Structure and decays of hidden heavy pentaquarks

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(Dated: March 18, 2022)

We study the hadronic molecular structure of hidden heavy pentaquarks — new exotic states composed of charmed/bottom baryons and $D(D')/B(B')$ mesons. Based on the observation of three pentaquark candidates $P_c^+(4312), P_c^+(4440)$, and $P_c^+(4457)$ by the LHCb Collaboration we consider the classification of possible flavor partners composed of charmed baryons and $D(D')$ mesons within the hadronic molecular approach. We extend this classification to the bottom sector. Using phenomenological Lagrangians we construct baryon-meson bound states governed by the Weinberg-Salam compositeness condition. As an application we consider strong two-body decays of the new exotic states into a light baryon and $V = J/\psi, \Upsilon$ or $P = \eta_c, \eta_b$ mesons. Results are presented in the heavy quark limit.

I. INTRODUCTION

In 2015 the LHCb Collaboration reported on the observation of resonances in the $J/\psi p$ decay channel consistent with possible pentaquark states in the full reaction $\Lambda_c^0 \to J/\psi K^- p$ decay \cite{1}. A model-dependent analysis of the invariant masses and angular distributions describing the $\Lambda_c^0$ decay lead to the claim of two pentaquark resonances, a broad state $P_c^+(4380)^+$ with mass $4380\pm8\pm29$ MeV and a width of $205\pm18\pm86$ MeV and the narrower $P_c^+(4450)^+$ state with mass $4449.8\pm1.7\pm2.5$ MeV and width $39\pm5\pm19$ MeV. Soon after in Ref. \cite{2} the LHCb Collaboration confirmed in a full amplitude analysis the consistency of data with the existence of the two exotic hadron structures $P_c(4380)^+$ and $P_c(4450)^+$. In a recent paper \cite{3} the LHCb Collaboration with the analysis of a much larger data sample confirmed the previously observed $P_c(4450)^+$ peak and resolved it into two narrow exotic baryon states $P_c(4440)^+$ and $P_c(4457)^+$. Furthermore, a narrow partner state $P_c(4312)^+$ has been claimed in \cite{3} while the existence of the $P_c(4380)^+$ can neither be confirmed nor excluded. The conclusion drawn from this analysis was: (1) the minimal quark content of the $P_c$-states is (diuuc$\bar{c}$), (2) since the $P_c(4312)^+, P_c(4440)^+$, and $P_c(4457)^+$ are narrow and below the $\Sigma^+_c\bar{D}^0$ and $\Sigma_c^+\bar{B}^0$ thresholds, these states are strongly correlated with baryon-meson bound state structures.

Recently, the GlueX Collaboration at JLab \cite{4} reported on the first measurement of the exclusive $J/\psi$ photoproduction cross section in the energy region from threshold up to $E_p = 11.8$ GeV using a tagged photon beam. Such a measurement is extremely important since it provides a crucial check for theoretical approaches to the gluonic structure of the proton at high $x$, but also to possible structure interpretations of the LHCb pentaquarks. At this stage the GlueX Collaboration did not see any evidence for the LHCb pentaquarks and set model-dependent upper limits on their branching fractions $\text{Br}(P_c^+ \to J/\psi p)$ with $4.6\%$ for $P_c(4312)^+$, $2.3\%$ for $P_c(4440)^+$, and $3.8\%$ for $P_c(4458)^+$ assuming spin-parity quantum numbers $J^P = \frac{3}{2}^-$ for each state.

The observation of the LHCb Collaboration stimulated extensive theoretical studies of hidden pentaquark structures using different scenarios and frameworks (see, e.g., Refs. \cite{5, 21} and recent overviews in Refs. \cite{3, 24, 25}). In particular, the composite structure of the new exotic states has been studied using coupled-channel dynamics \cite{6, 7}. An application of QCD sum rules to hidden charm pentaquark states has been done in \cite{8} using diquark-triquark-antiquark type interpolating currents, in \cite{9, 11} with meson-baryon molecular type currents, and the currents in form of product of two color-octet clusters of three light quarks and charm-anticharm pair \cite{12}. Double polarization observables in pentaquark photoproduction have been studied by JPAC Collaboration using reaction model in Ref. \cite{13}. Different types of potential models to explain the spectrum of LHCb pentaquarks has been developed in Refs. \cite{14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25}. Some of them are: a diquark-triquark potential model \cite{14, 15}, quark delocalization color screening potential model \cite{16}, nonrelativistic potential model \cite{17}, chiral quark model \cite{18}, color flux-tube model based on a five-body confinement potential \cite{19}, constituent quark model \cite{20}, diquark model derived using gauge/string duality \cite{21}, potential model based the Cornell-like potential \cite{22}, and a quasipotential Bethe-Salpeter equation approach \cite{25}. Using a simple phenomenological model based on the Gürsey-Radica mass formula was used to predict the masses of hidden charm and bottom pentaquarks in Ref. \cite{26}. In Refs. \cite{27} the hidden charm and bottom pentaquark states have been studied using chiral perturbation theory. Implications of $SU(3)$ flavor symmetry for heavy pentaquarks have been considered in Refs. \cite{28, 29, 30, 31}. A hadronic molecular approach based on a charmonium-nucleon structure of the hidden charm pentaquarks has been proposed in Ref. \cite{32}. An effective field theoretical approach incorporating heavy-quark spin symmetry has been suggested in Ref. \cite{33}. Using effective Lagrangian approach the production of the pentaquark states $P_c(4312), P_c(4440)$, and $P_c(4457)$ has been investigated in Refs. \cite{34}. A framework based on an effective-range expansion and resonance compositeness relations has been discussed in Ref. \cite{35}. Field-theoretical hadronic molecular approaches for heavy pentaquarks have been developed in
Refs. [35, 36]. In Refs. [38] the new LHCb peaks have been related to kinematical effects in the rescattering from $\chi_{c1}$ to $J/\psi$. Ref. [39] proposed a hadrocharmonium pentaquark scenario for the new states discovered by the LHCb Collaboration. Ideas of light- and heavy-flavor symmetries and their manifestation in properties of heavy hidden pentaquarks have been discussed in Refs. [40]-[42]. One should also note that for the identification of the hidden charm pentaquark states it is important to perform a complete analysis of the cascade decay $\Lambda_b \to \Lambda^*(4^-,1\frac{3}{2}^+) \to pK^- + J/\psi$ done for example in Ref. [43].

The main ideas in the application of quantum field theory to bound states using their compositeness have originally been laid out and formulated in Refs. [44]-[47]. Reference [44] contains the original application to the composite system of the deuteron - the canonical example of a hadronic molecule (HM). A extensive set of descriptions of hadronic molecules in the context of exotic heavy hadrons have been pursued by us for quite some time [48]-[57].

Next, we use recent results of the GlueX Collaboration [4] on upper limit of the branching fraction of the $D(4312)$ and $D(4440)$ states and present details of our formalism to set up these exotic states as hadronic molecules. We present a classification of these exotic states and calculate their strong two-body decays in the heavy quark limit. A first consideration of some of these states in a similar approach has been done recently [35]. In our numerical analysis we proceed as done for example in Ref. [43].

The main objective of the present paper is a self-consistent study of the hidden charm pentaquarks composed of charmed baryons and $D$ mesons in hadronic molecular picture based on the formalism proposed and developed in Refs. [48]-[57]. We present a classification of these exotic states and calculate their strong two-body decays in the heavy quark limit. A first consideration of some of these states in a similar approach has been done recently [35]. In our numerical analysis we proceed as follows: first we derive the results for the helicity amplitudes and decay rates in terms of two model parameters (size parameter $\Lambda$ and $D$-wave coupling of pseudoscalar heavy quarkonia with vector heavy-light meson and pair of light and heavy-light baryons). Next, we use recent results of the GlueX Collaboration [4] on upper limit of the branching fraction of the $P_c(4457)$ pentaquark to derive upper limit on its size parameter, which describes the distribution of the constituents in the pentaquark state.

The paper is structured as follows. In Sec. II we give a classification of hidden charm pentaquarks – exotic states composed of charmed baryons and $D$ mesons and present details of our formalism to set up these exotic states as hadronic molecules. We also discuss the extension to hidden bottom pentaquarks. In Sec. III we focus on the calculation of strong two-decays of hidden charm pentaquarks. We present a derivation of the corresponding matrix elements and the discuss numerical results. Finally, we summarize the results of the paper.

II. HADRONIC MOLECULAR STRUCTURE OF HIDDEN CHARM PENTAOQUARKS

For the spin-parity quantum numbers of the hidden charm pentaquark states we use one of the possible scenarios, which follows from the conjecture of the LHCb Collaboration [3] and the classification of some theoretical approaches: $J^P = 1^-$ for the $P_c(4312)^+$ and $P_c(4440)^+$ states and $J^P = 3^-$ for the $P_c(4457)^+$ state. We also base the following procedure on a base on a $SU_c(3)$ classification of hidden charm pentaquarks proposed in Ref. [29] and consider all together 8 hidden pentaquarks states coinciding with hadronic molecular states composed of charmed baryons and $D(D')$ mesons. In Table 1 we present the classification of $24 = 3 \times 8$ hidden charm pentaquarks composed by a single charmed baryon and $D(D')$ mesons. For each pentaquark discovered by the LHCb Collaboration we propose the existence of 7 partner states, which are composed of charmed baryons ($\Sigma_c, \Xi'_c, \Omega_c$) and $D(D')$ mesons. We specify isospin and spin-parity $J^P$, the interpolating currents in terms of the constituent fields, the constituent threshold $(\sum$ of the masses of the constituents), mass (if available from the LHCb Collaboration [3]), otherwise our conjecture as explained further on). For the case when the pentaquarks are mixed states of two components we indicate the constituent thresholds for both cases (the value for the second component is given in brackets). In our conjecture for the masses of the pentaquarks we suppose that the mass difference of two pentaquark states is roughly equal to the difference of the corresponding charm baryon + $D(D')$ meson thresholds. By analogy, one can also derive hidden bottom pentaquarks replacing $D(D') \to B(B')$ and the charmed baryons by the bottom one. Recently a five-flavor classification of hidden charm and bottom pentaquarks has also been proposed in Ref. [42]. Our scheme is differed since we only use nonstrange $D(D')$ and $B(B')$ heavy-light mesons in the construction of hidden heavy pentaquarks. We construct the pentaquark Fock states as eigenstates of the
isospin operator $|I, I_3\rangle$ and give an expansion in terms of the corresponding Fock states using standard $SU(2)$ couplings:

$$
|1/2, \pm 1/2\rangle = \pm \sqrt{2/3} \left[ |1, \pm 1\rangle \mp |1/2, \mp 1/2\rangle \right] \mp \sqrt{1/3} \left[ |1, 0\rangle \mp |1/2, \pm 1/2\rangle \right],
$$

$$
|1/2, \pm 1/2\rangle = |1/2, \pm 1/2\rangle \mp |0, 0\rangle, \quad |1, \pm 1\rangle = |1/2, \pm 1/2\rangle \mp |1/2, \pm 1/2\rangle,
$$

$$
|1(0), 0\rangle = \sqrt{1/2} \left[ |1/2, 1/2\rangle \mp |1/2, -1/2\rangle \right] \pm \sqrt{1/2} \left[ |1/2, -1/2\rangle \mp |1/2, 1/2\rangle \right].
$$

(1)

**TABLE I:** Classification of hidden charm pentaquarks composed by a single charm baryon and $D(D')$ mesons

| Pentauark | $I$ | $J^P$ | Interpolating current $J_F$ | Threshold (MeV) | Mass (MeV) |
|-----------|----|------|---------------------------|----------------|-----------|
| $P_c(4312)^+$ | $1\over 2$ | $-1\over 2$ | $\gamma^5 J^5 - \frac{1}{3} J^3 D^0 - \frac{1}{3} J^3 D^0$ | 4323.62 (4317.17) | 4311.9 ± 0.7\text{[3]} |
| $P_c(4312)^0$ | $1\over 2$ | $1\over 2$ | $\frac{1}{3} J^3 D^0 - \frac{1}{3} J^3 D^0$ | 4318.58 (4322.55) | 4312 |
| $P_c(4435)^+$ | $1\over 2$ | $1\over 2$ | $\Xi^+ D^0$ | 4422.23 | 4435 |
| $P_c(4435)^0$ | $1\over 2$ | $1\over 2$ | $\Xi^0 D^0$, $\Xi^0 D^0$ | 4447.05 (4443.63) | 4435 |
| $P_c(4420)^+$ | $0\over 2$ | $1\over 2$ | $\Xi^+ D^0 - \Xi^0 D^0$ | 4447.05 (4443.63) | 4420 |
| $P_c(4554)^0$ | $1\over 2$ | $1\over 2$ | $\Omega^0 D^0$ | 4560.03 | 45545 |
| $P_c(4554)^+$ | $1\over 2$ | $-1\over 2$ | $\Omega^+ D^0$ | 4564.85 | 4554 |

| Second Family $P_{c2}$ |
|------------------------|
| $P_c(4440)^+$ | $1\over 2$ | $-1\over 2$ | $\gamma^\nu \gamma^5 J^5 - \frac{1}{3} J^3 D^0 - \frac{1}{3} J^3 D^0$ | 4464.23 (4459.75) | 4440.3 ± 1.3\text{[4]} |
| $P_c(4440)^0$ | $1\over 2$ | $1\over 2$ | $-\gamma^\nu \gamma^5 J^5 - \frac{1}{3} J^3 D^0 - \frac{1}{3} J^3 D^0$ | 4460.60 (4463.16) | 4440 |
| $P_c(4560)^+$ | $1\over 2$ | $-1\over 2$ | $\gamma^\nu \gamma^5 \Xi^+ D^0 - \frac{1}{3} J^3 D^0$ | 4584.25 | 4560 |
| $P_c(4560)^0$ | $1\over 2$ | $1\over 2$ | $\gamma^\nu \gamma^5 \Xi^0 D^0$, $\Xi^0 D^0$ | 4587.66 (4585.65) | 4560 |
| $P_c(4560)^+$ | $0\over 2$ | $1\over 2$ | $-\gamma^\nu \gamma^5 \Xi^+ D^0 - \Xi^0 D^0$ | 4589.06 | 4560 |
| $P_c(4678)^0$ | $1\over 2$ | $1\over 2$ | $\gamma^\nu \gamma^5 \Omega^0 D^0$ | 4702.05 | 4678 |
| $P_c(4678)^+$ | $1\over 2$ | $-1\over 2$ | $\gamma^\nu \gamma^5 \Omega^+ D^0$ | 4705.46 | 4678 |

| Third Family $P_{c3}$ |
|------------------------|
| $P_c(4457)^+$ | $1\over 2$ | $-1\over 2$ | $\gamma^\nu \gamma^5 J^5 - \frac{1}{3} J^3 D^0 - \frac{1}{3} J^3 D^0$ | 4464.23 (4459.75) | 4457.3 ± 0.6\text{[5]} |
| $P_c(4457)^0$ | $1\over 2$ | $1\over 2$ | $-\gamma^\nu \gamma^5 J^5 - \frac{1}{3} J^3 D^0 - \frac{1}{3} J^3 D^0$ | 4460.60 (4463.16) | 4457 |
| $P_c(4575)^+$ | $1\over 2$ | $-1\over 2$ | $\gamma^\nu \gamma^5 \Xi^+ D^0 - \frac{1}{3} J^3 D^0$ | 4584.25 | 4575 |
| $P_c(4575)^0$ | $1\over 2$ | $1\over 2$ | $\gamma^\nu \gamma^5 \Xi^0 D^0$, $\Xi^0 D^0$ | 4587.66 (4585.65) | 4575 |
| $P_c(4575)^+$ | $0\over 2$ | $1\over 2$ | $\Xi^+ D^0 - \Xi^0 D^0$ | 4589.06 | 4575 |
| $P_c(4695)^0$ | $1\over 2$ | $1\over 2$ | $\gamma^\nu \gamma^5 \Omega^0 D^0$ | 4702.05 | 4695 |
| $P_c(4695)^+$ | $1\over 2$ | $-1\over 2$ | $\gamma^\nu \gamma^5 \Omega^+ D^0$ | 4705.46 | 4695 |

We next describe the hadronic molecular structure of the pentaquarks using phenomenological Lagrangians which involve the interpolating currents presented in Table I. Note that the three states $P_c(4312)^+$, $P_c(4440)^+$, and $P_c(4457)^+$ have already been considered in Ref. [35]. For all 24 pentaquark states the corresponding Lagrangians look as

$$
\mathcal{L}_{P_c}(x) = g_{P_c} \vec{P}_{c1}(x) J_{P_c}(x) + g_{P_c} \vec{P}_{c2}(x) J_{P_c}(x) + g_{P_c} \vec{P}_{c3}(x) J_{P_c}(x) + H.c.,
$$

(2)

where $P_{c1}$, $P_{c2}$, and $P_{c3}$, are the hidden charm pentaquark fields belonging to the first, second, and third family, respectively, $J_{P_c}(x)$, $J_{P_c}(x)$, and $J_{P_{c}}^\mu(x)$ are the nonlocal extension of the currents from Table I $g_{P_c}$, ($i = 1, 2, 3$) is the coupling constant. The
nonlocal pentaquark currents are written as (here flavor indices are suppressed)

\[
J_{P_{1s}}(x) = \int d^4y \Phi_{P_{1s}}(y^2) H_1(x + \omega_{D_{1}}y) \bar{D}(x - \omega_{H_{1}}y),
\]

\[
J_{P_{2s}}(x) = \frac{1}{\sqrt{3}} \int d^4y \Phi_{P_{2s}}(y^2) y_\mu y_5 H_2(x + \omega_{D_{2}}y) \bar{D}^{\mu}(x - \omega_{H_{2}}y),
\]

\[
J_{P_3}^\mu(x) = \int d^4y \Phi_{P_3}^\mu(y^2) H_3(x + \omega_{D_{3}}y) \bar{D}^{\mu}(x - \omega_{H_{3}}y).
\]

\(H_i\) denotes a single-charm baryon, \(\Phi_{P_i}(y^2)\) is a phenomenological correlation function describing the distribution of \(H_i \bar{D}(\bar{D}^*)\) in the pentaquark state \(P_{\alpha_1}\), \(\omega_{D_{1}} = M_{H_{1}}/(M_{H_{1}} + M_{D_{1}})\) and \(\omega_{D_{2}} = M_{D_{1}}/(M_{H_{1}} + M_{D_{2}})\) are the mass fractions of the constituent hadrons with \(\omega_{H_{1}} + \omega_{D_{2}} = 1\). Here we include an overall factor \(1/\sqrt{3}\) for the case of the interpolating current of pentaquark \(P_{1s,2}\) in order to have the same results for the couplings of all pentaquarks in heavy quark limit (HQQL), i.e., in the limit when heavy quark mass goes to infinity. To generate ultraviolet-finite Feynman diagrams, the Fourier transform of the correlation function \(\Phi_{P_{1s}}(y^2)\) should vanish sufficiently fast in the ultraviolet region of the Euclidean space. We use the Gaussian form for the correlation function \(\Phi_{P_{1s}}(p_E^2) \approx \exp(-p_E^2/\Lambda_{P_{1s}}^2)\), where \(p_E\) is the Euclidean Jacobi momentum and \(\Lambda_{P_{1s}}\) is a free size parameter.

| Pentaquark | \(I\) | \(J^P\) | Interpolating current \(J_P\) | Threshold (MeV) | Mass (MeV) |
|------------|-------|---------|-----------------------------|-----------------|-------------|
| \(P_1(11080)\) | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(\sqrt{2} \Sigma^0 \bar{B}^0 - \sqrt{2} \Sigma^0 \bar{B}^0\) | 11090.20 (11092.33) | 11080 |
| \(P_1(11080)\) | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(- \sqrt{2} \Sigma^0 \bar{B}^0 + \sqrt{2} \Sigma^0 \bar{B}^0\) | 11094.97 (11092.64) | 11080 |
| \(P_1(11215)\) | \(\frac{3}{2}\) | \(\frac{3}{2}\) | \(\Xi^0 \bar{B}^0\) | 11124.35 | 11125 |
| \(P_1(11215)\) | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(\Xi^0 \bar{B}^0 + \Xi^0 \bar{B}^0\) | 11124.66 (11124.35) | 11125 |
| \(P_1(11215)\) | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(\Xi^0 \bar{B}^0 - \Xi^0 \bar{B}^0\) | 11124.66 (11124.35) | 11125 |
| \(P_1(11135)\) | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(\Omega \bar{B}^*\) | 11135.26 (11137.70) | 11125 |
| \(P_1(11135)\) | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(\Omega \bar{B}^*\) | 11135.26 (11137.70) | 11125 |
| \(P_1(11135)\) | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(\Omega \bar{B}^*\) | 11135.26 (11137.70) | 11125 |
| \(P_1(11135)\) | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(\Omega \bar{B}^*\) | 11135.26 (11137.70) | 11125 |
| \(P_1(11135)\) | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(\Omega \bar{B}^*\) | 11135.26 (11137.70) | 11125 |
| \(P_1(11135)\) | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(\Omega \bar{B}^*\) | 11135.26 (11137.70) | 11125 |

| Pentaquark | \(I\) | \(J^P\) | Interpolating current \(J_P\) | Threshold (MeV) | Mass (MeV) |
|------------|-------|---------|-----------------------------|-----------------|-------------|
| \(P_1(11130)\) | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(\sqrt{2} \Sigma^0 \bar{B}^0 - \sqrt{2} \Sigma^0 \bar{B}^0\) | 11090.20 (11092.33) | 11080 |
| \(P_1(11130)\) | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(- \sqrt{2} \Sigma^0 \bar{B}^0 + \sqrt{2} \Sigma^0 \bar{B}^0\) | 11094.97 (11092.64) | 11080 |
| \(P_1(11215)\) | \(\frac{3}{2}\) | \(\frac{3}{2}\) | \(\Xi^0 \bar{B}^0\) | 11124.35 | 11125 |
| \(P_1(11215)\) | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(\Xi^0 \bar{B}^0 + \Xi^0 \bar{B}^0\) | 11124.66 (11124.35) | 11125 |
| \(P_1(11215)\) | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(\Xi^0 \bar{B}^0 - \Xi^0 \bar{B}^0\) | 11124.66 (11124.35) | 11125 |
| \(P_1(11215)\) | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(\Omega \bar{B}^*\) | 11135.26 (11137.70) | 11125 |
| \(P_1(11215)\) | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(\Omega \bar{B}^*\) | 11135.26 (11137.70) | 11125 |
| \(P_1(11135)\) | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(\Omega \bar{B}^*\) | 11135.26 (11137.70) | 11125 |
| \(P_1(11135)\) | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(\Omega \bar{B}^*\) | 11135.26 (11137.70) | 11125 |
| \(P_1(11135)\) | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(\Omega \bar{B}^*\) | 11135.26 (11137.70) | 11125 |
| \(P_1(11135)\) | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(\Omega \bar{B}^*\) | 11135.26 (11137.70) | 11125 |
The coupling $g_{P_c}$ is determined from the compositeness condition (see Refs. 44–47 and 48–57)

$$Z_{P_{c1/2}} = 1 - \Sigma_{P_{c1/2}}(M_{P_{c1/2}}) \equiv 0,$$

$$Z_{P_{c3}} = 1 - \Sigma_{P_{c3}}(M_{P_{c3}}) \equiv 0,$$

where $\Sigma_{P_{c1/2}}$ and $\Sigma_{P_{c3}}$ are the derivatives of the full and the transverse part of the mass operator of the $P_{c1/2}$ and $P_{c3}$ states, respectively. Here we have

$$\Sigma^\mu_{P_{c3}}(p) = g_{P_{c3}}^\mu(p) \Sigma^T_{P_{c3}}(p) + \frac{p^\mu p^\nu}{p^2} \Sigma^\nu_{P_{c3}}(p),$$

where $g_{P_{c3}}^\mu(p) = g^\mu_{P_{c3}} - p^\mu p^\nu / p^2$. The generic Feynman diagram representing the mass operator $\Sigma_{P_{c3}}$, which is generated by loop containing the $(H,D)$ or $(H,\bar{D}^*)$ constituents, is shown in Fig. 1 (left panel). Note that the compositeness condition gives a relation between the coupling constant $g_{P_{c3}}$ and the mass $m_{P_{c3}}$.

The extension to the bottom sector is straightforward. Now we have to construct phenomenological Lagrangians describing the coupling of the hidden bottom pentaquark to its constituents by the following replacements in Eqs. (4)-(5):

$$P_{c3} \rightarrow P_{b0}, \quad D(D^*) \rightarrow B(B^*), \quad \{\Sigma_1, \Xi_2, \Omega_3\} \rightarrow \{\Sigma_b, \Xi_b, \Omega_b\}.$$  

The corresponding Feynman diagram for the mass operator $\Sigma_{P_{b0}}$, which is generated by loop containing the $(H_bB)$ or $(H_bB^*)$ constituents, is shown in Fig. 1 (right panel).

The classification of the hidden bottom pentaquarks composed of single bottom baryons and $B(B^*)$ mesons is presented in Table I. For an estimate of their masses we use some typical values close to the corresponding thresholds.

The expressions for the mass operators $\Sigma_{P_0}$ ($Q = c, b$) are given by

$$\Sigma_{P_0}(p) = g_{P_0}^2 \int \frac{d^4k}{(2\pi)^4} \tilde{\Phi}_{P_0}(k) \left( - (k + p\omega)^2 \right) S_{H_0}(k + p) S_P(k),$$  

$$\Sigma_{P_0}(p) = g_{P_0}^2 \int \frac{d^4k}{(2\pi)^4} \tilde{\Phi}_{P_0}(k) \left( - (k + p\omega)^2 \right) \gamma_\alpha \gamma_5 S_{H_0}(k + p) \gamma_\alpha \gamma_5 S_{\nu}^\alpha(k),$$  

$$\Sigma_{P_0}(p) = - \frac{1}{3} g_{\omega \nu}(p) S_{P_0}^2 \int \frac{d^4k}{(2\pi)^4} \tilde{\Phi}_{P_0}(k) \left( - (k + p\omega)^2 \right) S_{H_0}(k + p) S_{\nu}^\alpha(k),$$

where

$$S_{H_0}(k) = \frac{1}{M_{H_0} - \not{k}}, \quad S_P(k) = \frac{1}{M_{P} - k^2}, \quad S_{\nu}^\alpha(k) = - \frac{g_{\omega \nu}^\alpha(k)}{M_{\nu}^2 - k^2}$$

are the propagators of the spin-$\frac{1}{2}$ baryon $H_0$, $P = D(B)$, and $V = D^*(B^*)$ mesons, respectively.

The leading diagrams contributing to the strong two-body decays $P_c \rightarrow HJ(\phi(\eta_c))$ and $P_b \rightarrow H\Upsilon(\eta_b)$, are shown in Fig. 2. Here $H$ is a light baryon octet state corresponding to the light quark flavor content of the decaying pentaquark. The two-body decays $P_c \rightarrow HJ(\phi(\eta_c))$ and $P_b \rightarrow H\Upsilon(\eta_b)$ proceed via the hadronic loops $H\bar{D}(\bar{D}^*)$ and $HB(B^*)$, composed of the constituents contained in the hidden heavy pentaquark state. To evaluate the diagrams in Fig. 2 we need to set up an interaction Lagrangian, which includes the coupling of the pentaquark constituents to the final hadrons $(H + M)$ pair, where $M = J/\psi(\eta_c, \Upsilon, \eta_b)$. We need to specify three types of interaction Lagrangians: (1) $\mathcal{L}_{H,H,P^V}$ – the coupling of heavy-light $H_i = H_c, H_b$ and light $H_f = H$ baryons to pseudoscalar $P = D, B, \eta_c, \eta_b$ and vector $V = D^*, B^*, J/\psi, \Upsilon$ mesons, (2) $\mathcal{L}_{H,H,P,P}$ – the coupling of $H_i$ and $H_f$ to two pseudoscalar mesons $P_1 = D, B$ and $P_2 = \eta_c, \eta_b$, (3) $\mathcal{L}_{H,H,V,V}$ – the coupling of $H_i$ and $H_f$ to two vector mesons $V_1 = D^*, B^*$ and $V_2 = J/\psi, \Upsilon$. 

![Figure 1: Diagrams representing the mass operator of the hidden charm $P_c$ and bottom $P_b$ pentaquarks.](image-url)
The $SU(4)$ symmetric form of the first phenomenological Lagrangian, $\mathcal{L}_{H,H,\rho \nu}$ was originally derived in Refs. [53, 54] and extensively employed in our formalism in Refs. [53, 54, 58]. Here we extend this Lagrangian to five flavors (we only display the part of the Lagrangian which contributes to current processes) with

$$\mathcal{L} = g H^{k[m} \gamma^5 [V_{\mu}, P]^{l]} H^{ln} + \text{H.c.}$$

(14)

The commutator of vector and pseudoscalar mesons is given by $[V_\mu, P] = V_\mu P - PV_\mu$, $H^{k[m}$ is the baryon field and $(k, m, n, l = 1, \ldots, 5)$ is the set of $SU(5)$ flavor indices. The effective coupling

$$g = -\frac{g_V}{f_P \sqrt{2}}$$

(15)

was already fixed in Refs. [53], where $g_V = g_{\rho NN} = 4.8$ [59] is the strong $\rho NN$ coupling constant and $f_P$ is the pseudoscalar decay constant, which for the charm and bottom sectors is identified with $f_{\eta_c}$ and $f_{\eta_b}$. The leptonic decay constants $f_{\eta_c}$ and $f_{\eta_b}$ have been evaluated in Lattice QCD [60]: $f_{\eta_c} = 438 \pm 5 \pm 6$ MeV and $f_{\eta_b} = 801 \pm 7 \pm 5$ MeV and in several phenomenological approaches (see, e.g., Refs. [59, 61]). In particular, in Ref. [61] the leptonic decay constants of heavy quarkonia have been predicted using the Royen-Weisskopf formula: $f_{\eta_c} = 420 \pm 52$ MeV and $f_{\eta_b} = 705 \pm 27$ MeV. In Ref. [59] these couplings have been estimated using the covariant confined quark model: $f_{\eta_c} = 427$ MeV and $f_{\eta_b} = 772$ MeV. In our calculations we will use averaged values of the three sources: $f_{\eta_c} = 430$ MeV and $f_{\eta_b} = 750$ MeV.

The expression of the physical meson and baryon states in terms of the tensors $V^{mn}$, $P^{mn}$, and $H^{mnk}$ are discussed in detail in Refs. [53, 54, 58]. Here we present some examples:

$$p = H_{11} = -2H_{12}, \quad n = H_{22} = -2H_{21}, \quad \Sigma^+_c = H_{14} = -2H_{41}, \quad \Sigma^+_b = H_{15} = -2H_{51},$$

$$J/\psi = V^{44}, \quad \Upsilon = V^{55}, \quad \eta_c = P^{44}, \quad \eta_b = P^{55},$$

$$\bar{D}^0 = P^{14}, \quad D^- = P^{24}, \quad \bar{D}^{*0} = V^{14}, \quad D^{*+} = V^{24},$$

$$B^{*+} = P^{51}, \quad B^0 = P^{52}, \quad B^{*0} = V^{51}, \quad B^{0*} = V^{52}.$$  

(16)

Part of this Lagrangian involving the coupling of the light-heavy light baryon pair to $DJ/\psi$, $D^*\eta_c$, $B\Upsilon$, and $B^*\eta_b$ pairs needed for our calculation reads

$$\mathcal{L} = g \left[ M_c^{\mu} J_{\mu}^{+} + \bar{M}_c^{\mu} J_{\mu}^{-} + M_b^{\mu} J_{\mu}^{+} + \bar{M}_b^{\mu} J_{\mu}^{-} \right] + \text{H.c.}.$$  

(17)

Here we introduce the notations

$$M_c^{\mu} = \eta_c D^{-}_\mu - J/\psi_{\mu} D^-, \quad \bar{M}_c^{\mu} = \eta_c \bar{D}^{*0}_\mu - J/\psi_{\mu} \bar{D}^0,$$

$$M_b^{\mu} = \eta_b B_{\mu}^{*+} - \Upsilon_{\mu} B^{*+}, \quad \bar{M}_b^{\mu} = \eta_b B_{\mu}^{0*} - \Upsilon_{\mu} B^{0*}.$$  

(18)

FIG. 2: Diagrams describing strong two-body decays $P_c \rightarrow HJ/\psi(\eta_c)$ and $P_b \rightarrow H\Upsilon(\eta_b)$, where $H$ represents a light baryon octet state.
and \( J_{\mu}^{c+/b-}, J_{\mu}^{0/b0} \) are the charged and neutral axial-vector currents composed of charm (bottom) and light baryon as

\[
J_{\mu}^{c+} = \frac{1}{4} \left[ \bar{p} \gamma_{\mu} \gamma_5 S_{c}^{++} - \frac{1}{\sqrt{2}} \bar{p} \gamma_{\mu} \gamma_5 S_{c}^{+} - \frac{1}{\sqrt{2}} \bar{p} \gamma_{\mu} \gamma_5 S_{c}^{-} - \frac{1}{2} \bar{p} \gamma_{\mu} \gamma_5 S_{c}^{0} + \frac{\sqrt{3}}{2} \bar{p} \gamma_{\mu} \gamma_5 S_{c}^{-} \right],
\]

\[
J_{\mu}^{c0} = \frac{1}{4} \left[ \bar{p} \gamma_{\mu} \gamma_5 S_{c}^{00} - \frac{1}{2} \bar{p} \gamma_{\mu} \gamma_5 S_{c}^{0+} - \frac{1}{2} \bar{p} \gamma_{\mu} \gamma_5 S_{c}^{0-} - \frac{1}{2} \bar{p} \gamma_{\mu} \gamma_5 S_{c}^{00} + \frac{\sqrt{3}}{2} \bar{p} \gamma_{\mu} \gamma_5 S_{c}^{0-} \right],
\]

\[
J_{\mu}^{b0} = \frac{1}{4} \left[ \bar{p} \gamma_{\mu} \gamma_5 S_{b}^{00} - \frac{1}{2} \bar{p} \gamma_{\mu} \gamma_5 S_{b}^{0+} - \frac{1}{2} \bar{p} \gamma_{\mu} \gamma_5 S_{b}^{0-} - \frac{1}{2} \bar{p} \gamma_{\mu} \gamma_5 S_{b}^{00} + \frac{\sqrt{3}}{2} \bar{p} \gamma_{\mu} \gamma_5 S_{b}^{0-} \right].
\]

Using Eqs. (19) and (20) we derive the couplings \( g_{H\rho J/\phi} = -g_{H\eta H^o} = c_{HH\rho} \) and \( g_{H\rho H^o} = -g_{H\eta H^o} = c_{HH\eta} \), where \( c_{HH\rho} \) and \( c_{HH\eta} \) are flavor factors shown in Table III. Note that above results for the couplings are consistent with recent predictions derived on the basis of heavy-flavor and spin symmetry in Refs. [40, 41]. We also derive the relations between couplings involving charm and bottom hadrons:

\[
\frac{g_{H\rho J/\phi}}{g_{H\rho H^o}} = \frac{g_{H\rho H^o}}{g_{H\rho H^o}} = \frac{c_{HH\rho}}{c_{HH\eta}}.
\]

In the next step we introduce the couplings of two pseudoscalar and two vector mesons to a baryon pair implementing the consequences of heavy-flavor and spin symmetry. We are therefore consistent with the results of Refs. [40, 41]. In this vein we derive the interaction Lagrangian required for the description of the decays of hidden flavor pentaquarks to \( J/\psi(\eta_c) + H \) and \( \Upsilon(\eta_c) + H \) pairs. The relevant Lagrangian reads:

\[
\mathcal{L}_{\text{int}} = g_{cHH} \bar{H} \left[ 5 J/\psi \mu + \eta \gamma^\mu \gamma^5 \right] D_\mu H_c + \left[ J/\psi \mu \gamma^\mu \gamma^5 - 3 \eta \gamma^\mu \gamma^5 \right] H_c
\]

\[
+ g_{cHH} \bar{H} \left[ 5 \Upsilon \gamma^\mu + \eta \gamma^\mu \gamma^5 \right] B_\mu + \left[ \Upsilon \gamma^\mu \gamma^5 - 3 \eta \gamma^\mu \gamma^5 \right] B_\mu + H.c.
\]

Note that again the coupling \( g \) is fixed [see Eq. (15)] and expressed in terms of well-known couplings/parameters. The \( P_{Q3} \) pentaquark cannot decay into \( \eta \) in an S-wave as was stressed in Ref. [40], while it can proceed in a D wave. We introduce an additional D-wave coupling and specify it by introducing an additional parameter \( \beta \) (for convenience we scale it by a factor \( M_{P_{Q3}} \)):

\[
\mathcal{L}_{\text{int}}^{D\text{-wave}} = \frac{\beta}{M_{P_{Q3}}} g_{cHH} \bar{H} \gamma^\mu \eta \gamma^5 \gamma^\nu D_\mu H_c + \frac{\beta}{M_{P_{Q3}}} g_{cHH} \bar{H} \gamma^\mu \eta \gamma^5 \gamma^\nu B_\mu H_c + H.c.
\]

Such an additional coupling leads to a suppression of the decay rates for \( P_{Q3} \to \eta \) by a factor \( 1/m_Q \) in comparison to the modes \( P_{Q1} \to \eta \) and \( P_{Q2} \to \eta \). It also gives a contribution to the matrix element of the \( P_{Q3} \to \eta \) transition at next-to-leading order in the heavy quark mass expansion, and, therefore, will be suppressed in comparison with the S-wave.

In the calculation of the two-body decays \( P_{ci} \to H + M (M = V, P \text{ and } V = J/\psi, \Upsilon ; P = \eta_c, \eta_b) \) of pentaquark \( P_{ci} \) with spin \( S_{P_c} \), we use the rest frame of the pentaquark with the final baryon moving in the positive z-direction. The 4-momenta of pentaquark \( (p_1) \), final baryon \( (p_2) \), and meson \( (p_3) \) are specified as

\[
p_1 = (M_{P_{ci}}, 0, 0), \quad p_2 = (E_{H_c}, 0, 0, |p_2|), \quad q = p_1 - p_2 = (E_{M_{H}}, 0, 0, -|p_2|).
\]

We will also use the following notations: \( M_{\pm} = M_{P_{ci}} \pm M_M, Q_{\pm} = M_{+}^2 - M_{-}^2 \) and \( \lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz \) is the Källen kinematical triangle function. Energy values and three-momenta of the decay products are defined as

\[
E_H = \frac{M_{P_{ci}}^2 + \lambda^2 - M_H^2}{2M_{P_{ci}}},
\]

\[
E_M = M_{P_{ci}} - E_H = \frac{M_{P_{ci}}^2 - \lambda^2 - M_M^2}{2M_{P_{ci}}},
\]

\[
|p_2| = \frac{\sqrt{Q_c Q_b}}{2M_{P_{ci}}},
\]

Due to Lorentz covariance and because of the transversity condition \( q_{\mu} E^\mu_{\nu} = 0 \) for the polarization vector of the \( V = J/\psi, \Upsilon \) mesons, the matrix elements of the \( P_{ci} \to H + V \) decay processes are in general expressed in terms of two form factors...
TABLE III: Flavor factors $c_{HH_i}$ and $c_{HH_b}$

| Flavor structure $HH_i$ | $c_{HH_i}$ | Flavor structure $HH_b$ | $c_{HH_b}$ |
|------------------------|------------|------------------------|------------|
| $p^{\Sigma^+_i}$      | $\frac{1}{2}$ | $p^{\Sigma^+_j}$      | $\frac{1}{2}$ |
| $n^{\Sigma^-}$        | $\frac{1}{2}$ | $n^{\Sigma^-}$        | $\frac{1}{2}$ |
| $p^{\Xi^-}$           | $\frac{1}{4}$ | $p^{\Xi^-}$           | $\frac{1}{4}$ |
| $n^{\Xi^-}$           | $\frac{1}{4}$ | $n^{\Xi^-}$           | $\frac{1}{4}$ |
| $\Sigma^+ \Xi^+_i$    | $\frac{1}{4}$ | $\Sigma^+ \Xi^+_j$    | $\frac{1}{4}$ |
| $\Sigma^0 \Xi^0$      | $\frac{1}{4}$ | $\Sigma^0 \Xi^0$      | $\frac{1}{4}$ |
| $\Sigma^- \Xi^-i$     | $\frac{1}{4}$ | $\Sigma^- \Xi^-j$     | $\frac{1}{4}$ |
| $\Xi^- \Omega^-i$     | $\frac{1}{4}$ | $\Xi^- \Omega^-j$     | $\frac{1}{4}$ |
| $\Xi^0 \Omega^0$      | $\frac{1}{4}$ | $\Xi^0 \Omega^0$      | $\frac{1}{4}$ |

TABLE IV: Flavor factors $d_H$

| Light baryon $H$ | $d_H$ |
|-----------------|-------|
| $p$             | $\frac{3}{4}\sqrt{6}$ |
| $n$             | $-\frac{3}{4}\sqrt{6}$ |
| $\Sigma^+$      | $\frac{1}{4}\sqrt{2}$ |
| $\Sigma^0$      | $-\frac{1}{4}\sqrt{2}$ |
| $\Sigma^-$      | $-\frac{1}{4}\sqrt{2}$ |
| $\Lambda^0$     | $-\frac{1}{4}$ |
| $\Xi^0$         | $\frac{1}{4}$ |
| $\Xi^-$         | $\frac{1}{4}$ |

$(F_i^V, i = 1, 2)$ for the $\frac{1}{2}^- \to \frac{1}{2}^+ + 1^-$ transitions and three $(F_i^V, i = 1, 2, 3)$ for the $\frac{3}{2}^- \to \frac{1}{2}^+ + 1^-$ transitions (see details in Refs. [43, 62]):

\[
M_{inv}(\frac{1}{2}^- \to \frac{1}{2}^+ + 1^-) = \bar{u}(p_2, s_2) \left[ \gamma_\mu F_1^V(M_2^2) - i\sigma_{\nu\mu} \frac{q_\nu}{M_{P_i}} F_2^V(M_2^2) \right] \gamma_\mu u(p_1, s_1) \epsilon_\nu^\mu(q),
\]  
(29)

transition $\frac{1}{2}^- \to \frac{1}{2}^+ + 1^-$:

\[
M_{inv}(\frac{3}{2}^- \to \frac{3}{2}^+ + 1^-) = \bar{u}(p_2, s_2) \left[ g_{\mu\nu} F_1^V(M_2^2) + \gamma_\mu \frac{P_{2\alpha}}{M_{P_i}} F_2^V(M_2^2) + \frac{P_{2\alpha}P_{2\mu}}{M_{P_i}} F_3^V(M_2^2) \right] \gamma_\mu u(p_1, s_1) \epsilon_\nu^\mu(q),
\]  
(30)

where $\sigma_{\nu\mu} = (i/2)(\gamma_\nu\gamma_\mu - \gamma_\nu\gamma_\mu)$ and all $\gamma$ matrices are defined as in the Bjorken-Drell convention. In our approach only one form factor $F_i^V$ contributes to the transitions $P_{c_1} \to H + V$ and $P_{c_2} \to H + V$, while the others vanish.

For the decay modes involving $P = \eta, \eta_b$ in the final state the matrix elements are expressed in terms of a single pseudoscalar form factor $F^P(M_P^2)$:

\[
M_{inv}(\frac{1}{2}^- \to \frac{1}{2}^+ + 0^-) = \bar{u}(p_2, s_2) F^p(M_P^2) u(p_1, s_1),
\]  
(31)

transition $\frac{1}{2}^- \to \frac{1}{2}^+ + 0^-$:

\[
M_{inv}(\frac{3}{2}^- \to \frac{3}{2}^+ + 0^-) = \bar{u}(p_2, s_2) F^p(M_P^2) i\gamma_\mu \frac{q_\mu}{M_{P\mu}} u(p_1, s_1).
\]
It is convenient to express the decay widths of the two-body decays \( P_{ci} \to H + V \) in terms of the helicity amplitudes \( H_{\lambda_{1,2},\lambda_{V}} \) \([43, 62]\), where \( \lambda_{V} = \pm 1, 0 \) and \( \lambda_{2} = \pm 1/2, \pm 3/2 \) are the helicity components of the \( V \) mesons and the final baryon \( H \), respectively. For our kinematics, helicity conservation reads: \( \lambda_{1} = \lambda_{2} - \lambda_{V} \), where \( \lambda_{1} \) is the helicity of decaying pentaquark. The helicity amplitudes are related to the sets of the previously introduced relativistic form factors \( F_{V}^{c} \) as \([43, 62]\):

transition \( \frac{1}{2}^{-} \to \frac{1}{2}^{-} + 1^{-} : H_{-\lambda_{z},-\lambda_{V}}^{V} = -H_{\lambda_{z},\lambda_{V}}^{V} \)

\[
H_{\frac{3}{2}^{-0}}^{V} = \sqrt{Q_{+}/M_{V}^{2}} \left( F_{1}^{V} \frac{M_{P} - M_{V}}{M_{P}} - F_{2}^{V} \frac{M_{P}}{M_{P}} \right),
\]

\[
H_{\frac{1}{2}^{-1}}^{V} = \sqrt{2Q_{+}Q_{-}} \left( -F_{1}^{V} + F_{2}^{V} \frac{M_{P} - M_{V}}{M_{P}} \right),
\]

(32)

transition \( \frac{1}{2}^{-} \to \frac{1}{2}^{-} + 0^{-} : H_{-\lambda_{z},\lambda_{V}}^{P} = -H_{\lambda_{z},\lambda_{V}}^{P} \)

\[
H_{\frac{3}{2}^{-0}}^{P} = \sqrt{Q_{+}} F_{c}^{P},
\]

(33)

transition \( \frac{1}{2}^{-} \to \frac{1}{2}^{-} + 1^{-} : H_{-\lambda_{z},-\lambda_{V}}^{V} = H_{\lambda_{z},\lambda_{V}}^{V} \)

\[
H_{\frac{3}{2}^{-0}}^{V} = \sqrt{Q_{+}Q_{-}3} \left( M_{c} M_{P} + M_{V}^{2} \right),
\]

\[
H_{\frac{1}{2}^{-1}}^{V} = \sqrt{Q_{+}} F_{1}^{V} H_{\frac{1}{2}^{-1}}^{V} = \sqrt{Q_{-}} F_{1}^{V},
\]

(34)

we omit the contribution of \( F_{2}^{V} \) and \( F_{3}^{V} \) since for the present calculation these form factors vanish; transition \( \frac{1}{2}^{-} \to \frac{1}{2}^{-} + 0^{-} : H_{\lambda_{z},\lambda_{V}}^{P} = H_{\lambda_{z},\lambda_{V}}^{P} \)

\[
H_{\frac{3}{2}^{-0}}^{P} = \sqrt{Q_{+}Q_{-}\frac{6}{M_{P}}},
\]

(35)

In the case of the \( \frac{1}{2}^{-} \to \frac{1}{2}^{-} + 0^{-} \) transition the helicity amplitude \( H_{\lambda_{z},\lambda_{V}}^{P} \) has an additional \( 1/m_{Q} \) suppression factor in comparison to the others. As already explained, the reason lies in the \( D \)-wave coupling of this transition.

The decay width of the two-body transition \( P_{ci} \to H + V(P) \) is calculated according to the formulas \([43, 62]\):

\[
\Gamma(P_{ci} \to H + V) = \frac{1}{8\pi(2S_{P_{ci}} + 1)} \frac{|p_{2}|^{2}}{M_{P_{ci}}^{2}} \mathcal{H}_{H_{\lambda_{z},\lambda_{V}}^{H}}, \quad \mathcal{H}_{H_{\lambda_{z},\lambda_{V}}^{H}} = \sum_{\lambda_{z},\lambda_{V}} |H_{\lambda_{z},\lambda_{V}}^{V}|^{2},
\]

\[
\Gamma(P_{ci} \to H + P) = \frac{1}{8\pi(2S_{P_{ci}} + 1)} \frac{|p_{2}|^{2}}{M_{P_{ci}}^{2}} \mathcal{H}_{H_{\lambda_{z},\lambda_{V}}^{P}}, \quad \mathcal{H}_{H_{\lambda_{z},\lambda_{V}}^{P}} = \sum_{\lambda_{z},\lambda_{V}} |H_{\lambda_{z},\lambda_{V}}^{P}|^{2},
\]

(36)

where \( S_{P_{ci}} = S_{P_{ci}} = 1/2 \) and \( S_{P_{ci}} = 1/2 \) are the spins of the pentaquarks, and \( \mathcal{H}_{H_{\lambda_{z},\lambda_{V}}^{P}} \) is the sum of the corresponding squared helicity amplitudes with:

\[
\mathcal{H}_{H_{\lambda_{z},\lambda_{V}}^{P}} = \frac{|H_{\lambda_{z}}^{0}|^{2}}{2} + |H_{\lambda_{z}}^{-40}|^{2} + |H_{\lambda_{z}}^{-42}|^{2} + |H_{\lambda_{z}}^{-4-1}|^{2},
\]

\[
\mathcal{H}_{H_{\lambda_{z},\lambda_{V}}^{H}} = |H_{\lambda_{z}}^{0}|^{2} + |H_{\lambda_{z}}^{-40}|^{2} + |H_{\lambda_{z}}^{-42}|^{2} + |H_{\lambda_{z}}^{-4-1}|^{2} + |H_{\lambda_{z}}^{-41}|^{2} + |H_{\lambda_{z}}^{-4-1}|^{2},
\]

\[
\mathcal{H}_{H_{\lambda_{z},\lambda_{V}}^{P}} = |H_{\lambda_{z}}^{P}|^{2} + |H_{\lambda_{z}}^{-4P}|^{2}. \quad (37)
\]

### III. RESULTS

We proceed with our calculation in the heavy quark limit (HQL) expanding the masses of the heavy hadrons around the corresponding heavy quark masses \( m_{Q}, Q = c, b \):

\[
M_{P_{ci}} = 2m_{Q} + O(1), \quad M_{J/\psi, \eta} = 2m_{c} + O(1), \quad M_{\Upsilon, \eta_{b}} = 2m_{b} + O(1),
\]

\[
M_{D,P} = m_{c} + O(1), \quad M_{B, P} = m_{b} + O(1),
\]

\[
M_{H_{c}, H_{b}} = m_{Q} + O(1), \quad M_{H_{c}, H_{b}} = O(1) \quad (38)
\]

(39)
We also introduce the heavy-flavor independent quantity $R$, the difference between the sum of the masses of the constituents and the pentaquark mass, with:

$$R = M_{H_0} + M_{M_0} - M_{P_0}$$

which at leading order is considered to be universal for all modes.

In the HQL the results become rather transparent. In particular, the coupling constants of pentaquarks to the constituents scale as $\sqrt{m_Q}$ and are given by

$$g_{P_{0i}} = g_{P_{02}} = g_{P_{03}} = 4\pi \sqrt{\frac{2m_Q}{\Lambda}} [I(r)]^{-1/2},$$

where $I(r)$, $r = R/\Lambda$ is the structure integral

$$I(r) = \int_0^\infty da_1 \int_0^\infty da_2 (a_1 + a_2) \exp\left[-\frac{1}{2}(a_1 - a_2)^2 - \frac{1}{2}(a_1 + a_2)r\right].$$

The couplings for all three types of pentaquarks are degenerate in the HQL. For these reasons we introduced an additional factor $1/\sqrt{3}$ in the interpolating current of the pentaquark $P_{Q3}$ [see Eq. (3)].

The transition amplitudes in the HQL are also given in terms of a single structure integral

$$J(r) = \int_0^\infty da_1 \int_0^\infty da_2 \exp\left[-(a_1 - a_2)^2 - \frac{1}{2}(a_1 + a_2)r\right].$$

Therefore, all helicity amplitudes only depend on the function $\xi(r) = J(r)/\sqrt{I(r)}$ involving the single parameter $r = R/\Lambda$. We find that $\xi(r)$ changes only slightly when varying the size parameter $\Lambda$ in the region 0.5-2 GeV. In particular, for $R \approx 10$ MeV and $\Lambda$ ranging from 0.5 to 2 GeV the quantity $\xi(r)$ changes from 0.7872 to 0.7905.

In the HQL results for the squared helicity amplitudes are simply expressed as:

$$H_{P_{0i}}^\nu = \frac{3}{25} H_{P_{02}}^\nu = \frac{3}{50} H_{P_{03}}^\nu = \frac{1}{5} H_{P_{01}}^\nu = H_{P_{02}}^\nu = 12 \left(\frac{g d H}{\pi}\right)^2 \Lambda^4 \xi^2(r) \left(r + \mu_H\right),$$

$$H_{P_{0i}}^p = \beta^2 \frac{m_H}{54 m_Q} H_{P_{01}}^\nu,$$

where $\mu_H = M_H/\Lambda$ is the mass of the light baryon in the final state rescaled by $\Lambda$. The flavor factors $d_H$ depend on the flavor of the final baryon and are summarized in Table IV.

In the HQL the helicity amplitudes $H_{P_{0i}}^\nu$ and $H_{P_{0i}}^p$ scale as $O(1/m_Q^2)$ since the heavy quarkonia decay constants, contained in the definition of the coupling $g$, have the scaling behavior $f_{H_{0i}} \sim \sqrt{m_Q}$. Therefore, the decay rates of pentaquarks into heavy charmonia and a light baryon scale as $1/m_Q^3$ (we take into account that $|p_i|$ behaves as $O(1)$ in the HQL). All relations between the helicity amplitudes (further below it will also be displayed for the decay rates) are consistent with the results reported previously in Refs. [40, 41]. In our approach the $P_{Q3}$ pentaquark decay into $\eta_Q H$ is restricted to an orbital $D$-wave, the $S$-wave is forbidden. The helicity amplitude for the decay $P_{Q3} \to \eta_Q H$ ($D$-wave) contains an arbitrary parameter $\beta$, which could be fixed from future experimental results. However $H_P^{\nu}$ has an additional suppression factor $1/m_Q$ in comparison with the other helicity amplitudes listed in Eq. (44) and the corresponding decay rate scales as $1/m_Q^4$, i.e. it is suppressed by a factor $1/m_Q$ relative to the other modes.

Neglecting the mass differences of $J/\psi - \eta_c$ and $\Upsilon - \eta_b$ we finally derive following relations for the pentaquark decay rates:

$$\Gamma(P_{ci} \to \eta_c p)/\Gamma(P_{ci} \to J/\psi p) = 3,$$

$$\Gamma(P_{ci} \to \eta_b p)/\Gamma(P_{ci} \to J/\psi p) = \frac{3}{25},$$

$$\Gamma(P_{ci} \to \eta_b p)/\Gamma(P_{ci} \to \Upsilon p) = \frac{3}{25},$$

$$\Gamma(P_{ci} \to \eta_c p)/\Gamma(P_{ci} \to \Upsilon p) = \left(\frac{m_b}{m_c}\right)^3, \quad i = 1, 2, 3.$$
and $\beta$ as

$$
\Gamma(P_{c1}^+ \rightarrow J/\psi p) = 4.12 \text{ MeV} \cdot \left(\frac{\Lambda}{1 \text{ GeV}}\right)^3,
$$
$$
\Gamma(P_{c2}^+ \rightarrow J/\psi n) = 39.82 \text{ MeV} \cdot \left(\frac{\Lambda}{1 \text{ GeV}}\right)^3,
$$
$$
\Gamma(P_{c3}^+ \rightarrow J/\psi \Sigma^+) = 80.04 \text{ MeV} \cdot \left(\frac{\Lambda}{1 \text{ GeV}}\right)^3,
$$
$$
\Gamma(P_{c1}^+ \rightarrow \eta_c p) = 14.81 \text{ MeV} \cdot \left(\frac{\Lambda}{1 \text{ GeV}}\right)^3,
$$
$$
\Gamma(P_{c2}^+ \rightarrow \eta_c n) = 5.45 \text{ MeV} \cdot \left(\frac{\Lambda}{1 \text{ GeV}}\right)^3,
$$
$$
\Gamma(P_{c3}^+ \rightarrow \eta_c \Sigma^+) = 0.13 \beta^2 \cdot \left(\frac{\Lambda}{1 \text{ GeV}}\right)^3.
$$

In Tables \(V\) and \(V\) we present the results for other decay modes. For many of the indicated states we use typical values for the masses. In all results we drop the factor \(\left(\Lambda/1 \text{ GeV}\right)^3\).

**TABLE V:** Two-body decay rates of hidden charm pentaquarks in MeV [results for decay rates should be multiplied by factor \(\left(\Lambda/1 \text{ GeV}\right)^3\)]

| Mode | Decay rate (MeV) | Mode | Decay rate (MeV) |
|------|------------------|------|------------------|
| $P_{c1}^+ \rightarrow J/\psi p$ | 4.12 | $P_{c1}^+ \rightarrow \eta_c p$ | 14.81 |
| $P_{c2}^+ \rightarrow J/\psi n$ | 4.11 | $P_{c2}^+ \rightarrow \eta_c n$ | 14.78 |
| $P_{c3}^+ \rightarrow J/\psi \Sigma^+$ | 1.01 | $P_{c3}^+ \rightarrow \eta_c \Sigma^+$ | 4.06 |
| $P_{c1}^+ \rightarrow J/\psi \Sigma^+$ | 1.00 | $P_{c2}^+ \rightarrow \eta_c \Sigma^+$ | 4.04 |
| $P_{c3}^+ \rightarrow J/\psi \Sigma^+$ | 0.98 | $P_{c3}^+ \rightarrow \eta_c \Sigma^+$ | 4.00 |
| $P_{c1}^+ \rightarrow J/\psi \Lambda^0$ | 7.13 | $P_{c2}^+ \rightarrow \eta_c \Lambda^0$ | 26.76 |
| $P_{c3}^+ \rightarrow J/\psi \Xi^0$ | 1.94 | $P_{c2}^+ \rightarrow \eta_c \Xi^0$ | 7.85 |
| $P_{c3}^+ \rightarrow J/\psi \Xi^+$ | 1.90 | $P_{c3}^+ \rightarrow \eta_c \Xi^+$ | 7.75 |
| $P_{c2}^+ \rightarrow J/\psi p$ | 39.82 | $P_{c1}^+ \rightarrow \eta_c p$ | 5.45 |
| $P_{c2}^+ \rightarrow J/\psi n$ | 39.77 | $P_{c2}^+ \rightarrow \eta_c n$ | 5.45 |
| $P_{c3}^+ \rightarrow J/\psi \Sigma^+$ | 1.09 | $P_{c3}^+ \rightarrow \eta_c \Sigma^+$ | 1.58 |
| $P_{c3}^+ \rightarrow J/\psi \Sigma^0$ | 1.03 | $P_{c3}^+ \rightarrow \eta_c \Sigma^0$ | 1.57 |
| $P_{c3}^+ \rightarrow J/\psi \Sigma^-$ | 1.04 | $P_{c3}^+ \rightarrow \eta_c \Sigma^-$ | 1.56 |
| $P_{c3}^+ \rightarrow J/\psi \Lambda^0$ | 72.09 | $P_{c3}^+ \rightarrow \eta_c \Lambda^0$ | 10.08 |
| $P_{c3}^+ \rightarrow J/\psi \Xi^0$ | 21.25 | $P_{c3}^+ \rightarrow \eta_c \Xi^0$ | 3.06 |
| $P_{c3}^+ \rightarrow J/\psi \Xi^+$ | 20.99 | $P_{c3}^+ \rightarrow \eta_c \Xi^+$ | 3.10 |

We can use the upper limit on the branching ratio of the $P_c(4457)^+$ recently set by the GlueX Collaboration at JLab \(4\) to constrain the size parameter $\Lambda$ of this LHCb pentaquark assuming spin-parity $J^P = \frac{1}{2}^-$ for this state. We do not consider $P_c(4312)^+$ and $P_c(4440)^+$ states in our estimate because they have spin-parity $J^P = \frac{1}{2}^+$ in our case, which is different from the assumption of the GlueX experiment. As pointed out by the GlueX Collaboration \(4\) their upper limits are sensitive to the spin-parity quantum numbers. In particular, the limits become by a factor of 5 smaller for the $J^P = \frac{1}{2}^-$ assignment. For an estimate of the branching of the $P_c(4457)^+$ state we use the central value for the decay width of the recent LHCb data analysis \(3\):

$$
\Gamma_{P_c(4457)^+} = 6.4 \text{ MeV}.
$$

(55)
We find the following constraint: $\Lambda_{P_1^{(4457)^*}} \leq 145$ MeV. Note that the $\Lambda$ parameter characterizes the binding forces acting on a charmed baryon and a meson in the bound state of a hidden charm pentaquark.

TABLE VI: Two-body decay rates of hidden bottom pentaquarks in MeV [results for decay rates should be multiplied by factor $(\Lambda/1 \text{ GeV})^3$]

| Mode | Decay rate (MeV) | Mode | Decay rate (MeV) |
|------|-----------------|------|-----------------|
| $P_{b1}^{\ast} \rightarrow \Upsilon \pi$ | 0.38 | $P_{b1}^{\ast} \rightarrow \eta_8 p$ | 1.20 |
| $P_{b1}^{\ast} \rightarrow \Upsilon \eta$ | 0.38 | $P_{b1}^{\ast} \rightarrow \eta_8 n$ | 1.20 |
| $P_{b1}^{\ast} \rightarrow \Upsilon \chi^+$ | 0.12 | $P_{b1}^{\ast} \rightarrow \eta_8 \chi^+$ | 0.38 |
| $P_{b1}^{\ast} \rightarrow \Upsilon \chi^0$ | 0.12 | $P_{b1}^{\ast} \rightarrow \eta_8 \chi^0$ | 0.38 |
| $P_{b1}^{\ast} \rightarrow \Sigma^++$ | 0.26 | $P_{b1}^{\ast} \rightarrow \eta_0 \Sigma^+$ | 0.38 |
| $P_{b1}^{\ast} \rightarrow \Sigma^0$ | 0.26 | $P_{b1}^{\ast} \rightarrow \eta_0 \Sigma^0$ | 0.38 |
| $P_{b1}^{\ast} \rightarrow \Pi^0$ | 0.24 | $P_{b1}^{\ast} \rightarrow \eta_0 \pi^0$ | 0.76 |
| $P_{b1}^{\ast} \rightarrow \Sigma^0$ | 0.24 | $P_{b1}^{\ast} \rightarrow \eta_0 \pi^0$ | 0.76 |

| Mode | Decay rate (MeV) | Mode | Decay rate (MeV) |
|------|-----------------|------|-----------------|
| $P_{b2}^{\ast} \rightarrow \Upsilon \pi$ | 3.27 | $P_{b2}^{\ast} \rightarrow \eta_8 p$ | 0.41 |
| $P_{b2}^{\ast} \rightarrow \Upsilon \eta$ | 3.27 | $P_{b2}^{\ast} \rightarrow \eta_8 n$ | 0.41 |
| $P_{b2}^{\ast} \rightarrow \Upsilon \chi^+$ | 1.03 | $P_{b2}^{\ast} \rightarrow \eta_8 \chi^+$ | 0.13 |
| $P_{b2}^{\ast} \rightarrow \Upsilon \chi^0$ | 1.03 | $P_{b2}^{\ast} \rightarrow \eta_8 \chi^0$ | 0.13 |
| $P_{b2}^{\ast} \rightarrow \Sigma^++$ | 1.03 | $P_{b2}^{\ast} \rightarrow \eta_0 \Sigma^+$ | 0.13 |
| $P_{b2}^{\ast} \rightarrow \Sigma^0$ | 1.03 | $P_{b2}^{\ast} \rightarrow \eta_0 \Sigma^0$ | 0.13 |
| $P_{b2}^{\ast} \rightarrow \Pi^0$ | 6.41 | $P_{b2}^{\ast} \rightarrow \eta_0 \Lambda^0$ | 0.78 |
| $P_{b2}^{\ast} \rightarrow \Sigma^0$ | 2.07 | $P_{b2}^{\ast} \rightarrow \eta_0 \pi^0$ | 0.26 |
| $P_{b2}^{\ast} \rightarrow \Sigma^0$ | 2.06 | $P_{b2}^{\ast} \rightarrow \eta_0 \pi^0$ | 0.26 |

| Mode | Decay rate (MeV) | Mode | Decay rate (MeV) |
|------|-----------------|------|-----------------|
| $P_{b3}^{\ast} \rightarrow \Upsilon \pi$ | 6.57 | $P_{b3}^{\ast} \rightarrow \eta_8 p$ | 0.004 $\beta^2$ |
| $P_{b3}^{\ast} \rightarrow \Upsilon \eta$ | 6.56 | $P_{b3}^{\ast} \rightarrow \eta_8 n$ | 0.004 $\beta^2$ |
| $P_{b3}^{\ast} \rightarrow \Upsilon \chi^+$ | 2.07 | $P_{b3}^{\ast} \rightarrow \eta_8 \chi^+$ | 0.002 $\beta^2$ |
| $P_{b3}^{\ast} \rightarrow \Upsilon \chi^0$ | 2.07 | $P_{b3}^{\ast} \rightarrow \eta_8 \chi^0$ | 0.002 $\beta^2$ |
| $P_{b3}^{\ast} \rightarrow \Sigma^++$ | 2.06 | $P_{b3}^{\ast} \rightarrow \eta_0 \Sigma^+$ | 0.002 $\beta^2$ |
| $P_{b3}^{\ast} \rightarrow \Sigma^0$ | 12.87 | $P_{b3}^{\ast} \rightarrow \eta_0 \Lambda^0$ | 0.009 $\beta^2$ |
| $P_{b3}^{\ast} \rightarrow \Pi^0$ | 4.15 | $P_{b3}^{\ast} \rightarrow \eta_0 \pi^0$ | 0.003 $\beta^2$ |
| $P_{b3}^{\ast} \rightarrow \Sigma^0$ | 4.13 | $P_{b3}^{\ast} \rightarrow \eta_0 \pi^0$ | 0.003 $\beta^2$ |

In conclusion, we presented a calculation for the strong two-body decays of hidden charm and bottom pentaquarks into pairs of a light baryon and heavy quarkonia. We evaluate the final results in the heavy quark limit using only one free parameter – the scale parameter $\Lambda$ characterizing the binding forces of the hadronic constituents in the hidden heavy pentaquark. Our predictions for the decay rates scale as $\Lambda^3$, therefore the results are very sensitive to the choice of this parameter. Using recent data of the LHCb Collaboration [1], [3] on the total widths of nonstrange hidden charm pentaquarks and of the GlueX Collaboration [4] on the upper limit of the partial two-body decay width of the $P_1^{(4457)^*}$ state with $J^P = \frac{3}{2}^-$ we derive preliminary result for the upper limit for our scale parameter $\Lambda$. Future and more precise experiments on the decay properties of hidden heavy pentaquarks can give strong constraints on the model parameter and insights into the predictions given in the last tables.

Acknowledgments

This work was funded by the Carl Zeiss Foundation under Project “Kepler Center für Astro- und Teilchenphysik: Hochsensitive Nachweistechnik zur Erforschung des unsichtbaren Universums (Gz: 0653-2.8/581/2)”, by CONICYT (Chile) under Grant PIA/Basal FB0821 and by FONDECYT (Chile) under Grant No. 1191103, by Tomsk State University Competitiveness Improvement Program and the Russian Federation program “Nauka” (Contract No. 0.1764.GZB.2017).

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