A new non-iterative self-referencing interferometer in optical phase imaging and holographic microscopy, HOLOCAM

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Phase retrieval and imaging phase measurements are fields of intense research. It has recently been shown that phase retrieval from self-referencing interferograms (SRI) can be reformulated leading to a stable, linear equation provided the amplitude of the field is known from prior measurement steps (HOLOCAM). Consequently, the numerical solution thereof is straightforward. This is a big achievement since convergence is otherwise not always guaranteed. Applications are expected in X-ray microscopy, general phase retrieval, holography, tomography and optical imaging.

I. INTRODUCTION

Measuring the phase of the electrical field is of primary importance in many areas such as X-ray microscopy, holographic (phase) imaging and tomography. A vast range of methods is currently discussed [1][2][3][4][5] which proves that the problem is of interest but has not been solved to satisfaction yet.

We will discuss possible applications of the self-referencing linear phase method [6] (HOLOCAM) which tackles current difficulties. The approach is a self-referencing interferometric (SRI) method. Instead of using a reference beam, it employs prior knowledge in the intensity which we do have. On that basis, the problem can be exactly reformulated as a linear equation in the unknown complex electric field. It has been proven that this method is stable for fields which have arbitrary pixel values in amplitude and phase [6]. No stabilizing or smoothing terms are needed. The complex field is obtained by solving a linear matrix equation. The method can be applied to a wide range of 2D and 3D interferometers including lenses, gratings, diffractive optical elements or mirrors. Even interferometers with more than two beams can be used.

In most digital holographic applications the phase is recovered by interference with a reference beam [2]. Although this is a perfect solution mathematically a reference beam must be available. In practice, the light from the illumination (usually a laser) is split into two beams, whereof one beam becomes the reference and the second beam illuminates the object (Fig. 1).

The setup in Figure 1 shows an interferometer with a reference beam. Since the object is part of one of the branches every change of the object, the illumination of the object or the location of the detector actually leads to a path change in the interferometer. Hence, it is sensitive to small path differences up to fractions of a wavelength. Thermal drifts and air fluctuations generate incoherent noise. Dust, scratches, defects produce further coherent noise which is system inherent. It is known that this limits the accuracy of the phase measurement, i.e. it blurs the image [4]. Many remedies have been developed such as active compensation [7] and short exposure times [8].

Yet this is encompassed by further unwanted physical complexity of the system.

The final results cannot be better than the initial data quality. Therefore, attempts are made to get rid of the reference beam. This is particularly addressed in former work on self-referencing interferometry (SRI) [3][4][5]. For SRI, it is vital to disentangle the two fields in the interference signal. For this purpose, a mode-filter synthesized reference beam [5] or a non-linear functional in a lateral shearing type interferometer [3] is used. This is restricted to special applications though.

The HOLOCAM has advantages compared to other known SRI methods: firstly the methodology is very simple and secondly it can be applied to many different interferometers not only to lateral shearing interferometers.

Lastly, the HOLOCAM can be designed as a compact detector box that measures the phase (Fig. 2). All interferometric parts of the system can be sealed in the detector housing. HOLOCAM is an artificial word created from HOLOgraphy and CAMera. The device is holographic since it uses a holographic type evaluation of holograms or interferograms and it measures some-
thing 'holographic', the complex amplitude. The device is also a camera such as a CCD camera which registers the locally resolved light intensity on a flat detector. The HOLOCAM does something similar but the registered quantity is the complex amplitude field. Both the CCD and the HOLOCAM do not need a reference beam.

The HOLOCAM can also make contributions to X-ray tomography. In the early days of physics, X-ray probing was used as a lensless system (Fig. 3). This is the honourful work of Laue, Debye and Scherrer. It is characterized by the measurement of diffraction intensities (Bragg reflections) on a screen. For real samples (in particular non-crystalline samples) the scattering intensities are more complicated including also light intensities besides the distinct Bragg peaks ('diffraction peaks midway between Bragg peaks'). The general view of a X-ray scattering process is a scattered wave emitted from the scatterer (Fig. 4). Like any other optical field, this wave possesses an intensity and a phase.

The purpose of X-ray scattering is the determination of the atomic structure of the scatterer ('the object', 'the sample'). The work on this exciting topic has led to discoveries such as the structure determination of the DNA [12], rewarded by the Nobel prize.

The problem is a so called 'inverse problem': the scattering structure is to be determined, knowing the intensities and the scattering process, but not the phase of the field [9]. The problem could be directly solved if not only the intensities but also the phases of the scattered field were known. This is not the case though. In fact, some additional knowledge is necessary. Different solution methods can be distinguished by the kind of prior knowledge needed [9]. It is indeed challenging to generate 'good' additional experimental knowledge ('experimental data'). No refractive materials as known from visible optics exist in the spectral X-ray region. Mirrors only work at grazing incidence. Until recently, it was not possible to produce useful X-ray lenses. Even though zone plates [10] are available nowadays it is still very difficult to do imaging with X-rays. Hence, the principal goal has remained unchanged: determine the structure of the object from intensity data of the scattering process. This is called 'X-ray imaging' since imaging is the purpose of these experiments.

In general, the needed prior knowledge has to be generated by additional measurements. One type of useful manipulation is shown in Figure 5. In many cases no lens is present between the object and the detector (so called 'lens-less imaging'). Thus, a possible approach is to vary the field that is incident on the object using either masks or a lens before the object (Fig. 5).

In conclusion, quite some effort is necessary to generate the additional information vital for the reconstruction process. In addition to that, the reconstruction process should work independently of the scattering object, at best with little or no adjustments. The reconstruction
process is usually based on numerically intensive, iterative steps. Only good data lead to a sharp object image.

That is where the HOLOCAM could come into play. Figure 6 shows a possible setup. It can be seen that the HOLOCAM needs a lens such as in any other common optical arrangement. This is certainly a drawback compared to lensless systems but there are two advantages which should compensate for the higher effort caused by a lens: For a HOLOCAM system, lens aberrations can be corrected numerically and the HOLOCAM gives directly non-iterative phase data. Lenses with sufficient quality are therefore available. Hence, the potential merit of the HOLOCAM is that it allows to improve the tomographic resolution up to the diffraction limit even if the lens is imperfect. The HOLOCAM can be built internally with grazing incidence mirrors (Fig. 7).

In Section II, we will give a brief summary of the theory involved. More details can be found in Reference [6].

In Section III, we will discuss some more aspects of the HOLOCAM applications in the visible spectrum. In Section IV this will be continued for X-rays.

II. THE PHASE MEASUREMENTS

A. The intensity based phase retrieval

We start by outlining the basics of phase retrieval. The light field is a complex field consisting of intensity and phase information. Physically, only intensities can be measured directly whereas phases must be determined indirectly. This is the origin of the phase retrieval problem, i.e. the task to determine the complex field $f$ from its intensity (or 'amplitude') $|f|$. Of course, stated that way, the problem is undetermined. The problem becomes tractable if some prior knowledge is available. Consequently, phase retrieval problems can be distinguished by the kind of prior knowledge used.

In optics, the scattering problem is of great importance since it allows the optical inspection of a specimen such as the structure of DNA [12]. In this case, the intensities of the Fourier coefficients are given. However, this does not improve the general situation as it merely reformulates the problem: Determine $f(x)$, a complex quantity, knowing $|F(k)|$:
The Fourier transformation is an injective mapping which immediately proves that this problem cannot have a unique solution except for cases of further prior knowledge. In the case of the Gerchberg and Saxton (GS) algorithm, this prior knowledge consists of two (instead of one) intensity measurements such as \( |F(k)| \) and \( |f(x)| \). Alternatively, the intensity can be measured with masks before or after the object (structured illumination, oversampling and sparsity) \[9\][11][12][15].

Phase retrieval techniques are collectively called 'intensity based phase retrieval'. The method has the following characteristics: No reference beam is needed but additional measurements (under different physical conditions). It is an iterative method and numerically very demanding.

B. The reference based phase retrieval

In this method, the interference \( IN_s \) between the light field and a reference beam is measured:\[17\]

\[
IN_s := (E_1 + E_2) \ast (E_1 + E_2) \tag{2}
\]

's' denotes some indexation of pixels of the image recording device. In this case, \( E_1 \) is the field of the unknown phase and \( E_2 \) is a field of known phase.

\[
IN_s = |E_1|^2 + |E_2|^2 + 2 \text{Re}(E_2 \ast E_1) \tag{3}
\]

Using known methods like 'phase shifting' or 'carrier phase'\[2\] the complex term \( IF \) is determined:

\[
IF_s := E_2 \ast E_1 \tag{4}
\]

If \( E_2 \) is known \( E_1 \) can be calculated directly. The knowledge of \( E_2 \) is a highly demanding type of prior knowledge since it involves the physical use of a reference beam of known properties.

The methods above will be called 'reference based phase retrieval'. The method has the following characteristics: A reference beam is needed with the advantage of a simple numerical evaluation.

C. The correlation based phase retrieval (or SRI methods)

Self-referencing interferometer (SRI) methods represent a third class \[3][4][5\] which includes all approaches that determine the phase by interferometric means and that do not use an 'external' reference beam. The spread of methods is quite large. The SRI approach is also called 'correlation based phase retrieval'.

One method is to generate a zero mode optical field, which serves as a synthetic reference beam \[5\]. This can be achieved by a spatial band pass such as an illuminated pinhole or single mode fiber leading to a loss of light though. Furthermore, the process generating the reference involves some intensity arbitrariness of the fields introducing noise sources in the overall process. This kind of SRI is thus not a really satisfying solution yet.

Another approach is to measure lateral shearing interferograms \[3][4\]. The phase is obtained by a iterative optimization of a non-linear functional

\[
\|IF - ET \ast S(E1)\| \rightarrow 0 \tag{5}
\]

\( S \) represents a lateral shear, either a \( x \) shear or a \( y \) shear. A convergence ensuring term is added which smooths the result. In general, convergence to a global minimum and stability are big challenges in phase reconstruction.

Lateral shearing interferometers do not generate particularly characteristic interferograms. This can be seen from the fact that the lateral shearing interferometer essentially measures the derivative of the phase \[16\]. As a consequence, plane waves show up as constant offsets in the interference signal which can only badly be resolved. In conclusion, incident optical plane waves cannot be well detected. Apart from that, it will be shown later in this article, that lateral shearing interferometers are difficult to calibrate since they do not have a fixed point. This will be explained in the context of the HOLOCAM method (Subsection \[ID\]).

This kind of SRI approaches are called 'classical SRI' in contrast to the HOLOCAM method which might be considered as a SRI technique, too.

'Classical SRI' has the following properties: No reference beam is needed. Depending on the particular approach the resolution of the method is limited.

D. The HOLOCAM phase measurement

Abstractly speaking, the method divides the field \( E \) into two parts \( E_1 \) and \( E_2 \) (Fig. \[8\]). The interferometers used for the HOLOCAM have the property that \( E_2 \) is a known linear function of \( E_1 \). This assumption is not very restricting. In some cases the mapping might even be very simple (Section \[III\] or Reference \[6\]). \( U \) is the back propagation of \( E_1 \) by branch to a plane before the HOLOCAM and subsequent forward propagation of this field by branch 2 to the detector plane. If the detector plane mapped by \( U \) corresponds to two conjugate planes, the map \( U \) becomes a geometric point mapping.

\[
E_2 = \sum U_{s,t} E_1 =: U(E_1) =: UE_1 \tag{6}
\]
FIG. 8. The HOLOCAM principle in general. The field propagation for the two branches is different which is symbolized by an optical system in the upper branch.

Using this the expression for IF can be reformulated.

$$IF_s = E^2_s * E_{1s} = E_{1s} * \sum_t U_{s,t} E_{1t}$$  \(7\)

Multiplying this equation with \(E_1\) yields

$$IF_s * E_1 = |E_{1s}|^2 * \sum_t U_{s,t} E_{1t}$$  \(8\)

Having prior knowledge in \(|E_{1s}|^2\) \(=: I_1\) this becomes a linear equation for \(E_1\):

$$\sum_t (I_1 U_{s,t} - IF_s * \delta_{s,t}) E_{1t} = 0$$  \(9\)

\(\delta_{s,t}\) is the Kronecker function. Equation \(9\) is a linear equation in \(E_1\) since \(U\) and \(I_1\) are known. This is called the fundamental equation for the HOLOCAM.

The intensity \(I_1\) can be measured by blocking branch 2 or by an additional beamsplitter which allows to measure the light \(E_1\) on a separate detector. For simplicity this almost trivial task will not be shown in the figures.

The dimension of the subspace of the zeroth eigenvectors of the fundamental equation must be checked for each case. For a unique solution the dimension has to be one. This can be achieved by an appropriately designed interferometer. Figure \(7\) shows an example which is also discussed in Reference \(6\). Another example is shown in Figure \(8\). It actually represents a HOLOCAM with two measured correlations.

The solution space of linear equations can be completely characterized. Hence, there is neither a convergence nor a stability problem \(6\).

The stability of the fundamental equation is of great importance. To some extent this can be verified by analyzing reference \(17\). It has been recognized by Osherovish et al. that prior knowledge in the intensities of interfering fields renders the phase determination more stable. This has been derived for an interference with a reference beam. Investigations on the HOLOCAM approach have verified that this is also true for SRI \(6\). This seems to be a cornerstone for the stability of the HOLOCAM method in which the intensity of the field is used as prior knowledge.

The HOLOCAM devices differ in their mapping \(U\). It is favorable to have a fixed point in \(U\). This entails the existence of a pixel with index \(s\) for which \(E_{1s}\) and \(E_{2s}\) correspond to the same point of the incoming wave front. An example is the rotation of an electric field where the center of the rotation is mapped onto itself. At the fixed point, the interference has a real value in IF. Hence, it can be used to determine the absolute phase of IF. Therefore, a fixed point in the mapping \(U\) allows an absolute calibration of IF. A pure lateral shearing interferometer has no fixed point which explains some of the difficulties with lateral shearing interferometers (Section \(4\)).

The HOLOCAM approach is quite general. To give an example, interferograms obtained by lateral shearing interferometers in ‘classical’ SRI can also be evaluated by the HOLOCAM method. As a consequence, the linear fundamental equation Eq. \(9\) has to be solved using an operator \(U\) which is a lateral shear (or shift). Furthermore, the intensity \(|E_{1s}|^2\) has to be known. Besides, the solution obtained by this method is the global minimum of the functional Eq. \(5\). Without noise, the solution of the fundamental equation is a pointwise solution of Eq. \(9\) and the functional Eq. \(5\) becomes even zero. This must be the global minimum of the optimization problem Equation \(5\). Of course the solution of a linear matrix is simpler than the search for a global minimum.

A drawback of lateral shearing is the lack of a fixed point calibration method. Calibration is an important aspect for all methods, not only for the HOLOCAM. Consequently, interferometers with fixed points should be used preferentially for the HOLOCAM method.

The HOLOCAM approach is also called ‘linear correlation based phase retrieval’.

In conclusion, phase retrieval can be divided in four categories:

- Intensity based phase retrieval (‘pure’ phase retrieval)
- Reference based phase retrieval (‘classical’ holography)
- Correlation based phase retrieval (‘classical’ SRI)
- Linear correlation based phase retrieval (HOLOCAM)

III. HOLOCAM VISIBLE IMAGING

Figure \(9\) shows a demonstration example of a HOLOCAM. A pair of lenses is inserted in one of the branches of a Mach-Zehnder interferometer in such a way that \(U\) becomes a point mapping. \(U\) introduces a radial shear with one fixed point (actually a beam expander). This configuration has been extensively investigated by numerical
FIG. 9. Design example for a HOLOCAM in the visible spectrum. The setup is conceptually simple and therefore useful as a demonstration. Advanced HOLOCAM systems might exploit the large design freedom for compactness. Furthermore, it is possible to choose a HOLOCAM design with equal path lengths.

This simple example is quite useful to get an understanding of the HOLOCAM concept. The interferometer in Figure 9 has unequal path lengths which is not a necessity but a particularity of this design.

Another example is a lateral shear interferometer with two recorded IF images for x and y shear respectively [3][16]. This configuration has no fixed points. The two branches are equal in path length. It has been shown that 10\(\mu\)m coherence length of a light emitting diode are sufficient to record good quality IF images [3]. The experimental proof was given for a method minimizing the functional Eq. 5 but is still valid for a HOLOCAM type evaluation, as shown at the end of Subsection II D.

The HOLOCAM method avoids an iterative solution process and can be applied to much more general configurations as shown by the 3D example in Figure 7. Multiple scattering devices and diffractive optical elements DOE can also be used.

Next, we will analyze the measurement of the complex interference term IF. Only real quantities can be measured. This sounds similar to the initial phase reconstruction problem but is in fact quite a different question compared to Section I. IF is, by definition, equal to \(E_2E_1\) which involves the phase of the product of the two fields (phase difference). The determination of a phase difference can be tackled by established methods such as ‘carrier phase method’ and ‘phase shifting method’ [8]. It is important to note that this can be done within one recorded real-valued interferogram (‘one shot technique’).

We also want to discuss the role of polarization. It can always be assumed that the field at the entry of the HOLOCAM is linearly polarized. If this should not be the case it can be enforced by a polarizer which might even probe both polarizations consecutively. Additional effects of polarization rotation might exist inside the HOLOCAM interferometer limiting the degree of interference. It is always possible to use a further polarizer just before the detector to ensure 100% interference though.

Polarization is interesting for another reason. The two light paths of the HOLOCAM can be designed to have different polarizations. Even if they occupy the same area in space they can be manipulated separately by polarization sensitive equipment allowing to design of ultra compact HOLOCAM devices.

The HOLOCAM strategy is mainly a two step process. In the first step, the complex phase of \(E_1\) is determined. This is the proper HOLOCAM step. In the second step, this information can be used for subsequent operations such as the propagation of the phase information to another imaging plane or as a part of a larger data series in a tomographic analysis. This stepwise approach allows us to monitor the quality within the process. An example is the size of the lowest eigenvalue of the fundamental equation Eq. (9). In an ideal case this eigenvalue should be zero but error influences such as detector quantization lead to a finite value [6]. The corresponding eigenvector is still the electrical field \(E_1\) [6]. Hence, every individual step of the HOLOCAM method can be calibrated and optimized independently. Another advantage is a better control of subsequent steps. For instance, the ‘twin problem’ known from holographic evaluations does not exist in this approach [18].

The HOLOCAM can be used in all known configurations in holography or diffraction tomography. Aberrations in optical components can be corrected numerically. Electronic focusing can be applied [18].

Nowadays, a reference beam based determination of \(E_1\) is mainly used in digital holography (Reference based phase retrieval or classical holography). Besides complexity and noise problems, this has the disadvantage that the detector cannot be displaced (Fig. 1). The HOLOCAM system does not have this limitation. To give an example, this would solve the known problem of the missing frequencies in tomography [19].

IV. HOLOCAM 3D INTERFEROMETER X-RAY IMAGING

Figure 4 has shown an example of a HOLOCAM for X-rays. The, to our knowledge, new development of 3D interferometers permits quite different design concepts of HOLOCAM interferometers. We recall that the appropriate design of \(U\) is of crucial importance for the successful application of the HOLOCAM method. The new 3D transport of fields in 3D interferometers is such a design concept (Fig. 7). To our best knowledge this type of
3D interferometers is a new design which is particularly useful for HOLOCAM applications.

The main difference between HOLOCAM applications in the visible spectrum and applications in the X-ray range is the availability of refractive materials. X-ray mirrors can usually only be used for grazing incidence. This can be respected for a HOLOCAM device so that the mirrors form some kind of grazing incidence waveguide. Two pairs of mirror denoted in Figure 7 as (1a,2a) and (1b,2b) form such a guiding structure. Since the guide is bent in 3D space the transported field is rotated. The two pairs have opposite bending properties. As a consequence, the field in branch 1 is mutually rotated with respect to branch 2. Such a rotation has a fixed point which is not moved by the mapping operation. The two branches in Figure 7 have equal length. U is therefore a point mapping (The field on one point is approximately mapped on another point, up to interpolation).

Figure 10 shows the motion of some arbitrary point of the field in the detector plane. The point moves on a circle around the fixed point (‘center of rotation’). Consequently, only correlations between field points on such concentric circles are formed in IF (defined by Eq. 7). Thus, the solution of Equation 9 is not unique. A second determining map is needed which is possible by conducting a second field transportation (called ‘#’) in Figure 10. The second superposition of the two fields has a different shear between the two fields and a different rotation angle. Figure 10 shows that the joint and multiple operation of both maps links all points, indeed. Hence, the fundamental equation (Eq. 9) now uniquely defines the field E1, provided both operations are evaluated.

The HOLOCAM detector in Figure 6 can be displaced. This enables us to measure the diffracted signal at very different locations while the illumination of the sample and the sample remain fixed in position.

The relationship between E1 and E2 expressed by $E2 = U(E1)$ is an essential basis for the HOLOCAM concept. This might include multiple scattering or different order contributions to E2. Even a general diffractive element can be used for the generation of E2. The only important point is the existence of a known linear relationship between E1 and E2. The requirements for the image guiding elements in the HOLOCAM are lower than for X-ray intensity based microscopes [10].

Hence, the HOLOCAM has all the potential to improve resolution and simplify X-ray imaging. The hitherto used phase retrieval from pure intensity based data is no longer needed. The image evaluation can be directly based on well known scattering formulas [20] which are independent of the actually probed sample.

V. CONCLUSION

The HOLOCAM readdresses the ‘reference free phase measurement’. It thereby reduces many disturbing factors known in reference beam based digital holography, such as environmental noise and mechanical instability. The drawback is a twisted path in the HOLOCAM. These expenses for the internals of a HOLOCAM setup are expected to be compensated by large savings in the overall ‘interferometric machinery’. To give an example the classical holographic microscope used for quantitative phase contrast or holographic tomography uses a design where the sample is part of the interferometer. As shown in the introduction (Section I) this setup is rather unreliable since it causes losses in the accuracy of the phase measurement. Using a HOLOCAM this is no longer the case. Thus, the same measurements can be done with greater precision.

For demonstration purposes the design example (Fig. 9) of the HOLOCAM has well separated explicit branches. The large design freedom allows internal reflection devices or twisted optical paths defined by polarization. This is for the moment reserved for subsequent development. The advantage of the HOLOCAM is the design freedom in the physical setup and the simplicity in the mathematical evaluation. Both should lead to compact device solutions.

Even X-ray imaging should benefit from these concepts. The HOLOCAM can be realized by state of the art grazing incidence mirrors. A lens is needed for the integration in the measurement system. This might be considered as a drawback compared to lenseless systems. Yet zone lenses are nowadays available and aberrations and other lens errors are not critical for the HOLOCAM since the effects can be corrected numerically. The HOLOCAM has all the potential to improve measurement quality in X-ray systems. This should be enough justification for the introduction of an additional lens to the system.
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