Reconstruction of tsunami source by a part of wave time series

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Abstract. Determination of the initial water surface displacement at tsunami source is an essential part for most of tsunami warning systems. Every additional minute is important for safety measures in case of a near field event. We propose to determine initial sea surface displacement at tsunami source by processing only a part of a measured wave profile – time series. The measured wave profile is an input data for initial wave shape determination algorithm. As is established, even a part of wave profile is enough to determine the wave key parameters with an appropriate precision. This paper provides numerical experiments that allow to determine the length of the necessary part of the measured profile. The results will help decreasing total time spent on tsunami forecast.

1. Introduction

Key characteristics of any tsunami warning system include: (1) Time, required to evaluate the wave parameters and, therefore, to make a decision about tsunami danger, and (2) Reliability in prediction of tsunami danger. The faster wave parameters are correctly evaluated, the more time is available for decision making, evacuation measures, and other safety activities. False alarms have at least two negative corollaries. First, economic losses could be very high due to evacuation measures, transportation delays, shutting down businesses, etc. Second, after several false warnings nobody will trust the system and so in case of a real danger no protection measures will be taken.

In this paper, we discuss one function of tsunami warning system, namely, determination of initial sea bed displacement at tsunami source. In fact, we rather determine the sea surface displacement based on the measurements of the sea level by the deepwater pressure detectors (or any other sensor system). These could be the DART buoys [1] or any other sensor. The so-called “calculation in advance method” is used, see [2]. This approach has been first applied in 1996 at Alaska-Aleutian subduction zone. The entire subduction zone was covered with 50 so-called “Unit Sources” (US’s) – 50x100 km rectangles (typical size of the seabed displacement area for tsunamigenic 7.5 M earthquake), figure 1. Then the following data base was developed and now covers all subduction zones of the Pacific. The selected shape of initial displacement was positioned at each of 50 US’s as the model tsunami source with the normalized amplitude of 0.57 m. Wave propagation over the entire Pacific Ocean was numerically calculated according to the linear approximation of the shallow water
system. As the governing equations are linear, the “complex” tsunami source that is the linear combination of several US’s, should generate the same linear combination of tsunami waves from these Unit Sources everywhere. The idea was to invert such scheme. First, at the given point (where tsunami wave is measured with the help of any sensor, DART station or other), we determine a linear combination of the calculated (as was mentioned above) “Unit Sources generated” tsunami waves, which provides the best mean square approximation of measured tsunami wave. Then we suppose that the same linear combination of normalized initial shapes optimally describes sea surface initial displacement at tsunami source. This technology, proposed and implemented by V. Titov, has been tested against many historical events and geo locations. Pretty good agreement with field observations was obtained.

Figure 1. 2D shaded and 3D images of the initial displacement at the Unit source used for aggregating composite sources.

In this paper we test numerically a very fast algorithm to determine coefficients of linear combination of USs, which provide the best approximation of the measured data by using only a part of measured wave profile. The first attempts to do this were dated by 2007 [3]. The algorithm, proposed in [4], is based on “orthogonal decomposition” of the measured wave profile with respect to simulated (calculated) waves from USs.

2. Mathematical model and calculation scheme

The information needed for the orthogonalization algorithm is the set of the tsunami wave profiles at certain geo-locations, obtained at the available sensor system. For example, at bottom based cable net (DONET – Dense Ocean-floor Network System for Earthquakes and Tsunamis – pressure gauge network of Japan, see [5]), or the time-series, obtained at DART buoys [1]. Figure 2 show the pressure sensors positions of different kinds all around Japan.

Figure 2. The ocean level observation network around Japan [5].
Aiming to avoid numerical solution to 2D inverse problem of a source term determination, following [4], [6], we treat the calculated wave profiles (generated at a given point by the normalized disturbance, located at different US’s) as a set of linearly independent basis functions. Thus, our problem is regarded as the classical mathematical problem of the best approximation of a given function (measured wave profile) by a linear combination of the given (finite) system of linearly independent basis functions (calculated profiles of the waves, generated by normalized disturbance from several US’s). Approximation is understood in the sense of $L_2$ space (mean square). This problem is well studied in mathematics. As was proved, the best values of coefficients of our linear combination are nothing but the corresponding Fourier coefficients, provided that our basis functions are orthogonal and normalized. The described idea was implemented in a software application [4] and then (see [6]) has been optimized and tested on synthetic (computed) data. A large tsunami source area was simulated, precisely 12 US’s – rectangles 50*100 km – arranged in two lines, was considered. In other words, the artificial tsunami source covers the area of up to 100*600 km. Even in case of a very small amplitude coefficient at one end of the source area (compared to 0.1 m), it was stably reconstructed at the over edge of water area considered, that is at the distance of 1000 km alongshore. At the same time, two neighbor US’s (one seaward and one shoreward) with rather different amplification coefficients (say, 0.1 m and 10 m) does not corrupt the information from each other.

Let $f(t)$ be the marigram – time series, obtained at any particular sensor (DART buoy, bottom cable sensor, or other). We will use notations $f_k(t), \ (k=1,\ldots,N)$ for simulated wave profiles, calculated (according to the linear or nonlinear shallow water approximation) at the same point of sensor location, by direct numerical modeling of the tsunami wave propagation. It is understood, that the subindex $k$ refers to a tsunami wave initiated by the normalized sea surface disturbance of a given shape from the $k$-th US. We assume that all these functions, $f_k(t)$, are linearly independent. We are looking for coefficients, $b_k$, for the linear combination of these functions, which provide the best approximation of the function $f(t)$ in $L_2$ norm:

$$\int_0^T (\sum_{k=1}^N b_k f_k(t) - f(t))^2 \, dt \to \min$$  \hspace{1cm} (1)

So, we consider the problem of optimal approximation of a given function, $f(t)$, by the linear combination of the finite subset of functions from the system $\{f_k(t)\}$. Suppose that the system is orthogonal and normalized according to:

$$\langle f_k(t), f_j(t) \rangle = \int_{t_0}^T f_k(t)f_j(t)dt = 0, \hspace{1cm} \langle f_j(t), f_j(t) \rangle = 1. \hspace{1cm} (2, 3)$$

As it is well known in the Fourier series theory, the coefficients of such an optimal approximation are nothing but the Fourier coefficients of expansion of $f(t)$ in a series with respect to $\{f_k(t)\}$. Suppose that the system is orthogonal and normalized according to:

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Based on the statement above, the following algorithm was proposed and tested in [6] for searching the coefficients of the linear combination allowing optimal approximation of the given function. It includes three steps. First, the calculated system of “marigrams”, $\{f_k(t)\}$, from the US’s should be orthogonalized and normalized, according to equations (2)-(3). Second, the measured marigram, $f(t)$, should be expanded to the Fourier series with respect to obtained orthonormal basis. Finally, the so-determined Fourier coefficients should be recalculated in terms of initial functions $\{f_k(t)\}$.

So, the proposed algorithm to approximate the given marigram, $f(t)$, by the linear combination of synthetic marigrams, $\{f_k(t)\}, \ k=1,\ldots,N$, in order to achieve minimal $L_2$ distance according to equation (1), is as follows. We first transform the system $\{f_k(t)\}$ to the orthogonal system, $\{e_k(t)\}$, and normalize it. Then the recorded marigram, $f(t)$, is decomposed to the part of Fourier series with respect to obtained orthogonal and normalized (see equations (2) and (3)) basis, $\{e_k(t)\}$. Finally, the obtained Fourier coefficients, $a_n$, are recalculated in terms of coefficients of the original linear combination (1).
3. Numerical experiments
We use the following steps to determine the Necessary Part of Wave Profile (NPWP), which is enough to reconstruct the sought coefficients:

(a) Preparation of the input data, namely digital bathymetry of the water area under study and a version of Initial Sea Surface Displacement (ISSD) as a set of amplification coefficients for the selected US’s.

(b) For each ISSD (set of amplification coefficients) the wave propagation was computed at the entire water area, included the points, where the artificial sensors are located.

(c) For each synthetic marigram (computed wave profile at given artificial sensor):
   (i) Determination of travel time (initial moment – wave arrival to artificial sensor moment);
   (ii) Calculation of ISSD by orthogonal decomposition using various parts of the wave profile;
   (iii) Identification of the minimal part of the wave profile, providing ISSD approximation with the reasonable accuracy (that is not more than 10% error at each point of an initial disturbance area).

Wave propagation for each set of amplification coefficients was computed with the help of specialized Calculator, which uses a version of Mac-Cormac scheme [8]. In order to determine the desirable NPWP, the “orthogonalization” algorithm (described above) was launched at different parts of the computed wave profile. We check wave profile from the beginning (time then wave approaches a given sensor) to the moment just after the highest maximum (see figure 2). Time step was adaptive, from 1000 sec to 1 sec. Thus, we first take 1000 sec interval, which include the entire wave profile in our cases. Then we take 1/10 of the initial time step (that is 100 sec) and determine a number of these 1/10th parts, which are enough to reconstruct the ISSD with the pointwise relative error less than 10%. Then the last part of 100 sec time step is divided into 10 substeps (10 sec each) and so on. As the result we obtain a part of the wave profile, which is enough to reconstruct ISSD within 10% pointwise relative error with the time step 1 sec. It means that first of all we find the minimal appropriate profile part value that is divisible by 1000, and then we repeatedly decrease the step value and search through values inside the last step to find more exact profile part value.

![Figure 3. Bottom relief of computational domain and location of the unit sources with observation network.](image-url)
Water area around the North-East coast of Japan was considered. Possible tsunami sources in this region are situated over the west slope of the Japan deep-water trough (white rectangles in figure 3). For our particular study, a regular rectangular calculation grid, connected with the geographical coordinates, has been created. This was based on the Japanese database [7], distributed by the Data On-line Service System (J-DOSS) of the Japan Oceanographic Data Center (JODC). The mesh size of this digital bathymetry is compared to 300m. Depths from this database were interpolated for regular grid, connected with the World geography. Connection with geography means that the East-West grid step slightly changes from southern edge of computation domain to its northern part. We account this fact in computations. The above bathymetry, visualization of which is given in figure 3, and the computational grid have the following characteristics:

- Mesh size is 3219 x 3219 points;
- Mesh steps are equal to 0.00248 arc degrees (which means 280 m in North-South direction and 226 m in East-West direction);
- Computation domain is between 140° and 147.9944° E, 34.0° and 41.9948° N.

We consider a system of US’s (50*100 km rectangles), covering the subduction zone (edge lines of the tectonic plates in the area), arranged in two lines (see figure 3). Each line contains 6 US’s. The entire set of 12 US’s, the location of which is given in figure 3, makes it possible to consider a variety of possible epicenter locations of an expected earthquake. Let us denote the unit sources as US₁ – US₁₂ and the receivers (artificial sensors) as R₁-R₃, see figure 3. For numerical tests we use the Composed Source (CS) containing 4 US’s, moving it along the lines of the US’s. CS positions are also indicated in figure 3. Each CS is a direct sum of a number of US’s with a certain amplification coefficients C₁=0.7, C₂=0.8, C₃=1.2, C₄=1.3. So, a CS represents a realistic sea bed displacement at tsunami source which profile is presented in figure 4. In numerical experiments such CS are turned by a negative wing towards the coast.

The large square of the initial sea bed displacement (nearly 100*200 km) is taken to check the idea of source reconstruction by only a part of a wave profile.

We will consider 5 such CS’s, shifting from the South to the North. At the same time, a set of coefficients for US’s will be the same to all considered five sources. For registration of a wave the three deep-water detectors which are most close located to an axis of a deep-water trench (figures 2 and 3) are used. During the numerical experiments it is required to demonstrate a possibility of correct reconstruction of the composite source before arrival of a tsunami to the closest site of the coast.

Figure 5 represents four calculated wave signals from composite sources: CS4 and CS5 by different censors R₁, R₂. Here the black vertical line on each mareogram designates length of wave time series, sufficient for the correct CS restoration. The gray vertical line designates time instance when the wave reaches the closest sites of the Japanese coast.
Lengths of a wave signal (as a percentage), necessary for the correct restoration of a composite source, for all numerical experiments are given in table 1.

Figure 5. Computed tsunami wave forms at deep-water sensor locations. Wave generated by CS4 and detected by R1 (a), CS4 by R2 (b), CS5 by R1 (c), CS5 by R2 (d).

Table 1. Lengths of wave signal, necessary for the correct restoration of a composite source.

| Composite source | Receiver | Period, sec | NPWP, sec | NPWP divided by period | Wave arrival time to the shore, sec |
|------------------|----------|-------------|-----------|------------------------|------------------------------------|
| CS 1             | R1       | 935         | 446       | 47,7%                  | 193                                |
| CS 1             | R2       | 861         | 444       | 51,6%                  | 193                                |
| CS 1             | R3       | 782         | 449       | 57,4%                  | 193                                |
| CS 2             | R1       | 958         | 481       | 50,2%                  | 306                                |
| CS 2             | R2       | 816         | 469       | 57,5%                  | 306                                |
| CS 2             | R3       | 875         | 471       | 53,8%                  | 306                                |
| CS 3             | R1       | 898         | 426       | 47,4%                  | 655                                |
| CS 3             | R2       | 911         | 443       | 48,6%                  | 655                                |
| CS 3             | R3       | 956         | 456       | 47,7%                  | 655                                |
| CS 4             | R1       | 797         | 522       | 65,5%                  | 810                                |
| CS 4             | R2       | 817         | 461       | 56,4%                  | 810                                |
| CS 4             | R3       | 783         | 456       | 58,2%                  | 810                                |
| CS 5             | R1       | 860         | 556       | 64,7%                  | 1094                               |
| CS 5             | R2       | 892         | 494       | 55,4%                  | 1094                               |
| CS 5             | R3       | 1053        | 238       | 22,6%                  | 1094                               |

4. Discussion
A series of numerical experiments with various location of CS showed that coefficients for US forming that source well determined by any of R1-R3 sensors taking into account only a part of the first period of a wave record. The difference consists is the following: for some CS positions tsunami reaches the coast earlier, then it is possible to restore coefficients for USs. In particular, the wave
generated by CS1 and CS2 reaches the next coast significantly earlier, then the sensor (even closest to CS) will be able to register the part of a wave signal sufficient for correct restoration of the initial water surface displacement in the source.

5. Conclusion
The proposed “orthogonal decomposition” method provides a possibility to reconstruct the approximation of the Initial Sea Surface Displacement at tsunami source using only a part of the measured wave profile. In case the “receiver” (DART buoys, DONET sensor, or other) is right in front of the ISSD zone, it may require only 20% of the wave profile for the source reconstruction. Rather complicated Composite Sources (CS) were used in numerical tests. In the future we are going to arrange extended numerical experiments to provide simple criteria to stop measurements and to optimize location of a sensor system to save more time for decision making and evacuation activities. As compared to the “brute force” method for determination of the coefficients in the linear combination of unit sources the method of “orthogonal decomposition” can help to restore a large tsunami source which consists of more than 5 unit sources at short time. Numerical experiments discover the higher efficiency of sensors R1, R2 for reconstruction of CS1-CS4 (see table 1). Sensor R3 is more effective for restoration of the Composite Source 5. It is possible to determine the unit-source coefficients using only one of these detectors. With the proposed “orthogonal decomposition” method it is possible to determine the initial sea surface displacement at tsunami source using only a part of the measured wave profile and, therefore, to save minutes for decision making and evacuation activities.

References
[1] DART (Deep-ocean Assessment and Reporting of Tsunamis) [Electronic resource]: https://nctr.pmel.noaa.gov/Dart/ (accessed 17.03.2018)
[2] Gica E, Spillane M C, Titov V V, Chamberlin C D and Newman J C 2008 Development of the Forecast Propagation Database for NOAA’s Short-Term Inundation Forecast for Tsunamis (SIFT)
[3] Bezhaev A Yu, Marchuk An G and Titov V V 2007 Estimation of initial elevation in the extended tsunami sources on the base of deep water wave measurements Proc. IX All-Russia conference “Modern methods for mathematical modelling of natural and anthropogenic disasters”, Barnaul, http://www.ict.nsc.ru/ws/hazards 2007/12510/Bezhaev full v2.pdf (accessed 09.11.2017)
[4] Romanenko A and Tatarintsev P 2013 Algorithm for reconstruction of the initial surface disturbance at the tsunami epicenter Vestnik NSU IT Ser. 11 113–23 [in Russian]
[5] Tsushima H and Ohta Y 2014 Review on near-field tsunami forecasting from offshore tsunami data and onshore GNSS data for tsunami early warning J. Disaster Research 9 339–57
[6] Lavrentiev M, Kuzakov D, Romanenko A and Vazhenin A 2017 Determination of Initial Tsunami Wave Shape at Sea Surface OCEANS 2017-Aberdeen, IEEE 1–7
[7] JODC 500 m digital bathymetry around Japan [Electronic resource]: http://jdoss1.jodc.go.jp/vpage/depth500_file.html (accessed 17.02.2019)
[8] Lavrentiev M, Lysakov K, Marchuk A, Oblaukhov K and Shadrin M 2019 Fast evaluation of tsunami waves heights around Kamchatka and Kuril islands Sci. Tsunami Hazards 38 1–13