GRAVITATIONAL DRESSING OF
AHARONOV-BOHM AMPLITUDES

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We investigate Aharonov-Bohm scattering in a theory in which charged bosonic matter fields are coupled to topologically massive electrodynamics and topologically massive gravity. We demonstrate that, at one-loop order, the transmuted spins in this theory are related to the ones of ordinary Chern-Simons gauge theory in the same way that the Knizhnik-Polyakov-Zamolodchikov formula relates the Liouville-dressed conformal weights of primary operators to the bare weights in two-dimensional conformal field theories. We remark on the implications of this connection between two-dimensional conformal field theories and three-dimensional gauge and gravity theories for a topological membrane reformulation of strings. We also discuss some features of the gravitational analog of the Aharonov-Bohm effect.

The Aharonov-Bohm effect is one of the most extensively studied problems in planar physics. Originally, the primary motivation for its investigation was the fact that this effect requires a physical, rather than mathematical, interpretation of the electromagnetic gauge potential. A second surge of interest in the subject was sparked by the realization that flux-charge composites (anyons) exhibit fractional statistics through the Aharonov-Bohm effect. Such composites can be described by coupling ordinary particles to a Chern-Simons gauge field, so that the Aharonov-Bohm effect is equivalently encoded in the scattering of a charged particle from a flux tube, the scattering of two flux-charge composites (two-anyon scattering), and the scattering of two particles coupled to a Chern-Simons gauge field (Aharonov-Bohm scattering). Substantial work has also been devoted to the development of successful perturbative approaches within non-relativistic and relativistic formalisms, in light of the intriguing observation that the perturbative expansion of the expression originally obtained by Aharonov and Bohm cannot be obtained within the ordinary Born approximation. Recently, we have also advocated the study of the (generalized) Aharonov-Bohm effect as a tool in the investigation of the relations between certain

*Presented by G. Amelino-Camelia as plenary talk at the Workshop on Low Dimensional Field Theory, Telluride, CO, 5-17 Aug 1996. To be published in the proceedings.
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three-dimensional gauge and gravity theories and some associated two-dimensional conformal field theories. Here we shall discuss Aharonov-Bohm scattering in a theory in which charged bosonic matter fields are coupled to topologically massive electrodynamics and topologically massive gravity, and use it to probe the relation between (two-dimensional) Liouville theory and topologically massive gravity.

The action of topologically massive gauge theory is

\[ S_{\text{T MGT}} = -\frac{1}{2e^2} \int_M d^3x \, \text{tr} \sqrt{g} \, g^{\mu\lambda} g^{\nu\rho} F_{\mu\nu} F_{\lambda\rho} + S_{CS} \]  

where

\[ S_{CS} = \int_M \frac{k}{4\pi} \text{tr} \left( A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right) \]  

is the topological, parity-violating Chern-Simons action, the three-dimensional space-time manifold \( M \) has metric \( g_{\mu\nu} \) of Minkowski signature, and \( A = A_\mu(x) dx^\mu = A_\mu(x) T^a dx^a \) is a gauge connection of a trivial vector bundle over \( M \) with \( T^a \) the anti-Hermitian generators of the compact gauge group \( G \). The first term in (1) is the usual Yang-Mills kinetic term for the gauge fields with \( F = dA + [A, A]/2 \) the curvature of \( A \). Unlike the pure Chern-Simons theory (2), the action (1) does not define a topological field theory because the \( F^2 \)-term depends explicitly on the three-dimensional metric \( g_{\mu\nu} \) and there are propagating degrees of freedom (massive vector bosons) with topological mass \( M = k e^2 / 4\pi \) so that the Hilbert space of this quantum field theory is infinite-dimensional.

The action of topologically massive gravity is

\[ S_{\text{T MG}} = \frac{k'}{8\pi} \int_M \left( \omega^a \wedge d\omega^a + \frac{2}{3} \epsilon^{abc} \omega^a \wedge \omega^b \wedge \omega^c \right) + S_E \]  

where

\[ S_E = \kappa \int_M e^a \wedge R^a \]  

is the three-dimensional Einstein gravity action, \( e^a = e^a_\mu dx^\mu \) are the dreibein fields which generate local SO(2, 1) rotations of the frame bundle of \( M \) (and are related to the metric of \( M \) by \( \eta_{\mu\nu} e^a_\mu e^b_\nu = g_{\mu\nu} \), with \( \eta^{ab} \) the local flat Minkowski metric of the tangent bundle \( TM \)), \( \kappa \) is the Planck mass, and

\[ R^a = R^a_{\mu\nu} dx^\mu \wedge dx^\nu = d\omega^a + \epsilon^{abc} \omega^b \wedge \omega^c \]  

is the curvature of the spin-connection \( \omega^a = e^{abc} \omega^b = \epsilon^{abc} \omega^b_\mu dx^a \) of the frame bundle of \( M \). The dreibein and spin-connection fields are related by the constraint

\[ \nabla e^a = de^a + 2\epsilon^{abc} \omega^b \wedge e^c = 0 \]
which ensures that $\omega^a$ is the Levi-Civita spin-connection for $g_{\mu\nu}$ (this means that we are working in the ‘minimal’ formalism of general relativity). Pure three-dimensional Einstein gravity ($k' = 0$ in (4)) can be viewed as a topological Chern-Simons theory with gauge group $ISO(2,1)$, but the full gravity theory (4) is not topological. The graviton degrees of freedom propagate with topological mass

$$\mu = \frac{8\pi\kappa}{k'}$$

(8)

The first term in (4) can be regarded as a Chern-Simons action for an $SO(2,1)$ gauge theory with connection $\omega$.

We shall be concerned with Aharonov-Bohm scattering in the theory in which charged, massive scalar fields are minimally coupled to topologically massive electrodynamics and topologically massive gravity, \textit{i.e.} the theory described by the action

$$S = S_{TMGT}^{U(1)} + S_{TMG} + \int_M d^3x \sqrt{g} \left( g^{\mu\nu}[(\partial_\mu - iA_\mu)\phi]^*[(\partial_\nu - iA_\nu)\phi] - m^2 \phi^*\phi \right)$$

(9)

where $g = \det[g_{\mu\nu}]$ and $g^{\mu\nu} = \eta^{ab}e^a_\mu e^b_\nu$. The Aharonov-Bohm amplitude resulting from the theory (4) can be expected, based on the expected relation between (dressing by) topologically massive gravity and (dressing by) Liouville theory, to be related to the ordinary Aharonov-Bohm amplitude in a simple way, dictated by the Knizhnik-Polyakov-Zamolodchikov (KPZ) scaling relations for primary fields of Liouville theory

$$\hat{\Delta} - \hat{\Delta}_0 = \frac{\hat{\Delta}(1 - \hat{\Delta})}{c + 2}$$

(10)

where $c$ is the central charge of the $SL(2,R)$ current algebra, $\hat{\Delta}_0$ is the bare conformal dimension of the primary field, and $\hat{\Delta}$ is its Liouville-dressed conformal dimension.

As discussed in detail in Ref.7 and references therein, the formal relation between topologically massive gravity and Liouville theory identifies the conformal dimensions of primary fields in the latter with the transmuted spins that charged particles acquire due to their interaction with the former.\(^a\) Moreover, the central charge $c$ of Liouville theory is related to the gravitational Chern-Simons coefficient in (4) by $c = -k' - 4$. Accordingly, from (10) it would follow that the transmuted

\(^a\)There are several results suggesting that topologically massive gravity might be related to Liouville theory. In particular, some evidence in support of this relation is provided in Ref.10 at the level of a (formal) path integral analysis. One can interpret the result we report in the present paper, as an indication that a suitable quantum measure does exist such that the results of the analysis of Ref.10 hold at the full quantum level.

\(^b\)In general, the transmuted spins that (three-dimensional) charged particles acquire due to their interaction with a Chern-Simons gauge field are equal to the conformal dimensions of the primary fields of a corresponding (two-dimensional) Wess-Zumino-Novikov-Witten model.
spins $\Delta$ of the theory (1) should be related to the “bare” transmuted spins $\Delta_0$ (the ones that charged particles acquire as a result of their interaction with the ordinary Chern-Simons gauge field) according to

$$\Delta - \Delta_0 = \frac{\Delta(\Delta - 1)}{k' + 2}$$

In turn this equation encodes the relation between the Aharonov-Bohm amplitude resulting from the theory (1) and the ordinary Aharonov-Bohm amplitude. In fact, the transmuted spins completely specify these amplitudes. To review this, we minimally couple charged particles in an irreducible unitary representation $R(G)$ of the gauge group $G$ to the Chern-Simons gauge field $A$ with a conserved current $J^\mu = J^\mu_a R^a$. Then the invariant amplitude for the scattering of two charged particles of initial momenta $p_1$ and $p_2$ represented by the current $J$ in the infrared limit $M \to \infty$ is [4] (Fig. 1)

$$A(p_1, p_2; q) \equiv \lim_{M \to \infty} i \text{tr} J^\mu(2p_1 - q)G_{\mu\nu}(q)J^\nu(2p_2 + q)$$

$$= -\frac{16\pi i}{k} \dim(G) T_R(G) f_G(k) \frac{\epsilon_{\mu\nu\lambda} p_1^\mu p_2^\nu q^\lambda}{q^2}$$

where

$$G_{\mu\nu}(p) = \langle A_\mu(p)A_\nu(-p) \rangle_A = -ie^2 \left( \frac{p^2 g_{\mu\nu}(p) + iM \epsilon_{\mu\nu\lambda} p^\lambda}{p^2(p^2 - M^2)} \right)$$

is the momentum space bare gluon propagator of the topologically massive gauge theory (1) in the transverse covariant Landau gauge, with $g_{\mu\nu}(p) = g_{\mu\nu} - p_\mu p_\nu/p^2$ the transverse projection operator on the momentum space of vectors, $q$ is the momentum transfer, $T_R(G)$ is the quadratic Casimir of $G$ in the representation $R(G)$, and $f_G(k) = \sum_{n \geq 0} f_n/k^n$ is a function whose coefficients $f_n$ (which can be computed perturbatively order by order in the Chern-Simons coupling constant $1/k$) depend only on invariants of the gauge group $G$. In the center of momentum frame the amplitude (12) is none other than the Aharonov-Bohm amplitude for the scattering of a charge of strength $\sqrt{T_R(G) f_G(k)}$ off of a flux of strength $(4\pi/k) \dim(G) \sqrt{T_R(G) f_G(k)}$. This is the standard argument for the appearance of induced fractional spin and statistics perturbatively in a Chern-Simons gauge theory and it leads to the spin factor (transmuted spin)

$$\Delta_G(k) = \frac{T_R(G)}{k} f_G(k)$$

which measures the anomalous change of phase in the Aharonov-Bohm wavefunction under adiabatical rotation of one charged particle about another in the gauge theory [3].

Eqs. (12) and (14) completely specify the relation between Aharonov-Bohm amplitudes and transmuted spins, and therefore in the following we report results on the Aharonov-Bohm amplitude by simply reporting the corresponding transmuted
Fig. 1. The scattering amplitude for two charged particles in topologically massive Yang-Mills theory. Here \( p_1, p_2 \) denote the incoming particle momenta and \( q \) is the momentum transfer. Straight lines denote external charged particles, wavy lines depict gluons, and the solid circles represent the minimal coupling of the particle current \( J^\mu \) to the gluon field.

spin. Since we intend to discuss the Aharonov-Bohm amplitude in the theory \( (9) \) up to one-loop order by viewing the topologically massive gravity theory as a quantum field theory on a flat space, we shift the dreibein fields as

\[
e^a_\mu \rightarrow e^a_\mu + \delta^a_\mu
\]

and expand the various metric factors in \( (9) \). The one-loop action is then

\[
S^{(1)}[A_\mu, \phi, e^a_\mu] = \int d^3x \left\{ \frac{1}{4e^2} F^2 + |(\partial - iA_\mu)\phi|^2 - m^2|\phi|^2 + \frac{k}{8\pi} AdA \\
- \frac{1}{4e^2} \left( \eta^{\mu\lambda} \eta^{\rho\sigma} e^\alpha_\lambda - 2\eta^{\mu\lambda} \eta^{\rho} e^\lambda_\rho - 2\eta^{\rho\lambda} \eta^{\lambda} e^\mu_\rho - \frac{3}{2} \eta^{\mu\lambda} \eta^{\nu\rho} e^\sigma_\rho e^\alpha_\sigma \\
+ \eta^{\mu\lambda} e^\rho_\alpha e^\sigma_\rho + \eta^{\nu\lambda} e^\rho_\alpha e^\sigma_\rho + 4\eta^{\lambda\rho} \eta^{\mu} e^\rho_\alpha e^\sigma_\rho \\
- 2\eta^{\mu\lambda} \eta^{\rho} e^\nu_\lambda e^\alpha_\rho - 2\eta^{\rho\lambda} \eta^{\mu\nu} e^\alpha_\rho + \frac{1}{2} \eta^{\mu\lambda} \eta^{\nu\rho} e^\alpha_\rho e^\sigma_\sigma \right) F^\mu_{\lambda\rho} F^\lambda_{\sigma\rho} \\
+ \left( \eta^{\mu\nu} e^\lambda_\lambda + 2\eta^{\nu} e^\mu_\sigma - \eta^{\mu\nu} e^\lambda_\sigma - e^\mu_\sigma e^\nu_\lambda \right) \left( \left[(\partial_\mu - iA_\mu)\phi\right]^* \left[(\partial_\nu - iA_\nu)\phi\right] \\
+ 2\eta^{\nu\lambda} e^\mu_\sigma e^\lambda_\rho + \frac{1}{2} \eta^{\nu\lambda} e^\rho_\sigma e^\mu_\rho \right) \\
- \frac{m^2}{4} \left( 2e^\mu_\mu + e^\mu_\mu e^\nu_\nu - 3e^\mu_\nu e^\nu_\mu \right) \phi^* \phi \right\} + S_{TMG}[e^a_\mu \rightarrow e^a_\mu + \delta^a_\mu]
\]

From \( (16) \) the Feynman rules associated with each of the graviton-matter interactions relevant for the total one-loop scattering amplitude can be easily written down. In particular, the bare propagator for the dreibein field in momentum space...
and in the Landau gauge is given by the Deser-Yang graviton propagator

\[ D^{ij}_{\mu\nu}(p) = \langle e_i^\mu(p)e_j^\nu(-p)\rangle \]

\[ = \frac{i}{\kappa} \left( \frac{\mu^2}{2p^2(p^2 - \mu^2)} \left\{ \left( \frac{p^2}{\mu^2} - 2 \right) \eta^\mu_\nu(p)\eta^{ij}(p) + \delta^i_\mu(p)\delta^j_\nu(p) + \delta^j_\mu(p)\delta^i_\nu(p) \right\} + \frac{i\mu}{4p^2(p^2 - \mu^2)} \left\{ \epsilon_i^{\mu\lambda\delta^j_\nu(p)} + \epsilon_j^{\mu\lambda\delta^i_\nu(p)} + \epsilon_i^{\nu\lambda\delta^j_\mu(p)} + \epsilon_j^{\nu\lambda\delta^i_\mu(p)} \right\} \right) \]

(17)

The Aharonov-Bohm amplitude for the scattering of charged particles in the topologically massive quantum field theories arises from the imaginary, parity-odd part of the propagator for the mediating bosons which has a singular pole term at zero momentum. We want to evaluate the one-graviton corrections to the ordinary Aharonov-Bohm amplitude, i.e. we wish to determine, to order \( 1/k' \), the transformation of the bare transmuted spin, which in the pure, topologically massive electromagnetic case is given by

\[ \Delta_0 = 1/k \]

and is exact at tree level, due to the gravitational dressing.

The one-loop gravitational dressing of the parity-odd part of the topologically massive photon propagator vanishes identically in the infrared regime of the theory. Consequently, the gravitational contributions to the vacuum polarization do not affect the Aharonov-Bohm interaction at one-loop order. Furthermore, a careful analysis of the gravitational dressing of the topologically massive electrodynamics in reveals that the standard set of Ward-Takahashi identities for pure quantum electrodynamics holds as well for the gravitational renormalizations, and this implies that the total one-loop gravitational renormalization of the Aharonov-Bohm amplitude comes from the contributions of the ladder diagrams depicted in Fig. 2. Actually, in Fig. 2 we have depicted only the topologically inequivalent Feynman diagrams. There are 9 ladder graphs in total. The “triangle” diagram in Fig. 2 has a counterpart corresponding to \( p_1 \leftrightarrow p_2, q \rightarrow -q \) and the “box” graph has a partner corresponding to \( p_2 \rightarrow -(p_2 + q) \). Also, with the exception of the last “circle” diagram, each diagram has a partner associated with interchanging the photon and graviton lines (equivalently the interchange of initial and final external particles in the diagrams).

The computation of the ladder amplitudes depicted in Fig. 2 are rather involved, even in the infrared regime \( \tilde{M}, \mu \rightarrow \infty \) of the quantum field theory. Each of the Feynman integrals for these ladder amplitudes converges in the infrared limit \( \epsilon^2, \kappa \rightarrow \infty \) and the final results for the functions \( f(k) \) defined in (12) are independent of any mass scale of the model. The leading order \( 1/k' \) coefficients of the functions associated with the three amplitudes in Fig. 2 are

\[ f_0^{(\Box)} = -\frac{14 + 13\pi^2}{2k'} \quad , \quad f_0^{(\triangle)} = \frac{29 + 26\pi^2}{4k'} \quad , \quad f_0^{(\odot)} = 0 \]  

(19)
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Fig. 2. The topologically inequivalent one-loop gravitational ladder (one-photon and one-graviton exchange) diagrams. The spiral lines depict the dreibein fields $e_{\mu}^a$.

Taking into account the remaining six ladder diagrams which come from permuting the external particle lines in the Feynman graphs in Fig. 2, the total conformal dimension up to one-loop order is then

$$\Delta_{\text{grav}}^{(1)} = \Delta_0 + (4f_0^{(\odot)} + 4f_0^{(\bigcirc)} + f_0^{(\triangle)})/k = 1/k + 1/kk'$$

which, taking into account (18) and selecting the branch of (11) with $\Delta(\Delta_0 = 0) = 0$, agrees with the leading orders of the (iterative) large-$k'$ expansion of the KPZ formula (11). Having established this relation at least to one-loop order of perturbation theory provides further evidence in support of the expected relation between topologically massive gravity and Liouville theory, and is encouraging for the topological membrane approach to string theory, in which the string world-sheet is filled in and viewed as the boundary of a three-manifold.

We conclude with a few comments concerning the possibility of a gravitational analogue of the Aharonov-Bohm effect, which could result from the parity-odd structure in (17) that has a simple pole at $p^2 = 0$. Recent work based on a non-perturbative analysis within an abelian, linearized approximation to the action, has advocated this effect based on the observation that spinless, non-dynamical point particles in the presence of a topologically massive gravity field acquire an induced spin $k'/32\pi^2$ under adiabatical rotation. However, our perturbative large-$k'$ analysis does not (and could not) expose this non-perturbative phenomenon. In particular, the “tree-level gravitational Aharonov-Bohm amplitude”, associated to the imaginary part of the Feynman diagram in Fig. 1 with the photon line replaced by a graviton line, vanishes identically for all ranges of momenta as a result of the property

$$\mathcal{T}_{\text{grav}}^{(p_1, p_2; q)^{\odot}} = i\mathbb{E}_i^{\mu}(p_1, p_1 - q)\mathbb{E}_j^{\nu}(p_2, p_2 + q)D^{ij}_{\mu\nu}(q)^{\odot} \equiv 0$$

where we have considered only the parity-odd “c-terms” of the full graviton propagator (17), and $\mathbb{E}_i^{\mu}(p, q; q)D^{ij}_{\mu\nu}(q)^{\odot}$ is the bare meson-meson-graviton vertex function.

*From the point of view of the two-dimensional Liouville theory, there is no immediate reason to choose the $\Delta(\Delta_0 = 0) = 0$ branch for the solutions of the KPZ scaling relations; however, our approach selects this branch automatically. It would be interesting to give a three-dimensional interpretation of the other branch of the scaling relation (11).*
determined from (16). This feature is also true of the higher-loop amplitudes involving only graviton lines, and is a result of the index contractions that occur in the integrands of the Feynman integrals. Moreover, our one-loop computations show that there is no induced gravitational Aharonov-Bohm effect from the interactions of the topologically massive graviton field with the meson and photon fields, i.e. we find that, for all ranges of momenta, the integrands of the corresponding Feynman integrals which come from contracting the parity-even part of the photon propagator with the parity-odd part of the graviton propagator vanish identically. Since the initial spin of the charged particles is zero if only gravitational interactions (no gauge fields) are turned on, this perturbative result is consistent with the KPZ formula (11) for the branch that has $\Delta(\Delta_0 = 0) = 0$, in which each term is at least of order $\Delta_0$, i.e. there are no terms at any given order of the $1/k'$ expansion which are not accompanied by factors of $\Delta_0$. It is only for particles with non-zero bare spin, such as fermions or vector bosons, that one could find perturbatively non-vanishing pure gravitational corrections. In these cases, the non-zero diagrams come from the coupling of the spinning fields to the spin-connection $\omega_\mu^a$ itself.

The framework of our analysis is different from the one of the analysis of the gravitational Aharonov-Bohm effect reported in Ref. 17, in which a mass source was coupled to gravity to yield a static space-time with conical singularities. Such a singularity acts as a source for an Aharonov-Bohm type amplitude, as illustrated in Ref. 18 within a study of gravitational Aharonov-Bohm scattering. It would be interesting to connect the low-energy amplitude obtained above with the conical structure of space-time, but this relation is still not understood. We saw above that the gravitational interaction changes the original Aharonov-Bohm amplitude in a flat space-time. From our calculation it is clear that this effect is a result of energy-momentum being induced by a charge which was a source for those “gravitons” and yielded an extra contribution to the Aharonov-Bohm amplitude. It would be interesting to find a suitable solution of the field equations for topologically massive gravity interacting with topologically massive gauge theory (see Ref. 20 for a discussion of this and some exact solutions in the pure Einstein theory). Because our result is valid only for distances much larger than the inverse graviton mass, i.e. the region of space-time where this renormalized amplitude is derived is the region of pure Einstein gravity, the space-time behaves like a cone. The source of mass inside the cone is the area filled by the electric and magnetic fields from the topologically massive gauge theory source, and as a result the deficit angle of the cone is proportional to the bare weight $\Delta_0$. Thus it appears that the renormalized weight $\Delta$ includes this cone angle. However, the problem now is that in order to find an appropriate solution of the field equations one has to match the outer region of the cone with the inner region where the full topologically massive gauge and gravity theory must be taken into account. Some interesting results along these lines have been obtained recently in Ref. 21, but the necessary solution has yet to be found.
Acknowledgements

G.A.-C. acknowledges very useful discussions with A. Cappelli, S. Deser, R. Jackiw, E. Mottola, M. Ortiz, and G. Zemba. The work of G.A.-C. was supported in part by PPARC. The work of R.J.S. was supported in part by the Natural Sciences and Engineering Research Council of Canada.

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