Study of $e^+e^- \rightarrow \Upsilon(1S,2S)\eta$ and $e^+e^- \rightarrow \Upsilon(1S)\eta'$ at $\sqrt{s} = 10.866$ GeV with the Belle detector

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I. INTRODUCTION

Bottomonium states (bound states of $b\bar{b}$) above the $B\bar{B}$ threshold have unexpected properties. For example, the $\Upsilon$(10860) resonance, commonly denoted as $\Upsilon$(5S), decays into $\Upsilon(nS)\pi^+\pi^-$ ($n = 1, 2, 3$) with widths around $300 - 400$ keV, about two orders of magnitude larger than those for similar decays of the $\Upsilon(2S) \rightarrow \Upsilon(4S)$ which have widths around $0.5 - 5$ keV. One possible interpretation of such behavior is the existence of a light-flavor admixture in the $\Upsilon$(5S) resonance, which leads to cancellation of the suppression caused by heavy quark gluon emission.

Observation by the Belle collaboration of unexpectedly large values for the ratios $\Gamma(\Upsilon(5S)\rightarrow b\bar{b}(1P)\pi^+\pi^-)/\Gamma(\Upsilon(5S)\rightarrow \Upsilon(1S)\pi^+\pi^-) = 0.46 \pm 0.08 \pm 0.07$ and $\Gamma(\Upsilon(5S)\rightarrow b\bar{b}(2P)\pi^+\pi^-)/\Gamma(\Upsilon(5S)\rightarrow \Upsilon(1S)\pi^+\pi^-) = 0.77 \pm 0.08 \pm 0.17$, while it was expected to be $O(10^{-2})$ due to heavy quark spin flip, has led to discovery of exotic four-quark bound states $Z_0(10610)$ and $Z_0(10650)$.

Another similar ratio $\Gamma(\Upsilon(4S,5S)\rightarrow \Upsilon(1S)\pi^+\pi^-)/\Gamma(\Upsilon(4S,5S)\rightarrow \Upsilon(2S)\pi^+\pi^-)$ is also expected to be $O(10^{-2})$ in the QCDME model, but has been measured to be $2.41 \pm 0.40 \pm 0.12$ for the $\Upsilon(4S)$ resonance. Moreover, the measurement of $B(\Upsilon(4S)\rightarrow \eta_b(1P)) = (2.18 \pm 0.11 \pm 0.18) \times 10^{-3}$, which violates naive quark-antiquark models like QCDME. Nevertheless, for bottomonium states below the $B\bar{B}$ threshold, the QCDME model predictions are consistent with measurements:

$\Gamma(\Upsilon(2S)\rightarrow \Upsilon(1S)\eta) = (1.64 \pm 0.25) \times 10^{-3}$ and

$\Gamma(\Upsilon(2S)\rightarrow \Upsilon(1S)\eta^{'}) < 2.3 \times 10^{-3}$.

Therefore, analysis of similar processes is crucial for better understanding of the quark structure of bottomonium states above the $B\bar{B}$ threshold.

This paper describes the study of hadronic transitions between bottomonium states with emission of an $\eta(1^3S_0)$ meson at $\sqrt{s} = 10.866$ GeV. The process $e^+e^\rightarrow \Upsilon(2S)\eta$ is studied in two different modes: the first decay chain $\Upsilon(2S) \rightarrow \Upsilon(1S)\pi^+\pi^-$, $\Upsilon(1S) \rightarrow \mu^+\mu^-$, $\eta \rightarrow \gamma\gamma$ denoted as $\Upsilon(2S)\eta[\gamma\gamma]$; the second decay chain $\Upsilon(2S) \rightarrow \mu^+\mu^-$, $\eta \rightarrow \pi^+\pi^-\pi^0$, $\pi^0 \rightarrow \gamma\gamma$ denoted as $\Upsilon(2S)\eta[3\pi]$. The process $e^+e^\rightarrow \Upsilon(1S)\eta'$ is studied in two different modes: the first decay chain $\Upsilon(1S) \rightarrow \mu^+\mu^-$, $\eta' \rightarrow \pi^+\pi^-\eta$, $\eta \rightarrow \gamma\gamma$ denoted as $\Upsilon(1S)\eta'[\pi\eta\eta]$; the second decay chain $\Upsilon(1S) \rightarrow \mu^+\mu^-$, $\eta' \rightarrow \rho^0\gamma$, $\rho^0 \rightarrow \pi^+\pi^-$ denoted as $\Upsilon(1S)\eta'[\rho\gamma]$ and is the only process with the $\mu^+\mu^-\pi^+\pi^-\gamma$ final state, while other processes lead to the $\mu^+\mu^-\pi^+\pi^-\gamma$ final state.

A first evidence for the $e^+e^\rightarrow \Upsilon(2S)\eta$ process has been reported in Ref. [1], where inclusive measurement with recoil mass distribution against $\eta$ meson was performed. The Born cross section (see eq. [3]) being $\sigma_B(e^+e^\rightarrow \Upsilon(2S)\eta) = 1.02 \pm 0.30 \pm 0.17$ pb and the upper limit $\sigma_B(e^+e^\rightarrow \Upsilon(1S)\eta) < 0.49$ pb being set at 90% confidence level. This analysis is exclusive in $\eta$ decays and independent from the latter one.

We use the data sample of 118.3 fb$^{-1}$ collected at the $\Upsilon$(5S) resonance and the data sample of 21 fb$^{-1}$ collected during the energy scan in the range from 10.63 GeV to 11.02 GeV by the Belle detector at the KEKB asymmetric-energy $e^+e^-$ collider in the energy range from 10.63 GeV to 11.02 GeV.

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II. EVENT SELECTION

The event selection is performed in two steps. First we require the presence of at least two oppositely charged muon and two oppositely charged pion candidates. Charged tracks must originate from a cylindrical region of length ±2.5 cm along the z axis (opposite the positron beam) and radius 2 cm in the transverse plane, centered on the $e^+e^-$ interaction point. Muon candidates are identified with a requirement on a likelihood ratio $P_\mu = \frac{e^{L_\mu}}{e^{L_\mu} + e^{L_\pi}} > 0.1$ (efficiency is ≈ 99.9%), where the likelihood $L_i$, with $i = \mu, \pi, K$, is assigned based on the range of the particle extrapolated from the CDC through KLM and on deviation of hits from extrapolated track. Every charged particle that is not muon or not well identified as an electron ($P_e < 0.99$) is considered as a charged pion candidate, where $P_e$ is a similar likelihood ratio based on CDC, ACC, and ECL information. Additionally, we require dion invariant mass $M_{\mu\mu} = \sqrt{(P_{\mu+} - P_{\mu-})^2}$ to be in the range from 8 GeV/c² to 12 GeV/c² and dipion invariant mass $M_{\pi\pi} = \sqrt{(P_\pi+ + P_\pi-)^2}$ less than 4 GeV/c², where $P_i$ is the reconstructed four-momentum of a particle $i$. At this stage no requirements on photon candidates are applied.

Final-state-specific requirements are applied at the second stage. The following set of selection variables are common to all processes: the angle $\Psi$ between the total momentum of the photons and the total momentum of the charged particles in the CM frame, the invariant mass of the muon pair $M_{\mu\mu}$ (corresponding to the $\Upsilon(1S,2S)$), and the total reconstructed energy of the final-state particles, $E_{\text{tot}}$. These variables are used to select exclusive decay chains that result in the same final states $\mu^+\mu^-\pi^+\pi^-\gamma(\gamma)$.

The signal region for $\Upsilon(1S) \rightarrow \mu^+\mu^-$ is defined to be 9.235 GeV/c² < $M_{\mu\mu}$ < 9.685 GeV/c² and that for $\Upsilon(2S) \rightarrow \mu^+\mu^-$ is 9.76 GeV/c² < $M_{\mu\mu}$ < 10.28 GeV/c². Four-momentum conservation requires the angle $\Psi$ to be equal to $\pi$ radian; however, it can deviate even for true candidates due to finite momentum and energy resolutions. For the $\Upsilon(2S)\eta(3\pi)$ mode this effect results in a less strict requirement on the angle $\Psi$ due to the low moment of the $\pi^0$. Selection criteria for the angle $\Psi$ are listed in Table I for all modes. In case of multiple decay candidates, usually due to additional photons in the event from background processes, the $\mu\pi\pi(\pi\eta)$ combination with $\Psi$ closest to $\pi$ radian is chosen as the best candidate. According to the simulation, fraction of multiple candidate events is about 24% for the $\Upsilon(2S)\eta(3\pi)$ mode and ranges from 3% to 12% for other modes. Finally, $E_{\text{tot}}$ is calculated as

$$E_{\text{tot}} = E_{\pi\pi\gamma(\gamma)} + \sqrt{M_{\Upsilon(1S,2S)}^2 + P_{\mu\mu}^2},$$  \hspace{1cm} (1)

where instead of the reconstructed value of the $\mu^+\mu^-$-pair invariant mass the world-average mass of the $\Upsilon$ meson is used. This approach allows one to improve the $E_{\text{tot}}$ resolution by removing a contribution of the $M_{\mu\mu}$ resolution, whose value is about 50 MeV/c² and comparable to the total contribution of all other terms in $E_{\text{tot}}$. Selection requirements on these common variables for all considered decay chains are summarized in Table II. Additional criteria for selection of specific modes are described below.

To reconstruct a neutral pion from the $\pi^0 \rightarrow \gamma\gamma$ decay in the $\Upsilon(1S,2S)\eta(3\pi)$ modes, the invariant mass $M_{\eta\gamma}$ should be in the signal range 110 MeV/c² < $M_{\eta\gamma}$ < 155 MeV/c², with resolution of 5.5 MeV/c². For the $\Upsilon(1S)\eta(\pi\pi\eta)$ mode the $\eta$ meson is reconstructed from the $\eta \rightarrow \gamma\gamma$ decay with a signal range 450 MeV/c² < $M_{\eta\gamma}$ < 625 MeV/c², with resolution of 12.3 MeV/c². For the $\Upsilon(1S)\eta(\rho\eta)$ mode the $\rho^0$ resonance is reconstructed from the $\rho^0 \rightarrow \pi^+\pi^-\gamma$ decay with a signal range 450 MeV/c² < $M_{\pi\pi\gamma}$ < 950 MeV/c², with root-mean-square of 56 MeV/c².

For the two-body $\Upsilon(5S) \rightarrow \Upsilon(2S)\eta(\gamma\gamma)$ decay the produced $\eta$ meson is monochromatic, with a CM momentum equal to 615 MeV/c. Thus, photons produced in the $\eta \rightarrow \gamma\gamma$ decay have an energy in the CM frame distributed in the range from 105 MeV to 715 MeV. We set a requirement on the minimum photon energy of 100 MeV that significantly reduces combinatorial background and has virtually no effect on signal events.

For the $\Upsilon(2S)\eta(\gamma\gamma)$ final state the $\Upsilon(2S)$ meson is reconstructed via its decay chain $\Upsilon(2S) \rightarrow \Upsilon(1S)\pi\pi\pi^-$ with $\Upsilon(1S) \rightarrow \mu^+\mu^-\pi^-$. The resolution of $M_{\mu\pi\pi}$ is approximately 50 MeV/c² and is dominated by the muon momentum resolution. To reduce this contribution we calculate the mass difference $M_{\mu\pi\pi} - M_{\mu\mu}$, where the correlated contributions to resolution from the muon momentum measurement substantially cancel. Due to the narrow width of the $\Upsilon(1S,2S)$ states, the $\delta M = M_{\mu\pi\pi} - M_{\mu\mu}$ peak position corresponds to $\Delta M = M_{\Upsilon(2S)} - M_{\Upsilon(1S)} = 562$ MeV/c² while the resolution is approximately 4.6 MeV/c², and a requirement of $|\delta M - \Delta M| < 18$ MeV/c² is applied.

The signal distribution for all modes is the $(M_{\mu\pi\pi})$ invariant mass, it having no peaking background (see Section II). The MC signal distribution is fitted by a sum of the Crystal Ball function and a Gaussian. The reconstruction efficiency $\varepsilon$ is then determined as $N_{\text{det}}/N_{\text{gen}}$, where $N_{\text{det}}$ is the integral of the fitted function and $N_{\text{gen}} = 10^6$. Results are summarized in Table II.

III. STUDY OF THE EXPECTED BACKGROUND

The most relevant background to this analysis comes from transitions between other bottomonium states with emission of an $\eta'(\eta)$ meson. These decays have an $\eta'(\eta)$ invariant mass distribution identical to our signal modes.

Due to the $\eta'$ mass and parity considerations, the $\eta'$ meson can originate only from the $\Upsilon(3S) \rightarrow \Upsilon(1S)\eta'$ de-
TABLE I: Selection criteria and reconstruction efficiencies, where $M_{\text{rec}}^{\text{SF}} = \sqrt{s + M_{\text{rec}}^2 - 2\sqrt{s}E_{\text{rec}}}$, $\delta M = M_{\mu\mu\pi\pi} - M_{\mu\mu}$, and $\Delta M_{2} = M_{\Upsilon(2S)} - M_{\Upsilon(1S)} = 562$ MeV/$c^2$, $\Delta M_{3} = M_{\Upsilon(2S)} - M_{\Upsilon(1S)} = 894$ MeV/$c^2$.

| Criterion | $\Upsilon(2S)\eta[3\pi]$ | $\Upsilon(2S)\eta[\gamma\gamma]$ | $\Upsilon(1S)\eta[3\pi]$ | $\Upsilon(1S)\eta[\gamma\gamma]$ | $\Upsilon(1S)\eta[\pi\pi\eta]$ | $\Upsilon(1S)\eta[\mu\mu\pi\pi]$ |
|-----------|----------------|---------------------|----------------|----------------|----------------|----------------|
| $M_{\mu\mu}$, GeV/$c^2$ | [9.76, 10.28] | [9.235, 9.685] | [9.235, 9.685] | [9.235, 9.685] | [9.235, 9.685] | [9.235, 9.685] |
| $\Psi$, rad. | $\geq 2$ | $\geq 2.8$ | $\geq 2.7$ | $\geq 2.8$ | $\geq 2.8$ | $\geq 2.8$ |
| $E_{\text{tot}}$, GeV | [10.775, 10.92] | [10.80, 10.955] | [10.75, 10.94] | [10.75, 10.94] | [10.75, 10.94] | [10.75, 10.94] |
| $M_{\gamma\gamma}$, MeV/$c^2$ | [110, 155] | [110, 155] | [450, 625] | | | |
| $\delta M$, MeV/$c^2$ | | | | | | |
| $\alpha_{\pi\pi}$, rad. | | | | | | |
| $E_{\gamma}$, MeV | | | | | | |
| $M_{\pi\pi}$, MeV/$c^2$ | | | | | | |
| $M_{\text{rec}}^{\pi\pi\pi\pi}$, MeV/$c^2$ | | | | | | |
| $\epsilon$, % | 10.25 $\pm$ 0.03 | 20.73 $\pm$ 0.04 | 17.02 $\pm$ 0.03 | 13.35 $\pm$ 0.03 | 29.25 $\pm$ 0.03 |

Decays of $\Upsilon(5S) \rightarrow \gamma$ or from $\Upsilon(5S) \rightarrow \pi\pi(1P)^0 \eta \gamma$ decays with a subsequent radiative decay of $\pi\pi(1P) \rightarrow \Upsilon(1S)\gamma$. The former is our signal and the latter is suppressed by the presence of an additional photon.

In contrast, the $\eta$ meson can also originate from $\Upsilon(5S) \rightarrow \Upsilon(1D)\eta$ [12] and $\Upsilon(5S) \rightarrow \Upsilon(2S,3S)\pi\pi$ followed by $\Upsilon(2S,3S) \rightarrow \Upsilon(1S)\eta$ decays. For the $\Upsilon(5S) \rightarrow \Upsilon(1D)\eta$ decay, the most relevant channels are those with $\Upsilon(1D) \rightarrow \chi_{b1}(1P) \rightarrow \Upsilon(1S)\gamma\gamma$ and $\Upsilon(1D) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decays. However, the first decay produces two extra photons and is suppressed by the requirement on $E_{\text{tot}}$. The second decay might produce a correct set of final-state particles (with $\eta \rightarrow \gamma\gamma$), but is significantly suppressed by the requirement on $M_{\mu\mu\pi\pi} - M_{\mu\mu}$: for the $\Upsilon(1D) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decay, this variable peaks at approximately 140 MeV/$c^2$, higher than for the $\Upsilon(2S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ with a resolution of about 5 MeV/$c^2$. Therefore, the $\Upsilon(1D) \rightarrow \Upsilon(1S)\pi^+\pi^-$ signal is completely eliminated.

$\Upsilon(2S,3S)\pi\pi$ mesons mostly originate from $\Upsilon(5S) \rightarrow \Upsilon(2S,3S)\pi^+\pi^-$ decays. With subsequent $\Upsilon(2S,3S) \rightarrow \Upsilon(1S)\eta$ and $\eta \rightarrow \gamma\gamma$ decays these channels produce the same set of final-state particles and the same signal distribution as the $\Upsilon(2S)\eta[\gamma\gamma]$ mode. However, branching fractions $B(\Upsilon(2S,3S) \rightarrow \Upsilon(1S)\eta)$ are small and with the current integrated luminosity the expected number of $\eta$ mesons produced by this mechanism is estimated to be 2 for the $\Upsilon(2S)$ and less than 1 for the $\Upsilon(3S)$. Contributions from these decays are also strongly suppressed to negligible level by the requirement on $M_{\mu\mu\pi\pi} - M_{\mu\mu}$: its mean value deviates from the $\Upsilon(2S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ signal window by 280 MeV/$c^2$ and 50 MeV/$c^2$ for $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-$ and $\Upsilon(5S) \rightarrow \Upsilon(3S)\pi^+\pi^-$ correspondingly.

A possible source of background for $\Upsilon(5S) \rightarrow \Upsilon(2S)\eta$ is the decay itself with $\Upsilon(2S) \rightarrow \Upsilon(1S)\eta$, where $\eta \rightarrow \pi^+\pi^-\pi^0$ or $\eta \rightarrow \pi^+\pi^-\gamma$. This final state is similar to the $\Upsilon(2S)\eta[\gamma\gamma]$ mode when a soft photon or $\pi^0$ is undetected. Nevertheless, this background is suppressed to negligible level by the intermediate branching-fraction ratio $\frac{B(\Upsilon(2S)\rightarrow\Upsilon(1S)\eta)}{B(\Upsilon(2S)\rightarrow\Upsilon(1S)\pi^+\pi^-)} \sim 4 \times 10^{-4}$ and requirements on $E_{\text{tot}}$.

Crossfeed between the signal modes is a background that passes through the common selection criteria but does not produce peaks in the signal distributions. For the $\Upsilon(1S)\eta[3\pi]$ and $\Upsilon(1S)\eta$ modes, there is such a background from the $\Upsilon(2S)\eta[\gamma\gamma]$ mode when $\Upsilon(2S) \rightarrow \Upsilon(1S)\pi^+\pi^-$. To reduce this background for the $\Upsilon(1S)\eta$ mode, we require $|M_{\mu\mu\pi\pi} - M_{\mu\mu} - (M_{\Upsilon(2S)} - M_{\Upsilon(1S)})| > 10$ MeV/$c^2$, which only slightly decreases the signal reconstruction efficiency and suppress this background to negligible level. The crossfeed between the $\Upsilon(1S)\eta[3\pi]$ and $\Upsilon(2S)\eta[3\pi]$ modes is efficiently removed by the common selection requirements.

Another significant part of the background is the nonpeaking combinatorial background. To evaluate the expected level of this background, we used a set of MC events six times larger than that in data including the following processes: $e^+e^- \rightarrow c\bar{c}, u\bar{u}, d\bar{d}, s\bar{s}; e^+e^- \rightarrow B_s^{(*)} B_s^{(*)}, B_s^{(*)} B_s^{(*)}$, and known decays of the $\Upsilon(5S)$. In addition, we performed simulation of $e^+e^- \rightarrow \tau^+\tau^-$ events with statistics equivalent to the integrated luminosity of our dataset. The only events remaining after application of the selection criteria originate from $\Upsilon(5S)$ decays to final states containing bottomonium. As an example, the dominant background to the $\Upsilon(2S)\eta[\gamma\gamma]$ comes from the $\Upsilon(2S)\pi^0\pi^0$ final state, which produces a broad peaking $M_{\gamma\gamma}$ distribution from 50 MeV/$c^2$ to 850 MeV/$c^2$ with a maximum near the signal $\eta$ peak position. To suppress this background we increased the requirement on the total reconstructed energy from 10.75 GeV to 10.80 GeV for the $\Upsilon(2S)\eta[\gamma\gamma]$ mode. This reduces the expected number of background events for the $\Upsilon(2S)\eta[\gamma\gamma]$ from 20 to 5 events and slightly decreases the detection efficiency.

For the $\Upsilon(1S)\eta$ mode, the MC study predicts high background from the $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-\eta$ decay, where $\Upsilon(2S) \rightarrow \mu^+\mu^-\gamma$. To reduce this background we set a veto on recoil mass $M_{\text{rec}}^{\pi\pi\pi\pi} > 20$ MeV/$c^2$. The MC study also predicts background contributions from decays with the $\Upsilon(2S,3S) \rightarrow \Upsilon(1S)\pi^+\pi^-\eta$ intermediate transition. Therefore, we reduced this background in the same way as for the $\Upsilon(1S)\eta[3\pi]$ and $\Upsilon(1S)\eta[\pi\pi\eta]$ modes by setting vetoes $|M_{\mu\mu\pi\pi} - M_{\mu\mu} - (M_{\Upsilon(2S,3S)} - M_{\Upsilon(1S)})| > 10$ MeV/$c^2$. Moreover, there are no requirements on photons except the general one.
from the four-momentum conservation, therefore we expect low-energy background photons. To suppress this background we set a minimum photon energy in the CM frame of $E_g^* > 80$ MeV. It does reduce reconstruction efficiency by factor of 1.12, but also greatly reduces background.

We find that all these sources predicted by the background MC account for less than 30% of the observed background in the side-band data. The rest of the background comes presumably from QED processes that in general have much higher cross sections, e.g. processes like $e^+e^- \rightarrow \mu^+\mu^-e^+e^-$. This $e^+e^-$ pair could be reconstructed as a pair of collinear pions. A selection requirement on the opening angle between two charged pion candidates of $\alpha_{\pi\pi} > 0.18$ radian for the $\Upsilon(1S)[3\pi]$ mode and of $\alpha_{\pi\pi} > 0.3$ radian for the $\Upsilon(2S)[3\pi]$ mode reduces this background substantially.

Finally, we tested for possible background from non-resonant $e^+e^- \rightarrow \mu^+\mu^-\eta^{(0)}$ decays using experimental data with a requirement on $M_{\mu\mu}$ shifted to a range from 8 GeV/c$^2$ to 9 GeV/c$^2$ that is lower than the ground bottomonium state. No evidence for such processes was observed.

**IV. DATA ANALYSIS**

**A. Cross section at the $\Upsilon(5S)$ resonance**

The signal yield is determined from a binned maximum likelihood fit to the invariant mass $M_{\eta^{(0)}} (M_{\pi\pi}, M_{\pi\pi\gamma})$ or $M_{\pi\pi\gamma}$) distribution (Fig. [1][2]), with the fitting function being the sum of the signal function and a background function $(x - p_1)^2e^{p_3x}$, where $p_1, p_2, p_3$ are floating parameters. All parameters of the signal function, except its normalization factor and the Crystal Ball peak position, are fixed to the values determined from the fit to the MC distribution, with a relative distance between Crystal Ball and Gaussian peaks being fixed.

The visible cross section is

$$\sigma_{\text{vis}} = \frac{N_{\text{sig}}}{L \cdot B \cdot \varepsilon}$$

where $N_{\text{sig}}$ is the fitted signal yield, $L$ is the integrated luminosity, $B$ is the product of the intermediate branching fractions for the process, and $\varepsilon$ is the reconstruction efficiency.

For the $\Upsilon(2S)[\gamma\gamma]$, $\Upsilon(2S)[3\pi]$, and $\Upsilon(1S)[3\pi]$ modes we evaluate the signal significance in standard deviations as $\sqrt{2 \log \left[ \mathcal{L}(N)/\mathcal{L}(0) \right]}$, where $\mathcal{L}(N)/\mathcal{L}(0)$ is the ratio between the likelihood values for a fit that includes a signal yield $N$ and a fit with a background hypothesis only. The calculated signals significance are 12.8$\sigma$, 10.5$\sigma$, and 10.2$\sigma$ respectively. Thus, we report the first observation of the $e^+e^- \rightarrow \Upsilon(2S)\eta$ process in both modes with the quadratically combined significance of 16.5$\sigma$, and the first observation of the $e^+e^- \rightarrow \Upsilon(1S)\eta$ process with the significance of 10.2$\sigma$ at $\sqrt{s} = 10.866$ GeV. Setting requirement $520 < M_{\eta} < 850$ MeV, we also confirm that there are clear peaks in $M_{\mu\mu}$ distributions (Fig. [3]) consistent with corresponding $\Upsilon(1S, 2S) \rightarrow \mu^+\mu^-$ for these modes.

For the $\Upsilon(1S)[\pi\pi\eta]$ and the $\Upsilon(1S)[\rho\gamma]$ modes the signal yield is $N_{\text{sig}} = -1.76 \pm 3.30$ and $N_{\text{sig}} = 3.30 \pm 4.41$ correspondingly; therefore, only upper limits are set using a pseudo-experiment method. Within the method we simulate $10^6$ trials, each having its own number of events sampled with the Poisson distribution having a mean of total events in the experimental signal distribution. For each event the value of $M_{\pi\pi\gamma(\gamma)}$ is sampled using the data background distribution. Then the obtained signal $M_{\pi\pi\gamma(\gamma)}$ distribution for each trial is fitted with the same procedure as the data and the obtained signal yield is recorded. We determine a confidence level ($CL$) on the upper limit as a ratio of the number of trials, which gave the signal yield from 0 to $N_{\text{sig}}$, to the total number of trials with $N_{\text{sig}}$ above 0. As a result, the 90% $CL$ upper limits for the $\Upsilon(1S)[\pi\pi\eta]$ and $\Upsilon(1S)[\rho\gamma]$ modes are $N_{\text{sig}} = 5.2$ and $N_{\text{sig}} = 5.6$, respectively.

Table [1] shows the signal yield, calculated visible cross section for all modes, and peak positions for the $\eta$ meson, which is consistent with the world-average value $M_\eta = 547.86 \pm 0.02$ MeV/c$^2$ within statistical uncertainty.

**B. Cross section outside of $\Upsilon(5S)$**

It is necessary to study the cross section behavior of the processes below the $\Upsilon(5S)$ to calculate radiative corrections. For that purpose we use 21 fb$^{-1}$ of the scan data collected in the energy range from 10.63 GeV to 11.02 GeV. We group the scan data into three ranges: 10.63 GeV – 10.77 GeV (below $\Upsilon(5S)$), 10.83 GeV – 10.91 GeV (at $\Upsilon(5S)$) and 10.93 GeV – 11.02 GeV (at $\Upsilon(6S)$). These data sets are analysed in the same way as the main one except for the requirement on $E_{\text{tot}}$, which is shifted to the corresponding energy. Analysis shows (Table [1]) that there are no signal events below the $\Upsilon(5S)$ resonance except one event for the $\Upsilon(2S)[3\pi]$ mode. Thus, we set upper limits for each mode $N_{\text{sig}} < 1$ corresponding to a $CL$ of 63%.

For $\Upsilon(1S)[3\pi]$, $\Upsilon(1S)[\pi\pi\eta]$ and $\Upsilon(1S)[\rho\gamma]$ modes, the upper limits are higher than values measured at the $\Upsilon(5S)$ resonance, while for the $e^+e^- \rightarrow \Upsilon(2S)\eta$ process the upper limit does not contradict resonance production of the final state. Other examples of resonance production are similar processes $e^+e^- \rightarrow \Upsilon(1S, 2S, 3S)\pi^+\pi^-$ [25][26] and $e^+e^- \rightarrow h_0(1P, 2P)\pi^+\pi^-$ [27]. Since the scan data results do not contradict this assumption, we use a resonance model to calculate a radiative correction for all modes as is described, for example, in [25], neglecting the possible energy dependence of the resonance width. For this calculation, the following $\Upsilon(5S)$ parameters are used: $M_{\Upsilon(5S)} = 10885.2$ MeV/c$^2$, $\Gamma_{\Upsilon(5S)} = 37$ MeV. The calcu-
lated radiative correction $1 + \delta$ varies from 0.624 to 0.628 for different modes. This correction is used to calculate Born cross section ($\sigma_B$) as

$$\sigma_B = \sigma_{\text{vis}} \frac{|1 - \Pi|^2}{1 + \delta},$$

where $|1 - \Pi|^2 = 0.929$ is the vacuum-polarization factor [12, 29].

C. Systematic uncertainties

The particle reconstruction efficiency and particle identification are important parameters whose values in simulation could deviate from those in experiment. According to independent studies, for example using the $D^*^- \rightarrow \pi^- D^0[K^0_S \pi^+ \pi^-]$ decay, the systematic uncertainty due to track reconstruction is 1% for pions and 0.35% for high-momentum muons [29]. The photon reconstruction uncertainty is 1.5%. The muon identification uncertainty is 1% according to analysis of $J/\psi \rightarrow \mu^+ \mu^-$ [30]. Therefore, the total systematic uncertainty for the $\mu^+ \mu^- \pi^+ \pi^- \gamma$ and $\mu^+ \mu^- \pi^+ \pi^- \gamma \gamma$ final states is 2.7% from charged track reconstruction, 1.5% or 3% from photons reconstruction respectively, and 2% from muon identification.

Another uncertainty can come from the accuracy of the PHOTOS module, which describes final-state radiation. To evaluate this uncertainty we simulate the $\Upsilon(2S)\eta[\gamma\gamma]$ and $\Upsilon(2S)\eta[3\pi]$ modes without the PHOTOS module. For both processes the cross section increases by 9% mostly due to absence of radiation by muons, which could account for hundreds of MeV of energy. Thus, the total influence of PHOTOS on the efficiency is 9% while its own uncertainty is a few percent [21]; therefore, the uncertainty of the detection efficiency appears in the next order and we take 1% as a conservative estimate.

Cross section dependence over energy could differ from the resonance one, leading to systematic uncertainty for the radiative correction. As alternative dependence we use the sum of the $\Upsilon(5S)$ Breit-Wigner and the constant contribution, whose amplitude is derived from the upper limit of 0.45 pb below $\Upsilon(5S)$ (see Table III). The upper limit of 0.45 pb below $\Upsilon(5S)$ corresponds to 0.58 pb after applying the correction for initial-state radiation. Con-
considering this 0.58 pb as a constant contribution into Born cross section and using visible cross section of 1.39 pb at $\sqrt{s} = 10.866$ GeV, one can estimate that the corrected cross section at $\sqrt{s} = 10.866$ GeV is 2.10 pb, implying that a relative amplitude of the constant contribution is 0.58/2.10 = 0.276. Using this cross section dependence we calculate a radiative correction for all modes. Its deviation from nominal values ranges from 4.3% to 5.7% and is referred to as a radiative correction uncertainty.

To estimate the influence of selection criteria we vary three unified requirements and check cross section stability. The width of the $E_{\text{tot}}$ signal range is symmetrically varied by $\pm 60$ MeV from the nominal value, the lower boundary for the angle $\Psi$ is varied from 2 radian to 2.8 radian, and the width of the $M_{\mu \mu}$ signal range is symmetrically varied by $\pm 200$ MeV/c$^2$ from the nominal value. The maximum cross section deviation from the nominal is taken as a systematic uncertainty. The total uncertainty due to selection criteria is a quadratic sum of these three contributions, and is shown in Table IV.

One more source of the simulation uncertainty is the deviation between simulated and experimental resolutions – usually experimental distributions are wider than those in simulation. To evaluate the deviation for systematic uncertainty, we choose events with the $\Upsilon(1S)$ originating from the $\Upsilon(2S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decay using the requirement $|M_{\mu\mu}-M_{\mu\mu}^\text{nominal}| < 18$ MeV/c$^2$. Parameterization of the experimental $M_{\mu\mu}$ distribution with a sum of a Gaussian and linear function finds a resolution of $54\pm 1.5$ MeV/c$^2$, which is larger

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FIG. 3: The experimental $M_{\mu\mu}$ distribution with requirement $520 < M_{\eta} < 580$ MeV for $\Upsilon(1S)\eta[3\pi]$ (a), $\Upsilon(2S)\eta[\gamma\gamma]$ (b), and $\Upsilon(2S)\eta[3\pi]$ (c) modes. No requirement on $M_{\mu\mu}$ is applied.

| Mode | $N_{\text{sig}}$ | $\sigma_{\text{vis}}, \text{pb}$ | $M_{\gamma^{'},\text{exp.}}, \text{MeV/c}^2$ |
|-----------------|-----------------|-----------------|-----------------|
| $\Upsilon(2S)\eta[\gamma\gamma]$ | 3.8 | 0.19 | 547.8 ± 2.0 |
| $\Upsilon(2S)\eta[3\pi]$ | 7.1 | 0.20 | 549.1 ± 1.5 |
| $\Upsilon(1S)\eta[3\pi]$ | < 5.2, $CL = 90\%$ | < 0.080, $CL = 90\%$ |
| $\Upsilon(1S)\eta'[\gamma\gamma]$ | < 5.6, $CL = 90\%$ | < 0.022, $CL = 90\%$ |

TABLE II: Signal yield, visible cross section and $M_{\gamma^{'}}$ peak position for all modes at $\sqrt{s} = 10.866$ GeV. The uncertainty is statistical only.

TABLE III: Results of the scan data analysis and comparison of the averaged upper limits on cross section below $\Upsilon(5S)$ with results from the main data set. $N_{\text{sig}}$ is the signal yield and $N_{\text{tot}}$ is the total number of events in the signal distribution.

| Mode | $\sqrt{s}$ range, GeV | $L$, fb$^{-1}$ | $N_{\text{sig}}$ | $N_{\text{tot}}$ | $\sigma_{\text{vis}}$ below $\Upsilon(5S)$, pb | $\sigma_{\text{vis}}$ at $\Upsilon(5S)$, pb |
|-----------------|-----------------|----------------|----------------|----------------|-----------------|-----------------|
| $\Upsilon(2S)\eta[\gamma\gamma]$ | 10.63 – 10.77 | 3.8 | 0 | 2 |
| $\Upsilon(2S)\eta[3\pi]$ | 10.83 – 10.91 | 10.1 | 2.0 ± 1.5 | 5 |
| 10.93 – 11.02 | 7.1 | 1.0 ± 1.0 | 2 |
| $\Upsilon(1S)\eta'[\gamma\gamma]$ | 10.63 – 10.77 | 3.8 | 0 | 3 |
| $\Upsilon(1S)\eta'[\gamma\gamma]$ | 10.83 – 10.91 | 10.1 | 0 | 8 |
| 10.93 – 11.02 | 7.1 | 0 | 8 |
| $\Upsilon(1S)\eta'[\gamma\gamma]$ | 10.63 – 10.77 | 3.8 | 0.8 ± 1.2 | 3 |
| $\Upsilon(1S)\eta'[\gamma\gamma]$ | 10.83 – 10.91 | 10.1 | 1.3 ± 1.8 | 18 |
| 10.93 – 11.02 | 7.1 | 1.3 ± 1.8 | 18 |
| $\Upsilon(1S)\eta[3\pi]$ | 10.63 – 10.77 | 3.8 | 0 | 1 |
| $\Upsilon(1S)\eta[3\pi]$ | 10.83 – 10.91 | 10.1 | 0.9 ± 1.1 | 11 |
| 10.93 – 11.02 | 7.1 | 1.0 ± 1.0 | 3 |
than the simulated resolution of 50 MeV/c^2 by 8%. This deviation is common for other distributions; therefore, we vary the resolution of the signal $M_{\mu\mu}$ distribution by ±10% to calculate the reconstruction efficiency and to fit experimental data. The maximum deviation of the cross section from the nominal one is referred to as a resolution uncertainty. Additionally, we verified that the data parameterization with unfixed resolution is consistent with the simulation within statistical uncertainty.

The signal lineshape uncertainty is taken as the maximum difference of the cross section between data fits with different signal parameterizations. The nominal lineshape is the sum of the Crystal Ball function and a Gaussian while two tested alternate parameterizations are a Gaussian only and a Crystal Ball only. The background lineshape is the sum of the Crystal Ball function and a Gaussian while two tested alternate parameterizations are a Gaussian only and a Crystal Ball only. The maximum difference of the cross section between data parameterizations taken over different ranges – in this way not every background event is included in the fit and the background lineshape.

The $\eta' \rightarrow \pi^+\pi^-\eta$ decay was simulated uniformly in phase space, which is not necessarily a correct representation of dynamics of this process. However, Ref. [31] shows that experimental Dalitz distributions are similar to those in the uniformly distributed over phase space model; thus, this source of uncertainty is neglected.

The nominal bin width for the experimental signal distribution is 10 MeV/c^2. Variation of the width affects the signal yield and leads to systematic uncertainty evaluated by refitting the data with bin widths of 5, 8 and 12 MeV/c^2.

Also, there is luminosity uncertainty of 1.4% and uncertainty of intermediate branching fractions (Table V). The total uncertainty is a quadratic sum of all sources. For the $\Upsilon(1S)\eta' [\pi\pi\eta]$ and $\Upsilon(1S)\eta' [\rho\gamma]$ modes, some of the uncertainties cannot be evaluated due to zero signal yield. Such uncertainties are assumed to be equal to those in the $\Upsilon(1S)\eta' [3\pi]$ mode.

### TABLE IV: Systematic uncertainties

| Uncertainty, % | $\Upsilon(2S)\eta[\gamma\gamma]$ | $\Upsilon(2S)\eta[3\pi]$ | $\Upsilon(1S)\eta[3\pi]$ | $\Upsilon(1S)\eta' [\pi\pi\eta]$ | $\Upsilon(1S)\eta' [\rho\gamma]$ |
|----------------|---------------------------------|--------------------------|--------------------------|----------------------------------|-------------------------------|
| Track reconstruction | 2.7                             |                          |                          |                                  |                               |
| Muon identification | 2.0                             |                          |                          |                                  |                               |
| Luminosity $L$ | 1.4                             |                          |                          |                                  |                               |
| PHOTOS | 1.0                             |                          |                          |                                  |                               |
| Radiative correction | 43.3                            | 5.1                      | 5.7                      | 5.7                              | 5.7                           |
| Photon correction | 3.0                             | 3.0                      | 3.0                      | 3.0                              | 1.5                           |
| Intermediate branchings | 2.5                             | 8.9                      | 2.4                      | 2.7                              | 2.4                           |
| Selection criteria | 6.0                             | 6.6                      | 5.6                      |                                  |                               |
| Resolution | 2.1                             | 1.4                      | 1.1                      |                                  |                               |
| Signal lineshape | 1.0                             | 1.4                      | 1.4                      |                                  |                               |
| Background lineshape | 1.5                             | 1.0                      | 1.1                      |                                  |                               |
| Binning | 0.3                             | 2.1                      | 0.8                      |                                  |                               |
| Total | 9.6                             | 13.4                     | 9.8                      | 10.0                             | 9.5                           |

### TABLE V: Branching fractions used in this work.

| Decay | Branching fraction [1], % |
|-------|---------------------------|
| $\Upsilon(1S) \rightarrow \mu^+\mu^-$ | 2.48 ± 0.05 |
| $\Upsilon(2S) \rightarrow \mu^+\mu^-$ | 1.93 ± 0.17 |
| $\Upsilon(2S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ | 17.85 ± 0.26 |
| $\eta \rightarrow \gamma\gamma$ | 39.41 ± 0.2 |
| $\eta' \rightarrow \pi^+\pi^-\pi^0$ | 22.92 ± 0.28 |
| $\eta' \rightarrow \pi^+\pi^-\eta$ | 42.5 ± 0.5 |
| $\eta' \rightarrow \rho^0\gamma$ | 29.5 ± 0.4 |
| $\pi^0 \rightarrow \gamma\gamma$ | 98.823 ± 0.034 |

### V. CROSSCHECK WITH

$\Upsilon(5S) \rightarrow \Upsilon(2S)\Upsilon(1S)\eta[\gamma\gamma]\pi^+\pi^-$

To validate the analysis procedure we measure the known process $e^+e^- \rightarrow \Upsilon(2S)\pi^+\pi^-$, where $\Upsilon(2S)$ is reconstructed via the decay chain $\Upsilon(2S) \rightarrow \chi_{bJ}(1P)\gamma$, $\chi_{bJ}(1P) \rightarrow \Upsilon(1S)\gamma$, $\Upsilon(1S) \rightarrow \mu^+\mu^-$ and $J = 0, 1, 2$. The cross section for this process is measured independently with the $\Upsilon(2S) \rightarrow \mu^+\mu^-$ decay where the statistics of signal events is much higher [30].

The analysis procedure is almost the same as for the other modes. Selection criteria for this process are based on the same set of common variables: an $\Upsilon(1S)$ meson is reconstructed by the $M_{\mu\mu}$ in the 9.235 GeV/c^2 < $M_{\mu\mu}$ < 9.685 GeV/c^2 range, the angle $\Psi > 2.6$ radian, and the total reconstructed energy 10.75 GeV < $E_{\text{tot}} < 10.94$ GeV. In addition, a requirement on the mass recoiling off two charged pions $M_{\pi\pi}^\text{rec}$ is applied as $|M_{\pi\pi}^\text{rec} - M_{\Upsilon(2S)}| < 30$ MeV/c^2. According to MC simulation, the resolution of $M_{\pi\pi}^\text{rec}$ is 6 MeV/c^2. This helps to reduce background from the $e^+e^- \rightarrow \Upsilon(1D)\pi^+\pi^-$ process, where $\Upsilon(1D) \rightarrow \chi_{bJ}\gamma$, $\chi_{bJ}(1P) \rightarrow \Upsilon(1S)\gamma$.

The signal distribution for this mode is the largest of two $M_{\mu\mu} - M_{\mu\mu}$ values. This variable reconstructs $\chi_{bJ}(1P) \rightarrow \Upsilon(1S)\gamma$, corresponds to $M_{\chi_{bJ}} - M_{\Upsilon(1S)}$, and reduces correlated contributions to the resolution from the measurement of muon momentum. The studied pro-
cess results in peaks at 399.1, 432.5, and 451.9 MeV/c² for $J = 0$, 1, 2, respectively. Distributions for each $\chi_{b1}(1P)$ are fitted to the sum of the Crystal Ball function and a Gaussian in the same way as for the other processes. Reconstruction efficiencies are $\varepsilon_{\chi_{b1}(1P)} = 28.12 \pm 0.04\%$, $\varepsilon_{\chi_{b1}(1P)} = 28.68 \pm 0.04\%$, and $\varepsilon_{\chi_{b2}(1P)} = 28.52 \pm 0.04\%$.

The known products of the intermediate branching fractions $B_{\chi_{b1}(1P)} = B(\Upsilon(2S) \to \chi_{b1}(1P)\gamma) \times B(\chi_{b1}(1P) \to \Upsilon(1S)\gamma)$ are $B_{\chi_{b1}(1P)} = (7.37 \pm 0.28) \times 10^{-4}$, $B_{\chi_{b1}(1P)} = (242 \pm 19) \times 10^{-4}$, and $B_{\chi_{b2}(1P)} = (128 \pm 9) \times 10^{-4}$ \cite{30}. The fraction of $\varepsilon_{\chi_{b1}(1P)} \times B_{\chi_{b1}(1P)}$ is equal to 0.029 : 1 : 0.527 ($J = 0$, 1, 2) and determines a relative contribution of each $\chi_{b1}(1P)$ to the total signal lineshape. The total MC signal lineshape is the sum of three contributions, with all parameters except normalization factor being fixed for the data analysis. The total branching fraction weighted with the efficiency is $B_{\Upsilon(2S)\pi^0} = B(\Upsilon(1S) \to \mu^+\mu^-) \sum \varepsilon_{\chi_{b1}(1P)} \times B(\Upsilon(2S) \to \chi_{b1}(1P)\gamma) \times B(\chi_{b1}(1P) \to \Upsilon(1S)\gamma) = (2.69 \pm 0.16) \times 10^{-4}$, and is used to calculate the cross section instead of $\varepsilon_B$ (Eq. 2).

Figure 4 shows the experimental $M_{J\mu_1\mu_2} - M_{J\mu_1\mu_2}$ distribution. The signal yield is determined from fitting the $M_{J\mu_1\mu_2} - M_{J\mu_1\mu_2}$ distribution, with the fit function being the sum of the total MC signal lineshape and a background function $(x-p_1)^2e^{-p_2x}$. We obtain $N_{sig} = 85.32 \pm 11.5$, resulting in the Born cross section $\sigma_{B}(e^+e^- \to \Upsilon(2S)\pi^+\pi^-) = 3.98 \pm 0.54$ pb (statistical uncertainty only). This value is consistent with the independent measurement $\sigma_{B}(e^+e^- \to \Upsilon(2S)\pi^+\pi^-) = 4.07 \pm 0.16 \pm 0.45$ pb \cite{30} within uncertainty.

![FIG. 4: The $M_{J\mu_1\mu_2} - M_{J\mu_1\mu_2}$ distribution for the $e^+e^- \to \Upsilon(2S)\pi^+\pi^-$ process, where $\Upsilon(2S) \to \chi_{b1}(1P)\gamma \to \Upsilon(1S)\gamma\gamma$. Data are shown as points, the solid red line shows the best fit to the data, and the dashed blue line shows the background contribution.](image)

VI. CONCLUSION

In summary, using the Belle data sample of 118.3 fb⁻¹ obtained at $\sqrt{s} = 10.866$ GeV, we report a measurement of the cross section for $e^+e^- \to \Upsilon(1S,2S)\eta$ processes, and set an upper limit on the cross section of the $e^+e^- \to \Upsilon(1S)\eta'$ process. The measured Born cross sections, with initial-state radiation being taken into account, are the following (Eq. 3):

- $\sigma_{B}^{\eta'\rightarrow\eta}(e^+e^- \to \Upsilon(2S)\eta') = 2.08 \pm 0.29 \pm 0.20$ pb,
- $\sigma_{B}^{\eta\rightarrow\eta}(e^+e^- \to \Upsilon(2S)\eta) = 2.07 \pm 0.30 \pm 0.28$ pb,
- $\sigma_{B}^{\eta'\rightarrow\eta}(e^+e^- \to \Upsilon(1S)\eta') = 0.42 \pm 0.08 \pm 0.04$ pb,
- $\sigma_{B}^{\eta'\rightarrow\eta}(e^+e^- \to \Upsilon(1S)\eta) < 0.130$ pb, $CL = 90\%$,
- $\sigma_{B}^{\eta\rightarrow\eta}(e^+e^- \to \Upsilon(1S)\eta') < 0.036$ pb, $CL = 90\%$.

The weighted averages for the corresponding modes are:

- $\sigma_{B}(e^+e^- \to \Upsilon(2S)\eta) = 2.07 \pm 0.21 \pm 0.19$ pb,
- $\sigma_{B}(e^+e^- \to \Upsilon(1S)\eta') = 0.42 \pm 0.08 \pm 0.04$ pb,
- $\sigma_{B}(\eta'\rightarrow\eta)(e^+e^- \to \Upsilon(1S)\eta') < 0.035$ pb, $CL = 90\%$.

Significance being $10.2\sigma$ and $16.5\sigma$ for the $e^+e^- \to \Upsilon(1S)\eta$ and $e^+e^- \to \Upsilon(2S)\eta$ processes respectively, we claim the first observation of these processes. For $e^+e^- \to \Upsilon(2S)\eta$ and $e^+e^- \to \Upsilon(2S)\eta$ processes, the measured cross section deviates from the inclusive measurement \cite{12} at $\sim 2.3\sigma$ considering both statistical and uncorrelated systematic uncertainties. For $e^+e^- \to \Upsilon(1S)\eta$, inclusive upper limit does not contradict our measurement.

Under the assumption that processes proceed only through the $\Upsilon(5S)$ meson, we calculate branching fractions with the formula $B(\Upsilon(5S) \to X) = \sigma_{vis}(e^+e^- \to X)/\sigma(e^+e^- \to \Upsilon(5S))$, where $\sigma_{vis}(e^+e^- \to \Upsilon(5S)) = 0.340 \pm 0.016$ nb \cite{22}:

- $B(\Upsilon(5S) \to \Upsilon(1S)\eta) = (0.85 \pm 0.15 \pm 0.08) \times 10^{-3}$,
- $B(\Upsilon(5S) \to \Upsilon(2S)\eta) = (4.13 \pm 0.41 \pm 0.37) \times 10^{-3}$,
- $B(\Upsilon(5S) \to \Upsilon(1S)\eta') < 6.9 \times 10^{-5}$, $CL = 90\%$.

Using $\sigma(e^+e^- \to \Upsilon(1S)\pi^+\pi^-) = 2.27 \pm 0.12 \pm 0.14$ pb, $\sigma(e^+e^- \to \Upsilon(2S)\pi^+\pi^-) = 4.07 \pm 0.16 \pm 0.45$ pb \cite{30} and the obtained Born cross section, we also calculate the width ratios between $\eta$ and dipion-transitions to be

$$\frac{\Gamma(\Upsilon(5S) \to \Upsilon(1S)\eta)}{\Gamma(\Upsilon(5S) \to \Upsilon(1S)\pi^+\pi^-)} = 0.19 \pm 0.04 \pm 0.01$$  \hspace{1cm} (4)

and

$$\frac{\Gamma(\Upsilon(5S) \to \Upsilon(2S)\eta)}{\Gamma(\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-)} = 0.51 \pm 0.06 \pm 0.04,$$  \hspace{1cm} (5)

where correlated systematic uncertainties are canceled out. These values are noticeably larger than the predicted values of $\sim 0.03$ for $\Upsilon(2S)$ and $\sim 0.005$ for $\Upsilon(1S)$, calculated in the QCDME regime, see Ref. \cite{5}, and are comparable to the $\frac{\Gamma(\Upsilon(4S) \to \Upsilon(1S)\eta)}{\Gamma(\Upsilon(4S) \to \Upsilon(1S)\pi^+\pi^-)} = 2.41 \pm 0.40 \pm 0.12$ \cite{8}, measured in a regime where QCDME is no longer valid. Similarly, our measured upper limit on the ratio between the $\eta'$ and $\eta$ transitions is

$$\frac{\Gamma(\Upsilon(5S) \to \Upsilon(1S)\eta')}{\Gamma(\Upsilon(5S) \to \Upsilon(1S)\eta)} < 0.09 \ (CL = 90\%),$$  \hspace{1cm} (6)

which is significantly smaller than the value of $\sim 12$ predicted by the naive QCDME model \cite{2}.
As shown in Refs. [2,3], one of the possible solutions is existence of a light-flavor admixture to the $bb$ state. Such a structure of the $\Upsilon(5S)$ resonance could increase the cross section of $e^+e^- \to \Upsilon(1S,2S)\eta$ and $e^+e^- \to \Upsilon(1S)\eta'$ processes and lead to dominance of the $e^+e^- \to \Upsilon(1S)\eta'$ process over $e^+e^- \to \Upsilon(1S)\eta'$ [3]:

$$\frac{\Gamma(\Upsilon(5S) \to \Upsilon(1S)\eta')}{\Gamma(\Upsilon(5S) \to \Upsilon(1S)\eta)} \approx \frac{p_{\eta'}}{2p_{\eta}} = 0.25,$$

that is much higher than the obtained limit. Such suppression also has been observed in Ref. [33], where $\frac{\Gamma(\Upsilon(4S) \to \Upsilon(1S)\eta)}{\Gamma(\Upsilon(4S) \to \Upsilon(1S)\eta')}$ is reported to be $0.20 \pm 0.06$, in agreement with the expected value in the case of an admixture of a state containing light quarks.

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