We investigate the influence of electron–electron interactions on the density of states of a ballistic two–dimensional electron gas. The density of states is determined nonperturbatively by means of path integral techniques allowing for reliable results near the Fermi surface, where perturbation theory breaks down. We find that the density of states is suppressed at the Fermi level to a finite value. This suppression factor grows with decreasing electron density and is weakened by the presence of gates.

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The suppression of electron tunneling into a conductor at low bias voltages, a phenomenon known as zero-bias anomaly (ZBA), has been under consideration theoretically and experimentally for more than 20 years [1–8]. This effect can be related to a reduction of the (tunneling) density of states (DOS) at the Fermi level, which is a clear signature of interaction effects that are otherwise often disguised in good conductors.

In the past the focus has been mainly on disordered systems with slow diffusive electron motion enhancing Coulomb effects so that strong ZBAs arise [8]. More recently, attention has shifted to one-dimensional ballistic wires [9], where the Tomonaga–Luttinger model leads to a power law suppression of the DOS [1]. Similar to disorder, the reduction of dimension hinders fast spreading and forces the particles to interact more strongly. Two–dimensional ballistic systems are a borderline case between strongly interacting one-dimensional fermions and weakly interacting three-dimensional systems. The DOS is also of special interest, since two-dimensional electron gases (2DEGs) arise in a variety of semiconductor microstructures, such as GaAs heterostructures and Si inversion layers that have attracted a lot of attention lately. Further, 2DEGs provide electrodes in experimental tunnelling setups measuring properties of 1D quantum wires. Since such experiments do not test the DOS of the wire alone, it is important to know the form of the DOS of the electrode.

In earlier work [10,12], based on perturbation theory in the Coulomb interaction, the change of the DOS of 2DEGs in the absence of a screening gate was found to show a cusp $\delta \nu(\epsilon)/\nu_0 = |\epsilon|/4e_F$ at the Fermi edge. This prediction is rather irritating since it is independent of the strength of the Coulomb interaction. Here we reexamine the problem employing nonperturbative path integral techniques and show that the suppression of the DOS at the Fermi level sensitively depends on the electron density and screening by gates. In contrast to the 1D case, in the absence of disorder, the DOS at the Fermi level remains finite also at zero temperature yet reduced from the bare density that is approached at larger energies.

The suppression increases with the effective interaction strength, i.e., with decreasing electron density.

The interacting electron gas can be described by the action for a fermion field $\psi$ coupled to an electric potential $\phi$

$$S[\psi^*, \psi, \phi] = \int_c dt \left\{ -\frac{e^2}{2} \int \frac{d^2q}{(2\pi)^2} \frac{\phi(-q)V_0^{-1}(q)\phi(q)}{\nu} + e \int \frac{d^2p}{(2\pi)^2} \frac{d^2p'}{(2\pi)^2} \bar{\psi}(p)\phi(p-p')\psi(p') \right\}$$

where $V_0$ is the Coulomb interaction potential and the operator $G_0^{-1}(p, t) = i\partial_t - e(p)$ (we set $\hbar = 1$) contains the electronic dispersion relation $\epsilon(p) = p^2/2m^*$ with the effective mass $m^*$. The time integration path is along the Keldysh contour. For simplicity we restrict ourselves to spinless Fermions but account for spin degeneracy by appropriate factors of 2. Further, we add a source term $J\bar{\psi} + \psi J$ to the action allowing us to calculate the Green function later.

The Fermion fields can be integrated out in the standard way [13], yielding an effective electromagnetic action

$$S_{\text{eff}}[\phi] = \int_c dt \left\{ -\frac{e^2}{2} \int \frac{d^2q}{(2\pi)^2} \frac{\phi(-q)V_0^{-1}(q)\phi(q)}{\nu} + \text{Tr} \ln \left[ G_0^{-1}(p)\delta(p-p') + e\phi(p-p') \right] \right\} + \int \frac{d^2p}{(2\pi)^2} \bar{\psi}(p)G[\phi](p)\psi(p) ,$$

where the trace means $\text{Tr} = \int \frac{d^2p}{(2\pi)^2} \frac{d^2p'}{(2\pi)^2} \delta(p-p')$ and $G[\phi] = [G_0^{-1} + e\phi]^{-1}$.

Expanding the logarithm in the action with respect to the Coulomb field to quadratic order, the first two terms in Eq. (1) can be combined to the Gaussian action of the electric potential field

$$S_F[\phi] = -\frac{e^2}{2} \int_c dt \int \frac{d^2q}{(2\pi)^2} \phi(-q)V^{-1}(q)\phi(q) ,$$

Tunneling Density of States of the Interacting Two–Dimensional Electron Gas

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where
\[ V^{-1}(q) = V_0^{-1}(q) + P_0(q) \]
(3)
is the dynamically screened interaction. Separating fields on the upper and lower parts of the Keldysh contour by introducing doublets, we can pass over to a convenient representation in terms of retarded and advanced functions (and mixtures thereof). Then \( P_0 \) becomes a matrix in Keldysh space, with a retarded part given by
\[
P_R^0(q, \omega) = \int \frac{d^2p}{(2\pi)^2} \frac{n_F(\epsilon(p + q)) - n_F(\epsilon(p))}{\omega + i\eta - \epsilon(p + q) + \epsilon(p)} ,
\] (4)
where \( n_F(\epsilon) = [1 + e^{\beta\epsilon}]^{-1} \) is the Fermi distribution function at inverse temperature \( \beta \). The Gaussian approximation made in deriving Eq. (2) is equivalent to the random phase approximation (RPA), which is well established for high electron densities \( n_s \) corresponding to small values of the Brueckner parameter \( r_s \) < 1, but usually gives still very reasonable results for larger \( r_s \). As standard, \( n_s = 1/\pi(r_s a_0)^2 \) with the effective Bohr radius \( a_0 = \varepsilon_d/m^* \varepsilon^2 \), where \( \varepsilon_d \) is the dielectric constant.

The quantity of interest here is the DOS
\[
\nu(\epsilon) = -\frac{2}{\pi} \int \frac{d^2p}{(2\pi)^2} \text{Im} G^R(p, \epsilon) ,
\] (5)
where \( G^R \) is the retarded Green function. The Keldysh matrix Green function is obtained as the mixed second order functional derivative of the partition function with respect to the sources yielding
\[
G(r - r', t - t') = \int D\phi \ G[\phi](r - r', t - t') \ e^{iS_F[\phi]}
\]
(6)
Following Schwinger [13, 14] we try to find a functional \( k[\phi](x,t) \) describing a local gauge transformation \( \psi(x,t) \to e^{i k[\phi](x,t)} \psi(x,t) \) such that
\[
G[\phi](x - x', t - t') = e^{i k[\phi](x,t)} G_0(x - x', t - t') e^{-i k[\phi](x',t')} .
\] (7)
Linearizing the dispersion relation near the Fermi surface, we can write for \( \omega \ll \epsilon_F \)
\[
[i \partial_t - \epsilon_F + i \mathbf{v} \cdot \nabla + e\phi(x,t)]G[\phi](x - x', t - t') = \delta(x - x', t - t').
\] (8)
where \( \mathbf{v} = \mathbf{p}/m^* \) is the Fermi velocity. Eqs. (6) and (7) determine \( k \) as a linear functional of \( \phi \). For the Green function \( G^> = -i \langle \psi(x,t)\psi(x',t') \rangle \) we then obtain
\[
G^>(x - x', t - t') = G_0^>(x - x', t - t') e^{J(x - x', t - t')}
\]
where \( G_0^> \) is the free Green function and \( J(x - x', t - t') \) is determined by the remaining Gaussian path integral over the \( \phi \) fields. This function, which we need for equal space arguments only, may be written as
\[
J(t) \equiv J(x = x', t) = \int \frac{d\omega}{\pi} \frac{e^{-i\omega t} - 1}{1 - e^{-\beta\omega}} \text{Im} Y(\omega) ,
\] (8)
where
\[
Y(\omega) = -\int \frac{d^2q}{(2\pi)^2} \frac{1}{(\omega + i\eta - \mathbf{v} \cdot \mathbf{q})^2} V_R^R(q, \omega) .
\] (9)
Now, from Eq. (7) we finally obtain the formal result
\[
\nu(\epsilon) = \nu_0 \int d\epsilon' \frac{1 + e^{-\beta\epsilon}}{1 + e^{-\beta\epsilon}} P(\epsilon - \epsilon') ,
\] (10)
where \( \nu_0 = m/\pi \) is the bare DOS and we introduced the spectral density
\[
P(\epsilon) = \int \frac{dt}{2\pi} e^{i\epsilon t} e^{J(t)} .
\] (11)
To evaluate the DOS explicitly, we first note that the polarization function \( \Pi \) reads for small \( q \)
\[
P_R^0(q, \omega) = -\nu_0 \int_0^{2\pi} \frac{d\theta}{2\pi} \frac{vq \cos \theta}{\Omega - vq \cos \theta} \]
(12)
where \( \Omega = \omega + i\eta \). If we introduce the function
\[
g(q) = [1 + \nu_0 V_0(q)]^{-1/2} ,
\]
which is unity in the noninteracting case and less than one in presence of interactions, Eq. (9) may be combined with Eqs. (9) and (12) to read
\[
Y(\omega) = -\frac{1}{2\pi} \int_0^{\infty} dq \frac{\Omega q^2 V_0(q)}{[\Omega^2 - (vq)^2] [\Omega + vq] \sqrt{\frac{2 - vq}{1 + vq} - [1 - g^2(q)]^2}} .
\]
The first factor in the denominator represents the particle pole and the second the plasmon pole, which is the solution \( q_{pl}(\omega) \) of the equation
\[
(vq)^2 - [1 - (1 - g^2(q))]^2 \omega^2 = 0 .
\]
As in 1D, the plasmon spectrum is gapless and makes an important contribution to low energy properties.

In Eq. (8) we need the imaginary part of \( Y(\omega) \), which has a \( \delta \)–function contribution at each of the poles of \( Y(\omega) \), but also a regular part, and we find correspondingly \( \text{Im} Y = Y^{(\text{par})} + Y^{(\text{plas})} + Y^{(\text{reg})} \) with the particle and plasmon contributions (for \( \omega > 0 \))
\[
Y^{(\text{par})}(\omega) = -\frac{1}{4\nu_0 v^2} ,
\]
\[
Y^{(\text{plas})}(\omega) = \frac{1}{2\nu_0 v^2} \left[ 1 + \frac{q[1 - g^2] g^2 \nu_0 d\epsilon}{2 - g^2} \right]^{-1} \]
where \( q = q_{pl}(\omega) \).
and the regular part
\[ Y^{(\text{reg})}(\omega) = -\frac{1}{4\pi \nu_0 v^2} \int_0^\infty dq \]

\[ \times \frac{2v^2 q \omega (1 - g^2)}{\pi \sqrt{(vq)^2 - \omega^2} \left[(vq)^2 - [1 - (1 - g^2)^2] \omega^2\right]} \].

To proceed, we consider a bare interaction \( V_0(q) \) of the form
\[ V_0(q) = \frac{2\pi e^2}{\varepsilon_d} (1 - e^{-2q\Delta}), \]
which is the 2D Coulomb interaction in presence of a screening gate at distance \( \Delta \). Then \( \text{Im} Y(\omega) \) contains three energy scales: \( \omega_0 = 2\pi \nu_0 v^2 = 4e_F \), \( \Omega_\kappa = \kappa v \), where \( \kappa = 2\pi \nu_0 v^2 / \varepsilon_d \) is the two-dimensional inverse screening length, and \( \Omega_\Delta = v / 2\Delta \). For the bare 2D Coulomb interaction, i.e. \( \Delta \to \infty \), only the two scales \( \omega_0 \) and \( \Omega_\kappa \) remain, that are related by \( r_s = 2\Omega_\kappa / \omega_0 \). For finite \( \Delta \) it is convenient to introduce the ratio \( \lambda = \Omega_\kappa / \Omega_\Delta = 2\kappa \Delta \).

In 2D disordered systems \( \text{Im} Y \) diverges at low frequencies which leads to a divergence of \( J(t) \) for \( t \to \infty \) implying a total suppression of \( \nu(\epsilon) \) for \( \epsilon \to 0 \). The same is true for 1D ballistic wires. In the 2D ballistic case considered here, \( \text{Im} Y \) remains finite or even vanishes in presence of a gate as shown in Fig. 1. Accordingly, at \( T = 0 \), \( J(t) \) approaches a constant \( J_{\text{as}} \) for \( t \to \infty \), which is given by
\[ J_{\text{as}} = -\int_0^\infty \frac{d\omega}{\pi} \text{Im} Y(\omega). \tag{13} \]

It is important to note that the explicit result for \( \text{Im} Y(\omega) \) given above does not suffice to determine \( J_{\text{as}} \) explicitly, since the integral also has contributions from frequencies that are not small compared to \( e_F \) where the true parabolic dispersion must be used, which leads to a faster decay of \( \text{Im} Y(\omega) \) at high frequencies. However, this high energy behavior chiefly affects the quantity \( J_{\text{as}} \), while for large times the remaining part \( J'(t) \) in the decomposition
\[ J(t) = J_{\text{as}} + J'(t) \]
is determined by the low energy behavior of \( Y(\omega) \).

Accordingly, the spectral density \( S(\omega) \) splits into
\[ P(\omega) = S\delta(\omega) + P'(\omega), \]
with the regular part
\[ P'(\omega) := S\theta(\omega) \int \frac{dt}{2\pi} e^{iEt} (e^{i\epsilon(t)} - 1), \]
where \( S = e^{J_{\text{as}}} < 1 \) is a suppression factor. Hence, the \( \delta(\omega) \)-function form of the spectral density of a noninteracting system partially survives in 2D ballistic electron systems.

From Eq. (10) the DOS now reads
\[ \nu(\epsilon) = S\nu_0 + \nu'(\epsilon), \]
where the energy dependent part is given by
\[ \nu'(\epsilon) = \nu_0 \int_0^{\epsilon} d\epsilon' P'(\epsilon'). \]
Clearly, \( \nu(\epsilon) \) has a nonzero value \( \nu(0) = S\nu_0 \) at \( \epsilon = 0 \), which depends on the integral property \( J_{\text{as}} \) and thus on the interaction strength as opposed to earlier predictions. Also \( \nu'(\epsilon) \) contains the suppression factor \( S \) as a prefactor. Further, the slope and curvature of \( \nu(\epsilon) \) at \( \epsilon = 0 \) are not universal but depend on the interaction strength \( r_s \).

To determine the value of \( J_{\text{as}} \) and \( S \) explicitly without artificial cutoffs, we either have to use the full quadratic dispersion or to approximate the less important regular part of \( Y(\omega) \). Since \( Y^{(\text{reg})}(\omega) \) approaches \(-1/4\pi \nu_0 v^2 \) for small and large frequencies and does not deviate much from this limiting value in between, we replace it by that constant. In this approximation, which does not necessarily capture the true high frequency behavior and thus gives only an estimate of \( J_{\text{as}} \) when inserted into Eq. (13), the function \( \text{Im} Y(\omega) \) is of the form \( \text{Im} Y(\omega) = \frac{\pi}{\omega_0} y(\omega / \Omega_\kappa, \lambda) \). From this scaling form we find
\[ J_{\text{as}} = -r_s \zeta(\lambda) \] with \( \zeta(\lambda) = 1/2 \int_0^\infty du y(u, \lambda) \), which indicates that the factor \( S = e^{-r_s \zeta(\lambda)} \) rapidly suppresses \( \nu(0) \) with increasing \( r_s \). With decreasing gate distances the suppression factor approaches 1, which means that interaction effects become weaker. These features of the DOS are also apparent from Fig. 2 which displays results for various values of \( r_s \) with and without gate. Without a gate the DOS is cusplike near \( \epsilon = 0 \), whereas in presence of a gate the slope at \( \epsilon = 0 \) vanishes. This latter result is in accordance with the perturbative analysis.

For finite temperatures and in the absence of a gate the denominator in Eq. 5 behaves singular at \( \omega = 0 \).
While $|J(t)|$ then exceeds $|J_{\text{as}}|$, it turns out that $\nu(0)$ always increases if the temperature is raised. This means that finite temperatures smear out the cusplike DOS and reduces the size of interaction effects near $\epsilon = 0$.

Now, two remarks are in order. Strongly interacting semiconductor systems, where values for $r_s$ up to 40 have been observed \[\text{[17]}\], are usually disordered, but a ballistic regime is feasible \[\text{[10]}\]. For large $r_s$, our analysis does not necessarily apply and, in particular, as was discussed above, we cannot obtain reliable values for $J_{\text{as}}$. However, if no scales other than $\epsilon_F$ and $\Omega_\kappa$ enter, $J_{\text{as}}$ should be of the form $J_{\text{as}} = -r_s\zeta(r_s, \lambda)$. Then $S$ will vanish for $r_s \gg 1$, provided $\lim_{r_s \to \infty} r_s\zeta(r_s, \lambda) = \infty$.

Second, in the absence of a gate we have $\nu(\epsilon) \approx \nu_0 S[1 + |\epsilon|/4\epsilon_F + O(\epsilon^2)]$. This differs from previous perturbative approaches giving $\nu(\epsilon) \approx \nu_0(1+|\epsilon|/4\epsilon_F)$ \[\text{[11, 12]}\] independent of $r_s$. The lack of a dependence on the interaction strength can be traced back to the fact that in these works $\Im Y(\omega)$ is effectively replaced by a constant and cut off at the Fermi level. Then, no interaction scale remains and therefore the authors obtain a universal (interaction independent) result. From Fig. 4 we see that this is not the case, but the dominant plasmon contribution to $\Im Y(\omega)$ falls off on the scale $\Omega_\kappa$, which depends on the interaction strength. Indeed, if we turn off the interaction, the corrections to the noninteracting DOS $\nu_0$ vanish.

Throughout this Letter, we have considered a pure 2DEG without disorder. In weakly disordered 2DEGs with elastic mean time $\tau$ electrons move diffusively on long time scales corresponding to excitation energies small compared to $1/\tau$. Then, at $T = 0$ the DOS drops down to zero for energies below $1/\tau$ where the behavior passes over to an Altshuler–Aronov ZBA \[\text{[3–6]}\]. Some authors have investigated the crossover from the diffusive to the quasiballistic limit perturbatively \[\text{[11, 12, 17]}\]. The clean limit of these treatments does not yield our results for the reasons discussed above.

Experimentally it is possible to realize very clean and strongly interacting 2DEGs (or 2D hole gases) \[\text{[17, 16]}\], but so far mainly transport measurements have been performed in the context of the metal–insulator transition. Tunneling experiments in samples with high $r_s$ have been suggested \[\text{[12]}\], but have not yet been performed, since they are more involved, because a counter electrode has to be added at a small distance without influencing the quality of the device. Nevertheless, tunneling experiments with a ballistic 2DEG seem to be feasible in the future. Interaction effects on the DOS in 2DEGs are also relevant for experiments with 2D–1D tunnel junctions \[\text{[18]}\], because there not only the DOS of the 1D wire is measured, but a convolution with the suppressed DOS of the 2DEG.

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