Boundary S-matrix in a \((2,0)\) theory of \(AdS_3\) Supergravity

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Abstract: We will discuss two inequivalent generalizations of the standard \((2,0)\) supergravity action \([1]\) to include gravitational Chern-Simons term. One is in the first order formalism where we treat \(\omega_{ab}^M\) as independent and the other is in the second order formalism where \(\omega_{ab}^M\) is determined in terms of other fields via a standard constraint equation. Both of them have different equations of motion and the equations of motion of the first order theory spans only a subset of the second order theory. We will be interested in computing the boundary S-matrix in a conformal field theory dual to the supergravity solutions in this sector and hence we use the equations of motion coming out of the first order theory. We restrict ourselves to the gauge+fermionic sector of the theory to compute the boundary S-matrix. We observe that we cannot directly extend the argument of \([2]\), for \((0,4)\) theories to argue for the non-renormalization of the boundary S-matrix in the presence of higher derivative terms. However, as shown in \([3]\), extended supersymmetry implies that the higher derivative terms could be removed by explicit field redefinition in the bulk. This, in turn implies that the boundary S-matrix should not get renormalized by these higher derivative terms. We therefore modify the arguments of \([2]\) using supersymmetry to argue for the non-renormalization of this boundary S-matrix.
1. Introduction

Three dimensional $AdS_3$ supergravity has always been an interesting area of study since the discovery of BTZ black holes \cite{1}. It has also played an important role in string theory since BTZ arises as a factor in the near horizon geometry of certain class of black holes arising out of string theory \cite{1}. The entropy of BTZ black holes has been carried out in two derivative theory of gravity \cite{5} as well as higher derivative theories of gravity \cite{6, 7, 8, 9, 10, 11, 12, 13}. The statistical analysis of the entropy \cite{6, 7, 8, 9, 10, 12} exploits the asymptotic symmetry of BTZ black hole and AdS/CFT correspondence \cite{14} whereas the gravity analysis in \cite{11} and \cite{13} applies Wald’s formula in the presence of gravitational Chern-Simons term. All the above analysis reproduce the same result for the entropy of BTZ black holes which has remarkable similarity with the Cardy formula for the degeneracy of states in the two dimensional conformal field theory.

Later, Kraus and Larsen argued using AdS/CFT correspondence that if the theory has extended supersymmetry then the entropy of such black holes is given completely in terms of the coefficient of the gravitational Chern-Simons term and the coefficient of the Chern-Simons term involving the R-Symmetry gauge field \cite{8, 9}. A bulk interpretation of their result was understood in \cite{2} for theories with (0,4) supersymmetry and higher. The main conclusion of \cite{2} was that the boundary S-matrix does not get renormalized by the addition of higher derivative terms. Since
the boundary S-matrix are the only perturbative observables in the bulk theory, their result implied that the bulk action even does not get renormalized in the bulk. This in turn implies the non-renormalization of the black hole entropy.

Later, in [3] Gupta and Sen came up with a result where the non-renormalization of the bulk action does not resort to the boundary S-matrix. They found that one can obtain a direct field redefinition in the bulk to absorb the higher derivative piece to bring the action to the standard form. The standard form of the action contains the Einstein-Hilbert piece with a negative cosmological constant term, gravitational Chern-Simons term (and its supersymmetrization), Chern-Simons term involving the gravitino. The standard form also has a Chern-Simons term involving the R-Symmetry gauge field if the theory has an extended supersymmetry. Such field redefinition does not affect the Chern-Simons terms but a priori it allows for the cosmological constant to change. But the cosmological constant is determined in terms of the coefficient of the gravitational and gauge Chern-Simons term in a theory with extended supersymmetry. Hence the cosmological constant even does not change by the field redefinitions.

Since the results of “Kraus and Larsen” and “Gupta and Sen” holds for all theories of gravity with extended supersymmetry, we would expect that the boundary S-matrix does not get renormalized even for theories with lesser extended supersymmetry (e.g. theories with (2, 0) supersymmetry). Let us now try to understand where does the argument for the non-renormalization of boundary S-matrix for (0, 4) case fails for (2, 0) case.

A theory of supergravity with (0, 4) supersymmetry has a SU(2) R-symmetry and a SU(2) gauge field $A_M^a$ corresponding to that. This gauge field sources the SU(2) R-symmetry current in the boundary CFT. One then works with the standard Chern-Simons action $S_0$ and truncates to just the bosonic part. The gauge field equation of motion in this case becomes $F_{a_{MN}}^a = 0$. In order to obtain the correlation function involving this current, first one obtains a solution to the gauge field equation of motion $F_{a_{MN}}^a = 0$ with a boundary condition specified on $A^a_2$. After putting this solution in the action $S_0$ one obtains a functional $I[A^{(0)a}_2]$ of the boundary values $A^{(0)a}_2$. Then according to AdS/CFT correspondence $e^{-I[A^{(0)a}_2]}$ is interpreted as the partition function for calculating the correlation functions in the boundary [13, 14].

One can then add higher derivative gauge invariant term to the action $S_0$ as

$$S = S_0 + \lambda \int d^3 x F_{a_{MN}}^a K^{aMN},$$

where $F_{a_{MN}}^a$ is the gauge covariant field strength. $K^{aMN}$ is some arbitrary term.
constructed out of field strengths, Riemann tensor etc. It is gauge covariant and hence it carries the gauge index $a$ which is required to make the full action gauge invariant. Thus $K^{aMN}$ should contain at least one power of $F^a_{MN}$. This implies that the additional terms in the action (1.1) contains at least two powers of $F^a_{MN}$ as a result of which it does not alter the equation of motion $F^a_{MN} = 0$ which is also the equation of motion without the additional term. As a consequence of this the additional term vanishes on-shell and hence does not alter the boundary S-matrix.

But on the contrary $(2,0)$ theories have a $U(1)$ R-symmetry and thus a $U(1)$ gauge field corresponding to that. Unlike $(0,4)$ case $F_{MN}$ does not carry a non-abelian group index and is gauge invariant by itself. We could add to the action

$$\lambda \int d^3x F_{MN} K^{MN} \sqrt{-g},$$

where $K^{MN}$ need not contain a power of $F_{MN}$. It can be a second rank antisymmetric tensor constructed purely out of Riemann tensor, its covariant derivative and metric.

In that case the equation of motion becomes:

$$\epsilon^{RMN} F_{MN} = 2 \lambda c D_M K^{MR},$$

where $c$ is a known constant appearing in the variation of $S_0$ as

$$\frac{\delta S_0}{\delta A_M} = c \epsilon^{MNP} F_{NP},$$

Thus we see that $F_{MN} = 0$ apparently ceases to be the equation of motion for $(2,0)$ theories. Hence the additional term does not vanish on-shell. Thus a direct extension of the argument of [2] to $(2,0)$ theories fails since the crux of their argument lies in the fact that $F_{MN} = 0$ still remains the equation of motion even in the presence of higher derivative terms and that these additional terms vanish as a consequence of this.

One has to find an alternative explanation to the argument of [2] for $(2,0)$ theories which is the goal of the present paper. For this purpose we have to first calculate the correlation function of the boundary currents from the standard Chern-Simons action $S_0$. But in this case we cannot just restrict ourselves to the gauge sector. This is because the gauge Chern-Simons action for a $U(1)$ gauge field is

$$S_{\text{gauge}} = -\frac{a_L}{2} \int d^3x \epsilon^{MNP} A_M \partial_N A_P$$

In this case we can arbitrarily scale $A_M$ to change the coefficient before it. This was not the case for $(0,4)$ theory in [3] because apart from the $A \wedge dA$ term in the action
there was a $A \wedge A \wedge A$ term which forbids the arbitrary scaling of coefficient before the gauge Chern-Simons term. So, for $(2,0)$ theory we need to have in the action a term which contains a power of $A$ different from two. And indeed there is such a term in the action which is the coupling of gravitino with the gauge field. Thus we need to work with gauge and fermionic sector simultaneously and calculate correlation function involving the R-symmetry current $J(z)$ and supersymmetry currents $G^+(z), G^-(z)$ from the standard action $S_0$. We then argue for the non-renormalization of these correlation functions in the presence of arbitrary higher derivative terms respecting supersymmetry.

We organize the paper as follows:

1. In section 2 we discuss the generalization of standard $\mathcal{N} = (2,0)$ supergravity action to include gravitational Chern-Simons term. We will see that we can have two inequivalent generalizations and we will discuss in great detail the differences between them.

2. In section 3 we obtain the supersymmetric covariant field strengths and Riemann tensor which will form the building blocks of constructing supersymmetric invariant higher derivative terms.

3. In section 4 we use the Euclidean gauge + fermionic Chern-Simons action for $\mathcal{N} = (2,0)$ supergravity in the background of Euclidean $AdS_3$ and describe the calculation of boundary correlators involving R-symmetry current $J(z)$ and supersymmetry current $G^+(z), G^-(z)$. In particular, we calculate the correlators $\langle J(z)J(w) \rangle$, $\langle G^+(z)G^-(w) \rangle$ and $\langle J(z)G^+(v)G^-(w) \rangle$.

4. In section 5 we discuss the implication of addition of higher derivative terms respecting supersymmetry on the boundary S-matrix calculated in section 4.

2. $\mathcal{N} = (2,0)$ supergravity action

We shall be working with a generalization of standard $(2,0)$ supergravity to include gravitational Chern-Simons term. Such a generalization has been obtained in for the simple case of $\mathcal{N} = 1$ supersymmetry. We shall be interested in the case having $(2,0)$ supersymmetry.

Before trying to obtain a $(2,0)$ supersymmetric generalization of cosmological topologically massive supergravity, let us first look at the standard $(2,0)$ supergravity

\footnote{Such a theory is also known as the cosmological topological massive supergravity because of the presence of a massive excitation}
without gravitational Chern-Simons term [3]. The field contents are the vierbeins $e_M^a$, $U(1)$ gauge field $A_\mu$, and a complex Rarita-Schwinger field $\psi_M$. The action written in terms of the fields $e_M^a$, $\psi_M$, $\bar{\psi}_M$, $A_\mu$ is

$$S_0 = \int d^3 x \left[ eR + 2m^2 e - \frac{1}{2m} \epsilon^{MNP} A_M \partial_N A_P + \frac{i}{4m} \epsilon^{MNP} (\bar{\psi}_M (D_N \psi_P) - (D_N \bar{\psi}_M) \psi_P) \right] \quad (2.1)$$

We follow the following convention for Riemann tensor, Ricci tensor and Ricci scalar

$$R_{MN}^{\ ab} \equiv 2\partial_{[M}\omega_{N]}^{\ ab} + 2\omega_{[M}^{\ ac} \omega_{N]}^{\ db} \eta_{cd},$$

$$R_M^a \equiv e^N_b R_{MN}^{\ ab},$$

$$R \equiv e^M_a e^N_b R_{MN}^{\ ab}, \quad (2.2)$$

$\omega_M^{ab}$ is the spin-connection and is determined in terms of $e_M^a$ and $\psi_M$ from the equation

$$\partial_{[N} e_{P]}^a + \omega_{[N}^{\ ab} e_{P]}^b = -\frac{i}{8m} \bar{\psi}_{[N} \gamma^a \psi_{P]} \quad (2.3)$$

and $e$ is the determinant of $e_M^a$ given by

$$e = \det (e_M^a) = \frac{1}{6} \epsilon^{MNP} \varepsilon_{abc} e_M^a e_N^b e_P^c, \quad \varepsilon^{012} = 1, \quad \varepsilon_{0\hat{1}\hat{2}} = 1 \quad (2.4)$$

$\mathcal{D}_M \psi_N$ and $\mathcal{D}_M \bar{\psi}_N$ are defined as

$$\mathcal{D}_M \psi_N = \partial_M \psi_N - \frac{1}{2} B_M^a \gamma_a \psi_N + \frac{i}{2} A_M \psi_N,$$

$$\mathcal{D}_M \bar{\psi}_N = \partial_M \bar{\psi}_N + \frac{1}{2} B_M^a \bar{\psi}_N \gamma_a - \frac{i}{2} A_M \bar{\psi}_N, \quad (2.5)$$

$B_M^a$ is given by

$$B_M^a = \frac{1}{2} \varepsilon^{abc} \omega_{Mbc} - e_M^a \quad (2.6)$$

$\gamma_a$ satisfies the algebra

$$\{\gamma_a, \gamma_b\} = 2\eta_{ab} \quad [\gamma_a, \gamma_b] = -2\varepsilon_{abc} \gamma^c \quad (2.7)$$

We can write the gravity part of the action (2.1) in a pure Chern-Simons form by defining the gauge field $B'_M^a$ in addition to $B_M^a$ as:

$$B'_M^a = \frac{1}{2} \varepsilon^{abc} \omega_{Mbc} + e_M^a \quad (2.8)$$

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2We will often refer to this as the cosmological topological massless supergravity because of the absence of a massive excitation

3We denote the curved space index $M$ by 0, 1, 2 and flat space index $a$ by $\hat{0}, \hat{1}, \hat{2}$. The epsilon symbol with curved space index is denoted by $\epsilon$ and the epsilon symbol involving flat index is denoted by $\varepsilon$. The index in both cases is raised and lowered using $\eta_{MN}$ and $\eta_{ab}$ and we follow the mostly positive convention i.e $\eta = \text{diag}(-1, 1, 1)$
Then the action (2.1) takes the following form \[ S_0 = -\frac{1}{m} \int d^3x \epsilon^{MNP} \left[ \frac{1}{2} B_M{}^a \partial_N B_P{}^b \eta_{ab} + \frac{1}{6} \varepsilon_{abc} B_M{}^a B_N{}^b B_P{}^c \right] \]
\[ + \frac{1}{m} \int d^3x \epsilon^{MNP} \left[ \frac{1}{2} B'_M{}^a \partial_N B'_P{}^b \eta_{ab} + \frac{1}{6} \varepsilon_{abc} B'_M{}^a B'_N{}^b B'_P{}^c \right] \]
\[ - \frac{1}{2m} \int d^3x \epsilon^{MNP} A_M \partial_N A_P \]
\[ + \frac{i}{4m} \int d^3x \epsilon^{MNP} \left( \bar{\psi}_M (D_N \psi_P) - (D_N \bar{\psi}_M) \psi_P \right) \]
(2.9)

The action (2.1), (2.9) is invariant under the supersymmetry transformations

\[ \delta^{(Q)}_{\bar{e}} e^a_M = -\frac{i}{8m} (\bar{\epsilon} \gamma^a \psi_P - \bar{\psi}_P \gamma^a \epsilon) \]
\[ \delta^{(Q)}_{\bar{e}} A_P = \frac{1}{4} (\bar{\epsilon} \psi_P - \bar{\psi}_P \epsilon) \]
\[ \delta^{(Q)}_{\bar{e}} \psi_P = D_P \epsilon, \]
\[ \delta^{(Q)}_{\bar{e}} \bar{\psi}_P = D_P \bar{\epsilon}, \]
(2.10)

Using (2.3) one can deduce the following supersymmetry transformation on the dependent gauge field \( \omega_M{}^{ab} \)

\[ \delta^{(Q)}_{\bar{e}} \omega_P{}^{ab} = -\frac{i}{8} \varepsilon^{abc} \left( \bar{\epsilon} \gamma^c \psi_P - \bar{\psi}_P \gamma^c \epsilon \right) \]
\[ + \frac{i}{16m} \left[ \bar{\epsilon} \gamma_P G_P{}^b - \bar{G}_P{}^b \gamma_P \epsilon \right] \]
\[ + \frac{i}{8m} \left[ \bar{\epsilon} \gamma^a [G_P{}^b] - \bar{G}_P{}^b [\gamma_P \epsilon] \right] \]
\[ \equiv -\frac{i}{8} \varepsilon^{abc} \left( \bar{\epsilon} \gamma^c \psi_P - \bar{\psi}_P \gamma^c \epsilon \right) + C_P{}^{ab}(\epsilon, G) \]
(2.11)

Where \( G_{MN} \) is the field strength associated with the Rarita-Schwinger field \( \psi_M \) defined in (2.13). The last line in the above equation defines \( C_P{}^{ab}(\epsilon, G) \). The supersymmetry transformations of the fields \( B_M{}^a \) and \( B'_M{}^a \) defined in (2.6, 2.8) takes the form

\[ \delta^{(Q)}_{\bar{e}} B_P{}^a = \frac{i}{4} (\bar{\epsilon} \gamma^a \psi_P - \bar{\psi}_P \gamma^a \epsilon) + \frac{1}{2} \varepsilon^{abc} C_P{}_{bc}, \]
\[ \delta^{(Q)}_{\bar{e}} B'_P{}^a = \frac{i}{2} \varepsilon^{abc} C_P{}_{bc}. \]
(2.12)

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One can check by expanding \( B'_M{}^a \) and \( B_M{}^a \) in the action (2.9) in terms of \( \omega_M{}^{ab} \) and \( e_M{}^a \) that one indeed gets back the action (2.1)
The equation of motion derived from the action (2.1) is

\[ R_a^M - \frac{1}{2} R e_a^M - m^2 e_a^M = 0, \]

\[ \hat{F}_{MN} \equiv 2 \partial_{[M} A_{N]} - \frac{1}{2} \psi_{[M} \psi_{N]} = 0, \]

\[ G_{MN} \equiv 2 \mathcal{D}_{[M} \psi_{N]} = 0 \]

(2.13)

where

\[ \mathcal{R}_{MN}^{ab} = R_{MN}^{ab} + i \frac{\varepsilon^{abc}}{4} \bar{\psi}_{[M} \gamma^c \psi_{P]}, \]

\[ \mathcal{R}_M^a = e_b^N \mathcal{R}_{MN}^{ab}, \]

\[ \mathcal{R} = e_a^M \mathcal{R}_M^a, \]  

(2.14)

\( \hat{F}_{MN} \) and \( G_{MN} \) as we will see later are supersymmetric covariant field strengths for the gauge field \( A_M \) and Rarita - Schwinger field \( \psi_M \) respectively and \( \mathcal{R}_{MN}^{ab} \) is the supersymmetric covariant Riemann tensor modulo some terms proportional to \( G_{MN} \).

It is straightforward to check from (2.13) that \( \mathcal{R}_{NP} \) satisfies

\[ D_M \mathcal{R}_{NP} = 0 \]  

(2.15)

Where \( D_M \) is the usual covariant derivative defined using the torsion free connections.

The field equation corresponding to \( \mathcal{R}_{MN}^{ab} \) can also be written in a more convenient way as

\[ \hat{\mathcal{R}}_M^a = 0 \]

where \( \hat{\mathcal{R}}_{MN}^{ab} = \mathcal{R}_{MN}^{ab} + 2 m^2 e^a_{[M} e^b_{N]} \)  

(2.16)

In the above analysis of the theory of supergravity without the gravitational Chern-Simons term, we treated \( \omega_{MN}^{ab} \) as defined through the equation (2.3) and the only independent fields in the theory were \( e_M^a, \psi_M \) and \( A_M \). This formulation is known in the literature as the “second order formulation” since it gives second order equations in the gravity sector. We will also resort to the supersymmetry transformations (2.11) and (2.12) for \( \omega_{MN}^{ab}, B_M^a \) and \( B'_M^a \) as the “second order supersymmetry transformations”.

In a different formulation we can treat \( \omega_{MN}^{ab} \) as an independent field in (2.1). The variation of the action with respect to this field will give rise to the constraint equation (2.3) and the variation with respect to other fields will give rise to the other equations in (2.13). All these equations are first order since we are treating \( \omega_{MN}^{ab} \) as independent. This formulation is known as the “first order formulation”.

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We now discuss the supersymmetry invariance of the action under supersymmetry transformation independently defined for $\omega_{M}^{ab}$

$$\delta^{(Q)}_{\epsilon} \omega_{P}^{ab} = -\frac{i}{8} \varepsilon^{abc} (\bar{\epsilon} \gamma_{P} \psi_{P} - \bar{\psi} \gamma_{P} \epsilon)$$  \hspace{1cm} (2.17)$$

This also implies the following supersymmetry transformation for the fields $B_{m}^{a}$ and $B_{M}^{a}$

$$\delta^{(Q)}_{\epsilon} B_{P}^{a} = \frac{i}{4} (\bar{\epsilon} \gamma^{a} \psi_{P} - \bar{\psi} \gamma^{a} \epsilon)$$

$$\delta^{(Q)}_{\epsilon} B_{P}^{a} = 0$$  \hspace{1cm} (2.18)$$

It is quite easy to check that the action (2.1), (2.9) is invariant with respect to this transformation along with (2.10) when we treat $\omega_{M}^{ab}$ as an independent gauge field. We will refer to the supersymmetry transformations (2.17) and (2.18) as the “first order supersymmetry transformations”. From (2.18) we see that the supersymmetry transformation of $B_{M}^{a}$ vanishes and this can be interpreted as belonging to the right sector of the theory where we do not have any supersymmetry whereas the rest of the fields $(B_{m}^{a}, \psi_{M}, A_{M})$ belongs to the left sector which has $N = 2$ supersymmetry. This justifies the (2, 0) nature of the theory.

This theory is supersymmetric in both the formulations. Both the formulations also give rise to the same equations of motion. In particular, the $\omega_{M}^{ab}$ equation of motion in the first order formulation is the same as the equation that determines $\omega_{M}^{ab}$ in terms of $e_{M}^{a}$ and $\psi_{M}$ in the second order formulation so that we can still interpret $\omega_{M}^{ab}$ as the spin connection.

Let us now try to make appropriate changes in (2.9) to obtain the gravitational Chern-Simons term and its supersymmetric generalization. Following [20, 2], we make the following changes

$$S_{0} = -a_{L} \int d^{3}x \varepsilon^{MNP} \left[ \frac{1}{2} B_{M}^{a} \partial_{N} B_{P}^{b} \eta_{ab} + \frac{1}{6} \varepsilon_{abc} B_{M}^{a} B_{N}^{b} B_{P}^{c} \right]$$

$$+ a_{R} \int d^{3}x \varepsilon^{MNP} \left[ \frac{1}{2} B_{M}^{a} \partial_{N} B_{P}^{b} \eta_{ab} + \frac{1}{6} \varepsilon_{abc} B_{M}^{a} B_{N}^{b} B_{P}^{c} \right]$$

$$- \frac{a_{L}}{2} \int d^{3}x \varepsilon^{MNP} A_{M} \partial_{N} A_{P}$$

$$+ \frac{ia_{L}}{4} \int d^{3}x \varepsilon^{MNP} (\bar{\psi}_{M}(D_{N} \psi_{P}) - (D_{N} \bar{\psi}_{M}) \psi_{P})$$

$$a_{L} = K + \frac{1}{m} \quad a_{R} = -K + \frac{1}{m}$$  \hspace{1cm} (2.19)$$
After replacing $B^a_M$ and $B'^a_M$ from (2.6) and (2.8) one can see that one indeed gets the gravitational Chern-Simons term

$$
S_0 = \int d^3x \left[ e R + 2m^2 e - \frac{a_L}{2} e^{MNP} A_M \partial_N A_P + \frac{i}{4} a_L e^{MNP} \left( \bar{\psi}_M (D_N \psi_P) - (D_N \bar{\psi}_M) \psi_P \right) \right] 
$$

$$
- K \int d^3x \ e^{MNP} \left[ \left( \frac{1}{2} \omega_{Mcd} \partial_N \omega_P^{dc} + \frac{1}{3} \omega_{Mbc} \omega_N^{cd} \omega_P^b \right) + m^2 e^a_M \left( \partial_N e^a_P + \omega_N e^c_P \right) \right],
$$

$$
a_L = K + \frac{1}{m}, \quad (2.20)
$$

In the first order formalism, when $\omega_M^{ab}$ is treated as an independent field, the first order supersymmetry transformation (2.18) and (2.10) of the left sector which comprises of $B_M^a$, $\psi_M$ and $A_M$ do not talk with the right sector comprising of $B'_M^a$. In the above action we have just changed the relative coefficients between the left and right sector fields without changing the relative coefficients between the fields of a particular sector. We therefore expect the above action to be invariant under the “first order” supersymmetry variations (2.17), (2.18), (2.11). We find that the action (2.19), (2.20) is indeed invariant under the first order supersymmetry transformations (2.17), (2.18), (2.11). In order to obtain the equation of motion we first vary the action (2.20)

$$
\delta S_0 = 2 \int d^3 x \ e^{MNP} \delta \omega^c_M \left[ \partial_N e^P c + \omega_N e^d_P d + \frac{i}{8m} \bar{\psi}_N \gamma^c \psi_P \right] 
$$

$$
- K \int d^3 x \ \delta \omega^c_M e \left[ \Re e^c_M + 2m^2 e^c_M - 2 \Re^M c \right] + \int d^3 x \ e^a_M \left[ \Re e^a_M + 2m^2 e^a_M - 2 \Re^M a \right] 
$$

$$
- 2m^2 \int d^3 x \ \delta e^a_M e^{MNP} \left[ \partial_N e^P a + \omega_N e^d_P d + \frac{i}{8m} \bar{\psi}_N \gamma^a \psi_P \right] 
$$

$$
- a_L \int d^3 x \partial_A e^{MNP} \left( \partial_N A_P - \frac{1}{4} \bar{\psi}_N \gamma_P \psi_P \right) + \frac{i a_L}{2} \int d^3 x \delta \bar{\psi}_M e^{MNP} D_N \psi_P 
$$

$$
+ \frac{i a_L}{2} \int d^3 x \ e^{MNP} D_N \bar{\psi}_P \delta \psi_M. \quad (2.21)
$$

In the first order formulation when we are treating $\omega_M^{ab}$ as an independent field, we should set all the variations independently to zero to get the equations of motion. We get

$$
2 \epsilon^{MNP} \left[ \partial_N e^c_P + \omega_N e^d_P d + \frac{i}{8m} \bar{\psi}_N \gamma^c \psi_P \right] - K e \left[ \Re e^c_M + 2m^2 e^c_M - 2 \Re^M c \right] = 0
$$

$$
- 2m^2 \epsilon^{MNP} \left[ \partial_N e^a_P + \omega_N e^d_P d + \frac{i}{8m} \bar{\psi}_N \gamma^a \psi_P \right] + e \left[ \Re e^a_M + 2m^2 e^a_M - 2 \Re^M a \right] = 0
$$

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\[ F_{MN} \equiv 2 \partial_{[M} A_{N]} = \frac{1}{2} \bar{\psi}_{[M} \psi_{N]}, \]
\[ G_{MN} \equiv 2 \mathcal{D}_{[M} \psi_{N]} = 0 \] (2.22)

When \( K_m \neq \pm 1 \) we can take linear combinations of first two equations and we get

\[
\partial_{[N} e_P^a + \omega_{[N}{}^{ab} e_{P]b} = -\frac{i}{8m} \bar{\psi}_{[N} \gamma^a \psi_{P]},
\]
\[
\mathcal{R}_a^M - \frac{1}{2} \mathcal{R}_{e_a}^M - m^2 e_a^M = 0, \tag{2.23}
\]

We thus see that when \( K_m \neq \pm 1 \) we get the same equation that we got in our analysis of the supergravity theory without gravitational Chern-Simons term. However, when \( K_m = \pm 1 \) the first two equations of (2.22) are degenerate and instead of two there is one equation governing \( e_M^a \) and \( \omega_M^{ab} \). This can also be seen from the action (2.19) written in terms of \( B_M^a \) and \( B'_M^a \). When \( K_m = \pm 1 \) either \( a_L \) or \( a_R \) vanishes. Then we have either \( B_M^a \) or \( B'_M^a \) present in the action describing the gravity sector. Thus the degree of freedom required for a theory of gravity is reduced and we cannot have a theory of gravity. Hence the first order formulation fails for \( K_m = \pm 1 \). However, at a generic point not satisfying \( K_m = \pm 1 \) we get a perfectly sensible first order theory which is supersymmetric and gives the equations of motion which coincides with the equation of motion derived for the theory of supergravity without gravitational Chern-Simons term.

We cannot add higher derivative terms to the supergravity action in the first order formulation. The equation for \( \omega_M^{ab} \) will become dynamical and there cannot be simple algebraic dependence of \( \omega_M^{ab} \) on \( e_M^a \) and \( \psi_M \) as in (2.3) and hence we cannot interpret \( \omega_M^{ab} \) as the spin connection. Addition of higher derivative terms will require us to go the second order picture.

In the second order formulation we work with \( e_M^a, \omega_M^{ab}, \psi_M \) and \( A_M \) with \( \omega_M^{ab} \) determined in terms of the other fields through (2.3). Substituting \( \omega_M^{ab} \) by the constraint equation (2.3) in (2.20) gives rise to the action

\[
S_0 = \int d^3x \left[ eR + 2m^2 e - \frac{a_L}{2} e^{MNP} A_M \partial_N A_P + \frac{i}{4} a_L \epsilon^{MNP} \left( \bar{\psi}_M (\mathcal{D}_N \psi_P) - (\mathcal{D}_N \bar{\psi}_M) \psi_P \right) \right]
\]

\[
- K \int d^3x \left\{ \frac{1}{2} \omega_{Mcd} \partial_N \omega_P^{dc} + \frac{1}{3} \omega_{Mw} \omega_N^{cd} \omega_P^{db} \right\} - \frac{i m}{8} e_a^M \bar{\psi}_N \gamma^a \psi_P \right], \tag{2.24}
\]

However, the action (2.19) or (2.24) is not invariant under the “second order supersymmetry transformations” (2.10), (2.11), (2.12). Varying the action (2.19) with
In order to get the equations of motion constructed out of $S_0$ in the second order formulation we can use the variation (2.21), but keeping in mind that $\delta \omega^{ab}_M$ is not independent and should be determined in terms of $\delta e^a_M$ and $\delta \psi_M$ from the constraint equation (2.3). After using the constraint (2.3), the variation (2.21) takes the form

$$\delta S_0 = -K \int d^3x \, \delta \omega^{ab}_M e \left[ \mathcal{R} e^c_M + 2m^2 e^c_M - 2 \mathcal{R}^c_M \right]$$

$$+ \int d^3x \, \delta e^a_M e \left[ \mathcal{R} e^c_M + 2m^2 e^c_M - 2 \mathcal{R}^c_M \right]$$

$$- a_L \int d^3x \delta A_M \epsilon^{MNP} \left( \partial_N A_P - \frac{1}{4} \bar{\psi}_N \psi_P \right) + \frac{i a_L}{2} \int d^3x \delta \bar{\psi}_M \epsilon^{MNP} D_N \psi_P$$

$$+ \frac{i a_L}{2} \int d^3x \epsilon^{MNP} D_N \bar{\psi}_P \delta \psi_M$$

(2.27)

To obtain the complete equation of motion systematically we should first express $\delta \omega^{ab}_M$ in terms of $\delta e^a_M$ and $\delta \psi_M$ and put it in the above variation. Then collect all the terms proportional to the variations of $e^a_M$, $\psi_M$ and $A_M$ and set them to zero. This will certainly not change the $A_M$ equations because $\delta \omega^{ab}_M$ does not contain $\delta A_M$ but it will certainly change the $e^a_M$ and $\psi_M$ equations of motion. The
equation, in general will be quite complicated involving the Cotton tensor \[21, 22\] and Cottino-vector spinor \[23\]. However, one can easily check that the equation of motion (2.13) obtained for the topologically massless case (which is also the equation of motion of \(S_0\) in the first order formulation) still extremizes the variation of \(S_0\) in the second order formulation and hence gives a sector of solution for the theory with gravitational Chern-Simons term in the second order formulation. This is contrary to the case of supergravity without gravitational Chern-Simons term where both the “first order” and “second order” formulation give rise to the same equations of motion. In the presence of gravitational Chern-Simons term, we saw that in the “first order” formulation the equations of motion are the same as that of the theory without gravitational Chern-Simons term. Whereas, in the “second order” formulation the equations get modified and the earlier equations span only a part of the full spectrum\(^5\).

We see two major differences in the first order and second order formulation of a theory of \((2, 0)\) supergravity with gravitational Chern-Simons term. One is in the supersymmetry invariance of \(S_0\). The other is in the equations of motion spanned by both of them. Such differences in both the formulations in a theory of topologically massive gravity (TMG) and topologically massive electrodynamics (TME), as far as equations of motion is concerned, has been previously observed in \[24\]. However, later in \[25\] it was argued that the definition of the “first order theory” in \[24\] is not the correct first order formulation of the “second order theory” because both of them give different equations of motion. The correct first order formulation will include several other non-dynamical fields. They found the additional fields required for the case of TME. Finding the additional fields required for the correct first order formulation of a theory of gravity in the presence of gravitational Chern-Simons term in general will be quite involved and as per our knowledge there is no such formulation yet.

However, we will keep our definition of first order and second order formalism without calling one as the equivalent of the other. As we have seen before, the supergravity spectrum spanned by our first order theory belongs to a subset of the second order theory. We will be interested in the correlation functions of conformal field theory operators dual to this sector and hence we will use the equations of motion in the first order theory will be the same as that of the theory without gravitational Chern-Simons term, we will still call this theory as the cosmological topological massless supergravity. The nontrivial massive excitation arises when we go to the second order formulation of the theory with gravitational Chern-Simons term and we will call this second order formulation as the cosmological topological massive supergravity.

\(^5\)Since the equations of motion in the first order theory will be the same as that of the theory without gravitational Chern-Simons term, we will still call this theory as the cosmological topological massless supergravity. The nontrivial massive excitation arises when we go to the second order formulation of the theory with gravitational Chern-Simons term and we will call this second order formulation as the cosmological topological massive supergravity.
motion (2.13) in our analysis of boundary S-matrix.

3. Supersymmetric covariant field strengths

In this section we will try to covariantize the various field strengths \((R^{ab}_{MN}, F_{MN}, G_{MN}, \tilde{G}_{MN})\) with respect to the second order supersymmetry transformations (2.10, 2.11, 2.12) considered in the previous section. First let us try to understand what do we mean by field strengths covariant with respect to supersymmetry. Normally the field strengths, for example Riemann tensor \(R^{ab}_{MN}\) or gauge field strength \(F_{MN} \equiv 2\partial_{[M}A_{N]}\), are not covariant with respect to supersymmetry i.e. under supersymmetry transformations (2.10, 2.11, 2.12) the above mentioned field strengths will give rise to terms proportional to partial derivatives of the supersymmetry transformation parameter. Thus we have to add certain terms to the above mentioned field strengths so that this non-covariant behavior gets canceled and we get perfectly covariant field strengths whose supersymmetry transformation gives rise to terms containing the supersymmetry transformation parameter and other supersymmetric covariant field strengths. As a result of this, if we add any term constructed out of these field strengths to the action, under supersymmetry it will give rise to terms involving the supersymmetry parameter and other super-covariant field strengths. This non-invariance under supersymmetry can be canceled by adding other terms to the action constructed out of the super-covariant field strengths such that its supersymmetry variation exactly cancels the supersymmetry variation of the term that we initially added to the action. Therefore, these supersymmetric covariant field strengths will form the basic building blocks for constructing supersymmetric invariant higher derivative terms. This observation will play a crucial role in the discussion of non-renormalization of the boundary S-matrix in section 4.

It is easy to see that \(G_{MN}\) and \(\tilde{G}_{MN}\) defined in (2.13) are already covariant with respect to the supersymmetry transformations (2.10). Hence, the fully covariantized Rarita-Schwinger field strength \(\tilde{G}_{MN}\) is the same as original Rarita-Schwinger field strength \(G_{MN}\)

\[
\tilde{G}_{MN} = G_{MN} \tag{3.1}
\]

The gauge field strength \(F_{MN} \equiv 2\partial_{[M}A_{N]}\) is not covariant with respect to the supersymmetry transformations (2.10). The covariantization of \(F_{MN}\) with respect to supersymmetry is obtained as

\[
\tilde{F}_{MN} \equiv 2\partial_{[M}A_{N]} - 2\delta^{(Q)}_{\frac{1}{2} \psi[M} A_{N]} \tag{3.2}
\]
Using the supersymmetry transformation (2.10), one finds that $\tilde{F}_{MN}$ is same as $\hat{F}_{MN}$ defined in (2.13), i.e

$$\tilde{F}_{MN} = \hat{F}_{MN} = F_{MN} - \frac{1}{2} \bar{\psi}_{[M} \psi_{N]}$$ (3.3)

Now we need to covariantize the Riemann tensor. The Riemann tensor $R_{MN\gamma}^{ab}$ (which is the field strength associated with $\omega_{M}^{ab}$) defined in (2.2) is not covariant with respect to the supersymmetry transformations (2.10) and we need to covariantize it. This is obtained as

$$\tilde{R}_{MN}^{ab} = R_{MN}^{ab} - 2\delta^{(Q)}_{\bar{\psi}_{[M} \psi_{N]}} \omega_{M}^{ab},$$ (3.4)

Using the supersymmetry transformations (2.11) we find

$$\tilde{R}_{MN}^{ab} = R_{MN}^{ab} + \frac{i}{16m} \left[ \bar{\psi}_{[M} \gamma_{N]} G^{ab} - \bar{G}^{ab}_{\gamma[N} \psi_{M]} \right]$$

$$+ \frac{i}{8m} \left[ \bar{\psi}_{[M} \gamma^{[a} G_{N]}^{b]} - \bar{G}_{\gamma[N}^{[b} \gamma^{a]} \psi_{M]} \right]$$ (3.5)

The prescription (3.2) and (3.4) for covariantization of the field strengths have been worked out by looking at the supersymmetry transformations considered in section 4. One can easily check that the field strengths reproduced by such prescription are indeed covariant with respect to the supersymmetry transformations (2.10, 2.11, 2.12).

4. Boundary S-matrix

In this section we shall evaluate the two and three point correlators involving the weight “1” R-symmetry current ($J(z)$) and the two weight “$3/2$” supersymmetry currents ($G^{(+)}(z)$ and $G^{(-)}(z)$) in the dual conformal field theory. Here the superscript “(+)” and “(−)” denote the charges of the corresponding operators with respect to the global $U(1)$, which is a symmetry of the theory.

There has been earlier works [26, 27, 28] which deals with Rarita-Schwinger fields in a general $AdS_{d+1}/CFT_{d}$ correspondence. All the above works considered free massless as well as massive Rarita-Schwinger fields without coupling to any other fields in the bulk apart from gravity. But in a theory of extended supergravity, the Rarita Schwinger field couples to gravity as well as gauge field in the bulk. Obtaining the solution to the coupled field equations in a general dimension will be a monstrous task. However, we will see that in 3 dimensions the field equations can be written in a form notation and hence solving the coupled equation becomes somewhat simple and the coupling to gauge field can be taken care of, in a order by order fashion.
We begin this section by writing the supergravity action in Euclidean space

\[ S = i a_L \int d^3x \epsilon^{MNP} \left[ \frac{1}{2} B_M^a \partial_N B_P^b \delta_{ab} + \frac{i}{6} \varepsilon_{abc} B_M^a B_N^b B_P^c \right] \]

\[ - i a_R \int d^3x \epsilon^{MNP} \left[ \frac{1}{2} B'_M^a \partial_N B'_P^b \delta_{ab} + \frac{i}{6} \varepsilon_{abc} B'_M^a B'_N^b B'_P^c \right] \]

\[ + i \frac{a_L}{2} \int d^3x \epsilon^{MNP} A_M \partial_N A_P \]

\[ + \frac{a_L}{4} \int d^3x \epsilon^{MNP} \left( \bar{\psi}_M (D_N \psi_P) - (D_N \bar{\psi}_M) \psi_P \right) \quad (4.1) \]

Where

\[ B_M^a = \frac{i}{2} \varepsilon^{abc} \omega_{Mbc} - m e_M^a, \]

\[ B'_M^a = \frac{i}{2} \varepsilon^{abc} \omega_{Mbc} + m e_M^a, \]

\[ D_M \psi_N = \partial_M \psi_N - \frac{1}{2} B_M^a \gamma_a \psi_N + \frac{i}{2} A_M \psi_N, \]

\[ D_M \bar{\psi}_N = \partial_M \bar{\psi}_N + \frac{1}{2} B_M^a \bar{\psi}_N \gamma_a - \frac{i}{2} A_M \bar{\psi}_N \quad (4.2) \]

The gamma matrices satisfies the algebra:

\[ \{ \gamma_a, \gamma_b \} = 2 \delta_{ab}, \]

\[ [\gamma_a, \gamma_b] = -2i \varepsilon_{abc} \gamma_c, \quad (4.3) \]

We will not be interested in obtaining any correlators involving the stress tensor. Therefore gravity just enters as a global \( AdS_3 \) background. The metric of Euclidean \( AdS_3 \) written in Poincare patch coordinate system is

\[ ds^2 = \frac{1}{m^2(x^0)^2} \left( (dx^0)^2 + (dx^1)^2 + (dx^2)^2 \right) \]

\[ = \frac{1}{m^2(x^0)^2} \left( (dx^0)^2 + dzd\bar{z} \right), \quad z = x^1 + ix^2 \quad (4.4) \]

The gauge field and Rarita Schwinger equations are the same as obtained in the Lorentzian case \( (2.13) \). The equations of motion can be written in a form notation

\[ dA = \frac{1}{4} \bar{\psi} \land \psi, \]

\[ d\psi = \frac{1}{2} B^a \gamma_a \land \psi = -\frac{i}{2} A \land \psi, \]

\[ d\bar{\psi} = \frac{1}{2} B^a \land \bar{\psi} \gamma_a = \frac{i}{2} A \land \bar{\psi} \quad (4.5) \]

Let us now outline the important steps involved in the calculation of the correlation function

---

Note the change in the definition of \( B_M^a \) and \( B'_M^a \) compared to the lorentzian case \( (2.6),(2.8) \)
1. Solve the equation (4.3) order by order i.e. first solve the leading order equations, where we set the RHS appearing in all the equations to zero. Then obtain the first order corrections by putting the solution obtained for leading order in the RHS and so on. For our purpose we will obtain just the first order correction. We will need to impose the following gauge conditions on $\psi$ and $\bar{\psi}$ to obtain the solution

$$\gamma^M \psi_M = 0 \quad \bar{\psi}_M \gamma^M = 0 \quad (4.6)$$

2. From the boundary behavior ($x^0 \to 0$), we need to figure out boundary values of which fields has the appropriate conformal dimension, so as to source the corresponding operators in the boundary and then impose boundary conditions on those fields.

3. Obtain the solutions obtained in (1) in terms of the boundary conditions imposed in (2) but upto terms quadratic in the boundary values.

4. We need to add boundary term to the action (4.1) for consistency requirements. The reason is the following. We have imposed boundary conditions on some components of the fields and left others to vary. The variation of the action (4.1) will have a boundary piece involving the variation of those components of the fields on which we have not imposed any boundary conditions and hence will not vanish. Therefore we need to add a boundary piece to the action such that its variation exactly cancels this contribution.

5. Evaluate the bulk action (4.1) as well as boundary action obtained in (4) as a functional of the boundary values but just upto terms cubic in the boundary values.

6. Obtain the correlation function in the boundary using the prescription given in [15, 16].

Before proceeding with the rest of the section, there is a subtle issue which we would like to address. $B^a_M$ and $B'^a_M$ defined in (4.2) has $\omega_M^a$ in it which satisfies the constraint equation (2.3). Because of the presence of $\bar{\psi}_M \gamma^a \psi_N$ in the right hand side of the equation (2.3), $\omega_M^a$ and hence $B^a_M$ and $B'^a_M$ do not have purely gravitational contribution. But the gravitational contribution and fermionic contribution to $\omega_M^a$ can be decoupled as $\omega_M^a = \omega^{(e)}_M + \omega^{(f)}_M$. 

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\( \omega_M^{(e)a} \) satisfies the standard torsionless constraint equation and \( \omega_M^{(f)a} \) contributes to the torsion part of the equation (2.3)

\[
d e^a + \omega^{(e)ab} \wedge e_b = 0, \\
\omega^{(f)ab} \wedge e_b = - \frac{i}{8m} \bar{\psi} \gamma^a \psi
\] (4.7)

Clearly \( \omega_M^{(f)a} \) will be \( \mathcal{O}(\bar{\psi} \gamma \psi) \) and its contribution to the equation of motion (4.3) through \( B_M^a \) and \( B'_M^a \) will be cubic in \( \psi \). Since we would like to obtain the solution upto terms quadratic in the fields, we will neglect this term and use the standard torsionless spin connection in \( B_M^a \) and \( B'_M^a \) for solving the equations of motion (4.5). Since \( \omega_M^{(f)} \) has a quadratic dependence on the fermions, \textit{a Priori} it seems that, in the evaluation of the action for the on-shell configuration of fields as prescribed in step 3, as if \( \omega_M^{(f)} \) will give a contribution to the on-shell action quadratic in the boundary values of the fermions through the dependence of the action (4.1) on \( B_M^a \) and \( B'_M^a \) as

\[
i a_L \int d^3x \epsilon^{MNP} \omega_M^{(f)a} \left[ \partial_N B^{(e)}_{P} + \frac{i}{2} \varepsilon_{abc} B^{(e)b}_{N} B^{(e)c}_{P} \right]
\]

\[- i a_R \int d^3x \epsilon^{MNP} \omega_M^{(f)a} \left[ \partial_N B^{(e)}_{P} \delta_{ab} + \frac{i}{2} \varepsilon_{abc} B^{(e)b}_{N} B^{(e)c}_{P} \right]
\] (4.8)

Where

\[
B^{(e)b}_{N} = \omega^{(e)b}_{N} - me_{N}^b, \\
B^{(e)b}_{N} = \omega^{(e)b}_{N} + me_{N}^b
\] (4.9)

But one can check that for \( AdS_3 \) background the terms inside the square bracket of the above integral (4.8) vanishes and hence will not give a quadratic contribution to the evaluation of the on-shell action prescribed in step 3. However, one has to be careful while obtaining quartic and higher contribution. One has to take the effect of the torsion in the spin connection into account. But since we are only interested in obtaining the on-shell action upto terms cubic in the fields, we will use the torsionless spin connection while solving the equation of motion (4.3) and while evaluating the action for the on-shell configuration of fields we will neglect contribution from the first two terms in the action (4.1).

The stepwise analysis of step 1 to step 5 has been given in Appendix A. But we will just outline the important results here and obtain the correlators. From step 2
we see that we need to impose the following boundary conditions

\[
\lim_{x^0 \to 0} A_\zeta = A_\zeta^0(\vec{z}), \\
\lim_{x^0 \to 0} (x^0)^\frac{1}{2} \bar{\psi}_\zeta^{(1)} = \Theta_\zeta^{-}(\vec{z}), \\
\lim_{x^0 \to 0} (x^0)^\frac{1}{2} \bar{\psi}_\zeta^{(2)} = \Theta_\zeta^{+}(\vec{z})
\] (4.10)

From step 4 we get the following boundary action

\[
S_{\text{bndy}} = S_{\text{bndy}}[\psi, \bar{\psi}] + S_{\text{bndy}}[A] \\
S_{\text{bndy}}[\psi, \bar{\psi}] = -\frac{i a L}{2} \int d^2 \vec{z} \left( \bar{\psi}_\zeta^{(1)}(x^0, \vec{z}) \psi_\zeta^{(1)}(x^0, \vec{z}) + \bar{\psi}_\zeta^{(2)}(x^0, \vec{z}) \psi_\zeta^{(2)}(x^0, \vec{z}) \right) \bigg|_{x^0=0} \\
S_{\text{bndy}}[A] = a L \int d^2 \vec{z} A_\zeta(\vec{z}, x^0) A_\zeta(\vec{z}', x^0) \bigg|_{x^0=0}
\] (4.11)

Here the superscripts (1) and (2) in the Rarita-Schwinger fields represents the spinor components. The measure \(d^2 \vec{z}\) is defined as

\[
d^2 \vec{z} = dx^1 dx^2
\] (4.12)

After obtaining the solution to the equations of motion in step 3 subject to the boundary conditions (4.10) upto terms quadratic in the boundary values, we proceed to step 5 where we evaluate the action (4.1) along with the boundary action (4.11) for these on-shell configuration of fields. We neglect the contribution coming from the first two terms in (4.1) as we have argued before. We get

\[
S[A_\zeta^0, \Theta_\zeta^+, \Theta_\zeta^-] = -\frac{a L}{\pi} \int d^2 \vec{z} d^2 \vec{w} \frac{1}{(z - w)^2} A_\zeta^0(\vec{w}) A_\zeta^0(\vec{z}) - \frac{2 i a L}{\pi} \int d^2 \vec{z} d^2 \vec{w} \frac{1}{(z - w)^2} \Theta_\zeta^+(\vec{w}) \Theta_\zeta^-(\vec{z}) - \frac{a L}{\pi^2} \int d^2 \vec{z} d^2 \vec{w} d^2 \vec{v} \frac{1}{(z - w)(z - v)(w - v)^2} A_\zeta^0(\vec{z}) \Theta_\zeta^+(\vec{v}) \Theta_\zeta^-(\vec{w})
\] (4.13)

Then according to \(AdS/CFT\) conjecture \([15, 16]\), we have

\[
\exp(-S(A, \psi, \bar{\psi})) = \left\langle \exp \left( \frac{1}{2 \pi} \int_\partial J(\zeta) A_\zeta^0(\zeta) + G^+(\zeta) \Theta_\zeta^-(\zeta) + G^-(\zeta) \Theta_\zeta^+(\zeta) \right) \right\rangle
\] (4.14)

\(^7\)The superscripts in the corresponding boundary values represents charge with respect to the \(U(1)\) symmetry in the theory
This implies the following two and three point correlation functions

\[ \langle J(\vec{z}_1)J(\vec{z}_2) \rangle = (2\pi)^2 \frac{\delta}{\delta A^0_1(\vec{z}_1)} \frac{\delta}{\delta A^0_2(\vec{z}_2)} e^{-S} \left|_{(A^0_1(\vec{z}), \Theta^+(\vec{z}), \Theta^-(\vec{z}))=0} \right. \]

\[ \langle G^{(+)}(\vec{z}_1)G^{(-)}(\vec{z}_2) \rangle = (2\pi)^2 \frac{\delta}{\delta \Theta^-(\vec{z}_1)} \frac{\delta}{\delta \Theta^+(\vec{z}_2)} e^{-S} \left|_{(A^0_1(\vec{z}), \Theta^+(\vec{z}), \Theta^-(\vec{z}))=0} \right. \]

\[ \langle J(\vec{z}_1)G^{(+)}(\vec{z}_2)G^{(-)}(\vec{z}_3) \rangle = (2\pi)^3 \frac{\delta}{\delta A^0_1(\vec{z}_1)} \frac{\delta}{\delta \Theta^-(\vec{z}_2)} \frac{\delta}{\delta \Theta^+(\vec{z}_3)} e^{-S} \left|_{(A^0_1(\vec{z}), \Theta^+(\vec{z}), \Theta^-(\vec{z}))=0} \right. \]

Using (4.13) in the above equation (4.15), we get

\[ \langle J(\vec{z}_1)J(\vec{z}_2) \rangle = 8iaL\pi \frac{1}{(z_1 - z_2)^2} \]

\[ \langle G^{(+)}(\vec{z}_1)G^{(-)}(\vec{z}_2) \rangle = 8iaL\pi \frac{1}{(z_1 - z_2)^3} \]

\[ \langle J(\vec{z}_1)G^{(+)}(\vec{z}_2)G^{(-)}(\vec{z}_3) \rangle = 8aL\pi \frac{1}{(z_1 - z_2)(z_1 - z_3)(z_2 - z_3)^2} \] (4.16)

These are the expected conformal field theory results.

5. Effect of Higher derivative terms

In this section we will look at the effect of higher derivative terms in the action on the correlation function. First let us consider the higher derivative terms appearing in \( S_1 \) which is needed for the supersymmetrization of the gravitational Chern-Simons term as argued in section 2. This term, as we saw in section 2, is constructed out of \( \hat{R}^{ab}_{MN}, \hat{F}_{MN} \) or \( G_{MN} \). So the contribution of this term to the equations of motion will be terms containing \( \hat{R}^{ab}_{MN}, \hat{F}_{MN}, G_{MN} \) and/or their super-covariant derivatives. Such terms will necessarily vanish when the original equation of motion (2.13)-(2.16) are satisfied. Therefore, the solutions obtained from \( S_0 \) will continue to hold and \( S_1 \) will vanish for such solutions and hence \( S_1 \) will not affect the boundary correlators obtained in section 4.

There can be other higher derivative terms that can be added to the action that are supersymmetric on their own. Such terms, as argued in section 3, should be constructed out of the supersymmetric covariant field strengths \( \hat{R}^{ab}_{MN}, \hat{F}_{MN}, G_{MN} \) and/or their supercovariant derivatives.

Our claim is that these higher derivative terms will change the original equations of motion (2.13) by terms which will vanish when the original equations of motion
are satisfied. Therefore, the solution obtained for the original equations of motion will still solve the equations of motion in the presence of these higher derivative terms. This, in particular means that \( \hat{F}_{MN} = 0 \) and \( G_{MN} = 0 \) will still continue to hold, as a result of which the higher derivative terms constructed out of these field strengths will vanish and hence will not affect the boundary correlators calculated in section 4.

Let us analyze our claim in a little detail. The higher derivative terms that we could add to the gauge+fermionic sector will be of the form

\[
\lambda_1 \int d^3x \, \hat{F}_{MN} K_1^{MN}[\hat{F}, G, \tilde{R}, D^{(n)}(\tilde{R}, \hat{F}, G)] + \lambda_2 \int d^3x \, \hat{F}_{MN} K_2^{MN}[\mathcal{R}, D^{(n)} \mathcal{R}] \\
+ \lambda_3 \int d^3x \, \tilde{G}^M \Gamma_{MN}[\hat{F}, G, \tilde{R}, D^{(n)}(\tilde{R}, \hat{F}, G), \gamma]G_N
\]

(5.1)

Let us understand the notations used in the above equation. \((D^{(n)})D^{(n)}\) are the \(n^{th}\) (super)covariant derivatives. \(G^M\) and \(\tilde{G}^M\) are defined as

\[
G^M = \frac{1}{2} \epsilon^{MNP} G_{NP} \quad \tilde{G}^M = \frac{1}{2} \epsilon^{MNP} \tilde{G}_{NP}
\]

(5.2)

\(\Gamma_{MN}\) is a bilinear, constructed out of various supercovariant field strengths, their supercovariant derivatives and gamma matrices.

In the first integral, \(K_1^{MN}\) is a function of \((\hat{F}, G, \tilde{R}, D^{(n)}(\tilde{R}, \hat{F}, G))\). Each term of \(K_1\) should have a factor of \(\hat{F}, G\) or their supercovariant derivatives. When there are no factors of \(\hat{F}, G\) or their supercovariant derivatives involved, need special care and hence has been written as a separate term in which \(K_2\) just depends on \((\mathcal{R}, D^{(n)} \mathcal{R})\). Since \(\hat{F}_{MN}\) is antisymmetric in \(M\) and \(N\), \(K_2^{MN}\) should also be antisymmetric in \(M\) and \(N\). This implies that we cannot construct \(K_2^{MN}\) purely out of \(\mathcal{R}_{MN}\) because of the symmetric nature of \(\mathcal{R}_{MN}\). We have to involve covariant derivatives of \(\mathcal{R}_{MN}\) in each term of \(K_2\). Therefore, it is obvious that the contribution of the higher derivative terms to the equations of motion of \(A_M\) and \(\psi_M\) will have a factor of \(\hat{F}_{MN}, G_{MN}, D_M \mathcal{R}_{NP}\) and/or their supercovariant derivatives in it. Similarly it can also be argued that the contribution to the \(\tilde{R}\) equation of motion from the higher derivatives terms should have the above factors. Just simply factors of \(\mathcal{R}_{NP}\) will not contribute to any equation of motion. It has to be accompanied by at least one of the above

\[\text{For supersymmetric invariance } K_2 \text{ should be constructed out of the supercovariant Riemann tensor } \mathcal{R} \text{ and its supercovariant derivatives } D^{(n)} \mathcal{R} \text{ but the difference between the supercovariant Riemann tensor } \tilde{R}, D^{(n)} \tilde{R} \text{ and } \mathcal{R}, D^{(n)} \mathcal{R} \text{ are terms proportional to } G_{MN} \text{ and we have included such terms already in } K_1^{MN}\]
factors\textsuperscript{9}. When the original equations of motion (2.13)-(2.16) are satisfied, all these additional contributions to the equations of motion vanish. This justifies our earlier claim that the solutions obtained for the original equations of motion will still solve the equations of motion in the presence of these higher derivative terms and that the correlators calculated in section 4 in the gauge+fermionic sector will not be affected by these terms. We now use the same supersymmetry argument of \cite{2} to argue for the non-renormalization of the stress tensor correlators. The crux of the argument is that the stress tensor correlators are related to the current correlators by supersymmetry. And since by our argument the current correlators does not get renormalized, this implies that the stress tensor correlators also does not get renormalized.

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**A. Boundary S-matrix analysis**

Here we will give stepwise analysis of the steps outlined in section 4 for obtaining the boundary S-matrix in the gauge+fermionic sector.

1. We will present the solution obtained for the equation (4.3) till first order iteration. For this we need to calculate the vierbeins and the torsionless spin connection from the $AdS_3$ metric (4.4) and put them in the expression for $B^a_M$.

\textsuperscript{9}The crucial observation in this analysis was that we cannot construct an antisymmetric $R^{MN}_2$ purely out of $R_{NP}$ because $R_{NP}$ is symmetric. We have to involve covariant derivatives of $R_{NP}$. The situation would have been drastically different if we were in more than three dimension. In that case we could construct an antisymmetric second rank tensor by using Riemann and Ricci tensors. But in three dimension Riemann tensor is not independent and is determined in terms of Ricci tensor and Ricci scalar.
We get
\[ B^0 = -\frac{1}{x^0} dx^0, \]
\[ B^i = -\frac{1}{x^0} dx^1 - \frac{i}{x^0} dx^2 = -\frac{1}{x^0} dz, \]
\[ B^\bar{\alpha} = \frac{i}{x^0} dx^1 - \frac{1}{x^0} dx^2 = \frac{i}{x^0} dz \]  \hspace{1cm} (A.1)

After putting (A.1) in the equations of motion (4.3), the leading order equations take the form
\[ dA^0 = 0, \]
\[ d\psi^0 + \frac{1}{2x^0} dx^0 \wedge \gamma_0 \psi^0 + \frac{1}{x^0} dz \wedge \gamma_\bar{z} \psi^0 = 0, \]
\[ d\bar{\psi}^0 - \frac{1}{2x^0} dx^0 \wedge \bar{\psi}^0 \gamma_\bar{0} - \frac{1}{x^0} dz \wedge \bar{\psi}^0 \gamma_\bar{z} = 0 \]  \hspace{1cm} (A.2)

The solution to the first equation is simple and the solution to the last two equations can be obtained after a little algebraic manipulation. We get
\[ A^0 = d\rho, \]
\[ \psi^0(1) = \left(x^0\right)^{-\frac{1}{2}} \left(d\eta + \phi dz\right), \]
\[ \psi^0(2) = \left(x^0\right)^{\frac{1}{2}} d\phi, \]
\[ \bar{\psi}^0(2) = \left(x^0\right)^{-\frac{1}{2}} \left(d\bar{\eta} - \bar{\phi} dz\right), \]
\[ \bar{\psi}^0(1) = \left(x^0\right)^{\frac{1}{2}} d\bar{\phi}, \]  \hspace{1cm} (A.3)

Where, as before the spinor index has been kept in the superscript and in () braces and the 0 to the left of it just denotes the leading order piece. Here \( \rho, \eta, \phi, \bar{\eta}, \bar{\phi} \) are some arbitrary functions of \((x^0, z, \bar{z})\). \( \rho \) is an ordinary function whereas \( \eta, \phi, \bar{\eta}, \bar{\phi} \) are grassman functions.

Now we obtain the first order correction \( A^1, \psi^1, \bar{\psi}^1 \) to the above equation as
\[ dA^1 = \frac{1}{4} \bar{\psi}^0 \wedge \psi^0, \]
\[ d\psi^1 + \frac{1}{2x^0} dx^0 \wedge \gamma_0 \psi^1 + \frac{1}{x^0} dz \wedge \gamma_\bar{z} \psi^1 = -\frac{i}{2} A^0 \wedge \psi^0, \]
\[ d\bar{\psi}^1 - \frac{1}{2x^0} dx^0 \wedge \bar{\psi}^1 \gamma_\bar{0} - \frac{1}{x^0} dz \wedge \bar{\psi}^1 \gamma_\bar{z} = \frac{i}{2} A^0 \wedge \bar{\psi}^0, \]  \hspace{1cm} (A.4)

We get
\[ A^1 = \frac{1}{4} \bar{\phi} d\eta + \frac{1}{4} \bar{\eta} d\phi + \frac{1}{4} \bar{\phi} \bar{\phi} dz, \]
\[ \text{We use } \gamma_0 = \sigma_3, \gamma_1 = \sigma_1, \gamma_2 = -\sigma_2, \gamma_\bar{z} = \frac{1}{2} (\gamma_1 - i\gamma_2), \gamma_\bar{z} = \frac{1}{2} (\gamma_1 + i\gamma_2) \]
\[ \psi^{1(1)} = -\frac{i}{2} (x^0)^{-\frac{1}{2}} \rho (d\eta + \phi dz), \]
\[ \psi^{1(2)} = -\frac{i}{2} (x^0)^{\frac{1}{2}} \rho d\phi, \]
\[ \psi^{2(1)} = \frac{i}{2} (x^0)^{-\frac{1}{2}} \rho (d\bar{\eta} - \bar{\phi} dz), \]
\[ \psi^{2(2)} = \frac{i}{2} (x^0)^{\frac{1}{2}} \rho d\bar{\phi}, \]

(A.5)

Thus the full solution till first order iteration is
\[ A = d\rho + \frac{1}{4} \rho d\eta + \frac{1}{4} \bar{\eta} d\phi + \frac{1}{4} \bar{\phi} d\bar{\eta}, \]
\[ \psi^{(1)} = (x^0)^{-\frac{1}{2}} \left( 1 - \frac{i}{2} \rho \right) (d\eta + \phi dz), \]
\[ \psi^{(2)} = (x^0)^{\frac{1}{2}} \left( 1 - \frac{i}{2} \rho \right) d\phi, \]
\[ \bar{\psi}^{(1)} = (x^0)^{\frac{1}{2}} \left( 1 + \frac{i}{2} \rho \right) d\bar{\phi}, \]
\[ \bar{\psi}^{(2)} = (x^0)^{-\frac{1}{2}} \left( 1 + \frac{i}{2} \rho \right) (d\bar{\eta} - \bar{\phi} dz) \]

(A.6)

After imposing the gauge condition (4.6), we get the following equations for \( \phi, \eta, \bar{\phi} \) and \( \bar{\eta} \)
\[ \partial_0 \eta + 2x^0 \partial_z \phi = 0 \]
\[ x^0 \partial_0 \phi - 2 (\partial_z \eta + \phi) = 0 \]
\[ \partial_0 \bar{\eta} - 2x^0 \partial_z \bar{\phi} = 0 \]
\[ x^0 \partial_0 \bar{\phi} + 2 (\partial_z \bar{\eta} - \bar{\phi}) = 0 \]

(A.7)

The above equations can be solved after decoupling them to obtain a second order equation and then using separation of variables to separate the \( x^0 \) equation and \((z, \bar{z})\) equation. The result is
\[ \eta = \frac{1}{2\pi} \int d^2 \bar{p} (x^0 \bar{p})^2 K_2(x^0 \bar{p}) e^{i\bar{p}.\bar{z}} A_\eta(\bar{p}), \]
\[ \phi = \frac{1}{2\pi} \int d^2 \bar{p} (x^0 \bar{p}) K_1(x^0 \bar{p}) e^{i\bar{p}.\bar{z}} (-2ip_z) A_\eta(\bar{p}), \]
\[ \bar{\eta} = \frac{1}{2\pi} \int d^2 \bar{p} (x^0 \bar{p})^2 K_2(x^0 \bar{p}) e^{i\bar{p}.\bar{z}} A_\eta(\bar{p}), \]
\[ \bar{\phi} = \frac{1}{2\pi} \int d^2 \bar{p} (x^0 \bar{p}) K_1(x^0 \bar{p}) e^{i\bar{p}.\bar{z}} (2ip_z) A_\eta(\bar{p}) \]

(A.8)

Where
\[ \bar{p} \equiv (p_z, p_{\bar{z}}) \quad \bar{p}^2 \equiv |\bar{p}|^2 \equiv 4p_z p_{\bar{z}} \quad \bar{p}.\bar{z} \equiv p_z z + p_{\bar{z}} \bar{z} \]

(A.9)
The measure $d^2\vec{p}$ is defined as
\[ d^2\vec{p} \equiv dp_1 dp_2 \quad (A.10) \]

Where $p_1$ and $p_2$ are related to $p_z$ and $p_{\bar{z}}$ as
\[ p_1 \equiv \frac{1}{2} \text{Re}(p_z), \quad p_2 \equiv -\frac{1}{2} \text{Im}(p_z) \]
i.e. $p_z = \frac{1}{2}(p_1 - ip_2), \quad p_{\bar{z}} = \frac{1}{2}(p_1 + ip_2), \quad \text{Hence } \quad p^2 \equiv 4p_z p_{\bar{z}} = p_1^2 + p_2^2 \]
And $d^2\vec{p} \equiv dp_1 dp_2 = \begin{vmatrix} \frac{\partial p_1}{\partial p_z} & \frac{\partial p_1}{\partial p_{\bar{z}}} \\ \frac{\partial p_2}{\partial p_z} & \frac{\partial p_2}{\partial p_{\bar{z}}} \end{vmatrix} dp_z dp_{\bar{z}} = -2idp_z dp_{\bar{z}} \quad (A.11) \]

Here $K_1$ and $K_2$ are the “modified Bessel’s functions” of order 1 and 2 respectively. They behave asymptotically ($x \to 0$) as
\[ K_2(x) = \frac{2}{x^2} \left[ 1 + \mathcal{O}(x^2) \right], \quad K_1(x) = \frac{1}{x} \left[ 1 + \mathcal{O}(x^2) \right] \quad (A.12) \]

It can be easily checked that the solutions (A.8) indeed satisfies the equations (A.7). For that, we need to use the following property of modified Bessel’s functions
\[ K_2'(x^0 p) = -K_1(x^0 p) - \frac{2}{x^0 p} K_2(x^0 p) \]
\[ K_1'(x^0 p) = -K_2(x^0 p) + \frac{1}{x^0 p} K_1(x^0 p) \quad (A.13) \]

2. We will now figure out the fields on which we should impose boundary conditions. Since we are working in (2, 0) theory, we should impose boundary conditions on the $\bar{z}$ components of the fields. The boundary conditions on the gauge field is unambiguous and is simply
\[ \lim_{x^0 \to 0} A_{\bar{z}}(x^0, \bar{z}) = A_{\bar{z}}^{(0)}(\bar{z}) \quad (A.14) \]

Before trying to figure out what boundary conditions we should impose on the Rarita-Schwinger fields, let us outline an important result of [15]. According to [15], if a p-form field $C$ behave as $(x^0)^{-\lambda}C_0$ near the boundary then the operator $\mathcal{O}$ that couples to $C_0$ in the boundary will have conformal dimensions $\Delta = d - p + \lambda$. Here $d$ is the dimensions of the boundary. For our case $d = 2$
and \( p = 1 \) and we want sources for weight \( \frac{3}{2} \) supersymmetry currents. This suggests \( \lambda = \frac{1}{2} \). This behavior near the boundary is there for the fields \( \psi^{(1)} \) and \( \bar{\psi}^{(2)} \) as can be seen from (A.3). Thus we impose the following boundary conditions on the Rarita-Schwinger fields

\[
\lim_{x^0 \to 0} (x^0)^{\frac{1}{2}} \psi^{(1)} = \Theta^{(-)}(\bar{z}), \\
\lim_{x^0 \to 0} (x^0)^{\frac{1}{2}} \bar{\psi}^{(2)} = \Theta^{(+)}(\bar{z})
\]  

(A.15)

Following [2], we take \( \rho \) appearing in the solutions (A.6) to be

\[
\rho = \int d^2\bar{w} \ K(\bar{z}, x^0, \bar{w}) B^{(0)}_{\bar{z}}(\bar{w}), \\
K(\bar{z}, x^0, \bar{w}) = \frac{1}{\pi} \left[ \frac{\bar{z} - \bar{w}}{(x^0)^2 + |z - w|^2} \right]
\]  

(A.16)

The function \( K(\bar{z}, x^0, \bar{w}) \) satisfies the following properties on the boundary

\[
\lim_{x^0 \to 0} K = \frac{1}{\pi} \frac{1}{z - w}, \\
\lim_{x^0 \to 0} \partial_{\bar{z}} K = -\frac{1}{\pi} \frac{1}{(z - w)^2}, \\
\lim_{x^0 \to 0} \partial_{\bar{w}} K = \delta^2(z - w),
\]  

(A.17)

3. We will use the above properties of \( K \) along with the behavior of the modified Bessel’s functions \( K_1 \) and \( K_2 \) near the boundary to fix \( B^{(0)}_{\bar{z}}(\bar{w}) \) appearing in (A.16) and \( A_{\eta}(\bar{p}) \) and \( A_{\bar{\eta}}(\bar{p}) \) appearing in (A.8) in terms of the boundary values so that the boundary conditions (A.14) and (A.13) are satisfied. We get

\[
B^{(0)}_{\bar{z}}(\bar{z}) = A_{\eta}(\bar{z}) + \frac{1}{4\pi} \int d^2\bar{w} \left( \frac{1}{(z - w)^2} \right) \Theta^{(+)}(\bar{w}) \Theta^{(-)}(\bar{z}) \\
A_{\eta}(\bar{p}) = \frac{1}{2\pi} \int d^2\bar{w} \left( \frac{1}{2ip_{\bar{z}}} \right) e^{-ip_{\bar{z}} \bar{w}} \Theta^{(-)}(\bar{w}) \\
+ \frac{i}{4\pi^2} \int d^2\bar{w} d^2\tilde{v} \left( \frac{1}{2ip_{\bar{z}}} \right) \left( \frac{1}{w - v} \right) e^{-ip_{\bar{z}} \bar{w}} A_{\eta}(\bar{v}) \Theta^{(-)}(\bar{w}) \\
A_{\bar{\eta}}(\bar{p}) = \frac{1}{2\pi} \int d^2\bar{w} \left( \frac{1}{2ip_{\bar{z}}} \right) e^{-ip_{\bar{z}} \bar{w}} \Theta^{(+)}(\bar{w}) \\
- \frac{i}{4\pi^2} \int d^2\bar{w} d^2\tilde{v} \left( \frac{1}{2ip_{\bar{z}}} \right) \left( \frac{1}{w - v} \right) e^{-ip_{\bar{z}} \bar{w}} A_{\bar{\eta}}(\bar{v}) \Theta^{(+)}(\bar{w})
\]  

(A.18)
The above result, then fixes the solutions completely in terms of the boundary values. We get\(^\text{11}\)

\[
A_z = -\frac{1}{\pi} \int d^2\bar{w} \frac{1}{(z - w)^2} A_z^{(0)}(\bar{w}) - \frac{1}{4\pi^2} \int d^2\bar{v} d^2\bar{w} \frac{1}{(z - w)^2(w - v)^2} \Theta_+^{(+)}(\bar{v}) \Theta_-^{(-)}(\bar{w}) - \frac{1}{2\pi^2} \int d^2\bar{v} d^2\bar{w} \frac{1}{(z - w)^3(z - v)} \Theta_+^{(+)}(\bar{v}) \Theta_-^{(-)}(\bar{w}) + \mathcal{O}((x^0)^2)
\]

\[
A_{\bar{z}} = A_{\bar{z}}^{(0)}(\bar{z}) + \mathcal{O}((x^0)^2)
\]

\[
(x^0)\frac{1}{2}\psi_z^{(1)} = \Theta_-^{(-)}(\bar{z}) + \mathcal{O}((x^0)^2)
\]

\[
(x^0)\frac{1}{2}\psi_{\bar{z}}^{(1)} = \mathcal{O}((x^0)^2)
\]

\[
(x^0)^{-\frac{1}{2}}\psi_z^{(2)} = -\frac{2}{\pi} \int d^2\bar{w} \frac{1}{(z - w)^3} \Theta_-^{(-)}(\bar{w}) - \frac{i}{\pi^2} \int d^2\bar{v} d^2\bar{w} \frac{1}{(z - w)^2(w - v)(z - v)} A_z^{(0)}(\bar{v}) \Theta_-^{(-)}(\bar{w}) + \mathcal{O}((x^0)^2)
\]

\[
(x^0)^{-\frac{1}{2}}\psi_{\bar{z}}^{(2)} = \mathcal{O}((x^0)^2)
\]

\[
(x^0)^{-\frac{1}{2}}\bar{\psi}_z^{(2)} = \Theta_+^{(+)}(\bar{z}) + \mathcal{O}((x^0)^2)
\]

\[
(x^0)^{-\frac{1}{2}}\bar{\psi}_{\bar{z}}^{(2)} = \mathcal{O}((x^0)^2)
\]

\[
(x^0)^{-\frac{1}{2}}\bar{\psi}_z^{(1)} = \frac{2}{\pi} \int d^2\bar{w} \frac{1}{(z - w)^2} \Theta_+^{(+)}(\bar{w}) - \frac{i}{\pi^2} \int d^2\bar{v} d^2\bar{w} \frac{1}{(z - w)^2(w - v)(z - v)} A_z^{(0)}(\bar{v}) \Theta_+^{(+)}(\bar{w}) + \mathcal{O}((x^0)^2)
\]

\[
(x^0)^{-\frac{1}{2}}\bar{\psi}_{\bar{z}}^{(1)} = \mathcal{O}((x^0)^2)
\]

\[\text{(A.19)}\]

4. Now we will obtain the relevant boundary action that we need to add for consistency requirements mentioned in section [4]. Varying the action \((1.1)\) we get

\[
\delta S = [0]_{\text{on-shell}} + \frac{i a_L}{2} \lim_{\epsilon \to 0} \int_{\mathcal{M}_\epsilon} d^2\bar{z} z \delta \psi_z^{(1)}(\epsilon, \bar{z}) \psi_z^{(1)}(\epsilon, \bar{z}) + \psi_{\bar{z}}^{(2)}(\epsilon, \bar{z}) \delta \psi_{\bar{z}}^{(2)}(\epsilon, \bar{z}) + \mathcal{O}(\epsilon)
\]

\[-a_L \lim_{\epsilon \to 0} \int_{\mathcal{M}_\epsilon} d^2\bar{z} \delta A_z(z, \epsilon) A_z(z, \epsilon) \]

\[\text{(A.20)}\]

Here \([0]_{\text{on-shell}}\) denotes the set of terms which vanish on-shell. In obtaining the above variation, we have used the fact that we have imposed boundary

\(^{11}\text{In order to arrive at the results (A.19) we needed to do a lot of “p” integrals. One of such integral is outlined in the end of the section. The others can be similarly obtained.}\)
conditions (A.14), (A.15) and hence
\[
\lim_{\epsilon \to 0} \delta A_\epsilon (\epsilon, \bar{z}) = \lim_{\epsilon \to 0} \delta \psi_\epsilon^{(1)} (\epsilon, \bar{z}) = \lim_{\epsilon \to 0} \delta \bar{\psi}_\epsilon^{(2)} (\epsilon, \bar{z}) = 0 \tag{A.21}
\]
Moreover from (A.19), we observe
\[
\lim_{\epsilon \to 0} \epsilon \bar{z} \psi_\epsilon^{(1)} (\epsilon, \bar{z}) = \lim_{\epsilon \to 0} \epsilon \bar{z} \bar{\psi}_\epsilon^{(2)} (\epsilon, \bar{z}) = \lim_{\epsilon \to 0} \epsilon \bar{z} \psi_\epsilon^{(2)} (\epsilon, \bar{z}) = 0 \tag{A.22}
\]
The relations (A.22) are valid on-shell since these are obtained from (A.19) which are the solutions to the equations of motion. This along with (A.21) has been used in arriving at (A.20). We need to add a boundary action \( S_{\text{bndy}} \) such that its variation exactly cancels the integrals in (A.20) and we get \( \delta (S + S_{\text{bndy}}) = [0]_{\text{on-shell}} \). We get
\[
S_{\text{bndy}} = S_{\text{bndy}}[\psi, \bar{\psi}] + S_{\text{bndy}}[A]
\]
\[
S_{\text{bndy}}[\psi, \bar{\psi}] = -\frac{ia_L}{2} \int d^2 \bar{z} \left( \bar{\psi}_\epsilon^{(1)} (\epsilon, \bar{z}) \psi_\epsilon^{(1)} (\epsilon, \bar{z}) + \bar{\psi}_\epsilon^{(2)} (\epsilon, \bar{z}) \psi_\epsilon^{(2)} (\epsilon, \bar{z}) \right) \bigg|_{x^0 = 0}
\]
\[
S_{\text{bndy}}[A] = a_L \int d^2 \bar{z} A_\epsilon (\bar{z}, x^0) A_\epsilon (\bar{z}, x^0) \bigg|_{x^0 = 0} \tag{A.23}
\]
5. Now we will put the solutions obtained in (A.19) in the bulk as well as boundary action and obtain the on-shell action as a functional of the boundary values. The contribution from the boundary action is straightforward to obtain. We get
\[
S_{\text{bndy}}[\psi, \bar{\psi}] = -\frac{2ia_L}{\pi} \int d^2 \bar{z} d^2 \bar{w} \frac{1}{(z - w)^3} \Theta_{\epsilon}^{(+)} (\bar{w}) \Theta_{\epsilon}^{(-)} (\bar{z})
\]
\[
- \frac{a_L}{2\pi^2} \int d^2 \bar{z} d^2 \bar{w} d^2 \bar{v} \frac{1}{(z - w)(z - v)(w - v)^2} A_\epsilon^{(0)} (\bar{z}) \Theta_{\epsilon}^{(+)} (\bar{v}) \Theta_{\epsilon}^{(-)} (\bar{w})
\]
\[
S_{\text{bndy}}[A] = -\frac{a_L}{\pi} \int d^2 \bar{z} d^2 \bar{w} \frac{1}{(z - w)^2} A_\epsilon^{(0)} (\bar{w}) A_\epsilon^{(0)} (\bar{z})
\]
\[
- \frac{a_L}{4\pi^2} \int d^2 \bar{z} d^2 \bar{w} d^2 \bar{v} \frac{1}{(z - w)^2(w - v)^2} A_\epsilon^{(0)} (\bar{z}) \Theta_{\epsilon}^{(+)} (\bar{v}) \Theta_{\epsilon}^{(-)} (\bar{w})
\]
\[
- \frac{a_L}{2\pi^2} \int d^2 \bar{z} d^2 \bar{w} d^2 \bar{v} \frac{1}{(z - w)^3(z - v)} A_\epsilon^{(0)} (\bar{z}) \Theta_{\epsilon}^{(+)} (\bar{v}) \Theta_{\epsilon}^{(-)} (\bar{w}) \tag{A.24}
\]
In order to obtain the contribution from the bulk action (4.1), we recall from section I that the first two terms in the action will not contribute towards the evaluation of two and three point functions. The last term vanish by equations
of motion. So the only term that contributes is

\[
S_{\text{bulk}} = \frac{a_L}{2} \int d^3x \epsilon^{MNP} A_M \partial_N A_P 
\]

\[
= \frac{a_L}{2} \int d^3x \epsilon^{MNP} A_M^0 \partial_N A_P^0 + i \frac{a_L}{2} \int d^3x \epsilon^{MNP} A_M^0 \partial_N A_P^1 
\]

\[
i \frac{a_L}{2} \int d^3x \epsilon^{MNP} A_M^1 \partial_N A_P^0 + i \frac{a_L}{2} \int d^3x \epsilon^{MNP} A_M^1 \partial_N A_P^1 
\]

(A.25)

In the above equation, we have broken the full solution to a sum of the leading solution and the first order correction. Since \( A_M^0 = \partial_M \rho \), this implies \( \epsilon^{MNP} \partial_N A_P^0 = 0 \). The last term in (A.25) gives a quartic contribution and hence can be neglected. Thus

\[
S_{\text{bulk}}[A, \psi, \bar{\psi}] = \frac{a_L}{2} \int d^3x \epsilon^{MNP} A_M^0 \partial_N A_P^1 
\]

\[
= i \frac{a_L}{2} \int d^3x \epsilon^{MNP} \partial_N \left( A_M^0 A_P^1 \right) 
\]

\[
= a_L \int d^2 \vec{z} \left( A_M^0(z) A_P^1(z) - A_M^0(z) A_P^1(z) \right) \bigg|_{x^0 = 0} 
\]

(A.26)

In order to evaluate the above contribution from the bulk action we need to know the leading order solution and first order correction separately for the gauge field \( A_M \).

\[
A^0_z = \partial_z \rho 
\]

\[
= A_z^0(z) + \frac{1}{4\pi} \int d^2 \vec{w} \frac{1}{(z - \vec{w})^2} \Theta_z^{(+)}(\vec{w}) \Theta_z^{(-)}(\vec{w}), 
\]

\[
A^0_\bar{z} = \partial_\bar{z} \rho 
\]

\[
= -\frac{1}{2\pi^2} \int d^2 \vec{v} d^2 \vec{w} \frac{1}{(z - \vec{w})^2} \Theta_z^{(+)}(\vec{v}) \Theta_z^{(-)}(\vec{w}) 
\]

\[
A_1^z = \frac{1}{4} \phi \left( \partial_z \eta + \phi \right) + \frac{1}{4} \bar{\eta} \partial_z \phi 
\]

\[
= -\frac{1}{2\pi^2} \int d^2 \vec{v} d^2 \vec{w} \frac{1}{(z - \vec{w})^2} \Theta_z^{(+)}(\vec{v}) \Theta_z^{(-)}(\vec{w}) 
\]

\[
A_1^\bar{z} = \frac{1}{4} \bar{\phi} \partial_{\bar{z}} \eta + \frac{1}{4} \bar{\eta} \partial_{\bar{z}} \phi 
\]

\[
= -\frac{1}{4\pi} \int d^2 \vec{w} \frac{1}{(z - \vec{w})^2} \Theta_z^{(+)}(\vec{w}) \Theta_z^{(-)}(\vec{w}) 
\]

(A.27)
Putting these solutions in (A.26), we get the contribution from the bulk action upto terms cubic in the boundary values as

\[
S_{\text{bulk}}[A, \psi, \bar{\psi}] = a_L \int d^2 \vec{z} \left( A_0^0(\vec{z}) A_1^1(\vec{z}) - A_1^0(\vec{z}) A_0^1(\vec{z}) \right) \\
= \frac{a_L}{4\pi^2} \int d^2 \vec{z} d^2 \vec{v} d^2 \vec{w} \frac{1}{(z-w)^2(w-v)^2} A_x^{(0)}(\vec{z}) \Theta_x^{(+)}(\vec{v}) \Theta_x^{(-)}(\vec{w}) \\
+ \frac{a_L}{2\pi^2} \int d^2 \vec{z} d^2 \vec{v} d^2 \vec{w} \frac{1}{(z-w)(z-v)} A_x^{(0)}(\vec{z}) \Theta_x^{(+)}(\vec{v}) \Theta_x^{(-)}(\vec{w})
\]

(A.28)

Combining this with the boundary contributions obtained in (A.24), we get

\[
S[A, \psi, \bar{\psi}] = S_{\text{bulk}}[A, \psi, \bar{\psi}] + S_{\text{bndy}}[\psi, \bar{\psi}] + S_{\text{bndy}}[A] \\
= -\frac{a_L}{\pi} \int d^2 \vec{z} d^2 \vec{w} \frac{1}{(z-w)^2} A_x^{(0)}(\vec{w}) A_x^{(0)}(\vec{z}) \\
- \frac{2ia_L}{\pi} \int d^2 \vec{z} d^2 \vec{w} \frac{1}{(z-w)^3} \Theta_x^{(+)}(\vec{w}) \Theta_x^{(-)}(\vec{z}) \\
- \frac{a_L}{\pi^2} \int d^2 \vec{z} d^2 \vec{w} d^2 \vec{v} \frac{1}{(z-w)(z-v)(w-v)^2} A_x^{(0)}(\vec{z}) \Theta_x^{(+)}(\vec{v}) \Theta_x^{(-)}(\vec{w})
\]

(A.29)

Obtaining the correlation functions from here on is straightforward and has been obtained in section 4.

In order to arrive at the solutions of the fields (A.19) in terms of the boundary values, we had to carry out a number of non-trivial “$p$” integrals. We conclude this section by outlining one of such integrals. The other integrals can be similarly worked out. One of such integrals is

\[
I_2(\vec{z} - \vec{w}) \equiv -\frac{i}{4\pi^2} \int d^2 \vec{p} \left( \frac{p_z^2}{p_\vec{z}^2} \right) e^{i\vec{p} \cdot (\vec{z} - \vec{w})} 
\]

(A.30)

In order to evaluate the above integral, we define

\[
\Upsilon \equiv p_z(z-w) \quad \bar{\Upsilon} \equiv p_\vec{z}(\vec{z} - \vec{w}) \\
\Rightarrow d^2 \vec{p} \equiv -2i dp_z dp_\vec{z} = \frac{-2i}{(z-w)(\vec{z} - \vec{w})} d\Upsilon d\bar{\Upsilon} \\
\equiv \frac{1}{(z-w)(\vec{z} - \vec{w})} d^2 \bar{\Upsilon}
\]

(A.31)

\[d^2 \bar{\Upsilon}, \text{ similarly to } d^2 \vec{p} \text{ in (A.10) and (A.11)}, \text{ can be expressed in terms of its real and imaginary parts}
\]

\[d^2 \bar{\Upsilon} \equiv d\Upsilon_1 d\Upsilon_2\]
where $\Upsilon_1 \equiv \frac{1}{2} \text{Re}(\Upsilon)$, $\Upsilon_2 \equiv -\frac{1}{2} \text{Im}(\Upsilon)$,

(A.32)

With all the above definitions, the integral (A.30) takes the following form

$$I_2(z - w) = -\frac{i}{4\pi^2} \frac{1}{(z - w)^3} \int d^2\Upsilon \left(\frac{\Upsilon^2}{\Upsilon}\right) e^{i(\Upsilon + \bar{\Upsilon})} \equiv \frac{c_2}{(z - w)^3}$$

(A.33)

The last line of the above equation defines $c_2$ as

$$c_2 \equiv -\frac{i}{4\pi^2} \int d^2\Upsilon \left(\frac{\Upsilon^2}{\Upsilon}\right) e^{i(\Upsilon + \bar{\Upsilon})}$$

(A.34)

Thus the “$p$” integral (A.30) now boils down to doing the “$\Upsilon$” integral (A.34). In order to do the integral (A.34) we define

$$\Upsilon_1 = R \cos \theta, \quad \Upsilon_2 = R \sin \theta$$

$$\Rightarrow d^2\Upsilon = R \, dR \, d\theta, \quad \Upsilon = \frac{1}{2} Re^{-i\theta}, \quad \bar{\Upsilon} = \frac{1}{2} Re^{i\theta}, \quad \Upsilon + \bar{\Upsilon} = R \cos \theta$$

(A.35)

Putting this definition in (A.34), we get

$$c_2 = -\frac{i}{4\pi^2} \frac{1}{2} \int_0^\infty R^2 \, dR \int_0^{2\pi} e^{i(R\cos\theta - 3\theta)} d\theta$$

$$= -\frac{1}{4\pi} \int_0^\infty R^2 J_3(R) \, dR = -\frac{2}{\pi}$$

(A.36)

In the last step, we have used the integral representation of Bessel’s function

$$J_n(x) = \frac{i^{-n}}{2\pi} \int_0^{2\pi} e^{i(x \cos \theta + n\theta)} d\theta$$

(A.37)

And

$$\int_0^\infty J_n(R) \, dR = 1 \quad \int_0^\infty R J_{n-1}(R) \, dR = n - 1 \quad \int_0^\infty R^2 J_{n-1}(R) \, dR = n(n - 2)$$

(A.38)

Therefore the integral (A.30) takes the form

$$I_2(z - w) = -\frac{2}{\pi(z - w)^3}$$

(A.39)
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