Revisiting the role of intermittent heat transport towards Reynolds stress anisotropy in convective turbulence

Subharthi Chowdhuri$^{1\dagger}$, Siddharth Kumar$^1$ and Tirtha Banerjee$^2$

$^1$Indian Institute of Tropical Meteorology, Pune, India
$^2$Department of Civil and Environmental Engineering, University of California, Irvine, CA 92697, USA

(Received xx; revised xx; accepted xx)

Thermal plumes are the energy containing eddy motions that carry heat and momentum in a convective boundary layer. The detailed understanding of their structure is of fundamental interest for a range of applications, from wall-bounded engineering flows to quantifying surface-atmosphere flux exchanges. We address the aspect of turbulence anisotropy associated with the intermittent nature of heat transport in thermal plumes by performing an invariant analysis of the Reynolds stress tensor in an unstable atmospheric surface layer flow, using a field-experimental dataset. Given the primary constitutive motions of these thermal plumes, the warm-updrafts and cold-downdrafts are asymmetrical, we formulate this problem in an event-based framework. In this approach, we provide structural descriptions of warm-updrafts and cold-downdrafts based on the concept of persistence in non-equilibrium systems. We investigate the degree of isotropy of the Reynolds stress tensor within these structures of different sizes, and find out that only a subset of these structures are associated with isotropic turbulence in highly-convective flow regimes. Additionally, intermittent extreme heat flux events are found to contribute substantially to Reynolds stress anisotropy under unstable stratification. Moreover, the sizes of the warm-updrafts and cold-downdrafts associated with the maximum value of the degree of isotropy (i.e. corresponding to least anisotropic turbulence) scale with a mixed-length scale. We hypothesize that this mixed-length scaling may reflect an interaction between two different physical processes, one of which is associated with the scale-invariance of the plume sizes and the other with non-Gaussian turbulence.

Key words:
there is a substantial size separation between the large and small scales. Consequently, isotropy is expected in the inertial and dissipation ranges and this is called Kolmogorov’s local isotropy hypothesis, where “local” refers to the small scales in the wavenumber space (Tennekes & Lumley 1972). Most of the studies related to isotropy in the high Reynolds number turbulence have focused on the spectral or the structure function methods. In this approach, a scale description of turbulence is obtained and the local isotropy hypothesis in the inertial wavenumber range is investigated by employing a few standard measures, such as: studying the $4/3$ ratio of spectral amplitudes; existence of the Kolmogorov $-5/3$ or $+2/3$ power laws in the spectra or structure functions; the rolling off of the momentum cospectra faster than the energy spectra; the Kolmogorov $4/5$ law in the third-order structure functions, and so on (Kaimal et al. 1972; Saddoughi & Veeravalli 1994; Katul et al. 1995, 1997b; Chamecki & Dias 2004). However, none of these measures conclusively show the evidence of local isotropy at small scales (Chamecki & Dias 2004).

There is another approach to study isotropy through bulk-statistics. In this approach, the anisotropic signatures of the energy containing motions at a point in the flow are studied via the time-averaged statistics of the Reynolds stress tensor (Krogstad & Torbergsen 2000; Smyth & Moum 2000; Antonia & Krogstad 2001; Pouransari et al. 2015), and for the fine-scale anisotropy dissipation tensor is used (Mansour et al. 1988; Antonia et al. 1991, 1994; Djenidi & Tardu 2012). The anisotropic Reynolds stress and dissipation tensors ($b_{ij}$ and $d_{ij}$) are defined in a Cartesian co-ordinate system as,

\[
\begin{align*}
    b_{ij} &= \frac{u'_i u'_j}{2q} - \frac{1}{3} \delta_{ij}, \\
    d_{ij} &= \frac{\varepsilon_{ij}}{\varepsilon_{ii}} - \frac{1}{3} \delta_{ij},
\end{align*}
\]

where overbar indicates averaging over time, $u'_i$ are the turbulent fluctuations in the velocity field ($i = 1, 2, 3$), $\delta_{ij}$ is the Kronecker delta, $q$ is the averaged turbulent kinetic energy, $\varepsilon_{ij}$ is the dissipation rate of $u'_i u'_j$, and $\nu$ is the kinematic viscosity of the fluid. Both of these tensors become zero in an isotropic turbulence and their anisotropy is quantified by using the invariants of $b_{ij}$ and $d_{ij}$, an approach pioneered by Lumley & Newman (1977) and Lumley (1979), known as invariant analysis. The combined assessment of the anisotropic Reynolds stress and dissipation tensors have been used to understand whether the anisotropy in the large-scale motions persists even at the smallest scales.

Except few studies (Browne et al. 1987; Gulitski et al. 2007; Longo et al. 2017), the measurements of all the nine components of the dissipation tensor are practically very hard to obtain from point based observations. Therefore, these are mainly evaluated from the direct numerical simulations of turbulent flows (Lee & Reynolds 1987; Mansour et al. 1988; Antonia et al. 1991; Smyth & Moum 2000; Liu & Pletcher 2008; Djenidi & Tardu 2012). On the other hand, the invariants of the anisotropic Reynolds stress tensor are relatively easy to compute from point measurements. These invariants have been used extensively to deduce the anisotropy of the energy-containing motions in the smooth and rough walls of a turbulent boundary layer (Shafi & Antonia 1995; Antonia & Krogstad 2001; Smalley et al. 2002; Ashrafian & Andersson 2006), as well as over the urban canopy and in the wake regions of wind turbines in an atmospheric flow (Castro et al. 2006; Boppana et al. 2014; Ali et al. 2018). Recently, Stiperski & Calaf (2018) and Stiperski et al. (2019) have evaluated the possibility of the dependence of Monin-Obukhov scaling relations (Monin & Obukhov 1954) with the three limiting anisotropic states of the Reynolds stress tensor in atmospheric surface layer (ASL) turbulence. Klipp (2018).
The role of intermittent heat transport towards Reynolds stress anisotropy has provided an alternate method to compute the friction velocity in an atmospheric flow by utilizing the concept of invariance associated with the Reynolds stress tensor. Vercauteren et al. (2019) have studied the anisotropy properties of the Reynolds stress tensor to relate those to the scale interactions associated with different flow regimes in a stable ABL. In atmospheric turbulence, some authors have also adopted a scale decomposition description of the Reynolds stress tensor, as discussed below.

By using this scale based decomposition, Klipp (2014) provided new insight into the description of the outer length-scale of the ABL related to the transition between isotropic and anisotropic turbulence. Using a similar approach, Liu et al. (2017) and Falocchi et al. (2019) investigated how atmospheric stability modifies isotropic and anisotropic states above an urban canopy and in a complex terrain for different scales of motions. Brugger et al. (2018) studied how the anisotropy produced at large scales relaxes to quasi-isotropic conditions at progressively smaller scales by following the trajectory on the anisotropic invariant maps for atmospheric and canopy surface layer flows.

Other than scale decomposition, there is an another approach where an event based description of turbulence is employed. This approach is based on the fact that coherent physical structures exist in a turbulent flow (Chapman & Tobak 1985; Narasimha & Kailas 1990; Kailas & Narasimha 1994; Högström & Bergström 1996; Baron & Quadrio 1997; Narasimha et al. 2007). Specifically, Narasimha et al. (2007) mentioned that in this approach, the turbulent field can be expressed in terms of events or flow structures, given that its types, magnitudes, arrival times, etc. are defined properly. The interest in the event based description of turbulence started with the flow visualization studies of Kline et al. (1967), Corino & Brodkey (1969), and Kim et al. (1971). They observed that the flow near the wall of a boundary layer was organized into streaks of high-and low-momentum fluid. The low-momentum streaks were seen to intermittently erupt away from the wall in a chaotic process named bursting. This accounted for much of the outward vertical transport of momentum and the production of turbulent kinetic energy in the boundary layer. These burst events were detected from the point measurements of velocity components by quadrant analysis (Lu & Willmarth 1973). A detailed review of different conditional sampling techniques to detect events in turbulence can be found in Antonia (1981) and Wallace (2016). The types of coherent structures whose signatures are associated with these events are reviewed in detail by Cantwell (1981), Robinson (1991), and Jiménez (2018).

Sreenivasan et al. (1979) first applied this event based description to investigate the non-zero temperature gradient skewness, which was in apparent violation of the isotropy of the fine-scale structures. Based on the premise that the fine structures were superposed on the large structures, Sreenivasan et al. (1979) extracted the coherent ramp-cliff patterns in a heated turbulent jet, and then subtracted these patterns from the signal to get the superposed fluctuations. Their focus was to show that the skewness in the temperature gradient vanishes for the fine structures, thus confirming the local isotropy. Recently following the work of Lozano-Durán et al. (2012), Dong et al. (2017) studied the connected regions of high-intensity momentum zones in three-dimensional simulations of homogeneous shear and channel flows and investigated the flow anisotropy. They quantified anisotropy by the invariants of the Reynolds stress tensor within these high-intensity momentum zones along with their sizes; where the size was defined as the box-diagonal of the parallelepiped which circumscribed these connected regions. Zhou & Xia (2011) attempted to disentangle the role of thermal plumes on the velocity field in a Rayleigh-Bénard convection, by studying separately the anisotropy in the inertial subrange of the positive and negative vertical velocity fluctuations. They showed that the negative fluctuations at small separations deviated from the Kolmogorov scaling, which
## Table 1: A brief summary of different approaches and statistic used to study anisotropy in a turbulent flow.

| Authors                  | Approach                           | Statistic                                      | Remarks                                           |
|--------------------------|------------------------------------|------------------------------------------------|--------------------------------------------------|
| Chamecki & Dias (2004)   | Scale-decomposition                | Spectra and structure functions                | Test of local isotropy hypothesis                |
| Kurien & Sreenivasan (2000) | Scale-decomposition                | SO(3) decomposition of structure functions     | Anisotropy in small scale motions                 |
| Djenidi & Tardu (2012)   | Bulk-statistics                    | Reynolds stress and dissipation tensors         | Large- and small-scale anisotropy                |
| Djenidi et al. (2009)    | Bulk-statistics                    | Taylor’s anisotropy coefficient                 | Anisotropy in energy-containing motions           |
| Salesky et al. (2017)    | Bulk-statistics                    | Vertical and horizontal velocity variance ratio | Anisotropy in energy-containing motions           |
| Liu et al. (2017)        | Bulk-statistics and scale-decomposition | Scale-decomposed Reynolds stress tensor        | Scale description of anisotropy in an urban surface layer |
| Dong et al. (2017)       | Event based description            | Reynolds stress tensor                          | Flow anisotropy associated with coherent structures |
| Zhou & Xia (2011)        | Event based description and scale-decomposition | Conditionally sampled structure functions       | Anisotropy in positive and negative velocity fluctuations |

they attributed to the presence of thermal plumes. To the best of our knowledge, very few studies have adopted an event-based approach to investigate the turbulence anisotropy directly associated with its coherent structures (see table 1 for a brief summary; for details see Biferale & Procaccia (2005)).

In buoyancy-driven turbulence, buoyant structures, such as thermal plumes, are predominant coherent structures that transport heat and drive the flow (Celani et al. 2001; Shang et al. 2003). These thermal plumes are well-organized structures of warm-rising (warm-updrafts) and cold-descending (cold-downdrafts) fluid which generate ramp-cliff patterns in temperature time series when passing a thermal probe (Zhou & Xia 2002). Shang et al. (2003) have shown that in turbulent Rayleigh Bénard convection, the time series of the instantaneous vertical heat flux associated with the thermal plumes displays intermittent characteristics. Intermittency is defined as a property of the turbulent signal which is quiescent for much of the time and occasionally burst into life with unexpectedly high values more common than in a Gaussian signal (e.g., Davidson 2015). However, the effect of this intermittent heat transport on the anisotropic fluctuations in the velocity field of a buoyancy-driven turbulence is not yet well understood, as acknowledged by Pouransari et al. (2015). This problem is particularly relevant for the surface layer of a convectively driven ABL, where the most prevalent coherent structures are the thermal plumes and the heat transport characteristics associated with these plumes appear to
The role of intermittent heat transport towards Reynolds stress anisotropy

be intermittent (Duncan & Schuepp 1992; Katul et al. 1994; Caramori et al. 1994; Chu et al. 1996; Katul et al. 1997a; Li & Bou-Zeid 2011).

The previous works on the surface layer plumes have focused on: deducing their detailed structures and dynamics (Wilczak 1984; Zhuang 1995); to identify the coupling between the surface and air temperatures (Garai & Kleissl 2011, 2013); and investigating the difference in the Monin-Obukhov similarity functions by conditioning on the updraft and downdraft motions (Li et al. 2018; Fodor et al. 2019). However, some early investigators noted that in an unstable ASL there are certain intermittent bursts in the upward heat flux, persisting for around 10-20 s of duration, which are associated with large downward momentum transport (Kaimal 1969; Kaimal & Businger 1970; Haugen et al. 1971).

Given that the averaged heat and momentum fluxes are almost uncorrelated in unstable conditions, these investigators concluded that the momentum transport is very sensitive to various convective circulations in the boundary layer. They commented that some of these circulations transport momentum downward in large bursts, while others transport it upward. Businger (1973) coined these intermittent momentum bursts associated with upward heat flux as “convection-induced stress”. Later, Maitani & Ohtaki (1987) observed that in an unstable surface layer over a plant canopy, the warm-updrafts were more efficient for the downward momentum flux. Recently Li et al. (2018) demonstrated from numerical simulations that in highly-convective conditions, the coherent warm-updrafts are associated with downward momentum transport, whereas the cold-downdrafts carried momentum both in upward and downward directions. Lotfy et al. (2019) also obtained the same conclusion from a field experiment in an ASL, where they observed that the persistent warm-updrafts of 10-20 s duration carried a large amount of momentum flux in the downward direction. By investigating the large eddy simulation results in convective conditions, Salesky & Anderson (2018) interpreted this phenomenon as a buoyancy-dominated scale modulation effect, where small-scale turbulence is excited in the warm-updraft regions contributing to the momentum flux.

From the discussion above, it becomes apparent that in an unstable surface layer the coherent heat transport events associated with the thermal plumes are related to intermittent bursts of momentum, either in upward or downward direction, depending upon the convective circulation patterns. Since only the anisotropic part of the velocity fluctuations can carry momentum (Dey et al. 2018; Könözsy 2019), it indicates that the flow anisotropy associated with these coherent heat flux events must be different from the averaged whole flow. Therefore, this can be systematically studied in an event based framework by performing an invariant analysis of the anisotropic Reynolds stress tensor.

Pouransari et al. (2015) emphasized that, a combined assessment of Reynolds stress anisotropy associated with intermittent fluctuations in the coherent structures is a state-of-the-art theoretical and experimental problem. However, there are no comprehensive studies based on an invariant analysis of the Reynolds stress tensor to quantify the flow anisotropy associated with the intermittent heat transport events in an unstable ASL. The present study is a humble effort to fill this gap, using a field-experimental dataset. We define our objectives as:

(i) To investigate the detailed correspondence between the heat transport events and Reynolds stress anisotropy in an unstable ASL.

(ii) To formulate a structural description of the thermal plumes associated with these heat transport events and investigate whether they have any characteristic length scales related to the degree of isotropy of the Reynolds stress tensor.

The present paper is organized in three different sections. In §2 we describe the dataset and methodology to develop various statistical measures to quantify the flow anisotropy
associated with heat transport events. In §3 we present and discuss the results and in §4 we conclude our findings and provide future directions for further research.

2. Data and Methodology

We have used the dataset from Surface Layer Turbulence and Environmental Science Test (SLTEST) experiment. The SLTEST experiment was conducted over a flat and homogeneous terrain at the Great Salt Lake desert in Utah, USA (40.14° N, 113.5° W), with the aerodynamic roughness length \( z_0 \) being \( z_0 \approx 5 \text{ mm} \) (Metzger et al. 2007). The SLTEST site characteristics and the high quality of the dataset have been documented in details in many previous studies (Hutchins & Marusic 2007; Hutchins et al. 2012; Chauhan et al. 2013; Marusic et al. 2013; Chowdhuri et al. 2019). In this experiment, nine north-facing sonic anemometers (CSAT3, Campbell Scientific, Logan, USA) were installed on a 30-m tower approximately logarithmically at \( z = 1.4, 2.1, 3, 4.3, 6.1, 8.7, 12.5, 17.9, 25.7 \text{ m} \), levelled to within ±0.5° from the true vertical. All CSAT3 sonic anemometers were synchronized in time and the sampling frequency was set at 20 Hz. The experiment ran continuously for nine days from 26 May 2005 to 03 June 2005.

2.1. Data Processing

The data were divided into 30-min periods containing the 20-Hz measurements of the three wind components and the sonic temperature from all the nine sonic anemometers. To select the 30-min periods for analysis, we followed these standard procedures listed below:

(i) The 30-min periods were selected from the fair weather condition during the daytime periods with no rain.

(ii) The time series of all the three components of velocity and sonic temperature were plotted and visually checked. No spikes were found in the data.

(iii) The horizontal wind direction sector was limited to \(-30° < \theta < 30°\) (where \( \theta \) is the horizontal wind direction from the North).

(iv) The coordinate systems of all the nine sonic anemometers were rotated in the streamwise direction by applying the double-rotation method of Kaimal & Finnigan (1994) for each 30-min period. The turbulent fluctuations in the wind components \( (u', v', w') \) in the streamwise, cross-stream, and vertical directions respectively, and in the sonic temperature \( (T') \) were calculated after removing the 30-min linear trend from the associated variables (Donateo et al. 2017).

(v) Only those 30-min periods were chosen when the surface layer was unstable, i.e. the sensible heat flux was positive at all the nine measurement heights, and the vertical variations in the 30-min averaged momentum and heat fluxes were less than 10 %.

Application of all these checks resulted in a total of 29 periods suitable for our analysis. For these periods \( \sigma_u/\overline{u} \) was less than 0.2, so the Taylor’s hypothesis could be assumed to be valid (Willis & Deardorff 1976). The Obukhov length \( (L) \) was calculated for each of these 30-min periods as,

\[
L = -\frac{u_*^3 T_0}{kg H_0},
\]

where \( T_0 \) is the mean temperature of the surface layer, computed from the mean sonic temperature at \( z = 1.4 \text{ m} \), \( g \) is the acceleration due to gravity (9.8 m s\(^{-2}\)), \( H_0 \) is the surface kinematic heat flux, computed as \( \overline{w' T'} \) at \( z = 1.4 \text{ m} \) (by constant flux layer assumption), \( k \) is the von Kármán constant (0.4), and \( u_* \) is the friction velocity computed
The role of intermittent heat transport towards Reynolds stress anisotropy

| Stability class | Number of 30-min runs | Heights |
|-----------------|-----------------------|---------|
| $-\zeta > 2$    | 55                    | $z = 6.1, 8.7, 12.5, 17.9, 25.7$ m |
| $1 < -\zeta < 2$ | 53                    | $z = 3, 4.3, 6.1, 8.7, 12.5, 17.9, 25.7$ m |
| $0.6 < -\zeta < 1$ | 41                    | $z = 2.1, 3, 4.3, 6.1, 8.7, 12.5, 17.9$ m |
| $0.4 < -\zeta < 0.6$ | 34                    | $z = 1.4, 2.1, 3, 4.3, 6.1, 8.7$ m |
| $0.2 < -\zeta < 0.4$ | 44                    | $z = 1.4, 2.1, 3, 4.3, 6.1$ m |
| $0 < -\zeta < 0.2$ | 34                    | $z = 1.4, 2.1, 3$ m |

Table 2: The six different stability classes formed from the ratio $-\zeta = z/L$ in an unstable ASL flow, from highly-convective ($-\zeta > 2$) to near-neutral ($0 < -\zeta < 0.2$). The associated heights with each of the stability classes are also given.

as,

$$u_\ast = (\overline{u'w'^2} + \overline{v'w'^2})^{\frac{1}{4}},$$ (2.2)

where $\overline{u'w'^2}$ and $\overline{v'w'^2}$ are the streamwise and cross-stream momentum fluxes respectively, computed at $z = 1.4$ m. Associated with $u_\ast$, the temperature scale ($T_\ast$) is defined as $H_0/u_\ast$ [Monin & Yaglom 1971].

The range of $-L$ values was between 2 to 20 m for these 29 periods suitable for our analysis. Since each 30-min period consisted of the nine level time-synchronized turbulence measurements from the CSAT3 sonic anemometers, a total of 261 combinations of the stability ratios ($\zeta = z/L$) were possible for these selected periods. The entire range of $-\zeta$ (12 $\leq$ $\zeta$ $\leq$ 0.07) was divided into six stability classes [Liu et al. 2011] and these were considered for the detailed analysis of the flow anisotropy associated with the heat transport events (see table 2). We discuss the analysis methods in the following sections.

2.2. Quadrant Analysis

The quadrant analysis is a conditional-sampling method of investigating the contributions to the turbulent transport of scalars and momentum in terms of the organized eddy motions present in the flow [Wallace 2016]. The four different quadrants of the $u'-w'$ and $T'-w'$ planes are defined in table 3. In the $T'-w'$ ($u'-w'$) quadrant plane, the warm-updrafts (I) (ejections (II)) and cold-downdrafts (III) (sweeps (IV)) are the co-gradient motions. Whereas the other two quadrants represent the counter-gradient motions generated due to the turbulent swirls in the flow [Gasteuil et al. 2007].

In the quadrant analysis method applied to the ASL, the momentum or heat flux fractions and time fractions from each quadrant of $u'-w'$ or $T'-w'$ are reported over smooth and rough surfaces [McBean 1974, Antonia 1977, Narasimha et al. 2007, Zou et al. 2017]. The flux fractions ($F_f$) and time fractions ($T_f$) for each quadrant (X) are
Table 3: The four quadrants of $u'-w'$ and $T'-w'$ in an unstable ASL.

| Quadrant name          | $u'-w'$ quadrant                        | $T'-w'$ quadrant                        |
|------------------------|----------------------------------------|----------------------------------------|
| Ejection               | $u' < 0, w' > 0$ (II)                  | $w' > 0, T' > 0$ (I) (warm-updraft)    |
| Sweep                  | $u' > 0, w' < 0$ (IV)                  | $w' < 0, T' > 0$ (III) (cold-downdraft) |
| Outward interaction    | $u' > 0, w' > 0$ (I)                   | $w' > 0, T' < 0$ (II) (cold-updraft)   |
| Inward interaction     | $u' < 0, w' < 0$ (III)                 | $w' < 0, T' > 0$ (IV) (warm-downdraft) |

evaluated as,

$$(F_f)_X = \frac{\sum[w'x']I_X}{\sum w'x'}, \quad (x = u, T) \quad (2.3)$$

and $N$ is the total number of points in a run.

However, following [Chowdhuri & Burman (2019)], we extend the quadrant analysis method to study the flow anisotropy and associated topology of the Reynolds stress tensor in relation to the heat transport events occurring in $T'-w'$ quadrant plane. We normalize $w'$ and $T'$ by their respective standard deviations, and use the symbol $\hat{x}$ to denote the turbulent fluctuations in $x$ normalized by its standard deviation ($\hat{x} = x'/\sigma_x$, where $x$ can be $u$, $w$, or $T$). Before describing the methodology, we give a short description of the Reynolds stress anisotropy.

2.2.1. Anisotropic Reynolds stress tensor

The anisotropic part of the Reynolds stress tensor is widely used to express the anisotropy in the energy-containing motions (Pope 2000), and is defined in the Cartesian tensor notation as:

$$b_{ij} = \frac{u_i'u_j'}{2q} - \frac{1}{3} \delta_{ij}, \quad q = \frac{u_i'u_i'}{2}, \quad (2.4)$$

where $i = 1, 2, 3$ denote the streamwise, cross-stream, and vertical directions, $q$ is the turbulent kinetic energy, and $\delta_{ij}$ is the Kronecker delta. Note that $b_{ij}$ is a symmetric and trace-less tensor, bounded between $-1/3 \leq b_{ij} \leq 2/3$, and equal to zero for isotropic turbulence (Könözsy 2019). From the Cayley-Hamilton theorem (Lumley 1979; Pope 2000), the two invariants $\xi$ and $\eta$ of $b_{ij}$ are defined as,

$$6\xi^3 = b_{ij}b_{jk}b_{ki}, \quad (2.5)$$

and

$$6\eta^2 = b_{ij}b_{ji}. \quad (2.6)$$

where $\xi$ represents the topology of the Reynolds stress tensor and $\eta$ represents the degree of isotropy.

The different realizable anisotropic states of turbulence are defined based on the values of $\xi$ and $\eta$ and are represented on the $\xi$-$\eta$ plane, known as the anisotropy invariant map.
This has three limiting anisotropic states based on the shape of the energy distribution in the three principle axes associated with the three eigenvectors of $b_{ij}$, also known as the componentality of turbulence (Kassinos et al. 2001, Simonsen & Krogstad 2005). These three limiting states of $b_{ij}$ are 1-component anisotropy (rod-like energy distribution), 2-component anisotropy (disk-like energy distribution), and 3-component isotropy (spherical energy distribution). An alternative to the anisotropy invariant maps is the barycentric map introduced by Banerjee et al. (2007), where every realizable anisotropic state is expressed as a linear combination of the three limiting states with coefficients $C_1$, $C_2$, and $C_3$ defined as:

$$
C_1 = e_1 - e_2
$$

$$
C_2 = 2(e_2 - e_3),
$$

$$
C_3 = 3e_3 + 1
$$

with

$$
C_1 + C_2 + C_3 = 1,
$$

where $e_1$, $e_2$, $e_3$ are the three eigenvalues of $b_{ij}$ in the order $e_1 > e_2 > e_3$ (Liu et al. 2017, Brugger et al. 2018). Given the linearity in the construction of the barycentric map, it provides a non-distorted visualization of anisotropy (Radenković et al. 2014). Banerjee et al. (2007) defined the coefficient $C_3$ as the degree of isotropy bounded between $0 \leq C_3 \leq 1$, such that the higher the value of $C_3$ the more isotropic turbulence is. The anisotropic states of $b_{ij}$ can be represented by the $RGB$ colour map of Emory & Iaccarino (2014) as,

$$
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix} = C_1 \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix} + C_2 \begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix} + C_3 \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix},
$$

such that the 1-component anisotropy is red, 2-component anisotropy is green, and the isotropic turbulence is blue, and all other states within the barycentric map are linear combinations of these three colours.

Since in this study we will be using the barycentric map to visualize the shape of the tensor $b_{ij}$ and its anisotropic characteristics, some details about its construction is appropriate here. The barycentric map is spanned by an Euclidean domain where the three limiting states of $b_{ij}$ (1-component, 2-component, and isotropic) are placed at the three vertices of an equilateral triangle having the coordinates $(0, 0)$ for the 2-component ($2C$) anisotropic state, $(1, 0)$ for the 1-component ($1C$) anisotropic state, and $(1/2, \sqrt{3}/2)$ for the isotropic ($3C$) state (Stiperski & Calaf 2018). For visualizing the anisotropic states, we have employed the $RGB$ colour map of Emory & Iaccarino (2014) (see (2.9)). The coordinate system $(x, y)$ of the barycentric map is defined as,

$$
x = C_1 x_{1C} + C_2 x_{2C} + C_3 x_{3C}
$$

$$
= C_1 + \frac{C_3}{2},
$$

and

$$
y = C_1 y_{1C} + C_2 y_{2C} + C_3 y_{3C}
$$

$$
= \frac{\sqrt{3}}{2} C_3,
$$

such that the distance from the base of the equilateral triangle is directly proportional to the degree of isotropy of $b_{ij}$ (Stiperski & Calaf 2018).
2.2.2. Representation of anisotropy on $\hat{T}$-$\hat{w}$ quadrant plane

To study the detailed correspondence between the anisotropic states of $b_{ij}$ and the heat transport events occurring in the $\hat{T}$-$\hat{w}$ quadrant plane, we first linearly bin $\hat{T}$ and $\hat{w}$ into a uniform 50x50 grid for each run belonging to a particular stability class. The widths of each grid are defined as,

$$d\hat{T} = \frac{\hat{T}_{\text{max}} - \hat{T}_{\text{min}}}{50}$$

and

$$d\hat{w} = \frac{\hat{w}_{\text{max}} - \hat{w}_{\text{min}}}{50}.$$  \hspace{1cm} (2.12)

We choose the maximum ($\hat{T}_{\text{max}}$, $\hat{w}_{\text{max}}$) and minimum values ($\hat{T}_{\text{min}}$, $\hat{w}_{\text{min}}$) over all the runs from a particular stability class to ensure the same grid for individual runs. Subsequently, we find the points lying between $\{\hat{T}_{\text{bin}}(m) < \hat{T} < \hat{T}_{\text{bin}}(m) + d\hat{T}, \hat{w}_{\text{bin}}(n) < \hat{w} < \hat{w}_{\text{bin}}(n) + d\hat{w}\}$, where $1 \leq m \leq 50$, $1 \leq n \leq 50$, and $\hat{T}_{\text{bin}}(m)$ and $\hat{w}_{\text{bin}}(n)$ are the edges of a particular $(m, n)$ grid. For these points, we construct the anisotropic Reynolds stress tensor at $(m, n)$ grid as,

$$b_{ij}\{\hat{T}_{\text{bin}}(m) < \hat{T} < \hat{T}_{\text{bin}}(m) + d\hat{T}, \hat{w}_{\text{bin}}(n) < \hat{w} < \hat{w}_{\text{bin}}(n) + d\hat{w}\} = \left(\begin{array}{c}
\frac{\sum u'_i u'_j}{\sum u'^2} - \frac{1}{3} \delta_{ij},
\end{array}\right)$$

and assign it to the value $\{\hat{T}_{\text{bin}}(m), \hat{w}_{\text{bin}}(n)\}$. In (2.14), the terms $(\sum u'_i u'_j)_{m,n}$ are the contributions to the Reynolds stress tensor from each $(m, n)$ grid. The trace of $b_{ij}$ from (2.14) can be written as,

$$\left[\frac{\sum u'^2}{\sum u'^2 + \sum u'^2 + \sum w'^2} - \frac{1}{3}\right]$$

$$+ \left[\frac{\sum v'^2}{\sum u'^2 + \sum v'^2 + \sum w'^2} - \frac{1}{3}\right]$$

$$+ \left[\frac{\sum w'^2}{\sum u'^2 + \sum v'^2 + \sum w'^2} - \frac{1}{3}\right].$$  \hspace{1cm} (2.15)

This sum goes to zero due to the kinetic energy term $(\sum u'_i u'_i)_{m,n}$ appearing in the denominator. Note that, this kinetic energy is the energy contained in each $(m, n)$ grid, rather than the total kinetic energy over the whole 30-min period. This formulation is similar to the scale decomposition of $b_{ij}$, where at each scale the anisotropic Reynolds stress tensor is normalized by the kinetic energy contained in that scale to make it trace-free [Yeung & Brasseur 1991, Liu et al. 2017, Brugger et al. 2018].

To assess the frequency of occurrences of these heat transport events, we also compute the joint probability density function (JPDF) between $\hat{T}$ and $\hat{w}$ [Tennekes & Lumley 1972] as,

$$P(\hat{T}_{\text{bin}}(m), \hat{w}_{\text{bin}}(n)) = \frac{N_{m,n}}{N \ d\hat{T} \ d\hat{w}},$$

where $N_{m,n}$ is the number of points lying in $(m, n)$ grid and $N$ is the total number of points in a 30-min run (36000 for SLTEST data). Clearly,

$$\int_{\hat{w}_{\text{min}}}^{\hat{w}_{\text{max}}} \int_{\hat{T}_{\text{min}}}^{\hat{T}_{\text{max}}} P(\hat{T}_{\text{bin}}(m), \hat{w}_{\text{bin}}(n)) \ d\hat{T} \ d\hat{w} = 1.$$  \hspace{1cm} (2.17)
Following Nakagawa & Nezu (1977), we also calculate the bivariate Gaussian JPDF for each grid as,

\[
G(\hat{T}_{\text{bin}}(m), \hat{w}_{\text{bin}}(n)) = \frac{1}{2\pi \sqrt{1 - R^2_{wT}}} \exp \left[ -\left( \frac{\hat{T}_{\text{bin}}^2(m) - 2R_{wT}\hat{T}_{\text{bin}}(m)\hat{w}_{\text{bin}}(n) + \hat{w}_{\text{bin}}^2(n)}{2(1 - R^2_{wT})} \right) \right],
\]

where \(R_{wT}\) is the correlation coefficient between \(w\) and \(T\) (\(w^T\sigma_{wT})\).

If the three eigenvalues of \(b_{ij}\) (as defined in (2.14)) are \(e_{1b}, e_{2b}, \) and \(e_{3b}\) respectively with \(e_{1b} > e_{2b} > e_{3b}\), we can calculate the degree of isotropy for \((m, n)\) grid as,

\[
C_3\{|\hat{T}_{\text{bin}}(m), \hat{w}_{\text{bin}}(n)\} = 3e_{3b} + 1,
\]

and the RGB colour map of the anisotropic states as,

\[
\begin{bmatrix}
\hat{R} \\
\hat{G} \\
\hat{B}
\end{bmatrix} = C_1\{|\hat{T}_{\text{bin}}(m), \hat{w}_{\text{bin}}(n)\} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + C_2\{|\hat{T}_{\text{bin}}(m), \hat{w}_{\text{bin}}(n)\} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + C_3\{|\hat{T}_{\text{bin}}(m), \hat{w}_{\text{bin}}(n)\} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},
\]

with

\[
C_1\{|\hat{T}_{\text{bin}}(m), \hat{w}_{\text{bin}}(n)\} = e_{1b} - e_{2b}
\]

\[
C_2\{|\hat{T}_{\text{bin}}(m), \hat{w}_{\text{bin}}(n)\} = 2(e_{2b} - e_{3b}).
\]

Since we construct the same linear grid values of \(\hat{T}\) and \(\hat{w}\) for all the runs belonging to a particular stability class, we take the average of the JPDF, \(C_3\), and the RGB colour matrices over all the individual periods. This averaging is necessary since it reduces the variability which exists from one run to another, due to the chaotic nature of turbulence. For a particular stability range, we can thus plot the averaged two-dimensional matrices of \(P(\hat{T}_{\text{bin}}(m), \hat{w}_{\text{bin}}(n)), C_3\{|\hat{T}_{\text{bin}}(m), \hat{w}_{\text{bin}}(n)\}, \) and the RGB colour maps for each \((m, n)\) grid of \(T-\hat{w}\) quadrant plane. The contour maps of these matrices help to assess the anisotropic characteristics of the Reynolds stress tensor associated with the heat transport events of varying intensities and frequency of occurrences. While presenting the results in \(3.2\) these averaged metrics are referred to as being associated with \((\hat{T}, \hat{w})\), without explicitly mentioning these are the binned values. It is worth to note that the results obtained from this method are not sensitive to the choice of the grid size. We verified this by choosing three different grid sizes of \(\hat{T}\) and \(\hat{w}\) (\(40\times40, 50\times50, \) and \(60\times60\)) and repeating the calculations, with no change being noticed in the results (not shown).

By performing the binning exercise in \(\hat{w}\) and \(\hat{T}\) as discussed above, we mask any time dependence and hence no information can be obtained about the time scales of the associated heat transporting events. Over the course of time these events from each quadrant can occur with a range of different time scales as they tend to exit and re-enter to their respective quadrant states (Kalmár-Nagy & Varga 2019). It is thus interesting to formulate a description of the distribution of Reynolds stress anisotropy associated with different time scales of these heat transporting events. To extract that information in addition to the quadrant analysis, we turn our attention to persistence analysis.
2.3. Persistence analysis

In non-equilibrium systems, persistence is defined as the probability that the local value of a fluctuating field does not change sign up to a certain time (e.g., Majumdar 1999; Bray et al. 2013; Ghannam et al. 2016). The concept of persistence has earlier been used by Chamecki (2013) to study the non-Gaussian turbulence in canopy flows. He showed that an asymmetric velocity distribution inside the canopy can have very different persistent time scales for ejection and sweep events. Chamecki (2013) also noted that the persistent time is equivalent to the inter-pulse periods between the subsequent zero crossings of the turbulent signal (Sreenivasan et al. 1983; Kailasnath & Sreenivasan 1993; Bershadskii et al. 2004; Cava & Katul 2009). We can apply this definition of persistence to the joint fluctuations in vertical velocity and temperature, to characterize the distribution of the time scales of the heat flux events from four different quadrants of $T'-w'$.

In order to implement our method, we choose the time series of $w'$ and $T'$ from any 30-min period belonging to a specific stability class (see table 2), and conditionally sample the events occurring in the four different quadrants of $T'-w'$ plane. The events conditionally sampled from each quadrant of $T'-w'$ (I, II, III, or IV) can either persist as a single pulse or as a block of many consecutive pulses with a certain duration $T_B$, before switching to another quadrant. The duration $T_B$ is computed as the number of points residing within a single block, multiplied by the sampling interval of 0.05 s. In figure 1 we provide a graphical illustration of this method by showing a segment of a time series belonging to a particular stability range ($-\zeta = 9$), sampled from the warm-updraft (I) and cold-downdraft (III) quadrants. The shaded blocks in figures 1a to b represent warm-updrafts (red) and cold-downdrafts (blue) respectively, which persist for around 10–20 sec of duration. Associated with these blocks of warm-updrafts (red) and cold-downdrafts (blue) we also show the horizontal velocity fluctuations ($u'$ and $v'$) in figures 1c and d.

We convert the block duration ($T_B$) to a streamwise length by using the Taylor’s hypothesis, that is multiplying $T_B$ with the mean wind speed ($\overline{u}$) computed over the 30-min period. We then scale $T_B\overline{u}$ with a relevant length scale. The possible candidates as the relevant length scales in an unstable ASL are the measurement height $z$ and the boundary-layer depth $z_i$. However, $z_i$ was not measured directly at SLTEST and hence an alternate large-eddy length scale $\lambda$ was used by Chowdhuri et al. (2019), where $\lambda$ was computed as the peak wavelength of the horizontal velocity spectrum at $z = 25.7$ m. This was based on the observation that the large-scale structures contribute directly to the horizontal velocity spectrum in the ASL (Busch & Panofsky 1968; Kaimal et al. 1976; Panofsky et al. 1977; McNaughton et al. 2007; Banerjee & Katul 2013; Banerjee et al. 2015). As discussed by Chowdhuri et al. (2019), a model spectrum of the form,

$$\kappa S_{uu}(\kappa) = \frac{ak}{(1 + bk)^{5/3}},$$

(2.22)

is fitted to the streamwise velocity ($u$) spectrum, where $\kappa$ is the streamwise wavenumber and $a$, $b$ are the best fit constants. By maximizing (2.22) with respect to $\kappa$, $\lambda$ is evaluated as $4\pi b/3$. The other details and the rationale behind the computation of $\lambda$ can be found in Chowdhuri et al. (2019).

The spectrum or the scalewise distribution of the normalized streamwise lengths of the blocks ($T_B\overline{u}/\ell$, where $\ell$ can be either $z$, $\lambda$, or the combination of the two) can be at least few decades wide, given the large variation in $T_B$, ranging from a minimum of 0.05 s (sampling interval) to few seconds. For the blocks associated with each $T'-w'$ quadrant, we thus logarithmically bin their scaled streamwise lengths ($T_B\overline{u}/\ell$) into 60
The role of intermittent heat transport towards Reynolds stress anisotropy

Figure 1: A 60-s long section of a time series of $w'$, $T'$, $u'$, and $v'$ for $-\zeta = 9$. The red- and blue-shaded regions show two particular blocks of warm-updrafts and cold-downdrafts respectively, which persist for a time $T_B$ (around 10-20 s).

bins, where the minimum and maximum are chosen over all the 30-min periods which fall within a particular stability class. Below we discuss the method to compute the Reynolds stress anisotropy associated with these blocks of different normalized streamwise lengths. Broadly speaking, this description can be considered to be a one-dimensional analogue of the analysis carried out by [Dong et al. (2017)]. Instead of defining the sizes of the structures as connected regions in a three-dimensional space, we define those as connected points in streamwise direction after converting the temporal signals into spatial signals using Taylor’s hypothesis.

2.3.1. The distribution of the Reynolds stress anisotropy

For any particular $T'$-$w'$ quadrant, we collect all the blocks of the events having their normalized streamwise lengths between

$$(T_B \overline{u}/\ell)_{\text{bin}} \{m\} < (T_B \overline{u}/\ell) < (T_B \overline{u}/\ell)_{\text{bin}} \{m\} + d \log (T_B \overline{u}/\ell),$$

where $(T_B \overline{u}/\ell)_{\text{bin}} \{m\}$ is the logarithmically binned value, $d \log (T_B \overline{u}/\ell)$ is the bin-width, and $m$ is the index of the bin ($1 \leq m \leq 60$). The bin-width is defined as,

$$d \log (T_B \overline{u}/\ell) = \frac{\log (T_B \overline{u}/\ell)_{\text{max}} - \log (T_B \overline{u}/\ell)_{\text{min}}}{60}.$$  

We construct the Reynolds stress tensor associated with these blocks as,

$$b_{ij} \left[ (T_B \overline{u}/\ell)_{\text{bin}} \{m\} < (T_B \overline{u}/\ell) < (T_B \overline{u}/\ell)_{\text{bin}} \{m\} + d \log (T_B \overline{u}/\ell) \right] =$$

\[
\sum_{i} w'_i w'_j - \frac{1}{3} \delta_{ij},
\]  

(2.24)
and assign it to a streamwise size of $(T_B \bar{u}/\ell)_\text{bin}\{m\}$. In (2.24), the terms $\sum u'_i u'_j$ are the contributions to the Reynolds stress tensor from all the blocks having their sizes between $(T_B \bar{u}/\ell)_\text{bin}\{m\}$ and $(T_B \bar{u}/\ell)_\text{bin}\{m\} + d\log (T_B \bar{u}/\ell)$.

Similar to \S 2.2.2 we calculate the degree of isotropy and the $RGB$ colour map of $b_{ij}(T_B \bar{u}/\ell)_\text{bin}\{m\}$ as,

$$C_3|(T_B \bar{u}/\ell)_\text{bin}\{m\} = 3\tilde{e}_{3b} + 1,$$

(2.25)

and

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = C_1|(T_B \bar{u}/\ell)_\text{bin}\{m\} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + C_2|(T_B \bar{u}/\ell)_\text{bin}\{m\} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + C_3|(T_B \bar{u}/\ell)_\text{bin}\{m\} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

(2.26)

respectively, with

$$C_1|(T_B \bar{u}/\ell)_\text{bin}\{m\} = \tilde{e}_{1b} - \tilde{e}_{2b}$$

$$C_2|(T_B \bar{u}/\ell)_\text{bin}\{m\} = 2(\tilde{e}_{2b} - \tilde{e}_{3b}),$$

(2.27)

where $\tilde{e}_{1b}$, $\tilde{e}_{2b}$, and $\tilde{e}_{3b}$ are the three eigenvalues of $b_{ij}(T_B \bar{u}/\ell)_\text{bin}\{m\}$ with $\tilde{e}_{1b} > \tilde{e}_{2b} > \tilde{e}_{3b}$.

Since we construct the same logarithmic grids of $(T_B \bar{u})/\ell$ for all the runs belonging to a particular stability class, we take the average of the degree of isotropy and the $RGB$ colour metrics over all these periods to reduce the run-to-run variability.

2.3.2. Probability and flux distributions

The probability density function (PDF) of the normalized streamwise lengths of the blocks belonging to any particular $T'-u'$ quadrant is calculated as,

$$P(T_B \bar{u}/\ell)_{\text{bin}}\{m\} = \frac{N}{N_{\text{tot}}} \frac{d}{d\log (T_B \bar{u}/\ell)},$$

(2.28)

where $N$ is the number of blocks lying between,

$$(T_B \bar{u}/\ell)_{\text{bin}}\{m\} < (T_B \bar{u}/\ell) < (T_B \bar{u}/\ell)_{\text{bin}}\{m\} + d\log (T_B \bar{u}/\ell),$$

and $N_{\text{tot}}$ is the total number of blocks detected over a 30-min period (from the same quadrant). Clearly,

$$\int_{(T_B \bar{u}/\ell)_{\text{min}}}^{(T_B \bar{u}/\ell)_{\text{max}}} \left[ P(T_B \bar{u}/\ell)_{\text{bin}} \right] d\log (T_B \bar{u}/\ell) = 1.$$  

(2.29)

The heat and momentum fluxes within these blocks are defined as,

$$H_f(T_B \bar{u}/\ell)_{\text{bin}}\{m\} = \frac{\sum u'T'}{36000 \times d\log (T_B \bar{u}/\ell)},$$

$$M_f(T_B \bar{u}/\ell)_{\text{bin}}\{m\} = \frac{\sum u'u'}{36000 \times d\log (T_B \bar{u}/\ell)},$$

(2.30)

These heat and momentum flux distributions are scaled by the product of the standard deviations such as $\sigma_u \sigma_T$ and $\sigma_u \sigma_w$, respectively. When these scaled flux distributions from (2.30) are integrated over the whole spectrum of $(T_B \bar{u})/\ell$, the results show the strength of the coupling between $u'$ and $T'$ ($u'$) from each quadrant X of $T'-u'$ (X = I,
The role of intermittent heat transport towards Reynolds stress anisotropy

II, III, or IV) as,

\[
\int_{(T_B\bar{\pi}/\ell)_{\min}}^{(T_B\bar{\pi}/\ell)_{\max}} \left[ \frac{H_f(T_B\bar{\pi}/\ell)_{\text{bin}}}{\sigma_u\sigma_T} \right] d\log (T_B\bar{\pi}/\ell) = \left( \frac{\bar{u}'T'}{\sigma_u\sigma_T} \right) X 
\]

\[
\int_{(T_B\bar{\pi}/\ell)_{\min}}^{(T_B\bar{\pi}/\ell)_{\max}} \left[ \frac{M_f(T_B\bar{\pi}/\ell)_{\text{bin}}}{\sigma_u\sigma_w} \right] d\log (T_B\bar{\pi}/\ell) = \left( \frac{\bar{u}'w'}{\sigma_u\sigma_w} \right) X. \tag{2.31}
\]

Similar to §2.3.1, we take the average of the PDFs and the heat and momentum flux distributions over all the 30-min periods belonging to a particular stability class. While presenting the results in §3.3, these averaged distributions of the degree of isotropy, probability, and fluxes are referred to being associated with \((T_B\bar{\pi})/\ell\) only, without explicitly mentioning these are the binned values. Apart from that, similar to quadrant analysis, the results obtained from this method too are not sensitive to the choice of the number of bins, verified with three different bin numbers 40, 50, and 60 (not shown).

3. Results and discussion

We begin with discussing the general characteristics of turbulence anisotropy with the change in the stability ratio \(\zeta\). We also highlight the correspondence between the intermittent nature of turbulent heat transport and turbulence anisotropy. By presenting the relevant results, this correspondence is further investigated in details, complemented with the quadrant and persistence analyses of the thermal motions. The possible physical interpretations of these results are also discussed.

3.1. Turbulence anisotropic characteristics with stability

We discuss the general effect of stability on the turbulence anisotropy associated with the whole flow in an unstable ASL. We establish that along with stability, the anisotropy in the whole flow is also related to the non-Gaussian nature of turbulent heat transport associated with the thermal plumes.

Figure 2a shows the degree of isotropy \(C_3\) (see (2.7)) of the anisotropic Reynolds stress tensor for the whole flow \(b_{ij}\) (see (2.4)) plotted on the barycentric map (see (2.9), (2.10), and (2.11)) with the stability ratio \(-\zeta\). As evident from figure 2a, the turbulence approaches towards an isotropic state \(C_3 \approx 0.6\) as \(-\zeta\) approaches the local free convection limit \((\zeta > 1)\). For small values of \(-\zeta\) \((0 < -\zeta < 0.2)\) the turbulence is in a 2-component anisotropic state with \(C_3 \approx 0.1\). From table 2, it is clear that the near-neutral stability class \((0 < -\zeta < 0.2)\) corresponds only to the lowest three levels of the SLTEST experiment \((z = 1.4, 2.1, 3 \text{ m})\), where due to the blocking of the ground the vertical velocity fluctuations are suppressed. Therefore, the turbulence very close to the ground is in a 2-component anisotropic state dominated by the horizontal velocity components. This is in agreement with the studies by Krogstad & Torbergsen (2000) and Ali et al. (2018).

From figure 2b, we also note that for \(0 < -\zeta < 0.2\), the 2-component anisotropic states are associated with strong horizontal wind shear \((\partial\bar{u}/\partial z)\). However, in the limit of local free convection \((\zeta > 1)\) the effect of horizontal wind shear is weak and the turbulence approaches an isotropic state. This result is consistent with the observations of Stiperski & Calaf (2018), and it suggests that in an unstable surface layer as we approach \(z \rightarrow 0\) (associated with small values of \(-\zeta\)) the anisotropic characteristics of turbulence is dominated by strong horizontal wind shear. However as the local free convection is approached \((\zeta > 1)\), the effect of horizontal wind shear weakens and the flow becomes more isotropic. A similar conclusion was reached by Jin et al. (2003), where
they showed both analytically and from numerical simulations that in a buoyant shear flow, the effect of increase (decrease) in buoyancy (shear) was to drive the turbulence towards isotropy.

To investigate this further, figure 2 shows the scatter plot of the scaled vertical and temperature standard deviations ($\sigma_w/u_*$ and $\sigma_T/T_*$) along with the correlation coefficient between $w$ and $T$ ($R_{wT}$) and the degree of isotropy ($C_3$), against the stability ratio $-\zeta$. The local free convection scalings for $\sigma_w/u_*$ and $\sigma_T/T_*$ are given as,

$$
\sigma_w \approx 1.8(-\zeta)^{1/3} \\
\sigma_T \approx 1.05(-\zeta)^{-1/3},
$$

where the coefficients are fitted from the data and match well with the values reported by [Wyngaard et al. 1971] and [Monin & Yaglom 1971]. It is interesting to note that after $-\zeta < 0.5$, the local free convection scaling does not hold for $\sigma_w/u_*$, but it extends for $\sigma_T/T_*$. [Khanna & Brasseur 1997] explained this as, the buoyancy-induced motions contribute more to the temperature fluctuations than the shear-induced motions.

In an unstable surface layer, the horizontal velocity variances are weakly dependent on the stability ratio $-\zeta$ [Monin & Yaglom 1971] Panofsky 1974 Panofsky et al. 1977.
The role of intermittent heat transport towards Reynolds stress anisotropy

Therefore, the degree of isotropy \( C_3 \) is mainly determined by the strength of the vertical velocity fluctuations \( \langle w' \rangle \), decreasing from \( C_3 \approx 0.6 \) to \( C_3 \approx 0.1 \) as \( \sigma_w / u^* \) decreases with \( -\zeta \) (see figure 2 and figure S1 in the supplementary material). From figure 2 and figure S1b (supplementary material), we also note that \( w' \) is more strongly coupled to \( T' \) than to \( u' \) in the local free convection \( (R_{wT} \approx 0.65 \) and \( R_{uw} \approx 0.05) \). However with decrease in \( -\zeta, w' \) gets coupled to \( u' \) \( (R_{uw} \approx 0.25) \), although the strength of this coupling remains weaker than with \( T' \) \( (R_{wT} \approx 0.4) \). This is also reflected in the transport efficiencies of heat \( (\eta_{wT}) \) and momentum \( (\eta_{uw}) \) which are defined as,

\[
\eta_{wx} = \frac{(\sum w'x')_{\text{co-gradient}} + (\sum w'x')_{\text{counter-gradient}}}{(\sum w'x')_{\text{co-gradient}}},
\]

where \( x \) can be either \( u \) or \( T \) \( \text{(Li \& Bou-Zeid 2011; Bou-Zeid et al. 2018)} \). From figure S1b (supplementary material), it is evident that in local free convection \( \eta_{uw} \rightarrow 0 \) whereas \( \eta_{wT} \) almost approaches a constant value of 0.9. However, with decrease in \( -\zeta, \eta_{uw} \) increases to \( \approx 0.6 \) and \( \eta_{wT} \) decreases to \( \approx 0.75 \).

The strong coupling of \( w' \) with \( T' \) \( (R_{wT}) \) along with enhanced heat transport efficiencies \( (\eta_{wT}) \) are typically associated with coherent thermal plumes in an unstable ASL flow \( \text{(Li \& Bou-Zeid 2011)} \). These thermal plumes contribute mainly to the production of the vertical velocity and temperature variances \( (\sigma_w^2 \) and \( \sigma_T^2) \) and their heat and momentum transport characteristics are affected by the change in \( -\zeta \) (figures 2 and S1). In a strongly sheared environment \( (0 < -\zeta < 0.2) \) these plumes carry heat and momentum almost equally efficiently and are associated with more anisotropic turbulence \( (C_3 \approx 0.1) \). In the local free convection \( (-\zeta > 1) \) where the effect of wind shear is weak, these plumes carry heat more efficiently and associated with more isotropic turbulence \( (C_3 \approx 0.6) \). We next investigate the PDFs of \( T', w' \), and \( u' \) to establish a correspondence between the turbulence anisotropy and the non-Gaussian characteristics (intermittency and asymmetry) associated with these thermal plumes.

Figures 3a and b show the skewness and kurtosis of the PDFs of \( T', w' \), and \( u' \) \( (x'^3 / \sigma_x^3 \) and \( x'^4 / \sigma_x^4, \) where \( x = u, w, T \) \) along with the degree of isotropy \( (C_3) \). The associated PDFs are shown in figure S2 (supplementary material). For a perfect Gaussian distribution, the skewness and kurtosis have values of 0 and 3 respectively \( (\text{e.g., Lumley 1970}) \). Physically, the skewness is associated with the asymmetry in the PDFs whereas the kurtosis is related to intermittency \( (\text{Tennekes \& Lumley 1972; Davidson 2015; Pouransari et al. 2015}) \).

From figures 3a and b, it is clear that the skewness and kurtosis of the temperature fluctuations are strongly non-Gaussian \( (\approx 1.5 \) and \( 5 \) respectively) in the local free-convection limit \( (-\zeta > 1) \). The strong non-Gaussian nature of temperature fluctuations in highly-unstable condition is remarkably consistent with the previous studies in the ASL \( \text{(Chu et al. 1996; Liu et al. 2011; Garai \& Kleissl 2013; Lyu et al. 2018)} \). Similar behaviour has also been observed in turbulent Rayleigh–Bénard convection experiments of Adrian et al. \( (1986) \), Balachandar & Sirovich \( (1991) \) and Wang et al. \( (2019) \). The strong non-Gaussianity in \( T' \) in highly-unstable conditions is caused due to the intermittent bursting of the warm-updraft plumes rising from the ground, interspersed with relatively more frequent quiescent cold-downdraft plumes bringing well-mixed air from aloft \( \text{(Adrian et al. 1986; Chu et al. 1996)} \). However, the skewness and kurtosis of \( T' \) become closer to Gaussian \( (0.5 \) and \( 3 \) respectively) for the near-neutral stability \( (0 < -\zeta < 0.2) \). The close-to-Gaussian characteristics of the \( T" \) PDFs in a near-neutral ASL are in agreement with \( \text{Chu et al. 1996} \) and with the pipe flow experiment of Nagano & Tagawa \( (1988) \) where temperature behaved more like a passive scalar.
Figure 3: The scatter plot of the (a) skewness and (b) kurtosis of the temperature, vertical velocity, and streamwise velocity fluctuations ($T'$, $w'$, and $u'$) are shown against $-\zeta$. The red, blue, and pink coloured open circles denote $T'$, $w'$ and $u'$ respectively, with their skewness and kurtosis being plotted on the left hand side of the $y$ axis. The black stars show the degree of isotropy ($C_3$, see (2.7)) with its values being plotted on the right hand side of the $y$ axis. The thick horizontal black lines denote the values of 0 and 3, which are the skewness and kurtosis for the Gaussian distribution.

On the other hand, the PDFs of $u'$ remain near-Gaussian for all the values of $-\zeta$ with its skewness and kurtosis approaching 0 and 3 respectively. For $w'$, the skewness stays almost constant at 0.4 to 0.5 for all the values of $-\zeta$, implying the consistent upward transport of vertical kinetic energy by the thermal plumes (Chiba 1978; Hunt et al. 1988). However, the kurtosis for $w'$ increases from 3 to 4 as $-\zeta$ decreases. This observation is consistent with Chu et al. (1996) where they found the kurtosis in $w'$ increased from 3.12 in highly-unstable conditions to 3.77 in near-neutral conditions. Chiba (1984) postulated that this increase in the kurtosis of $w'$ at small $-\zeta$ values is related to the increasing importance of the small-scale eddies near the ground. However, Hong et al. (2004) hypothesized it to be related to the low-speed streaks, initiating inactive and active turbulence interactions with increasing intermittency.

We note that the degree of isotropy ($C_3$) also decreases in a similar way as the skewness and kurtosis of the temperature fluctuations approach a near-Gaussian distribution with decrease in $-\zeta$ (figure 3). Katul et al. (1997a) demonstrated that the temperature skewness was directly related to the difference in the time fractions ($\Delta T_f$) of the warm-updrafts and cold-downdrafts (asymmetry) as,

$$\Delta T_f = \frac{Q_3}{3\sqrt{2\pi}},$$  

(3.3)
The role of intermittent heat transport towards Reynolds stress anisotropy

where \( Q_3 = \frac{T'\bar{w}^3}{\sigma_3^3} \), by assuming that the time fractions spent in the counter-gradient quadrants of \( T'\bar{w}' \) plane could be ignored. This implies that the asymmetry in the distributions of the warm-updrafts and cold-downdrafts associated with the skewness of the temperature fluctuations, has a strong correspondence with the anisotropy in the Reynolds stress tensor. In figure S3 (supplementary material) we show the heat flux fractions \( (F_f) \) and the time fractions \( (T_f) \) associated with each quadrant of \( T'\bar{w}' \) plane. It indicates that in highly-unstable conditions \((-\zeta > 1)\) the warm-updrafts carry more heat flux even though they spend less time than the cold-downdrafts (see figure S3c in supplementary material).

The same observation can also be made from figure 4a, where the strong non-Gaussianity in the temperature fluctuations in highly-unstable conditions \((-\zeta > 2)\) introduces a large asymmetry in the PDF of the scaled heat flux \( (P(\hat{w}\hat{T})) \). The intermittent bursts of warm-updraft plumes characterized by large kurtosis carry more heat flux than predicted by the distribution if \( \hat{w} \) and \( \hat{T} \) were both standard Gaussian random variables. According to Krogstad (2013), the PDF of the product of two standard Gaussian random variables \( \hat{x} \) and \( \hat{y} \) can be expressed as,

\[
P(\hat{x}\hat{y}) = \frac{K_0(|\hat{x}\hat{y}|)}{\pi},
\]

where \( K_0(|\hat{x}\hat{y}|) \) is the modified Bessel function of the second kind. However, this strong non-Gaussianity is not felt in the PDFs of the scaled momentum flux \( (P(\hat{u}\hat{w})) \) as the probability distributions of \( u \) and \( w \) fluctuations are closer to Gaussian compared to temperature (figures 4a and S2).

In a nutshell from figures 3 and 4 one can infer that the characteristics of the thermal plumes in an unstable surface layer is strongly (weakly) non-Gaussian for highly (feebly) convective conditions, associated with more isotropic (anisotropic) turbulence. This implies that there is a correspondence between the intermittent heat transport events associated with these thermal plumes and turbulence anisotropy.

So far we have analyzed the anisotropic characteristics of the whole flow, which comprises of all the heat flux events. Since these heat flux events occur intermittently, we turn our attention towards the quadrant analysis to investigate the flow anisotropy associated with their intermittency and intensity.

3.2. Quadrant analysis of turbulence anisotropy

From quadrant analysis, we study the detailed correspondence between the Reynolds stress anisotropy and the heat transport events of varying intensities with their frequency of occurrences. Figure 5 shows the RGB colour map computed by (2.20) with the superposed contours of degree of isotropy (see (2.19)) on \( \hat{T}\hat{w} \) quadrant plane. We also include the hyperbolic hole, defined as \( |\hat{T}\hat{w}| = 1 \), to identify the strong heat flux producing events which lie in the region outside of it (Perry & Hoffmann 1976; Smedman et al. 2007). The six different panels in figures 5a–f correspond to the six different stability classes as mentioned in table 2.

From figure 5a, we notice that in a highly-unstable surface layer \((-\zeta > 2)\), the regions outside of the hyperbolic hole (intense heat flux events, \( |\hat{T}\hat{w}| > 1 \)) are mostly associated with either isotropic or 1-component anisotropy as indicated by blue and red colours respectively. Apart from that, 2-component anisotropy (indicated by green) is associated with thermal motions which lie mostly within the hyperbolic hole (weak heat flux events, \( |\hat{T}\hat{w}| < 1 \)). This implies that only those thermal motions designated by a specific range in the vertical velocity and temperature fluctuations which reside within the blue regions
Figure 4: The PDFs of the product of the scaled (a) heat flux \((P(\hat{\omega}\hat{T}))\) and (b) momentum flux \((P(\hat{\omega}\hat{w}))\), are shown for the six different classes of \(-\zeta\) as indicated in the legend at the right most corner. The thick black curves denote the modified Bessel function of the second kind, which corresponds to the PDF of \(\hat{\omega}\hat{x}\) \((x = u, T)\), if \(\hat{u}, \hat{w}\), and \(\hat{T}\) were all standard Gaussian random variables (see (3.4)). The grey shaded portions show the hyperbolic hole, defined as \(|\hat{\omega}\hat{x}| = 1\) \((x = u, T)\).

of \(\hat{T}\)-\(\hat{w}\) quadrant plane, are associated to isotropic turbulence. We also notice from figure 5a that the zones of isotropic turbulence (blue regions) reside mainly within the warm-updraft and cold-downdraft quadrants (I and III respectively). The turbulence is also relatively more isotropic associated with the warm-updrafts \((C_3 \approx 0.5)\) than with the cold-downdrafts \((C_3 \approx 0.4)\). By comparing the features in figure 5a with the JPDF contours (see (2.16)) in figure 6a, we note that the JPDF between \(\hat{T}\) and \(\hat{w}\) departs significantly from the bivariate Gaussian distribution (see (2.18)) in highly-unstable case. We also notice from the figures 5a and 6a that the 1-component anisotropy zones (red regions) are associated with extremely low probability events of very high heat fluxes, located well beyond the hyperbolic hole \((|\hat{T}\hat{w}| > 1)\).

However as \(-\zeta\) becomes smaller, the JPDF contours become progressively close to bivariate Gaussian distribution (figures 6a to 6f), with the green regions (2-component anisotropy) being systematically more prominent (figures 5a to 5f). On the other hand, the blue regions (isotropic turbulence) become systematically less visible (figures 5a to 5f). This is consistent with figure 2 where the turbulence displays 2-component anisotropy in the whole flow as the near-neutral stability is approached. Furthermore, this is also in agreement with figure 3 where highly anisotropic turbulence is associated with almost symmetrical distribution of the warm-updrafts and cold-downdrafts in near-neutral stability, due to the small values of skewness in \(T'\) (see (3.3)). It is interesting to note that, the 1-component anisotropy indicated by the red regions in the \(\hat{T}\)-\(\hat{w}\) quadrant
The role of intermittent heat transport towards Reynolds stress anisotropy

Figure 5: The quadrant maps of the degree of isotropy (see (2.19)) plotted on the $\hat{T}$-$\hat{w}$ quadrant plane, are shown for the six different classes of the stability ratios as indicated in the legend at the right-most corner. The RGB colour maps indicate the three limiting states of the anisotropic Reynolds stress tensor such as 1-component anisotropy (red), 2-component anisotropy (green), and 3-component isotropy (blue) respectively (see (2.20)). The thick pink lines denote the hyperbolic hole $|\hat{T}\hat{w}| = 1$. The quadrants I and III represent the warm-updrafts and cold-downdrafts respectively.

plane does not appear to have a signature in the 30-min averaged Reynolds stress anisotropy (figure 2a). This is because this anisotropic state is associated with highly-intermittent low probability events of very high heat fluxes.

To analyze this more carefully, we can simplify the gridded Reynolds stress tensor $b_{ij}|(\hat{T}, \hat{w})$ (see 2.14) by retaining its diagonal part and making all the off-diagonal Reynolds shear stress components to be 0. Mathematically, this simplified gridded Reynolds stress tensor ($b_{ij,d}|(\hat{T}, \hat{w})$) can be expressed as,

$$b_{ij,d}|\{\hat{T}_{\text{bin}}(m) < \hat{T} < \hat{T}_{\text{bin}}(n) + d\hat{T}, \hat{w}_{\text{bin}}(n) < \hat{w} < \hat{w}_{\text{bin}}(n) + d\hat{w}\} = \left(\frac{\sum u'_i u'_j m_n}{\sum u'_i u'_i m_n} - \frac{1}{3}\right)\delta_{ij},$$

(3.5)

where the symbols mean the same as in (2.14). The construction of the RGB colour map for $b_{ij,d}|(\hat{T}, \hat{w})$ is similar to as described in (2.20), where instead of using the eigenvalues of $b_{ij}|(\hat{T}, \hat{w})$, the eigenvalues of $b_{ij,d}|(\hat{T}, \hat{w})$ are used.

The coordinate axes of a diagonal Reynolds stress tensor in which it is defined coincide with its principal axes (e.g., Klipp 2018). Therefore, comparing the anisotropic states of this simplified $b_{ij,d}|(\hat{T}, \hat{w})$ with the original $b_{ij}|(\hat{T}, \hat{w})$, indicates the misalignment of the principal axes with the reference co-ordinate frame. This is caused due to the presence of
Figure 6: The contour maps of the JPDFs between $\hat{T}$ and $\hat{w}$ (thick black lines, see (2.16)) and the bivariate Gaussian distribution (dotted red lines, see (2.18)) are shown for the six different classes of stability ratios as indicated in the legend at the right most corner. The same RGB colour map of figure 5 are shown here too, where red, green, and blue denote the three limiting states of the anisotropic Reynolds stress tensor such as 1-component anisotropy, 2-component anisotropy, and 3-component isotropy respectively. The thick pink lines denote the hyperbolic hole.

the off-diagonal Reynolds shear stress components. To illustrate this, figure 7 shows the comparison of the anisotropic states of gridded $b_{ij}|(\hat{T}, \hat{w})$ with $b_{ij,d}|(\hat{T}, \hat{w})$ for the three specific stability classes ($-\zeta > 2$, $0.4 < -\zeta < 0.6$, and $0 < -\zeta < 0.2$). The comparison shows, in the red regions (1-component anisotropy, $|\hat{T}\hat{w}| >> 1$) the off-diagonal elements of $b_{ij}|(\hat{T}, \hat{w})$ contribute significantly, whereas in the green (2-component anisotropy) and blue (isotropic) regions that contribution is significantly less. By investigating the contour maps of the three velocity variances (diagonal components of $b_{ij}|(\hat{T}, \hat{w})$), we have found that the green regions are dominated by the horizontal velocity components whereas all the three components contribute to the blue regions (not shown).

This points out that the highly-intermittent large heat flux events (red regions in $\hat{T}$-$\hat{w}$ plane) contribute significantly to the off-diagonal Reynolds shear stress components by making the turbulence anisotropic (1-component anisotropy) associated with these motions. We have also observed from figure 5 that not all the thermal motions associated with high heat flux events are isotropic, but only those residing in the blue regions of $\hat{T}$-$\hat{w}$ quadrant plane are associated to isotropic turbulence. This observation is non-trivial and this outcome would not be possible without an event based description. Since the bulk statistics based approaches would predict that higher convective conditions (high heat fluxes) are associated with more isotropy in the whole flow, this analysis shows that the
The role of intermittent heat transport towards Reynolds stress anisotropy

Figure 7: A comparison of the anisotropic states of the gridded Reynolds stress tensor corresponding to the three classes of stability ratios such as $-\zeta > 2$, $0.4 < -\zeta < 0.6$, and $0 < -\zeta < 0.2$ are shown for the illustration purpose. The upper panels (a, b, c) show the anisotropic states represented by the RGB colour map of the simplified gridded Reynolds stress tensor whose off-diagonal components are equal to 0 ($b_{ij, d} (\hat{T}, \hat{w})$, see (3.5)). The lower panels (d, e, f) show the same but for the original gridded Reynolds stress tensor ($b_{ij} (\hat{T}, \hat{w})$, see (2.14)).

connection between the heat transport and turbulence anisotropy is more intricate than that. However, the quadrant analysis of turbulence anisotropy in $\hat{T}$-$\hat{w}$ plane does not give information about the time scale or size of the thermal plumes associated with more isotropic or anisotropic turbulence. Therefore, one can ask “whether there are any characteristic sizes associated with these thermal plumes which are least anisotropic compared to the other plumes?” We thus focus our attention on the primary constitutive motions of these thermal plumes, the warm-updrafts and cold-downdrafts, and investigate the relation to turbulence anisotropy associated with their streamwise sizes.

3.3. Persistence analysis of turbulence anisotropy

In this section we employ persistence analysis to characterize the streamwise sizes of the thermal motions occurring in each quadrant of $T'$-$w'$. This is achieved by converting the persistent time $T_B$ to streamwise length from Taylor’s hypothesis. We begin with discussing the PDFs of the streamwise sizes to highlight the physical characteristics of the thermal motions and the aspect of non-Gaussianity. Along with that, we also investigate the turbulence anisotropy and the topology of the Reynolds stress tensor associated with the warm-updrafts and cold-downdrafts of different sizes. The detailed methodologies are described in §2.3.1 and §2.3.2.
Subharthi Chowdhuri, Siddharth Kumar and Tirtha Banerjee

Figure 8: The log-log plots of the PDFs of the normalized streamwise sizes \( (T_B \bar{u})/z \) (see (2.28)) corresponding to the warm-updrafts (blue open circles) and cold-downdrafts (black open squares), are shown for the six different stability classes as indicated in the legend placed at the right most corner. A distinct power-law of exponent \(-0.4\) is shown as a thick red line in all the panels, whereas the other two thick black lines correspond to the log-normal distribution.

3.3.1. PDFs of warm-updrafts and cold-downdrafts

Figures 8a–f show the PDFs of the normalized streamwise sizes \( (T_B \bar{u})/z \) separately for the warm-updrafts and cold-downdrafts, corresponding to the six different stability classes as outlined in table 2. We initially choose to normalize the streamwise sizes by \( z \), under the assumption that the thermal plumes grow linearly with height (Tennekes & Lumley 1972).

The most distinct feature what we notice from the highly-convective \( (-\zeta > 2) \) stability class (figure 8a) is that the PDFs of the normalized streamwise sizes of warm-updrafts and cold-downdrafts collapse with a power-law of an exponent \(-0.4\),

\[
P[(T_B \bar{u})/z] \propto [(T_B \bar{u})/z]^{-0.4},
\]
which approximately extends up to \( (T_B \bar{u})/z < 1 \). A similar power-law was reported by Chamecki (2013) for the persistent times of \( u \) and \( w \) fluctuations smaller than the integral time scale in a plant canopy. Apart from that, Yee et al. (1993) and Katul et al. (1994) also documented a power-law behaviour in the PDFs of the small sizes of the concentration and heat flux bursts from an unstable ASL, although the exponent they found was closer to \(-1.4\).

However, this power-law segment systematically disappears as we approach the near-neutral stability \( (0 < -\zeta < 0.2) \) and gets replaced by a log-normal distribution (figure
et al. (2006) commented that for an active scalar such as temperature in highly-convective turbulence, the PDFs of the inter-pulse periods followed a power-law. Whereas in a shear-driven turbulence when the temperature behaved more like a passive scalar, the PDFs followed a log-normal distribution. This is broadly consistent with our observations from figure 8, given the fact that in a highly-convective surface layer these plumes coincide with the low- and high-speed streaks in the streamwise velocity fluctuations and are associated with more passive turbulence (Khanna & Brasseur 1998; Salesky et al. 2017; Li et al. 2018). Additionally, we also show the PDFs of $(T_B \overline{u})/z$ for the motions occurring in the counter-gradient quadrants of $T'-w'$ in figure S4 of the supplementary material. We can notice that for small values of $(T_B \overline{u})/z < 0.2$ they all collapse for all the stability classes. This result is in agreement with the observations of Dong et al. (2017), where they found that the PDFs of positive and negative momentum-carrying motions of small sizes agreed with each other (see their figure 5b).

Interestingly, the PDFs of the streamwise sizes can also be transformed to an area fraction $(A_f[(T_B \overline{u})/z])$, by assuming each heat-carrying plume of size $(T_B \overline{u})/z$ has a circular cross-section. Under this assumption, the area fractions can be expressed as,

$$A_f[(T_B \overline{u})/z] \propto [(T_B \overline{u})/z]^2 P[(T_B \overline{u})/z].$$

(3.9)

In a highly-convective surface layer, for sizes $(T_B \overline{u})/z < 1$, the PDFs of the warm-updrafts and cold-downdrafts follow a power-law with an exponent $-0.4$ (figure 8a). From (3.6) and (3.9), this translates to a scale-invariance associated with the area fractions $(A_f \propto [(T_B \overline{u})/z]^{1.6})$ occupied by these plumes ($(T_B \overline{u})/z < 1$) and subsequently may relate to their fractal nature. The fractal characteristics of the plumes have been documented by the laboratory studies on Rayleigh Bénard convection (Sreenivasan 1991; Puthenveettil et al. 2005) and in an unstable ASL flow (Yee & Chan 1995).

For $(T_B \overline{u})/z > 1$, the PDFs of the warm-updrafts and cold-downdrafts significantly differ from each other in the highly-convective case ($-\zeta > 2$, figure 8b). However, they systematically agree with each other as the near-neutral stability ($0 < -\zeta < 0.2$) is approached (see figures 8b to 8d). As we will show later, this is related to the asymmetry in the distributions of the warm-updrafts and cold-downdrafts due to strong non-Gaussianity in temperature fluctuations in a highly-convective surface layer. As discussed by Chamecki (2013), these large values of $(T_B \overline{u})/z$ are exponentially distributed according to a Poisson type process which could be studied by considering the cumulative distribution functions of $(T_B \overline{u})/z$. As per Chamecki (2013), the cumulative distribution function $F[(T_B \overline{u})/z]$ is defined as,

$$F[(T_B \overline{u})/z] = \int_{[(T_B \overline{u})/z]_{max}}^{[(T_B \overline{u})/z]} P[(T_B \overline{u})/z] d \log [(T_B \overline{u})/z].$$

(3.10)
Figure 9: The log-linear plots of the cumulative distribution functions of $(T_B \bar{u})/z$ ((see (3.10)) corresponding to the (a) warm-updrafts and (b) cold-downdrafts are shown for the six different stability classes as indicated in the legend of panel (a). The open squares of different colours in panel (b) represent the same stability classes as in panel (a), but for the cold-downdrafts. The thin black lines on both the panels show the exponential distributions which represent a Poisson type process ((see (3.11)).

Figure 9 shows the cumulative distribution functions ($F[(T_B \bar{u})/z]$), separately for the warm-updrafts and cold-downdrafts in a log-linear co-ordinate system. The exponential decay,

$$F\left[\frac{(T_B \bar{u})}{z}\right] \propto \exp\left[-k\frac{(T_B \bar{u})}{z}\right],$$  \hspace{1cm} (3.11)

in such plots would appear as a straight line with the slope of $-k$. In figure S5 (supplementary material) we show the same in a log-log co-ordinate system to illustrate the statistical convergence in the tails. From figure 9, we notice that for larger values of $(T_B \bar{u})/z$, $F[(T_B \bar{u})/z]$ decays exponentially according to (3.11). Apart from that, the slopes of the straight lines ($-k$) on figure 9 do not change appreciably with the change in $-\zeta$ for both the warm-updrafts and cold-downdrafts. Sreenivasan et al. (1983) mentioned that the long intervals (large $(T_B \bar{u})/z$) are a consequence of large-scale structures passing the sensor and the short intervals (small $(T_B \bar{u})/z$) are a consequence of the nibbling small-scale motions superposed on the large-scale structures. From that perspective, we expect that the large-scale statistical characteristics of the warm updrafts and cold downdrafts do not change appreciably with stability in an unstable surface layer flow. Next we discuss the turbulence anisotropy and topology of the Reynolds stress tensor associated with these warm-updrafts and cold-downdrafts of different sizes.
3.3.2. Reynolds stress anisotropy associated with persistence

Before discussing anisotropy, to highlight non-Gaussianity we convert the PDFs in figure 8 to a distribution about the time fractions \( T_f \) spent in each quadrant of \( T' \)-\( w' \), by presenting the same in a premultiplied form. If from a particular quadrant of \( T' \)-\( w' \), \( N_{tot} \) number of blocks are being detected, with each \( N_i \)-th block containing \( n_i \) number of points, then we can write,

\[
\sum_{i=1}^{N_{tot}} N_i n_i \propto T_f,
\]

where \( T_f \) is the time fraction spent in that particular quadrant. Since,

\[
N_i \propto \left( P[(T_B \bar{u})/\ell] d \log \left[ (T_B \bar{u})/\ell \right] \right) \quad \text{and} \quad n_i \propto (T_B \bar{u})/\ell,
\]

we can write (3.12) as,

\[
\int_{\min}^{\max} \left( \frac{T_B \bar{u}}{\ell} \right) P\left( \frac{T_B \bar{u}}{\ell} \right) d \log \left( \frac{T_B \bar{u}}{\ell} \right) \propto T_f,
\]

where \( \ell \) is the relevant length scale to normalize the streamwise sizes of the plumes (\( z \), \( \lambda \), or the combination of the two). From (3.13) we can also write,

\[
\int_{\min}^{\max} \left( \frac{T_B \bar{u}}{\ell} \right) P\left( \frac{T_B \bar{u}}{\ell} \right) \left( \frac{\Delta T_f}{\bar{T}} \right) \propto \Delta T_f,
\]

where the subscripts I and III refer to the warm-updraft and cold-downdraft quadrants, and \( \Delta T_f \) is the difference in the time fractions spent in those quadrants. Since from (3.3) we know that \( \Delta T_f \approx \bar{T}^3 / \sigma^3_T \), we may rewrite (3.14) as,

\[
\int_{\min}^{\max} \left( \frac{T_B \bar{u}}{\ell} \right) P\left( \frac{T_B \bar{u}}{\ell} \right) \left( \frac{\Delta T_f}{\bar{T}} \right) \propto \frac{\bar{T}^3}{\sigma^3_T}.
\]

Figures 10a–f show the premultiplied PDFs of \( (T_B \bar{u})/z \) corresponding to the warm-updrafts and cold-downdrafts for the same six different stability classes, along with the degree of isotropy (see (2.25)). We observe from figure 10a that the premultiplied PDFs of the warm-updrafts and cold-downdrafts of sizes \( (T_B \bar{u})/z < 1 \) collapse onto each other with a power-law exponent of +0.6 in a highly-convective surface layer \((-\zeta > 2)\). However, for \( (T_B \bar{u})/z > 1 \), the premultiplied PDFs of the warm-updrafts and cold-downdrafts differ significantly from each other. In the premultiplied form we can relate the difference in the values between the warm-updrafts and cold-downdrafts to the non-Gaussianity through (3.15). Therefore, we claim that the effect of non-Gaussianity (asymmetry) in a highly-convective surface layer is only felt through those warm-updrafts and cold-downdrafts having sizes \( (T_B \bar{u})/z > 1 \). This also explains why at sizes \( (T_B \bar{u})/z > 1 \), the PDFs of the warm-updrafts and cold-downdrafts differ most for the highly-convective case.

From figure 10 we can also compare the distribution of the degree of isotropy between the warm-updrafts and cold-downdrafts associated with \( (T_B \bar{u})/z \) (see (2.25)). We observe that the warm-updrafts are relatively more isotropic than the cold-downdrafts as the values of \( C_3 \) are larger in general. Apart from that, there is a certain size of these plumes which are least anisotropic compared to the rest, and this critical size is larger for the cold-downdrafts compared to the warm-updrafts. Also the peak values of the degree of isotropy decreases systematically from 0.4 to 0.15 as the near-neutral stability is approached (see figures 10a to 10f). This is broadly in agreement with our previous
Subharthi Chowdhuri, Siddharth Kumar and Tirtha Banerjee

Figure 10: The log-log plots of the premultiplied PDFs of \(\frac{(TBu)}{z}\) (see (3.13)) corresponding to the warm-updrafts (blue open circles) and cold-downdrafts (black open squares) are shown for the six different stability classes indicated in the legend placed at the right-most corner. In all the panels, the right hand side of the \(y\) axis is linear and used to represent the distribution of the degree of isotropy associated with the warm-updrafts (thick blue line with open circles) and cold-downdrafts (thick black line with open squares) of different \(\frac{(TBu)}{z}\) (see (2.25)). The thick red line shows the same power-law as in figure 8, but owing to premultiplication the exponent changed to +0.6. The grey shaded region represents \(\frac{(TBu)}{z} < 1\), and the thick black line denotes the value of 1.

observations for the whole flow, where the degree of isotropy systematically decreased from highly-convective to near-neutral (figure 2). However in figure 10 with the change in stability, the peak positions of the degree of isotropy for the warm-updrafts and cold-downdrafts shift systematically from the region \(\frac{(TBu)}{z} < 1\) to the region \(\frac{(TBu)}{z} > 1\). In figure S6 of the supplementary material, we also show the degree of isotropy associated with the motions of different sizes occurring in the counter-gradient quadrants of \(T'\)-\(w'\). These motions are clearly more anisotropic than the warm-updrafts and cold-downdrafts.

Summarizing the figures 8–10, we have observed that the warm-updrafts and cold-downdrafts having sizes \(\frac{(TBu)}{z} < 1\) are scale-invariant owing to a power-law dependency in the highly-convective stability. This scale-invariant property disappears systematically as the near-neutral stability is approached. Apart from that, the effect of non-Gaussianity (Gaussianity) appears mostly at the sizes \(\frac{(TBu)}{z} > 1\) in a highly (weakly) convective surface layer. Therefore, the systematic shift in the peak positions of the degree of isotropy associated with warm-updrafts and cold-downdrafts from the region \(\frac{(TBu)}{z} < 1\) to the region \(\frac{(TBu)}{z} > 1\) might be related to an interaction between two different physical
processes. One of these processes might be associated with scale-invariance, while the other with non-Gaussianity.

3.3.3. Scaling of the degree of isotropy, heat, and momentum distributions

To get further insight into this systematic shift, we empirically investigated the distributions of the degree of isotropy associated with warm-updrafts and cold-downdrafts of different streamwise sizes (figure 11). It was done by normalizing the sizes with three different length scales, such as the large-eddy length scale $\lambda$ (see (2.22)), $z$, and the geometric mean of these two length scales $z^{0.5}\lambda^{0.5}$.

This third length scale $z^{0.5}\lambda^{0.5}$ is called a mixed length scale and has been reviewed in details by Buschmann et al. (2009), Gad-el Hak & Buschmann (2011), and Chowdhuri et al. (2019). The half powers in this mixed scale do not have any physical explanation except its empirical foundation, since the dimensional consistency only requires that the sum of the exponents to be 1 (Gad-el Hak & Buschmann 2011). In the context of event based analysis, Rao et al. (1971) discovered that the frequency of the burst events in a turbulent boundary layer scaled with a mixed time scale, involving both inner and outer variables. Similarly, Alfredsson & Johansson (1984) discovered from VITA analysis (an event based analysis) that the governing time scale of the near-wall region of a channel flow was a mixture of outer and inner scales. They interpreted this as a sign of the interaction of outer and near-wall flows. Recently, McNaughton et al. (2007) and Chowdhuri et al. (2019) noted that this mixed length scale could successfully collapse the peak positions of the temperature spectra in the local free convection layer.

From figure 11 it is evident that by normalizing the streamwise sizes by the mixed length scale could successfully collapse the peak positions of the degree of isotropy at $(T_B \bar{u})/(z^{0.5}\lambda^{0.5}) \approx 0.08$ and $(T_B \bar{u})/(z^{0.5}\lambda^{0.5}) \approx 0.15$ for the warm-updrafts and cold-downdrafts respectively. Upon close inspection, we note that these scaled peak positions of the degree of isotropy approximately occurs at the intersection of the two regions (see figure 12). The first region extends up to $(T_B \bar{u})/(z^{0.5}\lambda^{0.5}) \approx 0.1$, where the premultiplied PDFs of $(T_B \bar{u})/(z^{0.5}\lambda^{0.5})$ associated with the warm-updrafts and cold-downdrafts follow a power-law with the exponent +0.6 in highly-convective stability. This power-law region progressively diminishes as the near-neutral stability is approached (see figures 12a to 12f). The second region extends beyond $(T_B \bar{u})/(z^{0.5}\lambda^{0.5}) \approx 0.1$, where these premultiplied PDFs are widely separated in highly-convective stability, while agreeing with each other in near-neutral stability (see figures 12a to 12f). This is due to the transformation from non-Gaussian to Gaussian turbulence, as stability changes.

We also notice that for both the warm-updrafts and cold-downdrafts the mixed length scaled peak positions of the degree of isotropy are not exactly associated with that scale where the three velocity variances are equal to each other (see figure S7 in the supplementary material). The velocity variances associated with $(T_B \bar{u})/\ell$ ($\ell = z^{0.5}\lambda^{0.5}$) from each quadrant of $T’-w’$ are defined similarly as in (2.3.2) such that:

$$x'^2(T_B \bar{u}/\ell) = \frac{\sum x'^2}{36000 \times d \log (T_B \bar{u}/\ell)}, \quad (x = u, v, w)$$

(3.16)

with the property,

$$\int^{(T_B \bar{u}/\ell)_{\text{max}}}_{(T_B \bar{u}/\ell)_{\text{min}}} \left[ x'^2(T_B \bar{u}/\ell) \right] d \log (T_B \bar{u}/\ell) = \left( \sigma_x^2 \right)_X,$$

(3.17)

where X is any one quadrant of $T’-w’$ (I, II, III, IV).

From figure S7 we note that for small sizes of the warm-updrafts and cold-downdrafts ($(T_B \bar{u})/(z^{0.5}\lambda^{0.5}) < 0.08$ and $(T_B \bar{u})/(z^{0.5}\lambda^{0.5}) < 0.15$ respectively), the horizontal and
Figure 11: The distributions of the degree of isotropy ($C_3$) for the three different scalings of the sizes of the warm-updrafts and cold-downdrafts (see (2.25)) such as $(T_B\bar{u})/\lambda$ (a, b), $(T_B\bar{u})/z$ (c, d), and $(T_B\bar{u})/(z^{0.5}\lambda^{0.5})$ (e, f), are shown in the top and bottom panels respectively. The scale $\lambda$ is the large-eddy length scale obtained from (2.22). The different colours represent different stability classes as shown in the legend at the right-most corner. The open circles and squares represent the warm-updrafts and cold-downdrafts. The thick and dotted pink lines in panels (e) and (f) indicate the collapsed position of the peaks of the degree of isotropy associated with the warm-updrafts and cold-downdrafts, with the numbers shown below. The thick and dotted black lines in panels (c) and (d) indicate the collapsed position of the peaks of the heat flux distribution, as shown in figure 13.

vertical velocity variances are widely separated. Along with that, these small plumes do not carry much heat flux but occur more frequently, as shown in figures 8 and 13. This is consistent with our observation from quadrant analysis where the more frequently occurring weak heat flux events are associated with 2-component anisotropy (figures 5 and 9). We interpret this as, the small warm and cold plumes are embedded in the large-scale horizontal velocity field and do not interact strongly with the vertical velocity component, thus displaying a state of 2-component anisotropy. Later in figure 15, we show this explicitly from trajectory analysis of Reynolds stress anisotropy on a barycentric map.

It is intriguing to note that the same mixed scaling does not collapse the heat flux distributions (see (2.30)) associated with the warm-updrafts and cold-downdrafts of different streamwise sizes (figure 13). The $z$ scaling works better in this regard, and collapses the scaled heat flux peak positions at $(T_B\bar{u})/z \approx 2$ and $(T_B\bar{u})/z \approx 2.5$ for the warm-updrafts and cold-downdrafts respectively. We also note that in figures 13c and d, for $(T_B\bar{u})/z < 1$, the heat flux distribution follows a power-law scaling with an exponent of $4/3$. Interestingly, this exponent is equal to the slope of the premultiplied heat flux
The role of intermittent heat transport towards Reynolds stress anisotropy

Figure 12: Same as in figure 10, but for the sizes of the warm-updrafts and cold-downdrafts normalized with the mixed length scale, \((T_B\bar{u})/z^{0.5}\lambda^{0.5}\). The thick and dotted pink lines indicate the collapsed position of the peaks of the degree of isotropy associated with the warm-updrafts and cold-downdrafts, as in figure 11. The thick black line denotes the value of 0.1.

We compare these peak positions from the heat flux distribution in figures 13c and d with the degree of isotropy distribution in figures 11c and d. We notice that for the warm-updrafts (figure 11c) in a highly-convective surface layer \((-\zeta > 2\) the peak position of the degree of isotropy is located quite further away from the heat flux peak position. This is consistent with our observations from the quadrant analysis in figures 5–7. We have found from the quadrant analysis that the intermittent large heat flux events in warm-updrafts contribute significantly to the off-diagonal Reynolds shear stress components and are associated with 1-component anisotropic turbulence (see figures 6 and 7).

Figure 14 shows the distributions of the heat and momentum fluxes associated with the warm-updrafts and cold-downdrafts of different sizes, \((T_B\bar{u})/z\) (see (2.30)). It is clear that the heat flux peak position associated with warm-updrafts \((T_B\bar{u})/z \approx 2\), corresponds to significant amount of momentum in a highly-convective surface layer (figures 14a and c). However, for the peak position \((T_B\bar{u})/z \approx 2.5\) associated with the quiescent cold-downdrafts (carry less heat compared to warm-updrafts) the momentum transport is rather inefficient as there are both gradient and counter-gradient transport involved (figures 14b and d). Since the probability of occurrence of these large cold-downdrafts is higher than the large warm-updrafts (figures 8 and 10), this explains why the momentum transport is inefficient in a highly-convective surface layer. The association of highly inefficient momentum transport with the cold-downdrafts is also observed in the cospectra in the inertial subrange [Wyngaard & Coté 1972]. This correspondence needs further investigation which is beyond the scope of this paper.

We compare these peak positions from the heat flux distribution in figures 13c and d with the degree of isotropy distribution in figures 11c and d. We notice that for the warm-updrafts (figure 11c) in a highly-convective surface layer \((-\zeta > 2\) the peak position of the degree of isotropy is located quite further away from the heat flux peak position. This is consistent with our observations from the quadrant analysis in figures 5–7. We have found from the quadrant analysis that the intermittent large heat flux events in warm-updrafts contribute significantly to the off-diagonal Reynolds shear stress components and are associated with 1-component anisotropic turbulence (see figures 6 and 7).

Figure 14 shows the distributions of the heat and momentum fluxes associated with the warm-updrafts and cold-downdrafts of different sizes, \((T_B\bar{u})/z\) (see (2.30)). It is clear that the heat flux peak position associated with warm-updrafts \((T_B\bar{u})/z \approx 2\), corresponds to significant amount of momentum in a highly-convective surface layer (figures 14a and c). However, for the peak position \((T_B\bar{u})/z \approx 2.5\) associated with the quiescent cold-downdrafts (carry less heat compared to warm-updrafts) the momentum transport is rather inefficient as there are both gradient and counter-gradient transport involved (figures 14b and d). Since the probability of occurrence of these large cold-downdrafts is higher than the large warm-updrafts (figures 8 and 10), this explains why the momentum transport is inefficient in a highly-convective surface layer. The association of highly inefficient momentum transport with the cold-downdrafts is also observed in the
Figure 13: The heat flux distributions for the three different scalings of the sizes of warm-updrafts and cold-downdrafts (see (2.30)) such as \((T_B \bar{\nabla})/\lambda\) (a, b), \((T_B \bar{\nabla})/z\) (c, d), and \((T_B \bar{\nabla})/(z^{0.5} \lambda^{0.5})\) (e, f), are shown in the top and bottom panels respectively. The different colours represent different stability classes as shown in the legend at the right-most corner. The open circles and squares represent the warm-updrafts and cold-downdrafts. The thick and dotted pink lines in panels (e) and (f) indicate the collapsed position of the peaks of the degree of isotropy associated with the warm-updrafts and cold-downdrafts, as in figure 11. The thick and dotted black lines in panels (c) and (d) indicate the collapsed position of the peaks of the heat flux distribution, with the numbers shown below.

Numerical simulations of Li et al. (2018), Salesky & Anderson (2018) interpreted this as, under highly-convective conditions, the small-scale turbulence is excited in the updraft regions and suppressed in downdraft regions, leading to intermittent periods of small-scale excitation in the momentum fluxes.

Summarizing these observations, we note that there is a characteristic size of the warm-updrafts and cold-downdrafts which scales with a mixed length scale \(z^{0.5} \lambda^{0.5}\), and associated with least anisotropic turbulence. On the other hand, the sizes of the warm-updrafts and cold-downdrafts which carry the maximum heat are found to scale with \(z\). The mismatch in peak positions of heat flux and degree of isotropy is related to the fact that the large warm-updrafts which carry the maximum amount of heat are also associated with significant momentum transport (see figures 14a and c). However, for the cold-downdrafts these two peak positions almost coincide (see figure 11d). This might be related to inefficient momentum transport by the cold-downdrafts, unlike the warm-updrafts (see figures 14c and d). We next investigate the topology of the Reynolds stress tensor associated with the warm-updrafts and cold-downdrafts of different sizes.
The role of intermittent heat transport towards Reynolds stress anisotropy

3.3.4. The topology of the Reynolds stress tensor

The topology of the anisotropic Reynolds stress tensor can be assessed via its trajectory on the barycentric map as discussed in §2.2.1. Figure 15 shows the barycentric maps where the trajectories of the anisotropic states of the Reynolds stress tensor are shown as a function of the scaled sizes \((T_B \bar{u}) / (z^{0.5} \lambda^{0.5})\) (see (2.25) and (2.26)). For better visualization we restrict the vertical axes of the barycentric maps in figure 15 to the values of 0.5. We also include three perpendicular bisectors which divide the equilateral triangle into three equal portions. Each of these portions corresponds to the 2-component anisotropy (the left third), 1-component anisotropy (the right third) and close to isotropic states (the upper third) of the Reynolds stress tensor (see figure 2a).

From figure 15, we notice that in a highly-convective surface layer, the anisotropic state of the Reynolds stress tensor corresponds to 2-component anisotropy associated with the small values of \((T_B \bar{u}) / (z^{0.5} \lambda^{0.5})\) of warm-updrafts and cold-downdrafts \((T_B \bar{u}) / (z^{0.5} \lambda^{0.5}) < 0.08\). However, the Reynolds stress tensor approaches an isotropic state associated with the peak value of \((T_B \bar{u}) / (z^{0.5} \lambda^{0.5}) \approx 0.08\) for the warm-updrafts. It subsequently returns to an anisotropic state for sizes larger than this through 1-component anisotropy. This trend systematically disappears as we approach the near-neutral stability (see figures 15a to 15f). From figure 15, we note that the trajectories for all the sizes of the warm-updrafts and cold-downdrafts reside almost within the left third of the barycentric map, which represents 2-component anisotropy.

In summary, the results related to Reynolds stress anisotropy from the persistence
Figure 15: The barycentric map representation of the anisotropic states of the Reynolds stress tensor associated with the normalized streamwise sizes $(T_B \bar{u})/(z^{0.5} \lambda^{0.5})$ of the warm-updrafts and cold-downdrafts (see (2.25) and (2.26)), shown for the six different stability classes. The colour bar at the right most corner with different shades of grey represents the logarithmic values of $(T_B \bar{u})/(z^{0.5} \lambda^{0.5})$.

analysis complement the results obtained from the quadrant analysis. By comparing the figures 5a and 11a–f, from the highly-convective stability, we note that the maximum values of the degree of isotropy match reasonably well between these two analyses. From quadrant analysis, the maximum $C_3$ values are 0.5 and 0.4 respectively for the warm-updrafts and cold-downdrafts (figure 5a). On the other hand, from the persistence analysis we obtain the maximum $C_3$ values to be 0.4 and 0.3 respectively associated with the critical sizes of the warm-updrafts and cold-downdrafts (figures 11a and 11b). These critical sizes correspond to $(T_B \bar{u})/(z^{0.5} \lambda^{0.5}) \approx 0.08$ and $(T_B \bar{u})/(z^{0.5} \lambda^{0.5}) \approx 0.15$ respectively. Given the close similarity in the maximum $C_3$ values, we can also compare the topology of the Reynolds stress tensor from figures 5a and 15a (highly-convective stability). The comparison shows that the blue regions (isotropic state) in the anisotropy contour maps (figure 3a) broadly corresponds to those sizes of warm-updrafts and cold-downdrafts having $(T_B \bar{u})/(z^{0.5} \lambda^{0.5}) \approx 0.08$ and $(T_B \bar{u})/(z^{0.5} \lambda^{0.5}) \approx 0.15$ respectively. However, for the near-neutral stability (figures 5f and 15f) the 2-component anisotropy is associated with all the sizes of the warm-updrafts and cold-downdrafts.

To conclude the study, we note that a detailed comprehensive analysis of Reynolds stress anisotropy associated with the heat transport events of different sizes is presented here. We have found that there is a critical size of the warm-updrafts and cold-downdrafts associated with least anisotropic turbulence which scales with a mixed length scale. This mixed length scale probably reflects an interaction between two different physical processes, one of which is associated with the scale-invariance of the plume sizes and the
other with the non-Gaussianity in turbulence, although it requires further investigation. We present our conclusions in the next section.

4. Conclusions

We report novel comprehensive results of flow anisotropy associated with intermittent heat transport in an unstable ASL, from the SLTEST experimental dataset. We adopt an event-based description of the heat transporting events occurring intermittently and persisting over a wide range of time scales. The flow anisotropy is quantified by using a metric called degree of isotropy, computed from the smallest eigenvalue of the anisotropic Reynolds stress tensor. The important results from this study can be broadly summarized as:

(i) The flow approaches an isotropic (anisotropic) state as the surface layer becomes highly (weakly) convective. The degree of isotropy of the whole flow is governed by the strength of the vertical velocity fluctuations, which preferentially couple with the temperature fluctuations. These temperature fluctuations exhibit strong (weak) non-Gaussian characteristics in a highly (weakly) convective surface layer.

(ii) The flow anisotropy in an unstable surface layer is strongly related to the asymmetry in the distribution of the thermal plumes and intermittent heat transport, associated with non-Gaussianity in the temperature fluctuations.

(iii) By adopting an event based approach, it is discovered that not all the heat flux events are associated with same anisotropic state of turbulence. There are 1-component anisotropic states associated with intermittent large heat flux events. Whereas, 2-component anisotropic states are associated with more frequent but weak heat flux events. On the other hand, the isotropic turbulence is associated with moderate heat flux events which lie between these two extremes.

(iv) There is a critical size associated with the organized heat flux events (warm-updrafts and cold-downdrafts) which corresponds to the maximum value of the degree of isotropy (i.e. least anisotropic turbulence) and scales with a mixed-length scale \( z^{0.5} \lambda^{0.5} \), where \( \lambda \) is the large-eddy length scale and \( z \) is the measurement height. We hypothesize that this mixed-length scaling may reflect an interaction between two different physical processes, one of which may be associated with the scale-invariance of the plume sizes and the other with the non-Gaussianity in turbulence.

(v) By investigating the topology of the anisotropic Reynolds stress tensor, it is found that in a highly-convective surface layer, the warm-updrafts and cold-downdrafts smaller (larger) than the critical size, are associated with two (one) component anisotropy. However, in a near-neutral surface layer the turbulence is mostly 2-component regardless of the sizes of the warm-updrafts and cold-downdrafts.

(vi) The \( z \)-scaling is more successful in collapsing the peak positions of the heat flux distribution associated with the sizes of the warm-updrafts and cold-downdrafts. The sizes of the warm-updrafts corresponding to this peak scale also carry a significant amount of momentum. Due to their association with momentum, it causes a drop in the degree of isotropy associated with their sizes.

Note that the findings from this study should be verified from the field experiments in an unstable ASL flow conducted over the rough surfaces and in complex terrains. An inevitable limitation of this study is the unavailability of three-dimensional velocity and temperature information. Due to this constraint, the intermittent heat transport and the associated Reynolds stress anisotropy, cannot be connected to the three-dimensional topology of the thermal plumes in convective turbulence. In the future, we would address
this problem through large eddy or direct numerical simulations. This study also raises few important questions which deserve future attention:

(i) Why do the sizes of the warm and cold plumes associated with least anisotropic turbulence scale with a mixed length scale?

(ii) Is there a theoretical framework to explain the half powers in the mixed length scale?

(iii) What is the physical connection between the event based (related to flow structures) and scale based (related to harmonic analysis) description of turbulence anisotropy?

Acknowledgements

Indian Institute of Tropical Meteorology (IITM) is an autonomous institute fully funded by the Ministry of Earth Sciences (MoES). The author Tirtha Banerjee acknowledges the new faculty start-up funding from the Department of Civil and Environmental Engineering, the Henry Samueli School of Engineering, University of California, Irvine. The authors would like to thank Dr. Keith G McNaughton for letting them use the SLTEST dataset for this research. The computer codes used in this study are available to all the researchers by contacting the corresponding author.

Supplementary material

This paper has supplementary figures shown at the end.

REFERENCES

Adrian, RJ, Ferreira, RTDS & Boberg, T 1986 Turbulent thermal convection in wide horizontal fluid layers. Exp. Fluids 4 (3), 121–141.

Alfredsson, PH & Johansson, AV 1984 Time scales in turbulent channel flow. Phys. Fluids 27 (8), 1974–1981.

Ali, N, Hamilton, N, Cortina, G, Calaf, M & Cal, RB 2018 Anisotropy stress invariants of thermally stratified wind turbine array boundary layers using large eddy simulations. J. Renew. Sust. Energy. 10 (1), 013301.

Antonia, RA 1977 Similarity of atmospheric Reynolds shear stress and heat flux fluctuations over a rough surface. Boundary-Layer Meteorol. 12 (3), 351–364.

Antonia, RA 1981 Conditional sampling in turbulence measurement. Annu. Rev. Fluid Mech. 13 (1), 131–156.

Antonia, RA, Djenidi, L & Spalart, PR 1994 Anisotropy of the dissipation tensor in a turbulent boundary layer. Phys. Fluids 6 (7), 2475–2479.

Antonia, RA, Kim, J & Browne, LWB 1991 Some characteristics of small-scale turbulence in a turbulent duct flow. J. Fluid Mech. 233, 369–388.

Antonia, RA & Krogstad, PÅ 2001 Turbulence structure in boundary layers over different types of surface roughness. Fluid Dyn. Res. 28 (2), 139.

Ashrafian, A & Andersson, HI 2006 The structure of turbulence in a rod-roughened channel. Int. J. Heat Fluid Flow 27 (1), 65–79.

Balachandar, S & Sirovich, L 1991 Probability distribution functions in turbulent convection. Phys. Fluids 3 (5), 919–927.

Banerjee, S, Krahl, R, Durst, F & Zenger, Ch 2007 Presentation of anisotropy properties of turbulence, invariants versus eigenvalue approaches. J. Turbul. (8), N32.

Banerjee, T & Katul, GG 2013 Logarithmic scaling in the longitudinal velocity variance explained by a spectral budget. Phys. Fluids 25 (12), 125106.

Banerjee, T, Katul, GG, Salesky, ST & Chamecki, M 2015 Revisiting the formulations for the longitudinal velocity variance in the unstable atmospheric surface layer. Q. J. R. Meteorol. Soc. 141 (690), 1699–1711.
BARON, A & QUADRO, M 1997 Turbulent boundary layer over riblets: conditional analysis of ejection-like events. *Int. J. Heat Fluid Flow* 18 (2), 188–196.

BERSHADSKII, A, NIEMELA, JJ, PRASKOVSKY, A & SREENIVASAN, KR 2004 Clusterization and intermittency of temperature fluctuations in turbulent convection. *Phys. Rev. E* 69 (5), 056314.

BIFERALE, L & PROCACCIA, I 2005 Anisotropy in turbulent flows and in turbulent transport. *Phys. Rep.* 414, 43–164.

BOPPANA, VBL, XIE, ZT & CASTRO, IP 2014 Thermal stratification effects on flow over a generic urban canopy. *Boundary-Layer Meteorol.* 153 (1), 141–162.

BOU-ZEID, E, GAO, X, ANSORGE, C & KATUL, GG 2018 On the role of return to isotropy in wall-bounded turbulent flows with buoyancy. *J. Fluid Mech.* 856, 61–78.

BRAY, AJ, MAJUMDAR, SN & SCHEHR, G 2013 Persistence and first-passage properties in nonequilibrium systems. *Adv. Phys.* 62 (3), 225–361.

BROWNE, LWB, ANTONIA, RA & SHAH, DA 1987 Turbulent energy dissipation in a wake. *J. Fluid Mech.* 179, 307–326.

BRUGGER, P, KATUL, GG, DE ROO, F, KRÖNGER, K, ROTENBERG, E, ROHATYN, S & MAUDER, M 2018 Scalewise invariant analysis of the anisotropic Reynolds stress tensor for atmospheric surface layer and canopy sublayer turbulent flows. *Phys. Rev. Fluids* 3 (5), 054608.

BUSCH, NE & PANOFSKY, HA 1968 Recent spectra of atmospheric turbulence. *Q. J. R. Meteorol. Soc.* 94 (400), 132–148.

BUSCHMANN, MH, INDINGER, T & GAD-EL HAK, M 2009 Near-wall behavior of turbulent wall-bounded flows. *Int. J. Heat Fluid Flow* 30 (5), 993–1006.

BUSINGER, JA 1973 A note on free convection. *Boundary-Layer Meteorol.* 4 (1-4), 323–326.

CANTWELL, BJ 1981 Organized motion in turbulent flow. *Annu. Rev. Fluid Mech.* 13 (1), 457–515.

CARAMORI, P, SCHUEPP, P, DESJARDINS, R & MACPHERSON, I 1994 Structural analysis of airborne flux estimates over a region. *J. Clim.* 7 (5), 627–640.

CASTRO, IP, CHENG, H & REYNOLDS, R 2006 Turbulence over urban-type roughness: deductions from wind-tunnel measurements. *Boundary-Layer Meteorol.* 118 (1), 109–131.

CAVA, D & KATUL, GG 2009 The effects of thermal stratification on clustering properties of canopy turbulence. *Boundary-Layer Meteorol.* 130 (3), 307.

CELANI, A, MAZZINO, A & VERCASSOLA, M 2001 Thermal plume turbulence. *Phys. Fluids* 13 (7), 2133–2135.

CHAMECKI, M 2013 Persistence of velocity fluctuations in non-Gaussian turbulence within and above plant canopies. *Phys. Fluids* 25 (11), 115110.

CHAMECKI, M & DIAS, NL 2004 The local isotropy hypothesis and the turbulent kinetic energy dissipation rate in the atmospheric surface layer. *Q. J. R. Meteorol. Soc.* 130 (603), 2733–2752.

CHAPMAN, GT & TOBAK, M 1985 Observations, theoretical ideas, and modeling of turbulent flows—past, present, and future. In *Theoretical approaches to turbulence*, pp. 19–49. Springer.

CHAUHAN, K, HUTCHINS, N, MONTY, J & MARUSIC, I 2013 Structure inclination angles in the convective atmospheric surface layer. *Boundary-Layer Meteorol.* 147 (1), 41–50.

CHIBA, O 1978 Stability dependence of the vertical wind velocity skewness in the atmospheric surface layer. *J. Meteor. Soc. Japan* 56 (2), 146–142.

CHIBA, O 1984 A note on the height dependence of the skewness and the kurtosis of the vertical turbulent velocity in the neutral surface layer. *Boundary-Layer Meteorol.* 29 (4), 313–319.

CHOI, KS & LUMLEY, JL 2001 The return to isotropy of homogeneous turbulence. *J. Fluid Mech.* 436, 59–84.

CHOWDHURI, S & BURMAN, PKD 2019 Representation of the Reynolds stress tensor through quadrant analysis for a near-neutral atmospheric surface layer flow. *Environ. Fluid Mech.* pp. 1–25, https://doi.org/10.1007/s10652-019-09689-7

CHOWDHURI, S, McNAUGHTON, KG & PRABHA, TV 2019 An empirical scaling analysis of heat and momentum cospectra above the surface friction layer in a convective boundary layer. *Boundary-Layer Meteorol.* 170 (2), 257–284.

CHU, CR, PARLANGE, MB, KATUL, GG & ALBERTSON, JD 1996 Probability density functions
Subharthi Chowdhuri, Siddharth Kumar and Tirtha Banerjee

of turbulent velocity and temperature in the atmospheric surface layer. *Water Resour. Res.* **32** (6), 1681–1688.

Corino, ER & Brodkey, RS 1969 A visual investigation of the wall region in turbulent flow. *J. Fluid Mech.* **37** (1), 1–30.

Davidson, PA 2015 *Turbulence: An introduction for scientists and engineers.* Oxford University Press.

Dey, S, Ravi-Kishore, G, Castro-Orgaz, O & Ali, SZ 2018 Turbulent length scales and anisotropy in submerged turbulent plane offset jets. *J. Hydraul. Eng.* **145** (2), 04018085.

Djenidi, L, Agrawal, A & Antonia, RA 2009 Anisotropy measurements in the boundary layer over a flat plate with suction. *Exp. Therm. Fluid Sci.* **33** (7), 1106–1111.

Djenidi, L & Tardu, SF 2012 On the anisotropy of a low-Reynolds-number grid turbulence. *J. Fluid Mech.* **702**, 332–353.

Donateo, A, Cava, D & Contini, D 2017 A case study of the performance of different detrending methods in turbulent-flux estimation. *Boundary-Layer Meteorol.* **164** (1), 19–37.

Dong, S, Lozano-Durán, A, Sekimoto, A & Jiménez, J 2017 Coherent structures in statistically stationary homogeneous shear turbulence. *J. Fluid Mech.* **816**, 167–208.

Duncan, MR & Schuepp, PH 1992 A method to delineate extreme structures within airborne flux traces over the fife site. *J. Geophys. Res. Atmos.* **97** (D17), 18487–18498.

Emory, M & Iaccarino, G 2014 Visualizing turbulence anisotropy in the spatial domain with componentality contours. In *Center for Turbulence Research Annual Research Briefs*, pp. 123–138. Stanford University.

Falocchi, M, Giovannini, L, de Franceschi, M & Zardi, D 2019 A method to determine the characteristic time-scales of quasi-isotropic surface-layer turbulence over complex terrain: a case-study in the Adige valley (Italian Alps). *Q. J. R. Meteorol. Soc.* **145** (719), 495–512.

Fodor, K, Mellado, JP & Wilczek, M 2019 On the role of large-scale updrafts and downdrafts in deviations from Monin-Obukhov similarity theory in free convection. *Boundary-Layer Meteorol.* pp. 1–26.

Garai, A & Kleissl, J 2011 Air and surface temperature coupling in the convective atmospheric boundary layer. *J. Atmos. Sci.* **68** (12), 2945–2954.

Garai, A & Kleissl, J 2013 Interaction between coherent structures and surface temperature and its effect on ground heat flux in an unstably stratified boundary layer. *J. Turbul.* **14** (8), 1–23.

Gastelu, Y, Shew, WL, Gibert, M, Chilla, F, Castaing, B & Pinton, JF 2007 Lagrangian temperature, velocity, and local heat flux measurement in Rayleigh-Bénard convection. *Phys. Rev. Lett.* **99** (23), 234302.

Ghannam, K, Nakai, T, Paschalis, A, Oishi, CA, Kotani, A, Igarashi, Y, Kumagai, T & Katul, GG 2016 Persistence and memory timescales in root-zone soil moisture dynamics. *Water Resour. Res.* **52** (2), 1427–1445.

Gulitski, G, Khomymansky, M, Kinzelbach, W, Lüthi, B, Tsinober, A & Yorish, S 2007 Velocity and temperature derivatives in high-Reynolds-number turbulent flows in the atmospheric surface layer. part 1. facilities, methods and some general results. *J. Fluid Mech.* **589**, 57–81.

Gad-el Hak, M & Buschmann, M 2011 Effects of outer scales on the peaks of near-wall Reynolds stresses. In *6th AIAA Theoretical Fluid Mechanics Conference*, p. 3933.

Haugen, DA, Kaimal, JC & Bradley, EF 1971 An experimental study of Reynolds stress and heat flux in the atmospheric surface layer. *Q. J. R. Meteorol. Soc.* **97** (412), 168–180.

Högström, U & Bergström, H 1996 Organized turbulence structures in the near-neutral atmospheric surface layer. *J. Atmos. Sci.* **53** (17), 2452–2464.

Hong, J, Choi, T, Ishikawa, H & Kim, J 2004 Turbulence structures in the near-neutral surface layer on the tibetan plateau. *Geophys. Res. Lett.* **31** (15), 1–5.

Hunt, JCR, Kaimal, JC & Gaynor, JE 1988 Eddy structure in the convective boundary layer-new measurements and new concepts. *Q. J. R. Meteorol. Soc.* **114** (482), 827–858.

Hutchins, N, Chauhan, K, Marusic, I, Monty, J & Klewicki, J 2012 Towards reconciling the large-scale structure of turbulent boundary layers in the atmosphere and laboratory. *Boundary-Layer Meteorol.* **145** (2), 273–306.
The role of intermittent heat transport towards Reynolds stress anisotropy

Hutchins, N & Marusic, I 2007 Evidence of very long meandering features in the logarithmic region of turbulent boundary layers. J. Fluid Mech. 579, 1–28.

Jiménez, J 2018 Coherent structures in wall-bounded turbulence. J. Fluid Mech. 842.

Jin, LH, So, RMC & Gatski, TB 2003 Equilibrium states of turbulent homogeneous buoyant flows. J. Fluid Mech. 482, 207–233.

Kailas, SV & Narasimha, R 1994 Similarity in VITA-detected events in a nearly neutral atmospheric boundary layer. Proc. Roy. Soc. London Ser. A 447 (1930), 211–222.

Kailasnath, P & Sreenivasan, KR 1993 Zero crossings of velocity fluctuations in turbulent boundary layers. J. Fluid Mech. 842.

Jin, LH, So, RMC & Gatski, TB 2003 Equilibrium states of turbulent homogeneous buoyant flows. J. Fluid Mech. 482, 207–233.

Kailas, SV & Narasimha, R 1994 Similarity in VITA-detected events in a nearly neutral atmospheric boundary layer. Proc. Roy. Soc. London Ser. A 447 (1930), 211–222.

Kaimal, JC 1969 Measurement of momentum and heat flux variations in the surface boundary layer. Radio Sci. 4 (12), 1147–1153.

Kaimal, JC & Finnigan, JJ 1994 Atmospheric boundary layer flows: their structure and measurement. Oxford University Press.

Kaimal, JC, Wyngaard, JC, Haugen, DA, Coté, OR, Izumi, Y, Caughey, SJ & Readings, CJ 1976 Turbulence structure in the convective boundary layer. J. Atmos. Sci. 33 (11), 2152–2169.

Kaimal, JC, Wyngaard, JC, Izumi, Y & Coté, OR 1972 Spectral characteristics of surface-layer turbulence. Q. J. R. Meteorol. Soc. 98 (417), 563–589.

Kalmár-Nagy, T & Varga, Á 2019 Complexity analysis of turbulent flow around a street canyon. Chaos Soliton Fract. 119, 102–117.

Kassinos, SC, Reynolds, WC & Rogers, MM 2001 One-point turbulence structure tensors. J. Fluid Mech. 428, 612–620.

Katul, GG, Albertson, J, Parlange, M, Chu, CR & Strickler, H 1994 Conditional sampling, bursting, and the intermittent structure of sensible heat flux. J. Geophys. Res. Atmos. 99 (D11), 22869–22876.

Katul, G, Hsieh, CI, Kuhn, G, Ellsworth, D & Nie, D 1997a Turbulent eddy motion at the forest-atmosphere interface. J. Geophys. Res. Atmos. 102 (D12), 13409–13421.

Katul, G, Hsieh, CI & Sigmon, J 1997b Energy-inertial scale interactions for velocity and temperature in the unstable atmospheric surface layer. Boundary-Layer Meteorol. 82 (1), 49–80.

Katul, G, Kuhn, G, Schieldge, J & Hsieh, CI 1997c The ejection-sweep character of scalar fluxes in the unstable surface layer. Boundary-Layer Meteorol. 83 (1), 1–26.

Katul, GG, Parlange, MB, Albertson, JD & Chu, CR 1995 Local isotropy and anisotropy in the sheared and heated atmospheric surface layer. Boundary-Layer Meteorol. 72 (1-2), 123–148.

Khanna, S & Brasseur, JG 2019 A new hypothesis on the anisotropic Reynolds stress tensor for turbulent flows. Springer Nature.

Khanna, S & Brasseur, JG 2019 A new hypothesis on the anisotropic Reynolds stress tensor for turbulent flows. Springer Nature.

Krogstad, PÅ 2013 Development of turbulence statistics in the near field behind multi-scale grids. In TSFP Digital Library Online, pp. 1–6. Begel House Inc.

Krogstad, PÅ & Torbergsen, LE 2000 Invariant analysis of turbulent pipe flow. Flow Turbul. Combust. 64 (3), 161–181.

Kurien, S & Sreenivasan, KR 2000 Anisotropic scaling contributions to high-order structure functions in high-Reynolds-number turbulence. Phys. Rev. E 62 (2), 2206.
Lee, MJ & Reynolds, WC 1987 On the structure of homogeneous turbulence. In *Turbulent Shear Flows 5*, pp. 54–66. Springer.

Li, D & Bou-Zeid, E 2011 Coherent structures and the dissimilarity of turbulent transport of momentum and scalars in the unstable atmospheric surface layer. *Boundary-Layer Meteorol.* 140 (2), 243–262.

Li, Q, Gentine, P, Mellado, JP & McColl, KA 2018 Implications of nonlocal transport and conditionally averaged statistics on Monin-Obukhov similarity theory and Townsend’s attached eddy hypothesis. *J. Atmos. Sci.* 75 (10), 3403–3431.

Liu, H, Yuan, R, Mei, J, Sun, J, Liu, Q & Wang, Y 2017 Scale properties of anisotropic and isotropic turbulence in the urban surface layer. *Boundary-Layer Meteorol.* 165 (2), 277–294.

Liu, K & Pletcher, RH 2008 Anisotropy of a turbulent boundary layer. *J. Turbul.* (9), N18.

Liu, L, Hu, F & Cheng, XL 2011 Probability density functions of turbulent velocity and temperature fluctuations in the unstable atmospheric surface layer. *J. Geophys. Res. Atmos.* 116 (D12).

Longo, S, Clavero, M, Chiapponi, L & Losada, M 2017 Invariants of turbulence Reynolds stress and of dissipation tensors in regular breaking waves. *Water* 9 (11), 893.

Lotfy, ER, Abbas, AA, Zaki, SA & Harun, Z 2019 Characteristics of turbulent coherent structures in atmospheric flow under different shear–buoyancy conditions. *Boundary-Layer Meteorol*. pp. 1–27.

Lozano-Durán, A, Flores, O & Jiménez, J 2012 The three-dimensional structure of moment transfer in turbulent channels. *J. Fluid Mech.* 694, 100–130.

Lu, SS & Willmarth, WW 1973 Measurements of the structure of the Reynolds stress in a turbulent boundary layer. *J. Fluid Mech.* 60 (3), 481–511.

Lumley, JL 1970 *Stochastic tools in turbulence*. Academic Press.

Lumley, JL 1979 Computational modeling of turbulent flows. In *Advances in applied mechanics*, vol. 18, pp. 123–176. Elsevier.

Lumley, JL & Newman, GR 1977 The return to isotropy of homogeneous turbulence. *J. Fluid Mech.* 82, 15–44.

Lyu, R, Hu, F, Liu, L, Xu, J & Cheng, X 2018 High-order statistics of temperature fluctuations in an unstable atmospheric surface layer over grassland. *Adv. Atmos. Sci.* 35 (10), 1265–1276.

Maitani, T & Ohtaki, E 1987 Turbulent transport processes of momentum and sensible heat in the surface layer over a paddy field. *Boundary-Layer Meteorol.* 40 (3), 283–293.

Majumdar, SN 1999 Persistence in nonequilibrium systems. *Curr. Sci.* pp. 370–375.

Mansour, NN, Kim, J & Moin, P 1988 Reynolds-stress and dissipation-rate budgets in a turbulent channel flow. *J. Fluid Mech.* 194, 15–44.

Marusic, I, Monty, JP, Hultmark, M & Smits, AJ 2013 On the logarithmic region in wall turbulence. *J. Fluid Mech.* 716.

McBean, GA 1974 The turbulent transfer mechanisms: a time domain analysis. *Q. J. R. Meteorol. Soc.* 100 (423), 53–66.

McNaughton, KG, Clement, RJ & Moncrieff, JB 2007 Scaling properties of velocity and temperature spectra above the surface friction layer in a convective atmospheric boundary layer. *Nonlin. Process Geophys.* .

Metzger, M, McKeon, BJ & Holmes, H 2007 The near-neutral atmospheric surface layer: turbulence and non-stationarity. *Phil. Trans. R. Soc. Lond.* 365 (1852), 859–876.

Miller, SAE 2016 Noise from isotropic turbulence. *AIAA J.* 55 (3), 755–773.

Monin, AS & Obukhov, AM 1954 Basic laws of turbulent mixing in the surface layer of the atmosphere. *Akad. Nauk. SSSR, Geofiz. Inst. Trudy* 151, 163–187.

Monin, AS & Yaglom, AM 1971 *Statistical fluid mechanics: mechanics of turbulence*, vol. 1. MIT Press.

Nagano, Y & Tagawa, M 1988 Statistical characteristics of wall turbulence with a passive scalar. *J. Fluid Mech.* 196, 157–185.

Nakagawa, H & Nezu, I 1977 Prediction of the contributions to the Reynolds stress from bursting events in open-channel flows. *J. Fluid Mech.* 80 (1), 99–128.

Narasimha, R & Kailas, SV 1990 Turbulent bursts in the atmosphere. *Atmos. Environ.* 24 (7), 1635–1645.
The role of intermittent heat transport towards Reynolds stress anisotropy

Narasimha, R, Kumar, SR, Prabhu, A & Kailas, SV 2007 Turbulent flux events in a nearly neutral atmospheric boundary layer. Phil. Trans. R. Soc. A 365 (1852), 841–858.

Panofsky, HA 1974 The atmospheric boundary layer below 150 meters. Annu. Rev. Fluid Mech. 6 (1), 147–177.

Panofsky, HA, Tennekes, H, Lenschow, DH & Wyngaard, JC 1977 The characteristics of turbulent velocity components in the surface layer under convective conditions. Boundary-Layer Meteorol. 11 (3), 355–361.

Perry, AE & Hoffmann, PH 1976 An experimental study of turbulent convective heat transfer from a flat plate. J. Fluid Mech. 77 (2), 355–368.

Pope, SB 2000 Turbulent flows. Cambridge University Press.

Pouransari, Z, Biferale, L & Johansson, AV 2015 Statistical analysis of the velocity and scalar fields in reacting turbulent wall-jets. Phys. Fluids 27 (2), 025102.

Puthenveettil, BA, Ananthakrishna, G & Arakeri, JH 2005 The multifractal nature of plume structure in high-Rayleigh-number convection. J. Fluid Mech. 526, 245–256.

Radenković, DR, Burazer, JM & Novković, DM 2014 Anisotropy analysis of turbulent swirl flow. FME Transactions 42 (1), 19–25.

Rao, KN, Narasimha, R & Badri Narayanan, MA 1971 The ‘bursting’ phenomenon in a turbulent boundary layer. J. Fluid Mech. 48 (2), 339–352.

Robinson, SK 1991 Coherent motions in the turbulent boundary layer. Annu. Rev. Fluid Mech. 23 (1), 601–639.

Saddoughi, SG & Veeravalli, SV 1994 Local isotropy in turbulent boundary layers at high Reynolds number. J. Fluid Mech. 268, 333–372.

Salesky, ST & Anderson, W 2018 Buoyancy effects on large-scale motions in convective atmospheric boundary layers: implications for modulation of near-wall processes. J. Fluid Mech. 856, 135–168.

Salesky, ST, Chamecki, M & Bou-Zeid, E 2017 On the nature of the transition between roll and cellular organization in the convective boundary layer. Boundary-Layer Meteorol. 163 (1), 41–68.

Shafi, HS & Antonia, RA 1995 Anisotropy of the Reynolds stresses in a turbulent boundary layer on a rough wall. Exp. Fluids 18 (3), 213–215.

Shang, XD, Qiu, XL, Tong, P & Xia, KQ 2003 Measured local heat transport in turbulent Rayleigh-Bénard convection. Phys. Rev. Lett. 90 (7), 074501.

Simonsen, AJ & Krogstad, PÅ 2005 Turbulent stress invariant analysis: clarification of existing terminology. Phys. Fluids 17 (8), 088103.

Smailley, R, Leonard, S, Antonia, R, Djenidi, L & Orlandi, P 2002 Reynolds stress anisotropy of turbulent rough wall layers. Exp. Fluids 33 (1), 31–37.

Smedman, AS, Högström, U, Hunt, JCR & Sahli, E 2007 Heat/mass transfer in the slightly unstable atmospheric surface layer. Q. J. R. Meteorol. Soc. 133 (622), 37–51.

Smyth, WD & Moum, JN 2000 Anisotropy of turbulence in stably stratified mixing layers. Phys. Fluids 12 (6), 1343–1362.

Sreenivasan, KR 1991 Fractals and multifractals in fluid turbulence. Annu. Rev. Fluid Mech. 23 (1), 539–604.

Sreenivasan, KR, Antonia, RA & Britz, D 1979 Local isotropy and large structures in a heated turbulent jet. J. Fluid Mech. 94 (4), 745–775.

Sreenivasan, KR & Bershadskii, A 2006 Clustering properties in turbulent signals. J. Stat. Phys. 125 (5-6), 1141–1153.

Sreenivasan, KR, Prabhu, A & Narasimha, R 1983 Zero-crossings in turbulent signals. J. Fluid Mech. 137, 251–272.

Stiperski, I & Calaf, M 2018 Dependence of near-surface similarity scaling on the anisotropy of atmospheric turbulence. J. R. Meteorol. Soc. 144 (712), 641–657.

Stiperski, I, Calaf, M & Rotach, MW 2019 Scaling, anisotropy, and complexity in near-surface atmospheric turbulence. J. Geophys. Res. Atmos. 124 (3), 1428–1448.

Taylor, GI 1935 Statistical theory of turbulence. Proc. Roy. Soc. London Ser. A 151 (873), 465–478.

Tennekes, H & Lumley, JL 1972 A first course in turbulence. MIT press.

Vercauteren, N, Boyko, V, Faranda, D & Stiperski, I 2019 Scale interactions and anisotropy in stable boundary layers. Q. J. R. Meteorol. Soc. pp. 1–19.
Wallace, JM 2016 Quadrant analysis in turbulence research: history and evolution. *Annu. Rev. Fluid Mech.* **48**, 131–158.

Wang, Y, He, X & Tong, P 2019 Turbulent temperature fluctuations in a closed Rayleigh–Bénard convection cell. *J. Fluid Mech.* **874**, 263–284.

Wilczak, JM 1984 Large-scale eddies in the unstably stratified atmospheric surface layer. part I: Velocity and temperature structure. *J. Atmos. Sci.* **41** (24), 3537–3550.

Williams, AG & Hacker, JM 1993 Interactions between coherent eddies in the lower convective boundary layer. *Boundary-Layer Meteorol.* **64** (1-2), 55–74.

Willis, GE & Deardorff, JW 1976 On the use of Taylor’s translation hypothesis for diffusion in the mixed layer. *Q. J. R. Meteorol. Soc.* **102** (434), 817–822.

Wyngaard, JC 2010 *Turbulence in the atmosphere*. Cambridge University Press.

Wyngaard, JC, Coté, OR & Izumi, Y 1972 Cospctral similarity in the atmospheric surface layer. *Q. J. R. Meteorol. Soc.* **98** (417), 590–603.

Wyngaard, JC, Coté, OR & Izumi, Y 1971 Local free convection, similarity, and the budgets of shear stress and heat flux. *J. Atmos. Sci.* **28** (7), 1171–1182.

Yee, E & Chan, R 1995 Fractal characteristics of isoconcentration surfaces in plumes dispersing in the atmospheric surface layer. *Phys. Fluids* **7** (11), 2715–2724.

Yee, E, Kosteniuk, PR, Chandler, GM, Biltoft, CA & Bowers, JF 1993 Statistical characteristics of concentration fluctuations in dispersing plumes in the atmospheric surface layer. *Boundary-Layer Meteorol.* **65** (1-2), 69–109.

Yeung, PK & Brasseur, JG 1991 The response of isotropic turbulence to isotropic and anisotropic forcing at the large scales. *Phys. Fluids* **3** (5), 884–897.

Zhou, Q & Xia, KQ 2011 Disentangle plume-induced anisotropy in the velocity field in buoyancy-driven turbulence. *J. Fluid Mech.* **684**, 192–203.

Zhou, SQ & Xia, KQ 2002 Plume statistics in thermal turbulence: mixing of an active scalar. *Phys. Rev. Lett.* **89** (18), 184502.

Zhuang, Y 1995 Dynamics and energetics of convective plumes in the atmospheric surface layer. *J. Atmos. Sci.* **52** (10), 1712–1722.

Zou, J, Zhou, B & Sun, J 2017 Impact of eddy characteristics on turbulent heat and momentum fluxes in the urban roughness sublayer. *Boundary-Layer Meteorol.* **164** (1), 39–62.
Figure S1: The (a) three velocity variance components ($\sigma_x^2$, where $x$ can be $u$, $v$, or $w$) and the (b) correlation coefficients between $w$ and $x$ ($R_{wx}$, where $x$ can be $T$ or $u$) along with the transport efficiencies of heat and momentum ($\eta_{wT}$, $\eta_{uw}$) are plotted against the stability ratio $-\zeta$. In panel (b), the correlation coefficients are shown on the left $y$ axis, whereas the transport efficiencies are shown on the right $y$ axis. Note that the absolute values of $R_{uw}$ ($|R_{uw}|$) are plotted in panel (b) instead of their original negative values.
Figure S2: The PDFs of (a) $u' / \sigma_u (\hat{u})$, (b) $w' / \sigma_w (\hat{w})$, and (c) $T' / \sigma_T (\hat{T})$ are shown for the six different classes of the stability ratio as indicated in the legend on panel (c). The thick black line on all the panels is the Gaussian distribution.
Figure S3: The (a) heat flux fractions \(F_f\), see (2.3)) and (b) time fractions \(T_f\), see (2.3)) associated with the four different quadrants of \(T' - w'\) plane, as indicated in the legend in panel (a). The differences in flux fractions \(\Delta F_f\) and time fractions \(\Delta T_f\) between the warm-updrafts and cold-downdrafts are shown in panel (c).
Figure S4: The log-log plots of the PDFs of the normalized streamwise sizes \( \frac{(T_B \bar{u})}{z} \) (see (2.28)) corresponding to the warm-updrafts (blue open circles), cold-downdrafts (black open squares), cold-updrafts (blue crosses), and warm-downdrafts (black pluses) are shown for the six different stability classes as indicated in the legend placed at the rightmost corner. The thick red line is the same power law as shown in figure 8.
The role of intermittent heat transport towards Reynolds stress anisotropy

Figure S5: The log-log plots of the cumulative distribution functions of the normalized streamwise sizes $(T_B\bar{u})/z$ (see (3.10)) corresponding to the warm-updrafts (open circles) and cold-downdrafts (filled squares) are shown. The plots are shifted by at least a decade for visual representation and the different colours represent the six different stability classes as indicated on the figure.

Figure S5: The log-log plots of the cumulative distribution functions of the normalized streamwise sizes $(T_B\bar{u})/z$ (see (3.10)) corresponding to the warm-updrafts (open circles) and cold-downdrafts (filled squares) are shown. The plots are shifted by at least a decade for visual representation and the different colours represent the six different stability classes as indicated on the figure.
Figure S6: The distributions of the degree of isotropy ($C_3$) are plotted against the normalised sizes ($T_B u / z$ (see (2.25))) of the (a) warm-updrafts (open circles), (b) cold-downdrafts (open squares), (c) cold-updrafts (crosses), and (d) warm-downdrafts (pluses). The different colours represent the six different stability classes as indicated in the legend placed at the right most corner.
Figure S7: The distribution of the three velocity variances plotted against the normalized sizes $(T_B \bar{u})/(z^{0.5} \lambda^{0.5})$ (see (3.16)) of the warm-updrafts (top panels) and cold-downdrafts (bottom panels), are shown corresponding to three classes of stability ratios such as $-\zeta > 2$, $0.4 < -\zeta < 0.6$, and $0 < -\zeta < 0.2$, for the illustration purpose. The legends describing the markers are shown in panels (a) and (b). The thick and dotted pink lines indicate the collapsed position of the peaks of the degree of isotropy associated with the warm-updrafts and cold-downdrafts, as shown in figure 11.