All-optical controlled phase gate in quantum dot molecules

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Abstract
We propose a two-qubit optically controlled phase gate in quantum dot molecules via adiabatic passage and hole tunnelling. Our proposal combines the merits of the current generation of vertically stacked self-assembled InAs quantum dots and adiabatic passage. The simulation shows an implementation of the gate with a fidelity exceeding 0.98.

Keywords: medical imaging, coronary vessels, evaluation, software

1. Introduction
The spin of an electron, trapped in a self-assembled semiconductor quantum dot (SAQD) and manipulated by laser pulses, is believed to be a promising qubit candidate for quantum computation and quantum communication. Such qubits can be optically controlled at high speed [1, 2] and its coherence times have been prolonged to the order of microseconds [3]. Recently, there have been many experimental demonstrations of the key DiVincenzo requirements [4] for such qubits, for example, spin initialization [5–7], the coherent manipulation of electron spins [1, 2], and fast spin non-destructive measurement [8].

Entangling gates for two qubits form an essential ingredient of quantum computation. Significant effort has been invested in theoretical protocols for two qubit gate [9–17] and an experimental demonstration of optical entanglement control [18] between QDs. However, there has been no reported experimental realization of optically controlled phase gate between QDs. In this paper, we propose a two-qubit controlled phase gate in a vertical self-assembled quantum dot molecule (SAQDM) [19, 20] utilizing adiabatic passage and hole tunnelling. Compared with our previous scheme [16], the use of an adiabatic passage efficiently suppresses spontaneous decay from excited states [21–26] and need not precisely time the interval between the two pulses. Our proposal is based on the current generation of SAQDMs, and the simulation shows the gate can implemented with a fidelity exceeding 0.98.

2. The Basic model
Our model is based on the vertically coupled SAQDs [19] (see figure 1). The electrons or holes can tunnel between the two dots to create a quantum dot molecule (QDM). For the QDMs discussed here, the nominal height of dot 1, $h_1$, is greater than that of dot 2, $h_2$, so that dot 1 exhibits the lower transition energy. This allows the hole levels to be brought into resonance with a positive electric field applied along the $x$ axis (the growth direction), while the electron level of dot 2 is shifted to a much higher energy than that of dot 1. We use the spin of the electrons in each dot as the qubits. In the Faraday geometry, the energy levels for the single QD system in the presence of a small magnetic field $B_z$ along the $z$ axis (perpendicular to the $x$ axis) [7, 27]. The two eigenstate states of electron spin, i.e. $|↑\rangle$ and $|↓\rangle$, are split by the magnetic field $B_z$. The lowest-energy interband transition is to the trion state $|\uparrow\downarrow\uparrow\rangle$ (or $|\downarrow\uparrow\downarrow\rangle$), consisting of two electrons in a singlet state and a heavy hole. Optical selection dictates that the $\sigma^-$ ($\sigma^+$) polarization laser could couple the transition $|\downarrow\rangle \rightarrow |\downarrow\uparrow\downarrow\rangle$ ($|\uparrow\rangle \rightarrow |\uparrow\downarrow\uparrow\rangle$), and other transitions are forbidden.

3. Implementation of two qubits phase gate
The ideal phase gate aims to impose a phase change on the state $|\uparrow\downarrow\rangle$ without affecting the phase of the other three states.
It should also preserve phase coherence for a superposition of the four QD spin states. This operation can be characterized by the unitary transformation:

\[
\begin{array}{cccc}
\uparrow & \downarrow & \uparrow & \downarrow \\
\uparrow & \downarrow & \uparrow & \downarrow \\
\uparrow & \downarrow & \uparrow & \downarrow \\
\uparrow & \downarrow & \uparrow & \downarrow \\
\end{array}
\]

(1)

We use the convention that the vertical arrows on the left and right of the comma sign are, respectively, the directions of the spins in dot 1 and dot 2.

Figure 2 shows the level diagram of the two quantum dot and gate operation process, which includes three sequential steps:

Firstly, the QDM is illuminated with a \(\sigma^-\) circularly polarized continuous wave (CW) laser \(\Omega_1(t)\) propagating in the \(x\) direction. The laser is tuned such that it creates an exciton in the quantum dot 1 (only if its state is \(|\downarrow\rangle\)) without affecting the quantum dot 2. The Hamiltonian for the QDM under this laser excitation is

\[
\begin{equation}
H_1 = \Omega_1(t)|\uparrow, \downarrow\rangle \langle \downarrow, \uparrow| + \tau|\downarrow, \downarrow\rangle \langle \uparrow, \uparrow| + \text{H.c.},
\end{equation}
\]

where we assume \(\hbar = 1\), and \(\tau\) is the hole tunnelling rate between the two dots. The Hamiltonian has a dark state

\[
|D\rangle \propto \tau|\downarrow, \downarrow\rangle - \Omega_1(t)|\downarrow, \uparrow\rangle.
\]

(3)

If we turn on the \(\sigma^-\) circularly polarized laser and increase the \(\Omega_1(t)\) slowly, when \(\Omega_1(t) \gg \tau\) the state \(|\downarrow, \downarrow\rangle\) will be adiabatically transferred to the state \(|\uparrow, \uparrow\rangle\) almost without exciting the media state \(|\downarrow, \uparrow\rangle\). In this process, the states \(|\uparrow, \downarrow\rangle\) and \(|\downarrow, \uparrow\rangle\) are not affected by the CW laser \(\Omega_1\).

Secondly, we apply a \(\sigma^-\) circularly polarized pulse \(\Omega_2\) to couple the state \(|\uparrow, \downarrow\rangle\) to the state \(|\uparrow, \uparrow\rangle\). The Hamiltonian for the QDM under this laser excitation is

\[
\begin{equation}
H_2 = \Omega_2(t)|\uparrow, \downarrow\rangle \langle \downarrow, \uparrow| + \tau|\downarrow, \uparrow\rangle \langle \uparrow, \downarrow| + \text{H.c.},
\end{equation}
\]

(4)

Starting with the initial state \(|\uparrow, \downarrow\rangle\), the system evolves at the time \(t\) to

\[
|\psi(t)\rangle = \frac{1}{\tau^2 + \Omega_1^2}(|\uparrow, \downarrow\rangle + \Omega_1 \sqrt{\tau^2 + \Omega_1^2} \cos (\tau \sqrt{\tau^2 + \Omega_1^2})|\uparrow, \uparrow\rangle - \Omega_1 \sqrt{\tau^2 + \Omega_1^2} \sin (\tau \sqrt{\tau^2 + \Omega_1^2})|\downarrow, \uparrow\rangle + \tau \Omega_2 \cos (\tau \sqrt{\tau^2 + \Omega_2^2})|\downarrow, \downarrow\rangle + \text{H.c.}).
\]

(5)

At \(t \sqrt{\tau^2 + \Omega_2^2} = \pi\), \(|\psi(t)\rangle \approx |\uparrow, \downarrow\rangle - \frac{\Omega_2}{\tau^2 + \Omega_2^2}|\uparrow, \uparrow\rangle\).

In the case \(\Omega_2 \gg \tau\), \(t_1 \approx \pi/\Omega_2\), \(|\psi(t_1)\rangle \approx |\downarrow, \downarrow\rangle\). The state \(|\uparrow, \downarrow\rangle\)

Figure 2. The energy diagram of the two quantum dot ground states and the optically allowed transitions to the trion states. The large X indicates that the \(\Omega_2\) pulse does not affect this transition while performing a 2\(\pi\) rotation between \(|\uparrow, \downarrow\rangle\) and \(|\uparrow, \uparrow\rangle\).

Figure 1. Schematic of the vertically coupled quantum dot system. The height of the dot 1 is \(h_1\), that of dot 2 is \(h_2\), the interdot barrier is \(d\). A positive electric field along the \(z\) axis is applied to bring the hole levels into resonance.
acquiring a π phase, the transition $|\downarrow\uparrow, \downarrow\rangle \rightarrow |\uparrow\downarrow, \uparrow\rangle$ is blocked because of the Pauli exclusion principle.

Finally, we slowly turn off the CW laser $\Omega_1$ so that the state $|\downarrow\uparrow\downarrow, \downarrow\rangle$ can adiabatically transfer back to the state $|\downarrow, \downarrow\rangle$. The system returns to its original state and only the state $|\uparrow, \downarrow\rangle$ acquires the $-1$ factor, i.e. controlled phase gate.

4. Simulation and conclusion

To simulate the system’s dynamics, we employ a master equation of density matrix $\rho$ [28]

$$\frac{d\rho}{dt} = -i[H_1 + H_2, \rho] + L(\rho),$$

(6)

the superoperator $L$ is given by

$$L(\rho) = \frac{1}{2} \sum_{i=1}^{2} (2L_i \rho L_i^\dagger - L_i^\dagger L_i \rho - \rho L_i^\dagger L_i),$$

(7)

where $L_i = \sqrt{\gamma_i} c_i$ describes spontaneous photon decay in QD $i$.

We choose the laser Rabi frequencies as $\Omega_1 = \frac{5}{2\tau_0} \sqrt{\gamma_1} \exp[-(0.05 t)^2]$, $\Omega_2 = \frac{2}{\tau_0} \sqrt{\gamma_1} \exp[-(2t)^2]$, with $\tau_0 = 1$ ps, $\tau = 2$ meV, and $\gamma_1 = \gamma_2 = 1$ ns$^{-1}$. The dynamics of matrix elements $\rho_{10,01}^{(1)}, \rho_{20,01}^{(1)}, \rho_{01,10}^{(2)}, \rho_{01,11}^{(2)}$, and $\rho_{11,01}^{(2)}$ are shown in figure 3. $\rho_{11,01}^{(2)}$ does not change in this process. It shows that the time of implementation of the phase gate is $T_g \approx 100$ ps, if initial state $|\Psi(0)\rangle = \frac{1}{2} (|1,\uparrow\rangle + |1,\downarrow\rangle + |1,\downarrow\rangle + |1,\downarrow\rangle)$, we calculate the fidelity of the phase gate $F = 0.98$. The fidelity can be improved further if the lifetime of the state $|\downarrow\uparrow\downarrow\rangle$ is increased. Consider this system in a cavity, the state $|\downarrow\uparrow\downarrow\rangle$ will be off resonant with their cavity modes, $\gamma_2$ can be reduced to 1.25 ns$^{-1}$ [29], the fidelity can be as high as 0.998.

In conclusion, we have proposed an all-optical controlled phase gate which benefits from current generation of QDM and adiabatic passage. Our simulation shows the fidelity of the phase gate is exceeding 0.98 by using realistic values for all parameters.

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