A coupled lossy local-mode theory description of a plasmonic tip

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New Journal of Physics 14 (2012) 083041 (19pp)
Received 10 May 2012
Published 31 August 2012
Online at http://www.njp.org/
doi:10.1088/1367-2630/14/8/083041

Abstract. We investigate power propagation in a metal-coated tapered optical fiber. We analyze in detail the conversion from the fiber core guided mode to a surface plasmon polariton (SPP) confined at the tip apex. To this aim, we adapt coupled local-mode theory to include lossy modes. Two distinct regimes are identified. In the case of thin metal coating, a strong coupling regime occurs between a core guided mode and a SPP with good conversion efficiency. In the case of thick metal coating, a very weak coupling occurs. Finally, energy confinement and the role of Joule losses are discussed in the near-infrared and visible ranges. Moreover, the coupled equations derived for local lossy modes are not limited to plasmonic systems but also apply to any absorbing or leaky optical waveguide of arbitrary shape, so this formalism could find applications in various areas.

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1. Introduction

Nano-optics relies on the efficient confinement of the electromagnetic field on a strongly subwavelength scale. To this aim, surface plasmons polaritons (SPPs) appear as excellent candidates since they present extremely low modal volume. However, they suffer from low excitation efficiency, so specific strategies have to be developed to couple electromagnetic energy with localized plasmons. For instance, plasmonic antennas, consisting of a controlled arrangement of metallic nanoparticles, efficiently convert free-space propagative electromagnetic fields into confined SPPs [1]. It was also proposed to attach the sharp metallic nanostructures on a shear-force probe in order to control the position of the confined nanosource, forming thereby a plasmonic tip. When the elongated metallic tip is illuminated, a confined plasmon is generated at its apex [2]. A more efficient excitation of the tip apex can be obtained using a grating coupler lithographed on the massive metallic body. The grating couples free-space electric field to a delocalized SPP propagating at the metal/air interface. This mode is finally adiabatically converted to a localized SPP confined at the tip apex [3–5]. Another configuration for plasmonic tips couples the optical fiber mode to SPPs, offering a very versatile setup for light manipulation at the nanoscale [6–14].

In this work, we investigate a plasmonic tip made of an elongated optical fiber coated with a thin metal film (figure 1). When excited with the appropriate polarization, and for adiabatic tapering, the guided fiber mode couples to a guided SPP confined at the core/metal interface and could then be converted to a guided SPP confined outside the metal film (metal/air interface). This outer delocalized SPP is adiabatically transformed to a quasi-static localized SPP at the tip apex [7, 15, 16]. Optimal designing of the plasmonic tip shape (tapering angle, coating thickness, etc) necessitates a good understanding of the power transferred along the coated fiber. Recently, Ding et al [15] applied coupled local-mode theory to this configuration at telecom wavelength. Due to the limited metal losses in the near infrared, they introduced the propagation loss phenomenologically as a perturbation of the non-lossy coupled local-mode theory. Better
Figure 1. A tapered optical fiber tip of radius $R_1$ is coated with a metallic film of thickness $d = R_2 - R_1$. The core, metal and surroundings dielectric constants are $\epsilon_1$, $\epsilon_2$ and $\epsilon_3$, respectively.

SPP confinement at the tip apex could be achieved in the visible range but strong losses occur, so the coupled local-mode theory needs to be adapted to include lossy modes, as discussed in this paper. Although the achieved formalism is general, we apply it in the more specific context of plasmonic tips for the sake of clarity, but without loss of generality since coupled equations are derived without being restricted to a particular configuration.

In section 2, we extensively describe the modes supported by a metal-coated optical fiber. Then, in section 3, we develop coupled lossy modes theory and discuss power transfer along a tapered metal-coated fiber at telecom wavelength and in the visible range. We consider in the following a conical tip with a tapering angle $\alpha$. The generalization to an arbitrary tip profile, e.g. paraboloid, is straightforward.

2. Metal-coated fiber modes

2.1. Dispersion relations and mode profiles

The most efficient excitation of the radial guided plasmon at the tip apex is obtained with a transverse magnetic (TM) mode of the bare optical fiber (the magnetic field has no longitudinal component; $H_z = 0$) [7]. As a consequence, we consider only TM modes in the following. In coupled local-mode theory, the electromagnetic field on a given transverse section of the tapered fiber is written as a linear combination of the guided modes supported by the infinite fiber with the same cross section (local modes) [17]. Therefore, we analyze in this section the TM modes supported by a cylindrical metal-coated optical fiber. Their analytical expressions are gathered in table 1 [18]. The application of the boundary conditions at the core/metal and metal/background interfaces led to a system of linear equations for the unknown $a_i$. Non-trivial solutions exist when the determinant of this system is cancelled, defining the dispersion relation of the guided modes. The zeros of this determinant are numerically found in the complex plane of unknown $\beta$ using a Davidenko’s algorithm [19].

Figure 2 represents the dispersion relations calculated for the three first TM modes (TM$_{01}$, TM$_{02}$ and TM$_{03}$) at excitation wavelength $\lambda = 1550$ nm as a function of the coated fiber radius $R_2$. $n'_{\text{eff}} = \text{Re}(\beta/k_0)$ is the mode effective index (figure 2(a)). Bound guided modes are
Table 1. Components of the electromagnetic field. $a_i$ and $a'_i$ are complex coefficients, $\beta$ is the guided mode propagation constant and $k_i = (\epsilon_i k_0^2 - \beta^2)^{1/2}$, where $k_0 = \omega/c$. $I_0$, $J_0$ and $K_0$ refer to cylindrical Bessel functions.

| Core | Metal | External medium |
|------|-------|-----------------|
| $E_1^i(r) = a_1 J_0(k_1 r)$ | $E_2^i(r) = a_2 J_0(k_2 r) + a'_2 K_0(k_2 r)$ | $E_3^i(r) = a_3 K_0(k_3 r)$ |
| $E_1^r(r) = i \frac{\beta}{k_1} a_1 J_0(k_1 r)$ | $E_2^r(r) = i \frac{\beta}{k_2} (a_2 J_0(k_2 r) + a'_2 K_0(k_2 r))$ | $E_3^r(r) = i \frac{\beta}{k_3} K_0(k_3 r)$ |
| $H_0^i(r) = i \sqrt{\frac{\mu_0}{\mu_k}} \frac{\omega}{k_0} J'_0(k_1 r)$ | $H_0^r(r) = i \sqrt{\frac{\mu_0}{\mu_k}} \frac{\omega}{k_0} (a_2 J'_0(k_2 r) + a'_2 K'_0(k_2 r))$ | $H_0^r(r) = i \sqrt{\frac{\mu_0}{\mu_k}} K'_0(k_3 r)$ |

Figure 2. Dispersion curve as a function of the coated fiber radius $R_2$. The excitation wavelength is $\lambda = 1550$ nm and the metal coating thickness $d = 80$ nm. The core is glass ($\epsilon_1 = 2.25$), the coating metal is gold ($\epsilon_2 = -115 + 11i$) and the surrounding medium is air ($\epsilon_3 = 1$). The horizontal dotted line separates bound modes ($\text{Re}(\beta/k_0) > \sqrt{\epsilon_3}$) from leaky modes ($\text{Re}(\beta/k_0) < \sqrt{\epsilon_3}$).

characterized by effective indices above the outside medium optical index $n_3 = \sqrt{\epsilon_3}$. Effective indices below $n_3$ correspond to leaky modes. Leaky modes provide a good approximation of the radiation field near the fiber, except for the interface wave ($n_{\text{eff}} = \sqrt{\epsilon_3}$) that propagates along the core/clad interface that could contribute to the radiation field but is not described by a leaky mode [17]. We also report in figure 2(b) the imaginary part of the effective index $n''_{\text{eff}} = \text{Im}(\beta/k_0)$. Finite imaginary part indicates lossy modes with propagation length $L_e = \lambda/(4\pi n''_{\text{eff}})$. Mode losses originate from absorption into the metal (for either bound or leaky modes) or radiative leakages in the surrounding medium (leaky modes). Finally, strongly attenuated modes, for which $\text{Im}(\beta) > \text{Re}(\beta)$, are called reactive modes since their propagation length is smaller than their effective wavelength [19].

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Figure 3. Normalized mode intensity profiles for $R_1 = 1500$ nm. Electric field polarization is indicated by arrows. The modes are normalized according to equation (4). $TM_{01}$ is confined at the metal/core interface (inner SPP-like mode), $TM_{02}$ is confined at the core center (fiber-like mode) and $TM_{03}$ is confined at the metal/air interface (outer SPP-like mode).

The various TM modes of the coated fiber can be described as the hybridization of modes supported by a core (radius $R_1$) buried in metal and modes supported by a metallic cylinder (radius $R_2$) in air [15, 20]. A hollow cylinder supports both fiber guided modes and propagative SPPs confined at the core/metal interface (inner plasmon), whereas a metallic cylinder supports propagative SPPs confined at the metal/air interface (outer plasmons). We will discuss in the next section the coupling strength between these modes. We plot in figure 3 the mode profiles for core radius $R_1 = 1500$ nm and metal thickness $d = 80$ nm. These closely resemble the core guided mode or inner/outer SPP as expected from the hybridization scheme: $TM_{01}$ mode intensity is maximum at the core/metal interface (inner SPP-like mode); the $TM_{02}$ mode is concentrated at the core center (fiber-like mode) and the $TM_{03}$ mode intensity is maximum at the metal/air interface (outer SPP-like mode).
2.2. Coupling strength

Figure 2 shows the evolution of the mode effective index as a function of fiber radius. As discussed above, for a large fiber core, the supported modes resemble the core (TM$_{02}$), outer SPP (TM$_{01}$) or inner SPP (TM$_{03}$) guided modes. Their effective indices decrease as the fiber radius decreases so that they efficiently couple if their effective indices are similar [15, 20]. Indeed, near $R_2 = 1.27 \, \mu m$, we observe an anti-crossing between TM$_{02}$ and TM$_{03}$ effective indices associated with a crossing of their imaginary parts $n''_{\text{eff}}$. This clearly shows a strong coupling with energy exchange (since $n''_{\text{eff}}$ is related to mode losses) between the fiber mode and the outer SPP mode supported by the isolated hollow and metallic cylinders, respectively. Below $R_2 = 1.27 \, \mu m$, the so-called TM$_{02}$ mode (green curve) now presents the characteristics of an outer SPP mode. Accordingly, the imaginary part of its effective index strongly decreases at the coupling point since the mode field becomes confined outside the absorbing metal. Oppositely, TM$_{03}$ mode becomes leaky, hence the strong increase of $n''_{\text{eff}}$ (red curve). A similar behavior is observed between TM$_{02}$ and TM$_{01}$ modes near $R_2 = 500 \, \text{nm}$. This means that efficient coupling from a fiber guided mode to an outer SPP mode occurs, ensuring the formation of a highly confined mode at the tip apex. Figure 4 shows the mode profile modification at the anticrossing passage. Fiber core (bound or leaky) and outer SPP-like modes are clearly identified.

However, for a thicker coating $d = 90 \, \text{nm}$ (figure 5), we observe TM$_{01}$/TM$_{02}$ modes crossing, whereas the imaginary part of their effective indices presents a net anticrossing phenomenon. This means that the fiber mode and outer SPP only weakly couple, with practically no exchange of energy. A similar behavior is observed in the visible ($\lambda = 600 \, \text{nm}$) as shown in figure 6. For the coating metal thickness $d = 80 \, \text{nm}$ (figure 6(a)), we still observe a strong coupling phenomenon, although important modes losses as revealed by a rather low propagation length (high $n''_{\text{eff}}$). For thicker coating $d = 110 \, \text{nm}$ (figure 6(b)), we again observe mode crossing (weak coupling regime).

The coupling strength between two modes mainly depends on their overlapping. The radial penetration of the mode is estimated from $\delta = 1/\text{Im}(k_\perp) = 1/|\text{Im}(\epsilon_2 - n_{\text{eff}}^2)^{1/2}/k_0|$; $\delta = 23 \, \text{nm}$ at $\lambda = 1.55 \, \mu m$ and $\delta = 29 \, \text{nm}$ at $\lambda = 600 \, \text{nm}$. We observe that a strong coupling occurs for metal thicknesses below about $3.5\delta$.

3. Coupled lossy local-mode theory

The dispersion relations of metal-coated fiber clearly reveal energy transfer from the fiber mode to the outer SPP. In this section, we investigate the coupling mechanism and estimate the power transfer thanks to the coupled local-mode theory that we adapt to lossy systems. To this aim, we closely follow usual derivation (see, e.g., [17]) and discuss a simple but exact way to properly include losses in the coupled equations. We underline the main points in the text, with particular attention devoted to differentiate the lossy and non-lossy cases. For completeness, we present a detailed derivation of the equations in the appendix.

Coupled local-mode theory hypothesizes that the electromagnetic fields ($E$, $H$) propagating along the fiber are a linear combination of the so-called fiber local modes. Since local modes approximate the field of a finite section by the modal fields of an infinitely long fiber, it applies to slowly varying cross section. A qualitative criterion is that the fiber cross section must change over a distance large compared to the beat length between two modes [17]. The beat length between modes with propagation constant $\beta_1$, $\beta_2$ expresses $z_b = 2\pi/|\beta_1 - \beta_2|$.
and the fiber profile follows \( z = R / \tan(\alpha) \). The criterion for slow variation is therefore \( \alpha < \text{atan}(R|\beta_1 - \beta_2|/2\pi) \), where \( \beta_1 \) is the propagation constant of a guided mode and \( \beta_2 \) is the propagation constant of a guided, leaky or interface mode [17]. Equivalently, the fiber cross-section variation should be slow enough to permit adiabatic power transfer between two local modes [17]. Tapering angle of a few degrees satisfies the adiabatic criterion, except near the mode cutoff where the coupling to an interface (radiation) mode strongly constrains this criterion [15].

Practically, for adiabatic tapering, we express the field on each fiber cross section as

\[
E_t(r, \phi, z) = \sum_j [b_j(z) + b_{-j}(z)]e_{tj}(r, \phi, \beta(z)),
\]

\[
H_t(r, \phi, z) = \sum_j [b_j(z) + b_{-j}(z)]h_{tj}(r, \phi, \beta(z)),
\]

(1)

where \((e_j, h_j)\) is a normalized guided mode of a translationally invariant fiber with a cross section identical to the elongated fiber cross section at position \( z \). The subscript \( t \) denotes the

**Figure 4.** Normalized mode intensity profiles near the TM\(_{01}\)/TM\(_{02}\) anti-crossing point. Arrows represent the field polarization.
transverse component. The positions along the fiber are noted in the cylindrical coordinate \((r, z) = (r, \phi, z)\) with \(z\) being the position along the fiber axis and \(r = (r, \phi)\) the position on a given cross section. \(b_j\) is the complex coefficient of mode \(j\), and \(b_{-j}\) is associated with backward propagation.

In the case of a lossless waveguide, modes are orthonormalized with respect to their energy flux:

\[
N_{jk} = \frac{1}{2} \int_{A_{\infty}} (e_j \wedge h_k^*) \cdot z\, dA.
\]

However, this is no more valid for lossy modes and the factor of normalization is then defined as [17]

\[
N_{jk} = \frac{1}{2} \int_{A_{\infty}} (e_j \wedge h_k) \cdot z\, dA
\]

that becomes identical to (2) for lossless modes since then \(h_k^* = h_k\). Note that the integration surface \(A_{\infty}\) has to be deformed in the complex plane when leaky modes are considered [17]. We discuss later the introduction of leaky modes in the coupled modes equations.

The local modes \((e_j, h_j)\) satisfy relations of orthogonality:

\[
N_{jk} = |\mathcal{N}_{jk}| = \delta_{jk}.
\]

Introducing expression (1) into Maxwell’s equations and using the orthogonality condition (equation (4)), we derive the coupled local-mode equations (the prime denotes derivative with respect to \(z\))

\[
\begin{align*}
b_j' (z) - i \beta_j b_j (z) &= \Sigma_k C_{jk} b_k (z), \\
b_{-j}' (z) + i \beta_j b_{-j} (z) &= \Sigma_k C_{-j-k} b_{-k} (z),
\end{align*}
\]
Figure 6. Dispersion curve for \( \lambda = 600 \) nm (\( \epsilon_2 = -9.4 + 1.5i \)): (a) \( d = 80 \) nm and (b) \( d = 110 \) nm.

where the coupling coefficient \( C_{jk} \) expresses

\[
C_{jk} = \frac{1}{4N_j} \int_{A_{\infty}} \left( \mathbf{h}_j \wedge \frac{\partial \mathbf{e}_k}{\partial z} - \mathbf{e}_k \wedge \frac{\partial \mathbf{h}_j}{\partial z} \right) \cdot \mathbf{e}_z \, dA,
\]

(6)

\[
C_{jj} = 0.
\]

(7)
Equations (5) and (6) extend the coupled local-mode equations to absorbing waveguides. The only difference from the non-absorbing case is the introduction of the normalization factor $N_j^2$ [21].

The main difference between the absorbing and the non-absorbing case appears when the coupling coefficients $C_{jk}$ are expressed as a function of the mode overlap. To this aim, we closely follow the method described by Snyder and Love [17], except that we introduce the backward propagating mode $\tilde{e}_k = e_{-k} = e_{jk} - e_{jk}^*$ instead of the conjugated mode $e_k^*$. The calculation details are given in appendix A. As far as the TM mode is concerned, we achieve

$$C_{jk} = \frac{1}{4 N_j} \frac{k_0}{\beta_j - \beta_k} \sqrt{\frac{\varepsilon_0}{\mu_0}} \int_{A_\infty} \frac{\partial \varepsilon}{\partial z} e_j \cdot \tilde{e}_k \, dA. \quad (8)$$

Note again that $e_k^* = e_{-k} = \tilde{e}_k$ for the non-absorbing guide, so that we recover for lossless modes, as expected:

$$C_{jk} = \frac{1}{4} \frac{k_0}{\beta_j - \beta_k} \sqrt{\frac{\varepsilon_0}{\mu_0}} \int_{A_\infty} \frac{\partial \varepsilon}{\partial z} e_j \cdot e_k^* \, dA, \quad (9)$$

and our definition (equation (8)) is valid for both absorbing and non-absorbing waveguides (as discussed in the appendix, it is, however, demonstrated only for TM modes in the lossy case).

Although expression (8) closely resembles equation (9), it is worth noting that the propagation constant $\beta_{j,k}$ that appears in the denominator are complex numbers in the lossy case, where their imaginary part is related to mode losses. As a consequence, when two modes interact, namely as their effective index becomes similar, the difference $\beta_j - \beta_k$ does not cancel so that their coupling coefficient is finite even for $\text{Re}(\beta_j) = \text{Re}(\beta_k)$ (see, e.g., figures 5 and 6(b)).

We plot in figure 7 the coupling coefficient $C_{12}$ between the first two modes of the coated fiber at $\lambda = 1.55 \mu m$. Its real part closely follows lossless values. In contrast, its imaginary part is no longer negligible. Finally, our model takes into account the losses that occur during mode propagation along the fiber but also during modes coupling. Since the leaky mode profile only slightly differs from the bound mode, we use the same expression for the coupling coefficient.
with a leaky mode (i.e. we do not deform the integration surface $A_\infty$ into the complex plane). The coupling coefficient is set to 0 when $\text{Im}(\beta)$ exceeds 1% of $\text{Re}(\beta)$.

Here, we would like to point out an analogy between the coupled local-mode formalism and a molecular optics description. Indeed, in the presence of strong dissipation, the lossy coupled mode equations are the guided optics analogous to the dipole–dipole resonant energy transfer in an absorbing cavity where cooperative decay could occur [22–24]. In that case, the coupled system cannot be described as a function of the isolated systems alone, but should be described statistically using density matrix formalism. A similar description exists for guided optics [25] and its application to coupled local-mode theory would help in understanding the coupling process in absorbing waveguides. By analogy, we presume that the real part of the coupling coefficient $C_{jk}$ refers to the mode coupling efficiency, whereas its imaginary part would represent a cooperative dissipation due to the overlap between two lossy modes.

4. Application to light superfocusing

4.1. Mode propagation and transferred power

For adiabatically elongated fiber, the coupling to a backward propagating mode can be neglected. Coupled equations reduce then to

$$b'(z) - i \beta_j b_j(z) = \sum_k C_{jk} b_k(z),$$

with the transverse part of the electromagnetic field at any position along the fiber given by

$$E_t(r, \phi, z) = \sum_j b_j(z) e_t(r, \phi, \beta(z)),$$

$$H_t(r, \phi, z) = \sum_j b_j(z) h_t(r, \phi, \beta(z)).$$

The power at position $z$ can be calculated as

$$\mathcal{P} = \frac{1}{2} \int_{A_\infty} \text{Re}(E \wedge H^*) \cdot \text{d}A,$$

$$\mathcal{P} = \frac{1}{2} \text{Re} \int_{A_\infty} \sum_{j,k} b_j b_k^* (e_j \wedge h_k^*) \cdot \text{d}A.$$  

This expression cannot be further simplified $a$ priori. However, for a non-absorbing waveguide, the orthogonality relation (equation (2)) would lead to

$$\mathcal{P} = \frac{1}{2} \sum_j |b_j|^2.$$  

We numerically checked that expression (14) is a good approximation for the guided power (equation (13)) when losses remain weak (see appendix B). We therefore use the approximated form (equation (14)) to compute the power along the fiber since it is analytical, and it also indicates the weight of each local mode in the propagation mechanism. In the presence of strong losses, one has to use the exact form (13) only.

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The value of each modal amplitude $b_j$ is calculated solving coupled equations (10) with a Runge–Kutta algorithm. $b_j$ is set to 0 for a strongly dissipative mode ($\alpha''_{\text{eff}} > 1$).

### 4.2. Strong coupling regime

The modal weights are represented in figure 8 as a function of the position along the fiber. The coated fiber is excited with the TM$_{02}$ mode since it corresponds to the fiber-like mode that is efficiently excited by a mode propagating on the uncoated part of the fiber. At the telecom wavelength $\lambda = 1.55 \mu m$, we observe limited losses during propagation and 17% of the initial power is transferred to the outer SPP. In contrast, we observe significant losses at the visible wavelength $\lambda = 600$ nm. As shown by the fast decrease of the total power, this is mainly due to the limited propagation length. Moreover, a non-negligible part of the power is transferred into the TM$_{03}$ mode so that it cannot be transferred to the outer SPP at the tip apex. As a consequence, only 4% of the initial power remains at the tip extremity.

We present in figures 9(a) and 10 the field propagation along the elongated fiber. It is exact within ersatz (11) and neglecting reflected field. The passage from the core guided mode to the outer SPP confined at the tip apex is clearly visible. Moreover, although a rather low power is transferred at the tip apex at $\lambda = 600$ nm, the achieved field is strongly confined, which is of great interest for tip enhanced spectroscopies or optical trapping. Practically, a trade-off between mode confinement (i.e. electric field gradient) and transferred power has to be found to realize e.g. a low threshold optical trap.

#### 4.2.1. Excitation efficiency.

The transferred power strongly depends on the length of the coating area as shown in figure 11(a). For instance, when the coating begins at fiber radius...
Figure 9. Transverse component Re($H_\phi$) of the magnetic field propagating along the metalized tapered fiber. $d = 80$ nm and $\lambda = 1550$ nm. Leaky modes are cut off as soon as $n''_{\text{eff}} > 1$.

Figure 10. Transverse component Re($H_\phi$) of the magnetic field propagating along the metal-coated tapered fiber. $d = 80$ nm and $\lambda = 600$ nm. Leaky modes are cut off as soon as $n''_{\text{eff}} > 1$.

$R_2^0 = 2 \mu$m (coating length $l = R_2^0 / \sin \alpha = 11.5 \mu$m), 23\% of the initial power is transferred to the outer SPP at the tip extremity. For $R_2^0 = 1.6 \mu$m (coating length $l = 9.2 \mu$m), almost none of the initial power remains at the tip end, although we expect reduced losses during propagation. The oscillatory behavior reveals that the phase between propagating modes plays an important role in the coupling mechanism.

We plot in figures 11(b) and (c) the evolution of modal weight during the coupling for the two initial coating positions discussed above ($R_2^0 = 2 \mu$m or $R_2^0 = 1.6 \mu$m). The calculated phase shift $\Phi_2 - \Phi_1$ between modes $\text{TM}_{02}$ and $\text{TM}_{01}$ is also shown on the coupling interval. We observe that when the coated fiber mode $\text{TM}_{02}$ is excited at $R_0^2 = 2 \mu$m, the two modes $\text{TM}_{01}$ and $\text{TM}_{02}$ are in phase just before their coupling, leading to efficient energy transfer. Oppositely, when the coated fiber mode $\text{TM}_{02}$ is excited at $R_0^2 = 1.6 \mu$m, the two modes $\text{TM}_{01}$ and $\text{TM}_{02}$ are strongly dephased just before their coupling, hence the very low energy transfer.

We finally investigate the effect of both the tapering angle and the coating length in figure 12. We recover the oscillatory behavior of the transmitted power discussed above for a fixed tapering angle (see, e.g., $\alpha = 10^\circ$). Similarly, for a given initial coating position $R_2^0$, the
Figure 11. (a) Evolution of the outer SPP modal amplitude $|b_{\text{final}}|^2$ at the tip apex as a function of the position of the coating beginning. The excitation wavelength is $\lambda = 1.55 \, \mu m$ and the metal thickness is $d = 80 \, nm$. The final radius of the fiber is $R_2 = 40 \, nm$.

Figure 12. Evolution of outer SPP modal amplitude $|b_{\text{SPP}}|^2$ at the tip apex as a function of the position of the coating beginning $R_0^2$ and tapering angle $\alpha$. The excitation wavelength is $\lambda = 1.55 \, \mu m$ and the metal thickness is $d = 80 \, nm$. The final radius of the fiber is $R_2 = 100 \, nm$. 
transmitted power is an oscillatory function of the tapering angle. This is again governed mainly by the dephasing between two modes. Therefore, optimized conditions ($R_0^2$, $\alpha$) qualitatively follow a constant optical path $l = R_0^2 / \tan \alpha = \text{constant}$ (that is, a straight line $R_0^2 \propto \tan \alpha \approx \alpha$ for small angles). Additionally, the smaller the tapering angle is, the larger the propagation losses are and the transferred power at the tip apex is low. Finally, the plasmonics tip transmission efficiency reaches the value of 40% for $R_0^2 = 1.5 \, \mu m$ and $\alpha = 7^\circ$.

4.3. Weak coupling regime

Lastly, we present in figure 13 the modal amplitudes calculated using the coupled lossy local-mode equations at $\lambda = 600$ nm and for the coating thickness $d = 110$ nm, corresponding to a weak coupling regime (see figure 6(b)). Note that the non-absorbing expression (9) of the coupling coefficient diverges in this case.

5. Conclusion

We have extended the coupled local-mode theory to absorbing waveguides. The application to light superfocusing by a plasmonic fiber shows the effect of coupling strength and Joule losses on the power transfer efficiency. Moreover, the tapering angle and the coating length are critical parameters. In the near infrared, we demonstrated efficient energy transfer at the tip extremity although slightly confined. In the visible, light can be confined in a nanoscale area but at the price of a low energy transfer. Finally, a trade-off between efficient excitation and mode confinement at the tip apex has to be found for e.g. tip-enhanced spectroscopies or optical trapping, and coupled lossy local-mode theory would be helpful in designing plasmonic tips. We emphasize that the derived coupled local-mode equations are general and not limited to the plasmonic configuration investigated here.

Acknowledgment

This work was supported by the Agence Nationale de la Recherche (ANR) under the grant PlasTips (ANR-09-BLAN-0049).
Appendix A. Practical expressions for the coupling coefficient

A.1. The coupling coefficient as a function of the mode overlap

The coupling coefficient reads (see equation (6))

\[ C_{jk} = \frac{1}{4N_j} \int_{A_\infty} \left( \mathbf{h}_j \wedge \frac{\partial \mathbf{e}_k}{\partial z} - \mathbf{e}_k \wedge \frac{\partial \mathbf{h}_j}{\partial z} \right) \cdot \mathbf{e}_z \, dA. \]  

(A.1)

It is expressible in a simpler form, which introduces an overlap between the two modes. Such an expression presents a simple physical interpretation (the coupling strength is proportional to the mode overlap) and is also of practical interest for numerical evaluation. To this aim, we adapt the method described in standard textbooks (see, e.g., [17, section 31-15]) as is detailed below. We begin with the field expressions

\[ -i \left( \frac{\varepsilon_0}{\mu_0} \right)^{1/2} kn^2 \mathbf{e}_j = \nabla_t \wedge \mathbf{h}_j + i\beta_j \mathbf{z} \wedge \mathbf{h}_j, \]  

(A.2)

\[ i \left( \frac{\mu_0}{\varepsilon_0} \right)^{1/2} k \mathbf{h}_j = \nabla_t \wedge \mathbf{e}_j + i\beta_j \mathbf{z} \wedge \mathbf{e}_j. \]  

(A.3)

Let us note that \( \tilde{\mathbf{e}}_k = \mathbf{e}_{rk} - \mathbf{z} \mathbf{e}_{zk} \). We multiply equation (A.2) by \( \tilde{\mathbf{e}}'_k \) and equation (A.3) by \( \mathbf{h}'_k \), and subtract to obtain

\[ -ik \left[ \left( \frac{\varepsilon_0}{\mu_0} \right)^{1/2} n^2 \mathbf{e}_j \cdot \tilde{\mathbf{e}}'_k + \left( \frac{\mu_0}{\varepsilon_0} \right)^{1/2} \mathbf{h}'_k \cdot \mathbf{h}_j \right] 
\]

\[ = \tilde{\mathbf{e}}'_k \nabla_t \wedge \mathbf{h}_j - \mathbf{h}'_k \nabla_t \wedge \mathbf{e}_j + i\beta_j \left[ (\mathbf{h}'_k \wedge \mathbf{e}_j) - (\tilde{\mathbf{e}}'_k \wedge \mathbf{h}_j) \right] \cdot \mathbf{z}. \]  

(A.4)

A second equation is obtained from equations (A.2) and (A.3) by replacing the subscript \( j \) with \( k \) and differentiating with respect to \( z \). We then multiply the first equation by \( \tilde{\mathbf{e}}_j \) and the second by \( \mathbf{h}_j \) and subtract to obtain

\[ -ik \left[ \left( \frac{\varepsilon_0}{\mu_0} \right)^{1/2} n^2 \tilde{\mathbf{e}}'_j \cdot \tilde{\mathbf{e}}_k + \left( \frac{\mu_0}{\varepsilon_0} \right)^{1/2} \mathbf{h}'_k \cdot \mathbf{h}_j \right] - ik \left( \frac{\varepsilon_0}{\mu_0} \right)^{1/2} \frac{\partial n^2}{\partial z} \mathbf{e}_j \cdot \tilde{\mathbf{e}}_k 
\]

\[ = \tilde{\mathbf{e}}_k \nabla_t \wedge \mathbf{h}'_k - \mathbf{h}_j \nabla_t \wedge \mathbf{e}_j + i\beta_j \left[ \tilde{\mathbf{e}}_j (\mathbf{z} \wedge \mathbf{h}_k) - \mathbf{h}_j (\mathbf{z} \wedge \mathbf{e}_k) \right] 
\]

\[ - i\beta'_k \left[ \tilde{\mathbf{e}}_j (\mathbf{z} \wedge \mathbf{h}_k) - \mathbf{h}_j (\mathbf{z} \wedge \mathbf{e}_k) \right] \cdot \mathbf{z}. \]  

(A.5)

We now subtract equations (A.4) and (A.5)

\[ ik \left( \frac{\varepsilon_0}{\mu_0} \right)^{1/2} \frac{\partial n^2}{\partial z} \mathbf{e}_j \cdot \tilde{\mathbf{e}}_k = \tilde{\mathbf{e}}'_k \nabla_t \wedge \mathbf{h}_j - \mathbf{h}'_k \nabla_t \wedge \mathbf{e}_j - \tilde{\mathbf{e}}_j \nabla_t \wedge \mathbf{h}'_k + \mathbf{h}_j \nabla_t \wedge \mathbf{e}'_k 
\]

\[ - i\beta'_k \left[ \tilde{\mathbf{e}}_j (\mathbf{z} \wedge \mathbf{h}_k) - \mathbf{h}_j (\mathbf{z} \wedge \mathbf{e}_k) \right] 
\]

\[ + i(\beta_j - \beta'_k) \left[ (\mathbf{h}'_k \wedge \mathbf{e}_j) - (\mathbf{e}'_k \wedge \mathbf{h}_j) \right] \cdot \mathbf{z}. \]  

(A.6)

The key point is to gather the four terms involving \( \nabla_t \) so that the divergence theorem applies. We were not able to find a general way but achieve the expected result when considering TM
polarized modes. Indeed,
\[ \nabla_i \cdot (h_j \wedge \epsilon_j') = \epsilon_j' \cdot (\nabla_i \wedge h_j) - h_j \cdot (\nabla_i \wedge \epsilon_j'), \quad (A.7) \]
\[ \nabla_i \cdot (h_k' \wedge \tilde{e}_j) = \tilde{e}_j \cdot (\nabla_i \wedge h_k') - h_k' \cdot (\nabla_i \wedge \tilde{e}_j). \quad (A.8) \]
Moreover, as far as TM modes are concerned, we have
\[ \epsilon_j' \cdot (\nabla_i \wedge h_j) = -\tilde{e}_j' \cdot (\nabla_i \wedge h_j), \quad (A.9) \]
\[ h_k' \cdot (\nabla_i \wedge \tilde{e}_j) = -h_k' \cdot (\nabla_i \wedge e_j). \quad (A.10) \]
Finally, equation (A.11) becomes
\[ ik \left( \frac{\epsilon_0}{\mu_0} \right)^{1/2} \frac{\partial n^2}{\partial z} \epsilon_j \cdot \tilde{e}_j = \nabla_i(h_j \wedge \epsilon_j' + h_k' \wedge \tilde{e}_j) - i\beta_k' \left[ \tilde{e}_j(z \wedge h_k) - h_j(z \wedge \epsilon_k) \right] \\
+ i(\beta_j - \beta_k) \left[ (h_k' \wedge e_j) - (\epsilon_k' \wedge h_j) \right] \cdot z. \quad (A.11) \]
As a last step, we integrate over infinite cross section \( A_{\infty} \). Due to the local-mode orthogonality condition (equation (4)), terms multiplying \( \beta_k' \) vanish. Additionally, the two-dimensional divergence theorem transforms integration of the term \( \nabla_i(h_j \wedge \epsilon_j' + h_k' \wedge \tilde{e}_j) \) into a line integral at infinity that vanishes also since fields and their derivatives decrease to zeros exponentially.

It follows that
\[ C_{jk} = \frac{1}{4N_j} \frac{k_0}{\beta_j - \beta_k} \sqrt{\frac{\epsilon_0}{\mu_0}} \int_{A_{\infty}} \epsilon' \epsilon_j \cdot \epsilon_k \cdot dA. \quad (A.12) \]

A.2. Coupling coefficient computation

In the case of step profile waveguide, the computation of the coupling coefficient can be further simplified with the help of field boundary conditions [15]. The longitudinal component of the electric field and the radial component of the electric displacement \( d = \epsilon e \) are continuous across an interface. Therefore, we separate these two components in the expression for the coupling coefficient:
\[ C_{jk} = \frac{1}{4N_j} \frac{k_0}{\beta_j - \beta_k} \sqrt{\frac{\epsilon_0}{\mu_0}} \int_{A_{\infty}} \left( \frac{\epsilon' (\epsilon_{e_j} \cdot \epsilon_{e_k}) - \epsilon' e_{e_j} e_{e_k}}{\epsilon_k} \right) dA. \quad (A.13) \]
Moreover, owing to the cylindrical symmetry, this simplifies to
\[ C_{jk} = \frac{1}{4N_j} \frac{k_0}{\beta_j - \beta_k} \sqrt{\frac{\epsilon_0}{\mu_0}} \int_{0}^{\infty} \left[ \frac{\partial 1/\epsilon}{\partial z} d_{e_j} d_{e_k} - \epsilon' e_{e_j} e_{e_k} \right] 2\pi r \ dr. \quad (A.14) \]
For the specific case considered here (core/coating/clad), the dielectric permittivity reads
\[ \epsilon(r, z) = \epsilon_1 + (\epsilon_2 - \epsilon_1) \Theta(r - R_1(z)) + (\epsilon_3 - \epsilon_2) \Theta(r - R_2(z)), \quad (A.15) \]
where \( \Theta \) is the Heavyside distribution. Then, it follows that
\[ \epsilon'(r, z) = R_1'(\epsilon_2 - \epsilon_1) \delta(r - R_1(z)) + R_2'(\epsilon_3 - \epsilon_2) \delta(r - R_2(z)). \quad (A.16) \]
If we, moreover, assume the coating thickness constant over the elongated fiber, \( -dR_i/dz = \tan \alpha \) and
\[ \epsilon'(r, z) = \tan \alpha [(\epsilon_2 - \epsilon_1) \delta(r - R_1(z)) + (\epsilon_3 - \epsilon_2) \delta(r - R_2(z))]. \quad (A.17) \]
Similarly, the inverse of the dielectric permittivity reads
\[
\frac{1}{\epsilon(r, z)} = \frac{1}{\epsilon_1} + \left(\frac{1}{\epsilon_2} - \frac{1}{\epsilon_1}\right)\Theta(r - R_1(z)) + \left(\frac{1}{\epsilon_3} - \frac{1}{\epsilon_2}\right)\Theta(r - R_2(z))
\] (A.18)
that leads to
\[
\frac{\partial 1/\epsilon}{\partial z} = \tan \alpha \left[ \left(\frac{1}{\epsilon_2} - \frac{1}{\epsilon_1}\right)\delta(r - R_1(z)) + \left(\frac{1}{\epsilon_3} - \frac{1}{\epsilon_2}\right)\delta(r - R_2(z)) \right].
\] (A.19)

We now introduce expressions (A.17) and (A.19) in the coupling coefficient (equation (A.14)) to obtain
\[
C_{jk} = \frac{\pi \tan \alpha}{2N_j} \frac{k_0}{\beta_j - \beta_k} \sqrt{\frac{\epsilon_0}{\mu_0}} \left[ \epsilon_1 \left(\epsilon_1 - \epsilon_2\right)R_1 e_{rj}|_{r = R_1} \cdot e_{rk}|_{r = R_1} - R_1 e_{cj}|_{r = R_1} \cdot e_{ck}|_{r = R_1} \right.
\]
\[
+ \left. R_2 \left(\frac{1}{\epsilon_3} - \frac{1}{\epsilon_2}\right)\epsilon_1 e_{rj}|_{r = R_2} \cdot e_{rk}|_{r = R_2} - R_2 e_{cj}|_{r = R_2} \cdot e_{ck}|_{r = R_2} \right]
\] (A.20)
For numerical computation purposes, we use
\[
d_{rj}|_{r = R_1} = \epsilon_1 e_{rj}|_{r = R_1},
\] (A.21)
\[
d_{rj}|_{r = R_2} = \epsilon_3 e_{rj}|_{r = R_2}
\] (A.22)
to achieve
\[
C_{jk} = \frac{\pi \tan \alpha}{2N_j} \frac{k_0}{\beta_j - \beta_k} \sqrt{\frac{\epsilon_0}{\mu_0}} \left[ \epsilon_1 \left(\epsilon_1 - \epsilon_2\right)R_1 e_{rj}|_{r = R_1} \cdot e_{rk}|_{r = R_1} \right.
\]
\[
+ \left. R_2 \frac{\epsilon_2 - \epsilon_3}{\epsilon_2} \epsilon_1 e_{rj}|_{r = R_2} \cdot e_{rk}|_{r = R_2} \right].
\] (A.23)

Appendix B. Validity of equation (14)

See figure B.1.

**Figure B.1.** Comparison of the power at position \( z \), calculated using exact expression (13) (square symbols) or approximated form (14) (solid line). \( \lambda = 1.55 \mu m \) and \( d = 80 \) nm.
References

[1] Bharadwaj P, Deutsch B and Novotny L 2009 Optical antennas Adv. Opt. Photon. 1 438–83
[2] Novotny L, Bian R X and Xie S X 1997 Theory of nanometric optical tweezers Phys. Rev. Lett. 79 645–8
[3] Ropers C, Neacsu C C, Elsaesser T, Albrecht M, Raschke M B and Lienau C 2007 Grating-coupling of surface plasmons onto metallic tips: a nanoconfined light source Nano Lett. 7 2784–8
[4] Baida F I and Belkhir A 2009 Superfocusing and light confinement by surface plasmon excitation through radially polarized beam Plasmonics 4 51–9
[5] Sadiq D, Shirdel J, Lee J S, Selishcheva E, Park N and Lienau C 2011 Adiabatic nanofocusing scattering-type optical nanoscopy of individual gold nanoparticles Nano Lett. 11 1609–13
[6] Devaux E, Devaux E, Bourillot E, Weeber J C, Lacroute Y, Goudonnet J P and Girard C 2000 Local detection of the optical magnetic field in the near zone of dielectric sample Phys. Rev. B 62 10504–14
[7] Bouhelier A, Reuter J, Beversluis M and Novotny L 2003 Plasmon coupled tip-enhanced near-field microscopy J. Microsc. 210 220–4
[8] Janunts N A, Baghdasaryana K S, Nerkararyana Kh V and Hecht B 2005 Excitation and superfocusing of surface plasmon polaritons on a silver-coated optical fiber tip Opt. Commun. 253 118–24
[9] Issa N and Guckenberger R 2007 Fluorescence near metal tips: the roles of energy transfer and surface plasmon polaritons Opt. Express 15 12131–44
[10] Chang D E, Størensen A S, Hemmer P R and Lukin M D 2007 Strong coupling of single emitters to surface plasmons Phys. Rev. B 76 35420
[11] Tanaka K, Burr G W, Grosjean T, Maletzky T and Fischer U C 2008 Superfocussing in a metal-coated tetrahedral tip by dimensional reduction of surface- to edge-plasmon modes Appl. Phys. B 93 257–66
[12] Bortchagovsky E G, Klein S and Fischer U C 2009 Surface plasmon mediated tip enhanced Raman scattering Appl. Phys. Lett. 94 063118
[13] Chen X-W, Sandoghdar V and Agio M 2009 Highly efficient interfacing of guided plasmons and photons in nanowires Nano Lett. 9 3756–61
[14] Barthes J, Colas des Francs G, Bouhelier A, Weeber J-C and Dereux A 2011 Purcell factor for a point-like dipolar emitter coupling to a 2d-plasmonic waveguide Phys. Rev. B (Brief Reports) 84 073403
[15] Ding W, Andrews S R and Maier S A 2007 Internal excitation and superfocusing of surface plasmon polaritons on a silver-coated optical fiber tip Phys. Rev. A 75 063822
[16] Antosiewicz T J, Wróbel P and Szoplik T 2009 Nanofocusing of radially polarized light with dielectric–metal–dielectric probe Opt. Express 17 9191–6
[17] Snyder A and Love J 1983 Optical Waveguide Theory (London: Chapman and Hall)
[18] Schröter U and Dereux A 2001 Surface plasmon polaritons on metal cylinders with dielectric core Phys. Rev. B 64 125420
[19] Kim K Y, Tae H-S and Lee J-H 2003 Analysis of leaky modes in circular dielectric rod waveguides Electron. Lett. 39 61–2
[20] Novotny L and Hafner C 1994 Light propagation in a cylindrical waveguide with a complex, metallic, dielectric function Phys. Rev. E 50 4094–106
[21] Mu J and Huang W-P 2011 Complex coupled-mode theory for tapered optical waveguides Opt. Lett. 36 1026–8
[22] Agarwal G S 1997 Microscopic approach to coherent population trapping state and its relaxation in a dense medium Opt. Express 1 44–8
[23] Dung H T, Knöll L and Welsch D-G 2002 Resonant dipole–dipole interaction in the presence of dispersing and absorbing surroundings Phys. Rev. A 66 063810
[24] Colas des Francs G, Girard C and Martin O 2003 Fluorescent resonant energy transfer in the optical near-field Phys. Rev. A 67 53805
[25] Maeda K and Hamasaki J 1980 Density–matrix method applied to mode coupling in lens like fibers J. Opt. Soc. Am. 70 381–8

New Journal of Physics 14 (2012) 083041 (http://www.njp.org/)