The strange and charm quark contributions to the anomalous magnetic moment \( (g - 2) \) of the muon from current-current correlators

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The magnetic moment of the muon

- Magnetic moment of a lepton: $\vec{\mu}_l = g_l \frac{Q_e}{2m_l} \vec{s}$.
- Prediction from free Dirac theory: $g = 2$ for elementary fermions.
- In interacting quantum field theory: $g$ gets corrections.
- The fundamental vector interaction of the fermion with an external EM field:

$$\gamma_\mu \rightarrow \Gamma_\mu(q) = (\gamma_\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}}{2m} q_\nu F_2(q^2)).$$

- Definition: Anomalous magnetic moment of muon: $F_2(0) = \frac{g-2}{2} \equiv a_\mu$.
- $a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{had}$.

(T. Blum et.al., arXiv:1301.2607)
Hadronic Vacuum Polarization (HVP)

- g-2 discrepancy of $3.6\sigma$ between SM and experiment (Brookhaven E821):

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 25(9) \times 10^{-10}.$$  

- An exciting indication of new virtual particles ???
- Fermilab E989 goal to reduce experimental error to one-fourth.
- The current theoretical uncertainty dominated by that from the lowest order HVP contribution.

HVP contribution from dispersion relation + cross section for $e^+e^-$ (and $\tau$) $\rightarrow$ hadrons $\sim 700 \times 10^{-10}$ with 1% error (K. Hagiwara et al., J. Phys. G38, 085003(2011)).

- Our aim: To achieve precision at 1% level using first principle lattice QCD calculations.
Our method for calculating HVP from full lattice QCD
(B. Chakraborty et al., Phys. Rev. D 89, 114501 (2014))

Introduction
Lattice calculation
Results
Conclusions

Leading-order (one-loop) hadronic contribution

Contribution to $a_{\mu}$ from $q\bar{q}\gamma\gamma\mu\mu\gamma$ can be broken into the EM loop:

$$a_{\mu,\text{HVP}}^{(f)} = \frac{\alpha}{\pi} \int dq^2 f(q^2)(4\pi\alpha Q^2)\hat{\Pi}_f(q^2).$$

(T. Blum, '02)

- Renormalized vacuum polarization function: $\hat{\Pi}(q^2) \equiv \Pi(q^2) - \Pi(0)$.
- Integrand peaks at $q^2 \sim O(m_{\mu}^2)$.
- Previous methods calculated $\hat{\Pi}(q^2)$ at larger $q^2$ and extrapolated to zero. Gives large uncertainties.

Our method:
- We reconstruct $\hat{\Pi}$ from its derivatives at $q^2 = 0$.
- Derivatives calculated from time-moments of local-local vector current correlator at zero spatial momentum.
Our method for calculating HVP from full lattice QCD
(B. Chakraborty et al., Phys. Rev. D 89, 114501 (2014))

- For spatial currents at zero spatial momentum:

\[ \Pi^{ii}(q^2) = q^2 \Pi(q^2) = a^4 \sum_t e^{iqt} \sum_{\vec{x}} \langle j^i(\vec{x}, t) j^i(0) \rangle. \]

- Time moments of the correlator give the derivatives at \( q^2 = 0 \) of \( \hat{\Pi} \).

\[ G_{2n} \equiv a^4 \sum_t \sum_{\vec{x}} t^{2n} Z_V^2 \langle j^i(\vec{x}, t) j^i(0) \rangle = (-1)^n \frac{\partial^{2n}}{\partial q^{2n}} q^2 \hat{\Pi}(q^2) \bigg|_{q^2=0}. \]

- Defining \( \hat{\Pi}(q^2) = \sum_{j=1}^{\infty} q^{2j} \Pi_j \) where \( \Pi_j = (-1)^{j+1} \frac{G_{2j+2}}{(2j+2)!} \).

- Used 4th, 6th, 8th and 10th moments (i.e. \( j=1, 2, 3 \) and \( 4 \)).

- Only quark-line-connected contributions to the lowest order HVP considered.
Our method for calculating HVP from full lattice QCD
(B. Chakraborty et al., Phys. Rev. D 89, 114501 (2014))

- $\hat{\Pi}(q^2)$ replaced with its $[2, 2]$ Padé approximant derived from $\Pi_j$.

- Using Padé approximants instead of Taylor approximation allows us to deal with high momenta.

- Precision of different Padé approximants tested by comparing with the exact one-loop perturbative results for $a_\mu$.

- $q^2$ integral $\int dq^2 f(q^2)(4\pi\alpha Q_f^2)\hat{\Pi}_f(q^2)$ done numerically.
Lattice Configurations and Parameters in $a_\mu^s$ calculation

- 2+1+1 HISQ ensembles from MILC.
- $a \approx 0.15$ fm (very coarse), 0.12 fm (coarse), 0.09 fm (fine), determined using $w_0$ parameter (R.J.Dowdall et al., Phys.Rev. D88 (2013) 074504).
- Large box size: 5.6 fm on the finest lattices.
- Light (u/d) sea quark mass: $m_s/5$ and the physical value $m_s/27.5$.
- Test volume effect: At $m_l = m_s/10$, three different volumes, $M_\pi L = 3.2, 4.3$ and 5.4 (set 4, 5, 7).
- HISQ valence s quark masses accurately tuned to $m_\eta_s = 688.5$ Mev (R.J.Dowdall et al., Phys.Rev. D88 (2013) 074504).
- Test tuning effect: The valence s quark mass detuned by 5% (set 6).

| Set | $am_{\ell}^{\text{sea}}$ | $am_{s}^{\text{sea}}$ | $am_{s}^{\text{val}}$ | $am_{\eta s}^{\text{val}}$ | $Z_{V,\bar{s}s}$ | $L/a \times T/a$ | $n_{\text{cfg}}$ |
|-----|-----------------|-----------------|-----------------|-----------------|---------------|-----------------|----------------|
| 1   | 0.01300         | 0.0650          | 0.0705          | 0.54024(15)     | 0.9887(20)    | 16×48           | 1020           |
| 2   | 0.00235         | 0.0647          | 0.0678          | 0.526799(81)    | 0.9887(20)    | 32×48           | 1000           |
| 3   | 0.01020         | 0.0509          | 0.0541          | 0.43138(12)     | 0.9938(17)    | 24×64           | 526            |
| 4   | 0.00507         | 0.0507          | 0.0533          | 0.426369(58)    | 0.9938(17)    | 22×64           | 1019           |
| 5   | 0.00507         | 0.0507          | 0.0533          | 0.426369(58)    | 0.9938(17)    | 32×64           | 988            |
| 6   | 0.00507         | 0.0507          | 0.0507          | 0.41572(14)     | 0.9938(17)    | 32×64           | 300            |
| 7   | 0.00507         | 0.0507          | 0.0533          | 0.426369(58)    | 0.9938(17)    | 40×64           | 313            |
| 8   | 0.00184         | 0.0507          | 0.0527          | 0.423099(34)    | 0.9938(17)    | 48×64           | 1000           |
| 9   | 0.00740         | 0.0370          | 0.0376          | 0.313840(90)    | 0.9944(10)    | 32×48           | 504            |
| 10  | 0.00120         | 0.0363          | 0.0360          | 0.304800(40)    | 0.9944(10)    | 64×96           | 621            |
Non-perturbative renormalization \((Z_{V,\bar{ss}})\) of local vector current
(B.Chakraborty et al., PoS LATTICE2013, 309(2013))

- Local vector current not conserved in HISQ formalism.
- \(Z_{V,\bar{ss}}\) calculated from the normalization at zero momentum transfer:

\[
1 = Z_{V,\bar{qq}} \langle H_q | V_{qq} | H_q \rangle .
\]

- Need to use unstaggered (clover) spectator quark in the three point function with same meson \((\eta_s)\) at both ends.
- \(Z_{V,\bar{ss}}\) calculated completely non-perturbatively with 0.1\% precision on the finest \(m_l = m_s/5\) lattice.
Our results: $m_\phi - m_{\eta_s}$ and $f_\phi$ extrapolations

- The mass and decay constant of the $\phi$ meson extracted from the two-point correlators precisely.
- Our results in the continuum limit agree with the experimental results.

Disconnected diagrams are not included, but small contribution expected.
Our results: Connected contributions to $a_\mu^s$ from full LQCD

Our final result for the connected contribution for s quarks to $g - 2$ is:

$$a_\mu^s = 53.41(59) \times 10^{-10}$$

- **Blue points**: $m^\text{lat}_\ell = m_s/5$, **Red points**: $m^\text{lat}_\ell = m^\text{phys}_\ell$.
- Precision obtained at **1.1%** level.
- Lattice spacing error alone $\sim 1\%$, can be improved if better precision required.
The fit function

- We fit the results using \([2, 2]\) Padé approximant from each configuration set to a function of the form

\[
a_{\mu, \text{lat}}^s = a_{\mu}^s \times \left( 1 + c_{a^2} (a \Lambda_{\text{QCD}} / \pi)^2 + c_{\text{sea}} \delta x_{\text{sea}} + c_{\text{val}} \delta x_{\text{val}} \right)
\]

where \(\Lambda_{\text{QCD}} = 0.5\) GeV and

\[
\delta x_{\text{sea}} \equiv \sum_{q=u,d,s} \frac{m_q^{\text{sea}} - m_q^{\text{phys}}}{m_s^{\text{phys}}} m_s^{\text{phys}}
\]

\[
\delta x_s \equiv \frac{m_s^{\text{val}} - m_s^{\text{phys}}}{m_s^{\text{phys}}}.
\]

- Discretization effects are handled by \(c_{a^2}\).
Our result and result from ETM Collaboration, for \( a_{\mu}^s \), agree in the continuum limit.

It seems we have much smaller discretization error using HISQ formalism.
Comparison of our results for $a^s_\mu$ and $a^c_\mu$ with other results

- $a^c_\mu$ obtained from the previously calculated moments (G. Donald et al., Phys. Rev. D86, 095401(2012)).

$$a^c_\mu = 14.42(39) \times 10^{-10}.$$ 

- We could improve it by calculating $Z_{V,\bar{c}c}$ more precisely in the same way as before (will make no practical difference since error negligible).

|         | Results from dispersion + experiment | Our results | ETMC (preliminary) results |
|---------|--------------------------------------|-------------|----------------------------|
| $a^s_\mu$ | 55.3(8)$\times 10^{-10}$            | 53.41(59)$\times 10^{-10}$ | 53(3)$\times 10^{-10}$    |
| $a^c_\mu$ | 14.4(1)$\times 10^{-10}$           | 14.42(39)$\times 10^{-10}$ | 14.1(6)$\times 10^{-10}$ |

1. K. Hagiwara et al., J.Phys. G38, 085003(2011)
2. S. Bodenstein et al., Phys. Rev. D85, 014029(2012)
Preliminary results of the connected contribution to $a_{\mu}^{\text{light}}$

- Signal-to-noise ratio at large $t$ much worse.
- Calculated moments from the best fit parameters.

![Graph showing $a_{\mu}/e$ vs. $M^2$ (GeV$^2$)](image)

- 5-6% precision achieved using 1000 configs x 12 time sources for very coarse and 400 configs x 4 time sources for coarse (physical point).
- To achieve 1% precision, need 4 x time sources and up to 10 x configurations.
- Estimate of total $a_{\mu}^{\text{HVP,LO}} = a_{\mu}^{\text{light}} + a_{\mu}^{s} + a_{\mu}^{c} \sim 662(35) \times 10^{-10}$ (by averaging $a_{\mu}^{\text{light}}$ on physical point ensembles).

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The s and c quark contributions to muon g-2
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Conclusion

To summarize:

- 1\% precision for the HVP contribution to $a^{s}_\mu$ achieved from LQCD.

- $a^{s}_\mu = 53.41(59) \times 10^{-10}$ and $a^{c}_\mu = 14.42(39) \times 10^{-10}$ from connected pieces.

- $a^{\text{light}}_\mu$ currently gives a 5-6\% precision at the physical point.

- To achieve 1\% precision: We can gain statistics from more time sources on existing configurations and using smeared sources.

- More configurations can also be made for very coarse and coarse relatively cheaply.

- Estimation of total: $a^{HVP,LO}_\mu \sim 662(35) \times 10^{-10}$.

- The disconnected contributions need to be included.