A NEW UNIFIED EVOLUTION EQUATION

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We propose a new unified evolution equation for parton distribution functions appropriate for both large and small Bjorken $x$. Compared with the Ciafaloni-Catani-Fiorani-Marchesini equation, the cancellation of safe poles between virtual and real gluon emissions is made explicit without introducing infrared cutoffs, and next-to-leading contributions to the Sudakov resummation can be included systematically in this equation.

1 Introductions

As far as we know, the conventional evolution equations covering different kinematic regimes will be different equations. Such as: The DGLAP (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) equation governs the large logarithmic corrections due to high momentum transfer in the intermediate Bjorken $x$ region. The BFKL (Balitsky-Fadin-Kuraev-Lipatov) equation deals with the large logarithmic corrections coming from the small Bjorken $x$ region. The CCFM (Ciafaloni-Catani-Fiorani-Marchesini) equation, as a unified approach of these above two, is appropriate for both intermediate and small $x$ regime. But the derivation of these evolution equations based on the idea that the leading logarithm contribution can be produced by summing the ladder diagrams one by one to all orders. Also, by locating the momentum flows of the rungs of the radiated gluon ladder diagrams at different orderings, different evolution equations was obtained.

Recently, we develop the resummation idea of Collins-Soper-Sterman which was used to resum the double logarithmic correction of the parton distribution function in hard QCD scattering process to the conventional evolution equations. We can easily reproduce all these above equations basing on the resummation idea instead of the complex ladder diagram calculation. Also, we can introduce modified evolution equation (Modified BFKL equation) to consider extra kinematic variable dependence. In this short paper, a new unified evolution equation innovated by resummation idea is proposed to unify the kinematic regime that the DGLAP and BFKL work separately.

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2 From Resummation Idea To A New Unified Evolution equation

2.1 Resummation Idea

We study the unintegrated gluon distribution function in the axial gauge,

\[ F(x, k_T, p^+) = \frac{1}{p^+} \int \frac{dy^-}{2\pi} \int \frac{d^2y_T}{4\pi} e^{-i(xp^+ - k_T y_T)} \times \frac{1}{2} \sum_\sigma \langle p, \sigma | F_\mu^+(y^-, y_T) F_\mu^+(0) | p, \sigma \rangle , \]

where \( |p, \sigma\rangle \) denotes the incoming hadron with light-like momentum \( p^\mu = p^+ \delta^{\mu+} \) and spin \( \sigma \), and \( F_\mu^+ \) is the field tensor.

Because of the scale invariance of \( F \) in the gauge vector \( n \), \( F \) must depend on \( p^+ \) via the ratio \( (p \cdot n)^2/n^2 \). Hence, we have the chain rule relating \( p^+ d/dp^+ \) to \( d/dn \):

\[ p^+ \frac{d}{dp^+} F = -\frac{n^2}{v \cdot n} v_\alpha \frac{d}{dn_\alpha} F , \]

with \( v_\alpha = \delta_{\alpha+} \) a dimensionless vector along \( p \). The operator \( d/dn_\alpha \) applies to a gluon propagator, then the differentiated gluon attaches to a special vertex, which is read off from the Eqs. (2).

\[ \hat{v}_\alpha = \frac{n^2 v_\alpha}{v \cdot n n \cdot l} . \]

Employing the Ward identities, it reduces to our master equation of resummation idea with the special vertex at the outmost end. As shown in Fig. 1(a), we obtain:

\[ p^+ \frac{d}{dp^+} F(x, k_T, p^+) = 2 \tilde{F}(x, k_T, p^+) , \]

2.2 A New Unified Evolution Equation

First, we Fourier transform Eq. (4) into the \( b \) space, \( b \) being the conjugate variable of \( k_T \). The leading regions of the loop momentum flowing through the special vertex are soft and hard, so we factorize subdiagrams containing the special vertex order by order (Figs. 1(b) and 1(c)).

For the contribution from Fig. 1(b), We reexpress the function \( F \) as

\[ F(x + l^+/p^+, b, k^+) = \theta((1-x)p^+ - l^+) F(x, b, k^+) + [F(x + l^+/p^+, b, k^+) - \theta((1-x)p^+ - l^+) F(x, b, k^+)] . \]
Figure 1: (a) The derivative $p^+ dF/dp^+$ in the axial gauge. (b) The lowest-order subdiagrams for $K$. (c) The lowest-order subdiagrams for $G$.

The contribution from the first term of Eq. (5) is combined with virtual gluon correction, leading to

$$K(x, b\mu, \alpha_s(\mu)) = -\bar{\alpha}_s(\mu) \left[ K_0(2(1-x)p^+\nu b) + \ln \frac{b\mu}{2} + \gamma_E \right],$$

(6)

the other from the second term in Eq. (5) is denoted as:

$$\bar{F}'_s \approx \bar{\alpha}_s(k^+) \int_x^1 d\xi \frac{F(\xi, b, k^+) - F(x, b, k^+)}{1 - x/\xi}.$$  

(7)

While Fig. 1(c) gives

$$G(k^+/\mu, \alpha_s(\mu)) = -\bar{\alpha}_s(\mu) \left[ \ln \frac{2k^+}{\mu} + \ln \nu \right],$$

(8)

However, the ultraviolet divergence of $K$ and $G$ cancel each other, the sum of $K$ and $G$ will be RG invariant. Therefore, the RG solution will be

$$K(x, b\mu, \alpha_s(\mu)) + G(k^+/\mu, \alpha_s(\mu)) = K(x, 1, \alpha_s(k^+)) + G(1, \alpha_s(k^+)) - s(x, b, k^+),$$

(9)

with

$$K(x, 1, \alpha_s(k^+)) = \bar{\alpha}_s(k^+) \left[ \ln(1-x) + \ln(p^+b) + \ln 2\nu \right],$$

(10)

$$G(1, \alpha_s(k^+)) = -\bar{\alpha}_s(k^+) \ln 2\nu,$$

(11)

$$s(x, b, k^+) = \int_{1/b}^{k^+} \frac{d\bar{\mu}}{\bar{\mu}} \left[ \gamma_K(\alpha_s(\bar{\mu})) + \beta \frac{\partial}{\partial g} K(x, 1, \alpha_s(\bar{\mu})) \right],$$

(12)

Absorbing the term $\bar{\alpha}_s \ln(1-x)F(x, b, k^+)$, we arrive at the plus distribution $1/(1 - x/\xi)_+$ as splitting function. Our evolution equation becomes

$$p^+ \frac{d}{dp^+} F(x, b, k^+) = -2 \left[ s(b, k^+) - \bar{\alpha}_s(k^+) \ln(p^+b) \right] F(x, b, k^+).$$
\[ +2\alpha_s(k^+) \int_x^1 \frac{dz}{z} \frac{F(x/z, b, k^+)}{(1-z)^+}, \]  

By including the Sudakov form factor and splitting function, our new unified evolution equation, as a revised version of the CCFM equation, is very convincing.

3 Conclusion

In nature, the master equation of the Collins-Soper-Sterman resummation idea is equivalent to the evolution equation of the gluon distribution function. After carefully analysing the infrared and ultraviolet poles of evolution kernel, a new unified evolution equation can be proposed for both large and small \( x \). The infrared finiteness is explicit in both the Sudakov resummation and the splitting function. The next-to-leading logarithmic information are also included in the Sudakov form factor systematically.

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