Probing Majorana Physics in Quantum Dot Shot Noise Experiments

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We consider a quantum dot coupled to a topological superconductor and two normal leads and study transport properties of the system. Using Keldysh path-integral approach, we study current fluctuations (shot noise) within the low-energy effective theory. We argue that the combination of the tunneling conductance and the shot noise through a quantum dot allows one to distinguish between the topological (Majorana) and non-topological (e.g., Kondo) origin of the zero-bias conduction peak. Specifically, we show that, while the tunneling conductance might exhibit zero-bias anomaly due to Majorana or Kondo physics, the shot noise is qualitatively different in the presence of Majorana zero modes.

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Introduction. The search for topological superconductors hosting non-Abelian quasiparticles (defects binding Majorana zero modes) has become an active and exciting pursuit in condensed matter physics[1–3]. There has been enormous theoretical and experimental activity in this direction recently [4] fueled, in part, by the potential application of topological superconductors for the fault-tolerant topological quantum computation schemes[5]. A large number of theoretical proposals for engineering topological superconductors in the laboratory has been put forward [6–15], and there has been a significant amount of experimental activity in this area recently [16–25]. One of the simplest ways to detect the presence of Majorana zero modes (MZMs) in topological superconductors (TSC) is tunneling spectroscopy. Indeed, the presence of MZMs leads to a quantized zero-bias conductance $G = 2e^2/h$ [26–33]. The pioneering Majorana experiment based on a semiconductor/superconductor heterostructure proposal [10, 11] was performed in Delft [16] where the observation of zero-bias peak in a finite magnetic field was reported, consistent with the theoretical predictions[31]. However, other effects might also lead to the zero-bias anomaly which spurred the debate[34–38] as to the precise origin of the (un-quantized) zero-bias conductance peak observed in recent tunneling experiments [16–21]. Therefore, additional experiments testing other properties on MZMs[17, 36, 37, 39–50] are necessary in order to reach a consensus.

In this work, we propose to use a combination of the tunneling conductance and current fluctuations measurements to distinguish between the topological (Majorana) and other (non-topological) origin of the zero-bias peak. We consider a quantum dot (QD) coupled to a MZM and two normal leads, see Fig. 1. We show that the current between two normal leads and its fluctuations contain information about MZMs which allows one to distinguish it from the Kondo effect as well as resonant-tunneling physics. Indeed, while all the aforementioned phenomena exhibit zero-bias peaks in the tunneling conductance, their current fluctuations are qualitatively different suggesting that one could use shot noise as a diagnostic tool for MZMs. The physics of the QD coupled to the MZM has been discussed in Refs. [41, 44, 51, 52]. It has been shown that Majorana coupling significantly modifies the low-energy properties of the QD and drives the system to a new (different from Kondo) fixed point [44]. Building on top of the slave boson formalism developed in Ref. [44], we compute here the shot noise in the system shown in Fig. 1. It is well-known that noise measurements usually provide additional information for correlated systems [53–57] and often allow one to identify the nature of the charge carriers. Shot noise for the non-interacting systems such as the normal lead-TSC and non-interacting QD-TSC have been considered in Refs.[48, 58–61]. In this paper, we address this important and non-trivial question and obtain analytically the power spectrum of shot noise in the presence of the Coulomb interactions in QD and take into account the interplay between Kondo and Majorana physics.

Our main results are summarized in Table I. We find that in the case of symmetric couplings to the leads $\Gamma_L = \Gamma_R$ the shot noise power spectrum $P(\omega \to 0)$ in the presence of MZM coupling exhibits a universal value $e^2/2h$ which is independent of the QD energy level $\epsilon_d$. The shot noise power spectrum for the case of asymmetric couplings is depicted in Fig. 2. It is clear from the figure that the shot noise power spectrum exhibits the zero-bias anomaly for $\Gamma_L < \Gamma_R$ but not for $\Gamma_L > \Gamma_R$.
TABLE I. Shot noise power spectrum $P(\omega \rightarrow 0)$ and conductance $G$ \cite{40, 42, 43} for $\Gamma_L = \Gamma_R$.

|                  | spinless system (non-interacting $U = 0$) | spinful system: Kondo regime ($|\epsilon_d| \gg \lambda, \Gamma$) |
|------------------|-------------------------------------------|-----------------------------------------------------------|
| with MZM         | $P(0) = \frac{\pi}{2}$ and $G = \frac{\pi}{2}$, $P(0) = \frac{\pi}{2}$ and $G = \frac{\pi}{2}$, (independent of $\epsilon_d$) |
| without MZM      | $P(0) = 0$ and $G = \frac{\pi}{2}$, $P(0) = 0$ and $G = \frac{\pi}{2}$, for $\epsilon_d = 0$ |

This is to be contrasted with the case of the resonant Hall insulator [7], see also [62]. We assume here that the fluctuations can be easily accessed.

**Theoretical Model.** We consider a setup shown in Fig. 1 in which a QD is coupled to a MZM $\gamma_1$ localized at the domain wall between a magnetic insulator and an s-wave superconductor at the edge of a Quantum Spin Hall insulator [7], see also [62]. We assume here that the superconducting gap $\Delta$ is large, and develop an effective low-energy theory for the system valid at $E \ll \Delta$:

$$H = H_{\text{Leads}} + H_{\text{Dot}} + H_{L,D} + i\lambda(d_+^\dagger + d_+^\sigma)\gamma_1 + i\delta\gamma_1 \gamma_2.$$  

Here $H_{\text{Leads}} = \sum_{\alpha=L,R} \sum_{k,\sigma} \epsilon_k c_{k\sigma,\alpha}^\dagger c_{k\sigma,\alpha}$, $H_{\text{Dot}} = \sum_{\alpha=L,R} \sum_{k,k'} \epsilon_{kk'} c_{k\sigma,\alpha}^\dagger c_{k'\sigma',\alpha}$ (independent of $\epsilon_d$), $H_{L,D} = \sum_{\alpha=L,R} \sum_{k,k'} (\epsilon_{kk'} d_{k\sigma}^\dagger d_{k'\sigma'} + h.c.)$ describes the leads, QD, and the Lead-QD coupling, respectively. The operators $c_{k\sigma,\alpha}^\dagger$ create a spin-$\sigma$ electron in the $\alpha$-lead (the dot), $n_\sigma = d_{k\sigma}^\dagger d_{k\sigma}$ is the chemical potential of the QD, $U$ is the QD on-site Coulomb interaction, and $t_{kk'}(\lambda)$ is the tunneling coupling between the leads (TSC) and the QD. The splitting energy $\delta$ represents the finite overlap between two MZMs. We note that the time-reversal symmetry is broken by the magnetic insulator which also determines the spin polarization of the MZM. Without any loss of generality we assume $\gamma_1$ only couples to the spin up channel of the QD. The lead and QD Hamiltonians remain SU(2)-invariant under spin rotation.

We first integrate out Majorana operators $\gamma_1$ and $\gamma_2$, which leads to the self-energy $\Sigma(\omega)$ (defined below). We assume that the QD is in the single-occupancy regime $U \gg |\epsilon_d| \gg \lambda, \Gamma$ with $\Gamma$ being the broadening of the QD level due to normal leads $\Gamma = \Gamma_L + \Gamma_R$ with $\Gamma_\alpha = \pi |t_{kk'}|^2 \rho_{\alpha}$; here $\rho_{\alpha}$ is the density state of the leads at the Fermi level. In this limit, one can use a slave boson approximation for an infinite-U Anderson model [63, 64] where the double occupancy of the QD is suppressed. Following standard procedure [63, 64], one can introduce the auxiliary boson $b$ and fermion $f_\sigma$ in order to $d_\sigma \rightarrow d_\sigma f_\sigma b^\dagger$, with the constraint $b^\dagger b + \sum_\sigma f_\sigma^\dagger f_\sigma = 1$. Within the slave boson mean field approximation (SBMF), we replace the bosonic field and the Lagrangian multiplier $\eta$ by their expectation values. The mean field parameter $b$ and $\eta$ can be determined self-consistently by minimizing the free energy. The detail of SBMF calculation in the presence of a MZM can be found in Ref. [44] (also see [65]). The SBMF approach decouples the spin-up channel from the spin-down channel and allows one to compute various correlation functions.

**Shot noise calculation.** We now use the Keldysh path-integral formalism [66] to calculate the current fluctuations. Since two spin channels are decoupled within SBMF approximation, we drop the index $\sigma$ in this derivation. Given that MZM coupling breaks particle number conservation, the QD Green’s function now acquires an anomalous contribution (e.g. $i(T_d d(t) d(t'))$), and we need to work in the Nambu space. We also introduce a lead-QD basis $\Psi^\dagger = (\{c_{Lk}^\dagger, \bar{c}_{kL}, d_+^\dagger, d^\dagger, \{\bar{c}_{Rk}^\dagger, c_{kR}\})/\sqrt{2}$, and write the action in this new space $S$. The effective action can be written in terms of the full Green function $\hat{Q}$:

$$S = S_0 + S_{L-D} + S_{\text{source}},$$

where

$$S_0 + S_{L-D} = \int_C \int_C dt dt' \hat{\Psi}^\dagger(t) \hat{Q}^{-1}(t, t') \hat{\Psi}(t'),$$

and

$$\hat{Q}_{kk'} = \begin{pmatrix} Q_{Lk,Lk'} & Q_{Lk,d} & Q_{Lk,Rk'} \\ Q_{d,Lk'} & Q_{dd} & Q_{d,Rk'} \\ Q_{Rk,Lk'} & Q_{Rk,Rk'} & Q_{Rk,Rk'} \end{pmatrix}.$$ (4)

All matrix elements above have the same structure. For example, $Q_{d,d}(\omega)$ in the $N$ space is the Fourier transform of

$$Q_{d,d} = \begin{pmatrix} G_{dd} & F_{dd} \\ F_{dd} & G_{dd} \end{pmatrix} = \begin{pmatrix} i(T_d d(t) d(t')) & i(T_d d(t) d(t')) \\ i(T_c d(t) d(t')) & i(T_c d(t) d(t')) \end{pmatrix},$$ (5)

where $T_c$ denotes Keldysh contour ordering. After restoring the spin index, the retarded Green’s functions are given by

$$G_{dd,\sigma}^R(\omega) = \frac{\omega + \bar{\epsilon}_d + i\Gamma - \Sigma_{\sigma}(\omega)}{(\omega + i\Gamma - 2\Sigma_{\sigma}(\omega))(\omega + i\Gamma') - \bar{\epsilon}_d^2},$$ (6)

$$F_{dd,\sigma}^R(\omega) = \frac{-\Sigma_{\sigma}(\omega)}{(\omega + i\Gamma - 2\Sigma_{\sigma}(\omega))(\omega + i\Gamma') - \bar{\epsilon}_d^2},$$ (7)

where $\Sigma_{\sigma}(\omega) = \lambda^2 \rho_0^2 b^2 / (\omega - \lambda^2 \rho_0^2)$ with $\lambda = 0$ and $\lambda = 0$. The effective broadening and energy of the QD level will now be $\Gamma = \Gamma b^2, \bar{\epsilon}_d = \epsilon_d + \eta$. We refer the reader to SI [65] for other parts of $\hat{Q}_{kk'}$.

We now consider current fluctuations through the left junction. The current operator is given by

$$I_L = i e \frac{\hbar}{2} \sum_k \left( \bar{t}_{Lk} c_{Lk}^\dagger d - t_{Lk} c_{Lk}^\dagger d^\dagger \right) = \hat{\Psi}^\dagger M \hat{\Psi},$$ (8)
The 6-by-6 matrix $\hat{M}$ in $N \otimes S$ space for lead momentum $k$ is

$$\hat{M}_k = \frac{i e}{\hbar} \begin{pmatrix} 0 & M_{k}^{12} & 0 \\ M_{k}^{21} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (9)$$

where $M_{k}^{21} = (-\tilde{V}_{kk} - i \tilde{V}_{kk})$ and $M_{k}^{12} = (\tilde{V}_{kk} - i \tilde{V}_{kk})$. Then, the action for the source term is

$$S_{source} = - \int_{C} dt A(t) I_L(t) = - \int_{-\infty}^{\infty} dt \tilde{\Psi}_a \hat{A}_{ab} \hat{M} \tilde{\Psi}_b. \quad (10)$$

Here we rewrote the action in terms of the forward and backward components and performed Larkin-Ovchinnikov rotation\[66\]: $\Psi_{1,2} = (\Psi_{+} \pm \Psi_{-})/\sqrt{2}$ and $\Psi_{1,2}^{\dagger} = (\Psi_{+} \mp \Psi_{-})/\sqrt{2}$ and $A_{\sigma}^{\delta/\delta} = (A_{\sigma} \mp A_{-\sigma})/2$. As a result, the source $\hat{A} = A^{(a)} \tilde{\gamma}_{a}$ is now a matrix in Keldysh space, where $a = cl, q$ with $\tilde{\gamma}_{cl} = 1$ and $\tilde{\gamma}_{q} = \sigma_{1}$, see details in SI[65]. The generating function for this problem $Z[A] = \int D[(c_{\ell L}^{\dagger} c_{\ell L}) d^{d}d(c_{R}^{\dagger} c_{R})]$ can be obtained in the following way[66]: In $Z[A] = Tr \ln \left[ \tilde{I} - \hat{Q} \hat{A} \hat{M} \right]$, where the unit matrix $\tilde{I}$ and the Green function $\hat{Q}$ are defined in $N \otimes S \otimes \mathbb{K}$ space, $\hat{A}$ is in $\mathbb{K}$ space, and $\hat{M}$ is in $N \otimes S$ space. Finally, the symmetrized current noise for left junction can be written as

$$S_I(\omega, eV) = \int dt e^{i \omega t} \langle \delta I_L(t) \delta I_L(0) + \delta I_L(0) \delta I_L(t) \rangle$$

$$= - \frac{1}{4} \delta^{2} \ln Z[A] \bigg|_{A=0} \quad (11)$$

where $\delta I_L(t) = I_L(t) - \langle I_L \rangle$, and an extra factor 1/2 is to remove the doubling of the Hilbert space. The details of the evaluation of Eq.11 are presented in SI[65]. At zero temperature, the shot noise is given by

$$S_I(eV) = \sum_{\sigma} \int d\omega P_{\sigma}(\omega), \quad (12)$$

$$P_{\sigma}(\omega) = \frac{2e^2}{\hbar} \left( \tilde{A}_{N}(\omega) + \tilde{A}_{\lambda}(\omega) \right) \quad (13)$$

Here $P_{\sigma}(\omega)$ is the power spectrum of noise for each spin with $\tilde{A}_{N/\lambda}$ being the contributions to the noise from particle-particle (P-P)/particle-hole (P-H) channels, respectively. After tedious calculations (see SI[65] for details), one finds

$$A_{N}^{\delta} = 2\Gamma_{L} \Gamma_{R} \left[ |G_{dd,\sigma}^{R}|^2 + |G_{dd,\sigma}^{R}|^2 \right] + 4\Gamma_{L}^{2} |F_{dd,\sigma}^{R}|^2$$

$$- 8\Gamma_{L}^{2} |F_{dd,\sigma}^{R}|^2 \left( |G_{dd,\sigma}^{R}|^2 + |G_{dd,\sigma}^{R}|^2 \right) - 16\Gamma_{L}^{2} |F_{dd,\sigma}^{R}|^4$$

$$- 16\Gamma_{L}^{2} \Gamma_{R} \left[ (|G_{dd,\sigma}^{R}|^2 + |G_{dd,\sigma}^{R}|^2) |F_{dd,\sigma}^{R}|^2 \right] \quad (14)$$

where $G_{dd}^{R}$ can be obtained from $G_{dd}^{R}$ by $\epsilon_{d} \rightarrow -\epsilon_{d}$. The P-H contribution is vanishing at zero frequency $\tilde{A}_{N}(\omega) \sim \omega^2$. Here we assume a symmetric bias $V_L = -V_R$.

**Results and Discussions.** Before presenting the results for an interacting QD problem, it is instructive to consider first a non-interacting spinless model, for which the results can be easily obtained by setting $\eta = 0$ and $b = 1$ in $P_{\lambda}(\omega)(13)$. The power spectrum $P(0)$ at $T = 0$ and $\delta = 0$ is

$$P_{\lambda \neq 0}(0) = \frac{2e^2}{\hbar} \frac{\Gamma_{L} \Gamma_{R}}{\Gamma^2} = \frac{e^2}{2h} \left. \frac{1}{\Gamma_{L} = \Gamma_{R}} \right|_{\Gamma_{L} = \Gamma_{R}} \quad (16)$$

$$P_{\lambda = 0}(0) = \frac{2e^2}{\hbar} \frac{4\Gamma_{L} \Gamma_{R}}{\Gamma^2 + \epsilon_{d}^2} \left( 1 - \frac{4\Gamma_{L} \Gamma_{R}}{\Gamma^2 + \epsilon_{d}^2} \right) = \frac{e^2}{h} \frac{\Gamma^2 \epsilon_{d}^2}{(\Gamma^2 + \epsilon_{d}^2)^2} \left. \frac{1}{\Gamma_{L} = \Gamma_{R}} \right|_{\Gamma_{L} = \Gamma_{R}} \quad (17)$$

One can see that coupling to MZMs dramatically modifies the shot noise. For example, at the symmetric point
the shot noise power does not depend on $\epsilon_d$ and is given by $e^2/2h$ whereas without MZM $P_{\lambda=0}(0)$ depends on $\epsilon_d$ and is zero on resonance $\epsilon_d = 0$. By tuning the coupling asymmetry $\Gamma_L/\Gamma_R$ for the spinless case, one should observe a qualitative different behaviour with and without MZMs: in the former case, both tunneling conductance and shot noise are increasing and reaching the maximum at $\Gamma_L = \Gamma_R$ whereas in the latter case the increase of the conductance is accompanied with decreasing of $P(0)$. The shot noise at finite bias $eV$ is given by Eq.(12). In order to understand the $eV \neq 0$ results, we plot the power spectrum $P(\omega)$ in Fig. 2 (a), which shows a two-peak structure. For $\lambda \ll \Gamma$, we find that the width between the two peaks $\sim \lambda^2/\Gamma$, and for $\lambda \gg \Gamma$, this width becomes $\sim \Gamma$.

We now discuss results at finite splitting $\delta \neq 0$. As shown in Fig. 2 (b), the spectral function for a finite $\delta$ exhibits two peaks at small $\omega$. When $\lambda \ll \Gamma$, the position of the peak is at $\pm \delta$. Thus, in order to observe the predicted value $P(0) = e^2/2h$, one should adjust the voltage to be $\lambda^2/\Gamma \gg eV \gg \delta$. When $\lambda \gg \Gamma$, the width of the splitting is $\Gamma\delta^2/\lambda^2$. Thus, the condition to observe $P(0) = e^2/2h$ value is $\Gamma \gg eV \gg \Gamma\delta^2/\lambda^2$. We plot the shot noise as both a function of the $\lambda$ and $\delta$ in Fig. 3. One can see that the larger the splitting energy $\delta$, the larger Majorana coupling $\lambda$ is needed to observe the predicted value for the shot noise $P(0) = e^2/2h$. The effect of varying $\epsilon_d$ is discussed in [65].

The conclusion based on the results of the spinless non-interacting problem is that Majorana coupling qualitatively modifies the shot noise through the QD. Thus, the combination of the conductance and shot noise measurements allow one to clarify the nature of the zero-bias conduction feature, see Table I. Even though the spinful problem is more complicated, we show that this qualitative feature persists in the presence of interactions and allows one to distinguish between the Majorana and Kondo origin of the zero-bias feature in the tunneling conductance. We now consider the QD in the limit of single-occupancy $U \gg |\epsilon_d| \gg \Gamma, \lambda$ and $eV \ll T_K$ with $T_K$ being the Kondo temperature. We first analyze the case of no splitting $\delta = 0$. A recent study based on SBMF approach [44] shows that a crossover from Kondo- and Majorana-dominated regimes can be realized by tuning the coupling $\lambda$. For $\lambda \ll \lambda_c \equiv \sqrt{\Gamma/|\epsilon_d|}$, Kondo effect is important [44]: the renormalized coupling corresponds to Kondo temperature $\tilde{\Gamma} = \Gamma b^2 = T_K = \Lambda \exp(-\pi|\epsilon_d|/2\Gamma)$ and the renormalized energy level is $\tilde{\epsilon}_d = |\epsilon_d + \eta| \sim \Gamma b^4$ (Here $\Lambda$ is the bandwidth and $b \ll 1$ is the variational parameter). When $\lambda \gg \lambda_c$, the parameter $b \sim \lambda/|\epsilon_d|$ is determined by the Majorana coupling rather than the Kondo temperature. One can see that in the perturbative regime $|\epsilon_d| \gg \Gamma$, $\lambda$ corresponding to $b \ll 1$, the position of the renormalized level is close to the Fermi energy $e^{-\tilde{\epsilon}_d} \sim \Gamma b^4 \ll \tilde{\Gamma}$ [44]. In both cases the spin-down channel shows perfect transmission (i.e. linear conductance $G = e^2/h$), and, thus, its contribution to the shot noise is zero. On the other hand, the shot noise for the spin-up channel, due to the coupling to MZM, corresponds to the universal value $e^2/2h$ independent of $\epsilon_d$. The conductance $G$ and shot noise for spinful QD can be summarized as follows. The linear conductance for $|\epsilon_d| \gg \lambda, \Gamma$ reads

$$G = \frac{4e^2}{h} \frac{\epsilon^2}{1 + \frac{3e^2}{2}\frac{2\Gamma}{\epsilon^2}} = \frac{3e^2}{2h} \left(1 + \frac{1}{2} + 1\right)$$

which is consistent with the numerical renormalization group calculation [42]. The shot noise power is

$$P(0) = \frac{2\epsilon^2}{h} \frac{\epsilon^2}{1 + \frac{1}{2} + 1} = \frac{e^2}{2h}.$$  

The results beyond the $|\epsilon_d| \gg \lambda, \Gamma$ limit can be obtained numerically and are discussed in [65].

We now consider the effect of a finite energy splitting $\delta \neq 0$ and a finite bias $eV \neq 0$ which is important for
the experimental detection of the effect we predict here. The shot noise $S_I(eV)/eV$ as a function of $\lambda$ and $\delta$ is shown in the Fig. 4. One can see that in order to resolve the quantized value $P(\theta) = e^2/2h$, one has to satisfy the following conditions: a) in the regime $b\lambda \ll b\Gamma$, the voltage should be $\lambda^2/\Gamma \gg eV \gg \delta$; b) in the $\lambda \gg b\Gamma$ regime, the condition becomes $b^2\Gamma \gg eV \gg \delta^2/\lambda^2$. It is thus clear that in the Majorana-dominated regime, i.e. $\lambda \gg |\epsilon_d| \gg b\Gamma$, the voltage should satisfy condition b), in which case the shot noise power spectrum exhibits a plateau around $S_I(eV)/eV = e^2/2h$, see Fig. 4. One can also notice that the width of the plateau around $S_I(eV)/eV = e^2/2h$ gradually shrinks with increasing $\delta$. In the strong coupling limit $\lambda \sim |\epsilon_d|$, the renormalized energy level $\epsilon_d$ shifts away from the Fermi level since $b \sim 1$, which, in turn, suppresses the conductance at zero bias and enhances the shot noise, see [65].

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Supplementary Information for “Probing Majorana Physics in Quantum Dot Shot Noise Experiments”

In this supplementary information document we will provide 1) the details of the derivation of the shot noise formula $S_I(eV)$ defined in Eq.(11) of the main text; 2) the discussion of the effects of the dot energy level $\epsilon_d$; 3) the main steps in slave boson mean field approach.

The derivation of the shot noise $S_I(eV)$ in Eq.(12) of the main text

We first derive the full impurity Green function $Q_{d,d}$ in Eq. (5) of the main text. For the clarity of presentation, we drop the tilde index in $\Gamma$ and $\epsilon_d$. The dependence on $b$ and $\eta$ can be easily restored. For simplicity, we drop the spin index $\sigma$ in this derivation. Using the following convention for the Nambu spinors: $\Psi_{\alpha k}^\dagger = (c_{\alpha k}^\dagger, c_{\alpha k})/\sqrt{2}$ and $\Psi_d^\dagger = (d^\dagger, d)/\sqrt{2}$, where $\alpha = L,R$ is the lead index, the effective Keldysh action for each spin channel now reads

$$S = S_0 + S_{L-D},$$

where

$$S_0 = \sum_{kk',\alpha} \int_C \int_C dt dt' \Psi_{\alpha k}^\dagger(t) \tilde{Q}_{0,kk',\alpha}^{-1}(t,t') \Psi_{\alpha k}(t') + \int_C \int_C dt dt' \Psi_d^\dagger(t) \tilde{Q}_{0,d}^{-1}(t,t') \Psi_d(t'),$$

$$S_{L-D} = -\sum_{k\alpha} \int dt \left( t_{\alpha k} c_{\alpha k}^\dagger d + c.c. \right)$$

$$= -\sum_{k\alpha} \int dt (\tilde{\Psi}_{\alpha k}^\dagger(t) M_{T,\alpha k} \tilde{\Psi}_d(t) + h.c.),$$

are the actions for leads, QD, and Lead-QD coupling, and $M_{T,\alpha k} = (t_{\alpha k}^* 0 - t_{\alpha k}^{\dagger})$. Here the integration is over the Keldysh contour. The free lead Green’s function $\tilde{Q}_{0,kk',\alpha}$ and free QD Green’s function $\tilde{Q}_{0,d}$ (with MZM coupling) in the Nambu space $\mathbb{N}$ can be written as

$$Q_{0,kk',\alpha}(t-t') = \delta_{kk'} \begin{pmatrix} g_{\alpha k}(t-t') & 0 \\ 0 & \bar{g}_{\alpha k}(t-t') \end{pmatrix} , Q_{0,d}(t-t') = \begin{pmatrix} G_{0,dd}(t-t') & F_{0,dd}(t-t') \\ F_{0,dd}(t-t') & G_{0,dd}(t-t') \end{pmatrix} ,$$

where $\bar{g}_{\alpha k}(t-t')$ ($G_{0,dd}(t-t')$) is the P-H conjugation of $g_{\alpha k}(t-t')$ ($G_{0,dd}(t-t')$). We first perform Larkin-Ovchinnikov rotation [66]. The retarded, advanced and Keldysh components of the Green’s function are defined as

$$G_{ab}(t,t') = -i \langle \Psi_a(t) \Psi_b^{\dagger}(t') \rangle = \begin{pmatrix} G^R(t,t') & G^K(t,t') \\ G^A(t,t') & G^A(t,t') \end{pmatrix}$$

We remind the reader that the relationship between Keldysh Green’s function before and after LO rotation is

$$\begin{pmatrix} G^R(t,t') & G^K(t,t') \\ 0 & G^A(t,t') \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} G^T(t,t') & G^Z(t,t') \\ G^>(t,t') & G^< (t,t') \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$
After performing the Gaussian integration and Fourier transformation, one can obtain the full impurity Green function
\[ Q_{dd}(\omega)^{-1} = Q_{0,dd}(\omega)^{-1} - \left( \sum_{\alpha,k} |t_{\alpha,k}|^2 g_0^\alpha(\omega) \right) \frac{0}{\sum_{\alpha,k} |t_{\alpha,k}|^2 g_0^\alpha(\omega)}. \] (27)

The free QD Green function (with MZM coupling) can be written as
\[ Q_{0,dd}(\omega) = \left( \begin{array}{cccc}
G_{0,dd}^R & G_{0,dd}^K & F_{0,dd}^R & F_{0,dd}^K \\
F_{0,dd}^R & F_{0,dd}^K & G_{0,dd}^R & G_{0,dd}^K \\
0 & 0 & F_{0,dd}^A & F_{0,dd}^A \\
0 & 0 & F_{0,dd}^A & F_{0,dd}^A 
\end{array} \right), \] (28)

where
\[ G_{0,dd}^R(\omega) = [G_{0,dd}^A(\omega)]^* = \frac{\omega + i\eta + \epsilon_d - \Sigma(\omega)}{(\omega + i\eta - 2\Sigma(\omega))(\omega + i\eta) - \epsilon_d^2}, \] (29)
\[ G_{0,dd}^R_\alpha(\omega) = [G_{0,dd}^A_\alpha(\omega)]^* = \frac{\omega + i\eta - \epsilon_d - \Sigma(\omega)}{(\omega + i\eta - 2\Sigma(\omega))(\omega + i\eta) - \epsilon_d^2}, \] (30)
\[ F_{0,dd}^R_\alpha(\omega) = F_{0,dd}^R_\alpha(\omega) = [F_{0,dd}^A_\alpha(\omega)]^* = \frac{-\Sigma(\omega)}{(\omega + i\eta - 2\Sigma(\omega))(\omega + i\eta) - \epsilon_d^2}. \] (31)

Since we assume that \( E \ll \Delta \), all Keldysh components \( G_{0,dd}^K, G_{0,dd}^A, F_{0,dd}^K, F_{0,dd}^A \) are zero. The Green functions of the free lead electron are related to tunneling rate \( \Gamma_\alpha = \pi |t_\alpha|^2 \rho_F \) (\( \rho_F \) is the density of state of the leads near Fermi level)
\[ \sum_k |t_{\alpha,k}|^2 g_0^\alpha(\omega) = \left( \begin{array}{c}
-i\Gamma_\alpha & -2i\Gamma_\alpha(1-2n_\alpha) \\
0 & i\Gamma_\alpha
\end{array} \right), \] (32)
\[ \sum_k |t_{\alpha,k}|^2 g_0^\alpha(\omega) = \left( \begin{array}{c}
-i\Gamma_\alpha & -2i\Gamma_\alpha(1-2\bar{n}_\alpha) \\
0 & i\Gamma_\alpha
\end{array} \right). \] (33)

One notices that \( g_0^{0,R}(\omega) = -g_0^{0,A}(\omega) \) and \( g_0^{0,K}(\omega) = -g_0^{0,K}(\omega) \). Here, \( n_\alpha \) is the fermi distribution function of the \( \alpha \) lead with chemical potential \( \mu_\alpha \), and \( \bar{n}_\alpha \) corresponds to the fermi distribution function with \(-\mu_\alpha\). We consider a symmetric source-drain bias \((\mu_L = eV/2 \text{ and } \mu_R = -eV/2)\), and thus have \( \bar{n}_L = n_R \) and \( \bar{n}_R = n_L \). We note in passing here that for asymmetric couplings \( \Gamma_R \neq \Gamma_L \), the Keldysh Green’s function for the dot \( G_{dd}^K \) depends on coupling to MZM \( \lambda \).

To derive the shot noise, we then consider Eq. (8) of the main text. The source action on the Keldysh contour reads
\[ S_{\text{source}} = -\int_C dt A(t) I_L(t) = -\int_{-\infty}^\infty dt \left[ \tilde{\Psi}_+^\dagger A_+(t) \hat{M} \tilde{\Psi}_+ - \tilde{\Psi}_-^\dagger A_-(t) \hat{M} \tilde{\Psi}_- \right]. \] (34)

where \( \tilde{\Psi}_\pm \) and \( A_\pm(t) \) are the fermionic fields and source fields on the forward and backward branches of the Keldysh contour. We again perform Larkin-Ovchinnikov rotation [66]: \( \psi_{1,2} = (\psi_+ \pm \psi_-)/\sqrt{2} \) and \( \psi_{1,2}^\dagger = (\psi_+^\dagger \mp \psi_-^\dagger)/\sqrt{2} \) and \( A^{cl/q} = (A_+ \pm A_-)/2 \). Thus, the source term now becomes
\[ S_{\text{source}} = -\sum_{\alpha,b=1,2} \int_{-\infty}^\infty dt \tilde{\Psi}_\alpha^\dagger \hat{A}_{ab} \hat{M} \tilde{\Psi}_b, \] (35)

where \( \hat{A} = A^{cl}\hat{a}_0 \) is now a matrix in Keldysh K space, where \( \alpha = cl, q \) with \( \hat{c}_l \equiv 1 \) and \( \hat{c}_q \equiv \sigma_1 \). Using Eqs.(5), (10) and (11), one finds that the shot noise is given by
\[ S_I(eV) = \frac{1}{4} \int \frac{d\omega}{2\pi} \sum_{kk'} \text{Tr} \left\{ \hat{Q}_{kk'} (\hat{c} \hat{M}_{kk'} \hat{c} \hat{M}_{kk'}) \right\} \]
\[ = \frac{1}{4} e^2 \int \frac{d\omega}{2\pi} \sum_{kk'} \text{Tr} \left\{ Q_{L,k,d} (\hat{c} \hat{M}_{k21}^{cl}) Q_{L,k',d} (\hat{c} \hat{M}_{k21}^{cl}) + Q_{dL,k'} (\hat{c} \hat{M}_{k12}^{cl}) Q_{dL,k} (\hat{c} \hat{M}_{k12}^{cl}) \right. \]
\[ + Q_{L,k,k'} (\hat{c} \hat{M}_{k22}^{cl}) Q_{d,d} (\hat{c} \hat{M}_{k22}^{cl}) + Q_{d,d} (\hat{c} \hat{M}_{k22}^{cl}) Q_{L,k',L,k} (\hat{c} \hat{M}_{k12}^{cl}) \} \] (36)
where $Q_{\alpha k,\alpha k'}(\omega)$ and $Q_{\alpha k,d}(\omega)$ are Fourier transform of

$$
Q_{\alpha k,\alpha k'}(t-t') = \left( G_{\alpha k,\alpha k'} \begin{pmatrix} F_{\alpha k,\alpha k'} \\ F_{\alpha k,\alpha k'}^\dagger \end{pmatrix} G_{\alpha k,\alpha k'} \right) = - \left( i(T_{c\alpha k}(t)c_{\alpha k'}^\dagger(t')) i(T_{c\alpha k'}(t)c_{\alpha k}(t')) \right),
$$

$$
Q_{\alpha k,d}(t-t') = \left( G_{\alpha k,d} \begin{pmatrix} F_{\alpha k,d} \\ F_{\alpha k,d}^\dagger \end{pmatrix} G_{\alpha k,d} \right) = - \left( i(T_{c\alpha k}(t)d_{\alpha k'}^\dagger(t')) i(T_{c\alpha k'}(t)d_{\alpha k}(t')) \right),
$$

and one can define $Q_{d,\alpha k'}$ in a similar way. We apply the matrix product in $S$ space. For example, we expand the first term to find

$$
\text{Tr}\left\{ Q_{Lk,d} (\gamma^q M_{k'}^{21}) Q_{Lk',d} (\gamma^q M_{k'}^{21}) \right\} = \text{Tr}\left\{ t_{Lk'}^* t_{Lk'}^* G_{Lk,d} \gamma^q G_{Lk',d} \gamma^q + t_{Lk} t_{Lk'}^* G_{Lk,d} \gamma^q G_{Lk',d} \gamma^q + t_{Lk} t_{Lk'}^* G_{Lk,d} \gamma^q G_{Lk',d} \gamma^q + t_{Lk'} t_{Lk} F_{Lk,d} \gamma^q F_{Lk',d} \gamma^q + t_{Lk'} t_{Lk} F_{Lk,d} \gamma^q F_{Lk',d} \gamma^q \right\}
$$

Similarly, we expand the three other terms and obtain

$$
S_I(eV) = S_{I,N}(eV) + S_{I,A}(eV),
$$

where

$$
S_{I,N}(eV) = -\frac{1}{4} \left( \frac{e}{\hbar} \right)^2 \int \frac{d\omega}{2\pi} \sum_{kk'} \text{Tr}\left\{ t_{Lk}^* t_{Lk'}^* G_{Lk,d} \gamma^q G_{Lk',d} \gamma^q + t_{Lk} t_{Lk'}^* G_{Lk,d} \gamma^q G_{Lk',d} \gamma^q + t_{Lk} t_{Lk'}^* G_{Lk,d} \gamma^q G_{Lk',d} \gamma^q \right\} + \text{P-H conjugation}
$$

and

$$
S_{I,A}(eV) = -\frac{1}{2} \left( \frac{e}{\hbar} \right)^2 \int \frac{d\omega}{2\pi} \sum_{kk'} \text{Tr}\left\{ -t_{Lk}^* t_{Lk'} G_{Lk,d} \gamma^q F_{d,d} \gamma^q - t_{Lk} t_{Lk'}^* F_{d,d} \gamma^q G_{Lk',d} \gamma^q + t_{Lk}^* t_{Lk'}^* G_{Lk,d} \gamma^q F_{d,d} \gamma^q + t_{Lk}^* t_{Lk'}^* F_{d,d} \gamma^q G_{Lk',d} \gamma^q \right\}.
$$

The first (i.e. normal) contribution also appears in the case without Majorana zero mode (particle-particle and hole-hole channels) whereas the second contribution represents an anomalous part due to the MZM coupling (particle-hole channel).

![Diagram](image)

**FIG. 5.** Diagrammatic representation of the free electron Green function $g_{Lk}^0(t,t')$ and the full impurity Green function $G_{d,d}^0(t,t')$, $F_{d,d}^0(t,t')$ and $F_{d,d}^0(t,t')$.

We first consider the normal part. The Green function $G_{Lk,Lk'}$, $G_{Lk,d}$, and $G_{d,Lk}$ are related to the free electron Green function $g_{Lk}^0$ and the QD Green function $G_{d,d}$ via equations of motion, and thus can be written as

$$
G_{Lk,Lk'}(\omega) = g_{Lk}^0(\omega) \delta_{kk'} + t_{Lk}^* t_{Lk'}^* g_{Lk}^0(\omega) G_{d,d}(\omega) g_{Lk'}^0(\omega),
$$

$$
G_{Lk,d}(\omega) = t_{Lk} g_{Lk}^0(\omega) G_{d,d}(\omega),
$$

$$
G_{d,Lk}(\omega) = t_{Lk}^* G_{d,d}(\omega) g_{Lk}^0(\omega).
$$
We then insert those equations into Eq. (41), and obtain
\[
S_{I,N}(eV) = \frac{1}{4} \left( \frac{e}{h} \right)^2 \int \frac{d\omega}{2\pi} \left\{ \text{Tr} \left\{ \sum_k |t_{Lk}|^2 \gamma_q^q \tilde{G}_{d,d}(\omega) \gamma_q^q + G_{d,d}(\omega) \gamma_q^q \sum_k \bar{G}_{d,d}(\omega) \gamma_q^q \right\} + \text{P-H conjugation} \right\}
\]

In a diagrammatic representation defined in Fig. 5, the normal contribution to the shot noise can be described by the diagrams shown in Fig. 6. We insert the Green functions $G_{d,d}(\omega)$, $G_{d,d}(\omega)$, $\tilde{G}_{d,d}(\omega)$, and $\tilde{G}_{d,d}(\omega)$ from Eq. (27), (32), and (33) into Eq. (46). We choose a symmetric source-drain bias, and take the zero temperature limit. Then, we obtain the normal part of the shot noise for a spinless model, e.g. Eq. (12) and (14) of the main text:

\[
S_{I,N}(eV) = \frac{2e^2}{h} \int_{-eV/2}^{eV/2} A_N(\omega) d\omega,
\]

where

\[
A_N(\omega) = 2 \Gamma_L \Gamma_R \left( |G_{d,d}^R(\omega)|^2 + |G_{d,d}^R(\omega)|^2 \right) + 4 \Gamma_L^2 |F_{dd}(\omega)|^2 - 8 \Gamma_L^2 \Gamma_R^2 \left( |G_{d,d}^R(\omega)|^4 + |G_{d,d}^R(\omega)|^4 \right) - 16 \Gamma_L^4 |F_{dd}(\omega)|^4 - 16 \Gamma_L^3 \Gamma_R^2 \left( |G_{d,d}^R(\omega)|^2 + |G_{d,d}^R(\omega)|^2 \right) |F_{dd}(\omega)|^2.
\]

Here, to derive this compact form, we use the relations among $G_{d,d}$, $G_{d,d}$, $F_{dd}$, and $F_{dd}$, which can be obtained from Eq. (27).
We then consider the anomalous part $S_{I,A}(eV)$ in Eq. (42). We first write down equations of motion such that all Green function can be described by impurity Green function and free electron Green functions:

$$F_{Lk,Lk'}(\omega) = -t_k t_{k'} g_{Lk}^0(\omega) F_{d,d}(\omega) g_{Lk'}^0(\omega),$$  \hspace{1cm} (49)

$$F_{Lk,Lk'}(\omega) = -t_k^* t_{k'} g_{Lk}^0(\omega) F_{d,d}(\omega) g_{Lk'}^0(\omega),$$  \hspace{1cm} (50)

$$F_{Lk,d}(\omega) = t_k g_{Lk}^0(\omega) F_{d,d}(\omega),$$  \hspace{1cm} (51)

$$F_{Lk,d}(\omega) = -t_k^* g_{Lk}^0(\omega) F_{d,d}(\omega),$$  \hspace{1cm} (52)

$$F_{d,Lk}(\omega) = -t_k F_{d,d}(\omega) g_{Lk}^0(\omega),$$  \hspace{1cm} (53)

$$F_{d,Lk}(\omega) = t_k^* F_{d,d}(\omega) g_{Lk}^0(\omega).$$  \hspace{1cm} (54)

We then insert those relations into Eq. (42), and obtain

$$S_{I,A}(eV) = \frac{1}{2} \left( \frac{e}{\hbar} \right)^2 \int \frac{d\omega}{2\pi} \text{Tr} \left\{ -\sum_k |t_{Lk}|^2 g_{Lk}^0(\omega) F_{d,d}(\omega) \sum_{k'} |t_{Lk'}|^2 g_{Lk'}^0(\omega) \gamma^q F_{d,d}(\omega) \gamma^q -F_{d,d}(\omega) \gamma^q \sum_{k'} |t_{Lk'}|^2 g_{Lk'}^0(\omega) F_{d,d}(\omega) \gamma^q \sum_k |t_{Lk}|^2 g_{Lk}^0(\omega) \gamma^q +\sum_k |t_{Lk}|^2 g_{Lk}^0(\omega) F_{d,d}(\omega) \gamma^q \sum_{k'} |t_{Lk'}|^2 g_{Lk'}^0(\omega) F_{d,d}(\omega) \gamma^q +F_{d,d}(\omega) \sum_{k'} |t_{Lk'}|^2 g_{Lk'}^0(\omega) \gamma^q \sum_k |t_{Lk}|^2 g_{Lk}^0(\omega) \gamma^q \right\}. \hspace{1cm} (55)$$

In the diagrammatic representation, the anomalous contribution to the shot noise can be described by the diagrams shown in Fig. 7. We insert the Green functions $F_{d,d}(\omega)$, $F_{d,d}(\omega)$, $g_{k}^0(\omega)$, and $g_{k}^0(\omega)$ from Eq. (27), (32), and (33) into Eq. (55). In the $T = 0$ limit, we simplify the the anomalous part of the shot noise and obtain the result in the main text:

$$S_{I,A}(eV) = \frac{2e^2}{h} \int_{-eV/2}^{eV/2} A_A(\omega) d\omega, \hspace{1cm} (56)$$

where

$$A_A(\omega) = \frac{G_L^2}{2} \left[ (F_{dd}^R(\omega) + F_{dd}^A(\omega))^2 - 8(\Gamma_L^2 - \Gamma_R^2) \frac{|F_{dd}^R(\omega)|^2}{\Sigma(\omega)} (F_{dd}^R(\omega) + F_{dd}^A(\omega))^2 +16(\Gamma_L^2 - \Gamma_R^2)^2 (\Gamma_L + \Gamma_R)^2 + \epsilon_d^2 \frac{|F_{dd}^R(\omega)|^4}{\Sigma(\omega)^2} \right]. \hspace{1cm} (57)$$

The effect of the dot energy level $\epsilon_d$ for the spinless non-interacting model

![Figure 8](image_url)

**FIG. 8.** The power spectrum $P(\omega)$ (in units of $2e^2/h$) for (a) $\lambda/\Gamma = 0.1$, (b) $\lambda/\Gamma = 0.8$, and (c) $\lambda/\Gamma = 2.0$. We choose $\Gamma_L = \Gamma_R$, $\delta = 0.0$, and $\epsilon_d/\Gamma = -0.1, -0.5, -1.0, -1.5$.

Although the zero bias shot noise doesn’t depend on the change of the QD chemical potential $\epsilon_d$, i.e. $P(\omega) = A_N(\omega) + A_A(\omega)$ is independent of $\epsilon_d$ at $\omega = 0$, the change of $\epsilon_d$ can affect finite bias noise. The spectral function $P(\omega)$
for different $\epsilon_d$ and $\lambda$ are shown in Fig. 8. For small $\lambda$, the shape and the width of the central peak show large changes under varying $\epsilon_d$. For $\lambda \sim \Gamma$ and large $\lambda$, the width of the central regime becomes flatter. We plot the shot noise for a small finite bias $eV/\Gamma = 0.1$ as functions of $\lambda$ and $\delta$ in Fig. 9, and compare the $\epsilon_d = 0$ result with the $\epsilon_d / \Gamma = -0.6$ result. The shot noise $S_I(V)/eV$ shows crossover from non-universal value to $(e^2/2h)$ as $\lambda$ becomes large. As the dot energy $|\epsilon_d|$ increases, the crossover line shifts to the position with larger $\lambda$.

The Slave boson mean field approach

We outline here the main steps of SBMF approach, more details can be found in Ref. [44]. Following standard procedure [63, 64], one can introduce the auxiliary boson $b$ and fermion $f_{\sigma}$ to replace the impurity operator by $d_{\sigma} \rightarrow f_{\sigma}b^\dagger$, with the constraint $b^\dagger b + \sum_{\sigma} f_{\sigma}^\dagger f_{\sigma} = 1$. The Hamiltonian as shown in Eq. (1) of the main text becomes

$$H_{SBMF} = H_{\text{Leads}} + \sum_{\sigma} \epsilon_d f_{\sigma}^\dagger f_{\sigma} + i\lambda \gamma_1 (f_{\uparrow} b^\dagger + f_{\downarrow}^\dagger b) + \sum_{\alpha=L,R} \sum_{k,\sigma} t_{\alpha} (c_{k\sigma,\alpha}^\dagger f_{\sigma} b^\dagger + \text{h.c.})$$

$$+ i\delta \gamma_2 + \eta (b^\dagger b + \sum_{\sigma} f_{\sigma}^\dagger f_{\sigma} - 1).$$

where the lead Hamiltonian $H_{\text{Leads}}$ is unchanged, and the last term is Lagrangian multiplier which enforces the constraint on the Hilbert space.

We apply mean field approximation and replace the bosonic field and the Lagrangian multiplier by their expectation values. Because of a $U(1)$ gauge invariance, we choose $\langle b \rangle = \langle b^\dagger \rangle = b$ to be a real positive number. The mean field parameter $b$ and $\eta$ can be determined self-consistently by minimizing the free energy, and yield the saddle point relations [44]:

$$b^2 + \sum_{\sigma} \langle f_{\sigma}^\dagger f_{\sigma} \rangle = 1,$$

$$2b\eta + t \sum_{\alpha=L,R} \sum_{k,\sigma} (\langle f_{\sigma}^\dagger c_{k\sigma,\alpha} \rangle + \text{c.c.}) + i\lambda \langle \gamma_1 (f_{\uparrow}^\dagger + f_{\downarrow}) \rangle = 0.$$  

Here, we assume the $eV \ll \max \{T_K, \lambda\}$ and thus neglect the dependence of the $eV$ in the SMBF calculations. The effective coupling $\Gamma b^2$ and the renormalized energy level $|\epsilon_d + \eta|$ as a function of $\lambda$ are shown in Fig. 10 (a) and (b). For $\delta = 0$, we plot the SBMF results for the linear conductance and the shot noise (in the limit $eV \rightarrow 0$) in Fig. 10 (c) and (d), which show the crossover from universal values to non-universal ones as increasing the coupling $\lambda$. Note that the 1/4 values at even very tiny QD-MF coupling $\lambda$ is attributed to the $eV = 0$ limit. As shown in the main text, for $\lambda \ll b\Gamma$, the requirement to observe those half quantized values is $eV \ll \lambda^2/\Gamma$. In the discussion of the main text, we focus on the single-occupancy regime $|\epsilon_d| \gg \lambda, \Gamma$. Beyond this limit, the mean field parameter $b$ becomes large, and the effective energy level $\tilde{\epsilon}_d = |\epsilon_d + \eta|$ shifts away from Fermi level. In this

FIG. 9. The finite bias shot noise $S_I(eV)/eV$ (in units of $2e^2/h$) as a function of $\lambda$ and $\delta$. Left panel: $\epsilon_d = 0.0$; right panel: $\epsilon_d / \Gamma = -0.6$. We choose $\Gamma_L = \Gamma_R$ and $eV/\Gamma = 0.1$. 

$\epsilon_d / \Gamma = 0.0$

$\epsilon_d / \Gamma = -0.6$
FIG. 10. Dependence of various parameters on the Majorana coupling \( \lambda \). (a): The effective coupling in \( \Gamma b^2 \). (b): The renormalized energy level for \( |\epsilon_d + \eta| \). (c) and (d): The SBMF result for the linear conductance (c) and for the shot noise \( P(0) \). The starting point of \( \lambda \) is a very small non-zero number. We choose \( \Gamma_L = \Gamma_R, \epsilon_d/\Gamma = -10.0 \), and band width \( \Lambda/\Gamma = 30.0 \).

case, although the energy level shift does not affect the universal values (both linear conductance and shot noise) for spin-up channel (due to MZM coupling), this level shift will affect the spin-down channel if \( |\epsilon_d + \eta| > \Gamma b^2 \). The linear conductance and shot noise can be summarized as follows. The linear conductance reads

\[
G = \frac{e^2}{4\Gamma_L\Gamma_R/\Gamma} \begin{cases} 
\frac{\epsilon_d^2}{\hbar} \left( \frac{1}{2} + 1 \right) = \frac{3e^2}{2\hbar} & \text{for } |\epsilon_d + \eta| \ll \Gamma b^2, \\
\frac{\epsilon_d^2}{\hbar} \left( \frac{1}{2} + \frac{(\Gamma b^2)^2}{(\epsilon_d + \eta)^2 + (\Gamma b^2)^2} \right) & \text{otherwise},
\end{cases}
\]

which is consistent with the numerical renormalization group calculation [42]. The shot noise \((V \to 0 \text{ limit and } \Gamma_L = \Gamma_R)\) reads

\[
P(0) = \frac{2e^2}{\hbar} \left( \frac{1}{4} + 0 \right) \begin{cases} 
\frac{2e^2}{\hbar} \left( \frac{1}{4} + 0 \right) & \text{for } |\epsilon_d + \eta| \ll \Gamma b^2, \\
\frac{2e^2}{\hbar} \left( \frac{1}{4} + \frac{(\Gamma b^2)^2(\epsilon_d + \eta)^2}{((\epsilon_d + \eta)^2 + (\Gamma b^2)^2)^2} \right) & \text{otherwise}.
\end{cases}
\]

Here, the first term in the bracket corresponds to the spin-up channel (with MZM), and the second term corresponds to the spin-down channel (without MZM).

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