EMERGING PATTERNS IN INFLATION EXPECTATIONS WITH MULTIPLE AGENTS

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Abstract. Macroeconomic theory and central Banks’ policy recommendations have analyzed for decades the link between the expected value of future inflation and its subsequent realization. Agents’ inflation expectations have thus become a fundamental input of the economic policy: they allow to know if economic agents are synchronized with the policies and allow the Central Banks to anticipate the market trends. In this paper, we found evidence for the case of Uruguay of a discrepancy between the distribution of agents’ inflation expectations and the distribution expected by traditional models. A first consequence is an increase in uncertainty in the estimates; problems related to its asymptotic distribution and the assumptions that arise from this aggregate distribution are analyzed. Another consequence is related to the existence of a structure in the data and the notion of equilibrium in the model. It is concluded that a discussion regarding the nature of the economic phenomenon is essential for the correct specification of the model studied.

1. Introduction.

“...Since all models are wrong the scientist must be alert to what is importantly wrong. It is inappropriate to be concerned about mice when there are tigers abroad” [3]

For modern macroeconomic theory, there is a link between future inflation and the value forecasted by agents. This is why inflation expectations are one of the most important inputs for monetary policy, in particular, to know if market agents have their expectations anchored to the values wanted by the Central Bank.

Surveys are usually carried out to experts, firms and consumers and measures of central tendency - mean, median and mode - are used as a representative value of expectations. In the same way, the sample variance is used as a measure of the uncertainty of the estimation. This article will discuss the suitability of this approach, based on analyzing the generative models of this phenomenon and the evidence for different countries.

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The main argument is that there is evidence for the case of Uruguay, that the aggregate distribution of inflation expectations are far from the distributions expected by traditional models. The consequence is that the sample moments obtained from the surveys can be - in theory - not finite. A corollary of this process is that the sample mean converges - slowly - to the true value expected by the agents. Another consequence is the increase in uncertainty of the estimates. Finally, these results provide evidence in favor of the existence of a structure in the data.

Firstly, we will propose a basic macroeconomic model with inflation expectations. Then, we will develop what consequences this modeling has on the distribution of expectations. It will be shown that the type of links between the agents of the model influences the theoretical distribution of expectations. Then, the empirical distribution of firms' inflation expectations will be studied for a sample of firms in Uruguay between 2012-2017. This paper concludes with an analysis of the alternative generative models of distributions such as those found.

2. Models of inflation expectations. The analysis of the forecast of economic variables is fundamental since they allow us to understand the future behaviors of individuals. In particular, inflation expectations influence the consumption and savings decisions of households and firms.

Inflation expectations have been heavily studied in economic theory; it was not until [8] that an endogenous dynamic of expectations about future prices is introduced. [32] states that the individuals in the model use all available information to generate expectations. The hypotheses of adaptive and rational expectations then arise as different responses to ways of endogenizing price expectations.

One aspect to note is that the proposed model must admit heterogeneity of expectations among agents. That is, the expected value of the increase in prices by agents in the economy may differ. The first explanations on this fact were to link it to an informational problem, both from the frequency of information updates [30] and from the issues about agents’ restrictions for the processing of the information [39, 45] Under the presence of information costs, it is demonstrated in [6, 7] that the rational choice of individuals may be the non-use of rational expectations. The uncertainty about the future generated by the heterogeneity of the agents can generate, as [28] states, endogenous and unpredictable changes to the economic system.

Empirical analysis show evidence in favor of adaptive expectations, both on short and long term. The analysis of [4], [17] with data from the *Michigan Household Consumer Sentiment survey*¹ show this fact for consumer expectations surveys. In this regard, we must appreciate that when comparing the performance of inflation expectations indicated by experts with consumers inflation expectations, the latter achieves a better adjustment. This happens despite the greater knowledge of the market and the status of connoisseurs of the underlying model in the economy that experts usually have ².

The basic model admits other extensions, focused mainly on the models where the process of updating expectations is linked with learning of strategies from the economic environment. In this regard, [5, 25, 1] distinguish.

¹Available at http://www.sca.isr.umich.edu/.
²There is evidence for countries with a history of both low and high inflation. For OECD countries, observe [16, 13, 11]. For Uruguay, this fact is analyzed in [22]
2.1. **Basic model.** To characterize the basic economic problem, we will follow the model introduced in [41]. The same reasoning can be done for the different formulations of the Phillips curve outlined in [14, pp. 40].

Aggregate demand curve:

\[ y_t = -\beta r_{t-1} - \pi_t + (y_{t-1} - y^*_t) + z_t^d \] (1)

Phillips curve:

\[ \pi_t = \pi_t^r [1_t - 1] + \alpha (y_{t-1} - y^*_t) + z_t^s \] (2)

Interest rate rule:

\[ r_t = a (y_t - y^*_t) + b (\pi - \pi^*_t) + r_{t-1} \] (3)

Equations (1) and (2) have a strong tradition in economic models, while the rules on the interest rate have emerged in recent decades; see [42]. We assume that there are \( n \) individuals, where each one forms their expectations as follows:

\[ \pi_{et}^{i,t+1} = f(\pi_t, \pi_{et}^i) + \varepsilon_{i,t} \] (4)

With \( \varepsilon_{i,t} \sim N(0, \sigma_\varepsilon) \), therefore, \( \pi_{et}^{i,t+1} \sim N(\mu_i, \sigma_\varepsilon) \). If the expectations are IID, the central limit theorem states that when \( n \to \infty \), the sample average \( \overline{\pi}_{et}^{i,t+1} = \frac{1}{n} \sum_{i=1}^{n} \pi_{et}^{i,t} \) it is distributed:

\[ \frac{\overline{\pi}_{et}^{i,t+1} - \mu}{\sigma/\sqrt{n}} \xrightarrow{d} Z \sim N(0,1) \] (5)

Following this approach, we will analyze one of the basic assumptions: independence in the decision-making of economic agents. This assumption is linked not only to the underlying distributions of expectations but in the ontologies stated.

2.2. **Independence: Stable distributions.** If we assume that the sample is generated from IID observations, we can say that the distributions are stable [34]. We define a variable \( X \) as stable if and only if for \( n>1 \), the Equation 6 is fulfilled, where \( X_1 \) to \( X_n \) are independent and identically distributed.

\[ X_1 + ... + X_n \xrightarrow{d} c_n X + d_n \] (6)

This family of distributions - which includes Gaussian distributions - is a way of describing systems that are sums of IID terms, but it is empirically observed that their distribution may not be symmetric. There are examples in economics from [29, 19], for the analysis of market price variations. The way to describe all possible stable distributions is through the Fourier transformation 3.

These distributions can be defined as \( S = (\alpha, \beta, \gamma, \delta) \). In this case, \( \alpha \) informs us about the stability of the model \( (0 \leq \alpha \leq 2) \), \( \beta \) refers to bias, \( \gamma \) to the scale and \( \delta \) to the location. When \( \alpha = 2 \), the associated stable distribution is the Normal distribution.

What happens when \( \alpha \neq 2? \) [27] shows that the tails of the distribution in these cases asymptotically follow a power-law, with a parameter value equal to \( \alpha \).

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3In addition to the Normal distribution, the Cauchy distribution and the Lévy distribution, there are no other expressions of stable distributions with known density function. In any case, using numerical methods, the values of interest can be obtained.
2.3. **Dependency and contagion.** From this point of view, expectations are not generated independently, but from coordination and contagion between agents. The causal phenomena can be learning, feedback between individuals or contagion following a Boltzmann distribution.

The aggregate results, in this case, are not simply the sum of its components, but a series of emerging patterns. In particular, it is assumed that the variables $\pi_{e,i,t+1}$ and $\pi_{e,t+1}$ are not isomorphic, but have qualitatively different components. The system thus defined is self-organized; then the system will be organized around a critical point.

In economics, there are many ways to characterize the interaction between individuals. Can be seen forms of interaction in the models with quantal response choice [31], exploration and exploitation [35] and fairness and altruism [21]. It is important to note that interactions between individuals are an inherent characteristic of economic systems in reality.

Here we ask ourselves what should be the appropriate analytical instruments to analyze these types of problems. Complexity economics allows analyzing the aggregate results that arise from micro behaviors and interactions between individuals, allowing for self-organization.

3. **Background.** Since the first efforts to study expectations about future prices - the surveys of the Institute for Survey Research of the University of Michigan arise in the mid-twentieth century - the distribution of responses has been analyzed, in particular their bias. [43] states that while it is expected to find an unbiased distribution, given sustained increases in prices, its distribution may be skewed to the right. In his analysis of the different moments of the distribution of inflation expectations surveys, [10] shows that there is a right bias during periods of high inflation; in turn, it finds a greater kurtosis than under a Normal distribution. From these results and their subsequent analyzes, [40] argues that inflationary phenomena can be biased to the left or the right, therefore the imposition of a Normal distribution directs the subsequent analysis. [2] explains here that reducing the distribution of expectations to a Normal distribution simplifies the calculations since the distribution can be specified from $(\mu, \sigma)$.

From the background presented, there is evidence that the distribution of inflation expectations is not of the Gaussian type, but that it is biased and right-skewed.

4. **Data.** Data from the firms’ Inflation Expectations Survey, carried out by the Central Bank of Uruguay (BCU) and the National Institute of Statistics (INE), is used. More detailed information on the survey data is shown in Table 1.

Firstly, it is analyzed whether, for the number of observations for each month, the distribution of inflation expectations fits a normal distribution [38]. In case of rejection (for a p-value < 0.05), we will analyze if the distribution can be adjusted to a power-law distribution, following the procedure described by [33, 12]. That is, it is analyzed if for the entire dataset and for each month the distribution fits the Equation 7.

$$P(X > x) = a.x^{-\alpha}$$

The $\alpha$ coefficient is estimated by Maximum Likelihood and the p-value of the statistic for each month through bootstrap, through poweRlaw package [24] of R software. In Appendix I, we will refer to the results of the coefficients $\alpha$, $\beta$, $\gamma$ and $\delta$ calculated from the assumption of stable distributions, following the procedure set by [29].
Table 1. Information obtained from Firms’ Expectations Survey. Source: BCU-INE, Uruguay. Firms were asked about their inflation expectations in 18 months ahead until July 2013, and about their inflation expectations in 24 months ahead from July 2013. It is related to the change in the monetary policy horizon of the Central Bank of Uruguay.

Main results. From what is observed in Figure 1, where the log-log graph of the expected annual inflation is shown for the entire sample and each month of 2017, it can be seen that from a given threshold - with values between 5% and 10% - the tail of the distribution behaves following a power-law.

For each period we have a cross-section data set $X_t$, composed of the responses to the survey in time $t$. The following hypothesis tests are raised:

(a) $H_0 : X_t \sim PL(\alpha, x_{\text{min}})$
$H_1 : X_t \not\sim PL(\alpha, x_{\text{min}})$

(b) $H_0 : X_t \sim N(\mu, \sigma)$
$H_1 : X_t \not\sim N(\mu, \sigma)$
In the equation 8 we can see the hypothesis tests to evaluate, given the null hypothesis of Power-Law and Normal distributions, respectively. We will reject for a given month if his p-value is less than 5%.

We analyze in Table 2 the results of these tests, where the proportion of periods in which $H_0$ is rejected is reported. The results obtained indicate that (a) we cannot reject at a significance level of 95% that the data generating process in the time span evaluated is a power-law; (b) we reject in all the analyzed periods the hypothesis of Normal distribution of expectations, for different temporal frequencies studied.

As shown in Table 3 (a), the values of $\alpha$ found are higher than those usually found in financial series (see [29], among others). This first result indicates that the volatility associated with expectations is less than the volatility associated with price variations in the market. This leads us to analyze the underlying structure of links between price variations and expectations of price variations, in addition to the links between firms that generate these patterns at the aggregate level. A possible explanation for this phenomenon is the coexistence of different types of firms in the market, with differences in information processing and the type of information employed.

Then, in Table 3 (b) it is observed that the value of the expectations from which the power-law is fulfilled is 10%. In a country with a history of moderate to high inflation and annual inflation of 8% in the period analyzed, it seems clear that there are long-term processes in this system, which do not conform to the trend of recent years. The results obtained by [23, 36] show that inflation expectations are hardly modified by current inflation, while [18] shows on laboratory experiments that using long-term expectations, although they are associated with a greater heterogeneity and distance from the hypotheses of rational expectations, at the same time they are a successful strategy for agents since it protects them from coordinating at prices that are far away from the long-term trend. Although as mentioned in [22] economic agents are more informed of macroeconomic variables compared to other countries without a history of high inflation, there is a bias in the responses towards the right tail of the distribution. Finally, Table 3 (c) complements the information
in Table 2 (a): the null hypothesis of a power-law distribution is not rejected both on 12 months and 18 to 24 months. For example, only less than 2.5% of the periods analyzed with 12-month expectations have a p-value of less than 30.6%. In other words, it is reasonable to assume that we will not reject $H_0$ for reasonable p-values.

6. Conclusions. From these results, we cannot reject that the distribution of aggregate inflation expectations is distributed as a power-law, while we can reject that they follow a Normal distribution. This motivates further analysis to determine the nature of this phenomenon.

This result provides evidence to understand the expectations formation process as a network of links between agents. Expectations would not be stated an individual decision, but rather arise from the search for information and links at the local level, in order to obtain a benefit from this information. In this context, firms obtain information about their economic environment looking to obtain a better forecast; the aggregate expectations are qualitatively different from the individual ones, so they must be studied differently. Recent empirical evidence for US [15] as well as for Latin American countries [20] are in favor of understanding the evolution of expectations as an evolutionary mechanism, so this phenomenon is inherent to the expectation generation process.

The persistence of inflation at the aggregate level is another problem that arises as a consequence of the expectations formation process. From an econometric point of view, it has only been recently that long memory processes have been analyzed and spurious processes have been distinguished. A detailed review and results for European countries and the US can be found at [36]. Similar conclusions can be found in [44], where there is evidence for Latin American countries that inflationary processes perform as fractional Brownian noises.

On the other hand, if we assume that individual and aggregate expectations are distributed in the same way, this implies that the term $\varepsilon_{i,t}$ is biased and follows the same distribution as $\pi_e$. This has two fundamental consequences, one theoretical and one empirical. From the theoretical point of view, if we want to maintain independence in the observations we must explain the origin of the biased distribution of the errors; from the empirical point of view, it generates problems to analyze the coefficients associated with inflation expectations by econometric methods, due to bias.

Another aspect is the uncertainty associated with the estimates. Depending on the chosen distribution of expectations, it will be differences in uncertainty. In the present analysis, a power-law distribution is not rejected, but at the same time, there is important evidence in favor of assuming that the first moments of the distribution are finite. However, assuming that the underlying distribution is not Normal implies greater uncertainty about the future and the different moments of the distribution account for the relative importance of the average - or median - of expectations as the only relevant policy variable. Analyze the heterogeneity and the information transmission networks in the economy then become fundamental for a good performance of the macroeconomic variables.

Finally, we will refer again to the main topic of this article: on the link between the nature of the economic processes, the assumptions made from the theory, the empirical strategies used to capture these phenomena and the policy recommendations derived from the analysis. One possible approach, mentioned by [26],
indicates that the methods used must be adapted to the nature of economic phenomena. Another, more radical approach is the observation of [9]: the separation between economics and the economy. This has led us to the methods - in the form of constrained maximization - that define the economic discipline. In this way, the current problem in economic discipline lies in the imposition of preconceived models on reality, which prevents their full understanding. Advancing in a correct diagnosis of economic phenomena will generate the conditions for the use of appropriate techniques and more useful policy recommendations will be provided.

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Appendix I. This section shows the results of the estimation of the parameters under the hypothesis of IID observations, under generalization from stable distributions. The calculations were made through libstableR package [37] of R software.

As can be seen, the values found in Figure 2 are not consistent with a Normal distribution in any of the periods. It is observed in the graph of $\alpha$ that maximum values of 1.8 is reached, for this reason we reject the hypothesis that our dataset can be obtained from a Normal distribution during the analyzed period.

Bias is another characteristic of these data series. Since $-1 < \beta < 1$, where $\beta > 0$ implies right fat tails, while $\beta < 0$ implies left fat tails. It is observed that
Figure 2. Estimation of parameters $\alpha, \beta, \gamma$ and $\delta$. In dotted lines 95% confidence intervals, calculated using bootstrap.

during the period analyzed $\beta$ is between 0.6 and 1, which adds evidence in favor of a bias to the right, following the background set forth above.

The $\gamma$ graph shows that uncertainty in expectations (measured from the degree of disagreement) increases at the end of the analyzed period. This is linked to the $\delta$ graph - a proxy of the average distribution - that shows an increase in expectations in 2015 and a sharp decrease from mid-2016. This last graph is strongly linked to annualized inflation in each period.

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