CHARGED LEPTON G-2 AND CONSTRAINTS ON NEW PHYSICS

A.I. Studenikin

Department of Theoretical Physics, Physics Faculty,
Moscow State University, 119899 Moscow, Russian Federation

Abstract

A review of the theoretical and experimental values for the charged lepton (electron and muon) anomalous magnetic moment $a_l = (g_l - 2)/2$ is presented. Employing the most accurate value for the fine structure constant $\alpha^{-1} = 137.03599993(52)$ (0.0038 ppm) obtained \cite{6} from the electron $(g - 2)$ we find the new complete standard model prediction for the anomalous magnetic moment of the muon $a_{\mu}^{th} = 116591595(67) \times 10^{-11}$. The comparison of this theoretical value and the precise experimental result \cite{2} yields the estimation for the difference $\Delta a_{\mu} = a_{\mu}^{exp} - a_{\mu}^{th}$ at the 95 % confidence level: $-95 \times 10^{-10} \leq \Delta a_{\mu} \leq 236 \times 10^{-10}$. The implication of the expected a factor of about 20 increase of accuracy in the forthcoming Brookhaven National Laboratory measurement of $a_{\mu}$ implies $-47 \times 10^{-11} \leq \Delta a_{\mu} \leq 118 \times 10^{-11}$, (95 % C.L.). This interval is used to get constraints on the "new physics". The value of the one-loop contributions $a_{l}^{B_i}$ of different bosons predicted within extension of the standard model and coupled to a charged lepton are discussed. The dependence of $a_{l}^{B_i}$ on the masses of the bosons and leptons of the vacuum polarization loops are investigated. The constraints on "new physics" by requiring that the new contributions $a_{\mu}^{B_i}$ to the muon anomalous magnetic moment lie within the latter interval $\Delta a_{\mu}$ are obtained.

\cite{1}E-mail: studenik@srdlan.npi.msu.su
1 Introduction

There are two complementary approaches in exploration of frontiers of particle physics. One is based on experiments carried out at high energies on accelerators and storage rings. At present, various programmes of gaining on high energies are successfully realizing and even more powerful accelerators are being constructed. However, it is obvious that the traditional accelerators will not make it possible in the future to sustain the present rate of advance in experimental research towards higher energies. That is why it is important to develop another approach based on research of those elementary particle characteristics that can be measured in relatively low-energy experiments and calculated theoretically with high accuracy. A comparison of results of such experimental and high-precision theoretical studies establishes a non-accelerative method of getting information on properties of elementary particles and their interactions.

A unique example of such particle characteristics is provided by the anomalous magnetic moments of charged leptons, of the electron and the muon in particular. The anomalous magnetic moment of a charged lepton is proportional to the deviation of the so-called $g_l$-factor of the particle, that is the measure of the size of the magnetic dipole moment compared to its intrinsic angular momentum, from the value $g_l = g_0 = 2$.

Let me recall here the analogy with the classical motion of a charged particle. Classically the magnetic dipole moment $\mu_L$ can arise when a charged particle is orbiting on the circular trajectory with radius $r$. In this case the magnetic moment is associated with the kinetic orbital momentum $L = \vec{r} \times \vec{p}$ and given by $\mu_L = g_{cl} \frac{e}{2mc} \vec{L}$. Consequently, the $g$-factor for this classical motion is equal to $g = g_{cl} = 1$. Alternatively, for the point-like Dirac particle the relation between the magnetic dipole moment and the intrinsic spin moment takes the form

$$\vec{\mu}_m = g_l \frac{e_l}{2mc} \vec{S},$$

where $\vec{\mu}_m$ is the magnetic moment operator, $\vec{S} = \frac{\hbar}{2} \sigma$ is the spin operator, $\sigma$ are the Pauli matrices, $e_l$ and $m_l$ are the charge and mass of the particle. As it follows, the $g$-factor for this case is equal to

$$g_l = g_0 = 2.$$

This value for the $g$-factor corresponds to the particular case when the wave function of the particle obeys the Dirac equation and interaction with external electromagnetic field
is introduced via the minimal coupling by extension of the derivative: \( \partial_\mu \to \partial_\mu + ieA_\mu \). This non-trivial result of the Dirac theory can be also received in the frame of the non-relativistic approach based on the Pauli wave equation.

Note that the value \( g_l = 2 \) represents a fundamental property of the particle in respect to the electromagnetic interaction. If the particle participate in any other interaction which endow it with an internal structure then this departure from the point-likeness will be reflected on the value of \( g_l \)-factor. For example, for the proton \( g_\mu \)-factor is equal to 5.59 because of its internal structure.

It must be also mentioned that if the interaction with electromagnetic field \( F_{\mu\nu} \) is introduced in non-minimal way and the charged lepton wave function obeys the equation

\[
[(i\partial_\mu - eA_\mu)\gamma^\mu - m + \frac{\Delta g}{2} \frac{e}{4m} \sigma^{\mu\nu} F_{\mu\nu}] \Psi = 0, \quad \sigma^{\mu\nu} = \frac{i}{2}(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu),
\]

then the \( g \)-factor will be modified and equal to \( g = 2 + \Delta g \).

However, even in the case of point-like Dirac particle as it was shown first in [1] the quantum nature of minimal electromagnetic interaction shifts the value of the \( g_l \)-factor. That is why it is convenient to represent the charged lepton \( g_l \)-factor in the form

\[
g_l = 2 (1 + a_l),
\]

where \( a_l \) is equal to the so-called anomalous magnetic moment measured in units of the Bohr magneton \( \mu_0^l \)

\[
a_l = \frac{\Delta \mu_l}{\mu_0^l}, \quad \mu_0^l = \frac{e_l \hbar}{2m_l c},
\]

The most accurate experimental values for the electron AMM was obtained by the Washington University group [2] and the experimental value for the muon AMM was obtained in CERN [3]. If one compares the observed value of the electron AMM with the theoretical predictions one shall conclude that the main contribution is due to quantum electrodynamical processes, while in the case of the muon AMM the contributions of the strong and weak interactions are also substantial. The latter still remain outside the limits of experimental accuracy. However, with expected more than factor of 20 improvement in forthcoming results of the E821 Brookhaven National Laboratory measurement [4] of the muon AMM it will become possible to detect the contributions of the weak interactions. Within the achieved accuracy the theoretical and experimental values of the electron and muon AMM are in good agreement with the predictions of the standard Glashow-
Weinberg-Salam model. This provides both a sensitive verification to several orders in perturbation expansion of the QED and standard $SU(3)_C \times SU(2)_L \times U(1)$ model as well as put constraints on new physics beyond the standard model \cite{3, 4}. The expected improvements in accuracy of measurements of the leptons AMM \cite{4, 5, 6} would either allow for the new physics effects to be visible, or if no any extra contributions to the leptons AMM are seen, more severe bounds on alternative models will be received.

The paper is organized as follows. In the next Section 2 a brief review of the experimental values of the electron and muon AMM are given. In Sections 3 and 4 the present status of the theoretical values of the leptons AMM is discussed and the new theoretical value for the muon AMM is derived. In Section 5 we consider the present discrepancy between the experimental and theoretical values for the muon AMM and the one that could be achieved in the near future. The one-loop contributions of various types of bosons to the charged lepton AMM are considered in Section 6 on the base of calculation of the subsequent contributions to the lepton mass operator in external electromagnetic field. In Section 6 we also examine the dependence of these contributions to the lepton AMM on masses of involved particles. In Conclusion we derive constraints on couplings of the muon to hypothetical bosons and on masses of bosons.

2 Experimental values for anomalous magnetic moments of electron and muon

The permanently increasing accuracy of theoretical evaluations of charged leptons AMM is stimulated by the tremendous accuracy that is achieved in measurements of the electron and muon AMM. The latest experimental results \cite{2} of the Washington University group for the electron and positron AMM are \cite{3}

\[ a_{e^-}^{\text{exp}} = 1159652188.4(4.3) \times 10^{-12}, \quad a_{e^+}^{\text{exp}} = 1159652187.9(4.3) \times 10^{-12}. \]  

(6)

In these measurements a single electron and positron are confined within a Penning trap which constrains the position of a particle in a uniform magnetic field that is imposed by means of a hyperboloid cavity. The experimental uncertainties are dominated by a cavity shift effects of $\pm 4 \times 10^{-12}$, which arises from a lack of control over the resonant interaction of a particle with the electromagnetic modes of the surrounding microwave cavity of the Penning trap \cite{3}. There are also attempts to reduce this uncertainty \cite{4, 5, 6}.

The latest CERN storage ring measurements of the muon $\mu^+$ and $\mu^-$ AMM give the
results \[4\]

\[a_{\mu}^{\text{exp}} = 116593700(1200) \times 10^{-11}, \quad a_{\mu}^{\exp} = 116591100(1100) \times 10^{-11}, \quad (7)\]

which together provide the current experimental average for the muon AMM \[10\],

\[a_{\mu}^{\text{exp}} = 116592300(840) \times 10^{-11}. \quad (8)\]

The dominant error here is the statistical counting error of 7 ppm in the determination of the \((g_\mu - 2)\) precession frequency which is the difference between the spin precession frequency and the cyclotron frequency of the muon in the magnetic field of the ring. It should be possible to reduce this error \[11\] by at least a factor of 30 in the E821 BNL experiment because the intensity of the primary proton beam will be a factor of about 100 greater and the storage ring magnetic field a factor about of 3 greater than those used in the CERN experiment. The reduction of the systematic error somewhat below the expected level of the statistical error will be provided by the improvement of the homogeneity of the magnetic field and diminishing of deviations of muons orbits from the ideal reference orbit. Thus the present precision the measurement of \(a_\mu\) at the level of \(84 \times 10^{-10}\) is expected to be improved to the level of \(4 \times 10^{-10}\) or even less \((\sim 1 - 2 \times 10^{-10})\) \[12, 13\].

3 Anomalous magnetic moments of electron in standard model

In the quantum electrodynamics the AMM of an electron (as well as of the other charged leptons \(l\)) can be expressed as a perturbation series expansion in the fine structure constant \(\alpha\):

\[a_l = \sum_n \left(\frac{\alpha}{\pi}\right)^n A_n^{(l)} + \sum_n \left(\frac{\alpha}{\pi}\right)^n B_n^{(l)}. \quad (9)\]

The coefficients \(A_n^{(l)}\) are independent of the lepton mass, thus the first summ in (9) is identical for all charged leptons. The coefficients \(B_n^{(l)} = B_n^{(l)}(\frac{m_l}{m_{l'}})\) are functions of ratios of the mass of the external lepton \(l\) to those of leptons \(l'\) in the vacuum polarization loops.

In the case of the electron the mass independent terms has been evaluated to order \(\alpha^4\):

\[a_{e}^{QED}(\text{mass ind.}) = A_1\left(\frac{\alpha}{\pi}\right) + A_2\left(\frac{\alpha}{\pi}\right)^2 + A_3\left(\frac{\alpha}{\pi}\right)^3 + A_4\left(\frac{\alpha}{\pi}\right)^4, \quad (10)\]
where

\[ A_1 = 0.5, \quad A_2 = -0.328478965..., \]
\[ A_3 = 1.181241456..., \quad A_4 = -1.4092(384). \]  \hspace{1cm} (11)

The coefficients of the leading term \( A_1 \) correspond to the Schwinger value of the lepton AMM [1]. The coefficients \( A_2 \) and \( A_3 \) has been evaluated both in numerical [3, 14, 15] and analytical [16-18] technique, whereas \( A_4 \) has been evaluated only in numerical calculations [14].

Three most important mass dependent terms in (9) for the electron AMM are known analytically [19, 20, 6]

\[ B_2\left(\frac{m_e}{m_\mu}\right)\left(\frac{\alpha}{\pi}\right)^2 = 2.804 \times 10^{-12}, \]
\[ B_2\left(\frac{m_e}{m_\tau}\right)\left(\frac{\alpha}{\pi}\right)^2 = 0.010 \times 10^{-12}, \]  \hspace{1cm} (12)
\[ B_3\left(\frac{m_e}{m_\mu}\right)\left(\frac{\alpha}{\pi}\right)^3 = -0.924 \times 10^{-13}. \]

Other QED contributions to the electron AMM are too small to be accounted at present.

To get the theoretical value for the electron AMM, one must add also contributions of the lowest-order \( \mathcal{O}(\alpha^2) \) and of the order \( \mathcal{O}(\alpha^3) \) hadronic vacuum polarization and electroweak interaction [22, 6]:

\[ a_{e}^{had} = 1.635 \times 10^{-12}, \quad a_{e}^{EW} = 0.030 \times 10^{-12}. \]  \hspace{1cm} (13)

It has been shown that the two-loop electroweak contributions to a charged lepton AMM are rather important. The recent evaluation of the two-loop electroweak contributions to the electron AMM [21, 23, 12] amounts to \(-35\%\) of the one-loop term [28].

Finally, from eqs. (10)-(13) it is possible to obtain the theoretical value for the electron AMM [4]

\[ a_{e}^{th} = 1159652156.4(1.2)(22.9) \times 10^{-12}, \]  \hspace{1cm} (14)

where the value of \( \alpha \) based on the latest measurements of the quantum Hall effect [29] is used:

\[ \alpha^{-1}(Hall) = 137.0360037(27) \quad (0.020 \ ppm). \]  \hspace{1cm} (15)

The theoretical (14) and experimental (6) values for the electron AMM are in agreement within the 1.4 standard deviations level.

Note that the intrinsic theoretical uncertainty \( 1.2 \times 10^{-12} \) of \( a_{e}^{th} \) is much less than the uncertainty \( 22.9 \times 10^{-12} \) from the measurements of \( \alpha(Hall) \). It follows, that comparison
of the theoretical and experimental values of $a_e$ gives a more precise value of the fine structure constant $\alpha$ than one used for the derivation of $a_e^{th}$. The value of $\alpha$ determined from these arguments with the use of the average of $a_e^-$ and $a_e^+$ is 

$$\alpha^{-1}_{(g_e-2)} = 137.03599993(52) \quad (0.0038 \text{ ppm}),$$

where the most of the error comes from the experimental uncertainty in the measurements of $a_e$. We shall use this value for $\alpha$ when we get the value of the muon AMM.

### 4 Anomalous magnetic moment of muon in the standard model

The theoretical prediction for the muon AMM can be also divided into

$$a_\mu^{th} = a_\mu^{QED} + a_\mu^{had} + a_\mu^{EW}.$$  

At present the QED contribution is known to order $\alpha^5$ [30]

$$a_\mu^{QED} = \frac{\alpha}{\pi} C_1 + \left(\frac{\alpha}{\pi}\right)^2 C_2 + \left(\frac{\alpha}{\pi}\right)^3 C_3 + \left(\frac{\alpha}{\pi}\right)^4 C_4 + \left(\frac{\alpha}{\pi}\right)^5 C_5,$$

where

$$C_1 = 0.5, \quad C_2 = 0.765857381(51), \quad C_3 = 24.050531(140),$$

$$C_4 = 126.02(42), \quad C_5 = 930(170),$$

and in the calculation of the $\tau$ lepton loops was used $m_\tau = 1777$ MeV. Here coefficients $C_1$ and $C_2$ are known analytically whereas the other are derived by numerical integration (see ref. [3] for a review of the calculations of $a_\mu^{QED}$). Employing the value of the fine structure constant $\alpha^{-1}_{(g_e-2)} = 137.03599993(52)$ determined from the electron AMM [3] gives the new value

$$a_\mu^{QED} = 116584705(2) \times 10^{-11},$$

for the QED contribution to the muon AMM. This value is $1 \times 10^{-11}$ less than one used previously (see, for example, [31, 22]).

The muon AMM is more sensitive to processes at smaller distances than the electron AMM because of the large mass scale ($m_\mu \gg m_e$). That is why the effects of strong and electroweak interactions are much more important in the value of $a_\mu$ than in one of $a_e$. 

The hadronic contributions to the lepton AMM arises from two effects: hadronic vacuum-polarization and hadronic light-by-light scattering. The hadronic vacuum-polarization contributions to \( a_\mu \) can be evaluated within the dispersion theory using the experimental data on the total hadronic cross-section for the annihilation of electrons and positrons, \( \sigma_{e^+e^-\to \text{hadrons}} \), and perturbative QCD for the very high energies. The recent evaluation \cite{22} of the leading order \( \mathcal{O}\left(\alpha/\pi\right)^2 \) hadronic vacuum polarization contribution, \( a_\mu^{\text{had}}(\text{vac.pol.}, \alpha^2) = 6924(62) \times 10^{-11} \), together with the non-leading order vacuum contribution \cite{24, 25} \( a_\mu^{\text{had}}(\text{vac.pol.}, \alpha^3) = -100(6) \times 10^{-11} \) give
\[
a_\mu^{\text{had}}(\text{vac.pol.}) = 6824(62) \times 10^{-11}.
\] (21)

Following ref.\cite{22} we use for the light-by-light scattering contribution the average of results of refs. \cite{26, 27} that is
\[
a_\mu^{\text{had}}(\gamma \times \gamma) = -85(25) \times 10^{-11}.
\] (22)

The combination of (21) and (22) yields the final result for the hadronic contribution to the muon AMM
\[
a_\mu^{\text{had}} = 6739(67) \times 10^{-11}.
\] (23)

Note that the principal contribution as well as the error come from the hadronic vacuum polarization effects from the low energy region and the improvement in \( e^+e^- \) data near the \( \rho \) meson resonance energy \( \leq 1 \text{ GeV} \) could significantly reduce the uncertainty.

The one-loop electroweak \( a_\mu^{\text{EW}}(1 \text{ loop}) \) contributions to the charged lepton AMM have been calculated in \cite{28} (see also \cite{32, 33, 34, 35} for the evaluation of the one-loop Z, W, and Higgs bosons contributions to a charged lepton AMM exactly accounting for mass parameters \( (m_l/M_B)^2 \), where \( M_B = M_Z, M_W \) or \( M_{\text{Higgs}} \)):
\[
a_\mu^{\text{EW}}(1 \text{ loop}) = \frac{5}{3} \frac{G_\mu m_\mu^2}{8\sqrt{2}\pi^2} \left[ 1 + \frac{1}{5}(1 - 4\sin^2\theta_W)^2 + \mathcal{O}\left( \frac{m_\mu^2}{M_Z^2} \ln \frac{M_Z^2}{m_\mu^2} \right) + \mathcal{O}\left( \frac{m_\mu^2}{M_{\text{Higgs}}^2} \ln \frac{M_{\text{Higgs}}^2}{m_\mu^2} \right) \right] \approx 195 \times 10^{-11},
\] (24)

where \( G_\mu = 1.16639(1) \times 10^{-5} \text{ GeV}^{-1} \), and \( \sin^2\theta_W = 0.224 \). Note the presence of large logarithmic terms \( \ln(M_B^2/m_\mu^2) \) in the expansion of the neutral bosons contributions \( a_Z^{\text{had}}, H^{\text{had}} \) over parameters \( m_\mu^2/M_B^2 \).

The consideration \cite{21}, \cite{12}, \cite{23} of the two-loop electroweak contributions to the muon AMM leads to an overall 22.6% reduction of \( a_\mu^{\text{EW}} \) for \( M_{\text{Higgs}} \approx 250 \text{ GeV} \). Thus, the present prediction for the electroweak contribution to the muon AMM is \cite{12}
\[
a_\mu^{\text{EW}} = 151(4) \times 10^{-11}.
\] (25)
The error is due to uncertainties in $M_{Higgs}$ and quark two-loop effects, the current uncertainty in $\theta_W$ is about $0.05 \times 10^{-11}$.

Summing the different contributions (20), (23), and (25) and adding errors in quadratures, one can obtain for the theoretical value of the muon AMM

$$a^{th}_\mu = 116591595(67) \times 10^{-11}. \quad (26)$$

This our result for $a^{th}_\mu$ is less than one of ref. [24] because the new value [6] for $\alpha(g-2)$ is used for the evaluation of the QED contribution (20).

5 Discrepancy of experimental and theoretical values of muon anomalous magnetic moment

The uncertainty in (26) is mostly dominated by the hadronic lowest order contribution. The uncertainties in the QED (20) and electroweak (25) terms, $\Delta^{QED}_{\mu}$ and $\Delta^{EW}_{\mu}$, contribute on the level of a few percent that of hadronic term, $\Delta^{had}_{\mu}$ :

$$\Delta^{QED}_{\mu} + \Delta^{EW}_{\mu} < 10\% \, \Delta^{had}_{\mu}. \quad (27)$$

Remarkably, within the recently achieved improvement in computation of the hadronic contribution $a^{had}_{\mu}$ the overall error in $a^{th}_{\mu}$ becomes about 2.3 times less than the contribution of the electroweak interaction $a^{EW}_{\mu}$. Together with the anticipated precision of the E821 experiment the further reduction of errors in $a^{had}_{\mu}$ by a factor of about 2 will allow a direct detection of the electroweak contribution to $a_{\mu}$ and also will provide a test of new physics at the multi-TeV scale.

In the difference of the experimental (8) and theoretical (26) values

$$\Delta a_{\mu} = a^{exp}_{\mu} - a^{th}_{\mu} = 705(843) \times 10^{-11} \quad (28)$$

the dominated uncertainty is due to the experimental error. From (28) we can derive the 95 % confidence level limits on $\Delta a_{\mu}$:

$$-95 \times 10^{-10} \leq \Delta a_{\mu} \leq 236 \times 10^{-10}. \quad (29)$$

Let us suppose (see also ref. [31]) that after the expected a factor of about 20 increase of accuracy in the E821 experiment and the further improvement in the calculation of the
hadronic contribution the total uncertainty in $\Delta a_\mu$ will be again due to the experimental error. Then in the case when the central value for the deviation $\Delta a_\mu$ will be also shifted down by a factor of 20 (i.e., it is supposed that the deviations from the standard model will not be visible on the new level of accuracy) we can obtain the new limits

$$-47 \times 10^{-11} \leq \Delta a_\mu \leq 118 \times 10^{-11}$$

at the 95 % confidence level. These limits are used below in evaluation of constraints on interactions of the muon with hypothetical bosons that could be received from the forthcoming data of the BNL E821 experiment.

6 Different bosons contributions to charged lepton anomalous magnetic moment

In the various generalizations of the standard $SU(3)_C \times SU(2)_L \times U(1)$ model (such as, e.g., grand unified theories, technicolor models, composite models, models with horizontal symmetry, superstrings, etc) a rich spectrum of new bosons is predicted. The couplings of these new bosons $B_i$ to a charged lepton $l$ give the ”new physics” contributions $a^{B_i}_l$ to the lepton AMM via vacuum polarization loops. Constraints on the ”new physics” can be obtained by requiring that the new contributions $a^{B_i}_l$ to the lepton AMM lie within the discrepancy $\Delta a_l$ of the experimental and theoretical values.

6.1 Evaluation of different bosons contributions to anomalous magnetic moment of charged lepton

Let us briefly describe evaluation of various types of bosons contributions to a charged lepton AMM. The most systematic way to calculate contributions of various types of bosons $B_i$ to the lepton AMM is based on consideration [34, 36] of radiative corrections to the motion of the lepton in external electromagnetic field that appears due to the corresponding interactions of the lepton with bosons. We consider an equation analogous to the Schwinger equation in quantum electrodynamics, which describes the motion of the lepton in the presence of the electromagnetic field taking radiative $B_i$ boson effects into account

$$[(i\partial_\mu - eA^{\mu\tau}_\mu)\gamma^\mu - m] \Psi(x) = \int M^{B_i}(x', x)\Psi(x')dx',$$  

(31)
where $A^{ext}$ is the 4-potential of the external electromagnetic field and $M^{B_i}$ is the contribution to the lepton mass operator due to the corresponding vacuum polarization effects. As a starting point, we chose the external field to be constant crossed electric $\vec{E}$ and magnetic $\vec{B}$ field ($\vec{E} \perp \vec{B}$, $E = B$), which in the case of relativistic particles provides a good model for any constant electromagnetic field (a detailed discussion of this statement can be found in refs. [37, 38].

We consider the lowest-order contributions to the mass operator of a charged lepton, i.e. contributions of different virtual processes of the form:

$$l \rightarrow l' + B_i \rightarrow l.$$  \hspace{1cm} (32)

The lepton $l'$ in the virtual polarization loop may be not identical to the initial and final charged lepton $l$ (that is the case of the QED and the neutral couplings of the standard mode) which corresponds not only to the charged couplings $l^\pm - \nu_l - W^\pm$ of the standard model but, for example, to the flavour-changing neutral couplings $l^\pm - l'^\pm - B_i^0$ that are predicted in different models with the horizontal symmetry [34] or in the two-Higgs doublet extensions of the standard model [31, 40]. Note that the lowest-order slepton-photino and wino-sneutrino contributions of the SUSY extension of the standard model (see [41, 42] and references therein) are also of this type.

Let us specify the properties of the bosons $B_i$ and their couplings to leptons. We assume that interactions of a charged lepton $l$ with various bosons $B_i$ are given by the Lorentz-invariant couplings

$$\mathcal{L}^{B_i}_l = g_i \bar{\psi}_l \Gamma_i \psi_{l'} \phi_{B_i},$$  \hspace{1cm} (33)

where the index $i = 1, 2, ..., 6$ enumerates different types of interaction and corresponding different types of virtual processes (32) (for definiteness we consider the negatively charged lepton $l^-$):

$$\Gamma_1 = 1, \quad \Gamma_2 = \gamma_5, \quad \Gamma_3 = \gamma_\mu, \quad \Gamma_4 = \gamma_\mu \gamma_5, \quad \Gamma_5 = \gamma_\mu, \quad \Gamma_6 = \gamma_\mu \gamma_5.$$  \hspace{1cm} (34)

We also suppose that the other partner of $B_i$ in the vacuum polarization loop of the process (32) for the cases of the scalar and pseudoscalar neutral bosons, $B_1 = S^0$, $B_2 = P^0$, could be arbitrary charged leptons $l'^-$ with mass not indispensably equal to the mass of the initial lepton $l^-$ ($m_l \neq m_{l'}$).

In the cases of the vector and axial vector neutral bosons, $B_3 = V^0$, $B_4 = A^0$, it is supposed that leptons $l'^-$ in the vacuum polarization loops are equal to the initial leptons $l^-$, $m_l = m_{l'}$. In the cases of the charged bosons, $B_5 = V^-$ and $B_6 = A^-$, the virtual leptons are the massless neutrinos $l' = \nu_l$, $m_{l'} = 0$. This choice of pairs of particles in the
vacuum polarization loops of processes (32) allows to investigate different contributions to the charged lepton AMM not only in the standard electroweak model but also in the supersymmetrical extensions and in a wide class of other alternative models.

To the one-loop order the contributions to the mass operator of a charged lepton is given by

\[
M_i^{(2)}(x', x) = -ig_i^2 \Gamma_i G_i(x', x) \Gamma_i D_{B_i}(x', x),
\]

where \( G_i(x', x) \) and \( D_{B_i}(x', x) \) are propagators of the lepton \( l'_i \) and boson \( B_i \) accounting for the crossed electromagnetic field if \( l'_i \) or \( B_i \) are charged particles. The expressions for the charged lepton and boson propagators in the crossed external electromagnetic field are given in refs.\[38, 36\]. It is possible to find the contribution \( M_i^{(2)}(x', x) \) to the mass operator of the lepton in the presence of the electromagnetic field in the so-called \( E_p(x) \)-representation \[38\] (that is used instead the momentum \( p \) representation in calculations for the zero electromagnetic field case):

\[
M_i^{(2)}(x', x) = \int \frac{dp'dp}{(2\pi)^8} E(p', x') M_i^{(2)}(p', p) \bar{E}(p, x), \quad \bar{E}(p, x) = \gamma_0 E^+(p, x),
\]

where

\[
E(p, x) = (1 + e^{\hat{n}\hat{A}}) e^{-i(px - n(p, \phi))}, \quad \hat{n} = n\gamma^\mu, \quad \hat{A} = A_\mu\gamma^\mu,
\]

\[
\eta(p, \phi) = \int_0^\phi \left[ \frac{e^2 A^2(\rho)}{2np} - \frac{e p A(\rho)}{np} \right] dp, \quad \phi = n x. \tag{37}
\]

Here \( A_\mu(\phi) \) is the 4-potential of the crossed field, the reference frame is fixed by the condition \( n_\mu = (1, 0, 0, 1) \), then \( n^2 = n A = 0 \). The further details of calculations are discussed in \[36\], and for the contributions to the AMM of a charged lepton \( l \) of mass \( m_1 \), moving in the electromagnetic field, we have got \[34, 36, 42\]

\[
a_i^{B_i}(\chi) = \frac{g_i^2}{(2\pi)^2} \int_0^\infty \frac{du}{(1 + u)^2} \left( \frac{u}{\chi} \right)^\frac{\phi}{2} \Omega_i \Upsilon(z_i), \tag{38}
\]

where

\[
\Upsilon(z_i) = \int_0^\infty \sin(z_i x + \frac{x^3}{3}) dx.
\]

The arguments \( z_i \) of the upsilon function \( \Upsilon(z_i) \) for different contributions \( a_i^{B_i} \) can be obtained from the universal expression

\[
z = \left( \frac{u}{\chi} \right)^\frac{\phi}{2} \left[ -\frac{1}{u} + \frac{1 + u m_2^2}{u m_1^2} + \frac{1 + u m_3^2}{u^2 m_1^2} \right]. \tag{39}
\]
with the appropriate choice of the values for masses $m_{1,2,3}$. Here $m_1$ denotes the mass $m_l$ of the charged lepton $l$, $m_2$ and $m_3$ stand for the masses of charged and neutral particles in the vacuum polarization loop (32), correspondently.

Table 1.

| $i$ | $B_i$ | $\Gamma_i$ | $l'_i$ | $m_{\bar{A}}/m_1$ | $m_{\bar{P}}/m_1$ | $z_i$ | $\Omega_i$ |
|-----|-------|-------------|--------|-------------------|-------------------|-------|-----------|
| 1   | $S^0$ | 1           | $l^-$  | $m_{\bar{A}}/m_1$ | $m_{\bar{P}}/m_1$ | $z_1 = z$ | $\frac{1}{2} + \frac{1+u}{2} \frac{m_2}{m_1}$ |
| 2   | $P^0$ | $\gamma_5$ | $l^-$  | $m_{\bar{A}}/m_1$ | $m_{\bar{P}}/m_1$ | $z_2 = z$ | $\frac{1}{2} - \frac{1+u}{2} \frac{m_2}{m_1}$ |
| 3   | $V^0$ | $\gamma_{\mu}$ | $l^-$ | $m_{\bar{A}}/m_1$ | $m_{\bar{P}}/m_1$ | $z_3 = \left(\frac{u}{\chi}\right) \frac{3}{2} (1 + \frac{1+u}{u^2} \frac{m_2}{m_1})$ | 1 |
| 4   | $A^0$ | $\gamma_{\mu} \gamma_5$ | $l^-$ | $m_{\bar{A}}/m_1$ | $m_{\bar{P}}/m_1$ | $z_4 = \left(\frac{u}{\chi}\right) \frac{3}{2} (1 + \frac{1+u}{u^2} \frac{m_2}{m_1})$ | $-3 - \frac{4}{u} - 2u \frac{m_2^2}{m_3^3}$ |
| 5   | $V^-$ | $\gamma_{\mu}$ | $l^0$ | $m_{\bar{A}}/m_1$ | $m_{\bar{P}}/m_1$ | $z_5 = \left(\frac{u}{\chi}\right) \frac{3}{2} \left( - \frac{1}{u} + \frac{1+u}{u} \frac{m_2}{m_1} \right)$ | $2 + \frac{1}{u} - \frac{1}{u} \frac{m_1^3}{m_2^2}$ |
| 6   | $A^-$ | $\gamma_{\mu} \gamma_5$ | $l^0$ | $m_{\bar{A}}/m_1$ | $m_{\bar{P}}/m_1$ | $z_6 = \left(\frac{u}{\chi}\right) \frac{3}{2} \left( - \frac{1}{u} + \frac{1+u}{u} \frac{m_2}{m_1} \right)$ | $2 + \frac{1}{u} - \frac{1}{u} \frac{m_1^3}{m_2^2}$ |

In Table 1 columns denote, respectively, the types, charges and exact ratios of masses $m_2/m_1$ and $m_3/m_1$ (if they are not arbitrary) of bosons and leptons and the structure of their couplings $\Gamma_i$, as well as the arguments $z_i$ and functions $\Omega_i$ which determine the integrand in Eq.(38). Taking in mind the diversity in properties of scalar, $S^0$, and pseudoscalar, $P^0$, bosons, that are predicted within different alternative theoretical models, for these two cases we do not fix the mass parameters $m_2/m_1$ and $m_3/m_1$.

The obtained result (38) shows the dependence of $a_i^{B_l}$ on the characteristic dynamical
parameter
\[ \chi = \left[ -(e F^{\mu\nu} p_{\nu})^2 \right]^{\frac{1}{2}} m_l^{-3}, \]  
(40)
where \( F^{\mu\nu} \) is the tensor of external electromagnetic field and \( p_{\nu} \) is the momentum of the charged lepton \( l \).

Note that the analogous representation in terms of the function \( \Upsilon(z) \) of photon contribution in the lowest order of QED to the electron AMM was received in ref. [37] and derivation of the vector \( Z, W \), and scalar Higgs \( H \) bosons contributions in the standard electroweak model can be found in ref. [34] (see also references therein). The dependence of the photon contribution to the AMM of the electron on the strength of an external electromagnetic field was pointed out in ref. [43] and the dynamical nature of the AMM was demonstrated in ref. [44].

As it was mentioned above the received formula (38) for the AMM of the charged lepton moving in the crossed electromagnetic field gives also the expression for the AMM of the lepton moving with relativistic energy in a constant magnetic field. In this case the dynamical field parameter \( \chi \) takes the form
\[ \chi = \frac{B p_{\perp}}{B_0 m_l}, \]  
(41)
where \( p_{\perp} = \sqrt{2eBn} \) is the projection of the momentum of the lepton on the plane perpendicular to the vector \( \vec{B} \), \( n \) is the Landau levels number in the magnetic field and \( B_0 = \frac{m_l^2}{e} \) is the critical magnetic field that for the case of the electron is equal to \( B_e^c = 4.41 \times 10^{13} \text{ Gauss} \).

In the discussed above experiments on the charged leptons AMM the particles were moving in the presence of the magnetic field. The expected magnetic field induced shifts of contributions \( a_l^{B_i} \) for the reasonable strength of the field are rather small and are not accessible for observation in experiments on measurements of the electron and muon AMM of the types that have been already performed at CERN and the Washington University. However, with the further increase of accuracy in measurements of the electron and muon magnetic moments the electromagnetic field dependence could become important. To demonstrate the possible scale of influence of a magnetic field on the value of the charged lepton AMM let us consider [45] the conditions that can be realized, e.g., at the Stanford Linear Collider. Suppose that the typical electron beam energies are approximately 50 GeV and the effective magnetic fields around the electron bundle can approach \( 10^4 \text{ Gauss} \). Under such conditions the dynamical field correction to the electron AMM in the lowest-order of QED can reach the value of \( \Delta a_e = 0.6 \times 10^{-8} \) that exceeds the electroweak and
hadronic contributions as well as the $\alpha^4$ term of the vacuum (i.e., field independent) QED contribution.

On the basis of expression (38) we can obtain [34, 36] the asymptotic limits of the various contributions $a_{B_i}^l$, $B_i = S^0, P^0, V^0, A^0, V^-, A^-$, to the AMM of a charged lepton in electromagnetic field for small values of $\chi$ (that corresponds, for example, to the case of relatively weak magnetic fields, $B \ll B_0$, and not-too-large energies) and large values of $\chi$ (here we use the notation: $\lambda = m_{B_i}^2/m_1^2$, where $m_{B_i}$ denotes masses of different bosons $B_i$):

$$a_{B_i}^{S_0} (\chi) = \frac{g_1^2}{4\pi} \left\{ \frac{1}{\pi} k_1 + \frac{\lambda^2}{3\chi} + \frac{\lambda^2}{3} (\ln \lambda - \frac{257}{60}) \right\}, \quad \chi \ll \lambda$$

$$a_{B_i}^{P_0} (\chi) = \frac{g_2^2}{4\pi} \left\{ \frac{1}{\pi} (k_2 - \frac{2\lambda^2}{3\chi}), \quad \chi \ll \lambda \right\}$$

$$a_{B_i}^{V^0} (\chi) = \frac{g_2^2}{4\pi} \left\{ \frac{1}{\pi} k_3 + 2\lambda^2 (\ln \lambda - \frac{257}{60}) \right\}, \quad \chi \ll \lambda$$

$$a_{B_i}^{A^0} (\chi) = \frac{g_2^2}{4\pi} \left\{ \frac{1}{\pi} k_4 + \frac{\lambda^2}{9\sqrt{3}(\chi)^{3/2}} \right\}, \quad \chi \ll \lambda$$

$$a_{B_i}^{V^-, A^-} (\chi) = \frac{g_{5,6}^2}{4\pi} \left\{ \frac{1}{\pi} (k_{5,6} + \frac{\lambda^2}{10\chi^2}), \quad \chi \ll \lambda \right\} \quad \chi \gg \lambda^3$$

In evaluation of the terms depending on $\chi$ it was assumed that the bosons $B_i$ are heavy and the condition $\lambda \gg 1$ holds. We also set here the mass $m_2 = m_1$ for the $S^0$ and $P^0$ bosons contributions. However, the general case of $m_2 \neq m_1$ for these contributions is considered below.

From eqs.(42)-(46) it follows that for any type of considered bosons $B = S^0, P^0, V^0, A^0, V^-, A^-$ the dependence of the contributions to the lepton AMM on parameter $\chi$ is the same. For $\chi \ll \lambda$ small corrections quadratic in $\chi$ to the vacuum (i.e., field independent) contributions $a_{B_i}^{B_i}(0)$ arise. At large $\chi (\chi \gg \lambda^{3/2})$ all of the contributions explicitly demonstrate the dynamic nature of the lepton AMM being proportional to $\chi^{-2/3}$. The similar behavior of the lowest-order QED contribution to the lepton AMM was discovered in ref. [44, 37].
The formulas (42), (44), (45), and (46) allow us also to investigate the external field and energy dependence of the standard model \( Z, W, \) and \( H \) bosons contributions to the charged lepton AMM. The dependence of different contributions \( a_{iB} \) on the external electromagnetic field and the lepton energy could reveal itself and have to be accounted, for example, in the case of the motion of relativistic leptons in the vicinity of astrophysical objects like neutron stars, where strong magnetic fields of the order \( 0.1 \times B_0^\circ = 4.41 \times 10^{12} \text{ Gauss} \). Under such conditions the values of contributions as it follows from eqs.(42)-(46) could be substantially less than the vacuum one-loop terms \( a_{iB}(0) \).

6.2 Dependence of bosons contributions to lepton anomalous magnetic moment on masses of particles

Let us now turn to the vacuum contributions to a charged lepton AMM, for which we obtain \[ a_{iB} = \frac{g_i^2}{8\pi^2} k_{Bi}(\lambda_i), \] where the functions \( k_{Bi}(\lambda_i) = k_i(\lambda) \) depend on the only one mass parameter \( \lambda_i = m_{Bi}^2/m_l^2 \):

\[
k_1 = \left( \frac{1}{2} \lambda^3 - \frac{5}{2} \lambda^2 + 2\lambda \right) \epsilon^{-1} \ln K_1 + \left( \frac{1}{2} \lambda^2 - \frac{3}{2} \lambda \right) \ln \lambda - \lambda + \frac{3}{2}, \tag{48}
\]

\[K_i(\lambda_i) = K_i(\lambda) = \left| \frac{\lambda - \epsilon}{\lambda + \epsilon} \right|, \quad \epsilon = |\lambda(\lambda - 4)|^{\frac{1}{2}}, \quad (\lambda \neq 4), \tag{49}\]

\[
k_2 = \left( \frac{1}{2} \lambda^2 - \frac{3}{2} \lambda \right) \epsilon^{-1} \ln K_2 + \left( \frac{1}{2} \lambda^2 - \frac{1}{2} \lambda \right) \ln \lambda - \lambda + \frac{1}{2}, \tag{50}
\]

\[
k_3 = (\lambda^3 - 4\lambda^2 + 2\lambda) \epsilon^{-1} \ln K_2 + (\lambda^2 - 2\lambda) \ln \lambda - 2\lambda + 1, \tag{51}
\]

\[
k_4 = (\lambda^3 - 6\lambda^2 + 8\lambda) \epsilon^{-1} \ln K_2 + (\lambda^2 - 4\lambda + 2) \ln \lambda - 2\lambda + 5 - \frac{2}{\lambda}, \tag{52}
\]

\[
k_5 = k_6 = (2\lambda^2 - 5\lambda + 3) \ln L - 2\lambda + 4 + \frac{1}{2\lambda}, \quad L = \left| \frac{\lambda}{\lambda - 1} \right|. \tag{53}
\]

In Fig.1 the dependence on ratio \( m_{Bi}/m_l \) of the functions \( k_i \), which determine the vacuum contributions of the various types of bosons to the charged lepton AMM are
plotted. The number of the curve corresponds to the value of index \( i \). For instance, the curve (1) shows the dependence of the function \( k_1 \) on \( \sqrt{\lambda_1} \). The scalar, neutral vector, charged vector and axial vector bosons contributions to the negatively charged lepton are always positive, whereas the contributions of the pseudoscalar and neutral axial vector bosons are negative. The absolute values of all of the contributions go to zero with the increase of the masses of the bosons.

For the cases of light \((m_{B_i}/m_l \ll 1)\) and heavy \((m_{B_i}/m_l \gg 1)\) bosons it is possible to obtain from eqs.(47)-(53) the limiting values for the contributions to the negatively charged lepton AMM [36]. If the mass ratio squared, \( \lambda_i \), becomes small \((\lambda \ll 1)\) the functions \( k_i \) reduce to (here we again suppose that \( \beta_i = m_{2(i)}^2/m_l^2 = 1 \) for the scalar and pseudoscalar bosons contributions)

\[
\begin{align*}
    k_1 &= \frac{3}{2}, \quad k_2 = -\frac{1}{2}, \quad k_3 = 1, \quad k_4 = -\frac{2}{\lambda} \to -\infty, \quad k_5 = k_6 = \frac{2}{\lambda} \to \infty. \\
\end{align*}
\]

In the limit of large \( \lambda_i \) \((\lambda \gg 1)\) the functions \( k_i \) go to zero like

\[
\begin{align*}
    k_1 &= \frac{1}{\lambda} \ln \lambda - \frac{7}{6\lambda}, \quad k_2 = -\frac{1}{\lambda} \ln \lambda + \frac{11}{6\lambda}, \\
    k_3 &= \frac{2}{3\lambda} - \frac{2}{\lambda^2} \ln \lambda, \quad k_4 = -\frac{10}{3\lambda} + \frac{2}{\lambda^2} \ln \lambda, \\
    k_5 &= k_6 = \frac{10}{3\lambda} + \frac{2}{3\lambda^2}.
\end{align*}
\]

The expressions for the functions \( k_{1,2,4} \) and their limiting values correct mistakes that exist in papers of other authors (the detailed discussion on this item see in refs. [39]). As it follows from eqs.(54) the scalar, pseudoscalar, and neutral vector bosons contributions remain finite in the limit of massless bosons. Note that in this limit the vector boson contribution gives the lowest-order QED result for the lepton AMM: \( a_l^{QED} = \alpha/2\pi \), where \( \alpha = e^2/4\pi \). On the contrary, the absolute values of the neutral axial vector and charged vector and axial vector bosons contributions increase to infinity in this limit.

It is worth to note that in the limit of heavy bosons the scalar and pseudoscalar bosons contributions contain in addition to the term \( 1/\lambda_i \) (which is common for all of the contributions \( a_l^{B_i}(0) \)) an extra factor of \( \ln \lambda_i \) that can enlarge to some extent these two contributions to \( a_l \).


7 Conclusion

Substituting of the specific values for the coupling constants $g_i$ and masses of the bosons $B_i$ to the expressions (47)-(53) one can get the contributions to the AMM of the charged lepton of mass $m_l$ that could arise in various theories. Thus, with the combination of formulas (48), (49), (51)-(53) for the $S^0$, $V^0$, $A^0$, $V^-$, and $A^-$ bosons contributions we can find the vacuum contributions of the $Z$, $W$, and $H$ bosons of the standard model (see ref.34 and references therein).

Formulas (47)-(53) for the one-loop bosons contributions to the AMM of a charged lepton that exactly account for the masses of virtual particles can be used for getting constraints on parameters of theoretical models.

Let us discuss bounds from the muon AMM data on the existence of hypothetical forces which would couple to the muon. Here we reexamine constraints of ref.39 and predict new limits on $\alpha_{B_i\mu} = g_i^2/4\pi$ and masses $m_{B_i}$ of hypothetical bosons $S^0$, $P^0$, $V^0$, and $A^0$ that could be imposed by the improved measurements of the muon AMM at the BNL. The contributions of different bosons are considered separately. To get constrains we demand that each of the new effects could remove the discrepancy:

$$a^{B_{1,2,3,4}}_{\mu} = \Delta a_{\mu}.$$  \hspace{1cm} (56)

For $\Delta a_{\mu}$ we use the value of eq.(30).

The results as the exclusion plotts are presented in figures 2-5 that show the constraints on $\alpha_{B_i\mu}$ and $m_{B_i}$ in the decimal logarithmic scale. The regions above curves are excluded. The curves on fig.2 and 4 set the limits in the cases of bosons $S^0$ and $V^0$, respectively, and are governed by the equation (see limits (30) on $\Delta a_{\mu}$)

$$\alpha_{B_{1,3}} = 2\pi 1.18 \times 10^{-9}/k_{1,3}(\lambda_{1,3}^{(\mu)}).$$ \hspace{1cm} (57)

The curves on fig.3 and 5 corresponds to the cases of bosons $P^0$ and $A^0$ and are governed by the equation

$$\alpha_{B_{2,4}} = -2\pi 0.47 \times 10^{-9}/k_{2,4}(\lambda_{2,4}^{(\mu)}).$$ \hspace{1cm} (58)

Comparing the curves in figs.2, 3, and 4 for the scalar, pseudoscalar, and axial vector neutral bosons contributions with the similar curves on figures of ref.39 we can see that the constraints on $\alpha_{B_i}$ for the fixed values of $m_{B_i}$ obtained here with use of the new bounds (30) are about one order of magnitude as strong as those of ref.39.

The combined use of the $S^0$ and $P^0$ bosons contributions enables us to get 42 the slepton-photino and wino-sneutrino contributions to the lepton AMM in supersymmetric
models. The appropriate combination of the $S^0$ and $P^0$ bosons contributions gives also contributions to the lepton AMM in various models with the horizontal symmetry \cite{34}.

However, for these two cases we need to know the vacuum contributions of the scalar and pseudoscalar bosons in general form for arbitrary values of the mass parameter $\beta_i = m^2_{2(i)}/m^2_l \neq 1$. This can be done with the help of formulas (38), (39) and data of the first two lines of Table 1. For the functions $k_1(\beta_1, \lambda_1) = k_1(\beta, \lambda)$ and $k_2(\beta_2, \lambda_2) = k_2(\beta, \lambda)$ (here $\lambda_i = m^2_{3(i)}/m^2_l$) we get:

\begin{equation}
\begin{aligned}
k_{1,2}(\beta, \lambda) &= \frac{1}{2} \left[ ((\lambda - \beta)^2 - \beta) \ln \frac{\lambda}{\beta} + \left( \frac{(\lambda - \beta)^3}{\beta} - \frac{\lambda^2}{\beta} - \lambda + 2\beta - 1 \right) \epsilon^{-1} \ln K' + 2(\beta - \lambda) - 1 \right] \\
&\pm \frac{\beta - \lambda - 1}{2} \ln \frac{\lambda}{\beta} - \left( \frac{\lambda}{\beta} - 1 \right)^2 - \frac{2}{\beta} + \frac{1}{\beta^2} \right) \epsilon^{-1} \ln K' + \frac{2}{\beta} \right],
\end{aligned}
\end{equation}

where

$$\epsilon' = |\rho^2 - 4\lambda^2\beta|^{\frac{1}{2}}, \quad \rho = 1 + \frac{\lambda - 1}{\beta}, \quad K' = \left| \frac{\rho - \epsilon'}{\rho + \epsilon'} \right|.$$

This two parametric $(\beta, \lambda)$ analytic expression can be used to derive the value the scalar and pseudoscalar bosons contributions to the charged lepton AMM in different theoretical models in which such spinless bosons are predicted.

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References

[1] J.Schwinger, Phys.Rev 73 (1948) 416.

[2] P.Schwinberg, R.Van Dyck,Jr., H.Dehmelt, Phys.Rev.Lett. 59 (1987) 26.

[3] J.Bailey et al, Phys.Lett. 68B (1977) 191.

[4] V.Hughes, in ”Frontiers of High Energy Spin Physics”, ed. by T.Hasegawa et al, Universal Academy Press, Tokyo, 1992, p. 717.

[5] T.Kinoshita, W.Marciano, in Quantum Electrodynamics, ed. by T.Kinoshita, World Scientific, Singapore, 1990, p.419.

[6] T.Kinoshita, Rep.Prog.Phys. 59 (1996) 1459.

[7] R.Van Dyck,Jr., P.Schwinberg, H.Dehmelt, The Electron, ed. by D.Hestenes and A.Weingartshofer, Kluwer, Deventer, 1991.

[8] G.Gabrielse, J.Tan, Cavity Quantum Electrodynamics, ed. by P.Berman, Academic, New York, 1992.

[9] J.Bailey et al., Nucl.Phys. B 150 (1979) 1.

[10] Particle Data Group, Montanet et al., Phys.Rev.D 50 (1994) 1171.

[11] V.Hughes, T.Kinoshita, Comm.Nucl.Part.Phys. 14 (1985) 341.

[12] A.Czarnecki, B.Krause, W.Marciano, Phys.Rev. D52 (1995) R2619; Phys.Rev.Lett. 76 (1996) 3267.

[13] G.Bunce, as quoted in ref. [12].

[14] T.Kinoshita, CLNS96/1418, Cornell University, Ithaca, USA, 1996.

[15] T.Kinoshita, Phys.Rev.Lett. 75 (1995) 4728.

[16] A.Peterman, Helv.Phys.Acta 30 (1957) 407.

[17] C.Sommerfeld, Phys.Rev. 107 (1957) 328.

[18] S.Laporta, E.Remiddi, Phys.Lett. B379 (1996) 283.
[19] H.Elend, Phys.Lett. 21 (1996) 720.
[20] S.Laporta, Nuovo Cimento 106 (1993) 675.
[21] T.Kukhto, E.Kuraev, A.Schiller, Z.Silagadze, Nucl.Phys. B371 (1992) 567.
[22] M.Davier, A.Höcker, LAL 98-38, Orsay, France, 1998.
[23] S.Peris, M.Perrottet, E.de Rafael, Phys.Lett. B 355 (1995) 523.
[24] R.Alemany, M.Davier, A.Höcker, Europ.Phys.J. C2 (1998) 123.
[25] B.Krause, Phys.Lett. B390 (1997) 392.
[26] M.Haykawa, T.Kinoshita, Phys.Rev. D57 (1998) 465.
[27] J.Bijnene, E.Pallante, J.Prads, Nucl.Phys. B474 (1996) 379.
[28] R.Jackiw, S.Weinberg, Phys.Rev. D5 (1972) 2473; K.Fujikawa, D.Lee, A.Sanda, Phys.Rev. D6 (1972) 2923; I.Bars, M.Yoshimura, Phys.Rev. D6 (1972) 374; G.Altarelli, N.Cabbibo, L.Maiani, Phys.Lett. 40B, (1972) 415; W.Bardeen, R.Gastmans, B.Lautrup, Nucl.Phys. B46 (1972) 315.
[29] A.Jeffery et al., Conf. on Precision Electromagnetic Measurements (Braunschweig, Germany, 1996).
[30] T.Kinoshita, Phys.Rev. D47 (1993) 5013; T.Kinoshita, B.Nizic, Y.Okamoto, ibid. 41 (1990) 593; M Samuel, G.Li, ibid. 44 (1991) 3935; 48 (1993) 1879(E); S.Laporta, E.Remiddi, Phys.Lett. B301 (1992) 440; S.Laporta, ibid. 312 (1993) 495; S.Karshenboim, Sov.J.Nucl.Phys. 56 (1993) 80; M.Hayakawa, T.Kinoshita, A.Sanda, Phys.Rev.D54 (1996) 3137.
[31] M.Krawczyk, J.Zochowski, Phys.Rev. D55 (1997) 6968.
[32] I.Ternov, V.Rodionov, A.Studenikin, Sov.J.Nucl.Phys. 37 (1983) 1270.
[33] I.Ternov, V.Rodionov, A.Studenikin, Ann.der Phys. 46 (1989) 303.
[34] A.Studenikin, Sov.J.Part.Nucl. 21 (1990) 259.
[35] I.Obukhov, V.Peres-Fernandes, V.Khalilov, Sov.J.Nucl.Phys. 43 (1986) 137.
[36] A.Studenikin, Sov.Phys.JETP 70 (1990) 795.
[37] V.I.Ritus, Sov.Phys.JETP 30 (1969) 1181.

[38] V.I.Ritus, Ann.Phys. 69 (1972) 555.

[39] A.Studenikin, Phys.Lett. B267 (1991) 117.

[40] S.Nie, M.Sher, hep-ph/9805376.

[41] Y.Arnowitt, A.Chamseddine, P.Nath, Z.Phys. C26 (1984) 407.

[42] A.Studenikin, Nucl.Phys. B (Proc.Suppl.) 23A (1991) 133.

[43] S.Gupta, Nature 169 (1949) 163.

[44] I.M.Ternov, V.G.Bagrov, V.A.Bordovitsyn, O.F.Dorofeev, Sov.Phys.JETP 28 (1969) 1206.

[45] A.I.Studenikin, I.M.Ternov, Phys.Lett. B234 (1990) 367.
$x = \frac{\text{mass of boson}}{\text{mass of muon}}$

$y = k$

Fig. 1

$y$ vs. $x$ with curves labeled 1, 2, 3, 4, 5, 6.
Fig. 2 (decimal logarithmic scale)

$y = \alpha(S)$

$x =$ mass of scalar boson (MeV)
Fig. 3 (decimal logarithmic scale)

\( y = \alpha(P) \)

\( x = \text{mass of pseudoscalar boson (MeV)} \)
Fig. 4 (decimal logarithmic scale)

$y = \alpha(V)$

$x =$ mass of vector boson (MeV)
Fig. 5 (decimal logarithmic scale)

$y = \alpha(A)$

$x = \text{mass of axial vector boson (MeV)}$