A study of critical energy of phase transition from chemical potentials of light hadrons and quarks based on yield ratios of negative to positive high energy particles

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Abstract: We describe the transverse momentum (or mass) spectra of $\pi^{\pm}$, $K^{\pm}$, $p$, and $\bar{p}$ produced in central gold-gold (Au-Au), central lead-lead (Pb-Pb), and inelastic proton-proton (pp) collisions at different collision energies range from the AGS to LHC by using a two-component (in most cases) Erlang distribution in the framework of multi-source thermal model. The fitting results are consistent with the experimental data and the energy-dependent chemical potentials of light hadrons ($\pi$, $K$, and $p$) and quarks ($u$, $d$, and $s$) in central Au-Au, central Pb-Pb, and inelastic pp collisions from the yield ratios of negative to positive particles obtained from the normalization constants are then extracted. The study shows that most types of energy-dependent chemical potentials decrease with increase of collision energy over a range from the AGS to LHC. The curves of all types of energy-dependent chemical potentials, obtained from the linear fits of yield ratios vs energy, have inflection points at the same energy of 3.526 GeV, which is regarded as the critical energy of phase transition from a hadron liquid-like state to a quark gas-like state in the collision system and indicates that the hadronic interactions play an important role in this period. At the RHIC and LHC, all types of chemical potentials become small and tend to zero at very high energy, which confirms that the collision system possibly changes completely from the hadron-dominant liquid-like state to the quark-dominant gas-like state and the partonic interactions possibly play a dominant role at the LHC.

Keywords: transverse momentum spectra, yield ratios of negative to positive particles, chemical potentials of particles, critical end point of phase transition

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1 Introduction

The critical energy of phase transition \cite{1–4} is important for studying the quantum chromodynamics (QCD) phase diagram \cite{5,6} and the properties of quark-gluon plasma (QGP) \cite{7–9}, so more and more scientists devote to finding the critical energy. The experiments performed on the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC), especially the beam energy scan program at the RHIC, deal with a collision energy range from a few to several tens of GeV \cite{1,7,10,11}, which may contain the energy of the critical end point of hadron-quark phase transition \cite{1–4,12}. The STAR Collaboration found that the critical energy may be or below 19.6 GeV \cite{1}. One study based on yield ratio (the yield ratio of negative to positive particles) and the correlation between collision energy and transverse momentum indicated that the critical energy maybe range from 11.5 GeV to 19.6 GeV \cite{1,13–15}, while

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another study based on yield ratio showed that the critical energy may be about 4 GeV [12]. Studies about a striking pattern of viscous damping and an excitation function for \(R^{2}_{\text{out}} - R^{2}_{\text{side}}\) extracted for central collisions indicated the critical energy may be close to 62.4 GeV [16–18]. It is not hard to see that the value of critical energy has not been determined so far, so finding the critical energy arouses our great interest.

Lattice QCD [19–21], a powerful tool to investigate the QGP matter in high-temperature and high-density system, indicates that the critical end point (CEP) of phase transition on QCD phase diagram is a crossover at small chemical potentials or high collision energies [22, 23]. So it is important to study baryon chemical potential for finding the CEP on QCD phase diagram. When collisions occur at high energy, especially at RHIC and LHC, the collision system probably creates the QGP matter [24–26] where the partonic interactions play an important role, and the baryon chemical potential is small, even close to 1 MeV or zero [12, 27–29]. While when energy is not very high, the transition from hadron to quark has not yet taken place in the collision system, where the hadronic interactions play an important role [1, 13–15], and the value of baryon chemical potential is larger. We could predict that the chemical potential corresponding to the CEP should be an inflection point or abrupt change point in chemical potential-energy plane. It is therefore worthwhile to study the trend of chemical potential with energy.

The chemical potentials of light hadrons and quarks can be extracted from the yield ratios of negative to positive particles. Generally, one can get the yield ratios by two ways. One way is to directly collect the values of yield ratios from the productive international collaborations, which is a rapid and convenient method. The other one needs the aid of the extracted normalization constants in describing the transverse momentum spectra of negative and positive particles with consistent statistical law, but the workload is huge. In this paper, due to the fact that experiment data of some particles correspond to a narrow range of \(p_T\), we adopt the second method for the normalization constants being extracted from a wider range of transverse momentum (or mass) distribution to obtain a relatively accurate result.

In the present work, we describe the transverse momentum (\(p_T\)) or transverse mass (\(m_T\)) spectra of \(\pi^\pm\), \(K^\pm\), \(p\), and \(\bar{p}\) produced in central gold-gold (Au-Au), central lead-lead (Pb-Pb) and inelastic proton-proton (\(pp\)) collisions in mid-rapidity interval (in most cases) over a center-of-mass energy (\(\sqrt{s_{NN}}\)) range from the AGS to LHC [30–42] by using a two-component (in most cases) Erlang distribution [43, 44] in the framework of a multi-source thermal model [44–46], and obtain the yield ratios, \(k_\pi\), \(k_K\), and \(k_p\), of negative to positive particles according to the extracted normalization constants. Meanwhile, we collect the results of our previous work [28] directly, the yield ratios of \(k_\pi\), \(k_K\), and \(k_p\) in inelastic \(pp\) collisions at some energies, which were obtained by the same describing method as the present work, but with a Tsallis-Pareto-type function [47–49]. The energy-dependent chemical potentials of light hadrons (\(\pi, K\), and \(p\)) and quarks (\(u, d, \text{and } s\)) in central Au-Au, central Pb-Pb, and inelastic \(pp\) collisions are then extracted from the yield ratios.

## 2 The model and formulism

According to our method, to obtain the normalization constants, we need firstly to describe the \(p_T\) spectra of \(\pi^\pm\), \(K^\pm\), \(p\), and \(\bar{p}\) with a multi-component Erlang distribution [43, 44] which is in the framework of a multi-source thermal model [44–46]. The model assumes that many emission sources are formed in high energy collisions and are classified into a few groups due to the existent of different interacting mechanisms in the collisions and different event samples in experiment measurements. The sources in the same group have the same excitation degree and stay at a common local equilibrium state, which can be described by an Erlang \(p_T\) distribution. All emission sources in different groups result in the final-state distribution, which can be described by a multi-component Erlang \(p_T\) distribution.

The multi-component Erlang distribution based on the above multi-source thermal model has the following form. According to thermodynamic system, particles generated from one emission source obey
to an exponential distribution of transverse momentum,

$$f_{ij}(p_{tij}) = \frac{1}{\langle p_{tij} \rangle} \exp \left[-\frac{p_{tij}}{\langle p_{tij} \rangle}\right],$$  

(1)

where $p_{tij}$ is the transverse momentum of the $i$-th source in the $j$-th group, and $\langle p_{tij} \rangle$ is the mean value of $p_{tij}$. We assume that the source number in the $j$-th group and the transverse momentum of the $m_j$ sources are denoted by $m_j$ and $p_T$, respectively. All the sources in the $j$-th group then result in the folding result of exponential distribution

$$f_j(p_T) = \frac{p_T^{m_j-1}}{(m_j-1)!}\langle p_{tij} \rangle^{m_j} \exp \left[-\frac{p_T}{\langle p_{tij} \rangle}\right],$$  

(2)

which is the normalized Erlang distribution. The contribution of the $l$ group of sources can be expressed as

$$f(p_T) = \sum_{j=1}^{l} k_j f_j(p_T),$$  

(3)

where $k_j$ denotes the relative weight contributed by the $j$th group and meets the normalization $\sum_{j=1}^{l} k_j = 1$. This is the multi-component Erlang distribution.

In fact, in the present work, we describe the transverse momentum spectra of final-state light flavour particles by using a two-component Erlang distribution, where one component reflects the soft excitation process, while the other one reflects the hard scattering process. The soft process corresponding to low-$p_T$ region is regarded as the contribution of the interactions among a few sea quarks and gluons, and the hard process corresponding to high-$p_T$ region is regarded as originating from a harder head-on scattering between a few valent quarks. Due to the fact that the experimental data of some particles correspond to a narrow range of $p_T$, we adopt one-component Erlang distribution to fit these data.

Some experimental data we collect are about transverse mass distribution, not $p_T$ distribution, so we give the transformational relation between $p_T$ distribution and $m_T$ distribution based on the relation $m_T = \sqrt{p_T^2 + m_0^2}$, where $m_0$ is the rest mass of particle), i.e.

$$\frac{dN}{Ndm_T} = \frac{m_T}{p_T} \frac{dN}{Ndp_T}.  \tag{4}$$

The same as in our previous work [28], in the present work, we only calculate the chemical potentials of some light hadrons ($\pi$, $K$, and $p$), and some light quarks ($u$, $d$, and $s$). For the hadrons containing $c$ or $b$ quark, considering that there is a lack of the experimental data of $p_T$ spectra continuously varying with energy, we do not calculate the chemical potentials of the hadrons containing $c$ or $b$ quark, and $c$ and $b$ quarks. In addition, due to the lifetimes of the hadrons containing $t$ quark being too short to measure, we also cannot obtain the chemical potentials of the hadrons containing $t$ quark. According to the statistical arguments based on the chemical and thermal equilibrium within the thermal and statistical model [50], we can get the relations between antiparticle to particle (negative to positive particle) yield ratios and chemical potentials of hadrons to be [50–52]

$$k_\pi = \exp \left(-\frac{2\mu_\pi}{T_{ch}}\right),$$
$$k_K = \exp \left(-\frac{2\mu_K}{T_{ch}}\right),$$
$$k_p = \exp \left(-\frac{2\mu_p}{T_{ch}}\right),$$  

(5)

where $k_\pi$, $k_K$, and $k_p$ denote the yield ratios of antiparticles, $\pi^-$, $K^-$, and $\bar{p}$, to particles, $\pi^+$, $K^+$, and $p$, respectively, and $\mu_\pi$, $\mu_K$, and $\mu_p$ represent the chemical potentials of $\pi$, $K$, and $p$, respectively. In
addition, $T_{ch}$ represents the chemical freeze-out temperature of interacting system, and can be empirically obtained by the following formula

$$T_{ch} = T_{\text{lim}} \frac{1}{1 + \exp[2.60 - \ln(\sqrt{s_{NN}})/0.45]}$$

(6)

within the framework of a statistical thermal model of non-interacting gas particles with the assumption of standard Maxwell-Boltzmann statistics $[7, 8, 53]$, where the ‘limiting’ temperature $T_{\text{lim}}$ is 0.164 GeV, and $\sqrt{s_{NN}}$ is in the unit of GeV $[53, 54]$.

Assuming that $\mu_u, \mu_d,$ and $\mu_s$ represent the chemical potentials of $u$, $d$, and $s$ quarks, respectively, and according to Equation (5) and references $[12, 52, 55]$ under the same value of chemical freeze-out temperature, the yield ratios in terms of quark chemical potentials can be written as

$$k_\pi = \exp \left(-\frac{(\mu_u - \mu_d)}{T_{ch}}\right)/\exp \left(\frac{(\mu_u - \mu_d)}{T_{ch}}\right) = \exp \left[-\frac{2(\mu_u - \mu_d)}{T_{ch}}\right],$$

$$k_K = \exp \left(-\frac{(\mu_u - \mu_s)}{T_{ch}}\right)/\exp \left(\frac{(\mu_u - \mu_s)}{T_{ch}}\right) = \exp \left[-\frac{2(\mu_u - \mu_s)}{T_{ch}}\right],$$

$$k_p = \exp \left(-\frac{2(\mu_u + \mu_d)}{T_{ch}}\right)/\exp \left(\frac{2(\mu_u + \mu_d)}{T_{ch}}\right) = \exp \left[-\frac{2(2\mu_u + \mu_d)}{T_{ch}}\right].$$

(7)

Based on Equations (5) and (7), one can obtain the chemical potentials of hadrons and quarks in terms of yield ratios respectively,

$$\mu_\pi = -\frac{1}{2}T_{ch} \cdot \ln(k_\pi),$$

$$\mu_K = -\frac{1}{2}T_{ch} \cdot \ln(k_K),$$

$$\mu_p = -\frac{1}{2}T_{ch} \cdot \ln(k_p),$$

(8)

and

$$\mu_u = -\frac{1}{6}T_{ch} \cdot \ln(k_\pi \cdot k_p),$$

$$\mu_d = -\frac{1}{6}T_{ch} \cdot \ln(k_\pi^{-2} \cdot k_p),$$

$$\mu_s = -\frac{1}{6}T_{ch} \cdot \ln(k_\pi \cdot k_K^{-3} \cdot k_p).$$

(9)

In the present work, by describing the $p_T$ (or $m_T$) spectra of some light particles, $\pi^\pm$, $K^\pm$, $p$, and $\bar{p}$ in central Au-Au, central Pb-Pb, and inelastic $pp$ collisions in mid-rapidity interval at collision energy from the AGS to LHC with a two-component (in most cases) Erlang distribution, we obtain the yield ratios of $k_\pi$, $k_K$, and $k_p$ based on the extracted normalization constants, and the chemical potentials of light hadrons ($\pi$, $K$, and $p$) and light quarks ($u$, $d$, and $s$). Then the dependencies of chemical potentials on $\sqrt{s_{NN}}$ are analyzed.

What needs to be emphasized is that some $p_T$ spectra of light particles in inelastic $pp$ collisions at some energies ($\sqrt{s} = 62.4$, 200, 900, 2760, 7000, and 13000 GeV) have been described by a Tsallis-Pareto-type function in our previous work $[28]$, so we apply the results of yield ratios directly to our present work. It should be noted that the yield ratios based on extracted normalization constants are almost independent of statistical law, as long as it’s based on the suitable fits for experimental data. Therefore, the yield ratios and chemical potentials adopted directly are available, although the statistical laws used before and this time are different.

3 Results and discussion

Figure 1 shows the transverse mass distributions of (a)(b) $\pi^\pm$ and (c)(d) $K^\pm$ produced in central (0–5%) Au-Au collisions at mid-rapidity in the center-of-mass energy range from $\sqrt{s_{NN}} = 2.67$ to 4.84
GeV, where $dN/dy$ on axis denote the rapidity density. The experimental data represented by different kinds of symbols were measured by the E895 Collaboration [30] for $\pi^{\pm}$ at 2.67, 3.31, 3.81, and 4.28 GeV, and the E866 and E917 Collaborations [31,32] for $K^{\pm}$ at 3.31, 3.81, 4.28 and 4.84 GeV. The data at each energy are scaled by suitable factors for clarity. The plotted errors bars include both statistical and systematic uncertainties for $\pi^{\pm}$ and only statistical uncertainty for $K^{\pm}$. The solid curves are our results calculated by using the two-component Erlang distribution. The values of free parameters ($m_1$, $p_{t11}$, $k_1$, $m_2$, and $p_{t22}$), normalization constant ($N_0$), and $\chi^2$ per degree of freedom ($\chi^2$/dof) corresponding to the two-component Erlang distribution are listed in Table 1, where the normalization constant is for comparison between curve and data. One can see that the two-component Erlang distribution can well describe the experimental data of the considered particles in Au-Au collisions at the AGS. The values of $m_2$ corresponding to high-$p_T$ region for different particles are 2, which reflects that the hard process origins from a hard head-on scattering between two valent quarks, while the values of $m_1$ corresponding to low-$p_T$ region for different particles are 3, which reflects that the soft process origins from the interaction among a few sea quarks and gluons. The values of weight factor $k_1$ of soft excitation process are more than 50%, which shows that soft excitation is the main excitation process, and the normalization constants $N_0$ increases with increase of energy. It should be noted that the particle yield ratio is represented by $N_0$ from the spectrum of negative or positive particles. The relative value of $N_0$ is enough to obtain the particle yield ratio.

Figure 2 presents the transverse momentum spectra of $\pi^{\pm}$, $K^{\pm}$, $p$, and $\bar{p}$ in central $(0-5\%)$ Au-Au collisions at center-of-mass energy $\sqrt{s_{NN}} = (a)(d)\ 7.7, (b)(e)\ 11.5$, and (c)(f)\ 19.6 GeV. The symbols represent the experimental data recorded by the STAR Collaboration in the mid-rapidity range $|y| < 0.1$ [33]. The uncertainties are statistical and systematic added in quadrature. The curves are our results fitted by using the two-component Erlang distribution. The values of $m_1$, $p_{t11}$, $k_1$, $m_2$, $p_{t22}$, $N_0$, and $\chi^2$/dof corresponding to the two-component Erlang distribution are given in Table 1. It is not hard to see that the experimental data can be well fitted by the two-component Erlang distribution. Similarly, the values of $m_2$ are 2, and the values of $m_1$ are 2, 3, and 4. The values of weight factor $k_1$ are more than 50%, and $N_0$ in most cases increases with increase of collision energy.

Figure 3 gives the same as Figure 2 but for Au-Au collisions at $\sqrt{s_{NN}} = (a)(d)\ 27, (b)(e)\ 39$, and (c)(f)\ 62.4 GeV. All the experimental data were recorded by the STAR Collaboration [33,34]. The results calculated by using the two-component Erlang distribution are shown in the solid curves, where the values of corresponding free parameters, normalization constant, and $\chi^2$/dof are shown in Table 1. Obviously, the calculation results by the two-component Erlang distribution are in good agreement with the experimental data of the considered particles in Au-Au collisions. Once more, the values of $m_2$ are 2, and the values of $m_1$ are 2, 3, and 4. The values of weight factor $k_1$ are more than 50%, and $N_0$ in most cases increases with increase of collision energy.

The $p_T$ spectra of $\pi^{\pm}$, $K^{\pm}$, $p$, and $\bar{p}$ in $\sqrt{s_{NN}} = (a)(c)\ 130, (b)(d)\ 200$ GeV central $(0-5\%)$ Au-Au collisions are displayed in Figure 4. The symbols also denote the experimental data recorded by the PHENIX Collaboration [35,36]. The data for each type of particle are divided by suitable factors for clarity. The error bars indicate the combined uncorrelated statistical and systematic uncertainties for 130 GeV, and are statistical only for 200 GeV. The curves are the two-component Erlang model fits to the spectra. The values of all free parameters, normalization constant, and $\chi^2$/dof corresponding to the two-component Erlang distribution are listed in Table 1. Similarly, our calculation results with the two-component Erlang model are consistent with the experimental data. The values of $m_2$ are 2, and the values of $m_1$ are 2, 3, and 4. The values of weight factor $k_1$ are more than 50%, and $N_0$ in most cases increases with increase of collision energy.
Figure 1. Transverse mass spectra for (a)(c) positive ($\pi^+$, $K^+$) and (b)(d) negative ($\pi^-$, $K^-$) particles produced in $\sqrt{s_{NN}} = 2.67$ to 4.84 GeV central Au-Au collisions at mid-rapidity. The experimental data represented by the symbols are measured by the E895 Collaboration [30] for $\pi^\pm$ at 2.67, 3.31, 3.81, and 4.28 GeV, and the E866 and E917 Collaborations [31, 32] for $K^\pm$ at 3.31, 3.81, 4.28 and 4.84 GeV. The data at each energy are scaled by successive powers of 2 for clarity. The plotted errors bars include both statistical and systematic uncertainties for $\pi^\pm$ and only statistical uncertainty for $K^\pm$. The solid curves are our results calculated by using the two-component Erlang distribution.
Figure 2. Mid-rapidity transverse momentum spectra for (a)(b)(c) positive ($\pi^+$, $K^+$, $p$) and (d)(e)(f) negative ($\pi^-$, $K^-$, $\bar{p}$) particles produced in central Au-Au collisions at $\sqrt{s_{NN}}$ = (a)(d) 7.7, (b)(e) 11.5, and (c)(f) 19.6 GeV. The symbols represent the experimental data recorded by the STAR Collaboration [33]. The errors are the combined statistical and systematic ones, and the curves are our results by the two-component Erlang distribution.
Figure 3. Same as Figure 2 but for Au-Au collisions at $\sqrt{s_{NN}} = (a)(d) 27$, (b)(e) 39, and (c)(f) 62.4 GeV.

Figure 5 exhibits the $m_T$ spectra of $\pi^\pm$ at $0 < y < 0.2$, $K^\pm$ at $|y| < 0.1$, $p$, and $\bar{p}$ produced in central Pb-Pb collisions at $\sqrt{s_{NN}} = (a)(d) 6.3$, (b)(e) 7.7 and (c)(f) 8.8 GeV. The experimental data, represented by symbols, were taken by the NA49 Collaboration [37–39], where $p$ and $\bar{p}$ were done near mid-rapidity and covered the rapidity intervals of $1.5 < y < 2.2$ ($y_{c.m.} = 1.88$) for 6.3 GeV, $1.6 < y < 2.3$
\( y_{c.m.} = 2.08 \) for 7.7 GeV, and \( 1.9 < y < 2.3 \ (y_{c.m.} = 2.22) \) for 8.8 GeV. The error bars on the spectra points are statistical only. Because the original data are not found, we get the the data form the figures in the publication. Some error bars for \( \pi^\pm \) and \( K^\pm \) are smaller than the symbol sizes, so we take the symbol sizes as the corresponding statistical errors. The curves are fits of two-component Erlang function to the spectra. The values of free parameters, normalization constant, and \( \chi^2/d\text{of} \) are summarized in Table 2. We can see that the experimental data for all hadrons and energies are well described by the fit function. The values of \( m_2 \) are 2, and the values of \( m_1 \) are 2, 3, and 4. The values of weight factor \( k_1 \) are more than 50%, and \( N_0 \) increases with increase of collision energy.

Figure 6 presents the \( m_T \) and \( p_T \) spectra of \( \pi^\pm \), \( K^\pm \), \( p \), and \( \bar{p} \) in central Pb-Pb collisions at \( \sqrt{s_{NN}} = (a)(d) \ 12.3 \), (b)(e) \( 17.3 \) and (c)(f) \( 2760 \) GeV, where \( \sigma_{\text{trig}} \) on the vertical axis denotes the interaction cross section satisfying a T0 centrality trigger. The symbols represent the experimental data reported by the NA49 Collaboration for 12.3 GeV at mid-rapidity \( \mid y \mid < 0.1 \) for \( K^\pm \) [39], \( 2.2 < y < 2.6 \ (y_{c.m.} = 2.57) \) for \( p \) and \( \bar{p} \) [38], the NA44 Collaboration for 17.3 GeV near mid-rapidity \( (2.4 < y < 3.1) \) for \( \pi^\pm \), \( 2.4 < y < 3.5 \) for \( K^\pm \), and \( 2.3 < y < 2.9 \) for \( p \) and \( \bar{p} \) ) [40], and the ALICE Collaboration for 2760 GeV at mid-rapidity \( \mid y \mid < 0.5 \) [41]. Some data for different particles are divided by suitable factors for clarity. The errors for 12.3 GeV are statistical, where some error bars for \( \pi^\pm \) and \( K^\pm \) are smaller than the symbol sizes, so we take the symbol sizes as the corresponding statistical errors. The errors are systematic for 17.3 GeV, and are quadratic sum of statistical errors and systematic errors for 2760 GeV. The curves represent the two-component Erlang fits. The values of free parameters, normalization constant, and \( \chi^2/d\text{of} \) are summarized in Table 2. Obviously, the experimental data for all particles at all energies are in good agreement with the fits. The values of \( m_2 \) are 2, and the values of \( m_1 \) are 2, 3, and 4. The values of weight factor \( k_1 \) are more than 50%, and \( N_0 \) increases with increase of collision energy.

Figure 7 shows the \( p_T \) spectra of (a)(b) \( \pi^\pm \) and (c)(d) \( K^\pm \) produced in mid-rapidity \( y \approx 0 \) inelastic \( pp \) collisions at \( \sqrt{s} = 6.3, 7.7, 8.8, 12.3, \) and 17.3 GeV. The measurements were performed at the CERN-Super Proton Synchrotron (SPS) by the large acceptance NA61/SHINE hadron spectrometer [42]. Spectra at different energies are scaled by appropriate factors for better visibility. The error bars on data points correspond to combined statistical and systematic uncertainties. The curves are our fitting results by using the one- or two-component Erlang function. For some curves, we use one-component Erlang function because the number of corresponding experimental data points is small. Due to the proportion of the second component is small, it has little effect on the calculated particle ratio, despite the absence of the second component. The values of free parameters, normalization constant, and \( \chi^2/d\text{of} \) are given in Table 3. As can be seen, the fits for all hadrons at all energies are in good agreement with the experimental data. The values of \( m_2 \) and \( m_1 \) are 2 and 3, respectively. The values of weight factor \( k_1 \) are more than 50%. It should be noted that the dof for \( \pi^- \) at 12.3 GeV in Table 3 is zero, which means the dash curve in Figure 7(b) is drawn to guide the eye.

Note that the \( p_T \) spectra of \( \pi^\pm \), \( K^\pm \), \( p \), and \( \bar{p} \) produced in inelastic \( pp \) collisions at other energies (62.4, 200, 900, 2760, 7000 and 13000 GeV) have been fitted by using a Tsallis-Pareto-type function in our previous work [28]. The fitting results are directly used in our present work. In fact, only the normalization constant is used to extract chemical potential, so the fitting results are approximately independent of models, although the fits come from different models. One can even use the normalization constants from the data directly [12], while in the present work, we use the fitting results instead of the data due to the fact that the data in high \( p_T \) region in some cases are not available.
Figure 4. Same as Figure 2 but for Au-Au collisions at $\sqrt{s_{NN}} = (a)(c) 130, (b)(d) 200$ GeV.
Figure 5. Transverse mass spectra for $\pi^\pm$, $K^\pm$, $p$, and $\bar{p}$ at mid-rapidity in (a)(d) 6.3, (b)(e) 7.7 and (c)(f) 8.8 GeV central Pb-Pb collisions. The symbols represent the experimental data taken by the NA49 Collaboration [37–39]. The errors are statistical only. The curves are fits of two-component Erlang function to the spectra.
Figure 6. Transverse mass and momentum spectra for $\pi^\pm$, $K^\pm$, $p$, and $\bar{p}$ at mid-rapidity in central Pb-Pb collisions at (a)(d) 12.3, (b)(e) 17.3 and (c)(f) 2760 GeV. The symbols represent the experimental data reported by the NA49 Collaboration for 12.3 GeV at mid-rapidity $|y| < 0.1$ for $K^\pm$ [39], $2.2 < y < 2.6$ ($y_{c.m.} = 2.57$) for $p$ and $\bar{p}$ [38], the NA44 Collaboration for 17.3 GeV near mid-rapidity ($2.4 < y < 3.1$ for $\pi^\pm$, $2.4 < y < 3.5$ for $K^\pm$, and $2.3 < y < 2.9$ for $p$ and $\bar{p}$) [40], and the ALICE Collaboration for 2760 GeV at mid-rapidity $|y| < 0.5$ [41]. Some data for different particles are divided by suitable factors for clarity. The errors are statistical for 12.3 GeV, are systematic for 17.3 GeV, and are quadratic sum of statistical errors and systematic errors for 2760 GeV. The curves are fits of two-component Erlang function to the spectra.
Figure 7. Transverse momentum spectra for (a)(b) $\pi^\pm$ and (c)(d) $K^\pm$ in $y \approx 0$ inelastic $pp$ collisions at SPS energies (6.3, 7.7, 8.8, 12.3, and 17.3 GeV). The symbols represent the experimental data reported by the NA61/SHINE Collaboration [42]. Spectra at different energies are scaled by appropriate factors for better visibility. The errors are quadratic sum of statistical errors and systematic errors. The curves are fits of one- or two-component Erlang function to the spectra, where the dash curve in the figure is drawn to guide the eye.
According to Equations (5) and (7), and the extracted normalization constants from the above comparisons and our previous work [28], the yield ratios of negative to positive particles from different collision systems, $k_\pi$, $k_K$, and $k_p$ versus collision energy $\sqrt{s_{NN}}$ are obtained. The three yield ratios show regular trends with increase of collision energy. To see more clearly the dependences of three yield ratios on collision energy, we show the correlations between the logarithm of three yield ratios ($\ln(k_\pi)$, $\ln(k_K)$, and $\ln(k_p)$) and $1/\sqrt{s_{NN}}$ in Figure 8, where the black squares, red circles, and blue triangles denote the calculated results from $pp$, central Au-Au, and central Pb-Pb collisions at mid-rapidity, respectively. One can see that $\ln(k_\pi)$, $\ln(k_K)$, and $\ln(k_p)$ show obviously linear dependences on $1/\sqrt{s_{NN}}$, which are then fitted by linear functions for clarity. $\ln(k_K)$ and $\ln(k_p)$ from all collision systems mentioned above, decrease monotonously with increase of $1/\sqrt{s_{NN}}$, and can be described by the below linear functions of

\[
\ln(k_K) = (-7.837 \pm 0.178)/\sqrt{s_{NN}} + (-0.008 \pm 0.021),
\]

\[
\ln(k_p) = (-40.903 \pm 0.679)/\sqrt{s_{NN}} + (-0.031 \pm 0.047),
\]

respectively, with $\chi^2$/dof to be 4.947/26 and 3.888/17 respectively. While $\ln(k_\pi)$ displays different behavior from the above two ratios. With the increase of $\sqrt{s_{NN}}$, the $\ln(k_\pi)$ from inelastic $pp$ collisions increases obviously and that from nucleus-nucleus (Au-Au and Pb-Pb) collisions slightly decreases. The dependences of $\ln(k_\pi)$ with $1/\sqrt{s_{NN}}$ can also be described by the following linear functions of

\[
\ln(k_{\pi pp}) = (-2.784 \pm 0.202)/\sqrt{s_{NN}} + (-0.018 \pm 0.015),
\]

and

\[
\ln(k_{\pi NN}) = (1.185 \pm 0.107)/\sqrt{s_{NN}} + (-0.024 \pm 0.017),
\]

with $\chi^2$/dof to be 0.566/8 and 28.815/13 respectively, where $k_{\pi pp}$ and $k_{\pi NN}$ represent the $k_\pi$ from $pp$ and nucleus-nucleus (Au-Au and Pb-Pb) collisions, respectively. It is noticed that the values of intercepts of the above four lines are asymptotically 0, which means the limiting values of the three yield ratios are 1 at high energy.

It should be emphasized that at low energy (less than 10 GeV), the ratio $k_\pi$ diverges for originating from different collision systems which is different from $k_K$ and $k_p$. The reason we think is mainly about the effect of resonance decay [56]. When the collision energy is in the range from a few GeV to dozens of GeV, the existing of secondary cascade collisions between produced particles and subsequent nucleons in nucleus-nucleus collisions, which produces resonances and subsequent decays, mainly increases the $k_{\pi NN}$ ratio in nucleus-nucleus collisions, but has little effect on $k_K$ and $k_p$ ratios. As the collision energy increases, the effect on $k_{\pi NN}$ decreases dramatically, which results in the $k_\pi$ being almost the same in nucleus-nucleus collisions as it is in inelastic $pp$ collisions.

Based on the extracted yield ratios of negative to positive particles and Equations (8) and (9), the light hadron chemical potentials, (a) $\mu_\pi$, (b) $\mu_K$, and (c) $\mu_p$ of $\pi$, $K$, and $p$, and quark chemical potentials, (d) $\mu_u$, (e) $\mu_d$, and (f) $\mu_s$ of $u$, $d$, and $s$ quarks, which vary with collision energy, are obtained and shown in Figure 9 with symbols. The black squares, red circles, and blue triangles denote the calculated results from inelastic $pp$, central Au-Au, and central Pb-Pb collisions at mid-rapidity, respectively. The curves are the derivative results according to Equations (10–12) corresponding to the fitted lines in Figure 8. The red dashed curves in Figures 9(a), 9(d), 9(e) and 9(f) are the derivative results related to $k_{\pi NN}$ from central nucleus-nucleus collisions, and the black solid curves are the derivative results related to $k_{\pi pp}$ from inelastic $pp$ collisions or other yield ratios. One can see that, with the increase of $\sqrt{s_{NN}}$ from the AGS to LHC, $\mu_\pi$ in central nucleus-nucleus collisions increases obviously and that in inelastic $pp$ collisions decreases obviously, while $\mu_K$, $\mu_p$, $\mu_u$, $\mu_d$, and $\mu_s$ in both central nucleus-nucleus and inelastic $pp$ collisions decrease obviously. At the same energy, $\mu_K$ is larger than $\mu_\pi$ but less than $\mu_p$, and $\mu_u$ is
almost as large as $\mu_d$ but larger than $\mu_s$ due to the differences of different particle masses. The limiting values of the six types of chemical potentials in central nucleus-nucleus and inelastic $pp$ collisions are zero at very high energy.

\[ \sqrt{s_{NN}} \text{-dependent yield ratios of (a) } k_\pi, \text{ (b) } k_K, \text{ and (c) } k_p. \] The symbols denote the calculated results according to normalization constant $N_0$ and Equations (5) and (7), and the lines are the fits according to Equations (10–12), respectively.
Figure 9. Energy-dependent light hadron chemical potentials, (a) $\mu_\pi$, (b) $\mu_K$, and (c) $\mu_p$ for $\pi$, $K$, and $p$, respectively, and light quark chemical potentials, (d) $\mu_u$, (e) $\mu_d$, and (f) $\mu_s$ for $u$, $d$, and $s$, respectively. The symbols denote the calculated results according to the extracted yield ratios and Equations (8) and (9), and the curves are the derivative results based on the linear fits of Equations (10–12) in Figure 8.
| Figure | $\sqrt{s_{NN}}$ (GeV) | Particle | $m_1$ | $<p_T^{11}>$ (GeV/c) | $k_1$ | $m_2$ | $<p_T^{12}>$ (GeV/c) | $N_0$ | $\chi^2$/dof |
|--------|---------------------|---------|------|-------------------|------|------|-------------------|------|----------------|
| Figure 1 (a) | 2.67 | $\pi^+$ | 3 | 0.078 ± 0.001 | 0.74 ± 0.01 | 2 | 0.182 ± 0.012 | 12.051 ± 0.365 | 4.89/17 |
| Figure 1 (b) | 3.31 | $\pi^-$ | 3 | 0.061 ± 0.001 | 0.74 ± 0.02 | 2 | 0.163 ± 0.010 | 20.811 ± 0.365 | 1.30/24 |
| Figure 1 (c) | 3.81 | $\pi^+$ | 3 | 0.084 ± 0.002 | 0.68 ± 0.02 | 2 | 0.201 ± 0.010 | 27.819 ± 0.747 | 0.32/22 |
| Figure 1 (d) | 4.28 | $\pi^-$ | 3 | 0.190 ± 0.011 | 0.80 ± 0.06 | 2 | 0.202 ± 0.023 | 4.829 ± 0.265 | 0.08/5 |
| Figure 1 (d) | 4.48 | $\pi^-$ | 3 | 0.084 ± 0.002 | 0.60 ± 0.02 | 2 | 0.280 ± 0.010 | 39.064 ± 0.770 | 0.27/17 |
| Table 1 | | | | | | | | | |
| Figure 2 (a) | 7.7 | $\pi^+$ | 3 | 0.197 ± 0.033 | 0.94 ± 0.04 | 2 | 0.235 ± 0.030 | 7.717 ± 0.031 | 0.28/3 |
| Figure 2 (b) | 11.5 | $\pi^-$ | 3 | 0.197 ± 0.003 | 0.94 ± 0.02 | 2 | 0.192 ± 0.002 | 62.627 ± 0.744 | 2.53/34 |
| Figure 2 (c) | 19.6 | $\pi^+$ | 3 | 0.201 ± 0.003 | 0.93 ± 0.18 | 2 | 0.262 ± 0.048 | 10.06 ± 0.718 | 0.09/19 |
| Figure 2 (d) | 27 | $\pi^-$ | 3 | 0.192 ± 0.004 | 0.64 ± 0.07 | 2 | 0.292 ± 0.007 | 18.620 ± 0.585 | 0.18/20 |
| Figure 3 (a) | 39 | $\pi^+$ | 3 | 0.155 ± 0.008 | 0.54 ± 0.05 | 2 | 0.265 ± 0.004 | 185.159 ± 7.258 | 0.26/20 |
| Figure 3 (b) | 62.4 | $\pi^-$ | 3 | 0.232 ± 0.004 | 0.94 ± 0.10 | 2 | 0.274 ± 0.004 | 232.481 ± 1.320 | 0.82/20 |
| Figure 4 (a) | 130 | $\pi^+$ | 3 | 0.137 ± 0.003 | 0.69 ± 0.02 | 2 | 0.238 ± 0.004 | 288.147 ± 1.892 | 0.82/20 |
| Figure 4 (b) | 200 | $\pi^-$ | 3 | 0.256 ± 0.004 | 0.75 ± 0.04 | 2 | 0.256 ± 0.015 | 39.598 ± 0.232 | 0.01/9 |
| Figure 4 (c) | 1.03 | $\pi^-$ | 3 | 0.137 ± 0.003 | 0.69 ± 0.02 | 2 | 0.200 ± 0.004 | 314.409 ± 3.500 | 1.62/19 |
| Figure 4 (d) | 2.00 | $\pi^-$ | 3 | 0.208 ± 0.003 | 0.52 ± 0.03 | 2 | 0.205 ± 0.006 | 47.466 ± 0.793 | 1.13/10 |
| Figure 4 (e) | 3.00 | $\pi^-$ | 3 | 0.208 ± 0.003 | 0.52 ± 0.03 | 2 | 0.205 ± 0.006 | 47.466 ± 0.793 | 1.13/10 |
| Figure 4 (f) | 4.00 | $\pi^-$ | 3 | 0.208 ± 0.003 | 0.52 ± 0.03 | 2 | 0.205 ± 0.006 | 47.466 ± 0.793 | 1.13/10 |

Table 1. Values of free parameters, normalization constant, and $\chi^2$/dof corresponding to two-component Erlang ($p_T$ or $m_T$) distribution for Au-Au collisions in Figures 1–4.
### Table 2. Values of free parameters, normalization constant, and $\chi^2$/dof corresponding to two-component Erlang $p_T$ (or $m_T$) distribution for Pb-Pb collisions in Figures 5 and 6.

| Figure | $\sqrt{s_{NN}}$ (GeV) | Particle | $m_1$ | $<p_{T1}>$ (GeV/c) | $k_1$ | $m_2$ | $<p_{T2}>$ (GeV/c) | $N_0$ | $\chi^2$/dof |
|--------|---------------------|----------|------|---------------------|------|------|---------------------|------|------------|
| Figure 5(a) | 6.3 | $\pi^+$ | 3 | 0.095 $\pm$ 0.003 | 0.51 $\pm$ 0.03 | 2 | 0.228 $\pm$ 0.005 | 72.088 $\pm$ 2.379 | 0.228/10 |
| Figure 5(a) | 6.3 | $K^+$ | 3 | 0.194 $\pm$ 0.004 | 0.90 $\pm$ 0.10 | 2 | 0.295 $\pm$ 0.059 | 16.508 $\pm$ 0.644 | 0.295/4 |
| Figure 5(d) | 7.7 | $\pi^-$ | 3 | 0.082 $\pm$ 0.003 | 0.51 $\pm$ 0.03 | 2 | 0.225 $\pm$ 0.006 | 83.773 $\pm$ 2.765 | 0.225/10 |
| Figure 5(e) | 7.7 | $K^-$ | 3 | 0.179 $\pm$ 0.005 | 0.86 $\pm$ 0.09 | 2 | 0.328 $\pm$ 0.081 | 5.644 $\pm$ 0.186 | 0.408/4 |
| Figure 5(f) | 8.8 | $K^+$ | 3 | 0.203 $\pm$ 0.004 | 0.91 $\pm$ 0.05 | 2 | 0.205 $\pm$ 0.041 | 21.556 $\pm$ 0.625 | 0.205/4 |
| Figure 5(f) | 8.8 | $p$ | 4 | 0.214 $\pm$ 0.004 | 0.87 $\pm$ 0.12 | 2 | 0.420 $\pm$ 0.055 | 5.278 $\pm$ 0.306 | 0.420/8 |
| Figure 6(a) | 12.3 | $K^+$ | 3 | 0.204 $\pm$ 0.004 | 0.90 $\pm$ 0.07 | 2 | 0.220 $\pm$ 0.027 | 24.872 $\pm$ 0.696 | 0.196/8 |
| Figure 6(d) | 17.3 | $\pi^-$ | 3 | 0.192 $\pm$ 0.004 | 0.91 $\pm$ 0.01 | 2 | 0.330 $\pm$ 0.038 | 17.753 $\pm$ 0.781 | 0.330/20 |
| Figure 6(b) | 2760 | $\pi^+$ | 3 | 0.187 $\pm$ 0.009 | 0.90 $\pm$ 0.06 | 2 | 0.280 $\pm$ 0.031 | 8.062 $\pm$ 0.282 | 0.280/8 |
| Figure 6(c) | 2760 | $p$ | 4 | 0.305 $\pm$ 0.006 | 0.82 $\pm$ 0.07 | 2 | 0.407 $\pm$ 0.065 | 21.798 $\pm$ 0.602 | 0.407/24 |

### Table 3. Values of free parameters, normalization constant, and $\chi^2$/dof corresponding to one- or two-component Erlang $p_T$ distribution for inelastic $pp$ collisions in Figure 7.

| Figure | $\sqrt{s_{NN}}$ (GeV) | Particle | $m_1$ | $<p_{T1}>$ (GeV/c) | $k_1$ | $m_2$ | $<p_{T2}>$ (GeV/c) | $N_0$ | $\chi^2$/dof |
|--------|---------------------|----------|------|---------------------|------|------|---------------------|------|------------|
| Figure 7(a) | 6.3 | $\pi^+$ | 3 | 0.103 $\pm$ 0.006 | 0.77 $\pm$ 0.09 | 2 | 0.163 $\pm$ 0.011 | 0.538 $\pm$ 0.051 | 0.467/6 |
| Figure 7(c) | 6.3 | $K^+$ | 3 | 0.132 $\pm$ 0.012 | - | - | - | 0.056 $\pm$ 0.001 | 1.720/2 |
| Figure 7(b) | 7.7 | $\pi^+$ | 3 | 0.106 $\pm$ 0.002 | 0.75 $\pm$ 0.07 | 2 | 0.250 $\pm$ 0.060 | 0.067 $\pm$ 0.001 | 0.585/2 |
| Figure 7(d) | 7.7 | $K^-$ | 3 | 0.135 $\pm$ 0.008 | 0.80 $\pm$ 0.07 | 2 | 0.184 $\pm$ 0.020 | 0.016 $\pm$ 0.003 | 0.794/4 |
| Figure 7(a) | 8.8 | $\pi^+$ | 3 | 0.102 $\pm$ 0.008 | 0.76 $\pm$ 0.08 | 2 | 0.202 $\pm$ 0.011 | 0.698 $\pm$ 0.086 | 0.608/5 |
| Figure 7(c) | 8.8 | $K^+$ | 3 | 0.145 $\pm$ 0.013 | - | - | - | 0.066 $\pm$ 0.006 | 0.175/2 |
| Figure 7(b) | 8.8 | $\pi^-$ | 3 | 0.110 $\pm$ 0.042 | 0.73 $\pm$ 0.12 | 2 | 0.170 $\pm$ 0.012 | 0.482 $\pm$ 0.078 | 0.020/3 |
| Figure 7(d) | 8.8 | $K^-$ | 3 | 0.158 $\pm$ 0.017 | - | - | - | 0.028 $\pm$ 0.001 | 0.125/2 |
| Figure 7(a) | 12.3 | $\pi^+$ | 3 | 0.107 $\pm$ 0.004 | 0.80 $\pm$ 0.04 | 2 | 0.232 $\pm$ 0.015 | 0.711 $\pm$ 0.055 | 0.167/4 |
| Figure 7(c) | 12.3 | $K^+$ | 3 | 0.140 $\pm$ 0.004 | 0.75 $\pm$ 0.07 | 2 | 0.250 $\pm$ 0.060 | 0.067 $\pm$ 0.001 | 0.585/2 |
| Figure 7(b) | 17.3 | $\pi^+$ | 3 | 0.113 $\pm$ 0.005 | 0.75 $\pm$ 0.06 | 2 | 0.203 $\pm$ 0.018 | 0.461 $\pm$ 0.038 | 0.179/2 |
| Figure 7(d) | 17.3 | $K^-$ | 3 | 0.145 $\pm$ 0.007 | 0.78 $\pm$ 0.08 | 2 | 0.275 $\pm$ 0.090 | 0.030 $\pm$ 0.001 | 0.585/2 |
| Figure 7(a) | 17.3 | $\pi^+$ | 3 | 0.098 $\pm$ 0.009 | 0.51 $\pm$ 0.04 | 2 | 0.198 $\pm$ 0.008 | 0.796 $\pm$ 0.068 | 0.557/4 |
| Figure 7(c) | 17.3 | $K^+$ | 3 | 0.152 $\pm$ 0.005 | 0.76 $\pm$ 0.07 | 2 | 0.205 $\pm$ 0.030 | 0.076 $\pm$ 0.003 | 0.622/2 |
| Figure 7(b) | 17.3 | $\pi^-$ | 3 | 0.117 $\pm$ 0.009 | 0.80 $\pm$ 0.09 | 2 | 0.202 $\pm$ 0.016 | 0.631 $\pm$ 0.089 | 0.017/0 |
| Figure 7(d) | 17.3 | $K^-$ | 3 | 0.172 $\pm$ 0.004 | 0.78 $\pm$ 0.06 | 2 | 0.198 $\pm$ 0.030 | 0.044 $\pm$ 0.001 | 0.719/2 |

In Figure 8, due to the effect of resonance decay, the ratio $k_{\pi NN}$ from central nucleus-nucleus collisions for $\pi$ is greater than 1 in the energy range from a few GeV to dozens of GeV, which is different from other collision system ($pp$) and other particles ($K$ and $p$). Due to $k_{\pi NN} > 1$ and the application of Equation (8), the corresponding $\mu_{\pi NN}$ in Figure 9 is less than zero in the energy range from a few GeV to dozens.
of GeV, which is different from \( k_K \) and \( k_p \) and \( k_{\pi pp} \). These fully demonstrate the great role of resonant production of pions in central nucleus-nucleus collisions. While in the whole energy region, the trend of \( k_K \) (or \( k_p \)) from nucleus-nucleus collisions is close to that from inelastic \( pp \) collisions, and \( k_K \), \( k_p \), and \( k_{\pi pp} \) show similar trends, which indicates that resonance decay does not contribute much to \( K \), \( p \), and does not play a role in inelastic \( pp \) collisions. In addition, \( k_{\pi NN} \) (or \( \mu_{\pi NN} \)) in low energy region gradually decreasing (or increasing) and tending to \( k_{\pi pp} \) in high-energy region (RHIC and LHC), reveals that the effect of resonance decay on \( \pi \) occurs only in the low-energy region, and gradually decreases with the increase of \( \sqrt{s_{NN}} \), and disappears in high energy region.

In Figure 9, it should be noted that the derived curves of hadron and quark chemical potentials from the linear fits of the yield ratios in Figure 8 simultaneously show inflection points at around 4 GeV, which is not observed from the linear fits. In order to figure out the accurate energies at these inflection points, we made the following calculations according to Equations (6) and (8–12). From the linear fits of the yield ratios in Figure 8 simultaneously show inflection points at around 4 GeV, which is not observed from the linear fits. In order to figure out the accurate energies at these inflection points, we made the following calculations according to Equations (6) and (8–12). From the linear fits of the logarithms of three yield ratios with \( 1/\sqrt{s_{NN}} \) in Equations (10–12), one can see that all the intercepts on the vertical axes approximate to zero. We assume that all intercepts are zero for simplicity of calculation, then the chemical potential \( \mu_i \) can be given by

\[
\mu_i = T_{ch} \frac{A_i}{\sqrt{s_{NN}}} \quad (i = \pi, K, p, u, d, s), \tag{13}
\]

Where \( A_i \) is a constant. Let \( \frac{d\mu}{d\sqrt{s_{NN}}} = 0 \), we can obtain the energy values at all the inflection points and find all the energy values are the same, \( \sqrt{s_{NN}} = 3.526 \text{ GeV} \), which we think is the critical energy of phase transition from a hadron liquid-like state to a quark gas-like state in the collision system, where the liquid-like state and the gas-like state are the states in which the mean-free-path of interacting particles are relatively short and relatively long, respectively. In other words, at this special energy, the collision system starts to change initially its state from the liquid-like nucleons and hadrons to the gas-like quarks, and many properties of the system also change. The curves of chemical potentials having maximum values at these inflection points, indicates that the density of baryon number in nucleus-nucleus collisions has the largest value and the mean-free-path of particles has the smallest value at this energy, which means that the hadronic interactions play an important role. With the increase of \( \sqrt{s_{NN}} \), the chemical potentials gradually decrease, which indicates that the density of baryon number gradually decreases, the mean-free-path gradually increase and viscous effect gradually weakens. At the same time, the hadronic interactions gradually fade and the partonic interactions gradually become greater. When \( \sqrt{s_{NN}} \) increases to the RHIC, especially the LHC, all types of chemical potentials approach to zero, which indicates that the collision system possibly changes completely from the hadron-dominant state to the quark-dominant state and signifies that the partonic interactions possibly play a dominant role at the RHIC and LHC.

These results are consistent with our previous work [12, 28]. Our result (3.526 GeV) of the critical energy of phase transition is consistent with that (below 19.6 GeV) by the STAR Collaboration [1], and less than the result (between 11.5 GeV and 19.6 GeV) of a study based on the correlation between collision energy and transverse momentum [13–15] and the result (around 62.4 GeV) of the study based on a striking pattern of viscous damping and an excitation function [16].

4 Summary and Conclusion

The transverse momentum (or mass) spectra of final-state light flavour particles, \( \pi^\pm \), \( K^\pm \), \( p \), and \( \bar{p} \), produced in central Au-Au, central Pb-Pb and inelastic \( pp \) collisions at mid-rapidity over an energy range from the AGS to LHC, are described by a two- or one-component Erlang distribution in the frame of multi-source thermal model. The fitting results are in agreement with the experimental data recorded by the E866, E917, E895, NA49, NA44, NA61/SHINE, PHENIX, STAR, and ALICE collaborations.

From the fitting parameters, in most cases, the data of \( p_T \) (or \( m_T \)) spectra are suitable for the two-component Erlang distribution, where the first component corresponding to a narrow low-\( p_T \) (or \( m_T \))
region is contributed by the soft excitation process in which a few sea quarks and gluons take part in, and the second component corresponding to a wide high-$p_T$ (or $m_T$) region is contributed by the hard scattering process which is a more violent collision among two valent quarks in incident nucleons. The study shows that the contribution ratio of soft excitation process is more than 50%, which means the excitation degree of collision system is mainly contributed by soft excitation process.

The energy-dependent chemical potentials of light hadrons, $\mu_\pi$, $\mu_K$, and $\mu_p$, and quarks, $\mu_u$, $\mu_d$, and $\mu_s$, are extracted from the yield ratios of negative to positive particles based on the normalization constants in fitting the transverse momentum or mass spectra of final-state light flavour particles. With the increase of $\sqrt{s_{NN}}$ over a range from a few GeV to more than 10 TeV, the $\mu_K$, $\mu_p$, $\mu_u$, $\mu_d$, $\mu_s$ decrease obviously in central Au-Au, central Pb-Pb, and inelastic $pp$ collisions, while $\mu_\pi$ increases in central Au-Au and Pb-Pb collisions and it decreases in inelastic $pp$ collisions. When collision energy increases to the RHIC and LHC, all types of chemical potentials are small and the limiting values of them are zero in central Au-Au, central Pb-Pb, and inelastic $pp$ collisions at very high energy.

The logarithms of yield ratios, $\ln(k_\pi)$, $\ln(k_K)$, and $\ln(k_p)$, show obviously linear dependences on $1/\sqrt{s_{NN}}$. Base on the above linear relationships, we find that at 3.526 GeV, the derived curves of hadron and quark chemical potentials simultaneously show inflection points. The reason we think is that this energy is the critical energy of phase transition from a hadron liquid-like state to a quark gas-like state in the collision system, where the density of baryon number in nucleus-nucleus collisions has a large value and the hadronic interactions play an important role. When collision energy increases to the RHIC, especially the LHC, all types of chemical potentials approach to zero, which indicates that the collision system possibly changes completely from the hadron-dominant liquid-like state to the quark-dominant gas-like state and the partonic interactions possibly play a dominant role at the LHC.

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Data availability
All data are quoted from the mentioned references. As a phenomenological work, this paper does not report new data.

Conflicts of Interest
The authors declare that there are no conflicts of interest regarding the publication of this paper.

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