Basic oscillation measurables in the neutrino pair beam

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ABSTRACT

It is shown that the vector current contribution of neutrino interaction with electrons in ion gives rise to oscillating component, which is absent for the axial-vector contribution, when a single neutrino is detected in the recently proposed neutrino pair beam. CP violation measurements are thus possible with high precision along with determination of mass hierarchical patterns.

PACS numbers 13.15.+g, 14.60.Pq,

Keywords CP violation, CP-even neutrino pair beam, heavy ion synchrotron
**Introduction** A new strong source of neutrinos consisting of all flavor pairs of $\nu_a$ and $\bar{\nu}_a$ ($a = e, \mu, \tau$) was recently proposed to further accelerate neutrino physics experiments [1]. It was however pointed out in [2] that a single neutrino detection in the neutrino pair beam does not exhibit oscillation pattern, making detection of oscillation harder due to large backgrounds in double event detection. This disappearance of oscillation is based on (i) the unitarity of the $3 \times 3$ neutrino mixing matrix and (ii) the equality of pair emission amplitude squared that holds for the dominant axial vector contribution of light ions.

In the present work we show that the second condition, the equality of pair emission amplitude squared, does not hold in the vector current contribution (sub-dominant, when ionic electrons move with non-relativistic velocities, but may be comparable to the axial vector contribution in heavy ions) of pair emission amplitude, hence the emergence of oscillation patterns occurs from the vector contribution. When neutrino oscillation is made possible this way, the CP violating (CPV) parameter determination (the CPV phase $\delta$ common to both Dirac and Majorana cases) becomes possible.

We derive basic formulas for the three-flavor neutrino scheme including the earth matter effect and present numerical outputs of quantities for new experiments using the neutrino pair beam. It is found that oscillation patterns appear in all $\nu_a, \bar{\nu}_a, a = e, \mu, \tau$, but determination of CPV parameter is possible only by detection of $\nu_\mu, \bar{\nu}_\mu$ and tau-neutrinos. Electron neutrinos do not allow CPV determination. If the accelerator ring is placed in the underground of depth $d$, the neutrino pair beam appear on earth at a distance $\sim \sqrt{2dR}$ with $R$ the radius of the earth. This distance is $\sim 35$ km for $d = 100$ m. It is found that one can do sensitive CPV measurements at this distance.

Throughout this work we use the natural unit of $\hbar = c = 1$.

**How oscillation pattern appears in single neutrino detection** We shall follow notations of [2]. The probability amplitude of the entire process consists of three parts: the production, the propagation, and the detection due to charged current interaction (neutral current interaction is much smaller, hence not considered here), each to be multiplied at the amplitude level. Thus, one may write the probability for the $\nu_a$ neutrino quasi-elastic scattering (with $J^\alpha$ the nucleon weak current) as

$$
\sum_c \left( \frac{G_F}{\sqrt{2}} \right)^2 \bar{\nu}_a \gamma_a (1 - \gamma_5) l_a J^\alpha \bar{\nu}_a \gamma_\beta (1 - \gamma_5) \nu_a (J^\beta) \sum_b \langle \bar{c} | e^{-i\bar{H}L} | \bar{b} \rangle \langle a | e^{-iH_L} | b \rangle M_{bb}(1, 2) \right|^2 ,
$$

(1)

where $H$ ($\bar{H}$) is the hamiltonian for propagation of neutrino (antineutrino) including earth-induced matter effect [4], [5], [6], which is in the flavor basis

$$
H = U^* \begin{pmatrix}
\frac{m_1^2}{2E} & 0 & 0 \\
0 & \frac{m_2^2}{2E} & 0 \\
0 & 0 & \frac{m_3^2}{2E}
\end{pmatrix} U^T + \sqrt{2G_F} n_e \begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix},
$$

(2)

where $U_{ai}$ is the neutrino mixing matrix with $|a\rangle = \sum_i U_{ai}^* |i\rangle$, $a = e, \mu, \tau$, $i = 1, 2, 3$, and $n_e$ is the number density of electrons in the earth. $\bar{H}$ can be obtained by replacing $U \rightarrow U^*$ and changing the sign in the second term $\propto G_F$.

We shall denote three eigenvalues by $\lambda_i$ for neutrinos, and $\bar{\lambda}_i$ for anti-neutrinos. Let $V(\sim U)$ and $\bar{V}$ are unitary $3 \times 3$ matrices that diagonalize the hamiltonian $H$ for neutrino and $\bar{H}$ for anti-neutrino, including
the earth matter effect. The propagation amplitude is then
\[
\langle a | e^{-iH_L} | b \rangle = \sum_i V_{ai} V_{bi}^* e^{-i\lambda_i L}, \quad \langle c | e^{-iH_L} | \bar{b} \rangle = \sum_i \bar{V}_{ci}^* \bar{V}_{bi} e^{-i\lambda_i L}, \quad (3)
\]
\[
\sum_b \langle c | e^{-iH_L} | \bar{b} \rangle \langle a | e^{-iH_L} | b \rangle c_b = \sum_{ij} V_{ai} \bar{V}_{cj}^* \xi_{ij} \exp[-i(\lambda_i L + \bar{\lambda}_j \bar{L})], \quad \xi_{ij} = \sum_b c_b V_{bi}^* \bar{V}_{bj}, \quad . \quad (4)
\]
The factor $c_b$ arises from the production amplitude $M_{bb}(1, 2)$ and it is $(c_b^A)^2 = \frac{1}{2}(1, -1, -1)$ for the axial vector contribution and for the vector contribution,
\[
(c_b^V) = \left(\frac{1}{2} (1 + 4 \sin^2 \theta_w), -\frac{1}{2} (1 - 4 \sin^2 \theta_w), -\frac{1}{2} (1 - 4 \sin^2 \theta_w) \right), \quad (5)
\]
with the weak mixing angle $\theta_w$. The precise relation between neutrino and anti-neutrino eigenvalue problem is given by
\[
\bar{\lambda}(G_F) = \lambda(-G_F), \quad \bar{V}_{ai}(G_F) = V_{ai}(-G_F). \quad (6)
\]
The rate of neutrino $\nu_a$ detected and $\bar{\nu}_c$ undetected contains the squared propagation factor,
\[
\sum_{c} \left| \sum_{ij} V_{ai} \bar{V}_{cj}^* \xi_{ij} \exp[-i(\lambda_i L + \bar{\lambda}_j \bar{L})] \right|^2 = \sum_{ijkl} \sum_{c} V_{ai} V_{ak}^* \bar{V}_{cj}^* \bar{V}_{cl} \xi_{ij} \xi_{kl} \exp[-i(\lambda_i - \lambda_k) L] \exp[-i(\bar{\lambda}_j - \bar{\lambda}_l) \bar{L}] = \sum_{ik} V_{ai} V_{ak} p_{ik} \exp[-i(\lambda_i - \lambda_k) L], \quad p_{ik} = \sum_j \xi_{ij} \xi_{kj}. \quad (7)
\]
When $(|c_b^A|^2) = (1, 1, 1)/4 \propto 1$ for the axial vector contribution, $p_{jl} = \delta_{jl}/4$ and the detection probability becomes $1/4$, hence no oscillation pattern exits.

The relevant weak amplitude for the vector part gives oscillating components. Candidate ions for circulation that contribute to the vector current interaction are Be-like heavy ions of $2p2s^3P_1^-$ and Ne-like heavy ions of $2p^+3s^3P_1^-$ (electron-hole system).

**Basic measurable quantities in neutrino pair beam**  We first note
\[
p_{ik} = \sum_j \xi_{ij} \xi_{kj}^* = \sum_b |c_b^V|^2 V_{bi} V_{bk} = \frac{1}{4} (1 + 4 \sin^2 \theta_w)^2 V_{ei}^* V_{ek} + \frac{1}{4} (1 - 4 \sin^2 \theta_w)^2 (V_{\mu k}^* V_{\mu k} + V_{\tau k}^* V_{\tau k})
\]
\[
= \frac{1}{4} (1 - 4 \sin^2 \theta_w)^2 \delta_{ik} + 4 \sin^2 \theta_w V_{ei}^* V_{ek}. \quad (8)
\]
The detection probability of $\nu_a$ (when the other neutrino of the pair is undetected) is given by the oscillation formula based on the vector part of weak current,
\[
P_a(E, L; m_i, \delta) \equiv \frac{1}{3(1 - 4 \sin^2 \theta_w)^2 / 4 + 4 \sin^2 \theta_w}
\]
\[
\times \left( \frac{1}{4} (1 - 4 \sin^2 \theta_w)^2 + 4 \sin^2 \theta_w \sum_j U_{ej}^* U_{aj} \exp[-i \frac{m_j^2 L}{2E}] \right)^2, \quad (9)
\]
with $\sin^2 \theta_w \sim 0.231$. The formula (9) is valid when the earth matter effect is neglected. When the earth matter effect is included, one replaces $U \rightarrow V, m_j^2/2E \rightarrow \lambda_j$. The quantity $P_a(E, L; m_i, \delta)$ is the
normalized probability: $\sum_a P_a(E, L; m_i, \delta) = 1$. The oscillating component in eq. (9) is equivalent to the $\nu_e \rightarrow \nu_a$, $a = \mu, \tau$ appearance probability multiplied by

$$\frac{4 \sin^2 \theta_w}{3(1 - 4 \sin^2 \theta_w)^2/4 + 4 \sin^2 \theta_w} \sim 0.995.$$  \hspace{1cm} (10)

Thus, the constant off-set term $\propto (1 - 4 \sin^2 \theta_w)^2$ in eq. (9) is very small. In the limit of $\sin^2 \theta_w = 1/4$ there is no contribution to the vector part from Z-boson exchange. Due to the dominance of $\nu_e \rightarrow \nu_a$, $a = \mu, \tau$ in the oscillating term, oscillation patterns in the pair beam have similarities to the $\beta^- \beta^+$ beam [8].

The most striking feature of the neutrino pair beam is that circulating quantum ions produce coherent pairs of all flavors, $\nu_a \bar{\nu}_a$, $a = e, \nu, \tau$. When these pairs propagate, all mass eigen-states get involved, and relevant oscillation extrema at $L/E = 2\pi/\delta m^2_{ij}, (ij) = (12), (23), (13)$ may become relevant, making short baseline experiments a feasible approach.

![Figure 1](image-url)

Figure 1: $\nu_\mu$ component fraction of the pair beam, calculated by using eq. (9). A few choices of CPV parameter $\delta$ are taken: 0 in solid black, $\pi/2$ in dashed red, $\pi$ in dash-dotted blue, and $-\pi/2$ in dotted orange. In the left panel the neutrino energy is fixed at 200 MeV. In the right panel the distance is 50 km away from the ring and the lowest $\nu_\mu$ energy should be set at $\sim 200$ MeV to avoid $\nu_\mu \rightarrow \mu$ threshold effect.

We illustrate numerical results of oscillation patterns and CPV asymmetry in Figs. 1 and 2 respectively. We used neutrino data as determined from neutrino oscillation experiments [9]. CPV asymmetry here is defined by

$$A_a(E, L; m_i, \delta) = \frac{P_a(E, L; m_i, \delta) - P_a(E, L; m_i, -\delta)}{P_a(E, L; m_i, \delta) + P_a(E, L; m_i, -\delta)}.$$  \hspace{1cm} (11)

Experimentally, this quantity may be derived from measurements of both $\nu_a$ and $\bar{\nu}_a$ events.

We note a few important results: (i) CPV $\delta$ measurement is impossible for $\nu_e, \bar{\nu}_e$ events, because the oscillation probability appearing in eq. (9) depends on the quantity $|U_{ej}|^2$, hence is insensitive to $\delta$. (ii)
CPV asymmetry in $\nu_{\mu}$ component is small at the ion ring site due to the unitarity relation $\sum_{j} U_{ej}^{*} U_{\mu j} = 0$ valid at small $L$. (iii) Interestingly, as shown in Fig. 2, CPV asymmetry of $\mathcal{O}(0.1)$ can be obtained even if the distance is $L \sim 40$ km for $E = 200$ MeV. (iv) The determination of the mass hierarchical pattern, normal or inverted, namely NH/IH distinction is possible in the $\nu_e$ and $\nu_\mu$ components as seen in Fig. 3. The advantage of $\nu_e$ component was also pointed out in the reactor neutrino experiment \[10\].

Comparison with Z-decay pair may be of interest. In this case ($c_{\nu_{\mu}}^V = -1/2(1 - 4 \sin^2 \theta_w)(1, 1, 1)$. Hence, the neutrino-pair beam from Z-boson decay does not show the oscillation pattern when only one neutrino is detected \[11\].

To incorporate the earth matter effect, we numerically diagonalize the effective hamiltonian \[2\] \[13\]. The oscillation patterns including the earth matter effect are illustrated in Figs. 4 and 5. We took a pure SiO$_2$ model with density 2.8 g/cm$^3$ for the earth matter.

**Rates of neutrino-pair production from quantum ion beam** Calculations in \[2\] are for the axial-vector contribution. We now repeat calculations based on the vector contribution, following \[12\] of the spin current contribution derived for any ion velocity. In terms of two-component spinors, the axial vector matrix elements of electrons in ions have the form, $(0, 2\vec{S}_e)$ in the 4-vector notation, while the vector matrix elements have $(1, \epsilon_{eg}\vec{r}_{eg})$ with $\vec{r}_{eg}$ the transition dipole moment $\vec{d}_{eg}$ divided by the electric charge, although the time component is usually small due to the orthogonality of wave functions. The vector current
Figure 3: $\nu_e$ (left) and $\nu_\mu$ (right) component fractions of the pair beam, calculated by using eq. (9) for NH (solid red) and IH (dash-dotted blue). The distances are 50 km and 100 km away from the ring.

Figure 4: $\nu_\mu$ oscillation pattern with and without the earth matter effect (ME). The neutrino energy is fixed at 200 MeV. $\delta = 0$ with ME in solid black, without ME in dashed red, $\delta = \pi/3$ with ME in dash-dotted blue, and without ME in dotted orange.
Figure 5: $\nu_\mu$ oscillation pattern with the earth matter effect. The neutrino energy is fixed at 200 MeV. $\delta = \pi/4$ neutrino ($\nu_\mu$) in solid black, anti-neutrino ($\bar{\nu}_\mu = \nu_\mu b$) in dash-dotted blue, $\delta = \pi/2$ neutrino in dashed red, and anti-neutrino in dotted orange.

contribution is thus obtained by a simple replacement from the axial vector contribution:

$$4 S^2 \frac{1}{\gamma} \left(1 + \frac{2}{3} \beta^2 \gamma^2 \right) \to \frac{4}{3} \gamma^2 e_{eg}^2 g \frac{d_{eg}^2}{4 \pi \alpha}.$$  \hspace{1cm} (12)

We have replaced the flavor component fraction at production by a simplified result of $\sin^2 \theta = 1/4$, namely $(c_\psi)^2 = (1, 0, 0)$.

The differential energy spectrum of a single detected neutrino at the forward direction is, in the high energy limit of $\gamma \gg 1$,

$$\frac{d^2 \Gamma}{dy d\varphi} = \frac{8}{27 \sqrt{(2\pi)^4}} N \sqrt{\rho e_{eg} \alpha} \frac{G^2 e_{eg}^5}{\alpha} \gamma^{1/2} \varphi \int dy y \frac{1}{(y + y_2)^{1/4}} \left(4 \gamma^2 - y - y_2 - \gamma^2 y \varphi^2 \right)^{3/4}, \hspace{1cm} (13)

$$

$$\frac{8}{27 \sqrt{(2\pi)^4}} \sqrt{\rho e_{eg} \alpha} \frac{G^2 e_{eg}^5}{\alpha} \gamma^{1/2} \varphi \sim 7.1 \times 10^{10} \text{ Hz} \left(\frac{\rho e_{eg}}{10^{14}}\right)^{1/2} \left(\frac{e_{eg}}{10 \text{ keV}}\right)^5 \left(\frac{\gamma}{10^3}\right)^{1/2}, \hspace{1cm} y = \frac{E}{e_{eg}} \sqrt{\frac{1 - \beta}{1 + \beta}}$$  \hspace{1cm} (14)

Here $N$ is the available number of ions, $\rho$ is the radius of the ring, $\varphi$ is the effective angle of the neutrino pair beam.

The angular distribution is readily calculable, and is plotted in Fig. 6. The forward production rates are the most relevant to neutrino oscillation experiments away from the ring. The forward rate is estimated by taking the angular area $\pi / \gamma^2$ times the right hand side of eq. (13). The following figure Fig. 7 illustrates these rates. The forward rates scale with ion parameters $\propto A_{eg} e_{eg}^{5.5}$, and with the boost factor $\propto \gamma^{3.5}$. In order to detect $\nu_\mu$ events, neutrino energies larger than 200 MeV are desired, which gives a constraint on $2 \gamma e_{eg}$. 

\hspace{1cm}
Figure 6: Angular distribution of single neutrino for $\gamma = 2000$, $\rho \epsilon_{eg} = 10^{14}$, $N = 10^8$, $\epsilon_{eg} = 10$ keV: neutrino energy 150 MeV in solid black, 200 MeV in dashed red, 300 MeV in dash-dotted blue, and 400 MeV in dotted black.

Figure 7: Neutrino energy spectrum rate at the forward direction of solid angle area $\pi/\gamma^2$. Assumed parameters are $\rho \epsilon_{eg} = 10^{14}$, $N = 10^8$ and $\epsilon_{eg} = 10$ keV, $\gamma = 500$ in solid black, 1000 in dashed red, 2000 in dash-dotted blue. For reference the rate of $10^{11}$Hz is shown by the straight line.
Detection rates of neutrino events away from ion ring  We next estimate single neutrino event rates by a detector placed at 50 km away from the ion ring. 10 kt class of $^{56}\text{Fe}^{26}$ target is considered. We shall use PDG compilation [14] of neutrino quasi-elastic cross section extrapolated to lower neutrino energy region. Event rates for ideal detectors, namely the active target volume under full coverage by the neutrino pair beam, are

$$\nu_\mu(\text{Fe}); \ 6.2 \times 10^{-11} P_\mu \left( \frac{E}{1\ \text{GeV}} \right) \left( \frac{T}{10\ \text{kt}} \right) \frac{d\Gamma}{dE} \Delta E \ \text{Hz}, \quad (15)$$

$$\bar{\nu}_\mu(\text{Fe}); \ 1.8 \times 10^{-11} P_\mu \left( \frac{E}{1\ \text{GeV}} \right) \left( \frac{T}{10\ \text{kt}} \right) \frac{d\Gamma}{dE} \Delta E \ \text{Hz}. \quad (16)$$

Thus, if the production rate $\frac{d\Gamma}{dE} \Delta E$ presented in the previous section is larger than $10^{11}$ Hz, then events rates at experimental sites are larger than 1 Hz. $\Delta E$ is the energy bin taken at each energy. Prospects for high sensitivity experiments are bright.

In summary, we demonstrated that CPV parameter determination with a high precision is possible in a short baseline experiment using the neutrino pair beam. Clearly, both experimental R and D works of quantum coherent ion circulation and theoretical studies of candidate ions are required for further development of this new project.

Acknowledgements

One of us (M.Y.) should like to thank N. Sasao for discussions on experimental aspects of this work. This research was partially supported by Grant-in-Aid for Scientific Research on Innovative Areas ”Extreme quantum world opened up by atoms” (21104002) from the Ministry of Education, Culture, Sports, Science, and Technology, and JSPS KAKENHI Grant Numbers 15H01031(T.A.), 15H02093 (M.T. and M.Y.), 25400249 (T.A.), 25400257 (M.T.), and 26105508 (T.A.).

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For numerical analysis we use $s^2_{12} = 0.304, s^2_{23} = 0.452, s^2_{13} = 0.0218, \delta m^2_{21} = 7.50 \times 10^{-5} \text{eV}^2, \delta m^2_{31} = 2.457 \times 10^{-3} \text{eV}^2$ for the NH case, and $s^2_{12} = 0.304, s^2_{23} = 0.579, s^2_{13} = 0.0219, \delta m^2_{21} = 7.50 \times 10^{-5} \text{eV}^2, |\delta m^2_{32}| = 2.449 \times 10^{-3} \text{eV}^2$ for the IH case as determined by M. C. Gonzalez-Garcia, M. Maltoni and T. Schwetz, JHEP 1411, 052 (2014).

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We use the parametrization as given by

\[
(U_{ai}) = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & c_{13} & -s_{13}e^{-i\delta} \\
0 & -s_{13}e^{i\delta} & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
P,
\]

\[
P = \begin{pmatrix}
1 & 0 & 0 \\
0 & e^{i\alpha} & 0 \\
0 & 0 & e^{i\beta}
\end{pmatrix}, \quad a = e, \mu, \tau, i = 1, 2, 3,
\]

where $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$. The lightest neutrino mass is taken as 5 meV, on which the results in this article are independent.

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