The Torsion Bars System Reliability Analysis with Failure Mode in Crawler Vehicle

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Abstract. The traditional reliability analysis of mechanical systems was not considered the correlation between components, which leads to inaccurate analysis. In this paper, Copula function was applied to reliability analysis of the track vehicle torsion bar system, and a system reliability model which failure mode of a track vehicle torsion bar correlated was established. The relationship between trip and system reliability was obtained, and the superiority of Gumbel Copula system reliability model in analyzing the reliability of mechanical system was verified, which provided a new idea for reliability analysis and reliability sensitivity analysis of mechanical system.

1. Introduction
A torsion bar was an important buffer of a crawler vehicle suspension parts, due to installation error, unreasonable design, and the different hit from the road, lead to the failure of torsion bar rarely caused by a single failure mode in use process, was the failure mode mixing, and failure modes correlated. At present, in terms of reliability research and life prediction, the correlation between mechanical systems could not be accurately expressed, and the analysis results were deviated greatly by independent assumptions, so that the time of system failure could not be accurately predicted, which led to the delay of maintenance time and increased the probability of failure. Based on the analysis of the torsion bar reliability was introduced to describe the nonlinear correlation between mechanical parts of copulas function, for the reliability of mechanical system analysis provided a new method.

2. Determination of edge distribution function of torsion bar system
A crawler vehicle had 10 torsion bars, because of the bad working conditions of the crawler vehicle, torsion bar fracture was common failures encountered in engineering applications, the torsion bar failure mode was mainly by the high cycle fatigue damage and low cycle fatigue damage caused, for a crawler vehicle, a torsion bar fails, will made the crawler vehicle couldn't run normally, therefore, a torsion bar system could hypothesis for the series system. The structure diagram of the serial system was shown in figure 1.

![Figure 1. Series system structure diagram.](image)

The three-parameter weibull distribution had a good fitting effect on mechanical system parts. It was assumed that the life distribution function of the $i$ torsion bar was shown in equation (1).
\[ F_i(t) = 1 - R_i(t) = 1 - \exp\left(-\frac{t - \gamma_i}{\eta_i}\right)^{m_i} \]  

Where, \( m_i \) was the shape parameter of the \( i \) torsion bar, \( m_i > 0 \), \( \eta_i \) was the scale parameter of the \( i \) torsion bar, \( \eta_i > 0 \), \( \gamma_i \) was the position parameter of the \( i \) torsion bar, also known as the minimum life parameter, which meant that it will not fail before \( \gamma_i \), (when \( \gamma_i = 0 \), it degenerated into a two-parameter distribution). \( t_i \) was the working life, \( t_i \geq \gamma_i \). \( F_i(t) \) was the marginal distribution function of the \( i \) torsion bar.

3. Established the reliability model of torsion bar system

Copula connect function contained many distribution families, including Gumbel Copula connect distribution function had high sensitivity in the end, maintenance personnel wanted to get the system reliability near the tail, in order to determined whether you need vehicle maintenance, ensured reliability of the vehicle in the next run, thus the Gumbel Copula connect reliability model could meet the needs of the maintenance personnel.

The Gumbel Copula function took its generating function as \( \phi(u) = (-\ln(u))^{1/\theta} \). The multidimensional distribution function and density function could be obtained from formula (1), as shown in formula (2) and (3) respectively.

\[
C(u_i, u_2, \ldots, u_n; \theta) = \exp\left\{-\left[-\ln(u_i)^{1/\theta} + (-\ln(u_2)^{1/\theta}) + \cdots + (-\ln(u_n)^{1/\theta})\right]\right\} \\
\]

\[
c = \frac{\partial^2 C(u_i, u_2, \ldots, u_n; \theta)}{\partial u_i \partial u_2 \cdots \partial u_n} 
\]

Where, \( u_i \) was the failure distribution function \( F_i(t) \) of the driving gear at the \( i \) gear position; \( \theta \) was the relevant parameter of each random variable, \( \theta \in (0, 1) \). when \( \theta = 1 \), The variable \( u_i \) was independent, that was \( C(u_i, u_2, \ldots, u_n; 1) = \prod_{i=1}^{n} u_i ; \) when \( \theta \to 0, \lim_{\theta \to 0} C(u_i, u_2, \ldots, u_n; \theta) = \min(u_i) \). The variable \( u_i \) tended to be completely correlated. The established degree of reliability model for torsion bar of a crawler vehicle was shown in equation (4).

\[
R'(t) = P(T_1 > t, \ldots, T_{12} > t) = 1 - P\left(\bigcup_{j=1}^{12} T_j \leq t\right) = 1 - \sum_{j=1}^{12} P(T_j \leq t) + \sum_{1 \leq j < k \leq 12} P(T_j \leq t, \ldots, T_k \leq t) \\
= 1 - \sum_{j=1}^{12} F_j(t) + \sum_{1 \leq j < k \leq 12} (F_j(t) \cdots 0, \ldots 0,F_k(t)) - \cdots + (-1)^n \sum_{1 \leq j < k \leq 12} (F_j(t), \ldots, F_k(t)) \\
= 1 - \Delta'_1 \cdots \Delta'_2 C(F_1(t), F_2(t), \ldots, F_{12}(t)) 
\]

Where in (4), \( C(F_1(t), F_2(t), \ldots, F_{12}(t)) \) was Copula function; \( F_j(t) \) was the failure distribution function: \( F_j = 1 - R_j, \Delta \) According to difference.

Due to the malfunction of any torsion bar with a crawler vehicle will be unable to run normally, and in the process of investigation found no any two trouble occurs at the same time, and crawler vehicle working conditions bad, so the low cycle fatigue damage fracture of torsion bar accounted for 80% of torsion bar all trouble, so in this paper only considered the failure mode correlation between the torsion bar by the low cycle fatigue damage, system reliability model could be simplified as binary model, \( u_1(t) \) namely failure rate of torsion bar caused by low cycle fatigue damage fracture, \( u_2(t) \)
namely failure rate of the other torsion bars caused by low cycle fatigue damage fracture. The model could be simplified as shown in equation (5).

\[
R^*(t) = P(T_i > t, T_j > t) = 1 - P(T_j \leq t) = 1 - \sum_{j=1}^{n} P(T_j \leq t) + \sum_{1 \leq j \neq k \leq n} P(T_i \leq t, T_j \leq t) \\
= 1 - u_i(t) - C(u_i(t), u_j(t)) \\
= 1 - u_i(t) - u_j(t) + \exp\{[-\ln u_i(t)]^{\theta} + [-\ln u_j(t)]^{\theta}\}^\theta
\]

4. Parameter estimation of torsion bar system reliability model
The maximum likelihood estimation method was adopted for the failure distribution function of a single torsion bar, and the maximum likelihood estimation value of \( m_i \), \( \eta_i \), \( \gamma_i \), and the three parameters could be obtained by MATLAB. By solving the parameters of the distribution function model, the maximum likelihood estimation was shown in table 1.

| Failure Unit | \( \gamma_i \) | \( \eta_i \) | \( m_i \) |
|--------------|----------------|-------------|---------|
| \( u_i(t) \) | 566.79         | 13336.15    | 3.28    |
| \( u_i(t) \) | 378.46         | 15337.18    | 2.87    |

Correlation coefficient \( \theta \) could be estimated through MATLAB programming by using the estimation method of base-large likelihood function or non-parametric estimation method based on a large number of data collected in the survey. The results of parameter estimation were substituted into Gumbel Copula function respectively, and the relationship between torsion bar system reliability, mileage and parameters was obtained, as shown in figure 2. In this paper, the collected data were programatically estimated \( \theta = 0.38 \), and the relationship between its reliability and kilometer were shown in figure 2.

![Figure 2. Reliability relation diagram of torsion bar system.](image)

When \( \theta \to 1 \), the components of a system tend to be independent of each other; when \( \theta \to 0 \), Components tend to be completely correlation to each other. In the reliability analysis of the system, when the subsystems were completely correlated, the system had the highest consistency. When the subsystems were independent of each other, the system had the maximum randomness. For the series system, the better the consistency, the higher the reliability. Therefore, the reliability was the highest when the subsystems were completely correlated, while the reliability was the lowest when the subsystems were independent of each other, i.e:

\[
R^0(t) \leq R^\theta(t) \leq R^1(t)
\]
It could be seen from the figure that the system reliability model considering correlation could reflect the actual situation more accurately than the traditional reliability analysis.

![Scatter diagram of characteristic value of reliability of torsion bar system](image)

**Figure 3.** Scatter diagram of characteristic value of reliability of torsion bar system.

With the increase of mileage, assuming the components were independent of each other and assumptions components associated in full, torsion bar system reliability increased gradually, the deviation in the range of 12000 km, the reliability of the maximum deviation was 0.2201, if in accordance with the previous system components were independent of each other or completely correlation to system reliability analysis and forecast, the deviation between the two was more and more big, the resulting reliability maintenance personnel of the torsion bar without an accurate grasp, or the reliability of the same system prediction results were too good, or predict the results were too conservative, resulting in vehicle maintenance or less maintenance.

**5. Conclusion**

In this paper, the Copula function was introduced into the reliability analysis of the torsion bar system of crawler vehicles. The reliability model of the torsion bar system based on Gumbel Copula function was established. The complex multiple integral operations were replaced by simple Copula function operations, which reflected the practicability of the established system reliability model. The advantage of Gumbel Copula function to solve the problem of system reliability of components with correlation is verified, and the dynamic relationship between trip and system reliability is obtained. By comparing the prediction results with the survey results, it is verified that the Gumbel Copula system reliability model can solve the problem of large deviation between the reliability prediction of mechanical system and the actual situation, and provides an important technical means for the accurate solution of the reliability of mechanical system.

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