Neutron electric dipole moment and flavor changing interactions in supersymmetric theories

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Abstract
Supersymmetric contributions to the neutron electric dipole moment (EDM) are studied taking account of the flavor changing interactions. We found that the gluino contribution is sensitive to the flavor changing interaction. Enhancement of neutron EDM via flavor mixing effects is possible when the squark mass difference between the different generations is sizable. As an example, the results of the SUSY SU(5) GUT with right-handed neutrinos are briefly discussed.

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Supersymmetry (SUSY) provides an elegant solution to the gauge hierarchy problem. It is, however, known that SUSY must be broken softly at the Fermi scale, $G_F^{-1/2}$, since there has been no evidence of a boson whose mass is exactly same with one of known fermions and vice versa. Introducing soft SUSY breaking terms can increase the mass of SUSY particles, but its scale must be $O$(TeV) to make the electroweak scale stable against radiative corrections.

One of the serious problems of softly broken supersymmetric Standard Model (SUSY-SM) is a possibility of $CP$ violation beyond the Kobayashi-Maskawa mechanism. In general, a softly broken SUSY Lagrangian contains a lot of complex parameters. However, in a certain class of SUSY breaking scenarios, most of them could be removed by field re-definition. For example, in the gravity mediated SUSY breaking scenario which leads to the universal scalar mass, trilinear coupling and gaugino mass, it is known that there are two complex phases as physical degree of freedom, in addition to the Kobayashi-Maskawa phase [1]. Conventionally two complex phases are chosen as those of the scalar trilinear coupling $A_f$ and the higgsino mass $\mu$:

$$A_f = |A_f|e^{i\alpha_f}, \quad \mu = |\mu|e^{i\phi},$$

(1)

where $f$ denotes the flavor of (s)quarks or (s)leptons. Stringent constraints on the parameters $\alpha_f$ and $\phi$ are given by the upper bound of electric dipole moment (EDM) of neutron [2]:

$$|d_n| = 6.3 \times 10^{-26} e \cdot cm.$$  

(2)

The supersymmetric contributions to the neutron EDM consist of the following 1-loop diagrams: (i)squark-gluino, (ii) squark-chargino and (iii) squark-neutralino exchanges. It has been pointed out that a natural size ($\sim O(1)$) of $\alpha_f$ and $\phi$ is strongly disfavored from the neutron EDM measurements, unless the squarks are heavier than a few TeV [3]. However, so far constraints on squark masses from the neutron EDM have been studied without paying a special attention to the flavor off-diagonal interactions between quarks and squarks, though it is allowed in general. For example, some class of supersymmetric grand unified theories (SUSY GUT) predict that a large mixing of neutrinos leads to a sizable generation mixing of right-handed down-squarks, which could be observed as an enhancement of flavor changing interactions between quarks and (right-handed) down-squarks [4]. Also, recently reported deviation of $CP$ asymmetry in $b \rightarrow sq\bar{q}$ from the SM expectation at B-factories [5, 6] motivates us to study a possibility of $CP$ violation in SUSY flavor changing interactions.
In this paper we investigate effects of the flavor changing interactions between quarks and squarks to the neutron EDM model independently, when the SUSY CP phases are not suppressed. Our main interest is that if the lower mass bound on the SUSY particles from the neutron EDM is altered significantly due to the flavor changing interactions. The works in refs. [10] have been done in a similar direction with our study. They examined the squark flavor mixing based on some class of SUSY-GUT, emphasizing the anomaly in CP asymmetry of $B_d \to \phi K_S$. Contrary to the previous studies, we analyze the flavor mixing effect phenomenologically, allowing somewhat large flavor changing interactions. We find that the gluino mediated diagram is sensitive to the flavor mixing effect while the chargino and neutralino diagrams are not. However the flavor mixing effects are suppressed when the mass difference of squarks between different generations is small enough.

The squark mass matrix $M_\tilde{q}^2 (q = u, d)$ in the generation space is given by

$$M_\tilde{q}^2 = \begin{pmatrix} (M_\tilde{q}^2)_{LL} & (M_\tilde{q}^2)_{LR} \\ (M_\tilde{q}^2)_{RL} & (M_\tilde{q}^2)_{RR} \end{pmatrix},$$

where $(M_\tilde{q}^2)_{\alpha\beta}$ $(\alpha, \beta = L, R)$ is a $3 \times 3$ matrix. The generation mixing of $LL$ and $RR$ parts can be removed by transforming a basis of squark $\tilde{q}_a^0$ as

$$\tilde{q}_a^0 = \tilde{U}_a^0 \tilde{q}_a^0.$$

The off-diagonal elements, $(M_\tilde{q}^2)_{LR}$ or $(M_\tilde{q}^2)_{RL}$, are then given as follows:

$$(\tilde{U}_L^q)^\dagger (M_\tilde{q}^2)_{LR} \tilde{U}_R^q = (\tilde{U}_L^q)^\dagger U_L^q M_\tilde{q}^{\text{diag}} A_{\text{eff},q} (U_R^q)^\dagger \tilde{U}_R^q,$$

$$A_{\text{eff},q} = A_q^* - \mu T_q,$$

where $T_q$ is given as $T_u = \cot \beta$ and $T_d = \tan \beta$. $\tan \beta$ is defined as a ratio of $v_u$ and $v_d$ which are vacuum expectation values of two Higgs doublets, $H_u$ and $H_d$, respectively. The unitary matrices $U_L^q$ and $U_R^q$ diagonalizes the quark mass matrix as

$$(U_L^q)^\dagger M_q U_R^q = \text{diag}(m_{q1}, m_{q2}, m_{q3}),$$

where $(m_{q1}, m_{q2}, m_{q3}) = (m_u, m_c, m_t)$ for $q = u$ and $(m_d, m_s, m_b)$ for $q = d$. The Cabibbo-Kobayashi-Maskawa (CKM) matrix is defined as

$$V_{\text{CKM}} \equiv U_L^{u\dagger} U_L^{d}.$$
Since the off-diagonal parts of the mass matrix, eq. (6), is suppressed by a quark mass comparing with the diagonal parts when the soft SUSY breaking terms are around $O$(TeV). In general, the unitary matrices $\tilde{U}_q^{\alpha}$ and $U_q^{\alpha}$ are not coincide. This is a source of flavor changing interactions between quarks and squarks.

Next let us see the SUSY interactions relevant to the neutron EDM. There are two types of interactions – the gaugino-quark-squark interaction and the higgsino-quark-squark interaction. An example of the former is the interactions of gluino $\tilde{g}$ to quarks and squarks. After removing the generation mixing, the interaction Lagrangian is given by

$$
\mathcal{L} = -\sqrt{2} g_s \tilde{g}^a T^a \left\{ \bar{u}_L^u V_{LL}^u u_L - \bar{u}_R^u V_{RR}^u u_R \\
+ \bar{d}_L^d V_{LL}^d d_L - \bar{d}_R^d V_{RR}^d d_R \right\} + \text{h.c.,}
$$

(9)

where $g_s$ and $T^a$ are the SU(3)$_C$ gauge coupling and the color matrix, respectively. The unitary matrix $V_{\alpha\beta}^q$ is defined as

$$
V_{\alpha\beta}^q = \tilde{U}_q^{\alpha} U_q^{\beta}.
$$

(10)

Eq. (9) tells us that the gluino diagram can contribute to the neutron EDM only through the left-right mixing of squarks in the 1-loop propagator. The left-right mixing is induced by the off-diagonal element of squark mass matrix (3), which is proportional to the quark mass. Thus the gluino diagrams with flavor changing interactions could be enhanced by the left-right mixing of squark propagators beyond the 1st generation.

The chargino or neutralino can couple to the quarks and squarks through the Yukawa interactions, in addition to the gaugino-quark-squark interaction. The interaction of the right-handed down-quark to the up-squark and the charged higgsino is given as

$$
\mathcal{L} = -(f_d^{ij})^* \bar{u}_L^{ij} \tilde{H}_{dL} d_R^j + \text{h.c.,}
$$

(11)

where $i, j$ denote the generation index. Note that the Yukawa matrix $f_d$ is related to the quark mass matrix as $M_d = -(f_d^{ij}) v_d$. Using the unitary matrices $\tilde{U}_q^{\alpha}$ and $U_q^{\alpha}$, the interaction (11) is written as

$$
\mathcal{L} = (\bar{u}_L^u U_L^d) \frac{g_{\text{diag}}(m_d, m_s, m_b)}{\sqrt{2} m_W \cos \beta} \left\{ \bar{u}_L^{ij} U_{LL}^d \right\}_{ij} (d_R^j)
$$

+ \text{h.c.}

(12)

In the case of $d$-quark EDM ($j = 1$), therefore, the interaction is proportional to $m_d$. The complete set of the interactions will be shown explicitly in our subsequent paper [7].
It is well known that, in the flavor diagonal case, the GUT relation for the gaugino masses,
\[ M_3 : M_2 : M_1 = \alpha_3 : \alpha_2 : \frac{5}{3} \alpha_Y, \]
makes the chargino much lighter than the gluino so that the chargino diagrams give the dominant contributions \[3\]. The neutralino contribution is always small. We will see that the gluino contribution is quite sensitive to the flavor changing interaction while the chargino contribution is not. We show in Fig. 1 the 1-loop diagram which gives the leading contribution in the chargino exchanging diagrams. This is comprised of \( \tilde{W}_{\pm} - q_L - \tilde{q}_L' \) and \( \tilde{H}_{\pm} - q_R - \tilde{q}_L' \) vertices, the former is the SU(2)_L version of (9) and the latter is (12). As is already mentioned, the interaction (12) is always proportional to the 1st generation quark mass while (9) is not enhanced by the flavor off-diagonal components because of the gauge interaction. This is why the chargino contribution is insensitive to the flavor changing interactions. In this paper, therefore, we consider the flavor changing gluino interaction only, where the flavor mixing is parametrized by (10) with \( \alpha = \beta \).

![Feynman diagram of chargino contributions to the neutron EDM.](image)

FIG. 1: Feynman diagram of chargino contributions to the neutron EDM.

Throughout this paper, we will perform our numerical study to the two generation case. Then the flavor mixing matrix (10) is parametrized as
\[
V^q = \begin{pmatrix} \cos \theta_q & \sin \theta_q \\ -\sin \theta_q & \cos \theta_q \end{pmatrix}.
\] (13)

The extension to the three generation case is straightforward, and it will be shown in our next paper [7].

Next we show our numerical result for SUSY contributions to the neutron EDM taking account of the flavor off-diagonal interactions. It is already discussed that only the gluino contribution can give non-negligible contributions to the neutron EDM via flavor changing interactions. As is mentioned earlier, the transition between the left- and right-handed squarks are suppressed by the small quark mass as compared to the chirality preserved case. In our calculation, therefore, the effects of left-right mixing of squarks are taken into account using the mass insertion approximation, using (6) as an expansion parameter.
FIG. 2: Supersymmetric contributions to the neutron EDM for $\tan\beta = 5$. The flavor mixing angle is taken as $\pi/4$. The $CP$ violating phases are taken as $\alpha_f = \phi = \pi/4$. The gluino mass $M_3$ is 1 TeV. The chargino and neutralino contributions are obtained using the GUT relation and $\mu/M_2 = 1$.

In the following, we adopt the GUT relation for gaugino masses to reduce the number of parameters. The left- and right-handed squark masses are taken to be equal. We also assume all the squark flavor mixing angles between the 1st and 2nd generations are common, \textit{i.e.}, $\theta_{u\alpha} = \theta_{d\alpha} \equiv \theta_{12}$ for simplicity. We suppose that the scalar trilinear term $A_f$ is universal in the generation space and take as $A_f = m_\tilde{Q}/3$. In the numerical study, we introduce a parameter $\epsilon^d$,

$$\epsilon^d \equiv \frac{m_d^2 - m_s^2}{m_d^2}$$  \hspace{1cm} (14)

in order to take account of the mass difference between the 1st and 2nd generation down squarks. Another parameter $\epsilon^u$ is also introduced in the up squark sector. We will, however, take the same values for the up and down squarks for each generation, so that the parameters $\epsilon^u$ and $\epsilon^d$ are denoted by $\epsilon$ in the following.

Fig. 2 shows the neutron EDM as a function of the 1st generation squark mass, taking account of the flavor changing interactions. The gluino mass is fixed at 1 TeV and the flavor mixing angle is taken as $\theta_{12} = \pi/4$. The parameter $\epsilon$ is taken as 0, 0.2 and 0.4, as shown explicitly in the figure. The $CP$ violating phases $\alpha_f, \phi$ in (1) are fixed at $\pi/4$, and $\tan\beta$ is 5. The horizontal line denotes the upper bound of neutron EDM (2). The chargino and neutralino contributions are obtained using the GUT relation on the gaugino.
masses and $\mu/M_2 = 1$. We can see in the figure that the flavor off-diagonal interactions increase the neutron EDM as the violation of squark mass degeneracy increases. Note that the enhancement of the neutron EDM comes from the gluino diagram. It is interesting that, when the squark mass universality is sizably violated ($\epsilon = 0.4$), the flavor mixing effect somewhat increases the squark lower mass bound. It is known that the squark mass splitting between the first two generations is strongly constrained from the $K^0 - \bar{K}^0$ mixing as $\sin^2 \theta_\epsilon \left( \frac{30 \text{TeV}}{m_d} \right)^2 < 1$. Then, the mass splitting $\epsilon = 0.4$ with about 5 TeV squark mass is excluded from the $K^0 - \bar{K}^0$ mixing, when the flavor mixing angle is $\pi/4$. The reason why the flavor mixing effect is marginal in the universal limit of squark masses is as follows. In the gluino diagram, the flavor transition can occur at the quark-squark-gluino vertices and at the left-right mixing of squarks in the loop. Owing to the unitarity of the flavor mixing matrix, the diagrams proportional to the 2nd or the 3rd generation quark mass through the left-right mixing vanish when there is no mass difference of squarks among the different generation.

The flavor mixing angle dependence of the SUSY contributions to the neutron EDM is shown in Fig. 3 for $\tan \beta = 5$. The 1st generation squark masses are fixed at 4.5 TeV. In the figure, each line represents the violation of squark mass universality from $\epsilon = 0$ to 0.6. This figure tells us that there is a strong correlation between the flavor mixing effect and the violation of the squark mass universality. It is easy to see that the sensitivity of flavor mixing angle becomes strong as the mass difference of squark masses in the generation space

FIG. 3: Supersymmetric contributions to the neutron EDM as functions of the mixing angle for $\tan \beta = 5$. The squark mass is fixed at 4.5 TeV.
is large. In the figure, constraints on the mixing angle from the $K^0-\bar{K}^0$ mixing are shown explicitly for each value of $\epsilon$.

![Graph showing constraints on mixing angle](image)

**FIG. 4:** The neutron EDM in the SUSY SU(5) + $\nu_R$.

We have so far studied contributions due to the flavor changing interaction between quarks and squarks to the neutron EDM in general framework. The SUSY contribution is much enhanced by the large flavor mixing in the first two generations when the squark mass universality is violated significantly. However most of such parameter region is already disfavored from the $K^0-\bar{K}^0$ mixing. The similar enhancement of neutron EDM is possible for the flavor mixing between the 1st and 3rd generations, which is less constrained from the FCNC processes [7]. Then we might find severe constraints on so called “decoupling” solution to the SUSY FCNC problem (see, [11] and references therein).

The magnitude of the mixing angle is predicted when a specific SUSY breaking scenario is chosen. Here we would like to discuss consequences of the SUSY SU(5) GUT with the right-handed neutrinos, which is one of the well-motivated GUT models. In this model, the flavor off-diagonal interactions are induced through the radiative corrections to the soft SUSY breaking masses for the scalar fields. If the up-quark and the Dirac neutrino Yukawa couplings are sufficiently larger than the down-quark Yukawa couplings, we find that the $d_L-\tilde{d}_L$ transitions are parametrized by the CKM matrix while $d_R-\tilde{d}_R$ transitions are proportional to the MNS matrix in a good approximation [7]. On the other hand, $u_L-\tilde{u}_L$ and $u_R-\tilde{u}_R$ transitions are flavor diagonal. The contributions to the neutron EDM in this framework are shown in Fig. 4 for $\tan \beta = 5$. It can be seen in the figure that there is an enhancement via the flavor changing effects but its magnitude is small as compared to
This is because that, in this framework, the flavor changing interactions are allowed only in the $d$-quark EDM, and those in the gluino diagram is parametrized by the CKM and MNS matrix elements. The enhancement of the gluino contribution, therefore, is much suppressed as compared to the cases in Figs. 2 and 3.

The GUT relation for the gaugino masses makes the gluino mass heavier, in order to satisfy the lower mass bound on the chargino, $m_{\tilde{\chi}^-_1} > 104\text{GeV}$ \cite{8}. If, however, one allows more lighter gluino, e.g., as light as its lower mass bound from collider search experiments, $M_3 > 195\text{GeV}$ \cite{9}, the gluino diagram is overwhelmingly dominant in the SUSY contributions to the neutron EDM and makes the squark lower mass bound much severe.

We add a comment on the electron EDM. The lack of the gluino contribution makes the enhancement marginal. We will discuss this issue in ref. \cite{7} in more detail.

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