PROBING GRAVITY AT LARGE SCALES THROUGH CMB LENSING

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ABSTRACT

We describe a methodology to probe gravity with the cosmic microwave background (CMB) lensing convergence \( \kappa \), specifically by measuring \( E_G \), the ratio of the Laplacian of the gravitational scalar potential difference with the peculiar velocity divergence. Using \( \kappa \) from CMB lensing as opposed to galaxy-galaxy lensing avoids intrinsic alignments while also lacking a hard limit on the lens redshift or significant uncertainties in the source plane. We model \( E_G \) for general relativity and modified gravity, finding that \( E_G \) for \( f(R) \) gravity should be scale-dependent due to the scale-dependence of the growth rate \( f \). Next, we construct an estimator for \( E_G \) in terms of the lensing convergence-galaxy and galaxy angular power spectra, along with the RSD parameter \( \beta \). We also forecast statistical errors of \( E_G \) from the current Planck CMB lensing map and the CMASS and LOWZ spectroscopic galaxy samples measured from the Baryon Oscillation Spectroscopic Survey (BOSS), as well as BOSS spectroscopic quasars, from the Sloan Digital Sky Survey Data Release 11. We expect this experimental configuration to detect \( E_G \) at the level of 11\( \sigma \) with CMASS/LOWZ galaxies alone and 13\( \sigma \) when BOSS quasars are included. We also find that both upcoming spectroscopic and photometric surveys, each set combined with the final Planck lensing map, can measure precisely both the redshift- and scale-dependence of \( E_G \) out to redshifts \( z = 2 \) and higher, with photometric surveys having an advantage due to their large survey areas and high number densities. Tracing the CMB lensing field using Advanced ACTPol increases the sensitivity to \( E_G \) even further. Finally, we find that Advanced ACTPol cross-correlated with spectroscopic (photometric) surveys can differentiate between general relativity and modified gravity at the level of 3\( \sigma \) (13\( \sigma \)). Performing a < 1\% measurement of \( E_G \) requires precision in \( \beta \) on the order of 10\%, which is currently achievable with a spectroscopic survey but will be difficult with only a photometric survey.

Keywords: cosmology: theory, gravity, observations, large scale structure of the universe

1. INTRODUCTION

The discovery of cosmic acceleration [Riess et al. 1998 Perlmutter et al. 1999] has inspired numerous theoretical explanations for its existence. On one hand, the acceleration can be caused by a new, unknown force that exhibits dark energy, is a major component of the ΛCDM framework that explains the expansion history and the growth history of the Universe [Anderson et al. 2014] and the cosmic microwave background (CMB) [Bennett et al. 2013] Planck Collaboration et al. 2013]. On the other hand, it is possible that the dynamics of gravity deviate from general relativity (GR) on cosmological scales [Dvali et al. 2000] [Carroll et al. 2004], a concept called modified gravity. In observations of the universe’s expansion history, these two effects are degenerate with one another. However, we expect the growth of structure to differ between dark energy and modified gravity models such that the degeneracy is broken.

An observable that probes the expansion history and growth of structure simultaneously is \( E_G \) [Zhang et al. 2007], which is related to the ratio of the Laplacian of the difference between the two scalar potentials \( \nabla^2 (\psi - \phi) \) to the peculiar velocity perturbation field \( \theta \). The value of \( E_G \) depends on how gravity behaves on large scales. Traditionally, when measuring \( E_G \), \( \nabla^2 (\psi - \phi) \) is probed by a galaxy lensing correlation with a tracer of large-scale structure (LSS), while the peculiar velocity field is probed by either a galaxy-velocity cross-correlation or, equivalently, a galaxy autocorrelation times the redshift-space distortion (RSD) parameter \( \beta = f/b \), where \( f \) is the growth rate and \( b \) is the clustering bias of the galaxies. A major advantage of \( E_G \) over other observables is that it is independent of clustering bias on linear scales, reducing the model uncertainty. \( E_G \) was first measured in [Reyes et al. 2010], using galaxy-galaxy lensing exhibited by Sloan Digital Sky Survey (SDSS) [York et al. 2000] Luminous Red Galaxies (LRGs) [Eisenstein et al. 2001] to find \( E_G(z = 0.32) = 0.392 \pm 0.065 \), consistent with ΛCDM.

In this analysis we assess the possibility of using CMB lensing [Blanchard & Schneider 1987] [Cole & Efstathiou 1989] cross-correlated with galaxies [Hirata et al. 2004] [Smith et al. 2007] [Hirata et al. 2008] Planck Collaboration et al. 2014] as a probe of \( \nabla^2 (\psi - \phi) \) instead of the traditional method of using galaxy lensing. One advantage of using CMB lensing over galaxy lensing is that the CMB lensing kernel is very broad over redshift, allowing probes of \( E_G \) at much higher redshifts than with galaxy lensing. At these higher redshifts, CMB lensing also has the added bonus of probing more linear scales, reducing systematic effects due to nonlinear clustering. Also, since the CMB propagates throughout all of space, all of LSS sampled by the survey lenses the CMB, allowing us to measure the lensing part of \( E_G \) at much higher redshifts. We also do not have to worry about complex astrophysical uncertainties of the source galaxies, i.e. in-
trintrinsic alignments, since the CMB is simple. Finally, we know the CMB redshift, so we can avoid determining the photometric redshift distribution of the sources. We construct an estimator for $E_G$ in terms of the angular cross-power spectrum between the CMB lensing convergence $\kappa$ and galaxies, the angular auto-power spectrum of the same galaxies, and the RSD parameter $\beta$.

Next, we derive $E_G$ for general relativity (GR), as well as modified gravity using the $\mu\gamma$ formalism from Hojjati et al. (2011). While $E_G(z)$ can be written as $\Theta_{m,0}/f(z)$ for $\Lambda$CDM, where $\Theta_{m,0}$ is the matter density today relative to the critical density and $f(z)$ is the growth rate at redshift $z$, $E_G$ for modified gravity models is expected to differ from this value. We also found that $E_G$ for f(R) gravity (Carroll et al. 2004) and chameleon gravity (Khoury & Weltman 2004) can exhibit scale-dependence $(\phi_{ch}^2)$ gravity (Carroll et al. 2004) and chameleon gravity (Khoury & Weltman 2004) can exhibit scale-dependence.

Next, we consider the prospect of measuring $E_G$ with CMB lensing by forecasting errors for an $E_G$ measurement using the current Planck CMB lensing map (Planck Collaboration et al. 2014) along with the CMASS galaxy $(z = 0.51)$, LOWZ galaxy $(z = 0.32)$, and BOSS quasar (Pâris et al. 2014) spectroscopic samples from Data Release 11 (DR11) (Anderson et al. 2014) of the SDSS-III Eisenstein et al. (2011) Baryon Oscillations Spectroscopic Survey (BOSS) (Dawson et al. 2013). We also consider possibilities with upcoming surveys. First, we consider the final Planck lensing map cross-correlated with spectroscopic surveys, specifically the Dark Energy Spectroscopic Instrument (DESI) (Levi et al. 2013), Euclid (Laureijs et al. 2011), and the Wide Field InfraRed Survey Telescope (WFIRST) (Spergel et al. 2013). We find, however, that an $E_G$ measurement using the Planck CMB lensing map and upcoming spectroscopic surveys is not sensitive enough to differentiate between GR and $f(R)$ gravity at current limits, and that the new limits on chameleon gravity would be modest. We find that spectroscopic surveys with Advanced ACTPol can differentiate between GR and $f(R)$ gravity at the level of 3σ, with higher significances for the chameleon gravity model.

We also consider the CMB lensing maps cross-correlated with photometric surveys, specifically the Dark Energy Survey (DES) (The Dark Energy Survey Collaboration 2005), the Large Synoptic Survey Telescope (LSST) (LSST Science Collaboration et al. 2009), and Euclid. We find that for the scales which $E_G$ dominates, being small but still only quasi-linear, the lensing measurement dominates the error in $E_G$ as opposed to the RSD. Thus, reducing the shot noise by increasing the survey number density is more important than having more precise redshifts for RSD. We find DES is comparable in power to DESI, and LSST and photometric Euclid can discriminate between GR and $f(R)$ gravity at very high significance using lensing from Adv. ACTPol. However, this will require an RSD precision on the order of 10%, either from the photometric survey probing $E_G$ or through a spectroscopic survey overlapping in redshift. Photometric surveys combined with CMB lensing experiments can produce significant constraints to $E_G$ that could help uncover the true nature of gravity.

The plan of the paper is as follows: in Section 2, we write the theoretical $E_G$ for modified gravity, while Section 3 gives the estimator for $E_G$ in terms of CMB lensing. In Section 4 we construct forecasts for various experiment configurations, and in Section 5 we present conclusions. We assume the combined CMASS/Planck cosmology (Planck Collaboration et al. 2013, Anderson et al. 2014) with $\Omega_m h^2 = 0.1418$, $h = 0.676$, $\Omega_b h^2 = 0.0224$, $n_s = 0.96$, and $\sigma_8 = 0.8$.

2. THEORY

We begin with the definition of $E_G$ in Fourier space from Zhang et al. (2007), given by

$$E_G(k,z) = \frac{c^2 [\nabla^2 (\psi - \phi)]_k}{3H_0^2 (1 + z) \theta(k)} = \frac{c^2 [\psi - \phi]}{3H_0^2 (1 + z) \theta(k)} ,$$

(1)

where $H_0$ is the Hubble parameter today. Assuming a flat universe described by a Friedmann-Robertson-Walker (FRW) metric with perturbation fields $\psi$ in the time component and $\phi$ in the spatial component, as well as negligible anisotropic stress and non-relativistic matter species, the time-space and momentum Einstein field equations in general relativity (GR) can be written in Fourier space as (Hojjati et al. 2011)

$$k^2 \psi = -4\pi G a^2 \rho(a) \delta$$

$$\phi = -\gamma(k,a) \psi,$$

(2)

where $a$ is the scale factor, $\rho$ is the background matter density, and $\delta$ is the matter density perturbation. These equations are generalized to a modified gravity (MG) model such that

$$k^2 \psi = -4\pi G a^2 \mu(k,a) \rho(a) \delta$$

$$\phi = -\gamma(k,a) \psi,$$

(3)

where $\mu(k,a)$ and $\gamma(k,a)$ are arbitrary functions of $k$ and $a$, and $\mu = \gamma = 1$ for GR.

Using this formalism, we can write the numerator of $E_G$ in Eq. 1 as

$$k^2 (\phi - \psi) = -k^2 [1 + \gamma(k,a)] \psi$$

$$= 4\pi G a^2 \rho(a) \mu(k,a) [1 + \gamma(k,a)] \delta .$$

(4)

Substituting $\rho(a) = \rho_0 a^{-3}$ and $\Omega_m,0 = 8\pi G \rho_0 / 3H_0^2$, we find

$$k^2 (\phi - \psi) = \frac{3}{2} H_0^2 \Omega_m,0 (1 + z) \mu(k,a) [\gamma(k,a) + 1] \delta$$

(5)

The velocity perturbation $\theta$ can be written as $\theta = f \delta$ on linear scales. Combining this expression and Eq. 5 gives $E_G$ from Eq. 1 as

$$E_G(k,z) = \frac{\Omega_m,0 \mu(k,a) [\gamma(k,a) + 1]}{2f} .$$

(6)

This expression gives us the correct value in the GR limit, namely $E_G = \Omega_m,0 / f(z)$.

For $f(R)$ gravity (Carroll et al. 2004) using the Bertschinger and Zukin (BZ) parametrization (Bertschinger & Zukin 2008), $\mu$ and $\gamma$ are given by (Hojjati et al. 2011)

$$\mu^{BZ}(k,a) = \frac{1}{1 - B_0 a^{-3}} \left[ 1 + (2/3) B_0 k^2 a^3 \right] \left[ 1 + (1/2) B_0 k^2 a^3 \right]$$

$$\gamma^{BZ}(k,a) = \frac{1}{1 - B_0 a^{-3}} \left[ 1 + (2/3) B_0 k^2 a^3 \right] \left[ 1 + (1/2) B_0 k^2 a^3 \right] .$$

(7)
\[
\gamma^R(k, a) = \frac{1 + (1/3)B_0 \bar{k}^2 a^s}{1 + (2/3)B_0 k^2 a^s},
\]

(7)

where \( \bar{k} = k[2997.9 \text{ Mpc}/h] \), \( s = 4 \) for models that follow \( \Lambda \text{CDM} \), and \( B_0 \) is a free parameter which is related to the Compton wavelength of an extra scalar degree of freedom and determined by boundary conditions for \( f(R) \) gravity. Current measurements limit \( B_0 < 5.6 \times 10^{-5} \) (1σ) \( \text{[Xu 2014, Bel et al. 2014]} \). For this gravity model, \( E_G \) is given by

\[
E_G^{BZ}(k, z) = \frac{1}{1 - B_0 a^{s-1}/6} \Omega_{m0} f_{BZ}(k, z),
\]

(8)

where \( f_{BZ}(k, z) \) is the BZ growth rate, which is scale-dependent since \( \mu \) is scale-dependent \( \text{[Hojjati et al. 2011]} \).

Chameleon gravity \( \text{[Khoury & Weltman 2004]} \) is a Yukawa-type dark matter interaction equivalent to a class of scalar-tensor theories with a scalar field non-minimally coupled to the metric. For chameleon gravity, also using the Bertschinger and Zukin (BZ) parametrization \( \text{[Bertschinger & Zukin 2008]} \), \( \mu \) and \( \gamma \) are given by \( \text{[Hojjati et al. 2011]} \)

\[
\mu^Ch(k, a) = \frac{1 + \beta_1 \lambda_1^2 k^2 a^s}{1 + \lambda_1^2 k^2 a^s} \]

\[
\gamma^Ch(k, a) = \frac{1 + \beta_2 \lambda_2^2 k^2 a^s}{1 + \lambda_2^2 k^2 a^s}
\]

\[
\lambda_2^2 = \beta_1 \lambda_1^2
\]

\[
\beta_2 = 2 \beta_1 - 1,
\]

(9)

where the typical ranges for \( \beta_1 \) and \( s \) are \( 0 < \beta_1 < 2 \) and \( 1 \leq s \leq 4 \). We will relate \( \lambda_1 \) to \( B_0 \), a parameter similar to that for \( f(R) \) gravity but much less constrained due to the extra degree of freedom in the model. The typical range for \( B_0 \) in this model is \([0,1]\) and is related to \( \lambda_1 \) by

\[
B_0 = \frac{2\lambda_1^2 H_0^2}{c^2}.
\]

(10)

We calculate \( E_G(k, z) \) for GR, \( f(R) \) gravity, and chameleon gravity using MGCAMB \( \text{[Lewis et al. 2000, Hojjati et al. 2011]} \), which we plot in Fig. 1. In GR, \( E_G \) is scale-independent. However, in \( f(R) \) gravity \( E_G \) decreases at small scales by \( \sim 10\% \), with the decrease being more pronounced at higher redshifts and smaller scales. On the other hand, chameleon gravity shifts \( E_G \) to lower values with respect to GR. These results show that the ability to measure the scale-dependence of \( E_G \) will be advantageous for constraining MG models, particularly for \( f(R) \) gravity.

3. ESTIMATOR

Here we derive an estimator for \( E_G \) in terms of the galaxy-convergence cross-power spectrum \( C_{\kappa g} \), the galaxy auto-power spectrum \( C_{\kappa \kappa} \), and the RSD parameter \( \beta \). The estimator is similar to the expression in \( \text{[Reyes et al. 2010]} \) using the galaxy-galaxy lensing cross-power spectrum.

![Figure 1](image-url)
Starting with Eq. [11], $E_G$ can be estimated in terms of power spectra as
\[
\hat{E}_G(k,z) = \frac{c^2 P_{\nabla^2(\psi - \phi)}(k)}{3H_0^2(1+z)P_{\theta}(k)}, \tag{11}
\]
where $P_{\nabla^2(\psi - \phi)}$ is the galaxy-$\nabla^2(\psi - \phi)$ cross-power spectrum and $P_{\theta}$ is the galaxy-peculiar velocity cross-power spectrum. Projecting 3D power spectra into angular quantities, we can estimate $E_G$ as
\[
\hat{E}_G(\ell,\bar{z}) = \frac{c^2 \tilde{C}_\ell^{\phi g}}{3H_0^2(1+\bar{z})C_\ell^{\phi g}} , \tag{12}
\]
where $\bar{z}$ is the average redshift of the galaxy sample. $C_\ell^{\phi g}$ is the galaxy-convergence angular cross-power spectrum, given on small scales using the Limber approximation by
\[
C_\ell^{\phi g} = \frac{1}{2} \int_{\chi_1}^{\chi_2} d\chi W(\chi)f_g(\chi)\chi^{-2}P_{\nabla^2(\psi - \phi)} \left( \frac{\ell}{\chi} , \bar{z} \right) , \tag{13}
\]
where $f_g$ is the galaxy redshift distribution and $W(\chi) = \chi(1 - \chi/\chi(\text{CMB}))$ is the CMB lensing kernel. In order to match the kernel in the galaxy-convergence power spectrum, we define $C_\ell^{gg}$, the velocity-galaxy angular cross-power spectrum, as
\[
C_\ell^{gg} = \frac{1}{2} \int_{\chi_1}^{\chi_2} d\chi W(\chi)f_g(\chi)\chi^{-2}P_{\theta} \left( \frac{\ell}{\chi} , \bar{z} \right) . \tag{14}
\]
This cross-power spectrum is a construct used to measure $E_G$ without multiplicative bias, and is not equivalent to the RSD angular power spectrum derived in Padmanabhan et al. [2007].

In our analysis, as in Reyes et al. [2010], we assume that the RSD parameter $\beta$ will be measured separately, and we approximate the lensing kernel, the redshift distribution, and $\beta$ as constants over redshift within the integral. Also, we assume from linear theory $\theta = f\delta$. In that case, Eq. [13] can be written as
\[
C_\ell^{\phi g} \approx \frac{W(\chi)\Delta \chi}{2} \int_{\chi_1}^{\chi_2} f_g(\chi)\chi^{-2}\beta(z)P_{\theta} \left( \frac{\ell}{\chi} , \bar{z} \right) \nonumber \\
\approx \frac{W(\chi)\Delta \chi \beta(\bar{z})}{2} C_\ell^{gg} , \tag{15}
\]
where $C_\ell^{gg}$ is the galaxy angular auto-power spectrum. Thus, $E_G$ in this case can be written as
\[
\hat{E}_G(\ell,\bar{z}) = \frac{2c^2 \tilde{C}_\ell^{\phi g}}{3H_0^2(1+\bar{z})W(\chi)\Delta \chi \beta(\bar{z})C_\ell^{gg}} . \tag{16}
\]
and we can write the error of $E_G$ in terms of the errors of $\beta$ and $C_\ell^{gg}$ as
\[
\frac{\sigma^2[E_G(\ell,\bar{z})]}{E_G^2} = \left\{ \left( \frac{\sigma(C_\ell^{gg})}{C_\ell^{gg}} \right)^2 + \left( \frac{\sigma(\beta)}{\beta} \right)^2 + \left( \frac{\sigma(C_\ell^{gg})}{C_\ell^{gg}} \right)^2 \right\} . \tag{17}
\]
The uncertainty in $C_\ell^{\phi g}$ can be written as
\[
\frac{\sigma^2(C_\ell^{\phi g})}{C_\ell^{\phi g}^2} = \left( \frac{C_\ell^{\phi g}}{C_\ell^{gg} + \sigma^2} + \frac{N_\ell^{\phi g}(C_\ell^{gg} + N_\ell^{gg})}{(2\ell + 1)f_{\text{sky}}} \right) , \tag{18}
\]
where $C_\ell^{\phi g}$, the convergence auto-power spectrum, is computed from CAMB [Lewis et al. 2000], $N_\ell^{\phi g}$ is the noise in the convergence power spectrum computed using the formalism in Hu & Okamoto [2002], and $N_\ell^{gg}$ is the shot noise.

A precise measurement of $E_G$ will be slightly biased from the true value due to several reasons similar to those outlined in the Appendix of Reyes et al. [2010]. For one, the lensing kernel and galaxy redshift distributions are not constants, requiring us to account for the weightings within each redshift sample. Also, in order to extend our measurement of $E_G$ to small scales, we must correct for the scale-dependence of the bias due to clustering at nonlinear scales. We may also need to consider scale-dependent $\beta$ due to nonlinear velocity perturbations, although velocity perturbations tend to stay linear at smaller scales than for density perturbations. We expect these effects to be small and will neglect them in our forecasts.

4. FORECASTS

In this section we will predict the ability of current and future surveys to measure $E_G$ and differentiate between GR and MG models. In all our forecasts we will assume GR when calculating uncertainties. We describe the sensitivity of the measured $E_G$ with the signal-to-noise ratio (SNR) of $E_G$ marginalized over angular scale and redshift, given by
\[
\text{SNR}^2(E_G) = \sum_{\ell,z_i} \frac{[E_G^{GR}(z_i)]^2}{\sigma^2(E_G(\ell,z_i))} , \tag{19}
\]
where $z_i$ denotes redshift bins. We also calculate the $\chi^2$ value between GR and MG models to determine if a particular $E_G$ measurement could distinguish between GR and a given MG model. We write $\chi^2$ as
\[
\chi^2(E_G) = \sum_{\ell,z_i} \frac{[E_G^{GR}(\ell,z_i) - E_G^{MG}(\ell,z_i)]^2}{\sigma^2(E_G(\ell,z_i))} , \tag{20}
\]
where $E_G^{MG}(\ell,z_i)$ is the $E_G$ estimate for a given MG model, which is generally $\ell$-dependent. Throughout the section we quote $\chi_{\text{rms}} = \sqrt{\chi^2}$. Note that for the following limits, $f(R)$ gravity is set to its upper limit value $B_0 = 5.6 \times 10^{-5}$, and that chameleon gravity’s base set of parameters is $B_0 = 0.4$, $\beta_1 = 1.2$, and $s = 4$, unless otherwise stated.

4.1. Current Surveys

We begin with forecasts of $E_G$ measurements from the publicly available Planck CMB lensing map [Planck Collaboration et al. 2014] and the CMASS and LOWZ spectroscopic galaxy samples from BOSS DR11 [Anderson et al. 2014], as well as the spectroscopic quasar (QSO) sample [Paris et al. 2014] from DR11. We use the noise estimate given in the public Planck CMB lensing map. The total number of galaxies (or quasars) within each sample along with the survey area are listed in Table 1. For CMASS, we use the measurements of $b_s\delta_s$ and $\sigma_s$ from Samushia et al. [2014] to set $b(\text{CMASS}) = 2.16$ and $\sigma(\beta)/\beta \sim 10\%$. The corresponding values for the LOWZ sample have not been measured model-independently for DR11, so we assume a 10% measurement of $\beta$ and, as in Tojeiro et al. [2014], we assume $b(\text{LOWZ}) = 1.85$. We use Eq. [17] to calculate the $E_G$ uncertainty for these samples. For the BOSS quasar sample, we use the BOSS...
Table 1

Properties of the various spectroscopic surveys considered in our analysis.

| Survey         | z      | Area (deg²) | $N_{\text{col}}$ |
|----------------|--------|-------------|-----------------|
| BOSS CMASS     | 0.43-0.4 | 9400        | 830,000         |
| BOSS LOWZ      | 0.15-0.43 | 8300        | 360,000         |
| BOSS QSOs      | 2.1-3.5 | 10,200      | 204,000         |
| DESI ELGs      | 0.6-1.7 | 14,000      | $1.8 \times 10^7$ |
| DESI QSOs      | 0.6-1.2 | 14,000      | $4.1 \times 10^6$ |
| Euclid         | 0.5-2.0 | 20,000      | $1.7 \times 10^7$ |
| WFIRST         | 1.05-2.9 | 2000        | $2.5 \times 10^7$ |

DR9 bias measurements from White et al. (2012), assuming the average value $b_{\text{QSO}} = 3.83$. We also assume a 10% $\beta$ measurement for the BOSS quasars. This is a bit optimistic, considering that systematic issues on linear scales with measuring RSD from BOSS quasars. However, we confirm that even a measurement error of 100% $[\sigma(\beta)/\beta = 1]$ only increases the errors on $E_G$ by 5%.

We plot the signal-to-noise ratio (SNR) for $10 < \ell < 1000$ for the CMASS, LOWZ, and quasar samples in Fig. 2. The peaks of all three plots vary due to the redshift of each sample. We find that most of the signal for $E_G$ comes from linear to quasi-linear scales. We set the maximum wavenumber within quasi-linear scales to $k_{\text{NL}}$, where $\Delta(k_{\text{NL}}, z) = k_{\text{NL}}^3 P(k_{\text{NL}}, z)/(2\pi^2) = 1$. We find $k_{\text{NL}}[\text{LOWZ, CMASS}]=[0.354,0.466] h/\text{Mpc}$. For the following forecasts, we limit the angular modes used to $\ell < 100$ to avoid large-scale systematic effects. In Fig. 2 we also show how the SNR increases with $\ell_{\text{max}}$ for CMASS and LOWZ. We see that most of the sensitivity is obtained by $\ell \sim 500$ for CMASS and $\ell \sim 300$ for LOWZ. We set these as our limits in $\ell$ for CMASS and LOWZ, while for quasars we will use all scales $\ell < 1000$.

We also consider how our forecasts for $E_G$ are affected if we restrict the scales used to measure $E_G$ to only linear scales for CMASS and LOWZ. Note we define linear scales $\ell < k_{\text{lin}}$ as those for which the matter power spectrum $P(k)$ computed from N-body simulations differs from the linear $P(k)$ by less than a few percent. We determine which scales are linear using the linear and N-body $P(k)$ predictions from Fig. 2 of Vlah et al. (2014). We find that the purely linear scales for CMASS and LOWZ are limited to $\ell \lesssim 0.1h/\text{Mpc}$, which is significantly less than $k_{\text{NL}}$ determined above for these surveys. As can be seen in Fig. 2, the surveys each lose about half their signal to noise if the measurements are restricted to linear scales.\footnote{As seen in Fig. 2, we see that our LOWZ measurement would be comparable to that from Reyes et al. (2010), although this may be somewhat optimistic considering LOWZ may exhibit unforeseen systematics. However, combining all three measurements would give a SNR of 13, or an 8% measurement. This assumes we can measure RSD from quasars, which is very optimistic considering systematic errors that exist in quasars at large scales (Pullen & Hirata 2013). We also consider our fiducial model of $f(R)$ gravity corresponding to the upper limit on the BZ parameter $B_0$. We see that the $f(R)$ prediction differs from GR only at lower redshifts, greatly suppressing the utility of the quasar measurement and slightly increasing the utility of the LOWZ measurement. $\chi^{\text{rms}} = \sqrt{\chi^2}$ for CMASS and LOWZ are both less than unity, implying these surveys are not able to significantly tighten constraints on $B_0$. The sensitiv...}
Euclid consider the DESI emission line galaxy (ELG), luminous Pol are listed in Table 2. For spectroscopic surveys, we Planck than ACTPol will survey 20,000 deg$^2$ and its increased temperature and polarization sensitivity will create a CMB lensing map that is an order of magnitude more sensitive than Planck. The specifications we use for Adv. ACTPol are listed in Table 2. For spectroscopic surveys, we consider the DESI emission line galaxy (ELG), luminous red galaxy (LRG), and quasar surveys, as well as the Euclid H$\alpha$ survey and the WFIRST H$\alpha$ and OIII combined survey. The properties of the surveys are listed in Table 2. For DESI, we assume the same values as in the DESI Conceptual Design Report. $b_{LRG}D(z) = 1.7$, $b_{ELG}D(z) = 0.84$, $b_{QSO}D(z) = 1.2$, where $D(z)$ is the growth factor. We also assume a 4% error in $\beta$ within $\Delta z = 0.1$ bins. Note that Adv. ACTPol’s survey area overlaps with only $\sim 75\%$ of DESI’s area; we take this into account in our DESI forecasts. For Euclid and WFIRST ELGs, we assume $b(z) = 0.9 + 0.4z$, a fit (Takada et al. 2014) to semi-analytic models (Orsi et al. 2010) that compares well with data. We determine the redshift distribution of Euclid H$\alpha$ galaxies using the H$\alpha$ luminosity function from (Colbert et al. 2013) and assume a flux limit of $4 \times 10^{-16}$. This flux limit is in the middle of the range being considered, so the following Euclid forecasts can change accordingly. We also assume a 3% error in $\beta$ within $\Delta z = 0.1$ bins for Euclid and WFIRST (Amendola et al. 2013). For all subsequent forecasts, we assume $E_G$ measurements over angular scales $100 \leq \ell \leq 500$.

The forecasts are listed in Table 3, but here we list some highlights. Figs. 4 and 5 show that DESI and Euclid combined with Planck can each measure $E_G$ almost at the 2% level, unlike WFIRST which is limited by its small survey area. This should allow DESI and Euclid combined with Planck to produce constraints of some models, and $\beta_1$ constraints should get tighter than those from BOSS. For Adv. ACTPol, DESI should reach a 1% measurement of $E_G$, allowing it to differentiate GR and chameleon gravity with $\beta_1 > 1.1$ at the 5$\sigma$ level. DESI produces tighter constraints than Euclid due to its higher number density at low redshifts. Note that we use a moderate number of redshift slices for each survey, as seen in Figs. 4 and 5. The redshift accuracy of these spectroscopic surveys would allow us to use much smaller redshift bins in order to decrease errors in $E_G$. But each of these surveys are shot-noise dominated on the scales where the $E_G$ signal dominates, increasing the errors in the galaxy-CMB lensing cross-correlation. We also considered more pessimistic errors in $\beta$, finding that increasing the error in $\beta$ by a factor of 3 did not noticeably increase $E_G$ errors from Planck, while it increased $E_G$ errors from Adv. ACTPol by less than 2%.

4.3. Upcoming Photometric Surveys

In this section we consider measuring $E_G$ from upcoming photometric galaxy surveys. These surveys, which measure less precise redshifts than spectroscopic surveys, are tailored for measuring weak lensing and not RSD. However, the errors in $E_G$ are dominated by the CMB lensing at lower redshifts where the $E_G$ signal is highest, meaning that reducing shot noise in the lensing-galaxy cross-correlation through attaining higher number densities is be more important than having precise redshifts. Also, upcoming photometric surveys plan to approach redshift precisions of $\sigma_z/(1+z) \sim 0.05$. Recent work has shown that upcoming photometric surveys could measure RSD (Ross et al. 2011; Crocce et al. 2011; Asorey et al. 2014). This may cause photometric surveys to produce competitive $E_G$ measurements. It should be noted that Adv. ACTPol gets close to the lensing noise limit

![Figure 3. $E_G$ forecasts for BOSS galaxy surveys cross-correlated with the current Planck CMB lensing map, in comparison with the latest measurement of $E_G$ using galaxy-galaxy lensing (Reyes et al. 2010). Note that we do not translate their $E_G$ measurement from the WMAP3 cosmology (Spergel et al. 2007) to the cosmology we assume. The band around the GR prediction corresponds to the likelihood function of $E_G$ based on Planck and BOSS constraints on cosmological parameters. The $E_G$ predictions for $f(R)$ gravity and chameleon gravity are averaged over the wavenumber range at every redshift corresponding to $100 \leq \ell \leq 500$, the range used for CMASS. The dashed lines show chameleon gravity predictions for higher and lower values of $\beta_1$. These surveys are not sensitive enough to tighten constraints on $f(R)$ gravity set by current measurements.](http://desi.lbl.gov/cdr/)
where errors in RSD could begin to matter. An $E_G$ measurement from a future CMB experiment that surpasses Adv. ACTPol may reach the limit such that RSD errors may begin to dominate. Also, the photometric redshift errors and systematic errors within photometric surveys will make measuring RSD with photometric surveys more difficult than with spectroscopic surveys (Ho et al. 2012). We will construct forecasts for photometric surveys of DES, LSST, and Euclid. The properties of these surveys are listed in Table 4. For DES and LSST, we model the reduced redshift distribution in the same manner as Font-Ribera et al. (2014) as

$$f_g(z) = \frac{\eta}{\Gamma(\frac{\alpha+1}{\alpha})} \left(\frac{z}{z_0}\right)\alpha \exp\left(-\frac{z}{z_0}\right)\frac{\alpha}{\alpha+1},$$

(21)

where $\alpha = 1.25 (2.0), \eta = 2.29 (1.0),$ and $z_0 = 0.88 (0.3)$ for DES (LSST). For Euclid we use estimates of the redshift distribution based on the CANDELS GOODS-S catalog (Guo et al. 2013; Hsu et al. 2014). For all three surveys we assume $b(z) = 0.9 + 0.42$ (Takada et al. 2014; Orsi et al. 2010), as in the spectroscopic case. For RSD, we assume a 17% error in $\beta$ over $\Delta z = 0.1 (1 + z)$ bins for DES (Ross et al. 2011). Since Euclid and LSST will cover about 4 times the volume of DES, we expect the errors on $\beta$ to decrease by a factor of 2.

We find that photometric surveys can discriminate between gravity models more effectively than spectroscopic surveys, as can be seen in Fig. 6 and Table 3.

In this work we consider CMB lensing as a probe of $E_G$, a statistic that differentiates between gravity models on cosmological scales. We derive $E_G$ for the general MG parametrization described by $\mu(k, z)$ and $\gamma(k, z)$, as well as for the specific MG models of $f(R)$ gravity and chameleon gravity. We show that generally, $E_G$ for these models are scale-dependent, causing the scale-dependent $E_G$ to be useful for differentiating between MG models and GR.

We produce forecasts for current surveys, showing that BOSS spectroscopic galaxies and quasars combined with Planck CMB lensing each measure $E_G$ at the 8% level. Our results suggest that CMB lensing contributes most
Figure 4. $E_G$ forecasts for DESI galaxy surveys cross-correlated with the final Planck CMB lensing map and with the Advanced ACTPol lensing map. The points for Adv. ACTPol are shifted rightward by 0.02 for clarity. The $E_G$ predictions for $f(R)$ and chameleon gravity are averaged over the wavenumber range at every redshift corresponding to $100 < \ell < 500$. The dashed lines show chameleon gravity predictions for higher and lower values of $\beta_1$.

Figure 5. $E_G$ forecasts for Euclid and WFIRST galaxy surveys cross-correlated with the final Planck CMB lensing map and with the Advanced ACTPol lensing map. The points for WFIRST and Adv. ACTPol are shifted rightward by 0.02 for clarity. Note the Euclid-Adv. ACTPol forecasts contain 50 bins in redshift. The $E_G$ predictions for $f(R)$ gravity and chameleon gravity are averaged over the wavenumber range at every redshift corresponding to $100 < \ell < 500$. The dashed lines show chameleon gravity predictions for higher and lower values of $\beta_1$.

of the error, and that measuring $E_G$ on quasi-linear scales is required to produce significant constraints.

For upcoming surveys, we find that upcoming photometric surveys will outperform spectroscopic surveys due to their higher number densities, even at the expense of having less precise redshifts. Specifically, LSST and photometric Euclid should produce errors on $E_G$ less than 1%, and place very tight constraints on $f(R)$ and chameleon gravity. However, these measurements will be limited by how well these photometric surveys can identify and remove systematic errors. Also, since it is necessary to use $E_G$ from quasi-linear scales, measuring RSD effects at these scales will be challenging. Finally, it is possible that the underlying $E_G$ errors may change slightly. However, CMB lensing has the potential to probe $E_G$ with very high sensitivities without the astrophysical contaminants of galaxy-galaxy lensing, and reveal the nature of gravity.
Figure 6. $E_G$ forecasts for DES, LSST, and Euclid photometric galaxy surveys cross-correlated with the final Planck CMB lensing map and with the Advanced ACTPol lensing map. The points for Adv. ACTPol are shifted rightward by 2% for clarity. We also plot $E_G$ predictions for $f(R)$ gravity and chameleon gravity.

Figure 7. $E_G(k)$ forecasts for the LSST photometric galaxy survey in the redshift bin $z = 1 \pm 0.05$ cross-correlated with the final Planck CMB lensing map and with the Advanced ACTPol lensing map. The points for Adv. ACTPol are shifted rightward by 2% for clarity. We also plot $E_G$ predictions for $f(R)$ gravity and chameleon gravity.

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