Coherent Control of Optical Nonreciprocal Propagation in a V-Type Atomic System

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Abstract
Optical nonreciprocity and nonreciprocal propagation of light have attracted great research interest, due to not only their fundamental scientific significance, but also their extensive applications in lasing, quantum optical devices and quantum information. In this work, we theoretically and experimentally investigate nonreciprocal propagation of light in a V-type three-level thermal atomic system. By virtue of the EIT effect and the atom thermal motion, nonreciprocal propagation of light is achieved in the Rb87 warm atoms, where high transmission of the probe field is achieved in the co-propagation direction of the control field and the probe field is blocked in the opposite direction of the control field. Transmission and bandwidth for the nonreciprocal propagation of light can be enhanced and controlled by the control field in this system, where the nonreciprocal bandwidth can be broadened significantly in comparison with the Λ-type atomic system. In our experiments, we achieve ~60 MHz nonreciprocal bandwidth for the probe field. This work may have potential applications in quantum nonreciprocal devices such as optical isolator and circulator.

Introduction
Optical nonreciprocity and nonreciprocal devices, which supports drastically asymmetric propagation of light in two opposite directions, are essential in optical communications, laser systems and signal processing [1]. In recent years, optical nonreciprocity has attracted great research interest, and various strategies or physical mechanisms are proposed and studied for nonreciprocal transmission of light. Utilization of magneto-optic effect is a common approach to break the reciprocity [2-6]. However, response of magnetic materials often performs weak, implying bulky, costly and difficulty on integration of devices.

Great efforts dedicate to searching for alternative approaches and mechanisms to break reciprocity without the use of magnetism, especially those for suitable on-chip integration. A photonic band gap material with the combination of linear and nonlinear medium response previously proposed to support unidirectional propagation and optical diode [7]. Spatiotemporal modulation of refractive index of materials is one promising approach for this purpose, which generates optical nonreciprocity via introducing nonreciprocal phase transfer [8,9] and frequency conversion [10,11],
or establishing an angular momentum biasing [12-14]. Nonmagnetic optical nonreciprocity can also be achieved by optoacoustic effects [11,15], optical nonlinearity [16-19], and moving systems [20-22]. Great research interest were paid on the parity-time symmetry [23,24] recently. Using parity-time symmetric system, optical nonreciprocity [25] and phonon diode [26] have been studied. Due to the rapid development and the flexibility, optomechanical systems provide a good platform to support nonreciprocal transmission and create nonreciprocal devices such as optical isolator, optical circulator and optical router [27-34]. In chiral quantum physics, photons propagating in opposite directions are of spin-momentum or polarization locking, which drives emitters with different transition levels and rates [35]. It thus naturally offers a novel way to support nonreciprocal propagation of light even in the quantum regime [36-43]. The spin-orbit coupling channel so be used to realize optical nonreciprocity in low-dimensional materials [44-46].

Since the electromagnetically induced transparency (EIT) technology was introduced by Harris, et al. [47,48], many interesting quantum optical phenomena have been observed and realized in multi-level quantum systems based on the EIT effect, such as electromagnetically induced gratings (EIG) [49-51], four-wave and six-wave mixing [52,53], optical bistability and multistability [54,55], optical switching [56], Kerr nonlinearity enhancement [57-60], weak-light optical solitons [61-66] etc. Via inducing periodic structures by lasers in the EIT atomic systems, dipole soliton and optical vortices were generated and studied experimentally in thermal atoms [67,68]. The random motion of atoms often takes disadvantage on quantum coherence in warm atoms. However, utilizing the atom thermal motion and EIT effect, our group experimentally investigated and achieved optical nonreciprocity and isolation in a cavity-atom coupling system [69]. And soon, a scheme of unidirectional amplification of light was also proposed and demonstrated in an atomic system [70]. Utilizing the optical nonlinearity of cross phase modulation, Xia, et al. theoretically proposed a scheme for optical isolator and optical circulator in an N-type thermal atomic system [71]. We also proposed a scheme to theoretically achieve optical nonreciprocity via optical pump effect in multi-level atomic systems [72]. Our recent experiment demonstrated that, the nonreciprocal bandwidth can be broadened in the cavity-free N-type atomic system [73]. Three level V-type atomic system is a very common and frequently used quantum system. In this work, based on the EIT effect, we experimentally and theoretically investigate the nonreciprocal propagation of light in a warm Rb87 V-type atomic system. By adjusting the control field, transmission and bandwidth can be controlled and enhanced for nonreciprocal propagation of the probe field in this system. It is shown that, high transmission of the probe light is achieved in the co-propagation direction of the control field, while it is effectively blocked in the opposite direction. In addition, the V-type atomic system can provide a relatively wider bandwidth for nonreciprocal propagation of the probe light in comparison with the Λ-type atomic system. This work may provide reference for broadband applications of optical nonreciprocal devices.

Model and Theoretical Analysis

In this work, we consider a weak probe field of Rabi frequency $\Omega_p$, and a strong control field of Rabi frequency $\Omega_c$ interacting with a V-type atomic system (as shown in Figure 1). Under the slowly varying envelope and paraxial approximations, evolution of the probe field is governed by the following wave equation:

$$\frac{\partial E_p}{\partial z} = \frac{i}{2} \chi_p E_p^* .$$

(1)

Transmission of the probe field is determined by the macro susceptibility $\chi_p$, which can be derived by solving the motion equations of the density matrix elements under steady states. Under electric-dipole and rotating-wave approximation, the interacting Hamiltonian can be written in the interaction picture as

$$H_{\text{int}} = -\hbar (\Delta_p \sigma_{22} + \Delta_c \sigma_{31}) - \hbar (\Omega \sigma_{31} + \Omega \sigma_{-31} + \text{h.c.}),$$

(2)

Where $\Delta_p$ and $\Delta_c$ indicate the one-photon detunings respectively for the probe and control lasers. They are defined as $\Delta_p = \omega_p - \omega_{21}$ and $\Delta_c = \omega_c - \omega_{31}$ with $\omega_p$ and $\omega_c$ being the angular frequencies of the probe and control lights and $\omega_{ij}$ ($i = 31, 21$) the relevant transition frequency between states $|i\rangle$ and $|j\rangle$. $\Omega = \mu_0 \cdot E_i / 2\hbar (i = \{p,c\}, ij = (31,21))$ is the half Rabi frequency of the probe (control) field with the electric dipole momentum $\mu_0$ for the transition $|i\rangle \leftrightarrow |j\rangle$ and the slowly varying electric field amplitudes $E_i$ of the probe and control laser.
thermal motion. Considering all the atoms follow the Maxwell-Boltzman velocity distribution, we need to integrate the macro susceptibility $p\chi$ on all the velocities of atoms. Then the susceptibilities for the forward and backward probe light can be written respectively as:

$$p\chi^{(F,B)} = \int \frac{N|\mu_{13}|^2}{\hbar\varepsilon_0} \frac{\rho_{31}(\Delta_{p, B})}{\Omega_p} f(v)dv.$$  

**Figure 1:** (Color online) a) Coupling schemes of the V-type atomic model; b) Laser arrangement and experimental setup. PB: Polarized Beamsplitter; PD: Photoelectric Detector; HWP: Half-Wave Plate; PL: Probe Laser, CL: Control Laser; Rb: Rubidium Cell; OI: Optical Isolator; AR-Rb: Antireflection Coated Rb Vapor Cell; SAS: The Laser System Stabilized on the Saturated Absorption Lines of Rubidium.
Here $\Delta_j^{(F, B)} = \Delta_j + \vec{k}_j \cdot \vec{v}(i = p, c)$ is the effective detuning with the wave vector of the lasers $\vec{k}$ and the atom velocity $\vec{v}$. $f(v) = \exp(-v^2/v_p^2)/(\sqrt{\pi}v_p)$ represents the velocity distribution function of the atoms, where $v_p = \sqrt{2k_B T/M}$ is the most probable velocity with the Boltzmann constant $k_B$, the absolute temperature $T$, and the atom mass $M$. The superscript $(F, B)$ indicates the forward or backward propagation direction for short. Similarly, we can also obtain the susceptibility for the $\Lambda$-type atomic system. Transmission of the probe field in the two atomic systems can be calculated by Eqs. (1) and (3).

When the control field propagates along the forward direction, the atomic thermal motion gives rise to the same frequency shift on the forward probe field and opposite frequency shift on the backward probe field if so transmission of the backward probe field can be greatly suppressed due to the destruction of EIT, while transmission of the forward probe field can maintain a high level. Then nonreciprocal propagation of the probe field can be achieved in the two opposite directions. The nonreciprocal bandwidth is mainly determined by the EIT line width of the forward probe field and the Doppler line width of the warm atoms. Figure 2 shows the comparison of the transmission line width of the forward probe field in the $V$ and $\Lambda$-type warm atomic systems by solving the Eqs. (1) and (3) directly. In the calculation, the medium length is 5.0 cm and the temperature is 70 °C. The other parameters are $\Delta_2 = 0$, $N = 5.0 \times 10^{10}$ cm$^{-3}$, $\gamma = 5.746$ MHz and $\Gamma_{31} = \Gamma_{32} = \gamma$, $\gamma_{31} = \gamma_{21} = \gamma/2$ in the $V$-type atomic system while $\Gamma_{31} = \Gamma_{32} = \gamma$, $\gamma_{31} = \gamma_{21} = \gamma/2$, $\gamma_{23} = 0.001\gamma$ in the $\Lambda$-type atomic system, in which $\gamma_j$ and $\Gamma_j$ indicates the decoherence and population decay rates respectively. It can be seen in Figure 2 that, the transmission line widths $W_V$ and $W_L$ of the forward probe field increases with the Rabi frequency of the control field $\Omega_c$ (or the intensity of the control field), and the transmission line width $W_V$ can be much broader than $W_L$ especially for small $\Omega_c$. $W_V$ can be dozens or even hundreds of times larger than $W_L$ under the same $\Omega_c$.

**Experiment for Nonreciprocal Propagation of Light**

We experimentally investigate the nonreciprocal propagation of light in a warm $V$-type Rb87 atomic system. We select the levels $|5S_{1/2}, F = 2\rangle$, $|5S_{1/2}, F = 2\rangle$, and $|5S_{1/2}, F = 3\rangle$ of Rb87 atoms as the states $|1\rangle$, $|2\rangle$, and $|3\rangle$ and set laser couplings as shown in Figure 1a. The strong control laser $\Omega_c$ with wavelength of 780 nm is applied to drive the transition $|1\rangle \leftrightarrow |2\rangle$. A weak laser $\Omega_p$ with wavelength of 795 nm is used to probe the $|1\rangle \leftrightarrow |3\rangle$ transition. Such consideration of laser excitation forms a $V$-type configuration. The experimental setup and laser paths in experiment are laid out as shown in Figure 1b. With such arrangement of light paths, the coupling lasers $\Omega_c$ is vertically polarized and the probe laser $\Omega_p$ is parallel polarized when

![Figure 2](Image)

**Figure 2**: (Color online) Comparison of transmission bandwidth in $\Lambda$ and $V$-type thermal atoms. The parameters used in the calculation are $\Delta_2 = 0$, $N = 5.0 \times 10^{10}$ cm$^{-3}$, $\gamma = 5.746$ MHz and $\gamma_{23} = \gamma$ ($\gamma_{23} = 0.001\gamma$) in the $V$-type ($\Lambda$-type) atomic system.
they pass through the atoms. The cell length of Rb is about 5.0 cm and the temperature is set at 70 °C in experiment. The control laser is locked to be resonant with the transition |1⟩ ↔ |2⟩. For convenience, we define the path from left to right as the forward direction whereas the path from right to left as the backward direction. In our experiment, propagation direction of the control field is fixed to be along the forward direction.

Figure 3a and Figure 3b show the transmissions of the forward and backward probe fields versus the probe detuning and laser power $P_c$ of the control field respectively, where the average background noise has been erased. As shown in Figure 3a, we obtain high transmission for the forward probe field near the resonant frequency. The transmission and bandwidth depend sensitively on the power of the control field $P_c$. It is obvious that, with the increase of $P_c$, transmission and band width of the probe field are significantly enhanced (see Figure 3a). Contrarily, as the thermal motion of atoms induces opposite frequency shift for the backward probe field and breaks the EIT effect, backward transmission of the probe field is greatly suppressed (Figure 3b). The reason is that, thermal motion of atoms gives rise to the same frequency shift of the probe field in the forward direction but opposite shift in the backward direction compared with the control field. So, the EIT condition is always satisfied in the forward direction while broken in the backward direction. Forward and backward transmissions of the probe field at the resonant frequency ($\Delta p = 0$) are plotted in Figure 3c and Figure 3d respectively. The probe field obviously realizes high transmission in the forward direction because of the EIT effect (Figure 3c). While in the backward direction, transmission of probe field is very low under certain control powers (Figure 3d). Therefore, nonreciprocal propagation of probe field can be achieved in the two opposite directions.

We further measure bandwidth of the nonreciprocal propagation by evaluating the full width at half maximum (FWHM) of the forward transmission in the EIT window and show the results in Figure 4. It can be found that, with the increase of $P_c$ bandwidth of the nonreciprocal propagation

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Figure 3: (Color online) Transmissions of the probe field under different powers of the control field in the Rb87 thermal atoms. a) Forward and b) Backward transmissions versus the probe detuning and the control power; c) Forward and d) Backward transmissions at the resonant frequency ($\Delta p = 0$).
of the probe field can be broadened significantly, which provides a simple way to enhance the nonreciprocal bandwidth. As the backward transmission of the probe field is limited by the line width of the Doppler broadening of the thermal atoms, it grows evidently once the transmission line width becomes larger than the Doppler line width, which is undesirable for superior nonreciprocal propagation and nonreciprocal devices. Though limited by output power of the lasers, ∼60 MHz bandwidth of nonreciprocal propagation is achieved in the V-type atomic system in our experiments. Broadband nonreciprocal propagation may play important role in some quantum applications such as quantum communications and quantum information processing.

Conclusion

In conclusion, based on the electromagnetically induced transparency and the thermal motion of the atoms, we have investigated nonreciprocal propagation of light in a V-type Rb87 warm atomic system. Transmission of the probe field in two opposite directions can be controlled and enhanced by adjusting the control field. In the same propagation direction of the control field, the thermal motion of the atoms causes approximate frequency shift on the probe field, which guarantees the EIT condition and produce high transmission of the probe field. Conversely, in the opposite direction, the thermal motion induced frequency shift is nearly contrary between the probe field and the control field. So the probe field can be significantly blocked in the backward direction. In addition, bandwidth for nonreciprocal propagation of the probe field in the V-type atomic system can be enhanced and controlled by adjusting the control field. ∼60 MHz bandwidth (FWHM) for nonreciprocal propagation of the probe field is achieved in experiment. This work may provide references for wide band applications of nonreciprocal light propagation.

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Figure 4: (Color online) Variation of -3dB bandwidth for the forward probe field with the power of control field.
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