Two-Dimensional Dynamics of a New Configuration of Continuous Cislunar Payloads Transfer System

Nai-ming Qi, Yong Yang, Chang-zhu Wei,† and Jun Zhao

School of Astronautics, Harbin Institute of Technology, Harbin 150001, China

Based on the space tether, the concept of a motorized momentum exchange tether (MMET) using the momentum exchange principle has been proposed for many years. In this paper, a new symmetric configuration of MMET, referred to as HEX-MMET, is proposed. HEX-MMET has some unique advantages, such as higher payload transfer efficiency and shorter orbit reboost period compared to the MMET. In order to study the dynamic characteristic of HEX-MMET, using traditional Lagrange methods, a two-dimensional (2D) dynamic model of HEX-MMET is presented. Then, the numerical solutions based on MATLAB were implemented. The simulation results show that external torque has little evident influence on the orbital parameters, such as the radius of HEX-MMET’s center-of-mass (COM) and the true anomaly; however, there is an apparent effect on the attitude parameters, such as the angle between the line of two payloads and the line of local gravity gradient. Furthermore, the gravity gradient torque also affects the attitude of HEX-MMET periodically.

Key Words: Momentum Exchange, Payload Transfer, Tether, Dynamics, Cislunar

I. Introduction

In the coming several decades, with the development of exploring interplanetary space, the need for establishing permanent colonies in outer space, such as the moon or planets like Mars, etc. will increase significantly sooner or later. As a result, one of the problems of transferring massive materials (food, water, oxygen, etc.) to those colonies and the resources from these locations back to Earth must be solved. However, the disadvantages of traditional rocket systems, for example, the low transfer efficiency, high cost and non-reusability, are excessive. These missions will require innovative, propulsion-saving approaches and reusable vehicles. A space tether based on the momentum exchange principle and electrodynamic force has many unique advantages and is the subject of the paper.

In 1985, Carroll, Jeffrey and Kornuta, and Johnson et al. illustrated the possible applications of a tether, such as an electrodynamic tether to reboost orbit or remove debris, especially the application of a momentum exchange tether for trajectory payloads. Furthermore, Cartmell and McKenzie did some comprehensive research about space tethers. It was shown that the achievable orbit change scales with the tether length and tethered release operation could be a suitable choice. Additionally, the energy and momentum balance was mentioned in this literature.

Robert proposed the concept of momentum-exchange/electrodynamic-reboost (MXER) in 1980. MXER used the momentum exchange principle and combined the electrodynamic reboost tether to realize the full-reusability and lower propulsion cost. However, the MXER system has two shortcomings: firstly, MXER could only transfer one payload each time; and secondly, because of the payload being released from the tether tip, the chief satellite would lose partial energy and its orbit would be lowered partially. After this, MXER would have to reboost its orbit for a long time. In view of this, based on the MXER, a new kind of symmetrical configuration of space tether, denoted as the motorized momentum exchange tether (MMET) was proposed for the first time, and has been a hot topic over the last two decades and further investigated by Ziegler and Cartmell. In this literature, the conceptual design of the MMET system was proposed, comprising a physical model, including the propulsion tether, payload mass, launcher module and outrigger system. Compared to MXER, MMET has some advantages: since the structure of the MMET is symmetrical with its chief satellite, each time MMET transfers two payloads, one orbit needs to be lowered and the other needs to be reboosted. As a result, the chief satellite loses little energy and its orbit reboost period shorter than MXER.

In the almost two-decade history of studying the MMET, many scholars have conducted extensive work in theory and testing. Alpatov et al. studied the dynamics of tethers rigorously. In this literature, the equations of motion for flexible tethers with end masses and massless and massive variations were presented. Similar research was also done by Chen, Ismail, etc.

Furthermore, some programmes and research projects on tethers and the MMET system have been summarized. The three-dimensional dynamics of the system have been modelled by Lagrange equations; however, the literature does not consider the influence of the axial elastic effects or torsional oscillation on the dynamics of the system. In order to make the dynamic model of the MMET system more realistic, Chen and Cartmell presented a dynamic model of MMET with axial and torsional oscillation and did some re-
search about hybrid sliding model control strategies for the MMET.\textsuperscript{18,19)\textsuperscript{18}} Furthermore, Mouterde et al. investigated the feedback linearized control strategies for MMET.\textsuperscript{19)\textsuperscript{19}}

Recently, Murray and Cartmell did some different studies about the design of the MMET’s moon-tracking orbit.\textsuperscript{20,21)\textsuperscript{20}} It was illustrated that, if the nodes of the moon’s orbit could be tracked effectively and correctly, then an earth motorized momentum exchange tether (EMMET) system could launch payloads to the node of the moon’s orbit each time when the moon arrived at this point. Symmetrically, on the moon’s orbit, a similar Lunavator could achieve the same progress to deliver payloads from the moon to Earth continuously.

Though the MMET has some advantages compared to MXER, there are still some hidden deficiencies. Firstly, as we know, much debris exists in LEO orbit exists, which would endanger the propulsion tether if they collide with each other. If one of the propulsion tethers is cut down, the system would deviate its orbit. Additionally, Even if debris did not exist, and the MMET is intact in its useful life period, and the payload transfer efficiency is limited. After the MMET transfers two payloads (one to the moon and the other back to the Earth), it will have to reboost its orbit for a long time. In this paper, a new configuration of motorized momentum exchange tether, denoted as HEX-MMET is proposed for the first time. HEX-MMET is comprised of the chief satellite (denoted as \( O_M \)), six rigid rods with the same parameters (same length, cross-section area, and density) and six capture systems. Compared to the MMET, HEX-MMET has two obvious advantages:

- HEX-MMET’s security is better than that of MMET because there are three tether couples, and any two of those couples can be used as backup.
- The efficiency of transferring payloads by HEX-MMET is much higher than that by MMET. In each period, HEX-MMET has three chances to rendezvous with the coming payloads. Furthermore, the orbit adjustment period of HEX-MMET will be much shorter. The relative theoretical analysis is presented in Section 3.

The structure of this paper is described as follows. A brief introduction to space tethers and MMET is presented in Section 1. In Section 2, we explain why we chose hexagonal configuration of MMET, and analyze the advantages of HEX-MMET. In Section 3, the concept and schematic of the MMET and HEX-MMET are presented briefly, and the process and efficiency of payload transfer operation using HEX-MMET is analyzed theoretically. In Section 4, the two dimensional (2D) rigid dynamics model of HEX-MMET is presented by Lagrange methods. In Section 5, we solve the dynamic equations of the HEX-MMET by numerical simulations and investigate the coupling influences between the attitude and orbit compared to the MMET. Finally, conclusions are given in the last section.

2. Explanations for Choosing Hexagonal Configuration

We know that there are many different configurations with different numbers of tethers, such as MMET with four tethers, with eight tethers and so on. The schematics of the two typical configurations of MMET with four tethers and eight tethers are shown in Fig. 1. We can observe that for Quadra-MMET, there are only two couples of rigid rods. The distance between the adjacent rods tip points is \( \sqrt{2} \) times the length of the rods. For such long, strong tethers, it is very difficult to prevent them from “bulging” outward and affecting the dynamics of the satellite system. For the configuration of Octo-MMET with eight rigid rods, the advantage is that Octo-MMET has much more redundancy and much higher efficiency and can control the rotational angular velocity correctly. However, the total mass of the system is too heavy and it is too hard to deploy.

Accordingly, the configuration of a HEX-MMET with six rigid rods with the length of connecting high-strength tethers and the total mass of the system, as well as the deploying/retrieving difficulties, become the research topic of this paper.

3. Concepts of MMET and HEX-MMET and Payload Transfer Efficiency Analysis

The schematic of a MMET is shown in Fig. 2.\textsuperscript{10} We can observe that the MMET contains four main parts: (1) the chief satellite, which includes the control system, stator and rotor, (2) the outrigger system connected to the stator, with the mass to provide balance and stabilize the MMET, (3) the tether propulsion system, connected to the rotor and providing the payload required velocity increment for the orbital transfer, and (4) the two payloads. After the propulsion tether system is completely motorized, the outrigger will be retrieved in the chief satellite system.

A schematic of the full procedures for the MMET to transfer the payloads is shown in Fig. 3. There are three main steps in the procedures. Step 1, a payload is injected into an orbit which is immediately below the perigee of the MMET’s orbit, at the same time, one of the end effectors of the MMET reaches near the point and controls the effector to capture the payload while the other end-point effector of the MMET reaches the upper point of its orbit to rendezvous with the other payload from the moon. Step 2, the chief satellite’s orbit needs to be modified by the control system (by the electrodynamic force of the propulsion tether). Additionally, after some circles orbiting Earth, when the MMET reaches the perigee, the two payloads’ location have switched, the payload from Earth to the moon reaches the
Then, the mass of a high-strength tether made of CNTs is

given by

\[ m_{\text{CN}} = \frac{F_{\text{CN}} l_{\text{CN}}}{\sigma_{\text{CN}}} \]

where the subscript “\( \text{CN} \)” refers to the composite “CNTs.” For HEX-MMET, the angle between the adjacent rigid rods is 60 degrees, if the lengths of the rigid rods are \( l = 10^6 \) m, the length of the high-strength tether is \( l_{\text{CN}} = 10^6 \) m, \( F = 10^3 \) N, the mass \( m_{\text{CN}} = 0.06585 \) kg, the mass is very small compared to the total mass of the HEX-MMET. Furthermore, because we presume that the six rigid rods have no bending deformation or axial elasticity deformation, the lateral force \( F \) due to those rods is also very small. So the mass of these high-strength tethers can be overlooked.

Furthermore, when the two payloads move close to the perigee of each trajectory, the HEX-MMET needs to reach its perigee simultaneously. In general, the positions of the payloads will have little position error relative to the effector. At this moment, the effector can rotate around the tether tip and adjust its length to reach the payload and softly rendezvous with it.

We presume that the parking orbit of the HEX-MMET is elliptic. The velocity of the chief satellite at perigee is given by Liu and Zhao\(^{23,24} \)

\[ \upsilon_p = \sqrt{\frac{2 \mu}{r_p}} \sqrt{\frac{r_a}{r_a + r_p}} \]

where \( \mu = 3.988 \times 10^{14} \) m\(^3\)/s\(^2\) refers to the Earth’s gravitation constant, and \( r_a \) and \( r_p \) refer to the apogee and perigee of the initial parking orbit, respectively. The velocity that the payload requires to be injected into the cislunar transfer trajectory at the upper tip of the rigid rod needs is expressed by

\[ \upsilon_{p} = \sqrt{\frac{2 \mu}{r_{p'}}} \sqrt{\frac{R_m}{R_m + r_{p'} + l}} \]

where \( l \) refers to the length of the rigid rod and \( R_m = 3.844 \times 10^8 \) m refers to the radius of the moon orbits around Earth (the moon’s orbit is set to be a circle). Then, we can get the needed angular velocity of the HEX-MMET as follows.

\[ \omega_p = \psi = \frac{1}{l} \sqrt{\frac{2 \mu}{r_{p'} + 1}} \left( \frac{R_m}{R_m + r_{p'} + l} \right) - \frac{2 \mu}{r_p} \left( \frac{r_a}{r_a + r_p} \right) \]

On the other hand, while HEX-MMET orbits Earth \( m \) times, at the same time, the rigid rod must rotate around the chief satellite for \( n + 1/2 \) circles to ensure the line between the two payloads are co-linear with the local gravity gradient and the two payloads exchange position. In other words, the needed angular velocity \( \omega_p \) must also satisfy the equation as follows.

\[ \omega_p = \frac{2n + 1}{2m} \sqrt{\frac{\mu}{a^3}} \quad (m, n \in \text{positive integers}) \]

where \( a \) refers to the semi-major axis of the parking orbit. If the eccentricity of the parking orbit is \( e = 0.25 \), \( r_p = \]

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**Fig. 2.** Schematic of a MMET.\(^{10} \)

**Fig. 3.** Schematic of a MMET transferring payloads.

**Fig. 4.** Sketch of a HEX-MMET.
6.728 × 10^6 m, when HEX-MMET orbits Earth for 14 times, and the rigid rod rotates around the chief satellite for 396 times, the captured payloads have one chance to be released. After that, both the HEX-MMET and MMET need an interval to prepare for the next mission. For the HEX-MMET, the interval $\Delta t_{H}$ can be obtained by

$$\Delta t_{H} = \frac{(2p + k/3)\pi}{\bar{\omega}_{H,\psi}}$$

or

$$\Delta t_{H} = \frac{(2p + k/3 + 1)\pi}{\bar{\omega}_{H,\psi}},$$

(k = 0, 1, 2)

For the MMET, the interval $\Delta t_{M}$ can also be given by

$$\Delta t_{M} = \frac{2q\pi}{\bar{\omega}_{M,\psi}}$$

or

$$\Delta t_{M} = \frac{(2q + 1)\pi}{\bar{\omega}_{M,\psi}}$$

where $p$ and $q$ refer to the orbits of HEX-MMET and MMET in the interval and are positive integers, respectively. $\bar{\omega}_{H,\psi}$ and $\bar{\omega}_{M,\psi}$ refer to the mean angular velocities of HEX-MMET and MMET rotating around their chief satellites. It can be analyzed comparatively that the HEX-MMET has many more chances to prepare for the next mission than that of MMET.

Furthermore, in the process of capturing/releasing the payloads, the HEX-MMET loses only a little energy and its orbit will change slightly. In the interval of modifying the position of the two payloads (changing the relative positions while releasing the payloads), the rigid rods become the conductors to generate power and electrodynamic force to reboost the orbit and supply the energy for the satellite to elongate the service life of the HEX-MMET. However, the intention of this paper is to study the dynamics of the HEX-MMET compared to the MMET, therefore, more details about reboosting the orbit are not discussed.

4. Coordinates and Dynamics of HEX-MMET

Before building the 2D rigid dynamic model and motion equations of the HEX-MMET, we need to set the following relevant hypotheses:

1) The two payloads on the tether tips have the same shape and weight;

2) The bending deformation and axial elasticity deformation of the tether couples are ignored;

3) Third-body gravitational force and Earth oblateness are neglected, meaning that the forces acting on the HEX-MMET are only the gravitational force from the Earth and tension from the tethers.

4) The out-of-plane angle is zero. In other words, it is presumed that the rotational plane of the HEX-MMET coincides with the orbital plane.

Based on the above assumptions, some required coordinate systems are defined as follows:

1) For the geocentric equatorial inertial coordinate $O_E XYZ$ shown in Fig. 5, the origin is the Earth’s center-of-mass (COM), the axis $O_E X$ points in the direction of the vernal equinox in Earth’s equatorial plane, the axis $O_E Y$ is perpendicular to axis $O_E X$ in the Earth’s equatorial plane, and axis $O_E Z$ follows the right-handed rule about axis $O_E X$

and axis $O_E Y$. The unit vectors of each axis are presented as $\hat{i}, \hat{j}$ and $\hat{k}$.

2) For orbital plane coordinate $O_3x_3y_3z_3$, the origin refers to the Earth’s COM, axis $O_3x_3$ points to the perigee of HEX-MMET, $O_3y_3$ is perpendicular to axis $O_3x_3$ and lies in the orbital plane, and axis $O_3z_3$ follows the right-handed rule of the coordinate systems. The unit vectors of each axis are presented as $\hat{i}_3, \hat{j}_3$ and $\hat{k}_3$.

3) For movement coordinate $O_{M3}x_{o3}y_{o3}z_{o3}$, the origin is the COM of the HEX-MMET, axis $O_{M3}x_{o3}$ lies along the direction of HEX-MMET’s radius vector, axis $O_{M3}y_{o3}$ is perpendicular to axis $O_{M3}x_{o3}$ and lies in the orbital plane, and axis $O_{M3}z_{o3}$ follows the right-handed rule of the coordinate system. The unit vectors of each axis of $O_{M3}x_{o3}y_{o3}z_{o3}$ are given as $\hat{x}_o$, $\hat{\theta}_o$ and $\hat{n}_o$.

4) For body coordinate $O_{M3}x_{b3}y_{b3}z_{b3}$, the origin is also HEX-MMET’s COM, axis $O_{M3}x_{b3}$ lines along the direction of tether-span from payload $P_1$ to payload $P_4$, axis $O_{M3}y_{b3}$ is perpendicular to axis $O_{M3}x_{b3}$ and lies in the orbital plane, and axis $O_{M3}z_{b3}$ follows the right-handed rule of the coordinate system. Figure 6 shows the relationship of movement coordinates and body coordinates. The unit vectors of each axis of $O_{M3}x_{b3}y_{b3}z_{b3}$ are given as $\hat{i}_b$, $\hat{j}_b$ and $\hat{k}_b$.

In Fig. 6, $\theta$ refers to the true anomaly of the HEX-MMET, and $\psi$ refers to the yawing angle between the tether-span in orbital plane and direction of the HEX-MMET COM radius. Because the tether length is over dozens of kilometers, the length of the adjustable binding rods is several meters, the dimension and mass of adjustable binding rods can be ignored. Furthermore, as mentioned in Section 2, the high strength tethers also are ignored while building the dynamic model. From Fig. 6, it is assumed that the angle between the line of the two payloads and the line of local gravity gradient (i.e. $x_o$ axis) equals to the yawing angle $\psi$, the angle between
the \(i\)th rod and \(x_o\) axis is set to be \(\psi_i (i = 1, 2, \cdots, 6)\), which can be written as the function of \(\psi\) as follows:

\[
\psi_i = \psi + \frac{i - 1}{3} \pi, \quad (i = 1, 2, \cdots, 6)
\]  

(1)

From the above equation, we know that the angle between the \(i\)th rod and axis \(x_o\) equals to the angle between the \((i + 3)\)th rod and axis \(x_o\). Furthermore, the relation between \(\dot{\psi}\) and \(\dot{\psi}_i\) can also be given as

\[
\dot{\psi} = \dot{\psi}_i
\]  

(2)

In frame \(O_x\hat{x}, \hat{Y}, \hat{Z}\), the position of the chief satellite \(O_M\), payloads \(P_1\) and \(P_2\), and the position of each infinitesimal mass \(dm\) of the six rods distant \(x\) from the HEX-MMET COM (coinciding with the chief satellite) are as follows:

For the chief satellite:

\[
\mathbf{R} = R \cos \theta t + R \sin \theta j
\]  

(3)

\[
\mathbf{R}_{c, t} = [R \cos \theta + x \cos (\psi_i + \theta)]i + [R \sin \theta + x \sin (\psi_i + \theta)]j
\]  

(4)

For payloads \(P_1\) and \(P_4\) and effectors \(E_2, E_3, E_5\) and \(E_6\), because of ignoring the dimensions of adjustable binding rods, the distances of payloads and effectors from the chief satellite are equal to each other, and their positions in the orbital plane coordinate can be noted by \(R_{p1}\) and expressed by

\[
\mathbf{R}_{p1} = [R \cos \theta + l \cos (\psi_i + \theta)]i + [R \sin \theta + l \sin (\psi_i + \theta)]j
\]  

(5)

where \(R\) refers to the radius of the chief satellite in the orbital plane. Similarly, without considering the influence of Earth oblateness, third-body gravitation, etc., the orbit of the HEX-MMET can be determined after determining the orbit elements. Therefore, the first-order derivative of the unit vectors of the orbit plane coordinate are all zero. Combining Eqs. (3)-(5), we can get the velocity vectors of the components of HEX-MMET as follows:

\[
\dot{\mathbf{R}} = (\dot{R} \cos \theta - \dot{R} \sin \theta \sin \phi + \dot{\phi} \sin \theta \cos \phi - \dot{\phi} \cos \theta \cos \phi) \hat{i} + (\dot{\hat{R}} \sin \theta + \dot{\phi} \cos \theta \sin \phi + \dot{\phi} \cos \theta \cos \phi) \hat{j}
\]  

(6)

\[
\dot{\mathbf{R}}_{c, t} = [R \cos \theta - \dot{R} \sin \theta - x (\dot{\psi} + \dot{\theta}) \sin (\psi_i + \theta)]i + [R \sin \theta + \dot{R} \cos \theta + x (\dot{\psi} + \dot{\theta}) \cos (\psi_i + \theta)]j
\]  

(7)

\[
\dot{\mathbf{R}}_{p1} = [R \cos \theta - \dot{R} \sin \theta - l (\dot{\psi} + \dot{\theta}) \sin (\psi_i + \theta)]i + [R \sin \theta + \dot{R} \cos \theta + l (\dot{\psi} + \dot{\theta}) \cos (\psi_i + \theta)]j
\]  

(8)

Then, the translational energy of the HEX-MMET is given by

\[
T_{\text{trans}} = \frac{1}{2} m (R^2 + \dot{R}^2 \dot{\theta}^2)
\]  

(9)

where \(m\) is the total mass of the HEX-MMET and can be expressed as

\[
m = m_m + 2m_p + 4m_E + 6m_T
\]  

(10)

where \(m_m, m_p, m_E\) and \(m_T = \rho \Delta A\), refer to the masses of the chief satellite, payloads, effectors and rigid rods, respectively, \(\rho\) is the density of each rigid rod, and \(\Delta A\) is the cross-sectional area of the rigid rod.

It is presumed that the external dimensions of the chief satellite \(O_M\), payloads \(P_1\) and \(P_4\), effectors \(E_2, E_3, E_5,\) and \(E_6\) and the six rigid rods are cylinders with radii \((r_m, r_p, r_E,\) and \(r_T)\), and heights \((h_m, h_p, h_E,\) and \(h_T)\), respectively. The schematic of the components of the HEX-MMET is shown in Fig. 7.

Combining Fig. 7, the rotational kinetic energy of the HEX-MMET is given as

\[
T_{\text{rot}} = \frac{1}{2} I (\dot{\psi} + \dot{\theta})^2
\]  

(11)

where \(I\) refers to the total inertial moment of the HEX-MMET in the \(z_b\) axis and is expressed as

\[
I = \frac{1}{2} mm_r^2 + m_p (r_p^2 + 2l^2) + 2m_E (r_E^2 + 2l^2)
\]  

+ \(m_T \left(2l^2 + \frac{3}{2} \Delta r^2 \right)\)  

(12)

Combining Eq. (9) with Eq. (11), the total kinetic energy of the HEX-MMET \(T\) is given by

\[
T = \frac{1}{2} m (R^2 + \dot{R}^2 \dot{\theta}^2) + \frac{1}{2} I (\dot{\psi} + \dot{\theta})^2
\]  

(13)

The next step is to deduce the potential energy \(V\) of the HEX-MMET, because we do not consider the elastic energy. Therefore, the potential energy is made up only of gravitation. From Fig. 6, the potential energy of the components of HEX-MMET are given as follows:

1) For the chief satellite:

\[
V_M = -\mu \frac{m_m}{R}
\]  

(14)

2) For payloads \(P_1\) and \(P_4\):

\[
V_{p1} = -\mu \frac{m_p}{\sqrt{R^2 + l^2 + 2Rl \cos \psi}}
\]  

(15)

\[
V_{p4} = -\mu \frac{m_p}{\sqrt{R^2 + l^2 - 2Rl \cos \psi}}
\]  

(16)

3) For effectors \(E_2, E_3, E_5\) and \(E_6\):

\[
V_{Ej} = -\mu \frac{m_E}{\sqrt{R^2 + l^2 + 2Rl \cos \psi_j}}, \quad (j = 2, 3, 5, 6)
\]  

(17)

4) For tethers \(T\):

\[
V_T
\]
Using Tylor series expansion methods and ignoring those higher orders of \((s/R)^2, (l/R)^2\), the approximated expressions of the potential energy above can be written as follows:

\[
V_{p1} = -\mu \frac{m_p}{R} \left[ 1 - \left( \frac{l}{R} \right) \cos \psi + \frac{1}{2} \left( \frac{l}{R} \right)^2 (3 \cos^2 \psi - 1) \right]
\]

(19)

\[
V_{p4} = -\mu \frac{m_p}{R} \left[ 1 + \left( \frac{l}{R} \right) \cos \psi + \frac{1}{2} \left( \frac{l}{R} \right)^2 (3 \cos^2 \psi - 1) \right]
\]

(20)

\[
V_{e_j} = -\mu \frac{m_e}{R} \left[ 1 - \left( \frac{l}{R} \right) \sin \psi_j + \frac{1}{2} \left( \frac{l}{R} \right)^2 (3 \sin^2 \psi_j - 1) \right]
\]

(21)

\[
V_{t_i} = -\mu \frac{m_t}{R} \left[ 1 - \frac{1}{2} \left( \frac{l}{R} \right) \cos \psi_i + \frac{1}{6} \left( \frac{l}{R} \right)^2 (3 \cos^2 \psi_i - 1) \right]
\]

(22)

Combining Eqs. (19)–(22), the total potential energy of the HEX-MMET is given by

\[
V = -\mu \frac{m}{R} - \mu \frac{m_p}{R} \left( \frac{l}{R} \right)^2 (3 \cos^2 \psi - 1)
\]

\[-\mu \frac{m_e}{R} \left( \frac{l}{R} \right)^2 \left( 3 \sin^2 \psi - \frac{1}{2} \right) - \mu \frac{m_t}{2R} \left( \frac{l}{R} \right)^2
\]

(23)

Using Lagrange methods, combining Eq. (13) and Eq. (23), the Lagrange equation \( L \) of the HEX-MMET is written as

\[
L = \frac{1}{2} m \left( \dot{R}^2 + R^2 \dot{\theta}^2 \right) + \frac{1}{2} m_p \left( \dot{l}^2 \right) + \mu \frac{m}{R} \left( \frac{l}{R} \right)^2 (3 \cos^2 \psi - 1)
\]

\[+ \mu \frac{m_e}{R} \left( \frac{l}{R} \right)^2 \left( 3 \sin^2 \psi - \frac{1}{2} \right)
\]

(24)

Choosing \((R, \theta, \psi)\) as the generalized coordinates with generalized forces \((0, 0, 0)\), where \( \tau \) is external force exerted on the HEX-MMET system, and using the Lagrange equation as follows:

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i
\]

(25)

we can obtain the 2D dynamics of the HEX-MMET as follows:

\[
\ddot{R} - \frac{R \ddot{\theta}^2}{2} + \frac{\mu}{R} \left( \frac{l}{R} \right)^2 (3 \cos^2 \psi - 1)
\]

\[+ \frac{3 \mu m_t l^2}{m R^3} \left( 3 \sin^2 \psi - \frac{1}{2} \right)
\]

(26)

\[
\dot{\theta} + \frac{1}{L + m R^2} \ddot{\psi} + \frac{2m}{L + m R^2} R \ddot{\theta} = 0
\]

(27)

\[
\ddot{\psi} + \ddot{\theta} + \frac{3 \mu (m_p - m_e) l^2}{2LR^3} \sin 2 \psi \frac{\tau}{I} = 0
\]

(28)

In order to verify the correctness of this dynamic HEX-MMET model and analyze the dynamics characteristics, the ideal 2D rigid dynamic MMET model with a symmetrical tether length and payload mass is given by:

\[
\ddot{R} - \frac{R \ddot{\theta}^2}{2} + \frac{\mu}{R} \left( \frac{l}{R} \right)^2 (3 \cos^2 \psi - 1) = 0
\]

(29)

\[
\dot{\theta} + \frac{1}{m_1 R^2 + I} \ddot{\psi} + \frac{2 M_1}{(m_1 R^2 + I)} R \ddot{\theta} = 0
\]

(30)

\[
\ddot{\psi} + \ddot{\theta} + \frac{3 \mu I_{p1} \sin 2 \psi}{2 L R^3} \frac{\tau}{I} = 0
\]

(31)

where \( I_{p1}, I \) and \( m_1 \) are given as

\[
I_{p1} = (2 m + 2 \rho A) \frac{l^2}{3}
\]

(32)

\[
I = \frac{1}{2} m M^2 + m_p (r_p^2 + 2 l^2) + m_t \left( \frac{1}{2} r_t^2 + \frac{2 l^2}{3} \right)
\]

(33)

\[
m_1 = m_M + 2 m_p + 2 m_t
\]

(34)

5. Numerical Simulations

Table 1 shows the parameters of the HEX-MMET. Furthermore, the initial values of the generalized coordinates and generalized velocities are given as follows: \( R(0) = 6.728 \times 10^6 \text{ m}, R(0) = 0 \text{ m/s}, \theta(0) = 0 \text{ rad}, \dot{\theta}(0) = 0.00126 \text{ rad/s}, \psi(0) = 0 \text{ rad} \) and \( \dot{\psi}(0) = 0.0873 \text{ rad/s} \).

Figure 8 shows the time history of radius \( R \) of HEX-MMET’s COM and the radius of the MMET under the condition of different external torque \( \tau \).

| Table 1. Parameters of the HEX-MMET. |
|--------------------------------------|
| Parameters                        | Value | Unit |
|-----------------------------------|-------|------|
| Earth’s gravitation constant: \( \mu \) | 3.988 \times 10^{-11} | m^3/s^2 |
| Length of rigid rods: \( l \)      | 10000 | m    |
| Mass of chief satellite: \( m_M \) | 5000  | kg   |
| Mass of \( P_1 \) (\( P_2 \)): \( m_p \) | 1000  | kg   |
| Mass of effectors: \( m_e \)      | 200   | kg   |
| Density of rigid rods: \( \rho \)  | 970   | kg/m^3 |
| Cross-section area of rigid rods: \( A \) | 6.283 \times 10^{-5} | m^2   |
| Radius of chief satellite: \( r_M \) | 0.5   | m    |
| Radius of payloads: \( r_p \)     | 0.5   | m    |
| Radius of effectors: \( r_e \)    | 0.5   | m    |
| Radius of the rigid rods: \( r_t \) | 0.00447207 | m    |
|-----------------------------------|-------|------|
It is shown that even though the external torque has a different value, there is little influence on the orbital movement of the HEX-MMET from a macroscopic view. However, from a microscopic view, Fig. 9 shows a tiny radius error in the HEX-MMET COM and MMET COM with different external torque, where radius error is defined as

$$\Delta R = R_H - R_M$$  \hspace{1cm} (35)$$

where $R_H$ and $R_M$ refer to the radii of the HEX-MMET COM and MMET COM distant from Earth’s COM, respectively. From Fig. 9 we can observe that with an increase in external torque, the maximum amplitude of $\Delta R$ increases. It is illustrated that the external torque $\tau$ will affect the attitude movement of the HEX-MMET, and this has an impact on the orbital movement of the HEX-MMET due to the coupling effect between the attitude and orbit.

The initial angular velocity $\omega_0(0)$ (where $\omega_0 = \dot{\psi}$) is also very important to HEX-MMET if we want the effectors to rendezvous with the coming payloads. Therefore, studying the relation of the tether’s angular velocity and orbit parameter is necessary. Figure 10 shows the relationship of $\Delta R$ with different $\omega_0(0)$. It is shown that, while $\omega_0(0) \neq 0$ rad/s, if external torque exists, the change in the initial value of $\omega_0(0)$ has no influence on the difference in radius $\Delta R$. However, the changing trend of $\Delta R$ when $\omega_0(0) = 0$ rad/s is different than the case when $\omega_0(0) \neq 0$ rad/s.

From Table 1 we can see that the initial orbit of HEX-MMET’s COM is elliptical due to $\dot{\theta}(0) = 0.00126$ rad/s. Therefore, the radial velocity of HEX-MMET’s COM is not zero, except for the positions of perigee and apogee. The radial velocity $\dot{R}$ of HEX-MMET’s COM and radial velocity of MMET’s COM are shown in Fig. 11, with comparatively different $\tau$. Note that amplitude of $\dot{R}$ changes periodically and is not evidently affected by the external torque. The maximum of $|\dot{R}|$ is $1.481 \times 10^3$ m/s in the specified cases. Figure 12 shows the model error of radial velocity $\Delta \nu_1$ between the HEX-MMET and MMET with different external torque $\tau$. $\Delta \nu_1$ is defined by

$$\Delta \nu_1 = \dot{R}_H - \dot{R}_M$$ \hspace{1cm} (36)$$

where $\dot{R}_H$ and $\dot{R}_M$ refer to the radial velocities of HEX-MMET’s COM and MMET’s COM, respectively.

It can be observed in Fig. 12 that there is a tiny error of $\Delta \nu_1$ caused by external torque. With increasing $\tau$, the amplitude of $\Delta \nu_1$ increases nonlinearly. If we set relative error as

$$\xi = (\Delta \nu_1)_{max}/(\nu_1)_{max},$$

we can find that $\Delta \nu_1$ reaches its maximum and $\xi = 4.307 \times 10^{-4}$ when $\nu_1 = 0$ m/s (i.e., when the satellite moves to the apogee or perigee).

Another orbital element in the dynamic HEX-MMET dynamic model is true anomaly $\dot{\theta}$, which determines the orbital angular velocity of HEX-MMET’s COM.
Figure 14 shows the time history of the true anomaly error \( \Delta \phi \) for torque. It is presented that \( \tau \) has little influence on \( \theta \), which has similar variation trend with different torque values. Figure 14 shows the time history of the true anomaly error \( \Delta \theta \) under the condition of different external torque \( \tau \). \( \Delta \theta \) is defined as

\[
\Delta \theta = \theta_H - \theta_M
\]

where \( \theta_H \) and \( \theta_M \) refer to the true anomaly of HEX-MMET and MMET, respectively. It is shown that the variation period of \( \Delta \theta \) is the same as HEX-MMET’s orbit period. Furthermore, from Fig. 14 we can also find that the valley of \( \Delta \theta \) occurs at the apogee of the HEX-MMET and the peak value of \( \Delta \theta \) occurs at the perigee.

Next, we study the influence of the external torque \( \tau \) and initial conditions on the attitude elements of the HEX-MMET and MMET i.e., the yawing angle \( \psi \) between the tether-span, (that is the line between the two payloads) and the local gravity gradient. Figure 15 shows the relation of \( \psi \) over time.

It is shown that, if \( \tau = 0 \), without considering the outer perturbations, \( \psi \) increases linearly over time. However, if \( \tau \neq 0 \), with external torque \( \tau \) increasing, \( \psi \) increases exponentially over time. Figure 16(a) and (b) show the first-order derivative of \( \psi \) (i.e., the angular velocity \( \dot{\psi} \) of the HEX-MMET around its chief satellite, noted by \( \omega_\psi \)) over time. In general, if there are no other actual perturbations and external forces affecting the HEX-MMET, which will rotate with constant angular velocity by giving HEX-MMET an initial angular velocity value \( \omega_\psi(0) \), from Fig. 16, we can observe that \( \omega_\psi \) has a variation period that which coincides with the orbit period (i.e., when the HEX-MMET is located at perigee, \( \omega_\psi \) reaches its minimum, while when the HEX-MMET moves to apogee, \( \omega_\psi \) reaches its maximum. All of this is caused by the effect of gravity gradient torque, noted by \( \tau_G \). When the HEX-MMET locates at perigee, \( \tau_G \) reaches its maximum. This means that HEX-MMET’s rotation motion around its COM is obstructed the most. The result is that \( \omega_\psi \) reaches its minimum at this point. On the other hand, when the HEX-MMET locates at apogee, \( \tau_G \) reaches its minimum, and HEX-MMET’s rotation motion is obstructed the least, and \( \omega_\psi \) reaches its maximum at apogee. In addition, it is shown from Fig. 16 that the larger the external torque is, the faster the rotational angular velocity. This is because it is more evident that the external torque has a leading effect.

Figure 17 shows the difference in angular velocity between the HEX-MMET and MMET, denoted as \( \Delta \omega_\psi \) and defined as follows

\[
\Delta \omega_\psi = \omega_H - \omega_M
\]

where \( \omega_H = \dot{\psi}_H \) and \( \omega_M = \dot{\psi}_M \) show the angular velocities of the HEX-MMET and MMET, respectively.
It is shown that \( \Delta \omega_\psi \) has small-scale oscillation rectilinearly by \([-6.615 \times 10^{-6}, 4.083 \times 10^{-5}] \) (rad/s), when external torque \( \tau \) is zero. After that, as external torque takes on a much larger value, \( \Delta \omega_\psi \) decreases much quicker linearly in the same oscillation amplitude width. It can be explained that, while external torque is small, the gravity gradient torque affects the attitude movement of the propulsion tether much more. Therefore, each rotation of the propulsion tether around its chief satellite causes nonlinear oscillation in the orbit period. While the value of the external torque is much greater, the dominating effect of the gravity gradient torque decreases. The result is shown in Fig. 17(b).

Figure 18 shows the time history of HEX-MMET’s angular acceleration \( \alpha_\psi \) (where \( \alpha_\psi = \dot{\psi} \)) under the condition of different external torque \( \tau \). From Fig. 18(a) we can see that the two curves (one refers to angular acceleration of the MMET, the other refers to angular acceleration of the HEX-MMET by \( \tau = 0 \) N-m) have a similar trend in rectilinear change over time. However, they have different amplitudes: amplitude of \( \alpha_\psi \) related to the MMET is from \(-9.268 \times 10^{-7} \) rad/s\(^2\) to \(9.268 \times 10^{-7} \) rad/s\(^2\), and that of the HEX-MMET is from \(-2.006 \times 10^{-6} \) rad/s\(^2\) to \(2.006 \times 10^{-6} \) rad/s\(^2\). This is because the HEX-MMET has a much larger mass than the MMET, but has the same external torque. The result is that the amplitude of HEX-MMET’s angular acceleration is much smaller. From Fig. 18(b), we can observe that, when external torque is nonzero, as the external torque value increases, the two curves of angular acceleration shift upward overall. For example, when \( \tau = 250 \) kN-m, the amplitude of \( \alpha_\psi \) for the MMET is from \(-3.816 \times 10^{-7} \) rad/s\(^2\) to \(1.586 \times 10^{-6} \) rad/s\(^2\), and for the HEX-MMET the amplitude is from \(-1.053 \times 10^{-6} \) rad/s\(^2\) to \(3.116 \times 10^{-6} \) rad/s\(^2\). It is illustrated that the larger the external torque value is, the more apparent the effect of external torque on HEX-MMET.

It is concluded that, for such a large-scale satellite (caused by the length of the rigid rods), the rotational movement will yield a gravity gradient torque; the value of which is determined by the length of rods and its mass. The coupling effect of this gravity gradient torque will cause some difficulties in controlling the attitude movement. How to use it will be meaningful.

6. Conclusions

In this paper, a new configuration for a motorized momentum exchange tether, referred to as HEX-MMET, is proposed. The HEX-MMET is composed of six rigid rods having the same length, density and cross-section area. Compared to the MMET, HEX-MMET has some unique advantages: for example, if one tether of the MMET is broken, the MMET loses its payload transferring function. However, if some tether of the HEX-MMET is broken, HEX-MMET’s force sensor can detect the force unbalance, cut the symmetrical tether immediately and keep working using the other four tethers. Additionally, the HEX-MMET has much higher payload transferring efficiency than the MMET and has a much shorter orbit reboost period and more chances to begin a new payload capture mission than the MMET. Furthermore, because the HEX-MMET has six rigid rods, while reboosting the orbit by electrodynamic force, the HEX-MMET can make more power to modify the orbit.

Secondly, using Lagrange methods, a 2D rigid dynamic model of the HEX-MMET and a 2D rigid model of the MMET are compared. Numerical simulations show that the dynamics of the HEX-MMET are similar to those of the...
attitude-orbit coupling effect. However, those orbit parameters are slightly affected by the attitude-orbit coupling effect, and this kind of effect has a positive correlation with the value of external torque.

2) External torque has an apparent influence on the attitude parameters, just like the angle between the tether-span (i.e., the line linking the two payloads) and local gravity gradient, angular velocity and angular acceleration of the tether around the HEX-MMET’s COM. The larger the external torque value is, the faster the angle \( \psi \) increases. Furthermore, because of the existence of gravity gradient torque, the angular velocity and angular acceleration cause a trend of apparent periodic variation over time no matter if external torque exists or not.

Acknowledgments

This work is supported by the National Nature Science Fund of China (Grant No. 61403100), the Fundamental Research Funds for the Central Universities (Grant No. HIT.NSRIF.2015037) and the open National Defense Key Disciplines Laboratory of Exploration of Deep Space Landing and Return Control Technology, Harbin Institute of Technology (Grant No. HIT.KLOF.2013.079).

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H. Fujii
Associate Editor