Field integrals for the ATLAS tracking volume

V.I. Klyukhin  
*IHEP, Protvino, Russia*

A. Poppleton  
*CERN, Geneva, Switzerland*

J. Schmitz  
*NIKHEF-H, Amsterdam, The Netherlands*

1 Introduction

The ATLAS inner tracker measures charged track momenta from the deflection in a solenoidal magnetic field. The choice of a long tracking cavity, allied to the decision to build the coil inside the electromagnetic calorimeter, leads to a favourable solenoid aspect ratio which ensure a highly uniform field in the central tracking region. However the situation is not ideal for tracks with $|\eta| \gtrsim 1.8$, since $\sim 25$ cm before the end of the coil the magnetic field lines become parallel to the direction of stiff tracks from the vertex region. The subsequent track deflection is in the opposite sence to that experienced in the central region. As the last tracking detector lies about 20 cm beyond the end of the coil, almost 15% of the length of these forward tracks is subject to reverse bending. The purpose of this note is to compare the expected momentum resolution from the LOI magnet configuration with that from an ‘ideal’ (infinity long) solenoid with constant field in the axial direction.

The calculations were performed using a field map extracted from the POISSON program package [1]. The solenoidal coil was set to be 6.3 m long with a radius of 1.23 m. The number of Ampere-turns of the coil was tuned to give a magnetic flux density of 2 T at the centre of the solenoid. The cylindrical inner cavity for tracking detectors was taken to be 6.8 m long with 1.06 m radius. The field lines (assuming an iron return flux structure outside the hadronic calorimeter) are illustrated in Fig. 1. Fig. 2 and Fig. 3 show the variation of the magnetic flux density across radial and longitudinal sections of the tracking volume, respectively.

2 Magnetic field integrals and momentum resolution

To derive the connection between the momentum resolution of a particle passing through this solenoid and the field integral over the track length, consider the trajectory of a particle emitted, at angle $\theta$ to the beam axis, from the nominal beam crossing point (which is taken as the origin of the coordinate system). The $z$-direction is defined to be along the beam axis, and the transverse radius $r$ is the orthogonal distance from the $z$-axis.
For a small step $d\vec{l}$ along the direction of the particle motion in an ideal solenoid, $d\alpha$ (the change to the turning angle of the track) lies in the transverse plane and is given by

$$d\alpha = \frac{0.3}{p_T} B dl \sin \theta,$$

where $l$ is in meters, $p_T$ is the transverse momentum of the particle in GeV/$c$, and $\vec{B}$ is the vector of the magnetic flux density in Tesla. In general, for an inhomogeneous field, the track is turned according to, and in the direction of, the vector product $d\vec{l} \times \vec{B}$. However for tracks from the vertex region of the ATLAS solenoid, the majority of the tracking volume is contained in a good approximation to the field from an ideal solenoid, and furthermore $\vec{B}$ and $d\vec{l}$ are nearly parallel for the remaining trajectory (which is at large $|z|$). Thus the subsequent comparison is restricted to the transverse projection; firstly because it contains the dominant deflection component, and secondly because precision measurements to determine the track momentum will probably only be available in this plane.

For energetic particles the magnetic deflection is small compared to the track length, thus the distance along the trajectory can be approximated by $l = r/\sin \theta$ and small angle approximations are valid. At $l$ the relative angle $\alpha$ of the track with respect to its initial direction in transverse projection is given by

$$\alpha(l) = \frac{0.3}{p_T} \int_0^l B \sin \theta(\vec{d}l, \vec{B}) dl,$$

where $\theta(\vec{d}l, \vec{B})$ represents the longitudinal component of the angle between the track and field vectors. The total transverse deflection $x$ is obtained by integrating eq.(2) over $dr = dl \sin \theta$:

$$x(l) = \frac{0.3}{p_T} \int_0^l B \sin \theta(\vec{d}l, \vec{B}) \int_0^{r/\sin \theta} dl dr.$$

For the ideal solenoid, $x(l)$ is simply proportional to $l^2$.

Now consider a cluster of measurements, near the end of the tracking volume, providing a momentum measurement by their impact parameter at the beam axis. In this limiting case, the relative momentum precision for a real versus ideal solenoid is given by $R$, the relative values of $x(L)$, where $L$ is the total track length in the tracking volume. The degradation in $\Delta p/p$ is thus $1 - R$.

However a more precise momentum measurement can be obtained from measuring the track sagitta, which is $x(L)/2 - x(L/2)$. For the ideal solenoid the sagitta equals $x(L/2)$ since $x(L) = 4x(L/2)$ from eq.(3). The value of $x(L/2)$ is essentially the same for the ideal and ATLAS solenoids, for the reason given previously, so the ratio of the sagittas is given by $2R - 1$ and $\Delta p/p$ is degraded by $2(1 - R)$. The sagitta measurement is the least favourable extreme when taken in comparison with the ideal solenoid; to give an example: if the relative double field integral $R$ equals 0.85, then $\Delta p/p$ is degraded by 15% for an impact parameter measurement and 30% if the sagitta is measured.

In practice, measurements are taken along the track length causing degradation which lies between these two extremes; however the current ATLAS forward detector layout is close to the sagitta extreme, so $\Delta p/p$ is degraded by almost twice as much as the field integral.
Figure 1: top) Field lines for ATLAS solenoid with iron return flux, bottom) zoom onto inner tracking cavity.
Figure 2: Field components across radial sections of the tracking volume.

Figure 3: Field components across longitudinal section of the tracking volume.
The double field integral \( I_2 \) for the ideal (solid line) and inhomogeneous (dashed line) cases.

3 Results

To evaluate realistic ATLAS (inhomogeneous) fields integrals, the program POISGT [2] was used to extract magnetic field map from POISSON. This field map is meant to be linked to DICE for accurate simulations. The dependence of the double integral

\[
I_2 = \int_0^L \frac{r}{\sin \theta} \int_0^{\theta_{\text{max}}} B \sin \theta \left( d\vec{l}, \vec{B} \right) dldr
\]

versus \( \eta \) is shown in Fig.4 for the ideal (solid line) and inhomogeneous (dashed line) fields. Fig.5 shows \((1 - R)\), the degradation in this field integral, as a function of \( \eta \).

It can be seen that \( I_{2i} \) (\( I_2 \) inhomogeneous) is almost constant up to \(|\eta| = 0.6\) (it decreases by only 0.1% when compared to the ideal solenoid); the decrease reaches 1% by \(|\eta| = 1.1\), 5% at \(|\eta| = 1.6\) and 10% at \(|\eta| = 1.8\). Beyond the corner of the inner tracking cavity in the rz-plane (\(|\eta| > 1.88\)), the degradation only increases slowly from \(\sim 14\%\) at \(|\eta| = 1.88\) to \(\sim 15\%\) at \(|\eta| = 3.0\). Of course the magnitude of the field integral is falling rapidly in this region due to the reduced radial distance traversed. This directly effects the momentum resolution at a given \( p_T \) according to eq.(3).

An important question is whether the extra lever arm at the end of the tracking cavity is significantly improving the momentum resolution in this region (\(|\eta| > 1.88\)), since some free space in front of the end-caps might be desirable for installation convenience (e.g. to provide a patch-panel) and will be needed for the detector frames. For the ideal solenoid, moving the last chamber inwards by 20 cm would reduce the field integral by 11.4%; with the inhomogeneous field this reduction is less pronounced (\(\sim 7\%\)) but still significant, the \(1 - R\) degradation decreases accordingly.

\(^{1}\)However there is an additional \( \sin \theta \) term to be included when considering the momentum resolution for a fixed value of \( p \), which is perhaps more appropriate for extremely high momenta. From this viewpoint \( x(L) \), the field integral contribution to \( \Delta p/p \), rises from \( \eta = 0 \) to a maximum at \(|\eta| = 1.8\), then falls back to approximately the \( \eta = 0 \) value at \(|\eta| = 3\), i.e. although \( I_2, (\eta = 3) \sim 0.1I_2, (\eta = 0) \), a 1 TeV track at \(|\eta| = 3\) has \( p_T \sim 100 \text{ GeV} \), thus much the same \( \Delta p/p \) as for 1 TeV at \( \eta = 0 \) under the assumption of similarly precise measuring stations.
To conclude, the current geometry of the inner cavity of the ATLAS detector results in an ideal 2 T magnetic field for \(|\eta| < 0.6\). Beyond that rapidity, there is a loss of bending power which rises to a plateau for \(|\eta| \gtrsim 1.9\). Depending on the \(z\)-coordinate of the last tracking chamber, the field integral degradation for \(|\eta| \gtrsim 1.9\) is \(\sim 10 - 15\%\), resulting in a 20 – 30\% worsening of the \(p_T\)-resolution with respect to the value that would be obtained from an ideal solenoid field.

**References**

[1] R.F. Holsinger and C. Iselin. *The CERN-POISSON Program Package (POISCR) User Guide*. CERN Computer Center Program Library, T604, (1984).

[2] V.I. Klyukhin. *Interface Program for POISSON and GEANT Packages*. Proc. of the Conf. on Computing in High Energy Physics, 21-25 September 1992, Annecy, France.