We study a dark matter model with one singlet complex scalar and two Higgs doublets. The scalar potential respects a softly broken global symmetry, which makes the imaginary part of the singlet become a pseudo-Nambu-Goldstone boson acting as a dark matter candidate. The pseudo-Nambu-Goldstone nature of the boson leads to the vanishing of its tree-level scattering amplitude off nucleons at zero momentum transfer. Therefore, although the interaction strength could be sufficient large to yield a viable relic abundance via thermal mechanism, direct detection is incapable of probing this candidate. We further investigate the constraints from Higgs measurements, relic abundance observation, and indirect detection.
I. INTRODUCTION

Astrophysical and cosmological observations suggest that the majority of matter in the present Universe consists of a nonluminous component called dark matter (DM). In the conventional paradigm, dark matter is a thermal relic remaining from the early Universe, implying that the interaction strength between DM and standard model (SM) particles may be comparable to the strength of weak interactions [1–3]. However, null signal results from recent direct detection experiments have put rather stringent constraints on the DM-nucleon scattering cross section [4–6]. This has become a great challenge to the thermal DM paradigm.

A natural way out is to suppress DM-nucleon scattering in direct detection experiments without suppressing DM annihilation in the early Universe. One possibility is that there are some blind spots with particular parameters leading to the suppression of the DM couplings relevant to direct detection [7–11]. Additionally, the relevant DM couplings could vanish due to special symmetries [12–17]. Moreover, DM-nucleon scattering mediated by pseudoscalars can evade direct detection constraints [18–24]. Furthermore, the DM-nucleon scattering
amplitude could be greatly suppressed if the DM particle is a pseudo-Nambu-Goldstone boson (pNGB) protected by an approximate global symmetry [25–31].

In the last case, tree-level interactions of a pNGB are generally momentum-suppressed. As direct detection experiments essentially operate in the zero momentum transfer limit, the amplitude of pNGB dark matter scattering off nucleons vanishes at tree level [26]. Loop corrections could break the global symmetry, resulting in nonvanishing scattering. Nevertheless, a further investigation has shown that the DM-nucleon cross section at one-loop level is typically below $\mathcal{O}(10^{-50})$ cm$^2$, far away from the capability of current direct detection experiments [32]. Therefore, such a pNGB DM framework seems very appealing for thermal DM.

Previous studies in this framework assumed that the Higgs sector just involves one Higgs doublet as in the SM [25–32]. In this work, we would like to extend the study to two Higgs doublets [33]. A Higgs sector with two SU(2)$_L$ doublets has fairly good motivations. Firstly, two Higgs doublets are typically required for constructing realistic supersymmetric [34] and axion [35] models. Secondly, the flexible scalar mass spectrum and additional CP violation sources in two Higgs doublet models may be helpful for generating a desired baryon asymmetry of the Universe through the baryogenesis mechanism [36]. Finally, two Higgs doublets could provide an available portal to thermal dark matter with attractive phenomenological features [11, 18–21, 23, 24, 37–41].

In this paper, we consider that the scalar sector involves two SU(2)$_L$ Higgs doublets as well as a complex scalar $S$, which is an SM gauge singlet. Most terms in the scalar potential obey a global U(1) symmetry $S \to e^{i\alpha}S$. The exception is a quadratic term that softly breaks this symmetry and gives mass to the imaginary part of $S$, denoted as $\chi$. The real scalar $\chi$ is what we call pNGB dark matter. Its pNGB nature makes its scattering amplitude off nucleons vanish at tree level, evading direct detection constraints. Nonetheless, it is able to obtain an observed DM relic abundance via the thermal production mechanism. We will perform a random scan in the parameter space to investigate reasonable parameter points that satisfy current Higgs measurements at the Large Hadron Collider (LHC), observation of the DM relic abundance, and constraints from indirect detection experiments.

The paper is organized as follows. In Section II, we describe the details of the pNGB DM model with two Higgs doublets, including the scalar potential, mass eigenstates, four types of Yukawa couplings, the vanishing of the DM-nucleon scattering amplitude, and the alignment limit. In Section III, we perform a random scan in the parameter space and investigate phenomenological constraints from LHC Higgs measurements, relic abundance observation, and indirect detection. Section IV gives the conclusions and outlook. In Appendix A, we write down the scalar and gauge trilinear couplings. Appendix B gives some expressions for decay widths of the SM-like Higgs boson.
II. MODEL DETAILS

In this section, we study the model details. As explained above, we assume that the scalar sector involves two Higgs doublets and one SM gauge singlet, and there is a softly broken global U(1) symmetry leading to pNGB dark matter. The fermion content is assumed to be the same as in the SM. Analogous to generic two Higgs doublet models, there are four types of Yukawa couplings that do not induce flavor-changing neutral currents (FCNCs) at tree level. We find that these four types are all applicable to our purpose.

A. Scalar Potential

The two SU(2)\textsubscript{L} Higgs doublet fields are denoted as \(\Phi_1\) and \(\Phi_2\), both carrying hypercharge +1/2. The complex scalar \(S\) is an SU(2)\textsubscript{L} singlet and carries no hypercharge. For simplicity, we make two common assumptions for the scalar potential. The first assumption is that CP is conserved in the scalar sector, leading to only real coefficients. The second one is that there is a \(Z_2\) symmetry \(\Phi_1 \rightarrow -\Phi_1\) or \(\Phi_2 \rightarrow -\Phi_2\) forbidding quartic terms that are odd in either \(\Phi_1\) or \(\Phi_2\), but such a symmetry can be softly broken by quadratic terms.

Under these assumptions, the general terms in the scalar potential constructed with \(\Phi_1\) and \(\Phi_2\) are given by [33]

\[
V_1 = m_{11}^2 |\Phi_1|^2 + m_{22}^2 |\Phi_2|^2 - m_{12}^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) + \frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4 \\
+ \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \frac{\lambda_5}{2} [(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2].
\] (1)

And we can write down the potential terms that involve \(S\) and respect a global U(1) symmetry \(S \rightarrow e^{i\alpha} S\),

\[
V_2 = -m_S^2 |S|^2 + \frac{\lambda_S}{2} |S|^4 + \kappa_1 |\Phi_1|^2 |S|^2 + \kappa_2 |\Phi_2|^2 |S|^2 + \kappa_3 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) |S|^2.
\] (2)

In addition, we introduce a quadratic term softly breaking the global U(1) symmetry,

\[
V_{\text{soft}} = -\frac{m_S^2}{4} S^2 + \text{h.c.}
\] (3)

Note that even if \(m_S^2\) is complex, we can always make it real and positive by a phase redefinition of \(S\). Then \(V_2\) and \(V_{\text{soft}}\) respect a dark CP symmetry \(S \rightarrow S^*\) [26, 30]. The soft breaking term \(V_{\text{soft}}\) can be justified by treating \(m_S^2\) as a spurion, arising from a more fundamental theory that does not induce other soft breaking terms involving odd powers of \(S\) [26, 28].
Now the whole scalar potential is
\[ V = V_1 + V_2 + V_{\text{soft}}, \] (4)

In particular regions of the parameter space, \( \Phi_1, \Phi_2, \) and \( S \) develop nonzero vacuum expectation values (VEVs) \( v_1, v_2, \) and \( v_s. \) They can be expanded as
\[ \Phi_1 = \left( \frac{\phi_1^+}{(v_1 + \rho_1 + i\eta_1)/\sqrt{2}} \right), \quad \Phi_2 = \left( \frac{\phi_2^+}{(v_2 + \rho_2 + i\eta_2)/\sqrt{2}} \right), \quad S = \frac{v_s + s + i\chi}{\sqrt{2}}. \] (5)

By minimizing the potential, we find the following stationary point conditions:
\[ m_{11}^2 = \frac{v_2}{v_1} \tilde{m}_{12}^2 - \frac{1}{2} \lambda_1 v_1^2 - \frac{1}{2} \lambda_{345} v_2^2 - \frac{1}{2} \kappa_1 v_s^2, \] (6)
\[ m_{22}^2 = \frac{v_1}{v_2} \tilde{m}_{12}^2 - \frac{1}{2} \lambda_2 v_2^2 - \frac{1}{2} \lambda_{345} v_1^2 - \frac{1}{2} \kappa_2 v_s^2, \] (7)
\[ m_S^2 = -\frac{1}{2} \tilde{m}_S^2 + \frac{1}{2} \lambda_S v_s^2 + \frac{1}{2} \kappa_1 v_1^2 + \frac{1}{2} \kappa_2 v_2^2 + \kappa_3 v_1 v_2, \] (8)

where
\[ \tilde{m}_{12}^2 \equiv m_{12}^2 - \frac{1}{2} \kappa_3 v_s^2 \] and \( \lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5. \) (9)

Note that all terms in \( V_2 \) and \( V_{\text{soft}} \) are products of \(|S|\) or of \( S^2 + (S^*)^2. \) As their expansions are
\[ |S|^2 = \frac{1}{2} (v_s^2 + s^2 + \chi^2) + v_s s, \quad S^2 + (S^*)^2 = v_s^2 + s^2 - \chi^2 + 2v_s s, \] (10)
the real scalar \( \chi \) always appears in pair in the scalar potential. Therefore, \( \chi \) cannot decay, becoming a stable DM candidate.

### B. Mass Eigenstates

After the scalar fields obtain their VEVs, the mass squared of \( \chi \) is
\[ m_\chi^2 = -\tilde{m}_S^2 + \frac{1}{2} \lambda_S v_s^2 + \frac{1}{2} \kappa_1 v_1^2 + \frac{1}{2} \kappa_2 v_2^2 + \kappa_3 v_1 v_2 = m_S^2, \] (11)
where the terms with VEVs are totally canceled by the third stationary point condition (8). If \( m_S^2 = 0, \) there is no soft breaking term, and \( \chi \) is a massless Nambu-Goldstone boson. If \( m_S^2 > 0, \) \( \chi \) would have a physical mass \( m_\chi = m_S^2, \) behaving as a pseudo-Nambu-Goldstone boson. This is exactly what we want.

The mass terms for the charged scalars are derived as
\[ -\mathcal{L}_{\text{mass,}\phi} = \begin{bmatrix} \tilde{m}_{12}^2 - \frac{1}{2} (\lambda_4 + \lambda_5) v_1 v_2 \end{bmatrix} \begin{pmatrix} \phi_1^- \phi_2^- \end{pmatrix} \begin{pmatrix} v_2/v_1 & -1 & v_1/v_2 \end{pmatrix} \begin{pmatrix} \phi_1^+ \phi_2^+ \end{pmatrix}, \] (12)
while those for the CP-odd scalars are given by
\[ -\mathcal{L}_{\text{mass,}\eta} = \frac{1}{2}(\tilde{m}_{12}^2 - \lambda_5 v_1 v_2) \begin{pmatrix} \eta_1, \eta_2 \end{pmatrix} \begin{pmatrix} v_2/v_1 & -1 \\ -1 & v_1/v_2 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}. \] (13)

The above mass terms can be diagonalized by rotations
\[ \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix} = R(\beta) \begin{pmatrix} G^+ \\ H^+ \end{pmatrix}, \quad \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = R(\beta) \begin{pmatrix} G^0 \\ a \end{pmatrix}, \quad R(\beta) = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix}, \] (14)

where the rotation angle \( \beta \) satisfies
\[ \tan \beta = \frac{v_2}{v_1}. \] (15)

Now \( G^\pm \) and \( G^0 \) are massless Nambu-Goldstone bosons eaten by the weak gauge bosons \( W^\pm \) and \( Z \), while \( H^\pm \) and \( a \) are physical states with masses
\[ m_{H^+}^2 = \frac{v_1^2 + v_2^2}{v_1 v_2} \left[ \tilde{m}_{12}^2 - \frac{1}{2}(\lambda_1 + \lambda_5)v_1 v_2 \right], \quad m_a^2 = \frac{v_1^2 + v_2^2}{v_1 v_2}(\tilde{m}_{12}^2 - \lambda_5 v_1 v_2). \] (16)

The CP-even scalars \( \rho_1, \rho_2, \) and \( s \) mix with each other. Their mass terms are
\[ -\mathcal{L}_{\text{mass,}\rho s} = \frac{1}{2} \begin{pmatrix} \rho_1, \rho_2, s \end{pmatrix} \mathcal{M}^2_{\rho s} \begin{pmatrix} \rho_1 \\ \rho_2 \\ s \end{pmatrix}, \] (17)

where the elements of the \( 3 \times 3 \) symmetric mass-squared matrix \( \mathcal{M}^2_{\rho s} \) are given by
\[ (\mathcal{M}^2_{\rho s})_{11} = \lambda_1 v_1^2 + \frac{v_2}{v_1} \tilde{m}_{12}^2, \quad (\mathcal{M}^2_{\rho s})_{22} = \lambda_2 v_2^2 + \frac{v_1}{v_2} \tilde{m}_{12}^2, \quad (\mathcal{M}^2_{\rho s})_{33} = \lambda_5 v_s^2, \] (18)
\[ (\mathcal{M}^2_{\rho s})_{12} = \lambda_3 v_1 v_2 - \tilde{m}_{12}^2, \quad (\mathcal{M}^2_{\rho s})_{13} = \kappa_1 v_1 v_s + \kappa_3 v_2 v_s, \quad (\mathcal{M}^2_{\rho s})_{23} = \kappa_2 v_2 v_s + \kappa_3 v_1 v_s. \] (19)

\( \mathcal{M}^2_{\rho s} \) can be diagonalized by a \( 3 \times 3 \) real orthogonal matrix \( O \):
\[ O^T \mathcal{M}^2_{\rho s} O = \text{diag}(m_{h_1}^2, m_{h_2}^2, m_{h_3}^2). \] (20)

The mass eigenstates \( h_i \ (i = 1, 2, 3) \) are then related to the interaction eigenstates by
\[ \begin{pmatrix} \rho_1 \\ \rho_2 \\ s \end{pmatrix} = O \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}. \] (21)

One of \( h_i \) should behave like the SM Higgs boson in order to be consistent with observation.
Below we adopt a convention with \( m_{h_1} \leq m_{h_2} \leq m_{h_3} \).

From the covariant kinetic terms

\[
L_{\text{kin}} = (D^\mu \Phi_1)^\dagger D_\mu \Phi_1 + (D^\mu \Phi_2)^\dagger D_\mu \Phi_2,
\]

we derive the mass terms for the weak gauge bosons,

\[
L_{\text{mass}, WZ} = \frac{g^2}{4} (v_1^2 + v_2^2) W^{-\mu} W^\mu + \frac{1}{2} \frac{g^2}{4 c_W^2} (v_1^2 + v_2^2) Z^\mu Z_\mu,
\]

where \( c_W \equiv \cos \theta_W \) with \( \theta_W \) denoting the Weinberg angle, and \( g \) is the SU(2)_L gauge coupling.

Defining \( v \equiv \sqrt{v_1^2 + v_2^2} \), the masses of \( W \) and \( Z \) bosons become

\[
m_W = \frac{g v}{2}, \quad m_Z = \frac{g v}{2 c_W},
\]

just as in the SM. From the Fermi constant \( G_F = g^2/(4\sqrt{2} m_W^2) \), we obtain \( v = (\sqrt{2} G_F)^{-1/2} = 246.22 \) GeV. Note that \( v_1 \) and \( v_2 \) satisfy \( v_1 = v c_\beta \) and \( v_2 = v s_\beta \), where we have used the shorthand notations \( s_\beta \equiv \sin \beta \) and \( c_\beta \equiv \cos \beta \).

The scalar and gauge trilinear couplings of the scalar mass eigenstates can be found in Appendix A.

### C. Yukawa Couplings

Unlike the standard model, Yukawa couplings between the two Higgs doublets and SM fermions generally lead to tree-level FCNCs, which could cause phenomenological problems in flavor physics. This is because diagonalizing the fermion mass matrix cannot make sure that the Yukawa interactions are also diagonalized. Nevertheless, if all fermions with the same quantum numbers just couple to the one same Higgs doublet, the FCNCs will be absent at tree level [33, 42–44]. This can be achieved by assuming particular \( Z_2 \) symmetries for the Higgs doublets and fermions.

As a results, there are four independent types of Yukawa couplings without tree-level FCNCs, listed as follows.

- **Type-I:** \( L_{Y,I} = -y_e \bar{\ell}_i \ell_j R \Phi_2 - \bar{y}_d^i \tilde Q_i \tilde d_j^R \Phi_2 - \bar{y}_u^i \bar Q_i \tilde u_j ^R \tilde \Phi_2 + \text{h.c.} \)  
  (25)

- **Type-II:** \( L_{Y,II} = -y_e \bar{\ell}_i \ell_j R \Phi_1 - \bar{y}_d^i \tilde Q_i \tilde d_j^R \Phi_1 - \bar{y}_u^i \bar Q_i \tilde u_j ^R \tilde \Phi_2 + \text{h.c.} \)  
  (26)

- **Lepton-specific:** \( L_{Y,L} = -y_e \bar{\ell}_i \ell_j R \Phi_1 - \bar{y}_d^i \tilde Q_i \tilde d_j^R \Phi_2 - \bar{y}_u^i \bar Q_i \tilde u_j ^R \tilde \Phi_2 + \text{h.c.} \)  
  (27)

- **Flipped:** \( L_{Y,F} = -y_e \bar{\ell}_i \ell_j R \Phi_2 - \bar{y}_d^i \tilde Q_i \tilde d_j^R \Phi_1 - \bar{y}_u^i \bar Q_i \tilde u_j ^R \tilde \Phi_2 + \text{h.c.} \)  
  (28)

Here \( \tilde \Phi_2 \equiv i \sigma^2 \Phi_2^*, \bar{\ell}_i \equiv (\nu_{iL}, \ell_{iL})^T \), and \( Q_i \equiv (u_i' L, d_i' R)^T \). The down-type and up-type quark Yukawa matrices \( \bar y_d^i \) and \( \bar y_u^i \) can be diagonalized through \( (U_d)_i^{ij} \bar y_d^{jk} (U_d)_{kl} = y_d \delta_{ik} \) and \( (U_u)_i^{ij} \bar y_u^{jk} (U_u)_{kl} = y_u \delta_{ik} \).
TABLE I. Coefficients $\xi_{f_i}^f$ and $\xi_{a_i}^f$ in the four types of Yukawa couplings.

| Type-I | Type-II | Lepton-specific | Flipped |
|--------|---------|-----------------|---------|
| $\xi_{h_i}^{\ell_j}$ | $O_{2i}/\sin \beta$ | $O_{11i}/\cos \beta$ | $O_{12i}/\sin \beta$ |
| $\xi_{h_i}^{d_j}$ | $O_{2i}/\sin \beta$ | $O_{11i}/\cos \beta$ | $O_{12i}/\cos \beta$ |
| $\xi_{h_i}^{u_j}$ | $O_{2i}/\sin \beta$ | $O_{21i}/\sin \beta$ | $O_{11i}/\sin \beta$ |
| $\xi_{a_i}^{\ell_j}$ | $\cot \beta$ | $-\tan \beta$ | $-\tan \beta$ |
| $\xi_{a_i}^{d_j}$ | $\cot \beta$ | $-\tan \beta$ | $\cot \beta$ |
| $\xi_{a_i}^{u_j}$ | $-\cot \beta$ | $-\cot \beta$ | $-\cot \beta$ |

$(U_u)^1_{ij} \tilde{y}_{1k}^i (U_u)_{kl} = y_{ui} \delta_{il}$. Thus, the interaction eigenstates $u'_i$ and $d'_i$ are related to the mass eigenstates $u_i$ and $d_i$ via $d'_i = (U_d)_{ij} d_j$ and $u'_i = (U_u)_{ij} u_j$. The Cabibbo-Kobayashi-Maskawa matrix is defined as $V_{ij} \equiv (U_u)_{ik} (U_d)_{kj}$. As we would not discuss neutrino physics in this work, we assume the lepton sector is the same as in the SM.

After the scalars develop the VEVs, the Yukawa interactions provide mass terms to the fermions. For the mass eigenstates, the four types of Yukawa terms can be expressed in a same form,

$$
L_Y = \sum_{f=\ell, d, u} \left[ -m_f \bar{f} f - \frac{m_f}{v} \left( \sum_{i=1}^{3} \xi_{h_i}^f \bar{h}_i \bar{f} f + \xi_{a_i}^f \bar{a}_i \gamma_5 \bar{f} f \right) \right]
$$

$$
- \frac{\sqrt{2}}{v} [H^+(\xi_{a_i}^{\ell_i} m_{\ell_i} \bar{P_i} \ell_i + \xi_{a_i}^{d_i} m_{d_i} V_{ij} \bar{u}_i P_d d_j + \xi_{a_i}^{u_i} m_{u_i} V_{ij} \bar{u}_i P_L d_j) + \text{h.c.}],
$$

where $P_L$ and $P_R$ are the left-handed and right-handed projection operators, respectively. The coefficients $\xi_{h_i}^f$ and $\xi_{a_i}^f$ are listed in Table I.

### D. Vanishing of the DM-Nucleon Scattering Amplitude

In this subsection, we verify that the tree-level amplitude of DM scattering off nucleons vanishes at zero momentum transfer. In our case, DM-nucleon scattering are induced by DM-quark scattering. Therefore, we just need to prove that the DM-quark scattering amplitude vanishes in the zero momentum transfer limit.

From the $U(1)$ symmetric potential (2), we obtain the trilinear couplings for the DM candidate $\chi$ as

$$
L_{\text{tri}, \chi^2} = -\frac{1}{2} (\kappa_1 v_1 + \kappa_3 v_2) \rho_1 + (\kappa_2 v_2 + \kappa_3 v_1) \rho_2 + \lambda S v_s s) \chi^2 = \frac{1}{2} \sum_{i=1}^{3} g_{n_i} \chi^2 \, \chi_i \chi^2,
$$

(30)
where the coupling coefficients for the mass eigenstates are given by
\begin{equation}
gh = \frac{\gamma}{\varepsilon} \left[ \pm (m^2_{h_1} \pm m^2_{h_2} \mp m^2_{h_3}) \right] = \frac{\gamma}{\varepsilon} \left[ \pm (m^2_{h_1} \pm m^2_{h_2} \mp m^2_{h_3}) \right]. \tag{31}
\end{equation}

At tree level, only the CP-even Higgs bosons \(h_1\), \(h_2\), and \(h_3\) can mediate \(\chi\) scattering off quarks. The Feynman diagram is shown in Fig. 1.

Take the Type-I Yukawa couplings as an example. Defining a Lorentz invariant \(t \equiv p_\mu p^\mu\), where \(p_\mu\) is the 4-momentum of the mediator \(h_i\), we can write down the DM-quark scattering amplitude as
\begin{equation}
i\mathcal{M} = \frac{m_q}{v_{s\beta}} \bar{u}(k_2) u(k_1) \left( g_{h_1} \chi^2 - g_{h_2} \chi^2 + g_{h_3} \chi^2 \right) \left( M^2_{h} \right)^{-1} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \tag{32}\end{equation}
where \(u(k_1)\) and \(\bar{u}(k_2)\) are the wave functions for the incoming and outgoing quarks, respectively. In the zero momentum transfer limit, \(t \rightarrow 0\), and the above amplitude can be re-expressed as
\begin{equation}
i\mathcal{M} \rightarrow -i \frac{m_q}{v_{s\beta}} \bar{u}(k_2) u(k_1) \left( g_{h_1} \chi^2, g_{h_2} \chi^2, g_{h_3} \chi^2 \right) \left( M^2_{h} \right)^{-1} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \tag{33}\end{equation}
where \(\left( M^2_{h} \right)^{-1} = \text{diag}(m^2_{h_1}, m^2_{h_2}, m^2_{h_3})\) is the inverse of the diagonalized mass-squared matrix \(M^2_{h} \equiv \text{diag}(m^2_{h_1}, m^2_{h_2}, m^2_{h_3})\). From Eqs. (31) and (20), we have
\begin{equation}
\left( g_{h_1} \chi^2, g_{h_2} \chi^2, g_{h_3} \chi^2 \right) = \left( \gamma, \varepsilon \right) \left( \gamma, \varepsilon \right) \left( M^2_{h} \right)^{-1} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \left( \gamma, \varepsilon \right) \left( M^2_{h} \right)^{-1} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}. \tag{34}\end{equation}
Utilizing these equations as well as the orthogonality of \(O\), we obtain
\begin{equation}
i\mathcal{M} \rightarrow i \frac{m_q}{v_{s\beta}} \bar{u}(k_2) u(k_1) \left( g_{h_1} \chi^2, g_{h_2} \chi^2, g_{h_3} \chi^2 \right) \left( M^2_{h} \right)^{-1} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}. \tag{35}\end{equation}
This can be understood as the amplitude expressed in the interaction basis [26].

The inverse of $M^2_{\rho s}$ can be expressed as its adjugate $A$ divided by its determinant, i.e., 

$$(M^2_{\rho s})^{-1} = A/ \det(M^2_{\rho s}).$$ 

The relevant elements of $A$ are

$$A_{12} = -(\lambda_{345} v_1 v_2 - \tilde{m}^2_{12}) \lambda_S v_s^2 + (\kappa_1 v_1 + \kappa_3 v_2)(\kappa_2 v_2 + \kappa_3 v_1)v_s^2,$$  

$$A_{22} = (\lambda_1 v_1^2 + \tilde{m}^2_{12} \tan \beta) \lambda_S v_s^2 - (\kappa_1 v_1 + \kappa_3 v_2)^2 v_s^2,$$  

$$A_{32} = -(\lambda_1 v_1^2 + \tilde{m}^2_{12} \tan \beta)(\kappa_2 v_2 + \kappa_3 v_1)v_s + (\lambda_{345} v_1 v_2 - \tilde{m}^2_{12})(\kappa_1 v_1 + \kappa_3 v_2)v_s.$$

We then have

$$\left(\kappa_1 v_1 + \kappa_3 v_2, \kappa_2 v_2 + \kappa_3 v_1, \lambda_S v_s\right) (M^2_{\rho s})^{-1} \begin{pmatrix}0 \\ 1 \\ 0 \end{pmatrix} = \det^{-1}(M^2_{\rho s})[(\kappa_1 v_1 + \kappa_3 v_2)A_{12} + (\kappa_2 v_2 + \kappa_3 v_1)A_{22} + \lambda_S v_s A_{32}] = 0. \quad (39)$$

Therefore, we have proven that the tree-level DM-quark amplitude $iM$ vanishes in the zero momentum transfer limit for the Type-I Yukawa couplings. Similarly, we can prove this for the Type-II, lepton-specific, and flipped Yukawa couplings.

As the global U(1) symmetry is softly broken, loop corrections would give a nonvanishing DM-nucleon scattering cross section [26]. Nonetheless, we expect that the loop-induced cross section should be typically $\lesssim O(10^{-50})$ cm$^2$, as suggested by the one-loop evaluation in Ref. [32] where only one Higgs doublet is considered. Thus, current and near future direct detection experiments should not be able to probe our pNGB DM model.

### E. Alignment Limit

Current LHC Higgs measurements favor a 125 GeV SM-like Higgs boson. If one of the CP-even Higgs bosons mimics the SM Higgs boson, the constraints from Higgs measurements can be easily satisfied. For the two Higgs doublets, such a situation can be achieved by requiring the additional scalars are much heavier than the weak scale so that the lightest CP-even Higgs boson reproduces SM-like Higgs signals at the LHC. This is known as the “decoupling” limit [45]. In general, a particular parameter set or relation leading to a CP-even Higgs boson mimicking the SM Higgs boson is referred as an “alignment” limit. The decoupling limit is of course an alignment limit, but it is less interesting, as the new particles might be too heavy to be accessed at the LHC.

A more interesting possibility is alignment without decoupling [46, 47]. In order to find such a possibility, we may rotate the two Higgs doublets $\Phi_1$ and $\Phi_2$ into the Higgs basis [48, 49]

$$\begin{pmatrix} \Phi_h \\ \Phi_H \end{pmatrix} \equiv R^{-1}(\beta) \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix},$$

(40)
and have
\[ \Phi_h = \left( \frac{G^+}{(v + h + iG^0)/\sqrt{2}} \right), \quad \Phi_H = \left( \frac{H^+}{(H + ia)/\sqrt{2}} \right). \] (41)

Now \( \Phi_h \) gains a VEV \( v \) and contains a CP-even scalar \( h \) as well as the Nambu-Goldstone bosons, while \( \Phi_H \) have zero VEV and contains a CP-even scalar \( H \) and the physical states \( H^+ \) and \( a \). Consequently, the tree-level interactions of the CP-even scalar \( h \) with weak gauge bosons and SM fermions are totally identical to those of the Higgs boson in the SM. Therefore, the alignment limit means that \( h \) does not mix with \( H \) and \( s \).

In the Higgs basis, the potential terms (1) transform to
\[
V_1 = m_{hh}^2|\Phi_h|^2 + m_{HH}^2|\Phi_H|^2 - m_{hH}^2(\Phi_h^\dagger \Phi_H + \Phi_H^\dagger \Phi_h) + \frac{\lambda_h}{2}|\Phi_h|^4 + \frac{\lambda_H}{2}|\Phi_H|^4 + \tilde{\lambda}_3|\Phi_h|^2|\Phi_H|^2
+ \tilde{\lambda}_4|\Phi_h^\dagger \Phi_H|^2 + \frac{1}{2}[\tilde{\lambda}_5(\Phi_h^\dagger \Phi_H)^2 + \tilde{\lambda}_6|\Phi_h|^2\Phi_H^\dagger \Phi_h + \tilde{\lambda}_7|\Phi_H|^2\Phi_h^\dagger \Phi_H + \text{h.c.}],
\] (42)

where the new parameters are related to the previous parameters by [40]
\[
m_{hh}^2 = c_\beta^2 m_{11}^2 + s_\beta^2 m_{22}^2 - 2s_\beta c_\beta m_{12}^2, \quad m_{HH}^2 = s_\beta^2 m_{11}^2 + c_\beta^2 m_{22}^2 + 2s_\beta c_\beta m_{12}^2, \] (43)
\[
m_{hH}^2 = s_\beta c_\beta (m_{11}^2 - m_{22}^2) + (c_\beta^2 - s_\beta^2)m_{12}^2, \quad \lambda_H = c_\beta^4 \lambda_1 + s_\beta^4 \lambda_2 + 2s_\beta^2 c_\beta \lambda_{345}, \] (44)
\[
\tilde{\lambda}_3 = s_\beta^2 c_\beta (\lambda_1 + \lambda_2 - 2\lambda_4 - 2\lambda_5) + (s_\beta + c_\beta)\lambda_3, \] (45)
\[
\tilde{\lambda}_4 = s_\beta^2 c_\beta (\lambda_1 + \lambda_2 - 2\lambda_3 - 2\lambda_5) + (s_\beta + c_\beta)\lambda_4, \] (46)
\[
\tilde{\lambda}_5 = s_\beta^2 c_\beta (\lambda_1 + \lambda_2 - 2\lambda_3 - 2\lambda_4) + (s_\beta + c_\beta)\lambda_5, \] (47)
\[
\tilde{\lambda}_6 = -s_\beta^2 (c_\beta^2 \lambda_1 - s_\beta^2 \lambda_2) + s_\beta c_\beta c_\beta \lambda_{345}, \quad \tilde{\lambda}_7 = -s_\beta^2 (s_\beta^2 \lambda_1 - c_\beta^2 \lambda_2) - s_\beta c_\beta c_\beta \lambda_{345}. \] (48)

On the other hand, the potential terms (2) transform to
\[
V_2 = -m_S^2|S|^2 + \frac{\lambda_S}{2}|S|^4 + \tilde{\kappa}_1|\Phi_h|^2|S|^2 + \tilde{\kappa}_2|\Phi_H|^2|S|^2 + \tilde{\kappa}_3(\Phi_h^\dagger \Phi_H + \Phi_H^\dagger \Phi_h)|S|^2, \] (49)

where the new parameters are given by
\[
\tilde{\kappa}_1 = c_\beta^2 \kappa_1 + s_\beta^2 \kappa_2 + 2s_\beta c_\beta \kappa_3, \quad \tilde{\kappa}_2 = s_\beta^2 \kappa_1 + c_\beta^2 \kappa_2 - 2s_\beta c_\beta \kappa_3, \] (50)
\[
\tilde{\kappa}_3 = -s_\beta c_\beta (\kappa_1 - \kappa_2) + (c_\beta^2 - s_\beta^2)\kappa_3. \] (51)

Then the stationary point conditions for the scalar potential are
\[
m_{hh}^2 = -\frac{1}{2} \lambda_h v^2 - \frac{1}{2} \tilde{\kappa}_1 v_s^2, \quad m_{HH}^2 = \frac{1}{4} \tilde{\lambda}_H v^2 + \frac{1}{2} \tilde{\kappa}_3 v_s^2, \quad m_S^2 = -\frac{1}{2} m_S^2 + \frac{1}{2} \lambda_S v_s^2 + \frac{1}{2} \tilde{\kappa}_1 v^2. \] (52)
As a result, the mass-squared matrix for CP-even scalars \((h, H, s)\) is

\[
M_{hHs}^2 = \begin{pmatrix}
\lambda_h v^2 & \tilde{\lambda}_6 v^2 / 2 & \tilde{\kappa}_1 v v_s \\
\tilde{\lambda}_6 v^2 / 2 & m_{HH}^2 + (\tilde{\lambda}_{345} v^2 + \tilde{\kappa}_2 v_s^2) / 2 & \tilde{\kappa}_3 v v_s \\
\tilde{\kappa}_1 v v_s & \tilde{\kappa}_3 v v_s & \lambda_s v_s^2 \\
\end{pmatrix}.
\] (53)

In order to prevent \(h\)-\(H\) and \(h\)-\(s\) mixings, the off-diagonal terms \((M_{hHs}^2)_{12}\) and \((M_{hHs}^2)_{13}\) should be absent, corresponding to

\[
\tilde{\lambda}_6 = \tilde{\kappa}_1 = 0.
\] (54)

This is the alignment condition in our model. When this condition is satisfied, the tree-level couplings of \(h\) to SM particles are exactly the same as those of the SM Higgs boson.

### III. PHENOMENOLOGICAL CONSTRAINTS

In this section, we take the Type-I Yukawa couplings as an illuminating example to investigate the phenomenological constraints from Higgs measurements, relic abundance observation, and indirect detection.

#### A. Parameter Scan and Higgs Measurements

There are 13 free parameters in the model, which can be chosen as

\[
v_s, \ m_\chi, \ m_{12}^2, \ \tan \beta, \ \lambda_1, \ \lambda_2, \ \lambda_3, \ \lambda_4, \ \lambda_5, \ \lambda_S, \ \kappa_1, \ \kappa_2, \ \kappa_3.
\] (55)

In order to investigate the vast parameter space, we carry out a random scan within the following ranges:

\[
10 \text{ GeV} < v_s < 10^3 \text{ GeV}, \quad 10 \text{ GeV} < m_\chi < 10^4 \text{ GeV},
\] (56)

\[
(10 \text{ GeV})^2 < |m_{12}^2| < (10^3 \text{ GeV})^2, \quad 10^{-2} < \tan \beta < 10^2,
\] (57)

\[
10^{-3} < \lambda_1, \lambda_2, \lambda_S < 1, \quad 10^{-3} < |\lambda_3|, |\lambda_4|, |\lambda_5|, |\kappa_1|, |\kappa_2|, |\kappa_3| < 1.
\] (58)

Then we require the selected parameter points must give positive \(m_{h1,2,3}^2, \ m_{H^+}^2\), and \(m_a^2\), ensuring physical scalar masses. Moreover, one of the CP-even Higgs bosons \(h_i\) should have a mass within the 3\(\sigma\) range of the measured SM-like Higgs boson mass \(m_{h} = 125.18 \pm 0.16 \text{ GeV}\) [50]. We recognize this scalar as the SM-like Higgs boson, and denote it as \(h_{SM}\), and further examine if its properties are consistent with current measurements.

In the \(\kappa\) framework [51], the couplings of the SM-like Higgs boson to SM particles can be
expressed as

\[ L_{\text{hSM}} = \kappa_W g m_W h_{\text{SM}} W^+ W^- + \kappa_Z \frac{g m_Z}{2 c_W} h_{\text{SM}} Z \mu Z^\mu - \sum_f \kappa_f \frac{m_f}{v} h_{\text{SM}} \bar{f} f \]

\[ + \kappa g_{hgg}^{\text{SM}} h_{\text{SM}} G_\mu^a G^{\alpha \mu} + \kappa \gamma g_{h\gamma \gamma}^{\text{SM}} h_{\text{SM}} A_\mu A^{\mu} + \kappa Z g_{Z\gamma \gamma}^{\text{SM}} h_{\text{SM}} A_\mu Z^{\mu}, \]

(59)

where \( g_{hgg}^{\text{SM}} \), \( g_{h\gamma \gamma}^{\text{SM}} \), and \( g_{Z\gamma \gamma}^{\text{SM}} \) are the loop-induced effective couplings to \( gg, \gamma \gamma \), and \( Z \gamma \), respectively. \( \kappa \)'s are coupling modifiers, whose values are all equal to 1 in the SM. Eq. (A6) implies that \( \kappa_W \) and \( \kappa_Z \) are equal in our model, and we will use \( \kappa_V \) representing both of them. Assuming the SM-like Higgs boson is \( h_{\text{SM}} = h \), we have

\[ \kappa_V = c_\beta O_{1i} + s_\beta O_{2i}. \]  

(60)

The coupling modifiers for fermions can be read off from Table I. For the Type-I Yukawa couplings, all SM fermions have the same coupling modifier, given by

\[ \kappa_f = \frac{O_{2i}}{s_\beta}. \]  

(61)

It is also helpful to define another modifier \( \kappa_H \) as

\[ \kappa_H^2 \equiv \frac{\Gamma_{h_{\text{SM}}} - \Gamma_{h_{\text{BSM}}}}{\Gamma_{h_{\text{SM}}}}, \]

(62)

where \( \Gamma_{h_{\text{SM}}} \) is the Higgs total decay width in the SM, \( \Gamma_{h_{\text{SM}}} \) is the total decay width of the SM-like Higgs boson \( h_{\text{SM}} \), and \( \Gamma_{h_{\text{BSM}}} \) is the \( h_{\text{SM}} \) decay width into final states beyond the SM (BSM). Thus, \( \kappa_H \) indicates the deviation of the Higgs width decaying into SM final states and is also equal to 1 in the SM. In our model, \( \Gamma_{h_{\text{BSM}}} \) can be generally separated into two parts:

\[ \Gamma_{h_{\text{BSM}}} = \Gamma_{h_{\text{SM}}}^{\text{inv}} + \Gamma_{h_{\text{SM}}}^{\text{und}}, \]

(63)

\( \Gamma_{h_{\text{SM}}}^{\text{inv}} \) is the \( h_{\text{SM}} \) decay width into the invisible final state, \( i.e.\), a pair of the DM candidate \( \chi \). \( \Gamma_{h_{\text{SM}}}^{\text{und}} \) involves decay widths into all kinematically allowed BSM final states that are undetected in current LHC searches. Such final states may include \( aa, H^+ H^-, h_i h_j, a Z, \) and \( H^\pm W^\mp \). The expressions for these decay widths are listed in Appendix B. Once all the decay widths are evaluated, we can determine the invisible and undetected BSM branching ratios via \( \text{BR}_{\text{inv}} = \Gamma_{h_{\text{SM}}}^{\text{inv}} / \Gamma_{h_{\text{SM}}} \) and \( \text{BR}_{\text{und}} = \Gamma_{h_{\text{SM}}}^{\text{und}} / \Gamma_{h_{\text{SM}}} \), respectively.

We utilize a numerical tool \textit{Lilith} 1.1.4 [52] to study the constraints from current Higgs measurements. \textit{Lilith} is able to construct an approximate likelihood based on experimental results of Higgs signal strength measurements. For each selected parameter point in our random scan, we put the corresponding \( m_{h_{\text{SM}}}, \kappa_V, \kappa_f, \text{BR}_{\text{inv}}, \) and \( \text{BR}_{\text{und}} \) into \textit{Lilith}. Then \textit{Lilith} can evaluate \( \kappa_g, \kappa_\gamma, \) and \( \kappa_{Z\gamma} \) involving the loop contributions from SM fermions and gauge bosons whose couplings are modified by \( \kappa_f \) and \( \kappa_V \), including NLO QCD corrections.
Such an evaluation have neglected the loop contributions from the BSM scalars in our model. Nonetheless, these scalars are typically heavy and/or have small couplings. Therefore, their contributions are insignificant for most of the selected parameter points.

We further use Lilith to calculate the likelihood $-2 \ln L$ for each parameter point based on Tevatron data [53], ATLAS Run 1 data [54–61], CMS Run 1 data [62–66], ATLAS Run 2 data [67–75], and CMS Run 2 data [76–82]. We then transform $-2 \ln L$ to a $p$-value and require that the selected parameter points should give $p$-values larger than 0.05. This means that we have rejected the parameter points that are excluded by data at 95% confidence level (CL).

Now we can analyze the properties of the remaining parameter points. Fig. 2 shows the Lilith $p$-values of the selected parameter points projected in the $\tan \beta$-$\lambda_1$ and $\tan \beta$-$\lambda_2$ planes. We find that when $\tan \beta \lesssim 0.2$ ($\tan \beta \gtrsim 5$), $\lambda_1$ ($\lambda_2$) tends to converge on $\lambda_{SM} = m_h^2/v^2 \simeq 0.26$, which is the quartic Higgs coupling in the SM. This is because $\tan \beta \ll 1$ ($\tan \beta \gg 1$) leads to $v_1 \gg v_2$ ($v_2 \gg v_1$) and $\Phi_1 \simeq \Phi_h$ ($\Phi_2 \simeq \Phi_h$), i.e., $\Phi_1$ ($\Phi_2$) acting as the SM-like Higgs doublet. Since experimental data favors an SM-like Higgs boson, the corresponding quartic coupling would close to its SM counterpart.

Additionally, we project the parameter points in the $m_{h_{SM}}$-$m_{h_1}$ and $m_{h_2}$-$m_{h_3}$ planes in Figs. 3(a) and 3(b), respectively. In Fig. 3(a), the points with $h_{SM} = h_1$ align along a horizontal line with $m_{h_1} \simeq 125$ GeV, while the remaining points indicate that the SM-like Higgs boson is not the lightest CP-even Higgs boson $h_1$. On the other hand, two sets of aligned points in Fig. 3(b) correspond to $h_{SM} = h_2$ and $h_{SM} = h_3$.

The projection on the $m_{H^+}$-$m_a$ plane is presented in Fig. 4. From Eq. (16), we know that the difference between the masses of the charged Higgs boson $H^+$ and the CP-odd Higgs boson $a$ are due to the $\lambda_4$ and $\lambda_5$ couplings. If $\tilde{m}_{12}^2$ is much larger than the $\lambda_4$.
and $\lambda_5$ contributions, the difference would be negligible, as demonstrated in Fig. 4 for $m_{H^+}, m_a \gtrsim 500$ GeV.

Fig. 5(a) shows the projection on the $|\kappa_V| - |\kappa_f|$ plane. We find that the parameter points with $|\kappa_V| \simeq |\kappa_f| \simeq 1$ have the largest $p$-values, implying that current data still favor that the 125 GeV Higgs boson has SM-like couplings. Nonetheless, $|\kappa_V|$ may range from $\sim 0.85$ to $\sim 1$, and $|\kappa_f|$ may range from $\sim 0.6$ to $\sim 1.3$. In addition, there are two categories of parameter points approximately aligning along two outstanding lines.

- **Category 1**: One line in Fig. 5(a) corresponds to $|\kappa_V| \simeq |\kappa_f|$. Actually, the signs of $\kappa_V$ and $\kappa_f$ are the same for all selected parameter points. This line is thus related to $\kappa_V \simeq \kappa_f$. The main reason is that if $\tan \beta \gg 1$, we have $s_\beta \simeq 1$ and $c_\beta \simeq 0$,
and Eqs. (60) and (61) become $\kappa_V \simeq \kappa_f \simeq O_{2i}$, where $\kappa_V$ and $\kappa_f$ have a nearly total positive correlation. As $|O_{2i}| \leq 1$, in this case both $|\kappa_V|$ and $|\kappa_f|$ cannot exceed 1. Most of the parameter points in this category corresponds to the horizontal line with $|O_{2i}|/s_\beta \simeq 1$ in the $|O_{1i}|/c_\beta-|O_{2i}|/s_\beta$ plane shown in Fig. 5(b), while the rest gives $|O_{2i}|/s_\beta < 1$.

- **Category 2:** Another line in Fig. 5(a) corresponds to $|\kappa_V| \simeq 1$ with varying $|\kappa_f|$. This category is related to the vertical line with $|O_{1i}|/c_\beta \simeq 1$ in Fig. 5(b). From Eq. (60), we know that $|O_{1i}| \simeq c_\beta$ and $|O_{2i}| \simeq s_\beta$ could lead to $|\kappa_V| \simeq c_\beta^2 + s_\beta^2 = 1$. Nonetheless, the second relation $|O_{2i}| \simeq s_\beta$ is not important to keep $|\kappa_V| \simeq 1$ when $s_\beta \ll 1$. Therefore, in the case of $\tan \beta \ll 1$, $|O_{2i}|/s_\beta$ could deviate from 1, resulting in the vertical line in Fig. 5(b).

There are some scatter points not belonging in the two categories. Most of them correspond to $\tan \beta \sim 1$.

The dominant contributions to $\kappa_g$ come from the top and bottom loops, leading to a parametrization of [50]

$$\kappa_g^2 = 1.06\kappa_t^2 + 0.01\kappa_b^2 - 0.07\kappa_t\kappa_b.$$  \hfill (64)

On the other hand, $\kappa_\gamma$ is mainly contributed by the $W$ and top loops, resulting in [50]

$$\kappa_\gamma^2 = 1.59\kappa_W^2 + 0.07\kappa_t^2 - 0.66\kappa_W\kappa_t.$$  \hfill (65)

In both cases, the interference between the two contributions gives a term with negative coefficient. In Fig. 6(a), we project the parameter points in the $\kappa_g-\kappa_\gamma$ plane, where the points also align along two lines. One line implies a positive correlation between $\kappa_g$ and

![FIG. 5. Lilith $p$-values for the selected parameter points projected in the $|\kappa_V|-|\kappa_f|$ (a) and $|O_{1i}|/c_\beta-|O_{2i}|/s_\beta$ (b) planes.](image)
\( \kappa_\gamma \), corresponding to Category 1. This is because the relation \( \kappa_V \simeq \kappa_f \) gives rise to such a positive correlation via Eqs. (64) and (65). On the other hand, when \( |\kappa_V| \simeq 1 \), \( \kappa_\gamma \) is negatively correlated to \( |\kappa_f| \) as all selected parameter points satisfy \( \kappa_V \kappa_f > 0 \). As \( \kappa_g \) is positively correlated to \( |\kappa_f| \), Category 2 results in a second line with a negative correlation between \( \kappa_g \) and \( \kappa_\gamma \).

\( \kappa_{Z\gamma} \) is also dominantly contributed by the \( W \) and top loops, given by [50]

\[
\kappa_{Z\gamma}^2 = 1.12 \kappa_W^2 + 0.03 \kappa_t^2 - 0.15 \kappa_W \kappa_t. \tag{66}
\]

The correlations of \( \kappa_{Z\gamma} \) to \( \kappa_V \) and to \( \kappa_f \) are similar to those of \( \kappa_\gamma \). In addition, \( \kappa_H \) can be expressed as [50]

\[
\kappa_H^2 = 0.57 \kappa_b^2 + 0.06 \kappa_t^2 + 0.03 \kappa_c^2 + 0.22 \kappa_W^2 + 0.03 \kappa_Z^2 + 0.09 \kappa_g^2 + 0.0023 \kappa_\gamma^2, \tag{67}
\]

where all the coefficients are positive. Thus, \( \kappa_H \) is positively correlated to both \( |\kappa_V| \) and \( |\kappa_f| \). The projection in the \( \kappa_{Z\gamma}-\kappa_H \) plane are shown in Fig. 6(b). Analogous to Fig. 6(a), Category 1 leads to a line indicating a positive correlation between \( \kappa_{Z\gamma} \) and \( \kappa_H \) in Fig. 6(b).

Besides, parameter points in Category 2 roughly align along a second line with a negative correlation.

In Fig. 7(a), we show the projection in the \( m_\chi - \text{BR}_{\text{inv}} \) plane. When \( m_\chi > m_{h_{\text{SM}}}/2 \), we have \( \text{BR}_{\text{inv}} = 0 \), because the invisible decay \( h_{\text{SM}} \rightarrow \chi \chi \) is kinematically forbidden. When \( m_\chi < m_{h_{\text{SM}}}/2 \), the invisible branching ratio \( \text{BR}_{\text{inv}} \) could be as large as \( \sim 25\% \) and still consistent with data at 95\% CL. The projection in the \( \Gamma_{h_{\text{SM}}}-\text{BR}_{\text{und}} \) plane are presented in Fig. 7(b). We find that the undetected BSM branching ratio \( \text{BR}_{\text{und}} \) can be allowed up to
FIG. 7. Lilith $p$-values for the selected parameter points projected in the $m_{\chi}$-BR$_{\text{inv}}$ (a) and $\Gamma_{h_{\text{SM}}}$-BR$_{\text{und}}$ (b) planes.

(a) $m_{\chi}$-BR$_{\text{inv}}$ plane.
(b) $\Gamma_{h_{\text{SM}}}$-BR$_{\text{und}}$ plane.

FIG. 8. Lilith $p$-values for the selected parameter points projected in the $\tan \beta$-$\tilde{\lambda}_6$ (a) and $\tan \beta$-$\tilde{\kappa}_1$ (b) planes. The dashed lines indicate the alignment limit.

\(\sim 27\%\), while the total width $\Gamma_{h_{\text{SM}}}$ can range from $\sim 2$ MeV to $\sim 7$ MeV. There is a line implying a positive correlation between $\Gamma_{h_{\text{SM}}}$ and BR$_{\text{und}}$. This is reasonable, because opening new decay channels enlarges the total width.

In order to investigate the alignment limit, which corresponds to $\tilde{\lambda}_6 = \tilde{\kappa}_1 = 0$, the selected parameter points are projected in the $\tan \beta$-$\tilde{\lambda}_6$ and $\tan \beta$-$\tilde{\kappa}_1$ planes in Figs. 8(a) and 8(b), respectively. We find that most of the selected points satisfy $\tilde{\kappa}_1 \simeq 0$, showing no particular dependence on $\tan \beta$. On the other hand, $\tilde{\lambda}_6$ is typically close to 0 for $\tan \beta \gtrsim 20$ and $\tan \beta \lesssim 0.05$. For $0.05 \lesssim \tan \beta \lesssim 20$, there is no particular favor in the alignment limit.
B. DM Relic Abundance

The thermal relic abundance of dark matter is essentially determined by the total velocity-averaged annihilation cross section at the freeze-out epoch, which we denote as $\langle \sigma_{\text{ann}}v \rangle_{\text{FO}}$. In our model, the DM candidate $\chi$ has the following annihilation channels if kinematically allowed.

- Annihilation into a pair of fermions, $\chi\chi \rightarrow f\bar{f}$. This channel is mediated by $s$-channel CP-even Higgs bosons and suppressed by fermion masses. Thus, $t\bar{t}$ and $b\bar{b}$ are the important final states.

- Annihilation into a pair of weak gauge bosons, $\chi\chi \rightarrow W^+W^-, ZZ$. This channel is also mediated by $s$-channel CP-even Higgs bosons.

- Annihilation into a weak gauge boson and a Higgs boson, $\chi\chi \rightarrow W^\pm H^\mp, Za$, mediated by $s$-channel CP-even Higgs bosons.

- Annihilation into a pair of CP-even Higgs bosons, $\chi\chi \rightarrow h_ih_j$. This channel can be mediated by $s$-channel CP-even Higgs bosons, as well as by $t$- and $u$-channel $\chi$. Additionally, there are contributions from quartic scalar couplings.

- Annihilation into a pair of CP-odd or charged Higgs bosons, $\chi\chi \rightarrow aa, H^+H^-$. This channel is contributed by the mediation of $s$-channel CP-even Higgs bosons and quartic scalar couplings.

Some numerical tools are adopted to calculate the relic abundance. We implement the model with FeynRules 2.3.34 [83], and import the generated model files to a Monte Carlo generator MadGraph5_aMC@NLO 2.6.5 [84]. Then we utilize a MadGraph plugin MadDM 3 [85] to compute the relic abundance $\Omega_\chi h^2$ for each parameter point.

The relic abundance predicted by the selected parameter points is shown in Fig. 9, where the color bar denotes the freeze-out annihilation cross section $\langle \sigma_{\text{ann}}v \rangle_{\text{FO}}$. We find that the observed value $\Omega h^2 = 0.1186 \pm 0.0020$ given by the Planck experiment [86] corresponds to $\langle \sigma_{\text{ann}}v \rangle_{\text{FO}} \sim O(10^{-26})$ cm$^3$/s, which is typical for thermal dark matter. Increase in $m_\chi$ typically reduces the annihilation cross section, and hence increase the relic abundance. Consequently, if the DM candidate is too heavy, say $m_\chi \gtrsim 3$ TeV, the observed relic abundance could not be achieved.

In Fig. 9, the parameter points predicting $\Omega_\chi h^2$ over the observed value by $2\sigma$ are denoted with crosses. These points are considered to be excluded by data, because DM overproduction by the thermal mechanism contradicts standard cosmology. On the other hand, if the predicted thermal relic abundance is too low, there could be some nonthermal production occurring after DM freezes out.
C. Indirect Detection

In this subsection, we discuss constraints from γ-ray indirect detection experiments. There are couples of dwarf spheroidal galaxies discovered as satellites of the Milky Way Galaxy. They are considered as the largest substructures of the Galactic dark halo, predicted by the cold DM scenario [87, 88]. As known so far, they are the most DM-dominated systems [89]. Moreover, γ-ray emissions from typical astrophysical sources, such as neutral and ionized gases, and recent star formation activity, are expected to be rare in such dwarf galaxies [90–92]. These properties make them perfect targets for searching for γ-ray emissions from DM annihilation.

The DM velocity dispersion in dwarf galaxies is typically $\sim \mathcal{O}(10^{-5})$ [93], which is smaller than DM velocities at the freeze-out epoch by four orders of magnitude. Therefore, if the velocity dependence is significant in DM annihilation, the total velocity-averaged cross section in dwarf galaxies $\langle \sigma_{\text{ann}} v \rangle_{\text{dwarf}}$ could be much different from the freeze-out value $\langle \sigma_{\text{ann}} v \rangle_{\text{FO}}$.

We further use MadDM to calculate $\langle \sigma_{\text{ann}} v \rangle_{\text{dwarf}}$ for each parameter point assuming the average DM velocity is $2 \times 10^{-5}$. The ratio of $\langle \sigma_{\text{ann}} v \rangle_{\text{dwarf}}$ to $\langle \sigma_{\text{ann}} v \rangle_{\text{FO}}$ is demonstrated in Fig. 10(a), where the parameter points excluded by the Planck relic abundance measurement are not shown. Most of the parameter points give $\langle \sigma_{\text{ann}} v \rangle_{\text{dwarf}} / \langle \sigma_{\text{ann}} v \rangle_{\text{FO}} \sim 2$. Nonetheless, some points give the ratio away from $\mathcal{O}(1)$, indicating significant dependence on velocity. This is typically due to DM annihilation through the resonances of CP-even Higgs bosons, since the resonance effect extremely depends on the difference between the resonance location and the velocity-dependent center-of-mass energy [94, 95].

The vertical dashed line in Fig. 10(a) indicates the location of $m_\chi = m_{h\text{SM}}/2$, correspond-
Fig. 10. Parameter points with data-allowed relic abundance projected in the $m_{\chi}$-$\langle \sigma_{\text{ann}} v \rangle_{\text{dwarf}} / \langle \sigma_{\text{ann}} v \rangle_{\text{FO}}$ (a) and $m_{\chi}$-$\langle \sigma_{\text{ann}} v \rangle_{\text{dwarf}}$ (b) planes. The colors indicate the predicted relic abundance $\Omega_{\chi} h^2$. The dashed line in the left panel denotes the location of $m_{\chi} = m_{h_{\text{SM}}}/2$. The dot-dashed line in the right panel denotes the 95% CL upper limits from $\gamma$-ray observations of dwarf galaxies by Fermi-LAT and MAGIC [96].

Fig. 10(b) shows the projection of the parameter points in the $m_{\chi}$-$\langle \sigma_{\text{ann}} v \rangle_{\text{dwarf}}$ plane, as well as the 95% CL upper limits on $\langle \sigma_{\text{ann}} v \rangle_{\text{dwarf}}$ given by an analysis of Fermi-LAT and MAGIC $\gamma$-ray observations [96]. The analysis combined 6-year observations of 15 dwarf galaxies from the Fermi-LAT satellite experiment and 158-hour observations of a single dwarf galaxy Segue 1 from the MAGIC Cherenkov telescopes. The limits were obtained assuming that DM solely annihilates into $b\bar{b}$. However, there are various DM annihilation channels in our model. Fortunately, the $\gamma$-ray spectra yielded from these channels should be similar to the spectrum from the $b\bar{b}$ channel, because they are contributed by similar processes, such as hadronization, hadron decays, and final state radiation. Therefore, we have a good reason to expect that the $b\bar{b}$ limits are approximately applicable to our case.

From Fig. 10(b), we can observe that a large fraction of the parameter points with $m_{\chi} \lesssim 1$ TeV are ruled out, while the parameter points with $m_{\chi} \gtrsim 100$ GeV and $\Omega_{\chi} h^2 \sim 0.1$ are not excluded. Additionally, if $m_{\chi} \simeq m_{h_{\text{SM}}}/2$, the resonance effect could both yield a data-allowed relic abundance and lead to a small $\langle \sigma_{\text{ann}} v \rangle_{\text{dwarf}}$ evading the indirect detection constraint.
IV. CONCLUSIONS AND OUTLOOK

In this paper, we have studied the pNGB DM framework with two SU(2) \(_L\) Higgs doublets \(\Phi_1\) and \(\Phi_2\). The DM candidate \(\chi\) is the imaginary part of a complex scalar \(S\), which is an SM gauge singlet. Most of the scalar potential terms respect a global U(1) symmetry \(S \to e^{i\alpha}S\), expect for a soft breaking term giving mass to \(\chi\). As a result, \(\chi\) becomes a stable massive pNGB. Mass eigenstates in the scalar sector also include three CP-even Higgs boson \(h_i\), a CP-odd Higgs boson \(a\), and charged Higgs bosons \(H^\pm\).

There are four possible types of Yukawa couplings without tree-level FCNCs, just as in usual two Higgs doublet models. DM scattering off nucleons is mediated by the CP-even Higgs bosons. Because of the pNGB nature of \(\chi\), the scattering amplitude vanishes in the limit of zero momentum transfer for all the four Yukawa coupling types. Although loop corrections lead to a nonvanishing amplitude, the scattering cross section is expected to be below \(\sim O(10^{-50})\) cm\(^2\). Consequently, current and near future direct detection experiments are incapable of probing such a DM candidate.

Taking the Type-I Yukawa couplings as an example, we have performed a random scan in the 13-dimensional parameter space. The selected parameter points are required to provide an SM-like Higgs boson whose properties are consistent with current LHC Higgs measurements. We have found that for \(\tan \beta \gg 1\) or \(\tan \beta \ll 1\), one of the Higgs doublets acts as the SM-like Higgs doublet, and data favor the alignment limit. On the other hand, for \(\tan \beta \sim 1\) there is no preference to the alignment limit.

We have also calculated the relic abundance and annihilation cross sections predicted by the selected parameter points. For \(m_\chi \lesssim 3\) TeV, it is possible to achieve the observed relic abundance. Because of the resonance effect, the present velocity-averaged annihilation cross section at dwarf galaxies could be rather different from that in the freeze-out epoch. Fermi-LAT and MAGIC observations of dwarf galaxies have excluded a large fraction of parameter points with \(m_\chi \lesssim 1\) TeV. Nonetheless, for \(m_\chi \simeq m_{h_{\text{SM}}}/2\) or \(100\) GeV \(\lesssim m_\chi \lesssim 3\) TeV, it is still possible to simultaneously satisfy the constraints from the relic abundance observation and indirect detection.

Such a pNGB DM model is strongly related to Higgs physics. The proposed future Higgs factories, such as CEPC [97], ILC [98], and FCC-ee [99], would greatly improve the Higgs measurements. We expect that these measurements could significantly restrict the parameter space in our model. Nevertheless, Higgs measurements are not able to pin down the DM candidate mass \(m_\chi\), which is solely determined by the soft breaking term that does not affect the rest scalar masses. Thus, indirect detection experiments in the future are essentially important for exploring this model.
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Appendix A: Scalar and Gauge Trilinear Couplings

From the scalar potential (4), we derive the scalar trilinear couplings as

$$L_{\text{tri}} = \sum_{i=1}^{3} \left( \frac{1}{2} g_{h_i \chi^2} h_i \chi^2 + \frac{1}{2} g_{h_i \alpha^2} h_i \alpha^2 + g_{h_i H^- H^+} h_i H^- H^+ \right) + \sum_{i,j,k=1}^{3} g_{ijk} h_i h_j h_k, \quad (A1)$$

where $g_{h_i \chi^2}$ is already given by Eq. (31), and the other coupling coefficients are given by

$$g_{h_i \alpha^2} = -\left[ s_\beta^2 \lambda_1 + c_\beta^2 (\lambda_3 + \lambda_4 - \lambda_5) \right] v_1 - 2 s_\beta c_\beta \lambda_5 v_2 \right] O_{1i}$$

$$g_{h_i H^- H^+} = -\left[ s_\beta^2 \lambda_1 + c_\beta^2 (\lambda_4 + \lambda_5) \right] v_1 - 2 s_\beta c_\beta \lambda_5 v_1 \right] O_{2i} - (s_\beta^2 \kappa_1 + c_\beta^2 \kappa_2 - 2 s_\beta c_\beta \kappa_3) v_s O_{3i}, \quad (A2)$$

$$g_{ijk} = -\frac{1}{2} \left( \lambda_1 v_1 O_{1i} + \lambda_3 v_2 O_{2i} + \kappa_1 v_s O_{3i} \right) O_{1j} O_{1k} - \frac{1}{2} \left( \lambda_2 v_2 O_{2i} + \lambda_3 v_1 O_{1i} + \kappa_2 v_s O_{3i} \right) O_{2j} O_{2k} - \frac{1}{2} \left( v_2 O_{1i} + v_1 O_{2i} + \kappa_3 v_s O_{3i} \right) O_{1j} O_{2k} - \frac{1}{2} \left( v_2 O_{2i} + \kappa_3 v_1 O_{2i} \right) O_{2j} O_{3k} - \frac{1}{2} \left( \kappa_1 v_1 + \kappa_3 v_s O_{3i} \right) O_{1j} O_{3k} - \frac{1}{2} \left( \kappa_2 v_s O_{3i} O_{1i} O_{1k} - \frac{1}{2} \kappa_2 v_s O_{3i} O_{2j} O_{2k}. \quad (A4)$$

By expanding the Lagrangian (22), we obtain the gauge trilinear couplings for the scalars,

$$L_{\text{gauge}} = \sum_{i=1}^{3} \left( g_{h_i W^+} h_i W^{- \mu} W^{\mu^+}_\mu + \frac{1}{2} g_{h_i ZZ} h_i Z^\mu Z_\mu + i g_{Z_{abh}} Z_\mu a \partial^\mu h_i \right)$$

$$+ \sum_{i=1}^{3} \left( g_{W^+ H^- h_i} W^{\mu^+}_\mu H^- \partial^\mu h_i + i \frac{g}{2} W^{\mu^+}_\mu H^- i \partial^\mu a + \text{h.c.} \right)$$

$$+ e A_\mu H^- i \partial^\mu H^+ + \frac{g(c_W^2 - s_W^2)}{2 c_W} Z_\mu H^- i \partial^\mu H^+, \quad (A5)$$
where \( s_W \equiv \sin \theta_W \). The derivative symbol \( \partial^\mu \) is defined as \( F \partial^\mu G = F \partial^\mu G - G \partial^\mu F \). The coupling coefficients are given by

\[
\begin{align*}
    g_{h,WW} &= g m_W (c_\beta O_{1i} + s_\beta O_{2i}), \\
    g_{h,ZZ} &= \frac{g m_Z}{c_W} (c_\beta O_{1i} + s_\beta O_{2i}), \\
    g_{Z a h} &= \frac{g}{2c_W} (-s_\beta O_{1i} + c_\beta O_{2i}), \\
    g_{W^\pm H^\mp h_i} &= \frac{g}{2} (-s_\beta O_{1i} + c_\beta O_{2i}).
\end{align*}
\]  

(A6)

\[
\begin{align*}
    g^h_{a h_i} = g_{a h_i} m^3_{a h_i} \left( c_\beta O_{1i} + s_\beta O_{2i} \right), \\
    g^h_{W^\pm H^\mp h_i} = \frac{g}{2} (-s_\beta O_{1i} + c_\beta O_{2i}).
\end{align*}
\]  

(A7)

**Appendix B: BSM Decay Widths of the SM-like Higgs Boson**

This appendix gives the decay widths of the SM-like Higgs boson into two-body BSM final states when they are kinematically allowed. Assuming the SM-like Higgs boson is \( h^{SM} = h_i \), its invisible decay width at tree level is

\[
\Gamma(h_i \to \chi \chi) = \frac{g^{h_i,\chi^2}}{32 \pi m_{h_i}} \sqrt{1 - \frac{4m_\chi^2}{m_{h_i}^2}}.
\]  

(B1)

Moreover, its decay widths into \( aa \) and \( H^+ H^- \) are given by

\[
\begin{align*}
    \Gamma(h_i \to aa) &= \frac{g^{h_i,aa^2}}{32 \pi m_{h_i}} \sqrt{1 - \frac{4m_a^2}{m_{h_i}^2}}, \\
    \Gamma(h_i \to H^+ H^-) &= \frac{g^{h_i,12}}{16 \pi m_{h_i}} \sqrt{1 - \frac{4m_{H^+}^2}{m_{h_i}^2}}.
\end{align*}
\]  

(B2)

Furthermore, the \( h_i \to aZ \) decay width can be expressed as

\[
\Gamma(h_i \to aZ) = \frac{g^{a Z h_i}}{16 \pi m_Z^2} \lambda^{3/2}(1, m_a^2/m_{h_i}^2, m_Z^2/m_{h_i}^2),
\]  

(B3)

where the \( \lambda \) function is defined by

\[
\lambda(x, y, z) \equiv x^2 + y^2 + z^2 - 2xy - 2xz - 2yz.
\]  

(B4)

The decay width of \( h_i \to H^+ W^- \) is given by

\[
\Gamma(h_i \to H^+ W^-) = \frac{g^{h_i,12}}{16 \pi m_W^2} \lambda^{3/2}(1, m_{H^+}^2/m_{h_i}^2, m_W^2/m_{h_i}^2),
\]  

(B5)

which is equal to the decay width of \( h_i \to H^- W^+ \).

If \( h^{SM} = h_2 \) or \( h_3 \), it is possible to decay into \( h_1 h_1 \) and \( h_2 h_2 \), whose widths can be commonly expressed as

\[
\Gamma(h_i \to h_j h_j) = \frac{g^{h_i,12}_{ij}}{8 \pi m_{h_i}} \sqrt{1 - \frac{4m_{h_j}^2}{m_{h_i}^2}}.
\]  

(B6)

with \( g^{h_i,12}_{ij} = g^{h_i,12}_{ij} + g^{h_i,12}_{ji} + g^{h_i,12}_{jj} \). If \( h^{SM} = h_3 \), there is another possible decay channel into \( h_1 h_2 \).
The corresponding width is

\[
\Gamma(h_3 \to h_1 h_2) = \frac{\tilde{g}_{123}^2}{16\pi m_{h_3}} \lambda^{1/2}(1, m_{h_1}^2/m_{h_3}^2, m_{h_2}^2/m_{h_3}^2),
\]

(B7)

where \( \tilde{g}_{123} = g_{123} + g_{231} + g_{312} + g_{213} + g_{132} + g_{321} \).

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