Modeling Pulsar Gamma-Ray Light Curves Using Realistic Magnetospheric Geometries

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ABSTRACT
Gamma-ray emission from pulsars is thought to arise from accelerating regions in pulsar’s outer magnetosphere. The shape of the light curves is thus sensitive to the details of the magnetic geometry of the magnetosphere. In this work, we show the first calculations of light curves from the more realistic force-free field under the framework of conventional emission models. We compare the properties of gamma-ray emission between the commonly used vacuum dipole magnetic field and the new force-free field. We discuss the role of the polar cap shape and aberration effect on the appearance of the light curves as well as the formation of caustics on the sky map. With the force-free field, the double-peak pulse profile is best reproduced if the emission zone lies in a thin layer just outside the current sheet, and the peaks are mainly contributed from regions near the light cylinder. The conventional two-pole caustic model can produce up to four peaks in general, while the conventional outer-gap model can normally produce only one peak. These results will be useful for interpreting Fermi telescope observations.

Key words: gamma rays: theory, pulsars: general, stars: magnetic fields

1 INTRODUCTION
Pulsars are rotating neutron stars (NS) with strong magnetic field. Seven of them were detected as gamma-ray pulsars with EGRET (Thompson 2004). The light curves of these gamma-ray pulsars are typically double-peaked, and have substantial off-peak emission. Theoretical models have been developed in order to explain the nature of pulsar gamma-ray emission, namely, the polar-cap (PC) model (Daugherty & Harding 1982; Daugherty & Harding 1996), the slot-gap (SG, or two-pole caustic, TPC for short) model (?; Arons 1983; Dyks & Rudak 2003; Dyks et al. 2004), and the outer-gap (OG) model (Cheng et al. 1986; Romani & Yadigaroglu 1995; Cheng et al. 2000). In these models, particles are accelerated to ultra-relativistic energies in “gap” regions where strong electric fields are developed due to the deficit of charge. The gamma ray emission comes from curvature and inverse Compton (IC) radiation of these energetic particles. Based on vacuum magnetic field geometries, SG and OG models are more favored since they can reasonably well reproduce the double-peak light curves.

However, pulsar magnetosphere is filled with plasma (Goldreich & Julian 1969). The plasma is essentially force-free (FF), with $\rho E + j \times B/c = 0$, where $\rho$ and $j$ are charge and current density. In the presence of plasma, the magnetosphere consists of open and closed field line regions. Poloidal current flows out along open field lines and induces toroidal magnetic field, which dominates beyond the light cylinder (LC for short, $R_{\text{LC}} = c/\Omega$). Recently, Spitkovsky (2006) obtained the full 3D magnetospheric structure using FF simulations. The FF field geometry differs substantially from the vacuum field geometry (i.e., the retarded dipole field), which is commonly used in light curve calculations, especially around the LC. The light curve is very sensitive to the geometry of the emission zones, thus to the field geometry itself. Therefore, it is important to revisit pulsar high-energy emission models using the more realistic FF field geometry.

This paper is organized as follows. In section 2 we show the light curves from the vacuum field geometry, pointing out that previously calculated vacuum light curves should be modified when aberration effect is correctly treated. In section 3 we present the light curves from the FF field using the two-pole caustic model, and mention the applications to the outer-gap model.

2 RELATIVISTIC EFFECTS AND VACUUM FIELD RESULT CORRECTIONS
In order to calculate the light curve, we need to find the emitting region in the magnetosphere. In the TPC model, the emission zone is assumed to be along the last open field lines (LOFLs) starting from the polar cap. In the OG model, the radiation region is assumed to be beyond the null-charge surface and along open field lines. Technically, given a magnetic field geometry (either vacuum or FF field), field lines
are considered open if they can cross the LC in the lab (observer’s) frame (LF). We trace magnetic field lines in the LF and find the LOFLs. The polar cap is the region on the NS surface where open field lines originate. We calculate the magnetic colatitude of polar cap rim \( \theta_{m} \) at fixed magnetic azimuth \( \phi_m \) (the subscript "m" means w.r.t. magnetic axis rather than rotation axis), and define the open volume coordinate as \( r_{ov} = \theta_{m}/\theta_{m}^{\prime} \). In the TPC model (Dyks & Rudak 2003; Dyks et al. 2004) emission comes from a thin layer centered at LOFL \( (r_{ov} = 1) \) extended from the NS surface up to a certain radius \( (R \sim 0.75 R_{LC}, r \sim 1.0 R_{LC} \text{ where } r \text{ is distance to the NS center and } R \text{ is cylindrical radius}) \). In the OG model (Cheng et al. 2000) the emission comes from a thicker layer centered at \( r_{ov} \sim 0.90 \) extended from the null charge surface (where \( B_z = 0 \)) to the LC.

For curvature/IC radiation, the radiation direction should be along the direction of particle motion. An ultrarelativistic particle moves at the speed of light, and its velocity can be decomposed into corotation velocity (i.e., drift velocity) plus a component along the direction of the magnetic field. Thus, the radiation direction \( \vec{\eta} \) in the LF is determined by

\[
\vec{\eta} = f \mathbf{B} + \vec{\beta}, \tag{1}
\]

where \( \vec{\beta} = \vec{\Omega} \times r/c \) is the normalized corotation velocity, \( \mathbf{B} \) is the magnetic field in the LF, and \( f \) is a coefficient determined by the requirement that \( |\vec{\eta}| = 1 \).

This approach is different from earlier works on OG and TPC models with vacuum field geometries [e.g. Romani & Yadigaroglu (1995), Cheng et al. (2000), Dyks et al. (2004)]. In previous works, it was implicitly assumed that the vacuum field, which is the solution of vacuum Maxwell equations in the LF, is valid in the instantaneously corotating frame (ICF). In other words, in eqn.(1), \( \mathbf{B} \) was improperly replaced by \( \mathbf{B}' \), where \( \mathbf{B}' \) is obtained by the Lorentz transformation of \( \mathbf{B} \) from ICF to LF [see Bai & Spitkovsky (2009a) for a more detailed discussion]. Therefore, results from these models require revision.

Photons emitted from the emission zones are collected in the sky map \( (\phi, \xi_{\text{obs}}) \), where \( \phi \) is the phase of rotation with corrections for photon travel time (i.e., time delay effect), and \( \xi_{\text{obs}} \) is observer’s viewing angle. We assume constant emissivity along field line. The light curve seen by the observer is then obtained by cutting the sky map at a specified viewing angle \( \xi_{\text{obs}} \).

In Fig.1 we plot the sky map and light curve from TPC model for an oblique rotator with the inclination angle \( \alpha = 60^\circ \) using vacuum field geometry. The upper plot is a reproduction of Dyks et al. (2004)’s result, where the retarded vacuum field was implicitly assumed to be in the ICF. There are two strong caustics associated with the two poles. They are formed at modest distance from the NS (roughly \( 0.3 - 0.6 R_{LC} \)). In the light curve, two sharp peaks are thus produced. In the bottom plot we correct the aberration with eqn.(1). We find that the caustics in the upper plot are now much widened and weakened. There are still two peaks, but much wider, and a substantial contribution to the peaks comes from the overlap of emission from both poles, rather than from the caustics. This result also holds for other inclination angles. We conclude that the TPC model has difficulties in producing sharp peaks with corrected aberration. We also note that the correction of aberration formula also affect the conventional outer-gap models using the vacuum field (Bai & Spitkovsky 2009a).

### 3 RESULTS FROM FORCE-FREEFIELD GEOMETRY

The advent of FF field (Spitkovsky 2006) makes it possible to test theoretical models with more realistic magnetospheric geometries. The FF magnetosphere has \( \mathbf{E} \cdot \mathbf{B} = 0 \) everywhere and thus has no intrinsic particle acceleration. As long as pulsar's emission power is much smaller than the total spin down energy loss rate (which is almost always the case), the FF field provides a reliable field structure in the magnetosphere. The pulsar’s radiation comes from a small region of magnetosphere (i.e., the emission zone) where nonideal deviations from the FF condition makes particle acceleration possible. The current sheet is a key feature of FF field. Beyond the LC, it separates field lines from the two poles (Contopoulos et al. 1999; Bai & Spitkovsky 2009b), and it is also a likely place for dissipation and resistive heating (Gruzinov 2007). Consequently, the pulsar’s gamma-ray emission may be connected to the current sheet.

The sky map is determined by the field geometry as well as the shape of the polar cap. Although the FF field appears similar to the vacuum field near the NS surface, the shape of the polar cap is different, because the polar cap shape is sensitive to the field structure near the LC. The polar cap shape in the FF field is more circular and larger than in the vacuum field (Bai & Spitkovsky 2009b), and it will lead to fundamental differences in the sky maps, as we will illustrate here.

In Fig.2 we show the sky map of the conventional TPC model using the FF field which is calculated with \( r_{ov} = 1.0 \).
The emission zones are assumed to be extended from the polar cap to the LC along current sheet. Depending on observer’s viewing angle, the sky map can produce light curves with up to four widely separated peaks, which are inconsistent with the current observations. We have experimented with a number of other possible solutions, and find that the double-peaked pulse profile can be reproduced if we allow the emission zones to be located in a layer just outside the current sheet. In Fig.3 (upper plot) we show the sky map and light curve from the emission zone $r_{ov} = 0.90$. The double-peak pattern is very robust as we vary the inclination angle $\alpha$.

The launch of the Fermi Gamma-ray Space Telescope will significantly expand the sample of gamma-ray pulsars and will provide very accurate pulse profiles. Our results will be useful for the interpretation of Fermi telescope observations and for probing the physics of pulsar magnetospheres.

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**Figure 2.** Sky map from conventional two-pole caustic model $r_{ov} = 1.0$ using force-free field. Inclination angle $\alpha = 60^\circ$, viewing angle $\xi_{obs} = 80^\circ$.

**Figure 3.** Sky map (left) and light curve (right) from the annular gap model (a modified two-pole caustic model) with $r_{ov} = 0.90$ (up) and the conventional outer-gap model with $r_{ov} = 0.90$ (bottom) using force-free field. Inclination angle $\alpha = 60^\circ$, viewing angle $\xi_{obs} = 80^\circ$. 

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