The Color-Octet intrinsic charm in $\eta'$ and $B \to \eta' X$ decays

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Abstract

Color-octet mechanism for the decay $B \to \eta' X$ is proposed to explain the large branching ratio of $Br(B \to \eta' X) \sim 1 \times 10^{-3}$ recently announced by CLEO. We argue that the inclusive $\eta'$ production in $B$ decays may dominantly come from the Cabbibo favored $b \to (\bar{c}c)_8 s$ process where $\bar{c}c$ pair is in a color-octet configuration, and followed by the nonperturbative transition $(\bar{c}c)_8 \to \eta' X$. The color-octet intrinsic charm component in the higher Fock states of $\eta'$ is crucial and is induced by the strong coupling of $\eta'$ to gluons via QCD axial anomaly.

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Recently CLEO has reported [1] a very large branching ratio for the inclusive production of $\eta'$ in the $B$ meson decay:

$$ Br(B \rightarrow \eta'X; 2.2\, \text{GeV} \leq E_{\eta'} \leq 2.7\, \text{GeV}) = (7.5 \pm 1.5 \pm 1.1) \times 10^{-4}. $$

If $\eta'$ is regarded as a pure $SU(3)$ singlet meson made of $u$, $d$ and $s$ quarks, the process $B \rightarrow \eta' + X$ is Cabbibo suppressed by the factor $V_{ub}$. The conventional estimate of the branching ratio for this Cabbibo suppressed process is of order of $10^{-6}$ [2], which is over 2 order of magnitude smaller than the experimental result. To explain the large branching ratio of $Br(B \rightarrow \eta' + X)$, some suggestions have been made [2,3]. The authors of [3] give arguments for the need of new physics to enhance the $b \rightarrow s g^* \rightarrow \eta' g$. The latter step, $g^* \rightarrow \eta' g$ is controlled by the effective $\eta' - g - g$ coupling via the QCD anomaly [4]. The authors of [2] suggest that it is the very strong coupling between $\eta'$ and the color-singlet axial vector current of the charm quark that makes the dominant contribution to the large $\eta'$ production rate in the Cabbibo favored process $b \rightarrow \bar{c}c$. 

In this note, we will discuss another possibility. Utilizing the unique feature of $\eta'$, i.e., its strong coupling to gluons through QCD axial anomaly, and the large coefficient for the color-octet $\bar{c}c$ current in the $b \rightarrow \bar{c}c$ process. We will argue that the production of $\eta'$ in $B$ decays may dominantly come from the Cabbibo favored $b \rightarrow (\bar{c}c)_8 s$ process where $\bar{c}c$ pair is in a color-octet configuration followed by the color-octet $\bar{c}c$ pair nonperturbatively evolving into $\eta'$ via $(\bar{c}c)_8 \rightarrow \eta' + X$ due to the color-octet hidden charm component in the higher Fock states of $\eta'$. The essential point of our argument is that due to the strong coupling of gluons to the $\eta'$ there should be an appreciable color-octet $(\bar{c}c)_8$ component mixed into the higher Fock states of $\eta'$.

Before discussing this possibility, let us first recall the suggestion of [3], where a large magnitude of the color-singlet “intrinsic charm” of $\eta'$ is of critical importance. In our opinion, however, there could be two problems in this approach: (1) the mixed decay constant $f_{\eta'}^{(c)}$ suggested in [2] may be too large; (2) the color-singlet matrix element $|\langle \eta_c | \bar{c} \gamma_\mu \gamma_5 c |0\rangle|$ used in [2] may also be too large.

To see these, we write the axial anomaly relation for the charm quark

$$ \partial_\mu (\bar{c} \gamma_\mu \gamma_5 c) = 2i M_c \bar{c} \gamma_5 c + \frac{\alpha_s}{4\pi} G_{\mu\nu} \tilde{G}_{\mu\nu}, $$

and define
\langle 0 | \bar{c} \gamma_\mu \gamma_5 c | \eta'(p) \rangle = i f^{(c)}_{\eta'} p_\mu. \quad (3)

If the charm quark is infinitely heavy, the mixed decay constant \( f^{(c)}_{\eta'} \) should vanish. In the physical world, as argued in [3], the nonvanishing \( f^{(c)}_{\eta'} \) is due to the existence of the \( \bar{c}c \) component in the physical \( \eta' \) wave function, and then

\[ f^{(c)}_{\eta'} = O(\lambda_{\eta'\eta_c} f_{\eta_c}), \quad (4) \]

where \( \lambda_{\eta'\eta_c} \) is the mixing angle (in radian) between \( \eta' \) and \( \eta_c \), and \( f_{\eta_c} \) the decay constant of \( \eta_c \). Based on the axial anomaly and the gluonic matrix element of \( \eta' \), the mixing angle \( \lambda_{\eta'\eta_c} \) has been estimated to be [3]

\[ \lambda_{\eta'\eta_c} \approx 1.2 \times 10^{-2}. \quad (5) \]

It reproduces the experimental value of \( \Gamma(J/\psi \to \gamma \eta') \) via \( \Gamma(J/\psi \to \gamma \eta_c) \) and \( \eta' - \eta_c \) mixing, and is also consistent with most quark model calculations (see, e.g., [3]). With \( f_{\eta_c} \approx 400\,\text{MeV} \) [2], we find

\[ f^{(c)}_{\eta'} \ll f_{\eta'} \approx 130\,\text{MeV} \quad [7], \quad (6) \]

where \( f_{\eta'} \) is the flavor \( SU(3) \)-singlet decay constant of the \( \eta' \). Including the mixing of \( \eta' \) with other \( \bar{c}c \) states like \( \eta'_c, \eta''_c, \cdots \), will enhance \( f^{(c)}_{\eta'} \). As a safe estimate, we would expect

\[ \lambda_{\eta'-(\bar{c}c)} \approx (2 - 10)\%, \quad (7) \]

and then \( f^{(c)}_{\eta'} \lesssim 40\,\text{MeV} \), where \( \lambda_{\eta'-(\bar{c}c)} \) is the total mixed amplitude of the \( \bar{c}c \) component in \( \eta' \). In any case, we think that \( f^{(c)}_{\eta'} = 140\,\text{MeV} \), as suggested in [2], is too large. Also, even with \( f^{(c)}_{\eta'} = 40\,\text{MeV} \), we still have \( f^{(c)}_{\eta'} \ll f_{\eta'} \), and therefore in our estimate [5] the relation

\[ 2i M_c \langle 0 | \bar{c} \gamma_5 c | \eta' \rangle \approx -\frac{\alpha_s}{4\pi} \langle 0 | G_{\mu\nu} \tilde{G}^{\mu\nu} | \eta' \rangle \quad (8) \]

should be a fairly good and self-consistent approximation.

The second problem in [2] is the overestimate of the color-singlet matrix element \( |\langle 0 | \bar{c} \gamma_\mu \gamma_5 c | \eta_c \rangle| \), which is related to \( |\langle 0 | \bar{c} \gamma_\mu \gamma_5 c | J/\psi \rangle| \). The latter is determined in [2] by exhausting the experimental value of \( \Gamma(B \to J/\psi + X) \). However, it is well known that at the lowest order the color-singlet part of \( b \to \bar{c}cs \) process only contributes one third of the decay
width $\Gamma(B \to J/\psi + X)$ measured by CLEO [3,4]. The situation does not get better even if
the next-to-leading order (NLO) corrections in $\alpha_s$ are included in the nonleptonic effective
weak Hamiltonian for $B$ decays [10]. If the NLO results are used, even with the large value
of $f_{\eta'}^{(c)} = 140 MeV$, the estimated results of $\eta'$ production in $B$ decays in [4] will become

$$Br(b \to (\bar{c}c)_1 + X; (\bar{c}c)_1 \to \eta') = 0.12 \cdot 0.6 \cdot 0.9 \times 10^{-3} = 0.065 \times 10^{-3},$$  \tag{9}$$

which is much smaller than the experimental result Eq.(4).

Because of the two problems mentioned above, the color-singlet intrinsic charm mechanism
suggested in [2] may encounter difficulties in explaining the large $\eta'$ production rate in $B$ decays. We therefore consider another approach, i.e., the color-octet intrinsic charm of the $\eta'$. Our suggestion is motivated by two physical aspects: (1) the color-octet production for charmonium in $b \to \bar{c}cs$ process; (2) the appreciable color-octet intrinsic charm component in the higher Fock states of $\eta'$.

We now come to the production of color-octet components. It is known that based on
the NRQCD factorisation formalism [11] the theoretical investigations [8,12] show that the
color-octet contributions might account for the main part of $J/\psi$ production in $B$ decays.
Encouraged by the success of the color-octet mechanism in the explanation of $J/\psi$ production in $B$ decays, we conjecture that the color-octet contribution may also explain the large branching ratio of $\eta'$ production in $B$ decays. To see the importance of the color-octet component in the Cabbibo favored process $b \to \bar{c}cs$, we write explicitly the effective Hamiltonian of nonleptonic $B$ decays [3],

$$H_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \left( \frac{2C_+ - C_-}{3} \bar{c}\gamma_\mu(1 - \gamma_5) \bar{s}\gamma_\mu(1 - \gamma_5)b 
+ (C_+ + C_-)\bar{c}\gamma_\mu(1 - \gamma_5)T^a\bar{s}\gamma_\mu(1 - \gamma_5)T^a b \right),$$  \tag{10}$$

where $G_F$ is the Fermi constant and $V_{ij}$'s are KM matrix elements. The coefficients $C_+$ and $C_-$ are Wilson coefficients at the scale of $\mu = M_b$. To leading order of $\alpha_s(M_b)$ and to all orders of $\alpha_s(M_b)\ln(M_W/M_b)$, they are

$$C_+(M_b) \approx \left[ \frac{\alpha_s(M_b)}{\alpha_s(M_W)} \right]^{-6/23},$$  \tag{11}$$

$$C_-(M_b) \approx \left[ \frac{\alpha_s(M_b)}{\alpha_s(M_W)} \right]^{12/23}.  \tag{12}$$

In this effective Hamiltonian Eq.(10), the first term is the color-singlet term, while the second
term is the color-octet term. Numerically, taking $\alpha_s(M_W) = 0.116$ and $\alpha_s(M_b) = 0.20$, the coefficients in front of the two terms are then

$$C_1 = \frac{2C_+ - C_-}{3} = 0.13,$$

$$C_8 = C_+ + C_- = 2.2.$$  \hspace{1cm} (13)\hspace{1cm} (14)

The coefficient of the color-octet term is over 16 times larger than that of the color-singlet term. So the color-octet contributions to the decay width of $B$ meson in the $b \to \bar{c}cs$ process will have a factor of $\sim 280$ (squared the ratio of $C_8/C_1$) enhancement compared to the color-singlet contributions. This color-octet $b \to \bar{c}cs$ process may provide an explanation for the $n_c$ and $B_{s,l}$ problems in $B$ decays \cite{13}.

For $\eta'$ production, by including the intermediate $(\bar{c}c)_8$ contributions in the process $b \to \bar{c}cs \to \eta' + X$, one might explain the large branching ratio. The color-octet contributions come from the following process

$$b \to (\bar{c}c)[^3S_1^{(8)}]s; \text{ and nonperturbatively } (\bar{c}c)[^3S_1^{(8)}] \to \eta' + X.$$  \hspace{1cm} (15)

Here, the intermediate $\bar{c}c$ pair is in a color-octet $^3S_1$ configuration, which comes from the hidden charm component in the higher Fock states of $\eta'$. To realize the $(\bar{c}c)_8$ transition into $\eta'$, one must introduce the hidden charm component in the Fock states of $\eta'$. We write the Fock state expansion as

$$|\eta'\rangle = \Psi_{\bar{nn}}|(\bar{n}n)_1\rangle + \Psi_{\bar{n}ng}|(\bar{n}n)_8g\rangle + \Psi_{\bar{cc}g}|(\bar{c}c)[^3S_1^{(8)}]g\rangle + \cdots + \Psi_{\bar{cc}}|(\bar{c}c)_1\rangle + \cdots,$$  \hspace{1cm} (16)

where $|(\bar{n}n)_1\rangle = \frac{1}{\sqrt{3}}|(\bar{u}u + \bar{d}d + \bar{s}s)_1\rangle$ is the color-singlet and $SU(3)$ flavor-singlet state, which is the leading part of the $\eta'$ wave function (for convenience we neglect the $\eta - \eta'$ mixing). The mixing of charm in $\eta'$ can be understood by the transitions $(\bar{c}c)_1 \to$ two gluons $\to (\bar{n}n)_1$ and $(\bar{c}c)_8 \to$ one gluon $\to (\bar{n}n)_8$, as shown in Fig.1 (a) and (b) respectively. A naive counting rule for the quark-gluon vertex might indicate

$$|\Psi_{\bar{cc}g}| > |\Psi_{\bar{cc}}|,$$  \hspace{1cm} (17)

because the former has fewer vertex and then is less OZI suppressed than the latter. If this naive argument makes sense, the $\eta'$ will have more color-octet $\bar{c}c$ component than color-singlet $\bar{c}c$ component. Then, because the short distance coefficient (Wilson coefficient) of the
color-octet contributions is over 16 times larger than that of the color-singlet contributions, 
the former may dominate over the latter in the process \( b \to \bar{c} c \) with \( \bar{c} c \to \eta' X \).

We follow the calculations of \( J/\psi \) production in \( B \) decays \[8,12\] to estimate the production rate of \( \eta' \) in \( B \) decays via \( \bar{c} c \) transitions. The color-singlet contribution has been estimated before in Eq.\((9)\). For the color-octet contribution, the calculation is simple, and the result is

\[
\Gamma(B \to (\bar{c} c)^3 S_1^{(8)} + X \to \eta' + X) = \frac{\langle O_\eta' (\bar{c} c S_1) \rangle}{\langle O_\psi (\bar{c} c) \rangle} \frac{PS(\eta')}{PS(\psi)} \Gamma(B \to (\bar{c} c)^3 S_1^{(8)} + X \to J/\psi + X) = \frac{\langle O_\psi (\bar{c} c S_1) \rangle}{2M^2} (C_+ + C_-)^2 (1 + \frac{8M^2}{M_b^2}) \hat{\Gamma}_0, \tag{18}
\]

where

\[
\hat{\Gamma}_0 = |V_{cb}|^2 \left( \frac{G_F^2}{144\pi} \right) M_b^2 M_c (1 - \frac{4M^2}{M_b^2}) (1 - \frac{M_{\eta'}}{M_b}). \tag{19}
\]

Here, \( PS(\eta') \) and \( PS(\psi) \) are the phase space factors of processes \( b \to \eta' + X \) and \( b \to J/\psi + X \) respectively. In Eq.\((18)\), the notation \( \langle O_\psi (\bar{c} c S_1) \rangle \) is the long distance nonperturbative matrix element which represents the probability of the \((\bar{c} c)^3 S_1^{(8)} \) evolving into \( \eta' \). To account for the experimental measurement \( Br(B \to \eta' X) \approx 1 \times 10^{-3} \), the matrix element \( \langle O_\psi (\bar{c} c S_1) \rangle \) must be taken as \( 6.4 \times 10^{-3} GeV^3 \), which is three times smaller than the color-octet matrix element \( \langle O_\psi (\bar{c} c S_1) \rangle \) used in \[12\].

For the \( J/\psi \), the Fock state expansion \[11\]

\[
|J/\psi > = O(1)|\bar{c} c (3S_1, 1)\rangle + O(v)|\bar{c} c (3P_J, 8)g\rangle + O(v^2)|\bar{c} c (3S_1, 8 or 1)gg\rangle + \ldots.
\]

indicate the \( 3S_1 \) color-octet \( \bar{c} c \) component in the higher Fock states of \( J/\psi \) is of order \( O(v^2) \sim O(10^{-1}) \), where \( v \) is the charm quark velocity in the charmonium. On the other hand, \( 3S_1 \) color-octet \( \bar{c} c \) component in the higher Fock states of \( \eta' \) should be of order (see \(17)\)

\[
O(\Psi_{\bar{c}c g}) > O(\Psi_{\bar{c}c}) = (2 - 10) \times 10^{-2}, \tag{20}
\]

as indicated by \( \lambda_{\eta' - \bar{c} c} = (2 - 10) \times 10^{-2} \) in Eq.\((7)\). This may imply that due to QCD axial anomaly the \( \eta' \) strongly couples to gluons and then mixes substantially with \( |(\bar{c} c) s g\rangle \) and \( |(\bar{c} c)_1\rangle \) Fock states via intermediate gluons. Therefore, the color-octet \( \bar{c} c \) matrix element for the \( \eta' \) is not much smaller than the color-octet \( \bar{c} c \) matrix element for the \( J/\psi \). This means
that the obtained value for \( \langle \mathcal{O}_8 \rangle_{\eta'(3S_1[\bar{c}c])} \) by fitting the \( \eta' \) data can be understood on the basis of QCD anomaly.

To conclude, in order to explain the large production rate of \( \eta' \) in \( B \) decays within the Standard Model, the proposed color-singlet intrinsic charm mechanism may encounter difficulties. Therefore, we argue that the inclusive \( \eta' \) production in \( B \) decays may dominantly come from the Cabbibo favored \( b \to (\bar{c}c)_8s \) process where \( \bar{c}c \) pair is in a color-octet configuration, and followed by the nonperturbative transition \( (\bar{c}c)_8 \to \eta'X \). The color-octet intrinsic charm component in the higher Fock states of \( \eta' \) is crucial and is induced by the strong coupling of \( \eta' \) to gluons via QCD axial anomaly. Further investigations will be made to examine this mechanism.

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Figure Captions

Fig.1. Hidden charm mixing in $\eta'$ for the color-singlet $\bar{c}c$ pair (a) and for the color-octet $\bar{c}c$ pair plus a gluon (b).
Fig. 1  Hidden charm mixing in $\eta'$ for the color-singlet $\bar{c}c$ pair (a) and for the color-octet $\bar{c}c$ pair plus a gluon (b).