A Walking Dilaton Inflation

Hiroyuki Ishida\textsuperscript{a}, Shinya Matsuzaki\textsuperscript{b,*}

\textsuperscript{a}Theory Center, IPNS, KEK, Tsukuba, Ibaraki 305-0801, Japan
\textsuperscript{b}Center for Theoretical Physics and College of Physics, Jilin University, Changchun, 130012, China

Abstract

We propose an inflationary scenario based on a many-flavor hidden QCD with eight flavors, which realizes the almost scale-invariant (walking) gauge dynamics. The theory predicts two types of composite (pseudo) Nambu-Goldstone bosons, the pions and the lightest scalar (dilaton) associated with the spontaneous chiral symmetry breaking and its simultaneous violation of the approximate scale invariance. The dilaton acts as an inflaton, where the inflaton potential is induced by the nonperturbative-scale anomaly linked with the underlying theory. The inflaton potential parameters are highly constrained by the walking nature, which are evaluated by straightforward nonperturbative analyses including lattice simulations. Due to the pseudo Nambu-Goldstone boson’s natures and the intrinsic property for the chiral symmetry breaking in the walking gauge dynamics, the inflaton coupled to the pions naturally undergoes the small field inflation consistently with all the cosmological and astrophysical constraints presently placed by Planck 2018 data. When the theory is vector-like coupled to the standard model in part in a way to realize a dynamical electroweak symmetry breaking, the reheating temperature is determined by the pion decays to electroweak gauge bosons. The proposed inflationary scenario would provide a dynamical origin for the small field inflation as well as the light pions as a smoking-gun to be probed by future experiments.

Keywords: Small field inflation, Scale invariance, Many flavor QCD

1. Introduction

Exponentially expanded cosmological evolution, inflation, is one of the most attractive and plausible period of our Universe to dynamically and simultaneously solve crucial cosmological problems, such as the flatness, the homogeneous and isotropic, and the horizon problems. The dynamics itself would have no doubt that we have experienced such an epoch, however, details of inflation have not been revealed so far both theoretically and experimentally at all.

As one of candidate scenarios for the inflation, models having a log-type potential can be good targets. Since the log-type potential naturally has a plateau, it can be easily applied to the inflation. Such a log-type potential can be given in the context of the Coleman-Weinberg (CW) mechanism \cite{1} where quantum loop contributions modify the shape of the potential to be flatten around at the origin, which is called the CW inflation \cite{2}. However, it is in general quite difficult to satisfy all the cosmological parameters fitted by cosmic microwave background (CMB) observations such as the Planck satellite \cite{3}. Some recent developments have been made where the small field inflation (SFI) of the CW type can be achieved without conflicting with the CMB observations by introducing a linear term originated to a fermion condensation \cite{4} and a dynamical origin of the initial field value \cite{5}. Still, it is however unavoidable to naively assume the quartic coupling of the inflaton to be extremely small, in order to realize the observed amplitude of the scalar perturbation.

At this moment, one should notice that the inflaton quartic coupling $\lambda_\chi$ can be expressed by the ratio of the inflaton mass ($m_\chi$) to the vacuum expectation value of the inflaton ($v_\chi$), $\lambda_\chi \sim (m_\chi/v_\chi)^2$, with the stationary condition taken into account. This implies that the tiny enough $\lambda_\chi$ is directly linked to a large enough scale hierarchy between $m_\chi$ and $v_\chi$. Then, an interesting question would be raised: “Can this large hierarchy be physical in a sense of quantum field theory?”

It is a walking gauge theory that can naturally supply such a large scale hierarchy, which is characterized by the “journey-distance” during the walking (i.e. almost
scale-invariant) behavior, from an ultraviolet (UV) scale ($\Lambda_{\text{UV}}$) down to an infrared (IR) scale ($\Lambda_{\text{IR}}$).

For instance, in the case of many flavor QCD with the SU(N) group coupled to fermions ($F$) belonging to the fundamental representation, the walking regime can be established reflecting the perturbative IR fixed point, well known as the Caswell-Banks-Zaks IR fixed point [6, 7]. In that case, the fermion dynamical mass scale ($m_F$) is expected to be generated from the UV scale $\Lambda_{\text{UV}}$ having the characteristic scaling relation, called Miransky scaling [8] (sometimes also called Berezinsky-Kosterlitz-Thouless scaling) intrinsic to the conformal phase transition [9]: $m_F \sim \Lambda_{\text{UV}} e^{-\alpha / \sqrt{\alpha c - 1}}$ (in the chiral broken phase), where $\alpha$ is the fine structure constant of the gauge theory and $\alpha_c$ denotes the critical coupling above which the chiral symmetry breaking takes place. This scaling surely realizes a large scale hierarchy in the chiral broken phase during the walking regime, spanned by the IR $m_F$ and the UV $\Lambda_{\text{UV}}$ scales, where the $\Lambda_{\text{UV}}$ can be identified as the critical scale at which the $m_F$ is generated.

The lightest composite scalar, so-called walking dilaton, arises as the consequence of the spontaneous breaking of the (approximate) scale invariance, and simultaneously gets massive due to the explicit breaking induced by the scale anomaly [10] arising from the Miransky scaling above: $\beta(\alpha) = \partial \alpha / \partial \ln \Lambda_{\text{UV}} \sim -\frac{\pi^2}{2} \left( \frac{\alpha}{\alpha_c - 1} \right)^{3/2}$. QCD with eight flavors has been confirmed by lattice simulations to be walking with the chiral broken phase [11, 12, 13]. In that case, it has been observed on lattices [14, 15, 16] that the walking dilaton formed by a flavor singlet bilinear $\bar{F}F$ can be as light as or less than the chiral symmetry breaking scale $m_F$, in accordance with the expected particle identity as a pseudo Nambu-Goldstone (pNG) boson for the scale symmetry breaking. Furthermore, it has been found [14, 15] that the dilaton decay constant ($f_\chi$) can be much larger than the $m_F$, in accord with a many-flavor version of the Veneziano limit, dubbed anti-Veneziano limit [17]: $f_\chi \sim \sqrt{NN_F} m_F$ for $N\alpha$ fixed, $N_F/N = \text{fixed} \gg 1$, and $N, N_F \to \infty$. Hence the $f_\chi$, dictated by the scale-chiral breaking, would be on the order of the chiral-critical scale $\Lambda_{\text{UV}}$ above. Thus one would have a large scale hierarchy between $m_F \lesssim m_\chi$ and $f_\chi \sim \Lambda_{\text{UV}}$, via the Miransky scaling, $m_\chi \sim f_\chi e^{-\alpha / \sqrt{\alpha c - 1}}$, which has indeed been supported by some straightforward Schwinger-Dyson equation-analysis on many-flavor walking gauge theories [18].

Note that the dilaton potential is also generated by the scale anomaly driven by the chiral symmetry breaking, and is fixed by the anomalous Ward-Takahashi identity for the scale symmetry [17, 19, 20], to generically take the form of CW type [1]: like $V_\chi = \lambda_\chi \chi^2 \ln \chi$ with $\lambda_\chi \sim (m_\chi / v_\chi)^2$ and $v_\chi$ being identified as the dilaton decay constant $f_\chi$ above ($v_\chi \equiv f_\chi$). Thus, one undoubtedly expects that, with the walking dilaton identified as an inflaton, a tiny enough quartic coupling, $\lambda_\chi \sim (m_\chi / v_\chi)^2 \equiv (m_\chi / f_\chi)^2 \sim (m_\chi / \Lambda_{\text{UV}})^2 \ll 1$, desired for the consistent inflation, would naturally and dynamically be generated by the walking gauge dynamics, when the IR and UV scales are associated with $m_\chi$ and $v_\chi$, respectively.

A possible inflationary history goes like: We start from the chiral-scale broken phase in the vacuum for a many flavor walking gauge theory, where we have the walking dilaton and its potential of a CW type. Allowing some coupling between the walking fermions and some scalar, e.g. the standard model (SM)-like Higgs, a preheating mechanism [21] would work to trap the walking dilaton around the origin of the potential – ("approximate") chiral-scale symmetric point – due to an induced-finite matter-density effect (like a thermal plasma) triggered by parametric resonances, as proposed in [5]. As the temperature cools down, the walking dilaton is going slowly to roll the potential-down hill, i.e. undergoes the SFI due to the underlying walking nature, which governs the evolution of Universe at that time. After ending the inflation (almost in the same manner as in the CW inflation), the dilaton starts to drop down to the vacuum $\chi = v_\chi$, oscillate and will keep reheating until it decays. The reheating would work unless the created temperature gets higher than the critical temperature(s) for the chiral phase transition and/or deconfinement phase transition – expected to be around the temperature $\sim m_F$ – above which the walking dilaton gets dissociated to cease having the potential.

In this paper, we present a dynamical inflationary scenario of the CW-SFI type arising from eight-flavor QCD (a large $N_F$ walking gauge theory). The walking dilaton plays the role of an inflaton, where the inflaton potential parameters are highly constrained by the walking nature. We evaluate the potential parameters using outputs from straightforward nonperturbative analyses including lattice simulations. It is shown that with a tiny dilaton coupling to pions explicitly breaking the chiral-scale symmetry, the walking dilaton inflation resolves a well-known incompatibility intrinsic to the SFI of the CW type for realizing the desired e-folding number and

---

1 A similar composite dilaton (or glueball) potential of the CW type was discussed in a context different from the present interest in the SFI, where a large field inflation with non-minimal coupling to general relativity is assumed [18, 19, 20, 21, 22].
the observed spectral index \cite{2,29}. This makes the proposal in \cite{4} explicitized by a concrete dynamics as the many-flavor walking gauge theory.

To be more realistic, the inflationary scenario is fitted with all the cosmological and astrophysical constraints presently placed by Planck 2018 data \cite{3} and theoretical requirements on the walking dilaton and chiral pion physics. As a reference scenario, the hidden walking dynamics is vector-likely gauged in part by the electroweak (EW) charges and is coupled to a Higgs doublet in a scale-invariant way. Thus the EW symmetry breaking (EWSB) is triggered by the bosonic seesaw mechanism \cite{5} and \cite{6} as a smoking-and explanation for the tiny inflaton coupling and some-

- the deconfinement and or chiral phase transitions in the temperature (\(T_R\)) is then fixed by chiral pions decays to EW gauge bosons including photons to be \(\gtrsim 10^2\) GeV, where the masses of walking dilaton and pions are also constrained by several theoretical and astrophysical lim-

its to be \(\gtrsim 10^8\) GeV and \(\gtrsim 10^6\) GeV, respectively. Since \(T_R \ll m_F\), this reheating thus works consistently with the deconfinement and or chiral phase transitions in the walking gauge theory, as noted above.

Thus the presently proposed inflationary scenario of the CW-SFI type would provide the dynamical origin and explanation for the tiny inflaton coupling and some-
what light particles (the walking pions) as a smoking-gun to be probed by future experiments.

2. Walking dilaton potential

We begin by writing the dilaton potential induced from a generic many-flavor QCD, which takes the form including the CW type:

\[
V(\chi) = -\frac{C}{2N_F} \chi^a \text{tr}[U + U^\dagger] + \frac{\lambda}{4} \chi^a \left( \ln \frac{\chi}{v_F} + A \right) + V_0.
\]

\(V_0\) is the vacuum energy, which is determined by taking the normalization of the potential as \(V_0(v_\chi) = 0\).

The \(C\) term in Eq. (1) has come from the explicit-

- chiral breaking (flavor universal) mass term for the hidden QCD fermions,

\[
\mathcal{L}_m = -m_0 \sum_{\ell = 1}^{N_F} \bar{F}_F \chi^a F_F, \tag{2}
\]

(with \(m_0\) being real), by extracting the flavor-singlet component as

\[
\hat{F}_R F_{Lj} \approx \langle \hat{F}_R F_{Li} \rangle \cdot \left( \frac{\chi}{v_F} \right)^a \cdot U_{ij}, \tag{3}
\]

and its hermitian conjugate partner. \(U = e^{2\pi i / f_\pi}\) is the chiral field parametrized by the pion field \(\pi = \pi^a T^a\) with \(T^a\) being generators of \(\text{SU}(N_F)\) \((\alpha = 1, \ldots, N_F^2 - 1)\) normalized as \(\text{tr}[T^a T^b] = \delta^{ab}/2\), and the pion decay constant \(f_\pi\). The exponent \(a\) for the \(\chi\) controls the size of the overlap amplitude between the \(\chi\) and the composite operator \(\overline{F} F\) in the underlying theory, for which we will take \(a = 1\), so the \(\chi\) is allowed to linearly couple to the \(\overline{F} F\). Hence the parameter \(C\) is expressed in terms of the pion mass \(m_\pi\) and the decay constant \(f_\pi\) as

\[
C = N_F \frac{m_\pi^2 f_\pi^2}{2 v_F}, \tag{4}
\]

with the canonical form of the pion kinetic term being assumed. Note that the \(f_\pi\) can be related with the fermion dynamical mass \(m_1\) as

\[
f_\pi \approx \sqrt{N} \cdot \frac{m_1}{2 \pi}, \tag{5}
\]

which is based on a naive dimensional analysis regard-

---

\[\text{As discussed in the literature} \cite{12}, \text{when the walking dilaton } \chi \text{ purely arises as the } \overline{F} F\text{-composite scalar, the power parameter } a \text{ would be identical to } (3 - \gamma), \text{ the dynamical dimension of the } \overline{F} F\text{ operator with the anomalous mass dimension } \gamma, \text{ which is fixed by the anomalous Ward-Takahashi identity for the scale symmetry. However, possible mixing with other flavor-singlet scalars, like a glueball or tetraquark states, might be present, so in that sense the parameter } a \text{ would generically be undermined until fully solving the mixing structure, that is beyond the scope of the current interest. In the present work, thus, we will take a conservative limit with } a = 1, \text{ so that the } \chi \text{ couples to the } \overline{F} F \text{ as if it were a conventional singlet-scalar component as seen in the linear sigma model: the chiral } \text{SU}(N_F)\times \text{SU}(N_F)\text{ linear sigma-model field } M(M') \text{ is introduced as the effective local-operator description for the } \overline{F} F_{Lj} \text{(} \overline{F} F_{Li} \text{)}, \text{ and transforms as } M \rightarrow g_L M \xi_L, M' \rightarrow g_R M' \xi_R \text{ under the chiral symmetry with the transformation matrices } g_{L,R}. \text{ The current (universal) } \text{fermion mass } m_0 \text{ is therefore coupled to the } M(M') \text{ so as to respect the original form in Eq. (3) in a chiral invariant way, like } \text{tr}[\overline{M} M' + \overline{M} M'] \text{ with the mass-parameter spurious field } M \text{ transforming in the same way as } M, \text{ with the vacuum value } \langle M \rangle = m_0 \cdot \mathbb{1}_{N_F \times N_F}. \text{ In the chiral broken phase, the } M \text{ can generically be polar-decomposed by hermitian } (\hat{M}) \text{ and unitary } (\xi_{L,R}) \text{ matrices as } M = \xi_L \cdot \hat{M} \cdot \xi_R \text{ where } \xi_L^\dagger \xi_R = U. \text{ Supposing a low-energy limit where only the lightest scalar } (\chi) \text{ survives among the } N_F \text{ scalars in } M, \text{ one may write } M = \chi \cdot U. \text{ Plugging this approximated expression into the above } \text{M'}^M + \text{h.c. term, to get } \chi \cdot m_0 (\overline{U} + U^\dagger), \text{ the coupling form of which coincides with the conservative limit } a = 1. \text{ Thus, the choice } a = 1 \text{ is reasonable if the linear sigma model gives a good low-energy description for the underlying walking gauge theory in terms of the chiral-breaking structure.} \]
ing the definition of the pion decay constant. Then the parameter $C$ in Eq. (4) can be evaluated as

$$C \approx \frac{NN_F m_F^2 m_F^4}{8\pi^2 v_\chi^2}. \quad (6)$$

The quartic coupling $\lambda_\chi$ in Eq. (1) is set by the ratio of the dilaton mass $m_\chi$ to the dilaton decay constant $v_\chi$ as $\lambda_\chi = (m_\chi/v_\chi)^2$ (by taking into account the stationary condition) in the absence of the explicit chiral-scale breaking by the $C$ term (i.e. chiral limit). To this chiral-limit quantity, some straightforward computation of the scale anomaly in the many-flavor walking gauge theory (called ladder Schwinger-Dyson equation analysis), in combination with the partially-conserved dilatation-current (PCDC) relation, would give a constraint

$$m_\chi^2 v_\chi^2 = \frac{16NN_F}{\pi^4} m_F^4. \quad (7)$$

Thereby the $\lambda_\chi$ may be evaluated as

$$\lambda_\chi = \frac{16NN_F}{\pi^4} \left(\frac{m_F}{v_\chi}\right)^4. \quad (8)$$

The full walking dilaton mass ($M_\chi$) is given by (evaluating $V''(\chi) = \partial^2 V(\chi)/\partial \chi^2 |_{\chi=v_\chi}$) as the sum of the chiral limit value ($m_\chi$) and the correction from the C-term\(^3\)

$$M_\chi^2 = m_\chi^2 + \frac{3C}{v_\chi} \approx m_\chi^2 + \frac{3NN_F m_F^2}{2v_\chi}. \quad (9)$$

\(^3\) The Pagels-Stokar formula \(^4\) applying to the present walking gauge dynamics (with the nonrunning and ladder approximation taken) would yield \([17]\) $f_\chi = \sqrt{N/(2\pi^2)m_F}$, which is larger by about factor of $\sqrt{2}$.

\(^4\) The right hand side corresponds to the vacuum expectation value of the trace of (symmetric part of) energy momentum tensor, $\langle \sigma^\mu_{\mu} \rangle = 4E_{vac}$, where $E_{vac}$ denotes the vacuum energy in the walking gauge theory, which is dominated by the $F$-fermion loop contribution to the gluon condensate \([17]\), hence it scales solely with the dynamical mass $m_F$, like $E_{vac} \propto NN_F m_F^2$. On the other hand, the definition of the dilaton decay constant $f_\chi(= v_\chi)$ gives at the dilaton-soft mass limit $p^2 = m_\chi^2 \to 0$, $\langle 0|\phi^\mu(0)|\phi(p)\rangle = -m_\chi f_\chi$ (with $\phi = f_\chi \log(\chi/f_\chi)$), the left hand side of which can be evaluated by the $E_{vac}$ by using the PCDC: $\langle \phi^\mu(x) \rangle = -m_\chi f_\chi e^{-m_\chi^2\phi(x)}$ together with the standard reduction formula, and the Ward-Takahashi identity for the scale symmetry (the low-energy theorem), as $\langle 0|\phi^\mu(0)|\phi(p)\rangle = -4d_{\phi^\mu} \cdot E_{vac}/f_\chi$ with the scale dimension of $d_{\phi^\mu}$, $d_{\phi^\mu} = 4$. Thus one has $m_\chi f_\chi^2 = -16E_{vac}$.

For more detailed evaluation, see the literature \([17]\) and references therein.

This mass formula is precisely the same as the one (with a factor $(3-\gamma_\mu)(1+\gamma_\mu)$ taken to be 3) derived in the dilaton-chiral perturbation theory for the many-flavor walking gauge theory at the leading order of the derivative expansion \([17][19]\).

Through the stationary condition (evaluated at $\chi = v_\chi$), the parameter $\Lambda$ in Eq. (1) is given as a function in terms of the $C$ and $\lambda_\chi$ to be $\Lambda = -\frac{1}{2} + \frac{C}{4\pi^2 v_\chi^2}$.\(^5\)

Thus, the walking dilaton potential in Eq. (1) is highly constrained by nontrivial parameter correlations given by Eqs. (6) and (8). Then the potential $V(\chi)$ normalized to $v_\chi^2(= f_\chi^2)$ is essentially controlled by small underlying-theory parameters, $m_\chi/v_\chi \ll 1$ and $m_F/v_\chi \ll 1$ (with $m_\pi \ll m_F$, to be consistent with the nonlinear realization for the chiral-scale symmetry in which we are currently working, as seen from the potential form in Eq. (1)).

3. Small field inflation

The slow roll parameters ($\eta$ and $\epsilon$), the e-folding number ($N$) and the magnitude of the scalar perturbation ($\Delta_\chi^2$) are respectively defined as

$$\eta = M_{pl}^2 \langle V''(\chi)/V(\chi) \rangle,$$

$$\epsilon = M_{pl}^2 \langle V'(\chi)/V(\chi) \rangle^2,$$

$$N = \frac{1}{M_{pl}^2} \int_{\chi_{inf}}^{\chi_{end}} d\chi \langle V(\chi)/V'(\chi) \rangle,$$

$$\Delta_\chi^2 = \frac{V(\chi)}{24\pi^2 M_{pl}^2 \epsilon}. \quad (10)$$

with $M_{pl}$ being the reduced Planck mass $= 2.4 \times 10^{18}$ GeV. Since we work in an extremely tiny explicit-chiral (and scale) symmetry-breaking limit with $m_\pi \ll m_F(\ll v_\chi)$ and the magnitude of the dilaton potential during the inflation ($\chi \ll v_\chi$) can be approximated by the vacuum energy $V_0$ as in the CW-SFI case, the slow-roll

\(^5\) The second term in this stationary condition, which gives a scale invariant $\chi^4$ term proportional to the chiral explicit breaking $C$ parameter, plays the role of stabilization of the dilaton potential in the presence of the fermion current mass, as pointed out it is necessary to have in the literature \([25][20]\).
parameters are well approximately evaluated as
\[ \eta = \frac{M_{pl}^2 v^2}{V_0^0} \left( \frac{m_F^2}{v_F^2} \right)^2 \left( 1 - \frac{72}{\pi^2} \frac{\ln \lambda}{v_F^2} \right) \left( \frac{m_e^2}{v_e^2} \right)^2 \]
\[ + \frac{384}{\pi^2} \left( \frac{m_F^2}{v_F^2} \right)^2 \left( \frac{3}{2} \ln \lambda \right) \left( \frac{m_e^2}{v_e^2} \right)^2 + O \left( \frac{m_e^2}{v_e^2 m_F^2} \right), \]
\[ \epsilon = \frac{M_{pl}^2}{2V_0^0} \left( \frac{m_F^2}{v_F^2} \right)^4 v_F^4 \]
\[ \times \left( 1 - \frac{24}{\pi^2} \frac{\ln \lambda}{v_F^2} \right) \left( 1 - \frac{m_e^2}{v_e^2} \right)^2 \]
\[ + \left( \frac{192}{\pi^2} \frac{\ln \lambda}{v_F^2} \right) \left( \frac{m_F^2}{v_F^2} \right) \left( \frac{m_e^2}{v_e^2 m_F^2} \right)^2, \] (11)
with
\[ V_0^{LO} = \frac{24}{\pi^2} m_F^4. \] (12)

The SFI with the extremely small chiral-scale breaking by the \( m_e \) will give an overall scaling for \( \epsilon/\eta \) with the small expansion factors as \( \epsilon \sim \frac{m_e}{m_F} \left( \frac{v_F}{v_e} \right)^2 \). Hence the inflation would be ended by reaching \( \eta \approx 1 \), as long as \( v_F/v_e \gg (m_e/m_F)^2 \), as in the CW-SFI case, which indeed turns out to happen as will be seen later. In that case (with \( m_e \ll v_F \ll m_F \)), \( \eta \) and \( \epsilon \) as well as the \( \Delta_R \) and \( N \) can further be approximated to be
\[ \eta \approx 24 \frac{M_{pl}^2 \chi^2 \ln \frac{\lambda}{\epsilon}}{v_F^2 v_e^2}, \]
\[ \epsilon \approx \frac{\pi^2}{2} \left( \frac{M_{pl}}{v_F} \right)^2 \left( \frac{m_e}{m_F} \right)^4, \]
\[ \Delta_R \approx \frac{2}{\pi^{10}} \left( \frac{m_F^2}{v_F^2} \right)^4 \left( \frac{v_F}{M_{pl}} \right)^6 \left( \frac{m_F}{m_e} \right)^4, \]
\[ N \approx \chi_{end} \chi_{init} = 1, \] (13)

Note that the gigantic suppression factor \( \frac{1}{\pi^2} \left( \frac{m_e}{v_e} \right)^4 \) for \( \Delta_R \) in Eq.(13) shows up, corresponding to an extremely tiny quartic coupling \( \lambda_4 \), realized by the walking nature \( (m_e/v_e) \ll 1 \) as seen from the PCDC relation in Eq.(1) (also see Eq.(9)), which gets small enough to cancel the other factors coming from the small \( \epsilon \), to easily achieve the right small amount of the observed \( \Delta_R \) \( \sim 10^{10} \) at the pivot scale. Given the observed \( \Delta_R \), the pion mass \( m_e \) is actually written as a function of other potential parameters like
\[ m_e^2 \approx \sqrt{\frac{2}{\pi^{10}} \Delta_R^2 \left( \frac{m_F^2}{v_F} \right)^2 \left( \frac{v_F}{M_{pl}} \right)^3 m_F^2. \] (14)

Note also that the e-folding number \( N \) in Eq.(13) is set by the constant \( \epsilon \), in contrast to the case of the CW-SFI where it is instead set by \( \eta \) so that one would encounter the incompatibility between the \( N \) and the spectral index \( n_s \approx 1 + 2\eta \) in comparison with the observational values [29]. As discussed in the literature [4], a small enough tadpole term (corresponding to the \( C \)-term at present) helps avoid this catastrophe, which will be more concretely demonstrated by the present walking inflationary model later on.

4. Embedding into a dynamical scale genesis

To more realistically analyze the present walking dilaton inflationary scenario, we need to consider couplings between the walking gauge sector and SM sector, so that we can access the reheating temperature \( T_R \) and the e-folding number detected by the CMB photons at the pivot scale (\( k = k_{CMB} = 0.05 \text{Mpc}^{-1} \)) through the following relation [4]:
\[ N_{CMB} \approx 61 + \frac{2}{3} \ln \frac{V_0^{1/4}}{10^{16} \text{GeV}} + \frac{1}{3} \ln \frac{T_R}{10^{16} \text{GeV}}. \] (15)

As a benchmark model, we shall try to embed the present scenario into a dynamical scalegenesis [7] in which the EWSB is triggered by what is called the bosonic seesaw mechanism [30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42]. In that case, the hidden walking eight-flavor fermion fields \( F_i \) (\( i = 1, \ldots, 8 \)) are vector-like charged in part by the EW gauges. A possible charge assignment goes like
\[ \Psi_{U/R} = (F^i, F^j)^T \]
\[ \phi_{L/R} = F^{i=8} \]
under the SU(N) \( \times \) SU(3) \( \times \) SU(2) \( \times \) U(1) \( y \) symmetry.

\[ \text{Note also that the e-folding number } N \text{ in Eq.(13) is set by the constant } \epsilon, \text{ in contrast to the case of the CW-SFI where it is instead set by } \eta \text{ so that one would encounter the incompatibility between the } N \text{ and the spectral index } n_s \approx 1 + 2\eta \text{ in comparison with the observational values [29]. As discussed in the literature [4], a small enough tadpole term (corresponding to the } C \text{-term at present) helps avoid this catastrophe, which will be more concretely demonstrated by the present walking inflationary model later on.} \]
The gauge invariance as well as the classical scale-invariance allows us to introduce a Yukawa coupling between the $F$-fermion fields and a Higgs doublet field $H$ as
\[ L_{\chi H} = -\sum_{A=1}^{6} y_H^A (\bar{\Psi}_L H \psi_R^A + \bar{\psi}_R H \psi_L^A) + \text{h.c.}, \] (17)

(with the hidden-fermion parity invariance ensured by the underlying vectorlike gauge theory via Vafa-Witten theorem [78]), where the couplings $y_H$'s are assumed to be small enough (to be consistent with the fitting later).

After the chiral condensate (and confinement) develops, those $y_H$-Yukawa interactions generate a couple of mixings between the elementary $H$ doublet and composite Higgs doublets $\Theta^A - \bar{\psi}^A \psi$, to give the negative mass square of the SM-like Higgs field (arising as the lowest mass eigenstate from the mass matrix), that is called the bosonic seesaw mechanism [30,31,32,33,34,35,36,37,38,39,40,41], as follows:
\[ m_H^2 = \sum_{A=1}^{6} y_H^A \bar{y}_H^A m_F^2 \equiv -6y_H^2 m_F^2, \] (18)

where the $y_H^A$ couplings have been assumed to be flavor universal ($\bar{y}_H^A \equiv y_H$). Thus, with the quartic coupling for the $H$ at hand, the EWSB is dynamically achieved so that the SM-like Higgs acquires the EW vacuum expectation value ($v_{\text{EW}} \approx 246$ GeV) properly. Then, the $m_H$ scale is fixed by the 125 GeV Higgs mass as
\[ m_H^2 = -m_H^2/(2) \approx -(88 \text{ GeV})^2, \] so that the $y_H$ coupling is determined as a function of $m_F$ like
\[ y_H^2 = \left( \frac{36 \text{ GeV}^2}{m_F^2} \right)^2. \] (19)

Another important point arising from the $y_H$ interactions is that Eq.(17) explicitly breaks the $F$-fermion chiral $U(8)_L \times U(8)_R$ symmetry, even the vectorial part. The breaking effect of this non-vectorial type destabilizes the chiral manifold, yielding the instability for the pions (i.e. making the pions tachyonic) [35,36,37,38,39,40,41]. Although some pions charged by EW gauges get sizable enough masses on the order of $O(\alpha_w m_F^2)$ to safely overcome this instability, other chargeless pions can be unstable. The tachyonic correction to those pion masses are evaluated by using the current algebra, as done in the literature [36,37], to be
\[ m_{\pi}^2 = -(y_{H,\text{EW}})^2 \langle -\bar{F}F \rangle / f^2, \] (20)

where $\langle -\bar{F}F \rangle$ denotes the chiral condensate per flavor. This contribution has to be smaller than the pion-flavor universal $m_{\pi}$ term arising from the bare mass term for the $F$-fermions: $m_{\pi}^2 \ll m_F^2$. The chiral condensate in the many-flavor walking gauge theory can be evaluated by a straightforward nonperturbative computation based on the Schwinger-Dyson equation analysis (in the ladder approximation) [17]
\[ \langle -\bar{F}F \rangle_{m_F} \approx \frac{8N}{\pi^4} m_F^3, \] (21)
in which the renormalization scale has been set at the scale $m_F$ [9]. Using Eqs.(19) and (21) together with Eq.(5), the $m_{H,\pi}$ is completely fixed to a constant:
\[ m_{\pi}^2 \approx -(169 \text{ GeV})^2. \] (22)

Thus the walking pion (particularly for neutral pions) can safely be stabilized when
\[ m_{\pi} \gg 169 \text{ GeV}. \] (23)

In the present inflationary scenario, the reheating temperature is actually determined by the walking pion decays to EW bosons: since the walking dilaton as the inflaton predominantly decays to walking pion pairs with the strong coupling, which will be much faster than the Hubble evolution of the Universe at that time, the rate of the SM particle production is controlled by the pion decays to the EW bosons coupled to the vector-like charged $F$-fermion currents.

The interactions between the walking pions and EW bosons, relevant to the pion decay processes, are completely determined by a covariantized Wess-Zumino-Witten term [79,80] (in a way analogously to the three flavor case discussed in [36]):
\[ L_{\nu VV} = -\frac{N}{4\pi^2 f} e^{i\nu \pi} \text{tr} [\partial_\mu V_\nu \partial_\mu V_\nu \pi]. \] (24)
\[ V_\nu \text{ denotes the external gauge field of } 8 \times 8 \text{ matrix form, with the } SU(2)_W \times U(1)_Y \text{ gauge fields } (W^a_\mu, B_\mu), \text{ embedded following the charge assignment in Eq.(16), which is expressed as } \]
\[ V_\mu = \left( \begin{array}{cc} V_\mu^{\text{EW}} & 0_{2\times6} \\ 0_{6\times2} & 0_{6\times6} \end{array} \right), \]
\[ V_\mu^{\text{EW}} = \frac{g_W}{2} W^a_\mu \tau^a + \frac{g_Y}{2} B_\mu \cdot 1_{2\times2}, \] (25)

\[ ^9 \text{ The sign ambiguity for the } y_{H,\pi} \text{ has been fixed to be positive by the definition of the chiral condensate } (\bar{F}F) \text{ in Eq.(21).} \]
\[ ^10 \text{ Hence the } y_{H,\pi} \text{ coupling in Eq.(20) should also be interpreted as the one renormalized at the same scale } m_{\pi} \text{ so that the corresponding pion mass } m_{\pi} \text{ is independent of the renormalization scale, as it should be. Note also that the } y_{H,\pi} \text{ is not evolved by the renormalization below } m_F \text{ because of decoupling of the } F \text{-fermions, so the matching condition in Eq.(19), which should be set at the 125 GeV Higgs mass scale, can be read as } y_{H,\pi}(m_{\pi}) = y_{H,\pi}(m_{125}) = \text{ right hand side of Eq.(19).} \]
with the SU(2)$_W$ and U(1)$_Y$ gauge couplings, $g_W$ and $g_Y$, and normalized Pauli matrices $\tau^i$ with the normalization $\text{tr}[\tau^i \tau^j] = \delta^{ij}/2$. The walking pion field $\pi$ is parametrized in a way similar to the $V_\mu$ as

$$\pi = \begin{pmatrix} \pi^{\mu\nu}_{12} & \pi_{\rho\nu} \\ \pi_{\rho\nu} & \pi^{\mu\nu}_{56} \end{pmatrix}. \tag{26}$$

One can readily see that only the $\pi^{\mu\nu}_{12}$ component couples to the EW bosons. The EW charged pions in the $\pi^{\mu\nu}_{12}$ get large masses of $\mathcal{O}(g_W m_f)$, by the EW interaction, enough to close the decay channel of the walking dilaton into the pion pairs, as will be clarified later. Thereby, only the EW singlet one with the mass $m_\pi$, $\pi^{\mu\nu}_{56} \gg \frac{m_\pi}{2} \cdot 1_{56}$, contributes to the determination of the reheating temperature $\delta$. The $\pi^{\mu\nu}_{12}$ total width is computed to be

$$\Gamma_{\pi^{\mu\nu}_{12}} = \frac{m_\pi^3}{16\pi} \left( \frac{N}{4\pi^2 f_\pi} \right)^2 \left( \frac{g_Y(m_\pi)}{2} \right)^2 + 3 \left( \frac{g_W(m_\pi)}{2} \right)^2,$$ \tag{27}

where we have taken the EW bosons to be massless because $m_\pi \gg m_W$ as evident from Eq.\(23\). We also specified the renormalization scale for the EW gauge couplings at the $\pi^{\mu\nu}_{12}$ mass scale, which however turns out to be the same as the ones evaluated at the $Z$ boson pole, $g_W(m_\pi) = g_W(m_Z)$ and $g_Y(m_\pi) = g_Y(m_Z) = 0.13$ because possible renormalization corrections from EW-charged $F$-fermions, with the mass on the scale of $m_F$, necessarily get decoupled at the scale $m_\pi \ll m_F$. By equating the decay rate in Eq.\(27\) and the Hubble parameter with the radiation dominance, we determine the reheating temperature $T_R$ as follows:

$$T_R \approx 0.23 \left( \frac{100}{g_*(T_R)} \right)^{1/2} \sqrt{\Gamma_{\pi^{\mu\nu}_{12}} M_\text{pl}}, \tag{28}$$

with the effective degrees of freedom at $T_R$, $g_*(T_R)$, which is set to the SM value $= 106.75$ [81], as long as the walking dilaton and pion masses are larger than $T_R$, as in the present analysis to be clarified in the next section.

---

11 The possible mixing with the $\eta'$-like pseudoscalar would be small because of the large enough mass for the many-flavor walking $\eta'$ generated by the U(1) axial anomaly, $m_{\eta'} \sim \sqrt{N_F \langle m_{\pi} \rangle}$, due to the anti-Veneziano limit [77].

12 Those numbers have been estimated by using the electromagnetic couplings renormalized at the $Z$-boson mass scale ($m_Z = 91.2$ GeV [81]), $\alpha(m_Z) = \frac{\alpha_0(4\pi^2)}{\pi^2} = 1/128$ [81] and the (Z-mass shell) Weinberg angle quantity $\sin^2 \theta_W = m_0^2/m_Z^2 = 0.778$.

Thus the e-folding number for the CMB photons at the pivot scale ($N_{\text{CMB}}$) in Eq.(15) is evaluated as a function of $m_\pi$ and $m_F$, which has to be fitted to the theoretically predicted $N$ in Eq.(13), so as for the present inflationary model to surely generate the currently observed CMB photons.

5. Constraints and predictions

Using the observed data for $\Delta^2$ and $n_s(= 1 + 2\eta)$, $(\Delta^2 = 2.137 \times 10^{-9}$ and $n_s \approx 0.968$ [3]) with the initial stage of the inflation identified as the pivot scale, together with a couple of formulae derived in the previous sections, we now constrain the present walking-dilaton inflationary-scenarios embedded into a dynamical scale genesis. Figure [1] shows the exclusion plot on the parameter space spanned by $v_\chi$ and $V_0^{1/4}$ (instead of $m_F$ with Eq.\(12\) taken into account).

We first see from the figure that the $v_\chi$ has to be greater than $10^{11}$ GeV in order to realize the right amount of the e-folding number for the CMB photons today by the inflation (i.e. the bound from $N$ in Eq.(13) equals $N_{\text{CMB}}$ in Eq.(15)). Furthermore, the $v_\chi$ as well as the $V_0^{1/4}(= m_F)$ are highly constrained by the pivot instability set as in Eq.(23) to be $v_\chi \gtrsim 10^{14}$ GeV and $V_0^{1/4}(= m_F) \gtrsim 3 \times 10^{10}$ GeV.

When the EW phase transition is assumed to happen in the thermal history of our Universe, the reheating temperature $T_R$ is necessary to be $\gtrsim 100$ GeV. In that case, the bounds on the $v_\chi$ and $V_0^{1/4}$ get shifted further upward, so that we would have

$$v_\chi \gtrsim 1.7 \times 10^{15} \text{ GeV},$$
$$V_0^{1/4}(= m_F) \gtrsim 4.1 \times 10^{11} \text{ GeV},$$
$$T_R \gtrsim 10^2 \text{ GeV}. \tag{29}$$

Then the masses of the walking dilaton (in Eq.\(9\)) and pion (in Eq.\(14\)) at those lower bounds are estimated to be

$$M_\chi(= m_\pi) \approx 3.8 \times 10^8 \text{ GeV},$$
$$m_F \approx 6.7 \times 10^4 \text{ GeV}. \tag{30}$$

At this benchmark point, for other model parameters we have $\chi_{\text{init}} \approx 6.7 \times 10^9$ GeV, $\chi_{\text{end}} \approx 5.8 \times 10^{10}$ GeV, $\epsilon \approx 5.1 \times 10^{-21}$, $N \approx 51$, and $y_H \approx 6.5 \times 10^{-11}$. One can easily see from those reference outputs that the approximated analytic formulae in Eq.(13), derived for $m_F \ll \chi_{\text{init}}, \chi_{\text{end}} \ll m_F \ll v_\chi$, indeed work well, and surely reflects the large $N_F$ scaling (in the anti-Veneziano
limit \( \log_{10} V_{0} \) intrinsic to the underlying many-flavor walking gauge theory, \( v_{t} (\equiv f_{t}) \sim \sqrt{N_{F} m_{F}} \gg m_{F} \), and its associated pNG boson’s nature \( m_{s}, m_{z} \ll m_{F} \).

In terms of the bare fermion mass \( m_{0} \) in Eq. (2), one can also check the size of the explicit-chiral scale breaking regarding the \( m_{0} \) to be extremely tiny: using the current algebra for the pion mass together with Eqs. (5) and (21), one gets \( m_{0}/m_{F} = m_{0} f_{s}^{2}/(2(-\bar{F} F) m_{F}) \approx \pi^{2}/64(m_{z}/m_{F})^{2} \), which is \( \approx 4.1 \times 10^{-15} \) at the benchmark point above. This tiny \( m_{0} \) would be physical to be probed by detecting light walking pions with the mass \( \gtrsim 10^{3} \) GeV.

In addition, we observe that the non-Gaussianity is small enough to be consistent with the latest results by Planck satellite [82]. The non-Gaussianity can be evaluated as \( f_{NL} = (5/12)(n_{s} + f(k)n_{t}) \) where \( n_{s} \), \( n_{t} \) are the spectral indices for scalar and tensor modes, and \( f(k) \) is determined by the shape of triangle which has a range of values \( 0 \leq f \leq 5/6 \) [83]. Since \( n_{t} \) and the scalar to tensor ratio \( r \) have a relation: \( r + 8n_{t} = 0 \), the second term in the parenthesis gives a negative contribution to the non-Gaussianity. Namely, the maximal value of \( f_{NL} \) can be obtained to be roughly 0.4, which is within the current limit: \( f_{NL} = -0.9 \pm 5.1 \) (at the 68\% confidence level) [82]. Therefore, the non-Gaussianity is also consistent with the observation.

6. Summary and discussion

In summary, we have proposed an inflationary scenario of the CW-SFI type, dynamically arising from a large \( N_{F} \) walking gauge theory, what we called the hidden walking gauge theory. The inflation is played by the walking dilaton and the inflaton potential parameters are highly constrained by the walking nature. We have evaluated the potential parameters using outputs from our best knowledge based on straightforward non-perturbative analyses. We showed that due to the intrinsic feature of the large \( N_{F} \) walking dynamics, called the anti-Veneziano-large \( N_{F} \) scaling, the desired tiny inflaton quartic coupling can naturally be realized, and the inflaton coupled to the walking pions can survive the cosmological, astrophysical constraints, from which other models of the CW-SFI generically suffer.

For the inflationary scenario to be realistic, as a reference model, the hidden walking dynamics was vector-like gauged in part by the EW charges and was coupled to a Higgs doublet in a scale-invariant way, so that the EW symmetry breaking is triggered by the bosonic seesaw mechanism. This benchmark model has been fitted with all the cosmological and astrophysical constraints presently placed by Planck 2018 data and the-theoretical requirements on the walking dilaton and chiral pion physics. We found that the present inflationary scenario is highly constrained to give a stringent bound on the fermion dynamical mass scale to be \( > 10^{11} \) GeV, and the reheating temperature (\( T_{R} \)), which is fixed by chiral pion decays to EW gauge bosons including photons, to be \( \gtrsim 10^{2} \) GeV. It also turned out that the masses of walking dilaton and pions are constrained by several theoretical and astrophysical limits to be \( \sim 10^{6} \) GeV and \( \sim 10^{5} \) GeV, respectively.

In closing, we shall give comments on issues to be left in the future works.

The present walking inflationary scenario works for a so large dilaton decay constant \( v_{t} (\equiv f_{t}) \) compared to the pion decay constant \( f_{r} \), \( v_{t} \gtrsim 10^{4} f_{r} \). The current lattice simulations [15] have tried to observe those quantities with the current fermion mass \( m_{0}/m_{F} \approx 0.18 - 0.45 \), to give a preliminary result on the dilaton decay constant \( f_{d} (\equiv v_{t}) \approx 3.7 f_{r} \) based on the lowest-order formula in the dilaton-chiral perturbation theory [20], by simply fitting the dilaton mass formula as in Eq. (2) with the data on the slope with respect to the leading order \( m_{s}^{2} \) dependence on the dilaton mass \( (dM_{s}^{2}/dm_{s}^{2}) \). (This has been consistent with a direct measurement of the pole residue of the scalar current correlator with a simple-
minded linear fit assumed in the same range of the current fermion mass.). However, this result cannot simply be compared with our values at present: for our reference-current fermion-mass value $m_{\text{q,fermion}} \sim 10^{-15}$, the next-to-leading-order chiral-logarithmic correction would be significant for the estimates on the dilaton mass, and decay constant cannot be measured just by the slope $dM_\chi^2/dm_\chi^2$, as explicitly demonstrated in [20]. Future upgraded lattice setups, by which the chiral log corrections are visible, could check if our scenario is indeed viable on the ground of the realistic nonperturbative dynamics.

The phenomenological consequence for the present inflationary scenario, distinguishable from other inflation models in the huge ballpark, would be derived from the presence of walking pions with mass $\lesssim 10^7$ GeV (if an EW phase transition happens by supercooling like in the scenario discussed in the literature [84]). Such a sub TeV-walking pions can have several predictions for terrestrial and satellite experiments. At current or future hadron collider experiments, the walking pions are potentially produced via photon fusion process [36]. The detail analyses for these experimental signatures are to be discussed in future publications.

Moreover, actually we can have rich amount (35 kinds) of dark matter candidates (corresponding to the $\pi_{q\phi}$ states in Eq. [26]), except the heavy walking $q'$ decaying to EW bosons just like the $\pi_{q\phi}$'. Hence the thermal history and testability at underground and satellite experiments for those dark matter particles are deserved to be addressed as well.

As noted in the Introduction, the initial condition for the inflaton has been set to be away (to the left side) enough from the vacuum in the potential, by assuming some trapping mechanism to work there, as discussed in the literature [5] for the CW-SFI scenario. Actually, it would not be so plausible to apply the existing trapping mechanism: first, in our benchmark scenarios the inflaton gets coupled to the SM-like Higgs via a Higgs portal coupling as noted in [41], which would be crucial for the trapping to work [5]. Note the size of the portal coupling ($\lambda_{\text{mix}}$) turns out to be extremely smaller than the standard-Higgs quartic coupling of $O(10^{-1})$: $\lambda_{\text{mix}} = m_H^2/\chi^2 \lesssim 10^{-26}$ (see Eq. [29]), hence the Higgs parametric resonance may not work well for trapping the $\chi$ at the origin of the potential as argued in [5]. However, in contrast to the literature having arguments based on the potential for elementary scalar fields, another trapping possibility other than coupling to the SM Higgs might be present in the case of the composite inflaton we have employed, which could be closely tied with the underlying nonperturbative gauge dynamics. Though being beyond the current scope, this issue will involve some generic features and expected developments on the particle production mechanism, so it would be worth pursuing in the future.

Acknowledgements

We are grateful to Satoshi Iso and Kazunori Kohri for useful comments. We are also grateful to Seishi Enomoto for valuable discussions. H.I. thanks for the hospitality of Center for Theoretical Physics and College of Physics, Jilin University where the present work has been partially done. This work was supported in part by the National Science Foundation of China (NSFC) under Grant No. 11747308 and 11975108, and the Seeds Funding of Jilin University (S.M.). The work of H.I. was partially supported by JSPS KAKENHI Grant Number 18H03708.

References

[1] S. R. Coleman and E. J. Weinberg, Phys. Rev. D 7 (1973) 1888.
[2] G. Barenboim, E. J. Chun and H. M. Lee, Phys. Lett. B 730 (2014) 81 doi:10.1016/j.physletb.2014.01.039 [arXiv:1309.1695 [hep-ph]].
[3] N. Aghanim et al. [Planck Collaboration], arXiv:1807.06209 [astro-ph.CO].
[4] S. Iso, K. Kohri and K. Shimada, Phys. Rev. D 91 (2015) no.4, 044006 doi:10.1103/PhysRevD.91.044006 [arXiv:1408.2339 [hep-ph]].
[5] S. Iso, K. Kohri and K. Shimada, Phys. Rev. D 93 (2016) no.8, 084009 doi:10.1103/PhysRevD.93.084009 [arXiv:1511.05923 [hep-ph]].
[6] W. E. Caswell, Phys. Rev. Lett. 33, 244 (1974), doi:10.1103/PhysRevLett.33.244.
[7] T. Banks and A. Zaks, Nucl. Phys. B 196, 189 (1982), doi:10.1016/0550-3213(82)90035-9.
[8] V. A. Miransky and K. Yamawaki, Phys. Rev. D 55, 5051 (1997) Erratum: [Phys. Rev. D 56, 3768 (1997)] doi:10.1103/PhysRevD.56.3768, 10.1103/PhysRevD.55.5051 [hep-th/9611142].
[9] V. A. Miransky and K. Yamawaki, Phys. Rev. D 89, 055016 (2014), doi:10.1103/PhysRevD.89.055016 [hep-ph/1401.5936].
[10] C. N. Leung, S. T. Love and W. A. Bardeen, Nucl. Phys. B 273, 649 (1986), doi:10.1016/0550-3213(86)90382-2.
[11] Y. Aoki et al. [LatKMI Collaboration], Phys. Rev. D 87, no. 9, 094511 (2013) doi:10.1103/PhysRevD.87.094511 [arXiv:1302.6859 [hep-lat]].
[12] T. Appelquist et al. [LSD Collaboration], Phys. Rev. D 90, no. 11, 114502 (2014) doi:10.1103/PhysRevD.90.114502 [arXiv:1405.4752 [hep-lat]].
[13] A. Hasenfratz, D. Schaich and A. Veernala, JHEP 1506, 143 (2015) doi:10.1007/JHEP06(2015)143 [arXiv:1410.5886 [hep-lat]].
[14] Y. Aoki et al. [LatKMI Collaboration], Phys. Rev. D 89, 111502 (2014) doi:10.1103/PhysRevD.89.111502 [arXiv:1403.5000 [hep-lat]].
