The longstanding problem of the Hall resistivity $\rho_{xy}$ in the Hall insulator phase is addressed using four-lead Chalker-Coddington networks. Electron interaction effects are introduced via a finite dephasing length. In the quantum coherent regime, we find that $\rho_{xy}$ scales with the longitudinal resistivity $\rho_{xx}$, and they both diverge exponentially with dephasing length. In the Ohmic limit, (dephasing length shorter than Hall puddles’ size), $\rho_{xy}$ remains quantized and independent of $\rho_{xx}$.

This suggests a new experimental probe for dephasing processes.
where the vectors \( \Lambda^t \) and \( \Lambda^r \) project out the zero eigenvector \((1, 1, 1, 1)\) of the matrix \( 1 - T \) (matrix \( S \) is unitary). It is easy to show that the field-antisymmetrized Hall resistance is \( R_{xy} = (R_{xy} - R_{yy})/2 \). Numerically, we calculated the \( L \times L \) transfer matrix relating the amplitudes on \( L \) open wires at the left and right sides of the sample (see Fig. 1). We used the transfer-matrix technique with intermediate orthogonalizations \[22\] to reduce numerical errors for large systems. Then the closed edges were “linked” by eliminating the corresponding pairs of incoming and outgoing amplitudes. This resulted in a reduced \( 4 \times 4 \) transfer matrix, with which the full scattering matrix \( S_{ij} \) was computed. The numerical accuracy was controlled by checking the unitarity of \( S_{ij} \), and by test runs at quadruple accuracy.

To obtain the distributions of \( R_{xx} \) and \( R_{xy} \) for different values of \( \theta \) and the system size \( L \), we repeated each calculation up to \( 10^6 \) times, taking \((\tilde{R}_{xx}, \tilde{R}_{yy})\) as two different members which have the same \( R_{xy} \). In the insulating phase \( \theta > \theta_c, R_{xx} \) has a very wide, resembling log-normal, distribution over several decades, as generally expected in a localized insulator phase \[15\]. The Hall resistance \( R_{xy} \) is seen to have a much narrower distribution, but with mean and variance also increasing exponentially with the size of the system in the insulating phase.

Phase-incoherent network model. In the opposite limit we consider a network where inelastic mechanisms completely destroy quantum interference between different tunnel junctions. In this case different tunneling events happen independently, and the incoming and outgoing currents are related at each junction, \[ \begin{align*}
I^{\text{out}}_r &= \sum_{r'=p,q} |s^{(pq)}_{rr'}|^2 I^{\text{in}}_{r'}, \\
V^{\text{dis}}_{pq} &= R_{pq} I_{pq}, \\
V^H_{pq} &= \text{sign}(B) \left( h/\nu e^2 \right) I_{pq}.
\end{align*} \] where the tunneling matrix elements \( s^{(pq)} \) are given by Eq. \[5\]. According to Eq. \[4\], local chemical potentials at each edge are proportional to the corresponding currents. Thus, the dissipative and Hall voltages at each junction are related to the tunneling current \( I_{pq} \) as

\[ V^{\text{dis}}_{pq} = R_{pq} I_{pq}, \quad V^H_{pq} = \text{sign}(B) \left( h/\nu e^2 \right) I_{pq}. \]
lengths are consistent with the critical exponent of both correlation lengths as \( \theta \rightarrow \theta_c \). This form for \( \xi(\theta) \sim (\theta - \theta_c)^{-7/3} \) is consistent with previous studies. The fact that the correlation length for \( \rho_{xy} \) also diverges with the same exponent implies that the transition is actually governed by a single length scale \( \xi(\theta) \), and asymptotically \( \rho_{xy} \sim (\rho_{xy})^k \rightarrow \infty \) with \( k \approx 0.32 - 0.35 \) (Fig. 3). This divergence, one of the primary results of this paper, contradicts the expectations of finite Hall resistivity in the insulator at \( L \rightarrow \infty \). Our results agree with the conclusion of Ref. [15], although here we have used a different model, applicable for transport in quantizing magnetic fields.

Fig. 3 also shows \( \rho_{xy} \) in the metallic QH phase for \( \theta > \theta_c \). In this phase, as expected, \( \rho_{xy}(L) \) approaches the quantized value at large system sizes. We did not attempt to determine the corresponding correlation length since our geometry has narrow leads (see Fig. 3). We did, however, check that our results are not limited to systems with narrow leads by making a limited set of runs for a special self-dual network geometry, which remains identical under the interchange of QH and insulating regions and the replacement \( \theta \rightarrow 2\theta_c - \theta \).

We also attempted to suppress the quantum interference by calculating the ensemble averaged \( \langle T_{ij} \rangle \), which is formally equivalent to considering the same non-interacting system at very high temperatures. This averaging resulted in both Hall and longitudinal resistance of insulating phase much smaller than the average quantum values. However, the specific values of these resistances differed significantly from the values obtained numerically for the phase-incoherent networks of identical geometry; particularly, the Hall resistance was not quantized, with the offset increasing into the insulating phase. This demonstrates that at \( T > 0 \), without inelastic scattering quantum interference cannot be completely suppressed.

In the fractional Hall effect regime, electron-electron interaction effects are primarily threefold: (i) Stabilization of fractional \( \nu < 1 \) QH phases in the puddles, (ii) Renormalization of inter-puddle electron tunneling rates, and (iii) Dephasing of charge carriers by inelastic scattering. At low temperatures the second effect is expected to be finite, since infrared divergence of the tunneling probability is consistent with the critical exponent of both correlation lengths as \( \theta \rightarrow \theta_c \). This form for \( \xi(\theta) \sim (\theta - \theta_c)^{-7/3} \) is consistent with previous studies. The fact that the correlation

\[ \rho(L) \sim A_0 L^\gamma \exp \left[ L / \xi(\theta) \right], \]  

where \( \gamma \) is a \( \theta \)-independent exponent determined by the geometry of the system, and \( \xi(\theta) \) is the localization length (both quantities are defined separately for \( \rho_{xx} \) and \( \rho_{xy} \)). As illustrated in the inset in Fig. 2 the divergence of both correlation lengths as \( \theta \rightarrow \theta_c \) is consistent with \( \xi(\theta) \sim (\theta - \theta_c)^{-7/3} \). This form for \( \xi_{xx} \) is in agreement with previous studies.
nelling amplitudes is cut off by the finite size of the puddles. In this regime, the effective edge state transport theory for the fractional and integer cases is identical, up to factors of $\nu$ in the chemical potential relations. Thus it is legitimate to use the CC model as a model of coherent quantum transport on edges of fractional QH puddles. However the dephasing mechanism and determination of $l_\varphi$ remains an interesting open problem, for both integer and fractional cases.

**Quantum critical point and dephasing.** In a vicinity of the true quantum critical point, the effective dephasing length \( l_\varphi \) should diverge at small temperatures as

\[
l_\varphi(T) \sim T^{-1/z}.
\]

(9)

Experimentally, resistivity and non linear resistivity data at transitions between plateaux have been collapsed onto universal curves using $z \approx 1.0$ (and independently determined $\nu_\xi \approx 2.4$) \cite{23,24}. Based on our results, at a quantum critical point, and in the absence of additional phase-breaking mechanisms, one would also expect the Hall resistivity to diverge as

\[
\rho_{xy} \sim (h/e^2) \exp \left[ kT^{-1/z}(B-B_c)^{7/3} \right].
\]

(10)

Experiments, however, have reported a constant or weakly $B$-dependent Hall resistivity on the insulating side of the transition. For similar samples, resistivity saturation at low temperatures has been reported \cite{25}. Evidently, both effects are inconsistent with a true zero-temperature quantum Hall to insulator transition, characterized by a diverging dephasing length \( l_\varphi \).

We conclude that a nearly quantized Hall resistivity indicates a strongly dephased regime where, \( i.e., l_\varphi \leq l_V \). In contrast, the longitudinal resistivity in this regime is expected to diverge at large fields as \cite{19},

\[
\rho_{xx}(B) \sim (h/e)^2 \exp \left[ \nu \left( l_V/l \right)^2 (B/B_c - 1) \right],
\]

(11)

where $l$ is the Landau length. This expression allows an independent estimation of $l_V$, and an upper bound on the dephasing length in the limit of small temperatures. Experimentally, the interplay between $l_V$, the scale of the long-range potential fluctuations, and the phase breaking length $l_\varphi$ was noticed in Ref. \cite{23}.

It is not yet clear what mechanism can explain zero temperature finite resistivity in disordered QH systems, or why dephasing seems to be more pronounced in some particular experiments. One possible source might be a coupling of the edge excitations to nearby domains of compressible $\nu = 1/2$ phase.

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