Study on hysteretic deformation characteristics and energy dissipation of granite under cyclic loading

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Abstract. The cyclic load test of granite under different paths is conducted using the strain control method. The nonlinear hysteretic deformation behavior is analyzed, and the energy evolution law in the process is described in detail. Three specific discoveries were made. (1) The skeleton curve shape in the cyclic test is less affected by the increase in the upper load limit and shows good loading memory characteristics. (2) The hysteretic characteristics of the stress-strain curve characterize the energy dissipation process. Dissipated energy is divided into plastic hysteresis and damping energy consumption. Damping energy consumption accounts for a considerable proportion of the total energy consumption at all levels, and the changing trend of the two is positively correlated. The upper limit of the cyclic load can significantly change the plastic hysteretic energy. (3) The loading-unloading process is more prone to large hysteresis deformation, and the unloading-reloading process has an unloading threshold. When the unloading amount is less than the threshold, the unloading-reloading curve linearly coincides, and the energy is dissipated during the process. When the unloading amount is greater than the threshold, damping energy will be generated to increase energy dissipation, and the hysteresis characteristic will appear.

Key words. Nonlinearity; energy dissipation; memory; cyclic load; hysteresis threshold.

1. Introduction

Rock is a heterogeneous material composed of multi-component minerals, and many microscopic structures such as voids and fissures are often present in the rock. When subjected to external loads, its mechanical response exhibits nonlinear characteristics such as hysteresis and discrete memory [1]. Engineering practice shows that rock masses are often subject to cyclic loading during underground construction and operation activities such as underground cavern excavation and support, underground gas and fuel storage, mineral resource mining. Recently, many scholars have studied the rock mechanical response under cyclic loading. Xi et al. [2] conducted periodic load experiments at different frequencies under saturated and dry conditions and analyzed rock deformation lag characteristics under saturated conditions. Chen et al. [3] conducted multi-level cyclic loading experiments and found that the hysteresis loop in the stress-strain curve was sharp-lobed, and its position shifted toward the increasing strain with an increase in stress amplitude. He et al. [4–6] designed three cyclic load experiments under different stress paths to analyze the damping characteristics of rocks. Zhu et al. [7] found that when a dynamic load of 0.01 to 1 Hz disturbed the granite sample, the hysteresis loop area, dynamic elastic mode, and damping ratio
parameters increased with increasing frequency. Xiao et al. [8] conducted a constant amplitude cyclic load test of granite, and concluded that the stress and strain phases during loading and unloading could lag, equal, or lead with the increase in the number of cycles, and lead to different forms of hysteresis loops, such as elliptic, crescent, and long eggplant shapes. Nie et al. [9] evaluated the damping and dynamic elastic parameters of marble under different confining pressures and found that the damping ratio and damping coefficient decreased as the number of cycles increased.

Deng et al. [10] found that the hysteresis loop gradually filled with increasing frequency and gradually narrowed with increasing amplitude. The circulation frequency and amplitude have obvious effects on rock-damping characteristics. Liu et al. [11] studied the relationship between the dynamic elastic modulus and damping ratio as the dynamic strain changes. They found that the dynamic elastic modulus decreases linearly with the increase in dynamic strain, whereas the law of damping ratio increases linearly. Yang et al. [12] studied the correspondence between acoustic emission characteristics and non-uniform deformation of rock materials during cyclic loading and unloading. Song et al. [13] measured the damping ratio of gypsum samples under external excitation and studied the effect of strain amplitude, vibration frequency, and vibration waveform on the rock-damping ratio. Tutuncu et al. [14] obtained the cyclic stress-strain hysteresis curve characteristics of sedimentary rocks related to factors such as load frequency and strain amplitude. Liu et al. [15] established a damage structure model based on the law of energy dissipation in the circulation process.

Han et al. [16] conducted the cyclic test of increasing amplitudes of jointed limestone and granite samples. They found that rock joints have nonlinear mechanical behavior during loading and nearly linear behavior during unloading in the tensile stress-strain curves. Fu et al. [17] investigated the crack development and mechanical behavior of marble under three cyclic loadings. The results showed that cyclic loadings could improve the crack closure threshold. The crack initiation threshold, crack damage threshold, cohesion, and elastic modulus initially increase with the cycles, followed by a decrease as the rock heads toward failure.

Most results focus on the analysis of the influence of different control factors on the hysteresis loop shape and dynamic parameter changes. However, the separation of the hysteresis area in the stress-strain curve and the deformation mechanism reflected by it lack detailed analysis. The unload-reload curves do not coincide with the adjacent cyclic stages, the phenomenon of circle-in-loop is little discussed, and the significance of its representative is rarely described. This study studies the effects of strain amplitude and level on the nonlinear hysteretic deformation characteristics of granite samples by conducting cyclic loading and unloading tests under strain control. Each hysteresis area in the stress-strain curve is extracted, its energy evolution law is described, and the mechanism of rock hysteretic deformation under cyclic load from the perspective of energy is explained.

2. Test and analysis methods

2.1. Test samples and instruments

The granite was collected from Xinjiang, China. A standard cylindrical sample of Φ50 mm × 100 mm was drilled from the granite rock, and the mass and size were measured. The parallelism of the end face was controlled within ±0.02 mm. An ultrasonic testing instrument was used to measure the wave velocity, and the sample with a large wave velocity dispersion was eliminated, reducing the result data dispersion caused by the uneven occurrence of cracks in the sample. Table 1 shows the final selected samples divided into four groups and the basic physical information.

| NO | Size (mm × mm) | Quality (g) | Density (g/cm³) | Wave velocity (m/s) |
|----|----------------|-------------|-----------------|---------------------|
| A1 | Φ49.63 × 100.00 | 503.80      | 2.60            | 4494                |
| A2 | Φ49.66 × 99.44  | 503.39      | 2.62            | 4519                |
| A3 | Φ49.60 × 99.14  | 508.77      | 2.66            | 4260                |
| B1 | Φ49.57 × 98.82  | 509.28      | 2.67            | 4277                |
The four sample groups were evaluated according to the scheme listed in Table 2. The samples of group A were used to determine the average value of the stress-strain peak of granite. The amplitude of the cyclic test is divided as a reference, and the shapes of the uniaxial test curve and cyclic test skeleton curve are compared and analyzed. The samples of group B were subjected to constant amplitude cyclic tests to analyze the characteristics of hysteretic deformation under the condition of constant amplitude. Group C and group D were subjected to cyclic tests under different strain increments. Among them, the lower limit of cyclic strain at each level of the group C test was constant, and the upper limit of strain increased with the levels. The upper and lower limits of the cyclic strain at all levels in the test of group D increased with the levels to compare and analyze the effects of changes in the strain amplitude and level on hysteretic deformation.

Table 2. Experiment programs

| NO | Experiment programs                                      | Samples          |
|----|----------------------------------------------------------|------------------|
| ① | Uniaxial compression test                               | A1, A2, A3       |
| ② | Cyclic test of constant amplitude                       | B1               |
| ③ | Cyclic test of the upper limit increases and the lower limit is constant | C1, C2, C3 |
| ④ | Cyclic test of increasing upper and lower limits         | D1, D2, D3       |

2.2. Hysteresis characteristics and energy analysis methods

When calculating energy using the cyclic stress-strain curve, the area under the loading curve represents the total input strain energy, and the area under the unloading curve represents the recoverable strain energy of the sample after unloading (considered as elastic strain energy). The area of the hysteresis loop enclosed by the curve represents dissipated energy of the level.

Figure 2(a) and 2(b) shows that the loading-unloading curves of the linear elastic and perfect elastic materials coincide. Under cyclic loading, the external work done on the sample is converted into elastic strain energy for storage and release, and there is no energy dissipation during loading and unloading. The loading-unloading curves in Figure 2(c) do not coincide, and there is energy dissipation during loading and unloading. The residual strain (OB) is generated after the cycle ends.
The plasticity and frictional damping characteristics of the sample affect the shape of the hysteresis loop.

![Diagram of stress-strain curves](image)

**Figure 2.** Stress-strain curves of materials with different properties [18]

Some results show that the cyclic loading-unloading curve (L-U curve) of most rock samples under cyclic loading is not coincident, and the shape of the enclosed hysteresis loop is similar to Figure 2(c). The shape is open at the lower end, and its position gradually moves toward the increasing strain as the cycle number increases [19,20]. Furthermore, by extracting the unloading-reloading curve (U-R curve) between the adjacent cyclic series, the two do not coincide. In the hysteresis loop enclosed by the L-U curve, the phenomenon of circle-in-loop appears. Unlike the open hysteresis loop formed by the L-U curve, the circle-in-loop formed by the U-R curve is a closed area (Figure 3).

![Diagram of cyclic stress-strain curve](image)

**Figure 3.** Schematic diagram of cyclic stress-strain curve

The B-C-B process in Figure 3 undergoes cyclic behavior, does not contribute to the overall deformation, and has a certain area to characterize the existence of energy dissipation. This study defines it as the damping ring. The energy represented by area ① is defined as damping energy, and the energy represented by area ② is defined as plastic hysteresis energy because it promotes a new residual strain in the sample. Now, the energy dissipation in the cyclic process CDE can be divided into two, namely damping energy dissipation and plastic hysteresis energy dissipation. The author has conducted some research and analysis regarding the relationship between the two types of energy and deformation [21].

3. Experimental results and analysis

3.1. Constant amplitude cyclic test

Using the uniaxial compressive strength test of the loading rate of 0.004 mm/s, the average compressive strength of the granite samples is 117.42 MPa, and the peak strain is 1.34%. The rate of
group B under constant amplitude cyclic loading and unloading test is also set to 0.004 mm/s, the upper limit of cyclic stress is 50 MPa, and the lower limit is 1 MPa. Figure 4 shows the stress-strain curve of constant amplitude cyclic loading and unloading. The test was performed for 60 cycles, and the sample was not damaged. To facilitate the display of hysteresis loop morphological characteristics, Figure 4 shows only the first three cycles.

Figure 4 shows that the stress-strain curve shows obvious hysteresis characteristics. The area of the L-U curve of the first cycle is larger than the subsequent stages. The starting and ending points of the curves do not coincide, and there is a large residual strain. Some literature argues that it is because the initial stress causes the voids in the sample to compact [10,20]. Morphologically, except for the first-level, the subsequent L-U curves form an approximately closed hysteresis loop and show good memory characteristics. As the number of cycles increases, the hysteresis loop gradually moves to the right, but the deviation is small. During the process, the newly added residual strain at each level is low. After calculation, the difference between the hysteresis area in the second and subsequent cycles is within 2%, and the area of the damping ring accounts for more than 97% of the hysteresis loop of each stage. Therefore, the energy consumption at each level is relatively stable for the constant amplitude cyclic test with constant upper and lower limits at all levels. The constant amplitude slowly accumulates the damage of the sample, and the newly added plastic hysteresis energy consumption at each level accounts for a relatively small amount of energy dissipation. Energy loss is mainly based on the damping energy consumption.

3.2. Cyclic test with increasing upper limit and constant lower limit
Using strain as a control parameter, three groups of cyclic tests under increasing strain upper limit conditions were conducted. The average peak strain of the sample measured by the uniaxial test is 1.34%. Ensuring that the data are within the accuracy range of the equipment and that there is sufficient cyclic series for analysis, the upper limit increment of the cyclic test is set to 0.05%, 0.1%, and 0.2%. Table 3 shows the control parameters of group C, and Figure 5 shows the loading and unloading path of the C2 sample.

| Sample | Upper limit strain increment | Strain lower limit |
|--------|-----------------------------|-------------------|
| C1     | 0.05                        | 0.05              |
| C2     | 0.1                         | 0.1               |
| C3     | 0.2                         | 0.2               |
Figure 5. Cyclic test path with increasing upper limit and constant lower limit (C2)

Figure 6 shows the stress-strain curve of group C. The area of the hysteresis loop gradually increases as the number of cycles (upper limit of strain) increases, and the hysteresis effect gradually becomes more prominent. The deviation of the curve in the loading section is relatively small, whereas the degree of hysteresis in the unloading section gradually increases, which is the principal reason for the increasing hysteresis loop. Therefore, compared with group B, the increase in the upper limit of strain causes new damage to the sample during the loading of the new stage; therefore, more energy is consumed, and the hysteresis area enclosed by the L-U curve gradually increases. The intersection point of the U-R curve is near the upper strain point of the front stage. By connecting the upper limit points of the strain at all levels, a skeleton curve like the uniaxial compression curve path can be formed (Figure 7).

Figure 7 shows that by comparing the loading memory paths of different strain upper limit increments, the skeleton curve coincides with the uniaxial compression curve, and the hysteresis area changes caused by the increase in the upper limit of the cyclic strain negligibly influence the skeleton curve.

Figure 6. Stress-strain curve with increasing upper limit and constant lower limit

(a) C1 (Δ = 0.05%)  (b) C2 (Δ = 0.1%)  (c) C3 (Δ = 0.2%)
Figure 7. Skeleton curves of different strain increments

Figure 8 shows that by analyzing the changes in the hysteresis loops and damping cycles at various levels, the law of energy conversion with the increase in the strain amplitude is obtained. Figure 8(a) shows that as the upper limit of strain increases, the total strain, elastic strain, and dissipated energy show an increasing trend. The elastic strain energy increases rapidly, whereas the dissipated energy increases slowly, and the increasing strain amplitude has negligible effect on energy conversion. Figure 8(b) shows that both the damping and plastic hysteretic energy increase continuously with the increase in the upper limit strain. The growth of the two is slow before 0%–0.5% and relatively stable between 0.5% and 1.0%, the difference between the two is small. After that, the growth is rapid, and the fluctuation is large. The changing trend of damping energy dissipation is similar to the trend of total dissipation energy, and the influence of this energy on the total dissipated energy is more prominent.

Figure 8. The law of energy evolution in cyclic tests

Note: W is total strain energy at all levels, We is elastic strain energy at all levels, Wd is dissipated energy at all levels, Wp is plastic hysteretic energy at all levels, and Wd is damping energy dissipation at all levels.

Figure 9 shows that by analyzing the proportion of damping energy consumption and plastic hysteretic energy dissipation at all levels in the total dissipated energy, the proportion of damping energy dissipation in the total dissipated energy is greater than the plastic hysteresis dissipation as a whole. We found that when the upper limit of strain is increased by $\Delta = 0.05\%$, the proportion of
the two fluctuations is higher, and the plastic hysteretic energy consumption ratio of some cycles in the C1 sample is greater than the damping energy consumption. This study found that the main reason is the unstable development of the internal damage of the sample under cyclic loading and the problem of equipment control accuracy, reflecting that these two energies are more sensitive to the amplitude change.

![Figure 9](image_url)

**Figure 9.** Proportion relationship between damping energy and plastic hysteretic energy

### 3.3. Cyclic test with increasing upper limit and lower strain limits

Strain is also used as a control parameter for testing, and the cyclic loading and unloading rate is also 0.004 mm/s. The upper and lower limits of the cyclic strain at each level in group D increase with the cycle number. The unloading amount of the strain at each level is half of the loading amount. Table 4 shows the control parameters, and Figure 10 shows the test loading and unloading path of the D2 sample.

| Sample | Upper limit strain increment $\Delta'$ (%) | Lower limit strain increment $\Delta''$ (%) |
|--------|------------------------------------------|------------------------------------------|
| D1     | 0.05                                     | 0.05                                     |
| D2     | 0.1                                      | 0.1                                      |
| D3     | 0.2                                      | 0.2                                      |

![Table 4](image_url)

**Table 4.** Loading parameters of group D

![Figure 10](image_url)

**Figure 10.** Cyclic test path increasing upper and lower limits (D2)

Figure 11 shows the stress-strain curve of group D. The hysteresis of the L-U curve also occurs
throughout the test. When the curve is in a low strain stage (0%–0.5%), the L-U curve has a high degree of coincidence, the hysteresis is not obvious, and the curve is concave. When the upper limit of strain exceeds 0.5%, the sample is loaded into the elastic stage, and the hysteresis characteristics between the L-U curves gradually becomes more obvious as the upper limit of strain increases. Comparing the curves under three strain increments, it is found that with the increase in strain increments at all levels, the hysteresis characteristics between L-U curves will become more obvious.

Figure 11(a) and (b) show that when the unloading volume at each level is small, the U-R curve exhibits linear elastic characteristics (Figure 2(a)). The unloading and reloading paths overlap and merge along the same line. When the unloading amount exceeds a certain value, a microclosed damping ring is formed (Figure 11(c)) where the U-R curve does not coincide, but this area is also relatively small, and the hysteresis characteristics are not obvious. By comparing the curves of group B and group C, the unloading capacity of the two groups of circulation curves are much larger than that of group D. Therefore, the damping ring formed by the U-R curve is also larger, and the hysteresis characteristics are obvious.

The above results show that the size of the unloading amount significantly influences the hysteresis characteristics of the reloading process. An unloading hysteresis threshold exists in the granite sample. When the unloading amount is lower than the threshold, no obvious hysteresis in the U-R curve exists and it exhibits linear elastic characteristics. There will be no energy loss in the reloading process before the upper limit of the upper cycle is exceeded, and the main energy consumption will be mainly
plastic hysteresis energy after it is exceeded. When the unloading amount is greater than the threshold value, a hysteresis loop will occur in the U-R curve. Now, even if the reloading process does not exceed the upper limit of the upper cycle, a part of the energy will be converted into damping energy and be dissipated. The curve of group D shows that when the unloading strains are 0.05% and 0.1%, the U-R curve does not form a damping ring, and when it is increased to 0.2%, a microdamping ring appears. Therefore, the unloading hysteresis threshold of this batch of granite samples is 0.2% (approximately 15% of the average peak strain).

Figure 12 shows the changes in the modulus of the loading and unloading sections at all levels. As the number of cycles increases, both the loading and unloading modulus increase. The cyclic unloading modulus of each level is greater than the loading modulus, and the difference between the two gradually increases. The fitting curves of the two are in the form of a quadratic polynomial, tending to increase rapidly and then gradually slowing. The increase in the difference between the loading and unloading modulus is positively correlated with the hysteresis characteristics.

\[ y = \text{Intercept} + B_1 x^1 + B_2 x^2 \]

In summary, by analyzing and calculating the proportion of the damping ring formed by the U-R curve in the hysteresis loop formed by the L-U curve, the energy analysis is more refined. Through the opening and closing of existing cracks in the sample and the rolling friction of free particles in the cracks, damping energy consumes the energy input from the outside but does not affect the overall unrecoverable deformation. The plastic hysteretic energy consumption is related to the strain upper limit increment. When the sample has greater deformation, more internal damage will occur (new crack initiation accompanied by more free particles). In the tests of group C and group D, the upper limit of strain at each level increases with the cycle number. Now, new damage is continuously...
generated inside the sample, the destruction and friction behavior provide more channels for energy consumption, and the hysteresis phenomenon becomes more obvious. The results of the three sets of cyclic tests in this paper show that when it is less than the unloading threshold, no damping energy is generated during the reloading process. After the unloading threshold is exceeded, the increase in the unloading amount will cause the new loading behavior to consume more energy for conversion into damped energy. Because of the constant strain upper limit, only a small amount of plastic hysteretic energy can be generated in the constant amplitude-cycling process, whereas the increasing upper limit strain level significantly influences the plastic hysteresis. The continuous damage of the sample increases the plastic hysteresis energy and provides a source for the continuous accumulation of damping energy consumption. Therefore, under a certain loading and unloading amount, the ratio of damping energy becomes a representation of the destruction degree of the sample.

4. Conclusions
By conducting cyclic loading and unloading tests of three schemes, the hysteretic deformation characteristics of granite samples were discussed, and the energy evolution law during the tests was refined and analyzed.
(1) The skeleton curve of the graded cyclic loading and unloading test has good deformation memory characteristics. The changes in strain increments at all levels negligibly influence memory and has little effect on the evolution of energy distribution.
(2) Separating the damping ring area from the hysteresis loop, and defining the dissipated energy in each stage of the cycle comprises damping and plastic hysteretic energy, the energy dissipation analysis becomes more detailed. The damping energy dominates the energy dissipation in the granite sample constant amplitude cyclic test. In the staged cyclic test with a constant lower limit and increasing upper limit, damping energy dissipation accounts for a large proportion of the total dissipated energy, and the changing trends of the two are in good agreement.
(3) The hysteretic behavior of the granite sample is stable in the L-U stage, whereas the hysteretic behavior of the U-R stage is related to the unloading threshold. When it is less than the threshold value, the U-R curve coincides. When it becomes greater than the threshold value, the U-R curve forms a microclosed damping ring, and the energy input during the reloading phase will be partially converted into damping energy and consumed.

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