Estimation of the GNSS signal time-of-arrival in the presence of a Gaussian interference located in space

A P Rachitskaya and A M Oshuev

Peter the Great St. Petersburg Polytechnic University, Polytechnicheskaya 29, St. Petersburg 195251, Russia

E-mail: melikhova.a@spbstu.ru

Abstract. The problem of low positioning accuracy in the presence of interference is discussed in this work. As a way to solve this problem, spatial-temporal signal processing (STSP) is chosen when maximum likelihood (ML) optimisation criterion is used. The paper assumes that interference is a Gaussian random process with uniform (in the channel band) spectral noise density, and the additive components in each of the receiving channels are independent implementations of additive white Gaussian noise (AWGN). The commonly known form of likelihood function is transformed in the way to exclude interfering parameters of signal of interest. As a theoretical result, ML-estimator of time-of-arrival was obtained. The results are supported by a statistical simulation so that the efficiency of synthesized ML algorithm is shown. As implications for practice, the structure of navigation receiver implementation is proposed at the end of discussion.

1. Introduction
The presence of additive interference is one of the main problems of signal reception in radio systems [1-4]. At the same time, when we say about sources of interference which are located in space and have different angles-of-arrivals that signals of interest have, the most efficient way to deal with the problem of interference is using of spatial-temporal signal processing (STSP) [5-7].

The efficiency in current scope is the value of the ratio of the maximum allowable value of interference-to-signal ratio \( \gamma = \frac{P_f}{P_i} \) when the required accuracy of measurements is provided (vector \( \lambda \) of informative parameters). The use of STSP assumes involving an antenna array (AA) [8-10] and algorithms optimized in accordance with a particular criterion [11-13]. To achieve a high level of efficiency, the apparatus of statistical decision theory is usually used [15]. In this case the maximum likelihood (ML) is used as the optimisation criterion [15, 10]. The use of the ML criterion involves the calculation of the likelihood functions (LF) for the analyzed processes on the elements of the antenna array [10].

In the commonly known papers [11], the analytical form of the ML estimation algorithm contains interfering parameters \( \mu \), so that we have a case of composite hypotheses testing problem [15]. When the distribution of interfering parameters is unknown, their exclusion from the LF should be made in accordance to the generalized maximum likelihood criterion, which assumes the maximization of the LF by the unknown parameters [15]. Such maximization was dismissed in the mentioned papers, so that the given algorithm inapplicable in those practical situations when, for example, the initial phase and amplitude coefficient of the signal of interest are unknown.
Let’s perform an analytical maximization of the LF for processes on the elements of AA by the interfering parameters of the signal (initial phase and amplitude coefficient), and estimate the efficiency of the synthesized ML algorithm by statistical modeling. To obtain upper bounds for the effectiveness of such approach, we first consider the case where the angles-of-arrival of both signal and interfering are precisely known.

2. Theoretical calculations
The construction of the LF assumes involving the type and distribution of interference \( n(t) \), distribution of additive noise and the structure of signal of interest \( s(t, \lambda, \mu) \), where \( \mu \) is the vector of unknown (interfering) parameters of the useful signal and \( \lambda \) is the vector of information parameters. In case when the interference is a Gaussian random process with a uniform (in the channel band) spectral noise density, and the additive components in each of the receiving channels of the AA are independent implementations of additive white Gaussian noise (AWGN), the LF \( W(x(t) | \lambda, \mu) \) of vector \( x(t) \) of processes in channels of AA can be written as [11]:

\[
W(x(t) | \lambda, \mu) = \frac{\exp\left\{ -\text{Re}\left[ \begin{bmatrix} B^*(t_i) V(t_i)^{-1} B(t_i) \end{bmatrix} \right] \right\}}{(2\pi)^d |V|},
\]

where \( B(t) = F_s(t) - H F_i(t, \lambda, \mu) \); \( F_s(t) \) – vector of complex envelopes of process \( x(t) \) on elements of AA; \( H \) – signals steering vector [5]; \( n_0 \) – a column vector whose elements are the AWGN components of internal noise in the receive channels of AA; \( V \) – interference and noise covariance matrix; \( T_a \) – interval of analysis and \( \text{Re}[\cdot] \) – operator which returns the real part of the complex value.

3. General form of solving the likelihood equation
In this work we consider the case of using a wideband phase-manipulated signal \( s(t, \lambda, \mu) \), the modulation scheme of which \( \phi(t) \) is determined by a known pseudo-random sequence \( C(t) \). In many practical situations signal time delay \( \tau \) acts as an informative parameter when the initial phase \( \psi_0 \), the amplitude coefficient \( \alpha \) and the Doppler carrier frequency shift \( \Delta \omega_d \) act as interfering. The complex envelope of such a signal can be represented as follows:

\[
F_s(t) = \alpha \exp\left\{ j \phi(t - \tau) \right\} \exp\left\{ j \Delta \omega_d t \right\} \exp\left\{ j \psi_0 \right\},
\]

where \( \omega_0 \) – central frequency of emitted signal.

Assuming that the distribution of interfering parameters \( \mu = \{ \alpha, \psi, \Delta \omega_d \} \) is unknown, we use the generalized maximum likelihood criterion, which excludes these parameters from the likelihood function by replacing them with the general form of solving the likelihood equation to estimate these parameters [16].

Under the assumption of a known signal and interfering angle-of-arrival, after the corresponding mathematical transformations, the resulting form of the parameter estimation algorithm is as follows:
\begin{equation}
\hat{\tau} = \max_{\tau, \Delta \tau_0}^{-1} \left\{ \mathbf{H}^T \left( \mathbf{V}^{-1} \right) \left( \int_{(T)} F^* (t) C(t-\tau) e^{i\Delta \tau t} \, dt \right)^2 \right\}.
\end{equation}

4. Statistical simulation and algorithm efficiency

The efficiency of the synthesized STSP algorithm (3) can be measured during statistical simulation. We developed the MATLAB model which simulates the process of receiving and processing the phase-shift keyed signals of the global navigation satellite systems (GNSS) such as GPS L1. The model includes a navigation signal generator, a noise generator, simulators of receiving channels with additive white Gaussian noise in the signal band, and a block for the implementation of the STSP algorithms. The block diagram of the model is shown in figure 1.

![Figure 1. Block diagram of the MATLAB simulation model](image)

As a quantitative measure of the effectiveness of STSP algorithm, we selected the ratio of the maximum $\gamma$ when we use the STSP algorithm and when we do not, providing the required accuracy $\sigma$ of estimation of the signal time of arrival. The GPS signal of the L1 frequency band (1575.42 MHz) was selected as the navigation signal, in which the Gold code with a clock frequency of 1.023 MHz is used as the modulating sequence. When we simulated external interference, two types were used: a Gaussian wideband interference with a uniform power spectral density in the signal bandwidth, and a harmonic narrowband interference with a random initial phase and central frequency within the signal bandwidth. At the same time, an ideally calibrated three-element antenna array with a distance between the elements equal to the half of wavelength was used as an antenna array model.

5. Results and discussions

As a result of statistical modeling, the characteristics of the algorithm (3) were obtained depending on the interference to signal ratio $\gamma$. Fig. 2 illustrates the dependence between the estimation precision $\sigma$ and $\gamma$ when signal power $C_0$ to mean power spectral density $N_0$ ratio $C_0/N_0$ is equal to 35 dBHz, and Fig.3 illustrates the dependence of the same type when $C_0/N_0 = 45$ dBHz.

When interference-to-signal ratio is no more than 250 dB, ML-estimator provides an appropriate accuracy ($\sim 10^{-3}$ ms) of the time-of-arrival estimation. The results are obtained from known signal and interference source locations when antenna array is precisely calibrated.
Figure 2. Dependence between time-of-arrival estimation error and interference-to-signal ratio when $C_0/N_0 = 35$ dBHz, $M=3$ and the distance between them is half of the signals wavelength.

Figure 3. Dependence between time-of-arrival estimation error and interference-to-signal ratio when $C_0/N_0 = 45$ dBHz, $M=3$ and the distance between them is half of the signals wavelength.

Measured in accordance with the algorithm (3), signals times of arrival can be used to measure the coordinates of the objects. The block diagram of the navigation receiver implemented in accordance to the algorithm (3) is shown in Fig. 4. Such a receiver should include a multichannel analog part (radio receiver – RR) and a digital unit. Analog-to-digital converters (ADC) are used to convert an analog signal into a digital form. Complex envelop measurers (CEM) are then utilized to provide envelope estimation in digital form (2) and multi-channel processing in accordance with algorithm (3).

Figure 4. The structure of the multi-channel receiver with ML time-of-arrival estimator

6. Conclusion
When the interference-to-signal ratio is no more than 250 dB, the practical interest values $C_0/N_0$ ratio (35-45 dBHz) constructed form of ML-estimator implemented with 3-element AA provide an appropriate accuracy ($\sim 10^{-3}$ ms) of the time-of-arrival estimation. When GPS open access L1 signals are used and antenna array is precisely calibrated, the results are obtained from known signal and interference source locations.
Acknowledgments
The authors are grateful to Igor A. Tsikin for his help with the theoretical foundations, valuable consultations and comments.

References
[1] K Manolakis, W Xu and Ieee, "Time Synchronization for Multi-Link D2D/V2X Communication," IEEE Vehicular Technology Conference Proceedings, 2016
[2] Zavjalov S V, Tropkina I A and Mikhailov A S 2019 Effective choice of parameters of IR-UWB sensor network system J. Phys. Conf. Ser. 1236 012082
[3] Gelgor A and Gorlov A 2018 A performance of coded modulation based on optimal Faster-than-Nyquist signals 2017 IEEE Int. Black Sea Conf. Commun. Networking, BlackSeaCom 20172018-Janua 1–5
[4] Z Yao and M Q Lu, “Signal Multiplexing Techniques for GNSSs,” Ieee Signal Processing Magazine, vol. 34, no. 5, pp. 16-26, Sep, 2017
[5] H. L. Van Trees, Detection, estimation, and modulation theory, optimum array processing: John Wiley & Sons, 2004
[6] N Vagle, A Broumandan, A Jafarnia-Jahromi and G Lachapelle, “Performance analysis of GNSS multipath mitigation using antenna arrays,” The Journal of Global Positioning Systems, vol. 14, no. 1, pp. 4, November 04, 2016
[7] A Broumandan, A Jafarnia-Jahromi, S Daneshmand and G Lachapelle, “Overview of Spatial Processing Approaches for GNSS Structural Interference Detection and Mitigation,” Proceedings of the ieee, vol. 104, no. 6, pp. 1246-1257, Jun, 2016
[8] Tsikin I A and Shcherbinina E A. "Algorithms of GNSS signal processing based on the generalized maximum likelihood criterion for attitude determination." 2018 25th Saint Petersburg International Conference on Integrated Navigation Systems (ICINS). IEEE, 2018
[9] Shcherbinina E and Tsikin I "GPS antenna array calibration for attitude determination based on reference phase difference method." 2016 39th International Conference on Telecommunications and Signal Processing (TSP). IEEE, 2016
[10] Tsikin I, and Shcherbinina E "GNSS Attitude Determination Based on Antenna Array Space-Time Signal Processing." Internet of Things, Smart Spaces, and Next Generation Networks and Systems. Springer, Cham, 2016. 573-583
[11] Yefimenko V S Kharakteristiki optimal'noy prostranstvenno-vremennoy otsenki parametrov signalov. / V. S. Yefimenko, V. N. Kharisov. //Radiotekhnika. – 2006. – №. 7. – S. 71-74
[12] CMA Agee B A new approach to multipath correction of constant modulus signals. – IEEE Trans. Acoustics Speech Signal Processing: 459-472. 1983
[13] Al-Sadoon M and Abd-Alhameed R Weight Optimization for Adaptive Antenna Arrays Using LMS and SMI Algorithms. – WSEAS transactions on communica-tions. I.T.E. Volume 15. 2016
[14] Gecan A S and Zoltowski M D Advanced adaptive null steering concepts for GPS. –IEEE Military Communications Conference: Vol. 3. 1995
[15] Helstrom C W Statistical theory of signal detection: international series of monographs in electronics and instrumentation. – Elsevier, 2013. – T. 9
[16] Van Trees and Harry L Detection, estimation, and modulation theory, part I: detection, estimation, and linear modulation theory. John Wiley & Sons, 2004