Accretion of New Variable Modified Chaplygin Gas and Generalized Cosmic Chaplygin Gas onto Schwarzschild and Kerr-Newman Black holes

Jhumpa Bhadra$^*$ and Ujjal Debnath$^†$

Department of Mathematics, Bengal Engineering and Science University, Shibpur, Howrah-711 103, India.

(Dated: February 14, 2012)

In this work, we have studied accretion of the dark energies like new variable modified Chaplygin gas (NVMCG) and generalized cosmic Chaplygin gas (GCCG) onto Schwarzschild and Kerr-Newman Black holes. We find the expression of the critical four velocity component which gradually decreases for the fluid flow towards the Schwarzschild as well as Kerr-Newman black hole. We also find the expression for change of masses of the black hole in both cases. For the Kerr-Newman black hole which is rotating and charged we calculate the specific angular momentum and total angular momentum. We showed that in both cases due to accretion of the dark energy mass of the black hole increases and angular momentum increases in case of Kerr-Newman black hole.

PACS numbers: 04.70.Bw, 04.70.Dy, 98.80.Cq

Contents

I. Introduction

II. Accretion of dark energy onto Schwarzschild black hole
   A. Model I: New Variable Modified Chaplygin Gas as dark energy model
   B. Model II: Generalized Cosmic Chaplygin Gas as dark energy Model

III. General accretion of dark energy onto Kerr-Newman Black hole
   A. Model I: New Variable Modified Chaplygin Gas as dark energy model
   B. Model II: Generalized Cosmic Chaplygin Gas as the dark energy Model

IV. Discussions

References

I. INTRODUCTION

Different observational data together with observations of supernovae of type Ia [1–4], WMAP [5], Chandra X-ray Observatory [6] strongly indicate that our universe is undergoing an accelerating phase. Nonbaryonic matter recognized as dark energy having negative pressure and violate the strong energy [7–9] condition may explain this accelerated expansion. There are various candidates to play the role of the dark energy which is the dominant part of the universe. The dark energy candidates are cosmological constant [10], quintessence, K-essence [11], Chaplygin gas [12], its modification known as modified Chaplygin gas (MCG) [13], tachyonic field [14], DBI-essence [15] etc. The Chaplygin Gas (EoS $p = -\frac{B}{\rho}$, $B > 0$) [12] acts as pressureless fluid for small value of the scale factor and tends to accelerated expansion for large value of scale factor. Generalization of Chaplygin gas model known as Generalized Chaplygin gas which satisfies $p = -\frac{B}{\rho^\alpha}$, $0 \leq \alpha \leq 1$ [16–18]. This model also modified to Modified Chaplygin Gas (MCG) having the

$^*$ bhadra.jhumpa@gmail.com
$^†$ ujjaldebnath@yahoo.com, ujjal@iucaa.ernet.in
Eos $p = A\rho - \frac{B}{\rho^\alpha}$, $0 \leq \alpha \leq 1$, $A > 0, B > 0$ [13, 19, 20]. That illustrates a radiation era ($A = 1/3$) while the scale factor is vanishingly small and $\Lambda$CDM model for infinitely large scale factor. Further Guo and Zhang [21] established Variable Chaplygin Gas with Eos is $p = -\frac{B}{\rho}$, where $B = B(R)$, $R$ is the scale factor and $B$ is a positive function of the scale factor. Subsequently Debnath [22] provided Variable modified Chaplygin Gas with Eos is $p = A\rho - \frac{B(R)}{\rho^\alpha}$ for the accelerating phase of the universe. The another candidate of dark energy was introduced by Chakraborty et al [23], known as New Variable modified Chaplygin Gas (NVMCG) which follows the equation $p = A(R) - \frac{B(R)}{\rho^\alpha}$, $0 \leq \alpha \leq 1$ which gives interesting physical significance. In 2003 P.F. González-Díaz [24] gives the idea of another form of dark energy to the consequence of accelerating phase of universe namely Generalized Cosmic Chaplygin Gas (GCCG), This model is stable and free from unphysical behaviour even when the vacuum fluid satisfies the phantom energy condition.

A cosmological property in which there is an infinite expansion in scale factor in a finite time termed as ‘Big Rip’ [20]. In the phantom cosmology, big rip is a kind of future singularity in which the energy density of phantom energy ($\rho + p < 0$) will become infinite in a finite time. To realize the Big Rip scenario the condition $\rho + p < 0$ alone is not sufficient [27]. Distinct data on supernovas showed that the presence of phantom energy with $-1.2 < w < -1$ in the Universe is highly likely [28]. The accretion of phantom dark energy onto a Schwarzschild black hole was first modelled by Babichev et al [29]. They established that black hole mass will gradually decrease due to strong negative pressure of phantom energy and finally all the masses tend to zero near the big rip where it will disappear. Accretion of phantom like modified variable Chaplygin gas onto Schwarzschild black hole was studied by Jamil [30] who showed that mass of the black hole will decrease when accreting fluid violates the dominant energy condition and otherwise will increase. Also the accretion of dark energy with EoS $p = w\rho$ onto the Kerr-Newman black hole was studied by Madrid et al [31] and they obtained that if $w > -1$, mass and angular momentum increase. Mass of the black hole grows up unboundedly whereas the angular momentum increases up to a given level.

In the present work, we have studied accretion of dark energy namely new variable modified Chaplygin gas (NVMCG) and generalized cosmic Chaplygin gas (GCCG) onto Schwarzschild as well as most generalized Kerr-Newman black holes. For natures of black hole mass function with angular momentum have been analyzed when NVMCG and GCCG like dark energies accrete upon black holes.

II. ACCRETION OF DARK ENERGY ONTO SCHWARZSCHILD BLACK HOLE

Let us consider a spherically symmetrical accretion of the dark energy onto the black hole. We consider a Schwarzschild black hole (static) of mass $M$ which is gravitationally isolated (in geometrical units, $G = 1 = c$) [22, 30] described by the line element

$$ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \left(1 - \frac{2M}{r}\right)^{-1}dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

(1)

where, $r$ being the radial coordinate. Energy momentum-tensor for the DE, considering in the form of perfect fluid having the EoS $p = p(\rho)$, is

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}$$

(2)

where $\rho$, $p$ are the density and pressure of the dark energy respectively and $u^\mu = \frac{dx^\mu}{dt}$ is the fluid 4-velocity satisfying $u^\mu u_\mu = 1$. We assume that the in-falling dark energy fluid does not disturb the spherical symmetry of the black hole.

The relativistic Bernoulli’s equation after the time component of the energy-momentum conservation law $T^\mu_{\nu ; \nu} = 0$ provide the first integral of motion for stationary, spherically symmetric accretion onto BH which yields

$$(\rho + p)\left(1 - \frac{2}{x} + u^2\right)\dot{x}^2x = C_1$$

(3)
where \( x = \frac{r}{M} \) and \( u = \frac{\partial x}{\partial t} \) is the radial component of the velocity four vector and \( C_1 \) being the integrating constant. In the case of fluid flow directed towards the black hole, we must have \( u < 0 \).

Moreover, the second integration of motion is obtained from \( u_\mu T^\mu_\nu = 0 \), which gives

\[
u x^2 \exp \left[ \int_{\rho_\infty}^\rho \frac{d\rho}{\rho + p(\rho)} \right] = -A
\]

where, \( A(>0) \) is the integration constant, \( \rho_\infty \) is the dark energy density at infinity. Further value of the constant \( A \) is evaluated for different DE model.

Using the equations (3) and (4) we get

\[
\left( \rho + p \right) \sqrt{1 - \frac{2}{x} + u^2 \exp \left[ - \int_{\rho_\infty}^\rho \frac{d\rho}{\rho + p(\rho)} \right]} = C_2
\]

where \( C_2 = -C_1/A = \rho_\infty + p(\rho_\infty) \).

If \( n \) be the concentration of dark energy which satisfies the following equations

\[
n(\rho) = \frac{n(\rho)}{n_\infty} = \exp \left[ \int_{\rho_\infty}^\rho \frac{d\rho}{\rho + p(\rho)} \right]
\]

where \( n_\infty = n(\rho_\infty) \) being the concentration of the dark energy at infinity.

The constant value \( A \) can be determined by finding the critical point of the accretion using [32], then,

\[
u^2_1 = \frac{1}{2x_*} \quad c^2_\text{s}(\rho_*) = \frac{u^2_1}{1 - 3u^2_1}
\]

where \( c_\text{s} = \sqrt{\frac{\partial p}{\partial \rho}} \) is the usual speed of sound, \( u_* \) is the critical four velocity component and \( \rho_* \) is the density at the critical point \( x_* \). One may noted that for \( c^2_s > 0 \) or \( c^2_s < 1 \), no critical point exists outside the black hole (i.e., \( x_* > 2 \)). Using (5) and (7), we get the following relation

\[
\frac{\rho_* + p(\rho_*)}{\rho_\infty + p(\rho_\infty)} = \sqrt{1 + 3c^2_\text{s}(\rho_*) \exp \left[ \int_{\rho_\infty}^{\rho_*} \frac{d\rho}{\rho + p(\rho)} \right]}
\]

The rate of change of mass \( \dot{M} \) of the black hole is computed by integrating the flux of the dark energy over the entire horizon of the black hole i.e., \( \dot{M} = 4 \int T^r_t dS \). where \( T^r_t \) represents the radial component of the energy momentum densities and the surface element of the black hole horizon \( dS = \sqrt{-g} d\theta d\phi \).

Using the above equations we obtain the rate of change of mass as

\[
\dot{M} = 4\pi AM^2(\rho + p)
\]

Since the Schwarszchild black hole is static, so the mass of the black hole depends on \( r \) only. When some fluid accretes outside Schwarszchild the black hole, the mass function \( M \) of the black hole is considered as a dynamical mass function and hence it should be a function of time also. So \( \dot{M} \) of the equation (9) is time dependent and the increasing or decreasing of the black hole mass \( \dot{M} \) sensitively depends on the nature of the fluid which accretes upon the black hole.

At the black hole horizon \( (r = 2M \text{ i.e., } x = 2) \) the relation between four velocity \( u_H = u(\rho_H) \) and the energy density \( \rho_H \) at the black hole event horizon is given by
\[
\frac{A \rho_H + p(\rho_H)}{4 \rho_\infty + p(\rho_\infty)} = \frac{A^2}{16 u_H^2} = \exp \left[ 2 \int_{\rho_\infty}^{\rho_H} \frac{d\rho}{\rho + p(\rho)} \right]
\]

and velocity four component at the horizon of black hole is given by

\[
u_H = -\frac{A}{4} \exp \left[ \int_{\rho_\infty}^{\rho_H} \frac{d\rho}{\rho + p(\rho)} \right]
\]

### A. Model I: New Variable Modified Chaplygin Gas as dark energy model

We consider the background spacetime is spatially flat represented by the homogeneous and isotropic FRW model of the universe which is given by

\[
d s^2 = -dt^2 + R^2(t) \left[ dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right]
\]

where \( R(t) \) is the scale factor. We assume the universe is filled with New Variable Modified Chaplygin Gas (NVMCG) and the EoS is \([23]\) given by

\[
p = A'(R)\rho - \frac{B(R)}{\rho^\alpha} \quad \text{with} \quad 0 \leq \alpha \leq 1
\]

where \( A'(R), \) and \( B(R) \) are function of the scale factor \( R. \) For a particular choice \( A'(R) = A_0 R^{-n} \) and \( B(R) = B_0 R^{-m} \) with \( A_0, B_0, m, n \) are positive constants. For \( n = m = 0, \) this model reduces to modified Chaplygin Gas, and for \( n = 0 \) the model reduces to the variable modified Chaplygin gas model.

The Einstein’s equations for FRW universe are (choosing \( G = c = 1 \))

\[
H^2 = \frac{8\pi}{3} \rho
\]

\[
\dot{H} = -\frac{8\pi}{2} (p + \rho)
\]

Conservation equation satisfied by the dark energy model NVMCG is

\[
\dot{\rho} + 3H(\rho + p) = 0
\]

where \( H = \frac{\dot{R}}{R} \) is the Hubble parameter.

Expression for the energy density for NVMCG model is obtained from (12) and (15) as \([23]\)

\[
\rho = R^{-3} \exp \left( \frac{3A_0 R^{-n}}{n} \right) \left[ C_0 + \frac{B_0}{A_0} X^{\frac{3(1+\alpha)+n-m}{n}} \Gamma \left( \frac{m-3(1+\alpha)}{n}, X R^{-n} \right) \right]^{\frac{1}{1+\alpha}}
\]

where \( C_0 \) is an integration constant, \( \Gamma(s,t) \) is the upper incomplete gamma function and

\[
X = \frac{3A_0 (1+\alpha)}{n}
\]

Following \([32]\), the critical values for this model are as follows

\[
c_{ss}^2 = A' + \alpha \frac{B}{\rho_{s}^{\alpha+1}}
\]

\[
u_{s}^2 = \frac{A' \rho_{s}^{\alpha+1} + \alpha B}{\rho_{s}^{\alpha+1}(1+3A') + 3\alpha B}
\]

\[
x_{s} = \frac{\rho_{s}^{\alpha+1}(1+3A') + 3\alpha B}{2 [A' \rho_{s}^{\alpha+1} + \alpha B]}
\]

(17)
Also from equation (6) and (4), we obtain the ratio of number density of NVMCG near the horizon and at the infinity as in the following

\[
n(\rho) = \left[ \frac{B' - \rho^{\alpha+1}}{B' - \rho^{\alpha+1}_\infty} \right]^\mu
\]

where \( \mu = \frac{1}{(1+\alpha)(1+w)} \), \( B' = \frac{B}{1+A} \) and

\[
A = -\frac{1}{4} \left[ \frac{\rho^{\alpha+1}_\infty (1 + 3A') + 3\alpha B'}{A' \rho^{\alpha+1}_\infty + \alpha B} \right]^{3/2} \left[ \frac{B' - \rho^{\alpha+1}_\infty}{B' - \rho^{\alpha+1}_\infty} \right]^\mu
\]

Dark energy density \( \rho_H \) at the event horizon of the black hole satisfies the following equation

\[
\left[ \frac{\rho^{\alpha+1}_H - B'}{\rho^{\alpha+1}_\infty - B'} \right]^{2\mu - 1} = \frac{A}{4} \left( \frac{\rho_\infty}{\rho_H} \right)^\alpha
\]

Also the energy density can be expressed in terms of \( x \) as

\[
\left[ \left( 1 - \frac{2}{x} \right) + \frac{A^2}{x^4} \left( \frac{B' - \rho^{\alpha+1}_\infty}{B' - \rho^{\alpha+1}} \right) \right] \left( \frac{\rho_\infty}{\rho} \right)^{2\alpha} \left[ \frac{\rho^{\alpha+1}_\infty}{B' - \rho^{\alpha+1}_\infty} \right]^{2(1-\mu)} = 1
\]

Accreted fluid velocity with respect to radial coordinate \( x \) for NVMCG onto Schwarzschild black hole are drawn in figure 1. The fluid velocity decreases as \( x \) increases, i.e., accreted fluid velocity is high near the black hole and velocity is low when the fluid is far from the black hole. The black hole mass increases for the NVMCG dark energy type accreted fluid as time goes on, which is shown in figure 3. Also relative density \( \rho/\rho_\infty \) of accreted fluid increases when \( x \) increases from the black hole (figure 5).

**B. Model II: Generalized Cosmic Chaplygin Gas as the dark energy Model**

A new version of Chaplygin gas which is known as Generalized Cosmic Chaplygin Gas (GCCG)\(^{24,25}\) obeys the equation of state

\[
p = -\rho^{-\alpha} \left[ C + (\rho^{1+\alpha} - C)^{-w} \right]
\]

where \( C = \frac{A''}{(1+w)} - 1 \), \( A'' \) takes either positive or negative constant, \(-l < w < 0 \) and \( l > 1 \). The EOS reduces to that of current Chaplygin unified models for dark matter and dark energy in the limit \( w \to 0 \).
and satisfies the conditions: (i) it becomes a de Sitter fluid at late time and when \( w = -1 \), (ii) it reduces to \( p = w \rho \) in the limit that the Chaplygin parameter \( A'' \to 0 \), (iii) it also reduces to the EOS of current Chaplygin unified dark matter models at high energy density and (iv) the evolution of density perturbations derived from the chosen EOS becomes free from the pathological behaviour of the matter power spectrum for physically reasonable values of the involved parameters at late time. This EOS shows dust era in the past and \( \Lambda \)CDM in the future.

From the conservation equation we have the expression for energy density in the form \[25\]

\[
\rho = \left[ C + \left( 1 + \frac{B}{F^2(1+\alpha)(1+w)} \right)^{\frac{1}{1+w}} \right]^{-\alpha}
\]

Following \[32\], the critical values are obtained in the form

\[
\begin{align*}
\epsilon_s^2 &= \rho_s^{-1-\alpha} y \\
\epsilon_s^2 &= \rho_s^{1+\alpha} + 3y \\
x_s &= \frac{\rho_s^{1+\alpha} + 3y}{2y}
\end{align*}
\]

where \( y = \left[ C\alpha + (\rho_s^{1+\alpha} - C)^{-1-w} \{ \rho_s^{1+\alpha}(\alpha + w + \alpha w) - C\alpha \} \right] \).

From equations (6) and (4), we obtain the ratio of number density of GCCG near the horizon and at the infinity as in the following

\[
\frac{n(\rho)}{n_\infty} = \left( \frac{f_1(\rho)f_2(\rho)}{f_1(\rho_\infty)f_2(\rho_\infty)} \right)^\nu
\]

where,

\[
f_1(\rho) = \left[ \rho^{-\alpha} \{ C + (-C + \rho^{1+\alpha})^{-w} \} - \rho \right], \quad f_2(\rho) = \rho^{\alpha(1+w)} (C\rho^{-\alpha} - \rho)^w \quad \text{and} \quad \nu = \frac{1}{(1+\alpha)(1+w)} \]

Energy density can be expressed in terms of \( x \) as

\[
\left( 1 - \frac{2}{x} \right) + \frac{A^2}{x^4} \frac{n_\infty}{n(\rho)} = \left[ \frac{f_1(\rho)}{f_1(\rho_\infty)} \right]^{2(\nu-1)} \left[ \frac{f_2(\rho)}{f_2(\rho_\infty)} \right]^{2\nu}
\]

where \( A = -\frac{1}{4} \left( \frac{\rho_s^{1+\alpha} + 3y}{2y} \right)^{3/2} \frac{n(\rho_\infty)}{n_\infty} \).

Accreted fluid velocity with respect to radial coordinate \( x \) for GCCG onto Schwarzschild black hole are drawn in figure 2. The fluid velocity decreases as \( x \) increases, i.e., accreted fluid velocity is high near the black hole and velocity is low when the fluid is far from the black hole. The black hole mass increases for the GCCG dark energy type accreted fluid as time goes on, which is shown in figure 4. Also relative density \( \rho/\rho_\infty \) of accreted fluid increases when \( x \) increases from the black hole (figure 6).

III. GENERAL ACCRETION OF DARK ENERGY ONTO KERR-NEWMAN BLACK HOLE

For the rotating, charged, stationary, axisymmetric black hole, let us consider Kerr-Newman space-time metric (considering \( G = c = 1 \)) \[31\] prescribed by the line element
FIG. 3: Changes of the mass with respect to time of NVMCG onto Schwarzschild black hole.

FIG. 4: Changes of the mass with respect to time of GCCG onto Schwarzschild black hole.

FIG. 5: Relative density of accreted fluid with respect to radial coordinate $x$ for NVMCG onto Schwarzschild black hole.

FIG. 6: Relative density of accreted fluid with respect to radial coordinate $x$ for GCCG onto Schwarzschild black hole.

\[
ds^2 = \left(1 + \frac{Q^2 - 2Mr}{r^2 + a^2 \cos^2 \theta}\right) dt^2 + \frac{2a(2Mr - Q^2) \sin^2 \theta}{r^2 + a^2 \cos^2 \theta} dt d\phi - \frac{r^2 + a^2 \cos^2 \theta}{r^2 + a^2 + Q^2 - 2Mr} dr^2
- (r^2 + a^2 \cos^2 \theta) d\theta^2 - \left[ r^2 + a^2 + \frac{(2Mr - Q^2) a^2 \sin^2 \theta}{r^2 + a^2 \cos^2 \theta} \right] \sin^2 \theta d\phi^2 \tag{28}\]
where $M$ and $Q$ are respectively mass, electric charges of the black hole. Also $a = J/M$ is the specific angular momentum per unit mass and $J$ is the total angular momentum of the black hole. Since Kerr-Newman metric is static, the time evolution induced by accretion will be taken into account by the time dependence of the scale factor entering the integrated conservation laws and the rate equations for mass and angular momentum. Also the energy tensor $T_{\mu \nu}$ for dark energy satisfies the relation (2). The first integral of motion comes from the time component of the energy-momentum conservation law $T_{00}^{\mu \nu} = 0$ which can be put in the following form for Kerr-Newman metric:

\[
\frac{d}{dt} \left[ (p + \rho) \left( 1 + \frac{Q^2 - 2Mr}{r^2 + a^2 \cos^2 \theta} \right) \frac{dt}{ds} \right] + \frac{2r}{r^2 + a^2 \cos^2 \theta} (p + \rho) \left( 1 + \frac{Q^2 - 2Mr}{r^2 + a^2 \cos^2 \theta} \right) \frac{dt}{ds} \\
+ \frac{d}{d\theta} \left[ (p + \rho) \left( 1 + \frac{Q^2 - 2Mr}{r^2 + a^2 \cos^2 \theta} \right) \frac{dt}{ds} \frac{d\theta}{ds} \right] + \left[ \frac{\cos \theta}{\sin \theta} \right. \\
\left. \left( \frac{2a^2 \sin \theta \cos \theta}{r^2 + a^2 \cos^2 \theta} \right) (p + \rho) \left( 1 + \frac{Q^2 - 2Mr}{r^2 + a^2 \cos^2 \theta} \right) \right] = 0
\]

(29)

In general, $\rho$ and $p$ are functions of $t$, $r$ and $\theta$, but in our considered dark energy models $\rho$ and $p$ depend only on time $t$. Keeping $\theta$ constant and radial four velocity component $u = \frac{dr}{ds}$ we get the expression for rate of change of mass due to accretion of dark energy \[31\]

\[
\dot{M} = -\int T_{0}^{0} dS = \frac{4\pi A_{M} M^{3} r}{J} \arctan \left( \frac{J}{Mr} \right) (p + \rho)
\]

(30)

where $dS = r^2 \sin \theta d\theta d\phi$ with $r$ and $J$ are constants. Which gives the expression

\[
I_{M} = \int_{M_{0}}^{M} \frac{J dM}{M^{3} r \arctan \left( \frac{J}{Mr} \right)} = 4\pi A_{M} \int_{t_{0}}^{t} (p + \rho) dt
\]

(31)

with

\[
A_{M} = -\frac{u}{M^2} (r^2 + a^2 \cos^2 \theta) \exp \left[ \int_{\rho_{\infty}}^{\rho} \frac{d\rho}{\rho + p(\rho)} \right]
\]

(32)

When a fluid flow directed toward the black hole we have $u < 0$ and $A_{M} > 0$ is dimensionless constant.

Again keeping $r$ as constant and we get the rate of change of angular momentum of Kerr-Newman black hole as \[31\]

\[
\dot{a} = -\int r T_{0}^{\phi} dS = \frac{2\pi^2 A_{a} a r^2 (p + \rho)}{\sqrt{r^2 + a^2}}
\]

(33)

where $dS = r^2 \sin \theta d\theta d\phi$, $\theta$ constant. Equation (33) simplifies to

\[
I_{a} = \int_{a_{0}}^{a} \frac{\sqrt{r^2 + a^2}}{ar^2} da = 2\pi^2 A_{a} \int_{t_{0}}^{t} (p + \rho) dt
\]

(34)

with

\[
A_{a} = -\frac{1}{a} \omega \sin \theta (r^2 + a^2 \cos^2 \theta) \exp \left[ \int_{\rho_{\infty}}^{\rho} \frac{d\rho}{\rho + p(\rho)} \right], \quad \theta = \text{constant}.
\]

(35)

When a fluid flow is directed toward the black hole then $\omega = \frac{d\theta}{ds} < 0$ for $A_{a} > 0$.

Expression for the rate of change of total angular momentum, with $M$, $r$ constant, is given by \[31\]

\[
\dot{J} = -\int (Mr T_{0}^{\phi} + aT_{0}^{\phi}) dS = \pi (p + \rho) \left( 2\frac{J \pi A_{a} r}{\sqrt{1 + \frac{J^2}{M^2 r^2}}} + 4A_{M} M^{2} r \arctan \left( \frac{J}{Mr} \right) \right)
\]

(36)
Which reduces to
\[ I_J = \int_{J_0}^J \frac{dJ}{\frac{2J^2 A_0 r}{\sqrt{1+\frac{J^2}{M^2 r^2}}} + 4A_M M^2 r \arctan \left( \frac{J}{M r} \right)} = \pi \int_{t_0}^t (p + \rho)dt \] \hspace{1cm} (37)

We shall obtain the quantity \( \dot{M}, \dot{a}, \dot{J} \) for the following cosmological models:

**A. Model I: New Variable Modified Chaplygin Gas as dark energy model**

For the NVMCG satisfying the equations (11-15) we derive the following quantities

\[ \sqrt{\frac{3}{8\pi A_M^2}} I_M = \sqrt{\rho_0} - \sqrt{\rho} \] \hspace{1cm} (38)

\[ \sqrt{\frac{3}{2\pi^3 A_a^2}} I_a = \sqrt{\rho_0} - \sqrt{\rho} \] \hspace{1cm} (39)

\[ \frac{6}{\pi} I_J = \sqrt{\rho_0} - \sqrt{\rho} \] \hspace{1cm} (40)

where \( \rho \) satisfying the relation (16) and from this equation we obtain the present value of density \( \rho_0 \) which is given by

\[ \rho_0 = R_0^{-3} \exp \left( \frac{3A_0 R_0^{-n}}{n} \right) \left[ C_0 + \frac{B_0}{A_0} \left( \frac{3A_0 (1+\alpha)}{n} \right)^{\frac{3(1+\alpha)+n-3}{n}} \frac{\Gamma \left( \frac{m-3(1+\alpha)}{n}, \frac{3A_0 (1+\alpha)}{n} R_0^{-n} \right)}{\Gamma \left( \frac{m-3(1+\alpha)}{n} \right)} \right]^{\frac{1}{1+w}} \] \hspace{1cm} (41)

where \( R_0 \) is the present value of scale factor. Changes of the mass with respect to time of NVMCG onto Kerr-Newman black hole with the constant \( J \) and variable \( J \) are drawn in figures 7 and 8 and they are increasing. The angular momentum and the total angular momentum with time are shown in figures 9 and 10 and they are increasing with time.

**B. Model II: Generalized Cosmic Chaplygin Gas as the dark energy Model**

For the Generalized Cosmic Chaplygin Gas (GCCG) satisfying the equations (12-14) and (21,22) we derive

\[ R^{3(1+\alpha)(1+w)} = \frac{B}{1 - \left( \left( \sqrt{\rho_0} - \sqrt{\frac{3}{8\pi A_M^2}} I_M \right)^{2(1+\alpha)} - C \right)^{(1+w)}} \] \hspace{1cm} (42)

\[ R^{3(1+\alpha)(1+w)} = \frac{B}{1 - \left( \left( \sqrt{\rho_0} - \sqrt{\frac{3}{2\pi^3 A_a^2}} I_a \right)^{2(1+\alpha)} - C \right)^{(1+w)}} \] \hspace{1cm} (43)

\[ R^{3(1+\alpha)(1+w)} = \frac{B}{1 - \left( \left( \sqrt{\rho_0} - \sqrt{\frac{6}{\pi} I_J} \right)^{2(1+\alpha)} - C \right)^{(1+w)}} \] \hspace{1cm} (44)
FIG. 7: Changes of the mass with respect to time of NVMCG onto Kerr-Newman black hole with the constant $J$.

FIG. 8: Changes of the mass with respect to time of NVMCG onto Kerr-Newman black hole with $J = J(t)$.

FIG. 9: Changes of the specific angular momentum ($a$) with respect to time of NVMCG onto Kerr-Newman black hole with the constant $r$.

FIG. 10: Changes of the total angular momentum ($J$) with respect to time of NVMCG onto Kerr-Newman black hole.

with,

$$
\rho_0 = \left[ C + \left( 1 + \frac{B}{\rho_0^{\beta(1+\alpha)(1+w)}} \right)^{\frac{1}{1+w}} \right]^{\frac{1}{1+\alpha}}
$$

(45)
FIG. 11: Changes of the mass with respect to time of GCCG onto Kerr-Newman black hole with the constant $J$.

FIG. 12: Changes of the mass with respect to time of GCCG onto Kerr-Newman black hole with $J = J(t)$.

FIG. 13: Changes of the specific angular momentum ($a$) with respect to time of GCCG onto Kerr-Newman black hole with the constant $r$.

FIG. 14: Changes of the total angular momentum ($J$) with respect to time of GCCG onto Kerr-Newman black hole.

Changes of the mass with respect to time of GCCG onto Kerr-Newman black hole with the constant $J$ and variable $J$ are drawn in figures 11 and 12 and they are increasing. The angular momentum and the total angular momentum with time are shown in figures 13 and 14 and they are increasing with time.
IV. DISCUSSIONS

In this work, we have studied recently proposed two types of dark energy models like new variable modified Chaplygin gas (NVMCG) and generalized cosmic Chaplygin gas (GCCG). Accretion of NVMCG and GCCG onto the Schwarzschild and Kerr-Newman Black holes have been discussed. Our dark energy fluids violate the strong energy condition ($\rho + 3p < 0$ in late epoch), but do not violate the weak energy condition ($\rho + p > 0$). So the models drive only quintessence scenario in late epoch, but do not generate the phantom epoch (in our choice). We find the expression of the critical four velocity component which gradually decreases for the fluid flow towards the Schwarzschild as well as Kerr-Newman Black holes. Astrophysically, mass of the black hole is a dynamical quantity, so the nature of the mass function is important in our black hole models for different dark energy filled universe. Previously Babichev et al. [29] have shown that the mass of black hole decreases due to phantom energy accretion. We here found the expression for change of masses of the Schwarzschild and Kerr-Newman black holes in both the dark energy models and have seen that they are increasing in course of time. Since our considered dark energy candidates do not violate weak energy condition, so the dynamical mass of the black holes could not decaying by the accretion of dark energies, though the pressures of the dark energies are outside the black holes. The relative density $\rho/\rho_\infty$ increases as $r$ increases outside the black hole. For the most generalized Kerr-Newman black hole (which is rotating and charged) we have obtained the specific angular momentum ($a$) and total angular momentum ($J$). We showed that in both cases due to accretion of the dark energy mass of the black hole increases and angular momentum increases in case of Kerr-Newman black hole. The mass of the black hole increases for constant and variable $J$. The relative density shows the same nature as well as Schwarzschild black hole.

Acknowledgement:

One of the authors (JB) is thankful to CSIR, Govt of India for providing Junior Research Fellowship. The authors are thankful to IUCAA, Pune, India for warm hospitality where part of the work was carried out.

[1] N. A. Bachall, J. P. Ostriker, S. Perlmutter and P. J. Steinhardt, Science 284 1481 (1999).
[2] S. J. Perlmutter et al, Bull. Am. Astron. Soc. 29, 1351 (1997).
[3] S. J. Perlmutter et al, Astrophys. J. 517 565 (1999).
[4] A. G. Riess et al, Astron. J. 116, 1009 (1998).
[5] C. L. Bennett et al, Astrophys. J. Suppl. 148, 1 (2003).
[6] S. W. Allen et al, Mon. Not. Roy. Astron. Soc. 353, 457 (2004).
[7] V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. A 9, 373 (2000).
[8] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. 75, 559 (2003).
[9] T. Padmanabhan, Phys. Rept. 380, 235 (2003).
[10] Copeland, E. J., Sami, M. and Tsujikawa, S. Int. J. Mod. Phys. J 15, 1753(2006).
[11] C. Armendariz-Picon et al, Phys. Rev. D 63 103510 (2001).
[12] A. Kamenshchik et al, Phys. Lett. B 511 265 (2001).
[13] U. Debnath, A. Banerjee and S. Chakraborty, Class. Quantum Grav. 21, 5609 (2004).
[14] A. Sen, JHEP 065 0207 (2002).
[15] J. Martin and M. Yamaguchi, Phys. Rev. D 77 103508 (2008).
[16] V. Gorini, A. Kamenshchik and U. Moschella, Phys. Rev. D 67, 063509 (2003).
[17] U. Alam, V. Sahni, T. D. Saini and A. A. Starobinsky, Mon. Not. R. Astron. Soc. 344, 1057 (2003).
[18] M. C. Bento, O. Bertolami and A. A. Sen, Phys. Rev. D 66, 043507 (2002).
[19] H. B. Benaoum, hep-th/0205140.
[20] V. Sahni, T. D. Saini, A. A. Starobinsky and U. Alam, JETP Lett. 77, 201 (2003).
[21] Z.K. Guo, Y.Z. Zhang, Phys. Lett. B 645, 326 (2007).
[22] U. Debnath, Astrophys. Space Sci. 312, 295 (2007).
[23] W. Chakraborty, U. Debnath, Gravitation and Cosmology 16 223 (2010).
[24] P. F. González-Diaz, Phys. Rev. D 68 021303 (R) (2003).
[25] W. Chakraborty, U. Debnath and S. Chakraborty, Gravitation and Cosmology 13 293 (2007).
[26] R. R. Caldwell, M. Kamionkowski, and N. N. Weinberg, Phys. Rev. Lett. 91, 071301 (2003).
[27] B. McInnes, J. High Energy Phys. 0208, 029 (2002); M. Bouhmadi-Lopez and J. A. J. Madrid, JCAP 0505, 005 (2005).
[28] U. Alam, V. Sahni, T. D. Saini, and A. A. Starobinsky, Mon. Not. Roy. Astron. Soc. 354, 275 (2004).
[29] E. Babichev et al, 2004 Phys. Rev. Lett. 93, 021102; E. Babichev, V. Dokuchaev, Y. Eroshenko, J. Exp. Theor. Phys. 100 (2005) 528-538.
[30] M. Jamil, Eur. Phys. J. C62:609, 2009.
[31] José A. Jiménez Madrid, and Pedro F. González-Díaz, Grav. Cosmol. 14, 213 (2008).
[32] F. C. Michel, Astrophys. Space Sci. 15, 153 (1972).