Transfer information remotely via noise entangled coherent channels

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Abstract

In this contribution, a generalized protocol of quantum teleportation is suggested to investigate the possibility of remotely transfer unknown multiparities entangled coherent state. A theoretical technique is introduced to generate maximum entangled coherent states which are used as quantum channels. We show that the mean photon number plays a central role on the fidelity of the transferred information. The noise parameter can be considered as a control parameter only for small values of the mean photon number.

1 Introduction

Quantum information is one of the most important achievements of this century, where one can overcome the problems of handling confidential information. There are different ways to communicate the quantum information. As an example, quantum teleportation, which one can use it to send information remotely in a direct way [1, 2], quantum coding, where the given information is coded in a different state, which is send to the receiver who decodes the information [3, 4], quantum cryptography, which is another technique to distribute quantum key between two users to communicate safely [5, 6].

To achieve these tasks, one needs entangled pairs, which represent quantum channels between the sender and the receiver, local operations and measurements. Since entangled pairs are crucial resource in quantum communication, the preparation of maximally entangled states is a crucial task. There are several attempts carried out to generate entangled channels between different types (see [7, 8, 9, 10] for recent references).

One of the promising type of entangled states are the coherent states, which are used widely in the context of quantum information. As an example, Zhou and Yang [11] have used them to transfer entanglement between atomic state and two-modes cavity state. Also, they has been used to perform an optimal teleportation [12]. The coherent states are employed to implement a probabilistic teleportation scheme, in which the amount of classical information send by Alice is restricted to one bit [13]. Enk and Hirota [14], have studied another type of coherent states produced from Schrödinger cat states by using a 50/50 beam splitter. This class of coherent states which can be used to teleport one qubit where, a simple protocol that achieves this aim with a 50% probability of success is introduced. Also, Wang has
proposed a simple scheme to teleport both the bipartite and multipartite by using only linear optical devices such as beam splitters and phase shifters, and two-mode photon number measurements [15].

In reality, one can generate maximum entangled states, MES, but keeping them isolated is impossible task. Therefore, these MES turn into partially entangled states, PES due to the undesirable interactions. So, there are great efforts have been done to investigate the possibility of performing quantum information tasks by using these partially entangled states. In this context, Enk has considered the decoherence of multi-dimensional entangled coherent states due to photon absorption losses [19].

In this article, we introduce a general scheme of quantum teleportation by using maximum and partial entangled states. Also, this generalized protocol is employed to transfer quantum information through noise quantum channels. This paper is organized as follows: In Sec.2, we introduce a quantum teleportation scheme to teleport a tripartite entangled coherent state by using a quantum channel consists of four parties coherent state. This entangled channel is different from that has been used in [15]. The generalization of this teleportation protocol is the subject of Sec.3. Implementation of quantum teleportation via partially entangled state is described in Sec.4. Finally Sec.5, is devoted to discuss our results.

2 Teleportation scheme for tripartite state

Entangled coherent states have been proposed as a potential quantum channels to teleport unknown quantum states. These coherent states can be written as function of the Fock state [16](photon number state) $|n\rangle$, $$|\pm\alpha\rangle = \exp(-2|\alpha|^2) \sum_{n=0}^{\infty} \frac{(\pm\alpha)^n}{\sqrt{n!}} |n\rangle. \quad (1)$$ An entangled coherent state between two modes can be written as, $$|\alpha\rangle_{12} = (|\alpha,\alpha\rangle + |\alpha,-\alpha\rangle)/N_{\phi} \quad (2)$$ where $N_{\phi} = 2 - 2\exp[-4|\alpha|^2]$ and $\langle\alpha| - \alpha\rangle = \exp(-2(|\alpha|^2)) [15][19]$. This entangled state (2), has been used by Enk and Hirota to teleport a Schrödinger cat state [19]. Wang has used a tripartite entangled coherent state of the form, $$|\phi\rangle^\pm = N^\pm_{\alpha}(|\sqrt{2}\alpha,\alpha,\alpha\rangle_{123} \pm |\sqrt{2}\alpha,-\alpha,-\alpha\rangle), \quad (3)$$ where, $N^\pm_{\alpha} = [2(1 \pm e^{-8|\alpha|^2})]^{-\frac{1}{2}}$ is the normalized factor, to teleporte two qubits entangled coherent state [15]. Also, the same author has suggested a coherent state of four particles to transfer a tripartite entangled coherent state.
In this current protocol, we introduce a different class of entangled coherent states consists of four qubit as a quantum channel to teleport tripartite entangled coherent state. The advantage of this choice is: it can be generalized to multipartite entangled channels easily, where we have generalized our results theoretically to generate a family of maximum entangled coherent states of mode \( m \). This family can be used to transport a family of \((m - 1)\) modes of ECS by using the suggested generalized scheme. In the following subsections we discuss these phenomena by using maximum and partial entangled coherent states as quantum channels.

### 2.1 Using Maximum entangled as a quantum channel

Consider that Alice is given a tripartite coherent state defined as,

\[
\rho_u = |\kappa_1|^2 |\sqrt{2} \alpha, \alpha, \alpha\rangle_{123} \langle \sqrt{2} \alpha, \alpha, \alpha| + \kappa_1 \kappa_2^* |\sqrt{2} \alpha, \alpha, \alpha\rangle_{123} \langle -\sqrt{2} \alpha, -\alpha, -\alpha| \\
+ \kappa_1^* \kappa_2 | -\sqrt{2} \alpha, -\alpha, -\alpha\rangle_{123} \langle \sqrt{2} \alpha, \alpha, \alpha| + |\kappa_2|^2 | -\sqrt{2} \alpha, -\alpha, -\alpha\rangle_{123} \langle -\sqrt{2} \alpha, -\alpha, -\alpha|.
\]

The aim of Alice is sending this unknown state to Bob, who shares with her a maximum multiparities entanglement coherent state (ECS),

\[
\rho^\pm = \frac{1}{N_\pm^2} \left\{ |2\alpha, \sqrt{2} \alpha, \alpha\rangle_{4567} \langle 2\alpha, \sqrt{2} \alpha, \alpha| \pm |2\alpha, \sqrt{2} \alpha, \alpha\rangle_{4567} \langle -2\alpha, -\sqrt{2} \alpha, -\alpha| \\
\pm | -2\alpha, -\sqrt{2} \alpha, -\alpha, -\alpha\rangle_{4567} \langle 2\alpha, \sqrt{2} \alpha, \alpha| \\
+ | -2\alpha, -\sqrt{2} \alpha, -\alpha, -\alpha\rangle_{4567} \langle -2\alpha, -\sqrt{2} \alpha, -\alpha, -\alpha| \right\},
\]

where \( N_\pm = \sqrt{2(1 \pm e^{-16|\alpha|^2})} \) is the normalization factor. This density operator represents two class of states: maximum and partially entangled states. It behaves as MES between the system 4 and the systems 5, 6, 7, where the concurrence [18] in this case \( C_{4,567}^- = 1 \), while \( C_{4,567}^+ = tanh(8|\alpha|^2) \). On the other hand, the density operator \( \rho^- \) behaves as a PES for the other partitions, where

\[
C_{5,467}^\pm = \frac{\sqrt{1 - e^{-8|\alpha|^2}} \sqrt{1 - e^{-24|\alpha|^2}}}{1 \pm e^{-16|\alpha|^2}}, \quad C_{6,457}^\pm = \frac{\sqrt{1 - e^{-4|\alpha|^2}} \sqrt{1 - e^{-28|\alpha|^2}}}{1 \pm e^{-16|\alpha|^2}} = C_{7,456}^\pm
\]

Let us assume that, the partners use the maximum entangled state \( \rho^- \) as a quantum channel. Then the total state of the system is \( \rho_u \otimes \rho^-_{4567} \). We can summarize the steps of implementing the teleportation protocols as follows:

1. Alice mixes the unknown state \( \rho_u \), with the quantum channel \( \rho^-_{4567} \) by applying a series of operations defined by the beam splitters and phase shifts [15, 20]. In an explicit form

\[
\rho_u \otimes \rho^-_{4567} \rightarrow R_{34} R_{31} R_{32} \rho_u \otimes \rho^-_{4567} R^e_{32} R^e_{31} R^e_{34} = \rho_{out}
\]
where $R_{ij} |\mu\rangle |\nu\rangle = |\frac{\mu+i\nu}{\sqrt{2}}\rangle |\frac{\mu-i\nu}{\sqrt{2}}\rangle_j$. Theses operations separate the systems 1 and 2 from the initial system. So, the final output state $\rho_{out}$ is a direct product of the vacuum states $|0\rangle_1 |0\rangle_\otimes |0\rangle_2 |0\rangle$ and the final unnormalized state,

$$\rho_f = |\kappa_1|^2 |\psi_1\rangle \langle \psi_1| - |\kappa_1|^2 |\psi_1\rangle \langle \psi_2| - |\kappa_1| |\kappa_2\rangle \langle \kappa_1\rangle |\psi_3\rangle \langle \psi_1| + |\kappa_1| |\kappa_2\rangle \langle \kappa_2\rangle |\psi_4\rangle \langle \psi_1|$$

$$+ |\kappa_1|^2 |\psi_2\rangle \langle \psi_2| + |\kappa_1|^2 |\kappa_2\rangle \langle \kappa_2\rangle |\psi_3\rangle \langle \psi_2| - |\kappa_1| |\kappa_2\rangle \langle \kappa_2\rangle |\psi_3\rangle \langle \psi_2| + |\kappa_1| |\kappa_2\rangle \langle \kappa_1\rangle |\psi_3\rangle \langle \psi_2|$$

$$- |\kappa_2| |\kappa_2\rangle \langle \kappa_2\rangle |\psi_4\rangle \langle \psi_3| - |\kappa_2| |\kappa_1\rangle \langle \kappa_1\rangle |\psi_4\rangle \langle \psi_3|$$

$$- |\kappa_2| |\kappa_1\rangle \langle \kappa_1\rangle |\psi_4\rangle \langle \psi_3| - |\kappa_2|^2 |\psi_4\rangle \langle \psi_4| + |\kappa_2|^2 |\psi_4\rangle \langle \psi_4| \rangle \langle \psi_4|, \quad (8)$$

where,

$$|\psi_1\rangle = |2\sqrt{2}\alpha, 0, \sqrt{2}\alpha, \alpha, \alpha\rangle, \quad |\psi_2\rangle = |\sqrt{2}\alpha, -\sqrt{2}\alpha, -\alpha, -\alpha\rangle$$

$$|\psi_3\rangle = |0, -2\sqrt{2}\alpha, \sqrt{2}\alpha, \alpha, \alpha\rangle, \quad |\psi_4\rangle = |-2\sqrt{2}\alpha, 0, -\sqrt{2}\alpha, -\alpha, -\alpha\rangle. \quad (9)$$

2. Alice performs two photon number measurements on modes 3 and 4. The probability to find $l$ and $n$ photon, $P(l, n)$ in modes 3 and 4 is given by

$$P(l, n) = \left| \langle l, n | \rho_f | l, n \rangle \right|^2. \quad (10)$$

The probability, $P(l, n) = 0$, if both $l$ and $n$ are non zero. However, if $n \neq 0$ and $l = 0$, the state on Bob's side collapses into,

$$\rho_{Bob}^n = \lambda_1 |\sqrt{2}\alpha, \alpha, \alpha\rangle \langle \sqrt{2}\alpha, \alpha, \alpha| - \lambda_2 |\sqrt{2}\alpha, \alpha, \alpha\rangle \langle -\sqrt{2}\alpha, -\alpha, -\alpha| - \lambda_3 |-\sqrt{2}\alpha, -\alpha, -\alpha\rangle \langle \sqrt{2}\alpha, \alpha, \alpha| + \lambda_4 |-\sqrt{2}\alpha, -\alpha, -\alpha\rangle \langle -\sqrt{2}\alpha, -\alpha, -\alpha| \quad (11)$$

where $\lambda_1^{(n)} = \frac{|\kappa_1|^2}{N_1}$, $\lambda_2^{(n)} = \frac{\kappa_1 \kappa_2^*}{N_1} (-1)^n$, $\lambda_3^{(n)} = \frac{\kappa_1^* \kappa_2}{N_1} (-1)^n$, $\lambda_4^{(n)} = \frac{|\kappa_2|^2}{N_1} (-1)^{2n}$ and $N_1 = |\kappa_1|^2 + |\kappa_2|^2 - 2(-1)^n e^{-8|\alpha|^2} Re(\kappa_2^* \kappa_1)$ is the normalized factor.

3. Alice sends her results through classical channel to Bob, who performs a three $\pi$ phase shifters of modes 5, 6 and 7 local transformation on his state $\rho_{Bob}$, i.e., Bob applies the unitary operator,

$$U_p = e^{-i\pi(a_5^\dagger a_5 + a_6^\dagger a_6 + a_7^\dagger a_7)} \quad (12)$$

If the integer $n$ is odd, then Bob gets exactly the same input state $\rho_u$. However if $n$ is even, Bob performs an extra operation such that,

$$|\sqrt{2}\alpha, \alpha, \alpha\rangle \rightarrow |\sqrt{2}\alpha, \alpha, \alpha\rangle, \quad |-\sqrt{2}\alpha, -\alpha, -\alpha\rangle \rightarrow |-\sqrt{2}\alpha, -\alpha, -\alpha\rangle. \quad (13)$$

to get the teleported state exactly.

Now, if we assume that $n$ is an odd integer and $l = 0$, then the probability of successes (10) is given by

$$P(n, 0) = \frac{e^{-8|\alpha|^2} |2\sqrt{2}\alpha|^{2n}}{2n! (1 - e^{-16|\alpha|^2})}. \quad (14)$$
which is independent of the parameter $\kappa_{1,2}$. This probability becomes 0.5, when $|\alpha| \to \infty$.

Let us consider the case, $n = 0$ and $n \neq 0$. In this case the state of Bob collapses into,

$$
\rho_{Bob}^{(l)} = \lambda_1^{(l)} |-\sqrt{2}\alpha, -\alpha, -\alpha\rangle \langle -\sqrt{2}\alpha, -\alpha, -\alpha| - \lambda_2^{(l)} |-\sqrt{2}\alpha, -\alpha, -\alpha\rangle \langle \sqrt{2}\alpha, \alpha, \alpha| 
- \lambda_3^{(l)} |\sqrt{2}\alpha, \alpha, \alpha\rangle \langle -\sqrt{2}\alpha, -\alpha, -\alpha| + \lambda_4^{(l)} |\sqrt{2}\alpha, \alpha, \alpha\rangle \langle \sqrt{2}\alpha, \alpha, \alpha|,
$$

where,

$$
\lambda_1^{(l)} = \frac{|\kappa_1|^2}{N_2}, \lambda_2^{(l)} = \frac{\kappa_1\kappa_2^*}{N_2}(-1)^l, \lambda_3^{(l)} = \frac{\kappa_1^*\kappa_2}{N_1}(-1)^l, \lambda_4^{(l)} = \frac{|\kappa_2|^2}{N_1}(-1)^l \text{ and } N_2 = |\kappa_1|^2 + |\kappa_2|^2 - 2(-1)^l e^{-8|\alpha|^2} Re(\kappa_2^*\kappa_1) \text{ is the normalized factor.}
$$

Also, in this case the probability of successes $P(n, l) = P(n, 0)$.

2.2 Using Partially entangled state as a quantum channel

In sec.(2.1), we show that the density operator sometimes behaves as a non maximum entangled state, i.e., PES. Let us assume that Alice has the possibility of using only this class of states as quantum channels to teleportate a tripartite entangled. In this case the total state of the system is $\rho_u \otimes \rho_{4567}^+$. The partners perform the steps(1-3) as described in Sec.(2.1). If Alice measures $n$ photons in the mode 3 and $(l = 0)$ photons in mode 4, then the partners end the protocol with a similar state as (11). The only difference between them is the sign of the second and third terms are positive. To achieve this protocol with fidelity 1, one consider $n$ is an even number. In this case the probability of successes is given by,

$$
P = \frac{(1 - e^{-8|\alpha|^2})^2}{2(1 + e^{-16|\alpha|^2})}. \quad (16)
$$

It is clear that $P$ depends on the parameter $\alpha$, but is independent of the parameters $\kappa_{1,2}$. In the limit $|\alpha| \to \infty$, the probability of success becomes $\frac{1}{2}$ (one ebit) and $P < \frac{1}{2}$, in this case quantum channel is not a MES.

3 Generalized quantum teleportation Protocol

In this section, we generalized the results of Sec.2. For this aim, we define a non orthogonal maximum entangled as

$$
|\Psi\rangle_{0...m}^\pm = A_{m+1}^\pm \left( |2^{m-1}2^{m-1}| \alpha\rangle_0...|2^{m-1}2^{m-1}| \alpha\rangle_m - |2^{m-1}2^{m-1}| \alpha\rangle_0...|2^{m-1}2^{m-1}| \alpha\rangle_m \right),
$$

where $A_{m+1}^\pm = [2(1 \pm e^{-2m+1}|\alpha|^2)]^{-\frac{1}{4}}$, is the normalized factor. This class of states, which represent a quantum channel of $m + 1$ modes, can be used to teleporte a multiparities state
To evaluate the degree of entanglement contained in the general density operator \( \rho_{\text{gen}} = |\psi^{\pm}\rangle\langle\psi^{\pm}| \), where the state vector \( |\psi^{\pm}\rangle \) is given by (17), we evaluate the concurrence. For this aim, we divided the \( m + 1 \)-modes systems into two systems: the first, \( A \) is system 0 and second \( B \) is the all remaining system of modes \( m \), where each system is linearly independent with respect to \( \alpha \) and \(-\alpha\) which spanning a two dimensional subspaces of the Hilbert space. Consider a set of the orthonormal basis \( \{|0\rangle_x, |1\rangle_x\}, x = A, B \) which satisfy the Gram-Schmidt theorem. These basis could be written in the basis of the coherent state as following

\[
\begin{align*}
|0\rangle_A &= |2^{\frac{m-1}{2}}\alpha\rangle_0, \\
|1\rangle_A &= \frac{|-2^{\frac{m-1}{2}}\alpha\rangle_0 - 0(|2^{\frac{m-1}{2}}\alpha\rangle - 2^{\frac{m-1}{2}}\alpha\rangle_0)|2^{\frac{m-1}{2}}\alpha\rangle_0 \sqrt{1 - (a(2^{\frac{m-1}{2}}\alpha\rangle - 2^{\frac{m-1}{2}}\alpha\rangle_0)^2}}, \\
|0\rangle_B &= |2^{\frac{m-2}{2}}\alpha\rangle_1|2^{\frac{m-3}{2}}\alpha\rangle_2|\cdots|\alpha\rangle_m, \\
|1\rangle_B &= \frac{|-2^{\frac{m-2}{2}}\alpha\rangle_1|\cdots|\alpha\rangle_m - m_{\langle0\rangle_m}^\langle2^{\frac{m-2}{2}}\alpha\rangle_{1,\cdots,|\alpha\rangle_m}|2^{\frac{m-2}{2}}\alpha\rangle_1|\cdots|\alpha\rangle_m \sqrt{1 - (1(|2^{\frac{m-2}{2}}\alpha\rangle_{m,\cdots,|\alpha\rangle_m} - 2^{\frac{m-2}{2}}\alpha\rangle_1)^2}}.
\end{align*}
\]

(19)

Table 1: This table represents a simple scheme for generating a maximum entangled state of \( m \) modes.

| \( m \) | \( 0 \) | \( 1 \) | \( \sqrt{2} \) | \( 2 \sqrt{2} \) | \( 2^{\frac{m-1}{2}} \) |
|--------|--------|--------|--------|--------|--------|
| 0      | 1      |        |        |        |        |
| 1      | 1      | 1      |        |        |        |
| 2      | \( \sqrt{2} \) | 1      | 1      |        |        |
| 3      | 2      | 2\( \sqrt{2} \) | 1      | 1      |        |
| 4      | 2\( \sqrt{2} \) | 2      | \( \sqrt{2} \) | 1      | 1      |
| \( m + 1 \) | \( 2^{\frac{m-1}{2}} \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) |

This table represents a simple scheme for generating a maximum entangled state of \( m \) modes.

of \( m \) modes. To construct this maximum entangled multipartite state, one can use the following table, as an example for \( m = 4 \), the state vector is given by,

\[
|2\alpha, \sqrt{2}\alpha, \alpha, \alpha\rangle_{3210} = \prod_{3=m}^{1}|2^{\frac{m-1}{2}}\alpha\rangle_m|\alpha\rangle_0
\]

(18)

By using these new basis, state vector \( |\psi^{\pm}\rangle \) takes the form,

\[
|\psi^{\pm}\rangle = x_{00}|0\rangle_A|0\rangle_B + x_{01}|0\rangle_A|1\rangle_B + x_{10}|1\rangle_A|0\rangle_B + x_{11}|1\rangle_A|1\rangle_B,
\]

(20)
where,

\[
\begin{align*}
    x_{00} &= A_m (1 - e^{-2m+1} |\alpha|^2), \\
    x_{01} &= -A_m e^{-2m} |\alpha|^2 \sqrt{1 - e^{-2m+1} |\alpha|^2}, \\
    x_{10} &= -A_m e^{-2m} |\alpha|^2 \sqrt{1 - e^{-2m+1} |\alpha|^2}, \\
    x_{11} &= -A_m \sqrt{1 - e^{-2m+1} |\alpha|^2} \sqrt{1 - e^{-2m+1} |\alpha|^2}.
\end{align*}
\]

For this density operator the concurrence, \( C = 1 \). It is clear that the degree of entanglement is independent of the parameters \( \alpha \) and \( m \).

Now, we use the generalized density operator \( \rho^- = |\psi^\rangle \langle \psi^-| \), which is defined by the generalized state vector (17) with \( m+1 \) modes to transfer \( m \)-modes entangled coherent state. The partners, Alice and Bob apply the protocol which is described in the previous section and end the protocol with a probability of successes,

\[
\mathcal{P}_{n-\text{odd}} = \frac{e^{-2m} |\alpha|^2 |2^m \alpha|^{2n}}{2n! \left(1 - e^{-2m+1} |\alpha|^2\right)} \tag{21}
\]

for odd \( n \). However if \( n \) is even, then the probability of successes is,

\[
\mathcal{P}_{n-\text{even}} = \frac{(1 - e^{-2m} |\alpha|^2)}{2(1 + e^{-2m+1} |\alpha|^2)} \tag{22}
\]

From Eqs.(21) and (22), the probability of successes depends on both of \( \alpha \) and \( m \) and \( \mathcal{P} \to 0.5 \) as \( \alpha \to \infty \) or \( m \to \infty \).

### 4 Teleportation in the presences of noise

The realistic investigation of quantum systems for quantum information processing must take into account the decoherence effect [21]. The dynamical properties of coherent states in the presences of noise have received considerable attention [19, 22]. There are several ways that the noise affects the quantum channels, which may be used for the purposes of quantum information tasks. Among these method is decoherence due to the energy loss or the photon absorption [14, 19, 23].

Assume that we have a source supplies the partners, Alice and Bob with maximum entangled coherent states. These entangled coherent states propagate from the source to the locations of the partners. Due to the interactions with the environment, the maximum entangled coherent states turn into partial entangled states, where its degree of entanglement depends on the strength of the noise. Consider that the source produces MECS defined by the density operator, \( \rho^- \) given by Eq.(5). The effect of the noise is,

\[
\rho_{PE} = U_{AE} \otimes U_{BE} \rho^- U_{BE}^\dagger \otimes U_{AE}^\dagger.
\]
where $U_{IE} |\alpha\rangle |0\rangle_E = |\sqrt{\eta\alpha}\rangle_I |\sqrt{1-\eta\alpha}\rangle_E$, $I = A, or B$ and $|0\rangle_E$ referees to the environment state. This effect is equivalent to employing a half mirror for the noise channel [14]. In an explicit form, one can write the output density operator $\rho_{PE}$ as

$$
\rho_{PE} = \frac{1}{N_\alpha} \left[ |2\sqrt{\eta\alpha}, \sqrt{2}\sqrt{\eta\alpha}, \sqrt{\eta\alpha}, \sqrt{\eta\alpha}\rangle \langle 2\sqrt{\eta\alpha}, \sqrt{\eta\alpha}, \sqrt{\eta\alpha}, \sqrt{\eta\alpha}| + |-2\sqrt{\eta\alpha}, -\sqrt{2}\sqrt{\eta\alpha}, -\sqrt{\eta\alpha}, -\sqrt{\eta\alpha}\rangle \langle -2\sqrt{\eta\alpha}, -\sqrt{2}\sqrt{\eta\alpha}, -\sqrt{\eta\alpha}, -\sqrt{\eta\alpha}| + e^{-8|\alpha|^2} |2\sqrt{\eta\alpha}, \sqrt{2}\sqrt{\eta\alpha}, \sqrt{\eta\alpha}, \sqrt{\eta\alpha}\rangle \langle -2\sqrt{\eta\alpha}, -\sqrt{2}\sqrt{\eta\alpha}, -\sqrt{\eta\alpha}, -\sqrt{\eta\alpha}| + e^{-8|\alpha|^2} |-2\sqrt{\eta\alpha}, -\sqrt{2}\sqrt{\eta\alpha}, -\sqrt{\eta\alpha}, -\sqrt{\eta\alpha}\rangle \langle 2\sqrt{\eta\alpha}, \sqrt{2}\sqrt{\eta\alpha}, \sqrt{\eta\alpha}, \sqrt{\eta\alpha}| \right].
$$

(24)

where $N_\alpha = 2(1 - e^{-16|\alpha|^2})$ is the normalized factor. To investigate how much the two states $\rho^-$ (the input state) and the output state $\rho_{PE}$ are related to each other, we evaluate the fidelity $F$

$$
F = \text{tr} \{ \rho^- \rho_{PE} \} = \frac{(1 - e^{-16|\alpha|^2})(1 + e^{-16(1-\eta)|\alpha|^2})}{2(1 - e^{-16|\alpha|^2})}
$$

(25)

In Fig.(1), we plot the fidelity $F$, as a function of the parameter $\alpha$ and the noise strength $\eta$. Fig.(1a), displays the dynamic of the fidelity, $F$ for small value of $\alpha \in [0, 1]$, and $0 \leq \eta \leq 1$. It is clear that for small values of $\eta \leq 0.5$ i.e, the correlation between the noise and the ECS is vary strong, the fidelity of the output state is very small. However $F$, increases as the noise strength $\eta$ increases, which is means that the correlation between the coherent state and the environmental noise is weak. Fig.(1b) shows the behavior of the fidelity for different range of $\alpha \in [1, 3]$. In this case, we have two different effective range of the noise strength $\eta$. The first, when $0 \leq \eta \leq 0.5$, the fidelity increases and reach its maximum value ($F = 0.5$). The second one, for $0.5 \leq \eta \leq 1$, the fidelity decrease and reaches its minimum value ($F = 0.5$). From these figures, one can say that, the effect of the noise for small values of $\alpha$ is very small, but for a larger values of $\alpha$, the noise effect is almost constant.

Let us assume that the quantum channel suffering from environmental noise. So, the partners, Alice and Bob are forced to use the noise channel [24] to implement the quantum

Figure 1: The fidelity of the input state as a function of $\alpha$ and $\eta$. 
teleportation. If the given unknown state is of \( m \) modes coherent state. The partners use \((m + 1)\) modes coherent state as a quantum channel. They apply the steps which are described in the previous section and the final state at Bob’s hand is given by

\[
|φ⟩_{m+1..2m} = (κ_1 |2^{m+1} α⟩......|α⟩_{m+1..2m} - (−1)^n κ_1 |−2^{m+1} α⟩......|−α⟩_{m+1..2m})/\sqrt{N_1},
\]

where, \( N_1 = |κ_1|^2 + |κ_2|^2 - (−1)^n e^{2m+1}|α|^2 Re(κ_2^∗ κ_1) \) is the normalized factor. The fidelity of this state is,

\[
F_m = \frac{(1 + e^{-2m|η|^2})(1 - e^{-2m|α|^2})}{2(1 - e^{-2m|α|^2})},
\]

where \( η' = 1 - η \), and \( m \geq 1 \).

Fig.(2), shows the behavior of the fidelity \( F(α, η) \) for different classes of the teleported state. This behavior is displayed for small range of \( α \) and \( 0 \leq η \leq 1 \). In Fig.(2a), we set \( m = 2 \), i.e., the teleported state is a bipartite entangled coherent state, while the used quantum channel is a tripartite ECS. This figure shows, that for small values of \( α < 0.5 \), i.e., the mean photon number is very small, the tripartite ECS, can not teleport the bipartite ESC, even for large value of \( η \). However as the mean photon number increases the fidelity
Figure 3: The fidelity of the teleported state for where \( m = 3 \), for Figs. (a&c) and \( m = 5 \) for Fig. (b&d).

\( \mathcal{F} \) increases and reaches its maximum value (> 0.75) at \( \eta = 1 \). In Fig. (2b), we investigated the propagation of \( \mathcal{F} \), for \( m = 3 \), i.e., Alice, is asked to teleport a tripartite ECS to Bob via a four ECS as a quantum channel. In this case the intervals in which the quantum channel fails to teleport the required state is smaller than that depicted in Fig. (2a). On the other hand, \( \mathcal{F} \), increases gradually as \( \eta \) increases and for \( \alpha > 0.75 \), the fidelity of the teleport state is almost unity at \( \eta = 1 \).

These phenomena are clearly shown in Figs. (1c& 2d), where we plot the fidelity \( \mathcal{F} \) as a contour. It is clear that for small value of \( \eta \), the fidelity is almost zero which appears as a dark region. However, as one increases the noise strength, the fidelity increases and reaches its maximum value at \( \eta = 1 \). This behavior is shown in the bright region. The brightness (high fidelity) and darkness (low fidelity) regions can be determined simply from the figures, where the brightness appears for \( \eta \geq 0.5 \) for \( m = 2 \) (Fig. (2c)), while for \( m = 3 \), the brightness appears at \( \eta \geq 0.29 \). Also, the degree of brightness indicates the degree of fidelity.
The fidelity of a different class of ECS is displayed in Fig.(3a), where we consider \( m = 4 \). In this case, we can notice three different behaviors of the fidelity, \( F \). The first is \( F = 0 \), for smaller values of \( \alpha \). The second, for \( 0 \leq \eta < 0.5 \), the fidelity increases to reach its maximum value (0.5), while for \( 0.5 \leq \eta \leq 1 \), \( F \) decrease gradually to reach its minimum value(0.5). The same behavior is depicted in Fig.(3b), where we consider \( m = 5 \).

Also, In Fig.(3c), the fidelity \( F(\alpha, \eta) \) is plotted as a contour graph. It is shown as \( m \) increases the ability of transforming information is much better, where the dark region appears only for small range of the noise strength \( \eta \). This appears clearly by comparing Fig.(1c), where \( m = 2 \) and Fig.(3d), where \( m = 5 \).

For larger interval of the mean photon number \( \alpha \), the noise effect is very small comparing with that for small values of \( \alpha \). Theses results are shown in Fig.(4a) and Fig.(4b), where different classes of the teleported states are considered (\( m = 3 \) and \( m = 5 \)) respectively. From theses figures it is clear that the maximum value of \( F = 0.55 \) for tripartite ECS. As one increases \( m \), the maximum value is almost 0.5. We can summarize the preceding result as: It is possible to teleport a multipartite ECS with high degree of fidelity and efficiency. The mean photon number \( \alpha \) plays the central role on controlling the efficiency of transporting multipartite ECS. For small values of the mean photon number the effect of the noise channel appears clearly and it is considered the controller parameter.

5 Conclusion

In conclusion, we have proposed a general quantum teleportation protocol to teleport multipartite of entangled coherent states. In this scheme, one can generate the multipartite quantum channels by using a series of beam splitters and phase shifters. Also, we describe the theoretical technique to generate a multipartite quantum channels. It is shown that the probability of successes is 0.5 does not depend on the channel parameters. One of the most
The advantage of this protocol is, it works with the same efficiency even the modes are even or odd numbers. So it can teleport all class of multipartite coherent states with high efficiency. The possibility, of applying this protocol in the presences of noise quantum channels is investigated, where we consider the noise due to the photons absorption losses. The noise strength plays the central role on the fidelity of the teleported state for small value of the channel parameter $\alpha$. However, for larger value of $\alpha$, the fidelity decreases very fast reaches it minimum value, 0.5.

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