Flavour Physics and CP Violation in the SM

Andrzej J. Buras

Technische Universität München
Physik Department
D-85748 Garching, Germany

Abstract
We review the main aspects of Flavour Physics and CP Violation in the Standard Model. After presenting a grand view of the field including a Master Formula for weak decays we discuss i) Standard analysis of the unitarity triangle, ii) The ratio $\varepsilon'/\varepsilon$, iii) Rare decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$, iv) CP violation in B decays and v) Models with Minimal Flavour Violation. Our review ends with 20 questions that hopefully will be answered in the coming years.

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1 Introduction

The field of Flavour Physics and CP Violation constitutes an important part of the Standard Model (SM). It will certainly be one of the hot topics in particle physics during this decade. In this introductory lecture I will attempt to describe this field in general terms paying special attention to the theoretical framework and to a few selected topics which in my opinion are very important. Instead of an outlook I will provide a list of twenty questions for KAON 2001 and beyond, that will allow me to address other important topics. In view of considerable space limitations it is impossible to refer properly to the relevant literature. As a compensation, references to roughly 800 papers can be found in my Erice lectures [1].

2 Grand View

There are four basic properties in the SM that govern flavour physics and CP violation in this model. These are

- Breakdown of Parity: charged current interactions are only between left-handed quarks and between left-handed leptons.

- Quark Mixing: the weak eigenstates \((d', s', b')\) of quarks differ from the corresponding mass eigenstates \(d, s, b\):

\[
\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \equiv \hat{V}_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}.
\]

(2.1)

The unitary transformation connecting these states is the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

- GIM Mechanism: The unitarity of the CKM-matrix assures the absence of flavour changing neutral current transitions at the tree level. These processes can consequently appear first at the one-loop level and are very sensitive to short distance flavour dynamics.

- Asymptotic Freedom in QCD: Whereas strong interaction effects at short distance scales \(\mu_{SD} = \mathcal{O}(M_W, M_Z, m_t)\) can be treated by perturbative methods, at long distance scales, \(\mu_{LD} = \mathcal{O}(1–2 \text{ GeV})\), the use of non-perturbative methods becomes mandatory. The latter fact brings considerable uncertainties in the theoretical predictions. The appearance of two vastly different scales implies large \(\log(\mu_{SD}/\mu_{LD})\) multiplying \(\alpha_s\) that fortunately can be summed up to all orders of perturbation theory in \(\alpha_s\) by means of renormalization group methods.
According to the Kobayashi-Maskawa picture of CP violation, this phenomenon arises from a single complex phase $\delta_{KM}$ in the $W^\pm$–interactions of quarks. The CKM matrix can be parametrized by three mixing angles and $\delta_{KM}$ as described by the standard parametrization that is recommended by the Particle Data Group. While this standard parametrization should certainly be recommended for calculations, in a talk like this one, the Wolfenstein parametrization is certainly more useful:

$$
\hat{V}_{CKM} = \begin{pmatrix}
1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\varrho - i\eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
A\lambda^3(1 - \varrho - i\eta) & -A\lambda^2 & 1 \\
\end{pmatrix} + \mathcal{O}(\lambda^4).
$$

Including the most important $\mathcal{O}(\lambda^4)$ and higher order terms one finds then that to an excellent accuracy

$$
V_{us} = \lambda, \quad V_{cb} = A\lambda^2, \quad V_{ts} = -A\lambda^2, 
$$

$$
V_{ub} = A\lambda^3(\varrho - i\eta), \quad V_{td} = A\lambda^3(1 - \bar{\varrho} - i\bar{\eta})
$$

where $\lambda, A, \varrho, \eta$ are the Wolfenstein parameters and

$$
\bar{\varrho} = \varrho(1 - \frac{\lambda^2}{2}), \quad \bar{\eta} = \eta(1 - \frac{\lambda^2}{2})
$$

are the parameters introduced in \[2\]. The latter parameters describe the apex of the unitarity triangle (UT) shown in fig. 1 with the length CA, BA, and CB equal respectively to

$$
R_b = (1 - \frac{\lambda^2}{2}) \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right|, \quad R_t = \frac{1}{\lambda} \left| \frac{V_{td}}{V_{cb}} \right|, \quad 1.
$$

The angles $\beta$ and $\gamma$ of the UT determine the complex phases of the CKM-elements $V_{td}$ and $V_{ub}$:

$$
V_{td} = |V_{td}|e^{-i\beta}, \quad V_{ub} = |V_{ub}|e^{-i\gamma}.
$$

The apex $(\bar{\varrho}, \bar{\eta})$ of the UT can be efficiently hunted by means of rare and CP violating transitions as shown in fig. 2. Moreover the angles of this triangle can be measured in CP asymmetries in B-decays and using other strategies. This picture could describe in principle the reality in the year 2011, my retirement year, if the SM is the whole story. On the other hand in the presence of significant new physics contributions, the use of the SM expressions for rare and CP violating transitions in question, combined with future precise measurements, may result in curves which do not cross each other at a single point in the $(\bar{\varrho}, \bar{\eta})$ plane. This would be truly exciting and most of us hope that this will turn out to be the case. In order to be able to draw such thin curves as in fig. 2, not only experiments but also the theory has to be under control. Let me then briefly discuss the theoretical framework for weak decays.

3
3 Master Formula for Weak Decays

The present framework for weak decays is based on the operator product expansion (OPE) that allows to separate short and long distance contributions to weak amplitudes and on the renormalization group (RG) methods that allow to sum large logarithms log $\mu_{SD}/\mu_{LD}$ to all orders in perturbation theory. The full exposition of these methods can be found in [4, 5]. Here I just want to propose a master formula for weak decay amplitudes that follows from OPE and RG and goes beyond the SM. It reads:

$$ A(\text{Decay}) = \sum_{i} B_{i}^{\text{QCD}} C_{\text{CKM}}^{i} \left[ F_{i}^{\text{SM}} + F_{i}^{\text{New}} \right] + \sum_{k} B_{k}^{\text{New}} \left[ \eta_{QCD}^{k} V_{\text{New}}^{k} G_{\text{New}}^{k} \right]. \quad (3.1) $$
The non-perturbative parameters $B_i$ represent the matrix elements of local operators present in the SM. For instance in the case of $K^0 - \bar{K}^0$ mixing, the matrix element of the operator $\bar{s}\gamma_\mu(1-\gamma_5)d \otimes \bar{s}\gamma_\mu(1-\gamma_5)d$ is represented by the parameter $\hat{B}_K$. There are other non-perturbative parameters in the SM that represent matrix elements of operators $Q_i$ with different colour and Dirac structures. The objects $\eta_{QCD}^i$ are the QCD factors resulting from RG-analysis of the corresponding operators and $F_{SM}^i$ stand for the so-called Inami-Lim functions [3] that result from the calculations of various box and penguin diagrams. They depend on the top-quark mass. $V_{CKM}^i$ are the CKM-factors we want to determine.

New physics can contribute to our master formula in two ways. It can modify the importance of a given operator, present already in the SM, through the new short distance functions $F_{New}^i$ that depend on the new parameters in the extensions of the SM like the masses of charginos, squarks, charged Higgs particles and $\tan \beta = v_2/v_1$ in the MSSM. These new particles enter the new box and penguin diagrams. In more complicated extensions of the SM new operators (Dirac structures) that are either absent or very strongly suppressed in the SM, can become important. Their contributions are described by the second sum in (3.1) with $B_{New}^k$, $[\eta_{QCD}^k]_{New}$, $V_{New}^k$, $G_{New}^k$ being analogs of the corresponding objects in the first sum of the master formula. The $V_{New}^k$ show explicitly that the second sum describes generally new sources of flavour and CP violation beyond the CKM matrix. This sum may, however, also include contributions governed by the CKM matrix that are strongly suppressed in the SM but become important in some extensions of the SM. A typical example is the enhancement of the operators with Dirac structures $(V - A) \otimes (V + A)$, $(S - P) \otimes (S \pm P)$ and $\sigma_{\mu\nu}(S - P) \otimes \sigma^{\mu\nu}(S - P)$ contributing to $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mixings in the MSSM with large $\tan \beta$. The most recent compilation of references to existing NLO calculations of $\eta_{QCD}^i$ and $[\eta_{QCD}^k]_{New}$ can be found in [4].

Clearly the new functions $F_{New}^i$ and $G_{New}^k$ as well as the factors $V_{New}^k$ may depend on new CP violating phases complicating considerably phenomenological analysis. We will see this in Masiero’s lecture. In the present talk, that is dominantly devoted to the SM, I will only consider the simplest class of the extensions of the SM in which the second sum in (3.1) is absent (no new operators) and flavour changing transitions are governed by the CKM matrix. In particular there are no new complex phases beyond the CKM phase. I will call this scenario “Minimal Flavour Violation” (MFV) [7,8] being aware of the fact that for some authors MFV means a more general framework in which also new operators can give significant contributions. In the MFV models, as defined in [7,8], our master formula simplifies to

$$A(\text{Decay}) = \sum_i B_i \eta_{QCD}^i V_{CKM}^i [F_{SM}^i + F_{New}^i]$$ (3.2)

with $F_{SM}^i$ and $F_{New}^i$ being real.
4 Five Topics

4.1 Standard Analysis of the Unitarity Triangle

This analysis uses $\lambda = |V_{us}| = 0.222 \pm 0.002$,

$$|V_{cb}| = 0.041 \pm 0.002, \quad \frac{|V_{ub}|}{|V_{cb}|} = 0.085 \pm 0.018$$  \hspace{1cm} (4.1)

and the following three constraints:

- $\varepsilon_K$–Hyperbola (Indirect CP Violation in $K_L \rightarrow \pi\pi$):
  
  $$\bar{\eta} \left[(1 - \bar{\rho}) A^2 \eta_{QCD}^T F_{tt} + P_c(\varepsilon) \right] A^2 \hat{B}_K = 0.204,$$  \hspace{1cm} (4.2)

  where $\eta_{QCD}^T = 0.57 \pm 0.01$, $P_c(\varepsilon) = 0.30 \pm 0.05$ represents charm contribution and $F_{tt} = 2.38 \pm 0.11$ is the Inami-Lim ($t,t$) box diagram function, denoted often by $S_0(x_t)$.

- $B_d^0 - \bar{B}_d^0$–Mixing Constraint:
  
  $$R_t = 0.94 \sqrt{\frac{\Delta M_d}{0.487/\text{ps}}} \sqrt{\frac{15.0/\text{ps}}{\Delta M_s}} \left[ \frac{\xi}{1.15} \right], \quad \xi = \frac{\sqrt{B_s F_{B_s}}}{\sqrt{B_d F_{B_d}}}$$  \hspace{1cm} (4.3)

  where $A = 0.83 \pm 0.04$, $\Delta M_d = (0.487 \pm 0.009)/\text{ps}$ and $\eta_{QCD}^T = 0.55 \pm 0.01$.

- $B_s^0 - \bar{B}_s^0$–Mixing Constraint ($\Delta M_d/\Delta M_s$):
  
  $$R_t = 0.94 \sqrt{\frac{\Delta M_d}{0.487/\text{ps}}} \sqrt{\frac{15.0/\text{ps}}{\Delta M_s}} \left[ \frac{\xi}{1.15} \right], \quad \xi = \frac{\sqrt{B_s F_{B_s}}}{\sqrt{B_d F_{B_d}}}$$  \hspace{1cm} (4.4)

  where $\Delta M_s > 15.0/\text{ps}$ from LEP experiments.

The main uncertainties in this analysis originate in the theoretical uncertainties in the parameters $\hat{B}_K$ and $\sqrt{B_d F_{B_d}}$ and to a lesser extent in $\xi$:

$$\hat{B}_K = 0.85 \pm 0.15, \quad \sqrt{B_d F_{B_d}} = (230 \pm 40) \text{ MeV}, \quad \xi = 1.15 \pm 0.06.$$  \hspace{1cm} (4.5)

Also the uncertainties in $\hat{B}_K$, in particular in $|V_{ub}/V_{cb}|$, are substantial. Reviews of lattice results for the parameters in question can be found in [4].

One of the important issues is the error analysis of these formulae. In the literature five different methods are used: Gaussian approach [10], Bayesian approach [11], frequentist approach [12], 95% C.L. scan method [13] and the simple (naive) scanning within one standard deviation as used by myself. Interestingly, the last method gives ranges for the output quantities that
are similar to the 95% C.L. ranges obtained by the remaining methods. To this end the same input parameters have to be used and the implementation of the lower bound on $\Delta M_s$ has to be done in the same manner. Mele discusses these issues in his contribution. On my part I show in fig. 3 the result of my own analysis that uses naive scanning. The allowed region for $(\bar{\rho}, \bar{\eta})$ is the shaded area on the right hand side of the circle representing the lower bound for $\Delta M_s$, that is $\Delta M_s > 15/\text{ps}$. The hyperbolas in fig. 3 give the constraint from $\varepsilon$ and the two circles centered at $(0,0)$ the constraint from $|V_{ub}/V_{cb}|$. The circle on the right comes from $B_d^0 - \bar{B}_d^0$ mixing and excludes the region to its right. We observe that the region $\bar{\rho} < 0$ is practically excluded by the lower bound on $\Delta M_s$. It is clear from this figure that $\Delta M_s$ is a very important ingredient in this analysis and that the measurement of $\Delta M_s$ giving also lower bound on $R_t$ will have a large impact on the plot in fig. 3.

Table 1: Output of the Standard Analysis. $\lambda_t = V_{ts}^* V_{td}$.

| Quantity | Scanning | Bayesian I | Bayesian II |
|----------|----------|------------|-------------|
| $\bar{\rho}$ | 0.07 – 0.34 | 0.14 – 0.30 | 0.13 – 0.34 |
| $\bar{\eta}$ | 0.22 – 0.45 | 0.24 – 0.40 | 0.22 – 0.46 |
| $\sin(2\beta)$ | 0.50 – 0.84 | 0.56 – 0.82 | 0.52 – 0.92 |
| $\sin(2\alpha)$ | $-0.87 - 0.36$ | $-0.83 - 0.04$ | $-0.85 - 0.14$ |
| $\gamma$ | 37.7° – 75.7° | 42.8° – 67.4° | 41.8° – 67.6° |
| $\text{Im} \lambda_t / 10^{-4}$ | 0.94 – 1.60 | 0.93 – 1.43 | 0.91 – 1.55 |
| $| V_{td} | / 10^{-3}$ | 6.7 – 9.3 | 7.0 – 8.6 | 6.8 – 8.7 |

The ranges for various quantities found using the scanning method are compared in table 1 with the 95% C.L. ranges from Bayesian I of Ciuchini et al [11] that uses $|V_{cb}| = 0.0410 \pm 0.0016$ and $|V_{ub}/V_{cb}| = 0.086 \pm 0.009$ and Bayesian II with $|V_{cb}|$ and $|V_{ub}/V_{cb}|$ as in [4, 11] that are used in my analysis. I thank Stocchi for providing the latter numbers. My ranges are substantially larger than Bayesian I but only slightly larger than Bayesian II. This is partly related to a different treatment of the bound on $\Delta M_s$ done by Ciuchini et al and myself. My ranges are very close to the ones obtained using the frequentist approach [12].

One of the highlights of this year were the improved measurements of $\sin 2\beta$ by means of the time-dependent CP asymmetry in $B_d^0 (\bar{B}_d^0) \rightarrow \psi K_S$ decays

$$a_{\psi K_S}(t) \equiv -a_{\psi K_S} \sin(\Delta M_s t) = - \sin 2\beta \sin(\Delta M_s t)$$ (4.6)
with the last relation valid in those MFV models in which as in the SM $F_{tt} > 0$ \cite{14}. The most recent measurements of $a_{\psi K_S}$ from the BaBar and Belle Collaborations read

\[
(sin 2\beta)_{\psi K_S} = \begin{cases} 
0.59 \pm 0.14 \pm 0.05 & \text{(BaBar) \ [15]} \\
0.99 \pm 0.14 \pm 0.06 & \text{(Belle) \ [16]}
\end{cases}
\]

and establish confidently CP violation in the B system! A milestone in the field of CP violation. Combining these results with earlier measurements by CDF ($0.79^{+0.41}_{-0.44}$) and ALEPH ($0.84^{+0.82}_{-1.04} \pm 0.16$) gives the grand average

\[
a_{\psi K_S} = 0.79 \pm 0.10 .
\]

In view of the fact that the BaBar and Belle results are not fully consistent with each other, the averaging of these results and the grand average given above could be questioned. Probably a better description of the present situation is $a_{\psi K_S} = 0.80 \pm 0.20$.

In any case, these first direct measurements of the angle $\beta$ are in a good agreement (see fig. 3) with the results of the standard analyses of the unitarity triangle within the SM, even if the Belle result appears a bit too high. Clearly in view of a considerable difference between BaBar and Belle results and still sizable uncertainty in the error estimates of $(sin 2\beta)_{SM}$, there is a room for new physics contributions but the agreement of the prediction for $sin 2\beta$ from the fits of the unitarity triangle with the measured value of $a_{\psi K_S}$ is a strong indication that the CKM matrix could turn out to be the dominant source of CP violation in flavour violating decays.

In order to be sure whether this is indeed the case other theoretically clean quantities have to be measured. In particular the angle $\gamma$ that is more sensitive to new physics contributions
Figure 4: $\gamma$ as a function of $\Delta M_s/\Delta M_d$ for $\sin 2\beta = 0.6$ and different $R_{sd}$ [17, 18].

than $\beta$. In this context the measurement of the ratio $\Delta M_s/\Delta M_d$ will play an important role as for a fixed value of $\sin 2\beta$, the extracted value for $\gamma$ is a sensitive function of $\Delta M_s/\Delta M_d$ as shown in fig. [4]. The solid line, labeled by $R_{sd} = 1.0$ in the right plot, represents MFV models. The remaining lines, obtained in a general analysis in [17], represent generalized MFV models in which also significant contributions of new operators are possible. See the discussion below (3.1). In these models the expression for $R_t$ in (4.4) receives an additional factor $\sqrt{R_{sd}}$. For $R_{sd} > 1.2$, the angle $\gamma > 90^\circ$ is possible provided $\Delta M_s/\Delta M_d$ is not too large.

At this point I would like to stress the importance of the precise measurements of $a_{\psi K_S}$ and $\Delta M_s/\Delta M_d$ that should be available within the coming years. These two measurements taken together allow the determination of $\bar{\rho}$ and $\bar{\eta}$ through

$$\bar{\rho} \approx 1 - R_t \left[ 1 - \frac{a_{\psi K_S}^2}{8} \right], \quad \bar{\eta} \approx R_t \frac{a_{\psi K_S}}{2} \left[ 1 + \frac{a_{\psi K_S}^2}{8} \right]$$

(4.9)

with $R_t$ given by (4.4). Exact expressions can be found in [8, 14, 17]. The only theoretical uncertainty in these formulae resides in $\xi$ that should be known from lattice calculations within a few percent in the next years. There is another virtue of this particular determination of $\bar{\rho}$
and $\bar{\eta}$ that we will discuss in the context of the last topic on our list.

What about the angle $\alpha$? For a given $\sin 2\beta$ satisfying the CKM unitarity bound

$$\sin 2\beta \leq 2R_b \sqrt{1 - R_t^2}$$

there are two solutions for $\alpha$ with $\alpha < 90^\circ$ and $\alpha > 90^\circ$. However, if $\sin 2\beta$ saturates this bound, only $\alpha = 90^\circ$ is possible. An example of a corresponding triangle is shown in fig. 3. Such a possibility is hinted by a large value of $\sin 2\beta$ from Belle and has been advocated by Fritzsch and Xing for many years. For $\alpha = 90^\circ$ we simply have $(R_t < 1)$

$$\bar{\eta} = 1 - R_t^2, \quad \bar{\eta} = R_t \sqrt{1 - R_t^2}$$

with $R_t$ given by (4.3) or (4.4). Equivalently

$$\sin \beta = R_b, \quad \sin \gamma = R_t, \quad R_b = \sqrt{1 - R_t^2}.$$ (4.12)

Simultaneously the CP asymmetry $a_{\pi^+\pi^-}$ vanishes provided penguin pollution (see topic 4) can be neglected. Present BaBar data on $a_{\pi^+\pi^-}$ are consistent with $\sin 2\alpha = 0$.

### 4.2 The Ratio $\varepsilon'/\varepsilon$

The ratio $\varepsilon'/\varepsilon$ measures the relative size of the direct ($\varepsilon'$) and indirect ($\varepsilon$) CP violation in $K_L \to \pi\pi$ decays. One of the highlights of this year are the precise measurements of this ratio by NA48 and KTeV collaborations:

$$\text{Re}(\varepsilon'/\varepsilon) = \begin{cases} (15.3 \pm 2.6) \cdot 10^{-4} & \text{(NA48) [20]} \\ (20.7 \pm 2.8) \cdot 10^{-4} & \text{(KTeV) [21]} \end{cases}.$$ (4.13)

Combining these results with earlier measurements by NA31 collaboration at CERN ($(23.0 \pm 6.5) \cdot 10^{-4}$) and by the E731 experiment at Fermilab ($(7.4 \pm 5.9) \cdot 10^{-4}$) gives the grand average

$$\text{Re}(\varepsilon'/\varepsilon) = (17.2 \pm 1.8) \cdot 10^{-4}.$$ (4.14)

This is another milestone in CP violation.

On the theoretical side, the short distance contributions to $\varepsilon'/\varepsilon$ are fully under control but the presence of considerable long distance hadronic uncertainties precludes a precise value of $\varepsilon'/\varepsilon$ in the SM and its extensions at present. Consequently while theorists were able to predict the sign and the order of magnitude of $\varepsilon'/\varepsilon$, the range

$$(\varepsilon'/\varepsilon)_{\text{th}} = (5 \text{ to } 30) \cdot 10^{-4}$$ (4.15)

shows that the present status of $(\varepsilon'/\varepsilon)_{\text{th}}$ cannot match the experimental one.
It should be emphasized that the short distance contributions to $\varepsilon'/\varepsilon$ are governed by perturbative QCD and electroweak effects, that are very strongly enhanced through QCD renormalization group effects active in the range $1 \text{ GeV} \leq \mu \leq m_t$. Without these effects $\varepsilon'/\varepsilon$ would be a few $10^{-5}$. Consequently the short distance contributions determine the order of magnitude of $\varepsilon'/\varepsilon$.

On the other hand the long distance contributions govern the factor “$r$” in $\varepsilon'/\varepsilon = r \cdot 10^{-3}$. These contributions are not yet under control. This is clearly seen in an approximate formula

\[ \varepsilon'/\varepsilon = \text{Im} \lambda_t \cdot F_{\varepsilon'} \quad (\lambda_t = V_{ts}^* V_{td}) \] (4.16)

\[ F_{\varepsilon'} \approx 13 \cdot \left[ \frac{110 \text{ MeV}}{m_s(2 \text{ GeV})} \right]^2 \left[ B_6^{(1/2)}(1 - \Omega_{\text{IB}}) - 0.4 \cdot B_8^{(3/2)} \left( \frac{m_t}{165 \text{ GeV}} \right)^{2.5} \right] \left[ \frac{\Lambda_{\text{MS}}^{(4)}}{340 \text{ MeV}} \right] \] (4.17)

where $B_6^{(1/2)}$ and $B_8^{(3/2)}$ represent the hadronic matrix elements of the dominant QCD-penguin ($Q_6$) and electroweak-penguin ($Q_8$) operators, $\Lambda_{\text{MS}}^{(4)}$ is the QCD scale and $\Omega_{\text{IB}}$ are isospin breaking effects. The strange quark mass in this formula originates in the matrix elements of $Q_6$ and $Q_8$ evaluated in the large-N approach. The calculations of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ are based on three religions: Large-N approach, Lattice approach and the Chiral Quark Model. Large-N approach, formulated for weak decays in 1986 by Bardeen, Gérard and myself \[23\] and modified in various ways by different researchers, is used by the groups in Munich, Dortmund, Granada-Lund, Barcelona-Valencia, Beijing and Marseille. The lattice approach in connection with $\varepsilon'/\varepsilon$ is most extensively studied at present in Rome, Southampton, Brookhaven-Columbia, Geneva-Munich and by CPPACS but the early work of Gupta, Kilcup and Sharpe should not be forgotten. Chiral Quark Model in the context of $\varepsilon'/\varepsilon$ is the domain of the Trieste group. There are other small religions in Dubna-Zeuthen, Montpellier and Taipei. The results from various groups covering the range in (4.15) are listed in table 9 of \[1\]. The basic issues in these analyses are

- The values of $B_6^{(1/2)}$, $B_8^{(3/2)}$ and $m_s(2 \text{ GeV})$,
- Final State Interactions,
- Isospin Breaking effects and generally electromagnetic effects.

These topics are discussed extensively by Donoghue, de Rafael, Martinelli, Paschos, Colangelo, Golterman and Gardner in these proceedings and I will not elaborate on them here. They are also reviewed in \[1, 24, 25\]. Instead I would like to investigate what is the $(\varepsilon'/\varepsilon)_{\text{exp}}$ in (4.14) maybe telling us? To this end let us write

\[ (\varepsilon'/\varepsilon)_{\text{th}} = \text{Im} \lambda_t \left[ P^{1/2} - \frac{1}{\omega} P^{3/2} \right] \] (4.18)
with $\omega = 0.045$ representing the $\Delta I = 1/2$ rule. $P^{1/2}$ is dominated by QCD-penguins, in particular the operator $Q_6$. $P^{3/2}$ is governed by isospin breaking effects induced by the electric charge difference $\Delta e = e_u - e_d$ (electroweak penguins as $Q_8$) and the mass splitting $\Delta m = m_u - m_d$ represented by $\Omega_{\text{IB}}$ in (4.17).

As $\text{Im}\lambda_t$ is known from the analysis of the unitarity triangle, $(\epsilon'/\epsilon)_{\text{exp}}$ in (4.14) tells us that we are allowed to walk only along a straight path in the $(P^{3/2}, P^{1/2})$ plane, as illustrated in Fig. 5. This path crosses the $P^{1/2}$–axis at $(P^{1/2})_0 = 14.3 \pm 2.8$.

![Figure 5: $(\epsilon'/\epsilon)_{\text{exp}}$–path in the $(P^{3/2}, P^{1/2})$ plane](image)

As seen in (4.15), we are still far away from a precise calculation of $P^{1/2}$ and $P^{3/2}$. However, we know that isospin-symmetry and large-N limit represent two powerful approximations to study long-distance hadronic physics. So let us ask what we find when we go to the strict isospin-symmetry limit, setting in particular $\alpha_{\text{em}} = 0$, and take simultaneously the large N limit. As for $\alpha_{\text{em}} \to 0$ electroweak penguins disappear and $\Omega_{\text{IB}} = 0$ we land on the $P^{1/2}$–axis. On the other hand taking large N limit allows us to calculate $P^{1/2}$ as in this limit $P^{1/2}$ is given by the $Q_6$ penguin with $B_6^{(1/2)} = 1$ and the smaller $Q_4$ penguin. The only uncertainties in $P^{1/2}$ reside now in $m_s$ and $\Lambda_{\text{MS}}^{(4)}$ or equivalently $\alpha_s(M_Z)$. To our surprise, taking the central values $m_s(2\text{GeV}) = 110$ MeV and $\alpha_s(M_Z) = 0.119$, we find $P^{1/2} = 14.0$, landing precisely on the $(\epsilon'/\epsilon)_{\text{exp}}$-path. Equivalently

$$(\epsilon'/\epsilon)_0 = (17.4 \pm 0.7) \times 10^{-4} \quad (4.19)$$

where the error results from the error in $\text{Im}\lambda_t$ obtained with $\hat{B}_K = 3/4$ corresponding to the large
N limit. Clearly, as $\Lambda_{\text{MS}}^{(4)} = (340 \pm 40) \text{ MeV}$ and $m_s(2\text{ GeV}) = (110 \pm 20) \text{ MeV}$, improvements on these input parameters are mandatory.

Although this rather intriguing coincidence between (4.14) and (4.19) seems to indicate small $1/N$ and IB corrections, one cannot rule out a somewhat accidental conspiracy between sizeable corrections canceling each other that may also include new physics contributions:

$$\mathcal{O}(1/N) - \frac{1}{\omega}\mathcal{O}(IB) \approx 0.$$  \quad (4.20)

The latter equation describes the walking along the $(\varepsilon'/\varepsilon)_{\text{exp}}$-path.

### 4.3 $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$

The rare decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ are very promising probes of flavour physics within the SM and possible extensions, since they are governed by short distance interactions. They proceed through $Z^0$-penguin and box diagrams. As the required hadronic matrix elements can be extracted from the leading semileptonic decays and other long distance contributions turn out to be negligible [27], the relevant branching ratios can be computed to an exceptionally high degree of precision [28]. The main theoretical uncertainty in the CP conserving decay $K^+ \to \pi^+\nu\bar{\nu}$ originates in the value of $m_c(\mu_c)$. It has been reduced through NLO corrections down to $\pm 7\%$ at the level of the branching ratio. The dominantly CP-violating decay $K_L \to \pi^0\nu\bar{\nu}$ [29] is even cleaner as only the internal top contributions matter. The theoretical error for $Br(K_L \to \pi^0\nu\bar{\nu})$ amounts to $\pm 2\%$ and is safely negligible.

There are three virtues of these decays:

- $\text{Im} \lambda_t$ can be determined directly from $Br(K_L \to \pi^0\nu\bar{\nu})$ [30]:

$$\text{Im} \lambda_t = 1.36 \cdot 10^{-4} \left[\frac{170 \text{ GeV}}{m_t(m_t)}\right]^{1.15} \left[\frac{Br(K_L \to \pi^0\nu\bar{\nu})}{3 \cdot 10^{-11}}\right]^{1/2}$$  \quad (4.21)

without any uncertainty in $|V_{cb}|$. With $m_t(m_t)$ measured very precisely at Tevatron and later at LHC and future linear collider, (4.21) offers the cleanest method to measure $\text{Im} \lambda_t$ and effectively the Jarlskog invariant $J_{\text{CP}} = \text{Im} \lambda_t(1 - \lambda^2/2)\lambda$.

- $\sin 2\beta$ can be determined very cleanly once both branching ratios are known [30]. Measuring these branching ratios with 10\% accuracy allows to determine $\sin 2\beta$ with an error $\Delta \sin 2\beta = \pm 0.05$. Comparision of this determination with the one by means of $a_{\psi K_S}(t)$ is particularly well suited for tests of CP violation in the SM and offers a powerful tool to probe the physics beyond it [30, 33].
The unitarity triangle can be determined very cleanly with the main uncertainty residing in the value of the Wolfenstein parameter $A$ or equivalently $|V_{cb}|$. In particular $|V_{td}|$ can be determined to better than $\pm 10\%$.

At present we have:

$$Br(K^+ \rightarrow \pi^+\nu\bar{\nu}) = \begin{cases} (7.5 \pm 2.9) \cdot 10^{-11} & \text{(SM)} \\ (15^{+34}_{-12}) \cdot 10^{-11} & \text{(E787) [31] } \end{cases} \tag{4.22}$$

$$Br(K_L \rightarrow \pi^0\nu\bar{\nu}) = \begin{cases} (2.6 \pm 1.2) \cdot 10^{-11} & \text{(SM)} \\ < 5.9 \cdot 10^{-7} & \text{(KTeV) [32] } \end{cases} \tag{4.23}$$

where the errors in the SM branching ratios come dominantly from the uncertainties in the CKM parameters. The E787 result for $Br(K^+ \rightarrow \pi^+\nu\bar{\nu})$ is rather close to the SM expectations, excluding very large non-standard contributions. The KTeV result is still four orders of magnitude away from the SM prediction for $Br(K_L \rightarrow \pi^0\nu\bar{\nu})$. The latter branching ratio can be bounded in a model independent manner using isospin symmetry [33]:

$$Br(K_L \rightarrow \pi^0\nu\bar{\nu}) \leq 4.4 \cdot Br(K^+ \rightarrow \pi^+\nu\bar{\nu}) \leq 2 \cdot 10^{-9} (90\% \text{ C.L.}) \tag{4.24}$$

The experimental outlook for both decays has been reviewed by Littenberg in [34] and at this conference. See also [35]. We can hope that the efforts by experimentalists at Brookhaven, Fermilab and KEK will result in the measurements of both branching ratios with $\pm 10\%$ accuracy in the second half of this decade.

### 4.4 CP Violation in B Decays

CP violation in B decays is one of the most important targets of B-factories and of dedicated B-experiments at hadron colliders. The first results on $\sin 2\beta$ from BaBar and Belle, discussed already in Section 4.1, are very encouraging. These results should be improved over the coming years through the new measurements of $a_{\psi K_S}(t)$ by both collaborations and by CDF and D0 at Fermilab. An error for $\sin 2\beta$ of $\pm 0.08$ should be achievable by the next summer. Of interest is also the measurement of $\sin 2\beta$ through the CP asymmetry in the decay $B_d \rightarrow \phi K_S$ that proceeds dominantly through penguin diagrams.

In order to search for new physics it is mandatory to measure the angles $\alpha$ and $\gamma$ in the UT. Many strategies to measure these angles have been proposed in the last decade. Reviews can be found in [1, 36, 37, 38, 39]. Most of these strategies require simultaneous measurements of several channels in order to remove potential hadronic uncertainties present in non-leptonic B-decays. Prime example is the measurement of $\alpha$ through the CP asymmetry in $B_d^0 \rightarrow \pi^+\pi^-$. 


where the presence of penguin diagrams, in addition to the dominant tree diagrams, precludes a clean extraction of \(\alpha\) from \(a_{\pi^+\pi^-}(t)\). There are many ideas for determining or eliminating the penguin component. They are reviewed in [1, 36, 37, 38, 39]. None of them is straightforward and only time will show which of these methods will provide an acceptable determination of \(\alpha\). At present both BaBar and Belle make efforts to measure \(a_{\pi^+\pi^-}(t)\) that gives \((\sin 2\alpha)_{\text{eff}}\). The latter containing penguin contributions does not give the true angle \(\alpha\). Theorists are then supposed to translate \((\sin 2\alpha)_{\text{eff}}\) into the true \(\sin 2\alpha\). More about this issue can be found in Beneke’s talk. It should be emphasized that in view of a poor knowledge of \(\alpha\) at present (see table 1) even a rough measurement of this angle will have an important impact on the UT.

The theoretically cleanest and simultaneously experimentally feasible method for the determination of the angle \(\gamma\) is the full time dependent analysis of \(B_s \to D^{+}_s K^-\) and \(\bar{B}_s \to D^{-}_s K^+\) [40]. This method is unaffected by penguin contributions but the presence of the expected large \(B^{0}_s - \bar{B}^{0}_s\) mixing is a challenge for experimentalists. Yet, LHC-B should be able to measure \(\gamma\) in this manner with high precision [38]. Also \(B_c \to D D_s\) could be used for the extraction of \(\gamma\) at the LHC-B [11].

At present the most extensive analyses of the angle \(\gamma\) use the four \(B \to \pi K\) channels that have been measured by CLEO, BaBar and Belle. The main issues here are the final state interactions (FSI), SU(3) symmetry breaking effects and the importance of electroweak penguin contributions. Several interesting ideas have been put forward to extract the angle \(\gamma\) in spite of large hadronic uncertainties in \(B \to \pi K\) decays [42, 43, 44, 45, 46, 47]. Reviews can be found in [45, 48].

Three strategies for bounding and determining \(\gamma\) have been proposed. The “mixed” strategy [12] uses \(B^0_d \to \pi^0 K^{\pm}\) and \(B^\pm \to \pi^\pm K\). The “charged” strategy [17] involves \(B^\pm \to \pi^0 K^{\pm}\), \(\pi^\pm K\) and the “neutral” strategy [15] the modes \(B^0_d \to \pi^\pm K^\pm\), \(\pi^0 K^0\). Parametrizations for the study of the FSI, SU(3) symmetry breaking effects and of the electroweak penguin contributions in these strategies have been presented in [14, 15, 16]. Moreover, general parametrizations by means of Wick contractions [19, 50] have been proposed. They can be used for all two-body B-decays. These parametrizations should turn out to be useful when the data improve.

Parallel to these efforts an important progress has been made by Beneke, Buchalla, Neubert and Sachrajda [51] through the demonstration that in a large class of non-leptonic two-body B-meson decays the factorization of the relevant hadronic matrix elements follows from QCD in the heavy-quark limit. The resulting factorization formula incorporates elements of the naive factorization approach used in the past but allows to compute systematically non-factorizable corrections. In this approach the \(\mu\)-dependence of hadronic matrix elements is under control. Moreover spectator quark effects are taken into account and final state interaction phases can
be computed perturbatively. While, in my opinion, an important progress in evaluating non-leptonic amplitudes has been made in \[51\], the usefulness of this approach at the quantitative level has still to be demonstrated when the data improve. In particular the role of the $1/m_b$ corrections has to be considerably better understood. Recent lectures on this approach can be found in \[52\]. The techniques developed in \[51\] have been used for exclusive rare B decays \[53\].

An interesting proof of factorization for $B \to D\pi$ to all orders of $\alpha_s$ has been presented in \[54\].

There is an alternative perturbative QCD approach to non-leptonic decays \[55\] which has been developed earlier from the QCD hard-scattering approach. Some elements of this approach are present in the QCD factorization formula of \[51\]. The main difference between these two approaches is the treatment of soft spectator contributions which are assumed to be negligible in the perturbative QCD approach. While the QCD factorization approach is more general and systematic, the perturbative QCD approach is an interesting possibility. Only time will show which of these two frameworks is more successful and whether they have to be replaced by still more powerful approaches in the future.

Finally new methods to calculate exclusive hadronic matrix elements from QCD light-cone sum rules has been developed recently in \[56\]. This work may shed light on the importance of $1/m_b$ and soft-gluon effects in the QCD factorization approach. Reviews of QCD light-cone sum rules can be found in \[57\].

Returning to phenomenology, as demonstrated in \[42, 44, 45, 46, 47\], already CP-averaged $B \to \pi K$ branching ratios may imply interesting bounds on $\gamma$ that may remove a large portion of the allowed range from the analysis of the unitarity triangle. In particular combining the neutral and charged strategies \[45\] one finds that the existing data on $B \to \pi K$ favour $\gamma$ in the second quadrant, which is in conflict with the standard analysis of the unitarity triangle as we have seen in section 4.1. Other arguments for $\cos \gamma < 0$ using $B \to PP, PV$ and $VV$ decays were given in \[58\]. Also the analyses of $B \to \pi K$ in the QCD factorization approach \[59\] favour $\gamma > 90^\circ$.

In view of sizable theoretical uncertainties in the analyses of $B \to \pi K$ and of large experimental errors in the corresponding branching ratios it is not yet clear whether the discrepancy in question is serious. For instance sizable contributions of the so-called charming penguins to the $B \to \pi K$ amplitudes could shift $\gamma$ extracted from these decays below 90° but at present these contributions cannot be calculated reliably. Similar role could be played by annihilation contributions \[55\] and large non-factorizable SU(3) breaking effects \[45\]. Also, new physics contributions in the electroweak penguin sector could shift $\gamma$ to the first quadrant \[45\]. It should be however emphasized that the problem with the angle $\gamma$, if it persisted, would put into difficulties not only the SM but also the full class of MFV models in which the lower bound on $\Delta M_s/\Delta M_d$
implies $\gamma < 90^\circ$. On the other hand as seen in fig. 4 for sufficiently high values of $R_{sd}$, the angle $\gamma$ resulting from the unitarity triangle analysis in models containing new operators can easily be in the second quadrant provided $\Delta M_s/\Delta M_d$ is not too large. However, this does not happen in the MSSM in the large $\tan \beta$ limit, where the presence of new operators results in $R_{sd} < 1.0$ and in $\gamma$ that is generally smaller than in the SM.

Another interesting direction is the use of U-spin symmetry. New strategies for $\gamma$ using this symmetry have been proposed in [61]. The first strategy involves the decays $B_{d,s}^0 \to \psi K_S$ and $B_{d,s}^0 \to D_{d,s}^+ D_{d,s}^-$. The second strategy involves $B_d^0 \to K^+ K^-$ and $B_d^0 \to \pi^+ \pi^-$. These strategies are unaffected by FSI and are only limited by U-spin breaking effects. They are promising for Run II at FNAL and in particular for LHC-B provided the U-spin breaking effects can be estimated reliably [61]. A method of determining $\gamma$, using $B^+ \to K^0 \pi^+$ and the U-spin related processes $B_d^0 \to K^+ \pi^-$ and $B_d^0 \to \pi^+ K^-$, was presented in [62]. A general discussion of U-spin symmetry in charmless B decays and more references to this topic can be found in [63].

4.5 Minimal Flavour Violation Models

We have defined this class of models in section 3. Here I would like just to list four interesting properties of these models that are independent of particular parameters present in these models. These are:

- There exists a universal unitarity triangle (UUT) common to all these models and the SM that can be constructed by using measurable quantities that depend on the CKM parameters but are not polluted by the new parameters present in the extensions of the SM. The UUT can be constructed, for instance, by using $\sin 2\beta$ from $a_{\psi K_S}$ and the ratio $\Delta M_s/\Delta M_d$. The relevant formulae can be found in (4.9) and in [8, 14, 17], where also other quantities suitable for the determination of the UUT are discussed.

- There exists an absolute lower bound on $\sin 2\beta$ that follows from the interplay of $\Delta M_d$ and $\epsilon_K$. It depends only on $|V_{cb}|$ and $|V_{ub}/V_{cb}|$, as well as on the non-perturbative parameters $B_K$, $F_{B_d}\sqrt{B_d}$ and $\xi$ entering the standard analysis of the unitarity triangle. A conservative scanning of all relevant input parameters gives $\left(\sin 2\beta\right)_{\min} = 0.42$. A less conservative bound of 0.52 has been found in [53]. This bound could be considerably improved when the values of $|V_{cb}|$, $|V_{ub}/V_{cb}|$, $B_K$, $F_{B_d}\sqrt{B_d}$, $\xi$ and – in particular of $\Delta M_s$ – will be known better [1, 64].

- There exists an absolute upper bound on $\sin 2\beta$. It is simply given by

$$\left(\sin 2\beta\right)_{\max} = 2R_b^{\max}\sqrt{1 - (R_b^{\max})^2} \approx 0.82,$$

(4.25)
with $R_b$ defined in (2.4).

- For given $a_{\psi K_S}$ and $Br(K^+ \rightarrow \pi^+\nu\bar{\nu})$ only two values of $Br(K_L \rightarrow \pi^0\nu\bar{\nu})$ are possible in the full class of MFV models, independently of any new parameters present in these models [14]. Consequently, measuring $Br(K_L \rightarrow \pi^0\nu\bar{\nu})$ will either select one of these two possible values or rule out all MFV models. Taking the present experimental bound on $Br(K^+ \rightarrow \pi^+\nu\bar{\nu})$ and (4.25) one finds an absolute upper bound $Br(K_L \rightarrow \pi^0\nu\bar{\nu}) < 7.1 \cdot 10^{-10}$ (90\% C.L.) [14] that is stronger than the bound in (4.24).

5 Twenty Questions

1. What are the precise values of $V_{ud}$ and $V_{us}$?
The unitarity relation $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$ is violated by more than two standard deviations. The only hope is that our understanding of the errors for $V_{ud}$ and $V_{us}$ is still not what we think. Otherwise we have to conclude that some new physics is at work. The improved measurements of these two elements are mandatory.

2. How accurately can we determine $|V_{ub}|$ and $|V_{cb}|$?
Both determinations are subject to theoretical uncertainties. Recently, interesting new methods for the determination of $|V_{ub}|$ from inclusive B decays have been proposed [66]. They could provide, in conjunction with the future BaBar and Belle measurements, an improved measurement of $R_b$. Both elements are very important for the analysis of the unitarity triangle and for the predictions of rare decays as the latter are sensitive functions of the Wolfenstein parameter $A = |V_{cb}|/\lambda^2$.

3. How accurately can we calculate $B_K$, $\sqrt{B_dF_d}$, $\xi$ and determine $m_s$ and $m_c$?
All these low energy quantities enter the phenomenology of weak decays both in the SM and its extensions and they should be determined with a high precision. While ultimately lattice calculations should provide the most accurate numbers, QCD sum rule approach will also continue to be useful for some of these parameters [56, 57, 58].

4. What is the value of $\Delta M_s$?
Possibly we will know it already next summer if CDF and D0 are lucky and $\Delta M_s \leq 20/\text{ps}$ as indicated by LEP analyses. This will be a very important measurement, providing accurate values of $R_t$ and $|V_{td}|$. Simultaneously, combining this measurement with $a_{\psi K_S}$ will allow us to determine $\gamma$ in a clean manner as illustrated in fig.4. Moreover if $\sqrt{B_s F_{Bs}}$ can be calculated accurately by lattice or QCD sum rule methods, the measurement of $\Delta M_s$ with $|V_{ts}| \approx |V_{cb}|$ will shed some light on whether MFV is the whole story or whether new effective operators have to be taken into account [17].
5. Is the CKM matrix the only source of flavour and CP violation?
In order to answer this question the four properties of MFV models listed at the end of section 4 will be very useful. While last year the lower bound on $\sin 2\beta$ from MFV seemed to be a useful quantity for this purpose, the new BaBar and Belle results are well above this bound and possibly the upper bound on $\sin 2\beta$ in (4.25), almost violated by the Belle result, could turn out to be more interesting in the near future. To this end $R_\theta$ has to be better known. In the long run the last property of the MFV models that involves $a_{\psi K_S}$, $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$ should be very useful. There are of course other strategies to answer this question. See Masiero’s talk and [17].

6. What is the optimal error analysis of weak decays?
There have been already many suggestions in the literature but from my point of view none of them is fully convincing. It is important to make progress here, in particular if new physics contributions will turn out to be small.

7. What are the values of $B_6^{(1/2)}$, $B_8^{(3/2)}$ and $\Omega_{IB}$ and how important are FSI in $\epsilon'/\epsilon$?
These are very important questions. Personally, I doubt that these questions will be answered very soon but I hope that I am wrong. It will be interesting to see what new lattice calculations and new analytic efforts can contribute to this issue.

8. What is the value of $\epsilon'/\epsilon$ in the SM?
While this question is directly connected with the previous question, I pose it here as there are public statements by some speakers that $\epsilon'/\epsilon$ in the SM is fully under control, it is in perfect agreement with the experiment and the only remaining issue is the value of $m_s(2\text{ GeV})$. Such statements are totally misleading. While varying the relevant parameters one can certainly fit the experimental data, this is not what one wants to do. Until the question 7 is not answered satisfactorily we do not know the precise value of $\epsilon'/\epsilon$ in the SM. My favorite number is still the one of [4]. That is $(\epsilon'/\epsilon)_{SM} = 7.0 \cdot 10^{-4}$.

9. Which are the best decays to look for other signals of direct CP violation?
Clearly, in the field of B-decays, one should look at CP asymmetries in $B^\pm$ decays. Until now no effects have been found. There are also efforts by NA48 to measure direct CP violation in $K^{\pm}$ decays. Unfortunately none of these decays, similarly to $\epsilon'/\epsilon$, is theoretically clean. In this context one should ask the next question.

10. Who will give more money for $K^+ \to \pi^+ \nu \bar{\nu}$ and in particular for $K_L \to \pi^0 \nu \bar{\nu}$?
The nonvanishing branching ratio $Br(K_L \to \pi^0 \nu \bar{\nu})$ higher than $10^{-13}$ is a signal of CP violation in the interference of mixing and decay. But at the level of $3 \cdot 10^{-11}$, as expected in the SM, it is a clear signal of CP violation in the decay amplitude or equivalently of direct CP violation.
Higher branching ratios are still possible beyond the SM [68]. As $Br(K_L \to \pi^0 \nu \bar{\nu})$ in the SM and in its extensions is free of hadronic uncertainties it is exceptional in the field of weak decays and it would be a crime if one did not measure it. Similar comments apply to $K^+ \to \pi^+ \nu \bar{\nu}$. I am convinced that once both branching ratios have been measured to better than 10% accuracy our understanding of flavour violation and CP violation will improve considerably, independently of other measurements performed in this decade that in most cases suffer from substantially larger theoretical uncertainties than these two golden decays.

11. How large are CP-conserving and indirectly CP-violating contributions to $K_L \to \pi^0 e^+ e^-$?

This decay is believed to be dominated by the contribution from direct CP violation that can be calculated very reliably and is expected to give $Br(K_L \to \pi^0 e^+ e^-)_{\text{dir}} = (4.3 \pm 2.1) \cdot 10^{-12}$ where the error is dominated by the CKM uncertainties. In order to be able to compare this result with the future data both the CP conserving contribution (estimated to be well below $2 \cdot 10^{-12}$) and the indirectly CP-violating contribution (to be determined by KLOE at Frascati) have to be known. The most recent experimental bound from KTeV reads $Br(K_L \to \pi^0 e^+ e^-) \leq 5.1 \cdot 10^{-10}$ (90% C.L.) leaving considerable room for new physics contributions. A nice summary of the theoretical situation with the relevant references is given by D’Ambrosio and Isidori in [35].

12. Can we ever extract the short distance component of $K_L \to \mu^+ \mu^-$?

The absorptive part to this decay, determined from $K_L \to \gamma \gamma$, is very close to the experimental branching ratio: $(7.18 \pm 0.17) \cdot 10^{-9}$ from E871 at Brookhaven. In order to extract the short distance dispersive contribution (estimated to give $(0.9 \pm 0.3) \cdot 10^{-9}$ in the SM), the long distance dispersive contribution has to be known. There are different opinions whether the latter contribution can ever be reliably computed [39]. In the positive case one would get a useful measurement of the parameter $\bar{\rho}$.

13. How precisely can we determine $\alpha$ and $\gamma$ from B-decays before LHC-B and BTeV?

I have addressed this issue already in section 4.4. The answer to this question depends on the answer to the next question.

14. How can we get non-leptonic two-body B-decays fully under control?

Clearly, there has been a considerable progress in calculating branching ratios for these decays in the last two years. However, from my point of view the situation is far from satisfactory and I expect that it will take a considerable amount of efforts by experimentalists and theorists before the dynamics of these decays will be fully understood.

15. Is the angle $\gamma$ extracted from $B \to \pi K$ decays consistent with the UT fits?
This is an important question that requires some progress on the last question.

16. **What are the prospects for precise measurements of** $B \rightarrow X_{s,d}\gamma$, $B \rightarrow X_{s,d}\mu\bar{\mu}$ and $B \rightarrow X_{s,d}\nu\bar{\nu}$, for corresponding exclusive channels, $B_{s,d} \rightarrow \mu\bar{\mu}$ and related theory?

This is clearly an exciting field with new interesting theoretical papers on QCD factorization in exclusive decays $B \rightarrow K^{*}(q)\gamma$ and $B \rightarrow K^{*}\mu\bar{\mu}$\cite{53} and new very relevant analyses of $B_{s} \rightarrow \mu\bar{\mu}$ at large $\tan\beta$ in supersymmetry\cite{70}. The coming years should be very exciting for this field in view of the new data from BaBar, Belle, CDF and D0.

17. **Do we see any new physics in charm and hyperon decays?**

It is still too early to claim anything of this sort but these decays could be the first to provide some hints for new physics in spite of the fact that they are not theoretically clean. See the talks by Bigi and He in these proceedings\cite{71}.

18. **What are the lowest values of electric dipole moments still compatible with low energy supersymmetry?**

It will be exciting to follow the new experimental progress in this field and to see how far one can adjust various supersymmetric parameters in case no signal is found soon. As non-vanishing electric dipole moments signal CP violation in flavor diagonal transitions, that are very strongly suppressed in the SM, their observation will certainly signal the presence of new CP-violating phases and might help to explain the origin of matter-antimatter asymmetry in the universe.

19. **What are the prospects for leptonic flavour and CP violation?**

This field is experiencing a great push due to the discovery of neutrino oscillations. Every day several papers appear. Progress both in theory and experiment is to be expected in the coming years.

20. **What is the (indirect) impact of $(g-2)_{\mu}$ on weak decays?**

The possible discrepancy between the Brookhaven measurement\cite{74} and the SM\cite{73} is clearly one of the highlights of this year but I think we should not get overexcited in view of considerable theoretical uncertainties. New improved data as well as theoretical efforts will hopefully make the situation clearer. There is an avalanche of papers in this field, in particular in the framework of supersymmetry. In the context of this question, an interesting relation between $(g-2)_{\mu}$ and $B_{s} \rightarrow \mu\bar{\mu}$ at large $\tan\beta$ in supersymmetry has been pointed out in\cite{74}.

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