Properties of Axial-vector Mesons and Charmless $B$ Decays: $B \to VV, VA, AA$

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I introduce the properties of the light axial-vector mesons. The branching ratios, longitudinal fractions and direct $CP$ asymmetries of the related charmless two-body $B$ decays into final states involving two axial-vector mesons ($AA$) or one vector and one axial-vector meson ($VA$) are discussed within the framework of QCD factorization.

1. INTRODUCTION

The distribution amplitudes of an energetic light hadron moving nearly on the light-cone can be described by a set of light-cone distribution amplitudes (LCDAs). The LCDAs are governed by the special collinear subgroup $SL(2,\mathbb{R})$ of the conformal group. The conformal partial wave expansion of a light-cone distribution amplitude is fully analogous to the partial wave expansion of a wave function in quantum mechanics. Each conformal partial wave is labeled by the specific conformal spin $j$, in analogy to the orbital quantum number in quantum mechanics of having spherically symmetric potential [1].

There are two distinct types of (P-wave) axial-vector mesons, $^3P_1$ and $^1P_1$. Because of G-parity, the axial-vector (tensor) decay constants of $^1P_1$ ($^3P_1$) states vanish in the SU(3) limit. Nevertheless, the constituent partons within a hadron are actually non-localized. It is interesting to note that due to G-parity the chiral-even LCDAs of a $^1P_1$ ($^3P_1$) meson defined by the nonlocal axial-vector current is antisymmetric (symmetric) under the exchange of quark and anti-quark momentum fractions in the SU(3) limit, whereas the chiral-odd LCDAs defined by the non-local tensor current are symmetric (antisymmetric) [2]. The large magnitude of the first Gegenbauer moment of the mentioned antisymmetric LCDAs can have large impact on $B$ decays involving a $^3P_1$ or/and $^1P_1$ meson(s). The related phenomenologies are thus interesting [3-11]. Some $B$ decays involving an axial-vector meson were studied in [78] using the naive factorization approach.

2. POLARIZATION ANOMALY IN $B \to VV$ DECAYS

The $B$-factories have been measured the branching ratios and polarization fractions of charmless $\overline{B} \to VV$ decays, involving $\rho\rho$, $\rho\omega$, $\rho K^*$, $\phi K^*$, $\omega K^*$ and $K^*K^*$ in final states [9]. Theoretically, we naively expect that the helicity amplitudes $\bar{A}_h$ (with helicities $h = 0, -, +$) for $\overline{B} \to VV$ respect the hierarchy pattern [10][11]:

$$\bar{A}_0 : \bar{A}_- : \bar{A}_+ = 1 : \left(\frac{\Lambda_{QCD}}{m_b}\right) : \left(\frac{\Lambda_{QCD}}{m_b}\right)^2,$$

so that we have the following scaling law:

$$1 - f_L = \mathcal{O}\left(\frac{m_V^2}{m_b}\right), \quad \frac{f_L}{f_\parallel} = 1 + \mathcal{O}\left(\frac{m_V}{m_B}\right),$$

with $f_L$, $f_\perp$, and $f_\parallel$ being the longitudinal, perpendicular, and parallel polarization fractions, respectively. The large fraction of transverse polarization observed in penguin-dominated $K^*\rho$ and $K^*\phi$ modes poses a challenge for theoretical interpretation. To obtain a large transverse polarization in $B \to K^*\rho, K^*\phi$, this scaling law must be circumvented in one way or another. Various mechanisms such as sizable
penguin-induced annihilation contributions [11, 12], non-factorization of spectator-interactions [13,14], and new physics (where only models with large (pseudo)scalar or tensor coupling can explain the observation for \( f_\perp \sim f_\parallel \) respectively). We thus expect to have sizable new-physics effects contribute directly to the smallness of the decay amplitudes can be written as 

\[ A = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \langle p_1K | T_A^{h,p} + T_B^{h,p} | \bar{B} \rangle, \]  

where \( \lambda_p \equiv V_{ph}V_{pq}^* \) with \( q = s, d \), and the superscript \( h \) denotes the helicity of the final state meson. \( T_A \) accounts for topologies of the form-factor and spectator-scattering, while \( T_B \) contains annihilation topology amplitudes.

3.1. Tree-dominated \( B \rightarrow (a_1, b_1)(\rho, \omega) \)

Because of G-parity, the axial-vector (tensor) decay constants for \( ^1P_1 \) \((^3P_1)\) states vanish in the SU(3) limit. The amplitudes of \((a_1^-, b_1^-)(\rho^+, \rho^0, \omega)\) modes are proportional to \( f_{a_1} \) or \( f_{b_1} \) in factorization limit. The \( a_1^\pm \omega \) mode should have the rate similar to \( a_1^- \rho^0 \sim 23 \times 10^{-6}, b_1^- \rho^0 \) modes are highly suppressed by the smallness of \( f_{b_1} \). Since the \( a_1^- \pi^+ \) mode is also governed by \( f_{a_1} \), we anticipate that \( a_1^- \rho^+ \) and \( a_1^- \pi^+ \) have comparable rates.

The decays \( \bar{B}^0 \rightarrow (a_1^+, b_1^+)(\rho^-, \pi^-) \) are governed by the decay constants of the \( \rho \) and \( \pi \), respectively. We thus expect to have \( \bar{B}(\bar{B}^0 \rightarrow a_1^+ \rho^-) \simeq (f_\rho/f_\pi)B(\bar{B}^0 \rightarrow a_1^+ \pi^-) \) and \( B(\bar{B}^0 \rightarrow b_1^- \rho^-) \simeq (f_\rho/f_\pi)B(\bar{B}^0 \rightarrow b_1^- \pi^-) \) [16].

3.2. Penguin-dominated \( B \rightarrow (a_1, b_1)K^* \)

The potentially large weak annihilation contributions to the penguin-dominated decay \( \bar{B} \rightarrow M_1M_2 \) can be described in terms of the building blocks \( b_i^{p,h} \) and \( b_{i,i}^{p,h} \).

\[ \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \langle M_1M_2 | T_B^{h,p} | \bar{B} \rangle \]

\[ = i \frac{G_F}{\sqrt{2}} \sum_{p,i} \lambda_p f_B f_{M_1} f_{M_2} (d_i b_i^{p,h} + d_{i,i}^{p,h}), \]

where the coefficients \( d_i \) and \( d_{i,i} \) are process-dependent. The main contribution of annihilation amplitudes arises from the operator \(-2(\bar{q}_1b)_{s-P}(\bar{q}_2q_3)_{s-P}, \) and is denoted as \( A_3^{f,0}(h) \) (with the superscript \( f \) indicating the gluon emission from the final state quarks):

\[ A_3^{f,0}(^3P_1 V) \approx -18\pi\alpha_s(2X_A^0 - 1) \times \left[ a_1^- X_A^0 - 3 - r_\pi(X_A^0 - 2) \right], \]

\[ A_3^{f,-}(^3P_1 V) \approx 18\pi\alpha_s(2X_A^- - 3) \times \left[ \frac{m_V}{m_P} r_\pi(X_A^- - 1) - 3a_1^- X_A^0 - 1 \right], \]

\[ A_3^{f,0}(^1P_1 V) \approx 18\pi\alpha_s(X_A^0 - 2) \times \left[ r_\pi(2X_A^0 - 1) - a_1^- X_A^0 - 11 \right], \]

\[ A_3^{f,-}(^1P_1 V) \approx -18\pi\alpha_s(X_A^- - 1) \times \left[ -\frac{m_V}{m_P} r_\pi(2X_A^- - 3) + a_1^- X_A^- - 17 \right], \]

where the logarithmic divergences are simply parameterized as \( X_A^0 = (1 + \rho_A e^{i\phi_A}) \ln (m_B/\Lambda_b) \). It is interesting to note that the magnitude of the first Gegenbauer moments \( a_1^- X_A^0 \) and \( a_1^- X_A^- \) is of order 1. We use the penguin-annihilation parameters \( \rho_A = 0.65 \) and \( \phi_A = -53^\circ \) as the
Table 1
Branching ratios (B) in units of 10^{-6}, the longitudinal polarization fractions (f_L) in parentheses and direct CP asymmetries (A_{CP}) for decays B \to (a_1, b_1) K^* with a_1 = a_1(1260) and b_1 = b_1(1235). The central values for default inputs (left) refer to \rho_A = 0.65 and \phi_A = -53^\circ, and for results without annihilation (right) to \rho_A = -1. The first theoretical error corresponds to uncertainties due to variation of Gegenbauer moments, decay constants, quark masses, form factors, the \lambda_B parameter for the B meson wave function, and the second one to 0 \leq \rho_{A,H} \leq 1, arbitrary phases \phi_{A,H} for the left part (or 0 \leq \rho_H \leq 1, arbitrary phase \phi_H for the right part). For longitudinal polarization fractions and CPs, we consider only the latter one for the error. The light-cone sum rule results for form factors are used [1,6].

| Mode | (Default) B | f_L | A_{CP} | Mode | (\rho_A = -1) B | f_L | A_{CP} |
|------|-------------|-----|--------|------|----------------|-----|--------|
| a_1^- K^+- | 10.6^{+7.4+3.7}_{-4.0-8.1}(0.37-0.29) | 0.04+0.04 | a_1^+ K^+- | 3.6^{+1.6+0.5}_{-1.3-0.1}(0.68+0.06) | 0.07+0.01 |
| a_1^- K^+0 | 4.2^{+2.7+1.5}_{-1.9-4.2}(0.23+0.45) | 0.12+0.15 | a_1^+ K^+0 | 0.5^{+0.5+0.0}_{-0.4-0.0}(0.50+0.45) | -0.30+0.15 |
| a_1^- K^-0 | 11.2^{+6.1+3.9}_{-1.9-4.2}(0.37+0.48) | 0.005+0.001 | a_1^+ K^-0 | 4.1^{+2.0+1.7}_{-1.6-1.0}(0.62+0.13) | 0.01+0.00 |
| a_1^- K^+ | 7.8^{+3.2+1.6}_{-1.3-4.3}(0.52+0.42) | 0.005+0.003 | a_1^+ K^- | 4.4^{+1.1+0.0}_{-1.1-0.0}(0.75+0.06) | 0.15+0.04 |
| b_1^+ K^+ | 12.5^{+7.3+2.0+21.1}_{-9.0-8.2-41} | 0.44+0.03 | b_1^- K^- | 4.1^{+2.3+0.3}_{-2.0-0.3}(0.91+0.05) | 0.10+0.02 |
| b_1^- K^+0 | 6.4^{+2.4+8.8}_{-1.7-4.8} | 0.02+0.02 | b_1^- K^-0 | 2.4^{+3.3+0.5}_{-1.1-1.0}(0.88+0.04) | -0.12+0.07 |
| b_1^- K^-0 | 12.8^{+5.0+20.1}_{-9.6-9.6} | 0.02+0.02 | b_1^- K^- | 4.0^{+2.9+0.7}_{-2.5-0.6}(0.87+0.04) | 0.02+0.00 |
| b_1^- K^- | 7.0^{+2.6+12.0}_{-2.0-4.8} | 0.60+0.06 | b_1^- K^- | 2.4^{+1.2+0.3}_{-0.9-0.3}(0.92+0.01) | 0.24+0.08 |

default central values inferred from B \to K^*\phi decays as a guidance for annihilation enhancement in B \to V A, A A decays. We see from Table 1 that the branching ratios for a_1 K^* and b_1 K^* modes are substantially enhanced by penguin annihilation [6]. Due to the antisymmetric tensor and axial-vector distribution amplitudes for the a_1 and b_1, respectively, the direct CP asymmetry (A_{CP}) can reach 60% for b_1^- K^*+, 44% for b_1^+ K^*-, 12% for a_1^- K^+0, and -17% for b_1^- K^-0. Moreover, the branching ratios of these modes can be of order 10^{-5}. Here we adopt the convention for the A_{CP} to be

$$A_{CP}(\bar{f}) = \frac{B(\bar{B} \to \bar{f}) - B(B^0 \to f)}{B(\bar{B} \to \bar{f}) + B(B^0 \to f)}.$$  (9)

When penguin annihilation is turned off, we have alternative patterns for A_{CP}: A_{CP}(b_1^- K^*-) \sim 0.24, A_{CP}(a_1^- K^*+) \sim 0.10, A_{CP}(a_1^- K^-0) \sim -0.30 and A_{CP}(b_1^- K^-0) \sim -0.12. These can be easily accessible in present B-factories and LHCb. For the corresponding channels, we have the pattern

$$f_L(b_1 K^*) > f_L(\rho K^*) > f_L(a_1 K^*)$$  (10)

if \rho_A = 0.65 and \phi_A = -53^\circ for VA modes, but we have

$$f_L(b_1 K^*) > f_L(a_1 K^*) > f_L(\rho K^*)$$  (11)

if neglecting the penguin annihilation for VA modes. Experimentally, it is thus important to measure them to test the importance of the penguin annihilation mechanism [17].

3.3. Penguin-dominated B \to K_1\phi
The physical states K_1(1270) and K_1(1400) are the mixtures of K_{1A} (1^3P_1) and K_{1B} (1^3P_1) states. K_{1A} and K_{1B} are not mass eigenstates and can be mixed together due to the strange and nonstrange light quark mass difference. The physical states can be parametrized as

$$|K_1(1270)| = |\bar{K}_{1A}| \sin \theta_{K_1} + |\bar{K}_{1B}| \cos \theta_{K_1},$$  (12)

$$|K_1(1400)| = |\bar{K}_{1A}| \cos \theta_{K_1} - |\bar{K}_{1B}| \sin \theta_{K_1},$$  (13)

where the sign ambiguity for \theta_{K_1} is due to the fact that one can add arbitrary phases to |\bar{K}_{1A}| and |\bar{K}_{1B}|. This ambiguity can be further removed by fixing the signs for \bar{f}_{K_{1A}} and \bar{f}_{K_{1B}}, which do not vanish in the SU(3) limit. Following Ref. [2], we adopt the convention: \bar{f}_{K_{1A}} > 0, \bar{f}_{K_{1B}} > 0, which are defined by

$$\langle 0 | \bar{q} \gamma_{\mu} \gamma_5 \bar{s} | \bar{K}_{1A}(P, \lambda) \rangle = -i \bar{f}_{K_{1A}} m_{K_{1A}} \epsilon_{\mu}^{(3)}.$$  (14)
\[ \langle 0 | \bar{q} \sigma_{\mu \nu} s | K_1B(P, \lambda) \rangle = i f_{K_1B} \epsilon_{\mu \nu \alpha \beta} \epsilon^\alpha_{(\lambda)} P^\beta. \] 

From the study for \( B \to K_1(1270)\gamma \) and \( \tau \to K_1(1270)\nu \), we recently obtain \[ \theta_{K_1} = -(34 \pm 13)^\circ. \]

For \( B \to K_1\phi \), when the penguin annihilation is turned off, we find \( B(B^- \to K_1(1270)^-\phi) \approx 3 \times 10^{-6} > B(B^- \to K_1(1400)^-\phi) \approx 3 \times 10^{-7} \). This feature is dramatically changed in the presence of weak annihilation with \( \rho_A = 0.65 \) and \( \phi_A = -53^\circ \). Because \( \beta_3(K_{1A}\phi) \) and \( \beta_3(K_1B\phi) \) are opposite in sign, the interference between terms with \( \alpha_i \) of and \( \beta_i \) is destructive for \( B^- \to K_1(1270)^-\phi \), but constructive for \( B^- \to K_1(1400)^-\phi \). Therefore we have \( B(B^- \to K_1(1270)^-\phi) \approx 4 \times 10^{-6} < B(B^- \to K_1(1400)^-\phi) \approx 11 \times 10^{-6} \). If this relation is not borne out by experiment, this will indicate that the weak annihilation is negligible. For the recent measurement see \[ 16 \].

3.4. Tree-dominated \( B \to (a_1, b_1)(a_1, b_1) \)

Because \( f_{a_1} \) vanishes in SU(2) limit, it is expected that \( b_1b_1 \) channels are highly suppressed relative to \( a_1a_1 \). Only the color-allowed \( a_1^- b_1^+ \) and \( a_1^- b_1^0 \) modes, of which the decay amplitudes are proportional to \( f_{a_1} \), in large \( m_b \) limit, are comparable to \( a_1^- a_1^- \) and \( a_1^- a_1^0 \). We find that

\[ B(a_1^- b_1^+) > B(a_1^- a_1^-) \approx B(\rho^+ a_1^-) > B(\rho^- b_1^+) > B(\rho^+ \rho^-) \approx B(a_1^+ a_1^-). \]

These branching ratios are of order \((20 \sim 40) \times 10^{-6}\). Comparing with the \( \rho^+ \rho^- \) mode, we observe that \( f_L \) is enhanced by the replacement \( \rho \to b_1 \), but suppressed by \( \rho \to a_1 \), i.e.,

\[ f_L(b_1^+ \rho^-) > f_L(\rho^+ a_1^-) > f_L(a_1^+ a_1^-) \approx f_L(a_1^+ \rho^-). \]

4. CONCLUSION

Owing to the \( G \)-parity, the chiral-even two-parton LCDAs of the \( ^3P_1 (^1P_1) \) mesons are symmetric (antisymmetric) under the exchange of quark and anti-quark momentum fractions in the SU(3) limit. For chiral-odd LCDAs, it is other way around. Because the properties of LCDAs between axial-vector and vector mesons are different, the polarization puzzle can be further examined by studying hadronic B decays involving axial-vector mesons in the final states.

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