A novel robust and efficient algorithm for charge particle tracking in high background flux

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Abstract. The high luminosity that will be reached in the new generation of High Energy Particle and Nuclear physics experiments implies large high background rate and large tracker occupancy, representing therefore a new challenge for particle tracking algorithms. For instance, at Jefferson Laboratory (JLab) (VA,USA), one of the most demanding experiment in this respect, performed with a 12 GeV electron beam, is characterized by a luminosity up to \(10^{39} \text{cm}^{-2}\text{s}^{-1}\). To this scope, Gaseous Electron Multiplier (GEM) based trackers are under development for a new spectrometer that will operate at these high rates in the Hall A of JLab. Within this context, we developed a new tracking algorithm, based on a multistep approach: (i) all hardware - time and charge - information are exploited to minimize the number of hits to associate; (ii) a dedicated Neural Network (NN) has been designed for a fast and efficient association of the hits measured by the GEM detector; (iii) the measurements of the associated hits are further improved in resolution through the application of Kalman filter and Rauch-Tung-Striebel smoother. The algorithm is shortly presented along with a discussion of the promising first results.

1. The Tracking Challenge
We developed a new charged particle tracking algorithm, able to operate in high luminosity experiments such as those at Jefferson Laboratory (JLab) (VA,USA). In particular, the algorithm is designed for the experimental Hall A in JLab, where an hybrid tracker has been developed to optimally exploit the new 12 GeV, high intensity, electron beam. The tracker consists of six large GEM chambers [1] and two small planes of Silicon microstrip Detectors (SIDs). The GEM tracker will be placed in the Super Big Bite Spectrometer (SBS) (see, e.g., Fig. 1), after a momentum analyzing dipole, while the silicon tracker will be sitting very close to the scattering chamber, in order to increase the tracked flying path and the lever arm for better tracking. Each GEM chamber is composed (Fig. 2) by three \(40 \times 50 \text{cm}^2\) GEM individual modules, having three GEM foils, and with two-dimensional strip readout, with expected spatial resolution of about 80 \(\mu\text{m}\).

In the most demanding experiment (the measurement of the proton form factors at high momentum transfer [2]), we expect almost 20 kHz (the coincidence trigger rate of the elastically scattered electron and proton) of signal and 400 kHz/cm\(^2\) of background hits on each chamber (\(\gamma, e, \pi^+\pi^-\)); the hits rates in a cone of 10 cm\(^2\) in the first GEM plane are of 1 signal hit plus
Figure 1. Layout of the SBS spectrometer in Jefferson Lab Hall A.

Figure 2. Schematic diagram of the 3-GEM foils detector.

Figure 3. Time evolution of the charge coming from the strips of the GEM chamber for three different events: left a background particle uncorrelated to the trigger (starting at time = 0 ns); center a signal particle correlated to the trigger; right a combination of two particles (background and signal).

roughly 100 hits of background. To cope with these high rates of hits, we developed a new multistep tracking algorithm, based on (i) “hardware reduction”, (ii) a Neural Network (NN) for a fast “association of hits”, and (iii) Kalman filter for “precise tracking”, as explained in what follows.

Hardware Reduction We simulated the response of the electronics to charge deposits of GEM chambers. In particular, the protons represent the signal, whereas photons or charged particles (electrons or pions) account for background. In Fig. 3, the time evolution of the signals registered by the electronics is shown: the time evolution of proton signals develops around 0 ns (25 ns maximum jitter distribution is included in the simulation), having been synchronized with the trigger starting at 0 ns by definition. Background hits are uncorrelated with the trigger. This simulation has been performed with Geant4 [3], along with a custom, realistic digitization algorithm transforming the energy deposited by the particle into an electric discharge in the GEM chamber and then into an electronic signal. This temporal correlation, as well as x/y charge correlation, are used to significantly reduce the number of hits passed to the second step of the procedure.

Association of hits Lots of successful applications of artificial NN’s in High Energy and Nuclear Physics have been realized in the last two decades. The possibility to use an Hopfield-like
network in track finding problems has been firstly demonstrated in [4, 5]. In this proceeding, we present the preliminary results obtained with a NN defined within the framework of Mean Field Theory (MFT). Here we recall the essential features of a NN in MFT: neurons \( S_{ij} \) correspond to connections between two hits, \( i \rightarrow j \), belonging to two adjacent planes. \( S_{ij} \) assume binary values (0 - connection off, 1 - connection on), but, in MFT, their values are permitted to range continuously \( \in [0,1] \), and to make a clear distinction with the digital case, we use the notation \( V_{ij} \). In order to connect a neuron to another, namely \( i \rightarrow j \rightarrow l \), it is necessary to introduce the so called synaptic strengths, formally quadri-dimensional \( T_{ijkl} \), with the constraint \( j = k \). To this respect, two essential ingredients are the lengths of two adjacent segments \( r_{ij} \), \( r_{jl} \) and the angle between them \( \theta_{ijl} \). The general features of the MFT approach remain the same, but, in a bi-dimensional case, we can write

\[
V_{ij} = \frac{1}{2} \left[ 1 + \tanh \left( -\frac{\partial E}{\partial V_{ij}} \frac{1}{T} \right) \right].
\]

It is important to point out that the particular definition of the energy function \( E \) is a matter of choice and it depends strictly on the particular features of the problem. In our case, we have 6 subsequent tracking planes, spanning a certain angular region, which is crossed by a heavy number of charged particles, whose trajectories are approximately straight lines (magnetic field is negligible in the tracker region). We define an energy function able to describe and discriminate the trajectories even when they occur in a very small angular region (high background conditions). Given a hit \( i \) in a certain plane \( p \) (from first to penultimate, \( p = 1, ..., n - 1 \)), we consider connections \( i \rightarrow j \) to hits belonging to the next plane, where \( j \in p + 1 \), and from this to the subsequent plane \( (j \rightarrow l) \), where \( l \in p + 2 \). These requirements are encoded in the energy function as:

\[
E = -\sum_{ijkl} \frac{d_G \cos \theta_{ijl}}{r_{ij} + r_{jl}} \delta_{kj} V_{ij} V_{kl} + \sum_{ijkl} \frac{d_G \cos \theta_{ijl} \delta_{li} V_{ij}}{r_{ij} + r_{jl}} + \alpha \left( \sum_{ijkl} V_{ij} V_{kl} + \sum_{ijkl} V_{ij} V_{kj} \right),
\]

where \( d_G \) is the distance between two tracking planes, whereas \( r_{ij} \), \( r_{jl} \) and \( \theta_{ijl} \) are the segment lengths between the \( ij \), \( jl \) hits and the angle between them, respectively; the second term of the ‘cost’ part, comprises the so called ‘firing thresholds’ \( I_{ij} = (d_G \cos \theta_{ijl} \delta_{li} / (r_{ij} + r_{jl})) \), to correctly weight the connections (neurons) at the edge planes. In the ‘constraint’ part, we assume a bifurcation inhibitor by using a Lagrange multiplier \( \alpha \).

The energy function (2), and, in general, any cost term based on trigonometric functions, works well if one has to consider tracks separated enough in space and cases in which the occupancy of the planes (namely the density of hits) is not too high. On the other hand, these two conditions are not encountered in the high luminosity experiments. To recover the full discrimination power of the energy function even in the case of small spatial region, we propose to use a novel approach, based on an affine scaling transformation, that preserves the linearity of trajectories. This has been done by setting \( r_{ij}' \equiv S_{xy}(\xi) r_{ij} = \sqrt{\xi^2 \Delta x_{ij}^2 + \xi^2 \Delta y_{ij}^2 + \Delta z_{ij}^2} \), where \( \xi \) is a continuous parameter \( \in [1, +\infty) \). One has to apply this transformation to the cost term \( \cos \theta_{ijl} / (r_{ij} + r_{jl}) \) present in Eq. (2). In our case, the optimal value that distinguishes more effectively the ‘on’ and ‘off’ connections in the energy cost term, was found to be \( \xi \approx 100 \). The introduced affine scaling has revealed very effective, and guides the NN to fast convergence. As illustrated in Fig. 4, the energy decreases to a plateau value (and it behaves similarly to a Lyapunov function) as well as the convergence \( \sum_{ij} |V_{ij}(t + 1) - V_{ij}(t)| \), as a function of the number of sweep cycles \( t^1 \). Correspondingly, all the neurons saturate to values close to 0 (if connections are ‘off’) or

\(^1\) After each sweep cycle, a full asynchronous updating of all the neurons is completed.
close to 1 (if ‘on’), as shown in Fig. 5. Common difficulties in practical applications, arise from the arbitrary choice of the NN parameters (Lagrange multipliers, temperature, scaling variable). We provided general criteria which allow the convergence of the algorithm and an effective and robust association of the hits: for instance, we used a ‘Network digitalization’ (see, e.g., [6]) for setting the Lagrange multiplier, and a technique similar to an ‘a posteriori annealing’ (as suggested in [7]) to investigate the dependence on temperature of the system. We then applied

![Figure 4](image1.png)

**Figure 4.** (top) energy and (bottom) variation of the average variation of neuron activities as a function of the sweep cycles.

![Figure 5](image2.png)

**Figure 5.** Neuron activity. In MFT, values are continuous and range ∈ [0,1]. After a certain number of sweep cycles, all the neurons stabilize. A threshold of 0.5 is chosen, and the neurons are transformed to binary values, 0 or 1, depending on whether smaller or larger than the threshold.

our NN based tracking algorithm to the case of JLab12 experiments. As mentioned earlier, the typical rates expected in an area of 10 cm$^2$ in the first tracking plane are 1 signal (proton) track from the interaction vertex, 1 charged background particle, few background photons per plane, and about 100 ghost hits per plane, which arise from combinatorial x/y strips association. Notably, as shown in Fig. 6, a 97% tracking efficiency is obtained with 100 ghost hits per plane.

![Figure 6](image3.png)

**Figure 6.** Track association efficiency as a function of the number of ghost hits. The NN parameter settings were determined by optimizing the track finding efficiency for the case of 100 ghost hits (97%). In all the other cases, efficiency always exceeds 97% and it is larger than 99% below 50 ghost hits.

**Precise Tracking** Once the association is accomplished, one can provide even more accurate measurements of the associated hits through filtering techniques. We used both Kalman filter (KF) [8], to filter the tracks in the forward direction, and Rauch Tung Striebel (RTS) smoother [9], for the reverse backward filtering. It is worth mentioning here that these are basically
Bayesian algorithms, that combine both experimental and theoretical information, starting from a measured hit, and according to a certain evolution model, one can infer the predicted hit in the subsequent plane, and compare it to its effective measurement in this plane. In this way, one can easily obtain a posterior distribution, from which the most likely ‘true’ position of the hit is estimated. We simulated a large number of correctly associated tracks, in order to calculate the accuracy of the filters (KF+RTS) in rather realistic experimental conditions. As reported in Fig. 7, the high accuracy obtained by combining the filters method is $\sim 10 \mu m$, to be compared to the hit spatial resolution of about 80 $\mu m$.

![Figure 7](image_url)  
**Figure 7.** Distance between KF filtered and true hits (blue) and distance between KF+RTS filtered and true hits (gray) for 6000 hits belonging to associated tracks. The accuracy of the combined KF+RTS filters method is $\sim 10 \mu m$.

2. Conclusions
The overall procedure (NN+filtering) looks terrific promising, with high performances both in terms of association efficiency ($\geq 97\%$), provided by the NN, and of reconstruction accuracy ($\sim 10 \mu m$, a factor 8 smaller than GEM designed spatial resolution), this latter provided by the combined filters. Next development will concentrate on the comparison of NN with other “standard” algorithms. Furthermore, a study and optimization of the computational time is also compulsory. Multiple scattering is under implementation for a deeper testing of the KF+RTS filtering technique, which has proved to be effective on the associated hits of the NN algorithm. Study of the refined and optimized procedure on real data will be the ultimate goal.

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