Oscillations of magnetoResistance of 2DEG in a weak magnetic field under microwave irradiation

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Under microwave irradiation, in 2D electron systems with high filling factors oscillations of longitudinal magnetoResistance appear in the range of magnetic fields where ordinary SdH oscillations are suppressed. An unusual beat-like behaviour of these oscillations in weak magnetic fields \( B \leq 0.02 \) T was attributed to the zero spin splitting previously. We propose an alternate explanation of this beat-like structure on the basis of the Boltzmann kinetic equation, without any reference to the spin-orbit interaction.

The interest to theoretical investigations of non-linear transport phenomena in two-dimensional electron systems (2DES) has grown substantially due to new experimental results obtained in very clean 2DES samples. It has been discovered independently by two experimental groups [1, 2] that the resistance of two-dimensional high-mobility electron gas in GaAs/AlGaAs heterostructures reveals a series of new features in its dependence on magnetic field, temperature, radiation power, etc. Under microwave irradiation, in 2DES with high filling factors oscillations of longitudinal magnetoResistance appear in the range of magnetic fields where ordinary SdH oscillations are suppressed.

Among the new effects, an unusual behavior of longitudinal magnetoResistance in weak magnetic fields \( B \leq 0.02 \) T is worth mentioning. E.g., in experiments by Zudov [1] performed in GaAs/AlGaAs heterostructures (with electron mobility \( \mu = 25 \times 10^6 \) cm²/Vs and electron density \( n_e = 3.5 \times 10^{11} \) cm⁻²) vanishing of amplitude of magnetoResistance oscillations has been observed at \( B = 110 \) T. Upon further lowering of the magnetic field, the amplitude of oscillations grows again (see the inset in Fig. 1). This feature of magnetoResistance has been interpreted as a beat. The analogous effect has been also observed by Mani [3] in GaAs/AlGaAs heterostructures (with electron mobility \( \mu = 15 \times 10^6 \) cm²/Vs and density \( n_e = 3 \times 10^{11} \) cm⁻²). Vanishing of the amplitude of oscillations has been observed in the vicinity of \( B = 240 \) G (see Fig. 1). According to [3], the magnetic field at which the amplitude of magnetoResistance oscillations vanishes does not depend on microwave radiation frequency for the same sample, although the effect itself is visible only in the presence of radiation. The observed “beats” were associated by Mani with “zero-spin splitting” resulting from the spin-orbit interaction [4, 5].

In this paper we will show that the nature of the observed “beat” can be understood in a simple framework without the need for any spin-orbit interaction. First, let’s mention important conditions of the experiments discussed above: the effect is seen at

\[ \hbar/\tau \ll T \approx \hbar \omega_c \leq \hbar \omega \ll \zeta \]

It follows that the effect has a quasiclassical nature. Here \( \tau \) is the momentum relaxation time, \( \omega < \omega_c \) is the radiation and cyclotron frequencies respectively, \( \zeta \) denotes the Fermi energy, \( T \) is the temperature expressed in units of energy.

Since the “beat” expresses itself in the classical range of magnetic fields, we will start from the Boltzmann kinetic equation. Taking into account that the electrons...
are driven from the equilibrium both by external dc electric field and the electric field of radiation, we have:

\[
\frac{\partial f(p)}{\partial t}
\bigg|_{E_{dc}} = e E_{dc} \frac{\partial f(p)}{\partial p} - \frac{e}{mc} [p \times H] \frac{\partial f(p)}{\partial p} = I_{st}(f(p)).
\]

Here \(E_{dc}\) is the strength of the external dc field. The right hand side is the collision integral. We assume that the main electron scattering mechanism is the elastic scattering on impurities:

\[
I_{st}(f(p)) = \frac{-f(p) - f_0(p)}{\tau},
\]

where \(f_0(p)\) is the equilibrium distribution function, \(\tau\) is the relaxation time. \(\frac{\partial f(p)}{\partial t}\) is the rate of distribution function change due to transitions between Landau levels caused by microwave radiation:

\[
\frac{\partial f(p)}{\partial t}
\bigg|_{E_{ac}} = \sum_{p'} w_{pp'} (f(p') - f(p)),
\]

where \(w_{pp'}\) is the probability of the electron transition from the state with the kinetic momentum \(p\) to the state with the kinetic momentum \(p'\) in a unit of time due to microwave radiation. The non-equilibrium distribution function can be represented in the following form:

\[
f(p) = f_0(\varepsilon(p)) + pg(\varepsilon(p)),
\]

where \(f_0(\varepsilon(p))\) is the equilibrium distribution function and \(g(\varepsilon(p))\) is an unknown function that depends only on electron energy. Inserting (1) into the kinetic equation and linearizing upon \(E_{dc}\), we obtain:

\[
\sum_{p'} w_{pp'} (f_0(\varepsilon(p')) - f_0(\varepsilon(p)) + p' g(\varepsilon(p')) - pg(\varepsilon(p)) - \frac{e}{mc} E_{dc} p \frac{\partial f_0(\varepsilon(p))}{\partial \varepsilon} - \frac{e}{mc} [p \times H] g(\varepsilon(p)) = -\frac{pg(\varepsilon)}{\tau}.
\]

The main effect of the ac electric field amounts to changes in electron energy (and not its momentum). Taking all simplifications mentioned into account, we can express the kinetic equation in the following form:

\[
\sum_{p'} w_{pp'} (g(\varepsilon'(p')) - g(\varepsilon(p)) - \frac{e}{mc} E_{dc} p \frac{\partial f_0(\varepsilon(p))}{\partial \varepsilon} - \frac{e}{mc} [H \times g(\varepsilon)] + \frac{g(\varepsilon)}{\tau} = 0.
\]

We shall search for a solution of (6) in the form of a power series on \(w_{pp'}\). Up to the linear terms, we have:

\[
g(\varepsilon) = g_0(\varepsilon) - \frac{2\tau}{(1 + \omega_c^2\tau^2)^2} \frac{\varepsilon}{mc} \left[(1 - \omega_c^2\tau^2)E_{dc} + 2\tau[\omega_c \times E_{dc}\right] \times \sum_{p'} w_{pp'} \left(\frac{\partial f_0(\varepsilon(p'))}{\partial \varepsilon} - \frac{\partial f_0(\varepsilon(p))}{\partial \varepsilon}\right),
\]

where

\[
g_0 = \frac{\tau}{1 + \omega_c^2\tau^2} \frac{e}{mc} (E_{dc} + \tau[\omega_c \times E_{dc}]) \frac{\partial f_0(\varepsilon)}{\partial \varepsilon}.
\]

Here \(\omega_c = (eH)/(mc)\) and it has been taken into account that \(E_{dc} \perp H\).

Performing simple transformations of (6), we obtain:

\[
g(\varepsilon) = g_0(\varepsilon) - \frac{2\tau}{(1 + \omega_c^2\tau^2)^2} \frac{\varepsilon}{mc} \left[(1 - \omega_c^2\tau^2)E_{dc} + 2\tau[\omega_c \times E_{dc}\right] \times \sum_{p} w_{pp'} \left(\frac{\partial f_0(\varepsilon(p))}{\partial \varepsilon} - \frac{\partial f_0(\varepsilon(p))}{\partial \varepsilon}\right),
\]

where \(w_{p\pm} \propto E_{dc}^2 \ell^2 \left((n \pm 1|x|n)\right)^2 \propto E_{dc}^2 n/\omega_c \propto E_{dc}^2 \varepsilon_\pi/\omega_c^2\). \(E_{dc}\) is the amplitude of the ac electric field with the frequency of \(\omega\), \(\rho(\varepsilon)\) is the density of states, and \(\ell = \sqrt{\hbar/(m\omega_c)}\) is the magnetic length.

Using the correction to the distribution function calculated above, we find the current density caused by radiation:

\[
j = \frac{2e}{(2\pi\hbar)^2 \rho_0 m} \int_0^{2\pi} \int_0^\infty \rho' d\rho' \rho(\varepsilon') |p' g(\varepsilon')|,\]

where \(\rho_0 = m/(2\pi\hbar^2)\) is the density of states without the magnetic field for one spin direction.

Inserting the explicit form of \(g\) from (7) into (10), we obtain for diagonal components of the conductivity tensor:

\[
\sigma_{xx}^{ph} = -\frac{2e^2 \ell^2 (2\omega_c^2\tau^2 - 1)}{m (1 + \omega_c^2\tau^2)^2} \times \sum_{p=\pm} \int_0^\infty d\varepsilon \varepsilon w_{p\pm} \rho(\varepsilon)\rho(\varepsilon \pm \hbar\omega) \left(\frac{\partial f_0(\varepsilon \pm \hbar\omega)}{\partial \varepsilon} - \frac{\partial f_0(\varepsilon)}{\partial \varepsilon}\right).
\]

Taking the energy integral under the assumption of strong degeneracy of electrons, we have:

\[
\sigma_{xx}^{ph} \propto \frac{\ell^2 (2\omega_c^2\tau^2 - 1)}{\omega_c^2 (1 + \omega_c^2\tau^2)^2} \rho(\zeta) \times \sum_{\pm} \left(\zeta^2 \rho(\zeta \pm \hbar\omega) - (\zeta \mp \hbar\omega)^2 \rho(\zeta \mp \hbar\omega)\right),
\]

where \(\zeta\) is the Fermi energy. Using an explicit expression for the density of states \(\mathcal{F}\):

\[
\rho(\varepsilon) = \rho_0 (1 - \delta \cos(2\pi\varepsilon/\omega_c)),
\]

\[(\tau_f)\) is a single-particle lifetime without magnetic field, we obtain the following expression for diagonal photoconductivity:

\[
\sigma_{xx}^{ph} - \sigma_{xx}^{ph}, SdH \propto \frac{\ell^2 (2\omega_c^2\tau^2 - 1)}{\omega_c^2 (1 + \omega_c^2\tau^2)^2} \delta \cos(2\pi\omega/\omega_c).
\]
It follows from (14) that photon-assisted conductivity as a function of inverse magnetic field gives rise to slow oscillations: \( \propto \cos(2\pi \omega / \omega_c) \), whereas usual SdH oscillations denoted as \( \sigma_{\text{ph}, \text{SdH}}^{\text{xx}} \), contain factors like \( \cos(2\pi \zeta / \hbar \omega_c) \), are fast and their amplitude decreases rapidly when increasing temperature.

The dependency of photoconductivity upon \( \omega_c / \omega \), calculated according to Eq. (14), is plotted on Fig. 3.

It also follows from Eq. (14) that at \( \omega_c \tau = 1 \) the amplitude of radiation-induced magnetoresistance oscillations vanishes. This is the very fact responsible for the “beat” observed in experiments [1, 2]. Note that the observation of the node of the “beat” in stronger magnetic fields in experiments [2] as compared to [1] is (as one can easily verify) a direct consequence of the lower electron mobility in experiments [2].

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