State of the Art Study of Influence of Bed Roughness and Alluvial Cover on Bedrock Channels and Comparisons of Existing Models with Laboratory Scale Experiments

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Abstract. Several studies have implied towards the importance of bed roughness on alluvial cover, besides, several mathematical models have also been introduced to mimic the effect bed roughness may project on alluvial cover. Here, we provide a state of the art review of research exploring the relationship between alluvial cover, sediment supply and bed topography, thereby, describing various mathematical models used to analyse deposition of alluvium. In the interest of analysing the efficiency of various available mathematical models, we performed laboratory-scale experiments and compared the results with various models. Our experiments show that alluvial cover is not merely governed by increasing sediment supply, and, bed topography is an important controlling factor of alluvial cover. Testing experimental results with various theoretical models suggest a fit of certain models for a particular bed topography and inefficiency in predicting higher roughness topography. Three models efficiently predict the experimental observations, albeit their limitations which we discuss here in detail.

1 Introduction

Economic growth worldwide has fuelled the demand for the construction of straightened river channels, sabo dams, the collection of gravel samples for various research, etc.; leading to a decline in sediment availability and alluvial bed cover. Sediment availability has two contradicting effects on the river bed (Gilbert, 1877; Sklar and Dietrich, 1998; Johnson et al., 2010; Mishra et al., 2018). It acts as a tool and erodes the bedrock bed, known as tools effect. As sediment availability becomes larger than sediment transport capacity of the channel, the sediment starts settling down on the river bed providing a cover for the bed underneath from further erosion, known as the cover effect. Various studies have established the importance of alluvial cover in moderating bedrock incision and shape. Finnegan et al. (2007) conducted laboratory-scale experiments and studied the interdependence among incision, bed roughness and alluvial cover. Their results indicated that alluvial deposition on the bed shifted bed erosion to higher regions of the channel or bank of the channel. Cowie et al. (2008) and Mishra et al. (2018) showed that incision rate increased when the sediment supply rate of the channel became considerably smaller than the sediment carrying capacity of the channel. Johnson et al. (2009) surveyed the Henry Mountains in Utah and suggested that alluvial cover does not only inhibit bed erosion for a covered bed but also when the bedrock bed is exposed in some parts.
Flume studies conducted by Johnson et al. (2010) have shown the importance of alluvial cover in regulating the roughness of bedrock bed by providing a cover for the local lows and thereby inhibiting the erosion and focusing erosion on local highs. Yanites et al. (2011) studied the changes in Peikang River in central Taiwan triggered by the thick sediment cover introduced by landslides and typhoons during the 1999 Chi-Chi earthquake. Their results show slowed or no incision in high transport capacity and low transport capacity channels. Inoue et al. (2014) conducted experiments by excavating channel into natural bedrocks in Ishikari River, Asahikawa, Hokkaido – Japan. They suggested that the erosion rate in cover-free sections was positively associated with the sediment supply rate. They also advocated an interaction between alluvial cover and bed roughness such that when alluvial cover decreases the bed roughness increases.

According to saltation-abrasion model proposed by Sklar and Dietrich (2004) the bedrock exposure decreases linearly with the ratio of sediment flux to sediment transport capacity. Their cover model can be expressed as the following equation

\[ P_c = \begin{cases} \frac{q_{bs}}{q_{bc}} & \text{for } 0 \leq \frac{q_{bs}}{q_{bc}} \leq 1 \\ 1 & \text{for } \frac{q_{bs}}{q_{bc}} > 1 \end{cases} \]

where, \( P_c \) is the mean areal fraction of alluvial cover, \( q_{bs} \) and \( q_{bc} \) are the volume sediment supply rate per unit width and transport capacity, respectively. The value of \( P_c \) i.e. the alluvial cover ratio is 1 when the sediment supply is larger than the transport capacity, the gravel cover does not decrease as the sediment gets deposited on the bed, consequently, the bedrock is not exposed. If there is no sediment supply, the gravel deposit will disappear and eventually the bedrock bed will be completely exposed and \( P_c \) will be equal to 0.

Laboratory scale studies conducted by Chatanantavet and Parker (2008) and Johnson and Whipple (2010) mostly support the cover model proposed by Sklar and Dietrich (2004). Chatanantavet and Parker (2008) conducted laboratory-scale experiments in straight concrete bedrock channels with varying bedrock roughness and evaluated bedrock exposure with respect to sediment availability. In their experiments, alluvial cover increased linearly with increasing sediment supply in case of higher bed roughness, whereas in case of lower bed roughness and higher slopes, the bed shifted abruptly from being completely exposed to being completely covered. Hodge and Hoey (2016a) conducted laboratory-scale experiments in a 3D printed flume of natural stream Trout Beck, North Pennies-U.K. Their first set of experiments focused on quantifying hydraulic change with varying discharge, suggesting that hydraulic properties fluctuate more during higher discharge. Their second set of experiments (Hodge and Hoey, 2016b) concentrated on quantifying the sediment dynamics for varying discharge and sediment supply. They supplied 4 kg and 8 kg of sediment pulse to the channel and observed a similar alluvial pattern in both cases suggesting that the deposition of sediment on the bed may not only depend on the amount of sediment supplied, but may be strongly influenced by the bed topography. In order to express the non-linear relationship between \( P_c \) and \( \frac{q_{bs}}{q_{bc}} \), Turowski et al. (2007) proposed a model that considered the cover effect as an exponential function of the ratio of sediment flux to sediment transport capacity. Turowski et al.’s model can be expressed as the following equation

\[ P_c = 1 - \exp\left(-\varphi \frac{q_{bs}}{q_{bc}}\right) \]

where, \( \varphi \) is a dimensionless cover factor parameter and determines sediment deposition on covered areas for \( \varphi < 1 \) and deposition on uncovered areas for \( \varphi > 1 \) (Turowski et al., 2007; Turowski, 2009). Hodge et al. (2011) reported different types
of sediment cover from three rivers, River Calder, South Fork Eel River and Allt Dubhaig river. They contributed the formation and change in sediment cover to grain entrainment (E), translation caused by saltation (T) and deposition (D), combined together, cast E-T-D effect on a grain. They showed that grain deposition on a bedrock surface is unlikely and entrainment of grains on a bedrock surface occurs at a lower shear stress. Whereas, in case of alluvial surface grain deposition is more favourable. They advocated the need for models that focus on the cover extent and depth to understand sediment processes. Hodge and Hoey (2012) introduced Cellular Automaton Model that assigned an entrainment probability to each grain. The assigned probability of each grain was decided by the number of neighbouring cells containing a grain. If five or more of total eight neighbouring cells contained grain, the grain was considered to be a part of the cover, otherwise, it was considered an isolated grain. They suggested that runaway alluviation occurred only in cases when isolated grains were more than the cover on the bed. Also, they advised a sigmoidal relationship between \(q_{bs}/q_{bc}\) and \(1 - P_c\). Aubert et al., in 2016 proposed a Discrete-Element Model where \(q_{bs} = N[\pi(d/2)^2/wl]\) where \(N\) is the number of spherical grain, \(d\) is the grain size, \(w\) and \(l\) represent the width and length of the flume respectively. They determined \(P_c\) from the velocity distribution of the grains. If the velocity of a grain is 1/10th or lower than the maximum velocity, the grain settles as cover on the bedrock surface. The model, however cannot deal with non-uniform velocity fields and hence cannot predict results for varying alluvial cover. Recently, Johnson (2014) and Inoue et al. (2014) proposed physically-based models that could encompass the effects of bed roughness and sediment flux. Turowski and Hodge (2017) proposed probability based model that can deal with evolution of cover residing on the bed and the exposed bedrock.

In this study, we provide a detailed study of the similarities and differences among the models proposed by Sklar and Dietrich (2004), Turowski et al. (2007), Inoue et al. (2014), Johnson (2014) and Turowski and Hodge (2017). We compare the efficacy of these models from comparisons with our experimental results. For better insight, we also test these models for prediction of experiments conducted by Chatanantavet and Parker (2008).

1.1 Bedrock Models governing consequences of roughness and sediment flux

Inoue et al. (2014) conducted several experiments by excavating a channel into a natural bedrock at Ishikari River in Japan. They conducted experiments with different combinations of flow discharge, sediment supply rate, grain size and roughness. Their experiments advocated that the dimensionless critical shear stress is related to the roughness of the channel. Their experiments also suggested that with an increase in alluvial cover, the bedrock bed roughness decreases, also, erosion in areas with an exposed bed is proportionate to sediment flux. They also conceptualised ‘Clast Rough’ and ‘Clast Smooth’ bedrock surfaces. A bedrock surface is clast-rough when bedrock hydraulic roughness is greater than the alluvial bed hydraulic roughness (supplied sediment), otherwise, a surface is clast-smooth i.e. when the bedrock roughness is lower than the alluvial roughness.

These experimental results motivated their mathematical model formulating the interaction between alluvial cover, dimensionless critical shear stress, transport capacity and the ratio of bedrock hydraulic roughness to alluvial hydraulic roughness. They calculated total hydraulic roughness height \(k_s\) as a function of alluvial cover:
\[ k_s = \begin{cases} 
(1 - P_c)k_{sb} + (P_c)k_{sa} & \text{for } 0 \leq P_c \leq 1 \\
 k_{sa} & \text{for } P_c > 1 
\end{cases} \]  
(3)

where \( k_s \) is the total hydraulic roughness height of bedrock channel, \( P_c \) is the cover fraction calculated as proposed by Parker et al. (2013) depends on ratio \( \eta_d/L \) where \( \eta_d \) is the alluvial cover thickness and \( L \) is the macro-roughness height. \( k_{sb} \) and \( k_{sa} \) represent the hydraulic roughness height of bedrock and alluvial bed respectively. The total transport capacity per unit width \( q_{bc} \) in Inoue et al.’s model is calculated as follows:

\[ q_{bc} = \alpha (\tau_s - \tau_{sc})^{1.5} \sqrt{Rgd^3} \]  
(4)

\[ \tau_{sc} = 0.027(k_s/d)^{0.75} \]  
(5)

where \( \alpha \) is a bedload transport coefficient taken as 2.66 in this study, \( \tau_s \) and \( \tau_{sc} \) are the dimensionless shear stress and dimensionless critical shear stress, \( R \) is the specific gravity of the sediment in water (1.65), \( g \) is the gravitational acceleration and \( d \) is the particle size. In this model, \( P_c \) is back-calculated from Equations (3), (4) and (5) under the assumption that the sediment supply rate \( q_{bs} \) and the sediment transport capacity \( q_{bc} \) are balanced in dynamic equilibrium state.

The sensitivity analysis of bedrock roughness and sediment supply rate conducted by Inoue et al. (2014) showed that for a given sediment supply, the deposition is higher when bedrock roughness is larger. They also showed that clast-smooth surface shows a sudden transition from completely exposed bedrock to completely alluvial, also, clast-smooth surfaces show runaway alluviation. Johnson (2014) proposed a roughness model using the median diameter grain size. They also calculated the hydraulic roughness using the aerial alluvial cover fraction.

\[ k_{sa} = r_d d[1 + (k_{sb} - 1)P_c] \]  
(6)

where \( r_d = 2 \) is a coefficient and \( k_{sb} \) is called a non-dimensional alluvial roughness representing variations in topography. The bedrock hydraulic roughness \( k_{sb} = r_d r_{br} \sigma_{br} \) where \( r_{br} \) is a scaling parameter for bedrock roughness to grain roughness and \( \sigma_{br} \) is the bedrock surface roughness. Their model calculates bedrock shear stress using Wilcock and Crowe (2003) hiding/exposure function \( (b_r) \), modified to depend on a standard deviation of bedrock elevations and a bedrock roughness scaling parameter. Johnson (2014) calculated the total transport capacity using bedload equations proposed by Meyer-Peter and Müller (1948) and Wilcock and Crowe (2003). Here we introduce Meyer-Peter and Müller (MPM) based Johnson’s model:

\[ q_{bc} = (1 - P_c)q_{bc} + (P_c)q_{bca} \]  
(7)

\[ q_{bca} = \alpha (\tau_s - \tau_{sc})^{1.5} \sqrt{Rgd^3} \]  
(8)

\[ q_{bc} = \alpha (\tau_s - \tau_{sc})^{1.5} \sqrt{Rgd^3} \]  
(9)

\[ \tau_{sc} = \frac{\tau_{br}d^2}{d} \]  
(10)

\[ b_r = \frac{0.67}{1 + \exp(1.5 - d/r_{br}\sigma_{br})} \]  
(11)

where \( q_{bca} \) is the transport capacity per unit width for sediment moving on purely alluvial bed and \( q_{bc} \) is the transport capacity per unit width for sediment moving on purely bedrock bed. \( \tau_{sc} \) is the dimensionless critical shear stress for grains on bedrock portions of the bed.
The two models may seem rather similar in that they estimate the transport capacity of a mixed alluvial–bedrock surface. However, both models opt for different approaches when it comes to estimating hydraulic roughness. The model by Inoue et al. (2014) directly uses the hydraulic roughness, but the model by Johnson (2014) calculates the hydraulic roughness from the topographic unevenness of the bed surface. In the model by Inoue et al. (2014), the topographic unevenness (i.e., macro roughness) acts only when converting the alluvial layer thickness to the alluvial cover ratio. The macro roughness affects the temporal change of the alluvial cover ratio, but does not affect the alluvial cover ratio in the dynamic equilibrium state. In addition, in the model by Johnson (2014), first, the sediment transport capacities for the bedrock and alluvial bed are separately calculated, then total transport capacity is estimated using $P_c$. However, in the model by Inoue et al. (2014), first the total hydraulic roughness height is calculated using $P_c$, then total transport capacity is estimated using the total hydraulic roughness.

Turowski and Hodge (2017) proposed a probability based model for prediction of cover on bedrock channels. They proposed cover consisting of combined exponential and linear effects of sediment supply.

$$P_c = 1 - \left[1 + (1 - \omega)\ln\left\{1 - (1 - e^{-M_0 q_{bs}}) q_{bs}\right\}\right]$$  \hspace{2cm} (12)

where $\omega$ is the exponent, $M_0^*$ is the dimensionless characteristic sediment mass obtained as following:

$$M_0^* = \frac{3\sqrt{\tau_s \tau_c}}{2\pi} \frac{\tau_c / \tau_s - 1}{(\tau_c / \tau_s)^{0.3 - 0.7}}$$  \hspace{2cm} (13)

$M_0^*$ follows a linear relationship with $\tau_s$ for a high $\tau_s$. They suggested that on shorter time scales, the sediment cover follows a linear relationship with the sediment supply, but on longer time scales the channel adjusts its slope and width.

We hereafter refer Sklar and Dietrich (2004) model as linear model, Turowski et al.’s model (2007) as exponential model, Inoue et al.’s model (2014) as macro roughness model, Meyer-Peter and Müller (MPM) based Johnson’s model (2014) as surface roughness model and Turowski and Hodge’s model (2017) as probabilistic model.
2 Experimental Method

2.1 Experimental Flume

The experiments were conducted in a straight channel at the Civil Engineering Research Institute for Cold Region, Sapporo, Hokkaido, Japan. The experimental channel was 22m long, 0.5m wide and had a slope of 0.01. The width-depth ratio was chosen to achieve no-sandbar condition (i.e., small width-depth ratio, 6.1 to 8.3 in our experiments). Chatanantavet and Parker (2008) conducted several flume experiments with sandbar condition (i.e., large width-depth ratio, 11 to 30 in their experiments) and suggested that the alluvial cover increases linearly to the ratio of sediment supply and transport capacity of the channel when the slope is less than 0.015. In this study, we investigated the influence of bedrock roughness on the alluvial cover under conditions where the slope and width-depth ratio were small compared to the experiments of Chatanantavet and Parker (2008).

2.2 Bed characteristics and conditions

The channel bed consisted of non-erodible mortar. In order to achieve different roughness conditions, the beds in Run 1, Run 2 and Run 3 were embedded with gravel of different particle sizes. In Run 1, the embedded particle size is 30 mm, in Run 2 particle size of 50 mm is embedded and in Run 3, 5 mm particle size is embedded.

We performed an additional 2 cases with net-installation on the river bed. An installed net on the river bed can trap sediment during high flow, eventually protecting the bed from further erosion from abrading sediment (Kazuaki et al., 2015, in Japanese). A net of mesh size 30 mm X 30 mm was installed on the bed in Run 4 and Run 5. The height of the net was 4mm and 2 mm respectively. Figure 1 shows the experimental channel bed of all 5 runs.

Figure 1: Initial channel bed for each run.
2.3 Measurement of bedrock roughness

In order to measure the initial bed roughness (before supplying sand), a water discharge of 0.03 m³/s was supplied, and the water level was measured longitudinally at every 1 m at the centre of the channel. Manning’s coefficient was calculated using the water level. The Manning’s roughness coefficient and $U$ is the average velocity ($U = Q/wD$ where $U$ is the water discharge, $w$ is the channel width, $D$ is the water depth), $S_e$ is the energy gradient.

\[ n_m = \frac{1}{6}D^{2/3}S_e^{1/2} \]  

(14)

where $n_m$ is the Manning’s roughness coefficient and $U$ is the average velocity ($U = Q/wD$ where $U$ is the water discharge, $w$ is the channel width, $D$ is the water depth), $S_e$ is the energy gradient.

In order to compare the hydraulic roughness height and the riverbed-surface unevenness height, the riverbed height before water flow was measured with a laser sand gauge. The measurements were taken longitudinally at every 5 mm. The measurements were taken at three points: 0.15 m away from the right wall, the centre of the channel, and 0.15 m away from the left wall. The standard deviation representing the topographic roughness $\sigma_{br}$ was obtained by subtracting the mean slope from the riverbed elevation (Johnson and Whipple; 2010).

2.4 Measurement of dimensionless critical shear stress on bedrock

In order to measure the dimensionless critical shear stress of grains on completely bedrock portion, i.e. $\tau_{\ast cb}$, 30 gravels with diameter of 5mm were placed on the waterway floor at intervals of 10 cm or more to make sure that there was no shielding effect between the gravels (there was shielding effect due to unevenness of the bedrock). Next, water flow was supplied at a rate that no gravel moved, and was slowly increased to a rate at which all the gravels moved. The water level and the number of gravels displaced were measured and recorded for each flow rate.

The weighted average of the dimensionless shear stress $\tau_{\ast} = DS_e/Rd$ was obtained using the measured number of movement.

2.5 Validation of alluvial-cover

Different amounts of sand were supplied manually while the flow rate was kept constant at 0.03 m³/s. The alluvial cover ratio was measured once equilibrium state was achieved. The sediment supply amounts and other experimental conditions for various cases are provided in Table 1. Each run has multiple cases, each with different sediment supply and time duration.

Each case was performed unless the $P_c$ became constant. The gravels were supplied from Run-0 of no sediment to Run-4~5 of completely alluvial cover. The Run-0 with no sediment supply in each run represents the bedrock-roughness measurement experiment explained in section 2.3.

Equilibrium conditions were achieved after 2-4 hours of sand supply. The alluvial cover was calculated from black and white photographs of the flume by taking the ratio of the number of pixels. The water level was measured and recorded every hour at the centre of the channel in order to calculate the hydraulic roughness during and at the end of the experiment.
Table 1: Experimental Conditions

| Run   | $k_{sb}$ (mm) | $k_{sb}/d$ | $q_{bs}$ ($\times 10^{-5} m^2/s$) | Time (hour) | $P_c$ | $D$ | $U$ | $Fr$ |
|-------|----------------|------------|---------------------------------|-------------|-------|-----|-----|-----|
| Run1-0| 48.0           | 9.6        | 0.00                            | 0.25        | 0.00  | 0.082 | 0.74 | 0.82 |
| Run1-1| 24.8           | 5.0        | 0.00                            | 0.25        | 0.00  | 0.078 | 0.83 | 0.95 |
| Run1-2| 3.8            | 0.8        | 0.00                            | 0.25        | 0.00  | 0.063 | 0.95 | 1.21 |
| Run2-0| 36.3           | 7.3        | 0.00                            | 0.25        | 0.00  | 0.077 | 0.78 | 0.90 |
| Run2-1| 9.6            | 1.9        | 0.00                            | 0.25        | 0.00  | 0.068 | 0.88 | 1.08 |
| Run2-2| 5.60           | 2.00       | 0.00                            | 0.01        | 0.00  | 0.079 | 0.76 | 0.87 |
| Run3-0| 7.47           | 4.00       | 0.00                            | 0.10        | 0.00  | 0.078 | 0.76 | 0.87 |
| Run3-1| 4.67           | 6.00       | 0.00                            | 0.10        | 0.00  | 0.078 | 0.76 | 0.87 |
| Run3-2| 5.60           | 4.00       | 0.00                            | 0.10        | 0.00  | 0.078 | 0.76 | 0.87 |
3 Experimental results

3.1 Initial topographic roughness and hydraulic roughness

Figure 2 shows the relationship between the hydraulic roughness height of bedrock bed \( k_{sb} \) and the topographic roughness height of bedrock bed \( \sigma_{br} \). This figure suggests that Run 1 with 30 mm sized embedded gravel, has the largest hydraulic roughness and Run 3 with 5 mm sized embedded gravel has the lowest hydraulic roughness. Run 2 embedded with 50 mm gravel has large topographical roughness error bars for the reason that, the large gravels were embedded randomly in the bed, resulting in unintended spatial variation in the unevenness of the channel bed. Although the hydraulic roughness tends to increase with an increase in topographical roughness, it has large variation. This variation is due to the fact that the hydrological roughness height does not only depend on the topographical roughness but also on the arrangement of the unevenness.

![Figure 2: Relationship between initial bed hydraulic roughness height and topographic roughness height. The black circles in the image represent the average values measured on the three data collection lines, and the error bars represent the minimum and maximum value.](image-url)
3.2 Relative roughness, sediment supply and alluvial cover

Figure 3 shows the channel bed after the experiments of the Run 1 series (Run 1-1, Run 1-2, Run 1-3 and Run 1-4) with the highest relative roughness. Figure 4 shows the channel bed after the experiments of the Run 3 series (Run 3-1, Run 3-2, Run 3-3) which has the lowest relative roughness. In these two figures, we can compare Run 1-4 and Run 3-1 with equal sediment supply rates. It can be seen that the bed in Run 1-4 is completely covered with sediment whereas the bed in Run 3-1 has almost no accumulated sediment on the bed.

Figure 5 shows the relationship between alluvial-cover fraction $P_c$ and sediment supply per unit width $q_{bs}$. $P_c$ is obtained by dividing the sediment-covered area by the total area of the channel from photographs. The value of $P_c$ is 1 for a completely covered channel and 0 for a completely exposed bedrock bed. In Figure 5, if we compare Run 1-4, Run 2-4, Run 3-1, Run 4-4 and Run 5-1, the cases with equal sediment supply rate of $3.73 \times 10^{-5} \text{ m}^2/\text{s}$, it can be observed that alluvial-cover fraction is increasing with an increase in the bedrock roughness. Moreover, in Run 1 series, Run 2 series and Run 4 series with high relative roughness $k_{sb}/d$ (ratio of the hydraulic roughness height of bedrock bed $k_{sb}$ to the grain size $d$), $P_c$ is roughly proportional to the sediment supply rate $q_{bs}$. However, in Run 3 series and Run 5 series, which have lower $k_{sb}/d$ (relative roughness), $P_c$ shows hardly any increase when $q_{bs}$ is low (Run 3-0, Run 3-1, Run 3-2, Run 5-0, Run 5-1) and when sediment supply ($q_{bs}$) increases (Run 3-3, Run 5-2), the bedrock suddenly transitions to completely alluvial bed.

Figure 3: Bedrock exposure in Run 1 series at the end of the experiment. Initial bed had 30mm embedded particles.
Figure 4: Bedrock exposure in Run 3 series at the end of the experiment. Initial bed had 5mm embedded particles.

Figure 5: Variation in alluvial cover fraction (Pc) with sediment supply.
3.3 Relationship between gravel layer thickness and alluvial cover fraction

Various studies have employed alluvial cover as a function of gravel bed thickness (Zhang et al., 2015; Inoue et al., 2014; Parker et al., 2013; Tanaka and Izumi, 2013 (in Japanese)).

\[
P_c = \begin{cases} 
\frac{\eta_a}{L} & \text{for } 0 \leq \frac{\eta_a}{L} \leq 1 \\
1 & \text{for } \frac{\eta_a}{L} > 1
\end{cases}
\]  

(15)

here, \(\eta_a\) is the average thickness of the alluvial layer, \(L\) is the macro-roughness height of the bedrock bed. Parker et al. (2013) define \(L\) as the macroscopic asperity height of rough bedrock rivers \(L_b (\approx 2\sigma_{br})\). Tanaka and Izumi (2013) define \(L\) as the surface unevenness of alluvial deposits on smooth bedrock river \(L_a (\approx d)\). In this study, we define \(L = 2\sigma_{br} + d\) so that it can cope with both smooth and rough bedrocks. Figure 6 shows the relationship between relative gravel layer thickness \(\eta_a/L\) and alluvial cover ratio. The figure confirms that the alluvial cover ratio of the experimental result can be efficiently evaluated by Equation (15).

Figure 6: Relationship between relative gravel layer thickness and alluvial cover.
3.4 Time series change of relative roughness

Figure 7 shows the change in relative roughness with time in Run 1 and Run 3 series. The red and blue points in Figure 7 show the alluvial cover fraction after water supply in Run 1 and Run 3 series respectively.

In Run 1 series with a higher relative roughness, it can be seen that relative roughness decreased due to the increase in alluvial deposition and cover. In Run 3 series which has a lower initial relative roughness increased due to the increase in alluvial deposition and cover.

The relative roughness after the water supply is ~2 for both Run1-4 and Run3-3 while the alluvial cover fraction approaches 1. This value is almost the same as the relative roughness of flat gravel bed (about 1 to 4 times the particle size, generally about 2 times). This confirms that with an increase in alluvial cover, the relative roughness of the bed is determined by the gravel size.

![Figure 7: Change in Relative roughness with time.](https://doi.org/10.5194/esurf-2019-78)
Discussion and Comparison of the Existing Models with Experimental Results

4.1 Relative Roughness and dimensionless critical shear stress

Figure 8 shows the relationship between the ratio of the hydraulic roughness height of bedrock bed $k_{sb}$ to the grain size $d$ ($k_{sb}/d$: referred to as the relative roughness in section 3.2) and the dimensionless critical shear stress over bedrock bed $\tau_{cb}$. The Figure shows results obtained from Johnson (2014) (Eq. 10) and from Inoue et al. (2014) (Eq. 5) i.e. surface-roughness model and macro-roughness model respectively.

According to Figure 8, the non-dimensional critical shear stress depends on the relative roughness to the power of 0.6. Besides, the results obtained from Eq. (5) of macro-roughness model are not compatible with the experimental results in the region where relative roughness of the bedrock bed is small. In this study, we used the power approximation shown below instead of Eq. (5) in the macro roughness model by Inoue et al. (2014).

$$\tau_{sc} = 0.03(k_{s}/d)^{0.6}$$

(15)

Likewise, the results obtained from Johnson’s model (2014) (Eq. 10) (surface-roughness model) are consistent with our experimental results, but the model is inconsistent when the roughness is low.

![Graph showing the relationship between relative roughness and dimensionless critical shear stress.](https://doi.org/10.5194/esurf-2019-78)

Figure 8: Relationship between relative roughness and dimensionless critical bed shear stress. The black squares show the results of this experiment, the white circles show the results of investigation using the bedrock of Ishikari River in 2011 (Inoue et al., 2014), the grey rhombus represents a smooth aquifer floor (Inoue and Ito, 2013 (in Japanese)), the grey line shows the power approximation of all the experimental results. The dotted line shows the results from Eq. 5 proposed by Inoue et al. (2014). The black double dotted lines show the results obtained by Eq. 10 (Johnson, 2014).
4.2 Calibrating $k_{#D}$ and $r_{br}$

For the purpose of model comparisons with experimental results, we need to first calibrate Johnson’s model parameters $k_{#D}$ and rbr to minimize RMSD (root mean square deviation). When $k_{#D} = 1$, it means the alluvial hydraulic roughness is proportional to the grain diameter size and is independent of the cover fraction. For our calculations, we have used $k_{#D} = 4$ as applied in Johnson (2014). We also calibrate the exponential model’s parameter $\varphi$ (Turowski et al, 2007). Table 2 provides the calibration values for $r_{br}$ and $\varphi$ for comparison of the model with our experimental results.

| Run   | Observed $k_{sb}$ (mm) | Observed $\sigma_{br}$ (mm) | Adjusted $r_{br}$ ($k_{#D}=4$) | Calculated $k_{sb}$ (mm, $k_{sb}=r_d r_{br} \sigma_{br}$) | Adjusted $\varphi$ |
|-------|------------------------|----------------------------|-------------------------------|----------------------------------------------------------|-------------------|
| Run 1 | 48.0                   | 3.7                        | 3.0                           | 22.2                                                     | 3.1               |
| Run 2 | 24.8                   | 3.9                        | 2.1                           | 16.4                                                     | 1.1               |
| Run 3 | 3.8                    | 1.1                        | 3.0                           | 6.6                                                      | 0.4               |
| Run 4 | 36.3                   | 2.3                        | 4.6                           | 21.2                                                     | 2.2               |
| Run 5 | 9.6                    | 1.8                        | 2.6                           | 9.4                                                      | 0.9               |

4.3 Predicting experimental results using the models

Figure 9 shows the comparison among experimental results presented in this paper, Sklar and Dietrich’s linear model (2014), Turowski et al.’s exponential model (2007), Inoue et al.’s macro-roughness model (2014) and Johnson’s surface-roughness model (2014). This Figure suggests that the linear model is generally applicable to rough bed with relative roughness of 2 or more, but not to “smoother bed” with relative roughness less than 2 (Run 1, Run 2 and Run 4). As suggested by Inoue et al. (2014), in this study, “smooth bed” refers to the bed with roughness less than the roughness of supplied gravel and “rough bed” stands for the bed with roughness more that the roughness of the supplied gravel. The exponential model is also more suitable for a rough bed. The figure also shows that the macro-roughness model proposed by Inoue et al. (2014) can predict the increasing alluvial cover for cases with high relative roughness, as well as the runaway hysteresis for cases with lower relative roughness (Run 3 and Run 5), without adjusting the roughness. The surface-roughness model proposed by Johnson (2014) also shows good agreement in predictions of alluvial cover and runaway hysteresis if $k_{#D}$ and $r_{br}$ are adjusted.

In Run 3 and Run 5 with relatively smooth beds, a rather scarce deposition was observed when sediment supply was low, and runaway alluviation occurred when sediment supply exceeded the transport capacity of the channel i.e. the bed was suddenly completely covered by alluvium. The reverse-line slopes produced by macro-roughness and surface-roughness models depict similar hysteresis relationship between alluvial cover and sediment supply i.e. sediment deposition occurs only for a certain range of sediment supply. This phenomenon has also been observed in sufficiently steep channels, for slopes greater than 0.015.
by Chatanantavet and Parker (2008). Hodge and Hoey (2016b) also suggested a similar relationship between sediment cover and sediment supply. However, our study shows that rapid alluviation occurs irrespective of the slope steepness, if roughness of the bed is less than the roughness of supplied gravel, i.e. when relative roughness is less than 2.

In an effort to get a better understanding of the effectiveness of the two models: the macro-roughness model proposed by Inoue et al. (2014) and the surface-roughness model proposed by Johnson (2014), we also compared the model results with experimental results of Chatanantavet and Parker (2008). They conducted experiments in a metallic straight channel with three different types of bedrock bed surfaces namely Longitudinal Grooves (LG), Random Abrasion Type 1 (RA1) and Random Abrasion Type 2 (RA2), where RA1 is smoother than RA2. They performed various cases for each type with varying slope range of 0.0115 – 0.03. They also varied the sediment supply rate and grain size (2mm and 7mm). The major difference between their experiment and our experiments is the width–depth ratio. The width–depth ratios of their experiments were 11–30, and thus allowed for the formation of alternate bars. In contrast, the width–depth ratios of our experiments were 6.1 – 8.3, as a result alternate bars usually do not develop. Although we can see alternate alluvial patches in Figure 5, their thickness was less than 1 cm, and the patches did not progress to alternate bars with large wave height. Figure 11 shows the comparison among the two models and Chatanantavet and Parker’s experiment (2008). The experimental conditions are taken from Table 1 of Chatanantavet and Parker (2008). Figure 11a represents runs 2-C1 to 2-C4, Figure 10b represents runs 2-E1 to 2-E3, Figure 11c represents runs 3-A1 to 3-A5, Figure 11d represents runs 3-B1 to 3-B5, Figure 11e represents runs 1-B1 to 1-B4 (Chatanantavet and Parker 2008, Table 1). In case of the surface-roughness model, \( k_{bd} = 4 \) is used, the bedrock surface roughness required for calculations is taken as mentioned in Table 1 Johnson (2014), \( r_{br} \) is adjusted to minimize RMSD. In case of the macro-roughness model by Inoue et al. (2014), \( k_{sb} \) is adjusted to minimize RMSD. The two models can accurately predict the cover fraction and runaway alluviation for the experimental study conducted by Chatanantavet and Parker (2008). A very important point of interest is the adjustment of hydraulic roughness value of the bedrock surface \( k_{sb} \). In case of Chatanantavet and Parker’s experiment, \( k_{sb} \approx 0.4 \) mm to 3.5mm (Chatanantavet and Parker 2008, Table 1), whereas, in Johnson’s surface-roughness model (2014), \( k_{sb} \) can be as much as 13 – 27 mm. Also, in the case of Inoue et al.’s macro-roughness model \( k_{sb} \) is adjusted to 32 – 53 mm (Table 3).

Figure 10 shows the comparison of experimental results with Turowski and Hodge’s probabilistic model (2017). The model produces favourable results following some parameter adjustments. The values of exponent \( \omega \) and characteristic sediment mass \( M_0^* \) needs to be adjusted by trial and error. The value of \( \omega \) can be as high as 100 or 200 for Runs with runaway hysteresis, whereas it is as low as ~0.7 for other Runs.
Table 3: Parameter calibration values for comparison with experimental results of Chatanantavet and Parker (2008)

| Type | Slope  | Observed $k_{sb}$ (mm) | $\sigma_{br}$ (mm) | Adjusted $r_{br}$ for the surface-roughness model $k_{sb}=4$ | Calculated $k_{sb}$ in the surface-roughness model (mm, $k_{sb}= r_d r_{br}\sigma_{br}$) | Adjusted $k_{sb}$ for the macro-roughness model (mm) |
|------|--------|------------------------|-------------------|-----------------------------------------------------------------|---------------------------------------------------------------------------------|------------------------------------------|
| LG   | 0.02   | 0.4                    | 6.7               | 1.8                                                             | 24.1                                                                            | 42.0                                     |
| RA1  | 0.016  | 0.4                    | 2.4               | 5.3                                                             | 25.4                                                                            | 42.0                                     |
|      | 0.03   | 0.4                    | 2.4               | 5.7                                                             | 27.4                                                                            | 53.0                                     |
| RA2  | 0.0115 | 3.5                    | 2.7               | 2.5                                                             | 13.5                                                                            | 32.0                                     |
|      | 0.02   | 3.5                    | 2.7               | 4.3                                                             | 23.2                                                                            | 45.0                                     |

4.4 Differences and limitations

As mentioned earlier, the major difference between the two models employed in this study is the way the transport capacity is calculated. In case of the surface-roughness model (Johnson, 2014), first, the transport capacities for bedrock ($q_{bcB}$) and alluvial bed ($q_{bca}$) are separately calculated, then the total transport capacity ($q_{bc}$) is calculated for a range of cover fractions ($P_c$). Hence, in cases when $\tau_{sc} < \tau < \tau_{sec}$, the transport capacity over bedrock portion $q_{bcB} = 0$. As a result, bedrock roughness is hardly affected by the alluvial cover fraction which can also be the reason for inconsistency between the surface-roughness model results and experimental study for Runs 1 and 4 in Figure 9 and RA2 Slope = 0.0115 in Figure 11. Whereas, in the case of macro-roughness model (Inoue et al., 2014), the critical shear stress takes into account the value of total hydraulic roughness, which depends on cover fraction, alluvial hydraulic roughness and bedrock hydraulic roughness. Hence, even when $\tau_s$ is small, the bedrock roughness tends to affect the cover fraction.

Comparing the observed $k_{sb}$ with the adjusted $k_{sb}$ in the roughness models proposed by Inoue et al. (2014) and Johnson (2014), the adjusted $k_{sb}$ strongly depends on observed $k_{sb}$ in our experiments without alternate bars (Figure 12a), whereas, the adjusted $k_{sb}$ is not dependent on the observed $k_{sb}$ in experiments with alternate bars conducted by Chatanantavet and Parker (2008) (Figure 12b), suggests that bedrock roughness has a smaller effect on the alluvial cover in case of mixed alluvial – bedrock rivers with alternate bars. In such rivers, the bed slope may affect the alluvial cover fraction (Figure 12c). The roughness models are adjusted to produce the experimental results with alternate bars by fine-tuning $r_{br}$ and $k_{sb}$ values which have to be determined by trial and error method. While this method can be applicable to laboratory-scale experiments, the model calibration is unfeasible for a large-scale channel or natural rivers. In general, the formation of alternate bars is barely reproduced with a one-dimensional model as introduced in this study. In the future, research to incorporate the effects of bars into a one-dimensional model, or analysis using a two-dimensional planar model (e.g., Nelson and Seminara, 2012; Inoue et al., 2016, 2017) is expected.
Also, the probabilistic model proposed by Turowski and Hodge (2017) could reproduce experimental results but the model needed adjustment of $\omega$ and $M_0^*$ by trial and error, specially for cases involving runaway hysteresis.

Figure 9: Comparison of experimental results, linear model by Sklar and Dietrich (2004), exponential model by Turowski et al. (2007), roughness models by Inoue et al. (2014) and Johnson (2014). The $r_{br}$ for the surface roughness model and the $\varphi$ for the exponential model are adjusted to minimize RMSD.
Figure 10: Comparison of Experimental results with Turowski and Hodge’s probabilistic model (2017).
Figure 11: Comparison of the experimental results (Chatanantavet and Parker, 2008) with the macro roughness model (Inoue et al., 2014) and the surface roughness model (Johnson, 2014). RA1 in (a) and (b) represent Random Abrasion type 1, RA2 in (c) and (d) represent Random Abrasion type 2, LG in (e) represents Longitudinal grooves (Chatanantavet and Parker 2008). The \( r_{br} \) for the surface roughness model and the \( k_{sb} \) for macro roughness model are adjusted to minimize RMSD.
Figure 12: (a) Comparison between adjusted hydraulic roughness height of bedrock bed $k_{sb}$ and observed $k_{sb}$ for experiments conducted in this study by us. (b) Comparison between adjusted $k_{sb}$ and observed $k_{sb}$ for Chatanantavet and Parker’s experiment (2008). (c) Sensitivity of adjusted $k_{sb}$ to bed-slope $S$ for experiments conducted by Chatanantavet and Parker (2008).
Summary

Here we provide a review of models and studies focused at discovering the interaction between alluvial cover and bed roughness. In order to get an insight into the interaction between alluvial cover and bed roughness, we conducted laboratory-scale experiments with multiple runs of varying bed roughness and sediment supply. The experimental results show that the change in alluvial cover to the supplied sediment is controlled by bed roughness to a great extent. When the bedrock hydraulic roughness is higher than the hydraulic roughness of the alluvial bed, the alluvial cover increases proportionately with the increase in sediment supply and then reaches an equilibrium state. However, in cases where bedrock roughness is lower than the roughness of the alluvial bed, i.e. the bed is smoother than the sediment supplied, the deposition is insignificant unless sediment supply exceeds the transport capacity of the channel, after which, the bed abruptly becomes completely alluvial.

We have also implemented the cover-roughness interaction models: the linear model proposed by Sklar and Dietrich (2004), the exponential model by Turowski et al. (2007), the macro-roughness model by Inoue et al. (2014), the surface-roughness model by Johnson (2014), and the probabilistic model by Turowski and Hodge (2017) in order to predict the experimental results. The linear model and exponential model are inefficient for cases with a smooth-bed specifically, they cannot predict the hysteresis. The macro-roughness model (Inoue et al. 2014) and surface-roughness model (Johnson, 2014) can efficiently predict the runaway-alluviation for smooth-beds as well as the proportionate increase in alluvial cover for rough-bed. In particular, the macro-roughness model (Inoue et al. 2014) was able to reproduce the observed alluvial cover ratio without adjusting the parameters. The probabilistic model by Turowski and Hodge (2017) could also reproduce the laboratory experiments but for cases with runaway hysteresis, the model parameters had to be adjusted.

We also tested the macro-roughness model (Inoue et al. 2014) and surface-roughness model (Johnson, 2014) for their capability to predict the experimental results observed by Chatanantavet and Parker (2008), in which the bedrock surface has alluvial alternate bar formations. Both models were able to reproduce the alluvial cover fraction ensuing significant parameter adjustments. The two models do not include the 2-D effects caused by variable alluvial deposition and formation of bars on bedrock. Building a physical model that can predict alluvial cover fraction with bar formation represents an exciting challenge in the future.

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Notations:

- $b_r$: exposure function by Johnson (2014)
- $d$: particle size (m)
- $D$: water depth (m)
- $k_s$: hydraulic roughness height (m)
- $k_{sa}$: hydraulic roughness height of purely alluvial bed (m)
- $k_{sb}$: hydraulic roughness height of purely bedrock bed (m)
- $k_{#D}$: non-dimensional alluvial roughness
- $l$: flume length
- $L$: macro-roughness height of bedrock bed (m)
- $N$: number of spherical grain
- $M_0$: dimensionless sediment mass
- $n_m$: Manning’s roughness coefficient
- $P_c$: mean areal fraction of alluvial cover
- $q_{bs}$: sediment supply rate per unit width (m$^2$/s)
- $q_{bc}$: transport capacity per unit width (m$^2$/s)
- $q_{baa}$: transport capacity per unit width for sediment moving on purely alluvial bed (m$^2$/s)
- $q_{bcb}$: transport capacity per unit width for sediment moving on purely bedrock bed (m$^2$/s)
- $Q$: water discharge (m$^3$/s)
- $r_d$: scaling coefficient for $d$ and hydraulic roughness length
- $r_{br}$: fitting parameter that scales bedrock roughness to $d$
- $R$: specific gravity of sediment in water (1.68)
- $S$: Bed slope
- $S_e$: energy gradient
- $U$: depth averaged velocity (m/s)
- $w$: flume width
- $\alpha$: bedload transport coefficient
- $\eta_a$: average thickness of alluvial layer
- $\kappa$: Karman constant
- $\sigma_{br}$: topographic roughness height of purely bedrock bed (m)
\[ \tau_* \quad \text{dimensionless shear stress} \]
\[ \tau_{*c} \quad \text{dimensionless critical shear stress} \]
\[ \tau_{*ca} \quad \text{dimensionless critical shear stress for grains on purely alluvial bed} \]
\[ \tau_{*cb} \quad \text{dimensionless critical shear stress for grains on purely bedrock bed} \]
\[ \varphi \quad \text{cover factor proposed by Turowski et al. (2007)} \]
\[ \omega \quad \text{Exponent by Turowski and Hodge (2017)} \]
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