Contradictions in the traditional methods of electrodynamics teaching as a determinant of its update

O A Konoval¹, T I Turcot² and A O Solomenko³

¹ Kryvyi Rih State Pedagogical University, 54 Gagarin Ave., Kryvyi Rih, 50086, Ukraine
² Kherson Academy of Continuing Education, 41 Pokrysheva Str., Kherson, 73034, Ukraine
³ Kryvyi Rih Comprehensive School of I-III levels No. 89 of the Kryvyi Rih City Council in Dnipropetrovsk oblast, 1A Maliovnycha Street, Kryvyi Rih, 50054, Ukraine

E-mail: artemsolomenko@gmail.com

Abstract. The article deals with the theoretical analysis of the traditional approaches to electrodynamics teaching. The authors pay attention to the contradictions arising in the process of combined application of Biot–Savart and Coulomb’s laws. The thesis of some limitation of the traditional presentation of the course of electrodynamics is substantiated. In this connection the necessity of updating methodology of electrodynamics teaching is determined. As an innovative approach to solving this problem, it is proposed to analyze and address the existing contradictions of electrodynamics teaching using a specially developed (copyright) methodology – the methodology of the development of students’ critical thinking (MDCT). Using specific examples, the content of each of the stages of the proposed methodology, which can be used in the study of electrodynamics in secondary and higher school, is characterized.

1. Introduction and relevance of the problem
Modern socio-economic realities, rapid development of science and technology put forward a number of new requirements for future teachers training, because in accordance with the well-known pedagogical axiom, only the teacher, who is a professional, a creative person, capable of critical understanding the world around him, can bring up a creative, critically thinking student [13]. First of all, it concerns the students of physics departments – future physics teachers, whose pupils should become the driving force of scientific and technological progress in our country.

However, reference to teaching practice, modern manuals on theoretical physics [4], [8], [11], [17], general and school physics courses (for a review and analysis see [5], [6], [9], [10], [15], [18]) shows that in the methods of teaching electrodynamics during interpretation of certain laws there are contradictions and questions for discussion, which prevent the formation of students’ holistic picture of adequate ideas about the system of electrodynamics fundamental laws.

The presence of these contradictions logically leads to the thought that it is necessary to update the methods of teaching electrodynamics. It was determined by us as the starting point for: a) finding ways to resolve them; b) the development of such a methodology for teaching electrodynamics, which, using the content of contradictions and methods for resolving them, would contribute to the development of future physics teachers’ critical thinking.
In this regard, the purpose of the article is to identify the contradictions and problematic issues of electrodynamics as a science and academic discipline, the modern methodology of its teaching, and the rationale for solving these problems using the methodology for developing students’ critical thinking.

2. Research methods

Taking into consideration the limited volume of scientific article we shall concentrate more in detail on the analysis of the content of contradictions associated with Biot Savart Law [5], [6], leaving more thorough investigation of the causes of other contradictions and questions for discussion, taking place in the theory and methodology of electrodynamics teaching as the prospect of further research.

First of all let’s note the ambiguity in the treatment of the status of Biot Savart Law and also pay attention to a number of contradictions associated with the interpretation and application of this law:

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{\mathbf{r} \times \mathbf{E}}{r^3}$$

(1)

Despite understanding the fact that the law (1) in principle is unavailable to direct experimental verification, in the vast majority of modern textbooks Biot Savart Law is interpreted as an experimental law. But the analysis of the essence of this law and historical sources testifies about the incorrectness of such approach for the study and interpretation of the ratio of (1) [5], [6].

Secondly, our study has shown that in some cases the application of the law (1) leads to the results that are contrary to the main postulates of the relativistic electrodynamics [6].

Thus from (1) and the superposition principle we can conclude that the expression for magnetic field (MF) induction, which is created by a moving charged particle (CP) is [6], [18]:

$$\mathbf{B}_1 = \frac{\mu_0 q}{4\pi r^3} (\mathbf{v} \times \mathbf{r}) = \mu_0 \varepsilon_0 \left[ \mathbf{v} \times \frac{q\mathbf{E}}{4\pi \varepsilon_0 r^3} \right] = \mu_0 \varepsilon_0 \left[ \mathbf{v} \times \mathbf{E} \right],$$

(2)

where \(\mathbf{E} = \frac{q\mathbf{v}}{4\pi \varepsilon_0 r^3}\) is the strength of the electric field created by (in the non-relativistic approximation) a mobile CP at the corresponding point of the field; \(q\) – the magnitude of a moving charge; \(r\) – the radius vector drawn from the charge to a given point of the field; \(\mathbf{v}\) – the velocity of charges in a conductor with current (drift velocity of conduction electrons); \(\mu_0, \varepsilon_0\) – magnetic and electric constants.

We’ll show that the use of the expression (2) for the induction of the magnetic field of a moving charged particle \(\mathbf{B} = \varepsilon_0 \mu_0 [\mathbf{v}, \mathbf{E}]\), (which can be seen as a consequence of the law (1)) and \(\mathbf{E} = \frac{q\mathbf{v}}{4\pi \varepsilon_0 r^3}\) for the analysis of the same electrodynamic problem leads to the conclusions that contradict the principle of relativity.

3. Results

Indeed, let’s consider a thought experiment, shown schematically in figure 1.

According to the traditional interpretation of the phenomenon of electromagnetic induction, in the loop \(L\), due to time changes in the magnetic field at each point of a flat surface bounded by the loop \(L\) (and therefore the change of the magnetic flux in time), there will be EMF of induction. But this conclusion contradicts the principle of relativity: with the transition to the reference frame associated with CP, the loop \(L\) will be moving in a centrally symmetric Coulomb field.

Such field is a potential one, so there are no physical reasons that could generate voltage in the loop \(L\). Hence, the use of the law (1) and its equivalent – the expression for MF induction, which is generated by a separate mobile CP \(\mathbf{B}_1 = \varepsilon_0 \mu_0 [\mathbf{v}, \mathbf{E}] = \frac{\mu_0 q [\mathbf{v}, \mathbf{r}]}{4\pi r^3}\) – in such problems gives the result that contradicts physical reality.

Similarly, in the problem shown in figure 2 the use of the classical Biot Savart Law leads to the result that is contrary to the principle of relativity.

According to Biot Savart Law (1) and the traditional interpretation of the phenomenon of EMI (see the problems, shown in figure 3 and figure 4) we should expect a certain EMF of induction in the loop \(L_1\).
Figure 1. The scheme of a thought experiment to determine the EMF of induction in the loop \( L \). CP is moving with constant speed \( \vec{v} \) in the plane of the loop.

Figure 2. In the loop \( L_1 \) there is no EMF with the motion of the charged filament with the speed \( \vec{v} \).

Figure 3. In the wire in the loop \( L_1 \), which is in the vicinity of an infinitely long conductor with direct current there is no EMF.

Figure 4. In the wire loop \( L_1 \), which is located next to the DC circuit, EMF is absent.
This conclusion follows from the following considerations.

Each of the conduction electrons, moving with the drift velocity in the conductors of the electric circuit creates alternating magnetic field at each point of the flat surface bounded by the loop $L_1$. In its turn this alternating magnetic field creates a vortex electric field ($\text{rot} \vec{E} \neq 0$) at each point of the surface bounded by the loop.

Integral effect should manifest itself in the emergence of induced current in the loop $L_1$. But experiments show that the voltage in the circuit does not emerge.

So, Biot Savart Law and the formula $\vec{E} = \frac{q \vec{r}}{4\pi\varepsilon_0 r^3}$ (which is obviously a consequence of Coulomb's law) are not compatible for simultaneous use in the analysis of the same electrodynamic problem because they lead to principally false conclusions and contradictions [5].

Thus, the traditional method of studying electrodynamics, based, in particular, on the classical Biot Savart Law too cannot explain the absence of voltage in the loop $L_1$ in figures 1, 2, 3, 4. At the same time in standard textbooks and scientific and methodological publications these two formulas are used together, which is, from our point of view, the physical error [6].

4. Discussion

Our theoretical developments and practical experience [6] give us the opportunity to provide a new approach to the study of electrodynamics, allowing the use of the analysis of inconsistencies for developing students’ critical thinking. This approach is embodied in the methodology of the development of students’ critical thinking of (MDCT), which was tested in our practice when studying electrodynamics by physics students of the pedagogical university and deepening knowledge by teachers-practitioners in the process of independent work during training in the teacher-training courses at Kherson Academy of Continuing Education and at Kryvyi Rih State Pedagogical University. It is assumed that, in accordance with the principle of binary, teachers’ mastery of the methodology of the development of critical thinking will be used by them in the educational process of a comprehensive school. In the most general form, the methodology of the development of critical thinking consists of five main stages, a brief summary of which is proposed below.

The first stage is “Challenge”. The functions of the “Challenge” are: identifying the problem situation, understanding the importance of the information offered for study, encouraging schoolchildren (students) to study it using historical material, demonstrating interesting experiments, etc. In our example, drawing student’s attention to a number of contradictions connected with the interpretation and application of Biot Savart Law (1) can serve as the “Challenge”.

At the same time, the teacher should listen to all hypotheses with respect and attention, focusing on the value of ideas expressed by students.

The second stage – “Understanding” – involves independent classroom work of students on the material being studied, discussion, dispute of problematic issues, a comprehensive analysis of the revealed in these material contradictions. At this stage, the teacher can not be the only source of information, but first of all, he must offer methods of thoughtful familiarization with different sources of information, the formation of their own conclusions. During this period, it is necessary to recommend students to generalize their own thoughts, prepare for their argumentation with the possibility of using the texts of the originals. At the same time, it is advisable for the teacher to adhere to the rule: “The thought of everyone is a precious scientific value”.

The result of this step in our example may be a conclusion obtained, in particular, using figure 3 and figure 4, that Biot Savart Law and the formula $\vec{E} = \frac{q \vec{r}}{4\pi\varepsilon_0 r^3}$ as a consequence of Coulomb’s law are incompatible with each other for simultaneous use in the analysis of the same electrodynamic problem, as it leads to false conclusions and contradictions [5]. At the same time, in many manuals and in scientific and methodological publications, these two formulas are used together, which can be a physical error. The identification of such errors and contradictions contributes to the formation of students' needs for a thoughtful approach to the analysis of the content of school and university
textbooks, which is important for the formation of a critically constructive thinking style of a modern teacher.

The third stage – “Generalization” – is focused on collective analysis, systematization and generalization of knowledge gained. Taking into consideration the content of the topic, which is studied, students can make, for example, such inferences. From the undertaken analysis with reference to ([6], p. 158), we have at least 7 points of view regarding the status of Biot Savart Law. It should also be pointed out that many authors of the manuals and articles on this subject noted the inconsistency and inconsequence of these points of view and the corresponding methods for studying the magnetic field of constant and quasi-stationary currents. At the same time, they felt their own discomfort and some confusion when describing and explaining the basics of magnetostatics (as evidenced, in particular, by inconsistent and contradictory arguments to substantiate Biot Savart Law in such many ways).

And yet, Biot Savart Law is considered to be fundamental in the most of educational publications [3], [8], [14]. The main argument that reinforces this research position is that the results of calculations based on it are confirmed by experiments. But proceeding from the principle of fundamentality, the following problem arises: why should a relation that is not established by direct experiment, and which is a consequence of other principles, be extolled to the rank of fundamental and independent!? In our opinion, questions of such level of problematicity are the basis for the development of critical thinking of the teacher’s personality, and therefore make up the concept of the proposed MDCT. In the development of the discussion topic, the following questions can be offered to students as tasks for independent work and creative understanding [16]:

a) Should Biot Savart Law be granted the status of fundamental and independent one?

b) Some researchers consider the Biot Savart Law as a consequence of Maxwell’s equations ([6], p. 164; [7]). Reason the pros and cons of such a position.

The fourth step in the development of critical thinking should be the evaluative and resultant one – “Didactic reflection” stage. At this stage, students should analyze not only the results obtained, but also the process of cognitive activity. It is important that, under the guidance of a teacher, they conclude that: “Critical thinking is not a search for flaws in the conclusions of others, but primarily an objective assessment of the achievements gained, their negative and positive sides” [1].

The final (most important) stage of the proposed methodology is the organization of extracurricular independent work of students, focused on the further development of critical thinking skills.

For example, we give indicative questions for discussion and the content of the conversation with students in order to form motivation to continue working on a problem that began in the classroom.

– Is it possible to get the Biot Savart Law as a result of other initial provisions and principles (for example, based on independent and fundamental provisions: the principle of relativity and Coulomb’s law)?

– How to resolve the contradictions indicated and presented in figures 1, 2, 3, 4?

Regarding the last question, it should be remembered that electrodynamics is a relativistic theory. Therefore, the analysis of electrodynamic phenomena (tasks, processes) must be carried out exclusively and sequentially using the methods of the theory of relativity.

Based on the principle of relativity and Coulomb’s law it can be shown [6] that with uniform and rectilinear motion of a charged particle with an arbitrary velocity \( v < c \), the electric field and the magnetic field induction of a moving charged particle are determined according to the formulas:

\[
\vec{E}(\vec{r}, t) = \frac{q\vec{v}(1-v^2/c^2)}{4\pi \varepsilon_0 (x-ut)^2 + (y^2 + z^2)(1-v^2/c^2)^{3/2}}, \quad \frac{q\vec{v}(1-v^2/c^2)}{4\pi \sigma \rho^3 (1-v^2/c^2)^{3/2}}
\]

\[
\vec{B}(\vec{r}, t) = \frac{1}{c^2} \left[ \frac{\partial}{\partial t} E(\vec{r}, t), \right]
\]
speed; \( \mu_0, \varepsilon_0 \) – magnetic and electrical constants; \( c \) – speed of light in vacuum, \( \theta \) – angle between \( \vec{r} \) and charge velocity \( \vec{u} \).

Then the vortex electric field \( \vec{E}(\vec{r}, t) \) at any point in space at any moment of time is compensated by a vortex induction electric field \( \frac{\partial B(\vec{r}, t)}{\partial t} \): \( \text{rot} \vec{E}(\vec{r}, t) = -\frac{\partial B(\vec{r}, t)}{\partial t} \). And so the EMF in the circuit \( L_1 \) in figures 1, 2, 3, 4 in any inertial reference frame is always zero.

From the foregoing, two important consequences follow.

1. Faraday’s Law \( \text{rot} \vec{E}(\vec{r}, t) = -\frac{\partial B(\vec{r}, t)}{\partial t} \) can be justified on the basis of the Coulomb’s law and the principle of relativity.

2. Assuming that \( \vec{u} \) is the velocity of motion of charge carriers in the current element (drift velocity of conduction electrons in a conductor with current), in accordance with the principle of superposition, we obtain the Bio-Savart law in relativistic form:

\[
d\vec{B} = \frac{\mu_0}{4\pi} \frac{l|d\vec{r}|(1-\beta^2)}{r^2(1-\beta^2 \sin^2 \theta)^{3/2}} = \frac{\mu_0}{4\pi} \frac{l|d\vec{r}|(1-\beta^2)}{(\sqrt{\beta^2+(y^2+z^2)}(1-\beta^2))^{3/2}},
\]

where is \( l d\vec{l} \) – current element; \( \beta = \frac{\vec{u}}{c} \), \( \theta \) – angle between \( \vec{r} \) and current element \( l d\vec{l} \).

Obviously, with \( \beta = \frac{\vec{u}}{c} \ll 1 \), law (5) becomes the usual (classical) Biot-Savard law (1).

5. Conclusions and prospects of further researches

We understand that our reasoning can be perceived ambiguously. But the fact is that there are contradictions in physics and the methodology of its teaching, which have been shown by us using concrete examples in this article. It is desirable that these contradictions will become the subject of scientific and methodological discussion. We propose the use of these contradictions both to deepen the students’ knowledge and to develop their critical thinking [2], [6], [12]. The results of approbation in the system of higher and postgraduate education of the methodology for the development of critical thinking [17], which makes it possible to solve these problems in unity, are quite promising. Therefore, we consider promising to use the above-described MDCT on a wider scale (in particular, when studying other branches of physics, and possibly other natural-mathematical disciplines).

References
[1] Batrun I V 2018 Tekhnolohiia rozvytku krytychnoho myslenia i suchasnyi osvitnii protsess Zavuchu Use dla roboty 5/6 7–14
[2] Bensley D A and Spero R A 2014 Improving critical thinking skills and metacognitive monitoring through direct infusion Thinking Skills and Creativity 12 55–68 URL https://doi.org/10.1016/j.tsc.2014.02.001
[3] Fedorchenko A M 1992 Teoretychna fizyka vol 1 Klasychna mekhanika i elektrodynamika (Kyiv: Vyshcha shkola) p 535
[4] Feynman R, Leighton R and Sands M 1977 The Feynman Lectures on Physics vol 1 Mainly Electromagnetism and Matter (Boston: Addison-Wesley) p 592
[5] Konoval O A 2018 Debatable tasks in the traditional method of electrodynamics teaching Physical and mathematical education 15 113–17 URL http://fmo-journal.fizmatsspu.sumy.ua//journals/2018-v1-15/2018_1-15-Konoval_Scientific_journal_FMO.pdf
[6] Konoval O A, Turkot T I and Solomenko A O 2019 Metodyka rozvytku krytychnoho myslenia zdobuvachiv osvity (na prykladakh vyvchennia spetsialnoi teorii vidnosnosti ta elektrodynamiki) (Kryvyi Rih: KDU) p 232
[7] Landau L D and Lifshitz E M 2000 The Classical Theory of Fields (Course of Theoretical Physics Series vol 2) 4th ed (Oxford: Butterworth-Heinemann) p 444
[8] Levich V G 1962 Kurs teoreticheskoi fiziki vol 1 (Moscow: Fizmatgiz)
[9] Martinson M L and Nedospasov A V 1993 On the charge density inside a conductor carrying a
current *Phys. Usp.* 36 23–4 URL https://doi.org/10.1070/PU1993v036n01ABEH002062

[10] Nikolaev G V 1975 Paradoks Feinmana i asymmetryia laboratornoi i dvyzhushchisiasia system otscheta *Preprint* 1937-75 (Moscow: VINITI)

[11] Purcell E M and Morin D J 2013 *Electricity and Magnetism* 3rd ed (Cambridge: Cambridge University Press) p 833

[12] Saputra M D, Joyoatmojo S, Wardani D K and Sangka K B 2019 Developing Critical-Thinking Skills through the Collaboration of Jigsaw Model with Problem-Based Learning Model *International Journey of Instruction* 12 1077–94 URL https://doi.org/10.29333/iji.2019.12169a

[13] Savchenko S V, Shekhvatsova S O and Zaselskiy V I 2020 The development of students' critical thinking in the context of information security *CEUR Workshop Proceedings* 2731 383–99

[14] Savelev I V 1978 *Kurs obshechei fiziki* vol 2 Electrichestvo i magnetizm. *Voln. Optika* (Moscow: Nauka) p 480

[15] Ugarov V A 1977 *Spetsyalnaia teoryia otnosyelnosty* (Moskva: Nauka) p 384

[16] Willingham D T 2020 *How to Teach Critical Thinking* URL https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/education-for-a-changing-world/media/documents/How-to-teach-critical-thinking-Willingham.pdf

[17] Zangwill A 2013 *Modern Electrodynamics* (Cambridge: Cambridge University Press) p 964

[18] Zasiekina T M and Zasiekin D O 2018 *Fizyka (profilnyi riven)* 10th grade (Kyiv: Orion) p 304