Evaporation of liquid coating a fiber

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Abstract – We investigate theoretically and numerically the diffusion-limited evaporation of a liquid deposited on a fiber in two configurations: a sleeve and an axisymmetric barrel-shaped droplet. For a sleeve, the local flux depends on both the aspect ratio and the smallest length of the problem. By using analytical calculations and 3D finite elements simulations, we predict a divergence of this flux further localized at the edge as the aspect ratio increases. The evaporation of axisymmetric drops on a fiber is studied with numerical simulations. For sufficiently large volumes, we evidence that the evaporation rate is almost independent of the wetting properties of the liquid, even for small contact angles, and that the droplets evaporate as spheres of the same volume.

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Introduction. – The coating of liquid on a solid is a common operation in everyday life as well as in industrial processes (e.g., lubrication, painting). The first theoretical description of the deposited layer has been made almost simultaneously by Landau and Levich [1] and Derjaguin [2]. The coated thickness \( h \) is found to be proportional to the radius \( a \) of the fiber and depends in particular on the entrainment velocity \( V \), the liquid surface tension \( \gamma \) and the fluid viscosity \( \eta \). Thus, the thickness \( h \) can be written as \( h = a f(Ca) \), where \( f \) is a function which can be simplified under certain conditions on the capillary ranges. Coating of complex fluids also received some attention, such as non-Newtonian fluids or water-surfactant solutions [3,4].

Due to the fiber curvature, the liquid film is unstable; a mechanism that was first characterized by Plateau [5] and rationalized by Lord Rayleigh in the 19th century [6]. The liquid surface tension leads to the minimization of its surface area by breaking the film into a series of regularly spaced droplets. The characteristic growing time of the Rayleigh-Plateau instability \( \tau = 12\eta a^4/(\gamma h^3) \) depends significantly on the coating thickness. For non-Newtonian fluids, or for colloidal suspensions, the Rayleigh-Plateau instability can be delayed or suppressed [7–9].

Carroll [10] described the shape of axisymmetric drops on a fiber, which depends on the wetting properties of the material and the droplet volume with respect to the fiber size. Based on Carroll’s results, the drop height and the length of the wetted area can be used to characterize the wettability of the liquid on the fiber [11–14]. Beyond the axisymmetric shape, the equilibrium conformation of a drop on a fiber can also be a clam-shell: a drop sitting on a curved surface. Experimental observations have evidenced that both barrel and clam-shell conformations can coexist and Chou et al. studied in detail the phase diagrams of droplet-on-fiber with or without gravity [15].

The aim of the paper is to describe the different evaporation regimes of the fiber coatings when the evaporation is limited by the diffusion of the vapor in the atmosphere. Despite the fact that this is a model system with many industrial applications, there is a lack of theoretical rationalization on this subject.

In 1918, Langmuir [16] explained for the first time that the total diffusive evaporation flux of a spherical droplet of size \( R \) is not proportional to \( R^2 \), which means that, due to the curvature of the surface of the droplet, the evaporation flux is not proportional to the surface of the drop. The case of a sessile drop evaporation (see [17,18] for a review) is more complex because of the diverging evaporative flux at the triple line [19–21] and several studies have been devoted to the nature of the substrate. For instance, the case of a drop on a tilted surface [22], on crossed fibers [23], on superhydrophobic surfaces [24] or with complex wetting patterns [25] can be cited. The situation of a droplet on a curved surface such as a convex or concave surface [26,27] has been studied as well, the latter corresponding to a clam-shell on a fiber.
In this paper, we discuss the two limit cases of the diffusive evaporation of a liquid deposited on a fiber. In a first part, we study the evaporation of a liquid cylinder deposited on a fiber that we call the sleeve configuration. This configuration corresponds to a deposited layer observed at a time much smaller than the Rayleigh-Plateau characteristic time or when the instability is inhibited. To do so, we will develop an analytical model to calculate the evaporation, valid for small and large aspect ratios. These predictions are supplemented by a full 3D numerical simulation using finite element method. In the second part, we study the evaporation of a droplet on a fiber which corresponds to a destabilized liquid film coating the fiber. More precisely, we restricted the study to the axisymmetric barrel situation which is the common case, in particular for small contact angles of the liquid on the fiber. We first numerically calculate the shape of a droplet around a fiber using Surface Evolver [28], a surface minimization algorithm, whose results can be compared to the prediction by Carroll [10] and Chou et al. [15] in absence of gravity. By using finite element computations, we calculate the evaporation flux of the droplet, and show that except for very small liquid volume, the evaporation rate of the droplets is not significantly affected by the presence of the fiber, whatever the liquid contact angle is.

**Sleeve.** – We consider a sleeve of liquid of length $2L$ and of radius $a$ as depicted in fig. 1(a), such that the liquid cylinder has the same radius as the fiber. A natural geometrical parameter is $\lambda = L/a$. We assume that the liquid evaporation occurs in a diffusion-limited process in the stationary regime. Thus, the concentration field $c$ in the gas phase is the solution of the Laplace equation $\Delta c = 0$, which reads in cylindrical coordinates

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial c}{\partial r} \right) + \frac{\partial^2 c}{\partial z^2} = 0. \quad (1)$$

In this geometry, the boundary conditions are (a) a saturated vapor concentration $c_{\text{sat}}$ in the vicinity of the interface, i.e., $c(r = a, z) = c_{\text{sat}}$ for $|z| < L$, (b) the absence of evaporative flux at the surface of the fiber, i.e., $\left. \frac{\partial c}{\partial r} \right|_{r=a} = 0$ for $|z| > L$, and (c) a constant concentration far from the liquid, $\lim_{r \to \infty} c = c_{\infty}$.

We provide in the Supplementary Material Supplementarymaterial.pdf (SM) the derivation of analytical solutions to eq. (1) in the limits of small and large aspect ratios $\lambda$. The difficulty associated with this linear boundary-value problem (BVP) resides in the discontinuity of the boundary conditions (BC) at the inner boundary $r = a$. More precisely, two different types of BCs, namely a Dirichlet-type BC and a Neumann-type BC are applied on disjoint complementary subdomains of the cylindrical surface located at $r = a$. This difficulty prevents the use of the classical Fourier-Hankel analysis of the problem and, instead, the BVP is reduced to a set of dual integral equations. Following the theory of Sneddon [29], these coupled integral equations are then reduced to a single integral equation with a weak, logarithmic singularity that is solved analytically in the asymptotic cases $\lambda \ll 1$ and $\lambda \gg 1$. From the resulting concentration field $c(r, z)$, we can compute the local evaporative flux $j(z)$ defined as $j(z) = -D \frac{\partial c}{\partial r}|_{r=a}$, where $D$ is the diffusion coefficient of the vapor in the gas phase.

We also propose to solve eq. (1) by using the finite element method implemented in the proprietary software COMSOL Multiphysics using Transport of Dilute Species physics in axisymmetric 2D geometry. To compute accurately $j(z)$ while keeping the computational time reasonable, we divide the atmosphere in three concentric domains centered on the liquid. The maximum mesh size in these domains is chosen according to the variations of concentration: the area near the three-phase contact is meshed with more refinement to capture properly the divergence of the concentration gradient at the contact line, and as the distance to the fiber increases, the maximum mesh size allowance is increased. The size of the box describing the atmosphere is at least one hundred times larger than the largest of the lengths of the system in order to be considered as infinite with respect to the above boundary conditions.

*Results.* In the limit $\lambda \ll 1$, we obtained analytically the local evaporative flux

$$j_{\text{small}}(z) = j_{0}\left(1 - \frac{z^2}{L^2}\right)^{-1/2}, \quad (2)$$

Fig. 1: (a) Notations used to describe a sleeve of length $2L$ on a fiber of radius $a$. The dimensionless length is defined as $\lambda = L/a$. Boundary conditions used to solve eq. (1) as well as the coordinate system are also represented schematically. Vapor concentration field for (b) $\lambda = 5 \cdot 10^{-3}$ and (c) $\lambda = 500$, obtained from COMSOL for sleeves of radius $a = 125 \mu m$.\]
where $j_{0,\text{small}}$ is the local flux at $z = 0$ defined as

$$j_{0,\text{small}} = \frac{D(c_{\text{sat}} - c_\infty)}{L(1 - 2\gamma_c - \ln(\frac{1}{\lambda}))}, \quad (3)$$

with $\gamma_c \approx 0.577$, the Euler gamma constant.

Because the geometry is not reduced to a single length-scale, eq. (3) indicates that the flux depends both on the sleeve length $L$ and the aspect ratio $\lambda$, which is found numerically as shown in the inset of fig. 2(a). We find a good agreement between the numerical results and eq. (3).

Equation (2) indicates that the flux diverges at the edge between the liquid and the solid. This behavior is confirmed numerically as observed in fig. 2(b) where numerical results obtained for various $\lambda$ are compared to the analytical prediction of eq. (2) represented in solid black line. This divergence is also obtained for sessile droplets, which indeed exhibit the same diverging expression in the limit $\theta \to 0 \ [21]$.

We also solve analytically eq. (1) for infinitely long sleeves, i.e., $\lambda \to \infty$. Under this hypothesis, the system is invariant by translation along the fiber axis such that the local flux is uniform. We can write

$$j_{0,\text{large}} = \frac{D(c_{\text{sat}} - c_\infty)\pi}{2a(2 - 2\gamma_c + \ln 2 + \frac{\pi}{2}\ln \lambda)}, \quad (4)$$

We extend this result to large but finite aspect ratios by arguing that the local evaporative flux remains mostly uniform, such that $j_{0,\text{large}}(z) \approx j_{0,\text{large}}$.

Again, we observe a good agreement between numerical and analytical results for the local flux at the center of the sleeve $j_0$ as shown in fig. 2(a). As for small $\lambda$, because of the cylindrical geometry, numerical results (fig. 2(a)) and eq. (4) indicate that the flux depends on both the sleeve radius $a$ and the aspect ratio $\lambda$.

For $\lambda > 1$, fig. 2(b) shows that the increase of $\lambda$ leads to a localization of the divergence at the contact line. For the largest tested aspect ratios, the flux per unit surface is nearly constant along the sleeve except close to the contact line. However, as shown in the inset of fig. 2(b), we still find close to the contact line the classical minus one-half power-law divergence even for large aspect ratio. Indeed, mathematically, this divergence with a power $-1/2$ is the only one that is solution of eq. (1) to describe the divergence of the local flux at the contact line. For $\lambda = 5 \cdot 10^4$, numerical uncertainties prevent us from concluding on the expression of the divergence, but we expect that the classical square root law holds.

The total evaporative flux is defined as $Q = \int j(z) \, dS$ where the integral is taken over the liquid-vapor surface area. In the limit of small aspect ratios, the total flux writes

$$Q_{\text{small}} = \frac{2D(c_{\text{sat}} - c_\infty)\mu a \pi^2}{1 - 2\gamma_c - \ln(\frac{1}{\lambda})}. \quad (5)$$

In the limit of large aspect ratio, we neglect the contribution of the divergence. Indeed, this divergence is localized and contributes weakly once integrated to the total flux [31]. Thus, the total evaporative flux writes $Q_{\text{large}} = j_{0,\text{large}} 4\pi a L$, which gives

$$Q_{\text{large}} = \frac{2D(c_{\text{sat}} - c_\infty)\lambda a \pi^2}{2 - 2\gamma_c + \ln 2 + \frac{\pi}{2}\ln \lambda}. \quad (6)$$

As shown in fig. 3, where we plot the dimensionless total flux $Q/(2D(c_{\text{sat}} - c_\infty)\mu a)$ as a function of the aspect ratio $\lambda$, we observe an excellent agreement between the numerical results and the analytical model. The good agreement between numerical results and theory at large aspect ratio demonstrates the small effect of the edge contribution to the total flux, which is due to the localization of the

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**Fig. 2:** (a) Local evaporative flux at the center of the sleeve $a j_0$ as a function of the dimensionless sleeve length $\lambda = L/a$ obtained from numerical computations for various sleeves lengths $L$ and three different radii $a$ (see caption). The black solid line corresponds to eq. (3) and the black dashed line corresponds to eq. (4) both multiplied by $a$. In the inset, the local flux at the center of the sleeve $j_0$ is plotted as a function of $L^{-1}$ for the three sleeves radii mentioned above. (b) Dimensionless flux density $j/j_0$ as a function of dimensionless position $z/L$ along the sleeve. The points are obtained from numerical computation for $a = 125 \mu m$ and various sleeves lengths. The black solid line corresponds to eq. (2) nondimensionalized by eq. (3). The black dashed line corresponds to $j = j_0$ as predicted for infinitely long sleeve (eq. (4)). In the inset, the black line $j$ is plotted as a function of the distance to the contact line for $a = 125 \mu m$ and various sleeves lengths. The curves are arbitrarily shifted for clarity. The color gradient represents the length $L$ of the sleeve on both graphics.
gle barrel-shaped droplet and is stable for small contact analogy and a macroscopic axisymmetric drop with a vanishing put on a fiber does not necessarily adopt a sleeve morphol-
ture of the substrate, a droplet of perfectly wetting liquid
obtained for sleeves and spherical drops.

We then discuss these results and compare them to those
boundary conditions for axisymmetric droplets on fibers.

The numerical computation of eq. (1) with the associated

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Fig. 5: (a) Total evaporative flux of droplets on fiber as a function of $\Omega^{1/3}$. (b) Total dimensionless evaporative flux as a function of the contact angle. The points are numerical results for evaporating drops on fiber for different contact angles. Colors indicate the variation of the droplet aspect ratio $H = h_{0}/L$. The dashed black line is the evaporative flux of a sphere (eq. (7)) in the same condition as the numerical simulations.

The atmosphere is taken to be at least one hundred times larger than the longest lengths of the system to ensure $c = c_{\infty}$ far from the droplet.

**Results.** First, we want to understand how the presence of the fiber influences the evaporation speed. To do so, we obtained numerically the total evaporative flux $Q$ of barrel-shaped droplets on fibers of radius $a = 125 \mu m$. Numerical simulations are performed for various drop volumes $\Omega$ and contact angles $\theta$. We recall that we limit ourselves to cases of barrel-shaped droplets, which only exist for drops of sufficiently large volumes that are placed on hydrophilic substrates [15].

The results are shown in fig. 5(a) where we plot $Q$ as a function of $\Omega^{1/3}$ and in fig. 5(b) representing the dimensionless evaporative flux as a function of the contact angle. The total evaporative flux of a spherical drop [16],

$$Q_{\text{sphere}} = \frac{48\pi^2}{3}D(c_{\text{sat}} - c_{\infty})\Omega^{1/3},$$

is also represented in dashed black line for comparison. We observe that the evaporation rate of a barrel-shaped drop on a fiber is similar to that of a sphere for all the contact angles studied except for the smallest volumes tested. This means that varying the contact angle has a negligible effect on the evaporative flux except for small volume drops. We also note that, for small volumes, the deviation from the sphere is increasing as the contact angle is getting smaller, the largest variation being observed for the 0.01 $\mu$L drop having a contact angle of 10° whose profile is given in fig. 4(b). From fig. 5(b), we can quantify the deviation from the sphere, which varies from a few percent for $\Omega \geq 0.1 \mu L$ to around 35% for $\theta = 10°$, $\Omega = 0.01 \mu L$.

We can also note that decreasing contact angle has a small effect on the drop aspect ratio $H$ except for small volume drops. This is due to the curvature of the substrate that allows the existence of a macroscopic drop ($H \sim 1$) even for small contact angles.

To understand these observations in more details, we consider the local evaporative flux of barrel-shaped droplets. First, we focus on the effect of the contact angle. In fig. 6(a), we plot the local flux $j$ as a function of the position along the interface $z$ for a droplets of volume $1 \mu L$ with various contact angle corresponding to aspect ratios $H \simeq 1$. This figure shows that the flux $j$ diverges in close vicinity of the drop contact line. The decrease of the contact angle has a very small effect on the local evaporative flux and this effect is significant only near the contact line. We also compare the results obtained for barrel-shaped drops for which $\lambda \in [5; 10]$ with those of the sleeves having the same dimensionless length. These sleeves have an aspect ratio $\lambda > 1$, which means that the divergence of the local flux is localized as described in the previous section. The comparison between the drops and the sleeves highlights the effect of the drop profile curvatures. The longitudinal curvature of the surface is estimated by $H$, with $H = 1$ corresponding to a spherical drop whereas $H = 0$ is a sleeve. Figure 6(a) demonstrates that, due to the curvature of the drop surface, the divergence of the evaporative flux of a drop on a fiber is even more localized at the triple line than for the sleeve. We propose a phenomenological equation for the local flux $j(z)$ of drops on fibers defined as

$$j(z) = j_0 \left[ \beta \left( 1 - \frac{z^2}{L^2} \right)^{-\alpha} + (1 - \beta) \right],$$

where $\alpha$ and $\beta$ are positive adjustable parameters. Figure 4 provided in the SM shows fitted data for $1 \mu L$ drops for different contact angles. The good agreement between numerical results and fitted curves shows that, unlike sessile drops, the local evaporative flux of a drop on a fiber cannot be described by a simple power law. If we compare the local flux of barrel-shaped droplets to the local flux of a spherical droplet (fig. 2(a), (b) in the SM) we see that the local flux at the center of the barrel-shaped droplets is about 1.5 times smaller than the evaporative flux of a sphere. Thus, barrel-shaped drops are evaporating at the same speed as spherical droplets because the difference in local fluxes at the center of the drop compensates the localized divergence of the local flux of barrel-shaped droplets.
obtained for $\Omega = 1$ for the smallest volume tested to 0.5.

The evaporation rate of a liquid deposited on a fiber when the evaporation is isothermal and purely due to diffusion is significant for small volumes and small contact angles, i.e., when $\lambda \approx 3$. In this case, the geometry is similar to a liquid cylinder and we obtain results comparable to those expected for a sleeve. Nevertheless, the drop, which has a dimensionless length $\lambda \approx 2.3$ can be well compared to the sleeve for a greater aspect ratio, $\lambda = 5$. Although $h_0$ tends to 0, we interpret that the liquid thickness still has a significant effect on the evaporative flux.

Finally, to understand the difference observed between the total evaporative flux of a spherical drop and the one of a small volume wetting barrel-shaped drop on a fiber, we plot in fig. 6(b) the local evaporative flux as a function of the position along the interface for droplets of small contact angle $\theta = 10^\circ$ and different volumes. This difference is significant for small volumes and small contact angles, $\theta = 10^\circ$ and different volumes. This difference is significant for small volumes and small contact angles, i.e., when $H$ vanishes. In this case, the geometry is similar to a liquid cylinder and we obtain results comparable to those expected for a sleeve. Nevertheless, the drop, which has a dimensionless length $\lambda \approx 2.3$ can be well compared to the sleeve for a greater aspect ratio, $\lambda = 5$. Although $h_0$ tends to 0, we interpret that the liquid thickness still has a significant effect on the evaporative flux.

**Conclusion.** – In this paper, we studied the curvature effect on the evaporation rate of a liquid deposited on a fiber when the evaporation is isothermal and purely diffusive. Two particular situations were analyzed: a sleeve of liquid of size $L$ deposited on fiber of radius $a$ and a droplet of volume $\Omega$ and wetting contact angle $\theta$ on the same fiber. The sleeve is obtained when a liquid fiber is coated by a liquid layer before the onset of destabilization of the Rayleigh-Plateau instability. The droplet, and more particularly the axisymmetric barrel-shaped droplet that we studied here, is encountered in the late stage of destabilization when the liquid coating has been destabilized in a series of liquid pearls.

For the evaporation of the sleeve we obtained an analytical calculation of the local evaporative flux along the sleeves and the full evaporation rate in the case of small and large aspect ratios. For large aspect ratio, the evaporative flux is almost uniform along the sleeve except near the edges, where we recover the same power divergence as the one observed for sessile droplets at low contact angles.

For small aspect ratio, the role of the edges progressively becomes more and more important, the evaporative flux is varying significantly along the sleeve since the power law divergence invades all the sleeve. We compared our analytical calculation to finite element computations and showed that our asymptotic calculations capture quantitatively the simulations even in the regime where the aspect ratio is close to one.

For the evaporation of the liquid axisymmetric barrels, we performed numerical simulations in order to understand precisely the effect of the fiber on the evaporation rate. We have evidenced that, for drops of volume $\Omega \geq 0.1 \mu L$ corresponding to dimensionless volume $\Omega/a^3 \geq 50$, the evaporation rate is almost independent of the wetting contact angle and that the droplet evaporates as a sphere of the same volume. More precisely, the evaporative flux diverges near the triple line, but due to the barrel shape, the divergence is strongly localized close to the edges and the evaporative flux is nearly constant along the interface. The divergence close to the edge compensates fortunately the fact that the longitudinal curvature of the surface at the apex, estimated by $H$, is not equal to the one of a spherical drop.

This study provides precise calculations that capture the drying dynamics of the two important morphologies of a liquid on a fiber under the assumption of an isothermal evaporation process. They both emphasize the localization of the evaporation divergence close to the triple line, in contrast to sessile drops. To complete this analysis of the evaporation of a liquid coated on a fiber, it is necessary in a future work to focus more precisely on what happens after the destabilization of the sleeve into a series of regularly spaced drops. Indeed, additional studies would be necessary to understand and quantify the mutual influence of the drops on each other.

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