Parameters Optimization of a Direct Secondary Oblique Shock Wave Nozzle

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Abstract. The structural optimization design of the secondary shock wave channel in a convergent-divergent nozzle is carried out with the latin hypercube design method and response surface methodology. The multi-island genetic algorithm is used to find the optimal solution with four designed parameters including the wedge wall angle of secondary flow chamber δ1, the inclined wall deflection angle of secondary flow chamber γ, the width of front vacuum chamber w1 and the distance from the secondary flow chamber to nozzle outlet e. The numerical results indicate that the direct secondary shock wave channel can increase the efficiency of the secondary flow and improve the nozzle performance. The influence on the thrust pitching angle δp is w1, e, γ, and δ1 following in order of decreasing influence.

1. Introduction
Fluidic thrust vectoring is typically generated by time-periodic alternate ejection and suction of the working fluid through an orifice in the flow boundary [1]. The main flow interacts with the secondary flow to generate an oblique shock wave, and the main flow is forced to eject at a certain angle from the nozzle outlet when it passes the oblique shock wave. Shock vector control technology can create a large thrust vector with a simple structure, which is highly suitable for the development of innovative fighter aircraft [2-3].

Based on shock wave theory, theoretical analysis is used to establish the mathematics models. Maarouf et al. accomplished a modeling study to investigate the effects of a secondary injection in an axisymmetrical, convergent-divergent nozzle and focused on the influence of some parameters (pressure ratios, injection slot size and location, injected mass flow rates) for fluidic thrust vectoring purposes [4]. Computational fluid dynamics, which uses various discrete mathematical methods, can broaden the scope of experimental research and reduce expensive experimental work. Najar et al. employed k-ε and the S-A turbulence models and compared the parameters in the convergent-divergent nozzles to ascertain the most efficient design conditions [5]. The nozzle experiments can guide engineering practice, scientific research and provide experiment data by using Schlieren and background methods, etc.. Until now, a significant portion of research on the shock vector control nozzle was carried out by analyzing the complex flow field model in the nozzle and the relationship between the nozzle parameters and the thrust vector force [6-7].
Based on the shock wave theory and numerical simulations, the structural parameters of the secondary shock wave channel are designed and optimized. Additionally, the influence mechanism and the structural parameters of the secondary shock wave on nozzle performance are also investigated.

2. Nozzle geometric parameters

Since the nozzle is two-dimensional and the cross section is rectangular, the parameters of the main nozzle is shown in Table 1.

| Ma | d1/mm | d2/mm | d3/mm | r/mm | L/mm | α/(°) | β/(°) |
|----|-------|-------|-------|------|------|-------|-------|
| 2  | 71.61 | 0.02  | 0.03  | 0.07 | 0.08 | 55    | 4.78  |

Table 1. Geometric parameters of main nozzle.

In order to improve the secondary flow efficiently, a secondary shock wave channel is designed, as shown in Figure 1. The channel structure is composed of a front vacuum chamber 1, a secondary flow chamber 2, and a rear vacuum chamber 3.

The secondary flow chamber is designed to generate a secondary flow with a direct oblique shock wave. The front vacuum chamber is placed at upstream of the secondary flow chamber to prevent a large area of vortex from interfering with the oblique shock wave. The rear vacuum chamber is placed downstream of the secondary flow chamber to reduce the outlet back pressure of the secondary flow chamber. The configuration of the secondary flow chamber is similar to the main nozzle. The structural parameters of the secondary flow channel are shown in Table 2.

| M_{A2} | e/mm | w1/mm | h1/mm | e2/mm | d11/mm | d12/mm | d13/mm | r1/mm | L1/mm | δ1/mm | θ1/(°) | β1/(°) | α1/(°) | γ/(°) | w2/mm | e2/mm | h2/mm | w3/mm |
|--------|------|-------|-------|-------|--------|--------|--------|-------|-------|-------|--------|--------|--------|--------|-------|-------|-------|-------|
| 1.7    | 6.64 | 6     | 2     | 0.42  | 1.33   | 1      | 1.33   | 2.75  | 2     | 20    | 35.15  | 4.85   | 15     | 50    | 1.97  | 0.71  | 2     | 2.19  |

Table 2. Geometric parameters of secondary shock wave channel

3. Performance calculation

The thrust pitching angle \( \delta_p \) is an important parameter for evaluating the performance of the nozzle.

\[
\delta_p = \tan^{-1} \left( \frac{F_y}{F_x} \right)
\]  

The relationship between the secondary shock wave angle \( \beta_2 \) and the wedge angle \( \delta_1 \) is:

\[
\tan \delta_1 = 2 \cot \beta_2 \frac{Ma_1^2 \sin^2 \beta_2 - 1}{Ma_1^2 (k + \cos 2\beta_2) + 2}
\]  

The secondary shock wave intensity is expressed in \( p_2/p_1 \) as:

\[
\frac{p_2}{p_1} = \frac{2k}{k + 1} \frac{Ma_1^2 \sin^2 \beta - k - 1}{k + 1}
\]  

(3)
4. Parameters optimization

Based on the simulation results and using the detail optimization design method, variables with a large influence on \( \delta_p \) are retained as the input variables to the model. The design of experiment (DOE) method is investigated to select a certain number of samples to be able to calculate the function value of \( \delta_p \) for each group of samples, and then to establish the relationship between the design variables and the response values through the response surface methodology (RSM). Finally, the multi-island genetic algorithm (MIGA) is used to find the local maximum value of the objective function \( \delta_p \) \[8-9\].

4.1 DOE method

In this study, the geometric parameters of the secondary shock wave channel which have the greatest influence on \( \delta_p \) are selected as optimization variables, that is, \( e, \delta_1, \gamma \) and \( w_1 \) are input variables as constraints, \( \delta_p \) is the output variable as optimization target. A certain number of samples is selected using the latin hypercube design (LHD) method, which has an effective space filling capability and an adaptation of nonlinearity. In the \( n \)-dimensional space, each dimensional coordinate interval \([x_{i\text{min}}, x_{i\text{max}}] , k \in [1,n]\) is evenly divided into \( m \) intervals, and each cell is denoted as \([x_i^{e\text{min}}, x_i^{e\text{max}}], i \in [1,m]\).

The \( m \) samples are selected randomly to ensure that each level of a factor is only studied once, which is denoted as \( m \times n \) LHD.

According to the design requirements, the specific value ranges of the variables are determined as follows:

- \( 6.2 \text{ mm} \leq e \leq 12 \text{ mm} \);
- \( 45^\circ \leq \gamma \leq 60^\circ \);
- \( 16^\circ \leq \delta_1 \leq 30^\circ \);
- \( 0.4 \text{ mm} \leq w_1 \leq 0.6 \text{ mm} \).

Using RSM can reduce the size of the polynomial needed to approximate the response. The accuracy of the response surface depends on the number of points in the sample, the shape of the response surface being forced and the design space. In this paper, a second-order response surface is used with four variables and one response value.

4.2 Optimization method

The MIGA method is used to find the optimal solution, which can improve the optimization efficiency. As a pseudo-parallel genetic algorithm, it can quickly find the local best value and has a number of ‘islands’. There are some individuals on each island, and it is assumed that individuals can migrate between the islands. All the individuals with migration ability are excellent and have good genes that can help the algorithm jump out of the local best value and achieve global optimality.

The key items in the definition of MIGA are: subgroup = 40; island number = 40; genetic algebra = 10; crossover rate = 1.0; mutation rate = 0.01; migration speed = 50%; migration time interval = 5.

5. Optimization analysis

5.1 Parameters analysis on \( \delta_p \)

Table 3 compares configuration parameters of the secondary shock wave chamber and \( \delta_p \) of the original model, the pre-optimized model with superior thrust vector performance selected in multiple schemes, and the optimized model. It can be seen that \( \delta_p \) of the pre-optimized model has been improved greatly, but \( \delta_p \) can be further improved after optimization. The optimized \( \delta_p \) reaches 15.05\(^\circ\), an increase of 8.11% compared to the pre-optimization model, and 48.4% compared to the original model. Based on the DOE tests on the four parameters, it is found that the parameter that has the greatest influence on \( \delta_p \) is \( w_1 \), with \( e, \gamma \), and \( \delta_1 \) following in order of decreasing influence.

|          | \( e/\text{mm} \) | \( \delta_1/\text{°} \) | \( \gamma/\text{°} \) | \( w_1/\text{mm} \) | \( \delta_p/\text{°} \) |
|----------|------------------|------------------|---------------|------------------|------------------|
| Original model | 10              | 25               | 55            | 0.45             | 10.14            |
| Pre-optimized model | 6.64          | 20               | 50            | 0.6              | 13.92            |
| Optimized model   | 6.38           | 17.68            | 49.98         | 0.44             | 15.05            |
5.2 Pressure analysis of the upper and lower wall of the nozzle diverging part

Figure 2 shows the pressure distribution on the upper and lower wall of the nozzle diverging part under the conditions of NPR = 10 and SPR = 2 of the pre-optimized model and optimized model. The position ratio x/xt is the distance from the pressure point to the nozzle throat divided by the length of nozzle diverging wall. Due to the low back pressure, the pressure on the lower wall decreases gradually. At the beginning of the diverging part, the pressures on the upper wall and the lower wall are equal. When x/xt is around 0.635, the upper wall pressure begins to increase due to the influence of the secondary flow. The pressure on the upper wall reaches a maximum value at x/xt = 0.937, and then the pressure drops sharply to the level of the low back pressure.

(a) pre-optimized model
(b) optimized model

Figure 2. Pressure distribution of the upper and lower wall of the nozzle

Compared the Figure 2 (a) and (b), it is found that the pressure distribution on the lower wall has the same trend in the both models. However, the pressure distribution out of the secondary flow chamber of the optimized model is higher than that of the pre-optimized model, so the pressure difference at the diverging part of the optimized model is larger than that of the pre-optimized model which results in the larger δp.

6. Conclusions

A two-dimensional convergent-divergent nozzle with a direct secondary oblique shock wave is proposed in this paper. Based on the numerical simulations, the effects of channel geometric parameters on the thrust pitching angle are investigated.

In the optimized model, the MIGA is used to find the optimal solution after applying the LHD method. The relationship between the secondary shock wave channel parameters (e, δ1, γ and w1 and δp) is also analyzed. The parameter that has the greatest influence on δp is w1, e, γ and δ1 following in order of decreasing influence. The structural parameters of the optimized thrust vector control nozzle are: e = 6.38 mm, δ1 = 17.68°, γ = 49.98°, w1 = 0.44 mm.

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