A limit on the ABJ model

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Abstract.

There is a large amount of evidence that the ABJM model is integrable in the planar limit. Less clear is whether or not the ABJ model is integrable. Here we investigate a limit of the ABJ model in the weak coupling limit where one ’t Hooft parameter is much greater than the other. At the 4 loop level in the SU(2) × SU(2) sector the anomalous dimensions of single trace operators map to two Heisenberg spin chains with nearest neighbor interactions with an overall coefficient that is a function of one of the ’t Hooft parameters. We conjecture the form of this function and show that is consistent with observations about the ABJ model concerning unitarity and parity, including strong coupling statements.

The ABJM model [1] is a U(N)k × U(N)−k superconformal Chern-Simons theory with N = 6 supersymmetry. In the planar limit the two point functions of single trace operators map to a spin chain of alternating type that is believed to be integrable [2–5]. The integrability is remarkably similar to that found for N = 4 suer Yang-Mills. However, one important difference is the presence of an unknown function h2(λ), where λ is the ’t Hooft coupling λ = N/k. This function appears in the Bethe equations as well as the spin chain magnon dispersion relation

\[ E(p) = \sqrt{Q^2 + 4h^2(\lambda)} \sin^2 \frac{p}{2} - Q, \]

where Q is an R-charge. In N = 4 SYM, Q = 1 and h2(λ) = \( \frac{\lambda}{4\pi} \). For the ABJM model Q = 1/2, but h2(λ) is only known at its asymptotic limits, where for weak coupling h2(λ) = λ2 + O(λ4), while for strong coupling h2(λ) = \( \frac{1}{2\lambda} - \frac{\ln 2}{\sqrt{\pi}} + O(1) \).

The ABJ model [6] is an interesting extension of ABJM, where now the gauge group is U(M)k × U(N)−k but the amount of supersymmetry is unchanged. In the planar limit there are now two ’t Hooft constants, λ = M/k, λ = N/k, but whether or not ABJ is integrable is not clear. It turns out that at the two-loop level, the lowest order of perturbation theory, the anomalous dimension mixing matrix still maps to an integrable spin-chain [7,8]. However, the corresponding string dual for this theory is type IIA on AdS4 × CP3 with a b-flux through a CP1 on the CP3. This b-flux corresponds to a θ-angle for the world-sheet theory, where \( \theta = 2\pi(N - M)/k + \pi \) [9], which generically breaks parity. The θ-angle cannot affect the classical integrability [10,11] since it does not change the equations of motion. However, at the quantum level it is expected to have an effect. This is what happens for the O(N) model, where the two parity preserving values, \( \theta = 0, \pi \) are known to be integrable. However, the IR behavior for these two values of \( \theta \) are completely different, for example the \( \theta = 0 \) point has a gap while the \( \theta = \pi \) point is gapless. It is not clear at all how to smoothly interpolate between these two IR points by varying \( \theta \), leaving the integrability in doubt.

For the purpose of this note let us ask what the behavior would be if the planar ABJ model were integrable for any value of λ and \( \hat{\lambda} \). In this case it seems that the only possible modification to the ABJM results is...
with the dilatation operator $D$.

These terms arise from the permutation structures that sit in the spin chain Hamiltonian, or equivalently $\{ \}$. The notation $\{ \}$ describes the permutation structures, where

\[
\{a_1, a_2, \ldots, a_m\} = \sum_{i=1}^{L} P_{2i+a_1, 2i+a_1+2} P_{2i+a_2, 2i+a_2+2} \cdots P_{2i+a_m, 2i+a_m+2} ,
\]

Inspection of (6) shows that $h_4(\sigma)$ can be extracted from $D$ by considering those contributions that involve a single exchange. There are still dozens of Feynman diagrams to consider that are listed in [12,13]. Here we just report the result,

\[
h_4 = -16 + 4 \zeta(2) \approx -9.42 , \quad h_{4,\sigma} = -\zeta(2) .
\]
anomalous dimensions. Included in these is the single trace-operator $\text{Tr}[\ldots]$ that it changes sign as $\lambda$ passes through 1. But if the overall coefficient is negative, then so are the anomalous dimensions. Included in these is the single trace-operator $\text{Tr}[\ldots]$ which is consistent with the 't Hooft pair equivalence ($\lambda > \lambda' + 1$). In the limit where $\lambda \gg \lambda'$ this translates into $\lambda > 1$ violating the unitary bound. If we now consider the function in (10) we see that it changes sign as $\lambda$ passes through 1. But if the overall coefficient is negative, then so are the anomalous dimensions. Included in these is the single trace-operator $\text{Tr}[\ldots]$ which is consistent with the 't Hooft pair equivalence ($\lambda > \lambda' + 1$). In the limit where $\lambda \gg \lambda'$ this indicates an equivalence between the theory with $\lambda'$ and the theory with $1 - \lambda'$, which is consistent with the $f(\lambda)$ in (10).

Let us now motivate why $f(\lambda)$ in (10) has the right behavior.

First, as was argued in [6], unitary ABJ theories with gauge group $U(N + \ell)_{k} \times U(N)_{-k}$ do not exist if $\ell > k$. In terms of the 't Hooft parameters this corresponds to $\lambda > \lambda' + 1$. In the limit where $\lambda \gg \lambda'$ this translates into $\lambda > 1$ violating the unitary bound. If we now consider the function in (10) we see that it changes sign as $\lambda$ passes through 1. But if the overall coefficient is negative, then so are the anomalous dimensions. Included in these is the single trace-operator $\text{Tr}[\ldots]$ which is consistent with the 't Hooft pair equivalence ($\lambda > \lambda' + 1$). In the limit where $\lambda \gg \lambda'$ this indicates an equivalence between the theory with $\lambda'$ and the theory with $1 - \lambda'$, which is consistent with the $f(\lambda)$ in (10).

Of course more evidence is needed to confirm the form of (10). A six-loop calculation to compute the next term in the expansion is a daunting proposition but may be feasible, since the number of Feynman graphs that must be computed is quite restricted. We are presently investigating such a calculation.

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References

[1] O. Aharony, O. Bergman, D. L. Jafferis, and J. Maldacena, $N = 6$ Superconformal Chern-Simons-Matter Theories, M2-Branes and Their Gravity Duals, JHEP 10 (2008) 091, [arXiv:0806.1218].

[2] J. A. Minahan and K. Zarembo, The Bethe Ansatz for Superconformal Chern-Simons, JHEP 09 (2008) 040, [arXiv:0806.3951].

[3] D. Gaiotto, S. Giombi, and X. Yin, Spin Chains in $N = 6$ Superconformal Chern-Simons-Matter Theory, JHEP 04 (2009) 006, [arXiv:0806.4589].

[4] N. Gromov and P. Vieira, The All Loop $AdS_{4}/CFT_{3}$ Bethe Ansatz, JHEP 01 (2009) 016, [arXiv:0807.0777].

[5] D. Bak and S.-J. Rey, Integrable Spin Chain in Superconformal Chern-Simons Theory, JHEP 10 (2008) 053, [arXiv:0807.2063].

[6] O. Aharony, O. Bergman, and D. L. Jafferis, Fractional M2-Branes, JHEP 11 (2008) 043, [arXiv:0807.4924].

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[7] D. Bak, D. Gang, and S.-J. Rey, Integrable Spin Chain of Superconformal $U(M)\times U(N)$ Chern-Simons Theory, JHEP 10 (2008) 038, [arXiv:0808.0170].

[8] J. A. Minahan, W. Schulgin, and K. Zarembo, Two Loop Integrability for Chern-Simons Theories with $N=6$ Supersymmetry, JHEP 03 (2009) 057, [arXiv:0901.1142].

[9] O. Aharony, A. Hashimoto, S. Hirano, and P. Ouyang, D-Brane Charges in Gravitational Duals of 2+1 Dimensional Gauge Theories and Duality Cascades, arXiv:0906.2390.

[10] G. Arutyunov and S. Frolov, Superstrings on $\text{AdS}_4 \times \text{Cp}^3$ as a Coset Sigma-Model, JHEP 09 (2008) 129, [arXiv:0806.4940].

[11] B. Stefanski, Jr., Green-Schwarz Action for Type IIA Strings on $\text{AdS}_4 \times \text{Cp}^3$, Nucl. Phys. B808 (2009) 80–87, [arXiv:0806.4948].

[12] J. A. Minahan, O. Ohlsson Sax, and C. Sieg, Magnon Dispersion to Four Loops in the Abjm and Abj Models, arXiv:0908.2463.

[13] J. A. Minahan, O. Ohlsson Sax, and C. Sieg, Anomalous Dimensions at Four Loops in $\mathcal{N}=6$ Superconformal Chern-Simons Theories, arXiv:0912.3460.

[14] S. Minwalla, Restrictions Imposed by Superconformal Invariance on Quantum Field Theories, Adv. Theor. Math. Phys. 2 (1998) 781–846, [hep-th/9712074].