Atom-to-molecule conversion efficiency and adiabatic fidelity

Li-Hua Lu and You-Quan Li
Zhejiang Institute of Modern Physics and Department of Physics,
Zhejiang University, Hangzhou 310027, P. R. China
(Received 6 December 2007)

The efficiency of converting two-species fermionic atoms into bosonic molecules is investigated in terms of mean-field Lagrangian density. We find that the STIRAP technique aided by Feshbach resonance is more effective than the bare Feshbach resonance for $^6\text{Li}$ atoms rather than $^{40}\text{K}$ atoms. We also make general consideration on the symmetry and its relevant conservation law, which enable us to introduce a natural definition of adiabatic fidelity for CPT state. The calculated values of the fidelity then provide an interpretation on why the conversion efficiencies for $^{40}\text{K}$ and $^6\text{Li}$ are distinctly different.

PACS numbers: 03.75.Mn, 03.75.Nt, 03.75.Ss

I. INTRODUCTION

There has been much attention to the molecular Bose-Einstein condensate (BEC) which is a versatile playground not only for cold atomic physics experiments but also for other research areas, such as condensed matter theory as well as quantum field theory. It is no longer a pure Bose system as the molecular BEC can be a system of Bose-Fermi mixtures. This makes the theoretical structure rich and colorful. To realize the molecular BEC, one must create stable molecules with long lifetime.

In the recent experiments [1, 2, 3], the technique of Feshbach resonance plays an important role in the creation of molecules. Since the molecules created through such a technique usually suffer from fast decay due to the vibrational excitation, the stimulated Raman adiabatic passage (STIRAP) in photoassociation [4] has been regarded as an effective approach to create ground-state molecules. The success of STIRAP technique requires the existence of a coherent population trapping (CPT) state which can be followed adiabatically [5]. Such a condition can be fulfilled for linear $\Lambda$ systems by appropriately choosing laser frequencies. However, for the system with inter-particle interactions, the two-photon resonance condition dynamically changes when population is transferred from atomic states to molecular states. This makes the CPT state more difficult to be followed adiabatically. The adiabatic property was studied by means of adiabatic fidelity in a recent theoretical work [6] for a simplified model of monoatomic system without inter-particle interaction. It is worthwhile to appropriately define the adiabatic fidelity to study the adiabatic property for a more realistic system, such as the diatomic system which has been realized in several experiments [7, 8, 9, 10, 11, 12].

In this paper, we consider systems consisting of fermionic atoms in different hyperfine states and their compounded molecules coupled through the STIRAP technique aided by Feshbach resonance. We discuss systems of $^6\text{Li}$ and $^{40}\text{K}$ as concrete examples. In section II we model the systems with inter-particle interaction and derive the dynamical equations for them. We make a general consideration on the symmetry and the relevant conservation law, and then introduce the definition of fidelity for our system. In section III we look for solutions of the CPT state. In section IV, we calculate the conversion efficiency for $^{40}\text{K}$ atoms and $^6\text{Li}$ atoms, respectively, and discuss the corresponding features. In section V we study the relation between the atom-to-molecule conversion efficiency and the adiabatic fidelity for CPT state. We also study the effect of the decay of quasibound molecules and compare the difference between $^{40}\text{K}$ and $^6\text{Li}$ systems. Our results are briefly summarized in section VI.

II. MODEL AND ITS GENERAL PROPERTIES

We consider that two species of fermionic atoms are converted into stable molecules via the STIRAP technique aided by Feshbach resonance. Here we use $|a\rangle$ and $|b\rangle$ to stand for the states of the free atoms in the open channel, and $|m\rangle$ and $|g\rangle$ for the quasibound and ground molecular states in the close channel, respectively. A pair of fermionic atoms are coupled with the quasibound molecular state $|m\rangle$ through a Feshbach resonance, where the coupling strength is denoted by $\alpha'$ and the detuning by $\mathcal{E}'$. Additionally, the states $|m\rangle$ and $|g\rangle$ are coupled with each other through a laser field with the coupling strength $\mathcal{O}'$ and detuning $\Delta'$. Since the system is a Fermi-Bose mixture whose Hilbert space actually carries out the representation of a graded unitary group SU(2 | 2), it will be an arduous work to study the dynamics of the system by means of the Heisenberg equation of motion. Whereas, with the help of mean-field Langrange density [13], one can investigate the dynamics conveniently,

\[
L = \sum_i \left( \frac{i\hbar}{2} \left( \psi_i^* \frac{\partial \psi_i}{\partial t} - \psi_i \frac{\partial \psi_i^*}{\partial t} \right) - T_i \right) - \mathcal{E}'\psi_m^*\psi_m
- \alpha' (\psi_m^* \psi_b + \text{H.c.}) + \Omega' (\psi_m^* \psi_g + \text{H.c.}) - \Delta' \psi_g^* \psi_g
- \frac{1}{2} \sum_{i \neq j} \lambda_{ij} |\psi_i|^2 |\psi_j|^2 - \frac{1}{2} \left( \lambda_{mm} |\psi_m|^4 + \lambda_{gg} |\psi_g|^4 \right)
- \frac{3}{5} (A_{aa} |\psi_a|^{10/3} + A_{bb} |\psi_b|^{10/3}),
\]

(1)
where $T_i$ denotes the kinetic energy term of the $i$th component and $i, j = a, b, m, g$. Here the coefficients $\lambda'_{ij} = \frac{2\pi\hbar^2 a_{ij}/m_j}$ are the interaction strengths between particles with $m_{ij} = m_i m_j/(m_i + m_j)$ being the reduced mass and $a_{ij}$ the $s$-wave scattering length.

A. The conservation law

The Lagrangian \( \mathcal{L} \) is no more invariant under a simultaneous global phase transformation due to the presence of the atom-to-molecule conversion term. As the Lagrangian does not include the term flipping the two fermionic components into each other, their corresponding conjugations, we derive the following result, 

The system is invariant under the transformation Eq. (2) with

$$U(\partial_a, \partial_b) = \begin{pmatrix} e^{i\theta_a} 0 0 0 \\ 0 e^{i\theta_a} 0 0 \\ 0 0 e^{(\theta_a + \theta_b)} 0 \\ 0 0 0 e^{(\theta_a + \theta_b)} \end{pmatrix}.$$  

Following the formulism of Noether theorem, we evaluate the variation of the action caused by the infinitesimal phase transformation $U(\partial_a, \partial_b)$, 

$$\delta I = \int \left\{ \mathcal{L}(\psi + \delta \psi, \psi^* + \delta \psi, \partial_t \psi, \partial_t \psi^*) \right\} d(x)$$

For simplicity in the above expression, we omitted the subscripts of $\psi$ labelling different components, abbreviated $d(x)$ for $dt dx dy dz$ and $\partial_\mu$ for $\partial/\partial x_\mu$ with $\{ x_\mu \} = \{ t, x, y, z \}$. Because of, explicitly, $\delta \psi_a = i(\delta \partial_\mu) \psi_a, \delta \psi_b = i(\delta \partial_\mu) \psi_b, \delta \psi_m(\varphi) = i(\delta \partial_\mu) \psi_m(\varphi)$, and their complex conjugations, we derive the following result,

$$\delta I = \int \{ (\partial_\mu \psi^*_a) \delta \partial_\mu a + (\partial_\mu \psi^*_b) \delta \partial_\mu b \} d(x),$$  

where $J^\mu_A = J^\mu_a + J^\mu_m + J^\mu_g, J^\mu_B = J^\mu_b + J^\mu_m + J^\mu_g$ with

$$J^\mu_i = \psi_i \frac{\delta \mathcal{L}}{\delta(\partial_\mu \psi_i)} - \psi^*_i \frac{\delta \mathcal{L}}{\delta(\partial_\mu \psi^*_i)}, \quad i = a, b, m, g.$$  

The system is invariant under the transformation Eq. (2) such that $\delta I = 0$, which gives rise to two conserved currents, 

$$\partial_\mu J^\mu_A = 0, \quad \partial_\mu J^\mu_B = 0.$$  

In the present paper, we focus on uniform system ($\nabla \psi_i \approx 0$) and hence neglect the kinetic and trapping potential terms. Then the conservation law Eq. (5) reads 

$$\frac{d}{dt} \left( |\psi_a|^2 + |\psi_m|^2 + |\psi_g|^2 \right) = 0,$$  

which implies that 

$$|\psi_a|^2 + |\psi_m|^2 + |\psi_g|^2 = n_a,$$  

where the constants $n_a$ and $n_b$ are determined by $V$, the volume of the system, together with $N_a(0)$ and $N_b(0)$, the initial numbers of species $a$ and $b$, i.e., $n_a = N_a(0)/V, n_b = N_b(0)/V$. Here we assume there are no molecules at the initial time in the system.

To guarantee the compatibility with the constraints given by Eq. (6), we introduce two Lagrange multipliers $\mu_a$ and $\mu_b$ into the mean-field Lagrange density Eq. (7),

$$K = L + \mu_a |\psi_a|^2 + \mu_b |\psi_b|^2 + (\mu_a + \mu_b)(|\psi_m|^2 + |\psi_g|^2).$$  

Here the real parameters $\mu_a$ and $\mu_b$ can be identified as the chemical potentials of the corresponding components. Owning to Eq. (6), we can conveniently introduce new notations: $\phi_i = \psi_i/\sqrt{n_a + n_b}, \lambda_{ij} = \lambda_{ij}'(n_a + n_b)/\hbar, \alpha = \alpha'/\sqrt{n_a + n_b}/\hbar, \alpha_i = \alpha_i'(n_a + n_b)^{2/3}/\hbar, \Omega = \Omega'/\hbar, E = E'/\hbar, \Delta = \Delta'/\hbar$. Reexpressing equation (7) in terms of $\phi$'s and substituting it into the Euler-Lagrange equation, we obtain a set of equations

$$\frac{\partial \phi_a}{\partial t} = \sum_{i \neq a} \lambda_{ai} |\phi_i|^2 \phi_a + A_a |\phi_a|^{4/3} \phi_a + \alpha \phi_a^* \phi_m - \mu_a \phi_a,$$  

$$\frac{\partial \phi_b}{\partial t} = \sum_{i \neq b} \lambda_{bi} |\phi_i|^2 \phi_b + A_b |\phi_b|^{4/3} \phi_b + \alpha \phi_b^* \phi_m - \mu_b \phi_b,$$  

$$\nabla \phi_m = \sum_i \lambda_{mi} |\phi_i|^2 \phi_m + \alpha \phi_m, \quad \Omega \phi_m + \Delta \phi_m,$$  

where a phenomenological parameter $\gamma$ is introduced to characterize the decay of quasibound molecules. In terms of $\phi_i$'s, the conservation relations (3) turn to be

$$|\phi_a|^2 + |\phi_m|^2 + |\phi_g|^2 = (1 + \delta)/2,$$  

$$|\phi_b|^2 + |\phi_m|^2 + |\phi_g|^2 = (1 - \delta)/2,$$  

with $\delta = (n_a - n_b)/(n_a + n_b)$ characterizing the population imbalance between fermionic atoms in different states.

B. The definition of fidelity

Now we are in the position to introduce a proper definition of fidelity for our system. As our system is a four-component system of Bose-Fermi mixture which is related to a graded unitary group SU(2 | 2), we need to define the fidelity carefully as it must obey several basic properties. Equation (10) actually provides us the normalization condition

$$|\phi_a|^2 + |\phi_b|^2 + 2|\phi_m|^2 + 2|\phi_g|^2 = 1,$$

which can be expressed as the following form

$$\langle \phi | F^*(\phi) F(\phi) | \phi \rangle = 1,$$  

where $F(\phi)$ is the fidelity between the states $\phi$ and $\phi'$. The fidelity is a measure of how close two quantum states are to each other. It is a complex number that takes values in the interval $[-1, 1]$, with $|\phi | F^*(\phi) F(\phi) | \phi \rangle = 1$ indicating that the states are identical.
where \( \langle \phi | \) denotes \( (\phi'_a, \phi'_b, \phi'_m, \phi'_\gamma) \). One might think of a naive expression for the \( F \)-matrix, \( F = \text{diag}(1, 1, \sqrt{2}, \sqrt{2}) \). However, because the relation of Eq. (10) should be invariant under the transformation given in Eq. (2), just like that the conventional inner product in quantum mechanics is invariant under the \( U(1) \) transformation, the simplest correct expression of the \( F \)-matrix ought to be

\[
F(\phi) = \begin{pmatrix}
\phi_b / |\phi_b| & 0 & 0 & 0 \\
0 & \phi_a / |\phi_a| & 0 & 0 \\
0 & 0 & \sqrt{2} & 0 \\
0 & 0 & 0 & \sqrt{2}
\end{pmatrix}.
\] (11)

As a result, a nature definition of fidelity of a state labelled by \( \phi \) with that labelled by \( \phi' \) is given by

\[
f(\phi, \phi') = |\langle \phi | F^* (F' \phi') | \phi' \rangle|,
\] (12)

where the \( F \)-matrix was given in Eq. (11). Clearly, such a definition fulfills \( f(\phi, U(\varphi_a, \varphi_b) \phi') = f(U(\varphi_a, \varphi_b) \phi, \phi') = f(\phi, \phi') \), which means the phase transformation given in Eq. (2) does not vary the magnitude of fidelity; and the fidelity of a state with itself is always the unit \( f(\phi, \phi) = 1 \) which is just the normalization condition.

### III. COHERENT POPULATION TRAPPING STATES

Now we consider the stationary states where we neglect the decay of quasibound molecules \( (i.e., \gamma = 0) \). We know that the existence of stationary solutions of Eq. (8) requires the system satisfies the adiabatic approximation condition. Once the adiabatic approximation is valid, \( i.e., \partial \varphi_i / \partial t \approx 0 \), the time-evolution equations (8) become algebraic ones for \( \varphi \)'s, namely,

\[
\mu_a \varphi_a = \sum_{i \neq a} \lambda_{ai} |\varphi_i|^2 \varphi_a + A_a |\varphi_a|^{4/3} \varphi_a + \alpha \varphi'_b m,
\]

\[
\mu_b \varphi_b = \sum_{i \neq b} \lambda_{bi} |\varphi_i|^2 \varphi_b + A_b |\varphi_b|^{4/3} \varphi_b + \alpha \varphi'_a m,
\]

\[
(\mu_a + \mu_b) \varphi_m = \sum_i \lambda_{mi} |\varphi_i|^2 \varphi_m + \alpha \varphi_a \varphi_b - \Omega \varphi_g + \varepsilon \varphi_m,
\]

\[
(\mu_a + \mu_b) \varphi_g = \sum_i \lambda_{gi} |\varphi_i|^2 \varphi_g - \Omega \varphi_m + \Delta \varphi_g.
\] (13)

Although it is difficult to find the exact solutions of the above equations, one can easily obtain a set of steady state solutions for Eq. (13) by taking \( \varphi_m = 0 \). Such a state is called coherent population trapping (CPT) state which yields,

\[
|\varphi_0^0|^2 = \frac{(\delta - \Omega^2) + \sqrt{(\delta - \Omega^2)^2 + 2 \Omega^2 (1 + \delta)}}{2},
\]

\[
|\varphi_0^b|^2 = |\varphi_0^0|^2 - \delta, \quad |\varphi_0^g|^2 = \frac{1 + \delta}{2} - |\varphi_0^0|^2,
\]

\[
\mu_a = \lambda_{ab} |\varphi_0^0|^2 + \lambda_{ag} |\varphi_0^g|^2 + A_a |\varphi_0^a|^{4/3},
\]

\[
\mu_b = \lambda_{ab} |\varphi_0^0|^2 + \lambda_{bg} |\varphi_0^g|^2 + A_b |\varphi_0^b|^{4/3},
\] (14)

where \( \tilde{\Omega} = \Omega / \alpha \). The resonance condition corresponding to this solution is

\[
\Delta = (\lambda_{ab} - \lambda_{ag}) |\varphi_0^0|^2 + (\lambda_{ab} - \lambda_{bg}) |\varphi_0^g|^2 + (\lambda_{ag} + \lambda_{bg} - \lambda_{gg}) |\varphi_0^0|^2 + A_a |\varphi_0^a|^{4/3} + A_b |\varphi_0^b|^{4/3}.
\] (15)

We consider a laser pulse with \( \tilde{\Omega} \rightarrow \infty \) for \( t \rightarrow 0 \) and \( \tilde{\Omega} \rightarrow 0 \) for \( t \rightarrow \infty \). For such a laser field, at the initial time \( (t = 0) \), we have \( |\varphi_0^0|^2 = (1 + \delta)/2, |\varphi_0^b|^2 = 1 - \delta/2 \) and \( |\varphi_0^g|^2 = 0 \), which implies there are no molecules in the system at the initial time. At the final time \( (t \rightarrow \infty) \), \( |\varphi_0^0|^2 \rightarrow \delta, |\varphi_0^b|^2 \rightarrow 0 \) and \( |\varphi_0^g|^2 \rightarrow (1 - \delta)/2 \) for \( \delta > 0 \); whereas \( |\varphi_0^0|^2 \rightarrow 0, |\varphi_0^b|^2 \rightarrow -\delta \) and \( |\varphi_0^g|^2 \rightarrow (1 + \delta)/2 \) for \( \delta < 0 \). With the help of initial values of \( |\varphi_0^0|^2 \) and their asymptotic values at final time, it is easy to find that those fermionic atoms, in the presence of their counterparts, can be converted into molecules if the CPT states can be followed adiabatically. The residual atoms can not be converted into molecules due to the lack of counterpart atoms. After the numerical calculation in next section, we will go back to study whether the CPT state can be followed adiabatically with the help of the useful concept, adiabatic fidelity.

### IV. ON CONVERSION EFFICIENCIES

We know that there are two sorts of fermionic atoms \( ^6\text{Li} \) and \( ^{40}\text{K} \) in the group of alkali-metal atoms. They have been converted into molecules in experiments successfully through the bare Feshbach resonance \[1, 8, 9, 10\]. In those experiments, 60% to 80% of \( ^{40}\text{K} \) atoms and no more than 85% of \( ^6\text{Li} \) atoms can be converted into molecules. For \( ^6\text{Li} \) atoms, the atom-to-molecule conversion efficiency via bare Feshbach resonance is lower than that via STIRAP technique aided by Feshbach resonance, which is in contrast to the case for \( ^{40}\text{K} \) atoms. Due to the difference in atomic properties, the atom-to-molecule
conversion efficiency differs for different atoms even if the same technique is applied.

Now we evaluate the atom-to-molecule conversion efficiency, respectively, for $^6\text{Li}$ and $^{40}\text{K}$ atoms with concrete magnetic and laser fields. For these two sorts of atoms, we adopt the same time-dependent Rabi frequency,

$$\Omega(t) = \Omega_{\text{max}} \left[ 1 - \tanh \left( \frac{t - t_0}{\tau} \right) \right], \quad (16)$$

where the parameters $\Omega_{\text{max}}$, $t_0$ and $\tau$ are determined by the applied laser field that couples the two molecular states. In the numerical calculation, the detuning strength $\Delta$ is given by Eq. (15). We assume there are no molecules in the system at the initial time i.e., $\phi_{m, g} = 0$ at $t = 0$.

For $^{40}\text{K}$ atoms, we know that the Feshbach resonance occurs at a magnetic field strength of 202.1G and the resonance width is about 7.8G [12]. Then we can get the atom-to-molecule coupling strength $\alpha' = 16.6 \times 10^{-39} \text{J}$ according to Ref. [13]. Here we choose that the magnetic field is 201.7G and particle density $n_a + n_b$ is about $10^{20} \text{m}^{-3}$. It is easy to obtain $A_a = 0.16\alpha$, $\lambda_{ab} = 0.24\alpha$ and $\mathcal{E} = -4.4\alpha$. The time evolution of the corresponding population can be obtained by solving Eq. (8). We plot the numerical results in Fig. 1. From this figure, we can see that the conversion efficiency $2|\phi_g(t = \infty)|^2$ for $^{40}\text{K}$ atoms is less than 60%, lower than that via the technique of bare Feshbach resonance [10]. The low conversion efficiency implies that the CPT state can not be followed adiabatically, which will be confirmed confidently by evaluating the adiabatic fidelity in next section. Comparing the two panels in Fig. 1, we can know that the influence of $\gamma$ on the atom-to-molecule conversion efficiency is very small. This is due to that $|\phi_{m}|^2$ is close to zero at any time in contrast to the case for $^6\text{Li}$ atoms.

![FIG. 1: (color online) The time dependence of the population of particles for $^{40}\text{K}$ system for different $\gamma$. The parameters are $\delta = 0$, $\lambda_{ab} = 0.24$, $A_a = A_b = 0.16$, $\mathcal{E} = -4.4$, $\Omega_{\text{max}} = 200$, $t_0 = 120$, $\tau = 40$, and the other parameters are zero. Time is in unit of $1/\alpha$ and all other coefficients are in units of $\alpha$, where $\alpha = 16.6 \times 10^{-39} \text{J}$.](image)

For $^6\text{Li}$ atoms, the Feshbach resonance occurs at two distinct strengths of magnetic field (the called narrow and broaden Feshbach resonance respectively). In our calculation, we focus on the narrow Feshbach resonance for which the atom-to-molecule coupling strength $\alpha'$ is about $3.29 \times 10^{-27} \text{J}$. If the particle density $n_a + n_b$ is about $10^{20} \text{m}^{-3}$ and the magnetic field is about 543.6G, one can get $A_a = 0.055\alpha$, $\lambda_{ab} = 0.0027\alpha$ and $\mathcal{E} = -0.1125\alpha$. Figure 2 shows the time evolution of particle populations for the conversion of $^6\text{Li}$ atoms into molecules. From the top panel, we find that almost all of $^6\text{Li}$ atoms can be converted into molecules. For $^6\text{Li}$ atoms, the atom-to-molecule conversion efficiency via the STIRAP technique aided by Feshbach resonance is higher than that via bare Feshbach resonance [8, 9] which is in contrast to the case for $^{40}\text{K}$ atoms. From the bottom panel, we can see that the atom-to-molecule conversion efficiency decreases distinctly due to the existence of the quasibound molecular decay.

The dependence of the conversion efficiency on the decay rate $\gamma$ of quasibound molecules for both $^{40}\text{K}$ and $^6\text{Li}$ is plotted in Fig. 3. Clearly, the influence of the decay on the conversion efficiency for $^6\text{Li}$ atoms is more distinct than that for $^{40}\text{K}$ atoms, which confirms the previous interpretation.

![FIG. 2: (color online) The time dependence of the population of particles for $^6\text{Li}$ system for different $\gamma$. The parameters are $\delta = 0$, $\lambda_{ab} = 0.0027$, $A_a = A_b = 0.055$, $\mathcal{E} = -0.1125$, $\Omega_{\text{max}} = 200$, $t_0 = 120$, $\tau = 40$, and the other parameters are zero. Time is in unit of $1/\alpha$ and all other coefficients are in units of $\alpha$, where $\alpha = 3.29 \times 10^{-27} \text{J}$.](image)

V. ADIABATIC FIDELITY FOR CPT STATES

From the above numerical results, we can find that the atom-to-molecule conversion efficiencies of STIRAP technique aided by Feshbach resonance for $^{40}\text{K}$ and $^6\text{Li}$
are distinctly different although the CPT states are assumed to exist for the two systems. As we know that most of the atoms can be converted into molecules only when the CPT state is followed adiabatically, and the existence of the CPT state does not guarantee the state can be followed adiabatically. The adiabatic properties for the atom-to-molecule conversion have been studied in Ref. [4]. Whereas, we can not apply this method to our system since all the possible solutions of the stationary equation must be known in the approach and it is difficult to obtain the other solutions beyond the CPT-state solution in our system.

Recently, the fidelity was employed to characterize the adiabatic condition for systems with atom-to-molecule conversion. The key point is that the value of the fidelity should close to unity if the system can adiabatically evolve in the CPT state. Now we evaluate the adiabatic fidelity $f(\phi(t), \phi(t))$ for our model, in which $\phi(t)$ is the wave function obtained by solving the dynamical Eq. with $\gamma = 0$, and $\phi(t)$ is the wave function corresponding to the CPT state which is given in Eq. (14). The time evolution of the adiabatic fidelity for CPT state is plotted in Fig. (4) for $^{40}$K system. One can see that the magnitude of fidelity is about 1 at the initial time, which implies the system adiabatically evolves along CPT state. At the time $275/\alpha$, the fidelity diminishes to the minimal value 0.35, which implies the system deviates away from the CPT state distinctly at that time. Although the fidelity begins to fluctuate later on, its final value is still no more than 0.75. The above analysis implies that the CPT state can not be followed adiabatically, so the atom-to-molecule conversion efficiency is not high for $^{40}$K atoms. Fig. (4b) shows the time evolution of the adiabatic fidelity for CPT state corresponding to the conversion of $^6$Li atoms into molecules. Obviously, the fidelity for this system is very close to 1 at all the time, which implies that the system adiabatically evolves along the CPT state. This result is consistent with the fact that almost all of the $^6$Li atoms can be converted into molecules. Since the fidelity for CPT state can well reflect the atom-to-molecule conversion efficiency, one can improve the conversion efficiency by optimizing the parameters of the system to achieve a higher adiabatic fidelity.

VI. SUMMARY

With the help of mean-field Lagrangian density, we studied the conversion of two-species fermionic atoms into bosonic molecules via STIRAP technique aided by Feshbach resonance. We calculated conversion efficiencies for $^{40}$K and $^6$Li systems respectively, and found that almost all of the $^6$Li atoms can be converted into molecules, which implies that the STIRAP technique aided by Feshbach resonance is more effective than the bare Feshbach resonance for $^6$Li atoms rather than $^{40}$K atoms. We also compared the influence of the decay rate of quasibound molecules on the conversion efficiency for $^{40}$K and $^6$Li systems respectively, and found that there is a big difference between them. The success of STIRAP technique not only requires the existence of the CPT state but also requires that the system can adiabatically evolve within such a state.

The adiabatic fidelity was recently recognized to be a useful measurement for characterizing the adiabatic properties. Our analysis of the symmetry and the corresponding conservation law for the systems under consideration helped us to introduce an appropriate definition of adiabatic fidelity for CPT state. For $^{40}$K system, the CPT state can not be adiabatically followed since the fidelity was found to be less than 0.75 at final time, which is well consistent with the corresponding low conversion efficiency. In order to improve the conversion efficiency in $^{40}$K system, one should achieve a higher fidelity through optimizing the parameters of the system. Whereas, for $^6$Li system, the fidelity is very close to 1, hence the CPT state can be followed adiabatically, which is the reason for a high conversion efficiency. Our evaluation of the adia-
batic fidelity enable us to understand why the conversion efficiencies for $^{40}$K and $^6$Li are distinctly different.

The work was supported by Program for Changjiang Scholars and Innovative Research Team in University, and NSFC Grant No. 10674117.

[1] S. Jochim, M. Bartenstein, A. Altmeyer, G. Hendl, S. Riedl, C. Chin, J. H. Denschlag, and R. Grimm, Science \textbf{302}, 2101 (2003).
[2] M. W. Zwierlein, C. A. Stan, C. H. Schunck, S.M. F. Raupach, S. Gupta, Z. Hadzibabic, and W. Ketterle, Phys. Rev. Lett. \textbf{91}, 250401 (2003).
[3] M. Greiner, C. A. Regal and D. S. Jin, Nature \textbf{426}, 537 (2003).
[4] M. Mackie, R. Kowalski and J. Javanainen, Phys. Rev. Lett. \textbf{84}, 3803 (2000).
[5] G. Alzetta et al., Nuovo Cimento Soc. Ital. Fis., B 36, 5 (1976); G. Alzetta, L. Moi, and G. Orriols, ibid. 52, 209 (1979).
[6] S. Y. Meng, L. B. Fu and J. Liu, e-print \texttt{arXiv:0709.0359}.
[7] C. A. Regal, C. Ticknor, J. L. Bohn, D. S. Jin, Nature \textbf{424}, 47 (2003).
[8] K. E. Strecker, G. B. Partridge, R. G. Hulet, Phys. Rev. Lett. \textbf{91}, 080406 (2003).
[9] J. Cubizolles, T. Bourdel, S. J. J. M. F. Kokkelmans, G. V. Shlyapnikov, and C. Salomon, Phys. Rev. Lett. \textbf{91}, 240401 (2003).
[10] C. A. Regal, M. Greiner, and D. S. Jin, Phys. Rev. Lett. \textbf{92}, 040403 (2004).
[11] C. A. Stan, M.W. Zwierlein, C. H. Schunck, S.M. F. Raupach, and W. Ketterle, Phys. Rev. Lett. \textbf{93}, 143001 (2004).
[12] S. Inouye, J. Goldwin, M. L. Olsen, C. Ticknor, J. L. Bohn, and D. S. Jin, Phys. Rev. Lett. \textbf{93}, 183201 (2004).
[13] L. H. Lu and Y. Q. Li, Phys. Rev. A \textbf{76}, 053608 (2007).
[14] C. A. Regal, C. Ticknor, J. L. Bohn, and D. S. Jin, Phys. Rev. Lett. \textbf{90}, 053201 (2003).
[15] H. Y. Ling, H. Pu and B. Seaman, Phys. Rev. Lett. \textbf{93}, 250403 (2004).
[16] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information (Cambridge University Press, Cambridge, England, 2000), pp. 399-424.