Numerical analysis of biaxial elasto-plastic buckling of perforated rectangular steel plates applying the Constructal Design method

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Abstract. Thin plates are constantly used in many applications of the structural designs, like naval, aerospace, civil and offshore platforms. When the supported loading is compressive along the plane of plate, it can suffer from an instability phenomenon called buckling which can be elastic or elasto-plastic. In addition, many times the plated structures need to be designed with cutouts for the most varied reasons: access, maintenance, crossing pipes and reducing of weight, for example. The presence of holes affects the mechanical behavior of the plate and it must be investigated. This work used Finite Element Method associated to Constructal Design method and Exhaustive Search technique to find the optimal geometries, i.e. the ones that maximize the Normalized Ultimate Stress, for simply-supported rectangular steel plates under biaxial compressive loading. It was analyzed the size and geometry of the centered elliptical hole. It was observed that the presence of cutout reduces the mechanical strength of the plate and also, the same occurs when the hole size is increased. Constructal Design method has been proved as an effective method to analyze buckling problems conducting to the best geometry for each studied case. For the simulated cases it is evident the relevance of geometric evaluation of elliptical perforations, once the simply variation of geometry can provide differences until around 42% when comparing the optimum and the worst geometries, for the same hole size.

1. Introduction

Thin steel plates are structural components widely used in naval, aerospace, civil and automotive engineering. These elements are characterized by two dimensions on the plane (a and b) much bigger than the out-of-plane dimension, its thickness (t). For specific practical applications these plates must present a cutout insertion (passing pipes, inspection, weight reduction, etc.) modifying its mechanical behavior, due the stress redistribution caused by the presence of perforation [1].

It is well known that an instability phenomenon, called buckling, can occur when plates are under in plane compressive loading. However, plates do not enter in collapse soon after the elastic buckling occurrence, but it can support higher loads without deforming excessively. The load that defines the plate collapse, considering elasto-plastic behavior, is called post-critical or ultimate load ($P_u$) [2]. When the perforation is centralized to the plate, the buckling load is higher if compared with other
hole’s positions. Also, the post-buckling analysis shows that for rectangular plates with centralized perforation and biaxial compressive load the buckling resistance is less than a half of the resistance for the uniaxial loading [3]. It is important to mention that elasto-plastic buckling analysis of perforated plates is not a simple task, being the computational modeling via Finite Element Method (FEM) a consistent approach to solve these problems with accuracy.

Given the above, this work seeks to optimize the geometry of a centralized elliptical hole applied to a simply-supported rectangular steel plate subjected to biaxial elasto-plastic buckling. To do so, FEM is used simultaneously with Constructal Design method (CD) and Exhaustive Search technique (ES).

Using degrees of freedom, restrictions and a performance parameter, the CD is applied to define the search space, i.e., to define the different geometric configurations for an engineering system. Based on Constructal Law, this method is the philosophy of evolutionary design in engineering [4, 5]. Thus, all geometries are numerically simulated by FEM and the results compared among each other, allowing the definition of the optimized configuration by means the ES.

2. Buckling of Plates
Considering flat plates, the failure caused by buckling can occurs when it is subjected to compressive loads but, different of columns behavior, for plates it can happens over the critical load of elastic buckling [6-8]. When compressive loads are applied biaxially, the plate buckles with a single half-wave on $x$ and $y$-direction, being the buckling elastic for slender plates and elasto-plastic for sturdy plates, when the phenomenon occurs on plastic range [9].

The existence of holes in the plate redistribute de membrane stresses, what causes changes in the plate’s buckling characteristics and reduce its strength [9]. This redistribution, which is not considered in the theory of small displacements of elastic buckling, usually favors the less rigid regions of the plate and causes an increase in plate efficiency [2].

However, exact and/or experimental analysis of such structural components are complex; especially if a geometrical evaluation is intended to be performed. For this reason, the FEM has been widely used to obtain the ultimate buckling load of plates. To do so, [10] developed design equations to determine the ultimate load for biaxially compressed perforated square plates, through the FEM application. In addition, concluded that the ultimate load capacity is affected significantly by the hole size and the plate slenderness ratio.

It is important to highlight that in the present work the FEM and CD will be applied for the buckling analysis. The use of FEM and CD to evaluate the mechanical behavior of perforated thin plates, subjected to uniaxial elastic and elasto-plastic buckling, was performed by [5] and [11] analyzing different geometries and volume fractions ($\phi$) of hole. It was concluded that the redistribution of stresses, caused by the hole, does not affect only the resistance but also the buckling characteristics.

3. Computational Modeling
The computational modeling of the present work used the commercial software ANSYS®, which is based on FEM. The finite element adopted for thin plates analysis is SHELL281, having eight nodes with six degrees of freedom at each node, being: three rotations around $x$, $y$ and $z$-axes and three translations in $x$, $y$ and $z$-axes [12].

The computational domain for the numerical simulations is depicted in figure 1, showing the reference plate (with no hole) and the perforated plate, both simply supported and submitted to biaxial compressive loading.
Starting from the first elastic buckling mode configuration an initial imperfection is assumed, being the relation \( b/2000 \) the maximum value to be assumed. So, the ultimate load of the plate can be found using as reference the load \( P_y = \sigma_y t \), where \( \sigma_y \) represents the material yielding strength [13]. A more detailed explanation about the elastic and elasto-plastic buckling computational modeling can be found in [5].

A computational model validation was made for a simply-supported square plate, with centered circular hole, previously studied by [14]. Regarding figure 1b, a plate with \( a = b = 125 \text{ mm} \), \( t = 1.625 \text{ mm} \), circular hole with diameter \( d = 25 \text{ mm} \) and made by steel with \( \sigma_y = 323.3 \text{ MPa} \), the experiment resulted in the ultimate stress \( \sigma_u = 73.8 \text{ MPa} \) while the numerical solution resulted in \( \sigma_u = 77.6 \text{ MPa} \). An error of 5.13% was observed comparing results, indicating that the proposed computational model was validated.

4. Constructal Design

The CD is based on Constructal Law, which corresponds to the basis of Constructal Theory and say that “for a finite-size flow system to persist in time (to live), its configuration must evolve in such a way that provides greater and greater access to the currents that flow through it” [15-19]. According to [20], the Constructal Law can be understood as a unifying principle of design.

Regarding the CD application, according to [4], there is no “best” in evolutionary system, but there is “better” today, which turns out to be not as good tomorrow. Design with Constructal Theory is not a mathematical optimization method; then, if the studied system has many degrees of freedom, the CD method can be applied associated with some optimization method, such as the ES technique.

Considering the geometrical evaluation through the CD application, it is necessary to define at least one performance parameter (to be improved), the degrees of freedom (variables), and the geometrical restrictions (fixed parameters). The degrees of freedom are free to vary, since respecting the imposed constraints [5].

In this study the performance parameter is the Normalized Ultimate Stress (NUS), which must be maximized. The NUS factor is obtained through the ratio between the ultimate stress of perforated plate (\( \sigma_u \)) and the ultimate stress (\( \sigma_{ur} \)) of reference plate (without cutout). In its turn, the adopted degree of freedom is the ratio \( b_0/a_0 \) (see figure 1b). As restrictions, the dimensions \( a \), \( b \), and \( t \) were kept constant in all proposed geometric configurations. In addition, the volume fraction \( \phi \) (defined as the ratio between perforation volume and plate volume) assumed five different values which were kept constant during the cutout’s geometries changing.
It is interesting to mention that the CD have been largely employed in Heat Transfer and/or Fluid Mechanics applications, for instance in [21-23]; however one can find only some few works addressed to the application of CD in Structural Engineering, as proposed in the present study.

5. Results and Discussion

First of all, the reference plate (with no hole) was numerically simulated, being a simply-supported rectangular plate with $a = 2000$ mm, $b = 1000$ mm and $t = 12$ mm. The plate submitted to biaxial compressive loading on $x$ and $y$-direction, is made by AH-36 steel with $\sigma_y = 355$ MPa, Young’s modulus $E = 210$ GPa and Poisson’s ratio $\nu = 0.30$. The ultimate stress obtained through this analysis of $\sigma_{ur} = 596.40$ MPa was used to calculate the NUS factor.

In sequence, for the perforated plates, the elliptical hole insertion may respect the volume fractions $\phi = 0.025; 0.05; 0.10; 0.15$ and $0.20$. For each $\phi$, several $b_0/a_0$ ratios are tested. In order to evaluate the mechanical behavior of the perforated plates, the ultimate stresses $\sigma_u$ numerically obtained of each proposed geometric configuration was related with $\sigma_{ur}$, generating the NUS factor. All geometric configurations were numerically simulated with a converged SHELL281 mesh of 1298 quadrilateral finite elements of 50 mm side, and refined at line around the perforation.

The obtained results for the NUS factor of each studied perforated plate are presented in figure 2.

![Figure 2 Obtained NUS for each proposed geometric configuration.](image)

From figure 2 it is possible to observe that, for every simulated $\phi$, one maximized NUS is obtained indicating the optimum geometry, i.e., the geometric configuration that provides the higher value to the performance parameter. For $\phi = 0.025$ the higher NUS is 0.93, obtained for $b_0/a_0 = 0.707$. When $\phi = 0.05$ the higher NUS achieved is 0.86, for $b_0/a_0 = 0.509$. In these both cases the optimized geometric configuration is defined by an intermediate value of $b_0/a_0$. This fact was not repeated for $\phi = 0.10; 0.15$ and $0.20$, since in these cases the maximized NUS was achieved by geometries with the lower value of $b_0/a_0$, being, respectively: NUS = 0.75; 0.68 and 0.61; for $b_0/a_0 = 0.177, 0.170$ and $0.199$. From figure 2 one can also infer that as $\phi$ increases, the magnitude of NUS factor decreases; being an expected trend. Besides, in a general way, the increase of $b_0/a_0$ ratio promotes a reduction in the NUS value. Finally, for all perforated plates the obtained NUS is smaller than the reference plate, being a coherent behavior as previously noticed by [9].

To understand how the CD defines the optimized geometries, the von Mises stresses distributions are showed in figure 3, for some $b_0/a_0$ and $\phi = 0.20$. 


Figure. 3 Distribution of von Mises stresses being (a) the optimum geometry \( (b_0/a_0)_{opt} = 0.199 \) and maximized \( NUS_m = 0.61 \) and the geometries with inferior performance (b) \( b_0/a_0 = 0.629; NUS = 0.52 \) and (c) \( b_0/a_0 = 1.039; NUS = 0.43 \).

To analyze the von Mises stresses distribution (figure 3), the Principle of Optimal Distribution of Imperfections [19] was applied. In this work, the imperfections are the maximum stresses concentrations and the optimum geometry is the one that provides the best distribution of these imperfections. It is possible to notice that the optimized geometry (figure 3a) has more regions presented in red color, what represents the steel limit stress. In other hand, the other geometries besides having less regions in red color, also, present more areas in green and blue colors that represent the lower stress magnitudes. To the other \( \phi \), it is worth to highlight that the same trend was noted. Other works did notice analogous behaviors regarding the von Mises stress distribution when analyzing the elasto-plastic buckling of perforated plates [5,24] and stiffened plates [25].

Considering the figure 3, the optimum geometry \( (b_0/a_0)_{opt} = 0.199 \) (figure 3a) conducted to the maximized performance parameter \( NUS_m = 0.61 \), while the worst geometry \( b_0/a_0 = 1.039 \) (figure 3c) conducted to \( NUS = 0.43 \). It represents an improvement of 41.86% between the optimum configuration and the worst one for \( \phi = 0.20 \). Using the same comparison to the other \( \phi \), the obtained improvements are 10.71%, 17.81%, 13.64% and 30.77% for \( \phi = 0.025; 0.05; 0.10 \) and 0.15, respectively.

6. Conclusion
The obtained results showed that the perforation’s geometric evaluation is an important analysis to be done once it can vary the plate’s mechanical behavior under elasto-plastic buckling. Through the Principle of Optimal Distribution of Imperfections, the CD method, associated to ES technique and FEM, proved to be an effective tool for biaxial elasto-plastic buckling analysis once it has conducted to the optimum geometry which provided the highest performance parameter.

Furthermore, it is important to mention that was possible to obtain the limit stresses for each perforation geometry and volume fraction, from which might cause the collapse of plate submitted to biaxial elasto-plastic buckling. For all simulated cases the relevance of geometric evaluation of the perforations is evident, since the simply variation of geometric configuration can provide improvements of 10.71%, 17.81%, 13.64%, 30.77% and 41.86% for \( \phi = 0.025; 0.05; 0.10 \) and 0.15 and 0.20, respectively.

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References
[1] Shojaee T, Mohammadi B, Madoliat R and Salimi-Majd D 2019 Development of a finite strip method for efficient prediction of buckling and post-buckling in composite laminates containing a cutout with/without stiffener Composite Structures 210 pp 538–552
[2] Chajes A 1974 Principles of Structural Theory (New Jersey: Prentice-Hall Englewood Cliffs)
[3] Mohammadzadeh B, Choi E and Kim W J 2018 Comprehensive investigation of buckling behavior
of plates considering effects of holes Structural Engineering and Mechanics 68(2) pp 261–275

[4] Rocha L A O, Lorente S and Bejan A 2017 Constructal Theory in Heat Transfer. Handbook of Thermal Science and Engineering ed 1 1 pp 1–32.

[5] Helbig D, Silva C C C D, Real M D V, Dos Santos E D, Isoldi L A and Rocha L A O 2016 Study About Buckling Phenomenon in Perforated Thin Steel Plates Employing Computational Modeling and Constructal Design Method Latin American Journal of Solids and Structures 13(10) pp 1912–1936.

[6] Jones R M 2006 Buckling of bars, plates and shells (Blacksburg: Bull Ridge Publishing)

[7] Szilard R 2004 Theories and Applications of Plate Analysis: Classical Numerical and Engineering Methods (New Jersey: John Wiley & Sons)

[8] Trahair N S and Bradford M A 1988 The behavior and design of steel structures (New York: Springer)

[9] El-Sawy K M and Martini M I 2010 Stability of biaxially loaded square plates with single central holes Ship and Offshore Structures 5(4) pp 283–293.

[10] Shanmugam N E, Thevndran V and Tan Y H 1999 Design formula for axially compressed perforated plates Thin-Walled Structures 34 pp 1–20

[11] Helbig D, Cunha M L, Da Silva C C C, Dos Santos E D, Iturrioz I, Real M De V, Isoldi L A and Rocha L A O 2018 Numerical study of the elasto-plastic buckling in perforated thin steel plates using the constructal design method Research on Engineering and Structural Materials 4(3) pp 169–187.

[12] Ansys Mechanical APDL 2015 Version 15.0 – User’s Guide (USA: ANSYS Inc)

[13] El-Sawy K M, Nazmy A S and Martini M I 2004 Elasto-plastic buckling of perforated plates under uniaxial compression Thin-Walled Structures 42(8) pp 1083–1101

[14] Narayanan R and Chow F 1984 Strength of Biaxially Compressed Perforated Plates Int. Specialty Conf. on Cold-Formed Steel Structures pp 55–73

[15] Bejan A 2000 Shape and Structure, from Engineering to Nature (Cambridge: Cambridge University Press)

[16] Bejan A and Lorente S 2004 The constructal laws and the thermodynamics of flow systems with configuration. Int. J. Heat Mass Transfer 47 pp 3073–3083.

[17] Bejan A and Lorente S 2005 La Loi Constructale (Paris: L’Harmattan)

[18] Bejan A and Lorente S 2008 Design with Constructal Theory (New Jersey: Hoboken).

[19] Bejan A and Zane J P 2012 Design in nature: how the constructal law governs evolution in biology, physics, technology and social organization (New York: Doubleday).

[20] Rocha L A O, Lorente S and Bejan A 2013 Constructal Law and the unifying principle of design (New York: Springer)

[21] Hazarika S A, Bhanja D and Nath S, 2020 Fork-shaped Constructal fin array Design a better alternative for heat and mass transfer augmentation under dry, partially wet and fully wet conditions International Journal of Thermal Sciences 152, pp 106329.

[22] Ganjehkaviri A and Mohd Jaafar M N 2019 Multi-objective particle swarm optimization of flat plate solar collector using Constructal Theory Energy 194 pp 116846.

[23] Hermany L, Lorenzini G, Klein R J, Zinani F F, Dos Santos E D, Isoldi L A and Rocha L A O 2018 Constructual Design applied to elliptic tubes in convective heat transfer cross-flow of viscoplastic fluids International Journal of Heat and Mass Transfer 116 pp 1054–1063.

[24] Lorenzini G, Helbig D, Da Silva C C C, Real M V, Dos Santos E D, Isoldi L A and Rocha L A O 2016 Numerical evaluation of the effect of type and shape of perforations on the buckling of thin steel plates by means of the Constructal Design method International Journal of Heat and Technology 34 pp S29–S20.

[25] Lima J P, Cunha M L, Dos Santos E D, Rocha L A O, Real M V and Isoldi L A 2020 Constructual Design for the ultimate buckling stress improvement of stiffened plates submitted to uniaxial compressive load Engineering Structures 203.