LONGITUDINAL LOOP INTEGRALS IN THE GAUGE INVARIANT EFFECTIVE ACTION FOR HIGH ENERGY QCD

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We study integrations over light-cone degrees of freedom in the gauge invariant effective action for high energy processes in QCD. We propose a regularization which takes into account the signature of the reggeized gluon. For a test we apply it to the elastic and the production amplitude.

1 Introduction

In 1995 an effective action [1] for QCD scattering processes at high center of mass energies \( \sqrt{s} \) has been proposed by L.N. Lipatov, which describes the interaction of fields of reggeized gluons \( (A_\pm = -it^a A_\pm^a) \) with quark \((\psi)\) and gluon \((v_\mu = -it^av_\mu^a)\) fields, local in rapidity. The effective action is given by

\[
S_{\text{eff}} = \int d^4x (\mathcal{L}_{QCD}(v_\mu, \psi) + \text{tr}[(A_-(v) - A_-)\partial^2A_+ + (A_+(v) - A_+)\partial^2A_-])
\]

with

\[
A_\pm(v) = v_\pm D_\pm^{-1}v_\pm = v_\pm - gv_\pm \frac{1}{\partial_\pm}v_\pm + g^2v_\pm \frac{1}{\partial^2_\pm}v_\pm - \ldots
\]  

(1)

In the above, light-cone components are defined by \( k^\pm \equiv n^\pm \cdot k \) where \( n^\pm \) are the light cone directions associated with the scattering particles. The fields of the reggeized gluon obey the constraint \( \partial_\pm A_\mp = 0 \). The terms in \( \mathcal{L}_{QCD} \) which supplement the usual QCD-Lagrangian \( \mathcal{L}_{QCD} \) are called \textit{induced} terms and consist, apart from the kinetic term of the reggeized gluon \( -\partial_\mu A_\mu^a \partial_\mu A^a \) (the only part which is non-local in rapidity), of the \textit{induced} vertices. For the production of real particles within the Multi-Regge-Kinematics,

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it is then the induced vertex \(-gv_\pm \frac{1}{\sigma^2} v_\pm \partial^2 A_\pm\) which together with the three gluon vertex gives rise to the Lipatov production vertex \(C_\mu\). Higher induced vertices are needed if the production of real particles in the Quasi-Multi-Regge-Kinematics (QMRK) is considered, as demonstrated in [2]. From a point of view of the QCD action, these induced vertices correspond to gluon emissions from particles with a significantly different rapidity. As (1) is local in rapidity, double-counting due to the induced terms is absent for the real particle production amplitude. It seems appealing to use the effective action also to determine virtual corrections to Reggeon-Reggeon and Reggeon-particle vertices. Such corrections usually include an integration over light-cone degrees of freedom. One finds that for the parts of the diagrams connected with induced vertices, it is essential to impose some sort of cut-off to ensure locality in rapidity of the integrand. Moreover, reggeized gluons are Reggeons which carry negative signature. Consequently, positive and negative signatured parts of an amplitude are treated in different ways within the effective action. We show in the following how this can be done at the level of one-loop integrations by considering the elastic (Sec.2) and the production (Sec.3) amplitude.

2 The elastic amplitude

At one loop, the negative signatured part of the elastic amplitude is solely given (including the imaginary part) by diagrams that represent corrections to the exchange of one reggeized gluon i.e. quasi-elastic (e.g. fig[2]) and central rapidity (fig[1]) corrections. The latter leads to the dominant contribution proportional to the logarithm in \(s\) and will be discussed in the following.
2.1 The central-rapidity correction

In the relevant contribution of fig. 1, the reggeized gluon couples from above and from below by an induced vertex to the gluon loop resulting into the following expression

\[ iA_{2\rightarrow 2}^{(-)}(s, q_{\perp}^2) = s\Gamma_{AA'}^c 2g^2 N_c \int \frac{d^4k}{(2\pi)^4} D_R(s_A, q_{\perp}^2) \Gamma_{BB'}^c \frac{1}{k^+} \frac{q_{\perp}^2}{k^2} D_R(s_B, q_{\perp}^2), \tag{2} \]

where \( \Gamma_{II'}^c = 2igt_{I'I}^c \) is the coupling of the reggeized gluon to the external quarks \( I = A, B \) and \( D_R(s_I, q_{\perp}^2) \) the reggeized gluon’s propagator with \( s_I = (p_I + k)^2 \). The poles \( 1/k^+ \) and \( 1/k^- \) arise due to the induced vertices. In the corresponding QCD-diagram, the two gluons inside the loop couple directly to the quarks \( A \) and \( B \) and it is then the propagator of both quarks that reduces for large sub-center-of-mass-energy squared of the (quark, gluon) \( \rightarrow \) (quark, gluon) sub-amplitudes. The poles \( 1/k^+ \) and \( 1/k^- \) are larger than the typical transverse scale of the amplitude. Further, as the quarks interact with the gluon-loop by reggeized gluons, the corresponding (quark, gluon) \( \rightarrow \) (quark, gluon) sub-amplitudes should have negative signature in \( s_A \) and \( s_B \) respectively. The natural place to implement both requirements is the Reggeon propagator, as it is the only part of the effective action which is non-local in rapidity. Using the following definition, the negative signature constraint is there easily implemented. We define

\[ D_R(s_I, q_{\perp}^2) = -\frac{i}{2q_{\perp}^2} \lim_{\nu \to 0} \int_{-\infty}^{\infty} d\omega \frac{1}{4\pi i \omega + \nu} \left[ \left( \frac{-s_I}{\Lambda_I} \right)^{\omega_i} + \left( \frac{s_I}{\Lambda_I} \right)^{\omega_i} \right] = \theta(s_I - \Lambda_I) -\frac{i}{2q_{\perp}^2} \tag{3} \]

where \( \Lambda_I, I = A, B \) is a scale significantly smaller than \( s \), but larger than the typical transverse scale of the elastic amplitude. Note that \( -\nu \) has the interpretation of an infinitesimal small Regge-trajectory. Inserting (3) into (2) and performing the integration over \( k^+ \) and \( k^- \) we obtain

\[ iA_{2\rightarrow 2}^{(-)}(s, q_{\perp}^2) = s\Gamma_{AA'}^c \frac{1}{2} \ln \left( \frac{-sk^2}{\Lambda_A \Lambda_B} \right) + \ln \left( \frac{sk^2}{\Lambda_A \Lambda_B} \right) \beta(q) -\frac{i}{2q_{\perp}^2} \Gamma_{BB'}^c \tag{4} \]

which apparently has negative signature in \( s \) as required. \( \beta(q) \) is the well-known gluon trajectory function. Note, that (4) contains due to \( \ln(-s) = \ln|s| - i\pi \) also the corresponding imaginary part of the elastic amplitude. The dependence on the scales \( \Lambda_A \) and \( \Lambda_B \) in (4) can be shown to cancel with similar contributions arising from the quasi-elastic regions (e.g. fig 2) with rapidities close to the scattering particles \( A \) and \( B \).
2.2 Exchange of two reggeized gluons

For diagrams that contain two reggeized gluons in the $t$-channel (e.g. fig 3), the integrals over longitudinal loop momenta $k^+$ and $k^-$ factorizes, and it is sufficient to consider the 2 quarks-2 Reggeons vertex given by

$$A_{ij}^{\alpha_1 \alpha_2} = g^2 2 \sqrt{2} \int \frac{ds_I}{2 \pi i} \left( \frac{(t^{a_1} t^{a_2})_{IJ}}{-s_I + k^2 + i\epsilon} + \frac{(t^{a_2} t^{a_1})_{IJ}}{s_I + (k - q)^2 + i\epsilon} \right).$$  \hspace{1cm} (5)

The exchange of two reggeized gluons should yield now the positive signature part of the elastic amplitude. Indeed, for symmetric color quantum numbers which correspond to positive signature exchange, the integral over $s_I$, $I = A, B$ with $s_A = p_A^+ k^-$ and $s_B = p_B^+ k^+$ is convergent and we obtain the well-known QCD result. For anti-symmetric color quantum numbers corresponding to negative signature exchange the integral in (5) turns out to be divergent for large values of $s_I$. However as demonstrated in Sec 2.1 contributions with large $s_I$ and negative signature are taken into account by the central and quasi-elastic contributions. One therefore should isolate the part of the integral that was already contained in those contributions and cut off the integral for values of $s_I$ being larger than the scale $\Lambda_{A,B}$ introduced in the previous paragraph. This leads to convergence and furthermore vanishing of the two Reggeon exchange contribution with negative signature in accordance with the result that the negative signature part of the elastic amplitude is described by the exchange of a single reggeized gluon alone.

3 The production amplitude

The double-Regge limit of the five-point amplitude $A_{2\rightarrow3}$ (fig 4), where $s, s_{ab}, s_{bc} \rightarrow \infty$, $s_{ab}/s, s_{bc}/s \rightarrow 0$ and $\eta = s_{ab}s_{bc}/s, t_1, t_2$ are fixed, is known to possess the following analytic representation with $\tau_i, i = 1, 2$ the signature of the corresponding $t$-channel:

$$A_{2\rightarrow3}^{(\tau_1 \tau_2)} = \int \frac{dj_1 dj_2}{(2\pi i)^2} \left[ s_{ab}^{j_1} s_{bc}^{j_2} - j_1 \epsilon^{\tau_1 \tau_2} \eta^{j_1 j_2 j_1} F_{j_1 j_2}^L (t_1, t_2, \eta) + s_{ab}^{j_1} s_{bc}^{j_2} \epsilon^{\tau_2 \tau_1} \eta^{j_1 j_2 j_1} F_{j_1 j_2}^R (t_1, t_2, \eta) \right]$$

with $\xi_j^\tau = e^{-i\tau j} + \tau \sin \pi j$ and $\xi_{j_1 j_2}^{\tau_1 \tau_2} = \frac{e^{-i\tau (j_1 - j_2)} + \tau_1 \tau_2}{\sin \pi (j_1 - j_2)}$ \hspace{1cm} (6)

where $F_{j_1 j_2}^L$ and $F_{j_1 j_2}^R$ are real functions. In [3] they have been determined to leading accuracy considering discontinuities in $s, s_{ab}$ and $s_{bc}$. In the effective action the different signature contributions to the amplitude are determined as follows: For $\tau_1 = \tau_2 = +1$, one simple inserts the Lipatov-vertex $C_\mu$ into the elastic amplitude with positive signature.
exchange and one obtains the required expression. In the case where both t-channels carry different signatures, one encounters the Reggeon-particle-2 Reggeons vertex, which within the effective action was also addressed in [4]. Similar to Sec 2.2, the integration over $k^+$ and $k^-$ factorizes and can be carried out in a manner similar to Sec 2.2. If both $t$-channels carry negative signature, one has to determine the one-loop correction to the Lipatov production vertex $C_\mu$, which in principle involves next-to-leading accuracy calculations. Here we evaluate the resulting integrals using the methods of Sec 2.1 but restrict ourselves to the leading logarithmic part, while we keep track of all imaginary parts. For every combination of signatures, our result turns out to be in accordance with the one of [3].

4 Concluding remarks

We gave a prescription how longitudinal integrals at one loop can consistently be performed in the effective action. The prescription has been tested with the elastic and the production amplitude and we could reproduce correctly the leading logarithms and the corresponding energy discontinuities.

References

[1] L. N. Lipatov, Nucl. Phys. B 452 (1995) 369 [arXiv:hep-ph/9502308]; L. N. Lipatov, Phys. Rept. 286 (1997) 131 [arXiv:hep-ph/9610276].
[2] E. N. Antonov, L. N. Lipatov, E. A. Kuraev and I. O. Cherednikov, Nucl. Phys. B 721, 111 (2005) [arXiv:hep-ph/0411185].

[3] J. Bartels, Nucl. Phys. B 151 (1979) 293; J. Bartels, Nucl. Phys. B 175 (1980) 365.

[4] M. A. Braun and M. I. Vyazovsky, Eur. Phys. J. C 51 (2007) 103 [arXiv:hep-ph/0612323].