NEW CHANCE FOR RESEARCHES ON LORENTZ VIOLATION

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I present a brief review on the motivation for the study on Lorentz violation and on some of our studies with phenomenological analysis of Lorentz violation effects. I also discuss three effective field theory frameworks for Lorentz violation: the Coleman-Glashow model, the standard model extension (SME), and the standard model supplement (SMS). The situation of the OPERA “anomaly” is also briefly reviewed, together with some discussion on the superluminality of neutrinos within the effective field theory frameworks.

Keywords: Lorentz violation; effective field theory; superluminal neutrino

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1. Motivation for Lorentz Violation Study

In 1905, Einstein published his famous paper “On the Electrodynamics of Moving Bodies” and established the theory of special relativity. The special relativity offers a revolution to the concepts of space and time in Newton’s mechanics, and provides a proof to adopt the Lorentz transformation for the covariance of the equations of electrodynamics, to replace the traditional Galileo transformation in classical physics. There are two basic principles of special relativity:

• Principle of Relativity: the equations describing the laws of physics have the same form in all admissible frames of reference.
• Principle of constant light speed: the speed of light is the same in all directions in vacuum in all reference frames, regardless whether the source of the light is moving or not.

Through over one hundred years of investigations from various aspects, Einstein’s relativity has won great triumphs. It becomes one of the foundations of modern physics and has been proved to be valid at very high precision. Lorentz Invariance, the basic theoretical foundation of relativity, states that the equations describing the laws of physics have the same form in all admissible reference frames, or physical laws keep invariant under the Lorentz transformation. So we need to answer the question: why we seek for Lorentz violation?
The Lorentz symmetry is a symmetry related with space and time, therefore the Lorentz violation should be related to the basic understandings of space and time. From the viewpoint of physics, the origin for the breaking down of conventional concepts of space and time might be traced back to Planck. There are four basic constants before Planck’s creation of his quantum theory of black body in 1990: the Newton gravitational constant $G$, the light speed in vacuum $c$, the Boltzmann constant $k_B$, and the permittivity of free space $\epsilon_0$. In 1899, Planck introduced a new constant $\hbar$, for the purpose to construct a “God-given” unit system. Then he set the above five constants as bases, and elegantly simplified recurring algebraic expressions in physics. One year later Planck found that the constant $\hbar$ he introduced for his unit system is an indispensable constant for his new quantum theory. There are a number of basic quantities in this unit system, such as the Planck length $l_P \equiv \sqrt{G\hbar/c^3} \simeq 1.6 \times 10^{-35}$ m, the Planck time $t_P \equiv \sqrt{G\hbar/c^5} \simeq 5.4 \times 10^{-44}$ s, the Planck energy $E_P \equiv \sqrt{\hbar c^3/G} \simeq 2.0 \times 10^9$ J, and the Planck temperature $T_P \equiv \sqrt{\hbar c/2k_B} \simeq 1.4 \times 10^{32}$ K. Therefore one may suspect that conventional understanding of space and time might be breaking down at the Planck scale, i.e., at the Planck length $l_P$, or the Planck time $t_P$, or the Planck energy $E_P$, where new features of existence may emerge. The breaking down of continuous space-time was also conjectured.

Just recently, Xu and I provided a physical argument for the discreteness of space and time. From two known entropy constraints:

\[ S_{\text{matter}} \leq 2\pi ER, \quad \text{and} \quad S_{\text{matter}} \leq \frac{A}{4}, \]

combined with the black-body entropy,

\[ S = \frac{4}{45} \pi^2 T^3 V = \frac{16}{135} \pi^3 R^3 T^3, \]

we arrive at a minimum value of space

\[ R \geq \left( \frac{128}{3645\pi} \right)^{\frac{1}{2}} l_P \simeq 0.1l_P. \]

Thus we reveal from physical arguments that space-time is discrete rather than continuous. From another point of view, the newly proposed entropic gravity suggests gravity as an emergent force rather than a fundamental one. If gravity is emergent, a new fundamental constant should be introduced to replace the Newtonian constant $G$. It is natural to suggest a fundamental length scale, and such constant can be explained as the smallest length scale of quantum space-time. Its value can be measured through searches of Lorentz violation. The existence of an “æther” or “vacuum” can also bring the breaking down of Lorentz invariance.

Thus the research on the Lorentz violation may provide us the chance for new understanding of the nature of basic concepts, such as “space”, “time”, and “vacuum”, through physical ways, rather than from the viewpoint of metaphysics or philosophy.
Nowadays, there has been an increasing interest in Lorentz invariance Violation (LV or LIV) both theoretically and experimentally. The possible Lorentz symmetry violation (LV) effects have been sought for decades from various theories, motivated by the unknown underlying theory of quantum gravity together with various phenomenological applications. This can happen in many alternative theories, e.g., the doubly special relativity (DSR), effects in general relativity, non-covariant field theories, and large extra-dimensions. As examples, I list below some phenomenological consequences of the Lorentz violation effects studied by my students and I in the last a few years:

- The Lorentz violation could provide an explanation of neutrino oscillation without neutrino mass. We carried out Lorentz violation contribution to neutrino oscillation by the effective field theory for Lorentz violation and give out the equations of neutrino oscillation probabilities. In our model, neutrino oscillations do not have drastic oscillation at low energy and oscillations still exist at high energy. It is possible that neutrinos may have small mass and both Lorentz violation and the conventional oscillation mechanisms contribute to neutrino oscillation.

- The modified dispersion relation of the proton could increase the threshold energy of photo-induced meson production of the proton and cause an increase of the GZK cutoff energy. The earlier reports on super-GZK events triggered attention on Lorentz-Violation. The new results of observation of GZK cutoff put strong constraints on Lorentz violation parameters.

- The modified dispersion relation of the photon may cause time lag of photons with different energies when they propagate in space from far-away astro-objects. The Lorentz violation can modify the photon dispersion relation, and consequently the speed of light becomes energy-dependent. This results in a tiny time delay between high energy photons and low energy ones. Very high energy photon emissions from cosmological distance can amplify these tiny LV effects into observable quantities. We analyzed photons from γ-ray bursts from Fermi satellite observations and presented a first robust analysis of these taking the intrinsic time lag caused by sources into account, and gave an estimate to LV energy scale \( \sim 2 \times 10^{17} \text{ GeV} \) for linear energy dependence, and \( \sim 5 \times 10^{9} \text{ GeV} \) for quadratic dependence.

- We also studied recent data on Lorentz violation induced vacuum birefringence from astrophysical consequences. Due to the Lorentz violation, two helicities of a photon have different phase velocities and group velocities, termed as “vacuum birefringence”. From recently observed γ-ray polarization from Cygnus X-1, we obtained an upper limit \( \sim 8.7 \times 10^{-12} \) for Lorentz-violating parameter \( \chi \), which is the most firm constraint from well-known systems.
2. Lorentz Violation in Effective Field Theory Frameworks

Among many theoretical investigations of Lorentz violation, it is a powerful framework to discuss various LV effects based on traditional techniques of effective field theory in particle physics. The general effective field theory framework starts from the Lagrangian of the standard model, and then includes all possible terms containing the Lorentz violation effects. The magnitudes of these LV terms can be constrained by various experiments. In the following we briefly discuss three different versions of effective field theory frameworks for Lorentz violation: the Coleman-Glashow model, the minimal standard model extension (SME) and the newly proposed standard model supplement (SMS).

2.1. The Coleman-Glashow model

The Coleman-Glashow model is a simple version to include Lorentz violating terms into the standard model Lagrangian. Let $\Psi$ denote a set of $n$ complex scalar fields assembled into a column vector, one can add to the standard model Lagrangian the Lorentz-violating term:

$$L \to L + \partial_i \Psi \epsilon^{ij} \partial^j \Psi,$$

where $\epsilon$ is a Hermitian matrix that signals the Lorentz violation in the Coleman-Glashow model. In case for fermions, the wave function $\Psi$ denotes the Dirac spinor, and the LV parameter $\epsilon$ can be taken as a fixed scalar constant for the fermion under consideration.

We should stress that the Coleman-Glashow model can be only considered as a “toy model” for illustration as it does not meet the criterion of being an “exact” theory. The reason is that the simple form of the Lagrangian Eq. (4) can not be taken as invariant in all inertial frames of reference, but only valid in one inertial frame of reference the observer is working. This model can be adopted when the observer is focusing on the Lorentz violation effect within a certain frame such as the earth-rest frame, the sun-rest frame, or the CMB frame, and does not care about relationships between different frames. Otherwise the situation could become very complicated with different formalisms in different frames from the requirement of consistency, i.e., the absolute physical events should keep unchanged no matter observed from any reference frame. We can consider this requirement as a basic principle called absolute physical event consistency:

- For a physical process when the initial and final particles are experimentally produced and detected, such event is called an absolute physical event, and its existence does not change when viewed from another frame of reference.

For example, the decay of a physical particle, such as $\pi \to \mu + \nu_\mu$, if happens in one frame from an observer, it should also happen observed from another frame by another observer. However, the situation might be different for virtual processes. The virtual processes could be different when viewed from different frames.
Physically, the reason for the Lorentz violation of the Coleman-Glashow model is due to the existence of a “background” represented by the scalar constant $\epsilon$. This model can apply as a practical tool to demonstrate or estimate the magnitude of the Lorentz violation effect. It has been successfully applied in many phenomenological analysis to constrain the Lorentz violation effect in some physical processes.

2.2. The minimal standard model extension (SME)

In the standard model extension (SME), terms that violate Lorentz invariance can be added by hands, and then one can select the terms from some considerations such as gauge invariance, Hermitian, power-counting renormalizability and etc. In the minimal version of the SME\textsuperscript{31}, the LV terms are measured with several tensor fields as coupling constants, and modern experiments have built severe constraints on the relevant Lorentz violation parameters\textsuperscript{35}.

For example, the SME Lagrangian for massless neutrinos takes the form\textsuperscript{31,29,36},

$$\mathcal{L} = \frac{1}{2} \sigma^A \gamma^\mu \overleftrightarrow{D}_\mu \nu^B \delta_{AB} + \frac{1}{2} \epsilon_{\mu\nu} \nu^A \overleftrightarrow{D}_\mu \nu^B - a^\mu_{AB} \nu^A \gamma^\mu \nu^B + \cdots , \quad (5)$$

where $\sigma^A$ and $a^\mu_{AB}$ are Lorentz violation coefficients which can be thought as resulting from tensor vacuum expectation values in some kind of underlying theory, the $A, B$ are flavor indices, and the ellipsis denotes the non-renormalizable operators (eliminated in the minimal SME). The first term in Eq. (5) is exactly the SM operator, and the second and third terms (CPT-even and CPT-odd respectively) describe the contribution from Lorentz violation. The coefficients $\sigma^A$ and $a^\mu_{AB}$ serve as the background fields though they are not fields but general tensors with constant values in the observer working frame.

In SME, the background fields transform as tensors according to their Lorentz indices between different inertial frames of reference but keep unchanged within the same frame. It means that there exists a privileged inertial frame of reference in which the background can be considered as the “new æther”, i.e., the “vacuum” at rest. The “æther”, which is a collections of background fields, changes from one frame to another frame by Lorentz transformation. Within a same frame of reference, these background fields are just treated as fixed parameters. The Lorentz violation is due to the existence of the background fields. The standard model particles breaks the Lorentz invariance at a certain frame of the observer by taking these background fields as fixed. From a strict sense, there is no Lorentz violation for the whole system of the standard model particles together with the background fields.

2.3. The standard model supplement (SMS)

The standard model extension is an effective framework for phenomenological analysis. We still need a fundamental theory to derive the Lorentz violation terms from basic principles. In the Standard Model Supplement (SMS) framework\textsuperscript{32,33}, the LV
terms are brought about from a basic principle denoted as the physical independence or physical invariance:

- **Principle of Physical Invariance**: the equations describing the laws of physics have the same form in all admissible mathematical manifolds.\(^1\)

The principle leads to the following replacement of the ordinary partial \(\partial_\alpha\) and the covariant derivative \(D_\alpha\):

\[
\begin{align*}
\partial^\alpha & \to M^{\alpha\beta} \partial_\beta, \\
D^\alpha & \to M^{\alpha\beta} D_\beta,
\end{align*}
\]

where \(M^{\alpha\beta}\) is a local matrix. The Lorentz violation terms are thus uniquely determined from the standard model Lagrangian without any ambiguity\(^2\), and their general existence is derived from basic consideration rather than added by hand. The explicit form of the matrices \(M^{\alpha\beta}\) demands more basic theories concerning the true nature of space and time, and we suggest to adopt a physical way to explore these matrices through experiments rather than from theory at first. For more generality, we do not make any ad hoc assumption about these matrices. Thus these matrices might be particle-dependent corresponding to the standard model particles under consideration, with the elements of these matrices to be measured or constrained from experimental observations.

We separate \(M^{\alpha\beta}\) to two matrices like

\[
M^{\alpha\beta} = g^{\alpha\beta} + \Delta^{\alpha\beta},
\]

where \(g^{\alpha\beta}\) is the metric tensor of space-time and \(\Delta^{\alpha\beta}\) is a new matrix which is particle-type dependent generally. Since \(g^{\alpha\beta}\) is Lorentz invariant, \(\Delta^{\alpha\beta}\) contains all the Lorentz violating degrees of freedom from \(M^{\alpha\beta}\). Then \(\Delta^{\alpha\beta}\) brings new terms violating Lorentz invariance in the standard model and is called Lorentz violation matrix. The theory returns back to the standard model when these Lorentz violation matrices vanish. Thus one may consider the Lorentz violation matrices in the SMS framework as similar to the background fields in the minimal SME model.

For the electroweak interaction sector, the Lagrangian of fermions in the SMS framework can be written as\(^3\)

\[
L_F = i\bar{\psi}_{A,L} \gamma^\alpha \partial_\alpha \psi_{B,L} \delta_{AB} + i\Delta_{L,AB}^{\alpha\beta} \bar{\psi}_{A,L} \gamma_\alpha \partial_\beta \psi_{B,L} + i\bar{\psi}_{A,R} \gamma^\alpha \partial_\alpha \psi_{B,R} \delta_{AB} + i\Delta_{R,AB}^{\alpha\beta} \bar{\psi}_{A,R} \gamma_\alpha \partial_\beta \psi_{B,R},
\]

(7)

where \(A, B\) are flavor indices. The Lorentz violation terms are uniquely and consistently determined from the standard model by including the Lorentz violation matrices \(\Delta^{\alpha\beta}\), which are generally particle-dependent\(^3\) with flavor indices. For leptons, \(\psi_{A,L}\) is a weak isodoublet, and \(\psi_{A,R}\) is a weak isosinglet. After the calculation of the doublets and classification of the Lagrangian terms again, the Lagrangian can be written in a form like that of Eq. (7) too. Assume that the Lorentz violation matrix \(\Delta^{\alpha\beta}_{L,AB}\) is the same for the left-handedness and right-handedness, namely \(\Delta^{\alpha\beta}_{L,AB} = \Delta^{\alpha\beta}_{R,AB} = \Delta^{\alpha\beta}_{AB}\). Without considering mixing between flavors, one can rewrite Eq. (7) as

\[
L_F = \bar{\psi}_A (i \gamma^\alpha \partial_\alpha - m_A) \psi_A + i \Delta^{\alpha\beta}_{A,A} \bar{\psi}_A \gamma_\alpha \partial_\beta \psi_A,
\]

(8)
where $\psi_A = \psi_{A,L} + \psi_{A,R}$, i.e., the field $\psi_A$ is the total effects of left-handed and right-handed fermions of the given flavor $A$. When there is only one handedness for fermions, $\psi_A$ is just the contributions of this one handedness, which is the situation for neutrinos. The mass term in the Lagrangian $\mathcal{L}_F$ is included, one can let $m_A \rightarrow 0$ for massless fermions.

In similar to the above two frameworks, there also exists the question of how to understand and handle the Lorentz violation matrix $\Delta^{\alpha\beta}$. We list here three options for understandings and treatments:

- **Scenario I**: which can be called as fixed scenario in which the Lorentz violation matrices are taken as constant matrices in any inertial frame of reference the observer is working. It means that the the Lorentz violation matrices can be taken as approximately the same for any working reference frames such as the earth-rest frame, the sun-rest frame, or the CMB frame. This scenario can be only adopted as an approximation in similar to the Coleman-Glashow model, when the observer is focusing on the Lorentz violation effect within a certain frame and does not care about relationships between different frames. There will be the problem of inconsistency for an “absolute physical event” between different reference frames as pointed out for the Coleman-Glashow model, if one sticks to this scenario.

- **Scenario II**: which can be called as “new æther” scenario in which the Lorentz violation matrices transform as tensors between different inertial frames but keep as constant matrices within the same frame. This scenario corresponds to the same treatment of background fields as in the SME case. The Lorentz violation matrices play the roles as the background fields.

- **Scenario III**: which can be called as covariant scenario in which the Lorentz violation matrices transform as tensors adhered with the corresponding standard model particles. It means that these Lorentz violation matrices are emergent and covariant with their standard model particles. Such a scenario still needs to be checked for consistency and for applications.

Before accepting the SMS as a fundamental theory, one can take the SMS as an effective framework for phenomenological applications by confronting with various experiments to determine and/or constrain the Lorentz violation matrix $\Delta^{\alpha\beta}$ for various particles. So our idea is to reveal the real structure of Lorentz violation of nature from experiments rather than from theory. We consider this phenomenological way as more appropriate for physical investigations, rather than to derive everything from theory at first. As a comparison, the specific form of quark mixing matrix is determined from experimental measurements rather than derived from theory. Even after so many years of research and also the elements of CKM mixing matrix have been measured to very high precision, there is still no a commonly accepted theory to derive the quark mixing matrix from basic principles.
2.4. Some remarks

In the effective field theory frameworks, the standard particles transform according to the Lorentz symmetry between different momentum states. The background fields should also transform according to the Lorentz symmetry between different observer working frames from the requirement of consistency. From this sense, there is actually no Lorentz violation for the whole system of the standard model particles together with the background fields. The Lorentz violation exists for the standard model particles within an observer working frame, when these particles have different momenta between each other. From this sense, the Lorentz violation is due to the existence of the background fields, which are treated as fixed parameters in the observer working frame.

The above discussed three frameworks have their own advantages and disadvantages in formalisms and in phenomenological applications. The Coleman-Glashow model is the most simple and intuitive for fast applications to physical processes, for the estimation of the magnitude of the Lorentz violation effect. The SME is systematic with all possible terms that can serve as a useful tool to confront with various phenomenological constraints. The SMS is theory based with clear relationship between some general LV parameters in SME\textsuperscript{[39]}, and can be conveniently applied for phenomenological analysis\textsuperscript{[40]}. We would need more experimental investigations to check which one of them can meet the criterion of being able to provide a satisfactory description of the physical reality, with simplicity and beauty in formalism, together with the predictive power towards new knowledge for human being. It is also possible that the nature satisfies the Lorentz symmetry perfectly and we would be unable to find a physical evidence to support any of these theories.

3. The OPERA “Anomaly”

The report by the OPERA Collaboration for a faster-than-light speed of muon neutrinos has attracted the eyes by the physical society as well as the public society\textsuperscript{[37]}. We have witnessed three stages of the OPERA performance:

- The release of the first version of the OPERA report\textsuperscript{[41]} on September 22 of 2011, reporting that the muon neutrinos in the CERN-CNGS neutrino beam were detected by the OPERA detector over a baseline of about 730 km. Compared to the time taken for neutrinos traveling at the speed of light in vacuum, an earlier arrival time of $(60.7 \pm 6.9 \text{ (stat.)} \pm 7.4 \text{ (sys.)})$ ns was measured. The neutrino velocity $v$ is thus measured and its difference with respect to the vacuum light speed $c$ is $(v - c)/c = (2.48 \pm 0.28\text{ (stat.)} \pm 0.30\text{ (sys.)}) \times 10^{-5}$ at a significance of 6$\sigma$.
- To overcome the criticism that the long proton beam duration at CERN may introduce bias in the neutrino arrival time measurement, the OPERA collaboration released their revised report\textsuperscript{[42]} on November 17 of 2011. They repeated the measurement over the same baseline without any as-
sumptions about the details of neutrino production during the spill, such as energy distribution or production rate, by using a new CERN beam which provided proton pulses of 3 nanoseconds each with 524 nanosecond gaps. Without using the earlier statistical computation, the OPERA collaboration measured twenty events indicating neutrinos had traveled faster than light by 60 ns, with 10 ns uncertainty. The error bounds for the original superluminal speed fraction were tightened further to $(2.37 \pm 0.32 \text{(stat.)} + 0.34/ -0.24 \text{(sys.)}) \times 10^{-5}$, with the new significance level becoming 6.2σ.

- There was a news on February 22 of 2012 that the OPERA collaboration has identified two possible effects that could have an influence on its neutrino timing measurement. The first possible effect concerns an oscillator used to provide the time stamps for GPS synchronizations, and the second concerns the optical fibre connector that brings the external GPS signal to the OPERA master clock. The two effects could have led to an underestimate of the flight time of the neutrinos, and a re-measurement of the neutrino speed by the OPERA collaboration will be done in the near future.

Before the OPERA “anomaly”, there have been similar long baseline experiments on the speed of neutrinos. The first direct measurement of neutrino velocity was performed at Fermilab thirty years ago. Just a few years ago, the MINOS Collaboration reported a shift with respect to the expected time of flight of $\delta t = -126 \pm 32 \text{ (stat.)} \pm 64 \text{ (sys.)} \text{ ns}$, which corresponds to a constraint on the muon neutrino velocity, $(v_\nu - c)/c = (5.1 \pm 2.9) \times 10^{-5}$ at 68% confidence level. This 1.8σ signal was considered to be compatible with also zero, therefore the previous experimental data neither provide a strong evidence for the superluminality of neutrinos nor exclude it.

Besides the long baseline experiments, there are also measurable phenomenologies related to neutrino speed in astrophysics. For instance, one astronomical event was observed with neutrino emissions on 23 February 1987, 7:35:35 UT (±1 min) — the Supernova 1987A, which was later optically observed on 24 February 1987. More than ten neutrinos were recorded with a directional coincidence within the location of supernova explosion, several hours before the optical lights were observed. Because of weak interactions, neutrinos may leak out of the dense environment produced by the stellar collapse before the optical depth of photons becomes visible. Hence an early-arrival of neutrinos is expected. The journey of propagation of photons and neutrinos are of astrophysical distance ($\sim 51.4 \text{ kpc}$), hence it provides an opportunity to measure the speed of neutrinos to be within the light speed with a precision of $\sim 2 \times 10^{-9}$. This also neither provides an evidence for the superluminality of neutrinos nor excludes it.

However, as the OPERA “anomaly” seems to support the superluminality of neutrinos strongly, there has been a blossom of novel theories that can produce the superluminality of neutrinos. In fact, the effective field theory framework
can produce modification to the standard energy-momentum dispersion relation of a particle. One thus can calculate the particle velocity through the new dispersion relation, in which the LV parameters enter. The velocity of a particle could therefore be superluminal or subluminal by adjusting the LV parameters. By confronting with the OPERA reported “data”, the LV parameters were estimated in Ref. for the SMS framework and in Ref. for the minimal SME, indicating a magnitude of the order $10^{-5}$ for relevant LV parameters in both frameworks.

With a size of order $10^{-5}$ for the LV parameter $\epsilon$, Cohen and Glashow argued that the high energy muon neutrinos exceeding tens of GeVs cannot be detected due to the energy-losing process $\nu_{\mu} \rightarrow \nu_{\mu} + e^+ + e^-$ analogous to Cherenkov radiations through the long baseline about 730 km. Bi et al. also argued that the Lorentz violation of muon neutrinos of order $10^{-5}$ will forbid kinematically the production process $\pi \rightarrow \mu + \nu_{\mu}$ for muon neutrinos with energy larger than about 5 GeV. Their arguments provide demonstrations of adopting the Coleman-Glashow model for fast and intuitive illustrations of the LV effects, and the arguments have been also adopted as a reason to refute the OPERA “anomaly”. The conclusion of the Cherenkov-like radiations and the forbidden of the muon neutrino production processes can be also true in the SMS and SME frameworks with the LV parameters fixed in the Scenarios I and II as discussed in the last section. The rationality of superluminality of neutrinos seems to be only possibly in the covariant picture of Lorentz violation, such as in the Scenario III suggested as an option to handle the LV effect in the effective field theory framework. However, such a possibility can only be seriously considered when there will be strong evidence for the superluminality of neutrinos in future experiments.

4. Conclusion

Researches on Lorentz violation have been active for many years, with various theories have been proposed and many phenomenological studies have been performed to confront with various observations, though there is still no convincing evidence yet. However, there are new chances for Lorentz violation study due the availability of many new theoretical frameworks that can be applied to phenomenological analysis more conveniently and also due to the developments of high precision measurements for the future experimental investigations. We conclude that Lorentz violation is becoming an active frontier to explore both theoretically and experimentally.

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