New axially symmetric Yang-Mills-Higgs solutions with negative cosmological constant

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Abstract

We construct numerically new axially symmetric solutions of $SU(2)$ Yang-Mills-Higgs theory in $(3 + 1)$ anti-de Sitter spacetime. Two types of finite energy, regular configurations are considered: multimonopole solutions with magnetic charge $n > 1$ and monopole-antimonopole pairs with zero net magnetic charge. A somewhat detailed analysis of the boundary conditions for axially symmetric solutions is presented. The properties of these solutions are investigated, with a view to compare with those on a flat spacetime background. The basic properties of the gravitating generalizations of these configurations are also discussed.

1 Introduction

The study of topologically stable monopole [1, 2, 3, 4] solutions to the $SU(2)$ Yang-Mills-Higgs (YMH) equations with adjoint representation Higgs field, in flat spacetime, is a subject of long standing interest. Two types of solutions to this system are known: multimonopoles (MM) and monopole–antimonopole (MA) chains.

The first type comprise topologically stable multimonopole solutions with topological charge $n$, which for $n \geq 2$ cannot be spherically symmetric [5] and possess at most axial symmetry. For vanishing Higgs self interaction potential, these are solutions of first order Bogomol’nyi equations and the axially symmetric multimonopole solutions are minimal energy and topologically stable. In the presence of a Higgs potential, i.e. when the Higgs coupling constant $\lambda > 0$, like monopoles repel [6, 7] so we would expect that the solutions of these are saddle points of the energy functional. Ignoring this instability, we will include multimonopoles for YMH systems with $\lambda > 0$ in our definition for a MM solution.

Recently, further to these MM solutions, new types of configurations have been considered, which represent composite states of monopoles and antimonopoles in (topologically unstable) equilibrium. The existence of such solutions was first proven by Taubes for a model featuring no Higgs potential [8]. Notwithstanding the absence of the Higgs potential, such solutions are not self–dual and their energies exceed the Bogomol’nyi bound. Kleihaus and Kunz [9] (see also [10]) constructed numerically an axially symmetric solution with zero net magnetic charge, consisting of a MA pair for the YMH system with and without a Higgs potential. More complicated configurations consisting of MA chains and vortex rings were constructed subsequently [11, 12].

The main difference between the MM and MA solutions in flat space is characterised by their distinct boundary conditions at infinity. They have different boundary conditions also at the origin.

When gravity is switched on, or equivalently when the YMH system is considered on a curved background, the arguments leading to the saturation of a topological lower bound and hence to the first order Bogomol’nyi equations disappear and as a result only solutions to the second order Euler–Lagrange equations can be sought.

Naturally, the boundary conditions employed play a crucial role in the properties of these solutions. Even in flat spacetime, the main difference between the topologically stable axially symmetric MM solutions on the one hand, and the MA type on the other, is a result of the imposition of different boundary conditions. Otherwise,
both types of solutions obey the same axially symmetric YMH equations \(^1\). However, to date, all regular axially symmetric \(SU(2)\) YMH configurations of the types mentioned above have been studied on an asymptotically-flat spacetime. It is therefore important to analyse the effects on these solutions, of different asymptotic structures of the spacetime.

To this end, we are motivated to study the properties of these configurations for spacetimes with a cosmological constant \(\Lambda\), in particular. In what follows we consider the case of a negative \(\Lambda\), corresponding to an anti-de Sitter (AdS) geometry. This maximally symmetric spacetime has recently enjoyed a considerable amount of interest, motivated mainly by the proposed correspondence between physical effects associated with gravitating fields propagating in AdS spacetime and those of a conformal field theory (CFT) on the boundary of AdS spacetime \([13, 14]\). This adds further justification to the study of classical solutions of various field theories in AdS, the YMH case at hand being an obvious example.

Prominent amongst the remarkable features displayed by solutions of the gravitating Yang-Mills (YM) system with \(\Lambda < 0\) is the existence of stable particle-like and black hole solutions \([15, 16]\). This results from a particular behaviour of the YM field asymptotically, which feature is present also for the case of a fixed AdS background \([17, 18]\). One aim of this work is to inquire to what extent the features of YM systems with \(\Lambda < 0\), persist also for YMH systems?

Here we present numerical arguments for the existence of both MM and MA configurations, both in a fixed AdS spacetime background, and in the fully gravitating case. While the static solutions of the YM system are drastically different from those of the EYM ones, in the case of the YMH system the solutions with \(\Lambda < 0\) (with or without gravity) are found to be very similar to those of the corresponding Minkowski spacetime configurations. (The situation is the same also for asymptotically flat EYMH.)

As found in \([19, 20]\) for \(\Lambda = 0\), when gravity is coupled to YMH theory, a branch of gravitating solutions emerge from the flat spacetime configurations. This branch extends up to some maximal value of the gravitational strength. The YMH system with a negative cosmological constant has recently been studied \([21, 22]\), for the spherically symmetric, unit magnetic charge gravitating monopole. It was found \([21]\) that unlike in flat spacetime, in a fixed AdS background there are no analytic solutions that might be used as a guide. Moreover, when a cosmological constant is included (no matter how small this constant is) no solution close to the flat space BPS configuration can be found. Numerically constructed monopole solutions have been exhibited in \([22]\), where the effects of gravity are also included. A distinctive feature of AdS solutions concerns the asymptotic behaviour of the fields. When \(\Lambda < 0\), the Higgs field approaches its vacuum expectation value faster than in the flat space case. The radius of the monopole core decreases, as the magnetic field concentrates near the origin.

We aim to extend this analysis for axially symmetric solutions, by studying the basic properties for both types of configurations, MM solutions with nonvanishing magnetic charge and MA pairs with a zero net magnetic charge. As expected, we find that the basic properties of the gravitating AdS configurations configurations are similar to the asymptotically flat spacetime counterparts.

The paper is structured as follows: in the next Section we present the model and subject the system to axial symmetry. The imposition of axial symmetry is carried out in a uniform manner to include the two distinct boundary conditions pertaining to multimonopole and monopole–antimonopole solutions. Numerical solutions in a fixed AdS background are constructed in Section 3 for the two different sets of boundary conditions. Section 4 contains a discussion of the main properties of the gravitating counterparts of these solutions. We conclude with Section 5 where the results are summarised.

\(^1\)In the absence of a Higgs potential the MM solutions obey the first order Bogomol’nyi equations, while the MA solutions obey the second order Euler–Lagrange equations.

\(^2\)In Ref. \([17]\), a solution to the \(SU(2)\) YM system in a fixed dS background is found in closed form. Passing from dS to AdS by an appropriate change of sign in this solution, one discovers the same interesting feature mentioned above, which was discovered in \([15, 16]\) for the gravitating case.
2 The model

2.1 Action principle and field equations

We consider the action for an $SO(3)$ YM field $A_\mu^a$ coupled to a triplet Higgs field $\Phi^a$ with the usual potential $V(\Phi) = \frac{\lambda}{4}|(\Phi^a|^2 - \eta^2)^2$

$$S = \int d^4x \mathcal{L} = \int d^4x \sqrt{-g} \left[ \frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a + \frac{1}{2} D_\mu \Phi^a D^\mu \Phi_a + V(\Phi) \right],$$

where the field strength tensor and the covariant derivative are defined as

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \epsilon^{abc} A_\mu^b A_\nu^c,$$
$$D_\mu \Phi^a = \partial_\mu \Phi^a + \epsilon^{abc} A_\mu^b \Phi^c,$$

and $g = \det g_{\mu\nu}$, $g_{\mu\nu}$ being the metric of a fixed anti de Sitter (AdS) background. Subjecting the action (1) to the variational principle results in the Euler–Lagrange equations, and varying the Lagrangian $L$ with respect to the metric $g_{\mu\nu}$ yields the energy-momentum tensor

$$T_{\mu\nu} = F_{\mu\alpha}^a F_{\nu\beta}^b g^{\alpha\beta} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta}^a g^{\alpha\gamma} g^{\beta\delta} + \frac{1}{2} D_\mu \Phi^a D_\nu \Phi^a - \frac{1}{4} g_{\mu\nu} (D_\alpha \Phi^a D^\alpha \Phi^a + V(\Phi)) g_{\mu\nu}.$$ (2)

We are interested in static, purely magnetic ($A_t = 0$), axially symmetric finite energy solutions of the Euler–Lagrange equations. The energy density of a solution of the YMH equations is given by the $tt$-component of the energy-momentum tensor; integration over all space yields the total mass/energy

$$M = -\int T_{tt} \sqrt{-g} d^3x.$$ (3)

For the mass integral to converge, each term in the integrand of (3) must vanish at large $r$. This will give the required asymptotic behaviour of the gauge and Higgs field. The additional requirements to have a finite, locally integrable energy density impose boundary conditions at the origin and on the $z$-axis.

The static energy functional arising from (3) is bounded from below, in flat spacetime, by the topological charge

$$Q_m = \frac{1}{4\pi} \int d^3x \varepsilon_{ijk} F_{ij}^a D_k \Phi^a = \frac{1}{4\pi} \int dS^k \varepsilon_{ijk} F_{ij}^a \Phi^a,$$ (4)

which is the magnetic monopole charge, whose definition is valid also in curved spacetime.

2.2 Imposition of axial symmetry

We start by stating the line element for the fixed AdS background

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \frac{dr^2}{1 - \frac{\Lambda}{3}r^2} + r^2(d\theta^2 + r^2 \sin^2 \theta d\phi^2) - \left(1 - \frac{\Lambda}{3}r^2\right) dt^2,$$ (5)

where $r$, $\theta$, $\varphi$ are the usual spherical coordinates. The YMH system is defined by (1), with a metric tensor $g_{\mu\nu}$ given by (5).

We proceed in this section to present the usual axially symmetric Ansatz of Rebbi and Rossi [24], adapted in particular to solutions with boundary conditions supporting monopole-antimonopoles. Particular attention is given to finding the relevant asymptotic values. Note also that this analysis is valid in the limit $\Lambda = 0$.

Expressing the $SU(2)$ gauge connection $A_\mu^a$ and the algebra valued Higgs field $\Phi$ in terms of components labeled by the algebra indices

$$A_\mu = A_\mu^a \frac{\tau^a}{2}, \quad \Phi = \Phi^a \frac{\tau^a}{2}, \quad a = \alpha, 3 \equiv \alpha, z \quad ; \quad \alpha = 1, 2 \equiv x, y,$$
and splitting up the spacelike coordinates \( x_\mu = (x_i, x_3) \equiv (x_i, x_3) \), with \( x_i = (x_1, x_2) \equiv (x, y) \), (where \( r = \sqrt{\rho^2 + z^2} \), \( \theta = \arctan \frac{z}{\rho} \)) the axially symmetric Rebbi–Rossi Ansatz can be expressed as follows

\[
A_\alpha^r = -a_\rho \hat{x}_i (\varepsilon n)^\alpha + \left( \frac{\chi^1}{\rho} \right) (\varepsilon \hat{x})_i n^\alpha \\
A_3^r = \left( \frac{n + \chi^2}{\rho} \right) (\varepsilon \hat{x})_i \\
A_3^\alpha = -a_z (\varepsilon n)^\alpha , \quad A_3^z = 0
\]

for the gauge connection, and

\[
\Phi^\alpha = \eta \phi^1 n^\alpha , \quad \Phi^3 = \eta \phi^2
\]

for the Higgs field, \( \eta \) being the VEV of the Higgs field, with inverse dimension of a length. The six functions in \( \Phi^1 - \Phi^4 \) depend on \( \rho = \sqrt{x_\rho^2} \) and \( x_3 = z \), the unit vector in the azimuthal plane is

\[
n^\alpha = (\cos n\phi, \sin n\phi),
\]

and \( \varepsilon_{ACB} \) is the antisymmetric Levi-Civita symbol. This ansatz contains also an integer \( n \), which is the winding number.

Subject to the Ansatz (6) and (7), the static energy density functional (1) reduces to a 2 dimensional system in \((\rho, z)\) space which exhibits a residual \(U(1)\) gauge invariance. The two components of the gauge connection are \((a_\rho, a_z)\) appearing in the Ansatz (6), and the gauge arbitrariness can be removed by the gauge condition

\[
\partial_\rho a_\rho + \partial_z a_z = 0.
\]

Since it is more convenient to work in spherical coordinates, we replace the two functions \((a_\rho, a_z)\) by \((a_r, a_\theta)\) according to

\[
a_\rho = a_r \sin \theta + a_\theta \frac{\cos \theta}{r} , \quad a_z = a_r \cos \theta - a_\theta \frac{\sin \theta}{r}.
\]

With this replacement, and denoting \( \chi^A = (\chi^1, \chi^2) \), one can write

\[
\mathcal{L} = -g \left\{ g^{rr} g^{\theta\theta} f_r^2 + g^{r\theta} \left( g^{rr} |\mathcal{D}_r \chi^A|^2 + g^{\theta\theta} |\mathcal{D}_\theta \chi^A|^2 \right) \\
+ \eta^2 \left[ g^{rr} |\mathcal{D}_r \phi^A|^2 + g^{\theta\theta} |\mathcal{D}_\theta \phi^A|^2 + g^{r\theta} \left( \varepsilon_{AB} \phi^A \phi^B \right) \right] + V(\phi^A) \right\},
\]

which is manifestly a \(U(1)\) gauged Higgs model, with \(U(1)\) connection \((a_r, a_\theta)\) and Higgs field doublets \(\chi^A = (\chi^1, \chi^2)\), \(\phi^A = (\phi^1, \phi^2)\), and with the corresponding \(U(1)\) curvature and covariant derivatives in (6) defined by

\[
\mathcal{F}_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \\
\mathcal{D}_\mu \chi^A = \partial_\mu \chi^A + a_\mu \varepsilon^{AB} \chi^B \\
\mathcal{D}_\mu \phi^A = \partial_\mu \phi^A + a_\mu \varepsilon^{AB} \phi^B
\]

having denoted \( \mu = r, \theta \), not to be confused with the label \( \mu \) in (1).

We now consider the asymptotic conditions in the \( r > 1 \) region that must be satisfied for finite energy, and the conditions of analyticity at \( r = 0 \) and on the \( z \)-axis.

It is immediately obvious from the Ansatz (6), in this gauge, that when \( \rho \to 0 \)

\[
\lim_{\rho \to 0} \chi^1 = 0 \quad \lim_{\rho \to 0} \chi^2 = -n,
\]

which will be used presently to find the asymptotic values of the fields in the \( r > 1 \) region. In the same \((\rho \to 0)\) limit, differentiability sets the following condition on the Higgs field function \(\phi^1\) in (7)

\[
\lim_{\rho \to 0} \phi^1 \propto \rho^n \quad \text{i.e.} \quad \phi^1|_{r=0} = 0 \quad \text{and} \quad \phi^1|_{\theta=0, r} = 0.
\]

but no condition on the other Higgs field function \(\phi^2\). Thus the zeros of the Higgs field, i.e. when both \(|\phi^1|^2 = 0\) and \(|\phi^2|^2 = 0\) will not necessarily occur at the origin in general.
2.3 Asymptotics

In addition to the particular conditions (10) and (11) to be satisfied on the z-axis, the comprehensive set of asymptotic values will now be stated, both in the $r \gg 1$ and $r \ll 1$ regions, in that order (here also the analysis remains valid in the flat space limit).

Asymptotics in the $r \gg 1$ region

In the $r \gg 1$ asymptotic region on the other hand, the effect of a symmetry breaking Higgs potential, whether explicitly included in the action or not, results in the following finite energy condition on the Higgs field

\[ \lim_{r \to \infty} \phi^1 = \sin m \theta, \quad \lim_{r \to \infty} \phi^2 = \cos m \theta, \]  

\[ m \] being an integer. For $m = 1$, (12) are just the usual boundary conditions applying to (multi-) monopole solutions, and with $n = 1$ in (7), they are the boundary conditions for the spherically symmetric unit monopole. For $m \geq 2$, the solutions describe monopole–antimonopole chains, with magnetic charges equal to the winding of the asymptotic Higgs field (12),

\[ Q_m = 2\pi n [1 - (-1)^m]. \]  

Finiteness of the energy requires that both terms $|D_r \phi^A|^2$ and $|D_\theta \phi^A|^2$ vanish at infinity, yielding

\[ \lim_{r \to \infty} a_r = 0, \quad \lim_{r \to \infty} a_\theta = -m, \]  

while that for the last term in (9) describing the interaction between $\chi^A$ and $\phi^A$ yields

\[ (\varepsilon \phi)_A \chi^A = 0 \]  

in this limit, where we have expressed (12) as

\[ \lim_{r \to \infty} \phi^A = t^A, \]  

in terms of the unit 2-vector $t^A = (\sin m \theta, \cos m \theta)$. The $\theta$ derivative of (16) gives the condition

\[ (\varepsilon t)_A \partial_\theta \chi^A = m t_A \chi^A, \]  

which we will find convenient to use presently.

The corresponding condition for the vanishing of $|D_r \chi^A|^2$ in (9) at infinity implies that $\chi^A \to \text{const.}$ in this limit, but the actual asymptotic values of $\chi^A$ cannot be determined without analysing the corresponding Euler–Lagrange equations, since finiteness of energy does not require that the term $|D_\theta \chi^A|^2$ in (9) vanish at infinity. To leading order in this limit, this equation is

\[ \sin \theta D_\theta \left( \frac{1}{\sin \theta} D_\theta \chi^A \right)^A = 0. \]

Contracting this equation with the unit vector $t^A$ and using (16) and (17), it reduces to

\[ \frac{d}{d\theta} \ln \frac{t_A \partial_\theta \chi^A}{\sin \theta} = 0, \]

which is immediately integrated to give

\[ t_A \partial_\theta \chi^A = c_1 \sin \theta, \]  

$c_1$ being an integration constant. (12) and (16) together now result in

\[ \frac{d}{d\theta} (t_A \chi^A) + c_1 \sin \theta = 0, \]
yielding finally

\[ \chi^A = \left( c_2 + c_1 \cos \theta \right) t^A, \]

(20)
c_2 being the second integration constant.

It is easy to check that the asymptotic fields (20) as they stand, result in the cancellation of the singularities due to \( \frac{1}{\sin \theta} \) in (11). To fix the integration constants \( c_1 \) and \( c_2 \) in (20), additional constraints are needed. To this end we recall the analyticity conditions (10) on the \( z \)-axis. Substituting (20) in (10), we find that the first member of the latter is identically satisfied while the second yields

\[ \chi^2|_{\theta=0} = (c_2 + c_1) = -n \]
\[ \chi^2|_{\theta=\pi} = (-1)^m (c_2 - c_1) = -n, \]

yielding the values of \( c_1 \) and \( c_2 \) to be used in (20),

\[ c_1 = 0, \quad c_2 = -n, \quad m \text{ even} \quad \text{(21)} \]
\[ c_1 = -n, \quad c_2 = 0, \quad m \text{ odd} \quad \text{(22)} \]

Asymptotics in the \( r \ll 1 \) region

The \( r \ll 1 \) region is a special case of the \( \rho = 0 \) line, namely the \( z \)-axis. Starting from the constraints of analyticity on the \( z \)-axis, we have already (10)-(11). We restate these asymptotic behaviours in the \( \rho \to 0 \) limit, along with the other such conditions implied by the Ansatz (6), although we will be concerned almost exclusively with the actual asymptotic values. For the Higgs field functions \( \phi^A, A = 1, 2 \), these are

\[ \phi^1 = a_1(z) \rho^n, \quad \phi^2 = a_2(z), \]

(23)
and for the gauge field functions \( (a_\rho, a_z) \) and \( \chi^A, A = 1, 2 \),

\[ a_\rho = b_1(z) \rho^{n+1}, \quad a_z = b_2(z) \rho^n, \quad \chi^1 = c_1(z) \rho^{n+2}, \quad \chi^2 = -n + c_2(z) \rho^2. \]

(24)
It is clear from (24) that the asymptotic values of these functions are simply stated as

\[ a_\rho|_{r=0} = 0, \quad a_z|_{r=0} = 0, \quad \chi^1|_{r=0} = 0, \quad \chi^2|_{r=0} = -n. \]

(25)
To state the asymptotic values of the functions \( (\phi^1, \phi^2) \) on the other hand is much harder. It is well known that for axially symmetric multimonopoles (25), with \( m = 1 \), these are

\[ \phi^1|_{r=0} = 0, \quad \phi^2|_{r=0} = 0, \]

(26)
while for monopole-antimonopole-chain solutions with \( m \geq 2 \), the situation is more complex. Clearly, for monopole-antimonopole-chains on the \( z \)-axis, the function \( a_2(z) \) in (23) must have zeros away from \( r = 0 \), precluding the second member of (26). To date, where these zeros occur has not been determined analytically, but are found through the numerical process [11, 20].

For monopole-antimonopoles (26) is inadequate, so from (23) we infer the general, weaker, boundary conditions

\[ \phi^1|_{r=0} = 0, \quad \partial_\rho \phi^2|_{\rho=0} = 0. \]

(27)
We note that both (20) and (27) are consistent with the conclusions of (26), namely that there is only one zero of the Higgs field for an axially symmetric MM solution, and that if there are distinct zeros on the \( z \)-axis, either axially symmetry is violated or the charges corresponding to the distinct zeros have differing signs.

The parametrisation of Kleihaus and Kunz

This particular parametrisation of the Rebbi and Rossi Ansatz used by these authors, see e.g. [27], is very convenient and is employed in most works on axially symmetric YMH systems. Since the numerical integrations will be carried out for the variables \( (r, \theta) \) and not \( (\rho, z) \), it is useful to employ a parametrisation for which
the Ansatz (6)-(7) agrees with the unit magnetic charge monopole for \( n = 1 \). The functions \( (H_1, H_2, H_3, H_4) \) parametrising the gauge field are related to the functions in the axially symmetric Ansatz (6) as follows:

\[
\begin{align*}
H_1 &= r(a_\rho \sin \theta + a_z \cos \theta) \\
1 - H_2 &= r(a_\rho \cos \theta - a_z \sin \theta) \\
n \sin \theta H_3 &= \chi_1 \sin \theta + (n + \chi^2) \cos \theta \\
n \sin \theta(1 - H_4) &= \chi_1 \cos \theta - (n + \chi^2) \sin \theta,
\end{align*}
\]

and the functions \( (\Phi_1, \Phi_2) \) parametrising the Higgs field Ansatz (7),

\[
\begin{align*}
\Phi_1 &= \phi^1 \sin \theta + \phi^2 \cos \theta \\
\Phi_2 &= \phi^1 \cos \theta - \phi^2 \sin \theta.
\end{align*}
\]

In terms of this parametrisation, the boundary conditions for the monopole–antimonopole solutions at \( r \to \infty \), can be read off (14), (20)-(22), and (28). For odd \( m \) these are

\[
\begin{align*}
H_1 &= 0, \quad H_2 = -(m - 1), \quad H_3 = \frac{\cos \theta}{\sin \theta} \cos(m - 1)\theta - 1, \quad H_4 = -\frac{\cos \theta}{\sin \theta} \sin(m - 1)\theta.
\end{align*}
\]

and for even \( m \)

\[
\begin{align*}
H_1 &= 0, \quad H_2 = -(m - 1), \quad H_3 = \frac{1}{\sin \theta} \cos(m - 1)\theta - \cos \theta, \quad H_4 = -\frac{\sin(m - 1)\theta}{\sin \theta},
\end{align*}
\]

while

\[
\Phi_1 = \cos(m - 1)\theta, \quad \Phi_2 = \sin(m - 1)\theta,
\]

for any value of \( m \).

The corresponding boundary conditions for multimonopoles are simply given by the set for odd \( m \), with \( m = 1 \).

The boundary conditions for the gauge field functions at \( r \to 0 \) can be read off (25), for both multimonopoles and monopole-antimonopoles. The Higgs field boundary conditions for MM’s and MA’s on the other hand, are different, being stated respectively by (26) and (27). Using the relations (28)-(29), (25) and (26) yield the required boundary values at \( r = 0 \) for multimonopoles

\[
H_1 = H_3 = 0, \quad H_2 = H_4 = 1, \quad \Phi_1 = \Phi_2 = 0.
\]

For monopole-antimonopoles, the less stringent condition (27) must be used instead of (26). Following (26) we state the boundary values for monopole-antimonopoles as

\[
H_1 = H_3 = 0, \quad H_2 = H_4 = 1, \quad \cos \theta \partial_\theta \Phi_1 - \sin \theta \partial_\theta \Phi_2 = 0, \quad \sin \theta \Phi_1 + \cos \theta \Phi_2 = 0,
\]

the Higgs part of which is a weaker version of (27).

The above parametrisation is in fact employed in the literature for the numerical construction of axially symmetric MM solutions, e.g. in [27] and also for gravitating YM solutions [28]. In the corresponding construction of axially symmetric MA solutions however, these authors use a different parametrisation in Ref. [9], as well as in [20], where they have constructed the gravitating counterparts of the latter. However, the parametrisation used in the MA works [9, 20] is equivalent to that of [27], which is what we have finally used in the present work, unifying the treatments of axially symmetric MM and MA solutions.
Figure 1. The mass of solutions is plotted as a function of the cosmological constant for three values of the winding number $n$ and Higgs self-coupling constant $\lambda = 0.5$. Here and in Figures 2 and 4a the mass is given in units $4\pi\eta/g$.

3 Solutions in a fixed AdS background

We have performed our numerical computations using the parametrisation of Kleihaus and Kunz described above. All numerical calculations were performed with the software package CADSOL/FIDISOL, based on the Newton-Raphson method [29].

3.1 Multimonopole solutions

We start by discussing the simplest example of axially symmetric solution, describing multimonopoles containing only magnetic charges with the same sign ($m = 1$). The suitable boundary conditions for the Higgs field at infinity are $\Phi_1 = 1$, $\Phi_2 = 0$, while, in the same limit the YM potentials should vanish $H_i = 0$.

Given the parity reflection symmetry satisfied by the ansatz (28)-(29), we need to consider solutions only in the region $0 \leq \theta \leq \pi/2$; on the $z$- and $\rho$-axis the functions $H_1, H_3, \Phi_2$ and the derivatives $\partial_\theta H_2, \partial_\theta H_4$ and $\partial_\theta \Phi_1$ are to vanish. To fix the residual abelian gauge invariance we choose the usual gauge condition (8) which reads in terms of $H_i$ functions

$$ r \partial_r H_1 - \partial_\theta H_2 = 0. $$

(35)

For these boundary conditions, the configurations with $n = 1$ corresponds to spherically symmetric monopoles with $H_1 = H_3 = \Phi_2 = 0$, $H_2 = H_4 = w(r)$. $\Phi_1 = H(r)$, and generalize the ’t Hooft-Polyakov solution for a negative cosmological constant. Axially symmetric solutions are found for $n = 2, 3, \ldots$.

Dimensionless coordinates and Higgs field are obtained by rescaling

$$ r \rightarrow r/\eta, \quad \Lambda \rightarrow \Lambda \eta^2, \quad \Phi \rightarrow \eta \Phi. $$

(36)

Within this ansatz and gauge choice, we have solved the resulting set of six non-linear coupled partial differential equations, finding solutions for any considered value of the cosmological constant.
Figure 2. The mass per topological charge is plotted as a function of Higgs self-coupling constant $\lambda$ for MM solutions with $\Lambda = -0.5, -2$ and $n = 1, 2$.

Figure 3. The dimensionless energy density of a $n = 2$ MM solution in a fixed AdS background with $\Lambda = -1$ is plotted as a function of $\rho$ and $z$. The solution has been found for a Higgs coupling constant $\lambda = 10$ and has a mass $M = 3.998$.

Qualitatively, the behaviours of the Higgs field and Yang-Mills fields are very similar to that corresponding to Minkowski spacetime monopoles. In particular, we notice a similar shape for the functions $H_i$ and $\Phi_i$ and also for the energy density. Again, the functions $H_2$, $H_4$ and $\Phi_1$ present a small $\theta-$dependence. For this type of solutions, the nodes of the Higgs field are superposed at the origin.

In Figure 1 the mass of the solutions in units of $4\pi\eta/g$ is plotted as a function of the cosmological constant for various winding numbers and a fixed value of the coupling constant $\lambda = 0.5$.

The energy of the multimonopole is of the order of $n$ times the corresponding one-monopole energy. However, for any value of $\Lambda$, the ratio $M/n$ increasing with increasing $n$. Also, this quantity is greater than the mass of
the corresponding $n = 1$ spherically symmetric monopoles. For example, for solutions with $\Lambda = -4$ and $\lambda = 0.5$, we find $M(n = 2)/M(n = 1) = 2.466$, $M(n = 3)/M(n = 1) = 4.373$ while $M(n = 4)/M(n = 1) = 6.678$.

Most of our results have been obtained in the Prasad-Sommerfeld limit, but similar conclusions hold for a nonvanishing Higgs self-coupling constant $\lambda$. To demonstrate this dependence, we plot in Figure 2 the mass of solutions per topological charge as a function of $\lambda$ for two different values of the cosmological constant and winding numbers $n = 1, 2$.

For all configurations, the energy density $\epsilon = -T_t^t$ of the solutions has a strong peak along the $\rho$ axis, and it decreases monotonically along the symmetry axis. Individual unit charged monopole components of the MM do not feature distinct peaks (see Figure 3). Equal density contours reveal a torus-like shape of the solutions. For increasing $|\Lambda|$, these peaks increase in size and the energy density becomes localised in a decreasing region of space.

### 3.2 Monopole-antimonopole solutions

As discussed in [8] for $n = 1$ and $\Lambda = 0$, there exist also a different type of solutions of the second order Euler-Lagrange equations, which are not stable and represent saddle points of the energy, rather than absolute minima resulting from solving the Bogomol’nyi equations. Here we construct the simplest examples of such solutions, namely MA solutions obtained by taking $m = 2$ in the general Higgs field asymptotics at infinity, (31)-(32), which implies the asymptotic behaviour $H_1 = H_3 = 0$, $H_2 = H_4 = -1$. This corresponds to a MA configuration, with a vanishing net magnetic charge.

Similarly to the multimonopole case, we need to consider solutions only in the region $0 \leq \theta \leq \pi/2$; on the $z$- and $\rho$-axis the functions $H_1, H_3, \Phi_2$ and the derivatives $\partial_\theta H_2, \partial_\theta H_4$ and $\partial_\theta \Phi_1$ are to vanish. The same gauge condition, (35), fixes the residual Abelian gauge invariance.

A calculation similar to that done in [9] shows that this configuration possesses two magnetic charges of opposite sign, located on the positive and negative $z$–axis, respectively, with nonvanishing local density of magnetic charge. However, the integral (4), evaluated at infinity, for a surface enclosing both charges, yields a zero net magnetic charge. Also, the expansions of the matter functions at the origin and on the $z$–axis presented in Ref. [9] are still valid, and will not be given here.

The magnetic dipole moment $C_m$ of these solutions can be obtained from the asymptotic form of the non-abelian gauge field, after choosing a gauge where the Higgs field is asymptotically constant $\Phi \rightarrow \tau_3$, which yields

$$A_k dx^k = C_m \frac{\sin^2 \theta}{r^2} \frac{\tau_3}{2} d\varphi.$$  \hfill (37)

We have constructed MA solutions for a large range of the parameters ($\Lambda, \lambda$). For vanishing $\Lambda$, our results are in very good agreement with those of [9]. The main part of the numerical analysis has been done for the case $n = 1$, the case $n = 2$ being studied more briefly. However, most of the qualitative properties of the $n = 2$ solutions do not differ from those of the $n = 1$ case.

Similarly to the case of multimonopoles, we have found that a negative cosmological constant does not change the qualitative properties of the solutions. As expected, a negative cosmological constant affects the behaviour of the fields in the asymptotic region, where the modulus of the Higgs field, for example, reaches its asymptotic value faster.

In Figure 4 we present the the mass and the magnetic dipole moment of $n = 1$ MA solutions as a function of $\Lambda$ for three values of the Higgs self-coupling constant $\lambda$.

The energy density always possesses maxima on the positive and negative $z$-axis at the locations of the monopole and antimonopole and a saddle point at the origin. An increase of $|\Lambda|$ makes the maxima of the energy density higher and sharper. At the same time, the modulus of the Higgs field tends faster towards its vacuum expectation value. The typical energy density for nongravitating MA solutions have a similar form to that presented in Section 4 for gravitating configurations.
The mass and the magnetic dipole moment of $n = 1$ MA solutions is plotted as a function of $\Lambda$ for three values of the Higgs self-coupling constant $\lambda$.

The nodes of the modulus of the Higgs field correspond to the locations of the particles. With increasing $|\Lambda|$, the distance $d$ between the MA centers becomes smaller tending to a finite limit as $\Lambda \to -\infty$, making it more difficult to distinguish the monopole from antimonopole. Several values we found for solutions with $\lambda = 2$ are: $d(\Lambda = 0) = 3.24$, $d(\Lambda = -0.5) = 1.52$, $d(\Lambda = -1) = 1.22$, $d(\Lambda = -2.5) = 0.9$, the distance between the MA centers approaching for larger values of $|\Lambda|$ a limiting value $d \simeq 0.85$. The magnetic dipole moment $C_m$ decreases in the same limit.

Considering solutions with a fixed negative value of $\Lambda$, we find that the distance between the MA centers decreases as $\lambda$ increases and converges to a finite, nonzero value. For example, for MA solutions in an AdS spacetime with $\Lambda = -1$, $d$ decreases from $d(\lambda = 0) = 0.68$ to a limiting value $d \simeq 0.6$. This property is shared with the $\Lambda = 0$ case [9].
4 Inclusion of gravity

We now consider the effects of these axially symmetric configurations on the AdS spacetime, by including the Einstein gravity term with a negative cosmological constant $\sqrt{-g}(-R/(16\pi G) + 2\Lambda)$ in the action density $\mathcal{L}$.

The unusual mass formula $(40)$ reveals in this case the existence of a new dimensionless coupling constant $\alpha^2 = 4\pi G\eta^2$, where $G$ is the Newton constant. The complete classification of the solutions in the space of physical parameters $(\alpha, \Lambda, \lambda)$ is a considerable task which is beyond the scope of this paper. Instead, we analysed the situation for several values of $\Lambda$ which, hopefully, reflect all the properties of the general pattern. Also, for simplicity we will study in this section only solutions with no Higgs potential ($\lambda = 0$).

The asymptotically anti-de Sitter (AAdS) gravitating counterparts of the axially symmetric configurations discussed in Section 3 are found by using a metric of the form

$$ds^2 = \frac{m}{f} \left( \frac{dr^2}{1 - \frac{4}{3}r^2} + r^2 d\theta^2 \right) + \frac{l}{f} r^2 \sin^2 \theta d\phi^2 - f(1 - \frac{\Lambda}{3} r^2) dt^2,$$

with $f$, $l$ and $m$ being functions of $r$ and $\theta$. To obtain asymptotically AdS regular solutions with finite energy, the metric functions have to satisfy the boundary conditions $f = m = l = 1$ at infinity, and $\partial_r f = \partial_r m = \partial_r l = 0$ at the origin. The boundary conditions on the $z$-axis are $\partial \theta f = \partial \theta m = \partial \theta l = 0$, which are consistent with the requirement of invariance under reflection in the $\theta = \pi/2$ plane. Finally, regularity on the $z$-axis requires also $m = l$ for $\theta = 0$. These metric boundary conditions are valid for both MA and MM gravitating configurations. The boundary conditions for the gauge potentials and Higgs field are similar to the nongravitating case.

To solve the set of nine EYMH equations we employ the same numerical algorithm as for the YMH solutions in fixed AdS background. In the numerical procedure we use a suitable combination of the Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu},$$

such that the differential equations for metric variables $(f, l, m)$ are diagonal in the second derivatives with respect to $r$.

Similar to the asymptotically flat case, the value of the mass is encoded in the asymptotics of the metric functions $f, l, m$, whose expression, valid for large $r$, is

$$f = 1 + \frac{f_1 + f_2 \sin^2 \theta}{r^3} + O\left(\frac{1}{r^4}\right), \quad m = 1 + \frac{m_1 + m_2 \sin^2 \theta}{r^3} + O\left(\frac{1}{r^4}\right), \quad l = 1 + \frac{l_1 + l_2 \sin^2 \theta}{r^3} + O\left(\frac{1}{r^4}\right),$$

where $f_1$, $f_2$ are constants to be determined numerically, and

$$l_1 = m_1 = \frac{2f_1}{3}, \quad l_2 = \frac{6f_2}{17}, \quad m_2 = \frac{14f_2}{17}.$$  

(41)

The generalization of Komar’s formula for AdS asymptotics is not straightforward and requires the further subtraction of a background configuration in order to yield a finite result.

To compute the mass of these configurations, we employ the counterterm formalism proposed by Balasubramanian and Kraus [23]. This technique was inspired by AdS/CFT correspondence and consists of adding suitable counterterms $\Delta L$ to the total action of the theory. These counterterms are built of curvature invariants of a boundary metric and thus obviously do not alter the bulk equations of motion. However, they do regularise the boundary stress tensor and also the conserved charges. The details of the computation, which we omit here, are presented in [34], where an axially symmetric line element with the same parametrisation and the same asymptotics is considered.

The expression of the mass derived in this way is

$$M = \frac{\Lambda}{3G} \left( \frac{2f_1}{3} + \frac{8f_2}{17} \right).$$

(42)

The same form is obtained by using the Hamiltonian formalism of Henneaux and Teitelboim [32].
Figure 5. The metric function $f$ and the norm of the Higgs field $|\Phi| = \sqrt{\Phi_1^2 + \Phi_2^2}$ of typical AAdS MM solutions are shown as a function of $r$ for three values of the angle $\theta$ with $n = 2$, $\Lambda = -3$ and a sequence of values of $\alpha$.

The dimensionless mass is given by

$$\frac{\mu}{\alpha^2} = \frac{e}{4\pi \eta} M.$$  \hspace{1cm} (43)

4.1 Gravitating MM

A spherically symmetric line element is obtained for $l = m$, with $f$ and $m$ being functions of the coordinate $r$ only. Some properties of the corresponding $n = 1$ monopole solutions are discussed in \cite{22} by using Schwarzschild-like coordinates. Similar results are found also by using the above metric parametrisation.

As found in \cite{22}, when $\alpha$ is increased from zero, while $\Lambda$ is kept fixed, a gravitating monopole branch emerges
Figure 6. The mass per topological charge is plotted as a function of $\alpha$ for $n = 1, 2, 3$ (multi-)monopole solutions with $\Lambda = -10$ and $\lambda = 0$.

smoothly from the corresponding AdS space monopole solutions. The total mass of gravitating solutions decreases with increasing $\alpha$. A similar property has been noticed for asymptotically flat solutions too.

The fundamental $n = 1$ monopole branch extends up to a maximal value $\alpha_{max}$ of the coupling constant $\alpha$. This maximal value decreases with $|\Lambda|$; for example $\alpha_{max}(\Lambda = 0) \approx 1.4$, $\alpha_{max}(\Lambda = 0.1) \approx 1.28$, while $\alpha_{max}(\Lambda = -3) \approx 0.80$. As is known from the work of \cite{23}, for $\Lambda = 0$, and with the Higgs coupling $\lambda = 0$, the fundamental monopole branch bends backwards at $\alpha_{max}$, until a critical value $\alpha_{cr}$ corresponding to the extremal Reissner-Nordström (RN) solution is reached. For $\Lambda < 0$, we refer to such RN solutions as RNAdS. However, we have found that for large enough values of $|\Lambda|$, the maximal value $\alpha_{max}$ and the critical value $\alpha_{cr}$ seem to coincide, as they do also for the asymptotically flat case when $\lambda > 0$ \cite{24}.

Along the fundamental branch, the metric function $f(r)$ develops a minimum, which decreases asymptotically to zero, as $\alpha \to \alpha_{cr}$. The functions $\omega(r)$ and $H(r)$ parametrising the gauge and the Higgs fields respectively, approach their respective RNAdS values $\omega = 0$, $H = 1$ \cite{22}. It is also noticed that the critical value of $\alpha$ as well as the corresponding value of mass decrease with $\Lambda$. The limiting spacetime consists of an inner part with $r < r_c$ and an outer part with $r \geq r_c$. The exterior of the critical solution corresponds to the exterior of a RNAdS black hole solution with a degenerate horizon at $r = r_c$ and unit magnetic charge. A discussion of these solutions and typical profiles are presented in \cite{21, 22}.

Here we have constructed axially symmetric multimonopoles for $n > 1$, the metric function in this case displaying an angular dependence. Most of the properties of these solutions are similar to those corresponding to asymptotically flat space. In particular, for a given topological charge $n$, we find a branch of globally regular asymptotically AdS multimonopoles, emerging smoothly from the corresponding solutions in fixed AdS background.

In Figure 5 we show the metric function $f$ and the norm of the Higgs field $|\Phi| = \sqrt{\Phi_1^2 + \Phi_2^2}$ of gravitating $n = 2$ multimonopole solutions with $\Lambda = -3$, for several values of $\alpha$.

The fundamental branch extends up to some maximal value of $\alpha$, whose precise value depends on $\Lambda$. Along this fundamental branch, the mass of the solutions decreases monotonically. For the studied case $\lambda = 0$, the value of $\alpha_{max}$ decreases with increasing $\Lambda$. The value at the origin of the metric function $f$ decreases with increasing $\alpha$, corresponding as $\alpha \to \alpha_{cr}$ to the degenerate horizon of a RNAdS solution. At the same time the Higgs field function $\phi_1$ approaches the unit value. The exterior of the critical solution corresponds to the exterior of a degenerate horizon RNAdS black hole with magnetic charge $n$ (note that for the same value of $\alpha$ and $n_2 > n_1$, the mass of this RNAdS black hole satisfies the relation $M(n_2) > M(n_1)$).
Figure 7. The dimensionless energy density of a \( n = 1 \) gravitating MA solution with \( \Lambda = -0.04, \alpha = 0.2 \), is plotted as a function of \( \rho \) and \( z \).

The mass per unit charge of the multimonopole solutions decreases with increasing \( \alpha \). As discussed in [19], the mass per unit charge \( n \) of the \( \Lambda = 0 \) multimonopoles is smaller than the mass of \( n = 1 \) monopoles, i.e. that like monopoles attract. We have found however that this is not a generic property of AAdS solutions, and that for large enough values of \( \Lambda \) the monopoles of like charge in a gravitating Higgs model on AdS space repel for all values of \( \alpha \) (a typical situation is presented in Fig. 6).

4.2 Gravitating MA

For completeness, we present here some properties of the gravitating MA solution with \( \Lambda < 0 \). The field equations have been solved in this case by using the same metric ansatz [15], and taking again \( m = 2 \) in the matter field asymptotics at infinity, (31)-(32), for \( n = 1, \lambda = 0 \) and several values of \( \Lambda \).

For MA boundary conditions, we find that, similar to the MM case, a branch of gravitating MA solutions emerges smoothly from the MA solutions in fixed AdS background. The basic properties of these solutions are similar to those of the configurations in a fixed AdS background. For example, the modulus of the Higgs field always possesses two zeros on the \( z \)-axis, corresponding to the location of the monopole and the antimonopole.

With increasing \( \alpha \), the monopole and antimonopole move closer to the origin and the mass \( \mu \) of the solutions decreases. In Figure 7 we plot the energy density \( \epsilon = -T^1_1 \) of a typical MA solution as a function of the coordinates \( \rho, z \), for \( \Lambda = -0.04, \alpha = 0.2 \).

This fundamental branch of solutions ends at a critical value \( \alpha_{cr} \), when gravity becomes too strong for solutions to persist. As expected, this value decreases with \( |\Lambda| \) (for example \( \alpha_{cr}(\Lambda = 0) \approx 0.67 \) [20], \( \alpha_{cr}(\Lambda = -0.01) \approx 0.64 \) while for \( \Lambda = -1 \) we find \( \alpha_{cr} \approx 0.35 \)).

In asymptotically flat space, it has been found that at \( \alpha_{cr} \) a second branch of MA solutions emerges, extending back to \( \alpha = 0 \) [20]. Along this upper branch, the distance \( d \) between the MA centers shrinks to zero size in the limit \( \alpha \to 0 \). After a suitable rescaling, as \( \alpha \to 0 \) the configuration approaches the spherically symmetric Bartnik-McKinnon (BK) solution, with \( H_1 = H_3 = 0, \ H_2 = H_4 = \omega(r) \), with \( \omega(r) \to -1 \) asymptotically (thus \( \alpha \to 0 \) means here \( \eta \to 0 \) with fixed \( G \)). In AdS spacetime however, the situation is to be more complicated since the BK configuration is now replaced by a continuum of solutions. Here the picture depends crucially on the value of \( \Lambda \) (in particular, solutions with \( \omega \to -1 \) cease to exist for a large enough \( |\Lambda| \)), and the meaning of the limiting solutions is less clear. We hope to come back to this point in future.
5 Conclusions

We have studied, analytically and numerically, the basic properties of two types of solutions to the $SU(2)$ Yang-Mills-Higgs model in the presence of a negative cosmological constant. The first type of solutions studied are the axially symmetric multimonopoles (MM) with magnetic charge equal to the azimuthal winding number $n$. The second type are the axially symmetric monopole–antimonopole (MA) pairs with vanishing magnetic charge and azimuthal winding number $n$, and nonvanishing magnetic dipole moment. These are the simplest examples of MA chains and vortex lines, and are not topologically stable even in flat spacetime, unlike the MM solutions. Both MM and MA solutions obey the same Euler–Lagrange equations of this system, subjected to axial symmetry. The drastic difference in the said properties arises from the imposition of different boundary conditions at infinity, in each case, and while the MM solutions are labeled by an azimuthal winding number $n$, the MA solutions are further labeled by a number $m$ multiplying the boundary value of the polar angle, i.e. by the pair of integers $(n, m)$. We have emphasised this point and have presented a unified treatment of both types of solutions.

The spherically symmetric analogues of the MM solutions studied here were discussed in [21, 22], where unit magnetically charged monopoles on both fixed AdS backgrounds and gravitating ones were considered, but not MA type solutions which are at most axially symmetric.

Qualitatively, the behaviour of the AdS solutions we constructed were found to be very similar to that of corresponding Minkowski spacetime configurations. Thus, it seems that when studying gauge field systems containing a scalar field, the Higgs field in the case at hand, the solutions exhibit a generic behaviour for $\Lambda \leq 0$. A similar behaviour has been noticed for sphalerons [30].

Let us list some points of contrast and similarity of MM and MA solutions, between the $\Lambda = 0$ and $\Lambda < 0$ cases:

- With $|\Lambda| > 0$, the MM solutions satisfy the second order Euler–Lagrange equations and not the first order Bogomol’nyi equations, even when the Higgs potential is absent, i.e. when $\lambda = 0$ in (1). The immediate consequence of this is that in this limit ($\lambda = 0$), like charged monopoles are not non–interacting. It turns out that the mass of one $n = 2$ MM is larger than that of two $n = 1$ MM’s with their centres infinitely far apart. Thus, it looks as if the $n = 2$ axially symmetric MM is unstable since like monopoles repel each other. This feature holds for all $n$, and is in contrast to the MM solutions in flat spacetime, which are stable for any $n$ since they are all self–dual solutions.

- When $|\Lambda| > 0$, it turns out that like monopoles remain mutually repulsive even for positive Higgs coupling constant $\lambda > 0$ model. This property, which is not surprising, is similar to the flat spacetime, $\Lambda = 0$ case. Like in the case of flat background moreover, when $\lambda \gg 1$, the mass of the MM solution tends to a finite limiting value [31] which we have not estimated here. This may be an interesting detail to return to in future.

- It is known from [9] that in the flat space model the distance on the $z$-axis between the monopole and the antimonopole decreases as the Higgs coupling constant $\lambda$ increases, converging to a finite limit. In the present model with $|\Lambda| \neq 0$, this property is similar for the MA solutions. For a fixed negative value of the cosmological constant, this distance decreases as $\lambda$ increases and converges to a finite value. This is qualitatively the same as for the flat space model, but quantitatively this distance decreases faster with increasing $\lambda$.

- For a fixed value of the Higgs coupling constant $\lambda$, increasing $|\Lambda| \neq 0$ of the negative cosmological constant results in the decrease of the distance between the monopole and the antimonopole of the MA solution, converging to a finite value.

- Another one of the properties of in Minkowski space that persists in the AdS case, is that found by Houston and O’Raifeartaigh [23]: Any regular axially symmetric magnetic charge distribution can be located only at isolated points situated on the axis of symmetry, with equal and opposite values of the charge at alternate points. In particular, if only one sign of the charge is allowed, all the charge must be concentrated at a single point. The results of the present paper support an extension of this property to the AdS case.

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As expected, the inclusion of gravity does not change the general picture, since for $\Lambda < 0$, the gravitational field does not affect the behaviour of the YMH system at infinity. As it happens in asymptotically flat case, a critical value for the Newton constant exists above which no regular solution can be found. However, as in the spherically symmetric case, this critical value is smaller than the corresponding value of the asymptotically flat one.

For asymptotically flat solutions, the inclusion of gravity allows for an attractive phase of like monopoles not present in flat space. Here we have presented numerical arguments that for large enough values of $|\Lambda|$, only a repulsive phase exists for like monopoles.

Hairy black hole solutions of EYMH theory with a negative cosmological constant, generalising the asymptotically flat configurations discussed in [19] [27] can also be constructed numerically, but this was not done in the present work.

One may ask about the possible relevance of these solutions within the holographic principle and its AdS/CFT correspondence realisation. A scalar field has been discussed in this context by many authors. For example, the asymptotic behaviour of the bulk scalar field was directly related to one-point functions in the dual CFT [33]. The vortex solution of the Abelian Higgs model has also an interesting AdS/CFT interpretation, its mass density being dual to the discontinuity of the logarithmic derivative of the correlation function of the boundary scalar operator [36]. To date however the crucial interplay between the non Abelian gauge fields and a scalar multiplet (with its rich set of boundary conditions discussed in this paper) is not considered in this context in the literature.

On the other hand, the EYMH bulk action (or a suitable modification supporting the same set of asymptotic boundary conditions) does not seem to correspond to a known supergravity truncation, and in particular we do not know the underlying boundary CFT. We believe that further work in this direction will be of interest.

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