Distance dependent statistics in a P,T-invariant model

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Abstract

It is shown that in the P,T-invariant model with the mixed Chern-Simons term the interaction of charge carriers leads to effective changing of their statistics, which depends on distance between them. In particular, in the limit of large distances fermions effectively turn into bosons.

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The idea of fractional statistics in two dimensions [1, 2] and its possible relevance to real physical systems attract attention from different points of view. The standard mechanism [3, 4] leading to appearance of such statistics is associated with including the Chern-Simons term in the gauge field action. However, such term breaks P and T symmetry, which is a serious obstacle to potential realization of fractional statistics in nature. (As is well known, it is only in the systems of neutral kaons that a weak breakdown of T invariance has been discovered experimentally.) In essence, Laughlin’s anyon ansatz [5] in the explanation of fractional quantum Hall effect, where it is the external magnetic field which breaks P and T, remains the only application of the idea of fractional statistics which can be regarded as confirmed experimentally.

There had been numerous attempts to apply this idea to construct a theory of high-$T_c$ superconductivity, starting from the works by Laughlin [6]. However, such direct attempts apparently are not relevant, since the available experimental data do not confirm [7] the P and T breakdown effects predicted within the framework of the anyonic scenario of superconductivity. In connection with this, a P and T invariant model of superconductivity, which involves a mixed Chern-Simons term, was put forward [8]-[11]. The authors of [11] argue that in this model the charge carriers do not acquire fractional statistics, so that the model is not the one of “anyonic superconductivity”, despite the presence of the Chern-Simons term. In fact, as we will see, this statement holds only so far as the distance between the charges is kept either much more or much less than the characteristic scale of interaction determined by the coupling constants. In the present work, we will study the case of arbitrary distances and show that the model under consideration gives rise to effectively distance dependent statistics, which is in a certain sense a generalization of fractional statistics [12, 13]; moreover, it turns out that composites of arbitrary number of fermions at large distances behave as bosons. We reason that the discovered possibility of having such statistics in P and T invariant models makes it more likely to appear in real physical systems. We would like to note the difference of the case under consideration from the one of Ref.[14], where P and T are conserved only macroscopically; in the model at hand, P and T invariance is inherent in the microscopic equations of motion.

The Lagrangian of the model [11] is

$$L = -\frac{1}{4g^2}f_{\mu\nu}f^{\mu\nu} + \bar{\psi}(i\partial - \tau_3\beta - eA - \Delta)\psi - \frac{1}{4\sqrt{\partial^2}F_{\mu\nu}F^{\mu\nu} + \frac{\text{sign}(\Delta)e}{2\pi}\epsilon^{\mu\nu\rho}A_\mu f_{\nu\rho}.}$$

(1)

Here

$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

(2)

$A_\mu$ is the electromagnetic field and $a_\mu$ the so-called statistical gauge field; for the coupling constant $g$ one has $g^2 \sim J$, where $J$ is the parameter of the Hubbard model Hamiltonian. The model involves two species of fermions unified in a four-component bispinor $\psi = (\psi_1, \psi_2)$. The $\gamma$-matrices then form a reducible representation

$$\gamma_0 = \begin{pmatrix} i\sigma_3 & 0 \\ 0 & -i\sigma_3 \end{pmatrix}, \quad \gamma_1 = \begin{pmatrix} i\sigma_1 & 0 \\ 0 & -i\sigma_1 \end{pmatrix}, \quad \gamma_2 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix},$$

(3)
and

\[ \tau_3 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}. \]  

(4)

The parity operator acts on the four-component bispinor as

\[ P_4 \psi = \begin{pmatrix} 0 & P_2 \\ P_2 & 0 \end{pmatrix} \psi, \]  

(5)

so the mass term \( \Delta \bar{\psi}_1 \psi_1 - \Delta \bar{\psi}_2 \psi_2 \) is P-invariant.

The field equations corresponding to the Lagrangian (1) read

\[ \frac{1}{g^2} \partial_\mu f^{\mu\nu} + \frac{s e}{2 \pi} \epsilon^{\mu\nu\lambda} F_{\mu\lambda} = j^\nu_3, \]  

\[ \frac{1}{\sqrt{-\partial^2}} \partial_\mu F^{\mu\nu} + \frac{s e}{2 \pi} \epsilon^{\mu\nu\lambda} f_{\mu\lambda} = j^\nu, \]  

(6)

where

\[ j^\nu = \bar{\psi} \gamma^\nu \psi, j^\nu_3 = \bar{\psi} \gamma^\nu \tau_3 \psi, s = \text{sign}(\Delta). \]  

(7)

For our purposes it will be sufficient to restrict ourselves to a purely quantum mechanical treatment of the problem. Therefore one can simply impose the Lorentz gauge conditions \( \partial_\mu A^\mu = 0, \partial_\mu a^\mu = 0 \) so that (6) becomes

\[ \frac{1}{g^2} \partial^2 a^\mu + \frac{s e}{\pi} \epsilon^{\mu\nu\lambda} \partial_\nu a_\lambda = j^\mu_3, \]  

\[ \sqrt{-\partial^2} A^\mu + \frac{s e}{\pi} \epsilon^{\mu\nu\lambda} \partial_\nu a_\lambda = j^\mu. \]  

(8)

To solve these equations for given \( j^\mu, j^\mu_3 \), Fourier transformation may be applied:

\[ -\frac{p^2}{g^2} \tilde{a}^\mu(p) + \frac{i s e}{\pi} \epsilon^{\mu\nu\lambda} p_\nu \tilde{A}_\lambda(p) = \tilde{j}^\mu_3(p), \]  

\[ \sqrt{-p^2} \tilde{A}^\mu(p) + \frac{i s e}{\pi} \epsilon^{\mu\nu\lambda} p_\nu \tilde{a}_\lambda(p) = \tilde{j}^\mu(p), \]  

(9)

and after a straightforward calculation one comes to

\[ \tilde{A}^\mu(p) = \tilde{B}(p) \tilde{j}^\mu(p) - \frac{4 \pi}{e} f \tilde{D}^{\mu\nu}(p) \tilde{j}_3(\nu)(p), \]  

\[ \tilde{a}^\mu(p) = -\frac{4 \pi}{e} f \tilde{D}^{\mu\nu}(p) \tilde{j}_\nu(p) + g^2 \tilde{C}(p) \tilde{j}^\mu_3(p), \]  

(10)

where

\[ f = \left( \frac{e g}{\pi} \right)^2, \]  

(11)

\[ (f^{-1} \text{ is the characteristic scale of length}), \]  

\[ \tilde{B}(p) = \frac{1}{\sqrt{-p^2} + f}, \tilde{C}(p) = \frac{1}{\sqrt{-p^2} \left( \sqrt{-p^2} + f \right)}, \]  

(12)
\[
\mathcal{D}^{\mu\nu}(p) = \frac{i\epsilon^{\mu\nu\lambda}p_\lambda}{\sqrt{-p^2(\sqrt{-p^2} + f)}}. \tag{13}
\]

To investigate the effective change of statistics, consider a point source

\[
j^\mu(x) = ne\delta_0^\mu \delta^2(\vec{x}) , \quad j_3^\mu(x) = n_3\delta_0^\mu \delta^2(\vec{x}). \tag{14}
\]

Substituting the Fourier transforms \(\tilde{j}_0^\mu(p) = \frac{ne}{(2\pi)^3}\delta_0^\mu \delta(p_0)\), \(\tilde{j}_3^\mu(p) = \frac{n_3}{(2\pi)^3}\delta_0^\mu \delta(p_0)\) in (10) and performing the inverse transformation, we get

\[
A_0(r) = ne \left[ \frac{1}{2\pi r} - \frac{f}{4} u(fr) \right],
\]

\[
a_0(r) = \frac{n_3^2}{4} u(fr)
\]

for the temporal components, and

\[
A_\varphi(r) = -\frac{sn3}{2\pi r} v(fr),
\]

\[
a_\varphi(r) = -\frac{sn}{2\pi r} v(fr)
\]

for the angular components (\(\varphi\) is the polar angle). Here

\[
u(x) = H_0(x) - Y_0(x) = \frac{2}{\pi} x_1 F_2 \left(1; \frac{3}{2}, \frac{3}{2}; -\frac{x^2}{4}\right) - Y_0(x), \tag{17}
\]

\[
v(x) = x \int_0^\infty \exp(-x \sinh t - t) \, dt. \tag{18}
\]

The plots of \(u(x)\) and \(v(x)\) are displayed on Fig. 1. The temporal components correspond to the quasi-Coulomb interaction (note that for \(r \ll f^{-1}\), \(A_0\) behaves like \(1/r\) and \(a_0\) like \(\ln r\), as one should expect in accordance with the form of the Lagrangian). The effective change of statistics is due to the angular components.

For a general consideration, imagine a composite of \(n_1\) fermions of the first sort (\(\psi_1\)) and \(n_2\) of the second sort (\(\psi_2\)). According to (7) and (14), \(n = n_1 + n_2\) and \(n_3 = n_1 - n_2\). Interchanging two non-interacting such composites multiplies the wave function by the phase factor \(\exp[i\pi n^2]\), so that the composites themselves are fermions (bosons) for odd (even) \(n\). However, due to presence of the potentials (16) there appears an additional phase factor. If the composites are kept at a constant distance \(r\), then it equals \(\exp[i\pi \Delta(r)]\) where

\[
\Delta(r) = -\frac{1}{2\pi} [ne A_\varphi(r) \cdot 2\pi r + n_3 a_\varphi(r) \cdot 2\pi r]
\]

\[
= snn_3 v(fr)
\]

\[
= s(n_1^2 - n_2^2) v(fr). \tag{19}
\]

At small distances \((r \ll f^{-1})\) there is no effective change of statistics, since \(v(0) = 0\). At such distances one should in general remember about the Coulomb interaction;
however, its energy, being of the order of $e^2/r + g^2 \ln r$, can always be made small enough by the appropriate choice of parameters, while the function $v(x)$ is parameterless. Therefore, at least from the theoretical point of view, it is allowable not to take into account this interaction \[13\] .

On the contrary, for $r \gg f^{-1}$ the total phase change is $\pi m$, where $m = (n_1 + n_2)^2 + s(n_1^2 - n_2^2)$ is even for any integer $n_1$ and $n_2$. Therefore at large distances the considered composites always behave as bosons. At intermediate distances, the behaviour is in some sense intermediate between the two limiting cases; if $r$ is kept within a sufficiently narrow range in which $v(fr) \simeq \text{const}$, one effectively has anyons with the statistical parameter determined by (19). In the general case the situation is more complicated. Let $n$ be odd so that the composites themselves are fermions. At high temperatures such that $\lambda \ll \xi$ ($\lambda$ is the thermal wavelength and $\xi \sim \rho^{-1/2}$ is the average interparticle distance) a system of those behaves like Fermi gas for $\lambda \ll f^{-1}$ and like Bose gas for $\lambda \gg f^{-1}$ (although in both cases there is only a small deviation of the equation of state from that of the ideal gas) \[13\]. It seems plausible, however, that it is by the relation between $\lambda$ and $f^{-1}$ that the properties of the system are determined for $\lambda \gtrsim \xi$ as well. Therefore if the density is fixed and the temperature is being lowered, one should expect the behaviour of the system to change gradually from Fermi-like to Bose-like. It would be an interesting problem to study this process in more details. We emphasize once more that since the model is P and T-invariant, the possibility of its realization in nature appears considerably more actual than for the usual P and T-noninvariant Chern-Simons model.

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Figure Caption

Fig.1. The functions $u(x)$ and $v(x)$. 
