ORDERING POLICY FOR NON-INSTANTANEously DETERIORATING PRODUCTS UNDER PRICE ADJUSTMENT AND TRADE CREDITS

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Abstract. Non-instantaneously deteriorating products retain their quality for a certain period before beginning to deteriorate. Retailers commonly adjust their retail prices when products shift from a non-deteriorating state to a deteriorating state in order to stimulate demand. It is essential to consider this price adjustment for inventory models of non-instantaneously deteriorating products under trade credit, due to the fact that the calculation of earned interest is based on the retail price. This paper considers the problem of ordering non-instantaneously deteriorating products under price adjustment and trade credit. Our objective was to determine the optimal replenishment cycle time while minimizing total costs. The problem is formulated as three piecewise nonlinear functions, which are solved through optimization. Numerical simulation is used to illustrate the solution procedures and discuss how system parameters influence inventory decisions and total cost. We also show that a policy of price adjustment is superior to that of fixed pricing with regard to profit maximization.

1. Introduction. Trade credit is a popular form of payment in many businesses and an essential tool for financing growth. In practice, vendors commonly provide their buyers forward financing, which means that the vendor allows buyers a fixed period (a credit period) in which to pay the amount owed without interest. When the goods are delivered, trade credit is given for a specific number of days (usually 30, 60, or 90). In the jewelry businesses, credit is sometimes extended to 180 days or even longer. In other words, trade credit is the credit extended to retailers by suppliers, thereby allowing retailers to buy now and pay later.

A number of studies dealing with ordering problems under trade credit have been published over the years. Recently, Teng et al. [18] determined the optimal order quantity for a linear non-decreasing demand function under trade credit. Cheng et al. [2] considered the optimal ordering decisions of retailers under trade credit in various financial environments. Teng and Lou [17] determined the optimal credit...
period awarded by the seller and replenishment time in a supply chain with up-stream and down-stream trade credits. Yang et al. [21] developed optimal order and credit policies for retailers in the case where a supplier offers either a cash discount or a delay payment linked to order quantity. Ouyang and Chang [11] explored the effects of reworking items of compromised quality and trade credit when applied to a model of economic production quantity with imperfect production processes and complete backlogging. Teng et al. [19] extended constant demand to a linear increasing demand function of time and incorporated into the model a permissible delay in payment under two levels of trade credit. Ouyang et al. [12] investigated optimal replenishment decisions under trade credit policy with two levels depending on the size of the order. Lin et al. [8] and Liao et al. [7] considered the provision of trade credit from suppliers to retailers as well as from retailers to end-customers. Tsao [14] developed a piecewise nonlinear model for a production system under maintenance, trade credit, and limited warehouse space. Clearly, the issue of trade credit is attracting considerable attention among researchers in this field.

Inventory models used to deal with deteriorating items have also received considerable attention. Teng and Chang [16] established an economic production quantity model for deteriorating items under trade credit when the demand rate depends on on-display stock levels and the selling price. Dye et al. [4] used the discounted cash flow approach to model the deteriorating inventory model. Ouyang et al. [10] considered deteriorating items with partially permissible delays in payments linked to order quantity. Chang et al. [1] investigated the replenishment decisions of manufacturers in cases involving deteriorating items in a supply chain with up-stream as well as down-stream trade credits. Tsao [13] considered two-phase pricing and inventory management for deteriorating and fashion goods under trade credit. Wang et al. [20] sought to determine the seller’s optimal credit period and cycle time in a supply chain of deteriorating items with a maximum lifetime.

Ouyang et al. [9] developed the first inventory model for non-instantaneous deteriorating items under trade credit. Non-instantaneous deteriorating items are those that maintain quality or freshness for a limited duration. This model is usefully applied to items, such as fresh fruits, which do not deteriorate immediately. Chung [3], Dye and Hsieh [3], Dye and Hsieh [6] have consequently considered a variety of inventory problems involving non-instantaneous deteriorating items. Tsao [14] recently determined the joint location, inventory, and preservation decisions involving non-instantaneously deteriorating items under a delay in payment.

This paper also considers non-instantaneous deteriorating items under trade credit, specifically price adjustment from non-deteriorating to deteriorating periods. Many retailers reduce the price of products of decreased quality and those nearing their expiration date with the aim of stimulating demand. For trade credit inventory models involving non-instantaneously deteriorating items, consideration of price adjustment is essential because the calculations of earned interest are based on the retail price. This paper is the first to consider the ordering policy for non-instantaneously deteriorating products under price adjustment and trade credit. The objective was to determine the replenishment cycle time while minimizing total costs. We present three piecewise nonlinear optimization algorithms based on lemmas to solve this problem. Using numerical simulation, we illustrate the solution procedures and discuss how system parameters affect inventory decisions and total costs.
2. Model formulation. This study uses the following notations.

- $T$: replenishment cycle time (decision variable)
- $c$: unit purchasing cost
- $t_d$: the length of time in which the product has no deterioration (non-deteriorating period)
- $p_1$: retail price in the non-deteriorating period (before $t_d$)
- $p_2$: retail price in the deteriorating period (after $t_d$)
- $D_1$: annual demand rate in the non-deteriorating period
- $D_2 e^{-\lambda t}$: annual demand rate in the deteriorating period, where $\lambda > 0$. It decreases exponentially with time $t$
- $R$: ordering cost for distribution center (DC)
- $h$: inventory holding cost for DC
- $\theta$: deteriorating rate of the item
- $Q$: ordering quantity
- $t_C$: the length of credit period
- $I_e$: the interest earned per dollar
- $I_p$: the interest paid per dollar
- $I(t)$: the inventory level with respect to time $t$
- $TC$: total cost

The mathematical model in this study is based on the following assumptions:

1. Depletion of inventory can occur as a result of demand as well as through deterioration during the replenishment cycle after $t_d$.
2. Replenishments occur instantaneously and shortages are not allowed.
3. The retail price and demand rate vary according to whether they are in a non-deteriorating or deteriorating state.
4. The demand rate during the non-deteriorating stage is constant; i.e., $D_1$. The demand rate during the deteriorating stage decreases exponentially with time; i.e., $D_2 e^{-\lambda t}$.
5. The company receives a credit period $t_C$ from its supplier. Until the account is settled, the company deposits generated sales revenue into an interest-bearing account with the rate $I_e$. The account is settled at the end of the credit period and the company begins paying interest accumulated on the items in stock at the rate $I_p$.

This model calculates the components of the total cost as follows:

1. Ordering cost per unit time is $\frac{R}{T}$.
2. There are two cases for inventory holding costs.

   Case 1: when $t_d < T$
   
   Depletion of inventory can occur as a result of demand as well as due to deterioration during the replenishment cycle after $t_d$. When $0 \leq t \leq t_d$, the inventory level decreases due to demand $D_1$; when $t_d \leq t \leq T$, the inventory level decreases due to demand $D_2 e^{-\lambda t}$ and deterioration $-\theta I(t)$. Therefore, variations in the inventory level $I(t)$ with respect to time $t$ can be expressed as follows:

   $\frac{dI(t)}{dt} = \begin{cases} 
   -D_1, & 0 \leq t \leq t_d; \\
   -D_2 e^{-\lambda t} - \theta I(t), & t_d \leq t \leq T, 
   \end{cases}$

   where $\lambda > 0$.

   Using the boundary condition $I(T) = 0$, the solution to the above equation can be expressed as:

   $I_1(t) = D_1(t_d - t) + \frac{D_2 e^{-\lambda t_d}}{\theta - \lambda} [e^{(\theta - \lambda)t} - e^{(\theta - \lambda)t_d}]$,  
   $0 \leq t \leq t_d;$

   $I_2(t) = \frac{D_2 e^{-\lambda t_d}}{\theta - \lambda} [e^{(\theta - \lambda)t} - e^{(\theta - \lambda)t_d}]$,  
   $t_d \leq t \leq T.$
There are two cases for purchasing cost.

Case 1: when the credit period
\[ t_d < T, \]
the total inventory holding cost for Case 1 is
\[ Q = \int_0^T I(t) dt = \frac{h D_1 t_d}{T}. \]

Case 2: when \( t_d \geq T \)
When \( t_d \geq T \), no deterioration occurs in this model. Variations in the inventory level \( I(t) \) with respect to time \( t \) can be expressed as:
\[ \frac{dI(t)}{dt} = -D_1. \]
Using boundary condition \( I(T) = 0 \), the solution to \( \frac{dI(t)}{dt} = -D_1 \) can be expressed as \( I(t) = D_1 (T - t) \). Note that the quantity ordered during each replenishment cycle is \( Q = D_1 T \).

The total inventory holding cost for Case 2 is
\[ Q = \int_0^T I(t) dt = \frac{h D_1 T}{2}. \]

3. There are two cases for purchasing cost.

Case 1: when \( t_d < T \),
purchasing cost is
\[ Q = \int_0^T I(t) dt = \frac{h D_1 t_d}{T}. \]

Case 2: when \( t_d \geq T \),
purchasing cost is \( cD_1 \).

4. We discuss the interest accumulated under three different cases: \( t_C < t_d \), \( t_C > t_d \) and \( t_C = t_d \). For the accumulation of interest, the company can deposit generated sales revenue in an interest-bearing account at rate \( I_e \) before the credit period \( t_C \).

(4.1) When \( t_C < t_d \), there are three cases in which interest is earned.

Case 1: when \( t_C < t_d \leq T \),
the interest earned is
\[ \frac{D_1 L}{T} \int_0^{t_d} D_1 t dt = \frac{p_1 D_1 t_d^2}{2T}. \]
Case 2: when \( t_C \leq T < t_d \),
the interest earned is
\[ \frac{D_1 L}{T} \int_0^{t_d} D_1 t dt = \frac{p_1 D_1 t_d^2}{2T}. \]
Case 3: when \( T < t_C < t_d \),
In this subcase, the calculation of earned interest is based on accumulated sales revenue with demand rate \( D_1 \) from time 0 to \( T \) and the total sales revenue from time \( T \) to \( t_C \).

Interest earned is
\[ \frac{p_1 L}{T} \left[ \int_0^T D_1 t dt + D_1 T(t_C - T) \right] = \frac{p_1 L}{T} \left[ \frac{D_1 T^2}{2} + D_1 T(t_C - T) \right]. \]

(4.2) When \( t_C > t_d \), there are three cases in which interest is earned.

Case 1: when \( t_d < t_C \leq T \)
In this subcase, the calculation of earned interest is based on the accumulated sales revenue with demand rate \( D_1 \) from time 0 to \( t_d \), the total
5. We also discuss the interest paid under three different cases:

Interest earned = \( \frac{p_1}{T} \int_0^{t_d} D_1 dt + D_1 t_d(t_C - t_d) \)

\[ + \frac{p_2 I_c}{T} \int_{t_d}^T D_2 e^{-\lambda t} dt \]

\[ = \frac{p_1 I_c D_1 t_d^2}{2T} + \frac{p_1 I_c D_1 t_d(t_C - t_d)}{T} + \frac{p_2 I_c D_2[1 + t_d \lambda - e^{-T \lambda}(1 + T \lambda)]}{T \lambda} \]

Case 2: when \( t_d \leq T < t_C \)

In this subcase, the calculation of earned interest is based on the accumulated sales revenue with demand rate \( D_1 \) from time 0 to \( t_d \), the total sales revenue (generated in the non-deteriorating stage) from to time \( t_d \) to \( t_C \), the accumulated sales revenue with demand rate \( D_2 e^{-\lambda t} \) from time \( t_d \) to \( T \), and the total sales revenue (generated in the deteriorating stage) from to time \( T \) to \( t_C \).

Interest earned

\[ = \frac{p_1 I_c}{T} \int_0^{t_d} D_1 dt + D_1 t_d(t_C - t_d) \]

\[ + \frac{p_2 I_c}{T} \int_{t_d}^T D_2 e^{-\lambda t} dt + \frac{p_2 I_c D_2}{T} \int_{t_d}^T D_2 e^{-\lambda t} dt(t_C - T) \]

\[ = \frac{p_1 I_c D_1 t_d^2}{2T} + \frac{p_1 I_c D_1 t_d(t_C - t_d)}{T} + \frac{p_2 I_c D_2[1 + t_d \lambda - e^{-T \lambda}(1 + T \lambda)]}{T \lambda} \]

\[ + \frac{p_2 I_c D_2(e^{-t_d \lambda} - e^{-T \lambda})(t_C - T)}{T \lambda} \]

Case 3: when \( T < t_d < t_C \)

Interest earned = \( \frac{p_1 I_c}{T} \left[ \frac{D_1 t_d^2}{2} + D_1 T(t_C - T) \right] \).

(4.3) When \( t_C = t_d \), there are two cases in which interest is earned.

Case 1: when \( t_d = t_C \leq T \),

interest earned = \( \frac{p_1 I_c D_1 t_d^2}{2T} \).

Case 2: when \( T < t_d = t_C \),

interest earned = \( \frac{p_1 I_c}{T} \left[ \frac{D_1 t_d^2}{2} + D_1 T(t_C - T) \right] \).

5. We also discuss the interest paid under three different cases: \( t_C < t_d \), \( t_C > t_d \), and \( t_C = t_d \). The company starts paying the interest on the items in stock at rate \( I_p \) after credit period \( t_C \).

(5.1) When \( t_C < t_d \), there are three cases in which interest is paid:

Case 1: when \( t_C < t_d \leq T \)

In this subcase, the calculation of earned interest is based on inventory level \( I_1(t) \) from time \( t_C \) to \( t_d \) and based on the inventory level \( I_2(t) \) from to time \( t_d \) to \( T \).

Interest paid = \( \frac{c I_p}{T} \left( \int_{t_C}^{t_d} I_1(t) dt + \int_{t_d}^T I_2(t) dt \right) \)

\[ = \frac{c I_p}{T} \left\{ \frac{D_1(t_d - t_C)^2}{2} + \frac{D_2 e^{-\theta \lambda t_d}(t_d - t_C)[e^{\theta \lambda t_d} - e^{(\theta - \lambda) t_d}]}{\theta \lambda} \right\} \]

Case 2: when \( t_C \leq T < t_d \)

No deterioration occurs in this subcase. The calculation of earned interest is based on the inventory level \( I(t) \) from time \( t_C \) to \( T \).
Interest paid = \( \frac{c \cdot I_p}{T} \left( \int_{t_C}^{T} I(t) dt \right) \)
\[
= \frac{c \cdot I_p}{T} \int_{t_C}^{T} D_1(T - t) dt = \frac{c I_p D_1}{2T} (T - t_C)^2. 
\]

Case 3: when \( T < t_C < t_d \)

The credit period is longer than the replenishment cycle time.

Interest paid = 0.

(5.2) When \( t_d < t_c \), there are three cases in which interest is paid.

Case 1: when \( t_d < t_C \leq T \)

The calculation of earned interest is based on the inventory level \( I_2(t) \) from time \( t_C \) to \( T \).

Interest paid = \( \frac{c \cdot I_p}{T} \left( \int_{t_C}^{T} I_2(t) dt \right) \)
\[
= \frac{c \cdot I_p}{T} \left\{ \frac{D_2 e^{-\theta t_c}}{\theta \lambda (\theta - \lambda)} \left[ \lambda e^{(\theta - \lambda)T} - \theta e^{(\theta - \lambda)t_C} + e^{\theta t_C - \lambda T(\theta - \lambda)} \right] \right\}.
\]

Case 2: when \( t_d \leq T < t_C \),

interest paid = 0.

Case 3: when \( T < t_d < t_C \),

interest paid = 0.

(5.3) When \( t_C = t_d \), there are two cases in which interest is paid:

Case 1: when \( t_d = t_C \leq T \),

interest paid = \( \frac{c \cdot I_p}{T} \left( \int_{t_C}^{T} I_2(t) dt \right) \)
\[
= \frac{c \cdot I_p}{T} \left\{ \frac{D_2 e^{-\theta t_c}}{\theta \lambda (\theta - \lambda)} \left[ e^{(\theta - \lambda)T} - e^{(\theta - \lambda)t_C} + e^{\theta t_C - \lambda T(\theta - \lambda)} \right] \right\}.
\]

Case 2: when \( T < t_d = t_C \),

interest paid = 0.

Therefore, when \( t_C < t_d \), the total cost is

\[
TC(T) = \left\{ \begin{array}{ll}
TC_1(T), & \text{if } t_C < t_d \leq T; \\
TC_2(T), & \text{if } t_C \leq T < t_d; \\
TC_3(T), & \text{if } T < t_C < t_d,
\end{array} \right.
\]

where

\[
TC_1(T) = \frac{R}{T} + \frac{h}{T} \left\{ \frac{D_1 t_d^2}{2} + \frac{D_2 e^{-\theta t_d}}{\theta - \lambda} \left[ e^{(\theta - \lambda)T} - e^{(\theta - \lambda)t_d} \right] \right\} + c \left\{ D_1 t_d + \frac{D_2 e^{-\theta t_d}}{\theta - \lambda} \left[ e^{(\theta - \lambda)T} - e^{(\theta - \lambda)t_d} \right] \right\} - \frac{p_1 I_p D_1 t_C^2}{2T},
\]
\[
TC_2(T) = \frac{R}{T} + \frac{h D_1 T}{2} + c D_1 - \frac{p_1 I_p D_1 t_C^2}{2T} + \frac{c I_p D_1}{2T} (T - t_C)^2,
\]
\[
TC_3(T) = \frac{R}{T} + \frac{h D_1 T}{2} + c D_1 - \frac{p_1 I_p D_1}{T} \left[ \frac{D_1 T^2}{2} + D_1 (T - t_C) \right].
\]
When \( t_C > t_d \), the total cost is

\[
TC(T) = \begin{cases} 
TC_4(T), & \text{if } t_d < t_C \leq T; \\
TC_5(T), & \text{if } t_d \leq T < t_C; \\
TC_3(T), & \text{if } T < t_d < t_C,
\end{cases}
\]  
(5)

where

\[
TC_4(T) = \frac{R}{T} + \frac{h}{T} \left\{ \frac{D_1 t_d^2}{2} + \frac{D_2 e^{-t_d t_d}}{\theta - \lambda} \left[ e^{(\theta - \lambda)T} - e^{(\theta - \lambda)t_d} \right] + \frac{D_3 e^{-t_d t_d}}{\theta^2 (\theta - \lambda)^2} \left[ \lambda e^{(\theta - \lambda)T} - \theta e^{(\theta - \lambda)t_d} + e^{\theta t_d - \lambda T} (\theta - \lambda) \right] \right\}
\]

\[
+ c \left\{ D_1 t_d + \frac{D_2 e^{-t_d t_d}}{\theta - \lambda} \left[ e^{(\theta - \lambda)T} - e^{(\theta - \lambda)t_d} \right] \right\}
\]

\[- \frac{p_1 I_p D_1 t_d^2}{2T} - \frac{p_1 I_p D_1 t_d (t_C - t_d)}{T} - \frac{p_2 I_p D_2 \left[ e^{-t_d t_d} (1 + t_d \lambda) - e^{-T \lambda} (1 + T \lambda) \right]}{T \lambda^2} \]

\[- \frac{p_2 I_p D_2 \left( e^{-T \lambda} - e^{-T \lambda} \right) (t_C - T)}{T \lambda}.
\]  
(6)

When \( t_C = t_d \), the total cost is

\[
TC(T) = \begin{cases} 
TC_4(T), & \text{if } t_d < t_C \leq T; \\
TC_5(T), & \text{if } t_d = t_C > T.
\end{cases}
\]  
(7)

It is reasonable to use the Mcclaurin series to approximate \( e^{(\theta - \lambda)T} \approx 1 + (\theta - \lambda)T + (\theta - \lambda)^2 T^2 / 2 \) when \( (\theta - \lambda)T \) is very small. Wherein, Eqs. (2), (6), and (7) can be rewritten as follows:

\[
TC_1(T) = \frac{R}{T} - \frac{p_1 I_p D_1 t_d^2}{2T} + c \left\{ D_1 t_d + \frac{D_2 e^{-t_d t_d}}{\theta - \lambda} \left[ (\theta - \lambda) (T - t_d) + (\theta - \lambda)^2 (T^2 - T_d^2)/2 \right] + \frac{D_3 e^{-t_d t_d}}{\theta^2 (\theta - \lambda)^2} \left[ \lambda (1 + (\theta - \lambda)T + (\theta - \lambda)^2 T^2 - T_d^2) - \theta (1 + (\theta - \lambda)T_d + (\theta - \lambda)^2 T_d^2) + [1 + \theta T_d - \lambda T + \frac{\theta^2 + \lambda^2 T^2 - 2 T \theta T_d}{2} ] (\theta - \lambda) \right] \right\}
\]

\[
+ \frac{h}{T} \left\{ \frac{D_1 t_d^2}{2} + \frac{D_2 e^{-t_d t_d}}{\theta - \lambda} \left[ (\theta - \lambda) (T - t_d) + (\theta - \lambda)^2 (T^2 - T_d^2)/2 \right] + \frac{D_3 e^{-t_d t_d}}{\theta^2 (\theta - \lambda)^2} \left[ \lambda (1 + (\theta - \lambda)T + (\theta - \lambda)^2 T^2 - T_d^2) - \theta (1 + (\theta - \lambda)T_d + (\theta - \lambda)^2 T_d^2) + [1 + \theta T_d - \lambda T + \frac{\theta^2 + \lambda^2 T^2 - 2 T \theta T_d}{2} ] (\theta - \lambda) \right] \right\}
\]

\[- \frac{p_2 I_p D_2 \left[ e^{-t_d t_d} (1 + t_d \lambda) - e^{-T \lambda} (1 + T \lambda) \right]}{T \lambda^2} \]

\[- \frac{p_2 I_p D_2 \left( e^{-T \lambda} - e^{-T \lambda} \right) (t_C - T)}{T \lambda}.
\]  
(8)
$TC_4(T) = \frac{R}{T} + c \left\{ D_1 t_d + \frac{D_2 e^{-\theta t_d}}{T} \left[ (\theta - \lambda)(T - t_d) + \frac{(\theta - \lambda)^2(T^2 - t_d^2)}{2} \right] \right\}$

\[ + \frac{h}{T} \left\{ \frac{D_1 t_d^2}{2} + \frac{D_2 e^{-\theta t_d}}{\theta (\theta - \lambda)} \left[ (\theta - \lambda)(T - t_d) + \frac{(\theta - \lambda)^2(T^2 - t_d^2)}{2} \right] \right\} \]

\[ + \frac{P_1 I_e D_1 t_d^2}{2T} - \frac{P_1 I_e D_1 t_d(t_C - t_d)}{T} \]

\[ \frac{P_2 I_e D_2}{T \lambda^2} \left[ \frac{(1 - t_d \lambda + t_d^2 \lambda^2/2)(1 + t_d \lambda) - (1 - t_C \lambda + t_C^2 \lambda^2/2)(1 + t_C \lambda)}{T \lambda^2} \right] \]

3. Solution approach. In this section we solve three cases of the problem, when $t_C < t_d$, $t_C > t_d$, and $t_C = t_d$. For each case we provide the piecewise nonlinear optimization algorithm, which based on the corresponding lemma.

3.1. $t_C < t_d$.

When $t_C < t_d$, the goal is to determine the optimal joint replenishment cycle time $T^*$ in order to maximize the total cost $TC(T) = \{ TC_1(T), \text{ if } t_C < t_d \leq T; TC_2(T), \text{ if } t_C \leq T < t_d; TC_3(T), \text{ if } T < t_C < t_d, \}$ which is a three-branch function with a single decision variable.

For the case in which $t_C < t_d \leq T$, the second-order derivative of $TC_1(T)$ with respect to $T$ is

\[ \frac{d^2TC_1(T)}{dT^2} = e^{-\theta \theta}(2R + D_1[ht_d \lambda^2 + cI_P(t_C - t_d)^2 - I_e P_1 \lambda^2] - D_2 t_d \{ ht_d [1 + t_d (\theta - \lambda)] + cI_P t_d (\lambda - \theta) + t_d (\lambda - \theta) - 2 \}) \]

If $e^{-\theta \theta}(2R + D_1[ht_d \lambda^2 + cI_P(t_C - t_d)^2 - I_e P_1 \lambda^2] - D_2 t_d \{ ht_d [1 + t_d (\theta - \lambda)] + cI_P t_d (\lambda - \theta) + t_d (\lambda - \theta) - 2 \}) > 0$, then $TC_1(T)$ is a convex function of $T$. Therefore,
there exists a replenishment cycle time $T$ which minimizes $TC_1(T)$ as follows. By solving $\frac{dTC_1(T)}{dT} = 0$, we obtain

$$
T_1 = \sqrt{\frac{e^{t_d}{\{2R + D_1[ht_d^2 + cI_p(TC - t_d)^2 - I_cP_1t_d^2]\} + D_2t_d \{ht_d[1 + t_d(\theta - \lambda)] + cI_p[t_d^2 - 2]\}}{D_2\{h[1 + t_d(\theta - \lambda)] + cI_p[1 + t_d(\theta - \lambda) + TC(\lambda - \theta)]\}}}.
$$

(13)

To ensure $t_d \leq T_1$, we substitute Eq. (13) into $t_d \leq T_1$ to obtain that if and only if

$$
\begin{align*}
&= e^{t_d}\left\{2R + D_1 \left[ \frac{ht_d^2 + cI_p(TC - t_d)^2}{-I_cP_1t_d^2} \right] \right\} + D_2t_d \left\{ \frac{ht_d[1 + t_d(\theta - \lambda)] + cI_p\{t_d[1 + t_d(\theta - \lambda)]\}}{+tc_c[t_d(\lambda - \theta) - 2]} \right\} \\
&\geq t_d^2D_2\{h[1 + t_d(\theta - \lambda)] + cI_p[1 + t_d(\theta - \lambda) + TC(\lambda - \theta)]\},
\end{align*}
$$

(14)

then $t_d \leq T_1$.

Let $\Omega = e^{t_d}\left\{2R + D_1 \left[ \frac{ht_d^2 + cI_p(TC - t_d)^2}{-I_cP_1t_d^2} \right] \right\} + D_2t_d \left\{ \frac{ht_d[1 + t_d(\theta - \lambda)] + cI_p\{t_d[1 + t_d(\theta - \lambda)]\}}{+tc_c[t_d(\lambda - \theta) - 2]} \right\}.

The above analysis leads to the first lemma.

**Lemma 1.**

1. If $\Omega \geq t_d^2D_2\{h[1 + t_d(\theta - \lambda)] + cI_p[1 + t_d(\theta - \lambda) + TC(\lambda - \theta)]\}$, there exists an optimal replenishment cycle time $T^*_1 = T_1$ (as shown in Eq. (13)).

2. If $\Omega < t_d^2D_2\{h[1 + t_d(\theta - \lambda)] + cI_p[1 + t_d(\theta - \lambda) + TC(\lambda - \theta)]\}$, the minimal value of $TC_1(T)$ occurs at point $T^*_1 = t_d$.

In the case of $t_c \leq T < t_d$, the second-order derivative of $TC_2(T)$ with respect to $T$ is

$$
\frac{d^2TC_2(T)}{dT^2} = \frac{2R + D_1t_d^2(cI_p - p_1I_c)}{T^3}.
$$

(15)

If $2R + D_1t_d^2(cI_p - p_1I_c) > 0$, then $TC_2(T)$ is a convex function of $T$ and there exists replenishment cycle time $T_2$ which minimizes $TC_2(T)$ as follows. By solving $\frac{dTC_2(T)}{dT} = 0$, we obtain

$$
T_2 = \sqrt{\frac{2R + D_1t_d^2(cI_p - p_1I_c)}{D_1(cI_p + h)}}.
$$

(16)

To ensure $t_c \leq T_2 < t_d$, we substitute Eq. (16) into $t_c \leq T_2 < t_d$ to establish that, if and only if

$$
D_1t_d^2(cI_p + h) > 2R + D_1t_d^2(cI_p - p_1I_c) \geq D_1t_d^2(cI_p + h),
$$

(17)

then $t_c \leq T_2 < t_d$.

We develop Lemma 2 based on the above discussion.

**Lemma 2.**

1. If $2R + D_1t_d^2(cI_p - p_1I_c) > D_1t_d^2(cI_p + h)$, the minimal value of $TC_2(T)$ occurs at point $T^*_2 = t_d$.

2. If $D_1t_d^2(cI_p + h) > 2R + D_1t_d^2(cI_p - p_1I_c) \geq D_1t_d^2(cI_p + h)$, there exists an optimal replenishment cycle time $T^*_2 = T_2$ (as shown in Eq. (16)).

3. If $2R + D_1t_d^2(cI_p - p_1I_c) < D_1t_d^2(cI_p + h)$, the minimal value of $TC_2(T)$ occurs at point $T^*_2 = t_c$. 
In the case of \( T < t_C < t_d \), the second-order derivative of \( TC_3(T) \) with respect to \( T \) is

\[
\frac{d^2TC_3(n)}{dT^2} = \frac{2R}{T^3} > 0. \tag{18}
\]

From Eq. (18), we know that \( TC_3(T) \) is a convex function of \( T \). Thus, there exists replenishment cycle time \( T_3 \) which minimizes \( TC_3(T) \) as follows. By solving \( \frac{dTC_3(T)}{dT} = 0 \), we obtain

\[
T_3 = \sqrt{\frac{2R}{D_1(I_c p_1 + h)}}. \tag{19}
\]

To ensure that \( T_3 < t_C \), we substitute Eq. (19) into \( T_3 < t_C \) to establish that, if and only if

\[
2R - t_C^2[D_1(I_c p_1 + h)] < 0, \tag{20}
\]

then we obtain Lemma 3, based on the above analysis:

**Lemma 3.**

1. If \( 2R - t_C^2[D_1(I_c p_1 + h)] \geq 0 \), the minimal value of \( TC_3(T) \) occurs at point \( T_3^* = t_C \).
2. If \( 2R - t_C^2[D_1(I_c p_1 + h)] < 0 \), there exists an optimal replenishment cycle time \( T_3^* = T_3 \) (as shown in Eq. (19)).

Based on the above lemmas and the developed model, the optimal replenishment cycle time can be determined by the following piecewise nonlinear optimization algorithm:

**Algorithm 1.** (For the case of \( t_C < t_d \))

Step 1. Lemma 1 is used to determine the optimal value of \( T \) and calculate \( TC_1(T_1^*) \).

Step 2. Lemma 2 is used to determine the optimal value of \( T \) and calculate \( TC_2(T_2^*) \).

Step 3. Lemma 3 is used to determine the optimal value of \( T \) and calculate \( TC_3(T_3^*) \).

Step 4. Let \( TC(T^*) = \min\{TC_1(T_1^*), TC_2(T_2^*), TC_3(T_3^*)\} \).

### 3.2. \( t_C > t_d \)

When \( t_C > t_d \), the goal is to determine the optimal replenishment cycle time \( T^* \) in order to maximize the total cost \( TC(T) = \begin{cases} 
TC_4(T), & \text{if } t_d < t_C \leq T; \\
TC_5(T), & \text{if } t_d \leq T < t_C; \\
TC_3(T), & \text{if } T < t_d < t_C,
\end{cases} \)

which is a three-branch function with one decision variable.

In the case of \( t_d < t_C \leq T \), the second-order derivative of \( TC_4(T) \) with respect to \( T \) is

\[
\frac{d^2TC_4(T)}{dT^2} = e^{-\theta(t_d+t_C)} \frac{e^{\theta(t_d+t_C)}}{T^3} \lambda \{2R + D_1 t_d [ht_d - I_c p_1 (2t_C - t_d)]\} + D_2 \left\{ ce^{\theta(t_d+t_C)} \right. \\
\left. I_c p_2 (t_C - t_d)[4 + 3t_d \lambda + (t_C^2 + t_d^2) \lambda^2 + t_C \lambda (3 + t_d \lambda)] \right\} > 0, \tag{21}
\]

If \( e^{\theta(t_d+t_C)} \lambda \{2R + D_1 t_d [ht_d - I_c p_1 (2t_C - t_d)]\} + D_2 \left\{ ce^{\theta(t_d+t_C)} \right. \\
\left. I_c p_2 (t_C - t_d)[4 + 3t_d \lambda + (t_C^2 + t_d^2) \lambda^2 + t_C \lambda (3 + t_d \lambda)] \right\} > 0, \tag{21}
\]
that if and only if

\( t \) occurs at the point where

\[ D \{ e^{\theta t} I_p + e^{\theta t} h[1 + t_d(\theta - \lambda)] \} \]

To ensure \( t_C \leq T_4 \), we substitute Eq. (22) into \( T_C \leq T_4 \) to obtain establish that, if and only if

\[ e^{\theta(t_d+t_C)} \lambda \{2R + D_1 t_d[h t_d - I_e p_1(2t_C - t_d)] \}
\]

then \( t_C \leq T_4 \).

Let \( \Upsilon = e^{\theta(t_d+t_C)} \lambda \{2R + D_1 t_d[h t_d - I_e p_1(2t_C - t_d)] \}
\]

The above analysis leads to Lemma 4.

**Lemma 4.**

1. If \( \Upsilon \geq t_C^2 D_2 \lambda \{ e^{\theta t} I_p + e^{\theta t} h[1 + t_d(\theta - \lambda)] \} \), then there exists an optimal replenishment cycle time \( T_4 = T_4 \) (as shown in Eq. (22)).
2. If \( \Upsilon < t_C^2 D_2 \lambda \{ e^{\theta t} I_p + e^{\theta t} h[1 + t_d(\theta - \lambda)] \} \), the minimal value of \( T_C(T) \) occurs at the point where \( T_4 = t_C \).

In the case of \( t_d \leq T < t_C \), the second-order derivative of \( T_C(T) \) with respect to \( T \) is

\[ d^2 T_C(T) \frac{dT^2}{dt^2} = e^{-\theta t_d} \lambda \left\{2R + t_d D_1[h t_d - I_e p_1(2t_C - t_d)]\right\}
\]

If \( e^{\theta t_d} \left\{2R + t_d D_1[h t_d - I_e p_1(2t_C - t_d)]\right\}
\]

then \( t_d \leq T < t_C \). We substitute Eq. (22) into \( t_d \leq T < t_C \) to establish that if and only if

\[ t_C^2 D_2 \lambda \{ e^{\theta t} I_e p_2(1 + t_C \lambda) + h[1 + t_d(\theta - \lambda)] \} \geq e^{\theta t_d} \lambda \{2R + t_d D_1[h t_d - I_e p_1(2t_C - t_d)]\}
\]

then \( t_d \leq T < t_C \).
Let $\Lambda = e^{\theta d} \lambda \{2R + t_d D_1 [ht_d - I_c p_1 (2TC - t_d)]\} - D_2 t_d \left\{ ht_d \lambda [1 + t_d (\theta - \lambda)] + e^{\theta d} [2TC - t_d \lambda] \right\}$. We develop Lemma 5 based on the above discussion.

**Lemma 5.**

1. If $\Lambda > t_d^2 D_2 \lambda \{e^{\theta d} I_c p_2 (1 + t_c \lambda) + h[1 + t_d (\theta - \lambda)]\}$, the minimal value of $TC_5(T)$ occurs at the point where $T^*_5 = t_c$.
2. If $t_d^2 D_2 \lambda \{e^{\theta d} I_c p_2 (1 + t_c \lambda) + h[1 + t_d (\theta - \lambda)]\} > \Lambda \geq t_d^2 D_2 \lambda \{e^{\theta d} I_c p_2 (1 + t_c \lambda) + h[1 + t_d (\theta - \lambda)]\}$, there exists an optimal replenishment cycle time $T^*_5 = T_5$ (as shown in Eq. (25)).
3. If $\Lambda < t_d^2 D_2 \lambda \{e^{\theta d} I_c p_2 (1 + t_c \lambda) + h[1 + t_d (\theta - \lambda)]\}$, the minimal value of $TC_5(T)$ occurs at $T^*_5 = t_d$.

**Algorithm 2.** (For the case of $t_C > t_d$)

Step 1. Lemma 4 is used to determine the optimal value of $T$ and calculate $TC_4(T^*_4)$.
Step 2. Lemma 5 is used to determine the optimal value of $T$ and calculate $TC_5(T^*_5)$.
Step 3. Lemma 3 is used to determine the optimal value of $T$ and calculate $TC_3(T^*_3)$.
Step 4. Let $TC(T^*) = \min \{TC_4(T^*_4), TC_5(T^*_5), TC_3(T^*_3)\}$.

$3.3t_C = t_d$

When $t_C = t_d$, the goal is to determine the optimal joint replenishment cycle time $T^*$ in order to maximize the total cost $TC(T) = \begin{cases} \text{TC}_4(T), & \text{if } t_d = t_C \leq T; \\ \text{TC}_3(T), & \text{if } t_d = t_C > T, \end{cases}$ which is a two-branch function with one decision variable.

**Algorithm 3.** (For the case of $t_C = t_d$)

Step 1. Lemma 4 is used to determine the optimal value of $T$ and calculate $TC_4(T^*_4)$.
Step 2. Lemma 5 is used to determine the optimal value of $T$ and calculate $TC_5(T^*_5)$.
Step 3. Let $TC(T^*) = \min \{TC_4(T^*_4), TC_5(T^*_5)\}$.

**4. Numerical study.** This section presents a numerical example to illustrate the approaches used in the proposed solution. Numerical analysis provides several quantitative insights.

**4.1. Numerical example.** To illustrate the above algorithms described, we consider the following parameters: $D_1=50$; $D_2=30$; $p_1=20$; $p_2=15$; $c =$ $10 per unit item; $h =$ $2 per unit item; $R =$ $80$ per order; $\theta =$ 0.12; $\lambda =$ 0.2; $I_c =$ 0.1; $I_p =$ 0.15.

When $t_d = 0.5$ and $t_C = 0.3$ (when $t_d < t_C$), applying Algorithm 1, we find that the optimal replenishment cycle time is $T^* = 1.42$ and the total cost is $TC^* = TC^*_1 = $107.60. Figure 1 (a) illustrates $TC$ versus $T$ in various cases.

When $t_d = 0.3$ and $t_C = 0.5$ (when $t_d < t_C$), applying Algorithm 2, we find that the optimal replenishment cycle time is $T^* = 1.63$ and the total cost is $TC^* = TC^*_4 = $131.90. Figure 1 (b) illustrates $TC$ versus $T$ in various cases.

When $t_d = t_C = 0.5$ (when $t_d = t_C$), applying Algorithm 3, we find that the optimal replenishment cycle time is $T^* = 1.34$ and the total cost is $TC^* = TC^*_4 = $97.30. Figure 1 (c) illustrates $TC$ versus $T$ for different cases.

Table 1 illustrates how deviation from the optimal replenishment cycle time increases the total cost. These results were obtained by increasing the replenishment cycle time by $\pm 50\%$ of the optimal replenishment cycle time $T^*$. The total cost may increase by nearly 10% when the decision to replenish is not optimized. This illustrates the importance of optimizing the decision-making process and the effectiveness of the proposed approach in achieving this.
1. When \( t_d = 0.5 \) and \( t_C = 0.3 \)
2. When \( t_d = 0.3 \) and \( t_C = 0.5 \)
3. When \( t_d = 0.5 \) and \( t_C = 0.5 \)

Figure 1. The graphic illustrations of \( TC \) versus \( T \)

4.2. **Numerical analysis.** It is interesting to discuss the effects of system parameters on the behavior of retailers. In the following numerical analysis, we investigate the effects of \( h, R, \theta, I_v, I_p, \lambda, t_d, t_C, D_1 \) and \( D_2 \) on the optimal replenishment cycle time and the total cost in various cases. Tables 2 to 5 present our results related to cases where \( t_d > t_C, t_d < t_C \) and \( t_d = t_C \) respectively. In the following we present brief analysis of these results:
Table 1. Effects of replenishment cycle time on total cost

| Percentage | Cycle Time | Total Cost |
|------------|------------|------------|
| 50% T*     | t_C=0.3    | TC=441.46 (9.64%) |
| 75% T*     | t_C=0.3    | TC=446.18 (1.61%) |
| T*         | t_C=0.3    | TC=439.13 (0%) |
| 125% T*    | t_C=0.3    | TC=443.36 (0.96%) |
| 150% T*    | t_C=0.3    | TC=453.24 (3.21%) |

When t_d=0.5 and t_C=0.3

| Percentage | Cycle Time | Total Cost |
|------------|------------|------------|
| 50% T*     | t_C=0.5    | TC=481.46 (9.64%) |
| 75% T*     | t_C=0.5    | TC=446.18 (1.61%) |
| T*         | t_C=0.5    | TC=439.13 (0%) |
| 125% T*    | t_C=0.5    | TC=443.36 (0.96%) |
| 150% T*    | t_C=0.5    | TC=453.24 (3.21%) |

When t_d=0.5 and t_C=0.5

1. An increase in inventory holding cost h leads to a decrease in the optimal replenishment cycle time T*; however, total cost TC* increases. When the inventory holding cost increases, it is reasonable to assume that the company should decrease the replenishment cycle time in order to minimize the quantity of products being held.

2. An increase in ordering cost R leads to an increase in the optimal replenishment cycle time T* and total TC*. If the ordering cost increases, then the company should increase the replenishment cycle time in order to reduce replenishment frequency.

3. An increase in deterioration rate θ leads to a decrease in the optimal replenishment cycle time T* but an increase in total cost TC*. When the deterioration rate increases, the company should decrease the replenishment cycle time (order quantity) in order to reduce the amount of goods that are deteriorating.

4. Increasing of the value of λ means that the demand rate in the deteriorating stage decreases. When the value of λ increases, both the optimal replenishment cycle time T* and the total cost TC* increase.

5. The optimal replenishment cycle time T* and total cost TC* will decrease with an increase in interest earned I_e. When the interest earned increases, the company may shorten the replenishment cycle time in order to obtain the benefits of earned interest more frequently.

6. An increase in the interest paid I_p leads to a decrease in the optimal joint replenishment cycle time T*; however, the total cost TC* increases. When the interest paid increases, the company should decrease the replenishment cycle time in order to reduce the stock being held and reduce costs.

7. An increase in demand in the non-deteriorating stage D_1 leads to an extension of the optimal replenishment cycle time T*. An increase in demand in the deteriorating stage D_2 leads to a decrease in optimal replenishment cycle time T*. This also shows that the optimal replenishment cycle time T* decreases with a decrease in the difference between D_1 and D_2.

8. As shown in Table 5, an increase in the length of time in which the product does not deteriorate (non-deteriorating period; t_d) leads to an increase in the optimal joint replenishment cycle time T* and total cost TC*. When the length of the non-deteriorating period increases, the company is able to extend the replenishment cycle time in order to trade a greater quantity of stock.

9. An increase in the length of credit period t_C leads to a decrease in the optimal joint replenishment cycle time T* and the total cost TC*. When the credit period increases, it would be reasonable for the company to decrease the replenishment cycle time.
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Table 2. Effects of system parameters (when $t_d=0.5$ and $t_C=0.3$)

| Parameter | $T^*$ | $TC^*$ |
|-----------|------|-------|
| $h=1$     | 2.89 | 403.24 |
| $h=2$     | 2.31 | 439.13 |
| $h=3$     | 1.99 | 469.42 |
| $R=40$    | 2.06 | 420.81 |
| $R=80$    | 2.31 | 439.13 |
| $R=120$   | 2.53 | 455.65 |
| $\theta=0.05$ | 2.63 | 425.04 |
| $\theta=0.1$  | 2.31 | 439.13 |
| $\theta=0.15$ | 2.10 | 450.08 |
| $\lambda=0.1$ | 1.91 | 470.58 |
| $\lambda=0.2$ | 2.31 | 439.13 |
| $\lambda=0.3$ | 3.10 | 398.34 |
| $L_r=0.05$ | 2.32 | 440.10 |
| $L_r=0.10$ | 2.31 | 439.13 |
| $L_r=0.15$ | 2.30 | 438.15 |
| $I_P=0.10$ | 2.56 | 426.14 |
| $I_P=0.15$ | 2.31 | 439.13 |
| $I_P=0.20$ | 2.12 | 450.65 |
| $D_1=25$  | 1.33 | 367.97 |
| $D_1=50$  | 2.31 | 439.13 |
| $D_1=75$  | 2.98 | 488.22 |
| $D_2=15$  | 3.82 | 274.97 |
| $D_2=30$  | 2.31 | 439.13 |
| $D_2=45$  | 1.50 | 569.49 |

4.3. Extension: Pricing consideration. The model in Section 3 is used to determine the replenishment cycle time for non-instantaneously deteriorating products under trade credit and price adjustment. This model accounts for the fact that the interest earned over the replenishment period changes in cases where the firm implements a two-tiered pricing scheme, due to the effects of dealing with instantaneously deteriorating products. In this section, we extend the basic model to consider pricing policy. It is interesting to see how the profits and replenishment time of a product with two prices compares to the use of a single price in the non-deteriorating and deteriorating stages. Based on the determination of replenishment cycle time in the model in Section 3, the optimal retail prices in non-deteriorating and deteriorating stages can be determined in order to maximize the total profit.

The total profits for various cases are as calculated as follows:

When $t_C < t_d$, the total profit is

$$TP(p_1, p_2) = \begin{cases} 
TP_1(p_1, p_2) = \frac{p_1 D_1 t_d}{T} + \frac{p_2 D_2 (e^{\lambda T} - e^{\lambda t_d})}{\lambda T} - TC_1, & \text{if } t_C < t_d \leq T; \\
TP_2(p_1) = p_1 D_1 - TC_2, & \text{if } t_C \leq T < t_d; \\
TP_3(p_1) = p_1 D_1 - TC_3, & \text{if } T < t_C < t_d.
\end{cases}$$

(26)
Table 3. Effects of system parameters (when $t_d=0.3$ and $t_C=0.5$)

| Parameter | $T^*$ | $TC^*$ |
|-----------|-------|-------|
| $h=1$     | 2.40  | 382.19|
| $h=2$     | 1.91  | 412.67|
| $h=3$     | 1.63  | 438.08|
| $R=40$    | 1.61  | 389.91|
| $R=80$    | 1.91  | 412.67|
| $R=120$   | 2.17  | 432.30|
| $\theta=0.05$ | 2.16  | 399.33|
| $\theta=0.1$  | 1.91  | 412.67|
| $\theta=0.15$ | 1.74  | 423.22|
| $\lambda=0.1$ | 1.60  | 438.45|
| $\lambda=0.2$  | 1.91  | 412.67|
| $\lambda=0.3$  | 2.49  | 380.27|
| $L_e=0.05$ | 1.95  | 416.21|
| $L_e=0.10$ | 1.91  | 412.67|
| $L_e=0.15$ | 1.86  | 409.05|
| $I_p=0.10$ | 2.10  | 404.72|
| $I_p=0.15$ | 1.91  | 412.67|
| $I_p=0.20$ | 1.76  | 419.51|
| $D_1=25$  | 1.32  | 368.04|
| $D_1=50$  | 1.91  | 412.67|
| $D_1=75$  | 2.35  | 446.46|
| $D_2=15$  | 3.09  | 251.14|
| $D_2=30$  | 1.91  | 412.67|
| $D_2=45$  | 1.29  | 549.00|

When $t_C > t_d$, the total profit is

$$TP(p_1, p_2) = \begin{cases} 
TP_4(p_1, p_2) = \frac{p_1 D_1 t_d}{T} + \frac{p_2 D_2 (e^{\lambda T} - e^{\lambda t_d})}{\lambda T} - TC_4, & \text{if } t_d < t_C \leq T; \\
TP_5(p_1, p_2) = \frac{p_1 D_1 t_d}{T} + \frac{p_2 D_2 (e^{\lambda T} - e^{\lambda t_d})}{\lambda T} - TC_5, & \text{if } t_d \leq T < t_C; \\
TP_3(p_1) = p_1 D_1 - TC_3, & \text{if } T < t_d < t_C. 
\end{cases}$$

(27)

When $t_C = t_d$, the total profit is

$$TP(p_1, p_2) = \begin{cases} 
TP_4(p_1, p_2) = \frac{p_1 D_1 t_d}{T} + \frac{p_2 D_2 (e^{\lambda T} - e^{\lambda t_d})}{\lambda T} - TC_4, & \text{if } t_d = t_C \leq T; \\
TP_3(p_1) = p_1 D_1 - TC_3, & \text{if } t_d = t_C > T.
\end{cases}$$

(28)

Table 6 summarizes the results of cycle times and total profits for two-phase pricing and one-phase pricing for all cases. Clearly, two-phase pricing can increase total profit by nearly 14%. The superiority of two-phase pricing with regard to profit maximization demonstrates the importance of including price adjustment in models involving trade credit.
Table 4. Effects of system parameters (when $t_d=0.5$ and $t_C=0.5$)

| Parameter | $T^*$ | $TC^*$ |
|-----------|-------|--------|
| $h=1$     | 2.83  | 383.86 |
| $h=2$     | 2.27  | 429.09 |
| $h=3$     | 1.95  | 458.90 |
| $R=40$    | 2.01  | 410.40 |
| $R=80$    | 2.27  | 429.09 |
| $R=120$   | 2.49  | 445.90 |
| $\theta=0.05$ | 2.56 | 415.86 |
| $\theta=0.1$ | 2.27 | 429.09 |
| $\theta=0.15$ | 2.07 | 439.42 |
| $\lambda=0.1$ | 1.88 | 459.17 |
| $\lambda=0.2$ | 2.27 | 429.09 |
| $\lambda=0.3$ | 3.00 | 390.41 |
| $I_c=0.05$ | 2.30 | 431.83 |
| $I_c=0.10$ | 2.27 | 429.09 |
| $I_c=0.15$ | 2.23 | 426.31 |
| $I_p=0.10$ | 2.51 | 418.60 |
| $I_p=0.15$ | 2.27 | 429.09 |
| $I_p=0.20$ | 2.09 | 438.18 |
| $D_1=25$  | 1.33  | 359.50 |
| $D_1=50$  | 2.27  | 429.09 |
| $D_1=75$  | 2.92  | 477.32 |
| $D_2=15$  | 3.75  | 269.41 |
| $D_2=30$  | 2.27  | 429.09 |
| $D_2=45$  | 1.47  | 555.33 |

Table 5. Effects of different $t_d$ and $t_C$

| Parameters | $T^*$ | $TC^*$ |
|------------|-------|--------|
| $t_d=0.1$, $t_C=0.3$ | 1.58 | 407.73 |
| $t_d=0.3$, $t_C=0.3$ | 1.93 | 424.85 |
| $t_d=0.5$, $t_C=0.3$ | 2.31 | 439.13 |
| $t_d=0.7$, $t_C=0.3$ | 2.70 | 452.24 |
| $t_d=0.9$, $t_C=0.3$ | 3.10 | 464.39 |
| $t_d=0.5$, $t_C=0.1$ | 2.34 | 448.68 |
| $t_d=0.5$, $t_C=0.3$ | 2.31 | 439.13 |
| $t_d=0.5$, $t_C=0.5$ | 2.27 | 429.09 |
| $t_d=0.5$, $t_C=0.7$ | 2.22 | 415.92 |
| $t_d=0.5$, $t_C=0.9$ | 2.18 | 403.25 |

5. Conclusions. There exist important differences in retail prices and demand between non-deteriorating and deteriorating products. This paper presents an inventory model for non-instantaneously deteriorating products under price adjustment and trade credit. Our objective was to determine the optimal replenishment cycle time to minimize total cost. The problem is formulated as three piecewise nonlinear functions, which are solved through optimization. Numerical simulation
Table 6. Comparison of two-phase pricing and one-phase pricing

|                | When $t_d=0.5$ and $t_C=0.3$ | When $t_d=0.3$ and $t_C=0.5$ | When $t_d=0.5$ and $t_C=0.5$ |
|----------------|-------------------------------|-------------------------------|-------------------------------|
| Two-phase pricing | $p_1=39.22$  | $p_1=38.28$  | $p_1=38.81$  |
|                 | $p_2=23.77$  | $p_2=28.48$  | $p_2=23.25$  |
|                 | $T=1.19$     | $T=2.33$     | $T=1.10$     |
|                 | $TP=614.40$  | $TP=415.09$  | $TP=658.51$  |
|                 | (+14.05%)$^a$ | (+6.73%)     | (+14.13%)     |
| One-phase pricing | $p=30.28$    | $p=30.94$    | $p=30.39$    |
|                 | $T=1.46$     | $T=2.45$     | $T=1.35$     |
|                 | $TP=538.69$  | $TP=388.92$  | $TP=576.50$  |

$^a$ the percentage of profit increasing

illustrates the solution procedures and allows for a discussion of the influences of ordering cost, inventory holding cost, interest paid, deterioration rate, interest earned, interest paid, credit period and non-deteriorating period, as they pertain to inventory decisions and costs. This paper provides evidence that under trade credit, two-phase pricing schemes perform significantly better than does the one-price approach. Our results and the subsequent discussion serve as a useful reference for managerial decision-making. Extension of this work could include considerations of pricing decisions and multi-item replenishment.

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