GMM-based Codebook Construction and Feedback Encoding in FDD Systems

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Abstract—We propose a precoder codebook construction and feedback encoding scheme which is based on Gaussian mixture models (GMMs). In an offline phase, the base station (BS) first fits a GMM to uplink (UL) training samples. Thereafter, it designs a codebook in an unsupervised manner by exploiting the GMM's clustering capability. We design one codebook entry per GMM component. After offloading the GMM—but not the codebook—to the mobile terminal (MT) in the online phase, the MT utilizes the GMM to determine the best fitting codebook entry. To this end, no channel estimation is necessary at the MT. Instead, the MT’s observed signal is used to evaluate how responsible each component of the GMM is for the signal. The highest responsibility and the BS then employs the corresponding codebook entry. Simulation results show that the proposed codebook design and feedback encoding scheme outperforms conventional Lloyd clustering based codebook design algorithms, especially in configurations with reduced pilot overhead.

Index Terms—Gaussian mixture models, machine learning, feedback, codebook design, frequency division duplexing

I. INTRODUCTION

In multiple-input multiple-output (MIMO) communications systems, channel state information (CSI) has to be acquired at the BS in regular time intervals. In frequency division duplex (FDD) mode, the BS and the MT transmit in the same time slot but at different frequencies. This breaks the reciprocity between the instantaneous UL CSI and downlink (DL) CSI. Accordingly, acquiring DL CSI in FDD operation is difficult [1]. Possible solutions include to either extrapolate the DL CSI from the estimate of the UL CSI at the BS, or to transfer the DL CSI estimated at the MT to the BS directly or in a highly compressed version. However, the most common solution in practice is to avoid the direct feedback of the CSI and to use only a small number of feedback bits. For instance, as it is done in this paper, the feedback can be used as an index into a predefined codebook of precoders [2].

In recent years, machine learning techniques have been explored in the context of communications. However, these typically need a large dataset of DL channels for their training phases. This would require the users to collect large amounts of DL CSI and either to perform the training themselves or to share the collected data with the BS. The corresponding computation and/or signaling overhead involved in such a scheme is generally unaffordable in practice.

Recently, in [3] it has been shown that DL CSI training data can be replaced with UL CSI training data even for the design of DL functionalities. This completely eliminates the aforementioned overhead. UL CSI can, e.g., be acquired at the BS during the regular UL transmission. In [3], the observation has been made in the context of training autoencoders. Similar observations have since been made in [4] for DL channel estimation and in [5], [6] for codebook design. In this work, we also utilize the idea to centrally learn DL-related functionalities at the BS using UL training data.

The contributions of this work are summarized as follows. We propose a codebook construction and feedback encoding scheme which is based on GMMs. Since GMMs are universal approximators [7], we can use a K-components GMM to approximate the unknown channel probability density function (PDF). In the offline phase, we propose to fit the GMM centrally at the BS using solely UL data. Thereafter, we cluster the training data according to the GMM components and design a codebook entry for every component. In this way, the codebook is designed for a whole scenario, i.e., for the whole site in which the BS is located. The GMM is offloaded to every MT in the coverage area of the BS, with which the MT can then select the best fitting codebook entry in the online phase.

In conventional approaches, first the DL CSI is estimated and subsequently the best fitting codebook entry is determined. The technique proposed in this paper allows to bypass explicit CSI estimation and yields the best fitting codebook entry by using the receive signal to evaluate the GMM responsibilities. The kth responsibility of a GMM corresponds to the probability that the kth GMM component is responsible for the receive signal. The feedback then consists of the index k of the component with the highest responsibility. Finally, we make use of a Kronecker approximation to significantly reduce the number of GMM parameters such that the offloading overhead is smaller. In simulations, the proposed codebook design and feedback encoding scheme outperforms conventional Lloyd clustering based codebook design algorithms [8], [9].

II. SYSTEM MODEL

The DL received signal of a point-to-point MIMO system can be expressed as \( y' = H x + n' \), where \( y' \in \mathbb{C}^{N_t \times 1} \) is the receive vector, \( x \in \mathbb{C}^{N_t \times 1} \) is the transmit vector sent over the MIMO channel \( H \in \mathbb{C}^{N_r \times N_t} \), and \( n' \sim N_c(0, \sigma^2_\text{n} I_{N_r}) \).
denotes the additive white Gaussian noise (AWGN). In this paper, we consider system configurations with \( N_{tx} < N_{rx} \). The BS is equipped with a uniform rectangular array (URA) and the MT is equipped with a uniform linear array (ULA). If perfect CSI is known to both the transmitter and receiver, and assuming input data with Gaussian distribution, the capacity of the MIMO channel is [10], [11]:

\[
C = \max_{Q \in \mathbb{C}^{N_{rx} \times N_{tx}}} \log_2 \det \left( I + \frac{1}{\sigma_n^2} H Q H^H \right),
\]

where \( Q \in \mathbb{C}^{N_{rx} \times N_{tx}} \) is the transmit covariance matrix and the transmit vector is then given by \( x = Q^{1/2} s \) with \( E[ss^H] = I_{N_{tx}} \). The optimal transmit covariance matrix \( Q^* \) of the link between the BS and a MT achieves the capacity and can be obtained by decomposing the channel into \( N_{rx} \) parallel streams and employing water-filling [12].

Channel reciprocity can generally not be assumed in FDD systems, e.g., [2]. Therefore, only the MT could compute the optimal transmit covariance matrix \( Q^* \) if it estimated the DL CSI. This makes some form of feedback from the MT to the BS necessary. Ideally, the user would feed the complete DL CSI. This makes some form of feedback from the MT to the BS necessary. Only the MT could compute the error (MSE)-optimal estimator \( \hat{H} \) and to use it to estimate the channels distributed according to \( f(h) \). The motivation for this is that as an approximation of the true but unknown channel PDF arbitrary well [7]. Interestingly, GMMs can approximate any continuous PDF arbitrary well [7].

### IV. CHANNEL ESTIMATION

In the pilot transmission phase, the DL received signal is:

\[
Y = HP + N \in \mathbb{C}^{N_{rx} \times n_p},
\]

where \( n_p \) is the number of transmitted pilots and \( P = [p_1, p_2, \ldots, p_{n_p}] \in \mathbb{C}^{N_{rx} \times n_p} \). The pilot matrix \( P \in \mathbb{C}^{N_{rx} \times n_p} \) is a 2D-DFT (sub)matrix, constructed by the Kronecker product of two discrete Fourier transform (DFT) matrices, \( P = P_0 \otimes P_r \), where each column \( p_p \) of \( P \), for \( p \in \{1, 2, \ldots, n_p\} \), is normalized such that \( \|p_p\|^2 = \rho \), since we employ a URA at the BS [15]. In this work, we consider \( n_p \leq N_{tx} \). For what follows, it is convenient to vectorize (3):

\[
y = Ah + n,
\]

with the definitions \( h = \text{vec}(H) \), \( y = \text{vec}(Y) \), \( n = \text{vec}(N) \), \( A = P_0 \otimes I_{N_{tx}} \) and \( n \sim N(0, \Sigma = \sigma_n^2 I_{n_p \times N_{rx}}) \).

#### A. GMM based Channel Estimation

A GMM is a PDF of the form [16]

\[
f_h^K(h) = \sum_{k=1}^{K} p(k) N_C(h; \mu_k, C_k)
\]

where every summand is one of its \( K \) components. Maximum likelihood estimates of the parameters of a GMM, i.e., the means \( \mu_k \), the covariances \( C_k \), and the mixing coefficients \( p(k) \), can be computed using a training data set \( \mathcal{H} \) and an expectation maximization (EM) algorithm, see [16]. GMMs allow for the evaluation of responsibilities [16]:

\[
p(k \mid h) = \frac{p(k) N_C(h; \mu_k, C_k)}{\sum_{k=1}^{K} p(k) N_C(h; \mu_k, C_k)}.
\]

These corresponds to the probability that a given \( h \) was drawn from component \( k \). Interestingly, GMMs can approximate any continuous PDF arbitrary well [7].

With this GMM background, we now briefly recap the GMM channel estimator from [17]. Given a training data set of channels \( \mathcal{H} = \{ h_m = \text{vec}(H_m) \}_{m=1}^{M} \), the EM algorithm is used to compute a \( K \)-component GMM \( f_h^K \) as an approximation of the true but unknown channel PDF \( f_h \).

The idea in [17], [18] now is to compute the mean squared error (MSE)-optimal estimator \( \hat{h}_{GMM}^K \) for channels distributed according to \( f_h^K \) and to use it to estimate the channels distributed according to \( f_h \). The motivation for this is that
\( \hat{h}^{(K)}_{\text{GMM}} \) converges pointwise to the MSE-optimal estimator for channels distributed according to \( f_h \) as \( K \to \infty \) \cite{1}, \cite{2}.

The estimator \( \hat{h}^{(K)}_{\text{GMM}} \) can be computed in closed form:

\[
\hat{h}^{(K)}_{\text{GMM}}(y) = \sum_{k=1}^{K} p(k \mid y) \hat{h}_{\text{LMMSE},k}(y) \tag{7}
\]

with the responsibilities

\[
p(k \mid y) = \frac{p(k) \mathcal{N}_C(y; A\mu_k, AC_kA^H + \Sigma)}{\sum_{i=1}^{K} p(i) \mathcal{N}_C(y; A\mu_i, AC_iA^H + \Sigma)} \tag{8}
\]

and

\[
\hat{h}_{\text{LMMSE},k}(y) = C_kA^H(AC_kA^H + \Sigma)^{-1}(y - A\mu_k) + \mu_k. \tag{9}
\]

The estimator \( \hat{h}^{(K)}_{\text{GMM}} \) calculates a weighted sum of \( K \) linear minimum mean square error (LMMSE) estimators—one for each component. The weights \( p(k \mid y) \) are the probabilities that the current observation \( y \) corresponds to the \( k \)th component.

**B. Baseline Channel Estimators**

As a first baseline, we consider a sample covariance matrix based channel estimation approach, where we construct a sample covariance matrix \( C_s = \frac{1}{M} \sum_{m=1}^{M} h_my_m^* \) given the same set of training samples which is used to fit the GMM and calculate LMMSE channel estimates:

\[
\hat{h}_{\text{cov}} = C_sA^H(AC_sA^H + \Sigma)^{-1}y. \tag{10}
\]

Secondly, compressive sensing approaches commonly assume that the channel exhibits a certain structure: \( h \approx Dt \), where \( D = Dx \otimes (D_{x,h} \otimes D_{x,v}) \) is a dictionary with oversampled DFT matrices \( D_{x,h} \) and \( D_{x,v} \) (cf., e.g., \cite{19}), because we have a URA at the transmitter and a ULA at the receiver side. A compressive sensing algorithm like orthogonal matching pursuit (OMP) \cite{20} can now be used to obtain a sparse vector \( t \), and the estimated channel is then given by

\[
\hat{h}_{\text{OMP}} = Dt. \tag{11}
\]

Since the sparsity order is not known but the algorithm’s performance crucially depends on it, we use a genie-aided approach to obtain a bound on the performance of the algorithm. Namely, we use the true channel (perfect CSI knowledge) to choose the optimal sparsity order.

**V. CODEBOOK DESIGN**

**A. Proposed Codebook Construction and Encoding Scheme**

As explained around (7), the first step in computing channel estimates via \( \hat{h}^{(K)}_{\text{GMM}} \) consists of determining how likely it is that the current observation \( y \) corresponds to the \( k \)th component of the GMM \( f_h^{(K)} \), see the responsibility \( p(k \mid y) \) in (8). The idea of the proposed method now is to compute a codebook transmit covariance matrix \( Q_k \) for every component of the GMM and to use the responsibilities \( p(k \mid y) \) to determine the feedback index.

In detail, in an offline training phase, we take \( K = 2^B \) as the number of GMM components, use a training data set of channels \( H = \{h_m\}_{m=1}^{M} \) to fit a \( K \)-component GMM \( f_h^{(K)} \), and compute a codebook \( Q = \{Q_k\}_{k=1}^{K} \) of transmit covariance matrices—one matrix for every GMM component. We explain the codebook construction in another paragraph below. During the online phase, we bypass explicit channel estimation and directly determine a feedback index using the responsibilities computed via \( y \):

\[
k^* = \arg \max_k p(k \mid y). \tag{12}
\]

Thus, we compute the feedback index \( k^* \) without requiring (estimated) CSI. Note, we thereby also avoid the \( \log_2 \det \) evaluation in (2). Further, the knowledge of the codebook at the MT is not required. The MT only requires the GMM to compute (12).

We can think of \( p(k \mid y) \) as an approximation of \( p(k \mid h) \) from (6), because of the fixed noise covariance of every component. That is, since there is a true underlying channel \( h \) leading to the current observation \( y = Ah + n \), \( p(k \mid y) \) can be seen as an approximation of the probability \( p(k \mid h) \) that the channel \( h \) was generated from the \( k \)th GMM component. To both gauge the influence of using \( p(k \mid y) \) instead of \( p(k \mid h) \) and to evaluate the codebook itself, it is interesting to look at the performance of feedback information calculated as

\[
k^* = \arg \max_k p(k \mid h). \tag{13}
\]

Of course, this approach is not practically feasible because the channel \( h \) would have to be known.

**Codebook construction:** Once the training data set \( H = \{h_m\}_{m=1}^{M} \) has been used to fit a \( K \)-component GMM, we cluster the training data according to their GMM responsibilities. That is, we partition \( H \) into \( K \) disjoint sets

\[
\mathcal{V}_k = \{ h \in H \mid p(k \mid h) \geq p(j \mid h) \text{ for } k \neq j \} \tag{14}
\]

for \( k = 1, \ldots, K \). For a channel matrix \( H \) and a covariance matrix \( Q \), let

\[
r(H, Q) = \log_2 \det \left( I + \frac{1}{\sigma_n^2} HQH^H \right) \tag{15}
\]

be the spectral efficiency. We now determine the codebook \( Q = \{Q_k\}_{k=1}^{K} \) by computing every transmit covariance matrix \( Q_k \) such that it maximizes the summed rate in \( \mathcal{V}_k \):

\[
Q_k = \arg \max_{Q \geq 0} \frac{1}{|\mathcal{V}_k|} \sum_{H \in \mathcal{V}_k} r(H, Q) \tag{16}
\]

subject to \( \text{trace}(Q) \leq \rho \) and \( \text{rank} Q \leq N_{tx} \).

This optimization problem is solved via projected gradient descent (PGD), cf. \cite{5}, \cite{21}.

In summary, the GMM is used twice: Once for codebook construction (done offline) and thereafter to determine a feedback index (done online). For the latter, it is not necessary to estimate the channel and evaluating (2) is avoided.
B. Conventional Codebook Construction Methods

A standard codebook construction approach makes use of Lloyd’s algorithm [8], [9]. Given a training data set of channels \( \mathcal{H} = \{h_m\}_{m=1}^M \), the iterative Lloyd clustering algorithm alternates between two stages until a convergence criterion is met. We write \( \{Q_{k}^{(i)}\}_{k=1}^K \) for the codebook in iteration \( i \). The two stages in iteration \( i \) are:

1) Divide the training data set \( \mathcal{H} \) into \( K \) clusters \( \mathcal{V}_k^{(i)} \):

\[
\mathcal{V}_k^{(i)} = \{ h \in \mathcal{H} \mid r(H, Q_k^{(i)}) \geq r(H, Q_{j}^{(i)}) \text{ for } k \neq j \}.
\]

2) Update the codebook:

\[
Q_{k}^{(i+1)} = \arg \max_Q \frac{1}{|\mathcal{V}_k^{(i)}|} \sum_{H \in \mathcal{V}_k^{(i)}} r(H, Q)
\]

subject to \( \text{trace}(Q) \leq \rho \) and \( \text{rank}(Q) \leq N_{rx} \).

The optimization problem in stage 2) is again solved via PGD. To initialize the algorithm, stage 1) is replaced with a random partition of \( \mathcal{H} \) in the first iteration.

Lau’s heuristic: In order to avoid solving the costly optimization problem in stage 2) of every iteration, the authors of [9] provide a heuristic for the codebook update: A representative matrix \( S_k^{(i)} = \frac{1}{|\mathcal{V}_k^{(i)}|} \sum_{H \in \mathcal{V}_k^{(i)}} H H^H \) is calculated for every cluster \( \mathcal{V}_k^{(i)} \), and then the matrices \( S_k^{(i)} \) are decomposed into \( N_{rx} \) parallel streams and water-filling is employed, yielding the updated codebook entries \( Q_{k}^{(i+1)} \), see [9].

Analogously, we can replace the optimization problem in (16) with the described heuristic from [9] to compute a transmit covariance matrix for every GMM component. However, as the simulation results in Section VII show, the performance with PGD is better, especially in the high signal-to-noise ratio (SNR) regime.

VI. COMPLEXITY ANALYSIS

The responsibilities in (6) or (8) are calculated by evaluating Gaussian densities. A Gaussian density with mean \( \mu \in \mathbb{C}^N \) and covariance matrix \( C \in \mathbb{C}^{N \times N} \) can be written as

\[
\mathcal{N}_C(h; \mu, C) = \frac{\exp(- (h - \mu) H C^{-1} (h - \mu))}{\pi^N \det(C)}.
\]

Since the GMM covariance matrices and mean vectors do not change between observations, the inverse and the determinant of the densities can be pre-computed once. Therefore, the online evaluation is dominated by matrix-vector multiplications and has a complexity of \( O(N^2) \), where \( N = N_{tx} \times N_{rx} \) to evaluate \( p(k \mid h) \) in (6) (assuming perfect CSI), or \( N = n_p N_{rx} \) to evaluate \( p(k \mid y) \) in (8) using the observations \( y \).

A. Kronecker Approximation for Saving Complexity

In order for a MT to be able to compute feedback indices, the parameters of the GMM \( f_k^{(i)} \) need to be offloaded to the MT upon entering the BS’s coverage area. As demonstrated in a numerical example below, the number of GMM parameters can be quite large. This is mainly due to the large number of parameters of the GMM’s covariance matrices. As a remedy, we can constrain the GMM covariance matrices to a particular form with less parameters.

For spatial correlation scenarios, a well-known assumption is that the scattering in the vicinity of the transmitter and of the receiver are independent of each other, cf. [22]. This assumption leads to channel covariance matrices \( C \), which can be decomposed into the Kronecker product of a transmit and receive side spatial covariance matrix: \( C = C_{tx} \otimes C_{rx} \). As in [18], we use this assumption to construct a GMM consisting of Kronecker product covariance matrices \( C_k = C_{tx,k} \otimes C_{rx,k} \).

The procedure suggested thus far is to fit a single GMM using the vectorized channel training data \( \mathcal{H} = \{h_m = vec(H_m)\}_{m=1}^M \) of dimension \( N = N_{tx} N_{rx} \). This results in unconstrained GMM covariance matrices of dimension \( N \times N \). To achieve Kronecker product covariance matrices, a two stage procedure is used in [18]. First, we fit two independent “transmit and receive GMMs” with respective covariance matrices of dimensions \( N_{tx} \times N_{tx} \) and \( N_{rx} \times N_{rx} \). To this end, all rows of \( \{H_m\}_{m=1}^M \) are used to fit a \( K_{tx} \)-component transmit GMM, and all columns of \( \{H_m\}_{m=1}^M \) are used to fit a \( K_{rx} \)-component receive GMM. Thereafter, a \( K = K_{tx} K_{rx} \) component GMM with Kronecker covariance matrices of dimension \( N \times N \) is obtained by computing all Kronecker products \( C_{tx,i} \otimes C_{rx,j} \) of the transmit GMM covariance matrices \( C_{tx,i} \), and receive GMM covariance matrices \( C_{rx,j} \). Please refer to [18] for more details. The advantages of the Kronecker GMM are a lower offline training complexity, the ability to parallelize the fitting process, and the need for fewer training samples since the Kronecker GMM has much fewer parameters.

Numerical example: To illustrate the difference in the number of GMM parameters, we plug in the simulation parameters which we consider in Section VII. There, we have, \( N_{tx} = 32, N_{rx} = 16, K_{tx} = 16 \) and \( K_{rx} = 4 \), which yields \( N = N_{tx} N_{rx} = 512 \) and \( K = K_{tx} K_{rx} = 64 \). The normal GMM consists of \( K = 64 \) covariance matrices of dimension \( N \times N \) which means that it has \( K \frac{N(N+1)}{2} = 804992 \) covariance parameters (taking symmetries into account). By contrast, the Kronecker GMM has only \( K_{tx} N_{tx}(N_{tx}+1)/2 + K_{rx} N_{rx}(N_{rx}+1) = 8992 \) covariance parameters. Therefore, with the Kronecker GMM, the number of parameters which need to be offloaded is drastically reduced. For this reason, we consider the Kronecker GMM in Section VII.

VII. SIMULATION RESULTS

The BS equipped with a URA has in total \( N_{tx} = 32 \) antenna elements, with \( N_{tx,v} = 4 \) vertical and \( N_{tx,h} = 8 \) horizontal elements. At the MT we have a ULA with \( N_{rx} = 16 \). We consider \( B = 6 \) feedback bits and thus \( K = 2^B = 64 \).

We generate datasets with \( 30 \cdot 10^3 \) channels for both the UL and DL domain of the scenario: \( \mathcal{H}_{UL} \) and \( \mathcal{H}_{DL} \). The UL channels have a dimension of \( 32 \times 16 \) and the DL channels have a dimension of \( 16 \times 32 \). The data samples are normalized such that \( E[|h|^2] = N = N_{tx} N_{rx} \) holds for the vectorized channels. We further set \( \rho = 1 \) which allows us to define the SNR as \( \frac{1}{\rho} \).

We split the two sets \( \mathcal{H}_{UL} \) and \( \mathcal{H}_{DL} \) into a training set with \( M = 20 \cdot 10^3 \) samples, and the remaining samples constitute
an evaluation set: $H^\text{UL}_{\text{train}}$, $H^\text{UL}_{\text{eval}}$, $H^\text{DL}_{\text{train}}$, and $H^\text{DL}_{\text{eval}}$. However, the UL evaluation set $H^\text{UL}_{\text{eval}}$ is not relevant for our considerations and the following transmit strategies are always evaluated on $H^\text{DL}_{\text{eval}}$, i.e., in the DL domain. When we fit the GMM based on $H^\text{UL}_{\text{train}}$, we transpose all elements of the set to emulate a DL.

In the following, we depict the normalized spectral efficiency (nSE) as performance measure. The spectral efficiencies achieved with a given transmit covariance matrix are normalized by the spectral efficiency achieved with the optimal transmit covariance matrix which is given by decomposing the channel into $N_{\text{rx}}$ parallel streams and employing water-filling [12]. The empirical complementary cumulative distribution function (cCDF) $P(n\text{SE} > s)$ of the normalized spectral efficiency denoted by the variable $s$ (the corresponding random variable is simply denoted by nSE), is used to depict the empirical probability that the nSE exceeds a specific value $s$.

We consider the following baseline transmit strategies:

i) The curves labeled “uni pow cov” represent uniform power allocation where the transmit covariance matrix is given by $Q = \frac{\mathbf{I}}{N_{\text{rx}}}$. In this case, no CSI knowledge or codebook is used.

ii) Moreover, “uni pow eigsp” depicts the transmit strategy where a transmit covariance matrix is calculated by allocating equal power on the eigenvectors of the channel. That is, the channel is decomposed into $N_{\text{rx}}$ parallel streams and the equal power is allocated to each stream. Note, this approach is infeasible because the BS would require full knowledge of the DL channel (or its eigenvectors).

In Fig. 1(a), we set the SNR = 0 dB. The conventional codebook construction approaches are denoted by “Lloyd PGD UL/DL” and “Lloyd Lau UL/DL”, depending on whether PGD or Lau’s heuristic is used to update the codebook in the second stage of the iterative Lloyd clustering algorithm, and depending on whether $H^\text{UL}_{\text{train}}$ or $H^\text{DL}_{\text{train}}$ is used as training data to construct the codebooks. With these approaches, the codebook is known to the BS and the MT and additionally perfect CSI is assumed at the MT. Each user then selects the best possible codebook entry by evaluating (2). PGD seems to be slightly better than Lau’s heuristic. Further, using DL or UL training data results in approximately the same performance.

The proposed codebook construction and encoding scheme is denoted by “GMM PGD UL/DL” and “GMM Lau UL/DL”, again depending on whether PGD or Lau’s heuristic is used to construct the codebook. We either use $H^\text{UL}_{\text{train}}$ or $H^\text{DL}_{\text{train}}$ as training data to fit the GMM and to construct the codebook as described in V-A. With our proposed approach, the knowledge of the codebook at the MT is not required. After offloading the GMM to the MT and given perfect CSI knowledge, the MT can then simply determine the feedback index by evaluating (13). Again, PGD is slightly better than Lau’s heuristic, and using DL or UL training data results in approximately the same performance. The proposed GMM approach performs slightly worse in comparison to the conventional Lloyd clustering approach. In Fig. 1(b), we set SNR = 10 dB, and observe similar results. Interestingly, “GMM Lau UL/DL” performs better than “Lloyd Lau UL/DL”.

However, assuming perfect CSI at the MT is not feasible. In fact, it is desired to obtain relatively good system performances with estimated CSI, where typically only a fraction of the number of transmit antennas $N_{\text{tx}}$ is used as the number of pilots $n_p$, i.e., when considering systems with reduced pilot overhead. In the following, we consider the proposed codebook construction and encoding scheme and the conventional Lloyd clustering algorithm exclusively with PGD due to its superior performance. Additionally, we only consider UL training data in the remainder.

In Fig. 2(a), the SNR = 0 dB and we have $n_p = 8$. We depict results for the conventional Lloyd clustering approach, where we first estimate the channel either via OMP (11), or the sample covariance approach (10), or via the GMM estimator (7), and then select a transmit covariance matrix by evaluating (2) given the estimated channel: “Lloyd PGD, $\hat{h}_{\text{OMP}}$”, “Lloyd PGD, $\hat{h}_{\text{cov}}$”, and “Lloyd PGD, $\hat{h}_{\text{GMM}}$”. As can be seen, estimating the channel via the GMM estimator gives the best performance when considering the conventional approach.

By contrast, with our proposed approach denoted by “GMM PGD, $y$”, where we bypass channel estimation and directly evaluate (12) for determining a feedback index, we achieve an even better performance as compared to the conventional approach. With the curves “Lloyd PGD, $h$”, and “GMM PGD, $h$” we depict the case of assuming perfect CSI knowledge (this is a performance bound). A similar observation can also be made in Fig. 2(b), where the SNR = 15 dB and we only have $n_p = 4$ pilots.

In Fig. 3(a), we set SNR = 0 dB and in Fig. 3(b) we have SNR = 5 dB, we fix $s = 0.8$ and consider $P(n\text{SE} > 0.8)$ for a varying number of pilots $n_p$. We see, that our proposed
approach is especially beneficial in the low number of pilots regime and outperforms the conventional approach, which requires both channel estimation and the evaluation of (2).

VIII. CONCLUSION AND OUTLOOK

We proposed a codebook construction and feedback encoding scheme which is based on GMMs. The proposed approach involves an offline phase where a GMM is fitted and a codebook is constructed at the BS using solely UL data. In the online phase, the same GMM, which is offloaded to a MT upon entering the coverage area of the BS, was used for feedback encoding. Simulation results confirmed the validity of this approach, especially in configurations with a reduced pilot overhead. In future work, we will investigate the GMM based feedback encoding principle for systems with multiple users.

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