CKM - TRIANGLE ANALYSIS:
Updates and Novelties for Summer 2004

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Abstract

Using the most recent determinations of several theoretical and experimental parameters, we update the Unitarity Triangle analysis in the Standard Model. The basic experimental constraints come from the measurements of \(\nu_{e} \to \nu_{\mu} \times V_{ub} \times V_{cd}\) and \(m_t\), the limit on \(m_s\), and the measurement of the CP asymmetry in the B sector through the \(J = K^0\) channel. In addition we also include in our analysis the direct determination of \(\sin 2\theta_{\text{cp}}\), \(\sin (2\theta_{\text{cp}} + \phi_3)\) from the measurement of new CP-violating quantities, recently coming from the B-Factories. We also discuss the opportunities offered by improving the precision of the measurement of the various physical quantities entering in the determination of the Unitarity Triangle parameters.

The results and the plots presented in this paper can be also found at the URL http://www.utfit.org, where they are continuously updated with the newest experimental and theoretical results [1].

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1 Introduction

The analysis of the Unitarity Triangle (UT) and CP violation represents one of the most stringent tests of the Standard Model (SM) and, for this reason, it is also an interesting window on New Physics (NP). The most precise determination of the parameters governing this phenomenon is obtained using B decays, $B^0 - \bar{B}^0$ oscillations and CP asymmetries in the kaon and in the B sectors.

Up to now, the standard analysis [2, 3] relies on the following measurements: $V_{ub} = V_{cb}$, the limit on $m_s$, and the measurements of CP-violating quantities in the kaon ($V_K$) and in the B ($\sin 2\alpha$) sectors. Inputs to this analysis constitute a large body of both experimental measurements and theoretically determined parameters, where Lattice QCD calculations play a central role. A careful choice (and a continuous update) of the values of these parameters is a prerequisite in this study. The values and errors attributed to these parameters are summarized in Table 1 (Section 2).

The results of the analysis and the determination of the UT parameters are presented and discussed in Section 3 which is an update of similar analyses performed in [2] to which the readers can refer for more details.

New CP-violating quantities have been recently measured by the B factories, allowing for the determination of several combinations of UT angles. The measurements of $\sin 2\alpha$, $\sin 2\beta$, and $\sin 2\gamma$ are now available using B decays into $B^0 \bar{B}^0$ and $B^+ \bar{B}^-$ final states, respectively. These measurements are presented in Section 4 and their effect on the UT is discussed in Section 4.4.

Finally, we also discuss the perspectives opened by improving the precision in the measurements of various physical quantities entering the UT analysis. In particular, we investigate to which extent future and improved determinations of the experimental constraints, such as $\sin 2\beta$, $m_s$ and $m_d$, could allow us to invalidate the SM, thus signaling the presence of NP effects.

2 Inputs used for the \textit{Standard} analysis

The values and errors of the relevant quantities entering the standard analysis of the CKM parameters (corresponding to the constraints from $V_{ub} = V_{cb} m_d$, $m_s = m_d$, $V_K$ and $\sin 2\alpha$) are summarized in Table 1.

The novelties here are the new LEP/SLD likelihood from $m_s$, the value of $V_{ub} = V_{cb}$ from inclusive semileptonic decays [4], the new value of $\sin 2\beta$ and a new treatment of the non-perturbative QCD parameters as explained in the following section 2.1.

\begin{align*}
2.1 \text{ Use of } r_{B_s} B_{B_s} \text{ and } f_{B_s} B_{B_s} \text{ in } m_s \text{ and } m_d \text{ constraints}
\end{align*}

One of the important differences with respect to previous studies is in the use of information from non-perturbative QCD parameters entering the expressions of $m_s$ and $m_d$. The $B^0 - \bar{B}^0$ time oscillation frequency, which can be related to the mass difference between the light and heavy mass eigenstates of the $B^0_s - \bar{B}^0_s$ system, is proportional to the square of the $V_{ts}$ element. Up to Cabibbo suppressed corrections, $V_{ts}$ is independent of $\tau$ and $\nu$. As a consequence, the measurement of $m_s$ would provide a strong constraint on the non-perturbative QCD parameter $f_{B_s}^2 B_{B_s}$. 


Table 1: Values of the relevant quantities used in the fit of the CKM parameters. In the third and fourth columns the Gaussian and the at contributions to the uncertainty are given respectively (for details on the statistical treatment see [2]). The central values and errors are those adopted at the end of the CKM Unitarity Triangle Workshops ([5], [6]) and by the HFAG [4].
For this reason we propose a new and more appropriate way of treating the constraints coming from the measurements of $m_s$ and $m_d$. In previous analyses, these constraints were implemented using the following equations:

\[
\begin{align*}
    m_d & / \left[ (1 - \frac{q_2}{q_1})^2 + \frac{q_2^2}{q_1^2} \right] B_{B_d}^q \\
    m_s & / f_{B_s}^2 B_{B_s}^q = \frac{f_{B_d}^2 B_{B_d}^q}{2}
\end{align*}
\]

where $= f_{B_s} B_{B_s}^q f_{B_d} B_{B_d}^q$. In this case the input quantities are $f_{B_d} B_{B_d}^q$ and . The constraints from $m_s$ and the knowledge of are used to improve the knowledge on $f_{B_d} B_{B_d}^q$, which makes the constraint on $m_d$ more effective. The main problem of this method is that the quantity that we know best from lattice calculations is $f_{B_s} B_{B_s}^q$, whereas $^2$ and $f_{B_d} B_{B_d}^q$ are affected by large uncertainties coming from chiral extrapolations. We thus suggest to use a different method which consists in writing the constraints in the following way:

\[
\begin{align*}
    m_d & / \left[ (1 - \frac{q_2}{q_1})^2 + \frac{f_{B_d}^2 B_{B_d}^q}{2} \right] \\
    m_s & / f_{B_s}^2 B_{B_s}^q
\end{align*}
\]

At present, this new parameterization does not have a large effect on final results but, in the future, the measurement of $m_s$ will allow the elimination of a further theoretical parameter, $f_{B_s} B_{B_s}^q$, from the UTs. To obtain a more effective constraint on $m_d$, also the error on should be improved.

3 Determination of the Unitarity Triangle parameters

In this section, assuming the validity of the Standard Model, we give the results for the quantities defining the Unitarity Triangle: $^-, \sin 2^, \sin 2^, \sin (2 - ^)$, as well as other quantities such as $m_s, f_{B_s} B_{B_s}^q, B_K^q$ and . The inputs are summarized in Table 1.

3.1 Fundamental test of the Standard Model in the fermion sector

The most crucial test consists in the comparison between the $^- -$ region selected by the measurements which are sensitive only to the sides of the Unitarity Triangle (semileptonic B decays and $B^0 \rightarrow B^0$ oscillations) and the regions selected by the direct measurements of CP violation in the kaon ($^K$) or in the B ($\sin 2^$) sectors. This test is shown in Figure 1. It can be translated quantitatively through the comparison between the value of $\sin 2$ obtained from the measurement of the CP asymmetry in $J^ = K^0$ decays and the one determined from sides measurements:

\[
\begin{align*}
    \sin 2 & = 0.724 \pm 0.049 \pm 0.0803 \text{ at 95\% C.L.: sides only} \\
    \sin 2 & = 0.739 \pm 0.048 \pm 0.0787 \text{ at 95\% C.L.: } J^ = K^0.
\end{align*}
\]
The allowed regions for $-\beta$ and $\beta$ (contours at 68%, 95% probability ranges), as selected by the measurements of $|V_{ub}|=|V_{cb}| \cdot m_d$, and by the limit on $m_s=m_d$, are compared with the bands (at 68% and 95% probability ranges) from the measurements of CP-violating quantities in the kaon ($K$) and in the B ($\sin 2\beta$) sectors.

The spectacular agreement between these values illustrates the consistency of the Standard Model in describing CP violation phenomena in terms of one single parameter $\sin 2\beta$. It is also an important test of the Operator Product Expansion (OPE), the Heavy Quark Effective Theory (HQET) and Lattice QCD (LQCD) which have been used to extract the CKM parameters. It has to be noted that this test is even more significant because the errors on $\sin 2\beta$ from the two determinations are comparable.

As a matter of fact, the value of $\sin 2\beta$ was predicted, before its first direct measurement was obtained, by using all other available constraints, ($|V_{ub}|=|V_{cb}| \cdot m_d$ and $m_s$). The indirect determination has improved regularly over the years. Figure 2 shows this evolution for the indirect determination of $\sin 2\beta$ which is compared with the recent determinations of $\sin 2\beta$ from direct measurements.

### 3.2 Determination of the Unitarity Triangle Parameters

Using the constraints from $|V_{ub}|=|V_{cb}| \cdot m_d$, $m_s=m_d$, $K$ and $\sin 2\beta$, we obtain the results given in Table 2.

Figures 3 and 4 show, respectively, the pdf's for the main Unitarity Triangle parameters and the selected region in the $-\beta$ -- plane.

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1 In the following, for simplicity, we will denote as 'direct' (indirect) the determination of any given quantity from a direct measurement (from the UT without using the measurement under consideration).
Figure 2: Evolution of the "indirect" determination of $\sin^2 \theta$ over the years (until 2003). From left to right, they correspond to the following papers [3, 7]: DDGN 90, LMMR 92, AL94, CFMRS 95, BBL95, AL96, PPRS97, BF97, BPS98, PS98, AL99, CFGLM 99, CPRS99, M 99, CDFM PRS00, Be et al 00, HLLL00 and CFLPSS [2]. The dotted lines correspond to the 95% C.L. regions (the only information given in those papers). The larger bands (from year '99) correspond to values of $\sin^2 \theta$ from direct measurements (1).

### 3.3 Determination of other important quantities

In the previous sections we have shown that it is possible to obtain the p.d.f.'s for all the various UT parameters. It is instructive to remove from the fitting procedure the external information on the value of one (or more) of the constraints.

In this section we study the distributions of $m_s$ and of the hadronic parameters. For instance, in the case of the hadronic parameters, it is interesting to remove from the constraints on their values coming from lattice calculations and use them as one of
Figure 3: From top left to bottom, the p.d.f.’s for $\eta^-$, $\sin 2\alpha$, $\sin 2\beta$, and $\sin(2\beta + \gamma)$. The red (darker) and the yellow (lighter) zones correspond respectively to $68\%$ and $95\%$ of the normalised area. The following constraints have been used: $\mathcal{Y}_{ub} = \mathcal{Y}_{cb}$, $m_{\Lambda}$, $m_s$, and $\sin 2\gamma$ from the measurement of the CP asymmetry in the $J = K^0$ decays.
Figure 4: Allowed regions for $\tau$ and $\mu$ using the parameters listed in Table 1. The closed contours at 68% and 95% probability are shown. The full lines correspond to 95% probability regions for the constraints, given by the measurements of $y_{ub} = y_{cb} = y_{K} = m_{d}, m_{s}$ and $\sin 2\beta$ from the measurement of the CP asymmetry in the $J = K^{0}$ decays.

the free parameters of the $t$. In this way we may compare the uncertainty obtained on a given quantity through the UT $t$ to the present theoretical error on the same quantity.

3.3.1 The expected distribution for $m_{s}$

Removing the constraint coming from $m_{s}$, the probability distribution for $m_{s}$ itself can be extracted as shown in Figure 5. The results of this exercise are given in Table 3. Present analyses at LEP/SLD have established a sensitivity of $18.3 \, \text{ps}^{-1}$ and they show a higher probability region for a positive signal (see left plot in Fig. 5: a "signal bump" appears around $17.5 \, \text{ps}^{-1}") well compatible with the range of the $m_{s}$ distribution from the UT $t$ (see right plot in Fig. 5). Accurate measurements of $m_{s}$ are expected from the TeVatron in the next future.

3.3.2 Determination of $f_{B_{s}}$, $\hat{B}_{B_{s}}$, $\hat{B}_{K}$ and

To obtain the p.d.f. for a given quantity, we perform the UT $t$ imposing as input a uniform distribution of the quantity itself in a range much wider than the expected interval of values assumed by the parameter. Table 4 shows the results of the UT $t$ when one parameter at the time is taken out of the $t$ with this procedure (see Figure 6). The central value and the error of each of these quantities has to be compared to the current evaluation from lattice QCD, given in Table 1.

Some conclusions can be drawn. The precision on $f_{B_{s}}$, $\hat{B}_{B_{s}}$ obtained from the $t$ has an accuracy which is better than the current evaluation from lattice QCD. This proves
| Parameter | 68%      | 95%      | 99%      |
|-----------|----------|----------|----------|
| $-2$      | 0.348    | 0.028    | [0.293;0.403] |
| $-2$      | 0.172    | 0.047    | [0.082;0.270] |
| $\sin 2$  | 0.725    | 0.033    | [0.645;0.772] |
| $\sin 2$  | -0.16    | 0.26     | [-0.62;0.35] |
| $[\sin (2\theta + \delta)]$ | 61.5     | 7.0      | [47.5;76.6] |
| $\sin 2$  | > 0.94   | > 0.88   | > 0.84   |
| $\text{Im}^e[10^{-5}]$ | 13.5     | 1.0      | [1.15;15.3] |

Table 2: Values and probability ranges for the Unitarity Triangle parameters obtained by using the following constraints: $V_{ub}=V_{cb}$, $m_d=m_s$, $m_s=m_d$, $\kappa$ and $\sin 2$.

Figure 5: Left plot: combined results [4] from all analyses on the oscillation amplitude, as a function of $m_s$. The points with error bars are the data; the lines show the 95% C.L. curves (in dark the systematics have been included). The dotted curve corresponds to the sensitivity. Right plot: $m_s$ probability distributions, obtained without using the information from $B_s^0\rightarrow\bar{B}_s^0$ mixing.

that the standard $\mathcal{C}_{\text{KM}}$ is, in practice, weakly dependent on the assumed theoretical uncertainty on $f_{B_s}$.

The result on $B_K$ indicates that values of $B_K^\text{eff}$ smaller than 0.45 are excluded at 99% probability, while large values of $B_K^\text{eff}$ are compatible with the prediction coming from the UT using the other constraints. The present estimate of $B_K$ from lattice QCD, which has a 15% relative error (Table 1), is still more precise than the indirect determination from the UT. Likewise, the present best determination of the parameter comes from lattice QCD.
### Table 3: Central values and ranges for $m_s$ corresponding to defined levels of probability, obtained by including or not including the information from the experimental amplitude spectrum A ($m_s$).

| Parameter | 68%         | 95%         | 99%         |
|-----------|-------------|-------------|-------------|
| $m_s$ (including $m_s$) [ps$^{-1}$] | 18.3 (15.4-23.1) | 1.6 (15.1-27.0) |
| $m_s$ (without $m_s$) [ps$^{-1}$] | 21.1 (15.3-27.7) | 3.1 (13.7-29.9) |

### Table 4: Values and probability ranges for the non-perturbative QCD parameters, if the external information (input) coming from the theoretical calculation of these parameters is not used in the CKM fits.

| Parameter | 68%         | 95%         | 99%         |
|-----------|-------------|-------------|-------------|
| $q\bar{q}$ | $1.13^{+0.12}_{-0.09}$ | $[0.95;1.41]$ | $[0.92;1.57]$ |
| $f_{B_s} \hat{B}_{B_s} \text{ (MeV)}$ | 263 | $[236;290]$ | $[231;320]$ |
| $\hat{B}_{K}$ | 0.65 | 0.10 | $[0.49;0.87]$ | $[0.45;0.99]$ |

### Table 5: Values and probability ranges for the non-perturbative QCD parameters, if two external pieces of information (inputs) coming from the theoretical calculation of these parameters are not used in the CKM fits.

| Parameter | 68% | 95% | 99% |
|-----------|-----|-----|-----|
| $f_{B_s} \hat{B}_{B_s} \text{ (MeV)}$ | 252 | $[228;283]$ | $[219;325]$ |
| $\hat{B}_{K}$ | 0.63 | 0.10 | $[0.48;0.87]$ | $[0.44;1.02]$ |

### 4 New Constraints from UT angle measurements

The values for $\sin 2\beta$, $\gamma$, and $\sin(2\beta + \gamma)$ given in Table 2 have to be taken as predictions for future measurements. A strong message is given for instance for the angle $\beta$. Its indirect determination is known with an accuracy of about 10%. It has to be stressed
Figure 6: One- and two-dimensional 68% and 95% probability distributions for $\hat{B}_K$, and $f_{B_s \hat{B}_s}$ (see Sec. 3.3.2).
that, with present measurements, the probability for \( \sin(2\theta) \) to be greater than 90° is only 0.0055%.

Thank to the huge statistics collected at the B-Factories, new CP-violating quantities have been recently measured allowing for the direct determination of \( \sin(2\theta) \), \( \sin(2\phi) \), and \( \sin(2\phi) \). In the following we present the results including these new measurements and their impact on the plane.

4.1 Determination of the angle using DK events

Various methods using B \( \to \) DK decays have been used to determine the Unitarity Triangle angle \( \theta_{12} \) [8]. The basic idea in these methods is the following. A charged B meson can decay into a D^0 \( \bar{D}^0 \) final state via a \( V_{ub}(V_{ub}) \) mediated process. CP violation occurs if the D^0 and the \( \bar{D}^0 \) decay in the same final state. The measurement of the direct CP violation is thus sensitive to the \( V_{ub} \) and \( V_{ub} \) phase difference. The same argument can be applied to B \( \to \) DK decays.

The most important aspect of these decays is that they proceed only via tree-level diagrams, implying that the determination of \( \theta_{12} \) is not affected by possible New Physics loop contributions.

One of these methods is the GLW method and it consists in reconstructing D^0 mesons in a specific CP (even/odd) mode. The ADS method is, instead, based on the fact that D^0 \( \bar{D}^0 \) decays can reach the same final state through Doubly Cabibbo Suppressed (DCS) (Cabibbo Allowed (CA)) processes. The following observables are defined in these two methods:

\[
\begin{align*}
R_{CP} &= 1 + r_B^2 2r_B \sin \theta_B \sin \theta_D \\
A_{CP} &= 2r_B \sin \theta_B \sin \theta_D R_{CP}
\end{align*}
\]

\[
A_{CP} = \frac{A(B \to D^0K)}{A(B \to \bar{D}^0\bar{K})}
\]

\[
R_{ADS} = r_{DCS}^2 + r_{B}^2 + 2r_B r_{DCS} \cos \phi
\]

where

\[
r_B = \frac{A(B \to \bar{D}^0\bar{K})}{A(B \to D^0K)}
\]

\[
r_{DCS} = \frac{D^{0} \to K^{+}}{D^{0} \to K^{0}}
\]

and \( \theta \) (\( \phi \)) is the difference between the strong phases of the two amplitudes in the B system (B and D systems).

In [9] a new method based on the Dalitz analysis of three-body decays has been proposed and recent results using this new technique applied to the D^0 \( \to \) K\( _s \) decays have been published by the Belle Collaboration [10]. The advantage of this method is that the full sub-resonance structure of the three-body decay is considered, including interferences such as those used for GLW and ADS methods plus additional interferences because of the overlap of broad resonances in certain regions of the Dalitz plot. The same analysis can also be performed using B \( \to \) D^0K decays. The technique is identical to the one used in B \( \to \) D^0K decays but the values for \( r_B \) and \( r_D \) are different so they will be indicated as \( r_B \) and \( r_D \) in the following. It is also interesting to note that the Dalitz analysis has only a two-fold discrete ambiguity (\( +; - \)) and not a four-fold ambiguity as in case of the GLW and ADS methods. It has to be noted that experimental likelihoods have been
used to correctly implement the measurement of \( R_{ADS} \) and the Dalitz result. A summary of the experimental results is given in Table 6.

| Observable | World Average |
|------------|---------------|
| \( A_{CP} \) | 0.07 0.13 |
| \( A_{CP} \) | -0.19 0.18 |
| \( R_{CP} \) | 1.09 0.16 |
| \( R_{CP} \) | 1.30 0.25 |
| \( R_{ADS} \) | 0.0054 0.0124 |
| \( r_{B} (r_{B}) \) | 0.26^{+0.09}_{-0.14} 0.04 (0.20^{+0.03}_{-0.17} 0.02 0.04) |
| & \( 77^{+19}_{-17} 13 11 \) (Belle Dalitz) |

Table 6: Summary of the results obtained with \( B \to D^{(*)}K \) decays and using the GLW and the ADS methods. The last two lines give the results from the Dalitz analysis [9], for which the ambiguity is implicit.

All measurements in Table 6 are used to extract \( r_{B} \) \( (r_{B}) \) and the selected region in the \( r_{B} \) vs \( r_{B} \) plane are shown in Figure 7, where also the effect of this measurement in the \( r_{B} \) plane is shown.

The comparison between the direct and the indirect determination is:

\[
\left[ r_{B} \right] = 61 \pm 7 \pm [47.5;64] \text{ at } 95\% \text{ C.L.: direct} \\
\left[ r_{B} \right] = 73 \pm 27 \pm [4;133] \text{ at } 95\% \text{ C.L.: (107 27 [176;47]) direct} \quad (6)
\]

An important result of this analysis is also the p.d.f. for \( r_{B} \) from which the following result can be given:

\[
r_{B} = 0.105 \pm 0.065 < 0.22 \text{ at } 95\% \text{ C.L.: direct from } \text{DK} \quad (7)
\]

4.2 Determination of \( \sin 2 \) using and events

In the absence of contributions from penguin diagrams, the measurements of the parameter \( S \) of the time-dependent CP asymmetry for \( B^0 \to D^0 \to V^+V^- \) and \( B^0 \to D^{(*)} K \) give measurements of the quantity \( \sin \theta \). Even in the presence of penguins, one can use the SU(2) flavor symmetry to connect the measured value of \( S = \sin \theta_{\text{eff}} \) to the value of \( \sin \theta \), constraining the contribution from penguin diagrams using the Branching Fractions and the direct CP asymmetry measurements of all the \( B \to \ell^+\ell^- \) decays [11]. The decay amplitudes in the SU(2) limit and neglecting electroweak penguins can be written as:

\[
A^+ = Te^i + Pe^i \\
A^{+0} = \frac{1}{\sqrt{2}} e^i (T + T_c e^{i\tau_c}) \\
A^{00} = \frac{1}{\sqrt{2}} e^i T_c e^{i\tau_c} + P e^i 
\]

(8)
Figure 7: P.d.f.’s for $r_b$ from $D^0 K$ (top-left), for $r_b$ from $D K$ (center-left) and for (bottom-left) using the results from $B^+ D^{(*)} K$ decays. The plots on the right show the measurement of the angle in the $r_b^{(*)}$ (top, center) and (bottom) planes.

They can be expressed in terms of three independent hadronic amplitudes, the absolute values of which are denoted as $T$, $T_c$, and $P$. Similarly, $\rho$ and $\tau_c$ are the strong phases of $P$ and $T_c$, once the phase convention is chosen so that $T$ is real. It should be noted that these parameters are different for $B^+ D^{(*)} K$ and $B^+ D^{(*)} K$ decays. For the $B^+ D^{(*)} K$ decays we have in principle to further double the parameters for the longitudinal and the transverse polarization. On the other hand, the experimental measurements are compatible with decays which are fully longitudinally polarized. For this reason, in case of $B^+ D^{(*)} K$, we make the assumption of fitting the amplitude with only one set of parameters. Notice that the number of parameters exceeds the number of available measurements.
Table 7: Experimental inputs from Isospin Analysis in $B^+$ and $B^0$ decays [4]. The $B^0$ decays are assumed to be fully polarized, in agreement with available measurements.

| Observable | BaBar | Belle | Average | BaBar | Belle | Average |
|------------|-------|-------|---------|-------|-------|---------|
| C          | -0.19 | 0.20  | -0.58  | 0.17  | -0.46 | 0.13    |
| S          | -0.40 | 0.22  | -1.00  | 0.22  | -0.74 | 0.16    |
| $BR^+$ ($10^{-6}$) | 4.7   | 0.6   | 4.4    | 0.7   | 4.6   | 0.4     |
| $BR^{+0}$ ($10^{-6}$) | 5.5   | 1.2   | 5.0    | 1.3   | 5.2   | 0.8     |
| $BR^{00}$ ($10^{-6}$) | 2.1   | 0.7   | 1.6    | 0.6   | 1.9   | 0.5     |
| $BR^+$ ($10^{-6}$) | 4.7   | 0.6   | 4.4    | 0.7   | 4.6   | 0.4     |
| $BR^{+0}$ ($10^{-6}$) | 5.5   | 1.2   | 5.0    | 1.3   | 5.2   | 0.8     |
| $BR^{00}$ ($10^{-6}$) | 2.1   | 0.7   | 1.6    | 0.6   | 1.9   | 0.5     |

Table 8: Comparison of input and output values for the Branching Fractions of $B^+$ and $B^0$ decay modes.
Figure 8: Distribution of $\sin(2\alpha)$ (left) and relative bound on $-\frac{s}{c}$ plane (right), using BaBar only (top) and World Average (bottom) [4] values.

Figure 9: Plots showing the correlation of $\text{BR}(B^+ \to \pi^+ \pi^0)$ vs $\text{BR}(B^0 \to \pi^+ \pi^0)$ (left) and $\text{BR}(B^0 \to \pi^+ \pi^0)$ vs $\text{BR}(B^+ \to \pi^+ \pi^0)$ (right) as obtained from SU(2) parameterizations in UT$t$

4.3 Determination of $\sin(2\pi\alpha)$ using $D^0$ events

$\sin(2\pi\alpha)$ can be extracted from time-dependent asymmetries in B decays to $D^{(*)}$ final states, looking at the interference effects between the decay amplitudes implying $b \to c$
and $b^+u$ transitions. The time-dependent rates can be written as:

\[
R(B^0 \rightarrow D^{(*)+}) = N e^{t(1 + C \cos(m_d t) + S \sin(m_d t))}
\]

\[
R(B^0 \rightarrow D^{(*)+}) = N e^{t(1 + C \cos(m_d t) - S \sin(m_d t))}
\]

\[
R(B^0 \rightarrow D^{(*)+}) = N e^{t(1 + C \cos(m_d t) + \bar{S} \sin(m_d t))}
\]

\[
R(B^0 \rightarrow D^{(*)+}) = N e^{t(1 + C \cos(m_d t) + \bar{S} \sin(m_d t))}
\]

where $S$ and $C$ parameters are defined as

\[
S = 2r(1 + r^2) \sin(2 + )
\]

\[
\bar{S} = 2r(1 + r^2) \sin(2 + )
\]

\[
C = (1 - r^2)(1 + r^2)
\]

and $r$ and $\theta$ are the absolute value and the phase of the amplitude ratio $A(B^0 \rightarrow D^{(*)+}) = A(B^0 \rightarrow D^{(*)+})$. This ratio $r$ is rather small being of the order of $j_{(ub)}j_{(cb)}' \approx 0.02$.

There is a correlation between the tag side and the reconstruction side in time-dependent CP measurements at B-Factories [12]. This is related to the fact that interference between $b^+c$ and $b^+u$ transitions in $B^0 \rightarrow D X$ decays can occur also in the tag side. $S$ and $\bar{S}$ entering the time-dependent rates can be replaced by

\[
a = 2r \sin(2 + ) \cos(\theta)
\]

\[
b = 2r^* \sin(2 + ) \cos(\theta^*)
\]

\[
c = 2 \cos(2 + ) (r \sin(\theta) + r \sin(\theta^*))
\]

where $r^*$ and $\theta^*$ are the analogue of $r$ and $\theta$ for the tag side. It is important to stress that this interference on the tag side cannot occur when $B$ mesons are tagged using semi-leptonic decays. In other words, $r^* = 0$ when only semi-leptonic decays are considered. In the following we will consider the observables $a$, $c(\text{lepton})$, $a$ and $c$ (lepton), which are functions of $r^*$, $\theta^*$ and $2 +$.

BaBar and Belle provided three different measurements of this channel, with total (both) [13] or partial (BaBar only) [14] reconstruction of the final state, as summarized in Table 9.

| Parameters | HFAG average [4] |
|------------|------------------|
| $a$        | -0.038 0.021     |
| $a$        | 0.012 0.030      |
| $c$ (lepton)| -0.041 0.029    |
| $c$ (lepton)| -0.015 0.044    |

Table 9: Summary of the experimental results from BaBar and Belle, as reported in [4].

We use directly the four experimental quantities: $a^\prime$ and $c^\prime$ (lepton) defined before and we build a global p.d.f. as the product of the p.d.f.'s of these four quantities. We do not make any assumption on $r$ and $\theta$ which are extracted in the range [0,0,1]. The
Figure 10: Distributions for $r$ for $D$ (top-left), $r$ for $D$ (top-right), $\sin(2 + \gamma)$ for $D$ (middle-left), $\sin(2 + \gamma)$ for $D$ (middle-center) and $\sin(2 + \gamma)$ combined (middle-right) and bound from $\sin(2 + \gamma)$ on the $-\rho$ plane (bottom).
results on \( r, r', \sin(2 \gamma + \delta) \) and the impact of this measurement in the \( \beta - \gamma \) plane are shown in Figure 10.

The comparison between the direct and the indirect determination of \( \sin(2 \gamma + \delta) \) is given below:

\[
\sin(2 \gamma + \delta) > 0.94 \text{ at } 68\% \text{ C.L.} (> 0.88 \text{ at } 95\% \text{ C.L.}) \quad \text{indirect} \\
\sin(2 \gamma + \delta) > 0.28 \text{ at } 68\% \text{ C.L.} (> 0.08 \text{ at } 95\% \text{ C.L.}) \quad \text{D}^1 : \quad (13)
\]

4.4 Determination of the Unitarity Triangle parameters using also the new UT angle measurements

It is interesting to see the selected region in \( \beta - \gamma \) plane from the measurements of the UT angles in the B sector. The plot is shown in Figure 11. In Table 10 we report the results we get using these constraints.

The results given in Table 11 are obtained using all the available constraints: \( Y_{wb} = Y_{ub} \), \( m_s = m_d \), \( \sin 2 \gamma \), \( \sin(2 \gamma + \delta) \), and \( \sin 2 \beta \). Figure 12 shows the corresponding selected region in the \( \beta - \gamma \) plane.

5 Compatibility plots, or how to discover New Physics in the flavour sector

In this section we discuss the interest of measuring the various physical quantities entering the UT analysis with a better precision. We investigate, in particular, to which extent future and improved determinations of the experimental constraints, such as \( \sin 2 \beta \), \( m_s \),
Figure 12: Allowed regions for $-$ and $-$ using the parameters listed in Table 1. The closed contours at 68% and 95% probability are shown. The full lines correspond to 95% probability regions for the constraints, given by the measurements of $V_{ub} = V_{cb} = k$, $m_d$, $m_s$, and $\sin 2\beta$, $\sin 2\gamma$ and $\sin 2\alpha$.

| Parameter | 68%      | 95%      | 99%      |
|-----------|----------|----------|----------|
| $-\beta$  | $0.265^{+0.120}_{-0.070}$ | $0.165^{+0.869}_{-0.052}$ | $0.052^{+0.980}_{-0.005}$ |
| $\sin 2\beta$ | $0.315^{+0.040}_{-0.061}$ | $0.040^{+0.378}_{-0.004}$ | $0.004^{+0.437}_{-0.004}$ |
| $\sin 2\gamma$ | $0.733^{+0.049}_{-0.049}$ | $0.636^{+0.828}_{-0.606}$ | $0.606^{+0.858}_{-0.606}$ |
| $\sin 2\alpha$ | $-0.66^{+0.26}_{-0.26}$ | $<0.48$ | $<0.59$ |
| $\Im [10^5]$ | $50.0^{+9.2}_{-14.9}$ | $<61.5$ | $<79.6$ |

Table 10: Values and probability ranges for the Unitarity Triangle parameters obtained by using $\sin 2\beta$, $\sin 2\gamma$ and $\sin 2\alpha$.

and $\sin 2\gamma$, could allow us to possibly invalidate the SM, thus signaling the presence of NP effects.

5.1 Compatibility between individual constraints. The pull distributions.

In CKM fits based on $\chi^2$ minimization, a conventional evaluation of compatibility stems automatically from the value of $\chi^2$ at its minimum. The compatibility between constraints in the Bayesian approach is simply done by comparing two different p.d.f.'s. For example, compare the value for $\sin 2\beta$ obtained from the measurement of the sides of the Unitarity Triangle (the random variable $x_1$) with the one obtained from the direct measurement of the CP violation asymmetry (the random variable $x_2$). In this case the
### Table 11: Values and probability ranges for the Unitarity Triangle parameters obtained by using all the available constraints: $V_{ub} = V_{cd}$, $m_d = m_s = m_d$, $\sin^2 \theta_K$ and $\sin^2 \theta$.

| Parameter | 68%     | 95%     | 99%     |
|-----------|---------|---------|---------|
| $\theta_K$ | 0.324   | 0.020   | [0.283, 0.359] | [0.268, 0.373] |
| $\theta$  | 0.225   | 0.030   | [0.171, 0.288] | [0.151, 0.326] |
| $\sin^2 \theta$ | 0.710   | 0.032   | [0.645, 0.773] | [0.624, 0.792] |
| $\sin^2 \theta$ | -0.44$^{+0.07}_{-0.99}$ | [-0.68, -0.18] | [-0.81, -0.13] |
| $\text{Im} \left[ \sin^2 \theta \right]$ | 53.9$^{+5.5}_{-24}$ | [46.6, 62.4] | [41.1, 64.6] |
| $\text{Im} \left[ \cos^2 \theta \right]$ | 12.4    | 0.8     | [11.0, 14.0] | [10.5, 14.5] |

The distribution of the random variable $y = x_1 x_2$ has to be constructed and the integral of this distribution above (or below) zero gives the one side probability of compatibility. The advantage of this approach is that the full overlap between the p.d.f.'s is evaluated instead of a single number.

If two constraints turn out to be incompatible, further investigation is necessary to tell if this originates from a "wrong" evaluation of the input parameters or from a New Physics contribution.

#### 5.2 Pull distribution for $\sin^2 \theta$. Role of $\sin^2 \theta$ from Penguin processes.

We start this analysis by considering the measurement of $\sin^2 \theta$. The plots in Figure 13 show the compatibility ("pull") between the direct and indirect distributions of $\sin^2 \theta$, in the SM, as a function of the measured value (x-axis) and error (y-axis) of $\sin^2 \theta$.

From the left plot in Figure 13, it can be seen that, considering the actual precision of about 0.05 on the measured value of $\sin^2 \theta$, the 3 compatibility region is between [0.49-0.87]. Values outside this range would be, therefore, not compatible with the SM prediction at more than 3 level. To get these values, however, the presently measured central value should shift by more than 4.

The conclusion that can be derived from Figure 13 is the following: although the improvement of the error on $\sin^2 \theta$ has an important impact on the accuracy of the UT parameter determination, it is very unlikely that in the near future we will be able to use this measurement to detect any failure of the SM, unless the other constraints entering the picture substantially or, of course, in case the central value of the direct measurement move away from the present one by several standard deviations.

The right plot in Figure 13 shows the compatibility of the direct and indirect distributions of $\sin^2 \theta$ as a function of the measured value and error of $\sin^2 \theta$. The difference with respect to the left plot is that, in this case, all the available constraints have been used to obtain the indirect distribution of $\sin^2 \theta$, including the direct measurement of $\sin^2 \theta$.

---

2 In the Gaussian case it coincides with the pull which is defined as the difference between the central values of the two distributions divided by the sum in quadrature of the rms's of the distributions themselves.
Figure 13: The compatibility (\textit{pull}) between the direct and indirect determination of $\sin^2\beta$ as a function of the value and error of $\sin^2\beta$ measured from CP asymmetry in $J=K^0$ decays. The indirect distribution of $\sin^2\beta$ is computed without using the direct measurement (left plot) or using the measurement of $\sin^2\beta$ from CP asymmetry in $J=K^0$ decays (right plot). The compatibility regions from 1 to 6 are also displayed.

| Observable | BaBar | Belle | Average |
|------------|-------|-------|---------|
| $S_{K^0_S}$ | 0.47 $^{+0.08}_{-0.06}$ | -0.96 $^{+0.49}_{-0.11}$ | 0.02 0.29 |
| $C_{K^0_S}$ | 0.01 0.33 0.10 0.15 0.29 0.08 | 0.09 0.23 |

Table 12: Experimental inputs for $S$ and $C$ in $B^0 \to K^0_S$ and $B^0 \to K^0_S K^0_S$ decays [4].

It was pointed out sometime ago that the comparison of the time-dependent CP asymmetries in various $B$ decay modes could provide evidence of NP in $B$ decay amplitudes [15]. Since $\sin^2\beta$ is known from $J=K^0$, a significant deviation of the time-dependent asymmetry parameters of penguin dominated channels $B^0 \to K^0$ and $B^0 \to K^0 K^0$ from their expected values would indicate the presence of NP.

These asymmetries have been recently measured at B-Factories [16] and are reported in Table 12.

In a naive approach, one expects $S \sin^2\beta$ and $C \cos\beta$, but the theoretical uncertainties related to hadronic physics can change this expectation. Starting from the value of $\sin^2\beta$ of the standard analysis, we used the Charming Penguins model [17] to take into account these hadronic uncertainties and quantify the sensitivity of future measurements with the compatibility plots shown in Figure 14. It has to be noted that, including these hadronic uncertainties, the theoretical predictions of $S$ and $C$ in Table 12 have a typical uncertainty of $0.09$ [18].
5.3 Pull distribution for $m_s$

The plot in Figure 15 shows the compatibility of the indirect determination of $m_s$ with a future determination of the same quantity, obtained using or ignoring the experimental information coming from the present bound.

From the plot in Figure 15 it can be concluded that, once a measurement of $m_s$ with the expected accuracy of $1 \text{ ps}^{-1}$ is available, a value of $m_s$ greater than $32 \text{ ps}^{-1}$ would imply New Physics at 3 level.

5.4 Pull distribution for the angle

The plot in Figure 16 shows the compatibility of the indirect determination of the angle obtained from B decays. It can be noted that even in this case the angle can be measured with a precision of $10$ from B decays, the predicted region is still rather large, corresponding to the interval $[25-100]$. Values beyond 100 would clearly indicate physics beyond the Standard Model. The actual determination of the angle is not yet precise enough to test the validity of the Standard Model as shown.
Figure 15: The compatibility of the direct and indirect determination of $m_s$, as a function of the value of $m_s$, using (left) or ignoring (right) the present experimental bound.

Figure 16: The compatibility of the direct and indirect determination of $\gamma$, as a function of the value and the error of $\gamma$, using the UT fits results.
by the point with the error bar in Figure 16 and given in equation (6). Nevertheless, a direct determination of is of crucial importance to test NP models [19].

6 Conclusions

Flavour physics in the quark sector has entered its mature age. Today the Unitarity Triangle parameters are known with good precision. A crucial test has been already done: the comparison between the Unitarity Triangle parameters, as determined with quantities sensitive to the sides of the triangle (semileptonic B decays and oscillations), and the measurements of CP violation in the kaon ($K$) and in the B ($\sin^2$) sectors. The agreement is "Unfortunately" excellent. The Standard Model is "Standardissimo": it is also working in the flavour sector. This agreement is also due to the impressive improvements achieved on OPE, HQET and LQCD theories which have been used to extract the CKM parameters.

Many B decay branching Fractions and relative CP asymmetries have been measured at B-Factories. The outstanding result is the determination of $\sin^2$ from B hadronic decays into charmonium-$K^0$ final states. On the other hand many other exclusive hadronic rare B decays have been measured and constitute a gold mine for weak and hadronic physics, allowing in principle to extract different combinations of the Unitarity Triangle angles.

Besides presenting an update of the standard UT analysis, we have shown in this paper that new measurements at B-Factories start to have an impact on the overall picture of the Unitarity Triangle determination. In the following years they will provide further tests of the Standard Model in the flavour sector to an accuracy up to the percent level.

Finally, introducing the compatibility plots, we have studied the impact of future measurements on testing the SM and looking for New Physics.

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