Dicke Narrowing Effect for $r^{-\nu}$-type Collisional Potential

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Abstract. In this paper we present results of spectral profile shape calculations for the $r^{-\nu}$-type collisional potential. The attention is drawn to the Dicke narrowing, however we also refer to the speed-dependent effects. Our considerations were performed within the impact approximation. It was also assumed that dephasing and velocity-changing contributions to the collisions are statistically independent. We found that changing the potential type the collisional effects influence on the line shapes can vary by more than 10%. Therefore it is relevant not only for the ultraaccurate spectroscopy, where model accuracy up to $10^{-6}$ is required, but also in more common applications.

1. Introduction

There is a need for application of ab initio collision operators in the spectral line shapes modeling [1] including Dicke narrowing effect [2, 3] and speed-dependent effects [4, 5]. The class of $r^{-\nu}$-type collisional potentials applied to the line shape problem was introduced by Blackmore [1]. The billiard ball model (hard spheres collisions) based on results obtained by Lindenfeld [6] was applied to the case including speed-dependent collisional broadening and shifting [7]. We extended in Refs. [8, 9] the approach described in Refs. [7, 10] to the class of $r^{-\nu}$-type collisional potentials in the way similar to Blackmore [1]. Expressions for the matrix elements of the velocity-changing collision operators given by Lindenfeld and Shizgal [12] have been used. Contrary to conclusions in [1] we have found significant differences between profiles calculated for various potential powers $\nu$, especially when perturbers are much heavier than absorbers and collisional broadening is negligible. It was shown that, independently from potential power $\nu$, our results converge to the Galatry profile [13] in the soft-collision limit, where perturber to absorber mass ratio $\alpha$ goes to zero. The pure Dicke narrowing case [1] was compared with the case, where speed-dependent collisional broadening as well as Dicke narrowing were taken into account [7]. It was carefully analysed in which part of collisional operator the variation of the potential power $\nu$ has the biggest influence on the spectral line shape.

2. Model

In our consideration the attention is drawn to the system of absorbers highly diluted in a perturber bath. By absorbers we mean particles that resonantly or nearly resonantly interact with the light (here the interaction is approximated with two-level structure), while by perturbers we mean some other particles which are not influenced by the light. It is well known that to find the stationary state of such system taking into account both absorber-light interaction and
absorber-perturber collisions one has to solve transport/relaxation equation [1, 10, 11], which can be transformed to the following form [8, 9, 10]

\[
1 = -i(\omega - \omega_0 - \vec{k}\vec{v})h(\omega, \vec{v}) - (\hat{S}_{VC} + \hat{S}_{D})h(\omega, \vec{v}),
\]

where \(\omega\) and \(\vec{k}\) are frequency and wave vector of absorbed light, respectively. Unperturbed transition frequency is denoted by \(\omega_0\) and \(\vec{v}\) is an absorber velocity. The \(h(\omega, \vec{v})\) function, which related to optical coherence, allows us to calculate the normalized line shape profile

\[
I(\omega) = \frac{1}{\pi} \text{Re} \int f_m(\vec{v})h(\omega, \vec{v})d^3\vec{v},
\]

where \(f_m(\vec{v}) = (\sqrt{\pi}v_m)^{-3}e^{(-\vec{v}/v_m)^2}\) is a Maxwellian velocity distribution, \(v_m = \sqrt{2k_BT/m_1}\) is the most probable absorber speed, \(T\) is a temperature, \(k_B\) is the Boltzmann constant and \(m_1\) is an absorber mass.

The important task is to provide a description of absorber-perturber collisions for the repulsive power low potential \(V(r) = V_0(d/r)^\nu\). Since it is assumed that dephasing and velocity-changing contributions to collisions are statistically independent, see Refs. [14, 15, 16, 17, 18], the collisional operator in Eq. (1) can be written as a simple sum of dephasing operator \(\hat{S}_D\) and velocity-changing operator \(\hat{S}_{VC}\). Following Bermann as well as Ward et al. [4, 5] the dephasing operator \(\hat{S}_D\) for the \(r^{-\nu}\)-type collisional potential can be written as a confluent hypergeometric function of the first kind [19]

\[
\hat{S}_D = (\Gamma + i\Delta)(1 + \alpha)^{-\frac{\nu}{\nu - 2}} F_1 \left( -\frac{\nu - 3}{2\nu - 2}; \frac{3}{2}; -\alpha(x^2) \right),
\]

where \(\Gamma\) and \(\Delta\) are average collisional width and shift, respectively. The normalized absorber velocity is defined as \(x = \vec{v}/v_m\). It should be noted that the formula (3) has been derived within the straight line trajectories approximation.

To handle the velocity-changing contribution to collisions one has to take the Boltzmann collisional operator [1, 12, 20]

\[
\hat{S}_{VC}h(\omega, \vec{v}) = n_2 \int d^3\vec{v}_2 \int d\Omega \left( \frac{d\sigma}{d\Omega} \right) [\vec{v} - \vec{v}_2]f_{m_2}(\vec{v}_2) (h(\omega, \vec{v}_2) - h(\omega, \vec{v})),
\]

where \(d\sigma/d\Omega\) is a differential cross section, \(\vec{v}_2\) is a perturber velocity, \(f_{m_2}(\vec{v}_2)\) is a Maxwellian velocity distribution of perturbers, \(n_2\) is a concentration of perturbers and \(d\Omega\) is a scattering solid angle. The absorber velocity after collision is denoted by \(\vec{v}'\).

Equation (1) together with Eqs. (3) and (4) constitute an integral equation for the \(h(\omega, \vec{v})\) function. According to our knowledge the most efficient method of solving this problem is to expand the operators and unknown function \(h(\omega, \vec{v})\) in the Burnett functions basis [7, 12]. This technique has been already applied to analyse experimental spectral line shapes [21, 22, 23, 24]. That allows to convert the integral equation into an infinite set of self-coupled linear equations, which, however, may be truncated. Matrix elements of \(\hat{S}_D\) can be easily evaluated by performing one dimensional numerical integration [7]. More complex task is to evaluate \(\hat{S}_{VC}\) matrix elements where more sophisticated numerical techniques have to be applied [12] (in the case of \(\nu = \infty\) there is an analytical expression[12]).

3. Results

The model introduced in the previous section is characterized by a six physical parameters: the Doppler width \(\omega_D = kv_m\) (half width at 1/e of maximum), the collisional width \(\Gamma\) (half
width at half maximum - HWHM), the frequency of the velocity changing collisions $\nu_{\text{diff}}$, the perturber to absorber mass ratio $\alpha = m_2/m_1$, the exponent from the power-low potential $\nu$ and the spectral detuning $\Delta \omega = \omega - \omega_0$ or, alternatively, normalized spectral detuning defined as $u = (\omega - \omega_0)/\omega_D$. There is also collisional shift $\Delta$ from Eq. (3), but hereafter, it is assumed to be zero. Moreover, the convergence of numerical methods is determined by the Burnett basis dimensions $L_{\text{max}}$ and $N_{\text{max}}$, see notation in [7]. Within the following calculations it was assumed that $L_{\text{max}} = N_{\text{max}} = 10$. Blackmore [1] was the first who has provided calculations of line profiles for the $r^{-\nu}$-type collisional potential describing velocity-changing contribution to the collisions. Therefore we call such profiles the Blackmore profiles and denote by $B_\nu P$, where the subscript $\nu$ is the potential power. Consequently, when the model contains also speed-dependent dephasing part then we call the profiles speed-dependent Blackmore profiles and designate them by $SDB_\nu P$, where the subscript $h$ indicates that the hypergeometric speed-dependence, given by Eq. (3), is applied.

**Figure 1.** (a) The dashed line represents the Blackmore profile for the potential power $\nu = 4$, while the solid line represents the Blackmore profile for $\nu = \infty$ (e.i. billiard balls profile). (b) Difference between these two profiles ($I_{B_\nu P} - I_{B_\infty P}$). Above results were obtained for $\nu_{\text{diff}}/\omega_D = 0.8$, $\Gamma/\omega_D = 0.0$ , $\alpha = 20$ and for basis dimensions $N_{\text{max}} = L_{\text{max}} = 10$.

**Figure 2.** (a) The dashed line represents the speed-dependent Blackmore profile for the potential power $\nu = 4$, while the solid line represents the speed-dependent Blackmore profile for $\nu = \infty$ . (b) Difference between these two profiles ($I_{SDB_\nu P} - I_{SDB_\infty P}$). Above results were obtained for $\nu_{\text{diff}}/\omega_D = 0.8$, $\Gamma/\omega_D = 0.7$ , $\alpha = 20$ and for basis dimensions $N_{\text{max}} = L_{\text{max}} = 10$.

In the Fig. 1 the results for a zero dephasing part ($\Gamma = 0$) are presented. In this case theDicke narrowing contributes to the spectral profile significantly , because for $\Gamma = 0$ there is only a pure competition between Doppler broadening and velocity-changing collisions. If the profiles for $\nu = 4$ (dashed line) and for $\nu = \infty$ (solid line) are compared then it can be concluded that in this case the type of collisional potential have an influence on the line shape even higher then 10%. The difference between these profiles can be seen in more detail in the Fig. 1 (b). It should
be noted that such high differences can be observed when $\nu_{diff}/\omega_D$ ratio is close to one and this difference vanishes in two limit cases when the ratio $\nu_{diff}/\omega_D$ goes to zero or infinity. This conclusion is not strictly true for extremely high $\alpha$ [8].

The spectral profiles for the speed-dependent cases ($\Gamma = 0.7$) are given in the Fig. 2 (a). The nonzero dephasing collisional contribution suppresses the Dicke narrowing effect substantially (even in the cases when the dephasing is independent from the speed). Moreover, the presence of speed-dependency leads to the contrary result then in pure Dicke narrowing case. Finally the speed-dependent Blackmore profile for $\nu = 4$ is rather broadened then narrowed (as it was in the case from Fig. 1) with respect to the speed-dependent Blackmore profile for $\nu = \infty$, see the Fig. 2 (a). Again, the (b) part of the picture illustrates the difference between these two profiles. Note that the bigger narrowing for $\nu = \infty$ is related to more dramatic speed-dependence of collisional widths for this case comparing to $\nu = 4$ case [9].

To illustrate how the mass ratio $\alpha$ determines the potential power $\nu$ influence on the line shape the differences between the speed-dependent Blackmore profiles for $\nu = 4$ and $\nu = \infty$ normalized to the speed-dependent Blackmore profiles for $\nu = \infty$ were plotted, see Fig. 3. The solid line corresponds to the case without dephasing contribution to the collisions (compare with results from the Fig. 1), while the dashed line corresponds to the case with speed-dependent dephasing, where $\Gamma = 0.7$ (compare with results from the Fig. 2). As it is expected, in the limit of soft collisions ($\alpha \rightarrow 0$) both lines from the Fig. 3 converge to zero, because in this limit all profiles go to Galatry profile [13] independently from collisional potential related to the velocity-changing contribution. Note that here is no need to distinguish between Galatry [13] profile and speed-dependent Galatry [25] profile, since the hypergeometric speed-dependency given by Eq. (3) disappears for $\alpha = 0$.

4. Conclusions
In this paper we have shown that the type of collisional potential posses significant impact on the spectral profile shape and in particular cases it could be even higher then 10%. Moreover, it was presented that this impact, in the case of $\Gamma = 0$, is caused by the velocity-changing collisions, while for $\Gamma \neq 0$ is rather dominated by the speed dependency.

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