Design of a H∞ Robust Controller for a Small UAV

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Abstract. For Small Unmanned Aerial Vehicles (SUAV), because of its wide flight envelope and its own flight control system has many uncertainties, the aircraft itself is also vulnerable to strong interference such as atmospheric turbulence, so a robust controller is required to meet the specific performance requirements. In this work, a H∞ algorithm, based on a Linear Matrix Inequality (LMI), is used to the design of multivariable flight control system for the longitudinal motion mathematical model of a SUAV in a key laboratory. The simulation result demonstrated that the control system cannot only guarantee the stability of the closed-loop, but also has good robustness for internal and external disturbances.

Keywords: Small Unmanned Aerial Vehicle; longitudinal attitude control; H∞ control theory; Linear Matrix Inequality; Robustness.

1. Introduction

With the increasing complexity of its flight environment and the diversification of its mission, there are many uncertainties in Unmanned Aerial Vehicle (UAV) flight system, and the flight dynamics are easily affected by strong interference such as atmospheric turbulence [1-2]. Not only the classical feedback control theory and the modern control theory are difficult to meet the requirements of future UAV flight quality [3-4]. At present, many control algorithms have been applied to the design and research of UAV flight control system. Zhai Bin uses a PID control method to design the UAV flight controller [5], which can only design and adjust single loop at a time. When the flight conditions change, it is necessary to constantly change the parameters of the controller to meet the requirements of the flight conditions, so as to obtain the desired system performance and robustness. Cong Yuhua of Nanjing University of Aeronautics and Astronautics proposes the method of applying a Nonlinear Dynamic Inverse to design a flight controller [6]. Although the control effect is ideal, this method needs to get the linear model inverse system. When there is interference signal or unmodeled dynamic state in the system, the dynamic inverse controller is difficult to ensure the ideal control effect. Chen Jie of Northwestern Polytechnical University adopts a linear two times optimal control theory to analyze and design the longitudinal and lateral control rate [7], this method can solve an unique optimal controller, but it does not consider the influence of noise and uncertainty on the system. So its control effect needs to be further verified. He Hui of Nanjing University of Science and Technology applies a fuzzy control method to the UAV landing control process [8]. Although it has achieved certain control effect, in the algorithm implementation process, the output proportion factor needs to be constantly switched. It is easy to cause algorithm difficultly converging, and still can't guarantee better control accuracy and robustness.

In this paper, a H∞ control algorithm, based on a Linear Matrix Inequality (LMI), is applied to a longitudinal flight control strategy of a SUAV. At the same time, the H∞ norm from the disturbance input signals to the controlled outputs is optimized. By changing the controller parameters, the maximum gain
from the limited input energy to the output energy is optimized. Thus, the problem of robust stability and robust control is solved by considering the influence of external disturbance.

2. Dynamical Modelling of the SUAV
The establishment of UAV dynamic mathematical model is the premise and foundation of its controller design. The dynamical model of UAVs usually includes three force equations, three motion equations, three moment equations and three navigation equations [9]. Since the yaw angle $\psi$ can be decoupled by other state variables, the state variables related to the flight attitude are: $[V, \beta, \alpha, \phi, \theta, p, q, r]^T$, where $V$ is the flight speed, $\beta$ is the sideslip angle, $\alpha$ is the angle of attack, $\phi$ is the roll angle, $\theta$ is the pitch angle, $p$ is roll angle speed, $q$ is the pitch angle speed, $r$ is the yaw angle speed. When the flight state of the UAV meets the horizontal condition without a side slip $\phi=\beta=0$, the UAV motion equations can be further decoupled into longitudinal motion equations and lateral motion equations [9]. The following horizontal flight conditions are selected: altitude $1100 \text{ m}$, speed $20 \text{ m/s}$. Under this condition, the longitudinal nonlinear flight state equation of UAV represented is trimmed. On the basis of the equilibrium state, the linearization equation of longitudinal flight attitude can be obtained by applying the small disturbance principle:

$$
\begin{bmatrix}
V \\
\alpha \\
\theta \\
q
\end{bmatrix} =
\begin{bmatrix}
-0.136 & 11.52 & -9.8 & 0 \\
-0.0551 & -0.2059 & 0 & 0.98 \\
0.026 & -0.66 & 0 & -0.5534 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
V \\
\alpha \\
\theta \\
q
\end{bmatrix} +
\begin{bmatrix}
0.34 & 0.001 \\
-0.00167 & -0.63 \\
0 & 0 \\
0.000304 & -0.00985
\end{bmatrix}
\begin{bmatrix}
\delta_r \\
\delta_r
\end{bmatrix}
$$

(1)

$$
\begin{bmatrix}
V \\
\alpha \\
\theta \\
q
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
V \\
\alpha \\
\theta \\
q
\end{bmatrix} +
\begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\delta_r \\
\delta_r
\end{bmatrix}
$$

(2)

3. $H_\infty$ Controller Design Method

3.1. $H_\infty$ Standard Output Feedback Control
Based on the lateral motion model of the aircraft, the $H_\infty$ robust control system is designed according to Fig. 1 [10].

![Figure 1. Standard control system structure diagram](image)

where, $P(s)$ is a plant and $C(s)$ is a controller; $v(t)$ is an interference signal (including reference instruction signal, internal disturbance and sensor noises); $y(t)$ is a controlled output signal; $u(t)$ is a
feedback control signal; \( m(t) \) is a measured output signal and also a input signal of \( C(s) \). A state space realization of \( P(s) \) is represented as:

\[
\begin{align*}
\dot{x} &= A x + B_1 y + B_2 u \\
y &= C_1 x + D_{11} y + D_{12} u \\
m &= C_2 x + D_{21} y + D_{22} u.
\end{align*}
\]

A transfer function description of (3) can be represented as:

\[
\begin{bmatrix}
y(t) \\
m(t) \\
u(t)
\end{bmatrix} = P(s) \begin{bmatrix} v(t) \\
u(t)
\end{bmatrix}.
\]

Then the closed-loop transfer function matrix from the interference signal to the output can be described as a linear fractional transformation of \( P(s) \) with respect to:

\[
T_{sv}(s) = F_l(P(s), C(s)),
\]

The robust controller design problem can be described as follows: for the system shown in Fig. 1, try to find a real rational controller \( C(s) \) to make (5) and (6) internal stable and \( \|T_{sv}(s)\|_\infty \) minimal, that is to say

\[
\min_{C(s)} \|T_{sv}(s)\|_\infty.
\]

H\( \infty \) suboptimal control problem: given a small positive number \( r \), find a regular, real rational controller \( C(s) \), which makes the closed-loop system internal stable and \( \|T_{sv}(s)\|_\infty < r \).

3.2. \( H\infty \) State Feedback Solution Based on LMI

Generally, because the lateral states of the SUAV (can be measured directly, the aforegoing \( H\infty \) standard control problem can be simplified as the following state feedback controller design. Consider the following state space model [11]:

\[
\begin{align*}
\dot{x} &= A x + B_1 y + B_2 u \\
y &= C_1 x + D_{11} u,
\end{align*}
\]

where \( x(t) \in \mathbb{R}^n \) is the state vector, \( y(t) \in \mathbb{R}^r \) is the controlled output, \( u(t) \in \mathbb{R}^p \) is the control input [12], \( v(t) \in \mathbb{R}^q \) is the external disturbance. Because all states in the SUAV lateral open-loop control system can be observed and measured, a \( H\infty \) controller \( u = C x \) is designed to make the closed-loop system [13]:

\[
\begin{align*}
\dot{x} &= (A + B_2 C) x + B_1 y \\
y &= (C_1 + D_{12} C) x,
\end{align*}
\]

asymptotically stable for any given scalar \( \gamma > 0 \) and satisfies \( \|T_{sv}(s)\|_\infty < \gamma \).

Theorem 1 gives the design method of state feedback \( H\infty \) controller [14-15].

Theorem 1: for the closed-loop control system described in (7), any scalar \( \gamma > 0 \) is given, if and only if there are symmetric positive definite matrix \( X \) and matrix \( S \) satisfying [16]

\[
\begin{bmatrix}
AX + B_2 S + (AX + B_2 S)^T & B_1 \\
B_1^T & -I & D_{11}^T \gamma^2 I
\end{bmatrix} < 0
\]

\[
\begin{bmatrix}
C_1 X + D_{12} S \\
D_{11} \gamma^2 I
\end{bmatrix}
\]



\[
\begin{bmatrix}
B_1 \\
- I
\end{bmatrix}
\]

\[
\begin{bmatrix}
D_{11}^T \gamma^2 I
\end{bmatrix}
\]

\[
\begin{bmatrix}
C_1 X + D_{12} S \\
D_{11} \gamma^2 I
\end{bmatrix}
\]

\[
\begin{bmatrix}
B_1 \\
- I
\end{bmatrix}
\]

\[
\begin{bmatrix}
D_{11}^T \gamma^2 I
\end{bmatrix}
\]

\[
\begin{bmatrix}
C_1 X + D_{12} S \\
D_{11} \gamma^2 I
\end{bmatrix}
\]
Then the system has a state feedback $H_\infty$ control law $u = S^* (X^*)^{-1} x$, in which $X^*$, $S^*$, $Z^*$ are one set of feasible solutions to satisfy the above three matrix inequalities [12]. According to Theorem 1, as long as a group of feasible solutions are obtained, the state feedback $H_\infty$ control law of the linear time invariant system can be schemed [12]. In other words, a $H_\infty$ optimization problem can be constructed and solved with convex constraints [14].

4. Design of SUAV $H_\infty$ Optimized Controller

4.1. $H_\infty$ Controller Design

As shown in Eq. 3 and Eq. 4, the state space model of the UAV lateral motion is a MIMO system with four state variables, two inputs, and four outputs, which is constructed as shown Eq. 8. Herein,

$$A = \begin{bmatrix} -0.136 & 11.52 & -9.8 & 0 \\ -0.0551 & -0.2059 & 0 & 0.98 \\ 0 & 0 & 0 & 1 \\ 0.026 & -0.66 & 0 & -0.5534 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, B_2 = \begin{bmatrix} 0.34 & 0.001 \\ -0.00167 & -0.63 \\ 0 & 0 \\ 0.000304 & -0.00985 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$D_{12} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

![Figure 2](image2.png)

Figure 2. Response curve of closed-loop system to non-zero initial state (no noise)

![Figure 3](image3.png)

Figure 3. Response curve of closed-loop system under disturbance input ($w \sim (0, 0.01)$ Gaussian white noise is added to simulate a disturbance)
Take $\gamma = 0.02$. According to Theorem 1, a set of feasible solutions $X^*$, $S^*$ is obtained to satisfy the three linear matrix inequalities. Then a $H_\infty$ control law of the UAV's lateral motion state feedback can be designed as $u = S^* (X^*)^{-1} x$ where $C$ is calculated as a $2 \times 4$-order matrix.

$$
C = S^* (X^*)^{-1} = \begin{bmatrix}
-155.9232 & 12.6470 & -98.7778 & -155.3256 \\
28.7413 & 80.1207 & -185.1872 & -223.5787
\end{bmatrix}.
$$

4.2. Time Response Analysis of Control System

Set one nonzero initial state of the system as $X(0) = [5 \text{m/s}, -6^\circ, 2^\circ, -3^\circ / \text{s}]$ to simulate the designed $H_\infty$ closed-loop control system based LMI. The non-zero initial state response curve of the SUAV lateral motion control system is shown in Fig. 2. According to Fig. 2 and Fig. 3, the adjustment time of four outputs: sideslip angle, roll angle, roll angle speed, and yaw angle speed are 0.05 s, 0.9 s, 3.49 s, and 3.13 s, respectively. It can be seen that the lateral $H_\infty$ state feedback control system of UAV designed by the LMI method has a fast response speed and a satisfactory disturbance suppression effect in engineering.

5. Summary

In this paper, the $H_\infty$ control algorithm, based on LMI, is applied to the design of SUAV longitudinal flight attitude controller. The $H_\infty$ robust controller of the UAV's longitudinal flight control system is designed by using LMI sub-optimal method. The $H_\infty$ controller is applied to the disturbance-free model and disturbed model respectively for simulation verification. The simulation results show that the designed controller can not only make the closed-loop systems have good control performance, but also keep robust stability when the system has some external disturbances.

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