QCD condensate contributions to the effective quark potential in a covariant gauge

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Abstract

We discuss QCD condensate contributions to the gluon propagator both in the fixed-point gauge and in covariant gauges for the external QCD vacuum gluon fields with the conclusion that a covariant gauge is essential to obtain a gauge invariant QCD vacuum energy density difference and to retain the unitarity of the quark scattering amplitude. The gauge-invariant QCD condensate contributions to the effective one-gluon exchange potential are evaluated by using the effective gluon propagator which produces a gauge-independent quark scattering amplitude.

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I. INTRODUCTION

The quark potential model based on the one-gluon exchange approximation can correctly reproduce the baryon spectrum and the static properties of hadrons [1], especially the positive parity states; including the $q\bar{q}$ creation and annihilation terms in the one-gluon-exchange approximation, it can also give a description of meson-nucleon interactions [2]. Nevertheless, it is well understood that the one-gluon exchange can only generate the short-range part of the baryon-baryon interaction since the one-gluon exchange potential is the nonrelativistic reduction of the operator derived in the perturbative QCD scheme. It is clear that, to reflect medium- and long-range QCD, some nonperturbative effects induced by the complicated structure of the QCD vacuum should be taken into account. That the QCD condensates could produce the nonperturbative correction to the traditional potential model was first suggested by Shen et al. [3], with inspiration of QCD sum rules [4]. Several papers have appeared afterwards studying the QCD condensate contributions to the quark potential [5,6]. Possible effects of QCD condensates in heavy quarkonium spectra were also investigated by these authors. Their results indicate that the nonperturbative effects induced by QCD vacuum condensates play a significant role in the corrections to the $1/q^2$ behavior.

Unfortunately, the authors of the cited papers obtained the quark potential from a gauge dependent quark-quark scattering amplitude. Nonperturbative effects were phenomenologically considered in Refs. [3,5,6] by employing the vacuum condensate to modify the free gluon propagator in the fixed-point gauge, i.e., the non-local two-quark and two-gluon vacuum expectation values (VEVs) were calculated in the fixed-point gauge [7]. Although this gauge is extremely simple for many lowest order expansions [8], it violates translational invariance, and could in principle conflict with the covariant gauge used to formulate QCD. In particular, an explicit $\xi$-dependence (here and henceforth, we have specified the perturbative gauge dependence as $\xi$-dependence) in the transverse portion of the nonperturbative gluon propagator results in a $\xi$-dependent quark-quark scattering amplitude. The effective quark
potential obtained from a $\xi$-dependent amplitude is of course $\xi$-dependent too.

The aim of this paper is twofold: (i) to discuss QCD condensate contributions to the gluon propagator and, (ii) to evaluate the non-perturbative corrections to the one-gluon exchange quark potential in covariant gauges. This is then a natural continuation of the work done in Ref. [9]. We use here the nonperturbative gluon propagator, allowing for the presence of the ghost condensate which appears in covariant gauges.

In Sec. II, we discuss the nonperturbative gluon propagator both in the fixed-point gauge and in covariant gauges. In Sec. III, we present our main results of the gauge-invariant quark potential with the nonperturbative corrections of QCD vacuum condensates. The brief summary in Sec. IV contains some possible applications of the obtained quark potential.

II. NONPERTURBATIVE GLUON PROPAGATORS
IN DIFFERENT GAUGES

In this section, we discuss the nonperturbative gluon propagator in the fixed-point gauge and in covariant gauges for the external QCD vacuum gluon fields (nonperturbative ones). Recently, the operator product expansion (OPE) of the gluon propagator in QCD has been extensively studied [10–15].

In QCD sum rules for gauge invariant currents, the background field method is used, where the fixed-point gauge is generally employed for nonperturbative gluon fields, i.e.,

$$x_\mu B_\mu^a(x) = 0.$$  \hspace{1cm} (1)

But, for perturbative gluon fields, a covariant gauge is usually adopted, i.e.

$$iD_{\mu\nu}^{ab}(q) = i\delta_{ab} \left[ -\frac{g_{\mu\nu}}{q^2} + (1 - \xi) \frac{q_\mu q_\nu}{(q^2)^2} \right].$$  \hspace{1cm} (2)

In the fixed-point gauge, the non-local two-gluon VEV can be written as [4,16]

$$\langle 0| B_\mu^a(x) B_\nu^b(y) |0 \rangle = \frac{1}{4} x^\rho y^\sigma \langle 0| G_{\rho\mu}^a G_{\sigma\nu}^b |0 \rangle + \cdots$$

$$= \frac{\delta_{ab}}{48 (N_c^2 - 1)} x^\rho y^\sigma (g_{\rho\sigma} g_{\mu\nu} - g_{\rho\nu} g_{\sigma\mu}) \langle 0| G^2 |0 \rangle + \cdots.$$  \hspace{1cm} (3)
where
\[ \langle 0 | G^2 | 0 \rangle = \langle 0 | G_{\mu}^a G_{\rho}^{\beta \mu} | 0 \rangle. \] (4)

Obviously, the expansion (3) violates translational invariance since the right hand side (RHS) of (3) is a function of \( xy \) instead of \( x - y \).

To obtain the correct nonperturbative gluon propagator, it is essential to obtain the expansion of \( \langle 0 | B_a^a(x)B_b^b(y) | 0 \rangle \) with translational invariance. The basic requirements for translational invariance were studied before [17]. According to these requirements and the Lorentz gauge condition
\[ \partial_{\mu} B_{\rho}^a(0) = 0, \] (5)
the non-local two-gluon VEV can be expressed as
\[ \langle 0 | B_a^a(x)B_b^b(y) | 0 \rangle = \langle 0 | B_a^a(0)B_b^b(0) | 0 \rangle \]
\[ - \frac{\delta_{ab}}{2(N_c^2 - 1)} (x - y)^{\rho}(x - y)^{\sigma} \langle 0 | \partial_{\mu} B_{\mu}^d(0) \partial_{\sigma} B_{\nu}^d(0) | 0 \rangle + \cdots, \] (6)

where
\[ \langle 0 | B_a^a(0)B_b^b(0) | 0 \rangle = \frac{g_{\mu\nu}}{4} \frac{\delta_{ab}}{(N_c^2 - 1)} \langle 0 | B_{\mu}^d(0)B_{\nu}^d(0) | 0 \rangle \]
\[ = \frac{g_{\mu\nu}}{4} \frac{\delta_{ab}}{(N_c^2 - 1)} \langle 0 | B^2 | 0 \rangle \] (7)

and
\[ \frac{1}{2} \langle 0 | \partial_{\mu} B_{\nu}^a(0) \partial_{\sigma} B_{\rho}^a(0) | 0 \rangle = \left[ Sg_{\mu\nu}g_{\rho\sigma} + \frac{R}{2} (g_{\mu\nu}g_{\rho\sigma} + g_{\mu\sigma}g_{\rho\nu}) \right]. \] (8)

Contracting Eq. (8) with \( g^{\alpha \rho}g_{\mu \nu}, g^{\rho \mu}g^{\sigma \nu} \) and \( g^{\rho \mu}g^{\sigma \nu} \) leads to
\[ \frac{1}{2} \langle 0 | \partial^{\sigma} B_{\nu}^{\rho}(0) \partial_{\sigma} B_{\rho}^a(0) | 0 \rangle = 16S + 4R, \] (9)
\[ \frac{1}{2} \langle 0 | \partial^{\mu} B_{\nu}^a(0) \partial^{\sigma} B_{\rho}^a(0) | 0 \rangle = 4S + 10R \] (10)

and
\[
\frac{1}{2} \langle 0 | \partial^\nu B^\mu_a(0) \partial_\mu B^a_\nu(0) | 0 \rangle = 4S + 10R
\]  \hspace{1cm} (11)

respectively. According to the Lorentz gauge condition (5), (10) means

\[
R = -\frac{2}{5} S.
\]  \hspace{1cm} (12)

Furthermore, using the definition of the gluon field strength and by only retaining the contribution of the vacuum intermediate state [18], one can easily find that

\[
\langle 0 | G^a_{\rho\mu}(0) G^b_{\sigma\nu}(0) | 0 \rangle \\
\approx \left[ \langle 0 | \partial_\rho B^a_{\mu}(0) \partial_\sigma B^b_{\nu}(0) | 0 \rangle + \langle 0 | \partial_\mu B^a_{\rho}(0) \partial_\nu B^b_{\sigma}(0) | 0 \rangle \right] \\
- \left[ \langle 0 | \partial_\rho B^a_{\mu}(0) \partial_\sigma B^b_{\nu}(0) | 0 \rangle + \langle 0 | \partial_\mu B^a_{\rho}(0) \partial_\nu B^b_{\sigma}(0) | 0 \rangle \right] \\
+ \frac{\pi \alpha_s N_c}{3(N_c^2 - 1)^2} \delta_{ab} [g_{\rho\sigma} g_{\mu\nu} - g_{\mu\sigma} g_{\rho\nu}] \langle 0 | B^2 | 0 \rangle^2
\]  \hspace{1cm} (13)

which results in

\[
S \approx \frac{5 \langle 0 | G^2 | 0 \rangle}{288} - \frac{5N_c \pi \alpha_s}{72(N_c^2 - 1)} \langle 0 | B^2 | 0 \rangle^2.
\]  \hspace{1cm} (14)

In addition, combining (11) with (9) and (12), one can get

\[
S = \frac{5 \langle 0 | (\partial_\nu B^\mu_a - \partial_\mu B^\nu_a)^2 | 0 \rangle}{288}
\]  \hspace{1cm} (15)

Comparing (14) and (15) leads to

\[
\alpha_s \langle 0 | B^2 | 0 \rangle^2 \approx 0
\]  \hspace{1cm} (16)

provided that the approximation of the vacuum dominance in intermediate states is accepted, and that the equality of \( \langle 0 | (\partial_\nu B^\mu_a - \partial_\mu B^\nu_a)^2 | 0 \rangle \) and the Abelian part of \( \langle 0 | G^2 | 0 \rangle \) has been used [13]. The dimension-two condensate \( \langle 0 | B^2 | 0 \rangle \) is not gauge invariant. According to our estimate of its value, this term (maybe due to a spontaneous gauge symmetry breaking) is very small and can be omitted as compared with the gauge-invariant gluon condensate \( \langle 0 | G^2 | 0 \rangle \).

Therefore, by considering (12), (3) can also be rewritten as
\[ \langle 0 | B^a_\mu (x) B^b_\nu (y) | 0 \rangle = - \frac{\delta_{ab}}{(N^2_c - 1)} S \left[ (x - y)^2 g_{\mu \nu} - \frac{2}{5} (x - y)_\mu (x - y)_\nu \right] + \cdots \tag{17} \]

with manifest translational invariance. This result was used before by Bagan et al. \[15\] without looking at the condensate \( \langle 0 | B^2 | 0 \rangle \).

Now let us focus our attention on discussing the vacuum energy density difference and the quark scattering amplitude, which are related to the nonperturbative gluon propagator.

**A. \( \xi \)-dependence of the vacuum energy density difference**

The vacuum energy density difference (the effective potential of Coleman and Weinberg \[19\]) is defined as the perturbative contribution to the difference between the energy densities of the physical and bare vacua. The condensate contribution to the vacuum energy difference \[10\] is of interest because it can tell us something about the energy dependence on the condensates of the QCD vacuum. In order to calculate QCD condensate contributions to the the vacuum energy density difference, we need quark condensate contributions to the quark self-energy and the gluon vacuum polarization. They can be expressed as \[10\]

\[ \Sigma^{<\bar{q}q>} = \sum_f \frac{(N^2_c - 1) \pi \alpha_s \langle 0 | \bar{q}_f q_f | 0 \rangle}{2N^2_c q^2} \left[ 3 + \xi - \xi m_f \frac{\bar{q}}{q^2} \right] \tag{18} \]

and

\[ \Pi^{<\bar{q}q>}_{\mu \nu} = \sum_f \frac{4 \pi \alpha_s m_f \langle 0 | \bar{q}_f q_f | 0 \rangle}{N_c q^2} g^\perp_{\mu \nu} (q) \tag{19} \]

respectively, with \( g^\perp_{\mu \nu} (q) = g_{\mu \nu} - q_\mu q_\nu / q^2 \). The form of the gluon condensate contribution depends on the choice of the gauge for the external QCD gluon fields. In the fixed-point gauge, the contribution to the quark self-energy is \[16\]

\[ \Sigma^{<G^2>} = \frac{\pi \alpha_s \langle 0 | G^2 | 0 \rangle m_f (q^2 - m_f^2)}{N_c (q^2 - m_f^2)^3} \tag{20} \]

And, by employing Eq. (3), the gluon condensate contribution to the gluon polarization can be obtained as

\[ \Pi^{<G^2>}_{\mu \nu} = \frac{N_c \pi \alpha_s \langle 0 | G^2 | 0 \rangle}{12(N^2_c - 1)q^2} \left[ (36 + \xi) g^\perp_{\mu \nu} (q) - (12 + \frac{18}{\xi}) q_\mu q_\nu \right] \tag{21} \]
By using the above self-energies, we obtain the condensate contributions to the vacuum energy density in the fixed-point gauge as

\[
V(<\bar{q}q>, <G^2>) = \left[ \frac{9 \pi \alpha_s N_c \langle 0 | G^2 | 0 \rangle}{48} (10 - \xi) + \sum_f \frac{6 \pi \alpha_s (N_c^2 - 1) m_f \langle 0 | \bar{q}_f q_f | 0 \rangle}{N_c} \right]
\times \int_{q^2 < -\mu^2} \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2 + i\epsilon)^2},
\]  

(22)

which is \(\xi\)-dependent. The renormalization point \(\mu^2\) is used in QCD to divide the momentum range into a perturbative and a nonperturbative regions. As pointed out by Jackiw [20], any gauge dependence of the effective potential for a particular operator makes a physical interpretation questionable. This problem can be avoided in covariant gauges. Actually, Lavelle and Schaden [10] got a gauge invariant vacuum energy density difference by taking into account the ghost condensate contribution in covariant gauges. With our estimate of \(\langle 0 | B^2 | 0 \rangle\) given in Eq. (16), the vacuum energy density difference obtained in [10] is gauge invariant even if there exists a spontaneous gauge symmetry breaking.

**B. Unitarity of the quark scattering amplitude**

It is well known that the \(\xi\)-dependence of the perturbative gluon propagator does not carry through to the quark scattering amplitude. However, it is quite another story to prove this for the nonperturbative gluon propagator. In the fixed-point gauge, the QCD condensate contribution to leading order in \(\alpha_s\) to the gluon propagator can easily be obtained as

\[
iD_{\mu\nu}(q) = i \left\{ -\frac{1}{q^2} A_T g_{\mu\nu}^\perp(q) + A_L q_\mu q_\nu \right\}
\]  

(23)

where

\[
A_T = \frac{N_c \pi \alpha_s \langle 0 | G^2 | 0 \rangle}{12(N_c^2 - 1)q^4} (36 + \xi) + \sum_f \frac{4 \pi \alpha_s m_f \langle 0 | \bar{q}_f q_f | 0 \rangle}{N_c q^2 (q^2 - m_f^2)},
\]  

(24)

\[
A_L = \frac{N_c \pi \alpha_s \langle 0 | G^2 | 0 \rangle}{12(N_c^2 - 1)q^4} (18\xi + 12\xi^2).
\]  

(25)

The gluon vacuum polarization used in deriving Eq. (23) is given in Eq. (21). Notice that the longitudinal term in (21) is both nonzero and \(\xi\)-dependent. Hence, the unitarity in
the non-Abelian coupling amplitude cannot be satisfied, which stimulates us to try to use a covariant gauge for the external QCD vacuum gluon fields. At least, by introducing the covariant gauge for the gluon VEV, the difficulty of two potentially conflicting gauge conditions for the gluon VEV and the perturbative gluon fields in QCD can be avoided.

In covariant gauges, by using the non-local two-gluon VEV given in (17), the lowest-dimension gluon condensate contribution to the gluon propagator can be written as

$$i G^{<G^2>}_{\mu\nu}(q) = i \left\{ \frac{(25 - 3\xi)\pi\alpha_s(0|G^2|0)}{24q^6} g^\perp_{\mu\nu}(q) - \frac{3\xi^2\pi\alpha_s(0|G^2|0)}{8q^8} q_\mu q_\nu \right\}$$

(26)

There is also a longitudinal term in the gluon polarization of (26), i.e., the Slavnov-Taylor identities (STI) [21] is not fulfilled here. This problem can be fixed by allowing for the presence of the ghost condensate. In Ref. [12], Lavelle and Schaden got a transverse gluon vacuum polarization by taking into account mixing with equation of motion condensates. Their result for the gluon vacuum polarization in covariant gauges with the corrections of gluon and ghost condensates as depicted in Fig. 1(a-c) is

$$\Pi^{\mu\nu}(q^2) = -\frac{N_c\pi\alpha_s(68 + 3\xi)}{18(N_c^2 - 1)q^2} g^\perp_{\mu\nu}(0) (\partial_\nu B^a_\sigma - \partial_\sigma B^a_\nu)^2 |0\rangle,$$

(27)

where condensate terms which vanish due to the equation of motion are not shown. It is noteworthy that leading mixed condensate contributions to the gluon polarization (27) have also been taken into account (the diagrams for the mixed condensate contributions are not shown in Fig. 1, see Ref. [12]). Although the vacuum polarization (27) is transverse, it is also explicitly $\xi$-dependent. To obtain a gauge invariant effective gluon propagator, Lavelle [13] investigated the effects of the $\langle 0|G^2|0\rangle$ condensate on the effective gluon propagator in quark interactions by using the pinch technique (PT) in the context of QCD [22,23] (see diagrams in Ref. [13] for the PT). The effective gluon propagator was finally obtained as

$$i G^{T\mu\nu}(q) = i \left\{ \frac{1}{q^2} + \left[ \frac{34N_c\pi\alpha_s(0|G^2|0)}{9(N_c^2 - 1)q^6} \right] \right. - \frac{4\pi\alpha_s}{N_c q^4} \sum_f m_f \langle 0|\bar{q}_f q_f|0\rangle \left( \frac{1}{q^2 - m_f^2} + \frac{1}{2} \frac{m_f^2}{(q^2 - m_f^2)^2} \right) \right\} g^\perp_{\mu\nu}(q) - \xi \frac{q_\mu q_\nu}{q^4}.$$  

(28)
Here, following Ref. [13], we have identified $\langle 0|(\partial_\nu B^a_\sigma - \partial_\sigma B^a_\nu)^2|0\rangle$ with the Abelian part of $\langle 0|G^2|0\rangle$. The quark condensate contribution term differs from that in Eq. (23) due to the fact that the next-to-leading-order term in the full coefficient of the $\langle \bar{q}q \rangle$ component of the nonperturbative two-quark VEV [16] is retained.

After the above detailed discussion on the nonperturbative gluon propagator, we can now stress our main reason for deriving the quark interaction potential from the quark scattering amplitude in covariant gauges for the external QCD vacuum gluon fields. First, one can obtain a gauge invariant vacuum energy density difference in covariant gauges. However, the $\xi$-dependence of Eq. (21) in the fixed-point gauge brings about the explicit $\xi$-dependence in the vacuum energy density difference as shown in Eq. (22), which raises doubts about its physical validity. Second, the transverse portion of the nonperturbative gluon propagators in covariant gauges is $\xi$-independent. In contrast to this, the explicit $\xi$-dependence in the transverse portion of the nonperturbative gluon propagator in the fixed-point gauge results in a quark scattering amplitude with doubtful physical meaning. All of this motivate us to employ the nonperturbative gluon propagator in covariant gauges to derive the effective quark interaction potential.

III. QCD CONDENSATE CONTRIBUTIONS TO THE QUARK INTERACTION POTENTIALS

To derive the quark-quark interaction potential from the nonperturbative gluon propagator, we write down a proper scattering amplitude between two quarks as

$$ M = (-ig)^2 \bar{\psi}(p_1) \gamma^\mu \frac{\lambda^a}{2} \psi(p'_1) G^T_{\mu\nu}(q) \bar{\psi}(p_2) \gamma^\nu \frac{\lambda^a}{2} \psi(p'_2) $$

(29)

with $q = p_1 - p'_1 = p'_2 - p_2$ and the spinors $\psi(p_i)$ being the solution of free quarks. We then obtain the effective potential in momentum space by carrying out the Breit-Fermi expansion with the approximation $q_0 = 0$. Applying the three-dimensional Fourier transformation to convert the potential in momentum space to coordinate space, we finally obtain the total effective potential between quarks as
\[ U_{qq}(x) = U_{qq}^{\text{OGEP}}(x) + U_{qq}^{\text{NP}}(x) \]  

where \( U_{qq}^{\text{OGEP}}(x) \) is the perturbative one-gluon-exchange quark potential,

\[
U_{qq}^{\text{OGEP}}(x) = \delta(t) \frac{\lambda_1^a \lambda_2^a}{4} \alpha_s \left\{ \frac{1}{|\vec{x}|} - \frac{\pi}{m_1 m_2} \left( \frac{(m_1 + m_2)^2}{2m_1 m_2} + \frac{2}{3} \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right) \delta(\vec{x}) \right. \\
+ \frac{1}{m_1 m_2 |\vec{x}|^3} \left[ \frac{3}{|\vec{x}|^2} (\vec{\sigma}_1 \cdot \vec{x})(\vec{\sigma}_2 \cdot \vec{x}) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \right] \\
- \frac{1}{4m_1 m_2 |\vec{x}|^3} \left[ (2 + \frac{m_2}{m_1}) \vec{\sigma}_1 + (2 + \frac{m_1}{m_2}) \vec{\sigma}_2 \right] \cdot (\vec{x} \times \vec{p}) \left\} ,
\]

and \( U_{qq}^{\text{NP}}(x) \), the nonperturbative correction to the perturbative quark-quark interaction due to the quark, gluon and ghost condensates, can be expressed as

\[
U_{qq}^{\text{NP}}(x) = \delta(t) \frac{\lambda_1^a \lambda_2^a}{4} \pi \alpha_s^2 |A_3| |\vec{x}|^3 + (A_1 + 2C_1) |\vec{x}| + 2C_{-1} |\vec{x}|^{-1} \\
+ 2 \sum_f \left( \vec{C}_0^{(f)} + \vec{C}_{-1}^{(f)} |\vec{x}|^{-1} \right) e^{-m_f |\vec{x}|} .
\]

where

\[
A_3 = \frac{17N_c \langle 0 | G^2 | 0 \rangle}{108(N_c^2 - 1)} \left( 1 + \frac{\vec{p}^2}{m_1 m_2} \right) ,
\]

\[
A_1 = \frac{17N_c \langle 0 | G^2 | 0 \rangle}{72(N_c^2 - 1)} \left( \frac{1}{m_1} + \frac{1}{m_2} \right)^2 + \frac{17N_c \langle 0 | G^2 | 0 \rangle}{432m_1 m_2(N_c^2 - 1)} (8\vec{\sigma}_1 \cdot \vec{\sigma}_2 - S_{12}) \\
+ \frac{17N_c \langle 0 | G^2 | 0 \rangle}{144m_1 m_2(N_c^2 - 1)} \left[ \left( 2 + \frac{m_2}{m_1} \right) \vec{\sigma}_1 + \left( 2 + \frac{m_1}{m_2} \right) \vec{\sigma}_2 \right] \cdot (\vec{x} \times \vec{p}) ,
\]

\[
C_1 = \left( 1 + \frac{\vec{p}^2}{m_1 m_2} \right) \sum_f \frac{\langle 0 | \bar{q}_f q_f | 0 \rangle}{N_c m_f} ,
\]

\[
C_{-1} = \frac{1}{4N_c m_1 m_2} \sum_f \frac{\langle 0 | \bar{q}_f q_f | 0 \rangle}{m_f} \left\{ \frac{(m_1 + m_2)^2}{m_1 m_2} + \frac{S_{12}}{3} + \frac{4}{3} \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right. \\
+ \left. \left[ \left( 2 + \frac{m_2}{m_1} \right) \vec{\sigma}_1 + \left( 2 + \frac{m_1}{m_2} \right) \vec{\sigma}_2 \right] \cdot (\vec{x} \times \vec{p}) \right\} ,
\]

\[
\vec{C}_0^{(f)} = \frac{2}{N_c} \frac{\langle 0 | \bar{q}_f q_f | 0 \rangle}{m_f} \left[ \frac{1}{2m_f} \left( 1 + \frac{\vec{p}^2}{m_1 m_2} \right) \\
+ \frac{m_f (m_1 + m_2)^2}{16m_1^2 m_2^2} - \frac{m_f}{24m_1 m_2} S_{12} + \frac{m_f}{12m_1 m_2} \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right] .
\]
and
\[
\tilde{C}_{-1}^{(f)} = -\frac{2}{N_c} \frac{\langle 0|\bar{q}_f q_f|0 \rangle}{m_f} \left\{ \left( \frac{m_1 + m_2}{2m_1^2m_2^2} \right)^2 + \frac{S_{12}}{6m_1m_2} + \frac{1}{6m_1m_2} \bar{\sigma}_1 \cdot \bar{\sigma}_2 \right\} - \frac{3}{24m_1m_2} \left[ \left( 2 + \frac{m_2}{m_1} \right) \bar{\sigma}_1 + \left( 2 + \frac{m_1}{m_2} \right) \bar{\sigma}_2 \right] \cdot (\vec{x} \times \vec{p}) \right\} \tag{38}
\]

with \( S_{12} = 3(\bar{\sigma}_1 \cdot \bar{n})(\bar{\sigma}_2 \cdot \bar{n}) - \bar{\sigma}_1 \cdot \bar{\sigma}_2 \) and \( \bar{n} = \vec{x}/|\vec{x}| \).

For heavy quarkonium systems, where the potential concept is applicable, the quark-antiquark interaction can be obtained by taking the color generators for an antiquark as \(-\lambda^T\), i.e.,

\[
U^{\text{Direct}}_{qq}(x) = U_{qq}(x)|_{\lambda_1^q \lambda_2^q \rightarrow -\lambda_i^q (\lambda_2^q)^T}. \tag{39}
\]

However, in this case, if the quark and antiquark have the same flavor, the annihilation mechanism should also be taken into account. The condensate contributions to this mechanism can be sketched in Fig. 2 (as for leading mixed condensate contributions and diagrams for the PT, see Refs. [12,13]). The total \( q\bar{q} \)-pair annihilation potential can be obtained by summing up the contributions of all diagrams including nonperturbative ones in Fig. 2 and the corresponding perturbative one,

\[
U^{\text{Ann(Total)}}_{q\bar{q}}(x) = U^{\text{Ann}}_{q\bar{q}}(x) + U^{\text{Ann(NP)}}_{q\bar{q}}(x) \tag{40}
\]

where \( U^{\text{Ann}}_{q\bar{q}}(x) \), the perturbative \( q\bar{q} \) pair-annihilation potential in coordinate representation, is

\[
U^{\text{Ann}}_{q\bar{q}}(x) = \delta(t) \frac{\alpha_s}{4} \frac{\pi}{16N_c m^2} (\lambda_1 - \lambda_2^T)^2 (1 - \bar{\tau}_1 \cdot \bar{\tau}_2) \times \left\{ \left( \bar{\sigma}_1 + \bar{\sigma}_2 \right)^2 \left( 1 - \frac{1}{3m^2 \vec{\nabla}^2} \right) \delta(\vec{x}) - \frac{4}{m^2} [ (\bar{\sigma}_1 \cdot \vec{\nabla}) (\bar{\sigma}_2 \cdot \vec{\nabla}) - \frac{1}{3} \bar{\sigma}_1 \cdot \bar{\sigma}_2 \vec{\nabla}^2 ] \delta(\vec{x}) \right\}. \tag{41}
\]

and

\[
U^{\text{Ann(NP)}}_{q\bar{q}}(x) = \frac{\pi \alpha_s}{m^2} \left\{ \frac{N_c}{(N_c^2 - 1)} \frac{17 \langle 0|G^2|0 \rangle}{72m^2} + \frac{1}{N_c} \sum_j \frac{m_f \langle 0|\bar{q}_j q_j|0 \rangle}{(4m^2 - m_j^2)^2 (8m^2 - m_j^2)} \right\} U^{\text{Ann}}_{q\bar{q}}(x). \tag{42}
\]
IV. SUMMARY AND DISCUSSION

In this paper, we discussed the condensate contributions to the gluon propagator which is then used to derive the nonperturbative contributions to the quark potentials. To help the reader to understand what is new in this paper, we summarize some important points:

1. We estimated the value of \( \langle 0|B^2|0 \rangle^2 \) in the approximation of the vacuum dominance in intermediate states and found that this non-gauge-invariant condensate can be omitted as compared with the gauge-invariant gluon condensate contribution, which is of crucial importance for having gauge invariant vacuum energy density differences [10].

2. We gave a detail discussion about the nonperturbative gluon propagator and showed that it is essential to adopt covariant gauges in order to obtain a gauge invariant vacuum energy density difference and to retain the unitarity of the quark scattering amplitude.

3. The gauge-invariant nonperturbative contributions to the one-gluon exchange quark potentials were obtained by employing the nonperturbative gluon propagator, with the validity of the unitarity of the quark scattering amplitude from which the quark potential is derived.

In the fixed-point gauge, some uncertainties in the nonperturbative calculation such as three-point fermionic Green function \( G_{\alpha\beta}(p', p, q) \) are unavoidable [24]. To overcome this, one can complete the calculation in covariant gauges [25]. Of course, it is not enough to judge whether the result in any gauge for the external QCD vacuum gluon fields is better than the one in others only by calculating gauge-dependent objects such as quark-gluon n-point Green functions. However, we should accept the fact that the explicit \( \xi \)-dependence of the energy density difference in the fixed-point gauge is incompatible with the physical meaning of a gauge invariant object. In the context of the OPE, the fixed-point gauge is very convenient. But, as we have seen here, one should be very careful in using this gauge.

There are many possible applications of the obtained quark potentials, for instance, in the study of the nonperturbative effect in the spectra of \( J/\Psi \) and \( \Upsilon \), especially to improve the spin splitting for these systems. In addition, this work is intended to serve as a step forward...
in the direction of solving long-standing problems in light baryon spectroscopy such as the energy level order between the positive- and negative-parity partner states, in particular, Roper resonance puzzle, and the baryon spin-orbit structure puzzle.

As an extension of this work, we will verify whether the potentials obtained here can be used to improve the hadronic spectra and hadronic properties of $J/\Psi$ and $\Upsilon$ families by including perturbative closed-loop contributions in the same order of $\alpha_s$ as shown by Gupta et al. [26], Fulcher [27] and Pantaleone et al. [28]. Moreover, physically relevant results, such as the effective quark-quark interaction potential, should be gauge-independent, i.e., the result in the fixed-point gauge should be the same as that in covariant gauges. The discrepancy between the present result and that of Ref. [3] deserves further study. (1) We plan to consider all contributions from the gauge invariant set of diagrams such as gluon propagator, ghost propagator and quark-gluon vertex corrections; (2) We will verify whether the gauge-independent quark potential can be obtained by using the gauge invariant definition of the quark potential from the Wilson loop.

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FIG. 1. The Feynman diagrams for the contributions of the nonperturbative corrections to perturbative quark-quark potential in the one-gluon exchange approximation with the lowest dimensional gluon, ghost and quark condensates.
FIG. 2. The Feynman diagrams for the contributions of the nonperturbative corrections to perturbative $q\bar{q}$-pair annihilation potential in the one-gluon exchange approximation with the lowest dimensional gluon, ghost and quark condensates.