Improvements for multi-objective flow shop scheduling by Pareto Iterated Local Search

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1 Introduction

The flow shop scheduling problem consists in the assignment of a set of jobs \( J = \{J_1, \ldots, J_n\} \), each of which consists of a set of operations \( J_j = \{O_{j1}, \ldots, O_{joj}\} \) onto a set of machines \( M = \{M_1, \ldots, M_m\} \) [5, 18]. Each operation \( O_{jk} \) is processed by at most one machine at a time, involving a non-negative processing time \( p_{jk} \). The result of the problem resolution is a schedule \( x \), defining for each operation \( O_{jk} \) a starting time \( s_{jk} \) on the corresponding machine. Several side constraints are present which have to be respected by any solution \( x \) belonging to the set of feasible schedules \( X \).

Precedence constraints \( O_{jk} > O_{jk+1} \forall j = 1, \ldots, n, k = 1, \ldots, o_j - 1 \) between the operations of a job \( J_j \) assure that processing of \( O_{jk+1} \) only commences after completion of \( O_{jk} \), thus \( s_{jk+1} \geq s_{jk} + p_{jk} \).

In flow shop scheduling, the machine sequence in which the operations are processed by the machines is identical for all jobs, and for the specific case of the permutation flow shop scheduling the job sequence must also be the same on all machines.

The assignment of starting times to the operations has to be done with respect to one or several optimality criteria. Most optimality criteria are functions of the completion times \( C_j \) of the jobs \( J_j \), \( C_j = s_{joj} + p_{joj} \).

The most prominent optimality criteria are the maximum completion time (makespan) \( C_{max} = \max C_j \) and the sum of the completion times \( C_{sum} = \sum_{j=1}^n C_j \). Others express violations of due dates \( d_j \) of jobs \( J_j \). A due date \( d_j \) defines a latest point of time until production of a job \( J_j \) should be completed as the finished product has to be delivered to the customer on or up to this date. A possible optimality criteria based on tardiness of jobs is e.g. the total tardiness \( T_{sum} = \sum_{j=1}^n T_j \), where \( T_j = \max(C_j - d_j, 0) \).

It is known, that for regular optimality criteria at least one active schedule \( x \) does exist which is also optimal. The representation of an active schedule is possible using a permutation of jobs \( \pi = (\pi_1, \ldots, \pi_n) \), where each \( \pi_j \) stores a job \( J_k \) at position \( j \). The permutation is then decoded into an active schedule by assuming the starting times of all operations as early as possible with respect to the precedence constraints and the given sequence in \( \pi \). As a consequence, the search is, instead of searching all possible schedules, restricted to the much smaller set of active schedules only.
Multi-objective approaches to scheduling consider a vector \( G(x) = (g_1(x), \ldots, g_K(x)) \) of optimality criteria at once [21]. As the relevant optimality criteria are often of conflicting nature, not a single solution \( x \in X \) exists optimizing all components of \( G(x) \). Optimality in multi-objective optimization problems is therefore understood in the sense of Pareto-optimality, and the resolution of multi-objective optimization problems lies in the identification of all elements belonging to the Pareto set \( P \), containing all alternatives \( x \) which are not dominated by any other alternative \( x' \in X \).

Several approaches of metaheuristics have been formulated and tested in order to solve the permutation flow shop scheduling problem under multiple, in most cases two, objectives. Common to all is the representation of solutions using permutations \( \pi \) of jobs, as in previous investigations only regular functions are considered.

First results have been obtained using Evolutionary Algorithms, which in general play a dominant role in the resolution of multi-objective optimization problems when using metaheuristics. This is mainly due to the fact that these methods incorporate the idea of a set of solutions, a so called population, as a general ingredient. Flow shop scheduling problems minimizing the maximum completion time and the average flow time have been solved by NAGAR, HERAGU and HADDOCK [17]. In their work, they however combine the two objectives into a weighted sum. Problems minimizing the maximum completion time and the total tardiness are solved by MURATA, ISHIBUCHI and TANAKA [16], again under the combination of both objectives into a weighted sum. Later work on the same problem class by BASSEUR, SEYNAEVE and TALBI [4] avoids the weighted sum approach, using dominance relations among the solutions only.

Most recent work is presented by LOUKIL, TEGHEM and TYTTYTENS [14]. Contrary to approaches from Evolutionary Computations, the authors apply the Multi Objective Simulated Annealing approach MOSA of ULINGU, TEGHEM, FORTEMPS and TYTTYTENS [22] to a variety of bi-criterion scheduling problems.

Flow shop scheduling problems with three objectives are studied by ISHIBUCHI and MURATA [11], and ISHIBUCHI, YOSHIDA and MURATA [12]. The authors minimize the maximum completion time, the total completion time, and the maximum tardiness at once. A similar problem minimizing the maximum completion time, the average flow time, and the average tardiness is then tackled by BAGCHI [1, 2].

2 Pareto Iterated Local Search

The Pareto Iterated Local Search (PILS) metaheuristic is a concept for the solution of multi-objective optimization problems. It combines the two main driving forces of local search, intensification and diversification, into a single algorithm, and extends the work presented in [8]. The motivation behind this concept can be seen in the increasing demand for simple, yet effective heuristics for the resolution of complex multi-objective optimization problems. Two developments in local search demonstrate the effectiveness of some intelligent ideas that make use of certain structures within the search space topology of problems:

1. Iterated Local Search, introducing the idea of perturbing solutions to overcome local optimality and continue search in interesting areas of the search space [15]. After the pioneering work of BOESE [6], who investigated properties of the search space of the traveling salesman problem, this concept has been used with increasing success on problems where solutions of high quality can be found relatively concentrated in alternative space.
2. Second, Variable Neighborhood Search [10], combining multiple neighborhood operators into a single algorithm in order to avoid local optimality in the first place.

In the proposed concept, both paradigms are combined and extended within a search framework handling not only a single but a set of alternatives at once.

The main principle of the algorithm is sketched in Figure 1. Starting from an initial solution $x_1$, an improving, intensifying search is performed until a set of locally optimal alternatives is identified, stored in a set $P^{approx}$ representing the approximation of the true Pareto set $P$. No further improvements are possible from this point. In this initial step, a set of neighborhoods ensures that all identified alternatives are locally optimal not only to a single but to a set of neighborhoods. This principle, known from Variable Neighborhood Search, promises to lead to better results as it is known that all global optima are also locally optimal with respect to all possible neighborhoods while this is not necessarily the case for local optima.

![Figure 1: Illustration of the Pareto Iterated Local Search metaheuristic. The archive of the currently best solutions is updated during the search. Here, $G(x_4)$ dominates $G(x_2)$ which is going to be deleted from $P^{approx}$.](image)

After the identification of a locally optimal set, a diversification step is performed on a solution $x_2$ using a perturbation operator, continuing search from the perturbed solution $x_3$. The perturbation operator has to be significantly different from the neighborhoods used in intensification, as otherwise the following search would return to the previous solution. On the other hand however, the perturbation should not entirely destroy the characteristics of the alternative. Doing that would result in a random restart of the search without keeping promising attributes of solutions.

The PILS metaheuristic may be formalized as given in Algorithm 1. The intensification of the algorithm, illustrated in steps (1) and (3) of Figure 1 is between the lines 6 to 21, the description of the diversification, given in step (2) of Figure 1 is between the lines 22 to 26.

It can be seen, that the algorithm computes a set of neighborhoods for each alternative. The sequence in which the neighborhoods are computed is arranged in a random fashion, described in line 13 of Algorithm 1. This introduces an additional element of diversity to the concept, as otherwise the search might be biased by a certain sequence of neighborhoods.
Algorithm 1 Pareto Iterated Local Search

1: Initialize control parameters: Define the neighborhoods \( N_1, \ldots, N_k \)
2: Set \( i = 1 \)
3: Generate initial solution \( x \)
4: Set \( P^{\text{approx}} = \{ x \} \)
5: repeat
6: repeat
7: Compute \( N_i(x) \)
8: Evaluate \( N_i(x) \)
9: Update \( P^{\text{approx}} \) with \( N_i(x) \)
10: if \( \exists x' \in N_i(x) \mid x' \preceq x \) then
11: Set \( x = x' \)
12: Set \( i = 1 \)
13: Rearrange the neighborhoods \( N_1, \ldots, N_k \) in some random order
14: else
15: Set \( i = i + 1 \)
16: end if
17: until \( x \) locally optimal with respect to \( N_1, \ldots, N_k \), therefore \( i > k \)
18: Set neighborhoods of \( x \) as ‘investigated’
19: Set \( i = 1 \)
20: if \( \exists x' \in P^{\text{approx}} \mid \text{neighborhoods not investigated yet} \) then
21: Set \( x = x' \)
22: else
23: Randomly select some \( x' \in P^{\text{approx}} \)
24: Compute \( x'' = N_{\text{perturb}}(x') \)
25: Set \( x = x'' \)
26: end if
27: until termination criterion is met

3 Experiments on multi-objective flow shop scheduling problems

3.1 Algorithm configuration and experimental setup

In the following, the Pareto Iterated Local Search is applied to a set of benchmark instances of the multi-objective permutation flow shop scheduling problem. The first instances have been provided by BASSEUR, SEYNAEVE and TALBI [4], who defined due dates for the well-known instances of TAILLARD [20]. The instances range from \( n = 20 \) jobs that have to be processed on \( m = 5 \) machines to \( n = 100, m = 20 \). All of them are solved under the simultaneous consideration of the minimization of the maximum completion time \( C_{\text{max}} \) and the total tardiness \( T_{\text{sum}} \) and are referred to as ‘Ta \( n \times m \)’.

We also solved the benchmark instance ‘Ba 49 \( \times 15 \)’ by BAGCHI [1], consisting of \( n = 49 \) jobs on \( m = 15 \) machines. The three objective functions of the data set are the minimization of the maximum completion time \( C_{\text{max}} \), the minimization of the average completion time \( \frac{1}{n}C_{\text{sum}} \), and the minimization of the average tardiness \( \frac{1}{n}T_{\text{sum}} \).

Three operators are used in the definition of the neighborhoods \( N_1, \ldots, N_k \), described in the
work of Reeves [19]. First, an exchange neighborhood, exchanging the position of two jobs in \( \pi \), second, a forward shift neighborhood, taking a job from position \( i \) and reinserting it at position \( j \) with \( j < i \), and finally a backward shift neighborhood, shifting a job from position \( i \) to \( j \) with \( j < i \). All operators are problem independent operators for permutation-based representations, each computing \( \frac{n(n-1)}{2} \) neighboring solutions.

After a first approximation \( P_{\text{approx}} \) of the Pareto set is obtained, one element \( x' \) \( P_{\text{approx}} \) is selected by random and perturbed into another solution \( x'' \). We use a special neighborhood that on one hand leaves most of the characteristics of the perturbed alternatives intact, while still changes the positions of some jobs. Also, several consecutive applications of the neighborhoods \( N_1, \ldots, N_k \) would be needed to transform \( x'' \) back into \( x' \). This is important, as otherwise the algorithm might return to the initially perturbed alternative \( x' \), leading to a cycle in the search path. The perturbation neighborhood \( N_{\text{perturb}} \) can be described as follows. First, a subset of \( \pi \) is randomly selected, comprising four consecutive jobs at positions \( j, j+1, j+2, j+3 \). Then a neighboring solution \( x'' \) is generated by moving the job at position \( j \) to \( j+3 \), the one at position \( j+1 \) to \( j+2 \), the one at position \( j+2 \) to \( j \), and the job at position \( j+3 \) to \( j+1 \), leaving the jobs at the positions \( < j \) and \( > j+3 \) untouched. In brief, this leads to a combination of several exchange and shift moves, executed at once.

In order to analyze the quality of the approximations, we compare the results obtained by PILS to the approximations of a multi-objective multi-operator search algorithm MOS, described in Algorithm 2.

**Algorithm 2 Multi-objective multi-operator search framework**

1: Generate initial solution \( x \), set \( P_{\text{approx}} = \{ x \} \)
2: \textbf{repeat}
3: \hspace{1em} Randomly select some \( x \in P_{\text{approx}} \mid \) neighborhoods not investigated yet
4: \hspace{1em} Randomly select some neighborhood \( N_i \) from \( N_1, \ldots, N_k \)
5: \hspace{1em} Generate \( N_i(x) \)
6: \hspace{1em} Update \( P_{\text{approx}} \) with \( N_i(x) \)
7: \hspace{1em} \textbf{if} \( x \in P_{\text{approx}} \) \textbf{then}
8: \hspace{2em} Set neighborhoods of \( x \) as ‘investigated’
9: \hspace{1em} \textbf{end if}
10: \hspace{1em} \textbf{until} \( \forall x \in P_{\text{approx}} \mid \) neighborhoods not investigated yet
11: Return \( P_{\text{approx}} \)

The MOS Algorithm, taken from [8], is based on the concept of Variable Neighborhood Search, extending the general idea of several neighborhood operators by adding an archive \( P_{\text{approx}} \) towards the optimization of multi-objective problems. For a fair comparison, the same neighborhood operators are used as in the PILS algorithm. After the termination criterion is met in step 10, we restart search while keeping the approximation \( P_{\text{approx}} \) for the final analysis of the quality of the obtained solutions.

### 3.2 Results

The benchmark instances of Basseur and the one of Bagchi have been solved using the PILS algorithm. In each of the 100 test runs, the approximation quality of the obtained results has been
analyzed using the $D_1$ and $D_2$ metrics of Czyżak and Jaskiewicz [7]. The two metrics have been chosen for the analysis as they provide an interesting interpretation from an economical point of view. Based on a so called ‘achievement scalarizing function’, they compute the average ($D_1$) and the maximum ($D_2$) regret a decision maker would have to face when trying to select a certain most preferred alternative $x^* \in P$, approximated by the results in $P^{\text{approx}}$.

While for the smaller instances the optimal solutions are known, the analysis for the larger instances has to rely on the best known results published in the literature. Experiments have been carried out on a Intel Pentium IV processor, running at 1.8 GHz. Table 1 gives an overview about the number of evaluations executed for each instance. Clearly, considerable more alternatives have to be evaluated with increasing size of the problem instances to allow a convergence of the algorithm. Also, the running times, given in Table 1, too, increase with increasing size of the problem instances. No significant difference in running behavior can be found when comparing the two metaheuristics PILS and MOS. Apart from some minor differences around the perturbation neighborhood $N_{\text{perturb}}$, the approaches are identical with respect to the impact on the resulting running times as they use the same neighborhood operators.

| Instance $n \times m$ | No of evaluations | Eval. time | Neighbor. comp. time |
|-----------------------|-------------------|------------|----------------------|
| Ta 20 × 5 (#1)        | 1,000,000         | 42.7       | 0.5                  |
| Ta 20 × 5 (#2)        | 1,000,000         | 42.5       | 0.6                  |
| Ta 20 × 10 (#1)       | 1,000,000         | 78.0       | 0.6                  |
| Ta 20 × 10 (#2)       | 1,000,000         | 83.3       | 0.6                  |
| Ta 20 × 20            | 1,000,000         | 143.5      | 0.6                  |
| Ta 50 × 5             | 5,000,000         | 195.1      | 1.3                  |
| Ta 50 × 10            | 5,000,000         | 386.1      | 1.3                  |
| Ta 50 × 20            | 5,000,000         | 754.3      | 1.3                  |
| Ta 100 × 10           | 10,000,000        | 1459.7     | 2.5                  |
| Ta 100 × 20           | 10,000,000        | 2885.3     | 2.5                  |
| Ba 49 × 15            | 5,000,000         | 566.3      | 1.2                  |

Times are given in milliseconds.

An implementation of the algorithm has been made available within an integrated software for the resolution of multi-objective scheduling problems using metaheuristics. The system is equipped with an extensive user interface that allows an interaction with a decision maker and is able to visualize the obtained results in alternative and outcome space. The system also allows the comparison of results obtained by different metaheuristics.

The average values obtained by the investigated metaheuristics are given in Table 2. It can be seen, that PILS leads for all investigated problem instances to better results for both the $D_1$ and the $D_2$ metric. This general result is consistently independent from the actual problem instance and significant at a level of significance of 0.01. For a single instance, the ‘Ta 20 × 5 (#1)’, PILS was able to identify all optimal solutions in all test runs, leading to average values of $D_1 = D_2 = 0.0000$. Apparently, this instance is comparably easy to solve.

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Table 2: Average results of $D_1$ and $D_2$

| Instance $n \times m$ | PILS  | MOS  | PILS  | MOS  |
|-----------------------|-------|------|-------|------|
| Ta $20 \times 5$ (#1) | 0.0000| 0.0323| 0.0000| 0.1258|
| Ta $20 \times 5$ (#2) | 0.1106| 0.1372| 0.3667| 0.4249|
| Ta $20 \times 10$ (#1)| 0.0016| 0.0199| 0.0146| 0.0598|
| Ta $20 \times 10$ (#2)| 0.0011| 0.0254| 0.0145| 0.1078|
| Ta $20 \times 20$    | 0.0088| 0.0286| 0.0400| 0.1215|
| Ta $50 \times 5$     | 0.0069| 0.0622| 0.0204| 0.1119|
| Ta $50 \times 10$    | 0.0227| 0.3171| 0.0897| 0.4658|
| Ta $50 \times 20$    | 0.0191| 0.3966| 0.0616| 0.5609|
| Ta $100 \times 10$   | 0.0698| 0.3190| 0.1546| 0.4183|
| Ta $100 \times 20$   | 0.0013| 0.2349| 0.0255| 0.3814|
| Ba $49 \times 15$    | 0.0202| 0.2440| 0.0701| 0.3414|

While it was possible to show in [8] that the MOS algorithm is competitive to different Evolutionary Algorithms, iterating search in qualitatively good areas of the search space by PILS improves the results even further. Recalling, that with increasing problem size an increasing amount of time is needed to evaluate the alternatives, iterating in promising regions becomes even more interesting as opposed to restarting the search.

A deeper analysis has been performed to monitor the resolution behavior of the local search algorithms and to get a better understanding of how the algorithm converges towards the Pareto front. Figure 2 (a) plots with symbol $\times$ the results obtained by random sampling 50,000 alternatives for the problem instance ‘Ta $100 \times 10$’, and compares the points obtained during the first intensification procedure of PILS until a locally optimal set is identified. The alternatives computed starting from a random initial solution towards the Pareto front are plotted as $+$, the Pareto front as $\circ$. It can be seen, that in comparison to the initial solution even a simple local search approach converges in rather close proximity to the Pareto front. With increasing number of computations however, the steps towards the optimal solutions get increasingly smaller, as it can be seen when monitoring the distances between the $+$ symbols. After convergence towards a locally optimal set, overcoming local optimality is then provided by means of the perturbation neighborhood $N_{perturb}$.

An interesting picture is obtained when analyzing the distribution of the randomly sampled 50,000 alternatives for instance ‘Ta $100 \times 10$’. In Figure 2 (b), the number of alternatives with a certain combination of objective function values are plotted and compared to the Pareto front, given in the left corner. It turns out that many alternatives are concentrated around some value combination in the area of approximately $C_{max} = 6900$, $T_{sum} = 111500$, relatively far away from the Pareto front.

When analyzing the convergence of local search heuristics toward the globally Pareto front as well as towards locally optimal alternatives, the question arises how many local search steps are necessary until a locally optimal alternative is identified. From a different point of view, this problem is discussed in the context of computational complexity of local search [13]. It might be worth investigating this behavior in quantitative terms. Table 3 gives the average number of
evaluations that have been necessary to reach a locally optimal alternative from some randomly generated initial solution. The analysis reveals that the computational effort grows exponentially with the number of jobs $n$.

Table 3: Average number of evaluations until a locally optimal alternative is reached

| Instance $n \times m$ | No of jobs | No of evaluations | Instance $n \times m$ | No of jobs | No of eval. |
|-----------------------|------------|-------------------|-----------------------|------------|-------------|
| Ta 20 × 5 (#1)        | 20         | 3,614             | Ta 50 × 5             | 50         | 53,645      |
| Ta 20 × 5 (#2)        | 20         | 3,292             | Ta 50 × 10            | 50         | 55,647      |
| Ta 20 × 10 (#1)       | 20         | 2,548             | Ta 50 × 20            | 50         | 38,391      |
| Ta 20 × 10 (#2)       | 20         | 2,467             | Ta 100 × 10           | 100        | 793,968     |
| Ta 20 × 20            | 20         | 2,657             | Ta 100 × 20           | 100        | 479,420     |
| Ba 49 × 15            | 49         | 28,908            |

4 Conclusions

In the past years, considerable progress has been made in solving complex multi-objective optimization problems. Effective metaheuristics have been developed, providing the possibility of computing approximations to problems with numerous objectives and complex side constraints. While many approaches are of increasingly effectiveness, complex parameter settings are however required to tune the solution approach to the given problem at hand.

The algorithm presented in this paper proposed a metaheuristic, combining two recent principles...
of local search, Variable Neighborhood Search and Iterated Local Search. The main motivation behind the concept is the easy yet effective resolution of multi-objective optimization problems with an approach using only few parameters.

After an initial introduction to the problem domain of flow shop scheduling under multiple objectives, the introduced PILS algorithm has been applied to a set of scheduling benchmark instances taken from literature. We have been able to obtain encouraging results, despite the simplicity of the algorithmic approach. A comparison of the approximations of the Pareto sets has been given with a multi-operator local search approach, and, as a conclusion, PILS was able to lead to consistently better results. We had however to observe, that with increasing problem size, the number of iterations needed to identify an only locally optimal solutions grows exponentially. Nevertheless, the presented approach seems to be a promising tool for the effective resolution of multi-objective optimization problems. After first tests on problems from the domain of scheduling, the resolution behavior on problems from other areas might be an interesting direction for further research.

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