DUAL $N = 2$ SUSY BREAKING

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Abstract

We discuss spontaneous supersymmetry breaking in $N = 2$ globally supersymmetric theories describing abelian vector multiplets. The most general form of the action admits, in addition to the usual Fayet-Iliopoulos term, a magnetic Fayet-Iliopoulos term for the auxiliary components of dual vector multiplets. In a generic case, this leads to a spontaneous breakdown of one of the two supersymmetries. In some cases however, dyon condensation restores $N = 2$ SUSY vacuum. This talk is based on the work done in collaboration with H. Partouche [1].

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Recently there has been revived interest in $N = 2$ supersymmetry, in particular in the effective actions describing non-perturbative dynamics of non-abelian gauge theories. In general, these theories exist only in the Coulomb phase, with a number of abelian vector multiplets and possibly hypermultiplets, and their low energy effective actions can be determined exactly by using the underlying duality symmetries \cite{ref2}. These exact solutions rely heavily on the restrictions following from the general structure of $N = 2$ SUSY Lagrangians. In all known examples, non-perturbative effects preserve $N = 2$ SUSY, therefore massless vector multiplet interactions are fully described by the standard prepotential. However, a general $N = 2$ SUSY theory admits also some Lagrangian terms that lead to spontaneous breakdown of one or both supersymmetries.

Only one mechanism, based on $N = 1$ supersymmetric Fayet-Iliopoulos (FI) term \cite{ref3} has been known so far to break $N = 2$ supersymmetry. It can be realized in the presence of a $N = 2$ vector multiplet associated to an abelian gauge group factor. Decomposed under $N = 1$ supersymmetry, such a multiplet contains one vector and one chiral multiplet. A FI term is also equivalent to a superpotential which is linear in the chiral superfield. No other superpotential seemed to be allowed for chiral components of $N = 2$ vector multiplets.

Since we are interested in $N = 2$ SUSY theories viewed as low-energy realizations of some more complicated physical systems, we do not impose the renormalizability requirement and consider the most general form of the Lagrangian. The basic points of our analysis can be explained on the simplest example of $N = 2$ supersymmetric gauge theory with one abelian vector multiplet $A$ which contains besides the $N = 1$ gauge multiplet $(A_\mu, \lambda)$ a neutral chiral superfield $(a, \chi)$. For the sake of clarity, we begin with $N = 1$ superfield description and redervive our results later on by using the full $N = 2$ formalism. In the absence of superpotential and FI term, the most general Lagrangian describing this theory is determined by the analytic prepotential $\mathcal{F}(A)$, in terms of which the Kähler potential $K$
and the gauge kinetic function $\tau$ are given by:

$$K(a, \bar{a}) = \frac{i}{2}(a\bar{F}_\bar{a} - \bar{a}F_a) \quad \tau(a) = F_{aa}, \quad (1)$$

where the $a$ and $\bar{a}$ subscripts denote derivatives with respect to $a$ and $\bar{a}$, respectively. In $N = 1$ superspace, the Lagrangian is written as:

$$\mathcal{L}_0 = -\frac{i}{4} \int d^2\theta \tau \mathcal{W}^2 + c.c. + \int d^2\theta d^2\bar{\theta} K \quad (2)$$

where $\mathcal{W}$ is the standard gauge field strength superfield.

The Lagrangian $\mathcal{L}_0$ can be supplemented by a FI term which is linear in the auxiliary $D$ component of the gauge vector multiplet:

$$\mathcal{L}_D = \sqrt{2} \xi D, \quad (3)$$

with $\xi$ a real constant. It is well known that such a term preserves also $N = 2$ supersymmetry [3].

The Lagrangian $\mathcal{L}_0$ can also be supplemented by a superpotential term

$$\mathcal{L}_W = \int d^2\theta W + c.c. \quad (4)$$

In order to determine what form of the superpotential is compatible with $N = 2$ supersymmetry we will impose the constraint that the full Lagrangian,

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_D + \mathcal{L}_W, \quad (5)$$

be invariant under the exchange of the gaugino $\lambda$ with the fermion $\chi$. This condition is necessary for the global $SU(2)$ symmetry under which $(\chi, \lambda) \equiv (\lambda_1, \lambda_2)$ transforms as a doublet. It is easy to see that it is satisfied for [4]

$$W = ea + mF_a, \quad (6)$$

\[1\]We use the conventions of ref. [4].
up to an irrelevant additive constant. Here $e$ and $m$ are arbitrary real numbers. For $m = 0$ the above superpotential is equivalent to a FI term (3) with $\xi = e$.

After eliminating the auxiliary fields, $L_D + L_W$ gives rise to only two modifications in the original Lagrangian $L_0$. It induces the fermion mass terms mentioned before, $\frac{1}{2} \mathcal{M}_{ij} \lambda_i \lambda_j$, with

$$\mathcal{M} = \frac{i}{2} \tau_a \begin{pmatrix} e + m \bar{\tau} & i \xi \\ i \xi & e + m \bar{\tau} \end{pmatrix}$$

and the scalar potential

$$V_{N=1} = \frac{|e + m \tau|^2 + \xi^2}{\tau_2}$$

where $\tau = \tau_1 + i \tau_2$.

In order to prove that the full Lagrangian (5) is indeed invariant under $N = 2$ supersymmetry, we will rederive it by using the $N = 2$ superspace formalism. In this formalism, $N = 2$ vector multiplets are described by reduced chiral superfields. The reducing constraint

$$(\epsilon_{ij} D_i \sigma_{\mu \nu} D^j)^2 A = -96 \Box A^*$$

eliminates unwanted degrees of freedom, in particular by imposing the Bianchi identity for the gauge field strength. The auxiliary components of $A, Y_n, n = 1, 2, 3$, which form an $SU(2)$ triplet $\vec{Y}$ are also constraint by eq.(3):

$$\Box \vec{Y} = \Box \vec{Y}^*$$

The above equation imposes real $\vec{Y}$, modulo a constant imaginary part:

$$\vec{Y} = \text{Re} \vec{Y} + 2i \vec{M} ,$$

where $\vec{M}$ is an arbitrary real constant vector.

In terms of the reduced chiral superfield $A$, the Lagrangian $L_0$ can be written as

$$L_0 = \frac{i}{4} \int d^2 \theta_1 d^2 \theta_2 \mathcal{F}(A) + c.c.$$ 

These are related to the standard $N = 1$ auxiliary components $F$ and $D$ by $Y_1 + i Y_2 = 2i F, Y_3 = \sqrt{2} D$. 


As in the $N = 1$ case $\mathcal{L}_0$ can be supplemented with a Fayet-Iliopoulos term. Under $N = 2$ supersymmetry transformations, the auxiliary components of reduced vector multiplets transform into total derivatives. Hence a FI term, linear in $\vec{Y}$, can be added to the action:

$$\mathcal{L}_{FI} = \frac{1}{2} \vec{E} \cdot \vec{Y} + \text{c.c.},$$

where $\vec{E}$ is an arbitrary (complex) vector.

In order to make contact with the $N = 1$ Lagrangian (3), we perform an $SU(2)$ transformation which brings the parameters $\vec{M}$ and $\text{Re} \vec{E}$ into the form

$$\vec{M} = \begin{pmatrix} 0 & m \\ 0 & 0 \end{pmatrix}, \quad \text{Re} \vec{E} = \begin{pmatrix} 0 & e \\ \xi & 0 \end{pmatrix}.$$  

It is now straightforward to show that after elimination of auxiliary fields the Lagrangian $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{FI}$ coincides with (3) up to an additive field-independent constant. Indeed, the scalar potential is given by:

$$V = \frac{|\text{Re} \vec{E} + \vec{M} \tau|^2}{\tau_2} + 2 \vec{M} \cdot \text{Im} \vec{E} = V_{N=1} + 2m \text{Im} E_2.$$  

It can be easily shown that a non-zero parameter $\vec{M}$ generates a Fayet-Iliopoulos term for the dual magnetic $U(1)$ gauge field \[1\]. In fact, such a term can be obtained from the standard electric ($\vec{M} = 0$) FI term by a duality transformation. After performing a symplectic $Sp(2, R) \simeq SL(2, R)$ change of basis

$$(\begin{pmatrix} F_a \\ a \end{pmatrix}) \rightarrow \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} F_a \\ a \end{pmatrix} \quad \tau \rightarrow \frac{\alpha \tau + \beta}{\gamma \tau + \delta},$$

with $\alpha \delta - \beta \gamma = 1$, one obtains the same form of Lagrangian with new parameters $\vec{M}'$ and $\vec{E}'$ given by

$$(\vec{M}' \quad \text{Re} \vec{E}') = (\vec{M} \quad \text{Re} \vec{E}) \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \quad \text{Im} \vec{E}' = \frac{\vec{M} \cdot \text{Im} \vec{E}}{M'^2} \vec{M}'.$$  

We now turn to the minimization of the scalar potential (13). For $m \neq 0$, a stable minimum exists at\[^3\]

$$\tau_1 = -\frac{e}{m} \quad \tau_2 = \frac{\xi}{m}.$$  

\[^3\]Without losing generality we can choose $m, \xi \geq 0.$
In this vacuum, the complex scalar $a$ acquires the mass $\mathcal{M}_a = m|\tau_a|$. After diagonalizing the fermion mass matrix (18), we find one massless fermion $(\chi - \lambda)/\sqrt{2}$, and one massive spinor $(\chi + \lambda)/\sqrt{2}$, with the Majorana mass $\mathcal{M}_a$ equal to the scalar mass. This degeneracy is not accidental. As we explain below, the vacuum (18) preserves $N = 1$ supersymmetry, and the spectrum consists of one massless vector and one massive chiral multiplets.

In order to discuss supersymmetry breaking, it is sufficient to examine the auxiliary field dependence of fermion transformations under $N = 2$ supersymmetry:

$$\delta \lambda_i = \frac{i}{\sqrt{2}} Y_n \epsilon_{ij} (\sigma^n)^i_k \eta^k + \ldots$$

where $\sigma^n$ are the Pauli matrices and the spinors $\eta^k$, $k = 1, 2$, are the transformation parameters. As explained before, the effect of the magnetic FI term amounts to introducing a constant imaginary part (11) for the auxiliary field $\vec{Y}$. This constant $\text{Im} \vec{Y} = 2\vec{M}$ enters into the supersymmetry transformations (19) implying that generically both supersymmetries are realized in a spontaneously broken mode. However, at the minimum (18) the real part of $\vec{Y}$ acquires also an expectation value, so that:

$$\vec{Y} = -\frac{2}{\tau_2} (\text{Re} \vec{E} + \vec{M} \tau_1) + 2i\vec{M} = 2m(0 \ i \ -1)$$

As a result,

$$\frac{\delta \chi + \lambda}{\sqrt{2}} = 0 \quad \frac{\delta \chi - \lambda}{\sqrt{2}} = -2im(\eta^1 - \eta^2)$$

which shows that one supersymmetry, corresponding to the diagonal combination of the two, is preserved while the other one is spontaneously broken. The massless goldstino is identified as $(\chi - \lambda)/\sqrt{2}$, in agreement with the spectrum found before. Hence the vector multiplet contains the goldstino of the broken supersymmetry.

The partial breaking of extended supersymmetry might seem to contradict the algebra of supercharges:

$$\{\bar{Q}^i, Q_j\} = H\delta^i_j,$$
where \( H \) is the Hamiltonian and the spinor indices are contracted with the metric \(-\frac{1}{4}\sigma^0 = \frac{1}{4} \mathbf{1}\). It follows that when one supercharge annihilates the vacuum, \( Q_i|0\rangle = 0 \), then the vacuum energy vanishes and all supersymmetries remain unbroken, \( Q_i|0\rangle = 0 \). On the other hand if one of them is spontaneously broken then all remaining ones are broken as well. The loophole in this argument is that the local version of the above algebra, which is appropriate for studying spontaneously broken symmetries, is not the most general one \([6]\). The most general supercurrent algebra is:

\[
\{\bar{Q}^i, J^\mu_j(x)\} = T^\mu_0(x)\delta^i_j + \delta^\mu_0 C^i_j ,
\]

where \( J \) is the supercurrent, \( T \) is the energy-momentum tensor and \( C \) is a constant matrix. The presence of such a matrix does not affect the supersymmetry algebra (22) on the fields. However some supersymmetries, namely those associated with non-zero eigenvalues of \( C \), are realized in a spontaneously broken mode. In fact, as shown in ref. \([7]\), in the model under consideration

\[
C^i_j = 2\vec{\sigma}^i_j \cdot (\text{Re}\vec{E} \times \vec{M}) .
\]

For \( \xi = 0 \) the current algebra is not modified \([\text{Re}\vec{E} \parallel \vec{M}, \text{eq.}(14)]\) therefore partial supersymmetry breaking does not occur. In this case, the minimum (18) occurs at a point where the metric \( \tau_2 \) vanishes. This can happen either at “infinity” of the a-space or at finite singular points where massless particles appear. The quantum numbers of such states, including electric and magnetic charges, as well as quantization conditions, depend on details of the underlying theory. Its dynamics determines also the non-perturbative symmetries which form a (discrete) subgroup of \( Sp(2, R) \). These states cannot be vector multiplets since unbroken non-abelian gauge group is incompatible with FI terms. Hence we assume that they are BPS-like dyons which form \( N = 2 \) hypermultiplets and that the minimization condition (18) defines a point \( a = a_0 \) where one of these hypermultiplets becomes massless. This can happen only if the parameters \((m, e)\) are proportional to its magnetic and electric charges \((m_0, e_0)\), \((m, e) = c(m_0, e_0)\). In order to analyze the
behavior of the theory near \( a_0 \), one has to include the massless hypermultiplet in the effective field theory as a new degree of freedom. This can be done by performing the duality transformation \( A \rightarrow \tilde{A} = e_0 A + m_0 \mathcal{F}_A \), which makes possible local description of the dyon-gauge boson interactions. In \( N = 1 \) superspace the superpotential (3) becomes:

\[
W = c\tilde{a} + \sqrt{2}\tilde{a}\phi^+\phi^-,
\]

where \( \phi^\pm \) are the two chiral superfield components of the hypermultiplet, and \( \tilde{a} \) is the chiral component of \( \tilde{A} \).

The superpotential (25) describes \( N = 2 \) QED with a Fayet-Iliopoulos term proportional to \( c \) [3]. The minimization conditions of the respective potential are \( W_{\phi^\pm} = 0 \) which is automatically satisfied at \( \tilde{a} = 0 \) (\( a = a_0 \)) and

\[
W_{\tilde{a}} = c + \sqrt{2}\phi^+\phi^- = 0, \quad \tilde{D} = 0 = |\phi^+|^2 - |\phi^-|^2.
\]

As a result the dyonic hypermultiplet condenses in a \( N = 2 \) supersymmetric vacuum. For instance if \( e = 0 \), the dyonic state is a pure monopole and the VEV of the scalar field \( a \) is driven to the point where the monopole becomes massless and acquires a non-vanishing expectation value. Its condensation breaks the magnetic \( U(1) \) and imposes confinement of electric charges. This situation is similar to the case considered in ref. [4] in the context of \( SU(2) \) Yang-Mills with an explicit mass term for chiral components of gauge multiplets which breaks \( N = 2 \) supersymmetry explicitly to \( N = 1 \).

For \( m = 0 \), the scalar potential (15) has a runaway behavior, \( V \rightarrow 0 \) as \( \tau_2 \rightarrow \infty \). This case is equivalent by a duality transformation to the the case \( m \neq 0, \xi = 0 \) discussed above. The runaway behavior can be avoided if there are singular points corresponding to massless electrically charged particles. At these points the metric \( \tau_2 \) has a logarithmic singularity and the massless states have to be included explicitly in the low energy Lagrangian to avoid non-localities. A similar analysis of the effective theory shows that \( a \) is driven then to the points where the massless hypermultiplets get non-vanishing VEVs breaking the \( U(1) \) gauge symmetry while \( N = 2 \) supersymmetry remains unbroken.
In the context of string theory, this phenomenon is similar to the effect induced by a generic superpotential near the conifold singularity of type II superstrings compactified on a Calabi-Yau manifold. In this case, the massless hypermultiplets are black holes which condense at the conifold points. It has been shown that such a superpotential can be generated by a VEV of the 10-form which in four dimensions corresponds to a magnetic FI term, and that the black hole condensation at the conifold point leads to new $N = 2$ type II superstring vacua.

The model considered here cannot be coupled to supergravity in a straightforward way. In particular, partial breaking of $N = 2$ supergravity requires the existence of a hypermultiplet, necessary to provide the longitudinal degrees of freedom to the graviphoton which belongs to the massive spin $3/2$ $N = 1$ supermultiplet. An example of a construction leading to our model in an appropriate globally supersymmetric limit has been given recently in ref.

Still in the context of global supersymmetry, it is a very interesting question whether electric and magnetic Fayet-Iliopoulos terms described here can be generated dynamically, for instance by an underlying non-abelian gauge theory. It is clear that instantons do not generate them since they give rise only to correlation functions involving at least four fermions whereas FI terms are associated with fermion bilinears. Moreover, instantons respect the global $SU(2)$ symmetry while they break the standard $U(1)$ $R$-symmetry down to $Z_4$. On the other hand, a most general FI term violates both $SU(2)$ and $U(1)$ leaving unbroken only a single $Z_2$, as seen from the fermion mass matrix; only for $\xi = 0$, $SU(2)$ remains unbroken. In general, one cannot a priori exclude the existence of non-perturbative effects, possibly related to gaugino condensation, which could generate FI terms in the effective action. What seems to be most plausible is a dynamical generation of the superpotential with $\xi = 0$ which would not modify the supercurrent algebra while merely breaking the $U(1)$ $R$-symmetry down to $Z_2$ – a sort of “half-instanton” could do
this job.

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References

[1] I. Antoniadis, H. Partouche and T.R. Taylor, Phys. Lett. B 372 (1996) 83.

[2] N. Seiberg and E. Witten, Nucl. Phys. B 431 (1994) 484.

[3] P. Fayet, Nucl. Phys. B 113 (1976) 135.

[4] J. Wess and J. Bagger, *Supersymmetry and Supergravity* (second edition, Princeton University Press, 1992).

[5] R. Grimm, M. Sohnius and J. Wess, Nucl. Phys. B 133 (1978) 275; M. De Roo, J. W. Van Holten, B. De Wit and A. Van Proeyen, Nucl. Phys. B 173 (1980) 175.

[6] J. Hughes and J. Polchinski, Nucl. Phys. B 278 (1986) 147.

[7] S. Ferrara, L. Girardello and M. Porrati, hep-th/9512180.

[8] A. Strominger, Nucl. Phys. B 451 (1995) 96.

[9] J. Polchinski and A. Strominger, hep-th/9510227.

[10] N. Seiberg, Phys. Lett. B 206 (1988) 75.