Quantum Mechanics as a Classical Theory III: Epistemology

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Abstract
The two previous papers developed quantum mechanical formalism from classical mechanics and two additional postulates. In the first paper it was also shown that the uncertainty relations possess no ontological validity and only reflect the formalism’s limitations. In this paper, a Realist Interpretation of quantum mechanics based on these results is elaborated and compared to the Copenhagen Interpretation. We demonstrate that von Neumann’s proof of the impossibility of a hidden variable theory is not correct, independently of Bell’s argumentation. A local hidden variable theory is found for non-relativistic quantum mechanics, which is nothing else than newtonian mechanics itself. We prove that Bell’s theorem does not imply in a non-locality of quantum mechanics, and also demonstrate that Bohm’s theory cannot be considered a true hidden variable theory.

1 Introduction
The first two papers developed quantum mechanics within the fundamental principles of it’s mathematical formalism. For this, we use no more than classical mechanics and two rather natural postulates that do not modify its classical character.

We demonstrated in the first paper that the fundamental equations are those that involve the density function. We also demonstrated that the uncertainty relations are not valid for these equations and are, for this reason, an indication of the formalism’s limitation of using equations for the probability amplitudes. The uncertainty relations form the basis of the Copenhagen Interpretation[1, 2] and the contestation of their ontological character has profound implications for quantum mechanics epistemology.

A model in which the introduction of an observing system in a specific quantum problem is considered was constructed. This observing system has profound
differences from those introduced by quantum mechanics’ diverse measurement
theories[3]–[9].

In the present paper, we will develop a Realist Interpretation for quantum
mechanics based on the results obtained in the first and second papers (hereafter
abbreviated by (I) and (II)).

In the second section, we will quickly review the fundamental ideas of the
Copenhagen Interpretation.

In the third section, we will establish a Realist Interpretation for quan-
tum mechanics based on the results of this series’ first two papers[10]. Certain points
of the Copenhagen interpretation presented in the previous section will also be
criticized.

The fourth section will show that von Neumann’s demonstration of the im-
possibility of a hidden variable theory is not correct, independently of Bell’s
argument[11], which we prove inadequate. A local variable theory is then found
for the non-relativistic problem which is nothing more than Newtonian mech-

ics.

In the fifth section, we demonstrate that Bell’s theorem does not imply in a
non locality of quantum mechanics.

In the sixth section, we reinterpret Bohm’s theory[7, 12] and show that it
cannot be considered a true hidden variable theory.

In the last section, we make our final conclusions for this series of papers.

2 The Copenhagen Interpretation

We will present the main ideas of the Copenhagen Interpretation through axi-
oms[13]. We are not interested in the formalism, but in the interpretation which
accompanies it. For this reason, the formal apparatus will not be developed.

Let us first define the meaning of quantum state:

(def1) The state of a physical system is represented by \( \phi \), a function of \( s \)
coordinates. This function does not necessarily represent any distribution of
objects within space, because the \( s \) variables that index it possess no intu-
tive association with objects. It is not defined in terms of observables but
only as a function within configuration space. These \( s \) variables represent
the system’s degrees of freedom.

The Copenhagen Interpretation’s axioms are:

(CAx1) The function \( \phi \), which, in general, can be complex should be square
integrable.

(CAx2) The function \( \phi \) should be single valued.

(CAx3) For every observable \( p \) there is a single operator \( P \) acting upon the
state function. In particular, we obtain Schroedinger’s equation (for the
amplitudes) substituting, in the Hamiltonian function, the variables \( q \) and \( p \) by their respective operators.

**(CAx4)** The only possible values that can be observed when a measurement is made over an observable \( p \) are the eigenvalues of the following equation

\[
P\psi_\lambda = p_\lambda \psi_\lambda,
\]

where \( \psi_\lambda \) satisfies axioms (CAx1) and (CAx2).

**(CAx5)** When a system is in a certain state \( \phi \), the expected value of a series of measurements of the observable \( p \) is

\[
\bar{p} = \frac{\int \phi^* P \phi d\tau}{\int \phi^* \phi d\tau},
\]

where \( P \) is the operator which corresponds to \( p \).

All of quantum mechanical formalism can be obtained from these five axioms. We will not do this here. Nevertheless, one particular result which interests us will be presented, without demonstration, as the following theorem:

**(CT1)** If \( Q \) and \( P \) are canonically conjugated operators, then the dispersion associated with the simultaneous measurements of their eigenvalues is given by

\[
\left[ Q, P \right] = i\hbar.
\]

Even though this result is a theorem, it’s interpretation is fundamental for the Copenhagen Interpretation of quantum mechanics and it is called the Uncertainty Principle. Thus, according to the Copenhagen Interpretation\[14\], we have the following interpretative postulates:

**(CP1)** In a given experiment, it is not possible to measure with absolute precision both the position and the momentum of a given particle. The minimum dispersion in such a measurement is given by relation (3).

Because it adequately describes nature’s behavior, this postulate possess ontological status. Nevertheless, this description is not objective, but subjective, and entails:

**(CP2)** Any physical system’s attributes, for example, its trajectories, only come into existence when observed.

and more,

**(CP3)** An inevitable and uncontrollable interaction occurs between the measured object and the observer.
The dispersion relation \(3\) served as the mathematical element for Bohr to expose his ideas about the complementarity which he had been developing since his first contact with the dual wave-particle behavior of some experiments. Once it is assumed that there can be no space-time coordination, causality and the notion of a complete classical description become obsolete\[15\]. Thus

\(\text{(CP4)}\) Combining a classical observational system with a quantum system, one can only measure complementary values and, to express them in classical terms, the system must also be described in terms of classical figures which are also complementary.

Finally, to establish the state function’s referent, it is postulated that

\(\text{(CP5)}\) The wave function expresses our knowledge of events.

In relation to the influence of the act of observing on measurement, we can say that

\(\text{(CP6)}\) An unforeseeable and discontinuous reduction of the state vector, formally represented by a projection operation occurs during the act of measurement.

These are, shortly put, the main ideas which form the Copenhagen Interpretation. In the next section we will criticize these ideas through the results obtained in this series of papers.

3 The Realist Interpretation

In this section we will demonstrate how a realist view can be made compatible with quantum mechanics’ formalism as developed in the two previous papers.

First, it is important to note that the observer, massively discussed in the various quantum mechanical interpretations, does not play any role in it’s formalism. When any problem is to be solved, such as to find the energy levels of an atomic system for example, the possible interactions between this system and the external world are never formally taken into account. This can be seen in the present formalism through the derivations of the quantum equations from the Liouville’s equation for a closed system. The introduction of the observer into the orthodox epistemology is not only \(ad hoc\), it is also incompatible with quantum mechanic’s main postulate, which is Schroedinger’s equation, since a discontinuous reduction of the state vector, which does not satisfy this equation\[8\], must be considered when the observer is taken into account.

Here we wish to differentiate statements such as "the value of the property \(P\) of the physical object \(y\) is equal to \(x\)" from others such as: "the observer \(z\) found the value \(y\) for the property \(P\) of the physical object \(x\) using the measurement
technique \( t \) and the sequence of operations \( o \). In fact, while the first proposition can be mathematically represented as \( P(x) = y \), the second one should be given as \( P'(x, z, t, o) \), so that \( P \) and \( P' \) possess radically different referents\[16\].

We can give an example of these concepts using the treatment given to an external observer system as applied to Young’s interference experiment in (I). In the usual treatment given through Schroedinger’s equation, we have a state function which will depend only on the variables \( F(x_1, p_1; t) \) associated to the system’s internal degrees of freedom. In the treatment given in (I), we should consider not only \( F(x_1, p_1; t) \), but also the observing system’s probability density function \( F(x_2, p_2; t) \). Any variation of the pragmatic parameters which define this distribution can alter the probability distribution which we intend to calculate.

In the measurement theory associated to the Copenhagen Interpretation, the observer’s conscience must be postulated to avoid an infinite regression of reductions in the state vector\[3, 4, 5, 8\]. This infinite regression can, nevertheless, be easily understood through the present theory’s point of view. As was observed by Everett\[6\], the question about the infinite regression of reductions in the state vector is intimately linked to considerations of open and closed systems. When studying the Young’s interference experiment, we considered \( \text{system2} \) as an external observer and take into account its interaction with \( \text{system1} \) through the concept of scattering cross-section. If the \( \text{system1} \) and \( \text{system2} \) are to be considered as part of a greater system, then their interaction will be taken into account exactly and the equation representing the probability distribution of the whole system will be \( F(1, 2) \), which is, in general, very different from the uncoupled distributions \( F_1(1) \) and \( F_2(2) \) used to represent the open system.

In fact, the equation obtained in (I) for the probability density function of this experiment can no longer be transformed, through the Wigner-Moyal Infinitesimal Transformation, into the Schroedinger equation for the probability amplitude. Thus, the observed system’s wave-like behavior of the ensemble’s statistics no longer needs to manifest itself. It will manifest itself depending on the intensity of the perturbation introduced by the observer system.

Of course, this is not an uncontrollable interaction between the measuring apparatus and the observing system. In fact, this idea emerged from Bohr’s quantum postulate which saw the finite character of Plank’s constant, interpreted by him as associated with the minimal quantity of energy a system can emit or absorb, as representing the impossibility of analysis. Note that, as was shown in (I), Plank’s constant can’t be used as a characteristic entity of a quantum system since that which is conventionally called the classical limit is independent of it.

Schroedinger’s equation for the probability amplitudes represents systems in equilibrium. The absorption or emission of energy in these systems is done in such a manner that the system passes from one equilibrium state to another. It is this that we call the quantum jump. Of course, the non-equilibrium states are not subjected to such considerations (two-photon absorption is one example
of such a situation). Observing the equation in which the external observing system is introduced through a Boltzmann equation, it can be seen that the quantum equation obtained does not, in general, have eigenvalues. Therefore, it is neither associated with quantized values of energy nor equilibrium situations: it does not represent quantum jumps. *Natura non facit saltus.* It should be also mentioned that, in the second paper, it was shown that strong gravitational fields will, in general, deny equilibrium situations and no quantization is expected. So, quantization cannot be considered as a fundamental manifestation of Nature.

We note also that in this interpretation the problem of considering the frontier between the classical system, usually the external observer, and the observed quantum system, does not occur. In the same manner, there is a symmetry in treatment, because the observing system can be considered an observed system and *vice-versa,* this being an important property if we desire to make relativistic generalizations of our arguments.

It becomes clear from the discussion above that a general theory of measurement, as was proposed by von Neumann, has few chances of success since, to admit such a theory, supposing that the description of the observer should appear in the equations, we must also admit that there is a way to unify, under the same formal apparatus, the innumerable existing observational techniques. Having clarified the issue of the observer in quantum mechanics, we pass to a criticism of the postulates presented in the previous section:

(CP1) This postulate, as has been observed, has no ontological validity. Instead of determining a limitation to the classical concepts of space-time coordination, it determines a limitation of the quantum formalism given by Schrödinger’s Second Equation for the probability amplitudes.

(CP2) Without the uncertainty relations, this postulate suggests a confusion between the concepts of having a known value and having a definite value. Unlike what is stated by the Copenhagen Interpretation, the variables which index the density function do refer to the state of the ensemble’s constituent components. Even if we do not know these values in a specific experimental arrangement, and for this reason treat them statistically, we assume that these values are well defined. In fact, in the limit of dispersion free ensembles, we reobtain newtonian trajectories.

(CP3) We have seen that this postulate is associated to the fact that the Schrödinger equation for the probability amplitudes describes systems in an equilibrium situation and is therefore incapable, in this format, of giving information to the inter-phenomenon occurrences. In any manner, when we introduce the external observing system, we perceive that its interaction with the observed system is classical and controllable (deterministic), even if it is considered unknown and treated statistically.
(CP4) Once we have denied the ontological validity of the dispersion relations and assumed the figure of space-time coordination, we can no longer accept this postulate.

(CP5) There are no external psycho-physical observers. The density function supplies an objective description of the physical world and does not possess any relation with mental activities.

(CP6) See the criticism to postulate (CP3) and the discussion before it.

In relation to the wave-particle duality, it has been demonstrated that all the results in which the systems present wave-like behavior can be explained through the quantized interaction which occurs between them\cite{17, 18} through the use of the particle picture.

We are, at this point, ready to accept the definitions of reality and completion given by Einstein, Podolsky and Rosen\cite{19}. Yet there is, still, one problem. It has been recently proven that realist theories, called hidden variable theories, should possess non local behavior. Even if this behavior can be accepted by a realist theory, it contradicts the relativistic theories from which we derived quantum formalism presented in (II) (remember that, since there is no conscious observer in this theory, no distinction is made between signals and information as is usually done by those who wants to suppress the above cited contradiction). Such behavior is therefore, inconceivable within such formalism.

We will demonstrate, in the next sections, that quantum mechanics is a local theory.

4 Hidden Variables

In the previous section it was seen that, in sight of the results obtained in (I) and (II), quantum mechanics admits an epistemology completely different from that accepted as orthodox since the Solvay Congress in 1927. It has also been shown that this theory, of statistical character, is built upon a totally deterministic and local theory, which is no more than newtonian mechanics (in a non-relativistic approximation). In this manner, newtonian mechanics can be considered the quantum mechanic's hidden variable theory.

If this is true, we should demonstrate that von Neumann’s proof\cite{3} on the impossibility of such a theory is mistaken.

In this section and the following, we will demonstrate that newtonian mechanics can be considered the hidden variable theory for quantum mechanics and that von Neumann’s argument about the impossibility of a hidden variable theory is incorrect, independently of Bell’s argumentation\cite{11}.

Von Neumann’s axioms are

(A1) If an observable is represented by the operator $R$, then this observable’s function $f$ is represented by $f(R)$. 

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(A2) The sum of many observables $R, S,..$ is represented by the operator $R + S + ..$ whether they commute or not.

(A3) The correspondence between operators and observables is one to one.

(A4) If the observable $R$ is non negative, then its mean value $< R >$ is also non negative.

(A5) For arbitrary observables $R, S,..$ and arbitrary real numbers $a,b,..$ we have

$$\langle aR + bS + ... \rangle = a \langle R \rangle + b \langle S \rangle + ...,$$

for all the possible ensembles for which the mean values can be calculated.

From these axioms, von Neumann obtains the density operator $\rho$ by construction, together with it’s properties. Among these properties is it’s use for the calculation of the observable’s mean values

$$Tr (\rho R) = \langle R \rangle.$$  \hspace{1cm} (5)

Yet, von Neumann argues that, for any hidden variable theory, we should have dispersion free states for which

$$\langle R^2 \rangle - \langle R \rangle^2 = 0.$$  \hspace{1cm} (6)

Using the result (5) in (6) for the observable

$$R = |\phi \rangle \langle \phi|,$$

we reach

$$\langle \phi | \rho | \phi \rangle = \langle \phi | \rho | \phi \rangle^2,$$  \hspace{1cm} (8)

for every amplitude $|\phi \rangle$.

In this case, von Neumann concludes that $\rho = 0$ or $\rho = 1$. The first hypothesis has no physical meaning and the second does not imply a dispersion free state for vector spaces with more than one dimension. In fact, in case of a space of dimension $d$ we get

$$Tr (\rho) = Tr (1) = d,$$  \hspace{1cm} (9)

Thus, the expression to the left of (6) becomes

$$\langle R^2 \rangle - 2 \langle R \rangle^2 + \langle R \rangle^2 \langle 1 \rangle,$$  \hspace{1cm} (10)

which is not equal to zero if $d \geq 2$. Thus, it is not possible to construct a hidden variable theory compatible with quantum mechanics.

With the advent, in 1952, of a supposedly hidden variable theory totally compatible with quantum mechanics, it became necessary to demonstrate that von Neumann’s demonstration contained some mistake. To do this, Bell defended that the fifth axiom was responsible for this inconsistency; his argument was the following[20].
B-arg.: "At first sight the required additivity of expectation values seems very reasonable, and it is rather the non-additivity of allowed values (eigenvalues) which requires explanation. Of course the explanation is well known: A measurement of a sum of non commuting observables cannot be made by combining trivially the results of separate observations on the two terms - it requires a quite distinct experiment. For example the measurement of $\sigma_x$ for a magnetic particle might be made with a suitably oriented Stern-Gerlach magnet. The measurement of $\sigma_y$ would require a different orientation, and of $(\sigma_x + \sigma_y)$ a third and different orientation. But this explanation of the non-additivity of allowed values also established the non triviality of the additivity of expectation values. The latter is quite a peculiar property of quantum mechanical states, not to be expected a priori. There is no reason to demand it individually of the hypothetical dispersion-free states, whose function it is to reproduce the measurable peculiarities of quantum mechanics when averaged over."

Obviously this argument cannot be accepted by the present theory. For the present theory, measurements are done in quantum mechanics exactly as in classical statistical mechanics and therefore possess the same characteristics. The argument says, beyond this, that the non-triviality of measures is due to the non-commutativity of the observables which one is measuring. Since we have proven that this non-commutativity has no ontological validity, we should reject this argument.

It can, nevertheless, be shown that von Neumann's demonstration is incorrect. Let us now pass on to this demonstration:

The ensembles' state is determined, in phase space, by the density functions $F(x, p; t)$. For a realist theory, a dispersion free ensemble constituted by $N$ particle systems is represented by the product

$$ F(x_1, x_N; p_1, p_N; t) = \prod_{i=1}^{N} \delta (x_i - x_{0i}(t)) \delta (p_i - p_{0i}(t)), \quad (11) $$

where each pair of Dirac's delta functions determines the trajectory - deterministic, causal and given by newtonian mechanics - of one of the components of the ensemble's constituent systems.

Using the Wigner-Moyal Infinitesimal Transformation we obtain the density function

$$ \rho(x_1, \Delta x_1; \ldots; x_N, \Delta x_N; t) = \prod_{i=1}^{N} \delta (x_i - x_{0i}(t)) \exp \left[ \frac{i}{\hbar} \mathbf{p}_{0i}^\dagger (t) \cdot \Delta \mathbf{x}_i \right], \quad (12) $$

using $\Delta x$ in order not to confuse them with the Dirac's delta functions. Taking the limit $\Delta x \to 0$, we obtain the density

$$ \rho(x_1, \ldots, x_N; t) = \prod_{i=1}^{N} \delta (x_i - x_{0i}(t)), \quad (13) $$
as expected. Integrating this expression we find

$$\int \rho(x_1, \ldots, x_N; t) \, dx_1 \ldots dx_N = 1, \quad (14)$$

which is in clear contradiction with expression (10). In this manner we always have expression (10) equal to zero. It is important to note that the operations of taking the limit in (13) and integrating in (14) are equivalent to take the trace \( \text{Tr} (\rho) \).

We can therefore say that, from expression (10) and for dispersion free states, we cannot conclude, as von Neumann did, that \( \rho = 0 \) or \( \rho = 1 \).

Note that we demonstrated the incorrectness of von Neumann’s theorem above and, simultaneously showed that newtonian mechanics may be the hidden variable theory behind quantum mechanics. Yet this theory is local. Bell, extending the argument above, proved through a theorem, that every hidden variable theory should be non-local. We should, therefore, analyze his theorem.

5 Bell’s Theorem

Consider two meters measuring two particles which are the product of a physical system’s dissociation. The results given by these meters are represented by \( A(a, \lambda) \) and \( B(b, \lambda) \), where \( a \) and \( b \) are the meter’s orientations and \( \lambda \) is a set of hidden variable with a probability density \( \rho(\lambda) \), which determine the quantum state of each of the ensemble’s component systems. Writing the results in this manner we are assuming the locality thesis, since the values measured by a meter, \( A \) for example, do not depend on the other’s configuration (in this case the \( b \) orientation).

We now ask if the correlation

$$P(a, b) = \int \rho(\lambda) A(a, \lambda) B(b, \lambda) \, d\lambda, \quad (15)$$

where

$$\int \rho(\lambda) \, d\lambda = 1, \quad (16)$$

can be equal to the value obtained through the quantum mechanical calculation. Suppose, for generality

$$|A(a, \lambda)| \leq 1 ; \quad |B(b, \lambda)| \leq 1 ; \quad |P(a, b)| \leq 1, \quad (17)$$

We obtain, after some algebra that is independent of quantum mechanical considerations, the following inequality

$$|P(a, b) - P(a', b')| + |P(a', b') + P(a', b)| \leq 2, \quad (18)$$
which is called Bell’s inequality and should be obeyed by predictions of local theories.

From here Bell shows that the quantum correlation of spins does not obey this inequality. Bell’s theorem thus states that no local hidden variable theory can reproduce quantum mechanics’ results. His argumentation based on the experiment proposed by Bohm and Aharonov\(^\text{[21]}\), is the following:

Take an ensemble of systems initially in singlet form. The probability amplitude associated to this ensemble is given by

\[
|\Psi_S\rangle = \frac{|+\rangle |-\rangle - |-\rangle |+\rangle}{\sqrt{2}}.
\]  

(19)

At a given moment, this system dissociates itself into particle1 and particle2 which are measured by meter1 in direction a and meter2 in direction b. In this case

\[
\langle \Psi_S | \sigma_a \sigma_b | \Psi_S \rangle = \frac{1}{2} \left[ \langle + | \sigma_a | + \rangle \langle + | \sigma_b | + \rangle - \langle + | \sigma_a | - \rangle \langle - | \sigma_b | + \rangle - \langle - | \sigma_a | + \rangle \langle + | \sigma_b | - \rangle + \langle - | \sigma_a | - \rangle \langle + | \sigma_b | + \rangle \right] - \theta_{ab}.
\]

(20)

where \( \theta_{ab} \) is the angle between a and b.

Now placing a in direction z, we obtain

\[
\langle \Psi_S | \sigma_a \sigma_b | \Psi_S \rangle = -\cos \theta_{ab}.
\]

(21)

This result violates Bell’s inequality, when the arrangement

\[
\angle_{ab'} = 2\theta ; \quad \angle_{ba'} = 0 ; \quad \angle_{bb'} = \angle_{aa'} = \theta,
\]

(22)

is made along with the rotational symmetry of the calculation made in (21). Then Bell is in position to interpret this result as saying that quantum mechanics has a non-local character.

Let us now suppose that the state of the ensemble that is to be measured is prepared in such a way that only one of it’s component systems is measured at a time. To represent this system’s state after the separation we have

\[
|\Psi\rangle = |+\rangle |-\rangle \quad \text{or} \quad |\Psi\rangle = |\rangle |+\rangle.
\]

(23)

This must be so since, according to Born’s statistical interpretation, each system has probability one half to be in only one of the states above mentioned (in fact this question is related to that one about the state vector representing the state of one system - Schroendiger’s dead and alive cat - or representing an ensemble of states; we are clearly opting for the former interpretation). In this case the correlation is given by

\[
\langle \Psi_S | \sigma_a \sigma_b | \Psi_S \rangle = \frac{1}{2} \left[ \langle + | \sigma_a | + \rangle \langle + | \sigma_b | + \rangle + \langle - | \sigma_a | - \rangle \langle + | \sigma_b | + \rangle \right].
\]

(24)
For this correlation and for the configuration (22), we obtain the expressions

\[
P(a, b) = -\cos \theta; \quad P(a', b) = -\cos^2 \theta
\]

\[
P(a, b') = -\cos 2\theta; \quad P(a', b) = -\cos \theta \cos 2\theta
\]

(25)

which, when substituted in Bell’s inequality, give us the expression

\[
|\cos 2\theta - \cos \theta| + |\cos \theta| |\cos 2\theta \cos \theta| \leq 2
\]

(26)

which, it can be shown, is always satisfied.

It becomes clear from these two treatments that the difference is caused by the fact that we have not considered, in (24), the terms

\[
\langle + | \sigma_a | - \rangle \langle - | \sigma_b | + \rangle; \quad \langle - | \sigma_a | + \rangle \langle + | \sigma_b | - \rangle
\]

(27)

which represent interference effects acting upon only one system.

On the other hand, if we realize an experiment where many systems are measured simultaneously, then we expect the terms (27) to appear, even if representing a correlation between different systems. This correlation, evidently, cannot be used to probe the non-local character of a theory.

The discussion above shows that quantum mechanics is a local theory because it satisfies Bell’s inequality (in fact it remains to prove, by experiments, if the statistical interpretation adopted for the state vector is correct).

Many experiments were realized to demonstrate the violation of Bell’s inequality [22]-[30]; nevertheless, all these experiments, as far as we could see, perform measurements over various systems at a time, bringing results which agree perfectly well with quantum mechanics’ predictions, as expected [22]. We here propose that experiments be made in which the systems are measured one by one in order to confirm result (26) and validate the statistical interpretation of the state vector proposed. Some of these experiments have already been done. Indeed, experiments on Interrupted Florecence [31]-[34] show that the hypothesis made in expression (23) about the correct appearance of the probability amplitudes for a single system is adequate.

6 Bohm’s Theory

Bohm’s theory has been considered an authentic hidden variable theory ever since it was published in 1952. It was this theory that resurrected the discussion about the possibility of hidden variable theories.

In this section we will give an argument in order to show that this theory may not be considered a true hidden variable theory. For this, we will present it shortly.

From Schroedinger’s second equation, and writing the amplitude of probability as

\[
\Psi = R(x) \exp \left[ iS(x) / \hbar \right],
\]

(28)
where $R(x)$ and $S(x)$ are real functions, Bohm obtains, equating the real and imaginary terms to zero, the following equations

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{m} + V + Q = 0, \quad (29)$$

$$\frac{\partial P}{\partial t} + \nabla \left( \frac{P \nabla S}{m} \right) = 0, \quad (30)$$

where $V(x)$ is the classical potential (we have done this in reverse order in (I) and (II)). He calls $Q(x)$ the quantum potential, defined as

$$Q = -\frac{\hbar^2}{2m} \nabla^2 R \quad (31)$$

and

$$P(x; t) = R(x; t)^2 = \Psi^*(x; t) \Psi(x; t). \quad (32)$$

Using Hamilton-Jacobi formalism, the equation

$$m \frac{d^2 x}{dt^2} = -\nabla (V + Q), \quad (33)$$

is obtained as subjected to the initial condition

$$p = -\nabla S. \quad (34)$$

Now assuming that $x$ represents a single particle’s coordinate and $p$ its momentum, a spatial-temporal description similar to Newton’s is obtained.

Yet this association of meanings cannot be made within the realm of the present theory. In fat, the starting point was an equation that presents dispersion relations according to the uncertainty principle; this dispersion is included in the probability amplitude and shows up in the constituent terms of (28) and in equations (29) and (30). More still, in the first article of this series it was demonstrated that Schrödinger’s Second Equation was not satisfied for $s$ ensembles; for this reason equations (28) and (34) cannot be associated to these ensembles. It is symptomatic that Bohm’s theory cannot be obtained from the equation for the density function, which is dispersion free.\[35\]

As was said in the first article, the equations satisfied by the individual constituents of the systems are Newton’s equations themselves. The equations above represent no more than the representation of the ensemble’s behavior. The analysis of the double slit problem according to this theory demonstrates this behavior clearly; it is also important to note that the trajectories cannot be considered the real trajectories of the systems’ components, since we are using Schrödinger Second equation for the probability amplitudes which is adequate only to describe the passage from one equilibrium situation to another. According to equation (34), one cannot determine exactly the initial conditions
of one of the ensemble’s trajectory in particular, therefore leaving at least one hidden variable which the formalism is not capable of revealing.

The fact that it is not possible to find a well defined physical source for the quantum potential must also be stressed.

In this manner one concludes that Bohm’s theory is not a hidden variable theory and that the quantum potential should be interpreted as no more than a fictitious “potential” representing a statistical field associated to a particular problem’s specific configuration.

It’s non-local character is easily explained, once we have interpreted the potential \( Q(x) \) as a statistical potential. For Bohm’s theory for an ensemble of two particle systems, we have the following equations

\[
\frac{dX_1}{dt} = \rho (X_1, X_2)^{-1} Im \sum_{ij} \Psi_{ij}^* (X_1, X_2) \frac{\partial}{\partial X_1} \Psi_{ij} (X_1, X_2),
\]

\[
\frac{dX_2}{dt} = \rho (X_1, X_2)^{-1} Im \sum_{ij} \Psi_{ij}^* (X_1, X_2) \frac{\partial}{\partial X_2} \Psi_{ij} (X_1, X_2),
\]

where

\[
\rho (X_1, X_2) = \sum_{ij} |\Psi_{ij} (X_1, X_2)|^2
\]

and \( X_1 \) and \( X_2 \) represent the particles’ ”positions”. It is clear that, for such a system, every time that we cannot write the density as a product

\[
\Psi_{ij} (X_1, X_2) = \Phi (X_1) \Xi (X_2),
\]

we will have the equations for \( X_1 \) dependent upon \( X_2 \) and vice-versa, thus showing a non-local character.

As we have said, the potential \( Q(x) \) represents the statistical field associated to the ensemble. We should consider equations (35) and (36) as representing only the property of conditional probabilities. In other words, if we fix the statistical behavior for one of the particles, we will know the statistical behavior of the other.

7 Conclusions

In this series of papers we have presented a complete reconstruction of quantum mechanic’s principles. In this and the other papers, we demonstrated that it is possible to interpret quantum from a Realist point of view. It was also shown that the formalism itself obtained embraces that of usual quantum mechanics as a particular one. Quantum mechanics was shown to be local and, although statistical by principle, based on a deterministic theory that is nothing more than newtonian classical mechanics. This approach has also made possible for
us to obtain a general relativistic quantum theory for *ensembles* of systems with one particle.

All this collected, we are in position to maintain Einstein’s definition of reality\[17]. His affirmation about the incompleteness of quantum mechanics, with its *Complete Sets of Commuting Operators*, that is, based on the Schrödinger equations for the probability amplitudes, can be interpreted as a straightforward implication of the formalism connected with Heisenberg’s uncertainty relations.

Quantum mechanics, as represented by the first Schrödinger’s equations, was shown to be local and based on a deterministic Nature. Classical ontology must be reconsidered[39].

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