \text{done(body)} \); (last control label check)
- \( \text{instr(body)} \triangleq \text{instr(prelude)} \cup \text{instr(body)} \); (instructions)
- \( \mathbb{E}[body] \triangleq \mathbb{E}[body] \); (program events)
- \( S[body] \triangleq \{ \tau_{\text{start}} \times \prod_{p=1}^{n} \tau_{p} \times \tau_{\text{finish}} \times \tau \text{rf} \mid \text{wf(body)} (\tau_{\text{start}} \times \prod_{p=1}^{n} \tau_{p} \times \tau_{\text{finish}} \times \tau \text{rf}) \land \forall \tau \text{rf} \in \tau \text{rf} \text{-} \text{wf(body)} (\tau_{\text{start}} \times \prod_{p=1}^{n} \tau_{p} \times \tau_{\text{finish}} \times \tau \text{rf} \setminus \{\tau \text{rf}\}) \}
\]

\( (\text{i.e. rf is minimal}) \)

- \( S[body] = \{\} \)

Figure 3: LISA programs

4.2.2 Tags and scope trees

Instructions can bear tag sequences \( ts \), that are given a semantics within \( \text{cat} \) specifications (Alglave, Cousot, and Maranget, 2015c). Scope trees can be used to describe program architecture-dependent features for the \( \text{cat} \) specification (Alglave, Cousot, and Maranget, 2015c). These tag sequences and scope tree are not involved at all in the program anarchic semantics of LISA. They are added in the form \( ts \) to events forwarded to \( \text{cat} \). If there is no scope tree declaration, the trivial scope tree \( (\text{trivial P0 P1 ...P}_{n-1}) \) is used where \( n = \text{pid(body)} \) is the number of processes in the program. A process can appear at most once in a scope tree. Again scope trees are information for a \( \text{cat} \) specification, see (Alglave, Cousot, and Maranget, 2015c).

4.2.3 Parallel processes

Parallel processes are given in Figure 4. These process identifiers \( \text{pid} \) are defined to be 0 to \( n - 1 \) from left to right where \( n = \text{pid(body)} \) is the number of processes in the parallel program. The
program events $E[body]$ and well-formedness conditions $wf[body]$ are collected.

\[
\text{Parallel processes} \\
body \in \text{Body} \\
body ::= \text{proc} \\
\text{pid} \body \triangleq \text{pid} \proc \triangleq 0 \\
\Pi \proc \triangleq \Pi \body \\
\text{instr} \body \triangleq \text{instr} \proc \\
\text{done} \body(0) \triangleq \lambda \ell \cdot (\ell = \text{after} \proc) \\
\text{E} \body \triangleq \text{E} \proc \\
\text{wf} \body \triangleq \lambda \pi \cdot \text{wf} \proc \pi \\
\text{let } n = \text{pid} \body \text{ in} \\
\text{pid} \body \triangleq \text{pid} \proc \triangleq n + 1 \\
\Pi \body \triangleq \Pi \body, \Pi \proc \triangleq \Pi \body \\
\text{done} \body(p) \triangleq \lambda \ell \cdot \{ p \leq n \Rightarrow \text{done} \body(p) \ell \wedge (\ell = \text{after} \proc) \} \\
\text{instr} \body \triangleq \text{instr} \body \cup \text{instr} \proc \\
\text{E} \body \triangleq \text{E} \body \cup \text{E} \proc \\
\text{wf} \body \triangleq \lambda \pi \cdot \text{wf} \body \pi \wedge \text{wf} \proc \pi
\]

Figure 4: LISA processes

4.2.4 Processes

Processes are given in Figure 5. Each process $\text{proc}$ of the program body $\body$ has a unique process identifier attribute $p = \text{pid} \proc$. Each instruction $\text{instr}$ of a process has a label at $\text{instr}$ before and a label after $\text{after} \text{instr}$ that instruction. The label $\ell$ is after the last instruction of the list of instructions of the process. Each process with pid $p$ has a unique entry label $\text{at} \text{instr}$ which is the one of its first instruction and is where the process $p$ execution must start from, so $\text{at} \text{instr}$ is the label of the first control state.
Processes

\[ \text{proc} \in \text{Proc} \]
\[ \text{::=} \text{instrs} \]
\[
\begin{align*}
\text{let } p &= \text{pid} (\text{proc}) \text{ and} \\
\text{pid} (\text{instrs}) &\triangleq p \\
\text{Pl} (\text{instrs}) &\triangleq \text{Pl} (\text{proc}) \\
\text{instr} (\text{proc}) &\triangleq \text{instr} (\text{instrs}) \\
\text{E} (\text{proc}) &\triangleq \text{E} (\text{instrs}) \\
\text{after} (\text{proc}) &\triangleq \text{after} (\text{instrs}) \\
\text{wf} (\text{proc}) &\triangleq \lambda \pi \cdot \text{let } \pi = \text{start} \times \prod_{p \in \text{Pl}} \tau_p \times \text{finish} \times \text{rf} \text{ in } \text{Wf}^{\text{to}} (\pi) \\
\text{wf} (\text{instrs}) (\pi) &\triangleq \tau_{p_1} = \text{at} (\text{instrs}), \text{inf}_p \lambda x \in \mathcal{R} (p) \cdot 0, \emptyset
\end{align*}
\]

Figure 5: LISA processes

4.2.5 Lexems

For any process \( p \in [1, n - 1] \), where \( n = \text{pid} (\text{prog}) \) is the number of processes in the program \( \text{prog} \), registers \( r \in \mathcal{R} (p) \) cannot be shared variable identifiers \( x \in \mathcal{X} \), labels \( l, \ell \in \mathcal{L} (p) \) cannot be register or shared variable identifiers, and tags in tag sequences cannot be label, register, or shared variable identifiers. This informal context condition is easy to include in the attribute grammar by collecting these sets and checking that their pairwise intersections are empty.

4.2.6 Expressions

As shown in Fig. 6, an operation can be either an \( r \)-value, i.e., a register or immediate value, or the result of an arithmetic (e.g., \text{add}, \text{sub}, \text{mult}) or boolean (e.g., \text{eq}, \text{neq}) operator applied to a register \( r_2 \) and another \( r \)-value. The value \( \text{E} [r] \) of a register in process \( p \) is defined in next Sect. 4.2.7.
Register values

\[
\text{r-value } \in \text{ R-value} \\
\text{r-value ::= r} \\
\text{let } p = \text{pid (r-value)} \text{ in} \\
\text{r } \in \mathbb{R}(p) \\
E[r-value] \triangleq E[r] \\
| \text{d} \\
\text{d } \in \mathbb{D} \\
E[r-value] \triangleq E[d]
\]

Register operations

\[
\text{op } \in \text{ Op} \\
\text{op ::= add op}\|\text{op} \triangleq + \quad \text{integer addition} \\
| \text{sub op}\|\text{op} \triangleq - \quad \text{integer substraction} \\
| \text{mult op}\|\text{op} \triangleq \times \quad \text{integer multiplication} \\
| \ldots \\
| \text{eq op}\|\text{op} \triangleq = \quad \text{logical equality} \\
| \text{neq op}\|\text{op} \triangleq \neq \quad \text{logical disequality} \\
| \text{gt op}\|\text{op} \triangleq > \quad \text{logical strictly greater than} \\
| \text{ge op}\|\text{op} \triangleq \geq \quad \text{logical greater than or equal} \\
| \ldots \\
\text{operation } \in \text{ Operation} \\
\text{operation ::= op}\ r_2\ \text{r-value} \\
\text{let } p = \text{pid \{operation\} in} \\
\text{r_2 } \in \mathbb{R}(p) \\
\text{pid (r-value) } \triangleq p \\
E[\text{operation}](\rho, \nu) \triangleq E[r_2](\rho, \nu) \ op\|\text{op} \ E[\text{r-value}](\rho, \nu) \\
| \text{r-value} \\
\text{let } p = \text{pid \{operation\} in} \\
\text{pid (r-value) } \triangleq p \\
E[\text{operation}] \triangleq E[\text{r-value}] \\
\]

Figure 6: Syntax of LISA expressions
4.2.7 Local sequentiality

The interleaved trace semantics of a process \texttt{proc} of LISa is \textit{locally sequential}. This means that (1) the use of registers in a process are sequentially consistent in that the value of a local register \( r \) is its last assigned value and (2) that each process is executed in the process program order. Local sequentiality is much weaker than sequential consistency (SC) (Keller, 1976; Hennessy and Plotkin, 1979; Lamport, 1979) or sequential consistency per variable (SCPV) (Alglave, Maranget, and Tautschnig, 2014) which are relative to globally shared variables (and assume local sequentiality for local registers).

4.2.8 Events

All computation events collected in \( \mathcal{E}_{\llbracket \text{prog} \rrbracket} \) have the form \( e(\langle p, \ell, \text{u-instr}, \theta \rangle) \) or \( e(\langle p, \ell, \text{u-instr}, \theta \rangle, v) \) where \( e \) is the name of the event (\( r \) for read, \( w \) for write, etc.), \( p \) is the process, and \( \text{u-instr} \) is the instruction at label \( \ell \in L(p) \) of the process that gave raise to that event. The assignment \( a \), read \( r \), and write \( w \) events may also carry a computed or communicated value \( v \in \mathbb{Z} \). Stamps \( \theta \) are left unspecified but in case of an instruction \( \text{u-instr} \) executed several times in a loop the can be used to ensure that all the generated events are different (as required by the uniqueness condition \( \text{WF}_3(S) \)). The conditions \( \text{WF}_{\llbracket \text{body} \rrbracket} \) make sure that the trace events record a program execution. The traces may also include read-from events \( \text{rf} \) which are constraint by \( \text{WF}_6(S) \) to \( \text{WF}_9(S) \). It is checked in Fig. 3 that the events on a trace must be generated by a program execution and satisfy the constraints on communication event.

4.2.9 Instructions

Instructions are given in Figure 7 and sequences of instructions in Figure 8. Except for the branch instructions, for which the label must be provided in the program, program instructions may be unlabelled. In that case, an automatic labelling program transformation will add all missing labels. Therefore in the definition of a process \( \texttt{proc} \) in Fig. 4 all instructions are assumed to have been labelled.

A special case \( \text{reg-instrs} \) of sequences \( \text{instrs} \) of instructions is considered in Fig. 14 for the case where all instructions are register instructions in the read-modify-write instruction.
Unlabelled instructions

\[ u-instr \in U-instr \]

\[ u-instr ::= \text{reg-u-instr} | \text{w-u-instr} | \text{r-u-instr} | \text{rmw-u-instr} | b-u-instr | f-u-instr \]

For all these case \[ u-instr ::= \ldots | x-u-instr | \ldots \], we have

\[ \text{pid}(x-u-instr) \triangleq \text{pid}(u-instr) \]
\[ \Pi(x-u-instr) \triangleq \Pi(u-instr) \]
\[ \text{instr}(x-u-instr) \triangleq \text{instr}(u-instr) \]
\[ \mathbb{E}(u-instrs) \triangleq \mathbb{E}(x-u-instrs) \]
\[ \text{at}(x-u-instr) \triangleq \text{at}(u-instr) \]
\[ \text{after}(x-u-instr) \triangleq \text{after}(u-instr) \]
\[ \text{wf}(u-instr) \triangleq \text{wf}(x-u-instr) \]

Labelled instructions

\[ instr \in Instr \]

\[ instr ::= \text{u-instr} \]

\[ \text{pid}(u-instr) \triangleq \text{pid}(instr) \]
\[ \Pi(u-instr) \triangleq \Pi(instr) \]
\[ \text{instr}(u-instr) \triangleq \text{instr}(instr) \]
\[ \mathbb{E}(instrs) \triangleq \mathbb{E}(u-instrs) \]
\[ \text{at}(instr) \triangleq \text{at}(u-instr) \triangleq l \]
\[ \text{after}(u-instr) \triangleq \text{after}(instr) \]
\[ \text{wf}(instr) \triangleq \text{wf}(u-instr) \]

Figure 7: LISA instructions
Sequences of labelled instructions

\[
\text{instrs} \in \text{Instrs}\\
\text{instrs} ::= \text{instr}\\
\text{pid} (\text{instr}) \triangleq \text{pid} (\text{instrs})\\
\text{Pi} (\text{instrs}) \triangleq \text{Pi} (\text{instrs})\\
\text{instr} (\text{instrs}) \triangleq \text{instr} (\text{instrs})\\
\text{E} (\text{instrs}) \triangleq \text{E} (\text{instrs})\\
\text{at} (\text{instrs}) \triangleq \text{at} (\text{instrs})\\
\text{after} (\text{instr}) \triangleq \text{after} (\text{instrs})\\
\text{wf} (\text{instrs}) \triangleq \text{wf} (\text{instr})\\
\text{at} (\text{instrs}) \triangleq \text{at} (\text{instrs})\\
\text{after} (\text{instrs}) \triangleq \text{after} (\text{instrs})\\
\text{wf} (\text{instrs}) \triangleq \lambda \pi \cdot \text{wf} (\text{instrs}) \land \text{wf} (\text{instrs}) \pi\\
\]

Figure 8: LISA sequences of instructions
4.2.10 Markers

[Labelled] fences are given in Figure 9. Fences can only appear in processes (not in the program prelude or postlude). Fences can have different names and can be labelled, in which case the labels must all occur in the same process as the fence. In this last case, the fence is between any pair of actions with labels in the first and second set. Fences are just markers in the program and their semantics is defined by the \textit{cat} semantics.

\[
\text{f-u-instr} \in F-u-instr
\]

\[
\text{f-u-instr} ::= f[ts] \{ \llbracket l_0 \ldots l_m \rrbracket \llbracket l_0 \ldots l_q \rrbracket \}, \ m, q \geq 0
\]

let \( p = \text{pid}(f-u-instr) \) and \( \Pi = \Pi[f-u-instr] \) and \( \ell = \text{at}(f-u-instr) \) in

\[
\text{instr}[f-u-instr] \triangleq \llbracket (p, \ell, f[ts] \{ \llbracket l_0 \ldots l_m \rrbracket \llbracket l_0 \ldots l_q \rrbracket \}, \theta) | \theta \in \mathcal{P}(p) \rrbracket
\]

\[
\text{wf}[f-u-instr] \triangleq \lambda \pi \cdot \text{let } \pi = \tau_{\text{start}} \times \prod_{p \in \mathcal{P}} \tau_p \times \tau_{\text{finish}} \times \text{rf in } Wf(\pi)
\]

\[
\forall k \in [1, 1 + |\tau_p|], \forall \ell' \in [\mathcal{L}(p)]. \forall \rho, \rho' \in [\mathcal{E}(p)].
\]

\[
(\tau_{p_{k-1}} = s_\ell(\ell, \theta, \rho, \nu) \land \tau_{p_k} = m((p, \ell, f[ts] \{ \llbracket l_0 \ldots l_m \rrbracket \llbracket l_0 \ldots l_q \rrbracket \}, \theta)) \land
\]

\[
(\kappa' = \text{after}(f-u-instr) \land \theta' = \text{succ}(\theta) \land \rho' = \rho \land \nu' = \nu).
\]

([...]) indicates that ... is optional.

Figure 9: LISA fences

\textbf{rmw delimiters} beginrmw[ts] x and endrmw[ts] x in Fig. 14 are markers used to delimit read, modify, and write instructions \textit{rmw} which atomically update shared variable variable \( x \) as defined in Sect. 4.2.11. The fact that read, modify, and write instructions should be atomic will follow from the definition of their semantics in a \textit{cat} specification.

4.2.11 Actions

\textbf{Register instructions} are given in Fig. 10 where \( f[x := v](x) = v \) and \( f[x := v](y) = f(y) \) when \( y \neq x \). LISA register accesses are of the form \texttt{mov} \( r_1 \) \texttt{operation}. Namely, they move the result of an \textit{operation} into a register, e.g. \( r_1 \).

\textbf{Read.} In a read instruction of Fig 12 the \texttt{r-value} denotes the value assigned to register \( r \).
Register instructions

\[
\text{reg-u-instr} \in \text{Reg-u-instr}
\]

\[
\text{reg-u-instr} ::= \text{mov } r_1 \text{ operation}
\]

let \( p = \text{pid}(\text{reg-u-instr}) \) and \( \mathcal{P} = \mathcal{P}(\text{reg-u-instr}) \) and \( \ell = \text{at}(\text{reg-u-instr}) \) in

\[
r_1 \in \mathcal{R}(p)
\]

\[
\text{pid}(\text{operation}) \triangleq p
\]

\[
\text{instr}(\text{reg-u-instr}) \triangleq \{ \langle p, \ell, \text{mov } r_1 \text{ operation} \rangle \}
\]

\[
\mathcal{E}(\text{reg-u-instr}) \triangleq \{ (\langle p, \ell, \text{mov } r_1 \text{ operation} \rangle, \theta) \mid \theta \in \mathcal{P}(p) \land v \in \mathbb{Z} \}
\]

\[
\text{wf}(\text{reg-u-instr}) \triangleq \mathcal{W}^{12}(\pi)
\]

\[
\lambda \pi \cdot \text{let } \pi = \text{start} \times \prod_{p \in \mathcal{P}} \tau_p \times \text{finish} \times \text{rf} \text{ in}
\]

\[
\forall k \in [1, 1 + |\tau_p|]. \forall \kappa' \in \mathcal{L}(p). \forall p, \rho' \in \mathcal{E}(p). \forall v, \nu' \in \mathcal{V}(p). \forall \theta, \theta' \in \mathcal{P}(p). \forall v \in \mathbb{Z}.
\]

\[
\begin{align*}
\tau_{p_{k-1}} &= \tau(\ell, \theta, \rho, \nu) \\
\tau_{p_k} &= a(\langle p, \ell, \text{mov } r_1 \text{ operation} \rangle, \theta, \rho) \land \\
\tau_{p_k} &= a(\langle \kappa', \theta', \rho', \nu' \rangle) \Rightarrow (v = E(\text{operation} \rho, \nu) \land \kappa' = \text{after}(\text{reg-u-instr}) \land \\
& \theta' = \text{succ}(\theta) \land \rho' = \rho[r_1 := v] \land \nu' = v).
\end{align*}
\]

Figure 10: Syntax of LISA register accesses

Write. Write accesses are given in Figure 11. The value written is that of \textit{r-value}.

Read, modify, and write. Read-modify-write accesses are given in Figure 14. Read-modify-write instructions \texttt{rmw[ts] reg-instrs} can only appear in processes (not in the program prelude or postlude). The sequence of register accesses \texttt{reg-instrs} is defined in Fig. 10 as a sequence of (labelled or appropriately labelled by fresh labels) register accesses.

In a read-modify-write instruction \texttt{rmw[ts] reg-instrs r-value}, the last of the register instructions in \texttt{reg-instrs} should assign a value to register \texttt{r}.

The \texttt{rmw} instructions

\[
\texttt{rmw[ts] reg-instrs}
\]

are compiled into a sequence of concrete instructions delimited by \texttt{beginrmw[ts] x} and \texttt{endrmw[ts] x} markers as follows:
beginrmw[ts] x;  
reg-instrs;  
w[ts] x r;  
endrmw[ts] x

before being parsed by the attribute grammar (which therefore contains no \texttt{rmw[ts]} instruction). The last register instruction in \texttt{reg-instrs} must be an assignment to register \texttt{r}. A \texttt{cat} specification must be used to specify atomicity. When abstracting the traces to candidate executions for \texttt{cat} in Sect. 5.1, the abstraction $\alpha_\Xi(\tau)$ will replace the two consecutive events $m((p, \ell_1, \begin{texttt}beginrmw\end{texttt}[ts] x, \theta_1))r((p, \ell_2, \texttt{r[ts]} x, \theta_2), v)$ on $\tau$ by $r((p, \ell_2, \texttt{endrmw[ts]} x, \theta_2), v)$ and $m((p, \ell_1, \texttt{w[ts]} x r, \theta_1), v)m((p, \ell_2, \begin{texttt}endrmw\end{texttt}[ts] x, \theta_2), v)$ to conform to \texttt{cat} conventions (Alglave, Cousot, and Maranget, 2015c).

Write accesses

\begin{align*}
\texttt{w-u-instr} & \in \texttt{W-u-instr} \\
\texttt{w-u-instr} & ::= \texttt{w[ts]} \times \texttt{r-value} \\
\text{let } p = \texttt{pid(w-u-instr)} \text{ and } \Pi = \Pi[\texttt{w-u-instr}] \\
\text{and } \ell = \texttt{at(w-u-instr)} \text{ in} \\
\text{pid(\texttt{r-value})} & \triangleq p \\
\text{instr(\texttt{w-u-instr})} & \triangleq \{(p, \ell, \texttt{w[ts]} \times \texttt{r-value})\} \\
\text{\texttt{C(\texttt{w-u-instr})}} & \triangleq \{w((p, \ell, \texttt{w[ts]} \times \texttt{r-value} \theta), v) | \theta \in \mathbb{P}(p) \land v \in \mathbb{Z}\} \\
\text{\texttt{wf(\texttt{w-u-instr})}} & \triangleq W_{\texttt{w-u-instr}}(\pi) \\
\end{align*}

$\lambda \pi \cdot \text{let } \pi = \begin{texttt}start\end{texttt} \times \prod_{p \in \mathbb{P}} \texttt{tau}_p \times \texttt{finish} \times \texttt{rf} \text{ in} \\
\forall k \in [1, 1 + |\texttt{tau}_p|]. \forall \kappa' \in \mathbb{L}(p). \forall \rho, \rho' \in \mathbb{E}(p). \forall \nu, \nu' \in \mathbb{A}(p). \forall \theta, \theta' \in \mathbb{P}(p). \forall v \in \mathbb{Z}. \\
\begin{align*}
\tau_{p_k-1} &= s(\ell, \theta, \rho, \nu) \land \\
\tau_{p_k} &= m((p, \ell, \texttt{w[ts]} \times \texttt{r-value} \theta), v) \land \\
\tau_{p_{k+1}} &= s(\kappa', \theta', \rho', \nu') \\
(v = E \texttt{r-value}(\rho, \nu) \land \kappa' = \texttt{after(w-u-instr)} \land \\
\theta' = \text{\texttt{succ}}_p(\theta) \land \rho = \rho' \land \nu' = v). \\
\end{align*}

(this includes the initialization writes for $p = \begin{texttt}start\end{texttt}$)

Figure 11: LISA memory write accesses
Read accesses —

\[
\begin{align*}
    r-u-instr & \in R-u-instr \\
    r-u-instr & ::= r[ts] \ r_x \\
    \text{let } p = \text{pid}(r-u-instr) \text{ and } P_i = P_i[r-u-instr] \\
    \text{and } \ell = \text{at}(r-u-instr) \text{ in} \\
    x_1 & \in R(p) \\
    \text{let } p = \text{pid}(r-u-instr) \\
    \ell & = \text{at}(r-u-instr) \\
    \langle p, \ell, r[ts] \ r_x \rangle & \in \text{Wf}(p) \\
    \text{let } p = \text{pid}(r-u-instr) \\
    \ell & = \text{at}(r-u-instr) \\
    \langle p, \ell, r[ts] \ r_x \rangle & \in \text{Wf}(p) \\
\end{align*}
\]

Figure 12: LISA memory read accesses

\section*{Branch.}

Branches are given in Figure 13. The branch instruction \( b[ts] \) \textbf{operation \( \Leftrightarrow \) } branches to \( \Leftrightarrow \) if \textbf{operation} is true, else continues in sequence to the next instruction. The unconditional branching \( b[ts] \) \textbf{true \( \Leftrightarrow \) } will always branch to the next label \( \Leftrightarrow \). \( b[ts] \) \textbf{false \( \Leftrightarrow \) } is equivalent to \textbf{skip}. Branching can only be to an existing label within the same process.

\subsection*{4.2.12 Anarchic semantics of LISA}

The anarchic semantics of a LISA program \( P \) is

\[
S^a[P] \triangleq \{ \pi \in \Pi | \text{Wf}(\pi) \wedge \ldots \wedge \text{Wf}(\pi) \} .
\]  

\textbf{Example 4.1.} Consider the LISA LB (load buffer) program
Sequences of labelled register instructions

\[ \text{reg-instrs} \in \text{Reg-instrs} \]

\[ \text{reg-instrs} ::= \text{l-reg-u-instr} \]

\[ \text{pid}(\text{reg-u-instr}) \triangleq \text{pid}(\text{reg-instrs}) \]

\[ \text{Pi}[^{\text{reg-u-instr}}] \triangleq \text{Pi}[^{\text{reg-instrs}}] \]

\[ \text{instr}(\text{reg-instrs}) \triangleq \text{instr}(\text{reg-u-instr}) \]

\[ \text{E}[^{\text{reg-u-instrs}}] \triangleq \text{E}[^{\text{reg-u-instr}}] \]

\[ \text{at}(\text{reg-instrs}) \triangleq \text{at}(\text{reg-instr}) \triangleq \text{P} \]

\[ \text{after}(\text{reg-u-instr}) \triangleq \text{after}(\text{reg-instrs}) \]

\[ \text{wf}[^{\text{reg-instrs}}] \triangleq \text{wf}[^{\text{reg-u-instr}}] \]

\[ \text{at}(\text{reg-instrs}) \triangleq \text{at}(\text{reg-instr}) \]

\[ \text{after}(\text{reg-instrs}) \triangleq \text{after}(\text{reg-instr}) \]

\[ \text{wf}[^{\text{reg-instrs}}] \triangleq \lambda \pi. \text{wf}[^{\text{reg-instr}}](\pi) \wedge \text{wf}[^{\text{reg-instrs}}](\pi) \]

Figure 13: LISA sequences of labelled register instructions

\[
\{ x = 0; y = 0; \}
\]

\[ \text{P0} \mid \text{P1} ; \]

1: \text{r} x \mid 4: \text{r} 2 \ y ;

2: \text{w} y \mid 5: \text{w} x \ 1 ;

3: \mid 6: \exists(0:r1=1 \land 1:r2=1)
Read-modify-write accesses

\[\text{rmw-u-instr} \in \text{Rmw-u-instr} \quad \text{(originating form \text{rmw[u]}\text{|reg-instr|r-value})}\]

\[
\text{rmw-u-instr} \::= \begin{align*}
\text{beginrm}\text{w}[ts] & \times \text{let} \quad p = \text{pid}(\text{rmw-u-instr}) \text{ and } \Pi = \text{Pl}[\text{rmw-u-instr}] \text{ in } \\
\text{instr} & \triangleq \{(p, \ell, \text{beginrm}\text{w}[ts])\} \\
\mathcal{E} & \triangleq \{\text{m}(p, \ell, \text{beginrm}\text{w}[ts], \theta) | \theta \in \mathbb{P}(p)\} \\
\text{wf} & \triangleq \{m(\langle p, \ell, \text{beginrm}\text{w}[ts], \theta \rangle) | \theta \in \mathbb{P}(p)\} \\
\lambda \pi \cdot \text{let } & \pi = \text{start} \times \prod_{p \in \Pi} \tau_p \times \text{finish} \times \text{rf} \text{ in } \\
\forall k \in [1, 1 + |\tau_p|] \cdot \forall \theta \in \mathbb{P}(p), \forall \ell' \in \mathbb{L}(p), \forall \rho, \rho' \in \mathbb{E}(p), \forall \nu, \nu' \in \mathbb{V}(p), \forall \theta, \theta' \in \mathbb{P}(p), \\
(\tau_{p,k-1} = \mathbb{E}(\ell, \theta, \rho, \nu) \land \\
\tau_{p,k} = \text{m}(\langle p, \ell, \text{beginrm}\text{w}[ts], \theta \rangle) \land \\
(\kappa' = \text{after}(\text{rmw-u-instr}) \land \theta' = \text{succ}(\theta) \land \rho' = \rho \land \nu' = \nu) \Rightarrow \\
\text{endrm}\text{w}[ts] \times \text{let} \quad p = \text{pid}(\text{rmw-u-instr}) \text{ and } \Pi = \text{Pl}[\text{rmw-u-instr}] \text{ in } \\
\text{instr} & \triangleq \{(p, \ell, \text{endrm}\text{w}[ts])\} \\
\mathcal{E} & \triangleq \{\text{m}(p, \ell, \text{endrm}\text{w}[ts], \theta) | \theta \in \mathbb{P}(p)\} \\
\text{wf} & \triangleq \{m(\langle p, \ell, \text{endrm}\text{w}[ts], \theta \rangle) | \theta \in \mathbb{P}(p)\} \\
\lambda \pi \cdot \text{let } & \pi = \text{start} \times \prod_{p \in \Pi} \tau_p \times \text{finish} \times \text{rf} \text{ in } \\
\forall k \in [1, 1 + |\tau_p|] \cdot \forall \theta \in \mathbb{P}(p), \forall \ell' \in \mathbb{L}(p), \forall \rho, \rho' \in \mathbb{E}(p), \forall \nu, \nu' \in \mathbb{V}(p), \forall \theta, \theta' \in \mathbb{P}(p), \\
(\tau_{p,k-1} = \mathbb{E}(\ell, \theta, \rho, \nu) \land \\
\tau_{p,k} = \text{m}(\langle p, \ell, \text{endrm}\text{w}[ts], \theta \rangle) \land \\
(\kappa' = \text{after}(\text{rmw-u-instr}) \land \theta' = \text{succ}(\theta) \land \rho' = \rho \land \nu' = \nu) \Rightarrow \\
\text{Fig. 14: LISA memory read-modify-write accesses}\]

31
Branches

\[ b-u-instr \in B-u-instr \]

\[ b-u-instr ::= b[ts]\ operation[l] \]

let \( p = \text{pid}(b-u-instr) \) and \( \Pi = \Pi[b-u-instr] \) and \( \ell = \text{at}(b-u-instr) \) in

\[
\begin{align*}
\text{pid} & \triangleq p \\
\text{instr} & \triangleq \{ ((p, \ell), b[ts]\ operation[l]) \} \\
\mathcal{E}_{b-u-instr} & \triangleq \{ ((p, \ell), b[ts]\ operation[l], \theta)), \\
& t((p, \ell), b[ts]\ operation[l], \theta)) \mid \theta \in \mathcal{P}(p) \} \\
\text{wf} & \triangleq \end{align*}
\]

\[
\begin{align*}
\lambda \pi \cdot \text{let } & \pi = \text{start} \times \prod_{p \in \Pi} \tau_p \times \text{finish} \times \text{rf} \text{ in} \\
\forall k \in [1, 1 + |\tau_p|]. & \forall \kappa' \in \mathcal{L}(p). \forall \theta, \theta' \in \mathcal{P}(p). \forall \rho, \rho' \in \mathcal{E}(p). \\
\forall \nu, \nu' \in \mathcal{V}(p). & (\tau_{p_{k-1}} = \{\ell, \theta, \rho, \nu\}) \land \\
(\tau_{p_k} = t((p, \ell), b[ts]\ operation[l], \theta)) \land \\
(\tau_{p_k} = \text{after}(\kappa', \theta', \rho', \nu')) \implies \\
(sat(E\ operation[l], \rho, \nu) \neq 0) \land \kappa' = \{\} \land \\
\theta' = \text{succ}_p(\theta) \land \rho' = \rho \land \nu' = \nu) \land \text{Wf}_{16}(\pi) \land
\end{align*}
\]

Figure 15: LISA branches
The computational semantics $S[LB]$ of LB contains the following execution $\pi$ (stamps are useless since all events are different).

$$\pi = w(\text{start}, x, 0) \rightarrow w(\text{start}, y, 0) \rightarrow \cdots$$

- $\bullet (1: \emptyset, \emptyset) \xrightarrow{r(P0, x, \theta_1)} (2: \emptyset, \emptyset, \{\theta_1 = 1\}) \times (3: \emptyset, \emptyset, \{\theta_1 = 1\}) \times$
- $\bullet (4: \emptyset, \emptyset) \xrightarrow{r(P1, y, \theta_4)} (5: \emptyset, \emptyset, \{\theta_4 = 1\}) \times (6: \emptyset, \emptyset, \{\theta_4 = 1\}) \times$
- $\bullet (\text{finish}, y, 1) \rightarrow \bullet (\text{finish}, x, 1) \rightarrow \cdots$

Lemma 4.2 (Orderliness). The stamps of events in a trace $\tau \in \{\tau_p \mid p \in P\}$ of a Lisa program are in strictly increasing order.

Let $W_\tau$ be the condition:

$$\forall p \in \mathbb{P}, \forall i, j \in [1, 1+|\tau|]. \text{ (stamp}(\tau_i) \in C[p] \land \text{stamp}(\tau_j) \in C[p] \land i < j) \implies \text{stamp}(\tau_{i+1}) = \text{succ}_p[\text{stamp}(\tau_i)] \land \text{stamp}(\tau) \leftarrow \text{stamp}(\tau_j).$$

This condition $W_{\tau}$ enforces $W_{\tau}$. □

Example 4.3. The choice $\mathbb{P}(p) \triangleq \{p\} \times \mathbb{N}, \inf_p = (p, 1). \text{succ}_p(p, \theta) = (p, \theta + 1), \text{stamp}(\tau_k) = (p, k), k \in [1, 1+|\tau|], (p', \theta) \leq_p (p'', \theta') \triangleq p = p'' \land \theta < \theta'$ satisfies $W_{\tau}$ and $W_{\tau'}$. □

Proof of Lemma 4.2. The events $\tau_i$ have the stamp $\text{stamp}(\tau_i)$ of the state $\pi_i$ that generate them. By case analysis for Lisa instructions two successive states $\pi_i$ and $\pi_{i+1}$ generated by an instruction of a process $p$ have stamps $\text{stamp}(\pi_{i+1}) = \theta' = \text{succ}_p(\theta) = \text{stamp}(\pi_i)$ in $W_{\tau}$. It follows, by def. of $\text{succ}_p$ in Sect. 4.1.4 that $\theta \leq_p \theta'$. This extends along traces since stamps of process $p$ are not comparable with stamps of other processes. This implies $W_{\tau}$ since stamps for different processes are different and for the same process are comparable since $\leq_p$ is a total order and moreover in strictly increasing order. □

Intuition 4.4. The following Theorem 1 shows that in an execution consisting of a computation part and a communication part, the communication part provides enough information to rebuild the computation part. Otherwise stated, the abstraction $\alpha_L(L[P]) \triangleq \{rf \mid \zeta \times rf \in S[P]\}$ is an isomorphism. Note that Lisa is deterministic but a similar result would hold with random choices. The set of executions resulting from the random choices with a given communication relation $rf$ can be reconstructed from the communication relation $rf$. The importance of this result is to show that to put constraints on the computations it is enough to put constraints on communications. □

Theorem 1. In an anarchic execution $\zeta \times rf \in S[P]$, the communication $rf$ uniquely determines the computation $\zeta$.  

33
Proof of Theorem 1. Let \( \pi = \zeta \times \mathcal{R} = \langle \mathcal{S} \rangle \times \prod_{p \in \mathfrak{P}} \tau_p \times \text{finish} \times \mathcal{R} \), start depends on \( \mathfrak{P} \) (more precisely \( \text{loc} \mathfrak{P} \)) but not on \( \mathcal{R} \). Let \( p \in \mathfrak{P} \) and \( \tau_p = \langle \rightarrow_{\mathfrak{P}k} \tau_{p,k} | k \in [1, 1 + |\zeta|] \rangle \). The proof is by induction of \( k \). For \( k = 1 \), \( \tau_{p,k} = \mathcal{S} \) and \( \tau_{p,k} \) is defined by \( \mathcal{W}_f \mathcal{P}(\pi) \) so does not depend upon \( \mathcal{R} \). For the induction step, we have state \( \tau_{p,k-1} = \mathcal{S}(\ell, \theta, \rho, \nu) \) and must consider all possible instructions at \( \ell \) leading to the next event \( \tau_{p,k} \) and state \( \tau_{p,k} \).

- The semantics of the fence (\( \mathcal{W}_f \mathcal{P}(\pi) \)), register (\( \mathcal{W}_r \mathcal{P}(\pi) \)), write (\( \mathcal{W}_w \mathcal{P}(\pi) \)), RMW (\( \mathcal{W}_{\text{RMW}} \mathcal{P}(\pi) \)) and test (\( \mathcal{W}_t \mathcal{P}(\pi) \)) instruction, hence \( \tau_{p,k} \) and state \( \tau_{p,k} \), does not depend at all on the communication relation \( \mathcal{R} \).

- The semantics \( \mathcal{W}_r \mathcal{P}(\pi) \) of the read instruction is the only one depending on the communication relation \( \mathcal{R} \). The semantics \( \mathcal{W}_r \mathcal{P}(\pi) \) is completely determined by the choice of \( \mathcal{S}[\tau_{j,k}, \tau_{p,k}] \in \mathcal{R} \). By \( \mathcal{W}_f \mathcal{P}(\pi) \), \( \mathcal{W}_r \mathcal{P}(\pi) \) and \( \mathcal{W}_t \mathcal{P}(\pi) \), this choice is unique.

It remains to consider \( \text{finish} \). By \( \mathcal{W}_f \mathcal{P}(\pi) \), \( \text{finish} \) contains only read instructions, and so, by the above argument is uniquely determined by \( \mathcal{R} \). \( \blacksquare \)
5 The weakly consistent semantics of LISA defined by a cat communication specification

To be language independent, the cat communication specification \cite{Alglave, Cousot, and Maranget, 2015} does rely on an abstraction of executions called candidate executions. The abstraction essentially forget about values manipulated by programs and program instructions not related to communications. So a candidate execution records how communications are performed, not which values are communicated. See \cite{Alglave, 2015} for an introduction to the cat communication specification language and \cite{Alglave, 2015a} for models of architectures.

5.1 Abstraction to a Candidate Execution

The candidate execution abstraction \(\alpha : \Pi \rightarrow \Xi\) extracts a candidate execution \(\alpha(\pi) \in \Xi\) from an execution \(\pi \in \Pi\). This candidate execution \(\alpha(\pi)\) is used by the cat specification language semantics to decide whether that execution \(\pi\) is feasible in the weak consistency model defined by a cat communication specification.

5.1.1 Events of an execution.

The candidate execution abstraction extracts the computation events of an execution.

\[
\alpha_e(\tau) \triangleq \{ \epsilon \in \mathcal{E} | \exists \tau_1, \tau_2 . \tau = \tau_1 \xrightarrow{\epsilon} \sigma \tau_2 \}
\]

\[
\alpha_e(\text{start} \times \prod_{p \in \Pi} \tau_p \times \text{finish} \times \text{rf}) \triangleq \bigcup_{p \in \Pi \cup \text{start} \cup \text{finish}} \alpha_e(\tau_p)
\]

The events \(\alpha_e(\pi)\) of an execution \(\pi\) can be partitioned into write, read, branch, fence events, \textit{etc}.

5.1.2 Program order of an event trace.

The candidate execution abstraction extracts the program order of an execution, more precisely the program execution order, \textit{i.e.} the pair of events generated by execution of successive actions of a process.\(^4\) By convention, the initial write events \(w(\text{start})\times\) are before any process event or final read in the program order.

\(^4\)In addition, the cat language does not allow to refer to final reads in \(R(\text{finish})\).

\(^5\)By program order, one must understand order of execution of actions in the program, not necessarily the order in which they appear in the program text, although they are often the same. For a counter-example of the difference between the order of actions in the program and during execution, one can imagine a silly command \texttt{execute_next;a;b;c;} which semantics would to be to execute action \(b\), then \(a\), and then \(c\). So the program syntactic order is \(a\), then \(b\), and then \(c\) while the program execution order is \(b\), then \(a\), and then \(c\).
\[ \alpha_{po}(\tau) \triangleq \{ (\epsilon, \epsilon') \mid \exists \tau_1, \tau_2, \tau_3. \tau = \tau_1 \xrightarrow{\epsilon} \sigma_1 \xrightarrow{\epsilon'} \sigma' \tau_3 \} \]

\[ \alpha_{po}(\tau_{\text{start}} \times \prod_{p \in \Pi} \tau_p \times \tau_{\text{finish}} \times \tau_{rf}) \triangleq \bigcup_{p \in \Pi} \{ (\epsilon, \epsilon') \mid \epsilon \in \alpha_e(\tau_{\text{start}}) \land \epsilon' \in \alpha_e(\tau_p) \cup \alpha_e(\tau_{\text{finish}}) \}\]

5.1.3 Read-from relation.

The candidate execution abstraction extracts the read-from relation of an event trace modeling who reads from where.

\[ \alpha_{rf}(\tau_{\text{start}} \times \prod_{p \in \Pi} \tau_p \times \tau_{\text{finish}} \times \tau_{rf}) \triangleq \tau_{rf} . \]

5.1.4 Initial writes.

By the initialisation condition \( W_f(\pi) \), all shared variables are assumed to be initialised. The candidate execution abstraction extracts the initial writes of an execution.

\[ \alpha_{iw}(\tau_{\text{start}} \times \prod_{p \in \Pi} \tau_p \times \tau_{\text{finish}} \times \tau_{rf}) \triangleq \alpha_e(\tau_{\text{start}}) . \]

5.1.5 Final writes.

By the finalisation condition \( W_{f4f}(\pi) \) all the final values variables are assumed to be read upon program termination. The candidate execution abstraction extracts the final writes satisfying these final reads of an event trace.

\[ \alpha_{fw}(\tau_{\text{start}} \times \prod_{p \in \Pi} \tau_p \times \tau_{\text{finish}} \times \tau_{rf}) \triangleq \{ w \mid \exists r \in \alpha_e(\tau_{\text{finish}}) \cdot \tau f[w, r] \in \tau_{rf} \} . \]

5.1.6 \textbf{cat} candidate executions.

The \textbf{cat} candidate executions are

6The \textbf{herd7} tool considers the program order to be \( \alpha_{po}(\pi) \setminus (\times \times \text{finish}) \) instead.

7The initial writes are not ordered between themselves by the program order and similarly for the final reads. This is because if an execution of the semantics has the initial writes and final reads in some order, reshuffling them in any other order is also a valid execution of the semantics.
\[\Xi \triangleq \wp(\wp(\Sigma \times \wp(\Sigma \times \wp(\wp(\wp(\Sigma \times \wp(\Sigma)))))\\).
\]

\[\alpha_{\Xi} \in \Pi = \Xi\]

\[\alpha_{\Xi}(\pi) \triangleq \langle \alpha_{\Xi}(\pi), \alpha_{\text{po}}(\pi), \alpha_{\text{ff}}(\pi), \alpha_{\text{rf}}(\pi) \rangle \in \Xi\]

\[\alpha_{\Xi}(S) \triangleq \{\langle \pi, \alpha_{\Xi}(\pi) \rangle \mid \pi \in S \} .\]

**Example 5.1.** Continuing Ex. 4.1, we have

\[\alpha_{\text{po}}(t) = \{w(\text{start} x, 0), w(\text{start} y, 0), r(P0, x, \theta_1), w(P0, y, 1), r(P1, y, \theta_4), w(P1, x, 1), r(\text{finish} x), r(\text{finish} y, 1)\}\]

\[\alpha_{\text{po}}(t) = \{\langle w(\text{start} x, 0), r(P0, x, \theta_1)\rangle, \langle w(\text{start} x, 0), w(P0, y, 1)\rangle, \langle w(\text{start} x, 0), r(P1, y, \theta_4)\rangle, \langle w(\text{start} x, 0), r(\text{finish} x)\rangle, \langle w(\text{start} y, 0), r(P0, x, \theta_1)\rangle, \langle w(\text{start} y, 0), w(P0, y, 1)\rangle, \langle w(\text{start} y, 0), r(\text{finish} y, 1)\rangle, \langle w(\text{start} y, 0), w(P1, x, 1)\rangle, \langle w(\text{start} y, 0), r(\text{finish} x)\rangle, \langle w(\text{start} y, 0), r(\text{finish} y, 1)\rangle, \langle w(P0, x, \theta_1), w(P0, y, 1)\rangle, \langle r(P0, x, \theta_1), r(P0, y, 1)\rangle, \langle r(\text{finish} x), r(\text{finish} y, 1)\rangle, \langle r(P0, x, \theta_1), r(\text{finish} x)\rangle, \langle r(P0, x, \theta_1), r(\text{finish} y, 1)\rangle, \langle r(P1, x, \theta_4), w(P1, x, 1)\rangle, \langle r(P1, x, \theta_4), r(\text{finish} x)\rangle, \langle r(P1, x, \theta_4), r(\text{finish} y, 1)\rangle, \langle w(P1, x, 1), r(\text{finish} x)\rangle, \langle w(P1, x, 1), r(\text{finish} y, 1)\rangle\}\]

\[\alpha_{\text{ff}}(t) = \{\langle w(P1, x, 1), r(P0, x, \theta_1)\rangle, \langle w(P0, y, 1), r(P1, y, \theta_4)\rangle, \langle w(P1, x, 1), r(\text{finish} x)\rangle, \langle w(P1, x, 1), r(\text{finish} y, 1)\rangle, \langle w(P0, y, 1), r(\text{finish} x)\rangle, \langle w(P0, y, 1), r(\text{finish} y, 1)\rangle\}\]

\[\alpha_{\text{rf}}(t) = \{w(\text{start} x, 0), w(\text{start} y, 0)\}\]

\[\alpha_{\text{rf}}(t) = \{w(P1, x, 1), w(P0, y, 1)\}\]

This is a non-SC candidate execution because of its cycle in union of program order and communications (Alglave, 2015b):

![Diagram](image)

which would be invalid with the following cat specification

\[
\text{acyclic } (\text{po} \mid \text{rf})^+ \]

**5.2 Abstraction to a semantics with weak consistency model**

**5.2.1 The semantics of a cat weak consistency model specification.**
The semantics $\text{eval}_{\text{cat}}[H_{cm}]\Xi$ of a candidate execution $\Xi = \langle \varsigma, rf \rangle \in \Xi$ defined in (Alglave, Cousot, and Maranget, 2015d) returns a set of answers of the form $\langle j, f, \Gamma \rangle$ where $j = \{\text{allowed}, \text{forbidden}\}$, $f \in \mathcal{F}$ is the set of flags that have been set up on $\Xi$, and $\Gamma$ defines the communication relation for the execution to be allowed/forbidden. This is extended to a set $\mathcal{C} \in \wp(\Xi)$ of candidate executions as

$$\alpha_{\Xi}(\mathcal{C}) \triangleq \{\langle \Xi, \alpha_{\Xi}(\Xi) \rangle \mid \Xi \in \mathcal{C}\}$$

5.2.2 Computational semantics with weak consistency model.

The computational semantics $S$ restricted by a weak consistency model specified by cat specification $H_{cm}$ is then $S \triangleq \alpha_{\text{cat}}[H_{cm}] \circ \alpha_{\Xi}[S[P]]$ where

$$\alpha_{\text{cat}}[H_{cm}](\mathcal{C}) \triangleq \{\langle \varsigma, rf, \Gamma \rangle \mid \langle \varsigma, rf, \Xi \rangle \in \mathcal{C} \land \exists f \in \mathcal{F}. \langle \text{allowed}, f, \Gamma \rangle \in \text{eval}_{\text{cat}}[H_{cm}]\Xi\} \quad \text{(Def. 20)}$$

References

Jade Alglave. Modeling of architectures. In Marco Bernardo and Einar Broch Johnsen, editors, Formal Methods for Multicore Programming - 15th International School on Formal Methods for the Design of Computer, Communication, and Software Systems, SFM 2015, Bertinoro, Italy, June 15-19, 2015, Advanced Lectures, volume 9104 of Lecture Notes in Computer Science, pages 97–145. Springer, 2015a. ISBN 978-3-319-18940-6. doi: 10.1007/978-3-319-18941-3_3. URL http://dx.doi.org/10.1007/978-3-319-18941-3_3.

Jade Alglave. I can’t dance: adventures in herding cats. Lecture notes for Bertorino summer school, March 2015b.

Jade Alglave and Luc Maranget. herd7. virginia.cs.ucl.ac.uk/herd 31 August 2015.

Jade Alglave, Luc Maranget, and Michael Tautschnig. Herding cats: Modelling, simulation, testing, and data mining for weak memory. ACM Trans. Program. Lang. Syst., 36(2):7:1–7:74, 2014. doi: 10.1145/2627752. URL http://doi.acm.org/10.1145/2627752.

Jade Alglave, Mark Batty, Alastair F. Donaldson, Ganesh Gopalakrishnan, Jeroen Ketema, Daniel Poetzl, Tyler Sorensen, and John Wickerson. GPU concurrency: Weak behaviours and programming assumptions. In ASPLOS, 2015a.

Jade Alglave, Patrick Cousot, and Luc Maranget. La langue au chat: cat, a language to describe consistency properties. Unpublished manuscript, 31 January 2015b.

Jade Alglave, Patrick Cousot, and Luc Maranget. Syntax and semantics of the cat language. HSA Foundation, Version 1.1:38 p., 16 Oct 2015c. URL http://www.hsafoundation.com/?ddownload=5382.

Matthew Hennessy and Gordon D. Plotkin. Full abstraction for a simple parallel programming language. In Jirí Becvár, editor, Mathematical Foundations of Computer Science 1979, Proceedings, 8th Symposium, Olomouc, Czechoslovakia, September 3-7, 1979.
HSA F oundation. Hsa platform system architecture specifica tion 1.0. HSA-SysArch-1.01.pdf, cat_ModelExpressions-1.1.pdf, 15 January 2015.

Robert M. Keller. Formal verification of parallel programs. Commun. ACM, 19(7):371–384, 1976. doi: 10.1145/360248.360251. URL http://doi.acm.org/10.1145/360248.360251

James C. King. Symbolic execution and program testing. Commun. ACM, 19(7):385–394, 1976. doi: 10.1145/360248.360252. URL http://doi.acm.org/10.1145/360248.360252

Donald E. Knuth. The genesis of attribute grammars. In Pierre Deransart and Martin Jourdan, editors, Attribute Grammars and their Applications, International Conference WAGA, Paris, France, September 19-21, 1990, Proceedings, volume 461 of Lecture Notes in Computer Science, pages 1–12. Springer, 1990. ISBN 3-540-53101-7. doi: 10.1007/3-540-53101-7_1. URL http://dx.doi.org/10.1007/3-540-53101-7_1

Leslie Lamport. How to make a multiprocessor computer that correctly executes multiprocess programs. IEEE Trans. Computers, 28(9):690–691, 1979. doi: 10.1109/TC.1979.1675439. URL http://dx.doi.org/10.1109/TC.1979.1675439

Jukka Paakki. Attribute grammar paradigms - A high-level methodology in language imple-mentation. ACM Comput. Surv., 27(2):196–255, 1995. doi: 10.1145/210376.197409. URL http://doi.acm.org/10.1145/210376.197409