Domain wall solution for vectorlike model

Tomohiro Matsuda

National Laboratory For High Energy Physics (KEK)
Tsukuba, Ibaraki 305, Japan

Abstract

Domain wall solution for $N_c = N_f$ supersymmetric QCD is constructed. Astrophysical implications of the domain wall configuration is also discussed.
1 Introduction

When one extends the validity of the low energy effective field theory to energy scales much higher than its characteristic mass scale, one faces to a scale hierarchy problem. A typical example is the gauge hierarchy problem of the Standard Model of the strong and electroweak interactions, seen as a low-energy effective theory. When the Standard Model is extrapolated to cut-off scales $\Lambda \gg 1\text{TeV}$, there is no symmetry protecting the mass of the elementary Higgs field from acquiring large value, and therefore the masses of the weak gauge bosons, receive large quantum corrections proportional to $\Lambda$. The most popular solution to the gauge hierarchy problem of the Standard Model is to extend it to a model with global $\mathbb{N}=1$ supersymmetry, effectively broken at a scale $M_{\text{Soft}} \sim 1\text{TeV}$. These extensions of the Standard Model, for instance the Minimal Supersymmetric Standard Model (MSSM), can be safely extrapolated up to cut-off scales much higher than the electroweak scale, such as the supersymmetric unification scale $M_U \sim 10^{16}\text{Gev}$, the string scale $M_s \sim 10^{17}\text{Gev}$, or Planck scale $M_P = 2.4 \times 10^{18}\text{Gev}$. To go beyond MSSM, one must move to a more fundamental theory with spontaneous supersymmetry breaking which should be induced by the dynamical effects to explain the hierarchy. One of the candidate of the mechanism of supersymmetry breaking is the dynamical supersymmetry breaking (DSB) models in which supersymmetry is broken at a low energy scale ($\sim 10^7\text{GeV}$). A few years ago, a new mechanism for DSB was proposed by Yanagida et al. and then applied to many other models of dynamical symmetry breaking. Unlike other DSB models, supersymmetry breaking is realized in this model with a vectorlike gauge theory and only one gauge group is required in the breaking sector. It is also surprising that the scale of gaugino condensation is not fixed but dynamical in a sense that it explicitly depends on the vacuum expectation value of the additional singlets. To be more specific, let us consider the vectorlike supersymmetry breaking model of ref. Here we consider a supersymmetric SU(2) gauge theory with four doublet chiral superfields $Q_i$ and six singlets $Z^{ij} = -Z^{ji}$. Here, $i$ and $j$ denote the flavor indices ($i, j = 1, ..., 4$). We introduce a tree-level superpotential:

$$W_0 = \lambda_i^{k} Z^{ij} Q_k Q_i.$$  

(1.1)
In this model, supersymmetry is not broken classically but broken by the quantum deformation of the flat direction. According to Seiberg\cite{4}, holomorphy implies a constraint on the gauge-invariant degrees of freedom $V_{ij} = -V_{ji} \sim Q_i Q_j$:

$$PfV_{ij} = \Lambda^4.$$ \hfill (1.2)

With an additional chiral superfield $S$, we can impose this holomorphic constraint by means of the non-perturbative superpotential of the form:

$$W_{\text{dyn}} = S(PfV_{ij} - \Lambda^4) + \lambda_{ij}^{kl}Z^{ij}V_{kl},$$ \hfill (1.3)

where $PfV_{ij}$ denotes the Pfaffian of the antisymmetric matrix $V_{ij}$, and $\Lambda$ is a dynamical scale of the SU(2) gauge interaction. This effective superpotential yields conditions for supersymmetric vacua

$$PfV_{ij} = \Lambda^4$$

$$\lambda_{ij}^{kl}V_{kl} = 0.$$ \hfill (1.4)

These two conditions do not stand simultaneously. This mechanism of supersymmetry breaking is very similar to O’Raifeartaigh model. The relation to the ordinary supersymmetric QCD theory is established when we consider a limit

$$\lambda_{ij}^{kl} \to 0$$

$$\lambda_{ij}^{kl}Z^{ij} = m^{kl} \sim \text{const.}$$ \hfill (1.5)

In this paper we construct a domain wall solution for the vectorlike DSB sector described above and discuss its astrophysical implications.

\section{Domain wall configuration for the vectorlike sector}

Let us first consider an $SU(N_c)$ gauge theory with matter consisting of $N_f$ massive chiral superfields $Q_i$ and $\overline{Q}_i$ (massive SQCD) for $N_c = N_f$ and construct a domain wall solution\cite{6}. Here we consider a classical superpotential of the form:

$$W_0 = m^i_j Q_i \overline{Q}_j$$ \hfill (2.1)
The exact effective superpotential of the model may be written in terms of the gauge-invariant low-energy degrees of freedom $M_j^i, B$ and $\overline{B}$. Because we do not consider terms of the form $bB + \overline{bB}$, expectation values of lower components of chiral superfields is given by [4]:

$$M_j^i = < Q^i Q_j > \Lambda^{3N_c-N_f}/N_c (\det [m^i_j])^{1/N_c} (m^i_j)^{-1}$$

$$B = \overline{B} = 0$$

(2.2)

The phases from the fractional power $1/N_c$ corresponds to $N_c$ different ground states which is consistent to the arguments of the Witten index. The exact superpotential is now written by

$$W_{\text{eff}} = S (\det M_j^i - B \overline{B} - \Lambda^4) + m^i_j Q_i \overline{Q}^j$$

(2.3)

where $S$ is an additional chiral superfield which is introduced to make the holomorphic constraint manifest. For $N_c = 2$, the classical superpotential is written by

$$W_0 = m^i_j Q_i Q_j$$

(2.4)

and the explicit form of the constraint is given by [4]

$$V^{ij} = < Q^i Q^j > = \Lambda^2 ( Pf [m^i_j] )^{1/2} (m^i_j)^{-1}.$$  

(2.5)

As a result, the exact superpotential of the model is written by

$$W_{\text{eff}} = S ( Pf V_{ij} - \Lambda^4) + m^i_j V_{ij}$$

(2.6)

The effective scale $\Lambda_L$ of the low-energy SU(2) along this trajectory is given to all orders by the one-loop matching of the gauge coupling at the quark mass scale $m \sim ( Pf [m^i_j] )^{1/2}$ and leads [4]

$$\Lambda_L^6 = m^2 \Lambda^4.$$  

(2.7)

This relation is consistent to the Konishi anomaly if $\Lambda_L$ is regarded as the scale of gaugino condensation. In the pure SU(2) gauge theory an effective superpotential $\sim \Lambda_L^3$ is generated:

$$W_{\text{eff}} = m \Lambda^2.$$  

(2.8)

This effective superpotential may also be obtained by replacing the matter fields by their vacuum expectation values. In this model, contrary to the one we have discussed in
the previous section, supersymmetry is not broken but flat direction is modified by the dynamical effect. Recently, the domain wall solution for supersymmetric QCD theories are constructed by Shifman et al[6] and found that they are BPS-saturated. The key point is that the central extension is automatically zero for all spatially localized field configurations but need not necessary vanish for those field configurations that interpolate between distinct vacua at spatial infinities (domain wall). The central charge appears at both classical and at the one-loop level as a quantum anomaly. The explicit form is

\[
\{\mathcal{Q}_\alpha \mathcal{Q}_{\bar{\beta}}\} = 4\langle \bar{\sigma} \rangle_{\bar{\beta} \bar{\beta}} \int d^3x \nabla \left\{ \left[ W - \frac{N_c - N_f}{16\pi^2} Tr W^\alpha W_\alpha \right] + t.s.d. \right\}_{\theta = \bar{\theta} = 0}. \tag{2.9}
\]

where \(t.s.d.\) denotes total superderivatives. In our model, we should set \(N_c = N_f\) thus we can neglect the second term. The first term comes from the superpotential(2.8) and it parametrizes gaugino condensation. The domain wall configuration for the present model is almost the same as that was discussed in ref.[6]. The domain wall configuration appears because \(Z_{2N_c}\) symmetry is spontaneously broken by gaugino condensation, and have the energy

\[
E = 2A(W_{*1} - W_{*2}) \tag{2.10}
\]

where \(W_{*i}\) denotes the vacuum expectation value of the superpotential in the i-th domain. Now let us further discuss the domain wall configuration of the vectorlike sector, namely \(N_c = N_f\) supersymmetric QCD with singlets. The crucial difference from the ordinary supersymmetric QCD theory is the presence of the additional scalar potential of the form:

\[
V_{add} \sim |\lambda V|^2 \tag{2.11}
\]

which breaks supersymmetry when dynamical effects are included. To establish the contact with the original supersymmetric theory, here we consider a limit

\[
\lambda_{ij}^{kl} \rightarrow 0 \tag{2.12}
\]

while the lowest component of \(\lambda_{ij}^{kl}Z^{ij}\) remains finite:

\[
Z^{kl} \equiv \lambda_{ij}^{kl}Z^{ij}. \tag{2.13}
\]

In this limit, the vectorlike sector can be regarded as an ordinary supersymmetric QCD with a mass parametrized by \(Z'\). The domain wall configuration is constructed in the
same way. The energy of the field configuration $E$ now depends on $Z'$:

$$E \sim A\Lambda^2 Z'.$$

Because the energy of the domain wall configuration depends linearly on $Z'$, the direction of $Z'$ is not flat but stabilized in the presence of the domain wall configuration. Here we neglect other effects, such as one-loop corrections to the Kähler potential because they will be very small in the $\lambda \to 0$ limit. (See eq.(3.2).)

### 3 Cosmological implications of the domain wall

In the last section we considered the domain wall configuration for the vectorlike sector in the limit $\lambda \to 0$. In this section, we will consider some specific examples and discuss their astrophysical implications. The first example is the dynamical supersymmetry breaking model of ref.[2]. This model contains two kinds of singlets which are assumed to be stabilized by the effective Kähler potential of the form:

$$K = ZZ^* - \frac{\eta}{4\Lambda^2} \lambda^4 (ZZ^*)^2 + ...,$$

where $\eta$ is a real constant and the order of the coupling constant $\lambda$ is assumed to be $O(1)$. The effective scalar potential of the scalar $Z$ is given by

$$V_{eff} \sim \lambda^2 \Lambda^4 \left(1 + \frac{\eta}{\Lambda^2} \lambda^4 ZZ^* + ...ight).$$

The sign of $\eta$ is responsible for the vacuum expectation value of $Z$. When $\eta$ is positive, the above effective potential leads to $<Z> = 0$. Otherwise, when $\eta$ is negative, $<Z>$ may be the order of $\Lambda$. In ref.[2] it is assumed that one of the singlets is stabilized at the origin but the other is stabilized at $\Lambda \sim 10^7$ Gev. For this type of potential, domain walls can be produced during inflation, reheating or the parametric resonance[7]. It should be important to study the phenomenology of this domain wall production[8]. The second example is the dynamical unification model of ref.[3]. The tree level superpotential is given by

$$W_{tree} = g Str M_{ij} + \frac{g'}{2} Str \Sigma^2 + \frac{h}{3} Tr \Sigma^3$$

where $S$ is a singlet and $\Sigma$ is an adjoint of SU(5) GUT gauge group. Here $M_{ij}$ denotes $M_{ij} \equiv Q_i \bar{Q}_j$ where $Q$ is not charged under SU(5) but in the fundamental representation of
the vectorlike gauge sector SU($N_c$). The resulting dynamical superpotential which arises when we consider the full quantum moduli space of the vectorlike sector is:

$$W_{\text{dyn}} = A(\det M_{ij} - B\overline{B} - \Lambda^4) + S\left( gtr M_{ij} + \frac{g'}{2} Tr \Sigma^2 \right) + \frac{h}{3} Tr \Sigma^3. \quad (3.4)$$

The minimum of this dynamical superpotential does not break global supersymmetry. SU(5) gauge symmetry is broken to SU(3) × SU(2) × U(1) at the minimum:

$$< \Sigma > = \Lambda \sqrt{- \frac{2g}{15g'}} (2, 2, 2, -3, -3), \quad M^i_j = \delta^i_j \Lambda^2,$$

$$B = \overline{B} = 0, \quad S = \Lambda h \sqrt{- \frac{2g}{15}} g'^{-3/2}, \quad A = \Lambda^{-1} h \sqrt{\frac{2}{15} \left( \frac{g'}{g} \right)^{3/2}} \quad (3.5)$$

where the scale of $\Lambda$ is fixed at $\sim 10^{15}$GeV. A problem arises when we include supergravity effects. Because the scale of gaugino condensation is about the order of $gS\Lambda^2$, we should fine-tune the coupling constant $h < 10^{-9}$ so as not to break supersymmetry too much by gaugino condensation. With this fine-tuning, the vacuum expectation value of the singlet $S$ is estimated to be $\sim 10^6$GeV and the height of the potential is $V(0)^{1/4} \sim 10^{15}$GeV.

In this model, the domain wall production rate will be very small because the potential barrier is very high.

4 Conclusion and discussions

In this paper we constructed the domain wall solution for ordinary $N_c = N_f$ supersymmetric QCD and extend it for the vectorlike dynamical supersymmetry breaking model. Astrophysical implications of the domain wall configuration is also considered for two types of vectorlike sectors. We think it is very important to study the dynamics of the moduli dependent constants (gauge coupling or mass parameter) in the early universe. They may be displaced by fluctuations of any type and can trigger rich phenomena.

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