spl(2,1) dynamical supersymmetry and suppression of ferromagnetism in flat band double-exchange models

KARLO PENC\(^1\)(∗) and ROBERT LACAZE\(^1\)\(^2\)

\(^1\) Service de Physique Théorique, CEA-Saclay, 91191 Gif-sur-Yvette Cedex, France

\(^2\) ASCI, Bat. 506, Université Paris Sud, 91405 Orsay Cedex, France

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Abstract. – The low energy spectrum of the ferromagnetic Kondo lattice model on a \(N\)-site complete graph extended with on-site repulsion is obtained from the underlying spl(2,1) algebra properties in the strong coupling limit. The ferromagnetic ground state is realized for 1 and \(N+1\) electrons only. We identify the large density of states to be responsible for the suppression of the ferromagnetic state and argue that a similar situation is encountered in the Kagomé, pyrochlore, and other lattices with flat bands in their one-particle density of states.

It is believed that some aspects of the electronic properties of the strongly correlated transition metal oxides, like manganites\(^3\), can be revealed by considering the Kondo-lattice Hamiltonian \(\mathcal{H}_{KL} = \mathcal{T} + \mathcal{H}_{\text{int}}\), with

\[
\mathcal{T} = -\sum_{i,j,\alpha} t_{ij} c_{i\alpha}^\dagger c_{j\alpha}, \quad t_{ij} > 0, \tag{1}
\]

\[
\mathcal{H}_{\text{int}} = -\frac{J_H}{2} \sum_{j,\alpha,\beta} c_{j\alpha}^\dagger S_{cj} \sigma_{\alpha\beta} c_{j\beta} + U \sum_{j} n_{j\uparrow} n_{j\downarrow}, \tag{2}
\]

using standard notations. The kinetic part \(\mathcal{T}\) describes the hopping of conduction electrons on a lattice (\(\alpha = \uparrow, \downarrow\) is the spin index). The interaction part \(\mathcal{H}_{\text{int}}\) includes the intra-atomic ferromagnetic exchange between the conduction electrons and localized spin \(S_c\) core electrons\(^3\) with \(\sigma_{\alpha\beta}/2\) and \(S_{cj}\) as their respective spin operators (\(\sigma\) denotes a vector of Pauli matrices), and the on-site Coulomb repulsion \(U > 0\) between the electrons. While here we are interested in \(J_H > 0\) Hund’s coupling, the Kondo-lattice Hamiltonian has been extensively studied for \(J_H < 0\), as describing heavy fermion systems\(^3\).

Because the Hund’s coupling is typically larger than the hopping, the energetically unfavored low spin states are neglected (that is when electron and core spin are antiparallel), and

\(\ast\) on leave from Res. Inst. for Solid State Physics, Budapest, Hungary.
we get the quantum double exchange \[^4\, ^5\]. The next usually used approximation neglects the quantum spin fluctuations of the spinor core spin described by classical variables (spherical angles \(\theta\) and \(\phi\)). These two approximations lead to the double-exchange model \[^4\, ^5\, ^6\, ^7\] with Hamiltonian \(H_{DE} = -\sum_{ij} t_{ij} f_i^\dagger f_j\), which describes charges as noninteracting spinless fermions moving in a disordered background of classical spins with effective hopping amplitudes

\[
t_{ij} = t_{ij} \left[ \cos \frac{\theta_j}{2} \cos \frac{\theta_i}{2} + \sin \frac{\theta_j}{2} \sin \frac{\theta_i}{2} e^{i(\phi_i - \phi_j)} \right].
\]

The charges can freely propagate provided the core spins are aligned and therefore ferromagnetism is favored. The main effect of finite \(J_H\) is to introduce antiferromagnetic exchange between the core spins, which will hinder the free propagation of the charges, resulting in a competition between ferromagnetic and antiferromagnetic ordering. To check the scenario presented above, different numerical methods are applied and it has been found that on the cubic lattice the ferromagnetism is realized for a wide range of electron concentration\[^8\].

While the scenario described above is widely accepted, in this paper we would like to point out that the structure of the underlying lattice is important and we can expect suppression of the ferromagnetic phase when the noninteracting one-particle spectrum has nondispersing states forming the so called flat bands, like in Kagomé or pyrochlore lattice. This might appear surprising as, according to the Stoner criterion, the ferromagnetism goes hand in hand with large density of states. As it will turn out at the end, the reason behind this phenomenon is rather simple and will be demonstrated as follows: We first derive the low energy effective Hamiltonian in the strong coupling limit. Then the underlying spl(2,1) dynamical supersymmetry allows us to solve the effective model on the \(N\)-site complete graph referred as \(G(N)\) and defined by a infinitely long ranged hopping \(t_{ij} = t(1 - \delta_{ij})\) (on a lattice with \(N\) sites, each site has \(N - 1\) neighbors). The algebraic approach turns out to be a very useful tool, and in particular allow us to show that all the wave functions can be obtained from the noninteracting ones by an extended Gutzwiller projection. The breakdown of the standard scenario is already present in our toy model: the ground state is singlet apart from the case \(S_c = 1\) or \(N + 1\) electrons in the system, and the reason lies in the large density of states in the one–particle spectrum of \(G(N)\). Finally, we will present arguments that the same mechanism will destabilize the ferromagnetic phase in the lattices with flat bands.

To derive the effective Hamiltonian we start at \(t = 0\), where the different lattice sites decouple. An empty site has energy 0; one electron can form with the core spin either the high \((S_c + 1/2)\) or low \((S_c - 1/2)\) spin state, with energies \(-J_H S_c/2\) and \(J_H (S_c + 1)/2\), respectively; finally two electrons on a site results in a state with energy \(U\) and spin \(S_c\). This can be summarized by representing the high and low spin states using auxiliary fermions \(f\) and \(d\), respectively, and the spins by Schwinger bosons \(b\), so that \(S_j = \sum_{\alpha\beta} b_{j\alpha}^\dagger (\sigma_{\alpha\beta}/2) b_{j\beta}\) and

\[
\begin{align*}
  c_j^\dagger &= \frac{b_j^\dagger f_j^\dagger + b_j d_j^\dagger}{\sqrt{2S_c + 1}}, \\
  c_j &= \frac{b_j^\dagger f_j - b_j d_j^\dagger}{\sqrt{2S_c + 1}}.
\end{align*}
\]

The anticommutation relation for electrons requires the constraint \(\sum_{\alpha} n_{j\alpha} = n_j^f + n_j^d = 2S_c\) to be satisfied at each site and the interaction part becomes diagonal:

\[
H_{int} = \frac{J_H}{2} \sum_j \left[ (S_c + 1)n_j^d - S_c n_j^f - n_j^f n_j^d \right] + U \sum_j n_j^f n_j^d.
\]

Choosing \(U = J_H/2\) we can eliminate the four fermion term \(n_j^f n_j^d\). Now, using standard techniques\[^9\], we can apply a canonical transformation to get the effective Hamiltonian which

\[
H_{\text{eff}} = -\sum_{ij} t_{ij} \left[ \cos \frac{\theta_j}{2} \cos \frac{\theta_i}{2} + \sin \frac{\theta_j}{2} \sin \frac{\theta_i}{2} e^{i(\phi_i - \phi_j)} \right].
\]
is the expansion in $t/J_H$ around the atomic limit and in the lowest energy subspace, where we keep the $f$ fermions only, it reads (see also [10])

$$
\mathcal{H}_{\text{eff}} = -\frac{J_H N_c S_c}{2} - \sum_{i,j,\alpha} \frac{t_{ij}}{2S_c+1} f_i^{\dagger} b_{ia_j} f_j - \sum_{i,j,k,l,\alpha,\beta,\mu,\nu} \frac{t_{ijkl}}{J_H(2S_c+1)^3} (\delta_{\mu\nu}\delta_{\alpha\beta} - \sigma_{\mu\alpha}\sigma_{\nu\beta}) f_i^{\dagger} b_{ia\mu} b_{kl\nu} b_{j\beta} f_j + O\left(\frac{t^3}{J_H^3}\right). \tag{6}
$$

The first order term $\propto t$ in the expansion is equivalent to the quantum double-exchange Hamiltonian [4, 5]. It competes with the next order term $\propto t^2/J_H$ which is essentially an antiferromagnetic interaction between the spins, and there is no need to include higher order terms. The formula above is equally valid for $S_c = 0$, where it describes the large-$U$ Hubbard model, implying that some of the results below apply also to the $t$-$J$ model. The procedure can be repeated for general $U$, resulting in a more complicated $t^2/J_H$ term [5, 11].

When specializing to complete graph $G(N)$ the effective Hamiltonian can be written as

$$
\mathcal{H}_{\text{eff}} = N_c \varepsilon - \frac{t}{2S_c+1} \left(1 + \frac{2tN_c}{J_H(2S_c+1)}\right) \sum_\alpha F_\alpha^{\dagger} F_\alpha + \frac{2t^2}{J_H(2S_c+1)^3} \sum_{\alpha,\beta} F_\alpha^{\dagger} \left(\hat{Y}\delta_{\alpha\beta} + S\sigma_{\alpha\beta}\right) F_\beta + O\left(\frac{t^3}{J_H^3}\right), \tag{7}
$$

where $\varepsilon = t - J_H S_c/2$, and we introduced the operators $S = \sum_j S_j$, $\hat{Y} = (S_c + 1)N - \hat{N}_c/2$, $F_\alpha^{\dagger} = \sum_j b_{ja} f_j^{\dagger}$ and $F_\alpha = \sum_j b_{ja} f_j$. The nonvanishing anticommutation relations between the fermionic operators are

$$
\left\{F_\alpha, F_\beta^{\dagger}\right\} = \hat{Y}\delta_{\alpha\beta} + S\sigma_{\alpha\beta}. \tag{8}
$$

The bosonic spin operator $S$ satisfy the usual $su(2)$ spin algebra and commutes with $\hat{Y}$. The system closes with the commutation relations between the fermionic and bosonic operators

$$
[F_\alpha, S] = \frac{1}{2} \sum_\beta \sigma_{\alpha\beta} F_\beta, \quad [F_\alpha, \hat{Y}] = -\frac{1}{2} F_\alpha, \tag{9}
$$

with their conjugate. The set of relations (8, 9) define a spl(2,1) graded algebra spanned by $S$, $\hat{Y}$, $F_\alpha^{\dagger}$ and $F_\alpha$, $F_\alpha^{\dagger}$ and $F_\alpha$, while $S$ and $\hat{Y}$ span respectively the $su(2)$ and $u(1)$ subalgebras [12].

Both the electron number and total spin are conserved, but $\mathcal{H}_{\text{eff}}$ does not commute with the $F$’s - spl(2,1) is not a symmetry of eq. (6). Nevertheless $\mathcal{H}_{\text{eff}}$ can be expressed in terms of the Casimir operators of spl(2,1), $su(2)$ and $u(1)$ – the so called dynamical supersymmetry. Once the representations $[Y, S]$ of spl(2,1) are decomposed into multiplets $(S, Y)$ of $su(2) \times u(1)$, we know [12] on each multiplet the values of the different Casimir operators of spl(2,1), $su(2)$ and $u(1)$ and similarly to the $t$-model [5, 13] the spl(2,1) representations give the effective Hamiltonian.

The generic irreducible representation (irrep) $[Y, S]$ of the spl(2,1) algebra is $8S$ dimensional and can be labeled by two linearly independent Casimir operators [12]. It contains the $(S, Y)$, $(S - 1/2, Y + 1/2)$, $(S - 1/2, Y - 1/2)$ and $(S - 1, Y)$ spin multiplets. Special cases concern the irrep $[Y = S, S]$ which contains only the $(S, Y)$ and $(S - 1/2, Y + 1/2)$ spin multiplets (dimension $4S + 1$) and the irrep $[Y, S = 1/2]$ which do not contain the $(S - 1, Y)$ multiplet. Applying operators $F_\alpha$ and $F_\alpha^{\dagger}$ we can walk between the spin multiplets within an irrep.
The \((S-1/2, Y-1/2), (S, Y), (S-1, Y), \) and \((S-1/2, Y+1/2)\) multiplets of the irrep \([Y, S]\) are eigenstates of \(\sum_a F_a^\dagger F_a\) with the eigenvalues \(2Y - 1, Y + S, Y - S, \) and 0, respectively. As the second order of \(\mathcal{H}_{\text{eff}}\) is also quadratic in \(F\) operators and can be expressed in terms of the Casimir operators, the spin multiplets \((S, Y)\) in the irrep \([Y', S']\) are eigenstates of eq. (9) with \((\text{increasing})\) energies:

\[
E_{[Y+_Y^S]N}(S, Y) = N \varepsilon - 2 \left( t + \frac{2t^2 S_{\text{max}} + 1}{J_H (2S_e + 1)^2} \right) \left( N - \frac{S_{\text{max}}}{2S_e + 1} \right) + \frac{4t^2 S(S + 1)}{J_H (2S_e + 1)^2} + O(\frac{t^3}{J_H^2}),
\]

\[
E_{[Y]N}(S, Y) = N \varepsilon - \left( t + \frac{2t^2 S_{\text{max}}}{J_H (2S_e + 1)^2} \right) \left( N - \frac{S_{\text{max}}}{2S_e + 1} \right) + O(\frac{t^3}{J_H^2}),
\]

\[
E_{[Y, S+1]N}(S, Y) = N \varepsilon - \left( t + \frac{2t^2 S_{\text{max}} + S + 1}{J_H (2S_e + 1)^2} \right) \left( N - \frac{S_{\text{max}} + S + 1}{2S_e + 1} \right) + O(\frac{t^3}{J_H^2}),
\]

\[
E_{[Y-\frac{1}{2}, S+\frac{1}{2}]N}(S, Y) = N \varepsilon,
\]

where \(S_{\text{max}} = NS_e + N_e/2\) is the maximum total spin. The multiplicity \(M_{[Y, S]}^{(N)}\) of the irrep \([Y, S]\) in an \(N\)-site system is given by the number of times this irrep is contained in \([S_e + 1/2, S_e + 1/2])\). This is determined from the branching rule\([12]\), leading to the following recursion relation:

\[
M_{[Y, S+1/2]}^{(N+1)} = \sum_{y = (0, 1/2)}^S \sum_{s = -S_e - y}^{s - |S - y - S_e|} M_{[Y + y - S_e, s + 1/2]}^{(N)} - M_{[Y - S_e, S - s + 1/2]}^{(N)},
\]

with \(M_{[Y = N(S_e + 1/2), S]}^{(N)} = \delta_{N(S_e + 1/2)}\) as boundary condition. This formula along with the energy definition in eqs. (10)-(13) allows an iterative procedure to build the energy density distribution of the model.

In fig. 1 the energies given by eqs. (10)-(13) are compared with those of the Kondo lattice computed by exact diagonalization on a small \((N = 4)\) size. At \(J_H = \infty\), the observed lowest part of the spectrum is exactly the one predicted from analysis of spl(2,1) representations. Up to \(t/J_H = 0.05\) the effective Hamiltonian energies agree very well with the exact ones. For larger \(t/J_H\) values, higher correction terms need to be introduced in order to get a quantitative agreement. As the multiplicities of the levels shown at the right of fig. 1 do not depend on \(J_H/t\) and are those of the dynamical supersymmetry, these correction can be, in principle, calculable. To obtain this solution it is essential that the effective Hamiltonian can be expressed using the operators of the spl(2,1) superalgebra which, for finite \(J_H/t\), is possible for \(U = J_H/2\) only (see also [15]).

For \(N_e = 1\) the \((S, Y)\) spin multiplet of the \([Y + 1/2, S + 1/2]\) is missing and the ground state is the highest spin state in the \([Y, S]\) irrep. For \(2 \leq N_e \leq N\) the \(t^2/J_H\) correction in eq. (10) makes the lowest energy state to be the singlet \((S, Y)\) or doublet \((S, S)\) eigenstates of \(S(S + 1)\), like in the infinite range, antiferromagnetic Heisenberg model. The particle-hole transformation can be used for \(N_e > N\), and we get that for \(N_e = N + 1\) the ground state is again the highest spin state (like in the Hubbard model\([16, 17]\). To summarize, the model is ferromagnetic for \(1\) and \(N + 1\) electrons only.

While the algebraic approach gives the spectrum, it does not tell how to get the wave functions. For the case of the \(t\)-model it was shown\([18]\) by explicit construction that some of the wave functions can be obtained by Gutzwiller projecting the free fermion ones with \(\Pi_G = \prod_j (1 - n_j n_{j+1})\). Actually, more is true: for the quantum double exchange all the large-\(J_H\)
Fig. 1. – The low energy spectrum of the ferromagnetic Kondo lattice on a 4 site complete graph $G(4)$ with $S_c = 1/2$ and $N_e = 2$ for $U = J_H/2$. The solid straight lines show the energy of the effective Hamiltonian as given by eqs. (10)-(13) to be compared to the dotted lines from exact diagonalization. The multiplicity and the quantum numbers of the irrep the state belong to is also shown. At the upper right corner of the plot we can see states with low spin $d$ fermions to appear.

wave functions can be obtained by projecting out the $d$ fermions with $\hat{P} = \prod_j (1 - n^d_j)$ from a suitable set of the wave functions of the non-interacting Hamiltonian. To this end, one has to consider the states $|\Phi\rangle$ corresponding to the $(S - 1/2, Y + 1/2)$ of the $[Y, S]$ irrep. They turn out to be exact eigenstates of the Kondo lattice Hamiltonian for any value of $J_H$ with energy $N_e \varepsilon$ (eq. (13)). These states satisfy $F_\sigma |\Phi\rangle = 0$ and thus they are the states of the non-interacting Hamiltonian which do not contain zero momentum electron and are invariant with $\hat{P}$: $C_\sigma |\Phi\rangle = 0$ and $\hat{P}|\Phi\rangle = |\Phi\rangle$ (here $C_\sigma = \sum_j c_{j\sigma}$). Starting from these $|\Phi\rangle$ and adding zero momentum electrons, we obtain eigenfunctions of the $J_H = U = 0$ model. Next, using the identity $\hat{P} C_\sigma^\dagger = F_\sigma^\dagger \hat{P}$, the projected state remains in the same irrep with wave-functions which is eigenstate of the large-$J_H$ model. In other words, the extended Gutzwiller projection (as $\hat{P} = \hat{P}_G$ for $S_c = 0$) is exact for the model on $G(N)$.

At this point, it is instructive to study the classical ($S_c \to \infty$) and $t^2/J_H = 0$ limit of the model. In the language of Schwinger bosons $b_{j\uparrow}^\dagger \approx \sqrt{S_c} \cos(\theta_j/2)$ and $b_{j\downarrow}^\dagger \approx \sqrt{S_c} \sin(\theta_j/2) e^{i\phi_j}$ [9]. This immediately leads to the hopping amplitudes (3) of the double exchange model with the one-particle Hamiltonian $H_{1p} = \varepsilon - t(|c\rangle \langle c| + |s\rangle \langle s|)$ where $|c\rangle = \sum_j \cos \frac{\theta_j}{2} f_j^\dagger |0\rangle$ and $|s\rangle = \sum_j \sin \frac{\theta_j}{2} e^{i\phi_j} f_j^\dagger |0\rangle$. If we choose the $z$-axis to point in the direction of the total core spin, the $|c\rangle$ and $|s\rangle$ are orthogonal and eigenvectors of $H_{1p}$ with energies

$$\varepsilon_c = \varepsilon - t \left( N + \frac{S}{S_c} \right), \quad \varepsilon_s = \varepsilon - t \left( N - \frac{S}{S_c} \right).$$

Furthermore there are $N - 2$ states with energy $\varepsilon$. While in the lowest energy state the energy is linearly decreasing with $S$, and the fermions can freely propagate when the core spins are parallel, for $|s\rangle$ the tendency is reversed: energy is higher for larger $S$. Filling the one-particle
levels, the spectrum for $2 \leq N_e \leq N$ electron is $-Nt + N_e \varepsilon_c + (N_c - 1) \varepsilon_c$ and $N_e \varepsilon$. These energies are equal $O(1/S_c)$ to the energies $E_{1+2}^1$, respectively, and thus the correspondence between the semiclassical and quantum spectra for this model is established. The absence of the true ferromagnetic state is due to the cancellation of the contributions linear in $S$ for $N_e \geq 2$. Again, the projection operator establishes a relationship between $J_H = 0$ and $J_H \rightarrow \infty$ states: $|c\rangle = PC^\dagger_1|0\rangle$ and $|s\rangle = PC^\dagger_1|0\rangle$, and the role of the core spins is to act like an effective magnetic field which Zeeman splits the energy of $C^\dagger_0|0\rangle$ states.

Now, once we have seen that the ferromagnetism is suppressed in the model, we can search the reason of such a phenomenon. One of the peculiarities of our model is the large degeneracy of the one-particle spectrum when all the spin are aligned. Then $S$ is maximum and we have one state at $E = \varepsilon - tN$ and $N - 1$ at $E = \varepsilon$. Turning over one of the core spins the $S/S_c$ ratio is reduced by 2, increasing the lowest energy $\varepsilon_c$ by $t$, decreasing the energy of one of the levels in the degenerate manifold with the same amount $t$ (actually $\varepsilon_s$ in eq. (14) and leaving the other $N - 2$ states of the degenerate manifold with the same energy $\varepsilon$. In other words, we can gain antiferromagnetic energy without loosing total kinetic energy. For this to happen, the existence of the macroscopically large number of nondispersing states is necessary. This feature is also present in models defined on lattices of corner shared complete graphs (known as line graphs, see e.g. [20]). Let us call $G_\alpha(M)$ the $M$-site complete graphs centered around the point $r$ and first concentrate on the family where in $D$ dimensions $G_\alpha(D + 1)$ are sharing their corners, and (anti)periodic boundary condition are assumed. In $D = 2$ it is represented by the Kagomé and in $D = 3$ by the pyrochlore lattice. In momentum space they have two dispersing bands and just above them $D - 1$ flat bands. We start to fill the flat bands above the electron density $n_C = 2/(D + 1)$, and we can expect the mechanism we outlined above to act when we flip some of the corner spins. For $D \geq 3$ we can go even further following Ref. [2]: the Hamiltonian for $t^2/J_H = 0$ can be written as $H = \mathcal{E}(N_c) + \mathcal{R}$, where the number $\mathcal{E}(N_c)$ is $-J_H S_c N_c/2 - 4t(N - N_c)(1 + S_c)/(1 + 2S_c)$ while $\mathcal{R} = t/(2S_c + 1) \sum_{\alpha} F^\dagger_\alpha F^\dagger_\alpha \mathcal{F}^\dagger_\alpha \mathcal{F}^\dagger_\alpha$ is a positive semidefinite operator. The summation is over the centers of the $G_\alpha$ constituting the lattice with corresponding fermionic operator $F^\dagger_\alpha$. Choosing an initial state $|\Psi\rangle$ such that $F^\dagger_\alpha |\Phi\rangle = 0$ for any $\alpha$, the $|\Psi\rangle = \prod_\alpha F^\dagger_\alpha |\Phi\rangle$ is an exact ground state of the model with energy $\mathcal{E}(N_c)$, as $\mathcal{R} |\Psi\rangle = 0$. The number of good $|\Psi\rangle$ states is large (e.g. $(2S_c + 1)^N$ if it does not contain electrons) and $|\Psi\rangle$ will inherit its degeneracy and spin. Our wave function $|\Psi\rangle$ can be visualized as the product of suitable chosen ground states (eq. (14) of each graph $G_\alpha$. For finite $t^2/J_H$ the antiferromagnetic term will split the $|\Psi\rangle$ manifold and a high spin state will not be the ground state for electron densities larger than $n_Q = 4/(D + 1)$ (we added two electrons for each $G_\alpha$ by constructing $|\Psi\rangle$). The construction can be repeated for other line graphs as well. For example when $G_\alpha(2D)$ form a $D$-dimensional hypercubic lattice [3], we have one dispersing bands and $D - 1$ flat bands and, consequently, the critical densities are lower, $n_C = 1/D$ and $n_Q = 2/D$. It should be noted here that in the cubic lattice the ferromagnetism is suppressed near zero and half fillings due to lack of available carriers [4,8] and it is not the case here, where charges are available, however in dispersionless states.

From the arguments above the following speculative phase diagram emerges. (i) For densities below $n_C$ the usual double exchange mechanism will stabilize the ferromagnetic phase in a large parameter range. (ii) For densities between $n_C$ and $n_Q$ preliminary studies indicate a kind of ferrimagnetic state. (iii) Finally, if $n > n_Q$ the physics is governed by the antiferromagnetic term $\propto t^2/J_H$ in the effective Hamiltonian with a possible spin liquid ground state.
in the density of states can help the ferromagnetism to develop in some particular cases. Therefore one is tempted to generalize the conclusions drawn from the Stoner model. Here we have shown that the flat bands (at least if they are above the broad dispersing bands) inhibits ferromagnetism in the Kondo lattice. If we look more carefully, the fact that the driving force for finite magnetization is the kinetic energy gained by spin alignment, we do not expect from the beginning the Stoner criterion to apply.

To summarize, we have shown that the strong coupling limit of the Kondo lattice with infinite long range hoppings can be solved exactly using the underlying dynamical supersymmetry. We learned that on this particular lattice (i) an extended Gut willer projection becomes exact, (ii) the ferromagnetic ground state is not favored. Finally, extending our result to more general lattices we arrived at an interesting conjecture that large density of states is against ferromagnetism in the double exchange model.

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