Abstract

Feynman’s ratchet is a microscopic machine in contact with two heat reservoirs, at temperatures $T_A$ and $T_B$, that was proposed by Richard Feynman to illustrate the second law of thermodynamics. In equilibrium ($T_A = T_B$), thermal fluctuations prevent the ratchet from generating directed motion. When the ratchet is maintained away from equilibrium by a temperature difference ($T_A \neq T_B$), it can operate as a heat engine, rectifying thermal fluctuations to perform work. While it has attracted much interest, the operation of Feynman’s ratchet as a heat engine has not been realized experimentally, due to technical challenges. In this work, we realize Feynman’s ratchet with a colloidal particle in a one-dimensional optical trap in contact with two heat reservoirs: one is the surrounding water, while the effect of the other reservoir is generated by a novel feedback mechanism, using the Metropolis algorithm to impose detailed balance. We verify that the system does not produce work when $T_A = T_B$, and that it becomes a microscopic heat engine when $T_A \neq T_B$. We analyze work, heat and entropy production as functions of the temperature difference and external load. Our experimental realization of Feynman’s ratchet and the Metropolis algorithm can also be used to study the thermodynamics of feedback control and information processing, the working mechanism of molecular motors, and controllable particle transportation.

In his Lectures on Physics, Richard Feynman introduced an ingenious model to illustrate the inviolability of the second law of thermodynamics [1]. A ratchet and pawl are arranged to permit a wheel to turn in only one direction, and the wheel is attached to a windmill immersed in a gas. Random collisions of gas molecules against the windmill’s panes would then seemingly drive systematic rotation in the allowed direction, which could be used to deliver useful work in violation of the second law. As discussed by Feynman, this violation does not occur, because thermal fluctuations of the pawl occasionally allow the ratchet to move in the ‘forbidden’ direction. However, if the pawl is maintained at a temperature that differs from that of the gas, then the device is indeed able to rectify thermal fluctuations to produce work—in this case it operates as a microscopic heat engine. This model elegantly illustrates the idea that thermodynamic laws governing heat and work, originally derived for macroscopic systems, apply equally well at the nanoscale where fluctuations dominate.

Three essential features are needed to produce directed motion in Feynman’s model: (1) when the ratchet and pawl are engaged, the potential energy profile must be asymmetric; (2) the device must be in contact with thermal reservoirs at different temperatures; and (3) the device must be small enough to undergo Brownian motion driven by thermal noise. Feynman’s ratchet as a paradigm for rectifying thermal noise has inspired...
extensive theoretical studies [2–8] and related ratchet models [9] have been used to gain insight into motor proteins [10, 11]. Directed Brownian motion in asymmetric potentials, with a single heat reservoir, has been demonstrated experimentally in the presence of non-thermal driving [12–15]. Recently, a macroscopic (10 cm scale) ratchet driven by mechanical collisions of 4 mm diameter glass beads was demonstrated [16]. However, an experimental realization of Feynman’s ratchet, which rectifies thermal fluctuations from two heat reservoirs to perform work, has not been reported to date. A major challenge is to devise a microscopic system that is in contact with two heat reservoirs at different temperatures, without side effects such as fluid convection that can smear the effects of thermal fluctuations.

Here we realize Feynman’s two-temperature ratchet and pawl with a colloidal particle confined in a one-dimensional (1D) optical trap (figure 1). The particle’s 1D Brownian motion emulates the collision-driven rotation of the ratchet, and we use optical tweezers to generate both flat and sawtooth potentials, simulating, respectively, the disengaged and engaged modes of the pawl (figure 1(A)). In the flat potential, the colloidal particle moves freely. The water surrounding the colloidal particle provides a heat reservoir at temperature $T_A$. The other heat reservoir is generated by using feedback control of the optical tweezer array [17] to toggle between the disengaged and engaged modes of the pawl, implementing the Metropolis algorithm [18] as in reference [6] to satisfy the detailed balance condition at a chosen temperature $T_A$ (equation (1)). Our setup, inspired by theoretical models [3–6], captures the essential features of Feynman’s original model, with the pawl maintained at one temperature ($T_A$) and the ratchet at another ($T_B$). As described in detail below, both numerical simulation and experimental data clearly show the influence of the two heat reservoirs on the operation of the ratchet, in agreement with theoretical analyses [1, 6].

In our experiment, a silica microsphere with a diameter of 780 nm, immersed in deionized water, undergoes diffusion in a 1D optical trap created with an array of 19 optical tweezers. The array is generated by an acousto-optic deflector (AOD) controlled by an arbitrary function generator [17]. By tuning the power of each optical tweezer individually, the trap potential can be modified to be either flat or sawtooth-shaped. The water, at temperature $T_B = 296$ K, plays the role of the gas in Feynman’s model, providing thermal noise that drives the motion of the ratchet. We use feedback control to generate the effects of a second heat reservoir, at temperature $T_A$, that drives the pawl as it switches between its two modes: (1) disengaged and (2) engaged. Letting $U_i(x)$ and $U_j(x)$ denote the potential energies of these modes as functions of the particle location $x$, we generate attempted switches between modes at a rate $\Gamma$, and each such attempt is accepted with a probability given by the Metropolis algorithm [6, 18]:

$$P_{\text{switch}}(x) = \min\{1, \exp(-\Delta E(x)/k_B T_A)\},$$  

(1)

where $k_B$ is Boltzmann’s constant, and $\Delta E(x) = U_j(x) - U_i(x)$ for an attempted switch from mode $i$ to mode $j$.

By enforcing the detailed balance condition, equation (1) ensures that the toggling between the two modes consistently reflects the exchange of energy with a thermal reservoir at temperature $T_A$. When we set $T_A = T_B$ the system relaxes to equilibrium, but when $T_A \neq T_B$ we obtain a non-equilibrium steady state in which there is a net flow of energy between the system, the two reservoirs, and (as described later) an external work load.

To gain insight, we consider a simple model that couples the diffusive motion of the Brownian particle in the 1D trap to the stochastic switching between the engaged and disengaged potential modes. Letting $P(x, t)$ denote
the joint probability density to find the particle at position $x$ and the potential in mode $i \in \{1,2\}$ at time $t$, we construct the reaction-diffusion equation

$$\frac{\partial P_i(x, t)}{\partial t} = \frac{\partial}{\partial x} \left[ \gamma \frac{U_i(x)}{\gamma} P_i(x, t) \right] + D \frac{\partial^2 P_i(x, t)}{\partial x^2} + k_g(x) P_i(x, t) - k_q(x) P_i(x, t),$$

(2)

where $\gamma$ is the Stokes friction coefficient, $D = k_B T_B / \gamma$ the diffusion constant, and $k_q(i,j)(i \neq j)$ the switching rate from potential mode $i$ to $j$. This rate is the product of a constant attempt rate, $\Gamma$, and the Metropolis acceptance probability, equation (1):

$$k_q(i,j) = \Gamma \min \left[ 1, \exp \left( \frac{-U_i(x) + U_j(x)}{k_B T_A} \right) \right].$$

(3)

The total particle probability distribution is $P_i(x, t) + P_j(x, t)$.

To implement the ratchet dynamics, we first trap a silica microsphere with a single optical tweezer and position it near the middle of the trap, which corresponds to the potential minimum at the center of mode 2. The optical tweezer array is then turned on at mode 2, and we follow the diffusion of the microsphere as the potential switches between modes as described earlier (and in the method section at the end of this article).

Figures 2(B)–(D) show 60 s trajectories of the particle without external load for different heat reservoir temperatures $T_A$. The light lines display individual trajectories, and the thick blue lines are averages over these trajectories. We interpret positive displacements of the particle as clockwise rotation of the mechanical ratchet, and negative displacements as counter-clockwise rotation. As seen in figure 2(C), the average final displacement of the particle converges essentially to zero, $\langle \Delta x \rangle = -0.1 \pm 0.9 \mu m$, when the temperatures of the two reservoirs are equal ($T_A = T_B = 296$ K) even though the potential is asymmetric. This experimentally verifies Feynman’s prediction that the ratchet does not produce perpetual motion, as clockwise rotations are canceled by counter-clockwise rotations, on average. When $T_A = 30$ K ($< T_B$), the average final position of the microsphere is $\langle \Delta x \rangle = 2.1 \pm 0.6 \mu m$, indicating net clockwise rotation of the ratchet. When $T_A = 3000$ K ($> T_B$), the average final position is observed to be $\langle \Delta x \rangle = -4.5 \pm 0.9 \mu m$, corresponding to net counter-clockwise rotation. These results demonstrate that a temperature difference $T_B - T_A$ can give rise to unidirectional motion via the rectification of thermal noise [7]. We now investigate whether this motion can be harnessed to perform work against an external load.

To apply an effective external load to the ratchet, a slight linear slope $f$ is added to the potentials. Figure 3 shows the observed average particle velocity as a function of pawl temperature $T_A$, for $f = -0.05 k_B T_B / \mu m$, $f = 0$ and $f = 0.14 k_B T_B / \mu m$, corresponding to positive, zero and negative external loads, respectively. Each experimental data point is calculated from fifty 60 s trajectories. The lines show numerical simulation results, with each situation simulated over $5 \times 10^4$ times to achieve high accuracy. In these simulations we use the
overdamped Langevin equation \( \dot{x} = -\frac{1}{\mu} \frac{\partial U}{\partial x} + \xi(t) \), where \( \xi(t) \) is Gaussian random force satisfying \( \langle \xi(t) \rangle = 0 \), \( \langle \xi(t)\xi(t') \rangle = \frac{2k_B T}{\mu} \delta(t - t') \). The simulation time step is 2 ms. The simulations are performed with the same procedure as the experiment, described earlier. The simulation and experimental results show good agreement over a wide range of TA (figure 3).

By realizing an external load, we can interpret our system as a microscopic heat engine. The work produced by the engine is given by the product of the slope and the load, as seen in figure 2. For each trajectory, we can thus keep track of the heat absorbed by the system from the hot reservoir is delivered to the cold reservoir, with only a very small portion of the energy absorbed from the hot reservoir is delivered to the cold reservoir, with only a very small portion of the energy absorbed from the hot reservoir is delivered to the cold reservoir, with only a very small portion of the energy absorbed from the hot reservoir is delivered to the cold reservoir, with only a very small portion of the energy absorbed from the hot reservoir is delivered to the cold reservoir, with only a very small portion of the energy absorbed from the hot reservoir is delivered to the cold reservoir, with only a very small portion of the energy absorbed from the hot reservoir is delivered to the cold reservoir, with only a very small portion of the energy absorbed from the hot reservoir is delivered to the cold reservoir.

Changes in the potential energy of the particle due to diffusion are associated with heat exchange with the reservoirs, \( Q_A \) and \( Q_B \). For each trajectory, we can thus keep track of the heat absorbed by the system from the two reservoirs, \( Q_A \) and \( Q_B \). For the case of positive load, figure 4(B) plots the average values of these quantities. These data show that the system absorbs energy \( (Q > 0) \) from the hotter reservoir and releases energy \( (Q < 0) \) into the colder reservoir, in agreement with expectations. We have also computed the average entropy production \( \langle dS \rangle = -\frac{Q_A}{T_A} - \frac{Q_B}{T_B} \), shown in figure 4(D). As expected, \( \langle dS \rangle = 0 \) when \( T_A = T_B \) as the system is in equilibrium, but \( \langle dS \rangle > 0 \) when \( T_A \neq T_B \), in agreement with the second law. In our model, most of the energy absorbed from the hot reservoir is delivered to the cold reservoir, with only a very small portion converted to work—this explains the observation that the entropy production is largely independent of external load, as seen in figure 4(D).

Figure 3. Average velocity of the microsphere under different external loads at different temperatures \( T_A \). Blue squares, red circles and black triangles represent results with potential slopes of \(-0.05, 0 \) and \( 0.14 \, k_B T_B / \mu m \), respectively. The added slope to the original potential represents the external load. Every experimental data point represents the result of an average of 50 repetitions. Solid lines are simulation results. The vertical dashed line indicates \( T_B = 296 \, K \).
Figure 4 shows agreement between experiments (points) and simulations (lines), and demonstrates the operation of Feynman’s ratchet as an engine that rectifies thermal fluctuations to perform work. It is interesting to consider the thermodynamic efficiency of the engine, \( \eta = \frac{\langle W \rangle}{Q_{\text{high}}} \), where \( Q_{\text{high}} \) denotes the heat from the reservoir with a higher temperature. Although Feynman suggested that his ratchet could achieve Carnot efficiency \( \eta_C \), later authors argued that his analysis was incorrect \([3–5]\). The efficiency that we measured experimentally is \( \eta = 0.0015 \) for the data point labeled by a circled ‘5’ in figure 4(A), which is much lower than the corresponding Carnot efficiency \( \eta_C = 0.9 \), in agreement with the conclusions of references \([3–5]\). Feynman’s ratchet cannot achieve Carnot efficiency, as it is in contact with two heat reservoirs simultaneously and generates work by continuously rectifying fluctuations in a non-equilibrium steady state \([3]\). This mode of operation is fundamentally different \([4, 19]\) from the thermodynamic cycle of reversible expansion and compression that characterizes a Carnot engine. We note that micron-sized heat engines that operate in cycles have recently been implemented in experiments using colloidal particles \([19–22]\).

In conclusion, by combining the Metropolis algorithm with feedback control, we have realized Feynman’s two-temperature ratchet-and-pawl model using a silica microsphere confined in a feedback controlled 1D optical trap. We study the behavior of the ratchet over a range of temperatures and external loads, demonstrating that it rectifies thermal fluctuations to generate work while obeying the thermodynamic laws originally derived for macroscopic heat engines operating in cycles. When the temperatures of the two heat reservoirs are equal, we find no unidirectional average drift of the particle, in agreement with the second law of thermodynamics. When the temperatures differ, we demonstrate that thermal fluctuations can be rectified by the ratchet to generate work. Our system provides a versatile testbed for studying the non-equilibrium thermodynamics of microscopic heat engines and molecular motors \([7, 11, 23, 24]\). For instance, multiple heat reservoirs can be mimicked by using a position-dependent \( T_r(x) \) in the Metropolis algorithm. Moreover, although we have used feedback control to implement an effective heat bath, in an alternative scenario feedback control could be used to mimic the operation of Maxwell’s demon \([25]\), and thus to investigate issues related to the thermodynamics of information processing \([25–29]\). Our study may also have potential applications in particle transportation \([7]\) and separation \([30]\) induced by Brownian motion in asymmetric potential.
Method

The flat and sawtooth trapping potentials are created by 19 optical tweezers (Figure 5). To create them using a laser passing through an AOD, we drive the AOD with a combination of 19 radio frequency (RF) components at different frequencies. The power and position of each optical tweezer is controlled by adjusting the amplitude and frequency of each RF component. As shown in Figure 5, each optical tweezer (gray curve) has a Gaussian shape. The spacing between neighboring tweezers is smaller than the waist of each optical tweezer. Thus they overlap and create a continuous 1D trap. By carefully changing the power of each RF component, we can create a flat potential or a sawtooth potential. We only used the center-parts (inside dashed green boxes in Figure 5) of the potentials in the experiment. To create an effective periodic potential with an infinite length, we use an optical tweezer to drag the microsphere back to the center well every time when it reaches the bottom of the left well or the bottom of the right well of the sawtooth potential. The locations of the edges are chosen carefully to avoid bias.

The potentials \( U_1(x) \) and \( U_2(x) \) are determined from the measured equilibrium distribution of the particle in each mode. We fix the 1D trap in a particular mode and track the Brownian motion of the silica microsphere in the trap, recording its position every 5 ms. After more than \( 5 \times 10^5 \) data points are collected, the potential profile is extracted using the equation \( U(x) = -k_B T_B \ln[N(x)/N_{\text{total}}] \), where \( N(x) \) is the number of count at each point and \( N_{\text{total}} \) is the total count. The position \( x \) is discretized in bins of 52 nm corresponding to the pixel size of our camera. The measured flat and sawtooth potentials are shown in Figure 2(A) (in the main text). The depth (from trough to peak) of the sawtooth potential \( U_2(x) \) is about \( 4.8k_B T_B \). The sawtooth potential is described by an asymmetry ratio of approximately 1:3 (see Figure 2(A)); this is a key parameter for thermal ratchets. The flat potential \( U_1(x) \) has a standard deviation of about 0.15 \( k_B T_B \), which is small enough for the silica microsphere to diffuse freely. After measuring the potentials, we fit \( U_1(x) \) with a straight line and \( U_2(x) \) with an ideal sawtooth potential (Figure 2(A)). These fitted smooth potentials were used in our numerical simulations to avoid the statistical noise in the measurements.

To implement the ratchet dynamics, the position of the microsphere is recorded every 5 ms using a complementary metal-oxide semiconductor camera (Figure 1(B)). Every 200 ms (this time interval is equal to \( 1/\Gamma \)), an attempt to switch modes is made, and is accepted with the probability given by equation (1) in the main text. In our experiment, the sawtooth potential has finite length. To mimic the infinite nature of a rotating ratchet, the particle is dragged back to the trap center whenever it reaches one of the potential minima located on both ends. The trajectories of the microsphere are then connected end-to-end to create 60 s trajectories. The average velocities are independent of the length of each trajectory.

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Note added  We recently gave an oral presentation about this work in Conference on Lasers and Electro-Optics (CLEO) in May 2018 [31].

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