Review of the state of the art: empirical formulations for the estimation of rock mass deformability by means of dynamic testing

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Abstract. This paper is focused on the empirical formulations for the estimation of rock mass deformability dependent on data obtained from dynamic methods. The fundamentals of dynamic testing, as well as their advantages and limitations, are reviewed. Based on the results of recent studies, a discussion is made on the relationship between dynamic and static deformation moduli, and their dependency on factors such as lithology, weathering, or presence of discontinuity surfaces. The most significant empirical formulations published to this day are summarized, alongside a critical exposition of their characteristics, and a statistical analysis of their results is undertaken. As a result, the influence of the lithology as a crucial factor in the determination of the deformation modulus is highlighted. Thus, it must be properly considered when adopting a formulation for the estimation of the deformability behaviour of a rock mass.

1. Introduction
The geomechanical characterisation of rock masses in civil engineering projects has the aim of estimating their mechanical parameters, in relation with the strength and deformability behaviour.

In the specific case of the design of infrastructures such as dams, bridges and underground excavations, the estimation of the rock mass deformability is of the highest importance. However, due to the discontinuous nature of the rock mass, this analysis is complex. The presence of discontinuity sets with various orientations, openings, spacings and fillings, complicates the study of the deformability. To correctly consider the specific characteristics of each rock mass on their assessment, it must be based on measures from both laboratory and in situ testing.

The characterisation of the global behaviour of the rock mass cannot be solely based on laboratory tests made in samples, due to the scale effect and the limitations on the size of the testing equipment [1]. In order to obtain representative information on the deformability, in situ tests involving larger volumes of the rock mass are required. The discrepancy between the results of in situ and laboratory tests, due to the mentioned scale effect, has been assessed: the deformation moduli measured in situ are of 20% to 60% of the values obtained with laboratory tests [2].

On the other hand, field tests are not reliable unless there are enough of them to produce a representative database [3]. Their main disadvantage is that an exhaustive in situ investigation has a
high cost and is time consuming. For this reason, despite being the best tool for the assessment of rock mass deformability, in situ tests are not sufficient on their own. As a result, they are often used alongside laboratory tests.

In addition to this, empirical formulations for the estimation of the deformation moduli are widely used as a complement, mainly because of their simplicity. Their use on data from in situ tests enables the estimation of deformation moduli valid for the preliminary designs of infrastructures, even though their accuracy is not enough for detailed designs [4].

In this context, there is a wide variety of empirical relations that can be used, obtained from diverse databases and dependent on different parameters. Many of them assess the deformability of the rock mass based on classification indices (RMR, RQD, Q, GSI, etc.), thus considering the rock mass discontinuities. In some of these formulations, the characteristics of the intact rock are explicitly considered by means of the uniaxial compressive resistance or the deformation modulus of the intact rock. Some authors, such as Zhang [5], have published reviews on this kind of empirical formulae.

However, there are other methods to estimate the composite behaviour of the rock mass, including discontinuities and intact rock. One of the most common is the fitting of empirical formulations with data from dynamic tests, such as resonant column or sonic impulse tests. Another source of data are geophysical tests with geophones or seismographs, which enable fast and inexpensive testing of the full-scale rock mass [6]. The convenience of these dynamic methods for the assessment of the deformability of rock masses is the reason why they are used. For this reason, this paper aims to summarise the fundamentals of the empirical formulations dependent on dynamic parameters.

2. Fundaments and limitations

The empirical formulations for the estimation of rock mass deformability, when dependent on data from dynamic testing, can be based on one of the following parameters: the propagation velocity of longitudinal waves (p-waves), \( v_p \), or the dynamic deformation modulus of the rock mass, \( E_{dyn} \).

In accordance with Eissa and Kazi [7], the dynamic deformation modulus of the rock mass can be estimated with rebound hardness tests (i.e. Schmidt hammer), and wave propagation tests, either seismic (resonant column, geophysics) or ultrasonic. This paper is focused on the wave propagation tests. Table 1 summarises the formulations suggested by different authors for the estimation of the dynamic modulus by means of the propagation velocity results obtained from the dynamic tests.

| Parameter                          | Ultrasonic impulse tests [8]                                                                 | Geophysical tests [9]                        |
|------------------------------------|-----------------------------------------------------------------------------------------------|---------------------------------------------|
| Dynamic deformation modulus        | \[ E_{dyn} = \rho \cdot v_p^2 \cdot \frac{(1 - 2\nu)(1 + \nu)}{(1 - \nu)} \]                  | \[ E_{dyn} = 2 \cdot \rho \cdot v_s^2 \cdot (1 - \nu_{dyn}) \] |
| Dynamic Poisson coefficient        | \[ \nu_{dyn} = \frac{v_p^2 - 2v_s^2}{2(v_p^2 - v_s^2)} \]                                     | \[ \nu_{dyn} = 0.5 \cdot (v_p/v_s)^2 - 1 \] |
| Dynamic transversal deformation modulus | \[ G_{dyn} = \rho \cdot v_s^2 \]                                                               | \[ G_{dyn} = \rho \cdot v_s^2 \] |

Once a value of the propagation velocity of p-waves or the dynamic modulus has been determined, the static deformation modulus can be estimated by means of a fitted empirical formula. Savich [10] presented the general relation between static and dynamic moduli, which is a logarithmic expression. It can be written as follows:

\[ \log E_{st} = b_1 \cdot \log E_{dyn} - a_1 \]

Rearranging the formula:

\[ E_{st} = a \cdot E_{dyn}^b \]
The main limitation of the estimation of deformability parameters with dynamic methods lies in the characteristics of the behaviour of the rock masses: non-linear, inhomogeneous, and anisotropic [6, 9]. As a result, the difference between the static and dynamic modulus is of complex quantification.

Three main causes are found to be connected to the difference between both moduli. First, the variations between the samples used in different tests, in terms of porosity and orientation of the discontinuities [11]. Second, the dynamic and static tests originate different thermodynamic processes in the rock mass [12]. Third, the non-linearity of the rock mass stress-strain behaviour, which is usually the factor of greatest importance. Due to the nature of the tests, the deformation levels at which the static and dynamic moduli are measured in are completely different. On one hand, dynamic moduli are measured with deformation values within the $10^{-6}$ to $10^{-7}$ range, in which the behaviour of the rock mass is approximately elastic. The static moduli, on the other hand, are measured with deformation levels of $10^{-3}$ to $10^{-2}$. In this range, the behaviour of the rock mass is not elastic ([9],[12]).

Some authors present this last difference in terms of frequency: static tests are considered as dynamic tests with very low frequencies [11]. As in the case of deformations, the behaviour of the rock mass is non-linear on different frequencies.

As shown in Figure 1 [8], the dynamic deformation modulus has generally higher values than the static. However, it has been found that the difference between both parameters tends to be reduced for higher values of the moduli, in both linear and non-linear regressions [11].

![Figure 1. Relation between the static and dynamic moduli. Based on Martínez-Martínez et al [8]](image)

The empirical formulations published up to the present moment approach this matter by means of a correction of the relation between both moduli. This adjustment can adopt two different forms: other parameters can be included as correction factors, or the constants of the formula can be directly modified. In the next section, a brief summary of the main empirical models for the estimation of the static modulus is done.

3. Review of empirical formulations
The empirical relations published up to the present can be classified by distinguishing those dependent on the propagation velocity of longitudinal waves ($v_P$), and those based on the dynamic deformation modulus ($E_{dyn}$). All the mathematical expressions of the empirical formulations cited in this epigraph are summarised in Table 2. A review of their main characteristics is done in this section.

In the 70’s, Belikov et al [13] derived a linear empirical relation for the estimation of the static modulus with data of dynamic moduli. In the same decade, Grujic [14] published a formulation based on the propagation velocity of longitudinal waves, fitted with data obtained from ultrasonic testing and...
13 flat jack tests. All these tests were performed in the limestones in which the Mratinje dam (Montenegro) is founded.

Table 2. Summary of empirical formulations dependent on dynamic parameters for the estimation of the deformation modulus of a rock mass

| Authors                        | Year | Parameters | Formulation |
|-------------------------------|------|------------|-------------|
| Belikov et al                 | 1970 | $E_{dyn}$  | $E_{st} = 1.137E_{dyn} - 9.685$ |
| Grujic                        | 1974 | $v_p$      | $E_{st} = 15.3 + 6.59 \frac{v_p - 3660}{1000}$ |
| King                          | 1983 | $E_{dyn}$  | $E_{st} = 1.263 \cdot E_{dyn} - 29.5$ |
| Van Heerden                   | 1987 | $E_{dyn}, a, b$ | $E_{st} = a \cdot E_{dyn}$ |
| Eissa y Kazi                  | 1988 | $E_{dyn}$  | $E_{st} = 0.74 \cdot E_{dyn} - 0.32$ |
| McCann and Entwisle           | 1992 | $E_{dyn}$  | $E_{st} = 0.69 \cdot E_{dyn} + 6.40$ |
| McCann and Entwisle           | 1992 | $E_{dyn}$  | $\log_{10} E_{st} = 1.749 \log_{10} E_{dyn} - 1.075$ |
| Grasso et al                  | 1993 | $v_p$      | $E_{st} = 0.29 \cdot \exp(1.08 \cdot v_p)$ |
| Clerici                       | 1993 | $E_{dyn}, E_{st,i}$ | $E_{st} = E_{st,i} \cdot \frac{E_{dyn}}{E_{dyn,i}}$ |
| Jasarevic and Kovacevic       | 1996 | $v_p$ (km/s) | $E_{st} = 4.950 + 0.900 \cdot v_p$ |
| Starzec                       | 1999 | $E_{dyn}$  | $E_{st} = 0.48 \cdot E_{dyn} - 3.26$ |
| Oben                          | 2003 | $E_{dyn}$  | $E_{st} = 0.0158 \cdot E_{dyn}^{2.74}$ |
| Yasar and Erdogan             | 2004 | $v_p$ (km/s) | $E_{st} = 10.67 \cdot v_p - 18.71$ |
| Barton                        | 2007 | $v_p$ (km/s) | $E_{st} = 10^{1.5(v_p-3.5)/3}$ |
| Barton                        | 2007 | $\sigma_{ci}, v_p$ (km/s) | $E_{st} = 10^{\frac{1}{3}(v_p-2.5 \ln \sigma_{ci})}$ |
| Yilmaz and Yuksek             | 2009 | $v_p$ (km/s) | $E_{st, RM} = 6.8545 \cdot \exp(0.5561v_p)$ |
| Martínez-Martínez et al       | 2012 | $E_{dyn}, \alpha_S$ | $E_{st} = \frac{E_{dyn}}{3.8} \cdot \alpha_S^{0.68}$ |
| Rabbani et al                 | 2012 | $E_{dyn}$  | $E_{st} = 0.4145 \cdot E_{dyn} - 1.059$ |
| Brotons et al                 | 2014 | $E_{dyn}$  | $E_{st} = 0.867E_{dyn} - 2.085$ |
| Brotons et al                 | 2014 | $\rho, E_{dyn}$ | $E_{st} = 10^{1.275 \log_{10}(\rho \cdot E_{dyn})-4.714}$ |
| Pappalardo                    | 2015 | $v_p$ (km/s) | $E_{st} = 6.623 \cdot v_p - 22.64$ |
| Pappalardo                    | 2015 | $v_p$ (km/s) | $E_{st} = 5.076 \cdot v_p - 15.72$ |
| Sari                          | 2018 | $v_p$ (km/s) | $E_{st} = 1.578 \cdot \exp(0.623v_p)$ |
| Sari                          | 2018 | $v_p$ (km/s) | $E_{st} = 1.314 \cdot v_p^{0.73}$ |

King [15] published, in 1983, a study in which a linear relation between the static and dynamic moduli was suggested. It was fitted with experimental data obtained from igneous and metamorphic rock masses located in Canadian Shield. The database included 174 static moduli data, and it was completed with geophysical and wave propagation tests, which enabled the estimation of the dynamic moduli.

In the same decade, Van Heerden [6] suggested a power formulation, with the aim of covering a wide range of lithologies. For this reason, he used tests undertaken in ten different types of rock (such as sandstones, quartizes, magnetites and norites), with static moduli between 7 and 150 GPa. Those tests were done at different levels of confining stress, with the intention of resulting in a relation...
dependent on this variable. The outcome coefficients of the power formula are obtained from the charts depicted in Figure 2.

Figure 2. Coefficients dependent on the confining stress, a (a) and b (b) by Van Heerden [6]

Later on, Eissa and Kazzi [7] developed their empirical formulation, published in 1988. They used 76 datasets obtained from bibliographic sources, including data from resonant column and ultrasonic impulse tests. A linear correlation between the static and dynamic moduli was considered.

McCann and Entwisle [9] fitted an empirical relation with data from granitic and metasedimentary rock masses located in the UK. Samples were taken, and the static moduli were estimated by means of laboratory tests with strain gauges, whereas the dynamic moduli were calculated with s-wave velocity profiles. Both parameters were correlated with two formulations: linear and power.

In the decade of 1990, Grasso et al [16] published their formula. To derive it, they used data of p-wave velocity values in shales. The result was a power empirical relation between the static deformation modulus and the p-wave velocity. Later, in 1993, Clerici [17] suggested a formulation correlating the static modulus of the rock mass with the static and dynamic moduli of the intact rock, as well as the dynamic modulus of the rock mass.

The empirical formulation published by Jasarevic and Kovacevic [18] is a linear relation, dependent on the propagation velocity of longitudinal waves. They fitted their formula with monitoring data from infrastructures founded in the limestones of the coastal area of Croatia.

The propagation velocity of p-waves was also used in the study undertaken by Starzec [12], focused on the behaviour of crystalline rock masses. More specifically, he addressed igneous and metamorphic rock masses from Sweden. Dynamic moduli were estimated with ultrasonic tests, while the static moduli were obtained from laboratory tests with strain gauges, done in 300 samples of rock.

In 2003, Ohen [13] published a general study on dynamic modelling, applied to the oil and gas extraction facilities located in schists of the Mexican Gulf. Within the study, focused on the failure analysis of such infrastructures, he fitted an empirical relation between the static and dynamic moduli of the rock mass.

The investigation done by Yasar and Erdogan [20] includes a deformability assessment of carbonated rocks. Both laboratory tests and p-wave propagation tests were undertaken in Turkish dolomites, marbles and limestones. With the aim of eliminating the influence of anisotropy from the study, they selected rock masses with no stratification surfaces.

In 2007, Barton [21] published a study in which he offered, among other analyses, an empirical relation between the static deformation modulus and the propagation velocity of longitudinal waves, also including the uniaxial compression strength of the intact rock in the mathematical formulation. This formula was fitted with data obtained from metro stations in granites (South America) and tunnels from China, Norway and the UK.
After that, Yilmaz and Yuksek [4] studied the deformability of gypsum rock masses. They used data of velocity of propagation of longitudinal waves, as well as point load tests, to fit an empirical relation. All the tested samples were taken in gypsum rock masses located in the Sivas Basin (Turkey).

The study published by Martínez-Martínez et al [8] is focused on carbonated rock masses, with a wide variety of conditions of homogeneity, monolithism and weathering. Many lithologies were considered in the study, including limestones, travertines, marbles and dolomites. All the dynamic data for the study was obtained from ultrasonic impulse tests. They did an exhaustive assessment of the relation between the static and dynamic moduli and the factors that influenced it. In order to improve their formulation, the authors included a coefficient of spatial attenuation of the wave propagation ($\alpha_5$). This way, the ratio $E_{dyn}/E_{st}$ was corrected, considering the effect of discontinuities and cavities. As this factor is very sensitive to the presence of discontinuities, this formulation is specially oriented to the assessment of rock masses heavily fractured or with cavities.

In 2012, Rabbani et al [22] published an empirical relation estimated with a neural network (ANN). The ANN was adjusted with results of various acoustic geophysical tests (5000 datasets from oil wells in South Iran), as well as existing empirical relations.

Shortly after that, Brotons et al [11] published their research on calcarenites with non-destructive tests. Results from ultrasonic impulse test at different temperatures were used. The static moduli were estimated with uniaxial compressive strength tests, equipped with strain gauges. In order to correct the relationship between the static and dynamic moduli, the authors included the density of the rock mass as a parameter of the formulation.

Pappalardo [23] assessed the behaviour of Sicilian dolomites. This study was undertaken with data from geomechanical stations, geophysical tests, boreholes and laboratory tests. It resulted in two linear empirical correlations, dependent on the propagation velocity of p-waves.

The most recent publication regarding deformation modulus and dynamic testing was done by Sari [24], in 2018. The aim of this research was the development of a general formulation, valid for any type of rock mass. For this reason, the relation was fitted with a vast database, consistent on 4491 datasets obtained from bibliographical sources.

4. Discussion

Up to this point, a description of the existing empirical relations has been presented, with special attention to the origin of the data used for their derivation. Formulas developed for different types of rock masses are expected to have different results, and different applicability. As this matter is of great importance, a statistical assessment will be made, as it enables a direct comparison of the behaviour of all the formulations dependent on the same parameters.

Since empirical formulations are estimated with a set of data, their applicability is directly limited by the nature of those values used for its fitting. As a result, they can be used only for the assessment of rock masses with similar characteristics to those in which the data were obtained.

The previous epigraph includes equations developed for very specific rock masses (regarding their lithology, weathering and discontinuities), as well as other relations obtained from more diverse databases, meant to be used in a wider variety of rock masses. The former have a more restricted applicability, but their accuracy is generally higher. This happens because the scatter of the database used for their estimation is lower, and the differences between the rock masses assessed and those used for the fit, lesser.

Due to their local nature, there are many empirical relations. This enables a statistical assessment of those dependent on the same parameter. In this paper, the relations dependent of the velocity of propagation of longitudinal waves and the dynamic modulus (related by the formulations included in Table 1) have been analysed. These formulations are among those included in Table 2.

The resulting static deformation moduli of all these formulations are shown in Figure 3. The range of p-wave velocities used in this assessment has been determined based on the values presented by Fourmaintraux [25], which have been summarised in Table 3. Since that study was focused mainly on
hard rocks, the range has been expanded to longitudinal wave velocities from 1000 to 8000 m/s. These are equivalent to dynamic deformation moduli of approximately 5 to 400 GPa.

Table 3. Velocity of propagation of longitudinal waves in different lithologies [25]

| Lithology                  | p-wave propagation velocity $v_p$ (m/s) |
|----------------------------|----------------------------------------|
| Gabbro                     | 7000                                   |
| Basalt                     | 6500-7000                              |
| Limestone                  | 6000-6500                              |
| Dolostone                  | 6500-7000                              |
| Sandstone, quartzite       | 6000                                   |
| Granitic rocks             | 5500-6000                              |

Figure 3. Static deformation moduli obtained from various empirical relations

From the resulting data, the values of the upper and lower envelopes of static deformation moduli have been isolated. Additionally, the mean moduli for each value of p-wave velocity have been calculated. Thus, three sets of data are obtained, and subsequently used to fit the formulations of the envelopes and mean curves. The mathematical expressions, as well as their correlation coefficient, are summarised in Table 4. In accordance with the theoretical relation suggested by Savich [10], mentioned in section 2, power formulations have been considered. Their results are depicted in Figure 4 and Figure 5.

Table 4. Fitted equations of mean values and envelopes of the results of the empirical relations

| Curve            | Propagation velocity of p-waves $v_p$ | Dynamic deformation modulus $E_{ST}$ | $R^2$  | Formula                                      |
|------------------|--------------------------------------|-------------------------------------|--------|----------------------------------------------|
| Lower envelope   | $E_{ST} = 6.3727 \cdot v_p^{2.3652}$ | $E_{ST} = 0.7487 \cdot E_{DYN}^{1.1825}$ | 0.718  |                                              |
| Mean values      | $E_{ST} = 2.8834 \cdot v_p^{1.1601}$ | $E_{ST} = 0.4079 \cdot E_{DYN}^{1.0801}$ | 0.982  |                                              |
| Upper envelope   | $E_{ST} = 0.9004 \cdot v_p^{1.3938}$ | $E_{ST} = 0.2549 \cdot E_{DYN}^{0.6969}$ | 0.965  |                                              |
Figure 4. Fitted relations in terms of the dynamic deformation modulus

Figure 5. Fitted relations in terms of the propagation velocity of longitudinal waves

In order to compare the analysed empirical relations with the mean fitted curve, their mean square error (MSE) and Pearson correlation coefficient ($R^2$) have been calculated and presented in Table 5. It is observed that the relations closest to the mean curve are those of McCann and Entwisle [9], Eissa and Kazi [7], Starzec [12], Brotons et al [11], Barton [21], Sari [24] and Grasso et al [16]. It is worth mentioning that both Eissa and Kazi [7] and Sari [24] used data from a wide variety of rock masses, in order to obtain general relations. This explains the fact that their results are close to the average values. Besides, it can be seen that the relations estimated with data obtained from granites are closer to the average, while others (mainly those relations applicable to limestones and dolostones) stray further from the mean values. The results highlight, once more, the importance of the lithology in the assessment of rock mass deformability with dynamic parameters.
Table 5. Statistical errors between the fitted curve and the empirical formulations analysed

| Empirical relation                | MSE  | $R^2$ | Lithologies considered     |
|----------------------------------|------|-------|---------------------------|
| McCann and Entwisle (1992)       | 19   | 0.961 | Granites, metasedimentary  |
| Eissa and Kazi (1988)            | 20   | 0.952 | Various                   |
| Starzec (1999)                   | 35   | 0.624 | Igneous, metamorphic       |
| Brotons et al (2014)             | 43   | 0.835 | Calcarenites               |
| Barton (2007)                    | 45   | 0.754 | Granites                   |
| Rabbani et al (2012)             | 45   | 0.195 | Various                   |
| Sari (2018)                      | 46   | 0.511 | Various                   |
| Grasso et al (1993)              | 53   | 0.692 | Mudstones                  |
| Sari (2018)                      | 77   | NCa   | Various                   |
| Belikov et al (1970)             | 92   | 0.534 | Various                   |
| Yasar y Erdogan (2004)           | 95   | NCa   | Limestones, dolostones, marbles |
| Grujic (1974)                    | 104  | NCa   | Limestones                 |
| King (1983)                      | 107  | 0.471 | Igneous, metamorphic       |
| Yilmaz and Yuuksek (2009)        | 113  | 0.556 | Gypsum                     |
| Jasarevic and Kovacevic (1996)   | 117  | NCa   | Limestones                 |
| Pappalardo (2015)                | 137  | NCa   | Dolostones                 |
| Brotons et al (2014)             | 270  | NCa   | Calcarenites               |

* No correlation

The influence of lithology has already been assessed by other authors, as mentioned in this paper, but it is confirmed by the results of this analysis. However, it must be noted that the statistical assessment has not included any of the formulations that consider specific parameters, aside from the p-wave velocity and the dynamic deformation modulus. Two such cases are the relations published by Van Heerden [6] and Martínez-Martínez et al [8], that consider the influence of the in-situ stress state and the attenuation coefficient, respectively. None of those aspects are negligible for the characterisation of a rock mass, and they must be considered in the specific cases in which they have significant influence, such as the study of tectonized areas or rock masses with cavities.

5. Conclusions

Dynamic testing is an interesting tool for the assessment of rock masses, increasingly used in characterisation campaigns due to its many advantages: it is fast and inexpensive, and it also enables in situ testing. For this reason, various authors have published empirical formulations based on dynamic parameters, thus providing a tool for the estimation of the rock mass deformability during the preliminary stages of the design of infrastructures.

The greatest difficulty when correlating the static and dynamic moduli of rock masses derives from the influence of lithology, weathering, and other factors. The aforementioned researchers have concluded that the ratio between both moduli is close to 1 in monolithic, competent rock masses. Soft and weathered rocks, on the other hand, tend to have dynamic moduli much larger than their static moduli. The statistical analysis presented on this paper shows that there are significant differences in the results of the empirical relations, depending on the lithology of the rock masses considered for their derivation. For this reason, the lithology, weathering, and discontinuity structure of the rock masses must be considered when correlating its dynamic and static parameters, and when using correlations for the characterisation of a specific rock mass.

The presence of discontinuities or cavities is another fundamental factor in the dynamic behaviour of the rock masses, since it affects the propagation waves and, therefore, the results of the dynamic tests. The referred authors approach this problem by adjusting their correlation coefficients, or by considering correction parameters such as density or attenuation coefficients.

When the aforementioned aspects are properly considered, the resulting equations are simple, convenient and accurate enough to do a rough estimation of the static deformability behaviour of the rock mass.
Notation:

\( E_{st} \) Static deformation modulus of the rock mass
\( E_{dyn} \) Dynamic deformation modulus of the rock mass
\( E_{st,i} \) Static deformation modulus of the intact rock
\( E_{dyn,i} \) Dynamic deformation modulus of the intact rock
\( G_{dyn} \) Dynamic transversal modulus of the rock mass
\( \nu_{dyn} \) Dynamic Poisson coefficient of the rock mass
\( \nu \) Static Poisson coefficient of the rock mass
\( \rho \) Density of the rock mass
\( v_p \) Velocity of propagation of longitudinal waves \((p)\)
\( v_s \) Velocity of propagation of transversal waves \((s)\)
\( \sigma_{ci} \) Uniaxial compressive strength of the intact rock
\( \alpha_s \) Spatial attenuation coefficient of wave propagation
\( a_1, b_1 \) General coefficients of the logarithmic relation between the static and dynamic deformation moduli
\( a, b \) General coefficients of the exponential relation between the static and dynamic deformation moduli

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