QCD CORRECTIONS TO
TOPONIUM PRODUCTION
AT HADRON COLLIDERS$^{1}$

J.H. Kühn and E. Mirkes

Institut für Theoretische Teilchenphysik
Universität Karlsruhe
Kaiserstr. 12, Postfach 6980
7500 Karlsruhe 1, Germany

Abstract

Toponium production at future hadron colliders is investigated. Perturbative QCD corrections to the production cross section for gluon fusion are calculated as well as the contributions from gluon-quark and quark-antiquark collisions to the total cross section. The dependence on the renormalization and factorization scales and on the choice of the parton distribution functions is explored. QCD corrections to the branching ratio of $\eta_t$ into $\gamma\gamma$ are included and the two-loop QCD potential is used to predict the wave function at the origin. The branching ratio of $\eta_t$ into $\gamma Z$, $ZZ$, $HZ$ and $WW$ is compared with the $\gamma\gamma$ channel.

$^1$Supported by BMFT Contract 055KA94P
1 Introduction

The production of heavy quarks at hadron colliders has received a lot of attention from the experimental as well as from the theoretical side. Higher order QCD calculations for open top quark production \[1, 2, 3\] provide the theoretical basis for the current experimental limits \[4\] on the mass of the top quark. Given the current range for the top mass of 90 to 180 GeV deduced from electroweak radiative corrections \[5\] ongoing experimental studies at the TEVATRON should be able to discover the top quark in the near future.

Planned experiments at LHC or SSC will lead to a huge number of \(t \bar{t}\) events and may pin down the top quark mass with a remaining uncertainty of perhaps 5 GeV \[6\]. Aside from the intrinsic interest in the precise top mass determination, the aim to fix the input in one-loop electroweak corrections \[7\] constitutes one of the important motivations for the drive for future increased precision in the top mass.

In \[8\] it has been proposed to decrease the experimental error significantly through the study of \(\eta_t\) in its \(\gamma\gamma\) decay channel. Making use of the large luminosity of future hadron colliders, one should be able to overcome the tiny cross section multiplied by the small branching ratio into two photons. The excellent gamma energy resolution, originally developed for the search for a light Higgs boson, would in principle allow the measurement of the mass of the bound state at about 100 MeV accuracy. This would result in a determination of the top quark mass limited only by theoretical uncertainties. It was shown that the toponium signal could be extracted from the \(\gamma\gamma\) background with a signal to noise ratio decreasing from 2/3 down to 1/8 for \(m_t\) between 90 and 120 GeV.

The estimates of \[8\] for the production and decay rate were based on the Born approximation and on fairly crude assumptions on the bound state wave function which was derived from a Coulomb potential, neglecting QCD corrections. The signal to noise ratio and the statistical significance of the signal as estimated in \[8\] is just sufficient for the discovery. Therefore a more precise evaluation of the production rate is mandatory.

In lowest order the reaction is induced by gluon-gluon fusion (fig. 1a). QCD corrections will be calculated in this work. They affect and indeed increase the rate in this channel by about 50%. Quark-gluon scattering and quark-antiquark annihilation into \(\eta_t\) enter at order \(\alpha_s^3\). As we shall see the \(qg\) process increases the production cross section by about 8% whereas the effect of \(qq\) annihilation is negligible.

The production cross section for \(\eta_t\) with nonvanishing transverse momentum is infrared finite in the order considered here. The total cross section, however, exhibits singularities that have to be absorbed in appropriately chosen and defined
parton distribution functions. The dependence on the factorization as well as renormalization scales will be studied as well as the dependence on the choice on different parametrizations of parton distribution functions. A full NLO calculation requires the corresponding corrections for the physically interesting branching ratio. These are available and will be implemented.

The bound state production cross section and the branching ratio of $\eta_t$ into $\gamma\gamma$ depend (markedly!) on $R(0)^2$, the square of the bound state wave function at the origin and hence on the potential. The range of $R(0)$ compatible with the current knowledge of the perturbative two-loop QCD potential and the present experimental results for $\Lambda_{QCD}$ will be explored.

As we shall see in the following, fairly optimistic assumptions are necessary to open even a narrow window for toponium discovery in the mass range of $m_t = 90 - 110$ GeV. This is the consequence of the dominance single quark decays (SQD) and the small branching ratio into two photons. The situation improves dramatically if new hypothetical quarks are considered with suppressed single quark decay. Examples are $b'$ or isosinglet quarks with small mixing with ordinary $d, s$ or $b$ quarks. These would dominantly decay into gauge and Higgs bosons, leading to spectacular signatures.

The outlay of this paper is as follows: In section 2 the QCD corrections to gluon gluon fusion into $\eta_t$ and the cross section for the $q\bar{q}$ and $qG$ initiated processes will be derived. In section 3 the formulae for the decay rates of $\eta_t$ will be listed and the dependence of the wave function on the choice of the potential studied. Section 4 contains the numerical evaluation of the production cross section and the study of scale dependencies. Section 5 contains our conclusions. A brief account of this work has been presented in [9].

## 2 QCD corrections to hadronic $\eta_t$ production

For the complete evaluation of NLO corrections for $\eta_t$ production in gluon gluon fusion virtual corrections (fig. 1b) are required as well as corrections from real gluon radiation (fig. 1c,d). In addition one needs the cross section for quark-antiquark annihilation into $\eta_t$+gluon (fig. 2a) and the (anti-)quark-gluon (fig. 2b) initiated reaction into $\eta_t$+quark. Technically it is quite convenient to employ dimensional regularization for both ultraviolet and infrared singularities. The calculation of amplitudes for bound state production (or decay) can be performed in two different ways. One may either evaluate directly the amplitude for the production of a $(Q\bar{Q})$ bound state with the desired spin and orbital angular momentum configuration and arrive at an amplitude proportional to the wave function at the origin or its derivative \[10, 11\]; or alternatively one may evaluate the production rate for open $Q\bar{Q}$ and subsequently identify the rate at threshold.
with the rate for bound state production, substituting
\[ dP_S(n(k_1+k_2;k_3,\ldots,k_n,q,k_\bar{Q}) \to R(0)^2 \frac{2\pi M}{dP_{S-1}(k_1+k_2;k_3,\ldots,k_n,q) \right) \]

where \( M = 2m_t + E_{\text{Bind}} \) and \( k_Q = k_{\bar{Q}} = q/2 \). For the one particle final state relevant for resonant \( \eta_t \) production in gluon gluon fusion this implies (in \( 4-2\epsilon \) dimensions):
\[ dP_{S2}(k_1+k_2;k_Q,k_{\bar{Q}}) \Rightarrow \frac{R(0)^2}{2\pi M} \frac{1}{2\pi\delta(s-M^2)} \]
and for the \( \eta_t^+ \) parton configuration that will be of main concern in the following
\[ dP_{S3}(k_1+k_2;k_3,k_Q,k_{\bar{Q}}) \Rightarrow \frac{R(0)^2}{2\pi M} \frac{1}{8\pi s} \frac{4\pi s}{ut} \frac{1}{\Gamma(1-\epsilon)} dt \]

where the Mandelstam variables are defined by
\[ s = (k_1+k_2)^2 \quad t = (k_1 - q)^2 \quad u = (k_2 - q)^2 = M^2 - t - s \]

In the latter approach it is important to make sure that — in the limit of vanishing relative velocity — the cross section receives solely contributions from the \( Q\bar{Q} \) spin parity configuration corresponding to the bound state. It is therefore mandatory to restrict the evaluation of the amplitude to \( Q\bar{Q} \) color singlet configurations and in the reaction under consideration to those three gluon states that are totally antisymmetric with respect to color as well as to their Lorentz structure.

The former approach leads typically to more compact expressions\(^2\). However, for states with \( J^{PC} = 0^{-+} \) the formula for the bound state amplitude involves the \( \gamma^5 \) matrix whose formulation in the context of dimensional regularization introduces complications. This purely formal difficulty is absent in the second approach, which for this reason has been adopted in this calculation.

### 2.1 The hadronic cross section

In this section we define the general structure of the cross sections for \( \eta_t \) production in hadronic collisions
\[ h_1(P_1) + h_2(P_2) \to \eta_t + X \to \gamma + \gamma + X \]
within the framework of perturbative QCD. Here \( h_1, h_2 \) are unpolarized hadrons with momenta \( P_i \). The hadronic cross section in NLO is thus given by
\[ \sigma^H(S) = \int dx_1 dx_2 f_{a_1}(x_1, Q_F^2) f_{b_2}(x_2, Q_F^2) \hat{\sigma}^{ab}(s = x_1 x_2 S, \alpha_s(\mu^2), \mu^2, Q_F^2) \]

\(^2\) See for instance\([13]\) for particularly impressive examples.
where one sums over \(a, b = q, \bar{q}, g\). \(f^h_a(x, Q_F^2)\) is the probability density to find parton \(a\) with fraction \(x\) in hadron \(h\) if it is probed at scale \(Q_F^2\) and \(\hat{\sigma}^{ab}\) denotes the parton cross section for the process

\[
a(k_1 = x_1 P_1) + b(k_2 = x_2 P_2) \rightarrow \eta_t + X
\]  

from which collinear initial state singularities have been factorized out at a scale \(Q_F^2\) and implicitly included in the scale-dependent parton densities \(f^h_a(x, Q_F^2)\). We next specify the possible partonic subprocesses that contribute to \(\eta_t\) production in LO and NLO.

### 2.2 Gluon fusion

In Born approximation the cross section for \(gg \rightarrow \eta_t\) (fig. 1a) is given by \((N_C = 3)\)

\[
\hat{\sigma}^{gg}_{\text{Born}} = \frac{1}{2s} \frac{1}{64} \frac{1}{4(1 - \epsilon)^2} \sum |M_{\text{Born}}|^2 \frac{R(0)}{2\pi M^2} 2\pi \delta(s - M^2) 
\]

(8)

with

\[
|M_{\text{Born}}|^2 = \frac{N_C^2 - 1}{4N_C} g_s^4 16 (1 - 2\epsilon)(1 - \epsilon)
\]

(9)

The \(Q\bar{Q}\) state has been projected onto the color singlet configuration. The division by \(64 \cdot 4 \cdot (1 - \epsilon)^2\) is implied by the average over \(8^2\) gluon colors and \((n - 2)^2\) gluon helicities in \(n\) dimensions. \(g_s\) will be replaced by \(\alpha_s\) through \(g_s^2 = 4\pi \alpha_s \mu^{2\epsilon}\). The Born cross section can therefore be cast into the form

\[
\hat{\sigma}^{gg}_{\text{Born}} = \frac{\pi^2 \alpha_s^2 R(0)^2}{3s M^3} \mu^{4\epsilon} \frac{(1 - 2\epsilon)}{(1 - \epsilon)} \delta(1 - z) 
\]

(10)

where \(z = M^2/s\). Virtual corrections to the rate for \(Q\bar{Q}\) production close to threshold (fig. 1b) can be combined to yield\(^3\)

\[
\sum 2 |M_{\text{virtual}} \ast M_{\text{Born}}| = \sum |M_{\text{Born}}|^2 \left( \frac{4\pi \mu^{2\epsilon}}{M^2} \right)^\epsilon \Gamma(1 + \epsilon)
\]

\[
\times \frac{\alpha_s}{\pi} \left( C_F \frac{\pi^2}{v} + b_0 \frac{1}{\epsilon_{UV}} - N_C \frac{1}{\epsilon_{IR}} - b_0 \frac{1}{\epsilon_{IR}} + A \right)
\]

(11)

where

\[
A \equiv -T_F \frac{4}{3} \ln 2 + C_F \left( \frac{\pi^2}{4} - 5 \right) + N_C \left( \frac{5}{12} \pi^2 + 1 \right)
\]

\[
b_0 \equiv N_C \frac{11}{6} - n_f T_F \frac{2}{3}
\]

(12)

\(^3\)Here \(C_F = 4/3; N_C = 3;\) and \(T_F = 1/2.\)
$1/\epsilon_{UV}$ and $1/\epsilon_{IR}$ represent poles arising from ultraviolet and infrared divergent integrals respectively. The mass singularity $2 \ln(m_i/M)$ in $[12]$ has been replaced by $1/\epsilon_{IR}$. Since we are considering bound state production the $1/v$ term ($v$ denotes the relative velocity of the two heavy quarks) in eq. (11) is absorbed in the instantaneous potential and thus effectively dropped. The QCD coupling is renormalized in the $\overline{MS}$ scheme and is given by

$$\frac{\alpha_s}{\pi} = \frac{\alpha_{MS}}{\pi} \left(\mu^2\right) \left[1 - \frac{\alpha_{MS}}{\pi} b_0 \left(\frac{1}{\epsilon_{UV}} + \ln 4\pi - \gamma_E\right)\right]$$

(13)

Combining Born result and virtual corrections one thus obtains for resonant production

$$\hat{\sigma}_{gg}^{gg} = \sigma_0(\epsilon) \left[1 + \frac{\alpha_s}{\pi} \left(\frac{4\pi\mu^2}{M^2}\right)^\epsilon \Gamma(1 + \epsilon) \left(b_0 \ln \frac{\mu^2}{M^2} - \frac{1}{\epsilon_{IR}} - \frac{1}{\epsilon_{IR}} + A\right)\right] \delta(1 - \omega)$$

(14)

where the normalization factor

$$\sigma_0(\epsilon) = \frac{1}{s} \frac{\pi^2 R^2(0)}{3 M^3} \alpha_{MS}^2(\mu^2) \frac{\mu^4}{(1 - 2\epsilon)} \frac{1 - 2\epsilon}{1 - \epsilon}$$

(15)

has been split off for convenience.

In a second step the cross section for

$$gg \rightarrow gQ\bar{Q}$$

(16)

has to be calculated (fig. 1c,d). Only the kinematical situation will be treated where the relative momentum of $Q$ and $\bar{Q}$ vanishes. In an intermediate step the matrix element for

$$ggg \rightarrow Q\bar{Q}$$

(17)

will be considered (fig. 1e,f) and reaction (16) can be subsequently obtained through crossing. The $Q\bar{Q}$ state will be projected onto a color singlet configuration through $\delta_{ij}/\sqrt{3}$. The overall wave function of the gluons is of course totally symmetric. Requiring a totally antisymmetric configuration in color space implies, in turn, a totally antisymmetric wave function in Minkowski space. This latter condition secures $J^{PC} = 0^{-+}$ for the $Q\bar{Q}$ bound state. (Conversely, requiring symmetric wave functions in both color and Minkowski space would lead to $J^{PC} = 1^{--}$.)

The amplitude arising from the diagram fig. 1e is given by:

$$\hat{A}(k_1, k_2, k_3; \epsilon_1, \epsilon_2, \epsilon_3) = \bar{u} \left(\frac{(k_1 + k_2 + k_3)/2}{-ig f_1} i \left[\frac{(-k_1 + k_2 + k_3)/2 + m}{(-k_1 + k_2 + k_3)^2/4 - m^2}\right]\right) (-ig f_2)$$

$$\times \frac{i \left[\frac{(-k_1 - k_2 + k_3)/2 + m}{(-k_1 - k_2 + k_3)^2/4 - m^2}\right]}{\left((-k_1 - k_2 + k_3)^2/4 - m^2\right)} (-ig f_3)v \left((k_1 + k_2 + k_3)/2\right)$$

(18)
to be multiplied the color factor

$$\left( \frac{\lambda^a \lambda^a}{2} \right) \frac{\delta_{ij}}{\sqrt{3}} = \frac{1}{\sqrt{3}} \frac{1}{4} (i f_{a_1 a_2 a_3} + d_{a_1 a_2 a_3})$$  \hspace{1cm} (19)$$

As discussed above, only the $f$ term will be retained. The amplitude is obtained from the six permutations of the diagram without triple boson coupling

$$A = \frac{i}{\sqrt{3}} \frac{1}{4} f_{a_1 a_2 a_3} \sum_{i,j,m} \epsilon_{ijm} \tilde{A}(k_i, k_j, k_m; \epsilon_i, \epsilon_j, \epsilon_m)$$ \hspace{1cm} (20)$$

The second type of amplitudes, represented by fig. 1f, involve the triple gluon vertex. Their sum reads as follows

$$\tilde{B}(k_1, k_2, k_3; \epsilon_1, \epsilon_2, \epsilon_3) =$$

$$\bar{u} \left( \frac{(k_1 + k_2 + k_3)/2}{2} \left[ (-ig\gamma_{\alpha}) \frac{i}{((-k_1 - k_2 + k_3)/2 + m)} (-ig f_3)\right.ight.$$

$$+ (-ig f_3) \frac{i}{((-k_1 + k_2 - k_3)/2 + m)} (ig\gamma_{\alpha}) \left. \right] \nu \left( \frac{(k_1 + k_2 + k_3)/2}{2} \right)$$

$$\times \left( \frac{i}{(k_1 + k_2)^2} (-g) \right) \left( (k_1 - k_2)^\alpha \epsilon_1 \epsilon_2 + (k_1 + 2k_2) \epsilon_1 \epsilon_2 + (-2k_1 - k_2) \epsilon_1^2 \right)$$

with a color factor

$$\left( \frac{\lambda^b \lambda^a}{2} \right) \frac{\delta_{ij}}{\sqrt{3}} f_{a_1 a_2 b} = \frac{1}{\sqrt{3}} \frac{1}{2} f_{a_1 a_2 a_3}$$ \hspace{1cm} (22)$$

Three cyclic permutations are combined in this class of diagrams

$$B = \frac{i}{\sqrt{3}} \frac{1}{4} f_{a_1 a_2 a_3} \sum_{i,j,m(cyclic)} 2 \tilde{B}(k_i, k_j, k_m; \epsilon_i, \epsilon_j, \epsilon_m) \hspace{1cm} (23)$$

The ghost amplitude $C$ is easily obtained from eq. (21) through the replacement of the curly bracket $\{ \ldots \} \rightarrow k_3^\beta$, where $k_1, k_2$ and $k_3$ denote the momenta of the incoming ghost, antighost and gluon respectively.

The squared matrix element is finally obtained in $n = 4 - 2\epsilon$ dimensions

$$\sum |M|_R^2 = \sum |A + B|^2 - 2 \sum |C|^2 = \sum \frac{1}{48} f_{a_1 a_2 a_3}^2 g^6 128(1 - \epsilon)$$

$$\times \left( \frac{st + tu + us}{(s - M^2)(t - M^2)(u - M^2)} \right)^2 \left[ \frac{M^8 + s^4 + t^4 + u^4}{stu} - 4 \epsilon M^2 \right]$$

where

$$s = (k_1 + k_2)^2 \quad t = (k_1 + k_3)^2 \quad u = (k_2 + k_3)^2; \quad M = 2m + E_{Bind} \hspace{1cm} (25)$$
After crossing \((k_3 \to -k_3)\) this result is applicable to the reaction of interest \((16)\). In the \(\bar{Q}Q\) threshold region (with the color restriction discussed above) one obtains

\[
d\sigma(gg \to g\bar{Q}Q) = \frac{1}{2s} \sum |\hat{M}|^2 \frac{1}{dP S_3} \sum |\hat{M}|^2 \frac{1}{dP S_3}
\]

where gluon helicities and colors were averaged as usual. Replacing the three- by the two-particle phase space as indicated in eqs. \((1)\) and \((3)\) one obtains

\[
d\sigma(gg \to g\eta_t) = \sigma_0(\epsilon) \frac{\alpha_s}{\pi} \frac{3 M^2}{2 s} \left( \frac{M^8 + s^4 + t^4 + u^4}{stu} \right)^{\epsilon} \frac{dt}{\Gamma(1 - \epsilon)}
\]

The term proportional \(4\epsilon M^2\) in eq. \((24)\) without singular denominator will not contribute in the limit \(\epsilon \to 0\) and has therefore been dropped. In the limit \(\epsilon \to 0\) the result coincides with eq. \((8.45)\) of \([13]\). For the subsequent discussion it will be convenient to follow closely the treatment of NLO corrections for the Drell-Yan process presented in the classical paper by G.Altarelli et al. \([14]\). The invariants \(s, t\) and \(u\) are expressed in terms of \(M^2, y\) and \(z\) by

\[
s = \frac{M^2}{z}; \quad t = -\frac{M^2}{z}(1 - z)(1 - y); \quad u = -\frac{M^2}{z}(1 - z)y \quad (28)
\]

and one arrives at Gluon

\[
\sigma(gg \to g(Q\bar{Q})) = \sigma_0(\epsilon) \frac{\alpha_s}{\pi} \Gamma(1 + \epsilon) \left( \frac{4\pi \mu^2}{M^2} \right)^{\epsilon} \left( \frac{f_{gg}}{\Gamma(1 + \epsilon) \Gamma(1 - \epsilon)} \right) \int_0^1 f_{gg} z^\epsilon (1 - z)^{1 - 2\epsilon} y^{-\epsilon} (1 - y)^{-\epsilon} dy \quad (29)
\]

The normalization factors have been chosen to allow for easy combination with the virtual corrections and \(f_{gg}\) is given by

\[
f_{gg} = \frac{(1 - \epsilon)}{(1 - 2\epsilon)} M^2 |\hat{M}|^2
\]

\[
= \frac{z}{4} \left( \frac{(1 - z)(1 - y)y - 1}{(1 - z)[1 - y(1 - z)][(1 - z)y + z]} \right)^2
\]

\[
\times \left[ z^4 + 1 + (1 - z)^4((1 - y)^4 + y^4) \right] \left( \frac{1}{y} + \frac{1}{1 - y} \right) \quad (30)
\]

As a consequence of the symmetry of \(f_{gg}\) with respect to \(y \leftrightarrow (1 - y)\) it suffices to evaluate the \(1/y\) term. The \(y\) integration can be performed with the help of
the distribution identities

\[
\begin{align*}
x^{-1-\epsilon} &= -\frac{1}{\epsilon} \delta(x) + \left(\frac{1}{x}\right)_{+} - \epsilon \left(\frac{\ln x}{x}\right)_{+} \\
z^{\epsilon}(1-z)^{-1-2\epsilon} &= -\frac{1}{2\epsilon} \delta(1-z) + \left(\frac{1}{1-z}\right)_{+} + \epsilon \left(\frac{\ln z}{1-z}\right)_{+} - 2\epsilon \left(\frac{\ln(1-z)}{1-z}\right)_{+} \\
y^{-1-\epsilon}(1-y)^{-\epsilon} &= -\frac{1}{\epsilon} \delta(y) + \left(\frac{1}{y}\right)_{+} - \epsilon \left(\frac{\ln(1-y)}{y}\right)_{+} - \epsilon \left(\frac{\ln y}{y}\right)_{+}
\end{align*}
\]

\[
\frac{1}{\Gamma(1-\epsilon)} \frac{1}{\Gamma(1+\epsilon)} = 1 - \epsilon^2 \frac{\pi^2}{6} + \ldots
\]

Combining the Born result with real radiation and virtual corrections one obtains

\[
\sigma(gg \to \eta + X) = \sigma_0(\epsilon) \left\{ \delta(1-z) + \frac{\alpha_s}{\pi} \left(\frac{4\pi \mu^2}{M^2}\right)^\epsilon \Gamma(1+\epsilon) \right\} \times \left[ (b_0 \ln \frac{\mu^2}{M^2} - N_C \frac{\pi^2}{3} + A) \delta(1-z) - \frac{1}{\epsilon} P_{gg}(z) + N_C F(z) \right]\]

where \(b_0\) and \(A\) are given in eqs. (12) and

\[
F(z) = \Theta(1-z) \left[ \frac{11z^5 + 11z^4 + 13z^3 + 19z^2 + 6z - 12}{6z(1+z)^2} + 4 \left(\frac{1}{z} + z(1-z) - 2\right) \ln(1-z) + 4 \left(\frac{\ln(1-z)}{1-z}\right)_{+} \right. \\
+ \left. \frac{2(z^3 - 2z^2 - 3z - 2)(z^2 - z + 2)z \ln(z)}{(1+z)^3(1-z)} - 3 \right] \frac{1}{1-z}
\]

and

\[
P_{gg} = 2 N_C \left(\frac{1}{z} + \left(\frac{1}{1-z}\right)_{+} + z(1-z) - 2\right) + b_0 \delta(1-z);
\]

The \(1/\epsilon\) term can be absorbed in the gluon densities. If these are defined at a factorization scale \(Q_F^2\), one effectively subtracts from the above result the term

\[
\sigma_0(\epsilon) \frac{\alpha_s}{2\pi} \left(\frac{4\pi \mu^2}{Q_F^2}\right)^\epsilon \Gamma(1+\epsilon) \left( -\frac{1}{\epsilon} P_{gg}(z) + C_{gg}(z) \right)
\]

In the \(\overline{MS}\) scheme \(C_{gg} = 0\), in the DIS scheme

\[
C_{gg}(z) = -n_f \left[ (z^2 + (1-z)^2) \ln\left(\frac{1-z}{z}\right) + 8z(1-z) - 1 \right]
\]
The partonic cross section is then given by

\[ \hat{\sigma}^{gg} = \frac{1}{s} \frac{\pi^2 R(0)^2}{3 M^3} \alpha_{MS}^2(\mu^2) \left\{ \delta(1 - z) + \frac{\alpha_{MS}(\mu^2)}{\pi} \left[ \delta(1 - z) \left( b_0 \ln \frac{\mu^2}{M^2} + N_C \left( 1 + \frac{\pi^2}{12} \right) + C_F \left( \frac{\pi^2}{4} - 5 \right) - \frac{4}{3} T_f \ln(2) \right) - \ln \frac{Q^2}{M^2} P_{gg}(z) - C_{gg}(z) + N_C F(z) \right] \right\}. \] (38)

### 2.3 Gluon quark scattering

Gluon quark scattering \( g(k_1) + q(k_2) \rightarrow q(k_3) + \eta_t(P) \) proceeds evidently in leading order \( \alpha_s^3 \) through the diagrams in fig. 2b. Collinear singularities have to be absorbed in the structure functions. The (dimensionally regularized) squared matrix element for the production of \( Q \bar{Q} \) at threshold is given by

\[ |M^{gq}|^2(s, t, u) = 32 \left[ -\frac{s^2 + u^2}{(s + u)^2} \frac{1 + 2 \epsilon}{t} \right] (39) \]

and has to be supplemented by the color factor

\[ \sum \left| \frac{1}{\sqrt{3}} \delta_{ij} \left( \frac{\lambda^a \lambda^b}{2} \right)_{ij} \left( \frac{\lambda^b}{2} \right)_{kl} \right|^2 = \frac{1}{3} \] (40)

and the coupling constant \( g_s^6 \). After spin and color averaging the cross section can then be cast into a form identical to eq. (29) with \( f^{gg} \) replaced by

\[ f^{gq} = \frac{1}{9(1 - z) (1 - y)} \left[ \frac{1 + y^2 (1 - z)^2}{(1 - y (1 - z))^2} - \frac{\epsilon}{1 - 2 \epsilon} \right] \] (41)

The \( y \) integration is performed as before and only a single pole remains

\[ \sigma^{gq} = \sigma_0(\epsilon) \frac{\alpha_s}{2\pi} \Gamma(1 + \epsilon) \left( \frac{4\pi \mu^2}{M^2} \right)^\epsilon \left\{ -\frac{1}{\epsilon} P_{gq}(z) - \ln \left( \frac{(1 - z)^2}{z} \right) P_{gq}(z) + C_F \frac{2(1 - z)}{z} (1 - \ln z) + C_F z \right\} \] (42)

with

\[ P_{gq}(z) = C_F \frac{1 + (1 - z)^2}{z} \] (43)

The \( 1/\epsilon \) term can again be absorbed by the (quark) structure function. This amounts effectively to the subtraction of

\[ \sigma_0(\epsilon) \frac{\alpha_s}{2\pi} \left( \frac{4\pi \mu^2}{Q_F^2} \right)^\epsilon \Gamma(1 + \epsilon) \left( -\frac{1}{\epsilon} P_{gq}(z) + C_{gq}(z) \right) \] (44)
from the above result. The function $C_{gq}$ is again specific to the subtraction scheme: $C_{gq}$ vanishes is the $\overline{MS}$ scheme and in the DIS scheme it is given by

$$C_{gq}(z) = -C_{qq}(z)$$  \hspace{1cm} (45)$$

where

$$C_{qq}(z) = C_F \left[ \left(1 + z^2 \right) \left( \frac{\ln(1-z)}{1-z} \right)_+ - \frac{3}{2} \frac{1}{(1-z)_+} - \frac{1 + z^2}{1-z} \ln z \right] + 3 + 2z - \left( \frac{9}{2} + \frac{1}{3} \pi^2 \right) \delta(1-z)$$  \hspace{1cm} (46)$$

One therefore arrives at the following formula for the parton cross section

$$\hat{\sigma}^{q\bar{q}}(z) = \sigma_0(0) \frac{\alpha_s}{\pi} \Theta(1-z) \left[ \frac{1}{2} P_{gq}(z) \left( \ln \frac{M^2(1-z)^2}{Q_F^2 z} + 1 \right) - C_F \frac{1 - z}{2z} \ln z - \frac{1}{2} C_{gq}(z) \right]$$  \hspace{1cm} (47)$$

The $q(k_1) + g(k_2)$ and $g(k_1) + q(k_2)$ initiated reactions can evidently be deduced from the same partonic cross section, as well as the corresponding antiquark processes. All these will be included in our numerical analysis.

### 2.4 Quark-antiquark annihilation

Quark-antiquark annihilation (fig. 2a) can be obtained in a straightforward manner by crossing from the previous calculation:

$$|\mathcal{M}^{q\bar{q}}|^2(s, t, u) = |\mathcal{M}^{gq}|^2(u, t, s) \Rightarrow 32 \frac{u^2 + t^2}{(u + t)^2 s}$$  \hspace{1cm} (48)$$

No infrared divergency arises and hence $\epsilon$ can be put at zero from the beginning. The angular integration is trivial and one finds

$$\hat{\sigma}^{q\bar{q}}(z) = \sigma_0(0) \frac{\alpha_s}{\pi} \frac{32}{27} z(1 - z)$$  \hspace{1cm} (49)$$

This completes the evaluation of parton cross sections.

### 3 Bound state properties

The decay of $\eta_t$ are completely dominated by the single quark decay mode \cite{13, 14}. Decays into two photons constitute a tiny fraction of all events and the branching ratio is to a very good approximation given by

$$\text{BR}(\gamma\gamma) = \frac{\Gamma_{\gamma\gamma}}{\Gamma_{SQD}}$$  \hspace{1cm} (50)$$
Up to small boundstate corrections $\Gamma_{SQD}$ is given by twice the decay rate of a top quark $\Gamma_t$. For a complete NLO calculation of $\eta_t$ production and decay into $\gamma\gamma$ the QCD corrections to the decay rates must be included. In Born approximation

$$\Gamma_t \text{ (Born)} = \frac{G_F m_t^3}{8\sqrt{2}\pi} \frac{2p_W}{m_t} \left\{ \left[ 1 - \left( \frac{m_b}{m_t} \right)^2 \right]^2 + \left[ 1 + \left( \frac{m_b}{m_t} \right)^2 \right] \left( \frac{m_W}{m_t} \right)^2 - 2 \left( \frac{m_W}{m_t} \right)^4 \right\}$$

QCD corrections lead to a reduction by about 7%:

$$\Gamma_t = \Gamma_t \text{ (Born)} \left( 1 - \frac{2\alpha_s}{3\pi} f \right)$$

The complete formula for $f$ is given in [16]. For the decay rate into a real $W$, neglecting its width as well as the mass of the bottom quark, one obtains [16]

$$f = \frac{F_1}{F_0}$$

$$F_0 = 2(1 - y)^2(1 + 2y)$$

$$F_1 = F_0 \left[ \pi^2 + 2Li_2(y) - 2Li_2(1 - y) \right]$$

$$+ 4y(1 - y - 2y^2) \ln y + 2(1 - y)^2(5 + 4y) \ln(1 - y) - (1 - y)(5 + 9y - 6y^2)$$

$$\text{Br}(0^- \to \gamma\gamma) = \frac{1}{\Gamma_{SQD}} \frac{12Q_t^4 \alpha^2}{M^2} \left[ 1 + \frac{\alpha_s}{\pi} \left( \frac{\pi^2}{3} - \frac{20}{3} \right) \right]$$

The QCD corrections are negative in numerator and denominator and cancel to a large extent in the branching ratio. This completes our list of formulae for $\eta_t$ production and decay.

As stated in the introduction, the most important ingredients for the numerical evaluation are the wave function at the origin and the gluon distribution. The former are calculated for the two-loop potential $V_J$ [18]. The results for the dimensionless quantity $R(0)^2/M^3$ are displayed in fig. 3a for the potentials with the $\Lambda_{MS}^{(4)}$ values of 200, 300 and 500 MeV, corresponding to $\alpha_{MS}(M_Z) = 0.109, 0.1165$ and 0.127 respectively (and corresponding to $\Lambda_{MS}^{(5)} = 0.132, 0.207$ and 0.366 MeV). This choice for $\Lambda_{MS}^{(4)}$ covers well the present range $\alpha_{MS}(M_Z) = 0.117 \pm 0.07$ derived from a global fit to the present data [2]. In the subsequent discussions the results for $V_J$ with the values of 200 and 500 MeV for $\Lambda$ will be considered as indicative for the uncertainty from the wave function. The branching ratio of $\eta_t$ into $\gamma\gamma$ is shown in fig. 3b.

---

$^4$The numerical evaluation of $R(0)$ has been performed with the program BOUNDL [19].
In principle decays into massive gauge and Higgs bosons could be of interest and have been calculated in [15, 20, 21]. The decay rate is dominated entirely by the single quark decay. The branching ratio of $\eta_c$ into $\gamma Z$ (and correspondingly for $H_Z, ZZ$ and $WW$) is therefore given by the branching ratio into $\gamma \gamma$ as calculated before, multiplied by the ratio $\Gamma_{\gamma Z}/\Gamma_{\gamma \gamma}$. As shown in fig. 3c these latter ratios vary between 0.2 (for $\gamma Z$) and 40 (for $H_Z$) in the mass range of interest.

4 Numerical results

With these ingredients it is straightforward to arrive at numerical predictions for the production cross section. If not stated otherwise, these will be based on the parton distribution functions of MT (Morfin-Tung) set B1 [22]. $\alpha_s$ is chosen accordingly to the $\Lambda$ value consistent with the parton distribution functions with five flavours. All curves correspond to the DIS factorization scheme.

To investigate the stability of the predictions in a first step the dependence of the results on the renormalization scale $\mu$ and the factorization scale $Q_F$ will be explored. The resonance mass will be kept fixed at 240 GeV and $\mu$ and $Q_F$ will be chosen identically. The result of the variation for the $gg$ induced process alone is compared to the LO predictions in fig. 4a, the sum of all contributions in fig. 4b for $\sqrt{S}=16$ and 40 TeV. Varying $\mu = Q_F$ in the range $M/2$ to $2M$ induces an uncertainty of about $\pm 8\%$ for SSC and LHC. These curves have been obtained with $V_J(200\text{MeV})$. In the following discussion, both $\mu$ and $Q_F$ will be fixed to $M$.

The magnitude of the QCD corrected cross section including all subprocesses is compared to the Born cross section in fig. 5(a) for $\sqrt{S}=16$ and 40 TeV and (b) for $\sqrt{S}=1.8$ TeV. The size of the corrections amounts to 40-50% for LHC and SSC energies, whereas the corrections for Tevatron energies amounts to 80-90%.

To give an idea about the importance of different partonic processes, their relative contribution to the production rate is displayed in fig. 6. As anticipated in [1] $\eta_c$ production is dominated by gluon fusion with the remaining contributions amounting about 10% at $\mu = Q_F = M$. This holds true for all quark masses and energies of interest.

To study the dependence of the cross section on the choice of parton distribution functions, the predictions based on a variety of parametrizations (MT set SN [22], DFLM [23], MRS set B200 [24], KMRS set B [24], GRV [26]) are compared in fig. 7 to those based on MT set B1. $\alpha_s$ is chosen accordingly to the $\Lambda$ value consistent with the parton distribution functions with five flavours. The resulting difference amounts to less than 25% and is in any case small compared to the ambiguity from different choices of $\Lambda$ in the potential.

\footnote{The result for $\Gamma_{ZZ}$ in [21] (eq. 9.1c) should be multiplied by a factor $(\nu_a^2 + a_a^2)/16$.}
The final predictions for the cross section are shown in fig. 8a for LHC and SSC and in fig. 9 for 1.8 TeV. The solid line corresponds to $V_J(\Lambda^{(4)}_{\overline{MS}} = 300\text{MeV})$, parton distribution functions of MT set B1 and $\alpha_s$ derived from $\Lambda^{(5)}_{\overline{MS}}$. The dashed and dotted curves enclose the range of predictions resulting from the dominant uncertainty in the potential and correspond to $V_J(\Lambda^{(4)}_{\overline{MS}} = 500\text{MeV})$ to $V_J(\Lambda^{(4)}_{\overline{MS}} = 200\text{MeV})$. The corresponding predictions for the cross section times the branching ratio into $\gamma\gamma$ are displayed in fig. 8b.

The results are significantly below those of [8] even for an optimistic choice of the strong coupling constant.

**Acknowledgements:**

We would like to thank P.M. Zerwas for stimulating discussions which initiated this work and for informations about details of [27]. We thank M. Jezabek for providing the program BOUNDL.
References

[1] P. Nason, S. Dawson and R. K. Ellis, *Nucl. Phys.* B 303 (1988) 607; *ibid.* B327 (1989) 49.

[2] G. Altarelli, M. Diemoz, G. Martinelli and P. Nason, *Nucl. Phys.* B 308 (1988) 724.

[3] W. Beenakker, H. Kuijf, W. L. van Neerven and J. Smith, *Phys. Rev.* D 40 (1989) 54;
W. Beenakker, W. L. van Neerven, R. Meng, G. A. Schuler and J. Smith, *Nucl. Phys.* B 351 (1991) 507

[4] F. Abe et al. [CDF Coll.], *Phys. Rev. Lett.* 68 (1992) 447.

[5] J. Carter, Proceedings of the LP-HEP Conference, Geneva 1991. Eds. S. Hegarty et al., World Scientific.

[6] E. Reya, P.M. Zerwas [conv.] et al., in Proceedings ECFA LHC Workshop, Aachen 1990, CERN 90–10.

[7] D. Schaile, CERN-PPE-91-187.

[8] G. Pancheri, J.-P. Revol and C. Rubbia, *Phys. Lett.* B 277 (1992)

[9] J.H. Kühn and E. Mirkes, Karlsruhe preprint TTP92-26, to appear in *Phys. Lett.* B (1992)

[10] J. Kaplan, J.H. Kühn and E.G. Oudrhiri Safiani, *Nucl. Phys.* B 157 (1979) 125.

[11] B. Guberina, J. Kühn, R.D. Peccei and R. Rückl, *Nucl. Phys.* B 174 (1980) 317.

[12] K. Hagiwara, C.B. Kim and T. Yoshino, *Nucl. Phys.* B 177 (1981) 461

[13] R. Gastmans and T.T. Wu, The Ubiquitous Photon, Clarendon Press, Oxford (1990).

[14] G. Altarelli, R.K. Ellis and G. Martinelli, *Nucl. Phys.* B 157 (1997) 461

[15] J.H. Kühn, *Act. Phys. Pol.* B 12 (1981) 347.
J.H. Kühn, *Act. Phys. Austr.Suppl.XXIV* (1982) 203.

[16] M. Ježabek and J.H. Kühn, *Nucl. Phys.* B 320 (1989) 20.
[17] I. Bigi, Y. Dokshitzer, V. Khoze, J. Kühn and P.M. Zerwas, Phys. Lett. B 181 (1986) 157.

[18] K. Igi and S. Ono, Phys. Rev. D 33 (1986) 3349.

[19] We thank M. Jezabek for providing this program package.

[20] V. Barger et al., Phys. Rev. D 35 (1987) 3366.

[21] J. Kühn, Phys. Reports 167 (1988) 323.

[22] J. Morfin and Wu-Ki Tung, Z. Phys. C 52 (1991) 13

[23] M. Diemoz, F. Ferroni, E. Longo and G. Martinelli, Z. Phys. C 39 (1988) 21.

[24] A. Martin, R. Roberts and J. Stirling, Phys. Rev. D 43 (1991) 3645

[25] J. Kwieciński, A. Martin, R. Roberts and J. Stirling, Phys. Rev. D 42 (1990) 3645

[26] M. Glück, E. Reya and A. Vogt, DO-TH 91/07: see also Z. Phys. C 53 (1992) 127

[27] A. Djouadi, M. Spira and P. Zerwas, Phys. Lett. B 264 (1991) 440

Figure captions

Fig. 1 (a) Lowest order diagrams for \( \eta_t \) production,  
(b) Diagrams contributing from virtual corrections,  
(c,d) Diagrams contributing to corrections from gluon emission,  
(e,f) Diagrams for \( ggg \to Q\bar{Q} \).

Fig. 2 Diagrams contributing to corrections from (a) \( q\bar{q} \to \eta_t q \) and  
(b) \( qg \to \eta_t g \) gluon emission.

Fig. 3. (a) Predictions for \( R(0)^2/M^3 \) of the S–wave ground state and  
(b) branching ratio of \( \eta_t \to \gamma\gamma \) as functions of \( M \) for the potential \( V_J \) with \( \Lambda_{\text{MS}}^{(4)}=200 \) (solid), 300 (dotted) and 500 MeV (dashed).  
(c) Decay rates of \( \eta_t \) into \( \gamma Z, ZZ WW \) and \( HZ \) (for \( m_H = 70 \text{ GeV} \) normalizes to the \( \gamma\gamma \) channel.)
Fig 4. (a) Scale dependence for $pp \rightarrow \eta_t + X$ for the $gg$ induced process alone and (b) scale dependence for all contributions. Both LO (dashed) and NLO (solid) results are shown for $\sqrt{S}=16$ and 40 TeV. We use MT set B1 parton distributions with $\Lambda^{(4)}_{MS}=194$ MeV for four flavours and work in the DIS factorization scheme.

Fig 5. (a) Ratio between the radiatively corrected cross section for $pp \rightarrow \eta_t + X$ and the lowest order result for $\sqrt{S}=16$ and 40 TeV and (b) ratio between the radiatively corrected cross section for $p\bar{p} \rightarrow \eta_t + X$ and the lowest order result for $\sqrt{S}=1.8$ TeV. We use MT set B1 parton distributions with $\Lambda^{(4)}_{MS}=194$ MeV for four flavours and work in the DIS factorization scheme.

Fig 6. Relative contributions to $\eta_t$ production at $\mu = Q_F = M$ for $\sqrt{S}=16$ (dotted) and 40 (solid) TeV. LO contributions are (A) $gg \rightarrow \eta_t + X$, NLO contributions are from (B) $gg \rightarrow \eta_t + g + X$, (C) $gq \rightarrow \eta_t + q + X$, (D) $(q\bar{q} \rightarrow \eta_t + g + X) \times 100$.

Fig 7. Prediction for the production cross section of $\eta_t$ at LHC using different sets of parton distributions functions (MT set SN [22], DFLM [23], MRS set B200 [24], KMRS set B [25], GRV [26]) normalized to the predictions obtained with MT set B1 [22].

Fig 8. (a) Cross section for $\eta_t$ production including NLO corrections and (b) cross section multiplied by the two photon branching ratio at $\sqrt{S} = 16$ and 40 TeV for the potential $V_J$ with $\Lambda^{(4)}_{MS}=300$ (solid), $\Lambda^{(4)}_{MS}=200$ (dotted) and $\Lambda^{(4)}_{MS}=500$ MeV (dashed).

Fig 9. Same as fig. 8a for 1.8 TeV.