Understanding definite integral concepts of prospective teachers through actions and processes based on gender difference

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Abstract. This research was a qualitative research which aimed at describing the understanding of the integral concept of prospective teachers through actions and processes in terms of gender difference. The research subjects were one female student and one male student of Mathematics Education study program. Data were collected through assignment sheets and followed by interviews. After data were analyzed, it was found that the action of the female subject in determining the definite integral was certainly shown by completing the integral task procedurally using the appropriate formula. At the action stage, in determining the total area female subject directly used the integral formula regardless the form and location of the region formed by a curve. The action of male subject in determining the integral was certainly similar to the female subject. It was shown by calculating the integral using the appropriate formula. For the task of determining the total area under the curve, male subject first described the form of the curve and paid attention to the position of the region with respect to x-axis. At the process stage, female subject explained the steps to complete the integral. When explaining how to calculate the total area under the curve, the female subject explained the solution steps regardless the position of the region. Meanwhile, the process stage of male subject was shown by explaining the steps and revealing the reasons for using the formula he used after identifying the form of the given integral. The male subject gave attention to the position of the region with respect to x-axis when calculating the region.

1. Introduction
The concept of integral is one of the main topics in calculus given in every mathematics education study program. Although this material has been studied since high school, it is not easy to understand by prospective teachers. Many studies have revealed it, such as: Orton [1] has examined students’ understanding of integration and then categorized the students’ errors that arise. Maharaj [2] has found that students have a difficulty in applying the rules for integration. In addition, several studies reveal that students also experience difficulties in using integral concepts both in calculus classes and across calculus classes [3,4,5]. Yudianto in [6] also shows the result that prospective teachers in solving integral problems use predictions and are not detailed. It shows that the understanding of the integral concept still needs serious attention, especially for prospective teachers who will later become teachers and teach the material to their students.

The purpose of learning is to achieve an understanding of the concepts being studied. There are some factors that influence students’ understanding of a mathematical concept. One of the factors that can be considered is gender difference because male and female students tend to show different outcomes in
learning mathematics. Research conducted by Puloo et al. [7] shows that women’s abilities are better than men’s abilities in terms of verbal and visual solution methods in solving geometric problems. Another study highlighting gender differences is conducted by Sari et al. [8] that describe boys’ level of probabilistic thinking is higher than girls’ level of probabilistic thinking. Lestari et al. [9] revealed that female prospective teacher have better math literacy skills than male prospective teacher. It can be an illustration for conducting studies on understanding the definite integral concept of prospective teachers based on gender differences.

Students’ understanding in this research is reviewed through actions and processes which are parts of APOS (Action, Process, Object, and Scheme) theory. APOS theory was introduced by Dubinsky [10] that explains how a person constructs his or her knowledge through stages of action, process, object, and scheme. This research is focused on the action and process stages because this is the initial stage when someone constructs a concept. Dubinsky [11] describes that an action in APOS theory is an object transformation felt by individuals that is basically external in the sense that each step is still carried out explicitly and guided by external instructions. Similarly, Parragues & Oktac [12] talk about the stages of the action that is when the individual can perform calculations and mathematical object transformation as a result of an external stimuli where each step is triggered from its previous step. Planell & Delgado [13] state that at the action stage, individuals cannot anticipate or eliminate steps without doing them explicitly. In this research, the action is applied to see propesitve teachers’ understanding of the definite integral concept viewed from the steps to solve the problems given (calculate the integral and calculate the area).

The next stage is the process, that is an internal construction that is obtained after performing an action repeatedly, and it does not have to be controlled by external stimuli [11,14]. Likewise, Arnon et al. [15] also state that when an action is repeated and reflected, the individual moves from depending on external cues to having internal control over it. It is characterized by being able to do each one explicitly and to be able to pass steps and be able to reverse them [12, 16]. Individuals can imagine or can predict that an input can be done to produce output by explaining the steps without having implemented them explicitly. It means that the individual is no longer just following the steps as detailed as they have memorized but the transformation is within their control. Thus, to understand a concept, besides being able to solve a given problem explicitly, the individual then understands it by making predictions and estimating the output of a problem without having to solve it first, and being able to explain the steps to solve it along with an explanation of why and how it is. Two basic aspects in constructing this concept are indispensable to help prospective teachers later in understanding the definite integral concept.

2. Research method

This research was a qualitative research. In practice, the researcher acted as the main instrument because she could not be replaced by other people. Data were collected through assignment sheets accompanied by unstructured interviews. The research was conducted at Christian University of Indonesia Toraja in Mathematics Education Study Program. Subjects were selected based on gender, in order to obtain two research subjects consisting of one male prospective teacher (SL) and one female prospective teacher (SP). Subjects were given instruments that had been developed, namely integral arithmetic problems and the problem of determining the total area. Data collected were the results of written works and interviews.

Data were analyzed based on action and process indicators which were parts of the APOS theory. The description of the actions and processes that had been developed can be seen in Table 1 below.

| Step | Description |
|------|-------------|
| Action | - Determining the rules that can be used to solve the definite integral problem given.  
- Applying the prescribed rules to calculate a definite integral given.  
- Sketching the area in question.  
- Determining the area in question. |
| Step          | Description                                                                 |
|--------------|-----------------------------------------------------------------------------|
| Process      | - Explaining or determining the rules that are valid to an integration with  |
|              | the reasons.                                                                 |
|              | - Explaining the steps for calculating definite integrals.                   |
|              | - Explaining the steps to calculate the area.                               |

Data for action were written data that were the results of integral calculation and calculating the area. The problems to see "action" are as follow:

A1) Find the result of \( \int_{0}^{2} (x^2 - 1)^{3} 2x \, dx \)

A2) Draw and calculate the area of the region bounded by curve \( y = -x + 1 \), axis \( x \), \( x = 0 \) and \( x = 2 \).

The data for "process" was an explanation of the steps for calculating the integral and calculating the area. The problems to see "process" are as follow:

P1) How do you determine the result of \( \int_{a}^{b} g(x) k g'(x) \, dx \)?

P2) How do you calculate the area of the region bounded by curve \( y = f(x) \), axis \( x \), \( x = a \) and \( x = b \)?

3. Result and Discussion

After subjects worked on assignments accompanied by interviews, a description of the definite integral understanding of prospective teachers was obtained through the following actions and processes.

3.1. Understanding Definite Integral Concept through Action

3.1.1. Understanding Definite Integral Concept of Female Student through Action. Understanding definite integral concept of prospective teacher through action was explored using the task of calculating the definite integral value given and determining the total area. SP’s answer can be seen in figure 1 below.

![Figure 1. SP’s answer through action](image)

The female subject (SP) calculated the integral used the substitution rule by changing the integral from the function in \( x \) to the function in \( u \) with \( u = x^2 - 1 \) then integrating in \( u \) and substituting \( u \) into \( x \) again to determine the result of the definite integral by entering the limit value. SP did not find any obstacles in doing the work on the question until she gave the correct result. However, subject did not change the integral limit values in \( u \) when changing the function into \( u \). Subject did not know that the limit had to be changed as well. Furthermore, for the task of calculating the total area, SP made a sketch of the total area in question but the graph was only a sketch and was not in accordance with the given function. Then, subject wrote the definite integral form, namely \( \int_{0}^{2} -x + 1 \, dx \). As in the problem of calculating integrals, SP could solve the integration form that she made until she gave a result that was zero and it could be concluded that the total area was zero. When the researcher reconfirmed the results obtained, SP tried to describe the area in question and the figure remained in the form of a sketch like in Figure 1b. Following is the result of the researcher’s interview with SP.
P : Are you sure the form of the area is like that?
SP : I just give you a description, Ma’am, that the location of the area might be like that.

P : Why is the result zero while in your drawing there is a shaded area that will be calculated?
SP : (thinking and recalculating the integral she wrote) hm…. but I think the integral is correct ma’am.

The result shown by SP both through written answer and interview showed that SP did not understand the relationship between the sketch and the integral results she got. She only believed that to calculate the area was done by integrating the function with given limits. In this case, although SP was able to perform definite integral calculations, for the task of calculating the total area, SP was unable to correctly describe the location and form of the region referred to in the question and to interpret the relationship between the region and the determination its area with a definite integral. She did not realize the importance of determining the position of the area to be calculated using the integral. The understanding of SP was only limited to memorizing the rules for integrating. This is in accordance with the opinion of Parragues & Oktac [12] that at “action” stage individuals are very dependent on external or external stimuli. The action of SP in understanding the integral concept was certainly shown by the procedural ability to calculate integrals.

3.1.2. Understanding Definite Integral Concept of Male Student through Action. Next, the answer of the male subject (SL) in completing the task of calculating integrals and calculating the area will be shown. The written answer can be seen in Figure 2 below.

SL, in completing a task of calculating integrals was similar to that of the female subject. SL used the known substitution integration rules by changing the integral from the function in x to the function form in u so that the correct result was obtained. The mistakes made were also the same as for the female subject, namely not changing the limits in terms of u when the function was in u. Meanwhile, for the task of calculating the area, SL first tried to draw the location and form of the region referred to in the problem even though the graph he made was reversed. Next, he calculated the area of the region by applying definite integral. The researcher conducted an interview with SL regarding the answers that he had written.

P : Are you sure about the picture you made?
SL : Yes ma’am.

P : Then to calculate the area you use definite integral, which is the limit?
SL : From zero to 2 Ma’am.

P : Why is there a negative sign in front of the integral?
SL : It is because there is an area under x-axis, Ma’am.

P : So the location is also a matter right?
SL : Yes Ma’am ...
Why is the result zero even though in your picture there is a formed region that has area?

(Silent, not answering).

Based on the results of assignment and interview, it was known that after subject drew the graph and paid attention to the location of the graphic that was under \( x \)-axis, subject added a negative sign to the definite integral and calculated it. Even though the definite integration result got by the subject was zero, subject did not know the mistake.

SL mastered some integral formulas that were ready to use. When calculating integrals and when he was given the task of calculating area, SL used definite integral process with the limit of \( x \) value given in the problem. Although SL tried to describe the location and form of the region, it did not have much influence on his understanding of the total area. SL revealed that the area under the curve would be negative, but it was only a memory, and finally SL completed the task by relying on what he had memorized previously regarding the integration rules.

3.2. Understanding Definite Integral Concept through Process

3.2.1. Understanding Definite Integral Concept of Female Student through Process. Interview was conducted to see student’s understanding of the definite integral concept through process. There were two questions asked to subject, namely an explanation of how to take steps to solve the integral calculation and calculate the area. The following is the result of an interview with SP for the task of calculating integral.

P : Can you tell me your steps to solve this first problem? (Pointing at question \( \int_a^b g(x)^k g'(x) \, dx \))
SP : I distinguish first the function and the derivative.

P : In this question, which is the function and which is the derivative?
SP : \( g(x)^k \) is the function while \( g'(x) \) is the derivative of the function.

P : How about next?
SP : I suppose that \( g(x) \) is \( u \), then I derive it. The derivative of \( u \) is \( du \) that is \( g'(x) \). After that, I rewrite the problem as \( \int u^k \, du \). If we have this, then it is easy to integrate. The result will be \( \frac{1}{k+1} u^{k+1} \), but the value of \( u \) must be returned to its original form. For the last step, I enter the upper limit value which I then subtract with the lower limit value, so the result is obtained.

Interview with SP was continued for the second question, namely explaining the steps to determine the total area, as follows.

P : Try to explain how you determine the total area of the region by curve \( y = f(x) \), \( x \)-axis, \( x = a \), and \( x = b \).
SP : I use definite integral Ma'am.

P : Why using integral?
SP : Because one application of the integral is used to calculate the area under a curve.

P : Can you imagine the form or location of the region in question?
SP : (thinking for a moment) I'm not too sure Ma'am, but I can make it in definite integral to calculate the area.

P : Can you tell me the steps?
SP : The area can be written in the integral from \( f(x) \, dx \) with a lower limit \( a \) and an upper limit \( b \). Thus, the calculation result shows the area.
Based on the result of the interview with SP, it can be said that SP had mastered the rules on integration. It was proven by SP's ability in explaining the steps for integrating. For the case of calculating this integral, SP was able to recognize and explain the relationship between a function and its derivatives in an integral so that it can be said that SP has a relational understanding, namely being able to use mathematical rules along with the reasons for their use.

For the second task, namely calculating the area, SP could not explain the form and location of the area in question. Although SP did not experience problems in explaining the steps to integrate to determine the area in question, subject could not interpret and explain the relationship between the image or the graph location and the integral form to calculate the area. The process described by subject only valid to graphics over the x-axis entirely. The process shown by SP at the same time supported previous research which revealed the inability of individuals to apply the integral concept to calculate area.

3.2.2. Understanding Integral Concept of Male Student through Process. SL’s understanding of the definite integral concept is also traced through process, namely by asking SL to explain the steps to calculate the integral and calculate the area. Following is the result of an interview with SL for the task of calculating integral.

\[ P : \text{If I give the problem } \int_{a}^{b} g(x)^k g'(x) \, dx, \text{can you explain the steps to solve it?} \]
\[ SL : \text{Oh yes ma'am. It is an integral of a function with its derivative. So later, I will use an example, like the problem I worked on earlier.} \]
\[ P : \text{What is the example?} \]
\[ SL : \text{So later } g(x) \text{ is supposed by } u, \text{then } u \text{ is derived to get } du. \text{In this case } g'(x)dx \text{ is the same as } du. \text{So what I am integrating now is } \int u^k du, \text{and this is no longer difficult to integrate. If it has been integrated, then } u, \text{which is exemplified, must be returned to its original form. For the final step, enter the upper and lower limit values. The result will be substracted later.} \]

The second task given to SL was to explain the steps in determining the total area. Following is the result of an interview with SL.

\[ P : \text{Try to explain your steps in determining the area of the region bounded by curve } y = f(x), \text{x-axis, } x = a, \text{and } x = b. \]
\[ SL : \text{I can use definite integral Ma'am, that is } \int_{a}^{b} f(x) \, dx. \]
\[ P : \text{Can you imagine the form and location of the region in question?} \]
\[ SL : \text{Something that I imagine Ma'am, this region is located in the first quadrant. Curve } f(x) \text{ is above x-axis, then bounded by a straight vertical lines } x = a \text{ and } x = b \text{(drawing using fingers). It means it is true if using definite integral, } \int_{a}^{b} f(x) \, dx. \text{So the area will be positive because it is located above x-axis.} \]
\[ P : \text{Suppose that } f(x) \text{ curve intersects x-axis, so that there is a region that lies above x-axis and there is a region below x-axis. How do you calculate the area?} \]
\[ SL : \text{(Trying to draw using fingers). It means there is a curve intersection with x-axis. I name it n. Oh ... so the area will be the sum of the area of the region above x-axis and the area of the region below x-axis.} \]
\[ P : \text{How do you calculate it?} \]
\[ SL : \text{I still use definite integral, Ma'am, but I calculate it twice. Once for the area above x-axis and once for the area below x-axis. Oh ... in fact here is my mistake for the previous question (referring to the answer to the Action question).} \]
\[ P : \text{So how do you calculate it?} \]
\[ SL : \text{So the area is } \int_{a}^{n} f(x) \, dx + \int_{n}^{b} f(x) \, dx. \]

The result of this process shows that SL had a good ability in explaining the steps to calculate the integral. The first step taken by SL in carrying out integration is to identify the form of the integral given
to further decide the rules to be used. SL’s understanding of relational can be seen from his ability to explain the relationship between function and its derivative in an integral.

In the case of calculating the total area, SL also gave an explanation of the use of definite integral. SL’s explanation did not only focus on the integration rules used, but also considered the location and form of the region in question. Subject realized and interpreted the relationship between the graphic location and the integral form, namely by adding a negative sign to the region under x-axis. It means that SL did not only have instrumental understanding but also have relational understanding. SL’s explanation was in line with the description of the process in APOS theory stating that process can be characterized by the ability to do something explicitly and can pass the steps and be able to reverse them [16, 12]. SL could imagine and explain the process of determining the total area by drawing a graph, paying attention to the position of the region below or above x-axis to determine the definite integral form.

4. Conclusion
Based on the description above, it can be concluded that the understanding of female prospective teacher and male prospective teacher about the definite integral concept through action in calculating definite integrals tends to be the same. They solved integral arithmetic problems using correct rules and they can solve it even though there is a minor mistake that is not careful in changing the limit value when changing the function. However, there is a slight difference when solving the problem of calculating the area, namely that SP does not consider the form and location of the area referred to as the definite integral form used, while SL pays attention to and interprets the effect of the form and location of the region referred to as the integral used to calculate the area of the region.

There are differences in understanding the integral concept of female prospective teacher and male prospective teacher when viewed from the process. At the process stage, the female subject explains that the step in determining the total area is directly using the integral form of the given function and determine the limit of the problem given then solve the integral. Subject does not give steps to pay attention to the location of the area on x-axis. Meanwhile, the process stage for male subject that is shown by explaining the steps to calculate the total area with the definite integral is drawing the area in question, paying attention to the area position to determine the positive or negative of the integral, dividing it into several definite integral forms with the corresponding limits for the region below x-axis and the region above x-axis, then calculating the definite integral to get the area in question. Thus, the male subject understands and interprets the relationship between the calculated area of region and the integral form used.

Based on the results of this study, it is suggested that in determining the definite integral result is by using the substitution method, it is necessary to emphasize the meaning of the function change carried out and emphasize the need for limit changes in the definite integral after the function is substituted. Meanwhile, when studying the determination of the area with the definite integral method, it is necessary to emphasize the meaning and relationship between the function graph and the integral form used. It can be done by giving various examples of areas with various positions to make students aware of the relationship. Based on the results of the analysis of the actions and processes carried out by the subject, it can be followed up by determining learning strategies so that students construct concepts correctly and accurately, especially in determining the area of an area using integrals. This is in line with the results of research [17].

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