Thermalization Processes in Interacting Anderson Insulators

Z. Ovadyahu
Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel

Abstract

This paper describes experiments utilizing a unique property of electron-glasses to gain information on the fundamental nature of the interacting Anderson-localized phase. The methodology is based on measuring the energy absorbed by the electronic system from alternating electromagnetic fields as function of their frequency. Experiments on three-dimensional (3D) amorphous indium-oxide films suggest that, in the strongly localized regime, the energy spectrum is discrete and inelastic electron-electron events are strongly suppressed. These results imply that, at low temperatures, electron thermalization and finite conductivity depend on coupling to the phonon bath. The situation is different for samples nearing the metal-insulator transition; in insulating samples that are close to the mobility-edge, energy absorption persists to much higher frequencies. Comparing these results with previously studied 2D samples [Ovadyahu, Phys. Rev. Lett., 108, 156602 (2012)] demonstrates that the mean-level spacing (on a single-particle basis) is not the only relevant scale in this problem. The possibility of de-localization by many-body effects and the relevance of a nearby mobility-edge (which may be a many-body edge) are discussed.

PACS numbers: 72.80.Ng 73.61.Jc 72.20.Ee

INTRODUCTION

The question of how Coulomb interactions affect Anderson localization has been a challenging problem for decades. It was first addressed by Fleishman and Anderson [1] in the context of the stability of the insulating phase as well as the mechanism of system thermalization and energy exchange involved in hopping conductivity. Thermalization of Fermi-gas systems depends on inelastic scatterings of electrons. Energy-exchange via electron-electron scattering is required for establishing the Fermi-Dirac thermal-distribution, which defines the electronic temperature. Such events lead to decoherence of the electrons and thus also control the quantum effects exhibited by the system.

The most frequently encountered mechanisms for electronic energy-transfer in condensed matter systems are e-e and electron-phonon (e-ph) scatterings. The latter is needed to maintain steady-state conditions when the system is driven by an external source, and in particular, are responsible for the validity of Ohm’s law.

In metallic systems at sufficiently low temperatures the e-e inelastic-rate $\gamma_{\text{in}}^{e-e}$ is usually the main source of scattering [2]. Both, $\gamma_{\text{in}}^{e-e}$ and the e-ph inelastic-rate $\gamma_{\text{in}}^{e-ph}$ can be measured in the diffusive regime based on weak-localization effects [3]. While similar quantum effects sometimes extend into the hopping regime of the same system [4], a theoretical framework to analyze such data and obtain inelastic scattering rates is unfortunately not yet established. $\gamma_{\text{in}}^{e-e}$ in the hopping regime is defined in this work as the rate of energy-exchange $\delta E$ involved in electron-electron scatterings where $\delta E \neq 0$. These processes contribute to the overall life-time broadening of the electronic level, which is what our experiments are designed to capture. It should be remarked that the energy-state broadening includes the contribution of other mechanisms (such as electron-phonon inelastic scattering). In the diffusive regime the contribution of e-e inelastic-rate may be separated from the e-ph one as demonstrated by Bergmann [5]. This is not yet achievable by the methodology used here for the insulating regime. On the other hand, this method yields information on a wide frequency range (rather than just an average value for the inelastic-rate) and it can still be shown that relative to the diffusive regime $\gamma_{\text{in}}^{e-e}$ is dramatically suppressed.

It was recently shown that in the two-dimensional (2D) hopping regime of crystalline indium-oxide In$_2$O$_{3-x}$, $\gamma_{\text{in}}^{e-e}$ is suppressed by $\approx$six orders of magnitude relative to its value at the diffusive regime at the same temperature [5]. This was based on utilizing a unique property of electron-glasses [6]; using a non-Ohmic field to take the system out of equilibrium, endows the system with excess conductance that may be used as an empirical measure of the energy absorbed by the electrons from the field. This technique allows a measurement on systems with very small volume, it is sensitive enough to allow for weak absorption from electric fields, and can be carried over a wide frequency range.

Here we report on measurements performed on Anderson-localized amorphous indium-oxide films (In$_x$O) that exhibit three-dimensional (3D) hopping transport. The results of our measurements suggest that e-e energy exchange in In$_x$O is strongly suppressed relative to its value in the diffusive regime at the same temperature. Analysis of these results suggests that thermalization of the electronic system is governed by $\gamma_{\text{in}}^{e-ph}$ as was the case in the 2D crystalline version [5]. However, approaching the metal-insulator transition by reducing the quenched disorder, the perceived inelastic-rate tends towards the $\gamma_{\text{in}}^{e-e}$ value typical of the diffusive regime. This occurs while the system is still insulating; it exhibits variable-range-hopping transport and its disorder is as strong as that of the 2D samples where $\gamma_{\text{in}}^{e-e}$ was highly suppressed [6]. We point out some similarity of these observations with a peculiar temperature dependence of the conductivity in 3D systems near their metal-insulator transition,
which has been observed in several materials. The role of dimensionality, inherent inhomogeneities, many-body effects, and other issues that might be involved in bringing about an apparent de-localized behavior are discussed.

EXPERIMENTAL

Samples preparation and characterization

Three batches of In$_x$O$_{3-x}$ samples were used in this study. They were prepared by e-gun evaporation onto room-temperature substrates using 99.999% pure In$_2$O$_3$ sputtering target pieces. Substrates were either 1mm thick microscope glass-slides, or on 0.5µm SiO$_2$ layer thermally grown on <100> silicon wafers. Samples thickness d was 630Å or 1050Å for the glass-slides, and d=750Å for the Si wafers. Rate of deposition and thickness were measured by a quartz thickness monitor calibrated using optical interference measurements on thick MgF$_2$ films. Deposition was carried out at the ambience of (1-3)-10$^{-4}$ Torr oxygen pressure maintained by leaking 99.9% pure O$_2$ through a needle-valve into the vacuum chamber (base pressure ≃10$^{-6}$ Torr). Rates of deposition used for the samples reported here were typically 0.6-0.9 Å/s. Under these conditions, the In$_x$O$_3$ samples had carrier-concentration n in the range (7-8)-10$^{19}$cm$^{-3}$ as measured by Hall-Effect at room temperatures on samples that were patterned in a 6-probe configuration using stainless-steel masks. These samples were prepared during the same deposition as the strips used for the low temperature transport measurements. A standard Hall-bar geometry was used with the active channel being a strip of 1 mm wide, and 10 mm long. The two pairs of voltage probes (that doubled as Hall-probes), were spaced 3 mm from one another along the strip. This arrangement allowed us to assess the large scale uniformity of the samples, both in terms the longitudinal conductance and the Hall effect. Excellent uniformity was found on these scales; resistivities of samples separated by 1 mm along the strip were identical to within ±5%. No change (within the experimental error of 3%) was observed in the hall effect due to annealing (tested for samples with room temperature resistivity smaller than ≃0.4Ωcm which was the highest ρ in the samples studied in this work). On a mesoscopic scales (10-100nm) however, In$_x$O$_3$ films show compositional inhomogeneities; the various effects these may have on transport properties of these films were reported in [7].

As-deposited samples had room-temperature resistivity ρ in excess of 10$^5$Ωcm which, for the low temperature studies, had to be reduced by several orders of magnitude. This was achieved by thermal annealing at temperatures T$_a$<75 degree Celsius to prevent crystallization. For a comprehensive description of the annealing process and the associated changes in the material microstructure see [7].

Measurements techniques

Conductivity of the samples was measured using a two-terminal ac technique employing a 1211-ITHACO current pre-amplifier and a PAR-124A lock-in amplifier. Except when otherwise noted, measurements reported below were performed with the samples immersed in liquid helium at T=4.1K maintained by a 100 liters storage-dewar. This allowed long term measurements of samples as well as a convenient way to maintain a stable bath temperature. The ability to keep the sample at ≃4K for long times is essential for these studies where a typical series of measurements takes 4-6 weeks to accomplish. The ac voltage-bias, used during the off-stress periods, was small enough to ensure linear response conditions (judged by Ohm’s law being obeyed within the experimental error).

As measure of disorder we use the Ioffe-Regel dimensionless parameter, k$_Fℓ$=(9π$^4$/n)$^{1/3}$R$_Q$/ρRT, where R$_Q$=h/e$^2$ is the resistance quantum. This is based on free-electron expressions using the measured room-temperature resistivity ρ$_{RT}$ and the carrier-concentration n, obtained from the Hall-Effect measurements, as parameters.

Several sources were used for exciting the system by non-Ohmic fields; the internal oscillator of the PAR124A (up to 2 kHz and 10 Vrms) (Fluke PM5138A (dc and up to 10 MHz and 40 Vpp), and Tabor WS8101 (up to 100 MHz and boosted, when necessary, by Ophir 5084 RF power-amplifier). Complementary studies in the microwave regime employed the high-power synthesizer HP8360B. Care was taken in these experiments to use “RF-safe” components near the sample immediate vicinity to minimize spurious heating. For the same reason, it was ascertained, by performing four-probe measurements, that the contacts resistance was always negligible relative to the sample resistance.

Optical excitation was accomplished by exposing the sample to AlGaAs diode (operating at ≃0.88±0.05µm), placed ≃15mm from the sample. The diode was energized by a computer-controlled Keithley 220 current-source. The samples were attached to a probe equipped with calibrated Ge and Pt thermometers and were wired by triply-shielded cables to BNC connectors at room temperatures. The effective capacitance of the wires was ≤20pF. This allowed the use of 23-1500Hz ac technique without a significant phase shift for any of the samples used here.

Fuller details of measurement techniques are given elsewhere [8].
RESULTS AND DISCUSSION

Absorption measured via non-equilibrium transport

The main technique used in this study is the 'stress-protocol' previously used in aging experiments [9]. The procedure is composed of the following stages (see Fig.1 for details): After the sample is equilibrated at the measuring temperature (typically for 24 hours), its conductance is continued to be measured for few thousand s. Dashed lines delineate the equilibrium conductance G(F₀).

FIG. 1: Conductance versus time G(t) illustrating a typical run of a 'stress-protocol'. The sample here is a InₓO with d=630Å and resistivity ρ=21.4Ω·cm at T=4.1K. The Ohmic and stress fields here are F₀=100V/m and Fstress=10²V/m, both at 73Hz. The inset shows the logarithmic relaxation of δG(t) and the definition of δG₀. Dashed lines delineate the equilibrium conductance G(F₀).

REF: Absorption measured via non-equilibrium transport

The mechanism by which stressing the system with a non-Ohmic field increases the electronic energy is essentially Joule-heating; the energy absorbed by the electrons gives rise to an excess phonons, making it somewhat 'hotter' than the bath. A steady-state may be established, while the stress-field is on, by the flow of energy carried by the phonons in the sample into the thermal-bath. The increased density of high energy phonons (over the phonon population in equilibrium at the bath temperature), randomizes the charge configuration of the electron-glass in a similar vein that raising the bath-temperature would [11]. This produces the excess conductance that relaxes back to its near-equilibrium value once the stress is relieved.

It is therefore plausible to take δG₀ as a measure of the energy δε absorbed by the electronic system from the field. As long as δG₀/G(0)<1, δG₀ is arguably proportional to δε.

It is emphasized that the only assumption we make in this procedure is that δε enters the sample via the coupling of Fstress to the electronic system. No other
the excess conductance after the switch relaxes logarithmically held there for the reminder of the run. The inset shows that voltage, initially at -198V is switched to +198V in 2s, and is monitored for t=90s to establish a baseline G, the gate-paint coating on its back side acting as gate. After G(t) is on a 1mm thick glass substrate with a conducting silver-paint coating the assumption is made. In particular, the reason for the very sluggish release of this energy is not relevant for our considerations in this work. The logarithmic nature of δG(t) is taken as a convenient empirical fact that allows us to estimate the absorption via transport measurements. This rationale was used in [5] in the study of the frequency dependence of the electronic absorption of two-dimensional (2D) films of In\textsubscript{2}O\textsubscript{3-x}. In this study we extend the study to three-dimensional (3D) films.

Electronic absorption versus frequency and disorder

Our first goal here is to define a protocol that allows a meaningful comparison between energy absorption from the stress-field applied to the system at various frequencies. The natural choice is to normalize δG\textsubscript{dc} \textsuperscript{f} - the excess conductance measured under F\textsubscript{stress} at a frequency f by δG\textsubscript{dc} \textsuperscript{0} - the excess conductance measured with a dc field while keeping the same ΔG/G(0) and the same t\textsubscript{w} for each tested frequency of the applied stress. It was found that for f<30Hz δG\textsubscript{dc} \textsuperscript{f} was indistinguishable (within the experimental error) from δG\textsubscript{dc} \textsuperscript{0} and in the experiments reported here we used the result for F\textsubscript{stress} operating at f=11-23Hz as the normalizing value. For applying the stress-protocol at frequencies above ≈1kHz, the conductance was measured at f=23Hz and under low-bias conditions to ensure linear-regime measurement throughout the protocol. A stress-field F\textsubscript{stress} with frequency f was capacitively superimposed across the sample. The high frequency component of F\textsubscript{stress} was filtered out in the input to the current preamplifier (in addition to the band-pass filtering in the 124A lock-in amplifier) such that the conductance was measured at the low frequency. The amplitude of F\textsubscript{stress} was adjusted to achieve the desired ΔG/G(0) based on the conductance reading at f=23Hz. The relaxation part of the protocol was always measured under near-Ohmic conditions and at a low frequency (or at dc).

Results of absorption at different frequencies for three of the 3D samples studied with the protocol described above are shown in Fig.4. For comparison, the figure includes the 2D samples studied previously [5]. Looking at these data one notes the following:

1 The general trend is for the absorption to decrease with frequency, and in all three cases shown in Fig.4...
there is a faster decline of the absorption with $f$ above a certain roll-off frequency $f_{ROI}$. It is observed that $f_{ROI}$ is considerably lower than the respective electron-electron inelastic rate $\gamma_{in}^{e-e}$ of the material (based on measurements performed on a 3D $In_xO$ system in its diffusive regime at the same temperature).

2 The frequency range over which the absorption decays appears to be rather wide, extending over several decades. For comparing results under different conditions, we take $f_{ROI}$ to be the frequency where $\delta G_{0}^{f}/\delta G_{0}^{d}c=1/2$.

3 For the range of disorder shown, there seems to be no dependence on the disorder in either 3D or 2D: samples with different disorder essentially identical absorption versus f curves. However, as will soon transpire, this is only true for samples in the strongly-localized regime.

It has been shown [3] that the roll-off frequency in the two-dimensional $In_2O_{3-x}$ samples (lower graph in Fig.4) is consistent with the electron-phonon inelastic scattering-rate $\gamma_{in}^{e-ph}$ of the material at 4K. This estimate was based on the assumption that, while under a dc stress-field, the system reaches steady-state conditions in which case the energy absorbed by the electrons from $F_{stress}$ equals the energy dissipated into the bath. This may be described by the following expression:

$$ V^2/R(V) = C_{el}(T*) \cdot U \cdot \Delta T \cdot \gamma(T*) $$ (1)

The L.H.S of Eq.1 is the Joule-heating term; $V$ is the voltage across the sample, and $R(V)$ is its resistance under $V$. The R.H.S is the heat-removal rate while $F_{stress}$ is on, and assuming steady-state conditions. In this equation $C_{el}$ is the electronic heat-capacity, $U$ the sample volume, $T*$ is an "effective electron-temperature", $\Delta T$ is $T*-T_{bath}$ (the bath temperature). For our samples being macroscopic, $\gamma(T*)$ should coincide with the inelastic electron-phonon rate $\gamma_{in}^{e-ph}$.

The parameters needed for calculating, $\gamma(T*)$ are all obtained from measurements on the respective sample except for $C_{el}$ and $T*$. Following the procedure used in [12], $T*$ may be estimated from $G(T)$ data (the uncertainty associated with this procedure will be commented on below).

The electronic heat capacity $C_{el}$ is proportional to the temperature and to $\partial n/\partial \mu$, the thermodynamic density-of-states of the material. Since the carrier-concentration of the $In_xO$ samples used in this work is comparable to that of $In_2O_{3-x}$ (and therefore $C_{el}$ for them should be similar), we can estimate of the ratio between $\gamma_{in}^{e-ph}$ of $In_xO$ to that of the $In_2O_{3-x}$ samples by:

$$ \frac{\gamma_{in}^{a}(T*)}{\gamma_{in}^{c}(T*)} = \frac{V_{a}^2 \cdot R_{a}(V_{c}) \cdot U_{c} \cdot \Delta T_{c}}{V_{c}^2 \cdot R_{c}(V_{a}) \cdot U_{a} \cdot \Delta T_{a}} $$ (2)

where the subscripts ‘$a$’ and ‘$c$’ signify values for the amorphous and crystalline material respectively. The voltages used in the respective experiments are extracted from the conductance-voltage $G(V)$ measurements of the respective samples. For the samples we chose for the analysis below, these data are shown in Fig.5.

The conditions that were used in the stress-protocol for our experiments were $\Delta G(0)/G(0)=0.6$, and the associated $F_{stress}$ was applied for $t_{w}=1400$ seconds. To achieve this $\Delta G(0)/G(0)$ a voltage of 10.2V was needed for the $In_xO$ sample as compared to 0.5V for the crystalline sample $In_xO$, as can be read from Fig.5. Note that these values (with the respective resistances), mean that the power invested from the (dc) field is $\approx 1500$-times larger for the $In_xO$ sample. The volumes for these samples were: $10^{-14}m^3=U_{c}$ and $1.2 \cdot 10^{-13}m^3=U_{a}$. The “effective temperature” $T*$ for the $In_2O_{3-x}$ sample estimated in this study was $4.8K\approx T_{c}^e$. This was based on the measured $G(T)$ data for the sample under steady-state conditions. Using the same logic here, the respective $T^e$ for the $In_xO$ sample, based on its $G(T)$ data, is $5.3K\approx T_{a}^e$. The conductance versus temperature curves for this sample and two other samples of the same $In_xO$ batch with different degrees of disorder are shown in Fig.6 below.

It should be remarked that using the sample resistance as a thermometer to obtain $T*$ is a dubious procedure; in general, the resistance change due to the applied non-Ohmic field is not just a heating effect. In fact, deep in the hopping regime most of the resistance decrease is
The frequency dependence of the absorption results discussed above suggests that $\gamma_{in}^{e-e}$ (and, in some sense, also $\gamma_{in}^{e-ph}$) is suppressed relative to its typical values in the diffusive regime. In the case of $\gamma_{in}^{e-e}$ this reduction is by several orders of magnitude (see Fig.4). These rate being the main sources of decoherence in most disordered electronic systems the question arises: Does it also mean that there is less decoherence in the Anderson insulator phase?

If one ignores the possibility that the highly disordered phase may breed decoherence agents that were not present in the diffusive regime, like new types of two-level-systems or local magnetic moments, then the answer is yes; the insulating phase may have reduced inelastic rate relative to the diffusive phase. This however applies to the coherence time; the spatial extent of the electron coherence will be limited by the highly reduced diffusion constant. There is experimental evidence for quantum coherent effects in the Anderson localized regime that, in some respects, are more prominent than in the diffusive regime but the phase-coherent length is limited to the hopping-length. The most compelling evidence for quantum-coherent effects is the anisotropy of conductance-fluctuations produced by magnetic field at different orientations is mesoscopic samples as well as in the magnetoresistance of two-dimensional samples. The existence of quantum-coherent effects deep in the hopping regime ought not be surprising. The overlap between the initial and final state is affected by the interference between different spatial trajectories. The phonon involved in the actual transition does not destroy the interference once the phonon wavelength exceeds the hopping-length, which inevitably happens at sufficiently low temperatures.

The suppression of $\gamma_{in}^{e-e}$ may appear as a natural consequence of the spectrum discreteness associated with low-
calization \[20, 21\]. On closer examination, and taking into account the role of Coulomb interaction, this issue is more complicated and yet unresolved. The problem was first raised by Fleishman and Anderson \[1\]. They considered several scenarios by which interactions may modify the single-particle aspects of Anderson-localized systems, while noting that \( G(T) \) of these systems still conforms to variable-range-hopping law. It may be illuminating then to see what we can infer from the VRH conductivity of the samples on the spectrum discreteness.

The space-confinement due to localization forces a discrete energy spectrum with a mean level-spacing of order \( (\hbar \gamma /c)^{-1} \approx \xi^2 /\ell^3 \) (and in terms of bulk resistivity the disorder \( \xi = (\hbar /2\pi e^2 C)^{1/2} \)) where \( \kappa \approx 10 \) is the dielectric constant for indium-oxide \[22\], and \( \xi \approx 10\AA \) as will be shown below. Therefore, unless many-body physics plays a role, it would appear that the spectral discreteness is large enough to suppress electron-electron inelastic scattering. Thermalization of the electronic system and non-zero conductivity then depend on the existence of a continuous bath, presumably phonons. Fleishman and Anderson \[1\] reached this conclusion for the localized system in the limit of short range interaction. Our absorption versus frequency results (Fig.4) are consistent with their conclusions for a realistic interaction range, probably of the order of the hopping-length \( r(T) \approx \xi(T_0 / T)^{1/3} \) or \( \approx \xi(T_0 / T)^{1/4} \) in 2D or 3D respectively.

The suppression of \( \gamma_n \) of Anderson insulators turns out however to be true only for samples in the strongly disordered regime; things appear to be more complicated as the MIT is approached.

The frequency dependence of the absorption measured on two samples that were annealed to bring their \( k_F \ell \) to 0.19 and 0.21, is shown in Fig.7. These data exhibit a different trend than the more disordered samples shown in Fig.4 above; the absorption still decreases with frequency but it seems to extend to higher frequencies, possibly even surpassing the \( \gamma_n \) of the diffusive regime at this temperature. At the same time, these samples are on the insulating side of the MIT, and their mean-level-spacing is still much larger than both \( E_C \) and the bath temperature; their characteristic energies \( T_0 \) (defined by the VRH conductivity expression) are \( \approx 4300 \)K (Fig.6) and \( \approx 6600 \)K (Fig.9 below) for the sample with \( k_F \ell = 0.21 \) and 0.19 respectively.

More intriguing is the observation that the disorder of the samples in Fig.4 is considerably larger than that of the 2D samples \{Fig.3 in \[3\] and Fig.4\}; the disorder in the latter is comparable to that of these 3D samples in terms of \( \delta (\xi) \) (and in terms of bulk resistivity the disorder in the 2D films it is even much weaker than in the 3D samples). If the reason for the energy-absorption cutoff in the 2D samples is the spectrum-discreteness of the electronic states (having \( \delta (\xi) \gg k_B T \)) then why the same is not sufficient for 3D samples that exhibit equally large \( \delta (\xi) \)?

The ”T* problem”

A possible direction to look for the difference between the 2D and 3D results may be related to the materials; as noted above amorphous and crystalline versions of indium-oxide have their differences but none that seems relevant for this particular feature. On the other hand, there is reason to suspect that dimensionality plays a role: Just insulating 3D samples of \( \text{In}_2\text{O}_{3-x} \) exhibit transport peculiarities that are not observed in 2D \[24\]. The 3D samples exhibited insulating characteristics only when cooled below a disorder-dependent T* above this temperature, they show a metallic-like G(T) law. On the other hand, 2D samples of this material, with the same range of bulk resistivity exhibited insulating behav-
ior (σ→0 when T→0) over the same temperature range [24]. Three dimensional samples of In$_2$O also exhibited the same "T*-problem" [24]. To illustrate, an example of the "T*-problem" is shown in Fig.8 using one of the currently studied specimen. Note that, at low temperatures, G(T) exhibits VRH conductivity (inset to Fig.8). This implies G(T→0)=0, which means the system is insulating. However above T*=60K the conductance law changes, and if one has no knowledge of the behavior of G(T) at lower temperatures one will conclude, extrapolating along the dashed curve in the main figure that G(T→0)>0, that the system is actually a metal.

A similar G(T) anomaly appears in quite a few other 3D systems (note that this feature is easier to identify when G(T) rather than R(T) is plotted). Such a peculiar G(T) may be seen in a series of amorphous Mn$_3$Si$_{1-x}$ samples [23], in amorphous Si$_{1-x}$Ge$_x$ [20], in granular GeSb [25], in crystalline GeSb$_2$Te$_4$ [28], in a-Gd$_3$Si$_{1-x}$ samples [29]. We are not aware of any 3D system that was tested over a wide temperature range near its MIT without showing the ambiguous G(T) characterizing the T*-problem. This feature may be generic to 3D systems near their Anderson transition.

The phenomenology associated with this anomalous G(T), in particular the systematic increase of T* with disorder, and the effect in 2D [24], raised the possibility that T* signifies the mobility-edge; a threshold energy $E_C$ separating extended states for E>$E_C$ from localized states for E<$E_C$. This indeed might account for the experimental observations. However, the values for T* necessary for this line of explanation turned out to be smaller than what one (perhaps naively) anticipates. Note that near the metal-insulator transition the distance $\Delta E$ to the mobility-edge is expected to obey [30]:

$$\Delta E \equiv |E_C-E_F|=E^*_C(g_c-g_C)/g_C^*$$

where, $E_F$ is the Fermi-energy, g is the dimensionless conductance, $g_C$ is the dimensional-conductance value at the MIT transition, and the exponent $\nu$ is ≈1. To fit the G(T) data to Eq.5 it was necessary to use for $E^*$, a value considerably smaller than the Fermi energy of the material, which shed some doubts on the notion that T* reflects the mobility-edge [24].

The current results, and in particular the apparent role of dimensionality, instigated a renewed look at these phenomena. The absorption experiments (Fig.7) and the G(T) behavior of just insulating 3D samples have this in common: Both exhibit diffusive characteristics, a tendency that becomes more conspicuous as they further approach the MIT, and both involve probing the system away from its ground-state: The stress experiments take the system far from equilibrium, the T* problem is a finite temperature phenomenon. Indeed, a simple explanation of the G(T) anomaly might be related to a temperature-dependent probing length. An insulating sample will exhibit a diffusive G(T) law when, for example, $L_T<\xi$ where $L_T$=($L_0$T) is the inelastic diffusion length. This however cannot account for the experimental T*-problem unless one assumes either, an unusual energy dependence for $\xi$ or a specific $\xi$ - distribution [31]. A many-body scenario may have to be considered.

Let us examine the T*-problem in the context of the current issue assuming for the moment it is in fact a mobility-edge. If $E_C$ is not far above $E_F$, then new avenues for electron-electron energy exchange may become available. It is thus of interest to find out how close is a system with a given kp$\ell$ to the transition. This may be estimated from the dependence of the localization-length $\xi$ on the order-parameter kp$\ell$ of the sample in question. The localization length is evaluated using the G(T) data with Eq.4, this yields the $\xi(kp_\ell)$ plot shown in Fig.9.

The dependence of $\xi$ on kp$\ell$ in this figure fits reasonably well the expression: $\xi=\xi_0[(g_c-g_C)/g_C]^\nu$ which is a variation on on the scaling form [30] (with $\nu$=1):

$$\xi=\xi_0[(g_c-g_C)/g_C]^{-\nu}$$

where $\xi_0$ is the value of the localization-length far from the MIT. Fitting the dependence of $\xi$ on kp$\ell$ (Fig.9) yields $\xi_0$=11Å, which is of the order of the Bohr-radius, a$_B$ $\approx$15Å for indium-oxide, so this is a plausible result. From Eq.6a and Eq.5 one gets:

$$\Delta E=E^*_C(\xi_0/\xi)$$
that a specific value for \( \xi / \ell \) is singled-out by percolation
constraints \( k_F \ell \). One should not be surprised to find deviations from the predictions of these models even for some aspects of the dc conductivity and anyway, for measurements that involve the bulk of the sample, a realistic distribution of localization-lengths should be taken into account. At finite frequencies diffusive regions need not percolate in the system; it is only necessary that the applied \( F_{stress} \) induces dissipative currents in them thus generating phonons in excess of those present in equilibrium. This then will be effective in system-randomization with the ensuing (once Ohmic conditions are restored), slowly relaxing excess-conductance which contributes to the perceived absorption.

A pertinent consideration here is a conceivable modification of the wavefunctions due to correlation and many-body effects. The Coulomb interaction on a scale of \( \xi \) derived from the \( G(T) \) data is comparable with \( \Delta E \) of the samples in Fig.7 therefore hybridization with states above \( E_C \) cannot be ruled out. It is difficult to estimate the relevance of these processes nearer the transition where the contribution of electronic polarization to the dielectric-constant becomes dominant, so a self-consistent treatment must be invoked.

Another complication is that the states above \( E_C \) are actually localized in the ground-state: they just appear to be extended under finite temperature or non-Ohmic fields (at high frequencies the latter will be effective even when the associated conductance-change is smaller than that required by the relative freedom from percolation constraints). Note that the many-body density-of-states grows extremely fast with energy (unlike in a single-particle scenario where this change is algebraic), and delocalization or an increase of the localization-length may occur due to the excess energy supplied by the stress-field.

The combination of potential fluctuations, higher-order excitations, and extended states lying close to the Fermi energy may enhance the occurrence of metallic "puddles". Obviously, dimensionality plays a role in any of these scenarios. If however the observed \( E_C \) is a many-body mobility-edge it should also occur in 2D systems albeit probably at a considerably higher energy. More experiments are needed to elucidate the relative importance of these mechanisms. The appearance of the \( T^4 \)-problem in so many systems may be suggestive of an underlying physical effect relevant for the MIT problem. It clearly deserves to be addressed whether it signifies a real mobility-edge or it is just a consequence of a finite temperature measurement. The effectively low value of \( E^* \) (relative to \( E_F \)) means that, over a wide range of the physical parameter that is used to characterize the disorder, the system may be still within the ‘critical’ regime of the metal-insulator transition.
Summary

We have presented results of energy absorption from applied electromagnetic-fields in three-dimensional In$_x$O$_{1-x}$ samples. For Anderson-insulating samples that are far from the metal-insulator transition, the absorption appears to be limited to frequencies that are of the order of the electron-phonon scattering rates of the material. This suggests that the hopping process in the range of temperature and disorder studied is mediated by phonons. Likewise, thermalization of the electronic system then hinges on the presence of a phonon bath. This is in agreement with the conclusions reached by Fleishman and Anderson [1]. These authors also anticipated a change of behavior as the mobility-edge is approached from below and expected that this would be reflected in G(T) that ought to include contribution from activation to extended states. The experimentally observed change in G(T) as the mobility-edge is approached appears to be more complicated, and it would appear that more elaborate models of conductivity need to be developed for the critical transport regime. These may also shed light on the observation that absorption from ac fields for the critical transport regime. These may also shed light on the observation that absorption from ac fields is more sensitive than temperature to the proximity of a mobility-edge (whether real or apparent).

The effective range of the interaction, clearly important to these issues, was not explicitly dealt with in our experiments. At finite temperatures the range of the Coulomb interaction is limited by the finite conductivity for which the relevant scale is presumably the hopping-length r(T). At liquid helium temperatures and for the range of the disorder in the samples used in this study, r(T) is of order of few hundred angstroms. This is an order of magnitude larger than the inter-carrier separation $n^{1/3}$ suggesting that, for the more disordered samples, our experimental results are relevant for systems with long-range interaction.

As the disorder in our samples was reduced and the diffusive phase approached, the absorption extended to progressively higher frequencies. This was observed on 3D samples but not in 2D samples with comparable degree of disorder. The similarity of this observation with the T*-problem that appears to be a common feature in many 3D systems near their MIT was pointed out. It is hoped that this problem will receive due theoretical attention. Possible relevance of a nearby mobility-edge for bringing about this behavior as well as the diffusive-like absorption characteristics of just insulating 3D samples was discussed. Various effects that might contribute to these phenomena were mentioned including possible many-body effects. It would be interesting to extend the absorption study to include the dependence of the absorption on the amplitude of the stress-field near the transition as it may shed some light on the relative importance of many-body effects.

Acknowledgement 1 This research has been supported by a grant administered by the Israel Academy for Sciences and Humanities.

Summary

1. L. Fleishman and P. W. Anderson, Phys. Rev. B 21, 2366 (1980).
2. A. Mittal, Quantum Transport in Submicron Structures, Advanced NATO Proceedings Kluwer Academic, Dordrecht, (1996).
3. G. Bergmann, Physics Reports, 107, 1 (1984); S. Kobayashi and F. Komori, Prog. Theor. Phys. Suppl. 84, 224 (1985).
4. Z. Ovadyahu, Waves in Random Media, 9(2), 241 (1999) and references therein.
5. Z. Ovadyahu, Phys. Rev. Lett., 108, 156602 (2012).
6. J. H. Davies et al, Phys. Rev. Lett., 49, 758 (1982); M. Gruneiswald et al, J. Phys. C, 15, L153 (1982); M. Pollak and M. Ortuno, Sol. Energy Mater., 8, 81 (1982); M. Pollak, Phil. Mag. B 50, 265 (1984); G. Vignale, Phys. Rev. B 36, 8192 (1987); M. Müller and L. de Ioffe, Phys. Rev. Lett. 93, 256403 (2004); C. C. Yu, Phys. Rev. Lett., 82, 4074 (1999); Vikas Malik and Deepak Kumar, Phys. Rev. B 69, 153103 (2004); D. R. Grempel, Europhys. Lett., 66, 854 (2004); Eran Lebanon, and Markus Müller, Phys. Rev. B 72, 174202 (2005); Ariel Amir et al, Ann. Rev. Condens. Matter Phys. 2, 235 (2011); J. Bergli and Y. M. Galperin, Phys. Rev. B 85, 214202 (2012); For a recent review and relevant literature see: M. Pollak, M. Ortuno and A. Frydman, "The Electron Glass", Cambridge University Press, 2013.
7. U. Givan and Z. Ovadyahu, Phys. Rev. B 86, 165101 (2012).
8. A. Vaknin, Z. Ovadyahu, and M. Pollak, Phys. Rev. B 65, 134208 (2002).
9. V. Orlyanchik, and Z. Ovadyahu, Phys. Rev. Lett., 92, 066801 (2004).
10. A. Vaknin, Z. Ovadyahu, and M. Pollak, Phys. Rev. B 61, 6692 (2000); Ariel Amir et al, PNAS 109, 1850 (2012); Y. Meroz, Y. Oreg and Y. Imry, EPL, 105, 37010 (2014).
11. Z. Ovadyahu, Phys. Rev. B 73, 214208 (2006).
12. S. Marnieros, L. Bergé, A. Juillard, and L. Dumoulin, Phys. Rev. Lett. 84, 2469 (2000).
13. R. M. Hill, Philos. Mag. 24, 1307 (1971); B. I. Shklovskii, Fiz. Tekh. Poluprovodn. 6, 2335 (1972) [Sov. Phys. Semicond. 6, 164 (1973)]; B. I. Shklovskii, Phys. Tekh. Poluprovodn. 10, 1440 (1976) [Sov. Phys. Semicond. 10, 855 (1976)]; M. Pollak and I. Riess, J. Phys. C9, 2339 (1976).
14. Z. Ovadyahu, Phys. Rev. B 84, 165209 (2011).
15. P. M. Echternach, M. R. Thoman, C. M. Gould, and H. M. Bozler, Phys. Rev. B 46, 10339 (1992); M. E. Gershenson, D. Gong, T. Sato, B. S. Karasik and A. V. Sergeev, Appl. Phys. Lett. 79, 2049 (2001); A. Sergeev and V. Mitin, Europhys. Lett., 51, 641 (2000).
16. F. P. Milliken and Z. Ovadyahu, Phys. Rev. Lett., 65, 911 (1990).
17. A. Sergeev and V. Mitin, Phys. Rev. B 61, 6041 (2000).
18. Stefan Wessel, B. Normand, M. Sigrist, and Stephan Haas, Phys. Rev. Lett. 86, 1086 (2001); H. G. Schlager and H. v. Löhneysen, Europhys. Lett. 40 661 (1997).
An example of a possible scenario that formally accounts for the $T^*$-problem is based on assuming two distinct localization lengths $\xi_1$ and $\xi_2 > \xi_1$. Then insulating behavior will prevail at $T < T^*$ defined by $L_{\text{in}}(T^*) > \xi_2$ and an insulating behavior with a parallel "metallic" conductance at $T > T^*$ (when $\xi_1 < L_{\text{in}}(T) < \xi_2$).