Looking for a gift of Nature: Hadron loops and hybrid mixing

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We investigate how coupling of valence $q\bar{q}$ to meson pairs can modify the properties of conventional $q\bar{q}$ and hybrid mesons. In a symmetry limit the mixing between hybrids and conventional $q\bar{q}$ with the same $J^P C$ is shown to vanish. Flavor mixing between heavy and light $q\bar{q}$ due to meson loops is shown to be dual to the results of gluon mediated pQCD, and qualitatively different from mixing involving light flavors alone. The validity of the OZI rule for conventional $q\bar{q}$ and hybrid mesons is discussed.

I. INTRODUCTION

If hybrid mesons or glueballs are ever to be identified as states that are distinct from conventional $q\bar{q}$ with the same $J^P C$, or from the hadronic continuum, some gift of nature may be required. To illustrate what we mean, and to motivate the subsequent discussion, recall that the underlying valence $q\bar{q}$ structure to the light flavored mesons was historically identified because nature left some multiplets relatively clean. The $1^{--}$ and $2^{++}$ multiplets are to good approximation flavor eigenstates, the $I=0$ members being almost pure $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$ respectively. Had this not been the case, it is probable that the quark model for mesons would have been delayed until the subsequent discovery of another gift, namely the occurrence of heavy flavors and associated charmonium or bottomonium spectroscopies.

Glueballs and hybrids are expected to decay into channels that are also accessed by the decays of conventional mesons. Even when the gluonic hadrons have exotic $J^P C$, they can couple to pairs of mesons sharing these quantum numbers. In general therefore one may expect that diagrams involving meson loops will cause mixing between gluonic hadrons and conventional $q\bar{q}$. One of the questions that we shall study here is if there are circumstances where hybrid states, in particular, may decouple from conventional $q\bar{q}$ with the same $J^P C$. If such were to occur, this could be the sought for “gift of nature” enabling a clean hybrid signal to be identified.

A further question is whether there are $J^P C$ multiplets for hybrids which are expected to be ideal flavor eigenstates. In such cases, the correlations of mass and flavor throughout a nonet can distinguish between a hybrid nonet and a tetraquark or di-meson system: the former has the canonical degenerate $I = 0$ (isospin singlet) and $I=1$ (isospin triplet) low-lying with a single $I=0$ ($s\bar{s}$) above the strange members (as in the conventional $1^{--}$), whereas the latter has an inverted structure with the degenerate $I=0$ and $I=1$ heaviest, above the strange and a low-lying isolated $I=0$ state, as seen for the scalar mesons below 1 GeV[1, 2, 3].

These questions have a common feature: under what circumstances do hadron properties deduced from “valence” $q\bar{q}$ eigenstates avoid large corrections due to hadron loops? For example, what protects $\omega$ and $\phi$ from mixing through their common coupling to $K\bar{K}$? Discussion of the latter has a long history, as summarised for example in Refs. [4, 5, 6]. We shall assess the implications of these works for the flavor mixing in hybrid nonets. We find that flavor mixing for exotic $J^P C$ is likely to be suppressed but that the states are unlikely to be as pure flavor eigenstates as those of the conventional $q\bar{q}$ nonets with $J^P C = 1^{--}$ or $2^{++}$.

Recently some general theorems have been developed[7] regarding the effects of hadron loops on quark model predictions[4]. These theorems build on the factorisation of hadron and constituent spins in strong interaction vertices that are OZI-allowed[8], and become exact in a particular mass-degenerate limit. The theorems assumed that the creation of $q\bar{q}$, which triggered the OZI-allowed decay, is in spin-triplet[7, 8]; we show the theorem holds for either

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spin-triplet or spin-singlet. The generalisation confirms the result of Ref. [7] that mixing vanishes between $q\bar{q}$ with different orbital quantum numbers, such as $^3S_1$ and $^3D_1$, and we show also that within the same hypothesis, there is no mixing between these states and hybrid vector mesons.

These theorems will be violated by single gluon exchange. This has particular significance when heavy flavors are involved[12]. We show that flavor mixing of $c\bar{c}$ into the $\eta$ or $\omega$ is qualitatively different from the $s\bar{s}$-$q\bar{q}$ mixing of light flavors in those systems. In particular, the charmed hadron loops connecting $\psi - \omega$ or $\eta - \eta_c$ are dual to the mixing driven by gluon intermediate states in pQCD.

II. MESON LOOPS

Non-perturbative hadron loops, which are required by unitarity, can play an important role in shifting hadron masses and mixing higher Fock states into $q\bar{q}$ wavefunctions. They may also mix different $q\bar{q}$ configurations of the same $J^{PC}$. The formalism underlying this was developed long ago and an extensive set of applications made in a series of papers (Refs. 4,5, and references therein). Inter alia Ref. [4] noted that the contribution of a loop of hadrons spanning an SU(6)$_W$ (SU(2) spin $\otimes$ SU(3) flavour) multiplet could give sum rules whereby states in a flavor octet would be shifted uniformly in mass. When applied to charmonium, the masses of $\psi$ and $\eta_c$ were found numerically to be shifted by a similar amount[10]. In addition, for the $\chi_{cJ}$ states a similar size of mass shift was found for all $J^{PC}$.

Recently this numerical phenomenon has been rediscovered[5]. The theoretical underpinning of these results has been identified and theories developed[7] building on the factorisation of hadron and constituent spins in strong interaction vertices[8]. In particular these theorems showed that vector mesons in $^3S_1$ and $^3D_1$ do not mix in a certain symmetry limit. The purpose of the present paper is to build on these theorems, strengthen their validity and extend them to new situations. The salient features of Refs. 7 and 8 which form the point of departure for the present paper are as follows.

In Ref. 8 it was shown that OZI amplitudes can be factored into contributions depending on the total $J$, constituent spins $S$ and an overall spatial dependence including the angular momentum of the outgoing partial wave. The structure of the factored amplitude for the valence-continuum coupling in the particular case of $q\bar{q}$ creation in spin-triplet was written in Ref. 8. For a meson $A$ (orbital angular momentum $L_A$, spin $S_A$ and total angular momentum $J_A$) decaying to mesons $B$ ($L_B$, $S_B$, $J_B$) and $C$ ($L_C$, $S_C$, $J_C$), with $J_{BC} = J_B \otimes J_C$ and the final mesons in partial wave $L$, the result was:

$$
\langle (L \otimes ((L_B \otimes S_B)J_B \otimes (L_C \otimes S_C)J_C)J_{BC})J_A\rangle H\langle (L_A \otimes S_A)J_A\rangle = \\
\sum_{S_{BC},L_{BC},L_L} (-)^{L+L_{BC}+L_A+S_A+S_B} \Pi_X L_{BC} S_{BC} S_{BC} J_B J_B J_C J_C L_L S_A S_B S_C
$$

$$
\left\{ \begin{array}{c}
L \\
S_{BC} \\
J_A
\end{array} \right\} \left\{ \begin{array}{c}
L_B \\
L_C \\
S_A S_{BC} J_A
\end{array} \right\} \left\{ \begin{array}{c}
L_B \\
L_C \\
S_{BC} J_B
\end{array} \right\} \left\{ \begin{array}{c}
1/2 \\
1/2 \\
2/2
\end{array} \right\} \left\{ \begin{array}{c}
1/2 \\
1/2 \\
2/2
\end{array} \right\} \langle (L \otimes (L_B \otimes L_C)L_{BC})L_L\rangle \langle \psi|L_A\rangle
$$

(1)

with $\Pi_{ab} = \sqrt{(2a+1)(2b+1)}$. In the $^3P_0$ model where the $q\bar{q}$ pair is created in spin-triplet: $H = \sigma \cdot \psi$ where $\sigma$ is a vector in spin space, $X = 1$ and $\psi$ acts on the spatial (orbital and radial) degrees of freedom. If instead the $q\bar{q}$ pair is created in spin-singlet: $H = \sigma \psi$ with $\sigma$ a scalar in spin space, $X = 0$ and $\psi$ again acts on the spatial degrees of freedom.

Note that in the above expression it is the spatial matrix element $\langle (L \otimes (L_B \otimes L_C) J_{BC}) L_L\rangle |\psi]\langle L_A|$ that enforces parity conservation.

The mixing amplitude $a_{A\rightarrow A'}$ between initial $A$ and final $A'$ meson valence $q\bar{q}$ states when summed over a loop of degenerate mesons, labelled $BC$, is related to the mass shift $\Delta m(A)$. The diagram for mixing via loops is shown in Fig. 4 this consists of two decays of the same topology sewed together. Denoting

$$
\Psi = \int \frac{d^3p}{(2\pi)^3} \frac{1}{m_A - E_{BC}(p) + i\epsilon}
$$

as the common energy integral that may be taken outside the sums for intermediate states $BC$ with identical masses, then

$$
\Delta m(A) = \Psi \sum_{BC} \langle (L_A \otimes S_A)J_A|H|BC\rangle \langle BC|H|(L_A \otimes S_A)J_A\rangle
$$

(3)
and

\[(m_A - m_{A'})a_{A' A} = \Psi \sum_{BC} (|L_{A'} \otimes S_{A'}| J_{A'} |H| BC) \langle BC | H | (L_A \otimes S_A) J_A \rangle \] \hspace{1cm} (4)

Substituting the decay amplitudes from Eq. 4 into Eq. 3 with \( J_{A'} = J_A \) gives:

\[(m_A - m_{A'})a_{A' A} = \Psi \sum_{BC} \sum_{L_B S_B J_B L_C S_C J_C L_{BC} S_{BC} J_{BC}} (-)^{J_A + S_A + L_A' + S_{A'} + L_{BC} + L'_{BC}} \]

\[\Pi_{XX} L_{L_B L_{BC} L_{f}} S_{S_B S_{BC} S_{S_{BC} S_{BC}}} \]

\[\sum_{J_f A_f J_{BC}} \sum_{L_f A_f X} \sum_{J_f A_f X} \]

\[\langle (L \otimes (L_B \otimes L_C) L_{BC}) L_f | \psi \rangle |L_A, n_A \rangle \langle (L \otimes (L_B \otimes L_C) L_{BC}) L_f | \psi \rangle |L_{A'}, n_{A'} \rangle \dagger \] \hspace{1cm} (5)

where \( n_A \) and \( n_{A'} \) represent any other quantum numbers, such as radial excitation, of states \( A \) and \( A' \) respectively. The orthonormality of 6-j and 9-j symbols in Eq. 5 were shown in Ref. 7 to place significant constraints for mixings or energy shifts arising from the coupling between valence \( q \bar{q} \) and meson continuum states. In particular, if \( BC \) are OZI-allowed mesons, then while individual loop contributions \( q \bar{q} \to BC \to q \bar{q} \) may give large contributions to physical observables, simple closure relations were found when a degenerate subset of mesons was summed over. In particular Ref. 8 found that the masses of all \( \chi_{cJ} \) states are shifted by the same amount when the meson intermediate states \( D D^*; D^* D; D^* D^* \) are summed over, in the limit where \( M(D) = M(D^*) \). This explains the numerical results 8, 10. The theorem also explained the numerical result 7, 10 that \( \Delta m(\psi) \approx \Delta m(\psi^{'}) \) and \( \Delta m(\psi^{'}) \approx \Delta m(\psi^{'}) \).

The result in Eq. (A.7) of 7 from summing over what we shall call a semi-complete set of states (namely a set of degenerate states with all possible allowed \( S_B, S_C, J_B, J_C \) and \( J_{BC} \) for fixed \( L, L_B \) and \( L_C \) is:

\[(m_A - m_{A'})a_{A' A} = \frac{\delta_{L_A, L_{A'}} \delta_{S_A, S_A'} \delta_{J_A, J_{A'}}}{2L_A + 1} \sum_{L_{BC} L_f} \langle (L \otimes (L_B \otimes L_C) L_{BC}) L_f | \psi \rangle |L_A, n_A \rangle \langle (L \otimes (L_B \otimes L_C) L_{BC}) L_f | \psi \rangle |L_{A'}, n_{A'} \rangle \dagger \] \hspace{1cm} (6)

Although this result was derived for the \( ^3S_0 \) model (\( X = 1 \)) in Ref. 7, the derivation immediately generalises for \( X = 0 \) (the same properties of 6-j and 9-j symbols are used) giving the same final result.

If a complete set of degenerate intermediate states was summed over we would expect no mixing as the sum would just give an identity matrix. However, the complete set in practice would have to span the states in the particle data tables 11 and the concept of degeneracy would be grossly violated. The non-trivial result of Ref. 7 and here is that the above theorem requires a sum over only a subset of states. The practical implication of this is that for the subset the approximation of degeneracy can be a good first approximation 11.

We consider two classes of mixing following Eq. 4.
1. Mixing between states of the same flavor and application to hybrid mesons.

2. Mixing between states of different flavor. We shall draw attention to the qualitatively different way the loop sums conspire for light and heavy flavors.

A. Hybrid Mixing

The mixing between conventional $q\bar{q}$ of a given flavor in states with the same $J^{PC}$ but different $L$ and $S$ was analysed in Ref. [7]. We now apply such ideas to the mixing between a conventional $q\bar{q}$ and a hybrid meson of the same $J^{PC}$ for the traditional case of $S = 1$ pair creation. Summing over a semi-complete set of degenerate intermediate states, mixing between hybrids and conventional mesons only occurs if they have the same $L$, $S$ and $J^P$. However, such coincidences are not theoretically anticipated, at least for low-lying hybrids. For example, in the lattice (e.g. Ref. [15]) and models [13, 14, 16] if a conventional meson has $q\bar{q}$ spins in spin singlet (triplet), then in a gluonic hybrid with the same $J^{PC}$ the $q\bar{q}$ spins will be in spin triplet (singlet). Thus for example, the conventional $1^{\pm \pm}$ are $q\bar{q}$ triplets whereas their hybrid realisations are spin-singlets. Conversely, the conventional singlets $0^{-+}, 2^{-+}$ have hybrid configurations with the $q\bar{q}$ in spin-triplet. Eq. 6 therefore forbids these conventional mesons and hybrids to mix. A particular illustration of how the flavor-spin conspires to give destructive interference of hadron loops in hybrid-$q\bar{q}$ mixing is the $1^{-+}$ case in charmonium.

The decay to two charmed mesons involves one unit of angular momentum: this may be a relative P-wave between two charmed mesons with internal $L=0$ ($D, D^*$) (denoted here by $[L_B L_C; L] = [00; 1]$), or be an internal $L=1$ ($D_0, D_1, D_2$) where an orbitally excited charmed meson is produced with a $D$ or $D^*$ in relative S-wave (denoted by $[01; 0]$ which implies $[01; 0]$ and $[10; 0]$).

The decay of $^3S_1, ^3D_1$ excited charmonia to $[00; 1]$ is allowed but, as already illustrated (Ref. [2]), these states remain orthogonal in the limit where $D, D^*$ are degenerate. The relative couplings of these charmonia to $[01; 0]$ are given in Table I, where $D_{1L,1H}$ refer to the heavy quark basis where the light flavor quarks are respectively in $L = p_{1/2}$ or $p_{3/2}$. The $^3S_1 \rightarrow \sum_j DD_j \rightarrow ^3D_1$ amplitudes are seen to vanish here also in accord with the general theorem. This orthogonality survives for arbitrary mixing angle between the “light” and “heavy” axial mesons, $D_{1L}$ and $D_{1H}$ respectively, in the mass-degenerate limit.

From the general theorem, it might appear that conventional and hybrid mesons with the same $L$, $S$ and $J$ but different parity could mix. However, we emphasise that parity conservation is enforced by the spatial matrix element, for example in limiting the combinations of mesons and partial waves ($L$) that can appear as intermediate states.

Hybrid charmonia are forbidden to decay to $[00; L]$ charmed mesons in the mass degenerate symmetry limit because of a zero from the spatial matrix element. However, there is no mixing of hybrid with either $^3S_1$ or $^3D_1$ valence states through $[00; 1]$ loops. The first common channels are $[01; 0]$; the ground state $D, D^*$ produced in conjunction with orbitally excited $D_j$ states. Here too, Table I shows that the overlap between hybrid and $^3S_1$ or hybrid and $^3D_1$ vanish in the symmetry limit, again in accord with the general theorem.

| $\psi(^3S_1)$ | $\psi(^3D_1)$ | Hybrid |
|----------------|----------------|---------|
| $D^* D_0$     | $-1/2$         | $0$     | $1/\sqrt{6}$ |
| $D^* D_{1L}$  | $1/\sqrt{2}$  | $0$     | $1/\sqrt{3}$ |
| $D^* D_{1H}$  | $0$            | $1/4$   | $-1/2\sqrt{6}$ |
| $D^* D_2$     | $0$            | $1/4\sqrt{3}$ | $-2\sqrt{\frac{2}{3}}$ |
| $D D_{1L}$    | $1/2$          | $0$     | $-1/\sqrt{6}$ |
| $D D_{1H}$    | $0$            | $1/2\sqrt{2}$ | $1/2\sqrt{3}$ |

TABLE I: $\psi$ and hybrid vector decay amplitudes. Mixing amplitudes are proportional to sums over products of these numbers.

B. Flavor Mixing: light flavors

In Fig. 2 we show the $Q\bar{Q} \rightarrow q\bar{q}$ loop which mixes flavors. Ref. [4] noted that such hadronic mixing necessarily leads to breaking of the OZI rule. Contrast how the intermediate mesons in the loop combine in this configuration relative to the “connected” diagram (Fig. 1) for identical flavors. As for the “connected” diagram, this mixing diagram
consists of two decay diagrams sewed together. However, now the two diagrams are sewed together in a different way and this introduces an extra phase:

$(-1)^{L+S_A+S_B+S_C+1}$ (7)

in the mixing amplitude compared to Eq. 5. The $(-1)^{S_A+S_B+S_C+1}$ factor arises from the different spin recouplings in Fig. 2 relative to Fig. 1; a particular example being Eqs. 9 and 10. Integrating over the momentum flow in the two diagrams introduces the relative $(-1)^L$ factor.

Note that this diagram can also contribute to mixing between states of the same flavor, but is expected to be suppressed, being a disconnected diagram.

![FIG. 2: Flavor mixing via hadronic loops](image)

If we sum over a complete subset of intermediate states we obtain the following general expression:

$$(m_A - m_{A'}) a_{A'A} = \delta_{S_A',X} \delta_{J_{A'},X} \delta_{J_A,J_A'} (-1)^L$$

$$\sum_{L_{BC} L_f} \left\{ \frac{L_f}{X} \frac{X}{J_A} \frac{X}{L_{A'}} \right\} \langle (L \otimes (L_B \otimes L_C)_{L_{BC}})_{L_f} | \psi | L_A, n_A \rangle \langle (L \otimes (L_B \otimes L_C)_{L_{BC}})_{L_f} | \psi | L_{A'}, n_{A'} \rangle^\dagger$$ (8)

This should be compared with the analogous expression for mixing between states of a given flavor in Eq. 6. In the flavor mixing case, only spin-triplets mix in $S = 1$ pair creation models ($X = 1$), to which we shall restrict ourselves, but we can in general have $L_{A'} \neq L_A$.

We now illustrate the way that meson loops contribute to these two situations by considering $\omega$ and $\phi$ $1^{-+}$ mesons coupling to strange mesons. The relative amplitudes for $\phi$ decays to strange mesons are given in Table II. The squares of individual amplitudes give the relative rates (in the mass degenerate limit) for decays to $K\bar{K}; K\bar{K}^* + c.c; K^*\bar{K}^*$, which are in the ratio $1 : 4 : 7$. The flavor mixing involves the same amplitudes but folded together as in Fig. 2. The result is that $K\bar{K}$ and $K^*\bar{K}^*$ loops have the opposite sign relative to $K\bar{K}^*$ (and its charge conjugate), having opposite G-parity (the extra phase in Eq. 7). This is in accord with Refs. [4, 5], which argued that states of opposite G-parity interfere destructively in the flavor mixing. However, the above example shows that this is not in general a total destruction. The ratio of flavor mixing to total = -1/3 (i.e. - 1/12 + 4/12 - 7/12 which is just $\{-1 \ 1 \ 0\}$). Thus although there is a suppression arising from the different charge conjugation/G parity intermediate states, further suppression is required from the spatial wavefunction dependence of the amplitude[6].

Note that this contrasts with the total orthogonality that arose for mixing between states with the same flavor but different $L$, such as $^3S_1 - ^3D_1$, when the loops were summed corresponding to a “connected” topology. This flavour mixing configuration (that is dual to a disconnected topology) can also be considered for the $^3S_1 - ^3D_1$ situation and for states with the same flavor. States with the same $J, S$, flavor and parity, but different $L$, such as $^3S_1$ and $^3D_1$ receive the same 1/3 suppression from flavor-spin (rather than the exact cancellation in the “connected” topology) but the overall question of mixing depends on the spatial wavefunctions and the validity of the OZI rule in such multiplets.

Given that the OZI rule is empirically successful for the $^3S_1$ multiplet, this suggests that there will in practice be suppressed mixing between the $^3S_1$ and $^3D_1$ multipoles. However, this is just an empirical observation of the lack of a theoretical explanation of the OZI rule. Hence we return to this question for the case of the $\omega - \phi$ states.
The same spin-flavor destructive interference factor magnitude of $1/3 \left(\left\{ \frac{1}{3} \right\} \right)$ arises for coupling $\omega - \phi$ via $KK_j$ in $L = 0 \left(\left[01; 0\right]\right)$ intermediate states in the loop. Thus although there is indeed a flavor-spin suppression, this alone is insufficient to explain the OZI rule/lack of flavor mixing in the $1^{−−}$ $q\bar{q}$ multiplet. However, for $\omega - \phi$, the loops can couple to $\left[00; 1\right]$ or $\left[01; 0\right]$. The spatial amplitudes to $\left[00; 1\right]$ and $\left[01; 0\right]$ have a relative negative phase, the $(-1)^L$ factor in Eq. 7. Hence there is a destructive interference between these two multiplets in addition to the $1/3$ suppression from flavor-spin.

In general, summing over $\left[00; 1\right]$ and $\left[01; 0\right]$ will not give exact cancellation because of different counting factors ($\left[01; 0\right]$ can be $\left[01; 0\right]$ and $\left[10; 0\right]$) and different spatial matrix elements. In a specific model Geiger and Isgur\cite{6} noted that the spatial cancellation can be exact in an analytic limit which is near to the empirical set of parameters used in quark models. Even away from this ideal situation, they argued that summing over spatial excitations gives a destructive interference between $\left[00; 1\right]$ and $\left[01; 0\right]$ meson loops that can play a significant role in minimising loop corrections to the OZI rule in $\omega - \phi$.

For initial $q\bar{q}$ states with internal $L \neq 0$ the spatial analysis become model dependent. No general conclusions can be drawn other than if a complete set of states were summed over, in the $3^3P_0$ model the OZI rule would work for all $J^{PC}$ states other than $3^3P_0$.

| $J_{BC}$ | $\phi \to KK$ | $\phi \to KK^*$ | $\phi \to K^*K$ | $\phi \to K^*K^*$ |
|---|---|---|---|---|
| 0 | $1/2\sqrt{3}$ | $\omega$ | $\omega$ | $1/6$ |
| 1 | $-\phi$ | $1/\sqrt{6}$ | $-1/\sqrt{6}$ | $0$ |
| 2 | $-\phi$ | $-\phi$ | $-\phi$ | $-\sqrt{3}/3$ |

TABLE II: $\phi$ decay amplitudes ($L = 1$)

The above discussion contrasts with the case of the pseudoscalar $0^{−+}$. Denoting pseudoscalar and vector mesons by $P, V$ respectively, and the spin projections of the vector by $\uparrow, \downarrow, z$, the OZI creation of $q\bar{q}$ in spin-triplet gives in the symmetry limit the normalized state

$$\frac{1}{2} \left[ V_\uparrow \bar{P} + P \bar{V}_\downarrow + V_\downarrow V_\uparrow - V_\downarrow \bar{V}_\uparrow \right]$$

(9)

The amplitude for flavor mixing is then proportional to the overlap

$$\frac{1}{4} \left[ P \bar{V}_\uparrow + V_\uparrow \bar{P} - V_\downarrow V_\downarrow + V_\downarrow \bar{V}_\uparrow \bar{P} + P \bar{V}_\downarrow + V_\downarrow V_\downarrow - V_\downarrow \bar{V}_\uparrow \right] \equiv 0$$

(10)

In this case the sum over amplitudes for flavor mixing vanishes in the symmetry limit, as expected from Eq. 8. This is a result of the assumed $q\bar{q}$ creation being spin-triplet ($3^3P_0$) acting within a $q\bar{q}$ state ($0^{−+}$) which is spin-singlet.

We now consider the implications of these results for the case of hybrid mesons. Hybrid $1^{−−}$ have the $q\bar{q}$ coupled to $S = 0$ and hence the mixing is strongly suppressed; hybrid vector mesons should be ideal flavor states within the approximations employed here. While this may help distinguish a $q\bar{q}$ or hybrid vector nonet from a di-meson or tetraquark multiplet, it does not help distinguish a hybrid multiplet from a conventional $q\bar{q}$ nonet.

Hybrids with exotic $J^{PC} = 0^{++}, 1^{-+}, 2^{++}$ have the $q\bar{q}$ in spin-triplet. Ref.\cite{10} argued that for the conventional spin-triplet vector mesons, an essential source of the OZI rule for $\omega - \phi$ is $\left[00; 1\right]$ cancelling with $\left[01; 0\right]$. If this is indeed a major player in realising the OZI rule, then one would not anticipate this cancellation to occur for these exotic hybrids because the spatial matrix element to $\left[00; 1\right]$ is expected to vanish. In this case, there could be significant violation of the OZI rule, and flavor mixing in these exotic hybrids.

C. Flavor mixing: heavy flavors

When heavy flavors ($m_Q > \Lambda_{QCD}$) are involved, inter-flavor mixing is qualitatively different. Consider the case of $c\bar{c}$ mixing into the $\eta$ and $n\bar{n}$ into the $\eta_c$. This could occur via intermediate loops involving $DD^* + c.c. , D^*D^*$ ($DD$ being forbidden by parity). However, generically $c\bar{c} \to DD$ can occur by the factorising OZI creation of $q\bar{q}$, whereas $q\bar{q} \to DD$ requires the creation of $c\bar{c}$. In the latter, where $2m_c > \Lambda_{QCD}$, the OZI process would require the color fields of force to extend over excessive distances without having created light $q\bar{q}$. This is highly improbable and the OZI process is suppressed both theoretically and empirically\cite{12}. A way for such decays to be triggered is if the required energy to create $c\bar{c}$ is supplied by a hard process such as single gluon exchange.
The $q\bar{q} \rightarrow q\bar{q} + g \rightarrow q\bar{c}c\bar{q}$ leaves the light flavors in color-octet. For flavor mixing in a color-singlet meson, this will require the subsequent annihilation of the $q\bar{q}$ to mirror the former: $q\bar{c}c\bar{q} \rightarrow \bar{c}c + g \rightarrow \bar{c}c$. Hence the mixing amplitude, through an intermediate loop of charmed mesons, will necessarily be of $O(\alpha_s^2)$ and in accord with pQCD.

We see that even for light flavors the loop mixing is in effect $O(\alpha_s^2)$ for the $0^{++}$ case. In the limit where the intermediate vector and pseudoscalar mesons have equal mass ($m_V = m_P$), Eq. (10) shows the OZI loop amplitude vanishes. A non-zero overlap follows if $m_V \neq m_P$. In practice this is the case and arises due to the chromomagnetic interaction arising from one gluon exchange (OgE) [17]. Phenomenologically therefore, the non-zero mixing from OZI-generated loops is proportional to $O(\alpha_s^2)$, manifested by the vector-pseudoscalar mass splittings.

The mixings involving heavy flavors in spin-triplet, such as $\psi - \omega$, are required to involve perturbative gluons for analogous reasons to the $0^{++}$ case. However, charge conjugation plays a non-trivial role. The initial state $q\bar{q}$ has charge conjugation $C = -$. The initial step $q\bar{q} \rightarrow q\bar{q} + g \rightarrow q\bar{c}c\bar{q}$ converts the $q\bar{q}$ into $C = +$ and creates the $c\bar{c}$ with $C = -$. While the latter requires only color rearrangement to map onto the $C = -c\bar{c}$ final state, the subsequent annihilation of $q\bar{q}$ requires two gluons to satisfy charge conjugation, hence three gluons overall. Consequently for heavy flavors in general, meson loops are dual to pQCD expectations.

III. DISCUSSION AND CONCLUSION

In the limit where all mesons in a loop belong to a degenerate subset, we have extended an existing theorem of Ref. [7]. First, we have shown that that theorem is not restricted to $q\bar{q}$ creation in $S = 1$ but also applies for $S = 0$. The previous result that $^3S_1$ or $^2D_1$ states of given flavors remain orthogonal is found to apply also to hybrid mesons such that mixing of a hybrid vector with either $^3S_1$ or $^2D_1$ $q\bar{q}$ vanishes in these circumstances. Hence within these assumptions, hybrid vector $q\bar{q}$ mesons will decouple from their conventional $q\bar{q}$ counterparts.

We have extended the discussion to the case of loop-induced mixing between different flavors. For the case of heavy quarkonium the mixing with light $q\bar{q}$ through loops containing heavy-flavored mesons (for example $c\bar{c}$ mixing with $q\bar{q}$ via charmed mesons) is dominated by pQCD (gluon exchange). The meson loops are suppressed by $O(\alpha_s^{[2,3]})$ as in conventional pQCD of the form $c\bar{c} \to$ gluons $\to q\bar{q}$. For light flavors, by contrast, there is no connection between meson loops and pQCD, except perhaps insofar as degeneracy can be broken by OgE.

One of the motivations of this work was to examine the circumstances under which hybrid mesons would decouple from or be distinguishable from conventional $q\bar{q}$ and other exotics. We have shown that the vector hybrids decouple from the vector $q\bar{q}$ states; there is however also the question of flavor mixing.

If for some $J^{PC}$ a hybrid nonet were flavor ideal, this would enable a clear distinction from di-meson correlations with the same $J^{PC}$. The former would exhibit the familiar flavor triangle as in the $\phi - K^* - [\omega, \rho]$ as against the inverted structure for a tetraquark/di-meson nonet, e.g. as in the low-lying scalars $[f_0, a_0] - \kappa - \sigma$. To decide, a priori, which extreme is more likely requires understanding the origin of flavor mixing and the OZI rule for conventional $q\bar{q}$.

For light flavors where $q\bar{q}$ are in $S = 0$, the multiplets would be ideal within the approximations used here. However, these theorems can not immediately be applied to the ground-state $0^{++}$ even within a naive picture where they are $q\bar{q}$ states. First of all, the loops contain the mesons themselves. Therefore, to be consistent some iterative procedure, such as that suggested in [4], would need to be used and converge. Second, the assumption of mass degeneracy within a subset is empirically severely violated.

For the initial $q\bar{q}$ in $S = 1$ we demonstrated that coupling to loops gives a destructive interference that is 67% in amplitude; this quantifies a qualitative historical observation of Lipkin[6]. In the case of initial $q\bar{q}$ with internal $L = 0$, namely $1^{--}$, there is further destructive interference in the spatial overlaps of meson loops containing $[00; 1]$ and $[01; 0]$ (in the $[L_B L_C ; L]$ notation of Section[11]). The absolute cancellations are model dependent, as discussed by Geiger and Isgur [6]. Hence there is a qualitative understanding of the OZI rule for the $1^{--}$ nonet, but any quantitative description depends on the details of strong interactions, and is currently beyond lattice QCD.

This qualitative picture suggests that the flavor mixing for hybrid multiplets may be quite different from conventional $q\bar{q}$. For a hybrid vector with its $q\bar{q}$ coupled to $S = 0$ the multiplet should be ideal, at least in $S = 1 q\bar{q}$ pair-creation dynamics. Exotic $J^{PC}$, such as $1^{++}$, have the $q\bar{q}$ coupled to $S = 1$. While the flavor-spin suppression arises here, thereby giving a tendency towards an ideal OZI nonet as before, the spatial interferences are non-trivial. In a particular limit, discussed in Refs. [13] and [14], it is predicted that the spatial amplitudes for hybrid decays to $[00; 1]$ vanish. However, the amplitude to $[01; 0]$ is allowed and predicted to be the dominant decay channel. This implies that the pattern of spatial suppressions found for $\omega - \phi$ does not arise here.

In summary, the flavor-spin coupling favors ideal mixing for the $1^{++}$ hybrid nonet but of itself is not sufficient to expect a situation as ideal as that found for the conventional $1^{--}$ nonet. The ideal situation observed for the
conventional $1^{--}$ appears also to require spatial interferences, which are not anticipated to arise for the $1^{-+}$ hybrid if our current understanding of hybrid meson decays is any guide. In such a case, where decays to $[01;0]$ dominate, this will make it hard to distinguish a hybrid $q\bar{q}$ nonet from di-meson enhancements, though the decoupling from $[00;1]$ can give a characteristic signature. Conversely, if the predicted decouplings from $[00;1]$ are model-dependent artefacts, this may make an isolated hybrid harder to identify, but offers the possibility that $q\bar{q}$ $1^{-+}$ will be flavor-ideal and hence distinct from di-meson or tetraquark correlations.

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