True value, error, and measurement uncertainty: two views

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Abstract
Measurement uncertainty, as established in the Guide to the Expression of Uncertainty in Measurement (GUM), is a central concept in metrology. The GUM is known for its detailed discussions on the concepts of true value and error and their relation to measurement uncertainty. However, the GUM statements on true value and error have been a source of conceptual controversies, sometimes leading to inconsistent or unclear descriptions on true value, error, and uncertainty. Here, we discuss that such controversies arise from an unclear distinction between two views of measurement with fundamentally different premises. In one of the views, called the realist view, measurement is regarded as an activity of estimating or determining the true value, in which case measurement uncertainty represents the dispersion of reasonable estimates of true values. In the other view, called the instrumentalist view, measurement is regarded as an activity of assigning values to a measurand, in which case measurement uncertainty represents the dispersion of values that could reasonably be assigned to a measurand. By examining the philosophy of measurement in each view, we show that a clear understanding of the two views is critical for understanding the GUM.

Keywords Measurement uncertainty · True value · Error · GUM · Metrology

Introduction
The Guide to the Expression of Uncertainty in Measurement (GUM) [1] was published in 1993 as a result of efforts to provide a uniform and generally accepted approach for expressing uncertainty in measurement [2]. Since then, the concept of measurement uncertainty established in the GUM has become a de-facto international standard for scientific metrology.

The GUM aimed at addressing challenges that existed in the establishment of a general framework for the evaluation of uncertainty in measurement. One of the challenges was to arrive at a consistent rule for combining estimates of “systematic errors” and “random errors” to provide a single “overall” uncertainty [2]. Before the GUM, “systematic errors” and “random errors” of measured values were evaluated and reported separately or combined in an ad hoc way [3]. To address this challenge, the GUM argued that uncertainties arising from systematic effects and random effects are of the same nature and should be treated equally [1, E.3]. This aspect of the GUM is widely accepted in the literature.

Another challenge that the GUM needed to address was the unknowability of true values. True values were postulated to be the ultimate goal (target) of measurement that could be obtained by a “perfect” measurement [1, B.2.3]. At the same time, true values were regarded to be unknowable, because no measurement can be perfect [1, 3.2.1]. This created confusing metaphysical issues in metrology. If true values, the ultimate goal of measurement, are unknowable, accuracy and errors are also unknowable. Then, any statement on the quality of a measurement becomes unverifiable in principle.

To address this challenge, the GUM states that it adopted an operational view of measurement that focuses on the observables, rather than the unknowable true values.
and errors [1, E.5.4]. More specifically, the GUM states that it focuses on “the measurement result and its evaluated uncertainty rather than on the unknowable quantities “true” value and error” [1, E.5.1]. Further, it states that it does not use the term “true value” because the expressions “value of a mea- surand” and “true value of a mea- surand” are viewed as equivalent [1, 3.1.1 NOTE]. The GUM also states that its approach “makes any mention of error entirely unnecessary” [1, E.5.4]. These statements in the GUM have received wide attention in the literature. However, controversies persist on their interpretations [4–26].

The literature is divided on whether the GUM has avoided the concepts true value and error, or only the terms “true value” and “error” [5–8, 22]. Some insist that the GUM discarded the concepts true value and error [4, 21, 22]. Others insist that the GUM discouraged only the use of the term “true value”, and simply used “value” in place of “true value” [7, 8]. Some have further suggested that avoiding the expressions “true value” and “error” creates confusion with no clear benefit because true value and error are indispensable concepts in metrology [5–8]. Following such a view, recent authoritative publications use the expressions “true value” and “error” in association with measurement uncertainty [8, 27, 28]. This difference in the views towards true value and uncertainty is far-reaching for metrology. Unfortunately, the diverging views are not widely discussed and are becoming a source of confusion [14, 15].

Discussion on true value and uncertainty requires the identification and distinction of various philosophical standpoints of measurement. The philosophical standpoint that one assumes leads to fundamentally different attitudes toward the concepts [23] and becomes a source of miscommunication if not clarified. However, except for a few notable exceptions, such different standpoints are seldom discussed in the literature related to measurement uncertainty [23, 25, 29–31]. The third edition of the International Vocabulary of Metrology (VIM3), for instance, states that the existence of different philosophies of measurement posed difficulties in its development [32]. It does not, however, discuss the different philosophies in sufficient detail. Similarly, although the GUM states that it is consistent with different views [1, 2.2.4], it does not elaborate on the views. This lack of elaboration can lead to multiple interpretations of concepts and eventually cause controversies.

The existing views of measurement can broadly be stereotyped into two different views, which are referred to as the realist view and the instrumentalist view in this paper. The former view argues that measurement is aimed at revealing “truths” of the world, whereas the latter view argues that measurement is aimed at obtaining useful information for a purpose. In this paper, we describe how these different views lead to fundamentally different attitudes toward true value, error, and uncertainty. We further show the importance of clearly distinguishing the two views for understanding the GUM.

**Discussions**

**The realist view of measurement and uncertainty**

The realist standpoint claims that scientific theories are at least approximately true, and that entities discussed in scientific theories exist (are real) [33–35]. The realist, therefore, believes that quantities are natural attributes of the world that exist independently of the human perception [23, 31, 36–39]. The realist also believes that relations between quantities exist independently of the human mind. Based on these beliefs, the realist usually presumes the existence of mind-independent ratios of quantities and describes measurement as an activity to obtain information on the ratios. In the VIM3 terminology, these ratios would be referred to as the numerical values of true values. Therefore, to the realist, measurement is an activity aimed at determining true values [40, 41]. The realist, however, recognizes that imperfections present in any measurement generally prevent exact determination of true values. Thus, the realist also describes measurement as an activity of estimating true values, acknowledging that true values themselves are unknowable.

In the realist view, measurement uncertainty is directly related to true value and error. For elaboration, suppose that a calibration certificate states the mass of a calibration weight as 200.000 06 g and the associated expanded uncertainty as 0.000 03 g. With this information, the realist finds it natural to claim that the “true value” of the mass of the calibration weight is in the range 200.000 06 g ± 0.000 03 g. Therefore, the realist finds it natural to perceive measurement uncertainty as a measure related to how well the true value is known. More carefully, the realist recognizes that no measurement result is perfect; therefore, the realist describes measurement uncertainty as “a measure related to how well one believes one knows the true value” [8, 28]. The realist also finds measurement uncertainty to be related to error because measurement uncertainty appears to represent how large the deviation of a measured value from the true value could be [10].

The realist view is the traditional and intuitive view of measurement [36, 41–43]. However, there are a few conceptual issues with this view. One of the most important issues is epistemological. As discussed in the Introduction, if true values are the ultimate objective of measurement, a measurement result is accurate if it agrees with true values. However, true values are unknowable. This makes
The instrumentalist view of measurement

The instrumentalist standpoint claims that scientific theories are aimed at adequately describing the world for practical purposes [33–35]. The instrumentalist also claims that entities discussed in scientific theories are conceptual tools (instruments) for describing the world and do not necessarily correspond directly to concrete entities [43, 45]. Because historically many metrologists tended to favor the realist view [42, 43, 46], the instrumentalist view needs to be elaborated further.

Consider the concept “radius of the Earth.” At a first glance, “radius” appears to be a property of the physical entity Earth. However, because the Earth is not perfectly spherical, the concept “radius” cannot be validly applied to Earth. One could employ the concepts of equatorial and polar radii, but they also have limitations in describing the Earth because it is not a perfect ellipsoid. Does this mean that the Earth does not have a radius and such concepts should not be used? Instead of stating so to lose the advantage of the concept “radius”, it is possible to regard “radius” as an abstract concept that is used for describing the Earth. The instrumentalist believes that, though no concept will be perfect in describing reality, a concept is at any rate useful if the description is sufficiently good for a particular purpose. For instance, if one intends to understand how large the Earth is, using the concept “radius” will hardly be problematic. In such a case, asking whether it is a “true” attribute of the Earth is unimportant [45].

Concepts, though, are only “mental constructs which we have conceived in our mind” [47], and not a part of reality. Concepts need to be bridged to reality if they can be of any practical value. For example, to attribute a value (e.g., “6400 km”) to “the radius of the Earth”, operations should be available for obtaining values for the quantity [48].

The instrumentalist believes that it is ultimately such empirical operations that grant practical meanings to concepts.

To find the length of an object, we have to perform certain physical operations. The concept of length is therefore fixed when the operations by which length is measured are fixed: that is, the concept of length involves as much as and nothing more than the set of operations by which length is determined. In general, we mean by any concept nothing more than a set of operations; the concept is synonymous with a corresponding set of operations [53].

This view of measurement is well recognized in chemistry and materials science. ISO 17034:2016 [54] includes the term “operationally defined measurands”, defined as “measurand that is defined by reference to a documented and widely accepted measurement procedure to which only results obtained by the same procedure can be compared”, which is based on the idea. ISO 17034:2016 [54] and a report by a task group of the Consultative Committee for Amount of Substance (CCQM) [55] include crude fibre contents in food, impact toughness, enzyme activities, pH, amount of moisture in grain, etc. as examples of operationally defined quantities. For these quantities, specific sets of operations must be followed during measurement if the results for the quantities are to be comparable.

Many other quantities, however, are typically considered not to be operationally defined. Such quantities are usually quantities that are regarded to be part of a system of quantities (hereafter referred to as “systemic quantities” for conciseness). For these quantities, there exists not a single set of operations, but multiple sets of operations that can be used for measurement. For example, the amount of iron in a solution may be measured by titrimetry, spectrophotometry, or by mass spectrometry.

A reason that multiple measurement operations can be used for measuring systemic quantities is that they do not stand isolated, but are actually a part of a system [56]. In fact, systemic quantities are defined by their relations to other quantities of the system [57]. According to the instrumentalist, relations such as Newton’s Second Law of Motion (usually expressed symbolically as “F = ma”) are relations that define abstract concepts such as force, rather than laws.
of nature between fundamental properties of nature, as the realist may state [43, 46, 58]. As systemic quantities are defined by their relations to other quantities of the system, they are defined within a system. Therefore, the necessary and sufficient condition for valid measurement of a systemic quantity is the consistency of a measurement process within the system [30, 36]. For example, because both the ideal gas law and the Faraday’s laws of electrolysis are accepted in the International System of Quantities (ISQ), they can both be used for the measurement of amount of substance.

As remarked earlier, however, relations between quantities are abstract. How such relations should validly be realized in reality is not trivial by their abstract definitions. Again, it is the actual empirical measurement operations that are regarded to be fit for realizing the abstract concepts that bridge the abstract concepts to reality. Therefore, though it is rarely admitted, it could be stated that even systemic quantities are also operationally defined [59].

To summarize the instrumentalist position: quantities are not natural attributes of the world that exist independently of the human perception [43, 60]. Relations between quantities do not exist by themselves but are concepts within man-made models of the world. Numerical structures are abstract structures in scientific models that are used to represent empirical relations [61, 62], rather than an innate feature of concrete entities. Ratios of quantities, thus, are defined and exist only in the abstract system of quantities. Consider that it is in principle possible to define infinitely many quantities by multiplying and dividing other quantities. This does not necessarily mean that all such abstract quantities “exist.” It is the actual operations associated with the abstract concepts and the practical values they provide for particular purposes that ultimately allow them to “exist” [63].

The instrumentalist, therefore, does not describe measurement as an activity of determining or estimating pre-existing ratios, or true values, of nature [22]. Rather, the instrumentalist describes measurement as an activity of assigning values to an object following accepted rules [61, 62]. The purpose of measurement is to obtain information on an object for decision making, rather than to reveal independently existing “truths” of the world [64].

The instrumentalist may use the expression “true value” for convenience in communications, but it would be used in the sense of an ideal target to be pursued, rather than a pre-existing constant that transcends human perception [11, 65]:

This immediately raises the question: Just how is the “true value” of the magnitude of a particular property of something defined? In the final analysis, the “true value” of the magnitude or a quantity is defined by agreement among experts on an exemplar method for the measurement of its magnitude—it is the limiting mean of a conceptual exemplar process that is an ideal realization of the agreed-upon exemplar method. And the refinement to which one should go in specifying the exemplar process will depend on the purposes for which a determination of the magnitude of the quantity concerned is needed—not just the immediate purpose for which measurements are to be taken but also the other uses to which these measurements, or a final adjusted value derived therefrom, may possibly be put [65].

Note that the above quote regards agreement among experts to be of primary importance in metrology. This is because it is ultimately the consensus of the experts that determine the structure of an abstract system that describes the world, and the empirical procedures that should be used for realizing a quantity of the system [11, 66]. Therefore, the instrumentalist describes a measured value to be of high quality if the procedures leading to the value are broadly acceptable to contemporary experts. The instrumentalist employs such values as reference values for describing the quality of other measured values [22]. ISO 5725–1:1994 adopts this approach and defines “accuracy” and “bias” in terms of accepted reference value [67], which is in contrast to the VIM3, which defines “accuracy” in terms of true value [32].

Comparison of the two views

To clarify the differences between the two views, some statements in the literature that are consistent with the realist or the instrumentalist view are here quoted for comparison. Some statements that are consistent with the realist standpoint are as follows:

Nobody denies that every measurand has its own, unique value which really exists [5]. In physics, the equations between physical quantities would be impossible without the concept of a true value; indeed, physical equations would always be only approximately correct for obtained values of the quantities [6].

To summarise the realist position: understanding measurements under the umbrella of the realist concept of truth, commits us not just to the logically independent existence of things in space and time, but also to the existence of quantitatively structured properties and relations, and to the existence of real numbers, understood as relations of ratio between specific levels of such attributes. This position entails that measurement is the attempt to estimate the ratio between two instances of a quantitative attribute, the first being the magnitude measured, and the second being a known unit [39].
Some statements that are consistent with the instrumentalist view are as follows:

Now there is no assurance whatever that there exists in nature anything with properties like those assumed in the definition, and physics, when reduced to concepts of this character, becomes a purely an abstract science and as far removed from reality as the abstract geometry of the mathematicians, built on postulates. It is a task for experiment to discover whether concepts so defined correspond to anything in nature, and we must always be prepared to find that the concepts correspond to nothing or only partially correspond [53]. The Realists deal experimentally with energized electrical apparatus in the laboratory, the powerhouse, and the industrial plant. Through long familiarity they come to attribute to properties like current, inductance, magnetic field strength as much reality as to their machinery and raw materials. They quite overlook the fact that these electrical quantities are in truth only artificial concepts invented for convenience in describing the natural phenomena concerned [43]. Quantities do not exist in nature, cannot be observed in nature, and quantities are abstract mathematical concepts we use for describing nature [60].

Is there a “true value” in nature or in our experiments at all? The honest answer is: probably not! When we perform measurements, do we measure “true values” in the etymological sense of the word? No! We construct a model of reality in our minds, which we describe by an equation. Then we perform measurements, the results of which are more or less consistent with our model… Therefore, shouldn’t we forget about “true values?” [22].

On most occasions, the choice of a particular view does not lead to practical differences. However, sometimes differences arise. For example, in the realist view, the correctness of a measurement result may never be known because the true value is unknowable. This makes it logically impossible to disprove that a randomly chosen value is more “correct” than the best measurement result available. This is not the case in the instrumentalist view. In the instrumentalist view, the “correctness” of a measurement result is related to the consistency of the measurement procedure with the procedures accepted by contemporary experts, rather than to the deviation from an unknowable “true value.”

The instrumentalist view of measurement uncertainty

We now discuss measurement uncertainty from the instrumentalist viewpoint. To the instrumentalist, measurement is an activity of assigning values to a quantity following accepted rules. Often, there are multiple values that can be assigned to a measurand. Consider again the example of measuring length using a ruler. To the instrumentalist, if the concept of length allows the length of an object to be measured by reading a ruler, all values obtained by reading a ruler are values that could be assigned to the measurand. If there are variations each time one reads a ruler, then there are multiple values that can be assigned to the measurand.

For practical reasons, it is customary to present a single value that represents the set of values that can be assigned to a measurand (usually an average), rather than listing them all. However, because such a single value cannot deliver the information of all the reasonable values that can be assigned to the measurand, a parameter characterizing the dispersion of such values is required. This parameter is measurement uncertainty. In this view, measurement uncertainty can be described as a measure of how well a single value represents all the other values that can be assigned to the measurand, i.e., a measure of “doubt about which values should be reported in the measurement result” [29]. Using the VIM3 terminology, measurement uncertainty can be described as a measure of doubt on how well a measured value represents the measurement result. This description of uncertainty is consistent with the following statements of the GUM on true value, error, and uncertainty:

By taking the operational views that the result of a measurement is simply the value attributed to the measurand and that the uncertainty of that result is a measure of the dispersion of the values that could reasonably be attributed to the measurand, this Guide in effect uncouples the often confusing connection between uncertainty and the unknowable quantities “true” value and error [1, E.5.1]. Indeed, this Guide’s operational approach, wherein the focus is on the observed (or estimated) value of a quantity and the observed (or estimated) variability of that value, makes any mention of error entirely unnecessary [1, E.5.4].

These statements are hardly consistent with the realist view.

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1 As the VIM3 notes [32], the expression “result of a measurement” has been used in a number of different ways. In the GUM, “result of a measurement” is a single value attributed to a measurand with an evaluated measurement uncertainty, whereas in the VIM3, “measurement result” is a set of values attributed to a measurand. In the VIM3, a single value that is representative of a measurement result is a “measured value.”
Two views in the GUM

It is useful to recognize the existence of the two views of measurement for understanding the GUM. For example, the GUM states that it considers “value of a measurand” and “true value of a measurand” to be equivalent because the word “true” is viewed as redundant [1, 3.1.1, B.2.3, D.3.5]. In the realist view, a value obtained through measurement is a measured value and not necessarily the true value. Therefore, it is confusing to state that the two expressions are equivalent [7, 8]. On the contrary, in the instrumentalist view, values that have been obtained in a manner that is consistent with the definition are “true” values of a measurand in the sense that they are values that can be assigned to the measurand [1, D.3.4]. Other values should not be called values of the measurand at all, because they should not be assigned to the measurand. Therefore, it is acceptable in the instrumentalist view to state that “value of a measurand” and “true value of a measurand” are equivalent and that the word “true” is redundant.3

Though the GUM does not explicitly mention the two views discussed in this paper, they are implicated in several parts of the GUM and are critical for understanding it. Consider the following part of the GUM, where the GUM elaborates on its definition of uncertainty [1, 2.2.4]:

The definition of uncertainty of measurement given in 2.2.3 is an operational one that focuses on the measurement result and its evaluated uncertainty. However, it is not inconsistent with other concepts of uncertainty of measurement, such as

– a measure of the possible error in the estimated value of the measurand as provided by the result of a measurement;
– an estimate characterizing the range of values within which the true value of a measurable lies (VIM:1984, definition 3.09).

Although these two traditional concepts are valid as ideals, they focus on unknowable quantities: the “error” of the result of a measurement and the “true value” of the measurand (in contrast to its estimated value), respectively. Nevertheless, whichever concept of uncertainty is adopted, an uncertainty component is always evaluated using the same data and related information [1.2.2.4].

Without distinguishing the two views, the above part of the GUM appears confusing and inconsistent. The GUM seems to question the validity of the concepts “true value” and “error”, but at the same time regards the concepts to be not inconsistent with its concept of uncertainty. Here, the GUM is acknowledging the existence of two fundamentally different views of measurement, one with true value and error, and one without, and stating that its definition of uncertainty tries to accommodate both views.

The GUM definition of uncertainty is “parameter, associated with the result of a measurement that characterizes the dispersion of the values that could reasonably be attributed to the measurand” [1, 2.2.3]. This definition can accommodate both the realist and the instrumentalist views because it describes measurement as a process of attributing reasonable values to a measurand [7, 41]. The realist finds this description to mean that measurement is an activity of attributing reasonable estimates of true values to a measurand. The instrumentalist finds this description to recognize measurement as a process of assigning reasonable values to a measurand following accepted operations. In other words, the description of measurement as a process of value attribution allows the views with and without true value to coexist [41].

Similarly, consider the definition of “expanded uncertainty” in the GUM: “quantity defining an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand” [1, 2.3.5]. To the realist, this definition is unnecessarily complicated. The realist would find definitions such as “quantity defining an interval about the result of a measurement that may be expected to encompass the true value of the measurand at a high probability” to be clearer [7]. However, such a definition is unacceptable to the instrumentalist. In comparison, the GUM definition is acceptable to both the realist and the instrumentalist.

Allowing different views of measurement to coexist is reasonable because the choice of either view usually does not lead to practical differences. Other publications such as the VIM3 have also been written in a way to allow the different views to coexist [7, 41]. However, it is important to clearly distinguish the two views in conceptual discussions if confusion and miscommunication are to be avoided.

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3 In philosophy, this use of words is consistent with the redundancy theory of truth [23], which was pioneered by the Bayesian mathematician Frank Ramsey. According to the redundancy theory of truth, the phrase “it is true” is redundant because stating “it is true that Caes- sar was murdered” is identical to stating “Caesar was murdered” [68].
Availability of data and material Not applicable.

Declarations

Conflict of interest Not applicable.

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