ABSTRACT

In order to represent speech acts, in a multi-agent context, we choose a knowledge representation based on the modal logic of knowledge KT4 which is defined by Sato. Such a formalism allows us to reason about knowledge and represent knowledge about knowledge, the notions of truth value and of definite reference.

I INTRODUCTION

Speech act representation and the language planning require that the system can reason about intensional concepts like knowledge and belief. A problem resolver must understand the concept of knowledge and know for example what knowledge it needs to achieve specific goals. Our assumption is that a theory of language is part of a theory of action (Austin [4]).

Reasoning about knowledge encounters the problem of intensionality. One aspect of this problem is the indirect reference introduced by Frege ~ during the last century. Mc Carthy [15] presents this problem by giving the following example:

Let the two phrases: Pat knows Mike's telephone number (1) and Pat dialled Mike's telephone number (2)

The meaning of the proposition "Mike's telephone number" in (1) is the concept of the telephone number, whereas its meaning in (2) is the number itself.

Then if we have: "Mary's telephone number = Mike's telephone number",

we can deduce that:

"Pat dialled Mary's telephone number"

but we cannot deduce that:

"Pat knows Mary's telephone number"

because Pat may not have known the equality mentioned above.

Thus there are verbs like "to know", "to believe" and "to want" that create an "opaque" context. For Frege a sentence is a name, reference of a sentence in its truth value, the sense of a sentence is the proposition. In an oblique context, the reference becomes the proposition. For example the referent of the sentence p in the indirect context "A knows that p" is a proposition and no longer a truth value.

Mc Carthy [15] and Konolige [11] have adopted Frege's approach. They consider the concepts like objects of a first-order language. Thus one term will denote Mike's telephone number and another will denote the concept of Mike's telephone number. The problem of replacing equalities by equalities is then avoided because the concept of Mike's telephone number and the number itself are different entities.

Mc Carthy's distinction concept/object corresponds to Frege's sense/reference or to modern logicians' intension/extension.

Maida and Shapiro [13] adopt the same approach but use propositional semantic networks that are labelled graphs, and that only represent intensions and not extensions, that is to say individual concepts and propositions and not referents and truth values. We bear in mind that a semantic network is a graph whose nodes represent individuals and whose oriented arcs represent binary relations.

Cohen [5], being interested in speech act planning, proposes the formalism of partitioned semantic networks as data base to represent an agent's beliefs. A partitioned semantic network is a labelled graph whose nodes and arcs are distributed into spaces. Every node or space is identified by its own label. Hendrix [9] introduced it to represent the situations requiring the delimitation of information sub-sets. In this way Cohen succeeds in avoiding the problems raised by the data base approach. These problems are clearly identified by Moore [17,18]. For example to represent 'A does not believe P', Cohen asserts ~Believe (A,P) in a global data base, entirely separated from any agent's knowledge base. But as Appelt [2] notes, this solution raised problems when one needs to combine facts from a particular data base...
with global facts to prove a single assertion. For example, from the assertion:

\[ \neg \text{know} (\text{John},Q) \land \text{know} (\text{John},P \supset Q) \]

where \( P \supset Q \) is in John's data base and \( \neg \text{know} (\text{John},Q) \) is in the global data base, it should be possible to conclude \( \neg \text{know} (\text{John},P) \) but a good strategy must be found!

In a nutshell, in this first approach which we will call a syntactical one, an agent's beliefs are identified with formulas in a first-order language, and propositional attitudes are modelled as relations between an agent and an object in the object language, but Montague showed that modalities cannot consistently be treated as predicates applying to nouns of propositions.

The other approach no longer considers the intention as an object but as a function from possible worlds to entities. For instance the intention of a predicate \( P \) is the function which to each possible world \( W \) (or more generally a point of reference, see Scott [23]) associates the extension of \( P \) in \( W \).

This approach is the one that Moore [17,18] adopted. He gave a first-order axiomatization of Kripke's possible worlds semantics [12] for Hintikka's modal logic of knowledge [10].

The fundamental assumption that makes this translation possible, is that an attribution of any propositional attitude like "to know", "to believe", "to remember", "to strive" entails a division of the set of possible worlds into two classes: the possible worlds that go with the propositional attitude that is considered, and those that are incompatible with it. Thus "A knows that \( P \)" is equivalent to "\( A \) is true in every world compatible with what A knows".

We think that possible worlds language is complicated and unintuitive, since, rather than reasoning directly about facts that someone knows, we reason about the possible worlds compatible with what he knows. This translation also presents some problems for the planning. For instance to establish that A knows that P, we must make P true in every world which is compatible with A's knowledge. This set of worlds is a potentially infinite set.

The most important advantage of Moore's approach [17,18] is that it gives a smart axiomatization of the interaction between knowledge and action.

II PRESENTATION OF OUR APPROACH

Our approach is comprised in the general framework of the second approach, but instead of encoding Hintikka's modal logic of knowledge in a first-order language, we consider the logic of knowledge proposed by Mc Carthy, the decidability of which was proved by Sato [21] and we propose a prover of this logic, based on natural deduction.

We bear in mind that the idea of using the modal logic of knowledge in A.I. was proposed for the first time by Mc Carthy and Hayes [14].

A. Languages

A language \( L \) is a triple \((Pr,Sp,T)\) where:

- \( Pr \) is the set of propositional variables,
- \( Sp \) is the set of persons,
- \( T \) is the set of positive integers.

The language of classical propositional calculus is \( L = (Pr,\emptyset,\emptyset) \). So \( \emptyset \in Sp \) will also be denoted by \( 0 \) and will be called "FOOL".

B. Well Formed Formulas

The set of well formed formulas is defined to be the least set \( Wff \) such as:

\( \begin{align*}
(W_1) & \quad Pr \subseteq Wff \\
(W_2) & \quad a,b \in Wff \implies a \supset b \in Wff \\
(W_3) & \quad S \in Sp, t \in T, a \in Wff \implies (St)a \in Wff \\
\end{align*} \)

The symbol \( \supset \) denotes "implication".

\( (St)a \) means "\( S \) knows that \( a \) at time \( t \)"

\( (St)a \supset (St) \neg a \) means "\( a \) is possible for \( S \) at time \( t \)"

\( (St)a \land (St) \neg a \) means "\( S \) knows whether \( a \) at time \( t \)"
C. Hilbert-type System KT4

The axiom schemata for KT4 are:

A1. Axioms of ordinary propositional logic
A2. (St)a ⊃ a
A3. (Ot)a ⊃ (Ot)(St)a
A4. (St)(a ⊃ b) ⊃ ((Su)a ⊃ (Su)b), where t ≤ u
A5. (St)a ⊃ (St)(St)a
A6. If a is an axiom, then (St)a is an axiom.

Now, we give the meaning of axioms:

(A2) says that what is known is true, that is to say that it is impossible to have false knowledge. If P is false, we cannot say "John knows that P" but we can say "John believes that P". This axiom is the main difference between knowledge and belief.

This distinction is important for planning because when an agent achieves his goals, the beliefs on which he bases his actions must generally be true.

(A3) says that what FOOL knows at time t, FOOL knows at time t that anyone knows it at time t. FOOL's knowledge represents universal knowledge, that is to say all agents knowledge.

(A4) says that what is known will remain true and that every agent can apply modus ponens, that is, he knows all the logical consequences of his knowledge.

(A5) says that if someone knows something then he knows that he knows it. This axiom is often required to reason about plans composed of several steps. It will be referred to as the positive introspective axiom.

(A6) is the rule of inference.

D. Representation of the notion of truth value.

We give a great importance to the representation of the notion of truth value of a proposition, for example the utterance:

John knows whether he is taller than Bill (I)

can be considered as an assertion that mentions the truth value of the proposition P = John is taller than Bill, without taking a position as to whether the latter is true or false.

In our formalism (I) is represented by:

(John) P

This disjunctive solution is also adopted by Allen and Perrault [1]. Maida and Shapiro [13] represent this notion by a node because the truth value is a concept (an object of thought).

The representation of the notion of truth value is useful to plan questions: A speaker can ask a hearer whether a certain proposition is true, if the latter knows whether this proposition is true.

E. Representing definite descriptions in conversational systems:

Let us consider a dialogue between two participants: A speaker S and a hearer H. The language is then reduced to:

Sp = {O,H,S} and T = {I}

Let P stand for the proposition: "The description D in the context C is uniquely satisfied by E".

Clark and Marshall [5] give examples that show that for S to refer to H to some entity E using some description D in a context C, it is sufficient that P is a mutual knowledge; this condition is tantamount to (O)P is provable. Perrault and Cohen [20] show that this condition is too strong. They claim that an infinite number of conjuncts are necessary for successful reference:

(S) P & (S)(H) P & ... with only a finite number of false conjuncts.

Finally, Nadathur and Joshi [19] give the following expression as sufficient condition for using D to refer to E:

(S) B ⊃ (S)(H) P & ((S) B ⊃ (S)¬(O)P) where B is the conjunction of the set of sentences that form the core knowledge of S and ¬ is the inference symbol.

III SCHÜTTE - TYPE SYSTEM KT4'
cal proofs. He is the inventor of natural deduction (for classical and intuitionistic logics). Sato [21] defines Gentzen-type systems GT4 which is equivalent to KT4. We consider here, Schütte-type system KT4' [22] which is a generalization of S4 and equivalent to GT4 (and thus to KT4), in order to avoid the thinning rule of the system GT4 (which introduces a cumbersome combinatorial). Firstly, we are going to give some definitions to introduce KT4'.

A. Inductive definition of positive and negative parts of a formula F

Logical symbols are \( \land \) and \( \lor \).

a. \( F \) is a positive part of \( F \).

b. If \( \neg A \) is a positive part of \( F \), then \( A \) is a negative part of \( F \).

c. If \( \neg A \) is a negative part of \( F \), then \( A \) is a positive part of \( F \).

d. If \( A \lor B \) is a positive part of \( F \), then \( A \) and \( B \) are positive parts of \( F \).

Positive parts or negative parts which do not contain any other positive parts or negative parts are called minimal parts.

B. Semantic property

The truth of a positive part implies the truth of the formula which contains this positive part.

The falsehood of a negative part implies the truth of the formula which contains this negative part.

C. Notation

\( F[A^+] \) is a formula which contains \( A \) as a positive part.

\( F[A^-] \) is a formula which contains \( A \) as a negative part.

\( F[A^+,B^-] \) is a formula which contains \( A \) as a positive part and \( B \) as a negative part where \( A \) and \( B \) are disjoined (i.e., one is not a subformula of the other).

D. Inductive definition of \( F[..] \)

From a formula \( F[A] \), we build another formula or the empty formula \( F[..] \) by deleting \( A \):

a. If \( F[A] = A \), then \( F[..] = G[..] \).

b. If \( F[A] = G[A \lor B] \) or \( F[A] = G B \lor A \), then \( F[..] = G[B] \).

c. If \( F[A] = G[A \land B] \) or \( F[A] = G B \land A \), then \( F[..] = G[B] \).

E. Axiom

An axiom is any formula of the form \( F[P^+,P^-] \) where \( P \) is a propositional variable.

F. Inference rules

(R1) \( \frac{F[(A \land B)] \land \neg A}{\frac{F(A \land B)}{F(A \land B) \lor \neg A}} \)

(R2) \( \frac{F[(St)A] \lor \neg A}{\frac{F[(St)A]}{F[(St)A]}} \)

(R3) \( \frac{(Su)A \lor \ldots \lor (Su)Am \lor \neg (Ou)B \lor \ldots \lor \neg (Ou)Bn \lor C}{F[..]} \)

(R4) \( \frac{F_1[C^-], F_2[C^-]}{F[..] : V F_1[..] \lor F_2[..]} \)

(G. Cut-elimination theorem (Hauptsatz)

Any KT4' proof-figure can be transformed into a KT4' proof-figure with the same conclusion and without any cut as a rule of inference (hence, the rule (R4) is superfluous. The proof of this theorem is an extension of Schütte's one for S4'. This theorem allows derivations "without detour".

IV DECISION PROCEDURE

A logical axiom is a formula of the form \( F[P^+,P^-] \). A proof is a single-rooted tree of formulas all of whose leaves are logical axioms. It is grown upwards from the root, the rules (R1), (R2) and (R3) must be applied in a reverse sense. These reversal rules will be used as "production rules". The meaning of each production expressed in terms of the programming language PROLOG is an implication.

It can be shown [24] that the following strategy is a complete proof procedure:

The formula to prove is at the star-
• Queue the minimal parts in the given formula;
• Grow the tree by using the rule \((R1)\) in priority, followed by the rule \((R2)\), then by the rule \((R3)\).

The choice of the rule to apply can be done intelligently. In general, the choice of \((R1)\) then \((R2)\) increases the likelihood to find a proof because these (reversal) rules give more complex formulas. In the case where \((R3)\) does not lead to a loss of formulas, it is more efficient to choose it at first. The following example is given to illustrate this strategy:

**Example**

Take \((A4)\) as an example and let \(F_0\) denotes its equivalent version in our language (\(F_0\) is at the start node):

\[ F_0 = \neg(St)(\neg a \lor b) \lor (St)a \lor (Su)b \]

where \(t < u\)

\(P^+\) denotes positive parts and \(P^-\) denotes negative parts

\[ P^+ = \{\neg(St)(\neg a \lor b), \neg(St)a, (Su)b\}; \]

\[ P^- = \{(St)(\neg a \lor b), (Su)a\}; \]

By \((R3)\) we have (no losses of formulas):

\[ F_1 = \neg(St)(\neg a \lor b) \lor (St)a \lor b \]

\[ P^+_1 = \{\neg(St)(\neg a \lor b), (Su)a\} \]

\[ P^-_1 = \{(St)(\neg a \lor b), (Su)b\} \]

By \((R2)\) we have:

\[ F_2 = F_1 \lor (St)b \]

\[ P^+_2 = P^+_1 \lor \{\neg(\neg a \lor b)\}; \]

\[ P^-_2 = P^-_1 \lor \{\neg a \lor b\}; \]

By \((R1)\) we have:

\[ F_3 = F_2 \lor \neg a \]

\[ P^+_3 = P^+_2 \lor \{\neg a, a\}; \]

\[ P^-_3 = P^-_2 \lor \{a\}; \]

and \(\neg b\)

\[ F_4 = F_3 \lor \neg b \]

\[ P^+_4 = P^+_3 \lor \{\neg b\}; \]

\[ P^-_4 = P^-_3 \lor \{b\}; \]

\(F_5\) is a logical axiom because \(P^+_5 \cap P^-_5 = \{a\}\)

Finally, we have to apply \((R2)\) to the last but one node:

\[ F_6 = F_5 \lor (\neg a) \]

\[ P^+_6 = P^+_5 \lor \{\neg a\}; \]

\[ P^-_6 = P^-_5 \lor \{a\}; \]

\(F_6\) is a logical axiom because \(P^+_6 \cap P^-_6 = \{b\}\)

The generated derivation tree is then:
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