High-efficiency four-wave mixing in a five-level atomic system based on two-electromagnetically induced transparency in the ultraslow propagation regime

Zhi-Ping Wang and Shuang-Xi Zhang

Department of Material Science and Engineering, University of Science and Technology of China, Hefei, Anhui 230026, People’s Republic of China

E-mail: wzping@mail.ustc.edu.cn, zhipwang@126.com and shuangxi@mail.ustc.edu.cn

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Abstract
We have analyzed a five-level \( \Lambda \)-configuration four-wave mixing (FWM) scheme for obtaining high-efficiency FWM based on the two-electromagnetically induced transparency (EIT) in the ultraslow propagation regime. We find that the maximum FWM efficiency is nearly 30\%, which is orders of magnitude larger than previous schemes based on the two-EIT. Our scheme allows the possibility for new technological applications such as nonlinear spectroscopy at very low light intensity, quantum single-photon nonlinear optics and quantum information science.

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1. Introduction

As we know, light will be considerably absorbed when it passes through an optical medium, which is very bad for the conversion efficiency of nonlinear optical processes. However, the situation changed after Harris and his coworkers discovered the novel phenomenon of electromagnetically induced transparency (EIT) in a three-level atomic system in the 1990s [1]. Later, ways of using the EIT to control the absorptive and dispersive properties of an atomic medium attracted the attention of many researchers. Meanwhile, as one of the centerpieces of modern technology, the four-wave mixing (FWM) process based on the EIT in the ultraslow propagation regime [2–11] also attracted the attention of many researchers for its potential applications in nonlinear spectroscopy at very low light intensity, quantum single-photon nonlinear optics, high-efficiency generation of short-wavelength coherent radiation at pump intensity approaching the single-photon level and quantum information science. For instance, Harris et al [12] proposed the use of EIT to suppress the absorption of the short-wavelength light generated in an FWM scheme and showed that the FWM efficiency could be greatly enhanced. Deng et al [13] proposed an FWM scheme based on EIT and the associated slow light propagation, and their calculations showed that many orders of magnitude increase in the FWM efficiency may be obtained. Later on, Wu et al [14] investigated and discussed an FWM scheme in a five-level atomic system by the use of EIT, which led to the suppression of both two-photon and three-photon absorptions in the FWM scheme and enabled the FWM to proceed through real, resonant intermediate states without absorption loss; later, Wu and coworkers again analyzed a lifetime-broadened four-state FWM scheme in the ultraslow propagation regime and they put forward a new type of induced transparency resulting from multiphoton destructive interference [15].

On the other hand, recently, some researchers began to study the FWM schemes based on two-EIT in the ultraslow propagation regime [6, 16–18]. For example, Gong et al showed a tripod-configuration FWM scheme for enhancement of FWM efficiency, and they found that the conversion efficiency of FWM was enhanced several
orders by adjusting the intensity of the coupling fields [16]. Quite recently, Huang et al. proposed a scheme to obtain a highly efficient FWM in a coherent five-level tripod system by using a double-dark resonance and multiphoton destructive interference-induced transparency [17]. Zhang and Xiao reported an experimental observation of optical pumping-assisted FWM and two-EIT-assisted SWM. They found that the efficient SWM could be selected by the EIT window and controlled by the coupling as well as dressed field detuning and power. Owing to two-EIT and optical pumping assistance, the enhanced SWM signal was more than ten times larger than the coexisting FWM signal [18].

In this paper, we investigate a five-level $\Lambda$-configuration system based on two-EIT in the ultraslow propagation regime. The system of our study and the system of previous studies [14–18], however, are drastically different from each other. Firstly, we are interested in showing the effect of the external coherent driving field on both the relative intensity of the generated FWM field and the conversion efficiency of FWM. Secondly, we find that the maximum FWM efficiency is nearly 30%, which is orders of magnitude larger than that of previous schemes based on two-EIT. Thirdly, an important advantage of our scheme is that the high-efficiency FWM can be easily controlled by adjusting the external coherent driving field. Our paper is organized as follows: in section 2, we present the theoretical model and establish the corresponding Schrödinger–Maxwell equations. Our numerical results and physical analysis are shown in section 3. In section 4, some simple conclusions are given.

2. The model and the dynamic equations

We consider the five-level $\Lambda$-configuration system interacting with one weak, pulsed pump field, two cw laser pump fields and an external coherent driving field as shown in figure 1. The transitions $|3\rangle \leftrightarrow |1\rangle$ and $|1\rangle \leftrightarrow |2\rangle$ are mediated by two cw laser pump fields $\omega_1$ (Rabi frequency $2\Omega_1$) and $\omega_2$ (Rabi frequency $2\Omega_2$), respectively. A weak, pulsed pump field $\omega_p$ (Rabi frequency $2\Omega_p$) and an external coherent driving field $\omega_c$ (Rabi frequency $2\Omega_c$) are applied to the transitions $|0\rangle \leftrightarrow |3\rangle$ and $|3\rangle \leftrightarrow |4\rangle$, respectively. Then an FWM-generated pulsed field $\omega_m$ (Rabi frequency $2\Omega_m$) is achieved between $|2\rangle$ and $|0\rangle$.

In the interaction picture, with the rotating-wave approximation, the interaction Hamiltonian of the system can be written as follows (let $\hbar = 1$) [14, 19, 20]:

$$H_{int} = (\Delta_p - \Delta_1) |1\rangle \langle 1| + (\Delta_p - \Delta_1 + \Delta_2) |2\rangle \langle 2| + \Delta_p |3\rangle \langle 3| + (\Delta_p - \Delta_3) |4\rangle \langle 4| - (\Omega_1 e^{i\kappa_z t} \langle 3| \langle 1| + |1\rangle \langle 3| \Omega_2 e^{i\kappa_z t} |2\rangle \langle 2| + \Omega_p e^{i\kappa_z t} |3\rangle \langle 3| + \Omega_m e^{i\kappa_z t} |2\rangle \langle 2| + \Omega_m |0\rangle \langle 0| + \text{H.c.}),$$

(1)

where $\Delta_n$ ($n = 1, 2, p, c$) represent the respective single-photon detunings, $\kappa_n$ ($n = 1, 2, p, c, m$) is the respective wave vector, and we assume that $\Omega_n = \Omega_n^* = \Omega_n^0$ ($n = 1, 2, p, c, m$).

From the Schrödinger equation in the interaction picture, $i\hbar |\Psi(t)\rangle / dt = H_{int} |\Psi(t)\rangle$, and defining the atomic state as

$$|\Psi(t)\rangle = A_0 |0\rangle + A_1 e^{i(\kappa_z t - \kappa_1 t)} |1\rangle + A_2 e^{i(\kappa_0 t - \kappa_2 t) t} |2\rangle + A_3 e^{i(\kappa_1 t - \kappa_3 t) |3\rangle + A_4 e^{i(\kappa_2 t - \kappa_4 t) |4\rangle}.$$  

(2)

Under rotating-wave and slowly varying envelope approximations, the dynamics of the atomic response and the optical field is governed by the Maxwell–Schrödinger equations,

$$-i\hbar A_1 = (\Delta_1 - \Delta_p) A_1 + i \gamma_1 A_1 + \Omega_1^* A_3 + \Omega_2^* A_2,$$

$$-i\hbar A_2 = (\Delta_2 - \Delta_p - \Delta_3) A_2 + i \gamma_2 A_2 + \Omega_2^* A_1 + \Omega_3^* A_0,$$

$$-i\hbar A_3 = -\Delta_p A_3 + i \gamma_3 A_3 + \Omega_p A_0 + \Omega_1 A_1 + \Omega_4 A_4,$$

$$-i\hbar A_4 = (\Delta_4 - \Delta_p) A_4 + i \gamma_4 A_4 + \Omega_3^* A_3,$$

$$\frac{\partial \Omega_{p(m)}}{\partial z} + \frac{1}{c} \frac{\partial \Omega_{p(m)}}{\partial t} = \kappa_{03(02)} A_{3(2)} A_{0(m)}^*.$$  

(3)

In the above, $\gamma_n$ ($n = 1, 2, 3, 4$) are the decay rates. Here, $\kappa_{03(02)} = 2N|\alpha_{p(m)}|^2 |D_{\text{static}}|^2 / (\hbar c)$ with $N$ and $D_{\text{static}}$ being the atomic density and dipole moment, respectively. $\Omega_n$ and $\Omega_n^0$ are the respective Fourier transforms of $\Omega_n$, and the phase matching condition $\kappa_n = \kappa_1 + \kappa_2$.

Following the method described in [15, 20], we take the Fourier transform of equation (2) and the wave equations for the pulsed probe field $\Omega_p$ and the FWM-generated pulsed field $\Omega_m$, and using the undepleted ground-state approximation $A_0 \approx 1$, we obtain

$$F_1 \alpha_1 + \Omega_1^2 \alpha_2 + \Omega_3^2 \alpha_3 = 0,$$

$$F_2 \alpha_2 + \Omega_2^2 \alpha_1 + \Omega_m^2 \alpha_m = 0,$$

$$F_3 \alpha_3 + \Omega_3^2 \alpha_1 + \Omega_4^2 \alpha_4 + \Delta_p = 0,$$

$$F_4 \alpha_4 + \Omega_4^2 \alpha_3 = 0,$$

$$\frac{\partial \Omega_{p(m)}}{\partial z} - \frac{1}{c} \frac{\partial \Omega_{p(m)}}{\partial t} = \kappa_{03(02)} A_{3(2)} A_{0(m)}^*,$$

(4)

where $F_1 = \omega + \Delta_1 - \Delta_p + i \gamma_1$, $F_2 = \omega + \Delta_2 - \Delta_p - \Delta_3 + i \gamma_2$, $F_3 = \omega - \Delta_p + i \gamma_3$, $F_4 = \omega + \Delta_4 - \Delta_p + i \gamma_4$, and $\alpha_j$ ($j = 1, 2, 3, 4$) are the Fourier transforms of $\Omega_p$ and $\Omega_m$, $A_j$ ($j = 1, 2, 3, 4$), respectively.

Using the initial conditions $\Delta_p (0, \omega) \neq 0$, $\Omega_m (0, \omega) = 0$ and solving equation (3), it is easy to obtain the following
relations:

\[
\begin{align*}
\alpha_2 &= -\Omega_1^2 \Omega_2 F_4 + \frac{D_m}{D} \Lambda_m, \\
\alpha_3 &= -\Omega_1^2 \Omega_2 F_4 + \frac{D_p}{D} \Lambda_p, \\
\end{align*}
\]

\[
\Lambda_m(\omega, \omega) = \Lambda_p(\omega, \omega) \frac{\kappa_{02} \Omega_1^2 \Omega_2 F_1}{G} (e^{iKz} - e^{-iKz}),
\]

where \(K = (\omega/c) + ((\kappa_{03} D_p + \kappa_{02} D_m)/2D) \pm (\sqrt{G}/2D)\), and

\[
\begin{align*}
D &= F_1 F_2 F_3 F_4 + |\Omega_2|^2 |\Omega_4|^2 - F_1 F_2 |\Omega_2|^2 - F_2 F_3 |\Omega_4|^2, \\
D_m &= F_4 |\Omega_4|^2 + F_1 |\Omega_4|^2 - F_1 F_3 F_4, \\
D_p &= F_4 |\Omega_4|^2 - F_1 F_2 F_4, \\
\end{align*}
\]

\[
\begin{align*}
G &= (\kappa_{02} D_m - \kappa_{03} D_p)^2 + 4\kappa_{03} \kappa_{02} |\Omega_1|^2 |\Omega_2|^2 F_4^2.
\end{align*}
\]

Figure 2. Relative intensity of the generated FWM field \(|\Lambda_m(\omega, \omega)\Omega_p(0, 0)\tau^3/\tau|\) versus \(\omega\) for different EIT windows: (a) \(\Delta_1 = -\Delta_2 = -0.5\gamma\) and (b) \(\Delta_1 = -\Delta_2 = -\gamma\). Other parameters are \(\Omega_1 = 3\gamma\), \(\Omega_2 = 2\gamma\), \(\gamma = 0\), \(\Delta_1 = -\Delta_2 = 0\), \(\tau = 10^{-4}\) s\(^{-1}\), \(L = 1\) cm and \(\kappa_{02} = \kappa_{03} = 10^9\) m\(^{-1}\) s\(^{-1}\).

In what follows, as discussed in [15, 17], the FWM conversion efficiency can be defined as

\[
\eta \simeq \frac{\omega_m \kappa_{02} \kappa_{03} |\Omega_1|^2 |\Omega_2|^2 F_4^2}{\omega_p G} \exp(-2 \text{Im}[K_\omega(0)]L). \tag{6}
\]

3. Numerical results and physical analysis

Now, if we choose the initial incident pulse \(\Omega_p(0, t) = \Omega_p(0, t_0) \exp(-t^2/t^2)\), we can easily obtain \(\Lambda_p(0, \omega) = \Omega_p(0, t_0) \sqrt{\tau} \exp(-\omega t/4)\) (where \(\tau\) is the pulse width). It is well known that \(|\Lambda_m(\omega, \omega)|\) can be used to calculate the generated FWM intensity. Some numerical results about the relative intensity of the generated FWM field \(|\Lambda_m(\omega, \omega)\Omega_p(0, 0)\tau^3/\tau|\) (a dimensionless quantity) and the FWM conversion efficiency \(\eta\) will be given in figures 2-4.

In the following numerical calculations, all the parameters will be scaled by \(\gamma = \gamma_1\). The choices of some parameters might not be very reasonable in our paper; however, via properly adjusting other physical variables, we believe that some experimental scientists have adequate wisdom to deal with these problems.
In figure 2, we plot the relative intensity of the generated FWM field $|\lambda_m(z, \omega)/\Omega_p(0, 0)\tau\sqrt{\pi}|$ as a function of $\omega$ for different EIT windows. It can be easily seen from figure 2 that the relative intensities of the generated FWM field change dramatically for different EIT windows. The reason for the above result can be qualitatively explained as follows. Our atomic system is a kind of atomic configuration that possesses the property of double-dark resonances [21, 22]. Under this condition, the coherent interaction for this atomic system can lead to the emergence of sharp spectral features of interacting double-dark resonances that strongly modify the optical properties of the double EIT windows. Therefore, it can be seen that the FWM generated waves are clearly different for different EIT windows at that time.

In the following numerical calculations, we show the effect of the intensity of the external coherent driving field on the FWM conversion efficiency $\eta$ in figure 3. We find that, when the intensity of the external coherent driving field is small, the FWM conversion efficiency is low. However, with increasing intensity of the external coherent driving field, the FWM conversion efficiency becomes very large. The maximum FWM efficiency is greater than 25%, which is orders of magnitude larger than previous schemes based on two-EIT. In order to test the validity of the analysis described above, we carry out extensive numerical calculations in figure 4. We give the three-dimensional plot of the dependence of the FWM conversion efficiency $\eta$ on the two laser pump fields $\Omega_1$ and $\Omega_2$. Clearly, the three-dimensional plot also verifies the above comments.

Before ending this section, we would like to mention two key points of the present study. One of the major differences between our scheme and that of previous studies is that the maximum FWM efficiency (nearly 30%) is larger than that of previous schemes based on two-EIT, and the high-efficiency FWM is induced by an easily controlled coherent driving field. The second point is that the effect of the intensity of the external coherent driving field on the FWM conversion efficiency $\eta$ is monotonic when the coherent driving field is stronger (see figures 3 and 4). A reasonable explanation for this is that, in the presence of the external coherent driving field, we can obtain two quantum interference channels. With increasing intensity of the external coherent driving field, the destructive and constructive interferences induced via the two quantum interference channels will strike a balance, so the intensity of the coherent driving field will influence the FWM conversion efficiency $\eta$ monotonically at that time. In fact, this is a consequence of the competition between the two quantum interference channels under relevant parametric conditions.

4. Conclusions

To sum up, we have analyzed a five-level Λ-configuration FWM scheme for obtaining a high-efficiency FWM based on two-EIT in the ultraslow propagation regime. We find that the maximum FWM efficiency is nearly 30%, which is orders of magnitude larger than that of previous schemes based on two-EIT. We hope that our results will be helpful for experimental studies.

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