Special Relativity and possible Lorentz violations consistently coexist in Aristotle space-time

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Abstract: Some studies interpret quantum measurement as being explicitly non-local. Others assume the preferred frame hypothesis. Unfortunately, these two classes of studies conflict with Minkowski space-time geometry. On the contrary, in Aristotle space-time, Lorentz invariance, interpreted as a physical property applying to all phenomena actually satisfying this symmetry (as opposed to a geometrical constraint applying to an assumed pre-existing Minkowski space-time) consistently coexists with possible Lorentz violations. Moreover, as will be pointed out, the geometrical framework provided by Aristotle space-time is in fact necessary to derive the Lorentz transformations from physical hypotheses.

Keywords: Special Relativity, preferred frame, Aristotle space-time, quantum measurement.

1 Quantum Non-locality and quantum preferred frame

Percival proved realistic\(^1\) interpretations of quantum collapse to violate Lorentz invariance in Bell-type experiments [4]. Henceforth, as suggested by Bell, non-locally correlated quantum events can be interpreted as faster-than-light interactions [5, 3] complying with the causality principle provided it rests on the absolute chronological order associated with a quantum preferred frame [6]. Similarly, EPR experiments performed in Geneva by the Group of Applied Physics [7, 8] have been analyzed according to the non Lorentz invariant\(^2\) preferred frame hypothesis.

\(^1\)Realistic interpretations [1] assume quantum collapse to be an objective (i.e. observer independent) physical process. Contrary to the Everett Many Worlds Interpretation, severely criticized by Neumaier [2] and Bell [3], they conflict with Lorentz invariance.

\(^2\)Hence incompatible with Minkowski space-time.
2 Other reasons suggesting a preferred frame

Selleri argues that some superluminal effects strongly suggest the need for a preferred frame and its associated preferred chronology [9]. Moreover, the scalar theory of gravitation of Arminjon [10, 11], investigates a preferred frame gravitation approach as a possible way to make quantum and gravitation theories fit together. The preferred frame, formalized as a field of time-like unit vectors, is also used in the context of preferred frame theories of gravity by authors such as Will and Nordtvedt [12] Eling and Jacobson [13]. Moreover, some attention was devoted to the preferred frame hypothesis by Kostelecky as a consequence of possible Lorentz violations in High Energy Physics [14, 15].

Now, Aristotle space-time will provide us with a geometrical framework authorizing the peaceful coexistence of the preferred frame hypothesis with the ubiquitous Lorentz invariance. Let us now define Aristotle space-time’s symmetry group.

3 The Poincaré, Galilei and Aristotle groups

Thanks to Noether’s theorem, energy and linear momentum, as well as angular momentum conservation laws, arise from the invariance of the Lagrangian of dynamical systems respectively with regard to the group of space-time translations and the group SO(3) of spatial rotations. The semi-direct product group, arising from these two groups, is the so-called restricted Aristotle group [16]. It will be denoted SA(4).

This seven parameters group is also the direct product group of the Special Euclidean group SE(1) (i.e. the temporal translations of the 1D affine Euclidean space $E^1$) and the Special Euclidean group SE(3) (i.e. the direct spatial isometries of the 3D affine Euclidean space $E^3$). So, SA(4) is the intersection of the restricted Galilei and Poincaré groups. Hence, neither does it contain Galilean boosts, nor Lorentzian ones. Its relevance is the following:

• the invariance requirement of physical laws with regard to Galilean boosts conflicts with interactions propagating at a finite speed independent of the speed of their source, hence in particular with electromagnetism

• the invariance requirement of physical laws with regard to Lorentzian boosts conflicts with interactions propagating at infinite speed.

On the contrary, Aristotle group of symmetry complies with interactions propagating at the speed of light as well as possible faster-than-light interactions$^3$.

$^3$Or possible instantaneous actions at a distance caused by quantum measurements [17, 18, 6].
4 Aristotle space-time

4.1 Definition and foliation of Aristotle space-time

Aristotle space-time, arising from the Aristotle group, embodies only the Principle of Relativity with regard to space-time localization and spatial orientation. It is defined as a set denoted $A^4$ equipped with a bijection $f$ from $\mathbb{R}^4$ to $A^4$ providing it with an action $\Phi$ of the numerical restricted Aristotle group $SA(4)^4$ defined as:

$$\Phi : \left\{ \begin{array}{c}
SA(4) \times A^4 \rightarrow A^4 \\
(a, Z) \mapsto \Phi_a(Z) = f \circ a \circ f^{-1}(Z)
\end{array} \right.$$  \hspace{1cm} (1)

From now on, we will identify $SA(4)$ with its representation acting on $A^4$. So, for the sake of simplicity, $\Phi_a$, the image of $a$ by action $\Phi$, will be identified with $a$ and $\Phi_a(Z)$ will be referred to as an action $a$ of group $SA(4)$ on $Z$.

Aristotle space-time is endowed with 2 preferred foliations

- a 1D foliation of which the 1D leaves of absolute rest are the orbits of the time translation group, the invariant subgroup $SE(1)$ of $SA(4)$.
- a 3D foliation of which the 3D leaves of universal simultaneity are the orbits of the direct isometries group, i.e. the invariant subgroup $SE(3)$ of $SA(4)$.

As each one of these two foliations is a complete set of orbits of an Aristotle invariant subgroup, this foliated structure is preserved under Aristotle group actions. This foliation may be interpreted as the preferred inertial frame of Bell's realistic interpretation of quantum collapse and may also be helpful to account for possible Lorentz violations [14, 15].

4.2 Spatial and temporal metrics of Aristotle space-time

The quotient manifold of $A^4$ by its 1D foliation is diffeomorphic\(^5\) with the 3D leaves of universal simultaneity. This 3D manifold can be equipped with an action of $SE(3)$ (the invariant subgroup of spatial isometries). This provides

\(^4\)i.e. considered as a subgroup of $GL_4(\mathbb{R})$.

\(^5\)The chosen manifold structure of $A^4$ is that induced by $f$, i.e. diffeomorphisms of $A^4$ are bijections $F$ from $A^4$ to $A^4$ such that $f^{-1} \circ F \circ f$ are diffeomorphisms of $\mathbb{R}^4$. 

Figure 1: Aristotle space-time $A^4$
it with the metric structure of a 3D affine Euclidean space $E^3$. Similarly, the quotient manifold of $A^4$ by its 3D foliation is diffeomorphic with the 1D leaves of absolute rest and can be provided with the metric structure of a 1D affine Euclidean space $E^1$. Hence, Aristotle space-time can be identified as the Cartesian product $A^4 = E^1 \times E^3$. It is naturally equipped with two Euclidean metrics which are invariant with regard to the Aristotle group actions:

- a rank 1 temporal metric, which will be denoted $dT^2$
- a rank 3 spatial metric, which will be denoted $dL^2$.

4.3 Causal structure of Aristotle space-time

The principle of relativity of motion, embodied in Minkowski space-time geometry, forbids granting a privileged status to a preferred inertial frame. Now, the chronological order between space-like separated events depends on the rest inertial frame of the observer. This prevents Minkowski space-time complying with the existence of causal links spanning out of the light cone. On the contrary, Aristotle space-time foliation into 3D leaves of universal simultaneity enables us to define an objective chronology\(^6\) between any pair of events. This gives rise to a causal structure where possible faster-than-light interactions, comply with the principle of causality prevailing in this space-time.

5 Aristotle charts and Aristotle bases

Aristotle space-time is associated with a family of preferred coordinate systems, called Aristotle charts, preserving its foliated geometry and its metrics.

5.1 Aristotle charts

In Aristotle space-time $A^4 = E^1 \times E^3$, any event $Z$ reads: $Z = (T, R)$

- $T \in E^1$ denotes the moment when event $Z$ occurs.
- $R \in E^3$ denotes the localization where event $Z$ occurs.

Events $Z$ are localized in so-called Aristotle charts denoted $A$ such that $Z = A(z)$, where $z = (t, \underline{r}) = (t, x, y, z) \in \mathbb{R}^4$ are the so-called coordinates of $Z$ in Aristotle chart $A$. By definition, Aristotle charts are such that:

- they preserve the foliation of Aristotle space-time into 1D lines of absolute rest and 3D leaves of universal simultaneity. In particular, two events belonging to a same simultaneity leave (i.e. occurring at the same time $T$) have the same chronological coordinate $t$, i.e. $\exists T: \mathbb{R} \to E^1$ and $\exists R: \mathbb{R}^3 \to E^3$ such that $A(t, \underline{r}) = (T(t), R(\underline{r}))$

\(^6\)That is to say independent of the motion of inertial observers.
• the temporal metric be normalized, i.e. $dT^2 = dt^2$
• the spatial metric be Orthonormalized, i.e. $dL^2 = dx^2 + dy^2 + dz^2$.

Besides, $O = E^3 \times \{ R(0) \}$ will denote the motionless observer resting at the spatial origin of Aristotle chart $A$ and $\{ T(0) \} \times E^3$ is the 3D leaf of universal simultaneity passing through origin event $E = A(0)$ of chart $A$.

5.2 Aristotle bases

Any Aristotle chart $A$ is associated with a space-time basis $V = (\vec{t}, \vec{x}, \vec{y}, \vec{z})^7$ of the vector space $V^1 \oplus V^3$. Indeed, let us define

- $E = A(0)$ the so-called origin event of chart $A$
- Events $E_t = A(1, 0); E_x = A(0, 1, 0, 0); E_y = A(0, 0, 1, 0); E_z = A(0, 0, 0, 1)$
- Unit vectors $\vec{t} = EE_t; \vec{x} = EE_x; \vec{y} = EE_y; \vec{z} = EE_z$

$\vec{t}$ is a normalized vector of $V^1$ and $B = (\vec{x}, \vec{y}, \vec{z})$ an Orthonormalized basis of $V^3$.

5.3 Change of Aristotle charts

Let $A$ be an Aristotle chart. Let $\Phi$ denote the action (1) of the restricted Aristotle group $SA(4)$ on $A^4$. Any action $\Phi_a = \Phi(a)$ (denoted $a$ for the sake of simplicity), of $a \in SA(4)$ on $A^4$ entails an Aristotle chart change

$$(a, A) \rightarrow A_a = a \circ A = A \circ \varphi_a$$

This definition ensures coordinates’ covariance, i.e. the same system will be located by the same coordinates whenever observer and observed system both undergo the same chart change $A \rightarrow A_a$. Coordinates $z$ of the «new event» $Z = a(Z_0)$ in the «new chart» $A_a$ are the same as coordinates of the «old event» $Z_0$ in the «old chart» $A$. $\varphi_a$ is the numerical expression of action $a$ in chart $A$.

![Figure 2: Change of Aristotle chart](image)

\[ ^7 \text{From a differential geometry point of view, } V = dA \]
\[ ^8 V^1 \oplus V^3 \text{ is the tangent space to } A^4 = E^3 \times E^3 \]
6 Boosts and inertial charts in Aristotle space-time

So as to express Lorentz invariance of the phenomena that actually satisfy this symmetry, we have to define Lorentzian boosts, inertial charts and then to derive Lorentz transformations in Aristotle space-time framework.

6.1 Definition of boosts and pure boosts

6.1.1 Physical requirements applying to any boost

We define boosts as diffeomorphisms of $A^4$ having the following physical properties:

1/ When applied to motionless observers, boosts set them in motion with the same velocity $\vec{v}$ called velocity of the boost.

2/ The modification of durations and distances caused by the application of a boost in the vicinity of a boosted event «is the same» whatever this event.

3/ Freely moving observers keep on freely moving after the action of a boost.

4/ The covariance of boosts’ observed effect is satisfied under any change of Aristotle chart. Loosely speaking, a boost has the same effect whatever the Aristotle chart where it is applied. Actually, Aristotle covariance of boosts will be assumed to hold when, more generally, $a$ is any action of the complete Aristotle group $A(4)$.

5/ Symmetry of point of view between motionless and moving observers: loosely speaking, we cancel the effect of a boost of velocity $\vec{v}$, applied to Aristotle space-time $A^4$, by applying a boost of velocity $-\vec{v}$.12.

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9 They will be translated mathematically in sub-section 6.3

10 A freely moving observer is a line $D$ with a direction $D_0 = \{t\vec{v} + t\vec{v}/t \in \mathbb{R}\}$, where $||\vec{v}|| < c$ (the speed of light). $\vec{v} \in V^3$ is called the velocity of this observer so that the rest lines of Aristotle space-time are observers freely moving with a zero velocity.

11 This requirement expresses the homogeneity, the stationarity and the isotropy of Aristotle space-time physical properties with regard to boosts.

12 So, physical observers at rest in a boosted Aristotle chart (by a boost of velocity $\vec{v}$),
6/ The maximal propagation speed measured by a motionless observer is the same as that measured by an observer at rest in a boosted Aristotle chart.

6.1.2 Requirements applying specifically to pure boosts

So as to define the so-called pure boosts, i.e. boosts that are not combined with Aristotle group actions, we ask for the following additional properties:

7/ Any pure boost is endowed with at least one so-called origin event $E$, invariant under the pure boost action. Thus, a pure boost, combined with a spatial translation perpendicular to its velocity, is not anymore a pure boost.\(^\text{13}\)

8/ Any pure boost is completely determined given its velocity and an origin event. Together with property 4/ (the Aristotle covariance of boosts) this makes it possible to eliminate pure boosts combined with rotations.

9/ If $\{B_{E,\lambda\vec{v}}/\lambda \in \mathbb{R}\}$ is a family of pure boosts having a same origin event $E$ and velocities proportional to $\vec{v}$, $B_{E,\lambda\vec{v}} \to i_A^4$ when $\lambda \to 0$. Together with the other properties, in particular the symmetry of point of view, this will enable us to eliminate pure boosts combined with $P$ or $T$ symmetries.

6.2 Definition of inertial charts

With any pure boost $B_{\vec{v}}$ of velocity $\vec{v}$, of origin event $E$, and with any Aristotle chart $A$ of same origin event $E$, we associate a so-called inertial chart $A_{\vec{v}}$ moving with the velocity $\vec{v}$. The chart $A_{\vec{v}}$ is defined so as to ensure coordinates’ covariance with regard to boosts, i.e. if the «old» event $Z_0$ is localized by coordinates $\bar{z} = (t, r)$ in the «old» chart $A$, then the «new» event $Z = B_{\vec{v}}(Z_0)$ is also localized by these same coordinates in the «new» chart $A_{\vec{v}}$. So:

\[
\begin{align*}
\mathbb{R}^4 & \quad \mathbb{R}^4 \\
Z_0 = (t_0, r_0) & \Rightarrow \quad Z = (t, r) = B_{\vec{v}}(Z_0)
\end{align*}
\]

Figure 4: Definition of an inertial chart $A_{\vec{v}}$

\[
Z_0 = A(\bar{z}) \Rightarrow B_{\vec{v}}(Z_0) = Z = A_{\vec{v}}(\bar{z}). \quad \text{So that} \quad A_{\vec{v}} = B_{\vec{v}} \circ A
\]

Moreover $A_{\vec{v}}(0) = B_{\vec{v}}(A(0)) = B_{\vec{v}}(E) = E$, so that $A_{\vec{v}}$ has same space-time origin $E$ as $A$.

observing phenomena occurring in an Aristotle chart $A$, will observe the same effects as motionless observers (hence at rest in $A$) observing the same phenomena occurring in an Aristotle chart boosted with the velocity $-\vec{v}$. This assumption expresses the impossibility facing a steadily translating observer if he tries to detect his absolute motion when using measurements and phenomena that are Lorentz-covariant.

\(^{13}\)Under a space-time translation, of which the translation vector is included in the $(\vec{t}, \vec{v})$ plane, the origin event $E$ of a pure boost shifts but the new boost is still a pure boost.
The expression $b_v$ of boost $B_v$ in Aristotle chart $\mathcal{A}$ will be defined as follows:

$z_0 = (t_0, r_0) = b_v(z)$ are the coordinates of event $Z = B_v(Z_0)$ in the «old chart» $\mathcal{A}$ whereas $z$ are its coordinates in the «new chart» $\mathcal{A}_v$. The covariance of coordinates $z$ with regard to boost $B_v$ means that the passive transformation $\mathcal{A} \rightarrow \mathcal{A}_v$ (causing the change of coordinates $z_0 \rightarrow z$) caused by boost $B_v$ when applied to the observer only (i.e. to the observation frame $\mathcal{A}$ and not to the observed system) cancels the active transformation $Z_0 \rightarrow Z = B_v(Z_0)$ (causing the change of coordinates $z \rightarrow z_0$), i.e. the action of this same boost $B_v$ when applied to the observed system only. Besides, we notice that, thanks to the choice of a chart $\mathcal{A}$ that has same origin event $E$ as boost $B_{E'}$

$\frac{d}{dt} = \frac{d}{dt} \circ B_{E'} \circ A(t) = A^{-1} \circ B_{E'}(E) = A^{-1}(E) = 0$

6.3 Mathematical properties of pure boosts

Let us now translate mathematically the physical requirements of sub-section 6.1.1

1/ Motionless observers are set in motion with the velocity $\vec{v}$ of the boost.
If $\mathcal{M} = E^1 \times \{M\}$ (where $M \in E^5$) is a motionless observer and $B_v$ is a boost of velocity $\vec{v}$, then $B_v(\mathcal{M})$ is a line $\mathcal{D}$ of direction $D_v = \{tt + tv/t \in \mathbb{R}\}$

2/ The effect of boost $B$ in the vicinity of event $B(Z)$ does not depend on event $Z$. That is to say, $\forall Z_1, Z_1', Z_2, Z_2'$ such that $Z_1 Z_1' = Z_2 Z_2'$ and for any boost $B$:

$B(Z_1)B(Z_1') = B(Z_2)B(Z_2')$
∀ Z ∈ A⁴: B(T(Z)) = T′(B(Z)), i.e. B ∘ T = T′ ∘ B and it is easy to establish that: dB is constant (i.e. B is affine) and B ∘ T⁻¹ ∘ B⁻¹ = T⁻¹ dB(ΔZ)

It is worth noticing that the above equation may also be written:

\[ B_{\text{passive}} \circ B_{\text{active}}(T_{\Delta Z}) = T_{\Delta Z} \quad (4) \]

- the active transformation, \( B_{\text{active}}(T_{\Delta Z}) = T_{dB(\Delta Z)} \), of a space-time translation \( T_{\Delta Z} \) is a change of the observed space-time translation effect (applied to a given system) when the space-time translation vector \( \Delta Z \) as well as the observed system, both undergo the same boost B.

- The passive transformation, \( B_{\text{passive}}(T′) = B⁻¹ \circ T′ \circ B \), of a space-time translation \( T′ \) is a change of the observed translation effect when only the observer (i.e. the observation chart) undergoes boost B.

So, requirement 2/ amounts to the covariance of space-time translations observed effects under any boost B (i.e the invariance of these observed effects when the applied space-time translation, the observed system as well as the observer all undergo the same boost B). This proves requirement 2/ to express the principle of relativity of motion with regard to translation observed effects. As seen above, this causes boosts to be affine transformations.

Moreover, the expression \( b \) of a pure boost \( B \), in a chart \( \mathcal{A} \) that has the same origin event \( E \) as boost \( B \), satisfies \( b(0) = 0 \). Hence, \( b \) is linear.

3/ Any freely moving observer keeps freely moving when boosted: as boosts are affine transformations, they transform affine lines into affine lines of Aristotle space-time so that physical requirement 3/ of sub-section 6.1.1 is satisfied. Actually, there is an equivalence between the requirement 2/ and the requirement 3/ that lines of \( A^4 \) be transformed into lines of \( A^4 \) (we may have preferred to derive 2/ from 3/ instead of the other way around).

4/ Covariance of boosts observed effects under any change of Aristotle chart:

- Let us define the active transformation of a pure boost \( B = B_{E \vec{v}} \) of velocity \( \vec{v} \) and origin event \( E \) under an action \( a \in SA(4) \) as a pure boost of origin event \( a(E) \) and velocity \( da(\vec{v}) \), i.e.

\[ a_{\text{active}}(B_{E \vec{v}}) = B_{a(E)da(\vec{v})} \quad (5) \]

Physically, this transformation represents an action \( a \) both on the applied boost and on the observed system\(^{15}\).

- Let us now define a passive transformation of any boost \( B′ \) as:

\[ a_{\text{passive}}(B′) = a⁻¹ \circ B′ \circ a \quad (6) \]

Physically, this transformation represents an action \( a \) on the observer only, i.e. a change \( \mathcal{A} \rightarrow \mathcal{A}_a \) of Aristotle chart of observation.

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\(^{14}\)As concluded in sub-section 6.2.

\(^{15}\)But not on the observer.
The assumed covariance of observed boost effects when observer, observed system and applied boost all undergo the same Aristotle chart change $a$ reads\textsuperscript{16}:

$$a_{\text{passive}} \circ a_{\text{active}}(B) = B$$  

(7)

i.e. $a^{-1} \circ a_{\text{active}}(B) \circ a = B$, so that

$$a \circ B_{E\vec{v}} \circ a^{-1} = B_{a(E)dal(\vec{v})}$$  

(8)

Now, hypothesis 4/ of sub-section 6.1.1 requires that the above condition must hold for any action $a$ of the complete Aristotle group $A(4)$.

7 Derivation of Lorentz transformations

A rigorous derivation of Lorentz transformations from physical hypotheses (cf sub-section 6.1.1) needs using Aristotle space-time and its symmetries (cf sub-section 7.1, 7.2) with regard to boosts effects.

Let us consider a boost, denoted $B_{\vec{v}}$, of velocity $\vec{v}$ and origin event $E$.

Let us consider an Aristotle chart $A$ of origin event $A(0) = E$ and spatial origin $O$, having its vector $\vec{x}$ in the same direction as velocity $\vec{v}$ (i.e. $\vec{v} = v\vec{x}$).

7.1 Covariance of a boost under a $180^\circ$ rotation around $O\vec{x}$

As a $180^\circ$ rotation $R_{\pi\vec{x}}$ around $O\vec{x}$ neither changes $E$ nor changes $\vec{v}$, $dR_{\pi\vec{x}}(\vec{v}) = \vec{v}$ and

$$R_{\pi\vec{x}} \circ B_{\vec{v}} \circ R_{\pi\vec{x}} = B_{\vec{v}}$$  

(9)

So that, in $\mathbb{R}^4$: $r_{\pi\vec{x}} \cdot b_{\vec{u}} \cdot r_{\pi\vec{x}} = b_{\vec{u}}$. Now, in chart $A$, matrix $r_{\pi\vec{x}}$ of $R_{\pi\vec{x}}$ reads:

$$r_{\pi\vec{x}} = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & -1 \end{bmatrix}$$  

(10)

The right multiplication of matrix $b_{\vec{u}}$ (of boost $B_{\vec{v}}$) by matrix $r_{\pi\vec{x}}$ reverses the signs of columns $y$ and $z$ of $b_{\vec{u}}$. The left multiplication of matrix $b_{\vec{u}}$ by matrix $r_{\pi\vec{x}}$ reverses the signs of lines $y$ and $z$ of $b_{\vec{u}}$. Consequently, any off-diagonal $y$ and $z$ term of matrix $b_{\vec{u}}$ vanishes except the $yz$ terms.

\textsuperscript{16}To exemplify the physical meaning of Lorenzian boosts’ covariance property (with regard to any action $a$ of the Aristotle group), let us consider the special case of the covariance with regard to spatial rotations. So, let us consider, for instance, a strain tensor field induced in an isotropic 3D medium submitted to an homogeneous (but anisotropic) stress tensor field. If we rotate both the observer and the applied stress tensor field, then, the passive transformation (the rotation of the observer) cancels the active transformation (the strain field modification induced by the rotation of the applied stress tensor field). Because of this 3D medium isotropy, the rotated observer will observe the same effect as if neither himself, nor the stress tensor field had been rotated. It wouldn’t be the case if this medium were anisotropic. Similarly, we demand the invariance of space-time deformations under any Lorentzian boost, when the boost undergoes the combination of an active and a passive action of any Aristotle group action $a$. This amounts to require space-time behaving as an homogeneous, isotropic and stationary medium.
7.2 Covariance under a $90^\circ$ rotation around $O\vec{x}$ axis and under the $\Pi_{\vec{y}} = P_{\pi\vec{y}}$ plane symmetry

Similarly, we get $byy = bzz$ and $byz = bzy = 0$, so that we have:

$$\exists a, a', b', b'' \text{ and } e \in \mathbb{R} \text{ such that:}$$

$$\begin{align*}
    t_0 &= at + a'x \\
x_0 &= b't + b''x \\
y_0 &= ey \\
z_0 &= ez
\end{align*}$$

(11)

7.3 Symmetry between motionless and moving observers

- Applying boost $B_{-\vec{v}}$ erases boost $B_{\vec{v}}$, i.e. $B_{-\vec{v}} = B_{\vec{v}}^{-1}$,

- The maximum propagation speed is covariant with regard to any Aristotle group action, hence it is isotropic. Moreover, as far as Lorentz invariance is satisfied, it has the same norm $c$ in $A$ as in $A\vec{v}$.

For convenience, let us introduce speed $c$ in the previously stated equations:

$$\exists a, a', b, b' \text{ and } e \in \mathbb{R} \text{ such that:}$$

$$\begin{align*}
    ct_0 &= ac + bx \\
x_0 &= b'(ct) + a'x \\
y_0 &= ey \\
z_0 &= ez
\end{align*}$$

(12)

As $b_{\vec{v}}^{-1} = b_{-\vec{v}}$ and $b_{-\vec{v}} = \pi_{\vec{x}} \cdot b_{\vec{v}} \cdot \pi_{\vec{x}}$ (where $\pi_{\vec{x}}$ denotes the sign reversal of $\vec{x}$):

$$e^{-1} = e \text{ and }$$

$$\left[1/(aa' - bb')\right] \left\{ \begin{array}{cc}
    a' & -b \\
    -b' & a
  \end{array} \right\} = \left\{ \begin{array}{cc}
    a & -b \\
    -b' & a'
  \end{array} \right\}$$

(13)

Consequently $aa' - bb' = 1$, $a = a'$ and $e^2 = 1$ so that $e = \pm 1$. Actually $e = 1$. Indeed, according to requirement 9/ of sub-section 6.1.2, pure boost $b_{\vec{v}}$ is assumed to tend to the identity of $\mathbb{R}^4$ when $\vec{v}$ tends to $\vec{0}$. Now, as the origin of $A_{\vec{v}}$ (located at $x = y = z = 0$) moves with the velocity $\vec{v} = v\vec{x}$ we have $x_0 = vt_0$. As $x = y = z = 0$ we have: $ct_0 = a(ct)$ and $x_0 = b'(ct)$. Hence $a(vt_0) = ax_0 = ab'(ct) = b'(ct_0)$, so that

$$b' = av/c$$

(14)

Now, let us express the covariance of the relative speed $c$ of light:

$$x_0 = ct_0 \Rightarrow x = ct \text{ so that } x_0 = b'(ct) + a'x = ct_0 = a(ct) + bx \Rightarrow$$

$$b' + a' = a + b$$

(15)
As \( a = a' \) we get \( b = b' \). Hence \( aa' - bb' = 1 \) becomes \( a^2 - b'^2 = 1 \).

As \( b' = av/c \), this provides \( a^2 - (av/c)^2 = 1 \) so that \( a = \pm 1/(1 - v^2/c^2)^{1/2} \).

Now, we have excluded time reversal. Indeed, \( b_2 \) is assumed to tend to the identity of \( R^4 \) when \( \nu \) tends to 0 so that

\[
a = 1/\sqrt{1 - v^2/c^2}
\]

(16)

Finally, we get the Lorentz transformations:

\[
\begin{cases}
ct_0 = (ct + vx/c)/(1 - v^2/c^2)^{1/2} \\
x_0 = (vt + x)/(1 - v^2/c^2)^{1/2} \\
y_0 = y \\
z_0 = z
\end{cases}
\]

(17)

8 Conclusion

The present article exhibits Aristotle spacetime foliated structure, its causal structure and the peaceful coexistence, in this arena, of the phenomena actually satisfying Lorentz invariance with possible Lorentz violations. It provides a geometrical framework where realistic, hence explicitly non-local interpretations of quantum collapse, comply with the principle of causality and suggests the possibility of interpreting Lorentz invariance as a thermodynamical statistical emergence. Last but not least, Aristotle spacetime geometry modelizes the energy, linear and angular momentum conservation laws. This first step is in fact needed to derive rigorously the Lorentz transformations from the observed relativity of motion.

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