Hadrons as Signature of Black Hole Production at the LHC

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In models with several large extra dimensions and fundamental Planck scale of the order of 1 TeV, black holes can be produced in large numbers at LHC energies. We compute the charged hadron spectra obtained from the decay of black holes created in pp and Pb+Pb collisions at LHC. We show that hadrons from black hole decay dominate at transverse momenta $p_T \gtrsim 30 - 100$ GeV/c compared to usual QCD processes and black hole production signals are easy to identify in hadron transverse momentum spectra. Furthermore we show that a measurement of the charged hadron spectra probes Planck scales up to 5 TeV for any number of extra dimensions.

I. INTRODUCTION

In scenarios with large extra dimensions the fundamental Planck scale could be in the TeV range [1]. One of the most striking consequences of a low fundamental Planck scale is the possibility of producing black holes and observing them in future colliders or in cosmic rays/neutrino interactions [2, 3, 4, 5].

If gravity propagates in $d = 4 + n$ dimensions while the other fields are confined to a 3-brane, the 4-dimensional Planck scale is given by $M^2_P = M^2_P + 2V_n = G^{-1}_n V_n$, where $M_P$ is the fundamental Planck scale in $4 + n$ dimensions and $V_n = (2\pi R)^n$ is the volume of the n-torus that describes the compact space. For large size of the extra dimensions $R$, the fundamental scale $M_P$ can be as low as in the TeV range. The existence of large compact dimensions leads to deviations from Newtonian gravity at distances of the order of $R$, as well as strong effects of Kaluza-Klein excitations of the graviton on various processes at high energies. These effects impose constraints on the scale $M_P$, depending on the number of extra dimensions. For $n < 4$ the strongest limits are given by astrophysical processes like supernovae cooling and neutron star heating: $M_P \gtrsim 1500$ TeV for $n = 2$ and $M_P \gtrsim 100$ TeV for $n = 3$. Cosmological considerations give similar bounds for $n = 2$ and $n = 3$ and they imply $M_P \gtrsim 1.5$ TeV even for $n = 4$. These bounds contain, however, a larger degree of uncertainty and model-dependence. Non-observation of black hole production in cosmic ray interactions also imposes constraints for $n \geq 4$ at the level of $M_P \gtrsim 1$ TeV. Present collider limits are typically below 1 TeV for any number of large extra dimensions.

Given all these constraints, we will concentrate here on the scenario with $n = 6$ and $M_P \sim 1 - 5$ TeV. We will also discuss the dependence of our results on $n$ and $M_P$. One thing to note is that the constraints mentioned above are derived for a toroidal compactification where all the large extra dimensions have the same radius. For different types of compactification the limits could actually be considerably relaxed.

Black hole production and evaporation can be described semiclassically and statistically when the mass of the black hole is very large compared to the fundamental Planck mass. When the mass of the black hole approaches $M_P$ one expects quantum gravity effects to become important. We want to explore only the parameter space where the semi-classical treatment is justified. The total available energy at LHC is 14 TeV. Black holes with masses of this order can be produced in pp collisions. For Pb+Pb collisions, the black holes produced would have masses up to 5.5 TeV. These masses should be high enough above the Planck scales considered here for the semi-classical description to be valid [2].

In Ref. [6], it was suggested that the geometrical cross-section would be exponentially suppressed. Detailed subsequent studies [7, 8] did not confirm this proposal and showed that the geometrical cross-section is modified only by a numerical factor of order one. In Ref. [6], it was shown that even including the exponential suppression of the cross-section, the production rates are still high. We will use here the geometrical cross-section.

These black holes decay very rapidly. The decay occurs in several stages. For the purpose of detecting black hole events, the most important phase is the semi-classical Hawking evaporation, since it provides a large multiplicity of particles and a characteristic black-body type spectrum. Most of this Hawking radiation is on the brane [9], producing all Standard Model particles. Because most of the Standard Model degrees of freedom come from strongly interacting particles (quarks and gluons), hadrons will be the dominant signal for the events where black holes are formed.

In this paper we compute the transverse momentum distributions of charged hadrons at mid-rapidity obtained from the evaporation of black holes produced in pp collisions and Pb+Pb collisions at LHC energies. We show that in pp collisions the black hole events produce a large number of hadrons and dominate over the QCD background at transverse momenta above around 30-100 GeV/c, where they can be clearly measured. The results have a weak dependence on the number of large extra dimensions, but depend quite strongly on $M_P$. The signal is big enough to detect even for $M_P \sim 5$ TeV. For Pb+Pb collisions the energy available is lower, so one can only produce lower mass black holes and probe lower Plank scales. However, the rates could be higher due to the large number of binary collisions. We also discuss some
of the possible uncertainties that would affect our results.

II. BLACK HOLE PRODUCTION AND DECAY

In a high energy parton-parton collision, the impact parameter could be smaller than the Schwarzschild radius in $d$ dimensions for a black hole with mass $M_{BH}$:

$$r_h = \frac{1}{\sqrt{\pi} M_P} \left[ \frac{M_{BH}}{M_P} \left( \frac{8\Gamma\left(\frac{d+3}{2}\right)}{\pi (n+2)} \right) \right]^{\frac{1}{d-1}}. \quad (1)$$

This leads to the formation of a semi-classical $d$-dimensional black hole of size $r_h$ much bigger than the fundamental Planck scale $M_P$, but much smaller than the size of the large extra dimensions $R$.

With the above assumptions, the black hole production in a parton-parton collision is given by the geometrical cross-section $\sigma_{BH} = \pi r_h^2$. Then the cross-section in a $pp$ collision is obtained by folding in the parton densities:

$$\sigma(pp \rightarrow BH + X) = \frac{1}{\sqrt{\pi}} \sum_{a,b} \int_{x_{BH,\text{min}}}^{1} dM_{BH}^2 \times \int_{x_1,\text{min}}^{1} \frac{dx_1}{x_1} f_a(x_1, Q^2) \sigma_{BH} f_b(x_2, Q^2) \quad (2)$$

where $x_1$ and $x_2 = M_{BH}^2/(x_1 s)$ are the momentum fractions of the initial partons and $x_{BH,\text{min}} = M_{BH}^2/s$. The scale in the parton distribution functions $f(x, Q^2)$ is chosen to be $Q^2 = 1/r_h^2$. The results depend only weakly on the choice of this scale. We use CTEQ6M \cite{11} for the parton distribution functions.

The radiation rate into Standard Model particles is given by a thermal distribution in 4 dimensions:

$$\frac{dE}{dt} = \frac{1}{(2\pi)^3} \sum_{i} \int \frac{\omega g_i \sigma_{i,s} d^3k}{e^{\omega/T_{BH}} \pm 1} \quad (3)$$

with the black hole temperature:

$$T_{BH} = \frac{d-3}{4\pi r_h}, \quad (4)$$

where $d$ is the number of dimensions, $\omega$ is the sum over all Standard Model particles and $g_i$ is a statistical factor, counting the number of degrees of freedom. The sign in the denominator is $+$ for fermions and $-$ for bosons. $\sigma_{i,s}$ are the gray body factors, which depend on the spin $s$ of each particle. We first approximate these by $\sigma_{i,s} = \Gamma_s A_4$, where $\Gamma_s$ are constant \cite{11} ($\Gamma_{1/2} = 2/3, \Gamma_1 = 1/4, \Gamma_0 = 1$). A black hole acts as an absorber with a radius somewhat larger than $r_h$, such that $A_k$ can be written as \cite{12}:

$$A_k = \Omega_k - 2 \left( \frac{d-1}{2} \right)_{d-1} \left( \frac{d-1}{d-3} \right)_{d-1} \frac{k^{d-2}}{r_h^{d-2}} \quad (5)$$

with

$$\Omega_k = \frac{2\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d+1}{2}\right)}. \quad (6)$$

We compare these results with those obtained by using the recently computed gray body factors given in eq. (47) and (53) of Ref. \cite{12}. The differences in the final results are found to be around 30%.

For the emission into gravitons, which are $d$-dimensional, the rate is given by:

$$\frac{dE}{dt} = \frac{1}{(2\pi)^{d-1}} \sum_{i} \int \frac{\omega g_i \sigma_{d-1} d^3k}{e^{\omega/T_{BH}} \pm 1}, \quad (7)$$

where $\sigma_i \sim A_d$. This will be much smaller than the emission rate into SM particles \cite{12} and we can neglect it when calculating the lifetime.

The lifetime of the black hole is then obtained by integrating eq. (3). Assuming no mass evolution during the decay, we get:

$$\tau_{BH} = M_{BH} \left[ \frac{\pi^2}{30} \sum_f (\sum_{a,b} g_i) (\sum_{b} g_i \sigma_b) T_{BH}^4 \right]^{-1} = c M_{BH} \left[ \frac{\pi^2}{30} \sum_f (\sum_{a,b} g_i) (\sum_{b} g_i \sigma_b) T_{BH}^4 \right]^{-1} \quad (8)$$

III. HADRON SPECTRUM

We compute the cross section for inclusive charged hadron production from the partons produced in the black hole decay from

$$E \frac{d\sigma^h}{d^3p} = \frac{1}{(2\pi)^3} \sum_{a,b,c} \int_{M_{BH,\text{min}}}^{s} dM_{BH}^2 \int_{x_{BH,\text{min}}}^{1} \frac{dx_1}{x_1} \int_{z_{BH,\text{min}}}^{1} \frac{dz}{z^2} \times f_a(x_1, Q^2) \sigma_{BH} f_b(x_2, Q^2) E_c \frac{dN_c}{d^3p_c} \times D_c(z, Q_f^2), \quad (9)$$

where $z_{BH,\text{min}} = 2p/\sqrt{s}$ and $z = p/p_c$ and the decay distribution is:

$$E_c \frac{dN}{d^3p_c} = \frac{1}{(2\pi)^3} \frac{p_c^0 u_c y_c g_c \sigma_{BH}}{e^{\omega/T_{BH}} \pm 1} \quad (10)$$

where $\gamma$ is the Lorentz gamma factor and $u = (\gamma - 1, 0, 0, (x_1 - x_2)\sqrt{s}/(2Mf))$ takes into account that the black hole is not produced at rest, but with a small velocity. Here $p$ and $E$ refer to hadrons, while $p_c$ and $E_c$ are for partons.

We choose the scale of the fragmentation function $D^h_c(z, Q_f^2)$ to be the final transverse momentum $p_{Tf}$ of the hadrons. The KKP fragmentation function $\frac{13}{13}$ is used to get the final charged hadrons from the partons produced in the evaporation of the black hole. The KKP fragmentation function is only parametrized in the range of $0.1 \leq z \leq 1.0$ and $1.4 \leq Q_f \leq 100$ GeV. We need to access small values of the transverse momentum fraction $z = p_{hadron}/p_{parton}$, as well as large $Q_f$, for the partons from black hole decay. For the large $Q_f$, we explicitly
Black hole masses are integrated over $12 \text{TeV}$ collision at $\sqrt{s} = 14 \text{ TeV}$ compared to the QCD background as a function of $M_p$. Black hole masses are integrated over $12 \text{TeV} \leq M_{BH} \leq \sqrt{s}$. NLOpQCD predictions are plotted for the scales $Q = Q_f = 2p_T, p_T, p_T/2$.

We choose the KKP fragmentation function from the scale $Q_f = 100 \text{ GeV}$ up to the desired values (in this case up to $\sim 10 \text{ TeV}$) using DGLAP equations [14]. For the $z < 0.1$ range, we use small-$z$ fragmentation function by Fong and Webber [15] which is based on the coherent parton branching formalism, which correctly takes into account the leading and next-to-leading soft gluon singularities, as well as the leading collinear ones. It was found that the predicted energy dependence of the peak in the $\xi = \ln(1/z)$ distribution agrees very well with the $e^+e^-$ annihilation data up to c.m. energy of 200 GeV [16].

Fig. 1 shows the cross section for inclusive charged hadron production from black holes for several values of $M_p$ ranging from 1 to 5 TeV in $pp$ collision at LHC, compared to the expected spectrum of hadrons from QCD. LHC will be sensitive enough to detect the QCD hadrons up to $p_T$ around 400 GeV/c. The black hole signal is much bigger than the QCD one starting at $p_T \sim 50 - 200 \text{ GeV/c}$, depending on the Planck scale. It can be seen that even for $M_p$ as high as 5 TeV there is a considerable signal above background at $p_T \gtrsim 200 \text{ GeV/c}$. At higher $p_T$ the background is practically inexistent, while the black hole signal is still very large, as seen in Fig. 2.

We show in Fig. 2 the cross-section for inclusive charged hadron production from black hole decay for $M_p = 2 \text{ TeV}$, compared with the QCD background. It is reasonable to assume that black holes with masses only slightly higher than $M_p$ can be produced. However, as previously discussed, our semiclassical description of the black holes is only valid for black hole masses much bigger than the fundamental Plank scale. Integrating over the mass of the black hole starting at a low minimum value (close to $M_p$) would assume the validity of this treatment beyond its range of applicability. We choose to integrate starting at a much higher black hole mass. Clearly, black holes with masses lower than our cut-off can be produced, but results obtained with a cut-off at $M_p$ would have to take into account quantum gravity effects which are unknown. We show here results for $M_{BH}^{\text{min}} = 10 \text{ TeV}$ and $M_{BH}^{\text{min}} = 12 \text{ TeV}$. It can be seen that including lower mass black holes gives considerably higher rates. Consequently, we consider our approach to be a ‘conservative’ one: our results are an underestimate of the actual signal and our qualitative conclusions always hold, while the actual quantitative results could be much higher than our estimates, making the signal easier to detect. Even for high mass black holes the signal clearly dominates over the QCD background in the region above 100 GeV/c, where it can be easily seen in the experiments. Including lower mass black holes gives a bigger signal for all momenta and also drives the signal above the background even for lower momenta, of the order of tens of GeV.

We also study the dependence of the results on the number of extra dimensions and show that it is very small, as can be seen in Fig. 2. The QCD background [17] is shown for different choices of the scale used in the structure and fragmentation functions (we use $p_T, p_T/2, 2p_T$). The dependence on this scale is very weak in the high transverse momentum region. For the black hole signal, the same change in $Q_f$ leads to differences of up to a factor of 2 in the results, which does not change any of our conclusions.

We notice that even though there are significant changes in the overall rate of hadrons produced, the transverse momentum dependence of the hadrons does not change much when changing $M_p$ or $M_{BH}^{\text{min}}$. This is not the case at the parton level. Changing $M_p$ or $M_{BH}^{\text{min}}$ the temperature of the black hole is modified and consequently the spectrum of the emitted particles is differ-
ent. Even though we can still see this for partons, the hadronization washes out most of the effect. We conclude that we cannot get a direct determination of the temperature of the black holes from the hadron spectrum. One could attempt to do that by looking at the spectrum of photons and electrons in the black hole event, which preserves the black body radiation type of spectrum, but is considerably lower than the hadron signal because photons and electrons are only a small fraction of the particles produced in the black hole evaporation. In that case, one would be forced to consider black holes with lower masses in order to obtain a detectable signal.

We conclude from here that black hole events will be easily detected in the hadron spectra at high $p_T$. The values of $p_T$ for which the signal becomes higher than the background would give an indication on the values of $M_P$ and $M_{BH}$ that this signal corresponds to.

IV. BLACK HOLES IN Pb+Pb COLLISIONS

To compute the spectra in the case of Pb+Pb collisions, we multiply the expression in Eq. (2) by the Glauber profile density $T_{AA}(b) = \int d^2r T_A(r) T_A(|b - r|)$, where $T_A(r) = \int d\rho \rho(r, z)$, normalized such that $\int d^2r T_A(r) = A$, $A$ being the atomic mass number and $\rho(r, z)$ the nuclear density for which we take the Woods-Saxon distribution. This factor gives an enhancement for the production cross-section. For example, $T_{AA}(b = 2\text{fm}) = 28 \text{ mb}^{-1}$ in the case of Pb+Pb collisions at impact parameter $b = 2$ fm. However, $\sqrt{s_{NN}} = 5.5$ TeV in this case, so only lower mass black holes can be produced and a smaller parameter space can be probed. We do not include shadowing effects, since black holes dominate at high $x$, where these are negligible. In addition, it is demonstrated that nuclear modification of parton distribution is getting smaller when we go to larger scale from $Q^2 = 2.25 \text{ GeV}^2$ up to $Q^2 = (100\text{GeV})^2$ [18]. We have confirmed that this also holds true for much higher scales up to $Q = 30 \text{ TeV}$ by evolving the nuclear parton density in [18] and conclude they are negligible.

In Fig. 3 we show the results for Pb+Pb collisions for $M_P = 1 \text{ TeV}$ and $M_{BH}^{\text{min}} = 5 \text{ TeV}$. For these parameters, the signal is still significant and can be easily detected. Due to the fact that $\sqrt{s_{NN}}$ is only 5.5 TeV, the higher scales are no longer accessible in this type of experiment.

In Pb+Pb collisions, the black hole is expected to be produced in a dense medium of quarks and gluons, therefore we need to take into account the interactions of the partons produced in the decay of the black hole with the quarks and gluons around it, for example, as in Ref. [19]. The energy loss is expected to be small at high transverse momentum. At LHC energies, for $p_T \sim 100 \text{ GeV/c}$, where the black hole signal clearly dominates over the background, the energy loss was found to be small, (about 5% effect) [20].

However, energy loss has significant effects at $p_T$ below 10 GeV in the QCD spectrum at LHC [20]. For the hadrons coming from black holes we also expect the effect to be small in the high momentum region. However, there is a possibility to have enhanced particle yield around $p_T \sim 10 \text{ GeV/c}$, because the hadron spectra is much flatter than that of the QCD spectra and feedback from the emitted gluons could be non-negligible. If this is the case, the black hole signal could be also identified in the lower $p_T$ region, in addition to the high $p_T$ one. It could happen that the signal becomes higher than the background at these values of $p_T$. Even if the signal is somewhat lower than the background, it is still large, such that the experiment would detect a large number of hadrons from signal-background, even at $p_T$’s of tens of GeV.

We do not include interactions of the black hole itself with the surrounding particles. One can imagine taking into account possible absorption of these particles by the black hole, which would affect the decay of the black hole. This is an interesting issue and is presently under investigation [21].

V. CONCLUSIONS

In summary, we have computed the transverse momentum spectra for high $p_T$ charged hadrons from decay of black holes produced in $pp$ collisions at $\sqrt{s} = 14 \text{ TeV}$ as well as in central Pb+Pb collisions at $\sqrt{s_{NN}} = 5.5 \text{ TeV}$. We have shown that the hadrons from black holes are detectable and dominate the background for $p_T$ above about 100 GeV for fundamental Planck scales up to 5 TeV. Our results are conservative, as they only take into account very high mass black holes. Including black holes with lower masses gives even stronger signals. The value of $p_T$ at which the signal becomes bigger than the back-
ground is determined by $M_P$ and $M_{BH}$ considered.

We have neglected the evolution of the mass of the black hole during the decay. If we take into account that, as the mass decreases, the temperature increases, we would get a somewhat harder spectrum. In the same time, the lifetime would decrease compared to our estimate, so that our curves would move slightly down and to the right. However, all the qualitative features previously discussed will remain the same. The last stage of the decay, when the mass of the black hole has decreased to almost $M_P$ is not understood, since it requires a full quantum gravity description. This is why a full consideration of the mass evolution during decay is not really possible. We have not taken into account the angular momentum of the black hole and the phase of the decay when the black hole would just loose this angular momentum. Also, there are a few additional particles produced in the initial stage of the black hole decay, when the black hole looses the quantum numbers of the partons that produced it. In [8] it has been shown that classical gravitational radiation could be important and consequently the actual black hole mass is smaller than that of the center of mass energy of the parton collision. This would reduce the number of very high mass black holes produced. However, even in case of a small fraction of initial energy going into black hole production, as long as $M_{BH} \gg M_P$, there is still a large, observable signal since production of black holes with small mass is large.

If there are additional degrees of freedom around 100 GeV (new particles), so below $T_{BH}$, they should be taken into account and they would lead to a small decrease in the lifetime. Their contribution would be slightly suppressed, just as for the top quark, W’s, Z’s and Higgs, for which the masses are no longer much smaller than the temperature. These issues introduce some uncertainty in the numerical results, but our main conclusions are unaffected.

We would like to note that the QCD background we show in the graphs is computed at $y = 0$. For high rapidity this background is actually much smaller, while the black hole signal is the same for all rapidities. This would indicate that by looking in the high rapidity region one would enhance the signal to background ratio even further.

In conclusion, we have shown that the charged hadron distribution in $pp$ and Pb+Pb collisions at LHC energies provides an unique probe of black hole production and the physics of extra dimensions for Planck scale up to 5 TeV and for any number of extra dimensions.

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