The general theory of porcupines, perfect and imperfect

Latham Boyle
Perimeter Institute for Theoretical Physics, Waterloo, Ontario, Canada
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Porcupines are networks of gravitational wave detectors in which the detectors and the distances between them are short relative to the gravitational wavelengths of interest. Perfect porcupines are special configurations whose sensitivity to a gravitational plane wave is independent of the propagation direction or polarization of the wave. I develop the theory of porcupines, including the optimal estimator $\hat{h}^{ij}$ for the gravitational wave field; useful formulae for the spin-averaged and rotationally-averaged SNR$^2$; and a simple derivation of the properties of perfect porcupines. I apply these results to the interesting class of “simple” porcupines, and mention some open problems.

GENERAL FORMALISM

To fix notation, my fourier conventions are

$$g(t) = \int_{-\infty}^{\infty} df \, \tilde{g}(f) e^{-2\pi i ft}, \quad \tilde{g}(f) = \int_{-\infty}^{\infty} dt \, g(t) e^{+2\pi i ft}. \quad (1)$$

The lower case latin indices $\{i, j, k, l, m, n\}$ label the 3 spatial directions: $i, j, k, l, m, n=1, 2, 3$. The upper case latin indices $\{A, B\}$ label the 2 gravitational wave polarizations: $A, B = 1, 2$. The lower case greek indices $\{\alpha, \beta\}$ label the $N$ detectors in the network: $\alpha, \beta = 1, \ldots, N$. Indices in square braces $[\ldots]$ are symmetrized. I use the Einstein summation convention: repeated indices (one upper, one lower) are summed. I use hats in two different ways: (i) to denote variables with a random noise contribution (such as estimators); and (ii) to denote unit 3-vectors. I have tried to make the context clear enough to avoid confusion of these two meanings.

A gravitational wave on Minkowski space is described in transverse-traceless gauge $\{\bar{1}, \bar{2}\}$ by the line element

$$ds^2 = -dt^2 + [\delta_{ij} + 2h_{ij}]dx^i dx^j. \quad (2)$$

In this metric, worldlines with $\bar{x}$ = constant are geodesics; along such worldlines, the proper time $\tau$ coincides with the coordinate time $t$. When gravitational waves reach us from a distant astronomical source, they appear to us as plane waves travelling in the $\hat{n}$ direction

$$h^{ij}(\lambda) = P_A^{ij}(\hat{n}) h_A(\lambda) \quad (3)$$

where the polarization waveforms $h_A(\lambda)$ are arbitrary functions of $\lambda \equiv t - \hat{n} \cdot \bar{x}$, and the polarization tensors $P_A^{ij}(\hat{n})$ form an orthonormal basis on the 2-dimensional space of symmetric, transverse, traceless $3 \times 3$ matrices:

$$P_A^{ij}(\hat{n}) - P_A^{ji}(\hat{n}) = 0, \quad (4a)$$
$$\hat{n}_i P_A^{ij}(\hat{n}) = 0, \quad (4b)$$
$$\delta_{ij} P_A^{ij}(\hat{n}) = 0, \quad (4c)$$
$$P_A^{ij}(\hat{n}) P_B^{ij}(\hat{n}) = \delta^B_A. \quad (4d)$$

Let us imagine that all of the detectors are situated at $\bar{x} = 0$. The estimator $\hat{h}^\alpha$ represents the measured output

This decade, we hope and expect that gravitational waves will be directly detected for the first time; this will mark the beginning of what promises to be a long and fruitful era of gravitational wave astronomy. One can only guess what wonders will be revealed when this new window onto the universe is flung fully open. As attention shifts from gravitational wave detection to gravitational wave astronomy, so will it shift from individual gravitational wave detectors to networks of multiple detectors that function together as gravitational wave telescopes. (For previous work on gravitational wave networks, see $[3,4]$ and references therein.) There is an important regime in which the individual detectors and the distances between them are short relative to the gravitational wavelengths of interest, so that the arms of the various detectors in the network may be thought of as emanating from nearly the same point in space, like the famed quills of a frightened porcupine. In a previous paper $[5]$, such “porcupines” were motivated by recent ideas and developments in gravitational wave detection $[6,7]$; but it is worth reframing the motivation in more general terms: one may suspect that porcupines will play an important role in gravitational wave astronomy for the simple reason that astrophysical gravitational wavelengths are typically very long. One will often be interested in gravitational wavelengths that are longer than the practical size of one’s network (e.g. the diameter of the Earth, or the Solar System).

Ref. $[5]$ focused on “perfect porcupines”: special configurations with the defining property that their sensitivity to an incident gravitational plane wave is independent of the propagation direction or polarization of the wave. (A number of other nice properties follow automatically once this condition is satisfied.) Here I develop a more natural and general formalism for handling porcupines (perfect or imperfect). This formalism yields, for an arbitrary imperfect porcupine: (i) the minimum-variance unbiased estimator $\hat{h}^{ij}$ for the gravitational wave field; and (ii) useful expressions for the spin-averaged and rotationally-averaged SNR$^2$. It also yields a simple derivation of the properties of perfect porcupines. I apply these results to the interesting class of “simple” porcupines (defined below) and discuss open problems.
of detector $\alpha$, a sum of genuine gravitational wave signal $h^\alpha(t)$ and noise $\hat{n}^\alpha(t)$:

$$\hat{h}^\alpha(t) = h^\alpha(t) + \hat{n}^\alpha(t).$$

(5)

If we assume the network’s response is linear and time-translation invariant, we can write

$$h^\alpha(t) = \int_{-\infty}^{+\infty} dT W^\alpha_{ij}(T) h^j(t-T).$$

(6)

Since $h^\alpha(t)$ is real and $h^ij(t)$ is real and symmetric, $W^\alpha_{ij}(t)$ is also real and symmetric. We model the noise $\hat{n}^\alpha(t)$ as stationary and gaussian, with zero mean, so it is characterized by its correlation function $C^\alpha\beta(T)$ or, equivalently, its spectral density $S^\alpha\beta(f) = \hat{C}^\alpha\beta(f)$:

$$C^\alpha\beta(T) = \langle \hat{n}^\alpha(t + T) \hat{n}^\beta(t) \rangle,$$

(7a)

$$\delta(f - f')S^\alpha\beta(f) = \langle \hat{n}^\alpha(f) \hat{n}^\beta(f') \rangle.$$

(7b)

$S^\alpha\beta(f)$ induces a natural inner product on the space of signals (or noise) in the network:

$$(p|q) = \int_{-\infty}^{\infty} df \tilde{p}^\alpha(f) [S^{-1}(f)]^\alpha\beta \tilde{q}^\beta(f).$$

(8)

Using matched filtering, a given gravitational wave signal $h^j(t)$ can be detected by such a network with expected signal-to-noise (SNR) given by

$$\text{SNR}^2 = \langle \hat{h}|\hat{h} \rangle = \int_{-\infty}^{\infty} df \tilde{h}^\alpha(f) [S^{-1}(f)]^\alpha\beta \tilde{h}^\beta(f)$$

(9a)

$$= \int_{-\infty}^{\infty} df \tilde{h}^\alpha_{ij}(f) K_{kl}^{ij}(f) \tilde{h}^{kl}(f),$$

(9b)

where we have introduced a kernel

$$K_{kl}^{ij}(f) \equiv \tilde{W}^{ij}_{\alpha}(f)^* [S^{-1}(f)]^{\alpha\beta} \tilde{W}^\beta_{kl}(f)$$

(10)

with the following properties

$$K_{kl}^{ij}(f) = K^{ij}_{[kl]}(f) = K_{kl}^{ij}(f) = K_{kl}^{ij}(-f^*).$$

(11)

If a gravitational wave signal (which depends on various parameters $\mu^n$) is detected, and the likelihood function may be approximated as gaussian $\propto \exp[-(1/2)\mu^n T_{ab}^n \mu^b]$ near its peak, then the expected inverse covariance matrix is the Fisher information matrix, given by

$$\Gamma_{ab} = \left( \frac{\partial h}{\partial \mu^a} \right)^* \frac{\partial h}{\partial \mu^b}. $$

(12)

Now consider the angular resolution of this network. Let us define an orthonormal triad consisting of $\hat{n}$ and two additional unit vectors $\hat{m}_{\mu} (\mu = 1, 2)$. Under a rotation around the direction $\hat{m}_{\mu}$ by an infinitesimal angle $d\theta^\mu$, $h^{ij}(t)$ transforms as $h^{ij}(t) \rightarrow R^i_{\mu} R^j_{\nu} h^{kl}(t)$ where $R^i_{\mu} \equiv \delta^i_j - e^i_{jk} \hat{m}^k_{\mu} \theta^\mu$. So the angular part of $\Gamma_{ab}$ is given by

$$\Gamma_{\mu\nu} = \left( \frac{\partial h}{\partial \theta^\mu} \right)^* \frac{\partial h}{\partial \theta^\nu}.$$ (13)

where we have defined

$$\hat{L}^{ij}_{\mu\nu\rho\sigma}(f) \equiv 4K^{ik}_{\mu\nu}(f) e^{j\rho}_{kl} e^{\sigma\tau}_{ij} m^\rho_{\mu} m^\tau_{\sigma}$$

(14a)

$$L^{ij}_{\mu\nu\rho\sigma}(f) \equiv \hat{L}^{[ij]}_{\rho\sigma}(f).$$

(14b)

For fixed $\bar{\mu}$ and $\bar{\nu}$, $L^{ij}_{\mu\nu\rho\sigma}(f)$ has properties exactly akin to those in (11). The “off-diagonal” terms in the Fisher matrix, with one index $\bar{\mu}$ corresponding to an angular parameter $\theta^\bar{\mu}$, and the other index $\bar{\nu}$ corresponding to any other (non-angular) parameter is

$$\Gamma_{\bar{\mu}\bar{\nu}} = 2 \int_{-\infty}^{\infty} df \frac{\partial \hat{h}_{ij}(f)}{\partial \bar{\nu}} \hat{P}^{\alpha}_{ij}(f) C^{ij}_{kl}(f) e^{k}_{rs} m^r_{\bar{\mu}} m^s_{\bar{\nu}} h^{ls}(f).$$

(15)

**OPTIMAL ESTIMATOR FOR $h^{ij}$**

We want to find the minimum-variance unbiased estimator $\hat{h}^{ij}(f)$ and its covariance. Since $h^\alpha(f)$ is linearly related to $h^{ij}(f)$ $[\hat{h}^\alpha(f) = \hat{W}^\alpha f_{ij}(f)]$, $h^{ij}(f)$ is linearly related to $h^\alpha(f)$:

$$\hat{h}^{ij}(f) = \hat{V}^{ij}_\alpha(f) h^\alpha(f).$$

(16)

It is stationary and gaussian, with covariance

$$(\delta \hat{h}^{ij}(f) \delta \hat{h}^{kl}(f')) = S^{ij}_{kl}(f) \delta(f - f')$$

(17)

where $\delta \hat{h}^{ij}(f) \equiv \hat{h}^{ij} - \hat{h}^{ij}$. Our task is to express $\hat{V}^{ij}_\alpha$ and $S^{ij}_{kl}$ in terms of the given tensors $\hat{W}^{ij}_\alpha$ and $S^{\alpha\beta}$ that define the porcupine.

The estimator is unbiased if $\langle \hat{h}^{ij} \rangle = \hat{h}^{ij}$, and hence $\hat{h}^{ij} = \hat{V}^{ij}_\alpha \hat{W}^{ij}_\alpha$. Since this must hold for arbitrary $\hat{h}^{ij}$, we learn that

$$\hat{V}^{ij}_\alpha \hat{W}^{ij}_\alpha = I^{ij}_{kl}$$

(18)

where

$$I^{ij}_{kl} \equiv \delta^{ij}_{[k} \delta^{ij}_{l]}$$

(19)

is the identity operator on the 6-dimensional vector space of $3 \times 3$ symmetric matrices. The estimator $\hat{h}^{ij}$ only exists if the porcupine tensor $\hat{W}^{ij}_\alpha$ has a left-inverse $\hat{V}^{ij}_\alpha$ in the sense of (13). In a porcupine consisting of $N$ detectors: if $N < 6$, $\hat{V}^{ij}_\alpha$ does not exist; if $N = 6$ then, when $\hat{V}^{ij}_\alpha$ exists, it is specified uniquely by (13); and if $N > 6$ then, when $\hat{V}^{ij}_\alpha$ exists, it is not specified uniquely by (13). In this last case, the residual ambiguity in $\hat{V}^{ij}_\alpha$ may be fixed by using the method of Lagrange multipliers to minimize the total variance $I^{ij}_{kl} S^{ij}_{kl}$ of the estimator $\hat{h}^{ij}$, subject to the constraint (13); for help, see e.g. Appendix D in [13]. In this way we find

$$\hat{V}^{ij}_\alpha = (D^{-1})^{ij}_{kl} \hat{W}^{kl}_{AI} (S^{-1})^{\alpha}_{\beta}$$

(20)

and

$$S^{ij}_{kl} = (D^{-1})^{ij}_{kl}$$

(21)
where we have defined the tensor $D$ and its inverse $D^{-1}$
\[ D_{ij}^{kl} \equiv \bar{W}_{ij}^{\alpha} (S^{-1})_{\alpha \beta} W_{k\ell}^\beta, \] (22a)
\[ D_{mn}^{ij} (D^{-1})_{mn}^{ij} = I_{ij}^{kl}. \] (22b)

This completes our derivation of the minimum-variance unbiased estimator $\hat{h}_{ij}$ and its covariance $S_{ij}^{kl}$. Let us add a few remarks.

**Remark 1.** The minimum-variance unbiased estimator for the trace of $h_{ij}$ is $\hat{h} = \delta_{ij} h_{ij}$, with variance $\langle \hat{h}(f) \hat{h}(f^{'}) \rangle - \langle \hat{h}(f) \rangle \langle \hat{h}(f^{'}) \rangle = \delta_{ij} S_{ij}^{kl} (f) \delta_{ij} \delta(f - f^{'})$. If the porcupine detects a gravitational wave, $\hat{h}$ should be consistent with zero, to within this predicted uncertainty; this is an important observational test to distinguish genuine gravitational waves from spurious signals.

**Remark 2.** If $\delta^{ij} W_{ij} = 0$ (as in a network of equal-arm Michelson interferometers like LIGO/VIRGO [6]) the porcupine is insensitive to the trace of $h_{ij}$. It is convenient to introduce the notation $[\ldots]^T$ to denote the traceless part of a tensor. We must be careful, since this paper involves the relationship between several different linear spaces, each with its own trace. In particular, $[\ldots]^T$ will denote tracelessness with respect to 3-dimensional traces (i.e. those that can be performed using $\delta_{ij}$ and $\delta^{ij}$, but not $\delta^{ij}$). Explicitly:
\[ [T_{ij}^{kl}]^T = T_{ij}^{kl} - \frac{1}{3} \delta_{ij} \delta_{kl} T^{kl}, \] (23a)
\[ [T_{ij}^{kl}]^T = T_{ij}^{kl} - \frac{1}{3} \delta_{ij} \delta_{mn} T_{mn}^{kl} - \frac{1}{3} T_{ij}^{mn} \delta_{mn} \delta_{kl} + \frac{1}{9} \delta^{ij} \delta_{pq} T_{pq}^{mn} \delta_{mn} \delta_{kl}. \] (23b)

Thus, we want the minimum-variance unbiased estimator $[\hat{h}_{ij}]^T$ and its covariance. This is done by following the same steps as above; in fact, the answer is still given by Eqs. (16-22), as long as we replace each factor in these equations by their traceless part: e.g. $\hat{h}_{ij} \rightarrow \hat{h}_{ij}^T$, $\bar{V}_{ij}^{\alpha} \rightarrow \bar{V}_{ij}^{\alpha \alpha} T^{Tij}_{ij}^T$, $P_{ij}^{kl} \rightarrow [T_{ij}^{kl}]^T$, $S_{ij}^{kl} \rightarrow [S_{ij}^{kl}]^T$. Here
\[ [T_{ij}^{kl}]^T = \delta_{ij} [\delta^{kl}] - \frac{1}{3} \delta^{ij} \delta_{kl} \] (24)
is the identity operator on the 5-dimensional space of $3 \times 3$ symmetric traceless matrices. The estimator $[\hat{h}_{ij}]^T$ only exists if $W_{ij}^{\alpha}$ has a left-inverse $[\bar{V}_{ij}^{\alpha}]^T$ in the sense of (the traceless part of) Eq. (18); and this is only possible for $N \geq 5$ detectors.

**Remark 3.** Although the porcupine may be simultaneously immersed in many gravitational plane waves, travelling in many different directions, suppose that there is at most one plane wave in each frequency bin $f \pm \delta f$, travelling in an unknown direction $\hat{n}(f)$. Then, since the gravitational wave is transverse, we have $\hat{h}_{ij}(f) \hat{n}_{ij}(f) = 0$, which implies that $\text{Det}[\hat{h}_{ij}(f)]$ should vanish. We can check this observationally by confirming that the estimator $\text{Det}[\hat{h}_{ij}(f)]$ is consistent with zero, to within the predicted uncertainty. (Calculating the predicted variance of this estimator is an exercise in applying Wick’s Theorem.) Also note that if we choose two mutually orthogonal unit vectors $\hat{a}$ and $\hat{b}$, and use these to form the estimators $\hat{A} = \hat{h}_{ij} \hat{a}_i$ and $\hat{B} = \hat{h}_{ij} \hat{b}_j$, then $\hat{N} = (\hat{A} \times \hat{B})/(\hat{A} \times \hat{B})$ is an estimator for the zero-eigenvector of $h_{ij}$ (i.e. $\pm \hat{n}$, the propagation direction up to a sign).

**AVERAGE SNR$^2$**

Let us average the expected SNR$^2$ [13]: first over the polarization state, and then also over the direction $\hat{n}$ of the incident gravitational wave. First we average over the polarization, keeping $\hat{n}$ fixed, to obtain:
\[ \text{SNR}^2_{\hat{n}} = \int_{-\infty}^{\infty} df [\hat{h}(f)]^2 P_{ij}^{kl} (\hat{n}) K_{kl}^{ij}(f) \] (25)
where
\[ [\hat{h}(f)]^2 \equiv \hat{h}_{ij}(f) \hat{h}_{ij}(f), \] (26a)
\[ P_{ij}^{kl}(\hat{n}) = (1/2) \delta^{ij} P_{A}^{kl}(\hat{n}) P_{B}^{kl}(\hat{n})^* \] (26b)
and
\[ \Delta_{ij} \equiv \delta_{ij} - \hat{n}_i \hat{n}_j \] is the projector onto the plane perpendicular to $\hat{n}$. Now use Eq. (2.3b) in [12] to find
\[ \int \frac{d\Omega}{4\pi} P_{ij}^{kl}(\hat{n}) = \frac{1}{5} [T_{ij}^{kl}]^T \] (27)
and use this identity to obtain the SNR$^2$, fully averaged over both the polarization and direction $\hat{n}$ of the incident gravitational wave:
\[ \langle \text{SNR}^2 \rangle = \int_{-\infty}^{\infty} df [\hat{h}(f)]^2 \frac{1}{5} [T_{ij}^{kl}]^T K_{kl}^{ij}(f). \] (28)

**PERFECT PORCUPINES**

A perfect porcupine is a network for which the tensor $K_{ij}^{kl}(f)$ is rotationally invariant. The only objects from which we can build such a tensor are $\delta_{ij}$ and $\delta^{ij}$. Imposing the constraints [11] we find that the most general perfect porcupine is of the form
\[ K_{ij}^{kl}(f) = F(f) [I_{ij}^{kl}]^T + G(f) \delta^{ij} \delta_{kl} \] (29)
where $F$ and $G$ are real and symmetric under $f \rightarrow -f$
\[ F(f) = F(f)^* = F(-f), \quad G(f) = G(f)^* = G(-f). \] (30)
Then the expressions for the SNR$^2$, and the angular parts of the Fisher matrix, simplify beautifully
\[ \text{SNR}^2 = \int_{-\infty}^{\infty} df [\hat{h}(f)]^2, \] (31a)
\[ \Gamma_{\mu \nu} = (\text{SNR}^2) \delta_{\mu \nu}, \quad \Gamma_{\mu \chi} = \Gamma_{\chi \mu} = 0. \] (31b)
Thus, a perfect porcupine localizes a gravitational wave source within a circular spot (really two antipodal spots) of radius $1/\text{SNR}$, regardless of the source’s properties.
**SIMPLE PORCUPINES**

Suppose that the individual detectors in the network are identical to one another (up to spatial orientation), with noise that is uncorrelated between different detectors: \( S^β_\beta(f) = S(f)δ^α_\alpha \). Further suppose that each detector’s frequency response may be factored out from its tensor structure: \( \tilde{W}^α_\alpha(f) = W(f)A^α_\alpha \). (We can take the frequency-independent matrices \( A^α_\alpha \) to be normalized: \( A^α_\alpha A^{α′}_\alpha = 1 \) with \( α \) unsummed.) A network with these properties will be called “simple.”

The averaged SNR\(^2\) \( \langle \text{SNR}^2 \rangle \) for a simple porcupine is

\[
\langle \text{SNR}^2 \rangle = N \left( \frac{3 - |\text{Tr}A|^2}{15} \right) \int_{-\infty}^{\infty} df \frac{|\tilde{h}(f)|^2 |\tilde{W}(f)|^2}{S(f)} \quad (32)
\]

where \( \text{Tr}A = δ^α_\alpha A^α_α \).

A “simply perfect porcupine” is both simple and perfect. In this case, we can trace Eq. (29) in two inequivalent ways (by contracting with \( δ^α_\alpha δ^\beta_\beta \) or \( δ^α_\alpha δ^\beta_\beta \)) to find

\[
F(f) = N \frac{|\tilde{W}(f)|^2 (3 - |\text{Tr}A|^2)}{S(f)} \quad (33a)
\]

\[
G(f) = N \frac{|\tilde{W}(f)|^2 |\text{Tr}A|^2}{S(f)} \quad (33b)
\]

For such a porcupine, we see that the SNR\(^2\) expression \((33a)\) coincides with the averaged expression \((32)\) for a simple but imperfect porcupine. Note that these expressions depend *only* on \( N, S(f), \tilde{W}(f), \text{and Tr}A \); and *not* on the directional orientation of the individual detectors that make up the network. In other words, if two different simply perfect porcupines are built from the same collection (i.e., the same number and type) of individual detectors, then they will have identical properties, despite the fact that they may seem like entirely different network configurations.

Now consider the problem of finding simply perfect porcupine configurations explicitly. Two cases deserve particular attention. In the first case, the network is constructed from single-arm detectors (like the proposed AGIS detector \([7, 9, 10]\)); the arm of detector \( α \) lies along the direction \( \hat{k}_α \) so that \( A^α_α = \hat{k}_α \hat{k}_α \) and \( \text{Tr}A = 1 \). In the second case, the network is constructed from double-arm detectors (like the LIGO/VIRGO Michel-son interferometers \([6]\)); the two arms of detector \( α \) lie along the two perpendicular directions \( \hat{p}_α \) and \( \hat{q}_α \) so that \( A^α_α = (\hat{p}_α^2 \hat{q}_α^2 - \hat{q}_α^2 \hat{p}_α^2)/\sqrt{2} \) and \( \text{Tr}A = 0 \). In these two cases, Eq. (29) becomes

\[
\sum_α \hat{k}_α \hat{k}_α \hat{k}_α \hat{k}_α = \left( \frac{N}{15} \right) (δ^α_β δ^\gamma_\delta + δ^\gamma_\delta δ^α_β + δ^α_\gamma δ^\beta_\delta), \quad (34a)
\]

\[
\sum_α (\hat{p}_α^2 \hat{q}_α^2 - \hat{q}_α^2 \hat{p}_α^2) (\hat{p}_α^2 \hat{q}_α^2 - \hat{q}_α^2 \hat{p}_α^2) = \left( \frac{2N}{5} \right) |F^{ij}_{kl}|^2. \quad (34b)
\]

Note that both sides of the single-arm Eq. (34a) are completely symmetric under permutation of the indices \( \{i, j, k, l\} \); this gets us down to 15 independent equations, one of which (obtained by tracing both sides with \( δ^α_β δ^\gamma_\delta \)) is automatically satisfied. Thus we have 14 independent equations for 2\( N \) - 3 variables (2 angles for each \( \hat{k}_α \), minus 3 angles corresponding to an arbitrary rigid rotation of the entire network). So, for \( N \geq 9 \), Eq. (34a) will have a \((2N - 17)\) parameter family of inequivalent solutions. In addition, there is a unique smaller solution with \( N = 6 \) detectors along the 6 diameters of an icosahedron \([2, 3, 5]\). On the other hand, for the double-arm Eq. (34b), there are no solutions for \( N \leq 4 \), and continuous families of inequivalent solutions for all \( N \geq 5 \).

**DISCUSSION**

Let us highlight three open problems. (i) The first problem is to find the general solution of Eq. (34a) or Eq. (34b) [that is, the most general configuration of a (single-arm or double-arm) simply perfect porcupine]. Which among these configurations could be most practically situation on the Earth’s available land area? (ii) Whereas perfect porcupines have a completely isotropic sensitivity pattern, “pointed porcupines” lie at the opposite extreme: they are the porcupines with the sensitivity pattern that is most highly localized on the sky. The second problem would be to develop the theory of such pointed porcupines, particularly since they may be the configurations that maximize the expected event rate (for a fixed collection of detectors, and assuming the gravitational wave sources are Poisson distributed throughout space). The third problem is to extend the present formalism in a relativistically correct way to investigate networks in which each detector is allowed to follow its own (possibly accelerating and rotating) trajectory.

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