Axial Magnetostatics of a Ring Current in a Kerr Field

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Abstract

The electromagnetic fields generated by a ring current around a Kerr black hole have been found. The acceleration of a charged particle by a force electric field along the rotation axis is investigated in the constructed model, as applied to the astrophysics of quasars.

1 INTRODUCTION

Studying the interaction of electromagnetic fields with the gravitational field of a rotating black hole is of great importance in understanding the astrophysics of quasars. Quasars manifest themselves as compact, intense sources of electromagnetic radiation, which occasionally have huge narrowly directed jets and which are most likely active galactic nuclei. The BlandfordZnajek process [1] is one of the models that accounts for observational manifestations of quasars. A magnetohydrodynamic (MHD) model of plasma accreting onto a rotating black hole underlies this process. Through the BardeenPetterson process [2], accretion can proceed only from the equatorial plane; therefore, it makes sense to model accretion as a superposition of equatorial ring currents. Such modeling is proper if the pair production by an induced electric field gives rise to currents that are much weaker than the source ring current. The Hawking effect of particle production on the horizon is negligible in this model, because it gives a negligible correction when the Compton particle wavelength is much smaller than the radius of space curvature. For electrons, this corresponds to a black hole with a mass larger than about $10^{-16}M_{\odot}$.

In all the cited studies, calculations were performed by using 3 + 1 formalism. In contrast to these studies, we use the general covariant formalism of general relativity and do not use the approximation of MHD magnetic field line freezing-in in plasma, which leads to the condition for the scalar product of the electric and magnetic fields being equal to zero (force-free field). We introduce same notation for electromagnetic and gravitational quantities as in [3], (§90). [f]

1Below, we use the system of units $c = 1$ — the speed of light, and $G = 1$ — the gravitational constant, for convenience.
2 SPECIFYING BOUNDARY CONDITIONS

Let us specify the boundary conditions that must be imposed on the electromagnetic-field tensor components required to determine the latter. First, the classical boundary conditions must be satisfied: all field components must become zero at infinity. Second, the boundary conditions on the horizon must also be added to the classical ones. They are required so that, in the frame of reference associated with a freely falling observer (FFO), no anomalies arise in its motion as it flies up to the horizon due to the electromagnetic-field acceleration (anomalies result in the violation of the condition for the FFO electric charge being a test one.

Therefore, to find the conditions imposed on the field components on the horizon, we must write out the 4-vector of FFO acceleration and establish which of its components have the anomalies on the horizon related to the electromagnetic field. Below, we use the coordinates that are at rest with respect to an infinitely remote, static observer. The FFO 4-acceleration components are

\[ W^i = w^i + \left( \frac{e}{m} \right) F^{ij} u_j \]

Here, \( e \) and \( m \) are the FFO test charge and mass, respectively; \( w^i = -\Gamma^i_{km} u^k u^m \), where \( \Gamma^i_{km} \) are the Christoffel symbols; \( F^{ij} \) are the contravariant tensor components of the electromagnetic field; and \( u^k \) are the FFO 4-velocity components. Let us calculate the 4-acceleration for a radially falling FFO in the Schwarzschild metric. The components of the Christoffel symbols and the FFO 4-velocity for this metric are given in [3]. Denoting the Schwarzschild radius by \( r_g = 2M \), where \( M \) is the blackhole mass, we have

\[ u^0 = -g_{rr}; \quad u^r = -\sqrt{r_g/r}; \quad w^0 = -\frac{1}{r} (r_g/r)^{3/2}; \quad w^1 = \frac{r_g}{2r^2} (2 + g_{rr}). \]

The remaining \( u^i \) and \( w^i \) are zero. When approaching the horizon \( (r \to r_g) \), the metric tensor component \( g_{rr} \to -\infty \). Therefore, let us write out the asymptotics of the squares of the observed 4-acceleration components in the principal \( (in g_{rr}) \) order expressed in terms of the contravariant field components:

\[ W_0 W^0 \to -g_{rr} \left( \frac{e}{m} \right)^2 \frac{r_g}{r} (F^{r0})^2; \quad W_r W^r \to g_{rr} \left[ \frac{r_g}{2r^2} g_{rr} + \left( \frac{e}{m} \right) F^{r0} \right]^2; \]
\[ W_\theta W^\theta \to g_{rr}^2 \left( \frac{e}{m} \right)^2 \frac{r_g}{r} g_{\theta\theta} (F^{r\theta})^2; \quad W_\varphi W^\varphi \to g_{rr}^2 \left( \frac{e}{m} \right)^2 \frac{r_g}{r} g_{\varphi\varphi} (F^{r\varphi})^2. \]

We thus see that the singularity in the radial acceleration component does not result in the violation of the condition for the charge being a test one for the following reasons: (1) the gravitation near the horizon in the radial direction acts a factor of \( g_{rr} \) more strongly than the electromagnetic field; (2) the zero \( W \) component has a weak singularity; and (3) the tangential acceleration components have a strong

\[ ^2A \text{ more detailed discussion of this requirement can be found in [4, 5].} \]
\[ ^3\text{Unless stated otherwise, the Latin indices run the series 0, 1, 2, 3, while the Greek indices run the series 1, 2, 3.} \]
singularity, which can significantly change the FFO trajectory near the horizon and can violate the condition for the FFO charge being a test one: $e \ll m$. The contravariant components of the tangential magnetic field must be set equal to zero on the horizon, lest this happen. Similarly, the asymptotics of the squares of the 4-acceleration components dependent on the covariant field components is

$$W_0 W^0 \to -g_{rr} \left(\frac{e}{m}\right)^2 \left(\frac{r}{r_0}\right)^2 (F_{\theta 0})^2; \quad W_r W^r \to g_{rr} \left[\frac{r^2}{2rr} \right] g_{rr} + \left(\frac{e}{m}\right) F_{r0} \right]^2;$$

$$W_\theta W^\theta \to g_{rr} \left(\frac{e}{m}\right)^2 g_{\theta\theta} (F_{\theta 0})^2; \quad W_\phi W^\phi \to g_{rr} \left(\frac{e}{m}\right)^2 g_{\phi\phi} (F_{\phi 0})^2. \quad (3)$$

We thus see that the covariant components of the tangential electric field with the strongest singularity must become equal to zero on the horizon. However, the Kerr field rather than the Schwarzschild field is of physical interest. Let us write out the Kerr metric and its determinant in a nonrotating (relative to remote stars) frame:

$$ds^2 = (1 - \frac{2mr}{\rho^2}) dt^2 - \frac{dr^2}{\Delta} - \rho^2 (\sin^2 \theta) d\theta^2 - (r^2 + a^2) \sin^2 \theta d\phi^2 + \frac{2ra}{\rho} \sin^2 \theta d\theta d\phi,$$  

(here, $\rho^2 = r^2 + a^2 \cos^2 \theta$, $\Delta = r^2 + a^2 - r_g r$, and $a$ is the Kerr parameter). In this case, in the Kerr field, both the space itself and the FFO are drawn into rotation when approaching a black hole. The conditions for the tangential electromagnetic field components in the Schwarzschild field then change to the conditions for the same components in the frame of reference observers in the Kerr field. Expressions (2) and (3) give a summary of the boundary conditions for the electromagnetic field on the horizon in the Kerr field:

$$F''_{\theta 0} \to 0; \quad F''_{\phi 0} \to 0; \quad F''_{\mu \phi} \to 0; \quad F''_{\mu \phi} \to 0. \quad (5)$$

Let us write out formulas that can be of use in the subsequent analysis. Designating $\kappa = -g_{00}/g_{\phi\phi}$, $\kappa + \Omega^2 = \Delta \sin^2 \theta / g_{\phi\phi}$, we then have

$$g_{\phi\phi} = \frac{1}{g_{\phi\phi}}, \quad g_{rr} = \frac{1}{g_{rr}}, \quad g_{00} = -\frac{g_{\phi\phi}}{\Delta \sin^2 \theta},$$

$$g_{\phi\phi} = -\frac{g_{00}}{\Delta \sin^2 \theta}, \quad g_{\phi\phi} = \frac{g_{00}}{\Delta \sin^2 \theta}. \quad (6)$$

### 3 Passage to the Reference Frame

Since the current is axisymmetric and stationary, the system has two Killing vectors: along the translations in time and along the translations in angle $\phi$. Therefore, all fields in frame (4) are stationary. In general, this stationarity is not conserved when passing to a frame of reference rotating with an arbitrary angular velocity. To show this, let us write out the coordinate transformations to a frame of reference rotating with a spatially nonuniform angular velocity $\omega(r, \theta)$ required in the subsequent analysis:

$$dx^i = dx^j \delta^i_k + \omega \delta^i_\phi \delta^j_\phi + t \delta^i_\phi \delta^j_\phi \partial_\theta \omega \quad (7)$$

In what follows, by the reference frame we mean a frame in which $g_{\phi\phi}'' = 0$ (see [4, 5]). This is the frame in which the reference observers rotate with angular velocity $\Omega = -g_{00}/g_{\phi\phi}$ relative to remote stars and have a zero angular momentum.
The corresponding transformations of the contravariant field components (see [3]) are
\[
F'_{\alpha0} = F^{\alpha0}; \quad F'_{0\varphi} = F^{0\varphi} + t F^{\alpha0} \partial_\alpha \omega; \quad F'_{\alpha\varphi} = F^{\alpha\varphi} - \omega F^{\alpha0}; \quad F'_{r\theta} = F^{r\theta}.
\] (8)

For the covariant components, we derive
\[
F'_{\alpha\varphi} = F_{\alpha\varphi}; \quad F'_{\alpha0} = F_{\alpha0} + \omega F_{\alpha\varphi}; \quad F'_{0\varphi} = F_{0\varphi}; \quad F'_{r\theta} = F_{r\theta} + t e_{\alpha\beta\varphi} F_{\alpha\varphi} \partial_\beta \omega.
\] (9)

In expressions (8) and (9), \(\alpha\) runs the values of \(r\) and \(\theta\). We see from these expressions that only the frames of reference rigidly rotating relative to another stationary frame (relative to remote stars) are stationary. The stationary frame of reference rotating with the horizon angular velocity \(\Omega_H\) coincides with the FFO frame on the horizon. Therefore, the boundary conditions in this horizon frame, which below is marked by a tilde, are the same as (5):
\[
\tilde{F}_{\theta0} \to 0; \quad \tilde{F}_{\varphi0} \to 0; \quad \tilde{F}_{r\theta} \to 0; \quad \tilde{F}^{r\varphi} \to 0.
\] (10)

4 MAXWELL EQUATIONS

Suppose that the current has a density with a delta-shaped distribution function in the meridional plane:
\[
j^i(r, \theta) = J^\varphi \delta_\varphi \left[ \delta(r - r_0) \delta(\theta - \theta_0) \right] / \sqrt{-g}.
\] (11)

Since the other currents are assumed to be negligible and since the frame is stationary, the toroidal electromagnetic-field components are zero. Therefore, the axisymmetric Maxwell equations for the covariant field components outside the horizon are
\[
\begin{cases}
    e^{\alpha\beta\varphi} \partial_\beta F_{\alpha\varphi} = 0; \\
    e^{\alpha\beta\varphi} \partial_\beta F_{\alpha0} = 0.
\end{cases}
\] (12)

We thus see that the covariant tensor components of the electromagnetic field can be represented as
\[
F_{\alpha\varphi} = \partial_\alpha A_\varphi; \quad F_{\alpha0} = \partial_\alpha A_0.
\] (13)

Here, \(A_i\) — are the covariant components of the 4-vector electromagnetic-field components. Let us now write the axial Maxwell equations for the contravariant field components without electric charges:
\[
\begin{cases}
    \frac{1}{\sqrt{-g}} \partial_\alpha (\sqrt{-g} F^{\alpha0}) = 0, \\
    \frac{1}{\sqrt{-g}} \partial_\alpha (\sqrt{-g} F^{\alpha\varphi}) = 4\pi j^\varphi.
\end{cases}
\] (14)

The right-hand part of the second equation in (14) is given by expression (11).

5 \(e^{\alpha\beta\gamma} = e_{\alpha\beta\gamma}\) — is the LeviCivita symbol.

6 A tilde denotes the electromagnetic field components in a stationary frame of reference that passes to the horizon reference frame, i.e., to the frame rotating with the FFO angular velocity on the horizon, \(\Omega_H = \frac{a}{r_H r_\varphi}\).
5 DETERMINING THE MAGNETIC-FIELD COMPONENTS

Let us determine the magnetic field in the Schwarzschild metric \((a = 0\) in \([4]\)). Because of axial symmetry, the vector potential of the magnetic field has the toroidal component alone. According to (13), we have for its covariant part

\[ F_{\gamma \phi} = g_{\gamma \beta} g_{\phi \phi} \partial_\beta A_\phi. \]  

(15)

Denoting the tensor components in Euclidean space by the subscript \(0\), we then have, according to (6),

\[ F_{\gamma \phi} = g_{\gamma \beta}^0 \left( \delta_{\gamma \beta} - \frac{r_a}{r} \delta_{\beta r} \delta_{r \gamma} \right) \partial_\beta A_\phi = F_{\beta \phi}^0 \left( \delta_{\beta \gamma} - \frac{r_a}{r} \delta_{\beta r} \delta_{r \gamma} \right). \]  

(16)

We see from (15) and (16) that if the function \(A_j\) is smooth and has no singularities, then the boundary conditions (5) are satisfied with the required asymptotics (2). The second equation in (14) can then be rewritten as

\[ \frac{1}{\sqrt{-g}} \partial_\alpha (\sqrt{-g} F_{\alpha \phi}^0) = 4\pi j_\phi^\phi. \]  

(17)

where we designated \(j_\phi^\phi = j_\phi + \frac{4\pi r}{4\pi r^2} \partial_\phi (r F_{\phi \phi}^0)\). Next, let us introduce the physical vector components by the definition (see \([3]\))

\[ \hat{B}^\alpha = B^\beta \sqrt{|g_{\alpha \beta}|}. \]  

(18)

In these components, Eq. (17) has the form of the Poisson equation in Euclidean space for the vector potential \(\hat{A}^\gamma\) with source \(\hat{j}_\text{tot}^\phi\). The solution of this equation is known to be (see \([3]\))

\[ \hat{A}^\alpha (r, \theta) = \delta^\alpha_\phi \hat{e}^\phi \int \frac{\hat{j}_\text{tot}^\gamma \hat{e}_\gamma}{r^2 - r'^2} \sin \theta' \, d\theta' \, \bigg|_{0}^{\pi} \, d\phi' \bigg|_{-\pi}^{\pi}. \]  

(19)

Here, \(\hat{e}^\gamma\) — is a unit vector \(\hat{a}\) in the direction of angle \(\phi\), \(|\vec{r} - \vec{r}'|^2 = r^2 + r'^2 - 2rr'(\cos \theta \cos \theta' + \sin \theta \sin \theta' \cos \phi')\), \(\theta\) — is the inclination of vector \(\vec{r}\) to the z axis, \(\theta'\) is the inclination of vector \(\vec{r}'\) to the z axis, and \(\phi'\) is the angle between the projections of vectors \(\vec{r}\) and \(\vec{r}'\) onto the plane perpendicular to the z axis. The integration is performed in Euclidean space outside the sphere of radius \(r_g\). Solution (19) is obtained by iterations; we assume that \(r_g = 0\) in the initial iteration.

Let us determine the magnetic vector

\[ H_\lambda = -\frac{\sqrt{-g}}{2} e_{\lambda \beta \gamma} F_{\beta \gamma}. \]  

(20a)

or

\[ F_{\alpha \beta} = -\frac{1}{\sqrt{-g}} H_\lambda e^{\lambda \alpha \beta}. \]  

(20b)

\(\gamma \hat{j}_\text{tot}^\gamma \hat{e}_\gamma = \hat{j}_\text{tot}^\beta r^\beta \sin \theta' \cos \phi'.\)
According to (18) and (20), the components \( A_\varphi, H_r, \) and \( H_\theta \) are given by

\[
A_\varphi = -r \sin \theta \tilde{A}_\varphi, \quad H_r = \frac{\partial_\theta (\sin \theta \tilde{A}_\varphi)}{r \sin \theta}, \quad H_\theta / r = -\frac{1 - \frac{r_\alpha}{r}}{r} \partial_r (r \tilde{A}_\varphi).
\]  

(21)

For the initial iteration, according to (21), the following expressions can be derived for the magnetic-field components from (19) and because of the presence of delta functions in (11):

\[
\begin{align*}
H^0_r(r, \theta, \varphi) &= \left[ \frac{J^\varphi}{r_0} \right] \int_{-\pi}^{\pi} \cos \theta \cos^2 \varphi'(1 + x^2 - \sin \theta \cos \varphi') x \sin \theta (1 + x^2 - 2x \sin \theta \cos \varphi')^{3/2} \, d\varphi', \\
H^0_\theta(r, \theta, \varphi) / r &= \left[ \frac{J^\varphi}{r_0} \right] \int_{-\pi}^{\pi} \frac{\cos \theta \sin^2 \varphi' - \cos \varphi'}{x(1 + x^2 - 2x \sin \theta \cos \varphi')^{3/2}} \, d\varphi'.
\end{align*}
\]

(22)

Here, we designated \( x = r / r_0 \). In particular, we obtain

\[
\lim_{\theta \to 0} (H^0_\theta / r) = 0, \quad \lim_{x \to \infty} (H^0_\theta / r) = \frac{\pi J^\varphi \sin \theta}{r_0}, \quad \lim_{\theta \to 0} H^0_r = \frac{2 \pi J^\varphi \cos \theta}{r_0 (1 + x^2)^{3/2}}.
\]  

(23)

6 DETERMINING THE ELECTRIC POTENTIAL

To calculate the electric potential \( A_0 \), we use the formula

\[
F^{\alpha 0} = \{ F^{00}_\alpha + F^{\varphi \varphi} g^{\alpha 0} \} g^{0 \beta}.
\]  

(24)

However, the components \( F^{\alpha 0} \) must be expressed in terms of \( \tilde{F}^{00} \) and \( \tilde{F}^{\varphi \varphi} \), because we know the boundary conditions only for them in stationary frames of reference. For the poloidal electromagnetic-field components, using formulas (6), (8), and (9), and the expression

\[
g^{\alpha \beta} g_{\gamma \gamma} = g^{\alpha \gamma} g_{\gamma \beta} = \delta^{\alpha \beta},
\]

we obtain

\[
F^{\alpha 0} = L_f \left[ g^{\alpha 0} g^{0 \beta} \tilde{A}_0, \beta + F^{\alpha \varphi} (\Omega_H - \Omega) / (\kappa + \Omega^2) \right],
\]

(25)

where \( L_f = [1 - (\Omega - \Omega_H)^2 / (\kappa + \Omega^2)]^{-1} \) has the meaning of FFO Lorentz factor relative to the stationary frame of the horizon. Hence, in the second approximation in \( a^* = a^* / M = L / M^2 \), the dimensionless black-hole angular momentum \( 0 < a^* < 1 \), the first equation in (14) can be reduced to

\[
\Delta \tilde{A}_0 = -4\pi \left( f_{(H_0)} + U_{(\tilde{A}_0)} \right),
\]

(26)

Here, \( \Delta \) — is the Laplace operator in Euclidean space and \( H_\alpha \) — are the Schwarzschild magnetic-field components derived in the preceding section:

\[
U_{(\tilde{A}_0)} = -\frac{r_\alpha}{4\pi r^2} \partial_r (r^2 \partial_r \tilde{A}_0), \quad f_{(H_0)} = \frac{a^*}{4\pi r^2} \left( 1 - \frac{r_\alpha}{r} \right) \times \left( 1 + \frac{r_\alpha}{r} + \frac{r_\alpha^2}{r^2} \right) [2 \cos \theta H_r - 4\pi j^r r^2 \sin^2 \theta] - \sin \theta (2 + \frac{r_\alpha}{r}) H_\theta / r.
\]

The expression in round brackets on the right-hand side of Eq. (26) is an analog of

\footnote{For \( \alpha \) running the values of \( r \) and \( \theta \), we have \( g^{00} = 0 \) and \( g^{0 \alpha} = 0 \).}
the electric charge density in the Poisson equation. However, because of the second term, the \( \tilde{A}_0 \) — dependent function \( U \), it can be solved by the iteration method assuming that \( U = 0 \) in the initial iteration. According to the boundary conditions (10), solving Eq. (26) is equivalent to calculating the potential \( \tilde{A}_0 \), produced by the density of electric charge \( \rho = (f + U) \) around a conductive sphere of radius \( r_g \) in Euclidean space. This external problem for the Poisson equation can be solved by the image method [6]. The solution that satisfies the boundary conditions (10) is

\[
\tilde{A}_0(r, \theta) = \int \rho(r', \theta') \left\{ \frac{1}{|\vec{r} - \vec{r}'|} - \frac{r_g/r'}{|\vec{r} - \vec{r}'(r^2/r'^2)|} \right\} r'^2 \sin \theta' d\theta' \left[ d\varphi' \right]\left[ -\pi \right].
\]

(27)

The integration is performed in Euclidean space outside the sphere of radius \( r_g \) [see (19)]. According to (7), we obtain the potential \( A_0 \) from (27)

\[
A_0 = \tilde{A}_0 - \Omega \omega \tilde{A}_\varphi; \quad A_\varphi = \tilde{A}_\varphi.
\]

(28)

We can factor \( a J^z/r_0 \) outside integral (27). Denoting the magnetic-field strength at the center of the system in the absence of a black hole by \( H_0 = 2\pi J^z/r_0 \) [see (23)], we derive for \( \tilde{A}_0 \) and \( A_0 \):

\[
\tilde{A}_0 = \tilde{A}_0 - \Omega \omega \tilde{A}_\varphi; \quad A_0 = \tilde{A}_0 - \Omega \omega \tilde{A}_\varphi.
\]

(29)

Here, \( \tilde{A}_0 = \tilde{A}_0 - \Omega \omega \tilde{A}_\varphi \) and \( \tilde{A}_0 = \tilde{A}_0 - \Omega \omega \tilde{A}_\varphi \) are dimensionless functions, which can be numerically calculated using expressions (27) and (28). The corresponding results are shown in the figure. Some important characteristics of the solution can also be established analytically.

1). Since the system has a mirror symmetry relative to the inversion in the equatorial plane, the electric-field direction on the axis depends on the coincidence of the directions of black-hole angular momentum and ring-current moment.

2). We see from (27) that \( \tilde{A}_0 \to 0 \) when \( r \to r_H \) (the expression in curly braces becomes zero). This important result has a simple explanation: Since the total charge under the horizon is zero, the electric field must pierce the horizon in different directions. Therefore, there must be an equipotential surface that separates these directions, pierces the horizon, and goes to infinity. Since the potential \( \tilde{A}_0 \) changes neither on this surface, nor on the horizon (in view of the boundary conditions), the horizon potential is equal to the potential at infinity (zero). This conclusion has an important implication: the existence of a local extremum for the potential \( \tilde{A}_0 \). Similar reasoning also applies to \( A_0 \). It might seem that a nonzero electric charge density must arise near the extremum. In reality, however, this is not the case: the charge density is determined by the contravariant electric-field components \( F^{\alpha 0} \) rather than \( F_{\alpha 0} \), while the derivation of (27) is based on the first equation in (14), a zero charge density in the entire space. We emphasize that there is no extremum in the well-known solution of Wald [7] for a black hole placed in a

\( ^9 \)This can be seen from (26) and from the expression for \( f \) (\( \sin \theta \) and \( H_\theta \) are even, while \( \cos \theta \) and \( H_r \) are odd relative to the mirror inversion).
uniform magnetic field aligned along the symmetry axis of the black hole. This is because the field does not vanish at infinity in this solution, and this nonphysical boundary condition wipes out the extremum.

7 THE KINETIC ENERGY OF A CHARGED PARTICLE EMERGING ALONG THE AXIS

The conserved massenergy of a charged particle with a zero angular momentum component along the z axis is given by [4, 5]

$$E = m_0 \gamma_L \sqrt{\frac{\rho^2 \Delta}{(r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta} - q A_0} = m_0 \gamma_L \infty,$$

(30)

where $\gamma_L$ — is the Lorentz factor of the particle, and $q$ — is its charge. Consider the acceleration of an emerging charged particle along the axis by an electric field.
Since $qA_0 = -|q\tilde{A}_0|$ in this case, the kinetic energy of this particle at infinity with $\gamma_L$, $g_{00}$, and $A_0$ specified at any point of the $z$ axis is

$$E_k = E - m_o = m_o(\sqrt{g_{00}\gamma_L} - 1) + |qA_0|.$$  \hfill (31)

Since the electromagnetic energy of a charged particle for quasars is much larger than the corresponding rest energy, it is convenient to represent the result for $A_0$ as the energy (in electronvolts) that an elementary charge emerging along the axis acquires. For magnetic fields $H_0 \approx 10^4$ and mass $M \approx 10^8 M_\odot$ we obtain from (29) and (31):

$$E_k \approx a^* \cdot 10^{19} \cdot \text{Int} \left( \frac{r}{r_g}, 0 \right) [\text{eV}].$$

8 CONCLUSION

We have calculated the electric field generated by a rotating black hole that interacts with an external magnetic field. As can be seen from the above discussion, this field is quadrupole in nature, being actually an analog of the pulsar electric field (unipolar inductor model [8]). The situation on the $z$ axis was not chosen by chance. Because of strong magnetic fields, the Larmor radii of a charged particle must be of the order of the gravitational radius of the system, and the particle acceleration mechanism will be effective only in directions close to the $z$ axis, where the Lorentz force does not act. The reader may ask a legitimate question: Will a strong electric field produce electronpositron plasma near the horizon and will it destroy the force field $(E \cdot H) \neq 0$? At large gradients in $A_0$, this can actually happen, and the problem should then be solved in the force-free approximation (see review papers [911] on this subject). However, as numerical estimates show, the electric-field strength in the model does not exceed $10^7 \text{V cm}^{-1}$ while a strength of the order of $\sim 10^9 \text{V cm}^{-1}$ is required for the particle production. In addition, in the force-free approximation, particles cannot be accelerated along the $z$ axis, because there is no electric field in this direction. Besides, in any direction in a forcefree field, the electric and magnetic fields equally act on the particle. Therefore, even if its trajectory goes to infinity, the particle loses most of its energy in the process. Recall that jets are observed in quasars precisely in the directions along and opposite to the axis. For this reason, solving the problem under consideration in the force-field approximation is of considerable importance.

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\(^{10}\)The production of particles by inverse-Compton-type effects is not considered, because the particle number density around the black hole is assumed to be low.
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REFERENCES
1. R. D. Blandford and R. L. Znajek, Mon. Not. R. Astron. Soc. 179, 433 (1977).
2. J. M. Bardeen and J. A. Petterson, Astrophys. J. Lett. 195, L65 (1975).
3. L. D. Landau and E. M. Lifshitz, The Classical Theory of Fields (Pergamon, Oxford, 1975; Nauka, Moscow, 1988).
4. Black Holes: the Membrane Paradigm, Ed. by K. S. Thorne, R. H. Price, and D. A. Macdonald (Yale Univ. Press, New Haven, 1986; Mir, Moscow, 1988).
5. I. D. Novikov and V. P. Frolov, Physics of Black Holes (Nauka, Moscow, 1986).
6. L. D. Landau and E. M. Lifshitz, Course of Theoretical Physics, Vol. 8: Electrodynamics of Continuous Media (Nauka, Moscow, 1982; Pergamon, New York, 1984).
7. R. M. Wald, Phys. Rev. D 10, 1680 (1974).
8. P. Goldreich and W. Julian, Astrophys. J. 157, 869 (1969).
9. V. S. Beskin, Usp. Fiz. Nauk 167, 689 (1997) [Phys. Usp. 40, 659 (1997)].
10. V. S. Beskin, Ya. N. Istomin, and V. I. Parev, Astron. Zh. 69, 1258 (1992) [Sov. Astron. 36, 642 (1992)].
11. J. S. Heyl, Phys. Rev. D 63, 064028 (2001); gr-qc/0012007; A. Tomimatsu and H. Koyama, Phys. Rev. D 61, 124010 (2000); gr-qc/0002020.
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