A numerical application in four-point bent specimen

Lih-jier Young¹ and Bo-Han Yeh²

¹ Professor, Chung Hua University, Taiwan, R.O.C.
² Ph.D. Student, Chung Hua University, Taiwan, R.O.C.
E-mail: young@chu.edu.tw

Abstract. In this research a four point bent specimen with a crack will be employed to investigate the behavior of the crack by using the boundary element method numerically and the fracture mechanics theretically due to the industrious important. Boundary element is employed because the reduction of the computational dimension. The accuracy of this approach is as high as 98.62% and shows the potential application of the boundary element in four-point bent specimen.

1. Introduction

Crack problems are always exist in industry. It will also cause many serious industrial injuries in this sophisticated technological era such as nuclear power plant, tunnel and aircraft. Therefore, how to improve these complicated problem is the major purpose of this research. Nowadays several laboratories are now dealing with the crack problem. H.L. Oliveria, Edson Denner Leonel [1] model multiple crack propagation in two-dimensional domains which simulate localisation and coalescence phenomena. M. Munsche et al [2] solve the transient elastodynamic crack analysis in two-dimensional (2D), homogeneous, anisotropic, and linear elastic solids by a hypersingular time-domain boundary element method. Lih-jier Young [3] reduce the weight of rescue robot with a cracked lateral plate.

Due to the reason of crack growth not only by normal and shear forces but bending moment four-point bent specimen is employed in this research. Figure 1 shows the dimension of this specimen.

2. Four-point bent specimen
2.1 Boundary conditions and numerical implementation

Four-point bent specimen was first discussed in [4] which dealing with the mixed mode fracture of the specimen. As described in [5] the first step in BEM solution is to divided the homogeneous media into two bodies \( B_\gamma = (1, 2) \) along the plane of the crack referred to below as the interface of the center open crack problem as shown in Fig. 2. The interaction between two bodies is included through boundary conditions relating the displacements and stresses on either side of the interface. According to the different
boundary conditions of the cracked four-point bent specimen as in Fig. 2 there are totally five regions in the specimen, i.e., the open crack region (I_1), the ligament region (I_2), the loading region (I_3), the support region (I_4) and the free surface region (I_5). Let \( t_i \) and \( u_i \) (\( i = 1, 2 \) and \( i = 1, 2 \)) denote the \( i \)th boundary traction and displacement components, respectively, on the boundary \( B \). At points on this region the 2 displacement components and 2 stress components must be continuous. Therefore, the boundary conditions are \((2t_1)_{I_1} = -(1t_1)_{I_2}\) and \((2t_2)_{I_4} = -(1t_2)_{I_5}\), \((2u_1)_{I_1} = -(1u_1)_{I_2}\) and \((2u_2)_{I_4} = -(1u_2)_{I_5}\). This leaves \((1t_1)_{I_1}, (1t_2)_{I_4}, (1u_1)_{I_2}\) and \((1u_2)_{I_4}\) as
the unknowns. At points on open crack region $I_1$ and free surface region $I_4$ stresses are free, i.e., $(\gamma t_1)_{I_1}, (\gamma t_2)_{I_4} = (\gamma t_2)_{I_1}, (\gamma t_4)_{I_4} = 0$. The unknowns are, of course, $(\gamma u_1)_{I_1}, (\gamma u_2)_{I_1}$ and $(\gamma u_1)_{I_4}, (\gamma u_2)_{I_4}$, where $\gamma = 1$ or 2 depending on whether that portion of $I_4$ is in $B_1$ or $B_2$. At points on loading region ($I_3$) a concentrate load $P = 150$ Mpa is applied to the bar at the top of the specimen which lead two point loads $P_1 = \frac{b_1}{b_1 + b_2} P$ and $P_2 = \frac{b_2}{b_1 + b_2} P$, where $b_1 = 10.75$ mm and $b_2 = 5.75$ mm, i.e., $P_1 = 97.7273$ Mpa, $P_2 = 52.2727$ Mpa theoretically as shown in Fig. 2. The boundary conditions is therefore, $(t_1)_{I_1} = (t_1)_{I_4} = 0, (t_2)_{I_1} = -52.2727$ Mpa, $(t_2)_{I_4} = -97.7272$ Mpa. The unknowns are four displacement components, i.e., $(u_1)_{I_1}, (u_2)_{I_1}, (u_1)_{I_4}$ and $(u_2)_{I_4}$. The boundary conditions of the support region ($I_5$), i.e., two simply support points in the bottom of the specimen is $(u_1)_{I_5} = (u_2)_{I_5} = 0$. The unknowns are $(t_1)_{I_5}, (t_2)_{I_5}$, where $\gamma = 1$ or 2 depending on whether that portion of $I_5$ is in $B_1$ or $B_2$. Thus, at each pair of points on the ligament region ($I_2$) we have four conditions involving eight quantities. Four of those are eliminated algebraically using the boundary conditions, thus, leaving four unknowns at each point. Two coupled boundary integral integral equations, written as a function of position on the boundary of a body, enforce all of the field equations of elasticity for that body. The two equations for each of the two artificially divided bodies are applied to each discretized point on the interface, thus giving four equations and four unknowns at each pair of interface points. At each of the boundary points of either of artificially divided bodies consist of other than the common interface, i.e., the crack region ($I_1$), the loading region ($I_3$), the free surface region ($I_4$) and the support region ($I_5$), there are four boundary quantities to be accounted for. The BEM consists of the discretization of the boundary surfaces and the numerical approximation of the boundary quantities in the set of equation obtained from the boundary integrals. We model the boundary, using straight-line element, centered about nodes at which the integrals of the 2D Green’s function as in [5]. The final system of simultaneous linear algebraic equations for the unknown nodal displacements and stresses, can be obtained by using Gaussian elimination method.

2.2 Accuracy test

After plugging in the applied loading $P = 150$ Mpa in the BEM code with 200 elements for the non-crack specimen the numerical results show that $(t_2)_{I_1} = 97.0163$ Mpa
and \((s_1 t_2)_{t_0} = 52.997 \text{MPa}\), respectively. Compare the theoretical values as described above, the accuracy of present method is 98.62%. Figure 3 shows the normal and shear stress distributions of the interface. Figure 4 shows the normal and shear displacements of the interface. The displacement of the non-interface points are shown in Fig. 5.

![Normal and Shear Stress Distributions](image1)

![Normal and Shear Displacements](image2)

**Fig. 3** Normal (a) and shear (b) stress distributions of ligament portion of non-crack four-point bent specimen.

**Fig. 4** Transverse (a) and longitudinal (b) displacements of ligament portion of non-crack four-point bent specimen.
3. Numerical results of the cracked specimen

3.1 Open crack four-point bent specimen

After substitution all boundary conditions and parameters into the BEM code the unknowns can be reached by Gauss Elimination. In the present model stress intensity factors are calculated by fitting the standard square root form for crack displacement at the elastic crack tip, in terms of stress intensity factor shown inverted for $K$, i.e.,

$$K = \frac{2\mu(x_i)G}{\kappa+1} \sqrt{\frac{2\pi}{x_i}}$$

where $\kappa + 1 = 4(1 - \nu)$ for plane strain

$$\kappa + 1 = \frac{4}{(1 + \nu)}$$

for plain stress

2u is crack opening displacement (COD) for mode I or crack sliding displacement (CSD) for mode II which is calculated from the present BEM formulation,

$G$ is the shear modulus,

$x_i$ is the distance between crack tip and the nearest nodal point on the crack. The closer $x_i$ is to the crack tip the more accurate $K$ is. The result shows the calculated value of $K_I$ is 19.97 MPa $\sqrt{m}$. Compare with the analytical value 20.68 MPa $\sqrt{m}$ ($K_I = \frac{6M\sqrt{a}}{W^2}1.99$, when $\frac{a}{W} = 0.3, M=100$MPa) the accuracy is 96.57%.

Inspecting both displacements of the two surfaces of the crack we find that they are overlapped as shown in Fig. 6 which is impossible. Therefore, the boundary
boundary condition of the open crack should change into the partial contact boundary condition.

3.2 Partial contact four-point bent specimen

We divided the crack region $I_1$ into two subregions: the contact subregion $I_{11}$ and not contact subregion $I_{12}$. The boundary condition of $I_{11}$ is therefore, $(t_1)_{I_{11}} = -(z_1)_{I_{11}}, (u_1)_{I_{11}} = (u_1)_{I_{11}}$ with the unknowns $(u_1)_{I_{11}}, (u_2)_{I_{11}}, (t_1)_{I_{11}}, (z_2)_{I_{11}}$. However, the boundary condition of $I_{12}$ will be $(t_1)_{I_{12}} = (t_2)_{I_{12}} = 0, (z_1)_{I_{12}} = (z_2)_{I_{12}} = 0$ with the unknowns $(u_1)_{I_{12}}, (u_2)_{I_{12}}, (u_1)_{I_{12}}, (u_2)_{I_{12}}$. The nodal number is shown in Fig. 7. From the result of Sec. 2.3 points 15~17 are contact and points 18~20 are not. The stress distributions are

![Diagram](image)

**Fig. 6** Two surfaces overlapped of crack.

![Diagram](image)

**Fig. 7** Nodal number of the specimen.
shown in Fig. 8. The normal stress of the interface is similar to non-crack. There is no stress intensity occur at the crack tip. However, the shear force distribution is different. According to Fig. 8 (b) the shear stress of points 1~13 are the same with non-crack specimen. The shear stress near the crack tip is increase dramatically which is agreed with the theory of fracture mechanics, i.e, the stress concentration at the crack tip.

![Fig. 8 Normal (a) and shear (b) stress distributions of interface of partial contact specimen.](image)

The transverse displacement of the ligament region $I_2 (1\sim13)$ shown in Fig. 9 (a) are quite the same as the non-crack specimen. However, the crack region $I_1$ is slightly different. The subregion $I_{11} (15\sim17)$ keep the same as the non-crack specimen the subregion $I_{12} (18\sim20)$ does not, however, as shown in Fig. 9(b). It’s easy to understand that the crack open at point 18 which is not the crack tip. It means that the normal stress of this crack will not cause the stress intensity in normal direction at the crack tip and therefore the crack will not propagate.

![Fig. 9 Transverse displacement of ligament (a) and crack (b) portions of partial contact specimen.](image)
The longitudinal displacements of the ligament region $I_2$ is small (0.00137 mm~0.00138 mm) shown in Fig. 10 (a). On the contrary the displacement of the crack region $I_1$ are much different. It can be seen from Fig. 10 (b) that the changing rate of the displacement is increasing near the crack tip. Therefore, the crack will be propagate in shear direction because of the shear stress concentration at the crack tip. $K_{II}$ is the key factor which dominates the crack propogation of the four-point bent specimen.

Fig. 10 Longitudinal displacements of ligament (a) and crack portions (b) of partial contact specimen.

4. Conclusions

A 2-D BEM formulation predicting mode II intensity factor of the four-point bent specimen has been presented. Two differert boundary conditions of the crack were offered to match the pratical case. Accuracy test has been performed by comparison of the BEM model with analytical solutions. The numerical results show the potential application of the BEM model to the effects of mode II stress intensity factor $K_{II}$ which dominates the crack propogation of four-point bent specimen with a crack.

5. References

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