Tunable Nb superconducting resonators based upon a Ne-FIB-fabricated constriction nanoSQUID

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Hybrid superconducting–spin systems offer the potential to combine highly coherent atomic quantum systems with the scalability of superconducting circuits. To fully exploit this potential requires a high quality-factor microwave resonator, tunable in frequency and able to operate at magnetic fields optimal for the spin system. Such magnetic fields typically rule out conventional Al-based Josephson junction devices that have previously been used for tunable high-Q microwave resonators. The larger critical field of niobium (Nb) has allowed microwave resonators with large field resilience to be fabricated. Here, we demonstrate how constriction-type weak links, patterned into the central conductor of a Nb coplanar resonator using a neon focused ion beam (FIB), can be used to implement a frequency-tunable resonator. We study microwave transmission through the resonator and measure a quality factor of 30,000 and a frequency tuning range of 100 MHz over 10 μT of applied perpendicular magnetic field. The same resonator retains a quality factor of 9,100 at an applied perpendicular field of 0.48 mT. At 0.48 mT, due to flux-focusing effects, the local perpendicular fields around the resonator exceed 59 mT, suggesting that the resonator should be resilient to much larger applied fields when combined, for example, with patterned ground-planes. Our results demonstrate that resonators embedded with Ne-FIB-fabricated Nb nano-SQUIDs combine frequency tunability with magnetic-field resilience, fulfilling an important requirement for coupling to spin systems for quantum memory and quantum information applications.

I. INTRODUCTION

A large and growing variety of spin systems have been coupled to superconducting resonators, including ensembles of non-interacting spins in silicon [1], diamond [2] and other materials [3], magnons in ferrimagnets [4] and chiral magnetic insulators [5], and individual spins in quantum dot devices [6]. Motivations for such studies include long-range coupling of spin qubits [7], realisation and study of topological systems [8], long-lived microwave quantum memories for superconducting qubits [9, 10], and the demonstration of microwave-to-optical conversion at the single-photon level [11]. Planar superconducting circuits provide a robust, well-studied [12] and scalable architecture [13] for such hybrid systems, and superconducting resonators with Q-factors over 1 million have been achieved [14]. However, for the majority of the applications described above, externally applied magnetic fields from ~10 mT up to several 100 mT, or more, are required to bring the spin systems into a suitable regime of interest. Furthermore, control of the spin-resonator coupling, often on short timescales, is required in applications such as quantum memories, and may be achieved, for example, by frequency-tuning of the resonator.

Superconducting quantum interference devices (SQUIDs) act as flux-tunable inductors and have been successfully incorporated into resonators to provide frequency tunability [15]–[17]. The SQUID inductance is tuned from its minimum to maximum by an additional half flux quantum threading the SQUID loop, thus altering the resonator frequency. This means that small local fields provided by on-chip flux lines are able to tune SQUID-embedded resonators on timescales of a few nanoseconds [18]. Technologies which use DC currents to tune the kinetic inductance and hence resonant frequency of resonators have also been developed [18, 19]. Previous SQUID-tunable devices have been fabricated from aluminium with shadow-evaporated junctions [19] or used Nb/Al/AlOₓ/Nb trilayer junctions [15]. Al devices may suffer from the low critical field of Al and are expected to have poor magnetic-field resilience. AlOₓ tunnel junctions may introduce extra losses to resonators and limit quality factors. An alternative technology is based on nanoSQUIDs formed by superconducting-nanowire-based constriction junctions; these have already been shown to possess exceptional field resilience [20].

NanoSQUIDs are commonly fabricated [21] by a Ga-based focused ion beam (FIB); however, this technique has been shown to induce loss into superconducting resonators [22]. Here, we use a neon FIB (a technique shown to be compatible with high quality superconducting resonators [23]) to create constrictions within the centre conductor of a Nb superconducting λ/4 co-planar resonator. These constrictions have a width of 50 nm and are placed in parallel such that they complete a superconducting loop between the current antinode of the
resonator and the ground (Fig. 1). We study the microwave transmission of this device and demonstrate that it realizes a superconducting-nanowire-based frequency-tunable and field-resilient resonator with high quality factor, \( Q > 9 \times 10^3 \) at applied fields up to 0.48 mT, perpendicular to the resonator plane. We determine that flux-focusing increases the local field around the SQUID to approximately 59 mT, which provides evidence that our structures could withstand much higher applied fields with a different ground-plane design. Furthermore, the resilience to in-plane magnetic fields is expected to be much bigger, given that Nb resonators with \( Q \sim 2.5 \times 10^4 \) in 160 mT in-plane fields have already been demonstrated \[24\]. Overall, this suggests that such tunable Nb resonators are an attractive candidate technology for hybrid quantum systems \[17\] as well as for high-sensitivity electron spin resonance (ESR) \[25\].

II. EXPERIMENTAL

Co-planar resonators were fabricated by etching thin films of superconductor using a similar method to that described in Ref \[23\]. Here Nb is used, instead of NbN, because of its longer coherence length of 38 nm \[26\] which sets the length-scale for the width of constrictions needed to make junctions — Nb thus allows \( \approx 50 \) nm constrictions to be used, which is easier to achieve than the \( \approx 20 \) nm constrictions required in NbN. Nb also has a lower kinetic inductance (hence lower impedance and larger zero-point current fluctuations), which could enable stronger magnetic coupling of the Nb resonator to spins. Nb films were deposited on Si substrates by DC magnetron sputtering from a 99.99%-pure elemental Nb target in argon. The pressure before deposition was \( 6 \times 10^{-7} \) mbar and during deposition was \( 3.5 \times 10^{-3} \) mbar. The sputter power was 200 W, with the deposition timed to produce a 50-nm-thick film.

Quarter-wave (\( \lambda/4 \)) resonators (see Fig. 1b) with an embedded superconducting loop at the grounded end of the resonator were patterned by electron beam lithography (EBL) (Fig. 1a) in the same lithography step as a microwave feed line. This pattern was transferred from resist to the film by a reactive ion etch (RIE) process using a 2:1 ratio of SF\(_6\) to Ar, at 30 mbar and 30 W for 120 s. The RIE process additionally etches exposed Si to a depth of 500 nm, leaving areas with Nb raised above their surroundings. The superconducting loop has two constrictions (see Fig. 1b and c): broad constrictions are defined in the initial EBL exposure and subsequently narrowed to \( \approx 50 \) nm by Ne FIB milling, in which a beam of Ne ions, accelerated to 15 kV, mills through the Nb. A dose of \( \approx 2 \) nC\( \mu \)m\(^{-2} \) is used. On the same chip, 21 out of 22 constrictions milled were still intact after narrowing to a dimension approaching the coherence length, suggesting a high yield for this part of the processing; see supplementary materials for details.

Resonators are measured at a temperature \( T \approx 300 \) mK in a \(^3\)He cryostat with a heavily attenuated microwave in-line and a cryogenic high-electron-mobility transistor (HEMT) amplifier on the microwave out-line. The \( S_{21} \) transmission of microwaves through the feed-line—to which the resonator is capacitively coupled—is measured using a Rohde & Schwarz ZNB8 vector network analyzer. Perpendicular magnetic fields are applied by a superconducting magnet connected to a precision current source (Keithley 2400 SourceMeter). Samples are enclosed in a brass box lined with Eccosorb CR-117, a microwave absorber, to reduce the number of quasiparticles excited by stray IR photons, which we have previously shown can have a significant effect on superconducting constrictions \[23\].

III. RESULTS

A. Tunability

Fig. 2 shows the magnitude response of \( S_{21} \) from the \( \lambda/4 \) resonator with no applied magnetic field, measured at \( T = 307 \) mK and an applied microwave power \( P \approx -106 \) dBm. Using a traceable fit routine \[27\], based on fitting a circle to the resonance in the real–imaginary plane, we extract the resonator parameters including the internal quality factor (\( Q_i \approx 3.0 \times 10^4 \)), central frequency (\( \nu_0 = 7.417 \) GHz) and coupling quality factor (\( Q_c \approx 2.2 \times 10^5 \)). The asymmetry of the resonance, which persists down to single photon powers, is due to impedance mismatch and is fully captured by the fit routine.

To examine the field tuning of the resonator, we study its behaviour in a small perpendicular magnetic field...
The total impedance \( Z_T \) of a transmission line terminated by a SQUID is given at frequency \( \nu \) by

\[
Z_T = \frac{Z(Z_{\text{SQUID}} + jZ \tan(2\pi \nu d/v))}{(Z + jZ_{\text{SQUID}} \tanh(2\pi \nu d/v))}
\]

where \( d \) is the length of the transmission line, \( l_0 \) the inductance per unit length, \( Z \) is the impedance of the transmission line, \( Z_{\text{SQUID}} \) the impedance of the SQUID and \( v = \sqrt{1/l_0 c_0} \) the speed of light in the transmission line where \( c_0 \) is the capacitance per unit length. \( Z_T \) is real at resonance and the resonant frequencies \( \nu_i \) are therefore given by

\[
\tan \left( \frac{2\pi \nu_i d}{v} \right) = \frac{|Z_T|}{l_0 v},
\]

which may be solved numerically. The fundamental resonant frequency may be expressed approximately [29] in terms of the total inductance \( L \) and capacitance \( C \) of the distributed resonator:

\[
\nu_0(B) = \frac{1}{4\sqrt{(L_{\text{res}} + L_{\text{SQUID}} + \Delta L(B))C}},
\]

where \( L_{\text{res}} \) is the inductance of the resonator excluding the SQUID, \( L_{\text{SQUID}} \) is the zero-field inductance of the SQUID, \( \Delta L(B) \) is the change of inductance of the SQUID with field which, from Eq. [1] is equal to \( \Phi_0/4\pi I_{C0} \times (1/|\cos(f)| - 1) \). Assuming \( f \propto B \), an assumption which we examine further below, we can write

\[
\frac{\nu_0(B)}{\nu_0(0)} = \sqrt{1 + \frac{\Delta L(B)}{L_{\text{res}} + L_{\text{SQUID}}}} = \sqrt{1 - A + \frac{A}{\cos(KB)}},
\]

where \( A = \Phi_0/[4\pi I_{C0}(L_{\text{res}} + L_{\text{SQUID}})] \) and \( f = KB \) so that \( K \) scales field to \( f \). The observed \( \nu_0(B) \) dependence of the resonator in Fig. 3 fits well with Eq. 5 allowing determination of \( A \) and \( K \).

We next consider the relation between the field periodicity of the tuning behaviour and the flux quantum. SQUID behaviour is periodic in applied flux. The area of the SQUID loop is \( A_{\text{loop}} = 3.7\pm0.3 \, \mu m^2 \), such that 10 \( \mu T \) (the field required to maximally detune the resonator) corresponds to a flux \( BA_{\text{loop}} \approx 0.02\Phi_0 \). Assuming the tuning arises from the SQUID, the field required to maximally tune the resonator implies that the local flux density at the SQUID is much greater than the 10 \( \mu T \) applied by the magnet. This indicates substantial flux focusing due to flux expulsion from the superconducting ground plane surrounding the resonators. We return to the topic of flux focusing later.

As the resonator is detuned away from \( \nu_0 \), \( Q_i \) is found to drop from its maximum value, \( 2.8\times10^4 \), to \( 1.0\times10^4 \) when the resonator is maximally tuned. This phenomenon of decreasing \( Q_i \) has previously been observed and attributed to increasing thermal noise as the SQUID is detuned [15] or increased dissipation caused by a sub-gap resistance [16]. An alternative explanation could be that dilute surface spins [34] induce spectral broadening of the resonance lineshape by flux-noise-based frequency jitter in these flux-tunable resonators. However, even for the highest values of flux noise in Ref. [31] which correspond to \( \approx 100\mu \Phi_0 \), the corresponding frequency jitter would be too small to create sufficient spectral broadening to explain the drop in \( Q_i \) observed here. The source of these extra losses is the subject of ongoing work.

### B. Hysteresis and Premature Switching

When tuning the constriction-SQUID resonator over more than one period, as shown in Fig. 3, and d, a hysteretic behaviour is seen, similar to that previously reported in Al constriction SQUID resonators [17]. In addition, frequency jumps are observed at values of detuning...
Figure 3. (a) The tuning of resonant frequency (circles) and quality factor (triangles) as magnetic field is swept. The resonant frequency is fit by Eq. 5, with fitting parameters $A = 0.050$ and $K = 0.101 \ (\mu T)^{-3}$. For the data shown, the central value $\Delta B = 0$ corresponds to an applied field $B = 12.4 \ \mu T$. (b) Resonant frequency and (c) internal quality factor of the resonator as applied field is swept from 0 to 22 $\mu T$ (black), then down from 22 $\mu T$ to $-22 \ \mu T$ (red) and then back up from $-22 \ \mu T$ to 22 $\mu T$ (blue). The sweep directions are shown by arrows at the bottom of (b). (d) Detuning of the resonator when the field is changed in 0.1$\mu T$ steps from $B = 12.3 \ \mu T$. A maximal detuning of the resonator of $-101 \ MHz$ is obtained.

less than the maximum value (see for example the region between $-5 \ \mu T$ and $-20 \ \mu T$). We attribute jumps at non-maximal detuning to flux trapping as the field is ramped up and down. The oscillations in the internal quality factor (Fig. 3b) follow the frequency detuning of the resonator, showing that the degradation in $Q_i$ with magnetic field arises from the state of the SQUID and not from the properties of the resonator in a magnetic field.

The hysteretic tuning of the resonant frequency and internal quality factor may be explained by significant self-inductance of the superconducting SQUID loop. The SQUID has a characteristic parameter $\beta_L = 2L_{\text{loop}}/I_C/\Phi_0$ (where $L_{\text{loop}}$ is the inductance of the loop and $I_C$ is the Josephson critical current). When $\beta_L \gtrsim 1$, the SQUID behaviour becomes hysteretic with applied flux, as shown in Fig. 3a. The red path in Fig. 3a maps out the flux within the SQUID as the field is increased (assuming zero temperature, in the absence of fluctuations). At extremal points, the flux threading the SQUID exhibits discontinuous jumps as $\Phi_{\text{app}}$ is ramped upwards, occurring periodically with a period of $2\Phi_0$. At finite temperature, thermal fluctuations cause these jumps to occur at a temperature-dependent flux less than that at $T = 0$.

Using this $2\Phi_0$ periodicity, we are able to calibrate local fields at our device and experimentally quantify flux focusing in these hysteretic devices. jumps occur every 9.7 $\mu T$ (averaged over 4 consecutive jumps in Fig. 3a), which corresponds to $BA_{\text{loop}} \approx 0.016\Phi_0$. Identifying the jumps as $2\Phi_0$-periodic features, we infer a flux-focusing $\mathcal{F} \approx 124$, where we have defined $\mathcal{F}$ by $B_{\text{local}} = \mathcal{F}B$. Significant flux focusing from superconducting ground planes has recently been investigated theoretically and experimentally in Ref. 33 where simulations gave $\mathcal{F} \approx 27.5$. The extent of flux focusing is specific to device geometry; for example, features designed to trap flux can have large effects on $\mathcal{F}$.

The change in applied flux required to maximally detune the resonator frequency is determined by the $\beta_L$ value of the SQUID (see Fig. 3b). At finite temperature, the flux in the SQUID jumps before reaching the point of instability shown in Fig. 3a and so the experimentally measured jump positions provide a lower bound on $\beta_L$. We thus infer that $\beta_L > 3.4$ for our device. Using Eq. 1 and the fit parameter $A$ (which relates the inductance of the resonator and of the SQUID), we can calculate the expected tuning of the resonator in an applied magnetic field based on a sinusoidal current–phase relation (Fig. 3b). The smooth tuning shown in Fig. 3b (blue line in Fig. 3b,d) and jumps seen in Fig. 3d (red line in Fig. 3b,d) are qualitatively reproduced. The calculation, however, suggests that resonant frequencies should decrease significantly in the vicinity of hysteretic jumps due to the asymptote in $1/cos f$ at $f = 1/2$, whereas in practice these resonators tune by only ~1% of their untuned frequency. Numerical analysis based on Eq. 3 predicts such reduced tuning for resonators where the SQUID inductance or capacitance is a significant fraction of the total inductance or capacitance of the distributed resonator. We determine the untuned inductance of the SQUID to be approximately 10% of the total inductance of the distributed resonator (see supplementary materials for details), which in Ref. 33 is sufficient (even with small SQUID capacitance) to reduce the tuning of the resonator approaching $\Phi = \Phi_0/2$ to only around 30%. Additionally, asymmetry between junctions allows
switching at smaller detuning than perfectly symmetric devices as the narrower junction becomes maximally biased (and hence switches) before the wider junction becomes maximally biased.

C. Magnetic field resilience

In Fig. 5 the SQUID-containing resonator (black markers) and an identical resonator with no embedded SQUID (red markers) are measured as the applied perpendicular magnetic field is increased from zero to \(\approx 0.5 \text{ mT} \) which corresponds to a local field at the SQUID of \(\approx 60 \text{ mT} \) based on flux focusing factors calculated above. Even at these relatively large perpendicular magnetic fields, the internal quality factor of the resonators is still \(Q_i = 9.1 \times 10^3\) and the resonator remains tunable, demonstrating the field resilience of these devices.

The internal quality factors of the two resonators at zero applied field are similar: \(5.0 \times 10^4\) for the SQUID-embedded resonator vs. \(6.5 \times 10^4\) for the resonator without a SQUID \([35]\). Embedding a SQUID in our case therefore causes at most a minor degradation in the resonator \(Q\), though its effect may be more profound on resonators with larger internal \(Q \gg 10^4\). We also note that film inhomogeneity can lead to small differences in zero-field quality factors for different resonators on the same chip and therefore more statistics would be required to make quantitative statements regarding the impact of the SQUID on the resonator \(Q\)-factor at zero field.

For the resonator without an embedded SQUID, \(v_0\) and \(Q_i\) tune weakly and smoothly as the applied magnetic field is increased up to about 0.21 mT with the exception of abrupt drops in both \(v_0\) and \(Q_i\) at four field values up to 0.2 mT (see Fig. 5). This step-like response of \(Q_i\) vs \(B\) is consistent with the formation of vortices on the superconducting resonator’s central conductor \([36]\). For the nanoSQUID resonator, \(Q_i\) modulates by a factor of about 5 with field as the resonator tunes (as previously shown in Fig. 3). In addition to the modulation with tuning, the maximum \(Q_i\) also drops with applied field. The magnitude and field scale of this drop is similar to that of the bare resonator. The respective \(Q_i(B)\) dependences for the SQUID-embedded and bare resonators are consistent with the maximum of the field-modulated \(Q_i\) (in the SQUID-embedded resonators being limited, as \(B\) increases, by the same physics which causes the step-like reductions in \(Q_i\) for the bare resonators. The similarity is even more clearly seen in \(v_0(B)\), where the untuned resonant frequency of the nanoSQUID resonator jumps up at an applied field of 0.21 mT just as in the bare resonator. This suggests that the resonator’s field resilience is the limiting factor in the nanoSQUID device field performance, a conclusion that is also consistent with measurements of nanoSQUIDs successfully operating in fields up to 1 T \([20]\). Our results are therefore promising for future generations of devices where SQUIDs are embedded within field-resilient resonators. Importantly, these SQUID-embedded resonators operate at local fields comfortably above 30 mT, the first clock transition of bismuth spins.

In the literature, resonator tuning is typically given in units of flux quanta. This means that there are no comparable results on field resilience of tunable resonators to which this device can be compared. Therefore, we believe that this study is the first report addressing the field resilience of a tunable resonator.

IV. CONCLUSIONS

In conclusion, we have embedded constriction nanoSQUIDs in a Nb resonator, and demonstrated a tunability of \(>100\ \text{MHz}\) with \(Q_i \approx 3.0 \times 10^4\). The SQUID resonator is resilient to magnetic fields and maintains a \(Q_i\) of \(9.1 \times 10^3\) at an applied perpendicular field up to 0.5 mT. A number of modifications to the device and measurement setups may straightforwardly be made to improve field resilience, specifically: operating the device in parallel fields (which tune spins to the clock transition

Figure 4. (a) Variation with applied external flux \(\Phi_{\text{applied}}\) of flux \(\Phi\) threading the SQUID, for SQUIDs with \(\beta_L = 0\) (dashed), 0.5 (dot-dashed) and 1 (solid) lines. (b) Variation with applied flux of flux threading a \(\beta_L = 3.4\) SQUID. The black line shows the solution to the equation governing flux threading a hysteretic SQUID \([32]\). The red (blue) line indicates the flux jump threshold as in Fig. 3c,d. (c) The relationship between applied flux \(\Phi_{\text{applied}}\) and \(\Phi\) of the embedded SQUID. (d) Variation of resonant frequency of the SQUID with applied flux depending on how field is ramped, calculated using an inductance ratio between SQUID and resonator determined from the parameter \(A\) from the fit in Fig. 3(a). Colors of lines correspond to the same flux ramping as in (b).
Figure 5. Magnetic-field dependence of (a) resonant frequency (circles) and (b) internal quality factor (triangles) for a resonator with an embedded SQUID (black, unfilled symbols) and a resonator with no embedded SQUID (red, filled symbols). The local field $B_{\text{local}}$ is estimated based on measurements of flux focusing (see main text).

without applying large fields to the resonator), modifying the device design by the addition of anti-dots [37], patterning the whole ground-plane [24] and/or the use of resonator designs inherently more robust to external field [24, 37, 38]. These resonators hold great promise for future hybrid quantum system applications, in particular for applications which require magnetic field, such as for operating Bi impurities in Si at the clock transition.

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VI. SUPPLEMENTAL

In the main text of the article, we report measurements on two different resonators fabricated on the same chip. Six resonators were patterned on this chip by EBL. One device has no further constrictions embedded and is used as a reference (see Fig. 5 of the main text, red symbols). The remaining five resonators have constrictions embedded. Three resonators, including the resonator featured in the main text, have a single nanoSQUID embedded. The two other resonators have a chain of four SQUIDs is embedded in series.

In total, 11 nanoSQUIDs were fabricated on this sample, resulting in a total of 22 constrictions being milled by Ne FIB. Of these 21 were successfully fabricated and one constriction was destroyed during fabrication - a yield of 95%.

There is some distribution in the widths of constrictions which were fabricated; this is shown in Fig. 6. This distribution could likely be narrowed in future devices simply due to improved fabrication.

The resonators with chains of SQUIDs were fabricated because simple considerations suggest that this could increase the tunability of the resonator: for identical SQUIDs, the inductance of each SQUID should tune by the same amount, \( \Delta L \), implying that the total inductance of the resonator tunes by \( N \Delta L \) where \( N \) is the number of SQUIDs in series. Such behaviour, however, depends on the SQUIDs being identical: if the SQUIDs are not identical, switching in each individual SQUID will occur at different field values, meaning that it will not be possible to simultaneously maximally detune each SQUID at a single field value. This suggests that the total tuning will likely not increase significantly (and tuning will be much harder to control as there will be \( N \) SQUIDs, each of which may undergo a hysteretic jump in the flux threading it).

To demonstrate that behaviour similar to that presented in the main text is observed in other SQUID-embedded resonators, we present in Fig. 7 characterisation of one of the other resonators embedded with a single nanoSQUID. This resonator has an untuned resonant frequency of 3.825 GHz and \( Q_i \approx 3 \times 10^4 \). The \( S_{21} \) amplitude response shows hysteretic jumps just as in Fig. 3(a,b). The resonator may be tuned by >25 MHz. This resonator has a lower resonant frequency (higher inductance) than the resonator featured in the main text, so the same tunable inductor would be expected to tune the resonant frequency less. Analysis analogous to that presented in the main text gives \( \beta_L \approx 6 \) for this device.

The resonators reported in Fig. 5 (main text) have similar length by design and are designed to have 50Ω characteristic impedance and have the same capacitance and inductance per unit length. More refined calculations accounting for the width of the gap and central conductor, using \( \epsilon_r = 11.7 \) for Si at cryogenic temperatures, give an impedance of 51.1 Ω. Using the impedance of the bare resonator, its resonant frequency (7.84 GHz) and the design length (3.552 mm) of the two resonators we can calculate the total inductance \( L \) and total capacitance \( C \) of the resonator without the embedded SQUID from

\[
\nu = \frac{1}{4\sqrt{1/LC}} \quad \text{and} \quad Z = \sqrt{\frac{L}{\epsilon}}
\]

obtaining \( L = 1.63 \text{ nH} \) and \( C = 0.62 \text{ pF} \). By assuming the same capacitance and inductance per unit length in the SQUID-embedded resonator, we may calculate the inductance and capacitance coming from the resonator. At zero field, the inductance is given by \( L_{\text{tot}} = L_{\text{res}} + L_{\text{SQUID}}^0 \), so by taking the untuned frequency (7.42 GHz) we calculate the zero-field SQUID inductance \( L_{\text{SQUID}}^0 \approx 0.168 \text{ nH} \). We assume that this inductance comes from the loop inductance of the

![Figure 6. Histogram showing the width of 21 fabricated constrictions, each with a design width of 50nm. Widths were measured from He-ion micrographs.](image)

![Figure 7. S\(^{21}\) amplitude plot showing the tuning of a resonator in a global magnetic field from 22.32 µT to −16.74 µT. The resonator has untuned resonant frequency 3.825 GHz and \( Q_i \approx 30k \). Hysteretic jumps are found, as in Fig. 3(a,b).](image)
SQUID when untuned. We note that, when embedded within an resonator, the two arms of the SQUID are in parallel. This means that the inductance adds inversely $1/L_{tot} = 2/L_{arm}$. When considering the $\beta_L$ value in the SQUID, we must consider the total inductance in the loop which means that the arms of the SQUID are in series. Therefore, the series inductance is four times larger than the parallel inductance. Using the minimum bound on $\beta_L$ from the main text of 3.4 we can place a minimum bound on the critical current $I_C > 5 \mu A$. 
