Root-Mean Adjustment Threshold for the Improvement of Anomaly Detection Algorithm in Hyperspectral Images

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Abstract. Hyperspectral anomaly detection aims at identifying unique objects with different spatial and spectral appearances from its surrounding pixels. One of the widely used conventional methods is Collaborative-Representation-Based Detector (CRD). CRD approximates each pixel on the background that can be represented by neighboring pixels, while the anomaly pixels cannot be. The detection image generated from this method is quite satisfying since it can detect anomaly pixels quite accurately. However, the resulting detection images still contain a lot of pixels which are not anomaly even with tiny intensity values. In this study, we apply the Root-Mean (RM) adjustment threshold to filter the false-positive pixels in the output images of CRD, so that it gives more accurate results. In the experiment, six hyperspectral data with the corresponding ground truth images are used. The results show that after applying the RM threshold, the accuracy increases with the decrease of the six RMSEs in the range of 31.51 – 63.58%.

1. Introduction
Hyperspectral imagery (HSI) is a type of satellite imagery that contains hundreds of narrow and contiguous spectral bands making it possible to distinguish different objects based on their spectral signatures. Various applications of HSI have been provided and still drawn researchers’ attention. Over the past two decades, hyperspectral anomaly detection has indeed become a very interesting topic for researchers to develop under its high dimensionality and redundant information. Various algorithms in anomaly detection are very useful especially in civil and military affairs [1] - [4]. One algorithm that has been introduced is the Collaborative Representation-Based Detector (CRD). CRD has yielded satisfactory results so far, but there are still many unwanted pixels in the resulting image as false-positive pixels. Therefore, this study aims to create an appropriate threshold value to filter out these pixels. In this study, the Root-Mean (RM) adjustment threshold will be applied to the CRD results to improve the accuracy by filtering the false-positive pixels in the CRD output images. Then, the RM threshold adjustment performance will be evaluated by comparing the results to those of CRD with a threshold taken from the ROC curve analysis.

2. Anomaly Detection in Hyperspectral Images
Hyperspectral images that are rich in spectral information are unique to each other, facilitating various applications in remote sensing technology, which one of them is target detection. The target can be a recognized object or an anomaly contained in certain pixels in the image. Anomaly detection is conducted by distinguishing certain objects that do have significant differences with other pixels, which then these other pixels will be referred to as background.

Anomaly is defined by referring to a background modeling in which the background modeling uses reference data both from local data at certain pixels and overall data from the image. The pixels in the
background can be represented by certain mathematical modeling. There is a relationship between one pixel and another in the form of a linear equation. The pixels that are not related to the equation can be the pixels of the intended anomaly. Therefore, if the background is successfully identified, at the same time the anomaly can also be identified.

The purpose of anomaly detection is to detect pixels in an image that has a spectral different from its surroundings [5]. Based on an approach taken into the development of a method, there are five categories of anomaly detection methods, i.e., those that based on statistical modeling, Kernel, projection, segmentation, and representation [6]. The most popular statistical modeling-based algorithm is the one with the application of the Gaussian and RX distribution formulas, which is commonly used as a comparison in creating other algorithms. RX Global or also called Mahalanobis distance detector [7], models the background of all parts of the image with a Gaussian distribution formula. Similar to global, but the local difference is that the process occurs not all at once in the image as in global, but on a particular local bounded by a window. As for the widely used representation-based algorithm is Collaborative-Representation-Based Detector (CRD), which is based on the sparse representation of the anomaly pixels.

2.1. Collaborative-Representation-Based Detector

CRD is one method of target detection, in this case, an anomaly that also belongs to the unsupervised category, where the target object is unknown. The concept carried by CRD is to represent each pixel (normal) in the background, which is estimated from the surrounding spatial pixels. Representation is displayed with a linear combination of these pixels, in this case, pixels that cannot be represented will be considered as an anomaly or desired target. Therefore, once the background estimation can be represented, the desired anomaly pixel will also be obtained.

CRD applies two local windows to local pixels (dual-window strategy). The term window refers to the width of the local pixel matrix in the image. There is an inner window \( w_{in} \) and an outer window \( w_{out} \) whose size can be adjusted and measured which produces the most optimal anomaly pixels.

There are some uniqueness as well as advantages of the CRD method [8]. It is the method that was first applied to the detection of anomalies without estimating the covariant matrix in the background [11]. CRD applies a regularization matrix that allows pixels representing the background to be adaptive to conditions where there are also anomalous pixels on background pixels.

Given a three-dimensional (3D) hyperspectral image in the form \( H = \{ h \}_{i=1}^{n} \) in size of \( \mathbb{R}^d \), where \( d \) is the number of spectral bands of the image, \( n \) is the number of samples that will be used. Next, there is \( y \), a pixel size \( d \times 1 \), which is the pixel to be tested. This pixel as a central pixel, located in the middle of a collection of data that forms a matrix [8].

The matrix is a two-dimensional matrix \( H_s = \{ h_i \}_{i=1}^{s} \) the result of the transformation of the three-dimensional matrix, \( s \) is the number of samples whose size depends on the size of two windows (hereinafter referred to as the window), i.e. the inner window \( (w_{in}) \) and outer window \( (w_{out}) \). In this case, \( s = w_{out} \times w_{out} - w_{in} \times w_{in} \) and matrix \( H_s \) is contained in each pixel \( y \) (local) as illustrated in figure 1.

Refer to [9], the CRD algorithm is intended to find the weight vector \( \lambda \) obtained from minimizing the equation \( ||y - H_s \alpha||_2^2 \) and \( ||\lambda||_2^2 \). In the second term, a matrix regularization called distance-weighted Thikonov [12] is added which was previously applied to [9] and [10] shown in equation (1).

\[
\Gamma_y = \begin{bmatrix} \frac{1}{||y - h_1||_2} & \vdots & \frac{1}{||y - h_s||_2} \end{bmatrix}
\]

where \( h_1, h_2, \ldots, h_s \) is the columns of matrix \( H_s \). This regularization will calculate the euclidean distance between the middle pixels and each pixel \( H_s \) whose role is to compare how similar the values of neighboring pixels are to the central pixel. This matrix will give a new equation to be
Figure 1. Visualization of observed pixel $y$

$$\text{arg min}_\alpha ||\mathbf{y} - H_s \lambda ||_2^2 + \lambda^\top \mathbf{r} \lambda ||_2^2$$ (2)

which implies that the penalty value for each element of $\lambda$ depends on the similarity (shown by the distance) of these pixels to the central pixel. In other words, if the distance gets smaller on a certain pixel, it is almost the same as the central pixel (higher similarity), in this case, a relatively large coefficient value is allowed, and vice versa. Anomaly pixels can be obtained from the residual image, namely the reduction of the original image with a background representation as shown by the equation (3).

$$r = ||\mathbf{y} - \mathbf{y}^\ast||_2 = ||\mathbf{y} - H_s \lambda||_2$$ (3)

If $r$ is greater than a particular threshold value, the pixel $\mathbf{y}$ can be claimed to be an anomaly pixel. Conversely, if a value smaller than the threshold is obtained, the pixel $\mathbf{y}$ is considered a normal pixel representing the background pixel. The final results of this detection process will be accommodated in a matrix $\mathbf{Y} = \{\lambda_i\}_{i=1}^n$ where $\lambda_i$ is equal to $r$ for anomaly pixels and 0 for normal pixels, then the filter will proceed with the Root-Mean adjustment threshold.

2.2. Troc Threshold

This Troc Threshold adjustment threshold hereinafter referred to as $T_{roc}$, is obtained by taking a threshold or the best value limit from the ROC process. The ROC graph is represented by a false alarm rate on the $x$-axis and probability of detection on the $y$-axis. The algorithm of $T_{roc}$ is summarized in figure 2.

3. Root-Mean (RM) Threshold

Root-Mean Adjustment Threshold hereinafter referred to as RM is a value limit that will filter the pixels of the weight vector or output image from the CRD detector. The initial idea of this concept is that a boundary value that is sufficient to represent a value from group data is needed. In statistics, there are some centralized data values that can represent grouped data, one of which is Mean or average. Mean is believed to be more comprehensive than other data centralization because it involves and treats all data fairly so that one value is obtained as a representation of all the values that existed before. Because in this research the context is an anomaly, there will be several stages to get a fairly fair RM value. A group of data on the anomaly matrix consists of most of the zero pixels and pixels close to zero, while the anomaly pixels only amount to a few and as much as possible pixel value 1 or close to 1. Therefore, the Mean value of the anomaly matrix data is not directly used to filter the image CRD results but to filter pixels whose value close to 1 (pixels that are likely to be anomalous pixels).

The pixels obtained are then accommodated on a new matrix. This new matrix is whose mean value will be used to filter the CRD image. Because the mean value obtained is still relatively small compared to the anomaly pixel values which is relatively large, so it needs to be rooted to get better results. The value of RM is obtained from a mathematical equation which will be described below. Given a matrix $\mathbf{R} = \{t_i\}_{i=1}^m$ where $t_i$ is an element of output matrix $\mathbf{Y}$ which is less than $\bar{R}$. $\bar{R}$ is the average of $\mathbf{Y}$.

$$\bar{R} = \frac{\sum Y}{n} = \frac{\sum_{i=1}^n \lambda_i}{n}$$ (4)

$$t_i = r_i > \bar{R}$$ (5)
while \( m \) is the number of elements from \( t_i \), i.e. the number of elements of the output \( Y \) image that is greater than \( \hat{R} \). Next, the RM value of the \( R \) matrix, which is the root of the mean of \( R \).

\[
RM = \sqrt{\frac{\sum R}{m} - \frac{\sum t_i}{m}}
\]

(6)

The obtained RM value will be the limit in the filtering output image \( Y \). Matrix \( Y \) will have a new value. To make it easier to distinguish the original \( Y \) value from the new one, the results of this filter will be accommodated into a new matrix that is \( \hat{Y} = \{\lambda_j\}_{j=1}^{n} \), where \( \lambda_j \) is equal to 1 when \( \lambda_j \geq RM \), otherwise 0. The results of this CRD-RM are predicted to provide better image quality than the results of the CRD detection without RM and also the CRD results with the best threshold taken from the ROC. Figure 2 explains the concept of applying the threshold RM compared to the \( T_{roc} \) threshold.

| \( T_{roc} \) Threshold Algorithm | RM Threshold Algorithm |
|-----------------------------------|-----------------------|
| **Input** Matrix \( Y \)         | **Input** Matrix \( Y \) |
| **For all pixels** do            | Count mean \( \hat{R} \) |
| For each value of threshold on ROC curve, count \( T_{roc} = TPR - FPR \) | **For all pixels** do |
| **End for**                       | For each pixel \( Y \), compared with \( \hat{R} \) respectively. |
| **Find the maximum** \( T_{roc} \) | **End for**            |
| **For all pixel do**              | For pixels \( Y > \hat{R} \), saved in matrix \( R \) |
| 1. Compare each pixel in \( Y \) with the \( T_{roc} \) max. | Count mean of matrix \( R \) based on (6), resulting RM |
| 2. If the pixel is greater than or equal to \( T_{roc} \), the result will be 1 (white), otherwise 0 (black). | **For all pixels** do |
| 3. The pixels assemble matrix \( \hat{Y}_{T_{roc}} \) | • Compare each pixel in \( Y \) with the RM. |
| **End for**                       | • If the pixel is greater than or equal to RM, the result will be 1 (white), otherwise 0 (black). |
| **Output:** Image of anomaly detection result (matrix \( \hat{Y}_{T_{roc}} \) ) | • The pixels assemble matrix \( \hat{Y}_{RM} \) |
|                                   | **End for**            |
|                                   | **Output:** Image of anomaly detection result (matrix \( \hat{Y}_{RM} \) ) |

**Figure 2.** Comparison of \( T_{roc} \) and RM thresholding algorithm

### 4. Result Evaluation

To seek the performance and accuracy of CRD-RM, an experiment was conducted to six data\(^1\)[13] by comparing CRD-\( T_{roc} \) and CRD-RM based on visual quality and RMSE. Table 1 shows the Root Mean Square Error (RMSE) values for the two methods. RMSE shows the quantitative differences between the constructed anomaly map and the ground truth anomaly map by taking the root of square different average for all pixels.

From the table, CRD-RM shows improvement with lower RMSE in all data as well as reduces error in the range 31.51 – 63.58%. Moreover, in terms of visual quality, CRD-RM yields satisfactory results for all data as can be shown in figure 3. From the figure, one can see that CRD-\( T_{roc} \) results in many false-positive pixels (background pixels are detected as anomaly). Meanwhile, CRD-RM encounters the problem but raises some false-negative pixels (anomaly pixels are detected as background). However, the false-negative pixels in CRD-RM are still fewer than the false-positive pixels in CRD-\( T_{roc} \) which can be seen from figure 3. The errors in CRD-RM are caused by the pixels that is rounded up to 1 even though there are only one or two pixels that are not desired.

The value of RM is determined by taking one of the centralized data values which in statistical theory is a mean. The mean can be said to be the best representation of a group of data. However, because in this case, in the context of an anomaly where there are only a few anomalies compared to thousands of data (pixels), the total value is zero, the mean value taken is the mean value of data greater than \( \hat{R} \) as in

\(^1\) [xudongkang.weebly.com/data-sets.html](http://xudongkang.weebly.com/data-sets.html)
equation (5). Once again because of the anomaly context, the mean obtained will be squared so that it has a greater value and the undesirable pixels do not participate in the final output image.

Figure 3 shows the image detected in the two methods. It can be seen that the CRD-\(T_{roc}\) column gives results with a lot of false-positive pixels caused by the small threshold value. Meanwhile, CRD-RM gives more satisfying results which reduce RMSE.

Table 1. RMSE CRD-\(T_{roc}\) dan CRD-RM

| Algorithm  | Airport1 | Airport3 | Beach3 | Beach4 | Urban1 | Urban2 |
|------------|----------|----------|--------|--------|--------|--------|
| CRD-\(T_{roc}\) | 0.2317   | 0.1942   | 0.0316 | 0.1565 | 0.1667 | 0.1987 |
| CRD-RM     | 0.1166   | 0.1330   | 0.0200 | 0.0570 | 0.0663 | 0.1217 |
| Error reduction | 49.68%   | 31.51%   | 36.71% | 63.58% | 60.23% | 38.75% |

| Image   | Airport1 | Airport3 | Beach3 | Beach4 | Urban1 | Urban2 |
|---------|----------|----------|--------|--------|--------|--------|
| Original| ![Image](image1.png) | ![Image](image2.png) | ![Image](image3.png) | ![Image](image4.png) | ![Image](image5.png) | ![Image](image6.png) |
| Reference| ![Image](image7.png) | ![Image](image8.png) | ![Image](image9.png) | ![Image](image10.png) | ![Image](image11.png) | ![Image](image12.png) |
| \(T_{roc}\) | ![Image](image13.png) | ![Image](image14.png) | ![Image](image15.png) | ![Image](image16.png) | ![Image](image17.png) | ![Image](image18.png) |
| RM      | ![Image](image19.png) | ![Image](image20.png) | ![Image](image21.png) | ![Image](image22.png) | ![Image](image23.png) | ![Image](image24.png) |

**Figure 3.** Comparison of anomaly images of CRD-\(T_{roc}\) dan CRD-RM algorithm.

5. Conclusions
Based on experiments conducted in this study, we can conclude that Root-Mean algorithm that is applied to the output images of CRD anomaly detection significantly improves the accuracy in terms of quantitative (RMSE) and qualitative for the visual appearance, compared to the conventional thresholding method, i.e. \(T_{roc}\). It reduced error in the range 31.51 – 63.58% compared with the CRD-\(T_{roc}\). Future work may lay on the development of anomaly detection algorithm itself.

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