On rare core collapse supernovae inside planetary nebulae

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ABSTRACT

We conduct simulations of white dwarf (WD) - neutron star (NS) reverse evolution, and conclude that a core collapse supernova (CCSN) explosion might occur inside a planetary nebula (PN) only if a third star forms the PN. In the WD-NS reverse evolution the primary star evolves and transfers mass to the secondary star, forms a PN, and leaves a WD remnant. If the mass transfer brings the secondary star to have a mass of \( \gtrsim 8 M_\odot \) before it develops a helium core, it explodes as a CCSN and leaves a NS remnant. Using the Modules for Experiments in Stellar Astrophysics (MESA) we find that in the reverse evolution the time period from the formation of the PN by the primary star to the explosion of the secondary star is longer than a million years. By that time the PN has long dispersed into the interstellar medium. If we start with two stars that are too close in mass to each other, then the mass transfer takes place after the secondary star has developed a helium core and it ends forming a PN and a WD. The formation of a CCSN inside a PN (so called CCSNIP) requires the presence of a third star, either as a tertiary star in the system or as a nearby member in an open cluster. The third star should be less massive than the secondary star but by no more than few \( \times 0.01 M_\odot \). We estimate that the rate of CCSNIP is \( \approx 10^{-4} \) times the rate of all CCSNe.

Keywords: galaxies: active – (galaxies:) quasars: supermassive black holes – galaxies: jets

1. INTRODUCTION

The idea that some type Ia supernovae (SNe Ia) explode inside a planetary nebula (PN), so called a SN inside a PN (SNIP; e.g., Tsebrenko, & Soker 2013, 2015), is natural since SNe Ia are exploding white dwarfs (WDs). In the present study we ask whether a core collapse supernova (CCSN) might take place inside a PN, and if yes, then what are the conditions for that to occur. Such a scenario requires the WD to be born before the neutron star (NS), and therefore this binary evolutionary channel is termed a WD-NS reverse evolution. The initial more massive star in the binary system, the primary star, has a mass of \( M_{1,B,i} \lesssim 8 M_\odot \). It evolves first, transfers mass to its companion, and forms the WD. The mass-transfer brings the secondary to have a post-accretion mass of \( M_{2,B,i} \gtrsim 8 M_\odot \), and so it might end its life as a CCSN, leaving behind a NS remnant.

Many earlier studies considered a mass-transfer process that brings the secondary star to evolve toward a CCSN, leaving behind a WD-NS system, bound or unbound (e.g., Tutukov & Yungelson 1993; Portegies Zwart & Verbunt 1996; Tauris & Sennels 2000; Brown et al. 2001; Nelemans et al. 2001; Kim et al. 2003; Kalogera et al. 2005; van Haaften et al. 2013). The WD-NS reverse evolution might account for a massive WD in the binary radio pulsars PSR B2303+46 and PSR J1141-6545 (e.g., Portegies Zwart & Yungelson 1999; van Kerkwijk & Kulkarni 1999; Tauris & Sennels 2000; Brown et al. 2001; Davies et al. 2002; Church et al. 2006). Sabach, & Soker (2014) mentioned other outcomes of the WD-NS reverse evolution, including the merger of the WD with the core of the NS progenitor, and some types of bright transient events (intermediate luminosity optical transients; ILOTs). We note that in the case of a core-WD merger, the WD does not survive, and the system leaves only a NS.

The idea that CCSN occurs inside a dense circumstellar matter (CSM) shell is not new of course (e.g. Nelemans et al. 2001; Kalogera et al. 2005; Church et al. 2006; van Haaften et al. 2013). However, in most (or even all) of these cases the shell originates from the exploding star itself or from a companion that did not form a WD yet. A PN (by definition) is an expanding shell that the asymptotic giant branch (AGB) pro-
genitor of the central star formed. The central star is evolving to become a WD and it ionises the expanding shell. To have a a CCSN inside a PN, which we term here CCSNIP, we need to have a binary system where the PN was formed before the CCSN, namely, a WD-NS reverse evolution where the WD forms before the NS (e.g., Sabach, & Soker 2014). More than that, we need the explosion of the secondary star to take place before the PN is dispersed in the interstellar medium (ISM). The dispersion of such a PN takes place at a distance of \( \approx 5 \sim 10 \text{ km s}^{-1} \) the dispersion time is less than million years (e.g., Napierotzki 2001; Wu et al. 2011). Namely, for the formation of a CCSNIP we require the explosion to take place within about one million years (or even only \( 3 \times 10^6 \) yr) from the formation of the PN.

The unique signature of a CCSNIP is the collision of the CCSN ejecta with a relatively dense shell in a stellar population where the turnover mass is below the minimum for CCSN progenitor, \( \leq 8M_\odot \). Namely, in an environment where a single star evolution will not lead neither to a CCSN nor to a dense CSM. The question we examine is whether this might occur within a single binary system, or whether a CCSNIP might occur only when the explosion of the reverse evolution occurs within the PN of another (a third) star in a triple-star system or in a young open cluster. To answer this question we use MESA-BINARY and MESA-SINGLE (Modules for Experiments in Stellar Astrophysics, section 2) to simulate many WD-NS reverse evolution binary systems. We present our results in section 3, and conclude with our main findings in section 4.

2. NUMERICAL SETUP

We use MESA BINARY (version 10398; Paxton et al. 2011, 2013, 2015, 2018, 2019) to simulate the WD-NS reverse evolution. Conducting these simulations from start to end with MESA BINARY is not straightforward (e.g., Gibson & Stencel 2018). Gibson & Stencel (2018) use MESA BINARY to explore the evolutionary state of the epsilon Aurigae binary system. In about 60% of their simulations the time step became too small or MESA BINARY was unable to converge to an acceptable evolved stellar model. This large ratio of problematic simulations emphasises the difficulties in conducting such simulations.

To overcome the convergence problems we divide each run to two modes, the binary mode and the single star mode. Our subscript notation throughout this paper are as follows. The numbers 1 and 2 stand for the primary and secondary stars, respectively, B and S stand for the binary and single modes of MESA, respectively and i, and f for the initial and termination point, respectively. We also use the subscript ‘MT’ to indicate a quantity at the beginning of the mass-transfer during the binary mode. In the binary mode, we use MESA BINARY and terminate the run when the primary reaches a mass of \( M_{1,B,i} = 1.3M_\odot \). At this point the primary is already on its route to transform to a WD, and we avoid numerical convergence problems that would appear later. We tried other limits such as \( M_{1,B,i} = 1.1M_\odot \) and \( M_{1,B,i} = 1.2M_\odot \) for numerous simulations. Some encountered problems (such as convergence and limitation in the time step) but most simulations converged. The final values of different variables in the binary mode were different due to the above change in the mass limit \( M_{1,B,i} \) at the termination of the binary mode. However, the final outcome, after the single mode, of each simulation did not change much due to this change in \( M_{1,B,i} \). We encourage further study of this subject and perhaps developing a routine that does not require two modes.

In the single star mode we use MESA SINGLE and we simulate the secondary star that accreted mass from the primary star in the binary mode, and start the single mode with a mass of \( M_{2,S,i} \geq 8M_\odot \). We continue the single mode until the secondary star ends its evolution as a CCSN (leaving a NS), or as a WD if accretion takes place at late times (section 3). Specifically, we terminate the secondary star evolution by one of four conditions, as follows. (1) \( \log(T_c/K) > 9.1 \), where \( T_c \) is the core temperature; (2) \( \log(L_{\text{nuc}}/L_\odot) > 10 \) where \( \log(L_{\text{nuc}}) \) is the total power from all nuclear reactions; (3) \( \log L < 0.1L_\odot \); (4) \( \log R < 1R_\odot \). The first two conditions imply that there is oxygen burning in the core, and so the star is very close to core collapse and then NS formation. Conditions 3 and 4 indicate that the star is deep in the post-AGB track on its way to form a WD.

We hereby specify the general numerical details that hold for all simulations. In section 3 we list the unique properties for each run.

We consider only circular orbits, and in the simulations we present in section 3 we do not take into account tidal forces. We performed 4 tests where we did include tidal forces, and compared with simulations having the same initial conditions but that did not include tidal forces. The initial masses and orbital periods for these four cases are \( (M_{1,B,i}, M_{2,B,i}, P_i) = (7, 6, 98, 200), (6, 5, 100), (6, 5, 6, 100), \) and \( (7, 6, 4, 100) \), where masses are in \( M_\odot \) and period is in days. The first three simulations showed very minor variations from the runs without tidal forces. The fourth run did not converge. The investigation of this issue in full is beyond the scope of this paper.
In the MESA-BINARY mode we follow the *inlists* of Gibson & Stencel (2018) and adopt the mass-transfer scheme of Kolb & Ritter (1990). The mass-transfer efficiency scheme that we adopted is the one that Gibson & Stencel (2018) use (from Soberman et al. 1997), with the parameters: $\alpha = 0.1$ for the fractional mass-loss from the vicinity of the donor star, lost as fast wind; $\beta = 0.1$ for the fractional mass-loss from the vicinity of the accretor star, lost as fast wind; $\delta = 0.1$ for the fractional mass-loss from the circumbinary coplanar toroid, with a radius equal to $\gamma^2 a$, where $a$ is the binary semi-major axis. We further follow Gibson & Stencel (2018) and adopt $\gamma = 1.3$. For the initial equatorial surface rotation velocity of both stars we take $v_{1,e}, v_{2,e} = 2 \text{ km s}^{-1}$. All other parameters except the nuclear reaction, stellar masses and orbital periods, are as in the *inlists* of Gibson & Stencel (2018). For the nuclear reaction network we take the MESA-BINARY default; we compared two successful runs with the nuclear reaction of Gibson & Stencel (2018) to the default nuclear reaction of MESA, and found the differences to be less than 1%.

Other parameters that we do not mention here are set to their default option in MESA-BINARY or in MESA-SINGLE.

In the MESA-SINGLE mode, where we simulate the evolution of the secondary star from where it terminated the binary mode (in one case we also followed the primary star), we follow the parameters from the *inlist* of Gofman & Soker (2019). The initial structure (mass, radius, composition as function of radius) of the secondary star is taken directly from the last point of the binary mode, rather than starting at the main sequence or earlier.

3. RESULTS

3.1. Relevant simulations

In Table 1 we list the properties of the simulations that are relevant to our study. We perform many more simulations that we do not present in Table 1. Some of these simulations ended without mass-transfer and therefore are not relevant to our study. Others did not converge, either in the binary mode or in the single mode. Many simulations did converge and then terminated successfully according to one of the four conditions that we list in section 2, but did not give any added value to the results we present here.

Table 1 presents two sets of simulations, one with initial primary mass of $M_{1,B,1} = 6 M_\odot$ and one with $M_{1,B,1} = 7 M_\odot$. The second column lists the initial secondary masses.

Column 3 presents the initial orbital period and is termed $P_{B,1}$ (no primary or secondary notations are needed since this value is for the binary system).

Column 4 presents the final mass of the secondary star in the binary mode $M_{2,B,f}$, after it accreted mass from the primary star. By definition of our scheme, the initial secondary mass in the single mode is $M_{2,S,1} = M_{2,B,f}$.

Columns 5 and 6 are the masses of the different primary components at the end of the binary mode, of the hydrogen-rich envelope $M_{1,B,f,H}$ and of the mass of the core including the helium layer $M_{1,B,f,He}$, respectively. The helium core boundary is taken where the Hydrogen mass fraction is $\leq 0.01$ and Helium mass fraction is $\geq 0.1$.

As we terminate the binary evolution when the primary mass is $M_{1,B,f} = 1.3 M_\odot$, we have $M_{1,B,f,H} + M_{1,B,f,He} = M_{1,B,f} = 1.3 M_\odot$ (up to the accuracy of the calculations and of rounding numbers).

Column 7 is relevant to cases when the secondary star explodes as a CCSN and leaves a NS remnant (as we indicate by ‘WD, NS’ in the last column of the table). It presents the duration of the single mode $\Delta t_{CCSN} = t_{S,f} - t_{B,f}$. Namely, the time from the termination of the binary mode, $t_{B,f}$, i.e., when $M_{1,B,f} = 1.3 M_\odot$ which is very close to the time the primary forms a WD, and the time when the secondary star reaches a massive oxygen core, just before it explodes as a CCSN and leaves a NS remnant. At this time, $t_{S,f}$, we terminate the single mode for these cases.

We note the following. For numerical reasons we stop the binary mode when $M_{1,B,f} = 1.3 M_\odot$. By that time the primary star has a very massive core, so it is very luminous (see section 3.3), implying a very high mass-loss rate of $\dot{M}_1 \approx \text{few} \times 10^{-6} M_\odot$. Since the mass of the hydrogen-rich envelope is $M_{1,B,f,H} < 0.4 M_\odot$ (fifth column), the primary star will form a PN within $\approx 10^5$ yr. This is a very short time relative to the other evolutionary times that we list in Table 1, $\Delta t_{CCSN}$ and $\Delta t_{PN}$. The same consideration holds for the secondary star in cases where it forms a WD. Namely, the evolution time from when the secondary mass is $1.3 M_\odot$ to the formation of a PN is $\approx 10^5$ yr.

Column 8 refers only to cases where the secondary star leaves a WD remnant. It lists $\Delta t_{PN} = t(M_{1,B,f} = 1.3 M_\odot) - t(M_{2,S,f} = 1.3 M_\odot)$, which is the duration of time between when $M_{1,B,f} = 1.3 M_\odot$ in the binary mode (about the time when the primary forms a PN) and the time when the secondary mass reaches $M_{2,S,f} = 1.3 M_\odot$ (about the time the secondary star forms a PN).

Column 9 presents the orbital period of the binary system at the end of the binary mode $P_{B,f}$. In all cases it is much longer than the initial orbital period, by a factor of $\approx 5 - 7$ for $M_{1,B,1} = 6 M_\odot$, and by a factor of
The main points to take from Table 1 are as follows. (1) We learn that if mass-transfer takes place after the secondary star has developed a massive helium core, \( M_{2,\text{B,He}[\text{MT}]} \), the secondary star leaves a WD remnant, namely the binary system ends as a wide WD-WD binary system. On the other hand, if the secondary did not develop a helium core by the time mass-transfer is initiated, it explodes as a CCSN (as long as its mass after mass-transfer is \( M_{2,\text{B,He}} \geq 8M_\odot \), which holds for all the relevant cases here). For the two cases of initial primary masses we study here, the secondary star leaves a WD remnant for 5.5 \( M_\odot \leq M_{2,\text{B,He}} < M_{1,\text{B,He}} = 6M_\odot \) and 6.85 \( M_\odot \leq M_{2,\text{B,He}} < M_{1,\text{B,He}} = 7M_\odot \), respectively. (2) The time period \( \Delta t_{\text{CCSN}} \) that we list in column 7 of table 1 is the time from the formation of a PN by the primary star to the time when the secondary star explodes as a CCSN and leaves a NS remnant, as we list in the twelfth column.

3.2. The lessons from the simulations

The main points to take from Table 1 are as follows. (1) We learn that if mass-transfer takes place after the secondary star has developed a massive helium core, \( M_{2,\text{B,He}[\text{MT}]} \), the secondary star leaves a WD remnant, namely the binary system ends as a wide WD-WD binary system. On the other hand, if the secondary did not develop a helium core by the time mass-transfer is initiated, it explodes as a CCSN (as long as its mass after mass-transfer is \( M_{2,\text{B,He}} \geq 8M_\odot \), which holds for all the relevant cases here). For the two cases of initial primary masses we study here, the secondary star leaves a WD remnant for 5.5 \( M_\odot \leq M_{2,\text{B,He}} < M_{1,\text{B,He}} = 6M_\odot \) and 6.85 \( M_\odot \leq M_{2,\text{B,He}} < M_{1,\text{B,He}} = 7M_\odot \), respectively. (2) The time period \( \Delta t_{\text{CCSN}} \) that we list in column 7 of table 1 is the time from the formation of a PN by the primary star to the time when the secondary star explodes as a CCSN and leaves a NS remnant (last column). We see that in all cases this time period is longer than the expected maximum time a PN might preserve its identity, which is at most \( \approx 10^6 \) yr (e.g. Napiwotzki 2001; Wu et al. 2011). The conclusion is that the reverse evolution cannot lead a binary system to explode a CCSN inside a PN. Namely, a reverse evolution of a binary system cannot form a CCSNIP. In section 4 we
discuss the way to form CCSNIP with the presence of a third star.

(3) The hydrogen mass at explosion (not shown in table 1) in the different cases where the secondary star explodes as a CCSN is in the range of $M_{\text{2,H,exp}} = 3.7 - 5M_\odot$ (the hydrogen-rich envelope mass is about 1.5 times larger). This implies that in all cases the secondary star explodes as SNe II. Furthermore, since the mass that is lost at explosion, $M_{\text{ejecta}} > 5.4M_\odot$, is larger than the total mass of the WD+NS remnant, in all these cases the WD and NS will be unbound after the CCSN explosion.

(4) The time period $\Delta t_{\text{PN}}$ that we list in column 8 of table 1 is the time from the formation of a PN by the primary star to the formation of a PN by the secondary star when it leaves a WD remnant (last column). The values of $\Delta t_{\text{PN}}$ for the cases $(M_{1,B,i}, M_{2,B,i}) = (6, 5.95)$ and $(M_{1,B,i}, M_{2,B,i}) = (7, 6.98)$ show that a fine tuning (very close initial masses between the two stars) might lead to a PN that the secondary star forms inside the very old PN that the primary star had formed. We are not aiming at these cases.

3.3. Evolution on the HR diagram

We present the evolution on the HR diagram for two systems, one that ends with a CCSN that leaves unbound WD and NS remnants (Fig. 1), and one that leaves a wide WD-WD binary system (Fig. 2). The primary and secondary stars have complicated tracks on the HR diagram that include some loops. As our aim in this study is to explore the final outcome, we do not analyse that evolution (there are other studies of mass transfer in different kinds of stars in the literature, e.g., Poelarends et al. 2017; Yoon et al. 2017; Gibson & Stella 2018; Wu et al. 2018; Brinkman et al. 2019; Farrell et al. 2019; Gosnell et al. 2019).

The relevant properties to our study that Figs. 1 and 2 reveal are as follows.

(1) The mass-transfer that leads the secondary star to later explode as a CCSN takes place while the secondary star is still on the main sequence, and did not develop a helium core yet (blue triangle on the upper panel of Fig. 1). During and shortly after the mass-transfer process the secondary performs a large loop on the HR diagram (black line on the upper panel of Fig. 1), and returns to the main sequence as a more massive star (red diamond). It later evolves towards a CCSN (black line on the lower panel of Fig. 1).

(2) In cases where the secondary star explodes as a CCSN, it is a red supergiant at explosion (large open green pentagram on the lower panel of Fig. 1).

(3) If mass-transfer takes place after the secondary star has left the main sequence (blue triangle on the upper panel of Fig. 2), it ends as a WD. It contracts after accreting mass, and then resumes evolution to become an AGB star and to form a PN (lower panel of Fig. 2).

4. DISCUSSION AND SUMMARY

The question we raised at the beginning of this study was whether a single binary system that experiences the WD-NS reverse evolution, where the WD forms before the NS, might form a CCSN inside a PN, so called CCSNIP.

To perform a reverse evolution, the primary star of the system should have a zero age main sequence mass of $M_{1,B,i} \lesssim 8M_\odot$, such that it forms a PN and leaves a WD remnant. When the primary star evolves it transfers mass to the secondary star, such that after this binary interaction with mass transfer the secondary mass becomes $M_{2,B,f} \gtrsim 8M_\odot$. To explode as a CCSN, it is also necessary that the secondary star mass grows to $M_{\text{2,B,f}} \gtrsim 8M_\odot$ before it had developed a helium core (Table 1). We present one such case in Fig. 1.

In cases where the mass-transfer takes place after the secondary star has developed a helium core $(M_{2,B,\text{He[MT]}} > 0$ in Table 1) and left the main sequence, it forms a PN rather than a CCSN, and leaves a WD remnant, as we show for one case in Fig. 2. The remnant of such a binary system is a bound wide WD-WD binary system. When the initial masses of the two stars are very close, the time period from the formation of the first PN by the primary star to the formation of the second PN by the secondary star is $\Delta t_{\text{PN}} < 10^6$ yr (last two rows of table 1). Since some PNe can retain their identity for hundreds of thousands of years, it is possible that in such cases we form a PN inside a large and old PN. López et al. (2000) already suggested that KJPN 8 is composed of two consecutive PNe that originated from a binary system that had very similar initial masses. In the case of KJPN 8, however, the time period between the formation of the two PNe is only $\approx 10^4$ yr, which requires a hyper-fine-tuned set of initial masses.

In cases of reverse evolution we found that the secondary star explodes when it is a red supergiant, and it forms a SN II. The explosion unbinds the WD and the NS remnants.

Most relevant to our study is our finding that we could not bring the time period from the formation of the PN by the primary star to the explosion of the secondary star, $\Delta t_{\text{CCSN}}$, to be less than a few millions years (column 7 of Table 1). If we set the two initial masses to be too close to each other, the secondary leaves the main se-
Figure 1. The HR diagram for the system with $M_{1,B,i} = 6M_\odot$, $M_{2,B,i} = 5M_\odot$, and $P_i = 100$ days. The dotted red line and the solid black line present the evolution of the primary and of the secondary stars, respectively. The top panel presents the HR diagram of the binary mode from MESA BINARY, up to the formation of a primary star of mass $M_{1,B,f} = 1.3M_\odot$ that occurs at $t_{B,f} = 6 \times 10^7$ yr. We mark four points on the upper panel. (1) A red square at $t_{B,f}$ marks the termination point of the primary evolution in the binary mode, with a mass of $M_{1,B,f} = 1.3M_\odot$. (2) A red diamond at $t_{B,f}$ marks the termination point of the secondary evolution in the binary mode, with a mass of $M_{2,B,f} = 8.3M_\odot$ (see also Table 1). (3) A blue circle at $t_{MT} = 5 \times 10^7$ yr marks the location of the primary star on the HR diagram when mass-transfer starts. (4) A blue triangle at $t_{MT}$ marks the location of the secondary star on the HR diagram when mass-transfer starts. The bottom panel presents the HR diagram of the single mode for the primary star and for the secondary star. The starting points are the final points in the upper panel (a red square and a red diamond, respectively). We arbitrarily set the termination point of the primary star evolution when its radius is $R_1 = 1R_\odot$ (black pentagram), about the time it strongly ionises its PN. We end the evolution of the secondary star when $\log(L_{2,nuc}/L_\odot) = 10$ (section 2), very close to its explosion as type II CCSN (large open green pentagram).
Figure 2. The HR diagram for the binary system of $M_{1,B,i} = 7M_\odot$, $M_{2,B,i} = 6.9M_\odot$ and $P_i = 200$ days, which ends as a WD-WD wide binary system. The dotted red line and the solid black line present the evolution of the primary and of the secondary stars, respectively.

The top panel presents the HR diagram of the binary mode from MESA BINARY, up to the formation of a primary star of mass $M_{1,B,f} = 1.3M_\odot$ that occurs at $t_{B,f} = 4.3 \times 10^7$ yr. We mark four points on the upper panel. (1) A red square at $t_{B,f}$ marks the termination point of the primary evolution in the binary mode, with a mass of $M_{1,B,f} = 1.3M_\odot$. (2) A red diamond at $t_{B,f}$ marks the termination point of the secondary evolution in the binary mode, with a mass of $M_{2,B,f} = 10.9M_\odot$ (see also Table 1). (3) A blue circle at $t_{MT} = 4.19 \times 10^7$ yr marks the location of the primary star on the HR diagram when mass-transfer starts. (4) A blue triangle at $t_{MT}$ marks the location of the secondary star on the HR diagram when mass-transfer starts.

The bottom panel presents the HR diagram of the secondary star in the single mode from $t_{2,S,i} = t_{B,f}$, when the secondary mass is $M_{2,S,i} = 10.9M_\odot$ (red diamond), to the time when the secondary luminosity is $L_2 = 0.1L_\odot$, namely, when it is already on its WD cooling track (black pentagram). The secondary reaches the black pentagram location at $7.4 \times 10^7$ yr from the beginning of the binary mode (the zero age main sequence of the two stars).
quence before mass-transfer takes place, and it does not explode as a CCSN. Our finding that $\Delta t_{\text{CCSN}} \gg 10^6$ yr implies that by the time the secondary explodes as a CCSN, the PN had long dispersed into the ISM (section 3.2).

We conclude that the formation of a CCSNIP requires a third star in the system. The third star can be a very wide tertiary star in the system (such that it evolves independently), or can be a member in an open stellar cluster that is not too far from the binary system (a distance of $D_3 \lesssim 1$ pc). The binary system performs the WD-NS reverse evolution and leads the secondary star of initial mass of $M_{2,B,i} < 8M_\odot$ to explode as a CCSN. The third star has a mass that is in between the initial masses, $M_{2,B,i} < M_{3,i} < M_{1,B,i}$, and it forms a PN within few hundreds of thousands of years before the secondary star explodes as a CCSN, $\Delta t_{\text{PN3,CCSN}} \lesssim 5 \times 10^6$ yr. The PN might preserve its identity, although deformed by the ISM, by the time the secondary star explodes. In case the third star is a cluster member the CCSN might take place far from the center of the PN.

We crudely estimate the fraction of CCSNIP relative to all CCSNe as follows. Sabach, & Soker (2014) estimated that the event rate of all routes of WD-NS reverse evolution is $f_{\text{RE}} \approx 3 - 5\%$ of the CCSN rate. Note that we do not care whether the WD survives the evolution, as the cases we studied here, or whether in enters a common envelope with the secondary star as the later becomes a giant (Sabach, & Soker 2014; Soker 2019). From Moe & Di Stefano (2017) we find that for the initial mass range here of $M_{1,B,i} \approx 5.5 - 8M_\odot$ the fraction of single, binary, triple, and quadruple stars are about 0.24, 0.36, 0.27, and 0.13, respectively. This implies that on average each binary system has $f_{3,B,i,t} \approx (0.27 + 2 \times 0.13)/(0.36 + 0.27 + 0.13) = 0.7$ extra stars in triple (or quadruple) bound stellar system. It is more difficult to estimate the presence of an extra star from the open cluster, $f_{3,B,t,c}$. We simply assume that the cluster members add somewhat to this fraction that becomes larger than $f_{3,B,t} \approx 0.7$. We therefore crudely assume that on average each binary system has about one extra star that serves as the third star in the system that forms the PN shortly before the secondary star explodes as a CCSN, $f_{3,B} \approx 1$. Namely, the cluster members contribute only about 30%, $f_{3,B,c} \approx 0.3$, of the extra stars (as multiple systems contribute 0.7 extra stars for each binary system).

From the values of $\Delta t_{\text{PN}}$ in table 1 we estimate that the third star (tertiary or an open cluster member) should have an initial mass within $M_{2,3} \approx 0.02 - 0.05M_\odot$ of the secondary star to form a PN just before the secondary star explodes. Namely, $M_{2,B,i} - M_{2,3} < M_{3,i} < M_{2,B,i}$. With the upper limit for more massive stars. For a flat mass distribution of the tertiary star, the probability for this mass range is $f_{3,i} \approx \Delta M_{2,3}/M_{2,B,i} \approx (0.02M_\odot/6M_\odot) - (0.05M_\odot/7M_\odot) \approx 0.003 - 0.007$.

Overall, we crudely estimate the fraction of CCSNIP events from all CCSNe to be $f_{\text{CCSNIP}} \approx f_{\text{RE}} f_{3,B} f_{3,i} \approx 10^{-4} - 10^{-3.5}$. As future surveys aim at about $10^4$ CCSNe per year or so, we expect that about one to few of these will be CCSNIP, i.e., CCSN inside an old PN. The interaction of the ejecta with the relatively dense PN might take place tens to hundreds of years after explosion. The density of the PN is expected to be larger than that expected for CCSN in old open clusters. A more accurate estimate of the event rate of CCSNIPs and their possible observational signatures are the subjects of future studies.

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