The line shape of the radiative open-charm decay of $Y(4140)$ and $Y(3930)$

Xiang Liu
School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China

Hong-Wei Ke
Physical Department, School of Science, Tianjin University, Tianjin 300072, China

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In this work, we study the radiative open-charm decays $Y(4140) \to D_s^* D_s^* \gamma$ and $Y(3930) \to D_s^* D_s^* \gamma$ under the assignments of $D_s^* D_s^*$ and $D^* D^*$ as molecular states for $Y(4140)$ and $Y(3930)$ respectively. Based on our numerical result, we propose the experimental measurement of the photon spectrum of $Y(4140) \to D_s^* D_s^* \gamma, D_s^* D_s^* \gamma, D_s^* D_s^* \gamma$ and $Y(3930) \to D_s^* D_s^* \gamma, D_s^* D_s^* \gamma, D_s^* D_s^* \gamma$ can further test the molecular assignment for $Y(4140)$ and $Y(3930)$.

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In the past six years, experiments have announced a series of charmonium-like states $X(3872), X(3930)/Y(3930)/Z(3930), Y(4260), Z^+(4430)$ etc.. Recently the CDF Collaboration reported the observed mass of $Y(4140)$ by studying the invariant mass spectrum of $J/\psi \phi$ in $B^+ \to K^+ J/\psi \phi$. Its mass and width are $m = 4143.0 \pm 2.9 (\text{stat.}) \pm 1.2 (\text{syst.})$ MeV and $\Gamma = 11.7^{+8.5}_{-5.0} (\text{stat.}) \pm 3.7 (\text{syst.})$ MeV, respectively. $Y(4140)$ not only makes the spectroscopy of the charmonium-like state abundant, but also provides a good chance to further understand the property of the observed charmonium-like states.

In the observed charmonium-like states, $Y(3930)$ is a near-threshold $\omega J/\psi$ mass enhancement in the exclusive $B \to K \omega J/\psi$ decays, which was firstly observed by the Belle Collaboration and confirmed by the Babar Collaboration. Since both $Y(4140)$ and $Y(3930)$ were observed in the mass spectrum of $J/\psi + V$ of $B \to K J/\psi V$ channel ($V$ denotes light vector meson), $Y(4140)$ is similar to $Y(3930)$.

The authors of Ref. discussed the various possible interpretations of the $Y(4140)$, and further proposed that $Y(4140)$ is probably a $D_s^* D_s^*$ molecular state with $J^{PC} = 0^{++}$ or $J^{PC} = 2^{++}$ while $Y(3930)$ is its $D^* D^*$ molecular partner. Relevant dynamics calculation of $D_s^* D_s^*$ and $D^* D^*$ was performed in the potential model. Later N. Mahajan argued $Y(4140)$ to be a $D_s^* D_s^*$ molecular state or an exotic ($J^{PC} = 1^{--}$) hybrid charmonium.

Besides using the potential model to dynamically study $D_s^* D_s^*$ system, QCD sum rule (QSR) is applied to calculate the mass spectrum of $D_s^* D_s^*$ system. The mass of $D_s^* D_s^*$ system from the QSR calculation in Ref. is not consistent with the experimental value of $Y(4140)$. The authors in Ref. also used QSR to calculate the mass of $D_s^* D_s^*$ system. Their numerical results indicate the existence of a $D_s^* D_s^*$ bound state, which is consistent with CDF experimental observation and does not support the result in Ref.

By using one boson exchange model, Ding obtained the effective potential of $D_s^* D_s^*$ system. The result supports to explain $Y(4140)$ as a $D_s^* D_s^*$ molecular state with the quantum number $J^{PC} = 0^{++}$. Meanwhile, searching the $1^{--}$ and $1^{--}$ partners of $Y(4140)$ in $J/\psi \eta$ and $J/\psi \eta'$ were suggested.

As indicated in Ref., tetraquark $Y(4140)$ falls apart into a pair of charmed mesons very easily under the assignment of tetraquark. In general, the width of the tetraquark would be broad, which does not consist with the experimental value $\Gamma = 12$ MeV. Recently Stancu calculated the spectrum of $ccs \bar{s}$ by a quark model with chromomagnetic interaction, by considering a tetraquarks assignment ($ccs \bar{s}$) for $Y(4140)$, which favors $J^{PC} = 1^{++}$. In this assignment, the coupling constant of $Y(4140)$ with $VV$ channel is small, which can alleviate the contradiction between the small experimental width and the large width resulted from the fall apart mechanism.

There exist different understandings from the exotic explanation to the source of $Y(4140)$. In recent work, van Beveren and Rupp proposed that the $Y(4140)$ enhancement resulted from the opening of the $J/\psi \phi$ channel and that probably does not represent a resonance.

As an important and interesting topic, studying the decay of $Y(4140)$ can be helpful to reveal its underlying structure. By using an effective Lagrangian approach, Branz, Gutsche and Lyubovitskij calculated $Y(3930) \to J/\psi \omega$ and $Y(4140) \to J/\psi \phi$ strong decays and $Y(3930)/Y(3940) \to \gamma \gamma$ decays, which are induced by the hadronic loop effect. Furthermore, the molecular explanation for $Y(3930)$ and $Y(4140)$ is supported by the result of $J/\psi V (V = \omega, \phi)$ mode of $Y(3930)$ and $Y(4140)$.

In Ref., one of the authors of this work calculated the hidden charm decay of $Y(4140)$ in the assumption of the second radial excitation of the P-wave charmonium $\chi_{cJ} (J = 0, 1)$. Since the branching ratio of the
hidden charm decay $Y(4140) \rightarrow J/\psi \phi$ is of the order of $10^{-4} \sim 10^{-3}$ under the $\chi''_{cJ}$ assignment for $Y(4140)$, which disfavors the large hidden charm decay pattern indicated by the CDF experiment, the pure second radial excitation of the P-wave charmonium $\chi''_{cJ} (J = 0, 1)$ is problematic.

Besides investigating the hidden charm decay, the open charm decay and the double gamma decay of $Y(4140)/Y(3930)$, the radiative decay of $Y(4140)/Y(3930)$ can provide the useful information to distinguish the molecular explanation of $Y(4140)/Y(3930)$ from other assignments, which was indicated in Ref. [4]. The typical radiative decay modes are $D_s^+D_s^-\gamma$, $D_s\gamma\pi^0$, $D_s\gamma D_s\gamma$ for $Y(3930)$, and $D_s^+D_s^-\gamma$, $D_s\gamma D_s\pi^0$, $D_s\gamma D_s\gamma$ for $Y(4140)$.

Due to the suppressions from the phase space and the $\alpha = 1/137$ of the electromagnetic vertex, the width of the four-body radiative decay of $Y(4140)/Y(3930)$ is far less than that of the three-body radiative decay of $Y(4140)/Y(3930)$. Thus, in this work we will be dedicated to the study of the three-body radiative decays $Y(3930) \rightarrow D^+D\gamma$ and $Y(4140) \rightarrow D_s^+D_s\gamma$, which are named as the radiative open-charm decay. The radiative open-charm decays of $Y(4140)$ are $D_s^+D_s^-\gamma$ and $D_s^\pm D_s^-\gamma$. For $Y(3930)$, its radiative open-charm decays include $D_s^0D_s^\pm\gamma$, $D_s^0D_s^\mp\gamma$, $D_s^+D_s^-\gamma$ and $D_s^\pm D_s^\mp\gamma$. In the following, we will illustrate the radiative open-charm decays of $Y(4140)$ and $Y(3930)$ with $Y(4140) \rightarrow D_s^+D_s^-\gamma$ and $Y(3930) \rightarrow D^+D^\mp\gamma$ as the example.

The matrix element for $Y(3930) \rightarrow D_s^+D_s^-\gamma$ and $Y(4140) \rightarrow D_s^+D_s^-\gamma$ processes can be expressed as

$$\mathcal{M}[Y(3930)(Y(4140)) \rightarrow D_s^+D_s^-\gamma] = \langle D_s^-\gamma|H_2|D_s^+\rangle \langle D_s^+\gamma|H_1|Y \rangle,$$

where $H_1$ describes the collapse of $S$-wave $D_s^+D_s^-$ molecular state, $H_2$ denotes the interaction of $D_s^+D_s^-$ and $D_s^-\gamma$. For describing the decay amplitude, we adopt the same method as that in Refs. [16, 17] to study the radiative open-charm decays of $Y(4140)$ and $Y(3930)$. In Refs. [18, 19], Voloshin once assumed $X(3872)$ is a $DD^\ast + h.c.$ molecular state and explored the radiative decay $X(3872) \rightarrow DD\gamma$.

The matrix element $\langle D_s^+\gamma|H_1|Y \rangle$, describing the collapse of $Y(4140)$ into $D_s^+$, and $D_s^-$, can be repre-
\[|\langle D_s^{(s)}|H_1|Y\rangle = \alpha C\Psi(\bar{q}), \tag{2}\]

where \(\alpha\) is the weight of the corresponding component \(|D_s^{(s)}, D_s^{(s)}\rangle\) in the molecular wave function of \(Y(4140)/Y(3930)\) \[|\Psi(\bar{q})\rangle = \sqrt{\frac{1}{\pi\kappa^4}} e^{\frac{-\bar{q}^2}{2\kappa^2}}, \tag{5}\]

respectively. \(\Psi(\bar{q})\) is the wave function describing S-wave \(D_s^{(s)}D_s^{(s)}\) molecular state, which is of the form \([8, 10]\).

The amplitude describing \(D_s^{(s)} \rightarrow D_s^{(s)}\gamma\) reads as

\[A[D_s^{(s)} \rightarrow D_s^{(s)}\gamma] = g\epsilon_{ij}\epsilon_{13}, \tag{6}\]

where \(g\) is the effective coupling constant. \(\vec{k}\) and \(\vec{\epsilon}_1\) are the three momentum and the polarization vector of photon, respectively. \(\vec{\epsilon}_2\) denotes the polarization vector of \(D_s^{(s)}\).

Thus the amplitude of decay \(Y(4140) \rightarrow D_s^{(s)} + D_s^{(s)}\gamma\) or \(Y(3930) \rightarrow D_s^{(s)} + D_s^{(s)}\gamma\) can be written as

\[A[Y(3930)(Y(4140)) \rightarrow D_s^{(s)}D_s^{(s)}\gamma] = g\alpha \epsilon_{ij}\epsilon_{13}, \tag{7}\]

where \(\alpha = 1\) and \(\alpha = 1/\sqrt{2}\) correspond to \(Y(4140) \rightarrow D_s^{(s)}D_s^{(s)}\gamma\) and \(Y(3930) \rightarrow D_s^{(s)}D_s^{(s)}\gamma\) processes, respectively.

Finally one obtains the decay rate of \(Y(3930)(Y(4140)) \rightarrow D_s^{(s)}D_s^{(s)}\gamma\)

\[d\Gamma[Y(3930)(Y(4140)) \rightarrow D_s^{(s)}D_s^{(s)}\gamma] = \frac{g^2\alpha^2}{3(2\pi)^5} \Psi(\bar{q})^2 \delta(E_1 + E_2 + \omega_0 - M) \times \delta(p_1 + p_2 + \vec{k}) d^3p_1 d^3p_2 \frac{d^3k}{(2\pi)^3}. \tag{8}\]

After integration we obtain the differential decay rate in terms of the radiative width of \(D_s^{(s)}\)

\[\frac{d\Gamma[Y(3930)(Y(4140)) \rightarrow D_s^{(s)}D_s^{(s)}\gamma]}{d\omega} = \frac{g^2\omega^3\alpha^2}{3\pi} \left[\phi(2k + 2\bar{p})\right]^2 d^3p \frac{d^3k}{(2\pi)^3}, \tag{9}\]

where \(\bar{p}\) is the momentum of \(D_s^{(s)}\) in the c.m. frame of the \(D_s^{(s)}D_s^{(s)}\) system. \(\vec{k}\) denotes the momentum of the photon in the c.m. frame of \(Y(4140)/Y(3930)\). \(q_1^2(q_2^2)\) and \(E_1(E_2)\) are the momentum and energy of the final meson \(D_s^{(s)}D_s^{(s)}\). The photon energy \(\omega\) and the momentum \(\bar{p}\) are related by

\[\frac{d\Gamma[Y(3930)(Y(4140)) \rightarrow D_s^{(s)}D_s^{(s)}\gamma]}{d\omega} = \frac{\sqrt{|m_{12}^2 - (m_{D_s^{(s)} + m_{D_s^{(s)} + m_{D_s^{(s)}}})^2|}}}{2\sqrt{M^2 - 2M\omega}} \tag{10}\]

with the relation \(m_{12} = M^2 - 2M\omega\), where \(M\) denotes the mass of initial state \(Y(4140)/Y(3930)\). By using the above relation, we can replace \(dp\) with \(d\omega\) in eq. \(9\). Further we get the expression of \(d\Gamma[Y(3930)(Y(4140)) \rightarrow D_s^{(s)}D_s^{(s)}\gamma]/d\omega\). By this expression, we carry out the study of the line shape of the photon spectrum of \(Y(3930) \rightarrow D_s^{(s)}D_s^{(s)}\gamma\) and \(Y(4140) \rightarrow D_s^{(s)}D_s^{(s)}\gamma\) under the assumption of the \(D_s^{(s)}D_s^{(s)}D_s^{(s)}D_s^{(s)}\) molecular state for \(Y(4140)/Y(3930)\).
tion of the photon energy $\omega$ are presented. The line shape of the photon spectrum $Y(4140) \rightarrow D_s^+ D_s^- \gamma$ is similar to that of $Y(3930) \rightarrow D^+ D^- \gamma$. There exits a sharp peak near the large end-point of photon energy in Figs. 3 and 4. On the left side of the peak, the photon spectrum changes smoothly with the variation of the photon energy $\omega$ while on the right side of the peak the line shape of the photon spectrum goes down very rapidly with the increasing $\omega$. These features shown in our results can provide useful information for testing the molecular state structure assignment for $Y(4140)$ and $Y(3930)$.

In summary, as indicated in Ref. [4], the line shapes of the photon spectrum of $Y(4140) \rightarrow D_s^+ D_s^- \gamma$ and $Y(3930) \rightarrow D^+ D^- \gamma$ are crucial to test the molecular state assignment for $Y(4140)$ and $Y(3930)$. In this work, we study the photon spectrum of the radiative open-charm decays of $Y(4140)$ and $Y(3930)$ under the structure of the molecular state. Due to the peculiar characters of the line shape of the photon spectrum $Y(4140) \rightarrow D_s^+ D_s^- \gamma$ and $Y(3930) \rightarrow D^+ D^- \gamma$ shown in our numerical result, we suggest the experimentalist to carry out the measurement of the photon spectrum of the radiative open-charm decay of $Y(4140)$ and $Y(3930)$ in future experiment.

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