On the corona of magnetars

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ABSTRACT

Slow dissipation of non-potential magnetic fields in the magnetosphere of the magnetar is assumed to accelerate particles to hundreds MeV along the magnetic field lines. We consider interaction of fast particles with the surface of the magnetar. We argue that the collisionless dissipation does not work in the atmosphere of the neutron star because the two-stream instability is stabilized by the inhomogeneity of the atmosphere. Rather, the dominant dissipation mechanism is collisional Landau level excitations followed by pair production via the deexcitation gamma-rays ultimately leading to electrons with the energy below the Landau energy. We show that, because of the effects of the superstrong magnetic field, these electrons could emerge from the surface carrying most of the original energy so that a hot corona arises with the temperature of $1 \div 2$ MeV. This extended corona is better suited than a thin atmosphere to convert most of the primary beam energy to non-thermal radiation and, as we show, most of the coronal energy release is radiated away in the hard X-ray and the soft gamma-ray bands by Comptonization and bremsstrahlung. The radiation spectrum is a power-law with the photon index $1 < \alpha < 2$. The model may account for the persistent hard X-ray emission discovered recently from the soft gamma-ray repeaters and anomalous X-ray pulsars and predicts that the radiation spectrum is extended into the MeV band.

Subject headings: instabilities – plasmas – stars:magnetic field – stars:neutron

1. Introduction

It is now commonly accepted that Soft Gamma Repeaters (SGRs) and Anomalous X-ray Pulsars (AXPs) are magnetars, neutron stars with extremely high magnetic fields, $B \sim 10^{14} \div 10^{15}$ G (Duncan & Thompson 1992; Paczynski 1992; Thompson & Duncan 1995). Activity of these sources is fed by the energy of this magnetic field (see Woods & Thompson 2004 for a recent review). It was found recently (Kuiper et al. 2004, 2006;
Mereghetti et al. 2005, Molkov et al. 2005) that the persistent pulsed X-ray emission from these sources has a nonthermal spectrum extending up to \( \gtrsim 100 \) keV. The luminosity of this tail, \( L \sim 10^{36} \text{ erg/s} \), exceeds the thermal luminosity from the star’s surface. The spectra are exceptionally hard with photon indices typically in the range \( 1 \div 2 \) so that the luminosity peaks above 100 keV. In this paper, we discuss origin of this emission.

According to Thompson, Lyutikov & Kulkarni (2002), magnetic field in magnetar’s magnetosphere is generally non-potential. Slow untwisting of the magnetic field lines generates the electric field; particles are accelerated in this field until they produce electron-positron pairs filling the magnetosphere and providing charge carriers for the electric current (Beloborodov & Thompson 2007). Pairs are formed from gamma-photons produced by cyclotron scattering of the surface radiation of the neutron star on primary particles. In magnetar’s magnetic field, the energy of the first Landau level is high,

\[
\varepsilon_B = m_e c^2 \sqrt{1 + 2B/B_q} \approx 3.5 \sqrt{B_{15}} \text{MeV};
\]

so that the scattered photon is immediately converted into an electron-positron pair. Here \( B = 10^{15}B_{15} \) G is the magnetic field of the star, \( B_q = m_e^2 c^3/\hbar e = 4.4 \times 10^{13} \) G. If the surface emits radiation with the characteristic photon energy \( \varepsilon = 10\varepsilon_{10} \) keV, the cyclotron scattering occurs when the Lorentz factor of the electron reaches the value

\[
\gamma = \frac{B}{B_q} \frac{m_e c^2}{\varepsilon} = 10^{3}\frac{B_{15}}{\varepsilon_{10}}.
\]

Note that the recoil effect should be taken into account in this estimate (Thompson & Beloborodov 2005). Eventually the energy is deposited in pairs with the Lorentz factor something below (2) such that they already could not scatter the surface photons resonantly. Nonresonant scattering is strongly suppressed (Beloborodov & Thompson 2007) therefore these pairs freely flow along the magnetic field lines until they hit the surface of the neutron star where the energy is released. The observed hard X-ray persistent emission definitely comes from an optically thin, tenuous plasma; this suggests that the energy is released at a small depth, \( \lesssim 1 \text{ g/cm}^2 \).

It was assumed by Thompson & Beloborodov (2005) and by Beloborodov & Thompson (2007) that the two-stream instability develops in a very narrow upper layer of the atmosphere so that collisionless processes are responsible for deceleration of the beam and plasma heating. The observed hard radiation was attributed to bremsstrahlung from this atmosphere. However, the two-stream instability is quenched when the beam is spread into a plateau-like distribution so that only one half of the initial beam energy is released in the optically thin domain. The beam is in any case stopped finally at a larger depth by collisions with the background particles. So only one half of the beam energy could be radiated in
the hard band. Of this, one half is directed towards the star where it is mostly absorbed and reradiated back in the soft band. One concludes that collisionless dissipation in the atmosphere converts only 1/4 of the total energy to the hard radiation and could not account for the fact that most of the magnetars luminosity in the quiescent state is observed at $\varepsilon \gtrsim 100$ keV. Moreover, as demonstrated below, the inhomogeneity of the neutron star atmosphere stabilizes the two-stream instability so that collisionless heating of the atmosphere is quenched.

An alternative explanation is that the nonthermal tail is formed via resonant scattering of the thermal surface radiation at high altitudes, about 10 stellar radii, where the Landau energy is already nonrelativistic. This would mean that the persistent emission of the magnetar is associated only with a small fraction of the magnetic field lines (those rising into the resonant region) whereas the currents flowing along the most of magnetic field lines and carrying most of the energy do not show up. Moreover, careful calculations show that the resonant scattering cannot reproduce the observed rising energy spectra of the persistent emission (Fernández & Thompson 2007).

In this paper, we reanalyze interaction of the fast particle beam with the surface layers of the neutron star and show that, because of the extreme physics introduced by the ultrastrong magnetic field, a hot atmosphere could be formed via collisional processes and these difficulties avoided. It will be demonstrated below that even though these electrons appear at a relatively large depth, they could escape upwards forming a hot layer with the temperature $T \approx \varepsilon_B/2 \approx 1 \div 2$ MeV at the top of the cold atmosphere. The reason is that the Coulomb cross section sharply decreases when the electron energy becomes less than $\varepsilon_B$ and the electron is restricted to move only along the magnetic field like a bead on a wire. These electrons could carry away a significant fraction of their initial energy if the atmosphere is fully ionized so that there are no ionization losses. We show that helium atmosphere satisfies these conditions. Such an atmosphere is expected in view of spallation of heavy nuclei bombarded by high-energy electrons and positrons.

Bremsstrahlung radiation from this hot layer is hard enough to populate the magnetosphere by electron-positron pairs. This pair corona is also heated via collisionless relaxation of the primary beam because the corona, unlike the atmosphere, is extended enough for the two-stream instability to develop. The observed hard X-rays are radiated via Comptonization in the corona and bremsstrahlung in the hot atmosphere. Our model predicts a hard radiation spectrum extending to the MeV band. In the band $\lesssim 100$ keV, Comptonization forms a power-law spectrum with the photon slope $1 < \alpha < 2$ as is observed. Because the corona is extended, most of radiation is not intercepted by the surface of the neutron star but rather escapes to the infinity. Therefore the observed persistent radiation from magnetars is
dominated by hard emission.

The paper is organized as follows. In sect.2, we analyze the two-stream instability in the neutron star atmosphere and show that the instability is suppressed by the inhomogeneity of the atmosphere. In sect.3, we consider deceleration of the fast particle beam via resonant Coulomb scattering and development of the electron-positron cascade in the atmosphere (Kotov & Kelner 1985, Beloborodov & Thompson 2007); we show that the initial energy of the beam is eventually converted into electrons with the energy something below $\varepsilon_B$ at the depth of roughly 100 g/cm$^2$. In sect.4, we demonstrate that these electrons could escape upwards delivering most of the original energy into a hot layer at the top of the cold atmosphere. In sect.5, we argue that hard radiation from this hot layer populates the magnetosphere with electron-positron pairs and this pair corona emits hard radiation extended into the MeV band. Conclusions are presented in Sect.6. In Appendix A, we show that the energy the electron loses in the atmosphere for recoil of ions is small. In Appendix B, we demonstrate that helium is fully ionized in the atmospheres of magnetars. In Appendix C, we show that Comptonization of soft photons on mildly relativistic electrons in a super-strong magnetic field results in a power-law radiation spectrum with the photon index $\alpha \geq 1$, like in the non-magnetized case.

2. Inefficiency of the collisionless interaction of the beam with the atmosphere

Thompson & Beloborodov (2005) and Beloborodov & Thompson (2007) suggested that the energy of the magnetospheric currents is dissipated when the downward electron-positron beam with the Lorentz factor $\gamma$ is decelerated in a thin surface layer of the star where two-stream instability excites Langmuir turbulence thus providing the necessary relaxation mechanism. In this section, we check conditions for the development of the two-stream instability. One can conveniently express the plasma density in the atmosphere via the Thompson optical depth, $\tau$, as

$$n = \frac{\tau}{\sigma T H};$$

where

$$H = \frac{(1 + Z)T}{Am_p g} = 2.6 \frac{(1 + Z)T_f}{Ag_{14.5}} \text{cm}$$

is the hydrostatic height of the atmosphere, $g = 10^{14.5} g_{14.5} \text{cm/s}^2$ the surface gravity, $A$ and $Z$ the atomic weight and charge, correspondingly. The beam density may be expressed via the total energy release in the magnetosphere, $L = 10^{36} L_{36} \text{ erg/s}$, as

$$n_b = \frac{L}{4\pi R^2 m_e c^3 \gamma} = 4 \times 10^{15} \frac{L_{36}}{\gamma_{3}} \text{ cm}^{-3};$$
where $\gamma_3 = \gamma/10^3$.

The basic regimes of the two-stream instability for a relativistic beam propagating in a non-relativistic plasma were studied by Fainberg, Shapiro & Shevchenko (1970). For a monochromatic beam moving with the Lorentz factor $\gamma$ along the strong magnetic field, the growth rate is written in the hydrodynamic regime,

$$\kappa_{\text{hydr}} = \frac{\sqrt{3}}{2^{4/3}} \left(\frac{n_b}{n}\right)^{1/3} \frac{\omega_p}{\gamma};$$  \hspace{1cm} (6)$$

where

$$\omega_p = \sqrt{\frac{4\pi e^2 n}{m_e}}$$ \hspace{1cm} (7)$$
is the plasma frequency. This formula is valid provided all the particles in the beam are in resonance with the excited wave, $\delta v/c < \kappa/\omega_p$. For the beam with a large energy spread, $\delta \gamma \sim \gamma$, this condition is written as

$$\left(\frac{n_b}{n}\right)^{1/3} \gamma > 1.$$ \hspace{1cm} (8)$$

Making use of Eqs.(3-6), one can write this condition as

$$\gamma > 400 \left(\frac{A \tau g_{14.5}}{(1 + Z)T_7L_{36}}\right)^{1/2}.$$ \hspace{1cm} (9)$$

In the opposed limit, only some fraction of the beam particles are in resonance with the excited wave (kinetic regime). Then the growth rate is

$$\kappa_{\text{kin}} \approx \frac{n_b}{n} \frac{\omega_p}{\gamma^3 \delta v^2} \approx \frac{n_b \gamma}{n} \omega_p.$$ \hspace{1cm} (10)$$

Making use of these estimates, one easily finds that $\kappa H/c$ is very large in both regimes therefore at first glance, strong Langmuir turbulence should be excited. However, careful consideration shows that this is not the case because at $\kappa/\omega_p \ll 1$, the instability is easily stabilized by plasma inhomogeneity (e.g. Breizman & Ryutov 1974; Breizman 1990). Let us consider how inhomogeneity of the neutron star atmosphere affects development of the two-stream instability.

The instability excites the Langmuir waves with the dispersion relation

$$\omega = \omega_p \left(1 + \frac{3kT}{2m_e c^2} \frac{k^2 c^2}{\omega_p^2}\right);$$ \hspace{1cm} (11)$$

where $k$ is the wave number. The dispersion relation shows that the growth rate of the instability is proportional to $\kappa^2$. Therefore, the growth rate is very large in both regimes.
where $T$ is the plasma temperature, $k$ the wave vector. Within the atmosphere with the characteristic height (4), the plasma frequency varies with the depth, $z$, as

$$\frac{\delta \omega_p}{\omega_p} = \frac{\delta z}{2H}. \quad (12)$$

The frequency of the propagating wave remains constant whereas the wave vector varies to satisfy the dispersion equation (11):

$$\delta \omega_p + \frac{3kT}{m_e c} \delta k = 0. \quad (13)$$

Here we take into account that for the resonance wave, $\omega/k \approx \omega_p/k \approx c$. The waves are amplified only if their phase velocity is close to the beam velocity,

$$|\omega - vk| \lesssim \kappa. \quad (14)$$

In the hydrodynamic regime, when the beam velocity spread is less than the width of the resonance, the condition (14) yields $\delta k \lesssim \kappa_{\text{hydr}}/c$. Then one finds that the amplification stops after the wave propagates the distance

$$\delta z = \frac{6kT \kappa_{\text{hydr}}}{m_e c^2 \omega_p} H. \quad (15)$$

A significant fraction of the beam energy could be dissipated provided

$$\frac{\kappa_{\text{hydr}} \delta z}{v_g} > \Lambda; \quad (16)$$

where $\Lambda = 10 \Lambda_1$ is the logarithm of the ratio of the beam energy to the initial energy of the Langmuir oscillation (it is of the order of the Coulomb logarithm) and it is taken into account that the wave propagates with the group velocity

$$v_g = \frac{d\omega}{dk} = \frac{3kT}{m_e c^2}. \quad (17)$$

Now one finds making use of Eqs. (6), (15) and (17)

$$\frac{\kappa_{\text{hydr}} \delta z}{v_g \Lambda} = 1.3 \times 10^{-6}(1 + Z)T_7^{7/6}L_{36}^{2/3}A_{g_{14.5}}\gamma_3^{8/3}T_{1/6}^{1/6} \Lambda_1. \quad (18)$$

So the condition (16) is violated by a large margin.

In the kinetic regime, the condition (14) reduces to $\delta k/k < \delta v/c \sim (c\gamma)^{-1}$, which yields

$$\delta z = \frac{6kT}{m_e c^2 \gamma^2} H. \quad (19)$$
Then one finds
\[
\frac{\kappa_{\text{kin}} \delta z}{v_g A} = 4.3 \times 10^{-4} \frac{1}{\Lambda_1 \gamma^2} \sqrt{(1 + Z) T_7}.
\]
(20)

Again, the condition (16) is violated by a large margin. Therefore the two-stream instability does not develop in the atmosphere of the neutron star and the atmosphere could not be heated via collisionless dissipation.

It will be shown below that a hot atmosphere could arise in the case under consideration but as a result of collisional interaction of the beam with the surface layers of the star. Then the characteristic scale of the density variation significantly increases and moreover, the hot atmosphere emits hard X-rays therefore the Lorentz factor of the beam decreases according to Eq.(2). Substituting in Eq.(18) the characteristic temperature of the hot atmosphere, \( T = 10^{10} \, \text{K} \), and the Lorentz factor \( \gamma = 100 \), one can see that the two-stream instability could develop in the hot atmosphere. Therefore eventually some fraction of the beam energy could be released via collisionless dissipation. However, at least one half of the energy is in any case dissipated and delivered into the hot atmosphere by collisional processes; it is these processes that determine the temperature of the hot atmosphere.

3. Collisional deceleration of the beam

Consider deceleration of a relativistic electron-positron beam within the surface layers of the magnetar. Let a relativistic electron move in plasma along the strong magnetic field. If the electron energy exceeds the Landau energy (1), it efficiently scatters off ions into the first Landau level and then immediately falls back emitting a resonance photon; such a resonance Coulomb scattering is the main mechanism of deceleration of a relativistic electron-positron beam in the case under consideration (Kotov & Kelner 1985, Beloborodov & Thompson 2007). The cross section for the transition of an electron with the Lorentz factor \( \gamma \) and the momentum \( p \) from the ground to the \( j \)-th Landau level by scattering off an ion with the charge \( Ze \) is found by Bussard (1980) and Langer (1981):

\[
\sigma_{j0} = \frac{3}{16} \frac{B_q}{B} \frac{Z^2 \sigma_T}{(1 + \gamma)^2} \frac{1}{j!} \sum_{\pm} \frac{m_e^2 c^2}{|pp_\pm|} \times \left\{ \delta_{s', -1/2} \left[ (1 + \gamma)^2 + \frac{pp_\pm}{m_e^2 c^2} \right]^2 + \delta_{s', 1/2} \frac{2B}{B_q} \frac{p^2}{m_e^2 c^2} \right\} \epsilon_j \left( \frac{B_q (p - p_\pm)^2}{2B m_e^2 c^2} \right);
\]

\[
\epsilon_j(x) = \int_0^{\infty} \frac{t^j dt}{(t + x)^2} e^{-t}.
\]
Here $\sigma_T$ is the Thomson cross section, $s'$ the final spin of electron and summation is over the final electron momentums $p_{\pm} = \pm \sqrt{p^2 - 2m_ec^2jB/B_q}$. Even high energy electrons jump predominantly on the first Landau level therefore one can neglect excitations of higher levels. The cross section for the transition $0 \rightarrow 1$ is plotted in Fig.(1). In the limit $mc^2\gamma \gg \varepsilon_B$, it is reduced to

$$\sigma_{10} = \frac{3B_q}{4B} Z^2\sigma_T \ln \left(0.413 \frac{\gamma^2 B_q}{B}\right). \quad (22)$$

The electron with the Lorentz factor $\gamma > \sqrt{2B/B_q}$ jumps on the first Landau level having the Lorentz factor $\gamma_1 = m_ec^2\gamma/\varepsilon_B = 0.15B_{15}^{-1/2}$. The electron immediately falls back onto the background Landau level emitting a photon with the energy $(0.5 \div 1)\varepsilon_B$ as measured in the guiding center frame of the excited electron (frame moving with the Lorentz factor $\gamma_1$); due to recoil, the energy of the emitted photon depends on the emission angle. After the deexcitation, the Lorentz factor of the electron remains on average the same because in the guiding center frame, photons are emitted forward and backward with equal probability. Thus the electron retains only a fraction $\xi = \gamma_1/\gamma \approx 0.15B_{15}^{-1/2}$ of the total energy, most of the energy being taken away by a photon.

The fate of the photon depends on its polarization. Two polarization modes could propagate in the magnetized vacuum; the so called ordinary mode is polarized in the plane set by the propagation direction and the background magnetic field whereas the extraordinary mode is polarized perpendicularly to this plane. If an ordinary photon is emitted, it is immediately converted into an electron-positron pair provided its energy exceeds the threshold, $\varepsilon > 2m_ec^2/\sin \theta$. In magnetar’s field, this condition is satisfied for most $\theta$. In the frame of the scattered electron, the produced electron and positron have on average equal energies and move in opposite directions. In the laboratory frame, most of the energy is taken by the particle moving forward; only a fraction $1/\gamma_1^2$ of the total energy is taken by the second particle. This means that the energy of electrons in the beam decreases only by a fraction $\xi$ in one scattering. If this were the only process of the beam-plasma interaction, the energy of the particles in the beam would decrease $\exp (r\xi)$ times after $r$ scattering and the total number of scattering before the particle energy becomes less than the Landau energy would be large, $r \approx \xi^{-1} \ln(\gamma/\varepsilon_B) \approx 20 \div 30$. The full length of the avalanche, $l$, could be estimated by summing the free path lengths:

$$l = \sum_r \frac{1}{\sigma_{01}n_i} \approx \frac{4B}{3B_q Z^2\sigma_T n_i} \int_0^r \frac{dn}{\ln(0.4\gamma^2 B_q/B) - 2\xi n} \approx \frac{2B}{3\xi B_q Z^2\sigma_T n_i} \ln \left[ \ln \left(0.4\frac{\gamma^2 B_q}{B}\right) \right].$$
Here \( n_i \) is the number density of ions. For \( \gamma \sim 100 \div 1000 \), the corresponding Thomson depth is

\[
\tau \equiv \sigma_T Z n_i l \sim \frac{2}{Z} \left( \frac{B}{B_q} \right)^{3/2} = 200 Z^{-1} B_{15}^{3/2}.
\] (23)

An extraordinary photon does not produce pairs directly; it first splits into ordinary photons and only then pairs may be produced. Then the avalanche proceeds further. As the energy per particle decreases in this case roughly twice in each step, the avalanche penetrates the depth significantly less than that of Eq. (23) provided each emitted O-photon is converted into a pair. However, the energy of the photon, \( (0.5 \div 1) \varepsilon_B \approx 2 \div 3 \text{ MeV} \), is now distributed between two photons, both or one of them could fall below the threshold for the pair production. Then these photons either are converted into pairs in the Coulomb field of the nucleus or experience a Compton scattering off a background electron.

The cross section for the pair production at a nucleus is

\[
\sigma^{\pm}_Z = \frac{7}{6\pi} Z^2 \alpha \sigma_T \left[ \ln \left( \frac{2\varepsilon}{m_e c^2} \right) - \frac{109}{42} \right];
\] (24)

where \( \alpha \) is the fine structure constant. The Compton scattering of a photon moving along the superstrong magnetic field was studied by Gonthier et al. (2000). In this case, the resonance occurs only at the cyclotron fundamental \( \varepsilon = (B/B_q) m_e c^2 = 12 B_{15} \text{ MeV} \). Above the resonance, the scattering cross section is close to the Klein-Nishina one

\[
\sigma_C = \frac{3}{4} \frac{m_e c^2}{2\varepsilon} \left[ \ln \left( \frac{2\varepsilon}{m_e c^2} \right) + \frac{1}{2} \right].
\] (25)

The pair production at nuclei dominates at energies \( \varepsilon > 250 Z^{-1} m_e c^2 \). Note that this process comes into play only when the photon could not be converted directly into a pair. This occurs predominantly when E-mode resonant photons are emitted because these photons split into O-photons with lesser energies so that these O-photons could fall below the threshold for the direct pair production in the magnetic field. In this case, two pairs are produced in each step therefore the energy per particle decreases roughly four times in each step. This means that pairs are produced at nuclei only in the first few generations; after this, Compton scattering dominates. The corresponding depth varies from \( \tau_Z \sim 200/Z \) at small \( Z \), when the transition energy is high and the Compton scattering comes into play already after two generations, to \( \tau_Z \sim 800/Z \), when \( Z \) is large and three or four generations are necessary.

At the Compton stage, the avalanche proceeds further. The distribution of the scattered photons in their energies, \( \varepsilon', \) is

\[
d\sigma_C = \frac{3}{8} \frac{m_e c^2}{\varepsilon^2} \left[ \frac{\varepsilon}{\varepsilon'} + \frac{\varepsilon'}{\varepsilon} + \left( \frac{m_e c^2}{\varepsilon'} - \frac{m_e c^2}{\varepsilon} \right)^2 - 2 \left( \frac{m_e c^2}{\varepsilon'} - \frac{m_e c^2}{\varepsilon} \right) \right];
\] (26)
where
\[ \frac{\varepsilon}{1 + 2\varepsilon/m_e c^2} \leq \varepsilon' \leq \varepsilon. \]
The scattered photon takes on average a fraction \( 4/[3 \ln(2\varepsilon/m_e c^2)] \approx 0.2 \) of the initial energy. The photon is directed at the angle \( \sim \sqrt{4m_e c^2/\varepsilon} \) to the magnetic field therefore it is immediately converted into a pair. The rest 0.8 fraction of the energy is taken by the recoil electron, which emits a resonant photon via Coulomb scattering off an ion. If this photon is in O-mode, it is immediately converted into a pair; if it is in E-mode, it splits producing two photons, the energy of each of them being roughly \( (1/2) \cdot 0.8(1 - \xi) \approx 1/3 \) of the initial energy. These two photons either produce pairs directly or, if they are below the threshold for the direct pair production, experience Compton scatterings and the process repeats again. The Compton scattering comes into play predominantly if an E-mode photon was emitted because a resonant O-photon is converted into a pair immediately whereas the E-photon splits into two O-photons, which could fall below the threshold of the direct pair production. Therefore in the Compton channel, the energy per particle decreases by the factor of 3 in each generation. Then the energy per particle becomes less than \( \varepsilon_B \) in a few steps and the avalanche stops. As the Compton cross section grows rapidly with decreasing of the photon energy and cyclotron resonances only increase the cross section, the full length of the avalanche is determined by the free path of a photon with the initial energy \( \varepsilon = 250Z^{-1}m_e c^2 \). The corresponding depth varies from \( \tau_C \sim 100 \) at \( Z = 1 \) to \( \tau_C \sim 10 \) at \( Z = 26 \). We will argue in the next section that for our model to be self-consistent, the avalanche should develop in the medium composed from light elements. Spallation of nuclei by relativistic electrons and positrons from the avalanche seems to result in formation of a helium atmosphere; below we will consider only this case. Then the total depth the avalanche penetrates is estimated as \( \tau = \tau_Z + \tau_C \sim 300/Z \).

In real avalanches, ordinary and extraordinary photons are emitted alternately with approximately the same probability. As the Coulomb cross section \( (21) \) is significantly larger than \( (25) \) and \( (24) \), the longest parts of the chain is associated with emission of extraordinary photon with subsequent photon splitting. On the other hand, the total number of generations is also determined by emission of extraordinary photons because the energy per electron decreases significantly when the photon splits. Therefore the overall length of the avalanche is roughly the same as if only extraordinary photons were emitted. So finally the energy of the primary beam is converted into electron-positron pairs with the energy \( \sim \varepsilon_B/2 \sim 1/2 \) MeV at the depth of
\[ \tau_0 = \sigma_T Z n_i l \sim 300/Z. \]
Fig. 1.— Cross-section of the Coulomb excitation of the first Landau level; asymptotics is shown by dashed lines.

Fig. 2.— Coulomb cross-section for the electron on the ground Landau level, $B = 10^{15}$ G.
4. Formation of a hot atmosphere

Now let us consider the fate of the newly formed pairs with the energy less than $\varepsilon_B$. Their motion is purely one-dimensional and conservation of the energy and momentum implies that two colliding particles of equal mass may only exchange their energy and momenta. Therefore electron-electron collisions do not change the state of the system and may be neglected. Collision of a "hot" positron with a "cold" background electron results in a "cold" positron and "hot" electron. Collision frequency of cold particles is high therefore the cold positron does not escape but rather eventually annihilates with some background electron. However, the hot electron takes most of the energy in this case therefore most of the beam energy is eventually stored in electrons with the characteristic energy $\sim 1 \div 2$ MeV. They diffuse within the medium colliding with the background ions.

Collision of a one-dimensional electron with the ion may result either to the forward scattering, after which the electron energy remains the same to within a small recoil, or to reflection from the ion. The cross section for the Coulomb reflection is found from Eq. (21) as

$$\sigma_{00} = \frac{3}{4} Z^2 \sigma_T \frac{B_q}{B} \left( \frac{m_e c^2}{p} \right)^2 \epsilon_0 \left( \frac{2 B q p^2}{B m_e^2 c^2} \right) ; \quad (28)$$

this cross section is plotted in Fig. 2. For $p = 0.5 \varepsilon_B / c = \sqrt{B/2B_q m_e c}$ one gets

$$\sigma_{00} = 0.6 Z^2 \sigma_T (B_q/B)^2 = 0.001 Z^2 \sigma_T B_{15}^{-2}. \quad (29)$$

It was shown in the previous section that the avalanche penetrates the Thomson depth $< 27$ producing electrons with the energy $\sim \varepsilon_B / 2$. One sees that for these electrons, $\sigma_{00} n_i l = 0.3 B_{15}^{-2} \lesssim 1$ so they could easily escape upwards taking away most of the energy of the avalanche. The cross-section of the forward scattering is larger than (29) but because recoil is small, one can neglect this process (see Appendix A).

The above consideration assumes that the escaping electron does not lose energy on ionization so that the plasma in the atmosphere is fully ionized. It is shown in the Appendix B that under the condition of interest, helium is fully ionized. One can assume that the layer the avalanche penetrates composed mostly of helium because the avalanche electrons and gamma-quanta destroy heavy nuclei (Beloborodov & Thompson 2007). The helium is presumably preferable to hydrogen because does not require transformation of neutrons into protons.

One should check how much energy the electrons lose on bremsstrahlung before they escape. In the non-magnetized medium, a relativistic electron loses energy on bremsstrahlung
after passing the depth, in Thomson units,

\[
\tau_{0}^{br} = \frac{2\pi}{3\alpha Z(\ln 2\gamma - 1/3)}.
\]  

(30)

Unfortunately bremsstrahlung in the superstrong magnetic field has not been calculated yet however simple quasiclassical estimate shows that the rate of the energy losses decreases in this case.

In classical electrodynamics, the energy radiated by a relativistic electron may be written as (e.g. Landau & Lifshitz 1995)

\[
\delta E = \frac{2e^2}{3c^5} \int_{-\infty}^{\infty} \gamma^6[(\mathbf{v} \cdot \mathbf{w})^2 + c^2w^2/\gamma^2]dt;
\]

(31)

where \( \mathbf{w} \) is the electron acceleration. From the equation of motion

\[
m_e \frac{d\gamma \mathbf{v}}{dt} = eE
\]

one finds \( w_\parallel = eE_\parallel/m_e c\gamma^3 \) and \( w_\perp = eE_\perp/m_e c\gamma \) where the subscripts \( \parallel \) and \( \perp \) refer to the projections onto the direction of velocity and onto the perpendicular direction. In the non-magnetized case, both components of the acceleration are presented however the first term in the square brackets in Eq.(31) is \( \gamma^2 \) less than the second one and may be neglected.

For the electron passing the nucleus at the distance \( r \) one gets assuming that the motion is straightforward

\[
\delta E = \frac{\pi Z^2 e^4 \gamma^2}{12\gamma^3 m_e^2 c^4}.
\]

(32)

Integrating the obtained relation over \( r \) from \( r_{mn} = h\gamma/m_e c \) defined from the condition that the energy of the emitted quanta becomes equal to the electron energy, one gets Eq.(30) to within a numerical factor and the logarithmic term.

In the superstrong magnetic field, the electrons move only along the field therefore only the first term in the square brackets in Eq.(31) remains. Simple calculation shows that in this case \( \delta E \) is \( 3\gamma^2 \) less than that of Eq.(32). One can expect that the full bremsstrahlung cross section in the superstrong magnetic field decreases by the same factor. Of course quantum treatment of the process is necessary to rigorously justify this conclusion, but assuming that the above consideration is basically correct, the electron in the superstrong magnetic field loses the energy on bremsstrahlung after passing the depth \( \tau_{0}^{br} = 3\gamma^2 \tau_{0}^{br} \). For electrons with the energy \( \sim \varepsilon_B/2 \), one gets \( \tau_{0}^{br} = 6000Z^{-1}B_{15} \), which is significantly larger than the depth \( (27) \) the avalanche penetrates. This means that one can neglect bremsstrahlung energy losses.
So hot electrons freely escape upwards taking away most of the beam energy. Charge neutrality implies that the necessary amount of background ions rises together with the electrons so that a hot atmosphere with the temperature \( T \approx \frac{\varepsilon_B}{2} \approx 1 \div 2 \) MeV and the characteristic height \( H = 26(1 + Z)T_{10}/Ag_{14.5} \) m is formed.

5. Pair corona

The hot atmosphere is cooled via bremsstrahlung; radiation is dominated by photons with the energy \( \sim T \sim 1 \div 2 \) MeV. One half of this radiation is directed towards the star; it is absorbed and eventually is reradiated as thermal emission. The other half of radiation is radiated away. In the magnetosphere, hard photons could be converted into electron-positron pairs. The pair production rate may be written as \( \dot{N} = \zeta \frac{L}{m_e c^2} \) where the numerical factor \( \zeta < 1/2 \) takes into account uncertainties in the radiation spectrum and the field geometry. Within the light travel time, \( R_*/c \), the magnetosphere will be filled by pairs with the density \( n = 8 \times 10^{18} \zeta L_{36} \) cm\(^{-3}\). The characteristic inhomogeneity scale of the corona is of the order of the star radius, which is large enough for the two-stream instability to develop in the corona. The condition for the instability \( \text{(16)} \) is satisfied provided the left-hand side of Eq.\( \text{(18)} \) exceeds unity. Substituting in this relation \( H \) by \( R_*/ \) and making use of Eqs.\( \text{(6)} \), \( \text{(7)} \) and the above estimate for the density of the corona, one gets

\[
\frac{\kappa_{\text{kin}} \delta z}{v_g \Lambda} = 10^3 \frac{L_{36}^{1/2}}{\zeta^{1/6} \gamma_3^{8/3} \Lambda_1}.
\]

Note that the condition \( \text{(2)} \) is satisfied in the corona at any reasonable parameters therefore the instability is hydrodynamic. So after the corona is formed, the primary beam would experience collisionless relaxation. Then about one half of the beam energy is spent on heating of the corona whereas the other half will be delivered to the surface of the star where the the electron-positron avalanche is developed and the energy of the beam is eventually transferred to electrons with the energy \( \sim \frac{\varepsilon_B}{2} \sim 1 \div 2 \) MeV each. It was argued in the previous sections, that these electrons escape upwards delivering most of their energy into the tenuous, hot atmosphere.

The corona is efficiently cooled by Comptonization of the thermal emission from the surface. As the energy of the photons is well below the Landau energy, only O-mode radiation is scattered (polarized in the plane set by the direction of propagation and the magnetic field). Thermal energy stored in the star interior is transferred to the surface and radiated away by E-mode photons because their opacities are \( (Bmc^2/Bq\varepsilon)^2 >> 1 \) times less than those for O-mode photons. Soft O-mode photons could be emitted only if surface layers of the star are heated. As hard radiation from the hot atmosphere illuminates the underlaying cold
atmosphere, some fraction of this radiation will be absorbed and reradiated in the soft band. It is this thermal O-mode radiation that could be a soft photon source for Comptonization.

The scattering cross section for the O-mode photons is \( \sigma = \sigma_T \sin^2 \theta' \) where \( \theta' \) is the angle between the propagation direction and the magnetic field in the proper electron frame. The relativistic electron "sees" radiation at the angle \( \theta' \sim 1/\gamma \) therefore the cooling rate decreases \( \gamma^2 \) times as compared with the non-magnetized case and may be estimated as (Beloborodov & Thompson 2007)

\[
\left( \frac{d\mathcal{E}}{dt} \right)_C = -\sigma_T U c;
\]

where \( \mathcal{E} \) is the energy of the electron (assumed to be larger than \( m_e c^2 \)), \( U \) the radiation energy density. Writing the radiation energy density as \( U = L/4\pi R_s^2 c \), one finds that the electron cooling time

\[
t = 4 \times 10^{-5} \mathcal{E}_{\text{MeV}} L_{36}^{-1}
\]

is comparable with the light travel time.

The observed radiation is a superposition of the bremsstrahlung radiation from the hot atmosphere and the Comptonization radiation from the corona; the radiation spectrum extends to the characteristic particle energy of \( 1 \div 2 \) MeV. Bremsstrahlung has a flat intensity spectrum (the photon index 1) at \( \varepsilon \ll T \) whereas unsaturated Comptonization produces a power-law spectrum with the photon spectral index \( \alpha > 1 \) depending on the parameters of the system (see Appendix C). As the luminosity of the corona is larger than the thermal luminosity of the surface, which provides soft photons for Comptonization, the slope should be hard enough \( \alpha < 2 \). Therefore the low-frequency part of the spectrum is dominated by Comptonization and could exhibit a variety of spectral indices in the range \( 1 < \alpha < 2 \) as is observed. The observed high pulsed fraction is naturally explained by the fact that the scattering cross section for the ordinary mode photons is highly anisotropic.

When the pairs fill the magnetosphere, the energy release stops because the pair plasma shorts out the induction electric field. Energetic primary particles, which have already filled the magnetosphere, disappear at the star’s surface for about the light travel time. At the next stage, the hot atmosphere and the corona are cooled. The cooling time is of the order of the light travel time. An important point that positrons could not survive for the larger time because they are annihilated when hitting the surface of the star. When a positron falls from the corona onto the surface, it exchanges energy with an electron in the cold atmosphere and annihilates\[1\].

\[1\]Note that even though the annihilating particles are cold, the annihilation line is not formed. In the
The corona is expected to be extremely unsteady. Even though most of the energy is taken away by the electron, which reflects from an ion and goes upwards, this electron cannot rise directly into the corona because of the charge neutrality and would remain in the hot atmosphere. The corona is replenished by the hydrodynamic expansion of this hot atmosphere, and cooled by Compton losses. The Compton losses may or may not be compensated by collisionless interaction with the primary beam. Therefore the corona may be depleted by Compton cooling; in any case, we expect that half the primary beam energy makes it down to the hot atmosphere, so that new matter is constantly being added to the corona, and this very likely leads to a non-steady situation. Because the coronal matter can propagate the currents, the displacement currents and attendant primary beam are switched off, while Compton cooling continues. When the temperature of the hot atmosphere falls below $m_e c^2$, the pair production stops and the plasma density in the corona falls below the critical value necessary to maintain the magnetospheric currents. Then the displacement current arises again and the next cycle of energy release starts. So one can expect strong fluctuations of the radiation at the timescale of $\Delta t \geq 10^{-4}$ s. The possibility exists of revealing them by analysis of the photon statistics. If a source emits radiation in separate bursts of the characteristic duration $\Delta t \equiv 10^{-3} \Delta t_{-3}$, the probability distribution for a pair of photons to be detected within the time interval $t$ should differ significantly from a Poisson distribution at $t \sim \Delta t$. For a collecting area of $10^3 A_3 \text{cm}^2$, and a photon flux of $10^{-3} F_{-3} \text{cm}^{-2}$, an observation interval of $10^5 t_5 \text{s}$ should contain $10^5 A_3 F_{-3} t_5$ photons and, for Poisson arrival statistics, $10^2 A_3 F_{-3}^2 t_5 \Delta t_{-3}$ pairs of photons arriving within $\Delta t$ of each other. The non-steady nature of the hard coronal emission (this emission in fact dominates the thermal surface emission already at a few keV), which produces deviations from Poisson arrival statistics, is therefore detectable with a sufficiently powerful detector and large exposure times.

6. Conclusions

We considered dissipation of the energy released in the magnetosphere of the magnetar in the course of slow relaxation of non-potential magnetic fields. A basic physical picture was proposed by Thompson et al. (2002) and recently elaborated by Thompson & Beloborodov (2005) and Beloborodov & Thompson (2007). When the plasma density in the magnetosphere falls below a critical value necessary to maintain the magnetospheric currents, an induction electric field arises and initiates an electron-positron avalanche resembling that superstrong magnetic field, only longitudinal (along the field) component of the momentum is conserved therefore the two-photon annihilation results in photons with generally different energies, the annihilation spectrum being extended from 0 to 1 MeV (Kaminker, Pavlov & Mamradze 1987).
in pulsars. The fast electron-positron flow hits the surface of the star where the released energy is dissipated. The observed very hard spectra of the persistent X-ray emission from SGRs and AXPs (Kuiper et al. 2004, 2006; Mereghetti et al. 2005, Molkov et al. 2005) imply that most of the energy is released in a very hot and tenuous plasma. It was assumed by Thompson & Beloborodov (2005) and Beloborodov & Thompson (2007) that the flow loses a significant fraction of its energy in a thin surface layer where the two-stream instability develops so that the plasma is heated by collisionless processes. Collisionless heating is balanced by bremsstrahlung radiation and the equilibrium temperature about 100 keV is achieved.

We reanalyse interaction of the fast plasma flow with the surface of the magnetar and conclude that this mechanism is incapable of heating the atmosphere because strong density gradient in the atmosphere of the neutron star suppresses the two-stream instability. We propose, rather, that a hot, tenuous atmosphere/corona could arise due to specific properties of Coulomb scattering in the superstrong magnetic field, mainly due to one-dimensional character of the electron motion. Within the upper layers of the neutron star, the flow energy is transferred, via an electron-positron avalanche, to electrons with the energy something less than the Landau energy ($\sim 1 \div 2$ MeV in the magnetar's magnetic field). This occurs at a significant depth, $\sim 100$ g/cm$^2$, however these electrons are not thermalized but rather escape upwards taking away most of the initial flow energy. The reason is that collisions between electrons in one-dimension do not result in relaxation; after the collision, the two energies of the two electrons are the same as the initial energies. On the other hand, collisions with ions result only in nearly elastic reflection. These electrons form a hot atmosphere (the necessary amount of ions accompany the electrons in order to maintain charge neutrality) with the temperature $\sim 1 \div 2$ MeV, which is an order of magnitude larger than in the model by Thomson & Beloborodov (2005) and Beloborodov & Thompson (2007). Hard radiation from this atmosphere is generated via bremsstrahlung. Pairs are easily produced in this radiation field; they fill the whole magnetosphere forming a hot corona. Collisionless interaction of the primary beam with the pair plasma in the corona heats the pairs even more; they are cooled by Comptonization so that the overall spectrum of the source is a superposition of the bremsstrahlung radiation from the hot atmosphere and a Comptonization radiation from the corona.

The extended corona radiates in all directions so that only a small, $< 1/2$, fraction of the radiated energy is intercepted by the surface of the star. Therefore the observed luminosity is dominated by the hard radiation. The spectrum is extended to MeV band. Unsaturated Comptonization generates a power-law spectrum, which is generally steeper than the flat bremsstrahlung spectrum therefore radiation from the corona dominates in the range $\lesssim 100$ keV. The photon spectral slope is $1 < \alpha < 2$, as is observed. The energy release in the
magnetosphere occurs spasmodically at the time scale of at least the light travel time: the pairs short out the induction electric field in the corona and the energy release stops until the corona cools down, then the charge starvation necessitates the displacement current and the process starts again. Therefore one can expect strong fluctuations of the radiation at the time scale of $\geq 10^{-4}$ s.

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**Appendix A. Electron-ion collisions: recoil effect**

When the electron with the energy less than $\varepsilon_B$ passes an ion, the energy is transferred only due to recoil effect. In magnetar’s field, most of the ions are in the ground Landau state and a scattering occurs if the ion makes a transition to the first level with the energy

$$\varepsilon_{Bi} = \frac{ZeB}{m_ic} = 5\frac{Z}{A}B_{15} \text{ keV};$$

where $m_i = Am_p$ is the ion mass. The cross section for collisional transitions between the ion Landau levels was found by Langer (1981); for the transition between the ground and the first levels it is written as

$$\sigma = \frac{3B_B}{8B}Z\sigma_T \frac{m_e^2c^2\gamma\gamma'}{pp'}(\ln \Lambda - 0.577);$$

where

$$\Lambda^{-1} = \frac{B_B}{B} \left[ \left( \frac{p - p'}{m_e c} \right)^2 - (\gamma - \gamma')^2 \right].$$

The conservation laws imply

$$m_e c\gamma + m_i c = m_e c\gamma' + \sqrt{m_i^2c^2 + (p - p')^2 + 2\varepsilon_{Bi}m_i};$$

which yields

$$\gamma - \gamma' = \varepsilon_{Bi};$$

and

$$\Lambda^{-1} = \frac{\varepsilon_{Bi}}{m_i c^2} \left( \frac{m_e c\gamma}{p} - \frac{1}{4} \right).$$
As the electron looses only a small fraction of the energy in a scattering, $\gamma - \gamma' \ll \gamma$, one can conveniently define the effective cross-section for the energy loss

$$\tilde{\sigma} \equiv \frac{\gamma - \gamma'}{\gamma} \sigma = 2 \times 10^{-3} \frac{Z^2}{A \gamma} [1 + 0.08 \ln(A^2/ZB_{15})] \sigma_T.$$ 

It was shown in Sect. 3 that the electron-positron avalanche stops at the depth (27) forming eventually a cloud of electrons with the energy $\sim \varepsilon_B/2$. The electron with this energy loses the fraction of its energy

$$\frac{\Delta E}{E} = \tilde{\sigma} n_i l = 0.1 \frac{1}{A B_{15}^{1/2}} [1 + 0.08 \ln(A^2/ZB_{15})]$$

when rising from the depth (27). This estimate assumes that the electron moves straightforwardly but not diffuses upwards. This is justified because the layer of the depth (27) is transparent for the Coulomb reflection (see the estimate (29)). So the fraction of the energy the electron loses for recoil process is small.

**Appendix B. Ionization equilibrium.**

Ionization equilibrium in the super-strong magnetic field is still a subject of intense research, see recent reviews by Lai (2001) and Harding & Lai (2006) and references therein. Here we present rough estimates for the helium plasma. The ionization energy of the hydrogen-like ion is

$$Q = 0.16 Z^2 \left[ \ln \frac{\hbar^3 B}{m_e^2 e^3 c Z^2} \right]^2 \text{ a.u.}$$

For the helium ion, one gets $Q = 2.39 (1 + 0.086 \ln B_{15})$ keV. When the atom moves, the ionization energy decreases so that the above value gives the estimate of the ionization temperature from above.

The ionization equilibrium $\text{He}^{++} \rightleftharpoons \text{He}^+ + e$ is written as

$$\frac{n_e n_{++}}{n_+} = \frac{Z_e Z_{++}}{Z_+},$$

where $n_e$, $n_{++}$ and $n_+$ are the number densities of free electrons, $\text{He}^{++}$ and $\text{He}^+$, correspondingly, $Z_e$, $Z_{++}$, and $Z_+$ their partition functions. The partition function of the strongly magnetized electrons is

$$Z_e = \frac{eB}{2\pi \hbar c} \left( \frac{m_e T}{2\pi \hbar^2} \right)^{1/2}. \quad (B1)$$
The ratio of partition functions of He$^{++}$ and He$^+$ is dominated by the ionization energy and may be presented as

$$\frac{Z_{++}}{Z_+} \approx \exp\left(-\frac{Q}{T}\right).$$

Now the temperature of ionization (when $n_{++} = n_+$) is found as

$$T_{\text{ion}} = Q \left\{ \ln \left[ \frac{3eB}{2\pi n\hbar c} \left( \frac{m_e T}{2\pi \hbar^2} \right)^{1/2} \right] \right\}^{-1} = 1.7 \times 10^6 \frac{1 + 0.09 \ln B_{15}}{1 + 0.06 \ln (B_{15}/\tau_{g14.5})} \text{K}.$$ 

This means that helium is fully ionized in magnetar’s atmosphere with the temperature $T = 0.5 \div 1 \text{ keV}$.

### Appendix C. Comptonization in a superstrong magnetic field

Here we demonstrate that Comptonization of soft photons in the superstrong magnetic field results in a power-law spectrum with the photon index $\alpha \geq 1$, like in the non-magnetized case. Compton scattering of soft photons on hot electrons results in a photon flux over the spectrum from the initial energy, $\varepsilon_0$, to the spectral region $\varepsilon \sim T$. If the optical depth of the source is large enough, the photons are accumulated at $\varepsilon \sim T$ and the equilibrium Bose-Einstein spectrum, $N_{\text{BE}} = \{\exp[(\eta + \varepsilon)/T] - 1\}^{-1}$ is formed. This regime is called saturated Comptonization. In the medium of the moderate optical depth, photons are not accumulated but rather escape and therefore a power law spectrum could be formed in the range $\varepsilon_0 \ll \varepsilon \ll T$. Here we show that the same occurs also in the magnetic field so strong that the electrons populate only the ground Landau level. For nonrelativistic temperatures, Comptonization in the superstrong magnetic field was studied by Lyubarskii (1987a,b) in the Focker-Plank approximation. Here we allow relativistic temperatures but restrict ourselves only to low photon energies when one can neglect recoil.

Let us first consider scattering on electrons moving along the magnetic field with some momentum $p$; the number density of this electrons is $f(p)dp$; where $f(p)$ is the electron distribution function. The kinetic equation for photons is easily written in the proper frame of these electrons as

$$\left( \frac{\partial}{\partial t'} + c' \frac{\partial}{\partial r'} \right) n'(r', \varepsilon', l') = cf'dp' \int d\varepsilon d\Omega \delta(\varepsilon' - \varepsilon) \frac{\partial \sigma}{\partial \Omega} [n(r', \varepsilon', l') - n(r', \varepsilon, l')];$$

where $l$ is the direction of propagation of photons, $n(r, \varepsilon, l)$ the phase density of photons and prime marks quantities measured in the proper electron frame. The scattering cross-section is

$$\frac{\partial \sigma}{\partial \Omega} = \frac{3}{8\pi} \sigma_T \sin^2 \theta' \sin^2 \theta'_i;$$
where $\theta$ and $\theta_1$ is the angles between the photon direction and the magnetic field before and after the scattering, correspondingly. One can transform this equation to the laboratory frame taking into account that the distribution functions are relativistic invariants, $f'(p') = f(p)$, $n'(\varepsilon', l') = n(\varepsilon, l)$, as well as the expressions $\varepsilon d\varepsilon d\Omega$ and $\varepsilon (\partial / \partial t + l \partial / \partial r)$. Summation over all electrons yields

$$\frac{\partial n}{\partial t} + c \frac{\partial n}{\partial r} = \frac{3}{8\pi} e^6 \sigma_T \int [n(r, \varepsilon_1, l_1) - n(r, \varepsilon, l)] \times \delta[\varepsilon(c - v \cos \theta) - \varepsilon_1(c - v \cos \theta_1)] \frac{\sin^2 \theta \sin^2 \theta_1 f(p) d\varepsilon_1 d\Omega_1}{\gamma^6(c - v \cos \theta)(c - v \cos \theta_1)^3}.$$}

This equation describes Comptonization of photons with the energy larger than the energy of the seed photons, $\varepsilon_0$, but small enough for the recoil effect to be neglected, $\varepsilon_0 \ll \varepsilon \ll \min(T, m_e c^2 / T)$.

In the steady state case, $\partial / \partial t = 0$, solution to this equation has a power law form

$$n(r, \varepsilon, l) = J(r, l) \varepsilon^{-(2 + \alpha)};$$

where the spatial function $J$ satisfies the equation

$$l \frac{\partial J(r, l)}{\partial r} = \frac{3}{8\pi} e^5 \sigma_T \int \left[ \left( \frac{c - v \cos \theta_1}{c - v \cos \theta} \right)^{2+\alpha} J(r, l_1) - J(r, l) \right] \frac{\sin^2 \theta \sin^2 \theta_1 f(p) d\varepsilon_1 d\Omega_1}{\gamma^6(c - v \cos \theta)(c - v \cos \theta_1)^4}. \tag{B1}$$

The photon power index, $\alpha$, could be found as an eigenvalue of this equation. It is determined by the electron distribution function and by the geometry and the optical depth of the source. It is beyond the scope of the present paper to solve this equation (solution of a similar problem for a nonmagnetized plasma is given by Titarchuk & Lyubarskij (1995)). Let us only demonstrate that one can expect $\alpha > 1$.

In the infinite homogeneous medium, the left-hand side of Eq. (B1) is zero therefore one gets the equation

$$\int \left[ (c - v \mu_1)^{2+\alpha} J(\mu_1) - (c - v \mu)^{2+\alpha} J(\mu) \right] \frac{(1 - \mu^2)(1 - \mu_1^2)f(p) d \mu_1}{\gamma^6(c - v \mu)^{3+\alpha}(c - v \mu_1)^4} = 0; \tag{B2}$$

where $\mu = \cos \theta$. This equation has an evident solution $\alpha = -2$, $J(\mu) = \text{const}$, which is nothing more than the low frequency part of the equilibrium Bose-Einstein spectrum. We are interested in a solution with a non-zero photon flux over the spectrum; in the infinite medium, such a solution implies that except of the soft photon source at $\varepsilon = \varepsilon_0$, there is a sink at some large enough energy $\varepsilon_{\text{sink}}$; then Eq. (B2) describes the region $\varepsilon_0 < \varepsilon < \varepsilon_{\text{sink}}$. In a non-magnetized plasma, the solution with a non-zero photon flux over the spectrum is
\( n \propto \varepsilon^{-3} \) (Kats, Kontorovich & Kochanov 1978) so that the intensity spectrum is flat. In order to see that the same spectrum (\( \alpha = 1 \) in our notations) satisfies also Eq.(B2) note that at \( \alpha = 1 \), the integrand in the left-hand side of this equation is antisymmetric with respect to exchange \( \mu \leftrightarrow \mu_1 \). Therefore the integral from the left-hand side of Eq.(B2) over \( \mu \) vanishes identically. This means that a finite linear homogeneous set of equations, which could be obtained from Eq.(B2) by discrete approximation of the integral, is linearly dependent and therefore it has a nontrivial solution. (The formal proof. Denote the linear operator in the left-hand side of Eq. (B2) at \( \alpha = 1 \) as \( \mathcal{L} \) and introduce the standard notation for the scalar product of functions \( (\psi, \phi) \equiv \int_{-1}^{1} \psi \phi d\mu \). Now one can write that \( (e, \mathcal{L} J) = 0 \) for \( e = \text{const} \) and an arbitrary \( J \). Then \( (\mathcal{L}^* e, J) = 0 \) so that \( e = \text{const} \) is a nontrivial solution to the conjugate equation \( \mathcal{L}^* e = 0 \). In this case, the equation \( \mathcal{L} J = 0 \) also has by the Fredholm alternative a nontrivial solution.)

Thus the spectrum with the slope unity is formed in the infinite medium; then all photons produced at \( \varepsilon_0 \) reach \( \varepsilon_{\text{sink}} \). In the case of a finite optical depth, the photons escape so that the spectral photon density should decrease with the frequency faster than in the infinite medium. Therefore unsaturated Comptonization in the super-strong magnetic field generates power law spectra with the slope \( \alpha > 1 \) like in the non-magnetized plasma.

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