Experimental Results and Elasto-plastic Analysis of the Vertical Behavior of a Single Model Pile

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Abstract
This paper deals with the vertical behavior of a single model pile in sand in a pressurizing soil tank until the pile-soil system reaches an ultimate state, and presents the comparisons between the results of tests and the elasto-plastic analysis that was proposed by one of the authors. The analysis was based on the finite element method taking account of the elasto-plastic state corresponding to generalized strain in soil elements. On the other hand, the vertical loading tests were performed for three types of the pile model; the pile tip model subjected only to the bearing resistance at pile tip, the pile shaft model subjected only to skin friction and the entire pile model subjected to both types of resistance. As a result, it is found that prediction by the analysis is in fairly good agreement with the results of the tests.

Keywords: finite element method; elasto-plastic behavior; vertical loading test; pressurizing soil tank; sand bed; single model pile

Introduction
Although the analytical methods based on the finite element method to predict the behavior of vertically loaded piles have been presented by many investigators (Ellison et al. (1971), Desai (1974), Tsuchiya et al. (1986) and Baker et al. (1994)), most of them applied axial loads up to the normal design load, except for only a few of the studies that investigated the vertical behavior of the pile under excess of the normal load (Akino (1992) and so on).

On the other hand, a pile in field tests is rarely loaded until it reaches the ultimate state caused by whether the compression failure occurs in the pile shaft or the soil surrounding the pile fails. In most field tests, testing is usually terminated when the displacement of the pile exceeds the limit value while the load can still be borne. In the actual design of foundations, therefore, the ultimate bearing capacity of a pile is generally defined as the load at which the displacement amounts to a certain percentage of the pile diameter. As examples, "Recommendations for Design of Building Foundations" published by the Architectural Institute of Japan (1988) and "JSSMFE Standards for Vertical Load Tests of Piles" by the Japanese Society of Soil Mechanics and Foundation Engineering (1993) adopt 10 % of a pile diameter to the displacement. In consideration of these recommendations, this paper studies the behavior of piles until their displacement reaches 20 % of the pile diameter, double the above figure.

The authors have carried out experimental and theoretical studies focused on the vertical behavior of a pile until the pile-soil system reaches an ultimate state. This paper first presents the results of vertically loading tests on a model pile in a sand bed in a pressurizing soil tank. In order to examine factors having effects on the vertical behavior of the pile, the tests were performed under various conditions: for example, providing skin friction on the pile or not, providing base resistance at the tip or not, varying pressure confined to the sand bed, and varying the pile diameter. Secondly, this paper presents a method for evaluating the deformation modulus of the sand bed, which was used in the non-linear finite element analysis. In the analysis, however, the stress-strain relationship for the sand bed was assessed from the results of triaxial compression tests. The non-linear method incorporating the relationship was also presented. Finally, the analytical results were compared with the test results.

The load bearing mechanism of an axially loaded single pile in a sand layer was extensively discussed by Kishida and Takano (Kishida et. al (1979), Takano et al.
Their studies focused mainly on the point bearing mechanism and were made on the basis of results of model tests using a pressurizing soil tank and theoretical analysis based on the rigid-plastic theory. On the other hand, a series of model tests were performed on single piles in similar pressurizing soil tank for the experimental study in this paper. As stated above, the tests were carried out under three different pile conditions, skin friction resistance only, pile tip bearing only and a combination of these. The pile behaviors measured during these tests were also examined by the elasto-plastic finite element method, which takes account of the elasto-plastic state corresponding to generalized strain in soil elements.

A part of this paper has previously been published in other papers [Sahara et al. (1995, 1996a, 1996b, 1997, 1999, 2000)]. This paper was prepared by adding the results of a series of tests and analyses to what had been published.

**Outline of Model Test**

**Test Apparatus and Soil Condition**

The pressurizing soil tank was made by applying the structure of the triaxial compression test apparatus, and was designed to reproduce the stress condition corresponding to that at a given depth below the ground surface in a site. Fig. 1 gives an outline of the apparatus. The soil tank was cylindrical and had a length and diameter of 1.0 m. The outer surface of the soil tank was covered with a rubber membrane. It was possible to apply water pressure to the rubber membrane isotropically. Air-dried Toyoura standard sand was used for the test ground. The ground was made by spraying the sand in the air after placing a pile upright at the bottom of the soil tank. The relative density of the sand bed was 93 to 98%. The internal friction angle of the sand obtained from triaxial compression tests was 37 degrees. In order to grasp the deformation modulus of the test ground at a very small strain level, P- and S-waves were logged in the pressurizing soil tank and the elastic wave velocities in the sand bed were measured. An electromagnetic hammer and three acceleration sensors were installed in the sand bed. P- and S-waves were quaked by the electromagnetic hammer, and translation times to each sensor were measured. The elastic wave velocities were equal to the values dividing each translation time by the distances between the hammer and each sensor. The shear moduli estimated from the wave velocities are shown in Fig. 2, including the test results introduced by another investigator (Sasaki et al. (1985)). The shear moduli are evidently in proportion to the exponents of the confining pressure as shown by the existing test results.

**Test Program, Loading Procedure and Measurements**

The test cases and test conditions are shown in Table 1. Three types of pile models were used: the pile tip model subjected to bearing resistance at the tip only, the pile shaft model subjected to skin friction only and the entire pile model subjected to both types of resistance, as shown in Fig. 3. The pile shaft model assumed that a layer of soil with the pile shaft would be cut out to a certain depth. A hollow cap was placed at its tip to prevent pile tip resistance from occurring. The pile length for the pile shaft and entire pile models was 40 cm. Monotonic step loads were applied to the pile through load control. The load was maintained for one minute in each step. As shown in Table 1, the frequency of the test was three times for the pile tip model and once for the other types of model. The total frequency was 17 times.

Measurements were taken for the pile head load, pile head displacement, soil displacement around the pile and grading distribution in the soil around the pile tip. The

\[
I_s = \log_{10} \frac{D_{10}}{D_{10}'} + \log_{10} \frac{D_{20}}{D_{20}'} + \cdots + \log_{10} \frac{D_{60}}{D_{60}'}
\]

\[D_{10}, D_{20}, \cdots, D_{60} : \text{Grain size corresponded to each percentage of penetration particle before grain crushing} \]

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\[\sigma_3, \text{kN/m}^2 : \text{Confining pressure, } d \text{ cm; } \text{Pile diameter} \]

![Fig. 1. Pressurizing soil tank](image1)

![Fig. 2. Relation $\sigma_3$, and G](image2)
soil displacement around the pile was measured using a double-core tube. The measurements were taken at positions away from the pile surface by 0.5, 1.0, 1.5, 2.0, 3.0 and 5.0 times the pile diameter, at a depth level to the middle of the pile length.

Experimental Results

Pile Tip Model

The load-displacement curves obtained by the test of the pile tip model are shown in Fig. 4. The figure shows that the higher the confining pressure of the ground, the smaller the displacement under the same load, and the larger the gradient under the initial loading.

After completion of the test for the pile tip model, grading distribution of the soil sample obtained near the pile tip was measured to compare with the distribution before loading, and the areas with considerable grain crushing were found. The confining pressure during the test was 100 kN/m². Particle breakdown was expressed in its amount, $I_{cr}$ (Takano et al. (1979)). The $I_{cr}$, which is defined by Eq. (1), is increased in proportion to the energy required for breaking particles, and is in proportion to the shaded area in Fig. 5. The results of monitoring of $I_{cr}$ in the soil near the pile tip are shown in Fig. 6. It is evident that $I_{cr}$ is large in a cone-shaped sheared belt with a tip positioned below the pile center by about the pile diameter. The belt extends from the outer edge of the pile tip. The soil mass between the belt and the area right below the pile tip moved as a rigid body, and was not subjected to large shear, so the amount of grain crushing was assumed small in the area. Because most of the sand particles are silicate grains, the subject of the grain crushing area should appear whitish in Photo. 1. The white circular area in the photograph corresponds approximately to the area with a large $I_{cr}$ shown in Fig. 6.

Pile Shaft Model

Sand was attached to the surface of the pile so that a shear surface between the pile shaft and the soil might develop, not on the pile surface but in the soil around the pile. The pile was made of solid steel bar and had a large axial strength, so it could be handled almost as a rigid body under the level of applied load. The calculated and measured ultimate frictional loads are listed in Table 2. The ultimate frictional load was expressed as a product of multiplication of the vertical stress acting on the pile shaft, $\sigma$ (confining pressure), internal friction angle of the soil, $\phi$ (37 degrees), and the area of pile shaft, A. The calculated ($\sigma \cdot \tan \phi$) and measured values were almost identical to each other, so the conditions of a layer of the soil with the pile that is cut out, such as the application of no resistance at the pile tip and the encouragement of shear failure in the soil around the pile, were evidently satisfied.
Confining pressure was varied from 50 kN/m² to 100, 150 and 500 kN/m² where the pile diameter was 5.0 cm. The resultant relationship between the friction and displacement is shown in Fig. 7. The figure shows that the displacement, $S_u$, at which the friction was fully mobilized, was approximately constant at about 2 mm regardless of the confining pressure. The relationships between the friction and the displacement at the confining pressure of 100 kN/m² for pile diameters of 3 cm, 5 cm and 7 cm are shown in Fig. 8. It is found from this figure that the pile diameter has no affect on the displacement, $S_u$, the value of which is around 2 mm.

The displacement of the soil surrounding the pile was measured in a case of the diameter of 5.0 cm and the confining pressure of 100 kN/m². The distributions of the soil displacement are shown in Fig. 9. The abscissa presents the ratio of the horizontal distance from the pile surface to the pile diameter, so that a value on the ordinate at zero point on the axis means a displacement of the pile itself. The figure indicates that little soil displacement occurred at a point away from the pile surface by twice the pile diameter or more. It is also found from the figure that before the pile displacement of $S_u$ (see triangular mark in Fig. 9), the soil displacement occurred within the range of 0.5 to 1.5 times the pile diameter, but that after $S_u$, there was hardly any change in soil displacement within the range. Therefore, it is to be supposed that after the application of the ultimate frictional load, the soil displacement occurred following pile displacement within the range of half the pile diameter from the pile surface, where a shear surface may have developed.

**Entire Pile Model**

Figure 10 indicates the load-displacement curves obtained by the tests of the entire pile model with a series of open circles. In the figure, the curves for the pile tip model, for the pile shaft model and for the combination of both of them are also indicated with solid squares, triangles and circles, respectively. In Figs. 10 (a) and (b), the curves from the entire pile model show that the higher the confining pressure of the soil, the smaller the displacement under the same load and the larger the gradient under the initial loading.

On comparison of the curves for the entire pile model and the combination, Fig. 10 shows that at a high confining pressure of 500 kN/m², the former curve is in close agreement with the latter curve, but that at a low confining pressure of 100 kN/m², the former curve represents greater displacement than the latter curve under the same loading. It is to be supposed that the difference between the test results at the high and low confining pressure is mainly caused by restricting the soil movement, because the displacements of the soil...
surrounding the pile tip during the pile tip model test should be restricted by the rigid loading plate (steel pedestal) on the test ground, as shown in Fig. 3 (a). By confining the soil with the low pressure, especially the shear failure line reaching to the rigid loading plate affects the load-displacement curve, as shown in Fig. 11 (a) [proposed by Terzaghi (1943)]. On the contrary, in the case of the high confining pressure, the shear failure line at the ultimate state probably only occurs in limited area below the loading plate, as shown in Fig. 11 (b) [proposed by Vesic (1963)]. On the other hand, the difference of these shear failure lines should not occur in the entire pile model test, because in this case the pile tip was placed at a relatively deep level.

Although the aim of the pile tip model test was to investigate vertical behavior of the pile tip, we were unable to fully achieve this at the low confining pressure, \( \sigma_c = 100\text{kN/m}^2 \). As discussed above, this is the reason why the movement of the soil around the pile tip was restricted by the rigid loading plate placed on the soil surface. It is noticed that such a loading test should be performed on a pile tip model embedded a little in soil, with reduced skin friction along the embedded shaft.

### Elasto-plastic Analysis by the Finite Element Method

**Analysis Method**

An analysis based on the elasto-plastic finite element method was made regarding an axis-symmetric problem in a series of the test cases. Figs. 12 (a), (b) and (c) show finite element mesh models for the pile tip, pile shaft and entire pile models, respectively. In order to reflect the resistance conditions of these models in the analysis, the boundary conditions were given as shown in these figures, and the predetermined loads were then applied to the pile in stages. For the elasto-plastic analysis, the general-purpose structural analysis software ABAQUS (1996) was used.

For incorporation into the analysis, generalized stress, \( \sigma_c \), and strain \( \varepsilon_c \), expressed by Eqs. (2) and (3) were adopted, respectively. The stress-strain relationship is very simple as represented by Eq. (4) regardless of whether the analysis was elastic or elasto-plastic (Bezukhov (1974)). To represent the non-linearity of the sand bed in consideration of the effect of confining pressure, deformation modulus Es was expressed by the Eqs. (5) and (6).

\[
\begin{align*}
\sigma_c &= \frac{\sqrt{2}}{2} \sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)} \quad (2) \\
\varepsilon_c &= \frac{1}{2} \left( (\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2 + 1.5(\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{zx}^2) \right) = E_c \varepsilon_c \quad (3)
\end{align*}
\]

\[
\begin{align*}
\sigma_x &= E_e \varepsilon_x \quad (4) \\
E_s &= E_0 \left( \frac{\varepsilon_s}{10^{-n}} \right)^m \quad (5) \\
E_s &= E_0 \left( \frac{\varepsilon_s}{10^{-a}} \right)^b \quad (6)
\end{align*}
\]

Where \( \varepsilon_c \): Generalized stress, \( \varepsilon_e \): Generalized strain, \( \nu \): Poisson's ratio, \( E_0 \): Elastic component of generalized strain, \( E_r \): Plastic component of generalized strain, \( E_s \): Deformation modulus, \( E_0 \): Initial deformation modulus at a small strain level, \( m, n, a \) and \( b \): Constants.
curve. Eq. (6), however, relates deformation modulus to generalized strain while the Ramberg-Osgood equation relates shear modulus to shear strain. In the subsequent discussions, Eqs. (5) and (6) are referred to as the exponential equation and R.O. equation, respectively. The evaluation equation obtained by inputting m=0.40, n=0.55 and E_0 = 0.8E_0 is what one of the authors proposed based on the measurements of rebound and settlement of soil and structures in actual construction works. This is referred to as the existing equation below.

**Method for Evaluating Deformation Modulus of the Test Ground**

The stress-strain relationship in the test ground was evaluated based on the results of a triaxial compression test. The soil specimen for the test, like the soil made in the pressurizing soil tank, consisted of air-dried Toyoura standard sand with a relative density of 95%. The specimen had a diameter of 50 mm and a height of 100 mm. Loads were applied at a rate of 0.1% of the height per minute. Three levels of confining pressure, 100, 300 and 500 kN/m² were applied.

The apparatus used for the triaxial compression tests is outlined in Fig. 13. In order to grasp the axial strain from a very small level of 10⁻⁵ or less to the breaking strain exceeding 10⁻², continuously and accurately, a local deformation transducer [Goto et al. (1991)] was used for measuring small to medium strains, and an external deformation transducer was used for medium to breaking strains. The local deformation transducer is characterized by its ability to directly measure the axial strain of the specimen (Fig. 13), and thus, the effects of errors due to the compression of loose layers on the top and bottom edges of the specimen and incomplete contact between the loading plate and the specimen can be removed. These results of the triaxial compression test are discussed below.

The initial deformation modulus at a very small strain and the secant modulus of elasticity at a given strain are expressed by E₀ and E/sec, respectively, as shown in Fig. 14. The values of E₀ were estimated on the basis of the maximum secant line through the origin, when axial strain, ε₁, was in the range of 1.0 × 10⁻⁵ to 4.0 × 10⁻⁵. The relationships between deviator stress (σ_d - σ_3) and axial strain, ε₁, and those between E/sec/E₀ and ε₁ ob-
Table 3. Constants in equations and correlation coefficients

| Equations | m | n | a | b | R  |
|-----------|---|---|---|---|----|
| exponential | 3.81 | 3.87 | 3.95 | 0.48 | 0.57 | 0.61 |
| R. | 0.96 | 0.97 | 0.98 | 165.2 | 237.3 | 375.6 |
| R. | 0.99 | 0.99 | 0.99 | 65.2 | 237.3 | 375.6 |

Fig. 19. Material property of interface elements

Fig. 17. Comparison between measured and analytical results.

Comparison between Measured and Analytical Load-Displacement Behavior

Pile Tip Model

Comparisons between the predicted load-displacement curves and the measured curves are shown in Fig. 4 for the pile tip model tests. At the low confining pressure $\sigma_3$ of 100 kN/m$^2$, the exponential equation and the existing equation are highly applicable to the measured load displacement behavior. At $\sigma_3$ of 300 kN/m$^2$, the
exponential equation and R.O. equation are also in fair agreement with the measured behavior. Furthermore, the R.O. equation is highly applicable to the measured behavior at the high confining pressure $\sigma_3$ of 500 kN/m². The existing equation was formulated based on the rebound and settlement measurements of soil and structures in actual constructions. The depth of bedding subjected to the building load was about a dozen meters or less. The equation was proposed at a confining pressure of about 100 kN/m², so it was highly applicable at low confining pressure and vice versa.

Plastic component, $\varepsilon_p$, of the generalized strain was subtracted elastic component, $\varepsilon_e$, of the generalized strain from total generalized strain $\varepsilon$ (Eq. (3)). The distribution of $\varepsilon_p$ based on the results of analysis when the displacement of pile tip reached 10% of the pile diameter is shown in Fig. 21 which indicates that the greater the plastic component of the generalized strain is, the whiter its contrast becomes. It is evident that the distribution of $\varepsilon_p$ agrees fairly well with the distribution of $\varepsilon_e$ shown in Fig. 6.

**Pile Shaft Model**

For the pile shaft model, comparisons between the predicted and measured curves are shown in Fig. 22. On the basis of the above results for the pile tip model, the deformation moduli of the soil in the analysis using the exponential equation and R.O. equation were adopted at $\sigma_3$ of 100 kN/m² and 500 kN/m², respectively. The non-linearity of the interface element between the pile and the surrounding soil was approximated by a bilinear curve shown in Fig. 19. Adding the deformation of the pile due to shear deformation of the surrounding soil, the predicted curve is in a good agreement with the observed curve, as seen in Fig. 22. In view of this, the initial gradient and the upper limit shown in Fig. 19 are provided to the interface element adopted for the entire pile model described in the next section.

**Entire Pile Model**

Figure 23 shows comparison between the predicted load-displacement curves and the measured curves for the entire pile model. It is found from this figure that at $\sigma_3$ of 100 kN/m², the exponential and existing equations are highly applicable, and that at $\sigma_3$ of 500 kN/m², the R.O. equation is highly applicable.

**Conclusions**

Experimental studies were made of the vertical behavior of model pile, displacement of the soil surrounding the pile and grain crushing in the soil below the pile tip until the pile-soil system reached an ultimate state. The tests were performed in a pressurizing soil tank using such parameters as the resistance at the pile tip and around the pile, confining pressure of the test ground and the pile diameter. A method was then proposed for evaluating the deformation modulus of the test ground using the elasto-plastic finite element to predict the measured vertical behavior of the pile.

The following two major findings were obtained by this study:

1. A method was presented for evaluating the deformation modulus of the test ground that could express the non-linear behavior until the pile experienced large displacement, based on the results of the triaxial compression tests. The usability of the elasto-plastic finite element method incorporating the approximated evaluation equation was verified through a comparison with the test results.

2. The area with outstanding grain crushing caused by the failure of sand beneath the pile tip can be grasped by the distribution of a generalized plastic strain obtained from the analysis.
This paper discussed the tests and analyses for single piles only. The authors separately carried out model tests with groups of four piles under various pile spacing and confining pressure. These test results and comparison between the measured and analytical results will be reported on a near future occasion.

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