Radiative Corrections to the CKM Unitarity Triangles

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Abstract

The $3 \times 3$ CKM matrix defines six unitarity triangles in the complex plane, which will be carefully explored in the LHCb experiment and at the Super-$B$ factory. We calculate the running effects of nine different inner angles and eighteen different sides of the six triangles from the electroweak scale to a superhigh-energy scale by using the one-loop renormalization-group equations, and demonstrate that all the nine angles are stable against radiative corrections. In particular, we find that the CP-violating angle $\alpha$ is most insensitive to the changes of energy scales.

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The 10-year running of the BARBAR and Belle $B$-meson factories has greatly improved our knowledge on the Cabibbo-Kobayashi-Maskawa (CKM) matrix $V$, which describes the flavor-changing effects in weak charged-current interactions of $(u, c, t)$ and $(d, s, b)$ quarks. In particular, the observed CP-violating asymmetries in a number of $B$ decays have unambiguously and consistently verified the Kobayashi-Maskawa (KM) mechanism of CP violation in the standard model (SM). More extensive and precise studies of quark flavor mixing and CP violation will be done definitely in the LHCb experiment and hopefully at the super-$B$ factory, from which one may test the KM picture to an unprecedented degree of accuracy.

Theorists expect that the observed flavor puzzles at low energies, such as the strong hierarchies of quark masses and flavor mixing angles, should be resolved in a predictive flavor theory at a superhigh-energy scale $\Lambda$. Such a theory can be confronted with the experimental data via the renormalization-group equations (RGEs) which run its predictions from $\Lambda$ down to the electroweak scale. One may phenomenologically do the opposite — running the parameters of quark flavor mixing and CP violation from the electroweak scale up to $\Lambda$ and examining their sensitivities to radiative corrections. A comparison between theoretical predictions and experimental measurements will be available once the wanted flavor theory is in hand. So far a lot of works have been done to investigate the RGE running effects of the CKM matrix elements with or without the help of a specific parametrization.

The present paper aims to analyze how the unitarity triangles (UTs) of the CKM matrix $V$, both their sides and their angles, evolve with the energy scales. This analysis makes sense because the precision measurements to be done in the LHCb experiments and at the super-$B$ factory will probe all the six UTs or at least several of them, in order to test the KM mechanism of CP violation and explore possible new physics beyond it. We derive the one-loop RGEs for the $3 \times 3$ CKM angle matrix $\Phi$ proposed recently by Harrison et al., and demonstrate that its nine angles are all stable against radiative corrections. In particular, we find that the CP-violating angle $\Phi_{cs} = \alpha$ is most insensitive to the RGE running effect. We also present the one-loop RGEs for the eighteen sides of six UTs and for the Jarlskog invariant of CP violation. The running behaviors of these quantities are numerically illustrated by assuming $\Lambda \sim 10^{14}$ GeV, which is very close to the scale of grand unified theories or to the scale of conventional seesaw mechanisms in the framework of either the SM or the minimal supersymmetric standard model (MSSM). Our results are expected to be helpful for building quantitatively viable flavor models at superhigh-energy scales.
Because the $3 \times 3$ CKM matrix $V$ is unitary, its nine elements satisfy the following normalization and orthogonality conditions:

$$\sum_{\alpha} V_{\alpha i} V_{\alpha j}^* = \delta_{ij}, \quad \sum_{i} V_{\alpha i} V_{\beta i}^* = \delta_{\alpha \beta},$$

where the Greek and Latin subscripts run over $(u, c, t)$ and $(d, s, b)$, respectively. The six orthogonality relations geometrically define six UTs in the complex plane, as illustrated in Fig. 1. The six UTs have eighteen different sides and nine different inner angles [4], but their areas are all identical to $J/2$ with $J$ being the Jarlskog invariant of CP violation [6] defined through

$$\text{Im} \left( V_{\alpha i} V_{\beta j}^* V_{\alpha j} V_{\beta i}^* \right) = J \sum_{\gamma} \epsilon_{\alpha \beta \gamma} \sum_{k} \epsilon_{ijk}.$$

Nine inner angles of the CKM UTs can be defined as

$$\Phi_{\alpha i} \equiv \arg \left( -\frac{V_{\beta j} V_{\gamma j}^*}{V_{\beta k} V_{\gamma k}^*} \right),$$

where $\alpha, \beta$ and $\gamma$ run co-cyclically over $u, c$ and $t$, while $i, j$ and $k$ run co-cyclically over $d, s$ and $b$. Then one may write out the CKM angle matrix [5]

$$\Phi = \begin{pmatrix} \Phi_{ud} & \Phi_{us} & \Phi_{ub} \\ \Phi_{cd} & \Phi_{cs} & \Phi_{cb} \\ \Phi_{td} & \Phi_{ts} & \Phi_{tb} \end{pmatrix}.$$

Note that the angle $\Phi_{\alpha i}$ is just the inner angle shared by the UTs $\Delta_{\alpha}$ (for $\alpha = u, c$ or $t$) and $\Delta_{i}$ (for $i = d, s$ or $b$), as one can easily see in Fig. 1. Hence each row or column of the CKM angle matrix $\Phi$ corresponds to one UT, and its three matrix elements satisfy the normalization conditions

$$\sum_{\alpha} \Phi_{\alpha i} = \sum_{i} \Phi_{\alpha i} = \pi.$$

This result implies that one can have two off-diagonal asymmetries of $\Phi$ about its $\Phi_{ud}$-$\Phi_{cs}$-$\Phi_{tb}$ and $\Phi_{ub}$-$\Phi_{cs}$-$\Phi_{td}$ axes, respectively $^1$:

$$\mathcal{A}_L \equiv \Phi_{us} - \Phi_{cd} = \Phi_{cb} - \Phi_{ts} = \Phi_{td} - \Phi_{ub};$$

$$\mathcal{A}_R \equiv \Phi_{us} - \Phi_{cb} = \Phi_{cd} - \Phi_{ts} = \Phi_{tb} - \Phi_{ud}.$$

$^1$ The off-diagonal asymmetries of the CKM matrix $V$, given as $\Delta_{L} \equiv |V_{us}|^2 - |V_{cd}|^2 = |V_{cb}|^2 - |V_{ts}|^2 = |V_{td}|^2 - |V_{ub}|^2$ and $\Delta_{R} \equiv |V_{us}|^2 - |V_{cb}|^2 = |V_{cd}|^2 - |V_{ts}|^2 = |V_{tb}|^2 - |V_{ud}|^2$, have been discussed in Ref. [7]. We are able to write down the relations between these two types of off-diagonal asymmetries:

$$\sin \mathcal{A}_L = \frac{-\Delta_L J}{|V_{us}||V_{ub}||V_{cd}||V_{cb}||V_{td}||V_{ts}|}$$

and

$$\sin \mathcal{A}_R = \frac{-\Delta_R J}{|V_{ud}||V_{us}||V_{cd}||V_{cb}||V_{tb}||V_{ts}|}.$$
If $A_L = 0$ or $A_R = 0$ held, we would be left with three pairs of congruent UTs:

$$A_L = 0 \implies \triangle_u \cong \triangle_d, \ \triangle_c \cong \triangle_s, \ \triangle_t \cong \triangle_b;$$

$$A_R = 0 \implies \triangle_u \cong \triangle_b, \ \triangle_c \cong \triangle_s, \ \triangle_t \cong \triangle_d. \quad (7)$$

Interestingly, the same expectation would be true if one of the off-diagonal asymmetries of the CKM matrix $V$ itself (i.e., $\Delta_L$ or $\Delta_R$) were exactly vanishing.

In Ref. [5], the notations of $\Phi_{\alpha i}$ have been linked to the conventional ones used to denote the inner angles of the most popular UT $\Delta_s$ and some other CP-violating phases: $\Phi_{ud} = \beta_s = \chi$, $\Phi_{us} = \beta_1 = \phi_1$, $\Phi_{cd} = \gamma' = \gamma - \delta\gamma$, $\Phi_{cs} = \alpha = \phi_2$, $\Phi_{cb} = \beta + \delta\gamma$, $\Phi_{ts} = \gamma = \phi_3$ and $\Phi_{tb} = \beta_K = \chi'$. The present experimental data on CP violation yield

$$\Phi = \left(\begin{array}{ccc}
1.04^\circ \pm 0.05^\circ & 21.58^\circ \pm 0.86^\circ & 157.38^\circ \pm 0.89^\circ \\
66.82^\circ \pm 4.20^\circ & 90.60^\circ \pm 4.00^\circ & 22.58^\circ \pm 0.89^\circ \\
112.14^\circ \pm 4.21^\circ & 67.82^\circ \pm 4.22^\circ & 0.035^\circ \pm 0.003^\circ
\end{array}\right), \quad (8)$$

where the normalization conditions in Eq. (5) have been used. We can therefore obtain $A_L = \beta - \gamma + \delta\gamma \approx -45^\circ$ and $A_R = -\delta\gamma \approx -1^\circ$, in contrast with $\Delta_L \approx 6.4 \times 10^{-5}$ and $\Delta_R \approx 5.1 \times 10^{-2}$. In other words, the CKM matrix $V$ is almost symmetric about its $V_{ud}-V_{cs}-V_{tb}$ axis, while the CKM angle matrix $\Phi$ is approximately symmetric about its $\Phi_{ub}-\Phi_{cs}-\Phi_{td}$ axis. The nine angles $\Phi_{\alpha i}$ signify different CP-violating effects in a variety of $B-$, $D-$ and $K$-meson decay modes. So it is very desirable to determine $\Phi$ as precisely as possible in the upcoming LHCb experiment and at the future super-$B$ factory, in order to precisely test the CKM unitarity. In this sense we argue that the CKM angle matrix $\Phi$ is a useful phenomenological language to describe CP violation.

We proceed to derive the one-loop RGEs of the CKM angle matrix $\Phi$. The one-loop RGEs of the gauge couplings and charged-lepton and quark Yukawa couplings have already been calculated by several authors [10]. Here we make use of their results for the RGEs of the CKM matrix elements $|V_{\alpha i}|^2$ by taking account of $y_u^2 \ll y_c^2 \ll y_t^2$ and $y_d^2 \ll y_s^2 \ll y_b^2$.

Note that the unitarity of the CKM matrix $V$ allows one to determine the moduli $|V_{\alpha i}|$ in terms of the angles $\Phi_{\alpha i}$ (for $\alpha = u, c, t$ and $i = d, s, b$). Hence every pair of the UTs in Eq. (7) would not only be similar to each other but also be congruent with each other, if $A_L = 0$ or $A_R = 0$ were given.

We neglect the tiny RGE effects associated with three neutrinos by assuming that possible heavy degrees of freedom responsible for their mass generation (e.g., heavy particles in the seesaw mechanisms [11]) are decoupled at $\Lambda$ and thus do not affect the RGEs of charged-lepton and quark Yukawa couplings.
where $y_a$ and $y_t$ stand respectively for the eigenvalues of the Yukawa coupling matrices of up- and down-type quarks. In this excellent approximation we have [12]

$$16\pi^2 \frac{d}{dt} \left[ |V_{ud}|^2 |V_{us}|^2 |V_{ub}|^2 |V_{cd}|^2 |V_{cs}|^2 |V_{cb}|^2 |V_{td}|^2 |V_{ts}|^2 |V_{tb}|^2 \right]$$

$$= 2C y_b^2 \left[ |V_{ud}|^2 |V_{ub}|^2 |V_{us}|^2 |V_{ub}|^2 - |V_{ub}|^2 (1 - |V_{ub}|^2) \right]$$

$$+ 2C y_t^2 \left[ |V_{cd}|^2 |V_{td}|^2 |V_{cd}|^2 |V_{tb}|^2 - |V_{cd}|^2 |V_{tb}|^2 - |V_{cd}|^2 |V_{tb}|^2 - |V_{cd}|^2 |V_{tb}|^2 \right]$$

$$+ \frac{d}{dt} |\cos \phi| = \frac{d}{dt} \left( \frac{|V_{ub}|^2 |V_{ud}|^2 + |V_{tb}|^2 |V_{td}|^2 - |V_{cb}|^2 |V_{cd}|^2}{2 |V_{ub}| |V_{ud}| |V_{tb}| |V_{td}|} \right)$$

$$= \frac{1}{4} \left( \frac{|V_{ub}| |V_{ud}|}{|V_{ub}||V_{ud}|} + \frac{|V_{tb}| |V_{td}|}{|V_{tb}||V_{td}|} \right) \left( \frac{1}{|V_{ub}|^2 |V_{ud}|^2} \frac{d}{dt} |V_{ub}|^2 + \frac{1}{|V_{tb}|^2 |V_{td}|^2} \frac{d}{dt} |V_{tb}|^2 \right)$$

$$+ \frac{1}{|V_{ub}|^2 |V_{ud}|^2} \frac{d}{dt} |V_{ub}|^2 + \frac{1}{|V_{tb}|^2 |V_{td}|^2} \frac{d}{dt} |V_{tb}|^2$$

where $t \equiv \ln(\mu/M_Z)$, $C = -1.5$ in the SM and $C = +1$ in the MSSM. We first derive the RGE of $\Phi_{cs}$. This angle is related to the sides of the UT $\Delta s$ through the cosine rule $2 |V_{ub}| |V_{td}| |V_{tb}| |V_{td}| \cos \phi = |V_{ub}|^2 |V_{td}|^2 + |V_{tb}|^2 |V_{td}|^2 - |V_{cb}|^2 |V_{cd}|^2$. Therefore,

$$\frac{d}{dt} \cos \phi = \frac{1}{2 |V_{ub}| |V_{ud}| |V_{tb}| |V_{td}|} \left( \frac{1}{|V_{ub}|^2 |V_{ud}|^2} \frac{d}{dt} |V_{ub}|^2 + \frac{1}{|V_{tb}|^2 |V_{td}|^2} \frac{d}{dt} |V_{tb}|^2 \right)$$

Substituting the relevant expressions of $d|V_{\alpha}|^2/dt$ given in Eq. (9) into the right-hand side of Eq. (10), we immediately arrive at

$$16\pi^2 \frac{d}{dt} \Phi_{cs} = -16\pi^2 \frac{1}{\sin \Phi_{cs}} \frac{d}{dt} \cos \phi_{cs} = 0.$$  \hspace{1cm} (11)

This result is consistent with $d\alpha/dt = 0$ obtained in Ref. [12] from a slightly different way. With the help of Eqs. (9) and (11), we may easily calculate the RGE of the Jarlskog invariant $J$ from the relation $J = |V_{ud}| |V_{ub}| |V_{td}| |V_{tb}| \sin \Phi_{cs}$:

$$16\pi^2 \frac{d}{dt} J = 16\pi^2 \sin \Phi_{cs} \frac{d}{dt} (|V_{ud}| |V_{ub}| |V_{td}| |V_{tb}|)$$

$$= 8\pi^2 J \left( \frac{1}{|V_{ub}|^2 |V_{ud}|^2} \frac{d}{dt} |V_{ub}|^2 + \frac{1}{|V_{td}|^2 |V_{ub}|^2} \frac{d}{dt} |V_{td}|^2 + \frac{1}{|V_{tb}|^2 |V_{td}|^2} \frac{d}{dt} |V_{tb}|^2 \right)$$

$$= -2C J \left[ y_b^2 (|V_{tb}|^2 - |V_{ub}|^2) + y_t^2 (|V_{td}|^2 - |V_{td}|^2) \right].$$  \hspace{1cm} (12)
The one-loop RGEs for other angles of the CKM UTs can then be derived from Eqs. (9) and (12) by using the sine rule (and the cosine rule) repeatedly. For instance, the relationship $J = |V_{cs}| |V_{cb}| |V_{ts}| |V_{tb}| \sin \Phi_{ud} = |V_{cd}| |V_{cb}| |V_{td}| |V_{tb}| \sin \Phi_{us} = |V_{us}| |V_{ub}| |V_{ts}| |V_{tb}| \sin \Phi_{cd}$ allows us to separately calculate the RGEs of $\Phi_{ud}$, $\Phi_{us}$ and $\Phi_{cd}$. We find

$$16\pi^2 \frac{d}{dt} \Phi_{ud} = -2C \left( y_b^2 + y_t^2 \right) \frac{J}{|V_{cs}|^2},$$

$$16\pi^2 \frac{d}{dt} \Phi_{us} = -2Cy_b^2 \frac{J}{|V_{cd}|^2},$$

$$16\pi^2 \frac{d}{dt} \Phi_{cd} = -2Cy_t^2 \frac{J}{|V_{us}|^2}. \quad (13)$$

Taking account of Eq. (5) together with Eqs. (11) and (13), one may simply figure out the RGEs for the other five angles of $\Phi$. Our main results for the RGEs of $\Phi$ are summarized as

$$16\pi^2 \frac{d}{dt} \begin{pmatrix} \Phi_{ud} \\ \Phi_{us} \\ \Phi_{tb} \\ \Phi_{cd} \\ \Phi_{ts} \end{pmatrix} = 2CJ \begin{pmatrix} y_b^2 \\ |V_{cd}|^2 |V_{cs}|^2 \\ -|V_{cd}|^2 - |V_{cs}|^2 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} -|V_{us}|^2 & -|V_{cs}|^2 & 1 - |V_{cb}|^2 \\ 0 & 0 & 0 \\ |V_{cd}|^2 & |V_{cs}|^2 & |V_{cb}|^2 - 1 \end{pmatrix} \begin{pmatrix} y_b^2 \\ |V_{us}|^2 |V_{cs}|^2 \\ -|V_{us}|^2 \\ 0 \end{pmatrix} + \begin{pmatrix} y_t^2 \\ |V_{us}|^2 |V_{cs}|^2 \\ -|V_{cs}|^2 \\ 1 - |V_{ts}|^2 \\ 0 \end{pmatrix} \begin{pmatrix} -|V_{us}|^2 & 0 & |V_{us}|^2 \\ 0 & |V_{cs}|^2 & 1 - |V_{ts}|^2 \end{pmatrix}. \quad (14)$$

Some discussions are in order.

- The angle $\Phi_{cs} = \alpha$ is most insensitive to the RGE running effect. As shown in Eq. (8), current experimental data point to $\Phi_{cs} = \alpha = \pi/2$ to an excellent degree of accuracy, implying that the UTs $\Delta_c$ and $\Delta_s$ are almost the right triangles. This possibility was conjectured long time ago in an attempt to explore the realistic textures of quark mass matrices \[13\]. Some interest has recently been paid to whether there is a good reason for $\Phi_{cs} = \alpha = \pi/2$ in understanding quark flavor mixing and CP violation \[5, 12, 14\].

- Other angles of $\Phi$ receive negligibly small radiative corrections. $\Phi_{tb}$ should be most sensitive to the RGE running effect: $16\pi^2 d\Phi_{tb}/dt \approx -2CJ(y_b^2 + y_t^2)/|V_{us}|^2$ holds in the approximations of $|V_{cd}| \approx |V_{us}|$, $|V_{cs}| \approx 1$ and $|V_{cb}|^2 - 1 \approx |V_{ts}|^2 - 1 \approx -1$. Considering $J \approx 3.0 \times 10^{-5}$, $|V_{us}|^2 \approx 5.1 \times 10^{-2}$ and $y_b^2 \lesssim y_t^2 \sim O(1)$ in the SM or MSSM, we can roughly obtain $|\Phi_{tb}(\Lambda) - \Phi_{tb}(M_Z)| \lesssim O(10^{-5}) \ln(\Lambda/M_Z)$. Hence the RGE correction to $\Phi_{tb}$ is expected to be of $O(10^{-4})$ for $\Lambda \sim 10^{16}$ GeV.
The stability of $\Phi_{\alpha i}$ (for $\alpha = u,c,t$ and $i = d,s,b$) against radiative corrections indicates that the shape of every UT is almost unchanged from $M_Z$ to $\Lambda$ or vice versa. So one may directly confront the experimental data on these CP-violating angles at low energies with the predictions from a superhigh-energy flavor model.

Although the shape of each UT is insensitive to the changes of energy scales, its three sides may not be so. With the help of Eq. (9), it is straightforward to obtain the following approximate RGEs for eighteen sides of the six UTs:

$$\begin{align*}
\Delta_u & : \quad \frac{d}{dt} \ln |V_{cd}V_{td}^*| \approx \frac{d}{dt} \ln |V_{cs}V_{ts}^*| \approx \frac{d}{dt} \ln |V_{cb}V_{tb}^*| \approx - \frac{C (y_b^2 + y_t^2)}{16\pi^2}, \\
\Delta_c & : \quad \frac{d}{dt} \ln |V_{ud}V_{td}^*| \approx \frac{d}{dt} \ln |V_{us}V_{ts}^*| \approx \frac{d}{dt} \ln |V_{ub}V_{tb}^*| \approx - \frac{2C (y_b^2 + y_t^2)}{16\pi^2}, \\
\Delta_t & : \quad \frac{d}{dt} \ln |V_{us}V_{ub}^*| \approx \frac{d}{dt} \ln |V_{cs}V_{cb}^*| \approx \frac{d}{dt} \ln |V_{ts}V_{tb}^*| \approx - \frac{C (y_b^2 + y_t^2)}{16\pi^2}, \\
\Delta_d & : \quad \frac{d}{dt} \ln |V_{us}V_{ub}^*| \approx \frac{d}{dt} \ln |V_{cd}V_{cb}^*| \approx \frac{d}{dt} \ln |V_{td}V_{tb}^*| \approx - \frac{C (y_b^2 + y_t^2)}{16\pi^2}, \\
\Delta_s & : \quad \frac{d}{dt} \ln |V_{ud}V_{ub}^*| \approx \frac{d}{dt} \ln |V_{cd}V_{cs}^*| \approx 0, \quad \frac{d}{dt} \ln |V_{td}V_{ts}^*| \approx - \frac{2C (y_b^2 + y_t^2)}{16\pi^2}.
\end{align*}$$

(15)

This result is apparently consistent with the one obtained in Eq. (14) for nine angles of the UTs. For example, three sides of $\Delta_u$ run with energy scales in the same way, and thus its three inner angles keep unchanged from $M_Z$ to $\Lambda$ or vice versa. The UTs $\Delta_c$, $\Delta_d$ and $\Delta_s$ have the same RGE running behaviors as $\Delta_u$ does. As for the UT $\Delta_t$ or $\Delta_b$, its two long sides are stable against radiative corrections but its short side slightly changes with energy scales. Because the ratio of the short side to one of the long sides is of $\mathcal{O}(10^{-3})$ for either $\Delta_t$ or $\Delta_b$, in accordance with its smallest inner angle $\Phi_{tb} \approx 0.035^\circ$ at $M_Z$, the slight running of the short side has little effect on three inner angles.

To illustrate, we use the central values of the CKM angle matrix elements in Eq. (8) to calculate the RGE running effects of the Jarlskog invariant $\mathcal{J}$ and eighteen sides of six UTs from $M_Z$ up to $\Lambda \sim 10^{14}$ GeV. Our numerical results are shown in Fig. 2 and Table 1, where the Higgs mass $m_H = 140$ GeV in the SM and the parameter $\tan \beta = 10$ or 50 in the MSSM have typically been input. One can imagine that the shape of every UT expands for $\Lambda > M_Z$ in the SM, such that both its area ($= \mathcal{J}/2$) and sides become larger and larger when the energy scale increases. The running behaviors of six UTs are opposite in the MSSM: both the magnitude of $\mathcal{J}$ and those of eighteen sides become smaller and smaller when the energy
scale increases. But the nine angles of $\Phi$ are rather stable against radiative corrections either in the SM or in the MSSM. We have numerically confirmed that the values of $\Phi_{\alpha i}$ given in Eq. (8) keep unchanged even when the energy scale goes to $\Lambda \sim 10^{14}$ or higher $^4$.

In summary, we have calculated the one-loop RGEs for the CKM angle matrix $\Phi$ and the sides of six UTs. We have clearly demonstrated that all the nine angles of $\Phi$ are insensitive to radiative corrections. In particular, the angle $\Phi_{cs} = \alpha$ is most insensitive to the changes of energy scales. As for the UTs $\triangle_u$, $\triangle_c$, $\triangle_d$ and $\triangle_s$, we find that their sides have the same RGE running behaviors which are more or less different from those of the two sharp-angled UTs $\triangle_t$ and $\triangle_b$. Our results convincingly indicate that the experimental data on nine CP-violating angles $\Phi_{\alpha i}$ (for $\alpha = u, c, t$ and $i = d, s, b$), which will be well measured in the upcoming LHCb experiment and at the future super-$B$ factory, can directly be confronted with the predictions from a superhigh-energy flavor theory.

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$^4$ We find that this observation is also true when using the two-loop RGEs of charged-lepton and quark Yukawa couplings to do the numerical calculations $^{15}$. 

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FIG. 1: Schematic diagrams for six UTs of the CKM matrix in the complex plane, where each triangle is named by the index that does not manifest in its three sides.
FIG. 2: Running behaviors of the Jarlskog invariant $\mathcal{J}$ from $M_Z$ to a superhigh-energy scale in the SM or in the MSSM, where $\mathcal{J} = 2.95 \times 10^{-5}$ at $M_Z$ has been input.
TABLE I: Radiative corrections to the sides of six UTs at $\Lambda \sim 10^{14}$ GeV in the SM or MSSM.

| $\Delta$   | $M_Z$  | $\Lambda \sim 10^{14}$ GeV |
|------------|--------|-----------------------------|
|            | $|V_{cd}V_{td}^*|$ | SM ($m_H = 140$ GeV) | MSSM (tan $\beta = 10$) | MSSM (tan $\beta = 50$) |
| $\Delta_u$ | $1.95 \times 10^{-3}$ | $2.19 \times 10^{-3}$ | $1.74 \times 10^{-3}$ | $1.59 \times 10^{-3}$ |
|            | $3.94 \times 10^{-2}$ | $4.44 \times 10^{-2}$ | $3.52 \times 10^{-2}$ | $3.23 \times 10^{-2}$ |
|            | $4.12 \times 10^{-2}$ | $4.65 \times 10^{-2}$ | $3.68 \times 10^{-2}$ | $3.38 \times 10^{-2}$ |
| $\Delta_c$ | $8.40 \times 10^{-3}$ | $9.47 \times 10^{-3}$ | $7.50 \times 10^{-3}$ | $6.89 \times 10^{-3}$ |
|            | $9.14 \times 10^{-3}$ | $1.030 \times 10^{-2}$ | $8.16 \times 10^{-3}$ | $7.49 \times 10^{-3}$ |
|            | $3.51 \times 10^{-3}$ | $3.96 \times 10^{-3}$ | $3.13 \times 10^{-3}$ | $2.88 \times 10^{-3}$ |
| $\Delta_t$ | $0.219$ | $0.220$ | $0.220$ | $0.220$ |
|            | $0.219$ | $0.220$ | $0.220$ | $0.220$ |
|            | $1.45 \times 10^{-4}$ | $1.84 \times 10^{-4}$ | $1.16 \times 10^{-4}$ | $0.97 \times 10^{-4}$ |
| $\Delta_d$ | $7.93 \times 10^{-4}$ | $8.94 \times 10^{-4}$ | $7.08 \times 10^{-4}$ | $6.50 \times 10^{-4}$ |
|            | $4.02 \times 10^{-2}$ | $4.53 \times 10^{-2}$ | $3.59 \times 10^{-2}$ | $3.29 \times 10^{-2}$ |
|            | $4.05 \times 10^{-2}$ | $4.56 \times 10^{-2}$ | $3.61 \times 10^{-2}$ | $3.32 \times 10^{-2}$ |
| $\Delta_s$ | $3.42 \times 10^{-3}$ | $3.86 \times 10^{-3}$ | $3.06 \times 10^{-3}$ | $2.80 \times 10^{-3}$ |
|            | $9.30 \times 10^{-3}$ | $1.05 \times 10^{-2}$ | $8.31 \times 10^{-3}$ | $7.63 \times 10^{-3}$ |
|            | $8.62 \times 10^{-3}$ | $9.71 \times 10^{-3}$ | $7.69 \times 10^{-3}$ | $7.06 \times 10^{-3}$ |
| $\Delta_g$ | $0.220$ | $0.220$ | $0.220$ | $0.220$ |
|            | $0.220$ | $0.219$ | $0.220$ | $0.220$ |
|            | $3.49 \times 10^{-4}$ | $4.44 \times 10^{-4}$ | $2.78 \times 10^{-4}$ | $2.35 \times 10^{-4}$ |