The gravitational redshift of boson stars

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We investigate the possible gravitational redshift values for boson stars with a self-interaction, studying a wide range of possible masses. We find a limiting value of $z_{\text{lim}} \approx 0.687$ for stable boson star configurations. We compare theoretical expectation with the observational capabilities in several different wavebands, concluding that direct observation of boson stars by this means will be extremely challenging. X-ray spectroscopy is perhaps the most interesting possibility.

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I. INTRODUCTION

The idea of the boson star goes back to Kaup in 1968 [1]. A boson star is a gravitationally bound collection of bosonic particles, arising as a solution of the Klein–Gordon equation coupled to general relativity. Many investigations of the possible configurations have been carried out; for reviews see Ref. [2]. For non-self-interacting bosons of mass $m$, the mass of a typical configuration is of order $m^2\text{Pl}/m$, to be compared with a typical neutron star mass of $m^3\text{Pl}/m^2\text{neutron}$, which is about a solar mass. Here $m\text{Pl}$ is the Planck mass. Unless the bosons are extremely light, boson stars made from non-interacting bosons are orders of magnitude lighter than typical stars.

The situation is very different if the boson stars have even a very weak self-interaction; for a first investigation of self-interactions see Ref. [3]. Colpi, Shapiro and Wasserman [4] showed that the maximum mass of stable configurations is then of order $\lambda^{1/2}m^3\text{Pl}/m^2$, where $\lambda$ is the scalar field self-coupling, normally assumed to be of order unity. Then boson star configurations exist with mass (and radius) similar to that of neutron stars, if the bosons, like neutrons, have a mass around 1 GeV. They can also be much heavier, should the bosons be lighter. In this paper, we allow ourselves to consider a very wide range of possibilities for the boson star mass and radius.

It remains unknown whether or not boson stars could actually form in our own Universe, for example through a gravitational instability process, though there has been some discussion in the literature [5]. The most favourable situation would be for the dark matter in the Universe to be in the form of very weakly interacting bosons, a substantial fraction of which manage to form boson stars; such a process may be aided if the bosons have a reasonably strong coupling to each other, but not to conventional matter. If boson stars can form, they provide an alternative explanation for stellar systems in which an object is inferred to have a high mass; conventionally, a ‘star’ with mass greater than a few solar masses is assumed to be a black hole (since this is above the maximum permitted mass for a neutron star), but a boson star offers a more speculative alternative.

Dark matter is characterized by its extremely weak interaction with conventional matter; even though 90% of the mass of the galaxy is in this form, it is not directly visible. In this paper, we investigate the implications of assuming that the material from which boson stars are made is similarly extremely weakly interacting, so that its only interaction with neighbouring baryonic material and photons is gravitational, just as the relation between the visible galaxy and its dark matter halo. However, while a galaxy halo can be described using Newtonian theory, boson stars close to the maximum allowed mass are general relativistic objects. This gives a new characteristic of such objects, a gravitational redshift. In this paper, we examine whether radiating baryonic matter moving in the gravitational potential of a boson star could be used as an observational signal of boson stars.

II. THE EINSTEIN-KLEIN-GORDON EQUATIONS

The Lagrange density of a massive complex self-gravitating scalar field reads

$$\mathcal{L} = \frac{1}{2} \sqrt{|g|} \left[ \frac{m^2}{8\pi} R + \partial_{\mu} \Phi^* \partial^{\mu} \Phi - U(|\Phi|^2) \right],$$  \hspace{1cm} (1)

where $R$ is the curvature scalar, $g$ the determinant of the metric $g_{\mu\nu}$, and $\Phi$ is a complex scalar field with a potential $U$. We take $\hbar = c = 1$. Then we find the coupled system of equations

$\Phi$ An example already existing in the literature is the boson–fermion star [6], which is made up of bosons and neutrons interacting only gravitationally.
\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi}{m_{P1}^2} T_{\mu\nu}(\Phi), \quad (2) \]

\[ \Box + \frac{dU}{d|\Phi|^2} \Phi = 0, \quad (3) \]

where

\[ T_{\mu\nu} = (\partial_\nu \Phi^*)(\partial_\mu \Phi) - \frac{1}{2} g_{\mu\nu} [g^{\rho\sigma}(\partial_\rho \Phi^*)(\partial_\sigma \Phi) - U(|\Phi|^2)] \quad (4) \]

is the energy-momentum tensor and

\[ \Box = \partial_\mu \left( \frac{\sqrt{|g|} g^{\mu\nu} \partial_\nu} {\sqrt{|g|}} \right) \quad (5) \]

the generally covariant d’Alembertian. Within the model we want to have an additional global \( U(1) \) symmetry, so we can take the following potential

\[ U = m^2 |\Phi|^2 + \frac{\lambda}{2} |\Phi|^4, \quad (6) \]

where \( m \) is the scalar mass and \( \lambda \) a dimensionless constant measuring the self-interaction strength.

The method for finding solutions is well known \[1,2\]. For spherically symmetric solutions we use the static line element

\[ ds^2 = e^{\nu(r)} dt^2 - e^{\mu(r)} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2). \quad (7) \]

The most general scalar field ansatz consistent with this metric is

\[ \Phi(r,t) = P(r)e^{-i\omega t}, \quad (8) \]

where \( \omega \) is the frequency. Notice that to find finite solutions, the scalar field must carry a time-dependence of this form, which leaves the energy-momentum tensor, and hence the metric, time-independent.

The non-vanishing components of the energy-momentum tensor are

\[ T_{0}^{0} = \rho = \frac{1}{2} [\omega^2 P_2^2(r) e^{-\nu} + P_2^2(r) e^{-\mu} + U], \quad (9) \]

\[ T_{1}^{1} = \rho = \frac{1}{2} [\omega^2 P_2^2(r) e^{-\nu} + P_2^2(r) e^{-\mu} - U], \quad (10) \]

\[ T_{2}^{2} = T_{3}^{3} = p_\perp \quad \text{with} \]

\[ p_\perp = \frac{1}{2} [\omega^2 P_2^2(r) e^{-\nu} - P_2^2(r) e^{-\mu} - U] \quad (11) \]

where \( r' = dr/dr \). Note that the pressure is anisotropic; there are two equations of state \( p_r = \rho - U \) and \( p_\perp = \rho - U - P_2^2(r) e^{-\mu} \).

The decisive non-vanishing components of the Einstein equation are

\[ \nu' + \mu' = \frac{8\pi}{m_{P1}^2} (\rho + p_r) e^\mu, \quad (12) \]

\[ \mu' = \frac{8\pi}{m_{P1}^2} \rho e^\mu - \frac{1}{r} (e^\mu - 1), \quad (13) \]

the two further components giving degenerate equations because of the Bianchi identities.

The differential equation for the scalar field is

\[ P'' + \left( \frac{\nu' - \mu'}{2} + 2 \right) P' + e^{\nu-\nu} \omega^2 P - e^\mu \frac{dU}{d|\Phi|^2} P = 0. \quad (14) \]

For the rest of our paper, we employ the dimensionless quantities \( x = \frac{m r}{\sigma} \), \( \sigma = \sqrt{\frac{4\pi}{\Lambda}} \). In order to obtain solutions which are regular at the origin, we must impose the boundary conditions \( \sigma'(0) = 0 \) and \( \mu(0) = 0 \).

Once the fundamental parameter \( \Lambda \) has been fixed, the only remaining freedom is the value of the scalar field at the centre of the star (the scalar field mass simply contributing an overall scaling); the initial value \( \nu(0) \) is determined by the boundary condition of Minkowskian spacetime at infinity. It is well known that as the central scalar field value is increased, initially the inferred mass of the star (determined from the asymptotic form of the metric) increases up to a maximum, after which it exhibits oscillatory behaviour, shown in Figure 1. The configurations up to this maximum mass are dynamically stable, but those beyond it are unstable \[7,12\] and should not be considered physical.

### III. GRAVITATIONAL REDSHIFT

In this Section we calculate the redshift produced by the gravitational potential of a boson star. Under our assumption that the scalar particles have no interaction with the baryonic matter other than gravitationally, the baryonic matter can penetrate up to the center of the boson star. If the baryonic matter emits or absorbs radiation within the gravitational potential, the spectral feature will be redshifted.

The gravitational redshift \( z \) within a static line element is given by (see e.g. Ref. \[1\])

\[ 1 + z_{g} = \sqrt{\frac{e^{\nu(R_{\text{int})}}}{e^{\nu(R_{\text{ext})}}}} \quad, \quad (15) \]

where the emitter and receiver are located at \( R_{\text{int}} \) and \( R_{\text{ext}} \) respectively. The maximum possible redshift for a given configuration is obtained if the emitter is exactly at the center \( R_{\text{int}} = 0 \). The receiver is always practically at infinity, hence \( \exp[\nu(R_{\text{ext}})] = 1 \). For all other redshifts in between, we define the redshift function

\[ 1 + z_{g}(x) \equiv \exp \left( -\frac{\nu(x)}{2} \right). \quad (16) \]

Table \[1\] gives the results for the maximum mass boson star for different \( \Lambda \). The maximum mass gives the highest
IV. DETECTABILITY OF BOSON STARS

The mass $M$ of a boson star composed of non-self-interacting particles is inversely proportional to $m$, while the mass of a self-interacting boson star is proportional to $\sqrt{\lambda}/m^2$. Taking $\lambda \sim 1$, then for small $m$ (to be precise, provided $\Lambda \gg 1$) the self-interacting star is much more massive. For example, if we want to get a boson star with a mass of order $10^{35}$g (a solar mass), then we need $m \sim 10^{-10}$ eV for $\lambda = 0$, or $m \propto \lambda^{1/4}$ GeV if $\lambda \gg 10^{-38}$ (we see that the self-coupling has to be extraordinarily tiny to be negligible). In this example, the scalar particle has a mass comparable to a neutron, leading to a boson star with the dimensions of a neutron star.

If we reduce the scalar mass further, to $m \sim 1$ MeV, then we find $M \sim 10^{39}\sqrt{\lambda}$ g and $R \sim 10^6\sqrt{\lambda}$ km; this radius is comparable to that of the sun, but encloses $10^6$ solar masses. These parameters are reminiscent of supermassive black holes, for example as in Active Galactic Nuclei; the mass–radius relation is effectively fixed just by the objects being relativistic. An exceptionally extreme example is to take $m \sim 1$ eV, giving $M \sim 10^{18}\sqrt{\lambda}M_\odot$ and $R \sim 10^3\sqrt{\lambda}$ parsecs.

These numbers must be compared with the critical matter density in the Universe, around $10^{12}M_\odot$ Mpc$^{-3}$. This sets the scale of the typical distance expected to the nearest object of a given mass, provided such objects are common. The largest gravitationally bound objects in the present Universe are rich galaxy clusters, with masses of order $10^{15}M_\odot$, which require the assembly of material from a volume around 10 Mpc on a side.

In all cases, the density of the boson stars makes their direct detection difficult; in particular, they cannot be resolved in any waveband. The best optical spectroscopy has a resolution of order of an arcsecond, nowhere near good enough. The best imaging resolution is in the radio band; for example the VSOP program [8], which includes the recently launched Japanese HALCA satellite, offers resolution of around a milli-arcsecond, but this approach is limited because spectroscopy cannot be carried out. At a distance of 10 Mpc, we could resolve a boson star with a radius of 0.1 pc, corresponding to a mass of $10^{15}M_\odot$, but rich clusters cannot possibly be dominated by a single massive object of that sort. Any reduction of the boson star mass requires it to be closer, in proportion to the mass $M$, but the amount of mass available in that volume is decreasing as the cube of the distance. The situation therefore rapidly becomes worse at closer distances; the nearest object of a given size is expected to be far too distant to be resolvable. For example, to be resolvable at a distance of one kiloparsec, a boson star would need a radius of $10^{15}$ cm (roughly the Earth–Sun separation) requiring a mass of $10^{11}M_\odot$, almost the mass of an entire galaxy.

However, even if boson stars cannot be directly resolved, their influence might still be visible if material in their vicinity is sufficiently luminous. It is necessary to find a certain amount of luminous matter within the gravitational potential of the boson star. This could, for example, be HI gas clouds as seen in galaxies. From the rotation curves calculated below, one imagines that the hydrogen gas would probably be excited, hence HI, because of the kinetic energy gain in the gravitational
potential of the boson star. One might also expect accretion discs about boson stars, though there the luminosity could be dominated by regions outside the gravitational potential and the boson star would be indistinguishable from any other type of compact object. One should mention that bright HII regions are found within the host galaxies of quasars \[14\].

A more dramatic possibility would be to house a complete nuclear burning star within the gravitational potential of the boson star. Unfortunately that does not seem feasible, since we have already seen that to have a solar radius the boson star must weigh \(10^6 M_\odot\). A more generic situation therefore would be for lighter boson stars to be contained completely within conventional stars, influencing the stellar structure by means of their additional gravitational interaction. Such a situation has in fact been studied for a degenerate neutron core inside a supergiant, the so-called Thorne–˙Zytkow objects \[20\].

If the bosons are completely non-interacting, then the boson star might indeed be able to survive such an environment. With an interaction present, one would have to examine whether or not the boson star could be dissociated by high energy photons in the stellar interior, so that instead the scalar particles circulate freely within the star. Historically, such a situation was proposed as a possible resolution of the solar neutrino problem, under the name \textit{cosmion} \[21\].

The results from the last section showed that a boson star, like a conventional star, cannot be resolved in itself but rather would have to be part of a larger area. We know that the luminosity of the boson star will be redshifted. Hence we can distinguish between the luminosity of the boson star and other areas if there is a particularly strong spectral line. The luminosity \(L_{\text{boson}}\) could be derived from the redshifted tail of the emission line subtracting the ‘normal’ part. First, we assume that the gas distribution is everywhere homogeneous within the resolved area. This could be, in first order, valid for almost Newtonian boson stars. Let \(L\) be the complete measured luminosity; then we have \(L = L_{\text{normal}} + L_{\text{boson}}\), where we divided \(L\) into its two parts of the ‘normal’ non-redshifted luminosity and the one from the boson star area. Both parts are proportional to the product of the relation \(A\) of the boson star area to the total resolved area and the total luminosity \(L\), hence, \(L_{\text{boson}} = AL\) while \(L_{\text{normal}} = (1 - A)L\). The calculated value of \(A\) gives only an upper limit for the size of the boson star.

Given the inability to resolve boson stars, the most promising technique is to consider wave-bands where they might be extremely luminous. The best example is in X-rays. A very massive boson star, say \(10^6 M_\odot\) is likely to form an accretion disc and since its exterior solution is Schwarzschild it is likely to look very similar to an AGN with a black hole at the center. In X-rays it may be possible to probe very close to the Schwarzschild radius; it has been claimed by Iwasawa et al. \[22\] that using ASCA data they have probed to within 1.5 Schwarzschild radii.

Because this is inside the static limit for a non-rotating black hole, they conclude that a Kerr geometry is required. We note that a boson star configuration provides a more speculative alternative, giving a non-singular solution where emission can occur from arbitrarily close to the center. The signature they use is a redshifted wing of the Iron K-line, indicative of emission from deep within the gravitational potential well. If such techniques have their validity confirmed, it may ultimately be possible to use X-ray spectroscopy to map out the shape of the gravitational potential close to the Schwarzschild radius or boson star, and perhaps to differentiate between the theoretical predictions for a Kerr black hole and for a rotating boson star.

### V. Rotation Curves

We end by considering the rotation curves about a boson star in more detail. For the static spherically symmetric metric considered here circular orbit geodesics obey

\[
v^2 = r\nu' e^\nu / 2 = e^\nu (e^\mu - 1) / 2 + \frac{8\pi}{m^2_{\text{Pl}}} p_r r^2 e^{\mu + \nu} / 2
\]

\[
\simeq M(r) / r + \frac{8\pi}{m^2_{\text{Pl}}} p_r r^2 e^{\mu + \nu} / 2,
\]

which reduces for a weak gravitational field into the Newtonian form \(v^2_{\phi, \text{Newt}} = M(r) / r\) if \(p_r = 0\).

Figure 5 shows how the rotation velocities decrease by a Keplerian decrease. The possible rotation velocities circulating within this gravitational potential are quite remarkable. Their maximum reaches more then one-third of the velocity of light; if boson stars can form, then such enormous rotation velocities are not necessarily signatures of black holes. The matter possesses an impressive kinetic energy of about 6% of the rest mass. If we were to suppose that each year a mass of \(1M_\odot\) transfers this amount of kinetic energy into radiation, a boson star had a luminosity of \(10^{44}\) erg/s.

Figure 5 shows how the rotation velocities decrease with decreasing initial value \(\sigma(0)\) at fixed \(\Lambda\). Table 1 gives the maximal velocity. For Newtonian solutions the rotation velocity is low and quite constant over a larger interval.

### VI. Discussion

In studying a wide range of possible masses and coupling for the bosons, we have revealed a range of possibilities for boson stars to have an observational impact. In the literature it is common to consider boson stars to...
be like neutron stars, but this relies on specific choices for the boson mass for which there is no strong motivation. A boson star could have quite different dimensions both in size and mass. In analogy to the dark matter halos of galaxies, a boson star could also be transparent, i.e., without any electro-weak and strong interactions. Because of the general relativistic background of the boson star solutions, a gravitational redshift has to be taken into account. We find that for stable boson stars a certain redshift value cannot be exceeded. This is in good agreement with results for general fluids and especially the neutron star model where one finds similar redshift values. We mention that if one considers instead bosonic particles without mass or self-interaction, one finds solutions which can be applied to fit rotation curve data of spiral and dwarf galaxies, and which may also give very high redshift values [24].

It will be observationally very challenging to use the gravitational potential of these configurations to provide an observational test. Although radio offers by far the best imaging resolution, it seems that it may be spectroscopy using X-rays which offers the best opportunity to probe the central regions of strong gravitational sources.

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TABLE I. The redshift values $z$ for different self-interaction values $\Lambda$ and initial values of the scalar field $\sigma$. The addition (max) means the maximal central density up to which stable boson star states exist for the choice of $\Lambda$.

| $\Lambda$ | $\sigma(0)$ | $z$ |
|--------|-------------|-----|
| $-20$  | 0.067       | 0.0656984 (max) |
| $-15$  | 0.086       | 0.0869732 (max) |
| $-10$  | 0.122       | 0.129636 (max) |
| $-5$   | 0.195       | 0.237231 (max) |
| 0      | 0.025       | 0.0315 |
| 0      | 0.271       | 0.456516 (max) |
| 10     | 1.95        | 112.14 |
| 5      | 0.253       | 0.565177 (max) |
| 10     | 0.225       | 0.610195 (max) |
| 20     | 0.184       | 0.642457 (max) |
| 30     | 0.158       | 0.652836 (max) |
| 50     | 0.128       | 0.665739 (max) |
| 75     | 0.107       | 0.673652 (max) |
| 100    | 0.0935      | 0.674607 (max) |
| 200    | 0.0673      | 0.681385 (max) |
| 300    | 0.0552      | 0.682227 (max) |

$\Lambda \gg 1$ \quad \log (\omega/B) = 0.287 

0.687385 (max)
TABLE II. The maximal rotation velocities at $R$ for different initial values of $\sigma(0)$ for $\Lambda = 10$; cf. Fig. 5.

| $\sigma(0)$ | $x_{\text{max}}$ | $v_{\text{max}}$ (km s$^{-1}$) |
|-------------|-----------------|--------------------------------|
| 0.225       | 5.1             | 113276                         |
| 0.2         | 5.5             | 108377                         |
| 0.15        | 6.6             | 95641                          |
| 0.1         | 8.3             | 78105                          |
| 0.05        | 11.7            | 54049                          |
| 0.001       | 81.5            | 7338                           |
| 0.0001      | 257             | 2318                           |

FIG. 1. The mass $M$ (—) in units of $(m_{pl}^2/m)$ and the particle number $N$ (−−) in units of $(m_{pl}^2/m^2)$ as function of the central value $\sigma(0)$ for different values of $\Lambda := \lambda m_{pl}^2/4\pi m^2 = -5, 0, 5, 10$.

FIG. 2. The redshift $z$ depending on the self-interaction constant $\Lambda$ calculated at the first maximum in the $(M, \sigma(0))$ curve (see Figure 1) which gives the last stable boson star configuration.

FIG. 3. The redshift function (16) for different self-interactions.

FIG. 4. The rotation curves for the 'normal' non-interacting boson star and with a large value of the constant $\Lambda$. Both curves are taken for the last stable boson star configuration with the largest mass value.

FIG. 5. Rotation curves for different initial values $\sigma(0) = 0.225, 0.15, 0.05, 0.001$ of $\Lambda = 10$; largest initial value has largest maximal velocity.
