Symmetric pseudogap metal in a generalized $t−J$ model

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In this article we study a generalized $t−J$ model in which we dope spin-one doublons into a spin 1/2 Mott insulator. We generalize this model to $SU(N)$ case and obtain the ground state based on a three-fermion parton mean field calculation. Surprisingly, for almost every doping we find a fractional Fermi liquid (FL*) phase, which is a symmetric pseudogap metal in the sense that the total Fermi surface volume is equal to $x$, instead of $1+x$ as required by Luttinger constraint of a Fermi liquid. We then verify our theory in one dimension through DMRG simulation of the $t−J$ model. Indeed we find that the doped doublon forms a small Fermi surface on top of the "spinon Fermi surface" ($SU(2)_1$ CFT) inherited from the Mott insulator. This phase is clearly a one dimension analog of the FL* phase and we dub it as fractional Luttinger liquid (LL*). We also discover a continuous quantum phase transition from this LL* phase with small Fermi surface to conventional Luttinger liquid phase with large Fermi surface by tuning interaction strength in a two-orbital Hubbard model at fixed density. Lastly we discuss the experimental realizations of this generalized $t−J$ model and comment on possible connection to the recently discovered nickelate superconductor Nd$_{1-x}$Sr$_x$NiO$_2$.

Introduction. Despite intensive studies for several decades, there is still no well established theory for the pseudogap metal in hole doped cuprates[1, 2]. Especially there is debate on whether the pseudogap metal has a small or large Fermi surface[3]. Recent experiments find evidence of an unconventional metallic state with Hall number equal to $x$ instead of $1+x$ for doping level $x$ in the region $x < x^* \approx 0.19$[4]. Interestingly, no translation symmetry breaking order is found just below $x^*[2]$, suggesting the violation of the Luttinger theorem[5]. To have a translation invariant pseudogap metal with small carrier density, fractionalization is necessary. One candidate for such a symmetric pseudogap metal is the fractional Fermi liquid (FL*) phase[6–13]. However, to our best knowledge, FL* phase has not been found as a ground state in the numerical studies of the $t−J$ model and one-orbital Hubbard model[14, 15], even on a frustrated lattice[16]. This raises concern on whether this kind of exotic pseudogap metal phase can really exist as a ground state in a lattice Hubbard model or $t−J$ model.

The existence of a FL* phase is clear in a two-orbital model with a weak inter-orbital coupling[6, 7]. In this simple case, there can be an orbital-selective Mott transition (OSMT) if the density of one orbital is at $n_1 = 1$[17–19]. The other orbital can have a generic filling $n_2 = x$ and forms a small Fermi pocket. The spin moment of the Mott localized orbital couples to the itinerant Fermi pocket through a Kondo coupling $J_K[18]$. In the $J_K \rightarrow 0$ limit, the Fermi pocket decouples from the local spin, leading to a magnetic ordered metal or a FL* phase[6, 7, 20–23] depending on whether the localized spin moment orders or not. In contrast, the existence of a FL* phase at strong inter-orbital coupling regime is not obvious at all. For example, the conventional $t−J$ model for hole doped cuprates can be viewed as the $J_K \rightarrow +\infty$ limit of a Kondo-Heisenberg model as the doped hole enters the oxygen p orbital and has a large antiferromagnetic coupling to the spin moment from the $d_{z^2−y^2}$ orbital[24]. In this strong coupling limit there is no notion of two separate orbitals. Actually we should project to the restricted Hilbert space with itinerant hole and the localized spin to form a singlet (in hole doped cuprate, this is the famous Zhang-Rice singlet[24]). As a result, a simple orbital selective Mott transition picture or Kondo breakdown picture can not apply anymore and the existence of a FL* phase in the large $J_K$ limit is highly non-trivial. Indeed a FL phase with large Fermi surface is usually found in the large $J_K$ limit. The main question we want to ask in this paper is: can FL* phase be the ground state for a lattice model beyond the orbital decoupling limit?

We will provide a positive answer by studying a generalized $t−J$ model, which can be derived by taking $J_K \rightarrow −\infty$ limit of a Kondo-Heisenberg model. This new $t−J$ model can naturally arise in transition metal oxide with the two $e_g$ orbitals partially filled. Actually the model is first proposed by one of us[25] to describe the recently found nickelate superconductor[26]. Basically if we dope holes into the spin 1/2 Mott insulator in $d^9$ state (with one hole occupying the $d_{z^2−y^2}$ orbital), the resulting $d^9$ site (with two holes) may be in a spin-singlet or a spin-triplet state depending on the competition between the energy splitting of the two $e_g$ orbitals and the Hund’s coupling. If the Hund’s coupling $J_H$ wins, the doped holes enter the $d_{z^2}$ orbital and couple with the spin 1/2 moment from $d_{z^2−y^2}$ orbital through a large ferromagnetic Kondo coupling $J_K = −J_H$. We will take the $J_H \rightarrow 0$ limit and project to a restricted Hilbert space with three spin-triplet doublon states and two spin 1/2 singly occupied states[25].
To analyze the model, we first generalize it to a class of models with $SU(N)$ spin rotation symmetry. Then we perform a self-consistent mean field calculation based on a three-fermion parton theory with $U(2)$ gauge structure. Surprisingly, we find a FL* ground state for every doping on square lattice. The key point is that there can be two emergent spin 1/2 fermions fractionalized from the spin-one doublon state. Then one fermion become electron like and forms a small Fermi surface with volume $V_{FS} = x$, while the other fermion stays as a neutral spinon. These two emergent fermions are different from the two original microscopic orbitals and they are now only weakly interacting with each other. The emergence of two different effective orbitals at low energy is the key to have a stable FL phase in the strong coupling limit where there is no well-defined notion of microscopic orbital. We test our theory in one dimension through DMRG simulation of the $t$ – $J$ model. We find clear numerical evidence of a 1D version of FL* phase, which we dub as fractional Luttinger liquid (LL*). This result is striking given that only conventional Luttinger liquid has been found in the conventional $t$ – $J$ model[27, 28].

Another important theoretical question emerging in cuprate and heavy fermion systems is how to describe the large Fermi surface to small Fermi surface transition[7, 29–34]. The numerical discovery of a symmetric pseudogap metal in our generalized $t$–$J$ model offers a promising platform to study its transition towards a conventional phase with large Fermi surface. As a first attempt, we simulate a two-orbital Hubbard model in one dimension at a fixed density by tuning Hubbard $U$ and Hund’s interaction $J_H$ together. The large $U$ limit reduced to the generalized $t$ – $J$ model and thus is in the LL* phase, while the small $U$ limit is in a conventional LL phase. We find evidence for a continuous quantum phase transition between LL* and LL phase by increasing $U$. The charge compressibility diverges at the critical point. We expect more interesting physics and critical behaviors in higher dimension, which can be accessed by more advanced numerical techniques or real experiments.

Type II $t$ – $J$ model. We consider a generalized $t$ – $J$ model with $SU(N)$ symmetry. We will discuss physics of general $N$ and return to the experimentally relevant case with $N = 2$. We consider a Hubbard model with two orbitals $d_{1\alpha}$, $d_{2\alpha}$, where $\alpha = 1, 2, ..., N$. We imagine that $d_2$ has larger energy than $d_1$. The $n = 1$ Mott insulator is formed by one $d_1$ electron per site, with a $SU(N)$ magnetic moment. Then we dope the system to create doubly occupied site (doublon) with $n = 2$. We introduce a large Hund’s coupling between $d_1, d_2$, so that the doublon state consists of one $d_1$ electron and one $d_2$ electron, forming a symmetric representation of $SU(N)$ (two row, one column in Young tableau). For $N = 2$, this is just a spin triplet.

At each site, there are $N$ number of singly occupied (singlon) states and $\frac{N(N+1)}{2}$ number of doublon states. Thus the dimension of the Hilbert space at each site is $N + \frac{N(N+1)}{2} = \frac{N(N+3)}{2}$. The $n = 1$ state can be labeled as $|\alpha\rangle_i = d_{1\alpha}^\dagger |0\rangle$ with $\alpha = 1, 2, ..., N$. Similarly the doublon state is labeled as $|\alpha\beta\rangle_i = |\beta\alpha\rangle_i = \frac{1}{2} F_{\alpha\beta}(d_{1\alpha}^\dagger d_{1\beta}^\dagger - d_{1\alpha}^\dagger d_{1\beta}^\dagger |0\rangle$. $F_{\alpha\beta}$ is 1 when $\alpha = \beta$ and $F_{\alpha\beta} = \sqrt{2}$ when $\alpha \neq \beta$.

Next we need to project the physical operators into this restricted Hilbert space. After projection, $d_{1\alpha} = 0$ and $d_{2\alpha}$ becomes

$$c_{i,\alpha} = - \prod_{j<i}(−1)^{n_j} \sum_{\beta} G_{\alpha\beta} |\beta\rangle_i (\alpha\beta)_i (1)$$

where $G_{\alpha\beta} = 1$ when $\alpha = \beta$ and $G_{\alpha\beta} = \frac{1}{\sqrt{2}}$ when $\alpha \neq \beta$.

Here we define $c_{i,\alpha} = d_{2\alpha}$.

We can also define the spin operator for the singlon and doublon sites. For singlon, the spin operator is

$$s^\alpha_\beta(i) = |\alpha\rangle_i \langle \beta| (2)$$

For doublon, the spin operator is

$$S^\alpha_\beta(i) = P \left(d_{1\alpha}^\dagger d_{1\beta} + d_{1\beta}^\dagger d_{1\alpha}\right) P (3)$$

where $P$ is the projection operator to the doublon state. One can write down the terms after projection. Fortunately this is not necessary for our purpose.

With the above definition of Hilbert space and physical operators, the type II $t$ – $J$ model can be written as

$$H_{t-J} = H_t + H_J (4)$$

$$H_t = - \sum_{\langle ij \rangle} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \text{h.c.} (5)$$

$$H_J = \sum_{\langle ij \rangle} \frac{J_0}{2} s^\alpha_\beta(i) s^\alpha_\beta(j) + \frac{J_d}{2} S^\alpha_\beta(i) S^\alpha_\beta(j)$$

$$+ J' \left(s^\alpha_\beta(i) S^\alpha_\beta(j) + S^\alpha_\beta(i) s^\alpha_\beta(j)\right) (6)$$

The normalization factor in front of the spin-spin coupling is chosen so that the term reduces to the traditional $\vec{S} \cdot \vec{S}$ form for $N = 2$, as written in Ref. 25. For simplicity we will consider $J' = J_d = J$ in this paper.

Three-fermion parton theory. To deal with a restricted Hilbert space, it is easier to work with parton theory. In the conventional spin 1/2 $t$ – $J$ model, one can create the singlon state with an Abrikosov fermion operator $f_{1\alpha}^\dagger$ and create the spinless doublon with a slave boson operator $b_1^\dagger$. In our case, both the singlon and the doublon carry spin. The singlon state can still be generated by a fermion operator: $|\alpha\rangle_i = f_{1\alpha}^\dagger |0\rangle$. The doublon is in a representation with a huge dimension $d = \frac{N(N+1)}{2}$. This symmetric representation can be generated by two-orbital fermions with a $U(2)$ gauge constraint: $|\alpha\beta\rangle_i = \frac{1}{2} F_{\alpha\beta} \epsilon_{\alpha\beta\gamma} f_{1\gamma}^\dagger f_{2i}^\dagger |0\rangle$. 


Then the electron operator is
\[ c_{i;\alpha} = \frac{1}{2} \epsilon_{\alpha \beta \gamma} f_{i;\gamma}^\dagger \psi_{i;\alpha \beta} \psi_{i;\beta \gamma} \] (7)

The singlon spin operator and doublon spin operator can also be written as
\[
\begin{align*}
\sigma_\beta^a(i) &= f_{i;\alpha}^\dagger f_{i;\beta} \\
S^a_\beta(i) &= \sum_{a=1,2} \psi_{i;\alpha \alpha}^\dagger \psi_{i;\alpha \beta}
\end{align*}
\] (8)

For convenience we define a spinor \( \Psi_{i;\alpha}^0 = (\psi_{i;1\alpha}, \psi_{i;2\alpha})^T \) and label the Pauli matrices \( \tau_\alpha \) acting on this spinor. The above operators becomes the correct physical operator when we implement the constraint:
\[
\begin{align*}
f_{i;\alpha}^\dagger f_{i;\alpha} + \frac{1}{2} \Psi_{i;\alpha}^\dagger \Psi_{i;\alpha} &= 1 \\
\Psi_{i;\alpha}^\dagger \Psi_{i;\alpha} &= 0
\end{align*}
\] (9)

The above two constraints generate a \( U(1) \) and a \( SU(2) \) gauge symmetry respectively. The \( U(1) \) gauge symmetry acts as: \( f_{i;\alpha} \to e^{i \alpha_\alpha} f_{i;\alpha}, \Psi_{i;\alpha} \to e^{i \alpha_\alpha} \Psi_{i;\alpha} \).

The \( SU(2) \) acts as \( f_{i;\alpha} \to f_{i;\alpha}, \Psi_{i;\alpha} \to U_{i;\alpha} \Psi_{i;\alpha} \), where \( U_i \in SU(2) \). \( U(1) \) and \( SU(2) \) share a \( Z_2 \) center: \( f_{i;\alpha} \to f_{i;\alpha}, \Psi_{i;\alpha} \to -\Psi_{i;\alpha} \), so the final gauge structure is \((U(1) \times SU(2))/Z_2 = U(2)\). The \( U(2) \) has an Abelian subgroup \( U(1) \times U(1) \), which acts as \( \psi_{i;1\alpha} \to \psi_{i;1\alpha} e^{i \alpha_\alpha}, \psi_{i;2\alpha} \to \psi_{i;2\alpha} e^{i \alpha_\alpha} f_{i;\alpha} \to f_{i;\alpha} e^{i \alpha_{1\alpha} + i \alpha_{2\alpha}} \).

Basically if we label the corresponding two \( U(1) \) gauge fields as \( a_1 \) and \( a_2 \), then \( \psi_1 \) couples to \( a_1, \psi_2 \) couples to \( a_2 \) and \( f \) couples to \( a_1 + a_2 \).

Let us also discuss the coupling to the physical gauge field \( A \). We can assign charge in the following way: \( \psi_1 \) and \( \psi_2 \) carries \( \frac{1}{2} \) charge and \( f \) is neutral. So finally \( f \) couples to \( a_1 + a_2, \psi_1 \) couples to \( a_1 + \frac{1}{2} A \) and \( \psi_2 \) couples to \( a_2 + \frac{1}{2} A \). This charge assignment can be shifted if we redefine \( a_1 \) and \( a_2 \). Therefore the physical charge of the partons is not well-defined unless the internal \( U(1) \) gauge field is higgsed.

With the parton, we can rewrite the original Hamiltonian as shown in the supplementary.

**FL* phase.** We can write down mean field ansatz using the three-fermion parton theory by decoupling the original Hamiltonian to bilinear terms of the partons. For simplicity we focus on the translation-invariant ansatz.

\[
H_M = -t_f \sum_{\langle ij \rangle} (f_{i;\alpha}^\dagger f_{j;\alpha} + h.c.) - t_{\psi;ab} \sum_{\langle ij \rangle} \psi_{i;\alpha \alpha}^\dagger \psi_{j;\beta \alpha} + h.c.)
- \mu_f \sum_{i} f_{i;\alpha}^\dagger f_{i;\alpha} - \mu_{ab} \sum_{i} \psi_{i;\alpha \alpha}^\dagger \psi_{i;\beta \alpha}
- \Phi_0 \sum_{i} (f_{i;\alpha}^\dagger \psi_{i;\alpha \alpha} + h.c.) - \Phi_a \sum_{ij} (f_{i;\alpha}^\dagger \psi_{j;\beta \alpha} + h.c.)
\] (10)

In the above we did not include \( f_{i;\alpha}^\dagger \psi_{i;2\alpha} \) because we can always use the local \( SU(2) \) gauge transformation to remove it. Here we fix the gauge so that only \( \psi_1 \) has an on-site coupling to \( f \). \( \psi_2 \) can only hybridizes \( f \) through nearest neighbor coupling \( \Phi_2 \). Chemical potentials are introduced to fix \( \langle n_{\psi_1} \rangle = \langle n_{\psi_2} \rangle = 1 - \langle n_f \rangle = x \) and \( \langle \psi_{i;1\alpha}^\dagger \psi_{i;2\alpha} \rangle = 0 \).

We solve the self-consistent equations numerically on square lattice (please see details in the supplementary). At zero temperature, we find that \( \Phi_0 \neq 0, \Phi_1 \neq 0 \) and \( t_f \neq 0, t_{\psi,11} \neq 0, t_{\psi,22} \neq 0, \) but \( \Phi_2 = t_{\psi,12} = 0 \). A non-zero \( \Phi_0 \) and \( \Phi_1 \) higgs the \( U(2) \) gauge symmetry down to \( U(1) \). \( f \psi_1 \) couples to \( a_2 - \frac{1}{2} A \), hence its condensation locks \( a_2 = \frac{1}{2} A \). After that, \( \psi_2 \) couples to \( \frac{1}{2} a_2 + A \) and \( f, \psi_1 \) couples to \( a_1 = a_1 + \frac{1}{2} A \). \( a_1 \) remains deconfined and \( f, \psi_1 \) should be viewed as neutral spinons. In contrast, \( \psi_2 \) couples to \( A \) only and is identical to a physical electron. A more intuitive way to see this is from the original definition \( c_{i;\alpha} = \frac{1}{2} \epsilon_{\alpha \beta \gamma} f_{i;\gamma}^\dagger \psi_{i;\alpha \beta} \psi_{i;\beta \gamma} \). Because \( \frac{1}{2} \sum_{\alpha} \langle f_{i;\alpha}^\dagger \psi_{i;\alpha} \rangle = \sqrt{Z} \neq 0 \), we can identify \( c_{i;\alpha} = \sqrt{Z} \psi_{i;2\alpha} \). This implies the Green function \( G_{\alpha}(\omega, k) = Z G_{\alpha}(\omega, k) \) and \( Z \) can be identified as quasiparticle residue. In contrast, \( f \) and \( \psi_1 \) do not have overlap with \( c \) and remain neutral.

The final phase is a fractional fermi liquid. There is a small fermi pocket formed by \( \psi_2 \), whose volume is \( V_{FS} = \frac{\pi}{2} \) for each flavor. The violation of the Luttinger theorem \( V_{FS}^0 = \frac{1-x}{x} \) is compensated by the existence of a spin liquid formed by \( f, \psi_1 \). In our ansatz the spinons just form a spinon fermi surface coupled to a \( U(1) \) gauge field. The phase may be intuitively understood from orbital selective Mott transition. Starting from two microscopic orbitals \( d_1 \) and \( d_2 \), we can reach the FL* phase if only \( d_1 \) becomes Mott localized while \( d_2 \) remains to form a Fermi liquid with small pocket. However, we need to emphasize that this picture is not precise because the two orbitals \( d_1, d_2 \) feel an infinitely large Hund’s coupling and there is no well-defined notion of microscopic orbital in our \( t-J \) model. It is better to view \( \psi_1 \) and \( \psi_2 \) as emergent orbitals. \( \psi_2 \) has a finite overlap with the microscopic orbital \( d_2 \) only after the condensation of \( \Phi_2 \). We plot

![FIG. 1. Mean field ansatz at zero temperature with doping x for N = 2 and N = 10 on square lattice. We used the parameter t = 1, J = J_d = J' = 0.5. t_{eff} = t_{\psi,22} is the hopping of \( \psi_2 \) in unit of t. Z = \frac{1}{2} \sum_{\alpha} (f_{i;\alpha}^\dagger \psi_{i;1\alpha})^2.](image-url)
the quasi-particle residue $Z$ and the effective hopping $t_{eff} = t\psi_{22}$ of the Fermi pocket in Fig. 1, one can see that $Z$ is below 10%, suggesting that $\psi_2$ is not the same as the microscopic operator $d_2$. Besides, it is heavy because the effective hopping $t_{eff}$ is an order of magnitude smaller than the microscopic hopping.

**Numerical evidence of FL* in 1D.** We simulate the $t-J$ model with $N = 2$ in one dimension using DMRG. Figures 2 show the results at filling $x = \frac{1}{3}$ from infinite DMRG (iDMRG). The momentum distribution $n(k)$ clearly shows a small Fermi surface with size $2k_F^* = \frac{2}{3}2\pi$ [see Fig. 2(b)]. This small Fermi surface is further confirmed by the density-density correlation with discontinuities at $q = 2k_F^*$ in Fig. 2(d). We do not find any feature at $2k_F = 1+\frac{2}{3}2\pi$ corresponding to the large Fermi surface according to the Luttinger constraint. In Fig. 2(c), the spin-spin correlation shows two peaks at both $q = 2k_F^*$ and $q = \pi$. The first peak is apparently from the small Fermi surface. The mode at $q = \pi$ is charge neutral because it does not show up in the density-density correlation function and the electron distribution. Therefore we conclude that there is a small Fermi surface coexisting with another spin mode at $q = \pi$. In total there are three modes, consistent with the result of the central charge $c \approx 3.0$ fit from the entanglement entropy [see Fig. 2(a)].

In the Mott insulator at $x = 0$, there is a gapless spin mode at $q = \pi$, which is described by the $SU(2)_1$ conformal field theory (CFT) and can be thought as a spin liquid with "spinon Fermi surface" in one dimension. Our numerical results then suggest that the doped holes just form a small Fermi surface, which coexists together the "spin liquid" part in the Mott insulator. This is exactly the behavior of a fractional Fermi liquid described in the parton theory. Let us also comment on how this phase is compatible with the Lieb-Schultz-Mattis (LSM) constraint in one dimension [35, 36]. The LSM constraint says that for symmetric phase there must be a gapless model at crystal momentum $Q = 2\pi\nu$, where $\nu = \frac{1+\nu}{2}$ is the filling per spin [36]. In conventional Luttinger liquid, this gapless mode corresponds to $2k_F$ excitation. However, it is possible that this required gapless mode is fractionalized to two elementary modes. In our case, this required gapless mode is formed by a bound state of a neutral mode with momentum $\pi$ from the spin liquid part and the $2k_F^* = \frac{2}{3}2\pi$ of the small Fermi surface: $Q = 2\pi\nu = \pi + 2k_F^*$. One can see that a symmetric pseudogap metal with small Fermi surface is possible provided that there is a neutral spin liquid sector. We will call such a phase as fractional Luttinger liquid (LL*).

**Small to large Fermi surface transition.** The generalized $t-J$ model can be derived in the $U, J_H \gg t$ limit of a two-orbital Hubbard model (see the supplementary). In the weak coupling limit, the ground state must be a conventional Luttinger liquid (LL) phase with large Fermi surface. Therefore we can study a LL to LL* transition tuned by $U$.

We simulate the two-orbital Hubbard model using finite DMRG at $x = 0.2$ and show results in Fig. 3. At $U = 0$, there is a single Fermi surface with $2k_F^* = \frac{1+\pi}{2}$, formed mainly by $d_1$ orbital. Then when $U > 3$, the Fermi surface splits to two, presumably because the effective energy of the orbital $d_2$ is renormalized by the interaction.
teraction and becomes smaller. But the total Fermi surface volume still satisfies the usual Luttinger constraint. Then above a critical value $U_c$, one of the two Fermi surface becomes half-filled and gets a Mott gap, resulting a LL* phase.

![FIG. 4. $\partial E/\partial \mu$ from iDMRG with bond dimension $D = 5000$ for doping $x = 1/2$. The parameters is the same as in Fig. 3, we choose the doping $x = 1/2$ because it is easier for iDMRG. $\partial E/\partial \mu$ is continuous, implying a continuous phase transition. For $U < U_c$, $\partial E/\partial \mu$ can be fit with $A(U_c - U)\alpha + C$ with $\alpha \approx 0.64$.](image)

If we ignore the small pocket, the critical pocket goes through a chemical potential tuned Mott transition with chemical potential $\mu - \mu_c \propto -(U - U_c)$. It is known that for the chemical potential tuned transition in one-orbital Hubbard model[37], $\langle n \rangle = -\partial E/\partial \mu = \begin{cases} A\sqrt{\mu - \mu_c} + 1, & \mu > \mu_c, \\ 1 & \mu < \mu_c \end{cases}$. Therefore, in the orbital-selective Mott transition picture, we expect $\partial E/\partial \mu = \begin{cases} A(U_c - U)\alpha + C, & U < U_c, \\ C & U > U_c, \end{cases}$, where $\alpha = 1/2$. At the critical point, there is also a divergence of the charge compressibility $\kappa \sim \frac{1}{(U_c - U)}$. Next we check this directly in numerical simulation. As finite DMRG seems to suffer from a problem of discontinuous momentum jump due to finite size, we use iDMRG to examine the exponent around $U_c$. We indeed find a singularity for $\partial E/\partial \mu$ as shown in Fig. 4, but the fitted exponent is $\alpha \approx 0.64$, which is larger than that expected in the decoupling picture. Given the numerical noise in our calculation at the maximal bond dimension $D = 5000$ limited by our computational resource, it is not clear whether the discrepancy is just from numerical error or actually implies a new universality class. In our model, the two orbitals $d_1, d_2$ are strongly coupled by $J_H$, and thus it may be possible that the coupling to the small pocket modifies the exponent of the critical Fermi surface. We leave a systematic study to a future paper.

**Conclusion.** In summary, we find a symmetric pseudogap metal with small Fermi surface in a generalized $t - J$ model based on parton theory and DMRG simulation in one dimension. This generalized $t - J$ model can be realized in certain transition-metal-oxides, such as nickelates. It is known that Ni$^{2+}$ is in a spin-triplet state formed by the two $e_g$ orbitals in many cases, then doping spin $1/2$ Ni$^{1+}$ into a spin-one Mott insulator formed by Ni$^{2+}$ will realize our $t - J$ model. In one dimension, this can be achieved by doping the Haldane chain formed by $d^8$ state[38]. Recently, the superconductivity was found in a quasi 2D nickelate Nd$_{1-x}$Sr$_x$NiO$_2$[26], where presumably there are roughly $1 - x$ spin $1/2$ Ni$^{1+}$ states and $x$ Ni$^{2+}$ states. It is still not clear whether the Ni$^{2+}$ is in a spin singlet or spin triplet state. If spin-triplet is favored, then our model is relevant for Nd$_{1-x}$Sr$_x$NiO$_2$[25] and our theory predicts a non-trivial metallic phase with only small pocket above the superconductor $T_c$.

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Self consistent equations of the three-fermion parton theory

The original Hamiltonian of the type II $t - J$ model with $SU(N)$ spin can be rewritten using the three-fermion partons:

$$H = \frac{1}{4} \sum_{\langle ij \rangle} \epsilon_{ab} \epsilon_{a'b'} \psi_{i;\beta}^\dagger \psi_{i;\alpha}^\dagger \psi_{j;\beta'}^\dagger \psi_{j;\alpha'}^\dagger f_{i;\beta'}^\dagger f_{j;\alpha'}^\dagger + h.c.$$  

$$- \frac{1}{2} J \sum_{\langle ij \rangle} f_{i;\alpha}^\dagger f_{j;\alpha}^\dagger f_{i;\beta}^\dagger f_{j;\beta}^\dagger$$  

$$- \frac{1}{2} J_d \sum_{\langle ij \rangle} \psi_{i,\alpha}^\dagger \psi_{j,\beta}^\dagger \psi_{i,\alpha}^\dagger \psi_{j,\beta}^\dagger$$  

$$- \frac{1}{4} J' \sum_{\langle ij \rangle} (f_{i;\alpha}^\dagger \psi_{j,\alpha}^\dagger \psi_{j,\alpha}^\dagger f_{i;\beta} + \psi_{i,\alpha}^\dagger f_{j;\alpha}^\dagger f_{j;\beta}^\dagger \psi_{i,\alpha}^\dagger)$$  



The mean field ansatz can be determined from the self-consistent equations:

$$\Phi_0 = \frac{1}{4} t N^2 \sum_{j \sim i} \chi_{ji;22} \chi_{0;1}^\psi$$  

$$\Phi_a = \frac{1}{4} J' N \chi_{ji;2}^\psi f$$  

$$t_f = \frac{1}{2} J N \chi_{ji}^f$$  

$$t_{\psi;ab} = \frac{1}{2} J_d N \chi_{ji}^\psi + \frac{1}{4} t N^2 \chi_{0;1}^\psi |^2 \delta_{a2} \delta_{b2}$$
\[ \chi_{\alpha a}^{f} = \frac{1}{N} \langle \psi_{1,\alpha a}^{+} f_{1,\alpha} \rangle \]
\[ \chi_{ji}^{f} = \frac{1}{N} \langle \psi_{j,\alpha a}^{+} f_{1,\alpha} \rangle \]
\[ \chi_{ji;ab} = \frac{1}{N} \langle \psi_{j,\alpha a}^{+} \psi_{i,\beta b} \rangle \]
\[ \chi_{ji}^{f} = \frac{1}{N} \langle f_{j,\alpha a}^{+} f_{i,\alpha} \rangle \]

(13)

In the decoupling we did not include cross terms proportional to \( \delta_{\alpha\beta} \), which is smaller by a factor of \( 1/N \). This should be a good approximation at least at large \( N \).

**Two orbital Hubbard Model**

We consider a model with two orbitals (for example, the two \( e_g \) orbitals). A general lattice Hamiltonian is

\[ H = H_K + \frac{U_1}{2} \sum_i n_{1;i}(n_{1;i} - 1) + \frac{U_2}{2} \sum_i n_{2;i}(n_{2;i} - 1) \]
\[ + U' \sum_i n_{1;i} n_{2;i} - 2J_H \sum_i (S_{1;i} \cdot S_{2;i} + \frac{1}{4} n_{1;i} n_{2;i}) \]

(14)

where \( n_{\alpha;i} \) is the density of the orbital \( \alpha \) at the site \( i \). \( \alpha = 1, 2 \) denotes the \( d_{z^2-r^2} \) and the \( d_{z^*} \) orbital respectively. \( U_1, U_2 \) are intra-orbital Hubbard interaction. \( U' \) is the inter-orbital interaction. \( J_H \) is the inter-orbital Hund’s coupling.

We expect \( U_1 = U_2 = U \) and \( U - U' = 2J_H \).

The kinetic energy is

\[ H_K = \sum_i \varepsilon_{dd} n_{2;i} + V \sum_{i} (d_{i1}^{\dagger} d_{i2} + h.c.) + \sum_{(ij)} t_{1;ij} d_{1;i}^{\dagger} d_{1;j} + \sum_{(ij)} t_{2;ij} d_{2;i}^{\dagger} d_{2;j} \]
\[ + \sum_{(ij)} t_{12;ij} d_{1;i}^{\dagger} d_{2;j} + h.c. \]

(15)

where \( \varepsilon_{dd} \) is the splitting between the two \( e_g \) orbitals.

We consider the limit that \( U - U' + J_H > \varepsilon_{dd} \) and \( U, U', J_H >> t \) and \( \varepsilon_{dd} >> \frac{t^2}{U} \). In this limit, the Hilbert space at \( \nu_T = 1 \) is effectively an spin 1/2 chain formed by the \( d_1 \) orbital. Upon doping at \( \nu_T = 1 + x \), the doped site is in a spin-triplet state because of the Hund’s rule. The various inter-orbital hoppings \( t_{12}, V \) are not important at this limit because \( d_1 = 0 \) in the restricted Hilbert space.

In the DMRG simulation in the main text, we use \( t_1 = t_2 = V = 1, \varepsilon_{dd} = 2 \) and \( t_{12} = 0 \). We then tune \( U \) and \( J_H \) together while fixing the ratio \( U_1 = U_2 = U, U = 2U' = 4J_H \). In the electron picture, the filling is fixed to \( \nu_T = 3 - x \), or in the hole picture \( \nu_T = 1 + x \).

**Results from finite DMRG for the generalized \( t-J \) model**

Finite DMRG results of the generalized \( t-J \) model with spin-one doublon are shown in Fig. 5. We find that the LL* phase is unstable at exactly \( x = 0.5 \), at which there seems to be a SDW instability at momentum \( Q = 0.5\pi \). The LL* phase returns when \( x > 0.5 \) and survives at least to \( x = 0.7 \). When further increasing doping close to the spin one Haldane chain, the LL* phase may eventually be unstable to a different phase. We do not find singularity at first and second derivative of energy when \( x > 0.5 \), but a KT transition can not be ruled out. We leave it to future work to study the region close to \( x = 1 \).
FIG. 5. Results from finite DMRG with $L = 100$ and $D = 2000$. Doping is varied from 0 to 1 with $\delta x = 0.02$. (a)(b) show a first derivative jump of energy at $x = 0.5$. In (c) and (d) we show that the system is in a LL* phase for $x \leq 0.7$ except at $x = 0.5$. At $x = 0.5$ we find that $n(k)$ does not have sharp $k_F^*$, indicating that single electron is gapped. Meanwhile the peak at $q = \pi$ also disappears in spin-spin correlation, while the peak at $q = 2k_F^* = 0.25 \times 2\pi$ gets sharper, a SDW instability with $Q = 0.5\pi$. 