Effect of pseudospin polarization on wave packet dynamics in graphene antidot lattices (GALs) in the presence of a normal magnetic field

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We have investigated the role of pseudo-spin polarization in electron wave packet dynamics in pristine graphene and in a graphene antidot lattice subject to an external magnetic field. Employing a Green’s function formalism, we show that the electron dynamics can be controlled by tuning pseudospin polarization. In Landau quantized graphene, we find that an electron wave packet propagates in the direction of initial pseudospin polarization with no splitting. Zitterbewegung oscillations are found to persist. In the case of a graphene antidot lattice, the electron wave packet propagates along the axis of the antidot lattice when the initial pseudospin is parallel to this axis. We also show that the probability of finding an electron along the axis of the antidot lattice increases with the strength of the antidot potential. This suggests that a graphene antidot lattice can serve as a channel for electron transport with the possibility of tunability by means of pseudospin polarization, antidot potential and applied normal magnetic field strength.

I. INTRODUCTION

Over the past decade, tremendous work has been done to study and control charge transport in graphene in external fields and also by introducing external one dimensional potentials using nanopatterning.1–5 In the 1970s, it was realized that introduction of artificial one dimensional potentials can be used to modify the energy band structure.6 Potentials such as antidot lattices can be carved on graphene by various techniques and lattice parameters can be tuned.7,8 This engineering of transport properties in graphene can lead to possible graphene based logic devices,9,10 Analogous to physical spin, pseudospin is one of the unique properties carried by charge carriers in graphene due to presence of two triangular sublattices in graphene.2 Pseudospin appears in the wave function as an independent degree of freedom,11 hence giving birth to the field of pseudospintronics which has the potential for applications in logic devices and quantum computing similar to spintronics.12–17 Controlling pseudospin polarization with the help of external gates invites us to investigate wave packet dynamics in various graphene systems.13 Zitterbewegung (ZB), which is the oscillatory motion of the wave packet is another important phenomenon observed in Dirac materials.2 These oscillations have features which differ in the presence and in the absence of the magnetic field.18,19 Study of the dynamics of wave packets in monolayer graphene in the absence of a magnetic field with different pseudospin polarizations suggests that when pseudospin is perpendicular to the momentum of initial wave packet, then the wave packet splits into two parts and moves in the direction of the initial momentum of the packet with opposite velocities: However, when pseudospin is parallel to the the initial momentum of the packet then the packet does not split and moves in the direction of initial momentum and ZB oscillations are transient and disappear after some time.18 Also, ZB studies of graphene in a magnetic field suggest that ZB oscillations for B ≠ 0 are permanent and do not die with time.19 Pseudospintronics has much more to reveal, so to further investigate the dynamics of a wave packet in systems of monolayer graphene with initial pseudospin polarization, we have chosen two systems involving Landau quantized monolayer graphene: First is pristine graphene and the second is graphene having 1D antidot lattice, both placed in a perpendicular magnetic field. Along with analysis of the effect of pseudospin polarization on wave packet dynamics in these systems, we have studied the effect of tuning of antidot strength on motion of the wave packet.

II. GRAPHENE IN PERPENDICULAR MAGNETIC FIELD: GREEN’S FUNCTION

Graphene is a single layer of carbon atoms packed in a honeycomb lattice. The effective Hamiltonian of graphene has the form

\[ H = \gamma \sigma \cdot \mathbf{p} \]  

where \( \sigma = [\sigma_x, \sigma_y] \) and \( \sigma_x, \sigma_y \) are Pauli’s spin matrices, which act on the sublattice/pseudospin space and represent the sublattice degree of freedom of graphene’s honeycomb lattice structure, also \( 1_\nu = 1 \) or -1 for \( K \) or \( K' \) valleys, and \( \gamma = \frac{3}{2} \alpha d \approx 10^6 \text{ms}^{-1} \) plays the role of a density-independent Fermi velocity \( \alpha d \) is the hopping amplitude originating from the tight binding approximation in which the lattice spacing is \( d \).

We consider a graphene sheet placed on the \( xy \)-plane in a perpendicular and uniform magnetic field \( \mathbf{B} = B \hat{z} \) with vector potential \( \mathbf{A} = \frac{1}{2} (\mathbf{B} \times \mathbf{r}) \). The magnetic field is introduced by minimal substitution \( \mathbf{p} = \mathbf{p} - e \mathbf{A} \) in Eq. (1). The requirement of gauge invariance leads to

\[ \mathcal{G}(\mathbf{r}, \mathbf{r}'; t, t') = C(\mathbf{r}, \mathbf{r}') \mathcal{G}'(\mathbf{r} - \mathbf{r}'; t - t'), \]  

where \( C \) is a complex constant.
where the Green’s function $G'(r - r'; t - t')$ is spatially translationally invariant and gauge invariant and it is related to usual Green’s function $G(r, r'; t, t')$ with the help of Peierls phase factor $C(r, r')$, which carries all aspects of the lack of translational invariance in a magnetic field and all gauge dependence as

$$C(r, r') = \exp \left( \frac{ie}{2\hbar} \mathbf{r} \cdot \mathbf{B} \times \mathbf{r'} - \phi(r) + \phi(r') \right) \quad (3)$$

($\phi(r)$ is an arbitrary gauge function). The translationally invariant Green’s function is given by the equation of motion

$$\left[ \frac{\partial}{\partial T} - \gamma \sigma_{\nu} \cdot \left( \frac{1}{i} \frac{\partial}{\partial \mathbf{R}} - \frac{e}{2} \mathbf{B} \times \mathbf{R} \right) \right] G'(\mathbf{R}, T) = I_3 \delta^{2}(\mathbf{R}) \delta(T) \quad (4)$$

where $\mathbf{R} = \mathbf{r} - \mathbf{r'}$ such that $X = x - x'$, $Y = y - y'$ and $T = t - t'$. The diagonal elements of the Green’s function matrix ($\omega$ representation) for this system of monolayer graphene in uniform, constant perpendicular magnetic field are ($G'_{11}, G'_{22}$) collectively represents $G'_{11}, G'_{22}$

$$G'_{12}(\mathbf{R}; \omega) = \frac{1}{4\pi \hbar\gamma^2} \omega e^{-\frac{\omega^2}{\gamma^2}(X^2 + Y^2)}$$

$$\times \sum_{n=0}^{\infty} \frac{\omega^2}{\omega_g^2} \frac{1}{(n + 1/2)} \left( X^2 + Y^2 \right) \quad (5)$$

We introduced the notation

$$\omega_g = \gamma \sqrt{\frac{2eB}{\hbar}} \quad (6)$$

which is the cyclotron frequency for Dirac fermions. The poles of $G'_{11}(\mathbf{R}; \omega)$ show that the energy spectrum is Landau quantized, with Landau level index given by $n$. The off-diagonal matrix elements can be calculated using the relation ($\gamma \omega \equiv 1, \gamma$)

$$\omega_g G'_{12} = [\gamma \Pi_{XY} \pm i \eta_y \Pi_{YX}] G'_{12} \quad (7)$$

In the above equation, $\Pi_{XY} \equiv \frac{1}{i} \frac{\partial}{\partial X} + \frac{eB}{2} Y$ and $\Pi_{YX} \equiv \frac{1}{i} \frac{\partial}{\partial Y} - \frac{eB}{2} X$ are momentum operators and $G'_{12} = G'_{21}$ or $G'_{12}$ corresponds to the upper or lower $\pm$ signs elsewhere in the equations. Making use of Eq. (7), off-diagonal elements of Green’s function matrix for $K$ and $K'$ can be separately expressed as

$$K'(1, -1) : G'_{12}(\mathbf{R}; \omega) = \frac{\omega_g^2}{8\pi \hbar \gamma^3} e^{-\frac{\omega^2}{\gamma^2}(X^2 + Y^2)}$$

$$\times (ix \mp Y) \sum_{n=1}^{\infty} L_{n-1} \frac{\omega^2}{\omega_g^2} \left( X^2 + Y^2 \right) \quad (8)$$

Clearly, the off-diagonal elements interchange with the interchange of Dirac points ($K$ and $K'$).

The Green’s functions matrix in time representation can be obtained by Fourier transform of Eqs. (5), (8), (9) as

$$G'_{\mu\nu}(\mathbf{R}; t) = \int_{-\infty}^{\infty} d\omega \ e^{-i\omega t} \ G'_{\mu\nu}(\mathbf{R}; \omega) \quad (10)$$

where $\mu, \nu = 1, 2$ denote matrix indices.

Noting that $G'_{\mu\nu}(\mathbf{R}; \omega)$ has real energy poles at $\epsilon_K = \pm \omega_g \sqrt{n}; \epsilon_{K'} = \pm \omega_g \sqrt{n + 1}$ we employ contour integration with $\omega \to \omega + il' \tau$ for the retarded Green’s function with the contour closed in the lower half plane running clockwise from $-\infty$ to $+\infty$. For the $K$-point, we have (for the $K'$-point $\sqrt{n} \to \sqrt{n + 1}$)

$$G'_{12}(x, x'; y, y'; t) = -i \eta_y(t) \frac{\omega_g^2}{4\pi \gamma^2} e^{\frac{i}{2} \pi} \sum_{n=0}^{\infty} L_{n} \left[ \frac{1}{\gamma} \right]$$

$$\times \cos \left( \frac{\omega_g}{\gamma} \frac{t}{\tau_o} \sqrt{n} \right) \quad (11)$$

$$G'_{22}(x, x'; y, y'; t) = -\eta_y(t) \frac{\omega_g^2}{8\pi \gamma} e^{\frac{i}{2} \pi} \sum_{n=1}^{\infty} \frac{L_{n-1} \left[ \frac{1}{\gamma} \right]}{\sqrt{n}}$$

$$\times \sin \left( \frac{\omega_g}{\gamma} \frac{t}{\tau_o} \sqrt{n} \right) \quad (12)$$

Here, we have introduced an arbitrary constant length, $l$, chosen for convenience to be the width of an impressed wave packet, and $\eta_y(t) = 0,1$ for $t \leq 0, t > 1$ respectively, is the Heaviside step function; also, we have defined

$$\zeta = \frac{1}{4} \left( \frac{\omega_g}{\gamma} \frac{l^2}{\tau_o} \right) ^2 \left[ \left( x - x' \right)^2 + \left( y - y' \right)^2 \right]$$

and $\tau_o = l/\gamma$.

To study the wave packet dynamics, we take the initial wave function to be a Gaussian wave packet having non-vanishing average momentum $p_0 = h k_0$ and width $l$,

$$\psi(r, 0) = \frac{f(r)}{\sqrt{|c_1|^2 + |c_2|^2}} \left( \frac{c_1}{c_2} \right) \quad (13)$$

$$f(r) = \frac{1}{l \sqrt{\pi}} \exp \left( - \frac{x^2 + y^2}{2l^2} + il k_0 x \right)$$

where $c_1$ and $c_2$ are the coefficients which set the initial pseudospin polarization. Also $\psi(r, 0)$ can be taken
as a smooth enveloping function due to the reason that lattice period is much smaller than the width $l$ of the initial wave packet. The Green’s functions matrix elements $G_{\mu \nu}(r, r', t)$ helps us to determine the time evolution of an arbitrary initial state $\psi(r, 0)$; in Schrödinger representation it is given by

$$\psi_{\mu}(r, t) = \int dr' G_{\mu \nu}(r, r', t) \psi_{\nu}(r', 0)$$  \hspace{1cm} (14)

where $\mu, \nu = 1, 2$ denote the matrix indices, which corresponds to upper component $\psi_1(r, t)$ and lower component $\psi_2(r, t)$ of state $\psi_{\mu}(r, t)$. The probability density will be

$$\rho(r, t) = |\psi(r, t)|^2 = |\psi_1(r, t)|^2 + |\psi_2(r, t)|^2. \hspace{1cm} (15)$$

And to study ZB, the average value of coordinates can be represented as

$$\bar{x}_j = \int \psi_1^*(r, t) x_j \psi_1(r, t) d\mathbf{r} + \int \psi_2^*(r, t) x_j \psi_2(r, t) d\mathbf{r}$$

$$\hspace{1cm} (16)$$

where $j = 1, 2$ with $x_1=x$ and $x_2=y$. The Peierls phase factor defined in Eq. (3) for the choice $\phi(r) = \phi(r') \equiv 0$ has the form

$$C(r, r') = \exp \left[ \frac{i}{\hbar} \left( \frac{\omega g}{\gamma} \right)^2 \left( \frac{y'y'}{l^2} - \frac{xy'}{l^2} \right) \right].$$

III. DYNAMICS OF A GAUSSIAN WAVE PACKET WITH DIFFERENT PEUSODISPIN POLARIZATIONS IN LANDAU QUANTIZED GRAPHENE; ZITTERBEWEGUNG

To obtain results for the temporal evolution of the initial Gaussian wave packet, ZB oscillations and the effect of initial-pseudospin polarization, we have performed numerical calculations. To facilitate it, we have introduced following dimensionless variables:18,24

- A dimensionless parameter which is suitable to replace wave vector $k_{0x}$ is $a_0 = k_{0x} l$.
- Distance for propagation of the wave packet can be measured in units of initial width $l$ of wave packet.
- Time can be measured in $\tau_o = l/\gamma$ ($\gamma$ is Fermi velocity, $10^6$ m/s) units.
- Some other variables can be combined to produce dimensionless variables, e.g $b = \frac{\omega g}{\gamma} l$. The Landau level summation is performed up to the $10^{th}$ Landau level in all calculations since the results become convergent in this limit.

We consider four cases with different initial pseudospin polarizations $\{c_1, c_2\}$ for the Gaussian wave packet given in Eq. (13).

Case-1: $\{c_1, c_2\} = \{1, 0\}$ which corresponds to initial electron probability of one at the sites of sublattice A.

Case-2: $\{c_1, c_2\} = \{1, 1\}$ corresponds to the situation where the pseudospin is directed along the x-axis.

Case-3: $\{c_1, c_2\} = \{1, i\}$ this corresponds to the pseudospin directed along y-axis.

Case-4: $\{c_1, c_2\} = \{1, e^{i\pi/4}\}$ this implies that at time $t=0$, the pseudospin lies in $x$-$y$ plane making an angle of $45^o$ with x-axis.

The numerical results obtained from Eqs. (14), (15) are plotted in Fig.1.

In Fig.1, electronic probability density is plotted for parameters $t=1, 5, 10 \tau_o$, with momentum $k_{0x} = 0.6 \text{ nm}^{-1}$, width $l=2 \text{ nm}$ so that $a_0=k_{0x} l=1.2$ and $\tau_o=2 \text{ femtosecond}$ and $B=3.3$ in units of Tesla i.e. $\omega g \approx 1 \times 10^{14} \text{ Hz}$. Left panel shows the initial wave packet at a very small time $t=1 \tau_o$, and as we move from left to right the time evolution of initial Gaussian packet can be seen for different initial pseudospin polarizations. The strength of the electron probability density $\rho(x, y, t)$ is given by the color bar on the right side.

As one can see in Fig.1 (a), (b), (c), the wave packet spreads and propagates in the plane of graphene sheet in the form of rings for the pseudospin $c_1=1$ and $c_2=0$. Initially in Fig.1 (a), maximum probability of electron is located at the origin, but as time increases, the wave packet propagates and the electron density can be found at a radius of $r \simeq 24 \text{ nm}$ in 20 $fs$. Similarly for other three cases, the wave packet propagates with its maximum probability density in the direction of initial pseudospin polarizations but in the shape of incomplete rings, this is because the probability density gradually decreases in the directions away from the direction of pseudospin polarization. It propagates in the x direction when initial pseudospin polarization is along x axis (see Figs.1 (d), (e), (f)), in the y direction when initial pseudospin polarization is along y axis (see Figs.1 (g), (h), (i)) and it propagates in $(x, y) = (1, 1)$ direction when initial pseudospin is polarized along $x, y = (1, 1)$ direction (see Figs.1 (j), (k), (l)). This is because of the conservation of chirality in which momentum gets aligned with pseudospin and $\vec{p} = \vec{p}$ remains conserved. Also the distance covered by the wave packet in these three cases is $r \simeq 24 \text{ nm}$ in 20 $fs$. Hence the direction of propagation of a wave packet in Landau quantized graphene can be controlled using pseudospin polarization. Also, the wave packet propagates without any splitting; splitting was observed in the case of monolayer graphene in the absence of magnetic field.18 We propose that, experimentally, this type of controlled propagation of wave packet in any direction can easily be obtained using photonic graphene as test beds.14

Also note that the electronic probability densities plotted in Fig.1 are not symmetric with respect to both x and y axes: $\rho(x, y, t)_{\{c_1, c_2\}} \neq \rho(-x, -y, t)_{\{c_1, c_2\}}$ and $\rho(x, y, t)_{\{c_1, c_2\}} \neq \rho(x, -y, t)_{\{c_1, c_2\}}$ for any pseudospin polarization (subscript $\{c_1, c_2\}$ defines the corresponding pseudospin polarization): This means that center of the
wave packet is oscillating along both the $x$ as well as the $y$ directions; these oscillations can be readily recognized as Zitterbewegung oscillations.

To examine this trembling motion we use Eq. (16) and solve it numerically for expectation values of both $x$ and $y$ coordinates. Figs. 2 (a), (c), (e) and Figs. 2 (b), (d), (f) show the oscillations in the wave packet’s center along $x$ and $y$ directions respectively. Results are plotted for two values of initial momentum $k_{0x} = 0.6$ and 0.8 i.e $a_0 = k_{0x} b = 1.2$ and 1.6, given by solid and dotted lines for three different values of pseudospin polarization. It is clear from Fig. 2 that increase in momentum results in decrease in amplitude of Zitterbewegung oscillations without any other change in the behaviour of ZB. In further discussion, ZB oscillations corresponding to different pseudospin polarizations will be referred to as $\bar{x}_{(c_1,c_2)}$ and $\bar{y}_{(c_1,c_2)}$.

The ZB oscillations shown in Fig. 2 are of the order of nanometer (easily detectable) except $x(t)_{(1,0)}$ and $x(t)_{(1,i)}$ which are of the order of 0.1 pm. On comparing the ZB oscillations $x(t)_{(1,0)}$, $x(t)_{(1,1)}$, $x(t)_{(1,i)}$, we have concluded that when the direction of initial momentum ($x$-axis) is perpendicular to initial pseudospin polarizations ($z, y$-axis) then the ZB oscillations carry very small amplitude in the direction of initial momentum. In Figs. 2 (a)-(f) we have seen that initially the amplitude of the Zitterbewegung oscillations increases, then these oscillations seem to die out but reappear for all pseudospin polarizations. For example in Fig. 2 (c) these oscillations reappear at $t \simeq 18, 55, 84 \tau_0, \ldots$. Hence when there is a magnetic field applied to the system, Zitterbewegung oscillations are not transient; rather they are recurrent. Also, in the presence of the magnetic field, several ZB frequencies appear (see Figs. 2 (a)-(f)). This is different from the ZB phenomenon observed in monolayer graphene without magnetic field, in which ZB oscillations are transient having a single frequency. Finally, in Fig. 3 we have plotted the average coordinates $\bar{x}(t)_{(c_1,c_2)}$ and $\bar{y}(t)_{(c_1,c_2)}$ against each other to study the ZB trajectory of center of the wave packet corresponding to the Figs. 2 (e), (d). Initially due to large ZB, center of the packet sweeps a large area. For a better understanding, we have shown zoomed view of the ZB trajectory in Fig. 3 (b). Due to the presence of recurrent ZB, these trajectories do not disappear with time (infinite trajectories).
FIG. 3. Average coordinates \( \bar{x}(t) \) versus \( \bar{y}(t) \) corresponding to pseudospin polarization \( (c_1, c_2) = (1, 1) \) for momentum \( a_0 = 1.2 \) with \( B = 3.3 \text{Tesla} \). (a) ZB Trajectory for \( t = 0 \) to \( t = 100\tau_0 \). (b) ZB Trajectory for \( t = 7\tau_0 \) to \( t = 100\tau_0 \).

IV. GRAPHENE ANTIDOT LATTICE IN THE PRESENCE OF MAGNETIC FIELD: LANDAU MINIBANDS

We consider a two-dimensional graphene sheet having a one-dimensional lattice of quantum antidot potential barriers, with a quantizing magnetic field \( B \), which is perpendicular to the plane of graphene sheet. We model the antidot array as a row of Dirac delta functions. Following the Kronig-Penny model for a quantum antidot lattice, we introduce an infinite array of identical quantum antidot potential barriers periodically spaced along the \( x \)-axis at \( x_n = n'd, y \equiv 0 \) as

\[
U(\mathbf{r}) = U(x, y) = \alpha \sum_{n' = -\infty}^{\infty} \delta(x - n'd)\delta(y),
\]

where \( \alpha > 0 \), \( n' = -\infty \) to \( +\infty \) and

\[
\alpha = U_0 \frac{a^2}{d}
\]

where \( U_0 \) is the barrier height, \( a^2 \) is area and \( d \) is the uniform separation of the periodic quantum antidot potential barriers.

The Green’s function \( \mathcal{G}(x_1; x_2; 0, 0; \omega) \) for the two-dimensional Kronig-Penny-like model with a one-dimensional antidot lattice, for an electron propagating directly along the axis of the antidot lattice (\( y \equiv 0 \equiv y_1 \equiv y_2 \) and suppressing further reference to \( y \)) is given by

\[
\mathcal{G}(x_1; x_2; \omega) = \mathcal{G}^0(x_1; x_2; \omega) + \frac{\alpha d}{2\pi} \sum_{n' = -\infty}^{\infty} \mathcal{G}^0(x_1; n'd; \omega)
\times \int_{-\pi}^{\pi} dp \ e^{-ipn'd} \left[ 1 - \alpha \mathcal{G}^0(p; 0, 0; \omega) \right]^{-1} \mathcal{G}^0(p; x_2; \omega).
\]

(19)

Here, \( \mathcal{G}^0(\mathbf{r}_1, \mathbf{r}_2) \) is the Green’s function for graphene in a perpendicular magnetic field in the complete absence of quantum antidot potential barriers, which is given by Eqs. (5), (8) and (9), while the overhead dot represents the spatially translationally invariant Green’s function i.e. \( \mathcal{G}^0(m'm', n'n'; \omega) = \mathcal{G}^0([m' - n']d, \omega) \) where \( m' \) and \( n' \) are integers. Also, the Green’s function \( \mathcal{G}(p) \) can be expanded in a Fourier series due to the periodicity of the lattice, given by

\[
\mathcal{G}(p) = \sum_{r = -\infty}^{\infty} e^{iprd} \mathcal{G}(rd),
\]

(20)

where \( r = 0, 1, 2, 3... \) are integers, with

\[
\mathcal{G}(m'd) = \frac{d}{2\pi} \int_{-\pi}^{\pi} dp \ e^{-ipm'd} \tilde{\mathcal{G}}(p),
\]

(21)

and

\[
\tilde{\mathcal{G}}([m' - n']d) = \frac{d}{2\pi} \int_{-\pi}^{\pi} dp \ e^{-ip(m' - n)d} \tilde{\mathcal{G}}(p).
\]

(22)

It is important to note that the Peierls phase factor is \( C(\mathbf{r}, \mathbf{r}') = 1 \) for the case involving propagation directly along the axis of the antidot lattice (our choice \( y \equiv 0 \equiv y_1 \equiv y_2 \) results in \( r \parallel r' \)). Therefore \( C(\mathbf{r}, \mathbf{r}') \) does not appear in Eq. (19) and the eigen-energy spectrum given by poles of Eq. (19) is unaffected by \( C(\mathbf{r}, \mathbf{r}') \).

The energy spectrum of this system can be obtained from vanishing of the frequency poles of the Green's function of Eq. (19):

\[
\text{Det}(I_2 - \alpha \tilde{\mathcal{G}}(p; 0, 0; \omega)) = 0.
\]

(23)

Eqs. (5) and (8) taken jointly with Eq. (20) gives \( Y = 0, X^2 + Y^2 = X^2 = (rd)^2 \)

\[
\tilde{\mathcal{G}}_{12}(p; 0, 0; \omega)_K = \frac{1}{4\pi\hbar^2} \omega \sum_{r = -\infty}^{\infty} e^{iprd} e^{-\frac{\omega^2}{4\tau^2}(rd)^2}
\times \sum_{n=0}^{\infty} L_n \frac{\omega^2}{\pi^2} \left( \frac{rd}{\tau} \right)^2
\]

(24)

and

\[
\tilde{\mathcal{G}}_{11}(p; 0, 0; \omega)_K = \frac{i\omega^2}{8\pi\hbar^2} \sum_{r = -\infty}^{\infty} (rd) e^{iprd} e^{-\frac{\omega^2}{4\tau^2}(rd)^2}
\times \sum_{n=1}^{\infty} L_{n-1} \frac{\omega^2}{\pi^2} \left( \frac{rd}{\tau} \right)^2.
\]

(25)

for \( K \) point in graphene. It is clear from Eqs. (24) and (25) that \( \tilde{\mathcal{G}}_{11}(p; 0, 0; \omega)_K = \tilde{\mathcal{G}}_{22}(p; 0, 0; \omega)_K \) and \( \tilde{\mathcal{G}}_{12}(p; 0, 0; \omega)_K = \tilde{\mathcal{G}}_{21}(p; 0, 0; \omega)_K \). Hence Eq. (23) can be written as

\[
1 - 2\alpha \tilde{\mathcal{G}}_{11}^0 + \alpha^2 \left( \tilde{\mathcal{G}}_{11}^2 - \tilde{\mathcal{G}}_{12}^2 \right) = 0.
\]

(26)
Note that since the antidot radius is very small (i.e. $\frac{\omega_d}{\omega_g} \ll 1$), a root of Eq. (26) $\omega$ approaches the pole position i.e. $\omega \rightarrow \omega_n$, so that the $n$-th pole has the primary influence in determining the eigen-energy root $\omega_n$. Therefore, we can make a reasonable first approximation by dropping all other terms of the $n$-sum. Also, for $\omega_d > 1$, it suffices to keep only $r = -1, 0, 1$ terms of the $r$-sum in Eqs. (24) and (25). This imposes following condition on antidot spacing

$$d > \frac{145}{\sqrt{B} \ (T esa)} \ (nm). \quad (27)$$

Finally, Eq. (26) can be written as

$$1 - 2 \frac{\omega_g \Omega \omega_n}{\omega_n^2 - n \omega_g^2} + \frac{\omega_g^2 \Omega^2 \omega_n^2}{(\omega_n^2 - n \omega_g^2)^2} = \frac{- \omega_n^4 \kappa^2}{(\omega_n^2 - n \omega_g^2)^2} \quad (28)$$

where we have defined

$$\Omega = \frac{\alpha \omega_g}{4 \pi \hbar \gamma^2} \left[ 1 + 2 \cos pd \ e^{-\frac{\omega_g^2}{\gamma^2} d^2} L_n \left( \frac{\omega_g^2}{4 \gamma^2 d^2} \right) \right] \quad (29)$$

and

$$\kappa = \frac{\alpha \omega_g}{4 \pi \hbar \gamma^2} \left( \frac{\omega_g}{\gamma} \right) \sin pd \ e^{-\frac{\omega_g^2}{8 \gamma^2} d^2} L_n \left( \frac{\omega_g^2}{4 \gamma^2 d^2} \right). \quad (30)$$

Finally, four roots of Eq. (28) gives the energy spectrum at the $K$ point of graphene having a 1-D antidot lattice placed in a uniform normal magnetic field:

$$\omega_{n,K} = \frac{\Omega \pm \sqrt{\Omega^2 + 4(n \pm \kappa)^2}}{2} \ \omega_g. \quad (31)$$

Similarly, for $K'$ point, $n$ will be replaced by $n + 1$ on the right hand side of Eq. (31).

In this, we have the energy spectrum composed of broadened Landau levels (Landau minibands) for a graphene antidot lattice in a quantizing magnetic field. Each Landau level has split into two branches and each branch has broadened into a small continuous band (subband) of energy instead of a single energy. Fig. 4(a) and Fig. 4(b) show the Landau minibands at the location of $K$ and $K'$ points. The broadening is so small that it can not be observed with the naked eye, so we multiplied a broadening factor $\beta = 200$ with the oscillatory term $\cos(pd)$ and $\sin(pd)$. The parameter $\beta$ introduces an increase in the amplitude of the minibands to facilitate observation of the broadening of the Landau minibands; i.e we used

$$\Omega = \frac{\alpha \omega_g^2}{4 \pi \hbar \gamma^2} \left( 1 + 2 \beta \ \cos pd \ e^{-\frac{\omega_g^2}{\pi \gamma^2 d^2}} L_n \left( \frac{\omega_g^2}{4 \gamma^2 d^2} \right) \right)$$

and

$$\kappa = \frac{\alpha \omega_g}{4 \pi \hbar \gamma^2} \left( \frac{\omega_g}{\gamma} \right) \beta \ \sin pd \ e^{-\frac{\omega_g^2}{8 \gamma^2} d^2} L_n \left( \frac{\omega_g^2}{4 \gamma^2 d^2} \right).$$

Note that since the antidot strength $\alpha$ approaches zero (which means no antidot lattice), $\Omega$ and $\kappa$ also approaches zero, i.e

$$\lim_{\alpha \rightarrow 0} \Omega = 0 = \lim_{\alpha \rightarrow 0} \kappa.$$ 

In this case, Eq. (31) reduce to the case of discrete eigen-energy spectrum of graphene in a normal uniform magnetic field. The Width of Landau minibands for the $K$ point of graphene is

$$\Delta \omega_{n,K} = [\omega_{n,K}]_{p=\frac{2\pi}{\Lambda}} - [\omega_{n,K}]_{p=\frac{\pi}{\Lambda}}. \quad (32)$$

Evaluation of this expression for the width of Landau minibands yields

$$\frac{\Delta \omega_{n,K}}{\omega_g} = \alpha' \ \varrho \pm \frac{1}{2} \sqrt{\alpha'^2 (1 + \varrho)^2 + (2/\sqrt{n})^2}$$

$$+ \frac{1}{2} \sqrt{\alpha'^2 (1 - \varrho)^2 + (2/\sqrt{n})^2} \quad (32)$$

where we have defined

$$\alpha' = \frac{\alpha \omega_g}{4 \pi \hbar \gamma^2} \quad \varrho = 2 e^{-\frac{\omega_g^2}{8 \gamma^2 d^2}} L_n \left( \frac{\omega_g^2}{4 \gamma^2 d^2} \right).$$
Upper and lower signs are used for conduction and valence bands respectively. For the width of the Landau minibands for the $K'$ point, we have $(2\sqrt{n} + 1)^2$ in place of $(2\sqrt{n})^2$ in Eq. (32).

V. GREEN'S FUNCTIONS: FREQUENCY AND TIME REPRESENTATION

To evaluate the full Green’s function matrix $\mathcal{G}(x_1; x_2; \omega)_K$ given in Eq. (19), we have to find the $\mathcal{G}^0(x_1; x_2; \omega)_K$, $\mathcal{G}^0(x_1; n'd; \omega)$, $\hat{\mathcal{G}}^0(p; 0, 0; \omega)$ and $\mathcal{G}^0(p; x_2; \omega)$ matrices. These four matrix Green’s functions can be easily determined using $\mathcal{G}^0(x_1; x_2; \omega)$, which is the Green’s function in the presence of perpendicular and uniform magnetic field in absence of antidot lattice given in Eqs. (5) and (8).

For propagation along the antidot lattice i.e the x axis only ($y_1=0=y_2$, $X=x_1-x_2$ and $C(r_1; r_2)=1$), Eqs. (5) and (8) reduce to (for $K$-point)

$$\mathcal{G}^0_{12}(x_1, x_2; \omega)_K = \frac{1}{4\pi\hbar^2} \omega e^{-\frac{g^2}{\hbar}(x_1-x_2)^2} \times \sum_{n=0}^{\infty} L_n \left[ \frac{\sin^2}{\sqrt{\gamma} - n} \right] (x_1 - x_2)^2$$

and

$$\mathcal{G}^0_{11}(x_1, x_2; \omega)_K = \frac{i\omega^2}{8\pi\hbar^4} e^{-\frac{g^2}{\hbar}(x_1-x_2)^2} (x_1-x_2) \times \sum_{n=1}^{\infty} L_{n-1} \left[ \frac{\sin^2}{\sqrt{\gamma} - n} \right] (x_1 - x_2)^2$$

respectively. Matrix elements of $\mathcal{G}^0(x_1; n'd; \omega)_K$ can be obtained by taking $x_2=n'd$ in Eqs. (33) and (34), while matrix elements of $\mathcal{G}^0(p; 0, 0; \omega)_K$ are given in Eqs. (24) and (25).

Similarly, the matrix $\hat{\mathcal{G}}^0(p; x_2; \omega)_K$ can be obtained by using $x_1=r'd$ in Eqs. (33) and (34) jointly with Eq. (20). Note that for $\frac{\alpha\omega}{\hbar} > 1$, it suffices to keep only $r = -1, 0, 1$ of the $r$-sum in expressions of matrix elements of $\hat{\mathcal{G}}^0(p; 0, 0; \omega)_K$ and $\mathcal{G}^0(p; x_2; \omega)_K$ as discussed earlier. Above four matrix Green’s functions completely determine the full Green’s function $\mathcal{G}(x_1; x_2; \omega)_K$ given in Eq. (19).

As discussed earlier, for temporal evolution of the wave packet we require the time representation of the Green’s function. Hence, to find the time representation of the full Green’s function, we have to take the Fourier trans-
\[ \gamma_2 = \frac{i \alpha \omega^2 l}{8 \pi \hbar \gamma^3} \left( \frac{x_1 - n'd}{l} \right) e^{-\frac{y}{\tau}}, \quad (41) \]

and \( c_i(\eta) \) (where \( \eta = \frac{\omega}{\omega_0} \)) are matrix elements of (see Eq. (A5); notation \( q = pd \))

\[ \int_{-\pi}^{\pi} dq \ e^{-i\eta q'} \left[ I - \alpha \tilde{G}^0(p; 0, 0; \eta) \right]^{-1} \tilde{G}^0(p; x_2; \eta) \]

These considerations yield the time representation of the full Green’s function, i.e. \( \tilde{G}(x_1, x_2; t)_K \) for a Landau-quantized monolayer graphene having a one dimensional antidot lattice. In the following section, this Green’s function \( \tilde{G}(x_1, x_2; t)_K \) will be employed to study the temporal dynamics of a wave packet in the system.

VI. WAVE PACKET DYNAMICS ALONG A ONE DIMENSIONAL ANTIDOT LATTICE

With the solution of Eq. (35) in hand, in the form of Eqs. (36), (37), (38) and (39), the temporal study of an electron wave packet propagating along the axis of the antidot lattice given by

\[ \psi(r, 0) = \frac{f(r)}{\sqrt{\left| c_1 \right|^2 + \left| c_2 \right|^2}} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \quad (42) \]

\[ f(r) = \frac{1}{l \sqrt{\pi}} \exp \left( -\frac{x^2 + y^2}{2l^2} + ik_0 x \right) \delta(y) \]

can be made using Eqs. (14) and (15). In Fig. (5), results for the probability density \( \rho(x, 0, t) \) of an electron along the axis of the antidot lattice \( (x, y = 0) \) are plotted for four different cases to examine the effect of antidot lattice.

In Fig. 5, \( \rho(x, 0, t) \) is represented at times \( t = 1\tau_0 \) and \( t = 5\tau_0 \), with solid and dashed lines respectively. Moving from left to right, the three columns correspond to three different pseudospin polarizations \( \{c_1, c_2\} \), while from up to down the four rows represent the increase in antidot strength \( E_A = U_0 \) from zero meV to 330 meV with constant magnetic field strength \( E_B = 66 \) meV \( (E_B = \hbar \omega_0) \) at \( B = 3.3 \) Tesla. All three columns represent the propagation of the wave packet along \( x \) axis \( (y = 0) \). One can see that with the increase in antidot strength, the probability density of finding the electron starts increasing along the axis of antidot lattice. Row 1 corresponds to the situation when there is no antidot lattice and wave packet propagates in graphene under the effect of perpendicular magnetic field only. Similarly, rows 2, 3 and 4 correspond to \( E_A = U_0 = 13.2, 66 \) and 330 meV respectively. Clearly, the probability of finding an electron along the antidot direction increases with the introduction of the antidot lattice in a Landau quantized graphene sheet; this can be treated as the propagation of a wave packet through a quantum antidot wire, which is clearly supporting the propagation through it. Also, the probability density gets more confined with increase in antidot strength, this can be seen in rows 2, 3 and 4 of Fig. 5. Moving from up to down across the rows 2 to 4, the spread of the packet gets smaller with a clear increase in magnitude of probability density, which means that the probability of finding an electronic current along the antidot increases. This may be referred to as collimation of electronic beam along the axis of antidot lattice.

Further, we explore the effect of initial pseudospin polarization on the wave packet dynamics in the presence of antidot lattice shown in Fig. 5. In Fig. 5, it can be seen that propagation of the wave packet is strongly affected with the change of pseudospin polarization across the rows. Columns 1 and 3 correspond to pseudospin \( \{1, 0\} \) and \( \{1, i\} \) respectively and the propagation corresponding to these two columns is similar because the pseudospins are both perpendicular to the axis of the antidot lattice, and the center of the wave packet does not propagate with an increase in time, and is always located at \( x = 0 \) without being affected by...
antidot strength, as shown in Figs. 5 (a), (d), (g), (h) and Figs. 5 (c), (f), (i), (l). But, one can see in Figs. 5 (b), (e), (h), (k) (column 2) that the center of wave packet propagates along the axis of the antidot lattice when the initial pseudospin polarization is along the axis of the antidot lattice i.e. x axis, at \( t = 5\tau_o \) the dashed lines clearly show the propagation of the wave packet when compared with the solid lines at \( t = 1\tau_o \). Hence, the wave packet only propagates along the one dimensional antidot lattice when the initial pseudospin polarization is parallel to the axis of antidot lattice.

**Experimental relevance:** We now address the question of experimental relevance of this work. In this regard, we note that it is possible to initialize the pseudospin and study the dynamics of an electron wave packet in graphene by means of pump-probe laser spectroscopy. Experiments have been performed where pseudospin polarization and its subsequent relaxation have been probed on the femtosecond scale in graphene.\(^{26–28}\) Another possibility is the creation of artificial graphene (honeycomb lattices) in cold atom systems. In these systems, it is possible to generate effective fields and study transport in graphene-like structures.\(^{29–32}\) Recently it has become possible to treat pseudospin as a real measurable angular momentum at par with electron spin in photonic graphene (honeycomb array of evanescently coupled waveguides).\(^{14,30}\) These photonic systems allow great degree of control of initial conditions in the study of wave packet dynamics.

**VII. CONCLUSIONS**

We have studied the evolution of a two dimensional Gaussian wave packet in a graphene sheet placed in a uniform and perpendicular magnetic field. We have observed that the temporal dynamics of the wave packet strongly depends on the initial pseudospin polarization. We have shown that direction of propagation of the wave packet can be controlled through pseudospin polarization.

Further, we have studied the trembling motion (ZB) of the Gaussian wave packet in a graphene sheet placed in a quantizing magnetic field for non zero values of initial momentum \( k_{0z} \). Initially, these ZB oscillations seemingly die out but reappear i.e. the amplitude of the ZB oscillations starts to grow again without any fundamental frequency. So, for non zero magnetic fields, ZB oscillations have a recurrent character and they do not die with time. The quantized (discrete) energy spectrum, which is a consequence of the magnetic field, is the main reason for the recurrent character of ZB oscillations. This property is completely different from the zero magnetic field case, in which the energy spectrum is not quantized (discrete) and ZB of the wave packets has a transient character.\(^{33}\)

We have also studied wave packet dynamics in a one dimensional antidot lattice in graphene in the presence of the magnetic field. For this, we have determined the Green’s function matrices of the system both in frequency and time representations. From the poles of the Green’s function, we have found that along the axis of antidot lattice, the energy spectrum is composed of Landau minibands with a unit shift at \( K \) and \( K' \) points of graphene. In the time evolution of a wave packet, we find that wave packet dynamics is highly dependent on the initial pseudospin polarization; and the center of the Gaussian wave packet can be made to propagate along the antidot lattice by tuning the pseudospin parallel to the axis of antidot lattice. Also, when the strength of the antidot potential is greater than the strength of the magnetic field, the wave packet becomes more confined in space and the probability of finding it on the axis of antidot lattice significantly increases. Hence, we propose that quantum antidot channels can be made in which propagation of the wave packet can be controlled using pseudospin polarization and the strength of the antidot potential.

Finally, we would like to point out that this work may lead to new insights for controlling currents in both natural and artificial graphene systems by tuning initial pseudospin polarization, both in Landau quantized graphene and in a graphene antidot lattice in the presence of a magnetic field. This may lead to the preparation of graphene based nano gates in which two different pseudospin polarizations perpendicular to each other can be used to get ON and OFF states.

**Appendix A: Contour Integral**

To solve the second integral in Eq. (35), let us represent the integral by \( I \)

\[
I = \frac{\alpha d\omega_g}{2\pi} \sum_{n'=-\infty}^{\infty} \int_{-\infty}^{\infty} d\eta \ e^{-\omega_g n' t} G^0(x_1; n'd; \eta)
\]

\[
\int_{-\pi}^{\pi} dp \ e^{-\eta m'd} \left[I_2 - \alpha \tilde{G}^0(p;0,0;\eta)\right]^{-1} \tilde{G}^0(p;x_2;\eta).
\]

(A1)

\( I_2 \) is unit matrix of order 2) where

\[
\eta = \frac{\omega}{\omega_g} \quad \text{and} \quad d\omega = \omega_g d\eta
\]

Note that in the above integral, each Green’s function has a real pole at \( \eta = \pm \sqrt{n} \). The poles of the term

\[
T_1(\eta) = \left[I_2 - \alpha \tilde{G}^0(p;0,0;\eta)\right]^{-1} \tilde{G}^0(p;x_2;\eta)
\]

(A2)

can be seen to cancel by reexpressing the Green’s functions \( \tilde{G}^0(p;0,0;\eta) \) and \( \tilde{G}^0(p;x_2;\eta) \). For this purpose, by using Eqs. (24) and (25), one can easily write \( \alpha \tilde{G}^0(p;0,0;\eta) \) matrix as

\[
\alpha \tilde{G}^0(p;0,0;\eta) = \frac{1}{\prod_{n=0}(\eta^2 - n)} \begin{pmatrix}
a_{11} & a_{12} \\
a_{12} & a_{11}
\end{pmatrix}
\]

(A3)
where we have defined

\[ a_{11}(\eta) = a_1 \eta \sum_{m=0}^{\infty} L_m \left( \frac{\omega_T^2}{4 \gamma^2} \left( \frac{d}{l} \right)^2 \right)^2 \prod_{n=1, n \neq m}^{\infty} (\eta^2 - n) \]

and

\[ a_{12}(\eta) = a_2 \sum_{m=1}^{\infty} L_m^{1-m} \left( \frac{\omega_T^2}{4 \gamma^2} \left( \frac{d}{l} \right)^2 \right)^2 \prod_{n=1, n \neq m}^{\infty} (\eta^2 - n) \]

with

\[ a_1 = \frac{\alpha \omega_g}{4 \hbar \gamma^2} \sum_{j=-\infty}^{\infty} e^{ipjd} e^{-\frac{\omega_T^2}{\gamma^2} (j \frac{d}{l})^2} \]

and

\[ a_2 = \frac{i \alpha \omega_g l}{8 \pi \hbar \gamma^3} \sum_{j=-\infty}^{\infty} e^{ipjd} e^{-\frac{\omega_T^2}{\gamma^2} (j \frac{d}{l})^2} \]

Similarly, \( \tilde{G}^{0}(p; x_2; \eta) \) matrix can be written as

\[ \tilde{G}^{0}(p; x_2; \eta) = \prod_{n=0}^{\infty} (\eta^2 - n) \begin{pmatrix} b_{11} & b_{12} \\ b_{12} & b_{11} \end{pmatrix} \]  

(44)

where we have defined

\[ b_{11}(\eta) = \beta_1 \eta \sum_{m=0}^{\infty} L_m \left( \frac{\omega_T^2}{4 \gamma^2} \left( \frac{d}{l} \right)^2 \right)^2 \prod_{n=1, n \neq m}^{\infty} (\eta^2 - n) \]

and

\[ b_{12}(\eta) = \beta_2 \sum_{m=1}^{\infty} L_m^{1-m} \left( \frac{\omega_T^2}{4 \gamma^2} \left( \frac{d}{l} \right)^2 \right)^2 \prod_{n=1, n \neq m}^{\infty} (\eta^2 - n) \]

with

\[ \beta_1 = \frac{\omega_g}{4 \pi \hbar \gamma^2} \sum_{j=-\infty}^{\infty} e^{ipjd} e^{-\frac{\omega_T^2}{\gamma^2} (j \frac{d}{l} - x_2)^2} \]

and

\[ \beta_2 = \frac{i \omega_g l}{8 \pi \hbar \gamma^3} \sum_{j=-\infty}^{\infty} \left( \frac{j \frac{d}{l} - x_2}{l} \right) e^{ipjd} e^{-\frac{\omega_T^2}{\gamma^2} (j \frac{d}{l} - x_2)^2} \]

Note that in the above equations, \( n \) is Landau index and \( m \) is a dummy index for the Landau levels; the maximum value of \( n \) and \( m \) will be same.

Finally substituting Eqs. (43) and (44) in Eq. (42), matrix \( T_1 \) will become

\[ T_1(\eta) = \begin{pmatrix} \prod_{n=0}^{\infty} (\eta^2 - n) \\ \prod_{n=0}^{\infty} (\eta^2 - n) \end{pmatrix} \begin{pmatrix} a_{11}(\eta) & a_{12}(\eta) \\ a_{12}(\eta) & a_{11}(\eta) \end{pmatrix}^{-1} \begin{pmatrix} b_{11}(\eta) & b_{12}(\eta) \\ b_{12}(\eta) & b_{21}(\eta) \end{pmatrix}. \]

In the above expression, the real poles (\( \eta = \pm \sqrt{n} \)) cancel. The above expression can be solved numerically. The resultant 2 \( \times \) 2 matrix with \( c_{ij}(\eta) \) (where \( i, j = 1, 2 \)) as matrix elements can be written as (substituting \( q = pd \))

\[ Q(\eta) = \int_{-\pi}^{\pi} dq e^{-im\eta} T_1(\eta) = \begin{pmatrix} c_{11}(\eta) & c_{12}(\eta) \\ c_{21}(\eta) & c_{22}(\eta) \end{pmatrix}. \]  

(45)

Note that, above integral can be numerically calculated by applying trapezoidal rule in the limits \( -\pi \) to \( \pi \); this gives an accuracy up to five decimal points for each value of \( n' \).

Putting Eq. (45) in Eq. (41), integral \( I \) become

\[ I = \frac{\alpha \omega_g}{2 \pi} \sum_{n'=\infty}^{\infty} \int_{-\infty}^{\infty} d\eta e^{-i\omega_g \eta} G^{0}(x_1, n'd; \eta) Q(\eta). \]

(46)

Using Eqs. (33) and (34), matrix \( \alpha G^{0}(x_1, n'd; \eta) \) can be written as

\[ \alpha G^{0}(x_1, n'd; \eta) = \begin{pmatrix} \gamma_1 \eta & 0 \\ 0 & \gamma_1 \eta \end{pmatrix} + \sum_{n=1}^{\infty} \begin{pmatrix} \gamma_1 \eta & \gamma_1 \eta L_{n-1}[\Upsilon] \\ \gamma_1 \eta L_{n-1}[\Upsilon] & \gamma_1 \eta \end{pmatrix} \]

(47)

where \( \Upsilon \), \( \gamma_1 \) and \( \gamma_2 \) are defined in Eqs. (40) and (41).

Using matrices \( Q(\eta) \) and \( \alpha G^{0}(x_1, n'd; \eta) \) in Eq. (46), and breaking the matrix into two matrices, we get

\[ I = \frac{\omega_g}{2 \pi} \sum_{n'=\infty}^{\infty} \int_{-\infty}^{\infty} d\eta e^{-i\omega_g \eta} \begin{pmatrix} \gamma_1 \eta \frac{1}{n' \eta^2 - n} c_{11}(\eta) \\ \gamma_1 \eta \frac{1}{n' \eta^2 - n} c_{12}(\eta) \end{pmatrix} \]

and

\[ + \frac{\omega_g}{2 \pi} \sum_{n'=\infty}^{\infty} \int_{-\infty}^{\infty} d\eta e^{-i\omega_g \eta} \begin{pmatrix} M_{11}(\eta) & M_{12}(\eta) \\ M_{21}(\eta) & M_{22}(\eta) \end{pmatrix} \]

(48)

(First and second matrix corresponds to \( n = 0 \) and \( n > 0 \) Landau minibands respectively.) In above expression

\[ M_{11}(\eta) = \sum_{n=1}^{\infty} \begin{pmatrix} \gamma_1 \eta \frac{L_n[\Upsilon]}{n^2 - n} c_{11}(\eta) \\ \gamma_1 \eta \frac{L_n[\Upsilon]}{n^2 - n} c_{12}(\eta) \end{pmatrix} \]

and

\[ M_{12}(\eta) = \sum_{n=1}^{\infty} \begin{pmatrix} \gamma_1 \eta \frac{L_n[\Upsilon]}{n^2 - n} c_{21}(\eta) \\ \gamma_2 \frac{L_{n-1}[\Upsilon]}{n^2 - n} c_{21}(\eta) \end{pmatrix} \]

(49)

and

\[ M_{21}(\eta) = \sum_{n=1}^{\infty} \begin{pmatrix} \gamma_1 \eta \frac{L_n[\Upsilon]}{n^2 - n} c_{21}(\eta) \\ \gamma_2 \frac{L_{n-1}[\Upsilon]}{n^2 - n} c_{22}(\eta) \end{pmatrix} \]

In Eq. (48), the first matrix has a pole at \( \eta = 0 \), while the second matrix have poles at \( \eta = \pm \sqrt{n} \). We now use contour integration with Jordan lemma (solving the contour...
in lower half plane for \( t > 0 \) to evaluate the integrals. Results for the two terms in Eq. (A8) are

\[
\int_{-\infty}^{\infty} d\eta e^{-\omega g_l t / \gamma} \eta \frac{c_{ij}(\eta)}{\eta} = -i\pi \eta_+(t) [c_{ij}(\eta)]_{\eta=0}, \quad (A9)
\]

\[
\int_{-\infty}^{\infty} d\eta e^{-\omega g_l t / \gamma} M_{12}(\eta) = -i\pi \eta_+(t) \sum_{n=1}^{\infty} \cos \left( \frac{\omega g_l l \tau_0}{\gamma} \sqrt{n} \right)
\times \left[ \gamma_1 \mathcal{L}_n[\mathcal{Y}] c_{11}(\eta) + \frac{\gamma_2}{\sqrt{n}} \mathcal{L}_{n-1}[\mathcal{Y}] c_{22}(\eta) \right]_{\eta=\sqrt{n}} \quad (A10)
\]

and

\[
\int_{-\infty}^{\infty} d\eta e^{-\omega g_l t / \gamma} M_{21}(\eta) = -i\pi \eta_+(t) \sum_{n=1}^{\infty} \sin \left( \frac{\omega g_l l \tau_0}{\gamma} \sqrt{n} \right)
\times \left[ \gamma_1 \mathcal{L}_n[\mathcal{Y}] c_{21}(\eta) + \frac{\gamma_2}{\sqrt{n}} \mathcal{L}_{n-1}[\mathcal{Y}] c_{12}(\eta) \right]_{\eta=\sqrt{n}} \quad (A11)
\]

which we found during calculations. Hence Eq. (A9) along with Eqs. (A10) and (A11) gives the complete solution of the integral \( I \), which is the time representation of second term of full Green’s function \( G(x_1, x_2; t) \) given in Eq. (35).

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