Design and characteristic analysis of eccentric helical curve-face gear

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Abstract
A composite motion eccentric helical curve-face gear pair consisting of a helical non-circular gear and an eccentric helical curve-face gear is put forward based on the combination of eccentric gear and curve-face gear. This new gear pair can implement the rotation of intersected axes as well as the movement of output axe. Based on the spatial coordinate transformation theory, the coordinate system of eccentric helical curve-face gear pair is obtained and the pitch curve of eccentric helical curve-face gear is designed. The meshing equation and the tooth surface equation of eccentric helical curve-face gear are derived based on the spatial gear engagement theory and the conjugate surface theory. By changing the different parameters of eccentric helical curve-face gear pair, the influencing factors and variation of transmission ratio, pressure angle and kinematics are analyzed. According to the motion relationship of the gears and the method of generation, the establishment of solid model for the eccentric helical curve-face gear is presented with the application of SolidWorks. Furthermore, the correctness of the design theory of eccentric helical curve-face gear pair is verified by using the motion simulation and the experimental verification.

Key words: Composite motion, Eccentric helical curve-face gear, Characteristic analysis, Verification analysis

1. Introduction

Face gear drive is established by face gear meshed with involute cylindrical gear, commonly used to transfer motion and force of intersected axes, due to potential advantages in confluence-shunt transmission, face gear has been widely used in the main transmission system of helicopter. Compared with the tradition bevel gear, face gear has many advantages, such as it can’t be affected by axial installation misalignment, the contact ratio is larger and it has smaller vibration and lower noise (Gabiccini et al., 2004). Domestic and foreign scholars have done lots of researches on theoretical and engineering applications of face gear. Litvin et al studied the application and basic theory of face gear such as design method, processing method and stress analysis (Litvin et al., 1992, 2002, 2005). Tang Jinyuan and Li focused on meshing theory, contact analysis and finite element analysis et al (Tang et al., 2012. Li et al., 2009). In the case of crossed axis, Litvin et al analyzed the geometry and the contact performance of an orthogonal offset face gear (Litvin et al., 1998), and Li Daqing discussed the tooth geometry design and tooth width characteristics of offset helical face gear based on the spatial gear engagement theory (Li et al., 2011). These researches have highly improved the application of face gear.

With the rapid development of machinery industry, the requirement of gear mechanism is more and more complicated, such as variable transmission ratio, traditional face gear can no longer meet the needs. Therefore, based on the researches of face gear, Lin Chao et al have proposed a new type of gear pair named curve-face gear pair, it can transfer the movement of variable transmission ratio between intersected axes, and they have studied application and basic theory of the curve-face gear pair such as design method, processing method, measurement method, contact analysis et al (Lin et al., 2013, 2014a, 2015). Besides, Liu Dawei et al also put forward an irregular face gear, which can also transfer the movement of variable transmission ratio, and they focused on the geometrical model, tooth analysis and the basic meshing theory (Liu et al., 2015, 2016). These researches have filled up the vacancy of variable ratio gear mechanism.

Nowadays, the occasions that require speed variation and composite motion are increasing, and the types of motion
are also increasing. Scholars have developed many mechanisms to meet the demands, and non-standard gears are one of the most important parts. Mundo designed a non-circular gear planetary gear train, it can generate specific torque law curve or works as a function generator, and he presented an application in high performance bicycle (Mundo et al., 2006). Kang et al proposed a new type of uninterrupted transmission system that can accommodate various angular velocity ratios, method that realizes the desired rotation of the output shaft and solves the constraint problem between the angular velocity ratio and the number of teeth on the clutch was researched (Kang et al., 2012, 2013). Ottaviano et al analyzed the characteristics of non-circular gear transmission working as a function generator, and he studied its application in the blood pump system (Ottaviano et al., 2008). Hopwood et al studied the design method of non-circular gears working as function generator (Hopwood et al., 2008). Based on the non-circular bevel gear and curve-face gear, Lin Chao, Liu Yi et al have put forward two new types of gear pair named composite motion non-circular bevel gear pair and composite motion curve-face gear pair, and these gear pairs can transmit various speeds between intersected axes as well as generate periodic axial movement. They have studied the fundamental movement principle and the kinematics (Lin et al., 2014b, 2016). In order to perfect the study of curve-face gear and meet the requirement of composite motion and variable transmission ratio, based on the combination of eccentric gear and curve-face gear, this paper proposes the eccentric helical curve-face gear pair consisting of a helical non-circular gear and an eccentric helical curve-face gear. During the transmission process of eccentric helical curve-face gear pair, axes of the two gears are orthogonal, and the axis of the eccentric helical curve-face gear is fixed, the output shaft of helical non-circular gear can generate rotation and axial movement. In this paper, EHCF gear, HCF gear and HNC gear are stand for the eccentric helical curve-face gear, the helical curve-face gear and the helical non-circular gear, respectively.

2. Basic transmission principle

2.1 Coordinate system of EHCF gear pair

Based on the motion relationship of EHCF gear, the relative motion of pitch curves of gear pair at point $P$ is pure rolling during meshing process, so the coordinate system of gear pair is established as follows (Lin et al., 2013):

![Fig.1 The drive coordinate system](image)

As shown in Fig.1, $O_1X_1Y_1Z_1$, $O_2X_2Y_2Z_2$ and $O_3X_3Y_3Z_3$ are coordinate systems which are rigidly connected with the frame of the cutting machine. Coordinate $O_1X_1Y_1Z_1$, $O_2X_2Y_2Z_2$ and $O_3X_3Y_3Z_3$ are rigidly connected with the HNC, the fictitious HCF gear and the EHCF gear, respectively. The distance between plane $Y_2O_2Z_2$ and plane $Y_1O_1X_1$ is $e$ is the eccentricity of EHCF gear. According to the movement relationship between the EHCF gear and the HNC gear, the HNC gear rotates around axis $O_1Z_1$ with an angular velocity of $w_1$ in clockwise direction, and the HNC gear moves around axis $O_2Z_2$ with a velocity of $v_1$. Likewise the fictitious HCF gear rotates around axis $O_2Z_2$ with an angular velocity of $w_1$ in counter-clockwise direction, and the EHCF gear rotates around axis $O_3Z_3$ with an angular velocity of $w_3$ in counter-clockwise direction.

According to the gear engagement theory, the velocity of the tangent point of the two pitch curve must be equal, hence:

$$\overline{O_1P_1w_1} = r(\theta)w_1 = \overline{O_2P_2w_2} = Rw_2$$  \hspace{1cm} (1)

Where $r(\theta)$ is the pitch curve of HNC gear, and the radius can be expressed as: $R = \frac{r_2}{2\pi} \int_{0}^{\pi} r(\theta)d\theta$. Therefore, the...
transmission ratio and angular displacement of the fictitious HCF gear can be represented as follows:

\[
\begin{align*}
    i_{z2} & = \frac{w_1}{w_2} = \frac{R}{r(\theta)} \\
    \theta_z & = \int_0^{\theta_f} \frac{1}{i_{z2}} \, d\theta = \frac{1}{R} \int_0^{\theta_f} r(\theta) \, d\theta
\end{align*}
\]  

(2)

According to the meshing relationship of EHCF gear pair, the spatial geometry relation can be established as follows:

![Image of spatial geometry relation]

As shown in Fig.2, the polar radius of EHCF gear can be derived as follows:

\[ R_p = \sqrt{R^2 + e^2 - 2 \times R \times e \times \cos(\pi - \theta _2)} \]  

(3)

Then the angular displacement of the EHCF gear can be represented as follows:

\[
\begin{align*}
    \theta _2 & = \begin{cases} 
    2n\pi + \arccos \left( \frac{R_p^2 + e^2 - R^2}{R \times e} \right) / (2 \times R_p \times e) & \theta _2 \in (2n\pi, (2n+1)\pi) \\
    (2n+1)\pi - \arccos \left( \frac{R_p^2 + e^2 - R^2}{R \times e} \right) / (2 \times R_p \times e) & \theta _2 \in ((2n+1)\pi, (2n+2)\pi)
    \end{cases}
\end{align*}
\]  

(4)

2.2 The design of pitch curve

As shown in the Fig.1, the pitch curve of HNC gear is a modified elliptical curve. According to the theory of the modified elliptical curve, the equation of arbitrary modified elliptical pitch curve in polar coordinates can be represented as follows:

\[ r(\theta) = \frac{a(1-k^2)}{1-k \cos(n \theta)} \]  

(5)

Where the polar angle and the order of a modified elliptical curve are \( \theta \) and \( n \). The half of major axis and the deformation coefficient of a modified elliptical curve are \( a \) and \( k \). Therefore, when the HNC gear turns the angle \( \theta_1 \), the position vector \( r'_i \) of pitch curve of HNC gear is represented in \( O_1-X_iY_iZ_i \) by:

\[
\begin{bmatrix}
    r(\theta_1) \cos \theta_1 \\
    r(\theta_1) \sin \theta_1 \\
    0 \\
    1
\end{bmatrix}
\]  

(6)

As shown in Fig.1, on the basis of coordinate transformation theory, the transformation matrix used to transform the coordinate \( O_1-X_iY_iZ_i \) to the coordinate \( O_i-X_iY_iZ_i \) can be represented as follows:

\[
M_{O_1O_i} = \begin{bmatrix}
    \cos \theta_1 & \sin \theta_1 & 0 & 0 \\
    -\sin \theta_1 & \cos \theta_1 & 0 & 0 \\
    0 & 0 & 1 & R-R_p \\
    0 & 0 & 0 & 1
\end{bmatrix}
\]  

(7)

The transformation matrix form the coordinate \( O_1-X_iY_iZ_i \) to the coordinate \( O_i-X_iY_iZ_i \) can be represented as...
follows:

\[
M_{\theta\alpha} = \begin{bmatrix}
-sin\theta_1 sin\theta_1 & cos\theta_1 sin\theta_1 & -cos\theta_1 & (Rp - 2R_e cos\theta_1) \\
-sin\theta_1 cos\theta_1 & cos\theta_1 cos\theta_1 & sin\theta_1 & -(Rp - 2R_e sin\theta_1) \\
-cos\theta_1 & -sin\theta_1 & 0 & r(0) \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

(8)

Based on the theory of conjugated curves, the position vector \( r^p_e \) of pitch curve of EHCF gear is represented in \( O_1 - X_1Y_1Z_1 \) by:

\[
r^p_e = M_{\theta\alpha} r^p_e = \begin{bmatrix}
-Rp cos\theta_1 \\
Rp sin\theta_1 \\
0 \\
1
\end{bmatrix}
\]

(9)

According to the equation of pitch curve above, the pitch curve can be drawn by Matlab as shown in Fig.3.

3 The equation of tooth surface

3.1 The tooth surface of helical cutter

As shown in Fig.4, coordinate \( O_k - X_kY_kZ_k \) is rigidly connected with helical cutter, likewise \( O_k - X_kY_kZ_k \) is a dynamic coordinate system that performs a screw motion with respect to \( O_k - X_kY_kZ_k \). Parameters \( r_{bi} \), \( r_i \) and \( r_a \) are base circle radius, pitch radius and addendum radius of the helical cutter (Lin et al., 2015).

\[
r^i_i = \begin{bmatrix}
 r_{bi} cos(\theta_i + \theta_{ai}) + r_{bi} \theta_i sin(\theta_i + \theta_{ai}) \\
r_{bi} sin(\theta_i + \theta_{ai}) - r_{bi} \theta_i cos(\theta_i + \theta_{ai}) \\
0 \\
1
\end{bmatrix}
\]

(10)

The transformation matrix used to transform the coordinate \( O_i - X_iY_iZ_i \) to the coordinate \( O_k - X_kY_kZ_k \) can be represented as follows:
\[
M_{O_4O_3} = \begin{bmatrix}
\cos \psi & \sin \psi & 0 & 0 \\
-\sin \psi & \cos \psi & 0 & 0 \\
0 & 0 & 1 & p
\end{bmatrix}
\]

(11)

Where \( p \) is a spiral parameter and the angle of rotation in screw motion is designated by \( \psi \). There: \( p = H/2\pi \), \( \psi = u_0 \tan \beta / r_a \), \( H \) is the screw pitch and \( u_0 \) is a coefficient of tooth width. Therefore, the tooth surface vector of helical cutter is represented in \( O_4 - X'_4Y'_4Z'_4 \) by:

\[
\vec{r}_m(\psi, \theta) = M_{O_4O_3} \vec{r}_m = \begin{bmatrix}
r_a \cos(\theta + \theta_d - \psi) + r_a \theta_n \sin(\theta + \theta_d - \psi) \\
r_a \sin(\theta + \theta_d - \psi) - r_a \theta_n \cos(\theta + \theta_d - \psi) \\
p
\end{bmatrix}
\]

(12)

So the unit normal of tooth surface of helical cutter is derived in \( O_4 - X'_4Y'_4Z'_4 \) by:

\[
\hat{n}_m = \begin{bmatrix}
\cos \beta_n \sin(\theta + \theta_d - \psi) \\
-\cos \beta_n \cos(\theta + \theta_d - \psi) \\
-\sin \beta_n
\end{bmatrix}
\]

(13)

### 3.2 The trajectory of helical cutter

During the transmission process of EHCF gear, on the assumption that a helical cutter is internal meshing with the HNC gear, at this time, the helical cutter can be seen to externally mesh with the EHCF gear, so the trajectory of meshing process of EHCF gear can be represented as shown in Fig.5(a) (Lin et al., 2014a).

![Diagram](image1)

(a) The meshing process  (b) Space location of helical cutter

1- The initial position of HNC gear, 2- The instantaneous position of HNC gear
3- The initial position of helical cutter, 4- The instantaneous position of helical cutter
5- The position of EHCF gear, a- The pitch curve of HNC gear, b- The pitch curve of helical cutter

Fig.5 The space moving trace

On the assumption that the initial position of helical cutter is coincidence with the origin of coordinate \( O_4 - X'_4Y'_4Z'_4 \), so the space location of helical cutter can be represented as shown in Fig.5(b) (Lin et al., 2013). Coordinate \( O_4 - X'_4Y'_4Z'_4 \) is connected with the HNC gear. Coordinate \( O_4 - X'_4Y'_4Z'_4 \), \( O_4 - X'_4Y'_4Z'_4 \), and \( O_5 - X'_5Y'_5Z'_5 \) are connected with the helical cutter. Based on the coordinate transformation theory, the transformation matrix form the coordinate \( O_4 - X'_4Y'_4Z'_4 \) to the coordinate \( O_5 - X'_5Y'_5Z'_5 \) can be represented as follows:

\[
M_{ikr} = \begin{bmatrix}
\cos \lambda & \sin \lambda & 0 & L_i \cos(\theta - \lambda_i) \\
-\sin \lambda & \cos \lambda & 0 & L_i \sin(\theta - \lambda_i) \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(14)

Where, \( \lambda = \pi / 2 + \xi - \mu - \Theta \), \( \mu = \arctan(r(\theta)/r(\theta)) \), \( \lambda_i = \arccos(r^2(\theta_i) + L_i^2 - r_i^2)/2r(\theta_i)L_i \), \( L_i = \sqrt{(r^2(\theta + \Theta) + r_i^2 - 2r_2r(\Theta)\sin\mu} \).

According to the Eq. (7), the transformation matrix used to transform the coordinate \( O_4 - X'_4Y'_4Z'_4 \) to the coordinate \( O_4 - X'_4Y'_4Z'_4 \) can be represented as follows:

\[
M_{ikr} = M_{qik}M_{ikr} = \begin{bmatrix}
\cos(\lambda + \Theta) & \sin(\lambda + \Theta) & 0 & L_i \cos \lambda_i \\
-\sin(\lambda + \Theta) & \cos(\lambda + \Theta) & 0 & -L_i \sin \lambda_i \\
0 & 0 & 1 & R - Rp \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(15)
Based on the Eq. (8), the transformation matrix used to transform the coordinate \(O_z - X_z Y_z Z_z\) to the coordinate \(O_i - X_i Y_i Z_i\) can be represented as follows:

\[
M_{\alpha_i} = M_{\alpha_i} M_{\alpha_z} = \begin{bmatrix}
-sin(\theta_i + \lambda)sin\theta_i & cos(\theta_i + \lambda)sin\theta_i & -cos\theta_i \\
-sin(\theta_i + \lambda)cos\theta_i & cos(\theta_i + \lambda)cos\theta_i & sin\theta_i \\
-cos(\theta_i + \lambda) & -sin(\theta_i + \lambda) & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(16)

For simplification, let \(\psi_i = \theta_i + \theta - \psi - \theta_i\), so the unit normal of tooth surface of helical cutter is represented in \(O_i - X_i Y_i Z_i\) by:

\[
\vec{m} = \left[\cos\beta_i \sin\psi_i, -\cos\beta_i \cos\psi_i, -\sin\beta_i\right]^T
\]

(17)

### 3.3 Meshing equation and tooth surface equation of EHCF gear

As shown in Fig.5, during the meshing transmission process of EHCF gear pair, the tooth surface \(\sum_3\) of EHCF gear is enveloped by the tooth surface \(\sum_1\) of helical cutter, so the equation of the family of surface \(\sum_1\) in the coordinate \(O_i - X_i Y_i Z_i\) can be expressed as:

\[
\vec{r}_i (\theta_i, \psi, \theta) = M_{\alpha_i} \vec{r}_z (\psi, \theta)
\]

(18)

Based on the principle of tooth profile meshing, the meshing equation can be expressed as:

\[
f(\theta_i, \psi, \theta) = \vec{N}(t) \cdot \vec{v}_{t_1} = 0
\]

(19)

Where the superscript denotes that parameters are in the coordinate \(O_i - X_i Y_i Z_i\), and \(\vec{N}(t)\) can be replaced by the unit normal \(\vec{n}_i\), correspondingly, the relative velocity \(\vec{v}_{t_1}\) can be depicted as:

\[
\vec{v}_{t_1} = \left(\vec{w}_1 - \vec{w}_2\right) \times M_{\alpha_i} \vec{r}_z (\psi, \theta) - O_i \vec{r}_z (\psi, \theta) + \vec{v}_3
\]

(20)

According to Eq. (17), (19) and (20), the meshing equation can be deduced as:

\[
f(\theta_i, \psi, \theta) = \vec{N}(t) \cdot \vec{v}_{t_1} = -\cos\beta_i \left(L \cos(\lambda_i + \psi_i) + \psi_i \cos\beta_i \cos(\lambda_i - \psi_i)\right) + \sin\beta_i \left(L \sin(\lambda_i - \psi_i) + \psi_i \cos\beta_i \cos(\lambda_i - \psi_i)\right)
\]

(21)

According to the principle of gear engagement, the equation of \(\sum_3\) can be expressed as:

\[
\vec{r}_i (\theta, \psi, \theta) = \begin{cases}
(\cos\theta_i - \psi \cos\psi_i - L \sin\lambda_i \sin\theta_i - \psi_i \cos\beta_i \cos(\lambda_i - \psi_i)

(22)
\end{cases}
\]

The equation of \(\sum_3\) can be expressed by \(\vec{r}_i (\theta, \psi, \theta)\) when \(\psi\) is eliminated from Eq. (22).

### 4. Characteristic analysis

#### 4.1 Analysis of transmission ratio

The EHCF gear pair transfers variable transmission ratio movement between the two intersected axes, so it is very useful to analyze the transmission characteristics of EHCF gear pair by studying the rules of transmission ratio. According to the basic definition of transmission ratio, the transmission ratio of EHCF gear pair can be derived as
follows:
$$i_{i3} = \frac{w_i}{w_3} = \frac{d\theta_i}{dt}$$

Based on the Eq. (4), the angular velocity $w_3$ can be represented as:
$$w_3 = \frac{d\theta_i}{dt} = \frac{(R_p^2 - e^2 + R^2) \times e \times R \times \sin(\pi\theta_i) \times w_2}{4 \times R_p^2 \times e^2 - (R_p^2 + e^2 - R^2)^2 \times R^2}$$  \hspace{1cm} (24)

According to the Eq. (2), (23) and (24), the transmission ratio $i_{i3}$ can be expressed as follows:
$$i_{i3} = \frac{1}{i_{i3}} = \frac{w_i}{w_3} = \frac{(R_p^2 - e^2 + R^2) \times e \times R \times \sin(\pi\theta_i) \times w_2}{4 \times R_p^2 \times e^2 - (R_p^2 + e^2 - R^2)^2 \times R^2}$$

According to the above equations, changing the basic parameters of the EHCF gear pair transmission, diagram of the motion law of the transmission ratio can be obtained, and basic parameters of the EHCF gear pair are shown in Table 1.

| Parameters                        | Value                        |
|-----------------------------------|------------------------------|
| Eccentricity of HNC gear          | $k = (0.1, 0.2, 0.3, 0.4)$    |
| Helix angle of HNC gear           | $\beta = 15^\circ$           |
| Order of HNC gear                 | $n_1 = 2$                    |
| Normal module of HNC gear         | $m_n = 4$                    |
| Tooth number of HNC gear          | $z_i = 18$                   |
| Eccentricity of EHCF gear         | $e = (0, 10, 20, 30)$         |
| Order of EHCF gear                | $n_2 = (2, 3, 4, 5)$          |

Based on controlling variable method, the influence law curves of transmission ratio are depicted as shown in Fig.6.

As shown in Fig.6(a), with the increase of $k$, the average of transmission ratio becomes smaller, but the extent of the impact is not very obvious, and the periodic of the motion law is unaffected. Changing the eccentricity of EHCF
gear, when the eccentricity $e$ becomes larger, the volatility becomes larger, and maximum value of transmission ratio becomes larger and minimum value of transmission ratio becomes smaller. As shown in Fig.6(c), with the increase of $n_2$, the volatility becomes smaller, and the average of transmission ratio becomes smaller, and the periodic of the motion law is unaffected. Therefore, we can obtain the appropriate transmission ratio by changing the value of $e$ and $n_2$.

4.2 Analysis of pressure angle

The changes of the pressure angle directly affect the mechanical properties of gear pair, even self-locking phenomena may occur. In order to avoid the extremes, it is necessary to analyze the change law of pressure angle during the transmission process of EHCF gear, so based on the meshing process of EHCF gear, the pressure angle of pitch curve can be expressed as shown in Fig.7.

![Figure 7](image)

(a) Clockwise rotation of HNC gear  (b) Counter-clockwise rotation of HNC gear
1- Transverse tooth profile of HNC gear, 2- Transverse tooth profile of helical cutter
3- Tooth profile of EHCF gear, 4- Pitch curve of HNC gear
5- Pitch curve of helical cutter, 6- Pitch curve of EHCF gear

As shown in Fig.7, $p$ is the meshing point, and $\mu$ is the angle between the tangential direction and the radius vector based on the pitch curve of HNC gear. $F_t$ and $v$ are the transverse stress direction and the absolute speed of EHCF gear, respectively. Besides, $\alpha_{t1}$ and $\alpha_{t2}$ are the transverse pressure angle of helical cutter and EHCF gear based on the pitch curve, respectively. On the assumption that the HNC gear is clockwise rotation as shown in Fig.7(a), the left tooth surface of HNC gear becomes the tooth working area, so the transverse pressure angle $\alpha_{t2}$ can be derived as follows:

$$\alpha_{t2} = \alpha_{t1} - \mu + \pi / 2$$

(26)

On the assumption that the HNC gear is counter-clockwise rotation as shown in Fig.7(b), the right tooth surface of HNC gear becomes the tooth working area, so the transverse pressure angle $\alpha_{t2}$ can be derived as follows:

$$\alpha_{t2} = \alpha_{t1} + \mu - \pi / 2$$

(27)

Combined with the transmission characteristics of EHCF gear pair, and by changing the basic parameters of EHCF gear pair, as shown in Table 1, the change law of pressure angle of EHCF gear pair can be obtained. On the assumption that the HNC gear is clockwise rotation, and based on the data in Table 1, the influence laws of pressure angle curve are depicted as shown in Fig.8.

![Figure 8](image)

(a) Impact of $k$ to the pressure angle  (b) Impact of $e$ to the pressure angle
The parameter $\mu$ is constantly changing, so the pressure angle of EHCF gear is different. Along with the rotation of the gear pair, the pressure angle $\alpha_{2t}$ is cyclical change. As shown in Fig.8(a), the pressure angle of EHCF gear is positively correlated with the eccentricity of the HNC gear, and the periodic of the motion law is unaffected. It can be found from Fig.8(b) that the periodic of pressure angle and the volatility of pressure angle are not affected by parameter $\epsilon$, but the variation trend of pressure angle is affected by parameter $\epsilon$. As shown in Fig.8(c), with the increase of $n_2$, the fluctuation range, maximum value, minimum value and average value of $\alpha_{2t}$ experience no change, but the fluctuation cycle becomes shorter, and the fluctuation cycle can be expressed as: $2n_2\pi/n_1$. The load of gear shaft and bearing can be affected by the parameter $\alpha_{2t}$, and even self-locking phenomena may occur. Therefore, under the requirements of the design, by choosing smaller eccentricity of HNC gear, the transmission process can be more smoothly.

### 4.3 Kinematics analysis

During the meshing transmission process of EHCF gear, the variation law of axial movement of the EHCF gear can be represented as follows:

$$\Delta s = R + e - Rp \quad (28)$$

The HNC gear moves around axis $OZ_1$ with a velocity of $v_s$, derivate the displacement function $\Delta s$, the variation of velocity can be obtained as follows:

$$v_s(\theta_2) = \frac{d(\Delta s)}{d\theta_2} = \frac{R \times e \times \sin \theta_2}{\sqrt{R^2 + e^2} + 2 \times R \times e \times \cos \theta_2} \quad (29)$$

The HNC gear moves around axis $OZ_1$ with an acceleration of $a_s$, derivate the displacement function $v_s$, the variation of acceleration can be derived as follows:

$$a_s(\theta_2) = \frac{d(v_s(\theta_2))}{d\theta_2} = \frac{R \times e \times \cos \theta_2}{\sqrt{R^2 + e^2} + 2 \times R \times e \times \cos \theta_2} + \frac{R^2 \times e^3 \times (\sin \theta_2)^3 \sqrt{R^2 + e^2} + 2 \times R \times e \times \cos \theta_2} {\sqrt{R^2 + e^2} + 2 \times R \times e \times \cos \theta_2} \quad (30)$$

According to the above equations, changing the eccentricity of EHCF gear, the diagram of the motion law of the EHCF gear can be obtained as shown in Fig.9.
As shown in Fig.9, the axial displacement law of HNC gear is the undee change, and the change laws of velocity and acceleration are similarly sinusoidal change, and all are periodic change law. With the increase of the eccentricity of EHCF gear, the stroke of axial movement is positively correlated with the eccentricity of EHCF gear, and the amplitude of velocity and acceleration is also positively correlated with the eccentricity of EHCF gear. The periodic of the motion law is unaffected. So in order to change the stroke of the axial movement, eccentricity should be changed.

5 Modeling and validation
5.1 The establishment of solid model

Based on the machining principle of curve-face gear, according to the relative motion of pitch curves of EHCF gear pair at point is pure rolling during meshing process, on the assumption that a helical cutter is internal meshing with the HNC gear, at this time, the helical cutter can be seen to externally mesh with the EHCF gear, so the helical cutter completes the whole process of specific movement, and the profile of the blank enveloped by the profile of helical cutter is the profile of EHCF gear.

As shown in Fig.10, the distance between axis a and axis b is the eccentricity $e$. $O_1$-$X_1$,$Z_1$, $O_2$-$X_2$,$Z_2$ and $O$-$X$,$Y$,$Z$ are coordinate systems which are rigidly connected with the frame of the cutting machine, and the position 2 of helical cutter is the initial position of the processing. The simulation machining process is as follows: the helical cutter moves around axis $OZ_1$ with a displacement of $\Delta s$, so the helical cutter will be in the position 3, then the helical cutter moves and rotates based on the space moving trace in Fig.4, then the helical cutter arrives in the position 4 by rotating around axis $OZ_1$ with an angle of $\theta_1$ in clockwise direction, finally, the tooth surface of EHCF gear is generated by the Boolean between the helical cutter and the blank. Based on the above process, the simulated tooth surface is obtained as shown in Fig.11 (a), and then fitting profiles is depicted as shown in Fig.11 (b) by setting the layered form based on the simulated tooth surface. According to the surface modeling function, the fitting tooth surface of EHCF gear can be obtained as shown in Fig.11(c).
5.2 The motion simulation

According to the parameters of EHC gear pair shown in Table 2, and based on the above method of the establishment of solid model, the 3D model of EHC gear pair can be obtained as shown in Fig.12.

Table 2 Parameters of EHC gear pair

| Parameters                    | Value |
|-------------------------------|-------|
| Eccentricity of HNC gear      | \(k = 0.1\) |
| Helix angle of HNC gear       | \(\beta = 15^\circ\) |
| Order of HNC gear             | \(n_1 = 2\) |
| Normal module of HNC gear     | \(m_n = 4\) |
| Tooth number of HNC gear      | \(z_1 = 18\) |
| Eccentricity of EHC gear      | \(e = 8\) |
| Order of EHC gear             | \(n_2 = 4\) |
| Inner radius of EHC gear      | \(R_i = 76\) |
| Outer radius of EHC gear      | \(R_o = 86\) |

Based on the data in Table 2 and the 3D model, a curve of the transmission ratio of HNC gear to the EHC gear is derived through kinematical simulation in Fig.13 (a), which is consistent with that calculated by equation (18) in Fig.13 (b). The simulation results verify the correctness of the tooth model of the EHC gear. The slight fluctuation on the transmission ratio curve in Fig.13 (a) is related to the accuracy of the tooth surface. When the simulated tooth surface of EHC gear approaches the theoretical tooth surface sufficiently, the fluctuation on the transmission ratio curve tends to disappear.

5.3 The experimental verification

When the eccentricity of EHC gear is: \(e = 0\), the EHC gear becomes the HCF gear, so the experiment of HCF gear pair is built based on the complicated processing of EHC gear and the restrictive experimental device. Based on the 3D model of EHC gear pair, the theoretical and simulation transmission ratio of EHC gear can be comparative analysis with the theoretical, simulation and experimental transmission ratio of HCF gear. The parameters of HFC gear
and the experimental parameters are shown in the table 3.

| Parameters                        | Value |
|-----------------------------------|-------|
| Eccentricity of HNC gear          | $k = 0.1$ |
| Helix angle of HNC gear           | $\beta = 15^\circ$ |
| Order of HNC gear                 | $n_1 = 2$ |
| Normal module of HNC gear         | $m_n = 4$ |
| Tooth number of HNC gear          | $z_1 = 18$ |
| Tooth number of HCF gear          | $z_2 = 36$ |
| Order of HCF gear                 | $n_2 = 4$ |
| Inner radius of HCF gear          | $R_i = 70$ |
| Outer radius of HCF gear          | $R_o = 83$ |
| Input speed                       | $N_i = 150$ |
| Load torque                       | $T_i = 20$ |

The HCF gear pair is a new type of gear mechanism, so there is no special machine tool to realize the machining of the HCF gear pair, and a five-axis CNC machine tool will be used to machine this gear pair. Processing steps include blank rough machining, tooth surface semi-finishing and tooth surface finishing, as shown in Fig.14 and Fig.15.

(a) Blank         (b) Blank rough machining     (d) Semi-finishing        (e) Finishing
Fig.14 The processing of the HNC gear

(a) Blank         (b) Blank rough machining     (d) Semi-finishing        (e) Finishing
Fig.15 The processing of the HCF gear

In order to test the transmission ratio of HCF gear, the experiment device can be established as shown in Fig.16. Based on the parameters shown in the table 3 and the finished product of the HCF gear pair, the experimental results can be shown in Fig.17.

Fig.16 The experiment device of HCF gear pair
As shown in Fig. 17 (a), the simulation curve of transmission ratio of EHCF gear pair is well fitted on the theoretical one. The HCF gear is a special case of the EHCF gear, as shown in Fig. 17 (b), the simulation curve of transmission ratio of HCF gear pair is well fitted on the theoretical one, and the absolute deviation between experimental and theoretical results of transmission ratio of HCF gear pair is ranged from 0 to 0.025, and the error is controlled in 5%, so the experimental curve of transmission ratio of HCF gear pair is also well fitted on the theoretical one. Therefore, the correctness of theoretical design of EHCF gear pair is indirectly verified.

6. Conclusion

The results of performed research allow the following conclusions to be drawn:

1) A new type of composite motion EHCF gear pair based on the composite bevel gear has been proposed. The gear drive is formed by a HNC gear and an EHCF gear. It can transmit various speeds between intersected axes as well as generate periodic axial movement.

2) Based on the meshing process of EHCF gear pair, the drive coordinate system has been established, and the pitch curve has been designed. According to the spatial gear engagement theory and the conjugate surface theory, the meshing equation and tooth surface equation of EHCF gear have been developed. Besides, the changes raw of transmission ratio, pressure angle and kinematics characteristics of EHCF gear have been analyzed.

3) The 3D model of EHCF gear pair have been acquired on the basis of a modeling program, which is developed under the environment of VB and Solidworks (API). Through contrast to the transmission ratio curve of experimental, simulation and theoretical value, the correctness of the design and the transmission stability of EHCF gear have been verified.

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