ELECTROMAGNETIC STREAMING INSTABILITIES OF MAGNETIZED ACCRETION DISKS WITH STRONG COLLISIONAL COUPLING OF SPECIES

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ABSTRACT

Electromagnetic streaming instabilities of multicomponent collisional magnetized accretion disks are studied. Sufficiently ionized regions of the disk are explored where there is strong collisional coupling of neutral atoms with both ions and dust grains simultaneously. The steady state is investigated in detail and the azimuthal and radial background velocities of species are calculated. The azimuthal velocity of ions, dust grains, and neutrals is found to be less than the Keplerian velocity. The radial velocity of neutrals and dust grains is shown to be directed inward of the disk. The general solution for the perturbed velocities of species taking into account collisions and thermal pressure is obtained. The effect on the collisional frequencies, due to density perturbations of charged species and neutrals, is included. It is shown that dust grains can be involved in the fast electromagnetic perturbations induced by the ions and electrons through the strong collisions of these grains with neutrals that in turn have a strong collisional coupling with the ions. The dispersion relation for the vertical perturbations is derived and its unstable solutions due to different background velocities of ions and electrons are found. The growth rates of the streaming instabilities considered can be much larger than the Keplerian frequency.

Key words: accretion, accretion disks – instabilities – magnetic fields – plasmas – waves

1. INTRODUCTION

Protostellar and protoplanetary accretion disks are known to contain submicron- and micron-sized solid particles (dust grains; e.g., Beckwith & Sargent 1996; Isella et al. 2006; Besla & Wu 2007; Quanz et al. 2007; Pinte et al. 2007). The presence of dust grains is common in these and other astrophysical and cosmic objects, such as, molecular and interstellar clouds, planetary and stellar atmospheres, planetary rings, and cometary tails (Spitzer 1978; Goertz 1989; Havnes et al. 1996; Wardle & Ng 1999; Rotundi et al. 2000). Dust is contained in various plasmas of both research and technological importance (Merlino & Gore 2004). In space, dust grains vary widely in sizes, from submicrons up to 1 cm and more. Dust particles in both space and the laboratory acquire an electric charge due to cosmic-ray, radioactive or thermal ionization of the ambient gas, photoionization, absorption of electrons and ions from the background plasma, the presence of electron and ion currents, thermoionic emission, and secondary ionization (Hayashi 1981; Whipple 1981; Meyer-Vernet 1982; Chow et al. 1993; Barkan et al. 1994; Melzer et al. 1994; Mendis & Rosenberg 1994; Thomas et al. 1994; Horányi 1996; Fisher & Duerbeck 1998; Fortov et al. 1998). The polarity of charged grains in space and the laboratory can be negative or positive (Meyer-Vernet 1982; Chow et al. 1993; Mendis & Rosenberg 1994; Horányi 1996; Horányi & Goertz 1990; Havnes et al. 2001; Ellis & Neff 1991). The magnitude of charge grains in the laboratory can be very large (up to a few orders of magnitude of the electron charge). The polarity of charged grains depends on their size; large grains are usually negative and small grains positive (e.g., Chow et al. 1993; Mendis & Rosenberg 1994).

The dynamics of dust grains in protostellar and protoplanetary accretion disks determines the physical processes leading to the formation of planets. It is generally accepted that small solids (1 cm–1 m) can be formed due to chaotic motion and sticking of submicron- and micron-sized dust grains strongly collision coupled to the surrounding gas. These small solids in turn can form the kilometer-sized bodies (planetesimals), believed to be the building blocks for planet formation.

Various physical mechanisms have been discussed for the formation of planetesimals, such as, collisional coagulation (Adachi et al. 1976; Cuzzi et al. 1993; Weidenschilling 1995), gravitational instability (Safronov 1969; Goldreich & Ward 1973; Yamoto & Sekiya 2004), and the trapping of particles by large-scale persistent vortices (Barge & Sommeria 1995; Lovelace et al. 1999; Klahr & Bodenheimer 2003; Johansen et al. 2004; Petersen et al. 2007).

Recently the streaming instability arising from the difference between velocities of dust grains and gas has been suggested as a possible source contributing to planetesimal formation (Youdin & Goodman 2005). These authors have considered the hydrodynamic instability of the interpenetrating streams of dust grains and gas coupling via drag forces in the Keplerian disk. The particle density perturbations generated by this instability could seed planetesimal formation without self-gravity.

In the papers cited above, which deal with the dynamics of dust grains in accretion disks in connection with the problem of dust coagulation and planet formation, hydrodynamic models were applied in which the dust grains were considered as a neutral fluid or ensemble of individual particles interacting with the surrounding neutral gas. Thus, hydrodynamic processes were not involved. However, dust grains are generally charged (see references cited above). In the surface layers and near the star, the protostellar and protoplanetary accretion disks are multicomponent, containing electrons, ions, charged grains, and neutral gas. In the sufficiently dense inner regions, the disk matter likely contains only charged dust grains and gas (e.g., Wardle & Ng 1999). Dust grains can be of various forms. It is clear that electromagnetic forces could play an important role in dust coagulation and in collective processes within the disk involving the dust grains. The electromagnetic turbulence favors the coalescence of charged dust grains. The collisional coagulation due to electrostatic interaction of dust grains can lead to the formation of fractal aggregate structures (Matthews...
et al. 2007). These structures could build larger sized solid particles in the process of planet formation.

The necessity of taking into account electromagnetic phenomena in the disk dynamics is confirmed by observations, which show that astrophysical disks are turbulent (Hartmann 2000; Carr et al. 2004; Hersant et al. 2005). It is also known that accretion disks are threaded by the magnetic field (Hutawarakorn & Cohen 1999, 2005; Donati et al. 2005). In the latter, a direct detection of the magnetic field in the protostellar accretion disk FU Orionis, including the innermost and densest parts of the disk, is reported. The surface magnetic field was observed to reach strengths of about 1 kG close to the center of the disk and several hundred gauss in its innermost regions. The magnetic field is grossly axisymmetric and has both vertical and significant azimuthal components. The results of Donati et al. also suggest that magnetic fields in accretion disks could trigger turbulent instabilities that would produce enhanced radial accretion and drifts of ionized plasma through transverse field lines.

The electromagnetic dynamics of accretion disks is determined by physical parameters, such as, the magnetic field, density, temperature, the ionization degree, composition, and mass and size of charged dust grains, all of which vary significantly through the whole disk. So these would determine the disk dynamics on a local scale. The protostellar and protoplanetary accretion disks as well as cold molecular and interstellar clouds are weakly ionized (Wardle & Ng 1999; Norman & Heyvaerts 1985; Jin 1996; Bergin et al. 1999; Balbus & Terquem 2001; Desch 2004; Tscharnuter & Gail 2007). Dust grains are, possibly, the primary charge carriers in very dense, cool nebulae (Wardle & Ng 1999; Blaes & Balbus 1994).

One of the sources of electromagnetic turbulence in accretion disks may be electromagnetic streaming instabilities emerging due to the different velocities of species in equilibrium. We have studied this issue (Nekrasov 2007) for cold magnetized disks. The dust grains were considered to be magnetized and strong collisional coupling of neutrals with one of the charged species could be included. The thermal pressure was not taken into account. New compressible instabilities were found, with growth rates much larger than the Keplerian frequency. In a subsequent paper (Nekrasov 2008), the general theory for electromagnetic streaming instabilities in multicomponent weakly ionized regions of accretion disks embedded in the magnetic field has been developed. The compressibility, anisotropic thermal pressure, and collisions of charged species with neutrals were taken into account. The equilibrium state was found for the case in which the ions do not influence the motion of neutrals. However, the neutrals are assumed to have weak or strong collisional coupling with dust grains. In the perturbed state, the neutrals have been considered as immobile when the perturbation frequency is much larger than the collisional frequency of neutrals with charged species. A dispersion relation has been derived in the most general form for an arbitrary direction of the perturbation wave vector and for arbitrary strength of thermal effects. In particular, we found solutions of the dispersion relation for a case appropriate to define regions of the protostellar and protoplanetary disks, where the electrons can be considered as thermal and magnetized, and ions and dust grains as cold and unmagnetized. These solutions describe new instabilities of the weakly ionized disks due to collisions and differences between the stationary velocities of different charged species.

In the present paper, we study the electromagnetic streaming instabilities in the sufficiently ionized regions of the protostellar and protoplanetary magnetized accretion disks, where the neutrals have strong collisional coupling simultaneously with ions and dust grains both in the equilibrium and in the perturbed state. Dust grains and light charged components are respectively unmagnetized and magnetized, i.e., their cyclotron frequencies are respectively much smaller or much larger than their orbiting frequencies. The dust grains are treated as monosized with a constant charge. For the collisional regime under consideration, we find the azimuthal and radial stationary velocities of species. We consider the effect of the change in collisional frequencies due to density perturbations of species. Taking into account anisotropic thermal pressure, we derive the general expression for the perturbed velocity of any species that contains also the perturbed velocity of other species due to collisions. In the cold limit for horizontally elongated perturbations, we obtain the general solutions for the perturbed velocities of ions and dust grains incorporating their mutual influence on each other via collisions with neutrals. Further, we consider fast processes in which the electromagnetic dynamics of dust grains is determined by collisions with neutrals but not by their own motional frequencies. Finally, we investigate unstable perturbations due to different azimuthal velocities of ions and electrons.

The paper is organized as follows. In Section 2 the basic equations are given. In Section 3 we discuss in detail the collisional equilibrium state. The general solutions for the perturbed velocities and densities of species including thermal effects are obtained in Section 4. The perturbed velocities of species for vertical perturbations in the cold limit are considered in Section 5. Solutions for the perturbed velocities of species when the dynamics of dust grains is determined by collisions with neutrals are given in Section 6. In Section 7 we calculate the perturbed electric current. The dispersion relation in the general case is derived in Section 8. The dispersion relation in the case of strong collisional coupling of dust grains with ions through collisions with neutrals is considered in Section 9. Its unstable solutions are found in Section 10. Discussion of the obtained results is given in Section 11. The main points of the paper are summarized in Section 12.

2. FUNDAMENTALS

The fundamental equations in the inertial (nonrotating) reference frame are the following:

\( \frac{\partial \mathbf{v}_j}{\partial t} + \mathbf{v}_j \cdot \nabla \mathbf{v}_j = -\nabla U - \frac{\nabla \cdot P_j}{m_j n_j} + \frac{q_j}{m_j c} \mathbf{E} + \frac{q_j}{m_j c} \mathbf{v}_j \times \mathbf{B} - v_{jn}(v_j - v_n), \) (1)

\( \frac{\partial \mathbf{v}_n}{\partial t} + \mathbf{v}_n \cdot \nabla \mathbf{v}_n = -\nabla U - \frac{\nabla P_n}{m_n n_n} - \sum_j v_{nj}(v_n - v_j), \) (2)

the momentum equations,

\( \frac{\partial n_{j,n}}{\partial t} + \nabla \cdot n_{j,n} \mathbf{v}_{j,n} = 0, \) (3)

the continuity equation,

\( \nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \) (4)

\( \nabla \times \mathbf{B} = \frac{4\pi}{c} j, \) (5)
and Maxwell’s equations, where $j = \sum q_j n_j v_j$. Here the index $j = e, i, d$ denotes the electrons, ions, and dust grains, respectively, and the index $n$ denotes the neutrals. In Equations (1)–(5) $q_j$ and $m_{j,n}$ are the charge and mass of species $j$ and neutrals, $v_{j,n}$ is the hydrodynamic velocity, $n_{j,n}$ is the number density, $P_j$ and $P_n$ are the thermal pressure tensor of charged species $j$ and the thermal pressure of neutrals, respectively, $v_{jn} = \gamma_{jn} m_j n_j$ is the collisional frequency of charged species (neutrals) with charged species, where $\gamma_{jn} = (\sigma v_{jn} / m_j + m_n) / (\sigma v_{jn} / m_n)$ is the rate coefficient for momentum transfer, $U = -GM/R$ is the gravitational potential of the central object having mass $M$, $R = (r^2 + z^2)^{1/2}$, $G$ is the gravitational constant, $E$ and $B$ are the electric and magnetic fields, and $c$ is the speed of light in vacuum. We consider wave processes with typical timescales much larger than the time the light spends to cover the wavelength of perturbations. In this case one can neglect the displacement current in Equations (1)–(3), we do not take into account ionization and recombination processes. We comment on this point in Section 11. Self-gravity is not included in the present model.

3. EQUILIBRIUM

We suppose that $B_{\text{ext}}$ is axisymmetric and $B_{\text{corr}} = 0$. Such a configuration is typical of the magnetic field of the central star and/or of disks threaded by the vertical interstellar magnetic field. We neglect the radial component of the background magnetic field, considering the regions of the disk, where vertical components of both the external magnetic field and the magnetic field induced by the stationary azimuthal current are dominant.

Let us consider axisymmetric stationary flows of species $j$. We suppose that the vertical stationary velocity $v_{j,0r}$ is equal to zero. Then the $r$ and $\theta$ components of Equation (1) in the equilibrium take the form

$$\frac{\partial v_{j,0r}^2}{2 \partial r} - \frac{v_{j,0\theta}^2}{r} = \frac{F_{j,0r}}{m_j} + \omega_{cj} v_{j,0\theta} - v_{jn}^0 (v_{j,0r} - v_{n,0r}),$$

and

$$F_{j,0r} = -m_j \frac{\partial U}{\partial r} - \frac{1}{n_{j,0}} \frac{\partial p_{\perp,j,0}}{\partial r} + q_j E_{0r},$$

where $E_{0r}$ is the background electric field.

Equations (6) and (7) determine the stationary velocity $v_{j,0r}$ of different charged components due to the action of the electric (radial), magnetic (vertical), and gravitational fields as well as the thermal pressure and collisions with neutrals. Due to the latter effect the radial velocity emerges (Equation (7)). Solving Equations (6) and (7), we will consider two cases in which there are magnetized, $\omega_{cj} v_{j,0\theta} \gg v_{j,0r} / r$, and unmagnetized, $\omega_{cj} v_{j,0\theta} \ll v_{j,0r} / r$, charged species. The condition $\omega_{cj} \gg v_{j,0\theta} / r$ denotes that the cyclotron frequency is much larger (smaller) than the orbital frequency (we use below, as usual, the term “magnetized” (“unmagnetized”) also in the case in which $\omega_{cj} \gg (\ll) \nu_{j,n}^0$).

For magnetized species we have the following solutions of Equations (6) and (7):

$$v_{j,0\theta} = -\frac{\omega_{cj}}{\omega_{cj}^2 + v_{j,0\theta}^2 + \nu_{jn}^0} \frac{F_{j,0r}}{m_j} + \frac{v_{j,0\theta}^2}{\omega_{cj}^2 + v_{j,0\theta}^2 + \nu_{jn}^0} v_{n,0\theta} - \omega_{cj} v_{j,0\theta} v_{n,0\theta},$$

$$v_{j,0r} = \frac{F_{j,0r}}{\omega_{cj}^2 + v_{j,0\theta}^2 + \nu_{jn}^0} m_j + \frac{\omega_{cj} v_{j,0\theta}}{\omega_{cj}^2 + v_{j,0\theta}^2 + \nu_{jn}^0} v_{n,0\theta} + \frac{v_{j,0\theta}^2}{\omega_{cj}^2 + v_{j,0\theta}^2 + \nu_{jn}^0} v_{j,0r}.$$
\[ v_{d0r} = -v_{n0}^* r \left(1 - \frac{1}{x}\right). \] (15)

where \( x = v_{d0r}/v_{n0} \) and the index * denotes the modified collisional frequencies \( v_{n1,j}^* = v_{n1,j}/(1 - \alpha_1) \) and \( v_{n2}^* = v_{n2}/(1 - \alpha_2) \). The parameter \( \alpha_j \) in Equation (14) is
\[ \alpha_j = 1 + b_j = 1 + \frac{r}{v_{n0}^* v_{j0} \omega_{cj}^* + v_{jn}^*}. \] (16)

After substitution of Equations (14) and (15) into Equations (10) and (12), we obtain two equations containing only the azimuthal velocities of neutrals and dust grains (except for the terms proportional to \( \alpha_2 \)):
\[ -\frac{\nu_{n0}^2}{r} = \alpha_2 \frac{\nu_{n0}^2}{r} + F_{dn} - \frac{\nu_{nj}}{\omega_{cj}^* + v_{jn}^*} \times \left[ \frac{v_{jn} + \left(\frac{v_{n1,j}^2}{\omega_{cj}^* + v_{jn}^*} + v_{n2}^*\right)}{\omega_{cj}^* + v_{jn}^*} \right] \left(\frac{F_{f0r}}{m_j} + \omega_{cj} v_{n0}\right) + \nu_{nd} r (1 - x) \left[\frac{v_{n1,j}^2}{x} + \frac{1}{a_j} \left(\frac{v_{n1,j}^2 \omega_{cj}^*}{\omega_{cj}^* + v_{jn}^*} + v_{n2}^*\right)\right]. \] (17)

For simplicity, we have retained in these equations the radial velocities in terms proportional to \( \alpha_2 \). Multiplying Equation (17) by \( x^2 \) and equating it to Equation (18), we obtain the following equation:
\[ x^2 F_{n0r} = \frac{F_{n0r}}{m_n} + \frac{\nu_{nj}}{\omega_{cj}^* + v_{jn}^*} \left(\frac{F_{f0r}}{m_j} + \omega_{cj} v_{n0}\right) \times \left[x^2 v_{jn} + \left(x^2 \frac{v_{n1,j}^2}{\omega_{cj}^* + v_{jn}^*} + x^2 v_{n2}^*\right)\right] \left(\frac{F_{f0r}}{m_j} + \omega_{cj} v_{n0}\right) + r (1 - x) \left[\frac{v_{n1,j}^2}{x} + \frac{\nu_{nd} v_{nd}^*}{a_j} + \frac{v_{n2}^*}{x} + x^2 \frac{v_{n2}^*}{a_j}\right] \times \left(\frac{v_{n1,j}^2}{a_j} + v_{n2}^*\right) \times \alpha_2 \frac{v_{n0}^*}{r} + x^2 \alpha_2 \frac{v_{n0}^*}{r} = 0. \] (19)

In the paper (Nekrasov 2008), we have considered the case of the weak collisional coupling of neutrals with ions in the steady state, which is appropriate for weakly ionized disks. The parameter \( b_j \) (see Equation (16)) for ions has been supposed to be much less than unity, in which case \( a_i \simeq 1 \). This condition can be satisfied for magnetized, \( \omega_{cj} \gg v_{ni} \), or unmagnetized, \( \omega_{cj} \ll v_{ni} \), ions, if the orbital (Keplerian) frequency of neutrals \( v_{n0}/r \) is less than the collisional frequency of neutrals with ions \( v_{ni} \). In the case \( \omega_{cj} \sim v_{ni} \), it should be \( v_{ni} \ll v_{n0}/r \). In the present paper, we will consider the opposite case in which \( b_j \gg 1 \), i.e.,
\[ \frac{v_{n1,j} \omega_{cj}^*}{\omega_{cj}^* + v_{jn}^*} \gg \frac{v_{n0}}{r}. \] (20)

Thus, we explore here the regions of the disk, where the medium is sufficiently ionized and where a strong collisional coupling between neutrals and species \( j \) (ions or light dust grains) takes place.

We can find the analytical solution of Equation (19) in the case in which the azimuthal velocities of neutrals and dust grains are close to one another, i.e.,
\[ x = 1 + \delta, \delta \ll 1. \]

This condition is easily satisfied in disks, where the collisional coupling of dust grains and neutrals is strong (see below). We neglect further the contribution of terms proportional to \( \delta \) in the first and third terms on the left-hand side of Equation (19) as well as the contribution of two last terms in this equation. The corresponding conditions, taking into account the inequality in Equation (20), have the form
\[ v^2 \gg \frac{2}{r} \left[\frac{F_{n0r}}{m_n} + \frac{v_{nj} + v_{nd}}{v_{jn}} \left(\frac{F_{f0r}}{m_j} + \omega_{cj} v_{n0}\right)\right]; 2 \alpha_2 \frac{v_{n0}^*}{r^2} . \]
\[ \delta v^2 \gg \frac{1}{r^2} \left(v_{n0}^* - v_{d0r}^2\right), \] (21)

where \( v^2 \simeq v_{nd}^2 \left(v_{nj} + v_{nd}\right) \). We suppose that \( b_j v_{nj} \gg v_{nd} \). This condition is, obviously, satisfied. Conditions in Equation (21) imply strong dust-neutrals collisional coupling. Then we obtain from Equation (19) the following solution for \( \delta \):
\[ \delta v^2 = \frac{F_{n0r}}{m_n} - \frac{F_{n0r}}{m_d} + \frac{1}{v_{jn}} \left(v_{nj} + v_{nd} + v_{dn}\right) \left(\frac{F_{f0r}}{m_j} + \omega_{cj} v_{n0}\right). \] (22)

The second equation connecting \( \delta \) and \( v_{n0} \) is obtained from Equation (18) under conditions in Equation (20) and
\[ \delta v_{dn} v_{d0r}^2 \gg \frac{v_{d0r}^2}{r^2} + \alpha_2 \frac{v_{n0}^2}{r^2}. \] (23)

This inequality also implies strong collisional coupling of dust grains with neutrals. Then we have
\[ \delta r v_{dn} v_{d0r} = -\frac{F_{n0r}}{m_a} + \frac{v_{dn}}{v_{nj}} \left(\frac{F_{f0r}}{m_j} + \omega_{cj} v_{n0}\right). \] (24)

We can find from Equations (22) and (24) the expressions for \( v_{n0} \) and \( \delta \). The velocity \( v_{n0} \) is equal to
\[ -\omega_{cj} v_{n0} = \frac{F_{f0r}}{m_j} + \frac{v_{dn}}{v_{nj}} \left(\frac{F_{n0r}}{m_n} + v_{nd} \frac{F_{d0r}}{m_d}\right). \] (25)

It can be shown that the condition
\[ b_j \gg \frac{\omega_{cj}^2}{\omega_{cj}^* + v_{jn}^*} \] must be satisfied for the solution in Equation (25) to have the present form. It is obvious that this inequality can easily be realized, if \( b_j v_{dn} \gg v_{nj,d} \). The value \( \delta \), we find from Equations (24) and (25), is
\[ \delta r v_{dn}^* = -\frac{1}{v_{nj}} \left(\frac{F_{n0r}}{m_n} + \frac{v_{nj} + v_{nd}}{v_{dn}} \frac{F_{d0r}}{m_d}\right). \] (26)

The azimuthal velocity of dust grains is equal to
\[ v_{d0r} = v_{n0}(1 + \delta). \] (27)
The radial velocities of neutrals and dust grains are found from Equations (12) (or Equation (14) and (15):

\[ v_{e0r} = - \frac{F_{e0r}}{m_ev_{e0r}} - \frac{\delta_P v_{en}^*}{\omega_{c_e}}, \]
\[ v_{0r} = - \frac{\delta_P v_{en}^*}{\omega_{c_e}}. \]  

(28)

Let us now consider the velocities \( v_{j00}, r \). Substituting the velocities \( v_{j00}, r \) found above into Equations (8) and (9), we obtain \( v_{j00} \approx v_{j00} \) and \( v_{j0r} \approx v_{j0} \). We take our consideration the drift and internal gravity waves. We take into account the induced reaction of neutrals on the perturbed motion of charged species. This effect is important, if the collisional frequencies due to movement of particles along the background magnetic field we have \( \gamma_{ij} = 1 \). The neutrals are also considered as adiabatic. Then \( \nabla P_{n1}/m_n n_{n0} = v_{j1}^2 \nabla n_{n1}/n_{n0} \), where \( v_{j1}^2 = \gamma_n T_{n0}/m_n \), \( T_{n0} \) is the unperturbed temperature of neutrals.

The term \( R_{j1} \),

\[ R_{j1} = v_{j1}^{0} \left[ n_{j1} + \frac{n_{j1}}{n_{n0}} (v_{nj} - v_{j0}) \right], \]

describes the back reaction of charged components due to influence of the perturbed motion of neutrals and the term \( R_{n1} \),

\[ R_{n1} = \sum_j v_{nj}^{0} \left[ v_{j1} + \frac{n_{j1}}{n_{j0}} (v_{nj} - v_{n0}) \right], \]

leads to the induced perturbation of the velocity of neutrals. The collisional terms proportional to \( n_{n1} \) and \( n_{j1} \) take into account the dependence of the collisional frequencies \( v_{j1} \) and \( v_{nj} \) on the density perturbations of species.

In this section, we will consider the general case in which the wave vector \( k = (k_r, k_\theta, k_z) \) of perturbations has three components. We provide the normal mode analysis, assume the perturbations to be proportional to \( \exp(i(k_r r + \omega t + ik_z z - i\Theta)) \), and use the local approximation \( |k|r \gg 1 \). Then we find the Fourier amplitudes of the density perturbations from the linearized version of the continuity equation (3)

\[ \frac{n_{j,n1k}}{n_{j,n0}} = \frac{k v_{j,n1k}}{\omega_{j,n}}, \]

where \( \omega_{j,n} = \omega - k v_{j,n0} \), \( k = (k_r, \omega) \). Substituting \( n_{n1} \) into Equation (31) and solving this equation in the Fourier representation, we will find the velocity \( v_{nj1} \), whose components and the expression for \( n_{nj1} \) are given in the Appendix A.

In the protostellar and protoplanetary disks, the collisions of neutrals with charged species are frequent enough that the condition \( \omega_{j,n} \gg \Omega_{n} \) is satisfied (for systems with the weak collisions it should be supposed that \( \omega_{j,n} \gg \Omega_{n} \)). Then, using the equation for \( n_{nj1} \) from the Appendix A and the expression for \( R_{nj1} \), we obtain

\[ D_n \left[ \frac{n_{nj1}}{n_{nj0}} \right] = i\omega_{nj} \sum_j v_{nj}^{0} \left[ \frac{n_{j1}}{n_{j0}} \right]. \]  

(32)

In the present paper, we will further neglect the thermal effects of species when considering some specific cases. For neutrals, the necessary condition for this cold limit has the form (1) \( \delta_{lmm} \ll 1 \), where \( l, m = r, \theta, z \). In this case, the perturbed velocity and density of neutrals given in the Appendix A and Equation (32), correspondingly, will be equal to

\[ v_{nj1} = \frac{i}{\omega_{nj}} R_{nj1}, \]

(33)

and

\[ \frac{n_{nj1}}{n_{nj0}} = \frac{i}{\omega_{nj}} \sum_j v_{nj}^{0} \left[ \frac{n_{j1}}{n_{j0}} \right]. \]  

(34)

Let us substitute Equations (33) and (34) into \( R_{j1k} \). Then the collisional term \( C_{j1k} \) can be written in the form

\[ C_{j1k} = -v_{j1}^{0} v_{j1} + Q_{j1k}, \]

(35)
The general solutions for the perturbed electron velocity in the case \( k_z \neq 0, k_r = k_0 = 0 \) are given in the Appendix E. One can see that the collisions of neutrals with ions and dust grains influence the collisions of electrons with neutrals.

6. SOLUTIONS FOR \( v_{j,k} \) IN THE CASE OF NEGLECT THE OWN DYNAMICS OF DUST GRAINS

The own frequencies of dust grains, the plasma and cyclotron frequencies, are very low because of their large mass. Therefore, at the absence of collisions of species with dust grains the latter can stay immobile in the electromagnetic perturbations having sufficiently high frequency. However, for example, through collisions with neutrals, the dust grains can be involved in the fast electromagnetic perturbations induced by the ions and electrons when the ions have also strong collisional coupling with neutrals.

Let us neglect in Equation (D3) the terms proportional to \( \omega_{d1} \) and \( \omega_{d2} \) under conditions

\[
\omega_{vd}^2 \gg \left( 1 + \frac{\alpha^2}{\alpha^2_{ci}} \right) \omega_{d1}\omega_{d2}; \quad \mu_i \frac{\alpha_{d1,2}}{\alpha_{ci}}.
\]  

Note, that for frequencies of perturbations much less than the collisional frequencies, \( \omega_{d1} \gg \omega_{d2} \), these conditions can be written in the form \( \omega_{vd} \gg (1 + v_i/\omega_{e1})\alpha_{d1,2} \). We have taken into account that \( \omega_{ci} \gg \Omega \). Then the value \( A \) will be equal to \( A = -\Delta \), where \( \Delta = \omega_{vd} + \omega_{e1} \). In the case in Equation (37), one can also neglect the action of the electromagnetic forces on the dust grains. Then the Equations (D1) and (D2) take the form

\[
v_{1r} = \frac{\omega_{vd}}{\Delta} (\omega_{vd} \omega_{ci} G_{r1} + \lambda_{ae} G_{r1}),
\]

\[
v_{10} = \frac{\omega_{vd}}{\Delta} (-\omega_{vd} \omega_{ci} G_{r1} + \lambda_{ae} G_{r1}).
\]  

For the dust grains we have

\[
v_{d1r} = -\frac{\mu_i}{\omega_{vd}} v_{1r} - \frac{\mu_i}{\omega_{vd}} k_z w_r G_{11z},
\]

\[
v_{d10} = -\frac{\mu_i}{\omega_{vd}} v_{10} - \frac{\mu_i}{\omega_{vd}} k_z w_0 G_{11z}.
\]  

The vertical velocities \( v_{11z} \) and \( v_{d1z} \) are equal in the case under consideration,

\[
\lambda_{ae} v_{11z} = \omega_{vd} G_{11z},
\]

\[
\lambda_{ae} v_{d1z} = -\mu_i G_{11z}.
\]  

We can find now the perturbed electron velocity. Substituting the velocities \( v_{d1} \) defined by Equations (38) and (39) in the expressions for the components of the electron velocity given in the Appendix E, we obtain

\[
D_r v_{e1r} = -H_{e1r} + \frac{\eta_0}{\Delta} (\lambda_{ae} \omega_{ve} - i \omega_{vd} \omega_{ve} \omega_{ci}) G_{11r} + \frac{\eta_0}{\omega_{ae}} e_{er} k_z G_{11z},
\]

\[
D_r v_{e10} = -H_{e10} - \frac{\eta_0}{\Delta} (i \lambda_{ae} \omega_{ve} + \omega_{vd} \omega_{ve} \omega_{ci}) G_{11r} + \frac{\eta_0}{\omega_{ae}} e_{e0} k_z G_{11z},
\]

\[
v_{e1z} = \frac{i}{\omega_{ve}} G_{e1z} + \frac{\eta_0}{\omega_{ve} \lambda_{ae}} G_{11z}.
\]  

5. PERTURBED VELOCITIES OF SPECIES IN THE CASE \( k_z \neq 0, k_r = k_0 = 0 \): GENERAL SOLUTIONS

We have obtained the exact solutions of the system of equations for \( v_{1r,0} \) and \( v_{d1r,0} \) given in the Appendix C for the horizontally elongated perturbations when \( k_z \neq 0, k_r = k_0 = 0 \). These solutions and the general solutions for \( v_{11d} \) are given in the Appendix D.

If we set \( \mu_i = \mu_d = 0 \) in Equations (C1) and (C2), i.e., if we neglect the collisions of neutrals with ions and dust grains (\( \omega_n \gg \nu_n^0 \)), then we obtain \( D_{i,r} v_{i,r} = -H_{i,r} \), \( D_{i,0} v_{i,0} = -H_{i,0} \), and \( v_{i,1d} = (i/\omega_{vd}) G_{i,1d} \). In this case, the ions and dust grains move independently from each other under the action of the electromagnetic forces. However, in the case \( \omega_n \ll \nu_n^0 \) the movement of dust grains depends on the movement of ions, and vice versa (see below).
where
\[ \eta_0 = \rho_i \eta_{ei} - \omega_{ei} \eta_{de}, \]
\[ \varepsilon_{rr} = \omega_{ee} (v_{i0r} - v_{e0r}) + i \omega_{ee} (v_{i0e} - v_{e0e}), \]
\[ \varepsilon_{e0} = \omega_{ee} (v_{i00} - v_{e00}) - i \omega_{ee} (v_{i0e} - v_{e0e}). \]

7. PERTURBED ELECTRIC CURRENT: GENERAL EXPRESSIONS
In this section, we find the perturbed electric current
\[ j_1 = \sum_j q_j n_j \left( v_{j1} + \frac{k_i v_{j0}}{\omega_i} v_{j1z} \right). \]
Using Equations (38)-(41), we can write the components of this current in the form
\[ j_{1r} = \left( \frac{q_i}{m_i} W_{rr} + W_{zrr} \right) E_{1r} \]
\[ + \left( \frac{q_i}{m_i} W_{r0} + W_{zr0} \right) E_{10} + W_{zr z} E_{1z}, \]
\[ j_{10} = \left( \frac{q_i}{m_i} W_{0r} + W_{0zr} \right) E_{1r} \]
\[ + \left( \frac{q_i}{m_i} W_{00} + W_{z00} \right) E_{10} + W_{z0 z} E_{1z}, \]
\[ j_{1z} = W_{zrr} E_{1r} + W_{zr0} E_{10} + W_{z0 z} E_{1z}. \]
The expressions for \( W_{rr,0,0}, W_{rr,0,z}, W_{zrr,0,z}, \) and \( W_{zrr,0,z} \) in the general form are given in the Appendix F.

8. DISPERSION RELATION IN THE GENERAL FORM
From Maxwell’s equations (4) and (5) we have,
\[ n_i^2 E_{1r,0} = \frac{4\pi i}{\omega} j_{1r,0}, j_{1z} = 0. \]
Using the components of the perturbed electric current given in Section 7, we will find from the system in Equation (42) the dispersion relation
\[ (n_i^2 \varepsilon_{zz} - \varepsilon_{rr} \varepsilon_{zz} + \varepsilon_{rr} \varepsilon_{rr}) \left( n_i^2 \varepsilon_{zz} - \varepsilon_{00} \varepsilon_{zz} + \varepsilon_{zz} \varepsilon_{zz} \right) \]
\[ - (\varepsilon_{00} \varepsilon_{zz} + \varepsilon_{zz} \varepsilon_{rr}) (\varepsilon_{00} \varepsilon_{zz} + \varepsilon_{zz} \varepsilon_{zz}) = 0. \]
Here,
\[ \varepsilon_{rr} = \frac{4\pi i}{\omega} \left( \frac{q_i}{m_i} W_{rr} + W_{zrr} \right), \]
\[ \varepsilon_{r0} = \frac{4\pi i}{\omega} \left( \frac{q_i}{m_i} W_{r0} + W_{zr0} \right), \]
\[ \varepsilon_{0r} = \frac{4\pi i}{\omega} \left( \frac{q_i}{m_i} W_{0r} + W_{0zr} \right), \]
\[ \varepsilon_{00} = \frac{4\pi i}{\omega} \left( \frac{q_i}{m_i} W_{00} + W_{z00} \right), \]
\[ \varepsilon_{zz} = \frac{4\pi i}{\omega} W_{zrr}, \varepsilon_{zz} = \frac{4\pi i}{\omega} W_{zr0}, \varepsilon_{zz} = \frac{4\pi i}{\omega} W_{z0 z}. \]

9. DISPERSION RELATION IN THE CASE OF THE STRONG COLLISIONAL COUPLING OF SPECIES
We will further consider the case of the strong collisional coupling of neutrals with ions and dust grains and dust grains with neutrals. We also suppose that the collisional frequencies of electrons and ions with neutrals are much larger than the perturbation frequency. Thus, the case under consideration is the following:
\[ \nu_{en} \gg \omega_e, \nu_i \gg \omega_i, \nu_{d} \gg \omega_d, \min \left\{ \nu_{mi}, \nu_{nd} \right\} \gg \omega_n, \]
where
\[ \nu_i = \frac{\nu_{in}^0 \nu_{in}^0}{\nu_n^0}, \nu_d = \frac{\nu_{nd}^0 \nu_{nd}^0}{\nu_n^0}. \]
The collisional frequencies \( \nu_j \) (Equation (36)) are
\[ \nu_e = \nu_{en}^0, \nu_i = \nu_{0i}^0 - i \omega_n \frac{\nu_{in}^0 \nu_{in}^0}{\nu_n^0}, \nu_d = \nu_{0d}^0 - i \omega_n \frac{\nu_{nd}^0 \nu_{nd}^0}{\nu_n^0}. \]
Here we keep the corrections proportional to \( \omega_n \) because the main terms are cancelled when calculating some necessary expressions in the case of Equation (44). For example, the value \( \lambda_\omega \) has in this case the form
\[ \lambda_\omega = \alpha \frac{\nu_0^0 + \nu_i^0 + \nu_d^0}{\omega_n}. \]
where \( \nu_0^0 = \nu_{in}^0 \nu_{in}^0 / \nu_n^0 \).

Let us find \( W_{rr,0} \) and \( W_{0r,0} \) under the conditions in Equation (44). We will consider, as in the steady state, that the collisional frequencies \( \omega_{ce}^0 \gg \omega_n^0 \). This condition is generally satisfied. Using the condition of quasineutrality \( \sum_j q_j n_j = 0 \) and carrying out the calculations, we obtain the following expressions for \( W_{rr,0} \) and \( W_{0r,0} \):
\[ W_{rr} = \frac{\nu_{in}^0}{\nu_n^0} \nu_{in} \frac{q_i \nu_{i0r}}{q_i \nu_{i0r}^0} g, \]
\[ W_{0r} = \frac{\nu_{in}^0}{\nu_n^0} \nu_{in} \frac{q_i \nu_{i0r}}{q_i \nu_{i0r}^0} f. \]
The values \( g \) and \( f \) are equal to
\[ g = \nu_{en}^0 \nu_{in}^0 \left( \frac{1}{\nu_n^0} + \frac{1}{\nu_{in}^0} \right), \]
\[ f = \frac{1}{\nu_n^0} + \frac{1}{\nu_{in}^0}. \]

The values \( s_{r,0,zz} \) under conditions in Equation (44) and \( \omega_{ce}^0 \gg \omega_n \) are
\[ s_{zz} = i \frac{\nu_{en}^0}{\nu_{en}^0} \frac{\nu_{en}^0}{\nu_{en}^0} \left( \nu_{e00} - \nu_{e00} \right) - \nu_{en}^0 \nu_{en}^0 \frac{\nu_{en}^0}{\nu_{en}^0} f \nu_{e00}, \]
\[ s_{rzz} = \nu_{en}^0 \nu_{en}^0 \frac{\nu_{en}^0}{\nu_{en}^0} \nu_{en}^0 \frac{\nu_{en}^0}{\nu_{en}^0} f \nu_{e00}, \]
\[ \nu_{in}^0 \nu_{in}^0 \left( 1 + g \right) \left( \nu_{en}^0 \nu_{en}^0 \right) \frac{\nu_{en}^0}{\nu_{en}^0} \frac{\nu_{en}^0}{\nu_{en}^0} f \nu_{e00}. \]
Using the expressions for \( \varepsilon_{ij}, i, j = r, \theta, z \), given in Section 8 and taking into account Equations (45), (46), and the expressions for \( W_{rr,0,zz}, W_{zrr,0,z}, \) and \( W_{zrr,0,z} \) given in the Appendix F, we find the following relations containing in the dispersion...
relation in Equation (43):

\[ \epsilon_{rr} \epsilon_{zz} - \epsilon_{zz} \epsilon_{rr} = \tau \beta \sigma (ib_{zz} - 1) + \frac{\beta \sigma}{c^2} (b_{zz} v_{0e} - b_{rr}) (v_{0e} - v_{0e}) , \]

\[- \epsilon_{rr} \epsilon_{zz} + \epsilon_{zz} \epsilon_{rr} = -\tau \beta \sigma (ib_{zz} - 1) \times (ib_{zz} - 1) + \frac{\beta \sigma}{c^2} (b_{zz} - b_{zz} v_{0e}) (v_{0e} - v_{0e}) , \]

\[ \epsilon_{00} \epsilon_{zz} - \epsilon_{zz} \epsilon_{00} = \tau \beta \sigma (ib_{zz} - 1) + \frac{\beta \sigma}{c^2} (b_{zz} v_{00} - b_{rr}) (v_{00} - v_{0e}) , \]

\[- \epsilon_{rr} \epsilon_{zz} + \epsilon_{zz} \epsilon_{rr} = \tau \beta \sigma (ib_{zz} - 1) + \frac{\beta \sigma}{c^2} (b_{zz} - b_{zz} v_{0e}) (v_{0e} - v_{0e}) , \]

\[ + \frac{\beta \sigma}{c^2} (b_{rr} - b_{zz} v_{0e}) (v_{0e} - v_{0e}) . \]  \( (47) \)

where

\[ \tau = \frac{\omega_{pe}^2 \omega_{ci}}{\omega_{ce}^2}, \]

\[ \beta = \frac{\omega_{pe}^2}{\omega_{ce} \omega_{ci}}. \]

Substituting the expressions in Equation (47) and \( \epsilon_{zz} \) in Equation (43), we obtain the following dispersion relation:

\[ \omega^6 - \frac{\omega_{pe}^4 \omega_{ci}^2}{\omega_{ce}^5 \omega_{ci}} (\sigma_1^2 + \epsilon_{d1}^2) + 2 \omega_{pe}^2 \omega_{ce} - \sigma_1 k_z^2 c^2 \]

\[ + \omega^2 \left( k_z c \epsilon - \frac{\omega_{pe}^2}{\omega_{ce}^2} k_z^2 w_0^2 \right) \]

\[- \frac{\omega_{pe}^2 \omega_{ci}}{\omega_{ce}} \frac{1}{\sigma_1 - \epsilon_{d1} \frac{\omega_{ce}}{\omega_{ci}}} k_z^2 c^2 w_0^2 \]

\[ + i \omega^2 \omega_{pe} \omega_{ci} \sigma_1 \epsilon_{d1} k_z^2 w_0^2 \]

\[ + i \omega \omega_{pe} \epsilon_{d1} \sigma_1 \frac{\omega_{ce}}{\sigma_1 - \epsilon_{d1} \frac{\omega_{ce}}{\omega_{ci}}} k_z^2 c^2 w_0^2 = 0, \]

\( (48) \)

where \( \sigma_1 = \frac{\beta \sigma}{\omega_{ce}} - \frac{\omega_{ce}}{\omega_{ci}} g, \)

\[ \epsilon_{d1} = \frac{q_d n_d \omega_{ci}}{q_e n_e}, \]

\[ w_0^2 = (v_{0e} - v_{0e})^2 + (v_{00} - v_{0e})^2. \]

In Equation (48) the relationship \( ib_{zz} - 1 = -\frac{\omega_{ce}}{\omega_{ci}} (\sigma_1 - \epsilon_{d1} \frac{\omega_{ce}}{\omega_{ci}}) \) has been used. Below, we will find the unstable solutions of Equation (48), the growth rate of which is proportional to \( w_0. \)

10. SOLUTIONS OF THE DISPERSION RELATION

To solve the dispersion relation in Equation (48), we will consider the cases in which the unstable perturbations have the long and short wavelengths.

10.1. Long Wavelength Instabilities

We neglect the terms proportional to \( k_z^2 \) in Equation (48) under condition

\[ \omega^2 \frac{\omega_{pe}^2}{\omega_{ce} \omega_{ci}} \sigma_1 \gg k_z^2 c^2. \]

Then the dispersion relation takes the form

\[ \omega^4 (\sigma_1^2 + \epsilon_{d1}^2) - \omega_{ce}^2 w_0^2 + i \omega \omega_{ci} \frac{\sigma_1 \epsilon_{d1}}{\sigma_1 - \epsilon_{d1} \frac{\omega_{ce}}{\omega_{ci}}} k_z^2 w_0^2 = 0. \]

For frequencies in the range

\[ \omega_{ci} \gg \omega \frac{\sigma_1 \epsilon_{d1}}{\sigma_1 - \epsilon_{d1} \frac{\omega_{ce}}{\omega_{ci}}}, \]

the unstable solution of Equation (50) is the following:

\[ \omega = \frac{i (\omega_{ci} k_z w_0)^{1/2}}{\left( \sigma_1^2 + \epsilon_{d1}^2 \right)^{1/4}}. \]

In the case opposite to that in Equation (51) the instability is caused by the presence of dust grains:

\[ \omega = \left[ \frac{\sigma_1 \epsilon_{d1}}{\sigma_1 - \epsilon_{d1} \frac{\omega_{ce}}{\omega_{ci}}} \frac{k_z^2 w_0^2}{\left( \sigma_1^2 + \epsilon_{d1}^2 \right)^{1/4}} \right]^{1/3} \exp \left( -i \frac{\pi}{6} + i \frac{2p\pi}{3} \right), \]

where \( p = 0, 1, 2. \) Note, that the sign of \( \epsilon_{d1} \) depends on the sign of \( q_d. \)

10.2. Short Wavelength Instabilities

Now consider the case in which

\[ \omega^2 \frac{\omega_{pe}^2}{\omega_{ce} \omega_{ci}} \sigma_1^2 \ll k_z^2 c^2. \]

Then Equation (48) will be the following:

\[ \omega^2 - \frac{\omega_{pe}^2 \omega_{ci}}{\omega_{ce}^2 \omega_{ci}} \frac{1}{\sigma_1 - \epsilon_{d1} \frac{\omega_{ce}}{\omega_{ci}}} w_0^2 + i \omega \omega_{pe} \frac{\epsilon_{d1}}{\omega_{ce}} \frac{w_0^2}{\sigma_1 - \epsilon_{d1} \frac{\omega_{ce}}{\omega_{ci}}} \]

\[ \sigma_1 k_z^2 w_0^2 = 0. \]

For frequencies in the range

\[ \omega_{ci} \ll \omega \epsilon_{d1} \]

we find the following solution of Equation (55):

\[ \omega^2 = \frac{\omega_{pe}^2 \omega_{ci}}{\omega_{ce}^2} \frac{1}{\sigma_1 - \epsilon_{d1} \frac{\omega_{ce}}{\omega_{ci}}} \frac{w_0^2}{c^2}. \]

\( (57) \)

We see that when \( \sigma_1 > \epsilon_{d1} (v_{ce}^2 / \omega_{ce}) \) or \( \epsilon_{d1} > 0 (q_d < 0) \) there is the streaming instability.

In the case opposite to that in Equation (56),

\[ \omega_{ci} \ll \omega \epsilon_{d1}, \]

we obtain

\[ \omega = -i \omega_{pe} \frac{\epsilon_{d1}}{\omega_{ce}} \frac{w_0^2}{\sigma_1 - \epsilon_{d1} \frac{\omega_{ce}}{\omega_{ci}}} c^2. \]

\( (59) \)

This solution describes the instability when \( \epsilon_{d1} > 0 \) or \( \epsilon_{d1} (v_{ce}^2 / \omega_{ce}) > \sigma_1. \)
11. DISCUSSION

In the equilibrium and perturbation states we have not taken into account the collisions between ions and dust grains, supposing that \( v_{in} \gg v_{id} \) and \( v_{dn} \gg v_{di} \). These inequalities can be written in the form \( (\sigma v)_{in} \gg (m_i \rho_i / m_d \rho_n)(\sigma v)_{id} \) and \( (\sigma v)_{dn} \gg (\rho_i/\rho_n)(\sigma v)_{id} \), where \( \rho_i, \rho_n \) are the mass densities of ions and neutrals, respectively. Using Equation (60), we can write the condition where \( \Omega \) and \( \omega_{ci} \) are the Keplerian velocity and frequency, correspondingly. Using Equation (60), we can write the condition of the strong collisional coupling of neutrals with ions from Equation (20) in the form

\[
\frac{\omega_{ci}^2}{\omega_{ci}^2 + v_{in}^2} \gg \Omega^2_k.
\]  

We see from Equations (60) and (61) that \( v_{in}^0 \ll v_{K} \). As long as \( v_{in}^0 \ll v_{dn} \) (see Equation (27)), and \( v_{dn}^0 \ll v_{in}^0 \) (see Equation (28)), the species rotate in the magnetized regions of the disk with the velocity much smaller than the Keplerian velocity. This result agrees with observations (Donati et al. 2005). From Equations (26) and (28) we obtain the radial velocities of neutrals and dust grains under conditions at hand,

\[
v_{in}^0 \approx v_{in}^0 \approx -\frac{\Omega K}{\nu_{ni}} v_{K}.
\]  

Comparing Equations (60) and (62), we have \( v_{in}^0 \approx -v_{in}^0 \). Thus, for the magnetized (unmagnetized) ions, \( \omega_{ci} \gg (\ll) v_{in} \), the radial velocity of neutrals and dust grains is smaller (larger) than their azimuthal velocity. We see from Equation (62) that the radial velocity of neutrals and dust grains is directed inward of the disk.

It is followed from Equation (29) that the azimuthal electron velocity \( v_{e0} \ll v_{in}^0 \) and the radial electron velocity \( v_{e0} \approx (v_{en} / \omega_{ce}) v_{in}^0 \). The conditions in Equations (21) and (23) of the strong collisional coupling of dust grains with neutrals can be written in the form

\[
v_{dn} \gg \Omega K \left[ 1 + \frac{\Omega K}{\nu_{ni}} \left( 1 + \frac{v_{in}^2}{\omega_{ci}^2} \right) \right].
\]  

Let us consider some specific parameters for the protoplanetary disk with the solar mass central star at \( r = 1 \) AU, where \( B_0 \approx 0.1 \) G and \( \Omega K \approx 2 \times 10^{-7} \) s\(^{-1}\) (e.g., Desch 2004). We take \( m_i = 30 m_p \) and \( m_n = 2.3 m_p \) as the proton mass. Then the ion and electron cyclotron frequencies will be equal to \( \omega_{ci} \approx 32 \) s\(^{-1}\) (\( q_i = -q_e \)) and \( \omega_{ke} \approx 1.76 \times 10^6 \) s\(^{-1}\) (the sign \( \mid \) denotes an absolute value). The rate coefficients for momentum transfer by elastic scattering of ions and electrons with neutrals are \( (\sigma v)_{in} = 1.9 \times 10^{-9} \) cm\(^3\) s\(^{-1}\) and \( (\sigma v)_{en} = 4.5 \times 10^{-9} \) (T/30 K)\(^{1/2}\) cm\(^3\) s\(^{-1}\) (Draine et al. 1983). We take \( T = 300 \) K. Then the condition for neglecting collisions of neutrals with electrons has the form \( \omega_{ci}^2 / (\omega_{ke}^2 + v_{en}^2) \gg 1.89 \times 10^{-3} \) (Desch and Ciarcelluti 2004). Then the value \( v_{in} \) must satisfy the condition \( v_{in} < 7.36 \times 10^{-3} \) s\(^{-1}\). This inequality is satisfied for the neutral mass density \( \rho_n \approx 2.1 \times 10^{-11} \) g cm\(^{-3}\) or \( n_{en} < 5.38 \times 10^{13} \) cm\(^{-3}\). We will take the ionization degree as \( n_{en} = 10^{-3} \times n_{en} \) (Desch 2004). If we set \( \rho_n = 10^{-10} \) g cm\(^{-3}\), then we obtain \( v_{in} = 3.5 \times 10^{-3} \) s\(^{-1}\), \( v_{ni} = 4.5 \times 10^{-3} \) s\(^{-1}\), \( v_{en} = 3.65 \times 10^{-3} \) s\(^{-1}\), \( v_{ne} = 8.53 \times 10^{-3} \) s\(^{-1}\).

The azimuthal velocity of species using the parameters given above is equal to \( v_{in}^0 / \nu_{in} \approx 0.49 \nu_{K} \), where \( \nu_{K} \approx 30 \) km s\(^{-1}\) (see Equation (60)). The orbiting frequency of dust grains \( \Omega = 0.49 \nu_{K} \approx 0.98 \times 10^{-8} \) s\(^{-1}\). The dust grains will be unmagnetized, \( v_{id} \approx v_{d} \), if their mass \( m_d \approx 8 \times 10^{-15} \) g \( \approx 4.8 \times 10^{5} m_p \) for \( d_d = \pm q_e \). The dust grains, for example, with density of the material grains are made \( \sigma_d \approx 3 \) g cm\(^{-3}\) and with radius \( r_d > 8.6 \times 10^{-2} \) mm satisfy the condition of unmagnetization. At \( r_d = 0.1 \) mm, the mass of the grain is equal to \( m_d \approx 1.26 \times 10^{-14} \) g. The collisional frequency of dust grains with neutrals has the form \( \nu_{dn} \approx 6.7 \rho_n r_d v_{K} / m_d \) (e.g., Wardle & Ng 1999). Using parameters given above, we obtain \( v_{dn} \approx 0.53 \) s\(^{-1}\). For the solar abundance value \( \rho_d \approx 10^{-2} \) g cm\(^{-3}\), we have \( v_{dn} \approx 0.53 \times 10^{-2} \) s\(^{-1}\).

The radial velocity of neutrals and dust grains is equal to \( v_{dn} \approx 133 \) m s\(^{-1}\) (see Equation (62)). This velocity is directed inward. The conditions in Equations (61) and (63) are satisfied.

Above, we have considered only one set of disk parameters. Any other parameters can be treated also.

In the case in Equation (44) of the strong collisional coupling between ions, neutrals, and dust grains the perturbed velocity of dust grains is of the order of the perturbed velocity of ions, \( v_{pi} \approx v_{ni} \) (see Equations (39) and (40)). Thus, the dust grains can participate in the fast electromagnetic perturbations generated by the electrons and ions via collisions with neutrals and acquire the large velocities.

Let us now consider the obtained unstable solutions. For conditions used in this section the growth rate \( \gamma = \text{Im} \omega \) of the solution in Equation (52) can be estimated in the case \( \sigma_d \gg \varepsilon \text{d} \) as

\[
\gamma \approx (k, r)^{1/2} \Omega K.
\]

This growth rate is much larger than the Keplerian frequency as long as \( k, r \gg 1 \). The wave number \( k \) must satisfy the condition (see the inequality in Equation (49))

\[
k, r \ll \frac{q_e n_{en} \rho_n}{q_i n_{en} \rho_i} c_{Ai} \nu_{K}^2,
\]

where \( c_{Ai} = (\omega_{ci} / \omega_{pi}) c \) is the ion Alfvén velocity. For the parameters given above we have \( n_{en} = 2.56 \times 10^3 \) cm\(^{-3}\). In this case \( \omega_{pi} \approx 3.84 \times 10^4 \) s\(^{-1}\) and, accordingly, \( c_{Ai} \approx 250 \) km s\(^{-1}\). So far as \( \gamma \ll \nu_{ni} \), we have the second condition: \( (k, r)^{1/2} \ll \nu_{ni} / \Omega K \). It is interesting to note that the same growth rate also exists in the collisionless regime (Nekrasov 2007). The growth rate in Equation (53) is larger than the growth rate defined by Equation (64). This instability is possible when the density of dust grains is sufficiently large:

\[
\frac{v_{in}}{\omega_{ci}} > \left( \frac{v_{en}}{\omega_{ke}} \right)^{1/2} \frac{v_{ni}}{\Omega K} (k, r)^{-1/2}.
\]
The short wavelength perturbations considered in Section 10.2 have the following growth rates for conditions given above. An estimation for the growth rate in Equation (57) is
\[ \gamma \sim \left( \frac{|q_d| n_{e0} \rho_n}{q_i n_{i0} \rho_i} \right)^{1/2} \frac{v_K}{c_{AI}} \Omega_K. \] (65)
This growth rate is considerably larger than the Keplerian frequency in weakly ionized disks. For the parameters given above we have \( \gamma \sim 2.1 \times 10^{-5} \) s\(^{-1}\). The condition in Equation (54) at \( \sigma t \geq \varepsilon t \) is the following:
\[ k_\nu r \gg \frac{|q_d| n_{e0} \rho_n v_K}{q_i n_{i0} \rho_i c_{AI}}. \]
The growth rate in Equation (59) is larger than in Equation (65) and has the form (\( q_d < 0 \))
\[ \gamma \sim \frac{|q_d| n_{e0} \rho_n \Omega_k^2 v_K^2}{q_i n_{i0} \rho_i v_{ni} c_{AI}}. \]

For this instability the following condition must be satisfied (see the inequality in Equation (58)): \( q_d n_{e0}/q_i n_{i0} \gg v_{ni}/\gamma \gg 1 \).

Thus, in the collisional accretion disks there are possible the streaming instabilities involving the dynamics of neutrals and dust grains with growth rates much larger than the Keplerian frequency.

In the present paper, we did not take into account the ionization and recombination processes in the continuum and momentum equations which, in general, can be included (see, e.g., Pinto et al. 2008; Li et al. 2008). The main sources of ionization of accretion disks are Galactic cosmic rays and X-rays from the corona of the central star. These and other chemical processes can play an important physical role in the evolution of the disk. For example, the dead zones of the protostellar and protoplanetary disks may be enlivened due to turbulent transport of metallic ions, which are charged due to interaction with ionized gas in the surface layers, from the surface layers into these regions (Ilgner & Nelson 2008).

The ionization/recombination processes in the stationary continuity equations for species determine the ionization degree of medium. For the weakly ionized gases with large collisional frequencies, as it is the case for us, the contribution from the ionization source terms to the momentum equations can be ignored (e.g., Li et al. 2008). The chemical processes do not influence the dynamics of medium, for example, in the case of the ideal magnetohydrodynamics. In our case of multicomponent disks with strong collisional coupling of species, when the dynamics of each species is considered separately, the contribution of the ionization source term to the perturbed continuity equation can be neglected under condition \( \omega > \xi (n_{e0}/n_{i0}) \), where \( \xi \) is the ionization rate per H atom. The typical growth rates of the streaming instabilities considered in the present paper is \( \gamma \sim 10^{-5} \) s\(^{-1}\). If we take \( n_{e0}/n_{i0} \sim 10^9 \), then we can neglect the ionization for \( \xi < 10^{-14} \) s\(^{-1}\). Note, for example, that in the paper (Ilgner & Nelson 2008) the range of \( \xi \sim 10^{-15} - 10^{-19} \) s\(^{-1}\) is considered. For cosmic rays one obtains \( \xi \sim 3 \times 10^{-17} \) s\(^{-1}\) (e.g., Li et al. 2008). Thus, for not too high ionization rates and fast instabilities studied in the present paper, the ionization/recombination processes can be ignored.

12. CONCLUSION

In the present paper, the electromagnetic streaming instabilities of multicomponent collisional accretion disks have been studied. We have explored regions of the disk where the medium is sufficiently ionized and there is strong collisional coupling of neutrals with ions and dust grains simultaneously. We have included the effect of perturbation of collisional frequencies due to density perturbations of charged species and neutrals. This effect emerges when there are different background velocities of species, as occurs in accretion disks.

We have investigated in detail the steady state and found the azimuthal and radial velocities of species.

The general solutions for the perturbed velocities of species with collisional and thermal effects have been obtained. We have shown that dust grains can be involved in the fast electromagnetic perturbations induced by ions and electrons, through strong collisions with neutrals which have also strong collisional coupling with ions. The dust grains have been found to acquire the perturbed velocity of ions. This effect is important for their collisional coagulation and sticking.

We have derived the dispersion relation for the vertical perturbations and found the unstable solutions due to different background velocities of electrons and ions. It has been shown that the growth rates of these streaming instabilities can be much larger than the Keplerian frequency.

Electromagnetic streaming instabilities, with induced dynamics of neutrals can be a source of turbulence in sufficiently ionized regions of collisional accretion disks.

I thank the anonymous referee for his/her constructive comments.

APPENDIX A

SOLUTIONS FOR \( v_{n1k} \) AND \( n_{n1k} \)

The Fourier components \( v_{n1k} \) and \( n_{n1k} \) are the following (for simplicity, the index \( k \) is omitted here and below):

\[ D_n v_{n1r} = i R_{n1r} \omega_{n1} \left( 1 - \delta_{0\theta n} - \delta_{0\phi n} \right) \]

\[ + i R_{n1\theta} \left( \omega_{n1} \delta_{\theta n} + i/2 \Omega_n \left( 1 - \delta_{0\theta n} \right) \right) \]

\[ + i R_{n1\phi} \left( \omega_{n1} \delta_{\phi n} + i/2 \Omega_n \delta_{0\phi n} \right), \]

\[ D_n v_{n1\theta} = i R_{n1\theta} \left[ \omega_{n1} \delta_{\theta n} - i \frac{k_n^2}{2 \Omega_n} \left( 1 - \delta_{0\theta n} \right) \right] \]

\[ + i R_{n1\phi} \omega_{n1} \left( 1 - \delta_{\phi n} - \delta_{0\phi n} \right) \]

\[ + i R_{n1\phi} \left( \omega_{n1} \delta_{\phi n} - i \frac{k_n^2}{2 \Omega_n} \delta_{0\phi n} \right), \]

\[ D_n v_{n1\phi} = i R_{n1\phi} \left( \omega_{n1} \delta_{\phi n} - i \frac{k_n^2}{2 \Omega_n} \delta_{0\phi n} \right) \]

\[ + i R_{n1\theta} \left( \omega_{n1} \delta_{\theta n} + i/2 \Omega_n \delta_{0\theta n} \right) \]

\[ + i R_{n1\phi} \left[ \omega_{n1} \left( 1 - \delta_{0\phi n} \right) \right] - i \left( 2 \Omega_n - \frac{k_n^2}{2 \Omega_n} \right) \delta_{\theta n}, \]

\[ \frac{\omega_n}{\omega_{n1}} D_n n_{n1} = i R_{n1r} \left( k_r - i \frac{k_n^2}{2 \Omega_n \omega_{n1}} k_\theta \right) \]

\[ + i R_{n1\theta} \left( k_\theta + i \frac{2 \Omega_n}{\omega_{n1}} k_r \right) + i R_{n1\phi} \left( k_\phi - \frac{k_n^2}{\omega_{n1}^2} k_z \right). \]
The following notations are introduced here:

\[ D_n = \omega_n^2 (1 - \delta_n) - k_n^2 (1 - \delta|n|) - i \omega_n \left( 2 \Omega_n - \frac{k_n^2}{2 \Omega_m} \right) \delta_{\theta n}, \]
\[ \omega_v \equiv \omega_n + i \nu_v^0 = \omega - kv_f, \]
\[ \Omega_n = \frac{v_{n0}^0}{r^2}, k_n^2 = (2 \Omega_n/r) \partial (r^2 \Omega_n)/\partial r, k_0 = \frac{m}{r}, \]
\[ \delta_{lmn} = \frac{k_l k_m v_{l0}^2}{\omega_n v_{n0}}, \delta_{\perp} = \delta_{rr} + \delta_{\theta\theta} + \delta_{zz}, \delta_{n\perp n} = \delta_{zz} + \delta_{\perp n}, \]

where \( l, m = r, \theta, z. \)

**APPENDIX B**

**SOLUTIONS FOR \( v_{jlk} \) AND \( n_{jlk} \)**

The Fourier components \( v_{jlk} \) and \( n_{jlk} \) are:

\[ D_j v_{j1r} = i F_{j1r} \omega_v j (1 - \delta_{\theta j} - \delta|j|) + i F_{j10} \omega_v \delta_{\theta j} + i \omega_j j (1 - \delta_{|j|}) + i F_{j1z} \omega_v \delta_{|j|} \]
\[ D_j v_{j10} = i F_{j10} \omega_v \delta_{\theta j} + i \omega_j j (1 - \delta_{|j|}) + i F_{j1z} \omega_v \delta_{|j|}, \]

\[ D_j n_{j1} = i F_{j10} \omega_v \delta_{\theta j} + i \omega_j j (1 - \delta_{|j|}) + i F_{j1z} \omega_v \delta_{|j|}. \]

The following notations are introduced here:

\[ D_j = \omega_j^2 (1 - \delta_{\perp j} - \delta|j|) - \omega_j j (1 - \delta|j|) - i \omega_v (\omega_j j - \omega_j j) \delta_{\theta j}, \]
\[ F_{j1r} = G_{j1r} + Q_{j1r} = \frac{q_j}{m_j} E_{1r} \left( 1 - n_r \frac{v_{j0r}}{c} \right) + \frac{q_j}{m_j} E_{1r} n_r \frac{v_{j0r}}{c} + Q_{j1r}, \]
\[ F_{j10} = G_{j10} + Q_{j10} = \frac{q_j}{m_j} E_{10} \left( 1 - n_r \frac{v_{j0r}}{c} \right) + \frac{q_j}{m_j} E_{10} n_r \frac{v_{j0r}}{c} + Q_{j10}, \]
\[ F_{j1z} = G_{j1z} + Q_{j1z} = \frac{q_j}{m_j} E_{1z} \omega_j j + \frac{q_j}{m_j} E_{1z} n_z \frac{v_{j0r}}{c} + Q_{j1z}, \]

where \( l, m = r, \theta, z. \)

**APPENDIX C**

**THE SYSTEM OF EQUATIONS FOR PERTURBATIONS OF IONS AND DUST GRAINS**

Solutions given in the Appendix B upon taking into account collisions of neutrals with ions and dust grains simultaneously and neglecting the thermal effects, result in the following system of equations for perturbations of ions and dust grains:

\[ D_j v_{11r} + a_{dr} v_{1d} + a_{d01} v_{d10} - b_{dr} \frac{n_{d1}}{n_{d0}} = -H_{11r}, \]
\[ D_j v_{110} + a_{d02} v_{d1r} + a_{d0} v_{d10} - b_{d0} \frac{n_{d1}}{n_{d0}} = -H_{110}, \]
\[ \omega_j^2 v_{11z} + a_{dr} v_{1z} = i G_{11z} \omega_v, \]
\[ (k_i a_{dr} - k_i a_{d02}) v_{d1r} + (k_i a_{dr} + k_i a_{d01}) v_{d10} + \omega_j D \frac{n_{d1}}{n_{d0}} - \frac{\mu_i}{n_{d0}} = -kH_{11z}, \]

\[ D_j v_{d1r} + a_{d1r} v_{d1} + a_{d10} v_{d10} - b_{d1} \frac{n_{d1}}{n_{d0}} = -H_{d1r}, \]
\[ D_j v_{d10} - a_{d02} v_{d1r} + a_{d0} v_{d10} - b_{d0} \frac{n_{d1}}{n_{d0}} = -H_{d10}, \]

\[ \omega_{d01}^2 v_{d1z} + a_{d1r} v_{d1z} = i G_{d1z} \omega_v, \]
\[ (k_i a_{d1r} - k_i a_{d02}) v_{d1r} + (k_i a_{d1r} + k_i a_{d01}) v_{d10} + \omega_d D \frac{n_{d1}}{n_{d0}} - \frac{\mu_i}{n_{d0}} = -kH_{d1z}. \]

Here, \( D_j = \omega_j^2 - \omega_r d \omega_j d \omega_r d \). The following notations are introduced above:

\[ H_{11r} = -i \omega_v G_{11r} + \omega_r G_{11r}, H_{110} = -\omega_r G_{110} - i \omega_v G_{110}, \]
\[ H_{11z} = -i \frac{D_i}{\omega_v} G_{11z} + a_{dr} \frac{D_i}{\omega_r} v_{d1z}, H_{11z} = -i \frac{D_d}{\omega_r} G_{11z} + a_{dr} \frac{D_d}{\omega_v} v_{d1z}, \]

\[ a_{dr} = \omega_{dr} \mu_i, a_{d01} = i \omega_{d01} \mu_i, a_{d02} = i \omega_{d02} \mu_i, \]
\[ b_{dr} = -a_{dr} w_r, b_{d01} = -a_{d01} w_r + a_{d02} w_r, b_{d02} = a_{d01} \mu_d, a_{d01} = i \omega_{d01} \mu_d, a_{d02} = i \omega_{d02} \mu_d, \]

where \( w = v_{i0} - v_{d0} \) and \( \mu_i = \frac{v_{i0} v_{d0}}{\omega_v}, \mu_d = \frac{v_{i0} v_{d0}}{\omega_v}. \)

**APPENDIX D**

**SOLUTIONS OF THE SYSTEM OF EQUATIONS GIVEN IN APPENDIX C IN THE CASE \( k_z \neq 0, k_r = k_\theta = 0 \)**

The exact solutions of the system of equations given in the Appendix C for \( v_{11r,0} \) in the case \( k_z \neq 0, k_r = k_\theta = 0 \) have the
where the value \( \lambda_{\omega} \) is equal to
\[
\lambda_{\omega} = \omega_{vi} \omega_{vd} - \mu_i \mu_d.
\]

The value \( A \) (the determinant of the system is equal to \( A \omega_1^2 D_1^2 D_2^2 \)) has the form
\[
A = -\lambda^2_{\omega} - \omega_{vd}^2 \omega_{vi} \omega_{vd} - \omega_{i1} \omega_{vd} \omega_{i1} \omega_{vd} - \mu_i \mu_d \omega_{vd} \omega_{i1} \omega_{vd}.
\]

The solutions for \( v_{i1r} \) and \( v_{d1r} \) are obtained from Equations (D.1) and (D.2) under substitution the index \( i \) by the index \( d \), and vice versa.

The solutions for the velocities \( v_{i1z} \) and \( v_{d1z} \) are the following:
\[
\lambda_{\omega} v_{i1z} = \omega_{vd} G_{i1z} - \mu_d G_{d1z},
\]
\[
\lambda_{\omega} v_{d1z} = \omega_{vi} G_{d1z} - \mu_i G_{i1z}.
\]

**APPENDIX E**

**GENERAL SOLUTIONS FOR THE PERTURBED ELECTRON VELOCITY IN THE CASE \( k_z \neq 0, k_r = k_\theta = 0 \)**

From the solutions given in the Appendix B, we obtain the following expressions for the perturbed velocities of the cold electrons:
\[
D_e v_{i1r} = -H_{i1r} - \mu_d \omega_{ve} v_{i1r} - i \mu_i \omega_{ve} v_{i10}
- \mu_d \omega_{vd} v_{i1r} - i \mu_i \omega_{vd} v_{i10},
\]
\[
D_e v_{d1r} = -H_{d1r} + i \mu_i \omega_{ve} v_{d1r} - \mu_d \omega_{vd} v_{d1r}
+ i \mu_i \omega_{vd} v_{d10} - \mu_d \omega_{ve} v_{d10},
\]
\[
v_{i1z} = \frac{i}{\omega_{ve}} G_{i1z} + \frac{1}{\omega_{ve} \omega_{\omega}} \left[ (\mu_j \mu_{ed} - \omega_{vd} \omega_{ie}) G_{i1z}
+ (\mu_d \mu_{ei} - \omega_{vi} \mu_{ed}) G_{d1z} \right],
\]
where
\[
\mu_{ei} = \frac{v_{i1} v_{i0}}{2 \omega_{ve}}, \quad \mu_{ed} = \frac{v_{d1} v_{d0}}{2 \omega_{ve}}.
\]
