Parametric analysis of vehicle suspension based on air spring and MR damper with semi-active control

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Abstract. Three strategies are used in this paper to investigate suspension systems behaviour. Firstly, the steel coil passive spring (conventional spring) is replaced by air spring, which comprised of an airbag, a connection pipe with a control valve, and a reservoir tank system. The influence of air spring pressure, reservoir volume and connection pipe diameter on suspension system behaviour are investigated. Secondly, a logic control scenario for the oscillation mass of air between spring airbag and the reservoir is applied by using on/off valve controller to study the controlling valve influence on the whole vehicle dynamics responses. Thirdly, the MR damper is used to introduce a variable damping coefficient as a semi-active control (skyhook) strategy of the suspension system. To represent the suspension system with the earlier mentioned strategies, a mathematical model 2-DOF for a quarter car is adopted along with governing equations of the air spring and semi-active skyhook control under the Matlab/Simulink platform. The obtained result, i.e., suspension travel displacement, body acceleration, dynamic tire force, and damper dissipation energy, are calculated in terms of RMS to show up the dynamic behaviour of suspension system considering four configurations in various control styles. The models' responses were examined versus two road excitations pattern, including harmonic excitation and random road excitation. The results show that using air spring is considerably increased suspension quality, furthered more the suspension system is significantly enhanced when semi-active strategies are applied on air spring and MR damper.

1. Introduction
Improving road holding safety and ride comfort have led many researchers to develop new suspension techniques in facing vibrations that associated road unevenness. The suspension system consists of spring and damper installing in parallel; spring is absorbing the road excitations which is a transfer from road surface roughness and a road bump, the spring stores energy by compressing after that, the energy releases to the vehicle body by expanding gradually [1]–[3]. The damper function dissipates the energy of suspension vibrations where the damper absorbs a part of the shock energy directly when a vehicle goes over road excitation; it also guides spring action by dissipating the stored energy [4]. Depending on controlling approach, the suspension system can be classified into the passive system, semi-active system and active system [5], the passive suspension system is a traditional system where the spring stiffness and damping coefficient are constant, so that the suspension parameters have firm behaviour nevertheless road state [6]. Semi-active control systems represent a middle way between performance enhancement and implementation simplicity. A small amount of energy is needed to change system parameters damping/stiffness instantaneously [7]–[9].

In contrast with the passive system, the active system offers more holding road and riding quality. Mainly active suspension system uses controllers, sensors, and actuators to generate external power, which eliminates the vibration energy. The disadvantage of the active system is complexity and energy-consuming [10]. Air spring is used widely in heavy trucks and trains cars that due to a good behaviour
of using compressible fluid in the suspension system, thereafter a brief review of important configurations of air spring. Gao et al. using a fuzzy logic controller with air spring to make suspension height adjustment of the electric vehicles [11], Zhu et al. proposed a developed dynamic model of the bellow-pipe-tank pneumatic system considering bellow rubber friction effect on the suspension deflection [12]. Marzbanrad and Keshavarzi studied the dynamical behaviours of the nonlinear quarter car depending on gas spring with the auxiliary reservoir and the fluidic resistance [13]. Karimi Eskandary et al. developed a mathematical model of the air suspension system showed independent height and stiffness tuning [14]. Zhao et al. produced a new solution to control height tracking for an AAS control system [15]. Abid et al., the authors had been made a comparison study, including GENESIS model air spring and passive suspension system [16]. Gavriloski et al. proposed a modern air spring model that comprises spring bellow and reservoir take both of them filled by air and connected through a pipeline system. The fluidic resistance has been considered [17]. Moheyeldein et al. investigated the influence of spring parameters such us reservoir volume, spring dimensions by using a 2-DO model for a quarter vehicle with Matlab/Simulink [18].

Some semi-active control models [19]–[23] have also modified. Ketu et al. applied a semi-active control strategy on air spring by using “on-off” switch controllable throttle-valve between the air spring bellow and reservoir[19]. Yue et al. established a nonlinear model of hydropneumatic suspension for a heavy vehicle[20]; Ranjan et al. presented an adaptive hybrid control methodology for the damper that could be provided hybrid coefficient value dynamically [21]. Ahmadian and Pare Investigated experimentally three of semi-active control strategies, which termed as skyhook control, ground hook control and hybrid by using quarter-car test ring, the tuned magnetorheological damper was adopted to implement control policies [22]. Yıldız et al. submitted an adaptive control design for a quarter car model using MR damper with a semi-active approach. The suggested controller showed good stability for car body and wheel assembly comparing to passive (MR) damper [23].

This study aims to investigate the effect of using semi-active controls for air spring and MR damper on the ride comfort, road holding and the dissipation energy for a quarter car model. Four configurations of semi-active controls are considered. Moreover, An extensive comparison has been implemented to figure out the best configurations that contributed to improving the suspension performance over the passive one.

The paper is structured into the following parts, section 2 is included governing equations, mathematical models, parameters and performance criteria, while in section 3 the semi-active control configurations for the air spring and MR damper have been detailed, then section 4 is included modelling and numerical simulation under Simulink environment, Results and discussion are then stated in section 5. Finally, section 6 is dealing with the summarized conclusions of the achieved results.

2. Governing equations and mathematical models

2.1. Conventional suspension model

In this study 2-DOF of a quarter-car is considered to consist of sprung mass (vehicle body), unsprung mass (tire assembly), shock-absorber (damper ) and spring, these elements represented the conventional suspension system. The system layout is shown in figure 1.

Based on newton’s second law two differential equations, one can describe the dynamical model in figure 1, as follows

\[ M_b \ddot{Z} + F_s + F_d = 0 \]  
\[ M_t \ddot{Z} + F_s + F_d + F_t = 0 \]

Where \( F_s \), \( F_d \) and \( F_t \) are the spring force, damper force and dynamic tire force, they can obtain as below:

\[ F_s = K_s (Z - Z_2) \]  
\[ F_d = C_d (\dot{Z} - \dot{Z}_2) \]
\[ F_t = K_t (Z_2 - Z_t) \]  

Substitute equations (3),(4) and (5) in equations (1) and (2) to get

\[ M_b \ddot{Z} = -K_s (Z - Z_2) - C_s (\dot{Z} - \dot{Z}_2) \]  
\[ M_t \ddot{Z}_2 = K_s (Z - Z_2) + C_s (\dot{Z} - \dot{Z}_2) - K_t (Z_2 - Z_t) \] 

2.2. Air Spring model

Air suspension system configuration included three-parts, airbag, air reservoir, and connecting pipe with the control valve, the schematic structural model to be shown in figure 2.

The working medium of the air spring is compressed air. The air springs characteristics are depended on fluid dynamics & thermodynamics equations so that physical quantities such as pressure, temperature, and volumes have considerable influence on their behaviour. The addition or extraction volume of the accessory reservoir via the controlling valve produces changeable dynamic stiffness of the air spring; furthermore, the dynamic stiffness also depends on the other parameters, i.e., the airbag dimensions, reservoir volume, also diameter and length of connecting pipes. The force exerted by air spring is a function of internal airbag pressure where the compression and expansion can be considered as a polytrophic process.

![Figure 1. conventional model –quarter car][18].

![Figure 2. Structural model of air spring with MR damper.](image)

The dynamical model of air spring, which incorporates spring and damping characteristics, can be represented in figure 3 as in [17]. The air spring equivalent model comprises two linear springs \( K_1, K_2 \), mass \( M \) which indicates oscillation air mass and nonlinear damper \( C_x \) which came up due to oscillating air through fluidic resistance in the connecting pipes between airbag and reservoir. The governing equations of the dynamic model corresponding to a quarter vehicle can be represented according to newton’s second law as below:

\[ M_b \ddot{Z} + F_{s1} + F_{s2} + F_d + Fst = 0 \]  
\[ M \ddot{Z}_1 - F_{s2} + F_{ad} = 0 \]  
\[ M_t \ddot{Z}_2 - F_{s1} - F_d + F_{ad} - Fst + F_t = 0 \] 

Where: \( F_{s1}, F_{s2}, F_{ad} \) and \( Fst \) are linear springs forces, the nonlinear damper force and exerted pressure force respectively

\[ F_{s1} = K_1 (Z - Z_2) \]
\[ F_{s2} = K_2(Z - Z_1) \]  
\[ F_{ad} = C_z(\dot{Z}_1 - \dot{Z}_2)^2 \text{sign}(\dot{Z}_{12}) \]  

Substituting (11), (12), and (13) in (8), (9) and (10) to obtain:

\[ M_b \ddot{Z} = -K_1(Z - Z_2) - C_s(\dot{Z} - \dot{Z}_2) - K_2(Z - Z_1) - \text{fst} \]  
\[ M \ddot{Z}_1 = K_2(Z - Z_1) - C_z(\dot{Z}_1 - \dot{Z}_2)^2 \text{sign}(\dot{Z}_1 - \dot{Z}_2) \]  
\[ M \ddot{Z}_2 = K_1(Z - Z_2) + C_z(\dot{Z}_1 - \dot{Z}_2) - C_z(\dot{Z}_1 - \dot{Z}_2)^2 \text{sign}(\dot{Z}_2 - \dot{Z}_1) - K_1(Z_2 - Z_1) + \text{fst} \]

It can be noted that the force associated with \((C_z)\) will always be positive due to the related velocities being squared. So that the sign functions for relative velocities were multiplied by damping forces to give the proper directions to the forces [16]. Air spring parameters can be calculated as [17].

\[ K_1 = \frac{P_{bo} n A_{eff}^2}{V_{bo}^2 + V_{ro}^2} \]  
\[ K_2 = \frac{P_{bo} n A_{eff}^2 V_{ro}}{V_{bo}^2 + V_{ro}^2 V_{bo}} \]  
\[ M = \rho_{air} A_p l_p \left[ A_{eff} - \frac{V_{ro}}{V_{bo} + V_{ro}} \right]^2 \]  
\[ f_{st} = (P_{bo} - P_{at}) \cdot A_{eff} \]  
\[ C_z = 0.5 \rho_{air} A_p h_{st} \left[ \frac{A_{eff} \cdot V_{ro}}{V_{bo} + V_{ro}} \right]^3 \]

![Figure 3. A dynamical model of air spring [17.]](image)

2.3. Performance qualification
Since the aim of this study is evaluating suspension performance quality for studied models by analyzing vehicle’s dynamics parameters and power dissipation, therefore RMS(root mean square) values for four performance parameters are calculated as follow:

1) Ride Comfort
The criterion of ride comfort quality can be represented by the time domain RMS value of the sprung mass acceleration and displacement which can be calculated as in equations(22),(23)

\[ \text{RMS}_{acc} = \sqrt[2]{\frac{1}{T} \int_0^T \ddot{z}^2 \, dt} \]  

4
RMS\textsubscript{dis} = \sqrt{\frac{1}{T} \int_{0}^{T} Z^2 \, dt} \tag{23}

2) \textit{Dynamic tire force}

The dynamic tire force indicates the road holding and the vehicle’s safety on the road. The RMS time domain can be calculated by equation (24)

\[ \text{RMS}_{\text{TDF}} = K_t \cdot \sqrt{\frac{1}{T} \int_{0}^{T} (Z_t - Z_z)^2 \, dt} \tag{24} \]

3) \textit{Damper potential power}

The power dissipated against road excitation can be evaluated by damping coefficient times square suspension velocity; equation (25) represent potential power in RMS term [2]

\[ \text{RMS}_{\text{PP}} = C_s \cdot \sqrt{\frac{1}{T} \int_{0}^{T} \| (\dot{Z} - \dot{Z}_z)^2 \|^2 \, dt} \tag{25} \]

2.4. \textit{Road input Excitation models}

A quarter vehicle with an air suspension system is simulated by applying different types of road inputs. A harmonic sinusoidal wave uneven where it defined mathematically as in equation (26) [2].

\[ Z_t(t) = A \sin(2\pi f t + \phi) \tag{26} \]

Where \( A \) is wave amplitude; \( f \) is wave frequency, \( \phi \) is the phase.

Moreover, the suspension response behaviour has been investigated against a random road profile depending on ISO/TC108 [18]; the profile of the random road can be calculated based on roughness constant \( G_q \).

\[ \ddot{Z}_t(t) + 2\pi f_o \dot{Z}_t(t) = 2\pi n_o \sqrt{G_q(n_o)} u(t) \, w(t) \tag{27} \]

Where \( G_q(n_o) \) is the road roughness coefficient; \( n_o \) is a spatial frequency and equal to 0.1 m\(^{-1}\); \( f_o \) is a minimum boundary frequency and equal to 0.0628 Hz; \( u(t) \) is the vehicle velocity; \( w(t) \) is a Gaussian white noise signal.

2.5. \textit{Semi-active control strategies}

Improving suspension characteristics are the main reason behind using the adjusted suspension system, which is depending on the road condition. So that it is essential to find control strategies able to change the suspension state and hence produce the suit suspension parameter as per the instantaneous dynamical condition. That would be contributed to improving suspension performance finally. The controlling principle for both air spring system and shock absorber damper summarized by measuring relative velocity on both ends of the sprung mass, then on/off controlling action will be applied accordingly.

Table 2 includes the working parameters that are used in the model simulation. The parameters are depended from [18]:

1) \textit{Air spring semi-active control}

Logic control for air spring can be described as below [19]:

\[ d_p = \begin{cases} d_{\text{valve}} = d_{\text{min}} & \text{if } \dot{Z}(\ddot{Z} - \dot{Z}_z) > 0 \\ d_{\text{valve}} = d_{\text{max}} & \text{if } \ddot{Z}(\dot{Z} - \dot{Z}_z) \leq 0 \end{cases} \tag{28} \]

Where : \( d_{\text{max}} = d_p \); \( d_{\text{min}} = 0.5 d_p \)

That means the on-off controlling diameter valve of air spring is set as full opened “on state” (i.e \( d_{\text{valve}} = d_{\text{max}} \)) when the suspension velocities of sprung and unsprung masses in the opposite direction,
however the valve is partially closed “off state “ (i.e $d_{\text{valve}} = d_{\text{min}}$), when the suspension velocities in the same direction.

2) MR damper skyhook semi-active control

In this strategy, a damper generates the damping force always in the opposite direction of the suspension movement. When the generated force is not in the required direction, the damper needs to be shut off. The switching criteria which perform that function in the damper can be described as in equation [21],[22]:

$$C_S = \begin{cases} 
C_{\text{max}} & \text{if } \ddot{Z} \left( \dot{Z} - \dot{Z}_2 \right) > 0 \\
C_{\text{min}} & \text{if } \ddot{Z} \left( \dot{Z} - \dot{Z}_2 \right) \leq 0
\end{cases} \tag{29}$$

where $C_{\text{max}}$ and $C_{\text{min}}$ are the maximum and minimum damping coefficients, which can be produced by the skyhook controlled damper, respectively.

3. Suspension system control styles

To evaluate the suspension systems which are used different types of elements and control strategies, four simulated styles have been configured as below:

- Semi-active control air spring and semi-active control damper, which is denoted as (SAS+SD).
- Semi-active control air spring and passive damper, which is denoted as (SAS+PD).
- Passive air spring and semi-active damper, which is denoted as (PAS+SD).
- Passive air spring and passive damper, which is denoted as (PAS+PD).

The dynamic factors for the abovementioned styles are represented graphically together with that obtained from a conventional suspension system. On that basis, the performance of suspension systems can be evaluated.

**Table 1. Operation parameters for conventional and air suspension systems**

| Suspension system type | Parameter | Value          |
|------------------------|-----------|----------------|
| Passive suspension parameters | $M_b$   | 350 Kg         |
|                         | $M_t$   | 40 Kg          |
|                         | $K_s$   | 20 kN/m        |
|                         | $C_s$   | 1.5 kN.s/m     |
|                         | $K_t$   | 200 kN/m       |
| Air Spring parameters   | $P_{bo}$ | 2.5-8 bar      |
|                         | $P_{at}$ | 1 bar          |
|                         | $A_{\text{eff}}$ | 0.079 m$^2$ |
|                         | $V_{ro}$ | 0.00074 m$^3$ |
|                         | $V_{bo}$ | 0.00094 m$^3$ |
|                         | $d_p$   | 4 mm           |
|                         | $l_p$   | 2 m            |
|                         | $\rho_{\text{air}}$ | 4.23 Kg/m$^3$ |

4. Modelling and Numerical Simulation

The developed equations in the previous sections were modelled under the MATLAB-Simulink environment. The road input excitation including harmonic sine wave and random road class C with a vehicle’s speed 20m/s, also the semi-active control strategies which configured in four styles are studied.

Table 2 listed the roughness values for the typical road classes from A-F. The random road profile with a Gaussian white noise filter can be simulated as in [14].
5. Results and discussions

The suspension system performance has been investigated in this work by evaluating ride comfort, dynamic load, and damper dissipation energy. The simulation is conducted by using control strategies for the air spring and shock absorber damper with different configurations for suspension models. The results have been compared with conventional (coil spring) suspension systems. The results are termed in RMS to determine vehicle dynamic behaviour. The ride comfort is being enhanced at lower RMS values of the sprung mass acceleration. The dynamic tire force indicates the vehicle’s road-holding, increasing dynamic tire forces lead to a lack of road holding and safety due to poor contacting between the tire and road. However, the maximum RMS of harvesting potential power is preferred. Additionally, the time domain for accelerations with verify road inputs has been examined.

The influence of excitation frequency with 5mm amplitude on RMS of ride comfort, spring mass displacement, dynamic tire force, and potential power are shown in figure 4, figure 4-a shows the sprung mass acceleration is increased with the frequency. Although the maximum accelerations of SAS+SD and SAS+PD are less than other styles; however, curves history shows a lower bounce acceleration for the styles SAS+SD and PAS+SD at frequencies range below (9Hz). This is because of the skyhook damper strengths isolation of sprung mass. Figure 4-b represents the sprung mass displacement responses. For all styles the displacement rises at the frequencies lower than (1 Hz) and then it is sharply falling with increasing frequency, the maximum displacements reduced by (27%) for styles SAS+SD and SAS+PD also (16%) for styles PAS+SD and PAS+PD comparing with a conventional passive system. Figure 4-c shows the dynamic tire force history, generally the force proportional directly with the frequency at the range 1-9 Hz, then after the tire force trended to drop, the graph denoted clearly that when the frequency range in the high level, PAS+SD style produce worse road holding that because a high dynamic tire force, whereas the SAS+PD is superior to the others styles. Figure 4-d shows the potential power as a function of the excitation frequency; it can be seen that when a semi-active control scheme is used the potential energy is increased. This is because of skyhook damper dissipates more vibration energy to attain the sprung mass isolated on the contrary of semi-active air spring control scheme such as (SAS+PD and PAS+PD) which showed less dissipation power.

Figure 5 It is interesting to study dynamic coefficients and dissipation energy curves against excitation amplitude. The excitation frequency is fixed to (2Hz) and the other operations parameters for suspension system as it provided in the table(1). Generally, the responses are proportion linearly with the excitation amplitude. The high amplitude has an undesirable influence on ride comfort and road holding due to increasing vibration intensity and tire dynamic forces. It is clearly noticed that the suspension style PAS+SD provides a superior dynamic behaviour regarding ride comfort and road holding over than other styles. The potential power has a positive correlation with excitation amplitude as it can be seen in figure (5-d).

Figure(6) depicts the RMS of body dynamic parameters with various pressure values; generally, it was observed that the responses of dynamic parameters are showed an increase directly proportional to the pressure. That returns to the relevant dynamic stiffness is increased with pressure; this, in turn, leads to low isolation of the vehicle’s body. Until about 4.5 bar, SAS+SD and PAS+SD show better ride comfort over SAS+PD and PAS+PD for the same operational parameters, as shown in fig(6-a) and figure(6-b). However, SAS+SD style shows reducing in the ride comfort for the pressures higher than 4.5 bar because of the increasing nonlinear damping effect of air spring when the air spring control strategy is applied. It is evident that using softer spring combined with a semi-active control damper will give the best ride quality. Figure(6-c) results showed the passive air spring with semi-active damper (PAS+SD) produced superior road holding among all other suspension styles even at high-pressure state, The potential energy variation with air spring pressure has been shown in the figure 6-d. Pressure increasing leads to consequential increasing in potential energy, as it can be noticed the PAS+SD and PAS+PD are exposed more energy comparing other styles.

| Road Class | A     | B     | C     | D     | E     | F     |
|------------|-------|-------|-------|-------|-------|-------|
| $G_0(n_0)\times10^6$ m$^{-3}$ | 16    | 64    | 256   | 1024  | 4096  | 16384 |

Table 2. Road-roughness Values [14].
In Figure (7), the random road class C in accordance with ISO/TC108/SC2N67 was considered as excitation input. The curves illustrated graphically the suspension system dynamic response against vehicle speed. In the term of increasing vehicle’s speed, The models which used semi-active shock absorb damper produced better ride comfort comparing other models as it can be seen in PAS+SD and SAS+SD styles, on another side the worst behaviour was noticed in conventional styles.

The road-holding index was promoted with PAS+SD style while SAS+SD and PAS+PD were closed to each other. As it showed, The dissipation energy is proportional linearly with the vehicle speed and road roughness so that more harvesting energy can be gained with increasing vehicle speed or in the rough road due to increase vibration intensity.

To explore the suspension quality, figure 8 demonstrated a time-domain comparison between SAS+SD and conventional suspension systems excited by a random road profile. The response curves exhibit superior dynamical behaviour for the SAS+SD over a conventional system. It can be noticed that through reducing sprung mass accelerations and displacements as well as tire dynamic force. Figure 8-d showed relevant decreasing in the damper dissipation power exposed by the SAS+SD model comparing with conventional suspension that consequence of reducing vibration intensity.

Figure 9 illustrated the percentage improvement in dynamic indices against random road class C excitation. It can be noted that the ride comfort and road holding are considerably enhanced by using the PAS+SD style comparing with the conventional system. The improvements responses of the random road excitation in percentages are 23.58% and 18.69% respectively.

![Figure 4](image4.png)

**Figure 4.** Dynamic parameters and potential power versus harmonic excitation as a function of frequency, excitation amplitude 5mm.
Figure 5. Dynamic parameters and potential power versus harmonic excitation with various values of excitation amplitude, at 2 Hz.

Figure 6. Dynamic parameters and potential power versus air spring pressure, harmonic excitation at a 2 Hz frequency, and 5mm amplitude.
Figure 7. Dynamic parameters and potential power versus vehicle speed with random road excitation class C.

Figure 8. A time-domain for dynamic parameters and potential power versus a random road excitation, a road class C, vehicle speed 20m/s.
6. Conclusions

Air spring with MR damper has been simulated under quarter vehicle model. Four control configurations in addition to the conventional suspension system are used to investigate dynamic characteristics and damper potential power. The results have been termed in RMS to compare the suspension quality for using suspension styles. The achieved responses were against two kinds of input excitations, which including harmonic and random road profile excitations. Harmonic excitation responses demonstrated that the suspension styles SAS+SD, SAS+PD, PAS+SD, and PAS+PD reduced maximum sprung mass acceleration by 31.3%, 30%, 11.8%, and 13.2% respectively relative to the conventional system. Also, from results analysis, it can be considered that increasing spring pressure has a negative influence on dynamic factors. When the suspension system excited by random road profile, it can be concluded that the increasing in the vehicle’s speed, as well as road roughness, lead to reduce comfort ride and road holding. However, it is a benefit of potential power.

Nomenclature

\[ M_b \quad \text{body mass (sprung mass)} \]
\[ \dot{Z} \quad \text{body acceleration} \]
\[ Z \quad \text{body displacement} \]
\[ \ddot{Z}_2 \quad \text{unsprung mass displacement} \]
\[ Z_2 \quad \text{unsprung mass acceleration} \]
\[ Z_t \quad \text{road profile input} \]
\[ \ddot{Z}_1 \quad \text{acceleration of oscillation air mass} \]
\[ M \quad \text{oscillation air mass} \]
\[ Z_1 \quad \text{displacement of oscillation air mass} \]
\[ P_{bo} \quad \text{airbag pressure} \]
\[ n \quad \text{polytrophic index} \]
\[ V_{bo} \quad \text{airbag volume} \]
\[ V_{ro} \quad \text{Reservoir volume} \]
\[ A_{eff} \quad \text{airbag effective area} \]
\[ l_p \quad \text{connection pipe length} \]
\[ RMS_{acc} \quad \text{root mean square of body acceleration} \]
\[ RMS_{dis} \quad \text{root mean square of body displacement} \]
\[ RMS_{TDF} \quad \text{root mean square of Tire dynamic force} \]
\[ RMS_{pp} \quad \text{root mean square of damper potential power} \]
\[ G_0(n_0) \quad \text{road roughness coefficient} \]
\[ u(t) \quad \text{vehicle speed} \]
\[ w(t) \quad \text{road white noise signal} \]
\[ d_p \quad \text{diameter of connection pipe} \]
\[ SAS+SD \quad \text{Semi-active control air spring and semi-active control damper} \]
\[ SAS+PD \quad \text{Semi-active control air spring and passive damper} \]
\[ PAS+SD \quad \text{Passive air spring and semi-active control damper} \]
\[ PAS+PD \quad \text{Passive air spring and Passive damper} \]
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