Optimal power flow calculation for power system with UPFC considering load rate equalization

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Abstract. Unified power flow controller (UPFC) device can change system electrical quantity (such as voltage, impedance, phase angle, etc.) rapidly and flexibly under the premise of maintain security, stability and reliability of power system, thus can improve the transmission power and transmission line utilization, so as to enhance the power supply capacity of the power grid. Based on a thorough study of the steady-state model of UPFC, taking load rate equalization as objective function, the optimal power flow model is established with UPFC, and simplified interior point method is used to solve it. Finally, optimal power flow of 24 continuous sections actual data is calculated on a typical day of Nanjing network. The results show that the optimal power flow calculation with UPFC can optimize the load rate equalization on the basis of eliminating line overload, improving the voltage level of local power network.

1 Introduction
The power industry is an important basic industry of national economy. In recent years, the power system is developing towards the direction of ultra high voltage, long distance and large capacity. And the scale of system is more and more big, the structure is more and more complex. It becomes more and more important to ensure the safe and stable operation of power system[1]. In addition, the problem of uneven load distribution of transformers and lines exists generally. And because of the transmission bottleneck of some heavy equipment, the power supply capacity is difficult to significantly enhance. Therefore, it is urgent to put forward a method to solve the problem of heavy load, which can improve the power flow distribution and the power supply capacity.

At present, due to the rapid improvement of high power electronics technology and computer control technology, flexible AC transmission system (FACTS) technology comes into being[3]. UPFC is the third generation of FACTS devices, of which function is very powerful that can control the line impedance, voltage and power angle, and can also control the active and reactive power flow of transmission line[4-6]. It provides a new technical mean to solve the problem of uneven distribution of power flow and the lack of power flow control model. UPFC can be more comprehensive and flexible to meet the control requirements, it can control many kinds of steady state characteristics of the system at the same time, such as power flow and bus voltage, which can greatly improve the security and stability of the system and improve the power quality and transmission efficiency.

When system structure parameters and load are known, and by optimizing the control variables, we can find the power flow which can satisfy all the constraints and make the objective function optimal. And this power flow distribution is optimal power flow (OPF). UPFC hanging network to operate adds new means to the optimal control of power system, therefore, it is of great significance to study the OPF of the power system with UPFC[5]. Different steady state models of UPFC are discussed in [6], and the
advantages and disadvantages of each model are analyzed and compared. In [7], static model of UPFC is established based on interior point method, and the lecture studies the reactive power optimization problem of the system with UPFC. The OPF problem with UPFC is studied in [8], and the effect of UPFC on the network loss, generation cost, node voltage and line load are analyzed. Taking system network loss and static voltage stability margin as the objective function, multi reactive power optimization model with UPFC is established to analyze the influence of UPFC on the voltage stability of power system in [9]. According to the load rate of transmission lines in power system, the concept of power grid operation equilibrium is proposed in [10]. The influence of load rate distribution on self-organized criticality is analyzed in [11]. Although the OPF of power system with UPFC and load rate are studied in above documents, the OPF problem which takes load rate equalization as objective function of power system has not been studied.

Based on the above discussion, this paper takes load rate equalization as objective function, calculates OPF of the power system with UPFC, and tests in a real system. This paper is organized as follows. The steady state model of UPFC is introduced in Section 2, followed by the OPF model with UPFC introduction in Section 3. Section 4 presents simulation results on the 24 continuous sections actual data on a typical day of Nanjing network. Finally, conclusion is drawn in Section 5.

2 Steady state model of UPFC device

In steady state operation, UPFC is used as passive component, which does not have active exchange with the system, and the DC capacitor voltage remain constant. Therefore, it is necessary to ensure that the input active power of the parallel side is equal to the output power of the series side, that is, UPFC does not inject active power to the power system or absorb active power from the power system[12]. This is a constraint that must be considered in the establishment of the UPFC steady state model, and any steady state model of UPFC can not violate it. In the analysis of electric power system, there are several steady models, such as power injection model, independent branch model, and decoupled model. The power injection model does not need add new bus, and can achieve power flow calculation of power system with UPFC simply. The independent branch model is able to consider selflessness of UPFC, which can improve the accuracy of results. Because the independent branch model is more accurate, this paper adopts this model, and it is introduced as follow.

![Fig.1. Dual voltage sources model of UPFC](image)

The basic idea of the independent branch model of UPFC is to divide the branch of UPFC into the UPFC branch and the original branch, so that UPFC becomes an independent branch to participate in the power flow calculation and the optimal power flow calculation. Fig.1 represents the dual voltage sources model of UPFC. Bus $s$ is the original bus of branch and bus $r$ is a new bus which is added at the end of UPFC. UPFC consists of a shunt voltage source $\dot{V}_e$ and a series voltage source $\dot{V}_b$, where $R_e$, $R_s$ are the resistances of the series voltage source and the shunt voltage source, and $X_e$, $X_s$ are the reactance associated with the series and shunt voltage source. UPFC has powerful control function, which can adjust the bus voltage, bus phase and power flow at the same time.
Fig. 2 represents the independent branch model of UPFC, which is change from Fig. 1. The effect of UPFC is equivalent to the injection power of the two buses, and bus \( s \), bus \( r \) are no longer connected. According to Fig. 2, the injection power \( \hat{S}_s \), \( \hat{S}_r \) of bus \( s \) and bus \( r \) can be calculated as follow:

\[
\begin{align*}
\hat{S}_s &= P_s + jQ_s = \hat{V}_s \hat{I}_s = \hat{V}_s (I_s - I_r) \\
\hat{S}_r &= P_r + jQ_r = \hat{V}_r \hat{I}_r
\end{align*}
\]

From Fig. 1, we can assume:

\[
\begin{align*}
g_e + jh_e &= \frac{1}{R_e + jX_e} \\
g_b + jh_b &= \frac{1}{R_b + jX_b}
\end{align*}
\]

and from (1) and (2), we can get the injection power at both ends of branch:

\[
\begin{align*}
P_s &= (g_e + g_b)V_s^2 - V_s V_E \left[ g_e \cos(\theta_s - \theta_E) + b_e \sin(\theta_s - \theta_E) \right] \\
&\quad - V_s V_E \left[ g_b \cos(\theta_s - \theta) + b_b \sin(\theta_s - \theta) \right] \\
&\quad + V_s V_E \left[ g_b \cos(\theta_s - \theta_b) + b_b \sin(\theta_s - \theta_b) \right] \\
Q_s &= -(b_b + b_b) V_s^2 - V_s V_E \left[ g_e \sin(\theta_s - \theta_E) - b_e \cos(\theta_s - \theta_E) \right] \\
&\quad - V_s V_E \left[ g_b \sin(\theta_s - \theta) - b_b \cos(\theta_s - \theta) \right] \\
&\quad + V_s V_E \left[ g_b \sin(\theta_s - \theta_b) - b_b \cos(\theta_s - \theta_b) \right]
\end{align*}
\]

\[
\begin{align*}
P_r &= g_e V_r^2 - V_r V_E \left[ g_e \cos(\theta_r - \theta) + b_e \sin(\theta_r - \theta) \right] \\
&\quad - V_r V_E \left[ g_b \cos(\theta_r - \theta_b) + b_b \sin(\theta_r - \theta_b) \right] \\
Q_r &= -b_b V_r^2 - V_r V_E \left[ g_e \sin(\theta_r - \theta_E) - b_e \cos(\theta_r - \theta_E) \right] \\
&\quad + V_r V_E \left[ g_b \sin(\theta_r - \theta_b) - b_b \cos(\theta_r - \theta_b) \right]
\end{align*}
\]

where \( \theta \) represents the phase angle of the corresponding buses and voltage resources.

The expression of the active power \( P_e \) of the shunt side and the active power \( P_b \) of the series side of UPFC can be expressed as

\[
\begin{align*}
P_e &= -g_e V_e^2 + V_e V_E \left[ g_e \cos(\theta_e - \theta_E) + b_e \sin(\theta_e - \theta_E) \right] \\
P_b &= -g_b V_b^2 + V_b V_E \left[ g_b \cos(\theta_b - \theta_E) + b_b \sin(\theta_b - \theta_E) \right] \\
&\quad - V_b V_E \left[ g_b \cos(\theta_b - \theta) + b_b \sin(\theta_b - \theta) \right]
\end{align*}
\]

3 Optimal power flow model with UPFC

The optimal power flow problem with UPFC can usually be expressed as
\[
\begin{align*}
\min \ & f(x) \\
\text{s.t.} \ & h(x) = 0 \\
\ & g \leq g(x) \leq \bar{g}
\end{align*}
\]

this is a typical nonlinear programming problem, where \( f(x) \) is the objective function, \( h(x) \) is equality constraint, and \( g(x) \) is inequality constraint.

3.1 Objective function

In order to ensure the economy, the optimal power flow problem usually uses generator cost or system network loss as objective function. However, in order to improve the security of the system, make network have higher transmission capacity margin and meet the requirements of the power load change, this paper selects load rate equalization as objective function.

Generally, the ratio of active power flow of power system equipment to the ultimate active power flow of the equipment is defined as the load rate, which can reflect the reserve capacity of transmission lines to some extent. The smaller the load rate is, the greater the transmission margin of power transmission network is. The power flow regulation capability of the power grid is stronger. This illustrates the network can meet the load growth and system scheduling requirements in the future better. However, the load rate can only reflect the operation status of one transmission line, and can not reflect the overall load level of the network. Average load rate is the average load rate of all lines, although it can reflect the load level on the whole, it can not reflect the relationship between the load rate of transmission line and average load rate. There may appear some lines far greater than or less than the average load rate, so it is more reasonable to use the load rate equalization to reflect whether the branches of the power network are balanced, and whether the load rate is near the average load rate.

Therefore, the objective function can be expressed as

\[
\min \ \sqrt{\frac{1}{N_l} \sum_{i=1}^{N_l} \left( R_{i,j} - \mu \right)^2}
\]  \hspace{1cm} (10)

where

\[
R_{i,j} = \frac{P_{i,j}}{P_{l,ij,max}} \hspace{1cm} (11)
\]

\[
\mu = \frac{\sum_{i=1}^{N_l} R_{i,j}}{N_l} \hspace{1cm} (12)
\]

and \( N_l \) represents the number of system branches, \( R_{i,j} \) represents the load rate of branch \( l \), \( P_{i,j} \) represents actual transmission power of branch \( l \), \( P_{l,ij,max} \) represents transmission power limit of branch \( l \) and \( \mu \) is the average load rate of all branches.

3.2 Equality constraint

1) Power flow equation

When UPFC is not installed in the system, the equality constraints is ordinary power flow equations. While when UPFC is installed in the system, the power flow of the branch with UPFC changes. Compared with the original power flow equation, the additional injection power of UPFC is increased. The power equation of UPFC connected buses can be expressed as
\[
\begin{align*}
\Delta P_s &= P_{gs} - P_{ls} - V \sum_{j=1}^{n} V_j (G_{ij} \cos \theta_j + B_{ij} \sin \theta_j) - P_s' \\
\Delta Q_s &= Q_{gs} - Q_{ls} - V \sum_{j=1}^{n} V_j (G_{ij} \sin \theta_j - B_{ij} \cos \theta_j) - Q_s'
\end{align*}
\]

where \( P_{gs}, Q_{gs} \) represents the active and reactive power of generation connected to bus \( s \), \( P_{ls}, Q_{ls} \) represents the active and reactive injection power of bus \( s \).

2) Active power balance constraint of UPFC device
UPFC is neither absorb nor emit active power

\[ P_k + P_B = 0 \]  

(14)

3.3 Inequality constraint

1) Upper and lower bounds of variables
Compared with the traditional optimal power flow model, the UPFC control variables are added to the optimal power flow model with UPFC

\[
\begin{align*}
P_{11} &\leq P_{1i} \leq P_{11} & i \in (1, \ldots, N) \\
Q_{11} &\leq Q_{1i} \leq Q_{11} & i \in (1, \ldots, N) \\
V &\leq V_i \leq \bar{V} & i \in (1, \ldots, N) \\
V_e &\leq V_{ei} \leq V_{e1} & i \in (1, \ldots, N_{UPFC}) \\
V_{be} &\leq V_{bi} \leq V_{b1} & i \in (1, \ldots, N_{UPFC}) \\
P_{11} &\leq P_{1i} \leq P_{11} & i \in (1, \ldots, N_{UPFC})
\end{align*}
\]

(15)

2) UPFC capacity constraint

\[ 0 \leq S_i \leq \bar{S}_i & \quad i \in (1, \ldots, N_{UPFC}) \]

(16)

4 Numerical cases
In order to illustrate the performance of the model proposed in the previous sections, we make a test on Nanjing Network. The distribution of power generation and load of Nanjing Network is not balanced. The power flow distribution of transmission line is uneven, and the power supply capacity is limited. Above all, the UPFC project appears. The devices are installed at the north-side of Xiaozhuang-Tiebei double-line, as is shown in Fig.3.
In order to analyze and display the state of the power flow conveniently, this paper firstly makes an equivalent treatment on data and obtain Nanjing 117-buses equivalent system. The comparison of before and after equivalence is shown in Table 1.

| Bus number | Branch number | Generator number |
|------------|---------------|------------------|
| Before equivalence | 3251 | 5649 | 290 |
| After equivalence   | 117  | 194  | 13  |

Table 1. The comparison of before and after equivalence

The installation position of the two UPFC is consistent with the actual project, and the corresponding parameters are shown in table 2.

| Series Capacity /MVA | Shunt Capacity /MVA | $\sigma_x$ /pu | $\psi_x$ /pu | $\sigma_x/\psi_x$ | $x_x$ | $x_y$ |
|----------------------|---------------------|---------------|--------------|-------------------|-------|-------|
| 60                   | 60                  | 1.06          | 0.94         | 0.412             | 0     | 0.1   | 0.2   |

Table 2. Parameters of UPFC

This paper takes a typical day data of Nanjing network in 2016, per hour as 1 sections, and there are 24 sections totally. This paper tests in three control modes of UPFC. To describe convenience, UPFC not involved in optimization is defined as Model 1, UPFC control parameters optimize together is defined as Model 2, UPFC control parameters set in advance is defined as Model 3.

The voltage results of section 1 are shown in Fig 4. The results and conclusions of the other 23 sections are similar to those of the first section.

![Fig 4. The voltage results of the two models on section 1](image)

According to Fig 4, we have following conclusions: because of the strong ability to restrain the voltage of optimal power flow calculation, the voltage curves are maintained within ±5% under Model 2 and Model 3. And due to taking the load rate equalization as the optimization target, the power flow of the system changes greatly, so the voltage deviation is big before and after setting the control parameters of UPFC. But Model 3 can be used to control the voltage magnitude of UPFC parallel side bus at 1.0pu, which proves the powerful control function of UPFC.
Fig 5 shows load rate equalization of 24 sections under three control modes. According to Fig 5, we can see, because of the similarity and continuity of the power flow of the 24 sections, the optimal load rate equalization is similar, ranging from 0.09 to 0.12. The optimization results show that if the optimized scheduling based on this program, the power flow of power system will get great equilibrium. It can not only eliminate the hidden dangers of the line, but also guide the power grid planning and save the investment cost. Compared to Model 1, the load rate equalization of other two models all decreased, which shows that UPFC has the function of optimizing the load rate equalization of line. Because it takes the load rate equalization as the target, therefore, whether the UPFC is involved in the optimization and the setting of the control parameters will lead to large deviation. It is necessary to choose UPFC control mode according to different situations. When the load of line is light, it is better to take Model 2. When the load of line is heavy, UPFC needs to eliminate overloading firstly. So it is better to take Model 3 to reduce the load rate equalization. And it is acceptable in project to sacrifice economy so that the safety can be ensured.

5 Conclusion
In this paper, a optimal power flow model for power system with UPFC, which takes load rate equalization as objective function, has been proposed. In general, the optimal power flow is calculated by using the generator cost or the system network loss as the objective function, but this paper takes the load ratio equalization as the objective function, which is new, to study the optimal power flow of power system with UPFC. The new objective function can reflect the load level and safety of the whole power grid. Besides, we can analyze the regulating ability of UPFC from various angles by comparing the optimal power flow calculation results before and after the adjustment of UPFC, and obtain the control scheme of UPFC under different conditions. A test on 24 sections of Nanjing network has been carried out, and we can obtain the following conclusions:

1) Through the optimal power flow calculation, the system flow has been greatly balanced. By taking the load rate equalization as the optimization objective, the overall power flow of the power grid is adjusted greatly, and the power flow of the system is more sensitive.

2) UPFC has powerful control function and the control mode switching scheme is obtained by optimizing the three control modes of UPFC. Under the condition of light load, it is better to take the model UPFC control parameters optimize together. While under the condition of heavy load, the first target of UPFC is to eliminate overloading, it is better to take the model UPFC control parameters set in advance optimization, which can improve the safety of the power system.
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