SIMULTANEOUS ESTIMATION OF TIME DELAYS AND QUASAR STRUCTURE

Christopher W. Morgan and Michael E. Eyler
Department of Physics, United States Naval Academy, 572C Holloway Road, Annapolis, MD 21402; cmorgan@usna.edu, m082022@usna.edu

C. S. Kochanek and Nicholas D. Morgan
Department of Astronomy, Ohio State University, 140 West 18th Avenue, Columbus, OH 43210-1173; ckochanek@astronomy.ohio-state.edu, nmorgan@astronomy.ohio-state.edu

Emilio E. Falco
Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138; efalco@cfa.harvard.edu

AND

C. Vuissoz, F. Courbin, and G. Meylan
Laboratoire d’Astrophysique, École Polytechnique Fédérale de Lausanne (EPFL), Observatoire, 1290 Sauverny, Switzerland

Received 2007 October 9; accepted 2007 November 28

ABSTRACT

We expand our Bayesian Monte Carlo method for analyzing the light curves of gravitationally lensed quasars to simultaneously estimate time delays and the sizes of quasar continuum emission regions including their mutual uncertainties. We apply the method to HE1104−1805 and QJ0158−4325, two doubly imaged quasars with microlensing and intrinsic variability on comparable timescales. For HE1104−1805 the resulting time delay of $\Delta t_{AB} = t_A - t_B = 162.2^{+6.2}_{-5.9}$ days and accretion disk size estimate of $\log \left( \frac{r_d}{\text{cm}} \cos(i)/0.5 \right)^{1/2} = 15.7^{+0.4}_{-0.5}$ at 0.2 $\mu$m in the rest frame and for inclination $i$ are consistent with earlier estimates but suggest that existing methods for estimating time delays in the presence of microlensing underestimate the uncertainties. We are unable to measure a time delay for QJ0158−4325, but the accretion disk size is $\log \left( \frac{r_d}{\text{cm}} \cos(i)/0.5 \right)^{1/2} = 14.9 \pm 0.3$ at 0.3 $\mu$m in the rest frame.

Subject headings: accretion, accretion disks — cosmology: observations — dark matter — gravitational lensing — quasars: general

Online material: machine-readable table

1. INTRODUCTION

Variability in lensed quasar images comes from two very different sources. Changes in the quasar’s intrinsic luminosity are observable as correlated, achromatic variability between images, while microlensing by the stars in the lens galaxy produces uncorrelated, chromatic variability. Measurements of the time delays between the lensed images from the correlated variability can be used to study cosmology (e.g., Refsdal 1964 and recently Saha et al. 2006; Oguri 2007) or the distribution of dark matter in the lens galaxy (e.g., Kochanek et al. 2006; Poindexter et al. 2007; Vuissoz et al. 2007). The microlensing variability can be used to study the structure of the quasar continuum emission region, the masses of the stars in the lens galaxy, and the stellar mass fraction near the lensed images (Schechter & Wambsganss 2002; Wambsganss 2006). It is now possible to use microlensing to measure the correlation of accretion disk size with black hole mass (Morgan et al. 2007), the wavelength dependence of the size of the accretion disk (Poindexter et al. 2008), or the differing sizes of the thermal and nonthermal X-ray emission regions (Pooley et al. 2007; X. Dai et al. 2008, in preparation).

The challenge is that most lensed quasars exhibit both intrinsic and microlensing variability. To measure a time delay, one must successfully model and remove the microlensing variability such that only intrinsic variability remains. If the microlensing variability has a sufficiently low amplitude or long timescale, it can be ignored (e.g., PG 1115−080; Schechter et al. 1997), but this is a dangerous assumption for many systems. Eigenbrod et al. (2005) found that for an 80 day delay adding microlensing perturbations with an amplitude of 5% (10%) to a light curve increased the uncertainty in the time delay by a factor of 2 (6). Existing time delay analyses for lenses with microlensing (e.g., Ofek & Maoz 2003; Paraficz et al. 2006; Kochanek et al. 2006; Poindexter et al. 2007) depend on the intrinsic and microlensing variability having different timescales. These analyses also require that the microlensing variability can be modeled by a simple polynomial function. This approach will clearly fail if the two sources of variability have similar timescales or if the microlensing variability cannot be easily parameterized. In their analysis of HE1104−1805, Ofek & Maoz (2003) used simulations of the estimated microlensing variability to estimate its influence on the uncertainty in the delay measurement.

In this paper, we present a new technique for simultaneously estimating the time delay and size of the continuum emission region of lensed quasars that exhibit strong microlensing. In essence, we assume a range of time delays and then determine the likelihood of the implied microlensing variability using the Bayesian Monte Carlo method of Kochanek (2004; see also Kochanek et al. 2007). This allows us to estimate the time delays and the quasar structural parameters simultaneously and include the effects of both...
phenomena on the parameter uncertainties. We apply the method to the two doubly imaged lenses HE1104−1805 (Wisotzki et al. 1993) and QJ0158−4325 (Morgan et al. 1999). While HE1104−1805 has a well-measured time delay (Wyrzykowski et al. 2003; Ofek & Maoz 2003; Poindexter et al. 2007), the amplitude of the microlensing (~0.05 mag yr\(^{-1}\) over the past decade) and the fact that it exhibits variability on the 6 month scale of the time delay suggest that it is close to the limit where microlensing polynomial fitting methods (Burud et al. 2001; Kochanek et al. 2006) will break down. QJ0158−4325 clearly shows both correlated and uncorrelated variability, but the polynomial methods cannot reliably produce a time delay estimate. We describe the data and our models in § 2, our new approach in § 3, and the application to the two systems in § 4. In § 5 we discuss the results and their limitations. We assume a flat \(\Omega_0 = 0.3, \Lambda_0 = 0.7, H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}\) cosmology and that the lens redshift of QJ0158−4325 is \(z_L = 0.5\). Reasonable changes in this assumed redshift have negligible consequences for the results.

2. OBSERVATIONS AND MODELS

We monitored QJ0158−4325 in the R band using the SMARTS 1.3 m telescope with the ANDICAM (DePoy et al. 2003)\(^4\) optical/ infrared camera (0.369″ pixel\(^{-1}\)) and using the 1.2 m Euler Swiss Telescope (0.344″ pixel\(^{-1}\)). The Euler monitoring data were obtained as part of the COSMOGRAIL\(^5\) project. A full description of our monitoring data reduction technique can be found in Kochanek et al. (2006) but we provide a brief summary here. We model the PSF of each quasar image using three nested, elliptical Gaussian components, keeping the relative astrometry fixed for all epochs. We use relative photometry, comparing the flux of each image to the flux of reference stars in each frame. For QJ0158−4325, we used five reference stars located at (−120.6°, −35.0°), (−40.1°, +137.7°), (+5.2°, −125.1°), (+81.4°, +108.8°), and (−86.5°, −78.6°) relative to image A with relative fluxes of 1.0:0.568:0.437:0.136:0.0906, respectively. The lens galaxy flux is determined by optimizing its flux in observations with good seeing, and the light curves are then measured with the galaxy flux fixed to this optimal value. We eliminated all data points taken in seeing conditions worse than 1.7″. Three points satisfying the seeing conditions and reported in Table 1 were not used in the analysis because the sky was too bright to measure the flux of image B accurately. The monitoring data are presented in Table 1, and the light curves are displayed in Figure 1. For HE1104−1805, we use the composite R- and I-band light-curve data from Poindexter et al. (2007).

We created photometric models for the two systems from the WFPC2 and NICMOS \(\lambda V\) (F555W), I- (F814W), and H-band (F160W) observations of the two systems by the CfA-Arizona Space Telescope Lens Survey (CASTLES;\(^6\) Falco et al. 2001) following the methods of Lehár et al. (2000). The quasars are modeled as point sources and each lens galaxy as a de Vaucouleurs profile. We chose the de Vaucouleurs profile above other models since it provided the best fit. Table 2 summarizes the fits we use here, where the HE1104−1805 model is updated from that in Lehár et al. (2000) using a deeper H-band image obtained to study the quasar host galaxy (Yoo et al. 2006).

For each system we created a sequence of 10 lens models using the lensmodel software package (Keeton 2001). Each model is the sum of concentric Navarro-Frenk-White (NFW; Navarro et al. 1996) and de Vaucouleurs components, where the NFW component simulates the dark matter halo and the de Vaucouleurs component represents the galaxy’s stellar content. We parameterize the model sequence by \(f_{M/L}\), the mass of the stellar component relative to its mass in a constant mass-to-light ratio model with no contribution from the NFW halo. We generated model sequences covering the range 0.1 ≤ \(f_{M/L}\) ≤ 1.0. With their time delay measurement, Poindexter et al. (2007) constrained the stellar mass fraction of HE1104−1805 to \(f_{M/L} = 0.30_{-0.04}^{+0.04}\), but we chose not to apply these limits to \(f_{M/L}\) for our present calculations. From these models for the mass distribution we extract the convergence (\(\kappa\)), shear (\(\gamma\)), and stellar surface density (\(\kappa_s\)) for each image and then generate realizations of the microlensing magnification patterns at the location of each image using a variant of the ray-shooting (Schneider et al. 1992) method described in Kochanek (2004). A summary of the microlensing parameters

\[\text{Notes.—HJD is the Heliocentric Julian Day}−2,450,000 \text{ days. The number of degrees of freedom} N_{\text{df}} \text{ is set by the number of pixels used in the photometric measurement. Typical values are} N_{\text{df}} = 6003 \text{ for Euler and} N_{\text{df}} = 4482 \text{ for SMARTS/ANDICAM. The goodness of fit of the image,} x^2/N_{\text{df}}, \text{ is used to rescale the formal uncertainties when greater than unity. The QSO A and B columns give the magnitudes of the quasar images relative to the comparison stars. The} \langle\text{Stars}\rangle \text{ column gives the mean magnitude of the standard stars for that epoch relative to their mean for all epochs. A few points in the light curves (in parentheses) were not used in the analysis. Table 1 is published in its entirety in the electronic edition of the Astrophysical Journal. A portion is shown here for guidance regarding its form and content.}\]

\[\text{TABLE 1}\]

| HJD     | \(x^2/N_{\text{df}}\) | QSO A (mag) | QSO B (mag) | \langle\text{Stars}\rangle | Source |
|---------|-------------------|--------------|--------------|-----------------|--------|
| 2863.873 | 1.17              | 2.043 ± 0.010 | 2.631 ± 0.015 | −0.044 ± 0.003 | SMARTS |
| 2870.788 | 1.93              | 2.072 ± 0.013 | 2.585 ± 0.020 | −0.050 ± 0.003 | SMARTS |
| 2871.813 | 0.66              | 2.046 ± 0.014 | 2.609 ± 0.022 | −0.052 ± 0.003 | SMARTS |
| 2877.722 | 1.01              | 1.997 ± 0.008 | 2.600 ± 0.011 | 0.043 ± 0.002 | SMARTS |
| 2884.771 | 2.15              | 1.998 ± 0.007 | 2.600 ± 0.010 | 0.053 ± 0.002 | SMARTS |
| 2891.770 | 0.83              | 2.014 ± 0.013 | 2.537 ± 0.021 | −0.055 ± 0.003 | SMARTS |
| 2909.789 | 2.26              | 1.950 ± 0.010 | 2.524 ± 0.015 | −0.024 ± 0.003 | SMARTS |
| 2906.761 | 1.75              | 1.962 ± 0.007 | 2.483 ± 0.009 | 0.052 ± 0.002 | SMARTS |
| 2914.653 | 0.49              | 1.968 ± 0.018 | 2.417 ± 0.026 | −0.038 ± 0.004 | SMARTS |
| 2916.766 | 1.00              | 1.942 ± 0.008 | 2.429 ± 0.011 | 0.018 ± 0.003 | SMARTS |

\[4\] See http://www.astronomy.ohio-state.edu/ANDICAM/.

\[5\] See http://www.cosmograil.org/.

\[6\] See http://cfa-www.harvard.edu/glensdata/.
is presented in Table 3. We assume a stellar mass function of 
\( dN(M)/dM \propto M^{-3} \) with a dynamic range of 50, which approx-
imates the Galactic stellar mass function of Gould (2000). We
present the results either making no assumption about the mean
mass \( \langle M \rangle \) of the stars or by applying a prior so that it lies in the
range \( 0.1 \, M_\odot \leq \langle M \rangle \leq 1.0 \, M_\odot \). We used 4096^2 microlensing
magnification patterns with an outer scale of 20\( R_E \). We used a
prior for the relative motions of the observer, lens galaxy, lens
galaxy stars, and the source based on the projection of the CMB
dipole (Kogut et al. 1993) for the observer, the stellar velocity
dispersion of the lens set by its Einstein radius, and rms peculiar
velocities for the lens and source of \( \sigma_p = 235/(1+z) \) km s\(^{-1}\)
(Kochanek 2004). We modeled the continuum emission source as a face-on thin accretion disk (Shakura & Sunyaev 1973) with the
surface brightness profile

\[
I(R) \propto \left\{ \exp \left[ \left( R/r_s \right)^{3/4} \right] - 1 \right\}^{-1}.
\]

(1)

The scale radius \( r_s \) is the point where the disk temperature matches the rest-frame wavelength of our monitoring band, \( \delta T = h c (1+z)/(\Lambda_{\text{obs}}) \). We actually measure a projected disk area with

| TABLE 2 | HST Astrometry and Photometry of QJ0158−4325 and HE1104−1805 |
| --- | --- | --- | --- |
| **Lens** | **Component** | **Astrometry** | **Photometry** |
| | | \( \Delta \text{R.A.} \) | \( \Delta \text{decl.} \) | \( H = F160W \) | \( I = F814W \) | \( V = F555W \) |
| QJ0158−4325 | A | \( \equiv 0 \) | \( \equiv 0 \) | 16.47 ± 0.03 | 17.81 ± 0.04 | 18.10 ± 0.13 |
| | B | \(-1.156^\circ \pm 0.003^\circ\) | \(-0.398^\circ \pm 0.003^\circ\) | 17.27 ± 0.03 | 18.62 ± 0.11 | 18.91 ± 0.17 |
| | G | \(-0.780^\circ \pm 0.016^\circ\) | \(-0.234^\circ \pm 0.006^\circ\) | 16.67 ± 0.13 | 18.91 ± 0.06 | 20.36 ± 0.18 |
| HE1104−1805 | A | \( \equiv 0 \) | \( \equiv 0 \) | 15.91 ± 0.01 | 16.40 ± 0.03 | 16.92 ± 0.06 |
| | B | \(+2.901^\circ \pm 0.003^\circ\) | \(-1.332^\circ \pm 0.003^\circ\) | 17.35 ± 0.03 | 17.95 ± 0.04 | 18.70 ± 0.08 |
| | G | \(+0.965^\circ \pm 0.003^\circ\) | \(-0.500^\circ \pm 0.003^\circ\) | 17.52 ± 0.09 | 20.01 ± 0.10 | 23.26 ± 0.27 |
microlensing, so the true disk size will scale with inclination \(\cos i\), the surface brightness profile of the disk. For our analysis we must generate light-curve pairs with arbitrary delays. We always carry out the shifts on the less variable end of the observed light curve, we estimate the flux using linear interpolation between the nearest bracketing data points because they lead to a delay-dependent change in the statistical errors caused by the steadily growing probability distribution of the variance from the structure function. Given the delay range we wanted to test, we first found the limiting delay defined by the delay that yielded the minimum number of usable data points with our 7 day extrapolation limit, leaving 242 epochs for HE1104–1805 and 102 epochs for QJ0158–4325. We then restricted all of the trial light curves to use only the epochs permissible at the limiting delay. For the very long delays of HE1104–1805, the limiting delay is not the longest delay, because the longest delays shift curves completely through the interseason gaps. This forced us to restrict the HE1104–1805 analysis to the epochs permitted by both the limiting delay and the longest delay (219 total epochs), since some permissible times at the end of the limiting delay light curve are beyond the extrapolation limit in light curves with longer delays. We also experimented with simply allowing unlimited extrapolations; we compare the results for the two methods in § 4.

We then analyzed the light curves using the Bayesian Monte Carlo method of Kochanek (2004). In essence, we randomly select a time delay \(\Delta t\), a lens model, and a disk model (size), generate a microlensing light curve, and fit it to the microlensing light curve implied by the observed data and the selected time delay. This gives us a \(\chi^2\) statistic for the goodness of fit for the trial \(\chi^2(p, \Delta t)\) given the model parameters for the microlensing light curve \(p = (f_{\text{HL}}, r_s, \text{velocities, masses etc.})\) and the time delay. Figure 1 shows an example of a good trial light-curve fit to QJ 0158–4325 for a delay of \(\Delta t_{\text{AB}} = -20\) days. In essence, the probability of time delay \(\Delta t\) is the Bayesian integral

\[
P(\Delta t|D) \propto \int P(D|p, \Delta t)P(p)P(\Delta t)dp,
\]

where \(P(D|p, \Delta t)\) is the probability of fitting the data in a particular trial, \(P(p)\) sets the priors on the microlensing variables (see Kochanek 2004; Kochanek et al. 2007), and \(P(\Delta t)\) is the (uniform) prior on the time delay. The total probability is then normalized so that \(\int P(\Delta t|D)d\Delta t = 1\). We evaluated the integral as a Monte Carlo sum over the trial light curves, where we created four independent sets of magnification patterns for each of the 10 macroscopic mass models and generated \(4 \times 10^6\) trial light curves for each magnification pattern set.

4. RESULTS

Poindexter et al. (2007) recently estimated a time delay for HE1104–1805 of \(\Delta t_{\text{AB}} = t_A - t_B = 152.2^{+2.3}_{-0.5}\) days, in the sense that image A lags image B, improving on the earlier estimates by Ofek & Maoz (2003) and Wyrzykowski et al. (2003). The Poindexter et al. (2007) analysis used the Kochanek et al. (2006) polynomial method on light curves that combined the published data of Schechter et al. (2003), Ofek & Maoz (2003), and Wyrzykowski et al. (2003) with new R-band monitoring data. Poindexter et al. (2007) combined these data into a common light curve following the approach of Ofek & Maoz (2003). In this polynomial method, the source and microlensing variability are modeled as a set of Legendre polynomials that are then fit to the light curves. Ambiguities arise because the value of the delay depends weakly on the parameterization of the microlensing. Poindexter et al. (2007) used a Bayesian weighting scheme for the different polynomial orders but obtained 157.2 ± 2.6 days if they used the F-test to select among the different orders rather than a Bayesian weighting. The advantage of our present approach is that it uses a physical model for the microlensing rather than a polynomial parameterization of it.

We applied our joint Monte Carlo analysis technique to HE1104–1805 over a time interval range of 125 days ≤ \(\Delta t_{\text{AB}}\) ≤ 200 days with a sampling in 1.5 day intervals and no extrapolation of the light curves past 7 days. Figures 2 and 3 show the resulting probability distributions for the time delay and the disk points with our 7 day extrapolation limit.
size. We find a time delay of $\Delta t_{AB} = t_A - t_B = 162.2^{+6.3}_{-5.9}$ days (1 $\sigma$) that is in marginal agreement with the formal Poindexter et al. (2007) result but in better agreement with the F-test selection of the best polynomial model than with the Bayesian result. Scaled to the mean disk inclination angle, $\cos (i) = 1/2$, the disk size estimate of $\log \left( \frac{r_s}{\text{cm}} \right) \cos (i)/0.5^{1/2} = 15.7^{+0.4}_{-0.4}$ at 0.2 $\mu$m

The last stable orbit for a Schwarzschild black hole is at $3R_{\text{BH}}$. The Schwarzschild radius $R_{\text{BH}} = 2GM_{\odot}/c^2$ of the black hole, where the black hole mass $M_{\odot} = 2.37 \times 10^6 M_{\odot}$ was estimated by Peng et al. (2006) using the C iv emission line width. We also performed our analysis on a set of light curves in the rest frame is little changed from the estimates in Morgan et al. (2007) and Poindexter et al. (2008), which held the delay fixed to the Poindexter et al. (2007) value. The uncertainties in the size are a factor of 1.6 larger in the present analysis. The difference in the source sizes at the redshifted centers of the $V$ and $R$ bands is small enough that the use of a composite light curve had little effect on the analysis (see Poindexter et al. [2008] for a detailed discussion of the wavelength dependence of microlensing in HE1104–1805).
that interpreting varying amounts of the lower amplitude intrinsic variability as microlensing does not change the statistics of the microlensing enough to significantly affect the size estimate. One additional uncertainty in this result is that lens redshift of QJ0158−4325 is unknown. We experimented with running the Monte Carlo simulation at a range of lens redshifts (0.1 ≤ z_l ≤ 0.9), and we found that the resulting shifts in the r_s estimates were negligible relative to the size of the existing uncertainties.

5. DISCUSSION AND CONCLUSIONS

Peng et al. (2006) used the width of the C iv (λ1549 Å) emission line to estimate the black hole mass M_BH = 2.37 × 10^9 M_☉ in HE1104−1805 and the width of the Mg ii (λ2798 Å) emission line to estimate the black hole mass M_BH = 1.6 × 10^9 M_☉ in QJ0158−4325. Using these black hole masses, the quasar accretion disk size—black hole mass relation of Morgan et al. (2007) predicts source sizes at 2500 Å of log \( r_s/(\text{cm}) \cos (i)/0.5 \)\(^{1/2} \) = 15.9 ± 0.2 for HE1104−1805 and log \( r_s/(\text{cm}) \cos (i)/0.5 \)\(^{1/2} \) = 15.2 ± 0.2 for QJ0158−4325. If we scale our current disk size measurements to 2500 Å using the R_c ~ λ^2 scaling of thin disk theory, we find log \( r_s/(\text{cm}) \cos (i)/0.5 \)\(^{1/2} \) = 15.9 ± 0.5 for HE1104−1805 and log \( r_s/(\text{cm}) \cos (i)/0.5 \)\(^{1/2} \) = 14.8 ± 0.3 for QJ0158−4325, fully consistent with the predictions of the Morgan et al. (2007) accretion disk size—black hole mass relation.

The mixing of intrinsic and microlensing variability in lensed quasar light curves can be a serious problem for estimating time delays (e.g., Eigenbrod et al. 2005), and previous microlensing analyses have been restricted to lenses with known time delays. In HE1104−1805, which must be close to the limits of measuring time delays in the presence of microlensing, we confirm that the approach of fitting polynomial models for the microlensing works reasonably well. However, the dependence of the delay on the assumed model was a warning sign that the formal errors on the delays were likely to be underestimates, as was recognized by Ofek & Maoz (2003) and Poindexter et al. (2007). In our new, nonparametric microlensing analysis of HE1104−1805, we find a modestly longer delay of 162\(^{+6}_{-9} \) days that quantifies those concerns. Estimates of the quasar accretion disk size are little affected by these small shifts in the time delay. In QJ0158−4325, the microlensing amplitude is larger relative to the intrinsic variability, and traditional methods for determining delays fail. Our new method also fails to measure a delay, but it does allow us to measure the size of the quasar accretion disk despite the uncertainties in the time delay.

We thank S. Poindexter for helpful discussions about HE1104−1805 and D. Will for answers to countless cluster computing questions. M. E. E. is grateful for summer internship support from the USNA Bowman Scholar program. C. V. F. C., and G. M. acknowledge support from the Swiss National Science Foundation (SNSF). C. S. K. acknowledges support from NSF grant AST-0708082. This research made extensive use of a Beowulf computer cluster obtained through the Cluster Ohio program of the Ohio Supercomputer Center. Support for program HST-GO-9744 was provided by NASA through a grant from the Space Telescope Science Institute, which is operated by the Association of Universities for Research in Astronomy, Inc., under NASA contract NAS-5-26666.

Facilities: CTIO:2MASS (ANDICAM), HST (NICMOS, ACS), Euler1.2m
REFERENCES

Burud, I., Magain, P., Sohy, S., & Hjorth, J. 2001, A&A, 380, 805
DePoy, D. L., et al. 2003, Proc. SPIE, 4841, 827
Eigenbrod, A., Courbin, F., Vuissoz, C., Meylan, G., Saha, P., & Dye, S. 2005, A&A, 436, 25
Falco, E. E., et al. 2001, in ASP Conf. Ser. 237, Gravitational Lensing: Recent Progress and Future Goals, ed. T. G. Brainard & C. S. Kochanek (San Francisco: ASP), 25
Gould, A. 2000, ApJ, 535, 928
Keeton, C. R. 2001, preprint (astro-ph/0102340)
Kochanek, C. S. 2004, ApJ, 605, 58
Kochanek, C. S., Dai, X., Morgan, C. W., Morgan, N. D., Poindexter, S. A., & Chartas, G. 2007, in Statistical Challenges in Astronomy IV, ed. G. J. Babu & E. D. Feigelson (San Francisco: ASP), 43
Kochanek, C. S., Morgan, N. D., Falco, E. E., McLeod, B. A., Winn, J., Dembicky, J., & Ketzeback, B. 2006, ApJ, 640, 47
Kogut, A., et al. 1993, ApJ, 419, 1
Lehar, J., Falco, E. E., Kochanek, C. S., McLeod, B. A., Impey, C. D., Rix, H.-W., Keeton, C. R., & Peng, C. Y. 2000, ApJ, 536, 584
Morgan, N. D., Dressler, A., Maza, J., Schechter, P. L., & Winn, J. N. 1999, AJ, 118, 1444
Morgan, C. W., Kochanek, C. S., Morgan, N. D., & Falco, E. E. 2007, ApJ, submitted
Mortonson, M. J., Schechter, P. L., & Wambsganss, J. 2005, ApJ, 628, 594
Navarro, J. F., Frenk, C. S., & White, S. D. M. 1996, ApJ, 462, 563
Ofek, E. O., & Maoz, D. 2003, ApJ, 594, 101
Oguri, M. 2007, ApJ, 660, 1
Paraficz, D., Hjorth, J., Burud, I., Jakobsson, P., & Eliasdottir Á. 2006, A&A, 455, L1
Peng, C. Y., Impey, C. D., Rix, H.-W., Kochanek, C. S., Keeton, C. S., Falco, E. E., Lehar, J., & McLeod, B. A. 2006, ApJ, 649, 616
Poindexter, S., Morgan, N. D., & Kochanek, C. S. 2008, ApJ, 673, 34
Poindexter, S., Morgan, N. D., Kochanek, C. S., & Falco, E. E. 2007, ApJ, 660, 146
Pooley, D., Blackburne, J. A., Rappaport, S., & Schechter, P. L. 2007, ApJ, 661, 19
Refsdal, S. 1964, MNRAS, 128, 307
Saha, P, Coles, J., Macciò, A. V., & Williams, L. L. R. 2006, ApJ, 650, L17
Schechter, P. L., & Wambsganss, J. 2002, ApJ, 580, 685
Schechter, P. L., et al. 1997, ApJ, 475, L85
———. 2003, ApJ, 584, 657
Schneider, P., Ehlers, J., & Falco, E. E. 1992, Gravitational Lenses (Berlin: Springer)
Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337
Vanden Berk, D. E., et al. 2004, ApJ, 601, 692
Vuissoz, C., et al. 2007, A&A, 464, 845
Wambsganss, J. 2006, in Saas-Fee Advanced Course 33, Gravitational Lensing: Strong, Weak and Micro, ed. G. Meylan, P. Jetzer, & P. North (Berlin: Springer), 453
Wisotzki, L., Koehler, T., Kayser, R & Reimers, D. 1993, A&A, 278, L15
Wyzykowski, L., et al. 2003, Acta Astron., 53, 229
Yoo, J. Kochanek., C. S., Falco, E. E., & McLeod, B. A. 2006, ApJ, 642, 22