Two-field inflation with non-minimal coupling

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Received 4 October 2013, revised 11 April 2014
Accepted for publication 15 May 2014
Published 16 June 2014

Abstract
Motivated by the recent ‘Higgs-inflation’ scenario based on a single inflaton field, we consider more generic two-field inflation with non-minimal (NM) coupling term. The generic analytic expressions are derived for cosmological observables with the product-separable as well as additive-separable potentials when the NM coupling term is dominated by one of the two fields. A hybrid model of the inflaton potential $V = \mu^2 \phi^2 \left[ 1 + \cos (\chi \rho) \right]$ with a NM coupling $K = a \phi$, which guarantees the flatness, is closely examined as a concrete example. Compared to the minimal model with $K = 0$, the NM model is shown to provide better fit to the recent cosmological observation by WMAP9 and Planck2013 with the relatively lower field values during the inflationary epoch. Most interestingly, a small value of tensor-to-scalar ratio requires a large NM coupling in our scenario. The model produces non-observably small non-Gaussianity (NG) in most of parameter space while a large NG ($\sim \mathcal{O}(10)$) is obtainable only when the inflation takes place in a limited field space along the top of the potential.

Keywords: Higgs inflation, non-minimal coupling, multi-field inflation

(Some figures may appear in colour only in the online journal)

1. Introduction

The standard slow-roll inflationary paradigm, which can give rise to the primordial perturbations, has been emerged as a solution of cosmological problems, namely, flatness, homogeneity, and isotropy problems [1]. Observational precision has been dramatically improved recently [2–6] and the refined understanding of the origin of inflation would be hoped even though there exists degeneracy problem [7].
Some of the key measurements toward the origin of the inflation are the scalar spectral index, \( n_s \), the tensor-to-scalar ratio, \( r \), and the primordial non-Gaussianity (NG), \( f_{\text{NL}} \). The recent Planck data combined with the final nine-year WMAP data [4, 5] suggest that these observables are within a window:

\[
\begin{align*}
    n_s &= 0.9603 \pm 0.0073 \quad (68\% \, \text{CL}), \\
    r &= < 0.11 \quad (95\% \, \text{CL}), \\
    f_{\text{NL}}^{\text{local}} &= 2.7 \pm 5.8 \quad (68\% \, \text{CL}).
\end{align*}
\]

Combination of these measurements provide a powerful guidance to distinguish a specific inflationary model from the others. A large NG is not expected in single field models [8], and thus observations of the large NG would strongly disfavor single field models.

Recently, ‘Higgs-inflation’ scenario where the inflaton, \( \phi \), is identified with the electroweak Higgs boson in Jordan frame with non-minimal (NM) coupling, \( \xi \phi^2 R \), with the Ricci scalar, \( R \), [9] and its generalization to arbitrary monotonic function, \( \xi \phi^2 R \rightarrow K(\phi)R \), have been studied with one-field model [10]. The most interesting observation would be the fact that an asymptotically flat potential in Einstein frame is automatically obtained when the ratio between the scalar potential \( V(\phi) \) and the square of the NM coupling term \( K(\phi) \) approaches a constant,

\[
V \rightarrow C, \quad \phi \gg 1,
\]

where \( C \) is a positive constant\(^1\). The rather extensive study on NG in single-field inflation model with NM coupling term was recently performed [12], in particular, for the \( \lambda \phi^4 \) potential with \( \xi \phi^2 R \) term [13, 14].

In this paper we consider two inflaton fields aiming toward even further generic multi-field cases with NM couplings. Indeed multiple number of scalar fields and their NM couplings are naturally introduced in generic supergravity theories taking generic Kähler potential into account [15]. To be specific, we focus on the two-field inflaton potentials of the form (i) \( U_i(\phi)U_j(\chi) \) (product-separable) and (ii) \( U_i(\phi) + U_j(\chi) \) (sum-separable) in the presence of the NM coupling term dominated by one of the two fields, namely \( K(\phi, \chi) \approx K(\phi) \) and develop detailed analytic expressions for cosmological observations using \( \delta \) N-formalism. We reserve more generic cases for the future but still apply the developed techniques to a specific model of separable potential, \( V = \mu^2 \phi^2 \left[ 1 + \cos(\chi/\sigma) \right] \) with the NM term \( K = \alpha \phi \) as an example.

The rest of this paper is organized as follows: in section 2, we develop a set of tools in \( \delta \) N-formalism to analyze the two-field inflation with NM coupling term. The analytic expressions for physical quantities including power spectrum, spectral index, tensor-to-scalar ratio, and nonlinearity parameter are derived for a generic setup. In the subsequent section 3, as a concrete example, we apply our results to a specific model of separable potential and show the parameter space which is consistent with the current observation. Finally, the conclusion is given in section 4. In appendix, we provide useful analytic expressions for cosmological observables in various two-field cases with and without NM couplings, which are extensively used in the text.

\(^1\) It has been shown that the inflationary scenario with NM coupling term could be natural in the presence of hyperbolic extra dimensions [11].
2. General analysis of inflation with two fields and NM couplings

Having two scalar fields, \( \phi, \chi \), we write the generic (super)gravity action in Jordan frame as

\[
\frac{L}{\sqrt{-g}} = -\frac{1}{2} \left[ \frac{K (\phi, \chi)}{2} \right] + \frac{1}{2} \left( \partial \phi \right)^2 + \frac{1}{2} \left( \partial \chi \right)^2 - V (\phi, \chi),
\]

where the NM coupling term, in general, is expanded as

\[
K = K_0 + K_\phi \phi \chi + K_\chi \chi \phi + K_\phi \phi \chi \chi + \cdots
\]

and \( V (\phi, \chi) \) is the inflaton potential in the Jordan frame\(^2\). The constant term \( K_0 \) can be absorbed in the definition of \( M_P \) so is neglected in below. The specific forms of \( K \) and \( V \) depend on their origins.

For the given NM coupling term, we always find the conformal transformation leading to the Einstein frame:

\[
g_{\mu \nu} \rightarrow g^E_{\mu \nu} = \Omega^2 g_{\mu \nu},
\]

where the conformal factor, \( \Omega^2 \), is given by

\[
\Omega^2 = 1 + K (\phi, \chi).
\]

The resultant action in the Einstein frame is, then,

\[
\frac{L}{\sqrt{-g^E}} = -\frac{R}{2} + \frac{\Omega^2}{2} \left[ k_1 \left( \partial^\mu \phi \right)^2 + k_2 \left( \partial^\mu \chi \right)^2 + k_3 \left( \partial^\mu \phi \right) \left( \partial^\nu \chi \right) \right] - U (\phi, \chi),
\]

where

\[
k_1 \equiv 1 + \frac{3}{2} \Omega^2 \left( K_{\phi} \right)^2,
\]

\[
k_2 \equiv 1 + \frac{3}{2} \Omega^2 \left( K_{\chi} \right)^2,
\]

\[
k_3 \equiv 3 \Omega^2 K_{\phi} K_{\chi},
\]

and \( U \) is the potential in the Einstein frame, which is related to \( V \) by

\[
U \equiv \frac{V}{\left( 1 + K \right)^\frac{3}{2}}.
\]

Note that the subscript comma ‘,’ denotes the partial derivative, e.g., \( K_{\phi} \equiv \partial K / \partial \phi \).

Now a few comments on the general properties of the action are in order:

- Up to this point, the form of the Lagrangian is symmetric about \( \phi \leftrightarrow \chi \) if \( K (\phi, \chi) = K (\chi, \phi) \) and \( V (\phi, \chi) = V (\chi, \phi) \). The symmetry can be (explicitly) broken either by the NM coupling term and/or the potential.

- Suppose that \( K \gg 1 \) and \( V \gg 1 \) in the limit of large fields, \( \phi, \chi \gg 1 \). A generic condition for large field inflation can be easily noticed since the potential in the Einstein frame, equation (9), looks

\(^2\) Note that we are using \( M_P = 1/\sqrt{8\pi G} = 1 \) units.
\[
\lim_{\phi_j \to \infty} U = \frac{V}{K^2}.
\]  
(10)

If the asymptotic value is constant, as in equation (2), the potential involves a large plateau which may be responsible for large field inflation. Even when one of the two directions is asymptotically flat, it is still possible to have a slow-roll inflation along that direction. In this case, the condition for the flat potential at the particular field direction, \( \phi_j \) here, becomes
\[
\lim_{\phi_j \to \infty} \frac{V}{K^2} = \text{Const}.
\]  
(11)

These results are analogous to those in [10].

The kinetic term in equation (7) is not separable in general and thus is not canonically normalizable as far as \( k_1, k_2, \) and \( k_3 \) in equation (8) are mixed functions of \( \phi_j \) and \( \chi \). We restrict our interest to the cases of canonically normalizable two fields. To be specific, considering that the NM coupling is only a function of one of the two fields, \( K \left( \phi_j, \chi \right) = K \left( \phi_j \right) \) or the terms depending on \( \chi \) are negligibly smaller than the terms with \( \phi_j \), we have \( k_1 = k_1 \left( \phi_j \right) \), \( k_2 = 1 \), and \( k_3 = 0 \). More concretely,
\[
k_1 = 1 + \frac{3 \left( K \phi_j \right)^2}{2 \left( 1 + K \right)}.
\]  
(12)

Then the canonically normalized field in the Einstein frame is obtained by solving the equation:
\[
\frac{d \phi_{\text{E}}}{d \phi_j} = \sqrt{\frac{k_1}{1 + K}}.
\]  
(13)

The action in the Einstein frame, equation (7), is reduced to a well known form [16, 17]:
\[
\frac{\mathcal{L}_E}{\sqrt{-g^E}} = \frac{R}{2} + \frac{k_1}{2 \Omega} \left( \partial \phi_j \right)^2 + \frac{1}{2 \Omega^2} \left( \partial \chi \right)^2 - \frac{V \left( \phi_j, \chi \right)}{\Omega^2}
\]
\[
= \frac{R}{2} + \frac{1}{2} \left( \partial \phi_{\text{E}} \right)^2 + \frac{e^{2b}}{2} \left( \partial \chi \right)^2 - U \left( \phi_j, \chi \right).
\]  
(14)

where \( \phi_{\text{E}} \) is canonically normalized field of \( \phi_j, \chi_{\text{E}} = \chi \), and \( b = -\frac{1}{2} \log \left( 1 + K \right) \). One should notice that the ordinary minimally coupled case is recovered by letting \( b = 0 \) (or \( K = 0 \)).

If the potential is a product-separable in Jordan frame, as \( V = U_1 \left( \phi_j \right) U_2 \left( \chi \right) \), the potential in Einstein frame is also separable as \( U = \tilde{U}_1 \left( \phi_{\text{E}} \right) U_2 \left( \chi_{\text{E}} \right) \) where \( \tilde{U}_1 \left( \phi_{\text{E}} \right) = e^{ab} U_1 \). This type of model has been studied in [16] (and [17]) for (non-) minimal cases. For the brevity of the notation, we will drop subscript ‘E’ in the Einstein frame below.

Up to this far, all expressions are generically applicable for general two-field inflation models with one of the field non-minimally coupled, which is required to make the fields (analytically) canonically normalizable. From now on, however, we would be less ambitious and focus on analytically tractable cases. Indeed, we notice that the product-separable potential \( U \left( \phi_j, \phi_j \right) = U_1 \left( \phi_j \right) U_2 \left( \phi_j \right) \) and the additive-separable potential \( U \left( \phi_j, \phi_j \right) = U_1 \left( \phi_j \right) + U_2 \left( \phi_j \right) \) are tractable. Some detailed calculation are presented in the
appendix and here we will show some main results for the product-separable potential and see how the results could be applied for more realistic model in the subsequent section.

In the background Friedmann–Robertson–Walker metric, the equations of motion for the scalar fields are

\[
\ddot{\phi} + 3H\dot{\phi} + U_{,\phi} = b_\phi e^{2b}\ddot{\chi}^2,
\]

\[
\ddot{\chi} + \left(3H + 2b_\phi\dot{\phi}\right)\dot{\chi} + e^{-2b}U_{,\chi} = 0,
\]

(15)

where \( H \) is the Hubble parameter. From the Einstein equations we get

\[
H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 e^{2b} + U\right).
\]

(16)

Under the assumption of slow-roll, equations (15)–(16) become

\[
3H\dot{\phi} + U_{,\phi} = 0, \quad 3H\dot{\chi} + e^{-2b}U_{,\chi} = 0, \quad H^2 = \frac{1}{3}U.
\]

(17)

In slow-roll inflationary models, it is convenient to introduce slow-roll parameters\(^3\)

\[
e^\phi \equiv \frac{1}{2}\left(\frac{U_{,\phi}}{U}\right)^2, \quad e^\epsilon \equiv \frac{1}{2}\left(\frac{U_{,\chi}}{U}\right)^2 e^{-2b}, \quad e = e^\phi + e^\epsilon,
\]

\[
\eta^\phi \equiv \frac{U_{,\phi\phi}}{U}, \quad \eta^\epsilon \equiv \frac{U_{,\phi\chi}}{U} e^{-2b}.
\]

(18)

The slow-roll condition requires \( e^\phi \ll 1, e^\epsilon \ll 1, \eta^\phi \ll 1, \) and \( \eta^\epsilon \ll 1. \) It is also useful to introduce a dimensionless angle, \( \theta, \) defined by

\[
\cos \theta = \frac{\dot{\phi}}{\sqrt{\dot{\phi}^2 + e^{2b}\dot{\chi}^2}} e^\phi, \quad \sin \theta = \frac{\dot{\chi}}{\sqrt{\dot{\phi}^2 + e^{2b}\dot{\chi}^2}} e^\phi.
\]

(19)

and, in the slow-roll approximation,

\[
\cos^2 \theta \equiv \frac{e^\phi}{e}, \quad \sin^2 \theta \equiv \frac{e^\epsilon}{e}.
\]

(20)

For brevity of notations, we also introduce the following two parameters,

\[
e^b \equiv 8\left(b_\phi\right)^2, \quad \eta^b \equiv 16b_\phi e^\phi,
\]

(21)

which are functions of \( \phi, \) so that are not independent parameters but still useful for out analysis. In terms of the dimensionless angles, the cosmological observables are expressed as\(^4\)

\(^3\) Our \( \epsilon \) parameters and \( \eta \) parameters are respectively analogous to the slow-roll parameters and slow-turn parameters in [20–22].

\(^4\) Some details of the derivation are given in appendix.
\[ P_\gamma = \left( \frac{H_\gamma}{2\pi} \right)^3 e^{2\xi} \cos^2 \theta_\gamma \left( \mathcal{A} \tan^2 \theta_\gamma + \tan \theta_\gamma \right), \]

\[ n_\gamma = 1 - 2e_\gamma - 4e_\gamma \sin^2 \theta_\gamma \cos^2 \theta_\gamma \cos \theta_\gamma \frac{e^{-2\xi}}{\mathcal{A} \tan^2 \theta_\gamma \tan \theta_\gamma} - \frac{\cos^2 \theta_\gamma \left( \mathcal{A} \tan^2 \theta_\gamma - \tan^2 \theta_\gamma \right)^2}{12 \left( \mathcal{A} \tan^2 \theta_\gamma + \tan \theta_\gamma \right)} \left( \eta_\gamma + 2e_\gamma \right) e_\gamma + 2 \mathcal{A} \tan^2 \theta_\gamma \tan \theta_\gamma + 8e_\gamma \mathcal{A} \tan^2 \theta_\gamma \tan \theta_\gamma - s^\phi s^\phi \tan \theta_\gamma \tan \theta_\gamma \frac{2\mathcal{A} \tan^2 \theta_\gamma \tan \theta_\gamma - \tan^2 \theta_\gamma}{\mathcal{A} \tan^2 \theta_\gamma + \tan \theta_\gamma}, \]

\[ r = 16e_\gamma \sin^2 \theta_\gamma \cos^2 \theta_\gamma \mathcal{A}^2 \tan^2 \theta_\gamma \tan \theta_\gamma + \tan \theta_\gamma, \]

\[ f_{\text{nl}}^{(a)} = \frac{5}{6} \left[ 2J_p e_\gamma - F_p \eta_\gamma^\phi - G_p \eta_\gamma^\phi + s^\phi s^\phi K_p \mathcal{E}_\gamma^\phi \right. \]

\[ + \left. 2H_p \left( \eta_\gamma^\phi \sin^2 \theta_\gamma + \eta_\gamma^\phi \cos^2 \theta_\gamma - \frac{1}{2} s^\phi s^\phi \sqrt{\frac{\mathcal{E}_\gamma^\phi}{e_\gamma}} \cos \theta_\gamma + 4e_\gamma \cos^2 \theta_\gamma \sin^2 \theta_\gamma \right) \right], \quad (22) \]

where \( X \equiv 2b_\gamma - 2n_\gamma, \mathcal{A} \equiv e^{-\xi} \left[ 1 + \left( 1 - e^X \right) \tan^2 \theta_\gamma \right], \]

\[ s^{\phi \phi} = \begin{cases} +1, & \text{if } U_\gamma > 0 \\ -1, & \text{if } U_\gamma < 0 \end{cases}, \quad s^\phi = \begin{cases} +1, & \text{if } b' > 0 \\ -1, & \text{if } b' < 0 \end{cases}, \quad (23) \]

and

\[ J_p \equiv \sin^2 \theta_\gamma e^{-\xi} \mathcal{A} \tan^2 \theta_\gamma \tan \theta_\gamma \]

\[ F_p \equiv \frac{e^{-\xi}}{\cos \theta_\gamma} \left( \mathcal{A} \tan^2 \theta_\gamma + \tan \theta_\gamma \right)^2, \]

\[ G_p \equiv \frac{e^{-\xi}}{\cos \theta_\gamma} \tan \theta_\gamma \]

\[ K_p \equiv \frac{e^{-\xi}}{\cos \theta_\gamma} \tan^2 \theta_\gamma \tan \theta_\gamma \]

\[ H_p \equiv \tan^2 \theta_\gamma \left( \mathcal{A} \tan^2 \theta_\gamma - \tan^2 \theta_\gamma \right)^2. \quad (24) \]

Note that the sub(super)script * (e) indicates the value at the horizon exit (end of inflation) and the prime index denotes the derivative with respect to the corresponding fields as usual.

In order to see general tendency of the nonlinearity parameter we plotted \( J_p, F_p, G_p, K_p, \) and \( H_p \) in figure 1 with fixed \( X = 0.0 \) for numerical evaluation. Similar results are found for \( X = \pm \ 0.1, \) too. The general tendency is summarized as follows:

- Since the \( J_p \) is always less than unity and smaller than other parameters, we will ignore \( J_p \)
in evaluating cosmological observables.
The parameters, $F_p$, $K_p$, and $H_p$ can be significant when $e^\Phi \ll e^\chi$ or $(\theta_0, \theta_i) \sim (\pi/2, \pi/2)$. On the other hand, when $(\theta_0, \theta_i) \sim (0, 0)$ or $e^\Phi \gg e^\chi$, $G_p$ and $H_p$ are significant.

- Due to the large $K_p$-dependent term in equation (24), $e^\phi \ll e^\chi (e^\Phi \gg e^\chi)$ case produces large NG when $s^0\phi^0 > 0$ ($< 0$), respectively.

Details of the cosmological observables depend on the explicit realization of the model.

3. An explicit example: $V = \mu^2 \phi^2 [1 + \cos(\chi/\sigma)]$

In this section, we apply the developed formalism for the product-separable potential in the previous section and appendix to a two-field model and demonstrate how the formalism could be actually used. In particular, we are mostly interested in the case where two fields intimately interplay so that physics shows some essential differences from the case of single-field models.

As an example, we take a potential of the simple but still interesting form:

$$V = \mu^2 \phi^2 \left[ 1 + \cos \left( \frac{\chi}{\sigma} \right) \right], \quad K = \alpha \phi. \quad (25)$$
The potential and the typical paths of inflation are depicted in figure 2 for minimal ($\alpha = 0$, top row) and NM ($\alpha \neq 0$, bottom row) case, respectively. A different set of initial conditions presents a different behavior so that cosmological observables are also quite sensitive to the initial conditions. This potential is intriguing in several aspects. It is a hybrid of the chaotic inflation along $\phi$ direction [18] and the natural inflation along $\chi$ direction [19] but still those two fields are closely interconnected. As $V/K^2$ approaches asymptotically constant as we demand in equation (2), the potential in the Einstein frame involves a flat plateau along the $\phi$ direction\(^5\). As a result with the fact that the potential has a ‘valley’ of $V = 0$ along $\chi/\alpha = \pi$, it is possible to have an inflation along $\chi$ direction at a large $\phi$, which essentially reproduces the usual natural inflation with a single field [19]. If the initial field value of $\chi$ locates at the top of the ridge ($\chi = 0$), the inflaton rolls down along $\phi$ direction and again behaves as a single-field inflation (see the first column in figure 2). This case has been extensively studied in [10] where monomial potential $V \sim \phi^{2n}$ and NM coupling $K \sim \phi^n$ are examined with integer powers $n = 1, 2, \cdots$. In general, inflaton rolls down toward the valley not along a single field direction as shown in the second and third columns in figure 2 where we observe the collective behavior of the two fields.

3.1. Minimally coupled case: $K = 0$

In the minimally coupled case ($\alpha = 0$ or equivalently $b = 0$ in equation (14)), the potential in the Jordan frame and the Einstein frame become identical with $\phi_i^J = \phi$ and $\chi_i^J = \chi$:

$$U(\phi, \chi) = \mu^2 \phi^2 \left[ 1 + \cos \left( \frac{\chi}{\alpha} \right) \right].$$  \hspace{1cm} (26)

Here we drop the subscript, ‘E’. The shape of the potential and the typical trajectories of the inflaton are depicted in figures in the top row of figure 2. The potential may be understood as the mass term along the $\phi$ direction, which varies periodically along the $\chi$ direction.

\(^5\) A potential $V = \lambda \phi^4 (1 + \cos \chi)$ with $K = \xi \phi^2$ shows similar phenomenology even though the details can vary with different curvature of the potential and the initial conditions.
For each trajectory, the number of e-foldings, $N$, is found in a rather simple form:

$$N = - \frac{1}{4} \left( \phi_s^2 - \phi_e^2 \right) = - 2\sigma^2 \ln \left( \frac{\text{sin}(\chi_s/2\sigma)}{\text{sin}(\chi_e/2\sigma)} \right)$$

(27)

that determines the evolving field values of $\phi$ and $\chi$ for a given number of e-foldings:

$$\phi(N) = \sqrt{\phi_s^2 - 4N},$$

(28)

$$\chi(N) = 2\sigma \sin^{-1} \left[ \sin \left( \frac{\chi_s}{2\sigma} \right) \exp \left( \frac{N}{2\sigma^2} \right) \right].$$

(29)

where $\phi_s$ and $\chi_s$ are the initial values of inflaton fields. Here we only consider ‘large-to-small’ evolution along the $\phi$ direction and $\phi_s > 2\sqrt{N_0} \approx 15.5$ for reality of $\phi$ during inflation. In general, $\chi$ is a periodic function sitting in the cosine potential but we only restrict ourselves to the case where the inflation takes place within a single period of oscillation. Taking $N = 60$, we find $\phi_e$ and $\chi_e$ as the values of fields at the end of inflation,

$$\phi_e = \phi(N = 60), \quad \chi_e = \chi(N = 60).$$

(30)

The slow-roll parameters are obtained following the equation (18) with $b = 0$:

$$\epsilon^\phi = \frac{2}{\phi_s^2}, \quad \epsilon^\chi = \frac{1}{2\sigma^2} \tan^2 \left( \frac{\chi_s}{2\sigma} \right), \quad \epsilon = \epsilon^\phi + \epsilon^\chi,$$

$$\eta^\phi = \frac{2}{\phi_s^2}, \quad \eta^\chi = - \frac{1}{\sigma^2} \left[ \frac{\cos \left( \chi/\sigma \right)}{1 + \cos \left( \chi/\sigma \right)} \right].$$

(31)

The slow-roll conditions, $\epsilon^\phi, \epsilon^\chi \ll 1$, lead to the domain of the field values in

$$\phi \gg \sqrt{2}, \quad \chi \ll \pi \sigma.$$ 

(32)

Having all the slow-roll parameters in equation (30), we can find the analytic expressions of the power spectrum, spectral index, tensor-to-scalar ratio, and the nonlinearity parameter [38]. The results are presented in appendix. One immediately notices that the power spectrum $P_s \sim O(10^{-9})$ sets the value of $\mu$ parameter as $\mu \sim O(10^{-6})$. Given that the other parameter $\sigma$ has little effect for fixed initial conditions of the fields, we have set $\sigma = 10$.

In figure 3, we plotted the cosmological observables in $\phi_s \in (15, 30)$ and $\chi_s \in (10^{-3}, 10^2)$ with $\sigma = 10.0$. In the parameter space we typically found $f_{\text{NL}}^{(4)} \sim 0.002 - 0.006$. The cosmological observables in $\phi_e \in (15, 30)$ and $\chi_e \in (0, 10)$ are plotted in figure 4. Though we did not present numerical results for a different domain where the inflation takes place with $\chi_s < 10^{-14}$, we sometimes found larger values of $f_{\text{NL}}^{(4)} > 30$. However, this domain turns out to be largely in conflict with the measured cosmological observable: $n_s < 0.01$. Furthermore, if the inflation ends with $\epsilon = 1$, the model only produces non-observably small NG and the values of the tensor-to-scalar ratio, $r$, are turned out to be slightly large, $r > 0.1$. We summarize some representative results in table 1.
Finally, we introduce sizable NM couplings. With the NM coupling term, the inflaton potential in Einstein frame would be different from the one in Jordan frame. It results in the actual cosmological observables are modified. In figure 2 the potential in Einstein frame and the typical evolution of the inflaton fields are depicted for NM case ($\alpha \neq 0$, lower) in contrast to them in the minimal case ($\alpha = 0$, upper).

Defining the conformal transformation, $\Omega^2 = 1 + \alpha \phi$, and the $k$’s in equation (9) as

$$k_1 = 1 + \frac{3\alpha^2}{2(1 + \alpha \phi)}, \quad k_2 = 1, \quad k_3 = 0,$$

(33)

**Figure 3.** Cosmological observables $P_\zeta$, $n_\zeta$, $r$, and $f_{NL}^{(4)}$ are depicted for $\alpha = 0$ case. The large non-Gaussianity above 0.001 is hardly obtainable in this scenario when the observational constraints from Planck are taken into account.
Figure 4. Cosmological observables, $P_\zeta$, $n_\zeta$, $r$, and $f_{\text{NL}}^{(4)}$, are depicted for $\alpha = 0$ case. The large non-Gaussianity above 0.001 is hardly obtainable in this scenario when the observational constraints from Planck are taken into account.

Table 1. Results for the minimally coupled case with $\alpha = 0$ and $\sigma = 10$.

| $\sigma$ | e-foldings | $\phi_*$ | $\chi_\phi$ | $\chi_\zeta$ | $n_\zeta$ | $r$ | $f_{\text{NL}}^{(4)}$ | $\epsilon_\zeta$ |
|----------|------------|---------|-------------|-------------|---------|----|----------------|----------|
| 10       | 60         | 15.556  | 0.0005      | 1.4142      | 0.00067 | 0.9669 | 0.132 | 0.007 | 1          |
| 10       | 60         | 15.556  | 0.005       | 1.4142      | 0.0067  | 0.9669 | 0.132 | 0.007 | 1          |
| 10       | 60         | 15.556  | 0.5         | 1.4142      | 0.675   | 0.9670 | 0.131 | 0.007 | 1          |
| 10       | 60         | 15.556  | 2.872       | 1.4142      | 3.888   | 0.9676 | 0.105 | 0.010 | 1          |
| 10       | 60         | 15.556  | 8.812       | 1.4142      | 12.269  | 0.9684 | 0.0002 | 0.390 | 1          |
| 10       | 60         | 15.556  | 9.802       | 1.4142      | 13.771  | 0.9676 | 0.003 | -0.053 | 1         |
we find the action in Einstein frame following the equation (7):

\[
\mathcal{L}_E^\alpha = -\frac{1}{2} R + \frac{1}{2} \left( \frac{1}{1 + \alpha \phi^4} \right) \left( \partial \phi \right)^2 + \frac{1}{2} \left( \frac{1}{1 + \alpha \phi^4} \right) \left( \partial \chi \right)^2 - \frac{V}{\left( 1 + \alpha \phi^4 \right)^2}
\]

\[
= -\frac{1}{2} R + \frac{1}{2} \left( \partial \phi_E \right)^2 + \frac{1}{2} e^{2\phi_\chi} \left( \partial \chi \right)^2 - U \left( \phi_E, \chi_E \right),
\]

(34)

where \( \chi_E = \chi \) and \( \phi_E \) is canonically normalized by

\[
\frac{d \phi_E}{d \phi} = \sqrt{\frac{1 + \alpha \phi^4 + \alpha^2/2}{1 + \alpha \phi^4}}.
\]

(35)

The coefficient of the NM kinetic term is

\[
e^{2\phi_\chi} = \frac{1}{1 + \alpha \phi^4} \left( \phi_E \right),
\]

(36)

and the inflaton potential in the Einstein frame is

\[
U \left( \phi_E \left( \phi_1 \right), \chi \right) = \mu^2 \phi_1^2 \left( 1 + \cos \left( \frac{\chi}{\sigma} \right) \right).
\]

(37)

Hereafter, we will drop the subscript ‘E’ for brevity of the notation.

We are mostly interested in the large field limit, \( \phi_1 \gg 1/\alpha \), because the potential has a large flat plateau along the \( \phi \) direction as we discussed earlier. In this limit, the explicit form of the canonical field is easily obtained as

\[
\frac{d \phi}{d \phi_1} \approx \frac{1}{\sqrt{\alpha \phi_1}} \Rightarrow \phi \approx 2 \sqrt{\phi_1/\alpha} \Leftrightarrow \phi_1 \approx \frac{1}{4} \alpha \phi^2.
\]

(38)

In terms of the canonical field, the NM kinetic term looks

\[
e^{2\phi_\chi} = \frac{1}{1 + \alpha \phi^4} \approx \frac{4}{4 + \alpha^2 \phi^4},
\]

(39)

and the potential in equation (36) becomes

\[
U \left( \phi, \chi \right) \approx \mu^2 \alpha^2 \phi^4 \left[ 1 + \cos \left( \frac{\chi}{\sigma} \right) \right] \equiv U_1 \left( \phi \right) U_2 \left( \chi \right)
\]

(40)

which leads to

\[
U_1 \left( \phi \right) = \frac{\mu^2 \alpha^2 \phi^4}{4 + \alpha^2 \phi^4}, \quad U_2 \left( \chi \right) = 1 + \cos \left( \frac{\chi}{\sigma} \right)
\]

(41)

It is easily noticed that the potential becomes flat as \( U \rightarrow \mu^2/\alpha^2 \) in the large field limit. When \( \mu \ll 1 \) and we take \( \alpha \sim \mathcal{O} \left( 1 \right) \), the potential is clearly in the sub-Planckian domain.
The slow-roll parameters in equation (18) are, then, given by

\[ \epsilon = \frac{128}{\phi^2 (4 + \alpha^2 \phi^2)^2}, \quad \epsilon' = \frac{4 + \alpha^2 \phi^2}{8\sigma^2} \tan^2 \left( \frac{\chi}{2\sigma} \right), \]

\[ \eta = -\frac{48 \left( -4 + \alpha^2 \phi^2 \right)}{\phi^3 (4 + \alpha^2 \phi^2)^2}, \quad \eta' = -\frac{4 + \alpha^2 \phi^2}{8\sigma^2} \cos \left( \frac{\chi}{\sigma} \right) \sec^2 \left( \frac{\chi}{2\sigma} \right). \] (42)

It is convenient to express the evolution of the inflaton fields in terms of the number of e-foldings which is given by

\[ N = -\int_{\tau}^z \frac{U_1}{U_1'} d\phi = -\int_{\tau}^z e^{-\alpha \chi} \frac{U_1}{U_1'} d\chi, \] (43)

or more explicitly by

\[ N = \frac{1}{16} \left[ 2 \left( \phi_s^2 - \phi_e^2 \right) + \frac{\alpha^2}{4} \left( \phi_s^2 - \phi_e^2 \right) \right], \] (44)

from which we find

\[ \phi(N) = \frac{1}{\alpha} \sqrt{\alpha^4 \phi_s^4 + 8 \alpha^2 \phi_s^2 + 16 - 64 \alpha^4 N - 4}. \] (45)

The other field, \( \chi(N) \), is determined by the equation of motion, \( 3 \dot{H}_\chi \approx -e^{-\alpha \chi} U_\chi \),

\[ \int_{\tau}^z \frac{1 + \cos \left( \frac{\chi}{\sigma^2} \right)}{\sin \left( \frac{\chi}{\sigma} \right)} d\chi \approx \frac{1}{\sigma} \int_0^N \left( 1 + \frac{1}{4} \alpha^2 \phi_s^2(N) \right) dN \]

\[ = \frac{1}{4\sigma} \int_0^N \left( \alpha^4 \phi_s^4 + 8 \alpha^2 \phi_s^2 + 16 - 64 \alpha^4 N \right)^{1/2} dN, \] (46)

from which we find

\[ \chi(N) = 2\sigma \sin^{-1} \left[ \sin \left( \frac{\chi_s}{2\sigma} \right) \exp \left\{ \left( \frac{4 + \alpha^2 \phi_s^2}{768 \alpha^2 \sigma^2} \right) \left[ 1 - \left( 1 - \frac{64 \alpha^4 N}{\left( 4 + \alpha^2 \phi_s^2 \right)^2} \right)^{1/2} \right] \right\} \right]. \] (47)

Collecting all the above formulas, we can find cosmological observables. In appendix the results are presented in a generic form. The numerical results for \( P, n_s, r, \) and \( f_{\text{NL}}^{(i)} \) (local) are depicted in figure 5. We chose \( \mu \sim \mathcal{O} \left( 10^{-5} \right) \) to fit the observed data of the power spectrum of curvature perturbation, \( P_s \approx 2.5 \times 10^{-9} \). It is interesting to notice that \( \mu \sim 10^{-5} \sim M_{\text{GUT}} \), that is the scale of grand unified theory (figure 6).

Noting once again that the condition \( \epsilon_e = 1 \) and 60 e-folds give tight constraint on the initial values of \( \phi \) field, we plot the cosmological observables for a domain of \( \phi_s \in (7.63, 7.70) \) and \( \chi_s \in (1.5, 10) \). One may notice that the observational value of \( n_s \) requires \( \chi_s \lesssim 7 \).

One of the most distinctive features of this case with \( \alpha = 1.0 \) is a small value of tensor-to-scalar ratio, \( r \). This suppression of \( r \) may be understood as an effect of the flat potential in the Einstein frame which is originated from the NM coupling term. In addition, a large NG \( (\sim \mathcal{O}(10)) \) is obtainable when the inflation takes place in a specific field space,
\( \phi, \chi \sim (8.5, 10^{-4}) \), and the inflation ends before reaching \( \epsilon = 1 \). On the other hand, if the inflation ends with \( \epsilon = 1 \), the model only produces non-observably small NG.

We show the representative results in table 2.

4. Conclusion

In a single-field inducing inflation, a large NM coupling term of the inflaton field and Ricci scalar gives rise to a flat potential in Einstein frame. This potential is a desirable one in the slow-roll inflationary paradigm [10] and has been applied to ‘Higgs-inflation’ model [9]. In this paper, we enlarge the scope of the previous studies by considering two fields, one of which includes a significant NM coupling. We provide general analytic expressions for physical observables including power spectrum, spectral index, tensor-to-scalar ratio, and nonlinearity parameter with product-separable and additive-separable inflaton potentials. These results would be extremely useful for explicit comparison with the existing and forthcoming observational data and a specific choice of inflationary potential.

As a concrete example, we examine a hybrid potential, \( V = \mu^2 \phi^4 \left( 1 + \cos \chi / \sigma \right) \) assuming that \( \phi \) has a large NM coupling. This model is potentially realistic thus deserves further study especially when the NM coupling is involved as is summarized in table 2.
A small value of tensor-to-scalar ratio requires a large NM coupling (see table 1 for comparison). The model produces a non-observably small NG when it is required that the inflation ends at $\epsilon \sim 1$ in most of parameter space. We also notice that a large NG $\sim O(10)$ is obtainable when the inflation takes place in a limited field space, $(\phi, \chi) \sim (8.5, 10^{-4})$, near the top of the inflaton potential.

**Figure 6.** Cosmological observables, $P_T$, $n_s$, $r$, and $f_{NL}^{(4)}$, are depicted for $\alpha = 1.0$ case.

**Table 2.** Results for the non-minimally coupled case with $\alpha = 1.0$.

| $\alpha$ | $\sigma$ | e-foldings | $\phi_e$ | $X_e$ | $\phi_*$ | $X_e$ | $n_s$ | $r$ | $f_{NL}^{(4)}$ | $\epsilon$ |
|----------|----------|------------|----------|-------|-----------|-------|-------|----|--------------|-------|
| 1        | 10       | 60         | 7.638    | 0.0005 | 1.670     | 0.012 | 0.9736 | 0.010 | 0.011        | 1     |
| 1        | 10       | 60         | 7.638    | 0.05   | 1.670     | 1.168 | 0.9736 | 0.010 | 0.011        | 1     |
| 1        | 10       | 60         | 7.638    | 0.50   | 1.670     | 12.46 | 0.9714 | 0.010 | 0.008        | 1     |
| 1        | 10       | 60         | 7.638    | 3.54   | 1.664     | 31.42 | 0.9671 | 0.008 | -0.006       | 1     |
| 1        | 10       | 60         | 7.638    | 4.73   | 1.664     | 31.42 | 0.9630 | 0.008 | -0.005       | 1     |
| 1        | 10       | 60         | 7.638    | 6.60   | 1.664     | 31.42 | 0.9537 | 0.008 | -0.004       | 1     |
Acknowledgments

This work is supported by the project of Global PhD Fellowship which the National Research Foundation of Korea conducts from 2011 with grant number 2011-0008792 (JK) and by Basic Science Research Program through the National Research Foundation of Korea funded by the Ministry of Education, Science and Technology with grant numbers 2011-0011660 (YK), and also 2011-0010294 and 2011-0029758 (SCP).

Appendix: Cosmological observables with NM coupling

In this appendix, we derive analytic expressions of cosmological observables following the $\delta N$-formalism [23–27], which is powerful for the super-horizon evolution of the nonlinear curvature perturbation$^6$.

We explicitly present the generic results for product-separable potential, $U_i(\phi)U_i(\chi)$, and sum-separable potential, $U_i(\phi) + U_i(\chi)$, with NM coupling. The results nicely reproduce those for the minimal case [16, 17, 39] when $K = 0$ and also those in [40] where $e^{2\phi} = 1 - e^{-2\phi/\epsilon}$ is assumed.

- **Product-separable potential: $U_i(\phi)U_i(\chi)$** If the potential in Einstein frame (also in Jordan frame) is product-separable, the number of e-foldings is given by

$$N = - \int_U^\varepsilon \frac{U_i}{U_i} d\phi - \int_U^\varepsilon e^{2\phi} \frac{U_i}{U_i} d\chi.$$  \hspace{2cm} (A.1)

Using the constant of motion along the trajectory [39], $C_i = - \int e^{-2b_i} \frac{U_i}{U_i} d\phi + \int U_i d\chi$, we could read out first and second derivatives of $N$ as follows:

$$\frac{\partial N}{\partial \phi_\ast} = \frac{s^\phi}{2\epsilon_\ast^\phi} \left( 1 - \frac{e_{\phi}^\phi}{\epsilon_\ast} e^{2\phi_{\ast} - 2b_{\phi}} \right),$$

$$\frac{\partial N}{\partial \chi_\ast} = \frac{s^\chi}{2\epsilon_\ast^\chi} \left( \frac{e_{\phi}}{\epsilon_\ast} \right) e^{2\chi_{\ast} - 2b_{\chi}},$$

$$\frac{\partial^2 N}{\partial \phi_\ast^2} = \left( 1 - \frac{\eta_{\phi}}{2\epsilon_\ast^\phi} \right) \left( 1 - \frac{e_{\phi}^\phi}{\epsilon_\ast} e^{2\phi_{\ast} - 2b_{\phi}} \right) + \frac{1}{2} s^\phi \left( \frac{\epsilon_{\phi}}{\epsilon_\ast^\phi} \right) e^{2\phi_{\ast} - 2b_{\phi}} + e^{4\chi_{\ast} - 4b_{\chi}} \frac{1}{\epsilon_\ast^\chi} C,$$

$$\frac{\partial^2 N}{\partial \phi_\ast \partial \chi_\ast} = e^{2\chi_{\ast}} \left( 1 - \frac{\eta_{\phi}}{2\epsilon_\ast^\phi} \right) \frac{e_{\phi}^\phi}{\epsilon_\ast} + e^{4\chi_{\ast} - 2b_{\chi}} \frac{1}{\epsilon_\ast^\chi} C,$$

$$\frac{\partial^2 N}{\partial \phi_\ast^2} = - s^\phi \frac{1}{\sqrt{\epsilon_\ast^\phi \epsilon_\ast^\chi}} e^{4\chi_{\ast} - 3b_{\phi}} C,$$  \hspace{2cm} (A.2)

$^6$ There are other methods developed for nonlinear dynamics including perturbative approach [8, 28, 29] and the covariant formalism [30–34]. The advantage of the covariant formalism is that all the quantities are treated in a covariant manner. Physically, $\delta N$ formalism has been shown to be equivalent to the covariant formalism [35, 36]. In a recent work [37], an attractor behavior of NM coupling is shown in multi-field inflation. The corresponding ‘attractor behavior’ has been observed in an earlier work with a single-field inflation with NM couplings [10].
where \( C \equiv \frac{\dot{\phi}^2}{\dot{\eta}} \left[ \eta'' - 4 \frac{\dot{\phi}^3}{\dot{\eta}} - \frac{1}{2} \frac{\dot{\phi}^4}{\dot{\eta}^2} \right] \left[ \frac{\dot{\phi}^2}{\dot{\eta}} \right]^{1/2} \) and \( \eta'' \equiv \left( \eta^2 + \eta^4 \right)/\epsilon \) are used.\(^7\)

The power spectrum is given by

\[
P_s = P_s \sum_{J} N_J N_J G_{IJ},
\]

where \( P_s = \left( H_s/2\pi \right)^2 \) and \( G_{IJ} \) is defined in such a way that the multi-field action in the Einstein frame is written as of the form of

\[
\mathcal{L} = - \frac{R}{2} + \frac{1}{2} \sum_{IJ} \partial_\mu \phi_\mu \partial^\nu \phi^\nu - U \left( \phi \right),
\]

with \( \phi^\nu \)’s as inflaton fields. Substituting equation (A.2) into equation (A.3), one obtains

\[
P_s = \frac{1}{2} \left( \frac{H_s}{2\pi} \right)^2 \left( \frac{u^2 \alpha^2}{\epsilon_s^2} + \frac{v^2}{\epsilon_s^2} \right).
\]

The tensor-to-scalar ratio, \( r = 8 \mathcal{P}_s \), with \( \mathcal{P}_s = 8 P_s \) is

\[
r = \frac{16 e^{-4b_1 + 4b_2}}{u^2 \alpha^2 \epsilon_s^2 + v^2 \epsilon_s^2}.
\]

Since the scale-dependent part of the nonlinearity parameter is too small to be detectable [16, 17], we just take into account the scale-independent part only:

\(^7\) In \( C \) and \( B_s \), the terms \( \sqrt{\epsilon_s^2} \epsilon_s^2 \) and \( \sqrt{\epsilon_s^2} \epsilon_s^2 \) are read \( \sqrt{\epsilon_s^2} \epsilon_s^2 \) and \( \sqrt{\epsilon_s^2} \epsilon_s^2 \), respectively in [17]. Also the signs in \( \tilde{\alpha}_u \) and \( \tilde{\alpha}_v \) below equation (A.11) are opposite in [17].
\[ \frac{6}{5} \alpha^{(1)}_{\text{NL}} = \frac{2 e^{-2b_1 + 2b_2}}{(u^2 \alpha^2_{\phi} + \nu^2 / \epsilon^2_{\phi})} \left[ \frac{u^2 \alpha^2_{\phi}}{\epsilon^2_{\phi}} \left( 1 - \frac{\eta^2_{\phi}}{2\epsilon^2_{\phi}} \right) + \frac{\nu^2}{\epsilon^2_{\phi}} \left( 1 - \frac{\eta^2_{\phi}}{2\epsilon^2_{\phi}} \right) \right] + \frac{s^2 \nu^2 \nu^2 \alpha^2_{\psi}}{2 (\epsilon^2_{\phi})^2} \sqrt{\epsilon^2_{\phi} \epsilon^2_{\phi}} + \left( \frac{\nu^2 \alpha^2_{\psi}}{\epsilon^2_{\phi}} - \frac{\nu^2}{\epsilon^2_{\phi}} \right) e^{2b_1 - 2b_2} \mathcal{C} \right]. \tag{A.8} \]

When \( b = 0 \), then equations (A.5)–(A.8) reduce to the results of minimally coupled case \[17, 39\]. In terms of the dimensionless angle \( \theta \), the results of non-minimally coupled case in equations (A.5)–(A.8) are those in equations (22)–(24).

- Sum-separable potential: \( U_{\phi} (\phi) + U_{\chi} (\chi) \)

The number of e-foldings for the case of sum potential, \( U (\phi, \chi) = U_{\phi} (\phi) + U_{\chi} (\chi) \), is given by

\[ N (\phi_n, \chi_n) = - \int_n^\phi \frac{U_{\phi}}{U_{\phi}} d\phi - \int_n^\chi \frac{U_{\chi}}{U_{\chi}} e^{2b_1} d\chi. \tag{A.9} \]

Using the constant of motion along the trajectory, \( C_{\phi} = - \int e^{-2b_1} \frac{1}{2} d\phi + \int \frac{1}{2} d\chi \), we could read out first and second derivatives of \( N \) as follows:

\[
\begin{align*}
\frac{\partial N}{\partial \phi_n} &= \frac{s^f_{\phi}}{\sqrt{2e^{2\epsilon_{\phi}}}} U_{\phi} + Z_{\phi} - I_{\phi}, \\
\frac{\partial N}{\partial \chi_n} &= \frac{s^f_{\chi}}{\sqrt{2e^{2\epsilon_{\chi}}}} U_{\chi} - Z_{\chi} - J_{\chi}, \\
\frac{\partial^2 N}{\partial \phi_n^2} &= s_{\phi} \left[ 1 - \eta^2_{\phi} \frac{U_{\phi}}{2e^{2\epsilon_{\phi}}} + \frac{s_{\phi}}{U_{\phi} \sqrt{2e^{2\epsilon_{\phi}}} \partial \phi_n} \right], \\
\frac{\partial^2 N}{\partial \chi_n^2} &= e^{2b_1} \left[ 1 - \eta^2_{\chi} \frac{U_{\chi}}{2e^{2\epsilon_{\chi}}} - \frac{s_{\chi} e^{-b_1}}{U_{\chi} \sqrt{2e^{2\epsilon_{\chi}}} \partial \chi_n} \right], \\
\frac{\partial^2 N}{\partial \phi_n \partial \chi_n} &= \frac{s_{\phi}^2}{U_{\phi} \sqrt{2e^{2\epsilon_{\phi}}} \partial \phi_n} - \frac{s_{\phi}^2}{\sqrt{2e^{2\epsilon_{\phi}}} \partial \phi_n} \partial \chi_n - \frac{s_{\chi}^2}{U_{\chi} \sqrt{2e^{2\epsilon_{\chi}}} \partial \chi_n}, \tag{A.10} \\
\frac{\partial^2 N}{\partial \phi_n \partial \chi_n} &= s_{\phi} \left[ \frac{s_{\phi}^2}{U_{\phi} \sqrt{2e^{2\epsilon_{\phi}}} \partial \phi_n} - \frac{s_{\phi}^2}{\sqrt{2e^{2\epsilon_{\phi}}} \partial \phi_n} \partial \chi_n - \frac{s_{\phi}^2}{U_{\phi} \sqrt{2e^{2\epsilon_{\phi}}} \partial \phi_n} \right], \end{align*}
\]

where some useful parameters are introduced as \( Z \equiv \frac{f_{\phi} e^{-2b_1} - f_{\chi} e^{-2b_2}}{\epsilon_{\phi} - \epsilon_{\chi}}, \)

\( I \equiv \int_{\phi_n}^{\phi} \frac{1}{2} 2b_1 e^{2b_1} \frac{\partial \phi_n}{\partial \phi_n} d\chi \), and \( J \equiv \int_{\phi_n}^{\phi} \frac{1}{2} 2b_1 e^{2b_1} \frac{\partial \phi_n}{\partial \phi_n} d\chi \).

Cosmological observables, including power spectrum, spectral index, tensor-to-scalar ratio, and the nonlinearity parameter, are then given by
\[ \mathcal{P}_c = \mathcal{P}_v \left( \frac{u_2^2}{2e_{\phi}^v} \hat{a}_v^2 + \frac{\nu^2}{2e_{\phi}^v} \hat{a}_v \right), \]

\[ n_{c} - 1 = -2 \hat{e}_{\phi} - \frac{4}{(u^2 \hat{a}_v^2/e_{\phi}) + v^2 \hat{a}_v^2/e_{\phi})} - \frac{1}{12} \left( \eta_{e} + 2e_{\phi} \right) \left( \frac{u \hat{a}_v \sqrt{e_{\phi}/e_{\phi}^v} - \nu \hat{a}_v \sqrt{e_{\phi}/e_{\phi}^v}}{(u^2 \hat{a}_v^2/e_{\phi}) + v^2 \hat{a}_v^2/e_{\phi})} \right)^2 \]

\[ + 2 \left( \eta_{e} u \hat{a}_v^2/e_{\phi} + \eta_{e} v^2 \hat{a}_v^2/e_{\phi}) \right) (u^2 \hat{a}_v^2/e_{\phi}) + v^2 \hat{a}_v^2/e_{\phi}) \] \[ + \left( \frac{u \hat{a}_v \sqrt{e_{\phi}/e_{\phi}^v} - \nu \hat{a}_v \sqrt{e_{\phi}/e_{\phi}^v}}{(u^2 \hat{a}_v^2/e_{\phi}) + v^2 \hat{a}_v^2/e_{\phi})} \right). \]

\[ r = \frac{u \hat{a}_v^2/e_{\phi} + v \hat{a}_v^2/e_{\phi}}{2}, \]

\[ f_{\text{NL}}^{(4)} = \frac{5}{6} \left( u^2 \hat{a}_v^2/e_{\phi} + v^2 \hat{a}_v^2/e_{\phi}) \right) \]

\[ \times \left[ \frac{u \hat{a}_v \sqrt{e_{\phi}/e_{\phi}^v} - \nu \hat{a}_v \sqrt{e_{\phi}/e_{\phi}^v}}{\left( 1 - \eta_{e}/2e_{\phi} \right) + C_5} \right] \]

\[ + \left( \frac{u \hat{a}_v \sqrt{e_{\phi}/e_{\phi}^v} - \nu \hat{a}_v \sqrt{e_{\phi}/e_{\phi}^v}}{\left( \hat{A}_5 + \hat{B}_5 \right) - 2 s^h s^\ell \hat{a}_v \sqrt{e_{\phi}/e_{\phi}^v} \frac{\partial I}{\partial \eta_{e}} \right) \right] \]

(A.11)

where \( \mathcal{P}_c \equiv \left( H_0/2\pi \right)^2, \quad u \equiv (U_{z_0} + Z_{z_0})/U_n, \quad v \equiv (U_{z_0} - Z_{z_0})/U_n, \quad \hat{a}_v \equiv 1 - s^h s^\ell e_{\phi}/e_{\phi}, \) and \( \hat{a}_+, \equiv 1 - s^h s^\ell e_{\phi}, \)

It is useful to use \( s^h \sqrt{2e_{\phi}^v e_{\phi}} = 2U_n (\hat{A}_5 + \hat{B}_5 + C_5) \) and \( s^h \sqrt{2e_{\phi}^v e_{\phi}} e^{-h} e_{\phi} = -2U_n (\hat{A}_5 + \hat{B}_5), \) where

\[ \hat{A}_5 \equiv - \frac{U_{z_0}^2 e_{\phi}^v e_{\phi}}{e_{\phi}} \left( 1 - \frac{\eta_{e}}{e_{\phi}} - \frac{s^h s^\ell e_{\phi}^v e_{\phi}}{e_{\phi}^v} \right) e_{\phi}^{a_0 - 4b}, \]

(A.12)

\[ \hat{B}_5 \equiv \frac{s^h s^\ell e_{\phi}^v U_{z_0}}{2 e_{\phi} U_n} \sqrt{e_{\phi}^v e_{\phi}^v} Z e_{\phi}^{a_0 - 2b}, \]

(A.13)

\[ C_5 \equiv \frac{1}{2} s^h s^\ell Z_{z_0} / U_n \sqrt{e_{\phi}^v e_{\phi}^v}, \]

(A.14)

\[ \eta_{e} \equiv \frac{s^h e_{\phi}^v + \eta^v e_{\phi}^v}{e_{\phi}}. \]

(A.15)

When \( b = 0, \) the expressions in equation (A.11) reduce to the results of minimally coupled case in [16].

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