Effects of large CP violating phases on $g_\mu - 2$ in MSSM

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Abstract

Effects of CP violation on the supersymmetric electro-weak correction to the anomalous magnetic moment of the muon are investigated with the most general allowed set of CP violating phases in MSSM. The analysis includes contributions from the chargino and the neutralino exchanges to the muon anomaly. The supersymmetric contributions depend only on specific combinations of CP phases. The independent set of such phases is classified. We analyse the effects of the phases under the EDM constraints and show that large CP violating phases can drastically affect the magnitude of the supersymmetric electro-weak contribution to $a_\mu$ and may even affect its overall sign.
1 Introduction

As is well known supersymmetric theories contain many new sources of CP violation which mostly arise from the phases of the soft SUSY breaking parameters and that such phases contribute to the electric dipole moments (EDMs) of the electron and of the neutron. Experimentally the electron and the neutron EDMs have very strict limits, i.e., for the neutron the limit is

$$|d_n| < 6.3 \times 10^{-26} \text{ecm}$$

and for the electron the limit is

$$|d_e| < 4.3 \times 10^{-27} \text{ecm}$$

and these limits impose stringent constrains on particle physics models. In SUSY/string models one normally expects CP violating phases O(1) and phases of this size typically lead to EDM predictions in such models already in excess of the current experimental limits. Of the possible remedies to this problem the conventional approach has been to assume that the phases are small, typically, $O(10^{-2}-3)$, which, however, constitutes a fine tuning. Another possibility suggested is to assume that the SUSY spectrum is heavy in the several TeV region. Generally, a heavy spectrum may constitutes fine tuning except in certain limited domains of the parameter space. Further, such a heavy spectrum may lie outside the reach of even the Large Hadron Collider (LHC) and thus a disappointing scenario from the point of view of particle physicists. A third more encouraging possibility is that the large phases could indeed be there, but one escapes the experimental EDM constraints because of cancellations among the various contributions to the EDMs. This possibility was proposed in Ref. and there have been further verification and developments and applications such as in dark matter, low energy processes, and on other SUSY phenomena.

The cancellation mechanism opens a new window on the SUSY parameter space where large CP phases along with a light SUSY spectrum can co-exist. Thus significant effects on SUSY phenomena can result. One of the quantities affected by CP phases is $a_\mu = (g_\mu - 2)/2$, where $g_\mu - 2$ is the anomalous magnetic moment of the muon. This quantity is of considerable current interest since the new Brookhaven experiment will measure $a_\mu$ to an accuracy of better than a factor of 20. Further, recently there has been considerable progress in reducing the
hadronic error\textsuperscript{34, 35, 36}. With the reduced hadronic error the new $g_\mu$ experiment will test the Standard Model (SM) electro-weak correction\textsuperscript{37} which including the two loop SM corrections stands at\textsuperscript{38}

$$a_\mu^{SM} = 15.1 \times 10^{-10}$$

(3)

It turns out that the supersymmetric electro-weak corrections to $a_\mu$ can be quite large and these supersymmetric effects on $a_\mu$ have been investigated for many years\textsuperscript{39, 40, 41, 42}. However, the CP violating supersymmetric electro-weak effects on $a_\mu$ have been ignored for the reason that small CP phases or large CP phases with a heavy spectrum lead only to negligible effects on $a_\mu$.

With the cancellation mechanism the possibility of large CP phases along with a light spectrum arises and such a situation can lead to very significant effects on $a_\mu$. Indeed in a recent work\textsuperscript{43} the effects of CP phases on $a_\mu$ in the context of the minimal supergravity model (mSUGRA) were analysed and it was shown that CP violating phases can produce significant effects on $a_\mu$. In the absence of CP violating phases the soft SUSY breaking parameters at the GUT scale in mSUGRA\textsuperscript{44} consist of the universal scalar mass $m_0$, the universal gaugino mass $m_1$, the universal trilinear coupling $A_0$ and $\tan \beta = <H_2> / <H_1>$ where $H_2$ gives mass to the up quark and $H_1$ gives mass to the down quark. More generally the soft SUSY breaking parameters as well as the Higgs VEVs are complex and have phases. However, by a redefinition of fields it is easily seen that there are only two CP violating phases in mSUGRA. These can be chosen to be the phase of $A_0$ and the phase of $\mu_0$ where $\mu_0$ appears in the Higgs mixing term, i.e., in the term $\mu_0 H_1 H_2$ in the superpotential. In this paper we extend our analysis of the effects of CP violating phases on $a_\mu$ to supergravity models with non-universalities\textsuperscript{45, 46, 47, 48, 49, 50, 51, 52} and to the Minimal Supersymmetric Standard Model (MSSM) which has many more CP violating phases. The existence of a larger set of CP phases widens the region of the parameter space where cancellations can occur. The purpose of this paper is to derive the general one loop supersymmetric correction to $a_\mu$ with the most general set of CP violating phases allowed in MSSM and determine the numerical effects of these CP violating phases on $a_\mu$ under the experimental constraints on the electron and on the neutron EDM.

The outline of the rest of the paper is as follows: In Sec.2 we derive the general one loop formula for $a_f$ for the case of a fermion $f$ interacting with a fermion and a scalar in the presence of CP violating phases and without any approximation.

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on the relative size of the external and internal particle masses in the loop. In Sec.3 we apply this formula for the computation of the chargino and neutralino exchange contributions to $a_\mu$ for the most general allowed set of CP violating phases in this sector. In Sec.4 and in Appendix A we study the combination of CP phases that enter $a_\mu$ and compare them with the corresponding combinations that arise in the expressions for the electron and the neutron EDMs. In Sec.5 and in Appendix B the supersymmetric limit of our result is given and it is explicitly shown how the one loop Standard Model contribution to $a_\mu$ including the one loop QED correction, i.e. $\alpha_{em}/2\pi$, is cancelled by the supersymmetric contribution. In Sec.6 we give a discussion of the satisfaction of the EDM constraints. In Sec.7 we give an analysis of CP violating effects on $a_\mu$. Conclusions are given in Sec.8. Appendix C is devoted to a discussion of the vanishing CP violating phases and a comparison of our results with previous analyses.

## 2 CP Effects on $g-2$ in MSSM

We give here the general analysis for the CP effects on $g-2$ of a fermion. In general for the interaction of a fermion $\psi_f$ of mass $m_f$ interacting with a fermion $\psi_i$ of mass $m_i$ and a scalar $\phi_k$ of mass $m_k$, the vertex interaction has the general form

$$- \mathcal{L}_{int} = \sum_{ik} \bar{\psi}_f (K_{ik} \frac{1-\gamma_5}{2} + L_{ik} \frac{1+\gamma_5}{2}) \psi_i \phi_k + H.c.$$  (4)

This interaction violates CP invariance iff $\text{Im}(K_{ik}L_{ik}^*) \neq 0$. The one loop contribution to $a_f$ is given by

$$a_f = a_f^1 + a_f^2$$  (5)

where $a_f^1$ and $a_f^2$ arise from Fig. 1(a) and Fig. 1(b) respectively. $a_f^1$ is a sum of two terms: $a_f^1 = a_{f1} + a_{f2}$ where

$$a_{f1} = \sum_{ik} \frac{m_f}{8\pi^2 m_i} \text{Re}(K_{ik}L_{ik}^*) I_1 \left( \frac{m_f^2}{m_i^2}, \frac{m_k^2}{m_i^2} \right)$$  (6)

and

$$I_1(\alpha, \beta) = -\int_0^1 dx \int_{1-x}^1 dz \frac{z}{\alpha z^2 + (1-\alpha-\beta)z + \beta}$$  (7)

and where

$$a_{f2} = \sum_{ik} \frac{m_f^2}{16\pi^2 m_i^2} (|K_{ik}|^2 + |L_{ik}|^2) I_2 \left( \frac{m_f^2}{m_i^2}, \frac{m_k^2}{m_i^2} \right)$$  (8)
and
\[ I_2(\alpha, \beta) = \int_0^1 dx \int_0^{1-x} dz \frac{z^2 - z}{\alpha z^2 + (1 - \alpha - \beta)z + \beta} \]  
(9)

Similarly, \( a_f^2 \) consists of two terms: 
\[ a_f^2 = a_f^{21} + a_f^{22} \]  
where
\[ a_f^{21} = \sum_{ik} \frac{m_f}{8\pi^2 m_i} \text{Re}(K_{ik}L_{ik}^*) I_3 \left( \frac{m_f^2}{m_i^2}, \frac{m_k^2}{m_i^2} \right) \]  
(10)

and
\[ I_3(\alpha, \beta) = \int_0^1 dx \int_0^{1-x} dz \frac{1 - z}{\alpha z^2 + (\beta - \alpha - 1)z + 1} \]  
(11)

and where
\[ a_f^{22} = -\sum_{ik} \frac{m_f^2}{16\pi^2 m_i^2} (|K_{ik}|^2 + |L_{ik}|^2) I_4 \left( \frac{m_f^2}{m_i^2}, \frac{m_k^2}{m_i^2} \right) \]  
(12)

and
\[ I_4(\alpha, \beta) = \int_0^1 dx \int_0^{1-x} dz \frac{z^2 - z}{\alpha z^2 + (\beta - \alpha - 1)z + 1} \]  
(13)

In the above we have given the exact expressions for the integrals \( I_1 - I_4 \) rather than their approximate forms in the limit when one neglects terms of size \( m_f^2 \) relative to \( m_k^2 \) and \( m_i^2 \) which allows one to write simple closed form expressions for them. We will see that the general expressions are needed to discuss the supersymmetric limit of our results which provides the absolute check on our normalizations.

3 \( g_{\mu} - 2 \) with CP Violating Phases

We apply now the above relations for the computation of the chargino and the neutralino exchange contributions. We consider the chargino exchange contributions first. The CP violating phases enter here via the chargino mass matrix defined by

\[ M_C = \begin{pmatrix} \sqrt{2}m_W |\tilde{m}_2| e^{i\chi_2} & \sqrt{2}m_W \sin \beta e^{-i\chi_2} \\ \sqrt{2}m_W \cos \beta e^{-i\chi_1} & |\mu| e^{i\theta_\mu} \end{pmatrix} \]  
(14)

where \( \chi_1 \) and \( \chi_2 \) are phases of the Higgs VEVs, i.e., \( < H_i > = | < H_i > | e^{i\chi_i} \) (i=1,2). The matrix of Eq.(14) can be diagonalized by the biunitary transformation \( U^* M_C V^{-1} = \text{diag}(\tilde{m}_{\chi_1}^+, \tilde{m}_{\chi_2}^+) \) where U and V are unitary matrices. By looking at the muon-chargino-sneutrino interaction one can identify \( K_i \) and \( L_i \) and one finds

\[ a_{\mu}^{\chi^-} = a_{\mu}^{21} + a_{\mu}^{22} \]  
(15)
where $a_{\mu}^{21}$ and $a_{\mu}^{22}$ are given below. We exhibit these only in the limit where $I_3(\alpha, \beta)$ and $I_4(\alpha, \beta)$ have their first arguments zero and one may write

\[ I_3(0, x) = -\frac{1}{2} F_3(x), \quad I_4(0, x) = -\frac{1}{6} F_4(x) \]  

where

\[ F_3(x) = \frac{1}{(x-1)^3} (3x^2 - 4x + 1 - 2x^2 \ln x) \]  

\[ F_4(x) = \frac{1}{(x-1)^4} (2x^3 + 3x^2 - 6x + 1 - 6x^2 \ln x). \]

In the above approximation we have

\[ a_{\mu}^{21} = \frac{m_{\mu \alpha EM}}{4\pi \sin^2 \theta_W} \sum_{i=1}^{2} \frac{1}{M_{\chi_i}^2} Re(\kappa_{\mu} U_{i2}^* V_{i1}^*) F_3(\frac{M_{\mu}^2}{M_{\chi_i}^2}). \]  

and

\[ a_{\mu}^{22} = \frac{m_{\mu \beta EM}}{24\pi \sin^2 \theta_W} \sum_{i=1}^{2} \frac{1}{M_{\chi_i}^2} (|\kappa_{\mu} U_{i2}|^2 + |V_{i1}|^2) F_4(\frac{M_{\mu}^2}{M_{\chi_i}^2}). \]

where

\[ \kappa_{\mu} = \frac{m_{\mu}}{\sqrt{2} M_W \cos \beta} e^{-i\chi_1}. \]

Next we discuss the neutralino exchange contribution to $a_{\mu}$. CP violating effects here are all contained in the neutralino and smuon mass matrices. For the neutralino mass matrix the CP violating phases enter as below

\[
\begin{pmatrix}
|M_1| e^{i\xi_1} & 0 & -M_2 \sin \theta_W \cos \beta e^{-i\chi_1} & M_2 \sin \theta_W \sin \beta e^{-i\chi_2} \\
0 & |M_2| e^{i\xi_2} & M_2 \cos \theta_W \cos \beta e^{-i\chi_1} & -M_2 \cos \theta_W \sin \beta e^{-i\chi_2} \\
-M_2 \sin \theta_W \cos \beta e^{-i\chi_1} & M_2 \cos \theta_W \cos \beta e^{-i\chi_2} & 0 & |\mu| e^{i\theta_\mu} \\
M_2 \sin \theta_W \sin \beta e^{-i\chi_1} & -M_2 \cos \theta_W \sin \beta e^{-i\chi_2} & -|\mu| e^{i\theta_\mu} & 0
\end{pmatrix}.
\]

The neutralino mass matrix $M_{\chi} \equiv \tilde{m}$ is a complex non hermitian and symmetric matrix and can be diagonalized using a unitary matrix $X$ such that $X^T M_{\chi} X = \text{diag}(\tilde{m}_{\chi_1}, \tilde{m}_{\chi_2}, \tilde{m}_{\chi_3}, \tilde{m}_{\chi_4})$. Since the loop correction involving the neutralino exchange also involves the smuon exchange (see Fig.1a) the CP phases in the smuon (mass)$^2$ also enter the analysis.

The smuon (mass)$^2$ matrix is given by

\[ M_{\mu}^2 = \begin{pmatrix}
\frac{M_{\mu}^2}{\mu_1} & m_{\mu}(A_{\mu}^* m_0 - \mu \tan \beta e^{i(\chi_1 + \chi_2)}) \\
m_{\mu}(A_{\mu}^* m_0 - \mu \tan \beta e^{i(\chi_1 + \chi_2)}) & \frac{M_{\mu}^2}{\mu_2}
\end{pmatrix}, \]

This matrix is hermitian and can be diagonalized by the unitary transformation

\[ D^* M_{\mu}^2 D = \text{diag}(M_{\mu_1}^2, M_{\mu_2}^2). \]
The neutralino exchange contribution to $a_\mu$ is given by

$$a_\mu^{\chi^0} = a_\mu^{11} + a_\mu^{12}$$

(25)

where

$$a_\mu^{11} = \frac{m_\mu \alpha_{EM}}{2\pi \sin^2 \theta_W} \sum_{j=1}^4 \sum_{k=1}^2 \frac{1}{M_{\chi^0_j}} \text{Re}(\eta_{\mu j}^k) I_1 \left( \frac{m_\mu^2}{M_{\chi^0_j}^2}, \frac{M_{\tilde{\mu}_k}^2}{M_{\chi^0_j}^2} \right)$$

(26)

and

$$\eta_{\mu j}^k = -\left( \frac{1}{\sqrt{2}} \tan \theta_W |X_{1j}| D_{1k}^* \right)
+ \frac{1}{\sqrt{2}} \tan \theta_W |X_{2j}| D_{2k}^* + \kappa_\mu X_{3j} D_{1k}$$

(27)

and $a_\mu^{12}$ is given by

$$a_\mu^{12} = \frac{m_\mu^2 \alpha_{EM}}{4\pi \sin^2 \theta_W} \sum_{j=1}^4 \sum_{k=1}^2 \frac{1}{M_{\chi^0_j}^2} X_{\mu j}^k I_2 \left( \frac{m_\mu^2}{M_{\chi^0_j}^2}, \frac{M_{\tilde{\mu}_k}^2}{M_{\chi^0_j}^2} \right)$$

(28)

where

$$X_{\mu j}^k = \frac{m_\mu^2}{2M_W^2 \cos^2 \beta} |X_{3j}|^2
+ \frac{1}{2} \tan^2 \theta_W |X_{1j}|^2 (|D_{1k}|^2 + 4|D_{2k}|^2) + \frac{1}{2} |X_{2j}|^2 |D_{1k}|^2
+ \tan \theta_W |D_{1k}|^2 \text{Re}(X_{1j} X_{2j}^*)
+ \frac{m_\mu \tan \theta_W}{M_W \cos \beta} \text{Re}(e^{-i\chi_1} X_{3j} X_{1j}^* D_{1k} D_{2k}^*)
- \frac{m_\mu}{M_W \cos \beta} \text{Re}(e^{-i\chi_3} X_{3j} X_{2j}^* D_{1k} D_{2k}^*)$$

(29)

If one ignores the muon mass with respect to the other masses involved in the problem, the form factors $I_1(\alpha, \beta)$ and $I_2(\alpha, \beta)$ become

$$I_1(0,x) = \frac{1}{2} F_1(x), I_2(0,x) = \frac{1}{6} F_2(x)$$

(30)

where

$$F_1(x) = \frac{1}{(x-1)^3} \left( 1 - x^2 + 2x \ln x \right)$$

(31)

and

$$F_2(x) = \frac{1}{(x-1)^4} \left( -x^3 + 6x^2 - 3x - 2 - 6x \ln x \right).$$

(32)
4 The number of independent linear combinations of phases that enter $a_\mu$

Not all the phases that enter in the chargino, neutralino and smuon mass matrices are independent. We discuss here the set of independent phases that enter $a_\mu$. We consider the chargino contribution to $a_\mu$ first. Here the matrix elements of $U$ and $V$ as defined in the paragraph following Eq.(14) along with $\kappa_\mu$ as defined by Eq.(21) carry the phases $\xi_2$, $\theta_\mu$, $\chi_1$ and $\chi_2$. By introducing the transformation $M_C = B_R M_C^T B_L^\dagger$ and choosing $B_R = diag(e^{i\xi_2}, e^{-i\chi_1})$ and $B_L = diag(1, e^{i(\chi_2 + \xi_2)})$ we can rotate the phases so that $M_C'$ is given by

$$M_C' = \begin{pmatrix} |\tilde{m}_2| & \sqrt{2m_W} \sin \beta \\ \sqrt{2m_W} \cos \beta & |\mu| e^{i(\theta_\mu + \xi_2 + \chi_1 + \chi_2)} \end{pmatrix}$$ (33)

The matrix $M_C'$ can be diagonalized by the biunitary transformation $U_R^T M_C' U_L = diag (\tilde{m}_{1_1}, \tilde{m}_{3_3})$. It is clear that the matrix elements of $U_L$ and $U_R$ are functions only of the combination $\theta = \theta_\mu + \xi_2 + \chi_1 + \chi_2$. We also have $U^* M_C V^{-1} = diag(\tilde{m}_{1_1}, \tilde{m}_{3_3})$ where $U = (B_R U_R)^T$, and $V=(B_L U_L)^\dagger$. By inserting the new forms of $U$ and $V$ in the chargino contribution one finds (as shown in Appendix A) that $a_{\mu 21}$ and $a_{\mu 22}$ depend on only one combination, i.e., $\theta = \theta_\mu + \xi_2 + \chi_1 + \chi_2$.

Now we turn to the neutralino contribution, the phases that enter here are $\alpha_{A_\mu}$, $\xi_2$, $\chi_1$ and $\chi_2$ and they are carried by the matrix elements of $X$, $D_\mu$ and the phase of $\kappa_\mu$. Next we make the transformation $M_{\chi^0} = P_{\chi^0}^T M'_{\chi^0} P_{\chi^0}$ where

$$P_{\chi^0} = diag(e^{i\xi_1}, e^{i\xi_2}, e^{-i(\xi_2/2 + \chi_1)}, e^{-i(\xi_2/2 + \chi_2)})$$ (34)

After the transformation the matrix $M'_{\chi^0}$ takes the form

$$
\begin{pmatrix}
|\tilde{m}_1| & 0 & -M_z \sin \theta_W \cos \beta & M_z \sin \theta_W \sin \beta e^{-i\Delta \xi} \\
0 & |\tilde{m}_2| & M_z \sin \theta_W \cos \beta e^{i\Delta \xi} & -M_z \cos \theta_W \sin \beta \\
-M_z \sin \theta_W \cos \beta & M_z \cos \theta_W \cos \beta e^{i\Delta \xi} & 0 & -|\mu| e^{i\varphi} \\
M_z \sin \theta_W \sin \beta e^{-i\Delta \xi} & -M_z \cos \theta_W \sin \beta & -|\mu| e^{i\varphi} & 0
\end{pmatrix}.
$$ (35)

where $\theta' = \xi_1 + \xi_2 + \theta_\mu + \chi_1 + \chi_2$, and $\Delta \xi = (\xi_1 - \xi_2)$. The matrix $M'_{\chi^0}$ can be diagonalized by the transformation $Y^T M'_{\chi^0} Y = diag(\tilde{m}_{1_1}, \tilde{m}_{2_2}, \tilde{m}_{3_3}, \tilde{m}_{4_4})$ where $Y$ is a function only of $\theta'$ and $\Delta \xi/2$. Thus the complex non hermitian and symmetric matrix $M'_{\chi^0}$ can be diagonalized using a unitary matrix $X = P_{\chi^0}^T Y$ such that $X^T M_{\chi^0} X = diag(\tilde{m}_{1_1}, \tilde{m}_{2_2}, \tilde{m}_{3_3}, \tilde{m}_{4_4})$. As shown in Appendix A by applying the above transformations to each term of $\eta_{\mu j}^k$ and $X_{\mu j}^k$ one finds that the phase
combinations that enter here are $\theta', \Delta \xi/2$ and $\alpha_{A\mu} + \theta_{\mu} + \chi_1 + \chi_2$ from which we can construct the three combinations $\xi_1 + \theta_{\mu} + \chi_1 + \chi_2$, $\xi_2 + \theta_{\mu} + \chi_1 + \chi_2$ and $\alpha_{A\mu} + \theta_{\mu} + \chi_1 + \chi_2$.

By defining $\theta_1 = \theta_{\mu} + \chi_1 + \chi_2$ one finds that the $a_{\mu}$ dependence on phases from both the chargino and the neutralino exchanges consists only of the three combinations $\alpha_{A\mu} + \theta_1$, $\xi_1 + \theta_1$ and $\xi_2 + \theta_1$. One may compare these combinations with those that appear in the supersymmetric contribution to the electron and the neutron EDMs. In the analysis of Ref. \cite{10} we found that the electron and the neutron EDMs depend on the following combinations: $\xi_i + \theta_1$, $i = 1, 2, 3$ and $\alpha_{A_k} + \theta_1$ with $k = u, d, t, b, c, s; l$. We note that even though $a_{\mu}$ and the EDMs are very different physical quantities the linear combination of phases that enter in them are similar. In fact the phases that enter $a_{\mu}$ are a subset of phases that enter in the supersymmetric contributions to the EDMs.

5 The Supersymmetric Limit

To check the absolute normalization of our results we discuss now their supersymmetric limit. In Ref. \cite{43} the supersymmetric contributions to $a_{\mu}$ from the chargino sector in the supersymmetric limit were computed by going to the limit such that

$$U^* M_C V^{-1} = \text{diag}(M_W, M_W) \quad (36)$$

In this limit it was shown that the contributions from this sector was precisely negative of the contribution from the W exchange\cite{37}. The analysis of Ref. \cite{43} was carried out in the framework of mSUGRA with two CP violating phases. For the MSSM case being discussed here with many CP phases the structure of $U$ and $V$ matrices in susy limit will be modified so that

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{-i\chi_1} \\ -1 & e^{-i\chi_1} \end{pmatrix}, \quad V = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{-i\chi_2} \\ 1 & -e^{-i\chi_2} \end{pmatrix} \quad (37)$$

However, taking the phases $\chi_1$ and $\chi_2$ into account we find exactly the same result as in Ref. \cite{43} due to the appearance of the $\kappa_{\mu}$ phase. Thus the sum of the W exchange contribution and of the chargino exchange contributions cancel in the supersymmetric limit. Similarly it was shown in Ref. \cite{43} by making a unitary transformation that the neutralino mass matrix in the supersymmetric limit can be written in the form
\[ X^T M_{\chi} X = \text{diag}(0, 0, M_Z, M_Z) \] (38)

where the eigen-values are positive definite. It was then shown that the last two eigen-modes give a contribution which is negative of the contribution from the Z exchange in the Standard Model. In the case of MSSM we are discussing here the structure of the diagonalizing matrix \( X \) is now changed because of the \( \chi_1 \) and \( \chi_2 \) phases (see Appendix B). However, the final result we arrived at in Ref.\[43\] still holds due to the appearance of the \( \kappa_\mu \) phase once again. Thus the sum of the Z exchange and of the heavy neutralino exchanges exactly cancel in the supersymmetric limit.

We now turn to the supersymmetric limit of the contribution from the first two massless eigen states of the neutralino mass matrix. A direct sum over the first two eigen-modes for the case of Eq.(38) in the supersymmetric limit gives

\[ a_{\mu}^{\text{susy}}(\text{zero - modes}) = -\frac{\alpha_{em}}{2\pi} \] (39)

Thus the sum of the Standard Model contributions to \( a_\mu \) from the photon at one loop and form the Z and the W exchanges at one loop\[37\] is cancelled by the supersymmetric contributions from the neutralino and the chargino exchanges in MSSM at one loop in the supersymmetric limit, i.e., in the supersymmetric limit one has

\[ a_{\mu}^{\text{MSSM}} = 0 \] (40)

This result is consistent with the expectation on general grounds\[53, 54\]. The details of the derivation of Eq.(39) are given in Appendix B.

### 6 Satisfaction of EDM Constraints

Before proceeding to discuss the CP effects on \( a_\mu \) we describe briefly the EDM constraints on the CP violating phases. As is well known for the case of the neutron EDM there are three operators that contribute to the neutron EDM, namely, the electric dipole moment operator, the color dipole moment operator and the purely gluonic dimension six operator. Both the electric and color operators have three components each from the chargino, neutralino and gluino contributions. For the electron case we have only the electric dipole moment operator which has only two components, the chargino and the neutralino ones. Recently, it has been pointed
out that in addition to the above contributions certain two loop graphs may also contribute significantly in some regions of the parameter space. In our analysis here we include the effects of these contributions as well. However, the effect of these terms is found to be generally very small compared to the other contributions in most of the parameter space we consider. Satisfaction of the EDM constraints can be achieved in a straightforward fashion using the cancellation mechanism.

7 Analysis of CP Violating Effects

In the above we have given the most general analysis of $a_\mu$ within the framework of MSSM with inclusion of CP violating phases. Our results limit to those of Refs.\cite{39,40} in the limit when CP violating effects vanish (see Appendix C for details). For the case of the general analysis with phases in MSSM the number of parameters that enter $a_\mu$ along with the number of parameters that enter the EDM constraints which must be imposed on the CP violating phases is large. For the purpose of a numerical study of the CP violating effects on $a_\mu$ we shall confine ourselves to a more constrained set. Here we shall generate the masses of the sparticles at low energy starting with parameters at the GUT scale and evolve these downwards using renormalization group equations. At the GUT scale we shall use the parameters $m_0$, $m_{1/2}$, and $A_0$, and $\mu$ will be determined via radiative breaking of the electro-weak symmetry. We set $\chi_1 + \chi_2 = 0$ and choose the phases that we vary to consist of $\theta_\mu$, $\alpha_{A_0}$, and $\xi_i$ ($i = 1, 2, 3$). The choice of the above constrained set is simply for the purpose of reducing the number of parameters for the numerical study.

We begin our discussion of the numerical results by exhibiting the dependence of $a_\mu$ on the CP violating phases but without the imposition of the EDM constraints. The dependence of $a_\mu$ on $\theta_\mu$ and $\alpha_{A_0}$ was already studied in Ref.\cite{13} and we confine ourselves here to the dependence of $a_\mu$ on $\xi_1$ and $\xi_2$. In Fig.2 we exhibit the dependence of $a_\mu$ on $\xi_1$ and in Fig.3 the dependence of $a_\mu$ on $\xi_2$. From Figs.2 and 3 we find that $a_\mu$ is significantly affected by the dependence of both $\xi_1$ and $\xi_2$. However, a comparison of Fig.2 and Fig.3 shows that the dependence of $a_\mu$ on $\xi_2$ is much stronger than on $\xi_1$. The reason behind this difference is easily understood. The relatively weaker dependence on the $\xi_1$ phase arises because this phase appears only in the neutralino contribution while the $\xi_2$ phase appears both in the neutralino and in the chargino contributions to $a_\mu$.

We discuss now the effects of CP violating phases on $a_\mu$ under the EDM con-
straints. In Table 1 we show four points which lie on the curves of Fig.2 and Fig.3. As one can see the SUSY mass parameters \( m_0 \) and \( m_{1/2} \) are relatively small (i.e., \( m_0, m_{1/2} \ll 1 \text{ TeV} \)), the CP phases are large and there is compatibility with experimental constraints on the electron EDM and on the neutron EDM as a consequence of the cancellation mechanism. One also finds on comparing \( a_\mu \) with and without phases that the effects of CP violating phases on \( a_\mu \) are very significant.

Table 1:

|    | \( \theta_\mu \) | \( \alpha_{A_0} \) | \( d_h(10^{-26} \text{em}) \) | \( d_e(10^{-27} \text{em}) \) | \( a_\mu(\text{phases})(10^{-9}) \) | \( a_\mu(0)(10^{-9}) \) |
|----|-----------------|------------------|-----------------|-----------------|-------------------|-------------------|
| (1) | 2.35            | .4               | -3.08           | -0.86           | -4.8              | 7.45              |
| (2) | 1.98            | 0.4              | -0.34           | -1.67           | -7.8              | 11.7              |
| (3) | 1.2             | -1.5             | 1.87            | 2.24            | -3.25             | 5.6               |
| (4) | 2.7             | -0.4             | 1.87            | -0.03           | -15.5             | 3.15              |

Table caption: Parameters other than those exhibited corresponding to the cases (1)-(4) are: (1) \( m_0=70, m_{1/2}=99, \tan\beta=3, |A_0|=5.6, \xi_1=-1, \xi_2=1.5, \xi_3=0.62; (2) m_0=80, m_{1/2}=99, \tan\beta=5, |A_0|=5.5, \xi_1=-0.8, \xi_2=1.5, \xi_3=0.95; (3) m_0=75, m_{1/2}=132, \tan\beta=4, |A_0|=6.6, \xi_1=-1, \xi_2=1.78, \xi_3=2.74; (4) m_0=70, m_{1/2}=99, \tan\beta=6, |A_0|=3.2, \xi_1=0.63, \xi_2=0.41, \xi_3=0.47, where all masses are in GeV units and all phases are in rad.

In Fig.4 we exhibit \( a_\mu \) as a function of \( m_{1/2} \) where all points on these trajectories satisfy the experimental constraints on the electron EDM and on the neutron EDM by cancellation. One finds that the magnitude of the supersymmetric electro-weak contributions are comparable to and even larger than the Standard Model electro-weak contribution as given by Eq.(3)\[38\].

8 Conclusions

We have given in this paper a complete one loop analysis of the effects of CP violating phases on \( a_\mu \) with the most general set of allowed phases in MSSM in this sector. We have checked the absolute normalization of our results exhibiting the complete cancellation of the supersymmetric result in the supersymmetric limit with the Standard Model result including the qed one loop correction to \( a_\mu \), i.e., \( \alpha_{em}/2\pi \). A detailed numerical analysis of the CP violating effects on \( a_\mu \) for the regions which satisfy the EDM constraints is also given. Computations of \( a_\mu \) under the EDM constraints shows that the supersymmetric electro-weak effects can generate significant contributions to \( a_\mu \) even with moderate values of \( \tan\beta \).
\[ i.e., \tan \beta \sim 3 - 6, \text{ which can be comparable to the Standard Model electro-weak correction. Thus supersymmetric CP effects on } a_{\mu} \text{ are within the realm of observability in the new Brookhaven } g_{\mu} - 2 \text{ experiment.} \]

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**Appendix A:**

In this appendix we give the explicit derivation of the linear combinations of phases on which \( a_{\mu} \) depends. For the chargino contributions the phases are contained in the quantities \( \text{Re}(\kappa_{\mu}U_{i2}^{*}V_{i1}^{*}) \) and \( |\kappa_{\mu}U_{i2}^{*}|^2 + |V_{i1}|^2 \). By using \( U = (B_{R}U_{R})^{T} \) and \( V = (B_{L}U_{L})^{+} \) as defined in Sec.4, where \( U_{L} \) and \( U_{R} \) are functions of the combination \( \theta = \theta_{\mu} + \xi_{2} + \chi_{1} + \chi_{2} \), one finds

\[ U_{i2}^{*} = e^{i\chi_{1}}U_{R2i}^{*} \]  
(41)

and

\[ V_{i1}^{*} = U_{L1i} \]  
(42)

which leads to

\[ \kappa_{\mu}U_{i2}^{*}V_{i1}^{*} = |\kappa_{\mu}|U_{R2i}^{*}U_{L1i} \]  
(43)

and

\[ |\kappa_{\mu}U_{i2}^{*}|^2 + |V_{i1}|^2 = |\kappa_{\mu}|^2|U_{R2i}^{*}|^2 + |U_{L1i}|^2 \]  
(44)

Eqs.(43) and (44) show that the chargino contribution to \( a_{\mu} \) depends only on one combination of phases, i.e., \( \theta = \theta_{\mu} + \xi_{2} + \chi_{1} + \chi_{2} \).

For the case of the neutralino contribution to \( a_{\mu} \), the phases are contained in the quantities \( \text{Re}(\eta_{ij}^{k}) \) and \( X_{ij}^{k} \). We first consider the quantity \( \text{Re}(\eta_{ij}^{k}) \). It consists of six terms in the product

\[ \eta_{ij}^{k} = ([aX_{1j} + bX_{2j}]D_{1k}^{*} - \kappa_{\mu}X_{3j}D_{2k}^{*} \) \]

\[ (cX_{1j}D_{2k} + \kappa_{\mu}X_{3j}D_{1k}) \]  
(45)

where \( a, b \) and \( c \) are real numbers and independent of phases. The first term in the expansion of Eq.(45) is

\[ acX_{1j}^{2}D_{1k}^{*}D_{2k} = \pm acX_{1j}^{2} \cos \theta_{f} \sin \theta_{f} e^{i\beta_{f}} \]  
(46)

where \( +(-) \) sign is for \( k = 1(2) \) and where the following definitions are used

\[ \tan 2\theta_{f} = \frac{2m_{\mu}[|m_{0}A_{\mu}|^2 + |\mu R_{\mu}|^2 - 2|m_{0}A_{\mu}\mu R_{\mu}| \cos \alpha]^{1/2}}{M_{\mu11}^{2} - M_{\mu22}^{2}} \]  
(47)
Here \( R_\mu = \tan \beta e^{i(\chi_1 + \chi_2)} \) and \( \alpha = \alpha_{A_\mu} + \theta_\mu + \chi_1 + \chi_2 \). The phase \( \beta_f \) is defined such that
\[
\cos \beta_f = \frac{A}{|A^2 + B^2|^{1/2}}
\]
and
\[
\sin \beta_f = \frac{B}{|A^2 + B^2|^{1/2}}
\]
where \( A \) is defined by
\[
A = |m_0 A_\mu| \cos \alpha_{A_\mu} - |\mu R_\mu^*| \cos(\theta_\mu + \chi_1 + \chi_2)
\]
and \( B \) is defined by
\[
B = |m_0 A_\mu| \sin \alpha_{A_\mu} + |\mu R_\mu^*| \sin(\theta_\mu + \chi_1 + \chi_2)
\]
By using \( X_{ij} = Y_{ij} e^{-i\xi_1/2} \) where \( Y_{ij} \) are functions only of \( \theta' \) and \( \Delta \xi/2 \), one can write the first term as given by Eq.(46) as follows
\[
ac X_{ij}^2 D_{1k}^* D_{2k} = ac Y_{ij}^2 f_k(\alpha) e^{-i(\xi_1 - \beta_f)}
\]
where \( f_k(\alpha) \) are real functions of \( \alpha \). By using the definition of \( \beta_f \) as given by Eqs.(48) and (49) and by taking the real part of Eq.(52) we find that the right hand side of Eq.(52) contains the three combinations \( \theta' \), \( \Delta \xi/2 \) and \( \alpha \) which come from the first part of the right hand side of Eq.(52) and in addition it contains the following two combinations: \( \alpha_{A_\mu} - \xi_1 \) and \( \theta_\mu + \chi_1 + \chi_2 + \xi_1 \) which come from the exponent. But the latter two combinations are linear combinations of the first three combinations. Thus the left hand side of Eq.(46) or Eq.(52) will depend only on the combinations \( \theta' \), \( \Delta \xi/2 \) and \( \alpha \). The same analysis can be applied to the other five terms and each one of them will give us the same three linear combinations.

Next we consider \( X_{ij}^k \). It consists of six terms and the quantities in them which contain phases are \( |X_{3j}|^2, |X_{1j}|^2(|D_{1k}|^2 + 4|D_{2k}|^2), |X_{2j}|^2|D_{1k}|^2, |D_{1k}|^2 Re(X_{1j}X_{2j}^*), Re(e^{-i\chi_1}X_{3j}X_{1j}^* D_{1k} D_{2k}^*) \) and \( Re(e^{-i\chi_1}X_{3j}X_{2j}^* D_{1k} D_{2k}^*) \). The first one of them, i.e. \( |X_{3j}|^2 \), can be written in terms of the \( Y \) matrix as \( |Y_{3j}|^2 \) which depends only on the two combinations \( \theta' \) and \( \Delta \xi/2 \). The second expression can be written as \( |Y_{1j}|^2 g_k(\alpha) \) where \( g_k(\alpha) \) are real functions of \( \alpha \) as defined after Eq.(47). So this term will depend on the three combinations \( \theta' \), \( \Delta \xi/2 \) and \( \alpha \). The third expression is similar to the second one and will give the same combinations. The fourth expression can be written as \( h_k(\alpha) Re(Y_{1j}Y_{2j}^*) \) where \( h_k(\alpha) \) are real functions of \( \alpha \).
So this term also depends on the same three combinations. The fifth expression can be written as

\[ \text{Re}(e^{-i\chi_1 X_{3j}^* D_{1k} D_{2k}^*}) = \text{Re}(Y_{3j}^* Y_{1j}^* s_k(\alpha) e^{i(\xi_1 - \beta_j)}) \] (53)

where \( s_k(\alpha) \) are real functions of \( \alpha \). By treating the exponential term as we did in the first term of \( \eta^k_{\mu j} \), it will give us two extra combinations besides the usual three. These are \( \alpha A - \xi_1 \) and \( \alpha + \chi_1 + \chi_2 + \xi_1 \) which, however, are linear combinations of the usual three. Thus we end up here with the same three combinations. The sixth expression can be written as

\[ \text{Re}(e^{-i\chi_1 X_{3j}^* X_{2j}^* D_{1k} D_{2k}^*}) = \text{Re}(Y_{3j}^* Y_{2j}^* s_k(\alpha) e^{i(\xi_1 + \xi_2 - \beta_j)}) \] (54)

from which we can identify the usual three combinations and in addition one has the following combinations in the exponent: \( \alpha A, -\xi_1 \) and \( \alpha + \chi_1 + \chi_2 + \xi_1 \). These again are linear combinations of the usual three and thus we end up with only three phases in the neutralino contribution, i.e., \( \theta', \Delta \xi/2 \) and \( \alpha \).

**Appendix B**

In this appendix we discuss the supersymmetric limit in the massless sector. For this purpose we begin by exhibiting the unitary matrix \( X \) that diagonalizes the neutralino mass matrix in the supersymmetric limit such that the eigenvalues are arranged so that

\[ X^T M_{\chi^0} X = \text{diag}(0, 0, M_Z, M_Z) \] (55)

With the above ordering the unitary matrix \( X \) takes on the form

\[
\begin{pmatrix}
\alpha & \beta & \frac{\sin \theta_W}{\sqrt{2}} & \frac{i \sin \theta_W}{\sqrt{2}} \\
\alpha \tan \theta_W & \beta \tan \theta_W & -\cos \theta_W & -i \cos \theta_W \\
\alpha e^{i\chi_1} & \beta \sec^2 \theta_W e^{i\chi_1} & -\frac{1}{2} e^{i\chi_1} & -\frac{i}{2} e^{i\chi_1} \\
\alpha e^{i\chi_2} & \frac{1}{2} \beta \sec^2 \theta_W e^{i\chi_2} & \frac{1}{2} e^{i\chi_2} & -\frac{i}{2} e^{i\chi_2}
\end{pmatrix} \] (56)

where

\[
\alpha = \frac{1}{\sqrt{3 + \tan^2 \theta_W}}, \quad \beta = \frac{1}{\sqrt{1 + \tan^2 \theta_W + \frac{1}{2} \sec^2 \theta_W}}
\] (57)

Using these results it is easily seen that the sum over the first two neutralino mass eigenvalues gives

\[
a_{11}^{\text{zero}}(\text{modes}) = -\frac{\alpha_{EM}}{2\pi \sin^2 \theta_W} H \sum_{j=1}^{2} \sum_{k=1}^{2} \text{Re}(\eta_{\mu j}^k) \left( \frac{M_{\chi^0}}{m_\mu} \right) \to 0 \] (58)
where we set $M_{\tilde{\mu}k} = m_{\mu}$ and the factor $H$ is defined by

$$H = \int_0^1 dx \int_0^{1-x} dz \frac{z}{(z-1)^2}$$

Thus in the supersymmetric limit the entire supersymmetric contribution to $a_{\mu}^1$ from the masseless neutralino states comes from $a_{\mu}^{12}$. To compute this contribution we need the sum

$$a_{\mu}^{12} = \frac{m_{\mu}^2 \alpha_{EM}}{4\pi \sin^2 \theta_W} \sum_{j=1}^{2} \sum_{k=1}^{2} \frac{1}{M_{\chi_j^0}^2} X_{\mu j}^k I_2 \left( \frac{m_{\mu}^2}{M_{\chi_j^0}^2}, \frac{M_{\tilde{\mu}k}^2}{M_{\chi_j^0}^2} \right)$$

where the sum over $j$ runs only over the first two modes. In the supersymmetric limit we set $M_{\mu k}^2 = m_{\mu}^2$, $M_{\chi_j^0} \to 0$ ($j=1,2$), and $x_{\mu j} = \frac{m_{\mu}^2}{M_{\chi_j^0}^2} \to \infty$ ($j=1,2$) and

$$\frac{m_{\mu}^2}{M_{\chi_j^0}^2} I_2(x_{\mu j}, x_{\mu j}) \to -\frac{1}{2}$$

Now substitution of the explicit form of the $X$ matrix gives

$$\sum_{j=1}^{2} \sum_{k=1}^{2} X_{\mu j}^k = 4 \sin^2 \theta_W$$

Use of Eqs. (61) and (62) in Eq.(60) gives

$$a_{\mu}^{12} \text{(zero modes)} = -\frac{\alpha_{em}}{2\pi}$$

Thus we find that in the supersymmetric limit the exchange of two massless neutralinos gives a one loop contribution to $a_{\mu}$ which is exactly negative of the photonic one loop contribution. Thus in the supersymmetric limit the sum of the one loop contributions of the zero modes of the theory cancel. The cancellation provides an absolute check on the normalization of our supersymmetric result in this sector.

**Appendix C**

In this section we consider the limit of vanishing CP violating phases and compare our results with those of previous works. We first compare our results with those of Ref.[39]. We consider the chargino contribution first. Using Eq.(2.8) of Ref.[39] and noting that the free part of the Lagrangian density for the complex scalar fields in that work is given by

$$\frac{1}{2} (\partial_{\mu} z^* \partial^{\mu} z - m^2 z^* z),$$

we find that our $K_i$ and $L_i$ are related to the $A_L^\pm$ and $A_R^\pm$ of Ref.[39] as follows:

$$K_{1,2\nu} \to -i\sqrt{2} A_L^{+\nu}, \quad L_{1,2\nu} \to -i\sqrt{2} A_R^{+\nu}$$

$$15$$
Further, our form factors $F_3(x)$ and $F_4(x)$ are related to the form factors $F_1$ and $F_2$ of Ref. [39] as follows:

$$F_3(x) = -F_2(x), \quad F_4(x) = F_1(x)$$

(65)

Defining

$$g_1^\tilde{W} = 2a_{22}^\mu, \quad g_2^\tilde{W} = 2a_{21}^\mu$$

(66)

we find that our Eq.(12) in the limit of vanishing CP violating phases is given in the notation of Ref.[39] by

$$g_1^\tilde{W} = \frac{m_\mu^2}{24\pi^2} \sum_{a=1,2} \frac{A_R^{(a)} + A_L^{(a)}}{\tilde{m}_a^2} F_1(x_a)$$

(67)

and similarly our Eq.(10) in the same limit in the notation of Ref.[39] is given by

$$g_2^\tilde{W} = \frac{m_\mu^2}{4\pi^2} \sum_{a=1,2} \frac{A_R^{(a)} A_L^{(a)}}{\tilde{m}_a} F_2(x_a)$$

(68)

where

$$x_a = \frac{\tilde{m}_a^2}{\tilde{m}_a^2}, \quad a = 1, 2$$

(69)

Eqs. (67) and (68) agree precisely with Eqs.(2.6a) and (2.6b) of Ref. [39] to leading order in $\mu^2/\tilde{m}_a^2$ taking account of the typo in Eq.(2.6a) where $A_R^{(a)}$ should read $A_R^{(a)}$ and noting that $A_L^{2+,−}$ is proportional to $m_\mu^2/M_W^2$ and thus does not contribute to leading order.

We consider next the neutralino contribution. From the interaction Lagrangian Eq.(2.4) of Ref. [39] we find the transition from our notation to that of Ref.[39] as follows:

$$K_{kr} \rightarrow -\sqrt{2}i(O'_{1r}B_k^L - C_kO'_{2r}), \quad L_{kr} \rightarrow -\sqrt{2}i(O'_{2r}B_k^R + C_kO'_{1r})$$

(70)

where we identify $O'$ to be

$$O' = \begin{pmatrix} \cos \delta & \sin \delta \\ -\sin \delta & \cos \delta \end{pmatrix}$$

(71)

Noting that our form factors $F_1(x)$ and $F_2(x)$ are related to the form factors $G_2(x)$ and $G_1(x)$ of Ref.[39] by

$$F_1(x) = -G_2(x), \quad F_2(x) = -2G_1(x)$$

(72)

and defining $g_1^{\tilde{Z}} = 2a_{12}^\mu$ and $g_2^{\tilde{Z}} = 2a_{11}^\mu$ we find that our Eq.(8) gives precisely Eq.(2.10) of Ref.[39] taking account of the typos in Eq.(2.10a) in that $1/\tilde{\mu}_k$ should
read $1/\tilde{\mu}^2_1$ and $G_2(x_{2k})$ in the same equation should read $G_1(x_{2k})$. Further, our Eq.(6) agrees precisely with Eq.(2.12) of Ref.[39].

Next we compare our results with those of Ref.[40]. For this purpose in the chargino sector we identify $\tilde{W}_1$ and $\tilde{W}_2$ states with the states $\tilde{W}^-$ and $\tilde{H}^-$ of Ref.[40] in order to use Table 1 of Ref.[40]. With this identification in the limit of vanishing CP violating phases we find that in the chargino sector our matrices $V$ and $U$ are real and orthogonal and are related to the matrices $O_1$ and $O_2$ of Ref.[40] as follows

\[ V^*_{km} \rightarrow O_{1mk}, \quad U^*_{km} \rightarrow O_{2mk} \quad (73) \]

The analysis of Ref.[40] computes only the contribution $a_{\mu}^{21}$ of $a_{\mu}^{\tilde{\chi}^-}$ in their Eq.(5). Relating our $F_3(\eta)$ to their $F_{sv}(\eta)$ by $F_3(\eta) = -2F_{sv}(x)$, we find that our Eq.(19) can be written in the form

\[ 2a_{\mu}^{21} = -\frac{m_\mu}{4\pi^2\sin^2\theta_W} \sum_k \frac{m_\mu}{\sqrt{2}M_k m_W \cos\beta} O_{22k} O^T_{1k1} F_{sv}(\eta_{\nu k}) \quad (74) \]

which is exactly Eq.(5) of Ref.[40] on relating their $\sin \theta_H$ to our $\cos \beta$ by $\sin \theta_H = \cos \beta$. The consistency of the analysis of Ref.[40] with our analysis, however, requires that the sign of the terms with $M_W$ in the chargino mass matrix given by Eq.(3b) of Ref.[40] be reversed.

To compare our results to those of Ref.[40] in the neutralino sector we note that our $H_1^0$ and $H_2^0$ states are related to their $H^0$ and $H'^0$ states by $H_1^0 = H^0$ and $H_2^0 = H'^0$. In the limit of vanishing CP violating phases, our neutralino and smuon mass matrices become real, and the corresponding diagonalizing matrices $X$ and $D$ become orthogonal and can be identified with the real orthogonal matrices $O$ and $S$ of Ref.[40]:

\[ X \rightarrow O, \quad D \rightarrow S \quad (75) \]

The consistency of the analysis of Ref.[40] with our analysis, however, requires that the sign of the terms with $M_Z$ in the neutralino mass matrix given by Eq.(3a) of Ref.[40] be reversed. The analysis of Ref.[40] calculated only the part $a_{\mu}^{11}$ in their Eq.(6). To compare the result of $a_{\mu}^{11}$ of our Eq.(25) with their Eq.(6) we first note that our $F_1$ is related to their $F$ by $F_1(\eta) = -F(\eta)$. Second we need to identify the fields $\tilde{W}_i$ (i=1,2,3) in Eq.(6) of Ref.[40] in order to use Table 1 of Ref.[40] to write out in detail the interactions of Eq.(6). This identification is as follows:

\[ \tilde{W}_1 = \tilde{B}^0, \quad \tilde{W}_2 = \tilde{W}^0, \quad \tilde{W}_3 = \tilde{H}^0. \quad (76) \]
Further in Ref.[40] we identify \( L = 1 \) and \( R = 2 \) in their Eq.(6), and we need to complete their Table 1 since the term \( g(\mu_L H^0 s^R_\mu) \) is missing in Table 1 and one needs it to expand out Eq.(6). Here we find that the entry for the magnitude for this coupling in their Table 1 should be the same as the magnitude for the coupling \( g(\mu_R H^0 s^L_\mu) \) listed in Table 1 (see Eqs. (5.1) and (5.4) of Ref.[50]). Using the above correspondence we find that our result for \( 2a^{11}_\mu \) gotten from our Eq.(25) produces exactly Eq.(6) of Ref.[40] in the limit of vanishing CP phases.
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Figure 1: The one loop contribution to $g_\mu - 2$ from (a) neutralino exchange, and (b) chargino exchange diagrams.

Figure 2: Plot of $a_\mu^{SUSY}$ as a function of $\xi_1$ without the imposition of EDM constraints. The values of the other parameters for the curves (1)-(4) correspond the cases (1)-(4) in Table 1.
Figure 3: Plot of $a_\mu^{SUSY}$ as a function of $\xi_2$ without the imposition of EDM constraints. The values of the other parameters for the curves (1)-(4) correspond the cases (1)-(4) in Table 1.

Figure 4: Plot of $a_\mu^{SUSY}$ as a function of $m_{\chi}$, where all points on the trajectories satisfy the current experimental constraints on the neutron and on the electron EDM. The curves labelled (1)-(3) are drawn for the following set of data: (1): $|A_0|=6.5$, $\theta_\mu=2.92$, $\alpha_{A_0}=-0.4$, $\tan\beta = 4$, $\xi_1 = 0$, $\xi_2 = 0.2$, $\xi_3 = 0.065$; (2): $|A_0|=5.4$, $\theta_\mu=3.006$, $\alpha_{A_0}=-0.1$, $\tan\beta = 3.5$, $\xi_1 = 0.105$, $\xi_2 = 0.105$, $\xi_3 = 0.15$; (3): $|A_0|=2.9$, $\theta_\mu=3.02$, $\alpha_{A_0}=-0.5$, $\tan\beta = 2.6$, $\xi_1 = 0.19$, $\xi_2 = 0.19$, $\xi_3 = 0.41$. For all cases $50 < m_0 < 250$ (GeV) and all phases are in rad.