Baryon number diffusion with critical fluctuations

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Abstract

The description of dynamical fluctuations near the QCD critical point in heavy-ion collisions is crucial for understanding the existing and upcoming experimental data from the beam energy scan programs. In this talk we discuss the evolution of fluctuations of the net-baryon density as given by a stochastic diffusion equation. We study equilibrium as well as dynamical systems for which we can show the impact of nonequilibrium effects on the second-order moment.

Keywords: QCD phase diagram, QCD critical point, critical fluctuations, stochastic baryon number diffusion, real-time dynamics

1. Stochastic diffusion equation

Near the conjectured critical point of QCD the net-baryon density $n_B$ becomes the critical mode, which exhibits diffusive dynamics \cite{1,2,3}. It is therefore expected that for beam energies for which the created matter reaches temperatures and baryochemical potentials in the critical region, fluctuations in the net-baryon number are enhanced \cite{4,5,6,7}. Due to the fast dynamics of the expanding strongly interacting matter nonequilibrium effects become important for a quantitative description of the critical fluctuations \cite{8,9,10,11}. Dynamical models of fluctuations coupled to fluid dynamical expansions have so far considered only fluctuations in the chiral order parameter, see $N_f$FD \cite{12,13,14,15}, which imprint traces in the cumulants of the event-by-event net-proton distributions \cite{16,17}. In this talk we study a numerical implementation of the stochastic diffusion equation for net-baryon number near the critical point

$$\partial_t n_B(t, x) = \kappa \nabla^2 \left( \frac{\delta \mathcal{F}[n_B]}{\delta n_B} \right) + \nabla J(t, x).$$

Here, the free energy functional

$$\mathcal{F}[n_B] = T \int d^3 x \left( \frac{m^2}{2n_c} (\Delta n_B)^2 \right)$$

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for $\Delta n_B = n_B - n_c$, where $n_c$ is the critical net-baryon density, contains only a Gaussian term, such that Eq. (1) is linear in $n_B$. Criticality is included in the mass $m$ via the temperature dependence of the equilibrium correlation length $\xi = \xi(T)$, where $m^2 \propto 1/\xi(T)^2$ as shown in Fig. 1 (left) is motivated by the 3d Ising model equation of state [17].

The stochastic current $J$ describes the internal thermal fluctuations and is given by a Gaussian white noise variable $\zeta(t, x)$ and a variance determined from the fluctuation-dissipation theorem

$$J(t, x) = \sqrt{2T \xi \zeta(t, x)}.$$  

The present system is a simple, linear version of fluid dynamical fluctuations capturing the essential critical phenomena up to second-order moments of the fluctuations. A full implementation of fluid dynamical fluctuations including fluctuations in the energy-momentum tensor and the coupling of Eq. (1) to energy flow is a challenge for theory and numerical implementations [18].

In the following we present results for static and fully equilibrated systems in section 2 and for systems with evolving temperature in section 3.

2. Static equilibrium fluctuations

We solve Eq. (1) for a one-dimensional system of length $L$ with periodic boundary conditions in an implicit scheme considering first a fixed diffusion coefficient $D = 1$ fm expressed via $\kappa = D n_c / T$ and $n_c = 1/(3 \text{fm}^3)$. Equilibrium results are independent of the transport coefficient and solely depend on temperature and the lattice spacing $\Delta x = L/N_x$. The first quantity to verify our numerical algorithm against is the static structure factor $S(k) = \langle \Delta n_B(k, 0) \Delta n_B(-k, 0) \rangle$. Numerical results for the discrete structure factor for two temperatures are shown in Fig. 1 (right) for different lattice spacings. Within the statistical noise at small momenta, the theoretically expected perfectly flat behavior is reproduced. The implicit scheme applied here does not introduce any additional dependences on $\Delta x$ coming from the discretization.

In Fig. 2 we show the spatial correlation function $\langle \Delta n_B(r) \Delta n_B(0) \rangle = \int dk e^{ikr} S(k)/(2\pi)$ of density fluctuations for $T = T_c$ (left) and $T = 0.5$ GeV (right). We compare two different system sizes with the same resolution $\Delta x$. One observes that the fluctuations are uncorrelated over distances larger than $\Delta x$ and that the numerical results reproduce the analytical expectation $\langle \Delta n_B(r) \Delta n_B(0) \rangle = (n_c^2/m^2) \delta(r)$. Net-baryon number conservation in the finite-size system, which is satisfied in the numerics, results in a small but finite shift from this continuum expectation, $-n_c^2/(m^2 N_x)$, which vanishes in the thermodynamic limit. At $r = 0$, we obtain the local variance $\sigma^2$ of the density fluctuations, which is inversely proportional to the lattice spacing. Obviously, $\sigma^2$ is enhanced at the critical temperature compared to high temperatures.
In Fig. 3 (left) this variance is shown as a function of temperature. Again, the increase around $T_c$ is clearly visible and analytical expectations are perfectly reproduced. We also note that higher-order moments, like the kurtosis $\kappa$, are vanishing as expected for purely Gaussian models.

### 3. Dynamical fluctuations

Next, we turn to dynamical fluctuations by assuming a (spatially constant but) time-dependent temperature

$$ T(\tau) = T_0 \left( \frac{\tau_0}{\tau} \right)^{d/2} $$

with $d = 3$, $c_s^2 = 1/3$. The system is first equilibrated at high temperature $T_0 = 0.5$ GeV, before the cooling begins at $\tau_0 = 1$ fm/c. The critical temperature is reached at $\tau - \tau_0 = 2.33$ fm/c. In Fig. 3 (right) we compare the results for three different initial diffusion coefficients $D(\tau_0) = 1, 0.1, 0.01$ fm. For the fastest diffusion process the dynamical values of the variance come close to the equilibrium values around $T_c$. With decreasing diffusion strength nonequilibrium effects become stronger and lead to a decrease of the maximally achieved variance. A slight shift of the maximum to later times is also observed. This retardation effect is, however, rather weak in the present purely Gaussian model.
4. Discussion

We have benchmarked a numerical realization of the Gaussian stochastic diffusion equation versus analytical results for static and equilibrated systems. Because of the uncorrelated nature of the fluctuations in the net-baryon density, we find that the local variance depends on the lattice spacing. This unphysical situation will be cured in future work by the introduction of a surface tension term. We have observed an increase of the local variance near the critical temperature. We have additionally verified that higher-order moments, like the skewness and kurtosis, are vanishing, hence that propagated net-baryon number fluctuations are again purely Gaussian. Finally, we investigated systems in which the (spatially constant) temperature changes with time. In this case the dynamics is subject to nonequilibrium effects and becomes relaxational. We observe a reduction of the maximal variance and a shift of this maximum to later times corresponding to temperatures below $T_c$. Depending on the transport coefficient these nonequilibrium and retardation effects are more or less pronounced.

In ongoing work we include non-Gaussian couplings in the free energy functional motivated by the 3d Ising universality class and investigate in particular the real-time dynamics of non-Gaussian cumulants [19].

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