Can the Unruh–DeWitt detector extract energy from the vacuum?

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Abstract

The Unruh effect can be correctly treated only by using the Minkowski quantization and a model of a “particle” detector, not by using the Rindler quantization. The energy produced by a detector accelerated only for a short time can be much larger than the energy needed to change the velocity of the detector. Although the measuring process lasts an infinite time, the production of the energy can be qualitatively explained by a time-energy uncertainty relation.

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1 Introduction

A uniformly accelerated “particle” detector coupled to a quantum field behaves as if it were immersed in a thermal bath with a temperature proportional to the acceleration [1, 2]. Such a detector can spontaneously jump to a higher quantum level and produce a Minkowski particle. It is often argued that the energy needed for these two processes comes from the agency that accelerates the detector [3, 4]. However, a detector can respond even without acceleration if the detection lasts a finite time. This effect is naturally interpreted as a consequence of time-energy uncertainty relations [5, 6]. The aim of this letter is to demonstrate that even when the detection lasts an infinite time, the produced energy can be much larger than the energy needed to accelerate the detector. We do that by studying the response of a detector in a specific non-inertial trajectory, for which only a finite energy is needed for the acceleration. We find that this effect can also be qualitatively explained by a time-energy uncertainty relation.

There is an argument, based on the Rindler quantization, that the Unruh effect does not lead to a production of energy [7]. However, we show that the Rindler quantization is
not a correct approach. First, the particle-model approach is not equivalent to the Rindler-quantization approach. Second, the event horizon, which plays the essential role in the Rindler quantization, cannot play any physical role for a local non-inertial observer. After discussing it in more detail in Sec. 2, we study the response of a pointlike detector in Sec. 3. The conclusions are drawn in Sec. 4, where the implications on the properties of the vacuum are also shortly discussed.

2 Inappropriateness of the Rindler quantization

Let us first show that the “particle”-detector approach to the Unruh effect based on the Minkowski quantization is not equivalent to the Rindler-quantization approach. For definiteness, we use the model of a monopole detector described in [3]. Assuming that the detector and the field are in the ground state $|0, E_0\rangle$ initially, the first order of perturbation theory gives the amplitude for the transition to an excited state $|k, E\rangle$:

$$A(k, \Delta E) = \tilde{g} \int_{-\infty}^{\infty} d\tau e^{i\Delta E \tau} \langle k| \phi(x(\tau))|0\rangle,$$  

(1)

where $\tilde{g} = ig\langle E|m(0)|E_0\rangle$, $g$ is a real dimensionless coupling constant, $m(\tau)$ is the monopole moment operator, $x(\tau)$ is the trajectory of the detector, $\Delta E = E - E_0$, and

$$\langle k| \phi(x)|0\rangle = \frac{1}{\sqrt{(2\pi)^3 2\omega}} e^{i(\omega t - k \cdot x)}.$$  

(2)

We compare the predictions that can be obtained from this model with the predictions that result from the Rindler quantization [7, 8].

The Rindler-quantization approach predicts that the absorption of a Rindler particle by the accelerated atom will be seen by an inertial observer as an emission of a Minkowski particle only if the atom has actually jumped to the excited state. On the other hand, by putting $\Delta E = 0$ in (1), we see that the “particle”-detector approach predicts an emission of a Minkowski particle even if the transition to an excited state has not actually occurred. Nothing prevents the Minkowski particle produced in (1) from being observed by the accelerated observer, contrary to the prediction of the Rindler quantization. For a uniform acceleration, the two approaches agree in the prediction of a thermal distribution for $\Delta E$. However, even this partial agreement of the two approaches does not generalize when the uniform acceleration is replaced by a more complicated motion [9].

The Rindler quantization is unitarily inequivalent to the Minkowski quantization. However, this fact, being an artefact of the infinite volume [10], is only a technical problem. A more serious problem is the fact that the Rindler quantization cannot be applied to the whole space-time, but only to the left and right wedges bounded by the event horizon [11]. Below we show that the event horizon does not correspond to any physical entity that could influence the properties of the fields seen by an accelerated observer, making the Rindler quantization physically meaningless.

Let $x'$ be the Fermi coordinates of an observer at $x' = 0$ moving arbitrarily in flat space-time. If the observer does not rotate, the corresponding metric is given by $g'_{ij} = -\delta_{ij}$, $g'_{0i} = 0$.
and \[12\] \[13\]

\[g'_{00}(t', x') = (1 + a'(t') \cdot x')^2, \]

where \(a'\) is the proper acceleration. From (3) we see that the Fermi coordinates of an accelerated observer possess a coordinate singularity at a certain \(x'\). However, in general, this coordinate singularity does not correspond to any physical boundary. Only \(a'(\infty)\), defining the event horizon, defines a physical boundary. However, in real life, acceleration never lasts infinitely long. And even if it does, it does not have any physical influence on a measuring procedure that lasts a finite time. Actually, the correct interpretation of the Fermi coordinates, and therefore also of the Rindler coordinates as their special case, is purely local \[13, 14\], so they are not appropriate for quantization which requires a global approach to describe the EPR-like correlations.

### 3 The response of the detector

For a uniform acceleration, both the spent and the gained energy are infinite, so it is not easy to compare them. A suspicion that the accelerating agency is not the source of the produced energy comes, for example, from the response of a detector in a uniform circular motion. The gained energy is infinite \[15, 16\], whereas the accelerating force does not raise the kinetic energy of the detector. Nevertheless, one could still argue that the force, acting during an infinite time, somehow gives energy to the quantum states of the detector. Therefore, we study a trajectory for which the force does not act infinitely long.

We choose a simple trajectory with an instantaneous change of the velocity in the \(z\)-direction at the instant \(\tau_0\), after and before which the detector moves inertially. Explicitly, \(x = y = 0\) and

\[z(\tau) = \begin{cases} 0, & \tau \leq \tau_0, \\ v\gamma(\tau - \tau_0), & \tau \geq \tau_0, \end{cases} \]

\[t(\tau) = \begin{cases} \tau, & \tau \leq \tau_0, \\ \gamma \tau + (1 - \gamma)\tau_0, & \tau \geq \tau_0, \end{cases} \]

where \(\gamma = (1 - v^2)^{-1/2}\). Calculating the divergent integrals of the type \(\int_{\tau_0}^{\pm \infty} d\tau e^{i\Omega\tau}\) as

\[
\lim_{\epsilon \to \pm 0} \int_{\tau_0}^{\pm \infty} d\tau e^{i\Omega\tau} e^{\pm \epsilon\tau} = \frac{i}{\Omega} e^{i\Omega\tau_0},
\]

from (1), (2), (4) and (5) we find

\[A(k, \Delta E) = \frac{e^{i(\Delta E + \omega)\tau_0}}{i} \frac{\bar{g}}{\sqrt{(2\pi)^3 2\omega}} \left( \frac{1}{\Delta E + \omega} - \frac{1}{\Delta E + \gamma(\omega - k_z v)} \right). \]

For the case \(v = 0\), the amplitude vanishes, except for the trivial case \(\Delta E = \omega = 0\) (recall that \(\Delta E\) and \(\omega\) are non-negative). The physical quantity \(|A|^2\) does not depend on \(\tau_0\), just as we expect. Note also that if we replaced the instantaneous change of the velocity by a change that lasted a short but finite time, the result would not significantly change. We see that \(\omega\), and therefore the gained energy \(\omega + \gamma \Delta E\), can be arbitrarily large, although
the kinetic energy $M(\gamma - 1)$ spent for the change of the detector velocity is finite, provided that the mass $M$ of the detector is finite. Moreover, the averaged energy of the produced Minkowski particles
\[ \langle \omega \rangle = \int d^{3}k \omega |A|^2 \] (8)
is linearly divergent.

Although all further calculations can be performed using the exact expression (7), we find it more instructive to employ the non-relativistic limit $v \ll 1$. In this limit, (7) reduces to a simpler expression
\[ A(k, \Delta E) = ie^{i(\Delta E + \omega)\tau_0} \frac{\bar{g}}{\sqrt{(2\pi)^3 2\omega}} \frac{k_z v}{(\Delta E + \omega)^2} . \] (9)

Note that (9) implies that the average 3-momentum of the emitted Minkowski particles is zero, so one cannot object that the emission of the Minkowski particles affects the trajectory of the detector owing to the 3-momentum conservation. Assuming that the mass of the scalar field is zero and using $d^{3}k = d\varphi \sin \vartheta d\vartheta d\omega$ and $k_z = \omega \cos \vartheta$, we find
\[ \langle \omega \rangle = \frac{g^2 v^2}{3(2\pi)^2} \int_0^{\omega_{\text{max}}} d\omega \frac{\omega^4}{(\Delta E + \omega)^4} , \] (10)
where the cut-off $\omega_{\text{max}}$ is introduced only for the sake of regularization. Since the largest contribution to the divergent integral in (10) comes from large $\omega$, we can take $\Delta E = 0$, which leads to a simple expression
\[ \langle \omega \rangle = \eta E_{\text{spent}} , \] (11)
where $E_{\text{spent}} = Mv^2/2$ is the spent energy, while $\eta$ is the efficiency factor
\[ \eta = \frac{g^2}{6\pi^2} \frac{\omega_{\text{max}}}{M} . \] (12)

Even if one assumes that ultraviolet divergences, typical for quantum field theory, require introduction of a finite cut-off $\omega_{\text{max}}$, nothing prevents (12) from being larger than one.

If, as our results suggest, the energy of the accelerating agency is not the source of the produced energy $\omega + \Delta E$, then we must conclude that this disbalance of the energy is of quantum-mechanical origin. To support this conclusion, below we show that, although the measuring process lasts an infinite time, the disbalance of the energy can be qualitatively explained by a time-energy uncertainty relation. In our calculations we have assumed that the time $\tau_0$, at which the change of the velocity occurs, is known with certainty. On the other hand, if $\tau_0$ is completely unknown, then we must sum all the amplitudes (9) with different $\tau_0$, which gives a vanishing total amplitude (except for the trivial case $\Delta E = \omega = 0$). To explore an intermediate case of a finite uncertainty $\Delta \tau$ of the time at which the velocity changes, we introduce the averaged amplitude
\[ \bar{A} = \frac{1}{\Delta \tau} \int_{\tau_0-\Delta \tau/2}^{\tau_0+\Delta \tau/2} d\tau_0 A \]
\[ = -e^{i(\Delta E + \omega)\tau_0} \frac{\bar{g}k_z v}{\sqrt{(2\pi)^3 2\omega (\Delta E + \omega)^2}} \frac{\sin((\Delta E + \omega)\Delta \tau/2)}{(\Delta E + \omega)\Delta \tau/2} . \] (13)
Now the averaged energy of the produced Minkowski particles is

\[ \langle \omega \rangle = \int \frac{d^3 k}{(2\pi)^3} k^\omega |\vec{A}|^2 \]

\[ = \frac{\bar{g}^2 v^2}{3(2\pi)^2} \int_0^\infty d\omega \frac{\omega^4}{(\Delta E + \omega)^4} \frac{\sin^2(\Delta E + \omega \Delta \tau/2)}{[(\Delta E + \omega \Delta \tau/2)^2]} \]. (14)

If one expands the sine in (14) for small \( \Delta \tau \) and retains only the lowest contribution, then one recovers (10). However, (14) with a non-zero \( \Delta \tau \) is a finite quantity and is of the order

\[ \langle \omega \rangle \sim \bar{g}^2 v^2 \frac{\Delta \tau^2}{(\Delta E)^2} \]. (15)

This can also be written in the form (11), with

\[ \eta \sim \frac{\bar{g}^2}{(M \Delta \tau)(\Delta E \Delta \tau)} \], (16)

revealing that the produced energy is larger when \( \Delta \tau \) is smaller. Applying quantum kinematics to the motion of the detector, we estimate \( (\Delta \tau)^{-1} \sim M v^2/2 = E_{\text{spent}} \). Therefore, (16) gives

\[ \eta \sim \frac{\bar{g}^2 v^2 E_{\text{spent}}}{\Delta E} \]. (17)

This efficiency factor is finite. However, since for a typical case \( E_{\text{spent}} \gg \Delta E \), the efficiency factor can be much larger than one. Actually, from (14) we see that the case \( \Delta E = 0 \) is the most probable. From (17) we see that this case leads to the largest efficiency. The infrared divergence can be removed, for example, by taking the mass \( m \) of the scalar particle to be non-zero, which leads to

\[ \eta \sim \frac{\bar{g}^2 v^2 E_{\text{spent}}}{m} \]. (18)

Again, \( \eta \) can be much larger than one.

Equations (17) and (18) also suggest that \( \eta \) is much larger for relativistic velocities \( v \). This can also be explicitly shown by a similar calculation for the ultrarelativistic limit \( \gamma \gg 1 \) in (4).

4 Conclusion

Our analysis suggests that energy can be extracted from the vacuum. However, we stress that we have not found that the efficiency \( \eta \) must be much larger than one. The coupling constant \( \bar{g} \) or the velocity \( v \) can be so tiny that \( \eta \) is much smaller than one. Therefore, the fact that a large energy production has not yet been observed is not in contradiction with our results. Our results suggest only that a large energy can be extracted in principle. This is not in contradiction with the conservation of energy if one accepts the picture of the vacuum as a state of infinite (or large, owing to a large cut-off) energy density, as, indeed, quantum field theory suggests.
The only real problem with such a picture is to explain the smallness of the cosmological constant. A possible way out of this problem is to propose that only excited states, i.e. particles, contribute to $\langle T_{\mu\nu} \rangle$ in the semi-classical Einstein equation. The results of Sec. 2 and those of [4] suggest that there exist preferred coordinates with respect to which fields should be quantized and, consequently, that the notion of a particle does not depend on the observer, making such a proposal consistent. Another interesting possibility of resolving the cosmological constant problem is to adopt the DeBroglie–Bohm interpretation of quantum field theory (which also requires a preferred time coordinate [17]). Since the ground-state wave function is real, its energy is exactly canceled by the quantum potential [18, 19].

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