Fast Line Search for Multi-Task Learning

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Abstract

Multi-task learning is a powerful method for solving several tasks jointly by learning robust representation. Optimization of the multi-task learning model is a more complex task than a single-task due to task conflict. Based on theoretical results, convergence to the optimal point is guaranteed when step size is chosen through line search. But, usually, line search for the step size is not the best choice due to the large computational time overhead. We propose a novel idea for line search algorithms in multi-task learning. The idea is to use latent representation space instead of parameter space for finding step size. We examined this idea with backtracking line search. We compare this fast backtracking algorithm with classical backtracking and gradient methods with a constant learning rate on MNIST, CIFAR-10, Cityscapes tasks. The systematic empirical study showed that the proposed method leads to more accurate and fast solution, than the traditional backtracking approach and keep competitive computational time and performance compared to the constant learning rate method.

Introduction

Multi-task Learning (MTL) (Caruana 1997; Ruder 2017; Zhang and Yang 2017; Standley et al. 2019) is an approach to inductive transfer that improves generalization by using the domain information contained in the training signals of related tasks as an inductive bias. It does this by learning tasks in parallel while using a shared representation; what is learned for each task can help other tasks be learned better. Many MTL approaches have shown their efficacy and remarkable performance in many areas such as computer vision (Kokkinos 2017; Chenmputati et al. 2019), natural language processing (Subramanian et al. 2018; Collobert and Weston 2008), and speech recognition (Huang et al. 2015).

A neural network is a state-of-the-art model for solving various machine learning tasks, and multi-task learning is not an exception. Optimization of neural networks often utilizes gradient descent methods. Several methods were proposed to adapt gradient descent for multi-task problems. The first approach is scalarization (Johannes 1984) — a reduction of a multi-task problem to a single-task problem. The classical scalarization method is weighting — treating a multi-task problem as a weighted combination of single-task problems. This approach heavily relies on proper weight selection. Another approach that has no weight selection problem is treating multi-task learning as a multiobjective optimization problem. In this case, we search for direction minimizing all functions. Nevertheless, a problem of choosing step size arises in this approach.

Classical adaptive step size methods like ADAM (Kingma and Ba 2014), Nesterov Accelerated Gradient (Nesterov 1983), RMSProp (Tieleman and Hinton 2012), SAG (Schmidt, Le Roux, and Bach 2017) can be applied for multi-task learning, and they perform well in practice but often comes with little theoretical guarantees for convergence. Using line search methods as backtracking (Armijo 1966; Wolfe 1969; Vaswani et al. 2019, 2020) golden search and parabolic interpolation has theoretical guarantees for convergence but requires additional function calls, which in neural network case has a hefty cost. Especially it is impractical in multi-task learning when there are several task-specific modules, and each function call requires a forward step through a huge encoder.

Addressing the well-known problem of choosing learning
rate, we propose a novel optimization idea, which is based on structural properties of multi-task models with "hard shared parametrization" (Caruana 1997). Our idea is to do only task-specific model forward passes in backtracking line search (see Figure 1). At the same time, we use linear approximation, as it dramatically reduces line search time cost as the encoder containing most cost is inactive. That is why we name this idea Fast Backtracking Line Search (FBLS) later in the text.

We tested our method on Multi-MNIST (Sabour, Frosst, and Hinton 2017), CIFAR-10 (Krizhevsky, Nair, and Hinton 2014) and Cityscapes (Cordts et al. 2016) with fast backtracking, classical backtracking and constant learning rate stochastic gradient descent (SGD). Also, we compared these algorithms on the BERT (Devlin et al. 2018) model as an encoder in an artificial multi-task learning setting. We measured average epoch time for the algorithms on the subset of STSB dataset (Cer et al. 2017). The outcomes of our work are the following:

- We have proposed the Fast Backtracking Line Search method for multi-task learning and have proven its convergence to the Pareto stationary point.
- We have compared the proposed approach numerically with classical backtracking line search, SGD with a constant learning rate, and SGD in the latent space. The larger encoder is relative to the decoder size, the greater efficiency the proposed approach shows. It is worth noting that at a significant decrease of epoch time, in many experiments, final quality after the same number of epochs was not worse than SGD.

The work is organized as follows. In the first part, we introduce necessary preliminaries, which are specific for multiobjective optimization. In the second part, we introduce Fast Backtracking Line Search and provide the convergence theorem for it. Then, we show the results of our experiments for computer vision (CV) and natural language processing (NLP) models. In the end, we provide a review of related works in the field, followed by a conclusion.

**Preliminaries: Problem and Notation**

**Notation.**

Consider a multi-task model with encoder-decoder structure (see Figure 2). Let \( z(x, \theta^{sh}) \) be an encoder model. Let \( f^t(z(x, \theta^{sh}), \theta^t) \) be task-specific decoder for task \( t \). We use \( \{x_i, y_i^1, \ldots, y_i^T\}^{i=1}_1 \) to refer to a batch of objects and their corresponding labels.

Vectors of parameters \( \theta^{sh} \) lie in the parameter space, while the decoder outputs \( z(x, \theta^{sh}) \) belongs to the latent space. Let \( \hat{L}^t(\theta^{sh}, \theta^t) = \mathbb{E}_{x} \left[ \ell(z(x, \theta^{sh}), f(z, \theta^t)) \right] \) be a loss function for task \( t \). Let \( d_z \) and \( d_{sh} \) be some vectors in latent space and parameter space, respectively, which satisfies the following conditions:

\[
\forall t \in \{1, \ldots, T\} : (\nabla_z \hat{L}^t) \top d_z < 0
\]
\[
\forall t \in \{1, \ldots, T\} : (\nabla_{\theta^{sh}} \hat{L}^t) \top d_{sh} < 0
\]

So, applying gradient step with direction \( d_z \) in latent space or with direction \( d_{sh} \) in parameter space decreases all loss functions.

**Problem formulation.**

Consider multi-task learning problem with \( T \) tasks:

\[
\min_{\theta^{sh}, \theta^1, \ldots, \theta^T} \mathcal{L} (\theta^{sh}, \theta^1, \ldots, \theta^T),
\]

\[
\mathcal{L} = \left( \hat{L}^1(\theta^{sh}, \theta^1), \ldots, \hat{L}^T(\theta^{sh}, \theta^T) \right) \top
\]

To solve this multi-task learning problem, we treat it as a multiobjective optimization problem and introduce partial order - Pareto dominance.

**Definition 1.** (Pareto dominance). We say, that a vector \( \theta_1 \) is dominated by vector \( \theta_2 \) iff:

\[
\forall t \in \{1, \ldots, T\} : \hat{L}^t(\theta_2) \leq \hat{L}^t(\theta_1) \]

\[
\exists i : \hat{L}^i(\theta_2) < \hat{L}^i(\theta_1)
\]

In our case, we treat a multiobjective optimization as a problem of finding a point that is not Pareto dominated — a Pareto optimal point.

**Definition 2.** (Pareto optimality). We say, that a vector \( \theta \) in problem \( \mathcal{P} \) is Pareto optimal if and only if there \( \theta \neq \tilde{\theta} \) such that Pareto dominate \( \tilde{\theta} \).

In particular, if a point \( \theta \) is not Pareto optimal, there is at least one other point that should be better. (Boyd and Vandenberghe 2004)

Finding a Pareto optimal point could be a very challenging problem. However, we can try to satisfy the necessary condition for Pareto optimality - Pareto stationarity.
by Armijo rule (Armijo 1966) with parameter \( \beta \) using the beforementioned condition, it is usually replaced with constant step size or adaptive step-size gradient methods for making one gradient step.

The main idea of line search is to find step size \( \eta \) which satisfies the following condition \( \forall t \in \{1 \ldots T\} \):

\[
\hat{\mathcal{L}}^t(\theta - \eta \mathbf{d}) < \hat{\mathcal{L}}^t(\theta).
\]

This condition means that at each step, all functions are decreasing. If, in addition, the functions \( \mathcal{L} \) are bounded from below, the exact line search (like in steepest descent) guarantees convergence to a Pareto stationary point. In our paper, we consider only backtracking line search (BLS), but the idea works for an arbitrary line search method as well. The idea of backtracking is to reduce the step size by parameter \( \gamma \) until the line search condition is satisfied. Instead of using the aforementioned condition, it is usually replaced by Armijo rule (Armijo 1966) with parameter \( \beta \in [0,1] \),

\[
\begin{align*}
\theta^{sh} &\leftarrow \theta^t - \eta \cdot \nabla_{\theta^t} \hat{\mathcal{L}}^t, \\
\text{for } t &\leftarrow 1 \text{ to } T \\
\theta^t &\leftarrow \theta^t - \eta \cdot \nabla_{\theta^t} \hat{\mathcal{L}}^t \\
\text{end for}
\end{align*}
\]

until Stopping criterion

\[ \eta \leftarrow \gamma \cdot \eta \]

until Armijo condition \[ (\beta, \eta) \]

\[ \text{for } t \leftarrow 1 \text{ to } T \]

\[ \theta^t \leftarrow \theta^t - \eta \cdot \nabla_{\theta^t} \hat{\mathcal{L}}^t \]

\[ \text{end for} \]

\[ \theta_{\text{new}} \leftarrow \theta^{sh} \]

\[ \text{until Stopping criterion} \]

\[ \forall t \in \{1 \ldots T\}: \]

\[
\hat{\mathcal{L}}^t(\theta - \eta \mathbf{d}) \leq \hat{\mathcal{L}}^t(\theta) - \eta \beta \left( \frac{\partial \hat{\mathcal{L}}^t}{\partial \theta} \right)^\top \mathbf{d}
\]

So, to find the appropriate step size, we should use a line search. Nevertheless, it is computationally ineffective as on each iteration, we should perform forward and backward pass through the entire encoder and all decoders. We repeat this procedure till the Armijo rule has been fulfilled. It may take several steps and cause heavy wastage of computational resources since we do several forward and backward passes for making one gradient step.

The problem of substantial computational overhead for backtracking may be especially relevant for models with huge encoders (relatively to the decoder size) in terms of the number of parameters. An example of such a model is the Transformer (Vaswani et al. 2017; Liu et al. 2019; Dosenkov et al. 2020; Hu and Singh 2021; Zhang, Gong, and Chen 2021), which has become the de-facto standard for NLP tasks.

Generally, in NLP, a model has a heavy encoder to produce consistent word embeddings, while the decoder could be much smaller. In this case minimizing using encoder model can significantly improve computational speed. Therefore, in such models, latent space optimization could be of particular interest, as we only once use encoder just for making one gradient step.

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\[ \eta \leftarrow \gamma \cdot \eta \]

until Armijo condition \[ (\beta, \eta) \]

\[ \text{for } t \leftarrow 1 \text{ to } T \]

\[ \theta^t \leftarrow \theta^t - \eta \cdot \nabla_{\theta^t} \hat{\mathcal{L}}^t \]

\[ \text{end for} \]

\[ \theta_{\text{new}} \leftarrow \theta^{sh} \]

\[ \text{until Stopping criterion} \]

\[ \forall t \in \{1 \ldots T\}: \]

\[
\hat{\mathcal{L}}^t(\theta - \eta \mathbf{d}) \leq \hat{\mathcal{L}}^t(\theta) - \eta \beta \left( \frac{\partial \hat{\mathcal{L}}^t}{\partial \theta} \right)^\top \mathbf{d}
\]

Method

In this section, we propose an idea of using latent space for finding step size. The optimization in latent space allows not to use additional forward and backward passes through the
encoder. To establish the capabilities of the method, we experimented with the line search backtracking algorithm.

Consider gradients in the parameter and latent spaces: \( \nabla_{\theta^t} \hat{L}^t, z^t \nabla_{\theta^t} \hat{L}^t \). To obtain minimization direction \( d_{sh} \) or \( d_z \), one can use PCGrad [Yu et al. 2020], or EDM [Katrutsa et al. 2020], or MGDA (Desideri 2012; Sener and Koltn 2018).

As we get the minimization direction, one can consider the classical backtracking approach for multi-task learning. In this case, Armijo-rule can be represented as \( \forall t \in \{1 \ldots T\} \):

\[
\hat{L}^t (\theta^{sh} - \eta d_{sh}, \theta^t - \eta \nabla_{\theta^t} \hat{L}^t) \leq \\
\leq \hat{L}^t - \eta \beta \left\| \frac{\partial \hat{L}^t}{\partial \theta^t} \right\| ^2 - \eta \beta \left( \frac{\partial \hat{L}^t}{\partial \theta^{sh}} \right)^\top d_{sh}
\]

(4)

The complete classical backtracking algorithm is presented as Algorithm 1.

One can see that in the classical approach, one needs to update shared parameters several times on every iteration. We use the following local linear approximation:

\[
z(\theta^{sh} - \eta d_{sh}) \approx z - \eta d_z
\]

(5)

Then, one can get a new Armijo rule:

\[
\hat{L}^t (z - \eta d_z, \theta^t - \eta \nabla_{\theta^t} \hat{L}^t) \leq \\
\leq \hat{L}^t - \eta \beta \left\| \frac{\partial \hat{L}^t}{\partial \theta^t} \right\| ^2 - \eta \beta \left( \frac{\partial \hat{L}^t}{\partial z} \right)^\top d_z
\]

(6)

The complete fast backtracking algorithm is presented as Algorithm 2. In this Armijo rule, we do not have updates of encoder parameters \( \theta^{sh} \). So we can keep the encoder frozen during the step size search stage.

In the following theorem, we prove that the new algorithm converges to Pareto stationary point.

**Theorem 1.** Suppose loss functions \( \hat{L}^t \) are continuously differentiable and bounded below. Then, every accumulation point of sequence \( x_k = [\theta_k^{sh}, \theta_k^t, \ldots, \theta_k^T] \) produced by Algorithm 2 with following Armijo-rules is a Pareto stationary point \( \forall t \in \{1 \ldots T\} \):

\[
\hat{L}^t (z - \eta d_z, \theta^t - \eta \nabla_{\theta^t} \hat{L}^t) \leq \\
\leq \hat{L}^t - \eta \beta \left\| \frac{\partial \hat{L}^t}{\partial \theta^t} \right\| ^2
\]

(7)

\[
\ldots \leq \hat{L}^t - \eta \beta \left( \frac{\partial \hat{L}^t}{\partial \theta^t} \right)^\top d_z
\]

(8)

\[
\ldots \leq \hat{L}^t - \eta \beta \left( \frac{\partial \hat{L}^t}{\partial \theta^t} \right)^\top d_z - \eta \beta \left\| \frac{\partial \hat{L}^t}{\partial \theta^t} \right\| ^2
\]

(9)

We prove Theorem 1 in Appendix. Indeed, using a linear approximation of the encoder \( 5 \) instead of its full update allows us to keep the convergence properties for Armijo-like line search. In the next part, we conduct analysis FBLS, BLS, SGD, MGDA-UB methods.

## Experiments

The experiments can be divided into 4 groups by data type: MultiMNIST, CIFAR-10, Cityscapes, STS benchmark for BERT model.

| Alg / Data   | MultiMNIST | CIFAR-10 | w/o \( \eta \)? |
|--------------|------------|----------|-----------------|
| SGD          | 14.3 (100 %) | 550 (100 %) | x               |
| BLS          | 19.5 (136 %) | 650 (118 %) | ✓               |
| MGDA-UB      | 13.6 (95 %) | 80 (15 %) | x               |
| FBLS (Ours)  | 14.3 (100 %) | 85 (15 %) | ✓               |

Table 1: Average epoch time in seconds for different algorithms on MultiMNIST and CIFAR-10 datasets. By "w/o \( \eta \)?" we mean the ability to run an algorithm without selecting a learning rate \( \eta \). The lower, the better. We took SGD as the baseline - 100 %.

For comparison, we examined fast backtracking, classical backtracking, classical SGD, and Latent space SGD (MGDA-UB) (Sener and Koltn 2018). Experiments were conducted on MultiMNIST, CIFAR-10, and Cityscapes datasets. In practice for Algorithms 1, 2, we have set additional stopping criterion what \( \eta \) can't be lower some small \( \varepsilon \). For backtracking algorithms we have chosen learning rate lower bound \( \varepsilon = 10^{-10}, \beta = 0.1 \). The initial upper bound has been set as 1. At first experiments, we observed that from some iteration step size converged to upper bound. This behavior contradicts the theory of gradient algorithms. We decay step size upper bound by decay rate = 0.5 every \( N = 10 \) epochs to overcome this problem. Such version of algorithm is presented as “FBLS+decay” on the figures. Nevertheless, the time comparison, presented on the Table 1 was averaged over 10 epochs.

### MultiMNIST

Figure 3: Results of Backtracking Line Search performance and constant learning rate SGD after 100 epochs on MultiMNIST dataset.

On MultiMNIST, we use LeNet-5 architecture. The batch size has been set as 256. The training was conducted for 100 epochs. For comparison with standard gradient descent, we choose eight step sizes evenly from log-range between -3 and -1. As loss function, we chose cross-entropy loss, and as error, we consider (1 - accuracy). Results of SGD and FBLS were averaged over 3 experiments (Figure 3).
From Figure 3 we can see that as good as classical gradient descent. Also, it takes the same time per epoch as classic stochastic gradient descent (see Table 1).

**Classical backtracking** For comparison of classical and fast backtracking line search we examined their behaviour at different $\beta = \{0.1, 0.2, 0.3, 0.4\}$. Averaged over different $\beta$ results presented in Figure 4.

![Figure 4: Comparison of Classical Backtracking Line Search vs FBLS. Results averaged over four different $\beta$ parameters](image)

From Figure 4 one can see that fast backtracking overcome classical backtracking both on test and train. Also, fast backtracking is faster than classical backtracking by 36% on the MultiMNIST (see Table 1).

**CIFAR-10**

![Figure 5: Results of Backtracking Line Search performance and constant learning rate SGD after 50 epochs on MultiCIFAR10 dataset](image)

For CIFAR-10, we use ten classes for creating ten synthetic "one vs rest" classification problems. The batch size has been set equal to 256. As an encoder, we have chosen scratch ResNet-18 and as decoder one fully connected layer. We compare stochastic gradient and fast backtracking on these tasks (Figure 5). Eight step sizes for stochastic gradient descent have been chosen evenly from log-range from -3 to -1. As a loss, we chose cross-entropy, and as an error, we considered (1 - accuracy).

From Figure 5 one can see that fast backtracking has the same final test loss as classical gradient descent. Also, fast backtracking is faster than classical gradient descent (see Table 1).

**Cityscapes**

As an encoder, we have chosen ResNet-50 pre-trained on Imagenet, and as decoders, we use PSPNet for disparity estimation, instance segmentation, and semantic segmentation. We compare stochastic gradient descent and fast backtracking. For gradient we have chosen step sizes from $\{0.001, 0.01, 0.1\}$.

![Table 2: Time comparison for Cityscapes multi-task experiment. By "w/o $\eta$?" we mean the ability to run an algorithm without selecting a learning rate $\eta$. The lower, the better. We took SGD as the baseline - 100 %.

| Method | Average epoch time, s | w/o $\eta$? |
|--------|-----------------------|-------------|
| SGD    | 6.0 (100 %)           | x           |
| FBLS (Ours) | 7.7 (128 %)    | ✓           |

For this reason, we have decided to measure epoch time for the Transformer-like model in a multi-task setting. We used BERT (Devlin et al. 2018) language model as the encoder on the STS benchmark (Cer et al. 2017). While the original benchmark was single-task oriented, we made an artificial multi-task setting with $T = 3$ same tasks. It means that we studied three independent decoders of the same architecture. Two fully connected layers were used as the decoder.

The results of Table 3 clearly show that the use of latent space approaches (FBLS and MGDA-UB) gives a significant reduction in the time per epoch compared to approaches based on entire encoder computation (SGD and BLS). The fastest method by time per epoch is MGDA-UB, but it inherits the disadvantage of SGD - the need to adjust the learning rate in fast backtracking.
rate. As expected, FBLS is somewhat slower than MGDA-UB since some iterations of backtracking require several decoders forward/backward passes.

**Related work**

We recommend (Crawshaw 2020; Ruder 2017) for surveys of modern multi-task learning. A classical approach to solve multiobjective optimization is a scalarization (Johannes 1984). Scalarization is a reduction of a multiobjective optimization problem to a single objective optimization problem. There are several techniques to provide it. The first one is weighting (Das and Dennis 1997). In this case, one minimizes a weighted combination of objectives. The other approach is minimization on simplex constructed on minimums of every objective. These approaches included normal-boundary intersection method (Das and Dennis 1998), normal constraints method (Messac, Ismail-Yahaya, and Mattson 2003). Also, there are several evolutionary strategies (Knowles and Corne 2000; Deb et al. 2000).

The other approach is based on the min-max technique. (Fliege and Svaiter 2000) uses min-max to standard gradient descent $L^2$-proximal task and gain method which output vector minimizing all direction evenly. The min-max technique was further adapted to Newton method (Fliege, Drummond, and Svaiter 2009), stochastic gradient (Fliege and Xu 2011) and convergence rates were obtained (Fliege, Vaz, and Vicente 2019).

Initially, dual problem to aforementioned min-max approach convex hull norm minimization (Desideri 2012) became a new technique in multiobjective optimization. The method was modified in further works by adding the Gram-Schmidt process to gain solution (Desideri 2012), modification the Gram-Schmidt process to gain fast inexact solution (Desideri 2014) and second-order method through normalization (Desideri 2014). Also, it has been proved convergence in stochastic case (Mercier, Poirion, and Desideri 2018). In (Sener and Koltun 2018) MGDA method was applied to latent space rather than parameter space. The replacement plays the role of the upper bound, and the solution in this space implies a solution in a shared parameter space. Adding task losses normalization (Katrutsa et al. 2020) in norm minimization creates a more robust solution in case of unbalanced tasks. The last method that can be categorized as gradient choosing method is (Yu et al. 2020). In this method, we change the initial solution cone to a proper “semi-dual” cone where every vector from this cone minimizes all losses and choose the average vector.

Besides gradient methods, adaptive weighting techniques were proposed to overcome the necessity of proper weighting search. (Kendall, Gal, and Cipolla 2018) used uncertainty of likelihood as weights. (Chen et al. 2018) used loss term forcing gradient to update evenly. Also, (Liu, Johns, and Davison 2019) used weighting based on rates of loss on the previous step and current step.

The separate direction in multi-task learning is architecture choice. It is considered that the choice of appropriate architecture allows introducing inductive bias. In (Long et al. 2017; Yang and Hospedales 2016) it is assumed that shared parameters have hidden tensor structure and examined tensor factorization and tensor normal distribution approaches. In (Misra et al. 2016; Rosenbaum, Klinger, and Riemer 2017; Ruder et al. 2019) controlling units were used. These units control the rate of sharing between intermediate
encoder outputs. In (Lu et al. 2017; Standley et al. 2019) iterative neural architecture search was examined to build a multi-task model. In (Rebuffi, Bilen, and Vedaldi 2018; Meyerson and Mikkulainen 2017; Liu, Johns, and Davison 2019; Maninis, Radosavovic, and Kokkinos 2019) different attention mechanisms and adapters are used. These modules have fewer parameters than the main model, and they are used for solving multi-task and multi-domain problems. In (Zamir et al. 2018) exhaustive investigation of relationships between computer vision tasks was worked out.

Conclusion

In this work, we propose a novel optimization idea for multi-task learning. The idea is to use latent space to reduce the cost of line search. We examined this idea with a backtracking line search algorithm. We prove the convergence of the method theoretically. Also, we show the practical efficacy of the algorithm. The maximum efficiency of our method is achieved when the encoder is much larger than the decoder, which is valid on most NLP and CV models. The limitation of the Fast Backtracking Line Search is connected to SGD. We have to manually decay step size as it converges to the upper bound, but a more effective solution could exist.

References

Armijo, L. 1966. Minimization of functions having Lipschitz continuous first partial derivatives. Pacific Journal of mathematics, 16(1): 1–3.

Boyd, S.; and Vandenberghe, L. 2004. Convex Optimization. Cambridge University Press.

Caruana, R. 1997. Multitask learning. Machine learning, 28(1): 41–75.

Cer, D.; Diab, M.; Agirre, E.; Lopez-Gazpio, I.; and Specia, L. 2017. Semeval-2017 task 1: Semantic textual similarity-multilingual and cross-lingual focused evaluation. arXiv preprint arXiv:1708.00055.

Chen, Z.; Badrinarayanan, V.; Lee, C.-Y.; and Rabinovich, A. 2018. Gradnorm: Gradient normalization for adaptive loss balancing in deep multitask networks. In International Conference on Machine Learning, 794–803. PMLR.

Chennupati, S.; Sistu, G.; Yogamani, S.; and Rawashdeh, S. 2019. Auxnet: Auxiliary tasks enhanced semantic segmentation for automated driving. arXiv preprint arXiv:1901.05808.

Collobert, R.; and Weston, J. 2008. A unified architecture for natural language processing: Deep neural networks with multitask learning. In Proceedings of the 25th international conference on Machine learning, 160–167.

Cordts, M.; Omran, M.; Ramos, S.; Rehfelt, T.; Enzweiler, M.; Benenson, R.; Franke, U.; Roth, S.; and Schiele, B. 2016. The cityscapes dataset for semantic urban scene understanding. In Proceedings of the IEEE conference on computer vision and pattern recognition, 3213–3223.

Crawshaw, M. 2020. Multi-Task Learning with Deep Neural Networks: A Survey. arXiv preprint arXiv:2009.09796.

Das, I.; and Dennis, J. E. 1997. A closer look at drawbacks of minimizing weighted sums of objectives for Pareto set generation in multicriteria optimization problems. Structural optimization, 14(1): 63–69.

Das, I.; and Dennis, J. E. 1998. Normal-boundary intersection: A new method for generating the Pareto surface in nonlinear multicriteria optimization problems. SIAM journal on optimization, 8(3): 631–657.

Deb, K.; Agrawal, S.; Pratap, A.; and Meyarivan, T. 2000. A fast elitist non-dominated sorting genetic algorithm for multi-objective optimization: NSGA-II. In International conference on parallel problem solving from nature, 849–858. Springer.

Désidéri, J.-A. 2012. Multiple-gradient descent algorithm (MGDA) for multiobjective optimization. Comptes Rendus Mathematique, 350(5-6): 313–318.

Désidéri, J.-A. 2014. Multiple-gradient descent algorithm for pareto-front identification. In Modeling, Simulation and Optimization for Science and Technology, 41–58. Springer.

Devlin, J.; Chang, M.-W.; Lee, K.; and Toutanova, K. 2018. Bert: Pre-training of deep bidirectional transformers for language understanding. arXiv preprint arXiv:1810.04805.

Dosovitskiy, A.; Beyer, L.; Kolesnikov, A.; Weissenborn, D.; Zhai, X.; Unterthiner, T.; Dehghani, M.; Minderer, M.; Heigold, G.; Gelly, S.; et al. 2020. An image is worth 16x16 words: Transformers for image recognition at scale. arXiv preprint arXiv:2010.11929.

Fliege, J.; Drummond, L. G.; and Svaiter, B. F. 2009. Newton’s method for multiobjective optimization. SIAM Journal on Optimization, 20(2): 602–626.

Fliege, J.; and Svaiter, B. F. 2000. Steepest descent methods for multicriteria optimization. Mathematical Methods of Operations Research, 51(3): 479–494.

Fliege, J.; Vaz, A. I. F.; and Vicente, L. N. 2019. Complexity of gradient descent for multiobjective optimization. Optimization Methods and Software, 34(5): 949–959.

Fliege, J.; and Xu, H. 2011. Stochastic multiobjective optimization: sample average approximation and applications. Journal of optimization theory and applications, 151(1): 135–162.

Hu, R.; and Singh, A. 2021. Transformer is all you need: Multimodal multitask learning with a unified transformer. arXiv e-prints, arXiv–2102.

Huang, Z.; Li, J.; Siniscalchi, S. M.; Chen, I.-F.; Wu, J.; and Lee, C.-H. 2015. Rapid adaptation for deep neural networks through multi-task learning. In Sixteenth Annual Conference of the International Speech Communication Association.

Johannes, J. 1984. Scalarization in vector optimization. Mathematical Programming, 29(2): 203–218.

Katrutsa, A.; Merkulov, D.; Tursynbek, N.; and Oseledets, I. 2020. Follow the bisector: a simple method for multi-objective optimization. arXiv preprint arXiv:2007.06937.

Kendall, A.; Gal, Y.; and Cipolla, R. 2018. Multi-task learning using uncertainty to weigh losses for scene geometry and semantics. In Proceedings of the IEEE conference on computer vision and pattern recognition, 7482–7491.
Kingma, D. P.; and Ba, J. 2014. Adam: A method for stochastic optimization. arXiv preprint arXiv:1412.6980.

Knowles, J. D.; and Corne, D. W. 2000. Approximating the nondominated front using the Pareto archived evolution strategy. Evolutionary computation, 8(2): 149–172.

Kokkinos, I. 2017. Ubernet: Training a universal convolutional neural network for low-, mid-, and high-level vision using diverse datasets and limited memory. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, 6129–6138.

Krizhevsky, A.; Nair, V.; and Hinton, G. 2014. The cifar-10 dataset. online: http://www.cs.toronto.edu/hinton/cifar.html, 55: 5.

Liu, S.; Johns, E.; and Davison, A. J. 2019. End-to-end multi-task learning with attention. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, 1871–1880.

Liu, Y.; Ott, M.; Goyal, N.; Du, J.; Joshi, M.; Chen, D.; Levy, O.; Lewis, M.; Zettlemoyer, L.; and Stoyanov, V. 2019. Roberta: A robustly optimized bert pretraining approach. arXiv preprint arXiv:1907.11692.

Long, M.; Cao, Z.; Wang, J.; and Philip, S. Y. 2017. Learning multiple tasks with multilinear relationship networks. In Advances in neural information processing systems, 1594–1603.

Lu, Y.; Kumar, A.; Zhai, S.; Cheng, Y.; Javidi, T.; and Feris, R. 2017. Fully-adaptive feature sharing in multi-task networks with applications in person attribute classification. In Proceedings of the IEEE conference on computer vision and pattern recognition, 5334–5343.

Maninis, K.-K.; Radosavovic, I.; and Kokkinos, I. 2019. Attentive single-tasking of multiple tasks. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, 1851–1860.

Mercier, Q.; Poirion, F.; and Désidéri, J.-A. 2018. A stochastic multiple gradient descent algorithm. European Journal of Operational Research, 271(3): 808–817.

Messac, A.; Ismail-Yahaya, A.; and Mattson, C. A. 2003. The normalized normal constraint method for generating the Pareto frontier. Structural and multidisciplinary optimization, 25(2): 86–98.

Meyerson, E.; and Miikkulainen, R. 2017. Beyond shared hierarchies: Deep multitask learning through soft layer ordering. arXiv preprint arXiv:1711.00108.

Misra, I.; Shrivastava, A.; Gupta, A.; and Hebert, M. 2016. Cross-stitch networks for multi-task learning. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, 3994–4003.

Nesterov, Y. E. 1983. A method for solving the convex programming problem with convergence rate O (1/k^2). In Dokl. akad. nauk Sssr, volume 269, 543–547.

Rebuffi, S.-A.; Bilen, H.; and Vedaldi, A. 2018. Efficient parametrization of multi-domain deep neural networks. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, 8119–8127.

Rosenbaum, C.; Klinger, T.; and Riemer, M. 2017. Routing networks: Adaptive selection of non-linear functions for multi-task learning. arXiv preprint arXiv:1711.01239.

Ruder, S. 2017. An overview of multi-task learning in deep neural networks. arXiv preprint arXiv:1706.05098.

Ruder, S.; Bingel, J.; Augenstein, I.; and Sogaard, A. 2019. Latent multi-task architecture learning. In Proceedings of the AAAI Conference on Artificial Intelligence, volume 33, 4822–4829.

Savour, S.; Froiss, N.; and Hinton, G. E. 2017. Dynamic routing between capsules. In Advances in neural information processing systems, 3856–3866.

Schmidt, M.; Le Roux, N.; and Bach, F. 2017. Minimizing finite sums with the stochastic average gradient. Mathematical Programming, 162(1-2): 83–112.

Sener, O.; and Koltun, V. 2018. Multi-task learning as multi-objective optimization. In Advances in Neural Information Processing Systems, 527–538.

Standley, T.; Zamir, A. R.; Chen, D.; Guibas, L.; Malik, J.; and Savarese, S. 2019. Which Tasks Should Be Learned Together in Multi-task Learning? arXiv preprint arXiv:1903.07553.

Subramanian, S.; Trischler, A.; Bengio, Y.; and Pal, C. J. 2018. Learning general purpose distributed sentence representations via large scale multi-task learning. arXiv preprint arXiv:1804.00079.

Tieleman, T.; and Hinton, G. 2012. Lecture 6.5-rmsprop: Divide the gradient by a running average of its recent magnitude. COURSERA: Neural networks for machine learning, 4(2): 26–31.

Vaswani, A.; Shazeer, N.; Parmar, N.; Uszkoreit, J.; Jones, L.; Gomez, A. N.; Kaiser, Ł.; and Polosukhin, I. 2017. Attention is all you need. In Advances in neural information processing systems, 5998–6008.

Vaswani, S.; Kunstner, F.; Laradji, I.; Meng, S. Y.; Schmidt, M.; and Lacoste-Julien, S. 2020. Adaptive Gradient Methods Converge Faster with Over-Parameterization (and you can do a line-search). arXiv preprint arXiv:2006.06835.

Vaswani, S.; Mishkin, A.; Laradji, I.; Schmidt, M.; Gidel, G.; and Lacoste-Julien, S. 2019. Painless stochastic gradient: Interpolation, line-search, and convergence rates. In Advances in Neural Information Processing Systems, 3732–3745.

Wolfe, P. 1969. Convergence conditions for ascent methods. SIAM review, 11(2): 226–235.

Yang, Y.; and Hospedales, T. 2016. Deep multi-task representation learning: A tensor factorisation approach. arXiv preprint arXiv:1605.06591.

Yu, T.; Kumar, S.; Gupta, A.; Levine, S.; Hausman, K.; and Finn, C. 2020. Gradient surgery for multi-task learning. arXiv preprint arXiv:2001.06782.

Zamir, A. R.; Sax, A.; Shen, W.; Guibas, L. J.; Malik, J.; and Savarese, S. 2018. Taskonomy: Disentangling task transfer learning. In Proceedings of the IEEE conference on computer vision and pattern recognition, 3712–3722.
Zhang, T.; Gong, X.; and Chen, C. P. 2021. BMT-Net: Broad Multitask Transformer Network for Sentiment Analysis. *IEEE Transactions on Cybernetics*.

Zhang, Y.; and Yang, Q. 2017. A survey on multi-task learning. *arXiv preprint arXiv:1707.08114*. 
Appendix

Theorem proof

Theorem 1. Suppose loss functions $ \hat{L}^t $ are continuously differentiable and bounded below. Then, every accumulation point of sequence $ x_k = [ \theta_k^{h_1}, \theta_k^{h_2}, \ldots, \theta_k^{h_T} ]_{k=1}^{\infty} $ produced by Algorithm 2 with following Armijo-rules is a Pareto stationary point $ \forall t \in \{ 1 \ldots T \}$:

\[
\hat{L}^t(z - \eta d, \theta^t - \eta \nabla \theta^t) \leq \hat{L}^t(z) - \eta \beta \left\| u^t \right\|^2 - \eta \beta \left( \frac{\partial \hat{L}^t}{\partial z} \right)^T d_z.
\]

So $ d = 0 $ and $ \bar{\theta} $ — Pareto stationary point.

2. For this Armijo-rule, the proof can be obtained by the following modifications As $ \hat{L}(\theta_{k_j}) \rightarrow \hat{L}(\bar{\theta}) $, then $ \eta \beta \left( \frac{\partial \hat{L}^t}{\partial z} \right)^T d \rightarrow 0 $. In the first variant of alternative we have $ \forall t \left( \frac{\partial \hat{L}^t}{\partial z} \right)^T d = 0 $. As $ \left\| \frac{\partial \hat{L}^t}{\partial z} \right\|_{L_1} $ isn’t a singular matrix, then $ d = 0 $ and $ \bar{\theta} $ — Pareto stationary point (Sener and Koltun 2018).

In the second variant of alternative we can change all $ \eta \beta \left\| u^t \right\|^2 $ to $ \eta \beta \left( \frac{\partial \hat{L}^t}{\partial z} \right)^T d $ and the proof doesn’t change.

Remark 1. By the Bolzanno–Weierstrass theorem, under our conditions, there is at least one accumulation point. Thus, our algorithm’s result will always be a Pareto stationary point.

Cityscapes

This section presents the results on the Cityscapes dataset. Comparison of FBLS and SGD with fixed learning rate is presented on the Figure 7.
Figure 7: 
Top: Train losses on cityscapes dataset. 
Bottom: Test losses on cityscapes dataset.