Estimating the age of the Universe is an old problem. Rapid progress in observational cosmology in recent years has led to more accurate values of the fundamental parameters. The current most popular model is a flat Universe, with about 30% of the critical density in the form of matter (baryonic and non-baryonic) and 70% in the form of ‘dark energy’. These two densities, together with the Hubble constant (estimated to be about 70 km s\(^{-1}\) Mpc\(^{-1}\)), constrain the age of the Universe to be approximately 13 Gyr. This expansion age is uncomfortably close to the age of the oldest globular clusters (approximately 12.5 Gyr), in particular if they formed relatively recently. We review here proposed models of globular cluster formation and point out possible conflicts with cosmology.

In a Big Bang model with a cosmological constant \(\Lambda\) the age of the Universe is determined by three present-epoch parameters: the Hubble constant \(H_0 = 100h\) km s\(^{-1}\) Mpc\(^{-1}\) = (9.78 Gyr)\(^{-1}\)h, the mass density parameter \(\Omega_m\), and the scaled cosmological constant \(\Omega_\Lambda \equiv \Lambda/(3H_0^2)\). The cosmological constant tends to stretch the age of the Universe, but if the dark energy were in the form of ‘quintessence’ which decays with time, then \(t_H\) would be reduced. For this reason (and to abbreviate our discussion) we focus on the ‘optimistic’ case of constant \(\Lambda\). For a flat universe \((\Omega_m + \Omega_\Lambda = 1)\), supported by the recent Cosmic Microwave Background (CMB) experiments, the age depends only on two free parameters and is well approximated\(^{[1]}\) by:

\[
t_H \approx \frac{2}{3}H_0^{-1}\Omega_m^{-0.3}.
\]

This gives us an insight to the way the errors propagate in the determina-
tion of the cosmic age:
\[
\frac{\Delta t_H}{t_H} \approx \frac{\Delta H_0}{H_0} + 0.3 \frac{\Delta \Omega_m}{\Omega_m}.
\] (2)

This shows that the fractional error in \(H_0\) is three times as important as the fractional error in \(\Omega_m\). Typically the quoted error in \(H_0\) is 10\%\(^2\), while the recent range of \(\Omega_m\) is of order 50\%, so the expected fractional error in age is 25\% (e.g. for \(t_H \approx 13\text{ Gyr}\), \(\Delta t_H \approx 3\text{ Gyr}\)).

Numerous studies\(^3\)\(^4\)\(^5\) have compared and combined in a self-consistent way the most powerful cosmic probes: the CMB, galaxy redshift surveys, galaxy cluster number counts, type Ia supernovae, and galaxy peculiar velocities. These studies indicate that we live in a flat accelerating Universe, dominated by cold dark matter (CDM) and ‘dark energy’ (the cosmological constant \(\Lambda\) or some generalization such as ‘quintessence’\(^6\)). More precisely, the data are consistent with a \(\Lambda\)-CDM model with \(\Omega_m = 1 - \Omega_\Lambda \approx 0.3\) and \(h \approx 0.7\), which corresponds to an expansion age \(t_H = 13.5\text{ Gyr}\).

While the above \(\Lambda\)-CDM model is currently very popular, there is no simple theoretical explanation for the fortuitous near-equality of the present-epoch matter density \(\Omega_m\) and ‘dark energy’ density \(\Omega_\Lambda\), nor for the true nature of these components. As a further diagnostic, we would like to revive an old conundrum: is the age of the Universe compatible with the ages of the oldest objects within it? For consistency, the ages of globular clusters (\(t_{GC}\)) must satisfy the following relation with the epoch of their formation (\(t_f\)) and the age of the Universe:

\[
t_H = t_f + t_{GC}.
\] (3)

The age of the oldest GC is estimated to be \(t_{GC} = 12.5 \pm 1.2\text{ Gyr}\)\(^7\). Radioactive dating of a very metal-poor star in the Galaxy (using \(^{238}\text{U}\)) gives a similar age of \(12.5 \pm 3\text{ Gyr}\)\(^8\). Also, recently discovered Ly\(\alpha\) emitters\(^9\) and Ly break galaxies\(^10\) at redshift \(z \sim 2 - 3\) show an already evolved stellar population as old as 1 Gyr, comparable to the expansion age at that epoch.

We see that the ages of old objects might be uncomfortably close to the age of expansion. It is commonly assumed that \(t_f \lesssim 2\text{ Gyr}\)\(^2\), and hence \(t_f\) is neglected in eq. (3). However, we point out here that \(t_f\) is model dependent, and different models predict a wide range of ages.

Formation of globular clusters in the Milky Way intimately relates to the formation scenario of the Galaxy itself. For decades, two apparently
conflicting models dominated the thought on Galaxy formation: ‘monolithic collapse’\textsuperscript{11} versus highly fragmented star formation\textsuperscript{12}. The realization that galaxies are embedded into dark matter halos\textsuperscript{13} has led to a constructive synthesis with cosmology. According to the CDM model, galaxies form as a result of gravitational growth and interaction of primordial fluctuations. Small objects collapse first and merge into larger systems, extending the hierarchy to progressively higher masses. The present halo of the Milky Way formed in dozens of mergers of smaller progenitors.

Our understanding of star formation has also improved dramatically. Theoretical models still have many shortcomings\textsuperscript{14}, but the observational picture is becoming clearer. It looks that most stars form in clusters and associations of various sizes. The hierarchy of cluster masses ranges from the young OB associations to the old massive globular clusters, with no special scale between 10 and $10^6 \ M_\odot$: $dN/dM \propto M^{-\alpha}$, $\alpha = 1.5 - 2$.\textsuperscript{15,16} On a global scale, the efficiency of globular cluster formation remains low. In large and small elliptical galaxies, McLaughlin\textsuperscript{17} finds the same ratio of the mass of globular clusters to the total baryonic (stellar + gaseous) mass of their host galaxy, $\epsilon_{gc} \equiv M_{gc}/M_{bar} \approx 0.0026 \pm 0.0005$.

The observed distribution within galaxies and cluster mass function $dN/dM$ differ from the initial ones due to dynamical evolution. Small-mass clusters ($M < 10^5 \ M_\odot$) are gradually destroyed by stellar two-body relaxation and tidal interaction with the Galaxy\textsuperscript{18,19,20,21}, while very massive clusters sink to the center via dynamical friction. However, most of the high-mass clusters are essentially unaffected by the evolution, and therefore, preserve the shape of the initial mass function.

The suggested scenarios of globular cluster formation can be grouped into four main types, in decreasing order of the redshift of formation $z_f$. For reference, in our assumed cosmology ($\Omega_m = 0.3, \Omega_\Lambda = 0.7, h = 0.7$), redshifts $z_f = 7, 3, 1$ correspond to the formation epochs $t_f = 0.8, 2.1$ and 5.8 Gyr, respectively.

- **Cosmological objects**

  Peebles & Dicke\textsuperscript{22} were the first to propose that globular clusters are cosmological objects formed soon after recombination, with masses related to the Jeans mass (smallest mass of gas clouds able to collapse). However, including dark matter in the calculation would reduce the mass of proto-globular clouds. Also, the expected efficiency of the conversion of gas into star clusters would be much higher than
the observed $\epsilon_{GC}$.

- **Hierarchical population** ($z_f \sim 7 - 10$)

  A hierarchical formation is more promising within small galaxies – progenitors of the Milky Way. In progenitors just massive enough to be cooled by atomic hydrogen, the gas collapses into a small disk where gravitational instability causes fragmentation into clouds and formation of star clusters of progressively larger sizes. Each progenitor galaxy may form a few globular clusters, still remaining very gas-rich before merging into the Galactic halo. Similar ideas have been put forward in the literature\textsuperscript{23} \textsuperscript{24} \textsuperscript{25} \textsuperscript{26}. See Figure \ref{fig:1} for more detail.

- **Large galactic halo** ($z_f \sim 1 - 3$)

  Globular cluster formation as a result of thermal instability in hot gas at virial temperature $T_{\text{vir}} \sim 10^6$ K has been suggested by Fall & Rees\textsuperscript{27} (variations of this idea include \textsuperscript{28} \textsuperscript{29}). Dense clouds of metal-poor gas would cool to $T_c \approx 10^4$ K, confined by the pressure of diffuse gas, and would have the right Jeans mass to form massive globular clusters. The epoch of formation is constrained to be $z_f \lesssim 3$, when halos with large enough $T_{\text{vir}}$ have been assembled. On the other hand, massive gaseous clouds must not have disrupted the thin disk of the Galaxy (7-10 Gyr old), which demands $z_f \gtrsim 1$.

- **Mergers of disk galaxies** ($z_f \lesssim 1$)

  Finally, the observations of young massive star clusters in colliding galaxies prompted a “merger” scenario\textsuperscript{30}. In this model, a merger of two spiral galaxies leads to a burst of star formation producing a large population of metal-rich clusters, in addition to the older population associated with the original spirals. We include this model for completeness, although it has been originally designed to account for the bimodal metallicity distribution of globular clusters in elliptical galaxies and predicts that most of the clusters form fairly recently.

In the table we provide a comparison of the formation models, as they score against the observational properties of the Galactic globular clusters. The mark ✓ shows the model meets the observational constraint, ? indicates possible but not certain agreement, and X denotes clear disagreement with
the data. Hierarchical model seems to fare better in this comparison, but scenarios with later formation are still possible.

Figure 2 illustrates eq. (3) for the three possible values of the formation redshift. Early formation ($z_f = 7$) is favorite, $z_f = 3$ is marginally consistent with the errors of $H_0$ and globular cluster ages, but any lower redshift is ruled out in the flat Universe model with $\Omega_m = 0.3$.

Figure 3 shows the constraints on $H_0$ and $\Omega_m$, assuming that the absolute ages of globular clusters are fixed. Again, the current 'best-fit' values are consistent only with $z_f \gtrsim 3$.

We have thus compared cosmic ages as estimated by researchers in three different (somewhat unrelated) fields of astrophysics: the age of expansion of the Universe, the age of globular clusters, and the epoch of their formation. We find that the three are consistent (within the errors) only over a relatively small parameter space, and hence there might be a problem with any one of the estimates. This leads to several possible conclusions:

(i) If the currently popular cosmological model ($\Omega_m = 0.3, \Omega_\Lambda = 0.7, h = 0.7$) is correct and the age estimates of globular clusters are reliable, then one can put an extra constraint on the models of globular cluster formation, such that $z_f \gtrsim 3$.

(ii) If we trust the cosmology and wish to have a late globular cluster formation, then a revision of the globular cluster ages (which are somewhat model-dependent) is required.

(iii) If future research suggested that even old globular clusters formed at $z_f \lesssim 3$ and the age estimates of globular clusters are valid, then the above comparison indicates a problem for the 'standard' $\Lambda$-CDM model. This is because the time-lag between the Big Bang and the era of globular cluster formation would then be $> 2.1$ Gyr, larger than the estimated errors. The constraint is even more severe if the “dark energy” is time-dependent quintessence rather than a constant $\Lambda$. Our overall conclusion is that there is a strong cosmological (as well as astrophysical) motivation for firming up our understanding of globular cluster formation.

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Table 1: Scoreboard of globular cluster models

| Observational properties                                                                 | P&D | Hierarchy | F&R | Merger |
|------------------------------------------------------------------------------------------|-----|-----------|-----|--------|
| 1. Narrow range of metallicity within individual clusters \( \delta [\text{Fe/H}] \lesssim 0.1 \) | ✓   | ✓         | ✓   | ✓      |
| 2. Average metallicity \( Z \sim 0.03 \ Z_\odot \)                                       | \( \times^a \) | ✓         | ?\( ^b \) | \( \times^c \) |
| 3. Spherical geometry of globular cluster system                                           | ✓   | ✓         | ?\( ^d \) | \( \times^e \) |
| 4. Low efficiency \( \epsilon_{GC} \)                                                    | \( \times^f \) | ✓         | ✓   | ?\( ^g \) |
| 5. Spread of relative ages of oldest clusters \( \sim 2 \ \text{Gyr} \)                  | X   | ?\( ^h \) | ✓   | ✓      |
| 6. Young massive clusters have a broad mass function                                       | \( \times^i \) | ✓         | ?\( ^j \) | ?\( ^j \) |
| 7. Globulars have similar properties regardless of the size or morphology of the host galaxy | ✓   | ✓         | ?   | ?      |
| Overall\( ^k \)                                                                          | X   | ✓         | ?   | X      |

\( ^a \) in this model globular clusters form immediately after recombination when the intergalactic gas has few metals; self-enrichment is unlikely as globular clusters must collapse quickly enough to remain gravitationally-bound.

\( ^b \) also likely to have lower metallicity; pre-enriched gas would have cooled to a temperature lower than \( 10^4 \ \text{K} \).

\( ^c \) remaining gas in normal spiral galaxies is already enriched, \( Z \sim 0.1 - 1 \ Z_\odot \), overproducing metallicity in new GCs.

\( ^d \) spherical geometry is assumed, but cloud fragmentation is likely to proceed in highest density regions, i.e. in the disk.

\( ^e \) gas in mergers is contained within the orbital plane, and new GCs must have a highly flattened distribution.

\( ^f \) this model assumes almost 100% efficiency of gas conversion.

\( ^g \) observations suggest a possibly higher efficiency.

\( ^h \) age spread could be attributed to different time of virialization of the progenitors of different mass and the statistical variance of the progenitors of similar mass, creating a range of redshifts of formation.

\( ^i \) this model reproduces only the median mass scale, \( M \sim 10^5 \ M_\odot \).

\( ^j \) in these models cluster mass would depend on the virial temperature of the halo and the metallicity of cooling gas.

\( ^k \) note that these marks apply only to the formation of globular clusters in the Galaxy.
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Figure 1: In a hierarchical scenario that we propose, the oldest globular clusters form in small progenitor galaxies with the mass of the first-ranked cluster proportional to the amount of gas supply in the progenitor, \( f_b M \). Massive clusters are almost unaffected by dynamical evolution and serve as indicators of the initial distribution. The plot compares the mass function of large globular clusters (asterisks; data from [31]), renormalized by a factor \( (f_b \epsilon_{gc})^{-1} \approx 3000 \), with the cumulative number of virialized progenitors of the present Milky Way halo. For three different redshifts, \( N(> M) \) is calculated using the extended Press-Schechter formalism [32]. The two distributions match at \( z \approx 7 \), so that the first clusters might have formed soon thereafter. The virialization epoch of the progenitors is expected to have a statistical spread, which produces a corresponding spread of globular cluster ages, in agreement with observations. If the efficiency of cluster formation \( \epsilon_{gc} \) is not universal, i.e. significantly higher in some progenitors and lower in others, the distribution may shift towards higher redshift. For instance, filled circles demonstrate the case where 10% of the progenitors form clusters with the efficiency \( 10 \epsilon_{gc} \). The expected epoch of formation of oldest clusters thus follows to be \( z_f \approx 7 - 10 \).
Figure 2: The age of expansion vs the age of globular clusters, as a function of the formation redshift $z_f$ (eq. 3). Shaded box contains the current values of $t_H$ and $t_{GC}$ within the observational errors. In a flat Universe with $\Omega_\Lambda = 1 - \Omega_m = 0.7$, the age of expansion is $t_H = \frac{2}{3} H_0^{-1} \Omega_\Lambda^{-1/2} \ln[(1 + \Omega_\Lambda^{1/2})(1 - \Omega_\Lambda)^{-1/2}]$. 

$\Omega_m = 0.3 \quad \Omega_\Lambda = 0.7$

$H_0 = 65$

$H_0 = 75$

$z_f = 1$

$z_f = 3$

$z_f = 7$
Figure 3: The Hubble constant $H_0$ vs matter density $\Omega_m$, required to give the age of expansion consistent with the redshift of globular cluster formation $z_f$, assuming globular cluster ages are fixed at $t_{GC} = 12.5$ Gyr and the Universe is flat (cf Fig. 2). Dashed line shows the limiting case $t_H = t_{GC}$. Shaded box contains the 'best-fit' values.