Enhancing the sound transmission loss in double-leaf partitions with lateral local resonators substructure

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Abstract
We report theoretically on the sound transmission loss performance through a periodic double plate acoustic metamaterial made of lateral local resonance (LLR) substructure. The unit cell of the substructure consists of a four link mechanism, two lateral resonators, and a vertical spring. The combination of space harmonic expansion and Bloch-Floquet theorem are used to analyze this present study. Computed results show that high sound transmission loss (STL) up to 60 dB at 62 Hz is reached with the metamaterial plate while the mass spring mass resonant is observed for the conventional periodic double plate at the same frequency. The introduction of a negative stiffness spring causes an increased STL at low frequency. The potential of lateral resonant metamaterials to improve the sound transmission loss (STL) in the frequency region around the mass-spring-mass resonance for periodic double panel partitions is demonstrated.

Keywords
Metamaterial, sound transmission loss, double plate, local lateral resonators

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Introduction
The low-cost and high sound attenuation performance requirements in many engineering application such as aerospace, automotive, and mechanical industries has motivated research groups to develop a number of novel vibration isolation concepts. Due to low frequency noise tremendous penetrating strength, it has long been thought to be an uncontrollable environmental concern. The mass law, which states that sound transmission $T$ through a wall is inversely related to the thickness of the wall $L$, the mass density $\rho$, and the sound frequency $f$, has been recognized as governing the sound insulation efficiency at low frequencies. Therefore, increasing the wall thickness by two only adds 6dB of sound transmission loss (STL).¹ Preference is given to double panel leaf structures in noise reduction engineering. This is because of its advantage over single leaf structures in acoustic insulation enhancement.

Investigations reported that an increase in air gap thickness between double wall foster better sound insulation efficiency in low frequency range. However, there is a constraint of space in most applications, hence alternative means to achieve high STL without

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increasing the air gap thickness are areas under investigations.\textsuperscript{2,5} Wang et al.\textsuperscript{6} used stud to study the sound transmission through an intermittent double panel. The motion of the panels was modeled in space-harmonic expansion as described by Mead and Pujara.\textsuperscript{7} It should be noted that several authors have reported the limitations of double-wall structures at low frequency in the region of mass air mass resonance, at which the model for infinite panels delineate an out-of-phase motion.\textsuperscript{2,8,9} Thus, numerous methods are being investigated to enhance double walls for better STL. One of such methods adopted is the filling of the cavity of the double wall with filler materials.\textsuperscript{10–12} Claey\textit{s} et al.\textsuperscript{13} suggested the provision of a local resonant structures within the core of periodic sandwich structure. Panel structures lined with poroelastic materials,\textsuperscript{14} corrugated core,\textsuperscript{15,16} and rib stiffened core\textsuperscript{17} are various ways by which double panels STL have been enhanced.

The micro-perforated plate (MPP) is another technological approach for noise reduction that has been offered.\textsuperscript{18–21} Classical Helmholtz resonator is used as the design basis for the MPP structure.\textsuperscript{18,19} The MPP is made up of two parts which includes; a micro-perforated thin panel and an appurtenant air gap backed to the panel. The cavity in the air acts as a resonant cavity, and the circular micro perforations functions as mass blocks during the operation of the MPP. As the sound wave propagates, the surface of the plate produces viscous force, therefore reduces the surface wave velocity which in turn dissipates the sound energy. Studies on different layers of MPPS; both single\textsuperscript{22} and multi\textsuperscript{23} have been conducted. Despite the tremendous success achieved in STL performances through multilayer panel systems, a significant increase in the total thickness and weight of the whole system has been a major setback.\textsuperscript{20,21}

Liu et al.\textsuperscript{22} fabricated a lesser sub-wavelength required lattice constant that exhibits spectral band gaps. It should be noted that the acoustic metamaterial (AM) or locally resonant sonic crystal (LRSC) foster the use of a reduced-size system to allow for openings of acoustic band gaps in the sonic frequency spectrum. The simple discovery that composites with locally resonant structural units can display effective negative mass density at certain frequency ranges led to the development of these groundbreaking materials. Further experiment is performed on the unit cells of Liu’s work by Sheng et al.\textsuperscript{23} Notably, localized sonic resonances with micro-structure size in the range of millimeters to centimeters were displayed between 350 and 2000 Hz by the fabricated composite. The computation of mass density is generally obtained from the average of the volume of components’ densities. However, it was found that the static and dynamic mass densities possess significant differences.\textsuperscript{24} Thus, the structural configurations present viable evolution of designs of panels based on acoustic metamaterial to produce immense sound transmission loss with no change in the plate thickness. Acoustic metamaterials are typically composed of an intermittent distribution of low-frequency resonators with a stiff core coated with an immensely soft material, usually a soft polymer, and arranged in a rigid hosting matrix.

Assouar et al.\textsuperscript{25} investigated the acoustic characteristics of a plate type acoustic metamaterials in an airborne sound environment. Two models were studied; a square plate with arrays of cylindrical spring mass resonators and continuum model of replacing the spring mass with arrays of pillar resonators. The work demonstrated that when the incident wave pressure interacts with the metamaterial plate with a defined frequency coinciding with the resonance of the pillars or the spring mass, it excites the resonant mode which affects the sound transmission in the plate. The STL result of a plane wave for the acoustic metamaterial plate gave STL of 75 dB and that of the bare plate produced STL of 25 dB at 2000 Hz. In general, the metamaterial performed better in sound insulation at frequency lower than 2700 Hz.

Huang and Sun\textsuperscript{26,27} suggested LLR substructure should be made of a four-link mechanism, two-lateral resonators and a vertical spring. Computations of effective mass density and Young’s modulus were made for the LLR substructure. It was discovered that an usual elastic wave motion was exhibited by the elastic wave propagation around the area of the double negative characteristics. The attenuation of longitudinal wave propagation by the periodic organization of the LLR substructure was demonstrated by Huang and Sun.\textsuperscript{26,27} Recently, He et al.\textsuperscript{28} used a similar model in an operational feedback control system. According to the dynamic equivalent method, the metamaterial is equivalent to a single layer plate, when the dynamic effective parameter is used. Some parameters were explored in terms of sound transmission.

Oyelade\textsuperscript{29} recently looked into the impact of a negative stiffness module on a stiff double panel structure. The negative element component of the other set was relocated by an amount $q$. The negative element, engineering safety, offset, and elevation angle all had an impact on the results. Magnetic spring was generated through permanent magnets to improve the STL of double plate.\textsuperscript{30} The magnetic spring was modeled as a punctual translational spring at the center of the plates. The vibroacoustic equation coupled the effect of the spring and the resulting STL supersedes that of the double-plate without the effect of this negative magnetic stiffness. The use of foam core has been used to improve the STL of double panel partition.\textsuperscript{31} The foam core was modeled as a resonating mass. The resonant mass created an anti resonant at the mass-air-mass
resonant at 798–869 Hz. The potential of resonant metamaterials to improve the STL in the frequency region around the mass-spring-mass resonance for double panel partitions with a locally reacting core material was demonstrated.

However, to the best of the authors’ knowledge, the problem of sound transmission loss via double-wall panels has yet to be addressed theoretically, when considering a double panel with embedded lateral local resonators. In the spirit of previous works by Wang et al.,6 and Huang and Sun,26,27 the development of a theoretical model for STL across partitions stiffened with periodically placed studs with lateral local resonators is investigated in this study. Through parametric studies, the effect of resonating mass, elevation angles, and negative stiffness on STL are performed.

For improved sound transmission loss, we present an acoustic metamaterial double plate with LLR substructures in this study. The dynamic properties of an LLR substructure are described in Section 2 to study the mechanisms of wave transformation. The combination of the double plate and LLR are theoretically modeled in Section 3. The STL calculation is shown in Section 4. The results and discussions are given in Section 5. Conclusions on the exhibited work are eventually drawn in Section 6.

Configuration of the lateral local resonance substructure

The LLR substructure investigated here is of the Huang and Sun.26 The LLR substructure is made up of four-link mechanism (massless truss components), two lateral resonators ($m_1$), a vertical spring ($K_2$), and a horizontal spring ($K_1$), as shown in Figure 1. Only vertical vibration direction is permitted for the lateral resonators. As indicated in Figure 1, $H$ and $D$ are geometrical parameters. The gap $H$ also indicate the air gap separating the two panels. The governing equation for this lumped system is as 26,27,32:

$$F^L = K_1(w_2 - w_1) + \frac{H}{D}K_2(v_2 - v_1) \quad (1)$$

$$m_1\frac{\partial^2 v_2}{\partial t^2} = K_2(v_1 - v_2) \quad (2)$$

$$v_2 = \frac{v_1^L}{(1 - \frac{w_2^2}{w_0^2})} \quad (3)$$

where $w_1, w_2, v_1$, and $v_2$ stand for the displacement on plate 1, displacement on plate 2, displacement between the rigid and stiffness and displacement of $m_1$, respectively. $K_1, K_2$, and $D$ represent the linear spring connecting the plates, linear spring connecting $m_1$ to massless member, and the vertical distance of the truss member, respectively. Displacement of the lateral mass is represented by $v_2$, and the displacement of the vertical point of the truss by $v_1$.

where $\omega_0 = \sqrt{K_2/m_1}$. Based on the assumption of small displacements,33 we have

$$v_1^L = -\frac{H}{2D}(w_2 - w_1); \quad v_1^R = \frac{H}{2D}(w_1 - w_2) \quad (4)$$

Then equation (1) becomes

$$F^L = \left[ K_1 + \frac{H^2}{2D^2}K_2 \frac{\omega^2}{\omega^2 - \omega_0^2} \right] (w_2 - w_1) \quad (5)$$

In this same vein, the force to the right of the LLR becomes

$$F^R = \left[ K_1 + \frac{H^2}{2D^2}K_2 \frac{\omega^2}{\omega^2 - \omega_0^2} \right] (w_1 - w_2) \quad (6)$$

The force due to the effect of stiff spring to the left of LLR (connection between the massless spring and $K_1$) is given as:

$$F_1 = K_0(w_2 - w_1) \quad (7)$$

and to the right of LLR (connection between the massless spring and $K_1$) as:

$$F_2 = K_0(w_1 - w_2) \quad (8)$$

Theoretical models of acoustic metamaterials double plates

Fluid–structure coupling responses

Considering a double wall partition connected with elastic spring at periodic spacing $L$ as shown in
Figure 2. Double-panel with spring: (a) schematic representation model by Wang et al.\textsuperscript{6} and (b) Wang et al. model with LLR substructure.

Figure 2(a). The plates are assumed to be homogeneous and isotropic panels. The elastic spring provides a structural path for sound transmission, and the air space between the two plates can transfer sound waves between the plates. This work has been thoroughly investigated by Wang et al.\textsuperscript{6} The new proposed metamaterial double plate is illustrated in Figure 2(b). The model consists of double plate structure (Plate 1 and Plate 2). Spring $K_0$ connects Plate 1 and Plate 2 at $q$, spring constant $K_1$ connects Plate 1 and Plate 2 at periodic interval of $L$, and length $H$ (same as the air cavity of the plate) connected to four rigid and massless truss members. For the two panels, the equations of motion under sound excitation can be described as\textsuperscript{6,29}

\[
D_1 \frac{\partial^4 w_1}{\partial x^4} + m_{p1} \frac{\partial^2 w_1}{\partial t^2} - j\omega \phi_0 (\Phi_1 - \Phi_2) \\
- \sum_{n = -\infty}^{\infty} F_1 \delta(x - nL) - \sum_{n = -\infty}^{\infty} F^L \delta(x - (nL + q)) = 0
\]

\[
D_2 \frac{\partial^4 w_2}{\partial x^4} + m_{p2} \frac{\partial^2 w_2}{\partial t^2} - j\omega \phi_0 (\Phi_2 - \Phi_3) \\
- \sum_{n = -\infty}^{\infty} F_2 \delta(x - nL) - \sum_{n = -\infty}^{\infty} F^R \delta(x - (nL + q)) = 0
\]

where $\rho_0$ is the air density, $\Phi_i$ is the acoustic field velocity potentials, $m_{pi}$ ($i = 1, 2$) is the panel mass per unit area, and $D_i$ is the flexural stiffness of the panel;

\[
D_i = \frac{E_i h_i^3}{12(1 - \nu_i^2)} (1 + j\eta_i)
\]

Here $\eta_i$ is the loss factor of the panel material, $h_i$ is the thickness of the panel, and $E_i$, and $\nu_i$ are the Young’s Modulus and Poisson’s ratio of the panel material, respectively. The transverse displacement of the two panels takes the following forms

\[
w_i(x, t) = \sum_{n = -\infty}^{\infty} \alpha_{i, n} e^{-\beta_{i, n} t} e^{j\omega t} \]

$\alpha_{i, n}$ is the amplitude of the $n$th “space harmonic,” and $k_x$ is the wavenumber in the $x$ direction defined as

\[
k_x = k \sin \theta
\]

where $k = \omega / c_0$ is the acoustic wave number, $c_0$ is the speed of sound, and $\theta$ is the incidence angle. The velocity potential at the incident and reflected point is given by

\[
\Phi_1(x, y, t) = I e^{-j(k_x + \frac{2\pi}{L}y)} e^{j\omega t} \\
+ \sum_{n = -\infty}^{\infty} \beta_{i, n} e^{-\beta_{i, n} t} e^{j\omega t}
\]

where $I$ is the sound amplitude. The velocity potential in the cavity can be written as

\[
\Phi_2(x, y, t) = \sum_{n = -\infty}^{\infty} \epsilon_{n} e^{j(k_x + \frac{2\pi}{L}y)} e^{j\omega t} \\
+ \sum_{n = -\infty}^{\infty} \zeta_{n} e^{-j(k_x + \frac{2\pi}{L}y)} e^{j\omega t}
\]

In the transmitted area there is no reflected wave, hence the velocity potential at this point is given as:

\[
\Phi_3(x, y, t) = \sum_{n = -\infty}^{\infty} \xi_{n} e^{-j(k_x + \frac{2\pi}{L}y)} e^{j\omega t}
\]

$k_{yn}$ is the wavenumber in the $y$ direction given as:

\[
k_{yn} = \begin{cases} 
\sqrt{k_0^2 - (k_x + \frac{2\pi}{L})^2}, & k_0 \geq (k_x + \frac{2\pi}{L}) \\
-\sqrt{(k_x + \frac{2\pi}{L})^2 - k_0^2}, & k_0 < (k_x + \frac{2\pi}{L}), 
\end{cases}
\]

and $\beta_{i, n}$, $\epsilon_{n}$, $\zeta_{n}$, and $\xi_{n}$ are the modal amplitudes. At the air–panel interface the normal velocity is continuous, therefore the coupling of the equations take this form:

\[
-\frac{\partial \Phi_1}{\partial y} = j\omega w_1, \quad -\frac{\partial \Phi_2}{\partial y} = j\omega w_1 \quad \text{at} \quad y = 0,
\]

\[
-\frac{\partial \Phi_2}{\partial y} = j\omega w_2, \quad -\frac{\partial \Phi_3}{\partial y} = j\omega w_2 \quad \text{at} \quad y = H
\]

The relationships between the modal amplitudes are derived by substituting equations (13)–(15) into the continuity conditions equation (17), as\textsuperscript{6}:
\[ \beta_0 = 1 - \omega \frac{\alpha_{1,0}}{k_y} \quad n = 0 \]
\[ \beta_n = -\omega \frac{\alpha_{1,n}}{k_{yn}} \quad n \neq 0 \]
\[ \epsilon_n = \frac{\omega(\alpha_{2,n}e^{ik_yH} - \alpha_{1,n}e^{2ik_yH})}{k_{yn}(1 - e^{2ik_yH})} \]
\[ \xi_n = \frac{\omega(\alpha_{2,n}e^{ik_yH} - \alpha_{1,n})}{k_{yn}(1 - e^{2ik_yH})} \]
\[ \xi_n = \frac{\omega(\alpha_{2,n}e^{ik_yH} - \alpha_{1,n})}{k_{yn}} \]

(18)

The coefficients \( a_{i,n} \) can be found by solving the system equations derived using the principle of virtual work for one bay of the partition.\(^6\)

**Virtual work for the elements**

By applying the principle of virtual work thus\(^6,29\),

\[ \delta W_1 = \delta \alpha_{i,m} e^{-jk_y(k_x + \frac{2\pi}{L})} \Delta x \]

(19)

The total virtual work is the sum of the virtual work of the two panel elements, the virtual work by the translational and LLR components.

**Panel elements.** The equations governing the motion of the two plates are

\[ \int_0^L \left[ D_1 \frac{\partial^4 w_1}{\partial x^4} + m_{p1} \frac{\partial^2 w_1}{\partial t^2} - j\omega \rho_0 (\Phi_1 - \Phi_2) \right] \delta w_1 \, dx \]

\[ \int_0^L \left[ D_2 \frac{\partial^4 w_2}{\partial x^4} + m_{p2} \frac{\partial^2 w_2}{\partial t^2} - j\omega \rho_0 (\Phi_1 - \Phi_2) \right] \delta w_2 \, dx \]

(20)

(21)

where \( \delta \) represents the complex conjugate of the virtual displacement in equations (20) and (21), \( w_1 \) and \( w_2 \) are the transverse displacements. \( m_{p1} \) and \( m_{p2} \) are the mass per unit area, and \( D_1 \) and \( D_2 \) are the flexural rigidity of plate 1 and plate 2 respectively.

**Translational positive springs**

\[ \delta \Pi_{K_1} = K_0 (w_1(nL) - w_2(nL)) \delta \alpha_{i,m} \]

(22)

at panel one

\[ \delta \Pi_{K_0} = K_0 (w_1(nL) - w_2(nL)) \delta \alpha_{i,m} \]

(23)

**LLR.**

\[ \delta \Pi_{\psi_1} = \left[K_1 + \frac{H^2}{2D^2} K_2 \frac{\omega^2}{\omega^2 - \omega_0^2}\right] \times (w_1(nL + q) - w_2(nL + q)) \delta \alpha_{i,m} \]

(24)

\[ \delta \Pi_{\psi_1} = \left[K_1 + \frac{H^2}{2D^2} K_2 \frac{\omega^2}{\omega^2 - \omega_0^2}\right] \times \left(\sum_{n = -\infty}^{\infty} \alpha_{1,n} e^{-j(k_y + \frac{2\pi}{L})} \right) \delta \alpha_{i,m} \]

(25)

**Combined equation.** Finally, the total virtual work of the plate can be defined as;

\[ \delta \Pi = \left[D_1(k_x + \frac{2m \pi}{L})^4 - \omega^2 m_{p1}\right] - \frac{2j\omega \rho_0 e^{i\frac{2\pi}{L}}}{k_{ym}(1 - e^{2ik_yH})} \alpha_{1,m} \]

(26)
\( \delta I_2 = \left[ D_2 \left( k_x + \frac{2m\pi}{L} \right)^4 - \omega^2 m_p \right] + 2j\omega \rho_0 \left( 1 - 2j\omega/c_0 \right) \alpha_{2,m} + \frac{2j\omega^2 \rho_0 (1 - 2j\omega/c_0)}{k_{ym}(1 - 2j\omega/c_0)} \alpha_{1,m} + \frac{K_0}{L} \left( \sum_{n=-\infty}^{\infty} \alpha_{2,n} \right) + \frac{K_1}{L} \left( \sum_{n=-\infty}^{\infty} \alpha_{2,n} e^{-j\omega(k_x + \frac{2m\pi}{L})} \right) \]

\( + \frac{K_1 + G}{L} \left( \sum_{n=-\infty}^{\infty} \alpha_{1,n} e^{-j\omega(k_x + \frac{2m\pi}{L})} \right) \]

\( - \frac{K_0}{L} \left( \sum_{n=-\infty}^{\infty} \alpha_{1,n} \right) \]

\( - \frac{K_1}{L} \left( \sum_{n=-\infty}^{\infty} \alpha_{1,n} e^{-j\omega(k_x + \frac{2m\pi}{L})} \right) = 0 \) \tag{27}

where

\( G = \frac{H^2}{2D^2} \frac{\omega^2}{K_2} \frac{\omega^2 - \omega_0^2}{\omega^2} \) \tag{28}

Equations (26) and (27) give the coupled set of linear equations which determine the coefficients \( a_{i,m} \) of the vibroacoustic problem. Solving these coefficients from equations (26) and (27), the remaining unknown coefficients \( \beta_n, \epsilon_n, \xi_n, \) and \( \xi_n \) can be found by using equation (18). The power transmission coefficient is then defined as

\( \tau(\theta) = \frac{1}{|I|} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \xi_n^2 \text{Re}(k_{sn}) \) \tag{29}

The sound transmission loss, therefore, is calculated in decibel scale via

\( \text{STL}(\theta, \phi) = -10 \log[\tau(\theta, \phi)] \) \tag{30}

**Results and discussion**

Figure 3 shows the comparison of the STL–frequency spectrum of the present study with the theoretical STL spectra obtained by Wang et al.\(^6\) In the case of normal incidence, the agreement between the current model and Wang et al.\(^6\) theory is satisfactory for all configurations, especially at low frequencies below 1000 Hz. To investigate how lateral local resonators and other parameters of the wall affect the propagations of sound waves in the structure, numerical simulation is conducted in this section. For the simulations, the material parameters and corresponding lateral resonators dimensions are listed in Table 1.

**Effect of \( K_0 \) on STL**

The stiffener modeled as a punctual translational spring \( K_0 \) is assumed to connect the double plate at period \( L \). The variation of the translational spring as a function of frequency and STL is shown in Figure 4. The 3D plot for the translational stiffness versus frequency for the STL is given in Figure 4(a) and the three dimensional surface plot in Figure 4(b). It can be seen that, apart from the region where we have anti resonant frequency jump, other spring constants have mass in mass resonant dip. These dip tends to reduce as the value of translational springs increase in the system. Though for all the springs constants there is similar response at high frequency (150–200 Hz).

| Parameter                          | Value          |
|-----------------------------------|----------------|
| Stud spacing                      | L = 0.6 m      |
| Thickness                         | \( h_1 = h_2 = 12.5 \times 10^{-3} \) m |
| Material loss factors             | \( \eta_1 = \eta_2 = 0.1 \) |
| Poisson’s ratio                    | \( \nu_1 = \nu_2 = 0.3 \) |
| Young’s modulus                   | \( E_1 = E_2 = 7 \) GPa |
| Density                           | \( p = 1200 \) kg/m\(^3\) |
| Sound speed                       | \( c_0 = 342m/s \) |
| Offset from periodic length       | \( q = L/2 \) |
| \( K_0 \)                          | \( 6 \times 10^4 \) N/m |
| \( K_1 \)                          | \( 1000 \) N/m |
| \( K_2 \)                          | \( 1000 \) N/m |
| \( m_2 \)                          | 0.005 kg       |
| Lateral dimension of spring       | 0.005 kg       |

\( 2 \times 1000 = 0.6 \times 1000 = 0.2x10^3 = 1.2x10^3 \) m

\( \text{Wang et al. (2005)} \)
Effect of resonating mass on STL

Figure 5(a) presents the STL results for the double plate with three different lateral masses. It can be observed that the LLR has effect on the STL spectrum of the double panel. For illustration, when the LLR mass is $0.04 \text{g}$ there is a small increase in STL around the coincidence frequency, though the mass generated two resonances within this region. In case of mass $0.05 \text{g}$, there is little dip at the coincidence frequency and enhanced STL immediately after the coincidence frequency compared to the double panel without the lateral resonance mass. The coincidence frequency is one of the main limitations of double panel in sound insulation, but could be solved by using metamaterial plate. Figure 5(b) shows the result for STL spectrum for lateral resonating masses $m_1 = 0 \text{ g}$ and $m_1 = 0.065 \text{ g}$ on the double wall. The coincidence frequency matches exactly with the resonance frequency of the lateral spring-mass system at 62 Hz. The usual coincidence resonance dip seen in Figure 5(a) is completely abolished as a result of the lateral mass resonators’ impact, and the sound loss at this specific frequency is enhanced to nearly 60 dB. The lateral resonator provides means by which the double plate could be design so that their resonant frequency can match the coincidence frequency to overcome the plate systems limitations. This will pave ways for new innovations in the acoustic fields.

Variation of offset $q$ on STL

Figure 6 shows the sound transmission losses as one set of LLR is moved down the plate between a fixed set of $K_0$ positioned with periodic spacing $L$. It is shown that the variation of the offset can have a significant influence on the STL of the double plate. Having the offset of the LLR at $q = L/6$ produces 60 dB while when $q = L/2$, there is 83% reduction in STL. Therefore, the STL spectrum is influenced by the position of the LLR substructure in a double plate for a particular incidence angle.

Effect of elevation angle on STL

The influence of sound incident angles on the STL with LLR substructures is shown in Figure 7. The results demonstrate considerable influence of the incident angle on the structural STL of the periodic double plate system. As can be seen the incident sound waves with smaller elevation angles gives a larger STL values at low frequency than those with larger elevation angles.
For elevation angles between 0% and 65%, there is a spike in STL values around 55 Hz (Figure 7(a)). To clearly identify the resonance dips induced by the elevation angles, the elevation angle versus frequency curves of the metamaterial double plates are plotted in contour form in Figure 7(b). This increase in STL is absent in higher elevations angles. It has been previously reported that sound waves with large elevations angles are easier to transmit through double panel structure than lower elevation angles.34

**Effect of negative $K_1$ on STL**

The effect of having $K_1$ as a negative spring has important influences on sound transmission properties. Figure 8 illustrates the STL with three different negative spring values as $K_1 = -10^{3} \text{N/m}$, $K_1 = -10^{7} \text{N/m}$, and $K_1 = -10^{10} \text{N/m}$. We set the lateral mass, $m_1 = 0.075 \text{g}$ and $D = 0.0002 \text{m}$. In Figure 8, it can be seen that the first resonance of the double plate with negative stiffness $K_1 = -10^{3} \text{N/m}$ and $K_1 = -10^{7} \text{N/m}$ is the same. However, when the anti resonance $K_1 = -10^{10} \text{N/m}$ at 59 Hz, there is a small dip and an increase in STL for $K_1 = -10^{10} \text{N/m}$. Therefore, there is an advantage of an increased STL around this region (31 and 105 Hz) by the use of negative stiffness. Comparing a further reduction of the stiffness of the springs to $K_1 = -10^{10} \text{N/m}$, there is a shift of the first resonance to a higher frequency of 160 Hz. In general, for relatively high frequencies exceeding the mass-spring resonance frequency, improved sound insulation is demonstrated for double plates with high stiffness value.

**Effect of varying $D$ and holding $H$ constant on STL**

To demonstrate how the sound transmission performance of a double plate varies with the LLR substructures geometric parameters, the dimension of $D$ versus frequency curves for the periodic structure are plotted in Figure 9. Figure 9 shows the effects of varying the dimension of $D$ and holding $H$ constant. $H$ is held constant in order not to change the thickness of the air in the cavity. It is better to have the dimension of the D to be small in order to have high STL in the low frequency region. It is seen that the STL values increased in the
region of 50–150 Hz before the mass-spring resonance at 155 Hz. Dimension D greater than 0.1 mm makes the mass in mass resonance to occur at 50 Hz frequency.

**Conclusion**

This paper presents a new concept of the LLR substructures, which enables us to enhance the sound transmission loss in a double plate. The main findings are as follows:

(i) The LLR substructures help improve the coupling between the two panels. As a result, the LLR substructures improved the STL value of the double wall constructions at low frequencies, particularly at the mass-air-mass resonance, where the infinite panel model displays out of phase motion of the two panels.

(ii) Incorporating negative stiffness into the periodic double plate can also help overcome the coincidence frequency limit, which results in high sound transmission through double plates for sound shielding.

(iii) Larger elevation angles of incident sound waves are simpler to transfer through the double panel construction than smaller elevation angles.

(iv) Maximum STL can be obtained when the LLR substructure is positioned at offset $q = 1/6$ of the periodic length of the plate.

In summary, the LLR substructures can enhance the sound transmission loss and avoid the dip that is common in the first natural frequency of double plate in periodic structures.

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