Nucleosynthesis in slowly evolving Cosmologies

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Abstract

We explore aspects of Cosmological Nucleosynthesis in an FRW universe in which the scale factor evolves linearly with time: \( a(t) \sim t \). A high Lepton number density during the period when significant nucleosynthesis takes place would lead to a dominant screening of the Coulomb potential of colliding nuclei. This would lead to a significant enhancement of nucleosynthesis rates. We demonstrate how adequate amount of \(^4\text{He}\) and a collateral metallicity, close to the lowest metallicity observed in metal poor Pop II stars and clouds, can be produced with such an evolution.

1 Introduction:

In an earlier article [1] we reviewed Standard Big-Bang Nucleosynthesis [SBBN] and reported a study of nucleosynthesis in a universe in which the scale factor evolves linearly with time independent of the equation of state of matter. A strictly linear evolution of the cosmological scale factor is surprisingly an excellent fit to a host of cosmological observations. Any model that can support such a coasting presents itself as a falsifiable model as far as classical cosmological tests are concerned as it exhibits distinguishable and verifiable features. The motivation for such an endeavor has been discussed

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at length in a series of earlier articles [2, 3]. Linear Coasting turns out to be broadly concordant with Classical Cosmology tests. What makes linear coasting particularly appealing is a straightforward adaptation of standard nucleosynthesis codes to demonstrate that primordial nucleosynthesis is not an impediment for a linear coasting cosmology [4, 5]. A linear evolution of the scale factor radically effects nucleosynthesis in the early universe. With the present age of the universe some $15 \times 10^9$ years and the effective CMB temperature $2.73$ K, the universe turns out to be some 45 years old at $10^9$ K. With the universe expanding at such low rates, weak interactions remain in equilibrium for temperature as low as $\approx 10^8$ K. The neutron to proton ratio is determined by the n-p mass difference and is approximately $n/p \sim \exp[-15/T_9]$ as long as weak interactions are in equilibrium. This falls to abysmally low values at temperatures below $10^9$ K. Significant nucleosynthesis leading to helium formation commences only near temperatures below $\approx 5 \times 10^9$K. The low n/p ratio is not an impediment to adequate helium production. This is because once nucleosynthesis commences, inverse beta decay, which does not freeze at these temperatures, replenishes neutrons by converting protons into neutrons and pumping them into the nucleosynthesis network. For baryon entropy ratio $\eta \approx 7.8 \times 10^{-9}$, the standard nucleosynthesis network can be modified for linear coasting and gives $\approx 23.9\%$ Helium. The temperatures are high enough to cause helium to burn. Even in SBBN the temperatures are high enough for helium to burn. However, the universe expands very rapidly in SBBN. In comparison, the linear evolution gives enough time for successive burning of helium, carbon and oxygen. The metallicity yield is some $10^8$ times the metallicity produced in the early universe in the SBBN. The metallicity is expected to get distributed amongst nuclei with maximum binding energies per nucleon. These are nuclei with atomic masses between 50 and 60. This metallicity is close to that seen in lowest metallicity objects. The metallicity concommitantly produced with $\approx 23.9\%$ Helium is roughly $\approx 10^{-5}$ solar.

The only problem that one has to contend with is the significantly low residual deuterium in such an evolution. The desired amount would have to be produced by the spallation processes much later in the history of the universe. In [1] we demonstrated how observed abundances of light elements besides $^4He$ could be produced by spallation reactions in incipient Pop II stellar environments without a collateral overproduction of $^7Li$. It was demonstrated that the absence or deficiency of heavy nuclei in a target cloud and deficiency of alpha particles in the incident beam would clearly
suppress lithium production in typical spallation reactions. Such conditions are expected of the environments of incipient Pop II stars and easily circumvent the “no-go” concern of Epstein et al. related overproduction of $^7\text{Li}$ associated with collateral production of deuterium upto observed levels.

In SBB, hardly any metallicity is produced in the very early universe. Metal enrichment is supposed to be facilitated by a generation of Pop III stars. Pop III star formation from a pristine material is not well understood till date, in spite of a lot of effort that has been expanded to that effect recently. It is believed that with metallicity below a critical transition metallicity ($Z_{cr} \approx 10^{-4}Z_\odot$), masses of Pop III stars would be biased towards very high masses. Metal content higher than $Z_{cr}$ facilitates cooling and a formation of lower mass Pop II stars. In SBB, the route to Deuterium by spallation discussed in this article would have to follow a low metal contamination by a generation of Pop III stars. In SBB, large-scale production and recycling of metals through exploding early generation Pop III stars leads to verifiable observational constraints. Such stars would be visible as 27 - 29 magnitude stars appearing any time in every square arc-minute of the sky. Serious doubts have been expressed on the existence and detection of such signals. The linear coasting cosmology would do away with the requirement of such Pop III stars altogether.

Unfortunately, the baryon entropy ratio required for the right amount of helium in linear coasting cosmology corresponds to $\Omega_b \equiv \rho_b/\rho_c = 8\pi G \rho_b/3H_0^2 \approx 0.69$. This over-closes dynamic mass estimates by a little over a factor of two. This article demonstrates an easy and physically acceptable manner that one could get the observed amount of Helium with a baryon density significantly lower than $\Omega_b \approx 0.69$.

2 Coulomb Screening

In stellar nucleosynthesis, the effect of a screening charged cloud on the rate of thermonuclaeer reactions was investigated by Salpeter and others. The electrostatic potential of a bare nucleus induces a spherically symmetric polarization of the surrounding electrons and nucleii. Under typical conditions for the interior of main sequence stars, this screening leads to an increase of thermonuclear reaction rates up to a factor of two. The effect of screening is that the reaction rates between interacting nucleii is enhanced by a factor $exp(-U_0/kT)$, with $U_0$ the (negative) potential of the gas cloud.
at the origin (the location of the interacting nuclei). Although [9, 10, 11] give expressions for $U_0$ under varying conditions, it would be fair to say that there is no rigorous theory of screening to date. These papers are at best schematic and establish the effect in principle.

Under conditions expected to prevail in a linear coasting cosmology, the result of a typical run is reported in first two columns of Table 1. As seen in figures 3 & 4, significant nucleosynthesis takes place at temperatures between approximately $8 \times 10^9 K$ and $10^9 K$.

Figure 1 shows the variation of number density of electrons, positrons, baryons and protons with temperature. Figure 2 shows the variation of the ratio of electron to proton and positron to proton number densities with temperature. The electron number density drops from $\approx 10^{10}$ times the proton number density at $10^9 K$ to 1 as temperature falls below the electron positron annihilation temperature of $\approx 10^9 K$. The electron - positron cloud would therefore provide for a very strong screening of the Coulomb potential of the interacting nuclei.

As one is at a loss for a precise theory of screening, we were satisfied by playing around with the nucleosynthesis code and discovered that, among the reactions:

$\begin{align*}
3He + n &\rightarrow p + ^3H, \\
3H + p &\rightarrow \gamma + ^4He, \\
3H + D &\rightarrow n + ^4He, \\
3He + \alpha &\rightarrow \gamma + ^7Be, \\
7Be + n &\rightarrow \alpha + ^4He, \\
7Be + D &\rightarrow p + \alpha + ^4He, \\
3He + ^3He &\rightarrow 2p + ^4He, \\
2H + p &\rightarrow \gamma + ^3He,
\end{align*}$

which take part in the production of helium, the most effective reaction is

$$2H + p \rightarrow ^3He + \gamma.$$  (1)

An enhancement of rate of this reaction enhances the production of helium. The last two columns of Table 1 describes the result of the runs for Linear Coasting Cosmology. An enhancement of reaction rate of $D[p, \gamma]^3He$ by a factor of 6.7 gives the right amount of helium ($^4He$) for $\Omega_b = 0.28$ (i.e. $\eta = 3.159 \times 10^{-9}$). Figure 5 shows the variation of abundances of helium with the change in enhancement factor for the same value of $\eta$. In figures 3 & 4, the curve labeled “LC I” describes the profile of helium abundance with temperature in this case. In [9], for weak screening, the expression for the enhancement factor is given by $exp\left(-\frac{U_0}{kT}\right)$ with

$$-\frac{U_0}{kT} = \frac{Z_1Z_2e^2}{R(kT)}$$

where $R$ is the radius of charge cloud. An enhancement of this by a factor of 6.7 for $D[p, \gamma]^3He$, i.e. for $Z_1 = Z_2 = 1$, implies an enhancement $(6.7)^{Z_1Z_2}$.
for higher charged nuclei. This would lead to a significant creation of metallicity. However, the condition of weak screening no longer holds and we have therefore been content to report the plausibility of such a possibility.

We conclude that in principal it is possible to produce all light elements and metallicity seen in lowest metallicity objects in a linear coasting cosmology. With evolving electron, positron and proton number densities in the early universe, the screening enhancement factor would in general evolve with temperature. The purpose of this article is to make out a case for a better theory. Definitive results would have to therefore await a rigorous theory of screening that would play a vital role in such slowly evolving models.

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Table 1: Evolution of helium with temperature in Linear Coasting Cosmology: (1) without screening; $\eta = 7.80 \times 10^{-9}$ and (2) with screening; $\eta = 3.159 \times 10^{-9}$ and rate of $D[p, \gamma]^3He$ enhanced by factor 6.7.

| Temp($T_{\odot}$) | $^4He$       | Temp($T_{\odot}$) | $^4He$       |
|-------------------|--------------|-------------------|--------------|
| $1.000E + 02$     | $4.000E - 25$| $1.000E + 02$     | $4.000E - 25$|
| $1.000E + 02$     | $4.000E - 25$| $1.000E + 02$     | $4.000E - 25$|
| $1.000E + 02$     | $4.000E - 25$| $1.000E + 02$     | $4.000E - 25$|
| $1.000E + 02$     | $4.000E - 25$| $1.000E + 02$     | $4.000E - 25$|
| $9.999E + 01$     | $4.000E - 25$| $9.999E + 01$     | $4.000E - 25$|
| $9.876E + 01$     | $4.000E - 25$| $9.869E + 01$     | $4.000E - 25$|
| $6.590E + 01$     | $4.000E - 25$| $6.179E + 01$     | $4.000E - 25$|
| $3.148E + 01$     | $4.000E - 25$| $2.952E + 01$     | $4.000E - 25$|
| $1.504E + 01$     | $1.694E - 24$| $1.411E + 01$     | $1.327E - 24$|
| $7.645E + 00$     | $1.182E - 19$| $7.223E + 00$     | $1.439E - 19$|
| $5.710E + 00$     | $9.857E - 15$| $5.397E + 00$     | $1.188E - 14$|
| $4.766E + 00$     | $1.375E - 10$| $4.529E + 00$     | $1.562E - 10$|
| $3.783E + 00$     | $2.156E - 07$| $3.633E + 00$     | $2.436E - 08$|
| $3.693E + 00$     | $2.982E - 07$| $3.387E + 00$     | $6.010E - 08$|
| $3.066E + 00$     | $3.445E - 06$| $2.749E + 00$     | $1.202E - 06$|
| $2.383E + 00$     | $2.441E - 04$| $2.355E + 00$     | $2.095E - 05$|
| $1.790E + 00$     | $2.813E - 02$| $2.135E + 00$     | $1.798E - 04$|
| $1.495E + 00$     | $6.859E - 02$| $1.708E + 00$     | $1.980E - 02$|
| $1.279E + 00$     | $1.116E - 01$| $1.398E + 00$     | $7.419E - 02$|
| $1.091E + 00$     | $1.700E - 01$| $1.225E + 00$     | $1.195E - 01$|
| $9.273E - 01$     | $2.249E - 01$| $1.061E + 00$     | $1.806E - 01$|
| $7.622E - 01$     | $2.372E - 01$| $8.851E - 01$     | $2.324E - 01$|
| $5.835E - 01$     | $2.376E - 01$| $7.237E - 01$     | $2.384E - 01$|
| $4.940E - 01$     | $2.376E - 01$| $5.359E - 01$     | $2.386E - 01$|
| $3.637E - 01$     | $2.378E - 01$| $4.621E - 01$     | $2.386E - 01$|
| $2.360E - 01$     | $2.378E - 01$| $3.446E - 01$     | $2.386E - 01$|
| $1.560E - 01$     | $2.378E - 01$| $2.275E - 01$     | $2.386E - 01$|
| $9.085E - 02$     | $2.378E - 01$| $1.480E - 01$     | $2.386E - 01$|
| $4.694E - 02$     | $2.378E - 01$| $7.696E - 02$     | $2.386E - 01$|
| $2.313E - 02$     | $2.378E - 01$| $3.885E - 02$     | $2.386E - 01$|
| $1.105E - 02$     | $2.378E - 01$| $1.874E - 02$     | $2.386E - 01$|
Figure 1: Evolution of Number density of electrons, positrons, baryons and protons. The vertical lines mark the range of most effective nucleosynthesis in SBBN and LCN (Linear Coasting Nucleosynthesis).

Figure 2: Ratio of Number density of $e^-$ to $p$ and $e^+$ to $p$. The electron number density drops from $10^{10}$ times proton number density at $T \sim 10^{11} K$ to equal it around $T \sim 2.2 \times 10^8 K$. 

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Figure 3: Evaluation of helium abundance with temperature is shown for three cases: (1) Linear Coasting Cosmology with $\eta = 7.80 \times 10^{-9}$ (LC), (2) Linear coasting model with $\eta = 3.159 \times 10^{-9}$ with the reaction rate of $D[p, \gamma]^{3}He$ enhanced by a factor of 6.7 (LC_I) (3) Standard Big Bang Nucleosynthesis (SBBN).

Figure 4: The figure shows the temperature range in which significant helium formation occurs.
Figure 5: Evolution of helium with $T_9$ in Coasting Cosmology for $\eta = 3.159 \times 10^{-9}$ with reaction rate for $D[p, \gamma]^3He$ enhancing by varying factors ‘f’.