Quantum computation based on particle statistics

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Abstract

In spite of their evident logical character, particle statistics symmetries are not among the inherently quantum features exploited in quantum computation. A difficulty may be that, being a constant of motion of a unitary evolution, a particle statistics symmetry cannot affect the course of such an evolution. We try to avoid this possible deadlock by introducing a generalized (counterfactual, blunt) formulation where this type of symmetry becomes a watchdog effects shaping the evolution of a unitary computation process. The work is an exploration.

I. INTRODUCTION

This study deals with an alternative form of quantum computation. In this context, it is useful to identify the “boundary” of the usual approach.

Until now, quantum computation speed up\cite{1-5 among others} (with respect to the known classical algorithms) has taken advantage of a wide but incomplete set of inherently quantum features: effective reversibility, superposition, entanglement, interference and measurement. However, particle (fermionic, bosonic) statistics has never been exploited.

We should further note that all quantum algorithms found so far are based on the same form of computation, which is time-sequential, in the sense that a reversible Boolean network appears in the time-diagram of the logical transformations undergone by the register.

They are also based on a common strategy, as recently recognized\cite{6–8}; for example, they can be seen as sophisticated applications of the Mach-Zender interferometer.

Furtherly, this form of computation is suspected\cite{9} to be inherently unable of solving in polynomial time an NP-complete problem.\footnote{Solving in polynomial time (P) a problem formerly classified in NP (in particular, whose solution}
The alternative approach explored in this study is tailored to solve a particular NP-complete problem\cite{10} and is essentially based on the exploitation of particle statistics – under a generalized, counterfactual and \textit{blunt} interpretation thereof.

We give up time-sequential computation, by considering, in place of the register, a reversible Boolean network fully deployed in space (rather than appearing in the register time-diagram). Time is now orthogonal to the network lay-out (fig. 1). In this way, part of the network input and part of the output can both be constrained (fig. 1). Checking whether this network is satisfiable is a version of the NP-complete satisfiability (SAT) problem.

We shall describe this in some detail; in the meantime, given the novelty of the approach, we give an outline of the entire work.

![Fig. 1](image)

- \(a, b, ..., h\) (fig.1) labeling the network nodes, are Boolean variables. The overall network, or each gate, has its own input (left) and output (right) nodes-variables: \(a, b, f (c, g, h)\)

required exponential time with all formerly known algorithms), means declassifying to P that problem alone. Solving in polynomial time an NP-complete problem would declassify to P all NP-complete and all NP problems.
make the input (output) of the overall network; \( a, b (c, d) \) make the input (output) of gate \#1, etc. With time orthogonal to the network lay-out, the terms “input” and “output” loose part of their meaning but are kept out of habit.

- Each gate introduces a logical constraint between its input and output variables, by establishing a function between them. This function is given in table form: the gate “truth table”. For example table 1, associated with gate 1, gives the invertible XOR (exclusive or) function. Table 2, associated with gate 2, gives the NOT function (to spare notation, the two-node NOT gate will be used as a network wire – an inverting wire, without loss of generality). Tables 4, 5, 6 represent the (partial) network input/output constraints.

- Table \( N \) is introduced for convenience; it does not establish any constraint, its \( 2^8 \) rows are all the possible assignment of the network Boolean variables.

- The SAT problem is whether there is at least one Boolean assignment – one row of \( N \) – satisfying all the network constraints (all gates and wires and the input/output constraints).

- The above is readily translated in Hilbert space mathematics\(^{[10]}\). A specific assignment of all network Boolean variables (one row of \( N \) becomes a tensor product of corresponding qubit eigenstates, and a basis vector of the network Hilbert space \( \mathcal{H}_N \). The logical constraint established by each gate or wire (network element or N.E.) \( i \) can be seen as a projector \( A_i \) from \( \mathcal{H}_N \) on \( \mathcal{H}_i^{(c)} \). \( \mathcal{H}_i^{(c)} \) is a “constrained” Hilbert subspace, spanned by those basis vectors of \( \mathcal{H}_N \) which satisfy the constraint established by N.E. \( i \).

- Qubits are implemented as a distributed system of two-state identical particles. The particle distribution corresponding to the input/output nodes of N.E. \( i \) is subject to a Hamiltonian \( H_i \). Given a suitable form of \( H_i \), the projector \( A_i \) is physically “turned on” in the degenerate ground state of \( H_i \), more generally in any network state satisfying
Such projectors are shown to be epiphenomena of identical particle statistics, under the generalized interpretation\cite{11,12}. All $H_i$ commute pairwise and the network Hamiltonian is $H_N = \sum_i H_i$.

- The network is prepared in a specific ground state $|\psi (0)\rangle$ of $H_N$\footnote{The generic ground state of $H_N$ is $2^n$ degenerate, where $n$ is the number of input (or output) qubits.}. $\langle \psi (0) | H_N | \psi (0) \rangle = 0$ implies $\langle \psi (0) | H_i | \psi (0) \rangle = 0$ for all $i$, consequently all $A_i$ are “turned on” in $|\psi (0)\rangle$. Thus $|\psi (0)\rangle$ satisfies $\Pi_i A_i |\psi (0)\rangle = |\psi (0)\rangle$, namely all N.E. constraints (all $A_i$ are pairwise commuting). This preparation is also a linear combination of all $H_N$ basis vectors which satisfy the network input constraints, not necessarily the output constraints. Since the output constraints are removed in this phase, until now computation has solved a simple P (polynomial) problem.

- By suitably operating on the output qubits in order to bring them in match with their constraints, the network ground state, continuously submitted to the projection $\Pi_i A_i |\psi (t)\rangle = |\psi (t)\rangle$, is \textit{unitarily} brought from $|\psi (0)\rangle$ to a state $|\psi'(\tau)\rangle$ satisfying also the output constraints (if the network is satisfiable). This requires interpreting a particle statistics symmetry as a continuous watchdog effect shaping the network evolution\footnote{An example of a unitary evolution shaped by a watchdog effect, is the evolution of the polarization of a photon going through an infinite series of polarizing filters, each rotated by an infinitesimal constant angle with respect to the former one. In a way, we go back to the root of quantum computation (computation reversibility\cite{13,14}) and take an alternative branch, by exploring a strictly quantum form of reversible computation.}. It should be noted that $|\psi'(\tau)\rangle$ can be in superposition with an orthogonal excited state $|\psi''(\tau)\rangle$, which is an error state from a computational standpoint. However, the amplitude of this latter state should be small.
• Measuring the network after the above operations gives a Boolean assignment. If the network is satisfiable, this is likely to be a network solution (whether it is a solution is checkable off line in polynomial time). If the check gives a non-solution, the network is likely to be not satisfiable. By repeating the overall process for a sufficient number of times, it can be ascertained with any confidence level whether the network is satisfiable. Computation has now solved the hard, NP-complete, SAT problem. By the way, the constrained network of fig. 1 is satisfied by (only) \(|\Psi\rangle = |1\rangle_a |1\rangle_b |1\rangle_c |0\rangle_d |1\rangle_e |0\rangle_f |1\rangle_g |1\rangle_h\).

• The possibility that particle statistics yields a computation speed-up appears to be promising in principle.

II. A COUNTERFACTUAL INTERPRETATION OF PARTICLE STATISTICS

This work hinges on a counterfactual interpretation of particle statistics\[11,12,15\]. A particle statistics symmetry is usually considered to be an initial condition which is conserved as a constant of motion. This notion can be made more general by applying a counterfactual logic: the state of the system might violate the symmetry, but it cannot be so because of a special watchdog effect which would immediately project such a state on the symmetric subspace. This way of reasoning yields to effective consequences\[4\], diverging from the conventional application of quantum mechanics. Apropos of this, it is worth citing the following statement by Roger Penrose: “What is particularly curious about quantum theory is that there can be actual physical effects arising from what philosophers refer to as counterfactuals — that is things that might have happened, although did not in fact happen”\[16\].

\[4\] besides the fact that particle statistics symmetries should no more be put among the initial conditions.

\[5\] The relevance of counterfactual reasoning in sequential computation has been highlighted by Richard Jozsa\[17\].
By way of exemplification, let us consider a state which we assume to be symmetrical (for particle permutation) because of particle statistics:

$$|\Psi (0)| = \cos^2 \vartheta |0\rangle_1 |0\rangle_2 + \sin \vartheta \cos \vartheta (|0\rangle_1 |1\rangle_2 + |1\rangle_1 |0\rangle_2) + \sin^2 \vartheta |1\rangle_1 |1\rangle_2;$$  \hspace{1cm} (1)

1 and 2 label two identical two-state particles (which makes this an idealized situation). The above amplitudes are used for convenience with no significant restriction. Of course $|\Psi (0)|$ is symmetrical under the particle permutation $P_{12}$ iff it satisfies the equation $S_{12} |\Psi (0)| = |\Psi (0)|$, where $S_{12} = \frac{1}{2} (1 + P_{12})$ is the symmetrization projector. Let $\mathcal{H}_{12}$ be the Hilbert space of the two particles and $\mathcal{H}_{12}^{(s)}$ be the symmetric subspace:

$$\mathcal{H}_{12}^{(s)} = \text{span} \left\{ |0\rangle_1 |0\rangle_2, \frac{1}{\sqrt{2}} (|0\rangle_1 |1\rangle_2 + |1\rangle_1 |0\rangle_2), |1\rangle_1 |1\rangle_2 \right\}.$$

In order to introduce the counterfactual picture, we assume that $|\Psi (t)|$ might violate the symmetry. This is interpreted by taking, for all $t$, a $|\Psi (t)|$ which is a free normalized vector of $\mathcal{H}_{12}$. Namely, given:

$$|\Psi (t)| = \sum_{i,j \in \{0,1\}} \alpha_{ij}^{(t)} |i\rangle_1 |j\rangle_2,$$

with $\sum_{i,j \in \{0,1\}} \left| \alpha_{ij}^{(t)} \right|^2 = 1,$  \hspace{1cm} (2)

the $\alpha_{ij}^{(t)}$ are free complex variables independent of each other up to normalization for any time $t$. In other words, if $t_1 \neq t_2$, $|\Psi (t_1)|$ and $|\Psi (t_2)|$ are two independent free normalized vectors of $\mathcal{H}_{12}$.

The logic is that, at any time $t = t_-$, $|\Psi (t_-)|$ might violate the symmetry: it is thus a free vector of $\mathcal{H}_{12}$. Then $|\Psi (t_-)|$ should be projected on $\mathcal{H}_{12}^{(s)}$. This gives as a result the vector of $\mathcal{H}_{12}^{(s)}$ closest to $|\Psi (t_-)|$.

Let $|\Psi (t_+)|$ be a free normalized vector of $\mathcal{H}_{12}$ independent from $|\Psi (t_-)|$ ($t_+ > t_-$). The above projection can be obtained by submitting $|\Psi (t_+)|$ to the following conditions:

for all $t$:

i) $S_{12} |\Psi (t_+)| = |\Psi (t_+)|$,

ii) $|\langle \Psi (t_+ | \Psi (t_-) \rangle|$ must be maximum.
In this picture, $|\Psi (t_-)\rangle$ is the vector “immediately before projection” while $|\Psi (t_+)\rangle$, subject to conditions (i) and (ii), is the vector “immediately after projection”. This counterfactual logic will become clearer when applied to an evolution of the above triplet state.

Continuous projection can be interpreted as a continuous state vector reduction induced by particle statistics. To show this, it is convenient to rewrite (2) as follows:

$$|\Psi (t_-)\rangle = \alpha_{00}^{(t_-)} |0\rangle_1 |0\rangle_2 + \frac{1}{2} \left( \alpha_{01}^{(t_-)} + \alpha_{10}^{(t_-)} \right) (|0\rangle_1 |1\rangle_2 + |1\rangle_1 |0\rangle_2) + \frac{1}{2} \left( \alpha_{01}^{(t_-)} - \alpha_{10}^{(t_-)} \right) (|0\rangle_1 |1\rangle_2 - |1\rangle_1 |0\rangle_2) + \alpha_{11}^{(t_-)} |1\rangle_1 |1\rangle_2.$$  

This vector undergoes a partial state vector reduction on the subspace $\mathcal{H}_{12}^{(s)}$. This cancels the amplitude of the channel $(|0\rangle_1 |1\rangle_2 - |1\rangle_1 |0\rangle_2)$ and renormalizes the amplitudes of the other channels. This can be seen as a special form of interference (from now on: “interference”), destructive in amplitude cancelation and constructive in renormalization. Of course this form of state vector reduction is an interpretation of particle statistics and is not due to an interaction with an external apparatus.

$t_+$ might be as close to $t_-$ as desired, since state vector reduction does not involve the notion of elapsed time. However, we will take $t_-=t-dt$ and $t_+=t$, in view of applying (i) and (ii) to an evolution described by $|\Psi (t)\rangle$. The former conditions are thus rewritten:

for all $t$:

i) $S_{12} |\Psi (t)\rangle = |\Psi (t)\rangle$,

ii) $Max |\langle\Psi (t) | \Psi (t-dt)\rangle|$,  

where $|\Psi (t)\rangle$ is a free normalized vector of $\mathcal{H}_{12}$.

Now we go back to our system in a triplet state. Duly constrained, $|\Psi (t)\rangle$ will describe an evolution of that state. Conditions (i) and (ii) will be applied together with an operation

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It is essential that $\mathcal{H}_{12}^{(s)}$ has dimension higher than one.
performed on the system, say an identical rotation of the state of both particles of an angle \( \varphi (t) \) (where \( \varphi (t) \) is some function of time), starting from preparation (1).

Let us anticipate the result, by following the conventional approach. The above operation, symmetrical for particles permutation, commutes with \( S_{12} \), therefore symmetry is a constant of motion of the evolution; namely, it does not apply any constraint to it. The two particles can thus be rotated independently of each other. This yields of course:

\[
|\Psi (t)\rangle = \cos^2[\vartheta + \varphi (t)]|0\rangle_1|0\rangle_2 + \sin[\vartheta + \varphi (t)]\cos[\vartheta + \varphi (t)](|0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2) \\
+ \sin^2[\vartheta + \varphi (t)]|1\rangle_1|1\rangle_2
\]

(3)

Now we want to obtain the same result by ignoring the fact that symmetry is conserved as a constant of motion, and by resorting to the projection interpretation of particle statistics, that is to conditions (i) and (ii). We need to add a further condition to specify that the state of each particle has been rotated by \( \varphi (t) \) with respect to the initial state (1).

Doing this requires some attentions. Now the two particles cannot be considered independently of each other, since their overall state is submitted to the continuous projection, or watchdog effect, (i) and (ii). We must utilize the particle density matrices:

\[
\rho_i (t) = \text{Tr}_{3-i} [|\Psi (t)\rangle \langle \Psi (t)|],
\]

where \( \text{Tr}_{3-i} \) means partial trace over \( 3-i \), and \( i = 1, 2 \) is the particle label (if we use the method of random phases\(^{[18]} \), \( |\Psi (t)\rangle \) does not need to be a pure state – anyhow it will turn out to be that). Furthermore, the coherence elements of each density matrix – as overall entanglement – can be affected by the watchdog effect (in fact they will be determined by it). We only know that the diagonal of each density matrix must show a \( \varphi (t) \) rotation. In conclusion the further condition (iii) is:

\[
diag \rho_1 (t) = diag \{ \text{Tr}_2 [|\Psi (t)\rangle \langle \Psi (t)|] \} = \cos^2[\vartheta + \varphi (t)]|0\rangle_2 \langle 0|_2 + \sin^2[\vartheta + \varphi (t)]|1\rangle_2 \langle 1|_2.
\]

\[
diag \rho_2 (t) = diag \{ \text{Tr}_1 [|\Psi (t)\rangle \langle \Psi (t)|] \} = \cos^2[\vartheta + \varphi (t)]|0\rangle_2 \langle 0|_2 + \sin^2[\vartheta + \varphi (t)]|1\rangle_2 \langle 1|_2.
\]
It is readily seen that the simultaneous application of conditions (i) and (iii) yields:

$$|\Psi (t)\rangle = \cos^2 [\vartheta + \varphi (t)] |0\rangle_1 |0\rangle_2 + e^{i\delta_1} \sin [\vartheta + \varphi (t)] \cos [\vartheta + \varphi (t)] (|0\rangle_1 |1\rangle_2 + |1\rangle_1 |0\rangle_2)$$

$$+ e^{i\delta_2} \sin^2 [\vartheta + \varphi (t)] |1\rangle_1 |1\rangle_2 ,$$

where the phases \(\delta_1\) and \(\delta_2\) are still unconstrained; condition (ii), given the initial state (1), sets \(\delta_1 = \delta_2 = 0\) and keeps this result frozen throughout the evolution, as readily checked. This yields the desired evolution (3).

As readily seen, by removing either one of the two conditions (iii), evolution (3) is still obtained – the two rotations are redundant with respect to one another.

Therefore it is perfectly legitimate to say that the rotation of the state of only one particle, either one in an indistinguishable way, 

\textit{drags an identical rotation of the state of the other.}

It should be noted that applying condition (iii) to either part of the system does not mean applying the identity to the other part.\footnote{Which would mean: (a) applying an infinitesimal rotation to one particle and the identity to the other, (b) symmetrizing the result and (c) repeating the cycle. This would bring in a Zeno effect freezing the evolution in its initial state (1) – see ref. [12].} In the current formulation, the transformation of this latter part must be \textit{disregarded}, since it is already determined by the transformation of the former part \textit{and} the watchdog effect. In a sense, either part, being indistinguishable, represents the whole. This might seem a peculiar way of applying quantum mechanics but it is tautologically true here.

In this idealized picture (the two-state particles), particle statistics symmetry can be seen as an interaction-free constraint (working by way of “interference”) between the states of the two particles, \textit{before} being a constant of motion. Of course symmetry is conserved by the propagation (3), but this is due to the fact that the propagation is shaped in a symmetric way by the watchdog effect described by (i) and (ii).

This work explores the possibility of porting the above, generalized, interpretation of particle statistics to the case of \textit{identical} particles hosted by \textit{distinguishable} sites, provided
that the whole system is in the ground state of a suitable Hamiltonian. Section III gives a purely mathematical porting. The physical model will be the subject of Section IV.

III. THE NOT GATE AS A PROJECTOR

We consider a simple NOT gate (fig. 1, gate and table #2). When used as a network (inverting) wire it will be called “link”.

Mathematically, the previous model is readily ported to a pair of qubits \( r \) and \( s \). The constrained subspace, spanned by the basis vectors of \( H_{rs} \) which satisfy the NOT gate, is:

\[
H^{(c)}_{rs} = \text{span} \{ |0\rangle_r |1\rangle_s , |1\rangle_r |0\rangle_s \}.
\]

These two qubits will be implemented in a way that makes them distinguishable (Section IV).

The constrained subspace, spanned by the basis vectors of \( H_{rs} \) which satisfy the NOT gate, is:

\[
\mathcal{H}_{rs} = \text{span}\{|0\rangle_r |0\rangle_s , |0\rangle_r |1\rangle_s , |1\rangle_r |0\rangle_s , |1\rangle_r |1\rangle_s\}.
\]

The two qubits should be prepared in an initial state already belonging to \( H^{(c)}_{rs} \):

\[
|\Psi (0)\rangle = \cos \vartheta |0\rangle_r |1\rangle_s + \sin \vartheta |1\rangle_r |0\rangle_s.
\]  

(4)

The evolution we are looking for, supposedly obtained by operating only on qubit \( r \) and disregarding qubit \( s \) (see Section IV), is the solution of the following system of simultaneous equations:

For all \( t \):

\[
\text{For all } t:
\]
\begin{itemize}
  \item[i)] $A_{rs} |\Psi (t)\rangle = |\Psi (t)\rangle$,
  \item[ii)] $\text{Max} \left| \langle \Psi (t) | \Psi (t - dt) \rangle \right|,$
  \item[iii)] $\text{diag}\rho_r (t) = \text{diag} \left\{ \text{Tr}_s \left[ |\Psi (t)\rangle \langle \Psi (t)\rangle \right] \right\} = \cos^2 [\vartheta + \varphi (t)] |0\rangle_r \langle 0\rangle_r + \sin^2 [\vartheta + \varphi (t)] |1\rangle_r \langle 1\rangle_r,$
\end{itemize}

where $|\Psi (t)\rangle$ is a free normalized vector of $H_{rs}: |\Psi (t)\rangle = \sum_{i,j \in \{0,1\}} \alpha_{ij}^{(t)} |i\rangle_r |j\rangle_s$, with $\sum_{i,j \in \{0,1\}} \left| \alpha_{ij}^{(t)} \right|^2 = 1$.

As readily seen, conditions (i) and (iii) yield:

$$|\Psi (t)\rangle = \cos \left[ \vartheta + \varphi (t) \right] |0\rangle_r |1\rangle_s + e^{i\delta} \sin \left[ \vartheta + \varphi (t) \right] |1\rangle_r |0\rangle_s,$$

where $\delta$ is an unconstrained phase; condition (ii), given the link initial state (4), sets $\delta = 0$ and keeps it frozen throughout the evolution. This yields to the unitary evolution:

$$|\Psi (t)\rangle = \cos \left[ \vartheta + \varphi (t) \right] |0\rangle_r |1\rangle_s + \sin \left[ \vartheta + \varphi (t) \right] |1\rangle_r |0\rangle_s. \quad (5)$$

We must assume that condition (iii) represents some physical operation performed only on qubit $r$, under conditions (i) and (ii) – exactly like in Section II. The plausibility of this assumption will be discussed in Section IV. This would mean that a transformation operated on qubit $r$ drags an identical transformation of qubit $s$. Mathematically, this is true:

$$\rho_s (t) = T_r, \left[ |\Psi (t)\rangle \langle \Psi (t)\rangle \right] = \sin^2 [\vartheta + \varphi (t)] |0\rangle_s \langle 0\rangle_s + \cos^2 [\vartheta + \varphi (t)] |1\rangle_s \langle 1\rangle_s;$$

with respect to condition (iii), 0 and 1 are interchanged because qubit $s$ is the NOT of qubit $r$.

Interestingly, if all $H_{rs}^{(c)}$ basis vectors occur with amplitudes different from zero in the initial state (4), namely if $\vartheta \neq 0, \pi$, condition (i) is redundant with respect to condition (ii). In this case, condition (ii) alone implies $\alpha_{00}^{(t)} = \alpha_{11}^{(t)} = 0$, and this already satisfies condition (i).

On the contrary, condition (i) is not redundant if $\vartheta = 0$ or $\pi$. For example, if $\vartheta = 0$, i.e. $|\Psi (0)\rangle = |0\rangle_r |1\rangle_s$, condition (ii) implies $\alpha_{00}^{(t)} = 0$ and $\left| \alpha_{01}^{(t)} \right|^2 = \cos^2 [\vartheta + \varphi (t)]$, as needed,
while $\alpha^{(t)}_{10}$ and $\alpha^{(t)}_{11}$ are only subject to the constraint $|\alpha^{(t)}_{10}|^2 + |\alpha^{(t)}_{11}|^2 = \sin^2[\vartheta + \varphi(t)]$. Thus, disregarding condition (i) would allow for the existence of the forbidden state $\alpha^{(t)}_{11} |1\rangle_r |1\rangle_s$.

It is worth noting that the same evolution (5) can be obtained by applying the unitary operator $Q_{rs}[\varphi(t)]$ to the overall state $|\Psi(t)\rangle$:

$$Q_{rs}[\varphi(t)] = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \varphi(t) & \sin \varphi(t) & 0 \\
0 & -\sin \varphi(t) & \cos \varphi(t) & 0 \\
0 & 0 & 0 & 1
\end{pmatrix},$$

with $|0\rangle_r |1\rangle_s \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}_r \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}_s$, $|1\rangle_r |0\rangle_s \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}_r \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}_s$.

$Q_{rs}[\varphi(t)]$ operates on the overall state in a non-factorizable way, since at any time $t$ the transformation undergone by qubit $r$ ($s$) is conditioned by the state of qubit $s$ ($r$), because of entanglement:

$$Q_{rs}[\varphi(t)] |0\rangle_r |1\rangle_s = \cos \varphi(t) |0\rangle_r |1\rangle_s + \sin \varphi(t) |1\rangle_r |0\rangle_s,$$

$$Q_{rs}[\varphi(t)] |1\rangle_r |0\rangle_s = -\sin \varphi(t) |0\rangle_r |1\rangle_s + \cos \varphi(t) |1\rangle_r |0\rangle_s.$$

There is so to speak a “hidden” conditional logic: finding this evolution — i.e. $Q_{rs}[\varphi(t)]$ — amounts to solving a logical problem. This becomes NP-complete when the link or the NOT gate belongs to a generic Boolean network.

We have thus ascertained a peculiar fact. Our “operation on a part” [this is just the mathematical condition (iii) for the time being], blind to its effect on the whole, performed together with continuous $A_{rs}$ projection, generates a unitary transformation which is, so to speak, wise to the whole state, to how it should be transformed without ever violating $A_{rs}$ (i.e. the link or the NOT gate). Of course $A_{rs}$ ends up commuting with the resulting overall unitary propagator, but because this is shaped by $A_{rs}$.

8 the generator of $Q_{rs}[\varphi(t)]$ commutes with $H_{rs}$, the interaction Hamiltonian between qubits $r$ and $s$ which will be introduced in the next Section.
IV. TURNING ON THE NOT GATE

A_{rs} projection can be shown to be an epiphenomenon of particle (fermionic or bosonic) statistics “turned on” in a special physical situation. In the following, we will adopt fermionic statistics.

We submit a couple of identical fermions 1 and 2 to a suitable Hamiltonian. Each fermion must have two compatible, binary degrees of freedom \( \chi \) and \( \lambda \). Just for the sake of visualization (things should remain more abstract), we can think that each fermion is a spin 1/2 particle which can occupy one of either two sites of a spatial lattice. \( \chi \) can be the particle spin (say that \( \chi = 0, 1 \) correspond to spin = down, up) and \( \lambda = r, s \) the label of the site occupied by the particle. For example, \(|0\rangle_1 |1\rangle_2 |r\rangle_1 |s\rangle_2 \) reads: particle 1 spin = down (0), particle 2 spin = up (1), particle 1 site = \( r \), particle 2 site = \( s \). There are 16 combinations like this, which make up the basis of the Hilbert space \( \mathcal{H}_{12} \). However, there are only six \textit{antisymmetrical} combinations (not violating fermion statistics) which make up the basis of the \textit{antisymmetrical} subspace \( \mathcal{H}_{12}^{(a)} \).

These basis vectors are represented in first and second quantization and, when there is exactly one particle per site, in qubit notation (\( \chi \) and \( \lambda \) stand respectively for the qubit eigenvalue and label), \(|0\rangle\) is the vacuum vector:

\[
|a\rangle = \frac{1}{\sqrt{2}} (|0\rangle_1 |1\rangle_2 - |1\rangle_1 |0\rangle_2) |r\rangle_1 |r\rangle_2 = a^\dagger_{0r} a^\dagger_{1r} |0\rangle,
\]

\[
|b\rangle = \frac{1}{\sqrt{2}} (|0\rangle_1 |1\rangle_2 - |1\rangle_1 |0\rangle_2) |s\rangle_1 |s\rangle_2 = a^\dagger_{0s} a^\dagger_{1s} |0\rangle;
\]

\[
|c\rangle = \frac{1}{\sqrt{2}} (|0\rangle_1 |0\rangle_2 (|r\rangle_1 |s\rangle_2 - |s\rangle_1 |r\rangle_2) = a^\dagger_{0r} a^\dagger_{0s} |0\rangle = |0\rangle_r |0\rangle_s ,
\]

\[
|d\rangle = \frac{1}{\sqrt{2}} (|1\rangle_1 |1\rangle_2 (|r\rangle_1 |s\rangle_2 - |s\rangle_1 |r\rangle_2) = a^\dagger_{1r} a^\dagger_{1s} |0\rangle = |1\rangle_r |1\rangle_s ,
\]

\[
|e\rangle = \frac{1}{2} (|0\rangle_1 |1\rangle_2 + |1\rangle_1 |0\rangle_2) (|r\rangle_1 |s\rangle_2 - |s\rangle_1 |r\rangle_2) = \frac{1}{\sqrt{2}} (a^\dagger_{0r} a^\dagger_{1s} + a^\dagger_{1r} a^\dagger_{0s}) |0\rangle.
\]

\[
|f\rangle = \frac{1}{2} (|0\rangle_1 |1\rangle_2 - |1\rangle_1 |0\rangle_2) (|r\rangle_1 |s\rangle_2 + |s\rangle_1 |r\rangle_2) = \frac{1}{\sqrt{2}} (a^\dagger_{0r} a^\dagger_{1s} - a^\dagger_{1r} a^\dagger_{0s}) |0\rangle.
\]

\( a^\dagger_{\chi\lambda} \) creates a particle of spin \( \chi \) in site \( \lambda \); creation/annihilation operators are subject to:

\[
\{a_i, a_j\} = 0, \quad \{a^\dagger_i, a_j\} = \delta_{i,j}. \]
particle per site, they generate a qubit algebra.

Now we introduce the Hamiltonian

$$H_{rs} = E_a |a⟩ ⟨a| + E_b |b⟩ ⟨b| + E_c |c⟩ ⟨c| + E_d |d⟩ ⟨d|$$

or, in second quantization,

$$H_{rs} = -(E_a a_0^r a_1^r a_0^s a_1^s + E_b a_0^s a_0^r a_0^s a_0^r + E_c a_0^r a_0^r a_0^s a_0^s + E_d a_1^r a_1^s a_1^r a_1^s),$$

with $E_a, E_b, E_c, E_d \geq E$ discretely above 0. This leaves us with two degenerate ground eigenstates:

$$|c⟩ = \frac{1}{\sqrt{2}} (|0⟩_r |1⟩_s + |1⟩_r |0⟩_s) \text{ and } |f⟩ = \frac{1}{\sqrt{2}} (|0⟩_r |1⟩_s - |1⟩_r |0⟩_s).$$

The generic ground state is thus:

$$|Ψ⟩ = α |0⟩_r |1⟩_s + β |1⟩_r |0⟩_s, \text{ with } |α|^2 + |β|^2 = 1. \quad (6)$$

Of course $|Ψ⟩$ satisfies $A_{rs} |Ψ⟩ = |Ψ⟩$, and belongs to $H_{rs}^{(c)}$ (Section III), a subspace of $H_{12}^{(a)}$.

Let $A_{12} |Ψ⟩ = \frac{1}{2} (1 - P_{12})$ be the usual antisymmetrization projector. Due to the above anticommutation relations:

$$A_{12} |0⟩_r |1⟩_s = |0⟩_r |1⟩_s \text{ and } A_{12} |1⟩_r |0⟩_s = |1⟩_r |0⟩_s, \text{ moreover}$$

$$A_{12} |0⟩_r |0⟩_s = |0⟩_r |0⟩_s \text{ and } A_{12} |1⟩_r |1⟩_s = |1⟩_r |1⟩_s,$$

without forgetting that $|0⟩_r |0⟩_s = |c⟩$ and $|1⟩_r |1⟩_s = |d⟩$ are excited states.

The link can be implemented by suitably operating on the ground state (6). We assume this to be initially given by eq. (4), repeated here:

$$|Ψ (0)⟩ = \cos θ |0⟩_r |1⟩_s + \sin θ |1⟩_r |0⟩_s.$$

The transformation of qubit $r$ under (the equivalent of) a continuous $A_{rs}$ projection is described by:

for all $t$: 

15
i) $A_{12} |\Psi (t)\rangle = |\Psi (t)\rangle$,

ii) $\text{Max } |\langle \Psi (t)| \Psi (t - dt)\rangle|$, 

iii) $\text{diag } \rho_r (t) = \text{diag } \{ \text{Tr}_s \[ |\langle \Psi (t)\rangle \langle \Psi (t)\rangle| \] \} = \
\cos^2 [\vartheta + \varphi (t)] |0\rangle_r \langle 0|_r + \sin^2 [\vartheta + \varphi (t)] |1\rangle_r \langle 1|_r$,

iv) $\langle \xi_{rs} (t) \rangle = \langle \Psi (t)| H_{rs} |\Psi (t)\rangle = 0,$

where $|\Psi (t)\rangle$ is a free normalized vector of $\mathcal{H}_{12}$.

The solution of the above equations is the desired link evolution (5), repeated here:

$$|\Psi (t)\rangle = \cos [\vartheta + \varphi (t)] |0\rangle_r |1\rangle_s + \sin [\vartheta + \varphi (t)] |1\rangle_r |0\rangle_s.$$  

Conditions (i), (ii) and (iii) mean that the link state undergoes a transformation [driven by (iii)] under continuous state vector reduction on the antisymmetric subspace $\mathcal{H}_{12}^{(a)}$. If $\vartheta \neq 0, \frac{\pi}{2}$, namely if the preparation (4) comprises all the basis vectors of $\mathcal{H}_{rs}^{(c)}$, condition (ii) alone keeps the link evolution inside $\mathcal{H}_{rs}^{(c)}$ (Section III). The link state remains ground and consequently the link expected energy $\langle \xi_{rs} (t) \rangle$ is always zero. By excluding $\vartheta = 0, \frac{\pi}{2}$, condition (iv) is a consequence of the former conditions.

Mathematically, conditions (i) and (iv) give the constraint $A_{rs} |\Psi (t)\rangle = |\Psi (t)\rangle$. In conclusion the above conditions (i) through (iv) (which imply interpreting fermionic antisymmetry $A_{12}$ as continuous projection on the antisymmetric subspace) are equivalent to condition (i) through (iii) of Section III. This gives in fact the evolution (5).

Let us see how we could operate (in principle) on the state of qubit $r$, in order to implement the driving condition (iii). Since conditions (i) through (iv) generate (under the counterfactual interpretation of particle statistics) the projector $A_{rs}$, operating on qubits $r$ and $s$, the state of qubit $s$ is redundant with respect to the state of qubit $r$. In particular, operating only on qubit $r$ does not mean applying the identity to qubit $s$, it means disregarding it (we are now inside that “peculiar way of applying quantum mechanics” highlighted in Section II).
In order to operate on qubit $r$, we introduce the one-qubit Hamiltonian $H_r = E_r a_0^r a_0^r$ of ground state $|1\rangle_r$ and excited state $|0\rangle_r$. $H_r$ and $H_{rs}$ commute and the total expected energy of the system is $\langle \Psi (t) | (H_{rs} + H_r) | \Psi (t) \rangle$. This is conveniently split into a link internal energy $\langle \xi_{rs} (t) \rangle = \langle \Psi (t) | H_{rs} | \Psi (t) \rangle$ and a qubit $r$ internal energy $\langle \xi_r (t) \rangle = \langle \Psi (t) | H_r | \Psi (t) \rangle = E_r \cos^2 [\vartheta + \varphi (t)]$. One can see that the latter is in one-to-one correspondence with $\text{diag } \rho_r (t)$: the driving condition (iii) can thus be implemented by changing the internal energy of qubit $r$, provided that $\langle \xi_{rs} (t) \rangle = 0$ (see further below).

Qubit $r$ is now put in interaction with a heat-bath. This could also excite the link, yielding $\langle \xi_{rs} (t) \rangle = \langle \Psi (t) | H_{rs} | \Psi (t) \rangle > 0$. In order to take this possibility into account, it is convenient to split $|\Psi (t)\rangle$ into two orthogonal components:

$$|\Psi (t)\rangle = \alpha (t) |\Psi' (t)\rangle + \beta (t) |\Psi'' (t)\rangle,$$

with $|\alpha (t)|^2 + |\beta (t)|^2 = 1$, such that $\langle \Psi' (t) | H_{rs} | \Psi' (t) \rangle = 0$, $\langle \Psi'' (t) | H_{rs} | \Psi'' (t) \rangle = \langle \xi_{rs} (t) \rangle > 0$, $|\Psi' (t)\rangle$ belongs to $H_{rs}^{(c)}$ and $|\Psi'' (t)\rangle$ belongs to the orthogonal subspace.

The former component is hosted, so to speak, in a “good” Everett universe where $\langle \xi_{rs} (t) \rangle = 0$, $A_{rs}$ is turned on and the link works as required: the transformation performed on qubit $r$ drags a corresponding transformation of qubit $s$. From the standpoint of the link internal energy, this occurs in a reversible way with no free energy dissipation (nor increase).

The latter component is hosted in a “bad” Everett universe where the link state is a linear combination of the excited states $|0\rangle_r |0\rangle_s$ and $|1\rangle_r |1\rangle_s$ and the link (or NOT gate) logical constraint is not satisfied.

$\beta (0)$ was zero in the preparation (4). Let $T$ be the heat-bath temperature. We assume that $k_B T << E \leq E_a$, $E_b$, etc.. Thus $\beta (t)$ will remain close to zero after the heat-bath has been turned on. Therefore, the probability $|\alpha (t)|^2$ that measurement finds the link in the “good” Everett universe should remain very high.

We further assume that $E_r << E$ is of the order of $k_B T$. $\langle \xi_r (t) \rangle$ is driven to relax close to zero by reducing $T$. At a time $\tau$ such that $\langle \xi_r (\tau) \rangle = \cos^2 [\vartheta + \varphi (\tau)] \approx 0$, and in the “good”
Everett universe, the link state has the form $|\Psi'(\tau)\rangle \simeq |1\rangle_r |0\rangle_s$. The time required to reach $\langle \xi_r(\tau) \rangle \simeq 0$ is uniquely determined by the relaxation process of qubit $r$ – independently of the rest of the link.

This way of driving the link evolution, whose plausibility is based on heuristics for the time being, will be used to drive the evolution of the entire network (Section VI).

V. THE GENERIC GATE

To reduce notation, we shall work with a gate of three (coexisting) qubits, the minimum required to have a conditional logic. The result will be clearly generalizable to any number of qubits. We will consider the logically irreversible XOR gate, of inputs $t, u$, output $v$, and truth table:

$$
\begin{array}{c|c|c}
 t & u & v \\
 0 & 0 & 0 \\
 0 & 1 & 1 \\
 1 & 0 & 1 \\
 1 & 1 & 0 \\
\end{array}
$$

Since time is orthogonal to the network lay-out, the network evolution can be reversible even if gates are logically irreversible.

The gate mathematical model will be introduced first. Let

$$
\mathcal{H}_{tuv} = \text{span} \{ |0\rangle_t |0\rangle_u |0\rangle_v , |0\rangle_t |1\rangle_u |1\rangle_v, ... , |1\rangle_t |1\rangle_u |1\rangle_v \} 
$$

be the eight-dimensional Hilbert space of the gate three qubits. The constrained subspace (whose basis vectors map the rows of the gate truth table) is four-dimensional:

$$
\mathcal{H}^{(c)}_{tuv} = \text{span} \{ |0\rangle_t |0\rangle_u |0\rangle_v , |0\rangle_t |1\rangle_u |1\rangle_v, |1\rangle_t |0\rangle_u |1\rangle_v , |1\rangle_t |1\rangle_u |0\rangle_v \} ;
$$

The projector $A_{tuv}$ from $\mathcal{H}_{tuv}$ on $\mathcal{H}^{(c)}_{tuv}$ is defined by the (eight) equations:
\[ A_{tuv} |\chi_\alpha\rangle_t |\chi_\beta\rangle_u |\chi_\gamma\rangle_v = |\chi_\alpha\rangle_t |\chi_\beta\rangle_u |\chi_\gamma\rangle_v \] when \( t^t u^u v^v \) is a row of the gate truth table,

\[ A_{tuv} |\chi_\alpha\rangle_t |\chi_\beta\rangle_u |\chi_\gamma\rangle_v = 0 \] when \( t^t u^u v^v \) is not a row of the gate truth table (like e.g. \( t^0 u^1 v^0 \)).

Let us denote by \( |\Psi (t)\rangle \) the evolution of the gate state. \( |\Psi (t)\rangle \) is continuously projected on \( \mathcal{H}_{tuv}^{(c)} \). This is represented by:

for all \( t \):

i) \( A_{tuv} |\Psi (t)\rangle = |\Psi (t)\rangle \),

ii) \( \text{Max } |\langle \Psi (t) |\Psi (t - dt)\rangle| \),

where \( |\Psi (t)\rangle \) is an otherwise free normalized vector of \( H_{tuv} \); in fact conditions (i) and (ii) do not yet determine \( |\Psi (t)\rangle \): the “missing” driving condition (iii) will come out from the behaviour of the rest of the network (Section VI).

The extension to a gate with any number of qubits is obvious.

Now we should build the gate model Hamiltonian \( H_{tuv} \). We need three lattice sites \( t, u, v \) and three identical spin \( \frac{1}{2} \) particles 1, 2, 3. The unwanted states should all be excited eigenstates of \( H_{tuv} \). This has therefore the form:

\[ H_{tuv} = ... - E_\nu a_0^\dagger a_1^\dagger a_0 a_0 a_1 a_0 a_0 ... - E_\gamma a_0^\dagger a_1^\dagger a_0^\dagger a_0 a_1 a_0 a_0 ... \]

The first term corresponds to a state where each site is not occupied by exactly one particle. The second term corresponds to a state where each site is occupied by exactly one particle but is not a basis vector of \( \mathcal{H}_{tuv}^{(c)} \), etc.

Conditions (i), (ii) and (iv) of Section IV become now:

for all \( t \):

i) \( A_{123} |\Psi (t)\rangle = |\Psi (t)\rangle \), where \( A_{123} = \frac{1}{6} (1 - P_{12} - P_{13} - P_{23} + P_{13} P_{12} + P_{23} P_{12}) \) is the antisymmetrization projector,
ii) $\text{Max } |\langle \Psi (t) | \Psi (t - dt) \rangle|$, 

iv) $\langle \xi _{tuv} (t) \rangle = \langle \Psi (t) | H_{tuv} | \Psi (t) \rangle = 0$, 

where $|\Psi (t)\rangle$ is an otherwise free normalized vector of $\mathcal{H}_{123}$ (similar to $\mathcal{H}_{12}$ of Section IV).

The gate initial state should be a linear combination of all $\mathcal{H}^{(c)}_{tuv}$ basis vectors. Consequently, because of (ii), the gate state will remain in $\mathcal{H}^{(c)}_{tuv}$: condition (iv) is thus a consequence of (i) and (ii). (i) and (iv) are equivalent to $A_{tuv} |\Psi (t)\rangle = |\Psi (t)\rangle$. $A_{tuv}$ is thus “turned on” when $\langle \xi _{tuv} (t) \rangle = 0$.

VI. THE NETWORK

We must put together the stand-alone network elements of the previous Sections. The network lattice is the union of the lattices of the individual N.E. Of course not all sites are disjoint: for example a gate and a connected link share one lattice site. All N.E. Hamiltonians $H_i$ (where $i$ is a multiple index running over $de, abcd$, etc. – fig. 1) are diagonal in $\mathcal{H}_N$ basis thus pairwise commuting: $\forall i,j : [H_i, H_j] = 0$.

The overall network Hamiltonian is: $H_N = \sum_i H_i$, and the network expected energy $\langle \xi _N (t) \rangle$ is the sum of the expected energies of all N.E.: $\langle \xi _N (t) \rangle = \sum_i \langle \Psi (t) | H_i | \Psi (t) \rangle$, where $|\Psi (t)\rangle$ is the network state (i.e. a linear combination of the network basis vectors – each a tensor product of all qubit eigenstates).

Therefore, if $|\Psi (t)\rangle$ is a network ground state (this is highly degenerate, see Section I), then $\langle \xi _N (t) \rangle = 0$ and consequently $\langle \Psi (t) | H_i | \Psi (t) \rangle = 0$ for all $i$. Fermion statistics and these latter conditions work as conditions (i) and (iv) of Section V: all $A_i$ projectors are “turned on” when $\langle \xi _N (t) \rangle = 0$.

Of course $\forall i,j : [A_i, A_j] = 0$. $A_N = \Pi_i A_i$ projects $\mathcal{H}_N$ on the constrained subspace $\mathcal{H}^{(c)}_N$ spanned by the network basis vectors which satisfy all N.E. ($\mathcal{H}^{(c)}_N$ is the intersection of all
To sum up, any transformation of the network ground state $|\Psi(t)\rangle$ satisfies the following conditions:

for all $t$:

i) $A_N |\Psi(t)\rangle = |\Psi(t)\rangle$,

ii) $Max |\langle\Psi(t)|\Psi(t - dt)\rangle|$, 

iv) $\langle \xi_N (t) \rangle = 0$,

where $|\Psi(t)\rangle$ is an otherwise free vector of $\mathcal{H}_N$.

Of course, any tensor product of the network qubit eigenstates satisfying condition (i) satisfies all the N.E., not necessarily the input and output constraints.

The satisfiability problem requires checking whether such constraints can also be satisfied. Under the assumption that the network is satisfiable together with the input-output constraints, we proceed as follows.

1) We assume, with no restriction, that only one output qubit, say $z$, is constrained: $\rho_z = |1\rangle_z \langle 1|_z$. This output constraint is temporarily removed. Whereas we introduce one-qubit Hamiltonians in order to detain the constrained inputs in the required values. Let $h$ be an input qubit constrained by $\rho_h = |0\rangle_h \langle 0|_h$ ($\rho_h = |1\rangle_h \langle 1|_h$). This constraint is satisfied in the ground state of the one-qubit Hamiltonian $H_h = E_h a_h^{\dagger} a_h$ ($H_h = E_h a_h^{\dagger} a_0$), with $E_h$ discretely above zero (of the same order of $E$). We should note that these Hamiltonians commute with all the other N.E. Hamiltonians.

2) We wish to prepare the network in a state $|\Psi(0)\rangle$ which is a linear combination of all $\mathcal{H}_N$ basis vectors satisfying the input constraints, all the N.E., but not necessarily the

---

9We should note that the permutation of two identical particles belonging to any two different N.E. cannot set any further particle statistics constraint, since the two N.E. Hamiltonians commute.
output constraint (not implemented until now). This means satisfying the equations:

- \( \forall h : \langle \Psi (0) | H_h | \Psi (0) \rangle \), where \( h \) ranges over the labels of the input qubit and N.E. Hamiltonians;

- \( \forall k : \rho_k (0) = a_k |0\rangle_k \langle 0|_k + b_k |1\rangle_k \langle 1|_k \), with \( a_k, b_k \neq 0 \), where \( k \) ranges over the labels of the unconstrained input qubits; we should note that the coherence elements of any qubit density matrix are zero if, without any restriction, any two qubits are connected by a link (although this is not the case of fig.1).

The above conditions yield:

\[
|\Psi (0)\rangle = \cos \vartheta \sum_i \alpha_i |\text{prep}_i\rangle |0\rangle_z + \sin \vartheta \sum_j \beta_j |\text{prep}_j\rangle |1\rangle_z .
\]

with \( \sum_i |\alpha_i|^2 = \sum_j |\beta_j|^2 = 1 \); \( |\text{prep}_i\rangle, |\text{prep}_j\rangle \) denote tensor products of all qubit eigenstates but qubit \( z \).

The ground state (7) can be reached through quantum annealing, namely quantum ground state computation\(^{[11,19]}\). Since there is no output constraint, this amounts to solving a problem polynomial in network size. Therefore, for short, without entering into any detail, we assume that the time required to reach any desired probability of finding the network in the ground state (7) is polynomial in network size.

In the right side of eq. (7), each term \( i \) satisfies the input constraints, all N.E., not the output constraint, whereas each term \( j \) satisfies also the output constraint and is therefore a network solution.

\( \sin^2 \vartheta \) depends on the network and must decrease exponentially with network size, otherwise the problem would not be hard (solutions are “exponentially rare”). Of course:

\[
\text{diag}_z \rho (0) = \cos^2 \vartheta |0\rangle_z \langle 0|_z + \sin^2 \vartheta |1\rangle_z \langle 1|_z ,
\]

with \( \sin^2 \vartheta \) “extremely small”.

3) We operate now on qubit \( z \) as we did in Section IV, so that at some time \( \tau \):
diagρ_z (τ) ≃ |1⟩_z ⟨1|_z.

This is done by introducing the one-qubit Hamiltonian \( H_z = E_z a_z^\dagger a_z \), with \( E_z << E \) and ground state \(|1⟩_z\), and by putting the qubit in interaction with a heat-bath whose temperature is lowered down close to zero. The internal energy of qubit \( z \) is thus \( ⟨ξ_z (t)⟩ = ⟨Ψ (t)| H_z |Ψ (t)⟩ \). At some time \( t = τ \) ("after" relaxation), we have: \( ⟨ξ_z (τ)⟩ ≃ 0 \).

As in Section IV, the network evolution can be split into two orthogonal components:

\[
|Ψ (t)⟩ = \alpha (t) |Ψ ′(t)⟩ + \beta (t) |Ψ ″(t)⟩;
\]

- \( |Ψ ′(t)⟩ \) is ground (with reference to the network and the constrained input qubits internal energy, see Section IV), namely \( \sum_h ⟨Ψ ′(t)| H_h |Ψ ′(t)⟩ = 0 \), with \( h \neq z \) ranging over all the N.E. and the constrained input Hamiltonians; this state is hosted in the "good" Everett universe of amplitude \( \alpha (t) \);

- \( |Ψ ″(t)⟩ \) is excited and belongs to the subspace orthogonal to \( \mathcal{H}_N^c \); it is hosted in the "bad" Everett universe of amplitude \( \beta (t) \).

We assume to be in the "good" Everett universe (until different notice). Qubit \( z \) relaxation, under the continuous projection (i) and (ii) drives the transformation of the overall network state:

\[
|Ψ (t)⟩ = \cos [θ + φ (t)] \sum_i \alpha_i |prep_i⟩ |0⟩_z + \sin [θ + φ (t)] \sum_j \beta_j |prep_j⟩ |1⟩_z;
\]

The driving condition is \( E_z \cos^2 [θ + φ (t)] = ⟨ξ_z (t)⟩ \); we should keep in mind that, in current assumptions, qubit \( r \) relaxation is independent of the rest of the network. Condition (ii) keeps all \( \alpha_i \) and \( \beta_j \) unaltered throughout the evolution, as readily checked. At time \( t = τ \):

\[
|Ψ ′(τ)⟩ \simeq \sum_j \beta_j |prep_j⟩ |1⟩_z.
\]

By measuring the network at time \( τ \), and by repeating the overall process for a sufficient number of times, a straightforward application of probability theory shows that the number
of repetitions required to check whether the network is satisfiable (with any desired probability that this check is correct) grows polynomially with network size. Furthermore, $\tau$ is independent of network size. This would mean \( \text{NP-complete} = \text{P} \).

However, this result can be completely vanified if the probability of finding the network in the “good” universe decreases exponentially with network size. To find out how this probability decreases, we should compute how the qubit $r$-heat-bath interaction affects $\alpha(t)$ and $\beta(t)$. Doing this would require a level of formalization beyond the reach of the current study.

Anyway, one ingredient of the “quantum computation speed-up” consists of substituting interaction, which requires time, with interference, which requires no time. Particle statistics, in the current interpretation, works as interference and implements most of the network logical constraints. This, in a still generic way, should justify the hope of achieving a computation speed-up by using particle statistics.

**VII. DISCUSSION**

We have highlighted an alternative way of approaching quantum computation. With respect to time-sequential computation (Section I), it presents some significant differences:

- logical constraints are simultaneous in time rather than being mapped on the time evolution of the computation process;

- there is thus an analogy between such constraints and the simultaneous constraints (also logical in character) established by particle statistics;

- in order to make this analogy explicit, we must introduce a blunt interpretation of particle statistics. A particle statistics symmetry would no more be a passive constant of motion which does nothing to a unitary evolution, but an active watchdog effect shaping that evolution;

- the NP-complete SAT problem is in a way native in this approach.
The counterfactual interpretation of particle statistics might be interesting in itself. Checking it might not be out of reach. The link behaviour (Section IV) is similar to exchange interaction. One should devise an experimental situation, showing that an operation on one part drags the state of the other part according to the formulation propounded.

We should finally note that this formulation of particle statistics can be represented in a two-way (advanced and retarded) propagation model. This has been done in ref. [12] and can be outlined as follows. We have been dealing with state vector reduction on a predetermined subspace, namely always on the constrained subspace $H^c$, never on the orthogonal subspace violating the constraint. Although elusively (since we are in counterfactual reasoning), this means an evolution affected by a condition coming from the future.

Interestingly, a two-way propagation model of partial state vector reduction can also justify the speed-up of time-sequential computation. See ref. [12].

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Space Deployed Boolean Network

Fig. 1