Is quantum gravity necessary?

S Carlip

Department of Physics, University of California, Davis, CA 95616, USA
E-mail: carlip@physics.ucdavis.edu

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Abstract
In view of the enormous difficulties we seem to face in quantizing general relativity, we should perhaps consider the possibility that gravity is a fundamentally classical interaction. Theoretical arguments against such mixed classical–quantum models are strong, but not conclusive, and the question is ultimately one for experiment. I review some work in progress on the possibility of experimental tests, exploiting the nonlinearity of the classical–quantum coupling, which could help settle this question.

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The first attempts to quantize general relativity date back to the early 1930s [1]. In the 75 years that have followed, we have learned an enormous amount: gauge-fixing and Faddeev–Popov ghosts, background field methods, the effective action formalism, the canonical analysis of constrained systems, the investigation of gauge-invariant observables, and much of what we know about topology in quantum field theory grew out of attempts to quantize gravity. But despite the extraordinary work of a great many outstanding physicists, a complete, consistent and compelling quantum theory of gravity still seems distant [2].

In view of this history, we should perhaps consider the possibility that we are asking the wrong question. It could be that gravity is simply not quantum mechanical. The prospect of a fundamentally classical theory of gravity is unpalatable; in Duff’s words [3], it ‘seems to be the very antithesis of the economy of thought which is surely the basis of theoretical physics’. But the matter is ultimately one for experiment. As Rosenfeld has put it [4].

It is nice to have at one’s disposal such exquisite mathematical tools as the present methods of quantum field theory, but one should not forget that these methods have been elaborated in order to describe definite empirical situations, in which they find their only justification. Any question as to their range of application can only be answered by experience, not by formal argumentation. Even the legendary Chicago machine cannot deliver the sausages if it is not supplied with hogs.

There are old arguments that fundamentally classical fields are incompatible with quantum mechanics, in the sense that they could be used to violate the uncertainty principle [5]. Details
depend on how the classical field interacts with a quantum system. Eppley and Hannah [6, 7] have considered two cases:

1) A classical gravitational measurement collapses the quantum wavefunction: then momentum is not conserved. Consider a quantum object in a coherent state with a very small uncertainty in momentum and a correspondingly large uncertainty in position. Measure its position by scattering a very short wavelength gravitational wave, causing its state to change to one with a very small uncertainty in position and a large uncertainty in momentum. If gravity is classical, the gravitational wave can carry an arbitrarily small momentum, despite its short wavelength; yet by the uncertainty principle, the quantum system must sometimes experience a large change in momentum.

2) A classical gravitational measurement does not collapse the quantum wavefunction: then signals can be sent faster than light. Place a proton in a box, in a state in which it has an equal probability of being in the left or right half. Split the box into half and carry one half to a remote location. Monitor your half continuously with gravitational measurements, while a colleague performs a nongravitational measurement of the other half. Your colleague’s measurement will collapse the wavefunction, causing an instantaneous and detectable change in the half of the box you are monitoring.

Page and Geilker [8] added a third case:

3) Neither classical nor quantum measurements collapse the wavefunction (Everett interpretation): then gravitational fields will not be observed to have localized sources. Consider a gravitating mass in a superposition of two widely separated position eigenstates. If its classical gravitational field depends on its quantum wavefunction, its gravitational attraction should point toward some intermediate ‘average’ location [9, 10]. Page and Geilker tested this experimentally, but the outcome is already apparent in, say, the observed gravitational field of the moon.

But while such arguments are certainly suggestive, they are not really conclusive [4, 12, 7, 11]. For instance, there are inherent non-quantum limits to gravitational measurements [12, 13], whose implications for an Eppley–Hannah-type argument have yet to be fully explored. The general question of whether one can consistently couple classical and quantum systems is a matter of ongoing research—see, for example, [14–20]—and is not yet resolved.

The thought experiments of Eppley, Hannah and others do, however, suggest that a fundamentally classical theory of gravity is likely to require changes to quantum mechanics as well. As I shall argue below, once one allows a coupling between classical and quantum systems, quantum mechanics almost inevitably becomes nonlinear, suggesting the possibility of sensitive new experimental tests.

1. Semiclassical gravity and the Schrödinger–Newton equation

If we wish to couple classical gravity and quantum matter, we need field equations for gravity. The standard Einstein equations,

$$G_{ab} = 8\pi \tilde{T}_{ab},$$

no longer make sense, since they now equate a $c$-number with an operator. We might try to interpret (1.1) as an eigenvalue equation, but this picture fails: the components of the stress–energy tensor do not commute, and cannot be simultaneously diagonalized [10].
The obvious next step is to replace the right-hand side of (1.1) with an expectation value,

$$G_{ab} = 8\pi \langle \psi | \hat{T}_{ab} | \psi \rangle,$$

(1.2)

leading to the model of ‘semiclassical gravity’ first proposed by Møller [21] and Rosenfeld [4], and derived from an action principle by Kibble and Randjbar-Daemi [22]. Seen merely as a Hartree approximation to a full quantum theory of gravity, such a model seems uncontroversial. But as Kibble and Randjbar-Daemi emphasized [22], seen as a fundamental theory, the model implies nonlinearities in quantum mechanics: the Schrödinger equation for the wavefunction $|\psi\rangle$ depends on the metric, which now depends in turn on the wavefunction. Adler has observed that semiclassical gravity contains self-interaction terms that are not present in a Hartree approximation [23], further differentiating it from a mere approximation to a full quantum theory.

Several technical problems with semiclassical gravity have been pointed out in the literature. Field redefinition ambiguities can lead to inequivalent quantizations of the same classical theory [3]; renormalization may either require classical curvature-squared terms in the action that can lead to negative energies [24] or new matter vertices that imply noncausal behavior at short distances [25]; and it is not obvious that an abrupt change on the right-hand side of (1.2) due to wavefunction collapse can be consistent with conservation of the left-hand side [10]. Again, though, these objections do not seem conclusive. The nonlinearity of semiclassical gravity, on the other hand, suggests that experimental tests may be possible: gravity is very weak, but limits on nonlinearities in quantum mechanics are very strong [26].

To address this question, it is useful to start with the Newtonian approximation to (1.2), the Schrödinger–Newton equation [27, 28]

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi - m\Phi \psi, \quad \nabla^2 \Phi = 4\pi G m |\psi|^2.$$  

(1.3)

As in full semiclassical gravity, this model treats matter quantum mechanically, but describes gravity in terms of a classical Newtonian potential $\Phi$ sourced by the expectation value of the mass density. Despite the nonlinearities of the coupled system (1.3), the standard probability interpretation of the wavefunction remains consistent; in particular, the probability current continuity equation

$$\frac{\partial}{\partial t} |\psi|^2 = \vec{\nabla} \cdot \left[ \frac{i\hbar}{2m} (\psi^* \vec{\nabla} \psi - \psi \vec{\nabla} \psi^*) \right]$$  

(1.4)

still holds, and total probability is conserved. A number of authors have studied this system [29–32], and we know a good bit about the stationary states with low-energy eigenvalues, but time evolution has proven to be much more problematic [33–35].

2. Estimates and numerics

The question, then, is whether the nonlinearities in the Schrödinger–Newton equation (1.3) are large enough to lead to observable consequences. Let us begin with a rough estimate. Consider a particle of mass $m$ with a localized initial wavefunction—for simplicity, a Gaussian,

$$\psi(r, 0) = \left( \frac{\alpha}{\pi} \right)^{3/4} e^{-\alpha r^2 / 2}$$  

(2.1)

with width $\alpha^{-1/2}$. The time evolution of $\psi$ will depend on two competing effects, the quantum-mechanical spreading of the wavefunction and its Newtonian ‘self-gravitation’, the latter arising because semiclassical gravity treats a wavefunction as a distributed source. For

1 Dirac was also apparently aware of this; see [12], p 1.
a very low mass, self-gravitation should be negligible, while for a high enough mass, the wavefunction should undergo 'gravitational collapse'.

To estimate the critical mass at the boundary between wave packet spreading and collapse, note first that the peak probability density for a free particle occurs at

$$r_p \sim \alpha^{-1/2} \left(1 + \frac{\alpha^2 \hbar^2}{m^2 t^2}\right)^{1/2},$$

which 'accelerates' outward at a rate $$a_{\text{out}} = \ddot{r}_p \sim \hbar^2 / m^2 r_p^3$$. This will balance the inward gravitational acceleration $$a_{\text{in}} \sim Gm/r_p^2$$ at $$t = 0$$ when

$$m \sim \left(\frac{\hbar^2 \sqrt{\alpha}}{G}\right)^{1/3}.$$  

This is almost certainly an overestimate: as $$t$$ increases, $$a_{\text{out}}$$ drops more quickly than $$a_{\text{in}}$$, so even if wave packet spreading dominates initially, self-gravity may eventually win.

For more precise results, one must solve (1.3) numerically. Note that although the initial data (2.1) depend on two parameters, $$\alpha$$ and $$m$$, the Schrödinger–Newton equation is invariant under the rescalings

$$m \rightarrow \mu m, \quad \vec{x} \rightarrow \mu^{-3} \vec{x}, \quad t \rightarrow \mu^5 t, \quad \psi \rightarrow \mu^{9/2} \psi,$$

so it is enough to consider a one-parameter set of solutions. Peter Salzman and I have numerically simulated the evolution of an initial Gaussian wavefunction [36, 37]. We find the expected qualitative results:

1. For small masses, the behavior is virtually identical to that of a free particle, while as $$m$$ increases, the wave packet spreads more slowly.
2. In a transitional range of mass, the wave packet is unstable, fluctuating rapidly and developing growing oscillations. (A similar instability is seen in [33–35].)
3. For large masses, the wave packet undergoes 'gravitational collapse'.

Surprisingly, though, we find that the 'collapse' behavior occurs at considerably lower masses than estimate (2.3) suggests. For the initial width of $$\alpha = 5 \times 10^{16} \text{ m}^{-2}$$ used in the simulations, the mass (2.3) is on the order of $$10^{10} \text{ u}$$, while collapse first appears in the simulations for masses of about $$10^5 \text{ u}$$.

This result is somewhat unexpected, although not implausible in view of the highly nonlinear nature of the problem. Fortunately, it is now being tested by another group, using different, independently developed code.

Assuming the validity of our simulations, we can use the scaling behavior (2.4) to obtain the parameters for gravitational collapse. We find that a wave packet of initial width $$w = \alpha^{-1/2}$$ will shrink if its initial mass lies in a range $$m_-(w) < m < m_+(w)$$, with

$$m_-/1 \text{ u} = 1300(w/1 \text{ u m})^{-1/3}, \quad m_+/1 \text{ u} = 4.8 \times 10^{13}(w/1 \text{ u m})^{-1/3}. \quad (2.5)$$

(For $$m > m_+$$, we have not been able to run the simulation long enough to reliably determine the outcome.) The numerically obtained collapse times, in nanoseconds, are

$$t_-/1 \text{ ns} = 1.2 \times 10^{-4}(w/1 \text{ u m})^{-5/3} \quad t_+/1 \text{ ns} = 1.2 \times 10^{-2}(w/1 \text{ u m})^{-5/3}. \quad (2.6)$$

2 Anticipating a discussion of molecular interferometry, I am giving masses in unified atomic mass units.
3. Experimental tests

Are nonlinearities at the level described above experimentally testable? To get a measurable signal, one needs to use as large a mass as possible while still maintaining observable quantum behavior. The best bet seems to be molecular interferometry, where a ‘collapsing’ wave packet would lead to suppression of interference. At this writing, the heaviest molecule that has experimentally exhibited interference is fluorofullerene, $C_{60}F_{48}$, with a mass of 1632 u [38]. The grating slits in the fluorofullerene experiment have a width $w \sim 0.5 \mu m$. From (2.5), semiclassical gravity would predict a loss of interference for a wave packet of this width for masses greater than about $m \sim 1600$ u. Fluorofullerene lies just at the edge of this range.

Unfortunately, this is too optimistic an estimate: the wave packets in molecular interferometry experiments are not spherically symmetric Gaussians, and their effective width may be quite a bit larger. In [36], we estimate that the fluorofullerene experiment may fall short of a real test by a factor of about 500.

This leaves work for both experimentalists and theorists. On the theory side, assuming the results of [36, 37] are confirmed, we need to look at more realistic initial profiles. It will be important to see how sensitive the ‘collapse’ is to the shape of the initial wavefunction—we cannot yet rule out the possibility that the behavior we see is an artifact of the Gaussian initial conditions—and to see how the collapse time depends on the wave packet shape.

On the experimental side, some progress can come from reducing and better controlling the wave packet width, for example by using shutters to limit the longitudinal extent of the packet. The most important gain, though, will come from the move to heavier molecules. A number of experimentalists have predicted that with improved methods—optical gratings, for example—it should be possible to observe interference for molecules with masses as high as $10^6$ u [39–42]. If the next generation of molecular interferometry experiments can come even close to this limit, a clean test of semiclassical gravity should be well within reach.

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