AdS and dS Braneworld Kaluza-Klein Reduction

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ABSTRACT

We obtain new results for consistent braneworld Kaluza-Klein reductions, showing how we can derive four-dimensional \(N = 2\) gauged supergravity “localised on the AdS\(_4\) brane” as an exact embedding in five-dimensional \(N = 4\) gauged supergravity. Similarly, we obtain five-dimensional \(N = 2\) gauged supergravity localised on an AdS\(_5\) brane as a consistent Kaluza-Klein reduction from six-dimensional \(N = 4\) gauged supergravity. These embeddings can be lifted to type IIB and massive type IIA supergravity respectively. The new AdS braneworld Kaluza-Klein reductions are generalisations of earlier results on braneworld reductions to ungauged supergravities. The lower-dimensional cosmological constant in our AdS braneworld reductions is an adjustable parameter, and so it can be chosen to be small enough to be phenomenologically realistic, even if the higher-dimensional one is of Planck scale. We also discuss analytic continuations to give a de Sitter gauged supergravity in four dimensions as a braneworld Kaluza-Klein reduction. We find that there are two distinct routes that lead to the same four-dimensional theory. In one, we start from a five-dimensional de Sitter supergravity, which itself arises from a Kaluza-Klein reduction of type IIB\(^*\) supergravity on the hyperbolic 5-sphere. In the other, we start from AdS gauged supergravity in five dimensions, with an analytic continuation of the two 2-form potentials, and embed the four-dimensional de Sitter supergravity in that. The five-dimensional theory itself comes from an \(O(4,3)/O(3,2)\) reduction of Hull’s type IIB\(_{7+3}\) supergravity in ten dimensions.

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1 Introduction

An intriguing proposal that has attracted much attention in recent times is the suggestion by Randall and Sundrum that four-dimensional physics can effectively arise from a five-dimensional theory that admits anti-de Sitter spacetime, but not Minkowski spacetime, as a vacuum \([1]\). In the ground state four-dimensional Minkowski spacetime is embedded as a “3-brane” in AdS\(_5\) via the Poincaré or horospherical description, with

\[
ds^2_5 = dz^2 + e^{-2k|z|} dx^\mu dx^\nu \eta_{\mu\nu},
\]

where the AdS\(_5\) metric \(ds^2_5\) satisfies \(\hat{R}_{MN} = -4k^2 \hat{g}_{MN}\) in the bulk. The effective four-dimensional gravity can be investigated at the linearised level by replacing the Minkowski metric \(\eta_{\mu\nu}\) by \(\eta_{\mu\nu} + h_{\mu\nu}\), and studying the equation governing small fluctuations. This leads to the conclusion that gravity is localised on the brane.

One can consider the situation beyond the linearised level, by introducing a general four-dimensional metric \(ds^2_4\), and writing

\[
ds^2_5 = dz^2 + e^{-2k|z|} ds^2_4.
\]

One can easily show that in the bulk the Ricci tensor of \(ds^2_5\) satisfies the Einstein equation \(\hat{R}_{MN} = -4k^2 \hat{g}_{MN}\) if the Ricci tensor of the four-dimensional metric satisfies \(R_{\mu\nu} = 0\) (see, for example, \([2, 3, 4, 5, 6]\)). This approach was used, for example, in \([3]\) to study the global 5-dimensional geometry resulting from having a four-dimensional Schwarzschild black hole “on the brane.”

Equation (2) has very much the structure of a Kaluza-Klein reduction ansatz, and indeed one can view it as giving a consistent embedding of \(D = 4\) pure Einstein theory with equation of motion \(R_{\mu\nu} = 0\) in \(D = 5\) Einstein theory with a negative cosmological constant, with equation of motion \(\hat{R}_{MN} = -4k^2 \hat{g}_{MN}\). More precisely, the embedding is an exact one if the modulus sign is omitted in (2), giving

\[
ds^2_5 = dz^2 + e^{-2kz} ds^2_4.
\]

This now satisfies the five-dimensional equation not only in the bulk, but also at \(z = 0\) itself, whereas previously (2) would have given a delta-function contribution to \(\hat{R}_{MN}\), owing to the discontinuity in the gradient of the metric there.

This Kaluza-Klein theme was developed in \([4]\), where it was shown that the above embedding could be extended to a fully consistent Kaluza-Klein reduction of \(N = 4\) gauged supergravity in \(D = 5\) to give \(N = 2\) ungauged supergravity in \(D = 4\). The photon of
the four-dimensional supergravity theory arises in a somewhat unusual way; rather than coming from the $\hat{g}_{\mu 5}$ components of the five-dimensional metric, it comes instead from the reduction of the two 2-form gauge potentials $\hat{A}_1^{(2)}$ of the five-dimensional $N = 4$ gauged theory, with the ansatz \[ 4 \]

$$\hat{A}_1^{(2)} = \frac{1}{\sqrt{2}} e^{-k z} F^{(2)}, \quad \hat{A}_2^{(2)} = -\frac{1}{\sqrt{2}} e^{-k z} * F^{(2)},$$

where $*$ denotes the Hodge dual in four dimensions. The equations of motion of five-dimensional $N = 4$ gauged supergravity then imply and are implied by the equations of motion of four-dimensional $N = 2$ ungauged supergravity.

This consistent “braneworld Kaluza-Klein reduction” was also generalised in \[ 4 \] to give five-dimensional $N = 2$ ungauged supergravity from six-dimensional $N = 4$ gauged supergravity, and to give six-dimensional $N = 1$ chiral ungauged supergravity from seven-dimensional $N = 2$ gauged supergravity. Further generalisations, including four-dimensional ungauged $N = 4$ supergravity from five-dimensional gauged $N = 8$, and ungauged $N = 2$ in six dimensions from gauged $N = 4$ in seven, were obtained in \[ 7 \]. Some aspects of the fermion reduction in the four-dimensional $N = 2$ case were studied in \[ 8 \].

Two universal features in all the braneworld Kaluza-Klein reductions derived in \[ 4, 7 \] were the halving of supersymmetry in the reduction, as befits a theory localised on a brane, and also the fact that a gauged supergravity in the higher dimension was reduced to give an ungauged supergravity in the lower dimension. This latter feature was effectively built in from the outset, in the metric reduction ansatz \[ 4 \], which is based on the Poincaré or horospherical embedding of $(\text{Minkowski})_D$ spacetime in $\text{AdS}_{D+1}$.

Since one can also give a metric prescription for the embedding of $\text{AdS}_D$ in $\text{AdS}_{D+1}$, this naturally raises the question of whether one can generalise the constructions in \[ 4, 7 \], to obtain braneworld Kaluza-Klein reductions that yield gauged supergravities in the lower dimension from gauged supergravities in the higher dimension. This topic, which would be the Kaluza-Klein counterpart of the viewpoint taken in \[ 9 \], forms the subject of investigation in the present paper.

The embedding of the metric $ds^2_D$ of $\text{AdS}_D$ in the metric $d\hat{s}^2_{D+1}$ of $\text{AdS}_{D+1}$ proceeds as follows:

$$d\hat{s}^2_{D+1} = dz^2 + \cosh^2(k z) ds^2_D.$$  \[ 5 \]

A simple calculation shows that if $ds^2_D$ is Einstein, with Ricci tensor given by

$$R_{\mu \nu} = -(D - 1) k^2 g_{\mu \nu},$$

\[ 6 \]
then $d\hat{s}_{D+1}^2$ will be Einstein too, with Ricci tensor given by

$$\hat{R}_{MN} = -D k^2 \hat{g}_{MN}.$$  \hfill (7)

It should be emphasised that although it might appear from (6) and (7) that there is a close correlation between the scale-size of the cosmological constants in the higher and the lower dimension, this is in fact not the case. This can be seen by making the constant coordinate shift $z = \tilde{z} - c$, together with the constant scaling $ds_D^2 = 4e^{-2kc}d\tilde{s}_D^2$. If we now let $\lambda = e^{-2kc}$, and drop the tildes, we have the metric reduction ansatz

$$d\hat{s}_{D+1}^2 = dz^2 + (\lambda e^k z + e^{-k z})^2 ds_D^2,$$  \hfill (8)

for which we find that the $(D+1)$-dimensional and $D$-dimensional Ricci tensors are given by

$$\hat{R}_{MN} = -D k^2 \hat{g}_{MN}, \quad R_{\mu\nu} = -4\lambda (D-1) k^2 g_{\mu\nu}.$$  \hfill (9)

Thus by choosing $\lambda$ appropriately, one can make the cosmological constant of the lower-dimensional anti-de Sitter spacetime as small as desired, no matter how large the cosmological constant in the higher dimension. For example, we could arrange to have a four-dimensional cosmological constant that is compatible with experimental observation, even with a Planck-scale five-dimensional cosmological constant. (Of course an extreme example is to take $\lambda = 0$, in which case the AdS embedding (8) reduces to the Poincaré embedding (3) of Minkowski spacetime in the higher-dimensional AdS.) In everything that follows in the rest of this paper we shall similarly be able to tune the cosmological constant (or, equivalently, the supergravity gauge coupling constant) to any desired value in the lower dimension, regardless of the size of the higher-dimensional cosmological constant.\footnote{We are grateful to Hong Lü for emphasising this point to us.}

The embedding (3), with $ds_D^2$ taken to be a general $D$-dimensional metric, will form the metric reduction ansatz in our braneworld Kaluza-Klein reductions. We shall focus on two examples. In the first, we shall show that four-dimensional gauged $N = 2$ supergravity can be derived as a consistent braneworld reduction of five-dimensional gauged $N = 4$ supergravity. We shall also show that five-dimensional $N = 2$ gauged supergravity can be obtained as a consistent braneworld reduction of six-dimensional $N = 4$ gauged supergravity.

In each of our examples, the gauged supergravity that forms the higher-dimensional starting point can itself be obtained as a consistent Kaluza-Klein sphere reduction, from type IIB on $S^5$ in the case of the five-dimensional theory, and from massive IIA on a local
$S^4$ in the case of the six-dimensional theory \cite{12}. By combining these sphere reductions with the subsequent braneworld reductions, we can thereby obtain unusual embeddings of the final four-dimensional and five-dimensional gauged supergravities in type IIB and massive type IIA respectively.

There has also been recent discussion of de Sitter spacetime, and de Sitter braneworlds, in the literature \cite{13}-\cite{25}. We show how, by a process of analytic continuation, we can obtain from our first example an embedding of $N = 2$ gauged de Sitter supergravity in four dimensions as a braneworld reduction of $N = 4$ gauged de Sitter supergravity in $D = 5$. In fact in general one has the embedding

$$ds^2_{D+1} = dz^2 + \cos^2(kz) ds^2_D.$$  

of the $D$-dimensional de Sitter metric in $(D + 1)$-dimensional de Sitter spacetime, and this provides the starting-point for the Kaluza-Klein de Sitter braneworld reduction. The five-dimensional starting point for a braneworld reduction to four-dimensional de Sitter supergravity itself arises as a consistent Kaluza-Klein reduction on the hyperbolic space $H^5$ of the type IIB* theory, which was introduced by Hull \cite{10}. The type IIB* theory arises by performing a T-duality transformation of type IIA on a timelike circle, and is characterised by the fact that all the Ramond-Ramond fields have kinetic terms of the “wrong” sign. It has been argued that this notwithstanding, the theory is a valid sector within the web of dualities, with the subtleties of string theory serving to protect it from the usual problems of negative kinetic energies in field theory \cite{10}. Recently, de Sitter gauged supergravities related to the type IIA* and type IIB* theories have been discussed \cite{25}; they evade the usual “no-go” theorems for de Sitter supersymmetry precisely because of the reversed signs for some of the kinetic terms. As in the braneworld reductions to AdS supergravities, we can adjust the lower-dimensional cosmological constant at will, by performing rescalings of the type discussed above equation (8).

Our procedure for obtaining the de Sitter braneworld reduction consists of first noting that we can obtain the type IIB* theory by an analytic continuation of the usual type IIB theory, in which all the Ramond-Ramond fields $\Psi$ are transformed according to $\Psi \longrightarrow i \Psi$. We then implement this continuation in the usual $S^5$ reduction to $D = 5$, thereby obtaining the gauged de Sitter supergravity, with the $S^5$ analytically continuing to become $H^5$. Then, the analytic continuation of our previous braneworld reduction gives the embedding of
gauged $N = 2$ de Sitter supergravity on the brane in four dimensions. Of course the existence of de Sitter supergravities depends also upon the fact that certain fields have the “wrong” signs for their kinetic terms. By contrast, we find that our $D = 6$ to $D = 5$ AdS braneworld reduction does not admit an analytic continuation to a real reduction of the six-dimensional gauged de Sitter supergravity that can be obtained from reduction of the type IIA* theory.

We also show that the same four-dimensional de Sitter gauged supergravity can be obtained as a braneworld Kaluza-Klein reduction of a quite different type. This is based on the fact that $D$-dimensional de Sitter spacetime can be embedded within $(D+1)$-dimensional anti-de Sitter spacetime, with the metric given by

$$d\hat{s}_{D+1}^2 = dz^2 + \sinh^2(kz) ds_D^2.$$  \hspace{1cm} (11)

This has $\hat{R}_{MN} = -Dk^2 \hat{g}_{MN}$ if $R_{\mu\nu} = (D - 1) k^2 g_{\mu\nu}$, and again, with a shift and rescaling of the type discussed above equation (8), the lower-dimensional cosmological constant can be made arbitrary. Using an ansatz involving (11) for the metric reduction, we obtain a consistent braneworld Kaluza-Klein reduction from an anti-de Sitter gauged $N = 4$ supergravity in five dimensions, with an analytic continuation of the two 2-form potentials, to give de Sitter gauged $N = 2$ supergravity in four dimensions. This AdS$_5$ gauged supergravity can itself be obtained from the type IIB$_{7+3}$ ten-dimensional supergravity introduced by Hull [27], which has spacetime signature $(7,3)$, by reducing on the $(3,2)$-signature space $O(4,2)/O(3,2)$. The associated string theory is obtained from the usual type IIB theory by means of a sequence of T-duality transformations involving spacelike and timelike circles.

In the conclusions of this paper, we make some remarks about possible applications of our results, and about the relation between conventional braneworld scenarios and the Kaluza-Klein viewpoint.

2 Gauged $N = 2$, $D = 4$ supergravity from $N = 4$, $D = 5$

In this section, we show how gauged $N = 2$ supergravity in four dimensions arises as a consistent Kaluza-Klein braneworld reduction from gauged $N = 4$ supergravity in five dimensions. This generalises previous work in [4], where it was shown how the ungauged $N = 2$ theory could be obtained as a consistent Kaluza-Klein braneworld reduction from the same gauged $N = 4$ starting point in $D = 5$. Our results here can be viewed as an exact and fully non-linear extension of the discussion of supergravity fluctuations on an AdS$_4$ braneworld embedded in $D = 5$. This generalises the interpretation of the Kaluza-Klein
reduction in [4], which was the analogous non-linear extension of supergravity fluctuations around a flat Minkowski4 braneworld.

2.1 The bosonic fields

Our starting point is the $SU(2) \times U(1)$ gauged $N = 4$ supergravity in five dimensions. The bosonic sector of the five-dimensional theory comprises the metric, a dilatonic scalar $\phi$, the $SU(2)$ Yang-Mills potentials $\hat{A}^i_{(1)}$, a $U(1)$ gauge potential $\hat{B}_{(1)}$, and two 2-form potentials $\hat{A}^\alpha_{(2)}$ which transform as a charged doublet under the $U(1)$. The Lagrangian [28], expressed in the language of differential forms that we shall use here, is given by [11]

$$L_5 = \hat{R} \hat{\ast} 1 - \frac{1}{2} \hat{\ast} d\phi \wedge d\phi - \frac{1}{2} X^4 \hat{\ast} \hat{G}_{(2)} \wedge \hat{G}_{(2)} - \frac{1}{2} X^{-2} (\hat{\ast} \hat{F}^i_{(2)} \wedge \hat{F}^i_{(2)} + \hat{\ast} \hat{A}^\alpha_{(2)} \wedge \hat{A}^\alpha_{(2)})$$

$$+ \frac{1}{2g} \epsilon_{\alpha\beta} \hat{A}^\alpha_{(2)} \wedge d\hat{A}^\beta_{(2)} - \frac{1}{2} \hat{A}^\alpha_{(2)} \wedge \hat{\ast} \hat{B}_{(1)} - \frac{1}{2} \hat{F}^i_{(2)} \wedge \hat{F}^i_{(2)} \wedge \hat{B}_{(1)}$$

$$+ 4g^2 (X^2 + 2X^{-1}) \hat{\ast} 1,$$

(12)

where $X = e^{-\sqrt{6} \phi}$, $\hat{F}^i_{(2)} = d\hat{A}^i_{(1)} + \frac{1}{\sqrt{2}} g \epsilon^{ijk} \hat{A}^j_{(1)} \wedge \hat{A}^k_{(1)}$ and $\hat{G}_{(2)} = d\hat{B}_{(1)}$, and $\hat{\ast}$ denotes the five-dimensional Hodge dual. It is useful to adopt a complex notation for the two 2-form potentials, by defining

$$A_{(2)} \equiv A^1_{(2)} + i A^2_{(2)}.$$

(13)

Our bosonic Kaluza-Klein reduction ansatz involves setting the fields $\phi$ and $B_{(1)}$ to zero, together with two out of the three components of the $SU(2)$ Yang-Mills fields $A^i_{(1)}$. We find that the ansatz for the remaining bosonic fields, comprising the metric, the two 2-form potentials, and the surviving Yang-Mills potential, which we shall take to be $A^1_{(1)}$, is

$$d\hat{s}^2_5 = dz^2 + \cosh^2(kz) \, ds^2_4,$$

$$\hat{A}^1_{(2)} = -\frac{1}{\sqrt{2}} \cosh(kz) \, \ast F_{(2)}, \quad \hat{A}^2_{(2)} = \frac{1}{\sqrt{2}} \sinh(kz) \, F_{(2)},$$

$$\hat{A}^1_{(1)} = \frac{1}{\sqrt{2}} A_{(1)},$$

(14)

where $ds^2_4$ is the metric and $F_{(2)} = dA_{(1)}$ is the Maxwell field of the four-dimensional $N = 2$ supergravity, and $\ast$ denotes the Hodge dual in the four-dimensional metric.

We now substitute this ansatz into the five-dimensional equations of motion that follow from (12), namely [11, 11, 11]}

$$d(X^{-1} \hat{\ast} dX) = \frac{1}{2} X^4 \hat{\ast} \hat{G}_{(2)} \wedge \hat{G}_{(2)} - \frac{1}{4} X^{-2} (\hat{\ast} \hat{F}^i_{(2)} \wedge \hat{F}^i_{(2)} + \hat{\ast} \hat{A}^\alpha_{(2)} \wedge \hat{A}^\alpha_{(2)})$$

$$- \frac{1}{3} g^2 (X^2 - X^{-1}) \hat{\ast} 1,$$

$$d(X^4 \hat{\ast} \hat{G}_{(2)}) = -\frac{1}{2} \hat{F}^i_{(2)} \wedge \hat{F}^i_{(2)} - \frac{1}{2} \hat{\ast} \hat{A}_{(2)} \wedge \hat{A}_{(2)}.$$
\[
\begin{align*}
\text{d}(X^{-2} \hat{F}^i) &= \sqrt{2} g \epsilon^{ijk} X^{-2} \hat{F}^j \wedge \hat{A}^k_{(1)} - \hat{F}^i_{(2)} \wedge \hat{G}_{(2)}, \\
X^2 \hat{F}^i_{(3)} &= -ig \hat{A}^i_{(2)}, \\
\hat{R}_{MN} &= 3X^{-2} \partial_M X \partial_N X - \frac{4}{3} g^2 (X^2 + 2X^{-1}) \hat{g}_{MN} \\
&\quad + \frac{1}{2} X^4 (\hat{G}_M^P \hat{G}_{NP} - \frac{1}{2} \hat{g}_{MN} \hat{G}^2_{(2)}) + \frac{1}{2} X^{-2} (\hat{F}^i_M \hat{F}^i_{NP} - \frac{1}{2} \hat{g}_{MN} (\hat{F}^i_{(2)})^2) \\
&\quad + \frac{1}{2} X^{-2} (\hat{A}_M^P \hat{A}^P_N - \frac{1}{2} \hat{g}_{MN} |\hat{A}_{(2)}|^2),
\end{align*}
\]

where
\[
\hat{F}^i_{(3)} = D \hat{A}^i_{(2)} \equiv d \hat{A}^i_{(2)} - ig \hat{B}^i_{(1)} \wedge \hat{A}^i_{(2)}.
\]

In order to do this, it is useful to record that the Ricci tensor \(\hat{R}_{AB}\) for the five-dimensional metric \(d\hat{s}^2_5\) is related to the Ricci tensor \(R_{ab}\) of the four-dimensional metric \(ds^2_4\) by
\[
\begin{align*}
\hat{R}_{ab} &= \text{sech}^2(kz) R_{ab} - 3k^2 \tanh^2(kz) \eta_{ab} - k^2 \eta_{ab}, \\
\hat{R}_{55} &= -4k^2, \\
\hat{R}_{a5} &= 0,
\end{align*}
\]

where \(A = (a, 5)\), and we are using tangent-space indices. The following lemmata are also helpful:
\[
\begin{align*}
\hat{A}^i_{(2)} \wedge \hat{A}^i_{(2)} &= -\frac{1}{2} F^i_{(2)} \wedge F^i_{(2)}, \\
\hat{F}^i_{(2)} \wedge \hat{A}^i_{(2)} &= -\frac{1}{2} \ast F^i_{(2)} \wedge F^i_{(2)} \wedge dz, \\
\hat{F}^i_{(2)} &= \ast F^i_{(2)} \wedge dz, \\
\ast(F^i_{(2)} \wedge dz) &= \ast F^i_{(2)}.
\end{align*}
\]

After substituting the braneworld reduction ansatz, we find that all the five-dimensional equations of motion are satisfied, with an exact cancellation of all dependence on the fifth coordinate \(z\), if and only if the four-dimensional fields satisfy the bosonic equations of motion of four-dimensional gauged \(N = 2\) supergravity, namely
\[
\begin{align*}
R_{\mu\nu} &= \frac{1}{2} (F_{\mu\rho} F^\rho_{\nu} - \frac{1}{4} F^2_{(2)} g_{\mu\nu}) - 3k^2 g_{\mu\nu}, \\
d \ast F_{(2)} &= 0,
\end{align*}
\]

with
\[
k = g.
\]

It should be emphasised that the cancellation of the \(z\) dependence in the five-dimensional equations of motion is quite non-trivial, and it depends crucially on the details of the ansatz \((14)\). In particular, the presence of the four-dimensional \(U(1)\) gauge field both in the 2-form potentials \(\hat{A}_{(2)}\) and in a \(U(1)\) subgroup of the \(SU(2)\) Yang-Mills fields \(\hat{A}^i_{(1)}\) in five dimensions is essential for the matching of the \(z\) dependence to work. We should also emphasise that, as discussed in the introduction, the cosmological constant in the four-dimensional gauged
theory can be adjusted at will, whilst keeping the five-dimensional cosmological constant fixed, by shifting to a new coordinate $z = \tilde{z} - c$, together with the rescaling $ds^2_4 = 4e^{-2kc}d\tilde{s}^2_4$ of the four dimensional metric, and now, in addition, we rescale the four-dimensional gauge field so that $A_{(1)} = 2e^{-kc}\tilde{A}_{(1)}$. We shall return to this point in section 2.3.

### 2.2 The fermionic fields

Having obtained a consistent Kaluza-Klein reduction ansatz in the bosonic sector, we now turn to the fermions. In the five-dimensional $N = 4$ gauged theory there are spin-$\frac{1}{2}$ fields $\hat{\chi}_p$, where $p$ is a 4-dimensional $USp(4)$ index, and spin-$\frac{3}{2}$ fields $\hat{\psi}_{M\mu}$. Our fermionic ansatz will involve setting $\hat{\chi}_\alpha$ to zero, and so we must first verify that this is compatible with the expected surviving $N = 2$ supersymmetry in $D = 4$. Taking into account that certain of the bosonic fields have already been set to zero in the reduction ansatz, the remaining non-vanishing five-dimensional fields have the following contributions in the supersymmetry transformation rule for $\hat{\chi}_p$:

$$
\delta \hat{\chi}_p = -\frac{1}{4\sqrt{6}} \gamma^{MN} \left( F_{\mu\nu}^i (\Gamma_i)_{pq} + A_{\alpha}^\mu (\Gamma_\alpha)_{pq} \right) \hat{\epsilon}^q,
$$

(21)

where $\gamma_M$ are the five-dimensional spacetime Dirac matrices, and $(\Gamma_\alpha, \Gamma_i)$ are the five “internal” $USp(4) \sim SO(5)$ Dirac matrices. Substituting the bosonic ansatz into this, we find that for $\delta \hat{\chi}_p$ to vanish we must have

$$
\gamma^{\mu\nu} \left( F_{\mu\nu} \Gamma_3 - \cosh(kz) (\ast F)_{\mu\nu} \Gamma_1 + \sinh(kz) F_{\mu\nu} \Gamma_2 \right)_{pq} \hat{\epsilon}^q = 0 .
$$

(22)

Using $\gamma^{\mu\nu} (\ast F)_{\mu\nu} = i \gamma^{\mu\nu} \gamma_5 F_{\mu\nu}$, we therefore deduce that the five-dimensional supersymmetry parameters $\hat{\epsilon}_p$ must satisfy

$$
(\Gamma_3 - i \cosh(kz) \gamma_5 \Gamma_1 + \sinh(kz) \Gamma_2) \hat{\epsilon} = 0 ,
$$

(23)

where we have now suppressed the $USp(4)$ internal spinor index $p$. We shall return to this equation shortly.

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3In common with the majority of recent work in supergravity, we shall neglect quartic fermion terms in the Lagrangian, and their associated consequences in the supersymmetry transformation rules.

4The five-dimensional index range is spanned by $\alpha$, the doublet index on the 2-form potentials $\hat{A}_5^{\alpha}_{(2)}$, and $i$, the Yang-Mills $SU(2)$ triplet index on $\hat{A}_i^{(1)}$. In what follows, we shall take $\alpha = (1, 2)$, and $i = (3, 4, 5)$. Thus our previous bosonic ansatz with $\hat{A}_i^{(1)}$ non-zero will now be translated, in this fermionic discussion, to having a non-vanishing Yang-Mills term for the index value $i = 3$. 

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For the five-dimensional gravitino transformation rule, taking into account that certain bosonic fields are set to zero in the reduction ansatz, we shall have

$$\delta \hat{\psi}_M = \hat{\nabla}_M \hat{\epsilon}_p + \left( \frac{g}{\sqrt{2}} \hat{A}_i^{(i)} \Gamma_{12} - \frac{i g}{2} \gamma_M \Gamma_{12} - \frac{i}{12\sqrt{2}} \left[ \gamma_i^{PQ} - 4 \delta_M^P \gamma^Q \right] (\hat{F}^i_{PQ} \Gamma_i + \hat{A}_P^\alpha \Gamma_{\alpha}) \right)_{pq} \hat{\epsilon}^q. \quad (24)$$

Our fermion reduction ansatz will involve setting $\hat{\psi}_M$ to zero in the $M = 5$ direction. The resulting requirement $\delta \hat{\psi}_5 = 0$ for supersymmetry therefore leads to

$$\partial_z \hat{\epsilon} + \frac{i g}{2} \gamma_5 \Gamma_{12} \hat{\epsilon} = 0, \quad (25)$$

where we have again suppressed the $USp(4)$ index on $\hat{\epsilon} \gamma_5$ is $-\gamma_z$. Bearing in mind that $g = k$, we now find that (23) and (25) are solved by taking

$$\hat{\epsilon} = \left( \cosh(\frac{1}{2} k z) - i \sinh(\frac{1}{2} k z) \gamma_5 \Gamma_{12} \right) \epsilon, \quad (26)$$

where $\epsilon$ is the four-dimensional supersymmetry parameter, which must satisfy the constraint

$$\gamma_5 \Gamma_{13} \epsilon = i \epsilon. \quad (27)$$

This constraint has the effect of halving the supersymmetry. This is consistent with the fact that we are starting from the $N = 4$ gauged theory in $D = 5$, and ending up with the $N = 2$ gauged theory in $D = 4$.

By examining the components of $\delta \hat{\psi}_M$ when $M$ lies in the four-dimensional spacetime, we are now in a position to deduce the correct reduction ansatz for the gravitino. We find that it is

$$\hat{\psi}_\mu = \left( \cosh(\frac{1}{2} k z) - i \gamma_5 \Gamma_{12} \sinh(\frac{1}{2} k z) \right) \psi_\mu, \quad (28)$$

where $\psi_\mu$ denotes the four-dimensional gravitini, subject also to the constraint

$$\gamma_5 \Gamma_{13} \psi_\mu = i \psi_\mu. \quad (29)$$

Substituting (26) and (28) into (24), we find that the $z$ dependence matches on the two sides of the equation and we consistently read off the four-dimensional gravitino transformation rule

$$\delta \psi_\mu = D_\mu \epsilon - \frac{1}{2} k \gamma_\mu \epsilon + \frac{1}{8} F^{\nu\rho} \gamma_\mu \gamma_{\nu\rho} \Gamma_{123} \epsilon, \quad (30)$$

where

$$D_\mu \epsilon = \nabla_\mu \epsilon + \frac{1}{2} k A_\mu \Gamma_{123} \epsilon. \quad (31)$$
2.3 The $D = 4$ cosmological constant and the ungauged limit

If we implement the rescalings discussed in the introduction and at the end of section 2.1, namely

$$z = \tilde{z} - c, \quad g_{\mu\nu} = 4e^{-2k\tilde{c}} \tilde{g}_{\mu\nu}, \quad A_\mu = 2e^{-k\tilde{c}} \tilde{A}_\mu,$$

then after defining $\lambda \equiv e^{-2k\tilde{c}}$, and dropping the tildes on the rescaled four-dimensional fields, we see that the bosonic reduction ansatz (14) becomes

$$ds_5^2 = dz^2 + (\lambda e^{kz} + e^{-kz})^2 ds_4^2,$$
$$A_{(2)}^1 = -\frac{1}{\sqrt{2}} (\lambda e^{kz} + e^{-kz}) * F_{(2)}, \quad A_{(2)}^2 = \frac{1}{\sqrt{2}} (\lambda e^{kz} - e^{-kz}) F_{(2)},$$
$$A_{(1)}^1 = \sqrt{2\lambda} A_{(1)},$$

and the resulting four-dimensional equations of motion (19) become

$$R_{\mu\nu} = \frac{1}{2} (F_{\mu\rho} F_{\nu}^{\rho} - \frac{1}{4} F_{(2)}^2 g_{\mu\nu}) - 12\lambda k^2 g_{\mu\nu},$$
$$d*F_{(2)} = 0,$$

An analogous rescaling can also be performed in the fermionic reduction ansatz obtained in (2.2).

The above rescaling shows that by choosing the constant $\lambda$ appropriately, we can obtain any desired value of the cosmological constant in the four-dimensional gauged supergravity, whilst keeping the five-dimensional cosmological constant fixed. In particular, we could choose $\lambda$ so that the four-dimensional cosmological constant is compatible with observation, even if the five-dimensional cosmological constant is of Planck scale. Of course the standard supergravity relation between the cosmological constant and the gauge coupling constant in $D = 4$ will still hold. This can be seen, for example, from the gauge-covariant derivative (31), which now becomes

$$D_\mu \epsilon = \nabla_\mu \epsilon + k \sqrt{\lambda} A_\mu \Gamma_{123} \epsilon.$$

We may also note that we can recover the results in [4] for the brane-world reduction to ungauged $N = 2$ supergravity, by taking $\lambda = 0$. Note in particular that the cosmological constant in the Einstein equation scales to zero in this limit, as expected for the ungauged limit. A similar rescaling procedure applied to the fermionic sector shows that one also recovers the fermionic results discussed in [8].

---

\(^5\)The four-dimensional field strength here is the Hodge dual of the one arising in [4]. The choice of duality complexion was immaterial in [4], since the 1-form potential $A_{(1)}$ itself did not appear in the reduction ansatz there. In the AdS braneworld reduction we are considering in this paper the choice becomes important, because the ansatz (14) involves the explicit appearance of $A_{(1)}$ in the Yang-Mills sector, which was not excited in the ungauged case.
2.4 Lifting to type IIB in $D = 10$

The $SU(2) \times U(1)$ gauged $N = 4$ supergravity in $D = 5$ that formed the starting point for our braneworld Kaluza-Klein reduction to $D = 4$ can itself be obtained in a Kaluza-Klein reduction from type IIB supergravity. It is believed, although it has never been proved, that the maximal $SO(6)$-gauged $N = 8$ theory in $D = 5$ arises as an $S^5$ reduction from type IIB. For our purposes, it suffices to work with a truncated $S^5$ reduction in which only the fields of $N = 4$ supergravity in $D = 5$ are retained, and in the bosonic sector this was constructed in explicit detail in [11]. Using these results, we may therefore lift our braneworld Kaluza-Klein reduction to $D = 10$, giving a novel way of embedding four-dimensional $N = 2$ gauged supergravity as a consistent Kaluza-Klein reduction from type IIB supergravity.

There is no simple covariant Lagrangian for type IIB supergravity, on account of the self-duality constraint for the 5-form. However, one can write a Lagrangian in which the 5-form is unconstrained, which must then be accompanied by a self-duality condition which is imposed by hand at the level of the equations of motion [29]. This type IIB Lagrangian, in the notation we shall use here, is [11]

$$\mathcal{L}^{IIB}_{10} = \hat{R} \hat{\ast} 1 - \frac{1}{2} \hat{\ast} d \hat{\phi} \wedge d \hat{\phi} - \frac{1}{2} e^{2 \hat{\phi}} \hat{\ast} d \hat{\chi} \wedge d \hat{\chi} - \frac{1}{2} \hat{\ast} \hat{H} (5) \wedge \hat{H} (5) - \frac{1}{2} e^{\hat{\phi}} \hat{\ast} \hat{F}^2 \wedge \hat{F}^2 - \frac{1}{2} e^{\hat{\phi}} \hat{\ast} \hat{F}^1 \wedge \hat{F}^1 - \frac{1}{2} \hat{B} (4) \wedge d \hat{A}^1 \wedge d \hat{A}^2 ,$$

(36)

where $\hat{F}^2 = d \hat{A}^2 (2)$, $\hat{F}^1 = d \hat{A}^1 (2) - \hat{\chi} d \hat{A}^2 (2)$, $\hat{H} (5) = d \hat{B} (4) - \frac{1}{2} \hat{A}^1 (2) \wedge d \hat{A}^2 (2) + \frac{1}{2} \hat{A}^2 (2) \wedge d \hat{A}^1 (2)$, and we use hats to denote ten-dimensional fields and the ten-dimensional Hodge dual $\hat{\ast}$. The bosonic equations follow from the Euler-Lagrange equations, together with the self-duality constraint $\hat{H} (5) = \hat{\ast} H (5)$. The bosonic reduction ansatz is then given by [11]

$$d s^2_{10} = \Delta^{1/2} d s^2_5 + g^{-2} X \Delta^{1/2} d \xi^2 + g^{-2} \Delta^{-1/2} X^2 s^2 \left( d \tau - g B (1) \right)^2$$

$$+ \frac{1}{4} g^{-2} \Delta^{-1/2} X^{-1} c^2 \sum_i (\sigma^i - \sqrt{g} A^i_{(1)})^2 ,$$

$$\hat{G} (5) = 2 g U \xi_5 - \frac{3 sc}{g} X^{-1} \ast d X \wedge d \xi + \frac{c^2}{8 \sqrt{2} g^2} X^{-2} \ast F_{(2)} \wedge h^j \wedge h^k \xi_{ij}$$

$$- \frac{3 sc}{2 \sqrt{2} g^2} X^{-2} \ast F_{(3)} \wedge h^i \wedge d \xi - \frac{3 sc}{g^2} X^4 \ast G_{(2)} \wedge d \xi \wedge (d \tau - g B (1)),$$

$$\hat{A} (2) \equiv \hat{A}^1 (2) + i \hat{A}^2 (2) = - \frac{s}{\sqrt{2} g} e^{-i \tau} A (2) ,$$

$$\hat{\phi} = 0, \quad \hat{\chi} = 0 ,$$

(37)

where $\hat{H} (5) = \hat{G} (5) + \hat{\ast} \hat{G} (5)$, $h^i \equiv \sigma^i - \sqrt{g} A^i_{(1)}$, $\Delta \equiv X^{-2} s^2 + X c^2$, $U \equiv X^2 c^2 + X^{-1} s^2 + X^{-1}$, $\epsilon_5$ is the volume form in the five-dimensional spacetime metric $d s^2_5$, and $c \equiv \cos \xi$, $s \equiv \sin \xi$. The $\sigma_i$ are the left-invariant 1-forms of $SU(2)$, and the 5-sphere on which the reduction is
performed is described, if the Yang-Mills and scalar field are taken to vanish, by the round metric $ds^2 = d\xi^2 + s^2 d\sigma^2 + \frac{1}{4} d\sigma_1^2$. Thus $S^3$ is viewed as a foliation by $S^1 \times S^3$.

Owing to a lack of a variety of suitable adornments for fields, in the above we are using the “hat” to denote ten-dimensional quantities, while the unhatted quantities are five-dimensional. We now substitute our previous braneworld ansatz (14) into this (taking appropriate care over the change of rôles of hatted and unhatted fields), thereby obtaining a reduction ansatz from $D = 10$ to $D = 4$. This is therefore given by

$$
\begin{align*}
    ds^2_{10} &= dz^2 + \cosh^2(kz) ds^2_4 + g^{-2} \left( d\xi^2 + s^2 d\sigma^2 + \frac{1}{4} d\sigma_1^2 + (\sigma_3 - g A_{(1)})^2 \right), \\
    \hat{G}_{(5)} &= 4g \cosh^4(kz) dz \wedge \varepsilon_4 + \frac{c^2}{8g^2} F^{(2)} \wedge d\xi \wedge \sigma_1 \wedge \sigma_2 \\
    &\quad - \frac{sc}{4g^2} *F^{(2)} \wedge dz \wedge (\sigma_3 - g A_{(1)}) \wedge d\xi \\
    \hat{A}_{(2)} &= \hat{A}_{(2)}^1 + i \hat{A}_{(2)}^2 = \frac{s}{2g} e^{-i\tau} (-i \sinh(kz) F_2 + \cosh(kz) *F^{(2)}), \\
    \hat{\phi} &= 0, \quad \hat{\chi} = 0,
\end{align*}
$$

(38)

3 Gauged $N = 2$, $D = 5$ supergravity from $N = 4$, $D = 6$

In this section, we show how a similar consistent braneworld Kaluza-Klein reduction of the $SU(2)$ gauged $N = 4$ theory in six dimensions is possible, yielding $N = 2$ (minimal) gauged supergravity in $D = 5$. Again, this generalises a braneworld reduction in [1], where the ungauged $N = 2$ theory in five dimensions was obtained. Since the ideas used in the reduction to $D = 5$ are very similar to those for the reduction to $D = 4$ in the previous section, we shall give a rather briefer presentation of our results here.

3.1 The bosonic fields

The bosonic fields in the six-dimensional $SU(2)$ gauged theory comprise the metric, a dilaton $\hat{\phi}$, a 2-form potential $\hat{A}_{(2)}$, and a 1-form potential $\hat{B}_{(1)}$, together with the $SU(2)$ gauge potentials $\hat{A}_{(1)}^i$. The bosonic Lagrangian [30], converted to the language of differential forms, is [12]

$$
\begin{align*}
    \mathcal{L}_6 &= \hat{R} *1 - \frac{1}{2} \hat{d}\hat{\phi} \wedge d\hat{\phi} - g^2 \left( \frac{3}{2} X^{-6} - \frac{8}{3} X^{-2} - 2X^2 \right) *1 \\
    &\quad - \frac{1}{4} X^4 \hat{F}_{(3)} \wedge \hat{F}_{(3)} - \frac{1}{2} X^{-2} \left( \hat{F}_{(2)} \wedge \hat{F}_{(2)} + \hat{F}_{(2)}^i \wedge \hat{F}_{(2)}^i \right) \\
    &\quad - \hat{A}_{(2)} \wedge \left( \frac{1}{2} d\hat{B}_{(1)} \wedge d\hat{B}_{(1)} + \frac{s}{2g} \hat{A}_{(2)} \wedge d\hat{B}_{(1)} + \frac{2}{27} g^2 \hat{\hat{A}}_{(2)} \wedge \hat{A}_{(2)} + \frac{1}{3} \hat{F}_{(2)}^i \wedge \hat{F}_{(2)}^i \right),
\end{align*}
$$

(39)

where $X \equiv e^{-\hat{\phi}/(2\sqrt{2})}$. $\hat{F}_{(3)} = d\hat{A}_{(2)}$, $\hat{G}_{(2)} = d\hat{B}_{(1)} + \frac{2}{3} g \hat{A}_{(2)}$, $\hat{F}_{(2)}^i = d\hat{A}_{(1)}^i + \frac{1}{2} g \epsilon_{ijk} \hat{A}_{(1)}^j \wedge \hat{A}_{(1)}^k$, and here $*$ denotes the six-dimensional Hodge dual. The resulting bosonic equations of
motion are given in [12].

We find that the following bosonic ansatz yields a consistent braneworld Kaluza-Klein reduction:

\[
\begin{align*}
 ds_6^2 &= dz^2 + \cosh^2( kz) \, ds_5^2 , \\
 \hat{A}_{(2)} &= - \frac{1}{\sqrt{3}} k^{-1} \sinh(k z) \, F_{(2)} , \\
 \hat{A}^3_{(1)} &= - \sqrt{\frac{2}{3}} A_{(1)} , \\
 \hat{B}_{(1)} &= 0 , \quad \hat{A}^1_{(1)} = 0 , \quad \hat{A}^2_{(1)} = 0 , \quad \hat{\phi} = 0 , \\
\end{align*}
\]  

(40) 

where \( F_{(2)} = dA_{(1)} \) is the graviphoton of the five-dimensional \( N = 2 \) supergravity, and

\[
k = \sqrt{\frac{2}{3}} g.
\]  

(41) 

Note that although \( \hat{B}_{(1)} \) is set to zero, the field strength \( \hat{G}_{(2)} \), defined above, is non-zero, and is given by

\[
\hat{G}_{(2)} = - \sqrt{\frac{2}{3}} \sinh(k z) \, F_2 .
\]  

(42) 

It should also be noted that, as in the reduction from \( D = 5 \) to \( D = 4 \), we find it necessary for the five-dimensional graviphoton to appear not only in the ansatz for \( \hat{A}_{(2)} \), but also in one component of the \( SU(2) \) Yang-Mills fields in \( D = 6 \).

After substituting the ansatz (40) into the equations of motion in [12] that follow from (39), we find that all the \( z \) dependence matches in a consistent fashion, yielding the following five-dimensional equations of motion:

\[
\begin{align*}
 R_{\mu\nu} &= \frac{1}{2} (F_{\mu\rho} F_{\nu}^\rho - \frac{1}{6} F_{(2)}^2 g_{\mu\nu}) - 4 k^2 g_{\mu\nu} , \\
 ds^* F_{(2)} &= \frac{1}{\sqrt{3}} F_{(2)} \wedge F_{(2)} .
\end{align*}
\]  

(43) 

These are precisely the bosonic equations of motion of \( N = 2 \) (i.e. minimal) gauged supergravity in five dimensions. We may note that, as in section 2.3, we can make the analogous rescaling (32), so that the five-dimensional cosmological constant becomes freely adjustable. In particular, by setting \( \lambda = 0 \) we recover the braneworld Kaluza-Klein reduction to the ungauged supergravity that was obtained in [4].

3.2 The fermionic fields

The six-dimensional \( N = 4 \) gauged theory [31] has spin-\( \frac{1}{2} \) fields \( \hat{\chi} \) and spin-\( \frac{3}{2} \) fields \( \hat{\psi}_M \), where the fermions carry also \( USp(2) \times USp(2) \) indices, which we are suppressing. Thus we can think of the fermions as being tensor products of 8-component spacetime spinors...
with two-component \(USp(2) \sim SU(2)\) spinors. Now, we shall denote the six-dimensional spacetime Dirac matrices by \(\hat{\gamma}_A\). It is necessary to distinguish these hatted \(8 \times 8\) matrices from the \(4 \times 4\) Dirac matrices of the reduced theory in \(D = 5\), which will be denoted by \(\gamma_a\) without hats. We may take a basis where the spacetime Dirac matrices are related by

\[
\hat{\gamma}_a = \sigma_1 \times \gamma_a, \quad \hat{\gamma}_6 = \sigma_2 \times 1,
\]

where \(\sigma_1\) and \(\sigma_2\) are Pauli matrices. Note that the chirality operator in \(D = 6\) is given in this basis by

\[
\hat{\gamma}_7 = \sigma_3 \times 1.
\]

In the bosonic backgrounds that we need to consider, where the dilaton \(\hat{\phi}\) vanishes, the six-dimensional supersymmetry transformation rule for the spin-\(\frac{1}{2}\) fields is given by

\[
\delta \hat{\chi} = -\frac{i}{24} \hat{F}^{MNP} \hat{\gamma}_7 \hat{\gamma}_{MNP} \hat{\epsilon} - \frac{1}{8\sqrt{2}} \hat{\gamma}^{MN} (\hat{G}_{MN} + i \hat{F}_i^{ij} \hat{\gamma}_7 \tau_i) \hat{\epsilon},
\]

where \(\tau_i\) are the Pauli matrices associated with the internal \(USp(2)\) 2-component index. The \(D = 6\) gravitino transformation rule, after setting \(\hat{\phi}\) to zero, is

\[
\delta \hat{\psi}_M = \hat{D}_M \hat{\epsilon} - \frac{i}{2} k \hat{\gamma}_M \hat{\gamma}_7 \hat{\epsilon} - \frac{1}{38} \hat{\gamma}_7 \hat{F}_{NPQ} \hat{\gamma}^{NPQ} \hat{\gamma}_M \hat{\epsilon} - \frac{i}{16\sqrt{2}} (\hat{\gamma}_M^{PQ} - 6\delta_M^P \hat{\gamma}^Q) (\hat{G}_{PQ} + i \hat{F}_i^{ij} \hat{\gamma}_7 \tau_i) \hat{\epsilon},
\]

where

\[
\hat{D}_M \hat{\epsilon} = \hat{\nabla}_M \hat{\epsilon} - \frac{1}{2} g \hat{A}_M \tau_i \hat{\epsilon}.
\]

In our case where only one component is of the Yang-Mills fields \(\hat{F}^{\text{(2)}}_{\text{(2)}}\) is non-zero, we shall take it to be \(i = 3\).

Our fermionic ansatz involves setting \(\hat{\chi} = 0\), and so supersymmetry requires \(\delta \hat{\chi} = 0\), and hence, after substituting (40) into (46), we get

\[
\left( i \cosh(kz) \hat{\gamma}_7 \hat{\gamma}_6 + \sinh(kz) - i \hat{\gamma}_7 \tau_3 \right) \hat{\epsilon} = 0.
\]

Our reduction ansatz also involves setting the \(z\) component of the six-dimensional gravitino to zero. From the \(z\) component of the gravitino transformation rule (47), we find then that for surviving \(D = 5\) supersymmetry \(\hat{\epsilon}\) should satisfy

\[
\partial_z \hat{\epsilon} - \frac{1}{2} k \hat{\gamma}_6 \hat{\gamma}_7 \hat{\epsilon} = 0.
\]

From these equations, we find that the Kaluza-Klein ansatz for \(\hat{\epsilon}\) should be

\[
\hat{\epsilon} = \left( \cosh \left( \frac{1}{2} k z \right) - i \sinh \left( \frac{1}{2} k z \right) \hat{\gamma}_7 \hat{\gamma}_6 \right) \epsilon,
\]
where \( \epsilon \) is the five-dimensional supersymmetry parameter, which must satisfy the projection condition
\[
- \tau_3 \epsilon = \sigma_2 \epsilon .
\] (52)
This condition halves the number of components of supersymmetry in \( D = 5 \), as we should expect since we are ending up with \( N = 2 \) gauged supergravity.

We find that the Kaluza-Klein reduction ansatz for \( \hat{\psi}_\mu \) is
\[
\hat{\psi}_\mu = \left( \cosh \left( \frac{k}{2} z \right) - i \sinh \left( \frac{k}{2} z \right) \hat{\gamma}_7 \hat{\gamma}_6 \right) \psi_\mu ,
\] (53)
where \( \psi_\mu \) is the five-dimensional gravitino, which must also satisfy the projection condition
\[
- \tau_3 \psi_\mu = \sigma_2 \psi_\mu .
\] (54)
Substituting into the previous equations, this gives rise to the following five-dimensional gravitino transformation rule:
\[
\delta \psi_\mu = D_\mu \epsilon - \frac{1}{2} k \sigma_2 \gamma_\mu \epsilon - \frac{i}{8\sqrt{3}} F_{\nu\rho} (\gamma_\mu \gamma^\rho - 4 \delta_\mu^\nu \gamma^\rho) \epsilon ,
\] (55)
where
\[
D_\mu \epsilon \equiv \nabla_\mu \epsilon + \frac{i}{2} g \sqrt{\frac{2}{3}} A_\mu \tau_3 \epsilon .
\] (56)

### 3.3 Lifting to massive type IIA in \( D = 10 \)

The ansatz for obtaining the bosonic sector of six-dimensional \( SU(2) \) gauged \( N = 4 \) supergravity as a consistent Kaluza-Klein reduction from massive type IIA supergravity in \( D = 10 \) was found in [12]. The Lagrangian describing the bosonic sector of the massive IIA theory is
\[
\mathcal{L}_{10} = \hat{R} * 1 - \frac{1}{2} \hat{d} \hat{\phi} \wedge \hat{d} \hat{\phi} - \frac{1}{2} e^{\frac{\hat{2}}{2} \hat{\phi}} \hat{F}_{(2)} \wedge \hat{\bar{F}}_{(2)} - \frac{1}{2} e^{-\frac{\hat{2}}{2} \hat{\phi}} \hat{F}_{(3)} \wedge \hat{\bar{F}}_{(3)} - \frac{1}{2} e^{\frac{\hat{2}}{2} \hat{\phi}} \hat{F}_{(4)} \wedge \hat{\bar{F}}_{(4)}
\]
\[-\frac{1}{2} d \hat{A}_{(3)} \wedge d \hat{A}_{(3)} \wedge \hat{A}_{(2)} \wedge \hat{A}_{(2)} - \frac{1}{6} m \hat{A}_{(3)} \wedge (\hat{A}_{(2)})^3 - \frac{1}{20} m^2 (\hat{A}_{(2)})^5 - \frac{1}{2} m^2 e^{\frac{\hat{2}}{2} \hat{\phi}} * 1 ,
\] (57)
where the field strengths are given in terms of potentials by
\[
\hat{F}_{(2)} = d \hat{A}_{(1)} + m \hat{A}_{(2)} , \quad \hat{F}_{(3)} = d \hat{A}_{(2)} ,
\]
\[
\hat{F}_{(4)} = d \hat{A}_{(3)} + \hat{A}_{(1)} \wedge \hat{A}_{(2)} + \frac{1}{2} m \hat{A}_{(2)} \wedge \hat{A}_{(2)} ,
\] (58)
and in this subsection we are using the hat symbol to denote the ten-dimensional fields and Hodge dual.
It was shown in [12] that the consistent Kaluza-Klein reduction ansatz is

\[ d\hat{s}^2_{10} = s^{1/2} X^{1/2} \left[ \Delta^{1/8} ds_6^2 + 2g^{-2} \Delta^{-1/8} X^2 d\xi^2 + \frac{1}{2}g^{-2} \Delta^{-1/8} X^{-1} c^2 \sum_{i=1}^{3} (\sigma^i - g A_{(1)}^i)^2 \right], \]

\[ \hat{F}_4 = -\frac{\sqrt{2}}{6} g^{-3} s^{1/3} c^3 \Delta^{-2} U d\xi \wedge \epsilon_{(3)} - \sqrt{2} g^{-3} s^{4/3} c^4 \Delta^{-2} X^{-3} dX \wedge \epsilon_{(3)} \]

\[ -\sqrt{2} g^{-1} s^{1/3} c X^4 \ast F_{(3)} \wedge d\xi - \frac{1}{\sqrt{2}} s^{4/3} X^{-2} \ast G_{(2)} \]

\[ + \frac{1}{\sqrt{2}} g^{-2} s^{1/3} c F_{(2)}^i h^i \wedge d\xi - \frac{1}{4\sqrt{2}} g^{-2} s^{4/3} c^2 \Delta^{-1} X^{-3} F_{(2)}^i \wedge h^i \wedge h^k \epsilon_{ijk} , \quad (59) \]

\[ \hat{F}_{(3)} = s^{2/3} F_{(3)} + g^{-1} s^{-1/3} c G_{(2)} \wedge d\xi , \]

\[ \hat{F}_{(2)} = \frac{1}{\sqrt{2}} s^{2/3} G_{(2)} , \quad e^\hat{\phi} = s^{-5/6} \Delta^{1/4} X^{-5/4} , \]

where \( X \) is related to the six-dimensional dilaton \( \phi \) by \( X = e^{-\frac{1}{2\sqrt{2}} \sqrt{\Delta} \phi} \), and

\[ \Delta \equiv X^2 + X^{-3} s^2 , \quad U \equiv X^{-6} s^2 - 3X^2 c^2 + 4X^{-2} c^2 - 6X^{-2} . \quad (60) \]

We also define \( h^i \equiv \sigma^i - g A_{(1)}^i \), \( \epsilon_{(3)} \equiv h^1 \wedge h^2 \wedge h^3 \), and \( s = \sin \xi \) and \( c = \cos \xi \). The unhatted quantities, and Hodge dual \( \ast \), refer to the six-dimensional fields. These fields satisfy the equations of motion following from ([33]) (with the hats dropped on all the quantities in (39)), by virtue of the ten-dimensional equations of motion following from ([57]).

Substituting our braneworld reduction ansatz ([10]) into (59), we therefore find that the following gives a consistent braneworld Kaluza-Klein reduction from massive type IIA supergravity in \( D = 10 \) to \( N = 2 \) gauged supergravity in \( D = 5 \):

\[ d\hat{s}^2_{10} = s^{1/2} \left[ dz^2 + \cosh^2(kz) ds_6^2 + 2g^{-2} d\xi^2 + \frac{1}{2}g^{-2} c^2 \left( \sigma_1^2 + \sigma_2^2 + \sigma_3 + \sqrt{\frac{2}{3}} g A_{(1)}^2 \right)^2 \right] , \]

\[ \hat{F}_4 = -\frac{\sqrt{2}}{6} g^{-3} s^{1/3} c^3 d\xi \wedge \sigma_1 \wedge \sigma_2 \wedge \sigma_3 + \sqrt{\frac{2}{3}} g A_{(1)} + \sqrt{\frac{2}{3}} g^{-1} s^{1/3} c \cosh^2(kz) \ast F_{(2)} \wedge d\xi \]

\[ + \frac{1}{\sqrt{3}} s^{4/3} \sinh(kz) \cosh(kz) \ast F_{(2)} \wedge d\xi - \frac{1}{\sqrt{3}} g^{-2} s^{1/3} c F_{(2)} \wedge \left( \sigma_1 + \sqrt{\frac{2}{3}} g A_{(1)} \right) \wedge d\xi \]

\[ + \frac{1}{\sqrt{3}} g^{-2} s^{4/3} c^2 F_{(2)} \wedge \sigma_1 \wedge \sigma_2 , \quad (61) \]

\[ \hat{F}_{(3)} = -\frac{1}{\sqrt{3}} s^{2/3} \cosh(kz) dz \wedge F_{(2)} - \sqrt{\frac{2}{3}} g^{-1} s^{-1/3} c \sinh(kz) F_{(2)} \wedge d\xi , \]

\[ \hat{F}_{(2)} = -\frac{1}{\sqrt{3}} s^{2/3} \sinh(kz) F_{(2)} , \quad e^\hat{\phi} = s^{-5/6} , \]

where we recall that \( k = g \sqrt{2/3} \).

### 4 Braneworld reductions from type IIB* and IIA*

Hull has proposed that the theories one obtains by performing a T-duality transformation of type IIA or type IIB supergravity with a timelike reduction should be viewed as low-energy limits of consistent sectors of string theory ([11]). These theories, which he calls type IIB*
and type IIA* respectively, differ from the usual type IIB and IIA theories in that the signs of the kinetic terms of all the Ramond-Ramond fields are reversed. In fact, the type IIB* and type IIA* theories can be obtained from the usual type IIB and type IIA theories by making the replacements

$$ \Psi \rightarrow i \Psi, $$  \hspace{1cm} (62)

where $\Psi$ denotes the set of all Ramond-Ramond fields. Recently, gauged de Sitter supergravities that are related to the type IIA* and type IIB* theories were discussed [25].

In the rest of this section, we shall investigate the possibility of performing braneworld Kaluza-Klein reductions based on these type IIB* and IIA* theories. Since the theories themselves can be obtained by the analytic continuation (62), we can obtain the associated braneworld reductions by performing appropriate analytic continuations of our previous results. As we shall see, we can, by this means, obtain a braneworld reduction giving four-dimensional $N = 2$ gauged de Sitter supergravity on the brane. We also investigate the analogous procedure for a braneworld reduction to five-dimensional $N = 2$ gauged de Sitter supergravity, and show that in this case we cannot obtain a theory with real fields.

### 4.1 $N = 2$, $D = 4$ de Sitter supergravity on the brane

We first consider the braneworld reduction of gauged $N = 4$, $D = 5$ de Sitter supergravity. Specifically, this five-dimensional supergravity will itself be obtained as a reduction from $D = 10$, but now arising as a reduction of type IIB* supergravity on $H^5$, the hyperbolic 5-space. Implementing (62), we need to make the replacements

$$ \hat{H}_5 \rightarrow i \hat{H}_5, \quad \hat{A}^i_{(2)} \rightarrow i \hat{A}^i_{(2)}, \quad \hat{\chi} \rightarrow i \hat{\chi} $$  \hspace{1cm} (63)

in the type IIB Lagrangian [39]. Since our goal is to get a reduction to $D = 5$ after having made these replacements, it suffices for us to implement this directly in the reduction ansatz (37). We can achieve this with the following replacements:

$$ g \rightarrow i g, \quad \xi \rightarrow i \xi + \frac{1}{2} \pi, \quad \tau \rightarrow i \tau + \frac{1}{2} \pi, $$

$$ A^1_{(2)} \rightarrow i A^1_{(2)}, \quad A^i_{(1)} \rightarrow i A^i_{(1)}. $$  \hspace{1cm} (64)

It is easily seen that this gives a real Kaluza-Klein reduction ansatz under which the type IIB* theory reduces on $H^5$, to yield a de Sitter supergravity in $D = 5$ (see also [25]). In particular the metric reduction ansatz in (37) has become

$$ ds^2_{10} = \Delta^{1/2} ds^2_5 + g^{-2} X \Delta^{1/2} d\xi^2 + g^{-2} \Delta^{-1/2} X^2 \hat{c}^2 \left( d\tau - g B_{(1)} \right)^2 + \frac{1}{4} g^{-2} \Delta^{-1/2} X^{-1} s^2 \sum_i (\sigma^i + \sqrt{2} g A^i_{(1)})^2, $$  \hspace{1cm} (65)
where $\tilde{s} \equiv \sinh \xi$, $\tilde{c} \equiv \cosh \xi$, and $\Delta \equiv X^{-2} \tilde{c}^2 - X \tilde{s}^2$. Note that the level surfaces at constant $\xi$ in the internal dimensions are now $\mathbb{R} \times S^3$ (as opposed to $S^1 \times S^3$ before the analytic continuation), giving rise to an $H^5$ topology (as opposed to the $S^5$ topology before the analytic continuation). The five-dimensional theory is gauged $N = 4$ de Sitter supergravity, with $\mathbb{R} \times SU(2)$ Yang-Mills fields. We have seen that it is obtained by making the replacements

$$g \rightarrow i g, \quad \hat{A}_1^{(2)} \rightarrow i \hat{A}_1^{(2)}, \quad \hat{A}_i^{(1)} \rightarrow i \hat{A}_i^{(1)} \quad (66)$$

in the five-dimensional Lagrangian (12).

The next step is to perform the appropriate analytic continuation of the braneworld reduction (14), in order to obtain a reduction of the five-dimensional theory coming from (12) with the replacements (66). It is easily seen that in terms of the analytically continued quantities defined in (66) for the five-dimensional de Sitter supergravity, the braneworld reduction ansatz (14) will become

$$d\hat{s}_5^2 = dz^2 + \cos^2(kz) ds_4^2,$$

$$\hat{A}_1^{(2)} = -\frac{1}{\sqrt{2}} \cos(kz) * F^{(2)}, \quad \hat{A}_2^{(2)} = -\frac{1}{\sqrt{2}} \sin(kz) F^{(2)},$$

$$\hat{A}_1^{(1)} = \frac{1}{\sqrt{2}} A^{(1)}, \quad (67)$$

where we also have

$$k = g. \quad (68)$$

The equations of motion of the five-dimensional de Sitter supergravity imply the following four-dimensional equations

$$R_{\mu\nu} = -\frac{1}{2} (F_{\mu\rho} F^{\nu\rho} - \frac{1}{4} F^{(2)\mu\nu}) g_{\mu\nu},$$

$$d* F^{(2)} = 0, \quad (69)$$

which are the bosonic equations of motion for gauged $N = 2$ de Sitter supergravity in $D = 4$. Note that in comparison to the previous AdS braneworld reduction ansatz (14), the four-dimensional Maxwell potential $A^{(1)}$ has itself undergone an analytic continuation $A^{(1)} \rightarrow i A^{(1)}$. This has the consequence, apparent in the Einstein equation in (69), that it has the “wrong sign” for its kinetic term. This is indeed one of the features of $N = 2$ gauged de Sitter supergravity in $D = 4$. It should also be remarked that the cosmological constant in $D = 4$ can again be adjusted freely, whilst holding the five-dimensional cosmological constant fixed, by making rescalings as in (32). It is straightforward also to obtain the reduction ansatz for the fermionic fields, by implementing the analytic continuations on the reductions obtained in section 2.2. We shall discuss the fermions further in section 5.
4.2 \( N = 2, D = 5 \) de Sitter supergravity on the brane?

We can now repeat the analogous steps for the reduction massive type IIA* supergravity to give \( N = 4 \) gauged de Sitter supergravity in \( D = 6 \), and then investigate the possibility of a braneworld reduction to \( N = 2 \) gauged de Sitter supergravity in \( D = 5 \). The first part of this procedure is a straightforward generalisation of the one followed in the previous subsection. We make the following analytic continuation in the massive type IIA reduction ansatz (59),

\[
g \rightarrow i g, \quad \xi \rightarrow \frac{1}{2} \pi + i \xi, \\
B_{(1)} \rightarrow i B_{(1)}, \quad A_{(1)}^i \rightarrow i A_{(1)}^i, \\
(70)
\]

which leads to a consistent reduction to \( N = 4 \) gauged de Sitter supergravity in \( D = 6 \). Note that we still get a real theory after the reduction, even though fractional powers of \( \sin \xi \) appear in the reduction ansatz, since it is only the \( \cos \xi \) terms that pick up factors of \( i \), and these are all raised to integer powers. The resulting six-dimensional de Sitter supergravity is therefore described by the Lagrangian (39), after making the analytic continuations

\[
g \rightarrow i g, \quad \hat{B}_{(1)} \rightarrow i \hat{B}_{(1)}, \quad \hat{A}_{(1)}^i \rightarrow i \hat{A}_{(1)}^i. \\
(71)
\]

However, when we proceed to the next stage, of looking for an analytic continuation of the previous braneworld reduction ansatz (40), we encounter a difficulty. This can be attributed to the fact that compatibility between the \( SU(2) \) Yang-Mills ansatz in (40) and (71) will require that we must make the continuation \( A_{(1)} \rightarrow i A_{(1)} \) in the 5-dimensional Maxwell potential. However, this conflicts with the reality conditions for the remainder of the reduction ansatz. In fact the reason for this can be seen by looking at the equation of motion for the Maxwell field in (43); the non-linear term on the right-hand side means that we cannot perform the analytic continuation \( A_{(1)} \rightarrow i A_{(1)} \) and still get a real five-dimensional theory. This is quite different from the situation in four dimensions, where there is no analogous Chern-Simons term preventing one from sending \( A_{(1)} \) to \( i A_{(1)} \).

If we wanted to get a real de Sitter theory in \( D = 5 \) via a braneworld reduction from \( D = 6 \), we could consider the following \( D = 6 \) to \( D = 5 \) ansatz,

\[
d\hat{s}_6^2 = dz^2 + \cos^2(k z)\, dx_5^2, \\
\hat{A}_{(2)} = -\frac{1}{\sqrt{3}} k^{-1} \sin(k z) \, F_{(2)}, \\
\hat{A}_{(1)}^3 = i \sqrt{\frac{2}{3}} A_{(1)}, \\
\hat{B}_{(1)} = 0, \quad \hat{A}_{(1)}^1 = 0, \quad \hat{A}_{(1)}^2 = 0, \quad \hat{\phi} = 0. \\
(72)
\]
with \( k = \sqrt{2} g/3 \). This does formally provide a reduction to a real \( D = 5 \) de Sitter theory, although with the price that the ansatz in \( D = 6 \) is complex. Thus the situation is quite different from that in section 4.1, where we obtained a a completely real reduction to get \( N = 2 \) gauged de Sitter supergravity on the four-dimensional brane.

5 de Sitter supergravity on the brane from anti-de Sitter

It is also possible to embed the \( D \)-dimensional de Sitter metric in the \((D + 1)\)-dimensional anti-de Sitter metric, rather than in the de Sitter metric, by replacing \( \cosh(kz) \) by \( \sinh(kz) \) in (5):

\[
d\hat{s}^2_{D+1} = dz^2 + \sinh^2(kz) ds^2_D. 
\]

This was discussed in the context of four-dimensional braneworld models in [9].

We find that we can implement this as a consistent braneworld Kaluza-Klein reduction, starting from and \( N = 4 \) AdS gauged supergravity in five dimensions in which the two 2-form potentials are analytically continued by multiplication by \( i \), to obtain the same \( N = 2 \) gauged de Sitter supergravity in \( D = 4 \) that we discussed in section 4.1. After first replacing the potentials \( \hat{A}_\alpha^{(2)} \) in the usual \( N = 4 \) gauged supergravity in \( D = 5 \) by \( i \hat{A}_\alpha^{(2)} \), we obtain another real theory, for which the bosonic braneworld reduction ansatz is given by

\[
\begin{align*}
\hat{A}_{(2)}^1 &= \frac{1}{\sqrt{2}} \sinh(kz) \ast F_{(2)}, \\
\hat{A}_{(2)}^2 &= \frac{1}{\sqrt{2}} \cosh(kz) F_{(2)}, \\
\hat{A}_{(1)}^1 &= \frac{1}{\sqrt{2}} A_{(1)},
\end{align*}
\]

and this consistently yields the de Sitter gauged supergravity equations (69).

The above result can in fact be obtained directly from our previous anti-de Sitter braneworld reduction in section 2.1. This is done by making the coordinate transformation \( z \rightarrow z + i \pi/(2k) \), together with sending \( ds^2_4 \rightarrow -ds^2_4 \), in the expressions in section 4. This can indeed be seen to give a real embedding, provided that the 2-form potentials in five dimensions are analytically continued as described above, namely \( \hat{A}_\alpha^{(2)} \rightarrow i \hat{A}_\alpha^{(2)} \).

This five-dimensional AdS gauged supergravity can itself be obtained by dimensional reduction from a variant of type IIB supergravity. Specifically, we take as our starting point the type IIB\(_{7+3}\) theory introduced in [27]. This has spacetime signature \((7, 3)\), and it arises from a sequence of T-duality transformations of the usual type IIB theory, using timelike as well as spacelike circles in the various transformation steps. It has symplectic Majorana-Weyl spinors, and the kinetic terms for the two 2-form potentials are of the
non-standard sign, whilst all other bosonic fields have standard-sign kinetic terms. It can be reduced on the maximally-symmetric space $O(4,2)/O(3,2)$ of signature $(3,2)$, to give the non-standard AdS gauged supergravity in $D = 5$ that we invoked in our braneworld construction above. The reduction of the type IIB$_{7+3}$ theory on $O(4,2)/O(3,2)$ can be derived from the $S^5$ reduction of type IIB in (37), by first analytically continuing the 2-form potentials $\hat{A}^{\alpha}_{(2)} \rightarrow i \hat{A}^{\alpha}_{(2)}$, and then sending $\xi \rightarrow i \xi$. It is straightforward to extend the discussion to include the fermions too, to get the complete four-dimensional de Sitter gauged $N = 2$ supergravity.

One can also consider the possibility of a similar embedding of five-dimensional de Sitter gauged supergravity in six-dimensional anti-de Sitter gauged supergravity. However, in this case we can see from (37) that after sending $z \rightarrow z + i \pi/(2k)$ we would need also to send $A_{(1)} \rightarrow i A_{(1)}$ in order to keep $\hat{A}^{3}_{(1)}$ real, but this would then contradict the ansatz for the Yang-Mills potential $\hat{A}^{3}_{(1)}$, which requires that $A_{(1)}$ be kept untransformed. Thus we see in this approach, just like the de Sitter to de Sitter reduction considered in section 4.2, we cannot get a real embedding of five-dimensional de Sitter gauged supergravity on the brane.

6 Conclusions

In this paper, we have shown how the results in [4, 7] on the braneworld Kaluza-Klein reductions to ungauged supergravities can be extended, in certain cases, to braneworld reductions giving gauged supergravities. Specifically, we have constructed such reductions from five-dimensional $N = 4$ gauged supergravity to four-dimensional $N = 2$ gauged supergravity on the brane, and likewise from six-dimensional $N = 4$ gauged supergravity to five-dimensional $N = 2$ gauged supergravity on the brane. The lower-dimensional cosmological constant is freely adjustable, whilst holding the higher-dimensional one fixed, and so one can, for example, arrange to have a phenomenologically realistic cosmological constant in four dimensions “on the brane,” even if the five-dimensional cosmological constant is of the Planck scale. In each case, the higher-dimensional starting point can itself be obtained as a consistent Kaluza-Klein sphere reduction, from type IIB supergravity in the first example [11], and from massive type IIA in the second [12]. Thus one can lift the braneworld embeddings to the corresponding ten-dimensional theories.

The braneworld reductions allow one to obtain new explicit exact solutions of higher-dimensional supergravities, by starting from known solutions of the gauged supergravities on the brane. Thus, for example, we can consider a charged AdS black hole solution of
the $N = 2$ gauged supergravity in $D = 4$, and lift it to a solution of the $N = 4$ gauged supergravity in $D = 5$. The charged AdS$_4$ black hole is given by

$$ ds^2_4 = - H^{-2} f dt^2 + H^2 (f^{-1} dr^2 + r^2 d\Omega_2^2), $$

$$ A_{(1)} = \frac{1}{2} \sqrt{\epsilon} (1 - H^{-1}) \coth \beta dt, \quad H = 1 + \frac{\mu \sinh^2 \beta}{\epsilon r}, \quad f = \epsilon - \frac{\mu}{r} + 4g^2 r^2 H^4, $$

where $\mu$ and $\beta$ are constants, and $\epsilon = 1, 0$ or $-1$ according to whether the foliations in the space transverse to the black hole have the metric $d\Omega_2^2$, $T^2$ or hyperbolic space $H^2$. (In the case $\epsilon = 0$ one must scale $\sinh^2 \beta \to \epsilon \sinh^2 \beta$ before sending $\epsilon$ to zero.) (The AdS$_4$ black hole is written here in the notation of [33], and corresponds to setting all four charges equal in the four-charge solution given there.) Substituting the solution (75) into (14) gives an embedding of the AdS$_4$ black hole in five-dimensional $N = 4$ gauged supergravity, whilst substituting it into (38) gives an embedding of the AdS$_4$ black hole in type IIB supergravity. One can similarly embed an AdS$_5$ black hole [34] in six-dimensional $N = 4$ gauged supergravity using (40), and then in massive type IIA using (61).

We then showed that, based on the embedding $dS_D \subset dS_{D+1}$, the braneworld reduction to four-dimensional $N = 2$ gauged (anti-de Sitter) supergravity could be analytically continued to give a braneworld reduction to $N = 2$ de Sitter gauged supergravity. This theory, and its five-dimensional de Sitter supergravity progenitor, differ from normal gauged supergravities in having non-standard signs for the kinetic terms of certain gauge fields. (This is how they manage to evade the usual theorems about the non-existence of de Sitter supergravities.) We showed that the five-dimensional de Sitter supergravity in question can itself be obtained as a consistent reduction (on the hyperbolic space $H^5$) of the type IIB* supergravity discussed by Hull [10]. This theory has the non-standard sign for the kinetic terms of all the Ramond-Ramond fields, and was obtained by performing a timelike T-duality transformation of the type IIA theory [10]. It can equivalently be thought of as an analytic continuation of the usual type IIB theory in which all the Ramond-Ramond fields are multiplied by a factor of $i$.

A similar continuation yields a consistent reduction of massive type IIA* supergravity to give a de Sitter gauged supergravity in six dimensions. In this case, by contrast, we do not get a real embedding to give a five-dimensional $N = 2$ braneworld de Sitter supergravity.

We also showed that a very different braneworld Kaluza-Klein reduction to give four-dimensional de Sitter gauged $N = 2$ supergravity can be achieved, in which we start from five-dimensional anti-de Sitter gauged $N = 4$ supergravity, with an analytic continuation of the two 2-form potentials. This depends upon the fact that one can embed $dS_D$ in
AdS$_{D+1}$. The five-dimensional gauged theory can itself be derived from the type IIB$_{7+3}$ theory introduced in [27]. It is interesting that we can obtain the four-dimensional de Sitter gauged $N = 2$ supergravity by two different routes, one of which has its ultimate origin in the ten-dimensional type IIB* theory, and the other in the type IIB$_{7+3}$ theory.

In this paper we have obtained consistent braneworld reductions based on each of the embeddings AdS$_D \subset$ AdS$_{D+1}$, dS$_D \subset$ dS$_{D+1}$ and dS$_D \subset$ AdS$_{D+1}$. It is worth remarking that there is no fourth possibility, of having an embedding AdS$_D \subset$ dS$_{D+1}$. This can be seen easily from the fact that the isometry groups of AdS$_n$ and dS$_n$ are SO$(n-1,2)$ and SO$(n,1)$ respectively, and that SO$(D-1,2)$ is not a subgroup of SO$(D,1)$.

Several interesting questions about the braneworld Kaluza-Klein reductions we have considered in this paper arise. For example, we have seen that we obtain consistent reduction ansätze that give us the massless gauged supergravities “on the brane” in the lower dimension. In particular, these supergravities include, of course, the massless graviton in their spectrum of states. This appears at first sight to be at odds with the findings in [9] (see also [35]), where it is shown that there is no massless graviton in the effective spectrum of states on the brane. In fact it is not clear precisely what the relation between the two viewpoints is. This question already arose in the braneworld reductions to ungauged supergravity, constructed in [4, 7], although in a slightly less extreme form there since the existence of a massless graviton is a feature common to both viewpoints in those examples.

In the ungauged reductions the discrepancies between the standard Randall-Sundrum viewpoint and the braneworld reduction viewpoint show up, for example, in the discussion of the global structure of solutions “on the brane.” Thus, for instance, in [8] a four-dimensional Schwarzschild black hole was shown to give rise to a solution in $D = 5$ that was singular on the horizon of the AdS$_5$. The absence of supersymmetry in that example provided a possible mechanism, via a Gregory-Laflamme instability, for mitigating the effects of this singularity. However, in the braneworld reductions in [4, 7], the localised supergravities admit supersymmetric black hole solutions, which presumably are not subject to such an instability, and so understanding the singularities at the AdS$_5$ horizon becomes more pressing. A mechanism, within the Randall-Sundrum framework, for eliminating such singularities was proposed in [36]. It was argued that in practice the distribution of energy from a disturbance on the brane would be predominantly in very low mass, rather than massless, gravitons,

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6One does not need a singular or high-curvature solution on the brane in order to get such a curvature singularity on the AdS$_5$ horizon; even a mild gravitational disturbance on the brane will get “amplified” to give a singularity far from the brane.
and the effects of these would decay with distance from the brane. However, the existence of the exact Kaluza-Klein reductions obtained in [4, 7] still seems to require explanation, since these embeddings show that one can have arbitrarily large excitations of the massless fields on the brane that will never (classically, at least) “spill over” into the light massive modes.

It is interesting to note that the “amplification” effect that causes the diverging curvature in the bulk in the ungauged embeddings does not in fact occur in the gauged AdS embeddings, although it does for gauged de Sitter embeddings. To see this, we note that for a Kaluza-Klein metric reduction of the form $d\hat{s}^2 = dz^2 + f^2 ds^2$, where $f$ depends on $z$, the curvature 2-forms are given by

$$\hat{\Theta}_{0a} = -\frac{f''}{f} e^0 \wedge e^a, \quad \hat{\Theta}_{ab} = \Theta_{ab} - \frac{f'^2}{f^2} e^a \wedge e^b,$$

(76)

where $e^0 = dz$, $e^a = f e^a$, $ds^2 = e^a \otimes e^a$ and $\Theta_{ab}$ are the curvature 2-forms for the lower-dimensional metric $ds^2$. Thus for the ungauged, gauged anti-de Sitter and gauged de Sitter cases, the tangent-space components of the higher-dimensional Riemann tensor $\hat{R}_{ABCD}$ are given by

**Ungauged:**

$$f = e^{-kz}, \quad \hat{R}_{0a0b} = -k^2 \eta_{ab}, \quad \hat{R}_{abcd} = e^{2kz} R_{abcd} - k^2 (\eta_{ac} \eta_{bd} - \eta_{ad} \eta_{bd}),$$

**Gauged AdS:**

$$f = \cosh(kz), \quad \hat{R}_{0a0b} = -k^2 \eta_{ab}, \quad \hat{R}_{abcd} = \frac{1}{\cosh^2(kz)} R_{abcd} - k^2 \tanh^2(kz) (\eta_{ac} \eta_{bd} - \eta_{ad} \eta_{bd}),$$

(77)

**Gauged dS (1):**

$$f = \cos(kz), \quad \hat{R}_{0a0b} = k^2 \eta_{ab}, \quad \hat{R}_{abcd} = \frac{1}{\cos^2(kz)} R_{abcd} - k^2 \tan^2(kz) (\eta_{ac} \eta_{bd} - \eta_{ad} \eta_{bd}),$$

**Gauged dS (2):**

$$f = \sinh(kz), \quad \hat{R}_{0a0b} = -k^2 \eta_{ab}, \quad \hat{R}_{abcd} = \frac{1}{\sinh^2(kz)} R_{abcd} - k^2 \coth^2(kz) (\eta_{ac} \eta_{bd} - \eta_{ad} \eta_{bd}),$$

where $R_{abcd}$ are the tangent-space components of the Riemann tensor for the metric $ds^2$ in the lower dimension. The two de Sitter cases refer to the $dS_D \subset dS_{D+1}$ and $dS_D \subset AdS_{D+1}$ embeddings used in sections 4 and 5 respectively. Riemann tensor divergences in the higher-dimensional bulk resulting from curvature (even finite) on the brane therefore occur if the prefactor of $R_{abcd}$ diverges. This occurs on the higher-dimensional AdS horizon.
In the ungauged case \cite{4,7}, at $z = \pi/(2k)$ in the first gauged de Sitter case, and at $z = 0$ in the second gauged de Sitter case. But in the gauged AdS case, the prefactor $(\cosh(kz))^{-2}$ is always $\leq 1$, and no such divergence occurs.

In our present case, however, where gauged supergravities are obtained on the brane, we do have the remaining puzzle about the existence of the massless graviton. It may well be that a resolution of the ungauged braneworld reduction puzzles described above would also indicate the resolution of the massless graviton puzzle. In both cases, the differences between the usual effective-gravity discussions of, for example, \cite{4,7} and \cite{8}, and the Kaluza-Klein approach of \cite{4,7} and this paper, are concerned with whether one considers the distribution of excitations over massless and massive modes, or if, on the other hand, one considers only an exact embedding of the massless modes alone. It seems that more investigation is needed in order to reconcile the viewpoints. In the meantime we present our results for their intrinsic interest, since they provide new insights into the subject of consistent Kaluza-Klein reductions, and, in particular, they provide ways of constructing new exact higher-dimensional solutions from known lower-dimensional ones. For example, in a recent application the ungauged braneworld reductions of \cite{4,7} were used in order to construct exact multi-membrane solutions in seven-dimensional gauged supergravity \cite{37}.

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References

[1] L. Randall and R. Sundrum, An alternative to compactification, Phys. Rev. Lett. 83 (1999) 4690, hep-th/9906064.

[2] D. Brecher and M.J. Perry, Ricci-flat branes, Nucl. Phys. B566 (2000) 151, hep-th/9908018.

[3] A. Chamblin, S.W. Hawking and H.S. Reall, Brane world black holes, Phys. Rev. D61 (2000) 065007, hep-th/9909205.

[4] H. Lü and C.N. Pope, Branes on the brane, Nucl. Phys. B 598 (2001) 492, hep-th/0008050.
[5] A.H. Chamseddine and W.A. Sabra, *Einstein brane worlds in 5-D gauged supergravity*, Phys. Lett. **B517**, 184 (2001).

[6] A.H. Chamseddine and W.A. Sabra, *Curved domain walls of five-dimensional gauged supergravity*, hep-th/0105207.

[7] M. Cvetić, H. Lü and C.N. Pope, *Brane-world Kaluza-Klein reductions and branes on the brane*, hep-th/0009183.

[8] M.J. Duff, J.T. Liu and W.A. Sabra, *Localization of supergravity on the brane*, Nucl. Phys. B **605** (2001) 234, hep-th/0009212.

[9] A. Karch and L. Randall, *Locally localized gravity*, JHEP **0105**, 008 (2001), hep-th/0011156.

[10] C.M. Hull, *Timelike T-duality, de Sitter space, large N gauge theories and topological field theory*, JHEP **9807** (1998) 021, hep-th/9806146.

[11] H. Lü, C.N. Pope and T.A. Tran, *Five-dimensional $N = 4$ $SU(2) \times U(1)$ gauged supergravity from type IIB*, Phys. Lett. **B475** (2000) 261, hep-th/9909203.

[12] M. Cvetić, H. Lü and C.N. Pope, *Gauged six-dimensional supergravity from massive type IIA*, Phys. Rev. Lett. **83** (1999) 5226, hep-th/9906221.

[13] J. Maldacena and C. Nunez, *Supergravity description of field theories on curved manifolds and a no go theorem*, Int. J. Mod. Phys. **A16** (2001) 822, hep-th/0007018.

[14] S. Hawking, J. Maldacena and A. Strominger, *DeSitter entropy, quantum entanglement and AdS/CFT*, JHEP **0105**, 001 (2001); hep-th/0002143.

[15] R. Bousso, *Bekenstein bounds in de Sitter and flat space*, JHEP **0104**, 035 (2001), hep-th/0012052.

[16] J. Maldacena and A. Strominger, *Statistical entropy of de Sitter space*, JHEP **9802**, 014 (1998), gr-qc/9801096.

[17] T. Banks, *Cosmological breaking of supersymmetry or little Lambda goes back to the future. II*, hep-th/0007146.

[18] R. Bousso, *Holography in general space-times*, JHEP **9906**, 028 (1999), hep-th/9906022.
[19] V. Balasubramanian, P. Horava and D. Minic, *Deconstructing de Sitter*, JHEP **0105**, 043 (2001), [hep-th/0103171].

[20] R. Bousso, *Positive vacuum energy and the N-bound*, JHEP **0011**, 038 (2000), [hep-th/0010252].

[21] E. Witten, *Quantum gravity in de Sitter space*, [hep-th/0106109].

[22] A. Strominger, *The DS/CFT correspondence*, [hep-th/0106113].

[23] S. Nojiri and S. D. Odintsov, Phys. Lett. B **519**, 145 (2001), [hep-th/0106191] and [hep-th/0107134].

[24] D. Klemm, *Some aspects of the de Sitter/CFT correspondence*, [hep-th/0106247].

[25] C.M. Hull, *De Sitter space in supergravity and M-theory*, [hep-th/0109213].

[26] E. Cremmer, I.V. Lavrinenko, H. Lü, C.N. Pope, K.S. Stelle and T.A. Tran, *Euclidean-signature supergravities, dualities and instantons*, Nucl. Phys. **B534**, 40 (1998), [hep-th/9803259].

[27] C. M. Hull, *Duality and the signature of space-time*, JHEP **9811**, 017 (1998), [hep-th/9807127].

[28] M. Günyaydin, L.J. Romans and N.P. Warner, *Compact and non-compact gauged supergravity theories in five dimensions*, Nucl. Phys. **B272** (1986) 598.

[29] E. Bergshoeff, C.M. Hull and T. Ortin, *Duality in the type II superstring effective action*, Nucl. Phys. **B451** (1995) 547, [hep-th/9504081].

[30] L.J. Romans, *The F(4) gauged supergravity in six dimensions*, Nucl. Phys. **B269** (1986) 691.

[31] M.J. Duff and J.T. Liu, *Anti-de Sitter black holes in gauged N=8 supergravity*, [hep-th/9901149].

[32] W. Sabra, *Anti-de Sitter black holes in N=2 gauged supergravity*, [hep-th/9903143].

[33] M. Cvetič, M.J. Duff, P. Hoxha, J.T. Liu, H. Lü, J.X. Lu, R. Martinez-Acosta, C.N. Pope, H. Sati and T.A. Tran, *Embedding AdS black holes in ten and eleven dimensions*, Nucl. Phys. **B558** (1999) 96, [hep-th/9903214].
[34] K. Behrndt, M. Cvetić and W.A. Sabra, *Non-extreme black holes five-dimensional $N = 2$ AdS supergravity*, hep-th/9810227.

[35] M. Porrati, *Mass and gauge invariance. IV: Holography for the Karch-Randall model*, hep-th/0109017.

[36] S. B. Giddings, E. Katz and L. Randall, *Linearized gravity in brane backgrounds*, JHEP **0003**, 023 (2000), hep-th/0002091.

[37] J.T. Liu and W.Y. Wen, *Exact multi-membrane solutions in AdS$_7$*, hep-th/0110213.