High Precision Vertical Deflection Over China Marginal Sea and Global Sea Derived from Multi-Satellite Altimeter

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Abstract  On the basis of gravity field model (EIGEN_CG01C), together with multi-altimeter data, the improved deflection of the vertical gridded in 2′×2′ in China marginal sea and gridded in 5′×5′ in the global sea was determined by using the weighted method of along-track least squares, and the accuracy is better than 1.2″ in China marginal sea. As for the quality of the deflection of the vertical, it meets the challenge for the gravity field of high resolution and accuracy. It shows that, compared with the shipboard gravimetry in the sea, the accuracy of the gravity anomalies computed with the marine deflection of the vertical by inverse Vening-Meinesz formula is 7.75 m·s⁻².

Keywords  altimetry; along-track; the weighted least-squares; deflection of the vertical

CLC number  P223

Introduction

Classically, the vertical deflection are measured by the method of astronomical measurement and geodesy, and a great disadvantage of the method is that it is in need of much manpower and much operation time. There is no denying that the classical method can not meet the mobile demand of the neotype missile launching. For oceanic researches, Altimetry technique radically overcomes the obstacles of the classical method. With respect to the method of the vertical deflection calculated in oceanic area by the use of altimeter data, many domestic and international scholars carefully studied the problem[1-6]. Synthesizing domestic and international research achievements of ocean vertical deflection resolved by using altimeter data, the method of primary difference is the best one by which vertical deviation is resolved in oceanic areas by the use of satellite altimeter data, because it can minimize the effect of systematic errors, for example, orbit radial error and long wavelength sea surface topography error[7]. On the basis of gravity field model (EIGEN_CG01C), the improved vertical deflection gridded in 2′×2′ in China marginal sea, with high accuracy and resolution, were derived using the weighted method of along-track least squares, and the vertical deflection with the resolution of 5′×5′ in the global sea as well.

1  Data preparation

The vertical deflection were derived over China marginal sea and global sea, and the data used are

Received on July 25, 2008.

Supported by the National Nature Science Foundation of China(No. 40474030 , 40674013).

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listed as follows: ① T/P data(1993.10~2001.12, 2~333 cycle); ② ERS1/GM data(94.04~95.03, 10 cycle in all); ③ ERS1/ERM data (1992.10~1993.12, 1995.03~1996.06, 26 cycle in all); ④ ERS2/ERM data (1995.04~2000.12, 0~66 cycle); ⑤ GEOSAT/ERM data (1986.10~1989.12, 1~68 cycle); ⑥ GEOSAT/GM data (1985.03~1986.09, 0~7853 cycle). In order to combine multi-satellite altimeter, all the data we used were standardized to the frame of T/P reference ellipsoid; and the sea surface heights from T/P could serve as a stand surface because of its highly accurate orbit.

2 Basic principle and algorithm

As a result of the improvement in the gravity model, we can now obtain higher order and better accuracy in the gravity model, such as PGM2000A seriation, ITG seriation, GGM seriation, EIGEN seriation, etc. This research used a remove-restore procedure as reference, on account of is the existence of rich high-frequency component in vertical deflection, thus gravity field model is provided with comparative advantage nowadays. The gravity model of EIGEN-CG01C were resolved by combining with 200 days GRACE data, 860 days CHAMP data, altimeter data and gravity data in the surface, etc. Comparing geoid height from GPS station with that from two gravity models, the accuracy calculated by the model of EGM96 is better than that by the model of EGM96. As far as a remove-restore procedure was concerned, in this study, the research used the gravity model of EIGEN-CG01C and the sea surface topography model of EGM96-SST.

First, from sea surface heights, we subtracted the corresponding geoid derived by the EIGEN-CG01C model and the corresponding sea surface topography derived by the EGM96-SST model, and got residual geoidal undulations \( N_{res} \). Then, differentiate \( N_{res} \) for distance along satellite track, where the systematic errors, such as the orbits, can be reduced. From Eqs.(1) and (2), we got residual geoidal gradient \( \varepsilon \) and azimuth \( \alpha \):

\[
\varepsilon = \frac{dSSH}{dS} = \frac{\Delta N_{res}}{d} = \frac{N_{res2} - N_{res1}}{d}
\]

(1)

\[
\alpha = \arctan \frac{\Delta \lambda \cdot \cos \varphi_0}{\Delta \varphi}
\]

(2)

Here, \( \varepsilon \) position is located in the mid-position between two continuous observation points along the satellite track; \( \Delta \lambda \), \( \Delta \varphi \), \( \varphi_0 \) are longitude difference, latitude difference, latitude in the mid-position, respectively.

According to observation Eq.(3), the averaged meridional component and the prime vertical component in grid about residual geoid gradient were obtained:

\[
\varepsilon_i + v_i = \xi \cos \alpha_i + \eta \sin \alpha_i, \quad i = 1, \ldots, n
\]

(3)

Where \( n \) is the number of observation points which are part of the sea surface heights of the grid around, \( v_i \), \( \alpha_i \), and \( \varepsilon_i \) are the \( i \)th residual errors, azimuth and residual geoid gradient along track, respectively.

The weight of the geoid gradient in a typical cell size for testing for outliers with 4′×4′ is set to be:

\[
P_i = \frac{W_i(j)}{d_i^2 \sigma_i^2} \quad (j = 1, 2, \ldots, 6; i = 1, 2, \ldots, n)
\]

(4)

\[
\sigma_i = \frac{\sqrt{s_d \times 206.264.806}}{d_i}
\]

(5)

Where \( d_i \) is the distance of the discrete point to the proposed grid point; \( \sigma_i \) is the standard deviation of geoid gradient; \( s_d \) is the noise of sea surface heights; \( d_i \) is the spatial distance between two adjacent points in Eq.(1); and \( W_i(j) \) is the weight for the different satellite altimeter.

According to the different accuracy for the different satellite altimeter, we adopted the prior value which is provided with relative reliability, and got the weight of a different altimeter:

| Altimeter  | Weight |
|------------|--------|
| T/P        | 1/2.51 |
| ERS2       | 1/4.5  |
| ERS1/GM    | 1/8.6  |
| ERS1/ERM   | 1/6.7  |
| GEOSAT/GM  | 1/15.3 |
| GEOSAT/ERM | 1/8.0  |

According to the least squares theory \( \sum_{j=1}^{6} \sum_{i=1}^{6} P_i v_i^2 = \min \), the average value of the residual vertical deflection \((\xi, \eta)\) gridded in 2′×2′ was derived, and the specific derivation was:

\[
A = \sum_{j=1}^{6} \sum_{i=1}^{6} P_i v_i^2 = \sum_{j=1}^{6} \sum_{i=1}^{6} (\xi \cos \alpha_i + \eta \sin \alpha_i - \varepsilon_i)^2
\]

(6)
It is certain that $\frac{\partial A}{\partial \xi} = 0$ and $\frac{\partial A}{\partial \eta} = 0$, while $A$ is its minimal value. From Eq. (6), we can obtain:

$$\sum_{j=1}^{6} \sum_{i=1}^{n} (P_j \cos^2 \alpha_i) \xi + \sum_{j=1}^{6} \sum_{i=1}^{n} (P_j \cos \alpha_i \sin \alpha_i) \eta - \sum_{j=1}^{6} \sum_{i=1}^{n} P_j \epsilon_i \cos \alpha_i = 0$$

$$\sum_{j=1}^{6} \sum_{i=1}^{n} (P_j \cos \alpha_i \sin \alpha_i) \xi + \sum_{j=1}^{6} \sum_{i=1}^{n} (P_j \sin^2 \alpha_i) \eta - \sum_{j=1}^{6} \sum_{i=1}^{n} P_j \epsilon_i \sin \alpha_i = 0$$

From Eq. (7) and Eq. (8), the least square solution of the residual vertical deviation can be obtained as:

$$\bar{\xi} = \left[ \sum_{j=1}^{6} \sum_{i=1}^{n} (P_j \sin^2 \alpha_i) \cdot \sum_{j=1}^{6} \sum_{i=1}^{n} (P_j \epsilon_i \cos \alpha_i) - \sum_{j=1}^{6} \sum_{i=1}^{n} (P_j \cos \alpha_i \sin \alpha_i) \cdot \sum_{j=1}^{6} \sum_{i=1}^{n} (P_j \epsilon_i \sin \alpha_i) \right] / \left[ \sum_{j=1}^{6} \sum_{i=1}^{n} (P_j \cos \alpha_i \sin \alpha_i) \cdot \sum_{j=1}^{6} \sum_{i=1}^{n} (P_j \sin \alpha_i) - \sum_{j=1}^{6} \sum_{i=1}^{n} (P_j \sin^2 \alpha_i) \cdot \sum_{j=1}^{6} \sum_{i=1}^{n} (P_j \epsilon_i \cos \alpha_i) \right]$$

$$\bar{\eta} = \left[ \sum_{j=1}^{6} \sum_{i=1}^{n} (P_j \cos^2 \alpha_i) \cdot \sum_{j=1}^{6} \sum_{i=1}^{n} (P_j \epsilon_i \cos \alpha_i) - \sum_{j=1}^{6} \sum_{i=1}^{n} (P_j \cos \alpha_i \sin \alpha_i) \cdot \sum_{j=1}^{6} \sum_{i=1}^{n} (P_j \epsilon_i \sin \alpha_i) \right] / \left[ \sum_{j=1}^{6} \sum_{i=1}^{n} (P_j \cos \alpha_i \sin \alpha_i) \cdot \sum_{j=1}^{6} \sum_{i=1}^{n} (P_j \sin \alpha_i) - \sum_{j=1}^{6} \sum_{i=1}^{n} (P_j \sin^2 \alpha_i) \cdot \sum_{j=1}^{6} \sum_{i=1}^{n} (P_j \epsilon_i \cos \alpha_i) \right]$$

After the residual vertical deflection is derived, a way to avoid having low accuracy of the observed value is to make use of the theory of 3-times standard deviation. We replaced the value $(\bar{\xi}, \bar{\eta})$ in Eq. (3), in order to calculate the standard deviation $\bar{\sigma}$. If $v_j > 3\bar{\sigma}$, then the corresponding observed value was rejected. Afterwards, we recomputed the residual vertical deflection from the residual $\epsilon_j$, and restored the vertical deflection of gravity field model EIGEN_CGO1C. Lastly, the value of the vertical deflection was derived.

In this study, order to minimize the effect of high-frequency amplification while the primary difference between sea surface heights was used, we made use of Gaussian low-pass filter on the vertical deviation, and filter radius is set to 18 km by the resolution of altimeter data along track. At the same time, according to the theory mentioned above, we calculated the vertical deflection in the global sea. Only the vertical deviation gridded in $5' \times 5'$ in the global sea ($-80^\circ$-$80^\circ$N, $0^\circ$-$360^\circ$E) was calculated because of large amounts of data; meanwhile, the global area was divided into $4 \times 9 (40^\circ \times 40^\circ)$ area. Each $40^\circ \times 40^\circ$ cell was extended $5^\circ$ outwards, in order to avoid bad results at the edges.

### 3 Results and analysis

Fig.1 and Fig.2 show the gridded north-south and east-west vertical deflection component calculated over China marginal sea in this study. Over China marginal sea, the satellite-derived vertical deflection was compared with that from EIGEN_CGO1C, EGM96 gravity field model, the result of which is shown in Table 1. Moreover, the differences in the global sea between the vertical deflection in this research and that from two models is also shown in Table 1, where IGG_SN and IGG_WE are the gridded north-south and east-west vertical deflection component over China marginal sea, respectively; EIGEN1_SN and EIGEN1_WE are the vertical deflection

| Table 1 | In China Sea, comparison between the deflection of the vertical derived from altimeter and that from the model/arcsecond |
|---------|---------------------------------------------------------------------------------------------------------------------|
|         | min          | max          | mean         | $s_{ld}$         |
| China Sea |             |              |              |                 |
| IGG_SN-EIGEN1_SN | −6.06 | 5.77 | 0.18 | 0.99 |
| IGG_WE-EIGEN1_WE | −9.10 | 21.47 | 0.16 | 1.39 |
| IGG_SN-EGM96_SN | −6.78 | 8.98 | 0.12 | 1.01 |
| IGG_WE-EGM96_WE | −7.94 | 21.92 | 0.21 | 1.50 |
| Global sea |             |              |              |                 |
| IGG_SN-EIGEN1_SN | −122.32 | 15.97 | 0.01 | 1.11 |
| IGG_WE-EIGEN1_WE | −122.08 | 81.07 | 0.06 | 1.96 |
| IGG_SN-EGM96_SN | −121.97 | 15.83 | 0.01 | 1.12 |
| IGG_WE-EGM96_WE | −122.3 | 81.15 | 0.05 | 1.97 |
from the gravity field EIGEN_CGO1C; EGM96_SN and EGM96_WE are the vertical deflection from the gravity field EGM96.

In Fig.1, Fig.2 and Table 1, the max and the min are obviously too large, which could come from the reduced quality of altimeter data due to distorted altimeter waveforms returning from the ground. Moreover, the standard deviation of west-east vertical deflection is larger than that of north-south vertical deflection, which could come from the along-track azimuth trends towards the north-south, and it results in the higher accuracy of vertical deflection from north-south direction. Compared with the vertical deflection from gravity field EIGEN_CGO1C, as shown in Table 1, the general accuracy of the vertical deflection is \( \delta = \sqrt{\delta_x^2 + \delta_y^2} = 1.2'' \) in China Sea.

In order to objectively evaluate the satellite-derived vertical deflection accuracy in this study, we made use of vertical deflection data to calculate gravity anomaly which is compared with the shipborne measurements of gravity anomalies from the South China Sea. The shipborne track was shown in Fig.3.

Using a remove-restore procedure with the EIGEN_CGO1C gravity model, and the Vening-Meinsz formula as well, considering the effect in the inner circle, the gravity anomaly was derived from the vertical deflection over China marginal sea; and the satellite-derived gravity anomaly was compared with the shipborne gravity anomalies in South China Sea(Point, 52 741). Before the comparison, the satellite-derived gravity anomalies were adjusted to the ship gravity anomalies using the interpolation method of Shepard. The result of comparison is the 7.75 mgal standard deviation, which indicates that the satellite-derived gravity anomalies in this study corresponds with that from Hwang (GMCA97_G gravity anomalies gridded in 2'×2', and the standard deviation is 7.72 mgal).

Considering that the altimeter data size is enormous which needs long-time computer operation, only the vertical deflection with resolution of 5'×5' (IGG2006_DOV) was calculated in the global sea \((-80^\circ \sim 80^\circ \text{N}, \ 0^\circ \sim 360^\circ \text{E})\). Fig.4 shows the result.
of the north-south vertical deflection with the resolution of $5' \times 5'$ in the global sea.

4 Conclusion

In this study, the advantage of using the weighted method of along-track least squares is that the crossing point of altimeter data does not need to be calculated. Moreover, after the along-track discrete altimeter data were adjusted with the geophysical parameter, the along-track geoid gradient was processed by using the weighted method of least squares according to the different altimeter accuracy, and the bad geoid gradient data were rejected by using the theory of 3-times standard deviation. Thus, theoretically, by means of moving the selected cell in the grid, we can calculate the vertical deflection of the random point. Furthermore, the vertical deflection with high accuracy and resolution can be also derived. Making use of the vertical deflection, we can calculate another gravity-field parameter with high accuracy and resolution. Thus, we can use it for reference in the case of making scientific use of altimeter data to resolve the ocean gravity field. Over shallow waters, islands and ice cover, the altimeter data can be seriously affected, which results in low accuracy of the vertical deflection. Currently, many domestic and international scholars made a lot of research on the domain of vertical deflection, and it is common that the accuracy of vertical deflection can be improved farther by using waveform retracking technology, etc. over the off-shore sea and the shallow shoal.

Acknowledgements

Thanks to professor Hwang at the Department of Civil Engineering, National Chiao Tung University for altimeter data.

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