Multi-photon absorption in the channeling of electrons in an external field

VYaralov
Alikhanyan National Laboratory (Yerevan Physics Institute), Brothers Alikhanyan 2, Yerevan 0036, Armenia
E-mail: yaralov@mail.yerphi.am

Abstract. Following the methods developed for atom ionization by alternating electric field the probability of multi-photon absorption of photons of the strong external laser field by channeled electron (extraction of electron from the channel) have been calculated for different strengths of the monochromatic external field. The emission spectra of 54 MeV electron channeled in diamond crystal planes (110) are shown for different values of the resonant laser field of a frequency close to the transition frequency in the channel taking into account multi-photon absorption. It is shown that the multi-photon phenomena give some contribution to the total level width.

1. Introduction
The investigation of the interaction of a channeled electron and laser radiation is a natural development works associated with channeling [1-7]. In [8-11] the results of the theoretical predictions of splitting the spectrum of channeling radiation in a strong external field were presented. In quantum optics the effect of resonance external field on the spectrum of two-level atoms was sufficiently well studied both theoretically and experimentally. The spectrum is split into 3 peaks. The phenomenon is called dynamic Stark effect or "triplet Mollow" [12-14]. Nowadays this effect is being investigated by using ultra-short laser pulses with a field strength that exceeds the atomic [15-16]. Basically all the results were obtained in the semi-classical theory - a two-level quantum electron in a classical external field. The electron is described by the quantum-statistical method using the density matrix. For planar channeling the theoretical calculation is reduced to one-dimensional problem in the transverse direction with respect to the electron beam. In the theory of channeling were taken into account the effects leading to the departure of an electron from its level (de-channeling). That is: the scattering of electrons by atomic deviations from the ideal crystal structure; the scattering by the atom electrons; interaction with external field or vacuum field. The strong external field is another reason that reduces the lifetime of the considered two-level system. The number of photons is proportional to the square of the field strength. If the field is sufficiently strong, electron simultaneously interacts with many photons. The electron at the same time absorbs few resonant photons and escape from a potential well. The total energy of the photons must to exceed the extraction potential.

2. Multi-photon phenomena in the channeling
Photo-ionization of atoms was an object of investigation for a long time using different methods [17-20]. The theory developed in [17] combines multi-photon absorption with the tunnel effect. There is a certain probability of transition of the electron through the barrier. The given theory shows that the tunnel effect and multi-photon absorption are phenomena of the same nature. The input parameter is \( \gamma = \omega \sqrt{2 \text{m} \text{F}} / \text{eF} \) where \( \omega \) \ F – the frequency and strength of the external field, m, e – electron mass and charge, I – ionization potential (energy of the level). In the case of \( \gamma \ll 1 \) the general formula describes tunneling and for \( \gamma \gg 1 \) it is multi-photon absorption.
In the final state the interaction of the electron with the positively charged atomic ion was not taken into account. However this approach can be applied to calculate the probability of removal of an electron from a negative ion as well as to our case of channeling. In the final state only the external field affects the electron. There is no interaction between the ejected electron and averaged crystal potential. In [18] zero-radius-potential method is applied to calculate multi-photon absorption. For the short-range potential (smaller than the wavelengths of particles) the specific profile of potential is not essential and it is replaced by a one-dimensional delta function. Again, in the final state there is no interaction between the ejected electron and the positive residue. This one-dimensional model would be perfect for a plane-channeled electron. However, in the given model, the energy level of the electron in the channel (extraction potential) is significantly different from the real one. For a satisfactory agreement it is necessary to approximate the potential with the help of several delta functions. Therefore the calculations were made using formulas of work [17]. The probability of the process per unit time is as follows

\[
W = \omega \left( \frac{\hat{I}}{\hbar \omega} \right)^{1.5} \left( \frac{\gamma}{(1 + \gamma^2)^{0.5}} \right)^{2.5} S \left( \frac{\gamma}{\hbar \omega} \right) \exp \left\{ - \frac{2 \hat{I}}{\hbar \omega} \left[ \text{arsh} \, \gamma - \gamma \left( 1 + \gamma^2 \right)^{0.5} \right] \right\}
\]

\[
S \left( \frac{\gamma}{\hbar \omega} \right) = \sum_{n=0}^{\infty} \exp \left\{ -2 A_n \left( \text{arsh} \, \gamma - \frac{\gamma}{\sqrt{1 + \gamma^2}} \right) \right\} \Phi \left( \frac{2 \gamma}{\sqrt{1 + \gamma^2}} A_n \right)^{0.5}
\]

\[
\hat{I} = I \left( 1 + \frac{1}{2 \gamma^2} \right) \quad A_n = \text{int} \left( \frac{\hat{I}}{\hbar \omega} + 1 \right) - \frac{\hat{I}}{\hbar \omega} + n \quad \Phi(z) = \int_{0}^{z} \exp(y^2 - z^2) \, dy
\]

where \( I \) – extraction potential, i.e. the level energy taken with the opposite sign, \( \hat{I} \) – effective potential. Their difference is equal to the vibration energy of the electron in an external field. "int" means the integer part of the fraction.

3. Calculation results

All necessary parameters were calculated by the usual standard techniques developed in the theory of channeling. The calculated average potential of the plane-channeled electron is well approximated by an inverted parabola \( V(x) = V_0 \frac{x}{L} \left( 2 - \frac{x}{L} \right) \), where \( V_0 \) is well depth, \( L \) is half of the inter-planar spacing. The crystal field strength is \( \frac{2V_0}{\hbar^2 c^2} \left( 1 - \frac{x}{L} \right) \), \( e \) is the electron charge. Let us consider 54 MeV electron channelled in the (110) plane of the diamond crystal. In this case in \( h = c = 1 \) system of units \( V_0 = 23.6 \) (eV), \( L = 3.2*10^{-4} \) (eV)^{-1}, \( e = 0.0854 \). We are considering the radiation at the transition between the levels 4-3. The numbering of levels starts with the number 1. Transversal level energies \( E_\perp \) are respectively: -7.325, -3.908 (eV). The maximum value of the crystal field strength is \( F_{\text{max}} = 1.73*10^6 \) (eV)^2 = 7.5*10^8 (V/cm). The crystal field strength corresponding to level of 3.908 (eV) is equal to 6.85*10^9 (V/cm). These values greatly exceed the strength of the external field. Therefore we do not consider changes in the structure of the transverse electron levels. The transition frequency \( \Delta E_\perp \) between selected levels is equal to 3.417 (eV). The frequency and wavelength of the external control field must be equal respectively 8.26*10^{14} (1/s), 363 (nm). The required parameters of the laser field, however others. Let the electron moves at a speed of \( v = c \) against the laser wave \( E \cos[(n\omega/c)x + o \tau] \), where \( n \) is crystal refractive index. Suppose that at time \( t \) the electron is at the point \( x = ct \). The wave at the same point and at the same time takes the form \( E \cos[(n\omega/c)ct + o \tau] \). Thus the laser field acts on the electron with an effective frequency of \( \omega(1 + n) \). If the laser wave is directed at an angle of \( \theta \) to the direction of the electron, the effective frequency is equal of \( \omega(1 + n/cos \theta) \). Changing the angle \( \theta \) one can change and select the desired effective frequency of the control field in a wide range. Bloch-momentum averaged parameters considered a two-level system are given in table 1 in \( h = c = 1 \) system of units.
Table 1. The parameters of two-level quantum system.

| $\Delta E_L$ (eV) | $D$ (1/eV) | $G_{\text{scat}}$ (eV) | $G_{\text{spon}}$ (eV) |
|------------------|-------------|------------------------|-----------------------|
| 3.417            | 0.75*10^{-5}| 0.255                  | 3*10^{-8}             |

$D$ is dipole momentum of the transition. The probability of spontaneous transition $G_{\text{spon}}$ per unit time is much lesser than the probability of scattering $G_{\text{scat}}$. Therefore further spontaneous emission is not taken into account.

Let us now consider effective field for the selected pair of levels. Relevant parameters are given in table 2.

Table 2. The parameters of the interaction of channeled electron with effective laser field.

| External field strength (V/cm) | Intensity (W/cm²) | Rabi frequency (eV) | Width due multi-photon absorption (eV) | Multi-photon absorption frequency (1/s) |
|-------------------------------|-------------------|---------------------|---------------------------------------|---------------------------------------|
| 0.5*10^{8}                   | 3.3*10^{12}       | 0.17                | 0.258*10^{15}                         | 0.023                                 |
| 10^{8}                       | 1.3*10^{13}       | 0.35                | 0.532*10^{15}                         | 0.063                                 |
| 2*10^{8}                     | 5.3*10^{13}       | 0.69                | 1.048*10^{15}                         | 0.114                                 |
| 3*10^{8}                     | 1.3*10^{14}       | 1.04                | 1.58*10^{15}                          | 0.116                                 |

Two-level channeled electron under the influence of the resonant external field oscillates between the levels with the Rabi frequency which is of the order of magnitude greater than the frequency of the multi-photon absorption. Keldysh parameter $\gamma$ for various values of the field strength is respectively: 1.1; 0.55; 0.28; 0.18. Thus these external fields correspond to an intermediate region between a purely tunnel effect and a multi-photon absorption.

The spectra calculated with multi-photon absorption are shown in figure 1.

Figure 1. The spectral distributions of the channeling radiation in external intensive field, taking into account multi-photon absorption for various strengths of the external resonance field $F$.  

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Discussion and conclusion
The field parameters listed in table 2 correspond to the selected two-level system (table 1). As noted above, the laser frequency should be lower than the transition frequency between the transverse channeled electron levels to provide these conditions. Due to the crystal polarization, strength of the laser field, on the contrary, should be higher than shown in table 2. Furthermore, only the force component perpendicular to the direction of electron counts. At the direction of the laser field at an angle it also leads to the need to increase the field strength. One can get the required field in the crystal with help such adaptations. Made calculations show that multi-photon absorption is making some contribution to the broadening of the level (less than the scattering) and is not a fundamental limitation on the possibility of observing the splitting of the emission spectra in the channeling of electrons in a crystal in a strong field. It would be interesting to test experimentally these predictions, but the main objective is to enhance radiation. Although the increase in the intensity of the control field leads to reducing of the central peak, its distribution is strictly centered on the frequency of the control wave. If bunch of electrons is a small area compared to the wavelength of the control field, the electrons will radiate coherently.

Finally, it is appropriate to refer to the physical meaning of channeling radiation. Spontaneous radiation was omitted in the calculation. Allowance for spontaneous emission does not change the overall yield of radiation because of its smallness. The trajectory of the classical electron completely defines the radiation, regardless of the reasons for changing his movement. Similarly, the time-dependent density matrix of the channeling electron completely determines its radiation regardless of the reasons causing the change in its state. It turns out that the scattering plays a positive role in channeling radiation. This is the reason for the observed radiation. Multi-photon absorption destroying two-photon system plays only a negative role.

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