Decay Property of Axial-Vector Tetra-Quark Mesons

Kunihiko Terasaki

Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan

Decay property of hidden-charm tetra-quark mesons is studied. It is seen that estimated width of iso-triplet odd $C$ partners of $X(3872)$, although still crude, is compatible with the measured ones of $Z_c^{±,0}(3900)$. It is pointed out that confirmation of $\delta^0(3200)$ (an $\eta\pi^0$ peak around 3.2 GeV indicated in $\gamma\gamma$ collision) gives a clue to select a realistic model of multi-quark mesons.

Recently charged hidden-charm mesons $Z_c^±(3900)$ have been discovered in $\pi^±/J/ψ$ channels of $e^+e^-→Y(4260)→\pi^±\pi^0J/ψ$ [1], and just after the observation, $Z_c^{±}(3895)$ have been observed in the same type of reaction [2], and then, not only $Z_c^±(3900)$ but also a neutral $Z_c^0(3900)$ has been found in data on $e^+e^-→ψ(4160)→\pi^±\pi^0J/ψ$ and $π^0\pi^0J/ψ$ from CLEO-c [3]. When $Z_c^±(3900)$ and $Z_c^{±}(3895)$ are identified, the average values of their masses and widths are given by $m_{Z_c^±(3900)}=3894±5$ MeV and $Γ_{Z_c^±(3900)}=48±20$ MeV. Regarding $Z_c^0(3900)$, however, its observation has been reported only in [2], and its mass and width have been provided as $m(Z_c^0(3900))=(3907±12)$ MeV and $Γ(Z_c^0(3900))=(34±29)$ MeV. ($J/ψ$ is written as $ψ$ hereafter.) In addition, existence of an $Ωπ^0$ peak around 3.2 GeV (called as $δ^0(3200)$ in this short note) has been indicated in $γγ$ collision [3].

On the other hand, tetra-quark models [5, 6] predicted existence of iso-triplet ($I=1$) partners of $X(3872)$ with opposite charge-conjugation ($C$) property before the above observations of $Z_c^{±,0}(3900)$, and, after the observations, the prediction from the diquark-antidiquark model has been updated [7], and these mesons have been newly interpreted as $I=1$ opposite $C$ partners of $X(3872)$ from different pictures [8, 3]. Regarding $δ^0(3200)$, it can be considered as the neutral component of hidden-charm $I=1$ scalar mesons [10]. These mesons as well as the well-established [11] $D_{s0}^+(2317)$ and $X(3940)$ are considered as tetra-quark mesons from their decay properties, as seen below. The ratio of decay rates $R(D_{s}^{+}\gamma/D_{s}^{+}\pi^0)=(Γ(D_{s0}^{+}(2317)→D_{s0}^{+}\gamma)/Γ(D_{s0}^{+}(2317)→D_{s0}^{+}\pi^0))$ is experimentally constrained as [11] $R(D_{s}^{+}\gamma/D_{s}^{+}\pi^0)_{\text{exp}}<0.059$. From this fact, it is natural to consider that $D_{s0}^{+}(2317)$ is a member of $I=1$ states, because of the well-known hierarchy of hadron interactions [12]: $|\text{isospin conserving strong int.} (\sim O(1))|\gg|\text{radiative int.} (\sim O(\alpha^2))|\gg|\text{isospin non-conserving hadronic int.} (\sim O(\alpha^3))|$, where $\alpha$ is the fine structure constant.

Such a state cannot be any ordinary $\{\bar{s}s\}$ meson. When it is assigned to an iso-triplet scalar $F_{I}^{+} \sim \{|cn|\bar{s}n\}\{\bar{s}n\}^I_{I=1}$, $\{n=u,d\}$ meson, its narrow width is understood by a small overlap of color and spin wave functions (wfs.) [12], where the notation of tetra-quark states will be seen later. In addition, a recent lattice-QCD study on mass of the lowest-lying $I=0$ charm-strange ($C=S=1$) scalar-meson has reproduced [14] the measured one $m_{D_{s0}^{+(2317)}}$ of $D_{s0}^{+(2317)}$.

This suggests that there exists an iso-singlet charm-strange scalar meson which is (approximately) degenerate with $D_{s0}^{+(2317)}$, and implies that it is a compact object but not any extended object like a loosely bound DK molecule [15], i.e., there exist $D_{s0}^{+(2317)}$ as the iso-triplet $F_{I}^{+}$ and its iso-singlet $F^{+} \sim \{|cn|\bar{s}n\}\{\bar{s}n\}^I_{I=0}$ partner whose indication has been observed in $D_{s0}^{+}\gamma$ channel [16]. Regarding $X(3872)$, it is known that its $π^±π^±ψ$ decay proceeds through the intermediate $ρ^0ψ$ state [17, 18]. Nevertheless, $X(3872)$ is considered to be an $I=0$ state, because its charged partners have not been observed [19]. In addition, its spin ($J$), parity ($P$) and $C$-parity are given by $J^P^C=1^{++}$ [11]. Thus, quantum numbers of $X(3872)$ are the same as those of the charmonia $χ_{c1}(1P), χ_{c2}(2P), ⋯$. If it were a charmonium, however, the ratio of decay rates $R(γψ/π^±π^±ψ)=Γ(X(3872)→γψ)/Γ(X(3872)→π^±π^±ψ)$ would be much larger than unity [20], i.e., $R(γψ/π^±π^±ψ)_{\text{exp}}\gg1$, for the decay in the denominator would be suppressed because of the isospin non-conservation and the OZI-rule [21]. This result contradicts with the measurements,

$$R(γψ/π^±π^±ψ)_{\text{exp}}=0.33±0.12\text{ (Babar)}\text{ and }0.22±0.09\text{ (Belle)},$$

which have been provided in [22] and obtained by compiling the data in [23], respectively, and hence, it should be a multi-quark state. Here, an argument [24] that the measured cross section for prompt $X(3872)$ production [25] favors a compact object like a tetra-quark state over an extended one like a loosely bound meson-meson molecule should be noted. If it is the case, $X(3872)$ would be a tetra-quark meson. Concerning with $Z_c^{±,0}(3900)$, they cannot be charmonia and their neutral component $Z_c^0(3900)$ has an odd $C$-parity, so that they can be interpreted as iso-triplet opposite $C$ partners of $X(3872)$, as discussed before. As for $δ^0(3200)$, it will be an $I=1$ hidden-charm scalar meson.

The simplest way to understand it is to assign it to a tetra-quark state [10] $δ^0(3200)\sim\{|cn|\bar{s}n\}\{\bar{s}n\}^I_{I=1}$. In this case, its mass is estimated to be $m_{δ^0(3200)}\simeq3.3$ GeV, by using the quark counting in [26] and taking $m_{D_{s0}^{+(2317)}}$ as the input data. The result is close to $m_{δ^0(3200)}\simeq3.2$ GeV. In contrast, the diquark-antidiquark model [5] and the unitarized chiral one [27] have predicted that the mass of the lowest hidden-charm scalar (called as $X_0$ in these papers) is $m_{X_0}\simeq3.7$ GeV which is much higher than $m_{δ^0(3200)}\simeq3.2$ GeV. Therefore, $δ^0(3200)$ will provide an important clue to select a realistic model of multi-quark states.
We here review very briefly our tetra-quark model. Tetra-quark states can be classified into four groups,
\[
\{qq\bar{q}\bar{q}\} = [qq][\bar{q}\bar{q}] \oplus (qq)(\bar{q}\bar{q}) \oplus \{(qq)(\bar{q}\bar{q}) \oplus (qq)[\bar{q}\bar{q}]\},
\]
(2)
in accordance with difference of symmetry property of their flavor wfs., where parentheses and square brackets denote symmetry and anti-symmetry, respectively, of flavor wfs. under exchange of flavors between them \([28]\). Each term on the the right-hand-side (r.h.s.) of Eq. (2) is again classified into two groups \([28]\) with \(3c \times 3c\) and \(6c \times 6c\) of the color \(SU(3)\). Here, the former is taken as the lower-lying state in heavy mesons \([20]\). However, the second term \((qq)(\bar{q}\bar{q})\) on r.h.s. of Eq. (2) is not considered in this note, because no signal of scalar \((K\pi)_{I=3/2}\) meson which can arise from \((qq)(\bar{q}\bar{q})\) in the light flavor sector has been observed in a sufficiently wide enery region \(\lesssim 1.8\) GeV \([29]\). Regarding their \(J^P\), the first term and the last two on the r.h.s. of Eq. (2) have \(J^P = 0^+\) and \(1^+\), respectively, in the flavor symmetry limit, because \([qq]\) and \((qq)\) have \(J^P = 0^+\) and \(1^+\), respectively, in the same limit. Nevertheless, the flavor symmetry is broken in the real world, so that \([qq]\) and \((qq)\) can have both of \(J^P = 0^+\) and \(1^+\) in general, and hence each term on the r.h.s. of Eq. (2) can have all of \(J^P = 0^-,\) \(1^+\) and \(2^+\). Along with this line, the diquark-antidiquark model \([2]\) in which a large breaking of isospin symmetry is assumed has assigned axial-vector mesons to \([qq][\bar{q}\bar{q}]\) states (which disappear in the flavor symmetry limit), and hence, \(X(3872)\) to an element of \([cn][\bar{c}n]\). As the result, it predicts \(m_{X_{0}} \simeq 3.7\) GeV as the mass of the lowest hidden-charm scalar meson which is much higher than \(m_{exp}(3820)\), as discussed before, and therefore, the diquark-antidiquark model fails to understand it. It is because \(X(3872)\) has been assigned to an element of \([cn][\bar{c}n]\) with \(J^P = 1^+\) (which disappears in the flavor symmetry limit).

In contrast, we assign scalar and axial-vector tetra-quark mesons to different \([qq][\bar{q}\bar{q}]\) and \([\{qq\bar{q}\bar{q}\} \oplus \{(qq)(\bar{q}\bar{q})\}]\), respectively, and therefore, \(D_{s0}(2317)\) and \(\delta^0(3200)\) to \([cn][\bar{c}n]_{I=1}\) and \([cn][\bar{c}n]_{I=1}\), respectively \([10, 26]\). In the \(J^P = 1^+\) mesons, \([qq][\bar{q}\bar{q}]\) and \((qq)[\bar{q}\bar{q}]\) are not eigenstates of \(C\)-parity, so that they mix with each other to form eigenstates of \(C\)-parity. Therefore, we have pairs of hidden-charm axial-vector meson states with opposite \(C\)-parities, i.e., \(X(\pm) \sim \{[cn](\bar{c}n) \pm (cn)[\bar{c}n]\}_{I=0}\), \(X_{f}(\pm) \sim \{[cn](\bar{c}n) \pm (cn)[\bar{c}n]\}_{I=1}\) and \(X^{*}(\pm) \sim \{[cs](\bar{c}s) \pm (cs)[\bar{c}s]\}\), where \(\pm\) denote the \(C\)-parities (\(C\)-parities of their neutral components), and the ideal mixing among tetra-quark states is always assumed in this note. (In this scheme, scalar and axial-vector mesons survive even in the flavor symmetry limit.) Thus, \(X(3872)\) is assigned to \(X(+)\) with \(I = 0\) and \(C = +\). In this case, the measured ratio Eq. (1) can be easily reproduced, by assuming that the isospin non-conserving \(X(+) = X(3872) \to \pi^+\pi^-\psi\) decay in the denominator proceeds through the \(\omega^0\) mixing \([20]\) which plays important roles in the observed \(\omega \to \pi\pi\) decay \([11]\) and the isospin non-conservation in nuclear forces \([30]\). In addition, it has been argued \([31]\) that \(X_{f}(+)\) is considerably broad, when it is assumed (as an approximation) that its mass and spatial wf. are not very much different from those of \(X(+) = X(3872)\), i.e., \(m_{X_{f}(+)} \simeq m_{X(3872)}\) and the couplings of \(X_{f}(+)\) to ordinary mesons (up to the Clebsch-Gordan coefficients arising from the color and spin degree of freedom) are not very far from those of \(X(+)\). The above assumption seems to be natural, because these states belong to the same ideally-mixed \([\{cn][\bar{c}n] + (cn)[\bar{c}n]\}]\) multiplet. Concerning with \(X(-)\) and \(X_{f}(-)\), the measured mass values of \([39] (X(3872))\) are not very far from the measured one \([10]\) of \(X(3872)\) (\(= X_{f}(-)\) in the present scheme) are not very far from the measured one \([10]\) of \(X(3872)\) (\(= X_{f}(-)\) in our model), i.e.,

\[
\frac{m_{X_{f}(-)} - m_{X(+)} = m_{Z_{0}(3900)} - m_{X(3872)}}{m_{X(3872)}} \lesssim 1\%.
\]
(3)

Therefore, we again assume that spatial wfs. of \(X(-)\) and \(X_{f}(-)\) are not very far from those of \(X(+)\) and \(X_{f}(+)\). Under this condition, it has been intuitively expected that \(X(-)\) and \(X_{f}(-)\) also are considerably broad \([31]\).

To estimate numerically widths of \(X(-)\) and \(X_{f}(-)\), we first estimate phenomenologically the rate for \(X(3872) \to D^{0}\bar{D}^{*0}\). We here identify \([11]\) \(X(3872)\) with \(X(3875)\) which was observed in the \((D^{0}\bar{D}^{*0} + c.c. \to D^{0}\bar{D}^{*0})\) channel. Assuming that the total rate \(\Gamma_{X(3872)}\) is approximately saturated as

\[
\Gamma_{X(3872)} \simeq \Gamma(X(3872) \to \pi^+\pi^-\psi) + \Gamma(X(3872) \to \pi^+\pi^-\pi^0\psi) + \Gamma(X(3875) \to D^{0}\bar{D}^{*0} + c.c.),
\]
(4)
and taking the measured ratios of \([32]\), \(\Gamma(X(3875) \to D^{0}\bar{D}^{*0} + c.c.)/\Gamma(X(3872) \to \pi^+\pi^-\psi)\) \(= 9.5 \pm 3.1\) and \([\Gamma(X(3872) \to \pi^+\pi^-\pi^0\psi)/\Gamma(X(3872) \to \pi^+\pi^-\psi)]_{exp} = 0.8 \pm 0.3\), in addition to the measured width \([33]\) \(\Gamma_{X(3875)} = (3.9^{+2.8+0.2}_{-1.4-1.1})\) MeV, we obtain

\[
\Gamma(X(3872) \to D^{0}\bar{D}^{*0}) \sim (0.3 - 1.5)\ MeV.
\]
(5)

This result is consistent with an independent estimate \([34]\), \(\Gamma(X(3872) \to D^{0}\bar{D}^{*0})_{t\text{tensho}} \sim 1\ MeV\).

Next, we write the rate for the \(X(+) = X(3872) \to D^{0}\bar{D}^{*0}\) decay (as an example of \(1^+ \to 0^- + 1^-\) decays) as

\[
\Gamma(X(+) \to D^{0}\bar{D}^{*0}) = \frac{|X_{X(3872)D^{0}\bar{D}^{*0}}|^2}{24\pi m_{X(3872)}^2} \left\{ 2 + \frac{(m_{X(3872)}^2 - m_{D^{0}}^2 + m_{\bar{D}^{*0}}^2)^2}{4m_{X(3872)}^2 m_{D^{0}}^2} \right\},
\]
(6)
Table I. Rates for OZI-rule-allowed two- and three-body decays of hidden-charm partners of $X(3872)$ are listed, where it is assumed that the spatial wave functions of $X(\pm)$ and $X_I(\pm)$ are nearly equal to each other. The rate $\Gamma(X(+) \to D^0 D^{*0}) \sim (0.3 - 1.5)$ MeV which is given in the text is taken as the input data.

| Decay                                              | Rate (MeV) | Decay                                              | Rate (MeV) |
|----------------------------------------------------|------------|----------------------------------------------------|------------|
| $X_I^0(+) \to \rho^0 \psi \to \pi^+ \pi^- \psi$     | $\sim 20 - 200$ (†) | $X_I^0(+) \to D^0 D^{*0}$                         | $\sim 0.3 - 1.5$ |
| $X^-(\to \eta_\omega)$                            | $9 - 45$   | $X^-(\to \eta \psi)$                             | $\sim 7 - 35$ (+) |
| $X^0_I(\to \pi^0 \psi)$                           | $15 - 75$  | $X^0_I(\to \eta \rho^0 \to \eta \pi^+ \pi^-)$    | $\sim 6 - 30$ |

(†) Ref. \[31\]. (+) The $\eta \psi$ mixing with the mixing angle $\theta_F \simeq -20^\circ$ \[32\].

where $g_{X(+)D^0D^{*0}}$ denotes the $X(+)D^0D^{*0}$ coupling strength and $p_D$ is the size of the center-of-mass momentum of $D^0$ in the final state. To study two- and three-body decays (through quasi-two-body intermediate states) of $X^-$ and $X_I(\pm)$, we remember that we can decompose each of them into a sum of products of two $\{q\bar{q}\}$ pairs (and hence two ordinary mesons) as Eqs. (10) - (13) in \[31\]. Then, under the above assumptions (as an approximation) on spatial wfs. of $X^+(\to X(3872)$ and its hidden-charm partners $X_I(\pm)$ in addition to $X^-$, ratios of their coupling-strengths to ordinary mesons to $g_{X(+)D^0D^{*0}}$ are given by ratios of corresponding (Clebsch-Gordan) coefficients arising from color and spin degrees of freedom in the decompositions. In this way, rates for OZI-rule-allowed decays of $X_I(\pm)$ and $X^-$ are very crudely estimated as listed in Table I, where $m_{X(\pm)} \simeq m_{X_I(\pm)} = m_{X(3872)}$ has been assumed and Eq. \[35\] has been taken as the input data. Besides, the $\eta \psi$ mixing with the mixing angle \[33\] $\theta_F \simeq -20^\circ$ in $X^- \to \eta \psi$ and the broad width of $\rho^0$ meson in decays through intermediate $\rho^0$ states have been taken into account.

Full width $\Gamma_{X_I^-}$ is approximately given by the sum of rates for dominant two- and three-body decays, i.e., $\Gamma_{X_I^-} \simeq \Gamma(X_I^0(\to \pi^0 \psi)) + \Gamma(X_I^0(\to \eta \rho^0 \to \eta \pi^+ \pi^-)) \sim (20 - 100)$ MeV, and in a similar way, $\Gamma_{X^-} \sim (15 - 80)$ MeV. The results imply that the widths of $X_I^0(\pm)$ and $X^-$ are considerably broad as intuitively expected. Therefore, their detection in $B$ decays will require much higher statistics than those to observe $X(3872)$. Fortunately, however, $Z_c^{\pm,0}(3900)$ (= $X_I^-(\pm)$ in the present scheme) have been observed in $e^+e^- \to Y(4260) \to \pi^+\pi^-\psi$, and $e^+e^- \to \psi(4160) \to \pi^+\pi^-\psi$ and $\eta\pi^0\psi$, as discussed before. It should be noted that our estimate of $\Gamma_{X_I^-}$ is consistent with the measured widths of $Z_c^{\pm,0}(3900)$, and therefore, our assumption on spatial wfs. of $X(\pm)$ and $X_I(\pm)$ mesons seems to be feasible.

As seen above, our tetra-quark interpretation of $D_{s0}^+(2317)$, $\hat{D}_s^0(3200)$ and $X(3872)$ seems to be favored by experiments. In addition, the measured mass and width of $Z_c^{\pm,0}(3900)$ are consistent with our predictions. However, to establish our tetra-quark interpretation, observation of states which have been predicted in our model would be needed. In this sense, confirmation of existence of the $I = 0$ partner $\hat{F}^+ \sim \{[cn][\bar{s}\bar{n}]\}_{I=0}^+$ of $D_{s0}^+(2317) = \hat{F}_I^+ \sim \{[cn][\bar{s}\bar{n}]\}_{I=1}^{++}$ would take priority, because its indication has already been observed in the $D_{s}^{+\gamma}$ channel from $B$ decays \[16\]. (It should be noted that production of such a state in $e^+e^-$ annihilation is suppressed. This can be understood by considering their production in a framework of minimal $\{q\bar{q}\}$ pair creation \[30\].) In addition, our tetra-quark model has predicted \[26, 37\] existence of scalar and axial-vector mesons with exotic quantum numbers. Therefore, their observation is one of important options to establish our tetra-quark interpretation. Neutral and doubly charged partners $D_{s0}^0(2317)$ and $D_{s0}^{0\pm}(2317)$ of $D_{s0}^+(2317)$ are candidates of such states, where $D_{s0}^{0\pm,++}(2317) = \hat{F}_I^{0,++} \sim \{[cn][\bar{s}\bar{n}]\}_{I=1}^{++,++}$ in the present scheme \[26\]. Although they have not been observed in inclusive $e^+e^-$ annihilation \[38\], it does not necessarily imply their non-existence. In fact, it is expected that their production is suppressed in the inclusive $e^+e^-$ annihilation \[36\]. Therefore, they should be searched in $B$ decays, because branching fractions for their productions in $B$ decays have been estimated to be large enough to observe them \[36\], i.e., $Br(B_u^+ \to D^{(*)0}\to D_{s0}^{0\pm}(2317)) \sim Br(B_d^0 \to D_{s0}^{0\pm}(2317)) \sim (10^{-4} - 10^{-3})$.

Double-charm ($C = 2$) axial-vector mesons are $H_{Ac}^{++} \sim (cc)[u\bar{d}]$ and $K_{Ac}^{++,++} \sim (cc)[\bar{a}\bar{s}]^{++,++}$ in our model \[37\]. Their detection will be another option to establish our interpretation. Their masses have been very crudely estimated as $m_{H_{Ac}} \simeq 3.87$ GeV and $m_{K_{Ac}} \simeq 3.97$ GeV by using the same quark counting as before, where $m_{[cc][u\bar{d}]} \simeq m_{[cc][\bar{s}\bar{n}]} \simeq m_{X(3872)}$ has been assumed. These results are close to thresholds of possible OZI-rule-allowed two- and three-body decays of these mesons. Because deviations between the estimated masses and thresholds under consideration might be smaller than uncertainties involved in their estimated mass values, we here get rid of taking literally these values. Here it should be noted that the above $C = 2$ meson states might correspond to a part of $T_{cc}'$s in \[39\], although their mass values estimated in these two different models are not necessarily agree with each other. If $T_{cc}'$s are stable against their OZI-rule-allowed strong decays as discussed in \[34\], they should be very narrow and could be observed as sharp peaks in $DD_{s}^{0\pm}(2317)$ channels in $B_c$ decays and in inclusive $e^+e^-$ annihilation if their production rate is sufficiently high \[40\]. On the other hand, in our case, it is not very clear whether their OZI-rule-allowed strong decays are kinematically allowed or not, because the estimated mass values of these mesons are very close to corresponding thresholds of these decays. Even though their true masses are a little bit higher than the thresholds of
Table II. Rates for two-body decays of $E_{A(cs)}^0$ and $E_{A(cs)}^+$ are listed, where the rate $\Gamma(X(+) \rightarrow D^0 \bar{D}^{*0}) \sim (0.3 - 1.5)$ MeV which is given in the text is taken as the input data.

| Decay                        | Rate (MeV) | Decay                        | Rate (MeV) |
|------------------------------|------------|------------------------------|------------|
| $E_{A(cs)}^0 \rightarrow K^0 D^{*0}$ | $\sim 6 - 30$ | $E_{A(cs)}^0 \rightarrow K^- D^{*+}$ | $\sim 6 - 30$ |
| $E_{A(cs)}^0 \rightarrow K^0 D^0$ | $\sim 5 - 25$ | $E_{A(cs)}^0 \rightarrow K^+ D^+$ | $\sim 5 - 25$ |
| $E_{A(cs)}^+ \rightarrow \bar{K}^0 D^{*+}$ | $\sim 12 - 65$ | $E_{A(cs)}^+ \rightarrow \bar{K}^0 D^+$ | $\sim 10 - 50$ |

these decays, however, they would be narrow because of their small phase space volume. (We here do not estimate their widths, however, it is difficult to get a definite result under the present condition.) Regarding $K_{A(cs)}^{+,+}$, therefore, we expect intuitively that they will be observed as narrow peaks in $DD^*_\pm \gamma$ (and $DD^*_\pm \pi$ if kinematically allowed) channel(s) in $B^+_c$ decays. It is because branching fractions for $B^+_c$ decays producing them, which have been estimated very crudely as $\frac{Br(B^+_c \rightarrow \{ \bar{D}^{(*)} K_{A(cs)}^{\pm}\})}{\Gamma(X(+) \rightarrow D^0 \bar{D}^{0})} \sim (10^{-4} - 10^{-3})$, might be large enough to observe them. In contrast, observation of $H_{A(nc)}^+$ in $B^+_c$ decays would be not very easy, because its production in $B^+_c$ decays is CKM suppressed $^{37}$. Here, it should be noted that observation of double-charm mesons will exclude the diquark-antidiquark model.

Observation of exotic $C = -S = 1$ meson states is an additional option. However, the scalar $E^0 \sim [cs][\bar{u}\bar{d}]$ decays only through weak interactions $^{26}$, so that its detection might not be easy. Regarding axial-vector tetra-quark mesons $E_{A(cs)}^0 \sim (cs)[\bar{u}\bar{d}]$ and $E_{A(cs)}^0 \sim [(cs)(\bar{n}\bar{n})]^{\pm,0}$, their masses have been very crudely estimated $^{37}$ to be 2.97 GeV by using the same quark counting as the above. The above value is sufficiently higher than thresholds of their possible OZI-rule-allowed two-body-decays. Therefore, it can be intuitively expected that they are considerably broad, because of large phase space volume. To study numerically their decay rates, we here decompose $E_{A(cs)}^0$ as in $^{31}$,

$$E_{A(cs)}^0 = \frac{1}{4\sqrt{3}} \left\{ D^+ K^- + D^+ K^{*-0} - D^{*0} \bar{K}^0 - D^{0} \bar{K}^{*0} + K^{*0} D^0 + \bar{K}^0 D^{*0} - K^{*-0} D^- - \bar{K}^{*0} D^+ \right\} + \cdots, \quad (7)$$

where $\cdots$ denotes a sum of products of color octet $\{qq\}$ pairs. $E_{A(cs)}^{\pm,0}$ also can be decomposed in the same way. Here, we list only the decomposition of $E_{A(cs)}^{+}$ to save space,

$$E_{A(cs)}^{+} = \frac{1}{2\sqrt{6}} \left\{ D^+ \bar{K}^0 + D^+ \bar{K}^{*0} - K^{*0} D^+ - \bar{K}^{*0} D^+ \right\} + \cdots \quad (8)$$

Rates for the $E_{A(cs)}^0$ (or $E_{A(cs)}^+$) decays which are considered as their dominant ones are obtained by replacing $X(+)$, $D^0$ and $\bar{D}^{*0}$ in Eq. (8) in terms of $E_{A(cs)}^0$ (or $E_{A(cs)}^+$), $\bar{K}[D]$ and $D^*[\bar{K}^+]$, respectively. In this way, the ratios of rates $\Gamma(E_{A(cs)}^0 \rightarrow \bar{K} D^*[\bar{K}^+]) / \Gamma(X(+) \rightarrow D^0 \bar{D}^{*0})$ are given by the ratios of coupling strengths $\vert g_{E_{A(cs)}^0 \bar{K} D^*[\bar{K}^+]} \vert / \vert g_{X(+) D^0 \bar{D}^{*0}} \vert ^2$. The latter ratios are provided by the ratios of the Clebsch-Gordan coefficients in the above Eq. 7 to the ones of Eq. (10) in $^{31}$, under the assumption (as an approximation) that the spatial wf. overlaps among $E_{A(cs)}^0$, $\bar{K}[D]$ and $D^{*0}[\bar{K}^+]$ are very much different from that of $X(+)$, $D^0$ and $\bar{D}^{*0}$. In the same way, rates for two-body decays of $E_{A(cs)}^+$ also can be estimated as listed in Table II, where the rate in Eq. (5) has been taken as the input data. However, the results are still very preliminary, because they include large uncertainties.

Full widths $\Gamma_{E_{A(cs)}^0}$ and $\Gamma_{E_{A(cs)}^+}$ are approximately given by a sum of the rates for two-body decays listed in Table II, i.e., $\Gamma_{E_{A(cs)}^0} \simeq \Gamma_{E_{A(cs)}^+} \sim (20 - 100)$ MeV. From this, it is seen that they are much broader than $X(3872)$. On the other hand, branching fractions for $B$ decays which produce $E_{A(cs)}^0$ and $E_{A(cs)}^+$ have been estimated $^{37}$ to be $Br(B \rightarrow DE_{A(cs)}^0) \sim Br(B \rightarrow DE_{A(cs)}^+) \sim (10^{-3} - 10^{-4})$. Production of $E_{A(cs)}^{\pm,0}$ is also described by the same type of quark-line diagrams as those decibering $E_{A(cs)}^0$ production, so that the branching fractions for the $B$ decays producing them also are expected to be of the same order of magnitude, i.e., $Br(B \rightarrow DE_{A(cs)}^{\pm,0}) \sim (10^{-4} - 10^{-3})$. These results are not very much different from those of $B$ decays producing $D_s^+(2317)$ and $X(3872)$. Therefore, detection of $E_{A(cs)}^0$ and $E_{A(cs)}^{\pm,0}$ in $B$ decays will require much higher statistics than those to observe $D_s^+(2317)$ and $X(3872)$.

In summary, we have presented our tetra-quark interpretation of $D_s^+(2317)$, $X(3872)$ and $\delta(3200)$ (the $\eta\pi\pi$ peak around 3.2 GeV indicated in $\gamma\gamma$ collision), and have pointed out that confirmation of $\hat{S}^0(3200)$ can be called to select a realistic model of multi-charm mesons. Next, we have studied decay properties of tetra-quark partners of $X(3872)$. However, the hidden-charm and exotic $C = -S = 1$ partners are expected to be broad, and therefore, detection of them in $B$ decays will require much higher statistics than the ones to observe $X(3872)$. Therefore, to search for them
at the present experimental accuracy, some other processes should be studied. Fortunately, $Z_{\pm,0}^{\pm,0}(3900) (= X_{\pm,0}^{\pm,0}(-))$ in the present scheme have been observed in exclusive $e^+e^- \to \pi^\pm\pi^\mp\psi$ annihilations. However, decay property of $X^*(\pm)$ is left as one of our future subjects. Regarding $K_{\pm,++}^{\pm,++}$ with $C = 2$ and $S = 1$, they have been expected to be narrow. If their production rates are sufficiently high, therefore, they will be observed as narrow peaks in $(DD_s^{+}\pi$ and $DD_s^{+}\gamma$ channels in inclusive $e^+e^-$ annihilation. If not, however, they will be observed as narrow peaks in the same channels of the $B_s^{+}$ decays $B_s^+ \to \{D^{(*)K_{\pm,++}}\}^+ \to \{(D^{(*)DD_s^{+}\pi})^+\text{ and }\{D^{(*)DD_s^{+}\gamma})^+\}$.

In addition, confirmation of the iso-singlet partner of $D_{s0}^{+}(2317)$ in the $D_s^{+}\gamma$ channel, and observation of $D_{s0}^{+}(2317)$ in $D_s^{+}\pi^\pm$ channels from $B$ decays are awaited.

Acknowledgments

The author would like to thank Professor H. Kunitomo for careful reading of the manuscript.

[1] M. Ablikim et al., BESIII Collaboration, arXiv:1303.5949 [hep-ex].
[2] Z. Q. Liu et al., arXiv:1304.0121 [hep-ex].
[3] T. Xiao, S. Dobbs, A. Tomaradze and K. K. Seth, arXiv:1304.3036 [hep-ex].
[4] S. Uehara et al., Phys. Rev. D 80, 032001 (2009).
[5] L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. D 71 (2005), 014028.
[6] K. Terasaki, Prog. Theor. Phys. 118, 821 (2007); arXiv:0706.3941 [hep-ph].
[7] R. Faccini, L. Maiani, F. Piccinini, A. Pilloni, A.D. Polosa and V. Riquer, arXiv: 1303.6857 [hep-ph].
[8] For example, Q. Wang, C. Hanhalt and Q. Zhao, arXiv:1303.6355 [hep-ph] and F.-K. Guo, C. Hidalgo-Duque, J. Nieves, M. Pavon and Valderrama, arXiv:1303.6608 [hep-ph], as earlier works.
[9] M. B. Voloshin, arXiv:1304.0380.
[10] K. Terasaki, Prog. Theor. Phys. 121 (2009), 211; arXiv: 0805.4460 [hep-ph].
[11] J. Beringer et al. (Particle Data Group), Phys. Rev. D 86, 010001 (2012) and 2013 partial update for the 2014 edition.
[12] A. Hayashigaki and K. Terasaki, Prog. Theor. Phys. 114 (2005), 1191; hep-ph/0410393. K. Terasaki, Invited talk at the workshop on Resonances in QCD, July 11 – 15, 2005, ECT*, Trento, Italy; hep-ph/0512285.
[13] M. B. Voloshin, arXiv:1304.0380.
[14] Y. Namekawa, arXiv:1111.0142.
[15] T. Barnes, F. E. Close and H. J. Lipkin, Phys. Rev. D 68, 054006 (2003).
[16] P. Krokovny et al., Belle Collaboration, Phys. Rev. Lett. 91, 262002 (2003).
[17] S.-K. Choi et al., Belle Collaboration, Phys. Rev. Lett. 91, 262001 (2003).
[18] A. Abulencia et al., CDF Collaboration, Phys. Rev. Lett. 96, 102002 (2006).
[19] B. Aubert et al., Babar Collaboration, Phys. Rev. D 71, 031501 (2005).
[20] K. Terasaki, Prog. Theor. Phys. 122, 1285 (2009); arXiv: 0904.3368v2 [hep-ph]; Prog. Theor. Phys. Suppl. No. 186, 141 (2010); hep-ph/1005.5573.
[21] S. Okubo, Phys. Lett. 5 (1963), 165; G. Zweig, CERN Report No. TH401 (1964); J. Iizuka, K. Okada and O. Shito, Prog. Theor. Phys. 35 (1965), 1061.
[22] B. Aubert et al., Babar Collaboration, Phys. Rev. D 74 (2006), 071101(R); arXiv:0809.0042 [hep-ex].
[23] V. Bhardwaj et al., Belle Collaboration, Phys. Rev. Lett. 107, 091803 (2011); I. Adachi et al., arXiv:0809.1224 [hep-ex].
[24] C. Bignamini, B. Grinstein, F. Piccinini, A. D. Polosa, and C. Saballi, Phys. Rev. Lett. 103, 162001 (2009); arXiv:0906.0882 [hep-ph]; PoS-EPS-HEP 2009:074, 2009.
[25] A. Abulencia et al., CDF Collaboration, Phys. Rev. Lett. 98 (2007), 132002; CDF note 7159 (2004); URL http://www-cdf.fnal.gov.
[26] K. Terasaki, Phys. Rev. D 68 (2003), 011501(R); AIP Conf. Proc. 717, 556 (2004); hep-ph/0309279.
[27] D. Gamermann, E. Oset, D. Strottman, M. J. Vicente Vacas, Phys. Rev. D 76, 074016 (2007).
[28] G. A. Miller, A. K. Opper and E. J. Stephenson, Ann. Rev. Nucl. Part. Sci. 56, 253 (2006).
[29] I. Adachi et al., Belle Collaboration, arXiv:0810.0358 [hep-ex].
[34] F. Renga, Int. J. Mod. Phys. A26, 4855 (2011); arXiv:1110.4151 [hep-ph].
[35] K. Nakamura et al., Particle Data Group, J. Phys. G 37, 1 (2010).
[36] K. Terasaki, Prog. Theor. Phys. 116 (2006), 435; hep-ph/0604207. However, production mechanism of charm-strange tetraquark mesons in $e^+e^-$ annihilation has been revised in K. Terasaki, AIP Conf. Proc. 1030 (2008), 190; arXiv:0804.2295 [hep-ph].
[37] K. Terasaki, Prog. Theor. Phys. 125, 199 (2011); arXiv:1008.2992 [hep-ph]; AIP Conf. Proc. 1388, 352 (2011); arXiv:1102.3750 [hep-ph].
[38] B. Aubert et al., Babar Collaboration, Phys. Rev. D 74 (2005), 032007.
[39] S. H. Lee and S. Yasui, Eur. Phys. J. C 64, 283 (2009).
[40] T. Hyodo, Y.-R. Liu, M. Oka, K. Sudoh and S. Yasui, arXiv:1209.6207 [hep-ph].