Fermion localization on branes with generalized dynamics

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Abstract – In this letter we consider a specific model of braneworld with nonstandard dynamics diffused in the literature, specifically we focus our attention on the matter energy density, the energy of system, the Ricci scalar and the thin-brane limit. As the model is classically stable and capable of localize gravity, as a natural extension we address the issue of fermion localization of fermions on a thick brane constructed out from one scalar field with nonstandard kinetic terms coupled with gravity. The contribution of the nonstandard kinetic terms to the problem of fermion localization is analyzed. It is found that the simplest Yukawa coupling \(\overline{\Psi} \phi \Psi\) supports the localization of fermions on the thick brane. It is shown that the zero mode for left-handed fermions can be localized on the thick brane depending on the values for the coupling constant \(\eta\).

Introduction. – In the last decade, the braneworld scenario has attracted a lot of interest for it gives an effective way to solve the hierarchy problem by introducing two 3-branes which are embedded in a five-dimensional anti-de Sitter (AdS\(_5\)) space-time [1]. Parallel researchs on noncompact extra dimensions focused on trapping matter and model of the Universe as a 3-shell expanding in five-dimensional space-time [2] have also attracted interest. As another attractive property, the Newtonian law of gravity with a correction is also given in this braneworld scenario [3]. In the Randall-Sundrum (RS) model [1], we can further add scalar fields [4] with usual dynamics and allow them to interact with gravity in the standard way. In this scenario, the smooth character of the solutions generate thick branes with a diversity of structures [5–8]. In the braneworld scenarios, an important issue is how gravity and different observable matter fields of the Standard Model of particle physics are localized on the brane. It has been shown that, in the RS model in five-dimensional space-time, graviton and spin-0 field can be localized on a brane with positive tension [3,9,10]. Moreover, spin-(1/2) and -(3/2) can be localized on a negative-tension brane [10].

The localization problem of spin-(1/2) fermions on thick branes is interesting and important [9–22]. In order to achieve localization of fermions on a brane with positive tension, it seems that additional interactions except the gravitational interaction must be included in the bulk.

On the other hand, the first recent observations [23] have led us with the intriguing fact that the Universe is presently undertaking accelerated expansion. These information directly contributed to establish some important advances in cosmology, one of them being the presence of dark energy. The presence of dark energy has opened some distinct routes of investigations. In recent years, there appeared some interesting models with noncanonical dynamics with focus on early-time inflation or dark energy [24–27], as, for instance, the so-called k-fields, first introduced in the context of inflation [27] and the k-essence models, which were suggested to solve the cosmic coincidence problem [26–28]. The interaction between dark energy and fermion fields has already a precedent in the cosmology context [29]. In the context of the braneworld scenario, the effect of general brane kinetic terms for bulk scalars, fermions and gauge bosons in theories with and without supersymmetry has already been analyzed in [30]. We believe that the conditions for obtaining zero modes in braneworld models of scalar fields with generalized dynamics deserve to be more explored.

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The purpose of the present letter is to analyze the contribution of the nonstandard kinetic terms of a brane model to the problem of fermion localization. To achieve our goal, the braneworld model with nonstandard dynamics \( \mathcal{L} = K(X) - V(\phi) \), where \( K(X) = X + \alpha |X| X \) (type-I model in [24]) is considered. We will focus our attention mainly on the matter energy, the energy of system, the Ricci scalar and the thin-brane limit. As the model is classically stable and capable of localize gravity, additionally we address ourselves to the issue of fermion localization on a thick brane constructed out from one scalar field with nonstandard kinetic terms coupled to gravity. Following the same procedure of [24], we use the analytical expressions for small \( \alpha \) and investigate the contribution of this nonstandard kinetic terms to the problem of fermion localization. We find that the simplest Yukawa coupling \( \eta \bar{\psi} \psi \), where \( \eta \) is the coupling constant, allows left-handed fermions to possess a localized zero mode on the thick brane under some conditions on the value for the coupling constant \( \eta \). For \( \alpha \), which is not necessarily small, we cannot get any expressions for the solution of this model, for this case the numerical study is used. The numerical results bear out our results for small \( \alpha \). The organization of this paper is as follows: in the second section, we give a brief review of the model with generalized dynamics developed by Bazeia and collaborators [24]. In the third section, we study the localization of spin-(1/2) fermion for this model and we also analyze the essential conditions for the localization with the simplest Yukawa coupling. Finally, our conclusions are presented in the fourth section.

**Review of systems with generalized dynamics.**

The action for this kind of system is described by [24]

\[
S = \int d^5x \sqrt{|g|} \left[ -\frac{1}{4} R + \mathcal{L}(\phi, X) \right],
\]

where \( g \equiv \text{Det}(g_{ab}) \) and \( X = \frac{1}{2} \nabla^a \phi \nabla_a \phi \). The line element of the five-dimensional space-time can be written as

\[
dx^2 = g_{ab} dx^a dx^b = e^{2\mathcal{A}(\phi)} g_{\mu\nu} dx^\mu dx^\nu - dy^2,
\]

where we are using the five-dimensional Newton constant \( 4\pi G^{(5)} = 1 \), \( y = x^4 \) is the extra dimension (the Latin indices run from 0 to 4), \( g_{\mu\nu} \) is the Minkowski metric with signature \((+,-,-,-,-)\) and \( e^{2\mathcal{A}} \) is the so-called warp factor (the Greek indices run from 0 to 3). We suppose that \( A = A(y) \) and \( \phi = \phi(y) \).

One can determine that the static equations of motion for the above system are of the form

\[
(\mathcal{L}_X + 2X \mathcal{L}_{XX}) \phi'' - (2X \mathcal{L}_X \phi - \mathcal{L}_\phi) = -4\mathcal{L}_X A' \phi',
\]

\[
A'' + 2A' = \frac{2}{3} \mathcal{L},
\]

\[
A'^2 = \frac{1}{3} (\mathcal{L} - 2X \mathcal{L}_X),
\]

where the prime stands for derivative with respect to \( y \), \( \mathcal{L}_X = \partial \mathcal{L}/\partial X \) and \( \mathcal{L}_\phi = \partial \mathcal{L}/\partial \phi \). Furthermore, the matter energy density is given by

\[
\rho(y) = -e^{2\mathcal{A}(y)} \mathcal{L},
\]

and the scalar curvature (or Ricci scalar) is given by

\[
R = -4(5A'^2 + 2A'').
\]

The Lagrangian density \( \mathcal{L}(\phi, X) \) has the form

\[
\mathcal{L} = K(X) - V(\phi),
\]

where \( K(X) \) and \( V(\phi) \) are the nonstandard kinetic term and the potential, respectively. For this case, from eqs. (3), (4) and (5) the equations of motion can be expressed as

\[
(\phi'' + 2X K'') \phi'' - \phi = -4K' A' \phi',
\]

\[
A'' + 2A'^2 = \frac{2}{3} (K - V),
\]

\[
A'^2 = \frac{1}{3} (K - V - 2X K').
\]

These equations are the static equations of motion of a system with nonstandard dynamics. In [24], the authors present two explicit models for \( K(X) \), here we review one of these models.

The model: \( K(X) = X + \alpha |X| X \). This model is also considered in [31], where \( \alpha \) is a real non-negative parameter and \( X = -\frac{1}{2} \phi'^2 \). If \( \alpha = 0 \) the standard scenario is restored. For this model, the equations of motion are

\[
\phi'' + 4A' \phi' - V = -\alpha (3\phi'' + 4\phi' A') \phi'^2,
\]

\[
A'' + 2A'^2 = -\frac{1}{3} \left( 1 + \frac{\alpha}{2} \phi'^2 \right) \phi'^2 - \frac{2}{3} V,
\]

\[
A'^2 = \frac{1}{6} \left( 1 + \frac{3}{2} \alpha \phi'^2 \right) \phi'^2 - \frac{1}{3} V.
\]

It is possible to rewrite (13) and (14) as

\[
A'' = -\frac{2}{3} \phi'^2 \left( 1 + \alpha \phi'^2 \right).
\]

Now, to extend the first-order framework to the braneworld scenario, we follow the work in [5], and choose the derivative of the warp factor with respect to the extra dimension to be a function of the scalar field

\[
A' = \frac{1}{3} W(\phi).
\]

Substituting (16) into (15), we get

\[
\phi' + \alpha \phi'^3 = \frac{1}{2} W(\phi),
\]

this equation is a cubic equation in \( \phi' \), then the real solution to this cubic equation is given by

\[
\phi' = \frac{m(W_\phi)}{2\alpha} - \frac{2}{m(W_\phi)}.
\]

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where
\[ m(W_\phi) = \left(54\alpha^2 W_\phi + 6\sqrt{3}\left(16\alpha^2 + 27\alpha^4 W_\phi^2\right)^{1/2}\right)^{1/3}. \]  
It is instructive to note that eq. (18) is the first-order differential equation for the scalar field \( \phi \). The potential is obtained by substituting (16) in (14),
\[ V(\phi) = \frac{1}{2} \phi'^2 + \frac{3}{4} \alpha \phi^4 - \frac{1}{3} W(\phi)^2, \quad (20) \]
where \( \phi' \) is given by (18). On the other hand, we consider the energy functional [5]
\[ E[A, \phi] = \int dy (-L_{\text{system}}), \quad (21) \]
where \( L_{\text{system}} = \sqrt{|g| [-R/4 + L(\phi, X)]} \). This energy functional coincides with the Hamiltonian deduced from Einstein-Hilbert action (including a surface term), which is well defined for spatially noncompact geometries and it fully agrees with the definition of the total energy in general relativity, as was reported by Hawking [32]. In our case, (21) becomes
\[ E[A, \phi] = \int e^{4A} \left\{ \frac{1}{2} \phi'^2 - 3A'^2 + \frac{\alpha}{4} \phi^4 + V \right\} dy. \quad (22) \]
The Euler-Lagrange differential equations from the functional (22) are the static equations of motion (12) and (13), this result is valid for all \( \alpha \). Otherwise, substituting the potential (20) in (22), for \( \alpha \) which is not necessarily small, we cannot deduce the first-order differential equations (16) and (18) from the energy functional, therefore the solutions of the first-order differential equations not necessarily minimize the energy functional. At this point, it is also instructive to analyze the matter energy in the model with nonstandard dynamics. From (6), (8) and (20), we get
\[ E_\phi = \int e^{2A} \left\{ \phi'^2 + 4 \phi'^4 - \frac{1}{3} W^2 \right\} dy. \quad (23) \]
Finally, using (15) and (16) and integrating, we obtain
\[ E_\phi = \frac{1}{2} \left( e^{2A(\phi)} W(\phi(\infty)) - e^{2A(-\infty)} W(\phi(-\infty)) \right), \quad (24) \]
the value of the matter energy for all \( \alpha \). Note that the matter energy depends on the asymptotic behavior of the warp factor. Now, we follow the same procedure of [24] and let us focus our study in the case of \( \alpha \) very small. Thus, the solution of (17), up to first order in \( \alpha \) becomes
\[ \phi' = \frac{1}{2} W_\phi - \frac{\alpha}{8} W_\phi^3, \quad (25) \]
and substituting (25) into (20) we obtain the potential
\[ V(\phi) = \frac{1}{8} W_\phi^2 - \frac{\alpha}{64} W_\phi^4 - \frac{1}{3} W^2. \quad (26) \]
At this point we turn to examine the energy functional (22). Substituting the potential (26) in (22), we get
\[ E[A, \phi] = \int e^{4A(\phi)} \left\{ \frac{1}{2} \left( \phi'^2 - \frac{1}{2} W_\phi + \frac{\alpha}{8} W_\phi^3 \right) \right. \]
\[ - 3 \left( A' + \frac{1}{3} W \right)^2 \left. \right\} + \frac{\alpha}{8} \int e^{4A} \left( 2\phi'^4 + \frac{3}{8} W_\phi^4 \right. \]
\[ - W_\phi^3 \phi' \left. \right) + \frac{1}{2} \int_\infty^\infty \frac{dy}{\phi'} \left( W e^{4A} \right). \quad (27) \]
From (27) we get an important new result, the solutions of the first-order differential equations, (16) and (25), are those that minimize the energy functional. Thus, the first-order differential equations can be seen as the BPS equations and the energy functional could play the role of a BPS energy in such scenario. The value of the energy system for \( \alpha \) very small is given by
\[ E[A, \phi] = e^{4A(\phi(\infty))} W(\phi(\infty)) - e^{4A(-\infty)} W(\phi(-\infty)) \quad (28) \]
Again, the asymptotic behavior of the warp factor plays a leading role in the value of the energy functional. Now, we find explicitly the solutions for (16) and (25), which minimize the energy functional. Then, the solution for (25) becomes
\[ \phi(y) = \phi_0(y) - \frac{\alpha}{4} W_\phi(\phi_0(y)) W(\phi_0(y)), \quad (29) \]
where \( \phi_0(y) \) is the solution when \( \alpha = 0 \). From (16) and (29), we obtain
\[ A(y) = A_0(y) + \frac{\alpha}{12} W(\phi_0(y))^2, \quad (30) \]
where \( A_0(y) \) represents \( A(y) \) when \( \alpha = 0 \). The matter energy density given by (6) is
\[ \rho = e^{2A(y)} \left( \frac{1}{4} W_\phi^2 - \frac{1}{3} W^2 - \frac{\alpha}{16} W_\phi^4 \right), \quad (31) \]
substituting (29) and (30) in (31), we obtain
\[ \rho = \rho_0 - \frac{\alpha}{48} e^{2A_0(y)} G_0, \quad (32) \]
where
\[ \rho_0 = e^{2A_0(y)} \left( \frac{1}{4} W_\phi^2 - \frac{1}{3} W^2 \right)_{\phi = \phi_0}, \quad (33) \]
and
\[ G_0 = \left( 6 W_\phi W_\phi' W - 10 W^2 W_\phi + 3 W_\phi^4 + \frac{8}{3} W^4 \right)_{\phi = \phi_0}, \quad (34) \]
note that the energy density (32) is a little bit different from that given in [24]. To show the validity of the solutions, (29), (30) and (32), we consider the superpotential \( W(\phi) \) of the form [7]
\[ W(\phi) = 3 b c \sin \left( \frac{\sqrt{5}}{30} \phi \right), \quad (35) \]

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where $b$ and $c$ are positive parameters. The classical solutions for (29) and (30) are given by

$$\phi(y) = \frac{\sqrt{3}b}{2} \arcsin \left[ \tanh(cy) \right] - \frac{3\sqrt{6\alpha}}{4} b^{3/2} c^2 \tanh(cy),$$

and

$$A(y) = b \ln \left[ \text{sech}(cy) \right] + \frac{3\alpha}{4} b^2 c^2 \tanh^2(cy).$$

The profiles of the matter energy density and the Ricci scalar are shown in fig. 1 for $\alpha = 0.1$. A similar behavior is obtained for $\alpha = 1$ and $\alpha = 10$. Note that the presence of regions with positive Ricci scalar is connected with the localization of the brane. Also note that far from the brane, $R$ tends to a negative constant, characterizing the AdS$_5$ limit from the bulk. In the limit $|y| \to \infty$, the warp factor is $e^{3A} \to 0$, therefore the matter energy (24) and the energy of system (28) both are zero and the thick-brane solution coincides with the solution of RS model as $bc \to \text{const}$ [33].

Now on, we can analyze the exact solutions in the thin-brane limit ($c \to \infty$ and the product $bc$ is held fixed) [33, 34], we obtain

$$\phi(y) = \frac{\sqrt{6b}}{4} \pi \text{sgn}(y),$$

and

$$A(y) = -bc|y| + \frac{3\alpha}{4} b^2 c^2,$$

where the term $\frac{3\alpha}{4} b^2 c^2$ in (39) is the contribution of nonstandard kinetic term to the brane model geometry. From this we get another new result, the solutions of the braneworld model treated here is consistent, because when $\alpha = 0$ it reduces to the solutions of RS model (thin-brane model).

**Fermion localization.** – The action for a Dirac spinor field coupled with the scalar fields by a general Yukawa coupling is

$$S = \int \mathcal{D}^2 x \sqrt{|g|} \left[ i \bar{\psi} \Gamma^M \nabla_M \psi - \eta F(\phi) \psi \right],$$

where $\eta$ is the positive coupling constant between fermions and the scalar field. Moreover, we are considering the covariant derivative $\nabla_M = \partial_M + \frac{1}{4} \omega^A_M \Gamma_A \Gamma^B \Gamma_B$, where $\Gamma^A$ and $\Gamma^B$ denote the local Lorentz indices and the spin connection. Here we consider the field $\phi$ as a background field. The equation of motion is obtained as

$$i \Gamma^M \nabla_M \Psi - \eta F(\phi) \Psi = 0.$$  \hspace{1cm} (41)

We choose the irreducible representation $\Gamma^\mu = e^{-A} \gamma^\mu$ and $\Gamma^A = -i\gamma^5$. The equation of motion (41) becomes

$$[i\gamma^\nu \partial_\nu + \gamma^5 e^A(\partial_\mu + 2\partial_\nu A) - \eta e^A F(\phi)] \Psi = 0.$$  \hspace{1cm} (42)

Now, we use the general chiral decomposition

$$\Psi(x, y) = \sum_n \psi_L_n(x) \alpha_{L_n}(y) + \sum_n \psi_R_n(x) \alpha_{R_n}(y),$$

with $\psi_L_n(x) = -\gamma^5 \psi_{\bar{L}_n}(x)$ and $\psi_R_n(x) = \gamma^5 \psi_{\bar{R}_n}(x)$. With this decomposition $\psi_{\bar{L}_n}(x)$ and $\psi_{\bar{R}_n}(x)$ are the left-handed and right-handed components of the four-dimensional spinor field, respectively. After applying (43) in (42), and demanding that $i\gamma^\nu \partial_\nu \psi_{\bar{L}_n} = m_n \psi_{\bar{L}_n}$ and $i\gamma^\nu \partial_\nu \psi_{\bar{R}_n} = m_n \psi_{\bar{R}_n}$, we obtain two equations for $\alpha_{L_n}$ and $\alpha_{R_n}$:

$$[\partial_\nu + 2\partial_\nu A + \eta F(\phi)] \alpha_{L_n} = m_n e^{-A} \alpha_{R_n},$$

and

$$[\partial_\nu + 2\partial_\nu A - \eta F(\phi)] \alpha_{R_n} = -m_n e^{-A} \alpha_{L_n}.$$  \hspace{1cm} (44, 45)

The orthonormality condition for $\alpha_{L_n}$ and $\alpha_{R_n}$ is given by

$$\int_{-\infty}^{\infty} dy e^{3A} \alpha_{L_n} \alpha_{R_n} = \delta_{LR} \delta_{mn}.$$  \hspace{1cm} (46)

Implementing the change of variables

$$z = \int_0^y e^{-A(y')} dy',$$

we get

$$\alpha_{L_n} = e^{-2A} L_n$$

and $\alpha_{R_n} = e^{-2A} R_n$, we get

$$-L_n''(z) + V_L(z) L_n = m_n^2 L_n,$$

and

$$-R_n''(z) + V_L(z) R_n = m_n^2 R_n,$$  \hspace{1cm} (48, 49)

where

$$V_L(z) = \eta e^{2A} F(\phi)^2 - \eta \partial_z (e^{A} F(\phi)),$$

and

$$V_R(z) = \eta e^{2A} F(\phi)^2 + \eta \partial_z (e^{A} F(\phi)).$$  \hspace{1cm} (50, 51)

Using the expressions $\partial_z A = e^{A(y)} \partial_y A$ and $\partial_z F = e^{A(y)} \partial_y F$, we can recast the potentials (50) and (51) as a function of $y$ [21],

$$V_L(z(y)) = \eta e^{2A} \left[ F(\phi)^2 - \partial_y F - F \partial_y A(y) \right],$$

and

$$V_R(z(y)) = V_L(z(y)) |_{y \to -y}.$$  \hspace{1cm} (52, 53)

Now we focus attention on the calculation of the zero mode. Substituting $m_n = 0$ in (44) and (45) and using $\alpha_{L_n} = e^{-2A} L_n$ and $\alpha_{R_n} = e^{-2A} R_n$, respectively, we get

$$L_0 \propto \exp \left[ -\eta \int_0^y dy' F(\phi) \right],$$  \hspace{1cm} (54)
\[ R_0 \propto \exp \left[ \eta \int_0^\eta dy' F(\phi) \right]. \]  
\hfill (55)

This fact occurs also for the case of two-dimensional Dirac equation (isolated solutions)\textsuperscript{[35]}. In order to guarantee the normalization condition (46) for the left-handed fermion zero mode (54), the integral must be convergent, i.e,

\[ \int_\infty^{-\infty} dy \exp \left[ -A(y) - 2\eta \int_0^\eta dy' F(\phi(y')) \right] < \infty. \]  
\hfill (56)

This result clearly shows that the behavior of \( F(\phi(y')) \) plays a leading role for the fermion localization on the brane\textsuperscript{[21]}.

**Zero mode and fermion localization.** From now on, we mainly consider the simplest case \( F(\phi) = \phi \). First, we consider the normalizable problem of the solution. In this case, from eqs. (36) and (37) the integrand in (56) can be expressed as

\[ I = \exp \left[ -b \ln \left( \text{sech} \left( c y \right) \right) - \frac{3\alpha}{4} b^2 c^2 \tanh^2 \left( c y \right) \right. \]
\[ \left. -\eta \sqrt{6b} \bar{I}(y) - \frac{3\sqrt{6\alpha}}{2} b^{3/2} c \text{sech} \left( c y \right) \right], \]  
\hfill (57)

where \( \bar{I} = \int dy' \arcsin \left[ \tanh \left( c y' \right) \right] \). Following the same procedure of\textsuperscript{[15]}, we only need to consider the asymptotic behavior of the integrand. It becomes

\[ I \rightarrow \exp \left[ - \left( \eta \pi \sqrt{\frac{3b}{2}} - bc \right) |y| - \frac{3}{4} \alpha b^2 c^2 \right]. \]  
\hfill (58)

This result clearly shows that if \( \eta \geq \frac{\pi}{2} \left( \sqrt{\frac{2b}{c}} \right) \) the zero mode of the left-handed fermions is normalized. Note that the asymptotic behavior of the normalization condition for this case is independent of \( \alpha \). For the right-handed fermions, we can use the change \( \eta \rightarrow -\eta \) (that implies \( L_0 \rightarrow R_0 \)) in (58) and we can conclude that the right-handed fermions cannot be a normalizable zero mode. The shapes of the potentials for this case are shown in figs. 2 and 3 for some values of \( \alpha \). Figure 2 shows that the potential of left-handed fermions, \( V_L \), is indeed a volcano-like potential and that the depth of the well structure decreases as \( \alpha \) increases. From this, we can conclude that the ability to trap fermions of the effective potential \( V_L \) is inversely proportional to \( \alpha \). On the other hand, fig. 3 shows that the potential \( V_R \) is always positive (no bound fermions) and this effective potential has a maximum that decreases as \( \alpha \) increases. The analytic expressions for this model are only valid for small \( \alpha \). For \( \alpha \), which is not necessarily small, the numerical study is used. The numerical study done for a large range of values of \( \alpha \) bears out our results.

**Conclusions.** – We have considered the braneworld model with nonstandard kinetic terms \( \mathcal{L} \equiv K(X) - V(\phi) \), where \( K = X + \alpha |X|X \) (type-I model in\textsuperscript{[24]}). This letter completes the analysis of the meritorious research in\textsuperscript{[24]}. We showed that the equations of motion for all \( \alpha \)’s can be deduced from the functional \( E[A, \phi] \) (22), as done by Townsend\textsuperscript{[5]} in the case of standard dynamics. Furthermore, we showed that for small \( \alpha \) the solutions of the first-order differential equations, (16) and (25), are those that minimize the energy of system. In contrast, for \( \alpha \), which is not necessarily small, the solutions of the first-order differential equations not necessarily minimize the energy of system. Also, we showed that the value of the matter energy and the energy of system depends on the asymptotic behavior of the warp factor. We found an expression for the matter energy density that differs slightly from\textsuperscript{[24]}. The numerical study gives full support to our matter energy density expression for small \( \alpha \). Furthermore, we showed that the braneworld model with nonstandard dynamics treated here is consistent, because it reduces to the RS model (thin-brane model) for \( \alpha = 0 \). We also have investigated the localization problem of fermions for the type-I model. We have used the simplest Yukawa coupling \( \eta \Psi \phi \Psi \) between the scalar and the spinor fields. In order to guarantee the normalization condition for the zero mode, we showed that the zero mode of left-handed fermions is normalizable under the condition \( \eta \geq \frac{\pi}{2} \left( \sqrt{\frac{2b}{c}} \right) \) and it is independent of \( \alpha \). For this kind of solution, the effective potential of left-handed fermions \( V_L \) is a volcano-like potential. \( V_L \) has a minimum at the localization of the brane \( (y = 0) \), therefore the zero mode of the left-handed fermions is localized on the brane.
On the other hand, the value of $\alpha$ adjust the minimum of $V_L$, the depth of the well structure decreases as $\alpha$ increases. Therefore, we can conclude that the ability to trap fermions of $V_L$ is inversely proportional to $\alpha$. For $\alpha$ not necessarily small, the numerical study done for a large range of values of $\alpha$ bears out our results and conclusions. Finally, based on our results, we intend to expand the present study to analyze the interesting problem of localization of other spin fields.

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