A two-stage stochastic programming approach to integrated day-ahead electricity commitment and production scheduling

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Abstract

To ensure stability of the power grid, the electricity suppliers impose a daily day-ahead hourly electricity commitment to large consumers. In case the actual electricity consumption differs from the committed profile, the consumer is obliged to pay penalties. The challenge addressed in this work is to simultaneously determine the optimal day-ahead electricity commitment and the optimal production scheduling. Since the consumers have to commit themselves to the amount of energy they are going to purchase and use for a period of 24 hours one day before the actual electricity demand is realized, a major challenge lies in the uncertainty: equipment failures may reduce the production capacity and can make the actual electricity consumption drastically deviating from the day-ahead electricity commitment. For this purpose a two-stage stochastic mixed-integer linear programming model that considers equipment breakdowns is proposed. The application of the proposed approach to a continuous power intensive plant shows the improvement achieved by solving the stochastic model.

Keywords: Production scheduling, Two-stage stochastic programming, Demand Side Management, Load deviations, Conditional value-at-risk.

1. Introduction

Industrial Demand Side Response is a promising technology that is aiming at reducing operating costs for energy-intensive industries exploiting production flexibility to take advantage of time-sensitive electricity prices (Merkert et al., 2014). The volatility of the electricity prices, due to the availability of renewable energy sources, for electricity consumers constitutes a potential saving and for electricity suppliers an increased effort to match demand and supply. To ensure the stability of the power grid, the electricity suppliers impose a daily day-ahead hourly electricity commitment to large consumers and in case the actual consumption deviates from the pre-agreed values, financial penalties, which are often in the same range as the net electricity cost, are incurred. This is known in literature as the load-deviation problem. Nolde and Morari (2010) proposed a scheduling solution for electrical load tracking of a steel plant. The schedule is defined such that the total electricity consumption tracks the load curve as closely as possible while respecting all production constraints. In (Hadera et al., 2015), the authors take into account multiple electricity sources (Base load and Time-Of-Use power contracts, day-ahead market, onsite generation) and the load deviation problem to determine the optimal production schedule. However, in these contributions the load commitment
decisions are assumed as given and are not optimized. In this work, we address the challenge of determining simultaneously the optimal day-ahead electricity commitment and the optimal production scheduling. Since decisions regarding the electricity commitment have to be made before the actual electricity demand is known for the time horizon of interest, it is crucial to account for uncertainties in the decision-making process. To this challenge we propose a two-stage stochastic programming approach (Birge and Louveaux, 2011). The first-stage variables represent the day-ahead electricity commitment decisions. The second stage faces plant capacity uncertainty and second-stage variables for each breakdown scenario are the plant operating decisions (production levels, inventories…) and the electricity consumption deviations from the day-ahead commitment. A similar approach has not been considered in this context before. In Section 2, the problem statement is presented highlighting the uncertainty modelling strategy and the proposed two-stage MILP. Section 3 applies the proposed framework to a power-intensive process and the results are summarized in terms of the Value of the Stochastic Solution (VSS) for risk-neutral and risk-averse optimization. The main insights from the results are discussed in Section 4 before providing some final conclusion in Section 5.

2. Problem statement

2.1. Uncertainty modeling strategy

To integrate day-ahead electricity commitment and production scheduling, we propose a two-stage stochastic programming approach (Birge and Louveaux, 2011). In two-stage stochastic programming (2SSP), uncertainty is represented by discrete scenarios, and decisions are made at two different stages which are defined by the realization of the uncertainty. Therefore we can divide the decision variables in two sets: here-and-now (or first-stage) decisions that have to be made at the beginning and cannot be changed over the scheduling horizon, and wait-and-see (or second-stage) decisions that can be adjusted after the realization of the uncertainty. In the proposed approach, we consider equipment breakdowns as source of uncertainty and three different levels of uncertainty—low, medium, and high. The low, medium and high uncertainty levels represent respectively 10, 30 and 40 percent reduction of the maximum plant production capacity. The production capacity reduction is modelled by 8 breakdown scenarios: In scenario 1 no breakdown occurs, whereas in Schemes 2–8 the breakdown occur in periods 1–4,...,22–24, respectively. The probabilities are 50% for scenario 1 and (50/7)% for scenarios 2–8.

2.2. Plant model

To demonstrate the main features of the proposed approach, we apply it to a continuous-production plant. The plant is the same as in (Zhang et al., 2016). In the proposed mathematical formulation here-and-now decisions have no scenario subscript and wait-and-see decisions have the scenario subscript s. The plant produces two products \( P_1 \) and \( P_2 \) and it can operate in three different modes \( m: \) off, startup, and on. The possible mode transitions are off to startup, startup to on, and on to off and they can happen only after fixed period of time that have been spent in the modes (off: 8 h, startup:2 h, on:6 h) (Eq.6). The transitions between the modes of operation and the relations with the active modes are modelled by Eq.(7). The binary variable (Eq.(4)) is 1 if mode \( m \) is selected in time period \( t \) of the horizon \( T \). For each mode, the operating conditions are expressed as a convex combination of the extreme points \( v_{mi} \) of the feasible region of
operation (Eq. (2)). Eq. (1)-(2) define the hourly production levels \( \bar{PD}_{m,ts} \) for each product \( i \) and operating mode \( m \) and the aggregated production \( PD_{its} \).

\[
PD_{its} = \sum_{m} \bar{PD}_{m,ts} \quad \forall \ i, t \in T, s \tag{1}
\]

\[
\bar{PD}_{m,ts} = \sum_{m} \lambda_{m,ts} \cdot v_{mi} \quad \forall \ i, t \in T, s \tag{2}
\]

\[
\sum_{m} \lambda_{m,ts} = y_{mt} \quad \forall \ i, t \in T, s \tag{3}
\]

\[
\sum_{m} y_{mt} = 1 \quad \forall \ t \in T \tag{4}
\]

\[
EU_{its} = \sum_{m} \left( \delta_{m} \cdot y_{mt} + \sum_{i} y_{mi} \cdot \bar{PD}_{m,ts} \right) \quad \forall \ i, t \in T, s \tag{5}
\]

\[
\theta_{mm',t-k} \leq y_{mt} \quad \forall (m, m') \in M, t \in T \tag{6}
\]

\[
\sum_{k=1}^{\theta_{mm',t-k}} \sum_{m} x_{mm',t-1} = y_{mt} - y_{m',t-1} \quad \forall m, t \in T \tag{7}
\]

Eq. (8) defines the inventory level \( IV_{its} \) at time \( t \) as the sum of the inventory level at time period \( t-1 \) and the production at time \( t, PD_{its} \), minus the amount of product sold, \( SL_{its} \), and the amount of products wasted, \( PW_{its} \), at time period \( t \). Note that all these variables are wait-and-see variables and therefore they are defined for each scenario \( s \).

Eq. (9) sets upper and lower bounds of the inventory levels and Eq. (10) ensures that the demand of product \( i \) as the sum of the amount of product sold, \( SL_{its} \), and the amount of products purchased from other sources, \( PC_{its} \), is satisfied.

\[
IV_{its} = IV_{i,t-1,s} + PD_{its} - SL_{its} - PW_{its} \quad \forall \ i, t \in T, s \tag{8}
\]

\[
IV_{i,t}^{min} \leq IV_{i,t} \leq IV_{i,t}^{max} \quad \forall \ i, t \in T, s \tag{9}
\]

\[
SL_{i,ts} + PC_{i,ts} = D_{i,ts} \quad \forall \ i, t \in T \tag{10}
\]

Eq. (11) - (14) provide the initial and final condition of the plant in terms of inventory levels and active operating modes.

\[
IV_{i,0,s} = IV_{i,initial} \quad \forall \ i, s \tag{11}
\]

\[
y_{m,0} = y_{m,initial} \quad \forall \ m \tag{12}
\]

\[
IV_{i,final} \geq IV_{i,final} \quad \forall \ i, s \tag{13}
\]

\[
z_{mm',ts} = z_{mm',ts}^{ini} \quad \forall (m, m') \in M, s, -\theta_{max} + 1 \leq t \leq -1 \tag{14}
\]

2.3. Day-ahead electricity commitment

Eq. (15) defines the day-ahead electricity commitment \( ES_{t} \), the over-consumptions \( \Delta e_{t}^{+} \) and the under-consumptions \( \Delta e_{t}^{-} \). Since the consumers have to commit themselves to the amount of energy they are going to purchase for a period of 24 hours one day before the actual electricity demand is realized, the electricity commitment decisions are first-stage variables and the load deviations and the actual electricity consumptions are second stage variables.

\[
EU_{t,s} - ES_{t} = \Delta e_{t}^{+} - \Delta e_{t}^{-} \quad \forall \ t \in T, s \tag{15}
\]

2.4. Objective function

We solve the proposed stochastic formulation considering both risk-neutral and risk-averse optimization. The two approaches mainly differ in the objective functions. The
risk-neutral optimization minimizes the expected operating cost; the risk-averse optimization balances two conflicting objectives: the expected cost and the risk, defined as the expected cost over the worst scenarios.

2.4.1. Risk-neutral optimization

The model minimizes the total expected operating costs, \( z \), defined in Eq.(16) as the first-stage cost of day-ahead electricity commitment and the expected second-stage cost of deviation penalties and purchasing of products on the market.

\[
z = \sum_{t} p_t^{\text{day-ahead}} ES_t + \sum_{s} \varphi_s \left( p_t^+ \Delta e_t^+ + p_t^- \Delta e_t^- + \sum_{i} p_i PC_{its} \right)
\]  

(16)

where \( p_t^{\text{day-ahead}}, p_t^+, p_t^- \) represent the day-ahead electricity price, the penalty cost for over consumption and under consumption and the product purchasing price; \( \varphi_s \) denotes the probability of scenario \( s \).

2.4.2. Risk-averse optimization

Different risk measures have been presented in the literature: value-at-risk (VaR), downside risk, Conditional value-at-risk (CVaR) (Rockafellar and Uryasev (2000)). We adopted the CVaR (Rockafellar and Uryasev (2000)), since it is a coherent risk measure (it preserves convexity) and it is able to consider the tail of the probability density function. The CVaR is defined by Eq.(17)-(18).

\[
CV = k + \frac{1}{1 - \alpha} \sum_{s} \varphi_s \theta_s 
\]

(17)

\[
\sum_{t} \left[ p_t^{\text{day-ahead}} ES_t + p_t^+ \Delta e_t^+ + p_t^- \Delta e_t^- + \sum_{i} p_i PC_{its} \right] - k \leq \theta_s \quad \forall s 
\]

(18)

\[
\min \{ \delta \cdot CV + (1 - \delta) \cdot z \}
\]

(19)

where \( k, \theta_s \) are continuous variables with \( k \in R \) and \( \theta_s \geq 0 \). Eq.(19) defines the objective function as the weighted sum of the total expected cost and the CVaR (\( \delta = 0.5 \)).

3. Results

To measure the improvement that can be achieved by solving the stochastic model instead of its deterministic counterpart, we compute the value of the stochastic solution (VSS). The VSS and the relative \( \overline{VSS} \), defined in Eq.(20)-(21), show the impact of the uncertainty on the first-stage variables:

\[
VSS = z_{\text{det}}^* - z_{\text{stoc}}^* 
\]

(20)

\[
\overline{VSS} = \frac{z_{\text{det}}^* - z_{\text{stoc}}^*}{z_{\text{det}}^*}
\]

(21)

where \( z_{\text{stoc}}^* \) is the optimal solution of the stochastic problem and \( z_{\text{det}}^* \) is the optimal solution of the stochastic problem with first-stage variables fixed to the values at the optimal solution of the deterministic problem. Table 1 shows the VSS for risk-neutral and risk-averse optimization: the VSS can be quite significant and that it grows with the uncertainty level. The high values of the VSS demonstrate that there is a significant benefit from considering breakdown uncertainty in the integrated day-ahead electricity procurement and production scheduling. The VSS obtained for the risk-averse optimization are considerably higher than those obtained in risk-neutral optimization. Each model has up to approximately 6,500 continuous variables, 209 binary variables, and 3,900 constraints. All models were solved to zero integrality gap in less than 10 s.
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on an Intel(R) Core(TM) i7-4790 machine at 3.60 GHz with eight processors and 16 GB RAM. All models were implemented in GAMS 24.7.4 (GAMS Development Corporation, 2015), and the MILPs were solved applying the commercial solver CPLEX 12.6.3.0.

Table 1 Value of the stochastic solution for risk-neutral and risk-averse optimization for different levels of uncertainty

| Uncertainty level | Risk-neutral optimization | Risk-averse optimization |
|-------------------|---------------------------|--------------------------|
|                   | VSS [€]                   | VSS [%]                  | VSS [€] | VSS [%] |
| Low               | 84.4                      | 2.8%                     | 367.3   | 3.7%   |
| Medium            | 214.6                     | 6.4%                     | 1029.5  | 10.6%  |
| High              | 247.9                     | 7.0%                     | 1210.2  | 12.9%  |

4. Discussion

Fig.1 shows the electricity purchase profiles that were obtained solving the risk-neutral stochastic model and the deterministic model for the medium uncertainty level. TOU and base load profiles represent the amount of energy purchased from the power contracts. TOU and base load profiles are supposed to be given and therefore not optimized since contract related decisions have to be made before the time horizon of interest (1 week before for the TOU contract and 1 year before for the base load contract). The comparison between the two solutions shows the impact on the day-head electricity commitment (first-stage variables) of accounting for other scenario besides the expected one. In the deterministic solution (Fig.1b) the day-ahead commitment depends exclusively on the day-ahead electricity price: in case of price peak (at 9-10 h and 20-21 h) the electricity purchase is drastically reduced and the production is shifted to time periods when the electricity price decreases. In the stochastic solution (Fig.1a), instead, during the price peaks more electricity is purchased in order to able to compensate reductions of the production capacity. By doing so, lower penalties for deviating from the day-ahead commitment are incurred.

5. Conclusion

This work addresses the integrated day-ahead electricity commitment and production scheduling for continuous power-intensive processes. A two-stage stochastic programming approach is proposed to model uncertainty of equipment breakdowns. The application of the proposed approach to a power intensive plant shows the benefit of the stochastic model. Risk is taken into account by incorporating the CVaR into the optimization model.

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Figure 1. Electricity purchase profiles obtained solving the stochastic model (a) and the deterministic model (b) for the medium uncertainty level.