Alternative methods of verifying the reconstructed outline of a non-standard spur gear

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Abstract. Common methods of checking a gear that have been designed in reverse engineering are, for example, measurement with a modular caliper or disc micrometer. However, although these methods are among the most accurate, they allow only one or a few of the selected geometric parameters to be measured. The paper presents alternative methods of verification of the reconstructed outline of a very non-standard involute gear with the parameters \( m = 4.98 \), \( \alpha = 26.325 \, \text{deg} \), \( x = 0.0695 \), \( y = 0.795 \), \( c^* = 0.383 \). These methods are less accurate than the classic ones, but they allow for a comprehensive check of the entire outline of the reconstructed tooth. They are often used in industrial practice. However, here, in addition to the methodology, a short tolerance analysis was also carried out, which may to some extent compensate for the aforementioned measurement inaccuracy. The method consists in using the potential of a spreadsheet and CAD technique to generate an involute outline of a gear tooth whose geometry is recreated.

Keywords: gear design, gear geometry, gear reverse engineering, modification of gears, non-standard gear, involute gear, gear measurement, spur gear

Nomenclature

| Symbol | Description                                      | Unit   |
|--------|--------------------------------------------------|--------|
| \( \alpha \) | Operating pressure angle                        | deg    |
| \( \alpha \) as above |                                     | rad    |
| \( \alpha_{at} \) | Pressure angle at a tap diameter                | deg    |
| \( \alpha_{dg} \) | Pressure angle at a given diameter              | deg    |
| \( c \) | Clearance (slack)                               | mm     |
| \( c^* \) | Clearance factor                                 | –      |
| \( d_a \) | Tip diameter                                    | mm     |
| \( d_b \) | Base diameter                                   | mm     |
| \( d_f \) | Root diameter                                   | mm     |
| \( d_p \) | Pitch diameter                                  | mm     |
| \( d_s \) | Selected, any diameter                          | mm     |
| \( h \) | Total depth                                     | mm     |
| \( h_a \) | Addendum                                        | mm     |
| \( h_f \) | Dedendum                                        | mm     |
| \( \delta_a \) | Addendum modification                           | –      |
| \( \delta_{dp} \) | Tooth thickness at a pitch diameter             | mm     |
| \( \delta_{dg} \) | Tooth thickness at a given diameter             | mm     |
| \( m \) | Normal module                                   | mm     |

1. Introduction
Spare parts and their management \[1, 2\] play a huge role in the industrial process today, both on the side of the supplier of machines and spare parts for them, and on the side of the customers who purchased these machines. Unfortunately, sometimes the delivery of parts is either extended in time or simply too expensive - it is well known that manufacturers mainly earn from the service and sale of spare parts.

Therefore, more and more often customers use the practice of producing spare parts on their own, using reverse engineering \[3, 4\]. It is a type of research. It consists in recreating the geometry of the original part by means of measurements in order to create an drawing, on the basis of which the part will be made as a replacement, a substitute for the original part. Measurements can be made on a Coordinate Measuring Machine \[5, 6\], on a 3D optical scanner \[7, 8\] or using basic measuring instruments. Unfortunately, the average company cannot afford expensive modern metrology machines, let alone employing an operator (not to mention service and maintenance of the device).

The matter of making parts becomes more complicated when it is necessary to recreate the geometry of a gear with non-standard geometric parameters. Gear with a standardized module can be easily made using even the simplest measuring instruments, because in the course of calculations the theoretical, computational value of a module is always rounded to this value from a series of standardized nominal modules \[9\]. Then this module is the basis for determining most of the remaining parameters. In case the modulus is non-standard, such as the value of \( m = 4.98 \), it is very difficult to reverse engineer such a gear using inexpensive measuring techniques.

Other works by the authors are devoted to the methodology of reconstructing the outline itself, while this methodology shows an alternative way to verify the correctness of the reconstructed outline of any cylindrical gear with straight teeth. This method was often used in industrial practice by the author of the work himself, and with relatively correct initial measurements and correct calculations, the verification with the presented method gave the desired results. The results of the study are to serve average companies who cannot afford expensive techniques and measuring devices. Important note: the original gears were most likely (by the manufacturer) made in accuracy class 7 to 9. If the gears were more accurate, their reproduction and verification would most likely not be possible using the method below.

2. Presentation of an alternative method

Method no.1 consists in comparing the real circle to the drawing printout in a 1: 1 scale. But it's not as obvious as spelling. First of all, not everyone has access to the contour generator of involute gears. Not everyone can write, using the equation editor in the CAD system, a relationship describing the involute function, and then refer it to the sketch needed to extrude the 3D solid. However, some generators available, whether built into the CAD system, dedicated or run online, do not have enough options to generate the outline we want (e.g. no possibility to set a specific type of transition line, no possibility to set the head shortening factor) , it is not possible to set the blunting of the apex edges, the outline cannot be modified)

First, you need to create a spreadsheet in which the next geometric parameters of the gear will be calculated. An example with determining the basic parameters is shown in Figure 1. The formulas presented in column B are each time expressed by the available functions in the spreadsheet in column C. Dark blue cells are controlled values, and blue cells are dependent, result values.
The next step is to pick a few or a dozen of any diameters $d_x$ from the tip of the tooth $d_x = 98.25$ to the base of the involute $d_x = 98.25$ (the latter value is visible only in the next drawing). On these diameters, the arcuate tooth thickness $s_x$ will be calculated and a formula with an involute function will be used. The extremes will be the thickness of the top $s_a$ and the thickness at the base $s_b$. An example of the selection of diameters is shown in Figure 2. It also shows the calculated other geometric parameters.

Figure 1. Created a spreadsheet for an alternative verification method.

Figure 2. The next stage of activities in the spreadsheet: other geometric parameters and the selection of diameters on which the tooth thickness will be calculated.
Now we need to calculate the pressure angles $\alpha_{dx}$ on the appropriate diameters $d_x$, involute functions for these angles and the arc thicknesses of the tooth $s_x$. The example shown in Figure 3.

\[
\alpha_x = \cos^{-1}\left(\frac{d_x}{d_x'}\right)
\]

\[
\text{inv} \alpha_x = \tan^{-1}(\tan \alpha_x - \alpha_x)
\]

\[
\delta_x = d_x \cdot \left(\frac{d_x'}{d_x} + \text{inv} \alpha - \text{inv} \alpha_x\right)
\]

![Figure 3](image1.png)

**Figure 3.** The next stage of preparation for verification.

Now create the appropriate sketch to be extruded in the CAD program and draw the spline through the designated points. This curve is a polynomial function, which is close to the outline of a real involute - for the needs of industrial practice it is a sufficient approximation. The example shown in Figure 4 a.

The drawing must be prepared with the lowest possible line thickness in the CAD system. In the program used, it is 0.18 mm thick. Printout at a resolution of 600 DPI. It is also worth making the actual hole, because it will play a very important role in the final stage. The result is shown in Figure 4b.

![Figure 4](image2.png)

**Figure 4.** Work in the CAD system: a) created sketch in the model, b) 2D drawing for printing.
The most important stage that requires great care - visual inspection. The basic rule is that after roughly setting the tested gear on the printout, check each tooth on its left and right side and the center hole as perpendicularly as possible. This is because when you visually inspect, there will be some sort of perspective view in some places. Therefore, you need to check the gear one by one, choosing a certain area and verifying the convergence or possible discrepancy of the real gear with the printout, looking as if as perpendicularly possible from above. This methodology is presented in Figure 5 on the next page, and the description to the figure here: a) The undesirable effect of the perspective view is all the more evident due to the fact that the gear has a hub structure. In zone A, the apparent discrepancy between the real and the reconstructed root diameter is visible. However, subsequent inspection will show that the profile is convergent - correct. b) Inspection of the hole from above in zone B. Obviously there is a discrepancy in zone C due to perspective. This theme will be repeated in the following sections. However, the most important thing is to verify the test area selected by the inspector. c) Now checking the bottom of the hole profile in zone E. Zone D is bypassed. d) Correctness on the right in the G zone. The F zone on the left is invalid this time. e) Correctness on the left in zone H. Zone I on the right, this time invalid. In this way, it can be concluded that the gear is centered as if in relation to the printout. You should check your teeth now. f) It can be seen that in zone K the tooth contour is convergent, but of course it is divergent in zone J due to the perspective. g) In zone L, the same tooth has the correct outline on the opposite side. In this way, as stated in items f) and g), it is best to check cross-sectionally at least half the number of teeth. h) Check the diameter of the root in zone M. i) Check the diameter of the tip in zone N.

The issues of tolerance analysis in the case of reverse engineering of cylindrical gears are initially presented. Assuming that the designed gear was originally made in the 6th, 7th or 8th accuracy class, and that the measurement was performed with a digital caliper, the limit error of which is $G = 0.03$, an exemplary analysis is presented in Figure 6. Two extreme variants can be assumed. In the first variant, the diameter of the tip was made to the maximum and measured with the upper deviation of the limit error. In the second, the diameter of the tip was made to a minimum and measured with the lower deviation of the limit error. The calculation of the parameters may then result in a different involute profile, but it may be imperceptible to the naked eye with the presented method. However, the validation of the method in the industry shows that while maintaining the relatively effective life of spare parts made on one's own, it does not matter much for the accuracy classes 6, 7 or 8.

**Summary**

As a constructor of spare parts in an average company, making new gears in the expected 7th, 8th or 9th accuracy class, the above method can be used. It should be noted that in the company where the author worked, there were neither a modular caliper nor a disc micrometer. The developed method required only quite skillful handling of the geometry of the gears and functions in the spreadsheet, and a good printer. In this way, about 50 gears were drawn and made in one year.

The file from the CAD program can be treated as a template for subsequent simulations of the reproduced contour without the need to create such a tedious model in future studies. Additionally, any modifications to the gear structure can be carried out on it.

In addition, a spreadsheet created in this way allows you to quickly calculate the values for another geometric variant, if there is a need for another verification or comparative statement. It is enough to drag the appropriate functions of the spreadsheet with the mouse in the next column.

Further works by the authors will be based on a special measuring stand that uses the function of an SLR camera. The analysis of measurement tolerances and inaccuracies (measurement errors) will also be taken into account. It is expected that the above method will no longer be applicable for gears in 6th, 5th and lower accuracy class, but some refinement based on the results of planned analyzes is possible.
Figure 5. Verification of the by visual inspection (description in the text on the previous page).

Figure 6. Initial analysis of dimensional tolerances and measurement inaccuracies.
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