Sound Wave in Hot Dense Matter Created in Heavy Ion Collision

X. Sun$^{a,b}$, Z. Yang$^{c,d}$
$^a$Institute of High Energy Physics, Beijing 100049, China
$^b$Graduate University of the Chinese Academy of Sciences, Beijing 100049, China
$^c$Department of Engineering Physics, Tsinghua University, Beijing 100084, China
$^d$Center of High Energy Physics, Tsinghua University, Beijing 100084, China

A simple model is proposed to study the sound wave in hot dense matter created in heavy ion collisions by jet. The preliminary data of jet shape analysis of PHENIX Collaboration for all centralities and two directions is well explained in this model. Then the wavelength of the sound wave, the natural frequency of the hot dense matter and the speed of sound wave are estimated from the fit.

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I. INTRODUCTION

The energy loss of hard partons induced at the early stage of heavy ion collision has been regarded as a promising tool to study the properties of deconfined matter that created in the unrelativistic heavy ion reactions\cite{2,3,4,5,6,7}. While the energy and momentum that lost by a hard parton propagating through hot and dense matter would obviously be redistributed in the created medium, recent results\cite{8} of the measurements of two-particle correlations, which involve on hard trigger particle, have shown us a surprising features that double peaks are measured on the away side for all non-peripheral collisions for Au+Au collisions. This distinguished feature is quite different from that for Au+Au peripheral collisions and p-p or d-Au collisions.

This surprising feature attracts interests of many physicists and a lot of models have been proposed to explain this phenomenon, e.g., possible colored sound wave, the natural frequency of the hot dense matter and the speed of sound wave are estimated from the fit.

To study dependance of the angular correlation pattern to centrality i.e. the impact parameter, this letter derives the merit of conical flow model and uses a more microscopic and basic formalism of this effect. The collective exciting mode means the perturbation induced by the jet is not separately. There is correlation between contiguous perturbations. In our model, we introduce the intrinsic sound wave of relativistic hot dense matter which will be discussed in detail in the following to express the correlation between perturbations.

The letter is organized as follows: In Section II we explain basic framework and assumptions in our model. In Section III this result of the model is discussed and compared with experiment data. A summary is made in Section IV.

II. FRAMEWORK AND ASSUMPTIONS

For the shortage of knowledge of interaction between parton and gluon, it is difficult to know the behavior of hot and dense matter created in heavy ion collision. However, to study the energy losses of fast quenched parton, one treats two different types of cases separately: (i) the radiative losses, producing relativistic gluons; and (ii) the scattering/ionization losses, which deposit energy and momentum directly into the medium. Both of them have relations to the property of the hot dense matter created in heavy ion collision.

To overcome this shortage, one can understand this energy transportation on the point of view of a wave. Generally, a wave can travel through whatever matter which satisfies $\partial P/\partial V < 0$ with $P$ and $V$ being pressure and volume of the matter respectively, like lattice wave in solid, sound wave in liquid and gas, carrying energy.

It is reasonable to deem $\partial P/\partial V < 0$ is true for hot dense matter produced in heavy ion collision since it is wrong only for undetected dark matter which is important in cosmology. If a wave travelling the matter freely, the velocity and frequency of it are intrinsic properties of the matter. The hot dense matter created in heavy ion collision can be regarded as a kind of liquid or gas,
so we call the wave travelling through it as sound wave. And the perturbation induced by fast parton will propagate freely until it reaches the boundary of the source. So the frequency the sound wave should be the natural frequency of the hot dense matter.

As a standard wave equation always successes in describing wave phenomena in having equilibrated system, we assume it to be able to describe the sound wave in hot dense matter for a admittedly thermalization can be reached after heavy ion collision. The speed of sound $c_s$ is the unique parameter of the wave equation and one parameter in our model which should be invariant in every case of centricity. Then sound wave travelling the hot dense matter obeys a wave equation reading

$$\frac{\partial^2 u}{\partial t^2} - c_s^2 \Delta u = 0 \quad (1)$$

where $c_s$ is the speed of sound of the medium produced in heavy ion collision. We note that equation (1) is only a approximation of true one without considering anharmonic terms, which is reasonable where the anharmonic terms inducing dissipative effect in the medium is neglectible. This approximation is also necessary for Mach-Cone description and its reasonability should be tested in further study of non-perturbation QCD. However, equation (1) at least holds true where is away from the trajectory of quenched fast parton for a distance of order of sound attenuation length, $\Gamma = (4/3)\eta/(\epsilon + p)$, with $\eta$ being the shear viscosity.

On the other hand, as the quenched fast parton is constantly emitting gluons, which emit new ones etc., the whole shower is a complicated nonlinear phenomenon. The multiplicity of this shower grows nonlinearly with time. Eventually, the core may become a macroscopic co-moving body. All perturbations originating from this co-moving system produced by the quenched fast parton requires the consideration of Doppler effect. The velocity of the co-moving system $c_\nu$ is another parameter of our model which should be determined in experiment fit. However, it can also be discussed qualitatively. $c_\nu$ should increase with the increment of the quenched parton and decrease with the increment of the length of the hot dense matter through which the quenched parton travels. We note that the source is expanding instead of static, so the whole medium through which sound wave travels has a radial or elliptical velocity which should affect the energy distribution of the final-state particles. This effect is approximately negligible in center-center collision because a radial velocity field will not change the spatial angle distribution of energy. For a simple case, we adopt a radial velocity field in our discussion.

Now we depict our image of understanding the interference of perturbation induced by the quenched fast parton in Fig.1, where perturbations originate from line section $\overline{AB}$ with phase $2\pi ic_s/(V\lambda)$ where $l$ is the distance between $B$ and point located in line section $\overline{AB}$, $V$ is the velocity of the quenched fast parton, always taking $c$ for it, and $\lambda$ is the wavelength of the sound wave. The perturbations interfere with phase and produce the interference pattern, for example, in $D$ and $E$. This pattern represents the distribution of energy of perturbations and should be equivalent in shape to two-particle correlation about jet which is measured in experiment.

The momentum distribution of particles induced by the quenched fast parton is not treated in this model since it has relations to durative scattering rather than collective excitation mode. For this reason, we can only adjust the relative intensity between interference pattern of trigger and its counterpart according to experiment.

Conclusively, there are three parameters in our model totally, which are the speed of sound $c_s$, the velocity of co-moving system $c_\nu$ and the ratio of the size of the source to the wavelength of the sound wave $L_{\text{tot}}/\lambda$. Because the system in which we are interested evolves with a finite lifetime, these parameters should be regarded as time-weighted averages respectively.

### III. RESULT

In experiment, one can measure the azimuthal correlation around a high momentum trigger, of particles within a special momentum window, which should be reproduced by the diffraction pattern of the carrying energy sound wave in our model.

The perturbations induced by a same parton should be
coherent by phase, while these induced by different partons should be superposed by intensity. Thus the diffraction pattern for one parton reads

$$I(\theta) = A \left| \sin[\pi s(\cos \theta - c_s)/\lambda] \right|^2$$  \hspace{1cm} (2)

where $\theta$ is the azimuth angle in the frame of center of mass and $s$ is the length through which the fast parton passes and $\lambda$ is the wavelength of the sound wave. For showing the result in laboratory frame, one can substitute $\theta'$ for $\theta$ with,

$$\theta' = \begin{cases} \tan^{-1}\left( \frac{psin \theta}{gamma(p cos \theta - \beta E)} \right), & p\cos \theta > \beta E, \\ \tan^{-1}\left( \frac{psin \theta}{gamma(p cos \theta - \beta E)} \right) + \text{Sign}(\theta)\pi, & \text{otherwise}. \end{cases}$$  \hspace{1cm} (3)

where $\beta$ is $c_s$ and $\gamma = (1 - \beta^2)^{-\frac{1}{2}}$. $p$ and $E$ in (3) means the momentum and energy of observed particle in laboratory frame. The analytic form of $I(\theta')$ is difficult to write out, though it looks very easy.

The speed of sound $c_s$ should be a constant for a fixed temperature and the speed of co-moving system $c_s$ should increase obviously with the momentum of trigger particle. The length of trigger passing $s$ should be determined by the centrality of the collision, i.e. the multiplicity of the collision. Fig. 1 shows the comparison between preliminary data of jet shape analysis of PHENIX Collaboration[15] and our calculation. Different marks denote experiment results in different centralities, while different styles of lines denote our calculations using different parameters. The momentum of particle used in the calculation, 2.5 GeV , is the average of the momentum range of experimentally measured particles i.e. 2-3 Gev. The used speed of sound , much smaller than that used in Mach cone calculation in [14] where $c_s = 0.33c$, is 0.129c for the better fit, which is a constant in calculation. In this case the peak of induced particle should be at angles about 1.7 radian relative to the trigger. The normalization for all centralities has been performed.

The parameters in the calculation is listed below

| $s/\lambda$ | 0-10%↑ | 60-92%↑ | 30-40%↑ | 0-10%↓ | 30-40%↓ | 60-92%↓ |
|------------|--------|--------|--------|--------|--------|--------|
| $c_s$      | 0.15   | 0.08   | 0.05   | 0.6    | 0.4    | 0.07   |

The ↑ means the forward jet i.e. the jet caused by the trigger particle while ↓ means the backward jet which is caused by the counterpart of the trigger.

It is easy to understand the variation of the parameters. The centrality of the collision , relative to the source size, is reflected by the $s/\lambda$. As the increment of the source size, the co-moving velocity decreases. This reflects the energy loss of particle when traveling the source.

The intensity ratios between forward jet and backward jet are 0.60, 0.51 and 0.67 for 0−10%, 30−40%, 60−92% respectively, which come from data fitting and will not be discussed in our model as is above mentioned.

Now the picture of sound wave in hot dense matter is to some extend obvious. Let’s take double HBT radius [16] , 5fm, as the source size for center-center collision, the wavelength of sound wave in hot dense matter created in heavy ion collision is estimated as $\lambda = (2 \times 5 fm)/(0.6 + 0.15) = 13.3 fm$. The natural frequency of the hot dense matter is estimated as $c_s/\lambda = 2.91 \times 10^{21} Hz$. These quantities will be constants at fixed temperature.

But Making conclusion that we have catch signals about the intrinsic eigenvibration of hot dense matter is premature at present. After all, there must be other models being able to explain the same data. To test the existence of the sound wave, we can examine other behaviors of this model.

The velocity of co-moving system $c_s$ will increase as the increment of the momentum of the trigger. For higher momentum of trigger particle, the distance between two peaks in backward jet in center-center collisions will decrease since the Lorentz transformation in [14] , which can be tested in LHC where a lot of jets with high momentum trigger will be produced.

We can also make a more fine bins of centrality to draw the jet induced particle azimuth correlation again, which will give a more crashing test of this model.

Essentially, the properties of the sound wave are determined by the interaction between particles in hot dense matter. One can conceive that it is quite different for confined and deconfined scenarios. So it is meaningful to compare different scenarios to experiment, which will be a interesting tool to research QGP.

IV. CONCLUSION

We try to study the sound wave in hot dense matter created in heavy ion collisions in this letter by jet. The distribution of final-state particle is described by the interference of the sound wave induce by the quenched fast parton. The productions of a co-moving system around...
the quenched fast parton is introduced. The preliminary data of jet shape analysis of PHENIX Collaboration for all centralities and two directions is well explained in our model. From the parameter of the result, we estimate the wavelength of the sound wave is 13.3 fm, the natural frequency of the hot dense matter is $2.91 \times 10^{21} \text{Hz}$ and the speed of sound wave is 0.129c.

We predict that the distance between two peaks in backward jet in center-center collisions will decrease since the Lorentz transformation for higher momentum of trigger particle, which can be tested in LHC where a lot of jets with high momentum trigger will be produced in the future. Our model cares only collective property of the hot dense matter. Whether the sound wave can become a tool to test the creation of quark-gluon plasma depends on the difference between calculations of the sound wave in quark-gluon plasma scenario and hadron-gas scenario.

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