PROGRESS IN THE THEORY OF THE
ELECTROWEAK PHASE TRANSITION

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ABSTRACT

Recent progress in the theory of the electroweak phase transition is discussed. It is shown, that for the Higgs boson mass smaller than the masses of W and Z bosons, the phase transition is of the first order. However, its strength is approximately 2/3 times less than what follows from the one-loop approximation. This rules out baryogenesis in the minimal version of the electroweak theory with light Higgs bosons. The possibility of the strongly first order phase transition in the theory with superheavy Higgs bosons is considered.

We show that if the Yang-Mills field at high temperature acquires a magnetic mass \( \sim g^2 T \), then the infrared problem and the problem of symmetry behavior at high temperature effectively decouple from each other, no linear terms appear in the effective potential in all orders of perturbation theory and the symmetry in gauge theories at high temperatures is actually restored. Even though the last statement was never questioned by most of the authors, it was extremely difficult to come to a reliable conclusion about it due to the infrared problem in thermodynamics of non-Abelian gauge fields.

The phase transition occurs due to production and expansion of critical bubbles. A general analytic expression for the probability of the bubble formation is obtained, which may be used for study of tunneling in a wide class of theories.

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1 Introduction

This talk is based on the results obtained in our papers with Michael Dine, Patrick Huet, Robert Leigh and Dmitri Linde [1]. It contains also some more recent results on the infrared problem in the electroweak theory, on the existence of symmetry restoration at a high temperature and on the possibility of the first order phase transition in the theory with superheavy Higgs fields.

The existence of the phase transition in the electroweak theories was discovered by David Kirzhnits twenty years ago [2]. A detailed theory of the phase transition was proposed in 1974 by three groups of authors independently (by Weinberg, Dolan and Jackiw and by Kirzhnits and Linde [3]), and soon the theory of the electroweak phase transition became one of the well established ingredients of modern cosmology. Surprisingly enough, the theory of this phase transition is still incomplete.

In the first papers on this problem it was assumed that the phase transition is of the second order [2, 3]. Later Kirzhnits and Linde showed [4] that in the gauge theories with many particles, and especially with particles which are much more heavy than the Higgs boson \( \phi \), one should take into account corrections to the high temperature approximation used in [2, 3]. These corrections lead to the occurrence of cubic terms \( \sim g^3 \phi^3 T \) in the expression for the effective potential \( V(\phi, T) \). As a result, at some temperature, \( V \) acquires an extra minimum, and the phase transition is first order [4]. Such phase transitions occur through the formation and subsequent expansion of bubbles of the scalar field \( \phi \) inside the symmetric phase \( \phi = 0 \). A further investigation of this question has shown that the phase transitions in grand unified theories are always strongly first order [4]. This realization, as well as the mechanism of reheating of the universe during the decay of the supercooled vacuum state suggested in [4, 8], played an important role in the development of the first versions of the inflationary universe scenario [7]. (For a review of the theory of phase transitions and inflationary cosmology see Ref. [8].)

For a long time it did not seem likely that the electroweak phase transition could have any dramatic consequences. Even though the possibility of a strong baryon number violation during the electroweak phase transition was pointed out fifteen years ago [9, 10], only after the paper by Kuzmin, Rubakov and Shaposhnikov [11] was it realized that such processes do actually occur and may erase all previously generated baryon asymmetry of the universe.

Recently, the possibility that electroweak interactions may not only erase but also produce the cosmic baryon asymmetry has led to renewed interest in the electroweak phase transition. A number of scenarios have been proposed for generating the asymmetry [12] – [19]. All of them require that the phase transition should be strongly first order since otherwise the baryon asymmetry generated during the phase transition subsequently disappears. In all of these scenarios the asymmetry is produced near the walls of the bubbles of the scalar field \( \phi \). Therefore it is necessary to make a much more thorough analysis of the electroweak phase
transition than the analysis which is necessary for an approximate calculation of the critical temperature.

We will say that the phase transition is strongly first order if the ratio of the Higgs field $\phi$ inside the bubble to the temperature $T$ is larger than one, since otherwise the baryon asymmetry will be washed out by nonperturbative effects.

This condition was used in [12, 20] to impose a strong constraint on the Higgs mass in the minimal version of the electroweak theory, $m_H \lesssim 42$ GeV. This, of course, already contradicts the present experimental limits $m_H \gtrsim 57$ GeV [21]. However, more careful consideration of various theoretical uncertainties indicated that the constraint might be somewhat weaker, permitting $m_H$ up to 55 GeV, or possibly higher [22]. In multi-Higgs models [20, 15], the limits are substantially weaker.

Before one can discuss details of the process of baryogenesis, it is necessary to check that the results of our investigation of the phase transition are reliable. This is not a trivial issue even in the minimal electroweak theory. Indeed, as stressed in Refs. [4, 6], each new order of perturbation theory at finite temperature may bring a new factor of $g^2 T / m \sim g T/\phi$ for the theories with gauge boson masses $m \sim g \phi$. This means that the results of the one loop calculations may become unreliable at $\phi \lesssim g T$. Thus, it became very desirable to go beyond the one-loop approximation.

An example of such an approach is given by the self-consistent approximation elaborated in [4]. In this approximation, instead of the mass of a particle at zero temperature one uses its temperature-dependent mass, taking into account the contribution from the polarization operator. This method made it possible, in particular, to overcome unphysical difficulties related to imaginary masses of scalar particles at small $\phi$.

Recently this approach was reinvented by many authors. Some of the recent results obtained by this method were quite surprising. For example, it was claimed that higher order corrections lead to the appearance of a term in the effective potential $\sim -g^3 T^3$ [24, 25]. This term is linear in $\phi$; it is very large at small $\phi$. Depending on its sign, it either may remove the local minimum of $V(\phi, T)$ created by the cubic term $\sim -g^3 \phi T^3$, or it may make this minimum much more deep.

Our investigation of this problem shows that if one is careful with counting of Feynman diagrams, neither positive nor negative linear terms $\sim g^3 \phi T^3$ appear in the effective potential [1]. Even though now the authors of Refs. [24, 25] agree that the linear terms $\sim g^3 \phi T^3$ are absent, we will repeat our main arguments here, since these arguments may allow us to do much more than just say that there are no linear terms in order $g^3$. A generalization of these arguments allows us to formulate the conditions under which one can show, despite some uncertainties with higher order corrections, that the expectation value of the scalar field $\phi$ at high temperatures actually disappears, $\phi = 0$. Note, that this would be impossible in the presence of linear terms of any magnitude and sign.
However, higher order corrections do lead to a significant modification of the one-loop results. They lead to a decrease of the cubic term $g^3 \phi^3 T$ by a factor $2/3$. This effect decreases the ratio $\phi/T$ at the point of the phase transition by approximately the same factor $2/3$. This makes baryogenesis virtually impossible in the context of the minimal standard model with $m_W \gtrsim m_H \gtrsim 57$ GeV.

All results discussed above were obtained in the context of the theories with small coupling constants and light Higgs fields. However, we do not really know whether the Higgs boson is light or very heavy. If the Higgs boson is superheavy, $m_H \gtrsim 10^3$ GeV, the phase transition may become strongly first order. Even though this possibility is extremely speculative, it may lead to important consequences. Therefore we will discuss it in this paper.

Assuming that one knows the shape of the effective potential at small $\phi$, one should still work hard to determine the ratio $\phi/T$ at the point of the phase transition. One needs to know at what temperature the transition actually occurs, and some details of how it occurs. At very high temperatures the effective potential of the Higgs field, $V(\phi, T)$, has a unique minimum at the symmetric point $\phi = 0$. As the temperature is lowered, a second minimum appears. At a critical value $T_c$, this second minimum becomes degenerate with the first one. However, the phase transition actually occurs at a somewhat lower temperature, due to the formation of bubbles of true vacuum which grow and fill the universe. The usual way to study bubble formation is to use the euclidean approach to tunneling at a finite temperature [27]. One should find high-temperature solutions, which describe the so-called critical bubbles. Then one should calculate their action, which leads to an exponential suppression of the probability of bubble formation. Typically, these calculations are rather complicated, and analytic results can only be obtained in a few cases. One of these is the thin wall approximation, which is valid (as in the case of transitions at zero temperature) if the difference in depth of the two minima of $V(\phi, T)$ is much smaller than the height of the barrier between them. In this case the radius of the bubble at the moment of its formation is much larger than the size of the bubble wall, and the properties of the bubble can be obtained very easily. However, the thin wall approximation in our case leads to an underestimate of the tunneling action by a factor of two. Fortunately, we were able to obtain a simple analytic expression which gives the value of the euclidean action for theories with an effective potentials of a rather general type, $V(\phi, T) = a\phi^2 - b\phi^3 + c\phi^4$. We hope that this result will be useful for a future investigation of bubble formation in a wide class of gauge theories with spontaneous symmetry breaking.

On the other hand, validity of the standard assumption that the phase transition occurs due to formation of critical bubbles should be verified as well. Kolb and Gleiser [28] and, more recently, Tetradi [29] have argued that the phase transition may occur by a different mechanism, the formation of small (subcritical) bubbles. If this is the case, the transition

\[2\]

A similar result was obtained also by Carrington [26]. However, her original results were different from ours approximately by a factor of two. Consequently, they lead to an impression that modification of the one-loop results should lead to an increase of the strength of the first order phase transition. At present, there is no disagreement between our results.
is completed earlier and by a different mechanism than in the conventional picture. While this idea is very interesting, we will argue (see also [30] and the talk of Anderson at this Conference) that it is only relevant in cases where the transition is very weakly first order and the euclidean action corresponding to critical bubbles is not much larger than one. This is not the case for the strongly first order phase transitions, where the relevant value of the euclidean action at the moment of the transition is \( S \sim 130 - 140 \).

2 The Phase Transition

Let us consider the form of the effective potential at finite temperature. Contributions of particles of a mass \( m \) to \( V(\phi, T) \) are proportional to \( m^2 T^2 \), \( m^3 T \) and \( m^4 \ln(m/T) \). We will assume that the Higgs boson mass is smaller than the masses of W and Z bosons and the top quark, \( m_H < m_W, m_Z, m_t \). Therefore we will neglect the Higgs boson contribution to \( V(\phi, T) \).

The zero temperature potential, taking into account one-loop corrections, is given by

\[
V_0 = -\frac{\mu^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4 + 2Bv_o^2 \phi^2 - \frac{3}{2} B\phi^4 + B\phi^4 \ln\left(\frac{\phi^2}{v_o^2}\right). \tag{1}
\]

Here

\[
B = \frac{3}{64\pi^2 v_o^4}(2m_W^4 + m_Z^4 - 4m_t^4), \tag{2}
\]

\( v_o = 246 \text{ GeV} \) is the value of the scalar field at the minimum of \( V_0 \), \( \lambda = \mu^2/v_o^2 \), \( m_H^2 = 2\mu^2 \). Note that these relations between \( \lambda, \mu, v_o \) and the Higgs boson mass \( m_H \), which are true at the classical level, are satisfied even with an account taken of the one-loop corrections. This is an advantage of the normalization conditions used in [8]. An expression used in [23] is equivalent to this expression up to an obvious change of variables.

At a finite temperature, one should add to this expression the term

\[
V_T = \frac{T^4}{2\pi^2}(6I_-(y_W) + 3I_-(y_Z) - 6I_+(y_t)) \tag{3}
\]

where \( y_i = M_i\phi/v_o T \), and

\[
I_\pm(y) = \pm \int_0^\infty dx \, x^2 \ln(1 \mp e^{-\sqrt{x^2+y^2}}). \tag{4}
\]

The results of our work are based on numerical calculation of these integrals, without making any specific approximations [22]. However, in the large temperature limit it is sufficient to use an approximate expression for \( V(\phi, T) \) [2, 23],

\[
V(\phi, T) = D(T^2 - T_o^2)\phi^2 - ET\phi^3 + \frac{\lambda_T}{4}\phi^4. \tag{5}
\]
Here

\[ D = \frac{1}{8v_o^2}(2m_W^2 + m_Z^2 + 2m_t^2) , \]  

(6)

\[ E = \frac{1}{4\pi v_o^3}(2m_W^3 + m_Z^3) \sim 10^{-2} , \]  

(7)

\[ T_o^2 = \frac{1}{2D}(\mu^2 - 4Bv_o^2) = \frac{1}{4D}(m_H^2 - 8Bv_o^2) , \]  

(8)

\[ \lambda_T = \lambda - \frac{3}{16\pi^2v_o^4}\left(2m_W^4\ln\frac{m_W^2}{a_BT^2} + m_Z^4\ln\frac{m_Z^2}{a_BT^2} - 4m_t^4\ln\frac{m_t^2}{a_FT^2}\right) , \]  

(9)

where \( \ln a_B = 2\ln 4\pi - 2\gamma \simeq 3.91, \ln a_F = 2\ln \pi - 2\gamma \simeq 1.14. \)

It will be useful for our future discussion to identify several ‘critical points’ in the evolution of \( V(\phi, T) \).

At very high temperatures the only minimum of \( V(\phi, T) \) is at \( \phi = 0 \). A second minimum appears at \( T = T_1 \), where

\[ T_1^2 = \frac{T_o^2}{1 - 9E^2/8\lambda_T D} . \]  

(10)

The value of the field \( \phi \) in this minimum at \( T = T_1 \) is equal to

\[ \phi_1 = \frac{3ET_1}{2\lambda_T} . \]  

(11)

The values of \( V(\phi, T) \) in the two minima become equal to each other at the temperature \( T_c \), where

\[ T_c^2 = \frac{T_o^2}{1 - E^2/\lambda_T D} . \]  

(12)

At that moment the field \( \phi \) in the second minimum becomes equal to

\[ \phi_c = \frac{2ET_c}{\lambda_T} . \]  

(13)

The minimum of \( V(\phi, T) \) at \( \phi = 0 \) disappears at the temperature \( T_o \), when the field \( \phi \) in the second minimum becomes equal to

\[ \phi_o = \frac{3ET_o}{\lambda_T} . \]  

(14)

### 3 Infrared Problems and Reliability of the Perturbation Expansion

In our previous discussion, we have considered only the one loop corrections to the effective potential. In this section we discuss the role of higher order corrections.
It is well known that, in field theories of massless particles, perturbation theory at finite temperature is subject to severe infrared divergence problems. For small values of the scalar field, the gauge bosons (and near the phase transition, the Higgs boson) are nearly massless; as a result, as was pointed out in the early work on this subject \[4, 6\], one cannot reliably compute the effective potential for very small \(\phi\). The problem is that the higher order corrections in coupling constants may contain terms of the type of \(\left(\frac{g^2 T}{m}\right)^N\). As a result, higher order corrections go out of control for \(m < g^2 T\). For scalar particles this happens near the critical point only. Indeed, scalar particles have masses \(m \sim gT \gg g^2 T\) in the high temperature limit. However, if one takes into account gauge invariance, it can be shown that “magnetic” components of vector particles cannot acquire any contribution to their “masses” larger than \(g^2 T\).

At this point one should be more precise. The Green function of the vector field is singular at \(k^2 \sim g^2 T^2\). In this sense one may speak about the vector field mass \(\sim gT\). However, the Green function of the vector field at a finite temperature does not have a simple pole singularity. For example, in addition to the singularity at \(k^2 \sim g^2 T^2\), the Green function of a photon has a singularity at \(k_0 = 0, \vec{k} \to 0\). It is this singularity that is responsible for all infrared problems in quantum statistics of gauge fields, since the Green functions at \(k_0 = 0\) give the leading infrared divergent contribution to thermodynamical sums \[4, 33, 34\]. Investigation of the infrared problem in gauge theories without spontaneous symmetry breaking has shown that the “magnetic mass” (corresponding to the limit \(k_0 = 0, \vec{k} \to 0\)) may appear in the non-Abelian theories, but it cannot be larger than \(O(g^2 T)\).

Thus, in the absence of spontaneous symmetry breaking, or at \(\phi \lesssim gT\), when the magnetic mass of the vector particles become smaller than \(g^2 T\), perturbative results may become unreliable. One may wonder, therefore, is it possible that some unusual contribution to the effective potential at \(\phi \lesssim gT\) may alter our results.

Recently, in a very interesting paper, Brahm and Hsu found that at small \(\phi\), higher order corrections to the scalar field contribution to the effective potential may produce a large negative linear term \(-g^3 \phi T^3\), which eliminated any trace of a first order transition.

On the other hand, Shaposhnikov considered higher order corrections to the vector particle contribution to \(V(\phi, T)\) and found a large positive term \(+g^3 \phi T^3\) which made the phase transition strongly first order \((\phi/T > 1)\) even for \(m_H \sim 64\) GeV \[25\].

We will show that neither positive nor negative linear terms appear in the expression for \(V(\phi, T)\) if one studies higher order corrections paying particular attention to the correct counting of Feynman diagrams. \[4\]. (Additional information on this problem is contained in the talks by Michael Dine and Robert Leigh at this conference.)

We will consider here for simplicity the contribution of the scalar particles and the W bosons only; adding the contribution of Z bosons is trivial. As we have already noted, for questions of infrared behavior, fermions may be ignored. Coulomb gauge, \(\vec{\nabla} \cdot \vec{W} = 0\), is
particularly convenient for the analysis, though the problem can be analyzed in other gauges as well. In this gauge, the vector field propagator $D_{\mu \nu}$ after symmetry breaking (and after a proper diagonalization) splits into two pieces, the Coulomb piece, $D_{00}$, and the transverse piece, $D_{ij}$. For non-zero values of the discrete frequency, $\omega_n = 2\pi n T$, the Coulomb piece mixes with the 'Goldstone' boson. However, for the infrared problems which concern us here, we are only interested in the propagators at zero frequency. For these there is no mixing. One has \[ D_{00}(\omega = 0, \vec{k}) = \frac{1}{\vec{k}^2 + m_W^2(\phi)} \] and \[ D_{ij}(\omega = 0, \vec{k}) = \frac{1}{\vec{k}^2 + m_W^2(\phi)} P_{ij}(\vec{k}) , \] where $P_{ij} = \delta_{ij} - \frac{k_i k_j}{\vec{k}^2}$. The mass of the vector field $W$ at the classical level is given by $m_W = g v_o / 2$. Propagators of the Higgs field $\phi$ and of the 'Goldstone' field $\chi$ in this gauge are given by \[ D_\phi(\vec{k}) = \frac{1}{\vec{k}^2 + m_\phi^2} , \] \[ D_\chi(\vec{k}) = \frac{1}{\vec{k}^2} . \]

Let us review several ways of obtaining the standard one-loop expression for the cubic term in the effective potential, eq. (5). The most straightforward is to carefully expand eq. (3) for the effective potential in $y_W = m_W v_o / T = g \phi / 27$. Indeed, the contribution of $W$-bosons to the effective potential at $T > m_W(\phi)$ is given by

\[
V_W(\phi, T) = 2 \times 3 \times \left( -\frac{\pi^2}{90} T^4 + \frac{m_W^2(\phi)}{24} T^2 - \frac{m_W^2(\phi)}{12\pi} T + \cdots \right)
= 2 \times 3 \times \left( -\frac{\pi^2}{90} T^4 + \frac{g^2 \phi^2}{96} T^2 - \frac{g^3 \phi^3}{96\pi} T + \cdots \right) .
\] (19)

Here the expression in brackets coincides with the contribution of a scalar field with mass $m_W$; the factor 2 appears since there are two $W$-bosons with opposite charges, while the factor 3, which will be particularly important in what follows, corresponds to the two transverse and one longitudinal degrees of freedom with mass $m_W$.

Alternatively, we can obtain the cubic term by looking directly at the one-loop Feynman diagrams. For this purpose, it is only necessary to examine the zero frequency contributions. Certain diagrams containing four external lines of the classical scalar field naively give a contribution proportional to $g^4 \phi^4$; the cubic term arises because the zero frequency integrals diverge for small mass as $T/m_W \sim T/g\phi$.

Consider, in particular, the zero frequency part of the expression for the one loop free energy in momentum space. It is simplest to compute the tadpole diagrams for $dV/d\phi$ and
afterwards integrate with respect to $\phi$. The transverse gauge bosons give a contribution
\[
\frac{dV_{tr}}{d\phi} = 2 \times \frac{g^2 \phi T}{2} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{k^2 + m^2_W} = -2 \times \frac{g^2 \phi T}{8\pi} \sqrt{m^2_W},
\]
where, by keeping only the zero frequency mode, we have dropped terms which are analytic in $m^2$. The Coulomb lines give half the result of eq. (20). Integration of the total vector field contribution correctly represents the cubic term in (19).

A complete gauge boson contribution to the tadpole, including the non-zero frequency modes, is
\[
\frac{dV_W(\phi, T)}{d\phi} = 2 \times 3 \times \frac{g^2 \phi}{48} \left( T^2 - \frac{3m_W T}{\pi} + \cdots \right) = 2 \times 3 \times \frac{g^2 \phi}{48} \left( T^2 - \frac{3g\phi T}{2\pi} + \cdots \right). \tag{21}
\]
One can easily check that integration of this expression with respect to $\phi$ gives eq. (19).

With these techniques, we are in a good position to study higher order corrections to the potential. The authors of Refs. [24, 25] found a linear contribution to the potential by substituting the mass found at one loop back into the one loop calculation. The effective masses-squared of both scalar particles and of the Coulomb field contain terms of the form $\sim g^3 T \phi$, which, upon substitution in (34), give linear terms. But this procedure is not always correct. It is well known that the sum of the geometric progression, which appears after the insertion of an arbitrary number of polarization operators $\Pi(\phi, T)$ into the propagator $(k^2 + m^2)^{-1}$, simply gives $(k^2 + m^2 + \Pi(\phi, T))^{-1}$. Therefore one can actually use propagators $(k^2 + m^2 + \Pi(T))^{-1}$, which contain the effective mass-squared $m^2 + \Pi(\phi, T)$ instead of $m^2$. However, this trick with the geometric progression does not work for the closed loop diagram for the effective potential, which contains $\ln(k^2 + m^2)$. A naive substitution of the effective mass squared $m^2 + \Pi(\phi, T)$ instead of $m^2$ into $\ln(k^2 + m^2)$ corresponds to a wrong counting of higher order corrections.

This does not mean that there is no regular way to make this trick for the effective potential. One may add and subtract from the Lagrangian the term $-\frac{1}{2} \phi^2 \Pi(\phi(T), T)$ to the Lagrangian, and a similar term for the vector field as well. Here $\Pi(\phi(T), T)$ is the polarization operator with an account taken of all daisy and superdaisy diagrams, $\phi(T)$ is a classical field, not an operator. Then the effective mass (at zero momentum) becomes renormalized, $m^2 \to m^2 + \Pi(\phi, T)$, but one should add some extra diagrams containing the insertion $-\frac{1}{2} \phi^2 \Pi(\phi(T), T)$. These diagrams were not considered in [24, 25]. Here one can clearly understand the difference between calculating effective potential and tadpoles. If one uses this method to calculate the tadpole diagrams, insertions of $-\frac{1}{2} \phi^2 \Pi(\phi(T), T)$ self-consistently cancel the diagrams which would generate extraneous polarization operator corrections to already corrected effective mass $m^2(\phi(T), T) = m^2 + \Pi(\phi, T)$. This gives us the standard prescription of simply substituting $m^2(\phi(T), T) = m^2 + \Pi(\phi, T)$ instead of $m^2$ in all propagators $(k^2 + m^2)^{-1}$. 
Thus, the simplest way to take into account high temperature corrections to masses of vector and scalar particles without any problems with combinatorics is to compute tadpole diagrams for $dV/d\phi$; these are then trivially integrated to give the potential. One can easily check by this method that no linear terms appear in the expression for $V(\phi, T)$. Indeed, at a given temperature and effective mass, the tadpoles are linear in $\phi$ (see e.g. equation (21)). To take into account the mass renormalization in the tadpoles, one should substitute the effective mass squared $m^2 + \Pi(\phi, T)$ into the one-loop expression for the tadpole contribution; as we explained above (see also (3)), this is a correct and unambiguous procedure for tadpoles.

$$\frac{dV_{tr}}{d\phi} = 2 \times \frac{g^2 \phi T}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\vec{k}^2 + m^2 + \Pi(\phi, k, T)}.$$  \hspace{1cm} (22)

This expression could lead to a linear term in the effective potential only if the integral would behave as $\phi^{-1}$ in at small $\phi \rightarrow 0$. However, in our case this is impossible, since the polarization operator calculated in [24, 25] at small $k$ and $\phi$ is nonnegative, and the integral converges to a constant. Therefore its subsequent integration with respect to $\phi$, which gives the correction to the effective potential, is quadratic in $\phi$, i.e. it does not contain any linear terms.

We should emphasize that our approach effectively takes into account all diagrams considered in [24], [25], but with correct combinatorics. Note also, that this approximation works even if the polarization operator depends on the classical field $\phi$. An apparent triviality of the investigation with the help of tadpoles is not due to its incompleteness, but due to the power and simplicity of this method, which was elaborated in [4]. In particular, it was even unnecessary for us to use a particular expression for the polarization operator, as far as it is nonnegative. We will return to this issue shortly.

Even though there are no linear terms $\sim g^3 \phi T^3$, higher order corrections do have a dramatic effect on the phase transition. This effect is a modification of the cubic term.

As we have shown above, the cubic term appears due to the contribution of zero modes, $\omega_n = 2\pi n T = 0$. This makes it particularly easy to study its modification by high order effects. Indeed, it is well known that the Coulomb field at zero frequency acquires the Debye ‘mass’, $m_D^2 = \Pi_{00}(\omega_n = 0, \vec{k} \rightarrow 0) \sim g^2 T^2$. This leads to an important modification of the Coulomb propagator (15):

$$D_{00}(\vec{k} \rightarrow 0) = \frac{1}{\vec{k}^2 + m_D^2 + m_W^2(\phi)}.$$  \hspace{1cm} (23)

For the values of $\phi$ of interest to us, $m_D^2 \gg m_W^2(\phi)$. Thus, repeating the calculation of the cubic term, the Coulomb contribution disappears. However, the transverse contribution,

\footnote{We disagree with the recent claim made in [35] that our approximation works only if the polarization operator $\Pi$ does not depend on $\phi$, and that we do not take into account ‘subleading’ diagrams which give rise to dangerous linear terms obtained in [24], [25]. Indeed, in [4] we did not take into account diagrams of the type shown in Fig. 2; see their discussion below. However, these diagrams were not considered in [24], [25] as well, since they do not lead to the linear terms in the order $g^3$.}
which is two times larger than the Coulomb one, is unaffected at this order, due to the vanishing of the ‘magnetic mass’ \[6, 33, 34\]. As a result, the cubic term does not disappear, but it is diminished by a factor of 2/3:

\[
E = \frac{1}{6\pi v_0^2} (2m_W^3 + m_Z^3).
\] (24)

This small correction proves to be very significant. Indeed, eqs. (13), (14) show that the ratio of the scalar field \(\phi\) to the temperature at the moment of the phase transition is proportional to \(E\), i.e. to the cubic term. Actually, the dependence is even slightly stronger, since for smaller \(E\) the tunneling occurs earlier. Even before the reduction of the cubic term was taken into account, the ratio \(\phi/T\) for \(m_H \gtrsim 57\) GeV was slightly less than the critical value \(\phi/T \approx 1\). The decrease of this quantity by a factor of 2/3 makes it absolutely impossible to preserve the baryon asymmetry generated during the phase transition in the minimal model of electroweak interactions with \(m_H \gtrsim 57\) GeV.

Are these results completely reliable? The effective coupling constant of interactions between W bosons and Higgs particles is \(g/2\). In this case, a general investigation of the infrared problem in the non-Abelian gauge theories at a finite temperature suggests that the results which we obtained are reliable for \(\phi \gtrsim \frac{g}{2} T \sim T/3\) \[6\], \[33, 34\]. Thus, a more detailed investigation is needed to study behavior of the theories with \(m_H \gtrsim 10^2\) GeV near the critical temperature, since the scalar field, which appears at the moment of the phase transition in these theories, is very small. However, we expect that our results are reliable for strongly first order phase transitions with \(\phi \gtrsim T/3\), which is quite sufficient to study (or to rule out) baryogenesis in the electroweak theory. Recent investigation of higher order corrections to this theory indicates \[36\] that these results may be reliable even down to \(\phi \sim gT/10\).

Finally, we would like to address a fundamental question: since the theory for \(\phi \ll gT\) is infrared divergent, can we definitely establish that the symmetry is restored at high temperature, or is it possible that \(\phi\) always has some small, non-zero value? Indeed, if our approximation breaks down at \(\phi \lesssim gT\), how do we know that the symmetry restoration actually takes place, i.e. \(\phi = 0\) at \(T > T_o\)?

To address this question, we can work far away from the critical point, at \(T - T_o \gg T_o\). The best approach, as before, is to study all possible higher order tadpole diagrams, see Figs. 1, 2.

There are two different classes of diagrams to be considered. External line of the scalar field may split either into two lines of the vector field, Fig. 1, or into two lines of vector field and one line of scalar field, Fig. 2. All diagrams of the first type can be represented as the trivial one-loop diagram, plus the diagrams with an arbitrary number of polarization

\[\text{There was also a claim that the cubic term disappears completely [37], but recently this claim was withdrawn.}\]
operator insertions. The simplest diagram of that type is shown in Fig. 1. The black circle stands for the exact polarization operator \( \Pi(\phi, k, T) \), including all higher order corrections. The sum of all these diagrams gives us the one-loop diagram with an exact Green function of the vector field instead of the free field propagator. In other words, as usual, one must add polarization operator to the mass squared of the vector field.

The behavior of \( \Pi(\phi, k, T) \) at \( k > g^2 T \) is known perturbatively. It leads only to high-order corrections to \( V(\phi, T) \), which do not contain any nonanalytic terms, such as the dangerous linear terms in \( \phi \). The only possible source of problems is our absence of knowledge of \( \Pi(\phi, k, T) \) at \( \phi < \sim g T, k_0 = 0, |\vec{k}| < \sim g^2 T \). Indeed, in this domain all higher order corrections to \( \Pi(\phi, k, T) \) are equally important. However, the consequences of this uncertainty may appear not very significant.

Indeed, consider again the most dangerous part of the tadpole diagram, eq. (20), and add \( \Pi(\phi, k, T) \) to \( m_W^2(\phi) \):

\[
\frac{dV_{tr}}{d\phi} = 2 \times \frac{g^2 \phi T}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2 + \frac{g^2 \phi^2}{4} + \Pi(\phi, k, T)}.
\]  

(25)

The part of the integral in the domain of uncertainty, \(|\vec{k}| < g^2 T\), is given by

\[
\frac{g^2 \phi T}{2\pi^2} \int_0^{g^2 T} \frac{k^2 dk}{k^2 + \frac{g^2 \phi^2}{4} + \Pi(\phi, k, T)}.
\]  

(26)

On dimensional grounds, one expects that at \( g\phi \ll g^2 T \) and \( |\vec{k}| \ll g^2 T \) the polarization operator has some value of the order \( g^4 T^2 \) [6, 33, 34]. This follows from the fact that the most infrared divergent part of the theory corresponds to the three-dimensional theory, with \( g^2 T \) being the only mass (or coupling constant) scale.

One cannot exclude a possibility that the polarization operator is proportional to \(-g^2|\vec{k}|T\), or it is of the order \( g^4 T^2 \), but is negative, and the integral in (26) diverges in the limit \( \phi \to 0 \). One should keep this possibility in mind, since it may lead to interesting and unusual consequences, like Bose-condensation or even crystallization of the Yang-Mills field at high temperature [33]. Indeed, the tadpole integral has a simple interpretation in terms of integration over the occupation numbers of bose fields. Large contribution to this integral may be interpreted as a result of Bose condensation of particles in a state with a nonvanishing momentum. In our case, this effect may also lead to absence of a complete symmetry restoration at \( T > T_o \). Note, that this effect may occur only at \( \phi \ll g T \), as we anticipated. However, it is not quite clear that this possibility is physically viable. Whereas occupation numbers may be large, they can hardly be negative.

The standard (and most conservative) assumption is that \( 0 \leq \Pi(\phi = 0, k = 0) \leq g^4 T^2 \) [3, 33, 34]. This corresponds to generation of a magnetic mass \( 0 \leq \Lambda^2 \ll g^4 T^2 \). In such case
we may estimate the corresponding integral at small $\phi$ as

$$\frac{g^2 \phi T}{2\pi^2} \int_0^{g^2 T} \frac{k^2 dk}{k^2 + O(1) g^4 T^2} \sim g^4 \phi T^2.$$  \hfill (27)

This term after integration over $\phi$ does not give any linear terms in $\phi$. It gives just a small correction to the quadratic part of the effective potential, $\Delta V \sim g^4 \phi^2 T^2$. Such corrections do not alter our conclusions concerning symmetry restoration at high temperature. Note, that this term corresponds to the sum of all most dangerous contributions to the tadpole diagrams of the type of Fig.1, to all orders in $g^2$.

Now let us consider the diagrams which contain internal lines of scalar field, Fig. 2. These diagrams may be dangerous near the critical point, where the scalar fields are almost massless, but they are much less dangerous than the diagrams of the first class at the temperature much higher than critical. The reason is that at high temperature the scalar field acquires a large mass $m^2 \sim g^2 T^2 \gg g^4 T^2$. Therefore scalar particles by themselves do not lead to any infrared problems outside of a small vicinity of the critical point. The presence of such heavy particles effectively cuts infrared divergencies in the diagrams with vector particles as well. One can easily check that the diagram with one vector loop, Fig 2a, gives the contribution $g^4 \phi^2 T^2$ to the effective potential, the diagram with two vector loops gives $g^5 \phi^2 T^2$, the diagram with three vector loops gives $g^6 \phi^2 T^2$. Starting with this diagrams, infrared problem becomes manifest in that each diagram of this type with higher number of vector loops gives the contribution of the same order $g^6 \phi^2 T^2$.

Thus, the infrared problem in thermodynamics of gauge fields does not permit us to calculate the effective potential at high temperature to all orders of perturbation theory. The diagrams Fig. 2 contain uncertainties at the level of $g^6 \phi^2 T^2$; the diagrams Fig. 1 contain uncertainties at the level of $g^4 \phi^2 T^2$. However, neither of these diagrams produce linear terms in $\phi$, unless the Green functions of a massless Yang-Mills field has a pathological behavior at large temperature. As for the quadratic terms, they can be calculated at least with an accuracy up to $g^3 \phi^2 T^2$, or maybe even up to $g^4 \ln g \phi^2 T^2$. This is quite sufficient to calculate the critical temperature and to make a conclusion that at the temperature higher than critical the scalar field $\phi$ vanishes.

These considerations indicate that the situation with the phase transitions in the non-Abelian gauge theories is probably the same as in the standard case: infrared problems may prevent a simple description of the phase transition in a small vicinity of the critical point (unless the phase transition is strongly first order), but everywhere outside this region, the symmetry behavior of gauge theories can be described in a reliable way.
4 First order phase transitions with superheavy Higgs?

In our previous investigation we neglected the contribution of Higgs bosons to the one-loop effective potential. The reason was very simple: We have seen that the increase of the Higgs boson mass decreases the strength of the phase transition. This can be easily understood by considering a model of a single scalar field with the effective potential

\[ V_0 = -\frac{\mu^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4 \equiv -\frac{m_H^2}{4} \phi^2 + \frac{\lambda}{4} \phi^4. \] (28)

Near the point of the phase transition, where the high-temperature approximation works well, one may investigate the symmetry behavior in this theory in a self-consistent approximation suggested in [4], where only cactus diagrams should be evaluated. In this approximation the effective mass of the scalar field and the first derivative of the effective potential in its extremum are simply related to each other:

\[ m^2(T, \phi) = 3 \lambda \phi^2 - \mu^2 + \Pi(T, m(T, \phi)) \] ,

and

\[ \frac{1}{\phi} \frac{dV_1}{d\phi} = \Pi(T, m(T, \phi)). \] (30)

Here \( V_1 \) is the one-loop contribution to the effective potential,

\[ \frac{1}{\phi} \frac{dV_1}{d\phi} = \frac{3 \lambda}{2\pi^2} \int_0^\infty \frac{k^2 dk}{\sqrt{k^2 + m^2(T, \phi)}} \left( \exp \sqrt{k^2 + m^2(T, \phi)} - 1 \right). \] (31)

At the minimum of the effective potential with \( \phi(T) \neq 0 \)

\[ \frac{dV_1}{d\phi} + \lambda \phi^3 - \mu^2 \phi = 0, \] (32)

which, together with (29) and (30), gives \( m^2 = 2 \lambda \phi^2(T) \) and

\[ \frac{1}{\phi} \frac{dV_1}{d\phi} = \frac{3 \lambda}{2\pi^2} \int_0^\infty \frac{k^2 dk}{\sqrt{k^2 + 2 \lambda \phi^2(T)}} \left( \exp \frac{\sqrt{k^2 + 2 \lambda \phi^2(T)}}{T} - 1 \right) = \frac{\lambda T^2}{4} \left( 1 - \frac{3\sqrt{2}\lambda \phi(T)}{\pi T} + \ldots \right). \] (33)

On the other hand, the local minimum of \( V(\phi) \) at \( \phi = 0 \) disappears when \( m(T, 0) = 0 \). This gives the critical temperature

\[ T_0 = 2v, \] (34)

where \( v = \mu/\sqrt{\lambda} = 246 \text{ GeV}, v > \phi(T) \). For small \( \lambda \) the last term in eq. (33) is small at \( T \approx T_0 \), which implies that the phase transition is weakly first order. Actually, we cannot even say from eq. (33) whether the phase transition is second order or weakly first order, since near the critical point the higher order corrections are large [4].
Let us nevertheless use eq. (33) to make a bold estimate of conditions under which the phase transitions could be strongly first order. From eqs. (33), (34) it follows that the jump of the scalar field at the critical temperature (assuming that $T_o$ is close to the temperature of the phase transition) is given by $\frac{3\sqrt{2\lambda}}{4\pi}$. It is larger than the temperature $T$ (which is the condition for baryogenesis) if $\lambda > \sim 9$, or, equivalently,

$$m_H \gtrsim 10^3 \text{GeV}. \quad (35)$$

For obvious reasons, this estimate should not be taken as a serious indication of existence of strongly first order phase transitions and baryogenesis with superheavy Higgs bosons. However, the stakes are high, and the possibility to have a strongly first order phase transition and baryogenesis in the strong coupling regime with superheavy Higgs bosons (technicolour?) should not be overlooked.

5 Bubble Formation

In the previous section we noted that the two minima of $V(\phi, T)$ become of the same depth at the temperature $T_c$, eq. (12). However, tunneling with formation of bubbles of the field $\phi$ corresponding to the second minimum starts somewhat later, and it goes sufficiently fast to fill the whole universe with the bubbles of the new phase only at some lower temperature $T$ when the corresponding euclidean action suppressing the tunneling becomes less than $130 - 140 \text{ [14, 22, 23]}$. In [1] (see also [22]) we performed a numerical study of the probability of tunneling. Before reporting our results, we will remind the reader of some basic concepts of the theory of tunneling at a finite temperature.

In the euclidean approach to tunneling (at zero temperature) [32], the probability of bubble formation in quantum field theory is proportional to $\exp(-S_4)$, where $S_4$ is the four-dimensional Euclidean action corresponding to the tunneling trajectory. In other words, $S_4$ is the instanton action, where the instanton is the solution of the euclidean field equations describing tunneling. A generalization of this method for tunneling at a very high temperature [27] gives the probability of tunneling per unit time per unit volume

$$P \sim A(T) \cdot \exp(-\frac{S_3}{T}). \quad (36)$$

Here $A(T)$ is some subexponential factor roughly of order $T^4$; $S_3$ is a three-dimensional instanton action. It has the same meaning (and value) as the fluctuation of the free energy $F = V(\phi(\bar{x}), T)$ which is necessary for bubble formation. To find $S_3$, one should first find an $O(3)$-symmetric solution, $\phi(r)$, of the equation

$$\frac{d^2\phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} = V'(\phi), \quad (37)$$
with the boundary conditions \( \phi(r = \infty) = 0 \) and \( d\phi/dr|_{r=0} = 0 \). Here \( r = \sqrt{x_i^2} \); the \( x_i \) are the euclidean coordinates, \( i = 1,2,3 \). Then one should calculate the corresponding action

\[
S_3 = 4\pi \int_0^{\infty} r^2 dr \left[ \frac{1}{2} \left( \frac{d\phi}{dr} \right)^2 + V(\phi(r), T) \right].
\]  

(38)

Usually it is impossible to find an exact solution of eq. (37) and to calculate \( S_3 \) without the help of a computer. A few exceptions to this rule are given in Refs. \[8, 27\]. One of these exceptional cases is realized if the effective potential has two almost degenerate minima, such that the difference \( \varepsilon \) between the values of \( V(\phi, T) \) at these minima is much smaller than the energy barrier between them. In such a case the thickness of the bubble wall at the moment of its formation is much smaller than the radius of the bubble, and the action \( S_3 \) can be calculated exactly as a function of the bubble radius \( r \), the energy difference \( \Delta V \) and the bubble wall surface energy \( S_1 \):

\[
S_3 = -\frac{4\pi}{3} r^3 \Delta V + 4\pi r^2 S_1,
\]  

(39)

where

\[
S_1 = \int_0^{\infty} d\phi \sqrt{2V(\phi, T)}.
\]  

(40)

The radius of the critical bubble \( r \) can be found by finding an extremum of \( S_3(r) \). However, one must be very careful when using these results. Indeed, as can be easily checked, this extremum is not a minimum of the action, it is a maximum. Therefore, the action corresponding to the true solution of eq. (37) will be higher than the action of any approximate solution. As a result, one can strongly overestimate the tunneling probability by calculating it outside the limit of validity of the thin wall approximation. In our case the thin wall approximation underestimates the tunneling action by a factor of two, i.e. it gives the probability of tunneling about \( e^{-100} \) where the correct answer is \( e^{-200} \). If the only thing one wishes to know is the time when the tunneling occurs, this error is not very important. It leads only to a few percent error in calculation of the temperature of the universe at the moment of the phase transition, since the tunneling action is extremely sensitive to even very small changes of the temperature. Thus, one may argue that the thin wall approximation is still useful. (See also the talk of Anderson at this Conference.) However, it is possible to determine the time of the phase transition with an accuracy of few percent without any study of tunneling: It is enough to say that the phase transition happens in the middle of the interval between \( T_c \) and \( T_o \). In order to obtain a complete description of the phase transition, including a correct shape of the bubble wall, one should go beyond the thin wall approximation.

We would now like to obtain an analytic estimate of the probability of tunneling in the electroweak theory, which can be used for any particular numerical values of constants \( D, E \) and \( \lambda T \). As shown in Ref. \[23\], eq. (5) in most interesting cases approximates \( V(\phi, T) \) with an accuracy of a few percent. This by itself does not help very much if one must study tunneling anew for each new set of the constants. However, it proves possible to reduce this
study to the calculation of one function \( f(\alpha) \), where \( \alpha \) is some ratio of constants \( D, E \) and \( \lambda_T \). In what follows we will calculate this function for a wide range of values of \( \alpha \). This will make it possible to investigate tunneling in the electroweak theory without any further use of computers.

First of all, let us represent the effective Lagrangian \( L(\phi, T) \) near the point of the phase transition in the following form:

\[
L(\phi, T) = \frac{1}{2} (\partial_\mu \phi)^2 - \frac{M^2(T)}{2} \phi^2 + E T \phi^3 - \frac{\lambda_o}{4} \phi^4. \tag{41}
\]

Here \( M^2(T) = 2D(T^2 - T_c^2) \) is the effective mass squared of the field \( \phi \) near the point \( \phi = 0 \), \( \sim_t \) is the value of the effective coupling constant \( \lambda_T \) near the point of the phase transition (i.e. at \( T \sim T_t \), where \( T_t \) is the temperature at the moment of tunneling). With a very good accuracy, the constants \( \lambda_t, \lambda_{T_1}, \lambda_{T_2}, \lambda_{T_3} \) are equal to each other.

Defining \( \phi = \frac{M^2}{2E^2 T_2} \Phi, x = X/M \), the effective Lagrangian can be written as:

\[
L(\Phi, T) = \frac{M^6}{4E^2 T^2} \left[ \frac{1}{2} (\partial_\mu \Phi)^2 - \frac{1}{2} \Phi^2 + \frac{1}{2} \Phi^3 - \frac{\alpha}{8} \Phi^4 \right], \tag{42}
\]

where

\[
\alpha = \frac{\lambda_o M^2}{2E^2 T^2}. \tag{43}
\]

The overall factor \( \frac{M^6}{4E^2 T^2} \) does not affect the Lagrange equation

\[
\frac{d^2 \Phi}{dR^2} + \frac{2}{R} \frac{d\Phi}{dR} = \Phi - \frac{3}{2} \Phi^2 + \frac{1}{2} \alpha \Phi^3. \tag{44}
\]

Solving this equation and integrating over \( d^3 X = M^{-3} d^3 x \) gives the following expression for the corresponding action:

\[
\frac{S_3}{T} = \frac{4.85 M^3}{E^2 T^3} \times f(\alpha). \tag{45}
\]

The function \( f(\alpha) \) is equal \( \frac{M^2}{4E^2 T_2} \) to 1 at \( \alpha = 0 \), and blows up when \( \alpha \) approaches 1. In the whole interval from 0 to 1 this function, with an accuracy about 2%, is given by the following simple expression:

\[
f(\alpha) = 1 + \frac{\alpha}{4} \left[ 1 + \frac{2.4}{1 - \alpha} + \frac{0.26}{(1 - \alpha)^2} \right]. \tag{46}
\]

In the vicinity of the critical temperature \( T_o \), i.e. at \( \Delta T \equiv T - T_o \ll T_o \), the action \( \frac{S_3}{T} \) can be written in the following form:

\[
\frac{S_3}{T} = \frac{38.8 D^{3/2}}{E^2} \cdot \left( \frac{\Delta T}{T} \right)^{3/2} \times f\left( \frac{2 \lambda_o D \Delta T}{E^2 T} \right). \tag{47}
\]

Using these results, one can easily get analytical expressions for the tunneling probability in a wide class of theories with spontaneous symmetry breaking, including GUTs and the minimal electroweak theory.
6 Subcritical Bubbles

Despite our semi-optimistic conclusions concerning the infrared problem, it is still desirable to check that the whole picture of the behavior of the scalar field described above is (at least) self-consistent. This means that if the effective potential is actually given by eqs. (5), (6), (8), (9), then our subsequent description of the phase transition and the bubble formation is correct. Indeed, one would expect that the theory of bubble formation is reliable, since the corresponding action for tunneling $S_3/T$ is very large, $S_3/T \sim 130 - 140$. However, recently even the validity of this basic assumption has been questioned. Gleiser and Kolb [28] and Tetradis [29] have argued that in many cases phase transitions occur not due to bubbles of a critical size, which we studied in section 3, but due to smaller, subcritical bubbles. We believe that these authors raise a real issue. However, we will now argue that this problem only arises if the phase transition is extremely weakly first order.

The basic difference between the analysis of Ref. [28, 29] and the more conventional one is their assumption that at the time of the phase transition there is a comparable probability to find different parts of the universe in either of the two minima of $V(\phi, T)$. The main argument of Ref. [28, 29] is that if the dispersion of thermal fluctuations of the scalar field $<\phi^2> \sim T^2$ is comparable with the distance between the two minima of $V(\phi, T)$, then the field $\phi$ “does not know” which minimum is true and which is false. Therefore it spends comparable time in each of them. According to [28], a kind of equilibrium between the domains of the two types is achieved due to subcritical bubbles with small action $S_3/T$ if many such bubbles may appear within a horizon of a radius $H^{-1}$.

In order to investigate this question in a more detailed way, let us re-examine our own assumptions concerning the distribution of the scalar field $\phi$ prior to the moment at which the temperature drops down to $T_1$, when the second minimum of $V(\phi, T)$ appears. According to (11), the value of the scalar field $\phi$ in the second minimum at the moment when it is formed is equal to $\phi_1 = \frac{3ET}{2\lambda r}$. For $m_H \sim 60$ GeV (and taking into account the coefficient $2/3$ in the cubic term) one obtains $\phi_1 \sim 0.4T$. Thermal fluctuations of the field $\phi$ have the dispersion squared $<\phi^2> = T^2/12$. (Note an important factor $1/12$, which was absent in the estimate made in [28].) This gives dispersion of thermal fluctuations $\sqrt{<\phi^2>} \sim 0.3T$, which is not much smaller than $\phi_1$.

However, as the authors of [28] emphasized in their previous work [38] (see also [29]), the total dispersion $<\phi^2> \sim T^2/12$ is not an adequate quantity to consider since we are not really interested in infinitesimally small domains containing different values of fluctuating field $\phi$. They argue that the proper measure of thermal fluctuations is the contribution to $<\phi^2>$ from fluctuations of the size of the correlation length $\xi(T) \sim M^{-1}(T)$. This leads to an estimate $<\phi^2> \sim TM(T)$, which also may be quite large [38]. Here again one should be very careful to use the proper coefficients in the estimate. One needs to understand also why this estimate could be relevant.
In order to make the arguments of Ref. \[28, 29\] more quantitative and to outline the domain of their validity, it is helpful to review the stochastic approach to tunneling (see \[39\] and references therein). This approach is not as precise as the euclidean approach (in theories where the euclidean approach is applicable). However, it is much simpler and more intuitive, and it may help us to look from a different point of view on the results we obtained in the previous section and on the approach suggested in \[28, 29\].

The main idea of the stochastic approach can be illustrated by an example of tunneling with bubble formation from the point $\phi = 0$ in the theory (41) with the effective potential

$$V(\phi, T) = \frac{M^2(T)}{2} \phi^2 - ET \phi^3 + \frac{\lambda_o}{4} \phi^4.$$  

(48)

For simplicity, we will study here the limiting case $\lambda_o \rightarrow 0$.

At the moment of its formation, the bubble wall does not move. In the limit of small bubble velocity, the equation of motion of the field $\phi$ at finite temperature is simply,

$$\ddot{\phi} = \frac{d^2 \phi}{dr^2} + \left(\frac{2}{r}\right) \frac{d\phi}{dr} - V'(\phi).$$  

(49)

The bubble starts growing if $\ddot{\phi} > 0$, which requires that

$$|\frac{d^2 \phi}{dr^2} + \left(\frac{2}{r}\right) \frac{d\phi}{dr}| < -V'(\phi).$$  

(50)

A bubble of a classical field is formed only if it contains a sufficiently big field $\phi$. It should be over the barrier, so that $dV/d\phi < 0$, and the effective potential there should be negative since otherwise formation of a bubble will be energetically unfavorable. The last condition means that the field $\phi$ inside the critical bubble should be somewhat larger than $\phi_*$, where $V(\phi_*, T) = V(0, T)$. In the theory \[48\] with $\lambda_o \rightarrow 0$, one has $\phi_* = M^2/2ET$. As a simplest (but educated) guess, let us take $\phi \sim 2\phi_* = M^2/ET$. Another important condition is that the size of the bubble should be sufficiently large. If the size of the bubble is too small, the gradient terms are bigger than the term $|V'(\phi)|$, and the field $\phi$ inside the bubble does not grow. Typically, the second term in (50) somewhat compensates the first one. To make a very rough estimate, one may write the condition (50) in the form

$$\frac{1}{2} r^{-2} \sim \frac{1}{2} k^2 < \frac{1}{2} k_{\text{max}}^2 \sim \phi^{-1} |V'(\phi)| \sim 2M^2.$$  

(51)

Let us estimate the probability of an event in which thermal fluctuations with $T \gg M$ build up a configuration of the field satisfying this condition. The dispersion of thermal fluctuations of the field $\phi$ with $k < k_{\text{max}}$ is given by

$$<\phi^2>_{k<k_{\text{max}}} = \frac{1}{2\pi^2} \int_0^{k_{\text{max}}} \frac{k^2 dk}{\sqrt{k^2 + M^2} \left(\exp \frac{\sqrt{k^2 + M^2} \phi}{T} - 1\right)} \sim \frac{T}{2\pi^2} \int_0^{k_{\text{max}}} \frac{k^2 dk}{k^2 + M^2}.$$  

(52)
Note that the main contribution to the integral is given by $k^2 \sim k_{\text{max}}^2 \sim 4M^2$. This means that one can get a reasonably good estimate of $\langle \phi^2 \rangle _{k<k_{\text{max}}}$ by omitting $M^2$ in the integrand. This also means that this estimate will be good enough even though the effective mass of the scalar field $M^2(\phi) = V''(\phi)$ changes between $\phi = 0$ and $\phi$. The result we get is

$$\langle \phi^2 \rangle _{k<k_{\text{max}}} \simeq \frac{T}{2\pi^2} \int_0^{k_{\text{max}}} dk \frac{T k_{\text{max}}}{2\pi^2} = \frac{C^2 T M}{\pi^2}. \quad (53)$$

Here $C = O(1)$ is a coefficient reflecting the uncertainty in the determination of $k_{\text{max}}$ and estimating the integral.

Thus, we have a rough estimate of the dispersion of perturbations which may sum up to produce a field $\phi$ which satisfies the condition (51). We can use it to evaluate the probability that these fluctuations build up a bubble of the field $\phi$ of a radius $r > k_{\text{max}}^{-1}$. This can be done with the help of the Gaussian distribution\footnote{The probability distribution is approximately Gaussian even though the effective potential is not purely quadratic. The reason is that we were able to neglect the curvature of the effective potential $m^2 = V''$ while calculating $\langle \phi^2 \rangle _{k<k_{\text{max}}}$.

Thus, the probability distribution is approximately Gaussian even though the effective potential is not purely quadratic. The reason is that we were able to neglect the curvature of the effective potential $m^2 = V''$ while calculating $\langle \phi^2 \rangle _{k<k_{\text{max}}}$.}

$$P(\phi) \sim \exp\left(-\frac{\phi^2}{2 \langle \phi^2 \rangle _{k<k_{\text{max}}}}\right) = \exp\left(-\frac{M^2 \pi^2}{2 C^2 E^2 T^3}\right) \sim \exp\left(-\frac{4.92 M^3}{C^2 E^2 T^3}\right). \quad (54)$$

Note that the factor in the exponent in (54) to within a factor of $C^2 = 1.02$ coincides with the exact result for the tunneling probability in this theory obtained by the euclidean approach \cite{27} (see eq. (46)):

$$P \sim \exp\left(-\frac{4.85 M^3}{E^2 T^3}\right). \quad (55)$$

Taking into account the very rough method we used to calculate the dispersion of the perturbations responsible for tunneling, the coincidence is rather impressive.

As was shown in \cite{39}, most of the results concerning tunneling at zero temperature, at a finite temperature and even in the inflationary universe, which were obtained by euclidean methods, can easily be reproduced (with an accuracy of the coefficient $C^2 = O(1)$ in the exponent) by this simple method.

Now let us return to the issue of subcritical bubbles. As we have seen, dispersion of the long-wave perturbations of the scalar field, $\langle \phi^2 \rangle _{k<k_{\text{max}}} \simeq \frac{k_{\text{max}} T}{2\pi^2}$, is quite relevant to the theory of tunneling. Its calculation provides a simple and intuitive way to get the same results as we obtained earlier by the euclidean approach \cite{39}. To get a good estimate of the probability of formation of a critical bubble in our simple model one should calculate this dispersion for $k_{\text{max}} \sim 2M(T)$, which gives $\langle \phi^2 \rangle _{k<k_{\text{max}}} = T M/\pi^2$. Note, that this estimate is much smaller than the naive estimate $\langle \phi^2 \rangle \sim T M$.

The crucial test of our basic assumptions is a comparison of this dispersion and the value of the field $\phi$ at the moment $T = T_1$, when the minimum at $\phi = \phi_1 \neq 0$ first appears. Using
eqs. (5), (11), one can easily check that the mass of the scalar field at \( T = T_1, \phi = 0 \) is given by

\[
m = \frac{3ET}{2\sqrt{\lambda_T}}.
\]  

(56)

This yields

\[
\sqrt{\langle \phi^2 \rangle_{k<k_{\text{max}}}} \sim \phi_1 \frac{\lambda^{3/4}}{\pi^{1/2} \sqrt{3E/2}} \approx \phi_1 \frac{10\lambda^{3/4}}{\pi}.
\]  

(57)

For the Higgs boson with \( m_H \sim 60 \text{ GeV} \) one obtains

\[
\sqrt{\langle \phi^2 \rangle_{k<k_{\text{max}}}} \sim \frac{\phi_1}{5}.
\]  

(58)

Thus, even with account taken of the factor 2/3 in the expression for \( E \), the dispersion of long-wave fluctuations of the scalar field is much smaller than the distance between the two minima. Therefore, the field \( \phi \) on a scale equal to its correlation length \( \sim M^{-1} \) is not equally distributed between the two minima of the effective potential. It just fluctuates with a very small amplitude near the point \( \phi = 0 \). The fraction of the volume of the universe filled by the field \( \phi_1 \) due to these fluctuations (i.e. due to subcritical bubbles) for \( m_H \sim 60 \text{ GeV} \) is negligible,

\[
P(\phi_1) \sim \exp \left( -\frac{\phi^2}{2 \langle \phi^2 \rangle_{k<k_{\text{max}}}} \right) \sim \exp \left( -\frac{3E \pi^2}{4\lambda^{3/2}_T} \right) \sim e^{-12}.
\]  

(59)

Since we already successfully applied this method for investigation of tunneling, we expect that this estimate is also reliable. The answer remains rather small even for \( m_H \sim 100 \text{ GeV} \), when the phase transition is very weakly first order.

Moreover, even these long-wave fluctuations do not lead to formation of stable domains of space filled with the field \( \phi \neq 0 \), until the temperature is below \( T_c \) and critical bubbles appear. One expects a typical subcritical bubble to collapse in a time \( \tau \sim l_{\text{max}}^{-1} \); this is about thirteen orders of magnitude smaller than the total duration of the phase transition, \( \Delta t \sim 10^{-2}H^{-1} \sim 10^{-4}M_pT^{-2} \). We do not see any mechanism which might increase \( \tau \) by such a large factor.

Despite all these comments, we think that subcritical bubbles deserve further investigation. They may lead to interesting effects during phase transitions in GUTs, since the difference between \( T^{-1} \) and the duration of the GUT phase transitions is not as great as in the electroweak case. They may play an important role in the description of the electroweak phase transition as well, in models where the phase transition occurs during a time not much longer than \( T^{-1} \). This may prove to be the case for very weakly first order phase transitions with \( 10^3 \text{ GeV} \gg m_H \gtrsim 10^2 \text{ GeV} \), when the distance between the two minima of \( V(\phi, T) \) at \( T \sim T_1 \) is smaller than the dispersion \( \sqrt{\langle \phi^2 \rangle_{k<k_{\text{max}}}} \sim \sqrt{T/M}/\pi \).
7 Conclusions

One of the main consequences of our work [1] is that it is very difficult to generate baryon asymmetry in the standard model without expanding its Higgs sector. One the other hand, now we better understand what is necessary for the electroweak baryogenesis to work and how to calculate relevant quantities. Hopefully, this will help us to find a realistic theory of elementary particles where electroweak baryogenesis is possible.

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References

[1] M. Dine, R. Leigh, P. Huet, A. Linde and D. Linde, Stanford University preprints SU-ITP-92-6 (1992) (to be published in Physics Letters) and SU-ITP-92-7 (1992) (to be published in Phys. Rev.).

[2] D.A. Kirzhnits, JETP Lett. 15 (1972) 529; D.A. Kirzhnits and A.D. Linde, Phys. Lett. 42B (1972) 471.

[3] S. Weinberg, Phys. Rev. D9 (1974) 3357; L. Dolan and R. Jackiw, Phys. Rev. D9 (1974) 3320; D.A. Kirzhnits and A.D. Linde, JETP 40 (1974) 628.

[4] D.A. Kirzhnits and A.D. Linde, Ann. Phys. 101 (1976) 195.

[5] A.D. Linde, Phys.Lett. 99B (1981) 391.

[6] A.D. Linde, Rep. Prog. Phys. 42 (1979) 389.

[7] A.H. Guth, Phys. Rev. D23 (1981) 347; A.D. Linde, Phys. Lett. 108B (1982); 114B (1982) 431; 116B (1982) 335, 340; A. Albrecht and P.J. Steinhardt, Phys. Rev. Lett. 48 (1982) 1220.

[8] A.D. Linde, Particle Physics and Inflationary Cosmology (Harwood, Chur, Switzerland, 1990).
[9] A.D. Linde, Phys.Lett. 70B (1977) 306.

[10] S. Dimopoulos and L. Susskind, Phys. Rev. D18 (1978) 4500.

[11] V.A. Kuzmin, V.A. Rubakov and M.E. Shaposhnikov, Phys. Lett. B155 (1985) 36; P. Arnold and L. McLerran, Phys. Rev. D36 58187.

[12] M.E. Shaposhnikov, JETP Lett. 44 (1986) 465; Nucl. Phys. B287 (1987) 757; Nucl. Phys. B299 (1988) 797; A.I. Bochkarev, S.Yu. Khlebnikov and M.E. Shaposhnikov, Nucl. Phys. B329 (1990) 490.

[13] L. McLerran, Phys. Rev. Lett. 62 (1989) 1075.

[14] L. McLerran, M. Shaposhnikov, N. Turok and M. Voloshin, Phys. Lett. 256B (1991) 451.

[15] N. Turok and P. Zadrozny, Phys. Rev. Lett. 65 (1990) 2331; Nucl. Phys. B358 (1991) 471.

[16] M. Dine, P. Huet, R. Singleton and L. Susskind, Phys.Lett. 257B (1991) 351.

[17] A. Cohen, D.B. Kaplan and A.E. Nelson, Nucl. Phys. B349 (1991) 727.

[18] A. Cohen, D.B. Kaplan and A.E. Nelson, Phys.Lett. 263B (1991) 86.

[19] A. Cohen, D.B. Kaplan and A.E. Nelson, University of California, San Diego, preprint UCSD-PTH-91-20 (1991)

[20] A. Bochkarev, S. Kuzmin and M. Shaposhnikov, Phys. Lett. 244B (1990) 27.

[21] ALEPH, DELPHI, L3 and OPAL Collaborations, as presented by M. Davier, Proceedings of the International Lepton-Photon Symposium and Europhysics Conference on High Energy Physics, eds. S. Hegerty, K. Potter and E. Quercigh (Geneva, 1991), to appear.

[22] M. Dine, P. Huet and R. Singleton, Nucl. Phys. B375 (1992) 625; A.D. Linde and D.A. Linde, unpublished.

[23] G. Anderson and L. Hall, Phys. Rev. D45 (1992) 625.

[24] D. Brahm and S. Hsu, Caltech preprints CALT-68-1705 and CALT-68-1762 (1991).

[25] M.E. Shaposhnikov, Phys. Lett B277 (1992) 324.

[26] M.E. Carrington, Phys. Rev. D45 (1992) 2933.

[27] A.D. Linde, Phys.Lett. 70B (1977) 306; 100B (1981) 37; Nucl. Phys. B216 (1983) 421.
[28] M. Gleiser and E. Kolb, preprint FERMILAB-Pub-91/305-A (1991).
[29] N. Tetradis, preprint DESY 91-151.
[30] K. Enqvist, J. Ignatius, K. Kajantie, K. Rummukainen, Phys.Rev. D45 (1992) 3415.
[31] M. Sher, Phys. Rep. 179 (1989) 273.
[32] S. Coleman, Phys. Rev. D15 (1977) 2929.
[33] A.D. Linde, Phys. Lett. 93B (1980) 327.
[34] D.J. Gross, R.D. Pisarski and L.G. Yaffe, Rev. Mod. Phys. 53 (1981) 1.
[35] J.R. Espinosa, M. Quiros and F. Zwirner, preprint CERN-TH.6451/92 (1992).
[36] G. Boyd, D.E. Brahm and D.H. Hsu, preprint CALT-68-1795 (1992).
[37] T.S. Evans, Imperial/TP/91-92/23 (Apr. 1992).
[38] M. Gleiser, E. Kolb and R. Watkins, Nucl. Phys. B364 (1991) 411.
[39] A.D. Linde, Nucl. Phys. B372 (1992) 421.