Alleviating $H_0$ tension in Horndeski gravity

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We show that the $H_0$ tension can be alleviated in the framework of Horndeski/generalized galileon gravity. In particular, since the terms depending on $G_5$ control the friction in the Friedmann equation, we construct specific sub-classes in which it depends only on the field’s kinetic energy. Since the latter is small at high redshifts, namely at redshifts which affected the CMB structure, the deviations from ΛCDM cosmology are negligible, however as time passes it increases and thus at low redshifts the Hubble function acquires increased values in a controlled way. We consider two Models, one with quadratic and one with quartic dependence on the field’s kinetic energy. In both cases we show the alleviation of the tension, resulting to $H_0 \approx 74 \text{ km/s/Mpc}$ for particular parameter choices. Finally, we examine the behavior of scalar metric perturbations, showing that the conditions for absence of ghost and Laplacian instabilities are fulfilled throughout the evolution, and we confront the models with Supernovae type Ia (SNIa) and Cosmic Chronometer data.

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I. INTRODUCTION

The Standard Model of Cosmology, namely Λ-Cold
Dark Matter (ΛCDM) plus inflation, in the framework of general relativity, proves to be very efficient in describing the universe evolution, both at the background and perturbation levels [1, 2]. However, theoretical issues such as the cosmological constant problem and the non-renormalizability of general relativity, as well as the possibility of a dynamical nature for the late-time acceleration, led to the appearance of various extensions and modifications. In general these belong to two classes. In the first class one maintains general relativity as the underlying gravitational theory but adds extra components, such as the dark energy sectors [3, 4]. In the second class one constructs modified theories of gravity, which possess general relativity as a particular limit but which in general provide the necessary extra degree(s) of freedom that can drive the universe acceleration [5–7].

The last years there have appeared an additional motivation in favour of extensions/modifications of the concordance cosmology, namely the need to incorporate tensions such as the $H_0$ and $\sigma_8$ ones. The former arises from the fact that the Planck collaboration estimation for the present day cosmic expansion rate is $H_0 = (67.27 \pm 0.60) \text{ km/s/Mpc}$ [8], which is in tension at about 4.4σ with the 2019 SH0ES collaboration (R19) direct measurement, i.e. $H_0 = (74.03 \pm 1.42) \text{ km/s/Mpc}$, obtained using the Hubble Space Telescope observations of 70 long-period Cepheids in the Large Magellanic Cloud [9] (note that combination with gravitational lensing and time-delay data increases the deviation at 5.3σ [10]). Additionally, the $\sigma_8$ tension is related to the parameter which quantifies the matter clustering within spheres of $8h^{-1}\text{Mpc}$ radius, and the possible deviation between the Cosmic Microwave Background (CMB) estimation [8] and the SDSS/BOSS measurement [11–13]. If these tensions are not a result of unknown systematics, which at least concerning the $H_0$ one seems progressively less possible to be the case, then one should indeed seek for alleviation in extensions of the standard lore of cosmology.

In principle one has two main directions to alleviate the $H_0$ tension. On one hand he could alter the universe content and interactions while maintaining general relativity as the gravitational theory [14–50], and on the other hand he could seek for a solution in modified gravity. Since the second direction maintains the advantages that modified gravities bring related to renormalizability and early- and late-time acceleration, it might be preferable. Furthermore, since the $H_0$ tension implies that the universe expands faster than what ΛCDM cosmology predicts, in order to alleviate it one should seek for a modified gravity that qualitatively leads to “less gravitational power” at intermediate and late times. Hence, during the last years many models of modified gravity have been proposed as candidates for the potential alleviation of the $H_0$ tension [51–76].

In this work we are interested in alleviating the $H_0$ tension in the framework of Horndeski gravity. Horndeski gravity [77], which is equivalent to generalized Galileon...
theory [78–80], is the most general four-dimensional scalar-tensor theory with one propagating scalar degree of freedom, that has second-order field equations and thus is free from Ostrogradski instabilities [81]. Hence, by choosing suitable sub-classes of the theory we can obtain a cosmological behavior that is almost identical with that of ΛCDM at early times, but which at intermediate times deviates from it due to the weakening of the gravitational interaction, and thus alleviating the tension (see also [82, 83] for a different approach on the problem using cubic covariant Galileon formulation).

The plan of the work is the following: In Section II we present Horndeski gravity, providing the background cosmological equations as well as the conditions for pathologies absence at the perturbation level. In Section III we construct specific sub-classes of Horndeski gravity that can alleviate the $\mathcal{R}_0$ tension, we compare them to ΛCDM and we confront them with Supernovae type Ia (SNIa) and Cosmic Chronometer (CC) data. Finally, in Section IV we give a summary of the results and we conclude.

II. HORNDESKI GRAVITY

In this section we briefly review Horndeski gravity, or equivalently generalized Galileon theory. We first give the corresponding general action and applying it in a cosmological framework we extract the background Friedmann equations. Additionally, we give the perturbation equations around such background, and we provide the conditions for the absence of instabilities.

The most general Lagrangian with one scalar degree of freedom coupled to curvature terms, with second-order field equations is [77, 84, 85]

$$\mathcal{L} = \sum_{i=2}^{5} \mathcal{L}_i,$$

with

$$\mathcal{L}_2 = K(\phi, X),$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box \phi,$$

$$\mathcal{L}_4 = G_4(\phi, X) R + G_{4, X} \left[ (\Box \phi)^2 - (\nabla_\mu \nabla_\nu \phi) (\nabla^\mu \nabla^\nu \phi) \right],$$

$$\mathcal{L}_5 = G_5(\phi, X) G_{\mu \nu} (\nabla^\mu \nabla^\nu \phi) - \frac{1}{6} G_{5, X} \left[ (\Box \phi)^3 - 3(\Box \phi) (\nabla_\mu \nabla_\nu \phi) (\nabla^\mu \nabla^\nu \phi) + 2(\nabla^\alpha \nabla^\beta \phi) (\nabla_\alpha \nabla_\beta \phi) (\nabla_\gamma \nabla_\delta \phi) \right].$$

In the above expressions $R$ is the Ricci scalar and $G_{\mu \nu}$ the Einstein tensor, while the functions $K$ and $G_i$ ($i = 3, 4, 5$) depend on the scalar field $\phi$ and its kinetic energy $X = -\phi^{\prime \prime} \phi \partial_i \phi / 2$. Moreover, $G_{i, X}$ and $G_{i, \phi}$ ($i = 3, 4, 5$) denote the partial derivatives of $G_i$ in terms of $X$ and $\phi$, i.e. $G_{i, X} \equiv \partial G_i / \partial X$ and $G_{i, \phi} \equiv \partial G_i / \partial \phi$. Hence, the total action of the theory will be

$$S = \int d^4x \sqrt{-g} \left( \mathcal{L} + \mathcal{L}_m \right),$$

where $g$ is the metric determinant, and $\mathcal{L}_m$ accounts for the matter content of the universe, which corresponds to a perfect fluid with energy density $\rho_m$ and pressure $p_m$.

We consider an expanding Universe described by a flat homogeneous and isotropic Friedmann-Robertson-Walker (FRW) geometry with metric

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j,$$

with $a(t)$ the scale factor. Varying the action (6) with respect to the metric, and imposing the above FRW form we obtain the two generalized Friedmann equations:

$$2XK_{,X} - K + 6X\phi H G_{3, X} - 2XG_{3, \phi} - 6H^2 G_4 + 24H^2 X( G_{4, X} + XG_{4, XX} ) - 12HX\phi G_{4, \phi X}$$

$$- 6H\phi G_{4, \phi} + 2H^3 \phi (5G_{5, X} + 2XG_{5, XX})$$

$$- 6H^2 X(3G_{5, \phi} + 2XG_{5, \phi X}) = -\rho_m,$$

(8)

$$K \equiv 2X(3, \phi + \phi G_{3, X}) + 2(3H^2 + 2H) G_4$$

$$- 8HXG_{4, X} - 12H^2 XG_{4, XX} - 4H^3 XG_{4, X}$$

$$- 8HXG_{4, XX} + 2(\phi + 2H\phi) \rho_{4, \phi} + 4XG_{4, \phi \phi}$$

$$+ 4X(\phi - 2H\phi) G_{4, \phi X} - 4H^2 X^2 \phi G_{5, XX}$$

$$- 2X(2\phi^3 + 2H\phi + 3H^2 \phi) G_{5, X}$$

$$+ 4HX(\phi X - HX) G_{5, \phi X} + 4HX\phi G_{5, \phi \phi}$$

$$+ 2(2H + HX + H^2) X G_{5, \phi} = -p_m,$$

(9)

where dots mark derivatives with respect to $t$, and where we have defined the Hubble parameter $H \equiv \dot{a} / a$. Additionally, varying (6) with respect to $\phi(t)$ leads to its equation of motion, namely

$$\frac{1}{a^3} \frac{d}{dt} (a^3 J) = P_\phi,$$

(10)

where

$$J \equiv \dot{\phi} K_{,X} + 6HXG_{3, X} - 2\dot{\phi} G_{3, \phi} - 12HXG_{4, \phi \phi}$$

$$+ 6H^2 \phi (G_{4, X} + 2XG_{4, XX})$$

$$+ 2H^3 X(3G_{5, X} + 2XG_{5, XX})$$

$$+ 6H^2 \phi (G_{5, \phi} + XG_{5, \phi X}),$$

(11)

$$P_\phi \equiv K_{, \phi} - 2X(3G_{3, \phi} + \phi G_{3, X})$$

$$+ 6(2H^2 + H) G_{4, \phi} + 6H(\phi + 2HX) G_{4, \phi X}$$

$$- 6H^2 X G_{5, \phi} + 2H^3 X \phi G_{5, \phi \phi},$$

(12)

Finally, the system of equations closes by considering the matter conservation equation

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0.$$
Having obtained the background equations of motion, one can proceed to the investigation of perturbations [85–87]. In this work we are interested in the scalar perturbations, and specifically on the conditions of absence of ghosts and Laplacian instabilities, in order to ensure that our solutions are cosmologically viable. In particular, in order for Horndeski/generalized Galileon theory to be free from Laplacian instabilities associated with the scalar field propagation speed one should have [85]

\[ c_s^2 \equiv \frac{3(2w_1^2 w_2 H - w_1^2 w_4 + 4w_1 w_2 w_1 - 2w_1^2 w_2)}{w_1(4w_1 w_3 + 9w_2^2)} \geq 0. \]  

(14)

Similarly, for the absence of perturbative ghosts one should have [85]

\[ Q_S \equiv \frac{w_1(4w_1 w_3 + 9w_2^2)}{3w_2^2} > 0. \]  

(15)

In the above expressions we have set

\[ w_1 \equiv 2(G_4 - 2XG_{4,X}) - 2X(G_{5,X}\dot{\phi}H - G_{5,\phi}), \]  

(16)

\[ w_2 \equiv -2G_{3,X}X\dot{\phi} + 4G_4 H - 16X^2G_{4,X,X}H \]  

\[ + 4(\dot{\phi}G_{4,\phi}X - 4HG_{4,X})X + 2G_4 \dot{\phi} \]  

\[ + 8X^2 H G_{5,\phi,X} + 2H X (6G_{5,\phi} - 5G_{5,X}\dot{\phi} H) \]  

\[ - 4G_{5,X}X\dot{\phi} X^2 H^2, \]  

(17)

\[ w_3 \equiv 3X(K_{X} + 2X K_{X,X}) \]  

\[ + 6X \left( 3X \dot{\phi} H G_{3,XX} - G_{3,\phi,X}X - G_{3,\phi} + 6H \dot{\phi} G_{3,\phi} \right) \]  

\[ + 18H \left( 4H X^3 G_{4,XX,X} - 5X \dot{\phi} G_{4,\phi,X} + 7H G_{4,X}X \right) \]  

\[ - H G_{4} - G_{4,\phi} \dot{\phi} + 16H X^2 G_{4,XX,X} - 2X^2 \dot{\phi} G_{4,\phi,X} \]  

\[ + 6H^2 X \left( 2H \ddot{\phi} G_{5,XX} X^2 - 6X^2 G_{5,\phi,XX} - 18G_{5,\phi} \right) \]  

\[ + 13X H \dot{\phi} G_{5,XX} X - 27G_{5,\phi,XX} X + 15H \dot{\phi} G_{5,\phi} \right), \]  

(18)

\[ w_4 \equiv 2G_4 - 2X G_{5,\phi} - 2X G_{5,\phi}, \]  

(19)

We mention here that a negative sound speed square should be definitely avoided, however a sound speed square larger than one does not necessarily imply pathologies and acausal behavior [88, 89].

Lastly, we mention here that in Horndeski theories the gravitational-wave speed is in general different than 1, namely than the light speed. In particular, we have [85]

\[ c_T^2 \equiv \frac{w_4}{w_1} \geq 0, \]  

(20)

and as we can see from (16),(19) the \( G_5 \) terms may have an effect according to the cosmological evolution.

## III. ALLEVIATING THE \( H_0 \) TENSION

In the previous section we presented the cosmological equations in the framework of Horndeski/generalized Galileon gravity. In this section we desire to use particular sub-classes of the theory in order to obtain an alleviating of the \( H_0 \) tension. Our strategy is the following: since the simplest model in Horndeski cosmology is ΛCDM one, arising from \( G_4 = 1/(16\pi G) \), \( K = -2\Lambda = const \), and \( G_3 = G_5 = 0 \), we want to introduce deviations which will be negligible at high redshifts, in which CMB structure is formed, but that will play a role at low redshifts, in which direct Hubble measurements take place. In particular, since it is known that the terms depending on \( G_5 \) affect the friction term on the scalar field [90–97], we could consider \( G_5 \) functions depending only on the kinetic energy \( X \) in a way that their effect is negligible at high redshifts while being gradually important in a controlled way at low redshifts.

Having these in mind, in the following we will consider \( G_4 = 1/(16\pi G) \) and \( G_3 = 0 \), which are also the case in ΛCDM cosmology, we will impose a simple scalar field potential and standard kinetic term, hence \( K = -V(\phi) + X \), and we will consider the \( G_5 \) term to depend only on \( X \), namely \( G_5(\phi, X) = G_5(X) \). In this case, the Friedmann equations (8),(9) become

\[ H^2 = \frac{8\pi G}{3}(\rho_{DE} + \rho_m), \]  

(21)

\[ \dot{H} = -4\pi G (\rho_{DE} + p_{DE} + \rho_m + p_m). \]  

(22)

In these equations we have defined an effective dark energy sector with energy density and pressure respectively:

\[ \rho_{DE} = 2X - K + 2H^2 (\dot{\phi}(5G_{5,X} + 2XG_{5,XX})), \]  

(23)

\[ p_{DE} = K - 2XG_{5,X} \left( 2H^2 \ddot{\phi} + 2H \dot{\phi} + 3H^2 \dot{\phi} \right) \]  

\[ - 4H^2 X^2 \dot{\phi} G_{5,XX}, \]  

(24)

and thus the dark-energy equation-of-state parameter becomes

\[ w_{DE} \equiv \frac{p_{DE}}{\rho_{DE}}. \]  

(25)

Note that the scalar-field conservation equation (10) becomes simply

\[ \dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = 0. \]  

(26)

As we mentioned above we want to make our model to coincide with ΛCDM cosmology at high redshifts. Thus, it proves convenient to use the redshift \( z = -1 + a_0/a \) as the independent variable, fixing the current scale factor \( a_0 = 1 \) (therefore \( H = -1(z)H(z)H(z) \) where primes denote derivatives with respect to \( z \)). Introducing as usual the matter density parameter through \( \Omega_m = \frac{8\pi G \rho_m}{3H^2} \), we can express the Hubble function in the case of ΛCDM cosmology as

\[ H_{\Lambda\text{CDM}}(z) \equiv H_0 \sqrt{\Omega_{m0}(1 + z)^3 + 1 - \Omega_{m0}}, \]  

(27)

with \( H_0 \) the Hubble parameter at present and \( \Omega_{m0} \) the present value of the matter density parameter.
Hence, we want to suitably choose \( G_5(X) \) forms in order for the \( H(z) \) obtained from (21), (23) to coincide with \( H_{\Lambda \text{CDM}}(z) \) of (27) at \( z = z_{\text{CMB}} \approx 1100 \), namely \( H(z \rightarrow z_{\text{CMB}}) \approx H_{\Lambda \text{CDM}}(z \rightarrow z_{\text{CMB}}) \), but give \( H(z \rightarrow 0) > H_{\Lambda \text{CDM}}(z \rightarrow 0) \). In the following subsections we will consider two sub-cases of the \( G_5(X) \) term separately. For simplicity, from now on we focus on the dust matter case, i.e. we impose \( p_m = 0 \), while for the scalar-potential without loss of generality we choose \( K = -V_0 \phi + X \).

### A. Model I: \( G_5(X) = \xi X^2 \)

The first model we consider is the one with \( G_5(X) = \xi X^2 \), i.e \( G_5 \) has a quadratic dependence on the field’s kinetic energy. In this case \((23) \) and \((24) \) respectively become

\[
\rho_{DE} = \frac{\dot{\phi}^2}{2} + V_0 \phi + 7 \xi H^3 \phi^5, \tag{28}
\]

\[
p_{DE} = \frac{\dot{\phi}^2}{2} - V_0 \phi - \xi \phi^4 \left( 2 H^3 \phi + 2 H \dot{H} \phi + 5 H^2 \dot{\phi} \right). \tag{29}
\]

As described above we chose the model parameter \( V_0 \) and the initial conditions for the scalar field in order to obtain \( H(z_{\text{CMB}}) = H_{\Lambda \text{CDM}}(z_{\text{CMB}}) \) and \( \Omega_{m0} = 0.31 \) in agreement with [98], and we leave \( \xi \) as the parameter that determines the late-time deviation from \( \Lambda \text{CDM} \) cosmology.

![Figure 1](image.png)

**Figure 1:** The normalized \( H(z)/(1 + z)^{3/2} \) in units of \( \text{km/s/Mpc} \) as a function of the redshift, for \( \Lambda \text{CDM} \) cosmology (black - solid) and for Model I with \( V_0 = 0.08 \) and with \( G_5(X) = \xi X^2 \), for \( \xi = 1.5 \) (green - dotted), \( \xi = 1.3 \) (red - dashed) and \( \xi = 1 \) (blue - dashed-dotted), in \( H_0 \) units. We have imposed \( \Omega_{m0} \approx 0.31 \).

In Fig. 1 we depict the normalized \( H(z)/(1 + z)^{3/2} \) as a function of the redshift, for \( \Lambda \text{CDM} \) cosmology and for our model with various choices of \( \xi \). As we can see, indeed our model coincides with \( \Lambda \text{CDM} \) cosmology at high and intermediate redshifts, while at small redshifts the proposed Model I gives higher values. In particular, the present-day value \( H_0 \) depends on the model parameter \( \xi \) and it can be around \( H_0 \approx 74 \text{ km/s/Mpc} \) for \( \xi = 1.3 \) (in \( H_0 \) units, i.e. where the \( \Lambda \text{CDM} H_0 \) is 1). Specifically, the tension can be alleviated at \( 3 \sigma \) if \( 1.2 < \xi < 1.7 \). Hence, we can see that this particular sub-class of Horndeski/generalized Galileon gravity can alleviate the \( H_0 \) tension due to the effect of the kinetic-energy-dependent \( G_5 \) term. Specifically, at early times the field’s kinetic term is negligible and hence the \( G_5(X) \) terms do not introduce any deviation from \( \Lambda \text{CDM} \) cosmology, however as time passes they increase in a controlled and suitable way in order to make the Hubble function, and thus \( H_0 \), too, to increase. Note that, since \( H_0 \approx 10^{-61} \) in Planck units, the fact that \( V_0 = 0.08 \) and \( \xi = 1.3 \) in \( H_0 \) units implies that \( V_0 \approx 0.5 \times 10^{-61} \) and \( \xi \approx 8 \times 10^{365} \) in Planck units (in Planck units we obtain characteristic values of \( \phi \) and \( \dot{\phi} \) around \( 10^{-60} \)), which is the expected scale for the quantities of a scenario that describes the Universe acceleration (\( \xi \) has dimensions of \( [M]^{-9} \) i.e. \( \xi^{1/9} \sim 10^{60} \text{GeV}^{-1} \)).

Let us make a comment here on the specific mechanism behind the tension alleviation. In general, the alleviation of the \( H_0 \) tension or/and the \( \sigma_8 \) tension, is a complex issue, and it usually arises as a collective result of many effects. If one remains in the class of late-time modification (without examining possible early-time solutions, as it is the aim of this work) then one efficient mechanism is to have \( w_{DE} < -1 \) at some recent redshift, since a “phantom” dark energy implies “faster” expansion. Nevertheless, this requirement is efficient but not necessary, since the decrease of the effective Newton’s constant at intermediate redshifts is also an efficient mechanism [71, 99] (see also the discussion in the recent review [100]), since “weaker” gravity implies “faster” expansion. In the models proposed in the present work, although many terms are involved, the alleviation of the tension arises from such a decreased effective Newton’s constant, brought about in turn by the friction term. In particular, in Horndeski theories we have [101, 102]
we depict the corresponding solution, by investigating the sound speed square $c_s^2$ and the imposed initial conditions we can bring it to be almost 1 during the whole evolution. Additionally, the value of $Q_S$ for large $z$ is close to zero, however one can verify that it remains always positive, and for $z \gg 1$ (i.e. for $z \to 1000$) it is around 0.005.

Lastly, in Fig. 5 we depict the corresponding gravitational-wave speed square $c_T^2$ as a function of the redshift, for Model I with $V_0 = 0.08$ and with $\xi = 1.3$ in $H_0$ units.

Finally, we examine the stability of the obtained solution, by investigating the sound speed square $c_s^2$ given in (14) and the quantity $Q_S$ given in (15). In Fig. 4 we depict their evolution as a function of the redshift for the background solution given above. As we can see, the stability conditions are always satisfied, and hence the obtained solutions are free from ghost and Laplacian instabilities (we mention that for a general $G_5 \neq 0$ the $c_s^2$ is not identically 1, however for the chosen $G_5(X)$ with the chosen $\xi$ and the imposed initial conditions we can bring it to be almost 1 during the whole evolution). Additionally, the value of $Q_S$ for large $z$ is close to zero, however one can verify that it remains always positive, and for $z \gg 1$ (i.e. for $z \to 1000$) it is around 0.005.

The gravitational wave speed is very close to 1. Specifically, the numerical difference between $c_T$ and unity for small $z$ ($z < 0.5$) is less than $\lesssim 10^{-15}$, while for $z > 1$ it becomes larger, namely around $5 \times 10^{-10}$. However, this is not in contradiction with LIGO-VIRGO bounds [104], since both GW170817 and GW190425 neutron-star - neutron-star merger events are at very close distances, namely at redshifts around 0.01, and thus strictly speaking the observational verification of the gravitational-wave speed is only for very low redshifts (and also for the specific frequency range of LIGO-VIRGO). In summary, the present model is able to pass the LIGO-VIRGO bounds, however one could still try to construct Horndeski models that can alleviate the tension but have $c_T$ even more close to 1.
B. Model II: $G_5(X) = \lambda X^4$

The second model we consider is the one with $G_5(X) = \lambda X^4$, i.e. $G_5$ has a quartic dependence on the field’s kinetic energy. In this case (23) and (24) respectively become

$$\rho_{DE} = \frac{\dot{\phi}^2}{2} + V_0\phi + \frac{11}{2}\lambda H^3\dot{\phi}^2,$$

$$p_{DE} = \frac{\dot{\phi}^2}{2} - V_0\phi - \frac{\lambda\dot{\phi}^2}{2} \left(2H^3\dot{\phi} + 2H\ddot{\phi} + 9H^2\dot{\phi}\right).$$

In Fig. 6 we present $H(z)/(1 + z)^{3/2}$ as a function of the redshift for $\Lambda$CDM scenario and for our model with various choices of $\lambda$. The two models coincide at high and intermediate redshifts, but at small redshifts Model II gives higher value, while $H_0$ depends on the model parameter $\lambda$. Specifically, it can be around $H_0 \approx 74$ km/s/Mpc for $\lambda = 1$ in $H_0$ units, and in general the tension can be alleviated at $3\sigma$ if $0.5 < \lambda < 1.2$ (in Planck units we have $V_0 \sim 10^{-61}$ and $\lambda \sim 10^{510}$, and since $\lambda$ has dimensions of $[M]^{-17}$ we acquire $\lambda^{1/17} \sim 10^{300} GeV^{-1}$). Thus, we observe that this kinetic-dependent sub-class of Horndeski/generalized Galileon gravity can also alleviate the $H_0$ tension, since the $G_5(X)$ term that controls the friction term in the Friedmann equation is negligible at high redshifts, while it increases and plays a role at low redshifts. Lastly, we mention that the behavior of $\sigma^2$ and $Q_S$ is similar to the one of Fig. 4, i.e. the scenario at hand is free from ghost and Laplacian instabilities.

We close this section by confronting the models at hand with Supernovae type Ia (SNIa) and Cosmic Chronometer cosmological data. In particular, concerning SNIa it is known that

$$2.5 \log \left[ \frac{L}{l(z)} \right] = \mu \equiv m(z) - M = 5 \log \left[ \frac{d_{L}(z)_{\text{obs}}}{Mpc} \right] + 25,$$

with $l(z)$ and $m(z)$ the apparent luminosity and apparent magnitude, and $L$ and $M$ the absolute luminosity and magnitude, respectively, while $d_{L}(z)_{\text{obs}}$ is the luminosity distance. On the other hand, the theoretical value of the luminosity distance is

$$d_{L}(z)_{th} = (1 + z) \int^{z} \frac{dz'}{H(z')}.$$

Since we know the evolution of $H(z)$ in our models, as well as $H_{\Lambda\text{CDM}}(z)$, in Fig. 7 we depict the apparent minus absolute magnitude predicted theoretically for our models as well as $\Lambda$CDM cosmology, on top of the binned Pantheon sample SNIa data points from [105]. As we can see, the agreement is very good, and the proposed models have a slightly higher accelerating behavior, as expected. Additionally, the Cosmic Chronometer (CC) datasets is based on the measurements of $H(z)$ using the relative ages of massive and passively evolving galaxies and the corresponding estimation of $dz/dt$ [106]. In Fig. 8 we confront the theoretically predicted $H(z)$ behavior, as well as the one of $\Lambda$CDM cosmology, with the $H(z)$ CC data from [107] at $3\sigma$ confidence level. The agreement is very good, and the $H(z)$ evolution of the proposed models lies within the prediction of the direct measurements of the $H(z)$ from the CC data, having again a slightly higher accelerating behavior at low redshifts, for the parameter sets $\{\Omega_{m0}, V_0, \xi\} = \{0.31, 0.08, 1.3\}$ and $\{\Omega_{m0}, V_0, \lambda\} = \{0.31, 1, 1\}$.

In summary, there exist regions of the free parameters that are able to reproduce the observed Hubble function evolution and at late times potentially alleviate the $H_0$ tension, implying also the viability of the examined
models. Definitely, in order to conclude on whether a specific model can alleviate the cosmological tensions, a full confrontation with all observational datasets is required. The present work is just a first approach on the subject, in order to reveal the mechanism that is able to lift the present Hubble parameter value compared to ΛCDM scenario (following the general requirements of [99, 100]). The detailed verification of viability for the proposed models and their results, applying likelihood analysis and model selection criteria on full cosmological datasets, lies beyond the scope of the current work and will be presented in a forthcoming project.

\[ \lambda = 1 \]

Figure 8: The \( H(z) \) in units of Kms/Mpc as a function of the redshift, for Model I with \( V_0 = 0.08 \) and with \( \xi = 1.3 \) (red-dashed-dotted), and for Model II with \( V_0 = 1 \) and with \( \lambda = 1 \) (orange-dashed), in \( H_0 \) units, on top of the Cosmic Chronometers data points from [107] at 3σ confidence level. For comparison we depict the ΛCDM curve (black - solid) too. We have imposed \( \Omega_{\text{m}0} \approx 0.31 \).

IV. CONCLUSIONS

The \( H_0 \) tension, unless it is caused by some unknown systematics or is related with some basic data-handling error, may provide a strong indication towards the modification of Standard Model of cosmology. In the present work we investigated the possibility for its alleviation through Horndeski/generalized Galileon gravity.

In particular, knowing that the terms depending on \( G_5 \) control the friction term in the Friedmann equation, we constructed specific sub-classes depending only on the field’s kinetic energy \( X \). Since the kinetic energy is small at high redshifts, namely at redshifts which affected the CMB structure, the deviations from ΛCDM cosmology are negligible, however as time passes \( X \) increases in a controlled way and it leads to a decrease in the effective Newton’s constant, and thus at low redshifts \( H(z) \) acquires increased values.

We considered two Models, one with quadratic and one with quartic dependence on the field’s kinetic energy. In both cases we showed that at high and intermediate redshifts the Hubble function behaves identically to that of ΛCDM scenario, however at low redshifts it acquires increased values, resulting to \( H_0 \approx 74 \) km/s/Mpc for particular parameter choices. Hence, these sub-classes of Horndeski/generalized Galileon gravity can alleviate the \( H_0 \) tension. We mention that the above behavior is obtained without a tuning in the initial conditions of \( \phi \) and \( \dot{\phi} \) (we do not have much freedom since we set \( \Omega_{DE0} \approx 0.7 \) and moreover we desire to have \( w_{DE}(z = 0) \) around \(-1\)), however the amount of tuning comes mainly in the selection of the functions \( G_i \)’s and \( K(\phi, X) \), since only a small subclass of them can fulfill the above requirements.

As a self-consistency test we examined the behavior of scalar metric perturbations, showing that the conditions for absence of ghost and Laplacian instabilities are fulfilled throughout the evolution, and hence that the proposed solutions are stable and free from pathologies. Finally, for completeness we confronted the proposed models with SNIa and Cosmic Chronometer data, as a first evidence that they are viable and in agreement with observations.

In summary, in this pilot project we showed that the \( H_0 \) tension can be alleviated in the modified gravity framework of Horndeski/generalized Galileon theory, due to the weakening of gravity at low redshifts by the terms depending on the scalar field’s kinetic energy. Definitely, in order to obtain a more concrete verification of the above result one should perform a full observational confrontation, using the full datasets, namely data from SNIa, Baryonic Acoustic Oscillations (BAO), Cosmic Chronometers (CC), Redshift Space Distortion (RSD), Cosmic Microwave Background (CMB) shift temperature and polarization, and fR6 observations, performing also the comparison to ΛCDM concordance scenario using various information criteria. Such a full and detailed analysis, lies beyond the scope of this first work, and it is left for a future project.

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