Resilient Control under Quantization and Denial-of-Service: Co-designing a Deadbeat Controller and Transmission Protocol

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Abstract—This paper is concerned with the problem of stabilizing continuous-time linear time-invariant systems subject to quantization and Denial-of-Service (DoS) attacks. In this context, two DoS-induced challenges emerge with the design of resilient encoding schemes, namely, the coupling between encoding strategies of different signals, and the synchronization between the encoder and decoder. To address these challenges, a novel structure that is equipped with a deadbeat controller as well as a delicate transmission protocol for the input and output channels, co-designed leveraging the controllability index, is put forward. When both input and output channels are subject to DoS attacks and quantization, the proposed structure is shown able to decouple the encoding schemes for input, output, and estimated output signals. This property is further corroborated by designing encoding schemes as well as conditions that ensure exponential stabilization of a closed-loop system. On the other hand, when only the output channel is subject to network phenomenon, the proposed structure can achieve exponential stabilization without acknowledgment (ACK) signals, in contrast to existing ACK-based results. Finally, a numerical example is given to demonstrate the practical merits of the proposed approach as well as the theory.

Index terms— Denial-of-Service attacks, quantization, deadbeat control, acknowledgment-free protocol.

I. INTRODUCTION

Driven by recent advances in computing, communication, and networking technologies, modern engineering systems (e.g., [1]–[3]) have gradually shifted their computing and control workload to the cloud, and even edge with data transmitted over wired or wireless networks. Despite their flexibility, such network-based control systems (a.k.a., networked control systems) are known vulnerable to cyber threats [4], [5]. In fact, existing works have shown that malicious attacks can severely disrupt the control performance even render the system unstable [6]. Examples of such failures in widely used safety- and security-critical control systems nowadays could put our lives and even national infrastructure at risk [7].

Several types of cyberattacks have been studied, including replay attacks [8], [9], false-data injection attacks [10], [11], and Denial-of-Service (DoS) attacks [12]–[14]. Relative to the others, DoS attacks can cause jamming in communication channels with little knowledge of system dynamics. They are easy to launch and have received considerable attention [15]. For instance, the work [16] developed a general DoS framework, under which closed-loop system stability can be preserved via state-feedback control, provided certain DoS attack frequency and duration conditions are met. This result has been extended in several directions, e.g., via output-feedback control in [17], as well as considering multiple output channels in [18].

All the aforementioned works assumed that the communication channels have an infinite data-rate. Clearly for real-world engineering systems, this condition is difficult to be met. Systems with digital communication channels offer a basic paradigm. The problem of limited bandwidth have been studied by accounting for the effect of quantization. There is a great deal of research indicating that even without attacks, quantization can compromise system performance [19], which is often addressed by designing suitable encoding schemes and providing enough quantization levels. To name a few, for stabilization of systems with quantized measurements, [20] first introduced the so-called “zooming-in” and “zooming-out” method. Following this work, a number of stabilization encoding schemes have been designed for systems with quantized output feedback in [21], [22], and switched systems in [23]–[26]. Recently, a few works have considered these two factors (i.e., quantization and DoS attacks) simultaneously; see [27]–[31]. The trade-off between system resilience against DoS attacks and data-rate was analyzed in [28]. The minimum data-rate for stabilizing a centralized system and a multi-agent system were derived in [29] and [30], respectively. Capitalizing on the zooming-in and -out method, the work [31] designed a resilient output encoding scheme for systems whose output channel is subject to DoS attacks and limited data-rate.

The goal of this paper is to stabilize systems with both input (controller-to-plant) and output (plant-to-controller) channels subject to DoS attacks and limited bandwidth. To this aim, the quantizer encoding schemes should be carefully designed. In the absence of DoS attacks, the work [22] developed encoding schemes for signals transmitted through both input and output channels. However, their schemes cannot be applied here, due to the coupling between encoding strategies for different signals.
in the presence of DoS attacks. To overcome this challenge, we put forth a delicate structure, including a deadbeat controller and a transmission protocol. Our protocol requires signals transmitted through the input channel at a higher rate than those through the output channel. Precisely, their transmission rate ratio is exactly the controllability index of the system. Its efficacy is corroborated by the possibility to decouple design of different encoding schemes as, well as, establishing closed-loop stability conditions. We further apply this structure to stabilize systems with only output channel has network imperfections. In this scenario, it is proved that the proposed structure can secure the synchronization between encoder and decoder even without acknowledgments (ACKs), which are required by existing works, e.g., [28], [31].

In a nutshell, the main contributions of the present work are summarized as follows.

c1) To cope with the coupling and synchronization issues, a structure consisting of a deadbeat controller and a transmission protocol for input and output channels, co-designed in terms of the controllability index, is advocated.

c2) Under this structure, the input, output, and estimated data transmissions over the output channel occur periodically referred to as output channel and input channel. Specifically, different channels over a shared network, which are accordingly transmitted their quantized values to the plant periodically with interval \( \Delta \). At the plant side, the quantized control inputs are first decoded, then pass through a zero-order hold (ZOH) before entering the plant. To maintain the synchronization between input and output transmissions, we choose \( \delta = \Delta/b \) for some \( b \in \mathbb{N} \). For future reference, let

\[
   x_{q,k} := x(q\Delta + k\delta), \quad y_{q,k} := y(q\Delta + k\delta)
\]

for every \( q \in \mathbb{Z}_{\geq 0} \), and \( k = 0, \ldots, b-1 \), and

\[
   A_d := e^{A\delta}, \quad B_d := \int_0^\delta e^{A_s} B ds. \tag{2}
\]

Moreover, we use \( x_q \) to denote \( x_{q,0} \) for simplicity.

We make the following assumptions on system (1).

Assumption 1 (Controllability and observability). The pair \((A,B)\) is controllable, and the pair \((C,A)\) is observable.

Assumption 2 (Initial state bound). An upper bound on the initial state \( |x_0| \) is known.

Remark 1. Thanks to As. 1 it has been shown in [32] that if \( \delta \) is non-pathological, then \( (A_d,B_d) \) in (2) is controllable. Let \( \eta \) denote its controllability index, which can be computed by evaluating \( \text{rank}[B_d,\ldots,A_d^\eta B_d] = n_x \). Similarly, \( (C,A_d^\eta) \) is observable. An upper bound on the initial state in As. 2 can be derived via the zooming-out method; see [31] Sec. 4.

B. Denial-of-Service attack

In Fig. 1, since both input and output signals are transmitted periodically, we adopt the discrete-time DoS attack model in [31]. Under this model, attacks are launched only at output transmission instants, and each lasts for an output transmission period \( \Delta \). This model is general enough since it only poses requirements on the frequency and duration of DoS attacks. Here, DoS frequency is the number of DoS off/on switches over a fixed time interval, while DoS duration represents the total number of attacks.

Assumption 3 (DoS frequency). There exist constants \( \kappa_f \in \mathbb{R}_{\geq 0} \) and \( \nu_f \in \mathbb{R}_{\geq 2} \) such that DoS frequency satisfies

\[
   \Phi_f(q) \leq \kappa_f + \frac{q}{\nu_f} \tag{3}
\]

over time interval \( [0,q\Delta) \), where \( q \in \mathbb{Z}_{\geq 0} \).

Assumption 4 (DoS duration). There exist constants \( \kappa_d \in \mathbb{R}_{\geq 0} \) and \( \nu_d \in \mathbb{Z}_{\geq 1} \) such that DoS duration satisfies

\[
   \Phi_d(q) \leq \kappa_d + \frac{q}{\nu_d} \tag{4}
\]
over time interval \([0, q\Delta]\), where \(q \in \mathbb{Z}_{\geq 0}\).

Remark 2. Given its generality, this attack model has been widely used in e.g., [16]–[18], [28], [30], [31]. As pointed out in [53], \(\nu_f/\Delta\) in As. [3] can be regarded as the average dwell-time between two consecutive DoS attacks off/on switches. On the other hand, As. [4] indicates that, the average duration of DoS attacks does not exceed a proportion \(1/\nu_d\) of the time interval. Constants \(\kappa_f\) and \(\kappa_d\) are also known as chatter bounds. Conditions \(\kappa_d \geq 1\) and \(\nu_d \geq 2\) suggest that DoS attacks are not strong enough to prevent all packets from being transmitted, thus rendering it possible for the system to be stabilized by suitable control strategies.

III. NETWORKED PHENOMENA AT BOTH INPUT AND OUTPUT CHANNELS

This section aims to design resilient encoding schemes for stabilization of system (1) via a remote observer-based digital controller over communication channels subject to limited bandwidth and DoS attacks; see Fig. [1]. To this end, there are three signals that need to be quantized, i.e., the estimated output by observer \(\hat{y}_q\), the control input \(u_{q,k}\), and the plant output \(y_q\), with their quantized values denote by \(Q_1(\hat{y}_q)\), \(Q_2(u_{q,k})\), and \(Q_3(y_q)\), respectively. In addition, since the input and output channels share a communication network, we assume for simplicity that, once there is a DoS attack, neither the input nor the output signals will be received, and both of them are set to the default zero. In this manner, the decoder and encoder at both input and output sides can infer whether there is an attack. Further, their quantization ranges and centers are identical at every transmission instant. As a result, they can be synchronized even with an ACK-free protocol.

A. Controller architecture

To stabilize system (1), we put forth a two-stage observer-based controller by considering whether there is an attack or not. Specifically, in the absence of DoS attacks, both \(Q_3(\hat{y}_q)\) and \(Q_1(\hat{y}_{q-1,n})\) are available at the observer side, so we construct the following controller

\[
\begin{align*}
\hat{x}_{q,k+1} &= A_d \hat{x}_{q,k} + B_d u_{q,k}, \quad k \leq \eta - 1 \\
\hat{x}_q &= \hat{x}_{q-1,k} + M \left[ Q_3(y_q) - Q_1(\hat{y}_{q-1,n}) \right], \quad k = \eta \\
\hat{y}_{q,k} &= C \hat{x}_{q,k} \\
u_{q,k} &= K \hat{x}_{q,k}
\end{align*}
\]

where the initial condition \(\hat{x}_0\) is given by \(\hat{x}_0 = 0\), and \(\delta\) is chosen such that

\[
\delta = \frac{\Delta}{\eta}.
\]

Matrix \(M \in \mathbb{R}^{n_x \times n_y}\) can be regarded as an observer gain such that \(R := A_d^2 (I - MC)\) is schur stable, which always exists since \((C, A_d^2)\) is observable. Moreover, \((A_d, B_d)\) is controllable, thus a controller gain matrix \(K \in \mathbb{R}^n_{n_x \times n_u}\) can be designed such that

\[
R^n = (A_d + B_d K)^n = 0.
\]

Remark 3. Matrix \(K\) obeying (7) is also known as a class of deadbeat controller gain, since it assigns all the eigenvalues of \(A_d + B_d K\) to the origin. Solution of this eigenstructure assignment problem is non-unique, and can be obtained through several approaches, e.g., [34].

On the other hand, when there is a DoS attack, none of \(Q_1(\hat{y}_q)\), \(Q_2(u_{q,k})\), or \(Q_3(y_q)\) can be received, thus we simply employ an open-loop controller as follows

\[
\begin{align*}
\hat{x}_{q,k+1} &= A_d \hat{x}_{q,k} \\
\hat{y}_{q,k} &= C \hat{x}_{q,k} \\
u_{q,k} &= 0
\end{align*}
\]

with the initial estimated state \(\hat{x}_0 = 0\).

In addition, to apply the discrete-time signal \(Q_2(u_{q,k})\) to the continuous-time system (1a), a ZOH is used, and the control input is given by

\[
u(t) = Q_2(u_{q,k}), \quad q \Delta + k \delta \leq t < q \Delta + (k+1)\delta \]

where \(k = 0, \cdots, \eta - 1\).

B. Quantizer

We first design quantizers at the input channel. According to (5a) and (5b), \(u_{q,k}\) is needed for feedback control, whereas \(\hat{y}_{q,k}\), resetting the estimated state, is required at each successful transmission instant. Therefore, the controller sends \(u_{q,k}\) and \(\hat{y}_{q,n}\) to the quantizers periodically at a different rate. In more precise terms, periods for the former and the latter are \(\delta\) and \(\eta \delta\), respectively. Let \(E_{1,q} \geq 0\) and \(E_{2,q,k} \geq 0\) satisfy

\[
|\hat{y}_{q-1,n}| \leq E_{1,q}, \quad |u_{q,k}| \leq E_{2,q,k}.
\]

Suppose there are \(N_1 (N_2)\) levels for quantization of \(\hat{y}_{q,n}\) \((u_{q,k})\). Partition the hypercubes at the encoders

\[
\{y \in \mathbb{R}^{n_y} : |\hat{y}_{q-1,n}| \leq E_{1,q}\}, \quad \{u \in \mathbb{R}^{n_u} : |u_{q,k}| \leq E_{2,q,k}\}
\]

into \(N_1^{n_y}\) and \(N_2^{n_u}\) equal-sized boxes, respectively. In addition, each box is represented by a value in \(\{1, \cdots, N_1^{n_y}\}\), or \(\{1, \cdots, N_2^{n_u}\}\) following a bijection mapping. Indices that denote the partitioned boxes containing \(\hat{y}_{q,n}\) and \(u_{q,k}\) are then sent to the decoders. If \(\hat{y}_{q,n}\) and \(u_{q,k}\) are on the boundary of several boxes, then anyone of them can be chosen. At the decoders side, \(Q_1(\hat{y}_{q,n})\) and \(Q_2(u_{q,k})\) are recovered from the indices. This implies that the encoder and its corresponding decoder should share the same quantization ranges and centers. Since DoS attacks block both input and output signals from transmitting, encoders and decoders at both sides of input and output channels are naturally synchronized. The quantization errors of the aforementioned encoding schemes obey

\[
|\hat{y}_{q-1,n} - Q_1(\hat{y}_{q-1,n})| \leq \frac{E_{1,q}}{N_1}, \quad |u_{q,k} - Q_2(u_{q,k})| \leq \frac{E_{2,q,k}}{N_2}.
\]

Since \(\hat{x}_0 = 0\), we deduce that \(\hat{y}_0 = u_0 = 0\). Therefore, the initial bounds \(E_{1,0}\) and \(E_{2,0,0}\) can be set by

\[
E_{1,0} = 0, \quad E_{2,0,0} = 0.
\]

Moreover, as for the output \(y_{q}\), choose \(E_{3,q} \geq 0\) such that

\[
|y_{q} - Q_3(\hat{y}_{q-1,n})| \leq E_{3,q}.
\]
Let $N_3$ be the quantization level of $y_q$. The hypercube

$$\{y \in \mathbb{R}^n : |y_q - Q_1(\tilde{y}_{q-1,\eta})| \leq E_{3,q}\}.$$

is partitioned into $N_3^n$ equal-sized boxes with the center $Q_1(\tilde{y}_{q-1,\eta})$. Then, following the same procedure as the above two quantizers, $Q_3(y_q)$ is transmitted to the controller every $\Delta$ time. The quantization error satisfies

$$|y_q - Q_3(y_q)| \leq \frac{E_{3,q}}{N_3}.$$

Define error of the system $e_{q,k} := x_{q,k} - \tilde{x}_{q,k}$. Combining $e_0 = 0$ with As. 2, we deduce that the initial error obeys $|e_0| = |x_0|$. Thus it suffices to set $E_{3,0} := \|C\|_1|x_0|.$

C. Stability analysis

In this subsection, we start by presenting encoding schemes $\{E_{p,q,k}\} (p = 1, 2, 3),$ followed by formal stability conditions. Design $\{E_{1,q} : q \in \mathbb{Z}_{\geq 1}\}$ such that

$$E_{1,q} = E_{1,0}, \quad \forall q \in \mathbb{Z}_{\geq 1} \text{ (14)}$$

and let $\{E_{2,q,k} : q \in \mathbb{Z}_{\geq 1}, k = 0, \cdots, \eta - 1\}$ be updated by

$$E_{2,q,k} = \frac{N_3 - 1}{N_3} \|K \hat{R}^k M\| E_{3,q}, \quad \text{if} \ q\Delta = s_r \text{ (15)}$$

Moreover, the sequence $\{E_{3,q} : q \in \mathbb{Z}_{\geq 1}\}$ is given by

$$E_{3,q+1} = \begin{cases} \hat{\theta}_a E_{3,q}, & q\Delta \neq s_r, q\Delta = s_r \text{ (16)} \\ \hat{\theta}_a E_{3,q}, & q\Delta = s_r, q\Delta = s_r \end{cases}$$

where

$$\begin{align*}
\hat{\theta}_a &:= \|A_d^\ell\| \\
\hat{\theta}_0 &:= \alpha_0 \rho + \|C\|_1 a_1 \frac{N_3 - 1}{N_3} + \|C\|_1 a_2 \frac{N_3 - 1}{N_3} \\
\hat{\theta}_m &:= \rho + \|C\|_1 a_1 \frac{N_3 - 1}{N_3} + \|C\|_1 a_2 \frac{N_3 - 1}{N_3}
\end{align*}$$

with positive constants $a_0, a_1, a_2,$ and $0 < \rho < 1$ validating the following for all $\ell \geq 1$

$$\begin{align*}
\|R\ell\| &\leq \alpha_0 \rho^\ell, \\
\|R\ell A_d^\ell M\| &\leq \alpha_1 \rho^\ell \\
\sum_{i=0}^{\eta-1} \|R\ell A_d^{\ell-i-1} B_d\| K \hat{R}^i M\| &\leq \alpha_2 \rho^\ell.
\end{align*}$$

(17)

Since $R$ is schur stable, there always exist such constants.

Next, we show that our designed schemes above are resilient to DoS attacks, which is one of our main results too.

**Theorem 1.** Consider system (17) with the observer-based controller in (5) and (6), with $K$ obeying (7) and $M$ chosen such that $R$ is schur stable. Let As. 1, 4 hold. If i) the input and output transmission periods adhere to (6), ii) the number of quantization levels $N_1$ is odd, iii) DoS attacks satisfy

$$\frac{1}{\nu_d} \leq \frac{\log (1/\hat{\theta}_a)}{\log (\hat{\theta}_a/\hat{\theta}_m)} - \frac{\log (\hat{\theta}_0/\hat{\theta}_m)}{\log (\hat{\theta}_a/\hat{\theta}_m)} \nu_f \text{ (19)}$$

then the system is exponentially stable under the encoding scheme with error bounds $\{E_{p,q,k} : q \in \mathbb{Z}_{\geq 1}, k = 0, \cdots, \eta - 1\}$ for $p = 1, 2, 3$ constructed by the update rule in (14)-(16).

We begin proving Thm. 1 by giving a lemma demonstrating that the update rules in (14)-(16) satisfy (9) and (12).

**Lemma 1.** Consider system (17) with the controller in (5) and (6), where $K$ obeys (7). Let As. 1, 4 hold. If $\{E_{p,q,k} : q \in \mathbb{Z}_{\geq 1}, k = 0, \cdots, \eta - 1\}$ for $p = 1, 2, 3$ obey (14)-(16), then (9) and (12) hold true for all $q \in \mathbb{Z}_{\geq 1}$.

**Proof.** Encoding schemes for systems with quantized inputs and outputs in the absence of DoS attacks have been discussed in [22]. However, their methods cannot be directly applied due to the DoS-induced coupling between these schemes. This challenge is addressed through our carefully designed controller structure in (5)-(7). According to (5) and (7),

$$\tilde{y}_{q-1,\eta} = C \tilde{x}_{q-1,\eta} = C R^k \tilde{x}_{q-1} = 0$$

holds true irrespective of DoS attacks, which implies $E_{1,q} \geq |\tilde{y}_{q-1,\eta}| = E_{1,0}$ for all $q \in \mathbb{Z}_{\geq 1}$, so $E_{1,q}$ remains unchanged. This result further indicates that $Q_1(\tilde{y}_{q-1,\eta}) = 0$. Hence, it follows from (13) that the quantization center of $Q_3(y_q)$ is at the origin. On the other hand, if no DoS attacks occur within $[q_1 \Delta, (q_1 + 1)\Delta)$, then

$$\tilde{x}_{q_1,k} = (A_d + B_dK)^k \tilde{x}_{q_1} = \hat{R}^k (\tilde{x}_{q_1-1,\eta} + M[Q_3(y_{q_1}) - Q_1(\tilde{y}_{q_1-1,\eta})]) \text{ (20)}$$

hence $u_{q_1,k}$ can be expressed by $Q_3(y_{q_1}) - Q_1(\tilde{y}_{q_1-1,\eta})$. In addition, since

$$|Q_3(y_q) - Q_1(\tilde{y}_{q-1,\eta})| \leq \frac{N_3 - 1}{N_3} E_{3,q} \text{ (21)}$$

it follows that, in the absence of DoS attacks, $u_{q_1,k}$ satisfies

$$|u_{q_1,k}| \leq \frac{N_3 - 1}{N_3} \|K \hat{R}^k M\| E_{3,q} =: E_{2,q,k}$$

for all $q \geq 1, k = 0, \cdots, \eta - 1$. When DoS attacks occur, the plant cannot receive inputs from controller. In other words, $E_{2,q,k}$ only depends on the latest $E_{3,q}$, thus $E_{2,q,k}$ can remain unchanged during DoS attacks.

Following the definitions of $E_{1,q}$ and $E_{2,q,k}$, we are able to design sequence $\{E_{3,q} : q \in \mathbb{Z}_{\geq 1}\}$. First, in the absence of DoS attacks, the error just before each transmission instant, $e_{q-1,\eta} = x_q - \tilde{x}_{q-1,\eta}$, satisfies

$$e_{q-1,\eta} = A_{d}^\ell (I - MC)e_{q-1} - A_{d}^\ell M[Q_3(y_q) - y_q] + \sum_{i=0}^{\eta-1} A_{d}^{\ell-i-1} B_d Q_2(u_{q-1,i}) - u_{q-1,i} - A_{d}^{\ell}[\tilde{y}_{q-1,\eta} - Q_1(\tilde{y}_{q-1,\eta})]$$

which implies that $e_{q-1,\eta}$ generally relies on $\hat{y}_{q-1,\eta}, u_{q,k}$, and itself, thus introducing coupling in $E_{3,q}$ design. Here, this issue
is addressed by (7). To see this, recalling (21), \( \hat{y}_{q-1,j} = 0 \), and \( Q_1(\hat{y}_{q-1,j}) = 0 \), we have that

\[ e_{q+\ell-1,j} = R^\ell e_{q-1} + \sum_{j=0}^{\ell-1} R^j A^j_d M(Q_3(y_{q-j}) - y_{j-1}) \]

\[ + \sum_{j=0}^{\ell-1} R^j A^j_d \bar{B}_d \left[ Q_2(u_{q+\ell-1,i}) - u_{q+\ell-1,i} \right]. \tag{22} \]

Define \( E_{3,q} \) as follows

\[ E_{3,q} := a_0 \rho^\ell E_{3,q} + \sum_{i=0}^{\ell-1} \left( \frac{(N_3 - 1)a_2}{N_2 N_3} + \frac{|C|a_1}{N_3} \right) \rho^i E_{3,q-i}. \]

Hence, combining (21) with (22) yields

\[ |y_{q+1} - Q_1(\hat{y}_{q,j})| \leq |y_{q+1} - \hat{y}_{q,j}| + |\hat{y}_{q,j} - Q_1(\hat{y}_{q,j})| \leq |C||x_{q+1} - \hat{x}_{q,j}| \leq \theta_{n\alpha} E_{3,q} \leq E_{3,q+1}. \tag{23} \]

Moreover, since both the input and output channels are blocked in the presence of DoS attacks, and \( \hat{y}_{q,j} = 0 \), due to the property of \( \bar{R} \), it follows that

\[ |y_{q+1} - Q_1(\hat{y}_{q,j})| \leq \hat{\theta}_{q} E_{3,q} \leq E_{3,q+1} \]

and we complete the proof. \( \square \)

Next, we establish upper bounds on the sequences \( \{E_{p,q,k} : q \in \mathbb{Z}_{\geq 1}, k = 0, \ldots, \eta - 1\} \) \((p = 1, 2, 3)\), whose existence will imply the boundness of state trajectory.

**Lemma 2.** Consider system (1) with controller in (5) and (9), where \( K \) satisfies (7) and \( M \) is chosen such that \( \bar{R} \) is schur stable. Let the assumptions and conditions in Thm. 1 hold. If further \( \{E_{p,q,k} : q \in \mathbb{Z}_{\geq 1}, k = 0, \ldots, \eta - 1\} \) \((p = 1, 2, 3)\) obey (14) and (16), there exist \( \Omega_1 \geq 1 \) and \( \gamma \in (0, 1) \) such that

\[ E_{3,q} \leq \Omega_1 \gamma^{|x_0|}, \quad \forall k \in \mathbb{Z}_{\geq 1} \tag{24} \]

and \( E_{1,q} = E_{1,0} \) and \( E_{2,q,k} \leq \Omega_2 \gamma^{|x_0|} \).

**Proof.** Using (14), \( E_{3,q} \) remains unchanged within the considered interval, therefore, \( E_{1,q} = E_{1,0} \) holds for all \( q \in \mathbb{Z}_{\geq 1} \). The proof for \( E_{3,q} \leq \Omega_1 \gamma^{|x_0|}, \forall q \in \mathbb{Z}_{\geq 1} \) follows directly from that of Lemma 3.9 in [31], where \( \Omega_1 := \frac{\theta_{n\alpha}}{\theta_{n\alpha}} \left( \frac{\delta_{\sigma \alpha} - \bar{B}_d}{\theta_{n\alpha}} \right) \left( \frac{\delta_{\sigma \alpha} - \bar{B}_d}{\theta_{n\alpha}} \right) \). Moreover, applying (21), \( E_{2,q,k} \leq \Omega_2 \gamma^{|x_0|} \) can be verified with \( \Omega_2 := \frac{\delta_{\sigma \alpha}}{\theta_{n\alpha}} \left( \frac{\delta_{\sigma \alpha} - \bar{B}_d}{\theta_{n\alpha}} \right) \left( \frac{\delta_{\sigma \alpha} - \bar{B}_d}{\theta_{n\alpha}} \right) \).

We are now in a position to prove Thm. 1.

**Proof of Theorem 2** We first establish the bound of the state \( x \) at the transmission instants, i.e., \(|x(t)|\), then derive its bound at the sampling instants, i.e., \(|x(t\Delta + k\delta)|\). Finally, combining these two bounds to yield bound \(|x(t)|\) in the considered horizon.

First, according to (1), (3), and (9), one has

\[ x_{q,j} = R^q x_{q,k} + \sum_{i=0}^{\eta-1} R^i B_d K(x_{q-j-i-1} - \hat{x}_{q,j-i-1}) \]

\[ + \sum_{i=0}^{\eta-1} R^i B_d (Q_2(u_{q,j-i-1}) - K\hat{x}_{q,j-i-1}) \tag{25} \]

and

\[ |x_{q,j}| \leq \|R^q\||x_q| + \sum_{i=0}^{\eta-1} \|R^i B_d K\| |(x_{q,j-i-1} - \hat{x}_{q,j-i-1})| \]

\[ + \sum_{i=0}^{\eta-1} \|R^i B_d\| |Q_2(u_{q,j-i-1}) - K\hat{x}_{q,j-i-1}|. \tag{26} \]

Since (23), it follows that

\[ |x_q - \hat{x}_{q-1}| \leq \frac{E_{3,q}}{|C|}. \tag{27} \]

Noticing that \( \bar{R}^q = 0 \), substituting (15) and (27) into (26),

\[ |x_{q,k}| \leq \sum_{i=0}^{\eta-1} \frac{N_3 - 1}{N_2 N_3} \|R^i B_d\| |K R^{q-i-1} M| E_{3,q} \]

\[ + \sum_{i=0}^{\eta-1} \frac{\|R^i B_d K A_d^{q-i-1}\|}{|C|} E_{3,q} \]

\[ \leq \Omega_2 \gamma^{|x_0|} \] \tag{28}

where \( \Omega_2 := \sum_{i=0}^{\eta-1} \left( \frac{N_3 - 1}{N_2 N_3} \|R^i B_d\| |K R^{q-i-1} M| + \frac{\|R^i B_d K A_d^{q-i-1}\|}{|C|} \right) \), and the last inequality holds due to (24).

Since \( x_{q,k+1} = A_d x_{q,k} + B_d Q_2(u_{q,k}) \), we have that

\[ |x_{q,k}| \leq \|R^q\||x_q| + \sum_{i=0}^{\eta-1} \|R^i B_d Q_2\| E_{3,q} \]

\[ + \frac{\|R^i B_d K A_d^{q-i-1}\|}{|C|} E_{3,q} \]

\[ \leq \Omega_2 \max_{\ell} \sum_{i=0}^{\ell} \left( \frac{N_3 - 1}{N_2 N_3} \|R^i B_d\| |K R^{q-i-1} M| + \frac{\|R^i B_d K A_d^{q-i-1}\|}{|C|} \right) \]

\[ \leq \Omega_2 \gamma^{|x_0|} + \Omega_3 \gamma^{|x_0|} \leq \Omega_2 \gamma^{|x_0|}. \tag{29} \]

Finally, abiding by (1), \( x(t) \) satisfies

\[ x(t) = e^{A(t-q\Delta - k\delta)} + \int_{q\Delta + k\delta}^t e^{\sigma s} B Q_2(u_{q,k}) ds \]

for all \( t \in [q\Delta + k\delta, q\Delta + (k + 1)\delta] \). Combining Lem. 2 and (29), it follows that

\[ |x(t)| \leq \left( \|A_d\| |\Omega_2| + \frac{N_2 + 1}{N_2} \|B_d\| |\Omega_2| + \frac{\|R^i B_d K A_d^{q-i-1}\|}{|C|} \right) \]

where \( \sigma := \frac{1}{N_2} \log \frac{1}{N_2} \) and \( \Omega_2 := \|A_d\| |\Omega_2| + \frac{N_2 + 1}{N_2} \|B_d\| |\Omega_2| \). This implies exponential convergence of the state. \( \square \)

**Remark 4.** Leveraging the same technique as in Rmk. 3 one can also design \( M \) to nullify \( \bar{R}^q = 0 \), where \( \mu \) is the observability index of \( (C, A_d^q) \). A direct benefit from using the deadbeat observer gain is that the encoding schemes can be simplified, since \( R^q = 0 \) holds for all \( \ell \geq \mu \). However, the results in (22) indicate that despite exhibiting faster convergence and fewer quantization levels, due to the deadbeat property of matrices \( \bar{R} \) and \( R \), the quantization step size \( E_{p,q}/N_p \) is large, which leads to large quantization errors. Moreover, it was shown in (31) that if the quantization step size \( E_{p,q}/N_p \) grows slower during
DoS attacks, then the overshoot from an attack is smaller, and the level of system robustness is stronger. Therefore, instead of a deadbeat observer gain, a general one that can make \( R \) schur stable is employed in the present work.

IV. NETWORK PHENOMENA AT OUTPUT CHANNEL

In this section, we consider stabilizing linear systems over a communication network, where only the output channel is subject to DoS attacks, i.e., the input channel is assumed ideal; see Fig. 2. The transmission policy in the previous section is considered here; that is, the digital controller receives quantized output \( Q(y_q) \) from the plant with period \( \Delta \) and generates control input \( u_{q,k} \) with period \( \delta \). Notice that the decoder can recover the correct quantized value from the index sent by the encoder only if they share the same quantization ranges and centers. It is thus necessary to ensure that the encoder and the decoder are synchronized before designing encoding schemes.

A direct way to maintain synchronization is through using an ACK-based protocol; see Fig. 3, which has been adopted in previous studies, such as, [28], [31]. Nevertheless, in real-time applications, protocols without ACKs, e.g., UDP, are often preferred since the resulting implementation is simpler as well as saves the additional energy required for sending ACKs [35]. Hence, in the following, we first show that method for stabilizing systems with ACK-based protocols can no longer be used under ACK-free protocols. Then, we demonstrate that our proposed methods can inform the encoder of DoS attacks from zero inputs, thus the decoder and the encoder can be synchronized even without ACKs.

A. Controller under an acknowledgment-based protocol

Recall that \( \{s_r\}_{r \in \mathbb{N}_0} \) collects the sequence of successful transmission instants. Let \( \delta = \Delta \), and choose \( K \) such that \( \dot{R} = A_d + B_dK \) is schur stable.

We consider an observer-based controller described by
\[
\dot{x}_{q+1} = A_d\hat{x}_q + B_du_q + L(Q(y_q) - \hat{y}_q), \quad q\Delta = s_r \tag{30a}
\]
\[
\dot{\hat{x}}_{q+1} = A_d\hat{x}_q + B_du_q, \quad q\Delta \neq s_r \tag{30b}
\]
\[
\hat{y}_q = C\hat{x}_q \tag{30c}
\]
\[
u_q = K\hat{x}_q \tag{30d}
\]
where \( \hat{x}_q \in \mathbb{R}^{n_x}, \hat{y}_q \in \mathbb{R}^{n_y}, \) and \( Q(y_q) \in \mathbb{R}^{n_y} \) are the estimated state, the estimated output, and the quantized output, respectively. The initial condition is set to be \( \hat{x}_0 = 0 \). Since the input channel is ideal, it follows that
\[
u(t) = u_q, \quad q\Delta \leq t < (q + 1)\Delta, \quad q \in \mathbb{Z}_{\geq 0}.
\]
To design an encoding scheme such that the output \( y_q \) can be quantized without saturation, an error bound between the estimated output and the actual output, i.e., \( |e_q| := |x_q - \hat{x}_q| \leq E_q \), should be derived. Based on (1b) and (30b), it can be deduced that
\[
|y_q - \hat{y}_q| = |C(x_q - \hat{x}_q)| = |Ce_q| \leq \|C\|E_q. \tag{31}
\]
Let \( N \) denote the number of quantization levels of \( y_q \). Similar to the previous section, we partition the hypercube \( \{y \in \mathbb{R}^{n_y} : |y_q - \hat{y}_q| \leq \|C\|E_q\} \) into \( \mathbb{N}^{n_y} \) equal-sized boxes. The quantization error obeys \( |Q(y_q) - y_q| \leq \frac{\|C\|}{N}E_q \). According to Ass. [2], the initial value \( E_0 \) is given by
\[
|e_0| = |x_0|_{\Delta} = E_0. \tag{32}
\]
Sequence \( \{E_q, q \in \mathbb{Z}_{\geq 1}\} \) will be specified later. Notice that the hypercube center is \( \hat{y}_q \), which is generated by the predictor-based observer in (30). Therefore, this predictor should also be equipped at the encoder side. Under ACK-based protocol, the decoder sends ACKs to the encoder without delay at successful transmission instants; and when the encoder does not receive the ACKs, it infers that there is a DoS attack. In this manner, synchronization between these two predictors is ensured, which consequently implies that the quantization ranges and the centers at the encoder are identical to that of the decoder.

Before giving stability condition for ACK-based protocol case, we present an output encoding scheme. Let
\[
E_{q+1} := \begin{cases}
\theta_a E_q, & q\Delta \neq s_r \\
\theta_0 E_q, & (q - 1)\Delta \neq s_r, q\Delta = s_r \\
\theta_{na} E_q, & (q - 1)\Delta = s_r, q\Delta = s_r
\end{cases} \tag{33}
\]
with
\[
\theta_a := \|A_d\| \tag{34a}
\]
\[
\theta_0 := H_0 \rho + \frac{H_1 \|C\|}{N} \tag{34b}
\]
\[
\theta_{na} := \rho + \frac{H_1 \|C\|}{N} \tag{34c}
\]
where constants \( H_0, H_1, \) and \( 0 < \rho < 1 \) satisfy
\[
\|(A_d - LC)^\ell\| \leq H_0 \rho^\ell, \quad \|(A_d - LC)^\ell L\| \leq H_1 \rho^\ell.
\]
Theorem 2. Consider system \( \mathcal{L} \) with controller \( \mathcal{M} \), where \( M \) and \( K \) are chosen such that \( A_d - LC \) and \( A_d + B_d K \) are strictly stable. Under Ass. \([2]\) if \( i \) the quantization levels
\[
N > \frac{H_1||C||}{1 - \rho}
\]
and, ii) DoS attacks satisfy
\[
\frac{1}{\nu_d} \leq \frac{\log(1/\theta_{na})}{\log(\theta_a/\theta_{na})} - \frac{\log(\theta_0/\theta_{na})}{\log(\theta_a/\theta_{na})} \frac{1}{\nu_f}
\]
then the system is exponentially stable under the encoding scheme with error bounds \( \{E_q : q \in \mathbb{Z}_{\geq 1} \} \) constructed by the update rule \([30]\).

The proof is similar to that of \([31]\) Thm. 3.4 and is thus omitted here due to space limitations.

B. Controller under an acknowledgment-free protocol

In this subsection, we show that the aforementioned controller and encoding scheme cannot stabilize the system when the ACK-based protocol is replaced by an ACK-free protocol. This is because synchronization between the encoder and decoder is no longer guaranteed. To see this, consider controller \([30]\) with the encoding scheme in \([33]\) employing an ACK-free protocol. In this setting, predictors at the encoder and decoder sides may become asynchronized, since no matter whether DoS attacks happen or not, the decoder does not send ACKs to the encoder. When a DoS attack occurs, the predictor at the controller side switches to \([30b]\), whereas the predictor at the encoder side sticks to \([30a]\). Moreover, the update rule of sequence \( E_q \) at the decoder switches to \([34a]\), while adhering to \([34b]\) \([34e]\) at the encoder. As a result, their quantization ranges and centers may deviate, and the correct output value cannot be recovered by the decoder. We prove that even if one DoS attack occurs (i.e., decoder and encoder are asynchronized for only one transmission period), the state may diverge eventually.

To distinguish between predictors at the encoder and decoder, let \( \tilde{x}_q, \tilde{y}_q, \) and \( \tilde{Q}(y_q) \) denote the estimated state, estimated output, and quantized output at the controller side, and \( \hat{x}_q, \hat{y}_q, \) and \( \hat{Q}(y_q) \) denote their counterparts at the encoder side. In addition, let \( u_q \) stand for the input sent by the controller, and \( \hat{u}_q \) the estimated input generated by the predictor at the encoder side. Predictor at the controller side is described by
\[
\begin{align*}
\hat{x}_{q+1} &= A_d\hat{x}_q + B_d\hat{u}_q + L(\hat{Q}(y_q) - \hat{y}_q), \quad q\Delta = s_r \quad (37a) \\
\hat{y}_q &= C\hat{x}_q \\
\hat{u}_q &= K\hat{x}_q 
\end{align*}
\]
and predictor at the encoder side is described by
\[
\begin{align*}
\tilde{x}_{q+1} &= A_d\tilde{x}_q + B_d\tilde{u}_q + L(\tilde{Q}(y_q) - \tilde{y}_q) \quad (38a) \\
\tilde{y}_q &= C\tilde{x}_q \\
\tilde{u}_q &= K\tilde{x}_q 
\end{align*}
\]
where \( q \in \mathbb{Z}_{\geq 0} \). Similarly, let \( E_{d,q} \) and \( E_{e,q} \) denote the error bound at the decoder, and the encoder side, respectively
\[
\begin{align*}
E_{d,q+1} := \begin{cases} 
\theta_q E_{d,q}, & \text{if } q\Delta \neq s_r \\
(\theta_q E_{d,q} (q - 1)\Delta \neq s_r, & \text{if } q\Delta = s_r \\
\theta_{na} E_{d,q}, & \text{if } (q - 1)\Delta = s_r, & \text{if } q\Delta = s_r \n\end{cases}
\end{align*}
\]
\[
\begin{align*}
E_{e,q+1} := \begin{cases} 
\theta_q E_{e,q}, & q\Delta = 0 \\
\theta_{na} E_{e,q}, & q\Delta > 0 \n\end{cases}
\end{align*}
\]
where \( \theta_0, \theta_a, \) and \( \theta_{na} \) are defined in \([34]\). Accordingly, the errors at the encoder and decoder sides are \( e_{d,q} := x_q - \tilde{x}_q \) and \( e_{d,q} := x_q - \tilde{x}_q \). Moreover, the quantized outputs in \([37b]\) and \([38a]\) are
\[
\begin{align*}
\tilde{q}(y_q) &= \tilde{y}_q + Q_q ||C|| E_{d,q} N \\
Q(\tilde{y}_q) &= \tilde{y}_q + Q_q ||C|| E_{e,q} N
\end{align*}
\]
where \( Q_i^e \) denotes the quantization index transmitted from the encoder to the decoder.

Notice that the quantizer operates normally without saturation only if \( E_{e,q} \geq ||e_{d,q}|| = |x_q - \tilde{x}_q| \) and \( E_{e,q} \geq ||e_{d,q}|| = |x_q - \tilde{x}_q| \) hold for all \( q \in \mathbb{Z}_{\geq 1} \). If the quantizer saturates, the error between the actual output and the quantized output may become large, which consequently renders the system unstable. In the following, we assume that the quantizer is not saturated; that is, \( E_{e,q} \geq ||e_{d,q}|| \) and \( E_{d,q} \geq ||e_{d,q}|| \) for all \( q \in \mathbb{Z}_{\geq 1} \), and reach a contradiction. Since \( E_{e,q} + 1 = \theta_{na} E_{e,q} q > 0, \) and \( \theta_{na} < 1 \), sequence \( |x_q - \tilde{x}_q| \) is decreasing. Let \( \Pi_L := A_d - LC \) and \( \Pi_K := A_d + B_d K \). Combining \([39]\) and \([41]\) yields
\[
|\tilde{x}_{q+1} - \hat{x}_{q+1} | = |\Pi_L (x_q - \tilde{x}_q) - L(\tilde{Q}(y_q) - y_{q,a})| \\
\leq E_{e,q+1} = \Delta E_{e,q+1},
\]
Likewise,
\[
|\tilde{x}_{q+2} - \hat{x}_{q+2} | = |\Pi_L (x_q - \tilde{x}_q) - L(\tilde{Q}(y_q) - y_{q,a+1}) - BK L Q_{q,a} ||C|| E_{e,q+1} N \\
\leq E_{e,q+2} + \frac{1}{\theta_{na}} ||B KL Q_{q,a} ||C|| E_{e,q+1} N \\
\leq E_{e,q+2} + \frac{1}{\theta_{na}} ||B KL Q_{q,a} ||C|| E_{e,q+1} N
\]
will use a default zero. Then, it follows from (42a)-(42b) that
\[ |x_{q,a+\ell} - \hat{x}_{q,a+\ell}| \leq \hat{E}_{c,q,a+\ell} + \frac{1}{\theta_{na}} \| B_d K \Pi_{\ell-1} L Q_{\ell q} \| C \| E_{c,q,a} \| + \frac{1}{\theta_{na}} \| B_d K \Pi_{\ell-2} L Q_{\ell q} \| C \| (\theta_a - \theta_{na}) E_{c,q,a} \| + \frac{1}{\theta_{na}} \| B_d K \Pi_{\ell-1} L Q_{\ell q} \| C \| C R^{-1} \| \| \theta_{na} \| E_{c,q,a} \| + (\theta_a \theta_a - \theta_{na}^2) E_{c,q,a} \| \] Thus its quantization ranges can be updated following the same scheme with the decoder.

We have secured synchronization between the encoder and decoder. Now, what is left behind is the system stability analysis. Recall that \( A_d(M(I - M C)) \) is schur stable, there exist constants \( G_0, G_1, \) and \( 0 < \rho < 1 \) such that
\[ \| R^\ell \| \leq G_0 \rho^\ell, \quad \| R^\ell A_d^\ell M \| \leq G_1 \rho^\ell. \] (43)
Define constants \( \tilde{\theta}_a := |A_d^\ell|, \tilde{\theta}_0 := G_0 \rho + \frac{G_1 \| C \|}{N}, \tilde{\theta}_na := \rho + \frac{G_1 \| C \|}{N} \) and the error bound \( \{ E_q : q \in \mathbb{Z}_{\geq 1} \} \) is updated by
\[ E_{q+1} := \begin{cases} \tilde{\theta}_a E_q, & q + 1 \Delta \neq s_r, q\Delta = s_r. \\ \tilde{\theta}_0 E_q, & (q - 1) \Delta \neq s_r, q\Delta = s_r. \\ \tilde{\theta}_{na} E_q, & (q - 1) \Delta = s_r, q\Delta = s_r. \end{cases} \] (44)
The following result is an extension of Thm. 2 under an ACK-free protocol, whose proof follows from that of Thm. 2.

**Theorem 3.** Consider system (7) equipped with controller in (32), where \( M \) and \( K \) are chosen such that \( R \) is schur stable and (7) is met. Let Assumption 2 hold. If i) the output and input transmission periods satisfy (6), ii) the quantization levels \( N \) is even, and obey
\[ N > \frac{G_1 \| C \|}{1 - \rho} \] (45)
and, iii) DoS attacks satisfy
\[ \frac{1}{\nu_d} \leq \frac{\log (1/\tilde{\theta}_{na})}{\log (\tilde{\theta}_a/\tilde{\theta}_{na})} \frac{1}{\nu_f} \] (46) then the system is exponentially stable under the encoding scheme with error bound \( \{ E_q : q \in \mathbb{Z}_{\geq 1} \} \) constructed by (44).

V. NUMERICAL EXAMPLE
A linearized model of the unstable batch reactor in [31] is given by \( \dot{x}(t) = Ax(t) + Bu(t) \) and \( y = Cx(t) \), where
\[ A := \begin{bmatrix} 1.38 & -0.2077 & 6.715 & -5.676 \\ -0.5814 & -4.29 & 0 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & -1.343 & -2.104 \end{bmatrix} \]

![Fig. 4. Maximum norm of state x and its estimate \( \hat{x} \) with controller (4)].
This system \((A, B, C)\) is observable and controllable with \(\eta = \mu = 2\). Let the output transmission period \(\Delta = 0.2\), so \(\delta = \Delta/\eta = 0.1\). Choosing matrix \(K\), such that (7) is met, i.e.,

\[
K := \begin{bmatrix} 1.0106 & -1.5661 & 0.0385 & -4.0366 \\ 8.1074 & -0.0347 & 4.3337 & -3.6241 \end{bmatrix}.
\]

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K := \begin{bmatrix} 1.0106 & -1.5661 & 0.0385 & -4.0366 \\ 8.1074 & -0.0347 & 4.3337 & -3.6241 \end{bmatrix}.
\]

Calculating the gain of the steady-state Kalman filter

\[
M := \begin{bmatrix} 0.5534 & -0.0249 \\ -0.0287 & 0.396 \\ 0.1489 & 0.0892 \\ 0.0810 & 0.0931 \end{bmatrix}.
\]

We first present the time responses when both input and output channels suffer from the network phenomena. Applying Thm. 1 when both the quantization levels \(N_2\) and \(N_3\) go to infinity, the duration bound \(1/\nu_d\) and the frequency bound \(1/\nu_f\) of DoS attacks approach to the line \(\frac{1}{\nu_d} \approx -0.5544 \frac{1}{\nu_f} + 0.2707\). According to [19], if \(\frac{1}{\nu_d} < -2.0380 \frac{1}{\nu_f} + 0.2269\), then the closed-loop system with encoding schemes [13]–[16] is stabilized. Over a simulation horizon of 160s (800 time-step), DoS attacks (the gray shades) are generated randomly with \(\Phi_d = 47\) and \(\Phi_f = 44\). Setting \(\kappa_d = 3, \nu_d = 18, \kappa_f = 2, \nu_f = 19\), condition (19) holds, i.e., \(1/\nu_d = 0.056 < 0.119\). Figs. 4 and 5 illustrate the time response in this situation. Since the condition in Thm. 1 is satisfied, the maximum norm of the state converges, and the bound \(E_3, q\) exponentially decreases. Fig. 5 depicts that \(E_3, q\) shares the same trend with \(|y_q - Q_1(y_q)|\), and Fig. 6 demonstrates the evolution of the quantization step size \(E_2, q, k/N_2\), which jumps up and down within an output transmission period, and decreases in general. Difference between the trend of \(E_3, q/N_3\) and \(E_2, q, k/N_2\) lies in the property of \(\|R\|\) and \(\|\bar{R}\|\). Fig. 7 compares the quantization step size \(E_3, q/N_3\) of a general observer gain (blue line), such that \(R\) is schur stable, and the deadbeat observer gain (dot marked green line), namely \(R^e = 0\). This panel illustrates that although \(E_3, q\) responds faster under deadbeat observer, the large quantization step size results in large overshoot of the state; see Fig. 8 which confirms Rmk. 4.

Next, consider network phenomena only at output channel. From (45), the quantization levels satisfies \(N > 6.957\), also since \(N\) is even, we set \(N = 100\). Over a simulation horizon of 60s (300 time-step), generating DoS attacks randomly with \(\Phi_d = 27\) and \(\Phi_f = 25\). Setting \(\kappa_d = 1, \nu_d = 11, \kappa_f = 1, \nu_f = 11\), so condition (46) is met with \(1/\nu_d = 0.01 < 0.198\), and convergence of the state is presented in Figs. 9 and 10. Further, Fig. 11 shows that when a DoS attack happens, the control input is set to zero immediately, which verifies the effectiveness of our method.
IV. CONCLUSIONS

This paper considered the problem of stabilizing networked control systems in the presence of DoS attacks and limited data rates. To overcome the network-induced challenges, a structure consisting of a deadbeat controller and a transmission protocol which are carefully co-designed based on the system controllability index, was proposed to address the network-induced challenges. Specifically, when both input and output channels are subject to the network phenomena, the proposed structure was shown able to guarantee synchronization between the encoder and decoder under an ACK-free protocol. Finally, a numerical example was presented to verify the effectiveness of our approach as well as the correctness of our theory. Future developments will focus on generalizing the results to more general systems and controllers under ACK-free protocols.

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