A Two-layer Sequential State Estimation Method for electricity and heat Networks

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Abstract. As one of the main application forms of energy coupling utilization, the electric gas integrated energy system has been widely concerned. State estimation plays an important role in the operation monitoring and optimal scheduling of the electric gas integrated energy system. The state estimation is the basis of energy management of the electro-pneumatic coupling network, which can provide a high-precision global consistent solution for the subsequent optimal scheduling and safety evaluation of the electro-pneumatic coupling network. The steady state estimation model of gas network is established for grid connected operation or island mode, and the state estimation of complex gas network is realized. On this basis, the coupled state estimation model of electricity and gas is further established, and its performance in state estimation accuracy and bad data identification is tested.

1. Introduction

With the continuous development of social economy, the demand for energy is increasing. Due to the non-renewable nature of traditional energy (coal, oil, natural gas, etc.), and the single use of energy, the energy utilization rate is low [1-2]. Therefore, in order to improve the overall energy utilization efficiency and strengthen the utilization of renewable energy, it has become a research hotspot in recent years to integrate and interconnect all kinds of energy. Among them, the coupling of power system and natural gas system, i.e. integrated electricity gas system (IEGs), is one of the main application modes of energy master planning and design [4]. In order to realize the comprehensive, real-time and accurate perception of the running state of IEGs, state estimation is applied to IEGs [5-7]. In combinatorial analysis, two calculation techniques were developed. These are electricity-water-heat calculation techniques decomposed and integrated in the form of power flow and simple optimal scheduling. Using combinatorial analysis, the electrical and thermal network variables can be calculated. The results of decomposition and comprehensive calculation are very close.

When the scale of the system is large, the Jacobian matrix dimension of the energy flow model will be very high, resulting in poor convergence and long calculation time. Therefore, this paper uses the decomposition solution method to solve the energy flow model of electric thermal IES. According to the grid connected operation mode and island mode respectively, taking the electric thermal IES
working in the thermal fixed mode as an example, based on the electric gas thermal system model and the coupling mathematical model, it is divided into three parts. Firstly, it calculates the energy flow in the heating network interval; after the calculation convergence, it uses the electric thermal coupling element model to convert the energy; finally, it calculates the power flow in the grid interval until the convergence.

This project proposes a two-layer robust state estimation method for the electric-heat integrated energy networks. This paper linearizes the measurement equations in IEHS by introducing auxiliary variables, and constructs the IEHS-oriented linear weighted least absolute value (WLAV) SE method; and then obtains the original state of IEHS through a nonlinear transformation and linear WLS Estimated value of the variable. At the end of the article, a simulation example is used to test the effectiveness of the proposed method.

2. First Layer of state estimation Model

The objects of the first layer of SE modeling include power systems and hydraulic networks. The main purpose of modeling is to transform the traditional nonlinear non-convex SE model into a linear convex optimized RSE model.

2.1. Linearization of the measurement Equation Model

The traditional nonlinear non-convex SE method generally cannot guarantee the global optimal solution theoretically, and may have the problem of difficulty in convergence. The fundamental problem that causes the above problems is that the measurement equations in the SE model are nonlinear. This project realizes the linearization of the measurement equation by introducing appropriate auxiliary state variables and auxiliary quantity measurement.

(1) The auxiliary state variable in the power system is selected as \( x_a^i = \begin{bmatrix} V_i^a; R_y^a; X_y^a \end{bmatrix} \), Where, \( R_y^a = U_iU_j \cos \theta_y \), \( V_i^2 = U_i^2 \), \( X_y^a = U_iU_j \sin \theta_y \). And choose the auxiliary measurement as shown in equation (1), then the linearized measurement equation is expressed as (for a simple representation, the measurement noise is ignored here)

\[
z_a^i = \begin{bmatrix} U_i^2; P_i; Q_i; P_y; Q_y \end{bmatrix}^T
\]

\[
\begin{align*}
U_i^2 &= V_i^a \\
P_i &= \sum_{j \in N_i} (G_y R_y^a + B_y X_y^a) \\
Q_i &= \sum_{j \in N_i} (G_y X_y^a - B_y R_y^a) \\
P_y &= V_i^a (b_y + g_y) - g_y R_y^a - b_y X_y^a \\
Q_y &= -V_i^a (b_y + g_y) + b_y R_y^a - g_y X_y^a
\end{align*}
\]

(2) Choose the square root of the pressure head loss of the pipeline i-j \( \sqrt{S_{ij}P_{ij}} \) as the auxiliary measurement, \( x_a^i = \alpha_y^a \) as the auxiliary variable, where \( \alpha_y^a = \sqrt{S_{ij}P_{ij}} \). Then the measurement equation of auxiliary quantity measurement and linearization \( z_h^a \) is expressed as:

\[
z_h^a = \begin{bmatrix} m; m_y; \sqrt{S_{ij}P_{ij}} \end{bmatrix}
\]
At the same time, considering the thermal power measurement of the heating network node corresponding to the active power of the power system coupling node, the unified linear measurement model of the power system and the hydraulic network is expressed as:

\[
\begin{align*}
\dot{m}_i &= \frac{1}{\sqrt{K_{ji}}} \alpha_{ij}^a \\
\dot{m}_j &= \sum_{i=1}^{\text{N1}} \frac{1}{\sqrt{K_{ij}}} \alpha_{ij}^a \\
\sqrt{S_{ij} P_{ij}} &= \alpha_{ij}^a
\end{align*}
\]

(A4)

Among them, \(a_{ij}^a\) is a constant coefficient matrix, which is specifically composed of the following parts:

\[
H^a = \begin{bmatrix}
H^{ae} & 0 \\
0 & H^{ab} \\
c_{ab} H^{ae} & 0 \\
-ZH^{ae} & 0
\end{bmatrix}
\]

(6)

2.2. The establishment of linear second-order cone programming Model

The L-WALV model is expressed as:

\[
\begin{align*}
\min \ w(u + v) \\
\text{s.t.} \quad \begin{cases}
z^a - H^a x^a = u - v \\
u, v \geq 0
\end{cases}
\end{align*}
\]

(7)

For the power system, the introduction of auxiliary state variables increases the number of state variables by \(Be\) (\(Be\) is the number of branches in the power system), which reduces the measurement redundancy of the power system and affects the estimation accuracy.

Considering that the auxiliary state variables, \(R_i^a\), \(X_i^a\) and \(V_i^a\) in the power system satisfy the following relationship:

\[
(R_i^a)^2 + (X_i^a)^2 = V_i^a V_j^a
\]

(8)

Relax (8) to transform the quadratic equation into a second-order cone inequality constraint, and get:

\[
(R_i^a)^2 + (X_i^a)^2 \leq V_i^a V_j^a
\]

(9)

Add (9) to the model (7) to get the RSE model based on the second-order cone programming:
\[
\min \sum \lambda_{ij} R_{ij} - w (\mathbf{u} + \mathbf{v}) \\
\text{s.t.} \begin{align*}
\mathbf{z}^a - H^a \mathbf{x}^a &= \mathbf{u} - \mathbf{v} \\
\left( R_{ij}^a \right)^2 + \left( X_{ij}^a \right)^2 &\leq V_i^a V_j^a
\end{align*}
\] (10)

Among them, \( \lambda_{ij} \) is adjust the parameter, the value is equal to the weight value of the power measurement of the corresponding branch \( ij \) in the power system. It can be seen that the number of inequality constraints (9) is \( Be \). Therefore, the introduction of formula (9) equivalently increases the number of measurements in the power system part of the model (10) by \( Be \), so as to make up for part of the measurement redundancy lost by the model (7).

3. The establishment of the second layer RSE model based on L-WLAV

The modeling object of the second layer SE is the thermal network, and the modeling idea is the same as that of the first layer. Among them, through the state estimation of the first layer, the estimated value of the branch flow rate and the estimated value of the node injection flow rate are obtained, and the two are regarded as pseudo measurements for the SE of the second layer. The measurement equation of the thermal network itself is a linear equation, where, \( \mathbf{x}_t = [\mathbf{T}_t; \mathbf{T}_m] \), \( \mathbf{z}_t = [\mathbf{e}; \mathbf{T}_t; \mathbf{T}_m] \).

Therefore, the RSE model based on L-WLAV can be directly constructed:

\[
\min \ w_t (\mathbf{u}_t + \mathbf{v}_t) \\
\text{s.t.} \begin{align*}
\mathbf{z}_t - H_t \mathbf{x}_t &= \mathbf{u}_t - \mathbf{v}_t \\
\mathbf{u}_t, \mathbf{v}_t &\geq 0
\end{align*}
\] (11)

Among them, \( w_t \) is the measurement weight matrix of the thermal network quantity, \( \mathbf{u}_t \) and \( \mathbf{v}_t \) both are non-negative variables. \( H_t \) is a constant coefficient matrix, and its specific structure is:

\[
H_t = \begin{bmatrix}
C_f \bar{\mathbf{m}}_q & -C_p \bar{\mathbf{m}}_q \\
1 & 0 \\
0 & 1
\end{bmatrix}
\] (12)

The measurement redundancy of the thermal network itself is low (about 1.5). Therefore, when bad data is generated in the measurement, the state estimation result of the thermal network is easily affected. From the analysis in the first section, we can see that there are equality constraints between the node temperatures. Regarding the measurement estimates \( \bar{\mathbf{m}}_{ij} \) and \( \bar{\mathbf{m}}_q \) obtained from the first-level state estimation as pseudo-measurements, the corresponding linear equation constraints are obtained and added to the model (11) to obtain:

\[
\min \ w_t (\mathbf{u}_t + \mathbf{v}_t) \\
\text{s.t.} \begin{align*}
\mathbf{z}_t - H_t \mathbf{x}_t &= \mathbf{u}_t - \mathbf{v}_t \\
T_{\text{out}} &= (T_{\text{start}} - T_u) e^{-\frac{c_m}{C_p \bar{\mathbf{m}}_q}} + T_u \\
(\sum \bar{\mathbf{m}}_{\text{out}}) T_{\text{out}} &= \sum (\bar{\mathbf{m}}_{\text{in}} T_{\text{in}}) \\
\mathbf{u}_t, \mathbf{v}_t &\geq 0
\end{align*}
\] (13)
So far, the RSE model of the thermal network based on L-WLAV has been established.

4. Electrical-Hydraulic-Thermal State Estimation Model Calculation

The decomposition electrical-hydraulic-thermal calculation is based on the decomposition hydraulic-thermal calculation and the electrical power flow calculation. The RSE model (4-24) of the first layer is a linear SOCP model, and this paper uses the optimization software MOSEK to solve it. The RSE model of the second layer is a linear programming model. This paper uses CPLEX to solve. The estimated value of the state variable and in the thermal network can be directly obtained. Finally, the power flow model is solved.

In the grid connected mode, any surplus or deficiency of power is supplied by the main grid, and the power slack bus bar does not generate heat. Therefore, the independent hydraulic model, thermal model and power flow model are only calculated once.

In island mode, the independent hydro thermal model and electric model are solved in turn. This sequence will iterate until the solution converges to an acceptable tolerance.

The sequential state estimation WLS method of power system is extended to IEHS, and a WLS model for thermal electric coupling system is proposed

\[
\min J(x) = \left[ z - h(x) \right]^T R^{-1} \left[ z - h(x) \right] \\
\text{s.t. } b(x) = 0
\]

(14)

Where, \( b(x) = 0 \) is the zero injection equality constraint and coupling constraint are used in the electric thermal coupling system. \( z = [z_e; z_h] \), \( R = \text{diag}[R_e; R_h] \), \( h(x) = [h_e(x_e); h_h(x_h)] \).

The above model has high convergence and high estimation accuracy when there is no bad data, so it has important theoretical significance and application value.

5. Example Analysis

This paper uses a modified IEEE14-node power system and Bali 32-node thermal system coupled electric-heat integrated energy system as the simulation object. The electric-heat integrated energy system adopted in this project adopts island operation mode. Among them, the slack node 1 in the modified IEEE14-node power system is connected with the heat source node 31 in the Bali 32-node thermal system through a gas turbine, and the proportional coefficient; the PV node 6 in the power system and the slack node in the thermal system 1 is connected through a steam turbine, among which, the PV node 2 in the power system is connected with the heat source node 32 in the thermal system through an internal combustion engine, with a proportional coefficient.

The performance analysis indicators obtained by the three state estimation methods are shown in Table 1 and Table 2 respectively.

**Table 1. The maximum estimation error of state variables obtained by three state estimation methods**

| Method | Power system state variables | Thermal system state variables |
|--------|-----------------------------|-------------------------------|
|       | \( U / \text{p.u.} \)       | \( \theta / \text{rad} \)  | \( p / m \) | \( T_s / ^\circ \text{C} \) | \( T_r / ^\circ \text{C} \) |
| WLS    | \( 3.26 \times 10^{-4} \)    | \( 1.91 \times 10^{-3} \)  | \( 2.07 \times 10^{-3} \) | \( 2.35 \times 10^{-3} \) | \( 2.34 \times 10^{-3} \) |
| BRSE   | \( 3.73 \times 10^{-4} \)    | \( 3.66 \times 10^{-2} \)  | \( 2.47 \times 10^{-3} \) | \( 2.34 \times 10^{-3} \) | \( 2.35 \times 10^{-3} \) |
| TL-RSE | \( 3.67 \times 10^{-4} \)    | \( 1.72 \times 10^{-2} \)  | \( 2.46 \times 10^{-3} \) | \( 4.40 \times 10^{-4} \) | \( 2.13 \times 10^{-3} \) |

It can be seen from Table 1 that in the power system, the maximum estimation error of the state variables \( U \) and \( \theta \) obtained by WLS is the smallest among the three methods, and the estimation accuracy of TL-RSE is better than that of BRSE; in the thermal system, The maximum estimation error of the state variable \( p \) obtained by WLS is the smallest, and the maximum error between the state
variable $T_s$ and $T_r$ obtained by TL-RSE is the smallest, and the maximum error of $T_s$ has been significantly reduced.

This paper selects the average value $\bar{X}_{i, \text{error}}$ of the estimated error of the state variable, the maximum estimated error $\delta_{\text{max}}$ of the state variable, the statistical value $S_M$ of the measurement error, the statistical value of the estimated error $S_H$ and $\lambda$ the indicators for the performance analysis of the state estimation [18]. Denoted as:

$$\bar{X}_{i, \text{error}} = \frac{1}{T} \sum_{j=1}^{T} \left| \frac{x_{i, \text{true}} - \hat{x}_j}{x_{i, \text{true}}} \right|$$  \hspace{1cm} (15)

$$\delta_{\text{max}} = \frac{1}{T} \sum_{j=1}^{T} \max \left| \frac{x_{i, \text{true}} - \hat{x}}{x_{i, \text{true}}} \right|$$  \hspace{1cm} (16)

$$S_M = \frac{1}{T} \sum_{j=1}^{T} \left[ \frac{1}{m} \sum_{i=1}^{m} \left( \frac{z_{i,j} - h_i(x_{i, \text{true}})}{\sigma_i} \right)^2 \right]^{1/2}$$  \hspace{1cm} (17)

$$S_H = \frac{1}{T} \sum_{j=1}^{T} \left[ \frac{1}{m} \sum_{i=1}^{m} \left( \frac{h_i(\hat{x}) - h_i(x_{i, \text{true}})}{\sigma_i} \right)^2 \right]^{1/2}$$  \hspace{1cm} (18)

$$\lambda = S_H / S_M$$  \hspace{1cm} (19)

Among them, $x_{i, \text{true}}$ is the true value and $\hat{x}$ is the estimated value of the state variable respectively; $x_{i, \text{true}}$ is the true value and $\hat{x}_j$ is the estimated value of the power flow calculation of the i-th state variable in, respectively; $z_{i,j}$, $h_i(x_{i, \text{true}})$ and $h_i(\hat{x})$ are the measurement of the i-th measurement of the jth experiment, respectively value, true value of power flow calculation and measurement estimate; $\sigma_i$ is the standard deviation of the measurement error of the i-th measurement. $\lambda$ is used to evaluate the filtering effect of the state estimation method. The smaller the value, the better the filtering effect. $m$ is the total number of measurements, and $T$ is the number of Monte Carlo simulation experiments. This project performs Monte Carlo simulation experiments with $T=1000$ times.

**Table 2.** The results of performance analysis indicators obtained by three state estimation methods

| Method   | $S_M$  | $S_H$  | $\lambda$ |
|----------|--------|--------|-----------|
| WLS      | 25.273 | 25.234 | 0.998     |
| BRSE     | 25.143 | 25.127 | 0.999     |
| TL-RSE   | 25.162 | 9.614  | 0.382     |

It can be seen from Table 2 that the statistical value of the estimation error of the TL-RSE part is significantly smaller than that of WLS and BRSE, and the resulting $\lambda$ has been significantly reduced, which means that the filtering effect of TL-RSE is better than that of WLS and BRSE.

In summary, under normal measurement: (1) For the power system, the estimation accuracy of TL-RSE is higher than that of BRSE, which is consistent with the previous theoretical analysis; (2) For the thermal system, because the physical constraints between node temperatures are considered, The
estimation accuracy of TL-RSE for node heating temperature and node regenerative temperature is significantly better than WLS and BRSE.

6. Conclusion
A two-layer sequential state estimation method for electricity and heat system is proposed. Through the built IEHS example model, it is verified that the estimation accuracy of TL-RSE is better than BRSE under normal measurement, and at the same time, the estimation accuracy of the heating network is greatly improved. TL-RSE can effectively identify the general bad data and strong correlation bad data that appear in IEHS.

Acknowledgments
This work was financially supported by the science and technology project fund of State Grid Corporation of China (No.52110418002R) "Research on state estimation of comprehensive energy system considering different time scales"

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