Topology design of multi-row series-parallel multi-speed planetary gear automatic transmission based on extended lever method

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Abstract. With the urgent need of saving energy, reducing emissions, reducing shifting frustration and improving driving quality, the number of gears of automatic transmissions used in automobiles is increasing. However, the traditional lever method is insufficient in designing multi-row parallel-series multi-speed planetary gear automatic transmissions. In this study, an extended lever method is proposed, which can design and analyze the transmission state of planetary gears with multiple input elements, multiple output elements and no fixed elements. Then, based on the extended lever method, a series of multi-speed planetary gear automatic transmissions are designed, among which a twelve-speed planetary gear automatic transmission is selected, and the kinematics and dynamics are analyzed by using the lever method. The results show that the designed twelve-speed transmission has a wide range of speed ratios, with up to seven overdrive gears and a smaller speed ratio interval of each overdrive gear. The results further prove that the extended lever method is an important direction for the design of multi-row series-parallel multi-speed planetary gear automatic transmission.

1. Introduction
The planetary gear automatic transmission has both internal and external meshing, which has the characteristics of compact structure, balanced gear force and high element reuse rate, and is widely used in vehicle transmission parts [1]. With the continuous improvement of requirements such as reducing energy consumption, reducing emissions, reducing shifting frustration, and improving vehicle riding comfort [2-3], the demand for gear numbers of vehicles is getting higher and higher. At present, planetary gear automatic transmissions with 10 forward gears have appeared on mass-produced vehicles [4-5]. The single-row planetary gear row is a two-degree-of-freedom mechanism. In order to achieve more gears, a plurality of single-row planetary gears is generally combined in series and parallel. For example, the 4th and 5th gears generally adopt two planetary rows, the 6th to 8th gears generally adopt three planetary rows, and the 7th to 10th gears generally adopt four planetary rows [6]. With the increasing number of planetary rows, there are tens of thousands of structural forms and connection modes of planetary rows. Since the 1980s, the lever method has been widely used as a design method of planetary gear automatic transmission [7-9]. However, the traditional lever method requires three basic fulcrums: input, output and fixing. Yet, the multi-speed multi-planetary gear structure is in a state of double input or even multiple input in most gears, so the traditional lever method cannot adapt. Therefore, this study puts forward an extended lever method, and designs a series of multi-speed planetary gear automatic transmissions based on it, among which a twelve-speed planetary gear automatic transmission is selected and its mechanical transmission characteristics are analyzed.
2. The basic theory of the extended lever method

The single-row planetary gear row is divided into single-row single-stage and single-row double-stage, and the corresponding lever diagram is shown in Figure 1. For the single-row single-stage planetary gear row, the planetary carrier is located in the middle of the lever, and the gear ring and the sun gear are located at the two ends. The ratio of the corresponding line segments is $1: \alpha$. For the single-row double-stage planetary gear row, the ring gear is located in the middle of the lever, and the planet carrier and the sun gear are located at the two end points of the lever. The ratio of the corresponding line segments is $1:(\alpha - 1)$, and $\alpha$ is the ratio of the number of teeth of the ring gear to that of the sun gear.

In the traditional lever method, there are fixed points, input points and output points in the lever. For example, when the sun gear is fixed, the planetary carrier is the input and the gear ring is the output in Figure 1, according to the triangle similarity principle, the transmission ratio of the single-row single-stage planetary gear row $i_a$ and the transmission ratio of single-row double-stage planetary gear row $i_b$ are calculated as follows:

$$i_a = \frac{n_c}{n_o} = \frac{n_c}{n_r} = \frac{\alpha}{\alpha + 1}, \quad i_b = \frac{n_r}{n_o} = \frac{n_r}{n_s} = \frac{1+(\alpha - 1)}{\alpha - 1}$$

As shown in Figure 2, if the sun gear (with the same speed as the input shaft) and the planetary carrier are dual inputs and the ring gear is the output, the traditional lever method cannot be used for analysis and design. By making a dotted line parallel to the original vector lever at the end of the velocity vector line segment, the new lever fulcrums $S'$, $R'$, $C'$ are obtained respectively.

For the single-row single-stage planetary gear row and the single-row double-stage planetary gear row, there are

$$\frac{n_c}{n_r} = \frac{n_c - n_s}{n_r - n_s} = \frac{\alpha}{\alpha + 1}, \quad \frac{n_c}{n_r} = \frac{n_c - n_s}{n_r - n_s} = \frac{\alpha}{\alpha - 1}.$$

And then,

$$i_a = \frac{n_s}{n_r} = \frac{\alpha n_s}{\alpha n_s + (n_c - n_s)(\alpha + 1)}, \quad i_b = \frac{n_s}{n_r} = \frac{\alpha n_s}{\alpha n_s + (n_c - n_s)(\alpha - 1)}.$$

If any two of the three elements of the sun gear, the planetary carrier and the ring gear are the inputs, and the third is the output, the analysis is similar to this and will not be described in detail.
3. Topology design of multi-row series-parallel multi-speed planetary gear automatic transmission

In this section, the 4-fulcrum lever is taken as an example, and the speed vector diagram of each gear is designed by using the extended lever method, as shown in Figure 3. The four fulcrums ①, ②, ③ and ④ in the lever respectively represent the coaxial rotating elements in the double-row planetary gear mechanism, and the distance ratio between the corresponding fulcrums is $l_1:1:l_2$. There are three braking points $B_1(①)$, $B_4(②)$ and $B_7(④)$ and four input points, among which $K_1(①)$ and $K_2(④)$ have the same speed as the input shaft, and $K_3(④)$ and $K_4(②)$ can be input through the increasing of the single-row speed. Therefore, the combination of a single-row planetary gear row and a double-row planetary gear row can realize 12 forward gears and 1 reverse gear through 7 shift actuators with 4 clutches and 3 brakes.

Common double-row planetary gear mechanisms include the Ravigneaux type, the Simpson type, the CR-CR type, etc. Single-row planetary gear rows include single-row single-stage and single-row double-stage. According to the lever diagram in Figure 3, single-row single-stage plus Ravigneaux type, single-row single-stage plus Simpson type, single-row single-stage plus CR-CR type, single-row double-stage plus Ravigneaux type, single-row double-stage plus Simpson type and single-row double-stage plus CR-CR type can be designed. However, the connection mode of the corresponding points of the 3-fulcrum lever and the 4-fulcrum lever in each combination is not fixed, so it can actually be combined into a variety of planetary gear row connection modes. Now, two combinations are selected,
and the lever structure diagram is shown in Figure 4. In the diagram A, the 1st planetary row is single-row double-stage, and the 2nd and 3rd planetary rows are CC-RR structures with single-row double-stage and single-row single-stage combination respectively. In the diagram B, the 1st planetary row is single-row single-stage, and the 2nd and 3rd planetary rows are two single-row single-stage CR-CR structures.

![Lever structure diagram of multi-speed automatic transmission.](image)

**Figure 4.** Lever structure diagram of multi-speed automatic transmission.

### 4. Kinematics analysis of the new 12-speed automatic transmission based on extended lever method

With reference to Figure 3 and Figure 4(A), the 1st planetary gear row is a single-row double-stage 3-fullcrum lever. The 1st ring gear \( R_1 \) is connected with the input shaft, the 1st planetary carrier \( C_1 \) is connected with the shell (always in a fixed state), and the 1st sun gear \( S_1 \) is the output to the last two rows at a higher speed. The 2nd and 3rd planetary gear rows are 4-fullcrum levers of CC-RR type, the sun gear \( S_3(\overline{1}) \) in the 3rd row transmits the speed of the input shaft through \( K_1 \), the 2nd and 3rd planetary carriers assembly \( C_2C_3(\overline{2}) \) connect the 1st sun gear \( S_1 \) through \( K_4 \), the 2nd sun gear \( S_2(\overline{4}) \) connects the input shaft and the 1st sun gear \( S_1 \) through \( K_2 \) and \( K_3 \) respectively, and the 2nd and 3rd ring gears assembly \( R_2R_3(\overline{3}) \) serves as the output shaft. \( l_i = \alpha_i \), \( l_2 = (\alpha_2 - 1) \). According to the triangle similarity principle and trapezoidal arbitrary waist line calculation formula, combined with the size of each line segment in the figure, the transmission ratio of each gear can be calculated as follows, taking the gears D1 and D8 as examples:

\[
i_1 = \frac{n_{S_2}}{n_{R_2}} = \frac{(\alpha_2 - 1) + 1}{1} = \alpha_2
\]

Under the gear D8, \( n_s = n_{S_1} = \frac{[(\alpha_1 - 1)+1]}{1} n_{R_1} = \alpha_1 \). And it is calculated by trapezoidal arbitrary waist line formula as follow:

\[
n_{R_2} = n_{S_3} + \frac{1 + \alpha_3}{(\alpha_2 - 1) + 1 + \alpha_3} \times (n_{S_2} - n_{S_3}) = 1 + \frac{1 + \alpha_3}{\alpha_2 + \alpha_3} \times (\alpha_1 - 1) = \frac{\alpha_1 + \alpha_2 + \alpha_3 - 1}{\alpha_2 + \alpha_3}
\]

\[
i_8 = \frac{n_{R_1}}{n_{R_2}} = \frac{\alpha_2 + \alpha_3}{\alpha_1 + \alpha_2 + \alpha_3 - 1}
\]

According to the above method, the transmission ratios of other gears are:

\[
i_2 = \frac{\alpha_2 + \alpha_3}{\alpha_2 - 1}, \quad i_3 = \frac{\alpha_2}{\alpha_1}
\]

\[
i_4 = \frac{\alpha_2 + \alpha_3}{\alpha_3 + 1}, \quad i_5 = 1, \quad i_6 = \frac{\alpha_2 + \alpha_3}{\alpha_1(\alpha_3 + 1)}, \quad i_7 = \frac{\alpha_2}{\alpha_1(\alpha_2 - 1)}, \quad i_9 = \frac{\alpha_2}{\alpha_1\alpha_2 - \alpha_1 + 1}, \quad i_{10} = \frac{1}{\alpha_1}
\]
\[ i_{11} = \frac{\alpha_3}{\alpha_1 \alpha_3 + \alpha_1 - 1}, \quad i_{12} = \frac{\alpha_5}{\alpha_1 (\alpha_3 + 1)} \quad \text{and} \quad i_R = -\alpha_3. \]

Select \( \alpha_1 = 2 \), \( \alpha_2 = 4 \), and \( \alpha_3 = 4 \), then the transmission ratios of each gear are \( i_1 = 4 \), \( i_2 = 2.67 \), \( i_3 = 2 \), \( i_4 = 1.6 \), \( i_5 = 1 \), \( i_6 = 0.8 \), \( i_7 = 0.67 \), \( i_8 = 0.62 \), \( i_9 = 0.57 \), \( i_{10} = 0.5 \), \( i_{11} = 0.44 \), \( i_{12} = 0.4 \) and \( i_R = -4 \). This transmission has four reduction gears, one direct gear, seven overdrive gears and one reverse gear. What’s more, the speed ratio interval between overdrive gears is smaller, and the gear shift is smoother.

5. Dynamic analysis of the new 12-speed automatic transmission based on extended lever method

Taking the gear D6 of the new 12-speed automatic transmission (all three planetary gears participate in power transmission) as an example, the torque of each element in the planetary gear row is solved by the lever method. According to the force balance principle, \( M_i + M_O + M_B = 0 \), and \( M_O = -i_6 \times M_i \), so \( M_B = (i_6 - 1) \times M_i \). It can be known that there are two braking points \( C_1 \) and \( S_3 \) under the gear D6, and the braking torque is \( M_B = M_{B_1} + M_{B_3} \) (the specific value of \( M_{B_1} \) and \( M_{B_3} \) to be solved). Assuming that the input torque is known, the calculated output torque and braking torque are

\[ M_O = -i_6 \times M_i = -\frac{\alpha_2 + \alpha_3}{\alpha_1 (\alpha_3 + 1)} M_i \quad \text{and} \quad M_B = (i_6 - 1) \times M_i = \frac{\alpha_2 + \alpha_3 - \alpha_1 \alpha_3}{\alpha_1 (\alpha_3 + 1)} M_i. \]

Starting from the component whose external torque is known and receives only one internal torque, the corresponding internal torques of each component of the planetary row are solved.

In the 1st planetary gear row, the ring gear \( R_1 \) is connected with the input shaft, then there is the internal torque \( M_{R_1} = -M_i \). From the force balance of lever, the internal torque of the sun gear \( S_1 \) can be obtained: \( M_{S_1} = -\frac{1}{\alpha_1} M_{R_1} = \frac{1}{\alpha_1} M_i. \)

The internal torque of the planetary carrier \( C_1 \) is \( M_{C_1} = -\frac{\alpha_1 - 1}{\alpha_1} M_{R_1} = \frac{\alpha_1 - 1}{\alpha_1} M_i. \)

In the 2nd planetary gear row, the sun gear \( S_2 \) is connected with the 1st sun gear \( S_1 \), then there is the internal torque \( M_{S_2} = -M_{S_1} = -\frac{1}{\alpha_1} M_i \). From the force balance of lever, the internal torque of the sun gear \( S_2 \) can be obtained: \( M_{S_2} = -M_{S_1} = -\frac{1}{\alpha_1} M_i. \)

The internal torque of the planetary carrier \( C_2 \) is

\[ M_{C_2} = (\alpha_2 - 1) M_{S_2} = -\frac{(\alpha_2 - 1)}{\alpha_1} M_i. \]

In the 3rd planetary gear row, the planetary carrier \( C_3 \) is connected with the 2nd planetary carrier \( C_2 \), then there is the internal torque \( M_{C_3} = -M_{C_2} = \frac{(\alpha_2 - 1)}{\alpha_1} M_i \). From the force balance of lever, the internal torque of the gear ring \( R_3 \) can be obtained:

\[ M_{R_3} = -\frac{\alpha_3}{\alpha_1 + 1} M_{C_3} = -\frac{\alpha_3}{\alpha_1 + 1} \times \frac{(\alpha_2 - 1)}{\alpha_1} M_i = -\frac{\alpha_3 (\alpha_2 - 1)}{\alpha_1 (\alpha_3 + 1)} M_i. \]

The internal torque of the sun gear \( S_3 \) is

\[ M_{S_3} = -\frac{1}{\alpha_1 + 1} M_{C_3} = -\frac{1}{\alpha_1 + 1} \times \frac{(\alpha_2 - 1)}{\alpha_1} M_i = -\frac{\alpha_2 - 1}{\alpha_1 (\alpha_3 + 1)} M_i. \]
The clutch $K_3$ connects the 1st sun gear $S_1$ and the 2nd sun gear $S_2$ together, thus the coupling torque of the clutch is $M_{K3} = \left| M_{S1} \right| = \left| M_{S2} \right| = \frac{1}{\alpha_i} M_f$. The braking torques are $M_{B1} = -M_{C1} = -\frac{\alpha_1 - 1}{\alpha_i} M_f$ and $M_{B3} = -M_{S3} = \frac{\alpha_2 - 1}{\alpha_i(\alpha_2 + 1)} M_f$. Therefore, $M_{B1} + M_{B3} = M_B$. The calculated result is consistent with the previous one. The analysis of other gears is similar, so it will not be described in detail.

6. Conclusion
In this study, aiming at the shortcomings of the traditional lever method in designing the planetary gear automatic transmission, an extended lever method is proposed. Based on the analysis of its design principle, a kind of topological structure with 3 planetary rows and 12 speeds is designed by using the extended lever method, and its dynamic and kinematic characteristics are analyzed by using the extended lever method. The results show that the designed 12-speed transmission has a wide range of speed ratios, with up to seven overdrive gears, and the speed ratio interval of each overdrive gear is smaller. The results further prove that the extended lever method can quickly and efficiently design the multi-row series-parallel multi-speed planetary gear automatic transmission.

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