Stability Analysis of Steel Structure Model Based On the Finite Element Method

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Abstract. In this paper, the traditional method is used to solve the uncertainty of steel structure. Different stiffness of beams and columns are adopted. The lateral displacement modes of steel structure under symmetrical load are analysed, and the buckling load is obtained. The calculated length coefficient is further obtained. The results are compared with those obtained by traditional displacement method, which provides a reference for the application of finite element method and displacement method in the stability of steel structure.

1. Introduction
At present, the stability problem of steel structure is mainly solved in engineering design. The traditional method is based on displacement method, but this method is based on some uncertain assumptions and is commonly used in engineering.

In this paper, the finite element method is used to model the two-story and one-span steel structure with geometric rules. The lateral displacement modes of the steel structure under symmetrical loads are analyzed with different beam-column stiffness. The buckling loads are obtained. The calculated length is further obtained and compared with the solution obtained by the traditional displacement method. The feasibility of the application of finite element method in stability analysis of steel structure is demonstrated, which provides reference for engineering application design.

2. Double elastic single span frame antisymmetric buckling load of the theory
The displacement method is adopted for the anti-symmetric elastic buckling load of two-story and one-span steel structures, and the assumption of constant axial compression and small deformation is adopted for the modified method. When receiving the paper, we assume that the corresponding authors grant us the copyright to use the paper for the book or journal in question. Should authors use tables or figures from other Publications, they must ask the corresponding publishers to grant them the right to publish this material in their paper.
Using the following model: The corner of node B is $\theta_B$, node C corner is $\theta_C$; the lateral displacement angle of the two-story column is $\rho_1$, the lateral displacement angle of the first column is $\rho_2$, $\rho_1=(\Delta_1-\Delta_2)/l_1$, $\rho_2=\Delta_2/l_2$, $\Delta_1$ is the lateral displacement of a two-story column. $\Delta_2$ is the lateral displacement of a layer of columns. $l_1$ is the height of a one-story column, $l_2$ is the height of the lower column. Turning clockwise forward provision. Linear Stiffness of Columns and Beams

$$K_{1c} = \frac{EI_1}{l_1}, \quad K_{2c} = \frac{EI_2}{l_2}, \quad K_{1b} = \frac{EI_{1b}}{l_b}, \quad K_{2b} = \frac{EI_{2b}}{l_b}.$$ 

Node B uses the following equilibrium equation $0 = BEBCB + M_{BA}$, solve equations:

$$(K_{1c} + K_{2c})\theta_B + K_{1c} S_1 \theta_c - K_{1c} (C_1 + S_1) \rho_1 - K_{2c} (C_2 + S_2) \rho_2 = 0 \quad (2-1)$$

Node B uses the following equilibrium equation $M_{CB} + M_{CD} = 0$, solution:

$$K_{1c} S_1 \theta_B + (K_{1c} + 6K_{1b}) \theta_C - K_{1c} (C_1 + S_1) \rho_1 = 0 \quad (2-2)$$

The calculation model assumes that the sheer force of each column is zero, and the moment balance equation of the second and first columns is zero. $M_{AB} + M_{BA} + P\Delta_2 = 0 \text{ And } M_{BC} + M_{CB} + \alpha P(\Delta_1 - \Delta_2) = 0$, in the equation $M_{AB} = K_{2c} [S_2 \theta_B - (C_2 + S_2) \rho_2 ]$, $P = k_2^2 EI_2$

$\alpha P = k_1^2 EI_1$, the above equation into (2-1) and (2-2) the balance equation. Solve equations:

$$(C_2 + S_2) \theta_B - \left[2(C_2 + S_2) - (k_2 l_2)^2 \right] \rho_2 = 0 \quad (2-3)$$

$$(C_1 + S_1) \theta_B + (C_1 + S_1) \theta_C - \left[2(C_1 + S_1) - (k_1 l_1)^2 \right] \rho_1 = 0 \quad (2-4)$$

When $\alpha = 1.0$, $l_1 = l_2 = I_{c}$, $I_{1b} = I_{2b} = I_b$, $C_1 = C_2 = C$, $S_1 = S_2 = S$, $k_1 = k_2 = k$, then $K_{1c} / K_{c} = (l_{1c}/l_{c})/l_{1c}, l_{1c}$. The type (2-3) and (2-4) the expression, and then into the type (2-1) and (2-2), have:

$$2[(C + S)(C - S + 6K_1) - (C + 3K_1)(kl_2)^2] \theta_B - \left[(C + S)(C - S) + S(kl_2)^2 \right] \rho_2 = 0 \quad (2-5)$$

$$\left[(C + S)(C - S) + S(kl_2)^2 \right] \theta_B - \left[(C + S)(C - S + 12K_1) - (C + 6K_1)(kl_2)^2 \right] \rho_2 = 0 \quad (2-6)$$

The solutions of the homogeneous equation of Formula (2-5) and Formula (2-6) are equal to zero. However, the steel frame is not zero in the case of lateral displacement, which cannot satisfy the above assumptions. If the formula (2-5) and formula (2-6) are not zero and the homogeneous equation is required to be solved, the coefficient determinant of the equation system is equal to zero, and the equation is solved:

$$\left[2[(C + S)(C - S + 6K_1) - (C + 3K_1)(kl_2)^2][(C + S)(C - S) + S(kl_2)^2] \right] = 0 \quad (2-7)$$
Formula (2-7) has a parameter $P$. By solving the above equation, the buckling loads of two-story single-span steel structures can be obtained. The buckling equation of the two-story single-span steel structure of formula (2-7) is obtained.

After determining the linear stiffness $A$ of columns and beams, the buckling load $B$ of two-story single-span steel structures under symmetrical instability can be obtained. Buckling loads are calculated. In the design of instability, the effective length coefficient is used to calculate $C$. The formula is as follows (2-8). Using this formula, the effective length coefficient $C$ corresponding to different $D$ can be obtained. Table 3-1 shows the lateral displacement values corresponding to the linear stiffness of different columns and beams.

$$
\mu = \sqrt{\frac{\pi^2 EI}{l^2 P_{cr}}}
$$

(2-8)

### Tab 1 Calculated Length Coefficient

| $K_L$ | 0   | 0.5 | 1.0 | 2   | 5   | 10  | $\infty$ |
|-------|-----|-----|-----|-----|-----|-----|----------|
| $K_L$ |     |     |     |     |     |     |          |
| lateral displacement | 4.000 | 1.515 | 1.382 | 1.160 | 1.065 | 1.033 | 1.000    |

3. The finite element solution of elastic buckling equation.

#### 3.1. Finite element solution of buckling load

In the case of equilibrium stability, the equilibrium structural equation is:

$$
\begin{bmatrix}
K_E
\end{bmatrix} + \begin{bmatrix}
K_G
\end{bmatrix} \{U\} = \{P\}
$$

(3-1)

In the formula, $[K_E]$ is the element matrix; $[K_G]$ is the geometric matrix; $\{U\}$ is the node displacement vector; $\{P\}$ is the node load vector.

In the case of random equilibrium, the potential energy of the second-order variational system is equal to zero.

$$
\begin{bmatrix}
K_E
\end{bmatrix} + \begin{bmatrix}
K_G
\end{bmatrix} \{\delta U\} = 0
$$

(3-2)

From the above equation:

$$
\begin{bmatrix}
K_E
\end{bmatrix} + \begin{bmatrix}
K_G
\end{bmatrix} = 0
$$

(3-3)

In formula (2-11), the element matrix is known and the geometric matrix is unknown. When calculating buckling load, assuming load $\{P^0\}$ and geometric matrix $[K_G^0]$ corresponding to the given load $\{P^0\}$, assuming that external load $\{P^0\}$, $W$ is $1/\lambda$ times of buckling load. that is, $[K_G] = \lambda [K_G^0]$, the formula (2-11) is simplified as follows:
\[
\left[ [K_E] + \lambda [K_G] \right] = 0
\]  \hspace{1cm} (3-4)

Type (2-12) eigenvalue equation:

\[
(\left[ [K_E] + \lambda_i [K_G] \right] \{\phi_i\}) = 0
\]  \hspace{1cm} (3-5)

In the formula, \( \lambda_i \) represents the eigenvalue of order \( i \); \( \{\phi_i\} \) represents the corresponding eigenvector of \( \lambda_i \), i.e. instability mode

The eigenvalue buckling is analyzed by ANSYS software, and \( \lambda_i \), \( \{\phi_i\} \) and \( P_i^0 \) are obtained. The solution process assumes that \( P_i^0 \) is a unit force of 1, and then \( \lambda_i \) is a buckling load solution.

3.2. Finite element model calculation and analysis

Beam 3 element is used to calculate the effective length coefficient \( \mu \) of columns with different stiffness ratios of beams and columns. Different mesh element densities are divided in the calculation process, and the accuracy of finite element calculation is compared. Then the fourth-order buckling load and its modal diagram are obtained. Its finite element model is shown in Fig. 3-1:

Fig. 1 Finite Element Model

3.2.1. Calculations at \( K=1.0 \). The number of elements \( n \) is 60, 120 and 300, respectively. The corresponding buckling loads are:

| Cell number \( n \) | 60          | 120         | 300         |
|---------------------|-------------|-------------|-------------|
| buckling load \( P_{cr}(N) \) |             |             |             |
| 1 order buckling    | 0.49703E+10 | 0.49703E+10 | 0.49703E+10 |
| 2 order buckling    | 0.94694E+10 | 0.94694E+10 | 0.94694E+10 |
| 3 order buckling    | 0.18948E+11 | 0.18948E+11 | 0.18948E+11 |
| 4 order buckling    | 0.25351E+11 | 0.25350E+11 | 0.25350E+11 |

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From the above table, it can be seen that the number of meshes has a great influence on the calculation structure, but when the number of elements exceeds 60, the influence on the calculation structure decreases, and the graphical modal has no great difference. It is suggested that the case of 300 meshes be used in future engineering calculation. The following is the fourth-order buckling modes at n=60 as shown in Fig. 3-2.

From the above figure, it can be seen that the lateral displacement instability of two-story single-span steel structure is the most likely occurrence, and the first-order buckling load is the standard state of structural stability. At the same time, it is shown that the first-order case is usually taken into account in solving the stability of two-story single-span steel structures by theoretical method, and other cases are taken into account. Through the formula 2-8 of the first order critical load, the effective length coefficient $\mu$ of the column is deduced as follows:

$$\mu = \sqrt{\pi^2 EI / (l^2 P_{cr})} = 1.471$$

3.2.2. Calculations at $K=0.5$ When the number of meshes is n=300, the order buckling loads under the fourth mode are calculated as follows:

| Tab. 3 The order buckling loads under the fourth mode |
|------------------------------------------------------|
| Modal order | 1 | 2 | 3 | 4 |
| Buckling load $P_{cr}(N)$ | 0.36326E+10 | 0.86884E+10 | 0.16588E+11 | 0.21663E+11 |
The corresponding order modal diagrams of the values in the table are shown in Fig. 3-3.

From Figure 3-3, it can be seen that when $K_1 = 0.5$, the most prone form of buckling is lateral instability in the case of antisymmetry, which shows that the first-order buckling load used in engineering is reliable in the finite element state. The effective length coefficient $\mu = 1.720$ is obtained by substituting $K_1 = 0.5$ (2-8).

### 3.2.3. Calculations at $K=$.

When the linear stiffness of the column exceeds that of the beam, the number of meshes used in the calculation is $n=300$. Because the elastic modulus of the steel structure cannot be zero, $K_1$ cannot be adopted as zero value, but can only be infinitely close to zero. Taking $K_1$ as 0.00009853, the results of calculating the fourth-order buckling load are as follows:

**Tab. 4** The results of calculating the fourth-order buckling load

| Modal order | 1          | 2          | 3          | 4          |
|-------------|------------|------------|------------|------------|
| Buckling load $P_c (N)$ | 0.33860E+10 | 0.23026E+11 | 0.30348E+11 | 0.48913E+11 |

The corresponding order modal diagrams of the values in the table are shown in Fig. 3-4.
As can be seen from the figure above, the linear stiffness $K_1$ is close to zero. That is to say, the stiffness of the column far exceeds the stiffness of the beam, and the lateral displacement instability is most likely to occur in the case of anti-symmetry. According to formula (2-8), the effective length coefficient of the column at this time is $\mu = 3.965$.

3.2.4. Calculations at $K=10$. The fourth-order buckling loads obtained by using the number of elements $n=300$ are shown in the table below.

**Tab. 5** The fourth-order buckling loads

| Modal order | 1    | 2    | 3    | 4    |
|-------------|------|------|------|------|
| Buckling load $P_{cr}(N)$ | 0.42526E+10 | 0.52983E+10 | 0.17165E+11 | 0.19320E+11 |

The corresponding order modal diagrams of the values in the table are shown in Fig. 3-5.
When \( K_1 = 10 \), the lateral displacement instability is the most likely form of buckling under the same asymmetric condition. By substituting \( K_1=10 \) (2-8), the effective length coefficient of the column is \( \mu = 1.127 \).

4. Conclusion

In this chapter, by establishing the finite element model of double-deck single-span symmetrical rigid frame and changing the ratio of beam to column, the lateral displacement instability of double-deck single-span rigid frame under symmetrical load is studied, and the buckling load is obtained, and the effective length of the corresponding column is further obtained. The following conclusions are drawn.

(1) The first-order buckling modes of double-deck single-span symmetrical rigid frame based on finite element theory are the same as those of double-deck single-span symmetrical rigid frame based on displacement theory. However, based on the finite element theory, the buckling modes and corresponding buckling loads of double-deck single-span symmetrical frames can be obtained.

(2) Both the results of finite element analysis and theoretical analysis of displacement method show that the effective length of columns decreases with the increase of the ratio of beam to column rigidity, which is mainly due to the influence of beam restraint on column buckling.

(3) Because the calculation assumption of finite element simulation is less than that of displacement method, the result of finite element analysis is generally larger than that of displacement method, but the effect of axial deformation on buckling is considered in finite element simulation, which is closer to the actual situation. It also reflects that the first-order buckling value of double-deck single-span symmetrical rigid frame analyzed by displacement method is on the safe side.

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