Computations of Radiative Heat Transfer inside Buildings

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Computations of Radiative Heat Transfer inside Buildings

Tomas Ficker 1
1 Brno University of Technology, Faculty of Civil Engineering, Veveří 95, CZ-602 00 Brno, Czech Republic
ficker.t@fce.vutbr.cz

Abstract. The present conference contribution is devoted to the application of the theoretical model of radiative heat transfer formulated in our first conference contribution entitled “General Formalism for the Computation of Radiative Heat Transfer”. In that preceding contribution, a general model for radiative heat transfer in inner spaces of buildings has been developed and the present contribution illustrates its application to a simple room. The room consists of two surfaces, namely, the heated circular floor and arched ceiling (cupola). The processing of this two-surface room has been accomplished by two methods, i.e. by means of view factors along with radiosities and alternatively by means of mutual emissivities. The model based on radiosity is quite general and is not restricted to a certain number of surfaces whereas the second model based on mutual emissivities is restricted only to two-surface rooms. These two alternative procedures offer the comparison of their computational effectiveness. The behaviour of the radiative heat flow in closed two-surface envelopes is subjected to analysis and numerical computations have shown that the total radiative heat flows tend to reach the compensation state in which the positive flow of the warm surface is compensated by the negative flow of the cold surface.

1. Introduction
So far, the thermal building technology has investigated heat loss prevalently as simple heat conduction through building envelopes along with ventilation (infiltration or exfiltration) [1]. Such an approximation avoids considering an alternative procedure taking into account direct radiative and convective flows of heat from the interior heating system towards interior walls. Starting the computations from the heater, all the three kinds of heat transport processes (conduction, convection and radiation) directed to the walls can be accounted for in the thermal analysis [1 - 3]. The heat flow has to be in equilibrium with heat conduction through the envelope. Although such a model may seem to be rather complex, it would offer optimizing not only the surface temperatures but also the surface thermal resistances. The present paper deals with the first part of this complex model, namely, it discusses and computes the radiant heat transfer in interiors. The computations are based on the theoretical formalism developed in our first conference contribution called “General Formalism for the Computation of Radiative Heat Transfer”. A complete solution of radiative heat transfer in inner spaces of buildings is presented. A simple two-surface room has been chosen as a tested space (Figure 1). Two computational methods have been employed, namely, the method based on view factors along with radiosities [4 – 6] and the alternative method relying on mutual emissivity [3]. The procedure based on radiosities is general and is not restricted to a finite number of surfaces whereas the second model based on general mutual emissivity is restricted only to two-surface envelopes.
2. Radiative heat transfers in two-surface envelope

Figure 1 shows the model of two-surface envelope. The numbering of surfaces is as follows. The floor has no. 1 and the cupola above the floor has no. 2. Input data can be found in Table 1. The whole computational procedure is explained in our previous conference contribution entitled “General Formalism for the Computation of Radiative Heat Transfer and, thus, the derivation of the used equations and formulae are not repeated but their applications are straightforwardly presented.

Table 1. Input data for two-surface envelope.

| Parameter | Surface no.1 | Surface no.2 |
|-----------|--------------|--------------|
| S (m²)    | 80           | 240          |
| T (K)     | 300 (Heated floor) | 290 (Cold cupola) |
| ε         | 0.9          | 0.25         |
| ρ         | 0.1          | 0.75         |

\[ ε \cdot E_b = ε \cdot 5.67 \cdot \left( \frac{T}{100} \right)^4 \]

(Watt/ m²)

Matrix of view factors:

\[ \vec{F} = \begin{bmatrix} 0 & 1 \\ 1/3 & 2/3 \end{bmatrix} \] (1)

Radiosities \( W_1 \) and \( W_2 \) (system of two equations):

\[ W_1 = \varepsilon_1 E_{b1} + \rho_1 [F_{11}W_1 + F_{12}W_2] \] (2)

\[ W_2 = \varepsilon_2 E_{b2} + \rho_2 [F_{21}W_1 + F_{22}W_2] \] (3)

\[ W_1 = 413.34 + 0.1W_2 \] (4)

\[ W_2 = 100.257 + 0.75 \left( \frac{1}{3}W_1 + \frac{2}{3}W_2 \right) \] (5)
\[ W_1' = 456.2015 \text{ (Watt/m}^2\text{)} \] \hspace{1cm} (6)

\[ W_2' = 428.615 \text{ (Watt/m}^2\text{)} \] \hspace{1cm} (7)

**Heat fluxes** \( q_1 \) and \( q_2 \):

\[ q_i = \frac{\varepsilon_i}{\rho_i} (E_{\text{hi}} - W_i) = \frac{1}{\rho_i} (\varepsilon_i E_{\text{hi}} - \varepsilon_i W_i) \] \hspace{1cm} (8)

\[ q_1 = \frac{1}{0.1} (413.34 - 0.9 \cdot 456.2015) = +27.5865 \text{ (Watt/m}^2\text{)} \] \hspace{1cm} (9)

\[ q_2 = \frac{1}{0.75} (100.257 - 0.25 \cdot 428.615) = -9.1957 \text{ (Watt/m}^2\text{)} \] \hspace{1cm} (10)

**Heat flows** \( \Phi_1 \) and \( \Phi_2 \):

\[ \Phi_i = S_i \cdot q_i \]

\[ \Phi_1 = S_1 \cdot q_1 = 80 \cdot 27.5865 = +2206.9 \text{ (Watt)} \] \hspace{1cm} (11)

\[ \Phi_2 = S_2 \cdot q_2 = 240 \cdot (-9.1957) = -2206.9 \text{ (Watt)} \] \hspace{1cm} (12)

The floor emits heat (+2206.9 Watt) and the cupola absorbs the same portion of heat (−2206.9 Watt). This is in agreement with the fact that the floor is heated. In the closed envelope of a room, the total radiative heat transfer is compensated, i.e. \( \Phi_1 + \Phi_2 = 0 \).

The preceding computational procedure may be performed formally to obtain general expressions for radiative heat flows between surfaces 1-2 and 2-1. Let us term these flows as \( \Phi_{12} \) and \( \Phi_{21} \). They have the following forms:

\[ \Phi_{12} = S_1 C_{12} \left[ \left( \frac{T_1}{100} \right)^4 - \left( \frac{T_2}{100} \right)^4 \right] \]

\[ C_{12} = \frac{C_b}{1 + \frac{S_1}{S_2} \left( \frac{1}{\varepsilon_2} - 1 \right)} \] \hspace{1cm} (13)

\[ \Phi_{21} = S_2 C_{21} \left[ \left( \frac{T_2}{100} \right)^4 - \left( \frac{T_1}{100} \right)^4 \right] \]

\[ C_{21} = \frac{C_b}{1 + \frac{S_2}{S_1} \left( \frac{1}{\varepsilon_1} \right) - 1} \] \hspace{1cm} (14)

The symbols \( C_{12} \) and \( C_{21} \) may be termed as mutual emissivity and allow for geometrical arrangements of surfaces 1 and 2, i.e. they have a similar meaning as the view factors \( F_{12} \) and \( F_{21} \). Formulae (13) and (14) may be applied not only to the two-surface envelope shown in Figure 1 but also to other kinds of two-surface envelopes. For example, heat flows in the closed envelope shown in Figure 2 would obey the same formulae.
Let us calculate the heat flows $\Phi_{12}$, $\Phi_{21}$ and compare them with the foregoing results (11) and (12).

$$\Phi_{12} = S_1 C_{12} \left[ \left( \frac{T_1}{100} \right)^4 - \left( \frac{T_2}{100} \right)^4 \right] = S_1 \frac{C_b}{\varepsilon_1 + \frac{S_1}{S_2} \left( \frac{1}{\varepsilon_2} - 1 \right)} \left[ \left( \frac{T_1}{100} \right)^4 - \left( \frac{T_2}{100} \right)^4 \right] =$$

$$= 80 \frac{5.67}{0.9 + \frac{1}{3} \left( \frac{1}{0.25} - 1 \right)} [3^4 - 2.9^4] = +2207.05 \text{ (Watt)} \quad (15)$$

$$\Phi_{21} = S_2 C_{21} \left[ \left( \frac{T_2}{100} \right)^4 - \left( \frac{T_1}{100} \right)^4 \right] = S_2 \frac{C_b}{\varepsilon_2 + \frac{S_2}{S_1} \left( \frac{1}{\varepsilon_1} - 1 \right)} \left[ \left( \frac{T_2}{100} \right)^4 - \left( \frac{T_1}{100} \right)^4 \right] =$$

$$= 240 \frac{5.67}{0.25 + \left( \frac{1}{0.9} \right) - 1} [2.9^4 - 3^4] = -2207.05 \text{ (Watt)} \quad (16)$$

By comparing heat fluxes $\Phi$ obtained by means of radiosities $W$ (see Equations (11) and (12)) and those by means of the coefficient of mutual emissivity $C$ (see Equations (15) and (16)), it is obvious that the results (2206.9 Watt versus 2207.05 Watt) are almost the same. Small difference (0.15 Watt) has been caused by rounding operations performed during calculations (the so-called rounding errors).
3. Compensation principle

The zero total radiative heat flow $\Phi = \sum_{i} \Phi_i = 0$ seems to be a general property of all the closed envelopes since the portion of heat flows irradiated from warm surfaces are received by the cold surfaces of the envelopes. No portion of energy disappears provided the air inside the envelope is diathermal.

This fact may be easily proved just in the case of the two-surface envelope by using the concept of mutual emissivities. At first, it is shown that $S_1 C_{12} = S_2 C_{21}$:

$$C_{12} = \frac{S_2}{S_1} C_{12} = \frac{S_2}{S_1} \left[ \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} \right] = \frac{S_2}{S_1} \frac{1}{\varepsilon_1} \left( 1 + \frac{\varepsilon_2}{\varepsilon_1} - 1 \right) = \frac{S_2}{S_1} \frac{C_b}{\varepsilon_2} = \frac{S_2}{S_1} C_{21}$$  \hspace{1cm} (17)

$$S_1 C_{12} = S_2 C_{21}$$  \hspace{1cm} (18)

Now, the relation $\Phi = \sum_{i=1}^{2} \Phi_i = 0$ can be proved. This proof uses result (18):

$$\Phi_{12} = S_1 \cdot C_{12} \cdot \left[ \left( \frac{T_1}{100} \right)^4 - \left( \frac{T_2}{100} \right)^4 \right] = S_2 \cdot C_{21} \cdot \left[ \left( \frac{T_1}{100} \right)^4 - \left( \frac{T_2}{100} \right)^4 \right] =$$

$$= -S_2 \cdot C_{21} \cdot \left[ \left( \frac{T_2}{100} \right)^4 - \left( \frac{T_1}{100} \right)^4 \right] = -\Phi_{12}$$  \hspace{1cm} (19)

Equation (19) shows that both the radiative heat flows have the same absolute values but different mathematical signs:

$$\Phi_{12} = -\Phi_{21}$$  \hspace{1cm} (20)

$$\Phi = \sum_{i=1}^{2} \Phi_i = \Phi_{12} + \Phi_{21} = 0$$  \hspace{1cm} (21)

4. Conclusions

In this paper, the general formalism of radiative heat transfer has been applied to two-surface envelope that has been represented by the room with heated floor and arched ceiling in the form of cupola. The computations have been performed by two methods. The first method has employed view factors and radiosities whereas the second method has been based on the concept of mutual emissivity. The results of both the methods have been in good agreement and have shown that the heat flow of $+2207$ Watts has been emitted by the heated floor. The same portion of heat flow has been received by the cold cupola (-2207 Watts). Thus, the total sum of these flows has been equal to zero.

A mathematical proof concerning zero value of the total radiative heat flow in the two-surface envelope has been presented. The proof has been based on the concept of mutual emissivity. This property seems to be a general property not only of the two surface envelopes but also other envelopes consisting of many surfaces within the closed spaces of buildings.

The developed computational procedure of radiative heat transfer inside rooms is only a first part of a more general model focused on heat losses of building envelopes. The second part of this general
model will solve the convective heat transfer in interiors and in the third part of the model, the system of combined transcendental equations will be formulated. These equations will represent the thermal steady state between the interior and exterior. The computations will implicitly include an optimization procedure that will ensure attaining optimum surface temperatures and optimum surface thermal resistances, which will facilitate determining the most realistic value of heat loss.

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