Numerical simulation of helium injection into the accelerated turbulent xenon flow

A Yu Sakhnov
Kutateladze Institute of Thermophysics SB RAS,
1 Lavrentyev Ave., Novosibirsk, Russia.
E-mail: aleksei_sakhnov@mail.ru

Abstract. The paper reports on numerical simulation of a weakly accelerated turbulent boundary layer over the permeable wall. Helium-xenon mixture was injected into the xenon main flow after the laminar-turbulent transition. The finite difference method was applied to solve a system of boundary layer equations with variable gas properties supplemented by the k-ω-γ turbulence model. It has been shown that the light gas injection into the accelerated turbulent flow leads to arising of velocity maximum within the boundary layer and its partial laminarization.

1. Introduction
Helium-xenon mixtures have many applications due to their highly effective heat transfer properties [1]. At that, the mixing process of helium and xenon can be realized under various conditions. In the present paper we consider an injection of the helium-xenon mixture into the accelerated turbulent near-wall flow of xenon. Kays with some research groups carried out the most detailed and regular experiments with accelerated flows over the permeable surfaces [2]. These investigations showed that gas injection into the flow with favorable pressure gradient decreases skin-friction coefficient, and gas suction in contrast increases it. Most of experiments were realized for the turbulent regime. At the same time, Kays [3] demonstrated that turbulence is suppressed fully, and flow turns to the laminar regime at $K > 3.5 \times 10^{-6}$. Bourassa and Thomas [4] proved this critical value of acceleration parameter in their experiments as well. We showed [5] that if the favorable pressure gradient takes place from the leading edge of the wall then the laminar-turbulent transition is prevented at $K > 1 \times 10^{-6}$.

Gershbein [6] studied numerically and analytically an accelerated flow of a gas mixture around a body. The gas mixture consisted of hydrogen, nitrogen, and carbon dioxide. A similar gas mixture was injected into the flow through the body surface, but with other concentrations of the mixture components. Two types of flow with different density ratios were considered. The first flow was with $\rho_e / \rho_w = 0.28$; here, despite the presence of a favorable pressure gradient, the velocity profiles were less filled than the Blasius profile [7]. In the second flow considered, the density of blown mixture was 2.25 times lower than the gas density in the free stream. The streamwise velocity in the boundary layer was 45% greater than the main-stream velocity at the highest value of the favorable pressure gradient.

The goal of the present paper is the research of the accelerated turbulent boundary layer at light gas injection. We study the local laminarization arisen due to the overshoot phenomenon. The acceleration parameter has a low value of $K = 3 \times 10^{-7}$ in order to decrease the effect of favorable pressure gradient as the factor leading to flow relaminarization. Mass concentration of helium in the injected mixture has values from 0.05 to 0.5.
2. Flow configuration and modelling approach

We consider xenon flow in a plane convergent channel where the acceleration parameter 
\[ K = \left( \frac{\mu_x}{\rho U_x^2} \right) \frac{dU_x}{dx} \] remains constant over the entire channel length (figure 1). Through the lower wall of the channel at \( x = 0.2 \) m we set the helium-xenon injection with constant intensity 
\[ \bar{f}_x = \frac{\rho_x V_x}{\rho_x U_x} = \text{const} \] along the flow direction. The initial parameters such as velocity, turbulence intensity and scale were taken from T3A experiment (ERCOFTAC). They have the following values 
\[ U_0 = 5.2 \, \text{m} / \text{s}, \quad T_{U_0} = 3.5\% \quad \text{and} \quad R_{U_0} = 14. \] The sloped top wall of the channel was assumed to be located sufficiently far from the lower channel wall, i.e. \( h_0 >> \delta \), so that the analysis can be focused on the accelerated hydrodynamic and thermal boundary layers, developing over the bottom surface. All parameters across the boundary layer and dimensionless characteristics are based on main flow quantities designated by subscript “e”.

2.1. Equations and boundary conditions

The flow considered can be well approximated by the parabolized two-dimensional momentum, continuity, energy and mass concentration equations for a steady turbulent compressible boundary layer:

\[ \rho U \frac{\partial U}{\partial x} + \rho V \frac{\partial U}{\partial y} = - \frac{dP}{dx} + \frac{\partial}{\partial y} \left( \mu + \mu_t \frac{\partial U}{\partial y} \right), \] (1)

\[ \frac{\partial (\rho U)}{\partial x} + \frac{\partial (\rho V)}{\partial y} = 0 \] (2)

\[ c_p \rho U \frac{\partial T}{\partial x} + c_p \rho V \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left( \lambda_{eff} \frac{\partial T}{\partial y} \right) + U \frac{dP}{dx} + \mu \left( \frac{\partial U}{\partial y} \right)^2 + \rho D_{x_eHe} (c_{pHe} - c_{pHe}) \frac{\partial C_{x_e}}{\partial y} \frac{\partial T}{\partial y}, \] (3)

\[ \frac{\partial C_{x_e}}{\partial y} = - \frac{\partial C_{He}}{\partial y}, \quad \lambda_{eff} = \lambda + c_p \mu_t / P_r, \quad P_r = 0.9 \]

\[ \rho U \frac{\partial C_{x_e}}{\partial x} + \rho V \frac{\partial C_{x_e}}{\partial y} = \frac{\partial}{\partial y} \left[ \rho D_{x_eHe} + \frac{\mu_x}{S_{Cl}} \frac{\partial C_{x_e}}{\partial y} \right], \quad C_{He} = 1 - C_{x_e}, \quad S_{Cl} = 0.9. \] (4)

The diffusion coefficient of helium-xenon interaction was defined by the following formula:

\[ D_{12} = 0.6 \frac{RT}{P} \frac{A_{12} \cdot \mu_{12} (M_1 + M_2)}{M_1 M_2}, \quad A_{12} = 1.1. \] (5)
We used $k$-$\omega$-$\gamma$ turbulence model [8] to calculate the eddy viscosity $\mu$. This model is able to predict reasonably turbulization and laminarization of the boundary layer due to application of intermittency factor $\gamma$.

We simulated helium-xenon mixture properties depending on temperature and pressure using the method described in paper [9]. Pressure equals 0.1 MPa for all considered flow cases.

At the wall, the no-slip and constant temperature conditions were adopted:

$$y = 0: \quad U = 0, \quad V = \overline{j}, \quad \rho U_e / \rho, \quad T = T_e = \text{const}.$$  

At the outer edge of the boundary layer, the velocity $U_e$ was determined from the integration of the acceleration parameter

$$K = \left( \mu / \rho U_e^2 \right) \frac{dU_e}{dx}$$

accounting for variable viscosity and density of gas, with zero heat flux condition for the thermal field:

$$y \geq \delta: \quad U = U_e = -\left( \int_0^y \frac{\rho}{\mu} dx \right)^{-1}, \quad \partial T / \partial y = 0.$$  

2.2. Numerical method

The system of time-independent differential equations (1) – (4) was solved by a finite difference method using a fully implicit scheme, which is well-described in paragraph 7.3.3 of the book [10]. The accuracy of applied scheme is $o(\Delta x) + o(\Delta y)^2$. We used the Thomas algorithm to solve the derived system of linear algebraic equations. This algorithm is reported in paragraph 4.3.3 of the book [10]. Linearizing procedure was lagging the coefficients. Criterion for convergence of the system of algebraic equations was $\varepsilon < 10^{-5}$, where $\varepsilon$ is the relative difference between values of required parameters of the flow at two sequential iterations.

The length of the wall equaled 2.14 m with the modeling area height of 0.5 m. The number of grid nodes normal to the surface was 300 with compression to the wall. In the flow direction the grid step did not exceed $5 \times 10^{-5}$ m.

3. Results

Figure 2a presents dependencies of skin-friction coefficient on Reynolds number in the accelerated boundary layer over the impermeable wall ($\overline{j_w} = 0$), with homogeneous injection ($C_w(He) = 0$) and at four values of helium concentration in the injected mixture: 0.05, 0.11, 0.115 and 0.5. The increase of helium concentration decreases skin-friction coefficient. At that, there is a flow laminarization. At $C_w(He) = 0.115$ and $Re_x = 2 \times 10^6 - 10^7$ skin-friction coefficient coincides with line 3 that is calculated without any turbulence model for the same physical conditions. At $C_w(He) = 0.5$ skin-friction coefficient coincides with the laminar flow practically on the whole part of the injection. As we have mentioned in the introduction, Kays [3] demonstrated that the relaminarization of the homogeneous flow over impermeable wall takes place at $K > 3.5 \times 10^{-6}$. So, the light gas injection facilitates the flow laminarization significantly.

At the lower helium concentration of $C_w(He) = 0.11$ and $Re_x = 6 \times 10^6$ there is a second transition to turbulence, which is characterized by the increase in skin-friction coefficient. At $C_w(He) = 0.05$ skin-friction coefficient has a smooth change that points indirectly at keeping of turbulent regime. There is some increase in skin-friction coefficient at $Re_x > 10^6$ due to arising of streamwise velocity maximum within the boundary layer (figure 4). It is worth to note that skin-friction coefficient at the helium injection into the zero pressure gradient xenon flow (line 5) decreases for all considered Reynolds numbers.
Figure 2. Effect of helium-xenon mixture injection into the xenon flow on skin-friction coefficient and mass Stanton number. 1 – Blasius law \( c_f = 0.664 \text{Re}^{-0.5} \), 2 – turbulent flow \( c_f = 0.059 \text{Re}^{-0.2} \), 3, 4 – flow simulation without turbulence model for helium concentrations of 0.115 and 0.5 respectively; 5 – helium injection with \( C_w(\text{He})=0.05 \) and \( \overline{j_w} = 10^{-3} \) into zero pressure gradient flow of xenon.

Figure 2b shows dependencies of mass Stanton number on the integral Reynolds number. In paper [11] we demonstrated that in order to describe heat transfer in flows with the variable velocity of the main flow one has to use the integral Reynolds number that is \( \text{Re}_{int} = \text{Re}_L \int_0^\infty \frac{U_x}{U_0} d\overline{x} \), where \( \overline{x} = x/L, \text{Re}_L = \rho \overline{U}_L L / \mu_L, L \) is length of the wall. Also, it is valid for mass transfer and for the mass Stanton number which coincides with line 5. It means that flow acceleration has no effect on mass Stanton number. At \( C_w(\text{He}) = 0.11 \) and \( \text{Re}_{int} > 2 \times 10^6 \) mass Stanton number has the increase due to the second transition to turbulent regime. Unlike the skin-friction coefficient the mass Stanton number at \( C_w(\text{He}) = 0.115 \) and \( C_w(\text{He}) = 0.5 \) does not coincide with results of numerical modeling of the laminar boundary layer (lines 3 and 4). Moreover, the laminar boundary layer model gives the decrease in mass Stanton number at \( \text{Re}_w > 2 \times 10^6 \), whereas the consideration of the turbulence model leads to the constant value or to weak increase in mass Stanton number.

Figure 3a demonstrates profiles of turbulent kinetic energy \( k' = k / \overline{U}_r^2 \) at \( C_w(\text{He}) = 0.11 \) and various Reynolds numbers. Here \( y' = \rho U_r y / \mu \) and \( U_r = \sqrt{\overline{U}_w / \rho} \). Turbulent kinetic energy has lower values at \( \text{Re}_x = 5 \times 10^5 - 2 \times 10^6 \) than before injection part of the flow. However, turbulent kinetic energy at \( \text{Re}_x = 5 \times 10^6 \) exceeds the maximal value of this one in the accelerated turbulent boundary layer over the impermeable wall at \( \text{Re}_x = 2.5 \times 10^5 \). Distribution of turbulent kinetic energy at \( \text{Re}_x = 10^7 \) coincides practically with the profile at \( \text{Re}_x = 5 \times 10^6 \).

Figure 3b presents distributions of the intermittency coefficient \( \gamma \) for various Reynolds numbers. We have to remind that if \( \gamma = 0 \) then the flow has a laminar regime, and \( \gamma = 1 \) appropriates a fully turbulent flow. As it is seen, at \( \text{Re}_x = 2.5 \times 10^5 \) and \( \text{Re}_x = 10^7 \) values of the intermittency coefficient equal unity within the whole thickness of the boundary layer, i.e. the flow in these cross-sections is a
Figure 3. Profiles of turbulent kinetic energy (a), intermittency coefficient (b) and streamwise velocity (c) at helium-xenon mixture injection into xenon flow.

Figure 4. Streamwise velocity profiles at helium-xenon mixture injection into xenon flow.

fully turbulent one. At $Re_x = 5 \times 10^4$ the intermittency coefficient in the near wall area has a decrease to the value of 0.1 that means a local suppression of turbulence in this flow zone. At $Re_x = 2 \times 10^6$ intermittency coefficient near the wall equals zero, i.e. there is local relaminarization of the flow. While Reynolds number has an increase to the value of $5 \times 10^6$, where the second laminar-turbulent transition takes place, the intermittency factor near the wall increases up to the value 0.18. At the further increase in Reynolds number the flow restores the turbulent regime.

Figure 4 shows profiles of the streamwise velocity within the accelerated boundary layer. Increase of Reynolds number leads to arising of the overshoot phenomenon, when the velocity within the boundary layer exceeds the main flow velocity. This phenomenon exists due to density difference between the near wall gas and the main flow, which has a streamwise favorable pressure gradient [12]. It is obvious that the more filled velocity profiles facilitates local laminarization of the boundary layer. Nevertheless, turbulent regime can be realized at the presence of the velocity maximum within the boundary layer.
4. Conclusions
The paper presents numerical modeling of helium-xenon mixture injection into the accelerated turbulent xenon flow. We have shown that the light gas injection has the following influence on the boundary layer:

- With an increase in helium concentration in the injected mixture the skin-friction coefficient and mass Stanton number have a decrease;
- Local laminarization of the flow takes place at $C_w(He)=0.11$, after that the flow restores the turbulent regime with an increase in Reynolds number;
- At $C_w(He)\geq 0.115$, the area of the laminarized flow is expanded to the whole permeable part of the wall. At that, the skin-friction coefficient coincides with this one obtained for the laminar flow under the same conditions. However, the mass Stanton number exceeds the values calculated for the laminar flow;
- Turbulent kinetic energy retains sufficiently high values $k_{max}^+ \approx 2$ within the part of laminarized flow. At that, the intermittency coefficient has a low values only near the wall;
- Injection of gas mixture with the density lower than the main flow density and streamwise favorable pressure gradient lead to arising of the overshoot phenomenon, which facilitates the local laminarization of the boundary layer. However, the overshoot phenomenon can exist in the developed turbulent flow.

Acknowledgement
This research was supported by the Russian Scientific Foundation, project No. 14-19-00352.

References
[1] El-Genk M S, Tournier J-M 2008 Energy Conversion and Management 49 1881-1891
[2] Moffat R J, Kays W M 1984 Adv. Heat Transfer 16 242–365
[3] Kays W M 1966 Convective Heat and Mass transfer (McGraw-Hill, New York)
[4] Bourassa C, Thomas F O 2009 J. Fluid Mechanics 634 359–404
[5] Volchkov E P, Makarov M S, Sakhnov A Yu 2010 International Journal of Heat and Mass Transfer 53 2837–2843
[6] Gershbein E A 1971 Fluid Dynamics 6(3) 49–52 (in Russian)
[7] Schlichting H, Gersten K 2003 Boundary Layer Theory (Springer, Berlin)
[8] Ge X, Arolla S, Durbin P 2014 Flow Turbulence Combustion 93 37–61
[9] Tournier J-M P, El-Genk M S 2008 Energy Conversion and Management 49 469–492
[10] Anderson A D, Tannehill J C, Pletcher R H 1984 Computational Fluid Mechanics and Heat Transfer (Hemisphere Publishing Corp., New York)
[11] Volchkov E P, Makarov M S, Sakhnov A Yu 2012 International Journal of Heat and Mass Transfer 55 (4) 1126–1132
[12] Sakhnov A Yu 2015 International Journal of Heat and Mass Transfer 82 348–356