Higgs inflation at the critical point

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Abstract

Higgs inflation can occur if the Standard Model (SM) is a self-consistent effective field theory up to inflationary scale. This leads to a lower bound on the Higgs boson mass, $M_h \geq M_{\text{crit}}$. If $M_h$ is more than a few hundreds of MeV above the critical value, the Higgs inflation predicts the universal values of inflationary indexes, $r \simeq 0.003$ and $n_s \simeq 0.97$, independently on the Standard Model parameters. We show that in the vicinity of the critical point $M_{\text{crit}}$ the inflationary indexes acquire an essential dependence on the mass of the top quark $m_t$ and $M_h$. Thus the cosmological measurements of $r$ and $n_s$ different from the universal values lead to precise prediction of $M_h$ and $m_t$.

1. Introduction

The most economic inflationary scenario is based on the identification of the inflaton with the SM Higgs boson \cite{1} and the use of the idea of chaotic initial conditions \cite{2}. The theory is nothing but the SM with the non-minimal coupling of the Higgs field to gravity with the gravitational part of the action

$$S_G = \int d^4x \sqrt{-g} \left( -\frac{M_P^2}{2} R - \frac{\xi |H|^2}{2} R \right),$$  \hspace{1cm} (1)

Here $R$ is the scalar curvature, the first term is the standard Hilbert-Einstein action, $H$ is the Higgs field, and $\xi$ is a new coupling constant, fixing the strength of the “non-minimal” interaction. The presence of non-minimal coupling is required for consistency of the SM in curved space-time (see, e.g. \textsuperscript{3}). The value of $\xi$ cannot be fixed theoretically within the SM.

The presence of the non-minimal coupling insures the flatness of the scalar potential in the Einstein frame at large values of the Higgs field. If radiative corrections are ignored, the successful inflation occurs for any values of the SM parameters provided $\xi \simeq 47000\sqrt{\lambda}$, where $\lambda$ is the Higgs boson self-coupling. This condition comes from the requirement to have the amplitude of the scalar perturbations measured by the COBE satellite. After fixing the unknown constant $\xi$ the theory is completely determined. It predicts the tilt of the scalar perturbations given by $n_s \simeq 0.97$ and the tensor-to-scalar ratio $r \simeq 0.003$. After inflationary period, the Higgs field oscillates, creates particles of the SM, and produces the Hot Big-Bang with initial temperature in the region of $10^{13-14}$ GeV \cite{4,5}.

The quantum radiative corrections can change the form of the effective potential and thus modify the predictions of the Higgs inflation. The most significant conclusion coming from the analysis of the quantum effects is that the Higgs inflation can only take place if the mass of the Higgs boson is greater than some critical number $M_{\text{crit}}$ \cite{6–10}. Roughly speaking, the Higgs self-coupling constant must be positive at the energies up to the inflationary scale, leading to this constraint. In numbers \cite{11–13},

$$M_{\text{crit}} = \left[ 129.6 + \frac{y_t - 0.0361}{0.0058} \times 2.0 - \frac{\alpha_s - 0.1184}{0.0007} \times 0.5 \right] \text{GeV}. \hspace{1cm} (2)$$

Here $y_t$ is the top Yukawa coupling in $\overline{\text{MS}}$ renormalisation scheme taken at $\mu_t = 173.2$ GeV\textsuperscript{1} $y_t \equiv y_t(\mu_t)$ and $\alpha_s$ is the QCD coupling at the $Z$-boson mass. Thanks to complete two-loop computations of \textsuperscript{13} and three-loop beta functions for the SM couplings found in \textsuperscript{14–19} this formula may have a very small theoretical error, 0.07 GeV, with the latter number coming from an “educated guess” estimates of even higher order terms (see the discussion in \textsuperscript{11} and more recently in \textsuperscript{20}). The main uncertainty in determination of $M_{\text{crit}}$ is associated with experimental and theoretical errors in determination of $y_t$ from available data. Accounting for those, the value of $M_{\text{crit}}$ is about 2 standard deviations from the mass of the Higgs boson observed experimentally at CERN \textsuperscript{21,22}.

The determination of the inflationary indexes accounting for radiative corrections is somewhat more subtle and depends on the way the quantum computations are done

\textsuperscript{1}For precise relation between $\mu_t$ and the pole top mass $m_t$ see \textsuperscript{11}–\textsuperscript{13} and references therein.
(the SM with gravity is non-renormalizable, what introduces the uncertainty). In [3] we formulated the natural subtraction procedure (called “prescription I”) which uses the field independent subtraction point in the Einstein frame (leading to scale-invariant quantum theory in the Jordan frame for large Higgs backgrounds) and computed \( n_s \) and \( r \) for the Higgs masses that exceeded \( M_{\text{crit}} \) by just a small amount of few hundreds of MeV. We have shown that the values of \( n_s \) and \( r \) are remarkably stable in this domain and coincide with the tree estimates. However, we did not analyse what happens in the close vicinity of the critical point. Partially, this has been studied in [23], but the peculiar inflationary behaviour found in the present work was not discussed in [24].

The aim of the present paper is to study the behaviour of the inflationary indexes close to the critical point. In what follows we will use the prescription I. We expect to have qualitatively the same results in the prescription II, though the numerical values will be somewhat different. We will see that \( n_s \) and \( r \) acquire a strong dependence on the mass of the Higgs boson and the mass of the top quark. Thus, if the cosmological observations will show that one or both indexes do not coincide with those given by the tree analysis, they will pin down both the Higgs and the top quark masses.

2. The critical point

The behaviour of the scalar self-coupling constant \( \lambda \) as a function of the MS parameter \( \mu \) (energy) in the SM is very peculiar. If the mass of the top quark and of the Higgs boson are varied within their experimentally allowed intervals, it can be approximated in the region of Planck energies \( (M_P = 2.44 \times 10^{18} \text{ GeV}) \) with very good accuracy as follows:

\[
\lambda(\mu) = \lambda_0 + b \left( \frac{\mu}{qM_P} \right)^2 ,
\]

where \( \lambda_0 \), \( q \) and \( b \) are some functions of the top quark (pole) mass, Higgs mass, and the strong coupling constant \( \alpha_s \). These functions can be found from the analysis of the renormalisation group running for \( \lambda \). The fitting formulas are given below:

\[
\lambda_0 = 0.003297((M_h - 126.13) - 2(m_t - 171.5) + 0.555(\alpha_s - 0.1184)/0.0007),
\]

\[
q = 0.3 \exp(0.5(M_h - 126.13) - 0.03(m_t - 171.5)),
\]

\[
b = 0.0002292 - 1.12524 \times 10^{-6}((M_h - 126.13) - 1.75912(m_t - 171.5)),
\]

where \( M_h \) and \( m_t \) are to be taken in GeV.

It happens that \( \lambda_0 \) is small, \( \lambda_0 \ll 1 \) and \( q \) is of the order of one. To put it in words, both the value of \( \lambda \) and of its beta-function, \( \beta_\lambda = \mu \partial \lambda/\partial \mu \), are close to zero near the Planck scale. It is this fact that changes the behaviour of the inflationary indexes, as is demonstrated below.

The renormalisation group improved effective potential in the Einstein frame with an accuracy sufficient for the present discussion can be written as follows [9]:

\[
U(\chi) \approx \frac{\lambda(\mu)M_P^4}{4\xi^2} \left( 1 - e^{-\frac{2\chi}{\sqrt{\xi M_P}}} \right)^2 ,
\]

where \( \chi \) is the canonically normalised scalar field related to the original Higgs field by a known transformation [1], and the parameter \( \mu \) is related to the field \( \chi \) as

\[
\mu^2 = \kappa^2 \frac{[\mu(\chi)]^2 M_P^2}{2 \xi(\mu)} \left( 1 - e^{-\frac{2\chi}{\sqrt{\xi M_P}}} \right) .
\]

Here \( \kappa \) is some constant of the order of one. The expression (5) is valid for \( \xi > 1 \), and the scale dependence of \( \xi \) can be neglected.

Let us now consider the change of the form of the potential if \( \lambda_0 \), \( q \) and \( \xi \) are varying. For \( \lambda_0 \gg b/16 \) the potential is a rising function of the field \( \chi \), realizing the “tree” Higgs inflation (see Fig. 4 blue curve). If \( \lambda_0 = b/16 \), a new feature appears: the first and the second derivatives of the potential are equal to zero at some point (see Fig. 4 red curve). For \( \lambda_0 < b/16 \) but still close to \( b/16 \) we get a wiggle on the potential, which is converted into a maximum for somewhat smaller \( \lambda_0 \) (see Fig. 4 brown line). Decreasing \( \lambda_0 \) even further leads to the unstable electroweak vacuum, Fig. 5 (green line). Clearly, the necessary condition for inflation to happen in the slow-roll regime is to have \( dV(\chi)/d\chi > 0 \) for all \( \chi \), i.e. the absence of a wiggle. For \( \lambda_0 \gg b/16 \) all the potentials are very much similar, leading to the independence of inflationary indexes on the parameters, while if \( \lambda_0 \) is close to \( b/16 \), the form of the potential changes, and the dependence of \( r \) and \( n_s \) on \( \lambda_0 \) and \( q \) (and, therefore, on \( M_h \) and \( y_t \)) shows up.

The parameter \( q \) controls the value of \( \chi \) where the wiggle would appear for \( \lambda_0 = b/16 \); the parameter \( \lambda_0 - b/16 \) tells how close we are to the appearance of the feature, while a combination of \( \lambda_0 \) and \( \xi \) determines the asymptotic of the potential at large \( \chi \).

3. The inflationary indexes

Once the potential is known it is straightforward to determine inflationary indexes. We use exactly the same procedure and the same equations as in our previous work.
The results are summarized in Figs. 2 and 3 where we show the dependence of $n_s$, $dn_s/d\ln k$, and $r$ close to the critical point along the line determined by eq. (2) ($\alpha_s$ is set to 0.1184, and dependence on it can be easily reproduced as far as it always enters in the same combination with $m_t$ as in (2)).

The most important result is that the region in the $M_h-m_t$ plane where the indexes deviate considerably from the tree values is limited both in $M_h$ and $m_t$ directions. In other words, if at least one of the indexes is away from tree Higgs inflation prediction, the masses of the Higgs boson and of the top quark are pinned down. And this may appear to be the case if the results of [26] will be confirmed.

In Fig. 5 we present the dependence of the required non-minimal coupling on $M_h$ and $m_t$.

A very interesting feature of the inflation near $M_{\text{crit}}$ is the drastic decrease of the necessary non-minimal coupling $\xi$ down to a number of the order of ten. The large value of $\xi$, necessary for the Higgs inflation far from the critical point, effectively introduces a new strong-coupling threshold $\Lambda \sim M_P/\xi$ well below the Planck scale, if the scattering of the SM particles is considered around the EW vacuum [27, 28]. Though this fact does not invalidate the self-consistency of the Higgs inflation [29, 30] which occurs at large Higgs fields, it requires the UV completion of the SM or self-healing of high energy scattering [31] at energies much smaller than the Planck scale. The Higgs inflation at the critical point does not require any new cutoff scale, different from the Planck scale.

The evolution of the Universe after the Higgs inflation at the critical point is different from that for the case $\xi \gg 1$. If $\xi \gg 1$, the Universe after inflation is “matter dominated” due to oscillations of the Higgs field. The

Figure 1: The schematic change of the form of the effective potential depending on $\lambda_0$. For better visibility the values of $\xi$ are different for different lines. The horizontal axis corresponds to the canonically normalized field $\chi$, the vertical axis to the effective potential, all in Planck units.

Figure 2: The dependence of the inflationary indexes $n_s$ and $r$ on the Higgs boson and top quark masses.

Figure 3: The dependence of the running of the scalar spectral index $dn_s/d\ln k$ on the Higgs boson and top quark masses.

Figure 4: The form of the effective potential which leads to $r = 0.1$, $n_s = 0.96$.
transition to the radiation dominated Universe occurs due to particle production after some time, but not later than after $O(\xi/2\pi)$ oscillations \cite{4} \cite{5}. For $\xi \sim 10$ we have the radiation-dominated epoch right after inflation is finished.

4. Conclusions

The Higgs inflation for $M_h > M_{\text{crit}}$ is a predictive theory for cosmology, as the values of the inflationary indexes are practically independent of the SM parameters. Near the critical point (which is a region of parameters disjoint form the large mass regime) the situation completely changes, and we get a strong dependence of $n_s$ and $r$ on the precise values of the masses of the top quark and the Higgs boson. In this regime the Higgs inflation becomes a predictive theory for particle physics, as any deviation of inflationary indexes from the tree values tells that we are at the critical point, fixing thus the masses of the top quark and the Higgs boson. It is amazing that a possible detection of large tensor-to-scalar ratio $r$ in \cite{20} predicts the top quark and Higgs boson masses close to their experimental values. Another observation is that the running of the spectral index becomes relatively large and negative.

We conclude with a word of caution. All results here are based on the assumption of the validity of the SM up to the Planck scale. If this hypothesis is removed, the Higgs inflation remains a valid cosmological theory, but its predictability is lost even far from the critical point. For example, the modification of the kinetic term of the Higgs field at large values of $H$, leads to a considerable modification of $r$ \cite{32} \cite{33}. The change of the structure of the Higgs-gravity interaction to, for instance, 

$$M_P^2 R \sqrt{1 + \xi |H|^2/M_P^2},$$

will make the potential in the Einstein frame quadratic with respect to the field $\chi$ and thus would modify $r$ and $n_s$, making them the same as in the chaotic inflation with free massive scalar field.

While this paper was in preparation, the article \cite{35} appeared. Our results are in qualitative agreement with this work.

The authors would like to thank CERN Theory Division, where this paper was written, for hospitality. We thank Dmitry Gorbunov for helpful comments. The work of M.S. is supported in part by the European Commission under the ERC Advanced Grant BSMOXFORD 228169 and by the Swiss National Science Foundation.

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