Long term Throughput and Approximate Capacity of Transmitter-Receiver Energy Harvesting Channel with Fading

Jainam Doshi and Rahul Vaze

Abstract—We first consider an energy harvesting channel with fading, where only the transmitter harvests energy from natural sources. We bound the optimal long term throughput by a constant for a class of energy arrival distributions. The proposed method also gives a constant approximation to the capacity of the energy harvesting channel with fading. Next, we consider a more general system where both the transmitter and the receiver employ energy harvesting to power themselves. In this case, we show that finding an approximation to the optimal long term throughput is far more difficult, and identify a special case of unit battery capacity at both the transmitter and the receiver for which we obtain a universal bound on the ratio of the upper and lower bound on the long term throughput.

I. INTRODUCTION

Finding optimal power/energy transmission policies to maximize the long-term throughput in an energy harvesting (EH) communication system is a challenging problem and has remained open in full generality. Structural results are known for the optimal solution [1], however, explicit solutions are only known for a sub-class of problems, for example, binary transmission power [2], discrete transmission power [3], etc. Recently, some progress has been reported in approximating the per-slot throughput (or long term throughput) by a universal constant in [4], for an AWGN channel.

In this paper, we approximate the per-slot throughput of the EH system with fading by a universal constant for a class of energy arrival distributions. The fading channel problem is more challenging than the AWGN case, since the energy/power transmitted per-slot depends on the realization of the channel unlike the AWGN problem. Thus, finding an upper bound on the long term throughput is hard. We take recourse in Cauchy-Schwarz inequality for this purpose, and then surprisingly using a channel independent power transmission policy proposed in [4], show that the upper and lower bound on the per-slot throughput differ at most by a constant. Using the techniques of [4], we also show that our universal bound also provides an approximation of the Shannon capacity of the energy harvesting channel with fading up to a constant.

In addition to EH being employed at the transmitter, a more relevant or practical scenario is when EH is employed at both the transmitter and the receiver. The EH setting at the receiver is simpler than at the transmitter, since the only decision the receiver has to make is whether to stay on or not. In the on state, the receiver consumes a fixed amount of energy, so the energy consumption model at the receiver is Bernoulli.

With EH at both the transmitter and the receiver, there is inherent lack of information about the receiver energy levels at the transmitter and vice-versa. One can show that the optimal policy at both the transmitter and the receiver is of threshold type, but the thresholds depend on both the energy states in a non-trivial way. Because of the common fading channel state that is revealed to both the transmitter and receiver for each slot, both the transmitter and the receiver have some partial statistical information about others’ energy state which is important for finding the optimal policy.

We show that, in general, it is difficult to bound the gap between the upper and lower bound when EH is employed at both the transmitter and the receiver. Then we identify a special case of unit battery capacity at both the transmitter and the receiver, and where the transmitter operates with binary transmission power, for which we propose a strategy that achieves at least half of the upper bound on the per-slot throughput, giving a ratio bound.

II. SYSTEM MODEL

We consider slotted time, and a single transmitter-receiver pair, where the transmitter harvests energy from the environment. Let $E_t$ be the amount of energy harvested at time step $t$ which is stored in a battery of size $B_{max}$. The energy harvested at each time step $E_t$ is a discrete time ergodic and stationary random process. At time $t$, the fading channel between the transmitter and the receiver is $h_t$, where $h_t$ is assumed to be i.i.d. Rayleigh distributed for each $t$. Thus, $|h_t|^2 \sim \exp(1)$. For notational convenience we denote $|h_t|^2$ as $h_t$. Let $B_t$ be the battery energy level at time $t$. Let energy used at time $t$ given channel $h_t$ be $P_h(t) \leq B_t$, then the rate obtained at time $t$ is

$$r(t) = \frac{1}{2} \log(1 + h_t P_h(t)).$$

The energy state at the transmitter evolves as

$$B_t = \min \left\{ B_{t-1} + E_t - P_h(t-1) \bar{P}_{h(t-1)}\leq B_{t-1}, B_{max} \right\}.$$

The long-term throughput is defined as

$$T = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} r(t). \quad (1)$$
Our objective is to find optimal energy consumption $P_h(t)$ at the transmitter, given the energy neutrality constraint $P_h(t) \leq B_t$, that maximizes $T$, i.e. $T^* = \max_{P_h(t) \leq B_t} T$.

III. AN UPPER BOUND ON MAXIMUM ACHIEVABLE LONG TERM THROUGHPUT

In this section, we upper bound the maximum long term throughput $T^*$ (1).

Theorem 1: For any energy consumption policy $P_h(t)$ such that $P_h(t) \leq B_t$,

$$T^* \leq \frac{1}{2} \log \left( 1 + \sqrt{2 \frac{E[P_h^2]}{E[E_t^2]}} \right).$$  \hspace{0.5cm} (2)

Proof: By ergodicity, (1) is equal to

$$T = \mathbb{E} \left\{ \frac{1}{2} \log (1 + h_t P_h(t)) \right\}. \hspace{0.5cm} (3)$$

By Jensen’s inequality, for any $P_h(t)$,

$$\mathbb{E} \left[ \log (1 + h_t P_h(t)) \right] \leq \log \left( 1 + \mathbb{E} \left[ h_t P_h(t) \right] \right).$$  \hspace{0.5cm} (4)

Applying Cauchy-Schwarz inequality for any two random variables $X, Y$, $\mathbb{E}[XY]^2 \leq \mathbb{E}[X^2] \mathbb{E}[Y^2]$, we have

$$(\mathbb{E}[h_t P_h(t)])^2 \leq \mathbb{E}[h_t^2] \mathbb{E}[P_h^2].$$  \hspace{0.5cm} (5)

By energy neutrality constraint, we have,

$$\frac{1}{N} \left( \sum_{t=1}^N P_h(t) \right)^2 \leq \frac{1}{N} \left( \sum_{t=1}^N E_t \right)^2 \forall N.$$  \hspace{0.5cm} (6)

Hence, as $N \to \infty$, $\mathbb{E} \left[ P_h^2 \right] \leq \mathbb{E} \left[ E_t^2 \right]$, since $h_t$ and $E_t$ are assumed to be i.i.d. and hence the cross terms are independent. This is true for every feasible energy allocation strategy. Thus, from (4),

$$T^* \leq \frac{1}{2} \log \left( 1 + \sqrt{\mathbb{E}[h_t^2] \mathbb{E}[E_t^2]} \right).$$  \hspace{0.5cm} (7)

As $h_t$ is an exponentially distributed random variable with a mean of unity, we have that $\mathbb{E}[h_t^2] = 2$ and (6) can be expressed as $T^* \leq \frac{1}{2} \log \left( 1 + \sqrt{2 \mathbb{E}[E_t^2]} \right)$.

We denote this upper bound on the achievable long term throughput by $T_{ub}$. We next propose an energy allocation strategy, and compare the long term throughput obtained by it with $T_{ub}$.

IV. ACHIEVABLE STRATEGY

To build intuition on the proposed achievable strategy, we begin with the simple Bernoulli energy arrival process.

A. Bernoulli Energy Arrival

Let the energy arrival at time $t$ be i.i.d. Bernoulli with $E_t = E$ with probability $p$, and $E_t = 0$ with probability $1 - p$. First we look at the $E > B_{max}$ case, where it is sufficient to consider $E_t = B_{max}$. For $E_t = B_{max}$, each energy arrival (called epoch) completely fills up the battery, and hence the throughput obtained in each epoch is i.i.d. Thus, it is sufficient to consider any one epoch to lower bound the long term throughput.

We use the same strategy (call it Constant Fraction Policy (CFP)) as proposed in [4] for an AWGN channel, where

$$P_h(t) = p(1-p)^j B_{max}, \text{ for } j = 0, 1, 2, \ldots .$$  \hspace{0.5cm} (8)

where $j = t - \max \{ t' : E_{t'} = E, \forall t' \leq t \}$ is the number of slots since the last epoch. Note that between any two epochs $T_1$ and $T_2$, the energy used is $\sum_{t=T_1}^{T_2} p(1-p)^j B_{max} = B_{max}$, thus CFP satisfies energy neutrality constraint.

Note that CFP is independent of the channel fade state $h$, and hence seems to be highly sub-optimal. Any natural $h$ dependent policy, however, is not immediately amenable for analysis. Using the concavity of the log function, we show that even an $h$ independent policy has a bounded gap from the upper bound for a class of energy arrival distributions. For Bernoulli energy arrivals, we cannot bound the gap universally and the gap depends on the rate $p$.

Lemma 1: The per slot throughput obtained by CFP with Bernoulli energy arrival distribution is given by.

$$T_{lb} = \sum_{j=0}^{\infty} p(1-p)^j \int_0^\infty \frac{1}{2} \log(1+hP(j))e^{-h}dh.$$  \hspace{0.5cm} (9)

Proof: As stated before, we consider a single epoch to lower bound the per slot throughput of CFP. Let the first epoch be $T_1 = 0$ and $T_1$ be the time between the first two epochs. For ease of notation, let $P_h(t) = p(1-p)^j B_{max} = P(j)$ for $j = t = 0, \ldots, T_1 - 1$. Let $T_{lb}$ be the per slot throughput obtained by CFP. Then by renewal reward theorem,

$$T_{lb} = \frac{\mathbb{E} \left[ \sum_{j=0}^{T_1-1} \frac{1}{2} \log(1 + hP(j)) \right]}{\mathbb{E}[\tau_1]},$$  \hspace{0.5cm} (a)

$$= p \sum_{i=1}^{\infty} \mathbb{P}(\tau_1 = i) \int_0^\infty \frac{1}{2} \log(1 + hP(j))e^{-h}dh,$$

$$= p \sum_{j=0}^{\infty} \left( \sum_{i=j+1}^{\infty} (1-p)^{i-1}p \right) \int_0^\infty \frac{1}{2} \log(1 + hP(j))e^{-h}dh $$  \hspace{0.5cm} (b)

$$= p \sum_{j=0}^{\infty} \left( \sum_{i=j+1}^{\infty} (1-p)^{i-1}p \right) \int_0^\infty \frac{1}{2} \log(1 + hP(j))e^{-h}dh $$  \hspace{0.5cm} (9)
(c) satisfies the following,
\[
\sum_{j=0}^{\infty} p(1-p)^j \int_{0}^{\infty} \frac{1}{2} \log(1 + hp(1-p)^j B_{\text{max}}) e^{-h} dh.
\]
(10)
(a) follows because \( \tau_1 \) is a geometric random variable with parameter \( p \) with \( \mathbb{E}[\tau_1] = 1/p \), (b) follows since \( P(\tau_1 = i) = (1-p)^{i-1}p \), and thereafter interchanging the order of summations, and (c) follows using \( \sum_{i=j+1}^{\infty} (1-p)^{i-1}p = (1-p)^j \), and substituting back \( P(j) = p(1-p)^j B_{\text{max}} \).

Next, we bound the gap between upper bound \( T_{ub} \) (Theorem 1 and the \( T_{lb} \) of CFP from Lemma 1.

**Lemma 2:** \( T_{ub} - T_{lb} \leq \frac{1}{2} \log \left( 1 + \sqrt{2p} k \right) \),

where \( k \) satisfies
\[
\frac{1}{2} \log \left( 1 + \sqrt{2p} k \right) = 0.54 - \frac{1}{4} \log(p) + \frac{1}{2 \ln 2} \frac{1}{\sqrt{2p} k} + \frac{1}{2p} \log \left( \frac{1}{1 - p} \right).
\]
(12)

In general, for Bernoulli energy arrivals, the gap between the upper bound and the achievable long term throughput of the CFP is not universally bounded for all values of \( p \). This negative result is, however, only limited to Bernoulli energy arrivals. We show a universal bound on the gap between the upper bound and the achievable long term throughput of the CFP with other distributions using the bound on the \( T_{ub} - T_{lb} \) for the Bernoulli case with \( p = 0.5 \).

**Lemma 3:** For Bernoulli energy arrivals with \( p = 0.5 \), \( T_{ub} - T_{lb} \leq 1.41 \) bits.

**Proof:** Fixing \( p = 0.5 \), the solution to (12) is \( k = 6.05 \). Then, from Lemma 2, \( T_{ub} - T_{lb} \leq \frac{1}{2} \log \left( 1 + \sqrt{2p} k \right) = 1.41 \).

When the battery size \( B_{\text{max}} \geq E \), to derive an achievable strategy we assume the battery size \( B_{\text{max}} = E \) which is less than the actual battery capacity, \( B_{\text{max}} \), and use CFP clearly, it is a feasible strategy and one can easily obtain the same bounds as before for the \( B_{\text{max}} < E \) case.

**V. Generalization to Other Energy Profiles**

Let \( X_t \) denote the energy arriving in time \( t \), i.e. \( X_t = E_t \) with CDF \( F_X(\cdot) \). Let \( \delta \) be such that \( F_X(\delta) = 0.5 \). We will assume that \( \delta \leq B_{\text{max}} \). The case \( \delta > B_{\text{max}} \) needs some modifications but can be worked out similarly. Thus, we know that with probability \( p = 0.5 \), \( X_t \geq \delta \). We now propose to use CFP as if the energy arrival process were i.i.d. Bernoulli with fixed size \( \delta \) and \( p = 0.5 \). Thus, the actual energy stored in the battery is 0 if \( X_t \leq \delta \), and \( \delta \) if \( X_t > \delta \).

**Theorem 2:** The per slot throughput achieved by CFP \( T_{lb} \) satisfies the following,
\[
T_{lb} \geq T_{ub} - 1.67 - \frac{1}{4} \log \left( \frac{\mathbb{E}[X^2]}{(F_X^{-1}(0.5))^2} \right).
\]
(13)

**Corollary 1:** When \( E_t \) is uniformly distributed between 0 and \( B_{\text{max}} \), CFP achieves
\[
T_{lb} \geq T_{ub} - 1.76.
\]
(14)

**Proof:** The corollary follows by substituting \( F_X^{-1}(0.5) = B_{\text{max}} \) and \( \mathbb{E}[X^2] = B_{\text{max}}^2 \) for uniform energy arrivals between 0 and \( B_{\text{max}} \) in Theorem 2.

Using this universal bound, we can get bounds on the capacity of this channel similar to Theorem 9 of [4].

**Theorem 3:** The capacity \( C \) of the fading channel with EH is bounded by
\[
T_{lb} - \frac{1}{4} \log \left( \frac{\mathbb{E}[X^2]}{(F_X^{-1}(0.5))^2} \right) - c \leq C \leq T_{ub},
\]
(15)
where \( c \) is a constant that depends on the distribution of \( X \).

**VI. Receiver Energy Harvesting**

In this section, we assume that the receiver also uses energy harvesting to power itself. Compared to the transmitter, however, the receiver structure/decision is simpler; it only has to decide whether to stay on or off in any given slot. When the receiver is on, it consumes a fixed amount of energy. The receiver is assumed to have a finite battery of size \( B_{\text{max}} \), and energy arrival at time \( t \) is \( E_t \). Note that the transmitter and receiver are separated and do not have access to each others’ energy availability information. At each time \( t \), only the channel \( h_t \) is revealed to both of them, using which the transmitter has to decide how much power to transmit, and the receiver has to decide whether to stay on or not. Let \( 1_R(h_t, t) = 1 \) if receiver is on at time \( t \), otherwise 0. Then the rate obtained at time \( t \) is \( \tilde{r}(t) = \frac{1}{2} 1_R(h_t, t) \log (1 + h_t P_h(t)) \), and long-term throughput is
\[
\tilde{T} = \lim_{N \to \infty} \frac{1}{N} \sum_{t=1}^{N} \tilde{r}(t).
\]
(16)

Our objective is to find optimal \( P_h(t) \) and \( 1_R(h_t, t) \), given the energy neutrality constraint \( P_h(t) \leq B_t \), that maximizes \( \tilde{T} \), i.e. \( \tilde{T}^* = \max_{P_h(t) \leq B_t, 1_R(h_t, t)} \tilde{T} \).

We next show that with receiver energy harvesting, the problem of approximating \( \tilde{T}^* \) is much harder, and finding a universal bound on the difference of the upper and the lower bound is challenging, since the transmitter-receiver decisions about \( P_h(t) \) and \( 1_R(h_t, t) \) are intimately connected via the revealed channel \( h_t \).

If we use the Cauchy-Schwarz inequality twice, first on \( 1_R(h_t, t) \log (1 + h_t P_h(t)) \), and next on \( h_t, P_h(t) \), we get
\[
\tilde{T}^* \leq 2 \sqrt{\mathbb{E} \left[ \left( \tilde{E}_t \right)^2 \right]} \log \left( 1 + \sqrt{2} \sqrt{\mathbb{E} \left[ (E_t)^2 \right]} \right).
\]
(17)
In the simplest case, when the energy harvesting distribution at the receiver is Bernoulli with rate \( q \), we get
\[
\bar{T}^* \leq \bar{T}_{ub} = 2 \sqrt{q} \log \left( 1 + \sqrt{2} \sqrt{E[|E_i|]} \right).
\] (18)

For an achievable strategy, consider a simple receiver strategy, where \( 1_R(h_t, t) = 1 \), whenever receiver has sufficient energy, and \( 1_R(h_t, t) = 0 \) otherwise. From Lemma 1 the long term throughput achieved by using CFP at the transmitter is,
\[
\bar{T}_{lb} = q \int_{\gamma^*}^{\infty} \sum_{j=0}^{\infty} p(1-p)^j \log(1 + hp(1-p)^j B_{max}) e^{-h} dh.
\]

The main difference in \( \bar{T}_{ub} \) and \( \bar{T}_{lb} \) is the \( \sqrt{q} \) term and the \( \gamma^* \) term, respectively. Even if we use a channel dependent strategy where \( 1_R(h_t, t) = 1 \) if \( h > \gamma \), even then getting pre-log term of \( \sqrt{q} \) is not possible. Inherently in this case, the upper bound is too loose because of the coupled transmitter-receiver optimal decisions. We next present a special case for which we can bound the ratio of the upper and the lower bound with receiver energy harvesting.

A. \( B_{max} = \tilde{B}_{max} = 1 \)

In this section, we consider a special case of \( B_{max} = \tilde{B}_{max} = 1 \), and binary energy transmission policy at transmitter, i.e. \( P_h(t) \in \{0, 1\} \). Also, the energy arrivals at the transmitter and the receiver are assumed to be Bernoulli with parameter \( p \) and \( q \) respectively. Thus, \( \tilde{r}(t) = 1_T(h_t, t)1_R(h_t, t) \log(1 + h_t) \), and we want to maximize (16), with respect to \( 1_T(h_t, t) \) and \( 1_R(h_t, t) \) under the energy neutrality constraint at both the transmitter and the receiver.

**Lemma 4:** \( \bar{T}^* \leq \bar{T}_{ub} = \min \{p, q\} \int_{\gamma^*}^{\infty} \sum_{j=0}^{\infty} p(1-p)^j \log(1 + h) e^{-h} dh, \) where \( \gamma^* = -\ln \left( \min \{p, q\} \right) \).

**Proof:** Let us assume that \( p > q \). To upper bound \( \bar{T}^* \) assume that the transmitter always has energy to transmit, i.e. \( 1_T(h_t, t) = 1, \forall t \). Thus, we have a system with energy harvesting only at the receiver, for which the optimal transmission policy is known be of threshold type from [1], [2], i.e. for \( B_t = 1, 1_R(h_t, t) = 1 \) if \( h_t > \gamma_t \) and 0 otherwise.

Next, we argue that the optimal threshold \( \gamma^* \) will satisfy \( \mathbb{P}(h > \gamma^*) = q \).

- If \( \gamma > \gamma^* \), the energy arrival rate at the receiver is greater than the energy usage at the receiver, and essentially there is a wastage of energy resulting in sub-optimality.
- If \( \gamma < \gamma^* \), the energy usage rate is faster than energy arrival rate \( q \). Thus, the receiver will remain on for weak channel states, and will not have sufficient energy to transmit on the better channel gains.

Thus, the maximum achievable long term throughput is
\[
\bar{T}^* \leq \bar{T}_{ub} = q \int_{\gamma^*}^{\infty} \log(1 + h) f(h) dh.
\] (19)

The case of \( q > p \), can be proved by interchanging the role of the transmitter and the receiver.

Next, we propose a Common Threshold Policy (CTP) to lower bound the achievable rate. i) Transmitter Policy: The transmitter simulates an i.i.d. Bernoulli random variable \( X_1 \) for each slot with parameter \( q \). The transmitter waits till \( X_1 = 1 \), and thereafter transmits whenever \( h_t > \gamma^* \) if \( B_t = 1 \). ii) Receiver Policy: The receiver remains on whenever \( h_t > \gamma^* \) if \( B_t = 1 \).

**Theorem 4:** The long term throughput achieved by CTP, \( \bar{T}_{ub} \), satisfies,
\[
\bar{T}_{lb} \geq \frac{1}{2} \bar{T}_{ub}.
\] (20)

**Proof:** Consider the case of \( p > q \). Note that the receiver is following the optimal strategy with CTP. We need to show that with probability \( \frac{1}{2} \), the transmitter is on whenever receiver is. The random variable \( X_2 = \{1_{h_t > \gamma^*}\} \) is Bernoulli with parameter \( q \) as \( \mathbb{P}(h_t > \gamma^*) = q \). We call this random variable \( X_2 \). So whenever \( X_1 < X_2 \), the transmitter and receiver are on for slots where \( \{1_{h_t > \gamma^*}\} \). Since \( X_1 \) and \( X_2 \) are independent and identically distributed random variables, \( P(X_1 \leq X_2) \geq \frac{1}{2} \). Thus, CTP can achieve at least half of the long term throughput of the upper bound \( \bar{T}_{ub} \).

**VII. SIMULATIONS**

In Fig. 1 we plot the performance of CFP for uniform energy arrival process i.e., \( E_t \) is uniformly distributed between 0 and \( B_{max} \), for different values of battery size \( B_{max} \). We observe that the per slot throughput obtained by CFP is within bounded gap from the upper bound on maximum achievable per slot throughput in accordance with Theorem 2.

In Fig. 2 we consider the transmitter-receiver EH model and plot the performance of CTP for different values of \( q (p \geq q) \) always. We see that the long term throughput obtained by CTP is atleast half the upper bound on maximum achievable long term throughput as stated in Theorem 4.

**APPENDIX A**

**Proof:** The case \( B_{max} \leq k \) is trivial because \( T_{ub} \leq \frac{1}{2} \log \left( 1 + \sqrt{2p} B_{max} \right) \) for \( B_{max} \leq k \) and \( T_{lb} \geq 0 \).

For \( B_{max} > k \), \( T_{ub} - T_{lb} = \frac{1}{2} \log \left( 1 + \sqrt{2p B_{max}} \right) - \sum_{j=0}^{\infty} \frac{p(1-p)^j}{2} \int_{0}^{\infty} \log(1 + hp(1-p)^j B_{max}) e^{-h} dh, \) \( \leq \frac{1}{2} \log \left( 1 + \sqrt{2p B_{max}} \right) - \sum_{j=0}^{\infty} \frac{p(1-p)^j}{2} \int_{0}^{\infty} \log(hp(1-p)^j B_{max}) e^{-h} dh, \)
The solid curve represents the upper bound \( T_{ub} \) and the dashed curve corresponds to per slot throughput achieved by CFP \( T_{lb} \). 

\[ \frac{(d)}{2} \log \left(1 + \sqrt{2p} k \right). \]  

where (a) follows from the fact that removing 1 from the second log term results in an upper bound; (b) follows because \( \sum_{j=0}^{\infty} p(1-p)^j = 1 \) and \( \sum_{j=0}^{\infty} j p(1-p)^j = \frac{1-p}{p^2} \). Also \( \int e^{-h} dh = 1 \) and \( \int \log(h) e^{-h} = -0.29 \), (c) uses the identity \( \ln(1+x) \leq x \), and finally (d) follows from (12).

**APPENDIX B**

**Proof:** The proposed strategy views any i.i.d. energy arrival process as Bernoulli with packet size \( \delta \) and \( p = 0.5 \), and uses the CFP. By Lemma 4, we have the following,

\[ T_{lb} \geq \frac{1}{2} \log (1 + \delta) - 1.41 \]  

We next bound the difference between \( T_{ub} \) and the first term on the RHS of (22) as follows, \( T_{ub} - \frac{1}{2} \log (1 + \delta) \)

\[ \frac{a}{2} \frac{1}{2} \log \left(1 + \sqrt{2\mathbb{E}[X^2]} \right) - \frac{1}{2} \log (1 + \delta), \]

\[ = \frac{1}{2} \log \left(1 + \sqrt{2\mathbb{E}[X^2]} \right) - \frac{1}{2} \log (1 + \delta), \]

\[ \leq \frac{b}{2} \frac{1}{2} \log \left( \frac{\mathbb{E}[X^2]}{F^{-1}(0.5)^2} \right). \]

where (a) follows from Theorem 1, and (b) follows because the numerator is greater than the denominator.

Rearranging the terms in (23), we get,

\[ \frac{1}{2} \log (1 + \delta) \geq T_{ub} - 0.25 - \frac{1}{4} \log \left( \frac{\mathbb{E}[X^2]}{F^{-1}(0.5)^2} \right). \]  

From (22) and (24), we have,

\[ T_{lb} \geq T_{ub} - 1.67 - \frac{1}{4} \log \left( \frac{\mathbb{E}[X^2]}{F^{-1}(0.5)^2} \right). \]  

**REFERENCES**

[1] A. Sinha and P. Chaporkar, “Optimal power allocation for a renewable energy source,” in Communications (NCC), 2012 National Conference on. IEEE, 2012, pp. 1–5.

[2] N. Michelusi, K. Stamatou, and M. Zorzi, “On optimal transmission policies for energy harvesting devices,” in Information Theory and Applications Workshop (ITA), 2012. IEEE, 2012, pp. 249–254.

[3] R. Vaze and K. Jagannathan, “Finite-horizon optimal transmission policies for energy harvesting sensors,” in International Conference on Acoustics, Speech, and Signal Processing (ICASSP). IEEE, 2014.

[4] Y. Dong, F. Farnia, and A. Ozgur, “Near optimal energy control and approximate capacity of energy harvesting communication,” arXiv preprint arXiv:1405.1156, 2014.