A STANDARD MODEL
IN FOLDY-WOUTHUYSEN REPRESENTATION

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Abstract

The paper formulates a standard model in the Foldy-Wouthuysen representation using previously developed approaches as applied to quantum electrodynamics. The formulation of the theory in the FW representation does not require obligatory interaction of Higgs bosons with fermions for SU(2) invariance.

In this approach the Higgs boson spectrum is narrowed significantly: the Higgs bosons are responsible only for the gauge invariance of the theory and interact only with gauge bosons.

1 Introduction

Historically, the first equation to describe the interaction of the 1/2-spin particle with external electromagnetic field has been nonrelativistic Pauli equation

\[ p_0 \varphi(x) = \left( \frac{\vec{p} - e\vec{A}(x)}{2m} \right)^2 - \frac{e\vec{\sigma}\vec{B}}{2m} + eA_0(x) \varphi(x). \]  

In relation (1) and below the system of units with \( \hbar = c = 1 \) is used; \( x, p, A \) are 4-vectors; as usually, \( xy = x^\mu y^\mu = x^0 y^0 - x^k y^k, \mu = 0, 1, 2, 3, k = 1, 2, 3; p^\mu = i \frac{\partial}{\partial x^\mu}; \vec{B} = \text{rot}\vec{A}; \sigma^k \) are Pauli matrices; \( \varphi(x) \) is a two-component wave function.

Afterwards Dirac derived his famous relativistic equation describing the 1/2-spin particle motion. In the case of the interaction with electromagnetic field the Dirac equation is

\[ p_0 \psi_D(x) = \left[ \vec{\alpha} \left( \vec{p} - e\vec{A}(x) \right) + \beta m + eA_0(x) \right] \psi_D(x). \]  

Here \( \psi_D(x) \) is a four-component wave function; \( \alpha^i = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}, \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \) are Dirac matrices. Unlike equation (1), the Dirac equation is linear in impulse components \( p^\mu \) and can be straightforwardly written in its covariant form. The Pauli equation is a
nonrelativistic limit of the Dirac equation for the upper components of wave function \( \psi_D(x) \).

In principle, the relativistic generalization of the Pauli equation could be performed in another way, with using the relativistic particle energy-to-momentum ratio as a basis in the absence of any external fields. This was actually done by Foldy and Wouthuysen in their classical paper [1]. The Foldy-Wouthuysen equation for free motion is

\[
p_0 \psi_{FW} (x) = (H_0)_{FW} \psi_{FW} (x) = \beta E \psi_{FW} (x).
\]

In expression (3) \( E = \sqrt{\vec{p}^2 + m^2} \).

Solutions to equation (3) are plane waves of positive and negative energies,

\[
\psi^{(+)}_{FW} (x, s) = \frac{1}{(2\pi)^{3/2}} U_s e^{-ipx}; \quad \psi^{(-)}_{FW} (x, s) = \frac{1}{(2\pi)^{3/2}} V_s e^{-ipx}; \quad p_0 = \left( \vec{p}^2 + m^2 \right)^{1/2},
\]

where \( U_s = \begin{pmatrix} \chi_s \\ 0 \end{pmatrix} \), \( V_s = \begin{pmatrix} 0 \\ \chi_s \end{pmatrix} \) are two-component normalized Pauli functions. For \( U_s \) and \( V_s \) the relevant orthonormality and completeness relations are valid.

Hamiltonian \((H_0)_{FW}\) is related with free Dirac Hamiltonian \((H_0)_D\) by the unitary transformation. In equation (3), obvious asymmetry of space coordinates and time is seen, although it is relativistically invariant by itself.

In the general case of the interaction with electromagnetic field \( A^\mu (x) \) there is no exact unitary transformation transforming Dirac equation (2) to Foldy-Wouthuysen (FW) representation. In that case Foldy and Wouthuysen found Hamiltonian \( H_{FW} \) in the form of a series in terms of powers of \( 1/m \) [1].

Blount [2] found the Hamiltonian \( H_{FW} \) in the form of a series in terms of powers of smallness of fields and their time and space derivatives. Case [3] obtained an exact transformation in the presence of time-independent external magnetic field \( \vec{B} = rot \vec{A} \). In this case the Dirac equation is transformed to equation

\[
p_0 \psi_{FW} (x) = H_{FW} \psi_{FW} (x) = \beta \sqrt{\left( \vec{p} - e \vec{A} \right)^2 - e\vec{\sigma} \vec{B} + m^2} \psi_{FW} (x).
\]

In ref. [4] the author finds the relativistic Hamiltonian \( H_{FW} \) in the form of series in terms of powers of charge \( e \) in the general case of the interaction with external field \( A^\mu (x) \).

In all the cases considered, the wave function equations in the Foldy-Wouthuysen representation are of noncovariant form and their Hamiltonians are nonlocal. In the FW representation there is no relation between upper and lower wave function components, i.e. \( H_{FW} \) is in essence a two-component Hamiltonian.

As the relativistic Hamiltonian \( H_{FW} \) obtained in ref. [4] has become available the possibility appeared to consider quantum-field processes in the FW representation within the perturbation theory.

Section 2 of this paper briefly discusses quantum electrodynamics (QED) in the Foldy-Wouthuysen representation [5], [6], [7]. This Section introduces the conceptual apparatus
to be used in Section 3, which addresses the electroweak theory and quantum chromodynamics in the Foldy-Wouthuysen representation.

Section 3 also discusses the role of Higgs bosons as applied to the formulation of the theory in the new representation.

The formulation of the standard model in the Foldy-Wouthuysen representation does not require for the purposes of $SU(2)$ invariance of the theory the obligatory interaction of Higgs bosons with fermions; in this case the Higgs bosons are responsible only for the gauge invariance of the theory and interact only with gauge bosons $W^\pm, Z^\mu$.

## 2 Quantum electrodynamics in the Foldy-Wouthuysen representation

In notation of ref. [4] the Dirac equation for quantized electron-positron field in the $FW$ representation is written as

$$p_0\psi_{FW}(x) = H_{FW}\psi_{FW}(x) = (\beta E + K_1 + K_2 + K_3 + \ldots)\psi_{FW}(x);$$

$$K_1 \sim e, \quad K_2 \sim e^2, \quad K_3 \sim e^3. \quad (6)$$

Feynman propagator of the Dirac equation in the Foldy-Wouthuysen representation is

$$S_{FW}(x - y) = \frac{1}{(2\pi)^4} \int d^4 p \frac{e^{-ip(x-y)}}{p_0 - \beta E} =$$

$$= \frac{1}{(2\pi)^4} \int d^4 p e^{-ip(x-y)} \frac{p_0 + \beta E}{p^2 - m^2 + i\varepsilon} =$$

$$= -i\theta (x_0 - y_0) \int d\vec{p} \sum_s \psi^{(+)}_{FW}(x, s) \left(\psi^{(+)}_{FW}(y, s)\right)^+ +$$

$$+ i\theta (y_0 - x_0) \int d\vec{p} \sum_s \psi^{(-)}_{FW}(x, s) \left(\psi^{(-)}_{FW}(y, s)\right)^+ \quad (7)$$

Relation (7) implies the Feynman rule of pole bypass; $\theta(x_0) = \{1, \quad x_0 > 0; \quad 0, \quad x_0 < 0.$

The integral equation for $\psi_{FW}(x)$ is

$$\psi_{FW}(x) = \psi_0(x) + \int d^4 y S_{FW}(x - y)(K_1 + K_2 + \ldots)\psi_{FW}(y), \quad (8)$$

where $\psi_0(x)$ is a solution to the Dirac equation in the $FW$ representation in the absence of electromagnetic field ($A^\mu = 0$).

Expressions (7), (8) can be used to formulate the Feynman rules for writing of scattering matrix elements $S_{fi}$ and calculation of QED processes. In contrast to the Dirac representation, in the $FW$ representation there are infinitely many types of vertices of interaction with photons depending on the perturbation theory order: a vertex of interaction with one photon is correspondent with factor $-iK_{1\mu}$, a vertex of interaction with two
photons with factor $-iK_{2\mu}$, and so on. For convenience the parts of interaction Hamiltonian terms $K_1, K_2, \ldots$ without electromagnetic potentials $A^\mu, A^\mu A^\nu, \ldots$ are denoted by $K_{1\mu}, K_{2\mu}, \ldots$, respectively.

Each external fermion line is correspondent with one of functions (4). As usual, the positive-energy solutions correspond to particles, the negative-energy ones to antiparticles. The other Feynman rules remain the same as in the spinor electrodynamics in the Dirac representation.

When the external fermion line impulses lie on mass surface ($p^2 = m^2$), a feature of the theory under discussion is compensation of the contribution of fermion-propagator diagrams with that of the relevant terms in the diagrams with the higher-order vertices of the expansion in terms of powers of charge $e$ [7]. Vertex operators $K_n$ are therewith simplified significantly due to the law of conservation of energy-momentum. In view of the aforesaid, the scattering matrix expansion in terms of powers of $e$ can be performed, in which matrix elements $S_{fi}$ will contain no terms including electron-positron propagators. In this case the resultant relations are close in their structure to the relations of the “old”, noncovariant perturbation theory developed by Heitler in the Dirac representation [8]. A significant distinction of the relations in the FW representation from relations [8] is the absence of any interaction between real electrons and positrons because of the matrix structure of Hamiltonian $H_{FW}$. In this representation the electron-positron interaction can proceed only between real and intermediate virtual states.

The interaction Hamiltonian terms $K_n$ can only include an even number of operators relating the initial and final positive-energy states to the intermediate negative-energy states and vice versa.

To construct interactions of real particle-antiparticle in the theory, additional terms should be introduced to the Hamiltonian $H_{FW}$. A method to include the processes involving real electron-positron pairs ensuring proper results in the calculation of QED effects (for example, electron-positron pair annihilation cross section) is to introduce the interaction between positive-energy (negative-energy) states of equation (6) and negative-energy (positive-energy) states of equation (9) derived by the FW transformation of Dirac equation (2) with negative particle mass

$$p_0 \psi_{1FW} (x) = [\beta E + K_1 (-m) + K_2 (-m) + \ldots] \psi_{1FW} (x). \tag{9}$$

Equation (6) with the additional interaction can be written as

$$p_0 \psi_{FW} (x) = \beta E \psi_{FW} (x) + \left[ K_1 (m, I, m) + \frac{1}{2} K_2 (m, I, m; m, I, m) + \frac{1}{2} K_2 (m, I, m; m, I, m) + \ldots \right] \psi_{FW} (x) +$$

$$+ \left[ K_1 (m, \gamma_5 - m) + \frac{1}{2} K_2 (m, \gamma_5 - m; m, I, -m) + \frac{1}{2} K_2 (m, I, m; m, \gamma_5, -m) + \ldots \right] \psi_{1FW} (x). \tag{10}$$

In equation (10) the notation of terms $K_1, K_2, \ldots$ indicates the presence or absence of matrix $\gamma_5$ near potentials $A^\mu (x)$ and the mass sign on the left and on the right of fields.
$A^\mu (x)$. Factor $1/2$ of terms $K_2$ is introduced because of two possible methods of the transition to the final state of mass $+m$. Similar to (10), additional terms of interaction with field $\psi_{FW} (x)$ can be introduced to negative-mass equation (9).

It should be particularly emphasized that in the context under discussion the particle mass sign is some internal quantum number, which is not associated by the author in this paper with a gravitational interaction sign. Of course, the energy of negative-mass-sign particle is positive.

The considered interaction between equations (6), (9) can be formalized as follows. Introduce an eight-component field $\Phi_{FW} (x)$, in which the four upper components are solutions to equation (6) with positive mass $(+m)$, while the lower components are solutions to equation (9) with negative mass $(-m)$. In the case under discussion solutions (4) for free field are written as

$$\Phi_{FW}^{(+)} (x, s, +m) = \frac{1}{(2\pi)^{3/2}} \begin{pmatrix} U_s \\ 0 \end{pmatrix} e^{-ipx}; \quad (11a)$$

$$\Phi_{FW}^{(+)} (x, s, -m) = \frac{1}{(2\pi)^{3/2}} \begin{pmatrix} 0 \\ U_s \end{pmatrix} e^{-ipx}; \quad (11b)$$

$$\Phi_{FW}^{(-)} (x, s, +m) = \frac{1}{(2\pi)^{3/2}} \begin{pmatrix} V_s \\ 0 \end{pmatrix} e^{ipx}; \quad (11c)$$

$$\Phi_{FW}^{(-)} (x, s, -m) = \frac{1}{(2\pi)^{3/2}} \begin{pmatrix} 0 \\ V_s \end{pmatrix} e^{ipx}; \quad (11d)$$

The extension of the orthonormality and completeness relations to eight dimensions is quite evident.

Next, introduce matrices $8 \times 8$:

$$\beta_1 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \rho = \begin{pmatrix} 0 & \gamma_5 \\ \gamma_5 & 0 \end{pmatrix},$$

$$\alpha^k = \begin{pmatrix} \alpha^k & 0 \\ 0 & \alpha^k \end{pmatrix}, \quad \sigma^k = \begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix}, \quad \beta = \begin{pmatrix} \beta & 0 \\ 0 & \beta \end{pmatrix}.$$

$$\beta_1^2 = \rho^2 = I; \quad [\beta_1, \rho]_+ = [\beta, \rho]_+ = 0;$$

$$[\beta_1, \alpha^k]_- = [\beta_1, \sigma^k]_- = [\beta_1, \beta]_- = 0;$$

$$[\rho, \alpha^k]_- = [\rho, \sigma^k]_- = 0.$$

Having substituted $m \to \beta_1 m$, equations (6), (9) with using matrices $8 \times 8$ can be combined as

$$p_0 \Phi_{FW} (x) = \left[ \beta E + K_1 (\beta_1 m, (I + \rho), \beta_1 m) + \frac{1}{2} K_2 (\beta_1 m, (I + \rho), \beta_1 m); \quad \beta_1 m (I + \rho), \beta_1 m) + \frac{1}{4} K_3 (\beta_1 m, (I + \rho), \beta_1 m; \beta_1 m, (I + \rho), \beta_1 m; \right.$$  

$$\beta_1 m, (I + \rho), \beta_1 m) + \ldots ] \Phi_{FW} (x) \quad (12)$$
Equation (12) contains equation (10) for field $\psi_{FW}(x)$ and the relevant equation for field $\psi_{1FW}(x)$. Interaction $(I + \rho) A^\mu(x)$ enables coupling solutions (11a) and (11d), (11b) and (11c), whereas there is no coupling as previously between the other pairs of solutions (11a) and (11c), (11b) and (11d) as well as (11a) and (11b), (11c) and (11d).

The analysis suggests that coupling $(I + \rho) A^\mu$ alongside the inclusion of the real electron-positron pair interaction processes does not modify the physical results of QED processes [7]. Processes involving real negative-mass fermions appear in the theory.

Thus, with coupling $(I + \rho) A^\mu$ the theory in the $FW$ representation is symmetric about particle (antiparticle) mass sign, however the signs of masses in the particle and antiparticle interacting with each other must be opposite. Note that previously the conclusion of opposite signs of particle and antiparticle was made by Recami and Ziino [9] when analyzing a special relativity theory and conditions of “particle ↔ antiparticle” reversibility.

Another possible method to include processes with real particle-antiparticle pairs in the theory is coupling of the equations of motion for electron and positron in field $A^\mu(x)$. That coupling is not analyzed in this paper.

3 Electroweak theory and quantum chromodynamics in the Foldy-Wouthuysen representation.
Role of Higgs bosons in the Foldy-Wouthuysen representation

First write out the fermion and fermion-boson Hamiltonian of the standard model in the Dirac representation, which is responsible for free motion of the fermions and for the interaction of quarks and leptons with photons, $W^\pm$ and $Z^0$ particles, gluons, and Higgs bosons.

\[
H_D = \sum_{\nu, e, u, d} \left\{ (\psi_D)^+_f \left( \bar{\alpha} \gamma^\mu + \beta m_f \right) (\psi_D)_f + eQ_f (\psi_D)^+_f \alpha^\mu (\psi_D)_f A^\mu + \right.
\]
\[+ \frac{g_2}{\cos \theta_W} \left[ (\psi_D)_f^+ \alpha^\mu \left( \frac{I - \gamma_5}{2} \right) (\psi_D)_f \left( T^3_f - Q_f \sin^2 \theta_W \right) + \right. \]
\[+ (\psi_D)_f^+ \alpha^\mu \left( \frac{I + \gamma_5}{2} \right) (\psi_D)_f \left( -Q_f \sin^2 \theta_W \right) \]
\[\left. \right] Z^\mu \right\} + \]
\[+ \frac{g_3}{\sqrt{2}} \sum_{f=u,d} (\psi_D)^+_f \alpha^\mu \left( \frac{I - \gamma_5}{2} \right) (\psi_D)_d + (\psi_D)^+_d \alpha^\mu \left( \frac{I - \gamma_5}{2} \right) (\psi_D)_e \right] W^+_\mu + \text{Hermit.conj.} \] + \]
\[+ \frac{g_3}{\sqrt{2}} \sum_{f=u,d} (\psi_D)^+_f \alpha^\mu \lambda^a_{\alpha\beta} (\psi_D)_f G^a_{\mu} - \sum_{f=e,u,d} \frac{m_f}{\sqrt{2}} (\psi_D)^+_f \beta (\psi_D)_f h. \]

In (13), $(\psi_D)_f$ is the Dirac fermion field; $A^\mu$ is the electromagnetic field; $Z^\mu$, $W^\pm_\mu$ are the gauge boson fields; $G^a_{\mu}$ are the gluon fields; $h$ is the neutral Higgs boson field.
Besides, in \( \alpha^\mu = \begin{cases} 
1, & \mu = 0; \\
\alpha^k, & \mu = k = 1, 2, 3; \end{cases} \) \( Q_f \) is the fermion electric charge in the units of \( e \); \( T^3_f = 1/2 \) for \( f = \nu_e \), \( u \); \( T^3_f = -1/2 \) for \( f = e, d \); \( \theta_W \) is the electroweak mixing angle; \( g_2 = \frac{e}{\sin \theta_W} \); \( g_3 \) is the quantum chromodynamics coupling constant; \( \lambda^a \) are generators of group \( SU(3) \); \( m_f \) is mass of fermion \( f \) (in \( \lambda^3 \) \( m_{\nu_e} = 0 \) is assumed); \( v \) is the Higgs vacuum mean.

The Hamiltonian in \( \lambda^3 \) is written out only for the first lepton and quark family. For the second and third families it is necessary to make appropriate substitutions \( (\nu_e, e, u, d) \rightarrow (\nu_\mu, \mu, c, s) \) and \( (\nu_\tau, \tau, t, b) \) in \( \lambda^3 \) and introduce the quark mixing.

Previously, in ref. [7], the transition to the \( FW \) representation was discussed for the fifth term in Hamiltonian \( \lambda^3 \) responsible for charged current \( V-A \) interaction. It was shown that the transition to the Foldy-Wouthuysen representation can be performed by similar methods, like in quantum electrodynamics with the extension of the Dirac matrices to eight dimensions and replacement of even operator \( \gamma^5 \) in (13) and introduction of fermion fields \( \psi \).

In (14), (15), like previously, \( L = \frac{\beta \bar{\alpha} \bar{p}}{E + m} \); \( R = \sqrt{\frac{E + m}{2E}} \).

Of course, the final results of calculations of specific processes with the charged weak \( V-A \) interaction in the \( FW \) representation are the same as those in the Dirac representation [7].

In a more general case of Hamiltonian \( \lambda^3 \) the transition to the Foldy-Wouthuysen representation can be also performed using methods developed for quantum electrodynamics with the Dirac matrix extension to eight dimensions. As a result, in the Foldy-Wouthuysen representation Hamiltonian \( \lambda^3 \) can be written as follows:

\[
H_{FW} = \sum_{f = \nu_e, e, a, d} (\Phi_{FW})^*_f (\beta E_f + K'_1 + K'_2 + \ldots) (\Phi_{FW})_f
\]  

(16)

In \( \lambda^3 \), \( E_f = (\bar{p}_f^2 + m_f^2)^{1/2} \) is the fermion \( f \) free motion energy operator. Expansion \( \lambda^3 \) is in terms of powers of coupling constants \( e, g_2, g_3, \frac{m_f}{v} \) and their reciprocal products.

\(^3\)Hereinafter the even (old) operators are the ones that do not couple (couple) the upper and lower components of fermion fields \( \psi_{FW} \).
The operators $K_1', K_2', \ldots$ are similar in their structure to the operators $K_1, K_2, \ldots$ in quantum electrodynamics with $e\alpha^\mu A^\mu$ replaced by

$$eQ_f \alpha^\mu A^\mu + \frac{g_2}{\cos \theta_W} \left[ (T_f^3 - Q_f \sin^2 \theta_W) \alpha^\mu \left( \frac{I - \gamma_5}{2} \right) - Q_f \sin^2 \theta_W \alpha^\mu \left( \frac{I + \gamma_5}{2} \right) \right] Z_\mu +$$

$$+ \frac{g_2}{\sqrt{2}} \left[ \left( f = u \right) \alpha^\mu \left( \frac{I - \gamma_5}{2} \right) (f = d) + \left( f = \nu_e \right) \alpha^\mu \left( \frac{I - \gamma_5}{2} \right) (f = e) \right] W^+_\mu +$$

$$\text{Hermit.conj.} \right] + \frac{g_3}{2} \left[ (f = u, d) \alpha^\mu \lambda_\alpha^\beta (f = u, d) \beta G^a_\mu \right] - \frac{m_f}{\nu} \beta \hbar. \quad (17)$$

Besides, given expressions with operators $\left( \frac{I \pm \gamma_5}{2} \right)$ in the Dirac Hamiltonian, even operators $C^\mu$ should be replaced by operators $C^\mu \pm (N')^\mu$ and odd operators $N^\mu$ by operators $N^\mu \pm (C')^\mu$ in the relevant expressions in the FW representation as against the quantum electrodynamics case.

In (17) notations $(f = u), (f = d)$, etc. imply that spinor FW-fields of associated fermions will be located at specified places in the Hamiltonian.

By their construction Hamiltonians (13) and (16) are invariant under $SU(3) \times SU(2) \times U(1)$ transformations. Note that in (16) the Hamiltonian of free motion is invariant under $SU(2)$ symmetry as opposed to the Dirac Hamiltonian of free motion.

In their structure, in Dirac free Hamiltonian, terms $\sim \bar{\alpha} \vec{p}$ and $\sim \beta m$ are transformed in different ways in $SU(2)$ transformations, whereas in Foldy-Wouthuysen free Hamiltonian $\sim \beta \sqrt{\vec{p}^2 + m^2}$ both the subduplicates are transformed identically in the $SU(2)$ transformations. This can be shown straightforwardly.

In fact, for example, the left projection operator $(P_D)_L = \frac{I - \gamma_5}{2}$ in the FW representation is $(P_{FW})_L = \frac{1}{2} \left( I - \frac{\beta \vec{p}}{E} - \frac{\beta_1}{E} \gamma_5 \right)$; the even part of the operator is $(P_{FW})_L^{e} = \frac{1}{2} \left( I - \frac{\beta \vec{p}}{E} \right)$; similarly, the even part of the right projection operator is $(P_{FW})_R^{e} = \frac{1}{2} \left( I + \frac{\beta \vec{p}}{E} \right)$. Hamiltonian of free motion in the Foldy-Wouthuysen representation can be represented as

$$\psi_{FW}^+ \beta E \psi_{FW} = \psi_{FW}^+ \left[ (P_{FW})_L^{e} \beta E (P_{FW})_L^{e} + (P_{FW})_R^{e} \beta E (P_{FW})_R^{e} + \frac{1}{2} \frac{m^2}{E^2} \beta E \right] \psi_{FW} =$$

$$= \psi_{FW}^+ \left[ (P_{FW})_L^{e} \beta E \left( 1 + \frac{1}{2} \frac{m^2}{E^2} + \frac{1}{4} \frac{m^4}{E^4} + \ldots \right) (P_{FW})_L^{e} +$$

$$+ (P_{FW})_R^{e} \beta E \left( 1 + \frac{1}{2} \frac{m^2}{E^2} + \frac{1}{4} \frac{m^4}{E^4} + \ldots \right) (P_{FW})_R^{e} \right] \psi_{FW} =$$

$$= \left( \psi_{FW}^+_L \right) \frac{\beta E}{1 - \frac{1}{2} \frac{m^2}{E^2}} (\psi_{FW})_L + \left( \psi_{FW}^+_R \right) \frac{\beta E}{1 - \frac{1}{2} \frac{m^2}{E^2}} (\psi_{FW})_R. \quad (18)$$
In (18) \((\psi_{FW})_L = (P_{FW})_L^c \psi_{FW}; (\psi_{FW})_R = (P_{FW})_R^c \psi_{FW}\). From (18) one can see the desired invariance of the Hamiltonian. In view of the aforesaid an interesting observation can be made.

The mass term in the first term of Hamiltonian (13) and the last term in (13) appeared from the Higgs mechanism of the spontaneous break of symmetry. Introduction of the mass term to (13) without the Higgs mechanism would break the \(SU(2)\) symmetry of the standard model in the Dirac representation.

Nevertheless, regardless of the break of \(SU(2)\) symmetry, introduce the mass term to (13) without the Higgs mechanism and then transfer to the Foldy-Wouthuysen representation. As a result we will obtain Hamiltonian (16) invariant under \(SU(3) \times SU(2) \times U(1)\) transformations, but without the terms responsible for the interaction of fermions with scalar Higgs bosons. Thus, the formulation of the standard model in the Foldy-Wouthuysen representation requires no obligatory interaction of Higgs bosons with fermions for the purpose of the \(SU(2)\) invariance of the theory. In this case the Higgs boson sector gets narrower significantly: the Higgs bosons are responsible only for the gauge invariance of the theory and interact only with the gauge bosons \(W^\pm_\mu, Z_\mu\).

4 Conclusion

The consideration of the interacting field theory variations in the Foldy-Wouthuysen representation allows extraction of some new physical consequences as against the similar theory variations in the Dirac representation. When including interactions of real particles with antiparticles, fermions of negative mass sign (but positive energy) appear in the theory. The theory is symmetric about mass sign, but the particle and antiparticle masses should be of opposite signs. In the theory there is a possibility to relate break of \(CP\) symmetry to break of symmetry in particle (antiparticle) mass sign [7].

Finally, the standard model in the Foldy-Wouthuysen representation with preserved \(SU(3) \times SU(2) \times U(1)\) invariance can be formulated without the requirement of the interaction of Higgs bosons with fermions. Hence it appears that further theoretical studies of the Foldy-Wouthuysen representation and comparison of their results to available experimental data are needed.

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