Evaluation of the influence of directivity factor of directive elements of conformal and planar antenna arrays on the performances of azimuth-elevation DOA estimation

Sarmad K. D. Alkhafaji
Department of Information systems, Voronezh State University,
Universitetskaya sq. 1, Voronezh, 394036, Russia.
Sarmad_kad@yahoo.com

Abstract. The conformal and planar antenna arrays with directive elements are ones with antenna elements located on twisted and flat surfaces and are of great interest for communication systems. The research of direction-of-arrival estimation method with superresolution MUSIC on conformal and planar antenna arrays of different configurations depending on directivity ratio of elements has been fulfilled. The root mean square error rate for one and multiple incident signals has been also has been calculated by simulations. The directivity ratio has been changed from 1 (omnidirectional antenna) up to 30 (high directive antenna). The number of antenna elements was the same and equal to sixteen. Microstrip elements are used having the width of substrate equal to $\lambda/2$, the gap between the elements missed. The scenario with one incident signal source was researched; its azimuth coordinate was changed from 0° up to 180°. The average root-mean-square-error (RMSE) along all the estimated coordinates was calculated and also the minimum and maximum value of the errors was taken into account in order to realize the dependence of the RMSE on the azimuth coordinate. The RMSE of multiple signal sources estimation has been calculated.

1 Introduction
Direction-of-arrival estimation of signal sources takes a great interest in such tasks as radars, sonars and wireless communications [1-6] by using linear antenna arrays. The arrays are simple to implement and to understand, but they are not able to execute simultaneous direction-of-arrival estimation in three-dimensional space, i.e. azimuth and elevation [7]. Planar and conformal antenna arrays are capable of overcoming this problem [8-10]. Additionally, many papers assume that the antennas are isotropic [1-10]. Many conformal volume antenna arrays consist of rectangular patch-antenna elements as a rule, which have directivity factor greater than 1. So that considering the influence of the directivity factor of a particular antenna element on performances of DOA-estimation is of serious interest. Therefore, the paper focuses on researching three-dimensional conformal antenna arrays such as cylindrical, cubic, sphere and cone configurations using the MUSIC method. However, the results of comparative computer simulation can be interpreted as a special case, so, therefore, an instrument is highly needed such as the Cramer-Rao lower bound which is independent of implementation features.
2 Antenna Arrays

Figure 1 shows an array of N rectangular directional elements distributed in space, forming a cubic antenna array. Let us consider the narrow-band signal $s(t)$ on the carrier frequency of $\omega_0$ with the angular coordinates $\theta$ and $\phi$ with respect to the x-, y- and z-axes, respectively, i.e. $\theta$ is related to the azimuth and $\phi$ is related to the vertical planes. Thus, the task of radio direction-finding is to estimate $\theta$ and $\phi$ coordinates. It requires the model of the antenna array.

Let us denote $g_i(\omega, \theta, \phi)$ gain and AE phase depending on the frequency and direction. Then the analytical signal at the AA output is:

$$ x(t) = \left[ g_1(\omega, \theta, \phi)e^{j k_1} \quad g_2(\omega, \theta, \phi)e^{j k_2} \quad \cdots \quad g_N(\omega, \theta, \phi)e^{j k_N} \right] s(t) = a(\omega, \theta, \phi)s(t) $$

where $k = \frac{2\pi}{\lambda} (k_x, k_y, k_z) = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$ is the wave number, describing the rate of change of the phase of the propagating wave in directions $x$, $y$, $z$. $\mathbf{r}_n^T = (x_n, y_n, z_n)^T$ is the radius-vector to $n$-th AE and $g_n(\theta, \phi)$ is the gain coefficient of $n$ AE.

Figure 3 shows the antenna array in the form of a half-dodecahedron, the length of the face is equal to $\lambda/2$. The half-dodecahedron has two parameters: the radius of the incircle and the circumcircle, which are respectively equal to $r_{in}=1,1\lambda/2$ and $r_{out}=1,4\lambda/2$. In total, there are six elements, with five around the circumference. The angle between them is $72^\circ$. The angle between the upper face and any lateral one (antenna elements) is $116^\circ$.

Then, the directing vector of the antenna array can be expressed as:
The width of each radiator is the rotation matrix around a cylindrical. The location of other radiators is selected in a similar way. Figure 4 shows a cylindrical.
Here, the directing vector of \( m \)-th radio emission source on the \( k \)-th circular antenna array in terms of azimuth and angle of elevation, as well as the position of the array elements becomes:

\[
a(\theta, \varphi) = \begin{bmatrix}
g_1(\theta, \varphi) e^{j k R (0,0,0)}
g_2(\theta + \frac{2\pi}{N}, \varphi) e^{j k R (\frac{2\pi}{N})_{1}}
g_3(\theta + \frac{2\pi}{N}, \varphi) e^{j k R (\frac{2\pi}{N})_{2}}
g_4(\theta + 2\frac{2\pi}{N}, \varphi) e^{j k R (\frac{2\pi}{N})_{3}}
g_5(\theta + 4\frac{2\pi}{N}, \varphi) e^{j k R (\frac{2\pi}{N})_{4}}
g_6(\theta + 5\frac{2\pi}{N}, \varphi) e^{j k R (\frac{2\pi}{N})_{5}}
\end{bmatrix}
\]

\[
g_1(\theta, \varphi) e^{j k R (0,0,0)} = \begin{bmatrix}
g_1(\theta + \frac{\pi}{2}, \varphi) e^{j k R (\frac{\pi}{2})_{1}}
g_2(\theta + \frac{\pi}{2.5}, \varphi) e^{j k R (\frac{\pi}{2.5})_{1}}
g_3(\theta + \frac{2\pi}{2.5}, \varphi) e^{j k R (\frac{2\pi}{2.5})_{1}}
g_4(\theta + 2\frac{2\pi}{2.5}, \varphi) e^{j k R (\frac{2\pi}{2.5})_{1}}
g_5(\theta + 4\frac{2\pi}{2.5}, \varphi) e^{j k R (\frac{4\pi}{2.5})_{1}}
g_6(\theta + 5\frac{2\pi}{2.5}, \varphi) e^{j k R (\frac{5\pi}{2.5})_{1}}
\end{bmatrix}
\]

(3)

\[r_{circ} = \frac{\sqrt{3} \lambda}{2} \quad \text{.}\]

The planar antenna array with directive elements is similar to the half-dodecahedron (Fig. 4b). The main difference is the antennas placed on \( XY \) plane and their radiation patterns are rotated in elevation plane by \(+\frac{\pi}{2}\). Thus, the directing vector of the planar array becomes:

\[
a(\theta, \varphi) = \begin{bmatrix}
g_1(\theta, \varphi + \frac{\pi}{2}) e^{j k R (0,0,0)}
g_2(\theta + \frac{\pi}{2.5}, \varphi + \frac{\pi}{2}) e^{j k R (\frac{\pi}{2.5})_{1}}
g_3(\theta + \frac{2\pi}{2.5}, \varphi + \frac{\pi}{2}) e^{j k R (\frac{2\pi}{2.5})_{1}}
g_4(\theta + 2\frac{2\pi}{2.5}, \varphi + \frac{\pi}{2}) e^{j k R (\frac{2\pi}{2.5})_{1}}
g_5(\theta + 4\frac{2\pi}{2.5}, \varphi + \frac{\pi}{2}) e^{j k R (\frac{4\pi}{2.5})_{1}}
g_6(\theta + 5\frac{2\pi}{2.5}, \varphi + \frac{\pi}{2}) e^{j k R (\frac{5\pi}{2.5})_{1}}
\end{bmatrix}
\]

(4)

The following mathematical model of power directional pattern is used in the far zone relatively to the isotropic antenna, assuming that the antennas are perfectly matched and without losses [11]:

\[
G(\theta, \varphi) = \frac{D}{2^{2n}}\left(1 + \sin(\varphi - \gamma_n^\varphi)\right)^n\left(1 + \cos(\theta - \gamma_n^\theta)\right)^n, n = 0, 1, \ldots, N - 1
\]

(5)

where \( \gamma_n^\varphi \) and \( \gamma_n^\theta \) are a shift in elevation and azimuth planes of \( n \)-th AE, respectively, then:

\[
g = \sqrt{G(\theta, \varphi)} \quad , D \text{ is directivity factor.}
\]
3 The research of the efficiency of AA various configurations with directive radiators

Here we present some results of numerical simulation to illustrate the effectiveness of antenna arrays under study for the radio direction-finding task with super-resolution using MUSIC method. All sources are modeled as uncorrelated complex signals, and the additive noise in all channels of the array is modeled as a complex white Gaussian noise with the same variance. The ratio of the signal power to the noise power SNR (signal-noise ratio) is defined as $\text{SNR} = \frac{P}{\sigma^2}$, where $P$ is the power of one source and $\sigma^2$ is defined as the noise variance.

Thus, a sufficiently large number of images are required to obtain exactly these statistical properties. However, in some cases it is difficult to obtain enough readings from the outputs of the AA, for example, of a highly dynamic target, then it is accepted that $K=100$. At the same time, each step is followed by the calculation of the root mean square error (RMSE) of the coordinates determination by MUSIC method, the number of tests $L$ at a certain point $-500$, the space scanning is consistent:

$$\text{RMSE}_{\theta}(\phi_m, \theta_m) = \frac{1}{L-1} \sqrt{\sum_{l=1}^{L} (\theta - \hat{\theta}_l)^2}$$

$$\text{RMSE}_{\varphi}(\phi_m, \theta_m) = \frac{1}{L-1} \sqrt{\sum_{l=1}^{L} (\varphi - \hat{\varphi}_l)^2}$$

(6) (7)

where $\text{RMSE}_{\varphi}$ and $\text{RMSE}_{\theta}$ are RMSE direction-finding errors in azimuth and angle of elevation respectively, and $\hat{\theta}$ and $\hat{\varphi}$ are the coordinate estimates.

Let us consider the situation where there is one source of a complex radio signal, a random Gaussian process with zero mean, which coordinate in the angle of elevation is $\varphi=45^\circ$, with a shift in azimuth in the range $\theta=0^\circ - 360^\circ$, the signal-noise ratio is 10 dB. As mentioned earlier, many algorithms for estimating the coordinates of signal sources with super-resolution depend on certain statistical properties, for example, the covariance matrix (8).
Figure 3: RMSE of MUSIC while a signal source $\theta=0^\circ - 360^\circ$: a) D= 2, azimuth error, b) D=4, azimuth error, c) D=6, azimuth error. d) D= 2, elevation error, e) D=4, elevation error, f) D=6, elevation error. g) D= 2, sum error, h) D=4, sum error, i) D=6, sum error.

Figure 3 shows the diagrams of the root-mean square errors depending on the azimuthal coordinates of the signal source. It is worth noting, several cases of directional coefficients are considered: 2, 4, and 6 of each antenna element in the array. It is assumed that the antennas are perfectly matched and there is no mutual influence between the antennas. In figure 3 a, b, c the errors in estimating the azimuthal coordinates are shown, in figure 3 d, e, f - when estimating elevation coordinates, in figure 3 g, h, i - total errors.

As it can be seen from figure 3 a, b, c the errors of the azimuth estimates have significant fluctuations with a large amplitude, in particular for planar and half-sphere antennas. In most cases of the location of the signal source the RMSE of the circular array is less than that of the other two. In the case of coordinate estimates in the azimuthal plane, the opposite situation occurs, i.e. the scattering of the root mean square errors of the circular array becomes significant and exceeds the similar indicators of other arrays. If we analyze the sum errors in azimuth and elevation in figure 3 d, e, f, we can draw several conclusions. If the directivity factor of the antenna elements of the arrays is equal to 2, then the total errors are approximately the same, while the circular array has a uniform distribution. If the directivity factor of the antenna elements increase to 4 or 6, then the hemispherical array has the best accuracy.

Let us consider the situation where there is one source of a complex radio signal $\varphi_\text{m}$ above but coordinate in the angle of elevation is in the range $\varphi=0 - 90^\circ$, with azimuth fixed $\theta=0^\circ$, the signal-noise ratio is 10 dB.
Figure 4: RMSE of MUSIC while a signal source $\phi=0^\circ - 90^\circ$: a) D= 2, azimuth error, b) D=4, azimuth error, c) D=6, azimuth error. d) D= 2, elevation error, e) D=4, elevation error, f) D=6, elevation error. g) D= 2, sum error, h) D=4, sum error, i) D=6, sum error.

In figure 4 the diagrams of the standard deviation depending on the elevation coordinates of the signal source are depicted. In this case, the directional coefficient takes values 2, 4 and 6. It is assumed that the antennas are perfectly matched and there is no mutual influence between the antennas. In fig. 4 a, b, c the errors in estimating the azimuthal coordinates are shown, in figure 4 d, e, f - when estimating elevation coordinates, in figure 4 g, h, i - total errors.

From figure 4 a, b, c, it can be seen that the error distribution in the azimuth of a hemispheric array is the most uniform and smooth comparable to other arrays. However, the lowest values of the deviation has the planar array. When evaluating elevation coordinates, as it can be seen from figure 4 d, e, f, the hemispheric array has the smallest errors. At the same time, in the increase in the directivity factor up to 6, the RMSE errors of the planar and circular arrays increase to very significant values. Consider the total errors in figure 4 g, h, i, from which it can be seen that if the directivity factor of the antennas is equal to 2, then the best array is circular one. With an increase in the antenna directivity factor up to 4 and 6, the standard deviation of the hemispheric array is uniform, predictable, without significant bursts and has the smallest values.
4 Conclusion

In this paper, the method of estimating the coordinates of RMSE with MUSIC super-resolution in the structure of antenna arrays of various configurations was studied depending on the directivity of radiators. Circular (cylindrical), spherical and planar antenna arrays with the same number of antenna elements, as well as comparable area, were chosen for studying. Mean square deviations of errors of azimuth and angle of elevation coordinates were estimated, to take into account the shape of the antenna pattern of patch antennas. It is established that hemispherical antenna arrays have the minimal number of errors.

References

[1] Chetan R. Dongarsane and A. N. Jadhav. Intl. J. Tech. Eng. Sys. Vol. 2. No. 1. Mar. 2011. Pp 54-57.
[2] K. Ikeda, J. Nagai, T. Fujita, H. Yamada, A. Hirata, and T. Ohira. in proc. of Intl. Symp. on Antennas and Propagat. Aug. 2004. Pp. 45-48.
[3] Chen Sun, and N. C. Karmakar. Intl. J. Signal Process. Vol. 1. No. 3. 2004. Pp. 153-162.
[4] R. O. Schmidt. IEEE Trans. Antennas Propagat. Vol. 34. 1986. Pp. 276-280.
[5] F. A. Belhoud, R. M. Shubair, and M. E. Al-Mualla. IEEE Intl. Conf. on Electron., Circuits and Sys. Dec. 2003. Pp. 340-343.
[6] J. A. Cadzow. IEEE Trans. Acoust., Speech, Signal Process. Vol. 36. 1998. Pp. 965-979.
[7] A. Abouda, H. M. El-Sallabi, and S. G. IEEE Intl. Symp. on Personal, Indoor and Mobile Radio Comm. Vol.1. Sep. 2005. Pp. 568-572.
[8] Nechaev Yu.B., Peshkov I.V. 2016. №. 6. Pp. 137-142.
[9] B. Wu. The 4th Asia-Pacific Conf. on Environmental Electromagnetics. 2006. Pp. 908–912.
[10] Yu. Nechaev and I. Peshkov. Visn. NTUU KPI, Ser. Radioteh. radioaparatobuduv. Vol 67. 2016. Pp. 12-17. J.D. Kraus. Antennas. McGraw-Hill, 1988.