Application of Pentagonal Neutrosophic Number in Shortest Path Problem

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Abstract: Real-human kind issues have distinct sort of ambiguity and among them; one of the critical troubles is solving the shortest path problem. In this contribution, we applied the developed score function and accuracy function of pentagonal neutrosophic number (PNN) into a shortage path selection problem. Further, a time dependent and heuristic cost function related shortest path algorithm is considered here in PNN area and solved it utilizing an influx of dissimilar rational & pioneer thinking. Lastly, estimation of total ideal time of the graph reflects the importance of this noble work.

Keywords: PNN, Score and accuracy function, shortest path algorithm.

Introduction: The perception of fuzzy set was first manifested by Professor Zadeh [1] in 1965 to grip the uncertain data. Since then, the conception of fuzziness plays a most important feature to solve engineering and statistical problem. As the researches goes on, researchers from different areas published several articles in this areas and they extended the idea of fuzzy set in various fields according to their need. Recently, researchers invented the perceptions of pentagonal [2], hexagonal [3], heptagonal [4] fuzzy numbers and they applied it in distinct areas like operation research based EOQ, EPQ model, game theory, transportation problem etc. Further, in 1986 Prof. Atanassov [5] demonstrated the idea of intuitionistic fuzzy set which was the extension of Zadeh’s fuzzy set. Here, he considered both membership and non-membership function instead of Zadeh’s membership function. After that, a basic question grows up into our mind that how can we construct the mathematical model to deal with the idea of vagueness? Different kinds of methodologies were devised by using the researchers to describe intricately the conceptions of some new unsure parameters and to handle these complex problems, the selection makers placed forth their numerous thoughts in disjunctive areas. Later, Smarandache [6] in 1998 developed the remarkable idea of neutrosophic set which contains three disjunctive membership function namely i) truth ii) hesitation and iii) falsity. Actually, this is the extension of intuitionistic fuzzy set and general Zadeh’s fuzzy set. As researches goes on, researchers introduced triangular [7], trapezoidal [8] and very recently pentagonal [9] neutrosophic set and its classification in different cases. Recently, de-neutrosophic technique of pentagonal neutrosophic number [10], score function based application [11] and MCGDM problem [12] has been illustrated by Chakraborty et. al. Also, Chakraborty [13, 14] manifested the concept of cylindrical neutrosophic number in research domain and applied it in graph theory and MCGDM problem. A few novel works [15-25] are also comes out recently in neutrosophic area which plays an essential role in different fields like MCGDM, mathematical modeling, neutro-algebra, cryptography, linear programming, and topological spaces. In this current area, Shortest path search problem is one of the important problem in neutrosophic domain. Recently, Kumar et al [26] developed neutrosophic shortest path, [27] integer valued neutrosophic shortest path, [28] Gaussian based shortest path and [29] weighted arc length based shortest path in neutrosophic area. Also, Broumi et al [30] manifested bellman shaped shortest path problem which plays a vital role in graph theory research.

This paper deals with the conception of pentagonal neutrosophic number in different aspect. Nowadays, researchers are very much interested in doing shortest path problem in artificial intelligence problem in PNN environment. Here, we consider a shortest path problem in PNN area where we utilize the idea of our developed score and accuracy function for solving the problem.

1.1 Motivation

The idea of vagueness performs an important position in construction of mathematical modeling, economic problem and social real lifestyles hassle and so on. If, anyone consider a shortest path problem in PNN area then

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how he/she will manage it and solve it? How can we relate PNN and crispification result? From this aspect we actually try to develop this research article.

1.2 Novelties

Only some of the articles are published in PNN area till now. Although it can be applied in many fields and compute the results there. Our main focus is to apply the established PNN number in different areas.

(i) Develop score and accuracy function.

(ii) Application of our established score function in shortest path problem.

2. Preliminaries

2.1 Definition: Fuzzy Set: [1] Set \( M \) called as a fuzzy set when represented by the pair \((x, \mu_M(x))\) and thus stated as \( M = \{(x, \mu_M(x)) : x \in X, \mu_M(X) \in [0,1]\} \) where \( x \) is crisp set \( X \) and \( \mu_M(X) \in \) the interval [0,1].

2.2 Definition: Intuitionistic Fuzzy Set (IFS): [5] An fuzzy set \( \tilde{S}_x \) in the universal discourse \( X \), symbolized widely by \( x \) is referred as Intuitionistic set if \( \tilde{S}_x = \{(x; [\gamma(x), \delta(x)]) : x \in X\} \), where \( \gamma(x): X \to [0,1] \) is termed as the certainty membership function which specify the degree of confidence, \( \delta(x): X \to [0,1] \) is termed as the uncertainty membership function which specify the degree of indistinctness.

\( \gamma(x), \delta(x) \) exhibits the following relation: \( 0 \leq \gamma(x) + \delta(x) \leq 1 \).

2.3 Definition: Neutrosophic Set: [6] A set \( N_M \) in the universal discourse \( X \), figuratively represented by \( x \) named as a neutrosophic set if \( N_M = \{(x; [\lambda_M(x), \pi_M(x), \sigma_M(x)]) : x \in X\} \), where \( \lambda_M(x): X \to \rightarrow 0+1 \) is stated as the truthness function, which designates the degree of confidence, \( \pi_M(x): X \to \rightarrow 0+1 \) is stated as the hesitation function, which designates the degree of indistinctness, and \( \sigma_M(x): X \to \rightarrow 0+1 \) is stated as the falseness function, which designates the degree of deceptiveness on the decision taken by the decision maker.

\( \lambda_M(x), \pi_M(x), \sigma_M(x) \) displays the following relation:

\[ -0 \leq \text{Sup} \{\lambda_M(x)\} + \text{Sup} \{\pi_M(x)\} + \text{Sup} \{\sigma_M(x)\} \leq 3 + \]

2.4 Definition: Single-Valued Neutrosophic Set: [7] A Neutrosophic set \( N_M \) in the definition 2.3 is assumed as a Single-Valued Neutrosophic Set (\( S\overline{N}M \)) if \( x \) is a single-valued independent variable. \( S\overline{N}M = \{(x; [\lambda_{\overline{N}M}(x), \pi_{\overline{N}M}(x), \sigma_{\overline{N}M}(x)]) : x \in X\} \), where \( \lambda_{\overline{N}M}(x), \pi_{\overline{N}M}(x), \sigma_{\overline{N}M}(x) \) signified the notion of correct, indefinite and incorrect memberships function respectively.If three points \( d_0, e_0, f_0 \) exists for which \( \lambda_{\overline{N}M}(d_0) = 1, \pi_{\overline{N}M}(e_0) = 1 & \sigma_{\overline{N}M}(f_0) = 1 \), then the \( S\overline{N}M \) is termed neut-normal.

\( \overline{S}\overline{N}M \) is called neut-convex indicating that \( \overline{S}\overline{N}M \) is a subset of a real line by meeting the resulting conditions:

i. \( \lambda_{\overline{N}M}(\delta d_1) + (1-\delta)d_2) \geq min(\lambda_{\overline{N}M}(d_1), \lambda_{\overline{N}M}(d_2)) \)

ii. \( \pi_{\overline{N}M}(\delta d_1) + (1-\delta)d_2 \leq max(\pi_{\overline{N}M}(d_1), \pi_{\overline{N}M}(d_2)) \)

iii. \( \sigma_{\overline{N}M}(\delta d_1) + (1-\delta)d_2 \leq max(\sigma_{\overline{N}M}(d_1), \sigma_{\overline{N}M}(d_2)) \)

where \( d_1, d_2 \in \mathbb{R} \) and \( \delta \in [0,1] \)

2.5 Definition: Single-Valued Pentagonal Neutrosophic Number: [9] A Single-Valued Pentagonal Neutrosophic Number (\( \overline{M} \)) is defined as, \( \overline{M} = \left\{ ([s, t, u, v, w]; \mu), ([s', t', u', v', w']; \theta), ([s'', t'', u'', v'', w'']; \eta) \right\} \), where \( \mu, \theta, \eta \in [0,1] \). The truthness function \( \mu_{\overline{S}}: \mathbb{R} \to [0, \mu] \), the indeterminacy function \( \theta_{\overline{S}}: \mathbb{R} \to [\theta, 1] \) and the falsity function \( \eta_{\overline{S}}: \mathbb{R} \to [\eta, 1] \) are given as:

\[
\mu_{\overline{S}}(x) = \begin{cases} 
\mu_{\overline{S}}(x) & s \leq x < t^2 \\
\mu_{\overline{S}}(x) & t \leq x < u^2 \\
\mu_{\overline{S}}(x) & u \leq x < v^2 \\
0 & otherwise
\end{cases} \\
\theta_{\overline{S}}(x) = \begin{cases} 
\theta_{\overline{S}}(x) & s \leq x < t^2 \\
\theta_{\overline{S}}(x) & t \leq x < u^2 \\
\theta_{\overline{S}}(x) & u \leq x < v^2 \\
1 & otherwise
\end{cases}
\]

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3. Score Function: Score function actually relates any uncertain number and the crisp number in our real world. A score function is defined and developed in PNN $\tilde{W}_{pen} = (w_1, w_2, w_3, w_4, w_5; \pi, \mu, \sigma)$ as,

$$
\eta(x) = \begin{cases} 
\eta_wn(x) & \text{for } x \leq \mu^1 \\
\eta_wn(x) & \text{for } \mu^1 < x < \mu^2 \\
\eta_wn(x) & \text{for } \mu^2 < x < \mu^3 \\
\theta & \text{for } x = \mu^3 \\
\eta_hn(x) & \text{for } \mu^3 < x < \mu^4 \\
\eta_hn(x) & \text{for } \mu^4 < x < \mu^5 \\
1 & \text{otherwise}
\end{cases}
$$

Here, $(1 + \pi)$ is the beneficiary portions of PNN membership function and $(1 - \mu - \sigma)$ is the hesitation portions of PNN membership function. Also, we have the mean of the components as, $\frac{w_1 + w_2 + w_3 + w_4 + w_5}{5}$

Thus, Score function is described as $SC_{pen} = \frac{1}{15} (w_1 + w_2 + w_3 + w_4 + w_5) \times (2 + \pi - \mu - \sigma)$,

Accuracy function is described as $AC_{pen} = \frac{1}{15} (w_1 + w_2 + w_3 + w_4 + w_5) \times (2 + \pi + \mu + \sigma)$

3.1 Relationship between any two PNN:

Let us consider any two PNN defined as follows

$$W_{pen1} = (\pi_{pen1}, \mu_{pen1}, \sigma_{pen1}) \quad W_{pen2} = (\pi_{pen2}, \mu_{pen2}, \sigma_{pen2})$$

1) $SC_{pen1} > SC_{pen2}, W_{pen1} > W_{pen2}$
2) $SC_{pen1} < SC_{pen2}, W_{pen1} < W_{pen2}$
3) $SC_{pen1} = SC_{pen2}, W_{pen1} = W_{pen2}$

Then, if $AC_{pen1} > AC \quad W_{pen1} > W_{pen2}$

$AC_{pen1} < AC_{pen2}, W_{pen1} < W_{pen2}$

$AC_{pen1} = AC_{pen2}, W_{pen1} = W_{pen2}$

4. Shortest Path Search Algorithm under PNN Environment:

Here we consider a problem in PNN environment to compute the shortest path in a very simple way. Shortest path Search Algorithm is one of the best and popular skills used in path finding and graph traversals. Many games and web-based graphs are used here to compute the shortest path very efficiently. This Algorithm finds the shortest path through the search space using the hemistich function. It uses a best first search graph algorithm and finds a least cost path from current node to destination node. Consider a weighted graph in PNN area [10] whose weights and heuristic cost function are given as a pentagonal neutrosophic number with multiple nodes and we want to reach the target node to starting node as quick as possible. It defined a heuristic cost function $S(n) = p(n) + h(n)$ where $S(n) =$ estimated cost of the cheapest solution, $p(n) =$ cost to reach node $n$ from the starting position, $h(n) =$ estimated heuristic cost. Time and cost these are hesitation factors in case of real life problem. Here we consider the functions $p(n), h(n)$ are both pentagonal neutrosophic number. This algorithm expands less search tree and computes the optimal result faster.

4.1 Algorithm

Step 1: Convert all the PNN into crisp number using the established score value Section (3).

Step 2: Placed the starting node to open list

Step 3: Check whether the open list is empty or not, if the list is empty then stop the process.

Step 4: choose the node from the open list, which has the least value of estimation function $S(n)$, if node "n" is target node then back to success and stop.

Step 5: Expand node "n" and produce all of its successes and put ”n” in the closed list.

- For each successes "n", check whether “n” is already in the open or closed list.
- If not then compute evaluation function for “n” and placed it into open list.

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Step 6: Else if node “n” is already in open and closed then it should be attached to the back pointer which reflects the lowest $g(n')$ value.

Step 7: Stop.

4.2 Flowchart:

![Flowchart](image)

Figure 4.2.1: Flowchart for the problem

4.3 Illustrative Example: Find the shortest path from A to F of the following graph in PNN environment.

| Edges  | Optimistic Time                  | Stage | Heuristic Value                  |
|--------|----------------------------------|-------|----------------------------------|
| $E_1$  | $(2,3,4,5,6; 0.4,0.5,0.6)$       | A     | $(0.3,0.7,1.2,1.5,2; 0.6,0.5,0.3)$|
| $E_2$  | $(3,4,5,6,7; 0.6,0.3,0.4)$       | B     | $(0.5,0.8,1.4,2.4; 0.6,0.7,0.4)$   |
| $E_3$  | $(1,2,2.5,3,3.5; 0.6,0.4,0.5)$   | C     | $(0.2,0.4,0.6,0.8,1; 0.8,0.6,0.5)$  |
| $E_4$  | $(0,0.5,1.3,5; 0.3,0.2,0.6)$     | D     | $(0.8,1.3,1.8,2.4,3; 0.3,0.4,0.5)$  |
| $E_5$  | $(1.5,2.2,5,3,4.5; 0.3,0.4,0.3)$ | E     | $(0.7,1.5,2,2.5,3; 0.7,0.4,0.4)$   |
| $E_6$  | $(2,3,3.5,4,4.5; 0.7,0.2,0.4)$   |       |                                  |
### Step-1 Network Diagram:

![Network Diagram]

### Step-2 Crispification using the established Score function (3)

| Edges | Optimistic Time |
|-------|-----------------|
| $E_1$ | 1.73            |
| $E_2$ | 2.83            |
| $E_3$ | 1.36            |
| $E_4$ | 0.95            |
| $E_5$ | 1.44            |
| $E_6$ | 2.38            |
| $E_7$ | 2.80            |
| $E_8$ | 0.28            |

| Stage | Heuristic Value |
|-------|-----------------|
| A     | 0.68            |
| B     | 0.71            |
| C     | 0.34            |
| D     | 0.87            |
| E     | 1.23            |
| F     | 0.93            |

### Step-3 Here, $A$ is the starting node

- $A \rightarrow S(n) = p(n) + h(n) = 0 + 0.68 = 0.68$
- $A \rightarrow B \quad S(n) = 1.73 + 0.71 = 2.44$
- $A \rightarrow F \quad S(n) = 2.38 + 0.93 = 3.31$ (hold)
Step-4

\[ A \rightarrow B \rightarrow C \quad \frac{S(n)}{\text{hold}} = 1.73 + 2.83 + 0.34 = 4.9 \text{ (hold)} \]

\[ A \rightarrow B \rightarrow E \quad S(n) = 1.73 + 0.95 + 1.23 = 3.91 \]

Step-5

\[ A \rightarrow B \rightarrow E \rightarrow F \quad S(n) = 1.73 + 0.95 + 0.28 + 0.93 = 3.89 \]

\[ A \rightarrow B \rightarrow E \rightarrow D \quad S(n) = 1.73 + 0.95 + 1.44 + 0.87 = 4.99 \text{ (hold)} \]
SO, the final shortest path is \( A \rightarrow B \rightarrow E \rightarrow F \) and optimal cost is = 2.96 unit.

5. Conclusion and future research scope

The concept of PNN has an adequate scope of utilization in various studies in different domain. In this research article, we strongly erect the perception of score and accuracy function from different aspects. Additionally, we consider a shortest path problem in PNN environment and resolve the problem applying the idea of score function. Since, there may be no such articles is until now hooked up in PNN area, for this reason we cannot done comparison study of our work with the other established methods.

Further, researchers can immensely apply this idea of neutrosophic number in numerous flourishing research fields like engineering problem, mobile computing problems, diagnoses problem, realistic mathematical modeling, social media problem etc.

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