Single production of the vector-like top quark in the littlest Higgs model at TeV energy $e^−γ$ colliders

Yao-Bei Liu$^a$, Xue-Lei Wang$^b$, Yong-Hua Cao$^a$

a: Henan Institute of Science and Technology, Xinxiang 453003, P.R.China *

b: College of Physics and Information Engineering, Henan Normal University, Xinxiang 453007, P.R.China

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Abstract

The new colored vector-like heavy fermion $T$ is a crucial prediction in little Higgs models, which plays a key role in breaking the electroweak symmetry. The littlest Higgs model is the most economical one among various little Higgs models. In the context of the littlest Higgs model, we study single production of the new heavy vector-like quark via $e^−γ$ collisions and discuss the possibility of detecting this new particle in the TeV energy $e^+e^−$ collider(LC). We find that the production cross section can vary in a wide range($10^{−3}−10^1 fb$) in most parameter spaces. For the favorable parameter spaces, the possible signals of the vector-like top quark $T$ can be easily detected via $e^−γ$ collisions in future LC experiment with $\sqrt{s} = 3TeV$ and $L = 500fb^{-1}$.

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*E-mail:hnxxlyb2000@sina.com
1 Introduction

The standard model (SM) provides an excellent effective field theory description of almost all particle physics experiments. But in the SM the Higgs boson mass suffers from an instability under radiative corrections. The naturalness argument suggests that the cutoff scale of the SM is not much above the electroweak scale: New physics will appear around TeV energies. Recently, a new model, known as little Higgs model has drawn a lot of interest and it offers a very promising solution to the hierarchy problem in which the Higgs boson is naturally light as a result of nonlinearly realized symmetry [1, 2, 3, 4]. The key feature of this model is that the Higgs boson is a pseudo-Goldstone boson of an approximate global symmetry which is spontaneously broken by a vev at a scale of a few TeV and thus is naturally light. The most economical little Higgs model is the so-called littlest Higgs model, which is based on a $SU(5)/SO(5)$ nonlinear sigma model [4]. It consists of a $SU(5)$ global symmetry, which is spontaneously broken down to $SO(5)$ by a vacuum condensate $f$. In this model, a set of new heavy gauge bosons($B_H, Z_H, W_H$) and a new heavy-vector-like quark($T$) are introduced which just cancel the quadratic divergence induced by the SM gauge boson loops and the top quark loop, respectively. Furthermore, these new particles might produce characteristic signatures at the present and future collider experiments [5, 6, 7]. In little Higgs model, the new heavy vector-like top quark $T$ plays a key role in breaking the electroweak symmetry. Thus, studying the possible signatures of the new particle $T$ at future high energy colliders would provide significant information for $EWSB$ and test the little Higgs mechanism.

It is widely believed that the hadron colliders, such as Tevatron and future $LHC$, can directly probe possible new physics beyond the standard model (SM) up to a few $TeV$, while the TeV energy linear $e^+e^-$ collider (LC) is also required to complement the probe of the new particles with detailed measurement [8]. An unique feature of the LC is that it can be transformed to $\gamma\gamma$ or $e\gamma$ colliders with the photon beams generated by laser-scattering method. Their effective luminosity and energy are expected to be comparable to those of the $LC$. In some scenarios, they are the best instrument for the discovery of signatures of new physics. The $e^-\gamma$ collisions can produce particles which are kinematically not accessible in the $e^+e^-$ collisions at the same collider [9]. For example, Ref. [10] has recently discussed new single gauge boson production in the littlest Higgs model at this type of collider.
To avoid the fine tuning problems and produce a suitable Higgs mass, the mass of the new vector-like top quark $T$ should be about 2 TeV \cite{4}. In this case, the new particle $T$ can be produced at LHC via two mechanism: QCD pair production via the processes $gg \rightarrow T\bar{T}$ and $q\bar{q} \rightarrow T\bar{T}$; single production via $W$ exchange process $qb \rightarrow q'T$. Due to the large mass of $T$, the later process dominates over the QCD pair production processes. It has been shown that the new heavy top quark $T$ mass $M_T$ can be explored up to about 2.5 TeV via the $W$ exchange process \cite{5, 11}. Furthermore, Ref.\cite{12} has recently discussed single production of the new heavy vector-like top quark $T$ via the process $ep \rightarrow eb \rightarrow \nu_e T$ at the future linac-ring type $ep$ collider. In this letter, we will study single production of the new vector-like top quark $T$ predicted by the LH model via the process $e^-\gamma \rightarrow \nu_e b\bar{T}$ and see whether it can be detected in the future LC experiment with the c.m. energy $\sqrt{s} = 3 TeV$ and the integral luminosity $\mathcal{L} = 500 fb^{-1}$.

This paper is organized as follows, In section two, we first briefly introduce the littlest model, and then give the calculation process of the cross section. The numerical results and conclusions are given in section three.

2 The cross section of the process $e^-\gamma \rightarrow \nu_e b\bar{T}$ in the LH model

The littlest model is based on the $SU(5)/SO(5)$ nonlinear sigma model. At the scale $\Lambda_s \sim 4\pi f$, the global $SU(5)$ symmetry is broken into its subgroup $SO(5)$ via a vacuum condensate $f$, resulting in 14 Goldstone bosons. The effective field theory of these Goldstone bosons is parameterized by a non-linear $\sigma$ model with gauged symmetry $[SU(2) \times U(1)]^2$, spontaneously broken down to its diagonal subgroup $SU(2) \times U(1)$, identified as the SM electroweak gauge group. Four of these Goldstone bosons are eaten by the broken gauge generators, leaving 10 states that transform under the SM gauge group as a doublet $H$ and a triplet $\Phi$. This breaking scenario also gives rise to four massive gauge bosons $B_H, Z_H$ and $W^{\pm}_H$, A new vector-like top quark $T$ is also needed to cancel the divergence from the top quark loop. All of these new particles playing together can successfully cancel off the quadratic divergence of the Higgs boson mass.

In the LH model, the couplings of the heavy vector-like top quark $T$ to ordinary particles,
which are related to our calculation, can be written as [3]:

\[ g_{WLeV}^{\nu} = -g_{A}^{WLeV} = \frac{ie}{2\sqrt{2} s_{W}} [1 - \frac{v^{2}}{2f^{2}} c^{2}(c^{2} - s^{2})], \quad (1) \]
\[ g_{WHeV}^{\nu} = -g_{A}^{WHeV} = -\frac{ie c}{2\sqrt{2} s_{W} s}, \quad (2) \]
\[ g_{WLbV}^{\nu} = -g_{A}^{WLbV} = \frac{ie v}{2\sqrt{2} s_{W} s x_{L}}, \quad (3) \]
\[ g_{WHbV}^{\nu} = -g_{A}^{WHbV} = -\frac{ie v c}{2\sqrt{2} s_{W} s x_{L}}. \quad (4) \]

where \( f \) is the scalar parameter, \( v = 246 GeV \) is the electroweak scale, \( s_{W} \) represents the sine of the weak mixing angle, and \( c \) is the mixing parameter between \( SU(2)_{1} \) and \( SU(2)_{2} \) gauge bosons with \( s = \sqrt{1 - c^{2}} \). \( x_{L} \) is the mixing parameter between the \( SM \) top quark \( t \) and the vector-like top quark \( T \), which is defined as \( x_{L} = \lambda_{1}^{2}/(\lambda_{1}^{2} + \lambda_{2}^{2}) \), \( \lambda_{1} \) and \( \lambda_{2} \) are the Yukawa couplings parameters. We write the gauge boson-fermion couplings in the form of \( i\gamma^{\mu}(g_{V} + g_{A}\gamma^{5}) \). The mass of the heavy vector-like top quark \( T \) can be approximately written as:

\[ M_{T} = \frac{m_{t} f}{v} \left[ \frac{1}{x_{L}(1 - x_{L})} [1 - \frac{v^{2}}{2f^{2}} x_{L}(1 + x_{L})] \right]. \quad (5) \]

The number of up-type quarks is four in the LH model and thus the matrix relating the quark mass eigenstates with the weak eigenstates becomes a \( 4 \times 3 \) matrix. Compared to the \( CKM \) matrix in the SM, the extended \( CKM \) matrix has the fourth row elements \( V_{Td}, V_{Ts}, \) and \( V_{Tb} \). Thus, it is possible that there are the decay channels \( T \rightarrow ql\bar{\nu} \), which \( q \) is the down-type quark. However, their branching ratio are very small [13]. Thus, the dominate decay modes of the heavy vector-like top quark \( T \) are \( tH, tZ, \) and \( bW \) with partial widths in the ratio \( 1:1:2 \) [3] [14]. At the order of \( v^{2}/f^{2} \), the total width of the new vector-like top quark \( T \) can be written as:

\[ \Gamma_{T} = \frac{M_{T}}{8\pi} \left( \frac{m_{t} f}{v} \right)^{2} \frac{x_{L}}{1 - x_{L}}. \quad (6) \]

In this case, we can see that the new heavy vector-like top quark \( T \) can be single produced via the process \( e^{-\gamma} \rightarrow \nu_{e} bT \) via \( e^{-\gamma} \) collisions in the future \( LC \) experiment with \( \sqrt{s} = 3TeV \) and \( \mathcal{L} = 500 fb^{-1} \). The relevant tree-level Feynman diagrams of the process are shown in Fig.1.

In order to write a compact expression for the amplitudes, it is necessary to define the triple-boson couplings coefficient as:

\[ \Gamma^{\alpha\beta\gamma}(p_{1}, p_{2}, p_{3}) = g^{\alpha\beta}(p_{1} - p_{2})^{\gamma} + g^{\beta\gamma}(p_{2} - p_{3})^{\alpha} + g^{\gamma\alpha}(p_{3} - p_{1})^{\beta}, \quad (7) \]
Figure 1: Feynman diagrams of the process $e^-\gamma \rightarrow \nu_e b \bar{T}$ in the littlest Higgs model.

with all momenta out-going. The invariant production amplitudes of the process can be written as:

$$M = M_a + M_b + M_c$$

with

$$M_a = -\frac{i e^3}{8 s_{W}^2} \frac{v}{f} x_L \bar{u}(p_3) \gamma_\mu (1 - \gamma_5) \gamma_\rho \gamma_\mu (1 - \gamma_5) u(p_1) \{ G(p_3 - p_1, M_W) + \frac{c^2}{s^2} G(p_3 - p_1, M_{W_H}) \} \times G(p_4 + p_5, M_W) + \frac{c^2}{s^2} G(p_4 + p_5, M_{W_H}) \} \bar{u}(p_4) \gamma_\nu (1 - \gamma_5) v(p_5) \varepsilon^\rho(p_2),$$

$$M_b = \frac{i e^3}{8 s_{W}^2} \frac{v}{f} x_L \bar{u}(p_3) \gamma_\mu (1 - \gamma_5) \gamma_\rho \gamma_\mu (1 - \gamma_5) u(p_1) \Gamma^{\mu \nu \rho \sigma}(p_3 - p_1, -p_2, p_4 + p_5) \{ G(p_3 - p_1, M_W) \}
\times G(p_4 + p_5, M_W) + \frac{c^2}{s^2} G(p_3 - p_1, M_{W_H}) \} \bar{u}(p_4) \gamma_\nu (1 - \gamma_5) v(p_5) \varepsilon^\rho(p_2),$$

$$M_c = \frac{-i e^3}{8 s_{W}^2} \frac{v}{f} x_L \bar{u}(p_4) \gamma_\mu (1 - \gamma_5) v(p_5) \{ G(p_4 + p_5, M_W) + \frac{c^2}{s^2} G(p_4 + p_5, M_{W_H}) \} \bar{u}(p_4) \gamma_\nu (1 - \gamma_5) G(p_1 + p_2, 0) \gamma_\rho u(p_1) \varepsilon^\rho(p_2).$$

where $G(p, m) = 1/(p^2 - m^2)$ denotes the propagator of the particle.

The hard photon beam of the $e\gamma$ collider can be obtained from laser backscattering at the $e^+e^-$ linear collider. Let $\hat{s}$ and $s$ be the center-of-mass energies of the $e\gamma$ and $e^+e^-$ systems, respectively. After calculating the cross section $\sigma(\hat{s})$ for the subprocess $e^-\gamma \rightarrow \nu_e b \bar{T}$, the
total cross section at the $e^+e^-$ linear collider can be obtained by folding $\sigma(\hat{s})$ with the photon distribution function that is given in Ref[15]:

$$\sigma(tot) = \int_{M_T^2/s}^{x_{max}} dx \sigma(\hat{s}) f_\gamma(x),$$

(9)

where

$$f_\gamma(x) = \frac{1}{D(\xi)} \left[ 1 - x + \frac{1}{1 - x} - \frac{4x}{\xi(1 - x)} + \frac{4x^2}{\xi^2(1 - x)^2} \right],$$

(10)

with

$$D(\xi) = (1 - \frac{4}{\xi} - \frac{8}{\xi^2}) \ln(1 + \xi) + \frac{1}{2} + \frac{8}{\xi} - \frac{1}{2(1 + \xi)^2},$$

(11)

In the above equation, $\xi = 4E_e\omega_0/m_e^2$ in which $m_e$ and $E_e$ stand, respectively, for the incident electron mass and energy, $\omega_0$ stands for the laser photon energy, and $x = \omega/E_e$ stands for the fraction of energy of the incident electron carried by the backscattered photon. $f_\gamma$ vanishes for $x > x_{max} = \omega_{max}/E_e = \xi/(1 + \xi)$. In order to avoid the creation of $e^+e^-$ pairs by the interaction of the incident and backscattered photons, we require $\omega_0 x_{max} \leq m_e^2/E_e$, which implies that $\xi \leq 2 + 2\sqrt{2} \approx 4.8$. For the choice of $\xi = 4.8$, we obtain

$$x_{max} \approx 0.83, \quad D(\xi_{max}) \approx 1.8.$$  

(12)

For simplicity, we have ignored the possible polarization for the electron and photon beams.

With above production amplitudes, we can obtain the production cross section directly. In the calculation of the cross section, instead of calculating the square of the amplitudes analytically, we calculate the amplitudes numerically by using the method of the references [16] which can greatly simplify our calculation.

3 Numerical results and conclusions

From above discussions, we can see that the single $T$ production at TeV energy LC colliders comes from two processes: the SM gauge boson $W$ exchange and the new gauge boson $W_H$ exchange. The contributions of the former process mainly dependent on the free parameters $M_T$ and $x_L$, while those of the latter process mainly dependent on the free parameters $M_T$, $x_L$, $c$ and $W_H$. Taking into the precision electroweak constrains on the parameter space of the LH model, the free parameters $x_L$, $c$ and $W_H$ are allowed in the ranges of $0 < x_L < 1$, $0 \leq c \leq 0.5$, ...
and $1 TeV \leq M_{W_H} \leq 3 TeV$ \cite{17}. Observably, the cross section of single $T$ production mainly comes from $W$ exchange and is not sensitive to the free parameters $c$ and the $W_H$ mass $M_{W_H}$. So, we will take $c=0.3$ and $M_{W_H} = 2 TeV$ in our numerical calculation.

In Fig.2, we plot the single production section $\sigma(s)$ of the heavy vector-like top quark $T$ as a function of $T$ quark mass $M_T$ for three values of the mixing parameter $\chi_L$. One can see from Fig.2 that the cross section $\sigma(s)$ fall sharply with $M_T$ increasing and increases as the mixing parameter $\chi_L$ increases. For $\chi_L=0.5$, and $1 TeV \leq M_T \leq 2.5 TeV$, the value of the cross section $\sigma(s)$ is in the range of $3.24 fb \sim 1.7 \times 10^{-3} fb$. If we assume that the yearly integrated luminosity $L = 500 fb^{-1}$, then there will be several and up to more than one thousand a bit $\nu_e bT$ events to be generated per year.

To see the influence of the mixing parameter $\chi_L$ on the cross section, in Fig.3, we plot $\sigma(s)$ as a function of the mixing parameter $\chi_L$ for $M_T = 1 TeV, 1.5 TeV, 2 TeV, 2.5 TeV$, respectively. From Fig.3, one can see that the cross section increases as the mixing parameter $\chi_L$ increasing for fixed $T$ quark mass $M_T$. This is because the value of the production section is proportional to the mixing parameter $\chi_L$ which comes from the flavor change couplings $g^{W Tb}$. On the other hand, as long as the heavy vector-like top quark $T$ mass $M_T$ is smaller than $2 TeV$, the cross

Figure 2: The cross section $\sigma(s)$ of single $T$ production as a function of $M_T$ for $c=0.3$ and three values of the parameter $\chi_L$. 
Figure 3: The cross section $\sigma(s)$ of single T production as a function of $x_L$ for different values of $M_T$. The section can reach a few $fb$ even tens $fb$. If we take integral luminosity $\mathcal{L} = 500 fb^{-1}$, there are about $10^3 - 10^4 \nu_e b \bar{T}$ events to be produced. There will be a promising number of fully reconstructible events to detect single $T$ production via $e^-\gamma$ collisions in future LC experiment with $\sqrt{s} = 3 TeV$ and $\mathcal{L} = 500 fb^{-1}$.

Little Higgs theory has generated much interest as one kind of models of EWSB, which can be regarded as one of the important candidates of new physics beyond the SM. For all of the little Higgs models, at least one vector-like top quark $T$ is needed to cancel the numerically most large quadratic divergence coming from top Yukawa couplings. At the leading order, the heavy vector-like top quark $T$ predicted by the LH model mainly decays to the $tZ$, $tH$ and $bW$ modes, which can provide characteristic signatures for the discovery of the heavy vector-like quark $T$ in the future high energy collider experiments. It has been shown that the signal of the new vector-like quark $T$ might be detected via all of the three decay modes in the future LHC experiment and the linac-ring type $e p$ collider. In this paper, we study single heavy vector-like top quark $T$ production via the process $e^-\gamma \rightarrow \nu_e b \bar{T}$ in the future high energy LC experiment. With the favorable parameter spaces, the sufficient events can be produced to detected the single of the vector-like top quark $T$ at the $TeV$ energy $e^-\gamma$ colliders.
References

[1] N. Arkani-Hamed, A. G. Cohen, and H. Georgi, Phys. Lett. B513, 232(2001).

[2] N. Arkani-Hamed, A. G. Cohen, T. Gregoire, and J. G. Wacker, JHEP 0208 020(2002); N. Arkani-Hamed, A. G. Cohen, E. Katz, A. E. Nelson, T. Gregoire, and J. G. Wacker, JHEP 0208 021(2002).

[3] I. Low, W. Skiba, and D. Smith, Phys. Rev. D66, 072001(2002); M. Schmaltz, Nucl. Phys. Proc. Suppl. 117, 40(2003); W. Skiba and J. Terning, Phys. Rev. D68, 075001(2003).

[4] N. Arkani-Hamed, A. G. Cohen, E. Katz, A. E. Nelson, JHEP 0207 034(2002).

[5] T. Han, H. E. Logan, B. McElrath, and L. T. Wang, Phys. Rev. D67, 095004(2003).

[6] G. Burdman, M. Perelstein, and A. Pierce, Phys. Rev. Lett. 90, 241802(2003); T. Han, H. E. Logen, B. McElrath, and L. T. Wang, Phys. Lett. B563, 191(2003); G. Azuelos et al., hep-ph/0402037; H. E. Logan, Phys. Rev. D70, 115003(2004); G. Cho and A. Omete, Phys. Rev. D70, 057701(2004).

[7] S. C. Park and J. Song, Phys. Rev. D69, 115010(2004).

[8] T.Abe et al.[American Linear Collider Group], hep-ex/0106057; J. A. Aguilar-Saavedra et al. [ECFA/DESY Physics Working Group], hep-ex/0106315; K. Abe et al. [ECFA Linear Collider Working Group Collaboration], hep-ph/0109166.

[9] E. Boos et al., Nucl. Instrum. Methods A472(2001)100; B. Badelek et al. [ECFA/DESY Photon collider Working Group], hep-ex/0108012; S. J. Brodsky, Inter. J. of Mod. Phys. A18(2003)2871.

[10] Chongxing Yue and Wei Wang, Phys. Rev. D71, 015002(2005), hep-ph/0411266; Chong-Xing Yue, Feng Zhang, Li-Na Wang and Li Zhou, Phys. Rev. D72, 055008(2005), hep-ph/0508228; Xuelei Wang, Jihong Chen, Yaobei Liu, Suzhen Liu and Hua Yang, Phys. Rev. D74, 015006(2006), hep-ph/0606093.

[11] M. Perelstein, M.E.Peskin and A.Pierce, Phys. Rev. D69, 075002(2004); G. Azuelos, et al., hep-ph/0402037.
[12] Chong-Xing Yue, Feng Zhang, Wei-Wang, China.Phys.Lett 22,1083-1085(2005), hep-ph/05021797.

[13] Jaeyong Lee, JHEP 0412, 065(2004).

[14] H.E.Logan, hep-ph/0307340.

[15] G. Jikia, Nucl. Phys. B374, 83(1992); O. J. P. Eboli, et al. Phys. Rev. D47, 1889(1993); K. M. Cheung, ibid 47,3750(1993).

[16] K. Hagiwara and D. Zeppenfeld, Nucl. Phys. B313, 560(1989); V. Barger, T. Han, and D. Zeppenfeld, Phys. Rev. D41, 2782(1990).

[17] C. Csaki, J. Hubisz, G. D. Kribs, P. Meade and J. Tering, Phys. Rev. D68, 035009(2003); T. Gregoire, D. R. Smith and J. G. Wacker, Phys. Rev. D69, 115008(2004).