NUMERICAL SIMULATIONS OF EQUATORIALLY ASYMMETRIC MAGNETIZED SUPERNOVAE: FORMATION OF MAGNETARS AND THEIR KICKS

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Received 2007 February 8; accepted 2007 September 8

ABSTRACT

A series of numerical simulations of magnetorotational core-collapse supernovae are carried out. Dipole-like configurations which are offset northward are assumed for the initially strong magnetic fields, along with rapid differential rotations. The aim of our study is to investigate the effects of the offset magnetic field on magnetar kicks and on supernova dynamics. Note that we study a regime where the proto–neutron star formed after collapse has a large magnetic field strength approaching that of a “magnetar,” a highly magnetized slowly rotating neutron star. As a result, equatorially asymmetric explosions occur with the formation of the bipolar jets. We find that the jets are fast and light in the north and slow and heavy in the south for rapid cases, while they are fast and heavy in the north and slow and light in the south for slow-rotation cases. The resulting magnetar kick velocities are \( \sim 300–1000 \text{ km s}^{-1} \). We find that the acceleration is mainly due to the magnetic pressure, while the somewhat weaker magnetic tension works in the opposite direction, due to the stronger magnetic field in the northern hemisphere. Note that observations of magnetar proper motions are very scarce; our results supply a prediction for future observations. Namely, magnetars possibly have large kick velocities, several hundred \text{ km s}^{-1}, as ordinary neutron stars do, and in extreme cases they could have kick velocities up to 1000 \text{ km s}^{-1}. In each model, the formed protomagnetar is a slow rotator with a rotational period of more than 10 ms. It is also found that, in rapid-rotation models, the final configuration of the magnetic field in the protomagnetar is a collimated dipole-like field pinched by the torus of toroidal field lines, whereas in the protomagnetar produced in the slow-rotation model the poloidal field is totally dominant.

Subject headings: methods: numerical — MHD — pulsars: general — stars: magnetic fields — supernovae: general

Online material: color figures

1. INTRODUCTION

Soft gamma-ray repeaters (SGRs) and anomalous X-ray pulsars (AXPs) are candidates of so-called magnetars, highly magnetized slowly rotating neutron stars. This is based on the “magnetar model,” i.e., that their burstlike activity, persistent X-ray emission, and large spin-down rates originate from high magnetic fields, \( B \approx 10^{11}–10^{15} \text{ G} \) (Duncan & Thompson 1992; Paczynski 1992; Thompson & Duncan 1995, 1996). It is notable that such a large magnetic field may also power supernova explosions (magnetorotational supernovae). Thus, the study of magnetorotational supernovae may be important in the context of magnetar formation.

The first paper concerning magnetorotational supernovae written by LeBlanc & Wilson (1970) appeared about 30 years before the idea of the “magnetar model” arose. They found that a strong magnetic field combined with rotation produced an asymmetric supernova explosion. Only a few studies followed after this (Bisnovatyi-Kogan et al. 1976; Müller & Hillebrandt 1979; Ohnishi 1983; Symbalisty 1984), reflecting a feeling that magnetic fields large enough to drive supernovae were thought to be unrealistic. Now that such high magnetic fields are observationally supported, the magnetorotational supernova is attracting more attention than before. In the last several years some papers have been published on magnetorotational supernovae (e.g., Wheeler et al. 2002; Akiyama et al. 2003; Yamada & Sawai 2004; Kotake et al. 2004, 2005; Takiwaki et al. 2004; Adelman et al. 2005; Sawai et al. 2005; Obergaulinger et al. 2006; Nishimura et al. 2006; Moiseenko et al. 2006; Shibata et al. 2006; Burrows et al. 2007). Although with these studies we have developed our understanding of magnetorotational supernovae, many issues still remain to be investigated. For example, neutron star kicks in the context of magnetorotational supernovae (magnetar kicks) have not been fully studied numerically (see, however, Kotake et al. 2005; they computed the parity-violating effects in strongly magnetized supernova cores and expected that neutron star kicks would result). In fact, all numerical simulations of magnetorotational core collapse have so far assumed equatorial symmetry and have computed only a quarter of the meridional plane of the star.

Observed neutron stars are known to have generally larger proper velocities than those of their progenitor stars (Gunn & Ostriker 1970). Their three-dimensional velocities are typically several hundred \text{ km s}^{-1}, and some of them have a speed higher than 1000 \text{ km s}^{-1} (e.g., Cordes & Chernoff 1998; Hobbs et al. 2005). The origin of the proper velocities is still controversial. Although many researchers agree that an asymmetric supernova explosion is a more promising origin than a binary-system disruption by a supernova explosion, several different mechanisms have been proposed, which can be categorized into three main classes (Lai 2004).

The first mechanism is a hydrodynamically driven kick. Scheck et al. (2006) numerically studied hydrodynamic instabilities in neutrino-driven supernova explosions and showed that neutron stars were accelerated up to about 500–1000 \text{ km s}^{-1} due to the dominance of \( l = 1 \) mode. In their numerical simulations neutron stars with gravitating inner boundaries are substituted. Fryer & Warren (2004) performed three-dimensional smoothed particle...
hydrodynamics simulations and asserted that neutrino asymmetries decelerated a kicked neutron star significantly, while Sheck et al. (2006) argued that they were negligible. The second mechanism is a neutrino-magnetic-field-driven kick. Lai & Qian (1998) studied the possibility that an asymmetric field topology could produce a large kick when combined with neutrino emission. They pointed out problems in former similar works and improved on how to deal with the microphysics. The conclusion which they derived was that at least an $\sim 10^{16}$ G difference in the magnetic field strength between the north and south poles of a proto–neutron star is necessary to get a 300 km s$^{-1}$ kick velocity. Arras & Lai (1999) investigated parity-violation effects on a neutron star's kick also with improved microphysics. They argued that for the generation of a kick velocity of a few hundred km s$^{-1}$, at least $10^{15} - 10^{16}$ G for the dipole magnetic field is necessary in a proto–neutron star. The third mechanism is an electromagnetically driven kick, which was suggested by Harrison & Tademaru (1975). According to their analysis, a neutron star kick velocity of several hundred km s$^{-1}$ can be generated when the initial rotation period of the pulsar is $\sim 1$ ms, although with an initial rotation period of $\sim 10$ ms the result is 2 orders of magnitude smaller than the former velocity. This mechanism does not depend on the strength of the magnetic field. The main difference from the other two mechanisms is the timescale of the kick generation. With a surface field of $\sim 10^{12}$ G and an initial rotation period of $\sim 1$ ms, the acceleration timescale of a pulsar is approximately 5 yr.

In this paper we consider another class mechanism, the “magnetohydrodynamically driven kick.” Consider the situation in which the rapidly rotating core has a dipole-like magnetic field which is somewhat offset from the center of the core prior to collapse. Then, it is expected that a supernova with equatorially asymmetric bipolar jets will occur. This will lead to a kick of the neutron star. A magnetar-class magnetic field is required for this mechanism to work. If this is the kick mechanism of an ordinary neutron star, the magnetic field must decay from $\sim 10^{15} - 10^{16}$ G to $\sim 10^{12}$ G. (This is also the case for the second mechanism above.) Goldreich & Reisenegger (1992) estimated the decay timescales of the magnetic field in a neutron star due to ohmic dissipation or ambipolar diffusion. Their results imply that magnetar-class magnetic fields confined in a neutron star would require at least $\sim 10^6 - 3 \times 10^7$ yr to decay to the strength of a typical neutron star ($\sim 10^{12}$ G). This is much longer than the ages of no small number of ordinary pulsars. Hence, we ignore the kicks of ordinary pulsars here, and are concerned only with the kicks of magnetars.

To investigate the magnetohydrodynamically driven kick mechanism, we numerically study the core collapse of a massive star with off-center, strong, dipole-like magnetic fields and rapid rotation. The initial magnetic field strength chosen here corresponds to the magnetar class or larger ($\sim 10^{15} - 10^{16}$ G) when the core contracts to a typical neutron star radius. This means that we adopt the so-called fossil origin hypothesis of magnetic fields in magnetars. Although the origin of magnetic fields in magnetars is still a mystery, Ferrario & Wickramasinghe (2005) asserted that the magnetic flux of $\theta$ Orionis C measured by Donati et al. (2002), which is an O star, corresponds to that of magnetars and that the magnetism of a magnetar could be explained as the fossil of the progenitor, as in the case of white dwarfs (Wickramasinghe & Ferrario 2005). In fact, the magnetic flux of $\theta$ Orionis C is $\sim 10^{27}$ G cm$^2$, which would correspond to $\sim 10^{15}$ G for the surface field if a neutron star were formed with the frozen magnetic field. There are several other OB stars suggested observationally to possess magnetar-class magnetic fluxes. For example, the estimated magnetic flux of HD 191612 (Donati et al. 2006) corresponds to a surface field strength of $\sim 5 \times 10^{15}$ G, whereas several $10^{14}$ G for the surface magnetic fields would be expected for $\zeta$ Cassiopeiae (Neiner et al. 2003a), $\omega$ Orionis (Neiner et al. 2003b), $\xi$ CMa (Hubrig et al. 2006), and V2052 Ophiuchi (Neiner et al. 2003c) if their magnetic fields were compressed in the formation of a neutron star. Among them, $\theta$ Orionis C and HD 191612 are O stars, while the others are B stars. On the other hand, Thompson & Duncan (1993) proposed that strong magnetic fields of magnetars originate from the convective dynamo process, which requires a rotation period shorter than $\sim 30$ ms. However, whether such a process really operates in neutron stars is still uncertain. In this paper we take the scenario that some OB stars have large magnetic fluxes and end up as magnetars (fossil origin hypothesis). Although the initial magnetic fields assumed in this study are still a little larger than those implied by the above observations, we are exploring the extreme limit.

The offset of the magnetic field in our calculations can be supported by numerical simulations of the fossil magnetic field in A stars and white dwarfs carried out by Braithwaite & Spruit (2004). They found that initially random magnetic fields evolve into a coherent dipole-like field configuration stabilized by a torus of twisted magnetic field lines. This magnetic field diffuses within $\sim 10^9$ yr, which is much longer than the lifetime of a massive star. We would like to stress that the resultant magnetic field was usually somewhat offset from the center of the star. Note that they assumed the fossil origin hypothesis of the magnetic field, which we also adopt here.

The rotation period assumed in our rapid-rotation models (see §2) is an order of magnitude shorter than the result of Heger et al. (2005), who calculated the evolution of rotating massive stars. In spite of the importance of their study, it is still not the final answer. The angular momentum distributions at the precollapse stage should be studied further in the future. Our position is that they are not well known yet, and we consider just the extreme case.

Since observations of magnetar kick velocities are very scarce so far, our standpoint in this paper is to predict future observations of magnetar proper velocities. Proper motions have been measured for only two magnetar candidates (both are AXPs), in which just upper limits were determined; $\lesssim 1400$ km s$^{-1}$ for 4U 0142+61 (Hulleman et al. 2000; Woods & Thompson 2006) and $\lesssim 2500$ km s$^{-1}$ for 1E 2259+586 (Ogelman & Tepedelenlioglu 2005). Gaensler et al. (2001) examined the reliability of AXP/SGR association with supernova remnants (SNRs). The associations which they considered probable enable us to estimate the velocities of AXPs and SGRs from their positions in the SNRs: $< 500$ km s$^{-1}$ for AX 1E 1841–045, $< 400$ km s$^{-1}$ for AX 1E 2259+586, and $< 500$ km s$^{-1}$ for AX J1845–0258 (AXP candidate) (Gaensler et al. 2001 and references therein). Cline et al. (1982) evaluated the velocity of SGR 0526–66 assuming that it is associated with SNR N49 and derived $v_{\text{kick}} \sim 400 - 1400$ km s$^{-1}$. Corbel et al. (1999) derived an $\sim 800 - 1400$ km s$^{-1}$ velocity for SGR 1627–41 from the association with SNR G337.0–0.1. However, Gaensler et al. (2001) insisted that these two associations between the SGRs and the SNRs are not convincing.

Recently, we reported the results of a numerical study on whether the configurations of magnetic fields affect supernova dynamics significantly (Sawai et al. 2005). The effects of an offset dipole field on a supernova explosion is another subject in which we are interested in this paper. We particularly focus on how the dynamics and explosion energy are altered, as well as the north-south difference of the magnetic and rotational energies.

The rest of this paper is organized as follows. We describe the numerical models using basic equations, the equation of state (EOS), formulae for the magnetic field and rotation, and the
2. MODELS

We numerically simulate the core collapse of magnetized massive stars with the numerical code ZEUS-2D developed by Stone & Norman (1992). A 1.5 $M_\odot$ core in a 15 $M_\odot$ star (S. E. Woosley 1995, private communication) is chosen as the precollapse model, which gives spherically symmetric profiles for the density and specific internal energy. The magnetic field and rotation are just added to the core by hand (see § 2.3).

2.1. Basic Equations

The following ideal MHD equations are solved:

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{v} = 0,$$

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p - \rho \nabla \Phi + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B},$$

$$\rho \frac{D\Phi}{Dt} = -p \nabla \cdot \mathbf{v},$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}),$$

where $\rho$, $\mathbf{v}$, $p$, $\Phi$, and $\mathbf{B}$ are the density, velocity, internal energy density, gravitational potential, and magnetic field, respectively. We denote the Lagrangian derivative as $D/Dt$.

The computational region is half of the meridional plane with the assumption of axisymmetry. A grid of 200($r$) × 60($\theta$) extending to 2000 km is used initially. After the central density reaches $10^{12}$ g cm$^{-3}$, we change the radial mesh resolution and the radius of the outer boundary to 300(r) km and 1500 km, respectively. The radial grid spacing is not uniform, with a finer resolution for smaller radii, while the angular grid points are placed uniformly.

2.2. Equation of State

As in the previous papers by Yamada & Sawai (2004) and Sawai et al. (2005), we use a parametric EOS which was first introduced by Takahara & Sato (1984). Although we drastically simplify complicated microphysics such as the neutrino transport, it is not a serious concern since our computation is done only on the timescale of the prompt explosion, during which the neutrino transport is less important.

The parametric EOS we employ in this paper is

$$p_{\text{tot}} = p_c(\rho) + p_t(\rho, e_t),$$

$$p_c(\rho) = K\rho^\gamma,$$  

$$p_t(\rho, e_t) = (\gamma_t - 1)\rho e_t.$$

The pressure consists of two parts, the cold part ($p_c$) and the thermal part ($p_t$). The thermal part is a function of the density and the specific thermal energy, $e_t$, in which $\gamma_t$ is the parameter called the thermal stiffness. The cold part is a function of the density alone, where the two constants $K$ and $\gamma$ take account of the effect of the degeneracy of leptons and the nuclear force. We choose the same values for $\gamma_t$, $K$, and $\gamma$ as did Sawai et al. (2005), so that a numerical simulation with neither magnetic field nor rotation fails to explode, as in recent realistic simulations. Readers are referred to the paper by Sawai et al. (2005) for more details about this EOS.

2.3. Magnetic Field and Rotation

The initial magnetic field is assumed to have a dipole-like structure produced by the sum of line currents which are uniformly distributed in a spherical region within a 1500 km radius. The equatorially asymmetric dipole magnetic field is obtained by displacing an equatorially symmetric field along the rotation axis. Then the initial magnetic field in cylindrical coordinates is given by

$$B_z(z, \omega) = \frac{2I}{c} \sum_{\omega_j} \sum_{z_i} \frac{z'}{\sqrt{(\omega_j \omega - z^2 + z')^2 + z'}},$$

$$B_\omega(z, \omega) = \frac{2I}{c} \sum_{\omega_j} \sum_{z_i} \frac{1}{\sqrt{(\omega_j \omega - z^2 + z')^2 + z'}},$$

$$z' = z - z_j - z_{\text{off}},$$

where $R$ is the distance from the center and $(\omega_0, z_0)$ is the mesh point where the line currents exist. The variables $J$, $K$, $E$, $c$, and $z_{\text{off}}$ are the current, complete elliptical integral of the first kind, complete elliptical integral of the second kind, the speed of light, and the degree of offset, respectively. We compute models with changing magnetic field strength and degree of offset (see Table 1). Note that if the result of Braithwaite & Spruit (2004) is adopted for an iron core with a 2000 km radius, the degree of displacement is several hundred kilometers. Figure 1 shows the initial magnetic field configuration of models MR3 and SR10 (for the names of the models, see below).

The initial angular velocity in our simulations is assumed to have a differentially rotating distribution,

$$\Omega(r) = \Omega_0 \frac{r^2_0}{r^2 + r^2},$$

where $\Omega_0$ and $R_0$ are constants. With this distribution, rotation is faster at small radii. In all models, $R_0 = 1000$ km is chosen, with which the initial differential rotation is rather mild. We compute models for rapid-rotation and slow-rotation cases (see Table 1).

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**Table 1**

| Model     | $|E_{\text{kin}}|/W$ (%) | $|T|/W$ (%) | $B_0$ (G) | $\Omega_i$ (rad s$^{-1}$) | $z_{\text{off}}$ (km) |
|-----------|--------------------------|------------|-----------|--------------------------|---------------------|
| MR0       | 0.5                      | 0.5        | 5.6 × 10$^{13}$ | 3.9 × 10$^0$          | 0                   |
| MR3       | 0.5                      | 0.5        | 4.6 × 10$^{13}$ | 3.9 × 10$^0$          | 300                 |
| MS3       | 0.5                      | 0.005      | 4.6 × 10$^{13}$ | 3.9 × 10$^{-1}$       | 300                 |
| SR3       | 1.0                      | 0.5        | 6.5 × 10$^{13}$ | 3.9 × 10$^0$          | 300                 |
| SR10      | 1.0                      | 0.5        | 4.9 × 10$^{13}$ | 3.9 × 10$^0$          | 1000                |
| SS3       | 1.0                      | 0.005      | 6.5 × 10$^{13}$ | 3.9 × 10$^{-1}$       | 300                 |

**Notes.**—The columns are as follows: $|E_{\text{kin}}|/W$, the magnetic energy normalized by the gravitational energy; $|T|/W$, the rotational energy normalized by the gravitational energy; $B_0$, the initial maximum magnetic field; $\Omega_i$, the initial angular velocity at the center of the core; $z_{\text{off}}$, the degree of magnetic-field displacement (see eqs. [8] and [9]).
The rapid and slow rotation correspond to a neutron star rotating with a period of \( \sim 1 \) ms and \( \sim 10 \) ms, respectively, if angular momentum conservation is assumed.

In Table 1 six computed models are summarized. The name of each model consists of three parts, two characters and a number. The first character denotes the strength of the magnetic field: "moderate" or "strong." The second character represents the rotation type: "slow" or "rapid." The attached number stands for the degree of magnetic-field displacement. For example, model MR3 has a moderate magnetic field, rapid rotation, and a displacement of 300 km.

### 2.4. Definitions of a Protomagnetar and Its Velocity

We define a protomagnetar (or a proto–neutron star) in our analysis as a region which has a mass of 1.2 \( M_\odot \). The mass of each fluid element is summed up by descending order of density. This is almost the maximal mass of fluid which remains through the simulations in the numerically active region and does not go past the outer boundary. Neutron stars in binary systems typically have a mass of around 1.4 \( M_\odot \) (e.g., Taylor & Weisberg 1982; Thorsett & Chakrabarty 1999). However, the masses of magnetar candidates are unknown at present. Scheck et al. (2006) predicted that the resultant mass of a neutron star decreases as the explosion timescale becomes short. If the explosion occurs in a prompt manner, a 1.2 \( M_\odot \) magnetar may not be too unlikely.

The velocity of the protomagnetar is defined as

\[
v_{\text{NS}}(t) = \int_{t_0}^{t} \frac{F(t')}{M_{\text{NS}}} \, dt',
\]

where \( F(t) \) is the net torque acting on the protomagnetar. The magnetic field of C10 is parallel to the rotation axis, and its strength is stronger near the rotation axis, while the magnetic field of model MR0 is dipole-like, with a larger strength near the center. Namely, model MR0 is a more "centrally" concentrated counterpart of model C10. The dynamical evolution of model MR10 is described as follows. Model MR0 first bounces at 142.7 ms after the onset of collapse when the central density reaches \( 3.75 \times 10^{14} \) g cm\(^{-3} \). However, the first shock wave is generated not by the nuclear force but by the magnetic force 0.2 ms prior to the nuclear bounce around a radius of 30 km near the rotation axis, which was also the case for model C10 in Sawai et al. (2005). Indeed the magnetic field around the region is strong enough by the time of bounce due to compression and field wrapping that the magnetic pressure becomes much larger than the matter pressure. Then, the magnetic-force-dominant regions gradually spread outward, and shock waves propagate through the core accompanied by the formation of a bipolar jet. This occurs by transferring the magnetic energy to kinetic energy.

The velocity of the magnetar could also be defined in principle as the center-of-mass velocity. However, we do not adopt this definition here because it leads to enormous numerical errors in calculating the volume integral of momenta over the region of the protomagnetar. The velocity at each grid point is obtained by time integration of the volume forces (pressure gradient, Lorentz force, and gravity) exerted there and the advection term. Hence, the volume integral of the momentum over the protomagnetar is equal to the time integration of the volume integral of these forces and the advection term over the protomagnetar. The difficulty in the numerical estimation of the protomagnetar's kick velocity lies in the volume integral of the forces and the advection term. In fact, the error in calculating the protomagnetar's kick velocity in this way comes not only from the surface of the protomagnetar but also from its entire volume, and the problem is that the values of the integrand at the center are greater by a few orders of magnitude than those at the surface of the protomagnetar and, as a result, errors of a few percent around the central region of the protomagnetar correspond to errors of more than 100% around the surface. On the other hand, if we calculate the protomagnetar's kick velocity by time-integrating the surface forces, the resultant numerical errors originate from local errors in the integrals on the protomagnetar surface, and their magnitude will be comparable to the typical error, namely, a few percent. This corresponds to an error of a few 10 km s\(^{-1} \) in the protomagnetar's kick velocity in model MR3, for example. It should be noted that the employment of a conservative scheme does not guarantee the correct spatial distribution of the momentum. Moreover, since the self-gravity cannot be written in a conservative form, strict conservation of the total momentum cannot be expected after its inclusion.

### 3. RESULTS

We show the numerical results of our computation in this section. The explosion dynamics, explosion energies, final states of the magnetic field and rotation, and the magnetar kicks are given separately.

#### 3.1. Dynamics

We first describe the dynamics of each model. The important parameters for all models are summarized in Tables 2 and 3. Model MR0 has the same initial parameters as model C10 of Sawai et al. (2005) except for the magnetic field configuration. The magnetic field of C10 is parallel to the rotation axis, and its strength is stronger near the rotation axis, while the magnetic field of model MR0 is dipole-like, with a larger strength near the center. Namely, model MR0 is a more "centrally" concentrated counterpart of model C10. The dynamical evolution of model MR10 is described as follows. Model MR0 first bounces at 142.7 ms after the onset of collapse when the central density reaches \( 3.75 \times 10^{14} \) g cm\(^{-3} \). However, the first shock wave is generated not by the nuclear force but by the magnetic force 0.2 ms prior to the nuclear bounce around a radius of 30 km near the rotation axis, which was also the case for model C10 in Sawai et al. (2005). Indeed the magnetic field around the region is strong enough by the time of bounce due to compression and field wrapping that the magnetic pressure becomes much larger than the matter pressure. Then, the magnetic-force-dominant regions gradually spread outward, and shock waves propagate through the core accompanied by the formation of a bipolar jet. This occurs by transferring the magnetic energy to kinetic energy. As shown in Figure 2 (left) and its inset, the toroidal energy is produced by the rotational energy, especially around the time of the first bounce.
(−143 ms) and the subsequent second bounce (−147 ms). After amplification, the toroidal magnetic energy clearly decreases as the kinetic energy of matter whose radial velocity is positive (with outward kinetic energy) increases. Not only is the toroidal energy consumed to launch the jets, but it also continues to be produced by the rotational energy during this phase. This is why the rate of the outward kinetic energy increase is larger than that of the toroidal energy decrease. The poloidal field energy is also transferred into kinetic energy, although the total poloidal magnetic energy increases with time as the matter falls. The outermost shock front reaches a radius of 1500 km at 20 ms after the bounce (see Fig. 3, top left). The explosion energy at that time is $4.4 \times 10^{51}$ erg, which is almost twice as large as the most explosive model C10 in Sawai et al. (2005). This implies that centrally concentrated magnetic fields tend to explode supernovae more powerfully.

Table 3 shows that the mass ejection in the southern hemisphere is larger than that in the northern hemisphere for model MR3. This is explained by the effect mentioned above; i.e., the magnetic force prevents matter infall. The amount of ejected mass is determined by how much matter is accumulated around the outside of the boundary between the inner and outer cores when the shock waves are generated. In fact, it is the outer core where the braking of the infall by the magnetic forces works. Owing to the stronger magnetic field, the mass infall in the outer core is more effectively hampered in the northern hemisphere, although more gravitational energy is extracted in the northern hemisphere around the first bounce for models MR3 and SR3. This is because the stronger magnetic fields in the northern hemisphere extract the rotational energy more efficiently, which results in the centrifugal force decreasing and matter falling more easily. On the other hand, the same interpretation is not valid for model SR10, in which the collapse is deeper in the southern hemisphere in turn around the first bounce. It is considered that the magnetic field may not only encourage collapse by extracting rotational energy, but also discourage it. The latter effect occurs simply because the magnetic force precludes matter from falling where it is dominant, i.e., around the rotation axis at a radius of more than ∼100 km for the rapid-rotation models. Not that for every model computed here, there always locally exists a region where the magnetic pressure dominates the matter pressure during collapse, and so this effect is significant in spite of the fact that the total magnetic energy is much smaller than the rotational energy. This effect seems to work well in model SR10. It is likely that too strong a magnetic field is disadvantageous for the infall of matter. At the second bounce in each model, the magnetic field rather works to decrease the infall rate, which leads to deeper collapse in the southern hemisphere since the magnetic field grows larger than that at the first bounce. As a result, the rotational energy becomes larger in the northern hemisphere after the second bounce, and thus, the toroidal magnetic energy also becomes larger.

Table 3 shows that the mass ejection in the southern hemisphere is larger than that in the northern hemisphere for model MR3. This is explained by the effect mentioned above; i.e., the magnetic force prevents matter infall. The amount of ejected mass is determined by how much matter is accumulated around the outside of the boundary between the inner and outer cores when the shock waves are generated. In fact, it is the outer core where the braking of the infall by the magnetic forces works. Owing to the stronger magnetic field, the mass infall in the outer core is more effectively hampered in the northern hemisphere, although more gravitational energy is extracted in the northern hemisphere around the first bounce for models MR3 and SR3, and so at least the inner core collapses well in this half. As a result, mass ejection is smaller in the northern hemisphere. According to Yamada & Sato (1994), the amount of available gravitational energy is sensitive to the ejected mass for a prompt explosion with pure rotation. However, this cannot be said for the magnetorotational case.

The two bumps in the rotational energy evolution in each panel of Fig. 4 roughly show how deeply the core collapses, reflecting the gravitational energy evolution.

The ejected mass is defined as the sum of the mass of fluid elements with positive total energy. With this definition, a part of mass in the protomagnetar is also summed.
Fig. 2.—Time evolution of the rotational, kinetic, poloidal magnetic, and toroidal magnetic energy until the farthest shock reaches a radius of 1500 km. The left and right panels correspond to models MR0 and MS3, respectively. In both panels, the thick solid line shows the total magnetic energy, the thick dotted line the poloidal magnetic energy, the thick dashed line the toroidal magnetic energy, the thin solid line the total kinetic energy, the thin dotted line the outward kinetic energy, the thin short-dashed line the inward kinetic energy, and the long-dashed line the rotational energy. The outward kinetic energy and inward kinetic energy are defined as the kinetic energy of matter whose radial velocity is positive and negative, respectively. [See the electronic edition of the Journal for a color version of this figure.]

In summary, we find that the jets are light and fast in the northern hemisphere and heavy and slow in the southern hemisphere for the rapid-rotation cases.

The slow-rotation models, MS3 and SS3, produce order-of-magnitude weaker explosions than the rapid-rotation models. The expelled masses are also smaller by an order of magnitude (see Table 3). In each of the two models, the magnetic field only plays a dynamical role just behind the shock fronts as they expand outward. However, unlike in the rapid-rotation cases, the toroidal magnetic field plays almost no role in powering the shock, but probably the poloidal field single-handedly does. In fact, slow rotation cannot yield strong toroidal fields whose energy is enough to help the explosion (see Fig. 2). The farthest shock front reaches a radius of 1500 km about 70 ms after the bounce, which is 3.5 times longer than for model MR3. Contrary to the rapid-rotation models, the slow-rotation models expel more mass in the northern hemisphere (see Table 3). This is because the magnetic pressure during the bounce is not strong enough to hamper the collapse (see Fig. 2, right), and the magnetic field just acts to extract the rotational energy. Thus, for the slow-rotation models, the jets are heavy and fast in the northern hemisphere and light and slow in the southern hemisphere.

3.2. The Explosion Energy

The explosion energies for all models are given in Table 3. They are estimated by the sum of energies over the fluid elements with positive total energy when the shock front reaches a radius of 1500 km. It can be seen that for the rapid-rotation models the offset dipole field weakens the explosion in comparison of MR0 with MR3 and of SR3 with SR10. For model MR3 the explosion energy in the northern hemisphere is somewhat smaller than that of model MR0, while the explosion energy in the southern hemisphere is just slightly larger than that of model MR0, which makes model MR3 explode more weakly. These features are just reflected by the amounts of the ejected mass; i.e., the larger the mass ejected, the stronger the explosion becomes (see Table 3). Namely, in this case the explosion energy as the amount of ejected mass is also controlled by the degree of infall in the outer core. For the comparison between models SR3 and SR10, the situation is different. Although the expelled mass is larger for model SR10, the explosion energy is larger for model SR3. This is because the stored magnetic energy after the bounce for model SR3 is much larger than that for model SR10 in both the northern and southern hemispheres (see Fig. 4). For model SR10 the magnetic field is very sparse in the southern hemisphere from the beginning. Meanwhile, in the northern hemisphere, in spite of the magnetic field initially being very strong, it cannot be amplified that much since the increase of rotational energy due to collapse is very small (see § 3.1). Unlike the rapid-rotation models, the explosion energy is larger in the northern hemisphere for the slow-rotation models. This is simply because the mass ejection is larger in the north (see § 3.1). After all, what we find here is that the magnetic fields which induce the explosions also work against the energetic explosions.

3.3. Formation of a Protomagnetar

The protomagnetars which we discuss in this paper were defined in § 2.4. They have many different shapes between the rapid-rotation models and slow-rotation models. In the top panels of Figure 5, the final shapes of the protomagnetars for models MR3 and MS3 can be seen. For model MR3 this corresponds to the region whose density is more than $2.0 \times 10^7$ g cm$^{-3}$; the protomagnetar has a butterfly-like shape in the meridional plane, occupying a large fraction of the computational domain. On the other hand, the protomagnetar of model MS3, whose critical density is $7.8 \times 10^9$ g cm$^{-3}$, has a radius of only $\sim 100$ km, and its shape is an oblate ellipsoid.

The final rotation-period profiles for models MR3 and MS3 are also displayed in the top panels of Figure 5. For model MR3 the constant-$\Omega$ surface in the protomagnetar is cylindrical rather than spherical because the matter is expelled strongly toward both poles. There exists a slowly rotating torus with an $\sim 100$ km radius in the inner region of the protomagnetar. Starting from the surface of the torus, narrow zones where the angular velocities are also small run along the inner line of the wings of the “butterfly.” In the protomagnetar, there is almost no region whose
rotation period is smaller than 10 ms. In model MS3 the angular momentum distribution is in disarray, especially at small radii, although at large distances from the rotation axis it is marginally cylindrical. All fluid elements rotate with a rotation period of about a few seconds in the protomagnetar.

The magnetic field distribution at the end of the simulations is shown in the bottom panels of Figure 5. In model MR3, the region where the poloidal field is strong spindles along the rotation axis to the surface of the core due to the formation of the strong bipolar jet. The poloidal field has a collimated dipole-like shape around the rotation axis and is no longer as much off-center as before, while the toroidal field is somewhat equatorially asymmetric. Around the foot of the jets and the inner part of the protomagnetar, the toroidal fields are very weak since their energy is transferred into the kinetic energy of the jets. On the other hand, in the outer part of the protomagnetar (the wings of the “butterfly”), the toroidal field is dominant in most areas. The global magnetic field structure is roughly a dipole magnetic field pinched by toroidal magnetic field lines, defined by the region with more than $\sim 10^{13}$ G. How this magnetic field will evolve during the cooling of the protomagnetar to a magnetar of $\sim 10$ km radius is of additional interest, and should be investigated elsewhere in the future. In model MS3, the areas where the toroidal field is comparable to the poloidal field are very small. There is a cylinder
at the center of the core with \( \sim 1400 \) km length and \( \sim 400 \) km radius (Fig. 5, blue cavity) where the poloidal field is totally dominant. The protomagnetar is in the innermost part of this cylinder, in which the toroidal field is especially weak. The poloidal field is also not offset and is dipole-like, but is not as collimated as in the case of model MR3 owing to weakly ejected mass.

3.4. Kick Velocity of a Nascent Magnetar

We show in Figure 6 the time evolution of the velocities of the protomagnetars for all models. Models MR3, MS3, SR3, and SS3 produce kick velocities of around 350–500 km s\(^{-1}\), while model SR10 yields a velocity of more than 1000 km s\(^{-1}\). Note that in the equatorially symmetric model MR0, the velocity of the protomagnetar is less than 70 km s\(^{-1}\), which is due to unremovable numerical noise. The final velocities are estimated at 240 ms from the beginning of computations.

In model MR3, the protomagnetar is kicked southward and finally reaches a kick velocity of 512 km s\(^{-1}\). As shown in Figure 7, the driving force is the magnetic pressure. Since the magnetic pressure generally pushes the matter and the magnetic energy is superior in the northern hemisphere (see Fig. 4),\(^7\) the protomagnetar is pushed southward. Meanwhile, the magnetic tension which generally pulls the matter works northward again with the help of the stronger magnetic field in the northern hemisphere, although this is somewhat weaker than the magnetic pressure. Now we discuss what determines which of these two stresses becomes dominant. The z-components of the forces owing to magnetic pressure and tension of the poloidal field in cylindrical coordinates are

\[
F_{mp,z} = -\frac{1}{8\pi} \int_S B^2 dS_z = -\frac{1}{8\pi} \int_S B^2 \cos \theta_z dS, \tag{14}
\]

\[
F_{mt,z} = \frac{1}{4\pi} \int_S B \cdot \mathbf{B} dS = \frac{1}{4\pi} \int_S B^2 \cos \theta_1 \cos (\theta_1 - \theta_2) dS, \tag{15}
\]

\(^7\) This figure does not show just the magnetic energy around the protomagnetar but that integrated over the northern and southern hemispheres. Hence, it gives just a rough indication.
where $B^2 = B_z^2 + B_R^2$, and $\theta_1$ and $\theta_2$ are the angles of the magnetic field and surface element vector measured from the rotation axis, respectively. We consider here the range $90^\circ \leq \theta_1 \leq 2\theta_2 + 90^\circ$ and $-90^\circ \leq \theta_2 \leq 90^\circ$, i.e., the magnetic field is outward on the surface in the northern hemisphere. The contribution by the toroidal field is omitted for simplicity for the moment.

Where the magnetic tension is locally dominant, the following relation can be obtained from equations (14) and (15):

$$\frac{2B^2 \cos \theta_1 \cos(\theta_1 - \theta_2) - B^2 \cos \theta_2 = \cos(2\theta_1 - \theta_2)}{\theta_2} \geq 0,$$

(16)

and then $\theta_1$ satisfies

$$\frac{\theta_2}{2} - 45^\circ \leq \theta_1 \leq \frac{\theta_2}{2} + 45^\circ$$

(17)

(see Fig. 8). Regarding the northern hemisphere of model MR3 at 240 ms, Figure 5 indicates that $\theta_2$ is often less than $-60^\circ$, where the magnetic force strongly works. Hence, the magnetic tension is dominant when $-75^\circ \leq \theta_1 \leq 15^\circ$. If the effect of the toroidal field is taken into account, the upper critical angle becomes somewhat larger because the toroidal field is considered to be large in the southern hemisphere (see § 3.1). On the other hand, Figure 8 (bottom left) shows that $\theta_1$ is adequately more than $15^\circ$ almost everywhere. Therefore, the magnetic pressure is dominant there, and it becomes a leading force in accelerating
Fig. 7.—Time evolution of the accelerations acting on the protomagnetar star. The magnetic acceleration is divided into two parts: the tension and the pressure. From top to bottom and left to right, models MR3, SR3, SR10, MS3, and SS3 are displayed in sequence. The northward acceleration is taken to be positive. Solid lines denote the total acceleration, dotted lines the acceleration from pressure, short-dashed lines the acceleration from gravity, long-dashed lines the total magnetic stress, dot-short-dashed lines the magnetic tension, and dot-long-dashed lines the magnetic pressure. [See the electronic edition of the Journal for a color version of this figure.]
The protomagnetar. The same discussion can be done for the southern hemisphere. In fact, in model MR3 (and also in models SR3 and SR10), such a situation is common for almost the entire time of the computation.

Around 150–160 ms in model MR3, the magnetic acceleration decreases, which causes a kickback of the protomagnetar northward. This decrease is likely to be yielded by the predominant amplification of the toroidal field in the southern hemisphere around that time (see Fig. 4), which makes the magnetic pressure work northward. After kicking back, the magnetic stress comes back to life with even greater strength than before. This is because the difference of the magnetic stress between the north and south surfaces becomes larger, although the stresses themselves are weaker than before. Then the acceleration gradually decreases as the north-south difference in magnetic stress diminishes. At the end of the computation, the acceleration is quite small, and so the protomagnetar is no longer accelerated significantly. Compared with the magnetic stress, the pressure and gravity are less important for the acceleration of the protomagnetar except for the time of kicking back. After the northward shock front passes the north surface of the protomagnetar for the first time around 150 ms, the pressure is larger in the vicinity of the north surface. However, this is until the southward shock front passes the south surface of the protomagnetar with a few millisecond delay. After this, the pressure in the vicinity of the south surface is always larger because there is more matter or heat energy. Gravity pulls the protomagnetar in the direction of more matter. However, after 162 ms part of matter is ejected out of the numerical domain, but we neglect the gravity from the expelled matter. Hence, the value of the gravitational acceleration in Figure 7 is not valid after this time. Nevertheless, keeping in mind that the gravitational acceleration is comparable to the acceleration owing to the pressure, the effect of gravity on the kick velocity is not significant either way. The same can be said for models SR3 and SR10. For models MS3 and SS3, since the ejected mass is \( \approx 0.03 \, M_\odot \), the gravitational acceleration changes by at most \( \approx 10\% \), even if the ejected mass is taken into account.

In models SR3 and SR10, the time evolution of the magnetar velocities and the acceleration mechanisms are similar to that of model MR3. In model SR3, however, the magnetic stress which has accelerated the protomagnetar southward turns to work in the opposite direction around 210 ms, and the protomagnetar starts to accelerate. This is probably because the magnetic field becomes locally stronger around the south surface of the protomagnetar. At the end of the simulation, the magnetic stress still has a positive value. Thus, although the protomagnetar gets a final velocity of 353 km s\(^{-1}\), this decreases later to some degree. Model SR10 produces the highest velocity of the protomagnetar among all the models. In spite of the smaller stored magnetic energy than that in model SR3, a considerable north-south difference of the stored energy can produce a larger acceleration. In this model, kicking back occurs not because of the reduction of magnetic stress but because of pressure and gravity. At the end of the simulation the acceleration is still substantial. It is negligible around 300 ms if linearly extrapolated. The velocity of the protomagnetar around that time is \( \approx 1500–1600 \, \text{km s}^{-1} \).

The slow-rotation models, MS3 and SS3, show more complex time evolution of the protomagnetar acceleration (see Fig. 7). In each of these cases, the magnetic pressure alone is not always the leading driving force; magnetic tension, matter pressure, and gravity also play important roles. This is due to the low ratio of the magnetic pressure to the matter pressure in the vicinity of the protomagnetar. Although these models explode weakly, the final magnetar velocities do not differ significantly from those for stronger explosion models. This is because the kinetic energy of a protomagnetar whose velocity is approximately a few \( 100 \, \text{km s}^{-1} \) is just \( \approx 10^{48} \, \text{erg} \), which is even smaller than the explosion energies of models MS3 and SS3. The time evolution of acceleration still varies violently, even around the end of the computation. In order to get reliable final velocities, longer numerical simulations are necessary.

### 3.4.1. Uncertainties from the Definition of a Protomagnetar

As noted in §2.4, we assume a protomagnetar to be a region containing \( 1.2 \, M_\odot \), which is calculated by summing up the mass of each fluid element in descending order of the density. However, it is admitted highly uncertain at 240 ms which part of the gas is finally determined to be a protomagnetar. In fact, this is not known for sure until computations are done up to the time at which the fall-back of matter has occurred. According to Zhang et al. (2007), this time is estimated to be about \( 10^4 \, \text{s} \), which is far longer than our computational time, 240 ms, and is, unfortunately, too long to dynamically follow with numerical simulations like ours. What we can and should do instead is to make clear the uncertainty of the kick velocities associated with the definition of a protomagnetar. For this purpose, we have computed another model, which covers a greater portion of the star, and we have also calculated the kick velocities with several alternative definitions of a protomagnetar, taking model MR3 as an example.

We first investigate the uncertainties coming from the protomagnetar mass. Five different masses of protomagnetars, 1.2, 1.23, 1.26, 1.3, and 1.35 \( M_\odot \), are tried. We take here the maximal mass to be 1.35 \( M_\odot \), anticipating that a protomagnetar produced by a magnetic prompt explosion will be less massive than 1.4 \( M_\odot \), the canonical value for ordinary radio pulsars. We then introduce another criterion for a protomagnetar, by which fluid elements are summed up in the descending order of “pressure” instead of density. Several masses for the protomagnetar are tried again with this “pressure criterion.” In the study of protomagnetars more massive than 1.2 \( M_\odot \), a larger portion of the progenitor star needs to be computed. We thus run another numerical simulation which computes the evolution of the inner 2.0 \( M_\odot \) of a 15 \( M_\odot \) progenitor (S. E. Woosley 1995, private communication) for the...
same parameter values of \[|E_m/W|, |T/W|, \] and \(z_{\text{off}}\) as in model MR3, and obtain the kick velocities of 10 different protomagnetars. We name each model as MR3env[D or P] (protomagnetar’s mass], where “D” and “P” denote the density and pressure criteria, respectively. For example, a model for 1.23 \(M_\odot\) protomagnetar determined by the pressure criterion is referred to as MR3envP1.23.

The results are shown in Figure 9, which illustrates the time evolution of the velocity for each protomagnetar. We find that a difference in the criterion does not produce a significant variation in the kick velocities. On the other hand, it is found that a difference in the mass of the protomagnetar results in a noticeable variation in the kick velocities. In both series, MR3envD and MR3envP, the velocity tends to decrease with increasing mass. This is because a protomagnetar with a higher mass has a larger radius, and the magnetic field on the surface becomes weaker, which then leads to a smaller north-south difference in the magnetic forces. Note, however, that the most massive protomagnetar models in our analysis, MR3envD1.35 and MR3envP1.35, still have kick velocities of \(\sim 200\) km s\(^{-1}\).

The kick velocities obtained with several different definitions of a protomagnetar cover a range of \(\sim 200–500\) km s\(^{-1}\). We thus consider that the kick velocities we have procured in this study have an uncertainty of this degree. We can thus claim that a protomagnetar is possibly accelerated up to several \(100\) km s\(^{-1}\).

The off-center magnetic field weakens the explosion for the rapid-rotation case. In this case, the explosion energy is larger in the south since less matter is ejected or less magnetic energy is stored in the north. On the other hand, the explosion energy is larger in the north for the slow-rotation models.

3. The formed protomagnetars rotate slowly, with rotational periods of more than 10 ms. The final magnetic field around the protomagnetar has a collimated dipole-like configuration pinched by toroidal field lines for the rapid-rotation models, whereas in protomagnetars formed by the slow-rotation models the poloidal field is totally dominant.

4. If the initial magnetic field is stronger in the northern hemisphere, the protomagnetar is kicked southward with a velocity of several hundred km s\(^{-1}\), or in an extreme case, \(\sim 1000\) km s\(^{-1}\). In most cases they are accelerated mainly by magnetic pressure, while the somewhat weaker magnetic tension works in the opposite direction, due to the stronger magnetic field in the northern hemisphere.

From the results of the computations, we predict that magnetars also possibly have large velocities like ordinary pulsars, and in some extreme cases could have \(\sim 1000\) km s\(^{-1}\) velocities. Current observations show that at least some magnetar candidates have velocities of less than 500 km s\(^{-1}\) (see § 1), whereas Duncan & Thompson (1992) claimed that magnetars will have large kick velocities up to \(\sim 1000\) km s\(^{-1}\). It is notable that our results also show that some moderate initial conditions lead to velocities less than \(\sim 500\) km s\(^{-1}\), which is not inconsistent with observations. On the other hand, if the associations between SGR 0526-66 and N49 or SGR 1627-41 and G337.0-0.1 are true, these magnetar candidates should have extremely large kick velocities up to \(\sim 1000\) km s\(^{-1}\). In the context of our numerical simulations, these extraordinarily high velocities are interpreted as the products of extreme initial conditions, i.e., very strong and highly asymmetric magnetic fields as in model SR10.

Some recent observations imply that some magnetars originate from massive progenitors (\(\gtrsim 20–50\) \(M_\odot\); e.g., Gaensler et al. 2005; Figer et al. 2005; Munro et al. 2006). We use a 15 \(M_\odot\) stellar model, although our computation involves the formation of magnetars. This is because a 15 \(M_\odot\) star is the canonical progenitor of
core-collapse computations, and it is not yet known whether magnetars originating from massive stars are common. However, it is worth investigating in the future whether results similar to ours are yielded by numerical simulations with more massive progenitors. When massive progenitors are chosen, it is more natural for us to assume rapid rotation, since Heger et al. (2005) argued from their calculation of stellar evolution that more massive stars tend to rotate more rapidly. According to them, a 35 $M_\odot$ star will reach the rotation period of a neutron star of 4.4 ms in their standard model, although the magnetic flux of the star is far weaker than that of magnetars; again, this result is not conclusive.

The other kick mechanisms introduced in § 1 do not work in the situation considered here. For the hydrodynamically driven and neutrino-magnetic-field-driven mechanisms, this is because the kicks are accompanied by a delayed explosion, while in our simulation, a prompt explosion occurs with the help of a strong magnetic field and rotation. In an electromagnetically driven mechanism, the acceleration timescale is ~4 s if the field strength on the surface of the (proto)magnetar is $\sim 10^{16}$ G, which is much longer than that of the “magnetohydrodynamically driven kick.” Our computation shows that the initially rapid rotation becomes quite slow (rotation period more than 10 ms) in $\leq$ 100 ms after the bounce (see § 2.3). Note that this mechanism requires a rotation period of ~1 ms, so the protomagnetar cannot be substantially accelerated.

The above discussion is valid only when the initial conditions which we assume, strong magnetic fields, rapid rotation, and offset dipole fields, are achieved. Although the validity of these assumptions is discussed in § 2.3, they are far from solid. What will happen if our assumptions are partially fulfilled? With strong magnetic fields but a slow rotation, the magnetorotational explosion considered here cannot occur. In this case, the engine of the supernova might be heating by emitted neutrinos, and the second kick mechanism (neutrino-magnetic-field-driven) introduced in § 1 might work. We have no idea at the moment whether the hydrodynamical-instability-driven kick (the first mechanism) could operate or not in the presence of strong magnetic fields. This should be investigated in future works. If the initial magnetic field is weak and rotation is rapid, the magnetorotational explosion may ensue with the help of magnetorotational instability (MRI) or convective dynamo action (e.g., Akiyama et al. 2003; Ardeljan et al. 2005; Duncan & Thompson 1992). However, this needs more detailed study. Note that in order to reliably compute the growth of MRI from weak magnetic fields of $\sim 10^5$–$10^6$ G in the supernova core, the spatial grid size should be at least as small as $\sim 10^2$ cm, which no numerical simulation has yet achieved. Meanwhile, even in the present computations with $\sim 10^{12}$–$10^{13}$ G initially, the grid resolution is not fine enough by a factor of 10 in some important regions to resolve MRI. In this case, however, the compression and wrapping-up of the initial field amplify the field strength sufficiently before MRI sets in, and we expect MRI will not change the dynamics very much even if it occurs. But we do not intend to claim that MRI is not important. On the contrary, we are interested in MRI-related issues such as linear growth, nonlinear saturation, implications for explosion, pulsar kick and spin, and so forth. We think they should be addressed in a separate paper. When so doing, three-dimensional computations will be important, which we are now undertaking (H. Sawai et al. 2008, in preparation).

We find that a centrally concentrated magnetic field has an advantage in energetic explosion. Recently, Pian et al. (2006) reported that SN 2006aj, associated with X-ray flash 060218, is a factor of 2–3 more luminous than other normal Type Ic supernovae, although it is dimmer than the so-called hypernovae associated with gamma-ray bursts. Mazzali et al. (2006) claimed that the remnant of SN 2006aj is not a black hole but a neutron star, possibly a magnetar. If this is true, a highly energetic explosion induced by centrally concentrated magnetic fields could have been one of the origins of this middle-class supernova, although the mechanism to produce the X-ray flash is beyond the scope of this paper. Note that not every supernova accompanied by a magnetar should produce such an energetic explosion. It is reported that for some supernovae considered to be associated with magnetar candidates, the explosion energies are just as large as those of ordinary supernovae (Vink & Kuiper 2006; Sasaki et al. 2004).

Finally, how large a fraction do magnetars occupy among the whole population of isolated neutron stars? At present, the confirmed candidates of magnetars are up to 11, which is only ~1% of observed ordinary pulsars. However, it is too early to conclude that magnetars are such a minor population. Spin-down ages have been measured in three of all four known SGRs that are less than 1900 yr. This means a magnetar is born every ~600 yr, corresponding to ~15% of ordinary neutron stars. If the other magnetar, SGR 1627–41, has a similar spin-down age, the fraction of magnetars is estimated to be ~20% of that of ordinary pulsars. Here we omit AXPs since there may be a large number of AXPs which are too dim to detect (Woods & Thompson 2006). Woods & Thompson (2006) insisted that there may be a lot of magnetars which are too old to be detected and that the number of magnetars could be comparable to that of ordinary pulsars. If this is the case, the study of supernova theory in the strong magnetic field regime will have a more significant meaning.

H. S. thanks E. Müller and H.-T. Janka for useful discussions during his stay at the Max-Planck-Institute für Astrophysik. H. S. also thanks D. Lai for a helpful discussion. Some of the numerical simulations were done on the supercomputer VPP700E/128 at RIKEN and VPP500/80 at KEK (KEK supercomputer project 108). This work was partially supported by Japan Society for the Promotion of Science Research Fellowships (H.S.), Grants-in-Aid for Scientific Research (14079202 and 17540267) from the Ministry of Education, Science, and Culture of Japan, and Grants-in-Aid for the 21st century COE program “Holistic Research and Education Center for the Physics of Self-Organizing Systems.”

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