THE MODEL OF THE UNIVERSE WITH TWO SPACES

D.L.Khokhlov
Sumy State University, R.-Korsakov St. 2, Sumy 244007 Ukraine
e-mail: nik@demex.sumy.ua

The model of the homogenous and isotropic universe with two spaces is considered. The background space is a coordinate system of reference and defines the behaviour of the universe. The other space characterizes the gravity of the matter of the universe. In the presented model, the first derivative of the scale factor of the universe with respect to time is equal to the velocity of light. The density of the matter of the universe changes from the Plankian value at the Planck time to the modern value at the modern time. The model under consideration describes the universe from the Planck time to the modern time and avoids the problems of the Friedmann model such as the flatness problem and the horizon problem.

1. Introduction

As known [1, 2], the Friedmann model of the universe has fundamental difficulties such as the flatness problem and the horizon problem. These appears to be a consequence of that the space of the Friedmann universe, on the one hand, is defined by the gravity of the matter of the universe, and on the other hand, is a coordinate system of reference. The solution of the problem is to introduce the background space as a coordinate system of reference. In this case, the background space defines the behaviour of the universe, and the other space characterizes the gravity of the matter of the universe.

2. Theory

Let us consider the model of the homogenous and isotropic universe with two spaces. Let us introduce the background space as a coordinate system of reference. Then the evolution of the universe is described as a deformation of the background space. Let us take the homogenous and isotropic background space, with the spatial interval of its metric is given by

\[ d\hat{s}^2 = \frac{a^2 d\hat{l}^2}{\left[1 + \frac{k\hat{l}^2}{4}\right]^2} \]  

Suppose that the background space is defined by the total mass of the universe including the mass of the matter and the energy of gravity

\[ \hat{G}_{ik} = T_{ik}^{tot} = T_{ik} + t_{ik}. \]  

Let us consider the case when the total mass of the universe is equal to zero

\[ T_{ik}^{tot} = T_{ik} + t_{ik} = 0. \]  

Then eq. (2) take the form

\[ \hat{G}_{ik} = 0. \]
The solution of the equations (4) gives

\[ \frac{d^2a}{dt^2} = 0 \]  \hspace{1cm} (5)

\[ \frac{da}{dt} = \mathbf{c}. \] \hspace{1cm} (6)

Thus the second derivative of the scale factor of the universe with respect to time is equal to zero, and the first derivative of the scale factor is equal to the velocity of light. It should be noted that the scale factor of the universe coincides with the size of the horizon

\[ a = ct. \] \hspace{1cm} (7)

In the model (4), the laboratory coordinate system is synchronous. In the laboratory coordinate system, the background space is described by the flat metric

\[ ds^2 = c^2 dt^2 - a^2 d\tilde{l}^2. \] \hspace{1cm} (8)

Thus we arrive at the Milne model \cite{3} in which the size of the universe being the maximum distance between the particles coincides with the scale factor of the universe and coincides with the size of the horizon. In the universe with one space, the Milne model is derived from the condition that the density of the matter tends to zero \( \rho \to 0 \). Here the Milne model describes the background space of the universe, with the total mass of the universe being equal to zero \( m_{\text{tot}} = 0 \).

Let us determine the relationship between the lifetime of the universe and the Hubble constant. Since the Hubble constant is

\[ H = \frac{1}{a} \frac{da}{dt}, \] \hspace{1cm} (9)

so from (3), (7), (9) one can obtain

\[ t = \frac{1}{H}. \] \hspace{1cm} (10)

Let us estimate the size of the universe at the Planck time and at present. Remind that the size of the universe coincides with the scale factor of the universe. According to (4), at the Planck time \( t_{\text{Pl}} = (hG/c^5)^{1/2} \), the scale factor of the universe is equal to the Planck length \( l_{\text{Pl}} = (hG/c^3)^{1/2} \). According to (7), (10), for the modern Hubble constant \( H_0 \approx 3 \cdot 10^{-18} \text{ c}^{-1} \), the modern scale factor of the universe is \( a_0 \approx 10^{28} \text{ cm} \).

Let us determine the relationship between the mass of the matter and the scale factor of the universe at \( t = \text{const} \). The total mass of the universe is equal to zero, given the mass of the matter is equal to the energy of its gravity

\[ m = \frac{Gm}{c^2a}. \] \hspace{1cm} (11)

Allowing for (7) and (10), from (11) it follows that the mass of the matter changes with time as

\[ m = \frac{c^2a}{G} = \frac{c^3t}{G} = \frac{c^3}{GH}, \] \hspace{1cm} (12)
and the density of the matter, as

\[ \rho = \frac{3c^2}{4\pi Ga^2} = \frac{3}{4\pi Gt^2} = \frac{3H^2}{4\pi G}. \]  

(13)

According to (12), growth of the mass of the matter takes place during all the evolution of the universe. At the Planck time \( t_{Pl} \), the mass of the matter is equal to the Planck mass \( m_{Pl} = (\hbar c/G)^{1/2} \). At present, the mass of the matter is \( m_0 \approx 1.4 \cdot 10^{56} \) g, and the density of the matter is \( \rho_0 \approx 3.2 \cdot 10^{-29} \) g cm\(^{-3} \). Thus the model of the universe (3)-(6) provides growth of the mass of the matter from the Planckian value to the modern one.

### 3. Conclusion

We have considered the model of the homogenous and isotropic universe with two spaces, with the behaviour of the universe is defined by the background space. Unlike the Friedmann model, the presented model gets rid off the flatness and horizon problems.

Remind [1, 2] that the horizon problem in the Friedmann universe is that two particles situated within the horizon at present were situated beyond the horizon in the past. In the universe under consideration, all the particles are situated within the horizon during all the evolution of the universe, since the size of the universe being the maximum distance between the particles coincides with the size of the horizon. Hence the presented model avoids the horizon problem.

Remind [1, 2] that the essence of the flatness problem in the Friedmann universe is impossibility to get the modern density of the matter starting from the Planck density of the matter at the Planck time. In the presented theory, the density of the matter of the universe changes from the Planckian value at the Planck time to the modern value at the modern time. Hence the flatness problem is absent in the presented theory.

In order to resolve the above problems of the Friedmann universe an inflationary episode is introduced in the early universe [1, 3]. Since the presented model describes the universe from the Planck time to the modern time and avoids the above problems of the Friedmann universe, there is no necessity to introduce the inflationary model.

### References

[1] A.D. Dolgov, Ya.B. Zeldovich, M.V. Sazhin, Cosmology of the early universe (Moscow Univ. Press, Moscow, 1988, in Russian).

[2] A.D. Linde, Elementary particle physics and inflationary cosmology (Nauka, Moscow, 1990, in Russian).

[3] Ya.B. Zeldovich and I.D. Novikov, Structure and evolution of the universe (Nauka, Moscow, 1975, in Russian).

[4] L. Landau and E.M. Lifshitz, The classical theory of fields, 4th Ed. (Pergamon, Oxford, 1976).