Restoration of chiral and $U(1)_A$ symmetries in excited hadrons
in the semiclassical regime

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Restoration of chiral and $U(1)_A$ symmetries in excited hadrons is reviewed. Implications of the OPE as well as of the semiclassical expansion for this phenomenon are discussed. A solvable model of the ’t Hooft type in 3+1 dimensions is presented, which demonstrates a fast restoration of both chiral and $U(1)_A$ symmetries at larger spins and radial excitations.

Keywords: Chiral and $U(1)_A$ symmetry restoration; Excited hadrons

1. Introduction

There are some phenomenological evidences that the highly excited hadrons, both baryons $^1-^3$ and mesons $^4,^5$ fall into approximate multiplets of $SU(2)_L \times SU(2)_R$ and $U(1)_A$ groups, for a short overview see ref. $^6$. This is illustrated in Fig. 1, where the excitation spectrum of the nucleon as well as the excitation spectrum of $\pi$ and $f_0$ (with the $\bar{u}u + \bar{d}d$ content) mesons are shown. Starting from the 1.7 GeV region the nucleon (and delta) spectra show obvious signs of parity doubling. There are a couple of examples where chiral partners of highly excited states have not yet been seen. Their experimental discovery would be an important task. Similarly, in the chirally restored regime $\pi$ and $\bar{u}n$ $f_0$ states must be systematically degenerate. This phenomenon, if experimentally confirmed by discovery of still missing states, is referred to as effective chiral symmetry restoration or chiral symmetry restoration of the second kind.

By definition this effective chiral symmetry restoration means the following. In QCD hadrons with quantum numbers $\alpha$ are created when one applies an interpolating field (current) $J_{\alpha}$ with such quantum numbers on the vacuum $|0\rangle$. Then all hadrons that are created by the given interpolator appear as intermediate states in the two-point correlator,
Fig. 1. Left panel: excitation spectrum of the nucleon (those resonances which are not yet established are marked by two or one stars according to the PDG classification). Right panel: pion and $n\bar{n}$ $f_0$ spectra.

\[
\Pi = i \int d^4 x \, e^{iqx} \langle 0 | T \{ J_\alpha(x) J^\dagger_\alpha(0) \} |0 \rangle,
\]

where all possible Lorentz and Dirac indices (specific for a given interpolating field) have been omitted. Consider two interpolating fields $J_1(x)$ and $J_2(x)$ which are connected by a chiral transformation (or by a $U(1)_A$ transformation), $J_1(x) = U J_2(x) U^\dagger$. Then, if the vacuum was invariant under the chiral group, $U | 0 \rangle = | 0 \rangle$, it follows from (1) that the spectra created by the operators $J_1(x)$ and $J_2(x)$ would be identical. We know that in QCD one finds $U | 0 \rangle \neq | 0 \rangle$. As a consequence the spectra of the two operators must be in general different. However, it happens that the noninvariance of the vacuum becomes unimportant (irrelevant) high in the spectrum. Then the spectra of both operators become close at large masses and asymptotically identical. This means that chiral symmetry is effectively restored. We stress that this effective chiral symmetry restoration does not mean that chiral symmetry breaking in the vacuum disappears, but that the role of the quark condensates that break chiral symmetry in the vacuum becomes progressively less important high in the spectrum. One could say, that the valence quarks in high-lying hadrons decouple from the QCD vacuum.

2. Chiral symmetry restoration and the quark-hadron duality

There is a heuristic argument that supports this idea. The argument is based on the well controlled behaviour of the two-point function (1) at the large space-like momenta $Q^2 = -q^2$, where the operator product expansion
(OPE) is valid and where all nonperturbative effects can be absorbed into condensates of different dimensions. The key point is that all nonperturbative effects of the spontaneous breaking of chiral symmetry at large $Q^2$ are absorbed into the quark condensate $\langle \bar{q}q \rangle$ and other quark condensates of higher dimension. However, the contributions of these condensates to the correlation function are proportional to $(1/Q^2)^n$, where the index $n$ is determined by the quantum numbers of the current $J$ and by the dimension of the given quark condensate. Hence, at large enough $Q^2$ the two-point correlator becomes approximately chirally symmetric. At these high $Q^2$ a matching with the perturbative QCD (where no SBCS occurs) can be done.

Then we can invoke into analysis a dispersion relation. Since the large $Q^2$ asymptotics of the correlator is given by the leading term of the perturbation theory, then the asymptotics of the spectral density, $\rho(s)$, at $s \to \infty$ must also be given by the same term of the perturbation theory if the spectral density approaches a constant value (if it oscillates, then it must oscillate around the perturbation theory value). Hence both spectral densities $\rho_{J_1}(s)$ and $\rho_{J_2}(s)$ at $s \to \infty$ must approach the same value and the spectral function becomes chirally symmetric. This is definitely true in the asymptotic (jet) regime where the spectrum is strictly continuous. The conjecture of ref. 2 was that may be this is also true in the regime where the spectrum is still quasidiscrete and saturated mainly by resonances.

The question arises then what is the functional behaviour that determines approaching the chiral-invariant regime at large $s$? One would expect that OPE could help us. This is not so, however, for two reasons. First of all, we know phenomenologically only the lowest dimension quark condensate. But even if we knew all quark condensates up to a rather high dimension, it would not help us. This is because the OPE is only an asymptotic expansion. While such kind of expansion is very useful in the space-like region, it does not define any analytical solution which could be continued to the time-like region at finite $s$. This means that while the real (correct) spectrum of QCD must be consistent with OPE, there is an infinite amount of incorrect spectra that can also be consistent with OPE. Then, if one wants to get some information about the spectrum from the OPE side, one needs to assume something else on the top of OPE. Clearly a success then is crucially dependent on these additional assumptions, for the recent activity in this direction see refs. 8–10. This implies that in order to really understand chiral symmetry restoration one needs a microscopic insight and theory that would incorporate at the same time chiral symmetry breaking and confinement.
3. Restoration of the classical symmetry in the semiclassical regime

A fundamental insight into phenomenon can be obtained from the semiclassical argument. We know that the axial anomaly as well as the spontaneous breaking of chiral symmetry in QCD is an effect of quantum fluctuations of the quark field. The latter can generally be seen from the definition of the quark condensate, which is a closed quark loop. This closed quark loop explicitly contains a factor $\hbar$. The chiral symmetry breaking, which is necessarily a nonperturbative effect, is actually a (nonlocal) coupling of a quark line with the closed quark loop, which is a graphical representation of the Schwinger-Dyson (gap) equation. Hence chiral symmetry breaking in QCD manifestly vanishes in the classical limit $\hbar \to 0$.

At large $n$ (radial quantum number) or at large angular momentum $J$ we know that in quantum systems the semiclassical approximation must work. Physically this approximation applies in these cases because the de Broglie wavelength of particles in the system is small in comparison with the scale that characterizes the given problem. In such a system as a hadron the scale is given by the hadron size while the wavelength of valence quarks is given by their momenta. Once we go high in the spectrum the size of hadrons increases as well as the typical momentum of valence quarks. This is why a highly excited hadron can be described semiclassically in terms of the underlying quark degrees of freedom.

The physical content of the semiclassical approximation is most transparently given by the path integral. The contribution of the given path to the path integral is regulated by the action $S(\phi(x))$ along the path $\phi(x)$ (the fields $\bar{\psi}, \psi, A$ are collectively denoted as $\phi$) through the factor $\sim e^{i \frac{S(\phi(x))}{\hbar}}$. The semiclassical approximation applies when the action in the system $S \gg \hbar$. In this case the whole amplitude (path integral) is dominated by the classical path $\phi_{cl}(x)$ (stationary point) and those paths that are infinitesimally close to the classical path. In other words, in the semiclassical case the quantum fluctuations effects are strongly suppressed and vanish asymptotically. Then the generating functional can be expanded in powers of $\hbar$ as

$$W(J) = W_0(J) + \hbar W_1(J) + ..., \quad (2)$$

where $W_0(J) = S(\phi_{cl}) + J\phi_{cl}$ and $W_1(J)$ represents contributions of the lowest order quantum fluctuations around the classical solution (determinant of the classical solution). The classical path, which is generated by $W_0$, 

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is a tree-level contribution to the path integral and keeps chiral symmetries of the classical Lagrangian. Its contribution is of the order \((\hbar/S)^0\). The quantum fluctuations contribute at the orders \((\hbar/S)^1\) (the one loop order, generated by \(W_1\)), \((\hbar/S)^2\) (the two loops order), etc. The \(U(1)_A\) as well as the spontaneous \(SU(2)_L \times SU(2)_R\) breakings start from the one-loop order. However, in a hadron with large enough \(n\) or \(J\), where action is large, the loop contributions must be relatively suppressed and vanish asymptotically. Then it follows that in such systems both the chiral and \(U(1)_A\) symmetries should be approximately restored. This is precisely what we see phenomenologically.

While the argument above is solid, theoretically it is not clear \textit{a-priori} whether isolated hadrons still exist at excitation energies where a semiclassical regime is achieved. However, the large \(N_c\) limit of QCD, while keeping all basic properties of QCD like asymptotic freedom, confinement and chiral symmetry breaking, allows for a significant simplification. In this limit it is known that all mesons represent narrow states. At the same time the spectrum of mesons is infinite. Then one can always excite a meson of any arbitrary large energy, which is of any arbitrary large size. In such a meson the action \(S \gg \hbar\). Hence a description of this meson necessarily must be semiclassical. Actually we do not need the exact \(N_c = \infty\) limit for this statement. It can be formulated in the following way. For any large \(S \gg \hbar\) there always exist such \(N_c\) that the isolated meson with such an action does exist and can be described semiclassically. From the empirical fact that we observe multiplets of chiral and \(U(1)_A\) groups high in the hadron spectrum it follows that \(N_c = 3\) is large enough for this purpose.

4. A solvable model of the ’t Hooft type

While the argument presented above is general and solid enough, a detailed microscopical picture is missing. Then to see how all this works one needs a solvable field-theoretical model. Clearly the model must be chirally symmetric and contain the key elements, such as confinement and spontaneous breaking of chiral symmetry. Such a model is known, it is a generalized Nambu and Jona-Lasinio model (GNJL) with the instantaneous Lorentz-vector confining kernel \(^{15-17}\). This model can be considered as a generalization of the large \(N_c\) ’t Hooft model (QCD in 1+1 dimensions) \(^{18}\) to 3+1 dimensions. In both models the only interaction between quarks is the instantaneous infinitely raising Lorentz-vector linear potential. Then chiral symmetry breaking is described by the standard summation of the valence quarks self-interaction loops (the Schwinger-Dyson or gap equa-
tions), while mesons are obtained from the Bethe-Salpeter equation for the quark-antiquark bound states.

An obvious advantage of the GNJL model is that it can be applied in 3+1 dimensions to systems of arbitrary spin. In 1+1 dimensions there is no spin, the rotational motion of quarks is impossible. Then it is known that the spectrum represents an alternating sequence of positive and negative parity states and chiral multiplets never emerge. In 3+1 dimension, on the contrary, the quarks can rotate and hence can always be ultrarelativistic and chiral multiplets should emerge naturally. Restoration of chiral symmetry in excited heavy-light mesons has been previously studied with the quadratic confining potential. Here we report our results for excited light-light mesons with the linear potential.

An effective chiral symmetry restoration means that (i) the states fall into approximate multiplets of $SU(2)_L \times SU(2)_R$ and the splittings within the multiplets ($\Delta M = M_+ - M_-$) vanish at $n \to \infty$ and/or $J \to \infty$; (ii) the splitting within the multiplet is much smaller than between the two subsequent multiplets. Note that within the present model the axial anomaly is absent so the mechanism of the $U(1)_A$ symmetry breaking and restoration is exactly the same as of $SU(2)_L \times SU(2)_R$.

The condition (i) is very restrictive, because the structure of the chiral multiplets for the $J = 0$ and $J > 0$ mesons is very different. For the $J > 0$ mesons chiral symmetry requires a doubling of states with some quantum numbers. Given the set of quantum numbers $I, J^{PC}$, the multiplets of $SU(2)_L \times SU(2)_R$ for the $J = 0$ mesons are

\begin{align}
(1/2, 1/2)_a : & \ 1, 0^{-+} \leftrightarrow 0, 0^{++} \\
(1/2, 1/2)_b : & \ 1, 0^{++} \leftrightarrow 0, 0^{-+},
\end{align}

while for the mesons of even spin, $J > 0$, they are

\begin{align}
(0, 0) : & \ 0, J^{--} \leftrightarrow 0, J^{++} \\
(1/2, 1/2)_a : & \ 1, J^{-+} \leftrightarrow 0, J^{++} \\
(1/2, 1/2)_b : & \ 1, J^{++} \leftrightarrow 0, J^{-+} \\
(0, 1) \oplus (1, 0) : & \ 1, J^{++} \leftrightarrow 1, J^{--}
\end{align}

and for odd $J$ they are

\begin{align}
(0, 0) : & \ 0, J^{++} \leftrightarrow 0, J^{--} \\
(1/2, 1/2)_a : & \ 1, J^{+-} \leftrightarrow 0, J^{--} \\
(1/2, 1/2)_b : & \ 1, J^{--} \leftrightarrow 0, J^{++} \\
(0, 1) \oplus (1, 0) : & \ 1, J^{--} \leftrightarrow 1, J^{++}.
\end{align}
Restoration of the $U(1)_A$ symmetry would mean a degeneracy of the opposite spatial parity states with the same isospin from the distinct $(1/2, 1/2)_a$ and $(1/2, 1/2)_b$ multiplets of $SU(2)_L \times SU(2)_R$. Note that within the present model there are no vacuum fermion loops. Then since the interaction between quarks is flavor-blind the states with the same $J^{PC}$ but different isospins from the distinct multiplets $(1/2, 1/2)_a$ and $(1/2, 1/2)_b$, as well as the states with the same $J^{PC}$ but different isospins from $(0, 0)$ and $(0, 1) \oplus (1, 0)$ representations are degenerate.

In the Table below we present masses (in units $\sqrt{\sigma}$) of $I = 1$ mesons with $J = 0, 1$ and $J = 6$. A very fast restoration of both $SU(2)_L \times SU(2)_R$ and $U(1)_A$ symmetries with increasing $J$ and essentially more slow restoration with increasing of $n$ is observed.

| chiral multiplet | $J^{PC}$ | radial excitation n | 0  | 1  | 2  | 3  | 4  | 5  | 6  |
|------------------|----------|---------------------|----|----|----|----|----|----|----|
| $(1/2, 1/2)_a$   | 0$^{++}$ | 1.49                | 3.38| 4.72| 5.80| 6.74| 7.57| 8.33|
| $(1/2, 1/2)_b$   | 0$^{++}$ | 1.49                | 3.38| 4.72| 5.80| 6.74| 7.57| 8.33|
| $(1/2, 1/2)_a$   | 1$^{+-}$ | 2.68                | 4.03| 5.15| 6.14| 7.01| 7.80| 8.53|
| $(1/2, 1/2)_b$   | 1$^{+-}$ | 2.68                | 4.03| 5.15| 6.14| 7.01| 7.80| 8.53|
| $(0, 1) \oplus (1, 0)$ | 1$^{--}$ | 1.55                | 3.28| 4.56| 5.64| 6.57| 7.40| 8.16|
| $(0, 1) \oplus (1, 0)$ | 1$^{--}$ | 1.55                | 3.28| 4.56| 5.64| 6.57| 7.40| 8.16|
| $(1/2, 1/2)_a$   | 6$^{--}$ | 6.83                | 7.57| 8.26| 8.90| 9.52| 10.1| 10.7|
| $(1/2, 1/2)_b$   | 6$^{--}$ | 6.83                | 7.57| 8.26| 8.90| 9.52| 10.1| 10.7|
| $(0, 1) \oplus (1, 0)$ | 6$^{--}$ | 6.83                | 7.57| 8.26| 8.90| 9.52| 10.1| 10.7|
| $(0, 1) \oplus (1, 0)$ | 6$^{--}$ | 6.83                | 7.57| 8.26| 8.90| 9.52| 10.1| 10.7|

In Fig. 2 the rates of the symmetry restoration against the radial quantum number $n$ and spin $J$ are shown. It is seen that with the fixed $J$ the splitting within the multiplets $\Delta M$ decreases asymptotically as $1/\sqrt{n}$, dictated by the asymptotic linearity of the radial Regge trajectories. This property is consistent with the dominance of the free quark loop logarithm at short distances.

In Fig. 3 the angular Regge trajectories are shown. They exhibit deviations from the linear behavior. This fact is obviously related to the chiral symmetry breaking effects for lower mesons.

In the limit $n \to \infty$ and/or $J \to \infty$ one observes a complete degeneracy of all multiplets, which means that the states fall into

$$[(0, 1/2) \oplus (1/2, 0)] \times [(0, 1/2) \oplus (1/2, 0)]$$
Fig. 2. Mass splittings in units of $\sqrt{\sigma}$ for isovector mesons of the chiral multiplets $(1/2, 1/2)_a$ and $(1/2, 1/2)_b$ (circles) and within the multiplet $(0, 1) \oplus (1, 0)$ (squares) against $J$ for $n = 0$ (top) and against $n$ for $J = 0$ and $J = 1$, respectively (bottom). The full line is $0$.

Fig. 3. Angular Regge trajectories for isovector mesons with $M^2$ in units of $\sigma$. Mesons of the chiral multiplet $(1/2, 1/2)_a$ are indicated by circles, of $(1/2, 1/2)_b$ by triangles, and of $(0, 1) \oplus (1, 0)$ by squares ($J^{++}$ and $J^{--}$ for even and odd $J$, respectively) and diamonds ($J^{--}$ and $J^{++}$ for even and odd $J$, respectively).

A few words about physics which is behind these results are in order. In highly excited hadrons a typical momentum of valence quarks is large. Con-
sequently, the chiral symmetry violating Lorentz-scalar dynamical mass of quarks, which is a very fast decreasing function at larger momenta, becomes small and asymptotically vanishes $^1, ^{11}, ^{20}$. Consequently, chiral and $U(1)_A$ symmetries get approximately restored. Exactly the same reason implies a decoupling of these hadrons from the Goldstone bosons $^3, ^{20}$. Namely, the coupling of the valence quarks to Goldstone bosons is constraint by the axial current conservation, i.e. it must satisfy the Goldberger-Treiman relation. Then the coupling constant must be proportional to the Lorentz-scalar dynamical mass of valence quarks and vanishes at larger momenta. This represents a microscopical mechanism of decoupling which is required by the general considerations of chiral symmetry in the Nambu-Goldstone mode $^{21, 22}$.

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