Mathematical Model for Natural Frequency of Prestressed Concrete beam using STAAD.Pro.

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Abstract. The problem of assessing the variation in natural frequency of a prestressed and un-prestressed beam has been of considerable interest to Structural Engineers. The objective of the present work is to find the effect of pre-stressing on Natural frequency of a Concrete beam and thereby establishing a relationship between the natural frequency of 1st mode of vibration and other modes. Conventional methods were used to calculate the natural frequency of 1 to 5 modes of vibration for un-prestressed beams while STAAD.Pro was used for assessing the same for prestressed beams. A 20 m span beam with different live loads and cross-sections were considered for the study. From the results, it was observed that as the cross-section of the beam increases, the natural frequency decreases. Comparing the natural frequency of 1st mode of vibration for beams with and without pre-stressing, it can be observed that within the scope of this work, the natural frequency of the un-prestressed beam with live load 15 kN/m is more than that for the prestressed beam. For the other two load-carrying capacities, it is vice-versa. It was also observed that the % difference of Natural frequency of a particular mode of vibration w.r.t mode 1 natural frequency is almost constant. In other words, the effect of live load and cross-section is negligible on the % difference of Mode ‘i’ to Mode 1 natural frequency. Based on this characteristic, a mathematical model has been developed.

Keywords: Prestressed concrete; reinforced concrete; simply supported; natural frequency; modes of vibration; STAAD.Pro.;

1. Introduction
The application of the prestressed tendons made from the solid bars has an essential influence on the architectural expression for large-span structures. Historically, pre-stressing was developed simultaneously for concrete and steel structures (Trotsky, 1989). Prestressed members are widely being used presently in the construction industry. Because of the need for earthquake resistance of all constructions, determination of their natural frequency is necessary. Many empirical formulae are available but are based on Assumptions as per Literature. This paper demonstrates the application STAAD.Pro for finding the natural frequency of prestressed concrete beams. On comparing the natural frequency of prestressed and un-prestressed concrete beams, it is observed that just like in un-prestressed. Beams, there is a relationship between the natural frequency of the 1st mode of vibration and other modes. This phenomenon is applied to develop a mathematical model for finding the natural frequency of any mode of vibration if the natural frequency of 1st mode is known.
2. Literature Review

In the available literature concerning pre-stressed structures, it was found that the literature on the natural frequency of pre-stressed structures is less. However, a review of past studies on the analysis of the natural frequency of pre-stressed steel beams is presented. Kato and Shimada (1986) have found a decrease in natural frequency with damage in a bridge girder. They have attributed this to the loss of flexural rigidity due to opening up the cracks as the load is increased. One of the oldest papers on this topic has been written by Tang and Leu (1989). They studied a three-span countryside prestressed concrete bridge and concluded that the change of dynamic characteristics can be used as an indicator for damage detection of the bridge. It is therefore clear that prestressing force does not have the compression softening effect but influences the natural frequency of the prestressed beams differently. Prestressing force closes the cracks in concrete (microcracks as well as the structural cracks, if any). Cracks reduce the beam stiffness (natural frequency) and closing of cracks should increase the stiffness (and hence the natural frequency). However, there are serious difficulties in evaluating accurate values of modulus of elasticity and moment of inertia of concrete sections and hence the natural frequencies of vibration. Similar observations about the decrease in flexural stiffness due to damage, resulting in a reduction in natural frequency, have been reported by Mirza et al. (1990) and Ambrosini et al. (1999). Singh et al. (1991) treated the prestressed beam as a beam-column and took its theoretical basis for establishing the variation in the natural frequency with prestressing force. He carried out an experimental study on several prestressed girders to study the variation of natural frequency with loss of prestress and the presence of cracks. The natural frequency, obtained by ambient vibration measurements in the laboratory, increases with increase in prestress force up to a point; beyond it, the increasing rate is either very low or there is a decrease in frequency. The development of cracks in the beam results in a decrease of natural frequency. Saidi et al. (1994) recalled the well-known “compression softening” effect and reported the discrepancy between theoretical and experimental results. Saidi et al. (1994) carried out an experimental study of a prototype post-tensioned bridge with 155ft (47.2 m) span and 45 ft. width and a laboratory specimen. They found that the natural frequency of the bridge decreased with loss of prestress. The testing of the laboratory specimen also supported this result. The results available from the literature are different from the dynamic characteristics of an isotropic beam subjected to an axially compressive force which was put forward by Clough et al. (1995). The reason is that the beams were considered as isotropic, homogeneous, linear elastic material in the traditional analysis method. However, more accurate results are required in the analysis of frequency of PSC beam. The constitutive model of PSC member is analyzed based on the microstructure of concrete in this paper. The orthotropic linear elastic model is used to analyze the relationship between dynamic frequency and prestressing force of the concrete beam, at the same time the equivalent stiffness of prestressed tendon relating to the prestressing force is added to the bending deformation stiffness of the beam. The analytical value agrees well with the test result, indicating that the current analysis method in this paper is feasible to the full-prestressed concrete beam. Grace and Ross (1996) conducted an analytical study aimed at examining the influence on girders’ natural frequencies and mode shapes of vibration of many parameters like prestress force level and the shape of the bottom prestressing strands. Jain et al. (1996) conducted an experimental study to determine the effect of cracking and prestress loss of the concrete beam on their fundamental frequency has been carried out. It has been argued that the change in the natural frequency of the prestressed concrete beam is due to change in the stiffness of the beam caused by closing or opening of cracks as the prestressing force is applied or removed, not due to compression softening effect discussed by some researchers. The study aims to contribute towards a non-destructive test method to assess the soundness of the prestressed concrete (PSC) bridge girders. Jain and Goel (1996) have pointed out that prestressing force is an internal force of the prestressed girder system and therefore prestressing force does not cause compression softening effect. Dall’Asta and Leoni (1999) studied the free vibrations of a class of concrete beams prestressed using unbonded internal cables to test the influence of the elastic characteristic of the materials, as well as the influence of the path and stress of the prestressing cables on the beam free vibrations. Miyamoto et al. (2000) studied the effect of the external tendons on the frequency of vibration of prestressed concrete beams. Miyamoto et al. (2000) studied the pre-stressing technique using external
tendon to analytically investigate the dynamic behaviour of prestressed composite girder bridges. Hamed and Frostig (2006) presented a study dealing with the effect of the magnitude of the pre

![tress force on the Natural Frequencies of prestressed beams with bonded and unbonded tendons. Sunkyu et al. (2010) studied the flexural behaviour and strengthening effect of a bridge using a steel I-beam that had been externally prestressed with unbonded tendons. Zhang and Ruige (2007) conducted dynamic tests on three bonded and two un-bonded fully prestressed concrete beams. Marco Breeolotti et al. (2009) explored the effects of prestress forces on modal parameters of concrete beams. Vadims et al. (2012) presented the determination of natural frequencies (as mode shapes, of the model depending on a prestressing level) of a physical model of a prestressed suspension bridge.

3. Objective and Scope
The objective of the present work is
1. To find the effect of pre-stressing on Natural frequency of a Concrete beam using STAAD.Pro
2. To establish a relationship between the natural frequency of the 1st mode of vibration and other modes.
3. Compare the results with those of un-prestressed beams obtained by conventional methods.

The scope of the work is limited to the following.

Grade of concrete: M35
Modulus of Elasticity: $E = 5000 \sqrt{f_{ck}}$ Here, $f_{ck} = 35$ N/mm$^2 = 35000$ kN/m$^2$
Therefore $E = 5000 \sqrt{35} = 2.958 \times 10^7$ kN/m$^2$
Span of the beam: $L = 20$ m; Live load: $w = 5$ kN/m, 10 kN/m, 15 kN/m
$m$ = mass per unit length (including live load)
C/s of the beam: $B \times D = 300$ mm x 600 mm, 300 mm x 750 mm, 300 mm x 900 mm
Pre-stressing force: $P$ is calculated assuming that the stress at soffit is nullified at working loads after all the losses. % loss of prestressing is assumed as 25.
Eccentricity: $e = (\frac{D}{2} - 50)$ mm (maximum possible)
Mode of vibration: 1, 2, 3, 4 and 5.
The nomenclature for Un-prestressed / Reinforced concrete beam and Pre-stressed concrete beam that is presented in Table 1 and Table 2 is adopted for the study.

Table 1. Reinforced Concrete Beams

| Sl. No | Notation | Size of member (m) | Load kN/m |
|-------|----------|-------------------|-----------|
| 1     | C1R      | 0.3               | 0.6       | 5         |
| 2     | C2R      | 0.3               | 0.6       | 10        |
| 3     | C3R      | 0.3               | 0.6       | 15        |
| 4     | C4R      | 0.3               | 0.75      | 5         |
| 5     | C5R      | 0.3               | 0.75      | 10        |
| 6     | C6R      | 0.3               | 0.75      | 15        |
| 7     | C7R      | 0.3               | 0.9       | 5         |
| 8     | C8R      | 0.3               | 0.9       | 10        |
| 9     | C9R      | 0.3               | 0.9       | 15        |
Table 2. Prestressed Concrete Beams

| Sl. No | Nomenclature | B (m) | D (m) | P (kN) | e (m) | Load (kN/m) |
|-------|-------------|------|------|--------|------|-------------|
| 1     | C1P         | 0.3  | 0.6  | 1357   | 0.25 | 5           |
| 2     | C2P         | 0.3  | 0.6  | 2071   | 0.25 | 10          |
| 3     | C3P         | 0.3  | 0.6  | 2786   | 0.25 | 15          |
| 4     | C4P         | 0.3  | 0.75 | 1181   | 0.325| 5           |
| 5     | C5P         | 0.3  | 0.75 | 1736   | 0.325| 10          |
| 6     | C6P         | 0.3  | 0.75 | 2292   | 0.325| 15          |
| 7     | C7P         | 0.3  | 0.9  | 1068   | 0.4  | 5           |
| 8     | C8P         | 0.3  | 0.9  | 1523   | 0.4  | 10          |
| 9     | C9P         | 0.3  | 0.9  | 1977   | 0.4  | 15          |

4. Methodology and Results

To find Natural frequency of a prestressed beam, STAAD.Pro is used and for Concrete beams, the classical methods available are used.

4.1 The natural frequency of un-prestressed beams

The following Standard formula is considered for determination of the natural frequency of an RC beam.

\[
\omega_n = \frac{A}{L^2} \times \frac{\sqrt{EI}}{m}
\]

Where
- E - Young’s modulus
- I - area moment of inertia
- L - Length of beam
- m - mass per unit length of the beam
- \( A = (n \pi)^2 \) for Mode n, where n = 1, 2, 3, 4 and 5.

The 1st 3 modes of vibration for the simply supported beam are shown in figure 1.

![Figure 1. Natural vibration modes and frequencies of prismatic simply supported Un-prestressed beams](image)

The natural frequencies and modes of vibration of a prismatic beam simply supported at both ends with properties defined in Table 1 are determined in cycles/sec. and presented in Table 3.
Table 3. Natural Frequencies of un-prestressed Concrete Beams

| Types of Beams | Natural Frequencies of un-prestressed Concrete Beams (Cycles/sec) |
|----------------|---------------------------------------------------------------|
| Case           | C1R   | C4R   | C7R   | C2R   | C5R   | C8R   | C3R   | C6R   | C9R   |
| MODE 1         | 1.59  | 2.10  | 2.63  | 1.29  | 1.73  | 2.20  | 1.11  | 1.51  | 1.93  |
| MODE 2         | 6.38  | 8.43  | 10.54 | 5.16  | 6.95  | 8.83  | 4.45  | 6.05  | 7.75  |
| MODE 3         | 14.44 | 18.98 | 23.73 | 11.62 | 15.65 | 19.87 | 10.02 | 13.62 | 17.44 |
| MODE 4         | 25.53 | 33.74 | 44.18 | 20.67 | 27.82 | 35.33 | 17.82 | 24.22 | 31.00 |
| MODE 5         | 39.90 | 52.73 | 65.91 | 32.29 | 43.48 | 55.19 | 27.85 | 37.84 | 48.45 |

4.2 Model Calculation for natural frequency of un-prestressed beams (for C1 R Second Frequency)

\[ n = \frac{A}{l^8} \times \sqrt{\frac{EI}{m}} \]

\[ I = \frac{300 \times 600^2}{12} \]

\[ E = 5000\sqrt{35} = 29580.39 \text{ N/mm}^2 \]

\[ EI = 1.59 \times 10^{14} \text{ N/mm}^2 \]

\[ l = 20,000 \text{ mm} \]

For 2nd mode of vibration

\[ A \pi^2 = (2\pi)^2 = 4\pi^2 \]

\[ m = (5 + 0.3 \times 0.6 \times 25) \text{ kN/m} \]

\[ = (5 + 0.3 \times 0.6 \times 25) = 9.81 \times 10^3 \text{ kg/mm} \]

\[ f_2 = \frac{4\pi^2 \times \frac{1.59 \times 10^{14}}{9.683 \times 10^{-4} \times (20,000)^4}} \]

\[ = 40.12 \text{ Radian/sec} \]

\[ = \frac{4\pi}{2\pi} \times 40.12 = 6.385 \text{ cycles/sec} \]

4.3 Natural Frequency of Prestressed Beams

The natural frequencies for the first few modes of vibration for prestressed beams are computed for the flexural motion of a prestressed simply supported beam using STAAD.Pro. The simply supported beam is divided into spanwise beam elements. For example, if it is divided into 20 elements, STAAD.Pro can give the natural frequencies of the first 20 modes of vibration. At nodes 1 and 21, all degrees of freedom except the rotation about the Z-axis is restrained. For the remaining nodes, only the translation along Y and the rotation about Z are permitted. Both shear deformation and rotary inertia have been excluded from the model. The mass matrix is diagonal. The cross-section, material and loads are assigned as per requirements to all the 20 elements. While assigning loads, the prestressing force with corresponding tendon profile and eccentricity are also assigned to all the 20 elements. And lastly, in loads command, the following step is included for finding natural frequency as shown in figure 2.

Figure 2. The final step in Loads and Definitions for finding Natural Frequency of PSC beam
It is important to note that all the loads including self-weight, live load, prestress along with the command for modal calculations have to under only one load case. The STAAD.Pro analysis for all 9 cases is performed and the results are tabulated in Table 4.

Table 4. Natural Frequency of Prestressed Concrete Beams

| Type of Beams | Natural Frequency of Prestressed Concrete Beams (Cycles/sec) |
|---------------|----------------------------------------------------------|
| Case | C1P | C4P | C7P | C2P | C5P | C8P | C3P | C6P | C9P |
| MODE 1 | 1.58 | 2.24 | 2.58 | 1.28 | 1.85 | 2.35 | 1.18 | 1.61 | 2.06 |
| MODE 2 | 5.10 | 8.94 | 8.09 | 4.16 | 7.37 | 9.33 | 4.73 | 6.41 | 8.19 |
| MODE 3 | 8.74 | 19.96 | 13.64 | 7.14 | 16.45 | 20.76 | 10.59 | 14.32 | 18.22 |
| MODE 4 | 13.58 | 35.08 | 21.46 | 11.06 | 28.93 | 36.33 | 18.69 | 25.18 | 31.88 |
| MODE 5 | 22.06 | 54.01 | 35.21 | 17.90 | 44.54 | 55.61 | 28.93 | 38.76 | 48.80 |

Natural Frequency of Prestressed Concrete Beams with Different cross-sectional areas for different Modes of Vibration given in Table 4 is presented in Figure 3.

Figure 3. Natural Frequency of Prestressed Concrete Beams

Natural Frequency of un-prestressed Concrete Beams with Different cross-sectional areas for different Modes of Vibration given in Table 3 is presented in Figure 4.

Figure 4. Natural Frequencies of un-prestressed Concrete Beams
5. Discussions on Results
For both Prestressed Concrete and un-prestressed Concrete beams, the following observations can be made from the results. For the same cross-section, the increase in load decreases the natural frequency of the pre-stressed beam. However, as the cross-section of the beam increases, the natural frequency increases. Obviously, the natural frequency also increases with an increase in the mode of vibration. Comparing the natural frequency of the 1st mode of vibration for Prestressed Concrete and Reinforced Concrete, the following observations can be made.

Table 5. Comparison of Natural Frequencies of Prestressed Concrete and un-prestressed Concrete beam for 1st Mode of Vibrations

| Types of Beams | Natural Frequencies (Cycle/Sec) |
|----------------|---------------------------------|
|                | C1  | C4  | C7  | C2  | C5  | C8  | C3  | C6  | C9  |
| Prestressed    | 1.58| 2.24| 2.58| 1.28| 1.85| 2.35| 1.18| 1.61| 2.06|
| Un-prestressed | 1.59| 2.10| 2.63| 1.29| 1.73| 2.20| 1.11| 1.51| 1.93|

For the case study under consideration, the natural frequency of the un-prestressed Concrete beam with live load 5 kN/m is more than that for the prestressed beam. For the other two load-carrying capacities, it is vice-versa. The percentage difference of a mode of frequency to 1st mode is calculated for both prestressed and un-prestressed Concrete beams and presented in Table 6 and Table 7. For example, % diff. of Mode n to Mode 1, NF = (Mode n NF – Mode 1 NF) x 100/(Mode n NF).

Table 6. Percentage Difference of Natural Frequency of Mode i to Mode 1 for Prestressed Concrete Beams

| Beam           | Percent difference of Natural Frequency of Prestressed Concrete Beams |
|----------------|---------------------------------------------------------------------|
|                | C1P | C4P | C7P | C2P | C5P | C8P | C3P | C6P | C9P |
| % diff. of Mode 5 w.r.t Mode 1 NF | 95.82 | 95.89 | 95.75 | 95.82 | 95.89 | 95.75 | 95.82 | 95.89 | 95.75 |
| % diff. of Mode 4 w.r.t Mode 1 NF | 93.58 | 93.64 | 93.51 | 93.58 | 93.64 | 93.51 | 93.58 | 93.64 | 93.51 |
| % diff. of Mode 3 w.r.t Mode 1 NF | 88.73 | 88.79 | 88.66 | 88.73 | 88.79 | 88.66 | 88.73 | 88.79 | 88.66 |
| % diff. of Mode 2 w.r.t Mode 1 NF | 74.87 | 74.91 | 74.81 | 74.87 | 74.92 | 74.81 | 74.87 | 74.92 | 74.80 |

Table 7. Percentage Difference of Natural Frequency of Mode i to Mode 1 for un-prestressed Concrete Beams

| Beam           | Percent difference of Natural Frequency of un-prestressed Concrete Beams |
|----------------|----------------------------------------------------------------------------|
|                | C1R | C4R | C7R | C2R | C5R | C8R | C3R | C6R | C9R |
| % diff. of Mode 5 w.r.t Mode 1 NF | 96.00 | 96.00 | 96.00 | 96.00 | 96.00 | 96.01 | 96.01 | 96.00 |
| % diff. of Mode 4 w.r.t Mode 1 NF | 93.75 | 93.75 | 93.74 | 93.75 | 93.75 | 93.74 | 93.75 | 93.76 | 93.58 |
| % diff. of Mode 3 w.r.t Mode 1 NF | 88.89 | 88.88 | 88.88 | 88.89 | 89.05 | 88.88 | 88.91 | 88.91 | 88.90 |
| % diff. of Mode 2 w.r.t Mode 1 NF | 75.00 | 74.99 | 74.98 | 75.00 | 74.98 | 74.99 | 75.03 | 75.04 | 74.37 |
From the Tables 6 and 7, it can be observed that although the natural frequencies for Prestressed and un-prestressed Concrete beams are different, the % difference of Natural frequency of a particular mode of vibration to mode 1 natural frequency is almost constant. That is the effect of live load and cross-section is negligible on the % difference of Mode n to Mode 1 natural frequency.

The approximate values are observed as
- % diff. of Mode 5 to Mode 1 NF = 96
- % diff. of Mode 4 to Mode 1 NF = 93
- % diff. of Mode 3 to Mode 1 NF = 89
- % diff. of Mode 2 to Mode 1 NF = 75

Let
- $n_i =$ natural frequency of the $i^{th}$ mode for a given prestressed beam
- $n_1 =$ natural frequency of the 1st mode for given prestressed beam
- $k_i =$ (% difference of Mode i with respect to Mode 1 NF for rectangular beam)/100

For Example

$k_2 =$ % difference of Mode 2 with respect to Mode 1 NF of rectangular beam = 75/100 = 0.75.

This is calculated as

$$k_i = \left(\frac{n_i - n_1}{n_i}\right) \times 100$$

$$k_i = \left(\frac{n_i - n_1}{n_i}\right) \times 100 / 100$$

$$k_i = \left(\frac{n_i - n_1}{n_i}\right) \times 100$$

$$k_i = 1 - \frac{n_1}{n_i}$$

$$1 - k_i = \frac{n_1}{n_i}$$

$$(1 - k_i)^{-1} = \frac{n_i}{n_1}$$

$$n_1(1 - k_i)^{-1} = n_i$$

$$n_i = n_1(1 - k_i)^{-1}$$

6. Conclusion

For both types of Beams Prestressed Concrete and Reinforced Concrete, the following conclusions can be made.

1. For the same cross-section, the increase in load decreases the natural frequency of the pre-stressed beam.
2. As the cross-section of the beam increases, the natural frequency increases.
3. Comparing the natural frequency of 1st mode of vibration for beams Prestressed Concrete and un-prestressed Concrete, it can be observed that the natural frequency of the un-prestressed Concrete beam with live load 5 kN/m is more than that for the prestressed beam. For the other two load-carrying capacities, 5 kN/m and 10 kN/m, the natural frequency of Prestressed Concrete Beams is more than un-prestressed Concrete Beams.

Although the natural frequencies for Prestressed and un-prestressed Concrete beams are different, the % difference of Natural frequency of a particular mode of vibration w.r.t mode 1 natural frequency is almost constant. In other words, the effect of live load and cross-section is negligible on the % difference of Mode n to Mode 1 natural frequency.

The approximate values are observed as
- % difference of Mode 5 to Mode 1 NF = 96
- % difference of Mode 4 to Mode 1 NF = 93
% difference of Mode 3 to Mode 1 NF = 89
% difference of Mode 2 to Mode 1 NF = 75
4. Hence, if the natural frequency of the first mode of vibration of a prestressed beam is known along with the natural frequencies of first and i\textsuperscript{th} mode of vibration of the same cross-section when un-prestressed, then, the i\textsuperscript{th} mode of vibration of the prestressed beam can be calculated from
\[ n_i = n_1 (1 - k_1)^{-1} \]
Where
\[ n_i = \text{natural frequency of the i}^{\text{th}} \text{ mode of a prestressed concrete beam} \]
\[ n_1 = \text{natural frequency of the 1}^{\text{st}} \text{ mode of a prestressed concrete beam} \]
\[ k_1 = (\% \text{ difference of mode i to Mode 1 NF for an un-prestressed beam})/100 \]

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