Small Dirac Neutrino Masses in Supersymmetric Grand Unified Theories

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Abstract

A simple mechanism to generate Dirac masses for the neutrinos in SU(5) supersymmetric grand unified theory is proposed. The tiny Dirac masses are induced by the small mixing between the Higgs fields and another superheavy fields. The mixing terms are obtained by the same mechanism as the $\mu$-term generation of the order of the supersymmetry breaking scale, so that the mixing of order $\text{TeV}/M_{\text{GUT}} \sim 10^{-13}$ is realized. We consider the lepton flavor violating processes in this model. The branching ratios are directly related to the neutrino oscillation parameters and we can predict the $B(\tau \to \mu \gamma)/B(\mu \to e\gamma)$ ratio once the neutrino oscillation parameters are determined.

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The data from SuperKamiokande suggest the presence of tiny neutrino masses which clearly indicates a necessity of an extension of the lepton sector in the minimal Standard Model [1]. The easiest extension is to introduce the right-handed neutrinos and the Yukawa interaction terms such as $f_{ij} \bar{l}_i h \nu_{Rj}$, where $l, h$, and $\nu_R$ are the left-handed lepton doublets, the Higgs field and the right-handed neutrinos, respectively. These terms induce the Dirac masses for the neutrinos through the SU(2)$_L \times$ U(1)$_Y$ breaking effect. In that case, the coupling constants $f_{\nu}$ must be very small of order $10^{-13}$ in order to reproduce the tiny neutrino masses. Also, the neutrinos can acquire Majorana masses by adding terms as $\lambda_{ij}(\bar{l}_i h)(l_j h^\dagger)$. The parameters $\lambda_{ij}$ have mass dimension of $-1$ and should be of the order of $10^{-14}$ GeV$^{-1}$. In the framework of the seesaw mechanism, the smallness of $\lambda_{ij}$ is explained by large Majorana masses of the right-handed neutrinos $M_R \sim 10^{14}$ GeV [2].

In any case, we need somewhat unnatural parameters $f_{\nu} \sim 10^{-13}$ or $M_R \sim 10^{14}$ GeV. In Supersymmetric (SUSY) Grand Unified Theories (GUTs), there is a hint toward this problem. The SUSY GUT provides scales of $M_{\text{SUSY}} \sim$ TeV and $M_{\text{GUT}} \sim 10^{16}$ GeV to the theory, while the ratio $M_{\text{SUSY}}/M_{\text{GUT}}$ is adequate for the Dirac Yukawa couplings $f_{\nu} \sim 10^{-13}$. Several attempts to utilize the SUSY breaking scale have been made [3, 4]. Especially, in ref.[4], Borzumati et al. considered the possibility of explaining the LSND data [5] by introducing a sterile neutrino field ($S$) whose mass is given by the SUSY breaking effect at the suitable order ($M^2_{\text{SUSY}}/M_R \sim$ 1 eV). The Dirac mass terms between $S$ and the right-handed neutrinos $N$ are forbidden by $R$-symmetry at first, and given through a non-renormalizable interactions of $(W)SN/M^2_{\text{Pl}} \sim m_{3/2}SN$ where $W$ and $m_{3/2}$ are the superpotential and the gravitino mass, respectively. In supergravity scenario [6], the vacuum expectation value (VEV) of $W$ is necessary to cancel the cosmological constant caused by the SUSY breaking sector. They also pointed out the existence of the Dirac mass terms $\langle W \rangle LHS/M^3_{\text{Pl}}$. Although the Dirac Yukawa coupling constants from those terms are too small of order $m_{3/2}/M_{\text{Pl}} \sim 10^{-15}$, it is interesting that the replacement of $M_{\text{Pl}}$ to $M_{\text{GUT}}$ gives suitable magnitude for the neutrino oscillation.

Another type of the Dirac neutrino scenario have been proposed by Mohapatra and Valle in superstring models [7]. The small Dirac neutrino masses are obtained by hierarchy between VEVs of two distinct standard model singlet components in $27$ and $\bar{27}$.
representations of the $E_6$ group.

In this paper, we propose a mechanism to generate tiny Dirac Yukawa couplings $f_{\nu}$ in the context of SU(5) SUSY GUT. The mechanism is similar to the usual seesaw scenario or the Froggatt-Nielsen mechanism. In the Froggatt-Nielsen mechanism, the small mass parameters are explained by imposing U(1) symmetry and introducing a small VEV of a U(1) breaking field. In our case, as in ref. [4], we use $R$-symmetry as such U(1) symmetry and the small $R$-symmetry breaking terms are automatically supplied by the SUSY breaking effect as in the $\mu$-term generation mechanism [5, 6, 10]. By using the above mechanism, the tiny coupling of order $10^{-13}$ is naturally obtained as the ratio $M_{\text{SUSY}}/M_{\text{GUT}}$ through a mixing between the usual Higgs doublet and another superheavy Higgs doublet. We also consider the Lepton Flavor Violating (LFV) processes as a low energy prediction of our model.

We construct an SU(5) GUT model. We assign $R$-charges for the matter fields as follows:

\[
\bar{F} : (\bar{5}, 1) , \quad T : (10, 1) , \quad N : (1, -1) ,
\]

where the former and latter numbers are the dimension of the representation of the SU(5) group and the $R$-charge, respectively. The superfields $\bar{F}$ and $T$ represent the usual matter and $N$ is the right-handed neutrino superfield. In the Higgs sector, in addition to the usual Higgs fields, we introduce a pair of $5$ and $\bar{5}$ representation fields $H'$ and $\bar{H}'$ as follows:

\[
H : (5, 0) , \quad \bar{H} : (\bar{5}, 0) , \quad H' : (5, 2) , \quad \bar{H}' : (\bar{5}, 0) .
\]

The superpotential relevant to the mechanism is written as follows:

\[
W = \tilde{f}_{\nu}^{ij} \bar{F}_i H' N_j + M_{H'} H' \bar{H}'.
\]

The $H'\bar{H}$ term which is allowed by $R$-symmetry can be eliminated by the field redefinition of $\bar{H}$ and $\bar{H}'$. The Yukawa coupling constants $\tilde{f}_{\nu}$ and the mass parameter $M_{H'}$ are naturally taken to be of order unity and the GUT scale of $10^{16}$ GeV, respectively.

The essential point is that the SUSY breaking effect induces $R$-symmetry breaking (but $R$-parity conserving) terms for the combination of vanishing $R$-charge such as

\[
W_{R\text{-breaking}} = \mu H \bar{H} + \mu' H' \bar{H}' ,
\]
with the mass parameter $\mu$ and $\mu'$ of the order of SUSY breaking scale such as TeV. These terms are naturally induced by the Giudice-Masiero mechanism \cite{9} in the supergravity scenario, and other mechanisms have been considered in the literature \cite{3,10}. The tiny Dirac neutrino masses can be obtained with such terms. The existence of $\mu'$ term induce the tiny mixing $\delta_{H,H'}$ between $H$ and $H'$ fields as $\delta_{H,H'} \sim \mu'/M_{H'}$. It follows that the low energy effective superpotential are described in terms of the light Higgs multiplets $H_l$ and $\bar{H}_l$, which are almost $H$ and $\bar{H}$, as follows:

$$W_{\text{eff}} = \delta_{H,H'} \tilde{f}^{ij}_\nu \bar{F}_i H_l N_j + \mu H_l \bar{H}_l.$$  \tag{5}

The first term is the usual Dirac Yukawa interaction with coupling constants $f_\nu = \delta_{H,H'} \tilde{f}_\nu \sim 10^{-13}$ which is suitable for the neutrino oscillation data.

In general, if we assume the supergravity scenario, the small Majorana mass terms for $N$ may be induced by $\langle W \rangle^2 N^2/M_{Pl}^2$ term of the order of $m_{3/2}^2/M_{Pl}^2 \sim 10^{-13}$ eV \cite{4}. If we include this contribution, we have a possibility to realize the pseudo-Dirac scenario \cite{11}, where the large mixing of the neutrinos and the data from the LSND experiment \cite{5} can be naturally explained. However, the data from SNO experiments support the solar neutrino oscillation of $\nu_e$ to an active neutrino $\nu$, so that the Majorana masses for $N$ are strongly constrained. In order to escape this constraint, we need to assume that the neutrinos do not directly couple to the cosmological constant tuning sector. Another possible way to avoid small Majorana mass terms for $N$ is imposing global $B-L$ like symmetry with the charge of $F : -3$, $T : 1$, $N : 5$, $H : -2$, $\bar{H} : 2$, $H' : -2$, and $\bar{H}' : 2$. Also, in the gauge mediated SUSY breaking scenario \cite{13}, the contribution is negligible. Similarly, as mentioned before, the Dirac mass terms $\langle W \rangle \bar{F} H N/M_{Pl}^3 \sim 10^{-15} \bar{F} H N$ may arise \cite{4} in the supergravity scenario. These contributions to the Dirac masses are negligible compared to those from the above Higgs mixing effects.

As a prediction of our model, we consider LFV processes such as the $\mu \to e\gamma$ and $\tau \to \mu\gamma$ decays. An interesting point of the Dirac neutrino is that the Yukawa coupling constants are directly related to the neutrino oscillation parameters, namely

$$f^{ij}_\nu \propto U^{ij}_{\text{MNS}} m_{\nu j},$$  \tag{6}

where $U_{\text{MNS}}$ and $m_\nu$ are the Maki-Nakagawa-Sakata (MNS) matrix \cite{14} and the neutrino masses. In this case, the branching ratios of the LFV processes are strongly correlated
Figure 1: The ratio of $B(\tau \to \mu \gamma)/B(\mu \to e \gamma)$ is plotted. The horizontal axis is $U_{\text{MNS}}^{e3}$. The solid and dashed lines represent the large and small angle MSW solution to the solar neutrino problem, respectively.

with the neutrino oscillation parameters. In the minimal supergravity scenario, in which the slepton mass matrix is proportional to the unit matrix at the tree level, off-diagonal components of the slepton matrix are induced through the loop diagrams with the LFV interactions [15]. In the seesaw model, the relation between the neutrino oscillation parameters and the LFV processes have been investigated in detail [16, 17]. In that case, we need to assume the mass matrix of the right-handed neutrinos and the results depend on the pattern of the neutrino mass matrix, i.e., hierarchical or degenerate [17]. However, in our case, the prediction is directly related to the observables in the neutrino oscillation experiments, i.e., mass squared differences and mixing angles as we see in the following.

In the minimal supergravity scenario, the off-diagonal components of the left-handed slepton mass matrix are induced by the renormalization group running between the Planck scale to the GUT scale through the $\tilde{F}H'N$ interactions, and are approximately given by

$$ (m_{\tilde{f}}^2)_{ij} \simeq -\frac{1}{8\pi^2} \sum_k \tilde{f}_{ik}^{\nu*} \tilde{f}_{jk}^{\nu} (3 + |a_0|^2) m_0^2 \log \frac{M_{\text{GUT}}}{M_{\text{Pl}}}, $$

(7)

where $a_0$ and $m_0$ are the coupling constant of the universal three point scalar interaction and the universal scalar masses, respectively. From eqs. (8) and (7), we can describe the
Figure 2: The dependence of $B(\mu \to e\gamma)$ on the SUSY breaking parameters are plotted. We take $f^3_{\nu} = 1/\sqrt{2}$ and the neutrino oscillation parameters of the large mixing MSW solution and $U^{\nu 3}_{\text{MNS}} = 0$. The horizontal and vertical axes represent the universal scalar mass and the gaugino mass given at the Planck scale. The lines represent the branching ratio of $10^{-11}$, $10^{-12}$, $10^{-13}$, and $10^{-14}$ from inside. The value of the tan $\beta$, which is the ratio of the two VEVs of the Higgs fields, is taken to be $\tan \beta = 10$.

off-diagonal components as follows:

$$ (m^2_l)_{e\mu} \propto U^{\nu 1*}_{\text{MNS}} U^{\mu 1}_{\text{MNS}} \Delta m^2_{12} + U^{\nu 3*}_{\text{MNS}} U^{\mu 3}_{\text{MNS}} \Delta m^2_{32} , \quad (8) $$

$$ (m^2_l)_{\mu\tau} \propto U^{\mu 1*}_{\text{MNS}} U^{\nu 1}_{\text{MNS}} \Delta m^2_{12} + U^{\nu 3*}_{\text{MNS}} U^{\mu 3}_{\text{MNS}} \Delta m^2_{32} , \quad (9) $$

$$ (m^2_l)_{e\tau} \propto U^{\nu 1*}_{\text{MNS}} U^{\nu 1}_{\text{MNS}} \Delta m^2_{12} + U^{\nu 3*}_{\text{MNS}} U^{\nu 3}_{\text{MNS}} \Delta m^2_{32} , \quad (10) $$

where $\Delta m^2_{ij} = m^2_{\nu_i} - m^2_{\nu_j}$. From these equations, we can predict the ratio of the branching ratios such as $B(\tau \to \mu\gamma)/B(\mu \to e\gamma)$ once the neutrino oscillation parameters are determined. For example, in case of the large mixing MSW solution for the solar neutrino problem [18] and $U^{\nu 3}_{\text{MNS}} = 0$, the ratio of the branching ratios of $\mu \to e\gamma$ and $\tau \to \mu\gamma$ is
given by
\[
\frac{B(\tau \to \mu \gamma)}{B(\mu \to e \gamma)} \approx 0.35 \left( \frac{m_{\tau}^2}{m_{\mu}^2} \right)^2 \approx 0.35 \left( \frac{\Delta m_{32}^2}{\Delta m_{12}^2} \right)^2 \approx 10^3 .
\] (11)

In GUT models, there is another contribution to the off-diagonal components of the right-handed slepton mass matrix through the LFV interaction between the right-handed quarks and leptons with the CKM mixing \[19\]. However, those contributions are negligibly small compared to those from \(\bar{F}H'N\) interactions for \(\tilde{f}_{\nu}^{33} \sim 1\). For the case of large and small angle MSW solutions, we plot the ratio in Fig.1 as a function of \(U_{\text{MNS}}^{e3}\). We can see significant dependence on \(U_{\text{MNS}}^{e3}\). The ratio varies from 10 to 2000 for large angle MSW solution and from 10 to 30000 for small angle MSW solution. Dependence of \(B(\mu \to e \gamma)\) on the SUSY breaking parameters are given in Fig.2. We take the coupling constant \(\tilde{f}_{\nu}^{33} = 1/\sqrt{2}\) and neutrino oscillation parameter of the large angle MSW solution and \(U_{\text{MNS}}^{e3} = 0\). The parameter \(m_0\) and \(M_{1/2}\) are the universal scalar mass and the gaugino mass given at the Planck scale. In the wide region of the SUSY breaking parameters, \(B(\mu \to e \gamma)\) is greater than \(10^{-14}\) which is within the reach of the planned experiments at PSI \[20\] and JHF \[21\].

In conclusion, we proposed a simple mechanism to generate Dirac neutrino masses in SU(5) SUSY GUT in which the tiny coupling constants are induced by the mixing between the Higgs field and another superheavy Higgs field. The tiny mixing is realized by the hierarchy of the mass parameter \(M_{H'} \sim M_{\text{GUT}}\) and \(\mu' \sim M_{\text{SUSY}} \sim \text{TeV}\), and this situation is exactly the same as the \(\mu\)-problem \[22\]. Therefore using the same mechanism for solving the \(\mu\)-problem, we can obtain such hierarchy and the tiny Dirac neutrino masses are naturally explained. We emphasize that the small Dirac neutrino masses are obtained in SUSY GUT without introducing any new scale parameters. As an interesting feature of this model, the branching ratios of the LFV processes are directly related to the neutrino oscillation parameters. We calculated the branching ratios and found that \(B(\mu \to e \gamma)\) is large enough to be observable and the ratio \(B(\tau \to \mu \gamma)/B(\mu \to e \gamma)\) is predicted once the neutrino oscillation parameters are determined.

Finally, we would like to comment on an alternative model. If we add three pairs of \(F' : (5, -1)\) and \(\bar{F}' : (\bar{5}, 3)\) instead of the extension of the Higgs sector, the superpotential
is given by

$$W = y^i_j \bar{F}_i H N_j + M^i_j F'_i \bar{F}'_j + \mu H \bar{H} + \tilde{\mu}_{ij} F'_i \bar{F}'_j .$$  \hspace{1cm} (12)$$

By diagonalizing the mass matrix, we obtain small Dirac neutrino masses. However, in this case, the direct relation eq.(8–10) are lost because the low energy neutrino Yukawa coupling constants $f_{\nu}$ are not proportional to the original one $y_{\nu}$ in general.

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