A simple approach to $\alpha$-decay fine structure

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Abstract. We propose a simple method to evaluate $\alpha$-transition rates to low-lying excited states in even-even nuclei. For this a realistic $\alpha$-daughter double folding interaction is approximated by a parabola in the region where the decay process takes place. This allows us to evaluate the penetration probability analytically. The main experimental features of branching ratios to excited states are reproduced by this simple approach.

PACS numbers: 21.60.Gx,23.60.+e,24.10.Eq

Keywords: Alpha-decay, Fine structure, Double folding potential
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1. Introduction

Relative $\alpha$-decay rates are often explained in terms of a preformed $\alpha$-particle which penetrates through the Coulomb barrier \cite{1}. However the calculation of absolute half lives requires a knowledge of the formation probability of the $\alpha$-particle on the surface of the mother nucleus. This is a very difficult task. Therefore the formation probability is often replaced by effective parameters. Such parametrizations are rather successful in describing decay widths and, therefore, they are use extensively in $\alpha$-decay analyses (see e.g. Ref. \cite{2} and references therein). However, there are recent high precision measurements of relative decay rates (fine structure) from even-even emitters to excited states in the corresponding daughter nuclei \cite{3,4} which do not involve absolute decay widths. Their description is usually given within a rather involved coupled channels method by using a double-folding $\alpha$-core potential \cite{5}. This formalism is able to reproduce $\alpha$-decay intensities with a good accuracy.

The aim of this paper is to explain the gross features of the fine structure experimental data using a much simpler analytical approach. In Section 2 we give the necessary theoretical details how to estimate partial $\alpha$-decay widths, in Section 3 we systematize the experimental material concerning the $\alpha$-decay fine structure from even-even emitters and in the last Section we draw Conclusions.

2. Theoretical background

We start by noticing that a realistic $\alpha$-nucleus spherical potential reproducing well scattering data is given by the double folding procedure using the M3Y nucleon-nucleon interaction \cite{6,7,8}. The most relevant part of this potential in the case of $\alpha$-decay is the region between the innerst and outermost turning points, We found that in this region the potential can very well be approximated by the expression,

$$V(R) = c - a(R - R_0)^2, \quad R \leq R_m$$

$$= \frac{2Z e^2}{R}, \quad R > R_m,$$

where $Z$ is the charge number of the daughter nucleus. The parabolic form of the potential in the first line is defined by the parameters $R_0$, $c$ and $a$. These parameters are found by a fitting procedure and the matching radius $R_m$ is determined by imposing the continuity of the potential. An example of the quality of the fitting procedure can be seen in Fig. 1.

We will apply our method to $\alpha$-decay from even-even emitters to yrast states in the daughter nuclei for which there are experimental data \cite{3}. We found that in these cases the parameters of the analytic potential (2.1) are approximately given by,

$$R_0 = 3.24A^{0.234}, \quad \sigma = 0.0009$$

$$c = 0.97\frac{2Z e^2}{R_0}, \quad \sigma = 0.0054$$

$$a = 1.76A^{0.177}, \quad \sigma = 0.0198.$$
where \( Z (A) \) is the charge (mass) number of the daughter nucleus. The standard errors \( \sigma \) are also given.

\[\text{Figure 1.} \quad \text{Double folding potential (dashed line) and the parabolic fit of the internal region (solid line).}\]

We will evaluate the partial decay widths using the spherical semiclassical approximation, which is known to be 1-2\% accurate with respect to the exact solution \[9\]. The action integral for the nuclear interaction using the expression Eq. (2.1) can now be evaluated analytically. We found out that \( R_m \approx R_0 + 0.3 \text{ fm} \). We will evaluate the action integrals in the inner and outer intervals divided by the radius \( R_0 \). Thus, the inner part becomes

\[
K_{\text{int}}(Q) = \int_{R_1}^{R_0} \sqrt{\frac{2\mu}{\hbar^2} \left[ -a(R - R_0)^2 + c - Q \right]} dR
\]

\[
= \frac{1}{2} \sqrt{\frac{2\mu}{\hbar^2} (R_0 - R_1)} \sqrt{-a(R_0 - R_1)^2 + c - Q}
\]

\[
+ \sqrt{\frac{2\mu (c - Q)}{\hbar^2}} \frac{1}{2a} \tan^{-1} \left( \frac{\sqrt{a(R_0 - R_1)}}{\sqrt{-a(R_0 - R_1)^2 + c - Q}} \right),
\]

where \( Q \) is the Q-value of the process and \( R_1 \) is the innermost turning point, which has the value

\[
R_1 = R_0 - \sqrt{\frac{c - Q}{a}}. \tag{2.4}
\]

A similar action integral for the outer part gives

\[
K_{\text{ext}}(\chi, \rho_0) = \int_{\rho_0}^{\chi} \frac{1}{\rho} d\rho \quad \tag{2.5}
\]
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\[ \chi = \frac{4Ze^2\mu}{\hbar \kappa} \]

where the Coulomb parameter \( \chi \) and reduced radius \( \rho \) are given by,

\[ \chi = 4Ze^2\mu \]
\[ \rho = \kappa R , \quad \kappa = \sqrt{\frac{2\mu Q}{\hbar}} \]

The partial decay width to the excited state with spin \( J \) and excitation energy \( E_J \) is proportional to the exponent of the sum of the two action integrals [9],

\[ \Gamma_{J}^{(th)} = \exp \left\{ -2 \left[ K_{int} \left( Q - E_J - \frac{\hbar^2 J(J+1)}{2\mu R^2} \right) \right. \right. \]
\[ \left. \left. + K_{ext}(\chi_J, \kappa_J R_0) + \frac{J(J+1)}{\chi_J} \sqrt{\frac{\kappa_J R_0}{\chi_J} - 1} \right] \right\} , \]

where the centrifugal potential was evaluated at

\[ \bar{R} = \frac{R_1 + R_0}{2} = R_0 - \frac{1}{2} \sqrt{\frac{c}{a}} . \]

and the channel values of the Coulomb parameter and momentum, respectively, are given by,

\[ \chi_J = \frac{4Ze^2\mu}{\hbar \kappa_J} \]
\[ \kappa_J = \sqrt{\frac{2\mu(Q - E_J)}{\hbar}} . \]

The total decay width is given by the sum of the corresponding partial widths, i. e.,

\[ \Gamma^{(th)} = \sum \Gamma_{J}^{(th)} . \]

Let us stress on the fact that the parameters of the interaction potential [2,2] describe scattering data of α-particles. Thus, the α-particle is supposed to exist with the unity probability. In order to estimate total and partial α-decay formation probabilities we define total and partial spectroscopic factors respectively, as,

\[ S = \frac{\Gamma^{(exp)}}{\Gamma^{(th)}} , \quad S_J = \frac{\Gamma_J^{(exp)}}{\Gamma_J^{(th)}} . \]

3. Numerical application

We analyzed available experimental data concerning α-decays to excited states in even-even nuclei [3]. In Fig. 2 we plotted the total spectroscopic factor as a function of the neutron number for (a) \( N < 126 \) and (b) \( N > 126 \). One can see in this Figure the striking feature that the logarithm of the spectroscopic factor follows two separate lines depending upon whether the neutron number is larger or smaller than the magic number \( N = 126 \).
It is convenient to analyze the α-decay fine structure in terms of the so-called decay intensities \[ I_J = \log_{10} \frac{\Gamma_0}{\Gamma_J}. \] (3.1)

In Fig. 3 we plotted the intensities defined by the above relation versus the neutron number corresponding to (a) \( J = 2 \), (b) \( J = 4 \) and (c) \( J = 6 \). In this Figure the experimental values are represented by dark symbols while open symbols correspond to the results of our calculations. One sees that the experimental features are reasonable well reproduced by the theoretical estimates. In Fig. 4 we plotted the same values, but as a function of the excitation energy \( E_2 \). One notices the linear increasing trend of the intensity \( I_2 \), as predicted in Ref. [11].

In order to avoid the exponential influence of the penetrability in the decay process one usually defines the hindrance factor (HF) as \[ HF_J = \frac{S_0}{S_J} = \frac{\Gamma_0^{(exp)} \Gamma_J^{(th)}}{\Gamma_J^{(exp)} \Gamma_0^{(th)}}. \] (3.2)

The logarithm of this quantity can be written as a difference between experimental and theoretical decay intensities, i.e.

\[ \log_{10} HF_J = I_J^{(exp)} - I_J^{(th)}. \] (3.3)

This difference characterizes the other elements which we neglected in our simple approach, namely the deformation, given by the coupling between channels, and the clustering probability in the decay process.
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Figure 3. Intensity versus neutron number for $J = 2$ (a) $J = 4$ (b) and $J = 6$ (c).

Figure 4. Same as in Fig. 3 but versus the excitation energy $E_2$.

In Fig. 5 we plotted $\log_{10} HF_J$ as a function of the neutron number. For the states $J = 2^+$ one sees that the fine structure can be reproduced rather well in the region $N > 126$, while for $N < 126$ the above mentioned features are necessary in order to explain experimental data. For the transitions to the states $J = 4^+$ one sees a maximum in the region of Pu isotopes, as already found in Ref. [10]. This is connected to a subshell effect [13].
Figure 5. Logarithm of the hindrance factor \( \log_{10} H_F \) versus neutron number for \( J = 2 \) (a) \( J = 4 \) (b) and \( J = 6 \) (c).

Figure 6. Same as in Fig. 5 but versus the excitation energy \( E_2 \).

In Fig. 6, the hindrance factor is also shown as a function of the excitation energy \( E_2 \). As mentioned, the theoretical deviations are to be regarded as due to the influence of deformation and \( \alpha \)-clustering upon the fine structure.
4. Conclusions

In conclusion, we have analyzed the $\alpha$-decay fine structure to low-lying excited states in even-even nuclei by using an analytical semiclassical approach. We approximated the realistic $\alpha$-daughter double folding interaction by a parabola in the spatial region which is relevant in the decay process. Partial decay widths were estimated by using standard spherical semiclassical approach where the action integrals have close analytical forms. By analysing hindrance factors, we found that the main experimental features are reproduced by this simple method within one order of magnitude. Further improvement is due to the deformation effects induced by the coupling between multipoles and $\alpha$-particle formation probabilities.

Acknowledgments

This work was supported by the Royal Institute of Technology, Stockholm, the Swedish Research Council (VR) under Grants No. 621-2012-3805 and No. 621-2013-4323, and by the Grants of the Romanian National Authority for Scientific Research, CNCS-UEFISCDI, PN-II-ID-PCE-2011-3-0092 and PN-09370102.

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