Simulation and comparison of distributed target detectors in compound Gaussian background

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Abstract. In the non Gaussian background modelled by compound Gaussian distribution, a variety of distributed target detectors are addressed. Firstly, the detection statistics of different detectors are given. Then, the detection performance of five detectors is compared when the non Gaussian clutter covariance matrix is known. Secondly, when the non Gaussian clutter covariance matrix is unknown, a two-step architecture is adopted. Based on the same estimation method for non-Gaussian clutter covariance matrix structure, seven adaptive distributed target detectors are introduced and compared. Finally, through comparative analyses, the applicable conditions and application range of different detectors are summarized, which provide a theoretical reference for the practical application of radar target detection.

1. Introduction

With continuous progress of science and technology, the radar users put forward higher requirements for radar capability, not only hope to detect the presence of the target, but also hope to image the target and identify the target's category attributes. Compared with the traditional low range resolution radar, the high range resolution radar adopts the technologies of pulse compression, phase coding, frequency stepping or frequency agility, thus obtaining the ability of high range resolution. However, the corresponding distributed target detection problem has also attracted more attention in radar signal processing field [1].

On the one hand, the target echoes observed by the high range resolution radar are distributed in multiple radial range cells. If the point-target detector for the low resolution radar is still used, the energy of the strong scatterers from distributed target will leak into the adjacent range cells. This may form a "signal contamination" phenomenon, which reduces the detection ability [2]. On the other hand, with the increasing range resolution, the radar will receive spike clutter similar to the target, and the background clutter cannot be accurately represented by the traditional Gaussian model. Fortunately, the compound Gaussian distribution provides a feasible modelling approach [3]. On the distributed target detection in compound Gaussian background, many literatures have made positive and beneficial exploration [4-8], but there is a lack of systematic and in-depth comparative analysis between them.

In this paper, for known covariance matrix of non Gaussian clutter, the detection performance of a variety of distributed target detectors is compared. In view of unknown covariance matrix, a two-step detector architecture is adopted. Under the support of a unified covariance matrix structure estimation [9], the corresponding adaptive detectors are established and compared. Finally, through comparative
analysis, the application conditions and advantages of different detectors are discussed, which provides a good theoretical reference for the practical application of detectors.

2. Problem Description

Target detection has to be made across the $K$ test cells, i.e., $z_t$, $t=1,\cdots,K$. In addition, in the adaptive detection problem, it is usually assumed that the training data set, $z_t$, $t=K+1,\cdots,K+R$ is available, without target signal, and has the same clutter as the test cells. In this paper, the interference environment dominated by clutter is considered.

The testing problem can be expressed by the following binary hypothesis testing formula:

$$
H_0 : z_t = c_t, \quad t = 1,\cdots,K + R
$$

$$
H_1 : \begin{cases}
z_t = \alpha_t p + c_t, & t = 1,\cdots,K \\
z_t = c_t, & t = K + 1,\cdots,K + R
\end{cases}
$$

(1)

where, $p$ denotes known steering vector, $p^H p = 1$; $H$ indicates conjugate transpose; $\alpha_t$, $t=1,\cdots,K$ are unknown target latitudes. $N$ represents the coherent processing number. Let $R \geq N$.

The clutter echo is modelled by compound Gaussian distribution. Therefore, the $N$-dimensional compound Gaussian clutter vector of the range cell $t$ is

$$
c_t = \sqrt{r_t} \cdot \eta_t, \quad t = 1,\cdots,K + R
$$

(2)

where the texture component $r_t$ and the speckle component $\eta_t$ are independent of each other, and are also independent in different range cells. $r_t$ is a non negative random variable. $\eta_t$ is a zero mean complex Gaussian vector, and its corresponding covariance matrix structure is

$$
\Sigma = \mathbb{E} \left\{ \eta_t^H \eta_t \right\}, \quad t = 1,\cdots,K + R
$$

(3)

3. Detector Structures

Based on the generalized likelihood ratio test (GLRT) criterion [10], a variety of distributed target detectors have been proposed for compound Gaussian background. Among them, the detectors such as scatterer density dependent (SDD) detector, non-scatterer density dependent (NSDD) detector [4], order statistics (OS) detector, order statistics with dynamic threshold (DOS) detector [5], cascaded detector based on binary integration (BICD) [6], etc. are proposed for known clutter covariance matrix structure, and the implementation of the corresponding adaptive detectors depends on the non Gaussian clutter covariance matrix estimator. Moreover, the simplified modified generalized likelihood ratio test (SMGLRT) detector [7] and adaptive detector (AD) [8] can realize the adaptive detection of distributed targets without training data. For the convenience of comparison, different detectors mentioned above will be discussed.

The detection statistics of SDD [4] can be expressed as

$$
\lambda_{\text{SDD}} = \sum_{t=1}^{K} \ln \left[ 1 + \beta(1 - w_t)^{-N} \right]
$$

(4)

When the number of scatterers $h_0 \ll K$, $\beta \approx h_0/(K - h_0)$. In addition, $w_t$ is

$$
w_t = \frac{\left| p^H \Sigma^{-1} z_t \right|^2}{\left( z_t^H \Sigma^{-1} z_t \right) \left( p^H \Sigma^{-1} p \right)}, \quad t = 1,\cdots,K
$$

(5)

The detection statistics of NSDD [4] can be expressed as
\[ \lambda_{NSDD} = \sum_{t=1}^{K} u_t \]  

where \( u_t \) is 
\[ u_t = -2(N-1)\ln(1 - w_t), \quad t = 1, \cdots, K \]  

The detection statistics of OS [5] can be expressed as 
\[ \lambda_{OS} = \sum_{k=K-h_0+1}^{K} u_{(k)} \]  

where the ordered sequence of \( u_t, t=1,\cdots,K \) can be given by 
\[ 0 \leq u_{(1)} \leq \cdots \leq u_{(k)} \leq \cdots \leq u_{(K)} \]  

The detection statistics of DOS [5] can be expressed as 
\[ \lambda_{DOS} = \sum_{k=K-h_0+1}^{K} u_{(k)} \]  

where \( h_{eD} \) denotes the number of \( u_t, t=1,\cdots,K \) exceeding the first threshold \( T_1 \), and the threshold \( T_1 \) is determined by 
\[ h_e / K = 1 - F_{\chi^2}(T_1, 2) \]  

where \( F_{\chi^2}(\cdot, 2K) \) denotes the distribution function of the distribution \( \chi^2 \) with \( 2K \) degrees of freedom, and \( h_e \) is the estimated number of scatterers.

The BICD adopts a three-stage binary integration strategy [6]. Firstly, it detects the scatterer in a single range cell based on the statistics \( u_t \) indicated as (7). Then, the second-stage detector is a sliding window binary integrator. Finally, the third-stage detector also adopts a binary integrator. For the space consideration, the optimal threshold setting in each level can refer to [6].

The detection statistic of the adaptive detector SMGLRT [7] is 
\[ \lambda_{SMGLRT} = \frac{\det \left( \mathbf{Z}^\text{H} \mathbf{Z}^{-1} \right)}{\det_p \left( \mathbf{I}_N - \mathbf{pp}^\text{H} \right) \mathbf{Z}^\text{H} \mathbf{Z} \left( \mathbf{I}_N - \mathbf{pp}^\text{H} \right) \mathbf{Z}^\text{H} \mathbf{Z}^{-1} \} \]  

where \( \mathbf{Z} = [\mathbf{z}_1, \mathbf{z}_2, \cdots, \mathbf{z}_K] \) is the \( N \times K \) data matrix from test cells; \( \hat{T} = \text{diag}(\hat{\tau}_1, \hat{\tau}_2, \cdots, \hat{\tau}_K) \) is the estimation matrix of texture component; and the estimation of clutter texture component is 
\[ \hat{\tau}_t = \frac{1}{N} \mathbf{z}_t^\text{H} \left( \mathbf{I}_N - \mathbf{pp}^\text{H} \right) \mathbf{z}_t, \quad t = 1, \cdots, K \]  

The detection statistic of AD [8] is 
\[ \lambda_{AD} = -N \sum_{t \in \Theta_0} \ln \left( 1 - \frac{\left| \mathbf{p}^\text{H} \hat{\Sigma}_2 \mathbf{z}_t \right|^2}{\left( \hat{\Sigma}_2 \mathbf{z}_t^\text{H} \mathbf{p} \right) \left( \hat{\Sigma}_2 \mathbf{z}_t^\text{H} \mathbf{p} \right)} \right) \]  

where, the constrained recursive clutter-clustered estimation (CRCCE) [9] \( \hat{\Sigma}_1 \) is obtained for unknown clutter covariance matrix structure by using \( K \) data from test cells, and the initial estimation value \( \hat{\mathbf{w}}_{t}^{(1)} \).
for $w_t$ can be calculated by (5). The set of range cell subscripts corresponding to the smallest $K-h_0$ values in $\hat{w}_{(t)}$, $t=1,\cdots,K$ is given by $\Omega_{h_0}$, and the CRCCE $\hat{\Sigma}$ is calculated renewedly by using the range cell observations corresponding to $\Omega_{h_0}$. By substituting $\hat{\Sigma}$ into equation (5), $\hat{w}_{(t)}$ can be obtained again for $w_t$, and the set $\hat{\Theta}_{h_0}$ of range cell subscripts can be determined from the largest $h_0$ values in $\hat{w}_{(t)}$, $t=1,\cdots,K$.

4. Performance analyses

This section will compare the performance of different detectors. When the clutter covariance matrix structure is known, the detectors of SSD, NSDD, OS, DOS and BICD are mainly considered. When the clutter covariance matrix structure is unknown, the detection performance of adaptive SSD (ASDD), adaptive NSDD (ANSDD), adaptive OS (AOS), adaptive DOS (ADOS), adaptive BICD (ABICD), SMGLRT and AD are analyzed, by using the same estimator CRCCE.

In the simulation, the Toeplitz matrix is used in modeling the clutter covariance matrix structure [11], with the first-order exponential correlation coefficient $\gamma$. The texture components is modeled by Gamma distribution $f_\tau$, i.e.

$$f_\tau(x) = \frac{(L/b)^L}{\Gamma(L)} x^{L-1} e^{-(L/b)x}, \quad x \geq 0$$

where $L$ controls the deviation from the Gaussian distribution. $b$ is the mean value, usually set $b=1$.

All $K$ range cells have clutter components, while only $h_0$ range cells have signal components. The average target power and clutter power over $K$ range cells are $\sigma_s^2$ and $\sigma_c^2$, respectively. Hence, the signal to clutter ratio (SCR) is

$$SCR = \frac{\sigma_s^2}{\sigma_c^2} \frac{\mathbf{p}^H \Sigma^{-1} \mathbf{p}}{\sigma_c^2}$$

Firstly, different detectors are compared for known clutter covariance matrix structure. In the matched case of estimated scatterer number ($h_0=h_e=3$), the detection performance of NSDD, SDD, OS, DOS and optimal BICD is analyzed in Figure 1, for $K=15$, $N=2$, $L=1$, $\gamma=0$ and the false alarm probability $P_{fa}=10^{-4}$. It can be seen that the NSDD has obvious “collapse loss” and the detection performance is the worst. The accumulation of all range cells by the SSD also has certain performance loss, while the accumulation of possible scatterers by the OS effectively improves SCR. The detection performance of DOS is slightly better than that of OS because of the dynamic threshold processing. Interestingly, the detection performance of BICD is the best for low SCR. However, for the detection probability $P_d>0.9$, the BICD has a certain detection loss compared with OS and DOS. It should be noted that although the detection performance of OS, DOS and BICD is relatively good, they are all based on the known scatterer number or scatterer density; and the optimal parameter selection of BICD only aims at the special situation that the target scatterers have uniform energy distribution and Rayleigh fluctuation, which has some limitations.

Next, in the mismatched case of the estimated scatterer number ($h_0=3$, $h_e=1,2,3,4,7,15$), Figure 2 focuses on the performance comparison between DOS and BICD, both of which perform better in Figure 1. It can be seen from Figure 2 that, although the detection performance of DOS is not as good as BICD for low SCR in the matched case of scatterer number ($h_0=h_e$), the DOS shows better robustness for increasing mismatch degree of the scatterer number, while the BICD has basically lost its function for mismatched cases just as $h_e=1,7,15$.

Finally, for the unknown clutter covariance matrix structure, based on the same CRCCE estimation method, the detection performance of ANSDD, ASDD, AOS, ADOS and ABICD is evaluated with simulation data in Figure 3. In addition, the adaptive detectors SMGLRT and AD without training data...
are also compared with the abovementioned ones. It can be seen that the relative performance of ANSDD, ASDD, AOS, ADOS and ABICD is consistent with Figure 1. For the sparse-distributed-scatterer targets, the AD without training data is slightly worse than AOS but far better than ANSDD and ASDD. The possible reason is that, with the increase of scatterer density, less data can be used to estimate unknown clutter covariance matrix structure, and the detection performance of AD will decline sharply. However, the SMGLRT performs worst because it can not use any information of the clutter covariance matrix structure, but it does not need any training data and a priori information of target scatterer, which may provide a new way for the actual feasible environment.

All in all, the above detectors have their own advantages. The BICD performs best when the target scatterer density is known, but its optimal parameters are only selected for the special case of uniform energy distribution and Rayleigh fluctuation of each scatterer. The DOS is not as good as BICD when the target scatterer number is matched, but it has better robustness when the scatterer number is mismatched. The OS performs better than the existing NSDD and SDD, but performs slightly worse compared with DOS, and its computational complexity is also smaller. In the condition of no training data, the AD has good detection performance for sparse-distributed-scatterer target, but it needs the prior information of the scatterer number and only applies to the special case of sparse scatterers. The detection performance of SMGLRT is not as good as AD, but it is suitable for the case of no prior information of targets and does not need to estimate the clutter covariance matrix.

![Figure 1. Detection probability versus signal to clutter ratio of NSDD, SDD, OS, DOS and BICD for matched case](image1)

![Figure 2. Detection probability versus signal to clutter ratio of DOS and BICD for mismatched cases](image2)

![Figure 3. Detection probability versus signal to clutter ratio of ANSDD, ASDD, AOS, ADOS, ABICD, SMGLRT and AD for matched case](image3)
5. Summary
In this paper, under the background of compound Gaussian distribution, distributed target detectors are discussed in two cases of known and unknown covariance matrix of non-Gaussian clutter, and the application conditions and advantages of different detectors are summarized. The results show that BICD has the best detection performance when the target scatterer density is known, but its optimal parameter selection is only for the special case of the scatterer energy uniform distribution and Rayleigh fluctuation, which has some limitations. The DOS is not as good as BICD for the matched scatterer number, but it has better robustness for the mismatched scatterer number. The detection performance of OS is better than that of NSDD and SDD, and slightly worse than that of DOS, but its computational complexity is also less than DOS. Without the training data, the AD has a good detection performance for sparse-distributed targets, which is slightly worse than AOS, but much better than ANSDD and ASDD. It is also noted that the AD needs a priori information of the scatterer number and is only suitable for the special case of sparse scatterers. The SMGLRT has a poor detection performance than AD, but it is suitable for the case without a priori information and does not need to estimate the clutter covariance matrix.

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