Effects of Coriolis force on the nonlinear interactions of acoustic-gravity waves in the atmosphere

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ABSTRACT

The nonlinear theory of acoustic-gravity waves (AGWs) in the atmosphere is revisited with the effects of the Coriolis force. Previous theory in the literature [Phys. Scr. 90 (2015) 055001] is advanced. Starting from a set of fluid equations modified by the Coriolis force, a general linear dispersion relation is derived which manifests the coupling of the high-frequency acoustic-gravity waves (AGWs) and the low-frequency internal gravity waves (IGWs). The frequency of IGWs is enhanced by the Earth’s angular velocity. The latter also significantly modifies the nonlinear coupling of AGWs and IGWs whose evolutions are described by the Zakharov approach as well as the Wigner-Moyal formalism. The consequences of AGWs and the two equivalent evolution equations modified by the Coriolis force are briefly discussed.

1. Introduction

The propagation of acoustic-gravity waves (AGWs) has been known to play a significant role in the interpretation of a wide variety of wave phenomena in the atmosphere including those in the troposphere, as well as to describe the dynamics of ionospheric plasmas (Hines, 1960; Hooke, 1968). The atmospheric waves, whose frequency is of the order of the Brunt-Väisälä frequency or buoyancy frequency and for which the potential energy associated with the buoyancy frequency becomes almost equal to the kinetic energy plus the elastic energy of the acoustics, are termed as AGWs. The frequency of the latter is much lower than that human ears can detect it as sound waves. However, they have some visible impacts in the patterns of atmospheric clouds. Furthermore, the AGWs can be useful for predicting weather and climate phenomena for detecting and monitoring the nuclear detonations as well as to describe the dynamics of the global atmospheric turbulence. The importance of such AGWs has been recognized by a number of authors in the linear and nonlinear regimes of lower and upper atmospheres, as well as in the Earth’s E- and F-layers (Stenflo, 1987, 1998; Stenflo and Shukla, 2009; Kaladze et al., 2007, 2008; Mendonça et al., 2014; Roy et al., 2019). It has been investigated that the AGWs can also appear as a consequence of various meteorological and auroral conditions including the solar eclipses and earthquakes of shear flows (Jovanović et al., 2002). Other important consequences of the AGWs are the formation of localized solitary structures, solitary vortices (Kaladze et al., 2008) and the onset of turbulence due to the interactions of high- and low-frequency branches of AGWs (Mendonça and Stenflo, 2015).

Various appealing phenomena occur when the Coriolis force due to the Earth’s rotation with the angular velocity $\Omega_0$ is considered in the fluid dynamics. The Coriolis force not only gives rise to the coupling of high- and low-frequency AGWs but also modifies the resonance and cut-off frequencies of various other modes in the atmosphere. It has been shown that such a force in incompressible fluids can also lead to the evolution of solitary vortices (Kaladze et al., 2008).

Because of the existence of two frequency branches of AGWs, namely the high-frequency AGWs and the low-frequency internal gravity waves (IGWs), various nonlinear theories of wave-wave interactions have been explored in the context of Zakharov approach (Stenflo, 1986; Mendonça and Stenflo, 2015). The latter is not only useful for the evolution of solitons associated with the high-frequency wave fields but also for the description of chaos and fluid turbulence by the process of energy transfer in nonlinear media. An alternative approach, namely the wave-kinetic approach based on the Wigner-Moyal formalism, has also received considerable attention for the description of the nonlinear coupling of high- and low-frequency branches of AGWs (Mendonça et al., 2014; Mendonça and Stenflo, 2015; Mendonça, 2006b). Such an approach, based on two-fluid model, was first proposed by Tisza, and executed by Landau (Leggett, 2006). Later, this approach has been adapted in several fields including the atmospheric physics and the plasma physics (Mendonça, 2006a). Recently, Mendonça and Stenflo (Mendonça and Stenflo, 2015) developed the wave-kinetic theory of AGWs in the atmosphere starting from a set of Zakharov-like equations (Zakharov, 1972) without the influence of the Coriolis force. They remarked that the Zakharov and wave-kinetic approaches are nearly equivalent and they can provide two complementary views of the atmospheric turbulence.

In this work, our aim is to revisit the nonlinear theory of AGWs, especially to advance the work of Mendonça and Stenflo (Mendonça and Stenflo, 2015) with the influence of the Coriolis force in the fluid motion of charged particles. Starting from a set of nonlinear fluid equations for AGWs, we derive a set of modified Zakharov-type equations which govern the nonlinear interactions of two different frequency
branches of AGWs. Based on the Wigner-Moyal formalism, we also derive an equivalent coupled wave-kinetic equations that are modified by the Coriolis force. We find that the ponderomotive nonlinearity is enhanced and the Landau resonant velocity is up-shifted by the effects of the Coriolis force.

2. Theoretical Formulation

We consider the nonlinear propagation of AGWs in a weakly ionized atmospheric conducting fluid with density $\rho$, the pressure $p$ and the velocity $v$. We assume that the Coriolis force on the charged particles is due to the Earth’s rotation with the uniform angular velocity $\Omega_0 \equiv (0, 0, \Omega_0)$ along the vertical direction. It is further assumed that the atmospheric conducting fluid is unmagnetized for which there is no influence of the Ampère force. Such an assumption may be valid in the lower region of the Earth’s atmosphere, e.g., inospheric D-region (Kaladze et al., 2008). Also, we assume that the conducting fluid is quasi-neutral for which the inner electrostatic electric field can be neglected, i.e., $E = -\nabla \phi = 0$. Here, $\phi$ is the electrostatic potential. Thus, the dynamics of atmospheric fluids can be described by the following sets of equations.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0, \quad (1)$$

$$\frac{\partial v}{\partial t} + v \cdot \nabla v = -\nabla p - 2\Omega_0 \times v + g, \quad (2)$$

$$\left( \frac{\partial}{\partial t} + v \cdot \nabla \right) (\rho \gamma p) = 0, \quad (3)$$

where $g = (0, 0, -g)$ is the gravitational acceleration and $\gamma$ is the ratio of specific heats. At equilibrium, the background mass density and pressure can be assumed to vary as $\rho_0(z) = \rho_0(0) \exp(-z/H)$ and $p_0(z) = \rho_0(0) \exp(-z/H)$, where $p_0(\rho_0)$ is the background pressure (mass density) stratified by the gravitational field and $H$ is the reduced scale length of the atmosphere, i.e., $H = c_s^2/\gamma g$ with $c_s$ denoting the sound speed.

In what follows, we linearize Eqs. (1) - (3) by splitting up the physical quantities into their equilibrium (with suffix $0$) and perturbation (with suffix $l$) parts. Introducing a new variable $N \equiv \rho_l/\rho_0$ with $\rho_l = \rho - \rho_0$, and following Ref. (Stenflo, 1986) we obtain the following modified evolution equation for the density perturbation of AGWs.

$$\sqrt{\rho_0} \left[ \frac{\partial^4}{\partial t^4} + (\omega_a^2 - c_s^2 \nabla^2 + 4\Omega_0^2) \frac{\partial^2}{\partial z^2} \right] \sqrt{N} = 0, \quad (4)$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, and $\omega_a$ and $\omega_0$ are two characteristic frequencies, given by, $\omega_a^2 \equiv c_s^2/4H^2$ and $\omega_0^2 \equiv (\gamma - 1)c_s^2/\gamma^2 H^2$. In fact, these two frequencies define two distinct wave modes to be obtained shortly. Next, we derive the linear dispersion relation from Eq. (4) by assuming the density perturbations to vary as plane waves with frequency $\omega$ and the wave vector $k$, i.e., $N \propto \exp(ik \cdot r - i\omega t)$. Thus, we obtain (Kaladze et al., 2008)

$$\omega^4 - \omega^2 (\omega_a^2 + k^2 c_s^2 + 4\Omega_0^2) + c_s^2 \omega_0^2 k^2 + 4\Omega_0^2 (c_s^2 k_a^2 + \omega_a^2) = 0. \quad (5)$$

The dispersion equation (5) agrees with that obtained by Kaladze et al. for AGWs Kaladze et al. (2008). From Eq. (5) it is noted that the dispersion of AGWs is significantly modified by the Coriolis force ($\propto \Omega_0$). In fact, Eq. (5) gives two wave modes in two different frequency limits. In the limit of $\omega \gg \omega_a$ ($\omega_b > \Omega_0$) we obtain the high-frequency (with subscript $h$) acoustic-gravity mode, given by,

$$\omega_h^2 = \omega_a^2 + k^2 c_s^2, \quad (6)$$

while in the opposite limit, i.e., $\omega \ll \omega_a$, the low-frequency (with subscript $l$) internal gravity mode is obtained, i.e.,

$$\omega_l^2 = \frac{k^2 \omega_a^2}{k_a^2 + 1/4H^2 + \omega_a^2}. \quad (7)$$

Here, $\omega_a$ and $\omega_l$ = $2\Omega_0$, respectively, represent the cut-off frequencies corresponding to the high-frequency acoustic mode and the low-frequency internal wave. We note that while the high-frequency mode remains unaltered, the frequency of the internal wave mode and hence its phase velocity are increased by the effect of $\Omega_0$. However, these two modes can be nonlinearly coupled. In the following two sections 3 and 4, it will be shown that the Coriolis force significantly modifies the nonlinear coupling of the AGWs and IGWs.

3. Nonlinear evolution equations

From Eqs. (1)-(3) and following Refs. (Stenflo, 1986; Mendonça and Stenflo, 2015), the evolution equations for the high-frequency ($N_h$) and low-frequency ($N_l$) perturbations are obtained as

$$\sqrt{\rho_0} \left( \frac{\partial^2}{\partial t^2} + \omega_a^2 - c_s^2 \nabla^2 \right) N_h = \nabla \cdot \left( \rho_0 v_h \cdot \nabla v_l + \rho_0 v_l \cdot \nabla v_l - \sqrt{\rho_0} v_l \frac{\partial N_h}{\partial t} \right), \quad (8)$$

$$\sqrt{\rho_0} \left[ \frac{1}{4H^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \frac{\partial^2}{\partial z^2} - \omega_0^2 \nabla \cdot \left( \frac{\Omega_0^2}{H^2} \right) \right] N_l = S(N_h, v_h), \quad (9)$$
where $S(N_h, v_h) \approx S(\rho_{th}, v_h, \rho_{th}) \approx c_s^2 S$ and $c_s^2 S$ is given by

$$c_s^2 S \approx \frac{\sqrt{\rho_0}}{2} \left( \frac{1}{4H^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \frac{\partial^2}{\partial t^2} - \omega_k^2 \nabla^2 \right) \left( \frac{1}{\sqrt{\rho_0}} \left( v_h^2 - \frac{c_s^2 \rho_{th}^2}{\rho_0^2} \right) \right).$$

Next, introducing a new function $\phi = f^1 f^2 \, dt \, dz \rho_1$, the variables $N_h$, $v_h$, $N_i$ and $v_i$ are expressed as

$$N_i = -\frac{H}{\sqrt{\rho_0} c_s^2 (1 + 16 \Omega_0^2 H^2 / c_s^2)} \nabla^2 \left( \frac{\partial \phi_i}{\partial t} \right),$$

$$v_i = -\frac{1}{\rho_0} \left( 2 \nabla^2 \phi_i - \nabla \frac{\partial \phi_i}{\partial z} \right),$$

$$N_h = \frac{1}{c_s^2 \sqrt{\rho_0}} \frac{\partial}{\partial t} \left( \frac{\partial \phi_h}{\partial z} \right),$$

$$v_h = -\nabla \left( \frac{1}{\rho_0} \frac{\partial \phi_h}{\partial z} \right) - 2 \frac{H^2}{c_s^2} \Omega_0 \nabla \left( \frac{1}{\rho_0} \frac{\partial \phi_h}{\partial z} \right).$$

To simplify the formalism further, we again introduce the high-frequency variable $M_h \equiv \int \frac{1}{\rho_0} N_h(t, z) \, dt \, dz$ and the low-frequency variable associated with the current, i.e., $j_l \equiv \rho_0 v_l$. Thus, Eqs. (8) and (9) reduce to

$$\sqrt{\rho_0} \left( \frac{\partial^2}{\partial t^2} + \omega_k^2 - c_s^2 \nabla^2 \right) \frac{\partial M_h}{\partial t} = \nabla \cdot \left[ v_h \nabla j_l + j_l \nabla \cdot v_h + j_i \cdot \nabla v_h \right],$$

$$\frac{\partial j_l}{\partial t} = -\frac{c_s^4}{4H} \left( 1 + \frac{16 \Omega_0^2 H^2}{c_s^2} \right) \left( \frac{2 \Omega_0 H}{c_s^2} \right) \times \left[ \nabla^2 \left( \frac{\partial}{\partial z} - 2 \nabla^2 \right) \right] < M_h^2 >,$$

with

$$v_h = -c_s^2 \frac{M_h}{\sqrt{\rho_0}} - 2 \frac{H^2}{c_s^2} \Omega_0 \nabla \left( \frac{1}{\sqrt{\rho_0}} \frac{\partial M_h}{\partial t} \right).$$

Equations (15) and (16) are the desired Zakharov-like equations for the description of the nonlinear interactions of high-frequency AGWs and the low-frequency current density perturbations of IGWs that are driven by the ponderomotive force of the high-frequency density perturbations. The appearance of the new terms $\propto \Omega_0$ in $v_h$ indicates that the velocity component of the high-frequency field is enhanced by the influence of the Coriolis force. Furthermore, the latter not only modifies the local nonlinear coupling but also significantly enhances the ponderomotive nonlinearity. Such an enhancement may lead to an increase of the soliton amplitude to be formed in the coherently state as well as may favor the intermediate chaotic processes to develop faster than that in absence of the Coriolis force. We note that in absence of the effects of the Coriolis force, Eqs. (15) and (16) exactly agree with those in Ref. (Mendonça and Stenflo, 2015). In the next section 4, we derive an equivalent set of wave-kinetic equations for the nonlinear coupling of AGWs and IGWs, and show that the Landau resonance condition is also modified by the Coriolis force.

4. Wave-kinetic equations

We derive the wave-kinetic equations from Eqs. (15) and (16). Here, we describe the nonlinear coupling of the high- and low-frequency waves not in terms of the field amplitudes, but in terms of a quasi-probability where the high-frequency waves are described in terms of quasi-particles. Thus, the high-frequency perturbations can be described by a superposition of plane wave modes with amplitude $M_k$, given by,

$$M_h(r, t) = \int M_k \exp(ik \cdot r - i\omega t) \frac{dk}{(2\pi)^3},$$

where the wave frequency $\omega$ and the wave vector $k$ are related to the nonlinear dispersion relation, to be obtained from Eq. (15), as

$$[\omega^2 - (\omega_k^2 + c_s^2 k^2)] M_k = \frac{k}{\omega \sqrt{\rho_0}} \cdot [v_k \cdot \nabla j_l + j_l (i k \cdot v_k + i (j_l \cdot k) v_k)],$$

with

$$v_k = \bar{v}_k - \frac{2 H^2 \omega}{c_s^2} \Omega_0 \times \bar{v}_k,$$

and $k_0 = 1/2H$ is related to the scale length $H$ of the atmosphere.

Next, using Eqs. (20) and (21), Eq. (19) can be rewritten as

$$\omega^2 - (\omega_k^2 + c_s^2 k^2) = -\frac{c_s^2}{\rho_0} \mathcal{L}_k \bar{j}_l(r, t),$$

where the nonlinear coupling operator $\mathcal{L}_k$ is given by

$$\mathcal{L}_k = \frac{k}{\omega} \cdot \left\{ (i k + k_0 e_z) \right\} \cdot (i k + \nabla)$$

and (23)
Equation (22) can be stated as the local nonlinear dispersion relation of AGWs with the local nonlinear coupling \( \mathcal{L}_k J_l \) being associated with the slowly varying current density of IGWs. Note that this nonlinear coupling is significantly modified by the Coriolis force and without which Eq. (22) recovers the linear dispersion relation for the high-frequency branch (6). We note that this nonlinear coupling not only modifies the linear dispersion relation but also introduces a number of new effects including those leading to the collision and fusion among solitons to take place and the emergence of spatio-temporal chaos due to irregular interactions of high- and low-frequency wave fields for which energy can flow from unstable modes to high harmonic stable modes of AGWs. Thus, in order to take into account the exchange of energy among AGW spectrum and the flow due to the eventual occurrence of an instability, we consider the slow variations of both the high-frequency wave \( M_k \) and the low-frequency current \( j_l \), and thereby replacing \( \omega^2 \) by \( \omega^2 + 2i\rho \omega / \partial t \) in Eq. (19) and including the time dependence in \( M_k \) for consistency, we obtain (Mendonça and Stenflo, 2015)

\[
\left( \frac{\partial}{\partial t} + i\omega \right) M_k(t) + \int \frac{d\mathbf{q}}{(2\pi)^3} Q_k(\mathbf{q}) j_q(t) M_k(t) = 0. \tag{24}
\]

where \( j_q(t) \) is the spatial Fourier components of the nonlinear current \( j_l(\mathbf{r}, t) \), \( k' = k - q \) is the new wave vector and the expression \( Q_k(\mathbf{q}) \) is given by

\[
Q_k(\mathbf{q}) = \frac{i e_c^2}{2 \alpha_0^2 \rho_0} \mathbf{k} \cdot \left\{ \left( i k_0 \mathbf{e}_z - \mathbf{k}' \right) \right. \\
- \frac{2 H^2 \omega - \Omega_0 \times (i k_0 \mathbf{e}_z - \mathbf{k}')} {c_s^2} \left\{ \Omega_0 \times (i k_0 \mathbf{e}_z - \mathbf{k}') \right\} \left( \mathbf{k} \cdot \mathbf{e}_q \right) + (i k_0 k'_z - k'^2)(1 - i \frac{2 H^2 \omega}{c_s^2} \Omega_0) \mathbf{e}_q \left\} 
\]

with \( \mathbf{e}_q \approx j_q / |j_q| \) denoting the unit vector.

Using the standard Wigner-Moyal formalism and following the work of (Mendonça and Stenflo, 2015), we obtain the following wave-kinetic equation for the high frequency perturbations

\[
\left( \frac{\partial}{\partial t} + \mathbf{v}_{kq} \cdot \nabla \right) W = \int \frac{d\mathbf{q}}{(2\pi)^3} Q_k(\mathbf{q}) J_q(t) \times \left[ W^- - W^+ \right] \exp(i\mathbf{q} \cdot \mathbf{r}), \tag{26}
\]

where \( W \equiv W(\mathbf{r}, \mathbf{k}, t) \) is the Wigner function, given by

\[
W(\mathbf{r}, \mathbf{k}, t) = \int M_k(\mathbf{r} - s/2) M_{k'}(\mathbf{r} - s/2) e^{ik s} ds. \tag{27}
\]

\( W^\pm = W(\mathbf{r}, \mathbf{k} \pm q/2, t) \) and \( \mathbf{v}_{kq} = \partial \omega / \partial \mathbf{k} = c_s^2 \mathbf{k} / \omega \) is the group velocity of the high-frequency wave envelope. Equation (26) describes the evolution of the high-frequency quasiparticles interacting with low-frequency perturbations \( j_q(t) \).

Next, in the limit of \( |\mathbf{k}| \gg |\mathbf{q}| \) (Geometric optics approximation), i.e., if the typical scale length of low-frequency perturbations with wave vector \( q \) is much larger than that of the high-frequency oscillations with wave vector \( k \), the difference \( [W^- - W^+] \) in Eq. (26) can be Taylor expanded. Thus, retaining the lowest order of the Wigner function, Eq. (26) reduces to the form of a kinetic Vlasov equation, given by,

\[
\left( \frac{\partial}{\partial t} + \mathbf{v}_{kq} \cdot \nabla + \mathbf{F}_k \cdot \frac{\partial}{\partial \mathbf{k}} \right) W = 0. \tag{28}
\]

Here, \( W \) describes the distribution function for the high-frequency atmospheric quasiparticles or phonons, \( \mathbf{F}_k = -\nabla V_k \) is the effective nonlinear force acting on the phonons and \( V_k(\mathbf{r}, t) = Q_k(\mathbf{q}) J_q(t) \exp(i\mathbf{q} \cdot \mathbf{r}) \) is a nonlinear potential associated with the low-frequency perturbations described by the \( q \) spectrum.

In what follows, the evolution equation for the slowly varying current \( j_q(t) \) can be obtained from Eq. (16) in terms of the Wigner function as

\[
\frac{\partial}{\partial t} j_q = -\frac{c_s^4}{4 H} \left( 1 + \frac{16 \Omega_0^2 H^2}{c_s^2} \right) \left( 1 + \frac{2 \Omega_0 H}{c_s} \right) \times \left\{ \nabla_\perp \frac{\partial}{\partial z} - \varepsilon_z \nabla^2_\perp \right\} \int W(\mathbf{r}, \mathbf{k}, t) \frac{d\mathbf{k}}{(2\pi)^3}. \tag{29}
\]

Equations (26) and (29) are the desired wave-kinetic equations equivalent to Eqs. (15) and (16) for the nonlinear coupling of the high- and low-frequency AGWs that are modified by the Coriolis force. In absence of the latter, one can recover the same equations as in the work of (Mendonça and Stenflo, 2015). In order that Eq. (29) includes the linear internal gravity mode \( \Omega = \Omega_q \) [Eq. (7)] in absence of the ponderomotive nonlinearity for the low-frequency perturbations, i.e.,

\[
\Omega \sim \Omega_q \equiv \sqrt{\frac{\alpha_0^2 q^2}{q^2 + k^2_0} + 4 \Omega_0^2}, \tag{30}
\]

we replace \( \partial / \partial t \) by \( \partial / \partial t + i \Omega_q \) in Eq. (29) and rewrite it as

\[
\left( \frac{\partial}{\partial t} + i \Omega_q \right) j_q = -\frac{c_s^4}{4 H} \left( 1 + \frac{16 \Omega_0^2 H^2}{c_s^2} \right) \left( 1 + \frac{2 \Omega_0 H}{c_s} \right) \times \left\{ \nabla_\perp \frac{\partial}{\partial z} - \varepsilon_z \nabla^2_\perp \right\} \int W(\mathbf{r}, \mathbf{k}, t) \frac{d\mathbf{k}}{(2\pi)^3}. \tag{31}
\]

### 5. Nonlinear dispersion relation

In this section, we study the stability of large scale (\( |\mathbf{k}| \gg |\mathbf{q}| \) low-frequency perturbations by deriving an approximate nonlinear dispersion relation. To this end, we assume a low-frequency perturbation associated with the current of the form

\[
j_q(\mathbf{r}, t) = j_q \exp(i\mathbf{q} \cdot \mathbf{r} - i \Omega \Delta t) \]

and that this mode approximately satisfies the low-frequency dispersion equation (30). Thus,
from Eq. (31) we obtain (Mendonça and Stenflo, 2015)

\[
(\Omega - \Omega_q)\hat{q}_q = \frac{c_s^4}{4H} \left( 1 + \frac{\Omega_0^2 H^2}{c_s^2} \right) \left( 1 + \frac{2\Omega H}{c_s} \right) \times \left\{ q_\perp q_z - e_z e_q^2 \right\} \int W_q(r, k) \frac{d|k|}{(2\pi)^3},
\]

(32)

where \( W_q(r, k) \) denotes the modulation of the quasi-distribution function \( W_0(r, k, \tau) \) under the low-frequency plane wave perturbation. The value of \( W_q \) can be obtained by linearizing the wave kinetic equation (26) for high-frequency waves as

\[
W_q = Q_k(q) \frac{[W_{q^-} - W_{q^+}]}{\Omega - q \cdot v_k},
\]

(33)

where \( W_0 \) is the is the unperturbed quasi-particle distribution function and \( W_0^\pm = W_0(k \pm q/2) \). Thus, using Eq. (33), we obtain from Eq. (32) the following nonlinear dispersion relation for a low-frequency wave mode with frequency \( \Omega \) and wave vector \( q \) that is driven by the arbitrary spectrum of high-frequency perturbations.

\[
1 - \frac{\Omega_q}{\Omega} - i \frac{c_s^4}{4H\Omega} \left( 1 + \frac{\Omega_0^2 H^2}{c_s^2} \right) \left( 1 + \frac{2\Omega H}{c_s} \right) \times \left\{ q_\perp q_z - e_z e_q^2 \right\} \int Q_k(q) \frac{[W_{q^-} - W_{q^+}]}{\Omega - q \cdot v_k} \frac{d|k|}{(2\pi)^3} = 0,
\]

(34)

Here, \( e_q = \left\{ (q_\perp)/q_z - e_z e_q^2 \right\}/q_z \), indicating that the nonlinear current and the wave vector are perpendicular to each other. From Eq. (34), we note that the dispersion relation is significantly modified by the effects of the Coriolis force. Furthermore, the Landau resonance occurs when the group velocity \( v_{gk} \) of the high-frequency quasi-particles approaches the phase velocity \( \Omega/q \) of the low-frequency perturbations. It is also noticed that the resonant velocity of the quasi-particles is up-shifted by a quantity \( \propto \Omega^2 \) as the phase velocity \( \Omega/q \) is increased and \( v_{gk} \) remains unaltered by the influence of the Coriolis force [cf. Eqs. (6), (7)]. It follows that the wave-kinetic approach provides an alternative mechanism for the transfer of wave energy in the interactions of high- and low-frequency modes of AGWs in the atmosphere.

6. Conclusion

We have studied the influence of the Coriolis force on the nonlinear interactions of high- and low-frequency branches of AGWs in the atmosphere. Starting from a set of fluid equations modified by the Coriolis force the two linear dispersion branches are obtained in two different limits, namely \( \omega \gg \omega_0 \) (high-frequency) and \( \omega \ll \omega_0 \) (low-frequency). While the high-frequency acoustic mode remains unaltered, the low-frequency internal mode gets modified by the Earth’s uniform angular velocity. Following the work of Mendonça and Stenflo, (2015), the nonlinear coupling of these two modes are described by two-equivalent approaches: the Zakharov approach and the wave-kinetic approach. In the former, the ponderomotive nonlinearity, associated with the high-frequency fields, gets enhanced by the effects of the Coriolis force. This may eventually lead to an increase of the soliton amplitude to be formed due to the nonlinear interactions or a development of the chaotic aspects of the system. As a result, the energy transfer between the high- and low-frequency modes may become faster the larger is the possibility of the emergence of atmospheric turbulence. On the other hand, an approximate nonlinear dispersion relation for the low-frequency IGWs is obtained from the wave-kinetic equations in presence of an arbitrary spectrum of high-frequency atmospheric phonons, which indicates that the Landau resonance condition is modified by the Coriolis force, i.e., the resonant velocity of high-frequency quasi-particles gets up-shifted by a quantity \( \propto \Omega^2 \).

It is worthwhile to mention that the coupled high- and low-frequency modes of AGWs [also known as the inertio-gravity waves (Kaladze et al., 2007)] that are generated by the combined influence of the gravitational force and the Coriolis force can propagate in the regions of lower, middle or upper Earth’s atmosphere (e.g., ionospheric D, E or F layers). Such waves, while interacting with other waves or atmospheric charged particles, can break and produce different kinds of disturbances (Snively and Pasko, 2003; Chen Wei and Tabak, 2015). In presence of the geomagnetic field they may be dissipated [due to Pedersen conductivity (Kaladze et al., 2008)] which may, in turn, generate jet streams and change the heat balance in the upper atmosphere (Fritts et al., 2006; Karpov and Kshevetskii, 2017). Furthermore, the AGWs reaching the Earth’s ionosphere can influence the motion of plasma particles and hence the radio wave transmission.

It is to be noted that the Zakharov approach is more adequate than the wave-kinetic approach for the description of solitons where the formation of electrostatic or electromagnetic wave envelope is highly correlated with the density depletion (Banerjee et al., 2010). On the other hand, the wave-kinetic approach describes the energy exchange between low-frequency waves and high-frequency quasi-particles due to resonance with the group velocity (Mendonça and Stenflo, 2015).

To conclude, at high altitudes, the motion of atmospheric charged particles may be significantly influenced by the Ampère force (\( J \times B \) force). So, the inclusion of this force in the fluid dynamics may introduce a new physical effect to the nonlinear coupling of AGWs and IGWs. However, such an investigation is left for a future project.

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