Varying-coefficient functional additive models

Hidetoshi Matsui
Faculty of Data Science, Shiga University
1-1-1, Banba, Hikone, Shiga, 522-8522, Japan.
hmatsui@biwako.shiga-u.ac.jp

Abstract: We extend the varying coefficient functional linear model to the nonlinear model and propose a varying coefficient functional additive model. The proposed method can represent the relationship between functional predictors and a scalar response where the response depends on an exogenous variable. It captures the nonlinear structure between variables and also provides interpretable relationship of them. The model is estimated through basis expansions and penalized likelihood method, and then the tuning parameters included at the estimation procedure are selected by a model selection criterion. Simulation studies are provided to show the effectiveness of the proposed method. We also apply it to the analysis of crop yield data and then investigate how and when the environmental factor relates to the amount of the crop yield.

Key Words and Phrases: basis expansion, functional data analysis, regularization, varying-coefficient model

1 Introduction

Functional data analysis (FDA) is a widely applicable technique for analyzing longitudinally observed data, and is applied in various fields of data, such as bioinformatics, medicine and meteorology (Ramsay and Silverman, 2005; Kokoszka and Reimherr, 2017). In particular, functional regression analysis is one of the most useful techniques in functional data analysis. The basic idea behind functional regression analysis is to treat longitudinally observed data for predictors and/or responses as smooth functional data, and then to elucidate the relationship between them from estimated model and to predict the newly observed data.

There are various kinds of functional regression models according to the data structure of the predictor and the response. The most basic model is a functional linear model for a functional predictor and a scalar response, which is discussed in [Ramsay and Dalzell (1991), Cardot et al. (1999), and Goldsmith et al. (2010)]. On the other hand, the functional linear model for a functional predictor and a functional response is also considered in [Ramsay and Dalzell (1991), Yao et al. (2005b), Matsui et al. (2009)]. Numerous extensions and improvements of these models are reported; Morris (2015) and Reiss et al. (2017) extensively review several studies on functional regression models.

In this work we consider the situation where the predictor is a function while the response is a scalar, but it depends on another variable. The motivating data come from crop yield data of multi-stage tomatoes. A plant of the multi-stage tomatoes grows for a long time in a year, and fruits are harvested daily. In general, the amount of the crop yield depends on environmental factors such as the temperature and the amount of solar radiation. In addition, these effects may differ for the season of the year for the multi-stage tomatoes. Therefore we treat the seasonal time as an exogenous variable. For the analysis of such type of data, the varying-coefficient functional linear model (VCFLM) by [Cardot and Sarda (2008) and Wu et al. (2010)] can be applied by treating the seasonal time as an exogenous variable. The VCFLM
is an extension of the varying-coefficient model [Hastie and Tibshirani, 1993; Hoover et al., 1998] to the functional linear model framework, and it can represent the relationship between a functional predictor and a scalar response varying with the exogenous variable. Specifically, we can interpret the relationship by investigating coefficient functions of the model. Several refinements and application of the VCFLM are discussed in Peng et al. (2016), Li et al. (2017), and Davenport et al. (2018). However, the VCFLM captures only the “linear” relationship at fixed exogenous variable. In the case of crop yield data, if the temperature is moderately high, the yield will be high, but if the temperature is too high, the yield will decrease. It is difficult for the VCFLM to capture such relationship.

To solve this problem, we extend the VCFLM to the nonlinear model for a continuous response variable. On of the extensions of the traditional linear model to the nonlinear one is an additive model [Hastie and Tibshirani, 1990], and there are several extensions of the functional linear model to additive model frameworks. Müller and Yao (2008) proposed a functional additive model (FAM) that extend the linear term to the nonlinear one. Their approach uses Karhunen-Loéve (KL) expansions and local polynomial regression, whereas Mclean et al. (2014) directly applies the basis expansion technique to the nonlinear structure, and Fan et al. (2015) assumes an unknown link function between the linear predictor and the response. Among them, it is easier to interpret the Müller and Yao (2008)’s model in the viewpoint that how the functional predictor affect the response. In addition, Müller et al. (2013), Zhu et al. (2014), Han et al. (2018), and Wong et al. (2019) developed the FAM to several situations. Ivanescu et al. (2015), Scheipl and Greven (2010), and Scheipl et al. (2016) considered comprehensive functional regression model including functional additive models.

Using these ideas, we propose a novel varying-coefficient functional additive model (VCFAM). The VCFAM captures nonlinear relationship between functional predictors and a scalar response, where the response depends on an exogenous variable. Furthermore, we can interpret the relationship from the estimated model. We consider estimating the VCFAM by the penalized likelihood method along with the basis expansions. In order to select tuning parameters included in the penalty, we apply a model selection criterion for evaluating the estimated model, using the idea of Konishi and Kitagawa (2008). Simulation studies are conducted to evaluate the effectiveness of the proposed method. Then we apply the VCFAM to the analysis of crop yield data of multi-stage tomatoes to investigate how the temperature during the cultivation affect the crop yield. We also consider predicting future yields using the past temperature and yield data.

This paper is organized as follows. Section 2 briefly reviews existing models that relate to the proposed method, and then we introduce a VCFAM. Section 3 shows the method for estimating and evaluating the VCFAM. Simulation studies are given in Section 4, and then real data analysis are discussed in Section 5. Conclusions about our work are summarized in Section 6.
2 Model

Before introducing our model, we overview some existing models; the functional linear model, the functional additive model and the varying-coefficient functional linear model. Then we propose a novel varying-coefficient functional additive model.

2.1 Existing models

Suppose we have \( n \) sets of a functional predictor and a scalar response \( \{x_i(s), y_i; i = 1, \ldots, n, s \in S \in \mathbb{R}\} \), where \( x_i(s) \) and \( y_i \) are a functional predictor and a scalar response respectively. We also assume that the functional predictor \( x_i(s) \) is expressed by truncated Karhunen-Loéve (KL) expansions (Yao et al., 2005a):

\[
x_i(s) = \sum_{k=1}^{q} \xi_{ik} \phi_k(s),
\]

where \( \xi_{ik} \) and \( \phi_k(s) \) \( (k = 1, \ldots, q) \) are functional principal component (FPC) scores and corresponding eigenfunctions with eigenvalues \( \lambda_k \), and \( q \) is the truncation number of principal components. The FPC scores satisfy \( E(\xi_{ik}) = 0 \) and \( E(\xi_{ik}^2) = \lambda_k \), and the eigenvalues satisfies \( \lambda_1 > \cdots > \lambda_q > 0 \). In addition, the eigenfunctions are orthonormal basis, that is, \( \int \phi_k(s)\phi_l(s) ds = \delta_{kl} \), where \( \delta_{kl} \) is a Kronecker’s delta.

Although we can apply the well-known basis functions such as splines or radial basis functions (Green and Silverman, 1994) for \( \{\phi_k(s)\} \), the KL expansion can represent data with smaller number of basis functions.

The traditional functional linear model (FLM) is given in the form of

\[
y_i = \beta_0 + \int_S x_i(s) \beta_1(s) ds + \varepsilon_i,
\]

where \( \beta_0 \) is an intercept, \( \beta_1(s) \) is a coefficient function and \( \varepsilon_i \) are errors independently and identically distributed with mean zero and unknown variance. If we assume that the functional predictor and coefficient function are expressed by basis expansions (including KL expansion), the problem of estimating the model becomes that of estimating the ordinal linear model with predictors \( \xi_{ik} \) (Ramsay and Silverman, 2005).

The functional additive model (FAM) by Müller and Yao (2008) is given by

\[
y_i = \sum_{k=1}^{q} f_k(\xi_{ik}) + \varepsilon_i,
\]

where \( f_k \) are unknown functions. The mean structure of the traditional functional linear model is transformed into the ordinal linear model with predictors \( \xi_{ik} \) (Yao et al., 2005b), whereas the FAM (2) is a natural extension to the additive model. Therefore we can capture more complex relationship between the predictor and the response.

If the response depends on the exogenous variable \( t_i \), the following varying-coefficient functional linear model (VCFLM) is considered (Cardot and Sarda, 2008; Wu et al., 2010):

\[
y_i = \beta_0(t_i) + \int_T \beta_1(s, t_i) x_i(s) ds + \varepsilon_i,
\]
where \( \beta_0(t) \) is a baseline function and \( \beta_1(s,t) \) is a coefficient surface. Then we can represent the relationship between the response and the predictor with varying \( t \).

### 2.2 Varying-coefficient functional additive model

Again we denote \( n \) sets of observations as \( \{x_i(s), y_i, t_i; s \in S \subset \mathbb{R}\} \), where the response \( y_i \) depends on the exogenous variable \( t_i \) as well as a functional predictor \( x_i(s) \). In addition, \( x_i(s), y_i \) are supposed to be centered so that \( \sum_{i=1}^{n} x_i(s) = 0 \) and \( \sum_{i=1}^{n} y_i = 0 \). To express the relation of these variables, we extend the VCFLM \( (3) \) to the additive model framework \( (2) \). When applying the functional additive model, Zhu et al. (2014) proposed transforming the functional principal component (FPC) scores \( \lambda_k \) into \( \zeta_k \in [0,1] \) by using some monotonic function such as the cumulative distribution function \( \Phi(x; 0, \lambda) \) of the normal distribution \( N(0, \lambda) \). That is, \( \zeta_k \) is given by \( \zeta_k = \Phi(\xi_k; 0, \lambda) \).

Using these ideas, we model the relationship between the response and predictors as the following varying-coefficient functional additive model (VCFAM):

\[
y_i = \sum_{k=1}^{g} f_k(\zeta_{ik}, t_i) + \varepsilon_i ,
\]

where \( f_k \) is a nonlinear function of \( \zeta_{ik} \) and \( t_i \), here we assume that \( f_k \) satisfies \( E[f_k(\zeta_{ik}, t_i)|t_i] = 0 \). In addition, \( \varepsilon_i \) is an error that \( \varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)^T \) follows normal distribution with mean vector \( \mathbf{0} \) and variance covariance matrix \( \Sigma \). In our application described in Section 5 the index \( i \) corresponds to the observed time, so we assume that \( \varepsilon_i (i = 1, \ldots, n) \) depends on each other rather than i.i.d. Advantages of the VCFAM \( (4) \) is that we can consider the nonlinear relationship between the response and predictors at varying \( t \).

The VCFAM \( (4) \) can be extended to the situation where there are multiple functional predictors \( \{x_{i1}(s), \ldots, x_{ip}(s)\} \);

\[
y_i = \sum_{j=1}^{p} \sum_{k=1}^{q_j} f_{jk}(\zeta_{ijk}, t_i) + \varepsilon_i ,
\]

where \( f_{jk}(\cdot, \cdot) \) are nonlinear functions and \( \zeta_{ijk} \) are derived from FPC scores by the same strategies as \( \zeta_{ik} \).

### 3 Estimation

In order to estimate the unknown functions \( f_k \) in the VCFAM \( (4) \), we assume that this is expressed by basis expansions as follows.

\[
f_k(\zeta_{ik}, t_i) = \sum_{h=1}^{m_1} \sum_{l=1}^{m_2} \eta_{kh}(\zeta_{ik})\theta_{kl} \psi_{kl}(t_i) = \eta_k(\zeta_{ik})^T \Theta_k \psi_k(t_i),
\]

where \( \eta_k(\zeta) = (\eta_{k1}(\zeta), \ldots, \eta_{km_1}(\zeta))^T \) and \( \psi_k(t) = (\psi_{k1}(t), \ldots, \psi_{km_2}(t))^T \) are vectors of basis functions and \( \Theta_k = (\theta_{kl})_{hl} \) are \( m_1 \times m_2 \) matrices of unknown parameters. Using this assumption, the VCFAM
can be expressed as
\[ y_i = \sum_{k=1}^{q} \eta_k (\zeta_{ik})^T \Theta_k \psi_k(t_i) + \varepsilon_i \]
\[ = \sum_{k=1}^{q} \left\{ \psi_k^T(t_i) \otimes \eta_k^T(\zeta_{ik}) \right\} \text{vec} \Theta_k + \varepsilon_i, \]

Then the VCFAM is given by
\[ y = \sum_{k=1}^{q} X_k \text{vec} \Omega_k + \varepsilon = X \theta + \varepsilon, \]

where
\[ y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}, \quad X_k = \begin{pmatrix} \psi_k^T(t_1) \otimes \eta_k^T(\zeta_{1k}) \\ \vdots \\ \psi_k^T(t_n) \otimes \eta_k^T(\zeta_{nk}) \end{pmatrix}, \quad \varepsilon = \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_n \end{pmatrix}, \]
\[ X = (X_1, \ldots, X_q), \quad \theta = ((\text{vec} \Omega_1)^T, \ldots, (\text{vec} \Omega_q)^T), \]

and therefore the VCFAM has a probability density function
\[ p(y; \theta, \Sigma) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp \left\{ -\frac{1}{2}(y - X \theta)^T \Sigma^{-1}(y - X \theta) \right\}. \]

The unknown parameter \( \theta \) is estimated by maximizing the penalized log-likelihood function given by
\[ \ell_\lambda(\theta) = \log p(y; \theta, \Sigma) - n \theta^T \Omega_\lambda \theta, \]
\[ \Omega_\lambda = I_q \otimes \{ \lambda_\zeta (I_{m_2} \otimes P_{m_1}) + \lambda_t (P_{m_2} \otimes I_{m_1}) \}, \]

where \( \lambda_\zeta, \lambda_t > 0 \) are regularization parameters and \( P_{m_1} \) and \( P_{m_2} \) are \( m_1 \times m_1 \) \( m_2 \times m_2 \) non-negative definite matrices, respectively. The matrix \( \Omega_\lambda \) imposes penalties for the smoothness of the \( f_k(\zeta, t) \) with respect to \( \zeta \) and \( t \) directions, and the amounts of penalties are controlled by \( \lambda_\zeta \) and \( \lambda_t \), respectively. By maximizing the penalized log-likelihood function, a maximum penalized likelihood estimator of \( \theta \) is given by
\[ \hat{\theta} = \left( X^T \Sigma^{-1} X + n \Omega_\lambda \right)^{-1} X^T \Sigma^{-1} y, \]

where \( \hat{\Sigma} \) is an estimator of \( \Sigma \) and how to estimate it depends on the structure of \( \Sigma \). For details, see, e.g. [Fahrmeir et al. (2013)]. Then we have a statistical model for the VCFAM by plugging the estimators \( \hat{\theta} \) and \( \hat{\Sigma} \) into \( \ell_\lambda(\theta) \).

The VCFAM estimated by the above method depends on tuning parameters such as the numbers \( m_1, m_2 \) of basis functions for \( f_{jk} \) and the regularization parameters \( \lambda_\zeta, \lambda_t \). In order to select appropriate values of them, we use an AIC-type model selection criterion [Akaike 1974]. Using the result of [Hastie and Tibshirani 1990], the AIC for evaluating the statistical model is given by
\[ AIC = -2 \log p(y; \hat{\theta}, \hat{\Sigma}) + 2 \hat{df}, \]

where \( \hat{df} \) is an effective number of parameters obtained by
\[ \hat{df} = X(X^T \Sigma^{-1} X + n \Omega_\lambda)^{-1} X^T \Sigma^{-1}. \]

We select the values of the tuning parameters that minimize the AIC and treat the corresponding model as an optimal one.
4 Simulation

We conduct simulation studies to investigate the effectiveness of the proposed method. Here we referred the setting of the simulation study to Zhu et al. (2014). First, we set the eigenvalues of the functional predictors by $\lambda_k = 45.25 \times 0.64^k$, $k = 1, \ldots, q$, where the number of principal components is $q = 20$. Next we computationally generated the $k$-th PC scores of the $i$-th subject $\xi_{ik}$ from $N(0, \lambda_k)$. Here we used Fourier series for the eigenfunctions $\{\phi_k(s)\}$. Then the longitudinal data for the predictor are generated by

$$x_{i\tau} = \mu(s_{\tau}) + \sum_{k=1}^{q} \xi_{ik} \phi_k(s_{\tau}) + e_{i\tau},$$

where $\mu(s)$ is a mean function and here this is $\mu(s) = s + \sin(s)$, $s \in [0, 1]$ and $e_{i\tau} \sim i.i.d. N(0, 0.2)$. Furthermore, $\tau = 1, \ldots, r$ with $r = 21$ are the numbers of time points and we assume they are equally spaced on $[0, 1]$ and are the same for individual.

We set true unknown functions $f_k(\zeta, t)$ as follows:

$$f_1(\zeta, t) = \cos\{\pi(\zeta + t)\},$$
$$f_2(\zeta, t) = \sin\{2\pi(\zeta + t - 1/2)\},$$
$$f_3(\zeta, t) = \zeta^2 - 1/3,$$

and then the scalar response $y_i$ is given by

$$y_i = g_i + \varepsilon_i, \quad g_i = \sum_{k=1}^{q} f_k(\zeta_{ik}, t_i)$$

where $\zeta_{ik} = \Phi(\xi_{ik}; 0, \lambda_k)$, $\varepsilon_i \sim i.i.d. N(0, (\sigma R_y)^2)$ with $R_y = \max_i(g_i) - \min_i(g_i)$ and a standard deviation parameter $\sigma$. By the above setting, we can obtain a simulated dataset $\{x_{i\tau}, y_i; i = 1, \ldots, n, \tau = 1, \ldots, r\}$. Figure 1 shows simulated data for the predictor and the response.

For this dataset, we applied the proposed method and then estimated the unknown functions $f_k(\zeta, t)$ and evaluated the prediction accuracy. To do it, we transformed the data for predictor $\{x_{i\tau}\}$ into functional data $x_i(s)$, and then estimated the FPC scores $\xi_{ik}$ by applying the functional principal component
Table 1: Results for simulation studies. Values in the tables show averaged MSEs ($\times 10$) and their standard deviations ($\times 10^2$) are given in parentheses.

|          | VCFAM | VCFLM | FAM1 | FAM2 | FLM |
|----------|-------|-------|------|------|-----|
| n = 500  |       |       |      |      |     |
| $\sigma = 0.05$ | 0.230 | 3.005 | 4.402 | 5.435 | 6.138 |
|          | (1.247) | (1.880) | (2.734) | (3.194) | (3.326) |
| $\sigma = 0.1$ | 0.343 | 3.034 | 4.443 | 5.470 | 6.150 |
|          | (1.401) | (1.682) | (2.664) | (3.650) | (3.859) |
| n = 1000 |       |       |      |      |     |
| $\sigma = 0.05$ | 0.147 | 3.063 | 4.438 | 5.413 | 6.124 |
|          | (0.649) | (1.249) | (2.034) | (2.620) | (2.525) |
| $\sigma = 0.1$ | 0.207 | 3.098 | 4.450 | 5.421 | 6.108 |
|          | (0.787) | (1.364) | (2.080) | (2.642) | (2.536) |

We repeated this strategy for 100 times, and then calculated averages of 100 mean squared errors

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (g_i - \hat{y}_i)^2$$

where $\hat{y}_i = \sum_{k=1}^{q} \hat{f}_k(\zeta_{ik}, t_i)$. We compared the prediction accuracy of the proposed VCFAM with several other models; the VCFLM (3), FAM (2) with an additive nonlinear term for $t$ (denoted by FAM1), (2) itself (denoted by FAM2) and the functional linear model (FLM).

Table 1 shows results of the prediction. This shows that the proposed VCFAM minimizes the MSE for all cases, followed by VCFLM. FAM1 gives larger MSES compared to VCFAM and VCFLM, which indicates that the varying-coefficient model is more effective. Figure 2 shows true nonlinear functions $f_k(\zeta, t)$ and estimated functions obtained by averaging 100 estimated surfaces $\hat{f}_k(\zeta, t)$. These figures indicate that our method roughly reconstructs the true functions.

5 Real data analysis

We applied the proposed method to the analysis of the crop yield data for multi-stage tomatoes cultivated in a greenhouse on a farm in Kobe, Japan. Each seedling of the multi-stage tomatoes grows for about one year from August to next July, and are harvested almost every day from October to next July. In this study, we used daily yield data of single breed measured from October 2015 to July 2018, which is shown in Figure 3 left. However, the original data of the yield vary greatly because the yield is 0 when the farm is on a holiday, which makes the analysis too difficult. Therefore, we treat the moving averages of the yield up to 7 days from the harvest date as data. Environmental factors such as temperature and CO2 concentration are repeatedly measured by measuring equipment installed inside and outside the greenhouse, here we used the temperature inside the greenhouse as the data for environmental factor (Figure 3 right). It is considered that the growth of the tomato fruits are influenced by environmental factors during 80-day period before maturing the tomato. We used hourly averaged data for environmental factors as functional
data. Therefore, we constructed a regression model, treating the daily yield of tomatoes as a response and the temperature corresponding to 80 days before the maturing day as a functional predictor. A set of an yield of certain day and 80-day temperature before the day corresponds to an individual, as shown in Figure 4, and the sample size is $n = 836$. Readers may think that the analysis of this dataset can be applied by the function-on-function regression model with sparsely observed data discussed in Yao et al. (2005b). In our case, however, the number of time points for the response is one for individual, and is not included in the function-on-function regression model. If we have the crop yield data for decades of years, we may be able to apply the function-on-function regression models by treating the yearly crop yield data as individuals, but the observed period of the dataset is only three years. For such dataset, our VCFAM is applicable.

We transformed the data for the temperature into functional data and then calculated the FPCs, and then applied the VCFAM (4), where $s$ and $t$ correspond to the day before cultivation and the day of the year of the cultivation, respectively. The model is estimated by the penalized likelihood method and the tuning parameters are selected by AIC.

Figure 5 shows estimated first and second eigenfunctions $\phi_k(s)$ ($k = 1, 2$) of FPC and corresponding regression functions $f_k(\zeta, t)$. The eigenfunction for the first FPC means a high temperature especially at
Figure 3: Left: amount of crop yield for 3 terms, where the moving averages for 7 days are shown. Right: daily temperatures for 3 terms during the cultivation term.

Figure 4: Relation of the data for the analysis.

60 days before cultivation, and corresponding $f_k(\zeta, t)$ shows when and how this effect is high. The surface in the top right of Figure 5 shows positive at most area, which indicates that when the temperature at 60 days before cultivation is moderately, the crop yield is also high. In particular, in April, when temperature at that day before cultivation is moderately high, the crop yield increases. However, if the temperature is too high, the yield decreases. The second FPC shows the increase of the temperature in 80 days before cultivation. The corresponding regression function indicates that, for the crop yield in February and March, when the temperature decreases compared to 80 days before cultivation, the crop yield increases. On the other hand, in June, when the temperature increases compared to 80 days before cultivation, the crop yield increases.

We also performed the prediction of the crop yield. Here we predicted the weekly averaged yield for the $i_0 + 7$-th day using the data up to the $i_0$-th day, where $i_0$ is an index of the individual. The reason for predicting the crop yield for the $i_0 + 7$-th day instead of that for the $i_0 + 1$-th day is to prevent data leakage because the data for the response corresponding to $i_0$-th day consist of the average of the daily yield data from the $i_0$-th day to the $i_0 + 6$-th day. First, the data corresponding to the first two periods $\{x_i(s), y_i; i = 1, \ldots, i_0, i_0 = 550\}$ are used as training data. We applied the VCFAM to analyze this dataset, and then predicted the data for the $i_0 + 7$-th day $\{x_i(s), y_i; i = i_0 + 7\}$ as test data. We further
repeated this analysis by incrementing $i_0$ to $n - 7$ to predict the yield in the third period. Figure 6 shows the crop yield data for the third period and their predictions. The prediction results roughly capture the trends in the data. However, in some points they give large prediction errors. The reason for it seems that there are variations that were not seen in the two periods used as the training set. We also tried to apply the VCFLM as a similar way, but failed to predict since the tuning parameters are not selected appropriately by the model selection criterion.

6 Concluding Remarks

We have proposed a varying-coefficient functional additive model to capture the nonlinear structure of functional predictors and a scalar response varying with an exogenous variable. The proposed model is estimated by the penalized likelihood method, and then it is evaluated by a model selection criterion. Simulation studies show the effectiveness of our method in viewpoints of the prediction and the reconstruction of the effect of the predictor on the response. We also applied the proposed method to the analysis of crop yield data to investigate the effect of the environmental factors to the cultivation.

In this work we considered the situation with a single functional predictor, while the future work include the extension to that with multiple functional predictors. The crop yield is considered to be
affected by not only the temperature but also several other environmental factors such as CO\textsubscript{2} concentration and solar radiations in the greenhouse. In this case, we also want to know which combination of environmental factors relates to the crop yield. To do so, the application of the sparse regularization \cite{Hastie2015} to the VCFAM is considered. It also remains as a future work to introducing sparsity inducing penalties to the VCFAM framework, using the idea of \cite{Ravikumar2009} and \cite{Matsui2011}.

Acknowledgments

We would like to thank Higashibaba farm providing the data for cultivation of tomatoes. This work is supported by JSPS KAKENHI Grant Number 19K11858 and JST PRESTO Grant Number JPMJPR16O6.

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