Neutron Star Properties Viewed by the ENU Model

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Abstract

Up-to-now known characteristics of radio pulsars, such as mass limits, magnetic field intensity, rotational period, and maximum radiation are mainly of empirical nature. Applying the Expansive Non-decelerative Universe model (ENU) into the issue allows to offer a deeper theoretical explanation of the known parameters and to estimate their limits. Using the ENU approach the following values related to synchrotron radiation emitting radio pulsars were estimated: the lower and upper limits of magnetic field intensity are \( B_{P(\text{min})} \approx 8.5 \times 10^6 \, \text{T} \), and \( B_{P(\text{max})} \approx 4.4 \times 10^9 \, \text{T} \), respectively, the maximum rotation period reaches 3.9 s, the maximum radiation output of a pulsar is \( P_{P(\text{rad,max})} \approx 5.6 \times 10^{29} \, \text{W} \) (all the values relate to radio pulsars with 1.4 solar masses and radius \( r = 10^4 \, \text{m} \) and are mass and radius dependent). These values are in accordance with the experimental observations.

1 INTRODUCTION

Neutron stars are objects with the mass from about 1.4 (the Chandrasekhar limit) to 3.5 solar masses, diameter \( r \approx 10^4 \, \text{m} \), and density \( 10^{17} \, \text{kg} \, \text{m}^{-3} \). Spinning neutron stars emitting electromagnetic radiation from their poles are called pulsars. The radiation energy varies from radio waves to gamma rays. Until now investigated neutron stars are characterized [1 - 3] by the
magnetic field ranging from $10^7 \, \text{T}$ to $10^9 \, \text{T}$ and the rotation period from milliseconds to seconds. Rotating magnetic field influences the electrons being present in the environment, giving thus rise to the formation of synchrotron radiation. In addition to "classical" radio pulsars, soft gamma repeaters - magnetars - with magnetic fields up to $10^{11} \, \text{T}$ are known, their radiation is not, however, of synchrotron nature caused by electrons. Magnetars are outside the scope of this article.

Radiation output of radio pulsars reaches usually $10^{24} - 10^{28} \, \text{W}$, with $10^{30} \, \text{W}$ as a known maximum. The extremely strong gravitational field is able to attract and hold neutrons and electrons within the spheres above iron surface crust up about 1 cm or 10 m, respectively.

Owing to Vaidya metric application [4], the model of Expansive Nondecelerative Universe [5] enables to localize gravitational energy [6]. Stemming from a general formula [6], the absolute value of gravitational energy density $\varepsilon_g$ at a pulsar surface can be expressed as

$$|\varepsilon_g| = \frac{Rc^4}{8\pi G} \approx \frac{3m_Pc^2}{4\pi ar_P^2} \approx 4.6 \times 10^{12} \, \text{J/m}^3 \quad (1)$$

where $R$ is the scalar curvature ($R \neq 0$ in Vaidya metric [4, 6]), $m_P$ is the pulsar mass, $r_P$ is its radius, and $a$ represents the gauge factor (in the above and following equations the mass $2.8 \times 10^{30} \, \text{kg}$, radius $10^4 \, \text{m}$, and gauge factor $1.3 \times 10^{26} \, \text{m}$ were introduced).

It can hardly be a coincidence that the gravitational energy density is very close to (just about 1.4 times higher than) the electromagnetic energy density of hydrogen atom is.

Gravitational field may be described by a wave function [6]

$$\Psi_g = \exp(-i\omega_gt) \quad (2)$$

where $\omega_g$ is the frequency of gravitational wave.

At the pulsar surface

$$\omega_g = \left( \frac{m_Pc^5}{\hbar ar_P^2} \right)^{1/4} \approx 1.5 \times 10^{18} \, \text{Hz} \quad (3)$$

Based on the fact that the gravitational field of a pulsar is able to hold electrons up to 10 m distance from the surface it follows that the magnetic
moment vector of the electrons shall perform precessional motion with the frequency

\[ \omega_e = \frac{B \mu_e}{\hbar} = \frac{Be}{m_e} \]  

(4)

where \( B \) is the pulsar magnetic field intensity, \( \mu_e \) is the electron magnetic moment, \( m_e \) and \( e \) are the electron mass and charge, respectively.

The pulsar stability is preserved only when

\[ \omega_g \leq \omega_e \]  

(5)

In case of equality (5), stemming from (3) to (5) the lower limit of pulsar magnetic field intensity follows as

\[ B_{P(\text{min})} \approx 8.5 \times 10^6 \text{T} \]  

(6)

which is in excellent accord with the value obtained from experimental observations. The upper limit of pulsar magnetic field intensity can be estimated based on the Compton frequency of electron

\[ \omega_C = \frac{m_e c^2}{\hbar} \approx 10^{21} \text{Hz} \]  

(7)

where the limiting condition

\[ \omega_e = \omega_C \]  

(8)

\[ B_{P(\text{max})} \approx 4.4 \times 10^9 \text{T} \]  

(9)

Of course, the frequency \( \omega_e \) can approach but never reach the value of \( \omega_C \).

As to the structure and composition of neutron stars, various hypotheses have been formulated (from iron-like crust to quark-gluon plasmas). Further we show another mode to derive the value of \( B_{P(\text{max})} \). Suppose, whole pulsar consists of particle with the mass of electron. A number of electrons \( n(e) \) corresponding to a pulsar of the mass \( m_P \) is then given as

\[ n(e) = \frac{m_P}{m_e} \]  

(10)

In such a case it can be supposed that the maximum rotation energy of the pulsar is

\[ E_{P(\text{rot},\text{max})} = \frac{m_p \hbar \omega_{P(\text{max})}}{m_e} = \frac{m_p \hbar e B_{P(\text{max})}}{m^2_e} \]  

(11)
where $\omega_{P_{\text{max}}}$ is a maximum procession motion of the electron magnetic moment vector at the maximum magnetic field intensity $B_{P_{\text{max}}}$. The upper limit of rotational energy of the spherical bodies is expressed as

$$E_{(\text{rot, max})} = \frac{mc^2}{5}$$ \hspace{1cm} (12)

Putting (11) and (12) identical, it leads to

$$B_{P_{\text{max}}} \approx 6.7 \times 10^8 \text{T} \hspace{1cm} (13)$$

which is the value being in good agreement with expectations. It can be stated that there is no possibility to find a pulsar of a 1.4 solar masses having its magnetic field intensity higher than that given by (9) or (13).

The lower limit of pulsar rotational energy emerges when the electron mass in (11) is substituted for the neutron mass $m_n$ and the minimum value of the magnetic field intensity $B_{P_{\text{min}}}$ given by (6) is introduced. In such a case,

$$\frac{m_P \hbar e B_{P_{\text{min}}}}{m_n^2} = \frac{m_P r_P^2 \omega_{(\text{min})}^2}{5}$$ \hspace{1cm} (14)

The maximum rotation period of a neutron star following from (14) is then

$$t_{(\text{rot, max})} = \frac{2\pi}{\omega_{(\text{min})}} \approx 3.9 \text{s}$$ \hspace{1cm} (15)

Also this value is in excellent agreement with observation. At present, the maximum rotational period of 3.8 s is reported, it should be pointed out, however, that 1) the rotational period is mass and radius dependent, 2) it can change due to energy emission, 3) it can change due to mass transfer when existing in binaries. Longer rotational periods are usually ascribed to white dwarfs.

The ENU approach enable to evaluate the radiation output of pulsars $P_{P_{\text{rad}}}$. In order to secure a pulsar stability, its radiation output cannot exceed its gravitational output $P_{P_{(g)}}$, i.e.

$$P_{P_{(g)}} \geq P_{P_{(\text{rad})}}$$ \hspace{1cm} (16)

In the ENU model, generally [6]

$$|P_g| = \frac{d}{dt} \int \frac{Rc^4}{8\pi G} dV = \frac{mc^3}{a}$$ \hspace{1cm} (17)
Comparing eqs. (16) and (17) is follows that any pulsar formed from a star with the Chandrasekhar limit mass and radius $r \approx 10^4 \text{m}$ cannot have radiation output higher than

$$P_{P^{(\text{rad, max})}} \approx 5.6 \times 10^{29} \text{W}$$

(18)

which corresponds to the observed values.

1.1 Conclusions

Up-to-now known values of pulsar magnetic field intensity, rotational period, and maximum radiation output stem from experimental observation and are of empirical nature. Applying the ENU model into the matter allows to offer a deeper theoretical explanation of the known values and to estimate the limits for the mentioned parameters.

1.2 References

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