Supersymmetry and the Cosmic Ray Positron Excess

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Abstract

We explore several supersymmetric alternatives to explain predictions for the cosmic ray positron excess. Light sneutrino or neutralino LSP’s, and a fine-tuned model designed to provide a δ-function input, can give adequate statistical descriptions of the reported HEAT data if non-thermal production of the relic cold dark matter density dominates and/or if “boost factors” (that could originate in uncertainties from propagation or local density fluctuations) to increase the size of the signal are included. All the descriptions can be tested at the Tevatron or LHC, and some in other WIMP detecting experiments.

1 Introduction

The recent HEAT experiment [1] has confirmed an excess and possible structure around 8 GeV in the positron spectrum of cosmic rays [2][3]. Since it appears to be unlikely that conventional astrophysical phenomena can lead to such structure, and since it is plausible that the annihilation of WIMPs in the galactic halo can give rise to a high energy positron excess, this unexpected feature of the positron spectrum could be a major discovery. LSPs (the lightest supersymmetric particles) are stable particles predicted by supersymmetric standard model. They are natural candidates for the cold
dark matter which forms the galactic halo, and thus could be the needed WIMPs. Therefore, it is interesting to examine in detail whether LSPs can quantitatively explain the observations.

The conventional candidate for the LSP has been the lightest neutralino. The resulting positron spectrum from the annihilation of those neutralinos was studied [4, 5, 6, 7] and re-examined after the newest HEAT result was announced [8, 9]. The general result has been

1. If the mass of the LSP is less than the W mass, the cross section for a pair of neutralinos to annihilate into a pair of fermions tends to be very small due to the well known suppression proportional to the fermion masses. In this case the positron excess is far too small.

2. If the mass of the LSP is larger than the W mass (but smaller than the top mass), the annihilation to a pair of W’s always dominates. The positrons coming from the direct decay mode \( W^+ \to e^+ + \nu \) will produce a large excess at and below an energy of about half the W mass. Additional positrons come from secondary decays of b, \( \mu \), \( \tau \), etc. That excess, after propagation, can be extended substantially to lower energies. While it is hard to reproduce the apparent energy dependence of the HEAT data, it is possible to have an excess in the observed region.

3. The actual positron flux resulting from the annihilation is always too small to produce visible structure. The positron signal in the literature has been increased by a “boost factor” which is sometimes large. This boost factor may be explainable by the uncertainties in the propagation process or the clumpyness nature of the galactic halo [10]. The need for such a factor means the HEAT data alone cannot guarantee that superpartners and LSP cold dark matter have been observed, though if confirmed and not explained by conventional cosmic ray processes the superpartner discovery would be a favored option. At the same time, a need for a medium or large “boost factor” should not be viewed as excluding a particular LSP since large boost factors could actually be physical.

We use DARKSUSY [11] for calculating the positron flux. The results for the neutralino case are shown in Fig 1 for particular mass and type of LSP’s. Mainly bino LSP’s do not work since they do not annihilate to W’s. The
results are not too sensitive to the mass once it is above $m_W$, but extra kinetic
energy for the W’s will spread out the spectrum. We use the formula for
background positrons produced by conventional sources provided by [12] and
also used by [7, 8]. We also treat the overall normalization of the background
as a free parameter of $O(1)$ and include it in the combined fit performed
here, since the background parameterizations were previously done assuming
no new physics signal. In this and all the examples considered below, the
relic density is normalized to a local density $\rho = 0.3 \text{ GeV/cm}^3$.

Since the conventional scenario does not give a decisive answer, it is inter-
esting to explore alternative scenarios of the nature of the LSP and dark
matter in order to see if any provide better results. We explore sneutrino
LSPs in Fig 2, which leads to results similar to the conventional scenario if
$m_{\tilde{\nu}} \gtrsim m_W$. Since the HEAT data suggests structure in the energy spectrum,
we are even willing to try models with some extreme ideas. The best case
we could come up with is a sneutrino and a bino-like neutralino which are
degenerate in mass. Such a model does give more structure around 8 GeV,
also shown in Fig 4. We also check the two scenarios against all available ex-
perimental constraints, mainly from LEP, and examine their implications for
discoveries at the Tevatron. The sneutrino examples would be excluded by
the absence of observation in direct detection experiments, but such exclu-
sions are model dependent. For example, the models of ref.[13, 14, 15] evade
such constraints. Similarly, the limits could be evaded by right-handed sneu-
trinos.

It should be noted that it is very hard to obtain a “bump” like struc-
ture from any positron production mechanism. The best case would be a $\delta$
function as the initial positron energy distribution, but the energy loss will only
extend the distribution toward the low energy direction rather than spread
it out, as shown in Fig 3.

Our results for neutralinos differ somewhat from those of Baltz et al.[9]
since we do not force thermal equilibrium relic densities to account for the
dark matter. We allow non-thermal mechanisms [16, 17, 18, 19, 20, 21] to
dominate, and normalize to average local relic density. Similarly, some of our
examples will imply $\bar{p}$ rates somewhat above current measurements [22, 23, 24]
if naive $\bar{p}$ propagation were correct, but most people feel there are large
uncertainties that mean one should not take such results overly seriously.
Also, the $\bar{p}$ rate is somewhat sensitive to particle physics details, particularly
the neutralino annihilation mechanisms. Thus while the $\bar{p}$ rate should be
kept in mind we do not think it is a compelling constraint at the present
time.

2 Light $\tilde{\nu}$ LSP

For a sneutrino with mass larger than W boson, the annihilation to a pair
of W’s will dominate. The resulting positron spectrum will be similar to the
one we get from neutralino annihilation. Recent studies [25, 26] show that a
lighter sneutrino, close to the LEP lower bound, is actually favored by the
electroweak precision data. Therefore, we also explore the scenario in which
a light sneutrino with a mass just above the LEP lower limit is the LSP.

Similar to the neutralino annihilation, the annihilation of a pair of sneu-
trinos to a pair of fermions is still suppressed by the fermion masses or by
the p-wave scattering due to angular momentum conservation. However, if
$m_{\tilde{\nu}} \approx m_Z/2$ the s-channel annihilation is enhanced significantly by the Z-pole
contribution, leading to a larger excess of positrons. However, it turns out
that this is still not sufficient to produce an excess as large as that resulting
from W decays. Consequently a very large boost factor is needed.

2.1 Numerical Result

If $m_{\tilde{\nu}} < m_W$, the s-channel production through the Z-pole gives the dominant
contribution and t-channel exchange of a chargino is negligible. If $m_{\tilde{\nu}} \gtrsim m_W$,
the most important processes for the W pair production is the 4-point vertex
interaction and s-channel higgs boson exchange. We use CompHEP [27] for
calculating the cross sections. We included a boost factor in the combined
fit as described above. Both cases of sneutrino masses are studied. We
also assume here the dark matter only has one generation of sneutrinos, and
sneutrino and anti-sneutrino have the same number density. The results are
shown in Fig 2.

The result shows that

1. $m_{\tilde{\nu}} < m_W$. Although one can tune the mass of the sneutrino so the
annihilation cross section has a large Z-pole enhancement, one still
needs a large boost factor to have a sizable excess. Perhaps such a
large boost factor is unlikely, but we cannot be sure.
2. $m_\tilde{\nu} \geq m_W$. As expected, the result in this case is very similar to that of
the neutralino LSP scenario. It is not identical to the neutralino case
because the cross sections are somewhat different.

Notice that in both cases, there is no “bump”-like structure resulting
from the LSP annihilations, as expected since we do not produce anything
around 8 GeV in the first place. The excess is purely due to the energy loss of
the high energy positrons—from W and fermion decay, including secondary
positrons mainly from b, $\mu$ and $\tau$.

### 2.2 Phenomenology of This Scenario

The first question is whether the correct amount of relic sneutrinos can indeed
be produced, survive, and constitute the cold dark matter of our universe.
One possibility is standard thermal production. This always gives far too
few relic sneutrinos. In recent years, it has been realized that non-thermal
production mechanisms may dominate, and different mechanisms\[16, 17, 18,
19, 20, 21\] of non-thermal production have been proposed and studied. Al-
though most of them are in the context of non-thermal production of neu-
tralinos, they are still valid in our case since neutralinos will inevitably decay
into sneutrinos. Thus it is possible to produce sufficient sneutrinos by non-
thermal mechanisms, but the incomplete understanding of such mechanisms
means we cannot draw a definite conclusion.

Next, consider briefly the collider signatures.

1. Compatibility with LEP-II results \[28, 29, 31, 30\]. For the case of
$m_\tilde{\nu} < m_W$, we need the sneutrino mass to be half the Z mass (45.6
GeV) to get the maximum enhancement. This mass is consistent with
the invisible Z width measurement, which gives a model-independent
lower bound of 44.25 GeV. At tree level the associated slepton mass
is determined by $m_{\tilde{\nu}}^2 - m_{\tilde{l}_L}^2 = m_W^2 \cos 2\beta$. To be consistent with LEP
slepton results, we need larger $\tan \beta$ to make sleptons heavy. The lower
limit on the Higgs boson mass suggests $\tan \beta \geq 4$. When $\tan \beta \geq 4$,
$m_{\tilde{l}_L} = 91.2$ GeV. We must also take into account loop contributions,
so the slepton mass will be about 95 GeV\[32\]. This gives a 1$\sigma$ signal
expected at LEP (amusingly, a small excess of order 1$\sigma$ is observed
for smuons by OPAL and DELPHI.) If the LSP is a mixture of left-
handed and right-handed sneutrino, then the mass splitting of slepton and sneutrino will be larger than the D-term splitting\[4\].

Models with $\tilde{\nu}$ LSP and $m_{\tilde{\nu}} \geq m_W$ are consistent with current collider experiments. The $\tilde{\nu}$ could then be right-handed as well, so long as it can annihilate into W’s, but does not need to be.

2. Tevatron Signals. At the Tevatron, there can be $\tilde{\nu} L \pm$ and $\tilde{\nu} L \tilde{\nu}$ production. For the first case the signature is a single charged lepton plus transverse missing energy and the cross section for each family is about 246 fb for $m_{\tilde{\nu}} < m_W$ and 75 fb for $m_{\tilde{\nu}} \geq m_W$. For the second case the signature is a charged lepton pair with transverse missing energy and the cross section for each slepton generation is about 32 fb for $m_{\tilde{\nu}} < m_W$ and 15 fb for $m_{\tilde{\nu}} \geq m_W$. The light $\tilde{\nu}$ case can surely be observed with $\sim 2 fb^{-1}$ luminosity and presumably the heavier one with $5 - 10 fb^{-1}$. We use PYTHIA for calculating the cross sections. \[33\]

3. $\tilde{\nu}$, $\tilde{N}_1$ degeneracy

In this scenario, we made an attempt to generate some structure around 10 GeV in a special model. The best thing one can do is to have a positron production mechanism with a $\delta$-function distribution centering around 8 GeV. However, this does not give a bump but a structure such as that shown in Fig 3, which is the best possible. Such a source of positrons can only come from two-body direct production or direct decay from a particle that is almost at rest. Consider the decay first. Since LSP’s do not decay, we have to rely on the decay of a Standard Model particle, and the difference in masses of the decaying particle and the decay products other than the positron has to be around 8 GeV. There is no such particles. Therefore, we have to consider the possibility of production. The production of a pair of fermions will not work since there is no decaying particle with a mass of 16 GeV. The initial state of the annihilation must be neutral, and therefore also the final states. This naturally leads us to consider the final state $W^- + e^+$. Then R-parity conservation and lepton number conservation force us to choose the initial state $\tilde{N}_1 + \tilde{\nu}$. 

\[6\]
3.1 Numerical Results

The information needed from particle physics in this case are the mass of the sneutrino/neutralino and the content of the neutralino. It is impossible to have a wino/higgsino-like neutralino with the appropriate mass since it would imply a very light chargino which is not consistent with the LEP limit. Therefore, we are forced to use a bino-like lightest neutralino. We also include the boost factor and the overall normalization of the background positrons in our combined fit.

Notice that in this case, sneutrinos and neutralinos also annihilate among themselves. However, since their masses are less than $m_W$, those self-annihilation only give a tiny positron excess. We are fully relying on the coannihilation to produce the necessary excess. Therefore, it would be most efficient for the signal if both sparticles had nearly the same number density. In our calculation here, we assume the dark matter consists of neutralino and electron sneutrino and their number densities are equal. If we assume the neutralino and all three families of sneutrino are degenerate, the number densities for each generation of sneutrino, anti-sneutrino are equal, and the number densities of neutralino and sneutrino are equal, we need a boost factor three times larger the one we reported above.

The result is shown in Fig 2. We see that some structure near $8 - 10$ GeV is produced, though it does not resemble the data very well. The main reason is that although we manage to inject a $\delta$-function distribution into the spectrum before propagation, the one-sided character of energy loss cannot give us a spectrum just like the data.

3.2 Phenomenology of $\tilde{\nu}$-$\tilde{N}_1$-degenerate Scenario

This model is extremely fine-tuned since we require a degeneracy of masses $m_{\tilde{\nu}}$ and $m_{\tilde{N}_1}$ to an accuracy of less than a couple eV. It is of course very unlikely that such an accident can happen without a symmetry. To the best of our knowledge, a symmetry that can achieve this is unknown. In extended theories it is possible that the sneutrino and neutralino can appear in the same multiplet, so it is conceivable that such a symmetry could exist in the underlying theory. However, supersymmetry must be broken and the sneutrino and neutralino actually get mass from the supersymmetry breaking, so it is unlikely such a symmetry could be preserved. Without data as a
motivation we would not consider such a degeneracy.

Second, it is a more subtle question now if this special composition of the cold dark matter can be realized. Since the coannihilation is very effective, the thermal production will never give us enough relic abundance and non-thermal production is required. Mechanisms have been proposed [16, 17, 18, 19, 20, 21] to produce pairs of neutralinos from the decay of a moduli field. If mainly winos are produced, they will decay to binos through \( \tilde{W} \rightarrow \tilde{\nu} + \nu \), so the relative production of binos and sneutrinos is about what is needed for the model to be relevant. This may require a suppression of the coupling of the moduli to bino to avoid their overproduction.

Finally, we study the collider signature of this special scenario.

1. LEP-II. The constraint on the sneutrino mass is the same as in section 2.2. Here we have a light neutralino around 50 GeV. If the second lightest neutralino is also light such that \( e^+ e^- \rightarrow \tilde{N}_1 \tilde{N}_2 \) is kinematically allowed, then after \( \tilde{N}_2 \) decay, there will be an acoplanar lepton pair signal. To suppress this channel, the mass of the second neutralino should be above 155 GeV. A set of parameters that give a spectrum which is consistent with all LEP data is \( M_1 = 57 \text{ GeV}, M_2 = 400 \text{ GeV}, \mu = 180 \text{ GeV}, \tan \beta = 10 \).

2. Tevatron. The signature of sleptons are the same as in section 2.2. Now we also have \( \tilde{C}_1 \tilde{C}_1, \tilde{C}_1 \tilde{N}_1, \tilde{C}_1 \tilde{N}_2, \) and \( \tilde{N}_1 \tilde{N}_2 \) channels open with cross sections around 41fb, 19fb, 46fb and 0.7fb separately. The first one and the fourth one, after \( \tilde{C}_1^{\pm} \) or \( \tilde{N}_2 \) decay, can give a charged lepton pair plus transverse missing energy. The second one, after \( \tilde{C}_1^{\pm} \) decay, can give single charged lepton with transverse missing energy. The third one can give a trilepton signal, three charged lepton.

4 Detectability by various experiments

In Table 1, we list detectability by various experiments for our models. Detectable for the Tevatron means at least one superpartner of the spectrum containing the LSP would be produced in numbers large enough to observe. The measured antiproton flux in the energy region around 0.5 GeV is about \( 1.27 \times 10^{-6} \text{ cm}^{-2} \text{ sr}^{-1} \text{ sec}^{-1} \text{ GeV}^{-1} \). Estimates for the \( \bar{p} \) flux from our models
Table 1. Detectability by various experiments.

| Model                  | Collider | Direct Detector | $\bar{p}$ flux $(10^{-6} \text{ cm}^{-2} \text{sr}^{-1} \text{sec}^{-1} \text{GeV}^{-1})$ | underground neutrino |
|------------------------|----------|-----------------|---------------------------------------------------------------------------------|---------------------|
| mainly wino $\tilde{N}$ | Tevatron | detectable soon | $\sim 4.8$                                                                        | detectable soon     |
| mainly higgsino $\tilde{N}$ | Tevatron | no              | $\sim 4.1$                                                                        | detectable soon     |
| $\tilde{\nu}$ heavier than W | Tevatron | excluded for simple models | $\sim 0.7$                                                                        | excluded for simple models |
| $\tilde{\nu}$ lighter than W   | Tevatron | excluded for simple models | $\sim 6.0$                                                                        | excluded for simple models |
| degenerate $\tilde{\nu}$ $\tilde{N}$ | Tevatron | excluded for simple models | $\sim 0.8$                                                                        | excluded for simple models |

are shown in the table. Given the uncertainties we think none of these are excluded, and all are large enough to see a signal if qualitatively better measurements can be made\cite{34}.

5 Conclusions

We have studied whether positrons from LSP annihilation could account for the excess and structure in the positron spectrum reported by the HEAT Collaboration. Even normalizing the relic densities to the local galactic density, significant “boost factors” are sometimes required to get a sufficiently large signal. It is not known whether such boost factors can be explained by galactic propagation and local concentrations. Statistically reasonable descriptions of the data can be obtained for sneutrino or neutralino LSP’s heavier than W bosons, but they lead to a smooth energy spectrum rather than the apparent peaked structure of the data. In simple models, but not generally, sneutrinos are excluded by the absence of direct detection. Although no LSP annihilation approach could give the peaking suggested by the data since a $\delta$-function input becomes essentially a step function because of the energy loss, we also construct a fine-tuned model to generate a $\delta$-function positron energy input, by assuming degenerate sneutrino and neutralino LSP’s. The degenerate LSP model implies the observability of
charginos, neutralinos, and sleptons at the Tevatron, and the sneutrino and neutralino LSP interpretations both allow detection of signals at the Tevatron.

The description of the data with neutralinos is good, and rather natural if non-thermal LSP production gives the dominant contribution to the relic density. The light mass ties in well with indirect evidence for light superpartners. The sneutrino models seem less attractive, but we report them because we think assumptions made for theoretical simplicity should not be taken too seriously when examining potentially important data.

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Figure 1: Numerical results from different neutralino LSP models. The relic density is assumed to arise from non-thermal mechanisms and is normalized to the average local cold dark matter density. Bs is the boost factor defined in the text, possibly due to local dark matter concentrations and to uncertainties in propagation, and Bp the normalization factor multiplying the background from [12]. The $\chi^2$ values are given to help judge the quality of the description of the data. The two cases are for mainly Higgsino and mainly wino LSP’s; mainly bino cases need much more enhancement. These examples are for neutralinos that are consistent with all collider and direct detection data. The Higgsino-like case $M_1 = 500$ GeV, $M_2 = 400$ GeV, $\mu = -100$ GeV, $\tan \beta = 10$. The wino-like case $M_1 = 500$ GeV, $M_2 = 165$ GeV, $\mu = 225$ GeV, $\tan \beta = 10$. 
Figure 2: Numerical results from alternative LSP models. See Fig 1 caption. The cases displayed here are very light sneutrino ($m_{\tilde{\nu}} \sim \frac{1}{2} m_Z$), sneutrino LSP $m_{\tilde{\nu}} > m_W$, and sneutrino-neutralino degenerate models. All models are consistent with collider data.
Figure 3: A $\delta$-function source at 10 GeV will be spread out by propagation at lower energies as in the dashed curve, when added to the positron background it can give the solid line. The position of the $\delta$-function can be moved to higher energies – this figure is only to illustrate that such a result is the best one possible.