Model-independent constraints on $r^{-3}$ extra-interactions from orbital motions  

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Constraints on long-range power-law modifications $U_{\text{pert}} \propto r^{-3}$ of the usual Newtonian gravitational potential $U_N \propto r^{-1}$ are inferred from orbital motions of well known artificial and natural bodies. They can be interpreted in terms of a characteristic length $\ell$ which may be identified with, e.g., the anti-de Sitter (AdS) radius of curvature $\ell$ in the Randall-Sundrum (RS) braneworld model, although this is not a mandatory choice. Our bounds, complementary to those from tabletop laboratory experiments, do not rely upon more or less speculative and untested theoretical assumptions, contrary to other long-range RS tests proposed in astrophysical scenarios in which many of the phenomena adopted may depend on the system’s composition, formation and dynamical history as well.

Independently of the interpretation of $\ell$, the perihelion precession of Mercury and its radiotechnical ranging from the Earth yield $\ell \lesssim 10 - 50$ km. Tighter bounds come from the perigee precession of the Moon, from which it can be inferred $\ell \lesssim 500 - 700$ m. The best constraints ($\ell \lesssim 5$ m) come from the Satellite-to-Satellite Tracking (SST) range of the GRACE A/B spacecrafts orbiting the Earth: proposed follow-on of such a mission, implying a sub-nm $s^{-1}$ range-rate accuracy, may constrain $\ell$ at $\sim 10$ cm level. Weaker constraints come from the double pulsar system ($\ell \lesssim 80 - 100$ km) and from the main sequence star S2 orbiting the compact object in Sgr A* ($\ell \lesssim 6.2 - 8.8$ AU). Such bounds on the length $\ell$, which must not necessarily be identified with the AdS radius of curvature of the RS model, naturally translate into constraints on an, e.g., universal coupling parameter $K$ of the $r^{-3}$ interaction. GRACE yields $K \lesssim 1 \times 10^{16}$ m$^3$ s$^{-2}$.

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I. INTRODUCTION

In this paper, we deal with power-law modifications of the usual inverse-square law [1,2].

Extra dimensions, arising in string theory, supergravity, M-theory [3,4], and string inspired higher dimensional theories such as the braneworld models [4,5,7,8], have recently gained increasing importance in physics in the context of the search for a quantum-gravity theory. In particular, large, non-compactified extra dimensions could potentially solve the long-lasting hierarchy problem [4,5,7,8]. Indeed, if the standard model of particles and fields is restricted only on a (3+1)-dimensional brane, whereas gravity is allowed to propagate in the higher-dimensional spacetime, the effective Planck scale in the four-dimensional spacetime can be made significantly larger than the electroweak scale, matching the experimental requirements.

In the braneworld model by Randall and Sundrum (RS hereafter) [3], our usual four-dimensional spacetime is a brane, which the standard model fields are constrained to, embedded in a five-dimensional anti-de Sitter (AdS) spacetime. In it, the fifth spatial dimension can be infinite, with an AdS curvature scale $\ell$. Indeed, the RS model circumvents the need of compactifying all but the three observed spatial dimensions by including a bound state of the massless graviton on the brane [3] resulting from the curvature, rather than the size, of the extra dimension. At distances $r \gg \ell$, the RS model implies a correction $U_{\text{RS}}$ to the Newtonian gravitational potential of a body of mass $M$ at second post-Newtonian order (2PN). It is given by

$$U_{\text{RS}} = -k \frac{GM}{r} \left( \frac{\ell}{r} \right)^2,$$

where $G$ is the Newtonian constant of gravitation, and $k$ can assume different values depending on the schemes of regularization adopted [3]. E.g., it can be $k = 1$ [3], $k = 1/2$ [3], and $k = 2/3$ [3]. The occurrence of a correction to the Newtonian potential of the form of Eq. (1), which, however, is not necessarily limited to the RS model, being common to a wide class of power-law interactions [2], is important since, although it is only gravity that feels the presence of the extra dimensions, Eq. (1) allows for detectable effects on our (3+1)-brane that can be used in constraining the properties of the bulk. For upper limits on the brane parameter of other braneworld models, see, e.g., [12,13].

Several large-scale tests of the RS model [3] have been proposed so far in astrophysical scenarios; they claimed bounds on $\ell$ at $\sim 1 - 10$ $\mu$m level, which is the same order of magnitude reached in laboratory-scale experiments [14,15]. Anyway, such constraints typically depend on the particular interpretation of astrophysical observations, and suffer from large systematic effects whose accurate knowledge is often lacking. Moreover, they are often quite model-dependent in the sense that they heavily rely upon theoretical assumptions which are still speculative since they have not yet been tested independently with a variety of different phenomena, or have not yet been
directly tested at all. In our opinion, it is true also for those tests \cite{16, 19} requiring the least amount of information like, e.g., the evaporation of black holes \cite{21}. Indeed, they are based on the application of a concept like the antide Sitter space/conformal field theory (AdS/CFT) duality \cite{22} in braneworld gravity models that are asymptotically AdS (such as the RS models). Moreover, the consequent theoretical prediction for the black hole evaporation time \cite{22, 21} depends only on assumptions regarding the braneworld model and, in particular, on the validity and implementation of the AdS/CFT correspondence in view of the “no-hair” conjecture \cite{27, 28}, which is itself speculative and still awaits independent observational checks \cite{27, 28}. Suffice it to say that the basis of the previously cited calculation for the black hole evaporation time \cite{22, 21} has been recently challenged in Ref. \cite{29}. In addition to such theoretical considerations at fundamental level, it must also be remarked that the astrophysical phenomena themselves used to constrain $\ell$, like the orbital evolution of black-hole X-Ray binaries or the behavior of black holes in extragalactic clusters, are not lacking of uncertainties, both from a theoretical and observational point of view. E.g., they may crucially depend on the composition, formation and dynamical history of the systems considered. Last but not least, black holes may well not exist at all \cite{30, 31}; e.g., even in the case of the compact object in Sgr A*, there are not yet direct evidences that its $\sim 10^8M_\odot$ mass is actually concentrated within its Schwarzschild radius $R_s = 0.084$ AU \cite{32}. A signature for the absence of event horizons was even looked for by the authors of Ref. \cite{33}. In conclusion, to date, a definite proof for the existence of Kerr black holes is still lacking despite a wealth of observational evidence \cite{34}. Moreover, from a broader point of view, in correctly assessing the relevance of the laboratory-based tests of long-range modified models of gravity it should be mentioned that some authors \cite{35, 36} pointed out the necessity limited to the RS model \cite{5}, being valid also for other theoretical schemes yielding power-law interactions $\propto r^{-3}$ \cite{14, 35, 32}.

II. RS LONG-TERM ORBITAL EFFECTS AND COMPARISON WITH THE OBSERVATIONS

A. Analytical calculation of the secular precession of the pericenter

The long-period effects caused by Eq. (1) on the orbital motion of a test particle of mass $m$ can be computed perturbatively by adopting the Lagrange equations for the variation of the osculating Keplerian elements \cite{43}: their validity has been confirmed in a variety of independent phenomena. Generally speaking, they imply the use of a perturbing function $R$ which is the correction $U_{\text{pert}}$ to the standard Newtonian monopole term. In the case $U_{\text{pert}} = U_{\text{RS}}$, the average over one orbital revolution of the perturbing function $R$ is straightforwardly obtained by using the true anomaly $f$ as fast variable of integration: it is

$$\langle R \rangle = -\frac{GMk\ell^2}{a^3(1-e^2)^{3/2}}, \quad (2)$$

where $a$ is the semimajor axis and $e$ is the eccentricity of the test particle’s orbit. From Eq. (2) and the Lagrange equation for variation of the longitude of pericenter $\varpi$ \cite{44}

$$\langle d\varpi \rangle dt = -\frac{1}{n_b a^2} \left\{ \left[ \frac{(1-e^2)^{1/2}}{e} \right] \frac{\partial \langle R \rangle}{\partial e} + \frac{\tan(1/2)}{(1-e^2)^{1/2}} \frac{\partial \langle R \rangle}{\partial I} \right\}, \quad (3)$$

it turns out that $\varpi$ experiences a secular precession given by

$$\langle d\varpi \rangle dt = \frac{3}{a^7/2(1-e^2)^2} \langle GM \rangle^{1/2} k\ell^2. \quad (4)$$

In Eq. (4), $n_b = \sqrt{GM/a^3}$ is the Keplerian mean motion and $I$ is the inclination of the orbital plane to the reference $(x, y)$ plane. The longitude of pericenter $\varpi = \Omega + \omega$ is a “dogleg” angle since it is the sum of the longitude of the ascending node $\Omega$, which is an angle in the reference $(x, y)$ plane from a reference $x$ direction to the intersection of the orbital plane with the $(x, y)$ plane itself (the line of the nodes), and of the argument of pericenter $\omega$, which is an angle counted in the orbital plane from the order of $\ell \sim 1 \mu m$ from the event rate of stellar black holes inspiraling gravitationally into supermassive black holes, and of $\ell \lesssim 5 \mu m$ from the observation of individual galactic binaries containing a stellar mass black hole. In Sec. 11 we will also use the well known, and extensively studied, double pulsar binary system and the main sequence S2 star orbiting the compact object in Sgr A*. Finally, we stress that our results are not necessarily limited to the RS model \cite{5}, being valid also for other theoretical schemes yielding power-law interactions $\propto r^{-3}$ \cite{14, 35, 32}.
line of the nodes to the point of closest approach, usually
dubbed pericenter. The precession of Eq. (1), which is
an exact result in the sense that no a-priori assumptions
on & were made, agrees with the one obtained by Adkins
and McDonnel in Ref. [43] with a more cumbersome cal-
culation. To facilitate a comparison between such two
results, we note that, in general, the authors of Ref. [45]
work out the perihelion advance per orbit $\Delta \ell$; it corre-
sponds to $\langle \dot{\omega} \rangle P_0$, where $P_0 = 2\pi/n_0$ is the orbital period.
Moreover, in the potential energy $V(r) = \alpha_{-(j+1)}r^{-(j+1)}$
it must be posed $j = 2$ and $\alpha_{-3} \to -GMmk\ell^2$, while in
$\Delta \rho_0(\ell + 1)$ of Eq. (38) in Ref. [15] it must be
set $L \to a(1 - e^2)$, $\chi_2(e) = 6$. With such replacements,
it can be shown that the advance per orbit of Eq. (38)
in Ref. [45] corresponds just to our precession in Eq. (4).
Moreover, it turns out that the analytical result of
Eq. (4) is confirmed by a numerical integration of the
equations of motion for Mercury with, say, $L = 10^{-6}$ AU
and $k = 1/2$: both yield 5.4 milliarcseconds per century
(mas cty$^{-1}$ hereafter). The choice of the numerical value
adopted for $L$ is purely arbitrary, being motivated only
by the need of dealing with relatively small numbers.

B. Constraints from solar system planetary orbital
precession

The corrections $\Delta \dot{\omega}$ to the standard Newtonian-
Einsteinian secular precessions of the longitudes of the
perihelia are routinely used by independent teams of as-
tonomers [46, 47] as a quantitative measure of the maxi-
um size of any putative anomalous effect allowed by the
currently adopted mathematical models of the standard
solar system dynamics fitted to the available planetary
observations. Thus, $\Delta \dot{\omega}$ can be used to put constraints
on the parameters like $\ell$ entering the exotic models one
is interested in. From Eq. (4) it turns out that the tightest
constraints come from Mercury, which is the innermost
planet with $a = 0.38$ AU, for $k = 1$.

Fienga et al. [48], who used also a few data from the
three flybys of MESSENGER in 2008-2009, released an
uncertainty of 0.6 mas cty$^{-1}$ for the perihelion precession
of Mercury, so that it is $\ell \lesssim 34 - 48$ km for $k = 1 - 1/2$.
The uncertainty in the pre-MESSENGER Mercury’s per-
helion extra-precession by Pitjeva [49] is about one order of
magnitude larger (5 mas cty$^{-1}$).

Slightly tighter constraints on $\ell$ come from the inter-
planetary Earth-Mercury ranging. Indeed, according
to Table 1 of Ref. [47], the standard deviation $\sigma_{\Delta \rho}$ of
the Mercury range residuals $\Delta \rho$, obtained with the IN-
POP10a ephemerides and including also three Mercury
MESSENGER flybys in 2008-2009, is as large as 1.9 m.
A numerical integration of the equations of motion of
Mercury and the Earth yields a RS range signal with the
same standard deviation for $\ell \lesssim 13 - 18$ km ($k = 1 - 1/2$).
Figure 1 displays the case $k = 1/2$. It turns out that the
residuals of right ascension (RA) and declination (DEC)
in Ref. [47] yield much less tight constraints.

As far as natural bodies of the solar system are con-
cerned, the Moon yields better results. Its orbit is ac-
curately reconstructed with the Lunar Laser Ranging
(LLR) technique [48] since 1969; Figure B-1 of Ref.
[49] shows that the residuals of the Earth-Moon range
are at a cm-level since about 1990. The secular
precession of the lunar perigeoc is known with an accuracy
of about 0.1 mas yr$^{-1}$ [50, 51], so that Eq. (4) yields
$\ell \lesssim 524 - 741$ m ($k = 1 - 1/2$).

C. Constraints from the GRACE spacecraft
orbiting the Earth

Remaining within the solar system, tighter constraints
can be obtained from selected spacecrafts orbiting the
Earth. The Gravity Recovery and Climate Experiment
(GRACE) mission [52], jointly launched in March 2002
by NASA and the German Space Agency (DLR) to map
the terrestrial gravitational field with an unprecedented
accuracy, consists of a tandem of two spacecrafts mov-
ing along low-altitude, nearly polar orbits continuously
linked by a Satellite to Satellite Tracking (SST) mi-
crowave K-band ranging (KBR) system accurate to bet-
ter than 10 µm (biased range $\rho$) [53] and 1 µm s$^{-1}$ (range-
rate $\dot{\rho}$) [53, 54]. Studies for a follow-on of GRACE show
that the use of a interferometric laser ranging system may
push the accuracy in the range-rate to a $\sim 0.6$ nm s$^{-1}$
level. A numerical integration of the equations of motion
for GRACE A/B, including also the mismodelled signal
of the first nine zonal harmonics of geopotential [43],
according to the global Earth gravity model GOCC01S [55],
shows that the SST range is more effective than the SST
range-rate in constraining the RS parameter for which
it holds $\ell \lesssim 5$ m. Figure 2 depicts the numerically in-
tegrated GRACE SST range signal due to Eq. (1) and
the aforementioned mismodeled zonals. It can be shown
that a sub-nm s$^{-1}$ level of accuracy in the GRACE SST
range-rate would imply the possibility of constraining $\ell$
down to $\sim 10$ cm level.

D. Constraints from the double pulsar and Sgr A$^\ast$

Constraints comparable with the planetary ones can be
obtained from the periastron of the double pulsar PSR
J0737-3039A/B system [59, 60]. Indeed, the semima-
jor axis of its relative orbit amounts to just $a = 0.006$
AU [61]. The present-day accuracy in measuring the
secular precession of the periastron is $6.8 \times 10^{-4}$
degree per year (deg yr$^{-1}$ in the following) [61]; thus, a
straightforward application of it to Eq. (1) would give
$\ell \lesssim 11 - 16$ km ($k = 1 - 1/2$). Actually, the larger
uncertainty in the theoretical expression of the general
relativistic 1PN periastron precession must be taken into
account as well. It is as large as 0.03 deg yr$^{-1}$ [62], so
that it yields $\ell \lesssim 80 - 112$ km ($k = 1 - 1/2$).

The perinigricon of the S2 star, orbiting in 15.98 yr the
compact object hosted in Sgr A* with $M = 4 \times 10^6 M_\odot$ at $\langle r \rangle = 1433$ AU from it [64], yields much weaker constraints on the asymptotic AdS curvature. Indeed, an accuracy of 0.05 deg yr$^{-1}$ can be inferred for the perinigricon precession of S2: thus, $\ell \lesssim 6.2 - 8.8$ AU.

III. MODEL-INDEPENDENT CONSTRAINTS ON $r^{-3}$ INTERACTIONS

The validity of the previous results is not necessarily limited just to the RS braneworld model. Indeed, they are, in fact, quite model-independent in the sense that they hold for any long-range modification $U_{\text{pert}} \propto r^{-3}$ of the usual Newtonian potential. In particular, $\ell$ must not necessarily be identified with the AdS radius of curvature of the RS braneworld model.

By assuming that the putative, new interaction does not depend on the specific matter distribution generating the gravitational field, a universal parameter can be
introduced with the replacement

\[ kGM \ell^2 \to K, \quad [K] = L^5 \, T^{-2}. \]  

(5)

Thus, the bounds on \( \ell \) of Section 11B Section 11D straightforwardly translate into constraints on \( K \) itself.

It turns out that the tightest bounds come from the Earth-GRACE system yielding \( K \leq 1 \times 10^{16} \, m^2 \, s^{-2} \).

IV. SUMMARY AND CONCLUSIONS

Table II and Table III resume our findings.

As far as the \( \ell^2 \) form of the \( r^{-3} \) extra-potential is concerned, Table II tells us that, at the \( m \) level, come from the Satellite-to-Satellite Tracking ranging between the GRACE spacecrafts.

If we interpret them in terms of the RS model, from a general point of view, we find not entirely adequate arguing that Earth-based laboratory-scale tests of several modified models of gravity are in all respects superior since they can deliver much tighter constraints. Indeed, apart from the basic fact that it is important to scrutiny a theoretical paradigm in different, complementary scenarios, some authors also pointed out that there are theoretical reasons for wanting to test such foundational issues of gravity in space since extra gravitational degrees of freedom could have an environmental dependence. Moreover, about the seemingly superiority of certain constraints of the RS model inferred from some astrophysical systems, often they rely upon more or less speculative and untested theoretical assumptions, and the phenomena adopted may depend on the system’s composition, formation and dynamical history as well. Thus, we reiterate the importance of finding solid, well-understood means of constraining \( \ell \) like those proposed here. We also point out that the same method introduced here could yield another factor of \( 10^2 \) improvement, constraining \( \ell < 10 \, cm \), provided that the proposed improvement to satellite-to-satellite tracking is implemented for the follow-on of the GRACE mission.

Last but not least, we remark that, actually, our analysis is not necessarily limited to the RS model since it is valid for whatsoever theoretical scheme predicting \( r^{-3} \) corrections to the Newtonian potential. In this respect, the occurrence of a characteristic length scale \( \ell \) is common to a wide class of power-law interactions, so that it should not necessarily be thought of as the AdS radius of curvature in the RS model. Thus, the bounds of Table II have a wider range of applicability. Table III shows that GRACE yields the tightest bounds on a universal coupling parameter \( K \) which is constrained to \( \leq 10^{16} \, m^2 \, s^{-2} \). It could be pushed down to a \( 10^{12} \, m^2 \, s^{-2} \) level by GRACE follow-on.

[1] E. Fischbach, D. E. Krause, V. M. Mostepanenko and M. Novello, Phys. Rev. D 64, 075010 (2001).
[2] E. Adelberger, B. R. Heckel and A. E. Nelson, Ann. Rev. Nucl. Part. Sci. 53, 77 (2003).
[3] J. Polchinski, String Theory (Cambridge University Press, Cambridge, England, 1998), Vols. I and II.
[4] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999).
[5] L. Randall and R. Sundrum, Phys. Rev. D 62, 024012 (2000).
[6] T. Shiromizu, K.-I. Maeda, and M. Sasaki, Phys. Rev. D 59, 5038 (1999).
[7] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B 429, 263 (1998).
[8] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Lett. B 436, 257 (1998).
[9] E. Jung, S. H. Kim, and D. K. Park, Nucl. Phys. B 669, 306 (2003).
[10] A. O. Barvinsky, Phys. Usp. 48, 6 (2005).
[11] J. Garriga and T. Tanaka, Phys. Rev. Lett. 84, 2778 (2000).
[12] A. A. Abdusambarov and B. J. Ahmedov, Phys. Rev. D 81, 044022 (2010).
[13] A. I. Mamadjanov, A. A. Hakimov and S. R. Tojiev, Mod. Phys. Lett. A 25, 243 (2010).
[14] E. G. Adelberger, B. R. Heckel, S. Hocedl, C. D. Hoyle, D. J. Kapner, and A. Upadhye, Phys. Rev. Lett. 98, 131104 (2007).
[15] A. Kudinova, M. Eingorn and A. Zhuk, Odessa Astromerical Publications, 24, 46 (2011).
[16] D. Psaltis, Phys. Rev. Lett. 98, 181101 (2007).
[17] O. Y. Gnedin, T. J. Maccarone, D. Psaltis, and S. E. Zepf, Astrophys. J. Lett. 705, L168 (2009).
[18] T. Johannsen, D. Psaltis, and J. E. McClintock, 691, 997 (2009).
[19] T. Johannsen, Astron. Astrophys. 507, 617 (2009).
[20] J. Traschen, in Mathematical Methods in Physics, edited by A. Bytsenko and F. L. Williams (World Scientific, Singapore, 2000), p. 100.
[21] O. Aharon, S. S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, Phys. Rept. 323, 183 (2000).
[22] R. Emparan, J. Garcia-Bellido, and N. Kaloper, J. High Energy Phys. 1 (2003) 79.
[23] T. Tanaka, Prog. Theor. Phys. Suppl. 148, 307 (2003).
[24] R. Emparan, A. Fabbi, and N. Kaloper, J. High Energy Phys. 08 (2002) 042.
[25] P. T. Chrusciel, Contemp. Math. 170, 23 (1994).
[26] M. Heusler, Living Rev. Rel. 1, 6 (1998).
[27] C. M. Will, Astrophys. J. 674, L25 (2008).
[28] T. Johannsen, Adv. Astron. 2012, 486750 (2012).
[29] A. L. Fitzpatrick, L. Randall, and T. Wiseman, J. High Energy Phys. 11, 033 (2006).
[30] P. Ball, Nature doi:10.1038/news050328-8 (2005).
[31] G. Chapline, in Proceedings of the 22nd Texas Symposium on Relativistic Astrophysics at Stanford, eConf: C041213, edited by P. Chen, E. Bloom, G. Madejski, and V. Patrosian (Stanford University, Stanford, 2005) p. 101.
TABLE I. Summary of the upper bounds on $\ell$ inferred from the various scenarios discussed in the paper. In each space of the row for $\ell_{\text{max}}$, the first number from the left refers to $k = 1$, while the second one refers to $k = 1/2$.

| $\ell_{\text{max}}$ | Mercury ($\Delta \dot{\sigma}$) | Mercury ($\Delta \sigma$) | Moon ($\Delta \dot{\sigma}$) | GRACE ($\Delta \sigma$) | PSR J0737-3039 A/B ($\Delta \dot{\sigma}$) | S2-Sgr A* ($\Delta \dot{\rho}$) |
|----------------------|--------------------------------|--------------------------|--------------------------|-----------------------|--------------------------------|-------------------------|
| 34–48 km             | 34–48 km                       | 13–18 km                 | 524–741 m                | 5 m                   | 80–112 km                      | 6–8.8 AU                |

TABLE II. Summary of the upper bounds, in m$^3$ s$^{-2}$, on the model-independent parameter $K$ of a $c \propto r^{-3}$ modification of the $r^{-1}$ Newtonian potential inferred from the various scenarios discussed in the paper. They have been obtained from the values of $\ell_{\text{max}}$ in Table I with the replacement $kGM\ell^2 \rightarrow K$. $K$ has been assumed universal.

| $K_{\text{max}}$ | Mercury ($\Delta \dot{\sigma}$) | Mercury ($\Delta \sigma$) | Moon ($\Delta \dot{\sigma}$) | GRACE ($\Delta \sigma$) | PSR J0737-3039 A/B ($\Delta \dot{\sigma}$) | S2-Sgr A* ($\Delta \dot{\rho}$) |
|------------------|--------------------------------|--------------------------|--------------------------|-----------------------|--------------------------------|-------------------------|
| $1.5 \times 10^6$ | $2 \times 10^6$                 | $1 \times 10^{10}$       | $1 \times 10^{10}$       | $1 \times 10^{10}$     | $5 \times 10^{10}$              | $5 \times 10^{10}$       |

[32] A. Balbi, *Elementi di Astrofisica 2, Lezione 13* (Università degli studi di Roma “Tor Vergata”, Rome, 2010), p. 18.

[33] J. Barbieri and G. Chapline, Phys. Lett. B 709, 114 (2012).

[34] D. Psaltis, *Compact Stellar X-Ray Sources* (Cambridge University Press, Cambridge, 2006).

[35] J. Khouri and A. Veltman, Phys. Rev. Lett. 93, 171104 (2004).

[36] T. Clifton, P.G. Ferreira, A. Padilla and C. Skordis, Phys. Rept. 513, 1 (2012).

[37] S. T. McWilliams, Phys. Rev. Lett. 104, 141601 (2010).

[38] V. M. Mostepanenko and I. Yu. Sokolov, Phys. Lett. A 125, 405 (1987).

[39] F. Ferrer and J. A. Grifols, Phys. Rev. D 58, 096006 (1998).

[40] F. Ferrer and M. Nowakowski, Phys. Rev. D 59, 096006 (1999).

[41] B.A. Dobrescu and I. Mocioiu, J. High En. Phys. 11, 005 (2006).

[42] E.G. Adelberger, J.H. Gundlach, B.R. Heckel, S. Hoedl and S. Schlamminger, *Dynamic Planet*, edited by H. Dittus, A. Fienga, J. Laskar, P. L. Bender, J. E. Faller, X. X. Newhall, R. L. Ricklefs, J. G. Ries, P. J. Shelus, C. Veillet, A. L. Whipple, J. R. Want et al., Science 265, 482 (1994).

[43] W.M. Folkner, J.G. Williams, and D.H. Boggs, *The planetary and lunar ephemerides DE 421, Memorandum IOM 343R-08-003* (Jet Propulsion Laboratory, California Institute of Technology, 2008).

[44] J. Müller, M. Schneider, M. Soffel and H. Ruder, *Proc. of the International Astronomical Union, 5*, edited by S. A. Klioner, P. K. Seidelmann, and J. K. Stenbaek-Nielsen (Springer, Berlin, 2009), p. 170.

[45] A. Fioni, J. Laskar, P. Kuchynka, H. Manche, G. Desvignes, M. Gastineau, I. Cognard, A. G. Lyne, and B. M. Soffel (Cambridge University Press, Cambridge, 2009), p. 170.

[46] J.O. Dickey, P.L. Bender, J.E. Faller, X.X. Newhall, R.L. Ricklefs, J.G. Ries, P.J. Shelus, C. Veillet, A.L. Whipple, J.R. Want et al., *Science* 265, 482 (1994).

[47] W. M. Folkner, J. G. Williams, and D. H. Boggs, *The planetary and lunar ephemerides DE 421, Memorandum IOM 343R-08-003* (Jet Propulsion Laboratory, California Institute of Technology, 2008).

[48] J. Müller, M. Schneider, M. Soffel and H. Ruder, *Proc. of the International Astronomical Union, 5*, edited by S. A. Klioner, P. K. Seidelmann, and J. K. Stenbaek-Nielsen (Springer, Berlin, 2009), p. 170.