Evaluation of QCD sum rules for light vector mesons at finite density and temperature

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QCD sum rules are evaluated at finite nucleon densities and temperatures to determine the change of mass parameters for the lightest vector mesons $\rho$, $\omega$ and $\phi$ in a strongly interacting medium. For conditions relevant for the starting experiments at HADES we find that the in-medium mass shifts of the $\rho$ and $\omega$ mesons are governed, within the Borel QCD sum rule approach, by the density and temperature dependence of the four-quark condensate. In particular, the variation of the strength of the density dependence of the four-quark condensate reflects directly the decreasing mass of the $\rho$ meson and can lead to a change of the sign of the $\omega$ meson mass shift as a function of the density. In contrast, the in-medium mass of the $\phi$ meson is directly related to the chiral strange quark condensate which seems correspondingly accessible.

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\section{I. INTRODUCTION}

The in-medium modifications of the light vector mesons ($\rho$, $\omega$, $\phi$) receive growing attention both from theoretical and experimental sides. On the theoretical side there are various indications concerning an important sensitivity of vector mesons to partial restoration of chiral symmetry in a hot and dense nuclear medium. In particular, at finite temperature the vector and axial-vector correlation functions, which are related to the meson spectral densities, become mixed in accordance with in-medium Weinberg sum rules \cite{1}. At low temperature this mixing can be expressed directly via vacuum correlators in a model independent way \cite{2}. Additionally, as shown within lattice QCD \cite{3} and various effective model calculations \cite{4}, the chiral quark condensate as order parameter decreases with increasing temperature and baryon density. Within the QCD sum rule approach, considerable in-medium modifications of vector meson masses even at normal nuclear matter density have been predicted \cite{5,6}.

On the experimental side there is the idea to probe the in-medium modifications of vector mesons, in particular mass shifts, by measuring dileptons ($e^+e^-$) from meson decays. This is the primary motivation of the presently starting experiments with the HADES detector \cite{7} at the heavy-ion synchrotron SIS at GSI Darmstadt. At higher beam energies the heavy-ion experiments of the CERES collaboration \cite{8} evidenced already hints to a noticeable modification of the dilepton spectrum which can be reproduced under the assumption of a strong melting of the $\rho$ meson in a dense, strongly interacting medium at temperatures close to the chiral transition \cite{9,10}.

While numerous evaluations of QCD sum rules have been performed during the last decade, the majority of them deal either with cold nucleon matter or a hot pion medium. At the same time, to extract the wanted information on the behavior of in-medium mesons via measurements in heavy-ion collisions at SIS energies one has to study the case of finite baryon density and temperature. In this paper we present evaluations of the QCD sum rules for the light in-medium vector mesons in a density - temperature region which is relevant for experiments like the ones at HADES. We study systematically here the relative numerical contributions of the four-quark condensate to the QCD Borel sum rule to find out the general trends of the vector meson mass dependence on density and temperature with respect to variations of the poorly known four-quark condensate.

Our paper is organized as follows. In section 2 we recapitulate the general structure of the QCD Borel sum rule. The operator product expansion and evaluation of the needed condensates are summarized in section 3. In section 4 we discuss the form of the hadronic spectral density. The numerical evaluation of the sum rule is presented in section 5. The summary can be found in section 6.

\section{II. QCD SUM RULES AT FINITE DENSITY AND TEMPERATURE}

Following the approach developed in \cite{11,12} we employ for the QCD sum rules (QSR) at finite nucleon chemical potential $\mu_N$ and temperature $T$ the retarded current-current correlation function in a medium

$$ \Pi_{\mu\nu}^R(q; \mu_N, T) = i \int d^4 x \ e^{iqx} \langle R J_{\mu}(x) J_{\nu}(0) \rangle_{\mu_N, T}, \eqno(1) $$
where \( x = (x^0, \vec{x}) \), \( q = (q^0, \vec{q}) \) and \( \mathcal{R}J_\mu(x)J_\nu(0) \equiv \Theta(x^0)[J_\mu(x), J_\nu(0)] \) with the conserved vector current \( J_\mu; \langle \cdots \rangle_{\mu_N,T} \) denotes the grand canonical ensemble average.

We consider vector currents of QCD with isospin quantum numbers of the respective vector mesons specified as \( J_\mu = \frac{1}{2}(\bar{u}\gamma_\mu u \mp d\gamma_\mu d) \), where the negative (positive) sign is for the \( \rho (\omega) \) meson and \( J_\mu = \bar{s}\gamma_\mu s \) is for the \( \phi \) meson current. Since we focus on the limit \( \vec{q} \to 0 \) in the rest frame of the medium, only the longitudinal invariant \( \Pi_\mu^R = \Pi_\mu^R(\vec{q}/(-3q^2))\bar{q} \to 0 \) of the correlator (3) is needed in our analysis.

Due to the analyticity in the upper half of the complex \( q^0 \) plane the retarded correlation function \( \Pi_\mu^R \) satisfies the standard dispersion relation in a medium

\[
\Pi_\mu^R(q^0; \mu_N, T) = \frac{1}{\pi} \int_0^\infty ds \frac{\text{Im}\Pi_\mu^R(s; \mu_N, T)}{s - (q^0 + i\epsilon)^2},
\]

where a subtraction has been omitted. For large \( Q^2 \equiv -q^2_0 > 0 \) one can evaluate the correlator \( \Pi_\mu^R \) by expanding the product of currents in (3) means by means of the operator product expansion (OPE). The result can be written as

\[
\Pi_\mu^R(Q^2) = -C_0 \ln Q^2 + \sum_{n=1}^{\infty} \frac{C_n}{Q^{2n}},
\]

where the quantities \( C_n \) include the usual Wilson coefficients and the condensates as well. It is remarkable that in the considered region \( q^0 \neq \text{real} \) the OPE for \( \Pi_\mu^R \) is the same as the OPE for the causal (Feynman) correlator. This can be used to get an explicit expression of (\( \Pi_\mu^R \)) in a simple manner.

As usual within the QSR the spectral density \( \rho_{\text{had}}(s; \mu_N, T) = 4\text{Im}\Pi_\mu^R(s; \mu_N, T) \) is modeled by several phenomenological parameters related in particular to the forward scattering amplitudes of the external current \( J_\mu \) and the constituents of the medium.

Performing the Borel transformation with appropriate mass parameter \( M^2 \) (13) of the dispersion relation eq. (2) and taking into account the OPE (3) one gets the basic equation for the QSR evaluation in a medium as

\[
\int_0^{\infty} ds \rho_{\text{had}}(s)e^{-s/M^2} = M^2 \left( C_0 + \sum_{n=1}^{\infty} \frac{C_n}{(n-1)! M^{2n}} \right),
\]

which formally looks like the corresponding equation derived in (12) for the Borel sum rules at finite temperature.

III. OPE AND EVALUATION OF THE CONDENSATES AT FINITE \( \mu_N \) AND \( T \)

The coefficients \( C_n \), which define the r.h.s. of the basic equation (4), include the Wilson coefficients and the grand canonical ensemble average of the corresponding products of quark and gluon field operators. Their general structure up to \( n = 3 \) is given, for instance, in (13). For \( \rho \) and \( \omega \) mesons one has

\[
C_0 = \frac{1}{8\pi^2} \left( 1 + \frac{\alpha_s(\mu^2)}{\pi} \right),
\]

\[
C_1 = -\frac{3m_q^2}{4\pi^2},
\]

\[
C_2 = m_q(\bar{u}u)_{\mu_N,T} + \frac{1}{24}\alpha_s(\pi G^2)_{\mu_N,T} - i\frac{4q^\mu q^\nu}{Q^2}\langle \hat{S}\hat{T}(\bar{u}\gamma_\mu D_\nu u) \rangle_{\mu_N,T},
\]

\[
C_3 = -\frac{\alpha_s}{2}\left( (\bar{u}\gamma_\mu \gamma_5 \lambda^a u \mp d\gamma_\mu \gamma_5 \lambda^a d) \hat{\gamma}^2_{\mu_N,T} \right.
\]

\[
\left. -\frac{\alpha_s}{9}\left( (\bar{u}\gamma_\mu \lambda^a u \mp d\gamma_\mu \lambda^a d) \sum_{q=u,d,s} \bar{q}\gamma^\mu \lambda^a q \right)_{\mu_N,T} \right.
\]

\[
\left. + \frac{16}{3} q^\mu q^\nu q^\lambda q^\sigma \langle \hat{S}\hat{T}(\bar{u}\gamma_\mu D_\nu D_\sigma D_\lambda u) \rangle_{\mu_N,T} \right),
\]

where again the minus (plus) sign corresponds to \( \rho (\omega) \) meson; \( \alpha_s(\mu^2) \) is the strong coupling constant which is taken at the renormalization point \( \mu^2 = 1 \) GeV\(^2\) according to common practice; \( m_q = \frac{1}{2}(m_u + m_d) \) stands for the average light quark mass; \( G^2 = G_{\mu\nu}^a G_{\mu\nu}^a \) with the gluon field strength tensor \( G_{\mu\nu}^a \). The covariant derivative is defined
as \( D_\mu = \partial_\mu + \frac{i}{2} g A_\mu^a \lambda^a \) with SU(3) color matrices \( \lambda^a \) normalized as \( \text{Tr}(\lambda^a \lambda^b) = 2 \delta^{ab} \). The operator \( \hat{S}T \) creates a symmetric and traceless expression with respect to the Lorentz indices.

To get the \( \mu_N \) and \( T \) dependence of the condensates, entering the coefficients \( C_{2,3} \), we assume that the system, at small density and temperature, can be described by non-interacting nucleons and pions. For small enough \( \mu_N \) and \( T \) the particle gas is dilute and the grand canonical ensemble average of an operator \( \hat{O} \) can be approximated by

\[
\langle \hat{O} \rangle_{\mu_N, T} = \langle \hat{O} \rangle_0 + 3 \int \frac{d^3 p}{(2\pi)^3 2E_p} n_B(\pi(p)|\hat{O}|\pi(p)) + 4 \int \frac{d^3 k}{(2\pi)^3 2E_k} n_F(\pi(k)|\hat{O}|N(k)),
\]

where \( \langle \hat{O} \rangle_0 \) is the vacuum expectation value, and \( n_B = [e^{E_p/T} - 1]^{-1} \) and \( n_F = [e^{(E_k - \mu_N)/T} + 1]^{-1} \) are thermal Boson and Fermion distributions; we use the covariant normalization of the one-particle pion (nucleon) state: \( \langle \pi(p')|\pi(p)\rangle = (2\pi)^3 2E_p \delta(\vec{p} - \vec{p}') \langle N(k)|N(k') \rangle = (2\pi)^3 2E_k \delta^3(\vec{k} - \vec{k}') \), where \( E_p = \sqrt{\vec{p}^2 + m_\pi^2} \) (\( E_k = \sqrt{\vec{k}^2 + M_N^2} \)) with \( m_\pi \) (\( M_N \)) as pion (nucleon) mass; the vacuum is normalized as \( \langle 0|0 \rangle = 1 \).

Let us consider first the matrix elements of the scalar (Lorentz invariant) operators. For the vacuum quark and gluon condensates, \( \langle \bar{q} q \rangle_0 \) with \( q = u, d, s \) and \( \langle \bar{q} \gamma_\mu \gamma_5 q \rangle_0 \), we employ values presented in table 1. For the vacuum four-quark condensate we adopt the vacuum saturation hypothesis \([3] \) which is valid in the large-\( N_c \) limit \( (N_c \text{ is number of colors}) \) so that the four-quark condensates become proportional to the squares of the quark condensates. To control a deviation from the vacuum saturation we introduce the parameter \( \kappa_0 \) in the following manner

\[
\langle (\bar{q} \gamma_\mu \gamma_5 q)^2 \rangle_0 = -\langle (\bar{q} \gamma_\mu \gamma_5 \lambda^a u - \bar{d} \gamma_\mu \gamma_5 \lambda^a d)^2 \rangle_0 = -\frac{16}{9} \kappa_0 \langle \bar{q} q \rangle_0^2.
\]

The case \( \kappa_0 = 1 \) corresponds obviously to the exact vacuum saturation as used, for instance, in \([1] \).

The pion and nucleon matrix elements of scalar operators which contribute to \( \langle \bar{q} q \rangle_0 \) do not depend on the particle momenta and, therefore, can be calculated in the limit of vanishing momenta. The pion matrix elements of quark operators can be expressed via vacuum condensates by means of the soft pion theorem \([2] \)

\[
\langle \bar{u} u | \pi | \bar{u} u \rangle = \frac{1}{f_\pi} \langle \bar{u} u | \pi | \bar{u} u \rangle_0, \quad \langle \bar{s} s | \pi | \bar{s} s \rangle = 0
\]

with the vacuum pion decay constant \( f_\pi = 93 \text{ MeV} \).

Among four-quark operators in eq. \([3] \), only the term \( \langle \bar{u} \gamma_\mu \gamma_5 \lambda^a u - \bar{d} \gamma_\mu \gamma_5 \lambda^a d \rangle = \mathcal{O}_5^- \), which contributes to the \( \rho \) meson QSR, has a non-vanishing pion matrix element. Applying the soft pion theorem twice leads to

\[
\langle \pi | \mathcal{O}_5^- | \pi \rangle = -\frac{16}{3f_\pi^2} \langle \bar{u} \gamma_\mu \gamma_5 \lambda^a u \rangle_0^2.
\]

Using the vacuum saturation assumption with the corresponding parameter \( \kappa_0 \) in eq. \([10] \) one gets

\[
\langle \pi | \mathcal{O}_5^- | \pi \rangle = -\frac{256}{27f_\pi^2} \kappa_0 \langle \bar{u} u \rangle_0^2.
\]

The pion expectation value for the scalar gluon condensate is obtained as usual by employing the QCD trace anomaly of the energy-momentum tensor \([2] [3] \) and the Gell-Mann - Oakes - Renner relation

\[
\langle \pi | \bar{\alpha}_a \pi | \pi \rangle = -\frac{8}{9} m_\pi^2.
\]

By a comparison of the quark condensate in a nucleon medium obtained by means of the Feynman - Hellmann theorem, applied to the ground state of nuclear matter in Fermi gas approximation \([4] \),

\[
\langle \bar{q} q \rangle_{\mu_N} - \langle \bar{q} q \rangle_0 = \frac{1}{2} \frac{d e}{2 dm_q}, \quad e = 4 \int_{|k| \leq k_F} \frac{d^3 k}{(2\pi)^3} \sqrt{k^2 + M_N^2}, \quad k_F = \sqrt{\mu_N^2 - M_N^2}
\]

and eq. \([4] \) taken at \( T = 0 \) one can get the nucleon matrix element of the scalar quark operator as

\[
\langle N | \bar{q} q | N \rangle = \frac{M_N \sigma_N}{m_q}, \quad q = u, d.
\]
where $\sigma_N = m_q dM_N/dm_q$ is the nucleon sigma term. Similar steps for the $s$ quark condensate result in

$$\langle N|\bar{s}s|N\rangle = y \frac{M_N\sigma_N}{m_q}$$

(17)

with the parameter $y$ defined via $y\sigma_N = 2m_s dM_N/dm_s$. Our choice of the numerical values of parameters entering eqs. (16, 17) is given in table 1.

To get a proper ansatz for the nucleon matrix element of the scalar four-quark condensates we extend the widely used ground state saturation assumption for the in-medium four-quark condensate at $T = 0$ in the following way

$$\langle \bar{q}g_\mu\gamma_5\lambda^a q \rangle_{\mu_N} = -\langle \bar{q}g_\mu\lambda^a q \rangle_{\mu_N} = \frac{16}{9} \kappa(n_N) \langle \bar{q}q \rangle_{\mu_N}^2,$$

(18)

where the density dependent factor $\kappa(n_N)$ is introduced to control a deviation from the exact ground state saturation in analogy to the vacuum saturation in eq. (14). As pointed out in [15] the factor $\kappa(n_N)$ reflects the contribution from the scalar low-energy excitations of the ground state and seems to be weakly dependent on the nucleon density $n_N$, so that one can use $\kappa(n_N) = \text{const}$ as first approximation. In this case the density dependence of the four-quark condensate appears only via the density dependence of the quark condensate squared. In linear density approximation it is given by

$$\langle \bar{q}q \rangle_{\mu_N}^2 = \langle \bar{q}q \rangle_{\mu_0}^2 + \langle \bar{q}q \rangle_0 \langle N|\bar{q}q|N\rangle \frac{n_N}{M_N}.$$ 

(19)

We go beyond the above first approximation and assume here also a possible linear density dependence of $\kappa(n_N)$, keeping in mind that still $\kappa(n_N) = \kappa_0$ at $n_N = 0$ in accordance with the vacuum case in eq. (18). To leading order in the density this assumption can be expressed as correction to the second term of the r.h.s. of eq. (18) by a constant factor $\kappa_N$. As a result, our parameterization of the four-quark condensate at $T = 0$ has the form

$$\langle \bar{q}g_\mu\gamma_5\lambda^a q \rangle_{\mu_N} = \frac{16}{9} \langle \bar{q}q \rangle_{\mu_0}^2 \left(1 + \frac{\kappa_N}{\kappa_0} \frac{\langle N|\bar{q}q|N\rangle}{\langle \bar{q}q \rangle_0} \frac{n_N}{M_N}\right).$$

(20)

The limit $\kappa_N = \kappa_0 = 2.36$ is used in [15], while the parameterization $\kappa_N = 1.4$ and $\kappa_0 = 3.3$ with $\langle \bar{q}q \rangle_0 = (-230\text{MeV})^3$ corresponds to the choice in [6]. Below we vary the parameter $\kappa_N$ to estimate the contributions of the four-quark condensates to the QSR with respect to the main trends of the in-medium vector meson mass modification.

The needed ansatz for the nucleon matrix element of the scalar four-quark condensate can be extracted then from the direct comparison of our parameterization in eq. (20) and the general expression (11) for the condensates via the matrix elements (the latter ones to be taken at $T = 0$) as

$$\langle N|\bar{q}g_\mu\gamma_5\gamma_5q |N\rangle = \frac{32}{9} \langle \bar{q}q \rangle_0 \langle N|\bar{q}q|N\rangle \kappa_N.$$ 

(21)

Since the matrix element (21) does not depend on particle momenta and temperature it can be employed also with the approximation in eq. (11) for evaluating the four-quark condensates in the general case with $T \neq 0$ and $\mu_N \neq 0$.

The gluon condensate in the nucleon is obtained in the same manner as for pions by using the trace anomaly and eqs. (16, 17)

$$\langle N|\frac{\alpha_s}{\pi}G^2|N\rangle = -\frac{16}{9} M_N M_N^0,$$

(22)

where $M_N^0$ is the nucleon mass in the chiral limit; for numerical values see again table 1.

The pion matrix elements of twist-2 operators, which are symmetric and traceless with respect to Lorentz indices, can generally be written as

$$\langle \pi(p)|\hat{S}\hat{T}\bar{q}g_\mu D_\nu q|\pi(p)\rangle = -i \left(p_\mu p_\nu - \frac{1}{4} g_{\mu\nu} p^2 \right) A_{\pi,\pi}^\mu(\mu^2)$$

(23)

for $q = u, d, s$ and for mass dimension-4 operators, and

$$\langle \pi(p)|\hat{S}\hat{T}\bar{q}g_\mu D_\nu D_\lambda D_\sigma q|\pi(p)\rangle = i \left[p_\mu p_\nu p_\lambda p_\sigma + \frac{p^2}{48} (g_{\mu\lambda}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\lambda} + g_{\mu\nu}g_{\lambda\sigma}) + \frac{p^2}{8} (p_\mu p_\nu g_{\lambda\sigma} + p_\mu p_\lambda g_{\nu\sigma} + p_\nu p_\lambda g_{\mu\sigma} + p_\nu p_\mu g_{\lambda\sigma} + p_\lambda p_\mu g_{\nu\sigma} + p_\lambda p_\nu g_{\mu\sigma})\right] A_{\pi,\pi}^\nu(\mu^2)$$

(24)
for mass dimension-6 operators, respectively. The coefficients $A^q_{i,x}$ and $A^q_{i,s}$ are defined by

$$A^q_{i,x}(\mu^2) = 2 \int_0^1 dx \, x^{i-1} \left[ q_\pi(x, \mu^2) + \tilde{q}_\pi(x, \mu^2) \right], \quad i = 2, 4$$

(25)

where $q_\pi(x, \mu)$ is the quark distribution function inside the pion at the scale $\mu^2$.

The nucleon matrix elements for twist-2 operators of mass dimension-4, $\langle N(k)|\hat{S}\hat{T}q_\mu D_{\nu} q|N(k)\rangle$, and mass dimension-6, $\langle N(k)|\hat{S}\hat{T}q_\mu D_{\nu} D_{\lambda} D_\sigma q|N(k)\rangle$, have the same structure as in eqs. (23 - 25), but with the quark distribution $q_\pi(x, \mu)$ inside the nucleon at scale $\mu^2$ and $A^q_{i,N}(\mu^2) = 2 \int_0^1 dx \, x^{i-1} \left[ q_\pi(x, \mu^2) + \tilde{q}_\pi(x, \mu^2) \right]$. Our choice of the quantities $A^q_{i,\rho,\omega}(N)$ is listed in table 1.

Using the obtained pion and nucleon matrix elements of the corresponding operators and performing the grand ensemble average eq. (20) one finally gets the values of the coefficients $C_{2,3}$ entering the basic equation (3) for $\rho$ and $\omega$ mesons as

$$C_2 = q_2 + 2g_2 + a_2, \quad C_3 = q_4 + a_4,$$

(26)

$$g_2 = \frac{m_q}{24\pi} \alpha_s G^2(0) - \frac{4}{27} M_N M_{\rho} I_1^N - \frac{1}{18} m_\pi^2 I_1^\pi$$

(27)

$$q_2 = \frac{1}{81} \pi \alpha_s \kappa_0 \langle \bar{u} u \rangle_0 \left[ 1 - \frac{\kappa_N}{\kappa_0} m_q (\bar{u} u)_0 \right] Y I_1^N \left[ \frac{36}{7} \xi \pi I_1^\pi \right]$$

(28)

$$g_2 = \frac{112}{81} \pi \alpha_s \kappa_0 \langle \bar{u} u \rangle_0 \left[ 1 - \frac{\kappa_N}{\kappa_0} m_q (\bar{u} u)_0 \right] Y I_1^N \left[ \frac{36}{7} \xi \pi I_1^\pi \right]$$

(29)

$$a_2 = \frac{M_N^2}{3} A_{2,2}^{u+d} I_1^N + \frac{4}{3} A_{2,2}^{u+d} I_2^N + \frac{3}{4} m_\pi^2 A_{2,2}^{u+d} I_1^\pi + A_{2,2}^{u+d} I_2^\pi$$

(31)

$$a_4 = \frac{5}{3} M_N^4 A_{4,4}^{u+d} I_1^N - \frac{20}{3} M_N^2 A_{4,4}^{u+d} I_2^N - \frac{16}{3} A_{4,4}^{u+d} I_3^N$$

$$- \frac{5}{4} m_\pi^4 A_{4,4}^{u+d} I_1^\pi - 5 m_\pi^2 A_{4,4}^{u+d} I_2^\pi - 4 A_{4,4}^{u+d} I_3^\pi,$$

(32)

where $Y = 1$, $\xi^{\rho,\omega} = 1$ for $\rho$ and $\omega$ mesons, $\xi^\pi = 1$ for the $\rho$ mesons, and elsewhere $\xi^\pi = 0$. The sequence of replacements $m_q \rightarrow m_s$, $\langle \bar{u} u \rangle_0 \rightarrow \langle \bar{s} s \rangle_0$ and $Y \rightarrow y m_s/m_q$, $A_{i,N(\rho,\omega)}^{u+d} \rightarrow A_{i,\rho,\omega}(N)$ holds for the $\phi$ meson. The above integrals are

$$I_1^N = \int \frac{d^3 k}{(2\pi)^3} E_k k^{2n-2} n_F, \quad I_1^\pi = \int \frac{d^3 p}{(2\pi)^3} E_k p^{2n-2} n_B.$$

(33)

The density and temperature dependence of the scalar quark and gluon condensates $q_2$ and $g_2$ are exhibited in fig. 1 in [17]. One observes a striking linear density dependence, while for small temperatures there is only a tiny change of the condensate. A suitable approximation for $\langle \bar{q} q \rangle_{n_F,T}/\langle \bar{q} q \rangle_0$, $\langle \bar{s} s \rangle_{n_F,T}/\langle \bar{s} s \rangle_0$ and $\langle \bar{q} q G^2 \rangle_{n_F,T}/\langle \bar{q} q G^2 \rangle_0$ at small temperatures is $1 - \alpha n / n_0$ with $\alpha \approx 0.32$, 0.09 and 0.08, respectively (we use $n_0 = 0.17$ fm$^{-3}$).

### IV. HADRONIC SPECTRAL FUNCTION

To get insight for modeling the hadronic spectral density $\rho_{\text{had}}(s; \mu_N, T) = \frac{1}{\pi} \Im \Pi^R_{\mu\nu}(s; \mu_N, T)$, which enters the l.h.s. of the basic equation (3), one can decompose the in-medium correlator $\Pi^R_{\mu\nu}(q; \mu_N, T)$ in the same manner as in eq. (3) within the dilute gas approximation by restricting on not too large values of $\mu_N$ and $T$

$$\Pi^R_{\mu\nu}(q; \mu_N, T) = \Pi^R_{\mu\nu}(q; 0, 0) + 3 \int \frac{d^3 p}{(2\pi)^3} 2 E_p n_B T^\pi_{\mu\nu}(q; p) + 4 \int \frac{d^3 k}{(2\pi)^3} 2 E_k n_F T^N_{\mu\nu}(q; k),$$

(34)

where $\Pi^R_{\mu\nu}(q; 0)$ is the vacuum expectation and

$$T^\pi_{\mu\nu}(q; p) = i \int d^4 x \, e^{ipx} \langle \pi(x) | R J_\mu(x) J_\nu(0) | \pi(p) \rangle,$$

(35)

$$T^N_{\mu\nu}(q; k) = i \int d^4 x \, e^{ixk} \langle N(k) | R J_\mu(x) J_\nu(0) | N(k) \rangle.$$

(36)
are the forward scattering amplitudes of the external current $J_\mu$ for pion or nucleon, respectively.

For the vacuum QSR, only the first term in eq. (34) is obviously non-vanishing. The corresponding spectral density for a given vector meson channel was successfully modeled in [13] by means of a resonance peak and a continuum (perturbative QCD) contribution. In a medium one has to take into account also the contributions arising from the imaginary parts of the second and third scattering terms in eq. (34). To model such contributions we consider only absorption like processes related to the matrix elements $\langle \pi|J_\mu|\pi \rangle$ and $\langle N|J_\mu|N \rangle$. This is still in accordance with the dilute gas approximation when only the one-particle states are taken into account. In the limit $q \to 0$ the above scattering terms, called usually Landau damping terms, can be calculated exactly (cf. [6,11] for details). As a result, our parameterization of the hadronic spectral density in the vector meson channel $V$ is given by

$$\rho_{\text{had}}^V = \rho_{\text{Lan}}^V + \rho_{\text{res}}^V + \rho_{\text{con}}^V$$

with the usual splitting into the Landau term $\rho_{\text{Lan}}^V = (\rho_{\text{scatt}}^V + \rho_{\text{res}}^V)\delta(s)$ (specified further below), the resonance part $\rho_{\text{res}}^V$ and the continuum contribution $\rho_{\text{con}}^V = C_0 \Theta(s - s_{\text{res}}^V)^2$. The continuum threshold $s_{\text{res}}^V$ separates the resonance part of the spectrum from the continuum part.

Without specifying the resonance part one can define the resonance mass parameter $m_V$ as the normalized first moment via

$$m_V = \sqrt{\frac{\int_0^{s_{\text{res}}^V} ds \rho_{\text{res}}^V(s) e^{-s^2/M^2}}{\int_0^{s_{\text{res}}^V} ds \rho_{\text{res}}^V(s) e^{-s^2/M^2}}}$$

which is evident in the zero-width approximation where $\rho_{\text{res}}^V = F_V \delta(s - m_V^2)$. One should underline that, according to eqs. (34,28), only the weighted integrals over $\rho_{\text{res}}^V$ enter the QSR. That means that various shapes, such like the pole approximation or particular Breit-Wigner parameterizations or double-peak distributions etc., can deliver the same value of the mass parameter $m_V$ supposed the corresponding integrals over $\rho_{\text{res}}$ are fixed.

The above functions $\rho_{\text{scatt}}^N$ entering the Landau damping term in eq. (37) of the spectral density are given for the $\rho$ meson by

$$\rho_{\text{scatt}}^\omega = \frac{1}{48\pi^2} \int_{4m_\rho^2}^{\infty} d\omega^2 \frac{\hat{n}_F}{\sqrt{1 - \frac{4M_\omega^2}{\omega^2}}} \left[ 2 + \frac{4M_\omega^2}{\omega^2} \right]$$

and

$$\rho_{\text{scatt}}^\rho = \frac{1}{24\pi^2} \int_{4m_\rho^2}^{\infty} d\omega^2 \frac{\hat{n}_B}{\sqrt{1 - \frac{4m_\rho^2}{\omega^2}}} \left[ 2 + \frac{4m_\rho^2}{\omega^2} \right]$$

where $\hat{n}_F = [e^{(\omega - 2\mu_N)/2T} + 1]^{-1}$, $\hat{n}_B = [e^{\omega/2T} - 1]^{-1}$.

V. EVALUATION OF QCD SUM RULE

Inserting the hadronic spectral function eq. (37) in the basic equation (4), taking the derivative with respect to $M^{-2}$, and using the definition of the mass parameter $m_V$ in eq. (38) one finally gets the sum rule in the form

$$m_V^2 = M^2 \frac{C_0 \left( 1 - \left[ 1 + \frac{V_M}{2m_\rho^2} \right] e^{-s_{\text{fr}}^V/M^2} \right) - C_2 M^{-4} - \frac{1}{2} C_3 M^{-6} - (\rho_{\text{scatt}}^\omega + \rho_{\text{scatt}}^\rho) M^{-2}}{C_0 \left( 1 - e^{-s_{\text{fr}}^V/M^2} \right) + C_1 M^{-2} + C_2 M^{-4} + \frac{1}{2} C_3 M^{-6} - (\rho_{\text{scatt}}^\omega + \rho_{\text{scatt}}^\rho) M^{-2}}$$

with the above coefficients $C_{0-3}$ from eqs. (44,28,46,52). The identification of $m_V$ with the vector meson mass is suggestive, however, strictly speaking only valid in the limit of narrow resonances. Then the pole approximation is applicable and, as shown in [24], the width of the resonance is calculable from the pole residue $F_V$ afterwards. Having this in mind we focus on the in-medium change of the mass parameter $m_V$ which is associated to the vector meson mass.

The quantity $m_V(M^2)$ still needs to be averaged within a certain Borel window $M_{\min} \cdots M_{\max}$. The continuum threshold $s_{\text{fr}}^V$ is determined by requiring maximum flatness of $m_V(M^2)$ within the Borel window. We use a sliding Borel window determined by the "$10\% + 50\%$ rule" [20,21], i.e., the minimum Borel mass $M_{\min}^2$ is fixed such that the terms of order $O(1/M^6)$ on the OPE side contribute not more than $10\%$, while the maximum Borel mass $M_{\max}^2$ is
determined by the requirement that the continuum part equals to the resonance contribution in the spectral function. (A fixed Borel window with $M_{\text{min}} = 0.8$ GeV$^2$ and $M_{\text{max}} = 1.5$ GeV$^2$ (2.5 GeV$^2$ for the $\phi$ meson) delivers the same results.)

The results of our QSR evaluation for the density and temperature dependence of the resonance mass parameter $m_r$ are exhibited in figs. 1 - 5. As seen in figs. 1a and b the approximately linear dropping of $m_r$ with increasing density appears to be stable with respect to variations of the temperature in wide region $0 \leq T \leq 140$ MeV. In particular, if one parameterizes the four-quark condensate (see eq. (24)) by $\kappa_N = 1.4$ and $\kappa_0 = 3.3$ according to [3], the density dependence can be approximated by $m_r = m^0_r(1 - a_r n_N / n_0)$ with $a_r \approx 0.16$ remaining constant with respect to variations of the temperature within 30% accuracy. In fig. 1a one also observes that at fixed density the mass parameter $m_r$ suffers only small changes when increasing the temperature up to 100 MeV. This is in line with the Eletsky-Jaffe mixing theorem on the in-medium behavior of the vector and axial-vector correlation functions in the chiral limit [23,24,25]. Up to order $T^2$, at $n_N = 0$ the OPE eq. (3) for the $\rho$ meson correlator in a pion medium can be rewritten in the following form

$$\Pi_L^R(Q^2; T) = \frac{1}{3Q^2} \left[ \Pi_\rho(Q^2; 0) + \epsilon \left( \Pi_{\rho_s}(Q^2; 0) - \Pi_\rho(Q^2; 0) \right) \right], \quad (42)$$

where $\Pi_{\rho_s}(Q^2; 0) = \Pi_{\rho_s}(Q^2; 0)$ are the vacuum vector and axial-vector correlators and $\epsilon = \frac{T^2}{\pi^2} B \left( \frac{\rho}{\kappa_0} \right)$ with $B(x) = \frac{e}{4\pi} \int_x^\infty dy \sqrt{y^2 - x^2} / (\exp(y) - 1)$ (note that $B(0) = 1$). In deriving eq. (42) we use the dilute gas averaging [11] and the vacuum saturation hypothesis [30] which obviously give $\Pi_{\rho_s}(0) - \Pi_\rho(0) = \frac{\kappa_0}{\kappa_N} (q^4)_0^2/\kappa_0$. From eq. (42) one can conclude that the $\rho$ meson spectral function [27] used within the framework of the Borel QSR satisfies at the same time the following equation

$$\int_0^\infty ds \frac{\rho_{\text{had}}(s; T)}{s + Q^2} = \frac{1}{3Q^2} \left[ \Pi_\rho(Q^2; 0) + \epsilon \left( \Pi_{\rho_s}(Q^2; 0) - \Pi_\rho(Q^2; 0) \right) \right], \quad (43)$$

so that the in-medium modification of the $\rho$ meson up to order $T^2$ stems from the admixture of the vacuum axial-vector correlator, which is not considerable as long as the temperature is sufficiently small in comparison with chiral transition temperature.

The general trends of the $\rho$ meson mass dependence on the density in the region of low temperature, which is relevant for the heavy-ion experiments at HADES, can be understood by means of the following approximation relying on the pole approximation of $\rho_{\text{res}}^N$

$$F_\rho e^{-m^2_r/M^2} \approx C_0 (1 - e^{-s_0^2/M^2}) M^2 - \rho_{\text{scatt}}^N = \frac{a_2}{M^2} + \frac{q_4}{2M^4}, \quad (44)$$

where the four-quark condensate $q_4$, the non-scalar condensate $a_2$ and the Landau damping term $\rho_{\text{scatt}}^N$ are given by eqs. (30, 31, 28). Since the gluon condensate is weakly dependent on the density, and the quark condensate $q_2$ and the non-scalar condensate $a_2$ have rather small contributions, we have omitted the corresponding terms in eq. (11). It is clear from eq. (44) that in general the scattering term tends to increase the mass $m_r$, while the non-scalar condensate $a_2$ tends to decrease the value of $m_r$. When the value of the Borel mass $M^2$ is about in the center of the Borel window, i.e. $M \approx 1$ GeV, the numerical value of $\rho_{\text{scatt}}^N$ is very close to $a_2/M^2$, so that there is a cancellation of these terms in a wide range of nucleon densities. Therefore, the density dependence of $m_r$ is determined by the behavior of the four-quark condensate, in spite of the fact that the term $q_4/(2M^2)$ is much smaller than $\rho_{\text{scatt}}^N$ or $a_2/M^2$ separately. Actually just the linear dependence of the four-quark condensate on the density leads to the approximate linear decrease of $m_r$ with increasing density, as exhibited in fig. 1b.

Due to the crucial role of the four-quark condensate for the density dependence of $m_r$ it is useful to vary the parameters $\kappa_0$ and $\kappa_N$ which determine the numerical value of $q_4$ and its dependence on the density (see eq. (31)). As seen in fig. 2 the main trend in decreasing the mass $m_r$, obtained above for the parameterization $\kappa_N/\kappa_0 = 0.41$, is stable with respect to the variation of $\kappa_0$ and $\kappa_N$ within the reasonable numerical limits $2.0 \leq \kappa_0 \leq 3$ and $0 \leq \kappa_N \leq 3$. (Note that the value of $\kappa_0$ is usually adjusted to the vacuum mass of the $\rho$ meson and the ratio $\kappa_N/\kappa_0$ is restricted by the conditions $q_4 < 0$ and $q_4 \to 0$ with increasing density and temperature.) In accordance with eq. (30) for $q_4$ and eq. (1) the increase of the ratio $\kappa_N/\kappa_0$ leads to a stronger shift of $m_r$ at fixed density, as displayed in fig. 2. This further confirms the crucial role of the four-quark condensate for the in-medium behavior of the $\rho$ meson mass.

In a sufficiently dense nucleon medium one can expect a strong broadening of the $\rho$ meson width due to inelastic $\rho N$ scattering [25,26]. In principle, the inelastic processes should be included in the hadronic spectral function [25], but in this case the OPE side should also include the higher order effects related, in particular, to two-pion states.
Since such effects are beyond the often used dilute gas approximation we do not concentrate here on the in-medium modifications of the vector meson widths and insist to postpone this to a separate and self-consistent study with higher order effects in the OPE side for finite density and temperature.

The results of our QSR evaluation of the density and temperature dependence of the ω meson mass are exhibited in figs. 3a and b. Due to the comparatively large scattering term \( \rho_{\text{scatt}}^N \gg \rho_{\text{scatt}}^\rho \), the density and temperature behavior of the ω meson mass differs essentially from the corresponding in-medium modifications of the ρ meson. In particular, for the parameterization of the four-quark condensate with \( \kappa_N/\kappa_0 = 0.41 \), which is used above for the ρ meson, the mass of the ω meson increases with increasing density, as shown in fig. 3a for a wide region of the temperature. This can be still understood within the approximation \( \kappa_N/\kappa_0 \ll 1 \), but now one has to take into account that the numerically large term \( \rho_{\text{scatt}}^N \) overwhelms the contributions from the non-scalar condensate \( a_2 \). A similar behavior of the ω meson mass was obtained in [13] for the case \( T = 0 \).

We consider also the sensitivity of the density dependence of the ω meson with respect to various parameterizations of the \( q_4 \) term. In fig. 4 we plot the density dependence of the ω meson mass at fixed temperature \( T = 20 \) MeV for various values of the parameter \( \kappa_N \) which reflects the strength of the density dependence of the four-quark condensate. As seen in fig. 4 the in-medium modification of the ω meson changes even qualitatively under variation of the parameter \( \kappa_N \) (at fixed \( \kappa_0 = 3.0 \)): for \( \kappa_N \lesssim 2 \) the ω meson mass increases with increasing density, while for \( \kappa_N > 2 \) it drops. Such a drastic change of the density dependence happens since the behavior of the density dependent part of the combination \( -\rho_{\text{scatt}}^N + a_2/M^2 + q_4/(2M^4) \) in the QSR eq. (14) is governed by the parameter \( \kappa_N \). The obtained strong sensitivity of \( m_\omega \) on the four-quark condensate holds also for higher temperature. From the above considerations one can conclude that the sign of the in-medium mass shift is directly related to the density dependence of the four-quark condensate.

In contrast to the ρ and ω mesons the density dependence of the φ meson is governed mainly by the in-medium chiral condensate of strange quarks \( m_s \langle \bar{s}s \rangle_{\mu\nu,T} \), which has the dominant contribution in the OPE. As exhibited in figs. 3a and b, \( m_\phi \) drops with increasing nucleon density. (To adjust the vacuum value of the φ meson mass we employ \( \kappa_0 = 0.6 \) and the values of parameters as listed in table 1.) This is in agreement with results of [13] obtained at \( T = 0 \). Due to the weak dependence of the chiral quark condensate on temperature the linear decrease of \( m_\phi \) with increasing nucleon density holds in a wide range of temperatures up to 140 MeV. We also find a very tiny change of \( m_\phi \) by variations of the four-quark condensate due to the density dependence. From the above analyses one can conclude that just the measurement of the φ meson mass shift in a nucleon medium is most appropriate to search for in-medium modifications of the chiral quark condensate. In addition, the φ meson is expected to keep its quasi-particle character as narrow resonance [13]. Such measurements can be accomplished at HADES in heavy-ion experiments along with pion or proton induced φ production at nuclei.

VI. SUMMARY

In summary we present for the first time a systematic analysis of the QCD Borel sum rules for light vector mesons at finite nucleon density and temperature. Our approach is based on the dilute gas approximation for a nucleon medium which is appropriate at not too large nucleon density (say, \( n < 2n_0 \)) and temperature (say, \( T < 100 \) MeV). Such conditions are expected to be reached in heavy-ion experiments at HADES. We find that in-medium modifications of the ρ and ω meson masses are dominated, within the Borel QSR approach, by the dependence of the four-quark condensate \( \langle (\bar{q}q)^2 \rangle_{\mu\nu,T} \) on density and temperature. In particular, the numerical value of the parameter \( \kappa_N \), which describes the strength of the linear density dependence of \( \langle (\bar{q}q)^2 \rangle_{\mu\nu,T} \) governs the decrease of the ρ meson as a function on the density. At the same time an experimental identification of the ρ meson mass shift is closely related to the problem of the strong broadening of its in-medium width. This needs separate considerations with the Borel QSR including higher density and temperature effects into the OPE side. (For a possible strategy to get access to a strongly broadened ρ meson by means of the double differential \( e^+e^- \) spectra we refer the interested reader to [24].)

For the ω meson the sign of the in-medium mass shift is changed by variation of the parameter \( \kappa_N \). This points to the crucial role of the poorly known four-quark condensate and makes definite predictions difficult. A direct measurement of the in-medium mass shift can give, within the considered framework, an important information on the in-medium behavior of the four-quark condensate. Since the difference of the vector and axial-vector correlation functions is proportional to the four-quark condensate the measured sign of the vector meson mass shift can serve, via the four-quark condensate, as a tool for determining how fast the nucleon system approaches the chiral symmetry restoration with increasing density.

The in-medium φ meson mass shift is directly related to the behavior of the chiral strange quark condensate \( m_s \langle \bar{s}s \rangle_{\mu\nu,T} \) which decreases with density almost independently of the temperature in a wide region of density and temperature. This offers the chance to measure the density dependence of the chiral quark condensate via an in-
medium mass shift of the $\phi$ meson in heavy-ion experiments, supposed the strangeness factor $y$ is not too small \cite{25}. Since at nuclear saturation density the mass shift is already noticeable, one can also complementary search for a $\phi$ meson mass shift in hadron - nucleus reactions at the HADES detector under suitable kinematical conditions.

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| parameter | numerical value | reference |
|-----------|----------------|-----------|
| $\alpha_s$ | 0.38 | 26 |
| $m_q$ | 0.0055 GeV | 14 |
| $m_s$ | 0.130 GeV | 14 |
| $f_\pi$ | 0.093 GeV | 14 |
| $M_N$ | 0.938 GeV | 26 |
| $M_N'$ | 0.770 GeV | 3 |
| $\sigma_N$ | 0.045 GeV | 14 |
| $y$ | 0.22 | 3,14 |
| $m_\pi$ | 0.138 GeV | 14 |
| $(\bar{u}u)_0 = (\bar{d}d)_0$ | $(-0.245 \text{GeV})^2$ | c.f. 3 |
| $(\bar{s}s)_0$ | $0.7(\bar{u}u)_0$ | 26 |
| $(\bar{u}u'G^2)_0$ | $(0.33 \text{GeV})^4$ | 27 |
| $A_{2,N}^{+1/2}$ | 1.02 | 3 |
| $A_{2,N}^{+3/2}$ | 0.12 | 5 |
| $A_{2,N}^{+5/2}$ | 0.97 | 12 |
| $A_{3,N}^{+1/2}$ | 0.255 | 12 |
| $A_{3,N}^{+3/2}$ | 0.1 | 3 |
| $A_{4,N}$ | 0.004 | 3 |
| $A_{2,\pi}$ | 0.08 | 19 |
| $A_{4,\pi}$ | 0.008 | 19 |

TABLE I. Set of parameters used.
FIG. 1. The mass parameter $m_\rho$ as a function of temperature (a, upper panel) and density (b, lower panel). The solid (dashed, dotted) curves in (a) are for densities $0.01 \ (0.17, \ 0.34) \ fm^{-3}$, while the solid (dashed) curves in (b) are for temperatures $20 \ (140) \ MeV$. 

\[ T [MeV] \] 
\[ n [fm^{-3}] \]
FIG. 2. The mass parameter $m_\rho$ as a function of $\kappa_N$ for $\kappa_0 = 3.0$ and $2.0$ (solid and dashed curves, respectively) at $n_N = 0.15$ fm$^{-3}$ and $T = 20$ MeV.
FIG. 3. As in fig. 1 but for the $\omega$ meson. In (b) the temperatures are 20, 80 and 140 MeV (from top to bottom).
FIG. 4. The dependence of the $\omega$ meson mass parameter on the density at $T = 20$ MeV and $\kappa_0 = 3.0$. The values of $\kappa_N$ are 0, 1, 2, 3, 4 (from top to bottom).
FIG. 5. As in fig. 4 but for the $\phi$ meson.