Procurements with Bidder Asymmetry in Cost and Risk-Aversion

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\textbf{ABSTRACT}

We propose an empirical method to analyze data from first-price procurements where bidders are asymmetric in cost and risk-aversion. In particular, we consider a theoretical model of FPP with exogenous entry and type-symmetric Bayesian Nash Equilibrium bidding strategies. Here, a type of bidder refers to a pair of cost density and constant relative risk-aversion (CRRA) coefficient. For this setting, Campo (2012) identifies the model primitive (cost densities and CRRA coefficients) and proposes an indirect semiparametric estimation method. The empirical literature, however, has seldom considered asymmetry in both cost and risk-aversion. We, therefore, have limited empirical insights on the effects of asymmetric cost and risk-aversion. Our counterfactual study shows that choosing a type-specific cost-minimizing reserve price marginally reduces the procurement cost; however, inviting one more bidder substantially reduces the cost, by at least 5.5%. Furthermore, incorrectly imposing risk-neutrality would severely mislead inference and policy recommendations, but the bias from imposing homogeneity in risk-aversion is small.

\section{Introduction}

This article proposes a Bayesian method to analyze data from first-price procurements (FPP) with bidder-asymmetry in cost and risk-aversion. In particular, we consider a theoretical model of FPP with exogenous entry and type-symmetric Bayesian Nash Equilibrium bidding strategies. Here, a type of bidder refers to a pair of cost density and constant relative risk-aversion (CRRA) coefficient. For this setting, Campo (2012) identifies the model primitive (cost densities and CRRA coefficients) and proposes an indirect semiparametric estimation method. The empirical literature, however, has seldom considered asymmetry in both cost and risk-aversion. We, therefore, have limited empirical insights on the effects of asymmetric risk-aversion on procurement outcomes.

The main contribution of this article is to develop a novel empirical procedure that produces reliable inference for asymmetric FPPs by combining and extending several state-of-the-art methods in the literature. First, the procedure explores the posterior distribution over the space of the model primitive using the boundary-value method and integrates out unobserved heterogeneity through data augmentation. We study a new dataset from Russian government procurements focusing on the category of printing papers. We find that there is no unobserved heterogeneity (presumably because the job is routine), but bidders are highly asymmetric in their cost and risk-aversion. Our counterfactual study shows that choosing a type-specific cost-minimizing reserve price marginally reduces the procurement cost; however, inviting one more bidder substantially reduces the cost, by at least 5.5%. Furthermore, incorrectly imposing risk-neutrality would severely mislead inference and policy recommendations, but the bias from imposing homogeneity in risk-aversion is small.

1. Introduction

This article proposes a Bayesian method to analyze data from first-price procurements (FPP) with bidder-asymmetry in cost and risk-aversion. In particular, we consider a theoretical model of FPP with exogenous entry and type-symmetric Bayesian Nash Equilibrium bidding strategies. Here, a type of bidder refers to a pair of cost density and constant relative risk-aversion (CRRA) coefficient. For this setting, Campo (2012) identifies the model primitive (cost densities and CRRA coefficients) and proposes an indirect semiparametric estimation method. The empirical literature, however, has seldom considered asymmetry in both cost and risk-aversion. We, therefore, have limited empirical insights on the effects of asymmetric risk-aversion on procurement outcomes.

The main contribution of this article is to develop a novel empirical procedure that produces reliable inference for asymmetric FPPs by combining and extending several state-of-the-art methods in the literature. First, the procedure explores the posterior distribution over the space of the model primitive by a Markov chain Monte Carlo (MCMC) algorithm (Kim 2015). Second, we model the (procurement-specific) unobserved heterogeneity as additional latent components that are distributed jointly with the model primitive under the posterior so that we can integrate out the latent components via MCMC. This strategy to get around the difficulty of handling missing variables is known as data augmentation, which Li and Zheng (2009) and Aryal et al. (2018), among others, use to study auction markets. Third, we extend the boundary-value method of Fibich and Gavish (2011) to compute the type-symmetric equilibrium strategies for risk-averse bidders in FPPs and use it in our MCMC algorithm. Note that the literature has often used the backward-shooting method of Marshall et al. (1994) to compute asymmetric bidding strategies, which is, however, shown to be inherently unstable near the boundary of the support; see Fibich and Gavish (2011). The boundary-value method is reliable everywhere and, therefore, is more suitable for evaluating likelihoods and conducting policy simulations.

In addition, our Bayesian method provides a natural framework for formal decision-making under parameter uncertainty. In particular, we consider the policymaker who wishes to choose a reserve price to minimize the (expected) procurement cost when there is uncertainty about the model primitives. Our decision method computes the procurement cost under a model primitive at a given, possibly type-specific, reserve price, which to our knowledge is the first in the empirical auction literature; see Kotowski (2018) for theoretical developments on this topic. For this step, we further extend the algorithm of Fibich and Gavish (2011) to accommodate binding reserve prices and evaluate the procurement cost by simulating equilibrium bids. The decision method then integrates out the model primitive by the posterior, resulting in the (posterior) predictive procurement cost at the said reserve price. Finally, our method selects a reserve price that gives the smallest predictive cost.

This solution is also coherent under the rationality axioms for a decision-maker in Savage (1954) and Anscombe and Aumann (1963). See also Kim (2013), Aryal and Kim (2013), Kim (2015), and Aryal et al. (2018) for applications of statistical decision theory in empirical auction design problems. This article is the
first to conduct such detailed counterfactual simulations for FPPs with asymmetric costs and risk-aversion.

Using our empirical method, we study a new dataset with FPPs from Russian public procurements conducted in 2014. (Charankevich 2021) investigates a sample of “open auctions” in the Russian procurements, which is different from the sample of FPPs that we analyze here.) In Russia, all government units purchase a wide range of goods and services through public procurements, which accounts for 7% of Russian GDP in 2014. Since the economic transition beginning in the 1990s, Russia has been revising the procurement system through several legislative changes and technological innovations to establish transparent competition among the suppliers to reduce the government expenditure. It is, therefore, critical for the policymaker to learn the economic fundamentals of the procurement system and evaluate alternative policy options for further improvements. We illustrate how the policymaker can achieve the goals applying our method to the data.

The Russian procurement system practices several allocation methods and selects one of them, for example, FPP or “open auctions,” depending on the nature of the project and according to the relevant federal laws. For example, small projects (reserve price below 500,000) must use an FPP. In total, there are 102 different categories in FPPs based on different goods and services. The categories are separate, each with its own market and a different set of suppliers. We choose one category to analyze by applying a set of selection criteria mostly concerning the sample size. We divide bidders into three types: type 1 (frequent) bidders bid in at least 10% of the procurements and type 3 (fringe) bidders only once, and type 2 are the remainders. Among the four categories complying with our selection criteria, we choose to analyze the category of “printing papers,” as it has the largest sample with 411 procurements. In the “printing papers” category, 1, 171, and 402 unique bidders are of types 1, 2, and 3, and we observe a total of 58, 625, and 402 bids, respectively.

On average, type 1 bidder bids lower than type 2, and type 2 lower than type 3. The differences in bids may arise because of differences in costs or risk-aversion. The posterior of the model parameters (model primitive) reveals that the ordering of risk-aversion is the opposite of the observed bid pattern. In particular, the CRRA coefficient of type 1 (frequent) bidder is roughly 0.2, whereas the coefficients of type 2 and 3 bidders are respectively, 0.8 and 0.9. But, the predictive cost densities suggest that type 1 bidder tends to draw smaller costs than type 2, and type 2 smaller than type 3. Therefore, the difference between the cost densities is substantial enough to explain the bid pattern, despite the reversedly ordered risk-aversion. Our analysis also finds no variation in the unobserved heterogeneity, which is consistent with the job of supplying papers being routine.

Our estimates of the model parameters are coherent with our findings in the counterfactual policy analysis. All bidders, except the most frequent bidder, exhibit high risk-aversion. So, the policymaker should prefer FPP to second-price procurements (SPP) (Holt 1980) and choose a large reserve price in an FPP (Hu, Matthews, and Zou 2010). In particular, we find that when a single reserve price is used for all bidders, the current mechanism is cost-minimizing. Recall also that when bidders are asymmetric, choosing a single reserve price would be suboptimal. Although there is no closed-form expression for the reserve prices as in Myerson (1981) due to risk-aversion, our method allows evaluating the procurement costs at different type-specific reserve prices. It recommends lowering the reserve price for type 1 by 4% from the current reserve price, but leaving other types’ prices unchanged. In that case, however, our method predicts a cost reduction only of 0.2%, suggesting that the current mechanism is effectively cost-minimizing.

In addition, we consider a scenario where the policymaker may invite an additional bidder in the spirit of Bulow and Klemperer (1996). For symmetric FPPs, the article implies that an FPP without a reserve price generates lower expected costs than optimally chosen reserve price but with one less bidder. We find that this insight holds in our context with bidders who are asymmetric in both cost densities and risk-aversion. In particular, adding one bidder of type 1, 2, and 3 reduces the predictive procurement cost by 6.2%, 5.8%, and 5.5%, respectively. Thus, even one additional fringe bidder substantially lowers the cost.

Moreover, we investigate the implications on the procurement cost and efficiency of incorrectly assuming either risk neutrality or an identical CRRA parameter for all bidders. When one ignores risk-aversion, small bids in the data inflate the left tail of the cost densities, which then tilts cost-minimizing reserve prices toward zero. As a result, the misspecified model selects a small reserve price, predicting 14.0% of cost-reduction. But, this result is misleading because the model with asymmetric CRRAs predicts a 15.2% of increase in the cost at that price. The misspecified model predicts that the efficient bidder wins with 33.2% of probability, whereas the model with asymmetric CRRAs predicts that the probability would be 6.0%. We find that the model with a common CRRA overall approximates our analysis with asymmetric CRRAs, except for overestimating type 1 bidder’s CRRA coefficient.

Nevertheless, one should not conclude that the model with identical risk-aversion always approximates the model with asymmetric risk-aversion because the approximation quality may depend on the model primitive, a priori unknown to the researcher. Our method with asymmetric CRRAs is not computationally more expensive than the model with a common CRRA. Therefore, there is no reason to impose homogeneity in CRRA coefficients. Finally, our analysis is robust to alternative definitions of bidder types, the prior over the parameter that indexes the cost densities, and the density specification of unobserved heterogeneity.

The article proceeds as follows. Section 2 describes our model and its identification. Sections 3 and 4 present our data and econometric method, respectively. Section 5 discusses the empirical results and counterfactual analysis. Section 6 concludes with feasible extensions. The detailed information about data, computational detail, and additional results are in supplementary materials (Aryal et al. 2022).

2. Model and Identification

Consider a procurement that allocates a project to one of the bidders in the set \( I \) with \(|I| \geq 2\). Bidders submit their bids simultaneously, and the one with the lowest bid wins the project at a price equal to her bid, which we refer to as the first-price
procurement (FPP). Let bidder \( i \)'s cost be \( c_i \), which follows the distribution \( F_i(\cdot) \) and is independent of other bidders' costs. We make the following assumptions on the cost distributions.

**Assumption 1.** Bidder \( i \)'s cost distribution \( F_i(\cdot) \) has a density \( f_i(\cdot) > 0 \) on the support \([c_i, \infty) \subset \mathbb{R}_+\), and for two different bidders \( i \neq j \) it can be that \( F_i(\cdot) \neq F_j(\cdot) \) for some \( c \).

In addition, bidder \( i \) exhibits constant relative risk-aversion (CRRA) with coefficient, \( \eta_i \).

**Assumption 2.** Bidder \( i \)'s utility function is given by \( U_i(\xi) = \xi^{1-\eta_i} \) for consumption \( \xi \geq 0 \) with the parameter \( \eta_i \in (0, 1) \), and it can be that \( \eta_i \neq \eta_j \) if \( i \neq j \).

The CRRA specification has been widely used due to its convenient functional form in the auction literature, for example, Bajari and Hortacsu (2005), Lu and Perrigne (2008), Campo et al. (2011), and Aryal et al. (2018). Such convenience also allows the boundary-value method of Fibich and Gavish (2011) to accommodate bidder-specific risk-aversion in this article, and one may compare the coefficients with previous estimates due to its prevalence.

If bidder \( i \) wins the procurement at price \( b_i \), her utility is \((b_i - c_i)^{1-\eta_i} \) under **Assumption 2**. While the realizations of costs are bidders' private information, cost distributions and risk-aversion parameters, \( \{F_i(\cdot), \eta_i : i \in \mathcal{I}\} \), are assumed to be common knowledge. Bidder \( i \) with cost \( c_i \) chooses \( b_i \) to maximize her expected utility given everyone else's bidding strategy. Suppose all bidders other than bidder \( i \) use strictly increasing equilibrium bidding strategies \( \{\beta_i(\cdot | J) : j \neq i, j \in \mathcal{I}\} \). Then bidder \( i \) solves

\[
\max_{b_i \in \mathbb{R}_+} \left\{ (b_i - c_i)^{1-\eta_i} \prod_{j \in \mathcal{I} \setminus \{i\}} (1 - F_j(\beta_j^{-1}(b_i | J))) \right\}.
\]

Define \( \phi_i(b | J) := \beta_j^{-1}(b | J) \), the inverse bidding strategy of bidder \( j \). Then, the optimal bid \( b_i \) must satisfy the condition,

\[
1 - \eta_i = (b_i - c_i) \sum_{j \in \mathcal{I} \setminus \{i\}} \frac{f_j(\phi_j(b_i | J))}{1 - F_j(\phi_j(b_i | J))} \frac{1}{\phi_j'(\phi_j(b_i | J)) J},
\]

for all \( i \in \mathcal{I} \), implying a system of differential equations, which can be numerically solved with the boundary conditions \( \phi_i(c_i | J) = c_i \) and \( \phi_j(b_j | J) = c_j \) for all \( i \in \mathcal{I} \).

Since \( b_J = \beta_i(c_i | J) \) is unknown, the standard algorithm to solve (1), known as *backward-shooting*, starts with a guess of \( b_J \) and adjusts its guess at each iteration (Marshall et al. 1994). Fibich and Gavish (2011), however, show that the backward-shooting algorithm is inherently unstable near the boundary and propose the boundary-value method to overcome the problem. We use their boundary-value method in our empirical method, where \( \{\phi_i(\cdot | J)\} \) have to be evaluated at data points, including the ones near the boundary.

### 2.1. Identification

Campo (2012) uses the exogenous variation in the bidder configuration \( \mathcal{I} \) to identify the parameters of interest. For completeness, we present the core intuition of the identification strategy in Campo (2012). Since \( \{\phi_i : i \in \mathcal{I}\} \) are strictly increasing, the bid distribution of bidder \( i \in \mathcal{I} \) is \( G_i(b | J) = F_i(\phi_i(b | J)) \). Then, we can rewrite (1) as

\[
c_i = b_i - \frac{1 - \eta_i}{\sum_{j \in \mathcal{I} \setminus \{i\}} g_j(b_j | J) / 1 - G_j(b_j | J)},
\]

where \( g_j(\cdot | J) \) is the density of \( G_j(\cdot | J) \). Note that (2) in a procurement with \( \mathcal{I} = \{1, 2\} \) gives

\[
b_1 - \frac{1 - \eta_2}{g_1(b_1 | J) / 1 - G_1(b_1 | J)} = \xi = b_2 - \frac{1 - \eta_1}{g_2(b_2 | J) / 1 - G_2(b_2 | J)},
\]

which gives \( g_1(b_1 | J) \eta_1 - g_2(b_2 | J) \eta_2 = g_1(b_1 | J) - g_2(b_2 | J) \).

Then, the exogenous variation in \( \mathcal{I} \) is sufficient for identification. For example, if we observe FPPs with \( \mathcal{I}_1 = \{1, 2\}, \mathcal{I}_2 = \{1, 3\}, \) and \( \mathcal{I}_3 = \{2, 3\} \), we identify the CRRA coefficients as

\[
\begin{pmatrix}
\eta_1 \\
\eta_2 \\
\eta_3
\end{pmatrix} = \begin{pmatrix}
g_1(b_1 | J_1) \eta_1 - g_2(b_2 | J_1) \eta_2 & 0 \\
g_1(b_1 | J_2) \eta_1 & 0 & -g_2(b_2 | J_2) \eta_2 \\
g_1(b_1 | J_3) \eta_1 & g_2(b_2 | J_3) \eta_2 & 0
\end{pmatrix}^{-1}
\]

where the right-hand side depends only on the bid densities that are directly identified from the data. Then, by substituting \( \eta_i \) in (2), we identify the cost distributions.

### 2.2. Unobserved Heterogeneity

Bidders may observe some aspects of the project that affect their costs (and hence their bids), which the researcher does not observe. Let \( u \in \mathbb{R}_+ \) denote such unobserved characteristics.

**Assumption 3.** 1. \( u_i \overset{i.i.d.}{\sim} F_u \) with density \( f_u > 0 \) on the support \([c_i, \infty) \subset \mathbb{R}_+ \) with a location normalization, for example, \( \overline{u} \) is known.

2. In procurement \( t \), \( u_t \) is independent of bidder's private cost, that is, \( u_t \perp c_i, \) for all bidders.

3. The final cost for bidder \( i \) with \( c_i \) in procurement \( t \) is given by \( c_i' := u_t \times c_i \).

Let \( F_u^t \) be the distribution of \( c_i' \) for bidder \( i \in \mathcal{I} \) and \( \{\beta_i^t : i \in \mathcal{I}\} \) be the associated bidding strategies with the unobserved heterogeneity, \( u_t \). For \( c_i' \in [c_i, \overline{c}_i] \) and \( u_t \in [\overline{u}, \overline{u}], \beta_i^t(u_t, c_i') \mathcal{I} = u_t \times \beta_i(c_i' | J) \), where \( \beta_i(c_i | J) \) is bidder \( i \)'s equilibrium bidding strategy when \( u_t = 1 \); see Liu and Luo (2017). The identification argument combines Campo (2012) with Kotlarski (1966) as in Li and Vuong (1998) and Krasnkoutskaya (2011). We formalize this result below; see its proof in section S2 of the supplementary materials (S for the appendix, hereafter).

**Lemma 1.** Under Assumptions 1–3, \( \{F_i, \eta_i : i \in \mathcal{I}\} \) and \( F_u \) are identified by the data of all submitted bids \( \{(b_i)_h = \in \mathcal{I}_h\} \) and bidder configurations \( \mathcal{I}_h \) if every bidder \( i \) exogenously enters the procurement with a strictly positive probability (so, \( \mathcal{I}_h \) varies exogenously).
3. Russian Government Procurement

This section describes the institutional background of the government procurements in Russia, presents the dataset we analyze, and discusses the implications of the reserve price on our analysis in Sections 4 and 5. In particular, the background description here identifies a few cases where observed bids might not be competitive and justifies the data we study as equilibrium outcomes after excluding those suspicious cases. It also motivates us to take cost-minimization as the primary policy objective in counterfactual analysis.

3.1. Institutional Background

All government bodies and public units in Russia purchase goods and services through government procurements. Examples of potential buyers include federal public authorities, regional governments, city councils, public hospitals, and schools. Hundreds of goods and services, for example, car tires, hardcover textbooks, road maintenance, and printing papers, are traded via the official platform, “Unified Information System (UIS: zakupki.gov.ru).” (We use quotation marks to indicate field terms in their closest English translation. Moreover, procurement here is a general term referring to procuring something unless it comes with a technical qualifier, e.g., first-price procurement.) The public procurements are economically significant; according to the UIS, concluded contracts in 2014 add up to 5.47 trillion RUB ($2600), the procurer must select a supplier through a competitive procedure. The “maximum price” is then publicly announced, and any legal entity can participate with no entrance fee. The announcement should be placed at least four business days before the closing date if the “maximum price” is below 250,000 (and seven days if above) to prevent buyers from selecting an “insider” by setting a tight deadline.

In addition, to improve transparency, by Federal Law No. 44-FZ (January 1, 2014), the UIS publicly announces forthcoming procurements and maintains all procurement data, for example, participants’ identities, their offers, and the winner for every procurement, and the UIS provides a platform to run a procurement. The system practices several allocation methods, including negotiation and contest, and one of them is “sealed-bid auction” in the field term. When the latter is implemented, bidders submit their initial documents in a sealed envelope (or online, but rarely in 2014). In the presence of all participants, then, each bidder makes a final decision on her bid and, finally, the procurer opens all the bids (Federal Law No. 44-FZ, article 78 of paragraph 3). Note that the procurer, here, is a government agency running procurements on behalf of buyers, other government agents. The lowest bidder wins at a price equal to her bid, if not higher than the “maximum price.” The mechanism is, therefore, the first-price procurement (FPP), where bidders know whom they oppose, and the “maximum price” plays the role of a reserve price in FPPs.

FPPs are used for cases with a “maximum price” below 500,000, that is, small projects. But, the penalty for a supplier in corruption, for example, “insider,” is 500,000 plus a full reimbursement of expenses. The supplier is also publicly marked as “unreliable” for two years. In addition, the fine for the government agent in corruption is 50,000 with a three-year job suspension.

Despite all the legal devices to eliminate corruption, a buyer can still invite an “insider” only. For example, a buyer could deliberately make a typographical error, for example, replacing a Cyrillic letter with a similar Latin letter in keywords. Then, only the insider can easily search for it in the system. Therefore, the cases with only one bidder can be suspicious. Even when multiple bidders appear in a procurement, there can be corruption. For example, the agent could invite shill bidders who submit bids with no chance of winning, for example, bidding higher than the reserve price. Alternatively, the agent may manipulate submitted bids to increase the winning probability of the insider. Charankevich (2021) reports evidence of bid manipulation by exploring procurement outcomes and non-reported (or missing) bids in “open auctions,” which can be viewed as an oral-descending procurement. Considering all these, in our analysis, we exclude observations with only one bidder, bids above reserve prices, or missing bids to avoid those suspicious cases.

3.2. Data: Category “printing papers”

The procurements with FPPs provide an ideal setting to study asymmetry in both cost densities and risk-aversion for the following reasons. First, bid data from FPPs allow us to identify the risk-aversion parameters. Second, FPPs are less prone to collusion among the bidders than oral-descending procurements because bidders do not observe other bidders’ behavior...
(Robinson 1985). Third, since FPPs are used for projects with small budgets, they may attract small firms that are likely to be risk-averse (Herranz, Krasa, and Villamil 2015). Indeed, the test of Jun and Zinchenko (2022) rejects risk-neutrality in favor of risk-aversion ($p$-value $< 0.0001$) for each symmetric FPPs, that is, type 2 bidders only and type 3 bidders only with types defined below.

Finally, projects with small budgets are homogeneous and frequent, that is, the bid preparation cost would be minimal, and the jobs are routine. To this end, we fail to reject the independence of any two randomly selected bids in the same FPP ($p$-value $> 0.1$) via the test used by Krasnokutskaya (2011) with 1000 bootstrapped samples. For these reasons, the procurement-specific unobserved heterogeneity might not be substantial, which Section 5 shall confirm from the data.

The UIS provides data on 42,828 FPPs in 2014 across 102 categories of different goods and services; see section S3.1 for the list of the categories and some statistics. We consider each category as an independent set of procurements because they are separated by industries with different sets of suppliers. Now, we explain how we select a category to analyze; see also section S3.1.

In each category, first, we rule out all procurements with one bidder because they are vulnerable to corruption (Section 3.1) and are not even bidding competition. We then exclude procurements with missing bids, which may arise due to bid manipulation (Charankevich 2021). We also discard the procurements with bids larger than reserve prices because those bids could be shill bids as discussed or could signal that the reserve prices are set too low, violating the UIS protocols to select a reserve price.

After excluding these three cases, we group bidders into three types: in each category, type 1 bidders bid in at least 10% of the FPPs, type 3 bidders bid once, and type 2 bidders are all the others. We define the bidder types by the participation rate because it is the only exogenous bidder-specific attribute in the data, besides their identities. (Section 5.3 considers alternative definitions.) Since the identification strategy relies on the variation in the bid for each type and bidder configuration, we sort out categories with at least 50 bids for each type to estimate the type-specific parameters. Four categories satisfy this condition. Among them, we analyze the one with the largest sample, which is the category of "printing papers" with $T = 411$ procurements. (In the previous version of this article, Aryal et al. (2020) study the four categories separately, where the other ones have 228, 235, and 305 procurements.)

The category "printing papers" refers to white A4 paper for printing, copying, and faxing. A typical order specifies A4 white papers in packs of 500 sheets. The paper can vary from "regular white" of 92%–94% (ISO) to "premium white" above 98% (ISO). A standard contract includes the delivery term that can range from 5 to 30 days and provision for the fulfillment of incomplete orders and replacement or exchange of damaged goods within 3–10 days after delivery. As mentioned above, any government unit can purchase the products, and any legal entity that can supply papers may bid in this category.

Now, we document some descriptive statistics of the data from the "printing papers" category. This category initially has 536 FPPs. Among them, we exclude 114 for having only one bid, three for missing bids, and eight for bids above the reserve price. Among the eight bids, five (three) were submitted by bidders who bid once (twice). No bidder repeats bidding above the reserve price, and no one wins with such a bid. In the sample of 411 ($= 536 - (114 + 3 + 8)$) procurements, we have one type 1 bidder who bids 58 times, 171 type 2 bidders with 625 bids, and 402 type 3 bidders. The second and third frequent bidders appear 33 and 14 times in the data. To assess how the type definition affects our analysis, in Section 5.3 we change the definition of type 1 to include the second frequent bidder and then to also include the third frequent bidder. The entrance rate dramatically drops only for the first a few bidders; see the circles in Figure 1(a), suggesting that those bidders may differ from other bidders, that is, asymmetry in model primitives. Section 5.3 also considers alternative type definitions based on how often each bidder wins; see the crosses in Figure 1(a).

Figure 1(b) shows the histogram of bids from procurements with two bidders. It has 474 bids and (25,260,189) of them, that is, (5, 55, 40)% are submitted by type (1, 2, 3) bidders, respectively. The sample mean and standard deviation of the bids are 0.93 and 0.08. Panels (c)–(f) similarly show the bid data for procurements with 3–6 bidders.

Recall that bidders know their competitors when bidding; see Section 3.1. In the data, bidders appear to use the information on the competition. The bid distributions vary with the number of bidders, and there is a tendency that the average bid decreases in the number of bidders. We conduct the Kolmogorov–Smirnov (KS) test against the hypothesis that two marginal bid distributions are identical. The $p$ values are close to zero for all pairs involving the two bidder case (b) and for the pair of three bidder case (c) and four bidder case (d), rejecting the hypothesis of identical distributions. That is, we have some evidence that bidders appear to behave differently according to the competition level; Section S3.2 gives more evidence. As the number of bidders grows, however, the bid distributions get harder to distinguish statistically. That might be because the bid weakly converges to the cost with the competition level. The number of bidders also gets noisier in measuring the competition because the bidder configuration becomes more variable when bidders are asymmetric.

On average, type 3 bidders bid higher than type 2 and type 2 higher than type 1 (panels (g)–(i)). (For any pair of two types, the $p$-value of the KS test is close to zero.) This pattern arises if type $\tau$ is either more efficient or more risk-averse than type $\tau + 1$. But, the cause of the pattern cannot be detected by reduced form analysis, motivating a structural approach.

Figure 2 shows the bid histograms for symmetric procurements with only type 2 and type 3 bidders in the upper (lower) panels. Note that there is no symmetric procurements with type 1 bidder, as there is one type 1 bidder. The left and middle panels are for 2 and 3 bidder procurements and the right ones for the rest in the data. As discussed there, we conduct the KS test and get additional evidence that bidders’ behavior depends on the competition level, especially for the cases with sufficiently large bid samples.

Although we have excluded the procurements with bids above reserve prices, the high density near the reserve price may still seemingly suggest the presence of shill bids (no winning chance). According to the data, however, the bid of 0.99 gives a 14% chance of winning. Hence, the high density does not indicate shill bids. It is, instead, plausible that bidders’ optimal
Figure 1. Bid data, “printing paper.” Panel (a) shows the number of entrants (○) for the 10 frequent bidders and their numbers of wins (×). Panels (b)–(f) present bid histograms for different numbers of bidders and (g)–(i) for different types of bidders. All bids are divided by reserve prices. Panels (b)–(i) show the number and [average, standard deviation] of bids, and (b) to (f) counts bids with the proportion for each type in ( ). At the 1% level, the KS test rejects the homogeneity in bid distributions between (b) and any of (c) ~ (f), between (c) and (d), and for all pairs in {(g),(h),(i)}.

Figure 2. Bid data, symmetric procurements, “printing paper.” For types 2 (upper) and 3 (lower), the left/middle/right panel shows the histogram of bid data from symmetric procurements with 2/3/4+ bidders. All bids are divided by reserve prices. Each panel shows the number of bids and their [average, standard deviation]. For each type [For each number of bidders], the KS test rejects the distributional homogeneity against the alternative that the bid with the larger number of [of type 2] bidders is first-order stochastically dominated at the 5% level except for the pair of (b) and (c) [(c) and (f)].

bids are actually near the reserve price because the reserve price has to be justifiable as a market price as discussed in Section 3.1; see also Section 3.3.

Finally, a group of suppliers may collude (not necessarily involving the government agent) to submit noncompetitive bids. Empirical methods to detect a bidding ring require bidders to be
risk-neutral and a suspicious group of bidders to bid together in many procurements; see Schurter (2020) and references therein. It is infeasible to study a bidding ring using our data because even if the currently available methods may extend for risk-aversion, the dropping entrance rate in Figure 1(a) does not leave us any group of bidders jointly appearing in sufficiently many procurements. In particular, among 45 pairs of top 10 frequent bidders, only eight have ever bid in the same procurements, but rarely: type 1 bidder met the (2, 4, 6, 7, 9)th frequent bidders (5, 1, 1, 7, 4) times, respectively, and the second and sixth bidders met twice, the third and fifth bidders three times, and the fifth and tenth bidders five times.

### 3.3. Reserve Price

Recall that the reserve price is set sufficiently high to encourage suppliers’ participation and yet still justifiable as a market price to purchase comparable goods and services outside the procurement system. This description has three important implications on our analysis in the following sections. First, the procurement system does not use the reserve price to deter any bidder from entering, which alone might validate the reserve price as non-binding. Second, the description of the reserve price also implies that if a supplier incurred a cost higher than the reserve price, the cost is too high for the supplier to operate in the market, even outside the procurements. Hence, we do not consider such an inefficient supplier as a potential bidder in the procurement, and Section 4 specifies the cost densities to have their support below the current reserve price, that is, the latter is non-binding. Third, in our policy simulations, when no bidder can bid below the counterfactual reserve price, we assume that the procurer carries out the job at the current reserve price; see Section 5.2.

### 4. Inference Method

A set $\mathcal{I}_t$ of bidders who can beat the reserve price is exogenously given for each procurement $t \in \{1, \ldots, T\}$. Bidder $i \in \mathcal{I}_t$ with type $\tau(i) \in \{1, \ldots, \tau\}$ has the CRRA coefficient $\eta_{\tau(i)}$ and draws her cost $c_{it}$ from the cost distribution $F_{\tau(i)}(\cdot)$ with density $f_{\tau(i)}(\cdot)$ independently of other bidders. The cost density is strictly positive on $[0, 1]$, corresponding to the bid data normalized by the reserve price. Let $\eta := \{\eta_{\tau} : \tau \in \{1, \ldots, \tau\}\}$ and $\theta := (f, \eta)$. Let $\beta_{\tau(i)}(\cdot | \theta, \mathcal{I}_t)$ be the bidding strategy when $u_t = 1$. That is, bidder $i$ bids $b_{it} = \beta_{\tau(i)}(c_{it} | \theta, \mathcal{I}_t) \in [\underline{b}(\theta, \mathcal{I}_t), 1]$, where $\underline{b}(\theta, \mathcal{I}_t) := \beta_{\tau(i)}(0 | \theta, \mathcal{I}_t)$ for all $i \in \mathcal{I}_t$. When $u_t \neq 1$, bidder $i$ observes her cost $c_{it} \in u_t \times [0, 1] = [0, u_t]$ and bids $b_{it} = u_t \beta_{\tau(i)}(c_{it} | \theta, \mathcal{I}_t)$, which lies in $[u_t \underline{b}(\theta, \mathcal{I}_t), u_t]$. We model $u_1, \ldots, u_T \sim f_u(\cdot)$ with the support of $[0, 1]$ and $u > 0$. Note that $u_t \leq 1$ reflects the institutional feature that the reserve price does not exclude any bidder.

For estimation, we specify the cost densities as

$$f_{\tau}(c) := f(c | \psi_{\tau}) := \frac{0.01 + 0.99 \times \exp \left( \frac{\phi(\psi_{\tau})}{f_{\tau}\exp \left( \phi(\psi_{\tau}) \right) dc} \right)}{1 \times 1(c \in [0, 1]),}
$$

where $\psi_{\tau} \in \mathbb{R}^k$ is the vector of parameters and $\phi(\cdot)$ is the vector of the $k$ subsequent Legendre polynomials, defined on $[0, 1]$. Specifically, the $j$th entry in $\phi(\cdot)$ is given as $\phi(j(c)) := \sqrt{2j + 1} \times \phi_j(2u - 1)$, where $\phi_j(x) := \frac{\partial^j}{\partial x^j} (x^2 - 1)/(2j)$. The uniform component in (3) with the small weight (0.01) ensures the density is strictly bounded away from zero.

Note that $\psi_{\tau}$ has $j - 1$ extrema; see section S4 for graphs of $\psi_{\tau}(\cdot)$ for some $j$s. As $k$ increases, the density of $f_u(\cdot)$ defined in (3) can approximate more complicated densities, that is, the ones with many inflection points. Believing that the true cost densities are smooth, we put a smaller prior probability on $\psi_{\tau}$.

For the unobserved heterogeneity, we specify $f_u(\cdot)$ such that it implies

$$u_t \mid \sim \mathcal{N}(1, \sigma_u^2) \times \mathcal{U}(u_t \in [u(\sigma_u), 1]),
$$

where the lower bound $u(\sigma_u) := 1 - c_u \times \sigma_u$ with $c_u$ such that $Pr(\mathcal{N}(0, 1) \leq c_u) = 0.99$, and the upper bound is set to 1 because the reserve price is non-binding, larger than the upper bound of the cost. To ensure that $u_t$ is sufficiently larger than zero, we restrict $u(\sigma_u) \geq 0.1$. This restriction implies the upper bound of $u_t$, that is, $u_t \leq \sigma_u = (1 - 0.1) / c_u$. We then use the uniform prior $\pi_{u}(\sigma_u) \propto 1(0, \sigma_u)$. For the CRRA parameter, $\eta_{\tau}$, we use a uniform prior $\pi_{\eta_{\tau}}(\eta_{\tau}) \propto 1(0, 0.9)$, which excludes values near 1 to ensure that our computation does not fail due to the flat utility function, that is, $u(x) = x^{1-\eta} \rightarrow 1$ as $\eta \rightarrow 1$. We collect the priors by $\pi(\theta, \sigma_u) := \prod_{t=1}^{T} \pi_{\eta_{\tau}}(\eta_{\tau}) \pi_{u}(\sigma_u) \pi_{\eta_{\tau}}(\eta_{\tau})$, where $\theta := \{\theta_{\tau} \}_{t=1}^{T} \mid = \{\psi_{\tau}, \eta_{\tau}, \tau = 3 \}$.

Conditional on $(u_t, \theta)$, the bid $b_{it}$ in procurement $t$ has the density

$$g_{\tau(i)}(b_{i | t | u_t, \theta, \mathcal{I}_t}) := \frac{1}{u_t} \times \frac{f(\beta_{\tau(i)}^{-1}(b_{it} / u_t | \theta, \mathcal{I}_t) | \psi_{\tau})}{\beta_{\tau(i)}^{-1}(b_{it} / u_t | \theta, \mathcal{I}_t)} \times \mathcal{U}(b_{it} / u_t \in [b_{it}(\theta, \mathcal{I}_t), 1]),
$$

for which we need to evaluate $\beta_{\tau(i)}^{-1}(\cdot)$. To do so, section S5 modifies the boundary-value method to accommodate bidders with CRRA utility in CPPs, which Fibich and Gavish (2011) originally propose for risk-neutral bidders in high-bid auctions.

Then, the posterior density of the latent variables is given as

$$\pi(\theta, \sigma_u, (u_t)_{t=1}^{T} | z) \propto \pi(\theta, \sigma_u) \prod_{t=1}^{T} \left\{ f_u(u_t | \sigma_u) \prod_{i \in \mathcal{I}_t} g_{\tau(i)}(b_{it} | u_t, \theta, \mathcal{I}_t) \right\},
$$

where $z := \{(b_{it} \in \mathcal{I}_t, \mathcal{I}_t)_{t=1}^{T}$, including bids and bidder configurations in the dataset. Once we have the posterior of the structural parameter $\theta$, we are mostly interested in the posterior moments of important functions of $\theta$. For a measurable function $h(\theta)$, its rth posterior moment is $E[h(\theta)^r | z] := \int h(\theta)^r \pi(\theta | z) d\theta$. The posterior mean ($r = 1$) is often presented along with some uncertainty notions such as the posterior standard deviation, for which $r = 2$ is also used. To evaluate the moments we first draws $\theta^{(1)}, \ldots, \theta^{(M)} \sim \pi_0(\theta | z)$ by a standard
Markov chain Monte Carlo (MCMC) algorithm; see section S6. Then, we evaluate the moments by the MCMC draws, that is, 
\[ M^{-1} \sum_{m=1}^{M} h(\theta^{(m)}) \overset{a.s.}{\rightarrow} E[h(\theta)']|\mathbf{z}] \]

We conclude this section with a summary of section S7 that evaluates the performance of our method using simulated data. We consider three types of bidders, each with a different cost density and CRRA parameter, in FPPs with a substantial variation of bidder configurations: \( A \in \{(11), (111), (22), (222), (33), (333), (12), (13), (23), (123)\} \). We consider two cases: one with a substantial variation in the unobserved heterogeneity, \( \sigma_u > 0 \), and the other with no variation \( \sigma_u = 0 \). For each case, we consider two different sample sizes, \( T_A \in \{20, 100\} \) for all \( A \). Using those bid data we find that the MCMC traces are stable and converge quickly, and parameters are accurately estimated. As desired, the posterior of quantities of interests, such as \( f_c(\cdot|\theta) \) and \( f_u(\cdot|\sigma_u^2) \), becomes more precise around the true values, whether \( \sigma_u > 0 \) or \( \sigma_u = 0 \), as the sample size increases.

5. Inference and Counterfactual Results

This section first summarizes the posterior distribution of the parameters of the model with asymmetric risk-aversion, and discusses the bias from two model misspecifications: imposing symmetric risk-aversion or ignoring risk-aversion altogether. Then, the section predicts procurement costs under counterfactual scenarios, and also quantifies the impacts of model misspecification in terms of procurement costs. Finally, the section concludes with some sensitivity analysis.

5.1. Posterior Inference

5.1.1. Asymmetric Risk-Aversion

We sample \( (\theta, \sigma_u) \) from the posterior using the data discussed in Section 3, allowing for bidder asymmetry in cost density and risk-aversion. The top block of Table 1 shows the posterior mean, standard deviation, and a 95% credible interval (2.5 and 97.5 percentiles) for each CRRA coefficient \( \eta_t \) for \( t \in \{1, 2, 3\} \) and \( \sigma_u \).

The estimates suggest that risk-aversion varies across the three types. Type 1 (most frequent) bidder is the least risk-averse, and type 3 (one-time) bidders are the most risk-averse. The posterior of \( \eta_1 \) has a considerable variation, and its mean is substantially smaller than the ones of \( \eta_2 \) and \( \eta_3 \). The distribution of \( \eta_2 \) also differs from the one of \( \eta_3 \), where the latter is more precise and (slightly) larger. Formally, the \( p \) values of KS test are all close to zero, rejecting the hypothesis that \( \eta_1 \) and \( \eta_2 \) follow the identical marginal posterior distributions for \( t \neq \tilde{t} \). Table S8 discusses the use of KS test in Bayesian analysis.

In addition, there is no unobserved heterogeneity, \( \sigma_u \sim 0 \), which is consistent with supplying A4 papers being routine. If present, any measurable variation in the unobserved heterogeneity should have led to a nondegenerate posterior of \( u_t \), even if the parametric assumption in (4) was incorrect. Section S5.3 also obtains a degenerate posterior of \( u_t \) with a more flexible density of \( u_t \).

The top panels in Figure 3 show, for each type \( t \in \{1, 2, 3\} \), the posterior mean of the cost density (3) at every point \( c \in [0, 1] \) by a solid line and a 95% credible band around the predictive density by dotted lines. Bidders are asymmetric in the cost densities. Type 1 (frequent) bidder is more efficient in supplying papers than the other bidders, with type 3 (one-time) bidders being the least efficient. This prediction, especially for types 2 and 3, is precise, as indicated by the tight credible bands. Moreover, by the KS test, we reject the hypothesis that the posterior predictive cost distributions are identical for each pair of types; the relevant \( p \)-values are all close to zero. Note that we apply the KS test to cost samples drawn from the predictive cost densities; see section S8.

5.1.2. Homogeneous Risk-Aversion

We analyze the data again but imposing \( \eta = \eta_{\tilde{t}} \) for all \( t \in \{1, 2, 3\} \). The middle block of Table 1 summarizes the posterior distribution of \( \eta_t \), which precisely predicts that all bidders are highly risk-averse. Note that we reject the hypothesis that the posterior distribution of the constrained \( \eta \) is identical to the posterior of \( \eta_{\tilde{t}} \) from the upper block (heterogeneous CRRA) for each \( t \in \{1, 2, 3\} \). For this constrained model, there is no unobserved heterogeneity \( \sigma_u \approx 0 \) as with asymmetric risk-aversion above. The middle panels of Figure 3 show the posterior predictive densities (solid) and 95% credible bands (dotted), where the dashed lines copy the predictive densities from the upper block for comparison. Imposing homogeneity in risk-aversion generates a slightly different predictive cost density for type 1 bidder, but the other types remain the same. The KS test

| Table 1. Posterior distributions of CRRA coefficients and UH parameters. |
|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
|                             | Posterior                    | Posterior                    | Posterior                    | \( p \)-value of KS test    |
|                             | Mean (1)                     | SD (2)                       | 95% cred. int. (3)           | \( \eta_1 \) (4)            |
| Heterogeneous CRRA \( \eta \) | 0.196                        | 0.165                        | [0.005, 0.596]               | 0.000                      |
| \( \eta_2 \)                 | 0.828                        | 0.043                        | [0.729, 0.893]               | 0.000                      |
| \( \eta_3 \)                 | 0.891                        | 0.009                        | [0.867, 0.900]               | 0.000                      |
| \( \sigma_u \)               | 0.000                        | 0.000                        | [0.000, 0.000]               | 0.000                      |
| Homogeneous CRRA \( \eta \)  | 0.889                        | 0.009                        | [0.865, 0.900]               | 0.000                      |
| \( \sigma_u \)               | 0.000                        | 0.000                        | [0.000, 0.000]               | 0.000                      |
| No CRRA \( \sigma_u \)       | 0.066                        | 0.001                        | [0.064, 0.068]               | 0.000                      |

**NOTE:** The table shows the posterior distributions of CRRA parameters and the parameter of the distribution of the unobserved heterogeneity by their posterior means (1), standard deviations (2), and posterior 95% credible intervals (3). The columns (4)–(6) report the \( p \)-values of the KS test against the hypothesis that the two CRRA coefficients are drawn from the same (posterior) distribution.
Figure 3. Posterior predictive cost densities. The upper block shows the posterior predictive cost density (solid) and 95% credible band (dotted) for each type. The middle and lower repeat the exercise by imposing homogeneity in risk-aversion and imposing no risk-aversion, respectively. The dashed lines are the predictive densities pasted from the upper block.

with the 5% level rejects the hypothesis that the predictive cost distribution remains the same under the constraint for type 1 bidders.

5.1.3. Risk-Neutrality
We repeat the exercise but restricting $\eta_\tau = 0$ for all $\tau$. Table 1 and Figure 3 (bottom) suggest that this misspecification induces an overestimation of unobserved heterogeneity and substantial bias in predicting cost densities; the more risk-averse, the larger the bias. Once we impose risk-neutrality, smaller bids in our sample must be justified by other model components, causing the bias pattern as we observe. For all $\tau \in \{1, 2, 3\}$, the K-S test rejects, by $p$-value $\approx 0$, the hypothesis that the cost under $\eta_\tau = 0$ follows the same predictive distribution as the cost without the constraint.

5.2. Counterfactual Analysis
Russia has constantly been updating the procurement system mainly to reduce government spending, as we mentioned in Section 3.1. We, therefore, evaluate counterfactual scenarios by predictive procurement costs and investigate implications of misspecified risk-aversion. We also compute predictive costs and efficiency at other relevant policy options for comparison.

5.2.1. Decision Theoretic Approach
Since we study the policymaker’s decision problem, we use a statistical decision-theoretic approach; see Berger (1985) for a survey. Note that Kim (2013) introduces the approach for empirical auction design and Aryal and Kim (2013), Kim (2015), and Aryal et al. (2018) use or extend for different contexts.

If the policymaker knows $\theta$, he can choose an action $\rho \in A$, for example, a reserve price, to minimize the (expected) procurement cost $\Lambda(\rho, \theta)$. Let $\rho^*(\theta) := \arg\min_{\rho \in A} \Lambda(\rho, \theta)$. Choosing $\rho^*(\theta)$ is infeasible, however, as $\theta$ is uncertain. The posterior $\pi(\theta | z)$ represents the policymaker’s uncertainty about $\theta$, combining his prior and the sample $z$. Thus, he should choose

$$\rho_B(z) := \arg\min_{\rho \in A} \left\{ \int \Lambda(\rho, \theta) \pi(\theta | z) d\theta = E[\Lambda(\rho, \theta) | z] \right\}. \quad (6)$$

This is the idea of the Bayesian decision theory and it is similar to the usual expected utility theory. Choosing $\rho_B(z)$, known as a Bayes action, is rational under the axioms of Savage (1954) and Anscombe and Aumann (1963). A decision rule that maps every data $z$ to $\rho_B(z)$ is optimal under a frequentist perspective (Bayes risk principle).

This approach formally considers the structure of the procurement cost and uncertainty and, therefore, it may incur smaller procurement costs than a “plug-in” method. Since $\theta$ is unknown, the policymaker would choose some $\rho \neq \rho^*(\theta)$ in practice. Consider a cost function that drops sharply before $\rho^*(\theta)$ and slowly increases after $\rho^*(\theta)$. For this cost structure, the policymaker must prefer $\rho_{\text{large}} > \rho^*(\theta)$ to $\rho_{\text{small}} < \rho^*(\theta)$ for the same error, that is, $\rho_{\text{large}} - \rho^*(\theta) = \rho^*(\theta) - \rho_{\text{small}}$. Especially, if the cost is (almost) flat after $\rho^*(\theta)$, then, $\rho_{\text{max}} = \max A$ is (almost) equivalent to $\rho^*(\theta)$. The extent to which the policymaker prefers a large action must depend on the cost structure and amount of uncertainty. For example, if there

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is no uncertainty about θ, he would pick ρ∗(θ) regardless of the cost structure. Alternatively, if the cost is flat after ρ∗(θ) for all θ as in the case with a large number of bidders, he would choose ρmax. Solution (6) formalizes the idea of making a decision considering the cost structure and uncertainty. On the other hand, the plug-in approach, which is first popularized in empirical auctions by Paarsch (1997), chooses ρ∗(θ(z)). That is, the plug-in approach regards the estimate θ(z) as the true parameter uncertainty and the shape of the cost function (other than the fact that ρ∗(θ) is a minimizer of Λ(ρ, θ)).

After formalizing uncertainty by the data z and maintained assumptions (model and prior), ρB(z) is certainly the best action. So, no uncertainty notion comes with ρB(z). Specifically, a credible interval represents uncertainty associated with θ, but ρB(z) is the action after integrating out θ by π(θ|z). We instead provide a credible interval of procurement cost at ρB(z) or other notable actions, which is natural because the cost is the outcome of interest and is still uncertain at ρB(z). This is, however, different from the convention to construct a confidence interval around the plug-in estimate ρ∗(θ(z)) to consider the variation in random data z in repeated sampling. The confidence interval is designed for testing a hypothesis like ρ∗ = 0, which is a decision problem different from the policymaker’s problem.

We explain in section S9 our algorithm to evaluate the predictive procurement cost, E[Λ(ρ, θ)|z], which integrates out θ and I, respectively, by the posterior and the empirical distribution of I, considering (stochastic) refinements of I due to binding reserve prices.

### 5.2.2. Common Reserve Prices

We consider a situation where the policymaker wishes to choose one reserve price and apply it to all bidders regardless of their types, that is, ρ1 = ρ2 = ρ3. Figure 4(a) shows E[Λ(ρ, θ)|z] as a function of ρ (solid line) and its 95% credible band (dashed lines), that is, the 2.5 and 97.5 percentiles of Λ(ρ, θ) under the posterior at every ρ in the figure. The three dotted lines show the posterior predictive cost and its credible band if the policymaker implements the second-price procurements (SPP). When bidders are risk-averse, bidders bid more aggressively in an FPP than in an SPP. In particular, the figure shows that the FPP with ρc = (1, 1, 1) results in lower costs than the SPP with the cost-minimizing reserve price. Panels (b) and (c) show the predictive costs under the models with homogeneous risk-aversion and no risk-aversion, respectively. When bidders are modeled to be risk-averse, the predictive cost is minimized at ρc whether or not risk-aversion is type-specific. When risk-aversion is ignored, however, the method recommends a much smaller reserve price. This result follows from the fact that the cost densities in Figure 3 (bottom) falsely predict a large probability of small costs, especially for type 2 and 3 bidders, while these bidders would draw high costs more likely, Figure 3 (top).

The first block (common ρ) in Table 2 documents that the predictive procurement cost is 0.843 with a 95% credible interval of [0.835, 0.850] at the cost-minimizing reserve price ρ∗ B(z) := ρc, where the superscript C indicates that the Bayes action is selected under the restriction of common reserve price. The table also shows that the efficient bidder wins the procurement with a 99.2% chance at ρ∗ B(z). This prediction is similar even when risk-aversion is restricted to be homogeneous. Table 2 also shows that the model with risk-neutrality selects ρ∗ B(CRN) := (0.19, 0.19, 0.19) and predicts that the procurement cost would be 0.729 at ρ∗ B(CRN) := (0.971, 0.843, 0.848) of cost-reduction at ρ∗ B(CRN) := (0.971, 0.843, 0.848) of cost-reduction at ρ∗ B(CRN) := (0.971, 0.843, 0.848) of cost-reduction at ρ∗ B(CRN) := (0.971, 0.843, 0.848) of cost-reduction at ρ∗ B(CRN) := (0.971, 0.843, 0.848). At ρ∗ B(CRN) := (0.971, 0.843, 0.848), moreover, the model under risk-neutrality also predicts that the efficient bidder would win with a 33.2% of chance, but the chance of allocation is only 3.6% under the model with asymmetric CRRAs. That is, if one ignores risk-aversion, the proposed policy will substantially increase the procurement cost, and most procurements will fail to find a supplier.

### 5.2.3. Type-Specific Reserve Prices

Now, we consider the policymaker who can select type-specific reserve prices. The second block (type-specific ρ) in Table 2 shows that the model with asymmetric risk-aversion recom-

![Figure 4](image-url) Counterfactual analysis, common reserve price. Each panel shows the predictive procurement cost at each reserve price ρ (solid line) along with its 95% credible band (dashed) for the models with heterogeneous risk-aversion, homogeneous risk-aversion, and no risk-aversion. Panel (a) also shows the predictive cost under the second-price procurement (dotted).
Table 2. Counterfactual analysis.

| Cost min. reserve price (1) | Predictive procurement cost (2) | Prob. that lowest wins (3) | Prob. of transaction, if \( \neq 1 \) (4) |
|-----------------------------|---------------------------------|-----------------------------|---------------------------------------------|
| Common \( \rho \)          |                                  |                             |                                             |
| Heterogeneous CRRA (1.00, 1.00, 1.00) | 0.843 [0.835, 0.850] | 0.992 [0.989, 0.996] | 0.333 [0.303, 0.359] |
| Homogeneous CRRA (1.00, 1.00, 1.00) | 0.845 [0.837, 0.852] | 0.994 [0.993, 0.996] |                                             |
| No risk-aversion (0.19, 0.19, 0.19) | 0.722 [0.708, 0.754] | 0.332 [0.301, 0.358] |                                             |
| Type-specific \( \rho \) |                                  |                             |                                             |
| Heterogeneous CRRA (0.96, 1.00, 1.00) | 0.841 [0.834, 0.849] | 0.991 [0.989, 0.995] | 0.36 [0.30, 0.40] |
| Homogeneous CRRA (1.00, 1.00, 1.00) | 0.722 [0.703, 0.747] | 0.368 [0.335, 0.392] | 0.376 [0.344, 0.401] |
| No risk aversion (0.75, 0.20, 0.20) | 0.971 [0.952, 0.987] | 0.036 [0.016, 0.059] | 0.036 [0.016, 0.059] |
| For comparison |                                  |                             |                                             |
| Heterogeneous CRRA (0.19, 0.19, 0.19) | 0.965 [0.944, 0.980] | 0.060 [0.037, 0.087] | 0.063 [0.040, 0.091] |
| Homogeneous CRRA (0.19, 0.19, 0.19) | 0.848 [0.843, 0.851] | 0.985 [0.982, 0.988] |                                             |
| Additional bidder |                                  |                             |                                             |
| Type 1 bidder | 0.791 [0.775, 0.804] | 0.988 [0.981, 0.996] |                                             |
| Type 2 bidder | 0.794 [0.782, 0.805] | 0.996 [0.994, 0.998] |                                             |
| Type 3 bidder | 0.797 [0.784, 0.811] | 0.996 [0.993, 0.997] |                                             |

Note: Column (1) shows the Bayes action, that is, cost-minimizing reserve price, and columns (2)-(4) show predictive outcome variables along with 95% credible intervals: the procurement cost, the probability that the bidder with the lowest cost wins, and the probability that at least one bidder has a cost below the (or her own) reserve price. Column (4) shows the predictive probabilities only when the prediction is different from one.

It is unclear whether this finding would hold in our asymmetric setting. In a standard symmetric auction with risk-neutral bidders. But, revenue more than choosing a revenue-maximizing reserve price. Hence, we find that the insight of Bulow and Klemperer (1996) holds for the “printing papers” category of Russian procurements where bidders are asymmetric in both cost density and risk-aversion.

5.3. Sensitivity Analysis

This section examines how sensitive our empirical findings are to the definition of bidder types and prior specifications. In this section, the main specification refers to the one with heterogeneous risk-aversion outlined in Section 4, which gives the estimates in the top block of Table 1. We then consider six alternative specifications. The first (second) specification classifies the two (three) most frequent bidders as a type 1 bidder. Recall that the most (second-most) frequent bidder appears in the data 58 (33) times, but the frequency does not change much starting from the third frequent bidder, who appears 14 times; see Figure 1(a).

The type definitions here suggest that bidders’ entry depends on model primitives such as risk-aversion and cost density. To avoid resorting to any entry model, however, one might want to alternatively define the types, for example, by how often they win. In our data, the two most frequent entrants are also the most frequent (42 and 19 times) winners. Thus, the analysis remains the same even if one defines type 1 bidder based on the winning rate for the first two bidders. However, the fourth entrant is the third (8 times) winner. The third specification, 3
Table 3. Sensitivity analysis on latent variables.

| Specifications | Cost \{c_t\}, type τ = 1, 2, 3 | Unobserved heterogeneity \{u_t\} |
|----------------|--------------------------------|---------------------------------|
|                | KS test, p values              | KS test                        |
|                | Type 1 (1)                     | Type 2 (2)                     | Type 3 (3)                     | p value (4) | Predictive distribution (5) |
| (0) Main       | NA                             | NA                             | NA                             | 1.000 (0.000) [0.999, 1.000] |
| (1) 2 Type 1 bidders | 0.425 (0.048) [0.308, 0.542] | 1.000 (0.000) [1.000, 1.000] | 0.000 (0.000) [0.000, 0.000] |
| (2) 3 Type 1 bidders | 0.258 (0.097) [0.000, 0.548] | 0.999 (0.000) [0.999, 1.000] | 0.000 (0.000) [0.000, 0.000] |
| (3) 3 Type 1 bidders (wins) | 0.194 (0.152) [0.000, 0.548] | 0.999 (0.000) [0.999, 1.000] | 0.000 (0.000) [0.000, 0.000] |
| (4) Small prior \(V(\psi)\) | 0.105 (0.162) [0.008, 0.585] | 0.105 (0.008) [0.097, 0.113] | 0.000 (0.000) [0.000, 0.000] |
| (5) Large prior \(V(\psi)\) | 0.307 (0.176) [0.029, 0.692] | 0.105 (0.008) [0.097, 0.113] | 0.000 (0.000) [0.000, 0.000] |
| (6) Alternative \(f_{p1}\) | 0.988 (0.008) [0.982, 1.000] | 0.988 (0.008) [0.982, 1.000] | 0.000 (0.000) [0.000, 0.000] |

NOTE: Columns (1)–(3) show the p-values of the KS test against the hypothesis that the predictive cost distribution remains the same under the alternative specifications (within each type). Column (4) does similarly for the distribution of the unobserved heterogeneity. Column (5) summarizes the posterior of \(u_t\).

Table 4. Sensitivity analysis on posterior of CRRA coefficients.

| Specifications | \(\eta_1\) | \(\eta_2\) | \(\eta_3\) |
|----------------|----------|----------|----------|
|                | Posterior mean (standard deviation) [95% credible interval] |
| Main           | 0.196 (0.165) [0.005, 0.596] | 0.828 (0.043) [0.729, 0.893] | 0.891 (0.009) [0.867, 0.900] |
| 2 Type 1 bidders | 0.196 (0.152) [0.007, 0.548] | 0.819 (0.042) [0.736, 0.888] | 0.891 (0.008) [0.872, 0.900] |
| 3 Type 1 bidders | 0.350 (0.176) [0.029, 0.692] | 0.817 (0.042) [0.725, 0.886] | 0.892 (0.008) [0.870, 0.900] |
| 3 Type 1 bidders (wins) | 0.597 (0.127) [0.331, 0.823] | 0.865 (0.032) [0.779, 0.899] | 0.888 (0.012) [0.856, 0.900] |
| Small prior \(V(\psi)\) | 0.141 (0.118) [0.005, 0.443] | 0.798 (0.043) [0.704, 0.869] | 0.890 (0.010) [0.863, 0.900] |
| Large prior \(V(\psi)\) | 0.242 (0.197) [0.006, 0.720] | 0.842 (0.043) [0.740, 0.898] | 0.891 (0.008) [0.870, 0.900] |
| Alternative \(f_{p1}\) | 0.202 (0.162) [0.008, 0.585] | 0.819 (0.041) [0.727, 0.887] | 0.890 (0.010) [0.858, 0.900] |

NOTE: Each column shows the posterior mean of \(\eta_\tau\), standard deviation in (.), and a 95% credible interval in [ ].

Type 1 Bidders (win), defines type 1 bidders as the three most frequent winners. The winning rate does not drop after that; see Figure 1(a).

The prior variance of \(\psi\) of the fourth (fifth) specification is four times smaller (larger) than the main specification; see the cost density (3) where we introduce \(\psi\). The last one adopts a more flexible density for the unobserved heterogeneity \{\(u_t\)\}.

Extending (4), we specify \(u_t \sim \mathcal{N}(\mu_u, \sigma_u^2) \times 1(u_t \in [u_1, 1])\), where \(\sigma_u > 0, \mu > 0\), and \(\mu_u \in [u_1, 1]\) so that the distribution of unobserved heterogeneity is indexed by a three-dimensional parameter vector \((\mu_u, \sigma_u, \psi)\). Recall that the main specification indexes \(f_{p1}\) by a one-dimensional parameter \(\sigma_u\) with the restriction of \((\mu_u, \psi) = (1, 1 - c_u \times \sigma_u)\); see (4). Note that we use a flat prior for all the other parameters such as \(\sigma_u\) and \((\eta_1, \eta_2, \eta_3)\) in the main specification.

All the six alternative specifications produce predictive cost densities close to the ones under the main specification; see section S10.1. We test the hypothesis that the predictive distribution of type 1 bidder’s cost remains the same when we change the definition of type 1 bidder to include the second frequent bidder: Table 3 reports that the associated p-value is 0.425, and we fail to reject the hypothesis at any conventional level. Similarly, we conduct the hypothesis testing for other types. Then, we repeat it for the other specifications. In all cases, we fail to reject the hypothesis that the predictive cost distribution remains the same.

We also consider the predictive distribution of the unobserved heterogeneity \{\(u_t\)\}. The KS test strongly rejects, with \(p\) values close to zero, the hypothesis that the predictive distribution of \{\(u_t\)\} under the main specification is the same as the distribution under the alternative specification for each of the six cases; see column (4) of Table 3. The specifications, however, unanimously predict that \{\(u_t\)\} is practically degenerate at one: its mean and standard deviation are approximately one and zero; see column (5) of Table 3. This is an example to show that a statistically significant difference can be economically meaningless.

Table 4 summarizes the posterior of the type-specific CRRA coefficients, \(\eta_\tau\) for \(\tau = 1, 2, 3\). Including the second frequent bidder in type 1 does not change the prediction on \(\eta_1\), but the third bidder, when classified as type 1, inflates the prediction on \(\eta_1\). However, that should be natural if the third bidder is similar to type 2 bidders as suggested by the entrance rate (Figure 1(a)) because type 2 bidders are highly risk-averse. Similarly, the specification defining the three most winning bidders as type 1 bidder predicts a high \(\eta_1\), which should also be natural because type 1 bidder includes the fourth frequent entrant, who is a type 2 bidder with a high risk aversion under the main specification.

The stronger prior on \(\psi\) shrinks the prediction on \(\eta_1\) toward zero, but the weaker prior on \{\(u_t\)\} does not substantially change the prediction. The prediction on \((\eta_2, \eta_3)\) is more robust than \(\eta_1\) because the bid samples of those types are 7–10 times larger than type 1. Overall, all the specifications give qualitatively the same prediction on \((\eta_1, \eta_2, \eta_3)\): type 1 bidders are the least risk-averse, and the other bidders are highly risk-averse with \(\eta_2 < \eta_3\). Note that the KS test rejects, at any conventional level, the hypothesis that the posterior of \(\eta_1\) under the main specification equals the posterior of \(\eta_1\) under the alternative specification for each of the six cases.

However, the statistically significant differences in the posterior distributions of \(\eta_1\) between the specifications would not be large enough to induce an economically significant impact on the policymaker’s decision problem. When the policymaker applies a common reserve price to all bidders, our decision method selects the current reserve price as the cost-minimizing price under all the specifications, giving similar predictions on.
the procurement cost and the likelihood of the lowest bidder winning the procurement; see the upper block of Table 5. When the policymaker can choose bidder-specific reserve prices, our method selects different reserve prices for type 1 depending on the specification. As we discussed in the previous section, the predictive cost with the main specification is practically the same for \( \rho_1 \) near one at \((\rho_2, \rho_3) = (1, 1)\). All the alternative specifications produce similar predictive costs as a function of \( \rho_1 \) given \((\rho_2, \rho_3) = (1, 1)\). That is, the method could select any price \( \rho_1 \) near one, especially when the cost functions are evaluated by Monte Carlo, but all giving similar predictions on the outcome variables of interest; see the lower block of Table 5. All the specifications predict that the current mechanism is effectively cost-minimizing. Finally, we repeat the counterfactual analysis of inviting one additional bidder for each alternative specification and obtain predictions on the outcome variables of interest that are similar to the prediction under the main specification; see section S10.2.

6. Concluding Remarks

We conclude this article with a discussion about some possible extensions to our method. First, one may consider a non-separable unobserved heterogeneity instead of the separable one as introduced in Assumption 3-3. To be specific, if \( u \) is discrete with a finite support and if \( g_i(u) = \beta_i(u, \xi) \) is strictly increasing in \( u \) for all \( i \in I \), the bid distribution for \( \xi \) with \(|I| \geq 3\) identifies the conditional bid distribution given \( u \) for the given \( I \), following Hu, McAdams, and Shum (2013). Those identified conditional bid distributions at any \( u \) in its finite support then identify \( \{F_i, \eta_i\} \) with variation in \( I \) following Campo (2012). In our data, 237 procurements out of \( T = 411 \), approximately 58%, have only \(|I| = 2\) bidders, and we do not consider this specification for estimation.

Second, one may treat bidders’ type as an additional parameter to estimate instead of fixing a type for each bidder. For example, An (2017) proposes a method to estimate bidder types. Our sample, however, does not meet its requirement. In particular, every bidder must appear at least in three different procurements. But, this is not the case in our data, where 84% of bidders enter once or twice. In our Bayesian setting, alternatively, we may model each bidder’s membership to a type as a random variable following the Dirichlet prior. This approach is standard with a range of applications, for example, Dirichlet process mixture; see Ferguson (1973) and Escobar and West (1995) among others. If applied here, however, it would further complicate our method and substantially increase computing time.

Finally, a dataset may contain procurement-specific covariates, \( x_t \). We can also adapt the specification (3) to accommodate \( x_t \). In particular, let us use \( h(\cdot | \psi_t) \) to denote (3) and \( f \) for the cost density with \( x_t \). Let \( \tilde{f}(c(x_t, \gamma_t)) \) be a low-dimensional parametric density indexed by \( (x_t, \gamma_t) \), where \( \gamma_t \) is a vector of parameters. For example, \( \tilde{f}(c(x_t, \gamma_t)) \) can be the exponential density with the mean \( \exp(x_t^\top \gamma_t) \). Then, we may specify the CDF of the cost by \( \tilde{F}(c(x_t, \gamma_t)) = H(\tilde{F}(c(x_t, \gamma_t))) \psi_t \), where \( H \) and \( \tilde{F} \) are the CDFs of \( h \) and \( f \). To understand this specification, consider its log density, \( \log \tilde{f}(c(x_t, \gamma_t)) \approx \log f(c(x_t, \gamma_t)) + \psi_1 \phi_1(\tilde{X}_{t,t}) + \psi_2 \phi_2(\tilde{X}_{t,t}) + \cdots \), where \( \tilde{X}_{t,t} := \tilde{F}(c(x_t, \gamma_t)) \). That is, this specification first approximates the cost density by the parametric family and reduces the error by the additional terms. Therefore, if the parametric family offers a good fit, \( \psi_t \) need not be high dimensional for a given approximation quality. So, the specification is a parsimonious and yet flexible representation of the cost density with covariates. This approach has been used before, for example, see Kim (2013), Aryal and Kim (2013), Kim (2015), and Aryal et al. (2018).

### Supplementary Materials

See Aryal et al. (2022).

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The authors report there are no other competing interests to declare.

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