MODELING AND ANALYSIS

Mechanical behavior of hollow proppants based on finite element model

Zijia Liao1 | Zhaozhong Yang1 | Qi Xue2 | Xiaogang Li1 | Huabin Li3 | Wenhong Li4

1State Key Laboratory of Oil and Gas Reservoir Geology and Exploitation, Southwest Petroleum University, Chengdu, China
2School of New Energy and Materials, Southwest Petroleum University, Chengdu, China
3Pangzhihua BingYang Technology Co., LTD, Pangzhihua, China
4Beijing Gepetto Petroleum Technology Co. LTD, Beijing, China

Correspondence
Zhaozhong Yang and Xiaogang Li, State Key Laboratory of Oil and Gas Reservoir Geology and Exploitation, Southwest Petroleum University, Chengdu 610500, China.
Emails: yangzhaozhongswpu@126.com (Z.Y.); swpuadam@126.com (X.L.)

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Abstract
Proppants are being increasingly used in hydraulic fracturing, which keep the fractures open. Progress has been made in developing hollow proppants that can be transported to the far end of the fractures to improve propped fracture conductivity. Two primary mechanisms by which proppants are damaged under closure stress are embedment and crushing, leading to a remarkable reduction in fracture conductivity. In this study, finite element model (FEM), considering rock plasticity, was used to study the embedment and crushing of a hollow proppant. After comparison with the existing model and experimental data, the proposed FEM was employed to conduct sensitivity studies. The effects of the following parameters on embedment and crushing were analyzed: closure stress, distance coefficient, Young’s modulus of rock and proppant, proppant size, and hollow structure. The results show that the most sensitive environmental factor is the distance coefficient, followed by closure stress and Young’s modulus of rock. The embedment depth increases approximately linearly with the proppant size, and proppants with different sizes have similar crushing rates. The hollow structure has little effect on proppant embedment but significantly affects proppant crushing. Based on the data normalization method, the optimal ratio of hollow radius to proppant radius was designed to be 0.7. These results can guide the preparation of hollow proppants.

KEYWORDS
finite element model, hollow proppant, hydraulic fracturing, proppant crushing, proppant embedment

1 INTRODUCTION

Since its first successful testing in the United States in 1947,1 hydraulic fracturing has been widely used for enhancing oil and gas flow, especially in low-permeability fields. In hydraulic fracturing, high-pressure fluid is pumped into a wellbore to create fractures, and the proppants are then introduced into these fractures by the fracturing fluid. After the fracturing fluid is removed, proppants keep the fractures open (Figure 1). Proppants increase the conductivity of the flow channels, facilitating oil and gas extraction. Proppants should have a low density, to allow for their transportation to the far end of the fracture, especially for the hydraulic fracturing of shale, coal, and tight sands.

Main proppant materials include sand and ceramic. The relative density of sand is approximately 2.65 while that of...
the ceramic proppants can be as high as 3.9; hence, both materials are significantly heavier than the fracturing fluid. Consequently, the proppants settle rapidly, which leads to difficulty in filling more fractures. Field evidence and laboratory studies have proved that lightweight proppants are more beneficial for oil and gas production than basic proppants. In the past few years, many studies have focused on the preparation of lightweight proppants. For example, Wood stated that walnut hull-based ultra-lightweight proppants can be manufactured according to the following process: (a) grinding and sizing a walnut hull; (b) impregnating the hull with a strong epoxy or other resins; and (c) coating the hull with phenolic or other resins in a fashion similar to most resin-coated proppants. Wu et al. used fly ash from Inner Mongolia as the main raw material, clay as the plasticizer, bauxite to improve the molar ratio (Al₂O₃/SiO₂), and composite mineralizers to promote sintering and ceramic structure formation. The ideal sintering temperature was determined to be 1370°C, at which closely interlocking rod-like mullite crystals were observed on the proppant microstructure. Finally, they prepared a proppant with a relative density of 2.61, acid solubility of 5.7%, and crushing rate of 5.0% under 52 MPa. Zoveidavianpoor et al. described a lightweight proppant called chemically modified and reinforced composite proppant (CMRCP); it was synthesized as follows: First, raw coconut shells were crushed and sieved at different sizes. After surface modification, a reinforced natural fiber was formed over the substrate. Lignocellulosic material (e.g., flax fiber) was used to reinforce the substrates. Following curing of the reinforced layer, an outer coating was formed via a thermosetting polymer. Bestaoui-Spurr developed a new proppant that uses inorganic polymers as coatings on silica sand. Inorganic polymers are formed by the reaction between an alkali metal hydroxide/silicate solution and an aluminosilicate binder. Compared to the general polymer coating, inorganic coatings are stable and maintain their properties at much higher temperatures. This proppant may be manufactured via a two-step process by preheating the silica sand and mixing the inorganic polymers thoroughly until polymerization occurs. The resulting coated proppants have relative densities of 1.8-1.9 and an excellent tolerance to high closure stresses.

Hollow proppants are another type of lightweight proppants. Several methods have been employed for the development of hollow proppants. For instance, Jones et al. presented a hollow proppant manufacturing process that included mixing a fine polycrystalline ceramic powder (particle size < 25 µm in diameter) with water and organic binders; injecting the mixture through a nozzle into a spray dryer wherein the temperature is maintained between 100 and 400°C; passing the injected particles in the dryer against a hot air stream; and sintering the particles at a sufficient temperature to produce a proppant with a dense case and an average grain size of less than 30 µm in diameter and a central void greater than 5 vol% of the proppant. The resulting proppant has a relative density of less than 3.3, and a crushing strength equal to or greater than that of the Ottowa sand at closure stresses of above 34.5 MPa. Parse et al. developed the following method to form a single component hollow proppant: they developed a continuous hollow core tubular precursor that was largely cylindrical in shape; the precursor was shaped to form the proppant particle with ends; the ends were closed and sealed; and the sealed proppant particle was separated from the continuous hollow core tubular precursor to form a single component proppant particle. In our previous study, we used urea as the pore template, MnO₂ as the

FIGURE 1 Illustration of a hydraulic fracture

FIGURE 2 Hollow proppants supporting the fracture face (A) and embedded in fracture face (B)
sintering agent, and bauxite powder as the shell coating material, and a proppant with a controllable hollow radius was prepared. The relative density of the hollow proppant was 2.47, and the crushing rate was only 5.21% under 25 MPa. Similar approaches are reported in other studies.\textsuperscript{15}

Lightweight proppants exhibit good transportability, which is a key focus for engineers in hydraulic fracturing. However, proppant-related problems are significant, as they are used in complex underground environments. In addition to transportability, mechanical performance also plays an important role in hydraulic fracturing effects. An ideal proppant should exhibit low density, embedment depth, and crushing rate.

Many scholars have studied the mechanical performance of proppants. For example, proppants have been embedded in rock under closure stress, resulting in the loss of fracture width especially in the case of lightweight proppants which tend to be transported far end of the fractures to form a single layer (Figure 2). Lucy et al.\textsuperscript{16} experimentally observed the proppant embedment depth in soft sandstones. He found that embedment up to 300% in both fracture faces and propped fracture width reductions of 60% can be expected. Wen et al.\textsuperscript{17} studied the effect of closure stress, proppant concentration, and rock type on proppant embedment, and fracture conductivity. Their results showed that proppant embedment damaged the fracture conductivity up to 87.5% at a concentration of 5 kg/m\textsuperscript{2}. Li et al.\textsuperscript{18} developed a mathematical model to calculate the proppant embedment depth, and found that it increases with closure stress and proppant size and decreases with an increase in the Young's modulus of coal. Jiaxiang et al.\textsuperscript{19} established a new model to evaluate the proppant embedment depth based on contact mechanics and studied the effect of proppant size, fracture fluid loss, and closure stress on this parameter. Their results showed that the larger the proppant in the fracture, the deeper the embedment and the more obvious the nonlinear change under closure stress. The embedment depth with fracturing fluid loss was 2.47 times more obvious the nonlinear change under closure stress. The porosity of 5 kg/m\textsuperscript{2}. Freeman et al.\textsuperscript{21} reported that the crushing rate of lightweight ceramic is 8% higher than that of intermediate-density ceramic using the modified procedure. Crushing proppant fines have been known to migrate to the wellbore and accumulate,\textsuperscript{22} and approximately 14% additional crushed fine particles have been observed at the outlet of the conductivity cell compared with the inlet in the process of fracture conductivity test.\textsuperscript{23} Cutler\textsuperscript{24} and Swanson\textsuperscript{25} predicted that the hollow proppant can provide adequate strength while having a lower density.

In this study, the finite element method was used to calculate the embedment depth and crushing rate of the hollow proppant for the first time. The effects of closure stress, Young’s modulus of rock and proppant, proppant size, and hollow structure on the proppant embedment depth and crushing rate were analyzed. In addition, the hollow structure was optimized through data normalization considering the density of the hollow proppant. The simulation method and results are expected to be of significance for the hydraulic fracturing.

2 | FEM THEORY

2.1 | Model approach

2.1.1 | Geometric model

It is necessary to adequately simplify the geometric model to improve computing efficiency. The semi-proppant finite element model (FEM) was established considering the underground conditions (Figure 2); following assumptions were made: (a) the hollow part occurs at the center of proppant, (b) the proppant looks like a standard sphere, (c) the proppant exhibits elastic deformation, and (d) the proppant embedment depth is less than half of its size.

Proppant geometries for various cases are shown in Table 1. No. 1 was used to verify the finite element method. No. 4 is the basic case, which was used to analyze the influence of closure pressure, distance coefficient, and Young’s modulus of rock and proppant on embedment depth and crushing rate.

| Proppant geometries |
|---------------------|
| t | r | D | R | ξ |
| 1 (verification case) | 0.2 | 0 | 0.4 | 0.2 | 0 |
| 2 | 0.175 | 0.025 | 0.4 | 0.2 | 0.125 |
| 3 | 0.15 | 0.05 | 0.4 | 0.2 | 0.25 |
| 4 (base case) | 0.1 | 0.1 | 0.4 | 0.2 | 0.5 |
| 5 | 0.05 | 0.15 | 0.4 | 0.2 | 0.75 |
| 6 | 0.025 | 0.175 | 0.4 | 0.2 | 0.875 |
### 2.1.2 Interaction definition and boundary condition

The process of fracture closing can be regarded as quasi-static. The analysis model was assembled from the rock and proppant. Hard contact was used to simulate the contact relationship. To calculate the finite element, the central sphere and rock surface must be constrained. The stress was loaded on the bottom of the rock. The basic closure stress was set to 15 MPa, and a certain constraint condition was applied (Figure 3).

#### 2.1.3 Mesh

The created three-dimensional geometry model must be meshed into sufficient grid cells. Taking the base case as an example, the hollow proppant was divided by the medial axis algorithm for which seeds with an approximate global size of 0.01 were arranged. The rock specimen was divided by a structured grid method for which seeds with an approximate global size of 0.03 were arranged. A smaller seed size can yield more accurate results but will increase computational costs. When the seed size of the proppant is less than 0.01 or that of the rock is less than 0.03, the embedment depth tends to be stable (Figure 4); hence, the seed sizes stated here were selected.

The proppant was divided into 14,910 elements, and the rock specimen was divided into 8,000 elements. Both were assigned a C3D8R element type (8-node linear brick, reduced integration, hourglass control). The mesh was verified without errors or warnings. The FEM is shown in Figure 5.

#### 2.1.4 Material properties

Wu²⁶ reported that the rock can be assumed as an elastic-plastic material whose material property can be described by the Mohr-Coulomb model. The rock considered for the present study is sourced from coal rock in the southern Sichuan Basin, China. The fracturing proppant is composed of quartz sand, with its main mineral composition being SiO₂. The mechanical parameters of the materials are listed in Table 2.

### 2.2 Calculation of the embedment and crushing rate

In the FEM, the difference between the maximum and minimum displacements of the rock in the direction of closure stress is taken as the proppant embedment depth \( h \).

\[
h = h_{\text{max}} - h_{\text{min}}
\]

where \( h_{\text{max}} \) is maximum displacement of the rock; \( h_{\text{min}} \) is minimum displacement of the rock.

In addition, in the model described in Section 2.1, the brittle failure criterion was used to describe proppant crushing (Figure 6). The brittle failure criterion considers that...
when the element’s tensile stress exceeds a certain value, damage to the element begins to occur, after which, the element quickly fails. Thus, elements with tensile stress over $\delta n$ can be regarded as crushing elements. According to Cui,27 $\delta n$ can be considered as 100 MPa. When the proppant element’s tensile stress at the integral point exceeds 100 MPa, the element is considered to have been damaged. The crushing rate of the hollow proppant can be calculated by

$$\varphi = \frac{V_p}{V_b}$$  \hspace{1cm} (2)

where $\varphi$ is the crushing rate, $V_p$ is the sum of the crushing element volume, and $V_b$ is the total element volume.

### 2.3 Model verification

In this study, the results of embedment depth of the existing model and experimental data were used to verify the FEM. Few studies have evaluated the effects of rock plasticity and hollow structure on embedment depth. Li et al.18 developed a model to calculate the solid proppant embedment depth, ignoring rock plasticity. Hence, case 1, which ignores rock plasticity, was compared with Li’s model. Li’s model is expressed as

$$h = 1.04D(K^2P)^{\frac{1}{3}} \left[ \left( \frac{1 - v_2^2}{E_1} + \frac{1 - v_1^2}{E_2} \right)^{\frac{1}{3}} - \left( \frac{1 - v_2^2}{E_1} \right)^{\frac{1}{3}} \right] + D' \frac{P}{E_2}$$  \hspace{1cm} (3)

where $P$ is closure stress in MPa; $K$ is the distance coefficient (Figure 2), taken as 1.5 in the verification case; $D$ is the proppant size (mm); $D'$ is the rock thickness (mm); $E_1$ is the Young’s modulus of the proppant (MPa); $v_1$ is the Poisson’s ratio of proppant (dimensionless); $E_2$ is the Young’s modulus of rock (MPa); and $v_2$ is the Poisson’s ratio of rock (dimensionless).

The values under different closure stresses are shown in Figure 7. The embedment depth of both models increases approximately linearly with the increase in closure pressure, and the two results are very close to each other. Further, the FEM considering the rock plasticity was compared with the experiment. The proppant embedment depth test was conducted using the fracture conductivity unit (Figure 8). The closure stress was loaded on the rock-propellant stack through a metal loading ram controlled by a servo-controlled load frame, and two linear displacement sensors were used to measure the change in the fracture aperture, which is considered as the proppant embedment depth.

Figure 9 shows a comparison between the embedment depth as determined by the experiment and FEM under different closure stresses. The embedment depth calculated by the experiment and FEM considering rock plasticity increases more rapidly with the increase in closure stress. The difference between the model considering rock plasticity and the model ignoring rock plasticity is significant. Moreover, with the increase in closure stress, the difference also increases. This means that even a small closure stress can cause rock plastic deformation. As closure stress increases, rock plastic deformation increases, and proppant embedment is increasingly facilitated.

In addition, under the same closure stress, the experiment has a higher embedment depth value than FEM considering rock plasticity. One possible reason is that the quartz sand proppant used in the experiment have lower sphericity and roundness compared with the proppant of FEM, which resulted in nonuniform stress loaded on the proppant and embedded in the fracture face.

### 3 RESULTS AND DISCUSSION

#### 3.1 Effect of closure stress

The effect of closure stress on embedment depth is mentioned in Section 2.3. The embedment depth increases with

| Rock type | Poisson's ratio | Young's modulus (MPa) | Cohesion (MPa) | Angle of internal friction (°) |
|-----------|----------------|----------------------|----------------|-----------------------------|
| Coal      | 0.342          | 2632.6               | 6.91           | 16.22                       |
| Proppant  | 0.25           | 72 000               | /              | /                           |

FIGURE 5 Finite element model

TABLE 2 Mechanical parameters of the materials
an increase in the closure stress. In addition, the plastic strain distributions of rock under different closure stresses are shown in Figure 10; large plastic strain (color contours) occurs beneath the contact surface and surrounding areas. This results in 23, 49, 81, and 126 µm embedment depths under 5, 10, 15, and 20 MPa, respectively. Considering that the fracture aperture of a single layer of proppant is only 0.4 mm in the case 4, if the closure stress continues to increase, an almost complete closure of the fracture can be expected.

Closure stress is an important factor affecting proppant crushing, which has a further significant effect on proppant selection and usage in hydraulic fracturing. The effect of closure stress on the crushing rate is illustrated in Figure 11. The investigated closure stresses range from 5 to 20 MPa. The crushing rate results range from 1% to 40%. As shown in Figure 11, with an increase in closure stress, the crushing rate and incremental amplitude increased. This may explain why, for some experimental data, fracture conductivity decreases more rapidly as closure stress increases.

3.2 | Effect of distance coefficient

In addition, the distributions of tensile stress of the proppant under different closure stresses are shown in Figure 12; the empty areas represent the proppant-damaged element. With the increase in closure stress, the tensile stress of the proppant increases gradually. The minimum tensile stress always appears in the middle hollow position, and the maximum tensile stress appears in the contact area with the rock. This is because the stress first diffuses toward the center of the sphere and then spreads along the hollow sphere.
represents the distribution of proppant in fractures. Recently, some heterogeneous proppant placement techniques, such as the channel fracturing technique where the proppants are intermittently pumped, have been optimized to obtain high fracture conductivity. The range of the distance coefficient is 1-2. The results are shown in Figure 13. The distance coefficient has a significant influence on proppant embedment depth and crushing rate. The proppant embedment increases significantly with an increase in the distance coefficient. When the distance coefficient is 2, the fracture is almost completely closed. The proppant crushing rate increases slightly when the distance coefficient is less than 1.2 and increases considerably as the distance coefficient increases.

3.3 | Effect of Young’s modulus of rock

Young’s modulus of rock is an important environmental factor that affects the proppant performance. Figure 14 shows the effects of Young’s modulus ranging from 1 to 110 GPa. It can be observed that with an increase in Young’s modulus of rock, the embedment depth decreases rapidly and then slowly declines linearly. After hydraulic fracturing, the Young’s modulus of rock decreases because large amounts of fracture fluid interact with the rock at higher temperatures for an extended period of time. The softening rock may lead to a deep embedment depth when the Young’s modulus of rock reduces to less than 40 GPa, as suggested by our study.

In addition, the proppant crushing rate increases with an increase in the Young’s modulus when the value is less than 40 GPa and then remains stable as the Young’s modulus continues to increase. The reason for this is that the increasing Young’s modulus of rock decreases the proppant embedment depth as discussed above, which results in a smaller rock area coming into contact with the proppants. According to Chen, the contact stress of the proppant is inversely proportional to the contact area under the same conditions. Therefore, when the Young’s modulus of rock is lower than 40 GPa, the proppant tends to produce less damaged elements under the same closure stress, which decreases the risk of proppant spalling. Here, proppant embedment was considered the main factor when calculating fracture conductivity.
3.4 | Effect of Young's modulus of proppant

Optimizing Young's modulus of proppant is paramount for selecting the appropriate shell coating material. The simulation results under different proppant moduli are shown in Figure 15. The results show that the embedment depth increases when the Young's modulus of proppant is relatively small and then tends to be constant as the Young's modulus continues to increase. In contrast, as the Young's modulus of proppant increases, the crushing rate decreases sharply and then tends to become stable.

Combined with the analysis in Section 3.3, it can be inferred that when the Young's modulus of rock is less than 40 GPa, some proppants with low Young's modulus, such
as coalbed fly ash-based proppant, could be used in hydraulic fracturing, which reduce proppant embedment. Once the Young’s modulus of rock exceeds 40 MPa, the proppant should be prepared using materials with a high Young’s modulus to maintain a certain strength.

3.5 | Effect of proppant size

Optimizing proppant size is crucial for achieving ideal well productivity. Proppant grains with diameters ranging from 0.1 to 0.8 mm were considered in the present study, and the results are shown in Figure 16. The range of the embedment depth is 18-150 µm. The embedment depth increases approximately linearly as the proppant size increases. This may occur because a larger proppant resists more severe closure stress when the distance coefficient is constant, which results in a greater embedment depth.

In addition, it was observed that the proppant size has little influence on the proppant crushing rate. The proppant generates more damaged elements owing to the higher load when the distance coefficient is constant, which results in a higher crushing rate. However, in reality, the larger proppant increases the total element volume, as noted by \( V_b \) in Equation 2. Moreover, a larger proppant has a larger embedment depth and contact area with the rock, which results in a relatively small tensile stress. Consequently, similar crushing rates with different proppant sizes is due to the comprehensive effect of manifold causes, which reveal the possibility that the fracture filled with small proppant grains also has a higher conductivity. This is also one of the reasons for using a small-particle proppants in shale gas fracturing. A small-particle proppant can not only be transported to the far end of the fracture but can also maintain high conductivity.

3.6 | Effect of hollow structure

The ratio of hollow radius to proppant radius is an important characteristic for designing hollow proppants. The embedment depth and crushing rate with different ratios under 15 MPa are shown in Figure 17. The different hollow structure proppants have similar embedment depths, indicating that a hollow structure is not an important factor in proppant embedment. Thus, we concluded that the analytic method and results of embedment depth for solid proppants can be applied to hollow proppants. For the structural design of hollow proppants, the influence of hollow structures on other parameters, such as density and strength, rather than embedment depth, should be a more significant research focus.

When preparing the hollow proppant, the influence of the hollow structure on proppant strength must be considered, which is an important factor in choosing the amount of pore-forming material. Figure 17 also shows the relationship between different ratios and crushing rate. The calculated crushing rate ranges from 7% to 80%. It can be observed that
the crushing rate increases with increasing hollow radius in a nonlinear manner. Specifically, when the ratio is less than 0.25, the hollow structure has a marginal effect on strength. Hence, the hollow proppant could maintain a strength similar to that of the solid proppant with reduced density. When the ratio exceeded 0.25, it significantly influenced proppant crushing rate.

The ideal hollow proppant should meet the characteristics of lower density, embedment depth, and crushing rate. To this end, a reasonable hollow structure was designed based on the data normalization method. After obtaining the data of proppant embedment depth and crushing rate, the density of the hollow proppant was calculated and expressed as

$$\rho_o = \rho_s \left[ 1 - \left( \frac{r}{R} \right)^3 \right]$$

(4)

where $\rho_o$ is the density of the hollow proppant, $\rho_s$ is the density of the shell coating material, $r$ is the hollow radius, and $R$ is the proppant radius.

The data normalization method is expressed as

$$X^* = \frac{X - X_{\text{min}}}{X_{\text{max}} - X_{\text{min}}}$$

(5)

where $X_{\text{max}}$ is the maximum value and $X_{\text{min}}$ is the minimum value of the sample data.

Considering that the hollow structure has little effect on proppant embedment, only the density and crushing rate data were employed (Figure 18). The optimal hollow radius to proppant radius ratio was determined to be 0.7.

4 | CONCLUSIONS

Hollow proppants are an important type of lightweight proppant. Based on the FEM, we studied the mechanical behavior of a hollow proppant, including proppant embedment depth and strength. We performed model verification and sensitivity studies on some parameters based on the proposed model. The following conclusions can be drawn.

1. From the model verification, it was observed that the proppant embedment is mainly caused by the plasticity of the rock. The results of the FEM considering rock plasticity were much closer to the experimental data. Thus, the future models of proppant embedment and fracture conductivity should include the plastic parameters of the rock.

2. The most sensitive environmental factor affecting proppant embedment and strength is the distance coefficient, followed by closure stress and Young's modulus of rock.

3. The embedment can be significant when the rock Young's modulus is less than 40 GPa or the proppant Young's modulus is more than 5 GPa. On the contrary, when the rock Young's modulus is more than 40 GPa or the proppant Young's modulus is less than 5 GPa, the possibility of proppant breaking should be critically considered.

4. The embedment depth increases approximately linearly as the proppant size increases. Proppants with different sizes have similar crushing rates in the FEM, which indicates that the fracture filled with small proppant grains may also have a higher conductivity.

5. The hollow structure has a marginal effect on proppant embedment but a significant effect on proppant strength. The optimal ratio of the hollow radius to the proppant radius was determined to be 0.7.
radius is determined to be 0.7, which meets the characteristics of lower density and higher strength.

By systematically analyzing a series of complex factors influencing hollow proppant embedment and crushing, the proposed FEM is a useful tool for proppant selection in hydraulic fracturing design. These results can guide the preparation of hollow proppants.

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NOMENCLATURE

\( K \) Distance coefficient, dimensionless
\( \varphi \) Crushing rate, dimensionless
\( D \) Proppant diameter, mm
\( V_p \) Sum of the crushing element volume, mm\(^3\)
\( P \) Closure pressure, MPa
\( V_b \) Total element volume, mm\(^3\)
\( t \) Wall thickness, mm
\( \sigma \) Element's tensile stress, MPa
\( r \) Hollow radius, mm
\( \sigma_n \) Element's maximum tensile stress, MPa
\( R \) Proppant radius, mm
\( \delta \) Element's strain, dimensionless
\( \xi \) The ratio of hollow radius to proppant radius, dimensionless
\( D' \) Rock thickness, mm
\( h \) Embedment depth, \( \mu \)m
\( E_1 \) Young's modulus of proppant, MPa
\( \varphi \) Crushing rate, %
\( \nu_1 \) Poisson's ratio of proppant, dimensionless
\( h_{max} \) Maximum displacement of the rock, \( \mu \)m
\( E_2 \) Young's modulus of rock, MPa
\( h_{min} \) Minimum displacement of the rock, \( \mu \)m
\( \nu_2 \) Poisson's ratio of rock, dimensionless
\( \sigma_n \) Element's maximum tensile stress, MPa
\( \rho_o \) Density of the hollow proppant, g/cm\(^3\)
\( \delta_{fail} \) Element failure stress, MPa
\( \rho_s \) Density of the shell coating material, g/cm\(^3\)

ORCID
Zijia Liao https://orcid.org/0000-0003-2195-608X

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