Abstract—We study the two component Fermi plasma. Two components are electrons and ions. Using the Quantum-Hydrodynamic model (QHD), we study the linear properties of electrostatic wave. We derive the linear dispersion relation for the system from dynamical governing equations for the system. We study the dependence of linear-dispersion relation on various parameters of the system.

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Index Terms—Dispersion relation; Electrostatic wave; Fermi pressure; QHD model.

I. INTRODUCTION

In recent times, scientists believe that the universe consists of 69% of dark energy, 27% dark matter, and 1% normal matter, and all these celestial objects we see in the night sky are in the plasma state. Here, plasma (not the blood plasma) is an ionized gas that consists of electrons and ions. Plasma is called “The Fourth State of Matter” [1]. Nowadays, Quantum plasma has become a major topic for research. Its application is spreading from neutron stars, giant planets, dwarfs to laboratory plasmas, also in electron- hole plasma, electron gas etc. The basic condition for Quantum plasma is low temperature and sufficiently high density, unlike classical plasma [2]. In extreme densities when the distance between particles becomes as close as the quantum level, quantum tunneling plays a significant role. An electron feels a pressure due to its quantum effect. The examples of these pressures are Fermi pressure, relativistic pressure etc. [3]. In Quantum plasma field, many researchers (Haas et. al, 2003 [4]; Ali and Shukla, 2006 [5]; Manfredi, 2005 [6] ) have used Quantum Hydrodynamic Model (QHD). This QHD model treats plasma as a fluid and uses fluid equations for investigating this [7].

The electromagnetic fields in plasma have two parts- electrostatic and oscillatory. Waves are differentiated as electromagnetic or electrostatic according to there is any oscillatory magnetic field or not. And based on these, we can say, EWs must be longitudinal. So, If the magnetic field is zero then the wave is EW [1]. Now we will talk about dispersion properties. We know that plasma is a dispersive medium. Dispersion relation (DR) describes the dispersive effects on the properties of waves in plasma. It relates the frequency of wave with its wave number. Whenever dispersion is present, wave velocity cannot be defined separately, we need to define group and phase velocity. The linear and non-linear properties of electron-acoustic solitary waves with three-component Fermi plasma has been studied in [8]. For three-component Fermi plasma the Rouge-wave formation and dynamical properties of electron-acoustic solitary waves has been done in [9].

The paper is organized in the following structure: In Sec-[I], we have discussed about plasma waves and DR. In Sec-[II], we will set the governing equations using the QHD model and will normalize them with normalization schemes. In Sec-[III], we will derive the dispersion relation using normalized equations and standard reductive perturbation method. In Sec-[IV], we will discuss about plots of DR. And then, we will conclude the effects of dispersion relation on electrostatic waves in Sec-[V].

II. BASIC EQUATIONS

Let us consider the propagation of electrostatic waves in Quantum plasma consisting of electrons and ions. Also, assuming the plasma particles act like Fermi gas at zero temperature, the pressure term will be-

\[ P_j = \frac{m_j V_{F_j}^2}{3n_j^{\frac{3}{2}}} \]

where, \( j = e \) for electron \( j = i \) for ion

\( m_j = \text{mass} \) \( n_j = \text{Number density} \)

\( V_{F_j} = \sqrt{\frac{2k_B T_{F_j}}{m_j}} \) = Fermi speed

\( T_{F_j} = \text{Fermi temperature} \)

\( k_B = \text{Boltzmann’s constant} \)

So, the set of QHD equations governing the dynamics of Quantum plasma waves in a two-component plasma is listed below-

\[ \frac{\partial n_e}{\partial t} + \frac{\partial (n_e u_e)}{\partial x} = 0 \] (2)

\[ \frac{\partial n_i}{\partial t} + \frac{\partial (n_i u_i)}{\partial x} = 0 \] (3)

\[ \left( \frac{\partial}{\partial t} + u_i \frac{\partial}{\partial x} \right) u_i = \frac{1}{m_i} \left[ Q_i \frac{\partial \phi}{\partial x} + \eta_i \frac{\partial^2 u_i}{\partial x^2} \right] \] (4)
Now the normalization schemes for normalizing the governing equations which we have used are-

$$x \rightarrow x/x_p, t \rightarrow t/t_p, \phi \rightarrow e\phi/2k_B T_F,$$

where,

$$\omega_p = \sqrt{\frac{4\pi n_e e^2}{m_e}}$$ is the electron plasma wave frequency

$$V_F = \sqrt{\frac{2k_B T_F}{m_e}}$$ is Fermi speed

$$H = \frac{\hbar\omega_p}{2k_B T_F}$$

Using the normalization consideration, equation (2) to equation (6) can be rewritten as-

$$\frac{\partial n_e}{\partial t} + \frac{\partial (n_e u_e)}{\partial x} = 0$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i u_i)}{\partial x} = 0$$

$$\left( \frac{\partial}{\partial t} + u_i \frac{\partial}{\partial x} \right) u_i = -\mu \frac{\partial \phi}{\partial x} + \eta \frac{\partial^2 u_i}{\partial x^2}$$

$$0 = \frac{\partial \phi}{\partial x} - n_e \frac{\partial n_e}{\partial x} + \frac{H^2}{2} \frac{\partial}{\partial x} \left[ \frac{1}{\sqrt{n_e}} \frac{\partial^2 \sqrt{n_e}}{\partial x^2} \right]$$

$$\frac{\partial^2 \phi}{\partial x^2} = (n_c - n_i)$$

Here,

$$\mu = \frac{m_e}{m_i}$$

$$\eta = \frac{\eta V_F^2}{\omega_p}$$

$$H = \frac{\hbar\omega_p}{2k_B T_F}$$

H is a quantum parameter which is dimensionless and proportional to quantum diffraction. $\hbar\omega_p$ is the energy of initial oscillation of plasma waves and $k_B T_F$ is the Fermi energy.

III. Analytical Studies

In order to study linear properties of plasma waves we use the reductive perturbation expansion for the field quantities $n_e$, $n_i$, $u_e$, $u_i$ and $\phi$ about equilibrium values listed below-

$$\begin{bmatrix} n_j \\ u_j \\ \phi \end{bmatrix} = \begin{bmatrix} 1 \\ u_0 \\ \phi_0 \end{bmatrix} + \varepsilon \begin{bmatrix} n_{(1)} \\ u_{(1)} \\ \phi_{(1)} \end{bmatrix} + \varepsilon^2 \begin{bmatrix} n_{(2)} \\ u_{(2)} \\ \phi_{(2)} \end{bmatrix} + \cdots \quad (12)$$

Now substituting the expansion (12) in equations (7) to (11), and then linearizing and assuming all these field quantities changes like $e^{i(kx - \omega t)}$, we get $k$ which is wave number and $\omega$ which is normalized wave frequency. So, the derived dispersion relation is-

$$\omega = ku_0 \pm k \sqrt{\frac{\mu (4 + H^2 k^2)}{k^2 (4 + H^2 k^2) + 4}} \quad (13)$$

Where, $\mu = \frac{m_e}{m_i}$, $H = \frac{\hbar\omega_p}{2k_B T_F}$

The equation (13) represents the dispersion relation of electrostatic waves in Fermi plasma. This quadratic equation has two solutions-

$$\omega = ku_0 \pm k \sqrt{\frac{\mu (4 + H^2 k^2)}{k^2 (4 + H^2 k^2) + 4}}$$

$$\omega = ku_0 - k \sqrt{\frac{\mu (4 + H^2 k^2)}{k^2 (4 + H^2 k^2) + 4}}$$

During the calculation of dispersion relation, we have assumed the particles to be free from viscosity.

IV. Results and Discussions

The linear dispersion relation of electrostatic wave in Fermi plasma with the help of one dimensional QHD model and standard reductive perturbation method has been investigated. Keeping all the parameters within their range, we have plotted the dispersion curve for different quantum diffraction parameters (H) and different streaming velocities $u_0$. And we have got similar type of two dispersion curves that have been plotted below.

The dispersion curve corresponds to electrostatic waves, we have seen in both types of graphs keeping one parameter constant and varying the other parameter, the curves show the upward trend (We have plotted the dispersion curve with positive $\omega$ vs $k$).

In fig (1) and (2), keeping the streaming velocity $u_0$ constant and increasing the values of H we have plotted the graphs. An increase in H shows non-linearity in the graph. And after some region, $\omega$ attains maximum value. In fig (3) and (4), keeping H constant and increasing the values of $u_0$ we have seen the shift towards the upper direction in the wave frequency vs wave number graph.
V. CONCLUSIONS

The Fermi-plasma consists of non-relativistic electrons and ions. The dependency of wave frequency on Quantum diffraction parameter and streaming velocity are studied thoroughly. It is shown that these two components $H$ and $u_0$ have an important role in determining the linear properties of electrostatic waves. At high wave-number region, the dispersion curve with constant $H$ starts to split into different lines for different streaming velocities $u_0$ because at the high wave-number region the energy carrying capacity for different $u_0$ is different. In the graph of dispersion, with increasing $H$, the quantum effect is increasing and so the non-linearity is increasing as well. At high wave-number range, the frequency becomes high, and for this reason, energy becomes high. And because of the high energy, the curve becomes observable in the classical range. That’s why it is classically linear everywhere. At a higher range of wave-number, $\omega$ attains a maximum value. The newly gotten results will be useful for understanding the dispersion properties and obtaining the group velocities and phase velocities of the waves. And it will be also helpful for studying the instabilities of the waves in Fermi plasma.

REFERENCES

[1] F. F. Chen et al., Introduction to plasma physics and controlled fusion. Springer, 1984, vol. 1.
[2] S. Pramanik, S. Mandal, and S. Chandra, “International journal of engineering sciences & management,” Int. J. of Engg. Sci. & Mgmt.(IJESM), vol. 5, no. 2, pp. 89–97, 2015.
[3] J. Goswami, S. Chandra, J. Sarkar, S. Chaudhuri, and B. Ghosh, “Collision-less shocks and solitons in dense laser-produced fermi plasma,” Laser and Particle Beams, vol. 38, no. 1, pp. 25–38, 2020.
[4] F. Haas, L. Garcia, J. Goedert, and G. Manfredi, “Quantum ion-acoustic waves,” Physics of Plasmas, vol. 10, no. 10, pp. 3858–3866, 2003.
[5] S. Ali and P. K. Shukla, “Dust acoustic solitary waves in a quantum plasma,” Physics of plasmas, vol. 13, no. 2, p. 022313, 2006.
[6] W. Manfredi, “Fields institute communication!” Geometry and Topology of Manifolds, vol. 47, p. 264, 2005.
[7] S. Chandra, S. N. Paul, and B. Ghosh, “Electron-acoustic solitary waves in a relativistically degenerate quantum plasma with two-temperature electrons,” Astrophysics and Space Science, vol. 343, no. 1, pp. 213–219, 2013.
[8] S. Pramanick, A. Dey, and S. Chandra, “Electron-acoustic solitary waves in fermi plasma with two-temperature electrons,” Available at SSRN 3711910, 2020.
[9] T. Ghosh, S. Pramanick, S. Sarkar, A. Dey, and S. Chandra, “Dynamical properties and effects of quantum diffraction on the propagation of ea-solitary waves in three-component fermi plasma,” arXiv preprint arXiv:2012.13616, 2020.