Arbitrarily loss-tolerant verification of quantum steering without trustfulness

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We propose a method to verify quantum steering for two qubit states with an arbitrary amount of null measurement outcomes when both steering and steered parties cannot be trusted. We modify a score function that it may depend on the measurement efficiencies of both parties, the number of symmetrically placed measurement settings, and imperfection of the state preparation. The steering bound proposed in a recent work [Phys. Rev. X \textbf{2}, 031003 (2012)] plays an important role in our score function; thus, similarly, for null measurement outcomes obtained by the steering party with the ratio $1 - \eta$, the steering can be verified using the number of different measurement settings larger than $1/\eta$ and maximally steerable states. Furthermore, we show that, for null measurement outcomes at the steered party, the ratio of null measurement outcomes does not affect steerability unless it is 1. Our result will be helpful to enable loss-tolerant and device-independent steering tasks.

I. INTRODUCTION

Nonlocal correlations between distant parties are one of the most interesting features of quantum mechanics. One may classify nonlocal correlations into three categories: Bell nonlocality, steerability, and entanglement [1, 2]. The Bell nonlocality is the strongest non-classical property among the three that rejects any explanation of phenomena based on local realistic theory [2]. It enables one to perform various quantum information tasks such as secure communication protocol [3, 4], random number generation [5–10], and self-testing [11–14]. At the same time, while the Bell nonlocal state is very useful in non-trivial verifications of quantum operations, its implementation is highly demanding.

Entanglement may be understood as a nonlocal correlation invariant under local operations and classical communications. The requirement for a quantum state to be entangled is less strong compared with that of the Bell nonlocality, whereas entanglement does not always guarantee a successful test of the Bell nonlocality. In this respect, steerability is an intermediate property less demanding to implement for a reasonable scope of applications compared with the Bell nonlocality. Steerability is a nonlocality that rejects explanation of phenomena based on a combination of any local probability arrangements and local quantum models. From its definition, it is an intermediate nonlocality between entanglement and the Bell nonlocality, and thus has a wider range of applicability than entanglement and thus is less complicated to implement than the Bell nonlocality. Since the first formalization of steerability [1], many theoretical developments [1, 2, 15–31] and experimental verifications [22–24, 26, 32–39] have been studied, and their practical applications are still being investigated [33, 40–43].

One of the advantages of steerability is loss tolerance against null measurement outcomes [24]. In the literature [24], the steering bound for a bipartite qubit state is obtained as a function of the steering party’s heralding efficiency and the number of symmetrically placed measurement settings. Furthermore, it is shown that maximally steerable states with $n$ different measurement settings are sufficient to verify steering if the detection efficiency of the steering party is larger than $1/n$. This result can close the detection loophole in the steering process under an arbitrary amount of detectable errors on the steering party. However, this scheme is proposed under an original steering scenario, which is one-sided-device-independent (1s-DI), which indicates that we must assume that the measurement outcomes reported by the steered party are correct. Thus, if the steered party reports false outcomes or their measurement apparatus is imperfect, the assumption fails to be satisfied, which in turn yields a loophole in the determination of steerability. This 1s-DI property is an essential part of the steering scenario [1, 2], and has been pointed out as a weakness in its reliability as it restricts the applicability of steering only to the 1s-DI information processing tasks. To overcome the difficulty, a more elaborate device-independent (DI) steering scenario was proposed and shown to be equivalent to the original one [25] and its experimental verification has been demonstrated [26]. This DI steering scheme is called a quantum refereed steering (QRS) game, and it removes the assumption of trustfulness on the steered party, Bob in this case, by sending an information-encoded quantum state to Bob (steered party). However, in the QRS game, no scheme to overcome an arbitrary amount of null measurement outcomes is proposed yet and we discuss it herein.

In this paper, we show that the verification of quantum steering in a QRS game is possible under the scenario of arbitrary null measurement outcomes, which occur on both steering and steered parties. To this end, we first indicate the main differences between the Einstein-Podolsky-Rosen (EPR) steering scenario and QRS game, and propose a method to build the effective equivalence between the two scenarios in order to apply the results of the former to the latter. Subsequently, by using the results of the previous study [24], we obtain the steering
criteria in the QRS game, which is called a score function, in the canonical way [26]. As a last step, we consider the effect of our scheme on an honest steered party who suffers from null measurement outcomes, and determine the optimal strategy that can be adopted by them. Subsequently, the final form of the steering criteria shows that the measurement efficiency of the steered party does not affect the steerability, whereas that of the steering party does, with a corresponding steering bound. The result will enable loophole-free verification of steering with a two-sided loss-tolerant and device-independent property.

II. STEERING AND QRS GAME

We first review the QRS game introduced in Ref. [25], which is a DI steering scenario. We will not review the original steering task, which requires trust (or assumption) on the steered party, but only show its simplified figure to compare it with the QRS game (See Fig 1).

The detailed scenario of the QRS game is as follows. First, the steering party (say, Alice) and the steered party (say, Bob) initially share a bipartite quantum state. Second, a referee prepares sets of information \( \{j\} = J \), \( \{s\} = S \) with some probability distributions \( p(j) \) and \( q(s) \) and announces them. Third, the referee sets a payoff \( P(a, b, j, s) \) that Alice and Bob gain when Alice reports \( a \) and Bob reports \( b \), provided Alice receives \( j \) and Bob receives \( s \) from the referee. Subsequently, the sum of total payoff, or score, that Alice and Bob will gain in the game is expressed by

\[
Score = \sum_{j,s,a,b} p(j)q(s)P(a, b, j, s)P(a, b|j, s),
\]

where \( P(a, b|j, s) \) is a probability that Alice and Bob yield \( a \) and \( b \) when they receive \( j \) and \( s \), respectively. The goal of this game for Alice and Bob is to maximize their score. Therefore, having the shared state and knowing the payoff \( P(a, b, j, s) \) and probability distributions \( p(j) \) and \( q(s) \), Alice and Bob communicate to determine the optimal strategy to maximize their score.

Fourth, the referee provides information \( j \in J \) to Alice and encodes \( s \in S \) in non-orthogonal quantum states \( \omega_{j,s} \) and thereafter, provides it to Bob. Once the information is provided by the referee, no more communication from Alice to Bob is allowed. Note here that information transmission from Bob to Alice is still permitted because it is impossible for Bob to fully distinguish the value of \( s \) provided by the referee. Fifth, considering the information \( j \) and quantum state \( \omega_{j,s} \), Alice and Bob choose \( a \) and \( b \) according to their optimal strategy and thereafter send them back to the referee. For each payoff function and the sets of information \( J \) and \( S \), the highest score that can be achieved using unsteerable states is determined. Therefore, if some state strictly exceeds such a score, it indicates that the state is steerable, and thus, steering is verified.

III. LOSS TOLERANT QRS GAME

A. Loss tolerant scheme

In Ref. [26], a canonical way to convert steering inequality to the corresponding score function is proposed. The authors assumed that no communication between Alice and Bob is allowed, and they did not mention Bob’s heralding efficiency. However, one-way communication from Bob to Alice is one of the advantages of the QRS game, and considering Bob’s heralding efficiency is a necessary part of the DI scenario. Therefore, in order to fully exploit the advantages of the QRS game, we should not abandon these two aforementioned traits.

Here, we should emphasize that the verification of steering is accomplished by rejecting all explanations using unsteerable states. Thus, it is usual to consider an optimal strategy that dishonest Alice and Bob can adopt to deceive the referee by indicating that they share a steerable state while they do not, and check whether the steering criteria can detect their fraud. As the relaxed condition yields additional cheating strategies, there is a higher chance that the referee is deceived by employing the optimal strategy in the QRS game than that in EPR steering. Therefore, for the score function obtained from steering inequality to be the steering criterion, we must consider the optimal strategy of dishonest Alice and Bob with one-way communication. It is evident that the optimal cheating strategy for dishonest Alice and Bob using heralding efficiency is post-selection with perfect measurements. They may lie to the referee indicating that they failed to obtain valid measurement outcomes whenever their measurement results are unsuitable for them to obtain a high score. To be precise, provided...
that the heralding efficiency of Alice is $\eta = k/n$ and that
of Bob is $\eta' = k'/n'$, the best cheating strategy is that
Alice reports $k \cdot m$ and Bob reports $k' \cdot m'$ measurement
results of advantage, over a total $n \cdot m$ and $n' \cdot m'$
number of measurement trials with some positive integer
factors $m$ and $m'$, respectively. Note that the post-selection
scheme can be extended to obtain irrational numbers of
heralding efficiencies by probabilistically mixing deter-
minitistic post-selection strategies \cite{24}. Combined with
this post-selection, one-way communication may support
additional cheating strategies.

Thus, the question of how we can rule out all possibil-
ities of cheating arises. We do not have to determine the
optimal cheating strategy to prevent it; rather, we can
prevent it by imposing some constraint that Alice and
Bob must obey, which eliminates the deceit of Bob. We
propose a simple method. We prohibit Bob from post-
selecting the measurement outcomes so that the situation
is not significantly different from the lossy EPR steering
scenario. If we compel Bob to report any valid value
whenever measurement is performed, it forcibly adjusts
Bob’s heralding efficiency to 1, which blocks the post-
selection. Now that Bob cannot exploit a post-selection
scheme, he can use one-way communication and lying as
a cheating strategy. The best situation for Bob to take
advantage of these two properties is when his true effi-
ciency is perfect. It is well known \cite{19}, however, that
dishonest Alice and Bob with perfect measurement ap-
paratus without post-selection cannot cheat the referee
even with the aid of one-way communication, and hence,
one-way communication and lying are of no use under our
scheme. Thus, we can exclude all additional cheating
strategies in the QRS game, and be guaranteed to
obtain a credible score function from steering inequal-
ity. Furthermore, one can raise a question whether our
constraint affects the steerability of honest Bob who suf-
fers from null measurement outcomes. However, we will
show that this is an unnecessary concern in the following
section.

B. Modifying score function

Based on the foregoing argument, we intend to link a
result in the EPR steering to the QRS game. We start from the steering inequality proposed in Ref. \cite{24},

$$\frac{1}{n} \sum_{j=1}^{n} (a_j \sigma_j^B) \leq C_n(\epsilon), \quad (2)$$

where $a_j$ is Alice’s reporting value from $\{+1, -1\}$, $\sigma_j^B$ is
Bob’s measurement, $\epsilon$ is Alice’s heralding efficiency, and
$C_n(\epsilon)$ is the local hidden state (LHS) bound. Following
the methods in \cite{24}, we rewrite it as

$$\sum_{j=0}^{n} (a_j B_j) \leq 0, \quad (3)$$

by defining $a_0 = -nrC_n(\epsilon)$, $B_0 = I$, and $B_j = \sigma_j^B$
for nonzero $j$. For $s$ in a set $\{+1, -1\}$, let $\omega_{j,s} = \frac{1}{2}(I_2 + s \sigma_j)$
be a quantum state that the referee provides to Bob and
g$_{js}$ be coefficients satisfying $B_j = \sum_s g_{js} \omega_{j,s}$. Subse-
sequently, we have $g_{0s} = 1$ and $g_{js} = s$ for nonzero $j$. Note
here that $g_{0s}$ is determined even though $a_0$ is not de-
defined, to satisfy $B_0 = \sum_s g_{0s} \omega_{0,s}$. Now, we set the score
function as

$$P(r) = \sum_{j,s} s(ab)_{j,s} = \sum_{j \neq 0,s} s(ab)_{j,s} - \sum_s nrC_n(\epsilon) \langle b \rangle_{0,s}, \quad (4)$$

where $j$ runs from 0 to $n$, and $s$ is summed over $\{+1, -1\}$.

Argument $r$ in the score function is related to the ref-
eree’s imperfect state preparation, which will be ex-
plained later. To simplify Eq. (4), let \{$E_{0BC}^B$, $E_{1BC}^B$\} be a set of positive-operator valued measure (POVM) el-
ements that Bob performs on the composite system of his
state and Charlie’s state $\omega_{j,s}$, where value 0 is assigned
to the outcome of $E_{0BC}^B$ and 1 is assigned to the outcome of
$E_{1BC}^B$. Thus, we can express $\langle b \rangle_{j,s}$ for arbitrary $j$ from
0 to $n$ as

$$\langle b \rangle_{j,s} = \text{Tr}_{BC} [E_{1BC}^B (\rho_B \otimes \omega_{j,s})], \quad (5)$$

and hence, summation over $s \in \{+1, -1\}$ yields

$$\sum_s \langle b \rangle_{j,s} = \sum_s \text{Tr}_{BC} [E_{1BC}^B (\rho_B \otimes \frac{1}{2} (I_2 + s \sigma_j))] = \text{Tr}_{BC} [E_{1BC}^B (\rho_B \otimes I_2)], \quad (6)$$

which is independent of the index $j$. Therefore, we obtain

$$n \sum_s \langle b \rangle_{0,s} = \sum_{j \neq 0,s} \langle b \rangle_{j,s},$$

and Eq. (4) can be rewritten as

$$P(r) = \sum_{j,s} (s(ab)_{j,s} - rC_n(\epsilon) \langle b \rangle_{j,s}), \quad (7)$$

where $j$ runs from 1 to $n$.

Now, in a similar manner as \cite{19}, we show that the
score function in Eq. (8) cannot exceed zero if Alice and
Bob initially share an unsteerable state. As an unsteer-
able state is such that its measurement outcomes can be explained by the combination of any local probability
distributions and local quantum theories, the probability
distribution for obtaining $a$ and $b$ when Alice and Bob
operate measurement $A$ and $B$, respectively, is given by

$$p(a, b | A, B) = \sum_{\lambda} p(\lambda) p(a | A, \lambda) p_Q(b | B, \lambda), \quad (8)$$

where $p_Q$ denotes the probability distribution given by
quantum mechanics and $\lambda$ is a hidden variable with prob-
ability distribution $p(\lambda)$. Let $A_j$ be a measurement that
Alice chooses to perform when she receives informa-
tion $j$ from Charlie, \{$E_{0}^{jBC}$, $E_{1}^{jBC}$\} be a POVM as before, and
\( M_A \) be a set of measurement outcomes of \( A \). We then obtain
\[
\langle ab \rangle_{j,s} = \sum_{\lambda} p(\lambda)\langle a_j, s, \lambda | X^C X^C_{j,s} \rangle_{\lambda, \sigma, \omega_{j,s}}, \tag{9}
\]
where \( \langle a \rangle_{j,s} = \sum_{a \in M_A} a p(a | A_j, \lambda) \). To simplify the notations, let us define
\[
X^C := \text{Tr}_B[E^B_1 (\rho_\lambda \otimes I_2)], \\
\tau^C := X^C / \text{Tr}[X^C], \\
q(\lambda) := p(\lambda)\text{Tr}[X^C^\dagger] / N, \\
N := \sum_\lambda p(\lambda)\text{Tr}[X^C].
\]

We can thus rewrite Eq. (9) as
\[
\langle ab \rangle_{j,s} = \sum_{\lambda} p(\lambda)\langle a_j, s, \lambda | X^C X^C_{j,s} \rangle_{\lambda, \sigma, \omega_{j,s}} = N \sum_{\lambda} q(\lambda)\langle a_j, s, \lambda | \tau^C \rangle_{\lambda, \sigma, \omega_{j,s}}. \tag{10}
\]

Note that \( \sum_s s \omega_{j,s} = \sum_s \frac{1}{2}(I + s \sigma_j) = \sigma_j \); therefore,
\[
\sum_s s \langle ab \rangle_{j,s} = N \sum_{\lambda,s} \langle a_j, s, \lambda | \sigma_j \rangle_{\lambda, \sigma} = N \langle a_j | \sigma_j \rangle. \tag{11}
\]

By using the same notation above, the expectation value of \( b \) is obtained as
\[
\langle b \rangle_{j,s} = \sum_{\lambda} p(\lambda)\text{Tr}_C[P^C_{1}(\rho_\lambda \otimes X^C_{j,s})] = \sum_{\lambda} p(\lambda)\text{Tr}[X^C X^C_{j,s}] = N \sum_\lambda q(\lambda)\text{Tr}[\tau^C \langle j, s, \lambda | X^C \rangle_{\lambda, \sigma, \omega_{j,s}}]. \tag{12}
\]

Note that \( \sum_s \omega_{j,s} = \sum_s \frac{1}{2}(I_2 + s \sigma_j) = I_2 \), thus
\[
N \sum_{\lambda,s} q(\lambda)\text{Tr}[\tau^C \langle j, s, \lambda | X^C \rangle_{\lambda, \sigma, \omega_{j,s}}] = N \sum_\lambda q(\lambda)\text{Tr}[\tau^C \langle j, s, \lambda | X^C \rangle_{\lambda, \sigma, \omega_{j,s}}] = N. \tag{13}
\]

Thus, for the LHS model, Eq. (7) yields
\[
P(r) = 2N \sum_j [(a_j | \sigma_j) - rC_n(\epsilon)], \tag{14}
\]
which is bounded above by 0 from inequality (3) and the fact that \( r \geq 1 \). Note that, in Eq. (14), \( P(1) \leq 0 \) is equivalent to inequality (3). Factor \( r \) indicates how well the referee prepared the quantum state. If the referee provides an imperfect state to Bob, Alice and Bob may exploit this imperfection to elevate their score, and hence, even for an unsteerable state, \( \sum_j (a_j | \sigma_j) - rC_n(\epsilon) \) can be higher than \( C_n(\epsilon) \). \( r \) value suppresses their score so that \( \sum_j (a_j | \sigma_j) - rC_n(\epsilon) \) can be 0 again in spite of their optimal cheating strategy. Note here that there is no constraint that unsteerable states have to obtain a 0 score; rather, this is for mathematical convenience. For the method of calculating the \( r \) value precisely, refer to the method section in [20].

Note here that, until now, our modified score function can only be used in the case where Bob’s true efficiency is 1. We further determine Bob’s score with true inefficient measurements. If the null measurement outcomes on Bob’s party make the referee misjudge the steerable state as an unsteerable state, it would be unsatisfactory. However, we will show that this is not the case.

We first assume that Bob lost his information about a shared quantum state. Subsequently, the optimal strategy for Bob is to operate a POVM on the state provided by the referee, and send the result of his measurement to Alice. As the only information Alice can exploit is Bob’s measurement result, Alice’s reporting value \( a \) is determined by Bob’s reporting value \( b \). Therefore, for simplicity, we will say that Bob chooses both values \( a \) and \( b \). It is evident from Eq. (7) that reporting \( b = 0 \) contributes nothing to the score, and hence, we will only consider the case where Bob reports \( b = 1 \). Following the notations in [19], let \( p(\pm | s, j) \) be the probabilities of estimating a given \( s \) as \( \pm 1 \) when Alice receives \( j \). We simply write \( \pm 1 \) as \( \pm \). It is evident that Alice and Bob obtain 1 for \( s(ab) \) with probability \( p(\pm | s, j) \) and lose 1 for \( s(ab) \) with probability \( p(\pm | s, j) \) and lose 1 for \( s(ab) \) with probability \( p(\pm | s, j) \).

As \( p(\pm | s, j) \) is obtained as
\[
P(r) = 2 \sum_j [p(\pm | s, j) + p(-| s, j) - 1 - rC_n(\epsilon)]
\]

where \( p(\pm | s) \) denotes \( \frac{1}{2} \sum_j p(\pm | s, j) \). Recall that, if Bob reports \( b = 0 \), they obtain 0 as a payoff. Therefore, for reporting \( b = 1 \) to be meaningful, \( P(r) \) should be positive. Note that any POVM element on a qubit system can be written as \( \mu (I_2 + \tilde{m} \cdot \tilde{\sigma}) \) where \( \tilde{\sigma} \) is a pseudovector of Pauli operators, \( 0 \leq \mu \leq 1 \), and \( |\tilde{m}| \leq 1 \). Denoting \( \omega_{j,s} = \frac{1}{2}(I_2 + s \tilde{a}_j \cdot \tilde{\sigma}) \), we obtain
\[
p(\pm | s) = \frac{1}{2n} \sum_j \text{Tr}[\mu (I_2 + \tilde{m} \cdot \tilde{\sigma})(I_2 + \tilde{a}_j \cdot \tilde{\sigma})]
\]

\[
= \frac{\mu}{n} \sum_j (1 + \tilde{m} \cdot \tilde{a}_j), \tag{16}
\]

\[
p(\pm | s) = \frac{1}{2n} \sum_j \text{Tr}[(I_2 - \mu I_2 - \mu \tilde{m} \cdot \tilde{\sigma})(I_2 - \tilde{a}_j \cdot \tilde{\sigma})]
\]

\[
= \frac{1}{n} \sum_j ((1 - \mu) + \mu \tilde{m} \cdot \tilde{a}_j). \tag{17}
\]

Summing together, the score \( P(r) \) is given by
\[
P(r) = 2n \left( 2\mu \left( \frac{\sum_j \tilde{a}_j}{n} \right) \cdot \tilde{m} - rC_n(\epsilon) \right), \tag{18}
\]
which is bounded above by 0 as follows:

\[
\frac{P(r)}{2n} \leq 2\mu \left| \sum_n \frac{\vec{a}_n}{n} |\vec{\mu}| - n|\vec{\mu}| - (2\mu|\vec{\mu}| - 1)C_n(\epsilon) \right| \leq \left( \frac{2|\vec{\mu}|}{1 + |\vec{\mu}|} - 1 \right)C_n(\epsilon) \leq 0, \tag{19}
\]

where the first inequality is obtained by taking the absolute value of the first term and using \( r \geq 1 \) for the second term, the second inequality holds from the fact that \( \left| \sum_n \frac{\vec{a}_n}{n} \right| \leq C_n \leq C_n(\epsilon) \), and the third inequality holds from the property of PVM elements. Therefore, the optimal strategy for Bob when he reports \( b = 1 \) is no better than reporting \( b = 0 \). This indicates that Bob’s optimal strategy for null measurement is to report \( b = 0 \), which only reduces his total score by the factor of his measurement efficiency, say, \( \eta_B \). Therefore, Bob’s imperfect measurement reduces his score from \( P(r) \) to \( \eta_B P(r) \), where its sign is not changed, which indicates that considering Bob’s measurement efficiency does not make the referee misjudge steerability under our scheme. By adding all the results to the score function, it can be written as

\[
P(\eta, n, \epsilon, r) = \eta \sum_{j, s} s(ab)_{j,s} - rC_n(\epsilon)(b)_{j,s}, \tag{20}
\]

where \( \eta \) denotes Bob’s measurement efficiency, \( \epsilon \) denotes Alice’s measurement efficiency, \( n \) denotes the number of measurement settings for Alice, and \( r \) denotes the imperfection of state preparation. Note that \( C_n(\epsilon) < 1 \) if \( \epsilon > \frac{1}{\eta} \), and hence, for the number of measurement settings \( n \) larger than \( 1/\epsilon \), it is possible to verify steering with maximally steerable states, which yields \( s(ab)_{j,s} = 1 \).

IV. CONCLUSION

In this study, we have modified the score function in the QRS game to close the detection loophole by null measurement outcomes that may occur at both the steering and steered parties. In order to utilize the proposed loss-tolerant scheme in the EPR steering scenario, we analyzed the main differences between the EPR steering and the QRS game, and filled the gap between them by compelling the steered party to report any value whenever a measurement is performed. Consequently, we modified the score function to verify steering under arbitrarily inefficient measurements at both parties using maximally steerable states, without trust on any parties.

Two points should be noted here. First, if our modified score function is positive, then the state shared by the steering and steered parties is a steerable state. However, the converse is not generally true. Steerable states under the condition of the perfect measurement efficiency can be detected as unsteerable states if the measurement of the steering party is inefficient. Second, the measurement efficiency of the steered party does not affect the steerability. This shows another asymmetric property of the steering that was not observed in the EPR steering scenario.

The DI property enables one to perform quantum information tasks such as unconditionally secure communication, and the loss-tolerant property allows one to implement such tasks in a realistic environment. Furthermore, the asymmetric property that was not observed in the EPR steering scenario can be used for the asymmetric communication so that only one party is free from the threat of detectable information losses. Thus, our research may be used to realize useful and practical quantum information tasks.

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