On compatibility of the Kaluza-Klein approach with the COBE experiment *

Yuri Kubyshin †‡
Departamento de Física Teórica C-XI
Universidad Autónoma de Madrid
Cantoblanco, 28049 Madrid, Spain
and
Jérôme Martin §
Laboratoire de Gravitation et Cosmologies Relativistes
Université Pierre et Marie Curie
Tour 22/12, Boite courrier 142, 4 place Jussieu
75252 Paris Cedex 05, France

April 15, 1996

Abstract

Contributions of primordial gravitational waves to the large-angular-scale anisotropies of the cosmic microwave background radiation in multidimensional cosmological models (Kaluza-Klein models) are studied. We derive limits on free parameters of the models using results of the COBE experiment and other astrophysical data. It is shown that in principle there is a room for Kaluza-Klein models as possible candidates for the description of the Early Universe. However, the obtained limits are very restrictive. Assuming that the anisotropies are mostly due to gravitational waves, none of the concrete models, analyzed in the article, satisfy them. On the other hand, if the contribution of gravitational waves is very small then a string inspired model is not ruled out.

*Extended version of the contribution to the Proceedings of the Xth Workshop on High Energy Physics and Quantum Field Theory, Zvenigorod (Russia), 20-26 September, 1995.
†On leave of absence from the Institute of Nuclear Physics, Moscow State University, 119899 Moscow, Russia.
‡E-mail: kubyshin@delta.ft.uam.es
§E-mail: jmartin@ccr.jussieu.fr and jmartin@lca1.drp.cbpf.br
The idea of T. Kaluza and O. Klein [1], which was originally proposed as a mean of unification of gravity and electromagnetic interactions, has become later an important ingredient of many string and supergravity theories (see [2] for reviews) and in this way gave rise to Kaluza-Klein cosmological models [3], [4]. Thus, the idea has transformed into the physical hypothesis assuming the existence of additional (to the three known) spacelike dimensions of the space-time with a certain structure. It is quite interesting and it would be highly desirable to check if this hypothesis agrees or not with available observational data. Experimental tests of general relativity in higher dimensions were discussed in Ref. [5]. In this article we use the recent observational data on the anisotropies of the cosmic microwave background radiation (CMBR) provided by the satellite COBE [6] to obtain limits on the free parameters describing different cosmological scenarios.

We consider Kaluza-Klein cosmological models with the spacetime \( R \times M_3^1 \times M_2^d \), where the manifold \( R \times M_3^1 \) represents our four-dimensional Universe, which is assumed to be a Friedman-Lemaître-Robertson-Walker universe with flat space hypersurfaces, and where the \( d \)-dimensional manifold \( M_2^d \) represents the space of extra dimensions, often called internal space, which here is assumed to be compact. We restrict ourselves to the metrics of the form

\[
g = -dt \otimes dt + a^2(t)\tilde{g} + b^2(t)\hat{g},
\]

where \( a(t) \) and \( b(t) \) are scale factors of the spaces \( M_3^1 \) and \( M_2^d \) respectively. In the previous equation \( \tilde{g} \) is the three-dimensional metric on \( M_3^1 \) and \( \hat{g} \) is a \( d \)-dimensional metric on the internal space \( M_2^d \).

It is known that a multidimensional theory can be always re-written as an effective theory in four-dimensional space-time with an infinite tower of fields (Kaluza-Klein modes). This includes a zero mode and massive modes (often called pyrgons) with masses proportional to \( 1/b \) which form a discrete spectrum (since \( M_2^d \) is compact). From the absence of any signals of heavy Kaluza-Klein modes in present day high energy experiments it can be concluded that \( hc/b > (1 ÷ 10) \) TeV. Studies in Refs. [7] give strong bounds on the time variation of the scale \( b(t) \) since the time of the nucleosynthesis. They imply that nowadays the scale of the internal space is constant with very high precision. From these it follows that extra dimensions do not produce any significant effect now. Contrary to this, in the Early Universe, extra dimensions could have played an important role. Indeed, as many cosmological scenarios predict, the scales were of comparable size (\( a(t) \sim b(t) \)) and \( b(t) \) was changing rather rapidly. Therefore, experiments probing the Early Universe can shed light on the issue of the validity of the Kaluza-Klein hypothesis. The COBE experiment is the one of this kind: the temperature anisotropies, measured by COBE, carry certain imprints of that early stage of the evolution and, in this way, can constrain the topology of the Universe (see Ref. [8]) or give an evidence of possible existence of extra dimensions.

The effect we are going to calculate here is in many respects similar to the temperature anisotropies of the CMBR caused by the inflationary expansion of the spatial part of \( R \times M_3^1 \). For the case of the four-dimensional Universe the effect of production of gravitational waves during the inflationary stage was studied long ago in Ref. [9] and the effect of distortion of the CMBR by such waves with its consequent anisotropies was discussed in [10]. A quantum-mechanical mechanism of graviton creation (as amplification of the zero-point fluctuations) in four dimensions was proposed and developed in Ref. [11]. In Ref.
the temperature anisotropies due to the gravitational wave perturbations generated quantum-mechanically were studied and it was shown that a certain (although quite narrow) class of four-dimensional inflationary scenarios agrees with the observational data on the anisotropies of the CMBR. (See Refs. [4], [13] for reviews on these issues, Refs. [14] on effects of the gravitational perturbations in string cosmology.) In this article we calculate the anisotropies of the temperature of the CMBR due to the tensor perturbations (gravitational waves) within the Kaluza-Klein approach. Density perturbations and the spectrum of gravitational waves in multidimensional scenarios were studied in [15] - [17]. An analysis of a class of Kaluza-Klein models and their comparison with observations of the CMBR were carried out recently in [17].

In this article, we consider only physical gravitational waves, i.e. tensor type fluctuations on $M^4$, and assume that the only spatial dependence is given by the eigentensors of the Laplacian on $M^3$ labelled by the wavenumber $n$, that is we retain only the lowest (zero) mode on $M^d$. One can show that non-zero (massive) Kaluza-Klein modes do not produce any considerable contribution to the anisotropies of the CMBR: first, because their amplification during inflation is not sufficient and, second, because at time of the emission of the CMBR photons they are already heavy enough particles. In terms of the conformal time $\eta$ the perturbed metric is given by

$$ds^2 = a^2(\eta) \left[ -d\eta^2 + (\delta_{ij} + h_{ij}(\eta, x))dx^i dx^j \right] + b^2(\eta) \hat{g}_{ab} dy^a dy^b,$$

where the indices $i, j$ run from 1 to 3 and the indices $a, b$ from 1 to $d$; the wave can be expressed as

$$h_{ij}(\eta, x) = \sum_n \frac{\mu_n(\eta)}{f(\eta)} G_{n;ij}(x),$$

where $G_{n;ij}(x)$ are eigentensors of the Laplacian on $M^3$ and where the function $f(\eta)$ is defined by $f(\eta) \equiv a(\eta) b(\eta)^{d/2}$. The linearized Einstein equations with zero energy-momentum tensor give:

$$\mu''_n(\eta) + (n^2 - \frac{f''(\eta)}{f(\eta)}) \mu_n(\eta) = 0. \quad (1)$$

The physical interpretation of this equation is similar to that already given in the four-dimensional case [11], [19]: gravitational waves are parametrically amplified throughout the cosmic evolution. In the multidimensional case, Eq. (1) was first considered in Refs. [18, 15, 16].

In a quantum-mechanical treatment [11] (see also [19]) the perturbed metric $h_{ij}$ becomes an operator. If we require the amount of energy to be $\hbar \omega / 2$ in each mode, its general expression can be put under the form:

$$h_{ij}(\eta, x) = 4\sqrt{\pi} l_{pl}^{d/2} \frac{b_{KK}^{d/2}}{f(\eta)} \int_{-\infty}^{+\infty} d^3 n \sum_{s=1}^{2} p^{s}_{ij}(\eta) \frac{1}{\sqrt{2n}} (c^s_n(\eta)e^{i n \cdot x} + c^{s*}_n(\eta)e^{-i n \cdot x}).$$

We used the fact that the multidimensional gravitational constant $G^{(4+d)}$ is related to the four-dimensional one $G^{(4)}$ as $G^{(4+d)} = G^{(4)} V_d$ with the volume of the internal space $V_d$ evaluated for $b = b_{KK}$, the present day value of the scale factor of the internal space. The polarization tensor $p^{s}_{ij}(\eta)$ satisfies the relations: $p^{s}_{ij} n^j = 0$, $p^{s}_{ij} \delta^{ij} = 0$, $p^{s}_{ij} p^{s'ij} = 2 \delta^{ss'}$ and
The normalization is set. Then one can show that the function $P_{lj}^r(-\mathbf{n}) = \bar{P}_{lj}^r(\mathbf{n})$. The time evolution of the operator $h_{ij}(\eta, \mathbf{x})$ is determined by the time evolution of the annihilation and creation operators $c_{\mathbf{n}}^\dagger$ and $c_{\mathbf{n}}^\dagger$ which obey the Heisenberg equation:

$$\frac{dc_{\mathbf{n}}}{d\eta} = -i[c_{\mathbf{n}}, H], \quad \frac{dc_{\mathbf{n}}^\dagger}{d\eta} = -i[c_{\mathbf{n}}^\dagger, H].$$

The Hamiltonian $H$, providing a description in terms of travelling waves, is given by:

$$H = nc_{\mathbf{n}}^\dagger c_{\mathbf{n}} + nc_{\mathbf{-n}}^\dagger c_{\mathbf{-n}} + 2\sigma(\eta)c_{\mathbf{n}}^\dagger c_{\mathbf{-n}} + 2\sigma^*(\eta)c_{\mathbf{n}} c_{\mathbf{-n}},$$

where $\sigma(\eta) \equiv i f’/(2f)$. For $d = 0$ the expressions of Ref. [20] are recovered. In the multidimensional case, the second pump field $b(\eta)$ appears in the coupling function $\sigma(\eta)$ and, as a consequence, the production of gravitons will be affected by the dynamics of the internal dimensions. The form of the Hamiltonian (2) explicitly demonstrates that, while the Universe expands, the initial vacuum state evolves into a strongly squeezed vacuum state with characteristic statistical properties as discussed in Ref. [20]. The Heisenberg equations are resolved with the help of the standard Bogoliubov transformations: $c_{\mathbf{n}}(\eta) = u_{nc}(\eta) + v_{nc}(\eta)$ and similar one for $c_{\mathbf{n}}^\dagger(\eta)$. Here, $\eta_0$ is the beginning of inflation where the normalization is set. Then one can show that the function $\mu_n(\eta) \equiv u_n(\eta) + v_n^*(\eta)$ obeys the classical equation (3).

In order to derive bounds on parameters of cosmological models from COBE observational data we calculate the angular correlation function for the temperature variation $\langle \delta T/T \rangle(\hat{e})$ of the CMBR caused by the cosmological perturbations (Sachs-Wolfe effect [21]). This function depends only on the angle $\delta$ between the unit vectors $\hat{e}_1$ and $\hat{e}_2$, pointing out in the directions of observation, and can be expanded in terms of the Legendre polynomials $P_l$ as follows [22, 23]:

$$\langle 0 | \frac{\delta T}{T}(\hat{e}_1) \frac{\delta T}{T}(\hat{e}_2) | 0 \rangle = \sum_{l=2}^{\infty} C_l P_l(\cos \delta).$$

The contribution of the cosmological perturbations to the quadrupole moment is given by

$$Q_{rms-P_S} \equiv T_0(5C2/4\pi)^{1/2} \sim T_0 h_H/70,$$

where $T_0 = 2.7$ K and $h_H$ is the characteristic spectral component defined by $h(n; \eta) = l_P n |\mu_n(\eta)|/a(\eta)$ and evaluated at $n = n_H$, $\eta = \eta_R$. Here $\eta_R$ is the time at which photons of the CMBR were received, and $n_H = 4\pi$ is the wavenumber corresponding to the wavelength equal to the present Hubble radius $l_H$.

In this article we consider the following scenario for the behaviour of the scale factors:

1) Inflationary stage (I-stage): $\eta_0 < \eta < \eta_1 < 0$

$$a(\eta) = l_0|\eta|^{1+\beta}, \quad b(\eta) = b_0|\eta|^{\gamma}.$$

2) Transition stage: $\eta_1 < \eta < \eta_2$

$$a(\eta) = l_0 a_e(\eta - \eta_e), \quad b(\eta) = b_T(\eta).$$

3) Radiation-dominated stage (RD-stage): $\eta_2 < \eta < \eta_3$

$$a(\eta) = l_0 a_e(\eta - \eta_e), \quad b(\eta) = b_{KK}.$$
4) Matter-dominated stage (MD-stage): $\eta > \eta_3$

$$a(\eta) = l_0 a_m (\eta - \eta_m)^2, \quad b(\eta) = b_{KK}.$$ 

The scale factors $a(\eta)$ and $b(\eta)$ and their first derivatives $a'(\eta)$ and $b'(\eta)$ are assumed to be continuous throughout the evolution. This gives certain relations between the parameters of the scenario. All models with $1 + \beta < 0$ ($\eta$ must be negative in this case) describe inflationary expansion of the three-dimensional part of the Universe. It can be shown that the case $\beta = -2$ corresponds to the de Sitter expansion. This scenario is rather general since in most of the known models of Kaluza-Klein cosmology the behaviour of the scale factors at the inflationary-compactification stage is of the same type as the one described by Eqs. (5). The transition stage is added in order to assure the continuity of $b(\eta)$ and its first derivative. Its behaviour at this stage is characterized by some reasonable function $b_{Tr}(\eta)$ which describes the slowing of the evolution of the scale factor of extra dimensions in the process of compactification, that appears in many Kaluza-Klein cosmological models (see, for example, [23]). Thus, a realistic and physically interesting case is when the change of $b(\eta)$ during the transition stage is small, namely $r = (b(\eta_1)/b_{KK})^{d/2} \sim 1$. The explicit form of $b_{Tr}(\eta)$ will not be important for our results. For $\eta > \eta_2$ the function $b(\eta)$ is taken to be constant. This agrees with strong bounds on the time variation of the scale factor of extra dimensions during the RD- and MD-stages obtained in [7].

To calculate the angular variation of the temperature of the CMBR we need to solve Eq. (1). The initial conditions on the wave amplitude, corresponding to the vacuum spectrum of the perturbations characterized by ”a half of the quantum” in each mode, are the following: $\mu(\eta_0) = 1$, $\mu'(\eta_0) = -in$, where $\eta_0 < 0$ is such that $|\eta_0| \gg |\eta_1|$ [19]. Then the solution of Eq. (1) is equal to

1) I-stage:

$$\mu(\eta) = (n\eta)^{1/2} A H^{(2)}_{N+\frac{1}{2}}(n\eta),$$

where $H^{(2)}_{\nu}(z)$ is the Hankel function of the second kind, $N \equiv \beta + (\gamma d)/2$ and the constant $A$ given by the expression $A = -i\sqrt{\pi/2} \exp[i(n\eta_0 - \pi N/2)].$

2) Transition stage: for waves with long wavelengths

$$\mu(\eta) \sim B_1 \xi(\eta) + B_2 \xi(\eta) \int^{\eta} \frac{d\eta'}{\xi^2(\eta')},$$

where without loss of generality we wrote $b_{Tr}(\eta)$ as $b_{Tr}(\eta) = [\xi(\eta)/(a_e(\eta - \eta_e))]^{2/d}$, $\xi(\eta)$ being an arbitrary function (see details in Ref. [22]).

3) RD-stage:

$$\mu(\eta) = C_1 e^{-in(\eta - \eta_e)} + C_2 e^{in(\eta - \eta_e)}.$$ 

4) MD-stage:

$$\mu(\eta) = \sqrt{\frac{n}{2}} \left( D_1 J_\frac{1}{2}(z) + D_2 J_{-\frac{1}{2}}(z) \right),$$

where $z \equiv n(\eta - \eta_m)$. The coefficients $B_i$, $C_i$ and $D_i$ ($i = 1, 2$) are determined by matching the solution and its first derivative.
The expressions of $\mu(\eta)$ for the I-, RD- and MD-stages are exact, whereas the form of $\mu(\eta)$ for the transition stage is only valid for long wavelengths, namely for the wave numbers such that $n \leq n_H = 4\pi$. Such solution is sufficient for the calculation of the characteristic spectral component $h_H$.

To set the scale for $\eta$ it is convenient to choose $\eta_R - \eta_m = 1$. All realistic cosmological models should account for $a(\eta_E)/a(\eta_R) \approx 10^{-3}$ (\(\eta_E\) is the time at which the photons of the CMBR were emitted), $a(\eta_3)/a(\eta_R) \approx 10^{-4}$ and $a(\eta_i)/a(\eta_R) = k$, where $3 \cdot 10^{-32} < k < 3 \cdot 10^{-12}$. The lower bound on $k$ corresponds to the case when the radiation dominated expansion of the three-dimensional part of the Universe starts at the Planckian energy densities, whereas the upper one corresponds to the case when this process starts at the nuclear energy densities. Also we assume that the scale factor $a(\eta)$ has grown sufficiently during the I-stage: $a(\eta_i)/a(\eta_0) = E$ with $E \geq e^{70}$ \cite{24}. We obtain the following expressions for the parameters of the scenario in terms of $\beta$ and $k$.

\[
\begin{align*}
\eta_1 &= 50k(1 + \beta), \\
\eta_2 &= 50k\beta + 0.5 \cdot 10^{-2}, \\
\eta_3 &= 50k\beta, \\
\eta_4 &= -0.5 \cdot 10^{-2} + 50k\beta, \\
a_0 &= -(1 + \beta)(50k|1 + \beta|)\beta, \\
a_m &= 50|1 + \beta|(50k|1 + \beta|)\beta.
\end{align*}
\]

The characteristic scale $l_0$ in Eq. (5) is given by the relation

\[
\frac{l_{Pl}}{l_0} = 25 \left( \frac{l_{Pl}}{l_H} \right) (50k)^\beta |1 + \beta|^{(1+\beta)},
\]

where $l_H \equiv a^2(\eta_R)/a'(\eta_R)$ is the present day Hubble radius. We take it to be equal to $l_H = 10^{61}l_{Pl}$. For the sake of simplicity we also assume that at the beginning of the inflation the multidimensional Universe was symmetric with respect to all dimensions, i.e. $a(\eta_0) = b(\eta_0)$.

Let us now derive restrictions on the parameters of the scenario under consideration. One can show that

\[
S \equiv \frac{l_{Pl}}{b_{KK}} = \frac{\pi^2/d}{6} \left( \frac{l_{Pl}}{l_H} \right) \frac{1}{k/3} E^\rho,
\]

where $\rho = (1 + \beta - \gamma)/(1 + \beta)$. The only experimental bound on the size of $b_{KK}$ comes from the fact that no effects of extra dimensions are observed in high energy particle experiments. This, apparently, tells us that $hc/b_{KK} > (1 \div 10)$ TeV. On the other hand the classical description of the background dynamics can be trusted only if $b_{KK}$ is not much smaller than $l_{Pl}$. These arguments imply that $S$ in Eq. (7) should belong to the interval $10^{-16} < S < 1$.

To obtain restrictions given by the COBE experimental data we calculate the characteristic spectral component $h_H$ for the solution $\mu_\alpha(\eta)$ for the amplitude of the gravitational wave described above. We obtain:

\[
h_H = 25 \left( \frac{l_{Pl}}{l_H} \right) |\Psi(M)||1 + \beta|^M(50k)^M a_H^{2+M} \Pi,
\]

where $M = -1/2 - |1/2 + N|$ and $\Psi(M) \equiv -\exp(i\pi M/2)[\Gamma(-1/2 - M)]/[\sqrt{2\pi} 2^{M+1/2}]$. The factor $\Pi$ takes into account the effect of the transition stage. For $N < -1/2$ it is
equal to \(|1 + \beta|/r\), for \(N > -1/2\) it is given by some complicated expression which in the case of the transition stage to be short enough reduces to \((2\beta + 1 + d\gamma)r - (1 + \beta)/r\). It is easy to show that for further analysis the effect of the transition stage can be neglected if \(r \sim 1\). The most recent COBE experimental results give \(Q_{\text{rms-PS}} \sim 15 \cdot 10^{-6} \text{ K} [6]\). Since the contribution of tensor perturbations cannot exceed this value, from Eq. (4) it follows that \(h_H \leq 10^{-4}\).

Working out Eqs. (7) and (8), we find that

\[
\left| N + \frac{1}{2} \right| = -\frac{1}{2} - \frac{57 + \log h_H}{3 + \log(k/3)},
\]

(9)

\[
\rho = \frac{61 + \log(k/3) + \log S}{\log E},
\]

(10)

In addition \(\beta\) must satisfy \(\beta < -1\) to describe the inflationary expansion.

Let us first analyze the case when the observed anisotropies come mostly from gravitational waves, i.e. \(\log h_H = -4\) (see also the discussion in Ref. [26]). The allowed values of \(N\) and \(\rho\) are given by the regions bounded by dashed lines in Fig. 1. We see that

![Figure 1: The region of values of the parameters \(N\) and \(\rho\) given by the equations (9) and (10) with \(E = e^{70}\). The subregions bounded by dashed lines correspond to the case when the observed anisotropies are given by the gravitational waves only. \(N = -1/2\) is the symmetry axis of the whole region.](image)

there is a room for multidimensional cosmological models as candidates for the description of the Early Universe. However, the limits are rather restrictive, and we are unaware of any model of Kaluza-Klein cosmology which agrees with them. For example, among the models corresponding to our scenario, one finds that \(\beta = -5/4, \gamma = 1/4\) for \(d = 6\) \([\rho, N] = (2, -0.5)\) in the perfect-fluid-dominated model [27], \(\beta = -1.26, \gamma = 0.22\) in the \(D = 4 + d = 11\) supergravity with toroidal compactification \([\rho, N] = (1.85, -0.49)\) [28], \(\beta = -14/11, \gamma = 1/11\) for \(d = 22\) \([\rho, N] = (1.33, -0.27)\) in the model of string-driven inflation [24]. It is easy to check that none of these models satisfy the bounds with
\( \lg h_H = -4 \). In these examples the production of the gravitational waves is not sufficient to explain the quadrupole moment measured by COBE.

Now let us consider the case when contributions of density and rotational perturbations are not neglected, i.e. \( \lg h_H \leq -4 \). The allowed values of the parameters are given by the whole region presented in Fig. 1. Since \( \lg E > 30 \) the upper horizontal line is always below the limit \( \rho \sim 49/30 = 1.63 \). As a consequence, the perfect fluid model and the supergravity model do not account for sufficient inflationary growth of the three-dimensional part of the Universe within our scenario and are still ruled out whereas the string model now satisfies the bounds in principle. However, it requires \( \lg h_H \sim -40 \) that means that the contribution of the gravitational waves must be extremely small.

Taking into account further experimental restrictions (in particular, the tensor to scalar quadrupole ratio which is not determined by COBE) will allow to make the limits on multidimensional models more restrictive thus questioning the very validity of the Kaluza-Klein hypothesis.

Details of the calculations as well as results of more complete analysis of Kaluza-Klein models will be presented elsewhere [22].

**Acknowledgments**

We thank Leonid Grishchuk, Richard Kerner, Valery Rubakov and Gustavo Yepes for valuable discussions and useful comments. Financial support from M.E.C. (grants SAB94-0087 and SAB95-0224) and CIRIT and from the Ministère de la Recherche et de l’Enseignement (research grant) and the CBPF (postdoctoral grant) are acknowledged.

**References**

[1] T. Kaluza, Sitzungsber. Preuss. Akad. Wiss. Berlin, Math. Phys. K1 (1921) 966; O. Klein, Z. Phys. 37 (1926) 895.

[2] M.J. Duff, B. Nilsson and C. Pope, Phys. Rep. C130 (1966) 1; A. Salam and J. Strathdee, Ann. Phys. 141 (1982) 316; M.B. Green, I.H. Schwarz and E. Witten, *Superstring theory*, vols.I and II (Cambridge Univ. Press, 1987).

[3] A. Chodos and S. Detweiler, Phys. Rev. D21 (1980) 2167; P.G.O. Freund, Nucl. Phys. B209 (1982) 146; E. Alvarez and M.B. Gavela, Phys. Rev. Lett. 51 (1983) 931; S. Randjbar, A. Salam and J. Strathdee, Phys. Lett. 135B (1984) 388; D. Sahdev, ibid 137B (1984) 155; E. Kolb and R. Slansky, Phys. Lett. 135B (1984) 378.

[4] E. Kolb and M. Turner. *The Early Universe*. (Addison-Wesley Publishing Co, 1990).

[5] J.A. Casas, C.P. Martin and A.H. Vozmediano, Phys. Lett. B186 (1987) 29.

[6] G.F. Smoot et al., *Astrophys. J.* 396 (1992) L1; C.L. Bennett et al., *Astrophys. J.* 436 (1994) 423; C.L. Bennett et al., Preprint [astro-ph/9601067].
[7] E.W. Kolb, M.J. Perry and T.P. Walker, Phys. Rev. D33 (1986) 869; J.D. Barrow, Phys. Rev. D35 (1987) 1805.

[8] A. de Oliveira-Costa, G.F. Smoot and A.A. Starobinsky, Preprint astro-ph/9510109, 1995; N.J. Cornish, D.N. Spergel and G.D. Starkman, Preprint gr-qc/9602039.

[9] A.A. Starobinski, Pis’ma Zh. Exp. Teor. Fiz. 30 (1979) 719 [JETP Lett. 30 (1979) 682].

[10] V.A. Rubakov, M.V. Sazhin and A.V. Veryaskin, Phys. Lett. 115B (1982) 189; R. Fabbri and M.D. Pollock, Phys. Lett. 125B (1983) 445; A.A. Starobinsky, Pis’ma Astron. Zh. 9 (1983) 579 [Sov. Astron. Lett. 9 (1983) 302]; 11 (1985) 323 [11 (1985) 133]; L.F. Abott and M.B. Wise, Nucl. Phys. B244 (1984) 541; L.F. Abott and R. Schaefer, Astrophys. J. 308 (1986) 546.

[11] L.P. Grishchuk, Zh. Eksp. Teor. Fiz. 67 (1974) 825 (Sov. Phys. JETP 40 (1975) 409); L.P. Grishchuk and Y. Sidorov, Phys. Rev. D42 (1991) 3413.

[12] L.P. Grishchuk, Phys. Rev. D48 (1993) 3513.

[13] A. Linde, Particle Physics and Inflationary Cosmology (Harwood Publishing, 1990); J.E. Lidsey, A.R. Liddle, E.W. Kolb, E.J. Copeland, T. Barreiro and M. Abney, Preprint astro-ph/9508078, 1995.

[14] R. Brustein, M. Gasperini, M. Giovannini, V.F. Mukhanov and G. Veneziano, Phys. Rev. D51 (1995) 6744; R. Brustein, M. Gasperini, M. Giovannini and G. Veneziano, Preprint hep-th/9507017; M. Gasperini, Preprint hep-th/9509127, 1995.

[15] M. Demiański, A.G. Polnarev and P. Naselsky, Phys. Rev. D47 (1993) 5275; M. Biesiada, Class. Quantum Grav. 11 (1994) 2589.

[16] M. Gasperini and M. Giovannini, Class. Quant. Grav. 9 (1992) L137; Phys. Rev. D47 (1993) 1519.

[17] V. Faraoni, F.I. Cooperstock and J.M. Overduin, Int. J. Mod. Phys. D4, in press.

[18] R.B. Abbot, B. Bednarz and S.D. Ellis Phys. Rev. D33 (1986) 2147.

[19] L.P. Grishchuk, Class. Quant. Grav. 10 (1993) 2449.

[20] L.P. Grishchuk, Phys. Rev. Lett. 70 (1993) 2371.

[21] R.K. Sachs and A.M. Wolfe, Astrophys. J. 147 (1967) 73.

[22] Yu. Kubyshin and J. Martin, in preparation
[23] Yu.A. Kubyshin, V.A. Rubakov and I.I. Tkachev, *Int. J. Mod. Phys. A* 4 (1989) 1409; Yu. Kubyshin, E. Moreno and J.I. Pérez Cadenas, gr-qc/9411063. To appear in the Proc. of the Trobades Científiques de la Mediterrània E.R.E. 94 (Maó, Spain, 12-14 September 1994).

[24] M. Gasperini, N. Sanchez and G. Veneziano, *Nucl. Phys.* B364 (1991) 365.

[25] A. H. Guth, *Phys. Rev.* D23 (1981) 347.

[26] M. White, *Phys. Rev.* D46 (1992) 4198.

[27] R.B. Abbott, S.M. Barr and S.D. Ellis, *Phys. Rev* D30 (1984) 720; D. Sahdev, *Phys. Rev.* D30 (1984) 2495; D39 (1989) 3155.

[28] R.G. Moorhouse and J. Nixon, *Nucl. Phys.* B261 (1985) 172.