The comparative study for the isotropic and orthotropic circular plates

C Popa¹ and G Tomescu²

¹Mechanical Engineering Department, Valahia University, Targoviste, Romania
²SC UPET, Targoviste, Romania

E-mail: carменpopa2001@yahoo.com

Abstract. The aim of study is static bending analysis of an isotropic circular plate using analytical method i.e. Classical Plate Theory, Finite Element software ANSYS and experimental methods. The diameter of circular plate, material properties, like modulus of elasticity (E), poisson's ratio (μ) and intensity of loading is assumed at the initial stage of research work. In comparison with this plane plate we analyze a plate of same dimensions and charge, but having ribs, to see the advantage of the rigidify. The two plates are fixed supported subjected to uniformly distributed load.

1. Introduction

Plates are extensively used in many engineering applications like roof and floor of building, deck slab of bridge, foundation of footing, water tanks, turbine disks etc. The amount of deflection can be determined by solving the differential equations of an appropriate plate theory. The stresses in the plate can be calculated from these deflections. Once the stresses are known, failure theories can be used to determine whether a plate will fail under a given load [1].

Vanam and others [2] done static analysis of an isotropic rectangular plate using finite element analysis. The aim of study was static bending analysis of an isotropic rectangular plate with various boundary conditions and various types of load applications. Numerical analysis has been carried out by developing programming in mathematical software MATLAB. Later, for the same structure, analysis has been carried out using finite element analysis software ANSYS.

Kavade and others [3] analyzed circular plate with a hole by finite element method. In this research work they analyzed a plate with a circular hole subjected to a uniform stress, the effect of an initial stretching of a rectangular plate with a cylindrical hole on the stress and displacement distributions around the hole, which are caused by the additional loading, was studied using the finite element method. They analyzed a plate with a circular hole subjected to a uniform stress and observed the variation in the results obtained through various meshes.

Wang and others [4] studied three-dimensional solution of axisymmetric bending of functionally graded circular plates. They investigated the axisymmetric bending of transversely isotropic and functionally graded circular plates subject to arbitrarily transverse loads using the direct displacement method based on three-dimensional theory.

Kirstein and Woolley [5] studied symmetrical bending of thin circular elastic plates on equally spaced point supports. Solution for a thin circular elastic plate supported at points equally spaced along a concentric support circle and subjected to a transverse load which is symmetrically distributed over a concentric circular area is obtained.
Sukla [6] studied the finite element analysis on circular stiffened plates using ANSYS. He analyzed the structural behavior of a stiffened plate under static uniform loads. Analysis of a circular plate with radial stiffeners was done. The linear and nonlinear behavior of the beams is studied under static and dynamic loading. The simulation analysis is completed with a numerical analysis of the system using the ANSYS program.

Khobragade and Deshmukh [7] studied thermal deformation in a thin circular plate due to a partially distributed heat supply. In this research work, they developed an integral transform to determine temperature distribution in a thin circular plate, subjected to a partially distributed and axisymmetric heat supply on the curved surface and studied thermal deformation.

Meghare and Jadhav [8] presented a simple higher order theory for bending analysis of steel beams. A new shear deformation theory for the bending analysis of thick isotropic beams made up of steel is studied.

Manzoor and others [9] focuses on a numerical model developed for Hollow tube. Analytical results obtained are validated with the results of ANSYS software.

Gujar and Ladhane [10] studied the isotropic circular plates using the analytical and finite element analysis at simply supported and clamped boundary conditions subjected to uniformly distributed load and center concentrated /point load.

This article examines the state of deformations of a circular plane plate and a plate with radial ribs placed on one side of 650 mm diameter, under a pressure of 0.3 MPa. Some characteristics of material are: Poisson’s ratio of steel (μ) = 0.3; Young’s modulus of steel (E) = 200 GPa.

2. Plane plate analyze

2.1. Analytical method

![Figure 1. Geometrical characteristics of the circular plane plate.](image)

We analyze the plane plate, having 32 holes, and geometrical characteristics from figure 1, using analytical element method.

In this case the plane plate is considered fixed on the line of the centers of the holes, at \( r_\text{cr} = 285 \) mm. If it is supposed at a value of the pressure of 0,3 MPa, we obtain the maximal value of displacement on the centre of the plate \( w=1,624 \) mm [11].
2.2. Finite elements method

We analyze the same plane plate, having 32 holes, and geometrical characteristics from figure 1, using finite element method. We consider that the plate is fixed on circumference of the holes for the screws, which clamp this plate by the experimental recipient [12].

We present the calculus variant with finite elements, using Ansys program.

The plate displacements distribution, at 0.3 MPa pressure is presented in figure 2.

![Figure 2](image2.png)

**Figure 2.** The displacements of the embedded plane plate, using Ansys program.

Displacements from figure 3 are obtained at 0.3 pressure MPa, using a plan which pass through the centers of two exactly contrary holes, which are situated between two neighbor holes.

![Figure 3](image3.png)

**Figure 3.** The displacements of the embedded plane plate, after a plane which pass through the centers of two exactly contrary holes.
Figure 4. The displacements of the embeded plane plate, after a plane which pass through the centers of two exactly contrary holes.

Displacements values are given in figure 4 and table 1.

Table 1. The displacements of the embeded plane plate after a plane which pass through the centers of two exactly contrary holes.

| r [mm] | 0  | 7  | 21 | 36 | 50 | 66 | 80 | 95 | 110 | 125 |
|-------|----|----|----|----|----|----|----|----|-----|-----|
| w [mm]| 1,6227 | 1,611 | 1,6012 | 1,5789 | 1,5213 | 1,4323 | 1,3767 | 1,2766 | 1,1766 | 1,04332 |
|       | 138 | 152 | 167 | 175 | 194 | 211 | 225 | 239 | 251 | 262 |
|       | 273 | 285 | 0,92878 | 0,8233 | 0,68789 | 0,59898 | 0,4434 | 0,31234 | 0,20232 | 0,12345 | 0,0234 | 0,006 | 0 |

2.3. Experimental method

Figure 5. Experimental stall.
In the experimental case, we take into account the measuring of the linear deformations of the points of the plate, in the same conditions, using the comparators [13-15].

The components of the experimental stall are presented in figure 5. The zones where the comparators are fixed on the plate are presented in figure 6.

![Figure 6. The position of the comparators on the plane plate.](image)

The values of the displacements, which are theoretical and experimental calculated (MA), (MEF) and (ME), at 0, 3 MPa pressure, for the plane plate are presented in table 2.

| p [MPa] | r [mm] | w [mm] | MEF     | ME     |
|---------|--------|--------|---------|--------|
| 0,3     | 0      | 1,608  | 1,6227  | -      |
| 125     | 0,997  | 1,0433 | 1,039   |        |
| 175     | 0,413  | 0,5989 | 0,618   |        |
| 225     | 0,228  | 0,2023 | 0,221   |        |

3. Ribbed plate analyze

3.1. Elements finite method

We analyze the ribbed plate, having 32 holes, and geometrical characteristics from figure 7, using finite element method. We consider that the plate is fixed on circumference of the holes for the screws, like in the case of the plane plate.

We present the calculus variant with finite elements, using Ansys program, too.

The plate displacements distribution, at 0,3 MPa pressure is presented in figure 8. Displacements from figure 8 are obtained at 0,3 pressure MPa, using a plan which pass through the centers of two exactly contrary holes, which are situated between two neighbor holes. The values of the
displacements of the ribbed plate, after a plane which passes between two neighbor ribs are presented in figure 9 and table 3.

Figure 7. Geometrical characteristics of the circular ribbed plate.

Figure 8. The displacements of the ribbed plate, after a plane which passes between two neighbor ribs.
Figure 9. The displacements of the ribbed plate, after a plane which passes between two neighbor ribs.

Table 3. Displacements for the ribbed plate, embedded, using a plan which are situated between two neighbour holes.

| r [mm] | 0  | 7  | 21 | 36 | 50 | 66 | 80 | 95 | 110 | 125 |
|-------|----|----|----|----|----|----|----|----|-----|-----|
| w [mm]| 0,58993 | 0,58841 | 0,58317 | 0,57074 | 0,55412 | 0,53226 | 0,51285 | 0,4793 | 0,45098 | 0,4135 |
|       | 138 | 152 | 167 | 175 | 194 | 211 | 225 | 239 | 251 | 262 | 273 | 285 |
|       | 0,35876 | 0,3445 | 0,29899 | 0,27767 | 0,22109 | 0,16787 | 0,11307 | 0,0656 | 0,0345 | 0,01676 | 0,00876 | 0 |

Figure 10. The displacements of the ribbed plate, after a plane which passes in a rib.
Figure 11. The displacements of the ribbed plate, after a plane which passes in a rib.

Displacements from figure 10 are obtained at 0.3 pressure MPa, using a plan which pass through a rib. The values of the displacements of the ribbed plate, after this plane are presented in figure 11 and table 4.

Table 4. Displacements for the ribbed plate, embedded, after a plane which passes in a rib.

| r [mm] | 0   | 7   | 21  | 36  | 50  | 66  | 80  | 95  | 110 | 125 |
|--------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| w [mm]| 0.58993 | 0.588 | 0.58432 | 0.5723 | 0.554 | 0.53112 | 0.5054 | 0.47553 | 0.4409 | 0.40776 |

138  152  167  175  194  211  225  239  251  262  273  285
0.37667  0.34566  0.29887  0.2809  0.22343  0.17889  0.12792  0.0877  0.0434  0.0223  0.0056  0

3.2. Experimental method

Figure 12. The position of the comparators on the ribbed plate.
In the experimental case, we take into account the measuring of specific linear deformations of the external points of the plate, in the same conditions, like in the case of the plane plate. The zones where the comparators are fixed on the plate are presented in figure 12. We have been obtained the equivalent displacements values, for the ribbed plate, which are represented in the table 5. In table 6 the deformations for the plane and ribbed plate are given.

| Table 5. The values of the displacements, for the ribbed plate. |
|---------------------------------------------------------------|
| $p$ [MPa] | $r$ [mm] | $w$ [mm] | MA | MEF | ME |
| 0,3        | 0       | -      | 0,58993 | -   |
| 125        | -       | 0,4135 | 0,459 |
| 175        | -       | 0,2776 | 0,319 |
| 225        | -       | 0,12792 | 0,131 |

| Table 6. The values of the displacements, for the ribbed and plane plate. |
|---------------------------------------------------------------|
| $p$ [MPa] | $r$ [mm] | $w$ [mm] | Ribbed plate | Plane plate |
|---------------------------------------------------------------|
|---------------------------------------------------------------|
| $p$ [MPa] | $r$ [mm] | $w$ [mm] | Ribbed plate | Plane plate |
|---------------------------------------------------------------|
| 0,3        | 0       | -      | 0,58993 | -       | 1,608 | 1,6227 | - |
| 125        | -       | 0,4135 | 0,459 | 0,997 | 1,04332 | 1,039 |
| 175        | -       | 0,2776 | 0,319 | 0,413 | 0,59898 | 0,618 |
| 225        | -       | 0,12792 | 0,131 | 0,228 | 0,20232 | 0,221 |

4. Results comparison
We take into account the deformations values, which are produced in different points of the plate. As we infer from the comparison of the results, we observe the advantage of the rigidity of the plate. In this case the values of the displacements lower about three times. Closed values are obtained using the variants of calculus: analytical, finite elements and experimental methods.

5. References
[1] Timoshenko S and Woinowsky-Krieger S 2010 Theory of plates and shells (McGraw Hill Education)
[2] Vanam B, Rajyalakshmi M and Inala R 2012 Static analysis of an isotropic rectangular plate using finite element analysis (FEA) Journal of Mechanical Engineering Research 4 pp 148-162
[3] Mekalke G and Kavade M, Deshpande S 2012 Analysis of a plate with a circular hole by FEM Journal of Mechanical and Civil Engineering, pp 25-30
[4] Wang Y, Rongqiao X, Haojiang D 2010 Threedimensional solution of axisymmetric bending of functionally graded circular plates Elsevier Composite Structures 92 pp1683–1693
[5] Kirstein A and Woolley R 1967 Symmetrical bending of thin circular elastic plates on equally spaced point support Journal of Research of the National Bureau of Standards -C. Engineering and Instrumentation 71C
[6] Sukla B 2009 The FEM analysis on circular stiffened plates using ANSYS- Bachelor of Technology diss. (Rourkela: National Institute of Technology)
[7] Khobragade N and Deshmukh K 2005 Thermal deformation in a thin circular plate due to a
partially distributed heat supply Sadhana 30(4) pp 555–563

[8] Meghare T and Jadhav P 2015 A simple higher order theory for bending analysis of steel beams International Journal of Civil Engineering 2(4) pp 31-38

[9] Manzoor S and Vinay G 2015 A Study on Composite Steel Tubes International Journal of Civil Engineering, EFES

[10] Gujar P and Ladhane K 2015 Bending Analysis of Simply Supported and Clamped Circular Plate International Journal of Civil Engineering 2(5) pp 69-75

[11] Buzdugan Gh 1980 Rezistenţa materialelor (Bucureşti: Editura Tehnică)

[12] Hadar A, Marin C and Petre C 2005 Metode numerice in inginerie (Bucuresti: Politehnica Press)

[13] Iatan I R and Popa Carmen 2005 Determinarea tensiunilor şi deformaţiilor la plăcile circulare Buletinul Universităţii Petrol-Gaze LVII(2) pp 176

[14] Popa C 2004 Solicitation states at circular plates Modelling and optimization in the machines building field I pp 263-268

[15] Popa C 2006 Displacement analyze for circular plates Al III-lea Simpozion International “Mecatronica, Microtehnologii si Materiale Noi”, Targoviste