“Virtual Pole” method applied at the profiling of the rotary cutter tool for processing of ball screw

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Abstract. “Virtual pole” method represents a variant of the normals theorem, which can be used for profiling of tools that generate by enwrapping, by the rolling method. This variant of determining the enveloping condition allows to avoid the need to write, in explicit form, the relative movements between the piece and the tool, while remaining scientifically rigorous. In the paper, an application of the „virtual pole” method is presented, for the profiling of the rotary cutter tool designed to process a ball screw type piece. The specific form of the enwrapping condition was compared with a specific form obtained by a classical theorem.

1. Introduction
“Virtual pole” method represents a variant of the normals theorem, which can be used for profiling of tools that generate by enwrapping, by the rolling method [1-3].

This variant of determining the enveloping condition has the advantage that it allows to avoid the need to write, in explicit form, the relative movement equations of the piece towards the tool, while remaining scientifically rigorous.

The method can be applied to the profiling of tools of rack, gear shaped cutter, rotary cutter type and even tools that generate with single point contact, respectively hob mills. “Virtual pole” method starts from the premise that, in order to respect the Willis theorem, when a normal to the profile to be generated passes through the gearing pole, the respective normal is common for both the generated profile and the profile to be generated [2].

So, for the position discussed, three curves are in contact. These are the generated profile, the profile to be generated and the gearing curve which, by definition, is the geometric place of the tangency points between the two profiles mentioned.

The contact point coordinates depend on the absolute movements that the profile to be generated (the tool) and, respectively, the generated profile (the piece) execute [4].

In this way, the points on the tool profile can be successively determined, which will generate, by cutting, the known points on the piece profile.

In the paper, an application of the “virtual pole” method is presented, for the profiling of the rotary cutter tool designed to process a constant pitch helical surface, belonging to a ball screw piece type.

For this type of tool, the algorithm for applying the method was imagined and a dedicated calculation program was developed, which would allow the numerical determination of the searched profile.

By this application it is highlighted the simplification that the new variant of obtaining the enveloping condition brings, as well as the fact that this allows avoiding the determination of the
relative movements between the tool and the piece, known that the equations of the relative movements have complicated forms and represent potential sources of errors. It should be highlighted that the influence of these movements is not eliminated but only the need to know them effectively [2, 5, 6].

2. Rotary cutter tool profiling
For the rotary cutter tool profiling designed to process a constant pitch helical surface, three reference systems are used [2, 6]:

- \(xy\) - the fixed reference system;
- \(XY\) - the mobile reference system, joined with the profile to be generated and having, at the initial moment, the \(X\)-axis superimposed on the \(x\)-axis of the fixed system; the \(Y\)-axis of this system coincides with the \(C_1\) centrode associated with the piece and is, at the same time, parallel with the \(y\)-axis of the fixed system;
- \(\xi \eta\) - the mobile reference system, joined with the tool; initially, the axes of the system \(\xi \eta\) coincide with the axes of the fixed system but, during generation, it rotates around the origin with the angle \(\varphi\) (see figure 1); in this system, the profile of the piece and its centrode, circle \(C_2\) of radius \(R_{rp}\) are defined.

The equations of the profile to be generated are defined in the \(XY\) system and have the form:

\[
\begin{align*}
X(u) &= a - r \cdot \cos u; \\
Y(u) &= b - r \cdot \sin u,
\end{align*}
\]

\(a, b\) and \(r\) being constructive dimensions, and \(u\) a scalar variable parameter.

![Figure 1. Reference systems. The centrodes and the piece profile.](image)

The rolling condition, which connects the absolute movements of the piece and the tool, so that the two centrodes, \(C_1\) and \(C_2\), roll without sliding, has the form:

\[
\delta = R_{rs} \cdot \varphi.
\]
Considering a current point belonging to the profile $\Sigma$, the position vector of this point is of the form:

$$\mathbf{r} = (a - r \cdot \cos u) \cdot \mathbf{i} + (b - r \cdot \sin u) \cdot \mathbf{j}. \quad (3)$$

The normal at the profile $\Sigma$ is given by the equation:

$$\mathbf{N}_z = \lambda \cdot \left( \mathbf{X}_u \cdot \mathbf{i} - \mathbf{X}_v \cdot \mathbf{j} \right). \quad (4)$$

In (4), $\mathbf{X}_u$ and $\mathbf{X}_v$ represent the partial derivatives of the equations of the profile $\Sigma$ in relation with the variable $u$, that is:

$$\mathbf{X}_u = r \cdot \sin u; \quad \mathbf{X}_v = -r \cdot \cos u. \quad (5)$$

Therefore, the normal at the profile will have the form:

$$\mathbf{N}_z = \lambda \cdot \left( -r \cdot \cos u \cdot \mathbf{i} - r \cdot \sin u \cdot \mathbf{j} \right). \quad (6)$$

In equation (6), $\lambda$ represents the module of a vector with the origin at the current point $T$ belonging to the profile $\Sigma$ and the end on the centrode $C_1$, in the $P_v$ point. The $P_v$ point represents the virtual pole.

The position vector of the point $P_v$ can be determined by summing the vectors $\mathbf{r}$ and $\mathbf{N}_z$, see also figure 2:

$$\mathbf{r}_{P_v} = \mathbf{r} + \mathbf{N}_z = (a - r \cdot \cos u - r \cdot \lambda \cdot \cos u) \cdot \mathbf{i} + (b - r \cdot \sin u - r \cdot \lambda \cdot \sin u) \cdot \mathbf{j}. \quad (7)$$

Figure 2. Position vectors of points $T$ and $P_v$.

The scalar parameter $\lambda$ in equation (7) can be eliminated by giving the condition that the point $P_v$ also belongs to centrode $C_1$. The equations of this centrode are:

$$C_1: \begin{cases} X = \delta; \\ Y = 0, \end{cases} \quad (8)$$

so, the condition becomes:

$$\lambda = \frac{a - r \cdot \cos u - \delta}{r \cdot \cos u} = \frac{b - r \cdot \sin u}{r \cdot \sin u}. \quad (9)$$
or:

\[
\phi = \frac{b \cdot \cos u - a \cdot \sin u}{R_s \cdot \sin u}, \tag{10}
\]

According to virtual pole method, for a certain value of the parameter \(u\), \(\delta_u\) can be identified for which, during the absolute movement of the piece, the \(P_v\) point coincides with the gearing pole, \(P\):

\[
\delta_u = \frac{b \cdot \cos u - a \cdot \sin u}{\sin u}. \tag{11}
\]

The absolute movement of the profile \(\Sigma\) is given by:

\[
x = X + A; \quad A = \begin{pmatrix} -\delta \\ -R_s \end{pmatrix}. \tag{12}
\]

So, the current point coordinates, \(T\), in the fixed reference system, will be:

\[
T:\begin{align*}
x_T &= X_T - \delta_u; \\
y_T &= Y_T - R_s.
\end{align*} \tag{13}
\]

or, taking (11) into account:

\[
T:\begin{align*}
x_T &= 2 \cdot a - b \cdot \frac{\cos u}{\sin u} - r \cdot \cos u; \\
y_T &= b \cdot r \cdot \sin u - R_s.
\end{align*} \tag{14}
\]

When \(u\) varies from \(u_{\min}\) to \(u_{\max}\), the \(T\)-point go through the gearing curve. Therefore, the parametric equations of the gearing curve will be:

\[
C.A.: \begin{align*}
x(u) &= 2 \cdot a - b \cdot \frac{\cos u}{\sin u} - r \cdot \cos u; \\
y(u) &= b \cdot r \cdot \sin u - R_s. \tag{15}
\end{align*}
\]

The \(T\)-point, at the intersection between the profile \(\Sigma\) and the gearing curve, has the property of being a tangent point between \(\Sigma\) and the \(S\)-profile of the generating tool.

The coordinates of this point in the reference system of the tool are given by its absolute movement, namely:

\[
\xi = \omega \cdot \phi \cdot x, \tag{16}
\]

or, developed:

\[
\begin{pmatrix} \xi \\ \eta \end{pmatrix} = \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} x(u) \\ y(u) \end{pmatrix} = \begin{pmatrix} x(u) \cdot \cos \phi + y(u) \cdot \sin \phi \\ -x(u) \cdot \sin \phi + y(u) \cdot \cos \phi \end{pmatrix}. \tag{17}
\]

For a certain value of \(u\), the \(T\)-point coordinates are:

\[
\begin{align*}
x_T &= x_T \cdot \cos \varphi_u + y_T \cdot \sin \varphi_u; \\
y_T &= -x_T \cdot \sin \varphi_u + y_T \cdot \cos \varphi_u,
\end{align*} \tag{18}
\]

in which the value \(\varphi_u\) is given by (10).

Therefore, when \(u\) takes values from \(u_{\min}\) to \(u_{\max}\), the \(T\)-point will go through the whole profile \(S\).
3. Numerical application

A numerical application has been made for the profiling of the rotary cutter tool, designed to process a ball screw having the axial section shown in figure 3.

The dimensions for this profile are: $a=0.17$ mm; $b=0.155$ mm; $R_{rs}=50$ mm; $r=5.4$ mm.

The coordinates of the points belonging to the contact curve and the generating tool profile are presented in table 1.

Figure 4 shows, graphically, the profile of the generating tool and the contact curve.
Table 1. The coordinates of points belonging to generating tool and to the contact curve.

| Tool’s profile | Action line |
|----------------|-------------|
| $\xi$ [mm] | $\eta$ [mm] | x [mm] | y [mm] |
| -55.228 | -0.016 | -55.228 | 0.150 |
| -55.202 | 0.393 | -55.201 | 0.544 |
| -55.145 | 0.798 | -55.143 | 0.935 |
| -55.058 | 1.197 | -55.055 | 1.319 |
| -54.940 | 1.590 | -54.937 | 1.696 |
| -54.794 | 1.972 | -54.791 | 2.061 |
| -54.618 | 2.342 | -54.615 | 2.414 |
| -54.415 | 2.697 | -54.413 | 2.751 |
| -54.186 | 3.037 | -54.183 | 3.070 |
| -53.931 | 3.358 | -53.931 | 3.368 |
| -53.653 | 3.659 | -53.654 | 3.644 |
| -53.352 | 3.939 | -53.355 | 3.893 |
| -53.030 | 4.195 | -53.036 | 4.115 |
| -52.689 | 4.426 | -52.699 | 4.304 |
| -52.331 | 4.631 | -52.346 | 4.455 |
| -51.956 | 4.809 | -51.978 | 4.562 |
| -51.566 | 4.958 | -51.598 | 4.612 |
| -51.161 | 5.078 | -51.208 | 4.577 |
| -50.737 | 5.167 | -50.810 | 4.389 |
| -50.278 | 5.224 | -50.407 | 3.787 |
| -49.725 | 5.233 | -50.000 | 0.000 |

4. Conclusions

The “virtual pole” method, as a variant of the “normals theorem”, applied for the profiling of the cutting tools that generate by rolling, allows avoiding the need to write the relative movements that appear between the tool and the piece during the generating.

The advantage is that these movements have complicated equations, which can often be a source of error.

From analytical point of view, the method is rigorous, its accuracy being confirmed by the identity between the results obtained by the “virtual pole” method and those obtained by established methods.

5. References

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