In this paper we study the isotropic cases of static charged fluid spheres in general relativity. For this purpose we consider two different specialization and under these we solve the Einstein-Maxwell field equations in isotropic coordinates. The analytical solutions thus we obtained are matched to the exterior Reissner-Nordström solutions which concern with the values for the metric coefficients $e^\nu$ and $e^\mu$. We derive the pressure, density, pressure-to-density ratio at the centre of the charged fluid sphere and boundary $R$ of the star. Our conclusion is that static charged fluid spheres provide a good connection to compact stars.

Keywords: General Relativity; Charge Sphere; Compact Star

1. Introduction

The topic of static charged fluid spheres in general relativity is a challenging issue and has given rise to interesting studies, even if their number is not so high. Considering the research of the interior of stars in connection to their late stage evolution when the general relativistic effects play an important role, we notice the
interesting work of Tolman that yields a class of solutions for the static, spherically symmetric equilibrium fluid distribution. This important result was followed by some generalizations made by Wyman, Leibovitz and Whitman. After them, Bayin used the idea of the method of quadratures and gave new astrophysical solutions for the static fluid spheres.

In recent years, Ray and Das and Ray et. al. have developed some interesting solutions for the static charged fluid spheres in general relativity following the above line of thinking. Ray and Das performed the charged generalization of Bayin’s work related to Tolman’s type astrophysically interesting aspects of stellar structure. In this light, they actually considered that even in the case of stellar astrophysics there are physical implications of the Einstein-Maxwell field equations. In other papers Ray and Das performed a study of some previous solutions considering the phenomenological connection of the gravitational field to the electromagnetic field, and demonstrated the purely electromagnetic origin of the charged relativistic stars given by Tolman and Bayin. The existence of this type of astrophysical solutions is thought to be a probable extension of Lorentz’s conjecture that electron-like extended charged particle possesses only ‘electromagnetic mass’ and no ‘material mass’. In their latest work Ray et al. have considered Tolman-Bayin type static charged fluid spheres in general relativity and they have found out many interesting results, specially the cases which give support to the charged spherical models in connection to normal stars. It is argued by them that due to the inclusion of charge and by a suitable choice of charge part, pressure and density function could be a decreasing function of radius from centre to surface in contrary to Bayin’s case.

In this connection we also notice other two works, the first one elaborated by Varela which presents a neutral perfect fluid core bounded by a charged thin shell, and which demonstrates that it is possible to construct extended Reissner-Nordström sources with everywhere positive mass density, classical electron radius, electromagnetic mass, and everywhere non-negative gravitational mass. The other work was done by Ivanov that studied the interior perfect fluid solutions for the Reissner-Nordström metric using a new classification scheme. In addition, he found general formulae in some particular cases, presented explicit new global solutions and made a revision of some known solutions. In connection to the singularity problem it is argued by Ivanov that the presence of the charge function serves as a safety valve, which absorbs much of the fine-tuning necessary in the uncharged case. However, it is also believed by some authors that in the presence of charge the gravitational collapse of a spherically symmetric distribution of matter to a point singularity may be avoided through counterbalancing of the gravitational attraction by the repulsive Coulombian force in addition to the thermal pressure gradient due to fluid.

In continuation of the above theme, especially that of Ray et. al., in the present article we have tried to solve the Einstein-Maxwell field equations in isotropic coor-
ordinate system and derived expressions for pressure and density. We have found out conditions for the boundary of the charged sphere. The exterior Reissner-Nordström solution is compared and constants of integrations are expressed in terms of mass and radius. The mass-radius and mass-charge relations have been found out for various cases of the charged matter distribution along with the pressure-to-density ratio at the centre of the charged sphere.

Our paper is organized as follows: in Section 2 we introduce the Einstein-Maxwell field equations for the static charged fluid spheres in general relativity, we determine their analytical solutions making some assumption for the metric coefficients \( e^{\nu} \) and \( e^{\mu} \) and we consider two particular cases. In Section 3 we find out boundary conditions, which consist in a vanishing value for the pressure and a specific value for the boundary \( R \) of the star. The expression for the radius of the star \( R \) is established in Section 4 that is dedicated to a detailed analysis of the obtained solutions. The role of some parameters and integration constants in the evolution of pressure \( p \), density \( \rho \) and radius of the star \( R \) is discussed, with emphasis on some particular values. In Conclusions we enlighten the physical significance of the results.

2. The Einstein-Maxwell field equations and their analytical solutions

The static spherically symmetric matter distribution corresponding to the isotropic line element is given by

\[
\begin{align*}
    ds^2 &= e^{\nu} dt^2 - e^{\mu} \left( dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right),
\end{align*}
\]

where \( \nu \) and \( \mu \) are functions of the radial coordinate \( r \) only.

The Einstein-Maxwell field equations for the above line element can be written as

\[
\begin{align*}
    e^{-\mu} \left( \frac{\mu^{\prime^2}}{4} + \frac{\mu^{\prime} \nu^{\prime}}{2} + \frac{\mu^{\prime} + \nu^{\prime}}{r} \right) + \frac{q^2}{r^4} &= 8\pi p, \\
    e^{-\mu} \left( \frac{\mu^{\prime^2}}{2} + \frac{\nu^{\prime^2}}{2} + \frac{\mu^{\prime} + \nu^{\prime}}{2r} \right) - \frac{q^2}{r^4} &= 8\pi p, \\
    -e^{-\mu} \left( \frac{\mu^{\prime^2}}{4} + \frac{2\mu^{\prime}}{r} \right) - \frac{q^2}{r^4} &= 8\pi \rho,
\end{align*}
\]

where the total charge with a sphere of radius \( r \) in terms of the 4-currents \( J^i \) is

\[
q(r) = r^2 E(r) = 4\pi \int_0^r J^0 r^2 e^{(\mu+\nu)/2} dr,
\]

\( E \) being the intensity of the electric field. Here \( p \) and \( \rho \) are the pressure and matter-energy density, respectively. We have used prime to denote derivative with respect
Das, Ray, Radinschi, Rahaman and Ray

to radial coordinate \( r \) only. The field equations without the charge \( q \) have been studied by Buchdahl \[15\] and Bayin \[5\] who found physically meaningful solutions.

Equating equations (2) and (3) we get a differential equation by assuming \( e^{\nu} = A\Phi^{-a} \) and \( e^{\mu} = B\Phi^{b} \) in the form

\[
\Phi'' - c\frac{\Phi'^2}{\Phi} - \frac{\Phi'}{r} = \frac{4B}{b-a}\Phi^{b+1}\frac{q^2}{r^4},
\]

(6)

where \( \Phi = \Phi(r) \) and \( c = \left( \frac{1}{2}b^2 - \frac{1}{2}a^2 - ab + b - a \right)/(b - a) \).

Hence, to solve the above equation (6) for \( \Phi(r) \) we make use of the ansatz \( q(r)^2 = K^2\Phi(r)^d \) so that the above equation becomes

\[
\Phi'' - c\frac{\Phi'^2}{\Phi} - \frac{\Phi'}{r} = \frac{4B}{b-a}\Phi^{b-1}\frac{K^2}{r^4},
\]

(7)

with \( c = b+1+d \) where \( d \) behaves as the polynomial index. A direct solution for the above second order differential equation is difficult to find out. However, we can set different conditions for the parameters to get a solvable equation. Let us therefore consider the following two cases.

2.1. The case for \( c = 1 \) and \( d = -b \)

For the specification \( c = 1 \), when \( d = -b \), we get the following solutions

\[
\Phi = C_1e^{\left[\frac{b}{2(b-a)}\frac{K^2}{r^2} + C_0r^2\right]},
\]

(8)

\[
e^{\nu} = AC_1^{-a}e^{-a\left[\frac{b}{2(b-a)}\frac{K^2}{r^2} + C_0r^2\right]},
\]

(9)

\[
e^{\mu} = BC_1^{b}e^{b\left[\frac{b}{2(b-a)}\frac{K^2}{r^2} + C_0r^2\right]}.
\]

(10)

2.2. The case for \( c \neq 1 \) and \( d = -\frac{1}{2}(b^2 + a^2)/(b - a) \)

For this choice of \( c \) and \( d \) the solutions set becomes

\[
\Phi^{1-c} = \frac{B(1-c)}{2(b-a)}\frac{K^2}{r^2} + C_0r^2 + C_1,
\]

(11)

\[
e^{\nu} = A\left[\frac{B(1-c)}{2(b-a)}\frac{K^2}{r^2} + C_0r^2 + C_1\right]-a/(1-c),
\]

(12)

\[
e^{\mu} = B\left[\frac{B(1-c)}{2(b-a)}\frac{K^2}{r^2} + C_0r^2 + C_1\right]^{b/(1-c)}.
\]

(13)

We notice from the above two subcases that \( C_0 \) and \( C_1 \) represent integration constants and their expressions will be determined in Section 4 in terms of \( m \) and \( R \).
3. Boundary conditions

The exterior field of a spherically symmetric static charged fluid distribution described by the metric (1) in isotropic coordinates is the unique Reissner-Nordström solution

$$ds^2 = \left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)dt^2 - \left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

which by the radial coordinate transformation

$$r = r' \left(1 + \frac{m}{2r'} + \frac{q}{2r'} \right) \left(1 + \frac{m}{2r'} - \frac{q}{2r'} \right),$$

takes the form

$$ds^2 = \left(\frac{1 - \frac{m^2}{4r'^2} + \frac{q^2}{4r'^2}}{\left(1 + \frac{m}{2r'} - \frac{q}{2r'} \right)^2}\right)dt^2 - \left(\frac{1 + \frac{m}{2r'} - \frac{q}{2r'}}{\left(1 + \frac{m}{2r'} - \frac{q}{2r'} \right)^2}\right)dr^2 + r'^2 d\theta^2 + r'^2 \sin^2\theta d\phi^2.$$  

Therefore, matching at $r' = R$ we get

$$e^{\nu(R)} = \frac{\left(1 - \frac{m^2}{4R^2} + \frac{q^2}{4R^2}\right)}{\left(1 + \frac{m}{2R} - \frac{q}{2R} \right)^2},$$

$$e^{\mu(R)} = \left[\left(1 + \frac{m}{2R} \right)^2 - \frac{q^2}{4R^2}\right]^2,$$

where $R$ is the boundary of the star. At the boundary pressure is zero. Using this we will get an expression for $R$ in the next section.

4. The study of the solutions

4.1. The case for $c = 1$ and $d = -b$

$$8\pi p = \frac{C_1}{B} \left[ b(b-2a) \right] \left[ \left(\frac{B}{b-a} \right) \frac{K^2}{r^2} + 2C_0r \right]^2 + \left[ b(b-2a) \right] \left[ \left(\frac{B}{b-a} \right) \frac{K^2}{r^2} + 2C_0r \right] + \frac{BK^2}{r^2} \right) e^{-b\tilde{D}(r)},$$

$$8\pi \rho = -\frac{C_1}{B} \left[ b(b-2a) \right] \frac{K^2}{r^2} + 6bC_0 + \frac{K^2}{4} \left(\frac{B}{b-a} \right) \frac{K^2}{r^2} + 2C_0r^2 + \frac{BK^2}{r^2} \right) e^{-b\tilde{D}(r)},$$

where $\tilde{D}(r) = \frac{B(b-a)}{2} \frac{K^2}{r^2} + C_0r^2$.

In Fig. 1, Fig. 2 and Fig. 3 we plot the $8\pi p, 8\pi \rho$ and $[q(r)]^2$ against the parameter $r$ for some fixed values of the integration constants $C_0$ and $C_1$ and parameters $A$, $B$, $a$, $b$ and $K$.

Fig. 4 above indicates that $P + \rho$ is positive. That means though that the Weak Energy Condition (WEC) is violated in our case but obeys the Null Energy Condition (NEC) as well as the Strong Energy Condition (SEC) and the Dominant
Energy Condition (DEC). This is a very specific charged fluid solution. In reference to the figure for $p$ (Fig. 1), since $p = 0$ gives the radius of the charged fluid (rather we would say, $p(r = R) = 0$), one may conclude that the radius of the charged fluid falls within 3 to 4 Km. Thus our charged fluid is highly compact and we may consider our charged fluid as typically a highly compact strange star (we call it ‘strange’ in the sense that EOS components follow peculiar properties here i.e. they obey all energy conditions except WEC). In this context we would like to mention that in the original Bayin solutions, case V and case VI, $\rho$ is also negative.

Therefore, following Bayin, we are now in the position to find out pressure-to-density ratio at the centre of the charged sphere in the form

$$\frac{p_c}{\rho_c} = \frac{b - 2a}{b}.$$  

(21)

We notice that the pressure-to-density ratio at the centre of the charged sphere is entirely independent of the $A$ and $K$ parameters and integration constant $C_1$. In the case $b - 2a > 0$ with $b > 0$ we have $\frac{p_c}{\rho_c} < 0$ and this is not the case with simultaneous positive pressure and density at the origin. For $b - 2a < 0$ and $b > 0$ we obtain $\frac{p_c}{\rho_c} > 0$, which is a physically meaningful result for positive values for pressure and density at the origin. As a special case, if we choose $a = 0$, then we can easily recover the equation of state related to the Cases I and II for $n = 1$ of Ray et al. which refers to the ‘false vacuum’ or ‘degenerate vacuum’ or ‘$\rho$-vacuum’ equation of state.
Fig. 2. Bayin’s isotropic solutions with charge showing density vs radius plot for the given specifications in the figure (Case-4.1).

| Physical status       | a  | b      | b - 2a | p_c/p_c |
|-----------------------|----|--------|--------|---------|
| Dust                  | arbitrary | 2a | 0      | 0       |
| Vacuum fluid          | 0  | arbitrary | b       | -1      |
| Stiff fluid           | 1  | 1      | -1     | +1      |
| Radiation             | 2  | 3      | -1     | 1/3     |

Due to the repulsive nature of the pressure this provides a mechanism to avert the problem of singularity at the centre. On the other hand, if we choose $a = 2$ and $b = 3$ then we can recover the result of Ray et al. related to the Case I for $n = 3$ which refers to radiation. In Table 1 we have shown different possibilities for different values of the parameters $a$ and $b$.

Substituting $p(R) = 0$, we can get an expression for the boundary of the charged sphere as follows:

$$b(b - 2a)C_0R^8 + 2(b - a)C_0R^6 - \frac{b(b - 2a)BK^2C_0}{(b - a)}R^4 + \frac{b(b - 2a)BK^4}{4(b - a)^2}R^2 = 0.$$

Though it is a complicated power law equation for $R$, and hence very difficult to solve yet from this we get the same expression $R^2 = -\frac{2(b - a)}{5(b - 2a)C_0}$ which was obtained by Bayin (1978) in his non-charge case (see equation (4.23) there in). Again, comparing Reissner-Nordström metric in isotropic coordinates (16) [and
[\(C_1 - 1, C_0 - 1, B = 0.5, K = 1, a = 1\)]

Fig. 3. Bayin’s isotropic solutions showing charge vs radius plot for the given specifications in the figure (Case-4.1).

hence (17) and (18) we get

\[
AC_1^{-\alpha} e^{-\alpha \tilde{D}(R)} = \frac{\left(1 - \frac{m^2}{4R^2} + \frac{q^2}{4R^2}\right)^2}{\left(1 + \frac{m^2}{2R^2} \right)^2 - \frac{q^2}{4R^2}^2}, \quad (22)
\]

\[
BC_1 b e^{\tilde{D}(r)} = \left(1 + \frac{m}{2R}\right)^2 - \frac{q}{2R^2}\right)^2 \quad (23)
\]

so that

\[
C_0 = \frac{-G \pm (G^2 - 4FH)^{1/2}}{2F} \quad (24)
\]

where

\[
F = b(b - 2a)R^8, \quad (25)
\]

\[
G = 2(b - a)R^6 - \frac{b(b - 2a)}{(b - a)}BK^2R^4, \quad (26)
\]

\[
H = \frac{b(b - 2a)}{4(b - a)^2}P^2K^4 \quad (27)
\]
and

\[ C_1 = \frac{\left( 1 - \frac{m^2}{4R^2} + \frac{a^2}{4R^2} \right)^{-2/a}}{\left( 1 + \frac{m^2}{4R^2} \right)^{-2/a} A^{1/a} e^{-D(R)}}. \]  

(28)

Therefore, for \( B \) we get

\[ B = \frac{\left[ \left( 1 + \frac{m^2}{4R^2} \right)^{-2/a} \right]^{2(2a-b)/a}}{\left( 1 - \frac{m^2}{4R^2} + \frac{a^2}{4R^2} \right)^{-2b/a} A^{-b/a}}. \]  

(29)

Therefore, considering arbitrary values for \( a, b \) and \( A \) we get expressions for \( C_0, C_1 \) and \( B \) in terms of \( m \) and \( R \).

4.2. The case for \( c \neq 1 \) and \( d = -\frac{1}{2} (b^2 + a^2) / (b - a) \)

\[ 8\pi p = f \left[ \frac{b - 2a}{4(1-c)} \left( \frac{S'}{S} \right)^2 + \frac{b - a}{br} \left( \frac{S'}{S} \right) \right] + \frac{q^2}{r^4}, \]  

(30)

\[ 8\pi \rho = f \left[ -\left( \frac{S''}{S} \right) + \frac{b}{4(1-c)} \left( \frac{S'}{S} \right)^2 - \frac{2}{r} \left( \frac{S'}{S} \right) \right] - \frac{q^2}{r^4}. \]  

(31)
where

\[ f = \frac{bS^{-b/(1-c)}}{B(1-c)}, \]  

(32)

\[ S = \frac{B(1-c)K^2}{2(b-a)r^2} + C_0r^2 + C_1, \]  

(33)

\[ S' = -\frac{B(1-c)K^2}{b-a} r^3 + 2C_0r, \]  

(34)

\[ S'' = \frac{3B(1-c)K^2}{b-a} r^4 + 2C_0. \]  

(35)

The pressure-to-density ratio at the centre of the charged sphere in this case is as follows:\textsuperscript{5}

\[ \frac{p_c}{\rho_c} = -1. \]  

(36)

The same ‘false vacuum’ or ‘degenerate vacuum’ or ‘\(\rho\)-vacuum’ equation of state\textsuperscript{17,18,19,20} is achieved again without imposing the condition \(a = 0\) in the present case. Therefore, also due to the repulsive nature of the pressure this provides a mechanism to avert the problem of singularity at the centre.

The radius of the star is given by the expression obtained from the condition \(p(R) = 0\). However, here \(S\) and \(S'\) have to be taken at \(r = R\), so that the final expression for the radius \(R\) can be written as

\[ \frac{(b-2a)}{4(1-c)} \left( \frac{s'}{s} \right)^2 R^4 + \frac{b-a}{b} \left( \frac{s'}{s} \right) R^3 + \frac{1-c}{b} q^2 B s^{b/(1-c)} = 0 \]  

(37)

where we have used the symbol \(s = S(R)\).

We can find out the values of \(C_0, C_1\) and \(B\) in terms of \(M\) and \(R\) from the equation (37) and the following boundary conditions

\[ A s^{-a/(1-c)} = \left( \frac{1 - \frac{m^2}{2R^2} + \frac{q^2}{4R^2}}{\left( 1 + \frac{m^2}{2R^2} - \frac{q^2}{4R^2} \right)^2} \right)^{2}, \]  

(38)

\[ B s^{b/(1-c)} = \left( \frac{1 + \frac{m}{2R}}{2R} \right)^2 - \frac{q^2}{4R^2}. \]  

(39)

Due to the complicated nature of the solutions above, we could not discuss these analytically, especially we are unable to find out the polytropic relation as obtained by Bayin\textsuperscript{20} in his equation (4.43) for the classical limit \(P_c/\rho_c \rightarrow 0\).\textsuperscript{21} We expect to provide this study in a future project.
5. Conclusions

The study of static charged fluid spheres in general relativity is far from being complete, but some recent studies have motivated us to extend one previous work for improving the understanding of this topic. We found out the solutions of field equations in isotropic cases of charged fluid spheres in general relativity and performed a detailed study of the analytical solutions. In order to obtain the analytical solutions of the Einstein-Maxwell field equations, we made use of some assumptions and introduced two specialization, the first with the specification $c = 1$ and $d = -b$ and the second with the values $c \neq 1$ and $d = -\frac{1}{2}(b^2 + a^2)/(b - a)$. We calculated the corresponding expressions for the function $\phi(r)$ and for the metric coefficients $e^{\nu}$ and $e^{\mu}$, which depend on the parameters $A$, $B$, $a$, $b$ and integration constants $C_0$ and $C_1$. Matching the interior solution to the exterior Reissner-Nordström metric in isotropic coordinates at $r = R$ we obtained the formulae for $e^{\nu(R)}$ and $e^{\mu(R)}$.

We performed a deeper investigation of the solutions and for the case $c = 1$ and $d = -b$ we established the expression for the pressure-to-density ratio at the origin given by $\frac{p}{\rho} = \frac{-b - 2a}{1 - b}$ and the imposed conditions for $a$ and $b$ for obtaining a physically meaningful model for the star. For a zero value of $a$ we obtained the case of the false vacuum or degenerate vacuum or $\rho$-vacuum. In addition, we found out the explicit expression for the radius $R$ and discussed the physically reasonable cases. The radius $R$ for the charged sphere has a completed expression which, however, reduces to the same expression as in the non-charge case of Bayin. Finally, comparing Reissner-Nordström metric in isotropic coordinates we established the expressions for the integration constants $C_0$ and $C_1$ and $B$ parameter in terms of mass $m$ and the radius of the star $R$. On the other hand for the specification $c \neq 1$ and $d = -\frac{1}{2}(b^2 + a^2)/(b - a)$ we computed the pressure $p$ and the density $\rho$ that present a dependence on a new function $S(r)$ and its first and second derivatives with respect to the $r$ coordinate. In this case the expression for the pressure-to-density ratio at the origin can directly be given by $\frac{p}{\rho} = -1$. We also gave the expressions that allow to evaluate the boundary $R$ of the star and the values of $C_0$, $C_1$ and $B$ in terms of $m$ and $R$, but due to their complicated form we plan to perform a detailed study in a future work.

The most straightforward observation in the present work is that from our results we are able to recover the neutral cases of Bayin. We also observe that the models presented here have negative energy density and positive pressure and support the conclusion that static charged fluid spheres are connected to compact strange stars. We conclude that in the last years there is progress in the study of static charged fluid spheres which are connected to compact stars, but deeper and wider investigations for searching for new solutions need to be performed. One possibility will be to investigate our scenario based on different specialization and obtain new analytical solutions that can lead to other particular cases and more constrained connections with the involved parameters.
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