Cosmological Simulations and Models of Galaxy Formulation

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Abstract. Cosmological simulation has become the most efficient and advanced method to investigate the fundamental principle and physical model of the universe. Galaxy formulation is one of the most significant characters in this field, which attracts lots of attention. However, it is challenging to establish a proper model of galaxy simulation. Besides, a lot of investment is required to solve the problem of modelling dark matter, dark energy, and ordinary matter. This review introduces the most widely accepted cosmological models and galaxy formation procedures. Moreover, discussions and comparisons are presented for several advanced simulation methods. These results evaluate milestones in the development of cosmological numerical simulation objectively and will offer a guideline for the development of galaxy formation.

1. Introduction

Compared with other physics subjects, it is difficult to carry out traditional practical experiments for cosmological studies due to the macroscopic scale and eternal range. However, with the gradual deepening of basic theoretical research, many excellent physics theories can no longer be verified by experiments on the earth [1]. Contemporarily, the developments of computer science and semiconductor physics have a great contribution to cosmological simulations, which makes it possible to study and examine the fundamental theory of the universe on supercomputers. In recent decades, simulation models, computerized algorithms and simulation results have been widely proposed and discussed.

One significant and necessary topic of cosmological simulations is galaxy formulation [2]. A galaxy is a relatively independent mass aggregate, which is the basic unit to measure the interaction of matter in the universe. In order to figure out the cosmological model and its numerical methods, formulation and interaction model of galaxy is necessary to be established initially. Based on the hardware performance of the computer and the considered physics model consistent with important results in computational physics, there are many advanced cosmological galaxy formation models and are widely used.

According to previous literature, three essential problems are demonstrated in this review. The commonly used cosmological models are generally described in Sec. 2 as well as their principles and limitations. The numerical methods of cosmological simulations are introduced in Sec. 3 with related data and plots to illustrate the results. Section 4 contains the comparison of these simulation results and
the differences between the computer and observational data., and a summary of the achievement and limitation of cosmological simulation of the galaxy formulation as well as its future development tendency appears in Sec. 5.

2. Galaxy formulation model
Cosmological simulation is always based on the fundamental structure of the universe and the interaction between the particles. Therefore, it is necessary to figure out the cosmological model before diving into galaxy formulation.

2.1. Cosmological model
The most basic and important parameter in cosmology is mass. According to the research data [3], the most massive component in the universe is dark energy, which accounts for about ~ 68.3%, followed by dark matter, accounting for about ~ 26.8%. The last 5% is baryons, which are the particles defined from standard model [4]. It is worth mentioning that, according to the equation of conservation of mass and energy in Einstein’s special theory of relativity (Eq. 1), mass and energy are equivalent to this extent.

\[ E = mc^2 \]  (1)

In order to avoid complicated mathematical processes, an idealized model is proposed based on the mass distribution in the universe, which is ΛCDM model, or Lambda Cold Dark Matter model [5]. The first feature of this model is cold dark matter, which means the velocity of particles is much slower than the light where Newtonian dynamics can be applied. Besides, there would be no internal energy loss because of non-gravitational interaction. The second feature is that the universe is a curvature space-time coordinate [6] according to the Einstein Field Equation (Eq. 2). Moreover, in ΛCDM model, the curvature is considered as zero. The third feature is cosmological constant Λ, which is related to the dark energy, and its effects to the expansion of the universe [7].

\[ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \]  (2)

ΛCDM model is a widely accepted cosmological model, which can explain many important cosmological problems [3] (e.g., existence and structure of cosmic microwave background, distribution of the galaxies, accelerating expansion of the universe).

2.2. Galaxy formulation process
Galaxy formulation is one of the most significant and complicated topics in the cosmological simulation. Large-scale, multi-process, and multi-variability make it extremely difficult to model the formulation of galaxies. Here, we show the process of galaxy formation by introducing the influence of important physical parameters on the formation of galaxies [8].

2.2.1. Gravity. According to the cosmological model [8], gravity is the only force that can cluster galaxies. To be more technical, the formation of galaxies was originally caused by uneven distribution of mass (also known as density fluctuations), which is the result of quantum fluctuation. The shape and amplitude of fluctuations are related to the parameters and dark matter. According to standard paradigm, galaxies are formed in the dark matter halo, which is determined by the gravity. Besides, gravity also affects the interaction between galaxies and their morphological changes [8-10].

2.2.2. Thermal evolution. The formation and evolution of galaxies are usually accompanied by energy conversion, which makes it necessary to consider thermodynamic effects. When there is a collapse because of density change, the local entropy would change subsequently, leading to heat exchange and radiation, which are basically two-body radiation processes in galaxy formation. The action of gas is determined by its temperature. If the temperature of gas is higher than 107 K, the gas would fully ionize and cool mainly through bremsstrahlung radiation. If the temperature of gas is between 104 and 107 K,
the atoms of gas would decay to their ground state and there could be electrons combine with ions. If
the temperature of gas is lower than 104, the gas would cool by collision excitation and de-excitation of
heavy elements and molecular cooling [8-10].

2.2.3. Star formation and black hole. After galaxy formation, black holes may appear due to the
evolution of stars. If the mass of star is big enough, then after the end of this star, it could lead the
formation of black holes. Then in a local part of galaxy, there could be black holes merger, resulting to
supermassive black holes. Because of the unique and extreme physical properties of black hole, the
space-time, matter distribution, angular momentum and magnetic field around it will be changed. Meanwhile, black holes are also an important cause for the mass accumulation of galaxies and the
production of accretion disks [8-10].

2.2.4. Maturity period. After the basic framework of the galaxy is formed, there could be regular star
formulation and other interactivity in a huge range. The evolution of stars has influences on the
luminosity and chemical element distribution of the galaxy, and there must be active galactic nuclei
(AGN) during that period. Then the stellar population and chemical reaction might happen, and because
of the thermal process, radiation transfer would also be added into the process [9].

Figure 1. A simulation example Visualization of representative quantities computed by numerical
hydrodynamic simulations, from the Illustris project. From left to right, the dark-matter density,
gas density, gas temperature, and gas metallicity are shown at different cosmic times and from top
to bottom redshift z = 0, 1, 2, 4, respectively [8].
2.3. Equations and boundary conditions
Most cosmological simulation results are based on the ΛCDM model, and the most fundamental and necessary interaction between particles is gravitational interaction (Eq.3) [11-12].

\[ F = G \frac{m_1 m_2}{r^2}, \quad G = 6.67 \times 10^{-11} \]  

(3)

However, in galaxy formulation methods, this equation (Eq.3) is too limited to describe the function. Therefore, we have to consider gravity between a great number of particles, which is also known as N-body problem [2]. The answer is given by Poisson’s equation (Eq.4) and Boltzmann equation (Eq.5).

\[ \nabla^2 \phi = 4\pi G \int f \, dv \]  

(4)

\[ \frac{df}{dt} = \frac{\partial f}{\partial t} + v \frac{\partial f}{\partial r} - \frac{\partial \phi}{\partial r} \frac{\partial f}{\partial \phi} \]  

(5)

Besides, because of the expansion of the universe and the large scale of the cosmology, it is necessary to consider the Einstein Field Equation (Eq.2). Thus, one derives the Friedmann Equations (Eq.6) [9, 13].

\[ H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{k c^2}{a^2} + \frac{\Lambda c^2}{3} \]  

(6.a)

\[ \dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho + \frac{3p}{c^2} \right) + \frac{\Lambda c^2}{3} \]  

(6.b)

It is necessary to figure out the initial condition of these equations, and in classic dynamics, the initial conditions include position \( x \) and velocity \( v \) (Eq.7-8) [2].

\[ x = q + D(t) \Psi(q) \]  

(7)

\[ a(t) \dot{x} = a(t) \frac{dD(t)}{dt} \Psi(q) \]  

(8)

Here \( x \) is a vector of position of the particles while \( q \) is the unperturbed vector of position, but it is. \( D(t) \) is the linear growth factor, and \( a(t) \) is the scale factor which is related to the initial redshift (Eq.9). \( \Psi(q) \) is the curl-free displacement field given by the linearized continuity equation (Eq.10).

\[ \frac{1}{a(t)-1} \]  

(9)

\[ \nabla \cdot \Psi(q) = -\frac{\delta}{D(t)} \]  

(10)

3. Simulation methods
It is quite challenging to simulate the galaxies in the universe owing to the large scale of galaxies. Whereas, there are extremely complicated research objects and interactions that should not be ignored. Therefore, it is particularly important to figure out how to propose some simplified galaxy models under the premise of ensuring the accuracy of the algorithm and the accuracy of the physical principles.

3.1. N-body Method
The technique on how to simulate the formation of galaxies has skyrocketed to a great level in the past three decades. The most basic and fundamental algorithm in cosmological numerical simulation is to deal with multi-body problems, which is also known as N-body problems in physics [14].

3.2. Early algorithm
N-body methods are usually used to simulate the motion of particles that interact with others through physical forces. It is utilized to follow the non-collision dynamics of dark matter in the galaxy simulation.
For example, in an expanding universe, the dark-matter fluid could be illustrated by Boltzmann Equation and Poisson Equation (Eq.6).

In the 20th century, many numerical calculation methods have been proposed. For example, Von Hoerner and Sebastian proposed a multi-body model in 1960 to solve the problem of star clusters [15,16]. They used numerical integration and complex algorithms to simulate the dynamics of a fixed number of stars. There is an example of their work (Tab.1), they used velocity gradients and counting methods by specific rules to numerically integrate velocity. It is obviously that, the classifications and calculations become complicated as the number of stars increases.

Although they were quite successful in this condition, it reflects the shortcomings of numerical simulation in this period. Insufficient computer computing power makes it no choice but to simulate the model through extremely complicated mathematical work, which is quite difficult to accept. Besides, the calculation results are very unintuitive. Without mature drawing and rendering tools, many physical processes must be reflected in numbers. As a consequence, there is no need to comment and compare the complexity of the algorithm [17]. Therefore, advanced simulation algorithms become necessary.

Table 1. Table of velocity distribution and time. Columns represents a velocity range, and rows represents the speed count of the corresponding stellar sphere, where \( n \) is the number of stellar spheres [15].

| \( v \) | \( n=8 \) | \( n=12 \) | \( n=16 \) |
|-------|--------|--------|--------|
|       | 0 1 2 3 | 0 1 2 3 4 5 | 0 1 2 3 4 5 6 7 8 9 10 11 12 |
| 0.0 – 0.1 | 2 | | |
| 0.1 – 0.2 | 2 | 1 | 1 2 |
| 0.2 – 0.3 | 3 1 2 1 1 1 1 2 2 | 1 2 2 3 1 | 3 3 3 |
| 0.3 – 0.4 | 2 4 1 2 3 1 2 1 2 | 2 2 4 1 1 4 3 2 2 1 |
| 0.4 – 0.5 | 3 4 1 6 2 3 3 4 4 | 3 4 4 1 2 5 4 1 1 3 |
| 0.5 – 0.6 | 4 2 1 1 3 2 1 1 1 | 4 1 1 4 3 3 3 1 2 3 |
| 0.6 – 0.7 | 4 3 5 2 1 2 2 1 1 | 1 3 2 1 2 1 1 3 3 1 1 1 |
| 0.7 – 0.8 | 6 2 3 1 2 1 1 1 | 2 1 1 2 1 3 4 1 1 3 1 2 |
| 0.8 – 0.9 | 8 1 3 2 1 2 1 1 | 2 1 3 1 2 1 2 2 1 3 |
| 0.9 – 1.0 | 1.0 – 1.25 2 3 3 1 1 2 | 1 4 5 1 3 1 4 3 1 2 |
| 1.25 – 1.50 2 1 1 1 | 2 1 1 1 1 1 2 |
| 1.50 – 1.75 1 2 1 | |
| … | | 1 |
| 3.25 – 3.50 | | |
| >3.5 | | |

3.3. Gravity Pipe method

Gravity Pipe is a project of the University of Tokyo at the beginning of the 21st century, which mainly uses the gravitational model for calculation. The method of this project is Gravity Pipe Method (GRAPE). GRAPE meets the need of high accuracy for force calculation. On the other side of the continuum, GRAPE could be used by giving different types of timesteps for particles that have a large variety on density. The newest and most popular form should be GRAPE-6 system, which is amended from GRAPE-4 system. GRAPE-6 is mainly used for simulating collision systems, despite it also can simulate collision-free systems [18, 19].

3.4. Particle-Mesh Method

Particle-Mesh Method (PM) means to separate the space into meshes. Thus, nearby mesh’s vertices can divide particles with different conditions into spaces between them. As a result, the process can be much easier to sum the forces together. However, the method is limited by the mesh size. Sometimes scientists
use a smaller mesh or other algorithms to compute forces within small scales [20]. PM is an excellent solution to the N-body numerical simulation method, and one significant reason is the computational efficiency (Fig. 2), it is obvious that PM saves a lot of computing time under the premise of ensuring calculation accuracy.

![Figure 2. Comparison of CPU-time between PM and conventional methods with different accuracy levels. Attention to the value of the vertical axis, the CPU time is about $10^4$ (s) by using the conventional method and about $10^1$ (s) by PM [20].](image)

3.5. Tree-PM method
Tree Particle-Mesh method (Tree-PM) is a combination by not only tree algorithm but also with Particle Mesh method. The Particle Mesh method is the most common mean to simulate clustering of structures in large scales. Generally, it has a poor force resolution due to its origin though scientists have made some amendments so far (Particle-Particle Particle Mesh method). However, tree algorithm is a different approach on compute forces. Researchers can define the meaning of groups that are based on particles at a large distance to calculate the sum of forces. Currently, the most frequently used method is presented by Barnes & Hut (1986), which requires users to consider the periodic boundary conditions [21, 22]. There is an example of the result from Tree-PM method simulation as illustrated in Fig. 3. Generally, the difference between Tree-PM and PM methods is quite small, while there is still variation in local distribution, and the simulation result of Tree-PM seems to have higher resolution.
3.6. Hydrodynamic methods

Another type of simulating the universe is to describe it as an ideal fluid. Thus, we can solve the hydrodynamic equations: Euler equation (Eq. 11), fluid continuity equation (Eq. 12) and the first law of thermodynamics (Eq. 13) [23].

\[
\frac{d\bar{v}}{dt} = -\frac{\nabla P}{\rho} - \nabla \Phi \tag{11}
\]

\[
\frac{d\rho}{dt} + \nabla \cdot \bar{v} = 0 \tag{12}
\]

\[
\frac{du}{dt} = -\frac{\bar{p}}{\rho} \nabla \cdot \bar{v} - \frac{\Lambda(u, \rho)}{\rho} \tag{13}
\]

All equations listed above can be solved by the grid-based methods (based on structured of unstructured grids) and Smoothed-Particle Hydrodynamics methods (SPH, based on mass elements of the fluid instead of volume elements). 

Figure 3. A simulation result based on S-CDM Model (only consider dark matter and baryons). The top panel is from Tree-PM simulation and the bottom panel is from PM simulation [22].
3.6.1. Smoothed-Particle Hydrodynamic methods. SPH is a meshfree Lagrangian method, which makes it easier to implement. Besides, it can be expanded to many realms and combined with other types of modes. Nevertheless, there are some limitations. It is more difficult for scientists to set boundary conditions for SPH than for Grid-based method. In addition, the cost for computing SPH simulations is greater than that of Grid-based simulations, which means it is less efficient for the simulation of constant-density flows [24].

3.6.2. Grid-Based methods. Grid-based methods transforms one pair of map coordinates to another by using a low-order interpolation method [25]. Since it is quite challenging to solve the hydrodynamical partial differential equations, numerical calculation is quite suitable for this problem based on selected grids. Besides, in recent approaches, the simulation is based on the order of the grids and the interactions between nearby cells to form a hydrodynamical field.

4. Comparative Results between Model & Model, Model & Data
Numerical simulation has become an effective way to research astrophysical phenomena in the last few decades. Typically, two ways are effective to verify the correctness of numerical simulation: comparing a model with another and comparing a model with related data. In this paper, we call them ‘Model & Model’, ‘Model & Data’.

On the one hand, the comparison between models is a quick and easy way to search the best method. The correctness of the model can also be verified by comparing the results of various simulation methods. On the other hand, the interaction between numerical simulation and astronomical observation is mainly reflected in two aspects: to guide astronomical observation through simulation results; verify the correctness of numerical simulation, i.e., improve the method and precision of simulation. We can compare the model and observation data, to obtain more accurate data and better algorithms. In order to analyse the accuracy of numerical simulation, two examples are selected to discuss for each comparison method, respectively.

4.1. Comparison between Methods
In 1999, the Santa Barbara Cluster Comparison Project, presented by Frenk et al. [24], studied the different simulation codes. In this research, their code is based on the Hydrodynamic cosmological model. Then, they got some conclusions through the comparison of twelve different codes. All these codes in the project were tracking the evolution of forming galaxy clutter by cold dark matter model (Fig. 4). They used totally different methods to calculate. These methods can be roughly divided into several types, such as smoothed particle hydrodynamics and mesh-based techniques. The initial conditions of each code were given in an independent manner. The conclusion of these twelve models was acceptable [26]: the distribution of dark matter and gas is remarkably consistent. Nevertheless, there are still some problems we can find. Those two types of methods have a systematic deviation in the distribution of central entropy, which leads to the simulation in an error of the gas properties, especially to the core.

From Ref. [25-27], the central cluster entropy problem as mentioned above, is still an open question. In this case, one should aware that it is an important way to improve the verity of numerical simulation. If it can be figured out, the central accuracy of the simulation results will be greatly increased.
4.2. Comparison between Model and Data

The Kennicutt–Schmidt relation shows the interaction between gas surface density and stars. The traditional $H_2$ model cannot establish the relationship between star formation rate (SFR) and the dependence of the molecular fraction ($f_{\text{mol}}$) [28]. Since a lot of the parameters of the galaxy are related to the size, one is able to visualize it in the figure [28-34]. In this case, the method of spatially resolved manner is provided in Ref. [28] depicted in Fig. 5 for the galaxy TNG100.
Figure 5. The pictures of TNG 100 galaxy’s neutral, atomic and molecular gas in Kennicutt–Schmidt relation (from right to left). The outline in color of orange shows the positions of spatially determined observations of seven spiral galaxies, with horizontal and vertical dotted lines marking their sensitivity limits to star formation and $H_2$ surface densities [28].

The three graphs in Fig. 5 are histograms of $H_1+H_2$, $H_1$ and $H_2$ amount by pixels in TNG100. The orange curve is the image drawn from observational data. By comparing the curve and pixels, the differences can be investigated and analyzed between the observation results and the simulation data.

The graph on the left is the KS relation for neutral gas, which shows a good fit on the median. The simulation also matches the observational data around the area of $\Sigma_{H_1+H_2} = 1M_\odot/pc^2$. The relation tends to be steeper and lower than that of Kennicutt [28], which was consistent with the observation results. Then the graph on the center is for atomic gas, shows that the simulation result matches the outline of observational data. Other models show a trend that has analogous result. Especially in $\Sigma_{H_1} = 1M_\odot/pc^2$, it is perfectly suitable. Finally, the graph on the right matches the data at the intermediate densities of $\Sigma_{H_2} = 1M_\odot/pc^2$, which is about the KS relation for molecular. Nonetheless, the result presents a steep relationship at high density simulation.

In general, ‘Model & Model’ is more suitable for a preliminary test of the simulation method. Besides, the feasibility of an algorithm can be inferred by comparing the similarity of conclusions drawn from different algorithms. Meanwhile, a better algorithm will be found in the comparison of the simulation results. ‘Model & Data’ is a test of theory and practice. Multiple models can be compared to a set of observations to find the best one.

5. Conclusions & Perspective

In summary, galaxy formulation model, common simulation methods, and comparative results are described and argued meticulously. After extensively examining this field, galaxy simulation is of considerable importance in understanding the mechanism of the universe. In detail, according to the $\Lambda$CDM model, galaxies are initially formed from the quantum fluctuations by gravitational interaction, which leads to element synthesis, star formation, and galaxies interactions. Besides, N-body simulation methods, especially Gravity Pipe Method and Particle-Mesh Methods are of great importance for modeling galaxies, balancing algorithm complexity, and the accuracy of calculation result. Although there are differences between different simulation algorithms, they all restore the evolution of galaxies in the universe to a certain extent. Therefore, it undoubtedly has a huge impact on the understanding of the origination and destination of the universe. Based on the recent literature, two appealing aspects are large volume simulations and zoom simulations. However, the most urgent problem is to fully understand the physical process of galaxy formation. Moreover, the revolutionary improvement of computer computing power and algorithms would be needed continuously.
References

[1] Turok, Neil. (1987) Cosmic strings and the formation of galaxies and clusters of galaxies. Nearly Normal Galaxies. Springer, New York, NY., 431-450.

[2] Vogelsberger, Mark, et al. (2020) Cosmological simulations of galaxy formation. Nature Reviews Physics 2.1: 42-66.

[3] Aghanim, Nabila, et al. (2016) Planck 2015 results-XI. CMB power spectra, likelihoods, and robustness of parameters. Astronomy & Astrophysics 594: A11.

[4] Ooba, Junpei, Bharat Ratra, and Naoshi Sugiyama. (2018) Planck 2015 constraints on the non-flat $\Lambda$CDM inflation model. The Astrophysical Journal 864.1: 80.

[5] Cai, Rong-Gen, and Sang Pyo Kim. (2005) First law of thermodynamics and Friedmann equations of Friedmann-Robertson-Walker universe. Journal of High Energy Physics 2005.02: 050.

[6] Peebles, P. James E., and Bharat Ratra. (2003) The cosmological constant and dark energy. Reviews of modern physics 75.2: 559.

[7] Somerville, Rachel S., and Romeel Davé. (2015) Physical models of galaxy formation in a cosmological framework. Annual Review of Astronomy and Astrophysics 53: 51-113.

[8] Peacock, J. A. (1997) The evolution of galaxy clustering. Monthly Notices of the Royal Astronomical Society 284.4: 885-898.

[9] Benhaiem, David, Francesco Sylos Labini, and Michael Joyce. (2019) Long-lived transient structure in collisionless self-gravitating systems. Physical Review E 99.2: 022125.

[10] Sotiriou, Thomas P., and Valerio Faraoni. (2010) f (R) theories of gravity. Reviews of Modern Physics 82.1: 451.

[11] Bekenstein, Jacob D. (2004) Relativistic gravitation theory for the modified Newtonian dynamics paradigm. Physical Review D 70.8: 083509.

[12] Klypin, Anatoly, HongSheng Zhao, and Rachel S. Somerville. (2002) $\Lambda$CDM-based models for the Milky Way and M31. I. Dynamical models. The Astrophysical Journal 573.2: 597.

[13] Linder, Eric V. (2003) Exploring the expansion history of the universe. Physical Review Letters 90.9: 091301.

[14] Boylan-Kolchin, Michael, et al. (2009) Resolving cosmic structure formation with the Millennium-II Simulation. Monthly Notices of the Royal Astronomical Society 398.3: 1150-1164.

[15] Von Hoerner, Sebastian. (1960) Die numerische integration des n-körper-problemes für sternhaufen. i. Zeitschrift für Astrophysik 50.

[16] Von Hoerner, Sebastian. (1963) Die numerische Integration des n-Körper-Problems für Sternhaufen, II." Zeitschrift für Astrophysik 57.

[17] Reif, John H., and Stephen R. Tate. (1993) The complexity of N-body simulation. International Colloquium on Automata, Languages, and Programming. Springer, Berlin, Heidelberg.

[18] Aarseth, Sverre J. Gravitational N-body simulations: tools and algorithms. (2003) Cambridge University Press.

[19] Makino, Junichiro, and Makoto Taiji. (1998) Scientific simulations with special-purpose computers--the GRAPE systems.

[20] Essmann, Ulrich, et al. (1995) A smooth particle mesh Ewald method. The Journal of chemical physics 103.19: 8577-8593.

[21] Warren, Michael S., and John K. Salmon. (1992) Astrophysical N-body simulations using hierarchical tree data structures. SC 92: 570-576.

[22] Bagla, Jasjeet S. (2002) TreePM: A code for cosmological N-body simulations. Journal of Astrophysics and Astronomy 23.3: 185-196.

[23] Milne-Thomson, Louis Melville. (1996) Theoretical hydrodynamics. Courier Corporation.

[24] Frenk, C. S. , et al. (2009) The Santa Barbara Cluster Comparison Project: A Comparison of Cosmological Hydrodynamics Solutions. The Astrophysical Journal 525.2:554.
[25] Dolag, Klaus, et al. (2008) Simulation techniques for cosmological simulations. Space science reviews 134.1-4: 229-268.
[26] Debora, S., et al. (2012) Moving mesh cosmology: the hydrodynamics of galaxy formation. Monthly Notices of the Royal Astronomical Society 424.4:2999-3027.
[27] Sanderson, Ajr, and T. J. Ponman (2010). The Birmingham-CfA cluster scaling project - II. Mass composition and distribution. Monthly Notices of the Royal Astronomical Society 4:1241-1254.
[28] Diemer, B., et al. (2018) Modeling the atomic-to-molecular transition in cosmological simulations of galaxy formation. Astrophysical Journal Supplement 238.2.
[29] Daddi, E., et al. (2010) Different star formation laws for disks versus starbursts at low and high redshifts. The Astrophysical Journal 714.1.
[30] Krumholz, M. R., and T. A. Thompson. (2013) Numerical Simulations of Radiatively-Driven Dusty Winds. Monthly Notices of the Royal Astronomical Society 434.3:2329-2346.
[31] Krumholz, M. R. (2013) The Star Formation Law in Molecule-Poor Galaxies. Monthly Notices of the Royal Astronomical Society 436.3:2747-2762.
[32] Sternberg, A., et al. (2014) HI-to-H2 Transitions and H I Column Densities in Galaxy Star-Forming Regions. The Astrophysical Journal 790.1.
[33] Schmidt, M. (1963) The Rate of Star Formation. II. The Rate of Formation of Stars of Different Mass. The Astrophysical Journal 137.3:758.
[34] Daddi, E., et al. (2010) Different star formation laws for disks versus starbursts at low and high redshifts. The Astrophysical Journal 714.1.