Accuracy Measure On Fuzzy Near Sets

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Abstract. In this paper the concepts of fuzzy near set, fuzzy near neighborhood, lower and upper approximation of fuzzy near sets are introduced. Accuracy measures on fuzzy near sets are also established.

1. Introduction
The concept of classification of objects was introduced by Pawlak [1]. This generalization lead to the introduction of near set by James F. Peters [2]. Peters considered the problem of approximation of sets of perceptual objects that have matching descriptions. E.A. Marei [3] introduced two approaches to near sets by using topological structures and b-open sets and described some fundamental properties and characterizations of these.

Our aim in this paper is to introduce the notions of fuzzy near neighborhood, lower and upper approximation of fuzzy near sets. Accuracy measures on fuzzy near sets are also established.

2. Fuzzy Near Set

Notation. Throughout this paper \(U\) denotes a non-empty finite universe.

The definitions of fuzzy near sets, class of fuzzy near sets, lower and upper approximation and the boundary region on fuzzy near sets are given in [4].

3. Neighborhood of fuzzy near set

\textbf{Definition: 3.1} Let \(X\) be a non-empty set, \(A \subseteq X\) and \(FN_s(A)\) be a fuzzy near set. The neighborhood of an element \(x \in X\) with respect to the fuzzy near set is defined as
\[
(x)FN_s(A_\epsilon) = \{x' \in X : \left| \mu_{FN_s(A)}(x) - \mu_{FN_s(A)}(x') \right| \geq \epsilon \}
\]
where \(\epsilon\) is the length of neighborhood with respect to the fuzzy near set.

\textbf{Definition: 3.2} Let \(X\) be a non-empty set, \(A \subseteq X\) and \(FN_s(A)\) be a fuzzy near set. The union of the neighborhoods is denoted by \(\zeta(FN_s(A_\epsilon))\) and defined as
\[
\zeta(FN_s(A_\epsilon)) = \bigcup \{(x)FN_s(A_\epsilon) : x \in X\}.
\]

\textbf{Definition: 3.3} Let \(X\) be a non-empty set, \(A \subseteq X\) and \(FN_s(A_1), FN_s(A_2)\) be fuzzy near sets. \(\zeta\) with respect to two fuzzy near sets is defined as:
\[
\zeta(FN_s(A_1), FN_s(A_2)) = \zeta(FN_s(A_1)) \cap \zeta(FN_s(A_2)).
\]

Consequently, \(\zeta\) with respect to \(n\) fuzzy near sets is defined as
\[
\zeta(FN_s(A_1), FN_s(A_2), ..., FN_s(A_n)) = \zeta(FN_s(A_1)) \cap \zeta(FN_s(A_2)) \cap ... \cap \zeta(FN_s(A_n)).
\]

\textbf{Definition: 3.4} The family of neighborhood with respect to the fuzzy near sets is defined as:
\( \eta_1 = \bigcup \{ \zeta(FN_s(A_i)) \}, i = 1, 2, \ldots, n. \)
Similarly, \( \eta_2 = \bigcup \{ \zeta(FN_s(A_i), FN_s(A_j)) \}, i, j = 1, 2, \ldots, n, i \neq j \)
and \( \eta_3 = \bigcup \{ \zeta(FN_s(A_i), FN_s(A_j), FN_s(A_k)) \}, i, j, k = 1, 2, \ldots, n, i \neq j \neq k. \)

**Definition 3.5** Let \( X \) be a non-empty set, \( A \subseteq X \) and \( FN_s(A) \) be a fuzzy near set. Let \( \tau \) be the collection of all subsets of \( \zeta(FN_s(A_i)) \) satisfying the following axioms:
(i) \( \phi, X \in \tau. \)
(ii) The union of the elements of any sub collection of \( \tau \) is in \( \tau. \)
(iii) The intersection of the elements of any finite sub collection of \( \tau \) is in \( \tau. \)

\( \tau \) forms a topology called as the fuzzy near topology on \( X. \)

**Definition 3.6** The new upper approximations for the subsets of \( \zeta(FN_s(A_i)) \) by using the neighborhood with respect to the fuzzy near sets are defined as:
\[
FN_\eta(A) = \bigcap_{x:X \subseteq \{x \mid \zeta(FN_s(A_i))\}} [x \zeta(FN_s(A_i))].
\]

**Definition 3.7** Let \( X \) be a non-empty set, \( A \subseteq X \) and \( FN_s(A) \) be a fuzzy near set. The accuracy measure for any subset \( A \subseteq P(X) \) (where \( P(X) \) is the power set of \( X \)) with respect to the family of neighborhoods \( \eta_i \) and \( \eta \) is denoted by \( AM \) and is defined as:
\[
AM = |FN_s(A)|/|FN_s(A)|, \quad FN_s(A) \neq \phi.
\]

**Example 3.8** Let \( FN_s(A_1), FN_s(A_2) \) and \( FN_s(A_3) \) be three fuzzy near sets defined on a nonempty set \( X = \{\xi_1, \xi_2, \xi_3, \xi_4\} \) as in Table 1. If the length of the neighborhoods of the sets

| Table 1. Three fuzzy near sets |
|--------------------------------|
| \(X\) | \(FN_s(A_1)\) | \(FN_s(A_2)\) | \(FN_s(A_3)\) |
| \(\xi_1\) | 0.1 | 0.91 | 0.52 |
| \(\xi_2\) | 0.22 | 0.91 | 0.83 |
| \(\xi_3\) | 0.34 | 0.1 | 1 |
| \(\xi_4\) | 0.1 | 0.7 | 0.52 |
\[ \tau_3(FN_a(A1), FN_a(A2)) = \{\{\ell_1, \ell_2, \ell_4\}, \{\ell_2, \ell_3, \ell_4\}, X, \phi\} \]
\[ \tau_3(FN_a(A1), FN_a(A3)) = \{\{\ell_1, \ell_2, \ell_4\}, \{\ell_3, \ell_4\}, \{\ell_4, \ell_5\}, \{\ell_1, \ell_2, \ell_3\}, \{\ell_1, \ell_2, \ell_4\}, \{\ell_1, \ell_3, \ell_4\}, \{\ell_2, \ell_3, \ell_4\}, X, \phi\} \]
\[ \tau_3(FN_a(A2), FN_a(A3)) = \{\{\ell_1, \ell_2, \ell_4\}, \{\ell_3, \ell_4\}, \{\ell_4, \ell_5\}, \{\ell_1, \ell_2, \ell_3\}, \{\ell_1, \ell_2, \ell_4\}, \{\ell_1, \ell_3, \ell_4\}, \{\ell_2, \ell_3, \ell_4\}, X, \phi\} \]
\[ \eta_1 = \{\{\ell_1, \ell_2\}, \{\ell_3, \ell_4\}, \{\ell_1, \ell_2, \ell_3\}, \{\ell_1, \ell_4\}, \{\ell_1, \ell_2, \ell_4\}, \{\ell_1, \ell_3, \ell_4\}, \{\ell_2, \ell_3, \ell_4\}, X, \phi\} \]
\[ \eta_2 = \{\{\ell_1, \ell_2\}, \{\ell_3, \ell_4\}, \{\ell_1, \ell_2, \ell_3\}, \{\ell_1, \ell_4\}, \{\ell_1, \ell_2, \ell_4\}, \{\ell_1, \ell_3, \ell_4\}, \{\ell_2, \ell_3, \ell_4\}, X, \phi\} \]
\[ \eta_3 = \{\{\ell_1, \ell_2\}, \{\ell_3, \ell_4\}, \{\ell_1, \ell_2, \ell_3\}, \{\ell_1, \ell_4\}, \{\ell_1, \ell_2, \ell_4\}, \{\ell_1, \ell_3, \ell_4\}, \{\ell_2, \ell_3, \ell_4\}, X, \phi\} \]

| P(X)       | AM_1 | AM_2 | AM_3 | τAM_1 | τAM_2 | τAM_3 |
|------------|------|------|------|-------|-------|-------|
| {\ell_1}  | 0    | 0.33 | 0.33 | 0.25  | 0.25  | 0.25  |
| {\ell_2}  | 0.25 | 0.5  | 1    | 0.25  | 0.25  | 0.25  |
| {\ell_3}  | 0    | 0.33 | 0.33 | 0.25  | 0.25  | 0.25  |
| {\ell_4}  | 0    | 0.25 | 0.33 | 0.25  | 0.25  | 0.25  |
| {\ell_1, \ell_2} | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| {\ell_1, \ell_3} | 0.5 | 0.5 | 0.66 | 0.5 | 0.5 | 0.5 |
| {\ell_1, \ell_4} | 0.5 | 0.5 | 0.66 | 0.5 | 0.5 | 0.5 |
| {\ell_2, \ell_3} | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| {\ell_2, \ell_4} | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| {\ell_3, \ell_4} | 0.5 | 0.5 | 0.66 | 0.5 | 0.5 | 0.5 |
| {\ell_1, \ell_2, \ell_3} | 0.75 | 0.75 | 0.75 | 0.75 | 0.75 | 0.75 |
| {\ell_1, \ell_2, \ell_4} | 0.75 | 0.75 | 0.75 | 0.75 | 0.75 | 0.75 |
| {\ell_1, \ell_3, \ell_4} | 0.75 | 0.75 | 1   | 0.75 | 0.75 | 0.75 |
| {\ell_2, \ell_3, \ell_4} | 0.75 | 0.75 | 1   | 0.75 | 0.75 | 0.75 |

Table 2. Accuracy measure based on neighborhood and topological neighborhood
Figure 1. Accuracy measure based on neighborhood

Figure 2. Accuracy measure based on neighborhood using topology

Figure 3. Accuracy measure for new upper approximation
Table 3. Accuracy measure for new upper approximation

| P(X)      | AM₁ | AM₂ | AM₃ | τAM₁ | τAM₂ | τAM₃ |
|-----------|-----|-----|-----|------|------|------|
| {ℓξ₁₁}   | 0   | 0.5 | 0.5 | 1    | 1    | 1    |
| {ℓξ₂₂}   | 0.33| 1   | 1   | 1    | 1    | 1    |
| {ℓξ₂₃}   | 0   | 0.5 | 0.5 | 1    | 1    | 1    |
| {ℓξ₄₄}   | 0   | 0.5 | 0.5 | 1    | 1    | 1    |
| {ℓξ₁₁, ℓξ₂₂} | 0.66| 1   | 1   | 1    | 1    | 1    |
| {ℓξ₁₁, ℓξ₃₃} | 0.66| 0.66| 0.66| 1    | 1    | 1    |
| {ℓξ₁₁, ℓξ₄₄} | 0.5 | 0.66| 0.66| 1    | 1    | 1    |
| {ℓξ₂₂, ℓξ₃₃} | 0.66| 0.66| 1   | 1    | 1    | 1    |
| {ℓξ₁₁, ℓξ₄₄} | 0.5 | 1   | 1   | 1    | 1    | 1    |
| {ℓξ₂₂, ℓξ₄₄} | 0.5 | 0.66| 0.66| 1    | 1    | 1    |
| {ℓξ₁₁, ℓξ₂₂, ℓξ₃₃} | 1 | 1 | 1 | 1 | 1 | 1 |
| {ℓξ₁₁, ℓξ₂₂, ℓξ₄₄} | 0.75 | 1 | 1 | 1 | 1 | 1 |
| {ℓξ₁₁, ℓρ₃, ℓξ₄₄} | 0.75 | 0.75 | 0.75 | 1 | 1 | 1 |
| {ℓξ₂₂, ℓρ₃, ℓξ₄₄} | 0.75 | 1 | 1 | 1 | 1 | 1 |

References

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