Is Electron an Anyon with Spin-1/2?

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Abstract

It is argued that electron can be treated as an anyon which carries a charge (-e) and a magnetic flux $\pm \frac{\Phi_0}{2}$ in the presence and absence of a uniform external magnetic field. This flux is shown to arise due to the spin of the electron. The flux associated with the electron spin is calculated using a semi-classical model which is based on the magnetic top model. In accordance with spherical top model it is assumed that the spin angular momentum of the electron is produced by the fictitious point charge (-e) rotating in a circular orbit. It is shown that the flux through the circular orbit is independent of the radius and $\frac{\Phi_0}{2}$ for a spin down electron and $\frac{\Phi_0}{2}$ for a spin up one. Where $\Phi_0 = \frac{hc}{e}$ is the flux quantum.

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I. INTRODUCTION

In two dimensional space, quantum particle, can exhibit a continuous range of statistics interpolating between bosons and fermions \[1\, \text{[3]}\]. Such exotic particles are called anyons and they are believed to play a role in the Fractional Quantum Hall Effect and, perhaps, even in high-temperature superconductors \[4\]. Wilczek \[2\, \text{[3]}\] showed that, each anyon is taken to be a boson carrying a unit statistical magnetic flux, $\Phi$, concentrated at its location. When one anyon circles once around another in an anti-clockwise sense; the wavefunction picks up a Aharanov-Bohm phase: $\exp\left[-2\pi i \frac{2\Phi e}{hc}\right] \equiv \exp(2i\theta)$. The cases $\theta = 0, \frac{\pi}{2}$ and $\pi$ correspond to the particles being bosons, semions and fermions respectively \[5\]. Alternatively, we can take the anyons to be fermions carrying a statistical flux $2\theta$. Then $\theta = 0, \pi/2$ and $\pi$ correspond to fermions, semions and bosons. The purpose of this note is to present that electron itself may be considered as an anyon. Namely it carries a charge and a magnetic flux associated with its spin. We calculate the magnetic flux associated with the electron’s spin using by a semi-classical model which is based on the magnetic top model \[6-8\] that can be made equivalent to a circular motion of a point charge in two dimensions. We show that the magnetic flux associated with electron spin is $\frac{hc}{2e}$ for spin down electron and $-\frac{hc}{2e}$ for spin up one.

II. FORMALISM

Generally the Lagrangian of a non-relativistic electron with mass $m$ and electric charge $(-e)$ moving in a uniform magnetic field in z direction ($\vec{B} = B\hat{k}$) is given by

$$L = \frac{1}{2}m \vec{v}^2 - \frac{e}{c} \vec{v} \cdot \vec{A}(r)$$

where $\vec{r} = (x, y)$ is the position vector in 2D (two dimensions), $\vec{v} = \vec{\dot{r}}$ is the velocity vector and $\vec{A}$ is the vector potential ($\vec{B} = \nabla \times \vec{A}$). In the symmetric gauge, the vector potential can be written as
\[
\vec{A} = \frac{1}{2} \vec{B} \times \vec{r} = \frac{B}{2} (-y\hat{i} + x\hat{j})
\]  

(2)

where \(\hat{i}\) and \(\hat{j}\) are the unit vectors along the x and y axis respectively.

To calculate the flux in terms of the radius and the magnetic field; we write:

\[
\Phi = \oint \frac{\vec{B}}{2} \cdot (\vec{r} \times d\vec{r}) = \oint \frac{\vec{B}}{2} \cdot (\vec{r} \times \frac{d\vec{r}}{dt}) dt
\]  

(3)

If the electron is rotating in x-y plane in the counter clockwise direction with the angular frequency \(\omega_c = \frac{eB}{mc}\). The position vector is

\[
\vec{r} = r \cos \omega_c t \hat{i} + r \sin \omega_c t \hat{j}.
\]  

(4)

Taking the time derivative of Eq.(4) we get the velocity vector:

\[
\vec{v} = \frac{d\vec{r}}{dt} = -r \omega_c \sin \omega_c t \hat{i} + r \omega_c \cos \omega_c t \hat{j}.
\]  

(5)

Substitution of Eqs.(4),(5) and \(\vec{B} = B\hat{k}\) in Eq.(3) we get

\[
\Phi = \frac{B}{2} \omega_c r^2 \int_0^{T_c} dt = \frac{\pi}{\omega_c} r^2 B = \pi (x^2 + y^2)B
\]  

(6)

where \(\omega_c = \frac{2\pi}{T_c} = \frac{eB}{mc}\) is the cyclotron angular frequency. We note that the time integral in Eq.(6) has to be taken over one cyclic period \(T_c = \frac{2\pi}{\omega_c}\). This is the crucial point of our calculation.

By using Eqs.(6) and (2) the vector potential, \(\vec{A}\) in the symmetric gauge can be related to the magnetic flux \(\Phi\) by

\[
\vec{A}(r) = \frac{\Phi}{2\pi} (\frac{-y}{x^2 + y^2}\hat{i} + \frac{x}{x^2 + y^2}\hat{j}).
\]  

(7)

The canonical momentum \(\vec{p}\) can be derived from the Lagrangian given in Eq.(1):

\[
\vec{p} = \frac{\partial L}{\partial \dot{\vec{v}}} = m \vec{v} - \frac{e}{c} \vec{A}.
\]  

(8)

Because of the rotationally invariance of the Lagrangian the canonical angular momentum, \(J_c\) is constant:
\[ J_c = \vec{r} \times \vec{p} = \vec{r} \times (m \vec{v} - e \frac{\vec{A}}{c}) = \vec{r} \times m \vec{v} - \frac{e}{c} \vec{r} \times \vec{A} = J - \frac{e \phi}{2 \pi c} \] (9)

where \( J \) is the gauge invariant kinetic angular momentum. This conserved canonical angular momentum \( J_c \) has a conventional spectrum \([9,10]\): its eigenvalues are always integers in units of \( \hbar \). The difference between \( J_c \) and \( J \) is due to presence of the magnetic flux, and hence the magnetic field \( B \). Both in the absence and in the presence of the magnetic field \( B \), the canonical angular momentum is always represented by the quantum mechanical operator:

\[ J_c = -i\hbar \frac{\partial}{\partial \varphi} \] (10)

where \( \varphi \) is the polar angle on x-y plane.

The eigenvalues of \( J_c \) is \( m\hbar \) (\( m \in \mathbb{Z} \)) when it acts on single valued wavefunctions with angular dependence \( \exp(im\varphi) \). Therefore the kinetic angular momentum operator can be written as

\[ J = J_c + \frac{e\phi}{2\pi c} = -i\hbar \frac{\partial}{\partial \varphi} + \frac{e\phi}{2\pi c} \] (11)

which when acting on single-valued wavefunctions with angular dependence \( \exp(im\varphi) \) becomes:

\[ J = \hbar(m + \frac{e\phi}{\hbar c}) \quad (m \in \mathbb{Z}) \] (12)

which states that the spectrum of \( J \) consists of integers shifted by \( \frac{e\phi}{\hbar c} \) and is non-zero even for \( m=0 \). We believe that this non-zero contributions to the kinetic angular momentum comes from spinning motion of the electron. In the following part we calculate the magnetic flux associated with the electron’s spin using by a semi-classical model which is based on the magnetic top model \([6-8]\) that can be made equivalent to a circular current loop. Our calculations gives that magnetic flux associated with electron spin is \( \frac{\hbar c}{2e} \) for a spin down electron and \( -\frac{\hbar c}{2e} \) for spin up one.

As clearly stated by Barut et al \([8]\) the magnetic top is an adequate model of quantum spin, because the magnetic moment performs a simple precession around the magnetic field
although the top itself performs a complicated motion. it was also shown that by canonical and Schrödinger quantization, Pauli theory of spin was obtained. In the magnetic top model the electron is assumed to be a small sphere with the radius $r_e$ and the spin of the electron is assumed to be produced by the electron’s rotation about itself with an angular frequency $\omega_s$. Our model [11] is based on the magnetic top model which can be made equivalent to a circular current loop with the radius $R$ in $x$-$y$ plane. It will be shown that as far as the flux is concerned the radius $R$ of this loop is a phenomenal concept and it gets eliminated in the end. In this model the electron motion is considered in two parts namely an ”external” motion which can be interpreted as the motion of the center of mass (and hence the central of charge) and an ”internal” one whose average disappears in the classical limit. The latter is caused by the spin of the electron. The important thing is that although the average of the internal motion disappears the average of the flux associated with the internal motion does not. In accordance with the magnetic top model, we assume that the spin angular momentum of the electron is produced by the fictitious point charge (-e) rotating in a circular orbit with radius $R$ and with the same angular frequency $\omega_s$. It can be shown that the present calculations can be generalized to any kind of spherical charge distribution for electron. The details of the charge distribution determines the relation between the radius $r_e$ of the electron and the radius $R$ of the current loop. Further it is important to note that electron’s spinning frequency $\omega_s$ is very high compared to the cyclotron frequency $\omega_c$ (for $B=5 \times 10^3$ Gauss, $\omega_s \approx 10^{15}\omega_c$ ). As was shown in Eq.(6) for calculation of the magnetic flux the cyclotron period $T_c = \frac{2\pi}{\omega_c}$ is the important time interval. During the cyclotron period $T_c$, electron completes only one turn around its cyclotron orbit, but it spins ($\frac{\omega_s}{\omega_c} \gg 1$) times about itself. Although the radius of the electron ( and hence the radius of the loop) is very small, because of the rapid spinning, the total flux during the cyclotron period, $T_c$ will be comparable with the flux quantum, $\Phi = \frac{hc}{e}$. We will see that the total flux associated with the spin is exactly $\pm \frac{\Phi_0}{2}$, where (+) sign stands for spin down electron and (-) sign for spin up one.
In the above described model we define the vector going from origin to the fictitious point charge (-e) as
\[ \vec{r}' = \vec{r} + \vec{R} \] (13)
where \( \vec{r} \) is the vector going from origin to the centre of mass of the electron and \( \vec{R} \) is the vector going from the centre of mass to this fictitious point charge (-e) rotating in a circular orbit with a radius \( \vec{R} \) and an angular frequency \( \omega_s \). So the vectors \( \vec{R} (\uparrow) \) and \( \vec{R} (\downarrow) \) for spin up and down electrons read:
\[ \vec{R} (\uparrow) = R \cos \omega_s t \hat{i} - R \sin \omega_s t \hat{j} \] (14)
\[ \vec{R} (\downarrow) = R \cos \omega_s t \hat{i} + R \sin \omega_s t \hat{j} . \] (15)

From Eqs.(4) and (14) the vector \( \vec{r}' \) for spin up electron reads,
\[ \vec{r}' (\uparrow) = \vec{r} + \vec{R}(\uparrow) = [r \cos \omega_c t + R \cos \omega_s t] \hat{i} \]
\[ + [r \sin \omega_c t - R \sin \omega_s t] \hat{j} \] (16)

The time derivation of Eq.(16) gives
\[ \frac{d\vec{r}'}{dt} (\uparrow) = [-r\omega_c \sin \omega_c t - R\omega_s \sin \omega_s t] \hat{i} \]
\[ + [r\omega_c \cos \omega_c t - R\omega_s \cos \omega_s t] \hat{j} \] (17)

Analogously to Eq.(3) the total flux \( \Phi' (\uparrow) \) for spin up electron is
\[ \Phi' (\uparrow) = \oint \frac{\vec{B}}{2} [\vec{r}' (\uparrow) \times d\vec{r}' (\uparrow)] = \frac{\vec{B}}{2} \oint_0^{\frac{\tau_c}{}} \vec{r}' (\uparrow) \times \frac{d\vec{r}'}{dt} dt \] (18)
Substitution of Eqs.(16) and (17) into Eq.(18) gives:
\[ \Phi'(\uparrow) = \frac{B}{2} \int_0^{\frac{\tau_c}{}} (\omega_c r^2 - \omega_s R^2 + \text{cross terms}) dt \] (19)
where we used the identity: \( \sin^2 x + \cos^2 x = 1 \). Here the cross terms contain the product of different angular frequencies and phases, so the integral of these terms vanishes and Eq.(19) reduces to

\[
\Phi' (\uparrow) = \pi r^2 B - \frac{\omega_s}{\omega_c} \pi R^2 B
\]  

(20)

With a similar procedure the flux for spin down electron becomes

\[
\Phi' (\downarrow) = \pi r^2 B + \frac{\omega_s}{\omega_c} \pi R^2 B
\]  

(21)

In Eqs.(20) and (21) the first terms are the fluxes without the electron spin [Eq.(6)]. While the second terms are the spin contributions.

Now we want to calculate the second terms in terms of the flux quantum \( \Phi_0 \): The spin magnetic moment \( \vec{\mu} \) of a free electron is given by

\[
\vec{\mu} = -g \mu_B \vec{S}
\]  

(22)

where \( h \vec{S} \) is the spin angular momentum of the electron. When we introduce the magnetic field \( \vec{B} = B \hat{k} \), the z component of the magnetic moment becomes:

\[
\mu_z = \pm \mu_B = \pm \frac{e \hbar}{2mc}.
\]  

(23)

where we put \( g=2 \) for a free electron.

As we stated earlier we assume that the spin angular momentum of the electron is produced by the fictitious point charge (-e) rotating in a circular orbit with the angular frequency \( \omega_s \) and the radius R in x-y plane. In this case the z-component of this magnetic moment for spin down electron will be

\[
\mu_z = -IA = -\frac{e\omega_s R^2}{2c}
\]  

(24)

where we put \( I=\frac{e\omega_s}{2\pi} \) and \( A=\pi R^2 \).

If we compare Eqs.(23) and (24) we find

\[
\omega_s = \frac{\hbar}{mR^2}
\]  

(25)
which relates the spinning angular frequency $\omega_s$ to the radius $R$.

Substitution of Eq. (25) and $\omega_c = \frac{eB}{mc}$ in the second terms of Eqs. (20) and (21) we find

$$\pm \frac{\omega_s}{\omega_c} \pi R^2 B = \pm \frac{hc}{2e} = \pm \frac{\Phi_0}{2} \tag{26}$$

Eq. (26) states that the flux associated with the electron spin is independent of the radius of circular loop ($\propto$ the radius of the electron) and the mass of the electron. The (+) sign stands for spin down electron and (-) sign for spin up one.

Next we want to calculate the first terms of Eq. (20) and (21) in terms of the flux quantum. To do so we look at the Schrödinger equation for an electron in the presence of the uniform magnetic field $\vec{B} = B\vec{k}$. Let the center of the electron’s orbit be the center of a cylindrical coordinate system $(\rho, \phi, z)$ in which the z-axis is directed along the magnetic field $\vec{B}$. If we take the vector potential described in Eq. (2) namely $\vec{A} = \frac{1}{2} \vec{B} \times \vec{r}$, ie. $A_\phi = \frac{1}{2} B \rho$, $A_\rho = A_z = 0$. Then the Schrödinger equation in this gauge becomes:

$$-\frac{\hbar^2}{2m} \left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial \Psi}{\partial \rho} \right) + \frac{\partial^2 \Psi}{\partial z^2} + \frac{1}{\rho^2} \frac{\partial^2 \Psi}{\partial \phi^2} \right] - \frac{i\hbar B}{2mc} \frac{\partial \Psi}{\partial \phi} + \frac{e^2 B^2 \rho^2}{8mc^2} \Psi = E \Psi \tag{27}$$

Because of the cylindrical symmetry all terms containing $\frac{\partial}{\partial \phi}$ vanish and since the electron is assumed to be moving in x-y plane $\frac{\partial^2 \Psi}{\partial z^2} = 0$. So Eq. (27) is reduced to a simpler equation of polar coordinates:

$$-\frac{\hbar^2}{2m} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \Psi}{\partial r} \right) \right] + \frac{\hbar^2 r^2}{8m\lambda^2} \Psi = E \Psi \tag{28}$$

where

$$\rho \equiv r = \sqrt{x^2 + y^2} \tag{29}$$

The solutions of Eq. (28) has the form:

$$\Psi_n(r) \propto H_n(r)e^{-\frac{\rho^2}{4\lambda^2}} \tag{30}$$

where $H_n(r)$ is the $n^{th}$ Hermite polynomial. The eigenvalues corresponding to Eq. (30) are

$$E_n = (n + \frac{1}{2})\hbar \omega_c = (n + \frac{1}{2})\frac{\hbar eB}{mc} \quad (n=0,1,2,...) \tag{31}$$
which are the well-known Landau levels.

The probability density $P(r)$ in polar coordinates is given by:

$$P(r) = |\Psi_n(r)|^2 2\pi r$$

which has its maxima at

$$r_n = \sqrt{2n + 1}\lambda$$

From Eq.(33) the magnetic flux corresponding to these orbitals will be:

$$\Phi_n = \pi r_n^2 B = (n + \frac{1}{2}) \frac{hc}{e} = (n + \frac{1}{2})\Phi_0.$$  

(34)

If we compare Eqs.(31) and (34) we find that without spin, magnetic energy and the flux are proportional to each other:

$$\Phi_n = \frac{2\pi c}{e\omega} E_n.$$  

(35)

It is tempting to think that this proportionality is valid even in the presence of spin (ie. in the presence of Zeeman term in the energy). So in the presence of spin the additional Zeeman term ($\pm \frac{1}{2} g\mu_B B$) in the energy should produce an additional flux term $\frac{2\pi c}{e\omega} (\pm \frac{1}{2} g\mu_B B) = \pm \frac{\Phi_0}{2}$ which is exactly what we derived in Eq.(26). Another test of our model is that the definition of the spin of an anyon: Generally the spin of an anyon is defined $[2,3,9]$ by

$$s = \frac{J(m = 0)}{\hbar} = \frac{e\Phi}{hc}.$$  

(36)

If we substitute the value of $\Phi$ from Eq.(26), we find:

$$s = \frac{e\Phi}{hc} = \pm \frac{1}{2}$$

(37)

which is exactly equal to the electron spin.

So far we have considered only the flux associated with the external magnetic field. However even in the absence of the external field, because of its spin, electron will create and carry a magnetic flux with itself. In accordance with the magnetic top model, we can
assume that the spin angular momentum of the electron is produced by the fictitious point charge \((-e)\) rotating in a circular orbit with radius \(R\) and angular frequency \(\omega_s\) \(\text{[11]}\). If we take the spin vector in \(z\) direction then the magnetic field lines associated with the spin magnetic moment, \(-\vec{\mu}_e\), will be as in Fig.1a.

Outside the current loop the magnetic vector potential \(\vec{A}_e\) of the spin magnetic moment \(\vec{\mu}_e\) is given by

\[
\vec{A}_e = \frac{\vec{\mu}_e \times \vec{r}}{r^3} \quad r \geq R
\]  

(38)

The magnetic field \(\vec{B}\) of the electron is calculated through the relation:

\[
\vec{B} = \vec{\nabla} \times \vec{A}_e
\]  

(39)

In the \(x\)-\(y\) plane the \(x\) and \(y\) components of \(\vec{B}\) vanish and the \(z\) component takes the form

\[
B_z = -\frac{\mu_e}{r^3} \quad r \geq R
\]  

(40)

where \(r = \sqrt{x^2 + y^2}\).

We want to calculate the fluxes through the \(x\)-\(y\) plane outside and inside the current loop with radius \(R\): The related fluxes will be defined as follows:

\(\Phi_{\text{out}}(R)\) = The total flux through \(x\)-\(y\) plane outside the loop \((r \geq R)\)

\(\Phi_{\text{in}}(R)\) = The total flux through \(x\)-\(y\) plane inside the loop \((r < R)\)
The condition of no magnetic monopoles ($\nabla \cdot \vec{B} = 0$) requires that the number of magnetic field lines inside the loop must be equal to the number of magnetic field lines outside the loop. Therefore

$$\Phi_{in}(R) = -\Phi_{out}(R)$$  \hspace{1cm} (41)

$\Phi_{out}(R)$ can be calculated easily from eq.(40):

$$\Phi_{out}(R) = \int_{R}^{\infty} \vec{B} \cdot d\vec{a} = \int_{R}^{\infty} \frac{\mu_{e}}{r^{3}} 2\pi r dr = \frac{2\pi \mu_{e}}{R}$$  \hspace{1cm} (42)

where $d\vec{a} = -2\pi r dr \hat{k}$ is used to be consistent with the previous area vectors. From Eq.(41) $\Phi_{in}(R)$ will be

$$\Phi_{in}(R) = -\frac{2\pi \mu_{e}}{R}$$  \hspace{1cm} (43)

In the present current loop model it can be shown that inside the loop the z-component of the magnetic field is the same at every point and is equal to the magnetic field at the center, i.e. for $r < R$.

$$B_{z} = B_{0} = \frac{2\pi I}{cR} = \frac{2\mu_{e}}{R^{3}}$$  \hspace{1cm} (44)

where we used $I = \frac{\omega_{s} A}{2\pi}$ and $\mu_{e} = \frac{IA}{c} = \frac{I\pi R^{2}}{c}$. This can be proved just by calculating the flux of uniform field $\vec{B}_{0}$ through the loop:

$$\Phi_{in}(R) = \oint \vec{B}_{0} \cdot \frac{\vec{R}(\uparrow) \times d\vec{R}(\uparrow)}{2} = \oint \frac{B_{0}}{2} (\vec{R}(\uparrow) \times \frac{d\vec{R}(\uparrow)}{dt})dt$$  \hspace{1cm} (45)

Substitution of Eq.(14) and its derivative in Eq.(45) gives

$$\Phi_{in}(R) = -\frac{B_{0}}{2} \frac{\omega_{s}}{R^{2}} T_{s} = -\frac{2\pi \mu_{e}}{R}$$  \hspace{1cm} (46)

where we used Eq.(44) and put $T_{s} = \frac{2\pi}{\omega_{s}}$. Therefore as far as the magnetic flux is concerned the field lines inside the loop can be replaced by the field lines of the uniform magnetic field $\vec{B}_{0} = \frac{2\mu_{e}}{R^{3}} \hat{k}$. So in the equivalent picture of $\Phi_{in}(R)$ we have a uniform magnetic $B_{0}$ and a fictitious point charge ($-e$) rotating in the clockwise sense as in Fig.1b.
From Eq.(45) we can state that the flux $\Phi_{\text{in}}(R)$ is the one that corresponds to spinning period $T_s = \frac{2\pi}{\omega_s}$. On the other hand the uniform magnetic field $B_0$ inside the loop defines another period $T_c$ which is the cyclotron period:

$$T_c = \frac{2\pi}{\omega_c} = \frac{2\pi mc}{eB_0} = \frac{\pi mcR^3}{e\mu_e} \quad (47)$$

We have seen that for flux quantization the cyclotron period $T_c$ of the magnetic field is the important time interval. Therefore the total flux corresponding to $T_c$ is

$$\Phi(\uparrow) = \frac{T_c}{T_s} \Phi_{\text{in}}(R) = -\frac{T_c}{T_s} \frac{2\pi \mu_e}{R} \quad (48)$$

Substitution of Eq.(25) and (47) gives

$$\Phi(\uparrow) = -\frac{hc}{2e} = -\frac{\Phi_0}{2}. \quad (49)$$

With a similar procedure the flux associated with the spin down electron will be

$$\Phi(\downarrow) = \frac{hc}{2e} = \frac{\Phi_0}{2}. \quad (50)$$

Therefore even in the absence of the external magnetic field the flux associated with electron spin will be $\pm \frac{\Phi_0}{2}$.

III. RESULTS AND DISCUSSION

We have calculated the magnetic flux associated with the electron spin to show that electron itself can be treated as an anyon. Our model is based on the magnetic top model which can be made equivalent to a circular current loop with radius $R$. As far as the flux is concerned the radius $R$ of this loop is a phenomenal concept and is eliminated in the end. We distinguish the spin angular frequency, $\omega_s$ from the cyclotron angular frequency $\omega_c = \frac{eB}{me}(\omega_s >> \omega_c)$. It is shown that the calculation of the flux is led to a time integral. The crucial point is that, the limits of time is taken from zero to cyclotron period, $T_c$. During the cyclotron period $T_c$, electron completes one turn around the cyclotron orbit, but it spins $(\omega_s/\omega_c)$ times about itself. Although the radius of the electron is very small, because of the
rapid spinning, the total flux associated with the electron spin is comparable with the flux quantum. As far as the flux is concerned the mass of the electron is a parameter which is eliminated in the end. Therefore whether we take the relativistic mass or not the result does not change (to go from Eq.(22) to (26) we do not put any restriction on the electronic mass). Namely the flux is \( \pm \Phi_0 = \pm \frac{4e}{c} \). It can be shown that the present calculations can be generalize to any spherical charge distribution for electron. The details of the charge distribution determines the relation between the radius of the electron \( r_e \) and the radius \( R \) of the equivalent current loop. For a uniform charge distribution the magnetic moment of the spinning electron is found to be \( \mu = \frac{e\omega r_e^2}{5c} \) if we compare this with Eq.(24), the relation between \( R \) and \( r_e \) becomes: \( R = \sqrt{\frac{2}{5}} r_e \). We believe that the present study will bring a new insight to understand the area of physics which concerns the magnetic flux. A more complete report will be given in future.

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