A grey γ-ray transfer procedure for supernovae

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Abstract. The γ-ray transfer in supernovae for the purposes of energy deposition in the ejecta can be approximated as grey radiative transfer using mean opacities. In past work there is a single pure absorption mean opacity which is a free parameter. Accurate results can be obtained by varying this mean opacity to fit the results of more accurate procedures. In this paper, we present a grey γ-ray transfer procedure for energy deposition in which there are multiple mean opacities that are not free parameters and that have both absorption and scattering components. This procedure is based on a local-state (LS) approximation, and so we call it the LS grey γ-ray transfer procedure or LS procedure for short.

1. Introduction

A major source of observable supernova luminosity, and in the case of Type Ia supernovae (SNe Ia) virtually the only source, is the decay energy of radioactive species synthesized in the explosion. The overwhelmingly dominant decay chain for the observable period of most supernovae is $^{56}\text{Ni} \rightarrow ^{56}\text{Co} \rightarrow ^{56}\text{Fe}$ with half-lives of 5.9 and 77.27 days for the first and second decays, respectively (Huo 1992). The first decay releases almost all its energy in the form of γ-rays and the second in γ-rays and, in 19% of the decays, in positrons (Browne and Firestone 1986; Huo 1992). Until very late times almost all the positrons deposit their kinetic energy in the ejecta and annihilate to form more γ-rays.

The γ-ray and positron kinetic energy deposited in the ejecta is needed to calculate the supernova ultraviolet-optical-infrared luminosity. This deposited energy is effectively in the form of fast electrons. The usual assumption (which may not be entirely correct [e.g., Colgate et al. 1980; Chan and Lingenfelter 1993; Ruiz-Lapuente 1997; Milne et al. 1997]) is that the positron kinetic energy is deposited locally. For the γ-ray deposition, a γ-ray transfer procedure is needed. A full treatment of the γ-ray transfer requires computationally intensive procedures such as Monte Carlos. But if one just requires the energy deposited in the form of fast electrons without knowing the fast electron spectrum, then a simple, time-independent, static, grey (i.e., frequency-integrated) radiative transfer procedure using a single pure absorption mean opacity can be quite accurate as shown by Colgate et al. (1980), Sutherland and Wheeler (1984), Ambwani and Sutherland (1988), and Swartz et al. (1995, hereafter SSH). (Note there is no practical mean opacity that will reduce γ-ray transfer in supernovae to exact grey radiative transfer. Thus grey supernova γ-ray transfer procedures will always be approximations.)
The mean opacity used in past work is a parameter chosen by the comparison of the grey $\gamma$-ray transfer results to those of more accurate $\gamma$-ray transfer procedures. Only one value being used for all the ejecta. In the particular procedure of SSH, the mean opacity is explicitly varied to obtain optimum accuracy for each supernova epoch by comparison to Monte Carlo results. For the supernova model examined by SSH (SN Ia model W7 [Thielemann et al. 1986]), the optimized SSH procedure obtained an accuracy of a few percent locally and 2% globally. Despite the high accuracy achievable, the SSH procedure is not completely satisfactory because the optimization requires having done the full $\gamma$-ray transfer one wishes to avoid by using a grey $\gamma$-ray transfer procedure.

In this paper we briefly present a time-independent, static, grey $\gamma$-ray transfer procedure with no free parameters that we believe will be generally accurate at least globally to within a few percent. This procedure is based on a local-state (LS) approximation, and so we call it the LS grey radiative transfer procedure or LS procedure for short. A full presentation of the LS procedure is given by Jeffery (1998, hereafter J98).

In § 2 we discuss the opacities and mean opacities that are used in the LS procedure. Section 3 introduces the LS procedure itself. Conclusions are given in § 4.

2. Opacities

In the energy range $0.05\text{–}50\text{ MeV}$, $\gamma$-rays interact with matter principally through three processes: (1) pair production in the Coulomb field of a nucleus or an electron, (2) the photoelectric effect with bound electrons (which is just $\gamma$-ray photoionization of an atom or ion), and (3) Compton scattering off electrons (e.g., Davisson 1965, p. 37). Almost all the $\gamma$-rays from the $^{56}\text{Ni} \rightarrow ^{56}\text{Co} \rightarrow ^{56}\text{Fe}$ decay chain and other decay chains important in supernovae lie in the energy range $0.05\text{–}50\text{ MeV}$ and no $\gamma$-rays exceed $\sim 3.6\text{ MeV}$ in fact (Browne and Firestone 1986; Huo 1992). Thus the three mentioned processes determine $\gamma$-ray opacity in supernovae (see also SSH’s Fig. 1). Of the three opacities Compton opacity is dominant: in solar composition overwhelmingly so; in metal-rich composition (such as in SNe Ia) photoelectric opacity (a pure absorption opacity) is also important (e.g., J98).

Because of the dominance of Compton opacity, one expects scattered $\gamma$-ray fields (viz., 1st, 2nd, 3rd, etc. order scattered fields) to be of some importance alongside the initial (i.e., 0th order scattered) $\gamma$-ray field from the radioactive decays and positron annihilation. Nevertheless earlier grey $\gamma$-ray transfer procedures use a pure absorption mean opacity. The reason this works well is that for $\gamma$-rays of energy typical of the radioactive decays in supernovae a large component of the Compton opacity is nearly-forward, nearly-coherent scattering. From a radiative transfer point of view, this component is effectively almost neglectable. The non-neglectable component of Compton scattering is largely an absorption opacity. By using a pure absorption mean opacity the earlier procedures effectively neglected the nearly-forward, nearly-coherent scattering. This mean opacity incorporated all the absorption opacity encountered by the 0th order field plus an extra absorption component that accounted for the absorption of the relatively weak scattered fields arising from the 0th order non-forward, noncoherent scattering. This extra absorption component is the actual underlying free parameter of the SSH procedure.

For the LS procedure we choose to deal with the scattered fields explicitly albeit crudely. First, we will approximate the 0th order $\gamma$-ray field as a line spectrum.
(The decay γ-ray spectrum is almost exactly a line spectrum and we assume that the positron annihilation can be approximated as always going by the two-\(m_e c^2\)-photon process.) Second, we approximate the Compton opacity by subtracting the nearly-forward, nearly-coherent scattering component and treating the remaining component as isotropic scattering with an angle-independent energy loss. We call this approximated Compton opacity the iso-Compton opacity: see J98 for the exact prescription. The assumption of angle-independent energy loss ensures that the scattered fields also consist of line spectra.

Next we compute γ-ray energy \(E_{i,j}\) and fractional emissivity \(f_{i,j}\) for each order of scattering \(i\) and line \(j\) assuming an infinite, homogeneous, isotropic medium: see J98 for the procedure. We then use the \(E_{i,j}\)'s and \(f_{i,j}\)'s to construct total, absorption, and scattering mean opacities for each order. We adopt emissivity-weighted mean opacities since they offer a good prospect of accurately reducing the γ-ray transfer to grey radiative transfer for the purposes of energy deposition (J98). The mean opacity prescription is

### TABLE 1. Mean opacities for solar and mean model W7 compositions

| Order | \(E_i\) (MeV) | \(\kappa_i\) (cm² g⁻¹) | \(\xi^a_i\) | \(\xi^s_i\) | \(E_i\) (MeV) | \(\kappa_i\) (cm² g⁻¹) | \(\xi^a_i\) | \(\xi^s_i\) |
|-------|--------------|----------------------|----------|----------|--------------|----------------------|----------|----------|
| 0     | 1.24226      | 0.0547               | 0.7870   | 0.2130   | 1.24226      | 0.0318               | 0.7829   | 0.2171   |
| 1     | 0.23486      | 0.1307               | 0.3570   | 0.6430   | 0.23995      | 0.0827               | 0.4393   | 0.5607   |
| 2     | 0.15099      | 0.1637               | 0.2524   | 0.7476   | 0.15294      | 0.1306               | 0.4775   | 0.5225   |
| 3     | 0.11294      | 0.1873               | 0.1976   | 0.8024   | 0.11403      | 0.2006               | 0.5810   | 0.4190   |
| 4     | 0.09074      | 0.2052               | 0.1639   | 0.8361   | 0.09147      | 0.3028               | 0.6826   | 0.3174   |
| 5     | 0.07607      | 0.2194               | 0.1413   | 0.8587   | 0.07660      | 0.4459               | 0.7630   | 0.2370   |

\( ^{56}\text{Co} \)

| Order | \(E_i\) (MeV) | \(\kappa_i\) (cm² g⁻¹) | \(\xi^a_i\) | \(\xi^s_i\) | \(E_i\) (MeV) | \(\kappa_i\) (cm² g⁻¹) | \(\xi^a_i\) | \(\xi^s_i\) |
|-------|--------------|----------------------|----------|----------|--------------|----------------------|----------|----------|
| 0     | 0.53479      | 0.0807               | 0.5911   | 0.4089   | 0.53479      | 0.0494               | 0.6230   | 0.3770   |
| 1     | 0.18845      | 0.1443               | 0.3077   | 0.6923   | 0.19557      | 0.1043               | 0.4762   | 0.5238   |
| 2     | 0.12888      | 0.1749               | 0.2245   | 0.7755   | 0.13679      | 0.1573               | 0.5330   | 0.4670   |
| 3     | 0.09966      | 0.1964               | 0.1797   | 0.8203   | 0.10694      | 0.2298               | 0.6205   | 0.3795   |
| 4     | 0.08180      | 0.2128               | 0.1515   | 0.8485   | 0.08797      | 0.3328               | 0.7054   | 0.2946   |
| 5     | 0.06961      | 0.2259               | 0.1326   | 0.8674   | 0.07468      | 0.4764               | 0.7756   | 0.2244   |

\( ^{56}\text{Ni} \)

**NOTE.**—The solar composition is the solar system composition of Anders and Grevesse 1989. The mean model W7 composition (Thielemann et al. 1986) is the final mean composition after all radioactive species have decayed. \(\mu_e\) is the mean atomic mass per electron (e.g., Clayton 1983, p. 84).
\[ \kappa_i^R = \sum_j \frac{\kappa_{i,j}^R f_{i,j} E_{i,j}}{\sum_j f_{i,j} E_{i,j}}, \]

where \( \kappa_{i,j}^R \) is evaluated at energy \( E_{i,j} \). The superscript \( R \), here and elsewhere in this paper, is a variable that replaces a symbol designating a quantity as related to total (blank), absorption (“a”), or scattering (“s”) opacity. The \( \kappa_{i,j}^R \)'s are the sums of the iso-Compton, photoelectric, and pair production opacities or opacity components.

The mean \( \gamma \)-ray energy for an order is given by

\[ \bar{E}_i = \frac{\sum_j f_{i,j} E_{i,j}}{\sum_j f_{i,j}}. \]

The mean energy is not actually used in the LS procedure, but it is a useful diagnostic. It happens that on first scattering for both solar and metal-rich compositions that the \( {^{56}}\text{Co} \) and \( {^{56}}\text{Ni} \) mean energies are reduced by more than 80% and 60%, respectively. This result (which depends on the iso-Compton opacity approximation) shows that the importance of the nonzero order fields is relatively small.

Values for \( \kappa_i \) and, additionally, \( \bar{E}_i \) and \( \xi_i^R \equiv \kappa_i^R/\kappa_i \) (\( i \in [0, 5] \)) for \( {^{56}}\text{Co} \) and \( {^{56}}\text{Ni} \) in solar and mean model W7 compositions are given in Table 1.

3. The LS procedure

What we want from a grey radiative transfer procedure is the \( \gamma \)-ray energy deposition as function of position. Since supernova density varies widely with position and epoch, it is convenient to measure energy deposition by \( \epsilon_d \), the energy deposited per unit time per unit mass. The LS procedure expression for the energy deposition at a point is

\[ \epsilon_d = 4\pi \kappa_{\text{eff}} a_{\text{eff}} J_0, \]

where \( \kappa_{\text{eff}} \) is the effective absorption opacity and \( J_0 \) is the 0th order frequency-integrated mean intensity (or radiation field). The \( J_0 \) field is calculated from an numerical integral solution of the radiation transfer equation using the 0th order frequency-integrated source function (determined by the radioactive species) and the 0th order mean opacity \( \kappa_0 \).

The expression for \( \kappa_{\text{eff}} \) is

\[ \kappa_{\text{eff}} = \kappa_0 L, \]

where

\[ L = \left( \sum_{i=0}^{k-1} \xi_i \prod_{j=0}^{i-1} \xi_j \zeta_{j+1} \right) + \left( \prod_{j=0}^{k-1} \xi_j \right) \frac{\xi_k}{1 - \xi_k \zeta_k}, \]

is a series accounting for absorption from all orders. The \( \zeta \) factors in equation (4) are defined by

\[ \zeta_i = \oint \frac{d\Omega}{4\pi} \exp \left[ 1 - \exp \left( -\tau_i \right) \right] \approx \frac{1}{2} \left[ \left[ 1 - \exp \left( -\tau_{i,\text{out}} \right) \right] + \left[ 1 - \exp \left( -\tau_{i,\text{in}} \right) \right] \right], \]

where \( \Omega \) is solid angle and \( \tau_i \) is the \( i \)th order mean (total) opacity optical depth from the deposition point to the surface of the supernova. The second expression in equation (5) is a two-stream approximation to the first for spherically symmetric cases: \( \tau_{i,\text{out}} \) and \( \tau_{i,\text{in}} \) are the outward and inward radial \( \tau_i \) values.

We derived equation (4) making the LS approximation for the nonzero order fields by assuming the nonzero order fields could be approximated by their values at the deposition point itself. We also assumed that all the mean opacities from the \( k \)th order on could be approximated by the \( k \)th mean opacity. Using this assumption we summed the contributions from the \( k \)th term on analytically in the \( L \) series. Since most of the deposition comes from the lowest order fields (and thus the lowest order terms in the
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$L$ series), the crude approximation for terms $> k$ is adequate. For the cases we have studied, the difference in global energy deposition between choosing $k$ to be 2 and 5 turned out to be $\lesssim 0.6\%$. We assume $k = 5$ to be our fiducial $k$ value.

In summary, the whole LS procedure consists of doing the numerical $\gamma$-ray transfer for the 0th order field and then evaluating the effective absorption opacities and the energy deposition.

4. Conclusions

In this paper, we have outlined the LS procedure for grey $\gamma$-ray transfer and energy deposition in supernovae. A detailed derivation is given by J98. The LS procedure has the advantage over previous like procedures in that it has no free parameter.

J98 concluded that the LS procedure is modestly more reliable than the SSH procedure sans an optimized mean opacity and estimated that the maximum uncertainty in the LS procedure for global energy deposition may be of the order 6%. An extensive comparison of LS procedure results to those of a more accurate procedure is needed to definitively assess the accuracy of the LS procedure.

A computer code for the LS procedure can be obtained by request from the author.

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