Large angle QED processes at $e^+e^-$ colliders
at energies below 3 GeV

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Abstract
QED processes at electron–positron colliders are considered. We present differential cross–sections for large–angle Bhabha scattering, annihilation into muons and photons. Radiative corrections in the first order are taken into account exactly. Leading logarithmic contributions are calculated in all orders by means of the structure–function method. An accuracy of the calculation can be estimated about 0.2%.

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1 Introduction
At the existing VEPP–2M $e^+e^-$ collider and planned meson factories QED processes of the lowest order of the perturbation theory (PT) play an important role. These processes are to be considered as an essential background when extracting subtle mesons properties from experimental data. QED processes are used also for calibration and monitoring purposes. For instance, large–angle Bhabha scattering is used for a precise determination of luminosity at $e^+e^-$ colliders. That is the reason, why radiative corrections (RC) to QED processes are to be considered in detail.

A lot of attention was paid to the problem of radiative correction calculations to various QED processes, starting more than 50 years ago [1], concerning mainly the lowest order of
PT calculations. The accuracy requirements of modern experiments, however, exceed the ones provided by the first order RC. Unfortunately, calculations of radiative corrections in higher orders encounter tremendous technical difficulties. Nevertheless, powerful methods developed in quantum chromodynamics (see paper [2] and references therein) provide a possibility to improve essentially the results obtained earlier. At first we mean the methods based on the renormalization group ideas and on the factorization theorem. They allow us to get a differential cross-section of a certain process similar to the Drell–Yan process cross-section, and to consider leading logarithmic RC of higher orders. Meanwhile, nonleading contributions are to be taken from the lowest order PT calculations.

The aim of our paper is to provide relevant theoretical formulae for QED processes, which are required for experiments with CMD–2 and SND detectors at VEPP–2M (Novosibirsk) and collider, at DAΦNE (Frascati) and BEPC/BES (Beijing) machines. Formulae cited below may be applied also for higher energies (see the Conclusions), if one will take into account weak interaction and higher hadronic resonance contributions.

A cross-section calculated with an account of radiative corrections (RC) in the $n$-th order of perturbation theory (PT) contains enhanced contributions of the form $(\alpha/\pi)^n L^n$, where $L = \ln(s/m^2_e)$ is the large logarithm (for $s \sim 1$ GeV$^2$, $L \approx 15$). We call these contributions the leading ones. Nonleading contributions have the order $(\alpha/\pi)^n L^m$, $m < n$. The terms, proportional to $(\alpha/\pi)^n L^n$, can be calculated by means of the structure function method [6]. The structure function formalism, based on the renormalization group approach, permits one to keep all leading terms of order $(\alpha L/\pi)^n$, $n = 0, 1, 2, \ldots$ explicitly.

In the first order of PT the nonleading terms, proportional to $(\alpha/\pi)$, can be accounted by means of so-called $K$–factor, $K = 1 + (\alpha/\pi)K$. As for nonleading terms of the order $(\alpha/\pi)^2 L$, they can be correctly calculated in the two–loop approximation. We will consider them in further publications [7]. Fortunately, this second order nonleading RC are small: $(\alpha/\pi)^2 L \sim 10^{-4}$. So, the presented formulae guarantee theoretical precision of the order 0.2%. Considering hard photon emission in the first order of PT we distinguish the contributions due to the initial state radiation, the final state radiation, and their interference. The first one always contains large logarithms. The ones due to the final state radiation as well as the initial–final state interference do not enhance the corrections considerably, except the case of Bhabha scattering. In the case of hard photon radiation large logarithms come from kinematical regions, where the photon are emitted along electron and positron beams. We will call this kinematics as the collinear one.

Our work consist in explicit calculations of the Born and the first order RC contributions to differential cross–sections. We apply the structure function method to increase the accuracy.

The paper is organized as follows. In Section 2 we consider $\mu^+\mu^-$ production. Both charge–even and charge–odd contributions to the differential cross–section are evaluated. In Sect. 3 we consider the electron–positron scattering process at large angles. In Sect. 4 we investigate electron–positron annihilation into photons. In the Conclusions we discuss the results obtained and estimate the provided precision. Numerical illustrations are given for a realistic experimental set up.
2 Muon Pair Production

Consider the process

\[ e^+(p_+) + e^-(p_-) \rightarrow \mu^+(q_+) + \mu^-(q_-). \]

Taking into account the photon and Z–boson intermediate states, the differential cross–section in the Born approximation (in the framework of the Standard Model) has the form:

\[ \frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega} \{1 + K_W\}, \quad \frac{d\sigma_0}{d\Omega} = \frac{\alpha^2 \beta}{4s} \left(2 - \beta^2 (1 - \epsilon^2)\right), \]

\[ \beta = \sqrt{1 - 4m^2_\mu/s}, \quad c = \cos \hat{p} q, \quad s = (p_+ + p_-)^2 = 4\epsilon^2, \]

where the centre–of–mass reference system of the initial beams is applied.

Here \( K_W \) (we put the explicit expression for it in the Conclusions) represents contributions due to Z–boson intermediate states (see \[8\] for example), we will neglect them within the accepted precision: \( K_W \sim s/M_Z^2 \ll 10^{-3} \). We will drop also all other contributions due to weak interactions in higher orders.

Consider first the even, with respect to the \( c \leftrightarrow -c \) permutation, part of the one–loop virtual and soft radiative corrections. Using the known in the literature Dirac and Pauli muon form factors and the known soft photon contributions, we obtain

\[ \frac{d\sigma_{B+S+V}}{d\Omega} = \frac{d\sigma_0}{d\Omega} \left\{ 1 + \frac{2\alpha}{\pi} \left[ L - 2 + \frac{1 + \beta^2}{2\beta} l_\beta \right] \ln \frac{\Delta \epsilon}{\epsilon} \right. \]

\[ + \left. \frac{3}{4} (L - 1) + K_{even} \right\}. \]

\[ K_{even} = \frac{\pi^2}{6} - \frac{5}{4} + \rho \left( \frac{\frac{1 + \beta^2}{2\beta} - \frac{1}{2} + \frac{1}{4\beta}}{2 \beta^2 (1 - \epsilon^2)} \right) + \ln \frac{1 + \beta}{2} \left( \frac{1}{2\beta} + \frac{1 + \beta^2}{\beta} \right) \]

\[ - \frac{1 - \beta^2}{2\beta^2} \left( L - 2 - \frac{1 + \beta^2}{2\beta^2} \right) + \frac{1 + \beta^2}{2\beta} \left[ \frac{\pi^2}{6} + 2Li_2 \left( \frac{1 - \beta}{1 + \beta} \right) + \rho \ln \frac{1 + \beta}{2\beta^2} \right] + 2 \ln \frac{1 + \beta}{2\beta^2} \ln \frac{1 + \beta}{2\beta^2}, \]

\[ l_\beta = \ln \frac{1 + \beta}{1 - \beta}, \quad \rho = \ln \frac{s}{m^2_\mu}, \quad L = \ln \frac{s}{m^2_\epsilon}, \quad Li_2(x) \equiv - \int_0^x \frac{dt}{t} \ln(1 - t). \]

where \( \Delta \epsilon \ll \epsilon \) is the maximum energy of soft photon in the centre–of–mass system. \( \Pi(s) \) is the vacuum polarization operator including electron, muon, tau–meson and hadron contributions \[\mathbb{F}\] (see Appendix 1).

The odd part of the one–loop correction comes from the interference of Born and box Feynman diagrams and from the interference part of the soft photon emission contribution. It causes the charge asymmetry of the process:

\[ \eta = [d\sigma(c) - d\sigma(-c)]/[d\sigma(c) + d\sigma(-c)] \neq 0. \]
The odd part of the differential cross-section has the following form:

\[
\frac{d\sigma_{\text{odd}}^{S+V}}{d\Omega} = \frac{d\sigma_0}{d\Omega} \frac{2\alpha}{\pi} \left[ 2 \ln \frac{\Delta \varepsilon}{\varepsilon} \ln \frac{1 - \beta c}{1 + \beta c} + K_{\text{odd}} \right],
\]

\[
K_{\text{odd}} = \frac{1}{2} \beta^2 - L_- (\rho + l_-) + \text{Li}_2 \left( \frac{1 - \beta^2}{2(1 - \beta c)} \right) + \text{Li}_2 \left( \frac{\beta^2 (1 - c^2)}{1 + \beta^2 - 2\beta c} \right) - \frac{1 - \beta^2}{1 + \beta^2 - 2\beta c} (\rho + l_-) - \frac{1}{4} (1 - \beta^2) \left[ l_-^2 - 2L_- (l_- + \rho) \right] + 2\text{Li}_2 \left( \frac{1 - \beta^2}{2(1 - \beta c)} \right) + \beta c \left[ -\frac{\rho}{2\beta^2} + \left( \frac{\pi^2}{12} + \frac{\rho^2}{4} \right) \left( 1 - \frac{1}{\beta} - \frac{\beta}{2} + \frac{1}{2\beta^2} \right) \right] + \frac{1}{\beta} (-1 - \frac{\beta^2}{2} + \frac{1}{2\beta^2}) (\rho \ln \frac{1 + \beta}{2} - 2\text{Li}_2 \left( \frac{1 - \beta^2}{2} \right) - \text{Li}_2 \left( -\frac{1 - \beta}{1 + \beta} \right) ) - \frac{1}{2} l_-^2 + L_- (\rho + l_-) - \text{Li}_2 \left( \frac{1 - \beta^2}{2(1 - \beta c)} \right) \right] - (c \rightarrow -c),
\]

\[
f(x) = \left( \frac{1}{\sqrt{1 - x}} - 1 \right) \ln \frac{\sqrt{x}}{2} - \frac{1}{\sqrt{1 - x}} \ln \frac{1 + \sqrt{1 - x}}{2},
\]

\[
l_- = \ln \frac{1 - \beta c}{2}, \quad L_- = \ln \left( 1 - \frac{1 - \beta^2}{2(1 - \beta c)} \right).
\]

In the ultra-relativistic limit ($\beta \to 1$) for the even part we obtain:

\[
\left( \frac{d\sigma_{\text{even}}^{B+S+V}}{d\Omega} \right)_{\beta \to 1} = \frac{\frac{1}{1 - \Pi(s)} d\sigma_0}{d\Omega} \frac{1}{|1 - \Pi(s)|^2} \left\{ 1 + \frac{2\alpha}{\pi} \left[ (-2 + L + \rho) \ln \frac{\Delta \varepsilon}{\varepsilon} - 2 + \frac{3}{4} L + \frac{3}{4} \rho + \frac{\pi^2}{3} \right] \right\}.
\]

(5)

For the odd part in this limit we obtained the same as Khriplovich [11]:

\[
\left( \frac{d\sigma_{\text{odd}}^{S+V}}{d\Omega} \right)_{\beta \to 1} = \frac{\alpha^3}{s \pi} \left\{ 2(1 + c^2) \left[ \ln \text{ctg} \frac{\theta}{2} \ln \frac{\varepsilon}{\Delta \varepsilon} + \frac{1}{2} \ln^2 \left( \sin \frac{\theta}{2} \right) - \frac{1}{2} \ln^2 \left( \cos \frac{\theta}{2} \right) \right] - \frac{1}{4} \text{Li}_2 \left( \sin^2 \frac{\theta}{2} \right) + \frac{1}{4} \text{Li}_2 \left( \cos^2 \frac{\theta}{2} \right) + \cos^2 \frac{\theta}{2} \ln \left( \sin \frac{\theta}{2} \right) - \sin^2 \frac{\theta}{2} \ln \left( \cos \frac{\theta}{2} \right) \right\}.
\]

(6)

Consider now the process of hard photon emission

\[
e^+ (p_+) + e^- (p_-) \rightarrow \mu^+ (q_+) + \mu^- (q_-) + \gamma (k).
\]

(7)

It was studied in detail [13]. The photon energy is assumed to be larger than $\Delta \varepsilon$. The differential cross-section has the form

\[
d\sigma = \frac{\alpha^3}{2 \pi^2 s^2} R d\Gamma, \quad d\Gamma = \frac{d^3 q_+ d^3 q_- d^3 k}{q_0^2 q_+^0 q_-^0 k_0^0} \delta^{(4)} (p_+ + p_- - q_- - q_+ - k),
\]

(8)
The contribution due to the initial state radiation reads
\[ R_e = A_e + B_e, \]
\[ A_e = \frac{s}{s_1} \left\{ \frac{s}{\chi^+\chi^-} \left( \frac{1}{2}tt_1 + \frac{1}{2}uu_1 + sm_\mu^2 \right) + \frac{1}{\chi^+} (2m_\mu^2\chi_+ - u_1\chi'_+ - t_1\chi'_-) + \frac{1}{\chi^-} (2m_\mu^2\chi_- - u_1\chi'_- - t_1\chi'_+) + \frac{1}{2} \left( \frac{u_1 - t}{\chi^+} - \frac{t}{\chi^-} \right) (u - t_1) - \frac{1}{2} \left( \frac{t_1 - u}{\chi^+} - \frac{t}{\chi^-} \right) (t - u_1) \right\}, \]
\[ B_e = -\frac{s}{s_1} \left\{ \left[ \frac{m_\mu^2}{\chi^+} + \frac{m_\mu^2}{\chi^-} \right] \left( \frac{1}{2}tt_1 + \frac{1}{2}uu_1 + sm_\mu^2 \right) - \frac{m_\mu^2}{\chi^+} (2m_\mu^2\chi_+ - u_1\chi'_+ - t_1\chi'_+) \right\}, \]
\[ s = 2p_+p_-, \quad s_1 = (q_+ + q_-)^2, \quad t = -2p_-q_-, \quad t_1 = -2p_+q_+, \]
\[ u = -2p_-q_+, \quad u_1 = -2p_+q_-, \quad \chi_\pm = p_\pm k, \quad \chi'_\pm = q_\pm k. \]

The final state radiation and the interference of the initial and final state radiation contributions are
\[ R_\mu = A_\mu + B_\mu, \]
\[ A_\mu = \frac{1}{s} \left\{ \left( tt_1 + uu_1 + 2sm_\mu^2 \right) \frac{q_+ - q_-}{\chi^+\chi'_-} - \frac{4m_\mu^2\chi_+\chi_-}{\chi'_+\chi'_-} - \frac{t_1\chi_- + u\chi_+}{\chi^+} - \frac{u_1\chi_- + t\chi_+}{\chi^-} \right\} + \left( \frac{t_1 - u}{2\chi^+} - \frac{u}{2\chi^-} \right) (u - t_1) - \frac{q_+ - q_-}{\chi^+\chi'_-} [(u_1 + t_1)\chi_- + (u + t)\chi_+] \right\}, \]
\[ B_\mu = -\frac{tt_1 + uu_1 + 2sm_\mu^2}{2s} \left( \frac{m_\mu^2}{(\chi^+)^2} + \frac{m_\mu^2}{(\chi'_-)^2} \right) + \frac{1}{s} \left( \frac{m_\mu^2}{(\chi'_-)^2} (t_1\chi_- + u\chi_+) \right) + \frac{m_\mu^2}{(\chi^+)^2} (u_1\chi_- + t\chi_+), \]
\[ R_{e\mu} = -\frac{tt_1 + uu_1 + 2sm_\mu^2}{2s_1} \left( \frac{t}{\chi^-\chi'_-} + \frac{t_1}{\chi^+\chi'_-} - \frac{u}{\chi^-\chi'_-} + \frac{u_1}{\chi^+\chi'_-} \right) - \frac{2}{s_1} \left\{ -t - t_1 + u + u_1 - \frac{1}{2} uu_1 (\frac{p_-}{\chi^-} - \frac{q_+}{\chi'_-}) (Q\chi'_+ + P\chi_-) \right\} - \frac{1}{2} t_1 (\frac{p_-}{\chi^-} - \frac{q_+}{\chi'_-}) (Q\chi'_+ - P\chi_-) - m_\mu^2 (\chi_+ + \chi_-) (QP). \]


\[ R_c = \frac{s}{\chi^- - \chi^+} B - \frac{m_e^2}{2 \chi^2} \frac{(t_1^2 + u_1^2 + 2m_\mu s_1)}{s_1^2} - \frac{m_e^2}{2 \chi^2} \frac{(t^2 + u^2 + 2m_\mu s_1)}{s^2} + \frac{m_\mu^2}{s^2} \Delta_{ss_1}, \]

\[ R_{\mu} = \frac{s}{\chi^- \chi^+} B + \frac{m_\mu^2}{s^2} \Delta_{ss_1}, \quad B = \frac{u^2 + u_1^2 + t^2 + t_1^2}{4s_1}, \]

\[ \Delta_{s_1s_1} = \frac{1}{2} \left( \frac{t + u}{\chi^- \chi^+} \right)^2, \]

\[ \Delta_{ss} = \frac{u^2 + t_1^2 + 2sm_\mu^2}{2(\chi^2)} - \frac{u^2 + t^2 + 2sm_\mu^2}{2(\chi^2)} + \frac{1}{\chi^- \chi^+} (s_1 - s^2 + tu + t_1 u_1 - 2sm_\mu^2), \]

\[ \Delta_{ss_1} = \frac{s + s_1}{2} \left( \frac{u}{\chi^- \chi^+} + \frac{u_1}{\chi^- \chi^+} - \frac{t}{\chi^- \chi^+} - \frac{t_1}{\chi^- \chi^+} \right) + \frac{2(u - t)}{\chi^+} + \frac{2(u_1 - t)}{\chi^+}. \]

Note that from these expressions one may in a moment obtain the corresponding matrix element of the cross symmetrical process $e^- \mu^+ \rightarrow e^- \mu^+ \gamma$. To be rigorous, we have note that in the cross symmetrical channel one has to take into account some additional terms proportional to $m_e^2$. In our channel they can be shown as follows:

\[ R_\mu \rightarrow R_\mu + \frac{m_e^2}{s^2} \Delta'_ss, \]

\[ \Delta'_ss = \frac{2m_\mu^2 s_1}{\chi^- \chi^+} + \frac{t + u_1}{\chi^-} + \frac{t_1 + u}{\chi^+} + \frac{4m_\mu^2}{\chi^-} + \frac{4m_\mu^2}{\chi^+}. \]

We checked the matrix element by a comparison with the one used in FORTRAN program [14], describing electron–muon scattering.

The sum of the hard photon contribution, integrated over the photon phase volume with the condition $k^0 > \Delta_\varepsilon$, and the contribution due to the soft and virtual photon emission does not depend on the auxiliary parameter $\Delta = \Delta_\varepsilon / \varepsilon \ll 1$. The main contribution, proportional to the large logarithm, comes from the integration of $R_c$ in the case of collinear kinematics of photon emission. For definiteness let us consider the case when the photon moves close to the initial electron direction:

\[ \vec{p}_k \theta = \theta_0 \ll 1, \quad \theta_0 \gg \frac{m_e}{\varepsilon}. \]
Here we can use
\[
R_{e\nu|k\|p_\nu} = \frac{s^2}{s_1^2} \left\{ \frac{1 + (1 - x)^2}{x \sqrt{\Delta}} - \frac{m^2_e}{\sqrt{\Delta}(1 - x)} \right\} \frac{t_{1u} + uu_1 + 2s m^2_e}{2},
\]
where \( x \) is the energy fraction carried away by the emitted photon, \( x = k^0/\varepsilon = 1 - s_1/s \).
Performing the integration over the photon emission angles, we can present the corresponding part of the cross-section (analogous contribution of the hard photon emission along the positron is included below also) in the form
\[
\left( \frac{d\sigma}{d\Omega_-} \right)_{\text{coll}} = C + D,
\]
\[
C = \frac{\alpha}{2\pi} \left( \ln \frac{s}{m^2_e} - 1 \right) \int_\Delta \frac{1 + (1 - x)^2}{x} \left[ \frac{d\tilde{\sigma}_0(1 - x, 1)}{d\Omega_-} + \frac{d\tilde{\sigma}_0(1, 1 - x)}{d\Omega_-} \right],
\]
\[
D = \frac{\alpha}{2\pi} \int_\Delta \frac{1}{x} \left\{ 1 + (1 - x)^2 \ln \frac{\theta_0^2}{4} \right\} \left[ \frac{d\tilde{\sigma}_0(1 - x, 1)}{d\Omega_-} + \frac{d\tilde{\sigma}_0(1, 1 - x)}{d\Omega_-} \right],
\]
where \( d\tilde{\sigma}_0(1 - x_1, 1 - x_2)/d\Omega_- \) is the so-called shifted Born differential cross-section. It describes the process \( e^+(p_+ (1 - x_2)) + e^- (p_- (1 - x_1)) \rightarrow \mu^+(q_+) + \mu^-(q_-) \),
\[
\frac{d\tilde{\sigma}_0(z_1, z_2)}{d\Omega_-} = \frac{\alpha^2 \sqrt{z_1}^2 (Y_1 - y_1 c)^2 + z_2^2 (Y_1 + y_1 c)^2 + 8 z_1 z_2 m^2_e / s}{4 s z_1 z_2^2 [z_1 + z_2 - (z_1 - z_2) c Y_1 / y_1]},
\]
\[
y_{1,2}^2 = y_{1,2}^2 = \frac{4 m^2_e}{s}, \quad Y_{1,2} = \frac{q_0}{\varepsilon}, \quad z_{1,2} = 1 - x_{1,2}.
\]
Using the conservation laws
\[
z_1 + z_2 = Y_1 + Y_2, \quad z_1 - z_2 = y_1 c_+ + y_2 c_+,
\]
\[
y_1 \sqrt{1 - c_+^2} = y_2 \sqrt{1 - c_-^2}, \quad c_- \equiv c, \quad c_+ = \cos \hat{p}_- q_+,
\]
we obtain the energy fraction of the created muon
\[
Y_1 = \frac{4 m^2_e}{s} \frac{(z_2 - z_1) c}{2 z_1 z_2 [4 z_1^2 z_2^2 - 4 (m^2_e / s)(z_1 + z_2)^2 - (z_1 - z_2)^2 c^2)]^{1/2}} + \frac{2 z_1 z_2}{z_1 + z_2 - c(z_1 - z_2)^2}.
\]
Quantity \( C \), after adding the corrections due to soft and virtual photons, turns out to be the lowest order perturbative expansion of the convolution of the structure function \( D \) with the shifted Born differential cross-section. Quantity \( D \) plays role of a compensating term. Namely, in the sum with the contribution of the cross-section due to hard \( (k^0 > \Delta \varepsilon) \) photon emission at angles (with respect to the electron and positron) larger than \( \theta_0 \) the dependence on the auxiliary parameters will cancel.
Here we remind about experimental conditions of the final particles detection mentioned above. They are to be imposed explicitly by introducing the restriction of the following kind:
\[
\Theta(z_1, z_2) = \Theta(Y_1 - y_{th}) \Theta(Y_2 - y_{th}) \Theta(\cos^2 \Psi_0 - c_+^2) \Theta(\cos^2 \Psi_0 - c_-^2), \quad c_+ = \hat{p}_- q_+,
\]
where \( y_{th} \varepsilon = \varepsilon_{th} \) is the threshold of the detectors, angle \( \Psi_0 \) determines the dead cones, surrounding beam axes, unattainable for detection. More detailed cuts can be implemented in a Monte Carlo program, using the formulae given above.

The leading contributions to the cross-section, containing large logarithm \( L \), as may be recognized, combine to the kernel of Altarelli–Parisi–Lipatov evolution equation:

\[
\frac{d \sigma}{dz_1 dz_2 \mathcal{D}(z_1) \mathcal{D}(z_2)} = \frac{d \bar{\sigma}_0(z_1, z_2)}{|1 - \Pi(s z_1 z_2)|^2},
\]

This formula is valid in the leading logarithmic approximation. We will modify it by including nonleading contributions and using the smoothed representation for structure functions \( \mathcal{D} \):

\[
\mathcal{D}(z, s) = \mathcal{D}^\gamma(z, s) + \mathcal{D}^{e^+e^-}(z, s),
\]

\[
\mathcal{D}^{\gamma}(z, s) = \frac{1}{2} b \left(1 - z\right)^{\frac{b}{2} - 1} \left[1 + \frac{3}{8} b + \frac{b^2}{16} \left(9 - \frac{\pi^2}{3}\right)\right],
\]

\[
\mathcal{D}^{e^+e^-}(z, s) = \frac{1}{2} b \left(1 - z\right)^{\frac{b}{2} - 1} \left[\frac{b^2}{288} (2L - 15)\right],
\]

In comparison with the corresponding formula in ref. \[6\] we shifted the terms, arising due to virtual \( e^+e^- \) pair production corrections, from \( \mathcal{D}^\gamma \) into \( \mathcal{D}^{e^+e^-} \).

Finally, the differential cross-section can be presented in the form

\[
\frac{d \sigma^{e^+e^- \rightarrow \mu^+\mu^-}(\gamma)}{d \Omega} = \int_{z_{\min}}^{1} dz_1 \int_{z_{\min}}^{1} dz_2 \frac{\mathcal{D}(z_1, s) \mathcal{D}(z_2, s) \frac{d \bar{\sigma}_0(z_1, z_2)}{|1 - \Pi(s z_1 z_2)|^2}}{d \Omega} \left(1 + \frac{\alpha}{\pi} K\right)
\]

\[
+ \left\{ \frac{\alpha^3}{2 \pi^2 s^2} \int_{k^0 > \Delta c} \int_{k_p > \theta_0} R_{e|m_e=0} \frac{d \Gamma}{d \Omega} + \frac{D}{|1 - \Pi(s_1)|^2} \right\}
\]

\(8\)
\[ + \left\{ \frac{\alpha^3}{2\pi^2 s^2} \int_{k_0 > \Delta} \left( \frac{\text{Re} R_{\epsilon\mu}}{1 - \Pi(s_1) (1 - \Pi(s))^*} + \frac{R_{\mu}}{|1 - \Pi(s)|^2} \right) \frac{d\Gamma}{d\Omega_-} \right\} \\
+ \text{Re} \left\{ \frac{C_{\epsilon\mu}}{1 - \Pi(s_1) (1 - \Pi(s))^*} + \frac{C_{\mu}}{|1 - \Pi(s)|^2} \right\} \]

\[ C_{\mu} = \frac{2\alpha}{\pi} \frac{d\sigma_0}{d\Omega_{-}} \ln \frac{\Delta \varepsilon}{\varepsilon} \left( \frac{1 + \beta^2}{2\beta} \ln \frac{1 + \beta}{1 - \beta} - 1 \right), \quad z_{\text{min}} = \frac{2m_{\mu}}{2\varepsilon - m_{\mu}}, \]

\[ C_{\epsilon\mu} = \frac{4\alpha}{\pi} \frac{d\sigma_0}{d\Omega_{-}} \ln \frac{\Delta \varepsilon}{\varepsilon} \ln \frac{1 - \beta c}{1 + \beta c}, \quad K = K_{\text{odd}} + K_{\text{even}}, \]

where \( D, C_{\epsilon\mu} \) and \( C_{\mu} \) are compensating terms, which provide cancellation of auxiliary parameters \( \Delta \) and \( \theta_0 \) inside figure brackets. In the first term, containing \( D \) functions, we gather all leading terms. A part of nonleading terms proportional to the Born cross-section is written as the \( K \)-factor. The rest nonleading terms are written as two additional terms. The compensating term \( D \) (see Eq. (16)) comes from the integration in the collinear region of hard photon emission. Quantities \( C_{\mu} \) and \( C_{\epsilon\mu} \) come from the even and odd parts of the differential cross-section (arising due to soft and virtual corrections), respectively. Here we consider the phase volumes of two \((d\Omega_-)\) and three \((d\Gamma)\) final particles as the ones, which already include all required experimental cuts. Using the conservation laws Eq. (18) and concrete experimental conditions one can define the lower limits of the integration over \( z_1 \) and \( z_2 \).

There is a peculiar feature in the spectrum of hard photons. Namely, in the end of the spectrum the differential cross-section is proportional to the factor

\[ I(s_1) = \frac{2m_{\mu}^2 + s_1}{s_1^2} \sqrt{1 - \frac{4m_{\mu}^2}{s_1}}, \]

which defines a peak at \( s_1 \approx 5.6m_{\mu}^2 \). It comes from the Feynman diagrams describing the emission by the initial particles [15].

### 3 Large-Angle Bhabha Scattering

The cross-section of Bhabha scattering (corrected by the vacuum polarization factor), which enter into the Drell–Yan form of corrected cross-section, has a bit more complicated form, as far as the scattering and annihilation amplitudes and their interference are to be taken into account. We remind here the form of the Lorentz-invariant matrix element module squared in the Born approximation:

\[ R_0(s, t, u) = \frac{1}{16(4\pi\alpha)^4} \sum_{\text{spins}} \left| \mathcal{M}(e^-(p_-) + e^+(p_+) \rightarrow e^-(p'_-) + e^+(p'_+)) \right|^2 \]

\[ = \frac{s^2 + u^2}{2t^2} + \frac{u^2 + t^2}{2s^2} + \frac{u^2}{st}, \]

\[ s = (p_- + p_+)^2, \quad t = (p_- - p'_-)^2, \quad u = (p_- - p'_+)^2, \quad s + t + u = \mathcal{O}(m_e^2). \]

The first term in the right hand side describes the scattering–type Feynman diagram square. The second one corresponds to the square of the annihilation–type diagram. And the third
one deals with the interference of the two diagrams. A more compact representation of \( R_0 \) is also useful, \( R_0 = (1 + s/t + t/s)^2 \). The differential cross-section in the Born approximation has the form

\[
\frac{d\sigma_0^{\text{Born}}}{d\Omega_-} = \frac{\alpha^2}{4s} \left( \frac{3 + c^2}{1 - c} \right)^2.
\]  

(26)

We will need also quantity \( R \) for arbitrary energies of initial particles. Suppose that the initial electron and positron lost a certain energy fraction. The corresponding kinematics is defined as follows:

\[
e^-(z_1 p_-) + e^+(z_2 p_+) \rightarrow e^-(\tilde{p}_-) + e^+(\tilde{p}_+),
\]

\[
\tilde{s} = s z_1 z_2, \quad \tilde{t} = -\frac{1}{2} s z_1 Y_1(1 - c), \quad \tilde{u} = -\frac{1}{2} s z_2 Y_1(1 + c),
\]

\[
Y_1 = \frac{\tilde{p}^0}{\varepsilon} = \frac{2 z_1 z_2}{a}, \quad a = z_1 + z_2 - (z_1 - z_2)c.
\]

Here the shifted Born cross-section corrected by vacuum polarization insertions into virtual photon propagators reads

\[
d\tilde{\sigma}_0(z_1, z_2) = \frac{4\alpha^2}{s a^2} \left\{ \frac{1}{|1 - \Pi(t)|^2} \frac{a^2 + z_2^2(1 + c)^2}{2z_1^2(1 - c)^2} + \frac{1}{|1 - \Pi(s)|^2} \frac{z_1^2(1 - c)^2 + z_2^2(1 + c)^2}{2a^2} \right\}
\]

\[
- \frac{1}{(1 - \Pi(t))(1 - \Pi(s))^*} \frac{z_3^2(1 + c)^2}{a z_1(1 - c)} d\Omega_-.
\]  

(27)

Rewriting the known results \([11, 12]\) for the cross-section in the Born approximation with one-loop virtual corrections to it and with the other ones arising due to soft photon emission, we obtain

\[
\frac{d\sigma_{B+S+V}}{d\Omega_-} = \frac{d\tilde{\sigma}_0(1, 1)}{d\Omega_-} \left\{ 1 + \frac{2\alpha}{\pi}(L - 1) \left[ 2 \ln \frac{\Delta \varepsilon}{\varepsilon} + \frac{3}{2} \right] \right\}
\]

\[
- \frac{8\alpha}{\pi} \ln(\cot \theta_2) \ln \frac{\Delta \varepsilon}{\varepsilon} + \frac{\alpha}{\pi} K_{SV},
\]  

(28)

where

\[
K_{SV} = -1 - 2\text{Li}_2(\sin^2 \frac{\theta}{2}) + 2\text{Li}_2(\cos^2 \frac{\theta}{2}) + \frac{1}{(3 + c^2)^2} \left[ \frac{\pi^2}{3} (2c^4 - 3c^3 - 15c) \right.
\]

\[
+ 2(2c^4 - 3c^3 + 9c^2 + 3c + 21) \ln^2(\sin \frac{\theta}{2}) - 4(c^4 + c^2 - 2c) \ln^2(\cos \frac{\theta}{2})
\]

\[
- 4(c^3 + 4c^2 + 5c + 6) \ln^2(\tan \frac{\theta}{2}) + 2(c^3 - 3c^2 + 7c - 5) \ln(\cos \frac{\theta}{2})
\]

\[
+ \left( \frac{10}{3} c^3 + 10c^2 + 2c + 38 \right) \ln(\sin \frac{\theta}{2})
\]  

(29)

is the part of the \( K \)-factor coming from soft and virtual photon corrections,

\[
\frac{d\tilde{\sigma}_0(1, 1)}{d\Omega_-} = \frac{\alpha^2}{s} \left\{ 5 + 2c + c^2 \right\} \left[ \frac{1}{2(1 - c)^2|1 - \Pi(t)|^2} + \frac{1 + c^2}{4|1 - \Pi(s)|^2} \right].
\]
and the invariants are defined as
\[ s = 4\varepsilon^2, \quad t = -s \frac{1-c}{2}, \quad u = -s \frac{1+c}{2}, \quad c = \cos \theta, \quad \theta = \mathbf{p}_- \mathbf{p}'_-. \]

Quantity \( \Delta \varepsilon \) in Eq. (28) is the maximum energy of emitted soft photons. \( \Pi(s) \) and \( \Pi(t) \) are the vacuum polarization operators in the \( s \) and \( t \) channels. In the Conclusions we will estimate the contribution of weak interactions.

Consider now the process of hard photon (with the energy \( \omega = k^0 > \Delta \varepsilon \)) emission
\[ e^+(p_+) + e^-(p_-) \rightarrow e^+(p'_+) + e^-(p'_-) + \gamma(k). \]

We start with the differential cross-section in the form suggested by F.A. Berends et al. [1]
(which is valid for scattering angles being large compared with \( m_e/\varepsilon \)):
\[
d\sigma_{\text{hard}} = \frac{\alpha^3}{2\pi^2 s} R_{ee\gamma} \, d\Gamma, \quad \qquad d\Gamma = \frac{d^3 p'_+ d^3 p'_- d^3 k}{\varepsilon'_+ \varepsilon'_- k^0} \delta^{(4)}(p_+ + p_- - p'_+ - p'_- - k), \tag{31}
\]

where
\[
W = \frac{s}{\chi_+ \chi_-} + \frac{s_1}{\chi'_+ \chi'_-} - \frac{t_1}{\chi'_+ \chi_+} - \frac{t}{\chi'_- \chi_-} + \frac{u}{\chi_+ \chi_-} + \frac{u_1}{\chi'_+ \chi'_-},
\]
\[
T = \frac{ss_1(s^2 + s_1^2) + tt_1(t^2 + t_1^2) + uu_1(u^2 + u_1^2)}{ss_1tt_1},
\]

and the invariants are defined as
\[
s = 2p_- p_+, \quad s_1 = 2p'_- p'_+, \quad t = -2p_- p'_+, \quad t_1 = -2p_+ p'_-, \quad u = -2p_+ p'_-, \quad u_1 = -2p_+ p'_-,
\]
\[
\chi' = k p_+, \quad \chi'_+ = k p'_+.
\]

It is convenient to extract the contribution of the collinear kinematics. We do that for the following reasons. First, it is natural to separate the region with very a sharp behaviour of the cross-section and to consider it carefully. Second, we keep in mind the idea of the leading logarithm factorization, which is valid in all orders of the perturbation theory. We will evaluate the collinear kinematical regions in two different ways. The first one (the quasireal electron approximation) is suitable for a generalization in order to account higher order leading corrections by means of the structure function method. In this way we will obtain below the leading logarithmic contributions and the compensating terms, which will provide the cancellation of auxiliary parameters. The second one (the direct calculation) is more rigorous, it can be used as a check of the first one. We discuss it in detail in Appendix 2.

To obtain explicit formulae for compensators it is needed to consider four kinematical regions corresponding to hard photon emission inside narrow cones, surrounding the initial
and final charged particle momenta. The vertices of the cones are taken in the interaction point. We introduce a small auxiliary parameter \( \theta_0 \), it should obey the restriction
\[
m_e/\sqrt{s} \ll \theta_0 \ll 1.
\]

So, we define a collinear kinematical region, as the part of the whole phase space, in which the hard photon is emitted within the cone of \( \theta_0 \) polar angle with respect to the direction of motion of one of the charged particles.

Using the method of quasireal electrons \([14]\), the matrix element \( \mathcal{M} \) (squared and summed up over polarization states) of the process of hard photon emission can be expressed through a \textit{shifted} matrix element of the process without photon emission (see Eq. (4) in \([16]\)):
\[
\sum |\mathcal{M}(p_1, k, p'_1, \mathcal{X})|^2 = 4\pi\alpha \left[ \frac{1 + (1 - x)^2}{x(1 - x)} \frac{1}{k p_1} - \frac{m^2}{(k p_1)^2} \right] \sum |\mathcal{M}_0(p_1 - k, p'_1, \mathcal{X})|^2,
\]
\[
\sum |\mathcal{M}(p_1, p'_1, k, \mathcal{X})|^2 = 4\pi\alpha \left[ \frac{y^2 + Y^2}{\omega Y} \frac{\varepsilon}{k p'_1} - \frac{m^2}{(k p'_1)^2} \right] \sum |\mathcal{M}_0(p_1, p'_1 + k, \mathcal{X})|^2,
\]
where \( \mathcal{X} \) denotes the momenta of non–radiating incoming and outgoing particles in a concrete process. The integration over the phase volume of the emitted photon inside the narrow cone, surrounding its parent charged particle momentum, gives the following factors:
\[
\frac{4\alpha}{16\pi^2} \int \frac{d^3k}{\omega} \left[ \frac{1 + (1 - x)^2}{x(1 - x)} \frac{1}{k p_1} - \frac{m^2}{(k p_1)^2} \right] = \frac{\alpha}{2\pi} \frac{dz_1}{z_1} \left[ P^{(1)}_{\Theta}(z_1) \left( L - 1 + \ln \frac{\theta_0^2}{4} \right) + 1 - z_1 \right], \quad z_1 = 1 - x,
\]
\[
\frac{4\alpha}{16\pi^2} \int \frac{d^3k}{\omega} \left[ \frac{y^2 + Y^2}{\omega Y} \frac{1}{k p'_1} - \frac{m^2}{(k p'_1)^2} \right] = \frac{\alpha}{2\pi} \frac{dz_3}{z_3} \left[ P^{(1)}_{\Theta}(z_3) \left( L - 1 + \ln \frac{\theta_0^2}{4} \right) + 2 \ln z_3 \right] + 1 - z_3], \quad z_3 = 1 - \frac{\omega}{p'_1 + \omega} = 1 - \frac{x}{Y}.
\]
Note that the terms proportional to \((L - 1)\) contain the kernel \( P^{(1)}_{\Theta} \) (see Eq. (21)) of Altarelli–Parisi–Lipatov evolution equations (more precisely, they contain \( \Theta \)–part of the nonsinglet kernel):
\[
P^{(1)}_{\Theta}(z) = \frac{1 + z^2}{1 - z} \Theta(1 - z - \Delta).
\]
Collecting the contributions of the four collinear regions, we obtain
\[
\frac{d\sigma_{\text{coll}}}{d\Omega_-} = \frac{\alpha}{\pi} \int_\Delta \frac{dx}{x} \left\{ \left[ \left( 1 - x + \frac{x^2}{2} \right) \left( L - 1 + \ln \frac{\theta_0^2}{4} + 2 \ln(1 - x) \right) + \frac{x^2}{2} \right] \right.
\]
\[
\times 2 \frac{d\tilde{\sigma}_0(1, 1)}{d\Omega_-} + \left[ \left( 1 - x + \frac{x^2}{2} \right) \left( L - 1 + \ln \frac{\theta_0^2}{4} \right) + \frac{x^2}{2} \right] \right.
\]
\[
\left. \times \left[ \frac{d\tilde{\sigma}_0(1 - x, 1)}{d\Omega_-} + \frac{d\tilde{\sigma}_0(1, 1 - x)}{d\Omega_-} \right] \right\}. \quad (35)
\]
where the shifted Born cross-section is defined in Eq. (27).

Adding the contributions of virtual and soft photon emission, we restore the complete kernel. Generalizing the procedure for the case of photon emission by all charged particles, we come to the representation of the cross-section in the leading logarithmic approximation. The final expression for the cross-section therefore has the form

\[
\frac{d\sigma^{e^+e^-\rightarrow e^+e^-}(\gamma)}{d\Omega_-} = \int_{\bar{z}_1}^{Y_1} \int_{\bar{y}_1}^{Y_2} D(z_1)D(z_2) \frac{d\sigma_0(z_1, z_2)}{d\Omega_-} \left( 1 + \frac{\alpha}{\pi} K_{SV} \right) \Theta
\]

\[
\times \int_{y_{th}}^{Y_1} \int_{y_{th}}^{Y_2} D(y_1)D(y_2) \left[ \frac{\alpha}{\Delta} \left( \ln \frac{\theta_0^2(1-x)^2}{4} + \frac{x^2}{2} \right) \frac{4\alpha^2}{s(1-x)^2[2-x(1-c)]^4} \right.
\]

\[
+ \left( \frac{3-3x+x^2+2x(2-x)c+c^2(1-x+x^2)}{1-c} \right)^2 \frac{4\alpha^2}{s[2-x(1-c)]^4} \left( 3 - 3x + x^2 - 2x(2-x)c + c^2(1-x+x^2) \right)^2 \right] \}
\]

\[
\Theta - \frac{\alpha^2}{4s} \left( \frac{3+c^2}{1-c} \right) \frac{8\alpha}{\pi} \ln(\cot \frac{\theta}{2}) \ln \frac{\Delta \varepsilon}{\varepsilon} + \frac{\alpha^3}{2\pi^2s} \int_{\pi-\theta_0}^{\pi-\theta_0} \frac{W T}{4} \Theta \frac{d\Gamma}{d\Omega_-}, \tag{36}
\]

\[
Y_1 = \frac{2z_1 z_2}{z_1 + z_2 - c(z_1 - z_2)}, \quad Y_2 = \frac{z_1^2 + z_2^2 - (z_1^2 - z_2^2)c}{z_1 + z_2 - c(z_1 - z_2)},
\]

\[
\bar{z}_1 = \frac{y_{th}(1+c)}{2 - y_{th}(1-c)}, \quad \bar{z}_2 = \frac{z_1 y_{th}(1 - c)}{2z_1 - y_{th}(1 + c)}.
\]

The last term describes hard photon emission process, provided that the photon energy fraction \( x \) is larger than \( \Delta = \Delta \varepsilon/\varepsilon \), and its emission angle with respect to any charged particle direction is larger than some small quantity \( \theta_0 \). The sum of the last 3 terms in Eq. (36) does not depend on the auxiliary parameters \( \Delta \) and \( \theta_0 \), if they are sufficiently small. We omitted the effects due to vacuum polarization in the last three terms which describe real hard photon emission. Because the theoretical uncertainty, coming from this approximation, has the order \( \delta(d\sigma)/d\sigma \sim (\alpha/\pi)^2 L \lesssim 10^{-4} \). Nevertheless if the centre–off–mass energy is close to some resonance mass (say to \( m_\phi \)) the effect due to vacuum polarization may become visible. The differential cross-section of non–collinear hard photon emission, that takes into account vacuum polarization explicitly, is presented in Appendix 3.


\section{Annihilation of $e^+e^-$ into photons}

Considering the RC due to emission of virtual and soft real photons to the cross-section of two quantum annihilation process

$$e^+(p_+) + e^-(p_-) \rightarrow \gamma(q_1) + \gamma(q_2), \quad (37)$$

we will use the results obtained in papers \[17\]:

$$d\sigma_{H+S+V} = d\bar{\sigma}_0(1,1) \left\{ 1 + \frac{\alpha}{\pi} \left[ (L-1) \left( 2 \ln \frac{\Delta\varepsilon}{\varepsilon} + \frac{3}{2} \right) + K_{SV} \right] \right\}, \quad (38)$$

$$K_{SV} = \frac{\pi^2}{3} + \frac{1-c^2}{2(1+c^2)} \left[ \left( 1 + \frac{3}{2} \frac{1+c}{1-c} \right) \ln \frac{1-c}{2} \right] + \left( \frac{1-c}{1+c} + \frac{1+c}{2(1-c)} \right) \ln^2 \frac{1-c}{2} + (c \rightarrow -c),$$

$$d\bar{\sigma}_0(1,1) = \frac{\alpha^2(1+c^2)}{s(1-c^2)} d\Omega_1, \quad s = (p_+ + p_-)^2, \quad c = \cos \theta_1, \quad \theta_1 = \hat{q}_1 \hat{p}_-. \quad (39)$$

We assume that the two final photons are registered in an experiment and their polar angles with respect to the initial beam directions are not small ($\theta_{1,2} \gg m_e/\varepsilon$).

Consider the three-quantum annihilation process

$$e^+(p_+) + e^-(p_-) \rightarrow \gamma(q_1) + \gamma(q_2) + \gamma(q_3)$$

with the cross-section (see the paper by M.V. Terentjev \[17\])

$$d\sigma^{e^+e^-\rightarrow 3\gamma} = \frac{\alpha^3}{8\pi^2 s} R_{3\gamma} d\Gamma, \quad (40)$$

$$R_{3\gamma} = s \frac{\chi_2^2 + (\chi_3')^2}{\chi_1 \chi_2 \chi_1' \chi_2'} - 2m^2 \left[ \frac{(\chi_i^2 + \chi_i'^2)}{\chi_1 \chi_2 (\chi_3')^2} + \frac{(\chi_i^2 + \chi_i'^2)}{\chi_1 \chi_2' \chi_3'^2} \right]$$

+ two cyclic permutations,

$$d\Gamma = \frac{d^3q_1d^3q_2d^3q_3\delta(4)}{q_1^0q_2^0q_3^0}(p_+ + p_- - q_1 - q_2 - q_3),$$

where

$$\chi_i = q_ip_-, \quad \chi_i' = q_ip_+, \quad i = 1, 2, 3.$$}

The process can be treated as a radiative correction to the two-quantum annihilation.

In the same way as we have done before we will distinguish the contributions of the collinear kinematical region, when extra photons are emitted within narrow cones of the opening angle $2\theta_0 \ll 1$ to one of the charged particles and the semi-collinear ones, when extra photons are emitted outside these cones. This contribution can be obtained using the quasireal electron method \[16\]. It reads:

$$d\sigma_{\text{coll}} = \frac{\alpha}{\pi} \int_0^1 \frac{dx}{x} \left[ \left( 1 - x + \frac{x^2}{2} \right) \left( L - 1 + \ln \frac{\theta_0^2}{4} \right) + \frac{x^2}{2} \right] \left[ d\bar{\sigma}_0(1-x,1) + d\bar{\sigma}_0(1-x,1) \right], \quad (40)$$

$$\times$$
where the *shifted* cross-section has the form
\[
\frac{d\sigma_0(z_1, z_2)}{d\Omega_1} = \frac{2\alpha^2}{s} \frac{z_1^2(1-c)^2 + z_2^2(1+c)^2}{(1-c^2)(z_1 + z_2 + (z_2 - z_1)c)^2} d\Omega_1.
\] (41)

Again rearranging the separate contributions and applying the structure functions method, we obtain the *improved* cross-section
\[
d\sigma^{e^+e^→γγ}(γ) = \int_{\bar{z}_1}^{z_1} d\bar{z}_1 D(z_1) \int_{\bar{z}_2}^{z_2} d\bar{z}_2 D(z_2) d\bar{σ}_0(z_1, z_2) \left(1 + \frac{\alpha}{\pi} K_{SV}\right) \\
+ \frac{\alpha}{\pi} \int_{\Delta}^{1} \frac{dx}{x} \left[\left(1 - x + \frac{x^2}{2}\right) \ln \frac{\theta_0^2}{4} + \frac{x^2}{2}\right] \left[d\bar{σ}_0(1 - x, 1) + d\bar{σ}_0(1, 1 - x)\right] \\
+ \frac{1}{3} \int_{π - θ_0 ≥ θ_i ≥ θ_0} \left[\frac{4\alpha^3}{π^2 s^2} \left(\frac{z_i^2(1 + c_i^2)}{z_1^2 z_3^2(1 - c_1^2)(1 - c^2)} + \text{two cyclic permutations}\right)\right] dΓ, \quad (42)
\]

where lower limits \(\bar{z}_1, \bar{z}_2\) are defined in Eq. (36). The multiplier \(\frac{1}{3}\) in the last term takes into account the identity of the final photons. The sum of the last two terms does not depend on \(\Delta\) and \(θ_0\). Note that the annihilation process is a pure QED one, hadronic contributions as well as weak interaction effects are far beyond the required accuracy.

### 5 Conclusions

Thus we had considered the series of processes at electron–positron colliders of moderately high energies. We presented differential cross–sections to be integrated over concrete experimental conditions. The formulae are good as for semi–analytical integration, as well as for the creation of a Monte Carlo event generator [20]. In a separate publication we are going to present analysis of the effects of radiative corrections for the conditions of VEPP–2M (Novosibirsk), DAΦNE (Frascati) and BEPC/BES (Beijing). The idea of our approach was to separate the contributions due to \(2 → 2\) like processes and \(2 → 3\) like ones. The compensating terms allow us to eliminate the dependence on auxiliary parameters in both contributions separately.

Note that all presented formulae are valid only for large–angle processes. Indeed, in the region of very small angles \(θ \sim m_e/ε\) of final particles with respect to the beam directions there are contributions of double logarithmic approximation [9]. These small angle regions give the main part of the total cross–section. We suppose that this kinematics is rejected by experimental cuts.

In the Table and Figures we present some results of numerical calculations according to our formulae. We suppose that a process–event implies detecting of two final particles with the polar angles with respect to the beam axes more than some value \(Ψ_0\). The energies of the particles have to exceed some experimental threshold \(ε_{th}\). A cut–off on the acollinearity of the final particle momenta is possible. But we switched off it in the computations.
Table 1: The values of Bhabha cross–section and radiative corrections to it.

| $\theta_{\pm}$ | $\sigma_0^{\text{Born}}$ (mb) | $\delta_{\text{VP}}$ (%) | $\delta_{\text{ini SF}}$ (%) | $\delta_{\text{fin SF}}$ (%) | $\delta_{K}$ (%) | $\delta_{\gamma}$ (%) | $\sigma_{\text{tot}}$ (mb) |
|-----------------|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $0^\circ < \theta_{\pm} < 171^\circ$ | $3.77 \cdot 10^{-2}$ | 1.53 | 0.11 | -0.30 | -1.82 | 1.50 | $3.81 \cdot 10^{-2}$ |
| $1^\circ < \theta_{\pm} < 179^\circ$ | 3.08 | 0.80 | 0.70 | -0.17 | -3.5 | 4.74 | 3.16 |

In the Table we give the values of different RC contributions to large–angle Bhabha scattering cross–section. Here we switched off also the cut–off on the final particle energies (we used only kinematical restrictions). The contributions (see Eq.(36)) are defined as follows:

$$
\sigma_{\text{tot}} \equiv \int \frac{d\sigma^{e^+e^-\rightarrow e^+e^-(\gamma)}}{d\Omega} d\Omega = \int \frac{d\sigma_0^{\text{Born}}}{d\Omega} d\Omega \left[ 1 + \frac{1}{100\%} \left( \delta_{\text{VP}} + \delta_{\text{ini SF}} + \delta_{\text{fin SF}} + \delta_K + \delta_{\gamma} \right) \right], \quad (43)
$$

where $\delta_{\text{VP}}$ is due to the vacuum polarization being included into the Born level diagrams; $\delta_{\text{ini (fin)}}$ is due to the initial (final) state leading logarithmic corrections; $\delta_K$ shows the impact of the $K$–factor; $\delta_{\gamma}$ describes the contribution of one hard photon emission at large angles. The effect due to the width of $\phi$ meson is included as a part of vacuum polarization. It is small for the given integrated cross–sections. But for the description of a differential cross–section it is important, especially for large scattering angles (see [23, 24]).

In Figures 1 and 2 we illustrate the charge–odd part of the differential cross–section for the $e^+e^- \rightarrow \mu^+\mu^-$ process (see Eq.(23)). The quantity

$$
A_{FB} = \frac{d\sigma_{odd}}{d\sigma_{0}} \left( \frac{d\sigma^{e^+e^-\rightarrow e^+e^-(\gamma)}}{d\sigma^{e^+e^-\rightarrow e^+e^-(\gamma)}} \right) / dc \times 100\%, \quad (44)
$$

is shown there as a function of $c$. The short–dashed line represents the contribution due to soft photon emission and virtual corrections. The long–dashed line represents the corresponding odd contribution due to hard photon emission. It comes from the interference of the amplitudes due to initial and final radiation ($R_{e\mu}$). In the sum of the two contributions the dependence on the auxiliary parameter $\Delta$ disappears (it was chosen $\Delta = 0.01$). And we obtain an experimentally measurable asymmetry $A_{FB}$ (the solid line). There is also a contribution to the asymmetry due to electroweak interactions. Namely, due to the interference of the Born level amplitudes with $\gamma$ and $Z$ boson in the $s$–channel. It can be found from the weak $K$–factor (46). It is included in the total sums (solid lines). But for the chosen energies it is really small (it gives a maximal shift of about 0.01% for $E_{\text{beam}} = 0.51$ GeV and about 0.1% for $E_{\text{beam}} = 1.55$ GeV.

Let us discuss the accuracy, provided by our formulae. The contribution of weak interaction to the cross–section of muon pair production and Bhabha scattering was parameterized be so–called weak $K$–factors:

$$
K_W = \frac{(d\sigma)_{\text{EW}} - (d\sigma)_{\text{QED}}}{(d\sigma)_{\text{QED}}}, \quad (45)
$$

where quantities $(d\sigma)_{\text{EW}}$ and $(d\sigma)_{\text{QED}}$ are the cross–sections calculated in the Born approximation in the frames of the Standard Model and QED, respectively. The weak $K$–factors
(we used the results of papers [8, 18]) are

\[ K_{W}^{\ell\ell\rightarrow\mu\bar{\mu}} = \frac{s^2(2 - \beta^2)(1 - c^2)^{-1}}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \left\{ (2 - \beta^2)(1 - c^2)^2 \right\} \left\{ \left( 3 - 2 \frac{M_Z^2}{s} \right) + c_c^2 \right\}, \]  

\[ K_{W}^{\ell\ell\rightarrow\ell\bar{\ell}} = \frac{(1 - c)^2}{2(3 + c^2)} \left[ 4B_1 + (1 - c)^2B_2 + (1 + c)^2B_3 \right] - 1, \]  

\[ B_1 = \frac{s}{t} \left| 1 + (g_v^2 - g_a^2)\xi \right|^2, \quad B_2 = \left| 1 + (g_v^2 - g_a^2)\chi \right|^2, \]  

\[ B_3 = \frac{1}{2} \left| 1 + \frac{s}{t} + (g_v + g_a)^2 \left( \frac{s}{t} \xi + \chi \right) \right|^2 + \frac{1}{2} \left| 1 + \frac{s}{t} + (g_v - g_a)^2 \left( \frac{s}{t} \xi + \chi \right) \right|^2, \]  

\[ \chi = \frac{\Lambda s}{s - m^2 + iM_Z \Gamma_Z}, \quad \xi = \frac{\Lambda t}{t - M_Z^2}, \]  

\[ \Lambda = \frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha} = \frac{\alpha s}{\pi M_W^2}. \]  

It comes from one–loop electroweak radiative corrections. Another source of uncertainties comes from the approximation of collinear kinematics (or the approximation of quasireal electrons [10]). It can be estimated by the largest omitted terms

\[ \frac{\alpha}{\pi} \theta_0^2 \quad \text{and} \quad \frac{\alpha}{\pi} \left( \frac{m_e}{\varepsilon \theta_0} \right)^2. \]  

Really in the calculations we used the value of the parameter \( \theta_0 \) of the order \( 10^{-2} \) because of the restrictions \( \theta_0 \ll 1 \) and \( \varepsilon \theta_0/m_e \gg 1 \). Note that the coefficient before terms of that sort (calculable in principle) is the function of energy fractions and angles, they are of order of 1. For typical values of energy \( \varepsilon = 0.5 \) GeV this uncertainty is of order \( 2 \cdot 10^{-4} \) or less.

The third source of uncertainties is the error in the definition of the hadronic vacuum polarization. It has been estimated [9] to be of order 0.04%. For \( \phi \)–meson factories a systematic error in the definition of the \( \phi \)–meson contribution into vacuum polarization is to be added.

Next point concerns nonleading terms of order \( (\alpha/\pi)^2L \). There are several sources of them. One is the emission of two extra hard particles (for the case of Bhabha scattering it was considered in the series of papers [11]). Other are related to virtual and soft–photon
radiative corrections to single hard photon emission and Born processes. The most part of these contributions was not considered up to now. Nevertheless, we can estimate the coefficient before the quantity \((\alpha/\pi)^2 L \approx 1 \cdot 10^{-4}\) to be of order of unity. That was indirectly confirmed by our complete calculations of these terms for the case of small–angle Bhabha scattering [21].

Considering all mentioned above sources of uncertainties as independent, we conclude that the systematic error of our formulae does not exceed 0.2%. The main error is due to unknown second–order next–to–leading radiative corrections.

For precise luminosity measurements we suggest to use the large–angle Bhabha scattering process. It has a very large cross–section, a good signature in detectors, and the lowest theoretical uncertainty.

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Appendix 1

We present here leptonic and hadronic contributions into the vacuum polarization operator:

\[
\Pi = \Pi_l + \Pi_h, \quad (50)
\]

\[
\Pi_l(s) = \frac{\alpha}{\pi} \Pi_1(s) + \left(\frac{\alpha}{\pi}\right)^2 \Pi_2(s) + \left(\frac{\alpha}{\pi}\right)^3 \Pi_3(s) + \ldots
\]

\[
\Pi_h(s) = \frac{s}{4\pi \alpha} \left[ \text{PV} \int_{4m^2}^{\infty} \frac{\sigma^{e^+e^-\to\text{hadrons}}(s')}{s' - s} \, ds' - i\pi \sigma^{e^+e^-\to\text{hadrons}}(s) \right].
\]

The first order leptonic contribution is well known [1]:

\[
\Pi_1(s) = \frac{1}{3} L - \frac{5}{9} + f(x_\mu) + f(x_\tau) - i\pi \left[ \frac{1}{3} + \phi(x_\mu) \Theta(1 - x_\mu) + \phi(x_\tau) \Theta(1 - x_\tau) \right], \quad (51)
\]

where

\[
f(x) = \begin{cases} 
-\frac{5}{9} - \frac{x}{3} + \frac{1}{6}(2 + x)\sqrt{1 - x} \ln \left[ \frac{1 + \sqrt{1 - x}}{1 - \sqrt{1 - x}} \right] & \text{for } x \leq 1, \\
-\frac{5}{9} - \frac{x}{3} + \frac{1}{6}(2 + x)\sqrt{1 - x} \arctg \left( \frac{1}{\sqrt{x - 1}} \right) & \text{for } x > 1,
\end{cases}
\]

\[
\phi(x) = \frac{1}{6}(2 + x)\sqrt{1 - x}, \quad x_{\mu,\tau} = \frac{4m^2_{\mu,\tau}}{s}.
\]

In the second order it is enough to take only the logarithmic term from the electron contribution

\[
\Pi_2(s) = \frac{1}{4}(L - i\pi) + \zeta(3) - \frac{5}{24}. \quad (52)
\]
A discussion of the $\phi$-meson contribution to the vacuum polarization operator, which is important for $\sqrt{s} \approx m_\phi$, can be found in \[23, 24\].

**Appendix 2**

Here we present the direct evaluation of the collinear region contribution to the Bhabha scattering cross-section. Let us write the contribution of the collinear kinematics in the form:

$$ (d\sigma)_{\text{coll}} = d\sigma_{k\parallel p_-} + d\sigma_{k\parallel p_+} + d\sigma_{k\parallel p'_-} + d\sigma_{k\parallel p'_+} \equiv d\sigma_a + d\sigma_b + d\sigma_c + d\sigma_d. $$  \hspace{1cm} (53)

For the case of photon emission along the initial electron we have

$$ W_a = \frac{2}{\omega^2} \frac{1}{1 - \beta c_2}, \quad T_a = \frac{1 + (1 - x)^2}{1 - x} R(s_1, t^a, u^a), \quad d\Gamma_a = \frac{d^3k}{\omega} \frac{y^a_1}{a_a} d\Omega_-, \hspace{1cm} (54) $$

$$ c_2 = \cos(k\vec{p}_-), \quad \beta = \sqrt{1 - \frac{m_e^2}{\varepsilon^2}}, \quad \omega = k^0 = x\varepsilon, \quad s_1 = s(1 - x), \quad t^a = t^a(1 - x), $$

$$ u^a_1 = u^a(1 - x), \quad t^a = -s \frac{(1 - x)^2(1 - c)}{a_a}, \quad u^a = -s \frac{(1 - x)(1 + c)}{a_a}, \quad s = 4\varepsilon^2, $$

$$ a_a = 2 - x(1 - c), \quad y^a_1 = \frac{2(1 - x)}{a_a}, \quad c = \cos(\vec{p}_-\vec{p}'_+). $$

Performing the angular integration over photon angles inside the narrow cone, surrounding the direction of the initial electron beam, we get

$$ \int W_a d\Gamma_a = 4\pi \frac{d\omega}{\omega} d\Omega_- \int_{1-\theta_0^2/2}^{1} \frac{dc_1}{1 - \beta c_1} = 4\pi \frac{dx}{x} d\Omega_- \frac{y^a_1}{a_a^2} \left( L + \ln \frac{\theta_0^2}{4} \right) + O(\theta_0^2). $$  \hspace{1cm} (55)

We neglect the terms proportional to $\theta_0^2$. Collecting all the factors and reminding the contribution of the terms proportional to $m_e^2$ (see Eq. (51)), we obtain the contribution of the first collinear region:

$$ \frac{d\sigma_a}{d\Omega_-} = \frac{4\alpha^2}{s} \frac{\alpha}{\Delta} \frac{1}{x} \left[ \left( 1 - x + \frac{x^2}{2} \right) \left( L - 1 + \ln \frac{\theta_0^2}{4} \right) + \frac{x^2}{2} \right] \frac{1}{a_a^2} $$

$$ \times \frac{\left[ a_a^2 + (1 - c)^2(1 - x)^2 - a_a(1 - c)(1 - x) \right]^2}{a_a^2(1 - x)^2(1 - c)^2}. $$  \hspace{1cm} (56)

For the case of photon emission inside the narrow cone, surrounding $p_+$, in a similar way one gets

$$ \frac{d\sigma_b}{d\Omega_-} = \frac{4\alpha^2}{s} \frac{\alpha}{\Delta} \frac{1}{x} \left[ \left( 1 - x + \frac{x^2}{2} \right) \left( L - 1 + \ln \frac{\theta_0^2}{4} \right) + \frac{x^2}{2} \right] \frac{1}{a_b^2} $$

$$ \times \frac{\left[ a_b^2 + (1 - c)^2(1 - x)^2 - a_b(1 - c)(1 - x) \right]^2}{a_b^2(1 - c)^2}, \quad a_b = 2 - x(1 + c). $$  \hspace{1cm} (57)
Here we used the following formulae:

\[ W_b = \frac{2}{\omega^2} \frac{1}{1 - \beta c^2}, \quad T_b = \frac{1 + (1 - x)^2}{1 - x} R(s_1, t^b, u^b), \quad d\Gamma_b = \frac{d^3k}{\omega} \frac{y^b}{ab} d\Omega_-, \]

\[ y^b_1 = \frac{p^0}{\varepsilon} = \frac{1 - x}{ab}, \quad t^b = -s_1 \frac{1 - c}{ab}, \quad u_b = -s_1 \frac{(1 + c)(1 - x)}{ab}, \quad t^b + u^b + s_1 = 0. \]

For the cases \( k \parallel p'_- \) and \( k \parallel p'_+ \) quantity \( R \) (if suppose \( \Pi=0 \)) is simple:

\[ R_c = R_d = \frac{1}{4} \left( \frac{3 + c^2}{1 - c} \right)^2, \quad T_c = T_d = \frac{1 + (1 - x)^2}{1 - x} R_c, \]

\[ d\Gamma_{c,d} = \varepsilon^2 dx \ d\Omega_- \ d\phi_1 \ dc_1 \ \frac{xy_{c,d}^2}{2 - x + xc_1}, \quad c_1 = \cos k\hat{p'}_-, \]

\[ y^c = \frac{p^0}{\varepsilon} \left| k \parallel p'_{-} \right| \approx \frac{2(1 - x)}{2 - x + xc_1} \bigg|_{c_1 \rightarrow 1} = 1 - x, \]

\[ y^d = \frac{p^0}{\varepsilon} \left| k \parallel p'_{+} \right| \approx \frac{2(1 - x)}{2 - x + xc_1} + c_1 \frac{m_e^2}{4\varepsilon^2} \frac{x}{1 - x}, \]

\[ W_c \cdot (kp'_- ) = W_d \cdot (kp'_+ ) = \frac{2(1 - x)}{x}. \]

Note that \( kp'_+ = 2e^2(1 - y) \). In the evaluation of the rest multipliers for these cases one has to be careful:

\[ \int W_c d\Gamma_c \bigg|_{1 - \theta^2_{\Delta}/2 \leq c_1 \leq 1} = \int W_d d\Gamma_d \bigg|_{1 + \theta^2_{\Delta}(1 - x)^2/2 \geq c_1 \geq 1} = 2\pi \frac{dx}{x} d\Omega_- \frac{1 - x}{2} \left[ L + \ln \frac{\theta^2_{\Delta}}{4} + 2 \ln(1 - x) \right]. \]

Note that the collinear region \( d \) is defined by the condition \( 1 - \theta^2_{\Delta}/2 \leq \cos k\hat{p'}_+ \leq 1 \), which leads to the bounds on \( c_1 \) shown above. So, the contributions of these two collinear regions are

\[ \frac{d\sigma_c + d\sigma_d}{d\Omega_-} = 2 \alpha^2 \left( \frac{3 + c^2}{1 - c} \right)^2 \frac{\alpha}{\pi} \int_0^1 \frac{dx}{x} \left[ (1 - x + \frac{x^2}{2}) \right. \]

\[ \times \left. \left( L - 1 + \ln \frac{\theta^2_{\Delta}}{4} + 2 \ln(1 - x) \right) + \frac{x^2}{2} \right]. \]

Note that there is an asymmetry between the contributions due to the emission along the directions of the (initial or final) electron and the ones due to production of collinear photons along the positron momenta. The symmetry was broken when we decided to write a differential cross-section with respect to the electron scattering angles \( d\Omega_- = d\cos(\hat{p} - \hat{p'}_-) \ d\phi \). After an integration over a symmetrical angular acceptance the contributions would become equal. Compensating terms are to be extracted from the Eqs.\((56,57,58)\) by omitting the terms proportional to \((L - 1)\).
Appendix 3

The following expression could be used instead of $WT$ in Eq. (31) for more precise definition of the contribution due to non-collinear hard photon emission in large-angle Bhabha scattering:

\[
(WT)_\Pi = \frac{(SS)}{|1 - \Pi(s)|^2 s\chi_-'\chi_+} + \frac{(S_1S_t)}{|1 - \Pi(s_1)|^2 s_1\chi_-\chi_+} - \frac{(TT)}{|1 - \Pi(t)|^2 t\chi_-'\chi_+} \\
- \frac{(TT_1)}{|1 - \Pi(t_1)|^2 t_1\chi_-\chi_-} + \text{Re} \left\{ \frac{(TS)}{(1 - \Pi(t))(1 - \Pi(t_1))^{*t}t_1\chi_-\chi_+'} \right\},
\]

where

\[
(SS) = (S_1S_t) = t^2 + t_1^2 + u^2 + u_1^2, \quad (TT) = (T_1T_1) = s^2 + s_1^2 + u^2 + u_1^2,
\]

\[
(SS_1) = (t^2 + t_1^2 + u^2 + u_1^2)(t_+\chi_+' + t_1\chi_- \chi_-' - u\chi_+' - u_1\chi_-\chi_+)
\]

\[
(TT_1) = (s^2 + s_1^2 + u^2 + u_1^2)(u\chi_+' + u_1\chi_-\chi_+' + s\chi_-\chi_+' + s_1\chi_-\chi_+)
\]

\[
(TS) = -\frac{1}{2}(u^2 + u_1^2)[s(t + s_1) + t(s + t_1) - uu_1],
\]

\[
(TS_1) = -\frac{1}{2}(u^2 + u_1^2)[t(s_1 + t_1) + s_1(s + t) - uu_1],
\]

\[
(T_1S) = \frac{1}{2}(u^2 + u_1^2)[t_1(s + t) + s(s + t_1) - uu_1],
\]

\[
(T_1S_1) = \frac{1}{2}(u^2 + u_1^2)[s_1(s + t_1) + t_1(s + t) - uu_1].
\]

We checked analytically that for the switched off vacuum polarization the above formula is equivalent to the multiplication $WT$ in Eq. (31):

\[
(WT)_\Pi|_{\Pi=0} = WT.
\]

In the compensating terms, entering into Eq. (30) the vacuum polarization corrections, have to be inserted also. That can be easily done starting with Eq. (35). We get

\[
\frac{d\sigma_{\text{comp}}^{e^+e^-e^+e^-}}{d\Omega} = \frac{\alpha}{\pi} \int_\Delta \frac{dx}{x} \left\{ \left[ \left( 1 - x + \frac{x^2}{2} \right) \ln \frac{\theta_0^2(1 - x)^2}{4} + x^2 \right] \\
\times \frac{d\bar{\sigma}_0(1, 1)}{d\Omega} \left[ 1 + \frac{1}{(1 - x)^2} \right] + \left[ \left( 1 - x + \frac{x^2}{2} \right) \ln \frac{\theta_0^2}{4} + x^2 \right] \\
\times \left\{ \frac{d\bar{\sigma}_0(1 - x, 1)}{d\Omega} + \frac{d\bar{\sigma}_0(1, 1 - x)}{d\Omega} \right\} \\
- \frac{d\bar{\sigma}_0(1, 1) 8\alpha}{\pi} \ln(\text{ctg}^2 \frac{\theta}{2}) \ln \frac{\Delta \varepsilon}{\varepsilon} \right\} d\Omega.
\]

(61)
We note that quantity $R$ (see Eq. (9)) in the ultra–relativistic limit $s \gg m^2_\mu$ can be derived from Eq. (59). One has to omit there all terms except the ones proportional to $(SS)$, $(SS_1)$, $(S_1S_1)$ and divide by 4.

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Figure 1: The differential forward–backward asymmetry for $e^+e^- \rightarrow \mu^+\mu^-$. The parameters are $\varepsilon = 0.51$ GeV, $\Psi_0 = 10^\circ$, $\varepsilon_{\text{th}} = 0.2\varepsilon$.

Figure 2: The differential forward–backward asymmetry for $e^+e^- \rightarrow \mu^+\mu^-$. Beam energy $\varepsilon = 1.55$ GeV, other parameters the same as in Fig. 1.