Effect of quintessence on the Nature of Kerr-neuwan blackhole shadow with clouds of strings

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In this paper we have took reissner nordstrom blackhole with cloud of strings and surrounds it with quintessence. we processed the metric through newman janis algorithm to get its rotating counterpart. the blackhole in study now is a roating charged blackhole with clouds of string surrounded by quintessence. we studied its nature of effective potential and unstable photon orbits. Finally we have plotted the blackhole shadow for various variable profiles.

I. INTRODUCTION

From their early theoretical framework to the latest ob- serverence of the blackhole [1, 2]. It has remained the most enigmatic object in the entire cosmos. General Relativity establishes that the space time at the near vicinity of the black hole is so warped that the photon orbits around the black hole in an unstable circular orbit. There are plenty of research has been already done on both rotating and non-rotating cases [3-13], where the shadow is a perfect circle in later case and distorted in the former due to the frame-dragging effect. A blackhole with presence of charge has also been studied. In addition to this there are black holes with an additionnal dark energy quintessence field which was first proposed by kesilev. the quintessential field is present to explain the accelerated expansion of our universe [14-16].

In this paper, instead of assuming point particles, we consider the the black hole to be reside on on dimensional strings [16-26]. A static spherically symmetric spacetime metric for this black hole, has been studied in Ref. In our case we considered a Reissner nordstrom blackhole in a stingy universe with added quintessential field. using Newman Janis algorithm we have arrived at a metric for rotating charge blackhole with string cloud in pressece of quintessential field Fin

This Section 2 we discuss about this Kerr-Newman-Kiselev blackhole with string cloud, Section 3 discusses about the photon orbit and effective potential, we have plotted the black hole shadow in section 4 and summarized the paper in section 5.

II. KERR - NEWMAN - KISELEV BH WITH STRING CLOUD

The metric of Reissner-Nordstrom surrounded by quintessence and cloud of string [28] is given as,

\[ ds^2 = \left( 1 - b - \frac{2M}{r} + \frac{q^2}{r^2} - \frac{c}{r^{2\omega + 1}} \right) dt^2 + \left( 1 - b - \frac{2M}{r} + \frac{q^2}{r^2} - \frac{c}{r^{2\omega + 1}} \right)^{-1} dr^2 + r^2 d\Omega^2 \]

(1)

Following newman janis algorithm [27], we have our metric to be,

\[ ds^2 = -\left( 1 - \frac{2pr}{\Sigma} \right) dt^2 + \frac{\Sigma}{\Delta(r)} dr^2 + \Sigma d\theta^2 - \frac{4aprsin^2\theta}{\Sigma} dtd\phi + \sin^2\theta \left( r^2 + a^2 + \frac{2a^2 prsin^2\theta}{\Sigma} \right) d\phi^2 \]

(2)

Where,

\[ 2pr = r^2 + a^2 - \Delta(r) \]

(3)

\[ \Delta(r) = r^2 + a^2 - 2Mr - q^2 + cr^{2\omega + 1} + b r^2 \]

(4)

\[ \Sigma = r^2 + a^2 \cos^2\theta \]

(5)

In the above equations, q being the charge of the black hole, M and c term corresponds to mass and quintessence parameter, while a and b denotes the spin and string cloud parameter. The metric reduces to kerr newman kiselev blackhole when b = 0 and can be further reduce to kerr newman with b = c = 0 and to kerr with b = c = q = 0 and finally to schwarzschild by a = b = c = q = 0.

Solving \( \Delta(r) = 0 \) will give us the event horizon, the plot \( \Delta(r) \) vs r below gives us the comparison for different blackholes.

FIG. 1. Horizon plots for different blackholes.
III. PHOTON ORBITS

A. Unstable Circular Orbit

There is a particular unstable orbit in which a photon revolves around the blackhole, depending on the conditions of impact parameter, the would leave this orbit to either fall into the blackhole or escape to infinity. The equation of such orbit can calculated using Hamilton-Jacobi Variable separation method[29]. Hamilton - Jacobi equation in its general form is given as,

$$\frac{\partial S}{\partial \lambda} = -\frac{1}{2} g^{\mu \nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu}$$  \hspace{1cm} (6)

Where $\lambda$ is the affine parameter and $S$ is the action that is defined as,

$$S = \frac{1}{2} m^2 \lambda - E t + L \varphi + S_r (r) + S_\theta (\theta)$$  \hspace{1cm} (7)

where $m$ is the rest mass, $E$ and $L$ are energy and Angular momentum which are the constants of motion.

Solving the Hamilton jacobi equation will yield,

$$\frac{\partial S_r}{\partial r} = \sqrt{R(r)} \Delta(r)$$  \hspace{1cm} (8)

$$\frac{\partial S_\theta}{\partial \theta} = \sqrt{\Theta(\theta)}$$  \hspace{1cm} (9)

Where,

$$R(r) = (aL - (a^2 + r^2)E)^2 - (K + (aE - L)^2) \Delta$$  \hspace{1cm} (10)

$$\Theta(\theta) = K - \left( \frac{L^2}{\sin^2 \theta} - a^2 E^2 \right) \cos \theta$$  \hspace{1cm} (11)

where $K$ is the the separation constant. The trajectory of a photon is calculated with two impact parameter,

$$\xi = \frac{L}{E}$$

$$\eta = \frac{K}{E^2}$$

Rewriting $R(r)$ and $\Theta(\theta)$ in terms of impact parameters

$$R_p(r) = (a \xi - (a^2 + r^2))^2 - (\eta + (a - \xi)^2)$$  \hspace{1cm} (12)

$$\Theta_p(\theta) = \eta - \left( \frac{\xi^2}{\sin^2 \theta} - a^2 \right) \cos^2 \theta$$  \hspace{1cm} (13)

The equation of geodesic motion can be written the form

$$\Sigma t' = \frac{((r^2 + a^2)E - aL)(r^2 + a^2)}{\Delta(r)} a(aESin^2(\theta) - L)$$  \hspace{1cm} (14)

$$\Sigma r' = \sqrt{R(r)}$$  \hspace{1cm} (15)

$$\Sigma \theta' = \sqrt{\Theta(\theta)}$$  \hspace{1cm} (16)

$$\Sigma \varphi' = \frac{a((r^2 + a^2)E - aL)}{\Delta(r)} - \frac{aESin^2(\theta) - L}{\sin^2(\theta)}$$  \hspace{1cm} (17)

IV. EFFECTIVE POTENTIAL

The relationship between $r'$ and effective potential is given as,

$$V_{\text{eff}} = \frac{E^2}{2} - \frac{1}{2} r'^2$$  \hspace{1cm} (18)

Effective potential is the maximum potential energy for a photon to have zero radial velocity and zero radial acceleration and there by determining the photons’s stability.
From figure 2 - 4, we have plotted effective potential against r for various profiles such as by varying charge 1, quintessence parameter c and string parameter b. In fig 2, the maxima of the effective potential decreases with increase in charge, we can also notice that the peak is shifted toward the left as charge is being increased. In Fig 3, the maxima increases with increase in quintessence value and peak is shifted left. Similarly in fig 4, the maxima increases with string parameter b and peak shifting towards to left can be seen here too.

V. SHADOWS

For a photon to have spherical orbit, it should have null radial velocity and null radial acceleration, which means that,

\[ R_p(r) = 0 \]
\[ \frac{d R_p}{d r} = 0 \]

With this we can rewrite the impact parameter as,

\[ \xi = \frac{-4r\Delta(r) + a^2 \Delta'(r) + r^2 \Delta''(r)}{a \Delta'(r)} \] (19)

\[ \eta = \frac{r^2 \left( 16a^2 \Delta(r) - 16\Delta^2(r) + 8r\Delta(r) \Delta'(r) - r^2 \Delta'^2(r) \right)}{a^2\Delta'(r)} \] (20)

The shape of the black hole depends on the celestial coordinates \( \alpha \) and \( \beta \), which is given as

\[ \alpha = -r_0^2 Sin\theta \frac{d\phi}{dr} \] (21)
\[ \beta = -r_0^2 \frac{d\theta}{dr} \] (22)

Where \( \alpha \) and \( \beta \) have limit \( r_0 \) goes to infinity, using the geodesic equation the celestial coordinates can be written in terms of impact parameter.

\[ \alpha = \frac{-\xi}{\sin(\theta)} \] (23)

\[ \beta = \sqrt{\eta + a^2 \cos^2(\theta) - \xi^2 \cot^2(\theta)} \] (24)

The shadow can be obtained by plotting \( \alpha \) vs \( \beta \) in the equatorial plane.

VI. CONCLUSION

In this paper we have took reissner nordstrom blackhole with cloud of strings and surrounds it with quintessence. we processed the metric through newman janis algorithm to get its rotating counterpart. the blackhole in study now is a rotating charged blackhole with clouds of string surrounded by quintessence. we studied its nature of effective potential and unstable photon orbits. we have observed that the effective potential maxima is directly proportional to the increase in charge, quintessence and string parameter and the peak seems to incline towards the left.

Finally, we have plotted the blackhole shadow for various variable profiles. from figure 5, As expected the shadow shape starts to distort as we go for higher spin values. it also seems to a distorted circles for higher values of charge but upon closer observation, the circles are in deed concentric proving that only size of the shadow decreases with increase in charge. From figure 6, It is evident that the size of the shadow decreases with increase in quintessence and string parameters.
FIG. 5. Shadow plot fot KNK - cloud string black hole for variation in spin parameter(left) snd charge(right).

FIG. 6. Shadow plot fot KNK - cloud string black hole for variation in quintessense (left) snd string parameter(right).

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