Killing–Yano equations and $G$ structures

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Abstract
We solve the Killing–Yano equation on manifolds with a $G$ structure for $G = SO(n), U(n), SU(n), Sp(n) \cdot Sp(1), Sp(n), G_2$ and $Spin(7)$. Solutions include nearly-Kähler, weak holonomy $G_2$, balanced $SU(n)$ and holonomy $G$ manifolds. As an application, we find that particle probes on AdS$_4 \times X$ compactifications of type IIA and 11-dimensional supergravity admit a $W$ type of symmetry generated by the fundamental forms. We also explore the $W$ symmetries of string and particle actions in heterotic and common sector supersymmetric backgrounds. In the heterotic case, the generators of the $W$ symmetries completely characterize the solutions of the gravitino Killing spinor equation, and the structure constants of the $W$ symmetry algebra depend on the solution of the dilatino Killing spinor equation.

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1. Introduction

It has been known for sometime that spacetime forms of various degrees generate symmetries in particle and string supersymmetric worldvolume actions. In string theory, invariance of the worldsheet action requires that these forms are parallel with respect to a suitable connection leading to special holonomy manifolds [1]. In particular, the covariant constant forms of Berger manifolds are associated with the $W$ type of symmetries. In string theory the $W$ type symmetries are part of the chiral symmetry algebra and so are essential for the understanding of quantum theory [2–4].

The conditions for the invariance of the action of supersymmetric particles under symmetries generated by spacetime forms are somewhat different. To describe these symmetries, let $X$ be a superfield which is a map from the worldline supermanifold $\Xi^{[1]}$, with coordinates $(t, \theta)$, into the spacetime $M$. The transformation generated by a spacetime $(l + 1)$-form $\lambda$ is

$$\delta X^i = a_0 \lambda^i_{\gamma_1 \ldots \gamma_l} DX^{\gamma_1} \cdots DX^{\gamma_l},$$

where the index is raised using the spacetime metric $g$ and $a_0$ is an infinitesimal parameter. $D$ is the worldline superspace derivative $D^2 = i \partial_t$. Requiring that the worldline action$^1$ of [5]

$^1$ There are particle actions with one supersymmetry and additional couplings which can be found in [6].
written in superfields,

\[ I = -\frac{i}{2} \int dt \, d\theta g_{ij} \, DX^j \theta_i, \]  

(1.2)

be invariant under (1.1), one finds that the covariant derivative of the form \( \lambda \) coincides with the exterior derivative [7], \( \nabla \lambda = (l + 2)^{-1} d\lambda \), or explicitly,

\[ \nabla_{i_1 \ldots i_{l+2}} \lambda_{i_2 \ldots i_{l+2}} = \nabla_{[i_1 \ldots i_{l+2}]} \lambda_{i_2 \ldots i_{l+2}}. \]  

(1.3)

This condition is known as the Killing–Yano equation [8]. It has been extensively investigated in the context of black holes in relation to the integrability of the geodesic motion [9–11], and in relation to the separation of Klein–Gordon and Dirac equations [12–14], see also [15]. It is clear that if \( \lambda \) is a 1-form, then (1.3) implies that the associated vector field is Killing.

In this paper, we initiate the investigation of the question of whether the conditions imposed on the geometry of a background by the requirement of spacetime supersymmetry can be understood in terms of the conditions for the existence of symmetries in the worldvolume theories of particle and string probes. This will give an interpretation of the conditions for spacetime supersymmetry in terms of conditions for existence of worldvolume Noether symmetries, and so it may lead to an understanding of spacetime supersymmetry in terms of algebras.

It is implicit in the results of [1] that the geometry of supersymmetric supergravity backgrounds associated with a holonomy \( G \) (Berger manifolds), or equivalently a parallel \( G \) structure, can be encoded in terms of a \( \mathcal{W} \) algebra of a particle or a string worldvolume action. However supersymmetric backgrounds with fluxes exhibit \( G \) structure which are not parallel. For this, we solve (1.3) on manifolds that admit a \( G \) structure for \( G = SO(n)(n), U(n)(2n), SU(n)(2n), Sp(n) \cdot Sp(1)(4n), Sp(n)(4n), G_2(7) \) and \( Spin(7)(8) \), where in parentheses is the dimension of the manifold. This list includes most of the \( G \) structures of supersymmetric supergravity backgrounds. Assuming that \( \lambda \) is one of the fundamental forms of these manifolds, we shall solve (1.3). We shall show that in most cases (1.3) implies that \( \lambda \) is parallel with respect to the Levi-Civita connection, i.e. the manifold is of holonomy \( G \). However, there are some exceptions. In particular, we find that nearly Kähler, nearly parallel (weak) \( G_2 \) and balanced \( SU(n) \) manifolds arise as solutions. The results have been tabulated in tables 1, 2 and 3 for the \( U(n), SU(n) \) and \( G_2 \) cases, correspondingly. The remaining are stated at the appropriate section.

Next, we examine the relation between the geometric conditions that arise for the existence of \( \mathcal{W} \) symmetries in worldvolume actions and those imposed on a spacetime by the requirement of spacetime supersymmetry. In the heterotic case, there is a \( \mathcal{W} \) type of symmetry for every form constructed as a bilinear of the spinors that solve the gravitino Killing spinor equation. These forms completely characterize the solutions of gravitino Killing spinor equation. So there is a relation between the generators of \( \mathcal{W} \) algebra of a probe particle or string and the solutions of the gravitino Killing spinor equation. The dilatino Killing spinor equation imposes conditions on the generators of the \( \mathcal{W} \) algebra that appear on the right-hand side of commutators of the symmetries generated by the parallel forms. These are typically vanishing conditions of the Nijenhuis type of tensors. Similar observations are made for common sector backgrounds. In type II and 11-dimensional supergravity, nearly Kähler and nearly parallel \( G_2 \) manifolds appear in \( AdS_4 \times X \) compactifications, respectively. One therefore concludes that particle probes on such backgrounds admit \( \mathcal{W} \) symmetries.

This paper is organized as follows: in section 2, we solve the Killing–Yano equation on manifolds with a \( G \) structure, and in section 3, we explore the applications in the context of supersymmetric supergravity backgrounds.
### Table 1

| Symmetries | Conditions | Geometry     |
|------------|------------|--------------|
| $\omega$  | $W_2 = W_3 = W_4 = 0$ | Nearly Kähler |
| $\omega^k$ | $1 < k < n$ | $W_1 = W_2 = W_3 = W_4 = 0$ | Kähler |

#### 2. Solving the Killing–Yano equations

One way to solve (1.3) is to use the same technique as that one employs to classify $G$ structures [16]. For all the groups $G$ that are mentioned in the introduction, the Lie algebra of $G$, $\mathfrak{g} \subseteq \mathfrak{so}(d) = \Lambda^2(\mathbb{R}^d)$, where $d = \dim M$ is the dimension of the manifold. One can then write $\Lambda^2(\mathbb{R}^d) = \mathfrak{g} \oplus \mathfrak{g}^\perp$ and decompose the Levi-Civita connection as

$$\nabla = \pi(\nabla) + \sigma(\nabla) \quad (2.1)$$

where $\pi(\nabla)$ takes values in $\mathfrak{g}$ and $\sigma(\nabla)$ in $\mathfrak{g}^\perp$. If $\lambda$ is one of the fundamental forms of the $G$ structure, and so $G$ invariant, $\pi(\nabla)\lambda = 0$. Thus (1.3) does not depend on the $\pi(\nabla)$ components of the connection. So (1.3) turns to a condition on $\sigma(\nabla)$.

In practice, to carry out these calculations it is convenient to introduce an adapted frame to the $G$ structure. Then in most cases, it is straightforward to identify the components of the frame connection $\Omega$ in $\mathfrak{g}$ and those in $\mathfrak{g}^\perp$, and observe that (1.3) depends only on the latter. The conditions that (1.3) imposes on $\sigma(\Omega)$ can then be investigated in each case.

If $G = SO(n), d = n$, the fundamental form is the volume form of the Riemannian metric. This form is parallel with respect to the Levi-Civita connection and so (1.3) is satisfied without further restrictions.

2.1. $U(n)$

Let $M, d = 2n$, be a manifold with structure group $U(n)$. The fundamental form is the Hermitian form $\omega(X, Y) = g(X, IY)$ of an almost complex structure $I$ with compatible Hermitian metric $g$. In this case $\lambda$ can be identified with $\omega$ or its skew-symmetric powers $\omega^k = \wedge^k \omega$.

To identify $\sigma(\Omega)$ in this case choose a Hermitian frame, i.e. $g = \delta_{ij} e^i e^j = 2\delta_{ab} e^a e^b$ and $\omega = -i\delta_{ab} e^a e^b$. Then the $\sigma(\Omega)$ components of the frame connection $\Omega$ are $\Omega_{\alpha, \alpha\beta}$ and their complex conjugates. Observe that (1.3) does not depend on the components $\Omega_{\alpha, \alpha\beta}$ which are along $\mathfrak{u}(n)$. The different $U(n)$ structures can be found by decomposing $\sigma$ into four irreducible representations $W_1, \ldots, W_4$—the Gray–Hervella classes [16]. After some computation one finds that the invariance condition (1.3) implies that all these classes vanish apart from $W_1$, i.e. $W_2 = W_3 = W_4 = 0$. The Nijenhuis tensor $N(I)$ of such manifolds with a $U(n)$ structure does not vanish. In particular, it is a $(3,0)$ and $(0,3)$, and $\nabla$ parallel form. $M$ is a nearly Kähler manifold. If $\dim M = 4$, then $M$ is Kähler. If $\dim M = 6$ and $M$ is not Kähler, then the structure group reduces to $SU(3)$. An example of such a manifold is $S^6$. Similarly for $\dim M \geq 6$ and $M$ not Kähler, the structure group reduces to a proper subgroup of $U(n)$.

Next if $\lambda = \omega^k, 1 < k < n$ a rather lengthy but straightforward computation reveals that (1.3) implies that $M$ is Kähler. The conditions on the geometry of $M$ are summarized in table 1.
Table 2. The columns give the forms that are associated with the symmetries and the conditions for the transformations to leave the action invariant, respectively.

| Symmetries         | Conditions                        | Geometry           |
|--------------------|-----------------------------------|--------------------|
| \( \omega \)       | \( W_2 = W_3 = W_4 = 0 \)        | Nearly Kähler      |
| \( a^k \) \( 1 < k < n \) | \( W_1 = W_2 = W_3 = W_4 = 0 \) | Kähler             |
| \( \text{Re} \chi, \text{Im} \chi \) | \( W_4 = W_5 = 0 \) | Balanced Hermitian |
| \( a^k \) \( 1 < k < n \), \( \text{Re} \chi, \text{Im} \chi \) | \( W_1 = W_2 = W_3 = W_4 = W_5 = 0 \) | Special nearly Kähler |
| \( \text{Calabi–Yau} \) | |                   |

2.2. \( SU(n) \)

The fundamental \( SU(n) \) forms are the Hermitian form \( \omega \), as in the \( U(n) \) case, and the real, \( \text{Re} \chi \), and imaginary, \( \text{Im} \chi \), parts of a \((n,0)\)-form \( \chi \). Adapting a Hermitian frame to the \( SU(n) \) structure, \( \sigma \) is spanned by the component \( \Omega_{a}^a \) of the frame connection in addition to those described for the \( U(n) \) case. In fact \( \sigma \) is now decomposed in five \( SU(n) \) irreducible representations \( W_1, \ldots, W_5 \)—the \( SU(n) \) classes [17].

The investigation of the symmetries generated with the Hermitian form \( \omega \) and its skew-symmetric powers is identical to that we have done for the \( U(n) \) case. The conditions on the geometry are as those of the \( U(n) \) case above.

In the \( SU(n) \) case, additional transformations can be constructed from the real \( \text{Re} \chi \) and imaginary \( \text{Im} \chi \) components of the \((n,0)\)-form \( \chi \). For these transformations to be symmetries of action (1.2), the invariance condition (1.3) implies that \( W_4 = W_5 = 0 \) provided that \( n > 2 \). The \( SU(n) \) classes have been chosen as in [18]. For \( n = 1 \), the analysis is identical to the \( Sp(1) \) case that we shall explain below. The results are summarized in table 2.

2.3. \( Sp(n) \cdot Sp(1) \)

The \( Sp(n) \cdot Sp(1) \) fundamental is

\[
\chi = \wedge^2 \omega_I + \wedge^2 \omega_J + \wedge^2 \omega_K,
\]

where \( \omega_I, \omega_J \) and \( \omega_K \) are locally defined Hermitian forms associated with a quaternionic structure \( I, J \) and \( K \), \( K = IJ \).

To solve (1.3) for \( \lambda = \chi \), we adapt a Hermitian frame with respect to the \( I \) almost complex structure. Then \( \alpha_I \) becomes a locally defined \((2,0)\)- and \((0,2)\)-form. The components of frame connection along \( \sigma \) are spanned by \( \hat{\Omega}_{i,a}^\alpha \) and \( A\hat{\Omega}_{i,a}^\alpha \), where \( \hat{\Omega}_{i,a}^\alpha \) is the \( \omega_J \) traceless part of \( \Omega_{i,a}^\alpha \) and \( A\hat{\Omega}_{i,a}^\alpha \) is the component of \( \Omega_{i,a}^\alpha \) which satisfies

\[
A\hat{\Omega}_{i,a}^\alpha J^\beta J^\gamma = -A\hat{\Omega}_{i,a}^\alpha J^\gamma J^\beta , \quad A\hat{\Omega}_{i,a}^\alpha = 0.
\]

The components of \( \nabla \) along the \( \mathfrak{sp}(n) \oplus \mathfrak{sp}(1) \) directions are spanned by the remaining components of the frame connection. In particular the \( \mathfrak{sp}(1) \) directions are spanned by the \( \omega_J \) trace of \( \Omega_{i,a}^\alpha \) and the \( \omega_I \) trace of \( \Omega_{i,a}^\beta \). It is known that if \( n > 2 \), \( \sigma \) decomposes in six \( Sp(n) \cdot Sp(1) \) irreducible representations, and that if \( n = 2 \), \( \sigma \) decomposes in four irreducible representations [19]. It is also known that all these classes are determined by evaluating \( \nabla \chi \).

The invariance condition (1.3) imposes conditions on \( \sigma \), i.e., on \( \hat{\Omega}_{i,a}^\alpha \) and on \( A\hat{\Omega}_{i,a}^\beta \). After a long but straightforward computation, one finds that (1.3) implies that \( \sigma \) vanishes. Thus \( \chi \) is parallel and invariance of the action implies that \( M \) is a quaternionic Kähler manifold.
Table 3. The columns give the forms that are associated with the symmetries and the conditions for the transformations to leave the action invariant, respectively.

| Symmetries | Conditions | Geometry       |
|------------|------------|----------------|
| ϕ          | X₂ = X₃ = X₄ = 0 | Nearly Parallel |
| ⋆ϕ         | X₁ = X₂ = X₃ = X₄ = 0 | Holonomy G₂    |
| ϕ, ⋆ϕ      | X₁ = X₂ = X₃ = X₄ = 0 | Holonomy G₂    |

2.4. \(Sp(n)\)

The \(Sp(n)\) fundamental forms are the Hermitian forms \(ω_I, ω_J\) and \(ω_K\) of an almost hypercomplex structure \(I, J\) and \(K, K = IJ\). The conditions for the transformations generated by \(∧^kω_I, ∧^kω_J\) and \(∧^kω_K\), separately, to be symmetries are exactly as those we have found for the \(U(n)\) case associated with \(I\) or \(J\) or \(K\) almost complex structures.

We shall also investigate the conditions that are implied by taking \(λ = ω_I\) and \(ω_J\). For this we introduce a Hermitian frame with respect to \(I\) as in the \(Sp(n)\) case above.

However in this case, the components of the frame connection along \(σ\) are spanned by \(Ω_{αβ}^{1,i}\) and \(Ω_{α}^{1,i,β}\) and the components along the \(sp(1)\) directions, i.e. they are spanned by \(Ω_{αβ}^i\) and \(Ω_{αβ}^i\). For \(ω_I\) to satisfy (1.3), the non-vanishing components of the frame connection are \(Ω[α,β\iota]\). It remains to find the additional conditions that (1.3) imposes for \(λ = ω_I\) and \(λ = ω_J\) implies that \(M\) is a hyper-Kähler manifold.

2.5. \(G_2\)

The transformations are generated by either the \(G_2\) fundamental 3-form \(ϕ\) or its dual \(⋆ϕ\), or both. It is known that \(Λ^2(\mathbb{R}^7)\) decomposes in \(G_2\) representations as \(Λ^2(\mathbb{R}^7) = g_2 \oplus 7\). So the \(σ\) directions of the Levi-Civita connection are along the seven-dimensional representation. Adapting an appropriate frame, the \(σ\) directions of the frame connection can be written as

\[
σ(Ω)^{i,jk} = L_{im}ψ^{m,jk}.
\]

It is known that the \(G_2\) structures on a seven-dimensional manifold can be characterized by four classes [20]. Schematically, one has

\[
\nabla ϕ \iff X₁ + X₂ + X₃ + X₄
\]

where \(X₁\) is a singlet, \(X₂\) is in the 14 representation, \(X₃\) is in the 27 representation and \(X₄\) is in the 7 representation.

As we have explained (1.3) depends only on the \(σ\) component (2.4) of the frame connection. After some computation, one can show that \(ϕ\) solves (1.3), iff \(L_{ij} = fδ_{ij}\). This in turn implies that \(X₂ = X₃ = X₄ = 0\), i.e. \(M\) is nearly parallel or weak \(G_2\) manifold. For such \(G_2\) manifolds

\[
dϕ = X₁ ⋆ϕ.
\]

A similar computation reveals that \(⋆ϕ\) generates a symmetry iff \(M\) is holonomy \(G_2\). The results have been tabulated in table 3.

2.6. \(Spin(7)\)

The transformation in this case is generated by the fundamental \(Spin(7)\) self-dual 4-form \(ϕ\). Observe that (1.3) implies that \(ϕ\) is co-closed. Since \(ϕ\) is self-dual is also closed. In turn, (1.3)
implies that it is also parallel. Therefore $\phi$ generates a symmetry iff $M$ is a holonomy $\text{Spin}(7)$ manifold.

3. Supersymmetric backgrounds and $\mathcal{W}$ symmetries

3.1. Heterotic and common sector backgrounds

The geometry of all supersymmetric heterotic string backgrounds has been described in [21]. Such backgrounds admit $\tilde{\nabla}$ parallel forms of various degrees. These are constructed as form bilinears of the spinors $\tilde{\epsilon}$ that solve the gravitino Killing spinor equation $\tilde{\nabla}\tilde{\epsilon} = 0$, where $\tilde{\nabla}$ is a metric connection with skew-symmetric torsion$^2$. These forms have been given in [21].

The relevant part of heterotic string worldsheet Lagrangian is

$$L = (g + b)_{ij} D_i X^j \partial_{\sigma} X^i \quad (3.1)$$

where $X$ is a $(1,0)$ superfield. This Lagrangian apart from the metric coupling also contains a Wess–Zumino term. The transformation generated by a spacetime form $\lambda$ is

$$\delta X^i = a_{\lambda}^{i, j_1 \cdots j_l} D_{j_1} X^{j_2} \cdots D_{j_l} X^j. \quad (3.2)$$

This transformation leaves the action invariant, iff $\lambda$ is $\tilde{\nabla}$ parallel, $\tilde{\nabla}\lambda = 0$ [1]. Thus all forms constructed as $\tilde{\nabla}$ parallel spinor bilinears generate worldsheet $\mathcal{W}$ symmetries. So in this case there is a direct correspondence between the conditions on the spacetime that arise from the solution of the gravitino Killing spinor equation and those that arise for the existence of $\mathcal{W}$ symmetries in string worldsheet actions.

The commutator of two $\mathcal{W}$ symmetries (1.1) generated by $\lambda$ and $\mu$ has been given in [1]. Typically, the $\mathcal{W}$ algebra does not close on the original transformations of the form (3.2), and new generators can arise. Some of these new generators are transformations of type (3.2) associated with the Nijenhuis tensor $N(\lambda, \mu)$. The dilatino Killing spinor equation of the heterotic string imposes restrictions on these Nijenhuis tensors [21]. In particular in many cases, they are required to vanish. As a result, the conditions imposed by the dilatino Killing spinor equations can be interpreted as restrictions on the structure constants of the $\mathcal{W}$ algebra.

A similar analysis can be made for the supersymmetric backgrounds of the common sector, see also [1, 21]. The geometry of supersymmetric common sector backgrounds is not as well understood as that of the heterotic ones. For example the gravitino Killing spinor equation$^3$, which is two copies of the heterotic one, $\tilde{\nabla}\tilde{\epsilon} = 0$ and $\tilde{\nabla}\tilde{\epsilon} = 0$, has not been solved in all cases for more than two supersymmetries [18]. The analysis for the form bilinears constructed for either the $\tilde{\epsilon}$ or the $\tilde{\epsilon}$ parallel spinors is the same as that already given for the heterotic string. Supersymmetric common sector backgrounds admit additional form bilinears constructed from a $\tilde{\epsilon}$ and a $\tilde{\epsilon}$ parallel spinor. Such bilinears are not parallel with respect to either $\tilde{\nabla}$ or $\nabla$ connections. Requiring that these bilinears generate worldsheet $\mathcal{W}$ symmetries will impose additional conditions on the geometry of spacetime to those associated with gravitino Killing spinor equation.

The relevant worldline action suitable to investigating $\mathcal{W}$ symmetries for a particle in a heterotic or common sector background is that of [6] which contains an additional skew-symmetric coupling for the fermions. One can show that forms that are parallel with respect to a connection with skew-symmetric torsion generate $\mathcal{W}$ symmetries for the worldline action. The analysis is similar to that for the heterotic string and the common sector above.

$^2$ For conventions and notation see [18].

$^3$ We follow the notation of [18].
3.2. Type II backgrounds

The particle and string actions that probe generic type II backgrounds are of Green–Schwarz type. So the fermions are spacetime spinors rather than worldvolume spinors which transform as spacetime vectors, i.e. as those in (1.2) and (3.1) which we have used to construct the $W$ symmetries. Green–Schwarz actions can be rewritten in terms of worldvolume fermions after a gauge-fixing procedure to eliminate additional degrees of freedom, see e.g. [22]. However, this is done on a case-by-case basis and there is not a general procedure for generic supersymmetric type II backgrounds. So we cannot directly compare the geometry of supersymmetric type II backgrounds and the conditions that arise from the invariance of worldvolume actions under transformations generated by spacetime forms.

Nevertheless supersymmetric type II and 11-dimensional supergravity backgrounds admit forms of various degrees constructed from Killing spinor bilinears, see e.g. [23–25]. From the results of section 2 whenever the backgrounds are of holonomy $G$, the $W$ symmetries have been described in [1]. For the remaining cases, it is known that some $AdS_4 \times X$ compactifications of 11-dimensional supergravity require that $X$ be a nearly parallel $G_2$ manifold, [26], see also [27]. This condition coincides with the condition given in table 3 for the existence of a $W$ symmetry. Similarly, $AdS_4 \times X$ compactifications of IIA supergravity require that $X$ is a nearly Kähler manifold which coincides with the condition for the existence of a $W$ symmetry in tables 1 and 2. Therefore, we conclude that particle probes of such compactification exhibit $W$ symmetries.

In general, the conditions that one obtains from solving the Killing spinor equations of type II and 11-dimensional backgrounds are, so far, less stringent from those obtained by requiring invariance of worldvolume actions under a transformation generated by a spacetime form. This may change provided that the $W$ symmetries are formulated directly for Green–Schwarz actions. Consideration should also be given to probes whose dynamics is described in terms of field equations. In such a case, it has been shown that the requirement of worldvolume supersymmetry puts less restriction on the spacetime geometry [28]. For the particle case, action (1.2) that we have used for the analysis can be extended to include an additional 3-form and other couplings [6]. Then the conditions for the existence of $W$ symmetries change, see [29, 30]. In particular, they lead to a modification of Killing–Yano equations. The restrictions that these modified conditions impose on the geometry and their relation to those found by the requirement of spacetime supersymmetry on a spacetime will be investigated elsewhere.

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