Crossed Andreev reflection in a graphene bipolar transistor

J. Cayssol
Condensed Matter Theory Group, CPMOH,
UMR 5798, Université Bordeaux I,
33405 Talence, France

We investigate the crossed Andreev reflections between two graphene leads connected by a narrow superconductor. When the leads are respectively of the n- and p- type, we find that electron elastic cotunneling and local Andreev reflection are both eliminated even in the absence of any valley-isospin or spin polarizations. We further predict oscillations of both diagonal and cross conductances as a function of the distance between the graphene-superconductor interfaces.

Several decades after Einstein, Podolsky and Rosen raised their famous paradox [1], the successful implementation and study of polarization-entangled states of photons [2] has ruled out the possibility of simple local hidden-variables formulations of quantum physics [3]. In solid state physics, the controlled production and detection of charge- or spin-entangled electronic states remains a major challenge, regarding the fundamental concepts of quantum physics, as well as quantum processing and communication issues. Owing to the structure of their ground state, conventional singlet superconductors were suggested as natural sources of spin-entangled [4, 5, 6] or even momentum-entangled electrons [7]. Unfortunately, superconductors are also bad beam splitters since the electron-hole Andreev conversion is essentially a retroreflection in usual metals or semiconductors [8]. Strikingly Beenakker uncovered that Andreev reflection (AR) may even momentum-entangled electrons [7]. Unfortunately, superconductors are also bad beam splitters since the electron-hole Andreev conversion is essentially a retroreflection in usual metals or semiconductors [8]. Strikingly Beenakker uncovered that Andreev reflection (AR) may be specular in graphene [9, 10]. Therefore it should be possible to observe paired electrons along diverging trajectories within a single graphene flake connected to a large superconducting electrode. Nevertheless angular filtering is a rather difficult task in quantum electronics in contrast to optics. Accordingly a lot of theoretical [11, 12, 13] and experimental [14] efforts have been devoted to the crossed Andreev reflection (CAR) process by which a superconducting condensate (S) emits two quasiparticles in two normal metallic leads $N_1$ and $N_2$ where they can be probed separately. The main drawback of such $N_1SN_2$ junctions was identified as the ubiquitous presence elastic cotunneling (EC) and local AR. Indeed during the EC process an electron tunnels elastically from $N_1$ to $N_2$ through the superconductor without any Cooper pair transfer, while in AR the paired electrons are injected in the same lead. In standard nonrelativistic conductors with low transparency tunnel contacts, the cross conductances originating from CAR and EC cancel exactly each other in the noninteracting limit [11], and it is necessary to consider the noise properties to probe the CAR process [13].

In this Letter, we show that the unique relativistic band structure of graphene enables to observe a pure crossed Andreev reflection in a three-terminal n graphene/superconductor/p graphene ($G_1SG_2$) bipolar transistor, see Fig. 1. Accordingly the injected Cooper pair is splitted in electrons which further propagate in opposite directions within $G_1$ and $G_2$ respectively. Indeed both EC and local AR may be totally suppressed owing to the presence of Dirac points in the spectrum of $G_1$ and $G_2$. In contrast to the nonrelativistic case, a CAR dominated transport should be observed directly in the conductance measurements performed on such bipolar graphene transistor (see Fig. 2) without resorting to noise [13] or interaction effects [15, 16]. Similar phenomena in usual conductors are prohibited by the fact that the corresponding Fermi energies are always much larger than the superconducting gap. By studying the interplay of superconductivity [17] with the very special dynamics of massless relativistic quasiparticles at a bipolar pn junction [18, 19, 20, 21, 22, 23], we obtain the oscillatory behavior of both diagonal and cross conductances of the $G_1SG_2$ transistor as a function of the superconductor width.

We consider a graphene sheet occupying the $xy$ plane. A superconducting top electrode covers the region from $x = 0$ to $x = d$, creating a proximity induced superconducting barrier (S) between the normal leads $G_1$ ($x < 0$) and $G_2$ ($x > d$). Moreover it was argued recently that metal coating might also induce superconductivity in graphene [24]. Due to valley and spin degeneracy, one may use a four-dimensional version of the Dirac-Bogoliubov-de Gennes equation [9, 10]

$$
\begin{pmatrix}
\nu_p \sigma \cdot \mathbf{p} & + U(r) \sigma_0 & \Delta(r) \sigma_0 \\
\Delta^*(r) \sigma_0 & - \nu_p \sigma \cdot \mathbf{p} - U(r) \sigma_0
\end{pmatrix}
\Psi(r) = \varepsilon \Psi(r),
$$

where the 4-component spinor $\Psi(r) = (\Psi_{A+}, \Psi_{B+}, \Psi_{A-}, -\Psi_{B-})$ contains electron wavefunctions $\Psi_{A+}$, $\Psi_{B+}$ relative to one valley (+) and their time-reversed hole states $\Psi_{A-}$, $-\Psi_{B-}$ attached to the other valley (−). The indices A and B label the two sublattices of the honeycomb structure of carbon atoms. The kinetic Hamiltonian is given by $\nu_p \sigma \cdot \mathbf{p} = -i \hbar \nu_p (\sigma_x \partial_x + \sigma_y \partial_y)$ where the Pauli matrices $\sigma_x$ and $\sigma_y$ act in the sublattice space as well as the identity $\sigma_0$. The energy $\varepsilon$ is measured from the Fermi level of the superconductor and $\nu_p$ is the energy-independent Fermi velocity. The electrostatic potential $U(r)$ in leads...
FIG. 1: Top: Graphene-superconductor-graphene (G1SG2) transistor. We assume that a positive bias \( V_1 \) is applied to G1 while S and G2 are grounded. Bottom: Incident electron at energy \( \varepsilon = \mu \) in n-type graphene (G1). The Andreev reflected hole (●) in G1 and the transmitted electron (□) in p-type G2 are "blocked" at the Dirac points since \( k' = 0 \). Thus the incoming electron may only be reflected as an electron (□) in G1 or transmitted as a hole (●) in G2 for any incidence angle \( \alpha \). For \( \varepsilon \neq \mu \) (not shown), the elastic cotunneling and the local Andreev reflection are still blocked provided \( \alpha \) exceeds the critical angle \( \alpha_c(\varepsilon) = \arcsin([\mu - \varepsilon]/(\mu + \varepsilon)) \).

\( G_i \) (i = 1, 2) and in the central region may be adjusted separately using state-of-the-art local gates technology [20, 21, 22]. It is assumed that \( \Delta(r) = -\mu \) and \( \Delta(r) = 0 \) in G1, while \( \Delta(r) = -\mu_S \) and \( \Delta(r) = \Delta_0 e^{i\phi} \) is finite for \( 0 < x < d \). This square-well model is fully justified by the unusually large Fermi wavelengths in graphene leads, and the fact that the Fermi wavelength beneath the superconductor should be far smaller, namely \( |\mu_s| \ll \mu_S \).

In order to clarify the physics of such bipolar G1SG2 planar heterojunctions, we first give a simple argument based on the energy and transverse momentum conservation. Assuming \( \mu_1 = -\mu_2 = \mu > 0 \), a quasiparticle of energy \( \varepsilon \), in either G1 or G2, may only have \( k = (\mu + \varepsilon)/\hbar v_F \) or \( k' = |\mu - \varepsilon|/\hbar v_F \) as wavevector modulus. Conservation of the transverse wavevector \( k_y \) implies the Snell-Descartes law \( k_y = k \sin \alpha = k' \sin \alpha' \) between the incidence angle \( \alpha \) of the electrons and the reflection angle \( \alpha' \) of the holes in G1. Moreover \( \alpha' \) is also the refraction angle for transmitted electrons in G2. Since \( k' < k \), choosing incident electrons with \( \alpha \) above the critical angle \( \alpha_c(\varepsilon) = \arcsin((\mu - \varepsilon)/(\mu + \varepsilon)) \) yields a complete suppression of the Andreev reflection and electron transmission [25]. Thus the processes that are harmful for the CAR observation are both eliminated at once in channels with \( \alpha > \alpha_c(\varepsilon) \). In particular at \( \varepsilon = \mu \), this suppression holds in all channels since \( \alpha_c(\mu) = 0 \). Hence the whole current in G2 is purely carried by transmitted holes while the current in G1 is the superposition of the incoming and backscattered electronic currents.

In order to investigate quantitatively the consequences of the previous Snell-Descartes argument, we consider a scattering state with an incoming electron in the conduction band of G1 (\( \varepsilon > 0 \)) having energy \( \varepsilon \) and transverse momentum \( k_y \). Owing to translational invariance along the interfaces, all scattered quasiparticle wavefunctions are expressed as \( \Psi(x)e^{ik_y y} \).

We first consider channels with \( \alpha \) below the critical angle \( \alpha_c(\varepsilon) = \arcsin((\mu - \varepsilon)/(\mu + \varepsilon)) \), or equivalently \( k_y < k' \). In the n-type graphene lead G1, \( x < 0 \), the wavefunction is given by the following superposition of the incident electron, the reflected electron and the reflected hole

\[
\Psi(x) = (1, e^{i\alpha}, 0, 0)e^{ik\cos \alpha x} + r_{ee}(1, -e^{-i\alpha}, 0, 0)e^{-ik\cos \alpha x} + r_{he}(0, 0, 1, e^{i\alpha})e^{ik\cos \alpha' x},
\]

where \( r_{ee} \) and \( r_{he} \) are respectively the amplitude for ordinary and Andreev reflection at the G1-S interface. The index \( \sigma = \text{sign}(\mu - \varepsilon) \) indicates whether the hole belongs to the conduction (\( \sigma = + \)) or the valence band (\( \sigma = - \)).

In the p-type lead G2, \( x > d \), the wavefunction consists in the superposition of the transmitted electron and hole

\[
\Psi(x) = t_{ee}(1, e^{-i\alpha}, 0, e^{i\kappa\cos \alpha(x-d)}) + t_{he}(0, 0, 1, -e^{i\alpha})e^{ik\cos \alpha(x-d)},
\]

where \( t_{ee} \) and \( t_{he} \) are respectively the amplitudes for elastic cotunneling and Andreev transmission (CAR) through the superconducting barrier.

At incidence angles \( \alpha > \alpha_c(\varepsilon) \), namely for \( k_y > k' \), the expressions for the wavefunctions are still given by Eqs. [12] except for the hole in G1 which is described by the evanescent wave \( r_{he}(0, 0, 1, i\sigma\gamma) e^{\sqrt{k_y^2-k'^2}x} \) and for the electron in G2 described by \( t_{ee}(1, -i\sigma\gamma, 0, 0) e^{-\sqrt{k_y^2-k'^2}(x-d)} \), where \( \gamma = \exp(\arg(\cosh(k_y/k'))). \)

The wavefunction in the central superconducting barrier, \( 0 < x < d \), is the superposition of four kinds of waves given by \( a_{\pm,\rho}(e^{\pm i\beta}, \rho e^{\pm i\beta}, e^{-\rho}, \rho e^{-\rho}) e^{\rho(ik_0\pm k)x} \), with \( \rho = \pm 1 \), \( k_0 = \mu_S/\hbar v_F \gg k, k' \) and \( \kappa = \sqrt{k_y^2-\varepsilon^2}/\hbar v_F \). The phase \( \beta = \arccos(\varepsilon/\Delta_0) \) is intrinsically related to electron-hole conversion at a normal conductor-superconductor interface [8].

Demanding the continuity of the wavefunctions at \( x = 0 \) and \( x = d \) yields the scattering amplitudes \( r_{ee}, r_{he}, t_{he} \), and \( t_{ee} \) (and \( a_{\pm,\rho} \)) as functions of \( \varepsilon, \alpha, d \) and \( \mu \). In the limit \( d \rightarrow 0 \) we recover the expressions for the transmission and reflection amplitudes, \( t_{ee} \) and \( r_{ee} \), obtained so far in the study of the normal (non-superconducting) n-p junction [18, 19], while \( r_{he} = t_{he} = 0 \). In the opposite limit \( d \gg \xi_0 \), the expressions for Andreev and normal reflection amplitudes \( r_{he} \) and \( t_{ee} \) tend to those obtained

\[\varepsilon = \mu = \mu_1 = -\mu_2 = \mu > 0, \Delta(r) = -\mu \text{ in G1, while } \Delta(r) = -\mu_S \text{ and } \Delta(r) = \Delta_0 e^{i\phi} \text{ is finite for } 0 < x < d.\]
and holes transmitted in $G_1-S-G_2$ are deduced from an extended version of the Blonder-Tinkham-Klapwijk theory \cite{26}. In the following, we assume that a positive bias $V_1 = V$ is applied to the normal lead $G_1$ while the lead $G_2$ and the superconductor $S$ are grounded. Keeping in mind the critical angle effects discussed so far, the current $I_i$ in the graphene lead $G_i (i = 1, 2)$ is represented as the sum of the currents $I_{c_i}^c$ and $I_{c_i}^v$ carried by channels with $\alpha < \alpha_c(V/e)$ and $\alpha > \alpha_c(V/e)$ respectively.

We first obtain that the diagonal conductance $\partial I_d/\partial V$ is finite at $eV = \mu$ for thin superconducting barriers $d \sim \xi_0$, as shown in Fig. 2. In contrast, the main characteristic of the GS contacts with infinite superconductor is the vanishing of the differential conductance at $eV = \mu$.

We now consider the current $I_2$ carried by electrons and holes transmitted in $G_2$ when a positive bias is applied to $G_1$. Channels with $\alpha < \alpha_c(\varepsilon)$ contribute to the cross differential conductance as

$$\frac{\partial I_{c_2}^v}{\partial V} = \int d\varepsilon \left( -\frac{\partial f}{\partial \varepsilon} \right) g_\varepsilon \int_0^{\alpha_c(\varepsilon)} d\alpha \left( \frac{k'}{k} |t_{c_2}(\varepsilon)|^2 \cos \alpha' - |t_{h_2}(\varepsilon)|^2 \cos \alpha \right),$$

where $f = f(\varepsilon - eV_1) = 1/(e(eV_1+1)/T + 1)$ is the Fermi distribution of incident electrons in the lead $G_1$ at temperature $T$. The factor 4 in $g_\varepsilon = (4e^2/h)N_\varepsilon$ accounts for spin and valley-isospin degeneracy and $N_\varepsilon = (\mu + \varepsilon)W/(\pi \hbar v_F)$ for a graphene sheet of width $W$. In contrast, the contribution to the cross conductance arising from quasiparticles having $\alpha > \alpha_c(\varepsilon)$ is always negative

$$\frac{\partial I_{c_2}^v}{\partial V} = -\int d\varepsilon \left( -\frac{\partial f}{\partial \varepsilon} \right) g_\varepsilon \int_0^{\pi/2} d\alpha |t_{c_2}(\varepsilon)|^2 \cos \alpha,$$

since then the electrons are evanescent waves which do not carry current.

As shown in Fig. 2 the cross conductance $\partial I_2/\partial V$ exhibits a cusp at $\mu/e$ being negative between $V_{c_1}$ and $V_{c_2}$ and positive otherwise. This result may be understood further by comparing the cross differential conductances at Fermi bias $eV = \mu$, at zero bias and at large bias $eV \gg \mu$. First at $eV = \mu$, $\partial I_d/\partial V = \partial I_d^v/\partial V$ is negative for any width $d$ because the critical angle vanishes. For voltages slightly shifted from $\mu$, the contribution $I_d^v$ remains dominant over $I_d^v$ owing to the larger angular integration interval in Eq. (1) compared to Eq. (3). On the contrary at zero bias, the critical angle is maximal, $\alpha_c(0) = \pi/2$, yielding $\partial I_d/\partial V = \partial I_d^v/\partial V$. From the expressions of $t_{c_2}(0, \alpha, d)$ and $t_{h_2}(0, \alpha, d)$, one may show that the zero bias $\partial I_d^v/\partial V$ is always positive. In conclusion, the cross differential conductance has at least a zero at a finite voltage $V_{c_1}$ below $\mu/e$. A similar reversal of the cross conductance occurs at a voltage $V_{c_2}$ above $\mu/e$. The voltages $V_{c_1}$ and $V_{c_2}$ depend on the barrier width $d$, on $\mu$ and on $\mu_S$.

The cross conductance is finite and oscillates as a function of the superconductor size $d$ as shown in Fig. 3. Remarkably, the lengths for which the conductance maxima occur are almost independent of $\mu$. The experimental observation of these oscillations requires $\Delta d \ll k_f^{-1} \leq d \sim \xi_0$ where $\Delta d$ is the typical fluctuation on $d$ due to interface roughness. Owing to the good coupling between the superconductor and the atomic thick carbon layer, the Fermi wavelength $k_f^{-1}$ is likely to be quite small in comparison to $d \sim \xi_0$.

In addition, a recent experiment demonstrated that disorder may induce spatial fluctuations of the chemical potential $\mu$ \cite{27,28}. Since energy is still conserved, the general phenomena of AR and EC suppression at $eV = \mu$ should pertain although the wavefunctions are no longer plane waves. It should be very interesting to investigate the interplay of the AR and EC suppression with the formation of electron and hole puddles close to neutrality point \cite{28}.

Besides the intermediate energy regime $\mu \lesssim \Delta_0$ stud-
ied above, we now consider the extreme limits $\mu \gg \Delta_0$ and $\mu = 0$. Then the conductance $\partial I_1/\partial V$ of a thin superconducting barrier ($d \sim \xi_0$) oscillates as a function of the bias voltage (Fig. 4) due to the quasiparticles interferences inside the superconducting barrier. In contrast conductance oscillations in $G_1G_2S$ junctions [28, 30] are related to an interfacial barrier potential $G_2$ separating $G_1$ and $S$. Finally the cross conductance $\partial I_2/\partial V$ is always positive (EC dominated) because the phenomenon of EC suppression is lost at charge neutrality or when the Dirac points are largely outside the gap energy window.

In conclusion, we have demonstrated that the favorable kinematical conditions for splitting a Cooper pair towards two separate leads are met in a bipolar graphene transistor even in presence of weak disorder. This is the first step towards the realization of entangled states of massless electrons. Nevertheless clear-cut manifestation of entanglement depends on the actual relaxation and dephasing mechanisms originating from intrinsic effects in graphene as well as from the back action of the read-out devices. Finally, the proposed bipolar graphene transistor may serve as a very efficient Andreev beam splitter in Hanbury Brown-Twiss and Mach-Zender like experiments [31].

I am very grateful to A. Buzdin, J.N. Fuchs, M. Houzet, B. Huard, F. Konschelle, T. Kontos, G. Montambaux and F. Pistolesi for useful discussions. This work was supported by the Agence Nationale de la Recherche grant ANR-07-NANO-011: Electronic EPR source (ELEC-EPR).

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