Study of parametric pump performance curve based on surface fitting

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Abstract. By selecting the appropriate basis functions according to the variation of vane pump regulating performance curves, we analysed the relationship in blade angle, flow, and head (or shaft power) based on surface fitting, and we established a parametric model, proposed the solution of the problem. The calculation of the example shows that the model has a high correlation coefficient, and has a simple mathematical expression, which is helpful for further application and popularization.

1. Introduction

The operating effect of the pump station achieved optimum under the design working condition. However, in reality, it always deviates from the optimum somehow. It is a relatively economic adjustment method of working condition to adjust the blade angle, which can keep the pump’s running efficiency in a wide range of flow $^{[1]}$. The adjustment of the pump’s working condition is based on the general characteristic curve of the pump unit, which is generally figured out by converting and fitting the experimental data via the original model. While the adjustment method of the blade is commonly semi-adjusted limited to the cost when experimenting. There is a drawback of the semi-adjustment method that it can only adjust the blade to a certain integer angle, inaccessible to continuous data as a result. Therefore, in practical applications, the adjustment of the blade angle is limited to several discrete integer value, making it hard for the full blade-adjustment mechanisms, which are put into use by some pump stations, to exert the maximum utility. It also restricts the accuracy of the optimization results calculated by pump stations.

In this paper, according to the performance characteristics of blade adjustment, a parametric model of blade adjustment is established based on curved surface imitation theory. The solution method of the model is proposed as well. Through this model, the continuous data of pump operating condition can be fitted, which is a new exploration that compensate for the lack of precision in blade adjustment through the parametric performance curve.

2. Parametric model

In order to realize the parameterization of adjustable performance curve of the pump blade, firstly, it is necessary to know the correlations among the various working parameters. The adjustable performance curve is usually fitted by polynomial. However, the curve can only reveal the relationship, as the blade angle is fixed, between flow and lift, flow and power, flow and efficiency, and so forth, rather than reflect the adjustment of continuous operating conditions. For this reason, the blade angle is brought in, considered as a variable, to fit the correlations among the three working parameters, based on curved surface imitation theory.

For ease of use, the fitting expression is simplified as much as possible on the basis of fitting accuracy. A quadradic polynomial is chosen in consideration of previous experience...
and the arrangement regulation of performance curve in different angles.

Let \( \{x_i, y_i, z_i\} (i = 1, 2, \ldots, n) \) denote the \( n \) points coordinates in a space surface. The quadratic polynomial surface equation can be expressed as:

\[
z = f(x, y) = a_0 + a_1x + a_2y + a_3x^2 + a_4y^2 + a_5xy
\]  

(1)

So the relational expressions among parameters are:

\[
H = a_0 + a_1Q + a_2\beta + a_3Q^2 + a_4\beta^2 + a_5Q\beta
\]  

(2)

\[
P = a'_0 + a'_1Q + a'_2\beta + a'_3Q^2 + a'_4\beta^2 + a'_5Q\beta
\]  

(3)

\[
\eta = \frac{\rho QH}{P} = \frac{\rho Q[a_0 + a_1Q + a_2\beta + a_3Q^2 + a_4\beta^2 + a_5Q\beta]}{a'_0 + a'_1Q + a'_2\beta + a'_3Q^2 + a'_4\beta^2 + a'_5Q\beta}
\]  

(4)

where \( H \) is the pump lift, \( Q \) is the pump flow, \( \beta \) is the blade placement angle, \( P \) is the shaft power of pump, \( \eta \) is the pump efficiency, and \( a_k, a'_k (k = 0, 1, 2, 3, 4, 5) \) are fitting coefficients.

Combining formula (2), (3) and (4), the expressions of the commonly used performance curve can be realized only to input fixed blade placement angle, dispersing \( Q \) in its application range.

3. Model solution

The least square method is used to solve the model. This method is an approximation theory with a purpose to minimize the sum of squared deviation between fitting data and real data. That is

\[
\min \varphi = \sum_{i=0}^{n} [f(x_i, y_i) - z_i]^2 = \sum_{i=0}^{n} \left[ a_0 + a_1x_i + a_2y_i + a_3x_i^2 + a_4y_i^2 + a_5x_iy_i - z_i \right]^2
\]  

(5)

Obviously, through mathematical analysis, to minimize \( \phi(a_0, a_1, a_2, a_3, a_4, a_5) \), there must be

\[
\frac{\partial \varphi}{\partial a_0} = 0, \quad \frac{\partial \varphi}{\partial a_1} = 0, \quad \frac{\partial \varphi}{\partial a_2} = 0, \quad \frac{\partial \varphi}{\partial a_3} = 0, \quad \frac{\partial \varphi}{\partial a_4} = 0, \quad \frac{\partial \varphi}{\partial a_5} = 0
\]  

(6)

That is

\[
\begin{align*}
\frac{\partial \varphi}{\partial a_0} &= 2 \sum_{i=0}^{n} \left[ a_0 + a_1x_i + a_2y_i + a_3x_i^2 + a_4y_i^2 + a_5x_iy_i - z_i \right] = 0 \\
\frac{\partial \varphi}{\partial a_1} &= 2 \sum_{i=0}^{n} x_i \left[ a_0 + a_1x_i + a_2y_i + a_3x_i^2 + a_4y_i^2 + a_5x_iy_i - z_i \right] = 0 \\
\frac{\partial \varphi}{\partial a_2} &= 2 \sum_{i=0}^{n} y_i \left[ a_0 + a_1x_i + a_2y_i + a_3x_i^2 + a_4y_i^2 + a_5x_iy_i - z_i \right] = 0 \\
\frac{\partial \varphi}{\partial a_3} &= 2 \sum_{i=0}^{n} x_i^2 \left[ a_0 + a_1x_i + a_2y_i + a_3x_i^2 + a_4y_i^2 + a_5x_iy_i - z_i \right] = 0 \\
\frac{\partial \varphi}{\partial a_4} &= 2 \sum_{i=0}^{n} y_i^2 \left[ a_0 + a_1x_i + a_2y_i + a_3x_i^2 + a_4y_i^2 + a_5x_iy_i - z_i \right] = 0 \\
\frac{\partial \varphi}{\partial a_5} &= 2 \sum_{i=0}^{n} x_iy_i \left[ a_0 + a_1x_i + a_2y_i + a_3x_i^2 + a_4y_i^2 + a_5x_iy_i - z_i \right] = 0
\end{align*}
\]  

(7)

In general, the values of \( x \) and \( y \) are experimental data. If \( n \) values of \( x \) and \( y \) are known, the equations above can be regarded as a sixth-order linear equations regarding \( a_k (k = 0, 1, 2, 3, 4, 5) \), which can be solved by simple iterative method [2]. Then we can get an approximate fitting relationship \( f(x, y) \).
4. Application example

Parametric calculation was carried out by using the mathematical model above, using the test data of the TJ04-ZL-07 pump section of the South-to-North Water Diversion Project. The pump section has 3 rotors and 7 guide vanes, with a diameter of 298.5mm of the runner, a diameter of 120mm of the hub, a hub ratio of 0.4020 and a blade clearance of 0.4mm. The mechanical loss torque of the model pump is 2.34N·m. The original data refers to reference [1] and the results are as follows [3-5], surface fitting coefficient as shown in Table 1, parameterized calculation results with a blade angle of +1.5 as shown in Table 2.

Table 1. Surface fitting coefficients table.

| Coefficient | Fitted value | Standard  | Coefficient | Fitted value | Standard  |
|-------------|--------------|-----------|-------------|--------------|-----------|
| $a_0$       | 7.71245      | 0.70915   | $a'_0$      | 25.6329      | 2.7252    |
| $a_1$       | 0.00952      | 0.00428   | $a'_1$      | 0.03009      | 0.01643   |
| $a_2$       | -0.35444     | 0.06949   | $a'_2$      | 0.95959      | 0.26704   |
| $a_3$       | -5.93068×10^{-5} | 6.41392×10^{-6} | $a'_3$ | -1.68356×10^{-4} | 2.4648×10^{-5} |
| $a_4$       | -0.04225     | 0.00233   | $a'_4$      | -0.06946     | 0.00894   |
| $a_5$       | 0.0026       | 2.08146×10^{-4} | $a'_5$ | 0.00363       | 7.99882×10^{-4} |

R-squared 0.99374

Table 2. Parameterized calculation results with a blade angle of +1.5.

| Flow/m³/s | Lift/m | Power/kW | Efficiency/% |
|-----------|--------|----------|--------------|
| 200       | 7.397  | 27.288   | 53.13        |
| 220       | 7.168  | 26.584   | 58.13        |
| 250       | 6.734  | 25.276   | 65.27        |
| 300       | 5.774  | 22.423   | 75.71        |
| 320       | 5.307  | 21.046   | 79.08        |
| 350       | 4.518  | 18.728   | 82.75        |

The R-squared values of the basic mathematical model are 0.99374 and 0.99307 independently. Figure 1 and Figure 2 respectively show the $Q-H$ curve and $Q-P$ curve with a blade angle of -4°. It can be found from the comparison between the surface fitting data and the original experimental data that their quadratic polynomial fitting curves have little difference. So the model can be applied to actual operations.

5. Conclusions

Based on the least square surface fitting, a parameterized calculation model of the blade
adjustment performance curve is established, and the solution method of the model is given. Through the parametric model, continuous operating condition data can be obtained with different blade adjustment, which makes up for the lack of blade adjustment precision in the model test. In the calculation example, the correlation coefficient is high, with fitting correlation coefficient of $Q - \beta - H$ being 0.99374 and that of $Q - \beta - P$ being 0.99307. The basic functions applied in the parametric model are simple and clear. The model is easy to apply and popularize for there are fixed mathematical expressions.

Reference
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