QCD AND DIFFRACTION

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I review some of the main results on hard diffraction, in their relation to QCD,
and indicate some of the strengths and weaknesses of the arguments. I also make
some suggestions for future work.

1 Introduction

This talk summarized some ideas on how well we know properties of diffractive
and small-x physics as a consequence of QCD. The primary results concern the
separation of phenomena on different scales and in different rapidity ranges,
and it this separation or factorization that enables predictions and useful
models to be made despite our ignorance about non-perturbative QCD:

1. Ordinary hard-scattering factorization for $F_2^{\text{inclusive}}$, jet production, etc.
2. The same for diffractive hard processes.
3. BFKL, etc.
4. $k_T$-factorization, CCFM, etc.
5. Dipole model, etc.

There is also the subject of exclusive diffraction and skewed pdf’s, which I
will not discuss here. I will also not discuss the issue of Regge theory and
Regge factorization, except indirectly in the context of the BFKL equation.
Regge factorization does not follow in any demonstrated way from QCD, if one
includes non-perturbative phenomena, and in the context of hard scattering,
it clearly fails in its simplest form. However, this is an important area of
physics that needs to be properly elucidated.

In the general area of diffraction, a cultural divide exists between those
who work in the Breit frame (in DIS) and those who work in the target rest

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frame. The first group use a parton density language and the second tend to use the dipole model. The two frames lead to very different intuitions. But QCD is the same for all of us, and progress is to be made by understanding the compatibility of the two views.

My remarks are confined to lepto-production processes, even though the same issues extend to many other processes.

2 Parton viewpoint: hard-scattering factorization

The main ideas of the parton method are best understood in coordinate space in the Breit frame — Fig. 1. A highly time-dilated and Lorentz contracted proton arrives from the right. For numerical illustration, suppose that $Q^2 = 10 \text{ GeV}^2$ and $x = 2 \times 10^{-4}$. The hard interaction occurs with one constituent over a scale $1/Q$, about 0.1 fm. The struck quark has an energy of 1.6 GeV, and its scale of non-perturbative interactions is 5 fm, due to time-dilation. The valence interactions of the proton occur on a much longer scale $10^4$ fm.

This separation of scales leads to factorization, for $F_2$ and many other cross sections. The leading power result is

$$\text{hard-scattering coefficient} \otimes \text{parton density}. \quad (1)$$

This also applies to diffractive processes, where the parton density is replaced by a diffractive parton density (or extended fracture function), which is a number density of partons conditional on a diffracted proton in the final state.

The intuitive picture is clear, and is fully and provably realized in QCD. Results include: (a) the DGLAP evolution of pdf’s, (b) the need for higher-order corrections (non-leading logarithms) to the hard scattering and DGLAP kernels, and (c) methods to obtain these corrections from Feynman graphs. The successes of these methods is well-known.
However, there is a myth that has arisen and that is, I believe, hindering further progress. This is that hard scattering involves scattering of on-shell partons. Clearly this is false, since the partons are confined. Discussions predicated on the myth are misleading, if not incomprehensible, for students. New results in the field depend on a more exact treatment of parton kinematics, which can only be done correctly if the myth is removed.

Simple factorization breaks down when $Q$ is too small, and “higher twist” corrections come into play. More interestingly, it breaks down when $1/x$ is sufficiently large that “saturation” effects matter. Even if saturation does not yet occur, there are enhancements in higher order corrections at small $x$.

3 Small $x$ logarithms and saturation

At small $x$ two problems arise. One is that in gluon ladders there are integrals over the rapidities of intermediate gluons that lead in coefficient functions and in the DGLAP kernels to logarithms of $x$. Low-order perturbation theory for these quantities is no longer accurate. Possible remedies are (a) resummation of the large logarithms using the BFKL equation, (b) use of sufficiently many higher orders of perturbation theory, and (c) use of $k_T$ factorization. These are in principle compatible, but rely on approximations, and the $k_T$ factorization approach is probably the most unifying method.

A more fundamental breakdown of the methods occurs at sufficiently small $x$ that saturation occurs. The intuitive picture leading to factorization requires that the different partons in the target hadron be transversely separated at the moment of the hard scattering. This fails when the partons overlap transversely, when the packing fraction

$$\rho = \frac{\text{Area of partons}}{\text{Area of proton}} = \frac{\text{number of parton}/Q^2}{1/R^2}, \quad (2)$$

becomes larger than unity. Simple parton ideas fail, and a breakdown of the DGLAP prediction of the $Q$ fails even if calculations are carried to high orders of perturbation theory (non-leading logarithms).

In applying this idea quantitatively, one should know that at small $x$, $\overline{\text{MS}}$ pdf’s do not provide a good measure of the number of partons, since they are not equal to an appropriate integral of an unintegrated parton density.

4 $k_T$ factorization

A fundamental way of tackling these and other problems is to use “unintegrated parton densities” $P(x, k_T)$ that give the $k_T$ dependence of the partons.
At small $x$ and high $Q$ the LLA suggests that ladder-like graphical contributions dominate. As one goes from the virtual photon to the proton, the parton $x$ increases and the parton $k_T$ generally decreases. By locating where the first large gap in one of these variables occurs one obtains a factorization:

$$d\sigma = \text{coefficient function} \otimes P(x, k_T).$$

There are no large ranges of kinematics in the coefficient, so it is perturbatively calculable. But if the kinematic gap below it is in $x$, we must explicitly take account of parton $k_T$; hence the use of the unintegrated pdf.

A proper treatment needs care. For example, non-ladder graphs are important, but the need for most of them can be removed by the use of light-cone gauge. However, the light-cone gauge introduces divergences when the rapidities of virtual gluons go to $-\infty$, a region that is completely unrelated to the physics. There is no cancellation between real and virtual gluons in the situation we are considering, so it is compulsory to provide a cut-off: $P(x, k_T, y_c)$. Effectively, the BFKL and CCFM equations provide an equation for the dependence on the cut-off $y_c$.

Unfortunately, the cut-off is not explicit in public derivations, with the obvious resulting problems.

This impacts on the discussions of saturation. It is essential to discuss this in terms of $P(x, k_T, y_c)$. The relation to the MS pdf’s needs to be explicitly quantified. Many years ago, in other processes, Soper and I gave a suitable concrete definition, with the rapidity cutoff explicit. The definition of quark-parton state includes a certain amount of “wee glue”, and the equation for the $y_c$ dependence is a kind of renormalization-group equation with respect to the amount of wee glue.

Also, it is not obvious to me how the published derivations go beyond the leading-logarithm approximation; in particular, at sufficiently non-leading logarithmic accuracy, there is more than single-gluon exchange across kinematic gaps in $x$, contrary to what was needed to obtain Eq. (3). This matters if one wishes to use $k_T$-factorization as a property of full QCD, including non-perturbative effects.

## 5 Dipole model

A totally different is the dipole model. Its intuition comes from the target rest frame, where the time-scale of the virtual photon is much greater than the time for it to cross the target. If $Q^2 = 10 \text{GeV}^2$ and $x = 2 \times 10^{-4}$, as before, the photon energy is $2.5 \times 10^4 \text{GeV}$ and the $q\bar{q}$ component of the photon has a formation distance of around 500 fm, whereas the proton is just 1 fm across.
Clearly, it is a sensible first approximation to treat the photon’s $q\bar{q}$ component as frozen in transverse size while it crosses the target. This results in a lot of useful phenomenology. In particular inclusive $F_2$ and diffraction are related by applying the optical theorem to the dipole cross section.

It is important that the basic dipole, with a size of order $1 \text{ fm}$, is in the “aligned jet” configuration. One particle of the dipole has much higher energy than the other. The high-energy particle is the struck quark, which gives the high $p_T$ jet in the HERA laboratory frame. The other, “slow” quark is actually in the parton density: it is the quark of fractional momentum $x$ before it is struck, and the order of events for this quark is opposite than in the Breit frame. This change of time ordering between frames implies that the quark propagates over a space-like distance, and therefore that intuitive ideas may be somewhat misleading.

The correspondence with the factorization ideas is as follows. If the transverse momentum in the pair is small, then the dominant situation (to power-law accuracy) is the asymmetric aligned-jet configuration, with a size of $1/k_T$; this corresponds to the LO parton model. Provided we use the leading logarithm approximation in $x$, time dilation of the dipole system ensures that scattering occurs over a short distance scale, and the dipole idea is really valid.

If $k_T$ is large ($\sim Q$) we have a small size configuration, with the parton momenta not being greatly different. The lifetime of the state remains long (500 fm in the example). However, scattering can occur (in non-leading logarithm in $x$) by emission of a slow gluon. We then have a 3-body system, which is effectively an octet dipole, consisting of a gluon well separated from a small octet $q\bar{q}$ system.

Intermediate configurations are involved in giving the correct DGLAP evolution.

Clearly “QCD corrections” give non-trivial modifications to the dipole model, and it is questionable how well it is preserved beyond the perturbative domain for anything but qualitative purposes. The dipole model is most at home in a leading logarithm approximation. Quantitative comparisons of data and dipole model predictions can be quite tricky particularly to determine whether saturation occurs.

6 Relation of dipole-model to factorization

Clearly, there must be a relation between dipole model and pdf formalism. However, it is a non-trivial relation, because the time sequence of events is reversed between the natural reference frames for the two methods, and because of the need to treat non-leading logarithmic and non-perturbative
effects. Even so, the basic Feynman-diagrammatic view is the same, and the theory is the same. It is most natural, I think, to try to relate the dipole cross section to (unintegrated) pdf’s.

Some hints as to the details can be found in the work of Frankfurt, Strikman, et al. One definitely has to ask what happens beyond the leading approximation. The factorization point-of-view includes, in principle, all non-leading logarithms, and its pdf’s are fully non-perturbative.

7 Conclusions

I would like to know the answers to the following questions, which in my personal opinion have not been completely satisfactorily answered in the literature:

1. What precisely is the unintegrated pdf \( P(x, k_T, y_c) \)?
2. Explicitly, what is the rapidity cut-off on virtual gluons?
3. What is the relation to \( \sigma_{dipole}(x, r_T) \)? (What variables? Why?)
4. What is the quantitative relation to the \( \overline{\text{MS}} \) pdf’s?
5. We need a more complete derivation of the space-time structure. E.g.:
   - Where is diffracted proton formed?
   - What about Regge theory?

References

1. J.C. Collins, *Phys. Rev. D* **57**, 3051 (1998) [Erratum— *Phys. Rev. D* **61**, 019902 (1998)] [hep-ph/9709499].
2. E.g., M. McDermott, L. Frankfurt, V. Guzey and M. Strikman, *Z. Phys. C* **16**, 641 (2000) [hep-ph/9912547]; K. Golec-Biernat and M. Wusthoff, *Phys. Rev. D* **60**, 114023 (1999) [hep-ph/9903358].
3. S. Catani, M. Ciafaloni and F. Hautmann, *Nucl. Phys. B* **366**, 135 (1991).
4. J.C. Collins and D.E. Soper, *Nucl. Phys. B* **194**, 445 (1982).