Kinematics and dynamics analysis of a lower DOF spatial parallel mechanism

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Abstract: In order to control the rotation of the end effector in a long and narrow space, a spatial parallel mechanism that can realize 2 degrees of freedom rotation is proposed. In this article, the forward kinematics model is established according to the constraint conditions of the mechanism’s pose, and the closed vector method is used to obtain the inverse kinematics model. Then based on the kinematics results, the dynamic inverse solution model was established by Newton-Euler method, and the dynamic analysis of the whole mechanism was carried out. Finally, the theoretical calculation results are compared with the simulation results of Adams software to verify the accuracy of the solution process.

1. Introduction:
Parallel mechanisms have the characteristics of high stiffness, strong load-bearing capacity, and lightweight at the end. Compared with parallel mechanisms, lower DOF parallel mechanisms have the advantages of fewer driving parts, simple control, and lower price. The lower parallel mechanisms exhibit many special properties because the movement and rotation of the moving platform are restricted. Therefore, it has more complex motion characteristics than general 6-DOF parallel mechanisms. The kinematics of lower DOF parallel mechanisms can be achieved through methods such as the lower DOF kinematics model based on the screw theory, the virtual mechanism method based on the screw theory, and the vector method [1-3]. The dynamics of parallel mechanisms can be analyzed by Lagrangian method, Newton-Euler method, virtual work principle, Kane method and other methods [4]. Chen X.L. [5] used the Lagrangian method to analyze the rigid body dynamics of the 3-RRPAR spatial parallel mechanism with redundant structure; Hou Y.L. [6] used Newton-Euler method to establish a dynamic model of a 2-DOF decoupling parallel mechanism(RU-RPR); Zhu W. [7] used the principle of virtual work to establish the dynamic equation of a new type of 4-DOF high-speed parallel mechanism; Yang C. [8] analyzed the dynamics of a new type of wall-climbing robot over obstacles based on Kane's method.

Based on this, the author designs a spatial 2-DOF parallel rotating mechanism, which has a simple structure and strong load-bearing capacity. On the basis of kinematic analysis, this article used Newton-Euler method to establish the rigid body dynamics model of this parallel mechanism, and obtained its dynamic inverse solution, and laid the foundation for the follow-up research of such mechanisms.

2. Structure description
The structure of the parallel mechanism is shown in Figure 1. The moving platform is connected to the support shaft through the adapter and the yaw seat. The moving platform and the adapter, and the adapter and the yaw seat are all connected by internal pins. The yaw seat is fixed on the support shaft by screws. The moving platform is connected with the pitch slide through a hook hinge, a pitch link, and a rod end joint ball bearing. The adapter is connected with the yaw slider through a pin shaft and a yaw link. The
guidance of the pitch slide and the yaw slide are respectively realized by guide rails fixed on the support
shaft. When the mechanism is working, the yaw and pitch motors drive the respective screws to push
the yaw and pitch slides forward or backward. Then the two links drive the adapter and the moving
platform to rotate.

![Figure 1. 3D model of parallel mechanism](image)

3. Kinematics analysis of parallel mechanism

In order to describe the spatial pose of the moving platform, coordinate systems are established as shown
in Figure 2. The base coordinate system \( oxyz \) is established at the geometric center of the contact end
face of the support shaft and the yaw seat. The \( x \) axis is perpendicular to this end face. The \( y \) axis is
perpendicular to the side of the support shaft where the yaw slide is located, and the \( z \) axis is determined
by the right-hand rule. The origin \( o' \) of the moving coordinate system \( o'x'y'z' \) is located at the center of
mass of the moving platform. And the \( x' \) and \( y' \) axis are parallel to \( x \) and \( y \) axis respectively. The \( z' \) axis
is also determined by the right-hand rule.

According to the target pose of the moving platform, the two moving pairs produce a certain amount
of translation output under the drive of the motor. So the rotating pair \( R_1 \) rotates \( \alpha \) angle around the \( z_1 \)
axis, and the rotating pair \( R_2 \) rotates \( \beta \) angle around the \( y_2 \) axis. Finally the moving platform reaches the
required pose.

![Figure 2. Structure diagram and establishment of coordinate system](image)

According to the knowledge of matrix transformation, the pose transformation matrix between each
coordinate system can be established by using ZYX Euler angles as:

\[
\begin{bmatrix}
c_{\alpha}c_{\beta} & -s_{\alpha} & c_{\alpha}s_{\beta} & l_{0h} + l_{1h}c_{\alpha}s_{\beta} \\
c_{\alpha}s_{\beta} & s_{\alpha} & -c_{\alpha}c_{\beta} & l_{1h}s_{\alpha} + l_{1h}c_{\alpha}s_{\beta} \\
a_{\beta} & 0 & c_{\beta} & -l_{1h}c_{\beta} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\( (1) \)

3.1 Forward kinematics

Knowing the displacement input amounts \( s_0 \) and \( s_1 \) of the yaw slide and the pitch slide, we need to find
the coordinate \( (o', o'_x, o'_y) \) of the origin \( o' \) and the attitude angle \( (\psi, \gamma, \phi) \) of the moving platform.
From equation (1), we know that the six generalized coordinates of the moving platform are all
determined by the parameters \( \alpha \) and \( \beta \). Therefore, forward kinematics is the process of solving the
parameters \( \alpha \) and \( \beta \) with known inputs \( s_0 \) and \( s_1 \).
The yaw chain is similar to the pitch chain, so the forward kinematics takes the pitch chain as an example.

The structure diagram of the pitch chain in the base coordinate system is shown in Figure 3.

In the base coordinate system, \( \mathbf{U} = \left( l_{o_R} + r_z c \alpha + r_s c(\theta_1 + \beta) c \alpha, 0, -r_s s \theta \right) \), \( \mathbf{U}' = \left( r_z s \alpha + r_s c(\theta_1 + \beta) s \alpha, -r_s s(\theta_1 + \beta) \right) \),

\( S = (x_1, y_1, z_1) \), \( S' = (x_1 + s_1, y_1, z_1) \).

![Figure 3. Structure diagram of Pitch chain](image)

From the constraint condition of the yaw link length, we can get:

\[
l_2^2 = (l_{o_R} + r_z c \alpha + r_s s(\theta_1 + \beta) c \alpha - x_1)^2 + y_1^2 + (z_1 + r_s s \theta)^2 \tag{2}
\]

\[
l_2^2 = [l_{o_R} + r_z c \alpha + r_s c(\theta_1 + \beta) s \alpha - (x_1 + s_1)]^2 + [r_z s \alpha + r_s c(\theta_1 + \beta) s \alpha - y_1]^2 + [z_1 + r_s s(\theta_1 + \beta)]^2 \tag{3}
\]

Combining equations (2) and (3) leads to:

\[
\beta = \arcsin \left( \frac{A}{B} \right) - \arcsin t - \Theta_1 \tag{4}
\]

Where \( A = (x_1 + s_1 - l_{o_R} - r_z c \alpha)^2 + (y_1 - r_z s \alpha)^2 + r_s^2 + z_1^2 - l_2^2 \); \( B = 2r_z \sqrt{[l_{o_R} + (l_{o_R} - x_1 - s_1) c \alpha - y_1 s \alpha]^2 + z_1^2} \); \( \sin t = \frac{r_z + (l_{o_R} - x_1 - s_1) c \alpha - y_1 s \alpha}{\sqrt{[l_{o_R} + (l_{o_R} - x_1 - s_1) c \alpha - y_1 s \alpha]^2 + z_1^2}} \).

### 3.2 Inverse kinematics

This article mainly focuses on the inverse dynamics problem. When solving the inverse kinematics, the speed and acceleration of the sliders and the links will also be obtained. The pitch chain is similar to the yaw chain, so the inverse kinematics takes the yaw chain as an example.

In order to facilitate the analysis, the yaw chain is simplified as a planar crank slider mechanism in the plane. Its closed vector diagram is shown in Figure 4.

![Figure 4. Closed vector diagram of Yaw chain](image)

1) Position inverse solution

By the closed vector method, we can establish the closed vector equation as:

\[
q_i e_i + n_i + l_i u_i = o R_i + R_i R_i \tag{5}
\]

So the inverse solution of the yaw slider position is:
2) Slider’s speed
By differentiation of equation (5) with respect to time, we can get:

\[
v_{n} = \dot{q}_{i} + \omega_{i}l_{i}(\dot{R}_{i}R_{i}) = \ddot{q}_{i}(\dot{R}_{i}R_{i})
\]

(7)

Multiplying the two sides of equation (7) with \( u_{i} \) to get:

\[
\dot{q}_{i} = u_{i}^{T}(\dot{R}_{i}R_{i}) - \omega_{i}l_{i}u_{i}^{T}(\dot{R}_{i}R_{i}) = \dot{v}_{n} - \omega_{i}l_{i}u_{i}^{T}(\dot{R}_{i}R_{i})
\]

(8)

Where \( \dot{v}_{n} \) is the speed of the center of mass of the rotating pair \( R_{i} \); \( \omega_{i} \) is the angular velocity of the yaw link in the base coordinate system; \( \dot{E} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \); \( J_{i} \) is the Jacobian matrix of the yaw chain.

3) Link’s speed
Multiplying the two sides of equation (7) with \( \dot{E}u_{i} \), and substituting equation (8) leads to:

\[
2\dot{q}_{i} = \dot{u}_{i}^{T}(\dot{R}_{i}R_{i}) - \omega_{i}l_{i}u_{i}^{T}(\dot{R}_{i}R_{i}) = \dot{v}_{n} - \omega_{i}l_{i}u_{i}^{T}(\dot{R}_{i}R_{i})
\]

(11)

According to equation (7), the speed of the center of mass of the link is:

\[
v_{i} = v_{n} - \omega_{i}l_{i}\frac{1}{2}(\dot{E}u_{i}) = \begin{bmatrix} \dot{E}R_{i}R_{i} & 0 \\ -\frac{1}{2}(\dot{E}u_{i})J_{i} & \dot{E}R_{i}R_{i} \end{bmatrix} = J_{w_{i}}\dot{E}R_{i}R_{i}
\]

(10)

Where: \( J_{w_{i}} \) and \( J_{i} \) are the linear velocity Jacobian matrix and the rotational angular velocity Jacobian matrix of the link.

4) Slider’s acceleration
Differentiating both sides of equation (7) with respect to time leads to:

\[
\ddot{q}_{i} + \omega_{i}l_{i}(\ddot{R}_{i}R_{i}) - \omega_{i}^{2}l_{i}u_{i} = \ddot{q}_{i}(\dot{R}_{i}R_{i}) = \ddot{v}_{n} - \omega_{i}l_{i}u_{i}^{T}(\dot{R}_{i}R_{i})
\]

(11)

Multiplying the two sides of equation (11) with \( u_{i} \) to get:

\[
\ddot{q}_{i} = J_{w_{i}}\dot{E}R_{i}R_{i}
\]

(12)

5) Link’s angular acceleration

Multiplying the two sides of equation (11) with \( \dot{E}u_{i} \) to get:

\[
\ddot{\omega}_{i} = \frac{1}{2}(\dot{E}u_{i})^{T}(\dot{R}_{i}R_{i}) - \omega_{i}^{2}l_{i}u_{i}^{T}(\dot{R}_{i}R_{i}) = \frac{1}{2}(\dot{E}u_{i})^{T}(\dot{R}_{i}R_{i}) + \frac{1}{2}(D + D_{2})
\]

(13)

Where \( D = -\omega_{i}^{2}l_{i}u_{i}^{T}(\dot{R}_{i}R_{i}) \); \( D_{2} = -(\dot{E}u_{i})^{T}(R_{i}R_{i}) \).

6) Centroid acceleration analysis of link
Differentiating both sides of equation (10) with respect to time leads to:

\[
\ddot{v}_{i} = J_{i} \begin{bmatrix} \ddot{E}R_{i}R_{i} \\ 2(D + D_{2}) - \ddot{E}R_{i}R_{i} \end{bmatrix} + \frac{1}{2}(\ddot{q}_{i})
\]

(14)

4. Dynamics analysis of parallel mechanism

Without considering the friction, the dynamic model of the mechanism is obtained as shown in Figure 5. In the figure, \( m_{i} \) is the mass of the moving platform; \( m_{z} \) is the mass of the adapter; \( n_{i} \) is the external moment of the moving platform; \( f_{e} \) is the external force of the moving platform; \( m_{l} \) is the mass of the
link $i$; $m_i$ is the mass of the slider $i$.

In view of the simple structure of the parallel mechanism, and the Newton-Euler method is intuitive and easy to understand, this article adopts the Newton-Euler method to establish the dynamic model of the parallel rotating mechanism.

The force analysis of the yaw and pitch chains is shown in Figure 6 and 7. In the figures, $f_{yi}$ is the force vector of the moving platform acting on the yaw and pitch links; $f_{qi}$ is the force vector of the slider acting on the yaw and pitch links; $N$ is the force vector of the support shaft acting on the yaw and pitch sliders; $f_{ex}$ and $f_{ez}$ are the components of external force $f_e$ in the $xOy$ and $xOz$ planes; $n_{ex}$ and $n_{ez}$ are the components of the external moment $n_e$ in the $xOy$ and $xOz$ planes.

Based on the analysis of the forces on the yaw and pitch chains, the forces on the sliders and the links are decomposed as shown in Figures 8 and 9. In the figures, $f_{y1x}$, $f_{y1y}$, $f_{y1z}$, $f_{y2x}$, $f_{y2y}$, $f_{y2z}$ are the force components of $f_{qi}$ along the $x$, $y$, and $z$ directions; $f_{dx}$, $f_{dy}$, $f_{dz}$, $f_{dl}$ are components of the force $f_{di}$ along the $x$, $y$, and $z$ directions. $f_1$, $f_2$ are components of the driving force of the motors to sliders in the $x$ direction.

According to the force analysis shown in Figures 6 and 7, using Euler's method on the moving platform, the adapter and the moving platform respectively leads to:

$$R_i (\mathbf{o} \times f_{ex}) + R_i \times (-f_{di}) = n_{ex} + \mathbf{I} \ddot{\mathbf{a}}$$

(15)
\[ R_z \mathbf{o'} \times f_{e_z} + R_z U \times (-f_{e_z}) = n_z + I_z \beta \]  
(16)

Where \( I_1 = I_d + I_z \), \( I_d \) is the moment of inertia of the moving platform around the rotating pair \( R_1 \) in the \( xoy \) plane; \( I_z \) is the moment of inertia of the adapter around the rotating pair \( R_z \) in the \( xoy \) plane. \( I_z = I'_z \), \( I'_z \) is the moment of inertia of the moving platform around the rotating pair \( R_z \) in the \( xoz \) plane.

When the motion state of links is known, the Newton-Euler method is adopted for the yaw and pitch links as:

\[
\begin{align*}
\mathbf{f}_{ai} + \mathbf{f}_{qi} &= m_i \mathbf{v}_i \\
(\mathbf{f}_{ai} + \mathbf{f}_{qi})^T (\hat{\mathbf{E}u}_i / 2) &= I_i \omega_i
\end{align*}
\]  
(17)

Where \( I_i \) is the moment of inertia of the yaw and pitch links in the base coordinate system.

By using the Newton method for the yaw and pitch sliders, we can get:

\[
\mathbf{f}_i + (-\mathbf{f}_{qi}) = m_i \ddot{q}_i
\]  
(18)

By solving equations (15)~(17), the force of the yaw and pitch links on the respective sliders can be obtained. Finally, equation (18) can be used to obtain the driving force of motors to the sliders:

\[
\mathbf{f}_i = m_i \ddot{q}_i + \mathbf{f}_{qi}
\]  
(19)

5. Results & Discussion

According to the working conditions and actual requirements of the parallel mechanism, the parameters of the parallel mechanism are shown in Table 1:

| Parameters | Yaw chain | Pitch chain |
|------------|-----------|-------------|
| \( l_1 \) (mm) | 246 | 378.6 |
| \( m_i \) (kg) | 8.5 | 13.3 |
| \( m_z \) (kg) | 1.1 | 3.4 |
| \( I_z \) (g·m²) | 6.5 | 55.8 |

| \( m_z \) (kg) | 14.9 |
| \( m_z \) (kg) | 15.3 |
| \( I_z \) (g·m²) | 264.5 |
| \( I'_z \) (g·m²) | 79.3 |
| \( I_z \) (g·m²) | 155.7 |
| \( l_{ax} \) (mm) | 69 |
| \( l_{bx} \) (mm) | 111 |
| \( l_{cx} \) (mm) | 51 |

The external force acting on the moving platform is \( \mathbf{f}_e = [-892.5 \quad 715.3 \quad -1784.0]^T \) N; The external torque acting on the moving platform is \( \mathbf{n}_e = [72.3 \quad 143.1 \quad -241.6]^T \) N·m. According to the actual operation law of the moving platform, the motion trajectory is given as follows:

\[
\begin{align*}
\alpha &= 10 \sin(\pi t) \\
\beta &= 15 \cos(\pi t)
\end{align*}
\]  
(20)

By using the MATLAB software for numerical calculation, the velocity and acceleration curves of the slider are obtained from equations (8) and (12) as shown in Figure 10 and Figure 11. The driving force curves of the sliders obtained by equation (19) are shown in Figure 12 and Figure 13.
According to Figure 12, the calculated value of the yaw slider’s driving force is consistent with the simulated value when $0.4 \leq t < 0.8$ s. There are deviations between the calculated value and the simulated value in the rest of the time. The analysis of this phenomenon found that the angle $\theta_2$ between the pitching link and the $xoz$ plane of the base coordinate system is constantly changing in the actual movement process, which causes the pitch link to more or less share some yaw driving force. Therefore the calculated value deviates from the simulation value deviate most of the time. When $0 \leq t < 0.4$ s, $\theta_2$ is already small enough. The pitch link doesn’t share the yaw driving force in this case. Therefore the calculated value during this period is consistent with the simulation value.

According to Figure 13, the calculated value of the pitch slider’s driving force is consistent with the simulated value when $1.1 \leq t < 1.2$ s. There are deviations between the calculated value and the simulated value in the rest of the time. The reason for the difference between the theoretical calculation value and the simulation value in this figure is the same as that in Figure 12.

6. Conclusions

A spatial parallel mechanism with coupling effect is proposed. It’s driving part is located on the support shaft for reducing the movement mass of the mechanism. While reducing the radial dimension of the entire mechanism as much as possible, it ensures sufficient stiffness of the mechanism.

Comparing the numerical calculation results of the MATLAB software with the simulation results of the ADAMS virtual prototype, it verified the correctness of the dynamic model of the parallel mechanism, and laid the foundation for subsequent error analysis and optimization design.

When analyzing the problem, the model was partially simplified. Although the final calculation results can meet the actual engineering needs, follow-up research should strive to obtain accurate solutions.
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