Justification of the choice of numerical methods in the study of nonlinear micropolar mesh cylindrical panel’s oscillations

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Abstract. Based on the micro polar and the Kirchhoff-Love theories, the mathematical model of the cylindrical mesh panel’s oscillations is constructed. The panels are consisting of two families of mutually perpendicular edges. The scenarios of the transition of panel oscillations to chaos are investigated. To justify the reliability of the results obtained, the numerical implementation was carried out by fundamentally different numerical methods. The conclusion is drawn about the optimal combinations of methods for the numerical implementation of the task.

1. Introduction
Mesh cylindrical plates, panels and shells are widely used as elements of engineering structures in strategically important sectors of the economy: instrumentation, robotics, medicine. Therefore, many researches have been devoted to studying the effect of various loads on their behavior, but they do not take into account the large-scale effect that has a significant effect when reducing the size of the mechanical systems under consideration to a micro and nano level. Under certain parameters of external influences, a mechanical system can go into a chaotic dynamic mode, which, in turn, can lead to errors in the operation of devices and mechanisms, of which it is an element, or even to their destruction.

2. Problem formulation
The object of this study is a shallow cylindrical panel consisting of two families of densely spaced edges of the same material under the normal distributed load action of the form \( q_0 \sin(\omega_p t) \), where \( q_0 \) - is the intensity and \( \omega_p \) - is the frequency of the load. To take into account scale effects, the mathematical model of oscillations of the studied mechanical system is constructed on micro polar (asymmetric moment) theory basis with constrained particle rotation [1-6]. Geometric nonlinearity is taken into account according to T. von Karman theory. According to Pshenichnov continuum model [7] we will replace the regular system of edges with a continuous layer. As a result of this, the stresses arising in
the equivalent smooth panel associated with the stresses in the ribs constituting the angles \( \varphi_j \) with the abscissa axis will have the form:

\[
\begin{align*}
\sigma_{xx} &= \sum_{j=1}^{2} \frac{\sigma_j^i \delta_j \cos \varphi_j}{a_j}, \quad \sigma_{yy} = \sum_{j=1}^{2} \frac{\sigma_j^i \delta_j \sin \varphi_j}{a_j}, \quad \sigma_{xy} = \sum_{j=1}^{2} \frac{\sigma_j^i \delta_j \cos \varphi_j \sin \varphi_j}{a_j}, \\
m_{ix} &= \sum_{j=1}^{2} \frac{m_j^i \delta_j \cos \varphi_j}{a_j}, \quad m_{iy} = \sum_{j=1}^{2} \frac{m_j^i \delta_j \sin \varphi_j}{a_j}, \quad m_{ixy} = \sum_{j=1}^{2} \frac{m_j^i \delta_j \cos \varphi_j \sin \varphi_j}{a_j},
\end{align*}
\]

(1)

where \( a_j \) - is the distance between the ribs of the \( j \)th family, \( \delta_j \) - is the thickness of the \( j \)th family ribs, \( \varphi_j \) - is the angle between the axis and the \( j \)th family ribs, \( \sigma_{ij} \) and \( m_{ij} \) - are the stress tensor components and moment tensor of the smooth panel, the stresses with index \( j \) refer to the rods. The physical relationships for the mesh panel are determined based on the Lagrange multiplier method:

\[
\begin{align*}
\sigma_i^j &= \sigma_{xi} \cos^2 \varphi_j + \sigma_{yi} \sin^2 \varphi_j + \sigma_{xy} \cos \varphi_j \sin \varphi_j; \quad \tau_i^j = \tau_{xi} \cos \varphi_j + \tau_{yi} \sin \varphi_j; \\
m_{ix}^j &= m_{ix} \cos^2 \varphi_j + m_{iy} \sin^2 \varphi_j + m_{ixy} \cos \varphi_j \sin \varphi_j; \quad m_{iy}^j = m_{ix} \cos \varphi_j + m_{iy} \sin \varphi_j.
\end{align*}
\]

(2)

For the plate material, the defining relations are taken in the form:

\[
\sigma_{xx} = \frac{E}{1-\nu^2} \left[ e_{xx} + \nu e_{yy} \right], \quad \sigma_{yy} = \frac{E}{(1+\nu)} e_{yy}, \quad (m_{ix}, m_{iy}, m_{ixy}) = \frac{E^2 l^2}{1+\nu} (\chi_{xx}, \chi_{xy}, \chi_{yx}),
\]

(3)

where \( E \) - Young's modulus, \( \nu \) - Poisson's ratio, \( l \) - additional independent length parameter associated with the bending-torsion tensor \( \chi \). Strain tensor components \( e_{ij} \), taking into account the kinematic hypotheses of Kirchhoff-Love, will have the form:

\[
\begin{align*}
e_{xx} &= \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 - \frac{1}{2} \frac{\partial^2 w}{\partial x^2}, \quad e_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2 - \frac{1}{2} \frac{\partial^2 w}{\partial y^2}, \\
e_{xy} &= \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) - \frac{1}{2} \frac{\partial^2 w}{\partial x \partial y}.
\end{align*}
\]

(4)

The components of the torsion bending tensor \( \chi_{ij} \), under the assumption that the displacements and rotations fields are not independent, can be written as follows:

\[
\begin{align*}
\chi_{xx} = \frac{\partial^2 w}{\partial x \partial y}, \quad \chi_{yy} = -\frac{\partial^2 w}{\partial y \partial x}, \quad \chi_{xy} = \frac{1}{2} \left( \frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 w}{\partial x^2} \right), \quad \chi_{xx} = \frac{1}{4} \left( \frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} \right), \\
\chi_{yx} = \frac{1}{4} \left( \frac{\partial^2 v}{\partial y^2} - \frac{\partial^2 u}{\partial x^2} \right).
\end{align*}
\]

(5)
Here \( w \) - deflection, \( u, v \) - axial displacements of the plate middle surface in the directions \( x, y \) respectively, \( k_y \) - panel curvature parameter.

Balance equations, boundary and initial conditions for a smooth shell are obtained from the Ostrogradsky-Hamilton energy principle. Substituting expressions (1-2) into them and according to (3-5), and also setting \( \delta_1 = \delta_2 = \delta \), \( a_1 = a_2 = a \), \( \varphi_1 = 45^0 \), \( \varphi_2 = 135^0 \) we obtain the equations for a micro polar acclivous cylindrical panel consisting of two identical mutually perpendicular families of ribs (6). The system is reduced to dimensionless form in a standard way [8].

\[
2(\nu-1)\frac{\partial^2 u}{\partial y^2} - 2(3 + \nu)\frac{\partial^2 v}{\partial x \partial y} - 4(\nu + 1)\frac{h^2}{c^2} \frac{\partial^2 u}{\partial x^2} - 4k_y(\nu + 1)\frac{b}{h} \frac{\partial w}{\partial x} + 4(\nu-1)\frac{\partial^2 w}{\partial y^2} \frac{\partial w}{\partial x} - 8\frac{\partial w}{\partial y} \frac{\partial^2 w}{\partial x \partial y} - 4(1+\nu)\frac{b^2}{c^2} \frac{\partial^2 w}{\partial x^2} + + l^2(\nu-1)\left(-\frac{h^2}{b^2} \frac{\partial^2 u}{\partial x \partial y} - \frac{h^2}{c^2} \frac{\partial^2 v}{\partial x^2} + \frac{h^2}{b^2} \frac{\partial^2 u}{\partial x \partial y} + \frac{h^2}{c^2} \frac{\partial^2 v}{\partial x^2}\right) = \frac{8h^2a(\nu^2-1)}{c^2} \frac{\partial^2 u}{\partial t^2} - 4k_y(1+\nu)\frac{c^2}{bh} \frac{\partial w}{\partial y} - 2(3 + \nu)\frac{\partial^2 u}{\partial x \partial y} - 2(1-\nu)\frac{\partial^2 v}{\partial x^2} - 4(1+\nu)\frac{c^2}{b^2} \frac{\partial w}{\partial x} \frac{\partial^2 w}{\partial y^2} - 8\frac{\partial w}{\partial x} \frac{\partial^2 w}{\partial y^2} + l^2(\nu-1)\left(\frac{h^2}{b^2} \frac{\partial^2 u}{\partial x \partial y} - \frac{h^2}{b^2} \frac{\partial^2 v}{\partial x^2} + \frac{h^2}{c^2} \frac{\partial^2 u}{\partial x \partial y} - \frac{h^2}{c^2} \frac{\partial^2 v}{\partial x^2}\right) = \frac{h^28a(\nu^2-1)}{b^2} \frac{\partial^2 v}{\partial t^2}
\]

\[
(1 + v) + 6l(1 - v)\left[\frac{\partial^2 u}{\partial x \partial y} + 4(1+\nu)\frac{c^2}{b^2} \frac{\partial^2 w}{\partial x \partial y} + \left(1 + v\right) + 6l(1 - v)\right] = \frac{b^2}{c^2} \frac{\partial^2 w}{\partial x^2} + + 12k_y(1+\nu)\frac{c^2}{bh} \frac{\partial w}{\partial y} + 12k_y(1+\nu)\frac{b^2}{bh} \frac{\partial u}{\partial x} - 6k_y(1+\nu)\frac{c^2}{bh} \left(\frac{\partial w}{\partial x}\right)^2 - -12(1+\nu)\frac{c^2}{b^2} \frac{\partial w}{\partial y} - 12k_y(1+\nu)\frac{c^2}{bh} \frac{\partial^2 w}{\partial x \partial y} - 12(1+\nu)\frac{c^2}{b^2} \frac{\partial v}{\partial x} - 12(1+\nu)\frac{c^2}{b^2} \frac{\partial v}{\partial x} - 18(1+\nu)\left(\frac{\partial w}{\partial x}\right)^2 \frac{\partial^2 w}{\partial y^2} - -12(1+\nu)\frac{\partial^2 w}{\partial x \partial y} + 12(1+\nu)\frac{c^2}{bh} \frac{\partial^2 w}{\partial x \partial y} - 6k_y(1+\nu)\frac{b}{h} \left(\frac{\partial w}{\partial x}\right)^2 + 6(-5+3\nu)\frac{\partial^2 w}{\partial x \partial y} + -24\frac{\partial w}{\partial y} \frac{\partial^2 w}{\partial x \partial y} + 24(1+\nu)\frac{\partial u}{\partial x} \frac{\partial^2 w}{\partial x \partial y} + 24(1-\nu)\frac{\partial v}{\partial x} \frac{\partial^2 w}{\partial x \partial y} + 24(3-5)\frac{\partial w}{\partial y} \frac{\partial^2 w}{\partial x \partial y} + -12(1+\nu)\frac{\partial^2 u}{\partial x \partial y} + 12(\nu-1)\frac{b}{h} \left(\frac{\partial w}{\partial x}\right)^2 - 12(1+\nu)\frac{b}{h} \frac{\partial^2 w}{\partial x \partial y} - 12k_y(1+\nu)\frac{b}{h} \left(\frac{\partial w}{\partial x}\right)^2 + +6(3\nu-5)\left(\frac{\partial^2 w}{\partial x \partial y}^2 - 12(1+\nu)\frac{\partial w}{\partial x} \frac{\partial^2 w}{\partial x \partial y} - 18(1+\nu)\frac{b^2}{c^2} \left(\frac{\partial w}{\partial x}\right)^2 \frac{\partial^2 w}{\partial y^2} = \frac{24a}{c^2} (\nu^2-1) \left[ \frac{\partial^2 w}{\partial x \partial y} - \frac{\partial w}{\partial t} - q \right]
\]

Here \( c, b \) - are the linear panel dimensions along \( x \) and \( y \) accordingly, \( \varepsilon \) - is the dissipation coefficient of the medium. To the equations we attach zero initial conditions and boundary conditions for rigidly fixing the ends of the panel.

Due to the differential problem complexity describing the nonlinear dynamics of mesh cylindrical panels, it is not possible to find its analytical exact solution. The only way out is to solve such problems by numerical methods. But then the question arises of the reliability of the decisions obtained. To justify
the reliability of the results obtained, the numerical implementation in the problems of studying the vibrational modes of nonlinear micro polar mesh cylindrical panels in this work was carried out by fundamentally different numerical methods. To reduce the partial differential problem to the ordinary differential problem with respect to spatial variables, we used the Bubnov-Galerkin methods in higher approximations and finite differences method with a second-order approximation. The Cauchy problem was also solved by several methods: 4th and 2nd order Runge-Kutta (RK), 4th order RK Fehlberg method, 4th order Cash-Karp method, 8th order RK Dormand-Prince method, implicit RK method of the 2nd order and 4th order, as well as the Newmark method. The explicit method is characterized by the fact that the coefficients matrix has a lower triangular form (including the zero main diagonal) - in contrast to the implicit method, where the matrix has an arbitrary form. The methods of the RK Fehlberg, Cash-Karp, RK Prince-Dormand provide for automatic step change, as well as the ability to control the integration error. For each combination of methods, a scenario was constructed for oscillations transition of the cylindrical panel under consideration to chaos, where the control parameter was the intensity of the external normal distributed load $q_0$. To do this, for each fixed values $q_0$, signals, phase 2D and 3D portraits, Fourier power spectra, Morlet wavelets, cross-section and Poincare maps were constructed.

3. Numerical experiment

With the following experimental parameters: $\omega_p = 5$, $l = 0.5$, $v = 0.3$, $a = \delta = h = 0.002$, $\varepsilon = 1$, $k_p = 12$, $c = b = 1$, $q_0 \in [0;200]$, $t \in [0;512]$ for all the considered combinations of numerical methods, the Ruelle-Takens-Newhouse scenario was obtained, that is, the chaotic oscillations transition was carried out through two independent frequencies and their linear combinations.

The convergence of various numerical methods was studied to reduce the system of partial differential equations to the Cauchy problem. It was revealed that the finite difference method for a chaotic signal converges at 32 split points (32 x 32), the Bubnov-Galerkin method with the number of series members $N = 11$. The Bubnov-Galerkin method requires two times less machine time, however, difficulties arise when choosing an approximating system of functions satisfying the boundary conditions. The finite difference method allows us to consider a greater variety of boundary conditions and zero initial conditions.

4. Conclusion

Based on the numerical experiment results, the Newmark method was chosen from all the considered methods to solve the Cauchy problem. The solutions obtained by the Runge-Kutta methods higher the fourth order accuracy and the Newmark method are the same, but the latter requires significantly less machine time.

Acknowledgments

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