Mechanical Wear Contact between the Wheel and Rail on a Turnout with Variable Stiffness

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Abstract: The operation and maintenance of railroad turnouts for rail vehicle traffic moving at speeds from 200 km/h to 350 km/h significantly differ from the processes of track operation without turnouts, curves, and crossings. Intensive wear of the railroad turnout components (switch blade, retaining rods, rails, and cross-brace) occurs. The movement of a rail vehicle on a switch causes high-dynamic impact, including vertical, normal, and lateral forces. This causes intensive rail and wheel wear. This paper presents the wear of rails and of the needle in a railroad turnout on a straight track. Geometrical irregularities of the track and the generation of vertical and normal forces occurring at the point of contact of the wheel with turnout elements are additionally considered in this study. To analyse the causes of rail wear in turnouts, selected technical–operational parameters were assumed, such as the type of rail vehicle, the type of turnout, and the maximum allowable axle load. The wear process of turnout elements (along its length) and wheel wear is presented. An important element, considering the occurrence of large vertical and normal forces affecting wear and tear, was the adoption of variable track stiffness along the switch. This stiffness is assumed according to the results of measurements on the real track. The wear processes were determined by using the work of Kalker and Chudzikiewicz as a basis. This paper presents results from simulation studies of wear and wear coefficients for different speeds. Wear results were compared with nominal rail and wheel shapes. Finally, conclusions from the tests are formulated.

Keywords: wear; turnout; rail; stiffness; high-speed

1. Introduction

The study of wear in wheel-track systems is the subject of many works [1–11]. In these works, the task of wear is addressed by presenting different models of wheel–rail interaction. In most works, a constant value of normal force was assumed. In some works, simulations of rail vehicle movement on a straight track without a turnout were performed. Motion without a turnout and motion through a turnout were considered with track susceptibility (elasticity and viscous damping) as constant. According to the research conducted in [12], there are significant differences in the appearing vertical forces and normal forces when passing through a turnout with different values of the beam–subtrack system susceptibility. These forces affect the process of phenomena in wheel–rail contact and have an impact on the wear in the wheel–rail pair. The wear phenomenon was studied based on the works [13,14]. Simulation of rail vehicle movement on a straight track without a turnout and a track with a turnout was also shown.

The infrastructure of a rail transportation system consists of railroad tracks, curves, intersections, and turnouts. Turnouts are a complex structure of railroads. They connect neighbouring tracks and enable railway vehicles to change direction of travel. The basic type is the ordinary turnout consisting of switch blades (2), closure rails (2, 3), a crossing frog (4), turnout sleepers, and setting devices. The crucial element of each turnout is the
frog (4) that enables the wheels of railway vehicles to roll over the place of rail crossing. Due to difficult operating conditions characterised by high-dynamic loads generated by the wheels of rail vehicles, the crossbeams are particularly exposed to the destructive character of impact loads [15–20]. Crossings can have a fixed or movable bow. The subject of the analysis is a right-hand ordinary turnout with a radius of $R = 1200$ m, a fixed crossbeam of 1:9, and three setting closures with a holding force of 7.5 kN (each). The individual elements of this turnout are shown in Figure 1.

![Figure 1. Normal right turnout 1—Stock rail, 2—Switch blades, 3—Closure rails, 4—Frog, 5—Guardrail.](image)

The mechanical destruction process of the surface layer leads to undesirable changes in the dimensions and shape of the contacting rolling surfaces of the turnout and railroad wheel elements. Degradation of turnouts, especially crossbucks, contributes to the increase in dynamic interactions, which has an adverse effect on the cooperation of the wheel–rail system [21–26].

Railroad turnouts are particularly exposed to abrasive and fatigue wear, which causes shape changes that result from the impact of high-dynamic loads of cyclic nature that occur during the passage of rail vehicles [27,28].

Railway turnouts are important elements of railway infrastructure that ensure traffic runs smoothly between different branch tracks [29]. A turnout is a structure that allows railway vehicles to pass from one track to another while maintaining a certain speed [30]. One of the most common railway turnouts is the regular turnout (Rz) [31]. It consists of three basic units, such as the switch train assembly, the connecting rail assembly, and the crossover assembly. The switch assembly is a movable turnout unit that moves the switch blades by means of a drive. A smooth and safe track change depends on the correct execution of the initial part of the switch blade, which must have the appropriate shape in order to adequately adhere to the supporting rail in the switch. In turnouts, there are often two wheel–rail contact surfaces, as well as disturbances in the nominal wheel–rail contact conditions due to wheel movement from the main rail to the switch rail [32]. The dynamic interaction between a rail vehicle and a railway turnout is more complex than that of normal or curved tracks. Severe shock loads may occur during the passage through the turnout, generating severe damage to the surfaces of the turnout components [33].

Traffic of rail vehicles in regular operation may be considered as a source of influence of high-dynamic loads of cyclic nature, which translates into damages in the form of abrasive and fatigue wear and tear, as well as changes in the shape and dimensions of the outer layer [34–38]. Calculations of dynamic loads and resulting contact and internal stresses enable rational design of railway turnouts and correct selection of material to construct their elements [39]. The results described in [40] show that profile wear disturbs the distribution of wheel–rail contact points, changes the position of wheel–rail contact points along the longitudinal direction, and affects the dynamic interaction between the vehicle
and the turnout. Profile wear disturbs the normal contact situations between the wheel and switch rail and worsens the stress condition of the switch rail [41]. This model allowed the rational design of railway turnouts and the correct selection of material from which their components are made [42].

The development of turnout constructions also results from technological progress in the production of new steel grades for railway turnouts, the development of new material testing methods, and a better understanding of the phenomena occurring in wheel–rail interaction [43–48].

2. Mathematical Model of the Rail Vehicle–Track System

To determine the durability of individual elements of the railroad switch, it is necessary to calculate the characteristics of the load in the function of time and distance, originating from the wheelsets of the railroad vehicles acting along the switch. The method used to determine the distribution of forces along the switch is the simulation of mathematical models showing the dynamics of the rail vehicle–track system using computer software. When modelling the dynamics of the wheel–rail system, the most used programs are MATLAB and Universal Mechanism. The Universal Mechanism program provides greater capabilities in modelling dynamic phenomena.

In the modelling process, a high-speed train was used, the parameters of which were taken from the work [19].

The mathematical model of the rail vehicle was built based on linear and angular coordinate systems shown in Figure 2.

![Figure 2. Linear and angular coordinate system.](image)

This system is used as an inertial system associated with rigid solids, of which three groups can be listed in a vehicle (typical) as shown in Figure 3.
The vehicle has a body, two bogies, and four sets of wheels. The coordinate systems originate at the centre of mass of the individual solids, and the axes lie on the axes of symmetry. There is an identical system associated with the track that is called a non-inertial system. The matrix of directional cosines between the inertial and non-inertial systems was assumed to be zero–one [14,15].

In addition, the equations of the system were assumed according to the coordinate system and Figure 3.

The ties for analysing the system can be written as follows (Figure 4):

\[
\begin{align*}
\Phi &= \frac{z_p - z_l}{2b}, \\
z_{tl} &= z_l - z_{wl} - \left(y - \frac{y_{wp}}{2}\right)\sigma, \\
z_{tp} &= z_p - z_{wp} + \left(y - \frac{y_{wp}}{2}\right)\sigma, \\
\dot{z} &= -\frac{1}{r_s}\dot{y},
\end{align*}
\]

where \(2b\) is the distance between the contact points (wheel and rail) in the middle position of the wheelset; \(r_s\) is the radius of the wheel included in the wheelset, measured in the middle position; \(\sigma\) is the coefficient linking the angular and transverse displacement of a wheelset; and \(z_{tp}, z_p, z_{wp}, z_{tl}, z_l\), and \(z_{wl}\) are auxiliary nodes, used in the mathematical description of the movement of the railway vehicle.

The following assumptions were made in developing the equations of motion:

The vertical loads occurring on the rail will be a variable value and will be determined from the previous step of mathematical calculations performed to determine the train parameters (wheelset and bogie spacing).
The railroad track was modelled as a Euler–Bernoulli beam on which a wheel with velocity \( v \) is rolling (motion on the straight track and motion on the turnout return track were considered).

The contact between the rolling surfaces of the wheels and the rail heads is defined based on Kalker’s linear theory (defining ellipses with semi axes \( a \) and \( b \)).

In the wheel–rail contact area, the Coulomb kinetic sliding friction with a constant coefficient of friction is considered.

Such phenomena as adhesion, micro-slip, and material wear of the wheel and the rail were also considered in the dynamics of vehicle movement on the track.

In the model under consideration, the possibility of two contact ellipses occurring because of two-point rolling of the wheel on the rail within the turnout has been taken into account.

Suspension elements of the first and second stage were assumed to be linear for all assumed coordinates.

For the track without a turnout, the susceptibility was assumed according to the Voigt model (linear stiffness and linear damping). These quantities were determined by measurements on the actual object and consist of the stiffness coefficient, \( 0.2 \times 10^8 \text{ N/m} \), and damping coefficient, \( 3.2 \times 10^3 \text{ Ns/m} \) [16].

At the turnout, the stiffness coefficient was calculated according to the parameters shown in Figure 5.

![Figure 5](image-url)

Figure 5. The course of variation of the vertical stiffness coefficient of the rails in the switch with different values of the bedding coefficient (real measurements on CMK Idzikowice): 1—Inner track (with cross-brace), 2—Outer track [18].

Considering geometric and structural constraints, the rail vehicle has 27 degrees of freedom. The equation can be found in the work [19].
3. Results

The aim of this paper is to analyse the wear of a railway turnout with a radius of 3000 m, considering the change in contact area resulting from the variation of normal force. For the guidance mechanism of a wheelset on a through track, if the wheelset is offset to the side, the wheel radii at the wheel contact point are different. Due to the rigid coupling of speeds, one wheel becomes the driving wheel, and the other wheel becomes the braking wheel. This leads to a “steering motion” which drives the wheelset back to the centre of the track. The movement continues past the track axis until the same situation occurs in mirror image to the starting position; then the process starts again. It should be noted that during the passage through the crossing area, there are sleepers laid next to each other that are connected with a change in the substrate stiffness. The next stage of analysis on rail vehicle motion is the passage on a diverging track where, despite the rigid speed coupling between the wheels rolling on the inside and outside of the curve, the wheelset can turn without slipping on curves with large radii. This is possible because the lateral displacement towards the outer rail of the curve turnout results in a difference in wheel radii, which means that the peripheral velocity at the point of contact for the outer wheel is greater than that of the inner wheel. Bearing in mind the discussed phenomena, an attempt has been made to investigate the change in the value of normal force for wear that occurs in railway turnouts.

Using Universal Mechanism and MATLAB software, simulations were performed to determine vertical forces and normal forces for speeds from 100 km/h to 350 km/h, shown in Figures 6–17.

![Figure 6](image-url)  
**Figure 6.** Vertical force on wheels at 100 km/h on straight track through a turnout.
Figure 7. Vertical force on wheels at 150 km/h on a straight track through a turnout.

Figure 8. Vertical force on wheels at 200 km/h on a straight track through a turnout.
Figure 9. Vertical force on wheels at 250 km/h on a straight track through a turnout.

Figure 10. Vertical force at 300 km/h on a straight track through a turnout.
Figure 11. Vertical force at 350 km/h on a straight track through a turnout.

Figure 12. Normal force at 100 km/h on a straight track through a turnout.
Figure 13. Normal force at 150 km/h on a straight track through a turnout.

Figure 14. Normal force at 200 km/h on a straight track through a turnout.
Figure 15. Normal force at 250 km/h on a straight track through a turnout.

Figure 16. Normal force at 300 km/h on a straight track through a turnout.
As seen in the figures, the magnitudes of vertical and normal forces increase as velocity increases. Significant changes occur if speeds increase above 100 km/h. For speeds above 250 km/h, the increase in vertical and normal forces occurs mainly within the cross member. The change in these quantities is due to the change in track stiffness. For a straight track without a turnout, the stiffness of the track is usually assumed constant. For a track with a turnout, the stiffness changes along the length as shown in Figure 4. These quantities increase up to 1.5 times the static load.

Next, simulations were performed to determine the vertical and normal forces in straight-track traffic without a turnout. This was done for speeds from 150 km/h to 350 km/h and is shown in Figures 18–27. The value of the force due to the load per wheel is $8.1 \times 10^4$ N. The alternating loads fluctuate around this value.

From the results presented, there are oscillations of these forces in the movement on the track without turnout, but there is no significant difference. There is an increase of about $1 \times 10^4$ N in the normal force. These forces are the basis for determining the contact surfaces and the amount of wear.
Figure 18. Vertical force at 150 km/h on a straight track without a turnout.

Figure 19. Vertical force at 200 km/h on a straight track without a turnout.
Figure 20. Normal force at 150 km/h on a straight track without a turnout.

Figure 21. Normal force at 200 km/h on a straight track without a turnout.
Figure 22. Vertical force at 250 km/h on a straight track without a turnout.

Figure 23. Vertical force at 300 km/h on a straight track without a turnout.
Figure 24. Normal force at 250 km/h on a straight track without a turnout.

Figure 25. Normal force at 300 km/h on a straight track without a turnout.
Figure 26. Vertical force at 350 km/h on a straight track without a turnout.

Figure 27. Normal force at 350 km/h on a straight track without a turnout.
Next, we proceeded to determine the wear on the wheel and rail. Two works, [12] and [13], were used to consider this topic. Based on them, the following relations can be written:

\[ W_m = C \cdot W_f \]
\[ W_d = C_1 \cdot W_{f-na} \]  

(2)

where \( W_m \) is the mass consumed \([\mu \cdot g]\) per unit contact surface, \( W_d \) is the depth of the wear surface [mm], \( C \) is constant (for steel \( \approx 0.00124 \, \mu \cdot g/N \cdot mm \)), \( W_f \) is the work of friction forces [N \cdot mm], \( C_1 \) is the constant \( \approx 1.55 \times 10^{-7} [\text{mm}/\text{N}] \), and \( W_{f-na} \) is the work of friction forces per unit contact surface [mm\(^2\)/Nn].

\[ m_W = C \cdot W_f \]  

(3)

where \( m_W \) is the mass of consumed material per unit contact area \([\mu \cdot g/\text{mm}^2]\), \( C \) is the constant (for steel \( \approx 0.00124 \, \mu \cdot g/N \cdot mm \)), and \( W_f \) is the work done by friction force per unit area of contact ellipse [mm\(^2\)/N].

Using the presented relations, \( W_f \) was determined as the wear factor. The software used to perform the simulation makes it possible to determine the wear factor. Such a test was performed for a straight track with a turnout and for a straight track without a turnout. The test results are shown in Figures 28–37. The figures show wear factors and wheel and rail wear for the passage of a rail vehicle through a turnout with and without a turnout.

Figure 28. Wear coefficient at 100 km/h on a straight track with a turnout.
Figure 29. Wear coefficient obtained for a straight track without a turnout at 150 km/h.

Figure 30. Wear coefficient at 200 km/h on a straight track with a turnout.
Figure 31. Wear coefficient obtained for a straight track without a turnout at 200 km/h.

Figure 32. Wear coefficient obtained for a straight track with a turnout at 250 km/h.
Figure 33. Wear coefficient obtained for a straight track without a turnout at 250 km/h.

Figure 34. Wear coefficient obtained for a straight track at 300 km/h with a turnout.
**Figure 35.** Wear coefficient obtained for a straight track without a turnout at 300 km/h.

**Figure 36.** Wear coefficient obtained for a straight track at 350 km/h with a turnout.
Figure 37. Wear coefficient obtained for a straight track without a turnout at 350 km/h.

From the simulations presented, when passing through a turnout, the wear coefficients increase in the turnout entry area (needle and resistor) and in the crossover area. For traffic on a track without a turnout, the wear factors vary between 0.0005 and 0.05 N/mm², while for traffic through a turnout, the factors vary between 0.16 and 12 N/mm². In traffic on the track without a turnout, the maximum magnitudes come from the normal forces, which increase above the nominal force and decrease when the normal force decreases below the nominal force. The nominal force is $8.1 \times 10^4$ N.

Simulations of 20,000 train runs on a straight track through a turnout at 200 km/h were performed. The wear results are shown for the left wheel of the wheelset and the wear of the rail by the left wheel. The wear of the left wheel is shown in Figure 38, and the wear of the rail is shown in Figure 39. The wears for the other wheels and rails are identical.

For the same speed of 200 km/h, simulations were performed for wheel and rail wear when the rail vehicle moves on the track without turnout. Wheel wear is shown in Figure 40 and rail wear is shown in Figure 41 for the same conditions.
Figure 38. Wear of the first left wheel when a rail vehicle passes a turnout at 200 km/h on a straight track.
Figure 39. Wear of rail by the left wheel of the first set when the rail vehicle passes through the turnout at 200 km/h on a straight track.

Figure 39. Wear of rail by the left wheel of the first set when the rail vehicle passes through the turnout at 200 km/h on a straight track.
Figure 40. Wheel wear for a straight track without a turnout with constant stiffness obtained at 200 km/h. The maximum wear value is above 0.00014 mm.
Simulations were then performed for speeds of 300 km/h and 350 km/h for a straight track with a turnout and a straight track without a turnout. The results are shown in Figures 42–49.

Figure 41. Rail wear for a straight track without a turnout with constant stiffness obtained at 200 km/h. The maximum value of wear is above 0.0004 mm.
Figure 42. Wheel wear at 300 km/h for straight-track traffic through a turnout.
Figure 43. Wheel wear at 350 km/h for straight-track traffic through a turnout.
Figure 44. Rail wear at 300 km/h for straight-track traffic through a turnout.
Figure 45. Rail wear for a railroad turnout at 350 km/h for straight-track traffic through the turnout.
Figure 46. Wheel wear for a straight track without a turnout with constant stiffness obtained at 300 km/h.
Figure 47. Wheel wear for a straight track without a turnout with constant stiffness obtained at 350 km/h.
Figure 48. Rail wear for a straight track without a turnout with constant stiffness obtained at 300 km/h.
Figure 49. Rail wear for a straight track without a turnout with constant stiffness obtained at 350 km/h.

From the presented simulation results, it can be clearly seen that the wheel and rail wear when passing through a turnout is many times greater than on a track without a turnout. This coincides with the magnitude of forces and the magnitude of wear factors under the same conditions in relation to lower speeds.

This section may be divided by subheadings. It should provide a concise and precise description of the experimental results, their interpretation, as well as the experimental conclusions that can be drawn.

The wear phenomenon itself is related to the way in which the wheelsets are fitted into horizontal curves (circular curves and transition curves). The magnitude of the occurring wheel–track contact forces plays a decisive role. Of course, wear of the rails is accompanied by wear of the rims (Figures 38, 40, 42, 43, 46 and 47). In the presented wear diagrams, the distances between minima and maxima are within 2–3 m, while the distance between axles in the bogie is 1.9 m. The maximum lateral wear occurs at distances of 6–9 m from...
each other. These limits correspond to the distance between bogies in a wagon or between the last bogie in the front wagon and the first bogie in the next wagon. The changes in the amount of wear (between successive extremes) can be large and, as shown, occur at relatively short distances from each other. In this way, they can become the cause of sudden changes in track gauge. This, in turn, is directly related to the issue of driving safety. The rolling stock is at risk of derailment if the gradient of the track gauge exceeds the permissible value. During the wear of the side of the grooved head of the outer rail of a straight track turnout and the flange of the outer wheel, the clearance between the side of the inner rail guide and the inner side of the flange of the inner wheel decreases. In a certain state of wear, when the side surfaces rub together, there is simultaneous contact of the flanges of both wheels with the rails. The further interaction of the wheelset with the track depends on multiple factors, such as the position of the bogie when passing through the turnout crossings, the running angle, the value of the steering forces, and the degree of wear on the wheel flanges and rail heads. If the wear of the outer rail head increases, the guidance in the straight track movement of the railway turnout is taken over by the inner wheel, rubbing the inner side of the flange against the guideway, and then all these parameters exceed the acceptable criteria.

A significant influence on the nature and magnitude of lateral wear is exerted by the direction of the vehicle’s movement on the straight track, especially on the switch point and the frog, as well as the speed of travel. The obtained diagrams clearly show that wear increases in accordance with the direction of travel. The highest wear is observed on the crossing in the cross member. At this point, a clear irregularity in the path of the last wagons in the rail vehicle formation was observed. The pronounced projections (lateral vibrations) of the last carriage at these locations are the result of large increments in lateral acceleration. Hence, large lateral forces are transmitted to the outer rail in such a curve. The higher the speed of the vehicle on the straight track, the greater the acceleration. The higher the acceleration, the higher the speed of the vehicle on the straight track.

Apart from contact wear, there is also the phenomenon of corrugation, i.e., corrugated wear. According to the definition given by the UIC (International Union of Railways), corrugated wear is characterised by the occurrence of almost regular irregularities on the rail head surfaces and, on the wheel, running surfaces at intervals of 30 to 80 mm in the form of glossy wave ridges and darkening depressions. Wave wear is an additional source of noise. The course of this phenomenon varies greatly. The conditions conducive to its occurrence are the homogeneity of traffic flow, the type of traffic, and the variable speed of the rail vehicle. It often occurs at sections of rolling stock acceleration and at long straight sections. Railway track construction, as well as differences in the hardness of rails and wheel components of rolling stock, also influence the rate of increase in the phenomenon.

4. Conclusions

The intensity of lateral wear of the rails and the wheel when running on a straight track is considerably greater than on typical railway lines. This is due to the specific operating conditions and, as measurements have shown, the variability of the normal force and, most likely, the variability of the contact surface. The problem of wear must be looked at comprehensively. In order to reduce the intensity of lateral rail wear, it must also be borne in mind that rim wear must be reduced in parallel. This principle should be the basic assumption of any remedial measures taken.

According to the presented material, the mathematical modelling of vehicle motion on a track with a switch must consider the variation of stiffnesses in the area of entry to the spire and passage through the cross member. An increase in these stiffnesses causes an increase in vertical and normal forces, and thus, an increase in the wear process.

Simulation of different conditions of traffic on a straight track without a switch and on a straight track with a switch indicates that wear of both wheels and rails when a rail
vehicle passes through the switch at speeds exceeding 200 km/h causes very big increments of wear on both wheels and rails.

High-dynamic loads acting on the railroad turnout elements and variations in time lead to significant property changes, such as faster abrasive wear and local plastic deformation of the material in the rolling layer of the rail sections. The knowledge of the load sustained during the passage of the rail vehicle through the turnout makes it possible to determine the actual operating conditions of the turnout. By means of the digital simulation of mathematical models, it is possible to select the elements most exposed to destructive effects of dynamic loads.

The presented modelling and simulation process can be used for other conditions of rail vehicle passage through the turnout, i.e., turnouts with larger radii, e.g., turnouts with a radius of 3000 m or 10,000 m (the modelling and simulation has been carried out for the turnout of 1200 m).

In further work, the authors will carry out the study of traffic after passing through the turnout and the right and left turning track. A separate problem is the study of the movement of a rail vehicle on a curve.

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