Multi-UAV Objective Assignment Using Hungarian Fusion Genetic Algorithm

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ABSTRACT In the background of air combat, the situation between multiple unmanned aerial vehicle (multi-UAV) and objectives has a certain impact on the task assignment. In order to improve the efficiency of assignment and obtain the best assignment scheme during the process of performing tasks, this paper proposes a method to evaluate the situation at a certain time. This method is the basis for establishing a multi-UAV objective assignment model. For solving the model, this paper presents the Hungarian fusion Genetic Algorithm. It first uses the feasible solutions solved by the Hungarian algorithm as the elite individuals in the initial population of the genetic algorithm, and then uses the objective function in the assignment model as the fitness function to optimize the results. The algorithm solves the problem that the assignment result of the Hungarian algorithm is not unique, and optimizes the drawback that the traditional Genetic Algorithm is prone to fall into local optimum. The simulation verified the effectiveness of the situational assessment method and the improved algorithm.

INDEX TERMS Situational assessment method, objective assignment model, Hungarian algorithm, genetic algorithm.

I. INTRODUCTION

With the development of science and technology, unmanned aerial vehicle (UAV) has been extensively used in various domain. Because of its advantages of low cost, strong mobility, and low safety risk coefficient, it can play an important role in the military, such as monitoring, search and rescue [1], reconnaissance [2], [3], attack [4], and so on. However, since the mission execution capability of a single UAV is limited by the load, they cannot perform large-scale complex missions. An important solution to solve this problem is the use of multi-UAV with holistic combat capabilities.

On the air combat battlefield, the multi-UAV formation will face multiple objectives at the same time during the mission. In this scenario, in order to complete the mission in an orderly and efficient manner, it is necessary to quickly assign the objectives. Rapid and accurate objective assignment can avoid situations such as multi-UAV attacking one objective or missing targets, which can result in wasted resources or even mission failure. Therefore, multi-UAV multi-target assignment is critical to air warfare. The objective assignment belongs to NP-hard combinatorial optimization problem [5].

The idea of solving it is to construct an assignment model and then calculate the model. To address this problem, a large number of articles has been published, such as mixed integer linear programming [6], simulated annealing [7], particle swarm optimization (PSO) [8], Genetic algorithm [9], and expect system method, etc. Reference [10], starting from the model, uses the expert system method to tackle it. The paper established a rule base for the situational environment and built an objective assignment model based on a priori knowledge, which improved the efficiency of the assignment. Therefore, this method could only deal with the known cases, it could not achieve a completely independent assignment. Reference [11], starting from the solution method, integrates the simulated annealing algorithm with the discrete PSO. The paper solved the problem that the discrete PSO easily falls into the local minimum solution. Reference [12] designed an air warfare Nash equilibrium solution method using an improved particle swarm optimization algorithm (PP-PSO), but the method had to first decompose the problem into a game between single UAVs, which could not be applied to large-scale allocation. As a heuristic algorithm, GA has been applied to task allocation [13]. Reference [14] proposes a Discrete Genetic Algorithm (DGA) that used a discrete population generation algorithm to optimize
the population initialization mechanism and solve the task assignment. Reference [15] used the past cost and task cost as penalty factors to modify the objective function and then designed genetic codes to improve the GA based on physical significance. Reference [16] presents to combine SA and GA. The paper took advantage of the strong local search ability of SA to improve the GA search efficiency. Reference [17] proposes a fuzzy elite strategy genetic algorithm. The paper improved the gene delivery strategy and solved the deadlock problem in the stochastic optimization process.

Different from the above reference, in order to analyze the objective assignment in the air combat, the distance and angle between UAV and objective, survival probability, and value are added to the objective assignment function as influence factors; in order to improve the efficiency of assigning the assignment model and jump out of the local optimal solution, an objective assignment algorithm based on Hungarian fusion Genetic Algorithm (HGA) is proposed. This paper, firstly, builds an assignment model based on an air combat situation; secondly, uses the Hungarian Algorithm to calculate the objective function to obtain the initial feasible solution, and then incorporates the result into the initial population of the GA; finally, uses the objective function as the fitness function of GA and adopts the elite selection strategy to carry out the selection operation. Based on the above, the optimal assignment scheme with the highest reward value is obtained.

This paper is organized as follows. Section II introduces the air combat environment model. In section III, we use the air combat model as the basis for establishing the objective assignment model. Section IV explains the Hungarian fusion Genetic Algorithm in this paper. Section V demonstrates the effectiveness of the HGA and illustrates the advantages of the method via simulation and comparison. Finally, the concluding remarks are stated in Section VI.

II. PROBLEM FORMULATION
Multi UAV air combat environment is complex and dynamic, and the number, position of drones on both sides are constantly updated according to it. Therefore, the UAV should make online mission decisions based on the real-time situation. To facilitate the analysis of the confrontation task objective assignment for multi-UAV at a certain moment, the paper defines the adversarial model shown below. To simplify the problem, we model UAVs as point mass with zero width and make the following additional assumptions:

1. UAVs are at the same altitude, so the objective assignment in a two-dimensional Cartesian coordinate system is the focus of this paper.
2. UAVs fly at a constant speed.
3. UAVs are of the same type, have the same platform performance, and are capable of correctly identifying the enemy and friendly aircraft.
4. UAVs have entered an operational area that does not take into account any external features, such as terrain, obstacles, etc. [2]

The red has N drones, which are recorded as \( UAV_R^i = \{ UAV_R^1, UAV_R^2, \ldots, UAV_R^N \} \), where the \( i \)th red drone be denoted via \( UAV_R^i \). Similarly, the blue has \( N + 1 \) drones as well as a High Value Target (HVT) that needs to be protected. The UAVs of blue are denoted by \( UAV_B^i = \{ UAV_B^1, UAV_B^2, \ldots, UAV_B^{N+1} \} \), where \( UAV_B^i \) denotes the \( i \)th blue aircraft. The HVT denotes the High Value Target in the Blue, and it contains only location information \( x_{HVT} \).

In general, the state matrix of UAV is shown in Eq. (1):

\[
\xi_i^R(t) = [x_i^R(t), v_i^R(t), p_i^R(t), \theta_i^R(t)] \quad (1)
\]

The state matrix contains information on the velocity \( v_i^R(t) \), position \( x_i^R(t) \), survival probability \( p_i^R(t) \), and the number of missiles \( \delta_i^R(t) \), where \( t \) denotes the combat time, R and B denote the Red and Blue, respectively. The equation of state for each UAV can be expressed as:

\[
\begin{align*}
\dot{x}_i^R(t) & = v_i^R(t)  \\
\dot{v}_i^R(t) & = a_i^R(t)  \\
\dot{p}_i^R(t) & = p_i^R(t) A_i^R(t)  \\
\dot{\theta}_i^R(t) & = \delta_i^R(t) - n_i^R(t)
\end{align*}
\]

where \( a_i^R(t) \) denotes the acceleration at moment \( t \), which is used to update the speed and position of the UAV. This paper assumes that UAVs fly at a uniform speed, so it is equal to zero. \( A_i^R(t) \) denotes the survival probability from the current moment \( t \) to the next moment, depending on the enemy UAV’s attack on the drone at the current moment.

\[
A_i^R(t) = \prod_{j=1}^{N_B} [1 - P_{ji}^{BB}(t)]^{\delta_{ji}^{BB}(t)}
\]

where the parameters in Eq. (2) and (3) in conjunction with:

- \( N_R \): Number of UAVs on the Blue.
- \( \delta_{ji}^{BR}(t) \): Number of missiles launched by the \( j \)th UAV of the Blue against the \( i \)th UAV of the Red at time \( t \).
- \( P_{ji}^{BR}(t) \): Kill probability of missile, which is related to the combat environment, combat situation, and ideal kill probability of missile:

\[
P_{ji}^{BR}(t) = \beta S_{ji}^{BR} n_{ji}^{BR}
\]

\( 0 < \beta < 1 \) - Environmental impact factor, indicating the impact of the environment on missile kill probability.

\( S_{ji}^{BR} \): Situation matrix of the \( j \)th UAV of the Blue against the \( i \)th UAV of the Red.

\( n_{ji}^{BR} \): Ideal kills probability of missile.

What’s more, the number of missiles launched by Red during the period \( t \) is:

\[
\delta_{ji}^R(t) = \sum_{j=1}^{N_B} \delta_{ij}^{RR}(t) + \delta_{ib}^{RR}(t)
\]

where \( \delta_{ji}^R(t) \) and \( \delta_{ib}^{RR}(t) \) respectively denote the number of missiles launched by the \( i \)th UAV against the \( j \)th UAV and
HVT at the time \( t \).

\[
\begin{align*}
S_{ij}^{RB}(t) = 1, & \quad d_{ij}^{RB}(t) \leq r_{missile}, S_{ij}^{RB}(t) \geq S_T \\
S_{ij}^{RB}(t) = 0, & \quad \text{other}
\end{align*}
\]

\[
\begin{align*}
S_{ib}^{RB}(t) = 1, & \quad d_{ib}^{RB}(t) \leq r_{missile}, S_{ib}^{RB}(t) \geq S_{Tb} \\
S_{ib}^{RB}(t) = 0, & \quad \text{other}
\end{align*}
\]

where, \( d_{ij}^{RB}(t), d_{ib}^{RB}(t) \) - Respectively denote the distance of the \( i \)th UAV to the \( j \)th UAV and the HVT at the time \( t \).

\( S_{ij}^{RB}(t), S_{ib}^{RB}(t) \) - Respectively denote the situation matrix.

\( S_T, S_{Tb} \) - Respectively denote superiority threshold for attacking enemy fighters and HVT.

### III. OBJECTIVE ASSIGNMENT MODEL

#### A. SITUATIONAL ASSESSMENT METHODOLOGY

Situational assessment is the premise of UAV aerial combat. The key aspect of winning the battle is that UAVs infer the threat degree of both as well as the best feasible response depending on the situation.

The traditional angular dominance function model does not adequately capture the excellent off-axis launch and omnidirectional attack capability of third-generation air-to-air/ground missiles [18]. When the target azimuth is large, the UAV loaded with third-generation missiles still has the ability to enter the attack zone and constitute launch conditions by a large overload maneuver. When the azimuth is small, the missile attacks omnidirectionally with higher hit rates depending on the situation. Therefore, it is important to improve the situational assessment methods to evaluate the situation between UAVs and objectives on the air combat battlefield.

1) SITUATIONAL ASSESSMENT BETWEEN UAVs

According to the scenarios of the air-to-air game, the situation relationship among UAVs at moment is depicted in Fig. 1. Where \( \alpha_{ij} \) represents the azimuth angle between the Red UAV and the enemy aircraft, i.e., it is the angle between the Red UAV velocity vector \( v_i \) and the dual aircraft linkage, and the right of the line is the positive direction, satisfying \( \alpha_{ij} \in [-\pi, \pi] \).

1) The azimuth of red UAV is mainly considered, then the angle advantage function is designed as follows:

\[
S_{\text{angle}}^{ij} = \frac{v_i}{1000} e^{-\frac{|\alpha_{ij}|}{\alpha}} \quad \alpha_{ij} \in [-\pi, \pi] \quad (8)
\]

It can be seen from the formula that the smaller angle has a greater benefit than the larger one. Namely, the smaller the objective azimuth, the faster the UAV can reorient its movement toward the enemy aircraft.

2) Take into consideration the distance between UAVs, the maximum range of missiles \( d_{\text{missile}} \), and the maximum detection range of airborne radar \( d_{\text{radar}} \), all of which are sufficient to satisfy the criteria \( d_{\text{missile}} < d_{\text{radar}} \). The distance advantage function is designed as follows:

\[
S_{\text{dist}}^{ij} = \begin{cases} 
1 & d_{ij} < d_{\text{missile}} \\
1 - \frac{d_{ij} - d_{\text{missile}}}{d_{\text{radar}} - d_{\text{missile}}} & d_{\text{missile}} \leq d_{ij} \leq d_{\text{radar}} \\
0 & d_{ij} > d_{\text{radar}}
\end{cases} \quad (9)
\]

It is apparent from this formula that the shorter the distance between UAVs, then the greater the distance advantage.

To sum up, the comprehensive advantage index of the \( i \)th UAV to the \( j \)th UAV is \( S_{ij} \). It is equal to the sum of each dominance indices multiplied by the linear weighting of the degree of importance.

\[
S_{ij} = \tilde{\omega} S_{\text{angle}}^{ij} + \omega_1 S_{\text{dist}}^{ij} \quad (10)
\]

where \( \tilde{\omega} = \omega_1 + \omega_2 \) represents the weight vector, \( \omega_1 \) and \( \omega_2 \) respectively represent the weight of the dominant index, satisfying \( \omega_1, \omega_2 > 0 \).

2) SITUATIONAL ASSESSMENT BETWEEN THE UAV AND THE HVT

Fig. 2 shows the situational interaction between a UAV and an enemy HVT at a certain moment, such as bridges, dams, etc. Where \( d_{ib} \) represents the distance between the UAV and the HVT, \( \alpha_{ib} \) represents the azimuth angle between the UAV and the HVT, i.e., it is the angle between the velocity vector \( v_i \) of UAV and the line linking UAV to HVT, to the right of the line is the positive direction, satisfying \( \alpha_{ib} \in [-\pi, \pi] \).

1) The azimuth of red UAV is mainly considered, then the angle advantage function is designed as follows:

\[
S_{\text{angle}}^{ib} = \frac{v_i}{1000} e^{-\frac{|\alpha_{ib}|}{\alpha}} \quad \alpha_{ib} \in [-\pi, \pi] \quad (11)
\]
From the formula above, the angle advantage is greatest when the UAV is flying towards the HVT. The larger 
the UAV azimuth angle, the correspondingly smaller the 
angular dominance.

2) Referring to Eq. 9 to design the distance advantage 
function as:

\[ s_{\text{dist}}^{ib} = \begin{cases} 
1 & d_{ib} < d_{\text{missile}} \\
1 - \frac{d_{ib} - d_{\text{missile}}}{d_{\text{radar}} - d_{\text{missile}}} & d_{\text{missile}} \leq d_{ib} \leq d_{\text{radar}} \\
0 & d_{ib} > d_{\text{radar}} \end{cases} \] (12)

The formula illustrates that the closer the UAV is to the 
HVT, the larger the distance advantage.

Similarly, the comprehensive advantage index of the ith 
UAV to the HVT is \( S_{ib} \). It is equal to the sum of each 
dominance indices multiplied by the linear weighting of the 
degree of importance:

\[ S_{ib} = \omega_{1}^{b} S_{\text{angle}}^{ib} + \omega_{2}^{b} S_{\text{dist}}^{ib} \] (13)

where, \( \omega = \omega_{1}^{b} + \omega_{2}^{b} \) represents the weight vector, \( \omega_{1}^{b} \) and 
\( \omega_{2}^{b} \) respectively represent the weight of the dominant index, 
satisfying \( \omega_{1}^{b} + \omega_{2}^{b} = 1, \omega_{1}^{b}, \omega_{2}^{b} > 0 \). The HVT’s situation 
threat to the UAV can always be considered as zero, with no 
operational capability.

B. REWARD FUNCTION

While taking into account factors such as the survival 
probability \( p_{ib}^{B} \), value \( \text{val}_{ib}^{B} \) of the Blue targets, the situ-
tational advantage matrix \( S \) of the Red UAV over the 
Blue UAV as well as the HVT at a certain moment, the 
reward function of the ith UAV over the jth is designed as:

\[ R_{ij}^{RB} = [\mu S_{ij}^{RB} + (1 - \mu)p_{ij}^{B}] \cdot \text{val}_{ij}^{B} \] (14)

where \( \mu \) represents the weighting coefficient. The larger 
value \( \mu \) means the UAV is attacking the objective with the 
greater situation, and conversely, the smaller value of it means 
the UAV is attacking the objective with the greater probability 
of survival.

In the same way, the reward function for the ith UAV to the 
HVT is designed as:

\[ R_{ib}^{RB} = [\mu S_{ib}^{RB} + (1 - \mu)p_{ib}^{B}] \cdot \text{val}_{ib}^{B} \] (15)

where, \( p_{ib}^{B}, \text{val}_{ib}^{B} \) represent the survival probability and value of 
HVT, respectively.

C. OBJECTIVE FUNCTION

The objective function must be able to capture the whole 
scope of the problem’s complexity. Due to the difficulty of the 
study, this objective function can often be quite complicated.

In this paper, the objective function for the purpose of reward 
maximization is designed as follows:

\[
\begin{align*}
\max & \sum_{i=1}^{N_{B}} \sum_{j=1}^{N_{B}} [\mu S_{ij}^{RB} + (1 - \mu)p_{ij}^{B}] \cdot \text{val}_{ij}^{B} \cdot \phi_{ij}^{RB} \\
& + [\mu S_{ib}^{RB} + (1 - \mu)p_{ib}^{B}] \cdot \text{val}_{ib}^{B} \cdot \phi_{ib}^{RB} \\
\text{s.t.} & \sum_{i=1}^{N_{B}} \phi_{ij}^{RB} \leq \min \{1, \delta_{i}^{R} \} \\
& \phi_{ij}^{RB} \cdot \phi_{ib}^{RB} \in [0, 1]
\end{align*}
\] (16)

The first constraint indicates that each enemy aircraft can 
be assigned to at most one UAV; the second constraint indi-
cates that the number of targets assigned to the ith UAV is less 
than or equal to one and must not exceed the current number of 
remaining weapons.

IV. ALGORITHM SOLUTION

UAV objective assignment is a combinatorial optimization 
problem. On the one hand, Hungarian algorithm [19] is an 
overall assignment algorithm, which is used to rescue the 
objective assignment problem. However, in this paper, the 
reward function (Eq. 15) requires the participation of the sit-
tuational information between the UAV and the objective to 
solve, including the distance, angle, survival probability, and 
value. When using the Hungarian algorithm to solve the 
objective function based on this information, there will be a 
scenario where the assignment scheme is different, but the 
total reward value is the same. In this way, it is impossible 
to deal with the problem of the objective assignment in air 
combat.

On the other hand, the Genetic Algorithm is also a common 
solution method. The basic principle of Genetic Algorithms 
is to find the optimal solution by simulating the genetic 
strategy of a population in nature, using individuals to iter-
ate in the search space again [20]. The drawbacks of them 
are the tendency to fall into local optimal solutions and the 
slow convergence rate. This paper proposes the Hungarian 
fusion Genetic Algorithm (HGA) to overcome these prob-
lems. It optimizes the initial feasible solution obtained via 
the Hungarian algorithm as the elite individuals in the initial 
population of the genetic algorithm. The HGA effectively 
compensates for the shortcomings of both algorithms and 
obtains an objective assignment scheme with maximum total 
reward and uniqueness.

In the Hungarian fusion Genetic Algorithm, as shown in 
Fig.3, each assignment cycle is divided into an assignment 
phase and an optimization phase. In the assignment phase, 
the UAV performs the situational assessment base on the 
situation information, and then calculates the reward function 
for each objective to form the reward matrix. After that, HGA 
carries out the combined assignment of the reward matrix 
to obtain the assignment matrix. In the optimization phase,
we put the assignment matrix into the initial population as elite individuals, use the objective function of the assignment model as the fitness function, and apply the elite roulette selection strategy to genetically optimize the population.

### A. OBJECTIVE ASSIGNMENT PHASE

The classical Hungarian algorithm deals with the assignment problem by processing and solving the objective function as a whole. It is characterized with simple steps, the ability to obtain an overall optimal assignment scheme, and the absence of verification [21]. However, the application of it requires the following conditions to be satisfied:

1. The objective function is to find the minimum size;
2. The coefficient matrix is square;
3. Non-negative values of the elements of the coefficient matrix.

The objective function of this paper is to solve for the highest reward assignment scheme, in a way, it can be viewed as a variant of the assignment problem.

According to the objective assignment model, the reward function matrix is defined as $\text{Reward}$ with the value of the reward from the red UAV to the blue UAV and the HVT as elements:

$$
\text{Reward} = \begin{bmatrix}
R_{11}^{RB} & R_{12}^{RB} & \cdots & R_{1N}^{RB} \\
R_{21}^{RB} & R_{22}^{RB} & \cdots & R_{2N}^{RB} \\
\vdots & \vdots & \ddots & \vdots \\
R_{N1}^{RB} & R_{N2}^{RB} & \cdots & R_{NN}^{RB}
\end{bmatrix}
$$

The core idea of the Hungarian algorithm [22] is to subtract the same number simultaneously for a row or a column of the matrix without affecting the assignment result. According to it, we mathematically process the reward function matrix to transform the maximum value into a minimum value problem that can be solved by the Hungarian algorithm to get the assignment matrix $M$.

$$
M = \begin{bmatrix}
x_{11} & \cdots & x_{1j} & \cdots & x_{1n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
x_{i1} & \cdots & x_{ij} & \cdots & x_{in} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
x_{n1} & \cdots & x_{nj} & \cdots & x_{nn}
\end{bmatrix}
$$

where, the value $x_{ij}$ is equal to 0 or 1. 0 means that the objective is not selected, and 1 means that the objective is selected.

### Algorithm 1 Improved Hungarian Algorithm

**Input:** $\text{UAV}^R = \{\text{UAV}_1^R, \text{UAV}_2^R, \ldots, \text{UAV}_N^R\}$, $\text{UAV}^B = \{\text{UAV}_1^B, \text{UAV}_2^B, \ldots, \text{UAV}_N^B\}$, $\text{HVT}$

1. $R_{ij}^{RB} = \text{reward}(\text{UAV}_i^B, \text{UAV}_j^R)$ for $i=1,2,\ldots,N; j=1,2,\ldots,N-1$
2. An $n \times n$ matrix of reward
3. $\text{Reward} = \{R_{11}^{RB}, R_{12}^{RB}, \ldots, R_{N-1,N-1}^{RB}, R_{N,N-1}^{RB}\}$
4. $\text{Reward} \leftarrow \text{Reward} + \min(\text{Reward})$

**Output:** A complete matching, $M$

### B. OBJECTIVE OPTIMIZATION PHASE

1) **INITIAL POPULATION**

The size of the initial population is set to $\text{pop}$, and each assignment matrix is an individual. The $\lambda$ assignment matrices obtained by solving the Hungarian algorithm are added to the initial population as elite individuals, and the remaining individuals ($\text{pop} - \lambda$) in the population are generated randomly.

2) **FITNESS FUNCTION**

The fitness function is an important metric used to assess whether an individual has reached optimality. In this paper, the objective function proposed is used as the fitness function. The larger the fitness value is, the better the assignment scheme is.

3) **SELECTION OPERATOR**

The selection operation can effectively improve the convergence speed of the algorithm. The article uses the elite roulette for selection operation. This operation selects a subset of the best individuals (those obtained by solving the Hungarian algorithm) in the population to form an elite population to keep a backup. To prevent the elite individuals from being lost in the mutation operation, they are inherited directly to the offspring without participating in the mutation operation. The non-elite individuals calculate the probability of being selected in the offspring according to the formula $P_{\text{sel}}$, and form the offspring population based on it. From Eq.17, it can be seen that the higher the fitness value, the greater the probability that an individual will be selected. The selection steps are as follows.

1) Elite individuals are inherited directly to the next generation, and non-elite individuals calculate the probability of selection:

$$
P_{\text{sel}}(i) = \frac{R(i)}{\sum_{i=0}^{\text{pop}} R(i)} \quad (17)
$$
2) Individual selection of the initial population according to Eq.17. Selection results and elite individuals form new populations.

\[ d_p(k + 1) = \begin{cases} 
    d_p(k + 1) & \text{non-elite} \\
    d_p(k) & \text{elite}
\end{cases} \quad (18) \]

where, \( d_p(k + 1) \) is the \( p \text{th} \) individual in the \( k+1 \) iteration.

4) MUTATION OPERATOR

The mutation operation can increase the diversity of the population and prevent the algorithm from converging prematurely. The article uses chromosomes for mutation, where each individual randomly selects two rows or columns to swap with each other. Two positions within the parent chromosome Parent1 are randomly selected for swapping to produce the offspring Children1. The diagram of the mutation operation is as follows:

The mutation operation steps are as follows:

1) \( P_m \geq \text{rand} \)

\( P_m \) is the mutation probability and \( \text{rand} \) is a random number between 0 and 1. If the \( P_m \) is greater than the \( \text{rand} \) perform the mutation, otherwise don’t execute this operation.

2) Find the new population after mutation according to the \( P_m \).

\[ d_p(k + 1) = \begin{cases} 
    v_p(k + 1) & P_m \geq \text{rand} \\
    d_p(k) & \text{elite}
\end{cases} \quad (19) \]

V. SIMULATION

A. SMALL-SCALE UAVs

To verify the effectiveness of the designed situational assessment method as well as the HGA algorithm, a five-on-four small-scale air battle is conducted with four UA Vs and HVTs from the Blue side for the objective to be assigned and simulated in MATLAB.

Table 1 and Table 2 show the coordinates, velocity, and other information of the red as well as blue respectively. Where, \( x(m), y(m), \phi(\text{rad}) \), \( v(m/s) \) denotes the horizontal coordinates, vertical coordinates, heading angle and velocity of UA Vs, respectively. Table 3 is the weight coefficient table. Table 4 shows the situation values of the Red UAV against the Blue objective calculated according to the assessment method.

Table 5 shows the assignment schemes and the corresponding maximum fitness values obtained from the three algorithms. The rows of the assignment matrix M denote UAV_R, and the columns denote UAV_B.

From Table 5, it can be seen that different assignment results yield different objective function values, and the largest value is the assignment scheme solved by the HGA.
The variation curves of the best fitness for the three algorithms are shown in Fig. 4. The red dotted line is the solution result of HGA with the maximum value of 3.4297. The light blue solid line is MGA with the maximum value of 3.4004. The blue dashed line is GA with the maximum value of 3.3320. It can be concluded from Fig.4 that the optimal solution of HGA can obtain the maximum reward. Compared with GA, both HGA and MGA can suppress the problem that the algorithm is premature and easy to fall into local optimum, but the convergence speed of HGA is better than MGA. HGA converges to the optimal solution around the 7th step of the iteration, while MGA converges around the 14th step. It shows that HGA can effectively increase the convergence speed of the algorithm and also avoid the algorithm from falling into the local optimum.

Figure 5 shows the box plots of the optimal fitness values of the three algorithms. Comparing the box plots, it can be seen that the fitness values of both HGA and MGA are higher and less variable than GA, but there are discrete points in MGA, so its stability is slightly worse than HGA. It can be concluded that HGA can obtain a more stable solution.

### B. LARGE-SCALE UAVs

Under the same condition of the weight, the objective assignment problem is solved by HGA in the environment where the number of UAVs and objectives are changed.

Table 6 shows the optimal objective function values of the three algorithms for UAV swarms in different dimensions.

![FIGURE 5. Comparison of the best fitness.](image)

| Algorithm | Dimension 5 | Dimension 10 | Dimension 20 | Dimension 50 |
|-----------|-------------|--------------|--------------|--------------|
| GA        | 3.3320      | 4.9118       | 10.0795      | 22.8125      |
| MGA       | 3.4004      | 4.9466       | 10.1652      | 22.8488      |
| HGA       | 3.4297      | 4.9673       | 10.3985      | 23.9793      |

![FIGURE 6. Comparison of the optimal fitness box.](image)
operating environment, and their comparative relationship is reflected in Fig. 9.

It can be seen from the data in Fig. 9 and Table 8 that HGA takes less time to solve the problem than GA and MGA, which fully reflects the advantages of HGA. HGA effectively improves the efficiency of the algorithm, while overcoming the disadvantage of easily falling into local optimum to get the best assignment scheme.

TABLE 7. Running time (s).

| Algorithm | 5    | 10   | 20   | 50   |
|-----------|------|------|------|------|
| GA        | 1.257| 2.2193| 3.0575| 19.2743 |
| MGA       | 1.228| 1.8432| 2.8493| 18.6468 |
| HGA       | 0.102| 0.1325| 0.1697| 1.2692 |

VI. CONCLUSION

A new objective assignment algorithm for multi-UAV based on the Hungarian fusion Genetic algorithm is proposed in this paper namely HGA to solve the assignment model in the air combat at a certain time. Firstly, a multi-UAV countermeasure model is established under the battlefield environment and UAV status.; Secondly, an air combat situation assessment method is designed based on the countermeasure model for solving the air combat situation. Thirdly, the objective assignment model is established based on the situation assessment function. Finally, the Hungarian fusion Genetic algorithm is used to solve and optimize the objective assignment model.

In order to solve the assignment scheme that is not unique, HGA puts the feasible solution solved by the Hungarian algorithm into the initial population of the genetic algorithm as an elite individual, and then inheritance of it to offspring using the elite roulette selection. At the same times, the contribution of this study has been improved the defect of Genetic Algorithms that is trapped into local optimal solutions. Through analyzing the comparison experiments with traditional Genetic Algorithm, the effectiveness of the HGA algorithm is verified, and the assignment results with higher gain values are obtained; Through analyzing the comparison experiments with Multi population Genetic Algorithm, stability and fast convergence of the HGA algorithm is demonstrated. Through simultaneous analysis and comparison experiments of the two algorithms, HGA shortens the model solving time and improves the assignment efficiency.

Overall, the objective assignment model and solution method based on the situation between the UAVs and objectives in the air combat environment can effectively analyze the battlefield situation and obtain the best assignment plan. This has engineering significance for the actual air warfare environment, which lays the foundation for multi-UAV to perform missions.

A limitation of this study is that HGA can be used for the balanced objective assignment problem with the same number. But in actual air combat, the number of UAVs on each side is not necessarily equal. Further work needs to be done to deal with the unbalanced objective assignment problem with the inconsistent number of parties. The flight speed and altitude of the UAV are also a problem to be considered.
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