Fast Quasi-Optimal Power Flow of Flexible DC Traction Power Systems

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Abstract—This paper proposes a quasi-optimal power flow (OPF) algorithm for flexible DC traction power systems (TPSs). Near-optimal OPF solutions can be solved with high computational efficiency by quasi-OPF. Unlike conventional OPF algorithms, quasi-OPF does not utilize mathematical optimization algorithms but adopts a new methodology. First, we adopt a new modeling method and successfully reveal the physical meaning of OPF solutions in flexible DC TPSs. Then, by converting the physical meaning of OPF solutions into mathematical expressions, a simple mapping from the power flow solution to the near-optimal OPF solution is obtained, and the quasi-OPF algorithm is designed based on power flow and this mapping. Since calculating power flow is computationally cheap and the mapping is based on simple arithmetic, the quasi-OPF algorithm can solve OPF with much less execution time, achieving a speed-up of 57 times compared to the primal-dual interior point method. The effectiveness is verified by mathematical proofs and a case study with Beijing Metro Line 13. This study provides insight into the physical meaning of OPF solutions and a tool for analyzing the effects of coordinated control, design real-time coordinated control strategies, and solve operational problems in planning.

Index Terms—Computational efficiency, coordinated control, flexible DC, near-optimal, optimal power flow, traction power system, urban rail transit, voltage sourced converter.

NOMENCLATURE

Abbreviations

TPS Traction power system.
TSS Traction substation.
VSC Voltage sourced converter.
OPF Optimal power flow.
NDSS Natural distribution subsystem.
CCSS Coordinated control subsystem.

Variables

\( U_{si} \) VSC DC voltage of the \( i \)th TSS.
\( U_s \) Vector of VSC DC voltage \( U_{si} \).
\( U^*_s \) Reference order of \( U_s \), also the control variable of the OPF problem.
\( U_0^* \) Additional term to calculate \( U^*_s \).
\( U_v \) Voltage of the \( i \)th rail vehicle.
\( U_i \) DC voltage of the \( i \)th bus in the DC TPS.
\( U_{ci,cc} \) Catenary voltage between the \( i \)th and the \((i+1)\)th TSS in the CCSS.
\( U_{cc} \) Vector of \( U_{ci,cc} \).
\( U_{rl,cc} \) Rail voltage between the \( i \)th and the \((i+1)\)th TSS in the CCSS.
\( U_{cc} \) Vector of \( U_{rl,cc} \).
\( U_{bi,cc} \) Branch voltage between the \( i \)th and the \((i+1)\)th TSS in the CCSS, i.e., \( U_{ci,cc} + U_{rl,cc} \).
\( U_{bi,cc} \) Vector of \( U_{bi,cc} \).
\( U_{bi,cc}^* \) Reference order of \( U_{bi,cc} \).
\( I_{si} \) VSC DC current of the \( i \)th TSS.
\( I_s \) Vector of VSC DC current \( I_{si} \).
\( I_i \) Current of the \( i \)th rail vehicle.
\( I_{si,nd} \) Natural distribution component of \( I_{si} \), also the VSC DC current of the \( i \)th TSS in the NDSS.
\( I_{si,cc} \) Coordinated control component of \( I_{si} \), also the VSC DC current of the \( i \)th TSS in the CCSS.
\( I_s \) Vector of \( I_{si,cc} \).
\( I_{sc,cc} \) Reference order of \( I_{sc,cc} \).
\( I_{cc} \) Catenary current between the \( i \)th and the \((i+1)\)th TSS in the CCSS.
\( I_{cc} \) Vector of \( I_{cc} \).
\( I_{cc} \) Reference order of \( I_{cc} \).
\( I_{rl,cc} \) Rail current between the \( i \)th and the \((i+1)\)th TSS in the CCSS.
\( I_{rl,cc} \) Vector of \( I_{rl,cc} \).
\( I_c(d) \) Current in the catenary with respect to location \( d \) in the original system.
\( I_{c,nd}(d) \) Current in the catenary with respect to location \( d \) in the NDSS.
\( I_{c,cc}(d) \) Current in the catenary with respect to location \( d \) in the CCSS.
\( I_{si,lim} \) Maximum allowable current of VSCs in TSS \( i \).
\( I_{aux} \) TSS \( i \)’s station auxiliary load current.
\( P_{si} \) VSC DC power of the \( i \)th TSS.
\( P_{v1} \) Power of the \( i \)th rail vehicle.

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$P_{\text{cost}}$ Total power obtained from AC utility.
$P_{\text{loss}}$ Power loss in the DC TPS.
$P_{\text{loss_rad}}$ Power loss in the NDSS.
$P_{\text{loss_cc}}$ Power loss in the CCSS.
$P_{\text{loss_cr}}$ Cross term in $P_{\text{loss}}$ expression.
$d$ Location that varies from 0 to D.
$Y_{ij}$ $(i, j)$th element in node conductance matrix.

**Constant Coefficients**

$N$ Number of TSSs.
$M$ Number of rail vehicles.
$r_{ci}$ Catenary resistance between the $i$th and the $(i+1)$th TSS in the CCSS.
$r_{ri}$ Rail resistance between the $i$th and the $(i+1)$th TSS in the CCSS.
$r_i$ Branch resistance between the $i$th and the $(i+1)$th TSS in the CCSS, i.e., $r_{ci}+r_{ri}$.
$r$ Resistance per unit length.
$G$ Branch conductance matrix in the CCSS.
$R$ Branch resistance matrix in the CCSS.
$D$ Total length of the metro line.
$D_{ui}$ Location of TSS $i$ in the up-track.
$D_{di}$ Location of TSS $i$ in the down-track.
$P_{\text{auxi}}$ TSS $i$’s station auxiliary load power.
$T_{ai}$ Upper bound of VSC DC power in TSS $i$.
$P_{\text{ai}}$ Lower bound of VSC DC power in TSS $i$.
$U$ Upper bound of bus voltage.
$U_{\text{ai}}$ Lower bound of bus voltage.

**I. INTRODUCTION**

DC TRACTION power systems (TPSs) are special power systems that energize rail vehicles in urban rail transit systems. The requirements for reliable, low carbon, and economical operation of TPSs are increasingly high [1]. Flexible DC TPSs that introduce voltage sourced converters (VSCs) into DC TPSs are promising for achieving economic and technical merits based on state-of-the-art power electronics and system-level coordinated control. Flexible DC TPSs have high converter-level controllability [2], enable system-level coordinated control strategies to optimize power flow [3], [4], and facilitate the accommodation of renewable energy [5], [6]. Eleven countries with 12 metro lines adopt flexible DC TPSs to achieve better performance [7].

The optimal power flow (OPF) seeks optimal steady-state setpoints that minimize a certain objective in consideration of various constraints and is widely acknowledged as a powerful tool for analysis, operations, and planning [8], [9]. Since the power flow of flexible DC TPSs can be optimized by system-level coordinated control, OPF is a fundamental tool to analyze the effects of coordinated control and provide an important basis for the design of practical coordinated control strategies.

Flexible DC TPSs are special microgrids, and there are three peculiarities in flexible DC TPSs:

- **subsecond change in rail vehicles’ power demand.** The power demand is very volatile because of sudden acceleration, coasting, or braking.
- **time varying rail vehicle positions.** Unlike fixed load positions in normal power systems, the loads, i.e., rail vehicles, move along the rails.
- **long-chain topology of the DC network.** The transmission lines are laid along the rails to energize running rail vehicles. Rails serve as the negative return paths. Hence, transmission lines and rail form a long-chain topology.

The former two peculiarities both highlight the importance of OPF computational efficiency. First, the rapid change in rail vehicles’ power demand necessitates a computationally efficient algorithm for real-time coordinated control. OPF must be conducted at the subsecond level for proper application in coordinated control. Otherwise, the dated OPF solutions may deteriorate the control performance. Second, in system planning, large quantities of operating scenarios need to be analyzed by OPF in consideration of possible rail vehicle positions, various operating modes, and even uncertainty. Solving the planning problem in consideration of such large quantities of operating scenarios is too time-consuming. In conclusion, solving OPF with higher computational efficiency is significant for flexible DC TPSs.

The OPF of flexible DC TPSs is studied in [10] to achieve better energy management. Similarly, the OPF of DC TPSs considering the parallel running of diode rectifiers and VSCs is studied in [11], [12], [13]. In addition, VSCs can achieve reactive power compensation in AC networks that are connected to DC TPSs, and the relevant AC OPF is well addressed in [14], [15]. These studies effectively realize the reliable and economical operation of TPSs but do not focus on the computational efficiency problem, and information about the OPF execution time has not been reported.

Various methods have been proposed to speed up OPF in power systems. Parallel computing is utilized to accelerate the computing speed of various algorithms [16], [17]. A parametric distribution optimal power flow method is proposed in [18] to improve computational efficiency, and the optimal setpoints can be calculated by analytical functions of the volatile renewable output. Machine learning based data-driven methods are a promising approach to increase computational efficiency [19], [20], [21]; however, the TPS topology is time varying due to the rapid change in rail vehicle positions, and solving OPF at the subsecond level with time varying topologies is still in the initial stage of exploration [22]. Sparsity is exploited to accelerate the computing speed in [23]. Linear approximation methods are adopted in [24] and [25] to simplify the nonlinear OPF problem. Reference [26] proposes an interior point Powerball algorithm to improve the search directions and to accelerate the OPF solution process. Especially aiming at real-time control problems, computationally efficient OPF algorithms have been developed in consideration of real-time measurement and communication [27], [28], [29], [30].

Quasi-OPF is proposed in our study to achieve fast OPF solving in flexible DC TPSs. It is an algorithm that obtains near-optimal solutions for OPF without using mathematical optimization algorithms. It shares many key similarities with conventional OPF:

- time varying rail vehicle positions. Unlike fixed load positions in normal power systems, the loads, i.e., rail vehicles, move along the rails.
- long-chain topology of the DC network. The transmission lines are laid along the rails to energize running rail vehicles. Rails serve as the negative return paths. Hence, transmission lines and rail form a long-chain topology.

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Quasi-OPF is proposed in our study to achieve fast OPF solving in flexible DC TPSs. It is an algorithm that obtains near-optimal solutions for OPF without using mathematical optimization algorithms. It shares many key similarities with conventional OPF:
the solutions are the same as or very close to conventional OPF’s.

A new methodology is adopted to develop the quasi-OPF algorithm. First, we adopt a new modeling method and successfully reveal the physical meaning of OPF solutions considering the special long-chain topology in flexible DC TPSs. The physical meaning of OPF solutions can be interpreted as follows: only traction substations (TSSs) whose power violates peak power constraints or is transmitted back to AC utility require coordinated support, only neighboring TSSs provide the required coordinated support, and the required coordinated support is assigned to neighboring TSSs in a proportion that optimizes power loss. Second, by converting the physical meaning of OPF solutions into mathematical expressions, a simple mapping from the power flow solution to the near-optimal OPF solution is obtained, and quasi-OPF is designed based on power flow and the simple mapping. In essence, quasi-OPF converts the optimization problem of OPF into power flow and the mapping. Since power flow algorithms are mature and computationally inexpensive and the mapping is based on simple arithmetic, quasi-OPF can achieve fast OPF solving in flexible DC TPSs.

The twofold contributions can be summarized as follows:

First, we reveal the physical meaning of OPF solutions based on a new modeling method. The modeling method is based on the superposition principle, which decomposes an original system into the superposition of a coordinated control subsystem (CCSS) and a natural distribution subsystem (NDSS). The effect of coordinated control can be revealed from the complex power flow by this modeling, which helps to reveal the physical meaning of OPF solutions.

Second, we propose a quasi-OPF algorithm that solves near-optimal solutions for OPF with high computational efficiency. The effectiveness is verified by mathematical proofs and a case study with Beijing Metro Line 13, in which quasi-OPF achieves a speed-up of 57 times.

This study is beneficial to both real-time control and planning of flexible DC TPSs. Since quasi-OPF can generate steady-state near-optimal control orders rapidly, a practical real-time control strategy can be designed based on quasi-OPF. However, a real-time control strategy needs to handle many problems other than steady-state optimization, such as communication network deployment. Quasi-OPF cannot be used directly in real-time control but provides an important basis for it. Furthermore, many planning frameworks utilize OPF in lower-level optimization to solve the operational problem [31], [32], [33], [34]. Quasi-OPF can be utilized to accelerate the operational problem solving in planning.

This paper is organized as follows. In Section II, the flexible DC TPSs’ basic configuration and OPF are introduced as preliminaries. In Section III, the system modeling based on the superposition principle is elaborated, and the physical meaning of OPF solutions is interpreted. In Section IV, the quasi-OPF algorithm is proposed. In Sections V and VI, a case study and conclusions are provided, respectively. The mathematical proofs are elaborated in the appendix.
utility and increases power loss. For countries such as China, transit properties cannot obtain revenue and may even have to pay fines to transmit power back to AC utility. Only the energy obtained from AC utility is charged in most metro lines, and the \( \max(x,0) \) function in (1) represents this practical charge rule.

Reducing energy consumption is equivalent to two goals: to recuperate more rail vehicles’ regenerative power and to lower power loss. Lowering power loss is important but less significant than recuperating regenerative power because the power loss is considerably less than the regenerative power.

C. Operational Constraints

To ensure reliable operation, there are two main constraints: the peak power of VSCs and the range of DC voltage.

The peak power of VSCs is limited to ensure VSC safety. In normal cases, the constraint of VSC peak power is more rigid than the constraint of power line peak power. Hence, the latter is omitted.

The DC voltage is limited to ensure reliable operation of rail vehicles. In TPSs whose rated voltage is 750 V, the voltage range is usually 500 V–900 V. When braking rail vehicles regenerate power back to catenaries, the maximum allowable voltage is changed from 900 V to 950 V.

D. OPF

The objective is (1). Since the voltage of VSCs can be controlled flexibly, the control variable is the voltage of VSCs. \( U_s \) is defined as the vector of VSC DC voltage:

\[
U_s = [ U_{s1} \cdots U_{si} \cdots U_{sN} ]^T
\]

where \( U_{si} \) is the VSC DC voltage of the \( i \)th TSS.

The constraints of OPF include equality constraints, i.e., power balance equations, and inequality constraints about operational constraints: VSC power constraints and DC voltage constraints. The OPF problem is formulated as follows:

\[
\begin{align*}
\min & \quad P_{\text{cost}} = \sum_{i=1}^{N} \max(P_{si} + P_{aux}, 0) \\
\text{s.t.} & \quad P_{si} = U_{si} \sum_{j=1}^{N+M} Y_{ij} U_j, \quad i = 1, 2, \cdots, N \\
& \quad P_{vi} = U_{vi} \sum_{j=1}^{N+M} Y_{(i+N)j} U_j, \quad i = 1, 2, \cdots, M \\
& \quad \frac{P_{si}}{U_{si}} \leq P_{si} \leq \frac{P_{aux}}{U_{si}}, \quad i = 1, 2, \cdots, N \\
& \quad U \leq U_i \leq U, \quad i = 1, 2, \cdots, N + M
\end{align*}
\]

where \( P_{vi} \) and \( U_{vi} \) are the power and voltage of the \( i \)th rail vehicle, respectively, and \( P_{vi} \) is known as input; \( M \) is the number of rail vehicles; \( Y_{ij} \) is the \((i,j)\)th element in the node conductance matrix; \( U_i \) is the voltage of the \( i \)th bus; concerning node number, \( N \) TSS nodes are in front and \( M \) load nodes are behind; \( P_{si} \) and \( P_{aux} \) are the upper and lower bounds of VSC power in TSS \( i \), respectively; and \( U \) and \( U \) are the upper and lower bounds of bus voltage, respectively.

III. SYSTEM MODELING AND ANALYSIS

A. System Modeling

In system modeling, an original system is shown in Fig. 2(a). We assume rail vehicles as current sources and TSSs as voltage sources. For the \( i \)th TSS, its DC voltage and current are \( U_{si} \) and \( I_{vi} \), respectively. For the \( j \)th rail vehicle, its DC current is \( I_{vj} \). Note that the parallel up-track and down-track are omitted in Fig. 2 for simplicity.

Since the time scale of VSC control is much shorter than the time scale of power flow change, the detailed internal operation and control of VSCs are not considered in the OPF. In our study, VSCs are viewed as ideal voltage sources.

In normal OPF or power flow modeling, the loads are usually modeled as constant power nodes, and the power value is known by the input. However, constant power nodes introduce nonlinearity in the system modeling, which is difficult to conduct theoretical analysis. Therefore, based on the substitution theorem in circuit theory, we model the loads, i.e., the rail vehicles, as current sources, and the current value is determined to ensure that the load power is equal to the input value:

\[
I_{vi} = \frac{P_{vi}}{U_{vi}}
\]

Hence, the load current reflects the load power.

Therefore, there are two kinds of excitation sources in the original system, i.e., the voltage sources of TSSs and the current sources of rail vehicles. The voltages of TSSs are control variables that reflect the coordinated control effect, whereas the currents of rail vehicles reflect the power of loads.

Based on the superposition principle in circuit theory, the original system in Fig. 2(a) is equivalent to the superposition principle.
of the NDSS in Fig. 2(b) and the CCSS in Fig. 2(c). The NDSS only reserves the current sources of rail vehicles, whereas the CCSS only reserves the voltage sources of TSSs. Note that in Fig. 2, blue variables are related to the NDSS, whereas the red variables are related to the CCSS.

In the NDSS shown in Fig. 2(b), the “natural distribution” means the power flow exclusively excited by the current sources of rail vehicles. Since the currents of rail vehicles reflect the power of loads, the power flow of the NDSS represents the natural power flow distribution of loads.

In the CCSS shown in Fig. 2(c), the power flow is exclusively excited by the voltage sources of TSSs. Since the voltages of TSSs are control variables that reflect the coordinated control effect, the power flow of the CCSS is dependent on coordinated control, reflecting the coordinated control effect.

We use \( U_s \) and \( I_s \) to denote the vectors of TSS voltage and current, respectively. In the NDSS, the \( i \)th TSS’s current is \( I_{s_i, nd} \), which is the natural distribution component of TSS current \( I_s \). In the CCSS, the \( i \)th TSS’s current is \( I_{s_i, cc} \), which is the coordinated control component of TSS current \( I_s \). Based on the superposition principle, \( I_s \) can be decomposed into two components:

\[
I_s = I_{s, nd} + I_{s, cc}
\]

where \( I_{s, nd} \) and \( I_{s, cc} \) are the vectors of \( I_{s_i, nd} \) and \( I_{s_i, cc} \), respectively.

In the CCSS shown in Fig. 2(c), considering the branch between the \( i \)th TSS and the \( (i+1) \)th TSS, \( r_{c_i} \) and \( r_{c_i} \) denote the catenary and rail resistance, respectively; \( I_{c_{i, cc}} \) and \( U_{c_{i, cc}} \) denote the catenary current and voltage, respectively; \( r_{c_{i, cc}} \) and \( U_{c_{i, cc}} \) denote the rail current and voltage, respectively. Since the CCSS is a two-port network, based on Kirchhoff’s current law:

\[
I_{c_{i, cc}} = I_{r_{i, cc}}
\]

where \( I_{c_{i, cc}} \) and \( I_{r_{i, cc}} \) are the vectors of \( I_{c_{i, cc}} \) and \( I_{r_{i, cc}} \), respectively.

Define the branch voltage \( U_{b_{i, cc}} \) to simplify the analysis:

\[
U_{b_{i, cc}} = U_{c_{i, cc}} + U_{r_{i, cc}}
\]

where \( U_{b_{i, cc}} \), \( U_{c_{i, cc}} \), and \( U_{r_{i, cc}} \) denote the vectors of \( U_{b_{i, cc}} \), \( U_{c_{i, cc}} \), and \( U_{r_{i, cc}} \), respectively. Based on Kirchhoff’s voltage law:

\[
U_{b_{i, cc}} = \begin{bmatrix}
1 & -1 & 0 & \cdots & 0 & 0 \\
0 & 1 & -1 & \cdots & 0 & 0 \\
0 & 0 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & -1 & 0 \\
0 & 0 & 0 & \cdots & 1 & -1
\end{bmatrix}
\]

Define conductance matrix \( G \):

\[
G = \text{diag}(r_{c_1} + r_{r_1})^{-1}, (r_{c_2} + r_{r_2})^{-1}, \cdots, (r_{c(N-1)} + r_{r(N-1)})^{-1})
\]

Based on Ohm’s law:

\[
I_{c_{i, cc}} = GU_{b_{i, cc}}
\]

B. Physical Meaning of OPF Solutions

The physical meaning of OPF solutions can be illustrated by a typical OPF solution, as shown in Fig. 3. The parameters and descriptions of the case are given in the appendix. Based on the system modeling, the OPF solution can be interpreted from three aspects:

1) Limiting VSC Peak Power: \( I_{s, nd} \) is large, which means that the power demand of rail vehicles near TSS 2 is large. If there is no coordinated support, VSCs in TSS 2 would violate the peak power constraint.

The optimal TSS voltages form a hollow in the area of TSS 1-3 to limit the peak power of TSS 2. Since the voltages of TSS 1 and 3 are higher than TSS 2, TSS 1 and 3 output more power to energize rail vehicles near TSS 2, and hence, TSS 2 can lower its power output.

2) Recovering Regenerative Power: \( I_{s, nd} \) is negative and large, which means that there is large regenerative power near TSS 2. If there is no coordinated support, TSS 2 cannot recuperate all the nearby regenerative power and would transmit power back to AC utility.

The optimal TSS voltages form a peak in the area of TSS 19-21 to recuperate the regenerative power near TSS 20. Since the voltages of TSS 19 and 21 are lower than TSS 20, the braking rail vehicles can transmit their regenerative power to rail vehicles near TSS 19 and 21, and hence, TSS 20 can avoid transmitting power back to AC power sources.
III. QUASI-OPF ALGORITHM

A. Overview of the Algorithm

The flow chart of the algorithm is shown in Fig. 4. There are four steps in the algorithm. For clarity, the corner mark * is added as a suffix for variables as reference orders.

![Fig. 4. Flow chart of the quasi-OPF algorithm.](image)

3) Lowering Power Loss: As observed in Fig. 3, the coordinated support only comes from neighboring TSSs. In addition, the coordinated support is assigned to neighboring TSSs inversely proportional to the branch resistances between the target TSS and each TSS, i.e.,

\[
I_{s1,cc} = I_{s3,cc} \approx (r_{c2} + r_{r1}) : (r_{c1} + r_{r1}) \\
I_{s19,cc} = I_{s21,cc} \approx (r_{c20} + r_{r19}) : (r_{c19} + r_{r19})
\]

(12)
(13)

This pattern can be referred to as a proximity principle. The proximity principle can be understood from the aspects of power loss and voltage drop. If the proximity principle does not hold true, then the further the supportive TSS is, the more coordinated support it provides; then, \(I_{s,cc}\) would flow through a larger branch resistance, resulting in larger power loss and larger voltage fluctuation.

In conclusion, the coordinated control component of TSS current \(I_{s,cc}\) can be decomposed from TSS current \(I_s\) based on the proposed system modeling. \(I_{s,cc}\) reveals the essence of coordinated support. The optimal \(I_{s,cc}\) covers the demands to satisfy the peak power constraints and minimize the objective. The optimal \(I_{s,cc}\) can be realized by adjusting the control variable \(U_s\).

IV. QUASI-OPF ALGORITHM

A. Overview of the Algorithm

The input data include rail vehicle timetables, i.e., speed, location, and power of rail vehicles versus time, physical parameters, operational conditions, and constraints. Before step 1, the reference order \(U^\circ\) should be initialized. We use the nominal value of DC voltage to perform the initialization.

Step 1 is calculating the power flow, which can be conducted by mature algorithms [35], [36], such as the Newton–Raphson method.

Steps 2–4 utilize the modeling proposed in Section III-A. Step 2 calculates \(I_{s,nd}\) based on the power flow solutions. Based on the TSS current \(I_s\) and voltage \(U_s\) calculated in step 1, \(I_{s,cc}\) can be calculated using (8), (10), and (11), and then \(I_{s,nd}\) can be calculated by (5).

Step 3 calculates \(I_{s,cc}^*\) based on the \(I_{s,nd}\) from step 2, which is the most pivotal part of the algorithm. As explained in Section III-B, \(I_{s,cc}^*\) is the core variable revealing the essence of coordinated support. \(I_{s,cc}^*\) is the reference order for \(I_{s,cc}\). The mapping from \(I_{s,nd}\) to \(I_{s,cc}^*\) is obtained by converting the physical meaning of OPF solutions into mathematical expressions. Hence, the results of \(I_{s,cc}^*\) ensure that the solutions of quasi-OPF have the same physical meaning as OPF solutions: limiting VSC peak power, recuperating regenerative power, and lowering power loss.

Step 4 calculates the final reference order \(U_s^*\) based on \(I_{s,cc}^*\), which is basically the inverse process of step 2. It is worth mentioning that in some cases, the range of \(U_s^*\) may violate the DC voltage constraints because not all regenerative power can be recuperated. Then, some regenerative power can be allowed to be transmitted back to AC utility to ensure that \(U_s^*\) satisfies the DC voltage constraints.

The details of steps 3 and 4 are elaborated as follows.

B. Step 3: Calculation of Current Reference \(I_{s,cc}^*\)

Similar to the physical meaning interpretation in Section III-B, \(I_{s,cc}^*\) is solved considering three aspects:

1) Limiting VSC Peak Power: \(I_{s,lim}\) denotes TSS i’s VSC maximum allowable current which can be calculated by:

\[
I_{s,lim} = \frac{P_{si}}{U_{si}}
\]

(14)

The VSC peak power constraints relate to the upper bounds and the lower bounds. In this paper, we only consider the upper bounds in the \(I_{s,cc}^*\) calculation for simplicity, whereas the \(I_{s,cc}^*\) calculation concerning the lower bounds can be easily conducted in a similar way.

If \(I_{s,nd}\) is larger than \(I_{s,lim}\) and there is no coordinated support, the power of the VSCs in TSS i would violate the VSC power constraint. Hence, TSS i requires coordinated support. The supportive TSSs should increase their DC voltage and transmit more current to rail vehicles near TSS i. \(I_{s,cc}^*\) is determined by

\[
I_{s,cc}^* = I_{s,lim} - I_{s,nd}
\]

(15)

which means that the excessive power demand of rail vehicles near TSS i is energized by supportive TSSs, and the VSC power of TSS i is limited exactly at the upper bound.
2) Recuperating Regenerative Power: $I_{aux}$ is used to denote TSS $i$’s station auxiliary load current, which can be calculated by:

$$I_{aux} = P_{aux}/U_{si}$$

(16)

where $P_{aux}$ is TSS $i$’s station auxiliary load power.

Assuming $I_{aux}$ is lower than $-I_{aux}$, if there is no coordinated support, VSCs in TSS $i$ would transmit regenerative power back to AC utility. Hence, TSS $i$ requires coordinated support. The supportive TSSs should lower their DC voltage to ensure that the regenerative power near TSS $i$ is recuperated by rail vehicles near the supportive TSSs. $I_{s,cc}$ is determined by:

$$I_{s,cc} = -I_{auxx} - I_{s,nd}$$

(17)

which means that the excessive regenerative power of rail vehicles near TSS $i$ is transferred to other rail vehicles and that there is exactly no power transmitted from TSS $i$ to AC utility.

3) Lowering Power Loss: The $I_{s,cc}$ of TSSs that require coordinated support can be simply calculated by (15) or (17). The $I_{s,cc}$ of supportive TSSs should be optimized to lower power loss and is calculated based on the proximity principle. The proximity principle can be interpreted as follows: only neighboring TSSs provide coordinated support; the closer the supportive TSS is, the more coordinated support it provides. The effectiveness of the proximity principle can be proven by mathematical derivation, which is elaborated as follows:

**Proposition 1:** Minimizing $P_{loss}$ is approximately equivalent to minimizing $P_{loss,cc}$.

The proof is shown in the appendix, where $P_{loss}$, $P_{loss,nd}$, and $P_{loss,cc}$ denote the power loss in the original system, the NDSS, and the CCSS, respectively.

Proposition 1 helps simplify the problem. In Propositions 2 and 3, minimizing $P_{loss}$ is substituted by minimizing $P_{loss,cc}$. However, note that Proposition 1 is only approximately right. As shown in the proof of Proposition 1, we assume $P_{loss,nd}$ is independent of coordinated control. However, coordinated control actually has an indirect effect on $P_{loss,nd}$: $P_{loss,nd}$ is affected by $I_{c,nd}(d)$; $I_{c,nd}(d)$ is affected by load current; as (4) implies, the load current is affected by load voltage; and the load voltage is affected by coordinated control. Therefore, this assumption is not strictly right, and the assumption is strictly right only if the loads have a constant current. OPF lowers the power loss in the original system, whereas quasi-OPF actually lowers the power loss in the CCSS, leading to the difference between the quasi-OPF and the actual OPF solutions. In the case study, we analyze this difference at three typical instants.

**Proposition 2:** To minimize $P_{loss,cc}$, if TSS $(m-1)$ and $(m+1)$ do not need coordinated support, then $I_{s,m,cc} = 0$.

In other words, $I_{s,cc}$ is nonzero when and only when its neighboring TSSs need coordinated support. The proof is shown in the appendix. In the process of proof, a more powerful corollary can be derived:

**Corollary 1:** If TSS $(m-1)$ and $(m+1)$ do not need coordinated support, then branch currents $I_{c,m-1,cc}$ and $I_{c,m+1,cc}$ are 0, which means that the left side of TSS $(m-1)$ is isolated from the right side of TSS $(m+1)$.

The proof is shown in the appendix. Corollary 1 means that the TSSs on the left side of TSS $(m-1)$ and the TSSs on the right side of TSS $(m+1)$ are autonomous. The two parts do not affect each other at all. Then, the last proposition can be derived:

**Proposition 3:** To minimize $P_{loss,cc}$, if TSS $m$ needs coordinated support, and TSS $p$ and $q$ are supportive TSSs, then the coordinated support is assigned to TSS $p$ and $q$ reverse proportionally to the branch resistances between TSS $m$ and each TSS.

The proof is shown in the appendix.

In conclusion, based on Propositions 1, 2, and 3, we prove that adopting the proximity principle to calculate the $I_{s,cc}$ of supportive TSSs can approximately minimize power loss.

C. Step 4: Calculation of Voltage Reference $U^*$

$I_{cc}$ can be calculated based on (29). Then, $U_{b,cc}^*$ is calculated by:

$$U_{b,cc}^* = RI_{c,cc}^*$$

(18)

Then, $U^*$ can be calculated:

$$U^* = \begin{bmatrix} 1 & 1 & \cdots & 1 & 1 \\ 0 & 1 & \cdots & 1 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 \cdots & 1 & 1 \\ 0 & 0 \cdots & 0 & 1 \\ 0 & 0 \cdots & 0 & 0 \end{bmatrix} U_{b,cc}^* + U_0^*$$

(19)

$U_0^*$ is an additional term to ensure that $U^*$ satisfies the DC voltage constraints. $U_0^*$ can be flexibly chosen and can be determined by:

$$U_0^* = \overline{U} - \max \begin{bmatrix} 1 & 1 & \cdots & 1 & 1 \\ 0 & 1 & \cdots & 1 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 \cdots & 1 & 1 \\ 0 & 0 \cdots & 0 & 1 \\ 0 & 0 \cdots & 0 & 0 \end{bmatrix} U_{b,cc}^*$$

(20)

which renders the highest TSS voltage reaching the maximum allowable voltage to make the TSS voltage as high as possible to lower the power loss.

V. CASE STUDY

A. Comparisons in the Whole Running Cycle

Comparisons are conducted to verify the near-optimal performance of the proposed quasi-OPF algorithm. We run the simulation for 5439 seconds which is the whole running cycle of rail vehicles in Beijing Metro Line 13. The comparisons are recorded in Table I, where the energy consumption represents the objective in (1), and the regenerative power recuperation rate is defined as the ratio between the regenerative power recuperated by other rail vehicles and the total regenerative power.

The energy consumption optimization performance and voltage range are very close, and all operation constraints are satisfied, as shown in Table I, indicating that the proposed algorithm simulates the OPF successfully.
TABLE I

| Performance Index               | QUAS-OPF | OPF  |
|---------------------------------|---------|------|
| Energy consumption (10^4 kWh)   | 6,343   | 6,328|
| Regenerative power recuperation rate | 91.70% | 91.80%|
| Voltage range of TSSs (kV)      | 0.65-0.88 | 0.65-0.88 |
| Voltage range of rail vehicles (kV) | 0.58-0.94 | 0.58-0.94 |
| Maximum power of VSCs (MW)      | 11.0    | 11.0 |

B. Comparisons at Three Typical Instants

The comparisons of quasi-OPF solutions and OPF solutions at three typical instants are elaborated to further verify that quasi-OPF can solve near-optimal OPF solutions.

The optimization effects of limiting TSS peak power and lowering power loss can be shown by the solutions at typical instant 1. Typical instant 1 is the 3518th second in the whole running cycle. The solutions are shown in Figs. 5 and 6, and they are nearly identical.

As shown in Fig. 5 or Fig. 6, \( I_{s2_{nd}} \) and \( I_{s18_{nd}} \) are so large that the VSCs in TSSs 2 and 18 would violate the peak power constraints if there is no coordinated control. The power of the VSCs in TSSs 2 and 18 is limited at the maximum allowable value by the coordinated support from neighboring TSSs.

In addition, the effectiveness of the proximity principle is verified by the \( I_{s_{cc}^*} \) of OPF, as shown in Fig. 6. The power loss of quasi-OPF is 1.404 MW, whereas the power loss of OPF is 1.397 MW, indicating the small difference in the power loss lowering performance between the two algorithms.

The optimization effects of recuperating regenerative power can be shown by the solutions at typical instant 2. Typical instant 2 is the 2508th second in the whole running cycle. The solutions of quasi-OPF and OPF are shown in Figs. 7 and 8, respectively. The solutions are very close.

As shown in Fig. 7 or Fig. 8, \( I_{s3_{nd}} \), \( I_{s4_{nd}} \), \( I_{s17_{nd}} \), \( I_{s18_{nd}} \), and \( I_{s22_{nd}} \) are negative and lower than the corresponding \( I_{aux_{i}} \), indicating that TSS 3, 4, 17, 18, and 22 would not recuperate all the regenerative power if there is no coordinated control.

C. Computational Efficiency Comparisons

The computational efficiency comparisons are listed in Table II. The proposed quasi-OPF algorithm takes 1/57 of the time of OPF. All computations are carried out using MATLAB on a
The speed-up effect can be roughly interpreted as follows: Supposing there are $N$ TSSs and $M$ rail vehicles, the number of nodes is $(N+M)$. Usually, the calculation time of OPF is proportional to $(N+M)^3$. In the proposed algorithm, step 1 is a normal power flow calculation whose calculation time is proportional to $(N+M)^2$; the calculation time of steps 2, 3, and 4 is proportional to $N$. Hence, the calculation time of the proposed algorithm should be approximately $1/(N+M)$, i.e., $1/69$, of the OPF’s.

The frequency statistics for the number of iterations are shown in Fig. 11. Over 95% of the quasi-OPF calculation can be finished within 3 iterations, whereas OPF based on the primal-dual interior point method needs approximately 50 iterations to converge. It is worth mentioning that the iterations of the proposed quasi-OPF are not due to repeated adjustment of optimization directions because quasi-OPF does not utilize mathematical optimization algorithms. The iterations of quasi-OPF are due to the algorithm’s structure and the nature of the problem.

### TABLE II

| EXECUTION TIME (s) | QUASI-OPF | OPF   |
|-------------------|-----------|-------|
| Whole operation cycle (5439 instants) | 389.5     | 21650.7 |
| Average value for a single instant    | 0.07      | 3.98  |

Fig. 7. The quasi-OPF solution at typical instant 2.

Fig. 8. The OPF solution at typical instant 2.

Fig. 9. The quasi-OPF solution at typical instant 3.

Fig. 10. The OPF solution at typical instant 3.

**Fig. 11. Iteration times frequency statistics.**
to the constant source modeling for rail vehicles; we assume rail vehicle nodes are constant current nodes, as shown in Fig. 2, but actually rail vehicle nodes have constant power; therefore, the proposed algorithm needs two to five iterations to calculate the current value by (4) to ensure that the rail vehicle power is equal to the input value.

VI. CONCLUSION

Flexible DC TPSs have many economic and technical merits based on state-of-the-art power electronics and system-level coordinated control. Fast OPF calculation is required to achieve better analysis, operation, and planning of flexible DC TPSs. In this study, we propose the quasi-OPF algorithm to solve the problem of OPF computational efficiency. We first reveal the physical meaning of OPF solutions based on a new modeling method. The modeling method is based on the superposition principle, which decomposes the original system into the superposition of the CCSS and the NDSS. Second, the quasi-OPF algorithm is designed to solve near-optimal OPF solutions based on the physical meaning interpretation of OPF solutions. The effectiveness is verified by mathematical proofs and a case study with Beijing Metro Line 13. Quasi-OPF achieves a speed-up of 57 times and is solved in 0.07 s on average. Comparisons of quasi-OPF solutions and OPF solutions are conducted in the whole running cycle and at three typical instants to show the near-optimal performance of the quasi-OPF solutions.

APPENDIX

A. Parameters

As a typical case, Beijing Metro No. 13 is a subway line in the reconstruction stage, after which its conventional TPS will be substituted by a flexible DC TPS, and the proposed quasi-OPF algorithm is promising for use in the real-time control strategy design and planning of Beijing Metro No. 13.

The basic parameters are shown in Table III. There are 23 TSSs in the DC TPS. Selecting 2 minutes as the headway, there are 46 rail vehicles running in the metro line. The station auxiliary loads of different TSSs vary, as shown in Table IV. The peak power demand of station auxiliary loads is much less than that of rail vehicles.

B. Proof of Proposition 1

Use $d$ to denote the location that varies from 0 to $D$. $D$ is the total length of the metro line. $I_c(d)$, $I_{c,\text{nd}}(d)$, and $I_{c,\text{cc}}(d)$ denote the current in the catenary with respect to location $d$ in the original system, the NDSS, and the CCSS, respectively. Based on the superposition principle, $I_c(d)$ can be decomposed:

$$I_c(d) = I_{c,\text{nd}}(d) + I_{c,\text{cc}}(d)$$

(21)

The power loss $P_{\text{loss}}$ can be expressed as:

$$P_{\text{loss}} = \int_0^D r I_c^2 (d) \, \text{d}(d)$$

$$= \int_0^D r I_{c,\text{nd}}^2 (d) \, \text{d}(d) + \int_0^D r I_{c,\text{cc}}^2 (d) \, \text{d}(d)$$

(22)

where

$$P_{\text{loss,nd}} = \int_0^D r I_{c,\text{nd}}^2 (d) \, \text{d}(d)$$

$$P_{\text{loss,cc}} = \int_0^D r I_{c,\text{cc}}^2 (d) \, \text{d}(d)$$

(23)

$P_{\text{loss,nd}}$ and $P_{\text{loss,cc}}$ mean the power loss in the NDSS and the CCSS, respectively. $P_{\text{loss,cr}}$ is a cross term that does not have physical meaning. $r$ represents the resistance per unit length, including both catenary resistance and rail resistance.

$P_{\text{loss,cr}}$ can be further elaborated:

$$P_{\text{loss,cr}} = 2 \sum_{i=1}^{N-1} I_{c,\text{cc}}(d) \int_{D_{ui}}^{D_{ui}+1} I_{c,\text{nd}}(d) \, \text{d}(d)$$

(24)

where $D_{ui}$ and $D_{di}$ are the locations of TSS $i$ in the up-track and the down-track, respectively. Based on Kirchhoff’s voltage law in the NDSS:

$$\int_{D_{ui}}^{D_{ui}+1} I_{c,\text{nd}}(d) \, \text{d}(d) = 0, \int_{D_{di}}^{D_{di}+1} I_{c,\text{nd}}(d) \, \text{d}(d) = 0$$

(25)

Then,

$$P_{\text{loss,cr}} = 0$$

(26)
Since $I_{\text{rd}}(d)$ is the natural distribution of rail vehicles’ current in the catenary, $I_{\text{rd}}(d)$ mainly depends on load conditions. Therefore, $P_{\text{loss,rd}}$ mainly depends on load conditions and is less influenced by coordinated control. $P_{\text{loss,cc}}$ is the only term in (23) that is mainly influenced by coordinated control. By assuming $P_{\text{loss,rd}}$ is constant and independent of coordinated control, minimizing $P_{\text{loss}}$ is equivalent to minimizing $P_{\text{loss,cc}}$.

### C. Proof of Proposition 2

Based on Kirchhoff’s current law:

$$\sum_{i=1}^{N} I_{si,cc} = 0$$  \hspace{1cm} (27)

Then, the following equations can be derived:

$$I_{s(m-1),cc} = -I_{sm,cc} - I_{s(m+1),cc} - \sum_{i=1}^{m-2} I_{si,cc} - \sum_{i=m+2}^{N} I_{si,cc}$$  \hspace{1cm} (28)

Based on Kirchhoff’s current law:

$$I_{cc,cc} = AI_{s,cc}$$  \hspace{1cm} (29)

where the matrix $A$ is defined by:

$$A = \begin{bmatrix}
1 & 0 & 0 & \cdots & 0 & 0 & 0 \\
1 & 1 & 0 & \cdots & 0 & 0 & 0 \\
1 & 1 & 1 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
1 & 1 & 1 & \cdots & 1 & 0 & 0 \\
1 & 1 & 1 & \cdots & 1 & 1 & 0
\end{bmatrix}$$

Define the resistance matrix $R$:

$$R = \text{diag}(r_{c1} + r_{i1}, r_{c2} + r_{i2}, \ldots, r_{ci} + r_{zi}, \ldots, r_{c(N-1)} + r_{z(N-1)})$$  \hspace{1cm} (31)

For simplicity, we can use $r_{i}$ to represent $r_{c1} + r_{i1}, P_{\text{loss,cc}}$ can be calculated by:

$$P_{\text{loss,cc}} = I_{cc,cc}^{T} R I_{cc,cc} = I_{s,cc}^{T} R I_{s,cc}$$  \hspace{1cm} (32)

where

$$R = A^{T} R A$$  \hspace{1cm} (33)

Solving (36) based on (34) and (35), we can conclude:

$$\sum_{i=1}^{m-1} I_{si,cc} = 0, \sum_{i=1}^{m} I_{si,cc} = 0$$  \hspace{1cm} (37)

Based on (37), it is easy to conclude:

$$I_{sm,cc} = 0$$  \hspace{1cm} (38)

### D. Proof of Corollary 1

Based on (29), we can learn that

$$I_{c(m-1),cc} = \sum_{i=1}^{m-1} I_{si,cc}, I_{cm,cc} = \sum_{i=1}^{m} I_{si,cc}$$  \hspace{1cm} (39)

Based on (37) and (39), we can conclude:

$$I_{c(m-1),cc} = 0, I_{cm,cc} = 0$$  \hspace{1cm} (40)

### E. Proof of Proposition 3

Assume the TSSs from TSS $m$ to TSS $(m+z)$ need coordinated support. Based on Proposition 2, only TSS $(m-1)$ and $(m+z+1)$ provide coordinated support to these TSSs.

Based on (27), the following equations can be derived:

$$I_{s(m-1),cc} = -I_{s(m+z+1),cc} - \sum_{i=1}^{m-2} I_{si,cc} - \sum_{i=m+2}^{N} I_{si,cc}$$  \hspace{1cm} (41)

$$\frac{dP_{\text{loss,cc}}}{dI_{s(m+z+1),cc}} = -2 \left( \sum_{i=m+1}^{m+z} r_{i} \sum_{i=1}^{m-1} I_{si,cc} \right) + \sum_{j=m}^{m+z} \left( I_{sj,cc} \sum_{i=j}^{m+z} r_{i} \right)$$  \hspace{1cm} (42)

To optimize $P_{\text{loss,cc}}$, the following equation should be satisfied:

$$\frac{dP_{\text{loss,cc}}}{dI_{s(m+z+1),cc}} = 0$$  \hspace{1cm} (43)

Based on corollary 1, the TSSs from the first TSS to the $(m-2)$th TSS and the TSSs from the $(m+z+2)$th TSS to the $N$th TSS does not affect the area from TSS $(m-1)$ to TSS $(m+z+1)$. Based on (42) and (43), we can derive that

$$\sum_{i=m-1}^{m+z} r_{i} I_{s(m-1),cc} + \sum_{j=m}^{m+z} \left( I_{sj,cc} \sum_{i=j}^{m+z} r_{i} \right) = 0$$  \hspace{1cm} (44)

Then, we can conclude:

$$I_{s(m-1),cc} = - \sum_{j=m}^{m+z} \left( I_{sj,cc} \sum_{i=j}^{m+z} r_{i} \right) / \sum_{i=m-1}^{m+z} r_{i}$$  \hspace{1cm} (45)

which is the mathematical expression of Proposition 3.
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