Neutrino conversions in cosmological gamma-ray burst fireballs

H. Athar

Department of Physics, Tokyo Metropolitan University, Minami-Osawa 1-1, Hachioji-Shi, Tokyo 192-0397, Japan; E-mail: athar@phys.metro-u.ac.jp

We study neutrino conversions in a recently envisaged source of high energy neutrinos ($E \sim 10^6$ GeV), that is, in the vicinity of cosmological Gamma-Ray Burst fireballs (GRB). We consider the effects of flavor and spin-flavor conversions and point out that in both situations, a somewhat higher than estimated high energy tau neutrino flux from GRBs is expected in new km$^2$ surface area under water/ice neutrino telescopes.

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I. INTRODUCTION

Recently, cosmological fireballs are suggested as a possible production site for gamma-ray bursts as well as the high energy ($E \gtrsim 10^6$ GeV) neutrinos [4]. Although, the origin of these cosmological Gamma-Ray Burst fireballs (GRB) is not yet understood, the observations suggest that generically a very compact source of linear scale $\sim 10^7$ cm through internal or/and external shock propagation produces these gamma-ray bursts (as well as the burst of high energy neutrinos) [3]. Typically, this compact source is hypothesized to be formed possibly due to the merging of binary neutron stars or due to collapse of a supermassive star.

The main source of high energy tau neutrinos in GRBs is the production and decay of $D_S^\pm$. The production of $D_S^\pm$ may be through $p\gamma$ and/or through $pp$ collisions. In [3], the $\nu_e$ and $\nu_\mu$ flux is estimated in $pp$ collisions, whereas in [4], the $\nu_e$ and $\nu_\mu$ flux is estimated in $p\gamma$ collisions for GRBs. In $pp$ collisions, the flux of $\nu_\tau$ may be obtained through the main process of $p + p \rightarrow D_S^0 + X$. The $D_S^0$ decays into $\tau^+$ lepton and $\nu_\tau$ with a branching ratio of $\sim 3\%$. This $\tau^+$ lepton further decays into $\nu_\tau$. The cross-section for $D_S^\pm$ production, which is main source of $\nu_\tau$’s, is $\sim 4$ orders of magnitude lower than that of $\pi^+$ and/or $\pi^-$ which subsequently produces $\nu_e$ and $\nu_\mu$. The branching ratio for $\nu_e$ and/or $\nu_\mu$ production is higher up to an order of magnitude than that for $\nu_\tau$ production (through $D_S^\pm$). These imply that the $\nu_\tau$ flux in $pp$ collisions is suppressed up to 5 orders of magnitude relative to corresponding $\nu_e$ and/or $\nu_\mu$ fluxes. In $p\gamma$ collisions, the main process for the production of $\nu_\tau$ may be
\[ p + \gamma \rightarrow D_S^+ + \Lambda^0 + D^0 \] with similar relevant branching ratios and corresponding suppression for cross-section values. Here the corresponding main source for \( \nu_e \) and \( \nu_\mu \) production is \( p + \gamma \rightarrow \Delta^+ \rightarrow \pi^+ + n \). Therefore, in \( p\gamma \) collisions, the \( \nu_\tau \) flux is also suppressed up to 5 orders of magnitude relative to \( \nu_e \) and/or \( \nu_\mu \) flux. Thus, in both type of collisions, including the relevant kinematics, the intrinsic \( \nu_\tau (\bar{\nu}_\tau) \) flux is estimated to be rather small relative to \( \nu_e (\bar{\nu}_e) \) and/or \( \nu_\mu (\bar{\nu}_\mu) \) fluxes from GRBs, typically being, \( F_\tau^0 / F_{e,\mu}^0 < 10^{-5} \) [4].

In this paper, we consider the possibility of obtaining higher \( \nu_\tau \) flux, that is, \( F_\tau^0 / F_{e,\mu}^0 > 10^{-5} \), from GRBs through neutrino conversions as compared to no conversion situation. The present study is particularly useful as the new under ice/water Čerenkov light neutrino telescopes namely AMANDA and Baikal (also the NESTOR and ANTARES) will have energy, angle and flavor resolution for high energy neutrinos originating at cosmological distances [5]. Recently, there are several discussions concerning the signatures of a possible neutrino burst from GRBs correlated in time and angle [6]. In particular, there is a suggestion of measuring \( \nu_\tau \) flux from cosmologically distant sources through a double shower (bang) event [7] or through a small pile up of up ward going \( \mu \)-like events near \( (10^4 - 10^5) \) GeV [8].

The plan of this paper is as follows. In Sect. II, we briefly describe the matter density and magnetic field in the vicinity of GRBs. In Sect. III, we discuss in some detail, the range of neutrino mixing parameters that may give rise to relatively large precession/conversion probabilities resulting from neutrino flavor/spin-flavor conversions. In Sect. IV, we give estimates for separable but contained double shower event rates induced by high energy \( \nu_\tau \)'s originating from GRBs without/with conversions for km² surface area under water/ice neutrino telescopes for illustrative purposes and finally in Sect. V, we summarize our results.

**II. MATTER DENSITY AND MAGNETIC FIELD IN THE VICINITY OF GRB**

According to [1], the isotropic high energy neutrino production may take place in the vicinity of \( r_p \sim \Gamma^2 c \Delta t \). Here \( \Gamma \) is the Lorentz factor (typically \( \Gamma \sim 300 \)) and \( \Delta t \) is the observed GRB variability time scale (typically \( \Delta t \sim 1 \) ms). In the vicinity of \( r_p \), the fireball matter density is \( \rho \sim 10^{-13} \) g cm\(^{-3} \) [1]. In these models, the typical distance traversed by neutrinos may be taken as, \( \Delta r \lesssim (10^{-4} - 1) \) pc, in the vicinity of GRB, where 1 pc \( \sim 3 \times 10^{18} \) cm. Matter effects on neutrino oscillations are relevant if \( G_F \rho / m_N \sim \delta m^2 / 2E \). Using \( \rho \) from Ref. [1], it follows that matter effects are absent for \( \delta m^2 \gtrsim \mathcal{O}(10^{-10}) \) eV\(^2 \). Matter effects due to coherent forward scattering of neutrinos off the background for high energy neutrinos
originating from GRBs are not expected to be important in the neutrino production regions around GRBs and will not be further discussed here.

Taking the observed duration of the typical gamma-ray burst as, $\Delta t \lesssim 1 \text{ ms}$, we obtain the mass of the source as, $M_{\text{GRB}} \lesssim \Delta t / G_N$, where $G_N$ is the gravitational constant. For the relatively shorter observed duration of gamma-ray burst from a typical GRB, $\Delta t \sim 0.2 \text{ ms}$, implying $M_{\text{GRB}} \sim 40 M_\odot$ (where $M_\odot \sim 2 \times 10^{33} \text{ g}$ is solar mass). We use $M_{\text{GRB}} \sim 2 \times 10^2 M_\odot$ in our estimates.

The magnetic field in the vicinity of a GRB is obtained by considering the equipartition arguments \[^1\]. We use the following profile of magnetic field, $B_{\text{GRB}}$, as an example, for $r > r_p$ \[^3\]

$$B_{\text{GRB}}(r) \simeq B_0 \left( \frac{r_p}{r} \right)^2,$$

where $B_0 \sim L^{1/2} e^{-1/2} (r_p \Gamma)^{-1}$ with $L$ being the total wind luminosity (typically $L \sim 10^{51} \text{ erg s}^{-1}$).

**III. NEUTRINO CONVERSIONS IN GRB**

**A. Flavor oscillations**

In the framework of three flavor analysis, the flavor precession probability from $\alpha$ to $\beta$ neutrino flavor is \[^10\]

$$P(\nu_\alpha \to \nu_\beta) \equiv P_{\alpha\beta} = \sum_{i=1}^{3} |U_{\alpha i}|^2 |U_{\beta i}|^2 + \sum_{i \neq j} U_{\alpha i} U^*_{\beta i} U_{\alpha j} U^*_{\beta j} \cos \left( \frac{2\pi L}{l_{ij}} \right),$$

where $\alpha, \beta = e, \mu, \tau$. $U$ is the $3 \times 3$ MNS mixing matrix and can be obtained in usual notation through

$$U \equiv R_{23}(\theta_1) \text{diag}(e^{-i\delta/2}, 1, e^{i\delta/2}) R_{31}(\theta_2) \text{diag}(e^{i\delta/2}, 1, e^{-i\delta/2}) R_{12}(\theta_3),$$

thus coinciding with the standard form given by the Particle Data Group \[^11\]. In Eq. (3), $l_{ij} \simeq 4\pi E / \delta m^2_{ij}$ with $\delta m^2_{ij} \equiv |m_i^2 - m_j^2|$ and $L$ is the distance between the source and the detector. For simplicity, we will assume here a vanishing value for $\theta_{31}$ and CP violating phase $\delta$ in $U$.

At present, the atmospheric muon and solar electron neutrino deficits can be explained with oscillations among three active neutrinos \[^12\]. For this, typically, $\delta m^2 \sim \mathcal{O}(10^{-3}) \text{ eV}^2$
and \( \sin^2 2\theta \sim \mathcal{O}(1) \) for the explanation of atmospheric muon neutrino deficit, whereas for the explanation of solar electron neutrino deficit, we may have \( \delta m^2 \sim \mathcal{O}(10^{-10}) \text{ eV}^2 \) and \( \sin^2 2\theta \sim \mathcal{O}(1) \) [just so] or \( \delta m^2 \sim \mathcal{O}(10^{-5}) \text{ eV}^2 \) and \( \sin^2 2\theta \sim \mathcal{O}(10^{-2}) \) [SMA (MSW)] or \( \delta m^2 \sim \mathcal{O}(10^{-5}) \text{ eV}^2 \) and \( \sin^2 2\theta \sim \mathcal{O}(1) \) [LMA (MSW)]. The present status of data thus permits multiple oscillation solutions to solar neutrino deficit. We intend to discuss here implications of these mixings for high energy cosmic neutrino propagation.

In the above explanations, the total range of \( \delta m^2 \) is \( 10^{-10} \leq \delta m^2/\text{eV}^2 \leq 10^{-3} \) irrespective of neutrino flavor. The typical energy span relevant for possible flavor identification for high energy cosmic neutrinos is \( 2 \times 10^6 \leq E/\text{GeV} \leq 2 \times 10^7 \) (see Sect. IV). Taking a typical distance between the GRB and our galaxy as \( L \sim 1000 \text{ Mpc} \), we note that the cos term in Eq. (2) vanishes and so Eq. (2) reduces to

\[
P_{\alpha\beta} \simeq \sum_{i=1}^{3} |U_{\alpha i}|^2 |U_{\beta i}|^2. \tag{4}
\]

It is assumed here that no relatively dense objects exist between the GRB and the earth so as to effect significantly this oscillations pattern. Since \( P_{\alpha\beta} \) in above Eq. is symmetric under the exchange of \( \alpha \) and \( \beta \) indices implying that essentially no \( T \) (or \( CP \)) violation effects arise in neutrino vacuum flavor oscillations for high energy cosmic neutrinos [13].

Let us denote by \( F^0_\alpha \), the intrinsic neutrino fluxes. From the discussion in the previous Sect., it follows that \( F^0_e : F^0_\mu : F^0_\tau = 1 : 2 : \ll 10^{-5} \). For simplicity, we take these ratios as \( 1 : 2 : 0 \). In order to estimate the final flux ratios on earth for high energy cosmic neutrinos originating from cosmologically distant GRBs, let us introduce a \( 3 \times 3 \) matrix of vacuum flavor precession probabilities such that

\[
F_\alpha = \sum_{\beta} P_{\alpha\beta} F^0_{\beta}, \tag{5}
\]

where the unitarity conditions for \( P_{\alpha\beta} \) read as

\[
P_{ee} + P_{e\mu} + P_{e\tau} = 1, \\
P_{e\mu} + P_{\mu\mu} + P_{\mu\tau} = 1, \\
P_{e\tau} + P_{\mu\tau} + P_{\tau\tau} = 1. \tag{6}
\]

The explicit form for the matrix \( P \) in case of just so flavor oscillations as solution to solar neutrino problem along with the solution to atmospheric neutrino deficit in terms of \( \nu_\mu \) to \( \nu_\tau \) oscillations with maximal depth is
Using Eq. (7) and Eq. (5), we note that $F_e : F_\mu : F_\tau = 1 : 1 : 1$ at the level of $F^0_e$. Also, Eq. (8) is satisfied. The same final flux ratio is obtained in remaining two cases for which the corresponding $P$ matrices are [for SMA (MSW)]

$$P = \begin{pmatrix} 1/2 & 1/4 & 1/4 \\ 1/4 & 3/8 & 3/8 \\ 1/4 & 3/8 & 3/8 \end{pmatrix}. \tag{7}$$

whereas in case of LMA (MSW),

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{pmatrix}. \tag{8}$$

Thus, independent of the oscillation solution for solar neutrino problem, we have $F_e : F_\mu : F_\tau = 1 : 1 : 1$. Importantly, these ratios do not depend on neutrino energy or $\delta m^2$ at least in the relevant energy interval.

Summarizing, although intrinsically the high energy cosmic tau neutrino flux is negligibly small however because of vacuum flavor oscillations it becomes comparable to $\nu_e$ flux thus providing some prospects for its possible detection.

**B. Spin-flavor oscillations**

We consider here an example of a possibility that may lead to an energy dependence and/or change in the above mentioned final flux ratios. We consider spin-flavor oscillations resulting from an interplay of possible Violation of Equivalence Principle (VEP) parameterized by $\Delta f$ and the magnetic field in the vicinity of GRBs. We obtain the range of neutrino mixing parameters giving $F_\tau/F_{e,\mu} > 10^{-5}$. The possibility of VEP arises from the realization that different flavors of neutrinos may couple differently to gravity [14].

Consider a system of two mixed neutrinos ($\bar{\nu}_e$ and $\nu_\tau$) for simplicity. The difference of diagonal elements of the $2 \times 2$ effective Hamiltonian describing the dynamics of the mixed
system of these oscillating neutrinos in the basis $\psi^T = (\bar{\nu}_e, \nu_\tau)$ for vanishing vacuum and gravity mixing angles is \[\Delta H = \delta - V_G,\] whereas each of the off diagonal elements is $\mu B$ ($\mu$ is magnitude of neutrino magnetic moment). In Eq. (10), $\delta = \delta m^2 / 2E$, where $\delta m^2 = m^2(\nu_\tau) - m^2(\bar{\nu}_e) > 0$. Here $V_G$ is the effective potential felt by the neutrinos at a distance $r$ from a gravitational source of mass $M$ due to VEP and is given by \[V_G \equiv \Delta \phi \phi(r) E,\] where $\Delta \phi = f_3 - f_1$ is a measure of the degree of VEP and $\phi(r) = G_N M r^{-1}$ is the gravitational potential in the Keplerian approximation. In Eq. (11), $f_3 G_N$ and $f_1 G_N$ are respectively the gravitational couplings of $\nu_\tau$ and $\bar{\nu}_e$, such that $f_1 \neq f_3$. We assume here same gravitational couplings for $\nu$ and $\bar{\nu}$ for simplicity. This implies that the sign of $V_G$ is same for $\bar{\nu}_e \rightarrow \nu_\tau$ and $\nu_e \rightarrow \bar{\nu}_\tau$ conversions. If this is not the case then the sign of $V_G$ will be different for the two conversions and only one of the two conversions can occur. This may lead to a change in expected $\nu_e / \bar{\nu}_e$ ratio. We briefly comment on the possibility of empirical realization of this situation in next Sect..

There are at least three relevant $\phi(r)$’s that need to be considered. The effect of $\phi(r)$ due to supercluster named Great Attractor with $\phi_{SC}(r)$ in the vicinity of GRB; $\phi(r)$ due to GRB itself, which is, $\phi_{GRB}(r)$, in the vicinity of GRB and the galactic gravitational potential, which is, $\phi_G(r)$. Therefore, we use, $\phi(r) \equiv \phi_{SC}(r) + \phi_{GRB}(r) + \phi_G(r)$. However, $\phi_G(r) \ll \phi_{SC}(r), \phi_{GRB}(r)$ in the vicinity of GRB. Thus, $\phi(r) \simeq \phi_{SC}(r) + \phi_{GRB}(r)$. If the neutrino production region $r_p$ is $\lesssim 10^{13}$ cm then at $r \sim r_p$, we have $\phi_{GRB}(r) > \phi_{SC}(r)$. At $r \sim 6 \times 10^{13}$ cm, $\phi_{GRB}(r) \sim \phi_{SC}(r)$ and for $r \gtrsim 10^{14}$ cm, $\phi_{GRB}(r) < \phi_{SC}(r)$. If the supercluster is a fake object then $\phi(r) \simeq \phi_{GRB}(r)$. Here we assume the smallness of the effect of $\phi(r)$ due to an Active Galactic Nucleus (AGN), if any, nearby to GRB.

The possibility of vanishing gravity and vacuum mixing angle in Eq. (10) allows us to identify the range of $\Delta f$ relevant for the neutrino magnetic moment effects only. Latter in this Sect., we briefly comment on the ranges of relevant neutrino mixing parameters for non vanishing gravity mixing angle with vanishing neutrino magnetic moment.

The case of $\bar{\nu}_\mu \rightarrow \nu_\tau$ can be studied by replacing $\bar{\nu}_e$ with $\bar{\nu}_\mu$ along with corresponding changes in $V_G$ and $\delta m^2$. The intrinsic flux of $\bar{\nu}_\mu$ may be greater than that of $\bar{\nu}_e$ by a factor.
of $\sim 2$, thus also possibly enhancing the expected $\nu_\tau$ flux from GRBs through $\bar{\nu}_\mu \to \nu_\tau$. However, we have checked that observationally this possibility leads to quite similar results in terms of event rates and are therefore not discussed here further. We now study in some detail, the various possibilities arising from relative comparison between $\delta$ and $V_G$ in Eq. (10).

Let us first ignore the effects of VEP ($\Delta f = 0$). For constant $B$, the spin-flavor precession probability $P(\bar{\nu}_e \to \nu_\tau)$ is obtained using Eq. (10) as

$$P(\bar{\nu}_e \to \nu_\tau) = \left[ \frac{(2\mu B)^2}{(2\mu B)^2 + \delta^2} \right] \sin^2 \left( \sqrt{(2\mu B)^2 + \delta^2} \cdot \frac{\Delta r}{2} \right).$$  

(12)

We take $\mu \sim 10^{-12} \mu_B$ or less, where $\mu_B$ is Bohr magneton, which is less than the stringent astrophysical upper bound on $\mu$ based on cooling of red giants $[17]$. We here consider the transition magnetic moment, thus allowing the possibility of simultaneously changing the relevant neutrino flavor as well as the helicity. Therefore, the precessed $\nu_\tau$ is an active neutrino and interacts weakly. In Eq. (12), $\Delta r$ is the width of the region with $B$. If $\delta < 2\mu B$, then, for $E \sim 2 \times 10^6$ GeV and using Eq. (1), we obtain $\delta m^2 < 5 \times 10^{-8}$ eV$^2$. We take, $\delta m^2 \sim 10^{-9}$ eV$^2$, as an example and consequently we obtain from Eq. (12) an energy independent large ($P > 1/2$) spin-flavor precession probability for $\mu \sim 10^{-12} \mu_B$ with $10^{-4} \lesssim \Delta r/\text{pc} \lesssim 1$. This relatively small value of $\delta m^2$ is also interesting in the context of sun and supernovae $[18]$. Thus, for $\mu$ of the order of $10^{-12} \mu_B$, the $\nu_\tau$ flux may be higher than the expected one from GRBs, that is, $F_\tau/F_e > 10^{-5}$ due to neutrino spin-flavor precession effects. The neutrino spin-flavor precession effects are essentially determined by the product $\mu B$ so one may rescale $\mu$ and $B$ to obtain the same results. For $\delta \approx 2\mu B$ and $\delta > 2\mu B$, we obtain from Eq. (12), an energy dependent $P$ such that $P < 1/2$.

With non vanishing $\Delta f$ ($\Delta f \neq 0$), a resonant character in neutrino spin-flavor precession can be obtained for a range of values of relevant neutrino mixing parameters $[1]$. Two conditions are essential to obtain a resonant character in neutrino spin-flavor precession: the level crossing and the adiabaticity at the level crossing (resonance). The level crossing condition is obtained by taking $\Delta H = 0$ and is given by:

\[ \Delta H = 0 \]

\[ \frac{\Delta m^2}{2E} = \frac{\Delta r}{\text{pc}} \]

\[ \Delta m^2 \approx \frac{\Delta \tau}{\text{pc}} \]

\[ \frac{\Delta \tau}{\text{pc}} \approx \frac{\Delta r}{\text{pc}} \]

From above discussion, it follows that $E$ dependent/independent spin-flavor precession may also be obtained for non zero $\Delta f$, however, given the current status of the high energy neutrino detection, for simplicity, we ignore these possibilities which tend to overlap with this case for a certain range of relevant neutrino mixing parameters; for details of these possibilities in the context of AGN, see $[19]$. 

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\[ \delta m^2 \sim 10^{-3} \text{eV}^2 \left( \frac{\Delta f}{10^{-28}} \right). \] (13)

These \( \Delta f \) values are well below the relevant upper limits on \( \Delta f \) which are typically in the \( 10^{-20} \) range [24]. Conversely speaking, the prospective detection of high energy neutrinos from cosmologically distant GRBs may be sensitive to \( \Delta f \) values as low as \( \sim 10^{-28} \). The other essential condition, namely, the adiabaticity in the resonance reads [21]

\[ \kappa \equiv \frac{2(2\mu B)^2}{|dV_G/dr|} \gtrsim 1. \] (14)

Note that here \( \kappa \) depends explicitly on \( E \) through \( V_G \) unlike the case of ordinary neutrino spin-flip induced by the matter effects. A resonant character in neutrino spin-flavor precession is obtained if \( \kappa \gtrsim 1 \) such that Eq. (13) is satisfied. We notice that \( B_{ad}/B_{GRB} \lesssim 1 \) for \( \mu \sim 10^{-12} \mu_B \). Here \( B_{ad} \) is obtained by setting \( \kappa \sim 1 \) in Eq. (14). The general expression for relevant neutrino spin-flavor conversion probability is given by [22]

\[ P(\bar{\nu}_e \rightarrow \nu_\tau) = \frac{1}{2} - \left( \frac{1}{2} - P_{LZ} \right) \cos 2\theta_f \cos 2\theta_i, \] (15)

where \( P_{LZ} = \exp(-\frac{\pi}{4}\kappa) \) and \( \tan 2\theta_i = (2\mu B)/\Delta H \) is being evaluated at the high energy neutrino production site in the vicinity of GRB, whereas \( \tan 2\theta_f = (2\mu B)/\delta \) is evaluated at the exit. In Fig. 1, we plot \( P(\bar{\nu}_e \rightarrow \nu_\tau) \) given by Eq. (15) as a function of \( \Delta f \) as well as \( \delta m^2 \) with \( E \sim 5 \times 10^6 \) GeV. Four equi-\( P \) contours are also shown in Fig. 1. Note that the resonant spin-flavor precession probability is relatively small (\( P < 1/2 \)) for \( \Delta f \gtrsim 10^{-26} \) essentially irrespective of \( \delta m^2 \) values. The expected spectrum \( F_\tau \) of the high energy tau neutrinos originating from GRBs due to spin-flavor conversions is calculated as [22]

\[ F_\tau \approx P(\bar{\nu}_e \rightarrow \nu_\tau)F_0^{\mu}. \] (16)

The energy dependence in \( F_\tau \) is now evident [as compared to \( F_\tau \) given by Eq. (13)] when we convolve \( P(\bar{\nu}_e \rightarrow \nu_\tau) \) given by Eq. (15) with \( F_0^{\mu} \) taken from Ref. [1]. The degree of energy dependence clearly depends on the extent of spin-flavor conversions. With the improved information on either \( \Delta f \) and/or \( \mu \), one may be able to distinguish between the situations of resonant and non resonant spin-flavor precession induced by an interplay of VEP and \( \mu \) in \( B_{GRB} \).

Let us now consider briefly the effects of non vanishing gravity mixing angle \( \theta_G \) for vanishing neutrino magnetic moment. In the case of massless or degenerate neutrinos, the corresponding vacuum flavor oscillation analog for \( \nu_e \rightarrow \nu_\tau \) is obtained through \( \theta \rightarrow \theta_G \) and
$\frac{im^2}{4E} \rightarrow V_G$ in the standard flavor precession probability formula in 2 flavor approximation. For maximal $\theta_G$, the sensitivity of $\Delta f$ may be estimated by equating the argument of second sin factor equal to $\pi/2$ in the corresponding expression for $P$ [14]. This implies $\Delta f \sim 10^{-41}$ with $\phi(r) \simeq \phi_{SC}(r)$. This value of $\Delta f$ is of the same order of magnitude as that expected for neutrinos originating from AGNs. In case of non zero $\delta m^2$, a resonant or/and non resonant flavor conversion between $\nu_e$ and $\nu_\tau$ in the vicinity of a GRB is also possible due to an interplay of vanishing/non vanishing vacuum and gravity mixing angles. For instance, a resonant flavor conversion between $\nu_e$ and $\nu_\tau$ may be obtained if $\sin^2 2\theta_G \gg 0.25$ with $\Delta f \sim 10^{-31}$ ($\theta \rightarrow 0$). Here the relevant level crossing may occur at $r \sim 0.1$ pc with corresponding $\delta m^2 \sim 10^{-6}$ eV$^2$.

IV. SIGNATURES OF HIGH ENERGY $\nu_\tau$ IN NEUTRINO TELESCOPES

The km$^2$ surface area under water/ice high energy neutrino telescopes may be able to obtain first examples of high energy $\nu_\tau$, through double showers, originating from GRBs correlated in time and direction with corresponding gamma-ray burst or may at least provide relevant useful upper limits [7]. The first shower occurs because of deep inelastic charged current interaction of high energy tau neutrinos near/inside the neutrino telescope producing the tau lepton (along with the first shower) and the second shower occurs due to (hadronic) decay of this tau lepton.

The calculation of downward going contained but separable double shower event rate for a km$^2$ surface area under ice/water neutrino telescope can be carried out by replacing the muon range expression with the tau one ($\sim E(1 - y)\tau c/m_\tau c^2$) and then subtracting it from the linear size of a typical high energy neutrino telescope in the event rate formula while using the expected $\nu_\tau$ flux spectrum given by Eq. (5) and/or by Eq. (16). Here, $y$ is the fraction of the neutrino energy carried by the hadrons in lab frame. Thus, $(1 - y)$ is the fraction of energy transferred to the associated tau lepton having life time $\tau c$ and mass $m_\tau c^2$. We take here $y \sim 0.25$ [6]. The condition of containdness of the two showers is obtained by requiring that the separation between the two showers is less than the typical $\sim$ km size of the neutrino telescope. It is obtained by equating the range of tau neutrino induced tau leptons with the linear size of detector implying $E \lesssim 2 \times 10^7$ GeV. The condition of separableness of the two showers is obtained by demanding that the separation between the two showers is larger than the typical spread of the showers such that the amplitude of
the second shower is essentially 2 times the first shower. This leads to $E \gtrsim 2 \times 10^6$ GeV. Thus, the two showers may be separated by a $\mu$-like track within these energy limits. To calculate the event rates, we use Martin Roberts Stirling (MRS 96 R1) parton distributions and present event rates in units of yr$^{-1}$ sr$^{-1}$. We have checked that other recent parton distributions give quite similar event rates and are therefore not depicted here. Following [24,25], we present in Table I, the expected contained but separable double shower event rates for downward going $\nu_\tau$ in km$^2$ size under water/ice Čerenkov high energy neutrino telescopes for illustrative purposes. In Table I, the vacuum oscillation situation is essentially independent of the choice of the oscillation solution to solar neutrino problem. From Table I, we notice that the event rates for neutrino flavor/spin-flavor precession are up to $\sim 5$ orders of magnitude higher than that for typical intrinsic (no oscillations) tau neutrino flux.

The possibility of measuring the contained but separable double shower events may enable one to distinguish between the high energy tau neutrinos and electron and/or muon neutrinos originating from cosmologically distant GRBs while providing useful information about the relevant energy interval at the same time. The chance of having double shower events induced by electron and/or muon neutrinos is negligibly small for the relevant energies [7]. Collective information about directionality of the source, rate and energy dependence of neutrino fluxes will be needed to possibly isolate the mechanism of neutrino oscillation. The upward going $\bar{\nu}_e$ neutrinos at these energies may lead to a small pile up of upward going $\mu$-like events near $(10^4 - 10^5)$ GeV with fairly flat zenith angle dependence [8].

We now briefly discuss the potential of the underwater/ice high energy neutrino telescopes to possibly determine an observational consequence of neutrino spin-flip in GRB induced by VEP. In the electron neutrino channel, the $\bar{\nu}_e$ interaction rate (integrated over all angles) is estimated to be an order of magnitude higher than that of $(\nu_e + \bar{\nu}_e)$ per Megaton year [24]. This an order of magnitude difference in interaction rate of downward going $\bar{\nu}_e$ is due to Glashow resonance encountered by $\bar{\nu}_e$ with $E \gtrsim 10^6$ GeV when $\bar{\nu}_e$ interact with electrons inside the detector as compared to corresponding deep inelastic scattering. The upward going $\bar{\nu}_e$, on the other hand, while passing through the earth, at these energies, are almost completely absorbed by the earth mainly due to same resonant effect. Thus, for instance, if $E \sim 6.4 \times 10^6$ GeV, an energy resolution $\Delta E/E \sim 2\Gamma_W/M_W \sim 1/20$, where $\Gamma_W \sim 2$ GeV is the width of Glashow resonance and $M_W \sim 80$ GeV, may be needed to empirically differentiate between $\bar{\nu}_e$ and $(\nu_e + \bar{\nu}_e)$. The existing/planned high energy neutrino telescopes may thus in principle attempt to measure the $\nu_e/\bar{\nu}_e$ ratio in addition to
identifying \((\nu_{\tau} + \bar{\nu}_{\tau})\) and \((\nu_{\mu} + \bar{\nu}_{\mu})\) events separately.

This feature may be utilized, for instance, to explain a situation in which a change in \(\nu_{e}/\bar{\nu}_{e}\) ratio is observed as compared to GRB neutrino flux predictions in [1]. This situation, if realized observationally may be an evidence for the neutrino spin-flip in GRB due to VEP, provided if neutrinos and antineutrinos couple differently to gravity. This follows from the possibility discussed in previous Sect. that an interplay between VEP and neutrino magnetic moment in \(B_{GRB}\) may leads to conversions in either \(\nu_{e}\) or \(\bar{\nu}_{e}\) channel but not in both channels simultaneously.

V. RESULTS AND DISCUSSION

1. Intrinsically, the flux of high energy cosmic tau neutrinos is quite small, relative to non tau neutrino flavor, typically being \(F_{\tau}/F_{e,\mu}^{0} < 10^{-5}\) (whereas \(F_{e}^{0}/F_{\mu}^{0} \sim 1/2\)) from cosmologically distant GRBs.

2. Because of neutrino oscillations, this ratio can be greatly enhanced. In the context of three flavor neutrino mixing scheme which can accommodate the oscillation solutions to solar and atmospheric neutrino deficits in terms of oscillations between three active neutrinos, the final down ward going ratio of fluxes of high energy cosmic neutrinos on earth is \(F_{e} \sim F_{\mu} \sim F_{\tau} \sim F_{e}^{0}\), essentially irrespective of the oscillation solution to solar neutrino problem.

The (vacuum) flavor oscillations leads to an essentially energy independent flux of high energy neutrinos of all flavors originating from cosmologically distant GRBs at the level of electron neutrino flux, whereas spin-flavor precessions/conversions may lead to an energy dependence or/and change in this situation.

The spin-flavor conversions may occur possibly through several mechanisms. We have discussed in some detail mainly the spin-flavor precession/conversion situation induced by a non zero neutrino magnetic moment and by a relatively small VEP as an example to point out the possibility of obtaining some what higher tau neutrino fluxes as compared to no oscillations/conversions scenarios from GRBs.

The matter density in the vicinity of GRB is quite small (up to 4–5 orders of magnitude) to induce any resonant flavor/spin-flavor neutrino conversion due to normal matter effects. We have pointed out that a resonant character in the neutrino spin-flavor conversions may nevertheless be obtained due to possible VEP. The corresponding degree of VEP may be \(\sim (10^{-35} - 10^{-25})\) depending on \(\delta m^{2}\) value for vanishing gravity mixing angle.
3. This enhancement in high energy cosmic tau neutrino flux may lead to the possibility of its detection in km$^2$ surface area high energy neutrino telescopes. For $2 \times 10^6 \leq E/\text{GeV} \leq 2 \times 10^7$, the down ward going high energy cosmic tau neutrinos may produce a double shower signature because of charged current deep inelastic scattering followed by a subsequent hadronic decay of associated tau lepton.

The double shower event rate for intrinsic (no oscillations/conversions) high energy tau neutrinos originating from GRBs turns out to be small as compared to that due to precession/conversion effects up to a factor of $\sim 10^{-5}$. Thus, the high energy neutrino telescopes may possibly provide useful upper bounds on intrinsic properties of neutrinos such as mass, mixing and magnetic moment, etc.. The relevant tau neutrino energy range for detection in km$^2$ surface area under water/ice neutrino telescopes may be $2 \times 10^6 \lesssim E/\text{GeV} \lesssim 2 \times 10^7$ through characteristic contained but separable double shower events.

Observationally, the high energy $\nu_\tau$ burst from a GRB may possibly be correlated to the corresponding gamma-ray burst/highest energy cosmic rays (if both have common origin) in time and in direction thus raising the possibility of its detection. If the range of neutrino mixing parameters pointed out in this study is realized terrestrially/extraterrestrially then a relatively large (energy dependent) $\nu_\tau$ flux from GRBs is expected as compared to no oscillation/conversion scenario.

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TABLE I. Event rate ($\text{yr}^{-1}\text{sr}^{-1}$) for down word going high energy tau neutrino induced contained but separable double showers connected by a $\mu$-like track in various energy bins using MRS 96 $R_1$ parton distributions. For spin-flavor precessions, we use $\delta m^2 \lesssim 10^{-9}$ eV$^2$ and $\mu \sim 10^{-12} \mu_B$ [see Eq. (12) in the text], whereas for vacuum flavor oscillations, we used Eq. (5).

| Energy Interval | Rate ($\text{yr}^{-1}\text{sr}^{-1}$) |
|-----------------|----------------------------------|
|                | no osc | vac osc | spin-flavor precession |
| $2 \times 10^6 \lesssim E/\text{GeV} \lesssim 5 \times 10^6$ | $10^{-6}$ | $1 \times 10^{-1}$ | $0.5 \times 10^{-1}$ |
| $5 \times 10^6 \lesssim E/\text{GeV} \lesssim 7 \times 10^6$ | $2 \times 10^{-7}$ | $2 \times 10^{-2}$ | $10^{-2}$ |
| $7 \times 10^6 \lesssim E/\text{GeV} \lesssim 1 \times 10^7$ | $2 \times 10^{-7}$ | $2 \times 10^{-2}$ | $10^{-2}$ |
| $1 \times 10^7 \lesssim E/\text{GeV} \lesssim 2 \times 10^7$ | $2 \times 10^{-7}$ | $2 \times 10^{-2}$ | $10^{-2}$ |
FIG. 1. $P(\bar{\nu}_e \rightarrow \nu_\tau)$ using Eq. (15) as a function of $\delta m^2$ (eV$^2$) and $\Delta f$. 