Estimation of the value and localization of possible systematic errors in
determination of level density and radiative strength functions from the
\((n,2\gamma)\)-reaction.

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Abstract

Systematic error in determination of the absolute intensities of the two-step \(\gamma\)-cascades
after the thermal neutron capture and its influence on the value and localization of ex-
tracted from \((n,2\gamma)\)-reaction probable level densities and radiative strength functions of
dipole \(\gamma\)-transitions have been analysed. It was found that this error in limits of its
possible magnitude cannot change made earlier conclusions about the radiative strength
functions of \(E_1\) and \(M_1\) transitions at \(E_\gamma \simeq 3\) MeV and level density of heavy nucleus
below \(\simeq 0.5B_n\).

1 Introduction

Both radiative strength functions \(k\) of \(E_1\) and \(M_1\) transitions and density \(\rho\) of levels populated
by them are determined by inner nuclear processes at corresponding excitation energy. Below
the neutron binding energy \(B_n\), the parameters \(k\) and \(\rho\) are, in practice, the only probe for
nuclear models. Therefore, determination of these parameters with the maximum possible
reliability is one of the principle tasks of low-energy nuclear physics.

Very high level density of deformed, first of all, nuclei and a number of quanta following
\(\gamma\)-decay do not allow one to determine mentioned parameters by the direct methods of nuclear
spectroscopy above \(E_{ex} \simeq 2 - 3\) MeV. The only possibility to solve this task is the selection
of probable values of \(k\) and \(\rho\) which allow one to reproduce experimental spectra within the
experimental errors and uncertainties of the used model notions. The main source of systematic
errors of such procedure is the strong correlation between \(k\) and \(\rho\). This results from the fact
that the intensity of the primary production of the reaction is determined by a product of the
probability of emission of reaction product (including \(\gamma\)-quanta) and \(\rho\).

This causes a necessity to develop new experimental methods in which the observables are
connected by the mathematics relations of other type. The method [1] developed at the FLNP
JINR for simultaneous model-free estimation of the radiative strength functions \(k\) and density
of levels \(\rho\) by means of intensities \(I_{\gamma\gamma}\) of populating them cascades which follow thermal neutron
capture. These intensities are determined by energy dependence of \(k\) for \(E_1\) and \(M_1\) transitions
and, qualitatively, are inversely proportional to \(\rho\). As a consequence, the interval of \(k\) and \(\rho\)
values allowing reproduction of the experimental value of \(I_{\gamma\gamma}\) is narrow. This can promote
identification of false values of \(k\) and \(\rho\) determined [2] from the nucleon evaporation spectra or
spectra of primary \(\gamma\)-transitions [3]. Of course, the data obtained according to [1] contain some
statistic uncertainty caused by inadequacy of the model ideas of of the \(\gamma\)-decay process to the
experiment and error in the experimental data used for determination of \(I_{\gamma\gamma}\).
2 Conditions of the experiment

2.1 Model approaches

The sum intensities of the cascades whose intermediate levels are lying in the excitation energy interval $\Delta E$ are calculated under assumptions:

(a) the branching ratios for the decaying level $i$ do not depend on the mode of its population;

(b) at a given excitation energy, only the levels belonging to solely statistical ensemble exist. I.e., the mean reduced probability of their population by the primary $E1$ or $M1$ transitions is equal for any level in the spin window set by the selection rule and does not depend on the structure of the wave function the neutron resonance. On the basis of the mathematics theorem of the mean, any sum of partial widths is represented in calculation (like that used in [1]) by the product of their number and some mean partial width (which is determined through corresponding strength function);

(c) the energy dependence of $k$ (but not its absolute value) is equal for the primary and secondary cascade transitions.

One can suppose that the maximal discrepancy between the model notions and experiment is conditioned by assumptions (b) and (c). The truth of the assumption (c) now can be tested in direct experiment, but possibility to test thesis (b) is quite hypothetical.

In particular, the $k$ and $\rho$ values obtained according to [1] allow good reproduction of intensity distribution of cascades to the final levels with $E_{ex} \leq 1$ MeV and total experimental absolute intensities to the final levels with $E_{ex} \geq 2$ MeV excitation energy (without additional fit) in the $^{191,193}Os$ and $^{118}Sn$ nuclei. Somewhat different situation is observed in $^{185,187}W$ where analogous data allow possibility of some violation of condition (c).

Potentially, possibility of violation of condition (b) follows from qualitative interpretation of the results obtained in [1]. Change in the power of energy dependence in vicinity of $0.5B_n$ allows an assumption about the corresponding change in properties of levels excited by cascades in this region. If this change is spread over the excitation energy then violation of condition (b) in the vicinity of $0.5B_n$ can be significant. Although it should be noted that the approximation [4] of the experimental cumulative sums of cascade intensities shows that this distribution above $i_{\gamma\gamma} \approx 10^{-4}$ is well reproduced by the sum of only two distributions which correspond to primary $E1$ and $M1$ cascade transitions, respectively.

2.2 Errors of the experiment

In modern experiment [5] the error of cascade intensity $\delta I_{\gamma\gamma}$ it is mainly determined by the error of known [6] intensities of the high-energy primary transitions $i_1$ used for normalization of the $I_{\gamma\gamma}$ value. As it was shown in [5], uncertainty in determination [7] of dependence of $I_{\gamma\gamma}$ on the primary transition energy from the experimental spectra (each of them is the sum of two distributions) can be negligible if $HPGe$ detectors with 20-25% and higher efficiency are used in the experiment. This is true at least for even-odd deformed and even-even spherical nuclei. This conclusion unambiguously follows from the extrapolation [4] of intensity distribution of
cascades (resolved in the spectrum as pairs of peaks) to zero value $i_{\gamma\gamma} = 0$ — more than 95-99% of intensity of cascades with intermediate levels lying below $0.5B_n$ is determined in the experiment. So, at any energy of the cascade primary transition the shape of the functional dependence

$$I_{\gamma\gamma} = \sum_{\lambda,f} \sum_i \frac{\Gamma_{\lambda i}}{\Gamma_{\lambda}} \frac{\Gamma_{if}}{\Gamma_{if}} = \sum_{\lambda,f} \frac{\Gamma_{\lambda i}}{\Gamma_{\lambda} > m_{\lambda i}} \frac{\Gamma_{if}}{\Gamma_{if} > m_{if}}$$

(1)
is ascertained with relative error not more than several percent. Therefore, systematic error in determination of $k$ and $\rho$ from combination of $\Gamma_{\lambda}$ and $I_{\gamma\gamma}(E_1)$ (1) is practically caused by error in obtaining absolute $I_{\gamma\gamma}$ values, i.e., by error of $i_1$. Comparison [8] between the known [6] and measured in Budapest values of $i_1$ shows that the uncertainty of any arbitrary taken intensity $i_1$ for the data [6] can be estimated as 20%. Normalization of $I_{\gamma\gamma}$ to absolute value is made using 5-10 (supposedly non-correlated in errors) values of $i_1$. So, one can expect, that the total experimental intensity $I_{\gamma\gamma}$ differs from the actual value not more than by 40%, if $I_{\gamma\gamma}$ is normalized using data [6].

3 Uncertainties in determination of $k$ and $\rho$ and their localization

The values $k$ and $\rho$ resulting in a given magnitude of $\delta I_{\gamma\gamma}$, in practice, cannot be estimated using conventional in mathematical statistics procedure of error transfer. Therefore, these parameters were estimated in other manner. Varying the total experimental intensities of cascades from 70 to 120% in $^{185,187}W$ and from 50 to 100% in $^{191,193}Os$ we obtained an ensemble of functional dependences of $k$ and $\rho$ which permit reproduction of these $I_{\gamma\gamma}$ values. Variation coefficient $\kappa$ is stipulated by the fact that the total intensities of the observed primary transitions and two-step cascades to low-lying levels ($E_f < 0.5$ MeV) for isotopes of $W$ and $Os$ equal 65-70% and 92-95%, respectively. Besides, statistics of the accumulated coincidences and high energy resolution permit one to neglect the errors of procedure of decomposition of the experimental spectra into two components corresponding to solely primary and solely secondary cascade transitions.

Probable $\rho$ and $k$ values obtained in this manner are shown in Figs. 1-4 and 5-8, respectively. These data are compared in Figs. 9 and 10 with the model values of $\rho$ and $k$ in function of coefficient $\kappa$ for four excitation energies and four energies of $\gamma$-quantum. The main result of this analysis — $\rho$ in this interval of variation of $\kappa$ (probably overlapping the region of possible error of $I_{\gamma\gamma}$) weakly changes at the excitation energy below $\simeq 0.5B_n$. Variations of the energy dependence of $k$ in this case are much stronger. In the all considered cases, however, the sum of $k(E1) + k(M1)$ for $E1 < 2 - 3$ MeV does not exceed the sum of the calculated according to [10] value $k(E1)$ and value $k(M1) = const$ if their ratio is normalized to the experimental (approximated) value. Taking into account that the sign of derivative of $k^{exp}/k^{mod}$ in Fig. 10 differs for different energies $E_{\gamma}$ and its value is close to zero at 2.5-3.0 MeV for three nuclei discussed here and some bigger for $^{191}Os$ one can expect that the strength functions is estimated with the maximum possible reliability independently on $\delta I_{\gamma\gamma}$ at least at this $\gamma$-transition energy.
4 Discussion

In the framework of the enumerated above assumptions about the cascade $\gamma$-decay process one can conclude that significant decrease in the level density of heavy, at least, nuclei as compared with the predictions of the Fermi-gas model cannot be explained by some revealed up to now uncertainties of the method [1].

The observed results [1] on the level density can be qualitatively explained within modern nuclear models directly accounting [13] for dynamics of breaking of the nucleon pairs or within the generalized model of superfluid nucleus [2]. In both cases the corrections of no principle of these models are needed. In the former, the second pair must break at $\simeq 1$ MeV higher excitation energy as compared with [13]. In the latter, the temperature of the expected phase transition of a nucleus from superfluid to usual state should be decreased by a factor of about 1.5. Besides, the notions of the calculation of entropy below the point of the expected phase transition should be slightly corrected. This provides appearance of the clearly expressed step-like structure in energy dependence of level density that is observed in experiment [1]. Both conditions have analogies in the experimental data on super-conductivity (including high-temperature one) and super-fluidity of usual matter and, therefore, can be considered as quite possible.

Of course, one cannot exclude existence of unknown factors which distort the simple enough model [1] of the $\gamma$-decay process. First of all, it is necessary to test in direct experiment the possible and probably strong influence of the wave function structure of neutron resonance on the $\gamma$-decay process of high-lying states of a nucleus. Observation of strong correlation of reduced neutron width of resonance $\Gamma^0_n$ and partial widths of the cascade primary transitions to the levels with $E_{ex} > 1 - 3$ MeV would decrease discrepancy between the results [1] and notions of old models from one hand but require significant modification of the model ideas of a nucleus from other hand. These problems can be solved in experimental study of two-step cascades in different resonance of the same nucleus. This requires, however, to increase statistics of $\gamma - \gamma$ coincidences. Development of new of principle methods to study the $\gamma$-decay process can bring to the analogous result.

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Fig. 1. The mean value and interval of variations of level density \(1/2^\pm \leq J^z \leq 3/2^\pm\) populated by two-step cascades in \(^{185}W\) (points with bars). Line represents calculation according model [9]. Each variant of calculation is shifted relative to previous one by 1 MeV and is marked by value of \(\kappa\) (%).

Fig. 2. The same as in Fig. 1 for \(^{187}W\).
Fig. 3. The same as in Fig. 1 for $^{191}\text{Os}$.

Fig. 4. The same as in Fig. 1 for $^{193}\text{Os}$. 
Fig. 5. The mean value and interval of variations of radiative strength functions for $^{185}$W (points with bars). Predictions models [10] and [11] in sum with $k(M1) = const$ [12]. Experimental and model values of each variant are increased by a factor of 5.

Fig. 6. The same as in Fig. 5 for $^{187}$W.
Fig. 7. The same as in Fig. 5 for $^{191}$Os.

Fig. 8. The same as in Fig. 5 for $^{193}$Os.
Fig. 9. The ratio between the experimental and model level densities in function of parameter $\kappa$ for 4 excitation energies of studied nuclei.

Fig. 10. The ratio between the experimental and calculated within model $[11]$ and $[12]$ radiative strength functions.