Large-eddy simulation of turbulence-induced aero-optic effects in free shear flows

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Abstract. Formation, interaction and destruction of coherent structures play an important role in the propagation of the coherent beam through the environment and the occurrence of optical aberrations. The issues related to modeling and simulation of aero-optical effects in a free mixing layer and free round turbulent jet are considered. A semi-empirical model is developed to study the distortion of the phase function of the coherent beam, induced by turbulent flow fluctuations. Large-eddy simulation of free shear flows is performed. The results computed with the semi-empirical model and numerical results are compared with the experimental data and data computed with the Reynolds-averaged Navier–Stokes equations. The results obtained are beneficial for design and optimization of systems based on coherent optical adaptive technique.

1. Introduction

Optical aberrations induced by turbulent flows (turbulent boundary layers, mixing and shear layers, wakes) are a serious concern in airborne communications, imaging and optical systems because the quality of the beam degrades due to variations of the refraction index along its path. When an initially planar optical wavefront passes a compressible flow or thermal environment, different parts of the wavefront experience different density in the medium and have different propagation speeds (Figure 1). The consequences of such deformations include optical beam deflection (bore-sight error) and jitter, beam spread, and loss of intensity. Wavefront distortions cause reductions of resolution, contrast, effective range and sensitivity for airborne electro-optical sensors and imaging systems.

The distortions produced in the amplitude characteristics of the beam are negligibly small as compared to fluctuations of the refractive index (Sutton, 1985; Jumper & Fitzgerald, 2001). The semi-empirical correlations, analytical model and numerical model are developed to predict the variance of phase function. The semi-empirical correlations are based on engineering analysis of aero-optic aberrations induced in canonical flows and on post-processing of experimental data. Analytical model is developed from the theory of vortex flow, model of Lamb vortex and Biot–Savart–Laplace law. The results obtained are represented in terms of Zernike polynomials and with the use of the Fraunhofer approximation. Analytical descriptions of turbulence include proper orthogonal decomposition analysis, linear stochastic estimation, time series of multiple-view wavefronts. Numerical simulation is based on the solution of averaged or filtered Navier-Stokes equations.

Optical aberrations have been the subject of extensive experimental investigations, results of which are presented by Dimotakis, Catrakis & Fourguette (2001), Wyckham, Zaidi, Miles &
A number of semi-empirical models of different level of complexity have been developed. The most popular models are those described by Sutton (1985, 1994) for homogeneous and inhomogeneous turbulence. Limitations of these models are discussed by Tromeur, Garnier & Sagaut (2006). In the models developed by Sutton (1985, 1994) and Tromeur, Garnier & Sagaut (2006), the dispersion of density fluctuations is determined as the difference between the density in the inner region and the density in the outer region of the boundary layer. In the models developed by Truman (1992), Wei & Chen (1996), the density fluctuations are found by solving the transport equation of a passive scalar. In many models, pressure fluctuations are ignored, and density fluctuations are related to temperature fluctuations using the state equation of perfect gas (Truman 1992; Wei & Chen, 1996). The temperature fluctuations are computed using the Reynolds analogy between the momentum and energy transport, and the Prandtl mixing length model (Huang, Coleman & Bradshaw, 1995).

Semi-empirical models are developed based on the assumption of flow quasi-steadiness (Gordeyev & Jumper, 2003, 2005; Siegenthaler, Gordyeyev & Jumper, 2005). For the quasi-steadiness conditions, the response time of the system and the signal propagation time should not exceed the "frozen" turbulence time, which is of the order of $10^{-3}$ to $10^{-2}$ s.

Decomposition of the instantaneous velocity and pressure fields into a time-averaged quantity, a random (incoherent) component that describes small-scale motion, and a periodic (coherent) component corresponding to large eddies is used by Lifshitz, Degani & Tumin (2008). Large-eddy simulation (LES) of turbulent flows makes it possible to obtain the field of density fluctuations by solving the filtered Navier–Stokes equations (Jumper & Fitzgerald, 2001; Volkov & Emelyanov, 2008). Large-eddy structures carrying maximal turbulent shear stresses and governed by the boundary conditions are found from the Navier–Stokes equations. Small eddies have a universal structure and are simulated on the basis of a sub-grid scale (SGS) model. The results of numerical simulation of turbulent flows in a free turbulent jet and free mixing layer provide general statistics of optical distortions by examining the time-averaged intensity pattern and instantaneous flow field. The flow-induced optical aberrations are studied for different
optical wavelengths and propagation distances, and for different angles at which an optical beam is emitted from the surface with respect to the direction of the flow.

2. Phase function of wave front

The angle characterizing the direction of the beam that passed through a certain layer of the variable-density medium is determined as the derivative of the optical path of the beam

$$\alpha(x, t) = \frac{d\varphi(x, t)}{dt},$$

where $\varphi(x, t)$ is the phase function of the wave front. The main parameter of interest for applications is the difference between the instantaneous and mean values

$$\sigma_\varphi(x) = \varphi(x) - \langle \varphi(x) \rangle.$$

The transition from the time formulation of the problem to the space formulation is performed with the Taylor hypothesis on “frozen” turbulence.

The optical path length is determined by integrating the distribution of the refractive index along the beam propagation direction (Sutton, 1985)

$$\varphi(x) = \int \left[1 + n(x)\right] dl. \tag{1}$$

The turbulent mixing causes refractive index fluctuations in space and time. A relation between the refractive index and the gas density is based on the Gladstone–Dale law (Sutton, 1985; Jumper & Fitzgerald, 2001)

$$n(x) = 1 + G \left[ \rho(x) - \rho_\infty \right], \tag{2}$$

where $\rho_0$ is a characteristic density (e.g. the density of an undisturbed medium). The Gladstone–Dale constant, $G$, depends on the wavelength of the propagating beam and the working fluid. Weak dependence on wavelength is ignored, and $G = 0.223 \times 10^{-3}$ m$^3$/kg for air. Substitution of the equation (2) in the equation (1) gives

$$\varphi(x) = \int \left\{1 + G [\rho(x) - \rho_\infty] \right\} dl.$$

When a plane monochromatic wave passes through a layer with a varying refractive index, its amplitude remains almost constant and the phase is changed, so that $\varphi = \varphi_0 + \Delta \varphi$, where $\Delta \varphi$ is the phase shift due to the inhomogeneity of the medium. The field strength of a perturbed wave is obtained by multiplying the electric field strength by factor $\exp(i \Delta \varphi)$, where $\Delta \varphi(x, y)$ denotes the phase shift along the optical path length, $L = |z_2 - z_1|$. The phase distribution is found by integrating the distribution of the refractive index over the layer thickness

$$\Delta \varphi(x, y, t) = k \int_{z_1}^{z_2} \Delta n(x, y, z, t) dz,$$

where $\Delta n(x, y, z, t)$ is the variation of the refractive index along the propagation direction, $z$ (across the flow), and $k$ is the wave number. If the perturbations of the wave vector are small, the beam deflections is neglected and the integral appears in the form

$$\varphi(x, y, t) = k_0 \int_{z_1}^{z_2} n(x, y, z, t) dz, \tag{3}$$
where $\varphi(x, y, t)$ is the phase distribution in plane $(x, y)$ normal to the direction of wave propagation at time $t$. Subscript 0 refers to the reference state.

The results of these calculations are usually represented in the form (Dimotakis, Catrakis & Fourguette, 2001)

$$
\tilde{\varphi}(x, y, t) = \frac{\varphi(x, y, t)}{k_0 L \Delta n} \simeq \frac{1}{L \Delta n} \int_{z_1}^{z_2} \left[ n(x, y, z, t) - n_{\infty} \right] dz.
$$

Along the direction of the beam propagation, the equation (4) gives

$$
\psi(x) = L \frac{\partial \tilde{\varphi}}{\partial x} = \frac{1}{\Delta n} \int_{z_1}^{z_2} \frac{\partial n}{\partial x} dz,
$$

where $L$ is the size of the region of turbulent mixing. The spectrum follows the relationship

$$
S_\psi(k_x L) \sim (k_x L)^2 S_{\varphi}(k_x L).
$$

For the mixing layer, it is assumed that $\Delta n = |n_1 - n_2|$ and $L = 2\delta$, where $\delta$ is the local width of the shear region, and subscripts 1 and 2 refer to the flows involved in mixing. The values calculated for the jet are normalized to $\Delta n = |n_a - n_{\infty}|$ and $L = 2r_a$, where the subscripts $a$ and $\infty$ refer to the parameters at the nozzle outlet and in submerged space.

### 3. Dispersion of phase fluctuations

Dispersion of small-scale fluctuations of density, $\sigma_{\rho}^2$, and the correlation length scale, $l_\rho$, are related to the dispersion of wave phase, $\sigma_{\varphi}^2$, by the expression

$$
\sigma_{\varphi}^2 = \alpha \beta^2 \int_0^L \sigma_{\rho}^2 l_\rho dy, \quad \beta = \frac{2\pi}{\lambda} \frac{dn}{d\rho} = kG(\lambda),
$$

where $L$ is the optical path length, and the integral is taken across the flow. The constant, $\alpha$, depends on the form of the correlation function assumed for the density fluctuations (e.g. $\alpha = 2$ for the exponential correlation function and $\alpha = \pi$ for the Gaussian correlation function). According to the equation (5), this leads to an 11% difference in the dispersion of phase fluctuations.

Length scale $l_{\rho}$ is found by integrating the correlation function

$$
l_{\rho} = \int_{-\infty}^{+\infty} R_{\rho \rho}(y) dy.
$$

In the conditions of local equilibrium, the correlation length scale coincides with the correlation scale of velocity fluctuations, $l_{\rho} \sim l_u \sim k^{3/2}/\varepsilon$. This assumption is not applicable near the wall, where velocity fluctuations turn to zero, therefore $k = 0$ and $l_u = 0$, but $l_{\rho} \neq 0$.

To estimate the phase fluctuations in the near-wall region, the equation (5) is usually replaced with a semi-empirical correlation (Sutton, 1985)

$$
\sigma_{\varphi}^2 = \beta^2 l_9 \delta \sigma_{\rho}^2,
$$

where $l_9$ is the length scale.
where $l_y$ is the integral turbulence length scale in the direction normal to the wall. It is assumed that $l_y \sim 0.1 \delta$, where $\delta$ is the boundary layer thickness. The dispersion of the density fluctuations is estimated as

$$\sigma^2 = A^2 (\rho_w - \rho_\infty)^2,$$

where $\rho_w$ is the density at the wall, $\rho_\infty$ is the density in the free flow, and $A = 0.1 \div 0.2$. At $l_y \ll \delta$, the equation (6) is refined by integrating across the boundary layer (Sutton, 1985; Tromeur, Garnier & Sagaut, 2006)

$$\sigma^2 = \int_0^L \sigma^2(y) l_y(y) dy. \quad (7)$$

The results computed with semi-empirical correlations are compared with those based on vortex model of the flow and LES results.

4. Semi-empirical model

The flow is replaced by an infinite chain of vortices located on one line at an identical distance from each other and having a circulation $\Gamma$. The resultant velocity distribution is described by Batchelor (1967). Though the semi-empirical model constructed ignored the aperture size and is applicable at rather large (strictly speaking, infinite) values of the latter, the model is a useful physical simplification of subsonic shear flows.

The pressure inside the vortex is not constant, which is responsible for changes in density and refractive index. The pressure and velocity are related by the following equation (the viscous effects are neglected)

$$\frac{u_\theta^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r}.$$

A combined vortex consists from the inner core rotating as a solid with a constant vorticity, and the outer area where the vorticity tends to zero. The distribution of tangential velocity is described by the equations

$$u_\theta(r) = \begin{cases} \frac{U r}{R} & \text{if } r \ll R \\ \frac{U R}{r} & \text{if } r \gg R \end{cases}, \quad \omega = \begin{cases} \Omega & \text{if } r \ll R \\ 0 & \text{if } r \gg R \end{cases}. \quad (8)$$

where $R$ is the vortex radius and $U$ is the maximum velocity expressed via the circulation, $U = \Gamma/(4\pi R)$.

The velocity distribution described by (8) is not differentiable at the point $r = R$. To construct a smooth velocity distribution, the interpolation is used

$$u_\theta(r) = 2U \frac{r/R}{1 + (r/R)^2}, \quad (9)$$

which satisfies the equations (8). The difference between the velocity profiles described by equations (8) and (9) decreases with decreasing vortex radius.

The density distribution is found from the isentropic relations which gives

$$\rho(r) = \rho_\infty \left[ 1 - 2(\gamma - 1) \frac{(U/a_\infty)^2}{1 + (r/R)^2} \right]^{1/(\gamma - 1)}.$$


At $\Delta \rho/\rho_\infty \ll 1$, the density distribution is described by the formula

$$\frac{\rho - \rho_\infty}{\rho_\infty} = -2 \frac{(U/a_\infty)^2}{1 + (r/R)^2},$$

where $a_\infty$ is the speed of sound at infinity. As the density at the vortex center is smaller than that at the vortex periphery, optical aberrations appear.

Formation of large-scale coherent vortex structures structures leads to the emergence of optical aberrations, which follow a sinusoidal dependence over the space and time coordinates (Siegenthaler, Gordeyev & Jumper, 2005).

For isentropic flows, the change in density is directly proportional to the change in pressure and inversely proportional to the change in temperature. Assuming that the change in pressure in the mixing layer is approximately proportional to the squared characteristic velocity $U\lesssim (U_2 + U_1)/2$, where $U_1$ and $U_2$ are velocities of mixing flows, we obtain

$$\frac{\Delta \rho}{\rho_0} = \frac{1}{\gamma} \frac{\Delta p}{p_0} \approx \frac{1}{\gamma} \frac{\rho_0}{p_0} U_2^2 = M_c^2,$$

where $M_c = (U_1 - U_2)/(a_1 + a_2)$ is effective Mach number. The root-mean-square value of the optical aberrations is related to the integral of the change in density. Choosing the vorticity thickness as a characteristic scale, we obtain

$$\sigma_\varphi \approx G \Delta \rho \delta_\omega \approx G \rho_0 M_c^2 \delta_\omega.$$ (10)

In the equation (10), the vorticity thickness is calculated as (Papamoschou & Roshko, 1988)

$$\frac{d\delta_\omega}{dx} = 0.17 \frac{(1 - \lambda)(1 + s^{1/2})}{1 + \lambda s^{1/2}},$$

where $\lambda = U_2/U_1$, $s = \rho_2/\rho_1$. Taking into account that $\delta_\omega \sim x$, we obtain a linear dependence of the optical aberrations on the coordinate $\sigma_\varphi \sim \rho_0 M_c^2 x$.

5. Numerical calculations

The calculations are performed with in-house finite volume compressible CFD code designed with air as the working fluid (Volkov, 2010). The perfect gas law is used to link the density, pressure and temperature. An unsteady flow of a viscous compressible fluid is described by filtered Navier–Stokes equations. The re-normalization group sub-grid scale model is used to account for the effect of the small turbulent eddies on the flow field (Yakhot, Orszag, Yakhot & Israeli, 1986).

A velocity profile is specified at the nozzle section (at $|r| = r_a$) with random sinusoidal perturbations superposed

$$v_x(r,t) = \frac{u_a}{3} \left[ 1 + \tanh \left( \frac{0.5 - |r|}{2\delta} \right) \right] [1 + \alpha \sin(Sh t)],$$

where $\delta/r_a \sim 0.1$, $Sh = 0.45$, and $\alpha = 0.0025$. Small random perturbations are also imposed on the radial distribution of circumferential velocity

$$v_\varphi(r,t) = 0.025 \exp \left[ -3(1 - |r|)^2 \right] \varphi,$$

where $\alpha$ is a random number from the interval $[0.5, 0.5]$. The radial velocity at the nozzle outlet is $v_r(r,t) = 0$.  

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When mixing layer is simulated, velocity profile is specified at the inlet boundary

\[ u(y) = u_0 + \Delta u \tanh \left( \frac{y}{\delta} \right), \quad \delta = \frac{\Delta u}{(du/dy)_{y=0}}. \]

Non-reflecting boundary conditions are specified in the outlet section of the computational domain.

The code uses an edge-based data structure to give the flexibility to run on meshes composed of a variety of cell types (Volkov, 2010). The fluxes are calculated on the basis of flow variables at nodes at either end of an edge, and an area associated with that edge (edge weight). The edge weights are pre-computed and take account of the geometry of the cell. The non-linear CFD solver works in an explicit time marching fashion, based on a five-step Runge-Kutta stepping procedure and piecewise parabolic method. The governing equations are solved with Chakravarthy-Osher scheme for inviscid fluxes (Chakravarthy & Osher, 1985), and the central difference scheme of the 2nd order for viscous fluxes. Convergence to a steady state is accelerated by the use of multigrid techniques, and by the application of block-Jacobi preconditioning for high-speed flows, with a separate low-Mach number preconditioning method for use with low-speed flows. The sequence of meshes is created using an edge-collapsing algorithm.

6. Results and discussion

Two semi-infinite gas flows move in one direction with velocities \( U_1 \) and \( U_2 \) along the plane \( x < 0, y = 0 \). The flows contact each other at the point \( x = 0 \); further downstream (at \( x > 0 \)), there is turbulence on the boundary between these two flows.

Calculations for the mixing layer are based on the mesh \( 250 \times 80 \times 80 \) for \( Re_\delta = 2 \times 10^5 \) (the Reynolds number is calculated from the momentum thickness) within the range \( M_c = 0.15 \div 0.8 \), where \( M_c = (u_1 - u_2)/(a_1 + a_2) \), and \( a \) is the speed of sound. At \( M_c = 0.15 \), compressibility has no effect on the properties of the flow (\( \rho_2/\rho_1 = 1 \)), while at \( M_c = 0.9 \) the density ratio is \( \rho_2/\rho_1 = 4 \).

The vorticity thickness in the initial section is \( \delta_{\omega,0} = 0.02 \) m. The computational domain has a length \( 25\delta_{\omega,0} \) and width \( 5\delta_{\omega,0} \) (the \( y \) coordinate varies in the interval \(-3\delta_{\omega,0} \leq y \leq +3\delta_{\omega,0}\)). The \( z \) length of the computational domain is \( 5\delta_{\omega,0} \). The mesh nodes are clustered so that the mesh steep near the interface between the flows has the order of the Taylor length microscale. The time step is \( \Delta t = 1.5 \times 10^{-5} \) s.

The profiles of the density fluctuations in the mixing layer are shown in the Figure 2 at \( M_c = 0.8 \). The distributions of the density and pressure fluctuations in the cross sections of the mixing layer are similar with a maximum at the line separating the mixing flows. The profiles of pressure fluctuations are more filled, and the maximal amplitude of the pressure fluctuations is close to a linear dependence on the streamwise coordinate. Small deviations from the linear dependence occur only at \( x/L > 0.68 \). The dependence of the maximal density fluctuations on the \( x \) coordinate is non-monotonic (see Figure 3). At \( x/L < 0.6 \), it is close to a linear one and, shows a peak and a smooth decay.

The results presented in the Figure 4 are calculated for the mixing layer with the semi-empirical correlation (10) at \( M_c = 0.24 \), \( U_1 = 260 \) m/s, \( M_1 = 0.77 \), \( U_2 = 0.06 \) m/s, and \( M_2 = 0.06 \), which give \( d\delta_{w}/dx \approx 0.25 \). The results are in reasonable agreement with the experimental data of Siegenthaler, Gordyev & Jumper (2005) in the case of rather large aperture sizes (\( A > 20 \) cm) and with the LES results. Dashed line (line 2) shows the results obtained by Gordyev & Jumper (2003) with the use of a sinusoidal law for the deflection angle \( \alpha = \sin(2\pi ft) \), where \( f = U_c/A \) is the frequency of vortex formation, \( A \) is the vortex size, and \( U_c \) is the characteristic velocity. Such an approach allows obtaining the final dependence of the...
Figure 2. Density fluctuations in the cross section of the mixing layer.

Figure 3. Maximal density fluctuations in the mixing layer.

Figure 4. Root-mean-square values of optical aberrations in the mixing layer. Line 1 corresponds to the equation (10), line 2 corresponds to the calculation of Siegenthaler, Gordeyev & Jumper (2005), symbols ○ and • correspond to the experimental data with \( A = 20 \) and 30 cm, and symbols □ correspond to the LES results.

The density fluctuations grow with an increase in the effective Mach number. Calculations yields \( l_\rho \sim 4l_u \), where \( l_u = 0.2k^{3/2}/\varepsilon \).

In the intermediate range of wave numbers, the wave spectrum in the mixing layer follows the power dependence \( S_\varphi(k_xL) \sim (k_xL)^{-q} \), where \( q \sim 2 \) (Figure 5). The compressibility only has a weak influence on the spectrum behavior (this influence is largely displayed at large wave numbers).

Calculations of flow in the submerged round jet are carried out in the range \( 10^3 \leq \text{Re} \leq 10^5 \) (Reynolds number is found from the velocity at the nozzle outlet and the nozzle diameter). The generation of eddies is due to the Kelvin–Helmholtz instability of the shear layer. The maximum and minimum of the vorticity approximately correspond to the eddy centers. At low Reynolds numbers, \( \text{Re} \sim 10^3 \), the jet at the nozzle outlet is nearly axially symmetrical. As the Reynolds number increases with distance from the nozzle (to \( \text{Re} \sim 10^4 \)), a weak sinusoidal mode appears.

In the free jet at \( \text{Re} = 8 \times 10^4 \) and high wave numbers, \( 2r_uk_x > 1 \) (\( r_u \) is the nozzle-exit radius), follows the power dependence \( S_\varphi(k_xL) \sim (2r_uk_x)^{-q} \), where \( q \sim 2.5 \) (Figure 6). The
Figure 5. Spectrum of phase fluctuations in the free mixing layer.

Figure 6. Spectrum of phase fluctuations in the round jet.

results calculated for the jet are normalized to $\Delta n = |n_a - n_\infty|$ and $L = 2r_a$, where subscripts $a$ and $\infty$ refer to nozzle outlet and submerged space. The spectrum for the jet is steeper than that for the mixing layer because of a stronger turbulent mixing, and is almost independent on the Reynolds number.

7. Conclusion

The main source of large optical distortions is the mixing turbulent region which significantly degrade the performance of optical system.

Many experimental studies have been performed to develop high-speed wavefront measurement tools, study the refractive index structures, develop distortion scaling laws, and devise control techniques to suppress or modify optically important turbulence structures. The experimental methods provided no information concerning temporal frequencies. Most of the early computational studies were based on statistical analysis with simplifying assumptions such as homogeneous and isotropic turbulence, and therefore are not directly applicable in realistic aero-optical flowfields. The mesh resolutions were poor in the cases involving complex geometries, and the numerical schemes employed were either total variation diminishing or upwinding techniques that are highly dissipative.

The aero-optical aberrations produced by compressible shear layers are a direct consequence of the detailed physics in the flow. Time-accurate computational studies of aero-optical distortions are performed based on large-eddy simulation because of its ability to account for the optical phase errors due to anisotropy of the flow field and to resolve large-scale coherent structures at a reasonable computational costs.

The large-eddy simulation of aero-optic effects in a free mixing layer and round turbulent jet indicate that the spectrum of phase fluctuations is only a weak function of the input parameters (the dependence is stronger at large wave numbers). It is also shown that the use of semi-empiric models of turbulence leads to inaccurate values for the dispersion of phase fluctuations. Semi-empirical relations derived from the vortex model of the flow ensure reasonably accurate calculations of the root-mean-square values of optical aberrations. The dependence of the root-mean-square fluctuations of the wave phase on the dynamic pressure is linear beginning from the coordinate counted along the flow direction.
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