Dependence of static friction force on stiffness in a confined chain with the incommensurate and commensurate structure

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Abstract. It is used that a series of interacting atoms. They are confined between two sinusoidal substrate potentials. When the top substrate is driven by the external driving force, its velocity changes from zero to non-zero. The dependence of the static friction force on the strength is presented for the incommensurate (the spiral mean and the golden mean) and commensurate structure. It is found that incommensurate structures is more likely to produce superlubricate phenomenon more than commensurate structures under different stiffness coefficients.

1. Introduction

Seventy years ago, Dehlinger first used a one-dimensional Frenkel–Kontorova (FK) model to describe dislocation dynamics in crystals. Later, the FK model was widely used to study nonlinear physics, the problem of defects and crowdions in a metal[1,2], heat conduction, submonolayer films of atoms adsorbed on crystal surfaces[3,4], Josephson junctions[5,6,7] and so on. In particular, it is associated with solid friction phenomena, underdamped FK model has attracted more attention[8].

When two workpieces are in contact with each other, the minimum force required to achieve sliding is referred to as the maximum static friction. Especially, some researchers [9] have improved the one-dimensional FK model to more complex models to explain more complex physical phenomena. It is studied that the effect of system parameters on the maximum static friction force. A series of interacting atoms confined between two periodic sinusoidal substrate potentials is studied by O. M. Braun et al. [10,11]. When the upper layer was driven by a spring running at a constant speed, they found an interesting phenomenon. The golden mean incommensurability reveals a very regular time-periodic dynamics that has relatively high kinetic friction values compared with the spiral mean case. However, there are many problems which remain unsolved, especially the influence the chain stiffness for the incommensurate and commensurate structure on the static friction force is seldom studied.

In the present paper, it is used that a series of interacting atoms. They are confined between two sinusoidal substrate potentials. We have examined the influence of the chain stiffness for the incommensurate and commensurate structure on the static frictional force when the top substrate of mass $M$ is driven by the force $F_{ext}$. 
2. Model

![Schematic drawing of the model with three characteristic length scales](image)

Figure 1. Schematic drawing of the model with three characteristic length scales, the interatomic equilibrium \( b \), the top substrate period \( c \), the bottom substrate period \( a \).

The model consists of a chain of \( N \) harmonically interacting particles interposed between two rigid generally sinusoidal substrates externally driven by the force \( \text{F}_{\text{ext}} \) as shown in Fig. 1. The equations of motions of the \( i \)th particle are

\[
m \dddot{x}_i + \gamma \ddot{x}_i + \gamma (x_i - \bar{X}_{\text{top}}) + \frac{d}{dx_{ij}} \sum V'(|x_i - x_j|) + \frac{1}{2} \left[ \sin \frac{2\pi x_i}{a} + \sin \frac{2\pi (x_i - \bar{X}_{\text{top}})}{c} \right] = 0
\]

(1)

The top substrate of mass \( M \) is driven by the force \( \text{F}_{\text{ext}} \). It satisfies the following equations of motions:

\[
M \dddot{X}_{\text{top}} + \sum \gamma (\dot{X}_{\text{top}} - X_i) + \frac{1}{2} \left[ \sin \frac{2\pi (X_{\text{top}} - x_i)}{c} \right] = \text{F}_{\text{ext}} = 0
\]

(2)

Where \( x_i (i = 1, 2, 3...N) \) stands for position coordinates of \( N \) particles and \( X_{\text{top}} \) stands for the top substrate. The damping \( \gamma \) terms in Eqs. (1) and Eqs. (2) describe the dissipative forces. They are proportional to the relative velocities of the particles with substrate. In our system, we choose \( \gamma = 0.1 \), so that it is a underdamped model. Simulations have provided indirect evidence that such phenomenological viscosity terms serve well this purpose. We have used dimensionless units with substrate period \( a = 1.0 \) and chain atom mass \( m = 1 \). The fifth terms in Eqs. (1) represents the interaction between the particle and the bottom potential. Particle interaction [The fourth term in Eqs. (1)] is harmonic with equilibrium spacing \( b \) and strength \( k \). It is used that the fourth-order Runge-Kutta to solve Eqs. (1) and Eqs. (2). In the simulation, the time step we used is \( 0.02\tau \). The time interval for the system to reach equilibrium is \( 100\tau \). The force is varied with the step of \( 10^{-3} \).

3. Results and discussion
In the field of tribology research, maximum static friction is one of the main problems, which describes the minimum force that causes relative motion. Figure 2 shows the average velocities of the top substrate, \( v \), as a function of the external driving force \( F_{ext} \) for different structure of system. In Fig. 2(a), Golden incommensurability \( c = 144/89, b = 144/233, a = 1.0, N_c = 233, K = 1.0 \); (b) Commensurability \( c = 1.0, b = 0.5, a = 1.0, N_c = 233, K = 1.0 \); (c) Commensurability \( c = 1.0, b = 0.5, a = 1.0, N_c = 351, K = 1.0 \); (d) Spiral incommensurability \( c = 265/200, b = 265/351, a = 1.0, N_c = 351, K = 1.0 \).

Average velocity \( \bar{v} \) as a function of the external driving force \( F_{ext} \) for Commensurability \( c = 1.0, b = 0.5, a = 1.0, N_c = 233, K = 1.0 \). Under the same conditions, Golden incommensurability is presented. We can find that Golden incommensurability structure exhibit lower friction. In Fig. 2(c), Average velocity \( \bar{v} \) as a function of the external driving force \( F_{ext} \) for Commensurability \( c = 1.0, b = 0.5, a = 1.0, N_c = 351, K = 1.0 \). Under the same conditions, Spiral incommensurability structure is presented. We can find that Spiral incommensurability structure exhibit lower friction. Dependence of the maximum static friction force \( F_s \) on the chain stiffness \( K \) for the values of the different structure. Square represents commensurate structure; circles represents Golden Mean of incommensurate structure; triangles represents Spiral Mean of incommensurate structure is presented. We can find that when \( k \) is very small, the system exhibits a smaller maximum static friction force for incommensurate structure. However, under the same conditions, the commensurate structure exhibits greater static friction force.
4. Conclusions

We studied a one-dimensional system of two rigid sinusoidal substrates and a chain of interacting particles embedded between them. We have examined the influence of the chain stiffness for the incommensurate and commensurate structure on the static frictional force when the top substrate of mass $M$ is driven by the force $F_{ext}$. It is found that when the chain stiffness is very small, the system exhibits a smaller maximum static friction force for incommensurate structure. However, under the same conditions, the commensurate structure exhibits greater static friction force.

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