The confining string and its breaking in QCD

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Abstract

We point out that the world sheet swept by the confining string in
presence of dynamical quarks can belong to two different phases, de-
pending on the number of charge species and the quark masses. When
it lies in the normal phase (as opposed to the tearing one) the string
breaking is invisible in the Wilson loop, while is manifest in operators
composed of disjoint sources, as observed in many numerical experi-
ments. We work out an explicit formula for the correlator of Polyako
loops at finite temperature, which is then compared with recent lattice
data, both in the quenched case and in presence of dynamical quarks.
The analysis in the quenched case shows that the free bosonic string
model describes accurately the data for distances larger than \( \sim 0.75 \)
fm. In the unquenched case we derive predictions on the dependence
of the static potential on the temperature which are compatible with
the lattice data.

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1 Introduction

The confining interaction of a pair of static sources in non Abelian gauge theories is mediated by a thin flux tube, or string, joining the two sources. At large distance, the energy of the flux tube is proportional to its length, hence the static potential grows linearly.

When matter in the fundamental representation is added to the system, one expects that this potential flattens at some distance $r_o$, where the string breaks to form pairs of matter particles which screen the confining potential. The broken string state describes a bound state of a static color source (the fixed end of the string) and a dynamical matter field (the free end of the string). A similar screening is expected even in pure Yang-Mills theory for adjoint sources, since in this case the adjoint string may terminate on dynamical gluons.

Despite many efforts, these string breaking effects have proved elusive in standard analyses of large Wilson loops, both in QCD with dynamical fermions [1] and in pure Yang-Mills theory with adjoint sources in 3+1 dimensions [2] and in 2+1 dimensions [3].

On the contrary, clear signals of string breaking have been observed in studies where the basis of operators has been enlarged in order to find a better overlap to the true ground state, following a method originally advocated in Ref. [4]. In this way, fundamental string breaking has been found in SU(2) Higgs model in 2+1 [5] and 3+1 [6] dimensions, and adjoint string breaking in 2+1 SU(2) pure gauge theory [7, 8].

The outcome of these analyses is twofold. On one hand the Wilson loop appears to have in general very poor overlap on the broken string state, the only exception being observed up to now in the 2+1 dimensional SU(2) gauge theory with two flavors of staggered fermions [9]. On the other hand it turns out that the operators with a good overlap with the broken string state have as a common feature the presence of two disjoint source lines. This is not only true in the above mentioned cases at zero temperature, but it is also evident in the recent observation of string breaking at finite temperature QCD with dynamical fermions, where the static potential is extracted from the correlator of two disjoint Polyakov loops [10].

In this work we will show that in the usual string description of confinement [11] one can find a simple explanation of such a relationship between the overlap properties and the number of disjoint static sources. In particular we shall derive an asymptotic functional form of the unquenched finite temperature potential which is then successfully compared with the QCD data of Ref. [10].
A solvable prototype of string breaking suggests that the world sheet swept by the string in its time evolution can exist in two different phases, known as tearing and normal phases. The question of the existence of a finite overlap of the Wilson operator with the broken string state can be reformulated (see section 2) as the problem of finding the phase of the string world sheet. Indeed in the former phase the world sheet is torn by large holes corresponding to pair creation of charged matter, hence the Wilson loop is expected to fulfil the perimeter law at large distances. On the contrary, in the normal phase the string breaking is balanced by the inverse process of string soldering, so that adding dynamical matter to the system yields simply a renormalization of the string tension. In particular the Wilson loop behaves exactly like in the quenched model and no macroscopic string breaking is visible.

However, a simple topological argument shows that the situation drastically changes when one considers, instead of the Wilson loop, operators constructed out of disjoint static sources. Besides the usual contribution to the confining potential there is a new term which is necessarily absent in the quenched case and which survives at large source separations. This term makes string breaking effects easily detected on the lattice.

The remaining sections are organized as follows. In section 3 we review some known bosonic string formulae about the asymptotic infrared behaviour of gauge operators and derive the new term contributing to correlator of two Polyakov loops in the unquenched case. Since there are no recent studies on the string effects in the quenched approximation of QCD, we devote section 4 to an analysis of the quenched data of Ref. 10. It turns out that the bosonic string accurately describes the lattice data for distances larger than \( \sim 0.75 \) fm. In section 5 the analysis is extended to the unquenched case where we check that string predictions on the temperature dependence of the potential are compatible with the lattice data of Ref. 10. Finally in section 6 we draw some concluding remarks.

2 A topological argument

Let us start by considering the standard Wilson loop. In absence of charged dynamical fields this loop acts as a fixed boundary of the surface associated to the world sheet swept by the confining string. When matter fields are added, any number of holes of any size may appear on this surface, reflecting the pair creation of dynamical charged particles. If these are light, the holes behave like free boundaries. The total string contribution to the Wil-
son operator is then the sum over all possible insertions of multiconnected boundaries of the world sheet. It can be written diagrammatically as a loop expansion (see Fig. 1).

There are two quantities controlling the number and the size of these holes. One is the mass of the matter fields (it is easier to create lighter particles). The other is the number of charge species. For a theory with $N_f$ flavors and $N_c$ colors there is a factor of $N_f N_c$ for each hole. Likewise in the adjoint string every hole is accompanied by a factor of $N_c^2 - 1$.

For light particles and $N_f N_c$ large enough, we expect that the string world sheet is dominated by configurations with a large number of relatively small holes compared with the size of the Wilson loop.

These holes do not influence the area law of the Wilson loop and their effect can be absorbed into the renormalization of the string tension, according to a string mechanism originally proposed in Ref. [13]. Under the circumstances there is no way to observe the string breaking through the study of large Wilson loops. This may explain why this phenomenon has been so elusive both for fundamental and adjoint string in any dimension.

![Figure 1. Loop expansion of the rectangle.](image)

However it is worth noting, as mentioned in the introduction, that there is a solvable matrix model [12] describing idealized random surfaces with dynamical holes (or open strings embedded in zero dimension), where besides the above described ”normal” phase dominated by holes of small size there is also another phase, characterized by a spontaneous tearing of the surface, due to the formation of large holes growing with the size of the surface (see Fig. 2); as a consequence we expect that the associated Wilson loop should fulfil the perimeter law.

What is the phase of the world sheet of the confining string? The observed extremely poor overlap of the Wilson loop with the broken string state seems to suggest that it belongs to the normal phase, with a possible

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1. There is also a third phase separating the above two, where both the world sheet (the glue) and the holes (the charged matter) are large and in competition. This phase could play a role at the deconfining temperature.

2. Of course the numerical data cannot exclude the possibility that such a poor overlap
exception for the SU(2) gauge theory in 2+1 dimensions with dynamical fermions [9]. Perhaps in this case the quarks are not light enough.

For the operators with more than one connected source line the loop expansion has a topologically different form. Take for instance a pair of Polyakov loops of a gauge theory with dynamical fermions at finite temperature. From the point of view of the effective string, they are the two fixed boundaries of a cylinder (see Fig. 3). The loop expansion splits into two different series, because starting at two-loop level there are configurations in which the two fixed boundaries are not connected by the string world sheet.

If the system is in the normal phase the sum over the loop insertions can be still absorbed into the renormalization of the string tension, but now the loop expansion is the sum of two different terms. That with only one connected world sheet has no overlap to the broken string state but decays exponentially with the distance between the fixed boundaries, thus only the term with two disjoint pieces survives, which of course is expected to have a large overlap to the broken string state. In the case of light matter fields this term can be written as a square $[Z_{DN}(R_o, L)]^2$, where $Z_{DN}$ denotes the contribution of a cylindric world sheet with fixed (or Dirichlet) boundary conditions (b.c.) on the Polyakov line of length $L$ and free (or Neumann) b.c. on the other side (representing a dynamical charge) placed at a mean distance $R_o$ from the Polyakov loop. Clearly $R_o$ fixes the scale of the string breaking. Owing to the simple geometry of this term, we shall explicitly
evaluate its functional form in the next section.

Similar considerations apply to operators used to see string breaking at zero temperature. In such a case the fixed boundary is not necessarily closed because it may terminate on charged fields. The rule to associate a string world sheet to these operators is simply to connect the end points of the fixed boundaries (Wilson lines) with paths associated to the free end of the string, representing the world lines of the dynamical particles. For instance, using the diagrammatic representation of Ref. [6], the map to the string picture of other operators used to determine the static potential in the SU(2) Higgs model is drawn in Fig. 4.
3 String formulae

The main role of the string picture of confinement is to fix the functional form of gauge operators in the infrared limit. In particular it is known [14] that, according to the bosonic string model, a rectangular $R \times L$ Wilson loop should behave asymptotically as the partition function of $d - 2$ free two-dimensional bosonic fields describing the transverse oscillations of the world sheet with fixed b. c.:

$$\langle W(R, L) \rangle \propto e^{-\sigma_{RL}-p(R+L)} \left[ \frac{\sqrt{R}}{\eta(\tau)} \right]^{d-2}, \quad \tau = \frac{iL}{R},$$

where $d$ is the spacetime dimension, $\sigma$ the string tension, $p$ the perimeter term and $\eta$ is the Dedekind function

$$\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n), \quad q \equiv \exp(2\pi i \tau).$$

This in turn gives the static potential

$$V(R) = - \lim_{L \to \infty} \frac{1}{L} \log \langle W(R, L) \rangle = \sigma R - \frac{(d - 2)\pi}{24R} + \ldots$$

where dots indicate subleading terms. These predictions have been tested to high accuracy for the $Z_2$ gauge model in $d = 3$ in [15]. Good agreement with the string model predictions was also found for $SU(N)$ gauge theories in $d = 3$ [16] and for $SU(3)$ in $d = 4$ [17].

Similar considerations apply to the confining phase of finite temperature gauge theories. Here the relevant observable is the correlator of Polyakov loops. Since the lattice is finite and periodic in the time direction, the Polyakov loop correlation will be described by the partition function of $d - 2$ free bosons on a cylinder with periodic b.c. in the time direction and fixed b.c. on the two loops. The partition function is therefore:

$$\langle P(0)P^+(R) \rangle \propto Z_{DD}(R, L_t) \propto e^{-\sigma_0 RL_t} \frac{\eta(\tau)^{d-2}}{\eta(\tau)^{d-2}}$$

where $\sigma_0$ is the zero temperature string tension at the same coupling, $L_t$ is the lattice extension in the time direction, that is the inverse temperature, and now $\tau = \frac{iL}{R}$. Sec. [1] will be devoted to a comparison of Eq. (4) with lattice quenched QCD data at finite temperature.
We have seen in Section 2 that in presence of dynamical matter the Polyakov correlator contains an additional term $[Z_{DN}(R_o, L_t)]^2$, where $Z_{DN}$ is the partition function on a cylinder with fixed b.c. on the Polyakov loop side and free b.c. on the other side, corresponding to a dynamically generated loop. The functional form of $Z_{DN}$ is similar to that of Eq. (4), but now the the $\eta$ function is replaced by the function

$$\tilde{\eta}(\tau_o) = q^{-1/48} \prod_{n=1}^{\infty} \left(1 - q^{n-\frac{1}{2}}\right), \quad \tau_o = \frac{iL_t}{2R_o}, \quad q \equiv \exp(2\pi i \tau_o).$$

(5)

The form of the two functions $\eta$ and $\tilde{\eta}$ can be simply understood as the inverse of the partition function of the normal modes of vibration of the string. When both ends are fixed the allowed frequencies are $\omega_n = \frac{\pi}{R} n$ and these generate the infinite product of Eq. (2); if one end is free we have instead $\tilde{\omega}_n = \frac{\pi}{R} (n - \frac{1}{2})$ which generate the infinite product of Eq. (5). The prefactor $q^{\frac{1}{24}}$ in Eq. (2) is directly related to the zero point energy $E_o$ of the string

$$E_o = \sum_{n=1}^{\prime} \frac{h\omega_n}{2} = \frac{h\pi}{2R} \sum_{n=1}^{\prime} n = \frac{h\pi}{2R} \zeta(-1) = -\frac{h\pi}{24R},$$

(6)

where $\sum^{\prime}$ denotes the $\zeta$-function regularized sum. It is known that within such a regularization one can safely handle the series as they were finite sums, then we can write

$$\sum_{n=1}^{\prime} n = \sum_{n=1}^{\prime} 2n + \sum_{n=1}^{\prime} (2n - 1) = 2\zeta(-1) + 2 \sum_{n=1}^{\prime} (n - \frac{1}{2});$$

(7)

hence

$$\sum_{n=1}^{\prime} \frac{h\tilde{\omega}_n}{2} = \frac{h\pi}{2R} \sum_{n=1}^{\prime} (n - \frac{1}{2}) = \frac{h\pi}{48R}.$$  

(8)

This gives in turn the prefactor of $\tilde{\eta}$. The identity $\tilde{\eta}(\tau) = \frac{\eta(\tau)}{\eta(\tau/2)}$ allows us to write

$$Z_{DN}(R_o, L_t) = e^{-\sigma_0 R_o L_t} \left[ \frac{\eta(\tau_o)}{\eta(\tau_o/2)} \right]^{d-2}.$$  

(9)

Thus, assuming that the world sheet is in the normal phase, we would expect that the Polyakov loop correlation function in presence of dynamical quarks has the following effective string description:

$$\langle P(0) P^+(R) \rangle = c_1 Z_{DD}(R_o, L_t) + c_2 [Z_{DN}(R_o, L_t)]^2$$

$$= c_1 \frac{e^{-\sigma_0 R_o L_t}}{\eta(\tau)} + (N_c N_f)^2 e^{-2\sigma_0 R_o L_t} \left[ \frac{\eta(\tau)}{\eta(\tau/2)} \right]^{2(d-2)}.$$  

(10)
where $N_c$ and $N_f$ are respectively the number of colors and flavors of light quarks. The constants $c_1$ and $c_2$ and the length scale $R_o$ are not predicted by the model and will have to be determined numerically. Notice that the second term in Eq. (10) is just a constant at any fixed temperature: therefore the actual predictions of the model concern the temperature dependence of the potential. In Sec. 5 we will compare these predictions with lattice QCD data.

4 The static potential in quenched QCD at finite temperature

If no dynamical quarks are present, the effective string prediction for the Polyakov loop correlation is given by Eq. (4). For the static potential we have in $d = 4$

$$V(R) = -\frac{1}{L_t} \log \langle P(0)P^+(R) \rangle = \sigma_0 R + \frac{2}{L_t} \log \eta \left( \frac{iL_t}{2R} \right)$$

We will be interested in the $2R > L_t$ region, in which Eq. (11) is conveniently rewritten in the equivalent form

$$V(R) = \sigma_0 R - \frac{\pi R}{3L_t^2} + \frac{1}{L_t} \log \frac{2R}{L_t} + \frac{2}{L_t} \sum_{n=1}^{\infty} \log \left( 1 - e^{-\pi n R/L_t} \right)$$

Therefore at fixed temperature $1/L_t$ and asymptotically for large $R$ we have a temperature dependent string tension

$$\sigma(L_t) = \sigma_0 - \frac{\pi}{3L_t^2}$$

and a logarithmic term, plus exponentially suppressed subleading terms. It is perhaps worth stressing that there is no $1/R$ term in this regime.

We want to compare the prediction Eq. (12) with Monte Carlo data for quenched QCD at finite temperature. First let us stress some important points:

3The Lüscher term $\pi/12R$ appears in the opposite limit $2R \ll L_t$, as can be seen by rewriting Eq. (11) in the other equivalent form

$$V(R) = \sigma_0 R - \frac{\pi}{12R} + \frac{2}{L_t} \sum_{n=1}^{\infty} \log \left( 1 - e^{-\pi n L_t/R} \right)$$
1. The free string picture we are using is an effective *infrared* description: we expect it to describe the long distance behavior of the potential (for a detailed discussion of this point in the zero temperature case see Ref. [15]). Therefore we expect to find a minimum distance $R_{\text{string}}$ above which Eq. (12) describes the data accurately. Obviously there will be a distance $R_{\text{linear}}$ above which the data are well described also by the simple linear behavior $V \sim \sigma R$. The string model will be confirmed if $R_{\text{string}}$ is sensibly smaller than $R_{\text{linear}}$.

2. The string model gives the finite temperature potential in terms of the zero temperature string tension $\sigma_0$. Therefore the result of the fit of the finite temperature potential should be compared to the zero temperature string tension to confirm the string picture.

3. The free string picture must break down at temperatures near the deconfinement point where string interactions are believed to become important (see *e.g.* Ref. [19]). Therefore we need to use lattice data at temperatures not too close to the critical one.

4. The main effect of string interactions is to renormalize the value of the string tension $\sigma_0$ while leaving the functional form Eq. (12) approximately unchanged. Therefore we expect string interactions to introduce a systematic error in the evaluation of the zero temperature string tension from finite $T$ data.

Very precise data were obtained by the authors of Ref. [10] at three temperatures, with $T/T_c \sim 0.8$, 0.88 and 0.94. The first of these data samples is ideal for our purpose. We fitted those data to Eq. (11) and, for comparison, to a simple linear behavior $V \sim \sigma R$. Notice that these are both two parameter fits: the string contribution to the potential does not contain any adjustable parameters. Let us define $R_{\text{string}}$ as the value of $R$ above which the reduced $\chi^2$ of the fit is $< 1$, and $R_{\text{linear}}$ as the corresponding distance for the linear fit. The data are taken at $\beta = 3.95$ and $L_t = 4$. We obtain, in terms of the lattice spacing $a$,

$$R_{\text{string}} = 3.3a$$  \hspace{1cm} (14)

with $\chi^2 = 0.74$, and

$$R_{\text{linear}} = 4.5a$$  \hspace{1cm} (15)

If we tried to fit the data for $R > R_{\text{string}}$ to $V = \sigma R$ we would obtain $\chi^2 = 8.6$. 
The fit with the string model potential and $R > R_{\text{string}}$ gives

$$\sigma_0 = 0.22a^{-2}$$

(16)

This should be compared with the string tension at zero temperature at the same value of the coupling $\beta = 3.95$. Extrapolating the data of Ref. [7] to our $\beta$ we find $\sigma_0 = 0.24(1)a^{-2}$: we conclude that the systematic error discussed above is still rather small at $T/T_c = 0.8$ (the statistical error on $\sigma_0$ given by our fit is two order of magnitude smaller and hence not very meaningful).

It is interesting to express $R_{\text{string}}$ in physical units: using

$$\sigma_0 = (420 \text{ MeV})^2 = 4.41 \text{ fm}^{-2}$$

(17)

we obtain

$$R_{\text{string}} \sim 0.75 \text{ fm}$$

(18)

while using the string tension as the length scale

$$\sigma_0 R_{\text{string}}^2 \sim 2.4$$

(19)

If one studies the data samples closer to the critical temperature, at $T/T_c \sim 0.88$ and 0.94, one clearly sees the effect of string interactions discussed above: while the fit of the potential with the free string model potential is always very good, the systematic error in the determination of $\sigma_0$ increases: At $T/T_c \sim 0.88$ the fit to Eq. (12) gives $\sigma_0 \sim 0.18a^{-2}$ when the Wilson loop value is [17] 0.197(8)$a^{-2}$, and for $T/T_c \sim 0.94$ we obtain $\sigma_0 \sim 0.13a^{-2}$ instead of 0.173(5)$a^{-2}$.

## 5 String breaking

In this section we compare the prediction Eq. (10) to lattice data taken by the authors of Ref. [10] in finite temperature QCD with two flavors of staggered dynamical quarks. For the static potential Eq. (10) predicts, for $d = 4$,

$$V(R) = -\frac{1}{L_t} \log \langle P(0)P^+(R) \rangle =$$

$$-\frac{1}{L_t} \log \left\{ c_1 \frac{e^{-\sigma_0 R L_t}}{\eta(\tau)^2} + (N_c N_f)^2 c_2 e^{-2\sigma_0 R_o L_t} \left[ \frac{\eta(\tau_0)}{\eta(\tau_0/2)} \right]^4 \right\}$$

$$= -\log \left[ \frac{e^{-\sigma_0 R L_t}}{\eta(\tau)^2} + (N_c N_f)^2 c(R_o, L_t) \right] + A$$

(20)
where $A = - \frac{1}{L_t} \log c_1$ and

$$c(R_o, L_t) = \frac{c_2}{c_1} e^{-2\sigma_0 R_o L_t} \left[ \frac{\eta(\tau_0)}{\eta(\tau_0/2)} \right]^4$$  \quad \text{(21)}$$

The predictive content of the model lies in the dependence of $c$ on $L_t$. To verify this prediction we fit the Monte Carlo data for the static potential to the expression

$$V(R) = - \log \left[ \frac{e^{-\sigma_0 R L_t}}{\eta(\tau)^2} + (N_c N_f)^2 c \right] + A$$  \quad \text{(22)}$$

and verify that the dependence of $c$ on $L_t$ is actually described by Eq. (21).

We used data taken by the authors of Ref. [10] at $L_t = 4$ and four values of $\beta$ ranging from $\beta = 5.1$ to $\beta = 5.28$. The results of the fits of the four data samples to Eq. (21) are reported in Tab. 1, in lattice units:

| $\beta$ | $\sigma_0 a^2$ | $(N_c N_f)^2 c$ | $\chi^2$ |
|----------|---------------|----------------|---------|
| 5.10     | 0.594(80)     | 0.0151(68)     | 1.58    |
| 5.20     | 0.475(27)     | 0.0297(47)     | 1.11    |
| 5.25     | 0.407(19)     | 0.0653(72)     | 1.02    |
| 5.28     | 0.308(21)     | 0.189(25)      | 0.21    |

Table 1: Fit of the lattice static potential to Eq. (21) for the four values of $\beta$ considered

To verify Eq. (21) we need to trade the $\beta$ dependence at fixed $L_t$ for a temperature dependence, by determining the $\beta$-dependent lattice spacing $a(\beta)$. We chose to use the values of $\sigma_0$ derived from our fit to evaluate $a(\beta)$ (remember that $\sigma_0$ in Eq. (10) is the zero temperature string tension). Therefore fixing $\sigma_0$ using Eq. (17) we obtain the lattice spacing at the four $\beta$ values and therefore the inverse temperature in fm$^{-1}$. Using $T_c/\sqrt{\sigma_0} = 0.436(8)$ we can then determine the ratio $T/T_c$ for each $\beta$. These values are reported in Tab. 2. With the exception of the highest temperature, for which we cannot trust the free string picture for the reasons mentioned in the previous section, the values of $T/T_c$ are in good agreement with the ones quoted in Ref. [10].

Now we can compare our results with Eq. (21). The latter contains two free parameters, namely the length scale $R_o$ and the ratio $c_2/c_1$. It is clearly safer to disregard the highest temperature point; unfortunately in this way
| $\beta$ | $L_t$ (fm) | $T/T_c$   |
|--------|-----------|----------|
| 5.10  | 1.468(96) | 0.744(52) |
| 5.20  | 1.313(28) | 0.832(28) |
| 5.25  | 1.215(28) | 0.899(28) |
| 5.28  | 1.057(36) | 1.033(39) |

Table 2: *Inverse temperature in fm and the ratio $T/T_c$ as computed from the values of $\sigma_0$ for the four beta values.*

we have to perform a two parameter fit of three data. Therefore the best we can hope at present is to show that the Monte Carlo data are compatible with our model, while more data at temperatures not too close to $T_c$ would be needed for a more conclusive test of the model.

The fit of the three values of $c$ with Eq. (21) gives a satisfactory reduced $\chi^2$ of 0.64. The length scale $R_o$ is

$$R_o = 0.71(16) \text{ fm} \quad (23)$$

The fact that this length scale turns out of the same order of magnitude as the typical string breaking scale is of course encouraging. The constant $\frac{c_2}{c_1}$ is affected by an error which is bigger than the value of the constant itself: $\frac{c_2}{c_1} = 4.2(6.4)$.

If we push the model beyond its natural limits and use the parameters given by our fit to predict the value of $c$ at the highest temperature, we obtain for $\beta = 5.28$ the value $(N_f N_c)^2 c = 0.20$, again in good agreement with the numerical result. Even if this last result must be taken with all the necessary caution, it is nevertheless a significant hint that the model is realistic.

We can conclude that the free string model of string breaking Eq. (10) is compatible with lattice result for finite temperature QCD, and certainly reproduces at least the qualitative features of the phenomenon. More data at moderate temperatures are certainly needed to draw a definite conclusion about the correctness of the model.

### 6 Concluding remarks

In this paper we conjectured that the world sheet swept by the confining string in presence of matter fields belongs to the *normal* phase, characterized by a large number of microscopic holes produced by the matter fields, whose
net effect is just to renormalize the string tension. This must be contrasted with the tearing phase, in which large holes dominate and drive the string tension between static sources to zero.

This conjecture has two main consequences, both amenable to numerical verification:

- String breaking cannot be detected by studying the large distance behavior of Wilson loops: even in presence of dynamical matter fields the Wilson loop expectation value should behave at large distance like in the quenched case, that is according to Eq. (1).

- On the contrary, when one considers operators made of disjoint static sources a simple topological argument shows that even if the world sheet is in the normal phase string breaking can be easily detected.

Both of these are in agreement with most of the rather large body of lattice data on the subject that has been published recently [1] -[10]. The only exception is (2+1)-dimensional SU(2) theory with two flavors of staggered fermions, where string breaking effects in the Wilson loop have been observed [5]. We suggest that the mass of the quarks used in Ref. [6] is large enough to make the world sheet belong to the tearing phase. Indeed, the very same theory does not show any sign of string breaking in the adjoint Wilson loop [6, 9] where the “matter” is massless by definition.

Based on this conjecture, we have proposed an effective string model of string breaking for the simplest example of an operator constructed out of disjoint sources: the correlator of Polyakov loops in finite temperature QCD. While the predictions of this model appear to be compatible with the lattice data, more data would be needed for a conclusive assessment of its validity.

One can envisage a number of numerical experiments to further check the above conjecture and the effective string model. Perhaps the simplest could be the analysis of the Polyakov correlator in the adjoint representation at finite temperature. In this case one can work directly in the pure Yang Mills model and test accurately Eq. (10) as we did for Eq. (4) in the quenched case.

Also, it would be interesting to verify that the Wilson loop in presence of dynamical quarks actually behaves like in the quenched case, that is according to Eq. (1). It is however important to note that fuzzed Wilson loops should be avoided in this kind of check, because the string quantum fluctuations produce a strong shape dependence which is out of control in fuzzed loops. The \( \eta \) function of Eq. (1) accounts for these fluctuations only.
in the case of a rectangular shape. Likewise, analyses based only on the
asymptotic form of the potential given in Eq. (3) should be taken with
cautions, because the logarithmic correction generated by the $\sqrt{R}$ term is of
the same order of magnitude as the Lüscher term within the lattice size of
current simulations.

Acknowledgements We would like to thank the authors of Ref. [10] for
kindly providing us with their data, and M. Caselle, M. Hasenbusch, A.
Lerda and S. Vinti for useful discussions.

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