Unmanned Aerial Vehicle’s Formation Control-Based on Variable Structure

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Abstract. The paper deduces the state equation of line-of-sight angle based on the relative motion equation between the host and the wing plane. On this basis, a Formation-keeping controller is designed with variable structure control strategy, which controls the line-of-sight angle between the host and the wing man in a certain setting, in order to ensure that multiple UAVs fly in a fixed formation without collision. Fixed values are used to achieve the goal of UAV flying in a fixed formation. The effectiveness of the controller is proved in the framework of Lyapunov stability theory. The simulation results show that the designed controller can control the line-of-sight angle at a fixed value, and is robust to maneuvering and disturbance, thus achieving the goal of UAV flying in a fixed formation.

1. Introduction
In modern warfare, unmanned aerial vehicle (UAV) has become an important equipment to carry out combat tasks because of its advantages of zero casualties. In view of the complexity and diversity of combat tasks, multiple UAVs are often required to cooperate with each other to perform tasks. The formation control problem of UAVs needs to be solved urgently.

At present, formation control of UAV has become a hot research topic in the field of UAV, and a lot of research work has been carried out at home and abroad. The MICA project funded by DAR-PA focuses on improving UAV’s autonomous and cooperative control capability[1]; Galzi uses continuous high-order sliding mode control method to design formation controller, which has a certain ability to suppress external disturbances[2]; Jing Xiaonian and others use graph theory knowledge and multi-intelligence. The control model of cluster motion is established based on the unified consistency theory of energy system, and the dynamic projection control system is designed by Zhu Yi and others based on robust optimal theory and dynamic projection method. Shen Peijun studied the small high-speed UAV and its cooperative formation control. The improved robust LQR combined with model reference adaptive control was used to design the control law for UAV.

In this paper, based on the relative motion equation of the main aircraft and the wing man, the state equation of the line-of-sight angle change is deduced. The variable structure controller is designed to control the line-of-sight angle, and has strong robustness to disturbance and maneuver, thus the formation control of UAV is realized.

2. Problem Description and Modeling
According to different operational requirements, multiple UAVs are often required to perform operational tasks in clusters. How to ensure that multiple UAVs fly steadily in accordance with the set formation without collision is very important for UAVs formation control. The flying formations often used by UAVs are guided by following formations and left and right diamond formations. In this
paper, the left and right diamond formation is taken as the research object, and the state equation of the control object is established based on the relative motion relationship between the main engine and the bureaucrat, and the controller is designed on this basis.

The plane geometry relationship of formation flying is shown in Fig. 1.

In the oxy plane, the speed of the mainframe and the wingman is the same. As long as the angle of sight of the mainframe and the wingman is controlled at a certain angle and kept constant, the formation of the mainframe and the wingman can be guaranteed to remain unchanged.

The relative motion relationship between the long plane and the wing plane in the oxy plane can be described in a complete formula (1).

\[
\begin{align*}
R &= V \cos(q - \varphi) - V \cos(q - \varphi_m) \\
Rq &= -V \sin(q - \varphi) + V \sin(q - \varphi_m)
\end{align*}
\]  

(1)

In Formula (1), \(R\) represents the relative distance between the mainframe and the wingman, \(V\) represents the speed, \(q\) represents the line of sight angle, and \(\varphi, \varphi_m\) represent the speed direction angle of the mainframe and the wingman, respectively.

In order to facilitate the design of the controller, the state equation of the change of the line of sight angle of the mainframe and the wingman is deduced through the relative motion equation, as described in Formula (1). The deduction process is as follows.

After substituting \(V_R = \dot{R}, V_q = R \dot{q}\) in Formula (1), we can get as follows through differentiate Formula (1) for time.

\[
\begin{align*}
V_R &= \dot{q}(V \sin(q - \varphi) + V \sin(q - \varphi_m)) + [V \cos(q - \varphi) + V \varphi \sin(q - \varphi)] - [V \cos(q - \varphi_m) + V \varphi_m \sin(q - \varphi_m)] \\
V_q &= -\dot{q}[V \cos(q - \varphi) - V \cos(q - \varphi_m)] + [V \varphi, \cos(q - \varphi) - V \sin(q - \varphi)] - [V \varphi_m \cos(q - \varphi_m) - V \sin(q - \varphi_m)]
\end{align*}
\]  

(2)

(3)

Supposing:

\[
\begin{align*}
w_R &= V \cos(q - \varphi) + V \varphi \sin(q - \varphi) \\
u_R &= V \cos(q - \varphi_m) + V \varphi_m \sin(q - \varphi_m) \\
w_q &= V \varphi, \cos(q - \varphi) - V \sin(q - \varphi) \\
u_q &= V \varphi_m \cos(q - \varphi_m) - V \sin(q - \varphi_m)
\end{align*}
\]  

(4)

(5)

(6)

(7)

After plugging Formula (4-7) into Formula (2, 3), we can get as follows.

\[
\begin{align*}
V_R &= \frac{V_R^2}{R} + w_R - u_R \\
V_q &= -\frac{V_q V}{R} + w_q - u_q
\end{align*}
\]  

(8)

(9)
Plugging $V_\alpha = R \cdot V_q = R \cdot q$ into Formula (9), we can get as follows.

$$q = \frac{2R}{R} q + \frac{1}{R} w_q - \frac{1}{R} u_q \tag{10}$$

Supposing $x_1 = q, x_2 = q$, the formula (10) can be described by the following equation of state:

$$\begin{align*}
x_1 &= x_2 \\
x_2 &= \frac{2R}{R} x_2 - \frac{1}{R} u_q + \frac{1}{R} w_q 
\end{align*} \tag{11}$$

In Formula (11), $u_q = \frac{\dot{\varphi}}{\text{cos}(\varphi)} \cdot \text{sin}(\varphi - \varphi_0)$ can be designed as a controller because it is the amount related to the heading angle and speed of the wing aircraft and wing aircraft follows long aircraft. $w_q$ can be regarded as interference because of its relationship with long aircraft. The following is the controller design for the system described in equation of state (11).

3. Controller Design

Because variable structure sliding mode control has the advantages of good robustness and strong anti-jamming ability, this paper adopts variable structure control to design the controller for control variable $u_q$. Because the control objective is to keep the line of sight $x_1$ constant, which is controlled at 45 degrees, the sliding mode surface is taken as follows.

$$s = c(x_1 - \frac{\pi}{4}) + x_2 \tag{12}$$

Because $c > 0$, the line of sight angle $x_1 = q$ tends to be $\frac{\pi}{4}$ stably on the sliding surface $s = 0$. Select the Lyapunov function $V = \frac{s^2}{2}$, which will be obtained relative to the time derivative.

$$V = \dot{s} = s \cdot (c \cdot x_1 + x_2) = s \cdot (c x_2 - \frac{2R}{R} x_2 - \frac{1}{R} u_q + \frac{1}{R} w_q)$$

Design controller as follows $u_q = s + R \cdot c x_2 - 2 R x_3 + \varepsilon \cdot \text{sgn}(s)$, then

$$V = s[-s - \frac{\varepsilon}{R} \text{sgn}(s) + \frac{1}{R} w_q] = -\frac{1}{R} s^2 - \frac{s}{R} [\varepsilon \text{sgn}(s) - w_q]$$

Since $R > 0$ and the first item in the formula is obviously less than zero, and the second item is less than zero as long as $\varepsilon > |w_q|$ (if it is not satisfied, it can be obtained by increasing $\varepsilon$), so $\dot{V} < 0$. Because $V = \frac{s^2}{2} \geq 0$, $s \to 0$ when $V \to 0$.

Therefore, the system described by equation of state (11) converges steadily to the sliding surface $s = 0$ under controller $u_q = s + R \cdot c x_2 - 2 R x_3 + \varepsilon \cdot \text{sgn}(s)$, while the line of sight angle $x_1 = q$ tends steadily to 45 degrees on the sliding surface $s = 0$.

In order to eliminate the inherent chattering in variable structure control, supposing $\text{sgn}(s) = \frac{s}{|s| + \delta}$, it is theoretically proved that controller $u_q = s + R \cdot c x_2 - 2 R x_3 + \varepsilon \cdot \frac{s}{|s| + \delta}$ can control the sliding surface to a tiny neighborhood near zero, and is limited to space.

4. Simulation Analysis

The simultaneous equations are established by using the relative motion equation of the front host and the wingman, the system state equation and the controller equation as follows.
There are seven unknowns and seven equations in the system of equations (13), in which the rate is constant and the system has unique solutions. The four orders Runge-Kutta method is used to solve the equations. The initial conditions are as follows: the speed of the host and the wingman is $V = 30$ m/s, the initial line of sight angle is 35 degrees, the initial course angle of the host is 25 degrees, the initial course angle of the wingman is 15 degrees, the initial distance between the host and the wingman is 100m, the constant $c=2$, $\varepsilon = 10$, $\sigma = 0.01$, the simulation step is 0.001 s, and the simulation time is 30s. The simulation results are as follows:

**Figure 2.** Sight angle between main wing planes.

**Figure 3.** Angular velocity of line of sight.

**Figure 4.** Heading angle diagram of wing aircraft.

**Figure 5.** Distance between main wing planes.
From the simulation results, it can be seen that the line-of-sight angle between the main engine and the bureaucracy is 35 degrees at the initial time, which does not meet the requirements of formation formation 45 degrees. Under the control of the controller, the bureaucracy adjusts the course angle, which makes the line-of-sight angle between the main bureaucracies rapidly adjust to 45 degrees and remains stable. When the line-of-sight angle between the host and the wingman is controlled at 45 degrees, the distance between the host and the wingman remains unchanged, and the overall formation remains unchanged, which meets the requirements of formation control.

The controller designed in this paper also has strong robustness. The following example is simulated and validated by the direct flight of mainframe with constant speed after maneuvering through an obstacle. In the simulation, the sinusoidal function of the heading angle of the main engine in the initial period of time is used to simulate the maneuvering of the main engine.
From the simulation results, it can be seen that the course of the main engine changes when it passes through the obstacle, and the wingman changes the course under the control of the controller, keeping the formation unchanged as far as possible, so as to bypass the obstacle with the main engine. After the main engine and the wing plane bypass the obstacles, they continue to maintain the original formation flight. The angle of sight between the main engine and the wing plane fluctuates around the obstacle, and it keeps at 45 degrees after the obstacle is passed. From the modeling process of the previous control system, it can be seen that the controller treats the main engine maneuver as disturbance, and uses the robustness of variable structure control to suppress the disturbance, so as to achieve the control purpose. The simulation results also verify this point. When the main engine moves around obstacles, the stability of the controller cannot be changed.

5. Conclusion
Under the guidance of the idea that the UAV formation can remain unchanged as long as the line-of-sight angle between the host and the wingman can be controlled at a fixed angle, the state equation of the change of the line-of-sight angle between the host and the wingman is deduced through the relative motion equation between the host and the wingman, and the controller is designed by using the variable structure control method. The theoretical derivation and simulation show that the designed controller can ensure that the line of sight angle between the host and the bureau is controlled at the specified angle, and has strong robustness to maneuvering.

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