HANKEL DETERMINANT OF TYPE $H_2(3)$ FOR INVERSE FUNCTIONS OF SOME CLASSES OF UNIVALENT FUNCTIONS WITH MISSING SECOND COEFFICIENT

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ABSTRACT. In this paper we determine the upper bounds of $|H_2(3)|$ for the inverse functions of functions of some classes of univalent functions, where $H_2(3)(f) = a_3a_5 - a_2^2$ is the Hankel determinant of a special type.

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1. Introduction and preliminaries

Let $\mathcal{A}$ be the class containing functions that are analytic in the unit disk $D := \{ |z| < 1 \}$ and are normalized such that $f(0) = 0 = f'(0) - 1$, i.e.,

$$f(z) = z + a_2z^2 + a_3z^3 + \cdots .$$

By $S$ we denote the class of functions from $\mathcal{A}$ which are univalent in $D$.

Also, we need the classes of functions of bounded turning, of convex functions, of starlike functions, and of functions starlike with respect to symmetric points, subclasses of $S$, defined respectively in the following way

$$\mathcal{R} = \{ f \in \mathcal{A} : \Re f'(z) > 0, \ z \in D \} ,$$

$$\mathcal{C} = \{ f \in \mathcal{A} : \Re \left[ 1 + \frac{zf''(z)}{f'(z)} \right] > 0, \ z \in D \} ,$$

$$S^* = \{ f \in \mathcal{A} : \Re \frac{zf'(z)}{f(z)} > 0, \ z \in D \} ,$$

$$S^*_s = \{ f \in \mathcal{A} : \Re \frac{2zf'(z)}{f(z) - f(-z)} > 0, \ z \in D \} .$$

In his paper [4] Zaprawa considered the Hankel determinant of the type

$$H_2(3)(f) = a_3a_5 - a_2^2,$$
defined for the coefficients of the function \( f \) given by (1). The author treated bounds of \(|H_2(3)(f)|\) for the classes \( \mathcal{R}, \mathcal{C}, \mathcal{S}^\star \) and gave sharp results in the case \( a_2 = 0 \). He also investigated the general case of these classes. In the same paper it is proved that
\[
\max \{|H_2(3)(f)| : f \in \mathcal{S}\} > 1.
\]

The object of current paper is to obtained the bounds of the modulus of the Hankel determinant \( H_2(3)(f - 1) \) of coefficients of the inverse of function from the classes \( \mathcal{R}, \mathcal{C}, \mathcal{S}^\star \) and \( \mathcal{S}_s^\star \), defined before, as well as for the class \( \mathcal{S} \). In all cases we suppose that function \( f \) is missing its second coefficient, i.e., \( a_2 = 0 \).

Namely, for every univalent function in \( \mathbb{D} \) exists inverse at least on the disk with radius 1/4 (due to the famous Koebe’s 1/4 theorem). If the inverse has an expansion
\[
f^{-1}(w) = w + A_2w^2 + A_3w^3 + \cdots,
\]
then, by using the identity \( f(f^{-1}(w)) = w \), from (1) and (2) we receive
\[
A_2 = -a_2, \\
A_3 = -a_3 + 2a_2^2, \\
A_4 = -a_4 + 5a_2a_3 - 5a_2^3, \\
A_5 = -a_5 + 6a_2a_4 - 21a_2^2a_3 + 3a_3^2 + 14a_2^2.
\]
Especially, when \( a_2 = 0 \), we have
\[
A_2 = 0, \quad A_3 = -a_3, \quad A_4 = -a_4, \quad A_5 = -a_5 + 3a_3^2.
\]

So, in this case,
\[
H_2(3)(f^{-1}) = A_3A_5 - A_4^2 = a_3a_5 - a_4^2 - 3a_3^3,
\]
i.e.,
\[
H_2(3)(f^{-1}) = H_2(3)(f) - 3a_3^3.
\]

For our further consideration we need the next lemma given by Carlson [1].

**Lemma 1.** Let
\[
\omega(z) = c_1z + c_2z^2 + \cdots
\]
be a Schwartz function, i.e., a function analytic in \( \mathbb{D} \), \( \omega(0) = 0 \) and \(|\omega(z)| < 1\). Then
\[
|c_1| \leq 1, \quad |c_2| \leq 1 - |c_1|^2, \quad |c_3| \leq 1 - |c_1|^2 - \frac{|c_2|^2}{1 + |c_1|}, \quad \text{and} \quad |c_4| \leq 1 - |c_1|^2 - |c_2|^2.
\]
2. Main results

**Theorem 1.** Let \( f \in A \) is given by (1) and let \( a_2 = 0 \). Then

(a) \( |H_3(2)(f^{-1})| \leq \frac{28}{45} \) if \( f \in \mathcal{R} \);

(b) \( |H_3(2)(f^{-1})| \leq \frac{2}{45} \) if \( f \in \mathcal{C} \);

(c) \( |H_3(2)(f^{-1})| \leq 2 \) if \( f \in S^* \);

(d) \( |H_3(2)(f^{-1})| \leq 2 \) if \( f \in S^*_s \).

All these results are sharp.

**Proof.**

(a) Since \( f \in \mathcal{R} \) is equivalent to

\[
   f'(z) = \frac{1 + \omega(z)}{1 - \omega(z)},
\]

for certain Schwartz function \( \omega \), we receive that

\[
   f'(z) = 1 + 2\omega(z) + 2\omega^2(z) + \cdots. \tag{6}
\]

Using the notations for \( f \) and \( \omega \) given by (1) and (5), and equating the coefficients in (6), we receive

\[
   \begin{aligned}
   a_2 &= c_1, \\
   a_3 &= \frac{2}{3}(c_2 + c_1^2), \\
   a_4 &= \frac{1}{2}(c_3 + 2c_1c_2 + c_1^3), \\
   a_5 &= \frac{2}{5}(c_4 + 2c_1c_3 + 3c_1^2c_2 + c_2^2 + c_1^4).
   \end{aligned} \tag{7}
\]

Since \( a_2 = 0 \), by (7) we have \( c_1 = 0 \), and the appropriate coefficients have the next form:

\[
   a_3 = \frac{2}{3}c_2, \quad a_4 = \frac{1}{2}c_3, \quad a_5 = \frac{2}{5}(c_4 + c_2^2). \tag{8}
\]

Now, from (3) and (8), after simple computation, we obtain

\[
   H_3(2)(f^{-1}) = \frac{4}{15}c_2c_4 - \frac{1}{4}c_3^2 - \frac{28}{45}c_2^3.
\]
and further,
\[ |H_3(2)(f^{-1})| \leq \frac{4}{15} |c_2| |c_4| + \frac{1}{4} |c_3|^2 + \frac{28}{45} |c_2|^3. \]

Applying Lemma 1 (with \( c_1 = 0 \)) we receive
\[ |H_3(2)(f^{-1})| \leq \frac{4}{15} |c_2|(1 - |c_2|^2) + \frac{1}{4}(1 - |c_2|^2)^2 + \frac{28}{45} |c_2|^3. \]

and, finally,
\[ |H_3(2)(f^{-1})| \leq \frac{1}{4} + \frac{4}{15} |c_2| - \frac{1}{2} |c_2|^2 + \frac{16}{45} |c_2|^3 + \frac{1}{4} |c_2|^4 =: \varphi_1(|c_2|), \quad (9) \]

where \( 0 \leq |c_2| \leq 1 \). Since
\[ \varphi_1'(|c_2|) = \frac{4}{15} - |c_2| + \frac{16}{15} |c_2|^2 + |c_2|^3 \]
\[ = \frac{4}{15} (1 - 2|c_2|^2) + \frac{1}{15} |c_2| + |c_2|^3 > 0, \]
we have \( \varphi_1(|c_2|) \leq \varphi_1(1) = \frac{28}{45} \), and from (9),
\[ |H_3(2)(f^{-1})| \leq \frac{28}{45} = 0.622\ldots. \]

This result is best possible as the function \( f_1(z) = \ln \frac{1 + z}{1 - z} - z \) defined by \( f_1'(z) = \frac{1 + z^2}{1 - z^2} \), shows.

(b) We apply the same method as in the previous case. Namely, from the definition of the class \( \mathcal{C} \) we have
\[ 1 + z f''(z) = \frac{1 + \omega(z)}{1 - \omega(z)}, \]
where \( \omega \) is a Schwartz function, and from here
\[ (z f'(z))^2 = [1 + 2 (\omega(z) + \omega^2(z) + \cdots)] \cdot f'(z). \quad (10) \]

Using the notations (1) and (5), and comparing the coefficients in the relation (10), after some simple calculations, we obtain
\[
\begin{aligned}
\left\{ \begin{array}{l}
a_2 = c_1, \\
a_3 = \frac{1}{3} (c_2 + 3c_1^2), \\
a_4 = \frac{1}{6} (c_3 + 5c_1c_2 + 6c_1^3) \\
a_5 = \frac{1}{30} (3c_4 + 14c_1c_3 + 43c_1^2c_2 + 30c_1^4 + 6c_2^2).
\end{array} \right.
\end{aligned}
\]
If $a_2 = 0$, then by (11) we have $c_1 = 0$, which implies
\[
a_3 = \frac{1}{3} c_2, \quad a_4 = \frac{1}{6} c_3, \quad a_5 = \frac{1}{10} (c_4 + 2c_2^2).
\] (12)

Using (3) and (12) we obtain
\[
H_3(2)(f^{-1}) = \frac{1}{180} (6c_2c_4 - 5c_3^2 - 8c_2^3).
\]

From the last relation we get
\[
|H_3(2)(f^{-1})| \leq \frac{1}{180} (6|c_2||c_4| + 5|c_3|^2 + 8|c_2|^3),
\]
and further, after applying Lemma1 (with $c_1 = 0$),
\[
|H_3(2)(f^{-1})| \leq \frac{1}{180} (6|c_2|(1 - |c_2|^2) + 5(1 - |c_2|^2)^2 + 8|c_2|^3),
\]
i.e.,
\[
|H_3(2)(f^{-1})| \leq \frac{1}{180} (5 + 6|c_2| - 10|c_2|^2 + 2|c_2|^3 + 5|c_2|^4) =: \varphi_2(|c_2|), \quad (13)
\]
where $0 \leq |c_2| \leq 1$. Since
\[
\varphi_2'(|c_2|) = \frac{1}{90} (3 - 10|c_2| + 3|c_2|^2 + 10|c_2|^3),
\]
which, after considering this polynomial in the interval $[0, 1]$, gives $\varphi_1(|c_2|) \leq \varphi_2(1) = \frac{2}{45}$, and further, from (13),
\[
|H_3(2)(f^{-1})| \leq \frac{2}{45} = 0.044 \ldots
\]

The function $f_2(z) = \text{artanh} z$ satisfying $1 + \frac{zf''(z)}{f'(z)^2} = \frac{1+z^2}{1-z^2}$ shows that the result is the best possible.

(c) From the definition of the class $S^*$ we have that there exists a Schwartz function $\omega$ such that
\[
\frac{zf'(z)}{f(z)} = \frac{1 + \omega(z)}{1 - \omega(z)},
\]
and from here
\[
zf'(z) = \left[1 + 2 (\omega(z) + \omega^2(z) + \cdots)\right] \cdot f(z). \quad (14)
\]
As in the two previous cases ((a) and (b)), by comparing the coefficients in the relation (14), and some simple calculations, we have

\[
\begin{align*}
  a_2 &= 2c_1 \\
  a_3 &= c_2 + 3c_1^2 \\
  a_4 &= \frac{2}{3} \left( c_3 + 5c_1c_2 + 6c_1^3 \right) \\
  a_5 &= \frac{1}{2} \left( c_4 + \frac{14}{3} c_1c_3 + \frac{43}{3} c_1^2c_2 + 10c_1^4 + 2c_2^2 \right).
\end{align*}
\]

For the case \( a_2 = 0 \) we have the next

\[
\begin{align*}
  a_3 &= c_2, \\
  a_4 &= \frac{2}{3} c_3, \\
  a_5 &= \frac{1}{2} (c_4 + 2c_2^2).
\end{align*}
\]

(15)

So, from (3) and (15) we obtain

\[
H_3(2)(f^{-1}) = \frac{1}{18} \left( 9c_2c_4 - 8c_3^2 - 36c_2^3 \right),
\]

and from here

\[
|H_3(2)(f^{-1})| \leq \frac{1}{18} \left( 9|c_2||c_4| + 8|c_3|^2 + 36|c_2|^3 \right).
\]

Using estimates for \(|c_4| \) and \(|c_3| \) from Lemma 1 (with \( c_1 = 0 \)) from the last relation we receive

\[
|H_3(2)(f^{-1})| \leq \frac{1}{18} \left( 8 + 9|c_2| - 16|c_2|^2 + 27|c_2|^3 + 8|c_2|^4 \right) =: \varphi_3(|c_2|),
\]

(16)

where \( 0 \leq |c_2| \leq 1 \). Since

\[
\varphi'_3(|c_2|) = \frac{1}{18} \left( 9 - 32|c_2| + 81|c_2|^2 + 32|c_2|^3 \right)
\]

\[
= \frac{1}{18} \left[ 9(1 - 3|c_2|)^2 + 22|c_2| + 32|c_2|^3 \right] > 0,
\]

then \( \varphi_3(|c_2|) \leq \varphi_3(1) = 2 \), and from (16),

\[
|H_3(2)(f^{-1})| \leq 2.
\]

The result is the best possible as the function \( f_3(z) = \frac{z}{1-z^2} \) shows.
From the definition of the class $S^*_s$ we have that there exists a Schwartz function $\omega$ such that
\[
\frac{2zf'(z)}{f(z) - f(-z)} = \frac{1 + \omega(z)}{1 - \omega(z)},
\]
and from here
\[
2zf'(z) = [1 + 2(\omega(z) + \omega^2(z) + \cdots)] \cdot [f(z) - f(-z)]. \tag{17}
\]

Similarly as in previous cases, by comparing the coefficients in the relation (17), after some simple calculations, we receive
\[
\begin{aligned}
a_2 &= c_1 \\
a_3 &= c_2 + c_1^2 \\
a_4 &= \frac{1}{2} (c_3 + 3c_1c_2 + 2c_1^3) \\
a_5 &= \frac{1}{2} (c_4 + 2c_1c_3 + 5c_1^2c_2 + 2c_1^4 + 2c_2^3).
\end{aligned} \tag{18}
\]

For $a_2 = 0$, $(c_1 = 0)$, from (18) we get
\[
\begin{aligned}
a_3 &= c_2, \\
a_4 &= \frac{1}{2} c_3, \\
a_5 &= \frac{1}{2} (c_4 + 2c_2^2),
\end{aligned}
\]
and using (3),
\[
H_3(2)(f^{-1}) = \frac{1}{4} (2c_2c_4 - c_3^2 - 8c_2^3),
\]
and from here
\[
|H_3(2)(f^{-1})| \leq \frac{1}{4} (2|c_2||c_4| + |c_3|^2 + 8|c_2|^3).
\]

Using the estimates for $|c_4|$ and $|c_3|$ from Lemma 1 (with $c_1 = 0$) from the last relation we have
\[
|H_3(2)(f^{-1})| \leq \frac{1}{4} (1 + 2|c_2| - 2|c_2|^2 + 6|c_2|^3 + |c_2|^4) =: \varphi_4(|c_2|), \tag{19}
\]
where $0 \leq |c_2| \leq 1$. Since
\[
\varphi_4'(|c_2|) = \frac{1}{2} (1 - 2|c_2| + 9|c_2|^2 + 2|c_2|^3)
\]
\[
= \frac{1}{2} [((1 - |c_2|)^2 + 8|c_2| + 2|c_2|^3] > 0,
\]
then \( \varphi_4 \) is an increasing function and \( \varphi_4(|c_2|) \leq \varphi_4(1) = 2 \). So, from (19),

\[
|H_3(2)(f^{-1})| \leq 2.
\]

This result is the best possible as the function \( f_4 \) defined by

\[
\frac{2zf_4'(z)}{f_4(z) - f_4(-z)} = \frac{1 + z^2}{1 - z^2}
\]

shows.

Remark 1. From the relation (4) we get the following.

(a) For \( f \in \mathcal{R} \),

\[
|H_3(2)(f^{-1}) - H_3(2)(f)| = 3|a_3|^3 = 3\left(\frac{2}{3}|c_2|\right)^3 \leq \frac{8}{9},
\]

and the result is the best possible as the function \( f_1 \) shows (in this case \( H_3(2)(f_1) = \frac{4}{15} \) and \( H_3(2)(f_1^{-1}) = -\frac{28}{45} \)).

(b) For \( f \in \mathcal{C} \),

\[
|H_3(2)(f^{-1}) - H_3(2)(f)| = 3|a_3|^3 = 3\left(\frac{|c_2|}{3}\right)^3 \leq \frac{1}{9},
\]

and the result is the best possible as the function \( f_2 \) shows.

(c) For \( f \in \mathcal{S}^* \),

\[
|H_3(2)(f^{-1}) - H_3(2)(f)| = 3|a_3|^3 = 3|c_2|^3 \leq 3,
\]

and the result is the best possible as the function \( f_3 \) shows.

(d) For \( f \in \mathcal{S}_s^* \),

\[
|H_3(2)(f^{-1}) - |H_3(2)(f)| = 3|a_3|^3 = 3|c_2|^3 \leq 3,
\]

and the result is the best possible for the function \( f_4 \).
For obtaining the corresponding result for the whole class $\mathcal{S}$ we will use method based on Grunsky coefficients. In the proof we will use mainly the notations and results given in the book of N.A. Lebedev ([3]).

Here are basic definitions and results.

Let $f \in \mathcal{S}$ and let

$$\log \frac{f(t) - f(z)}{t - z} = \sum_{p,q=0}^{\infty} \omega_{p,q} t^p z^q,$$

where $\omega_{p,q}$ are the Grunsky’s coefficients with property $\omega_{p,q} = \omega_{q,p}$. For those coefficients we have the next Grunsky’s inequality ([2, 3]):

$$\sum_{q=1}^{\infty} \left( \sum_{p=1}^{\infty} \omega_{p,q} x_p \right)^2 \leq \sum_{p=1}^{\infty} \frac{|x_p|^2}{p}, \quad \text{(20)}$$

where $x_p$ are arbitrary complex numbers such that last series converges.

Further, it is well-known that if the function $f$ given by (1) belongs to $\mathcal{S}$, then also

$$\tilde{f}_2(z) = \sqrt{f(z^2)} = z + c_3 z^3 + c_5 z^5 + \cdots \quad \text{(21)}$$

belongs to the class $\mathcal{S}$. Then, for the function $\tilde{f}_2$ we have the appropriate Grunsky’s coefficients of the form $\omega_{2p-1,2q-1}^{(2)}$ and the inequality (20) has the form:

$$\sum_{q=1}^{\infty} (2q - 1) \left( \sum_{p=1}^{\infty} \omega_{2p-1,2q-1} x_{2p-1} \right)^2 \leq \sum_{p=1}^{\infty} \frac{|x_{2p-1}|^2}{2p - 1} \quad \text{(22)}$$

Here, and further in the paper we omit the upper index (2) in $\omega_{2p-1,2q-1}^{(2)}$ if compared with Lebedev’s notation.

If in the inequality (22) we put $x_1 = 1$ and $x_{2p-1} = 0$ for $p = 2, 3, \ldots$, then we receive

$$|\omega_{11}|^2 + 3|\omega_{13}|^2 + 5|\omega_{15}|^2 + 7|\omega_{17}|^2 \leq 1 \quad \text{(23)}$$

As it has been shown in [3, p.57], if $f$ is given by (1) then the coefficients $a_2$, $a_3$, $a_4$ and $a_5$ are expressed by Grunsky’s coefficients $\omega_{2p-1,2q-1}$ of the function $\tilde{f}_2$ given
by (21) in the following way:
\[
\begin{align*}
    a_2 &= 2\omega_{11}, \\
    a_3 &= 2\omega_{13} + 3\omega_{11}^2, \\
    a_4 &= 2\omega_{33} + 8\omega_{11}\omega_{13} + \frac{10}{3}\omega_{11}^3, \\
    a_5 &= 2\omega_{35} + 8\omega_{11}\omega_{33} + 5\omega_{13}^2 + 18\omega_{11}^2\omega_{13} + \frac{7}{3}\omega_{11}^4, \\
    0 &= 3\omega_{15} - 3\omega_{11}\omega_{13} + \omega_{11}^3 - 3\omega_{33}, \\
    0 &= \omega_{17} - \omega_{35} - \omega_{11}\omega_{33} - \omega_{13}^2 + \frac{1}{3}\omega_{11}^4. 
\end{align*}
\] (24)

We note that in the cited book of Lebedev there exists a typing mistake for the coefficient \(a_5\). Namely, instead of the term \(5\omega_{13}^2\), there is \(5\omega_{15}^2\).

**Theorem 2.** Let \(f \in S\) is given by (1) and let \(a_2 = 0\). Then
\[
|H_3(2)(f^{-1})| \leq \frac{\sqrt{3}}{6\sqrt{7}} + 2\sqrt{3} = 3.57321 \ldots 
\]

**Proof.** In the case when \(a_2 = 0\), from (24) we have \(\omega_{11} = 0\), and so
\[
\begin{align*}
    a_3 &= 2\omega_{13}, \quad a_4 = 2\omega_{33}, \quad a_5 = 2\omega_{35} + 5\omega_{13}^2, \quad \omega_{33} = \omega_{15}, \quad \omega_{35} = \omega_{17} - \omega_{13}^2. 
\end{align*}
\] (25)

Using (3) and (25), we have
\[
H_3(2)(f^{-1}) = 4\omega_{13}\omega_{35} - 14\omega_{13}^3 - 4\omega_{33}^2,
\]
and after applying the two last relations from (25),
\[
H_3(2)(f^{-1}) = 4\omega_{13}\omega_{17} - 18\omega_{13}^3 - 4\omega_{15}^2.
\]

From here we have
\[
|H_3(2)(f^{-1})| \leq 4|\omega_{13}||\omega_{17}| + 18|\omega_{13}|^3 + 4|\omega_{15}|^2,
\]
or finally, using \(|\omega_{17}| \leq \frac{1}{\sqrt{7}}\sqrt{1 - 3|\omega_{13}|^2 - 5|\omega_{15}|^2}\) (from (23)) we get
\[
|H_3(2)(f^{-1})| \leq \frac{1}{\sqrt{7}}|\omega_{13}||\omega_{13}|\sqrt{1 - 3|\omega_{13}|^2 - 5|\omega_{15}|^2} + 18|\omega_{13}|^3 + 4|\omega_{15}|^2
=: \frac{1}{\sqrt{7}}\psi_1(|\omega_{13}|, |\omega_{15}|) + 2\psi_2(|\omega_{13}|, |\omega_{15}|),
\] (26)
where
\[ \psi_1(y, z) = y \sqrt{1 - 3y^2 - 5z^2}, \quad \psi_2(y, z) = 9y^3 + 2z^2, \]
with \(0 \leq y = |\omega_{13}| \leq \frac{1}{\sqrt{3}}\) and \(0 \leq z = |\omega_{15}| \leq \frac{1}{\sqrt{5}} \sqrt{1 - 3y^2}\) (where we used the inequality (23)). It is easy to verify that for these range of \(y\) and \(z\), \(\psi_1(y, z) \leq \psi_1(1/\sqrt{6}, 0) = \frac{\sqrt{3}}{6}\) and \(\psi_2(y, z) \leq \psi_2(1/\sqrt{3}, 0) = \sqrt{3}\), so that from (26) we have
\[
|H_3(2)(f^{-1})| \leq \frac{\sqrt{3}}{6\sqrt{7}} + 2\sqrt{3} = 3.57321\ldots
\]

**Remark 2.** From the relation (4) we get for \(f \in \mathcal{S}\):
\[
|H_3(2)(f^{-1}) - H_3(2)(f)| = 3|a_3|^3 = 3|2\omega_{13}|^3 \leq 3 \left(2 \cdot \frac{1}{\sqrt{3}}\right)^3 = \frac{8}{\sqrt{3}} = 4.6188\ldots
\]

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