The Effects of Non-Local Interactions in Rare $B$ Decays, $B \to X_s l^+ l^-$

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Abstract

The effects of non-local interactions in rare $B$ decays, $B \to X_s l^+ l^-$, are investigated. We show the correlation between the branching ratio and the forward-backward asymmetry via two coefficients of the non-local interactions. This will certainly help us find any deviations from the standard model through the non-local interactions.
I Introduction

Flavor changing neutral current (FCNC) processes are possibly the most sensitive to the various theoretical extensions of the standard model (SM) because these decays occur in the SM only through loops. Non-standard model effects can manifest themselves in these rare decays through the Wilson coefficients, which can have values distinctly different from their standard model counterparts. (See, for example, [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11] for the model dependent analysis.) Previously we gave the model-independent analysis on rare $B$ meson decays $B \to X_s l^+ l^-$ in Refs. [12, 13], where we dealt with all the possible local interactions and investigated the property of these interactions. However, we did not include the non-local interactions, just for simplicity. These non-local interactions influence the process $B \to X_s \gamma$. This process is a kind of “rare” decay with quite large branching ratio, of order of $10^{-4}$ [14], and it has been studied extensively in Refs. [15, 16, 17, 18]. Compared to $B \to X_s \gamma$, the decay $B \to X_s l^+ l^-$ is much more sensitive to the actual form of the new interactions since we can measure experimentally various kinematical distributions as well as the total rate. While new physics can change only the systematically uncertain normalization for $B \to X_s \gamma$, the interplay of various operators will change the spectra of the decay $B \to X_s l^+ l^-$. E.g., the experimental observation of $B \to X_s \gamma$ restricts the absolute value of the Wilson coefficient $C_7^{eff}$ of the non-local interaction $O_7$, however, we cannot determine the sign of the $C_7^{eff}$ from the decay rate of $B \to X_s \gamma$. But, if we analyze the interference between the non-local interactions and the other operators in the process $B \to X_s l^+ l^-$, we can extract much more information about $C_7^{eff}$. Therefore, to search for new physics, it would be most interesting to have a model-independent study for the non-local interactions in $B \to X_s l^+ l^-$ decays.

Here we consider the branching ratio and the forward-backward (FB) asymmetry of inclusive $B \to X_s e^+ e^-$ or $B \to X_s \mu^+ \mu^-$ decay, which are functions of the twelve Wilson coefficients of four-Fermi interactions. The corresponding matrix elements [12, 13] are given as

$$M(B \to X_s l^+ l^-) = \frac{G_F \alpha}{\sqrt{2\pi}} V_{ts}^* V_{tb} \times$$

$$\left[ C_{SL} \bar{s}i\sigma_{\mu\nu} \frac{q^\nu}{q^2} (m_s L) b \bar{l} \gamma^\mu l + C_{BR} \bar{s}i\sigma_{\mu\nu} \frac{q^\nu}{q^2} (m_b R) b \bar{l} \gamma^\mu l \right. $$

$$+ C_{LL} \bar{s}L \gamma_\mu b_L \bar{l} L \gamma^\mu l_L + C_{LR} \bar{s}L \gamma_\mu b_L \bar{l} R \gamma^\mu l_R + C_{RL} \bar{s}R \gamma_\mu b_R \bar{l} L \gamma^\mu l_L + C_{RR} \bar{s}R \gamma_\mu b_R \bar{l} R \gamma^\mu l_R$$
Here, we represent the Wilson coefficients as $C_{XX}$'s. The $C_{SL}$ and $C_{BR}$ correspond to the non-local four-Fermi operators, and the other ten coefficients to the local operators. We choose the mass of $b$-quark, $m_b = 4.8$ GeV, as the renormalization scale $\mu$. The subscripts, $L$ and $R$, express chiral projection operators, $L = \frac{1}{2} (1 - \gamma_5)$ and $R = \frac{1}{2} (1 + \gamma_5)$, and thus correspond to the chirality of quark and lepton operators. Thus, there are two non-local interactions, $C_{SL}$ and $C_{BR}$ and ten local ones, i.e., four vector-type interactions $C_{LL}$, $C_{LR}$, $C_{RL}$ and $C_{RR}$, four scalar-type ones $C_{LRLR}$, $C_{LRRL}$, $C_{RLLR}$ and $C_{RLRL}$, and two tensor-type ones $C_T$ and $C_{TE}$. We note that two coefficients of the non-local interactions are also constrained by the experimental data of $B \to X_s \gamma$, which will be shown in Sec. II.

The SM predicts that:

• Both of the $C_{SL}$ and $C_{BR}$ are equal to $-2C_7^{\text{eff}}$, i.e.,

$$m_s^2 |C_{SL}|^2 + m_b^2 |C_{BR}|^2 = 4 \left( C_7^{\text{eff}} \right)^2 \left( m_s^2 + m_b^2 \right) \approx 4 \left( C_7^{\text{eff}} \right)^2 m_b^2. \quad (1.2)$$

• The $C_{LL}$ and $C_{LR}$ in vector parts are given in terms of $C_9^{\text{eff}}$ and $C_{10}$, that is,

$$C_{LL} = C_9^{\text{eff}} - C_{10} \quad \text{and} \quad C_{LR} = C_9^{\text{eff}} + C_{10}. \quad (1.3)$$

• The other coefficients are all negligible, and $\mathcal{M}(B \to X_s l^+ l^-)_{\text{SM}}$ becomes

$$\mathcal{M}(B \to X_s l^+ l^-)_{\text{SM}} \approx \frac{G_F \alpha}{\sqrt{2} \pi} V_{ts}^* V_{tb} \times$$

$$\left[ -2C_7^{\text{eff}} \bar{s} \gamma_\mu \frac{q^\nu}{q^2} (m_s L + m_b R) b \bar{l} \gamma^\mu l ight]$$

$$+ \left( C_9^{\text{eff}} - C_{10} \right) \bar{s} L \gamma_\mu b L \bar{l} \gamma^\mu l + \left( C_9^{\text{eff}} + C_{10} \right) \bar{s} L \gamma_\mu b L \bar{l} \gamma^\mu l R. \quad (1.4)$$

We incorporate the long distance effects of charmonium states $J/\psi$, $\psi'$ and higher resonances into the coefficient $C_9^{\text{eff}}$, following Refs. [19, 20].

• The three coefficients in the SM have been well studied [21, 22], and we follow Ref. [23, 24] for their choice and set

$$(C_7^{\text{eff}}, C_9^{NDR}, C_{10}) = (-0.311, 4.153, -4.546).$$
• We finally note that Eq. (1.1) is a model independent expression [12, 13] as a whole, even though the first two interaction terms (non-local interactions) imply the left-handed $b$-quark always comes with $m_s$ and the right-handed $b$-quark with $m_b$, e.g. the patterns similar to the SUGRA model. This is just for a convenient scaling to compare the Wilson coefficients of the non-local interactions $C_{SL}$, $C_{BR}$ with $C_{eff}^7$ of the SM and to get the constraints on them, i.e. the first two terms in Eqs. (1.1) and (1.4) compared as:

$$C_{SL}m_sL + C_{BR}m_bR \iff -2C_{eff}^7m_sL - 2C_{eff}^7m_bR,$$

as shown in Figure 1 and Eqs. (2.7) - (2.9). (We assume that interactions due to lepton chirality flip like $\bar{q}s\gamma_\mu b\bar{l}\sigma_\mu l$ is negligible as $m_l \to 0$.)

The prediction of the SM, as shown in Eq.(1.2), the interaction $\bar{s}i\sigma_\mu \gamma_7(q)(m_sL)b\bar{l}\gamma_\mu l$ is almost negligible in comparison with the other nonlocal interaction. However, in other models, it is not always so. (In fact, $m_sC_{SL} = m_bC_{BR}$ is the case in the left-right symmetric model.) One of our aim is that we know how much the former interaction gives influence to the precess $B \to X_s l^+ l^-$ in model-independent way, based on our previous works[12], where we examined new physics in the form of local interaction systematically. We will consider on the interference between such non-local interactions and other interactions, specially the SM interactions, and try to extract the above information.

The paper is organized as follows. In Sec. II, we study the effects due to the non-local interactions on the branching ratio and the FB asymmetry, which are derived from the most general effective Hamiltonian. In Sec. III, we give the correlation between the branching ratio and the FB asymmetry, which gives very useful information to understand the interaction from new physics. Conclusions are also in Sec. III.

II Branching Ratio and Forward-Backward Asymmetry of the Process, $B \to X_s l^+ l^-$

We calculate the branching ratio and the forward-backward (FB) asymmetry of the $B \to X_s l^+ l^-$ decay due to the new operators of the models beyond the SM, following the method [12, 13]. We first concentrate on the $B \to X_s \gamma$ decay to get the present constraints on the non-local Wilson coefficients of the $B \to X_s l^+ l^-$ decay. The effective Hamiltonian for
the $B \rightarrow X_s \gamma$ is given as

$$M(B \rightarrow X_s \gamma) = -\frac{4G_F}{\sqrt{2}} \frac{1}{V_{tb}^*V_{ts}} \sum_i^8 C_i(\mu) O_i(\mu),$$

(2.1)

where $C_i$'s and $O_i$'s are the relevant Wilson coefficients and the corresponding operators. We show only the $O_7$ explicitly, that is,

$$O_7 = \frac{e}{16\pi^2} \bar{s} \sigma_{\mu\nu} (m_b R + m_s L) b F^{\mu\nu},$$

(2.2)

where $e$ and $F^{\mu\nu}$ are the electromagnetic coupling constant and the electromagnetic field strength. The resultant branching ratio in the leading order is given as

$$B(B \rightarrow X_s \gamma) = B_0 \frac{32\pi}{3\alpha} \left| C_{eff}^{eff} \right|^2,$$

(2.3)

where $B_0$ is the normalization factor, normalized to the semi-leptonic branching fraction $B_{sl}(B \rightarrow Xl\nu)$ as

$$B_0 = B_{sl} \frac{3\alpha^2}{16\pi^2} \frac{|V_{ts}^*V_{tb}|^2}{|V_{cb}|^2} \frac{1}{f(\hat{m}_c)\kappa(\hat{m}_c)}.$$

(2.4)

Here $f(\hat{m}_c) = \frac{m_c}{m_b}$ and $\kappa(\hat{m}_c)$ are phase space factor and the $O(\alpha_s)$ QCD correction factor of a process $b \rightarrow c l\nu$ given by

$$f(\hat{m}_c) = 1 - 8\hat{m}_c^2 + 8\hat{m}_c^6 - \hat{m}_c^8 - 24\hat{m}_c^4 \ln \hat{m}_c,$$

(2.5)

$$\kappa(\hat{m}_c) = 1 - \frac{2\alpha_s(m_b)}{3\pi} \left[ (\pi^2 - \frac{31}{4}) (1 - \hat{m}_c)^2 + \frac{3}{2} \right].$$

(2.6)

For the numerical analysis, we set $|V_{ts}^*V_{tb}|^2 = 1$ and use $B_{sl} = 10.4\%$, the experimental value of semileptonic branching fraction of $B \rightarrow Xl\nu$. By measuring the branching fraction of $B \rightarrow X_s \gamma$, we can find present constraints on the theory describing the decay $B \rightarrow X_s l^+ l^-$, especially on $C_{eff}^{eff}$, which also appears as the coefficient of the non-local operators of the decay $B \rightarrow X_s l^+ l^-$. Based on the experimental values of the decay width of $B \rightarrow X_s \gamma$, which is consistent with the value of $C_{eff}^{eff}$ predicted by the SM as appearing in Eq. (1.2),

$$4 \left( C_{eff}^{eff} \right)^2 (m_b^2 + m_s^2) = m_s^2 |C_{SL}|^2 + m_b^2 |C_{RB}|^2,$$

(2.7)

we can easily find that the coefficients of two non-local operators are placed between two fans whose radii are about $2(m_b^2 + m_s^2)^{1/2}|C^{eff}_{eff}|$ in $(m_s |C_{SL}|, m_b |C_{RB}|)$ plain. And the recent result at CLEO for the branching ratio of the $B \rightarrow X_s \gamma$,

$$2.0 \times 10^{-4} < B(B \rightarrow X_s \gamma) < 4.5 \times 10^{-4} \text{ (95\% CL)},$$

where $C_{eff}$ and $O_i$'s are the relevant Wilson coefficients and the corresponding operators.
Figure 1: Constraint on $C_{SL}$ and $C_{BR}$ by rare $B$ decays $B \to X_s \gamma$. These coefficients can have values only within the region $II$. The mark $\times$ denotes the standard model point.

Figure 2:

\[
0.28 < |C_7^{eff}(m_b)| < 0.41.
\]

As shown in Figure 1, only the values of the coefficients $C_{BR}$ and $C_{SL}$ within the region $II$ can be permitted. Because $s$-quark mass is much less than $b$-quark mass, $m_s \ll m_b$, we may regard that the term $C_{SL}$, which is proportional to $m_s$, hardly contributes to the $B \to X_s l^+ l^-$ decay in the SM. This means that the SM point is placed near the $m_b |C_{BR}|$ axis in the $(m_s |C_{SL}|, m_b |C_{BR}|)$ plain, as shown in Figure 1. Assuming that there is no new phase from the non-local interactions, Eq. (2.7) gives, as $m_s \to 0$,

\[
-2C_7^{eff} \leq C_{BR} \leq 2C_7^{eff},
\]

and

\[
-2C_7^{eff} \leq C_{SL}^N \leq 2C_7^{eff}, \text{ where } C_{SL}^N \equiv \frac{m_s}{m_b} C_{SL}.
\]

Here, we denoted the normalized $C_{SL}$ as $C_{SL}^N$. Therefore, it is very important to know the branching ratio at 4 points

\[
(C_{SL}^N, C_{BR}) = (-2C_7^{eff}, 0), \ (2C_7^{eff}, 0), \ (0, -2C_7^{eff}), \ (0, 2C_7^{eff}).
\]

We show the branching ratio of $B \to X_s l^+ l^-$ for massless lepton case in Figure 2 in the absence of any new local interactions, but with new non-local interactions and the
already existing local operators of the SM. In this case, the branching ratio is given as

\[
\frac{d\mathcal{B}}{ds}(B \rightarrow X_s l^+ l^-) = \frac{1}{2m_b^2} B_0[S_1(s)m_b^2(|C_{SL}^N|^2 + |C_{BR}|^2) + 2S_2(s)m_b^2Re[C_{SL}^N C_{BR}^*] \\
+ 4S_3(s)m_b^2m_s Re[C_{SL}^N C_{9eff}^*] + 4S_4(s)m_b^2 Re[C_{BR} C_{9eff}^*] \\
+ M_2(s)\left( |C_{9eff}^* - C_{10}|^2 + |C_{9eff}^* + C_{10}|^2 \right),
\]

with \( s = (p_{l^+} + p_{l^-})^2 \), invariant mass-square of lepton pair. (The case with the all twelve operators is given in Appendix.) The \( S_n(s) \) and \( M_2(s) \) are given in Refs. [12, 13]. We note that the branching ratio is more sensitive to the change of \( C_{BR} \) than of \( C_{SL} \), as shown in Figure 2, because the interferences of the \( C_{SL} \) and the \( C_{BR} \) to the vector type interactions of the SM give

\[
\text{Tr}\{\bar{s}\sigma_{\mu\nu}Lb(\bar{s}_L\gamma_\mu b_L)^*\} \propto m_s,
\]

and \( \text{Tr}\{\bar{s}\sigma_{\mu\nu}Rb(\bar{s}_L\gamma_\mu b_L)^*\} \propto m_b, \)

respectively. The contribution from these coefficients oscillates as we vary the values of \((C_{SL}^N, C_{BR})\) within the region II in Figure 1, because of the constraint (2.7). However, if we account only for the \( C_{BR} \) (as in the SM), or equivalently, if we assume \( m_s = 0 \), then \( C_{BR} \) moves only between \(-2|C_{7eff}^e|\) and \( 2|C_{7eff}^e|\), and the branching ratio decreases monotonously. The points \((C_{SL}^N, C_{BR}) = (0,2|C_{7eff}^e|)\) and \((0,-2|C_{7eff}^e|)\) are the minimum and the maximum values. (The SM value corresponds almost to the point \((0,-2|C_{7eff}^e|)\).) This behavior reappears for the partially integrated branching ratio \( \mathcal{B} \equiv \int_1^8 ds \frac{d\mathcal{B}}{ds} \) [12] as shown in Figure 3.
Figure 3: Partially integrated branching ratio \( B \equiv \int_1^s ds \frac{d\mathcal{B}}{ds} \) and FB asymmetry \( \bar{A} \equiv \int_1^s d\mathcal{s} \frac{d\mathcal{A}}{ds} \). The angle \( \theta \) is defined by \( \frac{C_{\mathcal{N}}}{C_{\mathcal{BL}}} \equiv \tan \theta \). The coefficients \( C_{\mathcal{N}} \), \( C_{\mathcal{SL}} \), \( C_{\mathcal{BR}} \) move under the conditions (2.7).

Now we consider the forward-backward (FB) asymmetry defined as

\[
\frac{d\bar{A}}{ds} = \frac{d\mathcal{A}}{ds} = \frac{\int_0^1 dz \mathcal{A} \frac{d\mathcal{B}}{ds} - \int_{-1}^0 dz \mathcal{B} \frac{d\mathcal{A}}{ds}}{\int_0^1 dz \mathcal{B} \frac{d\mathcal{B}}{ds} + \int_{-1}^0 dz \mathcal{A} \frac{d\mathcal{A}}{ds}},
\]

(2.11)

where \( z \) is the cosine value of the angle between the momentum of \( B \) meson and that of \( l^+ \) in the laboratory frame. We also show the normalized FB asymmetry curves, \( d\bar{A}/ds \), at the four points \((C_{\mathcal{N}}, C_{\mathcal{BR}}) = (0, -2C_7^{\text{eff}}), (0, 2C_7^{\text{eff}}), (-2C_7^{\text{eff}}, 0), (2C_7^{\text{eff}}, 0)\) in Figure 2. The line for the \((0, -2C_7^{\text{eff}})\) corresponds to the SM result. The unnormalized FB asymmetry of \( B \to X_s l^+ l^- \) for massless lepton case in the absence of any new local interactions, but with new non-local interactions and the already existing local operators of the SM is given as

\[
\frac{d\mathcal{A}}{ds} = \frac{1}{2m_b^6} \mathcal{B}_0 u(s)^2 [8(\text{Re}\{(m_b^2 C_{\mathcal{BR}} + m_s m_s C_{\mathcal{N}}) C_{10}^{\ast}\}
+ 2s[|C_9^{\text{eff}} - C_{10}^\ast|^2 - |C_9^{\text{eff}} + C_{10}^\ast|^2]).
\]

(2.12)

(The case for massive lepton with the all 12 operators is given in Appendix.) We note that, as is the case for the branching ratio, the FB asymmetry is more sensitive to the change of \( C_{\mathcal{BR}} \) than of \( C_{\mathcal{SL}} \) as shown in Figure 2, and the oscillating behavior reappears.

To see the sensitivity of the asymmetry for each coefficient, we introduce the partially integrated (un)normalized FB asymmetry \( \bar{A} (A) \) defined as

\[
\bar{A} \equiv \frac{A}{B},
A \equiv \int_1^s ds \frac{d\mathcal{A}}{ds},
\]

8
where $B \equiv \int_1^8 ds \frac{dn}{ds}$. We present the influence of two non-local coefficients on the normalized FB asymmetry in Figure 3.

### III Discussions and Conclusions

In Sec. II, we investigated both the branching ratio and the FB asymmetry independently. As shown, both observables are more sensitive to changes of $C_{BR}$ than to those of $C_{SL}$, and oscillate as the non-local coefficients change. Now we show the correlation between the branching ratio and the FB asymmetry in Figure 4. It is very interesting to compare the correlation for various interactions, because the flows in the plane ($B$, $\bar{A}$) depend on interactions which we consider. We already investigated the correlation flows for the case of local interactions in [12]. The flows in the plain ($B$, $\bar{A}$) for the non-local interactions are quite different from the local ones. As found in Ref. [12], the standard model point is just near $(C_{SL}^N, C_{BR}) = (0, 2|C_7^{eff}|)$ in the plane, so that it is placed at the lowest point (marked as ◊) of the closed ellipse in Figure 4. Therefore, if there exist any non-local interactions in new theory beyond standard model, both the ratio and the FB asymmetry monotonically increase.

However, the vector-type interactions increase or decrease the branching ratio (the
Figure 5: Correlation as the Wilson coefficients $C_{LL}$ moves, receiving the effects of the interference between the new vector-type interaction and the new non-local interactions, whose Wilson coefficients are $(C_{N_{SL}}^N, C_{BR}^N) = (0, -2C_{eff}^7)$ (thick solid line), $(0, 2C_{eff}^7)$ (thick dashed line), $(-2C_{eff}^7, 0)$ (thin solid line) and $(2C_{eff}^7, 0)$ (thin dashed line). To refer, we also show the correlation as the set of the coefficients moves. The latter is also described in Figure 6 with the same notation.

FB asymmetry) as the values of vector-type coefficients increase (decrease) or decrease (increase). And the scalar-type and tensor-type interactions make no change for the unnormalized FB asymmetry. We can also understand the presented behavior for the non-local interactions with the following arguments: The branching ratio and the FB asymmetry change, for $C_{BR}$ and $C_{SL}$, only through the second term in Eq. (2.10) and the first term in Eq. (2.12), since the two coefficients cannot change simultaneously under the condition (2.7). If we leave the leading term in $m_s$, the partially integrated branching ratio ($B$) and the partially integrated forward-backward asymmetry ($A$) are expressed as

$$B = B_{c1} + B_{c2}[m_bC_{BR}(9 - 2m_b^2) + m_sC_{SL}^N(9 + 2m_b^2)] + O(m_s^2), \quad (B_{c2} > 0),$$

$$A = A_{c1} - A_{c2}(m_bC_{BR} + m_sC_{SL}^N) + O(m_s^2), \quad (A_{c2} > 0),$$

where $B_{c1}$, $B_{c2}$, $A_{c1}$ and $A_{c2}$ are independent of $C_{BR}$ and $C_{SL}$ and $m_s$. Under the condition $C_{SL} = \sqrt{4\left(C_{eff}^7\right)^2 - C_{BR}^2}$, $B$ and $A$ achieve the minimum and the maximum at $C_{BR} = 2\left|C_{eff}^7\right|$ and $-2\left|C_{eff}^7\right|$. Therefore we will be able to know the sign of the $C_{eff}^7$ and deviation from the predictions of the SM by using this correlation.

The non-local interactions are dominant as the momentum transfer from the $b$-quark to the $s$-quark gets comparable with the lepton mass, that is, as $\sqrt{s} \rightarrow m_l \sim 0$, because of the factor $1/s$. But, the sensitivity of the ratio to the changes of $C_{BR}$ and $C_{SL}$ is not so
large in comparison with the local interactions, because of the constraint (2.7). If there is a new local interaction in addition, understanding the interference between the non-local interactions and the new local interaction would be extremely important within the small and non-vanishing $s$ region, $1 < s < 8 \text{ GeV}^2$ [27]. For example, in the massless limit, the scalar- and tensor-type interactions cannot contribute to the FB asymmetry. Hence, if we find a deviation of the FB asymmetry from the SM prediction, we can infer that there are new non-scalar- or non-tensor-type interactions. To extend further our discussion, suppose that interactions which act on massless leptons are equal to the ones which act on massive leptons. If we cannot find the lepton longitudinal polarization asymmetry $\langle P_L^+ \rangle + \langle P_L^- \rangle$ from the precise experiments for the decays $B \to X_s \tau^+ \tau^-$, then we can conclude there is no scalar- or tensor-type interaction [13], and infer that there are vector-type interactions like the SM and non-local interactions as well. In such a case, we should consider the interference between the non-local interactions and the vector-type ones. In Figure 4, we show the flow as $C_{LL}$ moves when there are new non-local interactions, where, again, the branching ratio and the FB asymmetry are integrated over $s$ from 1 GeV$^2$ to 8 GeV$^2$.

To summarize, we investigated the effects of the non-local interactions in the rare $B$ decays $B \to X_s l^+ l^-$ in the model-independent way. In our model-independent analysis in this paper and Refs. [12, 13], we used all the operators which influence the process $B \to X_s l^+ l^-$, those are, ten local and two non-local four-Fermi operators. We, here, studied the sensitivity to the coefficients of the non-local interactions for the branching ratio and the forward-backward (FB) asymmetry. We note that both the ratio and the FB asymmetry are more sensitive to $C_{BR}$ than to $C_{SL}$. We did not use the $C_{SL}$ introduced at first in Eq. (1.1), instead we used the normalized Wilson coefficient $C_{SL}^N \equiv \frac{m_s}{m_b} C_{SL}$, in order not to mislead. Nevertheless, the interference terms between the $C_{SL}^N$ and the other local operators include an extra mass ratio $m_s/m_b$, compared to the interferences from the $C_{BR}$ and others, and therefore, the operator $C_{BR}$ gives greater influence on the ratio and the FB asymmetry than the $C_{SL}$. Consequently, the value of the $C_{BR}$ almost decides the size of the branching ratio and the FB asymmetry as the result of their correlation. If there is any new charged local interaction, which contributes to $B \to X_s l^+ l^-$, like in the minimal supersymmetric standard model, we must consider appropriate non-local interactions, because any charged particle’s interaction with photons yields non-local interactions. Especially, in the small invariant mass region, we cannot ignore the contribution from the non-local interactions. And our analysis would give very
useful help for the precise study of new physics in $B \rightarrow X_s l^+ l^-$ when such a new local interaction exists.

**ACKNOWLEDGMENTS**

We would like to G. Cvetic and T. Morozumi for very useful suggestions and comments. The work of C.S.K. was supported in part by BK21 Project, SRC Program and Grant No. 2000-1-11100-003-1 of the KOSEF, and in part by the KRF Grants (Project No. 1997-011-D00015 and Project No. 2000-015-DP0077). The work of T.Y. was supported in part by Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture of Japan and in part by JSPS Research Fellowships for Young Scientists.
APPENDIX

A  Branching Ratio and the Forward-backward Asymmetry with complete 12 Operators

In Ref. [12], we have already studied the differential branching ratio and the FB asymmetry for massless leptons without including two non-local interactions, and in Ref. [13] we investigated the differential branching ratio and the polarization asymmetries for massive leptons with including all 12 operators. Here we show the differential branching ratio for massless leptons by including two new non-local interactions,

\[
\frac{dB(s)}{ds} = \frac{1}{2m_b^8} B_0 \left[ S_1(s) \{m_s^2|C_{SL}|^2 + m_b^2|C_{BR}|^2\} \right. \\
+ S_2(s) \{2m_b m_s \text{Re}[C_{SL}C_{BR}^*]\} \\
+ S_3(s) \{2m_s^2 \text{Re}[C_{SL}(C_{LL}^* + C_{LR}^*)] + 2m_b m_s \text{Re}[C_{BR}(C_{RL}^* + C_{RR}^*)]\} \\
+ S_4(s) \{2m_b^2 \text{Re}[C_{BR}(C_{LL}^* + C_{LR}^*)] + 2m_b m_s \text{Re}[C_{SL}(C_{RL}^* + C_{RR}^*)]\} \\
+ S_5(s) \{2(m_s C_{SL} + m_b C_{BR}) C_T^*\} \\
+ S_6(s) \{4(m_b C_{BR} - m_s C_{SL}) C_{TE}^*\} \\
+ M_2(s) \{|C_{LL}|^2 + |C_{LR}|^2 + |C_{RL}|^2 + |C_{RR}|^2\} \\
- M_6(s) \{2 \text{Re}[C_{LL}^* C_{RL}^* + C_{LR}^* C_{RR}^*] \\
- \text{Re}[C_{LRLR} C_{RLLR}^* + C_{LRRL} C_{RLLL}^*]\} \\
+ M_8(s) \{|C_{LRLR}|^2 + |C_{RLRL}|^2 + |C_{LRRL}|^2 + |C_{RLLL}|^2\} \\
+ M_9(s) \{16|C_T|^2 + 64|C_{TE}|^2\}. \tag{A.1}
\]

The kinematic functions, \(S_n(s)\) and \(M_n(s)\), are all given in detail in Ref. [12, 13]. We find from Eq. (A.1) that \(S_2(s)\), which includes \(m_s\) as an overall factor, is multiplied by \(m_b\) and \(m_s\) and, therefore, it is negligible. We also show the most general form of the FB asymmetry in case of massive leptons, which includes all 12 operators. It is as follows:

\[
\frac{dA}{ds} = \frac{1}{2m_b^8} B_0 u(s)^2 \left[ -4(\text{Re}\{m_b^2 C_{BR} + m_s^2 C_{SL}\} (C_{LL}^* - C_{LR}^*)\} \\
+ 8m_b m_s \text{Re}\{(C_{BR} + C_{SL})(C_{RL}^* - C_{RR}^*)\} \\
+ 4m_s m_t \text{Re}\{C_{SL}(C_{RLLR}^* + C_{RLRL}^*)\} \\
+ 4m_b m_t \text{Re}\{C_{BR}(C_{RLRL}^* + C_{LRLR}^*)\} \\
+ 2s(|C_{LL}|^2 - |C_{LR}|^2 - |C_{RL}|^2 + |C_{RR}|^2) \right].
\]
\[-8s(Re\{C_{LRLR}(C_T^*-2C_{TE}^*)\} + Re\{C_{RLRL}(C_T^*+2C_{TE}^*)\})\]

\[-2m_b m_l(Re\{(C_{LL}+C_{LR})(C_{LRLL}^*+C_{LRLR}^*)\} + Re\{(C_{RL}+C_{RR})(C_{RLLR}^*+C_{RLRL}^*)\})\]

\[-2m_s m_l(Re\{(C_{LL}+C_{LR})(C_{RLLR}^*+C_{RLLR}^*)\} + Re\{(C_{RL}+C_{RR})(C_{LRLR}^*+C_{RLRL}^*)\})\]

\[+24(m_b+m_s)m_l Re\{(C_{LL}-C_{LR}+C_{RL}-C_{RR})C_T^*\}\]

\[+48(-m_b+m_s)m_l Re\{(C_{LL}-C_{LR}+C_{RL}-C_{RR})C_{TE}^*\}]\text{.}\quad (A.2)

If \(C_{BR} = C_{SL} = -2C_T^{eff}\), this corresponds with Eq. (A6) in Ref. [12].
References

[1] T. Goto, Y. Okada, Y. Shimizu, and M. Tanaka, Phys. Rev. D55 (1997) 4273; T. Goto, Y. Okada, Y. Shimizu, Phys. Rev. D58 (1998) 094006.

[2] J. L. Hewett and J. D. Wells, Phys. Rev. D55 (1997) 5549.

[3] L. T. Handoko, Phys. Rev. D57 (1998) 1776; L. T. Handoko, Nuovo.Cim. A111 (1998) 95.

[4] C. Greub, A. Ioannisian and D. Wyler, Phys. Lett. B346 (1995) 149.

[5] Y. Grossman, Z. Ligeti and E. Nardi, Phys. Rev. D55 (1997) 2768.

[6] T. G. Rizzo, Phys. Rev. D58 (1998) 114014.

[7] Ji-Ho Jang, Y.G. Kim and J. S. Lee, Phys. Rev. D58 (1998) 035006.

[8] P. Cho, M. Misiak, and D. Wyler, Phys. Rev. D54 (1996) 3329.

[9] Y.G. Kim, P. Ko and J.S. Lee, Nucl. Phys. B544 (1999) 64.

[10] C.-S. Huang, W.-J. Huo and Y.-L. Wu, Mod. Phys. Lett. A14 (1999) 2453.

[11] E. Lunghi, A. Masiero, I. Scimemi, L. Silvestrini Nucl. Phys. B568 (2000) 120.

[12] S. Fukae, C. S. Kim, T. Morozumi and T. Yoshikawa, Phys. Rev. D59 (1999) 074013.

[13] S. Fukae, C. S. Kim and T. Yoshikawa, Phys. Rev. D61 (2000) 074015.

[14] S. Ahmed et al. (CLEO Collab.), hep-ex/9908022 (1999).

[15] K. Chetyrkin, M. Misiak and M. Münz, Phys. Lett. B400, (1997) 207

[16] M. Ciuchini, G. Degrassi, P. Gambino and G. F. Giudice, Nucl. Phys. B527 (1998) 21.

[17] M. Misiak, Nucl. Phys. B269 (1991) 161.

[18] F. M. Borzumati and C. Greub, Phys. Rev. D59 (1999) 057501.
[19] C. S. Lim, T. Morozumi and A. I. Sanda, Phys. Lett. B218 (1989) 343; N. G. Deshpande, J. Trampetic and K. Panose, Phys. Rev. D39 (1989) 1461; P. J. O’Donnell and H. K. K. Tung, Phys. Rev. D43 (1991) R2067; N. Paver and Riazuddin, Phys. Rev. D45 (1992) 978.

[20] F. Krüger and L. M. Sehgal, Phys. Rev. D55 (1997) 2799.

[21] M. Misiak, Nucl. Phys. B393 (1993) 23 and erratum ibid B439 (1995) 461.

[22] A. J. Buras and M. Münz, Phys. Rev. D52 (1995) 186.

[23] A. Ali, G. Hiller, L.T. Handoko and T. Morozumi, Phys. Rev. D55 (1997) 4105.

[24] C. S. Kim, T. Morozumi and A. I. Sanda, Phys. Rev. D56 (1997) 7240; T. M. Aliev, C. S. Kim and M. Savci, Phys. Lett. B441 (1998) 410; T. M. Aliev, C. S. Kim and Y. G. Kim, hep-ph/9910501 (1999), to be published in Phys. Rev. D (2000).

[25] C. S. Kim and A. D. Martin, Phys. Lett. B225 (1989) 186.

[26] A. Ali, P. Ball, L.T. Handoko and G. Hiller, hep-ph/9910221.

[27] C. S. Kim, Y. G. Kim, Cai-Dian Lu and T. Morozumi, hep-ph/0001151 (2000), to be published in Phys. Rev. D (2000).

[28] A. Falk, M. Luke and M. J. Savage, Phys. Rev. D49 (1994) 3367.