Fundamental Limits of Full-Duplex Spectrum Sharing Under Peak Interference Power Constraint

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Accepted: 14 November 2021 / Published online: 2 December 2021
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Abstract

Cognitive radio (CR) and full-duplex (FD) have received extensive attention and research due to their high spectrum efficiency, which can effectively solve the problem of low spectrum efficiency in current communication systems. Based on CR and FD techniques, in this paper, a FD spectrum sharing CR networks is considered, in which both secondary users (SU1 and SU2) are each equipped with dual antennas, one antenna is used to transmit signals, and the other antenna is used to receive signals at the same time and frequency. Under peak interference power and peak transmit power constraints, we analysis the ergodic sum capacity and the outage probability based on the FD spectrum sharing CR networks and the conventional spectrum sharing CR networks. Furthermore, under no peak transmit power constrain and perfect self-interference cancellation (SIC), based on the FD spectrum sharing CR networks and the conventional spectrum sharing CR networks, the closed-form expressions of the theoretical performance upper bound of the ergodic sum capacity and the outage probability are derived by two lemmas and four propositions. Accurate mathematical analysis display, under the same bandwidth, the upper bound of ergodic sum capacity for the full-duplex spectrum sharing CR networks is twice as much as the traditional spectrum sharing CR networks, and the FD spectrum sharing CR networks based on SU1, also has better performance upper bound on the outage probability than the traditional spectrum sharing CR networks. Simulations results also validate that, the FD spectrum sharing CR networks obtains better communication performance than the conventional spectrum sharing CR networks, especially when the mean of self-interference channel power gain is small. Finally, we also can see that the simulation performance upper bound is completely consistent with the theoretical analysis performance upper bound, whether in the FD spectrum sharing CR networks or the conventional spectrum sharing CR networks. So also verifies the correctness of the theoretical performance upper bound derivation.

Keywords Cognitive radio · Spectrum sharing · Full-duplex · Outage probability · Ergodic sum capacity
1 Introduction

With the rapid growth of demand for high-speed real-time communication and wireless broadband video services, the current spectrum resources have become increasingly scarce. In order to effectively solve the problem of increasing shortage of spectrum resources, Cognitive radio (CR) is proposed to effectively solve the problem of lack of spectrum resources. This is mainly because CR can effectively improve the spectrum efficiency [1]. CR is mainly divided into two types of models, in which are referred to as spectrum sharing [2] and opportunistic spectrum access [3]. In a spectrum sharing scheme, the secondary user and the primary user use the same licensed spectrum. Hence, the secondary user must control its transmit power properly in order to achieve a reasonably high transmission rate without causing too much interference to the primary user. Alternatively, in an opportunistic spectrum access scheme, the secondary user only can communicate when the primary user does not use the licensed spectrum. Thus, the secondary user need do periodic spectrum sensing to avoid interference to the primary user. In this paper, our research is mainly based on the spectrum sharing scheme.

With the development of signal processing and integrated circuit techniques, the implementation of full-duplex (FD) communication technology becomes possible in the future wireless networks. Therefore, FD communication technology has received extensive attention and research in recent years [4–9]. Under perfect self-interference cancellation (SIC), compared with the half-duplex (HD) communication system, the FD communication system can obtain double spectrum efficiency. The SIC for FD mainly include analog-domain interference cancellation, propagation-domain interference cancellation, digital-domain interference cancellation, and propagation-domain interference cancellation [10]. Therefore, the combination of CR and FD has become an inevitable trend.

CR combined with FD current research works are mostly based on the opportunistic spectrum access model [11–19]. The research focus are mainly on spectrum sensing, optimal power allocation, network security and energy efficiency optimization et al. Compared with the FD opportunistic spectrum access CR networks, the traditional opportunistic spectrum access CR networks adopts HD communication mode. Thus, there are two major defects. First, the secondary user cannot sense and access the licensed spectrum at the same time. Therefore, the traditional HD opportunistic spectrum access CR networks adopt a two-stage communication pattern, where the secondary user is to sense the presence or absence of the primary user signal in the first stage and the secondary user chooses the opportunistic to access the licensed spectrum based on the sensing result in the second stage, so greatly reduces the ergodic capacity of secondary user due to time lost by doing spectrum sense. In addition, even if the primary user signal does not exist in the first stage and the secondary user senses correct. Thus, the secondary user uses the primary user’s spectrum in the second stage. However, once the primary user re-uses the licensed spectrum in the second stage, so the secondary user is likely to cause interference to the primary user. Second, in traditional HD opportunistic spectrum access CR networks, the secondary user needs to prepare additional bandwidth for spectrum sensing, thereby reducing spectrum utilization. The above two main shortcomings can be effectively overcome after adopting the FD communication mode. Therefore, application of FD to the opportunistic spectrum access CR networks has received extensive attention and research.

In addition, there are also some works of CR combined with FD based on the spectrum sharing model [20–33]. In the FD spectrum sharing CR networks, in which is divided into two categories according to whether with the help of the relay. The current research focus
is mainly on the FD spectrum sharing CR networks with relay assistance [20–28]. In the spectrum sharing CR networks, when considering FD relay to assist secondary user transmission, on the one hand, so increases the spectral efficiency and connectivity of the cognitive networks, but on the other hand, the relay also increases interference to the primary user. Hence, it is necessary to reasonably control the transmission power of secondary users and the relay. Based on this question, in [20], the authors proposed an optimal power allocation scheme based on the goal of minimum outage probability for secondary users under certain interference power constraint. As a more realistic consideration, the existence of residual self-interference is considered in [21], a hybrid relay transmission scheme was proposed, where the relay adaptive selected FD or HD transmission mode based on the degree of residual self-interference. Numerical results show significant performance improvement by using the hybrid scheme. Considering the improvement of networks performance by multi-relay selection diversity gain, multi-relay is also applied to the FD spectrum sharing CR networks [26–28]. In [26], an opportunistic FD relay selection scheme in spectrum sharing CR networks under different channel conditions was studied, in which FD relay adopted decode-forward (DF) protocol, and the out probability of the spectrum sharing CR networks is analyzed. Next, in Bin et al. [27], proposed a novel FD-based two-way amplify-and-forward (AF) relaying transfer protocol based on the same network model as [26]. Furthermore, in [28], the authors considered the situation of multi-primary users and multi-secondary user pairs, and utilized secondary transmitters as relays, where each primary user pair selected the best relay using a specific relay selection policy. Numerical results illustrate that the proposed schemes perform close to the exhaustive selection schemes.

Relatively few works based on the FD spectrum sharing CR networks without relay assistance [29–33]. In [29], the authors provided an optimal power allocation algorithm based on maximizing network capacity under primary outage threshold constraint. Simulation results show that, the proposed optimal power allocation scheme can greatly outperform the traditional equal power allocation scheme in terms of cognitive capacity. Zhang et al. [30] first investigated the secrecy outage performance of primary user system in spectrum sharing CR networks in the presence of the eavesdropping and interfering of secondary user. In addition, the authors in [31] studied the FD wirelessly powered spectrum sharing CR networks based on physical-layer security aspect. Furthermore, in [32], a novel polarization enabled full-duplex hybrid spectrum sharing scheme was proposed, and simulation results show that the proposed method guarantees sufficiently small collision ratio to primary users and secondary throughput are largely improved in comparison to the traditional schemes. Finally, in order to effectively improve energy efficiency, the authors in [33] proposed an adaptive spectrum sharing schemes, and theoretical analysis shows that the proposed adaptive spectrum sharing schemes improve the minimum energy efficiency in HD mode and improve the overall energy efficiency in FD mode. The research works mentioned above is not a study on the fundamental limits of the FD spectrum sharing CR networks. The fundamental limits of the communication system can be very intuitive to see the potential of the communication system, so it is necessary to study the fundamental limits of the communication system. Although, in Ghasemi et al. [3], studied the fundamental limits of the conventional HD spectrum sharing CR networks in fading environments. However, to the best of the authors knowledge, the fundamental limits of performance for the FD spectrum sharing CR networks has not been studied, which inspired the investigation of this paper.

In this paper, the fundamental limits of FD spectrum sharing CR networks under peak interference power constraint was studied. The main contributions of this paper are
as follows: (1) Under peak interference power and peak transmit power constraints, we analysis the ergodic sum capacity and the outage probability based on the FD spectrum sharing CR networks and the conventional spectrum sharing CR networks; (2) Under no transmit power constraint and perfect SIC, the performance upper bound closed expressions are obtained, based on the ergodic sum capacity and the outage probability for the FD spectrum sharing CR networks and the conventional spectrum sharing CR networks; (3) Accurate mathematical analysis display, under the same bandwidth, the upper bound of ergodic sum capacity for the FD spectrum sharing CR networks is twice that of the traditional spectrum sharing CR networks. In addition, compared with the traditional spectrum sharing CR networks, the FD spectrum sharing CR networks based on SU1, has a better upper bound performance on the outage probability; (4) Simulations results validate that, the FD spectrum sharing CR networks obtains better communication performance than the conventional spectrum sharing CR networks, and the simulation performance upper bound is completely consistent with the theoretical analysis performance upper bound, whether in the FD spectrum sharing CR networks or the conventional spectrum sharing CR networks. So also verifies the correctness of the theoretical upper bound derivation.

2 System Model

As shown in Fig. 1, we consider a FD spectrum sharing CR networks with two secondary users (SU1 and SU2) and one primary receiver (PR). SU1 and SU2 are each equipped with dual antennas, where one antenna is used to transmit signals and the other antenna is used to receive signals. Therefore, SU1 and SU2 adopt FD work mode. The channel coefficients for the link from SU1 to SU2, the link from SU2 to SU1, the link from SU1 to PR, and the link from SU2 to PR are denoted by \( h_{12}, h_{21}, h_{1P}, \) and \( h_{2P}, \) respectively. We assume that all channels obey independently and identically distributed Rayleigh fading channel, and all channel power gains obey exponential distributed. The mean of channel power gains \( g_{12}, g_{21}, g_{1P}, \) and \( g_{2P}, \) are defined as \( \lambda_{12}, \lambda_{21}, \lambda_{1P}, \) and \( \lambda_{2P}, \) respectively, where \( |h_{12}|^2 = g_{12}, |h_{21}|^2 = g_{21}, |h_{1P}|^2 = g_{1P}, |h_{2P}|^2 = g_{2P}. \) As a more practical consideration, we assume that SIC is imperfect. To facilitate analysis, furthermore, we also assume that self-interference channels obey independently and identically distributed Rayleigh fading channel, and the self-interference channel coefficients for the link from SU1 to SU1 and the link from SU2 to SU2 are denoted by \( h_{11} \) and \( h_{22}, \) respectively. Analogously, the mean

Fig. 1  FD spectrum sharing CR networks
of self-interference channel power gains $g_{11}$ and $g_{22}$ are defined as $\lambda_{11}$ and $\lambda_{22}$, respectively, where $|h_{11}|^2 = g_{11}$ and $|h_{22}|^2 = g_{22}$. Particularly, when $\lambda_{11} = \lambda_{22} = 0$, it means that the SIC is perfect. In addition, we also assume that all links noises are assumed to be independent random variables with the distribution $N(\mu, \sigma^2)$, where the mean $\mu = 0$ and the variance $\sigma^2 = 1$. Without loss of generality, we assume that the primary transmitter (PT) is far away from the secondary users, thus, the interference caused by PT to the secondary users is negligible. In addition, all channel state information (CSI) is assumed to be available at SU1 and SU2.

In the light of the spectrum sharing interference temperature concept in [2], we set up that PR can withstand peak interference power constraint for $Q_{pk}$, and SU1 and SU2 can be allowed peak transmit power constraint for $P_{pk}$. The transmit power of SU1 and SU2 are denoted as $P_1$ and $P_2$. Thus, in order to obtain the best of secondary users ergodic sum capacity and outage probability performance, under peak interference power and peak transmit power constraints, the corresponding optimal power allocation for SU1 and SU2 are respectively given by

$$P_1(g_{1p}) = \min \left\{ P_{pk}, \frac{Q_{pk}}{2g_{1p}} \right\}$$

$$= \begin{cases} \frac{Q_{pk}}{2g_{1p}}, & g_{1p} > \frac{Q_{pk}}{P_{pk}}, \\ P_{pk}, & \text{otherwise} \end{cases}$$

(1)

$$P_2(g_{2p}) = \min \left\{ P_{pk}, \frac{Q_{pk}}{2g_{2p}} \right\}$$

$$= \begin{cases} \frac{Q_{pk}}{2g_{2p}}, & g_{2p} > \frac{Q_{pk}}{P_{pk}}, \\ P_{pk}, & \text{otherwise} \end{cases}$$

(2)

For fair comparison with the conventional spectrum sharing CR networks, in FD transmission mode, the peak interference power constraint imposed by PR to each secondary user does not exceed $Q_{pk}/2$.

In order to obtain the best communication performance, SU1 and SU2 adopt the optimal transmission power, hence, the available signal to interference and noise ratio (SINR) are given by, respectively

$$SINR_1 = \frac{P_2(g_{2p})g_{21}}{P_1(g_{1p})g_{11} + \sigma^2},$$

(3)

$$SINR_2 = \frac{P_1(g_{1p})g_{12}}{P_2(g_{2p})g_{22} + \sigma^2}.$$  

(4)

For ease of understanding, we also give the conventional spectrum sharing CR networks as illustrated in Fig. 2, where SU1 and SU2 adopt HD transmission mode [2]. That is to say, SU1 and SU2 use different time slots to transmit information.
In this section, we give the detailed analysis to the ergodic sum capacity and the outage probability performance of between the FD spectrum sharing CR networks and the conventional spectrum sharing CR networks.

### 3.1 Ergodic Sum Capacity

In this subsection, we analyze the ergodic sum capacity of the FD spectrum sharing CR networks and the conventional spectrum sharing CR networks. However, it is extremely difficult to obtain closed expression of the ergodic sum capacity. Alternatively, based on no peak transmit power constraint and perfect SIC, an upper bounder closed-form expression for the ergodic sum capacity is derived, and the asymptotic characteristics of ergodic sum capacity is also discussed.

The ergodic sum capacity of the FD spectrum sharing CR networks can be expressed as

\[
    C_{FD,SS} = E\left[ \log_2(1 + SINR_1) + \log_2(1 + SINR_2) \right],
\]

where \( E[\cdot] \) represents statistical expectation.

In order to obtain the upper bound closed expression of the ergodic sum capacity for the FD spectrum sharing CR networks, we utilize the following two lemmas.

**Lemma 1** Assuming the random variables \( X \) and \( Y \) obey exponentially distributed and the mean are \( \lambda_x \) and \( \lambda_y \), respectively. The cumulative distribution function (CDF) of \( Z = \frac{aX}{Y}(a > 0) \) is given by

\[
    F_Z(z) = 1 - \frac{1}{\frac{\lambda_y}{a\lambda_x}z + 1}.
\]

**Proof** According to the definition of random variable \( Z \), its CDF can be given by

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Fig. 2 Conventional spectrum sharing CR networks
\[ F_Z(z) = P(Z < z) = P(\frac{aX}{Y} < z) = P(X < \frac{zY}{a}) = 1 - e^{-\frac{z}{\lambda_X}} \int_0^\infty \frac{1}{\lambda_X} e^{-\frac{y}{\lambda_X}} dy = 1 - \frac{1}{\frac{z}{\lambda_X} + 1}. \] (7)

The proof is completed. \(\square\)

**Lemma 2** Assuming the random variables \(X > 0\), the probability density function (PDF) and the CDF of can be expressed as \(f_X(x)\) and \(F_X(x)\), respectively. The following equation exists

\[ \int_0^\infty \log_2(1 + x)f_X(x)dx = \frac{1}{\ln 2} \int_0^\infty \frac{1 - F_X(x)}{1 + x} dx. \] (8)

**Proof** The proof process is as follows [5]

\[ \int_0^\infty \log_2(1 + x)f_X(x)dx = \int_0^\infty \log_2(1 + x)dF_X(x) = \log_2(1 + x)F_X(x) \bigg|_0^\infty - \int_0^\infty F_X(x)d\log_2(1 + x) \]

\[ = \log_2(1 + x) \bigg|_0^\infty - \int_0^\infty F_X(x)d\log_2(1 + x) = \int_0^\infty \log_2(1 + x) - \int_0^\infty F_X(x)d\log_2(1 + x) \]

\[ = \frac{1}{\ln 2} \int_0^\infty \frac{1 - F_X(x)}{1 + x} dx. \] (9)

The proof is completed. \(\square\)

**Proposition 1** When \(P_{pk} \to \infty\) (no peak transmit power constraint) and \(\lambda_{11} = \lambda_{22} = 0\) (perfect SIC), the upper bound of the ergodic sum capacity for the FD spectrum sharing CR networks is given by

\[ C_{\text{upper, FD, SS}} = \frac{m \ln m (m - 1) \ln 2}{2m^2 \lambda_{11}} + \frac{n \ln n (n - 1) \ln 2}{2n^2 \lambda_{12}}, \] (10)

where \(m = \frac{Q_{pk} \lambda_{11}}{2\sigma^2 \lambda_{1p}}\) and \(n = \frac{Q_{pk} \lambda_{12}}{2\sigma^2 \lambda_{1p}}\).

**Proof** Considering the proving process for ergodic capacity of SU1 and SU2 is similar, we first prove the ergodic capacity of SU1. Under \(P_{pk} \to \infty\) and \(\lambda_{11} = \lambda_{22} = 0\), use Lemma1 and Lemma2, the proving process for ergodic capacity of SU1 is given by
Similarly, we can prove $C_{upper, FD, SS, SU1} = E \left[ \log_2 (1 + \text{SINR}_1) \right] = E \left[ \log_2 (1 + \frac{P_1 (g_{2,p}) g_{21} q_{k21}}{\sigma^2}) \right] = E \left[ \log_2 (1 + \frac{q_{k21}}{2g_{2}^2 \sigma^2}) \right]$.

\[ = \frac{1}{\ln 2} \int_0^\infty \frac{1}{\ln (1+(x+1)(1+x))} \, dx \]

\[ = \frac{m}{\ln m} \int_0^\infty \frac{1}{(x+m)(x+1)} \, dx \approx \frac{n}{(m-1) \ln 2}. \quad (11) \]

Hence, Proposition 1 is proved. \qed

**Remark 1** From Proposition 1, we can see that, when $m \to \infty, n \to \infty$, the upper bound of the ergodic sum capacity $C_{upper, FD, SS} \approx \frac{\ln m}{\ln 2} + \frac{\ln n}{\ln 2}$. Namely, based on $P_{pk} \to \infty$ and $\lambda_{11} = \lambda_{22} = 0$, the upper bound of the ergodic sum capacity is determined by peak interference power constrain ($Q_{pk}$) and signal to leakage ratio of secondary users ($g_{21}/g_{2p}$ and $g_{12}/g_{1p}$).

In order to reflect ergodic sum capacity performance advantages of the FD spectrum sharing CR networks, as benchmark, the ergodic sum capacity of the conventional spectrum sharing CR networks is given by

\[ C_{HD, SS} = E \left[ \frac{1}{2} \log_2 (1 + \frac{P_{1, HD} h_{12}}{\sigma^2}) + \frac{1}{2} \log_2 (1 + \frac{P_{2, HD} h_{21}}{\sigma^2}) \right], \quad (12) \]

where $P_{1, HD} = \min \left\{ P_{pk}, \frac{Q_{pk}}{g_{1p}} \right\}$ and $P_{2, HD} = \min \left\{ P_{pk}, \frac{Q_{pk}}{g_{2p}} \right\}$, represent the optimal transmission powers for SU1 and SU2 of the conventional spectrum sharing CR networks.

In the conventional spectrum sharing CR networks, a complete transmission cycle need two transmission slots, where SU1 and SU2 need separately one transmission slot. In addition, we also assume that the mean of all channel power gains are equivalent for all compared networks.

To more intuitively show the theoretical upper bound of the ergodic sum capacity for the traditional spectrum sharing CR networks, here, we give Proposition 2.

**Proposition 2** When $P_{pk} \to \infty$, the upper bound of the ergodic sum capacity for the conventional spectrum sharing CR networks is given by

\[ C_{upper, HD, SS} = \frac{m \ln 2m}{(2m-1) \ln 2} + \frac{n \ln 2n}{(2n-1) \ln 2}. \quad (13) \]

**Proof** The proof is similar to the proof of Proposition 1. Hence, please refer to the proof process of Proposition 1. \qed

**Remark 2** From Proposition 2, we also can see that, when $m \to \infty, n \to \infty$, the upper bound of the ergodic sum capacity $C_{upper, HD, SS} \approx \frac{\ln 2m}{2 \ln 2} + \frac{\ln 2n}{2 \ln 2}$. Thus, we can also get the conclusions similar to remark 1.

**Remark 3** Combining Proposition 1 and Proposition 2, Compared with the traditional spectrum sharing CR networks, the FD spectrum sharing CR networks obtains the ergodic sum...
capacity gain \( d = \lim_{m \to \infty, n \to \infty} \frac{m \ln m}{m \ln m + n \ln n} \approx 2 \). That is to say, based on \( P_{pk} \to \infty \) and \( \lambda_1 = \lambda_2 = 0 \), under the same transmission bandwidth, the upper bound of the FD spectrum sharing CR networks ergodic sum capacity is twice that of the traditional spectrum sharing CR networks.

### 3.2 Outage Probability

In this subsection, we investigate the outage probability performance of between the FD spectrum sharing CR networks and the conventional spectrum sharing CR networks. Similarly, it is extremely difficult to obtain closed expression of the outage probability. Alternatively, based on no peak transmit power constrain and perfect SIC, an upper bounder closed-form expression for the outage probability performance of SU1 is derived, and the asymptotic characteristics of outage probability performance is discussed.

The outage probability of between SU1 and SU2 for the FD spectrum sharing CR networks are respectively given by

\[
P_{\text{out1}}^{FD, SS} = \Pr \{ \log_2 (1 + \text{SINR}_1) < R_1 \},
\]

\[
P_{\text{out2}}^{FD, SS} = \Pr \{ \log_2 (1 + \text{SINR}_2) < R_2 \}.
\]

where \( R_1 \) and \( R_2 \) are target rates for SU1 and SU2.

Next, in Proposition 3, we give the upper bound of the outage probability performance of SU1 for the FD spectrum sharing CR networks.

**Proposition 3** When \( P_{pk} \to \infty \) and \( \lambda_1 = \lambda_2 = 0 \), the upper bound of the outage probability performance of SU1 for the FD spectrum sharing CR networks is given by

\[
P_{\text{upper out1,FD,SS}} = \frac{1}{1 + \frac{m}{f_1}},
\]

where \( m = \frac{Q_{pk}}{2\sigma^2} \) and \( f_1 = 2^{R_1} - 1 \), \( f_1 \) denotes target SNR (signal to noise ratio) for SU1.

**Proof** Under \( P_{pk} \to \infty \) and \( \lambda_1 = \lambda_2 = 0 \), use Lemma2, the proving process for the upper bound of the outage probability performance of SU1 based on the FD spectrum sharing CR networks is given by

\[
P_{\text{upper out1,FD,SS}} = \Pr \{ \log_2 (1 + \text{SNIR}_1) < R_1 \}
\]

\[
= \Pr \{ \log_2 \left(1 + \frac{P_s g_1 g_2}{P_1 g_1 g_2 + \sigma^2 g_2} \right) < R_1 \}
\]

\[
= \Pr \{ \log_2 \left(1 + \frac{Q_{pk} g_2}{2\sigma^2 g_2} \right) < R_1 \}
\]

\[
= \Pr \{ \frac{Q_{pk} g_2}{2\sigma^2 g_2} < 2^{R_1} - 1 \}
\]

\[
= 1 - \frac{2^{R_1}}{Q_{pk} g_2} \left(2^{R_1} - 1\right) + 1
\]

\[
= 1 - \frac{1}{m + 1}.
\]
The proof is completed. □

**Remark 4** From Proposition 3, we can see that, under $P_{pk} \to \infty$ and $\lambda_{11} = \lambda_{22} = 0$, the upper bound of the outage probability performance of SU1 is determined by peak interference power constrain ($Q_{pk}$), target rate ($R_1$) and signal to leakage ratio of secondary users ($g_{21}/g_{2p}$ and $g_{12}/g_{1p}$), and the upper bound of the outage probability performance is monotonic with the above factors.

As benchmark, the outage probability of between SU1 and SU2 for the conventional spectrum sharing CR networks can be expressed as, respectively

\[
P_{\text{out1}}^{\text{HD}_{-}\text{SS}} = \Pr \left\{ \frac{1}{2} \log_2 \left( 1 + \frac{P_{2,\text{HD}g_{21}}}{\sigma^2} \right) < R_1 \right\},
\]

(18)

\[
P_{\text{out2}}^{\text{HD}_{-}\text{SS}} = \Pr \left\{ \frac{1}{2} \log_2 \left( 1 + \frac{P_{1,\text{HD}g_{12}}}{\sigma^2} \right) < R_2 \right\}.
\]

(19)

Next, in Proposition 4, we also give the upper bound of the outage probability performance of SU1 for the conventional spectrum sharing CR networks.

**Proposition 4** When $P_{pk} \to \infty$, the upper bound of the outage probability performance of SU1 for the conventional spectrum sharing CR networks is given by

\[
P_{\text{out1}}^{\text{upper}_{-}\text{HD}_{-}\text{SS}} = \frac{1}{1 + \frac{2m}{f_1(f_1+2)}}.
\]

(20)

The proofs of Proposition 4 and Proposition 3 are similar, please refer to the proof of Proposition 3.

**Remark 5** From Proposition 4, we can see that, under $P_{pk} \to \infty$, the upper bound of the outage probability performance of SU1 for the conventional spectrum sharing CR networks is determined by peak interference power constrain ($Q_{pk}$), target rate ($R_1$) and signal to leakage ratio of secondary users ($g_{21}/g_{2p}$ and $g_{12}/g_{1p}$). Similar to Remark 4, the upper bound of the outage probability performance is monotonic with the above factors.

**Remark 6** Combining Proposition 3 and Proposition 4, we can see that, compared with the traditional spectrum sharing CR networks, the FD spectrum sharing CR networks, has better upper bound performance on the outage probability of SU1. This is because two upper bounds satisfy

\[
\frac{1}{1 + \frac{m}{f_1}} < \frac{1}{1 + \frac{2m}{f_1(f_1+2)}}.
\]

Here, we did not analyze in detail the outage probability of SU2 mainly because SU1 and SU2 have similar outage probability performance.
4 Simulation Results

In this section, we evaluate the ergodic sum capacity and the outage probability performance of between the FD spectrum sharing CR networks and the conventional spectrum sharing CR networks, based on Monte-Carlo simulations and theoretical numerical analysis. Monte Carlo simulation is achieved through multiple random independent channel experiments. We assume that the noise variance is set to $\sigma^2 = 1$.

Under $P_{\text{pk}} \to \infty$ (no peak transmit power constraint) and fixed mean of self-interference channel power gain, Fig. 3 illustrates the ergodic sum capacity versus peak interference power constraint for the FD spectrum sharing CR networks and the conventional spectrum sharing CR networks. The Monte Carlo simulation parameters are set to: $\lambda_{1p} = \lambda_{2p} = 1, \lambda_{12} = \lambda_{21} = 5$. The number of Monte-Carlo simulations is set to 10,000. From this figure, we can clearly see that, the FD spectrum sharing CR networks has higher ergodic sum capacity than the conventional spectrum sharing CR networks. Especially when the mean of self-interference channel power gain is small. In addition, we also see that, whether the FD spectrum sharing CR networks or the conventional spectrum sharing CR networks, the simulation upper bound is in good agreement with the theoretical analysis upper bound based on ergodic sum capacity.

Figure 4 shows the ergodic sum capacity of the FD spectrum sharing CR networks as a function of peak interference power constraint under different peak transmit power constraint and perfect SIC. As shown in Fig. 4, when $Q_{\text{pk}}$ is small, the ergodic sum capacity of the FD spectrum sharing CR networks doses not change much with the change of $P_{\text{pk}}$. This is mainly because the smaller $Q_{\text{pk}}$ has become the bottleneck of ergodic sum capacity. That is to say, under smaller $Q_{\text{pk}}$, as $P_{\text{pk}}$ becomes larger, the ergodic capacity does not change significantly. In addition, we also see that, the ergodic sum capacity curve becomes flat as $P_{\text{pk}}$ becomes smaller. This is mainly because that
smaller $P_{pk}$ limits the increase of the ergodic sum capacity even when $Q_{pk}$ increases. That is to say, The smaller $P_{pk}$ has become the bottleneck of the ergodic sum capacity. Finally, from this figure also shows that, the ergodic sum capacity simulation curve continuously approaching the theoretical upper bound curve as $P_{pk}$ constantly increasing. So just verifies the correctness of the closed-form expression of the theoretical upper bound of ergodic sum capacity for the FD spectrum sharing CR networks.

Figure 5 depicts the ergodic sum capacity versus peak transmit power constraint under different peak interference power constraint and perfect SIC for the FD spectrum sharing CR networks. From the figure, we can see that, when $P_{pk}$ is small, the ergodic sum capacity doses not change much under different $Q_{pk}$. Similar to Fig. 4, this is due to that $P_{pk}$ limits the ergodic sum capacity of between SU1 and SU2. Besides, from this figure also shows that, when $P_{pk}$ is large enough relative to $Q_{pk}$, the ergodic sum capacity curve also becomes flat. Similarly, this is mainly because smaller $Q_{pk}$ limits the increase of the ergodic sum capacity even when $P_{pk}$ increases. Thus, it can be seen from above two figures that $P_{pk}$ and $Q_{pk}$ simultaneously affect and restrict the ergodic sum capacity of the FD spectrum sharing CR networks.

Figure 6 shows the ergodic sum capacity versus peak interference power constraint and peak transmit power constraint for the FD spectrum sharing CR networks. The simulation parameters are set to: $\lambda_{1p} = \lambda_{2p} = 1$, $\lambda_{12} = \lambda_{21} = 5$, $\lambda_{11} = \lambda_{11} = 0$. The number of Monte-Carlo simulations is set to 10,000. It can be clearly seen from the three-dimensional figure that the peak interference power constrain and peak transmit power constrain have a great impact on the ergodic sum capacity of the FD spectrum sharing CR networks. Namely, the ergodic sum capacity increases as $Q_{pk}$ and $P_{pk}$ increases at the same time. That is to say, unilaterally increasing $Q_{pk}$ or $P_{pk}$ cannot effectively improve the ergodic sum capacity of the spectrum sharing CR networks. Peak interference power
constrain and peak transmit power constrain jointly restrict the ergodic sum capacity performance of the FD spectrum sharing CR networks.

Figure 7 illustrates the outage probability of SU1 as a function of peak interference power constraint under $P_{pk} \to \infty$ (no peak transmit power constraint) for the FD spectrum sharing CR networks and the conventional spectrum sharing CR networks. The simulation
parameters are set to: $\lambda_{1p} = \lambda_{2p} = 1$, $\lambda_{12} = \lambda_{21} = 5$, $R_1 = 0.5 \text{bps/Hz}$. The number of Monte-Carlo simulations is set to 100,000. As shown in Fig. 7, we can see that, SU1 of the FD spectrum sharing CR networks has much better outage probability performance than SU1 of the conventional spectrum sharing CR networks, when the mean of self-interference channel power gain is small. However, SU1 of the conventional spectrum sharing CR networks has much better outage probability performance than SU1 of the FD spectrum sharing CR networks, when the mean of self-interference channel power gain becomes larger. Hence, effective control of SIC is particularly important in the design of the actual FD spectrum sharing CR networks. Considering SU2 and SU1 have similar outage probability performance, thus, in this paper, we do not analysis the outage probability of SU2.

Figure 8 illustrates the outage probability versus peak interference power constraint under different peak transmit power constraint and perfect SIC for SU1 of the FD spectrum sharing CR networks. From this figure, we can see that, when $Q_{pk}$ is small, the outage probability of SU1 does not change much under different $P_{pk}$. This is because that $Q_{pk}$ limits the outage probability of SU1. In addition, we also see that, when $Q_{pk}$ is large enough relative to $P_{pk}$, the outage probability curve becomes flat. This is mainly because that smaller $P_{pk}$ limits the drop of the outage probability of SU1 even when $Q_{pk}$ increases. Finally, from this figure also shows that, the outage probability of SU1 simulation curve continuously approaching the theoretical performance upper bound curve when peak transmit power constrain is constantly increasing.

Figure 9 depicts the outage probability versus peak transmit power constraint under different peak interference power constraint and perfect SIC for SU1 of the FD spectrum sharing CR networks. As shown in Fig. 9, when $P_{pk}$ is small, the outage probability of SU1 does not change much under different $Q_{pk}$. Similar to Fig. 8, this is mainly because that $P_{pk}$ limits the outage probability of SU1. In addition, from Fig. 9 also shows that, when $P_{pk}$ is large enough relative to $Q_{pk}$, the outage probability curve also becomes flat. This is
due to that smaller $Q_{pk}$ limits the reduce of the outage probability of SU1 even when $P_{pk}$ increases. It can be seen from Figs. 8 and 9 that $P_{pk}$ and $Q_{pk}$ simultaneously restrict and affect the outage probability of SU1.
Figure 10 depicts the Outage probability versus peak interference power constraint under different target rate, perfect SIC and $P_{pk} \to \infty$ for SU1 of the FD spectrum sharing CR networks. From this figure, we can see that, under fixed $R_1$, SU1 of the FD spectrum sharing CR networks has much better outage probability performance than SU1 of the conventional spectrum sharing CR networks. In addition, under fixed peak interference power and peak transmit power constraints, we can also see that, whether in the FD spectrum sharing CR networks or the conventional spectrum sharing CR networks, the outage probability of SU1 is reduced when the target rate $R_1$ decreases. This is because SU1 is less prone to interruption phenomenon when the target rate $R_1$ decreases, as a result, the outage probability performance is improved. Finally, we also can see that, based on the outage probability of SU1, the upper bound of the simulation is completely consistent with the theoretical upper bound of the analysis, whether in the FD spectrum sharing CR networks or the conventional spectrum sharing CR networks. So also verifies the correctness of the theoretical upper bound derivation for SU1 of outage probability.

5 Conclusion

In this paper, based on CR and FD technology, a FD spectrum sharing CR networks is considered, where SU1 and SU2 are each equipped with dual antennas, one of which is used to receive signals, and the other is used to transmit signals at the same time and frequency. We analysis the ergodic sum capacity and the outage probability based on the FD spectrum sharing CR networks and the conventional spectrum sharing CR networks. Furthermore, under no peak transmit power constraint and perfect SIC, based on the FD spectrum sharing CR networks and the conventional spectrum sharing CR networks, the closed-form
expressions of the theoretical upper bound performance of the outage probability and the ergodic sum capacity are derived by two lemmas and four propositions. Accurate mathematical analysis display, under the same bandwidth, the upper bound of ergodic sum capacity for the FD spectrum sharing CR networks is twice, compared with the traditional spectrum sharing CR networks. In addition, compared with the traditional spectrum sharing CR networks, the FD spectrum sharing CR networks based on SU1, has better performance upper bound on the outage probability. Simulations results validate that, the FD spectrum sharing CR networks obtains better performance than the conventional spectrum sharing CR networks, and the simulation performance upper bound is completely consistent with the theoretical analysis performance upper bound, whether in the FD spectrum sharing CR networks or the conventional spectrum sharing CR networks. So also verifies the correctness of the theoretical analysis upper bound performance derivation.

Acknowledgements This work has been supported by the Joint Foundation of Guizhou Province under Grant QianKeHeLHZ[2016]7106 and the Nature Science Foundation of Guizhou Province under Grant QianKeHeJiChu[2017]1070. In addition, the part work of this paper was presented at the International Conference on IEEE 20th International Conference on Communication Technology [34].

References

1. Mitola, J., & Maguire, G. Q. (1999). Cognitive radio: making software radios more personal. *IEEE Personal Communications*, 6(6), 13–18.
2. Ghasemi, A., & Sousa, E. S. (2007). Fundamental limits of spectrum-sharing in fading environments. *IEEE Transactions on Wireless Communications*, 6(2), 649–658.
3. Liang, Y.-C., Zeng, Y., Peh, E. C. Y., & Hoang, A. T. (2008). Sensing-throughput tradeoff for cognitive radio networks. *IEEE Transactions on Wireless Communications*, 7(4), 1326–1337.
4. Sultan, R., Song, L., Seddik, K. G., & Han, Z. (2017). Full-duplex meets multiuser MIMO: Comparisons and analysis. *IEEE Transactions on Vehicular Technology*, 66(1), 455–467.
5. Xie, X., Liu, J., Huang, J., & Zhao, S. (2020). Ergodic capacity and outage performance analysis of uplink full-duplex cooperative NOMA system. *IEEE Access*, 8, 164786–164794.
6. Riithonen, T., Werner, S., & Wichman, R. (2011). Hybrid full-duplex/halfduplex relaying with transmit power adaptation. *IEEE Transactions on Wireless Communications*, 10(9), 3074–3085.
7. Ahn, H., Park, Y., Kim, D., & Suh, Y. (2019). A full-duplex MAC protocol based on buffer status report for successive full-duplex link setup. *IEEE Communications Letters*, 23(9), 1506–1509.
8. Yilan, M., Ayar, H., Nawaz, H., Gurbuz, O., & Tekin, L. (2019). Monostatic antenna in-band full duplex radio: performance limits and characterization. *IEEE Transactions on Vehicular Technology*, 68(5), 4786–4799.
9. Kai, C., Huang, T., Wang, L., & Gu, Y. (2018). CSMA-Based utility-optimal scheduling in the WLAN with a full-duplex access point. *IEEE Access*, 6, 41399–41409.
10. Cheng, W., Zhang, X., & Zhang, Kai, H. (2013). RTS/FCTS mechanism based full-duplex MAC protocol for wireless networks. *IEEE GLOBECOM*, 5017–5022.
11. Bayat, A., & Aissa, S. (2018). Full-duplex cognitive radio with asynchronous energy-efficient sensing. *IEEE Transactions on Wireless Communications*, 17(2), 1066–1080.
12. Liao, Y., Wang, T., Song, L., & Han, Z. (2017). Listen-and-Talk: protocol design and analysis for full-duplex cognitive radio networks. *IEEE Transactions on Vehicular Technology*, 66(1), 656–667.
13. Boulogeorgos, A., Salameh, H., & Karagiannidis, G. (2017). Spectrum sensing in full-duplex cognitive radio networks under hardware imperfections. *IEEE Transactions on Vehicular Technology*, 66(3), 2072–2084.
14. Cheng, W., Zhang, X., & Zhang, H. (2015). Full-duplex spectrum-sensing and MAC-protocol for multichannel non-time-slotted cognitive radio networks. *IEEE Journal on Selected Areas in Communications*, 33(5), 820–831.
15. Shehata, H., & Khatib, T. (2019). Energy detection spectrum sensing in full-duplex cognitive radio: the practical case of rician RSI. *IEEE Transactions on Communications*, 67(9), 6544–6555.
16. Cirik, A., Wang, R., Rong, Y., & Hua, Y. (2015). MSE-based transceiver designs for full-duplex MIMO cognitive radios. *IEEE Transactions on Communications*, 63(6), 2056–2070.
17. Tan, L., & Le, L. (2015). Design and optimal configuration of full-duplex MAC protocol for cognitive radio networks considering self-interference. *IEEE Access, 3*, 2715–2729.
18. Towhidilou, V., & Bahaei, M. (2018). Adaptive full-duplex communications in cognitive radio networks. *IEEE Transactions on Vehicular Technology, 67*(9), 8386–8395.
19. Li, D., Cheng, J., & Leung, V. (2019). Polarization Jones vector distance statistics-based full-duplex primary signal extraction for cognitive radios. *IEEE Transactions on Communications, 67*(4), 2689–2701.
20. Kim, H., Lim, S., Wang, H., & Hong, D. (2012). Optimal power allocation and outage analysis for cognitive full duplex relay systems. *IEEE Transactions on Wireless Communications, 11*(10), 3754–3765.
21. Zheng, G., Krikidis, I., & Ottersten, B. (2013). Full-duplex cooperative cognitive radio with transmit imperfections. *IEEE Transactions on Wireless Communications, 12*(5), 2498–2511.
22. Nguyen, N., Kundu, C., Ngo, H., Duong, T., & Canberk, B. (2016). Secure full-duplex small-cell networks in a spectrum sharing environment. *IEEE Access, 4*, 3087–3099.
23. Shang, Z., Zhang, T., Cai, Y., Liu, Y., & Yang, W. (2019). Secure spectrum-sharing wiretap networks with full-duplex relaying. *IEEE Access, 7*, 181610–181625.
24. Choi, Y., & Kim, D. (2020). Optimal collaborative spectrum sharing for a cognitive multi-antenna relay with full-duplex capability and stability constraint. *IEEE Access, 8*, 68649–68667.
25. Long, R., Guo, H., Zhang, L., & Liang, Y. (2019). Full-duplex backscatter communications in symbiotic radio systems. *IEEE Access, 7*, 21597–21608.
26. Zhong, B., Zhang, Z., Chai, X., Pan, Z., Long, K., & Cao, H. (2015). Performance analysis for opportunistic full-duplex relay selection in underlay cognitive networks. *IEEE Transaction on Vehicular Technology, 64*(10), 4905–4910.
27. Zhong, B., & Zhang, Z. (2018). Opportunistic two-way full-duplex relay selection in underlay cognitive networks. *IEEE Systems Journal, 12*(1), 725–734.
28. Ali, B., Mirza, J., Zhang, J., Zheng, G., Saleem, S., & Wong, K. (2019). Full-duplex amplify-and-forward relay selection in cooperative cognitive radio networks. *IEEE Transactions on Vehicular Technology, 68*(6), 6142–6146.
29. Lan, P., Zhai, C., Chen, L., Gao, B., & Sun, F. (2018). Optimal power allocation for bi-directional full duplex underlay cognitive radio networks. *IET Communications, 12*(2), 220–227.
30. Zhang, C., Wang, D., Ye, J., Lei, H., Zhang, J., Pan, G., & Feng, Q. (2017). Secrecy outage analysis on underlay cognitive radio system with full-duplex secondary user. *IEEE Access, 5*, 25696–25705.
31. Zhang, J., Pan, G., & Wang, H. (2016). On physical-layer security in underlay cognitive radio networks with full-duplex wireless-powered secondary system. *IEEE Access, 4*, 3887–3893.
32. Li, D., Zhang, D., & Chen, J. (2019). A novel polarization enabled full-duplex hybrid spectrum sharing scheme for cognitive radios. *IEEE Communications Letters, 23*(3), 530–533.
33. Li, D., Cheng, J., & Leung, V. (2018). Adaptive spectrum sharing for half-duplex and full-duplex cognitive radios: from the energy efficiency perspective. *IEEE Transactions on Communications, 66*(11), 5067–5080.
34. Xie, X., Bi, Y., & Nie, X. (2020). Performance analysis of full-duplex spectrum sharing networks under peak interference power and peak transmit power constraints. *IEEE ICCT*, 153–156.

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