Neutrino oscillations as many-particle induced interference between distinguishable particles

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We investigate the causes of the curious property of neutrino oscillations of looking as an interference between fully distinguishable particles. The sources of this effect are identified as to be determined by the many particle nature of space of states of the quantum field theory. It is firstly underlined that in order to explain the observed oscillations, the neutrino subspace of states should not be interpreted as the direct product of the three neutrinos Fock spaces, which is equivalent to imposing a superselection rule. Further, it is argued that the linear completion of such a direct product space of states permits to describe the measured oscillations. Thus, the space of states of the Standard Model (SM) should be interpreted as the linear completion of the direct product of all the Fock spaces (boson or fermions ones) associated to the distinguishable particles. Then, it follows that the neutrino oscillations are determined as interference between independent particles, which are generated by states being outside the direct product of the neutrino Fock spaces. This interpretation seems to imply a large amount of interference observations between distinguishable particles in Particle as well as in Condensed Matter Physics. For seeking clearness, the discussion is done in the framework of a simple Quantum Field Theory (QFT) model of two relativistic free massive neutrinos.

I. INTRODUCTION

The interference effect between distinguishable particles had been observed long time ago and started to be investigated in the original works \cite{1-4}. The particular case of the neutrino oscillations has been and continues to be a relevant theme of research in Particle Physics. Since its prediction by B. Pontecorvo in references \cite{4-6}, the effect had been theoretically as well as experimentally intensively investigated. The discovery of the real occurrence in nature of these oscillations, in super Kamiokande and Solar neutrino observations was a breakthrough step \cite{7, 8}. After it, an enormous amount of investigations about this effect had been performed \cite{9-22}. The oscillations had been studied either through Quantum Mechanics (QM) and Quantum Field Theory (QFT) methods \cite{17-19, 21}. In our view the QFT methods had contributed to clarify some of the assumptions which have been done in the QM approaches. In particular the question about the possibility of defining a Fock space for the flavor eigenfunctions for the electron, muon and tau neutrinos as expressed as linear superpositions of the really stable propagating neutrino models, had been extensively discussed \cite{21}. This aspect constitutes an example of a question related with neutrino oscillation physics that today remain under discussions \cite{22}. One point which in our view deserves research attention is related with the fact that the neutrino oscillations look as a surprising interference effect, similar to the one occurring in the QM of a single particle, but occurring between fully distinguishable fermion particles, each one of them described by a wavefunction being in a separate Fock space \cite{4}. This is a peculiar effect if we consider the idea often used in QM presentations about that different (distinguishable) particles do not interfere between them. Thus, it seem of interest to identify the reason why the interference between neutrinos oscillations discards this frequent consideration adopted in QM.

In this work we address this question. For this purpose the related issue about the proper interpretation of the space of states in quantum field theory is examined. In particular we search for the possibility of describing the quantum oscillations involving different distinguishably particles in the many particle space of states. A model involving only two distinguishable types (flavors) of relativistic particles is considered for the sake of clearness. In first place, it is argued that to describe interference between two fermion flavors in analogy with the neutrino oscillations measurements, the space of states should not be defined as the direct product of the two Fock spaces associated to the two fermion particles. That procedure will be equivalent to impose a superselection rule for defining the allowed physical states. This is because to employing the direct product as the space of states leads to accept the nonphysical character of the state obtained after acting with the addition of two field operators (each one creating a particle the two separate Fock spaces) over the vacuum state. The explanation for this exclusion is the fact that the states
generated by the action of such additions do not pertain to the direct product of the two Fock spaces. 

Afterwards, it is considered that the space of the states of the model is the linear completion of the direct product of the two Fock states. In other words, the linear combination with arbitrary coefficients of the external products of the states in each of the two Fock spaces. It should be stressed that the use of this space of states is naturally suggested, if we do not assume the presence of any superselection rule to restrict the superposition principle of the quantum field theory. It is then verified that this space allows to define the addition of field operators acting in each separate Fock space, as generating new and physically allowed states. Further, rotated flavor creation and annihilation operators are defined by linear combinations of the creation and annihilation operators for stable propagating neutrino modes solving the Dirac equation. This procedure allows to argue that states generated by these rotated flavor fields at a given time, are linear combinations of the propagating particles modes. Moreover, it follows that under a measurement in such a propagating states the probability of its measurement oscillates. Note again, that after accepting the completed 

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the two Fock states. In other words, the linear combination with arbitrary coefficients of the external products of the 

The plan of the presentation is as follows. In Section 2, the two relativistic neutrino free QFT is presented. The 

expressed in terms of a four components $r = 1, 2, 3, 4$ fermion field $\psi(x,t)$

The fields in terms of the annihilation and creation operators $b_{\nu,s}(p), b_{\nu,s}^+(p)$, for the two types of particles $\nu = 1, 2$, having helicities $s = \pm 1$, and the corresponding annihilation and creation operators for their antiparticles $d_{\nu,s}(p), d_{\nu,s}^+(p), \nu = 1, 2$, have the usual expansions

$$
\psi_{\nu}(p) = \sum_{p, \nu, s} (\omega_{\nu,s}(p, x) b_{\nu,s}(p) + v_{\nu,s}(p, x) d_{\nu,s}^+(p)),
$$

$$
\psi_{\nu}^+(p) = \sum_{p, \nu, s} (\omega_{\nu,s}^*(p, x) b_{\nu,s}^+(p) + v_{\nu,s}^*(p, x) d_{\nu,s}(p)),
$$

in which the $\omega_{\nu,s}(p)$ are the positive energy solutions of the Dirac equation with helicity $s = \pm 1$ for each of the two
flavors $\nu = 1, 2$, defined as \[ \delta \]

\[ w_{\nu,\nu}(p, x) = \frac{\exp(i \cdot p \cdot x)}{\sqrt{L^3 \sqrt{2}} \sqrt{2} (n_3 + 1)} \frac{\sqrt{\epsilon_\nu(p) + m_\nu}}{\epsilon_\nu(p)} \left( n_3 + 1 \right) \right), \]

\[ w_{\nu,-}(p, x) = \frac{\exp(i \cdot p \cdot x)}{\sqrt{L^3 \sqrt{2}} \sqrt{2} (n_3 + 1)} \frac{\sqrt{\epsilon_\nu(p) + m_\nu}}{\epsilon_\nu(p)} \left( -n_3 + 1 \right) \right). \]

The antiparticle malfunctions $v_{\nu,+}(p)$ with helicity $s = \pm 1$ for each of the two flavors $\nu = 1, 2$, have the expressions

\[ v_{\nu,+}(p) = \frac{\exp(-i \cdot p \cdot x)}{\sqrt{L^3 \sqrt{2}} \sqrt{2} (n_3 + 1)} \frac{\sqrt{\epsilon_\nu(p) + m_\nu}}{\epsilon_\nu(p)} \left( \frac{n_3 + 1}{n_3 + 1} \right) \right), \]

\[ v_{\nu,-}(p) = \frac{\exp(-i \cdot p \cdot x)}{\sqrt{L^3 \sqrt{2}} \sqrt{2} (n_3 + 1)} \frac{\sqrt{\epsilon_\nu(p) + m_\nu}}{\epsilon_\nu(p)} \left( -n_3 + 1 \right) \right). \]

The energies associated to the two types of particles are

\[ \epsilon_\nu(p) = \sqrt{p^2 + m_\nu^2}, \quad \nu = 1, 2. \]

The field operators and the creation and annihilation ones for the two kinds of particles and antiparticles satisfy

\[
\begin{align*}
[\psi_\nu(x, t), \psi_\nu^+(x', t)]_+ &= I \delta_{\nu,\nu'} \delta(x - x'), \\
[b_{\nu,s}(p), b_{\nu',s'}^+(p')]_+ &= \delta_{\nu,\nu'} \delta_{s,s'} \delta_{p,p'}, \\
[d_{\nu,s}(p), d_{\nu',s'}^+(p')]_+ &= \delta_{\nu,\nu'} \delta_{s,s'} \delta_{p,p'}, \\
[b_{\nu,s}(p), b_{\nu',s'}(p')]_+ &= 0, \\
[b_{\nu,s}(p), b_{\nu',s'}^+(p')]_+ &= 0, \\
[d_{\nu,s}(p), d_{\nu',s'}(p')]_+ &= 0, \\
[d_{\nu,s}(p), d_{\nu',s'}^+(p')]_+ &= 0.
\end{align*}
\]

where, $\delta^{(K)}_{p,p'}$ is the Kronecker Delta

\[ \delta^{(K)}_{p,p'} = 1 \quad \text{if} \quad p = p', \]

\[ \delta^{(K)}_{p,p'} = 0 \quad \text{if} \quad p \neq p'. \]

and $p$ are the momenta satisfying periodicity conditions in a large cubic box having a length size $L$ and volume $L^3$. That is, if $L = Na$ and $N$ is even, the components of the momenta $p = (p_1, p_2, p_3)$ are given as

\[ p_1 = 2 \pi \frac{m_1}{a} \frac{N}{2} - \frac{N}{2} \leq m_1 < \frac{N}{2}, \]

\[ p_2 = 2 \pi \frac{m_2}{a} \frac{N}{2} - \frac{N}{2} \leq m_2 < \frac{N}{2}, \]

\[ p_3 = 2 \pi \frac{m_3}{a} \frac{N}{2} - \frac{N}{2} \leq m_3 < \frac{N}{2}. \]
III. THE SPACE OF STATES IN THE QUANTUM FIELD THEORY

After constructed the second quantization of the above defined simple non relativistic two massive neutrinos system, we will consider the main issue in this work: to investigate the influence of the space of states adopted for the theory, on the possibility for the description of the interference between different distinguishable as it occurs between the measured neutrino oscillations.

It can be started by remarking that in the literature, it has been discussed the possibility that when you have distinguishable particles the correct space of states of the combined system could be the direct product of the Fock spaces which is associated to each of the distinguishable particles. In connection with this view, it should be stressed that the this assumption is equivalent to establish a superselection rule over states not admitting a physical states, the addition of states of the different species. The establishment of superselection rules in QFT is allowed for sure in some cases [23]. That is the situation with respect to the electric charge in which you can adopt to not allow the superposition of states showing different amount of electric charge. However, in such cases the exclusion of these type of superpositions is "dynamically" excluded, since the interaction operators conserve the charge of the states over which they act. Therefore, if we assume that the initial states over which the evolution operator acts has a well defined amount of electric charge, any state after acting over it with evolution operator will have the same eigenvalue of the charge operator. However, in problems where the interaction operators have no property restricting the resulting states to the same physical subspace after their action, it seems not possible to impose such superselection rules. We had the impression about that this is the situation in the case of neutrino oscillations, and this idea motivated the present work.

Then, as it was mentioned, the space of states of the simple QFT model constructed here will be examined. The aim is to determine the conditions for being able to describe the observed neutrino oscillations. Below, in the context of the model constructed in the past section, it will be argued that the appropriate space of states for the QFT description should not be the direct product of the two Fock spaces defined. In place of it, the sates should be considered in the linear completion of the direct product of the two Fock spaces for each of the two distinguishable particles. The conclusion indicates that the superposition principle for states (the addition of physical states is a resulting physical state) should not have superselection rules, in order to describe neutrino oscillations.

Consider the two Fock spaces $F_{\nu}$ , $\nu = 1, 2$ generated by the before defined creation operators for each of the two particles. The states of a complete basis in the Fock spaces of each separate particles will be indicated as $|\Phi_{f_\nu}\rangle_{F_{\nu}}$, $\nu = 1, 2$ where $f_\nu$ is an index for any of the states in the Fock space of type $\nu$. Then, the states in the direct product of the two Fock spaces can be written as

$$|\Psi\rangle_{F_1 \otimes F_2} = \sum_{f_1} \sum_{f_2} C_{f_1} C_{f_2} |\Phi_{f_1}\rangle_{F_1} \otimes |\Phi_{f_2}\rangle_{F_2}$$

$$= \left( \sum_{f_1} C_{f_1} |\Phi_{f_1}\rangle_{F_1} \right) \otimes \left( \sum_{f_2} C_{f_2} |\Phi_{f_2}\rangle_{F_2} \right). \quad (19)$$

But as mentioned before, this class of states, for $a$ and $b$ different form zero constants, excludes superpositions of the form

$$a |\Phi_{f_1}\rangle_{F_1} \otimes |\Phi_{f_2}\rangle_{F_2} + b |\Phi_{f_1}'\rangle_{F_1} \otimes |\Phi_{f_2}'\rangle_{F_2},$$

if $(f_1, f_2)$ also differs form $(f_1', f_2')$. This exclusion is related with the fact that the direct product of Fock states is what is required for to implement a superselection rule. The scalar product of two states pertaining to the direct product can be defined as

$$\langle \Psi | \Psi' \rangle_{F_1 \otimes F_2} = \sum_{f_1} C^*_{f_1} C_{f_1} \times \sum_{f_2} C^*_{f_2} C_{f_2} \quad (20)$$

and normalized states for each component can be defined as separately satisfying

$$\sum_{f_1} C^*_{f_1} C_{f_1} = 1, \quad (21)$$

$$\sum_{f_2} C^*_{f_2} C_{f_2} = 1. \quad (22)$$
However, there is also the possibility of considering a wider space, the linear completion of the formerly defined direct product space. This linear completion, that will be called as \( C(F_1 \otimes F_2) \) can be defined as the set of states generated by the arbitrary coefficients \( C_{f_1,f_2} \) in the superposition of the form

\[
|\Psi\rangle_{C(F_1 \otimes F_2)} = \sum_{f_1,f_2} C_{f_1,f_2} |\Phi_{f_1}\rangle_{F_1} \otimes |\Phi_{f_2}\rangle_{F_2}.
\]  

(23)

It is clear that such states can not be always expressed in the form of a direct product of linear spaces

\[
\left( \sum_{f_1} C_{f_1} |\Phi_{f_1}\rangle \right)_{F_1} \otimes \left( \sum_{f_2} C_{f_2} |\Phi_{f_2}\rangle \right)_{F_2}.
\]  

(24)

The scalar product assumed that the basis states in both Fock spaces are normalized is defined as

\[
\langle \Psi | \Psi' \rangle_{C(F_1 \otimes F_2)} = \sum_{f_1,f_2} C^*_{f_1,f_2} C'_{f_1,f_2}
\]  

(25)

Normalized states satisfy

\[
1 = \sum_{f_1,f_2} C^*_{f_1,f_2} C'_{f_1,f_2}.
\]  

(26)

This definition of the space of states is avoiding a superselection rule not allowing the states being expressed as arbitrary linear combinations of two states in direct product of Fock spaces. Therefore, in general, but in the absence of superselection rules, the Fock space of any QFT of a system of a number of \( n_p \) distinguishable particles (being either bosons or fermions) should be interpreted as the whole set of states generated by the arbitrary coefficients \( C_{f_1,f_2,\ldots,f_{n_p}} \) of the form

\[
|\Psi\rangle_{C(F_1 \otimes \cdots \otimes F_{n_p})} = \sum_{f_1,f_2,\ldots,f_{n_p}} C_{f_1,f_2,\ldots,f_{n_p}} |\Phi_{f_1}\rangle_{F_1} \otimes |\Phi_{f_2}\rangle_{F_2} \otimes \cdots \otimes |\Phi_{f_{n_p}}\rangle_{F_{n_p}}.
\]  

(27)

\[
\langle \Psi | \Psi' \rangle_{C(F_1 \otimes \cdots \otimes F_{n_p})} = \sum_{f_1,f_2,\ldots,f_{n_p}} C^*_{f_1,f_2,\ldots,f_{n_p}} C'_{f_1,f_2,\ldots,f_{n_p}}
\]  

(28)

\[
1 = \sum_{f_1,f_2,\ldots,f_{n_p}} C^*_{f_1,f_2,\ldots,f_{n_p}} C'_{f_1,f_2,\ldots,f_{n_p}}.
\]  

(29)

IV. NEUTRINO OSCILLATIONS AND THE SPACE OF STATES

Let us now consider the QFT defined in section II. The only aspect of the theory which remains to be defined is the space of the physical states of the theory. In this case, the many states only including one the two types of particles (let say of flavor \( \nu = 1 \) or flavor \( \nu = 2 \)) are described by the Fock space \( (F_1 \text{ or } F_2) \) generated by the creation operators of the specific kind of flavor. Therefore, lets argue below that in the space of states defined by the linear completion of the direct product of the two Fock space \( C(F_1 \otimes F_2) \) the neutrino like oscillations can be effectively described. Conversely, the oscillations can not be directly explained by assuming the direct product of the two Fock space as defining the physical space of states for the system

A. Space of states \( C(F_1 \otimes F_2) \)

As they are well defined in this space, we will examine the states of the form

\[
|\Psi\rangle_{C(F_1 \otimes F_2)} = \sum_{s=\pm} C_{1,s} b^+_1 \sum_{t_1=1}^{1+1} |0\rangle_{F_1} \otimes |0\rangle_{F_2} + |\Phi_{f_1}\rangle_{F_1} \otimes C_{f_2} b^+_2 \sum_{t_2=1}^{1+1} |0\rangle_{F_2}.
\]  

(30)
describing states in which a one particle state with negative helicity \( s = -1 \) is created in the Fock space \( \mathcal{F}_1 \) (with zero particles in the Fock space \( \mathcal{F}_2 \)) is superposed with a zero particle created in \( \mathcal{F}_1 \) with one particle with helicity \( s = -1 \) created in \( \mathcal{F}_2 \). Both particles have the same momentum \( \mathbf{p} \). As it can be noted form the second line of the equation, these states are generated by a linear combination of field operators describing two different flavor modes both with a common value of the helicity and momentum. The form of these states was selected in order to more closely represent the situation for the neutrino oscillation measurement. The assumption of the physical nature of these states, then allow to define physical quantities (Hermitian operators constructed in terms of the employed fields) in terms of these superposition of fields, which create particles in different Fock spaces as the above defined ones.

B. Flavor rotated fields

It is possible to define now flavor rotated fields, as linear functions of the original fields in terms of which it is possible to define sets of physical quantities as Hermitian operator constructs. These definitions are here discussed in order to further consider measurements, describing quantum oscillations of amplitudes. Let us define the flavor rotated electron and muon like fields \( b_{\nu_e,s}(\mathbf{p}), b_{\nu_\mu,s}(\mathbf{p}) \) (which are not the stable neutrino fields \( b_{1,s}, b_{2,s} \)) as

\[
\begin{align*}
  b_{\nu_e,s}(\mathbf{p}) & = (\cos(\theta)b_{1,s}(\mathbf{p}) + \sin(\theta)b_{2,s}(\mathbf{p})) , \\
  b_{\nu_e,s}^+(\mathbf{p}) & = (\cos(\theta)b_{1,s}^+(\mathbf{p}) + \sin(\theta)b_{2,s}^+(\mathbf{p})) , \\
  b_{\nu_\mu,s}(\mathbf{p}) & = (-\sin(\theta)b_{1,s}(\mathbf{p}) + \cos(\theta)b_{2,s}(\mathbf{p})) , \\
  b_{\nu_\mu,s}^+(\mathbf{p}) & = (-\sin(\theta)b_{1,s}^+(\mathbf{p}) + \cos(\theta)b_{2,s}^+(\mathbf{p})) .
\end{align*}
\]

These operators, as the previous ones, also satisfy the following commutation relations

\[
\begin{align*}
  [b_{\nu_e,s}(\mathbf{p}), b_{\nu_e,s'}(\mathbf{p}')_+] &= \delta_{s,s'} \delta_{\mathbf{p},\mathbf{p}'} , \\
  [b_{\nu_\mu,s}^+(\mathbf{p}), b_{\nu_\mu,s'}^+(\mathbf{p}')_+] &= 0 .
\end{align*}
\]

We will call \( b_{\nu_e,s}(\mathbf{p}) \) as the electron neutrino field of helicity \( s \) and the \( b_{\nu_\mu,s}(\mathbf{p}) \) as the muon neutrino of helicity \( s \). The inverse transformation takes the form

\[
\begin{align*}
  b_{1,s}(\mathbf{p}) & = (\cos(\theta)b_{\nu_e,s}(\mathbf{p}) - \sin(\theta)b_{\nu_\mu,s}(\mathbf{p})) , \\
  b_{1,s}^+(\mathbf{p}) & = (\cos(\theta)b_{\nu_e,s}^+(\mathbf{p}) - \sin(\theta)b_{\nu_\mu,s}^+(\mathbf{p})) , \\
  b_{2,s}(\mathbf{p}) & = (\sin(\theta)b_{\nu_e,s}(\mathbf{p}) + \cos(\theta)b_{\nu_\mu,s}(\mathbf{p})) , \\
  b_{2,s}^+(\mathbf{p}) & = (\sin(\theta)b_{\nu_e,s}^+(\mathbf{p}) + \cos(\theta)b_{\nu_\mu,s}^+(\mathbf{p})) .
\end{align*}
\]

These new operators define creation and annihilation operators of the flavor rotated state over the vacuum. By example, the creation of a single particle state with rotated flavor \( \nu_e \) or \( \nu_\mu \), momentum \( \mathbf{p} \) and helicity \( s \) are defined by

\[
\begin{align*}
  b_{\nu_e,s}^+(\mathbf{p}) |0\rangle_{C(\mathcal{F}_1 \otimes \mathcal{F}_2)} &= b_{\nu_e,s}(\mathbf{p}) |0\rangle_{\mathcal{F}_1} \otimes |0\rangle_{\mathcal{F}_2} , \\
  b_{\nu_\mu,s}^+(\mathbf{p}) |0\rangle_{C(\mathcal{F}_1 \otimes \mathcal{F}_2)} &= b_{\nu_\mu,s}(\mathbf{p}) |0\rangle_{\mathcal{F}_1} \otimes |0\rangle_{\mathcal{F}_2} .
\end{align*}
\]

Now, it is possible to define the number of rotated flavor particles as the operator

\[
\begin{align*}
  q_\theta = \sum_{\mathbf{p}} \sum_{s=\pm1} (b_{\nu_e,s}^+(\mathbf{p})b_{\nu_e,s}(\mathbf{p}) - b_{\nu_\mu,s}^+(\mathbf{p})b_{\nu_\mu,s}(\mathbf{p})) ,
\end{align*}
\]

which has eigenvectors and eigenvalues

\[
\begin{align*}
  q_\theta b_{\nu_e,s}(\mathbf{p}) |0\rangle_{C(\mathcal{F}_1 \otimes \mathcal{F}_2)} &= b_{\nu_e,s}(\mathbf{p}) |0\rangle_{C(\mathcal{F}_1 \otimes \mathcal{F}_2)} , \\
  q_\theta b_{\nu_\mu,s}(\mathbf{p}) |0\rangle_{C(\mathcal{F}_1 \otimes \mathcal{F}_2)} &= -b_{\nu_\mu,s}(\mathbf{p}) |0\rangle_{C(\mathcal{F}_1 \otimes \mathcal{F}_2)} .
\end{align*}
\]

Since, the mentioned states are eigenfunctions of a physical observable (the Hermitian operator \( q_\theta \)), the result of the measurement of the rotated flavor eigenvalue should lead to the contraction of the wave-packet to one of the eigenstates of \( q_\theta \). Therefore, the probability of the measurement will be the square of the amplitude defined by the scalar product of those eigenstates and the eigenstate of the physical quantity being measured.

It can be remarked that a similar transformation can be also implemented for the antiparticle annihilation and creation operators \( d_{\nu_e,s}(\mathbf{p}) \) and \( d_{\nu_\mu,s}(\mathbf{p}) \).
Let assume that an electron neutrino with helicity $s = -1$ had been created over the vacuum at time equal to zero at the state
\[
|\phi_{\nu_e,-1}(0)\rangle = b^+_{\nu_e,-1}(p) |0\rangle_{C(F_1 \otimes F_2)}
= (\cos(\theta)b^+_{1,s}(p) + \sin(\theta)b^+_{2,s}(p)) |0\rangle_{C(F_1 \otimes F_2)}.
\]
(44)

Now, consider the evolution of the same electron neutrino density after a time $t$. Then, acting with the evolution operator over the created state at zero time, gives for the state at time $t$, the evolved state
\[
|\phi_{\nu_e,-1}(t, p)\rangle = U(t) |\phi_{\nu_e,-1}(0, p)\rangle
= \exp(-i H t) |\phi_{\nu_e,-1}(0, p)\rangle
= (\exp(-i\epsilon_1(p)t) \cos(\theta)a^+_{1,-1}(p) + \exp(-i\epsilon_2(p)t) \sin(\theta)a^+_{2,-1}(p)) |0\rangle_{C(F_1 \otimes F_2)}.
\]
(45)

where $\epsilon_1$ and $\epsilon_2$ are the energies of the mass eigenvalue neutrinos.

We can examine the projection amplitude of the above evolved state over the electron neutrino state. Then, it is needed to evaluate the scalar product
\[
C(F_1 \otimes F_2) \langle 0|b_{\nu_e,-1}(p') |\phi_{\nu_e,-1}(t, p)\rangle = C(F_1 \otimes F_2) \langle 0| (\cos(\theta)a_{1,-1}(p) + \sin(\theta)a_{2,-1}(p)) \times 
(\exp(-i\epsilon_1(p)t) \cos(\theta)a_{1,s}(p) + \exp(-i\epsilon_2(p)t) \sin(\theta)a_{2,s}(p)) |0\rangle_{C(F_1 \otimes F_2)}
= \cos(\theta)^2 \exp(-i\epsilon_1(p)t) + \sin(\theta)^2 \exp(-i\epsilon_2(p)t).
\]
(46)

Therefore, the probability for the detection of the electron neutrino mode at any time instant after its creation at zero time, becomes
\[
P_{\nu_e \rightarrow \nu_e}(t) = \left| C(F_1 \otimes F_2) \langle 0|b_{\nu_e,-1}(p') |\phi_{\nu_e,-1}(t, p)\rangle \right|^2
= \cos(\theta)^2 \exp(-i\epsilon_1(p)t) + \sin(\theta)^2 \exp(-i\epsilon_2(p)t)
= (\cos(\theta)^2)^2 + (\sin(\theta)^2)^2 + 
\cos(\theta)^2 \sin(\theta)^2 (\exp(-i\epsilon_1(p)t + i\epsilon_2(p)t) + 
\exp(i\epsilon_1(p)t - i\epsilon_2(p)t))
= (\cos(\theta)^2)^2 + (\sin(\theta)^2)^2 + 
2 \cos(\theta)^2 \sin(\theta)^2 \cos((\epsilon_1(p) - \epsilon_2(p)t)t)
= 1 - 2 \cos(\theta)^2 \sin(\theta)^2 (1 - \cos((\epsilon_1(p) - \epsilon_2(p)t))
= 1 - \frac{\sin(2\theta)^2}{2} (1 - \cos((\epsilon_1(p) - \epsilon_2(p)t)).
\]
(47)

Let us consider now the relativistic approximation
\[
|p| \gg m_1, m_2,
\]
(48)

which allows to derive the following relation
\[
\epsilon_2(p) - \epsilon_1(p) = \sqrt{m_2^2 + p^2} - \sqrt{m_1^2 + p^2}
= |p|(1 + \frac{m_2^2}{p^2} - \sqrt{1 + \frac{m_1^2}{p^2}})
= \frac{1}{2|p|}(m_2^2 - m_1^2) + ...
\]
(49)

Then, when the particles are ultra-relativistic, the propagation time for traveling along a distance $R$ is given as
\[
t = \frac{R}{c} = R,
\]
(50)
thanks to the natural units $c = 1$ being used. Henceforth, the probability formula for the transition between an electron neutrino state into another electron neutrino state as a function of the measurement distance $R$ gets the form

$$ P_{\nu_e \rightarrow \nu_e}(t) = \left| C(F_1 \otimes F_2) \langle 0 | b_{\nu_e,-1}(p') | \phi_{\nu_e,-1}(t, p) \rangle \right|^2 $$

$$ = 1 - \frac{\sin(2\theta)^2}{2} \left( 1 - \cos \left( \frac{1}{2} \frac{1}{|p|} (m_2^2 - m_1^2) R \right) \right) $$

$$ = 1 - \frac{\sin(2\theta)^2}{2} \left( 1 - \cos \left( -2\pi \frac{R}{L} \right) \right) $$

$$ L = \frac{4\pi|p|}{m_2^2 - m_1^2}. \quad (51) $$

which reproduces the usual formula for the neutrino oscillations in terms of the oscillation and observation distances $L$ and $R$, the momentum $|p|$ and the neutrino masses $m_1, m_2$.

In a similar way it can be evaluated the probability of measuring a muon neutrino in the same state resulting form creating an electron neutrino at zero time. The result is

$$ P_{\nu_e \rightarrow \nu_\mu}(t) = \left| C(F_1 \otimes F_2) \langle 0 | b_{\nu_\mu,-1}(p') | \phi_{\nu_\mu,-1}(t, p) \rangle \right|^2 $$

$$ = \frac{\sin(2\theta)^2}{2} \left( 1 - \cos \left( -2\pi \frac{R}{L} \right) \right) $$

$$ = 1 - P_{\nu_e \rightarrow \nu_\mu}(t). \quad (52) $$

Therefore, the discussion presented indicates that QFT or second quantization is able explain the interference between distinguishable particles which neutrino oscillation experiments show to exist. For this to happens the Hilbert space should be considered as the linear completion of the direct product of all the Fock spaces associated to each of the indistinguishable particles included in the physical system. But, the use of this space of states is naturally suggested if we do not assume the presence of any superselection rule to restrict the superposition principle of the theory.

V. SUMMARY

The question about why the neutrino oscillations effects curiously seems to indicate the possibility of interference between distinguishable particles is investigated. The compatibility of this effect with the quantum theory defined by a QFT like the SM, is shown after a proper interpretation of the space of states in the considered quantum field theory. The analysis was suggested by the measured neutrino oscillations. A model involving only two distinguishable types (flavors) of particles is formulated, in order to consider the study within a simple framework. It consists of two types of spin one-half relativistic particles satisfying the Dirac equation.

The formulation of the space of the states of the system is examined. It is argued that in order to describe interference between two fermion flavors in analogy with the neutrino oscillation measurements, the space of states should not correspond to the direct product of the two Fock spaces associated to the two fermion particles. It is underlined that to employ the direct product as the space of states leads to accept the unphysical character of the addition of two field operators, each one creating a particle in the two separate Fock spaces. The reason for this is that the states generated by the action of such additions do not pertain to the direct product of the two Fock spaces. To exclude these states from the space of physical states becomes equivalent to establish the validity of a superselection rule.

Therefore, we passed to consider as the space of the states the linear completion of the direct product of the two Fock states. That is, the linear combination with arbitrary coefficients of the external products of the states in each of the two Fock spaces. It is then verified that this space allows to define the addition of field operators acting in each separate Fock space, as generating new physical states. Further, rotated flavor creation and annihilation operators are defined by linear combinations of the creation and annihilation operators for propagating modes solving the Dirac equation. This procedure allows to argue that states generated by these rotated flavor fields at a given time, are physical linear combinations of the propagating particles modes. In addition, it follows that under a measurement in such a propagating states (defining after it a contracted flavor rotated state) the probability of the measurements oscillates. The evaluated expression for the transition probabilities exactly coincide with ones describing the observed neutrino oscillations.
The analysis clarifies that the usual representation of QFT allows to consistently predict quantum interference between two distinguishable particles, explaining in this way how the quantum theory which is a QFT, can explain the interference between different particles, at variance with what is sometimes been considered in QM. It also suggests the validity of what we consider a relevant property of QFT generalization of the Quantum Mechanics: the physical possibility of observing interference between many kinds of distinguishable particles. This indication comes from the fact that the wider superposition principle in QFT (with respect to QM) allows to add states having components in different Fock spaces (distinguishable particles) and then, interference effects should be expected to be observable in seemingly many processes. These issues will be further explored elsewhere.

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