Driving Reinforcement Learning with Models

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Abstract

Over the years, Reinforcement Learning (RL) established itself as a convenient paradigm to learn optimal policies from data. However, most RL algorithms achieve optimal policies by exploring all the possible actions and this, in real-world scenarios, is often infeasible or impractical due to e.g. safety constraints. Motivated by this, in this paper we propose to augment RL with Model Predictive Control (MPC), a popular model-based control algorithm that allows to optimally control a system while satisfying a set of constraints. The result is an algorithm, the MPC-augmented RL algorithm (MPCaRL) that makes use of MPC to both drive how RL explores the actions and to modify the corresponding rewards. We demonstrate the effectiveness of the MPCaRL by letting it play against the Atari game Pong. The results obtained highlight the ability of the algorithm to learn general tasks with essentially no training.

1 Introduction

Driven by its recent impressive results, see e.g. [Mnih et al., 2015], [Silver et al., 2018], reinforcement learning (RL) has become a popular paradigm to make agent achieve optimal policies [Sutton and Barto, 1998]. On an intuitive level RL manages to find the optimal policy by exploring the state space and by learning which actions are best for each scenario, on the basis of function called reward function, which rewards or punishes the agent for its behaviour. Although, it can be proven that in the long term a RL algorithm manages to reach optimal control, the trade-off is that during the learning process the agent might end up in unsafe situations [Berkenkamp et al., 2017]; this, especially in applications where agents need to physically interact with their environment, might end up being infeasible due to safety requirements. Take autonomous driving, as an example: a key requirement would be that, while learning a policy for certain critical functionalities, safety needs to be guaranteed at all times (to e.g. avoid crashes). Furthermore, on this note we point out that it has been recently found that even using a dataset with 30 million driving examples is inadequate to train policies that are safe in any operational condition: no dataset exists that can exhaustively cover all situations that might arise when driving [Bansal et al., 2018].

Motivated by this, we propose in this paper an algorithm that complements the capabilities of RL with those of a well known and established model-based control method such as the Model Predictive Control (MPC). In so doing, we present the MPC-augmented RL (MPCaRL) that: (i) for certain critical functionalities, for which a dynamical model can be identified, it computes a control action...
optimizing a given cost function; (ii) for non-critical functionalities it uses RL to compute a policy from data; (iii) it makes use, when possible, of the MPC to both provide a guidance for the state-space exploration and to fine-tune the rewards obtained by the RL.

1.1 Related Work

This work is related to a number of threads of recent work in RL. We now briefly review some related works on physics simulation, model-based and safe RL.

**Physics simulation.** The idea of using models and simulation environments to develop intelligent reinforcement learning agents has recently been attracting a lot of attention from academia. For example, in [de Avila Belbute-Peres et al., 2018] it is shown how a physical simulator can be embedded in a deep network, enabling agents to both learn parameters of the environment and improve the control performance of the agent. Essentially, this is done by simulating rigid body dynamics via a linear complementarity problem (LCP) technique [Cottle, 2009][Cline, 2002]. LCP techniques are also used within other simulation environments such as MuJoCo [Todorov et al., 2012], Bullet [Lee et al., 2018], and DART [Hermans et al., 2014]. As remarked in [de Avila Belbute-Peres et al., 2018], in these environments gradients are computed through finite differences and this, in turn, implies evaluating the results of the simulations multiple times. In order to improve the computational burden of this approach, LCP techniques have also been developed which involve analytic differentiation. See, for example [Degrave et al., 2019] and, more recently, [Lerer et al., 2016]. Furthermore, a complementary body of literature investigates the possibility of integrating into networks the mechanisms inspired by the intuitive human ability to understand physics. See e.g. [Chang et al., 2016][Battaglia et al., 2016][Battaglia et al., 2013], which leverage the ideas of [Hamrick et al., 2015][Smith and Vul, 2013][Werbos, 1989].

**Model-based RL.** Although model-free methods, have achieved considerable successes in the recent years (see for example [Mnih et al., 2015]), many works suggest that a model-based approach can potentially achieve better performances. Among those, the interested reader can refer to [Berkenkamp et al., 2017][Atkeson and Santamaria, 1997]. [Kurutach et al., 2018]. Model-free approaches (see e.g. [Mnih et al., 2015]), in fact, tend to provide optimal policies only on the long term and employing previous knowledge of domain, such as a mathematical model, can considerably decrease the learning time of a model-based RL algorithm (with respect to a model-free one). Based on these premises, the research in model-based RL is a very active area and it is focused on two main settings. The first one makes use of neural networks and a suitable loss function to simulate the dynamics of interest [Nagabandi et al., 2017][Ljung, 2017], whereas another approach makes use of more classical mathematical models and closely resembles classic system identification [Pecka and Svoboda, 2014]. Recently, an orthogonal stream of literature (see e.g. [Herzallah, 2015][Kárny et al., 1996]) has also emerged and focuses on designing of optimal decision makers based on a pure probabilistic knowledge (in the form of a probability density function) of the system of interest and dynamic programming [Bertsekas, 1995].

**Safe RL.** The design of safe RL algorithms is a key research topic and different definitions of safety have been proposed in the literature (e.g. [García and Fernández, 2015][Corraluppi and Marcus, 1999] and references therein). As an example, in [Geibel and Wysotzki, 2011] the authors relate safety to a set of error states associated to dangerous/risky situations while in [Tamar et al., 2014] risk aversion is specified in the reward. Other works, [Wiesemann et al., 2010][Herzallah] define safety, by using accurate probabilistic model of the system or [Pegueroles and Russo, 2019] where a notion of safety for closed-loop probabilistic systems is defined. A complementary approach is the one of model-based RL where safety is formalized via state space constraints. Examples of this approach include [Mnih et al., 2015][Berkenkamp et al., 2017], where Lyapunov functions are used to show forward invariance of the safety set defined by the constraints. Finally, we note that other approaches include [García and Fernández, 2012], where a priori knowledge of the system is used, in order to craft safe backup policies or [Sadigh and Kapoor, 2016] in which authors consider uncertain systems and enforce probabilistic guarantees on their performance.

1.2 Contributions

Our key contributions can be summarized as follows. We introduce an algorithm, the MPCaRL that combines RL and MPC in a novel way so that they can augment each other’s strengths. The
MPCaRL is inspired by the fact that, often, complex tasks can be fulfilled by combining together a set of functionalities and, in turn, each functionality can be either critical or non-critical. For example, in automated driving, critical functionalities are those directly associated to the prevention of crashes. At each iteration, MPCaRL identifies for critical functionalities, a mathematical model. Once the model is identified, a control action is computed via MPC so as to optimize a given cost function. For all the other non-critical functionalities the algorithm makes use of a RL algorithm (in particular, we will use Q-Learning) to compute a policy from data. Moreover, MPC and RL interact within the MPCaRL and, in particular, MPC both drives the state-space exploration and tunes the RL rewards in order to speed the learning process. Finally, in order to illustrate the effectiveness of the MPCaRL, we show its performances at playing the Atari game Pong. In these contexts, the critical functionality is the defense (i.e., when the ball is bounced back to the player) which, if not performed correctly, leads to the loss of the round.

Interestingly the experiments highlight the ability of the algorithm to learn tasks with essentially no training.

2 Background

We start with outlining the two building blocks composing MPCaRL, i.e. the Model Predictive Control algorithm and the Q-Learning. We refer the reader to [García et al., 1989] and [Sutton and Barto, 1998] for a detailed description of MPC and Q-Learning.

Model Predictive Control. Model Predictive Control (MPC) is a model-based control technique. At each iteration, the algorithm computes a control action by solving an optimization problem having as constraint the (discrete-time) dynamics of the system being controlled. In addition to the dynamics, other system requirements (e.g. safety or feasibility requirements) can also be formalized as constraints of the optimization problem [García et al., 1989]. Let $x_k \in \mathbb{R}^n$ be the state variable of the system at time $k$, $u_k \in \mathbb{R}^m$ be its control input and $\eta_k$ be some noise. In this paper we consider discrete-time dynamical systems of the form $x_{k+1} = A_k x_k + D_k + C_k u_k + \eta_k$ with initial condition $x_{initial}$ and where the time-varying matrices have appropriate dimensions. For this system, formally the MPC algorithm generates the control input $u_k$ by solving the problem

$$\arg\min_{x_0:T \in X} \sum_{t=0}^{T} J_t(x_t, u_t) \text{ subject to } x_{t+1} = A_t x_t + D_t + C_t u_t, \quad x_0 = x_{initial},$$

with $x_{0:T} (u_{0:T})$ denoting the sequence $\{x_0, \ldots, x_T\}$ (resp. $\{u_0, \ldots, u_T\}$) and where: (i) $A$ and $X$ are sets modelling the constraints for the valid control actions and states; (ii) $\sum_{t=0}^{T} J_t(x_t, u_t)$ is the cost function being optimized. See [Giorgetti et al., 2006], [Borrelli et al., 2006] for more details on this subject.

Q-Learning and Markov Decision Processes. Q-Learning (Q-L) is a model free RL algorithm, whose aim is to find an optimal policy with respect to a finite Markov Decision Process (MDP). We adopt the standard formalism for MDPs. A MDP [Sutton and Barto, 1998] is a discrete stochastic model defined by a tuple $\langle S, A, P, \gamma, R \rangle$, where: (i) $S$ is the set of states $s \in S$; (ii) $A$ is the set of actions $a \in A$; (iii) $P(s'|s, a)$ is the probability of transitioning from state $s$ to state $s'$ under action $a$; (iv) $\gamma \in [0, 1]$ is the discount factor; (v) $R(s, a)$ is the reward of choosing the action $a$ in the state $s$. Upon performing an action, the agent receives the reward $R(s_t, a_t)$. A policy, $\pi$, specifies (for each state) the action that the agent will take and the goal of the agent is that of finding the policy that maximizes the expected discounted total reward. The value $Q^\pi(s, a)$, named Q-function, corresponding to the pair $(s, a)$ represents the expected future reward that can be obtained from $(s, a)$ when using policy $\pi$. The objective of Q-learning is to estimate the Q-function for the optimal policy $\pi^\star$, $Q^\pi(s, a)$. Define the estimate as $Q(s, a)$. The Q-learning algorithm works then as follows: after setting the initial values for the Q-function, at each time step, observe current state $s_t$ and select action $a_t$, according to policy $\pi(s_t)$. After receiving the reward $R(s_t, a_t)$ update the corresponding value of the Q-function as follows:

$$Q(s_t, a_t) \leftarrow (1 - \alpha)Q(s_t, a_t) + \alpha \left[ R(s_t, a_t) + \gamma \max_a Q(s_{t+1}, a) \right],$$

where $\alpha \in [0, 1]$ is called the learning rate. Notice that the Q-learning algorithm does not specify which policy $\pi(\cdot)$ should be considered. In theory, to converge the agent should try every possible
action for every possible state many times. For practical reasons, a popular choice for the Q-learning policy is the $\epsilon$-greedy policy, which selects its highest valued (greedy) action, $\pi(\mathbf{s}_t) = \text{argmax}_a \ Q(\mathbf{s}_t, a_t)$, with probability $1 - \epsilon (k - 1)/k$ and randomly selects among all other $k$ actions with probability $\epsilon/k$ [Wunder et al., 2010]. However, in this paper, due to the nature of the proposed algorithm and the way rewards are provided, we implemented a greedy policy, with $\epsilon = 0$. The interested reader can refer to [Peng and Williams, 1994] and [Wunder et al., 2010] for further information on this topic.

3 The MPCaRL Algorithm

We are now ready to introduce the MPCaRL algorithm, the key steps of which are summarized as pseudo-code in Algorithm 1. The main intuition behind this algorithm is that, in many real world systems, tasks can be broken down into a set of critical and non-critical functionalities. Often, critical functionalities are associated to safety constraints and, for these functionalities, it often makes sense for the designer to build a mathematical model. Given this intuition, the MPCaRL aims at combining the strengths of MPC and Q-L. Indeed: (i) within MPCaRL, MPC can directly control the agent in performing critical functionalities and, at the same time, it drives the state exploration of Q-L. This can be used to e.g. prevent the agent to enter in states that might be unsafe; (ii) on the other hand, Q-L handles non-critical functionalities, for which e.g. no mathematical model is available and hence for which a classic control algorithm could not be used.

The MPCaRL takes as input the following design parameters: (i) the set of allowed actions, $A$; (ii) the time horizon and cost function used is $f$; (iii) a recording window used to identify the dynamics in $1$; (iv) a positive and a negative reward, $r$ and $r^-$ used by MPC to drive the exploration of Q-L; (v) an initial matrix $Q(s, a)$. Given this set-up, following Algorithm 1, the following steps are performed:

Step 1: at each time-step, MPCaRL determine whether the task functionality is a critical or a non-critical functionality. As we will see in Section 4, the critical functionality in the Pong game is associated to the agent’s defense strategy;

Step 2a: if the functionality is critical, then the agent’s action is computed via MPC. Even if the action applied by the agent is given by MPC, the action that would have been obtained via Q-L is also computed. If the action from MPC and Q-L are the same, then the $Q(s, a)$ matrix is updated by using the positive reward $r$. On the other hand, if the actions from MPC and Q-L differ from one another the $Q(s, a)$ matrix is updated with a non-positive reward $r^-$;

Step 2b: if, instead, the functionality is non-critical (or a model cannot be identified) then, the agent’s action is generated by Q-L;

Step 3: all relevant quantities are saved within the main algorithm loop

4 Simulations

We now illustrate the effectiveness of MPCaRL by making it play against the Atari game Pong. Pong is a 2 player game where each player moves a paddle in order to bounce a ball to the opponent. A player scores a point when the opponent fails to bounce the ball back and the game ends when a player scores 21 points. In what follows, we give a thorough description of how Algorithm 1 has been implemented in order to allow MPCaRL to play against Pong.

4.1 The environment and data gathering

The environment of the game was set-up using the OpenAI gym library in Python. In particular, we used the PongDeterministic-v4 (with 4 frame skips) configuration of the environment, which is the one used to assess Deep Q-Networks, see e.g. [EndtoEndAI] and [Mnih et al., 2013]. The configuration used has, as observation space, Box(210, 160, 3) [1]. Within our simulations, we first removed part of the images that display the points earned by the players and this yielded an observation space of Box(160, 160, 3) and then we down-sampled the resulting image to get a reduced observation space of Box(80, 80, 3) so that each frame consists of a matrix of $80 \times 80$ pixels. Within the image,
Algorithm 1 MPCaRL Algorithm

1: Inputs:
2: Allowed actions, \( A \)
3: Time horizon, \( T \), and cost function \( \sum_{t=0}^{T} J_t(x_t, u_t) \)
4: Recording window \( T_w \)
5: Rewards \( \tau \) and \( r \)
6: Initial matrix \( Q(s, a) \)
7: Main loop:
8: for \( k = 0, \ldots \) do
9: Determine functionality (and whether model in (1) can be identified)
10: if Functionality critical then
11: Get \( s_k \) and \( x_k \)
12: if \( k \geq 1 \) then
13: if \( a_{k-1} = u_{k-1} \) then
14: \( Q(s_{k-1}, u_{k-1}) \leftarrow (1 - \alpha)Q(s_{k-1}, u_{k-1}) + \alpha (\tau + \gamma \max_a \{Q(s_k, a)\}) \)
15: else
16: \( Q(s_{k-1}, u_{k-1}) \leftarrow (1 - \alpha)Q(s_{k-1}, u_{k-1}) + \alpha (r + \gamma \max_a \{Q(s_k, a)\}) \)
17: end if
18: end if
19: if \( k - T_w > 0 \) then
20: Given \( x_{k-T_w:k} \) identify the model in (1) and compute \( u_k \) with \( x_{init} = x_k \)
21: else
22: Given \( x_{0:k} \) identify the model in (1) and compute \( u_k \) with \( x_{init} = x_k \)
23: end if
24: Compute \( a_k \) using Q-L (Section 2)
25: Apply \( u_k \)
26: Save \( x_k, s_k, a_k, u_k \)
27: else
28: Apply \( a_k \) computed via Q-L
29: Save \( s_k, a_k \)
30: end if
31: \( k \leftarrow k + 1 \)
32: end for

a coordinate system is defined within the environment, with the origin of the \( x \) and \( y \) axes being in the bottom-right corner.

Given the above observation space, both the position of the ball and the vertical position of the paddle moved by MPCaRL were extracted from each frame (see also Figure 1, left panel). In particular:

- At the beginning of the game, the centroid of the ball is found by iterating through the frame to find the location of all pixels with a value of 236 (this corresponds to the white color, i.e. the color of the ball in Pong). Then, once the ball is found the first time, the frame is only scanned in a window around the position of the ball previously found (namely, we used a window of 80 × 12 pixels, see Figure 1, right panel);

- Similarly the paddle’s centroid position is found by scanning the frame for pixels having value 92 (this corresponds to the green color, i.e. the color of the MPCaRL paddle).

4.2 Definition of the task and its functionalities

The agent’s task is that of winning the game, which essentially consists of two phases: (i) defense phase, where the agent needs to move the paddle to bounce the ball in order to avoid that the opponent makes a point; (ii) an attack phase, where the agent needs instead to properly bounce the ball in order to make the point. When implementing MPCaRL to play Pong, we associated defense to critical functionalities, while the attack functionalities were defined as non-critical. Explicit conditions identifying critical and non-critical functionalities are given in the next sub-sections.
We describe the MPC implementation used within MPCaRL to play Pong by first introducing the mathematical model serving as the constraint in (1). This model describes both the dynamics of the ball and of the paddle moved by MPCaRL.

**Ball dynamics.** We denote by $x^{(b)}_t$ and $y^{(b)}_t$ the $x$ and $y$ coordinates of the centroid of the ball at time $t$. The mathematical model describing the dynamics of the ball is then:

$$
x^{(b)}_{t+1} = x^{(b)}_t + v_{t,x}, \quad y^{(b)}_{t+1} = y^{(b)}_t + v_{t,y},
$$

where $x^{(b)}_{t+1}$ and $y^{(b)}_{t+1}$ are the predicted next coordinates at time $t + 1$ and where the speeds at time $t$, i.e. $v_{t,x}$ and $v_{t,y}$ are computed from the positions extracted from the current and the two previous frames, i.e. $x^{(b)}_{t-2}, x^{(b)}_{t-1}, x^{(b)}_t$ and $y^{(b)}_{t-2}, y^{(b)}_{t-1}, y^{(b)}_t$ (that is, $T_w = 2$ in Algorithm 1). In particular, this is done by first computing the quantities $v_{t,1}, v_{t,2}$ and $v_{y1}, v_{y2}$ as follows:

$$
\begin{bmatrix}
  v_{x1} \\
  v_{y1}
\end{bmatrix} = \begin{bmatrix}
  x^{(b)}_1 \\
  y^{(b)}_1
\end{bmatrix} - \begin{bmatrix}
  x^{(b)}_{-1} \\
  y^{(b)}_{-1}
\end{bmatrix}, \quad \begin{bmatrix}
  v_{x2} \\
  v_{y2}
\end{bmatrix} = \begin{bmatrix}
  x^{(b)}_2 \\
  y^{(b)}_2
\end{bmatrix} - \begin{bmatrix}
  x^{(b)}_{-2} \\
  y^{(b)}_{-2}
\end{bmatrix}.
\tag{4}
$$

Consider now the speed along the $x$ axis. If there is no impact between the ball and the paddle, we set $v_{t,x} = 0.5(v_{x1} + v_{x2}) + \text{var} \left( \begin{bmatrix} v_{x1} \ v_{x2} \end{bmatrix} \right) = \bar{v}_{t,x} + \eta_{t,x}$ (where var$(a)$ denotes the variance of the generic vector $a$). Instead, along the $y$ axis, we have $v_{t,y} = 0.5(v_{y1} + v_{y2}) + \text{var} \left( \begin{bmatrix} v_{y1} \ v_{y2} \end{bmatrix} \right) = \bar{v}_{t,y} + \eta_{t,y}$ if there has been no impact and $v_{t,x} = v_{x1}$ if there has been an impact of the ball with one of the walls.

**Paddle dynamics.** In the gym environment used for the experiments, the only control action that can be applied by MPCaRL at time $t$, i.e. $u_t$ is that of moving its paddle. In particular, the agent can either move the paddle up ($u_t = 1$) or down ($u_t = -1$) or simply not moving the paddle ($u_t = 0$). It follows that, given the vertical position of the centroid of the paddle at time $t$, say $y^{(p)}_t$, its dynamics can be modeled by

$$
y^{(p)}_{t+1} = y^{(p)}_t + u_t.
$$

**The MPC model and the cost function.** Combining the models in (3) - (5) yields the dynamical system serving as constraint in (1). Note that the resulting model can be formally written as the system in (1) once the state $x_t$ is defined as $x_t = [x^{(b)}_t, y^{(b)}_t, y^{(p)}_t]^T$. Finally, in the implementation of the MPC algorithm, we used as cost function $\sum_{t=0}^T J_t = \|y^{(b)}_{t+T} - y^{(p)}_{t+T}\|^2$. That is, with this choice of cost function, the goal is to minimize the distance between the predicted trajectory of the ball at each step and the trajectory of the paddle. This ensures that the agent maximizes the expected return in the Pong environment.
We are now ready to present the results obtained by letting MPCaRL play Pong (larger versions of

We implemented the Q-L algorithm outlined in Section 4. The set of actions available to the agent

As a first experiment, we implemented an agent that would only use MPC or the untrained Q-L

was able to beat the opponent; the maximum value that can be obtained is

When MPCaRL is not able to score any point. Viceversa, a positive game reward means that the agent

whereas all the others are considered non-critical). Note that: (i)

Finally, we now describe the conditions that we used in the experiments to allow MPCaRL to
discriminate between critical and non-critical functionalities. Intuitively, critical functionalities were
associated to the defense phase of the game. Therefore, all the functionalities performed when the
ball was coming towards the MPCaRL paddle and, at the same time, the future vertical position of the
ball (predicted via the model) was far from the actual position of the MPCaRL paddle were
defined as critical. That is, the states that are defined as critical are the ones that satisfy the conditions

Finally, the values of the Q-table were initialized to 0. In the experiments, the state-action pair was
updated whenever a point was scored and a greedy policy was used to select the action. Also, in the
experiments we set both α and γ in (2) to 0.7. Moreover, following Algorithm 1 the Q-function was
even updated when non-critical functionalities were performed by the agent. In particular, within
the experiments we assigned: (i) a positive reward, r, whenever the action from Q-L and MPC were
the same; (ii) a non-positive reward, r, whenever the actions were not the same. In this way, within
MPCaRL, the Q-L component is driven to learn to take actions similar to MPC.

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cost function the algorithm seeks to regulate the paddle’s position so that the distance between
the position of the ball at time t and the position of the MPCaRL paddle at time t + T is minimised.
Note that the time horizon, T, used in the above cost function is obtained by propagating the ball
model in order to estimate after how many iterates the ball will hit the border protected by the
MPCaRL paddle.

4.4 Implementing Q-L

We implemented the Q-L algorithm outlined in Section 4. The set of actions available to the agent
were \( \alpha_t \in \{-1, 0, +1\} \), while the state at time \( s_t \) was defined as the 5-dimensional vector containing:
(i) the coordinates of the position at time \( t \) of the ball (in pixels); (ii) the velocity of the ball
(rounded to the closest integer) across the \( x \) and \( y \) axes; (iii) the position of the paddle moved by
MPCaRL. The reward was obtained from the game environment: our agent was given a reward of
+1, each time the opponent missed to hit the ball, and −1 each time our agent missed to hit the ball.
Finally, the values of the Q-table were initialized to 0. In the experiments, the state-action pair was
updated whenever a point was scored and a greedy policy was used to select the action. Also, in the
experiments we set both \( \alpha \) and \( \gamma \) in (2) to 0.7. Moreover, following Algorithm 1 the Q-function was
also updated when non-critical functionalities were performed by the agent. In particular, within
the experiments we assigned: (i) a positive reward, \( r \), whenever the action from Q-L and MPC were
the same; (ii) a non-positive reward, \( r \), whenever the actions were not the same. In this way, within
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\[ v_{t,x}^{(b)} < 0, \| y_{t+T}^{(b)} - y_{t}^{(p)} \| > H_y, \] (whereas all the others are considered non-critical).

Note that: (i)

\[ v_{t,x}^{(b)} \] is estimated from the game frames as described above (in the environment negative velocities
along the \( x \) axis mean that the ball is coming towards the green paddle); (ii) the computation of \( y_{t+T}^{(b)} \)
relies on simulating the model describing the ball’s dynamics (3); (iii) \( H_y \) is a threshold and this is a
design parameter”.

4.5 Results

We are now ready to present the results obtained by letting MPCaRL play Pong (larger versions of
the figures of this Section are provided in the Supplementary Materials). The results are quantified by
plotting the game reward as a function of the number of episodes played by MPCaRL. An episode
consists of as many rounds of pong it takes for one of the players to reach 21 points, while the game
reward is defined to be the difference between the points scored by MPCaRL within the episode and
the points scored by the opponent within the episode. Essentially, a negative game reward means that
MPCaRL lost that episode; the lowest possible value that can be attained is −21 and this happens
when MPCaRL is not able to score any point. Viceversa, a positive game reward means that the agent
was able to beat the opponent; the maximum value that can be obtained is +21, when the opponent
did not score any point.

As a first experiment, we implemented an agent that would only use MPC or the untrained Q-L
algorithm (see e.g. the implementation in [Karpathy]). We let this agent play Pong for 50 episodes
and, as the left panel in Figure 2 shows, as expected, the Q-L agent did not obtain good rewards in the
first 50 episodes, consistently loosing games with a difference in the scores of about 20 points.
Instead, when the agent used the MPC described in Section 4 better performance were obtained.
These performance, however, were not comparable with those obtained via a trained Q-L agent
and the reason for this is that, while MPC allows the agent to defend, it does not allow for the
learning of an attack strategy to consistently obtain points. Using MPCaRL allowed to overcome the
shortcomings of the MPC and Q-L agents. In particular, as shown in the left panel of Figure 2 when
the MPCaRL agent played against Pong, it was able to consistently beat the game (note the agent
never lost a game) while quickly learning an attack strategy to obtain high game rewards. Indeed,
note how the agent is able to consistently obtain rewards of about 20 within 50 episodes.
Figure 2: Left panel: episode rewards as a function of the number of episodes played when either MPC or Q-L are used. Right panel: episode rewards as a function of the number of episodes played by MPCaRL. Parameters of MPCaRL were set as follows: $\tau = 0.1$, $\zeta = 0$, $H_y = 5$.

In order to further investigate the performance of MPCaRL, we also evaluate its performance sensitivity to the parameters $\tau$, $r$, and $H_y$. We first take $H_y$ in consideration; $H_y$ directly affects the definition of critical functionalities. As Figure 3 (left panel) illustrates, MPCaRL is still able to consistently obtain high rewards when $H_y$ is perturbed, i.e. $H_y \in \{4, 5, 6\}$. Notice that when $H_y = 4$, while MPCaRL is still able to beat the game, it also experiences some drops in the rewards. This phenomenon, which will be further investigated in future studies, might be due to the fact that restricting the space of movements of the Q-L component of the algorithm also restricts its learning capabilities. Instead, when the manoeuvre space of the Q-L is bigger ($H_y = 6$), the algorithm, due to the increased flexibility in the moves is able to learn faster. As a further experiment, we fixed $H_y = 5$ and perturbed the algorithm parameter $\tau$ so that $\tau \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$. The results of this experiment are shown in the middle panel of Figure 3. It can be noted that smaller values of $\tau$ (i.e. $\tau = 0.1$ and $\tau = 0.3$) lead to a more consistent performance as compared to the higher values of $\tau$ which lead to negative spikes in the performance. This behaviour might be due to the fact that higher values of $\tau$ essentially imply that MPCaRL trusts more the MPC actions than Q-L actions, hence penalizing the ability of MPCaRL to quickly learn the attack strategy. Intuitively, simulations show that, while the MPC component is important for enhancing the agent’s defense, too much influence of this component on the Q-L can reduce the attack performance of the agent (this, in turn, is essential in order to score higher points). Consistently, the same behaviour can be observed when $r$ is perturbed ($H_y = 5$ and $r \in \{-0.1, -0.3, -0.5, -0.7, -0.9\}$), as shown in Figure 3 (right panel), where it can be observed that the most negative values $r$ lead to dips in performance.

Figure 3: Left panel: rewards obtained by the MPCaRL when $H_y$ is perturbed. All the other parameters were kept unchanged (i.e. $\tau = 0.1$, $\zeta = 0$). Middle panel: rewards obtained by MPCaRL when $\tau$ is perturbed. In this experiment, $H_y = 5$ and $r = 0$. Right panel: rewards obtained by MPCaRL when $\zeta$ is perturbed. In the experiment, $H_y = 5$ and $\tau = 0.1$. 
5 Conclusions and Future Work

We investigated the possibility of combining Q-L and MPC so that they can augment each other’s capabilities. In doing so, we introduced a novel algorithm, the MPCaRL that: (i) leverages MPC and mathematical modelling for certain, critical, functionalities; (ii) makes use of Q-L to both perform non-critical functionalities; (iii) uses MPC to drive the state-space exploration of Q-L. The effectiveness of our algorithm has been illustrated by letting it play against Pong. Its performance are analysed when the parameters are perturbed. Interestingly, the experiments highlight the ability of the algorithm to quickly learn general tasks with essentially no training. In future works, we will focus on implementing the MPCaRL in different settings (e.g., Hide and Seek with multiple agents might be a good candidate) and we will explore its convergence and stability properties.

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