Analysis of $N^*$ spectra using matrices of correlation functions based on irreducible baryon operators

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We present results for ground and excited-state nucleon masses in quenched lattice QCD using anisotropic lattices. Group theoretical constructions of local and nonlocal straight-link irreducible operators are used to obtain suitable sources and sinks. Matrices of correlation functions are diagonalized to determine the eigenvectors. Both chi-square fitting and Bayesian inference with an entropic prior are used to extract masses from the correlation functions in a given channel. We observe clear separation of the excited state masses from the ground state mass. States of spin $\geq \frac{5}{2}$ have been isolated by use of $G_2$ operators.

1. INTRODUCTION

Reproducing the spectrum of baryon resonances with spin-$1/2$ and spin-$3/2$ and both parities is an important test of lattice QCD. For that we require three-quark operators that transform irreducibly under the spinorial rotation group of the lattice [1]. Local and nonlocal straight-link operators corresponding to irreducible representations (IRs) $G_{1g,u}$, $H_{g,u}$ and $G_{2g,u}$ are constructed to obtain suitable sources and sinks [2]. Here we analyze $N^*$ spectra using these operators.

To determine the $N^*$ excited states, a matrix of correlation functions is computed in the quenched approximation to QCD using irreducible baryon interpolating operators $\overline{B}_i(\vec{x},t)$ of definite quantum numbers,

$$C_{ij}(t) = \sum_{\vec{x}} \langle 0|T(\overline{B}_i(\vec{x},t)\overline{B}_j(0,0))|0\rangle. \quad (1)$$

The $C_{ij}(t)$ matrices for $G_1$ and $H$ states for each parity are constructed using local operators, smeared local operators and smeared straight-link operators. For $G_2$, we have one operator which is of smeared straight-link type (Tables 1, 3 in [2]).

The computation of masses of the lowest-lying resonances is based on the variational method applied to the matrix of correlation functions. In this paper we solve the generalized eigenvalue equation,

$$C_{ij}(t) V_j^{(\alpha)}(t) = \lambda^{(\alpha)}(t, t_0) C_{ij}(t) V_j^{(\alpha)}(t) \quad (2)$$

and determine eigenvectors $V^{(\alpha)}(t)$ for each $t$, with $t_0$ close to the source time. Then the masses of $N^*$ states correspond to the eigenvalues of Eq. (2): $\lambda^{(\alpha)}(t, t_0) \rightarrow e^{-m^{(\alpha)}(t-t_0)} \Rightarrow$. The effective masses are determined from

$$m^{(\alpha)}_{\text{eff}} = \ln \left[ \frac{\lambda^{(\alpha)}(t, t_0)}{\lambda^{(\alpha)}(t+1, t_0)} \right] \Rightarrow e^{-m^{(\alpha)}(t-t_0)}. \quad (3)$$

Another way to extract spectrum information is to calculate the spectral mass density $\rho(\omega)$ from

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Figure 1. $G_{1g}$ effective masses for a selected few low-lying states.

Figure 2. Lowest positive parity effective masses for the IRs $G_{1g}$, $H_g$ and $G_{2g}$.

G_{1g}$ matrices with $t_0 = 2$. In Fig. 4 we choose to show a few low-lying states that are clearly separated. However, the details of the states above the ground state are under study. The plot shows a good plateau for the ground state and statistically significant splittings for a couple of excited states.

In Figs. 2 and 3 we have collected the effective mass plots of the lowest states of both parities for $G_{1g}$, $G_{2g}$ and $H$. The ratio of lowest masses for $G_{1g}$ and $G_{1u}$ is roughly in accordance with experiment, for spin-1/2 states, the $G_{1u}$ mass being higher. The effective masses for $H_{g,u}$ are obtained using a $7 \times 7$ matrices of correlation functions. The lowest negative parity $H_u$ state has smaller mass than the lowest positive parity $H_g$ state. This is compatible with the pattern found in nature for spin-3/2. However, the masses of $H_u$ and $G_{1u}$ overlap within errors. Our result for $G_2$ masses also reveals reasonable separations of the $G_{2g}$ and $G_{2u}$ masses, $G_{2g}$ being lower. From Fig. 2 it is evident that the effective mass for $G_{2g}$ (allowed spin 3/2$, 5/2$, · · ·$)$ is very similar to that for $G_{2g}$ (allowed spin 5/2$, 7/2$, · · ·$)$. These states are orthogonal. One possibility for this is that the lowest $H_g$ state has spin-3/2$^+$ and its mass is accidentally close to that of the lowest $G_{2g}$ state. Another possibility is that the lowest $H_g$ state is spin-5/2$^+$, in which case the same

lattice correlation functions using the Maximum Entropy Method (MEM) [4].

$$C(t, t_0) \rightarrow \int d\omega \rho(\omega)e^{-\omega(t-t_0)}.$$  (4)

One of the advantages of MEM is that it utilizes data on a wide range of available time slices and has been shown to yield results even for noisy data [4]. This feature may be helpful in extracting masses of excited states.

2. RESULTS

We use an ensemble of 287 quenched, anisotropic $16^3 \times 64$ lattices with renormalized anisotropy $\xi = 3.0$ and $\beta = 6.1$, corresponding to $a_\pi^{-1} = 6.0$ GeV [5]. We use the anisotropic Wilson action. The parameters of the Wilson fermion action are tuned nonperturbatively so as to satisfy the continuum dispersion relation $E(p)^2 = E(0)^2 + c(p)^2p^2$ at a pion mass $m_\pi \simeq 500$ MeV. To improve the coupling of operators to the lower mass states we employ gauge-covariant smearing of the quark fields on both source and sink: $\psi(x) = (1 + \sigma^2\Delta(\tilde{U})/4N)^N\psi(x)$, where $\Delta(\tilde{U})$ is the three dimensional Laplacian and $\tilde{U}$ denotes APE-smear $SU(3)$ link variables. The parameters used to smear the quark fields are $\sigma = 3.6$ and $N = 32$.

The effective masses are calculated from $10 \times 10$ $G_{1g}$ matrices with $t_0 = 2$. In Fig. 4 we choose to show a few low-lying states that are clearly separated. However, the details of the states above the ground state are under study. The plot shows a good plateau for the ground state and statistically significant splittings for a couple of excited states.

In Figs. 2 and 3 we have collected the effective mass plots of the lowest states of both parities for $G_1$, $G_2$ and $H$. The ratio of lowest masses for $G_{1g}$ and $G_{1u}$ is roughly in accordance with experiment, for spin-1/2 states, the $G_{1u}$ mass being higher. The effective masses for $H_{g,u}$ are obtained using a $7 \times 7$ matrices of correlation functions. The lowest negative parity $H_u$ state has smaller mass than the lowest positive parity $H_g$ state. This is compatible with the pattern found in nature for spin-3/2. However, the masses of $H_u$ and $G_{1u}$ overlap within errors. Our result for $G_2$ masses also reveals reasonable separations of the $G_{2g}$ and $G_{2u}$ masses, $G_{2g}$ being lower. From Fig. 2 it is evident that the effective mass for $G_{2g}$ (allowed spin 3/2$, 5/2$, · · ·$)$ is very similar to that for $G_{2g}$ (allowed spin 5/2$, 7/2$, · · ·$). These states are orthogonal. One possibility for this is that the lowest $H_g$ state has spin-3/2$^+$ and its mass is accidentally close to that of the lowest $G_{2g}$ state. Another possibility is that the lowest $H_g$ state is spin-5/2$^+$, in which case the same
state must be present in $H_g$ and $G_{2g}$, but not in $G_{1g}$. Study over different values of lattice spacing is required to decide.

Finally, we present the $G_{2g}$ MEM spectral function in Figure 4. We find that the peak of the spectral density roughly corresponds to the effective mass value.

In Table 1 we summarize our preliminary estimates of the lowest masses for the different representations extracted from single-exponential fits to the $\lambda(\alpha) (t, t_0)$ of Eqn. 1. The effective masses for the lowest states of $G_1$, $H$ and $G_2$ for both parities, whether obtained from the variational method or preliminary MEM analysis, show a spectrum of distinct $N^*$ masses. However, the behavior of the spectrum with $m_\pi$ and the sensitivity of the spectrum to variations in the lattice volume has yet to be studied.

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Table 1

| IRs | Fit range | $m_{\text{eff}}$ | $\sim$ Mass (MeV) |
|-----|-----------|-----------------|------------------|
| $G_{1g}$ | 9 – 20 | 0.208 (4) | 1250 |
| $G_{1u}$ | 8 – 12 | 0.321 (4) | 1930 |
| $H_g$ | 9 – 12 | 0.410 (2) | 2460 |
| $H_u$ | 6 – 15 | 0.315 (4) | 1890 |
| $G_{2g}$ | 9 – 14 | 0.409 (7) | 2450 |
| $G_{2u}$ | 8 – 15 | 0.475 (7) | 2850 |

Figure 3. Lowest negative parity effective masses for the IRs $G_{1u}$, $H_u$ and $G_{2u}$.

Figure 4. Example of a spectral density function from a MEM analysis of $G_{2g}$ correlation functions.