Analysis of the collapse mode classification in case of circular tubes

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Abstract. In this paper, the authors present an analysis of the collapse mode of tubes with circular cross-section under axial loads. Considering that requirements related to pollution are becoming more stringent, modern transportation systems require lightweight materials. These materials should have a high degree of protection. To reduce the CO₂ emissions value, a vehicle is powered by an engine with a small displacement. Furthermore, to achieve good performances in terms of acceleration and maximum speed, the total mass of the vehicle should be decreased. In case of a collision, this energy is consumed, mainly, by controlled deformation of the sacrificial structures. During an impact, the dimensions of the cockpit must not change. The same time is necessary to obtain a minimal acceleration to ensure the health of the passengers. This requirement is not compatible with a rigid structure and leads to a high acceleration level. Sacrificial structures are defined by tubes with closed profiles. Studies related to thin-walled structures with the circular cross-section under axial loads have a strong background in the work of Guillow and Andrews. The paper proposes to add complemental information these charts for another type of material, and the result obtained are correlated with the limits from the other studies existing. The research is focused on the structures with the height correlated to the structures used in the automotive industry. The analysis of global buckling for the tall structures is completed by a theoretical evaluation using Euler’s equation.

1. Introduction
Safety, performance, and comfort are the attributes that dominate the present of the automotive industry. Like any domain of science and technology, there is a permanent trend of new concerns aimed to achieve new performances regarding the safety and comfort of the passengers.

The actual trend is to minimize the costs using lightweight materials, but at the same time, it’s desired that the chosen material has to respond to all the requirements. After a more in-depth analysis, the materials should absorb the energy and also have a high degree of strength. The researchers have studied a long time the behavior of a thin-walled tube under axial load. \( F_{max} \) represents the maximum force value and can be identified as the maximum value from the first fold.

According to the geometrical parameters like \( D/t \) and \( L/D \) where \( D \) (diameter), \( L \) (length), \( t \) (thickness of walled), and material properties, from available literature, three main types of collapse modes are known.

- Axi-symmetric mode (concertina) – the folds are similar
- Non-symmetric mode (diamond) – the folds have a different number of corners
- Mixed mode (concertina + diamond) – a combination of two previous modes
The foundations of the subject were laid in 1959 by Alexander[1], who developed the collapse model of thin-walled circular tubes and derived the first set of equations to predict the crushing parameters. These parameters were investigated by Abramowicz and Jones[2], Wierzbicki, and Abramowicz[3], Hayduk, and Wierzbicki[4] who investigated the rectangular thin-walled structures. Since 2000, structures with different geometric shapes (circular tubes, frusta shape, square tubes, honeycomb shape, multi-cell structures, and other shapes) were filled with a wide variety of materials (aluminium+alloy, stainless steel, wood, polyurethane foam, polymer materials, fiberglass and the other)[5]. Fan et al. were focused on hollow tubes with six (hexagon) to sixteen shaped (star) cross-section[6]. Since 2006 the researchers focused on the multi-cell structures [7][8][9]. Starting with the year 2011, rapid prototyping offers an excellent method to analyze a model and to create a product. X. Zhang and H.Zhang[10] show that the multi-cellular metal structures are more efficient than single-cell structures and developed the mean crushing force for multi-cellular square structures.

Many studies have approached the topic of thin-walled structures with a circular cross-section. The most used material is an aluminum alloy[11].

Based on the previous researches the tube can be considered a base element also used in other domains. As a result, if the tube can be filled with various materials (polyurethane foam, metallic foam, ABS, PA, or PP insert made by rapid prototyping) and it is under axial load, much energy is consumed by axisymmetric mode. The paper is analyzing the concertina and diamond mode for a series of tubes made from a drawn pipe by turning.

2. Experimental procedure

Like the present paper, the researchers studied the behavior of circular tubes with thin-walled under quasi-static axial loading.

Alexander is the first author who studied the behavior of axially loaded thin-walled tubes, but at the same time, researches continued in other papers with a significant impact. Alexander proposed a simple model for axial crushing based on experiments with metal tubes.

The following figure presents a generic crushing model applied for circular and rectangular tubes.

Two mechanisms consume the energy: one of them is bending energy consumed along the lines of plastic hinges $E_b$ and, the other is membrane energy $E_m$ that considers the change of the shape of the initial structure by compressing or stretching to the final shape:

$$ P_m \cdot 2H = E_b + E_m $$(1)

Effective crush length can be considered being the above equation, but the factor $\kappa$ is taken into account, so the equation becomes:

$$ \kappa \cdot \frac{P_m \cdot 2H}{2} = E_b + E_m $$
Considering that L is the length of the cross-section and $M_0$ is fully plastic bending moment per unit length, then bending energy $E_b$ is defined by:

$$E_b = 2 \cdot \pi \cdot L \cdot M_0$$  \hspace{1cm} (4)

The membrane energy results from compression in the hoop direction for the circular tube and it over a complete fold formation in case of a circular tube is:

$$E_{m,tube} = 2 \cdot \pi \cdot N_0 \cdot H^2 = 8 \cdot M_0 \cdot H^2 \cdot \frac{1}{t}$$  \hspace{1cm} (5)

The mean crushing force was obtained by Alexander [4]:

$$P_m = 22.27 \cdot M_0 \cdot \sqrt{\frac{D}{t}}$$  \hspace{1cm} (6)

Where:
- $M_0$ – fully plastic bending moment per unit length
- D – Diameter
- t – Wall thickness

A few years later, Andrew et al. performed a series of tests on aluminum alloy tubes covering a wide range between D/t (4 ÷ 60) and L/D (0.2 ÷ 8.8) and after that developed a chart which predicted the mode of collapse[12].

Another studied was made by Grzebieta, which used a strip method to analyze concertina and diamond mode. The concertina mode represents a fold consisted of three equal lengths. One of them is a straight line segment, and the others are the curves of equal radius. For the diamond mode, Grzebieta carried out static and dynamic tests on steel tubes with D/t=30 ÷ 300.

### 2.1. Material and specimens

The material of these thin-walled structures with the circular cross-section is aluminum, but the type of the material is unknown. We used for experiments the tubes with different height and different thicknesses.

The height is 60 mm, 80 mm, and 100 mm, and the wall thickness is 0.9 mm, 1.2 mm respectively 1.5 mm. Each tube was marked with a horizontal line which represents 60% of the tube height, and a number representing the order number (see table 1).

The specimens were extracted from a pipe manufactured by extrusion. The tests were conducted under quasi-static conditions and axial load. As mentioned above, the tubes tested were made from aluminum alloy. Information on the material type and grade were obtained by tensile testing and XRF analysis.

#### Table 1. Details of circular tubes tested under axial load

| No. | Length L [mm] | External Diameter D [mm] | Wall Thickness t [mm] | D/t | L/D | Fm [kN] | Rm [MPa] |
|-----|---------------|--------------------------|-----------------------|-----|-----|--------|---------|
| 1   | 60            | 27.8                     | 0.9                   | 30.8| 2.1 | 26.14  | 344     |
| 2   | 60            | 28.4                     | 1.2                   | 23.6| 2.1 | 44.91  | 438     |
| 3   | 60            | 29                       | 1.5                   | 19.3| 2.0 | 45.11  | 348     |
|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| 4 | 80 | 35.8 | 0.9 | 39.7 | 2.2 | 19.55 | 198 |
| 5 | 80 | 36.5 | 1.2 | 30.4 | 2.1 | 30.56 | 230 |
| 6 | 80 | 37   | 1.5 | 24.6 | 2.1 | 42.74 | 255 |
| 7 | 100| 27.8 | 0.9 | 30.8 | 3.5 | 25.82 | 339 |
| 8 | 100| 28.4 | 1.2 | 23.6 | 3.5 | 34.80 | 339 |
| 9 | 100| 29   | 1.5 | 19.3 | 3.4 | 47.45 | 366 |
| 10| 60 | 40   | 1.23| 32.5 | 1.5 | -     | -    |
| 11| 80 | 39   | 0.9 | 43.3 | 2.0 | -     | -    |
| 12| 40 | 27.8 | 0.9 | 30.8 | 1.4 | 26.81 | 268 |

Figure 2. Specimens with different height

2.2. Testing machines
Tests were carried out on a universal testing machine. The microcomputer-controlled electronic universal testing machine is capable of developing a displacement rate of 500 mm/min while the data are recorded at 1000Hz. The data set was processed by Matlab.
3. Results and discussion

The resulting curves from the experimental data, which were obtained with a universal testing machine, are analyzed below.

The experimental data start with the tube with a circular cross-section with \( L = 60 \) mm, \( D = 27.8 \) mm, and the wall thickness is 0.9 mm. The following graphic shows \( F_{\text{max}} = 26.14 \) kN (PCF – peak crushing force) and with a discontinuous line is marked MCF – mean crushing force, which is 12.59 kN. From experimental data can be observed that the tube under axial load generates a series of corners.

![Figure 4. Force displacement curve for small structure \( L = 60 \) mm; \( t = 0.9 \) mm](image)
These corners appear only in non-symmetric mode. It’s observed that if the ratio D/t is higher, then the number of lobes is increasing. If the ratio (D/t) is more than 200, then the number of lobes can be different from 3 ÷ 5 corners, but in some cases, the lobes were simply incompletely formed.

Guillow present in his paper that the ratio $F_{av}/M_p$ is empirically dependent on $(D/t)^{0.32}$ for concertina mode, but for diamond mode exist a wide variety of theories. The ratio between maximum axial force for the first peak and mean crushing force varies as a function of D/t.

It was observed that for tubes with the same height $L=60$mm, but different thicknesses, there was a different behavior.

Of the three samples, the collapse modes for 1st and 3rd samples were non-symmetric mode (diamond mode). In contrast, the second sample has a combination of axisymmetric and non-symmetric mode,
which means that it is a mixed-mode. It can be said that during testing, the samples were broken due to the rough aluminum alloy.

**Figure 8.** Force displacement curve for small structure with a different wall thickness

The above chart specifies Peak Crushing Force and Mean Crushing Force for all the tubes with L=60mm.

The next set of the experimental test was performed on tubes with a length of 80mm. It can be observed that the last tube with a thickness of 1.5mm undergoes two deformations. At the beginning of the crushing process, there is a symmetric fold created at the bottom. After that, the bending starts from above and continues.

**Figure 9.** Axial crushing of tubes with L=80mm (thickness 0.9mm, 1.2mm, 1.5mm)
Figure 10. Force displacement curve for medium structure L=80mm

Due to the material properties, ratio D/t and, L/D, the above case presents two samples with one symmetrically fold. The collapse mode starts from the first defect encountered in the tube structure. These non-conformities come from the processing of the pipes, defects of the material or defects that come from the preparing the samples.

The next set of the experimental test was performed on tubes with a length of 100mm.

Figure 11. Axial crushing of tubes with L=100mm (thickness 0.9mm, 1.2mm, 1.5mm)
Figure 12. Force displacement curve for high structure L=100mm

For circular tubes, the parameters used to define the geometrical model of the structures[11] can be adapted to the input values $\theta_e = \pi/2$ and a high number of corners $N_c \Box$. In this case for small angles:

$$\sin \theta_e \approx \theta_e; \cos \theta_e \approx 1$$  \hspace{1cm} (7)

The tube dimensions (L-length and t-thickness) were selected in order to collapse in diamond mode, concertina mode or mixed mode considering the criteria defined in the paper of Andrews et al. [1] and Guillow et al. [2]. For the experiments, three sets were chosen:

- The first set of tubes with 60 mm length and 0.9 mm, 1.2 mm and, 1.5 mm wall thickness;
- The second set of tubes with 80 mm length and 0.9 mm, 1.2 mm and, 1.5 mm wall thickness;
- The third set of tubes with 100 mm length and 0.9 mm, 1.2 mm and, 1.5 mm wall thickness;
- The fourth set of tubes with different lengths and 0.9 mm, 0.9 mm and, 1.2 mm wall thickness.

From the last set of data (tube with the following details, L=40 mm, D=27.8, T=0.9 mm) was obtained collapse in concertina mode.

Figure 13. Collapse mode – concertina
The tube dimensions (L-length and t-thickness) were selected in order to collapse in diamond mode, concertina mode or mixed mode considering the criteria defined in the paper of Andrews et al. [1] and Guillow et al. [2].

![Figure 14. Chart of collapse mode and solution of the experiments.](image)

According to Guillow et al.[13] and Andrew et al.[12], some tubes collapse in mixed mode, but other tubes collapsed directly in diamond mode. The experiments were performed on a computerized universal testing machine (class 0.5/max. load 50 kN), and the displacement rate was set to 5 [mm/min]. The stars marked with purple color are outside from area defined by Andrew et al. and Guillow et al. Both tubes are presented below:

![Figure 15. Axial crushing of tubes, star from Fig. 14](image)

Knowing that the tubes were made of aluminum, after processing the tubes of different sizes (60 mm, 80 mm, 100 mm) from the rest of the aluminum pipe, tensile samples were made. These tensile samples were subjected to experimental tests to observe the tensile strength ($R_m$) and yield strength ($R_p$).

To choose a material, we must have primary information. This information can be extracted from the chart of collapse mode (Figure 14) and was made some additions with materials that were studied. The experiments were performed on a universal testing machine, and on the tensile sample was
mounted an electronic extensometer (YYU) mainly used to determine the axis strain (modulus of elasticity E, tensile strength, yield strength, and other parameters). Two sets of specimens were checked from two different aluminum pipes (tensile samples with thickness 2 mm and 3 mm), and three types of parameters were found.

| Thickness [mm] | R_m [MPa] | R_p [MPa] | E [GPa] |
|----------------|-----------|-----------|---------|
| 2 mm           | 361 – 371 | 329 – 337 | 69 – 71 |
| 3 mm           | 269 – 273 | 239 – 250 | 64 – 76 |

After these tests, the type of aluminium was identified based on the information presented in Table 2, therefore, an XRF test was performed to identify the type of material by the reverse method. Between mechanical test (tensile samples) and chemical test (XRF) can be confirmed that the results are matched. To analyze another structure should know the behavior of the basic structure.

X-ray fluorescence is a non-destructive analytical technique used to determine the elemental composition of materials. Using the SPECTRO X-LabPro program, it was possible to correctly identify the type of aluminum used because the sum of concentration was 99.33%, respectively 99.36%.

The aluminum alloy was of two types:
- EN AW-6003 (EN AW-AlMg1Si0.8)
- EN AW-6015 (EN AW-AlMg1Si0.3Cu)

4. Conclusion
Depending on geometrical parameters such as D/t (diameter/thickness), L/D (length/diameter), and material properties, there are a variety of possible modes of collapse. The material of tubes was a rough aluminum alloy, so under axial crushing, the material breaks.

The axial crushing profile was asymmetric (diamond mode). The experimental data are in good agreement marking similar peak and mean values of the crushing force. The thickness of the samples was 0.9 mm, 1.2 mm, and 1.5 mm, but the thinner section of the structure acted as a trigger, in case of axial crushing.

The values obtained were compared with the information available from literature, and the analysis continued, considering different wall thicknesses, different lengths of tubes, and unknown types of aluminum alloy. Same time supplementary information on the existing data can be extracted in terms
of category results or by pointing to a specific item in a data set. In conclusion, it was made results correlation between a mechanical test (tensile sample) and a chemical test (XRF analysis). Moreover, the chart of collapse mode was updated because the initial chart was developed only for some type of materials. On the chart of collapse modes, areas that point to similar failure modes can be identified. An experimental test was performed on AW-6003 and AW-6015 material. The analysis revealed the dimensions of the structures (length, diameter, thickness) have a greater contribution to the collapse pattern that the material. The present chart of collapse modes is useful for the selection of base structures when advanced structures are designed. These advanced structures can be in the form of foam-filled or nested structures that show higher stiffness and improved energy absorption [14].

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Acknowledgments
All authors have an equal contribution to the present work.