Faster flicker of buoyant diffusion flames by weakly rotatory flows

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Abstract  Flickering buoyant diffusion methane flames in weakly rotatory flows were computationally and theoretically investigated. The prominent computational finding is that the flicker frequency nonlinearly increases with the nondimensional rotational intensity $R$ (up to 0.24), which is proportional to the nondimensional circumferential circulation. This finding is consistent with the previous experimental observations that rotatory flows enhance flame flicker to a certain extent. Based on the vortex-dynamical understanding of flickering flames that the flame flicker is caused by the periodic shedding of buoyancy-induced toroidal vortices, a scaling theory is formulated for flickering buoyant diffusion flames in weakly rotatory flows. The theory predicts that the increase of flicker frequency $f$ obeys the scaling relation $(f - f_0) \propto R^2$, which agrees very well with the present computational results. In physics, the external rotatory flow enhances the radial pressure gradient around the flame, and the significant baroclinic effect $\nabla p \times \nabla \rho$ contributes an additional source for the growth of toroidal vortices so that their periodic shedding is faster.

Keywords  Flickering flame · Laminar diffusion flame · Weakly rotatory flow · Flicker frequency · Toroidal vortex

List of symbols

\begin{itemize}
\item $A$  Control mass
\item $\partial A$  Material contour of $A$
\item $B(t)$  Time-dependent control volume enclosing the toroidal vortex
\item $C$  Constant threshold for vortex shedding
\item $C_j$  Constant pre-factor relating to the initial fuel jet
\item $C_r$  Constant pre-factor relating to rotational flow
\item $C_{jr}$  $C_j + C_r$
\item $C_\theta$  Constant pre-factor relating to azimuthal velocity within the vortex core
\end{itemize}

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$C_h$  Constant pre-factor relating to the vertical motion of toroidal vortex
$C_\rho$  $\rho_\infty / \rho_f$ ranging from 4 to 8
$D$  Diameter of fuel inlet, fixed at 10 mm
$D_i$  Diffusion coefficient of species $i$
$f$  Flickering frequency
$\Delta f$  $f(R) - f(R = 0)$
g  Acceleration due to gravity
$h(t)$  Height of the upper boundary of $B(t)$
$\bar{H}$  Time-averaged height of growing toroidal vortex, $\bar{H} = \hat{\tau}^{-1} \int_0^{\hat{\tau}} \hat{h}(t) dt$
$Q$  Total heat release rate
$R$  Nondimensional rotational intensity
$ra$  Radius of vortex core
$rc$  Radial position of vortex layer
$r(t)$  Width of the right boundary of $B(t)$
$\bar{R}$  Time-averaged width of growing toroidal vortex, $\bar{R} = \hat{\tau}^{-1} \int_0^{\hat{\tau}} \hat{r}(t) dt$
$\mathcal{R}$  Universal gas constant
$t$  Time
$\hat{t}$  Dimensionless time, $t \sqrt{g/D}$
$t^*$  Normalized time, $t f_0$
$U_0$  Inlet velocity of fuel jet, fixed at 0.165 m/s
$U$  Magnitude of the inlet airflow, $|U|$
$Y_i$  Mass fraction of species $i$
$u$  Velocity magnitude
$h$  Helicity density, $u \cdot \omega$
$h_s$  Sensible enthalpy
$p$  Pressure
$\dot{q}''$  Heat release per unit volume
$\dot{q}'$  Heat flux vector
$Fr$  Froude number, $U_0^2 / g D$
$Re$  Reynold number, $U_0 D \nu_F$
$Ri$  Richardson number, $(C_\rho - 1) g D / U_0^2$
$a$  Acceleration
$r$  Unit radial vector
$s$  Unit tangential vector along the contour $\partial A$
$z$  Unit normal vector
$u$  Velocity vector
$U$  Inlet velocity of airflow on the wind wall
$U_\perp$  Normal component of $U$
$U_\parallel$  Tangential component of $U$
$\sim$  Dimensionless quantity
-  Perturbation quantity

**Greek symbols**

$\alpha$  Angle between the $U_\perp$ and $U_\parallel$, fixed at 45°
$\nu$  Viscosity
$\rho$  Density
$\sigma$  Viscous stress
$\tau$  Periodic time, $1/f$
$\omega$  Vorticity magnitude
$\omega$  Vorticity vector

**Subscripts**

0  Quiescent environment ($R = 0$)
1 Introduction

In nature and various domestic and industrial applications, diffusion flames could be unstable under buoyancy [1–18]. Some buoyant diffusion flames possess a well-known phenomenon where flame vibrates due to the periodic shedding of its upper portion, and the phenomenon is often referred to as “the flicker of luminous flames” [1,2] or “fire puffing” [3,9]. A similar phenomenon is also observed for the oscillatory behavior of buoyant jets and plumes [19–22] and the flickering wake flame of the droplet and porous sphere [14,15,23,24].

Many studies [3,7,10,11,13] have substantiated that the flame flicker is a self-exciting flow oscillation and that the flickering frequency is proportional to \( \sqrt{g/D} \), where \( g \) is the acceleration due to gravity and \( D \) is the diameter of the flame burner (or fire pool). The experiment of Chen et al. [4] clearly illustrates that the shear layer around a diffusion flame rolls up due to the instability of the buoyancy-induced flow (a hydrodynamic global instability [25]), evolves into a toroidal vortex and eventually sheds off downstream, being synchronized with the flame flicker. Following the physics picture, Xia and Zhang [13] established a vortex-dynamical scaling theory to predict the flicker frequency of various pool and jet flames reported in the literature. They confirmed and extended the correlation between the periodicity of toroidal vortices and the flickering buoyant diffusion flame. In addition, some previous studies reported the effects of the transient dimension on the flickering phenomenon, for example, the characteristic length scale of a freely falling droplet flame is dynamic [14,24]. Pandey et al. [23] reported that the periodic shedding of toroidal vortices could be altered by external perturbation in acoustically excited droplet flames.

The coupling interaction of multiple flickering flames can generate richer dynamical phenomena. In the system of dual flickering candle flames, Kitahata et al. [26] reported two distinct dynamical modes, namely the in-phase and anti-phase synchronizing modes, at relatively small and large gap distances between flames, respectively. Similar phenomena can appear in not only candle flames [26,27] but also diffusion flames [28,29], pool flames [30], and plumes [31]. Physically, the distinct modes are due to the interaction of the vortices generated around each flame [30], similar to the mechanism causing flow transition in the wake of a bluff body and forming the Kármán vortex street. The triple flickering flames [32–34] with various arrangements exhibit more complex dynamical modes. From the perspective of vorticity reconnection and vortex-induced flow, Yang et al. [34] recently interpreted four typical dynamical modes (in-phase, flickering death, rotation, and partially in-phase) of triple flickering buoyant diffusion flames in an equilateral triangle arrangement.

All the flickering flames mentioned above were produced in a quiescent environment, and their interaction with externally forced flows is apparently of interest but insufficiently investigated. The stabilization of flames under rotatory flows is a long-lasting and practically relevant problem. In the past decades, many experiments [35–39] studied the influence of burner rotation on flame stabilization with an emphasis on the dynamical behaviors of buoyancy-induced flame oscillation. Gotoda, together with coworkers [37–39], reported that the periodic flame flickering could retain at low rotational speeds but transition into low-dimensional deterministic chaos (flames exhibit spiral oscillation) at sufficiently large rotation speeds. Another experimental approach is to tangentially import airflow from the ambient into the central flame region [40–45]. This approach was usually used for producing a fire whirl [46], where whirling eddies of air, like a tornado flow, suck the ground fuel and form a slender fire downstream. Lei et al. [42] observed higher pulsation frequencies of small-scale buoyant flames in rotatory flows than in the quiescent environment. Recently, Ju et al. [45] investigated that the characteristic frequency of a pulsating fire whirl increases with the imposed circulation. In addition, Coenen et al. [43] found that the puffing instability of pool fires is suppressed under sufficiently strong rotating flows, and helical instability appears instead.

All the experimental studies mentioned above suggest that the flame oscillation frequency would be changed with the intensity of external rotatory flows to a certain extent. However, a clear physics interpretation from the perspective of vortex dynamics is not available in the literature. In this study, we attempted to answer the following questions motivated by the previous experiments:

(1) How does a flickering buoyant diffusion flame behave when it is subject to an external rotating flow?
(2) How does its flickering frequency vary with the rotating flow speed?
What is the vortex-dynamical origin of the observed frequency variation?

It should be emphasized that the present computational and theoretical investigations were limited to the situation of weakly rotatory flows so that the vortex breakdown does not appear in the present problem [47], although it certainly merits future studies.

The present paper is organized as follows. First, external rotatory flows are generated by using four forced-ventilation walls, and the rotational intensity is controlled by adjusting the ventilating velocity; flickering buoyant diffusion flames of methane gas are computationally produced and validated. Then, by comparing the evolution of toroidal vortices around flames under different rotational intensities, the effects of the rotatory flows on flickering flames are identified and analyzed. Finally, following Xia and Zhang’s work [13], we established a vortex-dynamical scaling theory for flickering flames in weakly rotatory flows; the computational and theoretical results are compared and analyzed.

2 Computational methodology and validation

2.1 Computational setup of flickering buoyant diffusion flames

In the present study, flickering buoyant diffusion flames were computationally produced using Fire Dynamics Simulator (FDS).[48, 49], which is a widely used open-source code for solving the unsteady, three-dimensional, low-Mach, and variable-density flow. In the past decades, the code has been demonstrated to be a reliable computational platform for studying unsteady processes in fire-driven flows [30,34,50–52]. Based on FDS, our previous computational works successfully captured the dynamical behaviors of flickering buoyant diffusion flames in a quiescent environment and reproduced a variety of dynamical modes in the dual- and triple-flame systems [30,34]. In the present study, the thermally driven flow was computed by the governing equations of continuity, species concentration (mass fraction), momentum, energy (sensible enthalpy), and state for an ideal gas:

\[
\begin{align*}
\frac{\partial}{\partial t} (\rho) + \nabla \cdot (\rho \mathbf{u}) &= 0 \\
\frac{\partial}{\partial t} (\rho Y_i) + \nabla \cdot (\rho Y_i \mathbf{u}) &= \nabla \cdot (\rho D_i \nabla Y_i) + \dot{m}_i'''
\end{align*}
\]

\[
\begin{align*}
\frac{\partial}{\partial t} (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{uu}) &= -\nabla \tilde{p} - \nabla \cdot \sigma + (\rho - \rho_{\infty}) \mathbf{g}
\end{align*}
\]

\[
\begin{align*}
\frac{\partial}{\partial t} (\rho h_s) + \nabla \cdot (\rho h_s \mathbf{u}) &= \frac{D \tilde{p}}{Dt} + \dot{q}''' - \nabla \cdot \dot{q}''
\end{align*}
\]

\[
\rho = \frac{\tilde{p} W}{RT}
\]

where \(\rho\) is the density, \(\mathbf{u}\) the velocity vector, \(Y_i\) and \(D_i\) are the mass fraction and diffusion coefficient of species \(i\), respectively, \(\dot{m}_i'''\) is the mass production rate per unit volume of species \(i\) by chemical reactions, \(\tilde{p}\) is the pressure perturbation, \(\sigma\) is the viscous stress, \(\rho_{\infty}\) is the background density, \(\mathbf{g}\) is the gravity vector, \(h_s\) is the sensible enthalpy under low Mach number approximation, \(\tilde{p}\) is the back pressure, \(\dot{q}'''\) is the heat release per unit volume, \(\dot{q}''\) is the heat flux vector, \(W\) is the molecular weight of the gas mixture, \(\mathcal{R}\) is the universal gas constant, and \(T\) is the temperature. Spatial integration was carried out using a kinetic-energy-conserving central difference scheme, and an explicit second-order predictor/corrector scheme advanced the time integration. More details of the numerical scheme are given in. [49].

As shown in Fig. 1a, the computational domain is a square column with 16\(D\) side and 24\(D\) height, where \(D = 10\) mm is the fixed diameter of the fuel inlet. A uniform structured mesh of 160 \(\times\) 160 \(\times\) 240 is utilized as a balance of the computational cost and the grid independence, which was discussed in detail in the previous paper [30]. It should be noted that the present mesh is sufficiently fine to resolve the large-scale toroidal vortices outside the luminous flame, which are responsible for the flame flicker [4,6]. Ignoring the small vortices inside the flame does not influence our results and avoids unnecessary computational costs. A typical simulation of the present problem carried out on the mesh takes about 3800 core hours on 32 cores of the Intel®Xeon®Gold 5120 CPU @ 2.20GHz. An impermeable, non-slip, and adiabatic solid-wall boundary is used at the bottom ground (the grey area), while a central circle (yellow area) is open for the fuel inlet. The ambient air is the...
density of $\rho_\infty = 1.20 \text{ kg/m}^3$ at room temperature. The methane gas jet of the density of $\rho_F = .66 \text{ kg/m}^3$ is ejected out at the uniform velocity of $U_0 = 0.165 \text{ m/s}$ to sustain a laminar diffusion flame with $Re = 100$ and $Fr = 0.28$, where $Re = U_0D/\nu_F$ is the fuel-jet Reynold number ($\nu_F = 1.65 \times 10^{-5} \text{ m}^2/\text{s}$ is the viscosity of methane gas) and $Fr = U_0^2/gD$ is the Froude number ($g = 9.8 \text{ m/s}^2$ is the acceleration due to gravity).

For the flames in the quiescent environment, the top and lateral sides of the computational domain are set as the open boundary condition (the lateral sides will be changed as fixed inflow conditions for the flames with rotatory flows), through which gaseous products are allowed to flow in or out depending on the local pressure gradient. This is a simple upwind boundary condition. Specifically, the Poisson solver requires a Dirichlet condition for the constant quantity $\Omega = \hat{\rho}/\rho + |\mu|^2/2$ along a streamline. [49]. Thus, for incoming flow, $\Omega_{\text{ext}}$ at the boundary equals that at infinity of the ambient flow, whereas for outgoing flow, $\Omega_{\text{ext}}$ is set to the value of its adjacent interior grid cell. The temperature and species mass fractions are calculated similarly at an open boundary.

To generate a rotatory flow environment for the flickering flame, forced-ventilation walls can be specified at four lateral sides of the computational domain for tangentially ejecting airflows into the central region. The generated rotatory flows will be illustrated and analyzed shortly in Sect. 2.2. In the present study, the fundamental conservation equations governing fluid dynamics are solved based on explicit, second-order, kinetic-energy-conserving numerics and shown in detail in the previous paper [30]. Turbulence modeling is not needed in the present laminar flows. The mixing-limited, infinitely fast reaction of lumped species is used for modeling the present diffusion flames being far from extinction, as the dynamic structures of flickering flames are unaffected by the fast-chemistry assumptions [53]. In the lumped species approach [49], the one-step overall reaction is given as Fuel + Air $\rightarrow$ Products, where the fuel is usually a single gas species, and the air and products are lumped species that are reacting scalar quantities. The heat release rate per unit volume is defined by summing the lumped species’ mass production rates times their respective heats of formation. According to the thermal diffusivity (about 20–140 mm$^2$/s at 300–2100K) and the characteristic time of the present flickering flames (flame frequency about 10 Hz), the diffusion zone of the flame is evaluated to be 1.4–3.7 mm and can be resolved by the present mesh size, while the reaction zone is of vanishing thickness (due to the infinitely fast reaction rate approximation) and its resolution is unnecessary. The soot formation and radiation are not modeled in the present small-scale flames. The assumptions that small-scale flickering buoyant diffusion flames are laminar, mixing-limited, and insignificantly affected by soot or radiation facilitate the present simulation, theory, and quantitative comparison. The present computational validation of flickering buoyant diffusion flames will be expounded shortly in Sect. 2.3.

It should be noted that the applicability of the computational approaches to the present problem relies on the two essential features of the small-scale flickering buoyant diffusion flames concerned. First, the flicker of the flames is due to the periodic shedding of toroidal vortices, not by the local extinction/re-ignition, and the dominant flickering frequency is insensitive to the fuel types and the chemistry [11,54]. In the scenario, the characteristic time scales of chemical reactions are significantly smaller than those of the flow. Consequently, a complex reaction mechanism is not indispensable for computationally reproducing the flame flicker, and a simplified mixing-limited chemical reaction mechanism of methane/air combustion is adequate instead. Second, the flames concerned are of relatively small sizes, and the flow remains laminar almost everywhere except the far downstream of the flame, where the transitional or turbulent flow characteristics have, however, little influence on the vortices and the flame upstream. Consequently, more sophisticated computational models for turbulent flows, for example, the large eddy simulation (LES), are not necessary for the present problem. The present computational results, to be shown shortly in the next sections, substantiate the reliability of the adopted computational methodology and models for small-scale flickering diffusion flames in weakly rotatory flows. For those flames of large scales or in strongly rotatory flows, the LES approach with finite-rate chemical reaction modeling is needed.

2.2 Computational setup of rotatory flows

To computationally set up the external rotatory flow, four wind walls at the lateral edges of the computational domain are loaded to eject horizontal airflows, as shown in Fig. 1b. Relevant information for setting up the present boundary conditions in FDS is provided in the Supporting Material. The inlet velocity on the wind wall is $U = U_\perp + U_\parallel$, where $U_\perp$ and $U_\parallel$ are the normal and azimuthal velocity components, respectively, and the angle between them is $\alpha$. The approach is similar to the way of arranging some vanes [55] or fins [56] to adjust the swirl of the incoming air by changing the angle. In addition, the square domain has been widely used to
generate external rotatory flows in previous experimental and numerical studies [57–59]. In the present study, \( \alpha \) is fixed at 45\(^\circ\). Consequently, the inlet airflows form a rotatory flow in the central region, and the rotation intensity of the central vortical flow can be controlled by adjusting the magnitude of inlet airflow \( U = |U| \).

To facilitate the following presentation and discussion of results, \( D, \sqrt{gD} \), and \( \rho_\infty \) are used to nondimensionalize all kinematic and dynamic flow quantities. As a result, the dimensionless Cartesian coordinates are \((\hat{x}, \hat{y}, \hat{z}) = (x, y, z)/D\), the dimensionless time is \( \hat{t} = t\sqrt{g/D} \), the dimensionless velocity components are \((\hat{u}_x, \hat{u}_y, \hat{u}_z) = (u_x, u_y, u_z)/\sqrt{gD}\), the dimensionless vorticity components are \((\hat{\omega}_x, \hat{\omega}_y, \hat{\omega}_z) = (\omega_x, \omega_y, \omega_z)/\sqrt{D/g}\). To measure the intensity of the rotatory flow, a dimensionless parameter \( \hat{R} = \hat{U}/\hat{U}_0 = U/U_0 \) is introduced. In addition, the coordinate transformation of \( \hat{r} = \sqrt{\hat{x}^2 + \hat{y}^2} \) and \( \hat{\theta} = \tan^{-1}(\hat{y}/\hat{x})/\pi \) is used for the cylindrical polar coordinates of \((\hat{r}, \hat{\theta}, \hat{z})\), where the velocity \((\hat{u}_r, \hat{u}_\theta, \hat{u}_z)\) and the vorticity \((\hat{\omega}_r, \hat{\omega}_\theta, \hat{\omega}_z)\) are calculated correspondingly [60].

To examine the generated rotatory flows, we simulated a few testing cases of non-reacting flows with different \( R \), as shown in Fig. 2. The defined rotational intensity \( R \) varying from 0.09 to 0.26 is significantly smaller than unity, which justifies the "weakness" of the rotatory flows concerned. As shown in Fig. 2a, the rotatory flow resembles a quasi-cylinder vortex, where the range and intensity of the vertical component \( \hat{\omega}_z \) of vorticity vector remains approximately unchanged above \( \hat{z} = 3 \). Figure 2b shows that the magnitude of \( \hat{\omega}_z \) of the generated vortical flows at \( \hat{z} = 9 \) increases with \( R \), but the size of those vortex cores (illustrated by the streamline-encompassed area) keeps nearly the same. It should be noted that there are no vortical structures at the four corners in the present study. The flow converges gradually from the lateral sides of the square to the central region, as shown in Fig. S1 in the Supporting Material. Because toroidal vortices in the flickering flames finish their whole shedding-off process within \( \hat{z} = 6 \sim 9 \), the vorticity contours at \( \hat{z} = 9 \) are chosen, for example, to show the rotational intensity increase with \( R \). To quantify the vortical flow fields, we plotted the radial profiles of azimuthal velocity \( \hat{u}_\theta \) for four cases in Fig. 2c. It is seen that \( \hat{u}_\theta \) linearly increases up to a maximum value (referred to as \( \hat{u}_{\theta,\text{max}} \)) at \( \hat{r}_1 \) and then gradually decays along the radial direction, which is similar to the experimentally measured transverse flow before the vortex breakdown occurs [44].
The vertical component $\hat{\omega}_z$ of vorticity of a non-reactive methane jet in rotatory flows: a longitudinal section of $R = 0.17$ and b horizontal section of $R = 0.09, 0.13, 0.17,$ and $0.26$, where central vortical flows within the square region of $|\hat{x}| \leq 4$ and $|\hat{y}| \leq 4$ are shown at $\hat{z} = 9$. The whole rotatory flow refers to Fig. S1 in the Supporting Material. c The radial profiles of azimuthal velocity $u_\theta$ in the four cases. The eight geometrical symbols denote different azimuth angles distributing uniformly along a circle. The blue and red solid lines denote the vertical positions of $\hat{z} = 6$ and 9, respectively. The corresponding Rankine vortex approximation is plotted in the dashed line. d The maximum azimuthal velocity $\hat{u}_{\theta, \text{max}}$ and e the radial location $\hat{r}_a$ of vortex cores generated at different $R$ (colour figure online).

In addition, Fig. 2b, c clearly shows that the radial profiles of $u_\theta$ at eight azimuth angles, which distribute uniformly along a circle, keep the same within the range of $\hat{r} \leq 3$. Therefore, it is believed that the symmetry of the central vortical flows is hardly affected due to the square computational domain when $R \leq 0.26$. In fact, the following results of flickering buoyant diffusion flames in weakly rotatory flows ($R < 0.24$) show that the axis symmetry of flickering flames is retained well.

To facilitate the following comparison of computational and theoretical results, we approximated the calculated vortical flow as the Rankine vortex \([60]\), which has an azimuthal velocity profile as $\hat{u}_\theta(\hat{r}) \sim \hat{r}$ within a vortex core of radius $\hat{r}_a$ and $\hat{u}_\theta(\hat{r}) \sim 1/\hat{r}$ outside the vortex core. As shown in Fig. 2d, the present results show that $\hat{u}_\theta(\hat{r}_a) = \hat{u}_{\theta, \text{max}} \simeq 3.5\hat{U}$ is linearly proportional to $R$, with the proportionality being about 1.9. In addition, Fig. 2e shows that $\hat{r}_a$ is nearly a constant about 0.9, which indicates that the flickering flame is almost completely resided inside the vortex core.

It should be noted that $R$ is proportional to the nondimensional circumferential circulation in the present study. Specifically, the circulation imposed by the rotatory flow is evaluated as $2\pi \hat{r}_a u_{\theta, \text{max}}$. According to the results discussed above that $\hat{r}_a$ is approximately a constant and $u_{\theta, \text{max}}$ is proportional to $\hat{U}$, the nondimensional circulation is proportional to $R$.

### 2.3 Computational validation

To validate the present computational methodology and models for capturing the flickering phenomenon of buoyant diffusion flames, we conducted a number of simulation runs with different $D$ and $g$ in the quiescent environment (i.e., $R = 0$). For example, a snapshot of the flickering buoyant diffusion flame at $D = 10$ mm and $g = 9.8$ m/s$^2$ is illustrated in Fig. 3a, b. Around the flame, the outside shear layer (denoted by the vorticity contour) rolls up to form the toroidal vortex (denoted by the curly streamlines). The flame (denoted by the isoline of heat release) is pinched off into two parts (the main flame anchored to the ground and the separated flame bubble) by the vortex at $\hat{z} = 6$. The dynamical process of flame flicker will be expounded in the next section.
Fig. 3 The contours of (a) vorticity \( \hat{\omega}_\theta \) and (b) temperature \( T \) of a flickering buoyant diffusion flame. The black curly lines are the streamlines. The flame is denoted by the orange isoline of heat release at 5 MK/m\(^3\). (c) The validation of single flickering jet flames (red filled circle, orange filled circle, blue filled circle) with the scaling law of \( f_0 = 0.4 \sqrt{g/D} \) and experiments obtained by Durox et al. [61] (black filled down-pointing triangle), Fang et al. [62] (black filled diamond), Cetegen and Ahmed [5] (black filled up-pointing triangle), and Hamins et al. [63] (black filled square). The present simulations of flickering flames are in the range of \( Re = 100 \sim 120 \) and \( Fr = 0.05 \sim 0.56 \), of which detailed parameters are provided in Table S1 in the Supporting Material (colour figure online).

In Fig. 3c, the present results clearly show that the calculated frequencies of flickering flames agree fairly with previous experiments [5,61,63] at a small error range (< 10%) and particularly predict the famous scaling relation of \( f_0 \sim \sqrt{g/D} \) [3,13], where the proportionality factor of 0.4 is close to 0.48 obtained from plenty of experimental data reported in the literature [13]. In addition, we found that the gravity changes of \((0.5g - 1.5g)\) can significantly affect the flickering frequency, confirming that the buoyancy is predominant in the present diffusion flames. The boundary flow near the bottom wall has negligible influence on the flickering of laminar diffusion flames, as the growth and shedding of the toroidal vortex occur downstream far from the bottom. Carpio et al. [64] reported that, in a strongly rotatory flow, the bottom corner flow might result in the lift-off of the flame base, which is absent in the present problem focusing on weakly rotatory flows.

3 Results and discussion

In this section, we will answer the three questions raised in the Introduction. Particularly, we will clarify the influences of the external rotatory flow on the buoyancy-induced toroidal vortex and the variation of the flickering frequency and will theoretically establish the underlying mechanism of flickering flames in weakly rotatory flows.

3.1 Phenomenological description

In flickering buoyant diffusion flames, the buoyancy-induced shearing between the flame and the surrounding air is the precursor of the toroidal vortex [13,30,34]. During the lifecycle of the vortex, its formation, growth, and shedding correspond to the stretching, necking, and pinch-off of the flame, respectively. It can be seen in Fig. 4a for the case of \( R = 0 \) (without external rotatory flow) that the vorticity layer forms at the flame base to stretch the flame (the normalized time \( t^* = \hat{t} \hat{f}_0 = 0 \sim 0.3 \), where \( \hat{t} = t/\sqrt{D/g} \) and \( \hat{f}_0 = f_0/\sqrt{g/D} \), curlicly grows along and necks the flame (\( t^* = 0.3 \sim 0.8 \), and sheds off (\( t^* = 0.8 \sim 1.0 \)) to pinch off the flame. In this way, the flame performs a periodic flickering process. During \( t^* = 1.0 \sim 1.1 \), the shedding vortex moves downstream, and the carried flame bubble burns out soon, while a new vortex generates at the flame base, and a new cycle starts.
A similar evolution of the toroidal vortex can also be observed in Fig. 4b for the typical case of \( R = 0.17 \), where the stretching, necking, and pinch-off of the flame remain in close associations with the formation, growth, and shedding of the vortex. An interesting observation is that the flame tends to be pinched off at a further downstream location with increasing \( R \). Specifically, the flame pinch-off occurs at \( \hat{z} = 6.0 \) for the case of \( R = 0 \) but at \( \hat{z} = 7.2 \) for the case of \( R = 0.17 \). This observation can be understood by that the external rotatory flow induces an additional vertical flow, which convects the vortex downstream. To substantiate the understanding, the central axial profiles of the vertical velocity \( \hat{u}_z \) in the two cases are plotted in Fig. 4c, d, respectively. It is seen that \( \hat{u}_z \) rapidly increases from the fuel inlet and reaches the first peak at the position of the toroidal vortex. The comparison of \( \hat{u}_z \) clearly indicates that the external rotation enhances the vertical flow (with higher peak values) and thus contributes to the additional motion of the vortex (more downstream position of the first peak). The most interesting observation is that the flame pinch-off tends to occur earlier with increasing \( R \). Specifically, the flame is pinched off at \( t^* = 0.75 \) for the case \( R = 0.17 \) compared with that at \( t^* = 0.85 \) for the case of \( R = 0 \). To further quantify and interpret, this observation will be the focus of the following subsections.

The present results indicate that the dynamics of the toroidal vortex is affected due to the rotation of ambient flow. To clearly show the effects of rotatory flow, we plotted streamlines around the flames and colored the local helicity density \( \hat{h} = \hat{u} \cdot \hat{\omega} \) along the streamlines in Fig. 5a, b for the cases of \( R = 0 \) and \( R = 0.17 \), respectively. The zero value of \( \hat{h} \) means that streamline is locally orthogonal to the vorticity line, while the nonzero \( \hat{h} \) can be used to quantify the local geometrical helix. Specifically, it can be seen in Fig. 5a that the flow around the flickering flame in a quiescent environment is two-dimensional axisymmetric because \( \hat{h} \) is zero everywhere, and the streamlines are restricted in the \( Y-Z \) plane due to the axis symmetry. As the ambient flow is rotaring, the flow is not restricted in the two-dimensional plane and is twisted toward the circumferential direction so that the streamlines tilt out of the \( Y-Z \) plane to form a spiral ring, as shown in Fig. 5b. Nonzero \( \hat{h} \) appears in the vorticity layer around the flame and increases with the rotational intensity \( R \). Regardless of the local helix of the flow field, the flame morphology retains the approximate axis symmetry to a certain extent. Therefore, in the present problem concerning weakly rotatory flows, there is only moderate symmetry breaking to the flame shape and surrounding shear layer, which will be used as a useful approximation to simplify our theoretical modeling to be expounded in the next section.

To estimate the extent to which the presence of flame affects the rotatory ambient flow, we replotted the radial profiles of azimuthal velocity \( \hat{u}_\theta \) at different streamwise locations of \( \hat{z} = 3, 6, \) and 9. It is clearly seen that, although the profile of \( \hat{u}_\theta \) changes with time due to the flame flicker, it retains the similarity to the Rankine vortex flow as \( \hat{u}_\theta \) increases linearly with \( \hat{r} \) within a certain range (i.e., the vortex core) that is sufficiently large to include the flame and its surrounding shear layer. It is also found that the decrease of \( \hat{u}_\theta \) with \( \hat{r} \) outside the
vortex core is slower than the trend of $1/\hat{r}$ due to the additional flow induced by the thermal expansion around the flame. This does not have significant effects on the evolution of toroidal vortices within the vortex core. In addition, the decrease of $\hat{u}_\theta/\sqrt{\hat{z}}$ along the axial direction indicates that the circumferential motion caused by the external rotatory flow becomes weaker downstream. Consequently, the buoyancy-induced vortex flow still plays the predominated role in the present flickering flames in weakly rotatory flows, which will facilitate the following modeling in the next section.

The previous study [13] has shown that the pressure gradient $\nabla p$ can be negligible for the vorticity generation in flickering flames if the ambient flow is quiescent and the Froude number is small. In the present problem, we hypothesized that the formation and evolution of the toroidal vortex around the flame are affected by the baroclinity $\nabla \rho \times \nabla p$ due to the external rotatory flow. To verify this hypothesis, we plotted the radial pressure gradient $\partial \hat{p}/\partial \hat{r}$ around the flames in the cases of $R = 0$ and $R = 0.17$, as shown in Fig. 6a. It is seen that the magnitude of $\partial \hat{p}/\partial \hat{r}$ is not negligible in almost the entire flow domain and is significant in the domain close to the flame. Besides, Fig. 6b shows that the azimuthal pressure gradient $(\partial \hat{p}/\partial \hat{\theta})/\hat{r}$ has much smaller magnitude than $\partial \hat{p}/\partial \hat{r}$ so that we can neglect the azimuthal component in the following analysis. More details on their instantaneous comparisons during the periodic flickering process are provided in Fig. S2 in the Supporting Material. Consequently, the pressure gradient along the radial direction should be considered in the present problem concerning the influence of the rotatory flows. It should be noted that the radial pressure gradient is assumed as the time-averaged dominant term, which mostly acts to balance the Centrifugal force. The unsteady significance of pressure gradients deserves further work.

3.2 Influence of $R$ on flickering frequency

To determine the frequency of flame flicker, it is required to choose an appropriate quantity that can characterize the dynamical behavior of flickering flames. In the previous experiments, there is quite a freedom in selecting and acquiring characteristic (either local or global) flame quantities, for example, the pressure or temperature of flickering flame at a certain position [5,65], the flame luminosity at a certain height [66], and the flame morphology information (e.g., the flame height [1], width [39], size [67] and brightness [26,33]) obtained
The radial component of the pressure gradient $|\partial \hat{p} / \partial \hat{r}|$ in the cases of $R = 0$ and $R = 0.17$, corresponding to the left and right subfigures, respectively. The instantaneous radial component $|\partial \hat{p} / \partial \hat{r}|$ (the red line) and azimuthal component $|\left(\partial \hat{p} / \partial \hat{\theta}\right) / \hat{r}|$ (the blue line) of pressure gradient at $\hat{z} = 6$ in the case of $R = 0.17$ (colour figure online).

from high-speed images. In the present computational work, we choose the total heat release rate $Q$ and the vertical velocity $u_z$ and temperature $T$ at a fixed point to determine the frequency of flame flicker, as shown in Fig. 7a–c, respectively. The former one can be treated as a representative global quantity, and the latter two are representative local quantities. The present results show that all the quantities exhibit periodic behaviors with almost the same frequency for the ambient flow being rotatory or not, while there are slight phase differences among them. This result is consistent with previous experimental observations [5,33] that the dominant frequencies based on different quantities in the flickering flame have negligible differences. For simplicity and consistency, the global quantity $Q$ will be adopted in the present study for analyzing the frequency variation in different cases.

It is seen in Fig. 7a that the periodic wave of $Q$ for the case of $R = 0$ oscillates at the frequency of $f_0 = 9.6$ Hz, which is smaller than 10.6 Hz for the case of $R = 0.17$. The frequency comparison further confirms the observation in Sect. 3.1 that the external rotatory flow causes the flame to be pinched off early. Figure 7d shows a monotonically increasing trend of the frequency $f$ with the rotational intensity $R$, and the trend is noticeably nonlinear. It should be noted that the present study focuses on the weakly rotatory flows with $R < 0.24$, beyond which the stronger rotational flow may lead to the occurrence of vortex breakdown and local flame extinction [44,64]. More sophisticated computational approaches are needed to deal with these emerging phenomena in strongly rotatory flows and merit future work.

3.3 Theoretical model of flickering flame in a weakly rotatory flow

The above computational results hitherto have demonstrated the influence of weakly rotatory flows on the formation of the toroidal vortex and the frequency augmentation. Next, we will reveal the underlying mechanism. Following the theory of Xia and Zhang [13], the present theoretical model for flame flicker is based on its connection with the shed-off of the toroidal vortex. To facilitate the comparison with the theory of Xia and Zhang [13], we adopted the same notation within this subsection. The central idea of the theory is that the flame flicker occurs when the circulation of the toroidal vortex reaches a threshold, which is an approximately universal constant [68–70] and presumably does not have a significant change in the present weakly rotatory flows.
environments. The frequency relation for buoyancy-driven diffusion flames in weakly rotatory flows will be theoretically derived through three steps as follows.

In the first step, the generation rate of total circulation $\dot{\Gamma}$ inside a control mass $A$ is formulated based on the approximation of axis-symmetry, as discussed in Sect. 3.1. As illustrated in Fig. 8a, a vortex layer segment around the flame is encircled by the red dashed box, which is the material contour $\partial A$ of a control mass $A$. The zoomed subfigure clearly shows that the domain of $A$ is vertically between $S_{v1}$ and $S_{v2}$ and radially between $S_{r1}$ and $S_{r2}$, where a material line element is represented as $s ds$ and $s$ is the unit tangential vector along the contour $\partial A$. According to Kelvin’s circulation formula [71], the rate of total circulation change is

$$
\dot{\Gamma} = \oint_{\partial A} \mathbf{a} \cdot ds
$$

(6)

where $\dot{\Gamma} = d\Gamma/dt$ is the change rate of circulation, $\mathbf{a} = Du/Dt = - [\nabla p + (\rho_\infty - \rho) \mathbf{g}] / \rho + \nu \nabla^2 \mathbf{u}$ is the acceleration of control mass [13]. As the diffusion term is not a source of vorticity production and vanishes on $\partial A$, the dimensionless $\hat{\mathbf{a}} = (Du/Dt)/\rho$ can be expressed as

$$
\hat{\mathbf{a}} = -\frac{\nabla \hat{p}}{\hat{\rho}} + \left(1 - \frac{1}{\hat{\rho}} \right) \hat{\mathbf{g}}
$$

(7)

For the pressure gradient term $\nabla \hat{p}$, we only consider the radial component $\partial \hat{p} / \partial \hat{r}$ as discussed in Sect. 3.1. Considering that the present rotatory flows resemble the Rankine vortex, we use the relationship between the azimuthal velocity and the radial pressure gradient

$$
\frac{\partial \hat{p}}{\partial \hat{r}} = \hat{\rho} \hat{u}_\theta \hat{\rho}^{-\frac{3}{2}}
$$

(8)

where the azimuthal velocity $\hat{u}_\theta$ has a linear correlation with $\hat{r}$, namely $\hat{u}_\theta = C_\theta \hat{r}$ with a measurable quantity $C_\theta = \hat{u}_\theta (\hat{r}_a) / \hat{r}_a$ for a given rotatory flow. Applying Eq. (8) and Eq. (7) in Eq. (6), we have

$$
\hat{\Gamma} = \oint_{\partial A} -C_\theta \hat{r} \hat{\rho} \cdot s ds + \oint_{\partial A} \left(1 - \frac{1}{\hat{\rho}} \right) \hat{g} \cdot s ds
$$

(9)
Thus, we have Eq. (11), then we have vortex is formed at the flame base, and a new cycle starts. It can be seen in Fig. 8b that the increment of \( t \) can be further simplified as

\[
\dot{\bar{t}} = -[2C_{df}^2 \dot{\bar{r}} \Delta \hat{r} + (C_{\rho} - 1) \hat{\bar{g}} \Delta \hat{z}]
\]

where \( \dot{\bar{r}} \) is the radial position of the vortex layer and close to the flame sheet; the density ratio \( C_{\rho} = \rho_{\infty} / \rho_f \) is a measurable quantity for a given flame, for example \( C_{\rho} \) is about 7.5 for the present computational methane/air flames, \( \Delta \hat{r} \) and \( \Delta \hat{z} \) are the radial and vertical unit lengths of the vortex layer associated with the control mass \( A \). It should be noted that the first term of Eq. (11) is attributed to the external rotatory flow, which is absent in the theory of Xia and Zhang [13].

In the second step, the periodic formation process of a toroidal vortex associated with the flame flicker is established. Figure 8b illustrates the evolution of a toroidal vortex in the moving control volume \( B \). During \( t_1 \sim t_2 \), a new vortex core of the toroidal vortex generates near the base of the flame and then grows downstream under the buoyancy-induced convection. During \( t_3 \sim t_2 \), the vortex head region rolls up outward, and the central vortex core moves downstream. During \( t_2 \sim t_3 \), the vortex fully develops and detaches. Meanwhile, a new vortex is formed at the flame base, and a new cycle starts. It can be seen in Fig. 8b that the increment of \( \Delta \hat{z} \) is much larger than that of \( \Delta \hat{r} \) during a periodic motion, and \( \dot{\bar{r}} \) of the vortex core is approximately constant. For the total circulation \( \dot{\bar{r}} \) of the moving vortex enclosed in \( B \), its change rate should include an additional term \( \dot{\bar{r}}_{z=0} = -C_{j} \dot{U}_0^2 \) [13, 72], which is influenced by the fuel inlet (jet or pool) and the fuel type (gaseous or liquid). Specifically, the initial velocity and the evaporating rate should be considered in \( \dot{\bar{r}}_{z=0} \). According to Eq. (11), then we have

\[
\dot{\bar{r}}_{B}(\bar{t}) = -2C_{df}^2 \dot{\bar{r}}(\bar{t}) - (C_{\rho} - 1) \hat{\bar{g}} \hat{\bar{h}}(\bar{t}) - C_{j} \dot{U}_0^2
\]

where \( \dot{\bar{r}}(\bar{t}) \) is the width of the right boundary of \( B \), \( \hat{\bar{h}}(\bar{t}) \) is the height of the upper boundary of \( B \), and \( -C_{j} \dot{U}_0^2 \) enters the system through the lower boundary of \( B \). The constant \( C_{j} \) relies on the configuration and the jet inlet condition.

Focusing on the formation of the toroidal vortex from the new to the fully developed, we integrate Eq. (12) in a period \( \bar{t} = U_0 \bar{t} / D \) and have

\[
\dot{\bar{r}}_{TV} = \int_{0}^{\bar{t}} \hat{\bar{r}}_{B} d\bar{t} = -2C_{df}^2 \dot{\bar{r}} \bar{R} \bar{t} - (C_{\rho} - 1) \hat{\bar{g}} \bar{H} \bar{t} - C_{j} \dot{U}_0^2 \bar{t}
\]

where \( \bar{R} = \bar{t}^{-1} \int_{0}^{\bar{t}} \dot{\bar{r}}(\bar{t}) d\bar{t} \) and \( \bar{H} = \bar{t}^{-1} \int_{0}^{\bar{t}} \hat{\bar{h}}(\bar{t}) d\bar{t} \) represent the time-averaged width and height of the growing toroidal vortex, respectively. According to the present computational results that the width of the toroidal vortex is nearly unchanged, and the vortex core moves close to the flame sheet, we have \( \bar{R} \approx 1.9 \) and \( \bar{H} \approx 0.6 \). In addition, \( \hat{\bar{h}}(\bar{t}) \) can be roughly estimated as \( \sqrt{gDt} \), as the toroidal vortex is driven downstream by buoyancy. Thus, we have \( \bar{H} = C_{h} \bar{t} / \dot{U}_0 \), where \( C_{h} \) is a constant pre-factor. With being scaled by \( -\dot{U}_0 \), Eq. (13) can be rewritten as

\[
\dot{\bar{r}}_{TV} = C_{h} \bar{R} \bar{t}^2 + (C_{j} + C_{r}) \sqrt{Fr} \bar{t}
\]

where \( Fr = (C_{\rho} - 1) \hat{\bar{g}} / \dot{U}_0^2 = (C_{\rho} - 1) gD / \dot{U}_0^2 \) is the Froude number, \( Fr = \dot{U}_0^2 / gD \) is the Froude number, and \( C_{r} = 2C_{df}^2 \dot{\bar{r}} \bar{R} / \dot{U}_0^2 \) is a pre-factor for characterizing the external rotational flow.
In the third step, a threshold for the accumulation of the circulation inside the toroidal vortex [68,70] is applied to obtain the frequency relation. Applying \( \dot{\Gamma}^*_{TV} = C \) to Eq. (14), we have

\[
\frac{f}{\sqrt{g/D}} = \frac{\dot{U}_0}{\epsilon} = \frac{1}{2C} \left( C_{jr} Fr + \sqrt{C_{jr}^2 Fr^2 + CC_\theta C_\rho} \right)
\]

(15)

where \( C_{jr} = C_j + C_r \) is a pre-factor for the combined contributions of the initial jet flow and the external rotatory flow. Hereto, we complete the derivation of the frequency relation for flickering buoyant diffusion flames in rotatory flows. For the case of \( R = 0 \), the pre-factor \( C_{jr} \) degenerates to \( C_j \), and the frequency of flickering flames in a quiescent environment is obtained as

\[
\frac{f_0}{\sqrt{g/D}} = \frac{1}{2C} \left( C_j Fr + \sqrt{C_j^2 Fr^2 + CC_\theta C_\rho} \right)
\]

(16)

which is exactly the same as the scaling formula obtained by Xia and Zhang [13]. It is interesting to see that Eqs. (15) and (16) have the same functional form and differ in only the pre-factor of the Fr-term. The underlying physics is that the external rotatory flow plays a similar role as that of the initial jet for enhancing the vortex growth.

3.4 Comparison between computation and theory

Next, we compare the above theoretical formula with the present computational results. Based on Eq. (15), we can have the frequency increase of flickering buoyant diffusion flames due to the additional rotation of the ambient flow

\[
\Delta \hat{f} = \hat{f}(R) - \hat{f}(R = 0) = \frac{f(R) - f(R = 0)}{\sqrt{g/D}} = \frac{5\Phi}{C} R^2
\]

(17)

where \( \Phi = 1 + (C_j + C_{jr}) / \left( \sqrt{C_j^2 + CC_\theta C_\rho/ Fr^2} + \sqrt{C_{jr}^2 + CC_\theta C_\rho/ Fr^2} \right) \) is a constant factor of about 1 \( \sim \) 2, and the approximation of \( C_r = 10R^2/\dot{U}_0^2 \) is used due to \( C_\theta = 2.11 R \) and \( \dot{U}_0 \). Therefore, we obtain a scaling law of \( \Delta \hat{f} \sim R^2 \) from Eq. (17). As shown in Fig. 9, the scaling law agrees very well with the
Fig. 9 Comparison between the correlation of $\Delta \hat{f} \sim R^2$ with the present numerical results. The circles correspond to the cases of $\alpha = 45^\circ$ in Fig. 7d. The squares are the cases of $\alpha = 27^\circ$.

present computational results. Particularly, the two cases of $\alpha = 27^\circ$ in Fig. 9 show that the angle between the normal and azimuthal velocity components of $U$ does not affect the finding. According to Eq. (17), $\alpha$ is embedded in the constant factor $\Phi$ by the coefficient of $C_\theta$ with $R$. The effect of the angle $\alpha$ on flames in strongly rotatory flows merits further parametric studies.

It should be emphasized that the present theoretical analysis is focused on the weakly rotatory flow. Consequently, the $\Delta \hat{f} \sim R^2$ scaling law is limited to sufficiently small $R$. When $R$ is larger than 0.24 in the present study, the flame flicker may not be clearly observed, and the flame oscillates in its tip with a slight swing. This result implies that the circumferential motion of the toroidal vortex is not negligible, and the axis-symmetry approximation becomes questionable. This was confirmed by the experimental observation that free buoyant pool fires tilt increasingly at a higher rotating speed [42]. The available experimental data are very limited to make a quantitative comparison with the numerical results and the scaling relation. The relevant experiment validations merit future work. In addition, the sufficiently large $R$ is likely to cause the occurrence of vortex breakdown and local flame extinction. This problem requires a completely different computational and theoretical framework and merits further research.

4 Concluding remarks

Flickering diffusion flames in a quiescent environment have been extensively studied in the literature, but their characteristics in complex external flows, particularly in external rotatory flows, were investigated inadequately. The present study presents computational and theoretical investigations on the small-scale flickering buoyant diffusion flames in weakly rotatory flows, and the conclusions are summarized as follows.

First, four lateral forced-ventilation walls were computationally imposed to generate a rotatory flow with variable rotational speed $U$ around a methane jet diffusion flame with a fixed initial velocity $U_0$ (Re = 100 and Fr = 0.28). The rotational intensity is controlled by the defined dimensionless parameter $R = U/U_0$, which is proportional to nondimensional circumferential circulation. The generated rotatory flow is well symmetric within the relatively wide range ($\hat{r} \leq 3$), and found to resemble the Rankine vortex as it has a linear velocity profile within a vortex core whose size slightly changes with $R$. As a validation, flickering buoyant diffusion flames of methane gas in a quiescent environment were computationally reproduced, and the calculated flicker frequencies $f_0$ agree well with the famous scaling relation of $f_0 \sim \sqrt{g/D}$.

Second, the present computational results show that the external rotatory flow enhances the periodic flickering motion and accord with the experimental observations reported in the literature [42]. Furthermore, the flicker frequency $f$ is found to nonlinearly increases with $R$ up to 0.24. By analyzing the flow and pressure fields around the flame, we found that there is only slight axis-symmetry breaking to the flame shape and surrounding shear layer in weakly rotatory flows and that the radial pressure gradient is significantly increased compared with that at $R = 0$. The approximate axis-symmetry was subsequently used to simplify our theoretical
investigation, and the significant radial pressure gradient implies that the baroclinic effect must be taken into account in our theory.

Third, we formulated the scaling theory for interpreting the frequency increase of flickering buoyant diffusion flames in weakly rotatory flows. This theory can degenerate into that of Xia and Zhang [13] at $R = 0$. This theory clearly shows that the effect of external rotatory flows can be characterized in the pre-factor of the Fr-term, in which the rotatory flow has a similar role as the initial jet in enhancing the vortex growth. The predicted frequency correlation of $\Delta \hat{f} = \hat{f} - \hat{f}_0 \sim R^2$ agrees very well with the present computational results. The present finding that flicker frequency $\hat{f}$ increases with rotation intensity $R$ agrees with previous experimental observations [42,45]. The relevant experiment validations merit future work. The underlying physics can be understood as that the external rotatory flow enhances the pressure gradient in the radial direction, and the significant baroclinic effect $\nabla p \times \nabla \rho$ contributes an additional source for the growth of toroidal vortices. Consequently, the toroidal vortices reach the threshold of circulation for shedding at an early time.

It should be noted that the present study is concerned with the weakly rotatory flows, as a sufficiently large $R$ tends to cause the occurrence of vortex breakdown and even local flame extinction. The flame phenomena in strongly rotatory flows are governed by different physical mechanisms and require differential computational and theoretical treatment, which are of interest for future work. In addition, the flickering spray diffusion flames may merit future studies, where droplet vaporization and combustion would change the local flow field and therefore affect the vortex ring evolution.

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Data availability The data that support the findings of this study are available from the corresponding author upon reasonable request.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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