Supersymmetry and Inflation

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Abstract
Inflation is a promising solution to many problems of the standard Big-Bang cosmology. Nevertheless, inflationary models have proved less compelling. In this chapter, we discuss why supersymmetry has led to more natural models of inflation. We pay particular attention to multifield models, both with a high and a low Hubble parameter.

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Supersymmetry and Inflation

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Inflation is a promising solution to many problems of the standard Big-Bang cosmology. Nevertheless, inflationary models have proved less compelling. In this chapter, we discuss why supersymmetry has led to more natural models of inflation. We pay particular attention to multifield models, both with a high and a low Hubble parameter.

1 Introduction

Supersymmetric cosmology is necessarily a speculative subject, since the evolution of the universe is sensitive not only to the observed light degrees of freedom and their superpartners, but also to the as yet undetected heavy particle spectrum. The heavy degrees of freedom only decouple at low temperatures; in the early universe they can be very relevant. In fact, degrees of freedom which are heavy could have been light in the early universe, and vice versa. Furthermore, the vacuum structure of supersymmetric and superstring theories can be very rich and complex; we do not yet know how our vacuum is determined. Nevertheless, despite our ignorance of many aspects of high-energy particle physics, there are certain features of supersymmetric theories which have been shown in recent years to be relevant to cosmology. If the world is supersymmetric, it is clearly important for cosmology, both because of the many new particles which would be present and because of the many flat direction (moduli) fields. These are fields which have no potential in the supersymmetric limit. They can however get a small potential due to supersymmetry breaking, higher dimension operators, or interactions with other fields. These flat directions which only occur naturally in supersymmetric theories can provide large amounts of energy as they will almost certainly not start their evolution from their minimum. Many recent models of inflation are based on this observation, though often in different contexts. In this chapter, we will see how supersymmetric theories might provide more compelling models of inflation. We will consider some examples which demonstrate that supersymmetric theories might provide viable inflaton candidates. Even without knowing the correct particle physics model at high energy, we can identify what might be desirable features of this model if they are to simultaneously account for an earlier epoch of inflation.

In this chapter we will briefly review the motivation for inflation and the
requirements for a successful inflationary cosmology. We will then discuss in some detail the multifield models of inflation which potentially succeed in meeting the requirements of inflation with little or no fine-tuning. We will discuss several particular models, but cannot attempt a complete enumeration of all models to date—this list is changing very rapidly! We will instead focus on what we think are the additional requirements of the supersymmetric inflation models, their possible predictions, and important questions which remain and attempts to address them.

The standard Big Bang cosmology has many important successes. Most notable are the measured Hubble expansion of the universe, the predictions of the light element abundances from nucleosynthesis in the early universe, and the prediction of the $2.7^\circ$ microwave background radiation spectrum. More recently, the measurement of the anisotropy in the cosmic microwave background is an indication that theories of structure formation are on the right track. The standard cosmology is simple and successful, but very likely incomplete. As with the standard model of particle physics, the major reason this is believed is that the model as it stands is unnatural, in that it requires very fine-tuned initial conditions.

The shortcomings of the standard cosmology are the problems of the large-scale smoothness of the universe, the spatial flatness problems, the origins of small inhomogeneities, and the potential presence of unwanted relics. Inflationary cosmology successfully resolves the first two problems for a sufficiently long-lived inflationary phase. If inflation involves the correct mass scales and/or parameters, inflation can also lead to the observed density perturbations. For relics which do not get produced late in the universe, inflation can solve the problem of unwanted relics, although it should be noted that the problem of unwanted relics is a serious consideration for most supersymmetric theories, even with an early inflationary epoch.

Most successful inflationary models are based on slow-roll inflation. In its earliest implementation it is phenomenologically successful as a model of inflation but requires fine-tuning, either of the potential or of the initial conditions. The requirements on the potential for the inflaton field $\phi$ for slow roll to be valid are $|V''(\phi)| < 9H^2$ and $|V'M_P/V| < \sqrt{48\pi}$. While these constraints are met, the potential is approximately constant as is the Hubble parameter, $H = \sqrt{8\pi V/3M_P^2}$. During the period of slow-roll, the universe can expand by an exponential factor, the value of which is determined by the time for which the slow-roll conditions are valid. Inflation ends when these

\textsuperscript{b}There is debate over whether an initial condition should be considered fine-tuned, particularly in an eternal inflationary scenario. We will not discuss this here but refer the reader to Ref. [4].
conditions cease to apply.

So far, we see that the important condition for inflation is to have an approximately constant energy density for a finite interval. This in and of itself is not a serious constraint, particularly in a supersymmetric theory for which many light or massless scalars might be present. What makes the construction of inflationary models tricky (or fine-tuned) is that inflation needs to end, so that reheating can produce the known matter content of the universe. With the further requirement that inflation accounts for the density fluctuations in the microwave background (here given in the slow-roll approximation), \( V^{3/2}/V'M^2 = 5.4 \cdot 10^{-4} \), one is led to the introduction of small parameters. These problems have been reviewed elsewhere.\(^3\) In general, \( \frac{\delta \rho}{\rho} \sim \delta N \sim \frac{H^2}{\phi} \sim \frac{H^3}{V'} \sim \frac{H^3}{m^2 \phi} \) (1)

where use has been made of the slow-roll equation of motion \( 3H \dot{\phi} = -V'(\phi) \) and \( N \) is the number of e-folds. It is important to notice that the Hubble parameter which will give the correct magnitude of density perturbations is determined not only by \( m \), the mass of the inflaton, but also by \( \phi \), which in this case means the magnitude of the field \( \phi \) during the time density fluctuations are formed. Therefore, although it is conventionally assumed that \( H \) during inflation is determined to get the density fluctuations right, this is not necessarily the case. By constructing an inflationary model with a different value of \( \phi \), one can obtain more than one scale for \( H \) which can yield sensible density perturbations.

We note that it is not an essential requirement that density fluctuations formed during inflation account for the observed structure. Other suggestions for producing density perturbations have been given.\(^6\) However, it would certainly be more economical and greatly desirable to have density perturbations taken care of during inflation, since inflation automatically produces fluctuations. This is the assumption which we make here. Moreover, recent papers indicate a substantial vector and tensor contribution to the CMBR implying too low anisotropies on small angular scales in cosmic defect models.\(^7\) Ultimately, measurements should conclusively distinguish inflationary perturbations from others.\(^8\) Current evidence seems to favor inflation.\(^\text{a} \)

In this chapter, we will give an overview of some recent ideas for implementing inflationary models in the context of supersymmetry. Most of them are based on “hybrid”\(^4\) inflationary models, although there have been a few recent suggestions which try to implement slow-roll in the context of a single inflaton field. We will first review the motivation behind multifield inflation
models, and discuss some examples. We will then briefly discuss recently suggested single field models.

2 Hybrid Inflation and Supersymmetry

Before introducing supersymmetry into our discussion, let us first consider the potential advantage to multifield inflation models. To do so, we consider a toy model \(^{[1]}\) with potential

\[
V = (\phi^2 - M^2)^2 + \lambda \phi^2 \psi^2 + m^2 \psi^2
\]

(2)

Notice that the \(\psi\) potential is minimized when \(\psi = 0\), at which point the \(\phi\) potential is minimized with \(\phi = M\), where the potential energy \(V = 0\). On the other hand, it is unlikely that \(\psi\) starts at its vacuum value. In fact, when the temperature exceeds the \(\psi\) mass, the potential is negligible and \(\psi\) evolves extremely slowly towards its minimum. Therefore in the early universe, one can reasonably expect large values of \(\psi\). If \(\psi > \psi_c = \sqrt{2}M/\sqrt{\lambda}\), and \(\phi\) is sufficiently small, \(\phi\) will rapidly move towards the origin where it will sit leading to a vacuum energy of approximately \(V = M^4\) (assuming this dominates the \(\psi\) contribution to the energy). This nonzero vacuum energy permits an inflationary stage.

In this model, inflation will end at around the time when the \(\phi\) potential turns over, when \(\psi = \psi_c\). In other implementations of “hybrid” inflation, \(^{[4]}\) it could be that inflation ends when the \(\psi\) potential ceases to correspond to a slow-roll situation. We will see an example of this shortly.

The above toy model is fine as a model of inflation. In fact, multifield models seem to resolve very nicely one of the major problems with the standard slow-roll potentials; how can the potential be very flat and then give rise to a rapid end and reheat? Having two fields to control inflation solves this problem beautifully. One field controls the vacuum energy, whereas the other field essentially acts as a “switch” for inflation.

However, there are several obvious questions. First, why should the mass parameters be small? And what sets the mass scales in the first place? Non-supersymmetric field theory cannot in general address this question. Only for a Goldstone boson is there a reason to believe the mass of a scalar is small; in general we would not expect a slow-roll potential for the \(\psi\) field.

Why can supersymmetry change this picture? First of all, flat directions are natural in supersymmetric theories. Not only does supersymmetry protect

\(^{[4]}\)I will generally use the GRS\(^{[3]}\) conventions for the slow-rolling inflaton field \(\psi\) and the field which controls the energy density will be denoted \(\phi\). The reader should be aware of other conventions existing.
against radiative corrections; supersymmetric theories in general have a large moduli space of flat directions which need not be put in by hand. We should qualify what we mean by flat directions. In general, we are referring to fields with no potential in the supersymmetric limit, with no other fields away from their vacuum expectation value, and with nonrenormalizable terms neglected. The presence of any of these terms will in fact generate a potential, but one which can in general be consistent with the requirements of inflation if the curvature of the potential is sufficiently small (when compared to the scale set by the vacuum energy).

The other interesting aspect of supersymmetric theories is that in general they require at least one mass scale which is distinct from the Planck scale. This is necessary to account for the supersymmetry breaking scale, which is lower than the Planck scale if the standard low-energy picture of supersymmetry breaking as accounting for stabilization of the electroweak scale is correct. The precise value of this scale is model dependent. In hidden sector models, the new scale will be of order $10^{11}$ GeV, while in models of supersymmetry breaking based on more direct communication, the supersymmetry breaking scale will be lower. The supersymmetry breaking scale, and in particular, the intermediate scale, seems to be an obvious candidate for application to inflationary models since it is associated with nonvanishing vacuum energy density. In some supersymmetric models, other scales can appear. Notable among these is the Grand Unification (GUT) scale. Many models try to associate directly particle physics models which incorporate a grand unified gauge theory to inflation.

To account for density perturbations, it is clear that there needs to be some small number in the particle physics theory, which could be a ratio of masses. Much of the work on supersymmetric inflation has been focussed on trying to exploit these mass scales to realize the necessary requirements of inflation.

It is useful to divide these efforts into two categories. In one class of theories, the density fluctuations are roughly of order $(M_G/M_P)^2$, whereas the second class of theories only exploits the intermediate scale, and obtains density fluctuations either as $(M_I/M_P)$ or as a result of various parameters which might appear. Here $M_G \approx 10^{16}$ GeV is the GUT scale and $M_I \approx \sqrt{M_W M_P}$ is the intermediate scale of order $10^{11}$ GeV which determines the soft supersymmetry breaking parameters in a hidden sector scenario for the communication of supersymmetry breaking. Notice that the first class of models involves the scale $M_G$, which is the VEV of some field and may or may not set the magnitude of the potential energy density. In the second case, $M_I$, we know it is associated with a vacuum energy density. We will discuss each of these models in turn.

Many of the models we discuss are given in the context of global supers-
symmetry, although some models are incorporated into a supergravity theory. Before proceeding, we mention two potential problems with supergravity inflaton models; only some of the models presented below address these issues. The first issue is that if $W$ is the superpotential and $K$ is the Kahler potential, the potential takes the form

$$e^{[K]} \left( (W_i + K_i W) K^{-1}_{ij} (W_j + K_j W) - 3|W|^2 \right) + D - \text{terms}$$

Since during inflation, the term in parentheses is nonzero if inflation is due to nonvanishing $F$ terms, one will in general find a large potential for any field appearing in the Kahler potential, which will of course include the inflaton. This can destroy the flatness of the inflaton potential and thereby destroy inflation.

The other potential problem is that in the supergravity theory where the cosmological constant at the desired minimum is cancelled by a constant, one generically finds a deeper minimum out at values of the field larger than $M_p$. However, if the superpotential is purely cubic in the fields this problem will not arise. In general though, it is difficult to know how to take this problem, as one is generally treating the potential as a Taylor expansion, and at field values beyond $M_p$ the theory is presumably no longer valid. Furthermore, it is not clear that our world is in the global minimum of the potential.

### 3 Hybrid Inflation and High Scale Models

The idea of hybrid inflation in supersymmetric theories was studied in a seminal paper by Liddle, Lyth, Stewart, and Wands. Dvali, Shaeffer, and Shafi [DSS] pointed out the importance of considering quantum supersymmetry breaking effects during inflation, which they then exploited to introduce an interesting model of inflation. However, their model still required arbitrary mass scales. There were subsequent models in which the authors tried to identify an appropriate mass scale. None of these models are perfect, but might nonetheless have the germ of truth.

We first discuss the DSS model. They have the superpotential

$$W = \kappa S \bar{\phi}\phi - \mu^2 S$$

where $S$ is a singlet and $\phi$ and $\bar{\phi}$ transform under a GUT group. Notice that when $\phi$ and $\bar{\phi}$ vanish, the $S$ field is a flat direction. The model also contains

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\[d\] I thank Paul Langacker for stressing this problem.

\[e\] I thank Gia Dvali for this comment.

\[f\] Classical supersymmetry breaking effects during inflation had been pointed out in Ref. [3] and [4].
an $R$-symmetry under which $S$ transforms which forbids an $S^3$ term in the superpotential. This is important as it is essential that $S$ is a flat direction.

Let us now consider the potential for this model. We have

$$V(S, \phi, \bar{\phi}) = \kappa^2 |S|^2 (|\phi|^2 + |\bar{\phi}|^2) + |\kappa \phi - \mu|^2 + D - \text{terms}$$

(5)

where the $D$-terms depend on the gauge representation of $\phi$ and $\bar{\phi}$.

Now at the supersymmetry preserving minimum, the $D$-term requirement imposes $\phi = \bar{\phi}$, whereas the superpotential imposes $\phi = \bar{\phi} = \mu/\sqrt{\kappa}$ and $S = 0$. However, in the early universe it is very likely that not all fields were at their supersymmetry-preserving minimum. In fact, $S$ might have started off at a value $S > S_c = \mu/\sqrt{\kappa}$, in which case the $\phi$ potential is minimized at vanishing $\phi$ and $\bar{\phi}$, where $V = \mu^4$. In other words, this is looking precisely like a hybrid inflation model, where $S$ plays the role of $\psi$ and $\phi$ and $\bar{\phi}$ play the role of $\phi$ in our toy model.

Now naively it looks like $S$ is exactly flat, which would be bad, since there would be no potential driving $\phi$ and inflation would never end. However, this neglects the fact that supersymmetry is broken during inflation! Here the nonzero breaking is due to the nonvanishing $F$ term; however we know this is generally true since inflation relies on nonvanishing vacuum energy. In fact, in general this can be a problem in models with more than one mass scale. Since supersymmetry is broken during inflation, this can be unnatural as in a nonsupersymmetric model. However, in this model, the quantum corrections introduce a potential for the $S$ field which is desired.

The consequence of the nonvanishing $F$ term and the breaking of supersymmetry is that a potential for the $S$ field will be generated through radiative corrections. The one-loop effective potential for $S$ is

$$\Delta V(S) = \sum \frac{(-1)^F}{64\pi^2} M_i(S)^4 \log \left( \frac{M_i(S)}{\Lambda} \right)^2$$

(6)

Here $M_i(S)$ are the $S$-dependent masses of the fields. This effective interaction introduces a slope to the $S$ potential. In fact, in this type of model, inflation generally ends when slow-roll ceases to apply, rather than when the "$\phi$" potential turns over.

Let us consider this in more detail. Because of the supersymmetry breaking $F_S$, the $\phi$ and $\bar{\phi}$ spectrum do not respect supersymmetry. The scalars have mass $\kappa^2 S^2 \pm \kappa \mu^2$, whereas the fermion has mass $\kappa S$. Substituting these $S$-dependent masses into the effective potential, one derives the $S$ potential at one-loop to be

$$V_{eff}(S) = \mu^4 + \frac{\kappa^2 \mu^4}{32\pi^2} \left( \log \frac{\kappa^2 S^2}{\Lambda^2} + \frac{3}{2} \right)$$

(7)
This model succeeds as a hybrid inflation model. However, there are some important open questions. First, what is the origin of the scale \( \mu \). To get the correct magnitude of density fluctuations, it turns out that \( \mu \) is of order the GUT scale. This means one might want to tie \( \phi \) to the field with GUT mass and VEV. However, if we take \( \phi \) to be an adjoint, there is too much global symmetry and one obtains too many Goldstone bosons. Interactions which violate this symmetry can also destroy inflation. Alternatively, one can take a GUT group like SU(6) and let \( \phi \) and \( \bar{\phi} \) be Higgs fields in the 6 and \( \bar{6} \). However, the VEV of this field is likely to be too low for a successful GUT model. So in summary, although the fact that the scale \( \mu \) is of order the GUT scale is intriguing, in this basic model it is tricky to realize the connection.

Another potential problem when the scale \( \mu \) is high is that the reheat temperature is likely to be too high, and can cause problems with the gravitino constraint\(^{27}\). This is readily seen by a simple estimate assuming instantaneous reheat. Reheat occurs when the Hubble parameter \( H \) is of order of the inflaton width \( \Gamma \). Since \( H^2 \sim \rho/M_p^2 \sim T_R^4/M_p^2 \sim \Gamma^2 \), we find \( T_R \sim \sqrt{\Gamma/M_p} \). The bound on the reheat temperature depends on the mass of the gravitino, but is generally of order \( 10^{10} \) GeV. If the inflaton decays perturbatively, one expects \( \Gamma \sim \frac{1}{\alpha^2} M_{\text{inf}} \). If \( M_{\text{inf}} \sim M_G \), this is clearly too big. Even if the reheat occurs through higher dimension Planck-suppressed operators, the reheat temperature is probably too high, since it is of order \( \sqrt{M_G^3/M_p} \). This is not necessarily an insuperable problem, but it generally requires a more complicated model. In the context of reheat, it should be mentioned that there is still debate over the role of parametric resonance in the decay of the inflaton; however this would generally only increase the reheat temperature. A further point is that the reheat bound assumes only gravitino couplings suppressed by \( M_{P_L} \). If the inflaton decays to particles in the sector in which supersymmetry is broken, the rate for gravitino production can be even larger and the reheat bound even stricter. One can also estimate a reheat bound in gauge-mediated models of supersymmetry breaking\(^{18}\). One generally finds even more stringent bounds in this case, since the gravitino is more strongly coupled.

A third problem is that our discussion so far has been in the context of global supersymmetry. Planck-suppressed operators however cannot be neglected, since the Hubble parameter itself is Planck-suppressed. Without some tuning, supergravity corrections can invalidate the conditions for slow-roll.

Subsequent models have tried to address the first type of problem, namely the origin of the scale \( \mu \) and some have also addressed the third problem. Various authors\(^{17}\) (see also\(^{19}\)) suggested D-term inflation. The idea is to generate the scale “\( \mu \)” though a Fayet-Iliopoulos D-term. They envision a model with an anomalous field content in the low-energy theory, with \( n_+ \) fields
of charge 1 and $n_-$ fields of charge -1. The model contains a superpotential

$$W = \lambda_A X \phi^A_+ \phi^A_-$$

The potential for this model is then

$$V = \lambda^2 |X|^2 (|\phi_-|^2 + |\phi_+|^2) + \lambda^2_A |\phi_+ \phi_-|^2 + \frac{g^2}{2} (|\phi^A_+|^2 + |\phi^A_-|^2 - |\phi^A_+|^2 + \xi)^2$$

If one looks at the potential along $\phi_+ = 0$, one sees that this potential takes precisely the form required for a successful hybrid inflation model. Here, $X$ plays the role of the $\psi$ field, and $\phi_-$ (or some linear combination) the role of the $\phi$ field. One can work out the requirements for sufficiently long slow-roll and for sufficient density fluctuations.

In this model, the vacuum energy density during inflation is given by

$$V = \frac{g^2}{2} \xi^2$$

This model has the nice feature that because it is $D$-term inflation, one can control supergravity corrections which would destroy slow-roll. Recall that there is no symmetry to prevent the quadratic terms in the Kahler potential, which, in the presence of a nonzero $F$-term, will generate an inflaton mass. The situation is better with $D$-term inflation; however Lyth has pointed out that even in this case, there can be large corrections is there is no symmetry preventing quadratic corrections to the holomorphic function of fields appearing in the gauge kinetic term.

One problem with $D$-term inflation is that it is difficult to get the scale right. If the parameter $\xi$ arises due to the Green-Schwartz mechanism, it is about $g^2 \text{Tr} Q M^2_{Pl}/192 \pi^2$, and is probably too big for the scale set by density fluctuations. One would need a small parameter to get the correct mass scale.

Some authors have tried to address the question of getting the correct size of the $D$-term. One possible solution is that the $D$-term is generated at a scale below the Planck scale. However, it is difficult to see how this can be done without large $F$-term contributions to the energy as well, destroying the initial motivation for these models.

Matsuda pointed out that the strength of the gauge coupling determines the magnitude of the $D$-term, and if this coupling is dynamically determined, the $D$-term at the time of inflation might be of a different size. He shows various ansatze for the dependence of the coupling on mass scale; unfortunately however these are not motivated by any underlying physics. However, a realistic model of the scale dependence of the coupling would require a solution to the problem of dilaton stabilization.
March-Russell\cite{3} suggests that in a model in which the string scale is reconciled with the GUT scale, one could obtain better numbers for the size of the $D$-term.

Lyth and Riotto\cite{4} also tried to address the discrepancy of scales required for $D$-term inflation. They point out that the normalization of the magnitude of the $D$-term which is required to agree with density perturbations depends on the slope of the potential at the end of inflation. In some cases, this slope is given by the one-loop effective potential, so by altering the number of fields coupled to the inflaton, the slope can be increased. However adjusting the slope by this (or any other mechanism) will only buy you at most an order of magnitude (once consistency with the observations on the spectral index $n$ are imposed) if one does not take the coupling $g$ to be small.

Another interesting attempt to tie the $\mu$ scale to a physical scale (here a GUT scale) in the problem was made by Dimopoulos, Dvali, and Rattazzi\cite{5}. Their model is based on a quantum corrected moduli space, where the strong interaction scale $\Lambda$ provides the scale for the overall energy density. The model they give has a gauged SU(2) group with four flavors, so there is a quantum modified moduli space. The superpotential, including the constraint, is

$$W_{\text{eff}} = A(\text{Det}M - \bar{B}B - \Lambda^4) + S(\text{Tr}M + \frac{g'}{2} \text{Tr}\Sigma^2) + \frac{h}{3} \text{Tr}\Sigma^3$$  \hspace{1cm} (11)$$

where the last terms arise due to a tree-level superpotential, and $Q\bar{Q}$ has been replaced by the confined meson field $M$. If the field $\Sigma$ is the adjoint field which breaks $SU(5)$ down to the standard model, the scale $\Lambda$ should be of order the GUT scale, which works well for producing density fluctuations.

Inflation does not involve the $\Sigma$ field until the end. Initially, the model works as with other hybrid inflation models. $S$ is a flat direction. When $S$ is big, there is a mass, and $M$ sits at zero, so there is nonzero vacuum energy. In this model, inflation ends at a nonzero value of $S$ and $\Sigma$ and SU(5) is broken. In principle, inflation could also end with $\Sigma$ zero although the authors of Ref.\cite{5} argue that this is not the case.

There are other models which try to incorporate hybrid inflation into a GUT model. For example, Covi, Mangano, Masiero, and Miele\cite{6} implement hybrid inflation in an SU(5) model with an additional singlet, and an arbitrary parameter $\mu$ which they need to take of order the GUT scale. Another model is given by Lazarides, Panagiotakopoulos, and Vlachos\cite{7}, who use a nonrenormalizable potential to introduce a slope to the inflaton field (which was given by supersymmetry breaking parameters in other models).

One potential worry with any model based on SU(5) is that most such models do not solve the doublet-triplet splitting problem, and are therefore
unrealistic unless a severe fine-tuning is imposed. Since the point of inflationary model building is to eliminate small parameters, this is a less than satisfactory situation.

An even more severe problem in models which really try to tie inflation to an SU(5) GUT of the real world was pointed out by Dvali, Krauss, and Liu. They point out that in SU(5) models with an adjoint which gets a nonzero VEV, there will be two choices of vacua, one in which $SU(4) \times U(1)$ is preserved, and one in which $SU(3) \times SU(2)$ is preserved. They parameterize the vacua with three parameters, the overall scale of the symmetry breaking, and two angles (or orbit parameters). When the inflaton field (here we mean the field whose potential generates the vacuum energy density) begins to evolve away from its inflationary value, only one potential minimum is present, namely that corresponding to the bad $SU(4) \times U(1)$ vacuum. They argue further that the field will never make its way to the desired $SU(3) \times SU(2)$ vacuum. Furthermore, whatever vacuum is chosen, the transition happens after inflation, so the monopole problem is not solved. So without embellishment, the simplest hybrid inflationary models based on SU(5) GUTS are not successful. These authors suggest possible resolutions which involve somewhat more complicated theories. It is also possible that in a model such as that of Ref. 24 that a noncanonical Kahler potential invalidates the energy argument and that the suitable vacuum is obtained.

Most authors do not address the question of reheat. Lazarides suggests a decay to a second generation neutrino to avoid too big renormalizable couplings. It is hard to think of a natural decay mode without small coupling which can avoid a high reheat temperature and overproduction of gravitinos. An alternative proposal of Dimopoulos and Dvali is that reheat is delayed by rolling along a flat direction; however it is necessary to ensure that no other dangerous perturbations will be produced. It could be that there is some late entropy release which invalidates the gravitino bound; this might be required to solve the Polonyi problem in any case. It is clear that the question of the high reheat temperature should be addressed in these high scale models.

So to summarize, models of hybrid inflation based on a high $H$ scale seem close to working. $D$-term inflation doesn’t quite get the scale right, but is close. Models based on SU(5) generically suffer from the problem outlined in which is unfortunate since it makes the very nice coincidence of scales less useful. However, it is not impossible to make these models work, and further advances might be forthcoming.
4 Low-Scale Models

We now go on to discuss another very promising class of models, which do not introduce a high scale Hubble parameter, but try to implement successful inflation only by assuming the existence of soft-supersymmetry breaking in a hidden sector. These models are intended as illustrations of how the moduli fields can be employed in a hybrid inflation scenario without strong or unnatural assumptions on the particle physics model. With the specific cases that were discussed, one can identify distinguishing characteristics of this class of model, which should be testable.

The goal of Randall, Soljačić, and Guth (RSG) was to construct models of inflation using only the intermediate mass scale $M_I$ and the Planck scale $M_p$. The aim was to see whether a natural model could be simply constructed which employed moduli fields with soft supersymmetry breaking masses. The essential observation is that the temperature fluctuations observed by COBE do not necessarily require high scale inflation, since the formula for density fluctuations actually depends both on the magnitude of the potential and its slope. Taking the potential quadratic at the time density fluctuations relevant to physical scales are formed, the formula for density fluctuations is

$$\frac{\delta \rho}{\rho} \approx \frac{H^2}{\psi} \approx \frac{H^3}{m^2 \psi}$$

where $\psi$ is the slow rolling field. Now if the energy density during inflation is $M^4$, the Hubble parameter is of order $M^2/M_p$. On the other hand, if the $\psi$ mass arises from hidden sector supersymmetry breaking, it is $m \approx M_I^2/M_pl$. The assumption in this class of models is that $M \sim M_I$, in which case

$$\frac{\delta \rho}{\rho} \approx \frac{H}{\psi}$$

From this equation, it is clear that the magnitude of density fluctuations depends on the value of $\psi$ when inflation ends, which for hybrid inflation models is essentially $\psi_c$.

In Ref. [1], two types of potentials were considered. The first class of models assumed the fields were coupled through a higher dimension operator derived from the superpotential

$$W = \frac{\phi^2 \psi^3}{2M'}$$

where $M'$ is a relevant physical mass scale. Perhaps the most natural possibility is $M_p$. However, it is conceivable there are heavy particle exchanges so that $M'$ can be identified with the GUT scale $M_G$ or $M_I$. 

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The other class of model assumed there was a potential coupling the two
fields involved in hybrid inflation of the form

\[ V = \frac{\lambda}{4} \phi^2 \psi^2 \]  

(15)

Such a coupling could arise for example if the superpotential coupled together
three fields \( W = \chi \phi \psi \), where \( \chi = 0 \) during inflation. This is in fact something
which happens quite naturally, even in the context of the MSSM. For example,
if \( \psi = \bar{u} \bar{d} \bar{d} \) and \( \phi = H_u H_d \), \( W = \lambda u Q u H_u \), one realizes this situation. In fact,
nonstandard GUT models can realize this potential in a way consistent
with the inflationary constraints. In this model, the magnitude of density
fluctuations will be set by a Yukawa coupling; it is important to recognize that
this might well be significantly less than unity.

The complete specification of the model requires the potential for the
\( \phi \) and \( \psi \) fields (apart from their mutual coupling). Note that these potentials arise
due to soft supersymmetry breaking and therefore should be characterized
by potentials of the form \( M_I^4 g(\phi/M_p) \) where \( g \) is a function with a Taylor
expansion with coefficients of order unity. To realize the hybrid inflationary
scenario, the potential for \( \phi \) is taken as \( V = M^4 \cos^2(\phi/\sqrt{2}f) \) and the potential
for \( \psi \) is taken as \( \frac{1}{2} m_\psi^2 \psi^2 \). To be consistent with the requirement that these
are moduli fields with a potential generated by soft supersymmetry breaking,
we would want to find \( M \sim M_I \) and \( m_\psi \sim M^2 f/M_p \). There has been much
confusion over the very specific-looking form taken for the \( \phi \) potential. Indeed,
all that is relevant to inflation are the first two terms in the Taylor expansion!
Only when inflation ends and \( \phi \) moves from zero are the other terms relevant.
This is simply a compact way of writing a function which has the correct
negative curvature at the origin and zero vacuum energy for the true vacuum.
It is interesting however that exactly such a potential could be produced by a
nontrivially coupled pseudo-Goldstone mode. However, this precise form of
the potential is not at all essential, so the field \( \phi \) can be any moduli field with
negative mass squared and a supersymmetry-breaking source for its potential.

This model realizes very nicely the hybrid inflation scenario. Depending
on the form of the soft-supersymmetry breaking potential and the couplings
between moduli fields, it is very likely one can find suitable candidates for
inflation. The major distinguishing characteristic of this type of model is that
the \( \phi \) field is light. Therefore, the dynamics controlling the end of inflation is
very different. In many other models, the \( \phi \) mass is large, so it very quickly rolls
to its true minimum once inflation has stopped. In the RSG models, the field \( \phi \)
spends more time moving primarily due to de Sitter fluctuations, subsequent to

\footnote{I thank Gia Dvali for sharing this observation of Lawrence Krauss and himself.}
which it rolls classically. Because of the initial motion when the field is moving relatively slowly, there is a spike in the density fluctuation spectrum. This spike can be interesting (or dangerous) in that it will lead to more structure on small scales. However, it is too large to be present in observed density fluctuations on scales from about 1 Mpc to $10^4$ Mpc, which gives a constraint on how quickly inflation must end. Including this constraint as well as the constraint from density fluctuations, one finds that the parameters of this model are such that it works rather well, with mild tuning depending on the particular model (numbers as small as 0.01 might be required; see Ref. 31 for details). That is, the original goal, to motivate the parameters by supersymmetry breaking scales, can be reasonably well accommodated.

It should be noted that the derivation of the parameters of the spike in the density perturbation spectrum is subtle. In Ref. 31, the calculation was based on using the Fokker-Planck equation to establish the $\phi$ mean $\dot{\phi}$ distribution and the time delay given by a fluctuation in the $\phi$ field. Garcia-Bellido, Linde, and Wands 34 objected to the calculational method of 31 and instead calculated the fluctuations associated with each element of the ensemble assuming it was classical. However, it can be shown 35 that the method used is not valid, though the original 31 calculation needed to be improved to account for deviation from slow-roll and for a more exact calculation of the fluctuations for a massive field.

Stewart 36, 37 has also constructed low-scale models for which the small tuning required to get a sufficiently flat potential is not required and for which the spike will have different properties. He points out that once quantum corrections are incorporated, there can be a special point (or more than one) where the potential is particularly flat. Initial conditions are probably different for this class of model; one relies on entering a phase of eternal inflation from which one will enter the desired hybrid inflationary phase.

In summary, the low-scale inflation models can have quite distinctive features, and do not have the problems associated with introducing a high scale in a particle physics context. However, they might involve a small amount of fine-tuning; on the other hand they might also involve a small parameter which is present. The spike is generically a test of the models; however if the field is rolling quickly at the phase transition, as is true for Ref. 37, this might be lessened; further work is needed for these models to establish the detailed form of the spike. In general, the low-scale models are well motivated and worthy of further investigation.

Before discussing further models, it is worth noting a distinguishing feature of many hybrid inflation models, those with a mass for the inflaton (like the GRS models), namely the fact that the index $n$ is generally bigger than 1. Furthermore, by measuring the ratio of tensor to scalar perturbations, one
can in principle (because it is a difficult measurement) distinguish high and low scale models. The deviation of the index $n$ from 1 measures the scale dependence of density fluctuations. It can be determined from the potential at the time the relevant perturbation leaves the horizon from the formula

$$n = 1 - 3 \left( \frac{V'}{V} \right)^2 + 2 \frac{V''}{V}$$

(16)

whereas $R$, the ratio of tensor to scalar perturbations, is given by

$$R \approx 6 \left( \frac{V'}{V} \right)^2$$

(17)

where in these equations we have set $M_p$ to unity. One can obtain interesting qualitative information from these formulae. First consider the quantity $R$. If we can approximate $V'$ by a mass term near the end of inflaton, we have $V'/V \sim m^2 \psi M_p / H^2 M_p^2$. So if $H \sim m$, which is often the case, we find that $R$ is negligible unless $\psi \sim M_p$. As we have argued, models with low $H$ can achieve adequate density perturbations if $\psi$ is small (much less than $M_p$) at the end of inflation. We conclude that these type of models will always have negligible $R$. For other models, with $\psi$ closer to $M_p$, it is a detailed question whether $R$ can be measurable.

Notice also that when $V'/V$ is negligible, the sign of the mass squared term at the end of inflation determines whether $n$ is bigger, or less than unity. So models for which the inflaton field rolling towards the origin will have $n$ bigger than 1. The RSG models are of this type, as are hybrid inflation models with a mass term determining the evolution of the inflaton.

It should be noted that there are many models where the potential for the inflaton is not determined by a mass term. An interesting example of hybrid inflation which he dubbed “Mutated Hybrid Inflation” for which the index $n$ is less than 1 was given by Stewart. He considers a toy model where inflation occurs along a nontrivial trajectory in field space. The net result is that along this trajectory, the potential can be written as a polynomial function of the inverse field, and the index $n$ can be shown to be less than 1. Generalizations of this idea and other mechanisms for producing an index less than 1, again in toy models, was given in Ref.

5 Single Field Models

Aside from the many models of hybrid inflation based on supersymmetry, there are a couple of single field inflationary models worthy of note. By single field,
we do not mean there is only one field in the potential, but that the inflationary dynamics can be viewed in terms of a single field (as opposed to hybrid inflation models). Garcia-Bellido\textsuperscript{40} observed that the potential given by an $N = 2$ SU(2) gauge theory with supersymmetry breaking\textsuperscript{41} incorporated takes a form which looks remarkably like a slow-roll potential along a particular trajectory in field space. However, the scale of supersymmetry breaking which is required has no particle physics motivation, so it is not yet clear if this can be tied to a particle physics theory of our world.

Another single field model is that of Adams, Ross, and Sarkar\textsuperscript{42}. They are interested in the problem of the large quadratic terms which can be present in supergravity theories. They argue that there can be special points where the quadratic terms vanish, and these can be quasi-fixed points in the evolution of the field.

Another paper which addresses the issue of large supergravity corrections is by Gaillard, Murayama, and Olive\textsuperscript{43}. They observe that at tree-level, the mass term for the inflaton which occurs generically in supergravity theories is absent if there is a Heisenberg symmetry. Although there is no symmetry reason, it is claimed that gravitational interactions preserve the symmetry (based on a one-loop calculation), so that the potential can be calculated from gauge and superpotential interactions. A more complete realization of this scenario could be interesting.

Stewart\textsuperscript{16} also addressed the issue of large supergravity corrections to the inflaton mass. He identified conditions, which when imposed on the superpotential, guarantee the absence of such corrections. These conditions are $W = W_{\psi} = \phi = 0$ (in the GRS naming convention) during inflation, as well as some conditions on the Kahler potential. He argues that such potentials might arise naturally in superstring theories.

In fact, Copeland, Liddle, Lyth, Stewart, and Wands\textsuperscript{14} had initially pointed out that supergravity corrections to the mass can cancel if there is a minimal Kahler term. Linde and Riotto\textsuperscript{44} make the assumption that nonminimal terms which would destroy this cancellation are small, and then consider the model with both one-loop and gravitational effects taken into account.

In summary, there are currently many ideas on how to use the naturalness property of supersymmetry to provide candidates for inflaton fields. There are some clever ideas involved in high scale models; however the $D$-term models generally give too large density fluctuations while the GUT models often lead to the wrong vacuum following inflation. These models might however be incorporated into more complete and realistic models in the future. The low scale inflation models have the advantage that they require no new mass scales aside from that which was already required to give supersymmetry breaking.
parameters of order the weak scale in a hidden sector scenario. They provide an interesting signature of a spike in the spectrum so they should be subject to experimental verification in the future. Other objections might include the fact that this inflation is relatively late, so this might mean some prior nonstandard evolution (like a previous inflationary phase) is required. These models require mild tunings; presumably even this is not necessary if one will compromise with more complicated scenarios. It is intriguing that new models of particle physics might also lead to new potentials that could provide slow-roll. Exact superpotentials in the strong interaction regime often take nonpolynomial forms which would not have been anticipated on the basis of weakly coupled renormalizable field theories. Other models which fall outside the range of the supersymmetry-motivated field theories we have considered here include dilaton-based inflation \cite{Dvali} and string-theory-motivated-domain walls as the seed for inflation \cite{Rattazzi}.

Although there are many ideas, it should be remembered that there are many requirements for a good inflationary model. Given the indirectness of many cosmological constraints, it is remarkable how constrained models are. Requirements include a sufficiently long period of inflation, a mechanism for ending inflation sufficiently quickly to reheat to temperatures higher than the weak scale (this might be too stringent but in alternative scenarios one needs a mechanism for baryogenesis), a reheat temperature sufficiently low not to overproduce gravitinos, consistency with a spectral index which does not deviate by more than 20% from 1 (the exact constraint is subject to interpretation), and hopefully no ad hoc scales or small numbers. There are only a few models which meet all these criteria. And we have not even addressed the many issues of how inflation fits into a more complete picture of the cosmology of the early universe which includes baryogenesis \cite{March-Russell} and a solution to the Polonyi problem \cite{Polonyi}. So despite the many recent advances, the field still remains fertile, and it would not be surprising to see new and more compelling models of inflation in the future.

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