Tools for solving the problem of optimal regulation of airline fleet schedules

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Abstract. The problem of optimal regulation of fleet assignments for airline flights is considered. The optimal scheduling consists in such scheduling or change of schedules that minimize the increase in the total length of the schedule or the losses of the system from violations of the set departure schedules. This is an NP-hard problem and has no effective algorithms for exact solution. A detailed review of approaches to solving its modifications and related fleet management tasks is given. Original formal setting and solutions are presented. The statistics of the testing of its software solution means modifications are presented, which proves the actual effectiveness of the developed tools.

1. Introduction

The problem of appointing aircraft lies in deciding which side should be assigned for each flight (the so-called segment or leg) in the schedule. The segments assigned to one aircraft are a set of consecutive flights with connections at the airports of departure and arrival, and are called the schedule of movement of aircraft. At the time of the decision on the designation of the aircraft, the type of aircraft for each flight has already been determined. The distribution of aircraft types by direction is a separate task that is solved at the stage of strategic planning of the schedule, based on the expected demand for the direction and restrictions on the fleet (Fleet Assignment Modeling). The main restrictions that must be taken into account when assigning an aircraft for flights are restrictions on the maintenance frequency and restrictions associated with the availability of permissions for flights of one or another type of aircraft at a given airport.

The problem of assigning aircraft for flights is only one of the large and complex planning tasks that commercial airlines solve on a regular basis. So, one of the medium-sized air carriers of the Russian Federation performs and plans about 360 flights per day in 80 cities of 40 countries, using about 100 planes of 11 different types.

The problems associated with planning and managing airline schedules can be divided into phases [1]:

- Schedule synthesis
- Assignment of aircraft types in directions
- Aircraft routing
- Crew planning
The process of assigning aircraft for flights is carried out, as a rule, at the stage of formation of a tactical flight plan (aircraft routing). However, when there is a deviation of the actual from the planned flight schedule, it is necessary to promptly decide on the restoration of the schedule. At this stage, software that optimizes the solution is indispensable.

Section 1 provides an overview of studies of problems associated with optimal schedule management.

Sections 2 and 3 present a meaningful and formal statement of the problem.

Section 4 shows the results of testing the algorithm for solving the problem.

2. Approaches to solving problems related to the reassignment of aircraft and mitigation of planned schedule violations

The study of the tasks of restoring flight schedules in cases of deviation from a set plan began in the 1980s. Teodorovic and Guberinic were among the first who in 1984 investigated the task of assigning aircraft for flights and restoring the schedule deviating from the planned schedule in order to minimize passenger delays. The solution was based on the application of the branch-and-bound method [2].

In 1990, Teodorovic & Stojkovic applied a greedy heuristic algorithm to formulate a new daily flight plan taking into account the problem of aircraft shortage by minimizing the number of canceled flights and delays [3]. They developed a heuristic algorithm that was tested on a small sample of 14 planes and 80 flights. After 5 years (in 1995, [4]), the authors expanded the problem to include crew restrictions. They developed a heuristic algorithm based on the FIFO principle in flight planning and dynamic programming.

In 1993, Jarrah et al. developed a decision support system for air traffic controllers at the United Airlines FOCC [5]. The basis were two network models containing arcs corresponding to flights, on-site maintenance of aircraft, as well as overnight stays at base airports. One model was used to determine the need to cancel a flight, and the other to determine the change in departure and arrival times. The decision was based on minimizing the costs associated with flight delays and cancellations. The ideas of Jarrah are continued in the works of Cao and Kanafani, who were engaged in the development of a real-time decision-making system [6]. The approach to the solution was to find a compromise between canceling and delaying the flight, taking into account the function of maximizing revenue minus the costs of delays and cancellations. When deciding, options were considered with replacing the type of flight aircraft or the need to transfer the aircraft to the destination from another airport.

We should note a Talluri’s research [7] made in 1996 (developing a heuristic decision-making algorithm for replacing the type of aircraft on a flight according to the criterion of minimizing costs), Yan and Yang (creating a flight schedule after a schedule violation using the simplex method and Lagrangian relaxation [8]), as well as Yan and Tu (studying violations of the planned schedule when performing multi-leg flights for a fleet with several types of aircraft [9]).

One of the interesting works from the point of view of the chosen objective function was presented in 1997 by Lou and Yu [10]. They dealt with the problem of regularity and formulated it as an integer programming problem according to the criterion of minimizing the percentage of flights with a delay of more than 15 minutes. The solution was based on the use of LP relaxation.

In 1997, Argüello et al. developed a heuristic algorithm for the greedy random search for new aircraft routes in the event of a delay in order to minimize the costs associated with reassignment and cancellation of flights [11]. A network containing two dimensions was laid in the basis—time intervals and airports (Time-Band Network). In 2000, the authors used the branch and bound method and LP relaxation to minimize costs associated with delays and cancellations [12]. In the same year, part of the research team (Bard, Yu), co-authored with Thengvall, presented yet another approach to solving the problem of restoring the schedule, in which delays and cancellations are used to solve the problem of shortage of aircraft so that a significant part of the initial routes of the aircraft remains intact. The model was unique in that part that allowed users to set their own goals. For the solution, the LP relaxation method was used; in the case when the optimal solution was not found, heuristics were used. Testing
showed that in most cases the solutions were close to optimal [13].

In 2003, Rosenberger et al. [14] proposed another heuristic algorithm for synthesizing aircraft motion graphs within each aircraft type separately. The algorithm was termed as ASR (Aircraft Selection Heuristic). The objective function was to minimize the costs associated with reassignments, delays and cancellations of flights.

In 2004, Andersson and Varbrand [15] presented a solution to the problem of managing flight delays in order to maximize revenue from ticket sales. For the solution, the column generation method was used. The testing used the data of a real Swedish airline and the results showed that the system can be used to make real-time decisions.

However, it should be noted that most of the early studies focused on individual tasks were usually aimed at minimizing costs in the event of a deviation from the planned schedule. Further research is aimed at solving complex problems, which are aimed not only at constructing an optimal schedule for the aircraft, but also at resolving problems with passenger connections, crew availability, etc.

In 2009, Eggenberg et al. [16] applied dynamic programming and the column generation method to solve the problem of synthesizing aircraft traffic schedules, taking into account maintenance plans and planned passenger connections according to the criterion of minimizing costs (operating costs for failure situations).

In 2010, Jafari et al. [17] presented the mixed integer programming problem, the goal of which was to simultaneously resolve the issues of restoring the schedule and passenger connections after a failed situation. It was possible to solve the problem only on a small test data set, which contained data on 13 aircraft of 2 types.

In 2012, Bisaillon et al. [18] developed a heuristic algorithm for solving the problem of managing schedules in a failure situation, combining the redistribution of both aircraft and passengers with one goal—to minimize operating costs and impact on passengers. The solution approach was improved in later works by Sinclair [19–20].

One of the latest is the work of Hu et al. As early as 2011, Hu et al. [21] formulated an integer programming problem aimed at minimizing delays and costs associated with canceling flights and servicing passengers in failure situations. The solution was carried out by LP relaxation with the subsequent application of a heuristic algorithm. This model was tested on data that included 16 aircraft and 70 flights. In 2019, an article was published with an approach to solving the complex task of restoring the schedule and passenger routes (provided that the passenger can choose to return the ticket or change the route). As the objective function, the minimization of the total costs associated with the reassignment of aircraft for flights and the costs associated with changing the routes for passengers was proposed [22].

Based on the analysis of existing approaches to solving the problem, we can conclude that existing approaches to solving do not guarantee an optimal solution, even with a separate consideration of the reassignment of aircraft. There is no algorithm for finding the optimal assignments for each aircraft with proven effectiveness, which indicates the relevance of this study.

3. Statement of the problem of optimal regulation of airline fleet schedules

The need for regulation of schedules arises due to the appearance of unforeseen flight delays. The applied task of optimal (operational) regulation of the current schedules of departure of aircraft is daily relevant for any airline and requires multiple solutions during the day in real time. In contrast to the general task of scheduling departures for a relatively long period, the task of optimally regulating the current schedule has, on the one hand, a smaller dimension, but on the other hand, much more stringent time frames are set to regulate the schedule. Its meaningful statement can be described as follows.

Consider the set of planned airline flights, which must be reassigned to a variety of aircraft of different types with known (different) durations of all operations (including airport services and flights) in such a way as to minimize the total deviation of all flights from the planned schedule (or minimize the maximum deviation for all flights from the original planned schedule).

At the time of schedule adjustment, the flight times and current delays in the start of flights are
known. The magnitude of such delays is different for different aircraft and different flights. Substantially, the tasks of scheduling departures for the season and the task of optimal operational regulation of the current schedule differ little from one another. Differences relate to more solutions, to ensure proper accuracy and speed. Therefore, the formal statement of the problem considered below, in essence, is universal both for the case of compiling a seasonal schedule and for the case of operational regulation of the current schedule.

There are several formal statements and approaches to solving this problem, including the formalization proposed by the authors and an algorithm for solving a close problem with recursions in the conditions [23].

This article describes an alternative new approach based on the formalization of a problem with disjunctions in constraints. The following also shows its significant advantages in speed and accuracy (proximity to the optima of the obtained schedules) in comparison with other existing approaches.

We accept the following notation

\[ s. \]

Let us use \( t_{i,j} \) denote the given maintenance time of flight \( i \) of aircraft \( j \):

\[ Tt_{i,j} = \begin{cases} 1, & j \in J \end{cases}, \]

\[ t_{i,j} \]

Besides, let \( b_j \) and \( b_j \) are respectively minimum and maximum numbers of flights assigned to aircraft \( j \). Hereinafter \( \| \) means the vector, matrix or tensor corresponding to the context of the dimensions.

Let us denote the delay of the \( i \)-th flight by aircraft \( j \) as \( \tau_{i,j}^0 \).

If we arrange the flights for every airline aircraft in ascending order \( \tau_{i,j}^0 \), then \( \tau^0 = \| 0, i \|, j = 1, J, i = 1, I, \) \((I\) is the total number of flights, \( J \) is the number of aircraft) can be interpreted as the schedule at the input of every aircraft.

Using \( x_{i,j} \) let us denote Boolean variables: destinations of flight \( i \) of aircraft \( j \) to be determined.

Let us introduce continuous variables \( C_{i,j} \geq \tau_{i,j}^0 \) as departure time of flight \( i \) when assigned to aircraft \( j \), and determine the number of dummy final flight \( \bar{\tau} \) for every aircraft \( j \), \( j = 1, J \), as well as Boolean variables \( w_{i,k,j} \) with the true condition of flight \( k \) following immediately after flight \( i \) for aircraft \( j \).

\[ \sum_{j=1}^{J} x_{i,j} = 1, \ i = 1, I \]  

(1)

\[ b_j \leq \sum_{i=1}^{I} x_{i,j} \leq b_j, \ j = 1, J \]  

(2)

\[ x_{i,j} = \begin{cases} 1, & \text{if flight } i \text{ was assigned to aircraft } j, \\ 0 & \text{in other cases,} \end{cases} \]  

(3)

\[ C_{i,j} = C_{k,j} + t_{i,j} x_{i,j} - M w_{i,k,j} \leq 0, \ C_{i,j} \geq \tau_{i,j}^0 \]  

(4)

\[ C_{k,j} = C_{i,j} + t_{k,j} y_{k,j} + M w_{i,k,j} \leq M, \ i \neq k, i, k = 1, I, j = 1, J, \]  

(5)

\[ w_{i,k,j} = \begin{cases} 1, & \text{if flight } i \text{ preceeds flight } k \text{ for aircraft } j, \\ 0 & \text{in other cases.} \end{cases} \]  

(6)

Conditions (1)–(3) are characteristic of the assignment problem. (1) ensure the assignment of any
flight to a single aircraft. (2) destination of at least $b_j$ and no more than $\bar{b}_j$ flights to any aircraft $j$.

Constraints (4)–(6) determine the choice of the shortest paths by defining $w_{i,k,j}$ (taking into account destinations $x_k, j$) on the complete links between flights for each aircraft.

Conditions (4) and (5) determine that flights $i$ and $k$ are operated at the same time by aircraft $j$. With their help, logical OR operations are realized when choosing options for sequences of flights. In (4) and (5), $M$ is a large positive number.

For the final flight of each aircraft we will have:

$$C^*_T, j + T_j x^*_T, j \leq C_{\text{max}} \cdot j = \overline{1, J},$$

where $T$ is the number of final flight for aircraft $j$,

$$C_{\text{max}} \rightarrow \min.$$  

Condition (8) defines a criterion for the effectiveness of the schedule, known as the criterion for the speed of a system of parallel unconnected devices with a common designation $C_{\text{max}}$.

The schedule synthesized by solving problem (1)–(8) is completely determined by the optimal assignments $x^*_{i,j} = I, J \text{ and the actual values of the departure time of each flight for such appointments, calculated as } C_{i,j} x^*_{i,j}$, where $C^*_{i,j}$ are the values of $C_{i,j}$ in the optimal solution.

Let us define relevant to practice class and dimensions of the task of operational regulation of the airline fleet departure schedules.

It is not difficult to determine the class of the problem, because even with a substantial simplification by eliminating conditions (4)–(6) and, accordingly, variables $w_{i,k,j}$, we obtain the problem of optimizing the schedules of unconnected parallel machines according to the criterion $C_{\text{max}}$. Such a simplified problem, however, is NP-hard, as shown, for example, in [2, 6, 7].

Problem (1)–(8) contains $I \cdot J$ Boolean variables $x_{i,j}$ and $I \cdot J$ continuous variables $C_{i,j}$. (4), (5) and (6) add $I \cdot J \cdot \binom{2}{j} = J \cdot I^2 \cdot (I - 1)/2$ variables $w_{i,k,j}$ and $2 \cdot I \cdot J \cdot \binom{2}{j} = J \cdot I^2 \cdot (I - 1)$ constraints. The above estimates of the number of variables and the constraints of the formal statement with disjunctions in the constraints show that even without taking into account the complexity of conditions (4)–(6), they greatly complicate the initially difficult problem (1)–(3), (7)–(8).

Regarding the actual dimensions of the implementations of the optimal operational regulation of the schedule, we give the following estimates. For the average airline size, the actual dimensions of the implementations of the problem in question lie in the intervals: $J \ [10 \div 30]$, $I \ [100 \div 300]$. Whence the estimate of the number of Boolean variables $J \cdot I^2 \cdot (I - 1)/2 + J \cdot I = 30 \cdot 300^2 \cdot 299/2 + 30 \cdot 300 = 403,659,000$, i.e. over 400 billion.

For this reason, the use of statement (1)–(8) for scheduling even after eliminating some of the variables using exact algorithms is very difficult, and for problems of real dimension it is impossible due to the virtually infinite calculation time, which is confirmed by computational experiments with IBM CPLEX.

4. Relaxation with a priori assignment of service sequences

For each aircraft, we arrange flights according to an increase in the set delays $\tau^0_{i,j}$. We consider any such sequence as the sequence of possible flights for each aircraft.

Instead of (4)–(6), we apply the following conditions:

$$C_{i,j} \geq \tau^0_{i,j} x_{i,j},$$  

(9)
where $C_{i,j}$ is the departure time of flight $i$ of aircraft $j$, $i = 1, I$, $j = 1, J$,

$$C_{i,j} + t_{i,j}x_{i,j} \leq C_{k,j}, \quad \text{(10)}$$

if flight $i$ immediately precedes $k$. We denote the formulated problem as (1)–(3), (7)–(10).

Let us prove the equivalence of the statements of problem (1)–(8) and (1)–(3), (7)–(10) provided that flights are arranged for all aircraft according to the increase in initial delays $\tau_{i,j}^0$.

Consider the case $\tau_{i,j}^0 = 0$, $i = 1, I$, $j = 1, J$. Under such conditions, the optimal solution $C_{\max} = \lambda$, $x_{i,j}^*$, $i = 1, I$, $j = 1, J$ does not depend on the sequence of flights, since restrictions (10) and (7) for $x_{i,j}^*$ are valid for any sequence, and (9) mean $C_{i,j} \geq 0$.

Indeed $C_{i,j} + t_{i,j}x_{i,j}^* = C_{k,j}$, if flight $i$ immediately precedes $k$. And then the completion time of the last flight of any aircraft $j = 1, J$ will be $C_j = \sum_{i=1}^I t_{i,j}x_{i,j}^* \leq C_{\max}$. This directly means that for $\tau_{i,j}^0 = 0$, $i = 1, I$, $j = 1, J$, the solutions to problems (1)–(3), (4)–(8) and (1)–(3), (7)–(10) coincide.

Consider a single-aircraft system and arrange the list of flights in ascending order of magnitude $\tau_{i,j}^0 > 0$, $i = 1, I$ (0 $\leq \tau_{i,j}^0 \leq \tau_{i,j}^0 \leq \cdots \leq \tau_{i,j}^0 \leq \cdots \leq \tau_{i,j}^0$). Consider successively the coordinates of the solution $C_i = \tau_{i,j}^0$, $C_2 = \tau_{i,j}^0 + t_i x_i$, $C_3 = \max \{\tau_{i,j}^0 C_2 + t_i x_i \}$, $C_{i+1} = \max \{\tau_{i,j}^0 C_i + t_i x_i \}$, $i = 1, I$. In general, if flight $i$ immediately precedes $k$, $\max \{\tau_{i,j}^0 C_i + t_i x_i \}$, obviously for the final flight $I$, $C_i = C_{\max}$.

Let us change the order of flights $i$ and $k$ to the opposite ($k$ immediately precedes $i$). Without loss of generality, we assume that the conditions of $\tau_{i,j}^0 \geq t_{i,j}^0$ are satisfied. Then $C_i = \tau_{i,j}^0 + t_i x_i = C_k$, $C_{i+1} = C_i + t_i x_i$, from where directly follows $C_{\max} = C_{\max} + t_i x_i$.

If $C_i > \tau_{i,j}^0 \geq t_{i,j}^0$, then the values of $\tau_{i,j}^0$ and $t_{i,j}^0$ can conditionally be considered equal to zero. As shown above, in this case, a change in the sequence of flights (10) does not lead to a change in the value of $C_{\max}$ ($C_{\max} = C_{\max}$).

Thus, any replacement of the sequence of aircraft flights with a sequence other than that specified as $0 \leq \tau_{i,j}^0 \leq \tau_{i,j}^0 \leq \cdots \leq \tau_{i,j}^0 \leq \cdots \leq \tau_{i,j}^0$ can only lead to an increase in the total service time $C_{\max}$.

Let us extend the result to the general case $\tau_{i,j}^0 \geq 0$, $i = 1, I$, $j = 1, J$. Let us arrange delays for each aircraft $0 \leq \cdots \leq \tau_{i,j}^0 \leq \tau_{i,j}^0 \leq \cdots \leq \tau_{i,j}^0 \leq \cdots \leq \tau_{i,j}^0$, $j = 1, J$. Then, for any aircraft, replacing the flight sequence with a different one, as it follows from item 2, can only lead to an increase in the total length of the schedule $C_{\max}$. This directly implies the equivalence of formulations (1)–(3), (4)–(8) and (1)–(3), (7)–(10).

In the conclusion of the section, we note a decrease by many orders of magnitude of the dimensions of the problem for $J \in [10–30]$, $I \in [100–300]$: $I \cdot J = 300 \cdot 30 = 9000$.

5. Means of implementation and decision algorithms

There are many algorithms for solving modifications of the considered problem using several formal statements. Various approaches are used to find solutions, from varieties of local search [24, 25, 26], approximation algorithms [27, 28], programming in constraints [29] to exact algorithms, including dynamic programming and branch and bound (B&B) methods [23, 30, 31]. However, despite belonging to the class NP, the use of exact numerical optimization methods shows good results. This applies in particular to the binary B&B algorithm developed by the authors [32, 33], and the set of “classic” milp
solutions implemented in the IBM CPLEX optimization studio and GURUBI optimization. The practical results of using binary B&B to solve the problem of optimizing the schedules of unconnected parallel devices with delays in starting service according to the criterion in the formulation with recursions for relatively small dimensions (from 5x100 to 30x100) are presented in [31]. In [31], a comparison of binary B&B applications with parametric algorithms based on dynamic programming with screening out of the locally worst schedules at each step of dynamic programming is also given, which revealed the full advantage of binary B&B on problems of such dimensions. In the same article, the results of testing the model in statement (1)–(3), (7)–(10) using the set of IBM CPLEX optimization studio components are presented below.

Model testing for compiling optimal airline fleet schedules was carried out on the generated source data as close as possible to the real dimensions. We used 2 groups of tests of 10 in each group. The first group contained data for 500 flights operated by 10 aircraft, the second one for 300 flights operated by 30 aircraft. The bounds of 10 and 30 aircraft correspond to the average and maximum number of aircraft of one type of a real airline. The upper bounds of 300 and 500 flights significantly exceed the existing requirements of operational regulation of a real airline. The tests took into account that aircraft can have the same capacity, but different flight times due to differences in speed and maintenance time on the ground.

The structure of the data and the results of the test count can be described by means of a small illustrative example of a test for scheduling 10 flights with 3 aircraft.

Table 1. Data on aircraft flight time $t_{i,j}$.

| $j \backslash i$ | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
|----------------|----|----|----|----|----|----|----|----|----|----|
| 1              | 2  | 4  | 1  | 9  | 7  | 9  | 6  | 7  | 3  | 3  |
| 2              | 1  | 4  | 3  | 5  | 3  | 8  | 3  | 4  | 5  | 3  |
| 3              | 6  | 1  | 3  | 6  | 3  | 4  | 6  | 9  |    |    |

Table 2. Data on light delays $\tau_{i,j}^0$ without ordering.

| $j \backslash i$ | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
|----------------|----|----|----|----|----|----|----|----|----|----|
| 1              | 6  | 10 | 14 | 2  | 6  | 13 | 11 | 5  | 12 | 3  |
| 2              | 9  | 10 | 14 | 7  | 3  | 3  | 6  | 14 | 7  | 0  |
| 3              | 12 | 11 | 0  | 10 | 13 | 14 | 12 | 11 | 15 |    |

Table 3. Order $L_j = \|l\|_j$ of flights in ascending order.

| $j \backslash i$ | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
|----------------|----|----|----|----|----|----|----|----|----|----|
| 1              | 4  | 10 | 8  | 1  | 5  | 2  | 7  | 9  | 6  | 3  |
| 2              | 10 | 5  | 6  | 7  | 4  | 9  | 1  | 2  | 3  | 8  |
| 3              | 3  | 4  | 2  | 9  | 1  | 7  | 8  | 5  | 6  | 10 |
Table 4. Optimal assignments of flights $x_{i,j}^*$.

| $j \backslash i$ | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
|----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1              | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 1   | 1   | 0   |
| 2              | 1   | 0   | 0   | 0   | 1   | 1   | 0   | 0   | 0   | 1   |
| 3              | 0   | 1   | 1   | 1   | 0   | 1   | 0   | 0   | 0   | 0   |

Table 5. Optimal departure time $C_{i,j}^*$

| $j \backslash i$ | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
|----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1              | 12  | 15  | 0   | 12  | 12  | 12  | 5   | 12  | 5   |     |
| 2              | 14  | 12  | 15  | 14  | 3   | 6   | 14  | 15  | 14  | 0   |
| 3              | 12  | 0   | 10  | 15  | 12  | 15  | 12  | 15  |     |     |

Table 6. Time of assigned flights $C_{i,j}^*, x_{i,j}^*$

| $j \backslash i$ | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
|----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1              | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 5   | 12  | 5   |
| 2              | 14  | 0   | 0   | 3   | 6   | 0   | 0   | 0   | 0   |     |
| 3              | 0   | 11  | 0   | 6   | 0   | 0   | 0   | 12  | 0   | 0   |

Test results of the schedule optimization software model

Table 7. Results of the test tasks calculations (500 flights 10 aircraft)

| Test no. | Test name | Time (h:mm:ss) | $C_{\text{max}}$ | Max. dev. (%) |
|----------|-----------|----------------|------------------|---------------|
| 1        | 500101    | 0:11:39        | 727              | 0.5           |
| 2        | 500102    | 0:25:10        | 717              | 0.5           |
| 3        | 500103    | 0:01:42        | 709              | 0.5           |
| 4        | 500104    | 0:00:20        | 733              | 0.5           |
| 5        | 500105    | 0:09:01        | 709              | 0.5           |
| 6        | 500106    | 0:01:00        | 700              | 0.5           |
| 7        | 500107    | 0:03:14        | 697              | 0.5           |
| 8        | 500108    | 0:02:30        | 723              | 0.5           |
| 9        | 500109    | 0:00:35        | 704              | 0.5           |
| 10       | 500110    | 0:02:36        | 710              | 0.5           |
Table 8. Results of the test tasks calculations (300 flights 30 aircraft).

| Test no. | Test name | Time (h:mm:ss) | $C_{\text{max}}$ | Max. dev. (%) |
|----------|-----------|----------------|------------------|--------------|
| 1        | 300301    | 0:00:07        | 225              | 0.5          |
| 2        | 300302    | 0:47:18        | 170              | 2.5          |
| 3        | 300303    | 0:30:49        | 163              | 2.2          |
| 4        | 300304    | 0:28:44        | 172              | 2.4          |
| 5        | 300305    | 0:16:09        | 171              | 2.5          |
| 6        | 300306    | 0:12:29        | 166              | 0.5          |
| 7        | 300307    | 0:00:01        | 172              | 0.5          |
| 8        | 300308    | 0:08:00        | 160              | 0.5          |
| 9        | 300309    | 0:56:38        | 167              | 2.7          |
| 10       | 300310    | 0:37:36        | 174              | 1.7          |

6. Conclusion
The main results were the development, software implementation and comparative analysis of the properties of modifications of algorithms for finding approximations to optimal solutions to the NP-hard problem of scheduling a system of unconnected parallel devices with delays in starting service according to the performance criterion.

The results of a study of software implementations of algorithms on tests close to real dimensions allow us to conclude that the developed tools are directly applicable to the needs of practice.

7. Acknowledgments
This work was supported by the Russian Foundation for Basic Research (RFBR), as part of the research project 19-37-9001219.

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