Interedge Phase Coherence in Quantum Hall Line Junctions

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Kang et al. have recently observed a remarkable zero-bias anomaly in the spectrum for electron tunneling between two 2D electron gases separated laterally by a narrow but high barrier in the presence of a perpendicular quantising magnetic field. We argue that this is a result of interedge phase coherence analogous to the interlayer phase coherence seen in quantum Hall (QHE) bilayer systems. The disruption of the QHE by the barrier is ‘healed’ by strong coulomb exchange enhancement of the weak tunneling through the barrier.

Kang et al. have recently observed a remarkable zero-bias peak of height $G \sim 0.1 e^2/h$ in the differential tunneling conductance between separate two-dimensional electron gases (2DEGs) for certain values of magnetic field in the integer quantum Hall (QHE) regime (see Fig. (1)). They used cleaved-edge overgrowth to construct a new sample geometry in which two adjacent 2DEGs in GaAs quantum wells lying in the same plane are separated horizontally by a narrow (88 Å) Al$_x$Ga$_{1-x}$As barrier forming a ‘line junction’ as illustrated in Fig. (2). In previous experiments, the barrier was defined electrostatically using top gates or narrow wires and so was much too thick (0.1 – 10 μm) to allow coherent tunneling.

A cleaved-edge barrier was used by Chang and collaborators to tunnel from a metallic electrode into a quantum Hall edge state. For the case of integer Landau level filling factor the differential conductance is flat and independent of voltage at small bias (as in a Fermi liquid), while for fractional filling factor $\nu = 1/3$, chiral Luttinger liquid physics yields an orthogonality catastrophe and hence a differential conductance which vanishes as a power law in bias voltage. The resonance-like peak observed near zero bias by Kang et al. for tunneling between two 2DEGs is quite different from either of these two behaviors. A vastly sharper zero-bias anomaly was discovered recently by Spielman et al. in interlayer tunneling in a bilayer QHE system in which the 2DEGs are separated vertically (i.e. in the MBE growth direction) rather than horizontally by a Al$_x$Ga$_{1-x}$As barrier. This feature is believed to be due to a novel broken symmetry producing spontaneous interlayer phase coherence.

In this paper we argue that the zero-bias anomaly seen by Kang et al. is due to the analogous effect of interedge phase coherence. Because the barrier thickness is actually smaller than the spacing between electrons in the plane, strong Coulomb correlations between the two sides can ‘heal’ the disruption in the QHE state caused by the barrier. We develop a bosonization scheme with parameters derived from Hartree-Fock estimates to describe this process.

Tunneling in the absence of a magnetic field is a relatively simple matter. Electrons approaching the barrier attempt to tunnel and either succeed or are reflected and never return again to the barrier. The Landauer conductance is $G_0 = 2e^2/N_\perp |T|^2$ where the number of transverse channels for the electron waves in a sample of width $W$ is $N_\perp = k_F W/\pi$ and the transmission probability (averaged over transverse channels) is $|T|^2$. Kang et al. found $1/G_0 \approx 450 k\Omega$ for a sample with density $2 \times 10^{11} cm^{-2}$ and $W = 100 \mu m$. Using this and solving the Schrödinger equation for the transmission we estimate the height of the 88 Å barrier to be 212 meV which is in reasonable agreement with the barrier of 232 meV expected for Al$_x$Ga$_{1-x}$As if we use the average value $x = 0.3$ appropriate for the particular ‘digital’ barrier in this sample.

The physics of tunneling through a line junction in the presence of a quantizing magnetic field is quite different. The magnetic field traps the electrons into skipping orbits moving along the edge of the barrier in opposite directions on the two sides, an effect previously observed in 3D samples. Each electron thus makes not one, but many, attempts to tunnel and can form a coherent superposition of states on opposite sides. Kane and Fisher have modeled this situation assuming that the tunneling is a random function of position along the line and showed that for integer filling fractions, random tunneling is equivalent to random back scattering in a Fermi liquid with repulsive interactions and hence the states become strongly localized. However because the barrier material in this particular case is grown by MBE (during the first growth prior to cleavage), disorder effects can be neglected in the barrier, at least to a first approximation (see below however). Strong justification for this assumption is provided by vertical tunneling experiments (at $B = 0$) which clearly demonstrate nearly perfect momentum conservation for electrons passing through such alloy barriers.

To understand the electronic structure and particle transport in the vicinity of the barrier we begin by assuming that the barrier is infinitely long in the $y$ direction and presents a very high potential $V_B(x)$ which is
symmetric about x = 0. Choosing the Landau gauge $A = eB x \hat{y}/c$, the single particle wave functions are of the form $\psi(x) = e^{i k_x x} \phi_k(x)$ where $\phi_k(x)$ is an eigenfunction of the following Hamiltonian

$$H_k(x) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + \frac{1}{2} m \omega^2_c (x - k t)^2 + V_B(x)$$  \hspace{1cm} (1)$$

For each value of momentum $k$, there is a solution on the left side of the barrier and another on the right side of the barrier. For the special value of $k = 0$, $H_k$ is reflection symmetric in $x$ and the two solutions will be degenerate in energy. If the barrier height is large but finite, then these solutions will mix weakly and a gap $\Delta$ will open in the spectrum in the vicinity of the degeneracy point as illustrated in Fig. (2). Away from the degeneracy point it is not possible to tunnel through the barrier because states at the same momentum have very different energies.

In QHE bilayer systems it has proven extremely useful to introduce a pseudospin 1/2 to represent the layer index degree of freedom. The analog here represents the edge mode label. The low-energy physics of the edge modes in this language is captured (for the non-interacting case) by the 1D Hamiltonian

$$H = \left( \begin{array}{cc} -i \hbar v_0 \partial_y & t_0 \\ t_0 & +i \hbar v_0 \partial_y \end{array} \right)$$  \hspace{1cm} (2)$$

where $v_0 \sim 1.3 \omega_c \ell$ is the bare edge mode velocity, $\ell$ is the magnetic length, $\omega_c$ is the cyclotron frequency and for an 88A barrier of height 212 meV, the bare gap is $\Delta_0 = 2t_0 = 0.52K$. The ground state pseudospin orientation as a function of momentum $k$ determined from the eigenfunctions of $H$ are illustrated in Fig. (3). In the middle of a gap $\Delta$, the decay length for the evanescent wave $\xi = 2\hbar v_0/\Delta$ which is on the scale of 1$\mu$m for the bare gap and therefore much smaller than the $\sim 100\mu$m barrier length in the Kang et al. samples. We assume a picture in which the transport current is carried by chiral edge modes which feed into the barrier modes. If the chemical potential lies within the gap for the barrier modes, the edge modes of the Hall bar fail to propagate along the barrier and must be perfectly transmitted along the edge of the Hall bar (since the mode is chiral). On the other hand, if the chemical potential lies outside the gap, the edge mode connects smoothly onto the barrier mode and is perfectly transmitted along the barrier to the chiral edge mode on the opposite edge of the sample as illustrated in Fig. (3). This picture, which assumes a QHE plateau and $\sigma_{xy} = 0$ in the bulk, predicts a Landauer conductance peak of $e^2/h$. The observed peak value of $0.1e^2/h$ represents an enhancement of about a factor of 2 over the $B = 0$ conductance but is considerably smaller than the ideal Landauer result. This may be a consequence of bulk transport which puts the system in an intermediate regime between the $B = 0$ limit and the idealized edge state picture of transport and allows electrons to leak away from the barrier before they have had many opportunities to tunnel back and forth to establish the coherent resonance gap. Unfortunately the contact geometry in the experiment did not permit a determination $\sigma_{xx}$ in the QHE regime. We also note in passing that the conductance reported by Kang et al. used the voltage drop between points V1 and V2 shown in Fig. (2) instead of between points V1 and V4. The distinction is not crucial when the peak height is much less than $e^2/h$ however. As we discuss further below, inhomogeneities in the donor density provide another possible explanation of the reduced height and large width of the conductance peak.

A second peculiarity of the experimental data is that the zero-bias peak occurs over a rather significant range $\delta \nu \sim 0.3$ centered at filling factor $\nu^* \approx 1.35$. The width $\delta \nu$ is well more than an order of magnitude wider than the expected value $\delta \nu \sim \Delta_0/\hbar \omega_c$ given the tiny bare gap of 0.52K. Takaguchi and Ploog (TP) have modeled the barrier transport using a non-interacting electron tight binding model with lattice constant $a = \ell/4$ and a near-neighbor hopping strength of $t = \hbar^2/(2m_0) = 8\hbar \omega_c$. In a purely phenomenological fit to the data, the barrier is modeled simply as a line of bonds with reduced strength $t' = 0.02t \sim 20K$. This is two orders of magnitude larger than the bare tunnel amplitude $t_0 = 0.26K$ computed from first principles above. The tight-binding model used by TP puts the level crossing slightly below the second Landau level, however for non-interacting electrons with a realistic treatment of the barrier, we find the crossing above the second Landau level.

In an attempt to resolve these paradoxes we have investigated the role of strong Coulomb interactions along and across the barrier. We find in a self-consistent Hartree calculation that the uncompensated background charge underneath the barrier lowers the level crossing to below the second Landau level which is roughly consistent with the experimental value of $\nu^*$ (assuming, as TP do, complete spin polarization). Using the self-consistent Hartree orbitals we further find that Coulomb exchange leads to a large enhancement of the gap in the Hartree-Fock (HF) spectrum to $\sim 10K$ (which is roughly consistent with the observed peak width). However by bosonization (mapping the system onto a Luttinger liquid) we find that the true gap is reduced by quantum fluctuations.

The HF variational ansatz for the many-body wave function is

$$|\Psi\rangle = \prod_k (\cos \frac{\theta_k}{2} e^{ik y} + \sin \frac{\theta_k}{2} e^{ik y^*})|0\rangle$$  \hspace{1cm} (3)$$

The order-parameter representing inter-edge phase coherence in this state

$$\langle \Psi_\uparrow(y) \Psi_\downarrow(y) \rangle = e^{i \alpha y} \frac{1}{4\pi} \int dk \sin \theta_k$$  \hspace{1cm} (4)$$
tumbles at rate $\alpha$ along the $y$ direction.
We define a dimensionless variable $\varphi(y)$ that represents the phase of the order-parameter as it tumbles. Evaluating the expectation value of the number density operator in the HF state yields

$$\delta \rho(y) = \frac{1}{2\pi} \alpha = \frac{1}{2\pi} \partial_y \varphi$$

which is the usual Luttinger liquid result.

Computing the expectation values of the kinetic, tunneling, Hartree, and exchange energies for the HF ansatz states yields the following effective Hamiltonian density in terms of the order parameter field $\varphi(y)$ and the charge density imbalance $m_z \equiv n_\uparrow(y) - n_\downarrow(y)$

$$H_{\text{eff}} = \frac{\rho_s}{2} (\partial_y \varphi)^2 + \frac{\Gamma}{2} m_z^2 - t_0 \cos \varphi(y)$$

We quantize this by making use of the pseudo-spin commutation relations $[S_y, S_z] = i\hbar S_z$, which in terms of the operators $\varphi$ and $m_z$, may be written as (assuming $\varphi$ is small) $[\varphi, m_z/2] = i\hbar$. Thus, $\varphi$ and $m_z/2$ are canonically conjugate to each other and the lagrangian density may be written as

$$L = \frac{1}{8\pi g} \left[ \frac{1}{c} (\partial_t \varphi)^2 - c (\partial_y \varphi)^2 \right] - t_0 \cos \varphi(y)$$

In the absence of the tunneling term, this is the lagrangian density of a Luttinger liquid with interaction parameters $g = \sqrt{\Gamma/\rho_s/2\pi}$ and collective mode velocity $c = 2\sqrt{\rho_s\Gamma}$. In this language, the tunneling term is equivalent to $2k_F$ backscattering. An important property of our Luttinger liquid action is the anisotropy of the coulomb interactions in the pseudospin space which destroys the galilean invariance in the problem.

Our Hartree-Fock theory enables us to explicitly calculate the parameters $\rho_s$ and $\Gamma$ and hence the Luttinger liquid parameter $g$. The calculations have been done for a long range coulomb potential, so the parameters $\rho$ and $\Gamma$ are momentum dependent, with a $\ln k$ singularity in $\rho_s$, and under these conditions the resulting action is not strictly a Luttinger liquid. To regulate this weak divergence we assume metallic screening from nearby gates (or from the finite density of states in the bulk, although this would violate our assumption of $\sigma_{xx} = 0$) and in that limit we have a Luttinger liquid whose parameters depend on the Thomas-Fermi screening length. Table I lists the $k \rightarrow 0$ values for the Luttinger liquid parameter for different Thomas-Fermi screening lengths. These calculations have been done for the experimental barrier width of 88Å and also for a narrower barrier width of 52Å, the latter having a non-interacting tunnel splitting which is almost 10 times that of the former.

In order to estimate $t_0 \ell$, we choose the configuration of the pseudo-spin corresponding to $\varphi = 0$ everywhere and evaluate the total tunneling energy in the corresponding HF state $t_0 \ell = \frac{1}{2\pi} \int dk \ell \Delta_k \sin \theta_k$ where $\Delta_k/2$ is a measure of the amount by which the energy is lowered as a result of tunneling. At $k = 0$, $\Delta_0$ is the bare tunnel gap. The variational parameter $\sin \theta_k$ is unity at $k = 0$, but falls very rapidly to zero (within $|k| < 0.1$) because the high velocity in the region of the barrier makes it energetically costly to have a coherent superposition of left and right orbitals for large $k$. Thus we can safely make the approximation, $t_0 \ell = \frac{1}{2\pi} \Delta_0 \int dk \ell \sin \theta_k = 0.0053 \Delta_0$.

Our HF calculations show an exchange enhanced gap of $\sim 10K$, some $\sim 20$ times larger than the bare gap. While this indicates the importance of inter-edge coulomb interactions, the HF gap represents the excitation gap when the order parameter is held fixed. The true low-energy charge excitations come from fluctuations of the order-parameter field. In the absence of tunneling, the resulting U(1) symmetry of Eq.(7) guarantees that the charge excitations are gapless even though the HF gap survives the limit of zero tunneling. We see from Eq.(6) and Eq.(7) that in the presence of tunneling, the charged excitations are solitons having non-trivial topological charge $\varphi(+\infty) - \varphi(-\infty) = \pm 2\pi$.

The classical expression for soliton energy (which becomes exact in the strong interaction limit $g \rightarrow 0$) is

$$\Delta_0 = 16\sqrt{\rho_p} s$$

The sine-gordon model in Eq.(7) is integrable and the expression for the soliton mass including all quantum corrections is exactly known and in the field theory limit $t_0/\hbar c \Lambda^2 \ll 1$ where $\Lambda$ is the momentum cutoff. The momentum cut off for the sine-gordon theory that naturally emerges from our HF calculations is $\Lambda = \frac{1}{2} \int d\sin \theta_k$. Physically, this corresponds to the cutoff being proportional to the HF excitation gap, i.e., determined non-perturbatively by the interactions themselves. Mathematically, we arrive at this expression by comparing the coefficient of the cos $\varphi(y)$ term in the lagrangian obtained from our Hartree-Fock analysis, and that obtained from bosonising the tunneling operator. This procedure is not exact but can be justified in the limit of small $g$.

The charge gap calculated for two different barrier widths is shown in Table I. For the barrier of width 88Å we find that this number is about 1.3K. This is smaller than the classical value of 4.8K but more than twice the size of the bare non-interacting value. It is much too small to explain the large width of the zero-bias conductance peak seen in the experiment.

These results imply that it is necessary to invoke disorder. We imagine that the MBE barrier is still smooth and momentum conserving, but note that the random variations in the donor density which broaden the Landau levels in the bulk would lead to a random potential which would enter the Luttinger liquid action as forward scattering terms of the form $V_+(y) \partial_y \varphi + V_-(y) m_z$. While this model ultimately flows under renormalization onto the Kane and Fisher model of random tunneling along the barrier, we note that, paradoxically, the initial effect of this type of disorder is to enhance rather than destroy the propagation along the barrier since even if the chemical potential is centered on the nominal charge gap, back scattering is effective only in small regions where the random potential happens to pass through zero. This strong
enhancement of the localization length could simultane-
ously explain both the reduced height and the large ob-
served width of the zero-bias peak. Some back scattering
will occur over a range of chemical potential determined
by the strength of the random potential rather than the
nominal size of the charge gap.

Our results suggest that a second generation of sam-
pies with lower disorder and a somewhat thinner barrier
could show the idealized behavior modelled here. In such
samples dynamical effects associated with the interedge
coherence should be observable. While the dephasing
time may be too short to observe the AC oscillations sug-
gested by Ho, microwave resonance absorption similar
to that already seen in 3D is likely to be visible.

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TABLE I. Numerical results for model parameters for two values of Thomas-Fermi screening constant $q_{TF}$ and barrier thickness $w_b$. Calculations were done for a magnetic field of $B = 6T$, magnetic length $\ell = 105\,\text{Å}$ and cyclotron energy $\hbar\omega_c = 10.4\text{meV}$. $\Delta_0$ and $\Delta_c$ are the bare and renormalized charge gaps. $\Gamma_0$ and $\Gamma_{\text{int}}$ are the bare and interaction contributions to the Luttinger liquid stiffness parameter $\Gamma$. $\rho_0^s$ and $\rho_{\text{int}}^s$ are the bare and interaction contributions to the Luttinger liquid compressibility parameter $\rho_s$. The parameter $g$ is the Luttinger exponent, $\Lambda$ is the estimated cutoff wavevector, and $c$ is the collective mode velocity in the absence of tunneling.