Elliptic Flow Suppression due to Hadron Mass Spectrum

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In this letter we show that the presence of Hagedorn resonances in the equation of state of quantum chromodynamics determined by lattice QCD calculations at temperatures $T \sim 100 – 155$ MeV in heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) is only created transiently. Experimentally, it is only possible to measure hadrons, leptons, and photons produced throughout the collision with most of the hadrons being formed in the final stages of the collision.

A simulation that includes at least items 2) and 4) is usually referred to as hybrid model (see, for instance, [21]), which couples fluid dynamics to hadronic transport simulations. These are thought to provide a more reliable description of the hadronic matter formed at the late stages of the collision which would then remove uncertainties in the extraction of thermodynamic and transport properties of the QGP.

In practice, in hybrid models the transition between the fluid degrees of freedom to the hadronic ones is implemented via the Cooper-Frye method [22]. Usually this procedure employs an isothermal space-time hypersurface, with the distribution of hadrons computed in such a hypersurface being used as an initial condition and also boundary condition for the hadron cascade simulation.

Recently, LQCD thermodynamics at temperatures $T \sim 100 – 155$ MeV has been shown to be compatible with calculations performed using Hadron Resonance Gas (HRG) models where the hadron density of states increases exponentially $\rho(m) \sim g(m) \exp((m/T_H)^{1/2})$ with $m/T_H \gtrsim 1$, $g(m) = 0$ [23, 24]. The main parameter that characterizes the increase of the hadron density of states in this case is $T_H$, which is the Hagedorn temperature [25, 26], which is an energy scale of the order of the QCD (pseudo) phase transition temperature [27, 28]. In this paper we investigate the effects of an exponentially increasing hadron mass spectrum (Hagedorn spectrum) on the azimuthal anisotropy of the rapidly expanding matter formed in ultrarelativistic heavy ion collisions. If the temperature at which the conversion from fluid degrees of freedom to hadrons is sufficiently close to the Hagedorn temperature, the production of Hagedorn resonances suppresses the differential elliptic flow of all hadron species.

PACS numbers: 12.38.Mh, 24.10.Pa, 24.85.+p, 25.75.Dw
hadrons species. The effect will be more significant in hybrid models in which the fluid degrees of freedom are usually converted into hadronic ones at isothermal hypersurfaces with temperatures where Hagedorn resonances are highly populated. This introduces a new source of uncertainty to the determination of the viscosity effects in the QGP formed in ultra-relativistic heavy ion collisions.

The thermodynamics of a resonance gas with Hagedorn-like resonances has been studied a long time ago [39–42]. The equilibrium pressure of such a system at temperature $T$ is given by (we use classical statistics throughout this paper, for simplicity)

$$p(T) = \frac{T^2}{2\pi^2} \int_{m_{\text{min}}}^{M_{\text{max}}} dm \, m^2 \rho(m) K_2(m/T),$$

where the integral is limited from below by a mass scale $m_{\text{min}}$ (taken here to be zero) and from above by $M_{\text{max}}$. The standard HRG pressure computed using the discrete set of hadron states from the particle data group can be obtained from this continuum model by taking the appropriate discrete limit of the integral above. The effect of Hagedorn resonances on the thermodynamics can be seen in Fig. 1. Following [23, 24], the trace anomaly obtained from lattice calculations [3] is compared to that found in Hagedorn temperature, this unusual way to redistribute energy via the production of heavier resonances will affect the case of a gas with Hagedorn resonances, any extra energy given to the system is used to produce more and larger particle number density. However, in the typical switching temperature is known for quite some time that for heavy hadrons the range for hydrodynamical behavior in ultra relativistic heavy ion collisions is $p_T^{\text{max}} < 3$ GeV. Very heavy resonances with transverse masses $m_T = \sqrt{p_T^2 + m^2}$ a few times larger than $p_T^{\text{max}}$ contribute to make the overall distribution more isotropic since $e^{-p_T u^\mu / T_{\text{sw}}} = e^{-m_T \gamma / T_{\text{sw}} (1 - |p_T \cdot \nabla / m_T)} \sim e^{-m_T / T_{\text{sw}}}$. In fact, it has been known for quite some time that for heavy hadrons the

$$dN/dy_T dp_T d\phi = \frac{g_a}{(2\pi)^3} \int_{\Sigma} d\Sigma \mu p^\mu e^{-p_T u^\mu / T_{\text{sw}}},$$

and degeneracy $g_a$ is given by

$$dN_a = \int_{\Sigma} d\Sigma \mu p^\mu e^{-p_T u^\mu / T_{\text{sw}}},$$

where $\psi_2(p_T)$ is the event plane angle. The relevant $p_T$ range for hydrodynamical behavior in ultra relativistic heavy ion collisions is $p_T^{\text{max}} < 3$ GeV. Very heavy resonances with transverse masses $m_T = \sqrt{p_T^2 + m^2}$ a few times larger than $p_T^{\text{max}}$ contribute to make the overall distribution more isotropic since $e^{-p_T u^\mu / T_{\text{sw}}} = e^{-m_T \gamma / T_{\text{sw}} (1 - |p_T \cdot \nabla / m_T)} \sim e^{-m_T / T_{\text{sw}}}$. In fact, it has been known for quite some time that for heavy hadrons the

**FIG. 1:** Trace anomaly in the hadron resonance model. The black solid curve denotes the result obtained in the standard HRG model. The short dashed blue curve was computed using the model defined by the exponentially rising hadron mass spectrum $\rho_1(m)$ while the long dashed red curve was computed using $\rho_2(m)$ (both defined in the text). The data points correspond to $N_t = 10$ lattice data [3].
differential elliptic flow generally increases slower with \( p_T \) in comparison to that found for light hadrons \([11, 40]\). Therefore, as \( T_{sw} \) is brought closer and closer to \( T_H \), more of these heavy states are emitted and this "isotropization" mechanism induced by heavy resonances should lead to a suppression of the overall differential elliptic flow of the matter.

Moreover, note that as we increase \( M_{max} \), more states are produced and the total number of particles increases. The \( p_T \) spectrum is also enhanced and this effect becomes more significant at high \( p_T \). This occurs because the exponential term \( e^{m/T_H} \) in the density of states compensates the Boltzmann factor \( e^{-m + \gamma / T_{sw}} \) for very heavy states and one obtains considerably more particles in the spectrum at high \( p_T \) by increasing \( M_{max} \) (heavy particles should have flatter \( p_T \) spectra in comparison to light particles). Also, while the velocity field in the hydrodynamic calculation is not particularly sensitive to the change in the EOS due to Hagedorn resonances, note that conservation of energy and momentum through the isothermal hypersurface implies that these heavy states must be produced when converting the fluid degrees of freedom into hadrons if the switching temperature is around 155 MeV.

We tested these arguments by computing the elliptic flow coefficient of hadrons emitted from an isothermal hypersurface of temperatures, \( T_{sw} = 130 \) and 155 MeV. The isothermal hypersurfaces were computed by solving (boost invariant) relativistic ideal fluid dynamics. We used a single averaged optical Glauber initial condition \([17]\) to describe RHIC’s 20-30% most central events at \( \sqrt{s} = 200 \) GeV \([18]\). We further assumed that the initial transverse flow of the system is zero. This will be sufficient to understand the effects of the Hagedorn spectrum on the particle spectrum and elliptic flow, although event-by-event simulations would be required to investigate higher order Fourier coefficients. The equations of boost invariant ideal hydrodynamics are solved for this initial condition using a Smooth Particle Hydrodynamics (SPH) algorithm \([50]\). For simplicity, particle decays are not included and we fix the value of the energy density at the initialization time (1 fm/c) to match the expected number of direct pions for a given choice of the switching temperature. The equation of state used in this calculation is the one presented in \([3]\), whose low temperature behavior was shown to be compatible with a hadron resonance gas that includes a Hagedorn spectrum \([23, 24, 51]\).

In principle, the hadrons and resonances emitted from the isothermal hypersurface would rescatter and also decay, leading to further changes in the momentum distribution of hadrons and its anisotropy. This effect is not included in this work since our purpose is not to make realistic predictions for the values of flow coefficients but rather to estimate the effect the inclusion of Hagedorn resonances can have on such observables. Note, however, that one would expect that particle decays should not enhance the overall anisotropy of the expanding matter.

In Figs. 2 and 3 we show the differential elliptic flow of all hadron species at isothermal hypersurfaces of temperatures \( T_{sw} = 0.130 \) GeV and \( T_{sw} = 0.155 \) GeV, respectively. We assume that \( T_{sw} = 0.155 \) GeV is the largest temperature at which one can still reliably say that the Hagedorn gas describes the lattice data. We used the previously defined density of states \( \rho_1(m) \) and \( \rho_2(m) \). In both plots, the solid black curve corresponds to the value of elliptic flow in the case of an ordinary HRG without Hagedorn resonances while the short-dashed blue curve and the long-dashed red curve corresponds to the values of \( v_2 \) computed including Hagedorn resonances according to \( \rho_1 \) and \( \rho_2 \), respectively. The maximum mass of the Hagedorn resonances was taken to infinity although we have verified that the results already saturate when \( M_{max} \approx 10 \) GeV.

One can see in both plots that the addition of Hagedorn resonances leads to a suppression of \( v_2(p_T) \) with respect to the HRG calculation. Also, note that the \( v_2 \) computed using \( \rho_2 \) is smaller than that computed using \( \rho_1 \), which was expected since \( T_{H2} < T_{H1} \). In fact, note that the elliptic flow suppression for \( T_{sw} = 0.155 \) GeV is larger than that obtained when \( T_{sw} = 0.130 \) GeV. This confirms our expectation that the production of heavy Hagedorn resonances leads to a suppression of elliptic flow if the switching temperature is sufficiently close to the Hagedorn temperature. We checked that the suppression does increase even further if \( T_{sw} \) is taken to be 0.165 GeV. Also, we verified that the spectrum becomes flatter due to the effect of resonances in the \( p_T \) range where the suppression of \( v_2(p_T) \) is more pronounced.

The elliptic flow suppression discussed here can be of the same order of the typical elliptic flow reduction obtained due to the inclusion of \( \eta/s \sim 1/4\pi \) viscous effects (see, for instance, \([17]\)). It would be interesting to investigate the interplay between the suppression of elliptic flow induced by heavy resonance production and that coming from viscous hydrodynamic effects. Also, the effects of these heavy resonances on the dynamics of the hadronic phase as described by current hadronic cascade simulations remains to be studied.

In conclusion, in this letter we showed that resonance production according to a Hagedorn spectrum leads to a significant suppression of the differential elliptic flow of all hadron species in ultrarelativistic heavy ion collisions if the switching temperature used in the conversion from fluid to hadronic degrees of freedom is close to the Hagedorn temperature. The isotropization mechanism implied by heavy resonance production should amount to a reduction of the higher flow harmonics as well, which can be verified by extending the study done here using event-by-event calculations \([18]\). Our results indicate that the inclusion of Hagedorn resonances in the description of the hadron rich phase formed in heavy ion collisions may be needed to improve the current estimates of the viscous effects in the QGP.
This work was supported by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) and Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq). G. S. Denicol is supported by the Natural Sciences and Engineering Research Council of Canada and thanks the University of São Paulo for the hospitality provided during the completion of this work. C. Greiner thanks the Helmholtz International Center for FAIR within the framework of the LOEWE program launched by the State of Hesse.

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