Value of structural health monitoring quantification in partially observable stochastic environments

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Abstract

Sequential decision-making under uncertainty for optimal life-cycle control of deteriorating engineering systems and infrastructure entails two fundamental classes of decisions. The first class pertains to the various structural interventions, which can directly modify the existing properties of the system, while the second class refers to prescribing appropriate inspection and monitoring schemes, which are essential for updating our existing knowledge about the system states. The latter have to rely on quantifiable measures of efficiency, determined on the basis of objective criteria that, among others, consider the Value of Information (VoI) of different observational strategies, and the Value of Structural Health Monitoring (VoSHM) over the entire system life-cycle. In this work, we present general solutions for quantifying the VoI and VoSHM in partially observable stochastic domains, and although our definitions and methodology are general, we are particularly emphasizing and describing the role of Partially Observable Markov Decision Processes (POMDPs) in solving this problem, due to their advantageous theoretical and practical attributes in estimating arbitrarily well globally optimal policies. POMDP formulations are articulated for different structural environments having shared intervention actions but diversified inspection and monitoring options, thus enabling VoI and VoSHM estimation through their differentiated stochastic optimal control policies. POMDP solutions are derived using point-based solvers, which can efficiently approximate the POMDP value functions through Bellman backups at selected reachable points of the belief space. The suggested methodology is applied on stationary and non-stationary deteriorating environments, with both infinite and finite planning horizons, featuring single- or multi-component engineering systems.

Keywords: Structural Health Monitoring (SHM); Value of Information (VoI); Partially Observable Markov Decision Processes (POMDP); sequential decision-making; point-based value iteration; inspection and maintenance planning

1. Introduction

The development of new monitoring technologies, data acquisition techniques and information processing methodologies further encourages the use of Structural Health Monitoring (SHM) in supporting management of critical infrastructure and deteriorating systems [1, 2]. These new possibilities come with relevant questions related to the actual value and necessity of increased quality measurements or continuous SHM information in facilitating optimal actions. SHM frameworks seek to determine appropriate mappings from raw response measurements to condition and performance indicators, which can, in turn, support decision-making towards cost-effective intervention and maintenance actions that increase safety and mitigate risks [3]. Decision-making pertains to the type and sequence of actions that are selected in order to optimize some overarching predefined life-cycle objective. As such, when the objective is to maximize long-term safety and resilience, and to effectuate preventive maintenance actions, SHM typically constitutes a natural choice, as it can be used to diagnose faults and even determine the root cause of the fault process, e.g. [4]. However, to what measurable extent is the acquired information able to support improved policy-planning in a specific engineering environment, and how can we objectively quantify the resulting gains?

An important discussion in this respect is whether the benefits of the various observational strategies, e.g. SHM-aided plans, or in situ visual and specialized non-destructive evaluation inspections, can be quantified in terms of life-cycle value-based metrics, and whether these benefits are comparable to the costs associated with the use of SHM equipment (acquisition, installation, operation, maintenance costs, etc.). The question that summarizes this discussion is how much is information worth or, similarly, how much is a SHM system worth investing? [5]. In response, recent research efforts focus on quantifying the Value of Information (VoI) and, similarly, the Value of Structural Health Monitoring (VoSHM) within rigorous mathematical frameworks. Following the definitions in [6], VoI may be defined in pertinence to inspection and maintenance planning, and may as such be devised along the lines of pre-posterior engineering decision problems [7, 8, 9]. The concept of VoI can be utilized to (i) evaluate the amount the decision-maker is willing to pay for information prior to a single decision step of the decision process, either considering the long- or short-term information benefits, e.g. [10] or [11], respectively; or (ii) to quantify the overall gain that information may yield regarding a fixed inspection and/or monitoring policy, applied over the entire life-cycle of a system, e.g. [12]. The latter measure of VoI may be used to assess whether it is worth adopting a certain observational strategy over others, from the beginning up to the end of the system’s operational life, and this is the approach followed in this paper. Similarly, within the context of SHM, VoI may be quantified as the difference between the expected cost of maintaining the system in absence of SHM information, and the cost given availability of monitoring information [13, 14, 15, 16]. Along these lines, within the context of Partially Observable Markov Decision Processes (POMDPs), VoI analysis and quantification approaches have been also developed in [17, 18, 19]. VoSHM is herein defined as a more specialized definition of VoI, describing relative costs between intermittent/optional observational schemes, e.g. periodic or non-periodic inspection visits, and SHM-aided plans, where the flow of
observations is typically continuous [20].

As already mentioned, the VoI and VoSHM metrics may be quantified as per their impact in rendering infrastructure management more effective. One approach to quantifying these metrics is to formulate an optimization problem which seeks to determine optimal sequences of actions (policies) and their respective life-cycle costs for different observational scenarios. Key to the success of such optimization formulations is (i) incorporation of environment stochasticity, (ii) long-term optimality of decisions, and (iii) integration of dynamic, real-time, noisy observations. Numerous formulations exist in the literature dealing with the issue of decision-making for optimal management of infrastructure [21]. Typically, the objective function pertains to various life-cycle conditions, reliability, risk and cost measures, which are sought to be optimized by the decision-maker. These, as well as the employed optimization approaches and environment simulators may vary depending on the system specifications. Dynamic Bayesian networks are utilized in [22], to determine the underlying structural deterioration process. Based on the established dependencies, the cumulative life-cycle cost is evaluated and the policy space can be subsequently searched through optimization heuristics [10, 23, 24]; genetic algorithms [25], or other relevant optimization solvers. POMDPs are also built within dynamic Bayesian network premises, so the two approaches can be seen as equivalent in terms of how the environments are simulated, however, their adopted optimization approaches and capabilities are completely different. Renewal processes can also be utilized in this regard, with various extensions and refined formulations to account for multi-threshold and multi-level action plans, or even integrated resilience considerations [26, 27, 28]. Within this context, direct search of the discretized decision variable space can be conducted to determine the best strategy, or even analytical and gradient-based approaches, if applicable. Multi-criteria objectives have been also examined in [29, 30], to account for a diversified quantification of risk, in its socioeconomic and environmental constituents, or to seek optimized values of competitive cost and performance indicators [31]. In such cases, optimization can be efficiently conducted using heuristics, such as genetic algorithms.

In this work, the inspection and maintenance optimization problem is addressed within the framework of POMDPs. Primarily developed in the field of robotics over the past years for stochastic optimal control in partially observable dynamic environments, POMDPs provide a well-suited mathematical framework for sequential decision-making, with sound life-cycle optimality guarantees and convergence properties [32], which can be conveniently lent to the class of structural inspection and maintenance problems [33]. POMDPs extend Markov Decision Processes (MDP) to partially observable environments, where the decision-maker/agent seeks to optimize a policy maximizing the collected rewards over time (or minimizing costs), without knowing the exact state of the system. In [34, 35], POMDPs are adopted for decision-making for highway-pavements. The use of POMDPs has been also applied in [36, 37] for bridge inspection planning, whereas point-based solutions for stochastic deteriorating systems using POMDPs have been presented in [38, 39, 40, 41]. A continuous formulation for problems described by linear and/or nonlinear transition functions is presented in [42], whereas specialized cases of mixed observability are also presented in [43]. Exploiting VoI, POMDPs can further be extended to tackle inspection and maintenance problems at the system level as in [18]. Recent frameworks within the premises of deep reinforcement learning, particularly efficient in addressing the curse of dimensionality and model unavailability issues in large-scale POMDP system applications, are developed in [44, 45]. Regardless of the adopted numerical solution scheme (i.e., alpha-vector value iteration, point-based value iteration, reinforcement learning, etc.), POMDPs are particularly favorable for decision-making formulations in infrastructure management and have been demonstrated to significantly outperform conventional fixed inspection and maintenance policies [44, 46]. This is particularly true in the presence of discrete or discretized spaces, where exhaustive evaluation and search of policy subspaces, evolutionary approaches, or gradient methods may be impractical or ineffective, if at all applicable. Moreover, regarding the step-wise definition of VoI, i.e., the amount the decision-maker is willing to pay prior to each decision, it is worth noting that POMDPs inherently and straightforwardly leverage VoI, if observation actions are introduced as separate decision variables, since the Bellman equation of optimality that POMDP solutions satisfy, minimizes the future cumulative cost at each decision step after considering all possible alternative observational choices.

The developed method for calculating VoI and VoSHM in this work adopts their life-cycle concepts, thus targeting decision-support for selection of life-cycle observational plans, among various alternatives. Detailed definitions of the above value metrics are also devised and discussed, and the underlying steps for their computation are demonstrated in numerical experiments of deteriorating engineering systems operating in partially observable stochastic environments. Quantification is based on solutions derived through POMDP formulations for the inspection and maintenance optimization problem, however, the applicability of the method is not particular to POMDPs and can be easily adjusted to the needs and outcomes of other optimization schemes. An infinite horizon three-component POMDP system and a larger finite horizon POMDP problem, modeling a deteriorating port deck structure, are analyzed under two different inspection scenarios; one with optional inspection visits and one with continuous availability of observations, resembling a SHM system. The underlying POMDPs are solved using various point-based value iteration algorithms, which are shown to provide particularly effective solutions for the proposed framework. The described VoI and VoSHM analysis provides the respective expected gains in terms of a life-cycle metric of interest, e.g. cost, thus answering the previously posed question of how much is inspection or monitoring information eventually worth. In particular, VoI quantifies the value that is added by the availability of observation choices over the system lifetime, whereas the VoSHM quantifies the possible benefits of adopting a monitoring system from the beginning of the planning horizon, over following a plan based on optional inspection visits.

2. Partially Observable Markov Decision Processes

POMDPs provide an adept framework for stochastic optimal control. They are established within the premises of dynamic programming, thus providing strong global optimality guarantees for long-term decision problems described by stochastic environment dynamics POMDPs in non-Markovian environments, as the latter can be pro-
perly transformed to fit Markovian assumptions through state augmentation [38]. POMDPs generalize Markov Decision Processes (MDPs) to partially observable environments, i.e., to cases where observations are unable to reveal the actual state of the system with certainty. This feature, along with their neat mathematical formulation of POMDPs, is suitable to efficiently describe inspection and maintenance planning problems in structural and generic engineering settings, where the inspection techniques and monitoring devices deployed, typically provide incomplete information about the system condition (states), which evolves according to an underlying stochastic deterioration process.

According to the POMDP problem statement, the decision-maker/agent starts at a state, \( s_t = s \in S \) at every decision step, \( t \), takes an action, \( a_t = a \in A \), receives a reward, \( r = r(s,a,t) \), transitions to the next state, \( s_{t+1} = s' \), according to a Markovian transition probability model conditioned at the current state and action, \( p(s'|s,a) \), and receives an observation, \( o_{t+1} = o \in \Omega \), based on its state and action, according to the probability defined by an observation model, \( p(o|s',a) \). This process is schematically depicted in Fig. 1. More formally, a POMDP is a 7-tuple \( \mathcal{L} = \langle S, A, P, \Omega, O, R, \gamma \rangle \) where \( S \) and \( \Omega \) are finite sets of states, actions and possible observations, respectively. \( P, O \) are the 3-dimensional Markovian state transition and observation probability matrices, respectively, whereas \( R \) is the reward matrix, defined as:

\[
\begin{align*}
P &= [p]_{s,a} = \sum_{s' \in S} p(s'|s,a) \in [0,1]^{S \times A} \\
O &= [o]_{s,a} = \sum_{o' \in \Omega} o(o'|s,a) \in [0,1]^{\Omega \times S \times A} \\
R &= [r]_{s,a} = \sum_{s'} r(s',a) \in [0,1]^{S \times A} 
\end{align*}
\]

(1)

As a result of partial observability, at every decision step \( t \), the agent cannot be fully aware of its state, \( s_t \) (shaded nodes in Fig. 1), which may only be perceived through an observation \( o_t \) that is a noisy indicator of that state [47].

Starting with an initial distribution of state \( s_0 \) over \( S \), the objective of the agent is to determine a sequence of actions that maximizes the expected return, i.e., the expected total cumulative future reward. This is accomplished by executing an optimal policy \( \pi^* = \pi \), which maps the history of actions and observations up to time \( t \), to the current action \( a_t \), such that:

\[
\pi^* = \arg \max_{\pi} \mathbb{E}_{o_t \in \Omega} \left[ \sum_{t=0}^{\infty} \gamma^t r(s_t,a_t) \right] = \pi(a_0,o_1,...,a_{t-1},o_t)
\]

(2)

where \( \gamma \) is the discount factor, a positive scalar less than 1, associated with the present value of future rewards. In the context of inspection and maintenance planning, rewards are typically negative quantities describing costs.

Although the agent cannot observe the exact state with certainty as a result of partial observability, it can form a belief \( b_t = b \in B \) about its state, where \( b \) is a probability distribution over set \( S \), of all possible discrete states. Space \( B \) is a \( (|S| - 1) \)-dimensional simplex. The new belief \( b_{t+1} = b' \), i.e., the posterior state distribution for a given action and observation, can be readily computed through a Bayesian update [43]:

\[
b'(s') = p(s'|o,a,b) = \frac{\sum_{s} p(s'|s,a) p(o|s',a) b(s)}{\sum_{s} p(o|s',a) b(s)}
\]

(3)

where \( p(o|s',a) \) is the standard normalizing constant, given as:

\[
p(o|b,a) = \sum_{s} p(o|s',a) \sum_{s} p(s'|s,a) b(s)
\]

(4)

Following Eq. (3), beliefs can be updated as new actions are performed and new observations are collected, essentially encoding the information of the entire history of actions and observations up to the current time step \( t \). As such, a new belief \( b' \) is a sufficient statistic of the history of actions and observations up to \( t \). Namely, by forming a belief about its state, using Eq. (3), the agent has all the information required for deciding on an action. The policy in Eq. (2) can then be equivalently expressed as a mapping from beliefs to actions, \( \pi: B \rightarrow A \).

It also follows from Eq. (3) that the agent moves from one belief to another based on the selected action and received observation. We can, thus, define the transition probability from belief \( b \) to belief \( b' \) as [47]:

\[
p(b'|b,a) = \sum_{o \in \Omega} p(o|b,a)
\]

(5)

where \( \Omega \subset \Omega \) is the subset of observations leading to \( b' \), when starting at belief \( b \) and taking action \( a \). Owing to Eq. (5), a POMDP can be seen as a belief-MDP, where transitions pertain to belief points, instead of states. For a given observation, which depends on the actual system state, the respective probabilistic graph is shown in Fig. 2. The belief-MDP reward \( r = r(b,a) \) is the expected reward at
the current step, which in the context of inspection and maintenance planning can be defined as [39]:

\[
\begin{align*}
    r_\delta (b, a) &= \sum_{s \in S} b(s) r(s, a) \\
    &= \sum_{s \in S} b(s) (r_\delta + r_\alpha + r_\gamma) \\
    &= b \cdot R_\delta + b \cdot R_\alpha + b \cdot R_\gamma
\end{align*}
\] (6)

where reward \( r \) (reward matrix \( R \)) is decomposed into \( r_\delta, r_\alpha \) and \( r_\gamma \) \( (R_\delta, R_\alpha \) and \( R_\gamma) \), which are the maintenance, observation and damage state rewards (negative to reflect costs, respectively). Expected damage cost in Eq. (6) depends merely on the current state distribution (belief), and may be decomposed into more components pertaining for example to economic losses due to system downtime or shutdown, or costs related to various societal and environmental metrics (energies, consumption, CO2 equivalent emissions, among others, e.g. in [29]).

The expected return under any policy, \( \pi \), defines the value function, \( V^\pi \), whereas the expected return under the optimal policy defines the optimal value function, \( V^\star \). Exploiting the concept of belief-MDPs, we can use the Bellman equation [48], expressing the optimal value function as [43]:

\[
V^\star (b) = \max_{a \in A} \left[ r_\delta (b, a) + \gamma \sum_{s \in \Omega} \alpha(b, a) V^\star (b') \right]
\] (7)

It should be noted that Eq. (7) is defined over the continuous space of the belief simplex, \( B \), which essentially consists of an infinite number of beliefs. However, it has been proven that the optimal value function is piece-wise linear and convex, and can thus be described by a finite number of affine hyperplanes [49]. This important result reduces the decision problem to determining a finite set of vectors, also known as the \( \alpha \)-vectors:

\[
V^\star (b) = \max_{a \in A} \left[ \sum_{s \in S} b(s) \alpha(s) \right]
\] (8)

where \( \Gamma \) is the set comprising all \( \alpha \)-vectors. Substituting Eqs. (4), (8) in Eq. (7) we obtain the detailed expression of the POMDP optimal value function:

\[
V^\star (b) = \max_{a \in A} \left[ \sum_{s \in S} b(s) (r_\delta + r_\alpha + r_\gamma) + \gamma \sum_{s \in \Omega} \max_{s' \in S} \sum_{s' \in S} b(s) p(s' | s, a) \alpha(s') \right]
\] (9)

Eq. (9) can be solved using value iteration on the space of \( \alpha \)-vectors. However, performing exact value iteration on the vector space is generally impractical, except for small POMDP problems, since the new set of alpha vectors generated at every iteration step scales exponentially with the cardinality of the observation set, \( 1 \Omega \) [50].

2.1. Point-Based POMDP Algorithms

Point-based solvers adopt the concept of belief-MDPs and manage to alleviate the POMDP complexity by avoiding the exponential increase of \( \alpha \)-vectors. The idea is to restrict value iteration operations to a meaningful collection of discrete belief points, i.e., to perform \( \alpha \)-vector backups on a finite subset of the belief space, \( \hat{B} = \{b_1, b_2, ..., b_m \} \subseteq B \), which is considered to be able to sufficiently approximate the original continuous (\( |\Omega| \)-1)-dimensional simplex. Point-based algorithms take advantage of the fact that despite the continuity of the belief space, in practice there is only a finite number of belief points that are actually visited. These belief points lie in a reachable subset of the belief space, with respect to an initial (root) belief \( b_0 \). At each iteration step, new \( \alpha \)-vectors are generated merely based on these points, forming a set \( \hat{\Gamma} \subseteq \Gamma \) that can efficiently recover the true value function over the entire belief space, with the aid of the max operator of Eq. (8). Of course, since the \( \alpha \)-vectors in \( \hat{\Gamma} \) cover the entire space, \( B \), one can also compute an estimate of the value function for non-reachable beliefs, however, this estimate may be expected to be of lower accuracy. At each iteration, \( \hat{\Gamma} \) is updated through every \( b \in \hat{B} \), or a subset of it, based on the backup operator defined as:

\[
\text{backup}(\hat{\Gamma}, b) = \arg \max_{a \in A} \sum_{s \in S} b(s) \alpha(s)
\] (10)

\[
\alpha(s) = r_\delta + r_\alpha + r_\gamma + \gamma \sum_{s' \in \Omega} \sum_{s \in S} p(s' | s, a) \alpha(s')
\] (11)

\[
\alpha(s') = \arg \max_{a \in A} \sum_{s' \in \Omega} \sum_{s \in S} p(s' | s, a) \alpha(s')
\] (12)

All point-based solvers maintain a lower bound on the value function, which is updated throughout the iteration process, as described in Eqs. (10)-(12), e.g. [50, 51, 52, 53]. This lower bound consists of the linear hyperplanes defined in Eq. (8), and is typically initiated by evaluating a simple policy. Modern point-based algorithms also compute, maintain and update an approximate upper bound on the value function. This bound allows these algorithms to employ more efficient strategies for belief space exploration, as well as to monitor convergence over the course of the iterative procedure.

ZMDP with its Heuristic Value Iteration (HSVII) and Focused Real-Time Dynamic Programming (FRTDP) variants [52, 54], as well as Successive Approximation of the Reachable Space under Optimal Policies (SARSP) [53] belong to this class of algorithms. The upper bound is typically initiated with optimistic values and, similarly to the lower bound, should be constructed as a piece-wise linear and convex function. However, it is not possible to update or evaluate the upper bound over the entire belief simplex using Eqs. (10)-(12), due to the presence of the max operator. Thus, the upper bound can be maintained by point-wise value estimates at visited beliefs and the formed convex hull that they support, which is determined through linear programming. Point-based solvers avoid solving this expensive linear program however and, instead, determine the upper bound using a much faster sawtooth approximation, since as the number of beliefs supporting the upper bound estimates increases, the linear program becomes considerably difficult to solve [32].

The points of \( \hat{B} \) are either collected through randomly sampled belief trajectories, i.e., based on random sequences of actions and observations, or through more focused and informed search heuristics. The Point-Based Value Iteration (PBVI) algorithm [50], the first point-based algorithm, iterates between backup and belief space expansion steps. PBVI proposes an exploration strategy which
expands over the existing points of \( \overline{B} \), at every iteration. For every existing belief point, its successor is added to \( \overline{B} \) such that the new set spreads as sparsely as possible over \( B \). PBVI updates \( \alpha \)-vectors over all collected beliefs. The Perseus algorithm [51], traverses a series of path trials based on randomly sampled action and observation histories, in order to form \( \overline{B} \), at the beginning of the solution procedure. This set of collected points remains unchanged during the \( \alpha \)-vector backups. Perseus also performs asynchronous randomized backups, i.e., it does not perform backups over all beliefs in \( \overline{B} \), but instead selects randomly which belief values to update at every iteration step. Beliefs whose value is improved by \( \alpha \)-vectors supporting previously selected beliefs, are not updated in the current step. ZMDP and SARSOP utilize both the lower and upper bounds to inform the exploration of the belief space, choosing actions based on the upper bound and observations based on the maximum lower-upper bound gap. Both algorithms perform asynchronous bound updates over the visited beliefs.

In addition to their advanced exploration strategies, HSVI, FRTFP and SARSOP also apply pruning techniques to reduce the complexity and memory requirements related to the expansion of the \( \alpha \)-vectors set, removing vectors from \( \Gamma \) that are considered to be suboptimal under certain criteria. HSVI and FRTDP prune vectors that do not support at least one of the collected belief points and their immediate successors. The above algorithms can also optionally implement a masking technique which essentially tries to create compressed representations of the \( \alpha \)-vectors, by maintaining and updating \( \alpha \)-vectors entries that are not zero or not close to zero. Similarly, SARSOP prunes vectors that either do not support at least one of the collected belief points or are dominated by other \( \alpha \)-vectors within a predefined neighborhood. SARSOP also prunes beliefs that are considered to be suboptimal based on the current information provided by the upper and lower bounds. Thereby, the entire tree of successors under these beliefs is pruned and exploration is restricted to more optimally reachable belief subspaces.

A detailed overview on point-based solvers along with their application in various robotic tasks can be found in [32]. Their insights and application details in structural inspection and maintenance planning can be found in [43, 41], where different point-based approaches are tested. Among them, the three most competitive are identified and used herein. Overall, it is demonstrated that point-based solvers can provide comprehensive and efficient near-optimal solutions in problems with thousands of states and a much lower number of actions and observations. In cases featuring more complex POMDP settings, deep reinforcement learning actor critic architectures have been shown to have significant success, as presented in [44]. The Deep Centralized Multi-agent Actor Critic (DCMAC) approach developed in [44] combines belief-MDPs with deep reinforcement learning concepts and is able to learn detailed non-stationary life-cycle inspection and maintenance policies for engineering system settings with multiple components, operating in extremely large state, action and observation spaces.

3. Life-Cycle Gain from Changing the Control Setting

The expected life-cycle gain of one control setting versus another can be expressed as the value difference between the two settings, when different control action sets are available for each setting, but these apply to the same system, i.e., the two settings have the same state space and the same deterioration dynamics (transition model for the uncontrolled case), as well as the same discounted horizon. To quantify the value of expected life-cycle reward (or cost) of these two settings, we consider two tuples that define the following distinct POMDP problems:

\[
\begin{align*}
L_1 &= \left\{ S, A'_d \times A'_b P_{aO}^{1,2} \left[ O_v^{1,2} \right]_{aO} \cdot \Omega_v^{1,2} \cdot R_u^{1,2} + R_v^{1,2} + \gamma \right\} \\
L_2 &= \left\{ S, A'_d \times A'_b P_{aO}^{1,2} \left[ O_v^{1,2} \right]_{aO} \cdot \Omega_v^{1,2} \cdot R_u^{1,2} + R_v^{1,2} + \gamma \right\} 
\end{align*}
\]

where \( A'_d \) is the set of maintenance actions, \( A'_b \) is the set of observation actions, \( P_{aO} \) is the transition model for different maintenance actions, and \( \Omega_v \) is the observation model for different observation actions, for \( i = 1, 2 \). If a no-maintenance action is taken \( P_{aO} = P_{aO}^{0,0} \), which together with \( S, r, \gamma \) define the same stochastic system. State transitions \( P_{aO} \) only depend on maintenance actions, meaning that only maintenance actions can change the state of the system. Observation actions can only change the agent’s perception about the state of the system, thus they suffice to define the observation model \( \Omega_v \). The observation model does not depend on the maintenance actions. Note that the tuples in Eq. (13) follow the formal POMDP 7-tuple definition introduced in Section 2, decomposed as per the specific inspection and maintenance problem requirements.

Then, the expected life-cycle gain, \( G_{\Delta, \xi} \), from following the optimal policy in \( L_2 \) versus \( L_1 \), starting at any belief \( b \in B \), is computed as:

\[
G_{\Delta, \xi} (b) = V_{L_2} (b) - V_{L_1} (b)
\]

where \( V_{L_2}, V_{L_1} \) are the optimal value functions of each tuple. Equivalently, Eq. (14) describes the expected benefit from changing a control scheme from \( L_1 \) to \( L_2 \) at belief \( b \).

To assess the expected life-cycle gain of one observational scheme versus another (e.g. SHM, inspection visits, etc.), the tuple elements related to maintenance actions have to be the same, thus one has to apply Eq. (14), considering the following POMDP problems:

\[
\begin{align*}
L_1 &= \left\{ S, A'_d \times A'_b P_{aO}^{1,2} \left[ O_v^{1,2} \right]_{aO} \cdot \Omega_v^{1,2} \cdot R_u^{1,2} + R_v^{1,2} + \gamma \right\} \\
L_2 &= \left\{ S, A'_d \times A'_b P_{aO}^{1,2} \left[ O_v^{1,2} \right]_{aO} \cdot \Omega_v^{1,2} \cdot R_u^{1,2} + R_v^{1,2} + \gamma \right\}
\end{align*}
\]

For Eqs. (14), (15), \( G_{\Delta, \xi} \) is the expected life-cycle gain of two control settings which are merely discerned by their observation actions. In this case, Eq. (14) quantifies potential benefits as a result of different sources and/or accuracy of information. In the remainder of this section, we elaborate on special cases of Eqs. (14), (15) to derive the gains related to different observational schemes and their relation to VoI and VoSHM.

3.1. Value of Information

Considering Eq. (15), suppose \( A'_b \) is a unit set, with only the ‘no observation’ action available. Then, \( R_v = 0 \) and \( \Omega_v \), can be defined
by a unit set as well, meaning that from all states only one observation is possible (uninformative observation). In this case, tuple \( \mathcal{L}_2 \) defines the \textit{blind (or prior) control problem} of \( \mathcal{L}_2 \), i.e., \( \mathcal{L}_2 = \mathcal{L}_2 \text{blind} \), thus Eq. (14) gives the VoI of the observational scheme adopted in \( \mathcal{L}_2 \) [12]:

\[
G_{\mathcal{L}_2,\text{blind}}(b) = \text{Vol}_{\mathcal{L}_2}(b) = V_{\mathcal{L}_2}(b) - V_{\mathcal{L}_2,\text{blind}}(b) 
\]

(16)

On top of the previous assumption, let us assume that \( \mathcal{A}_2 \) is a unit set with only the \textit{observation} action available at no cost, and \( |\mathcal{O}_2| = |S| \) with \( O_2 = I \) (identity matrix). In this case, the agent operates under perfect information at every decision step of \( \mathcal{L}_2 \). Technically, this makes the POMDP defined by \( \mathcal{L}_2 \) a MDP problem, thus \( \mathcal{L}_2 = \mathcal{L}_2 \text{MDP} \). Under these assumptions, using Eq. (14) we obtain the Value of Perfect Information (VoPI):

\[
G_{\mathcal{L}_2,\text{MDP}}(b) = \text{VoPI}_{\mathcal{L}_2}(b) = V_{\mathcal{L}_2}(b) - V_{\mathcal{L}_2,\text{blind}}(b) 
\]

(17)

It can be readily noticed that the VoPI is an upper bound of the VoI since \( V_{\mathcal{L}_2,\text{MDP}} \geq V_{\mathcal{L}_2} \). If the value functions in Eqs. (16) and (17) include the cost related to observational actions, then, according to [5], they can also be associated with the net VoI.

3.2. Value of Structural Health Monitoring

The VoSHM refers to the possible gains from investing in life-long SHM devices and practices, instead of, or in addition to, planning inspection visits at discrete times during the structural life-cycle. As such, the VoSHM relates to the critical decision, either at the design stage or later, on whether a monitoring scheme is worth to be adopted, and if so, of which type. VoSHM in essence quantifies the benefits of continuous data collection and information inflow in the decision-support system.

In this work, to quantify the VoSHM, we examine another special case of Eq. (15). We assume that \( \mathcal{A}_2 \) contains at least two available actions. One action is the \textit{no observation} action, whereas the rest may describe various observation choices. The \textit{no observation} comes at a zero cost. Conversely, \( \mathcal{A}_2 \) contains only one \textit{observation} action which is, however, considered costless at this stage, i.e., \( R = 0 \), so that its value is appropriately quantified through this analysis. For the two POMDPs, the observation actions may follow different observation models, whereas the no observation actions give an uninformative observation, as previously discussed. Thereby, \( \mathcal{L} = \mathcal{L}_2 \text{opt} \) corresponds to the scenario of optional inspection visits, whereas \( \mathcal{L} = \mathcal{L}_2 \text{perm} \) corresponds to an alternative observational scheme with permanent characteristics, as this provided by an SHM system. Along these lines, the VoSHM is defined as:

\[
G_{\mathcal{L}_2,\text{opt } \mathcal{L}_2,\text{perm } (b) = \text{VoSHM }_{\mathcal{L}_2,\text{opt } \mathcal{L}_2,\text{perm } (b) = V_{\mathcal{L}_2,\text{perm } (b) - V_{\mathcal{L}_2,\text{opt } (b)} (18)
\]

It should be noted that the expected life-cycle gain of VoSHM defined in Eq. (18) cannot be strictly seen as VoI as it can also take negative values. A VoSHM value lower than the cost of a SHM system (including acquirement, installation, maintenance, and operation costs, etc.) simply suggests that there is no reason for the decision-maker to invest in SHM but, instead, optimal planning with selected inspection visits should be preferred. However, if the \textit{observation} actions in \( \mathcal{L}_2 \text{opt } \), \( \mathcal{L}_2 \text{perm } \) share the same observation model, then \( \mathcal{L}_2 \text{perm } = \mathcal{L}_2 \text{perm } \), which gives a non-negative value in Eq. (18), thus the VoSHM can be defined in this case as the Relative Value of Continuous Information (RVoCI):

\[
\text{VoSHM }_{\mathcal{L}_2,\text{opt } \mathcal{L}_2,\text{perm } (b) = R\text{VoCI}_{\mathcal{L}_2}(b) = V_{\mathcal{L}_2,\text{perm } (b) - V_{\mathcal{L}_2,\text{opt } (b)} (19)
\]

Using Eqs. (18), (19) we can compute the VoSHM at every possible belief point that the system can visit throughout the planning horizon. Typically, the belief of foremost interest is the root belief, \( b_0 \), which reflects the probability distribution over all possible states at the initial conditions, i.e., for time step \( t=0 \). In this case, the VoSHM quantifies the life-cycle value of the monitoring system. For \( t>0 \), which usually corresponds to \( b_0 \neq b_0 \), Eqs. (18), (19) describe the remaining VoSHM from that time onward. The notion of remaining VoSHM can be of particular practical importance in cases where the optimal salvage time of the SHM system needs to be determined.
TABLE 1

| Condition levels | 1 | 2 | 3 |
|------------------|---|---|---|
| Maintenance rewards (r_m) | 1: Do nothing | 0 | 0 | 0 |
| Observation rewards (r_o) | 1: No observation | 0 | 0 | 0 |
| Damage rewards (r_d) | 0 | -5 | -12 |

4. Numerical Applications

We consider two inspection and maintenance problems and assess the VoSHM for the underlying systems as discussed in Section 3. The first problem pertains to a stationary three-component system, whereas the second to a single-component structure deteriorating according to a non-stationary corrosion model. For the reported results the point-based algorithms of FRTDP, SARSOP and Perseus have been implemented to solve the POMDP problems and to determine the optimal life-cycle strategies.

4.1. Three-Component Deteriorating System

4.1.1. Environment and description of control settings

For the purposes of a parametric numerical investigation in the presence of various observability accuracy levels, we consider a small three-component system. An infinite horizon case with γ=0.95 is analyzed. Stochastic deterioration of the components, for all \( i \in \{1,2,3\} \), is defined by independent transition matrices, \( P_{i,0} \), whereas whenever a repair action is taken the components share the same transition matrix \( P_{i=1,2,3,rep} \):

\[
P_{i,0} = \begin{bmatrix} 0.82 & 0.13 & 0.05 \\ 0.87 & 0.13 & 0.05 \\ 0.79 & 0.17 & 0.04 \end{bmatrix}, \quad P_{2,0} = \begin{bmatrix} 0.72 & 0.19 & 0.09 \\ 0.78 & 0.22 & 0.01 \\ 0.90 & 0.10 & 0.70 \end{bmatrix},
\]

\[
P_{3,0} = \begin{bmatrix} 0.85 & 0.15 & 0.04 \\ 0.80 & 0.20 & 0.05 \\ 0.70 & 0.30 \end{bmatrix}, \quad P_{i=1,2,3,rep} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (20)
\]

As indicated by Eq. (20), each component is described by three condition levels with stationary transition dynamics, i.e., transition from condition level \( k \) to \( j \) is independent of time, component age or deterioration rate. For example, for component 3, the transition probability from state 1 to state 3 is 0.04. Overall, the examined system can be fully specified by 27 states. Markovian transition probabilities of structural systems can be constructed based on simulated or real data of longitudinal responses, system conditions, rankings, etc., e.g., in [33, 55, 56], either through maximum likelihood estimation, or expectation-maximization schemes in the presence of latent state variables.

In order to quantify the VoSHM for this three-component system, two POMDP control settings are evaluated. For Setting 1, 4 observation and maintenance control actions are available for each component, including the possibility of structural inspection visits at belief points suggested by the POMDP solution. These actions are ‘no observation and no repair’, ‘observation and no repair’, ‘no observation and repair’, and ‘observation and repair’. The total number of system actions is 64. For Setting 2, observations are available by default at every decision step, corresponding to a permanent monitoring observational scheme. Accordingly, 2 maintenance control actions are available, i.e., ‘no-repair’, and ‘repair’. Based on the possible action combinations, 8 system actions are available for Setting 2. Observation matrices, for all components, are given as:

\[
O_{i=1,2,3} = \begin{bmatrix} p & (1-p)/2 & (1-p)/2 \\ (1-p)/2 & p & (1-p)/2 \\ (1-p)/2 & (1-p)/2 & p \end{bmatrix} \quad (21)
\]

Eq. (21) assigns an observation accuracy of \( 0 \leq p \leq 1 \) every time an ‘observation’ is taken, meaning that the correct state is observed with
probability $p$, whereas either one of the other states is observed uniformly at random with probability $1-p$.

Negative rewards (or costs) for individual components are given in Table 1 for different states and actions. System level interdependence among components is established though the reward function, with certain penalties added to the cumulative component costs at different system state configurations. That is, for system states \{(2,2,1), (2,2,2),(1,2,3),(2,2,3)\}, \{(3,3,1),(3,3,2)\}, and \{(3,3,3)\}, penalties are -5.0, -10.0, -14.0, and -18.0, respectively, where vector $(i,j,k)$ denotes component condition level combinations, i.e., (3,3,1) indicates that there are 2 components in condition level 3 and one component in condition level 1.

4.1.2. Evaluation of optimal policies and VoSHM

For both POMDP settings, FRTDP, SARSOP and Perseus point-based algorithms are implemented. As shown in the analysis results presented in Figs. 3 and 4, for $p=0.90$, Setting 1 practically converges after 1,000s, whereas Setting 2 after 110s for all algorithms. It can be seen that the precision of the solution of Setting 1 is somewhat lower than the precision of Setting 2, for FRTDP and SARSOP. This can be attributed to the fact that the system in Setting 1 operates in a much more challenging POMDP environment with more actions and, consequently, larger reachable belief space. Apart from that, low precision can also be triggered by a rough approximation of the upper bound. As discussed in Section 2.1, FRTDP and SARSOP utilize approximate upper bounds, determined by a sawtooth approximation. The bound that actually contains all the information of the optimal policy is the lower bound and this is shown to be reached with great agreement among the different algorithms. Overall, in Fig. 3 SARSOP converges faster, thus exhibiting a better anytime performance, as also discussed in [43]. Perseus, although starting from a cruder initial lower bound, eventually reaches the best value, slightly outperforming its counterparts. The same features are also noticed in Fig. 4, where the overall convergence is much faster for all algorithms, due to the simpler nature of the decision problem. SARSOP demonstrates considerable strengths in early convergence, practically converging before 10s. Perseus has an anytime performance advantage compared to FRTDP, whereas all solvers reach identical lower bounds after 3,600s.

A realization of the converged policy is shown in Figs. 5 and 6.

For Settings 1 and 2, each component needs to perform different policies in order for their combined behavior to collectively minimize the total expected cost of the system. In Fig. 5, depicting a policy realization for the case of optional inspections, component 1 requires an inspection visit roughly every two years, whereas its 'repair' actions are mostly taken at the inspection times. Component 2 requires inspections at almost every decision step (all time steps except $1=10$ in the realization of Fig. 5. Component 3 policy combines features of the other two policies, choosing frequent observations, with a few 'no observation and no repair' actions. These policy patterns are intuitively anticipated as the transition dynamics of component 3 are in-between the other two cases defined by components 1, 2. Fig. 6 illustrates a life-cycle policy realization for the case of permanent monitoring (Setting 2). In this POMDP setting, observations are always available at no cost due to the permanent monitoring system assumption, as explained in Section 3.2.

The converged value functions and VoI for each setting, as well as the VoSHM are shown as functions of the observability accuracy level, $p$, in Fig. 7. VoSHM equals the VoCl, as Settings 1 and 2 share the same observation matrices for their observation actions. It can be observed that as the observation accuracy increases, the VoSHM increases and is concave down, reaching a plateau at higher levels of accuracy. The VoSHM of the system ranges from ~3% to ~11% of the value of Setting 1, for $p=0.50$ to $p=1.00$, respectively. This means that any permanent monitoring system with lifetime cost lower than these amounts should be preferred, in place of any

![Fig. 7](image-url) Optimal value functions of three-component system settings 1 and 2 and respective VoSHM, for different observability levels.

![Fig. 8](image-url) Transition probabilities between adjacent structural conditions as functions of the deterioration rate.

| Table 2 |
| --- |
| Condition levels | 1 | 2 | 3 | 4 |
| Maintenance rewards ($\alpha$) | 1: Do nothing | 0 | 0 | 0 | 0 |
| | 2: Repair | -60 | -110 | -160 | -280 |
| | 3: Major repair | -105 | -195 | -290 | -390 |
| | 4: Replace | -820 | -820 | -820 | -820 |
| Inspection rewards ($\beta$) | 1: No observation | 0 | 0 | 0 | 0 |
| | 2: Visual obser. | -4.5 | -4.5 | -4.5 | -4.5 |
| | 3: Monitoring obser. | -7.5 | -7.5 | -7.5 | -7.5 |
| Damage rewards ($\gamma$) | -5 | -40 | -120 | -250 |
inspection plan, including the optimal one. The VoI also increases with increased observability, for both settings, however it is concave up. This pattern is more prominent for the value function of Setting 1, where a plateau is practically reached for \( p < 0.60 \). This indicates that the observation quality is quite poor at this region, so the decision-maker does not choose to pay for inspection and, consequently, the value of Setting 1 becomes equal to the value of the optimal blind policy. The VoI is \(~25\%\) of the optimal blind policy cost and, by definition, is reached by the VoI of Setting 2, for \( p = 1.00 \).

### 4.2. Corroding Deck Structure

#### 4.2.1. Environment and description of control settings

The concrete port structure under corrosion presented in [33] is studied in this example. The original deteriorating environment for this system is described by 4 condition levels and 83 corrosion rates. The discount rate for this problem is set \( \gamma = 0.95 \). The Markovian deterioration of the system, corresponding to the uncontrolled system evolution, is computed by a physically-based corrosion model [57], and is of the following form:

\[
P_0 = \begin{bmatrix} p(x_{i+1} = j, x_i = i, \tau = \tau') \end{bmatrix}_{x \in [1, \ldots, 83]} = \begin{bmatrix} p_{r,11} & p_{r,12} \\ p_{r,21} & p_{r,22} \\ p_{r,31} & p_{r,32} \\ p_{r,41} & p_{r,42} \end{bmatrix}
\]

(22)

where \( x \) is the condition level and \( \tau \) is the deterioration rate. The values for the probabilities in Eq. (22) are shown in Fig. 8. To account for finite horizon policies we appropriately augment the state space with respect to different time steps, thus finally the structure is defined by 14,009 states in total. There are 4 available actions related to maintenance interventions or replacements, namely ‘no repair’, ‘minor repair’, ‘major repair’ and ‘replace’. The transition for the ‘no repair’ action follows Eq. (22). The ‘minor repair’ action influences only the condition level transition of the system, whereas the ‘major repair’ action influences both the condition level and the deterioration rate transition (deterioration rate is reduced by 3 steps). The full transition probability matrices for all maintenance actions can be found in [33].

For Setting 1 (optional inspections), the decision maker has 3 available inspection actions, ‘no observation’, ‘visual observation’ and ‘monitoring observation’. For the complete action set, including the maintenance and the observations, we form 10 actions, instead of 12, since due to the nature of transition probabilities of the ‘replace’ action, possible replacements do not need to be combined with observations. The observation matrices for the two nontrivial observations are:

\[
O_{vis} = \begin{bmatrix} 0.63 & 0.37 \\ 0.10 & 0.63 & 0.27 \\ 0.10 & 0.63 & 0.27 \\ 0.20 & 0.80 \end{bmatrix}
\]

(23)

\[
O_{max} = \begin{bmatrix} 0.80 & 0.20 \\ 0.05 & 0.80 & 0.15 \\ 0.05 & 0.80 & 0.15 \\ 0.10 & 0.90 \end{bmatrix}
\]

(24)

The values of the observation matrices reflect the probability (likelihood) of receiving an observation (columns), given a state (rows). It should be noted that the number of observations is not necessarily equal to the number of states. The typically used probability of detection (PoD) for example, e.g. [22, 23], can be given by an \( n \)-by-2 observation matrix, where \( n \) is the number of states and 2 is the number of observations, e.g. defect detection or not. More details about the selected observation matrices in Eqs. (23) and (24) can be found in [39]. The relevant negative rewards (costs) are shown in Table 2. For Setting 2 we consider a permanent observational scheme, which is assumed to capture the flow of information provided by an SHM system. The respective observation matrix is also described by Eq. (24). As discussed in Section 3.2., this is a default observation at no cost for the purposes of evaluating the VoSHM.
4.2.2. Evaluation of optimal policies and VoSHM

The analysis results during the value iteration are shown for Settings 1 and 2 in Figs. 9 and 10, respectively. In Fig. 9, where the optional inspection setting is considered, we can observe that SARSOP has very good early performance, however FRTDP eventually converges after about 24h. This can be attributed to the masking technique of FRTDP which exploits the sparsity of \( \alpha \)-vectors, thus accommodating a better sparse environment like the one considered in this example. Fig. 10, shows the convergence of the point-based solvers for the permanent monitoring case (Setting 2). Similar performance is observed regarding the solvers comparison, however, a very good near-optimal solution is discovered much faster. Indicatively, FRTDP converges after about 300s. Fast convergence in Setting 2 is anticipated, since the problem comprises only 4 actions, compared to 10 in Setting 1, so the overall reachable belief space is less extensive.

Using the converged lower bound, we also show two realizations of the optimal policies of Settings 1 and 2, in Figs. 11 and 12, respectively. In both Settings 1 and 2, the decision-maker starts taking nontrivial maintenance and observation actions after \( \sim 30 \) time steps. This happens because severe deterioration has not typically started until these time steps. In Setting 1, the agent mostly takes ‘visual observation’ actions and a few ‘monitoring observation’ actions, which are combined with ‘no repair’ actions. Regarding maintenance actions, ‘minor repair’ actions are shown to suffice for optimal control, along with a sparse selection of ‘major repair’ actions. In Setting 2, the agent has a better understanding of its state, due to the presence of the permanent monitoring system. This gives the agent the opportunity to avoid taking any maintenance actions unless it is necessary due to expectation of high cost states. When this time comes, after \( \sim 30 \) time steps, it starts with ‘major repair’ actions and then it proceeds with ‘minor repairs’, until stopping taking actions, after \( \sim 90 \) time steps. At the final steps of the realizations, in both settings, no maintenance and observation actions are selected. This happens as a result of the finite horizon policy, which means that the agent knows its exact time step, in addition to a belief over possible condition level and the exact deterioration rate, before taking an action. As such, when approaching the final zero-valued absorbing state, which signifies end of the planning horizon, future cumulative state costs start becoming less significant, so no state corrections or better understanding of the system condition is required.

The results of the life-cycle cost estimates based on the policy described by the lower bound of the converged value functions, along with the respective VoSHM estimates, are shown in Table 3. Results of all the utilized point-based POMDP algorithms are shown with a maximum analysis time of 24h. It can be seen that the VoSHM is in the order of \( \sim 7\% \) of the life-cycle cost estimate of Setting 1. As mentioned in the example of Section 4.1., this amount indicates the maximum cost the decision-maker should plan invest at the beginning of the control horizon, in order to acquire, install, operate and maintain a SHM system.

5. Conclusions

A methodology for quantifying the Value of Information (VoI) and the Value of Structural Health Monitoring (VoSHM) is presented in this work. The two metrics are defined over the life-cycle of the system, quantifying the expected life-cycle gains upon availability of inspection or monitoring information. POMDPs are employed to handle the optimal decision-making problem, which is essential for the assessment of the relevant life-cycle costs. The POMDP framework is chosen on the basis of its concrete mathematical properties for optimality in long-term sequential decision-making problems that feature stochastic environments with uncertain action outcomes and noisy observations. Optimal POMDP solutions are derived with the aid of point-based algorithms that can efficiently

| Algorithm | Setting 1 | Setting 2 | VoSHM |
|-----------|-----------|-----------|-------|
| FRTDP     | \(-198.253 \pm 1.042\) | \(-181.126 \pm 3.830\) | \(17.127 \pm 4.872\) |
| SARSOP    | \(-198.549 \pm 2.512\) | \(-184.437 \pm 2.187\) | \(14.112 \pm 4.699\) |
| Perseus   | \(-199.015 \pm 2.829\) | \(-183.043 \pm 2.168\) | \(15.972 \pm 4.997\) |

Fig. 11: Two policy realizations of corroding deck structure, for Setting 1 (optional inspection setting).

Fig. 12: Two policy realizations of corroding deck structure, for Setting 2 (permanent monitoring setting).
explore and evaluate the reachable belief space, from an initial system state distribution. Based on the above, VoI and VoSHM estimates are obtained based on pairs of different POMDP settings. These settings share the same state space, with the same stochastic deterioration properties, and operate over the same discounted horizon, having identical sets of maintenance actions. For the quantification of VoI, the first setting involves optimization of maintenance actions for the blind problem (no observations available), whereas the second setting optimizes both maintenance and observation actions. For the quantification of VoSHM, the first setting corresponds to an observational scheme with optimal inspections, whereas the second setting operates under the assumption of continuous observations throughout the entire operational life, thus representing a permanent monitoring system. The results presented in a three-component deteriorating system and a single-component structure under corrosion indicate that the proposed approach provides a straightforward way to quantify the expected gains of different observational alternatives. The outcome of this analysis is a quantitative answer to the question of how much information is from inspections and/or monitoring worth. Potential applications and extensions of the present work include consideration of different types of inspection and monitoring observation models directly calibrated based on real data, integration of advanced learning techniques with the decision-making process for online extraction of efficient damage and condition indicators from high-dimensional and heterogeneous SHM data, as well as VoSHM utilization for design of SHM systems and sensor placement.

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