Sequential nature of \((p,3p)\) two-proton knockout from neutron-rich nuclei

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Twenty-one two-proton knockout \((p,3p)\) cross sections from neutron-rich nuclei at \(\approx 250\) MeV/nucleon were measured. For the first time, the angular distribution of the three emitted protons were measured in coincidence with the tracker and hydrogen target MINOS, demonstrating that the \((p,3p)\) kinematics is consistent with two sequential proton-proton collisions within the projectile nucleus. Ratios of \((p,3p)\) over \((p,2p)\) inclusive cross sections follow the trend of other many-proton (neutron) removal reactions, further reinforcing the sequential nature of \((p,3p)\) in neutron-rich nuclei.

The nuclear shell model, wherein nucleons layer onto quantized energy orbitals grouped into shells, has been remarkably successful in describing the overall properties of atomic nuclei [1–3]. Similarly to electrons in atoms, shell closures give rise to enhanced stability. However, contrary to the atomic case, the nuclear shell structure is not universal, but changes for neutron-rich nuclei [4]. This shell evolution originates in the properties of the many-body nuclear interactions and is currently the focus of intense study, motivating the development of dedicated new generation radioactive beam facilities [5–9] and experimental methods in inverse kinematics. In particular,
one-nucleon knockout reactions at intermediate energies, above $\sim 50$ MeV/nucleon, have been extensively and successfully used for the spectroscopy of unstable isotopes, accessible only at low intensities, both with heavy targets [10, 11] and more recently, proton targets [12, 13]. In one-nucleon knockout, the reaction mechanism is understood as a one-step collision between the removed nucleon and the target, the rest of the projectile nucleus being a spectator, and much theoretical effort has been devoted to describe the process [11, 12, 14–16]. Recent spectroscopic results can be found in [17–19]. Given the success of one-nucleon knockout reactions, two-nucleon knockout has naturally garnered much interest, as it allows to explore further nuclear properties, such as nucleon-nucleon correlations [20–24]. Two-proton knockout from neutron-rich nuclei has been used to populate very exotic species, in a process that can be assumed to be direct, due to the high energy required for proton evaporation in these nuclei [25–29]. In particular, two-proton knockout on a proton target, $(p, 3p)$, is a promising reaction to explore nucleon-nucleon correlations. Remarkably, the reaction $^{80}$Zn$(p, 3p)$ has recently been able to populate new $^{78}$Ni states, inaccessible to one-nucleon knockout [19]. This noteworthy result reinforces the potential of $(p, 3p)$ reactions as spectroscopic tools on exotic nuclei and highlights the need of a proper understanding of their reaction mechanism, to allow for the extraction of quantitatively reliable information from them.

In this work we demonstrate that the $(p, 3p)$ reaction at intermediate energies takes place mainly through two sequential proton-proton collisions. We analyze for the first time twenty-one $(p, 3p)$ reactions on neutron-rich nuclei, using a unique setup [30] to detect the three emitted protons originating from the knockout on an event-by-event basis, thus measuring their angular distributions as well as the inclusive cross sections.

The measurements were conducted at the Radioactive Isotope Beam Factory (RIBF), operated jointly by the RIKEN Nishina Center and the Center for Nuclear Study of the University of Tokyo. They were divided into two consecutive experimental campaigns, containing three and four settings each. A $^{238}$U beam was accelerated to 345 MeV/nucleon and impinged onto a 3 mm beryllium target. The average beam intensity was 12 pnA for the first campaign and 30 pnA for the second. Fragments were produced via in-flight ablation-fission and were identified in an event-by-event basis with the so-called $B\rho - \Delta E - \text{TOF}$ method through the BigRIPS spectrometer [31]. The magnetic rigidity ($B\rho$), energy loss ($\Delta E$) and time of flight (TOF) were obtained by parallel plate avalanche counters [31], ionization chamber and plastic scintillators, respectively. The secondary fragments impinged with an energy of $\sim 240$ MeV/nucleon onto the liquid hydrogen (LH$_2$) target of 102(1) and 99(1) mm thickness for the first and second experimental campaigns, respectively [30]. The LH$_2$ target was contained with 110$\mu$m entrance and 150$\mu$m exit Mylar windows. Emitted protons from the $(p, 3p)$ reactions were detected using MINOS [30], a vertex tracker consisting of a time projection chamber (TPC) surrounding the target as shown in Fig. 1. Protons traversing the TPC ionised the 82%Ar-3%isobutane-15%CF$_4$ gas mixture. An applied electric potential of 180 V/cm caused the generated electrons to drift towards the anode. The drift velocity ($\sim 4$ cm/$\mu$s) of the electrons was determined from experimental data. A full reconstruction of the track was performed through a Hough transformation [32–34], yielding the three-dimensional vertex position as well as the angles between the protons and the beam. The combined angular resolution for individual tracks was $\sim 7^\circ$ (FWHM) while the efficiency for the detection of three tracks in a $(p, 3p)$ event was $\sim 35\%$. The vertex distribution along the beam axis was validated by the length of the LH$_2$ target. Reaction channels were identified via the detected fragments. The energy of the outgoing protons was not measured. After losing 70 – 100 MeV/nucleon in the target area, the secondary fragments were identified via $B\rho - \Delta E - \text{TOF}$ and separated by the ZeroDegree spectrometer [35], operated in the large momentum acceptance mode $\pm 3\%$. Thus, only bound final states were measured. Details of the experimental campaigns can be found, for example, in [19, 36–43]. The inclusive $(p, 3p)$ and $(p, 2p)$ cross sections $\sigma$ were evaluated with:

$$\sigma = \frac{1}{n_{H_2} \cdot \tau \cdot N_i} \left( \frac{1}{1 + \nu} \right)$$

with the number of identified particles in BigRIPS and ZeroDegree, $N_i$ and $N_0$, which were selected in momentum to pass through ZeroDegree. $n_{H_2}$ was the areal density of the liquid hydrogen target, $\tau$ the transmission from...
the beam trigger detector at the end of BigRIPS to the end of ZeroDegree and \( \nu \) the areal density ratio of the material downstream the LH\(_2\) target and the LH\(_2\) target. \( \nu \) was 4.8\% for the first campaign, and 4.4\% for the second. The target density was calculated through the density of the LH\(_2\), and determined to be 70.97(3) g/l for the first campaign, and 73.22(8) g/l for the second, the areal density \( n_{\text{H}_2} \) was 4.32(4) and 4.33(4)·10\(^{22}\) atoms/cm\(^2\), respectively [36]. Only fully stripped ions identified in ZeroDegree were considered. For \((p, 3p)\) reactions, the number of events where two consecutive \((p, 2p)\) reactions take place inside the target have been estimated and subtracted. Experimental uncertainties were dominated by statistics of the secondary fragments and the charge state subtraction. For \((p, 3p)\) the removal of \((p, 2p)\) events can also be a significant contributor to the uncertainty.

As the reaction products were shifted partly out of momentum acceptance in ZeroDegree for both \((p, 3p)\) and \((p, 2p)\) reaction channels, the determination of the transmission was done by simulations with LISE++ [44]. It comprised the second half of the BigRIPS spectrometer, the LH\(_2\) target and the ZeroDegree spectrometer. The simulation takes into account the orientation of the pair, the proton (neutron) separation energy and \( S_p (n) \) being the Coulomb barrier [46]. The general trend of the data presented in [45] (shown with empty diamonds) is followed by the present ratios (full diamonds). This trend for proton-deficient nucleons (\( \Delta C > 0 \)) has been understood from comparison to nuclear-cascade simulations as uncorrelated multiple nucleon knockout. For \( \Delta C < 0 \) a thorough explanation can be found in [45]. Thus the \((p, 3p)\) reaction is likely to be of sequential nature.

In order to further constrain our understanding of the reaction mechanism, we compare angular correlations of the emitted protons to three kinematical models, with different descriptions for the reaction mechanism:

(i) Sequential: The reaction takes place through two sequential and independent proton-proton collisions, each following the free proton-proton cross section. Isotropic emission of the colliding protons in their center of mass is assumed, while the cross-section energy dependence is taken from [47]. Test calculations with anisotropic proton-proton cross sections from [48] show the same features as those with isotropic proton-proton cross sections. Apart from the projectile momentum, the protons inside the nucleus are assumed to have an intrinsic momentum \( \sqrt{p^2} = p_{\text{inc}} \).

(ii) Pair breakup: The two removed protons are assumed to form part of a correlated pair, with each proton having a momentum \( p_{\text{inc}} \), and with the overall pair having zero total momentum in the projectile rest frame. The orientation of the pair is assumed to be isotropic.

FIG. 2. (Left) Experimentally deduced cross sections. All \((p, 2p)\) cross sections are about two orders of magnitude larger than the \((p, 3p)\) cross sections. (Right) Ratio of \((p, 3p)\) over \((p, 2p)\) for the same incident nucleus, plotted against the evaporation cost asymmetry \( \Delta C \). Figure and references adapted from [45].
During the collision, the target proton interacts with only one of the protons of the pair, following the free proton-proton cross section as in the sequential model. The other proton of the pair acts as a spectator and escapes the nucleus with its original momentum.

(iii) Pair knockout: The two removed protons are assumed to be in a correlated pair as in the pair breakup case. During the collision, the target proton interacts with the whole pair, following the elastic \( p,d \) angular distribution, taken from [49]. Afterwards, the protons in the pair are each emitted with their intrinsic momentum plus half the momentum of the pair after the collision.

All models consider the interaction between the protons and the residual nucleus through the following prescription: We assume that if any of the protons has an energy lower than \( E_{\text{thresh}} \) in the projectile rest frame it will not exit the core, and the process will not contribute to the reaction. Furthermore, the potential between the residual nucleus and the protons will deflect their trajectories on the way out of the nucleus. We model this deflection by adding a random momentum to each of the protons, taken from a Gaussian distribution with a width of \( p_{\text{def}} \). For the components perpendicular to the beam direction the mean value of the Gaussian is assumed to be 0, while for the direction of the beam, the mean value is set to \( +p_{\text{def}} \), to reflect the pull of the residual nucleus (which keeps the speed of the beam) over the proton (which has lost speed in the collision).

Each Proton track contributes one angle \( \theta \) between itself and the beam axis and one angle \( \lambda \) between itself and all other proton tracks. The three parameters \( p_{\text{nuc}} = 200 \text{ MeV}/c, E_{\text{thresh}} = 30 \text{ MeV} \) and \( p_{\text{def}} = 18 \text{ MeV}/c \) have been adjusted to reproduce experimental relative \( \lambda \) and beam \( \theta \) angle (see Fig. 1) distributions for the two outgoing protons in \( ^{81}\text{Ga}(p,2p) \), assuming a quasifree model. We note that the values for \( p_{\text{nuc}} \) and \( E_{\text{thresh}} \) are close to the standard values for the Fermi momentum and the potential well in the INCL model: 270 MeV/c and 40 MeV, respectively [50].

Events generated with each model are then used as an input for a \textsc{Geant4} simulation [51], so that the same analysis is applied to the simulated events and the experimental data.

The observables we have chosen to explore are \( \theta \), the angle between the outgoing protons and the beam, \( \lambda \), the relative angle between each pair of protons, and \( \varphi_{s} \) and \( \varphi_{m} \), the smallest and second smallest angles that each pair of protons form in the plane perpendicular to the beam. For the \( (p,3p) \) reaction \( \varphi_{l} \), the largest angle, is defined by \( \varphi_{s} + \varphi_{m} + \varphi_{l} = 360^\circ \). An illustration of these observables is given in Fig. 1.

The chosen observables have not been found to vary significantly from one projectile to another. As such, we have chosen to focus on \( ^{81}\text{Ga}(p,3p)^{79}\text{Cu} \) and \( ^{81}\text{Ga}(p,2p)^{80}\text{Zn} \), for which statistics were the largest. Events exceeding a vertex uncertainty of 10 mm have been omitted.

In Fig. 3 the double distribution for the \( \varphi_{s} \) and \( \varphi_{m} \) angles is plotted for the \( ^{81}\text{Ga}(p,3p) \) and the three models. All models have been normalised to the experimental number of counts. The phase space is restricted to a triangle, as \( \varphi_{l} > \varphi_{m} > \varphi_{s} \). The experimental data shows that the angular distribution is largely symmetric: \( \varphi_{m} \sim 180^\circ - \varphi_{s}/2 \), but has a wide spread to smaller \( \varphi_{s}, \varphi_{m} \). The sequential model reproduces the features of the experiment very well. Most events are symmetric and there is an accumulation at \( \varphi_{s} = \varphi_{m} \approx 120^\circ \). The distribution, however, does not extend to small \( \varphi_{s}, \varphi_{m} \) as the experimental one.

The pair breakup model shows events following a curved line from \( (\varphi_{s}, \varphi_{m}) = (0^\circ, 180^\circ) \) to \( (100^\circ, 100^\circ) \), which can be understood as the distortion due to \( p_{\text{nuc}} \) of the \( (\varphi_{s}, 180^\circ - \varphi_{s}) \) line expected from a back-to-back emission, with the spectator proton emitted in a random direction. The pair knockout model presents a maximum at \( 85^\circ, 135^\circ \), which is not observed in the experimental data.

The comparison between the quasifree model and \( (p,2p) \) data is presented in the insets of Fig. 4. The assumed quasifree model reproduces remarkably well the data for all observables. The value of \( p_{\text{def}} \) is considerably smaller than \( p_{\text{nuc}} \) and the beam momentum per nucleon, so its effect on the observables is only moderate, although it is essential to reproduce the position of the peak of the
Counts

The pair breakup model agrees similarly well, but fails to reproduce the tail above 105°. The pair knockout model features a slow rise followed by a strong peak, whose position has a large uncertainty and does not reproduce the data.

The middle panel of Fig. 4 shows the θ angles. MI-NOS acceptance cuts angles below 10°. Experimental data exhibit a steep slope and peak at 20°. They then fall linearly, reaching zero at 65°. The sequential model reproduces remarkably the experimental shape, as opposed to the other models. The pair breakup model features a strong peak at 15°, caused by the spectator proton, whose intrinsic momentum $p_{\text{nuc}}$ is much smaller than the beam momentum, so its beam angle is severely restricted, while the other two protons show a mostly statistical shape. The pair knockout model has the same steep slope at low angles as the data, with a double peak structure, not seen in the data, whose most prominent peak is shifted by 17° when compared to the data and it presents a sharp decrease after the peak. This decrease is caused by the limitations of the $p, d$ elastic scattering in inverse kinematics (which is assumed in the pair knockout model), so the target proton cannot leave with an angle larger than 30°.

For λ data show a rather symmetric peak at 55°, with a linear falloff to the sides of the maximum with very few events for λ > 100°. None of the models describes fully the experimental distribution, with the sequential model showing the best agreement, although the peak position is shifted by 10°, and the distribution is more asymmetric than the data. The pair breakup model produces a markedly different shape, with a prominent peak at 85°, easily understood as the quasifree peak between the two colliding protons. The pair knockout model increases with a larger slope than the data, peaking at 45°, and then drops off for smaller angles than the data.

For the three exclusive observables considered, the best agreement has been obtained with the sequential model. As the half-transparent bands for the $(p, 3p)$ models in Fig. 4 show, a variation on the parameters of the toy models, constrained by $(p, 2p)$ observables, does not significantly modify the features in $(p, 3p)$ observables, showing the robustness of these conclusions with respect to the parameters of the models. A fit of the models in $\varphi_s$ and θ simultaneously[52] yields contributions of $86^{+10}_{−9}$° for sequential, $14^{+7}_{−6}$% for pair knockout and $0^{+5}_{−3}$% for pair breakup processes with a reduced $\chi^2 = 0.79$, demonstrating the sequential nature of this reaction. This result sets a solid basis for a quantitative description of the $(p, 3p)$ reaction at intermediate energies, opening a new probe for nuclear structure, in particular, the population and description of two-particle two-hole configurations in neutron-rich nuclei. Previous works have shown a relative importance of correlated pairs of around 50% in two-nucleon knockout at lower energies (93 MeV/nucleon) using heavier targets [20]. We have not found such a strong contribution of correlated pairs, possibly due to

FIG. 4. Distribution for the projected angle $\varphi_s$ (Top), the beam angle θ (Middle) and the angle between the scattered protons λ (Bottom) for $^{81}\text{Ga}(p,3p)$ and $^{81}\text{Ga}(p,2p)$ (in insets). The predictions from pair breakup, pair knockout and sequential model are also displayed. Half-transparent bands show the dependence on the parameters of the model (See text for more information). In all three cases, especially for $\varphi_s$ and θ, the sequential model describes the data best. $p_{\text{nuc}}, E_{\text{thresh}}$ and $p_{\text{def}}$ have been fitted to reproduce $(p, 2p)$ data.

$(p, 2p)$ relative angles λ. As a test of the sensitivity to the parameters of the models, we have performed four different fits to $(p, 2p)$ observables: fitting λ and θ separately, letting $p_{\text{def}}$ vary or fixing it to 18 MeV/c for each observable. The envelope of all of these results is shown as the half-transparent band around the best fit line in Fig. 4, both for $(p, 2p)$ and $(p, 3p)$ reactions.

In the main panels of Fig. 4, the distributions for the smallest projected angle $\varphi_s$ (top), the beam angles θ (middle) as well as the angles between the scattered protons λ (bottom) are shown for $^{81}\text{Ga}(p, 3p)$. The experimental data for the $\varphi_s$ angles (top) feature a steep slope at 10°, reaching a large plateau up to 95° and falling to zero at 120°. The sequential model follows the data very closely over the full range. The pair breakup model
the higher beam energies in this work, which result in a larger mean free path of the recoil protons in the nucleus.

In conclusion, we have presented twenty-one new $(p, 3p)$ cross sections on neutron-rich medium-mass nuclei at energies of $\sim 250$ MeV/nucleon. Our measurement shows that the ratios $\sigma_{(p,3p)}/\sigma_{(p,2p)}$ follow the systematics of [45], which points to the sequential nature of the $(p, 3p)$ process. For the first time, the angular distributions of the three outgoing protons have been measured thanks to the unique combination of the MINOS charged-particle tracker and elongated liquid hydrogen target. These angular distributions were compared to three kinematical models, obtaining for the sequential target. These angular distributions were compared to three kinematical models, obtaining for the sequential model a very good agreement, while other models show poor reproduction of the data. The combined results for inclusive cross sections and angular distributions prove the kinematics of the reaction mechanism to be of sequential nature, $86^{+10}_{-6} \%$ within our kinematical framework. The sequential description of the $(p, 3p)$ reaction from neutron-rich nuclei at intermediate energies is therefore a reliable approximation for a quantitative description opening new opportunities to explore nuclear structure towards the neutron dripline.

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