Market interaction structure and equilibrium price heterogeneity in monopolistic competition

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Abstract
This paper investigates the extent to which the structure of the interaction network between suppliers and buyers affects equilibrium price heterogeneity. An incomplete interaction structure leads to uneven information flows and different information bases for consumers which is then taken into account by price setting producers. This results in heterogenous prices even if producers are identical in all other respects. The complete interaction network serves as a special case resembling standard monopolistic competition models. We show that a slight deviation from the complete network results in heterogeneous prices, although this heterogeneity becomes economically significant only under sparse or small networks. Sparsity as the main determinant of price heterogeneity dominates network asymmetry: relatively dense networks show minimal price dispersion even if its degree distribution follows a power law.

Keywords Network structure · Price dispersion · Monopolistic · Competition · Optimal pricing

1 Introduction

Price dispersion has been the subject of strong academic interest in recent decades. As standard general equilibrium models assuming perfect information yield competitive equilibria with competing producers/sellers setting identical prices, researchers have come up with approaches which are able to reproduce price dispersion as an equilibrium phenomenon [36].

The two main lines in this respect are either suggesting the extension of the standard monopolistic competition model or the use of a search-theoretic approach [10]. Along the first line, we can find heterogeneous marginal costs as the most trivial solution but also heterogeneous demand elasticities resulting from heterogeneous
visiting costs [40]. The second line emphasizes that there is some incompleteness in price information on the side of consumers which is assumed to arise from positive search costs (see e.g. [11] for a comprehensive review of this literature).

In this paper, we join to this vein of literature, but take an approach to consumer search and price heterogeneity from the viewpoint of complex systems. Instead of starting from a detailed account of how consumers are limited in attaining complete information, we rather take the structure of information flows as given and view this structure as a complex system of market interactions between producers/suppliers and consumers/buyers. As a result, we are able to infer on how the structure of the network connecting market actors shape price heterogeneity.

Our approach from the side of complex systems is also supported by recent work which argues that the analysis of increasingly complex economic systems requires the integration of aspects and methods from the toolbox of complex system research [23, 24]. On the other hand, the results of network science indicate that the aggregate performance of a system is inherently related to the structure of connections among agents in the system (see e.g. [7, 16, 28, 34]).

Several contributions have recently shown how economic micro-structure affects aggregate performance. Acemoglu et al. [2] and [1] e.g. demonstrate that the asymmetric distribution of intersectoral transactions can lead to heavy-tailed aggregate fluctuations. Other papers show that accounting for the structure of connections between market actors can significantly change predictions from standard economic models [5, 37, 38]. Further studies focus on connections between financial institutions and reveal that the structure of these connections contribute to systemic risk [3, 4, 19].

Some other studies have explicitly taken into account the relationship between network structure or network formation and the pricing behavior of firms. However, these contributions typically refer to network externalities where the connection structure is assumed to play a role between consumers [8, 30, 31, 33], to situations where firms can engage in some kind of information exchange [12] or markets where networks (like telecommunication networks) compete with each other [27]. Some of these contributions assume targeted pricing (price discrimination) on the side of producers, where consumer-specific pricing depends on the level of influence a given consumer has on others [21, 22], while network structure among consumers is taken into account in these papers. Other studies, like the authors in [43], have focused on pricing in supply chain networks, where the structure of interaction among production sites is explicit, but modeling the demand from outside the supply network remains limited.

A relatively recent line of research focuses on network externalities from a structural perspective. These contributions explicitly consider the structure of network connections among consumers (local network externalities) and present how specific connection structures affect pricing behavior and may lead to market segmentation [6, 8, 13, 14, 18, 29]. A similar approach is followed by [41], who introduce explicit network structures among productive subsidiaries of decision making firms with strategic substitutability among the products of the subsidiaries. However, there is no attempt up to our knowledge which examines the role of explicit network structures between buyers and sellers, and how this structure affects price dispersion.
Although by taking a different perspective, our model is a close relative to search-cost models, like [39] or [17], where consumers are not perfectly informed about all prices on the market. We introduce a specific version of a standard model of monopolistic competition with finite actors, where the interaction network among market actors is given explicitly and this network represents the biased information flows on the market. In this sense, this paper assumes some sort of biased information flows, represented by a specific network structure, and then continues to analyze the role of this structure in shaping price dispersion. This is in contrast with the standard approach where consumers are assumed to be aware of the true price index as made up of all prices in the market. Instead, our more realistic extension assumes that this information basis is selective and consumer-specific, which producers take into account when making their decisions.

From a methodological point of view, our approach is similar to [42] and [41]: instead of assuming a given network structure, we also derive an existence condition without a fixed structure and then analyze the effect of some specific network arrangements using network models. Also, our approach builds on that of [38] by using the standard symmetric monopolistic competition model and develop it in a way that explicit supplier and buyer relationships are taken into account.

The contribution of our paper with respect to these closely related papers is two-fold. First, we analyze the explicit connection structure between suppliers and buyers while previous papers focused on network externalities between consumers (demand side) of given firm or subsidiaries (supply side). Thus, our results contribute to previous research by pointing to the effects of the connection structure between the supply and demand side. Second, it releases the implicit symmetry- and representativity condition used in [38] by allowing for arbitrary network structures within the same model framework of monopolistic competition.

The paper is structured as follows. In the next section, we construct the model of optimal pricing, with an incomplete market interaction structure. The following section then derives the condition under which optimal pricing is possible and economically feasible. Then, a separate section is devoted to the analysis of the optimal price distribution under different assumptions about the structure of the market interaction network. First, we consider a complete network as a baseline case and then explore the properties of the price distribution if the network structure is symmetric (Erdős–Rényi type) and if it is asymmetric. We conclude the paper with a summary of the main results and pointing out some areas for further research.

## 2 The model

Assume an economy with $N \geq 2$ actors. These actors interact with each other by selling and purchasing their differentiated products and labor force (hours). Every actor, engaged in productive activities, produces one type of differentiated product. All actors are perfectly informed, so they possess all relevant information to make optimal decisions. We disregard transaction costs and assume the products to be infinitely divisible. Resulting from differentiation, product markets are monopolistic while the labor market is set to be perfectly competitive. The latter implies that
wages in the economy are identical across producers. Finally, we do not consider market dynamics explicitly; so the model is static.

The key element of the model is that it explicitly takes into account a possibly non-complete network of interactions between producers and consumers. In other terms, a given producer does not necessarily sell to all consumers on the market whereas a given consumer does not necessarily buy from all producers. This interaction structure is assumed to be exogenous and it is described by the adjacency matrix where the cell in row $i$ and column $j$ is $a_{ij} \in \{0, 1\}$ and it describes whether actor $j$ consumes/buys from actor $i$ ($a_{ij} = 1$) or not ($a_{ij} = 0$). We assume that producers are informed about the structure of this interaction network.

Although the model is general in the sense that an actor can be both a producer and buyer, for an easier exposition, we clearly distinguish between the two types of actors. Producers are indexed with $\{1, 2, ..., M\}$ and consumers with $\{M + 1, M + 2, ..., N\}$, where $N > M \geq 2$. As a result, $A$ contains a relevant block of rows 1 to $M$ and columns $M + 1$ to $N$, all other entries are zero. This corresponds to a bipartite graph where nodes are grouped into the set of producers and consumers while edges are defined exclusively between the two groups of nodes. In what follows, we only refer to this relevant block of the adjacency matrix $A$, which is then $A \in \{0, 1\}^{M \times (N - M)}$.

### 2.1 The demand side

We assume a linear demand structure as e.g. in [12] or [32]. Actor $j$’s demand for the product of actor $i$ is denoted by $x_{ij}$ and it is defined as:

$$x_{ij} = b_{ij} \left[ \gamma_j - \epsilon \left( p_i - \bar{p}_j \right) \right], \forall i, j$$

where $\gamma_j > 0$ is a consumer-specific intercept of the demand function, $\epsilon > 0$ is an elasticity parameter reflecting the substitutability of product varieties, $p_i$ is the price charged by producer $i$ and $\bar{p}_j$ is the price index perceived by consumer $j$. The scaling constant $b_{ij}$ is constructed from the adjacency matrix as

$$b_{ij} = \frac{a_{ij}}{\sum_k a_{kj}} = \frac{a_{ij}}{d_j}, \forall i, j$$

where $d_j = \sum_i a_{ij}$ is the degree of consumer $j$, reflecting the number of consumed product variants. Matrix $B$, constructed from the elements $b_{ij}$, is the column-standardized version of $A$. As a result, it holds that $0 \leq b_{ij} \leq 1$ and $b_{ij}$ reflects the share of product $i$ in the consumption-basket of consumer $j$. If consumer $j$ buys only from one producer $k$ then $b_{ij} = 1$ for $i = k$ and $b_{ij} = 0$ for all $i \neq k$. The more varieties consumer $j$ is choosing from, the lower $b_{ij}$ will be for those $i$ such that $a_{ij} = 1$. Generally, the $j$th column of $B$ contains zeros and $1/d_j$ exactly $d_j$ times. In the extreme case when consumer $j$ buys from all suppliers, we have $b_{ij} = 1/M$ for all $i$. Finally, this scaling constant in Eq. (1) ensures that the demand of consumer $j$ is equally distributed among producers if the latter charge the same price.

The perceived price index is defined as
which means that the perceived price index of consumer \( j \) is the average price of the producers it is connected to. This is a key point in our model. While standard monopolistic models assume that consumers are aware of the true price index which reflects all prices in the economy, in our model the selective (non-complete) interaction network constrains this information and consumers only take into account those prices which are relevant to them.

At this point, it is important to highlight some limitations of the demand function in Eq. (1). This demand function is not defined if

1. \( \varepsilon \to 0 \), i.e. when consumers are irresponsive to prices;
2. \( \varepsilon \to \infty \), i.e. when consumers are infinitely responsive to prices;
3. if the number of product varieties consumed by consumers grows infinitely.

The last two conditions resemble perfect competition with infinitely elastic demand and continuum producers. As a result, our model is not suitable to describe perfect competition on the product market. On the other hand, the first condition resembles monopoly to some extent where consumers do not face a variety to choose from. Moreover, we are going to introduce a further condition later on which excludes monopolies. Thus, our model is able to reproduce oligopolistic situations (finite number of producers and at least some competition), so the results are valid for this market structure.

This oligopolistic structure ensures that all consumers buy at least two product varieties, so \( d_j \geq 2 \) for all \( j \) which results in \( b_{ij} \leq 1/2 \) for all \( i \) and \( j \).

### 2.2 The supply side and profit maximization

We assume that output depends on labor use according to the following production function:

\[
y_i = \alpha \ell_i, \quad \forall i
\]

where \( \ell_i \geq 0 \) is the labor units used by producer \( i \), \( \alpha > 0 \) is a constant productivity parameter and \( y_i \) is the output of producer \( i \).

Given that producers are well informed, they know the structure of demand against their products, along with the connection structure of consumers. As a result, they produce exactly the demanded quantity. It follows that

\[
y_i(p) = \sum_j x_{ij} = \sum_j b_{ij} y_j - \varepsilon \sum_j b_{ij} p_j + \varepsilon \sum_j \sum_k b_{ij} b_{kj} p_k, \quad \forall i
\]

This means that the output of a given producer depends on the prices set by all other producers as well (reflected by the price vector \( p \) on the left-hand side). Also, the formula in Eq. (5) shows that the structure of the market interaction network affects the way these other prices influence the demand for producer \( i \), hence its output.
Producers maximize profits by setting their price. Price determines demand, hence output according to Eq. (5), however, the prices of all other producers influence price setting and profit maximization. The profit function of producer $i$ can be written as

$$\pi_i(p) = p_i y_i(p) - \omega \ell_i = y_i(p) \left( p_i - \frac{\omega}{\alpha} \right), \ \forall i$$

where the presence of the price vector ($p$) refers to the interrelatedness of prices through profit maximization. $\omega$ labels nominal wage and the second equation uses the inverse production function from Eq. (4).

### 3 Equilibrium prices

#### 3.1 Profit maximization

The key question is whether producers can set optimal prices given the non-complete market interaction structure, fixed by $A$. Equilibrium price setting requires the producers to simultaneously solve the maximization problem composed of the objective function (6) subject to the demand functions (5). As mentioned before, these individual profit maximization problems are not independent from each other, even if we assume that producers take other prices as given, so that $\partial p_i/p_j = 0$ for all $j \neq i$.

The first order condition for the profit-maximizing price of producer $i$ is

$$\frac{\partial \pi_i}{\partial p_i} = y_i + p_i \frac{\partial y_i}{\partial p_i} - \frac{\omega}{\alpha} \frac{\partial y_i}{\partial p_i} = 0, \ \forall i$$

It is helpful at this point to focus on the price-sensitivity of output/demand of a given producer $i$:

$$\frac{\partial y_i}{\partial p_i} = -\epsilon \sum_j b_{ij}(1 - b_{ij}), \ \forall i$$

The parameter $\epsilon$ enters naturally in this expression: the less substitutable product varieties are (the lower $\epsilon$), the lower this price sensitivity will be. Also, because $0 \leq b_{ij} \leq 1/2$, the right-hand side of (8) is non-positive which is in line with standard expectations about the price sensitivity of demand. In addition to $\epsilon$, price sensitivity is also affected by the summed expression on the right-hand side. For a given consumer $j$ we have $b_{ij}(1 - b_{ij})$, which is quadratic in $b_{ij}$, with a maximum at $1/2$. As a result, on the relevant interval $[0, 1/2]$ this is monotonically increasing in $b_{ij}$. The term $b_{ij}$ reflects the share of producer $i$ in the consumption basket of consumer $j$. The larger this share, the larger the expression in the sum becomes for consumer $j$. As we sum over consumers, the right-hand side of (8) reflects the extent to which the consumers of producer $i$ are exposed to this producer in the sense that they have less alternative varieties to choose from.
To sum up, the expression in (8) decomposes the monopoly power of producer \( i \) into two elements. The first element \( (\epsilon) \) reflects the overall substitutability of the product varieties (a kind of technological or preference issue), while the second element (the sum) reflects market structure, i.e. the extent to which producers can rely on their monopoly power. The latter is a function of the network structure of market interactions.

The first order condition in (7) can be extended using (5) and (8) as

\[
\frac{\partial y_i}{\partial p_i} = \sum_j b_{ij} \gamma_j + \epsilon \left[ \sum_j \sum_k b_{ij} b_{kj} p_k - p_i \sum_j b_{ij} - p_i \sum_j b_{ij} (1 - b_{ij}) \right] + \frac{\epsilon \omega}{\alpha} \sum_j b_{ij} (1 - b_{ij}) = 0, \forall i
\]

Given the interrelated pricing decisions, it is straightforward to compress the \( M \) first order conditions in (9) into matrix notation:

\[
B \Gamma 1_{N-M} + \epsilon (BB^T - \tilde{B} - C)p + \frac{\epsilon \omega}{\alpha} C1_M = 0_M,
\]

where \( B \) is the \( M \times (N - M) \) matrix of \( b_{ij} \)'s, \( \Gamma \) is the \( (N - M) \times (N - M) \) diagonal matrix with \( \gamma_j \) along its main diagonal, \( \tilde{B} \) and \( C \) are \( M \times M \) diagonal matrices with \( \sum_j b_{ij} \) and \( \sum_j b_{ij} (1 - b_{ij}) \) on their main diagonals, respectively. \( 1_M \) and \( 1_{N-M} \) are column vectors of ones with the respective sizes while \( 0_M \) is a column vector of zeros with the respective size. Finally, \( p \) is the vector of prices and its size is \( 1 \times M \).

Let us use the following substitution: \( Q = \tilde{B} + C - BB^T \). Then, (10) can be rearranged to express \( p \) in a more compact way:

\[
p = \frac{1}{\epsilon} Q^{-1} \left( B \Gamma 1_{N-M} + \frac{\epsilon \omega}{\alpha} C1_M \right)
\]

This shows that the solution for the optimal price vector hinges on matrix \( Q \) and whether it is invertible or not. As the previous definition shows, \( Q \) is a function of matrix \( B \), which describes network structure. \( \tilde{B} \) contains the sums \( \sum_j b_{ij} \) on its diagonal. These values reflect the extent to which consumers of given products are exposed to them. Based on our previous discussion, matrix \( C \) reflects a similar exposition of consumers through the sums \( \sum_j b_{ij} (1 - b_{ij}) \). Finally, the matrix product \( BB^T \) reflects the connectedness of producers. As producers are not connected directly, this connectedness is defined through consumers. Unfolding the general element of this matrix product, we get \( \tilde{b}_{ik} = \sum_j b_{ij} b_{kj} \). One element in this sum reflects a given consumer \( j \) and the product within the element shows the extent to which this consumer is exposed to both producers \( i \) and \( k \). \( b_{ij} \) is higher if consumer \( j \) is more exposed to producer \( i \) in the sense that there are less alternative varieties to choose from. This means more market power towards consumer \( j \) from the perspective of producer \( i \). Now if both producers \( i \) and \( k \) have this strong market power towards consumer \( j \), this means that they are competing for the same consumer to a large extent or in other terms, they are sharing markets. The more consumers they share, the higher \( \tilde{b}_{ik} \) will be.
It follows, that the matrix product $BB^\top$ can be interpreted as an adjacency matrix describing the (indirect) connectedness of producers. It is a weighted adjacency matrix where the weights reflect the extent to which two producers share the same consumers and these consumers are exposed to them with a low number of alternative varieties at their reach. This adjacency matrix nature dominates matrix $Q$, while its two other determinants $\tilde{B}$ and $C$ affect only its diagonal.

The parentheses on the right-hand side of Eq. (11) contain non-negative constants by definition, so the existence of the optimal price vector hinges on the existence of $Q^{-1}$.

Once the price vector is given, the $\mathbf{y}$ vector of producer outputs $y_i$ can be calculated as

$$y = B\Gamma 1 + \varepsilon BB^\top p - \varepsilon \tilde{B}p = B\Gamma 1 + \varepsilon (BB^\top - \tilde{B}) p$$

### 3.2 Existence and uniqueness

In order to prove the existence of the matrix inverse $Q^{-1}$, we use a corollary of Gershgorin’s theorem [25]. This corollary states that the inverse of a matrix exists if diagonal dominance applies, i.e., all elements on the main diagonal have a strictly higher absolute value than the sum of the absolute values of all off-diagonal elements in the given row. If we denote the general element of $Q$ by $q_{ik}$ then formally we must have $|q_{ii}| > \sum_{k,k\neq i} |q_{ik}|$. This condition requires the following inequality to hold:

$$2 \sum_j b_{ij} (1 - b_{ij}) > \sum_{k\neq i} \sum_j b_{ij} b_{kj}, \forall i$$

(13)

The right-hand side of (13) can be rewritten as

$$\sum_{k\neq i} \sum_j b_{ij} b_{kj} = \sum_{k\neq i} \sum_j \frac{a_{ij}}{d_j} \frac{a_{kj}}{d_j} = \sum_j \frac{a_{ij}}{d_j^2} \sum_{k\neq i} a_{kj} = \sum_j \frac{a_{ij} (d_j - a_{ij})}{d_j^2}$$

$$= \sum_j \frac{a_{ij}}{d_j} \left(1 - \frac{a_{ij}}{d_j}\right) = \sum_j b_{ij} (1 - b_{ij}), \forall i$$

(14)

It follows that it is enough to require the following condition to hold for $Q^{-1}$ to exist:

$$\sum_j b_{ij} (1 - b_{ij}) > 0, \forall i$$

(15)

The condition in (15) holds if there is at least one $b_{ij}$ different from 1 and 0 for any $i$. In other terms, there has to be at least one entry in every row of $B$ which is different from 0 and 1. Given the discrete nature of the adjacency matrix and the possible number of connections, the second-highest possible value of $b_{ij}$ is 1/2 when $d_j = 2$. Taken this together, condition (15) holds if there is at least one consumer $j$ for every producer $i$ for which $d_j \geq 2$. So every producer must be connected to at least one such consumer who is connected to at least one other producer. To put it differently,
this condition rules out isolated producers \( (b_{ij} = 0 \text{ for all } j) \) and producers without competition (monopolies).

The economic intuition behind this result can be summarized as follows. Standard monopoly pricing is not ‘available’ for producers in this model as in that case consumers would become indifferent about prices according to the demand function (1). In other terms, this demand function is expressed in relative prices. As producers are assumed to rely on the knowledge of these demand functions when making their price decisions, they need at least one reference price relative to which they can set their price optimally.

Finally, the existence condition (15) does not require the whole network between producers and consumers to be connected. There may exist different components, within which (15) still holds, but these components (groups of producers and consumers) are not connected.

In Appendix A we also show that the elements of the optimal price vector are positive, thus economically meaningful. In addition, the structure of the profit-maximization problem and the previous derivation ensure that the optimal price vector is unique.

The previous discussion can be summarized in the following proposition.

**Theorem 1** The profit maximization problem in (6) subject to (5) has a unique solution if (15) holds, i.e., if every producer is connected to at least one such consumer which consumes at least one additional product variant.

It is important to note that in the present model setup we assume \( d_j \geq 2 \) for all \( j \), which is required for the demand functions (1) to be meaningful. In this case the existence condition (15) only requires that every producer has at least one consumer which is a non-binding condition.

### 3.3 Second-order condition and feasibility

The previous discussion focused on the first-order condition of profit maximization, which should be augmented by the analysis of the second-order (sufficiency) condition of profit-maximization, i.e., if the stationary point in (11) indeed maximizes profits. The sufficiency condition can be analyzed through the second-order derivative

\[
\frac{\partial^2 \pi_i}{\partial p_i^2} = -2 \epsilon \sum_{j=1}^{N-M} b_{ij}(1 - b_{ij}), \forall i
\]

(16)

If this expression is negative for all \( i \), then the optimal prices in (11) are indeed maximizing profits. Because we have \( d_j \geq 2 \) for all \( j \), it follows that \( b_{ij} \leq 1/2 \) for all \( i \) and \( j \). From this, it is apparent that the second-order partial derivative in (16) is negative for all \( i \), hence we have profit maximizing prices. This is summarized in the following proposition.
Theorem 2 The solution (11) of the profit maximization problem in (6) subject to (5) is a profit maximum if \(d_j \geq 2\) for all consumers \(j\).

In order to wrap up this discussion with the economic feasibility of the optimal price vector, first we express prices from the first order condition (9) as

\[
p_i = \frac{\sum_j b_{ij}y_j}{2\epsilon \sum_j b_{ij}(1-b_{ij})} + \frac{\sum_j \sum_{k\neq i} b_{kj}b_{ji}p_k}{2 \sum_j b_{ij}(1-b_{ij})} + \frac{\omega}{2\alpha}, \quad \forall i
\]

(17)

It is interesting to devote some words to the structure of this solution. The right-hand side consists of three terms, all reflecting a specific element behind price-determination. The last term reflects marginal costs. Total production cost of producer \(i\) is \(\omega \epsilon_i = \omega y_i / \alpha\), the derivative of which with respect to \(y_i\) is \(\omega / \alpha\). This is naturally the basis of pricing as zero profit would imply \(p_i = \omega / \alpha\) (recall that the specific production function (4) implies that marginal cost equals average cost). The first term on the right-hand side reflects monopoly power, which arises from imperfect substitutability of product varieties and it is primarily reflected by \(\epsilon\). The higher \(\epsilon\) is, the easier it gets to substitute varieties, so monopoly power and markups over marginal cost rise. The middle term reflects the effect of competitor prices: the higher the prices are that are charged by competitors on average, the higher \(p_i\) will be. In addition to these intuitive effects, it is also obvious that the network structure of market interactions plays a role in pricing behavior. This structure is reflected by matrix \(B\) and it affects both the role of monopoly power and competitor prices in optimal pricing. This means that the extent to which producers are able to set markups above marginal costs is affected by the network structure connecting producers and consumers. It follows that price heterogeneity may arise from incomplete (asymmetric) network connections even if other underlying parameters (wages, productivity, substitutability) are identical across producers. Further properties of the price Eq. (17) is going to be explored in Section 4.1.

Using the price equation in (17), we are going to put forward a proposition which ensures that the optimal price vector derived in (11) is economically feasible. We have already shown (see Appendix A) that the resulting price vector is positive. However, it is still to be shown that the allocation of output is economically meaningful, i.e., \(x_{ij} \geq 0\) for all \(i\) and \(j\).

Lemma 1 Given the demand functions (1), the production functions (4) and the profit function (6), the output-allocation \(x_{ij}\) is non-negative for all \(i\) and \(j\) if \(p_i \geq \omega / \alpha\).

The proof in Appendix B shows with simple rearrangements that \(x_{ij} \geq 0\) follows from \(p_i \geq \omega / \alpha\). As noted before, \(\omega / \alpha\) is the (identical) marginal cost of producers. Thus, the condition \(p_i \geq \omega / \alpha\) requires individual producers’ profit \(\pi_i\) to be positive, which is in line with the assumption that producers have (some) monopoly power arising from differentiated products. Given Lemma 1, the question is whether the equilibrium prices in (11) indeed exceed the marginal cost \(\omega / \alpha\). The following proposition, the proof of which can be found in Appendix C,
ensures that \( p_i \geq \omega/\alpha \) indeed holds for all \( i \), thus the output allocation is feasible, i.e. non-negative.

**Theorem 3** Given the demand functions (1), the production functions (4) and the profit functions (6), the condition \( p_i \geq \omega/\alpha \) holds for all \( i \).

### 4 Network structure and price heterogeneity

The previous sections introduced a model of simultaneous price setting under incomplete market interaction network, and discussed the conditions for the existence of an economically feasible equilibrium price vector. In this section we use this model to analyze the properties of the price distribution arising under this equilibrium. We organize these analyses around different assumptions about the underlying network structure \( A \). We recall here that (17) shows that there is a role for network structure in shaping prices and possibly price heterogeneity.

First, we show that under a complete network prices are homogeneous, then we proceed along two random network structures: the symmetric structure of [20], and the asymmetric or scale-free structure as in [9]. The intuition behind using these specific structures is to have a baseline case with the complete network and then to follow the logic of e.g. [2], who show that non-trivial aggregate behavior emerges from asymmetric network structure at the microeconomic level.

#### 4.1 A baseline case – the complete network and homogeneous prices

Assuming a complete interaction network, we have \( b_{ij} = 1/M \) for all \( i \) and \( j \). Substituting this into (17) we have

\[
p_i = \frac{M}{2\varepsilon N(M - 1)} \sum_j \gamma_j + \frac{1}{2(M - 1)} \sum_{k \neq i} p_k + \frac{\omega}{2\alpha}, \quad \forall i
\] (18)

If we assume homogeneous consumer budgets \( \gamma_j = 1 \) for all \( j \), then (18) can be simplified to:

\[
p_i = \frac{M}{2\varepsilon (M - 1)} + \frac{1}{2(M - 1)} \sum_{k \neq i} p_k + \frac{\omega}{2\alpha}, \quad \forall i
\] (19)

This equation resembles (17), but the network-related parts become simplified. It is evident from (19) how network size \( M \) affects the three main drivers of price setting. While the role of marginal cost is not affected by network structure, the first and second terms on the right-hand side are (the monopoly power arising from imperfect substitutability of product variants and the role of competitor prices). As the number of producers \( M \) grows from 2 to infinity, the first term decreases from \( 1/\varepsilon \) to \( 1/(2\varepsilon) \) showing a lower markup under more intensive competition. On the other hand, the effect of competitor prices vanishes as the number of producers grows.
Going one step further, under a complete network the diagonal elements of matrix $Q$ are $2N(M - 1)/M^2$, while the off-diagonal elements are $-n/M^2$. As the elements of $B$ and the diagonal elements of $C$ are identical, it can be easily shown that the price vector as defined in (11) contains identical elements as well, so prices are homogeneous. Using identical prices $\hat{p}$ in (19) we get

$$\hat{p} = \frac{M}{\epsilon(M - 1)} + \frac{\omega}{\alpha}$$

This version clearly shows that price setting follows markup pricing over marginal costs in this model. In addition to the substitution parameter $\epsilon$, the number of producers also shape the markup. As $M$ grows, the markup decreases.

The discussion above shows that as a special case, the complete network brings back standard pricing behavior, while the general price Eq. (17) reveals that incomplete network structure modifies optimal prices both through an additional effect on market power (perceived substitutability) and through the way competitors’ prices are taken into account. We explore the properties of price heterogeneity under incomplete network structures in the following subsection. The reasoning above is summarized in the following proposition.

**Theorem 4** Given the demand functions (1), the production functions (4) and the profit functions (6), the complete market interaction network, i.e. $a_{ij} = 1$ for all $i$ and $j$, results in homogeneous prices where $p_i = p_k = \hat{p}$ for all $i$ and $k$.

There are two clear economic intuitions behind homogeneous prices under a complete market interaction network. First, we have symmetric and complete information-distribution among producers in this case. Not only are they completely informed about the prices of other consumers (this is required as well if the network is incomplete), but the structure of the network forces them to consider all prices. Consumers are linked to all producers, so all producers take this information into account and calculate the prices of others as direct competitors. Second, all actors are structurally equivalent (have identical positions) in this network structure which also points to identical behavior, hence homogeneous prices. As a result, this special case resembles standard monopolistic competition with a finite set of producers/varieties where this complete information and identical price-setting is assumed rather than derived.

### 4.2 Price heterogeneity in symmetric network structures

In this section, we move away from the extreme case of complete network structure. In order to do this, we employ the random network model of [20] to fill up the adjacency matrix $A$. This model assumes an identical and independent probability $r$ of an edge appearing between any two nodes $i$ and $j$. The resulting network is symmetric in the sense that nodes have very similar positions (degrees) in the network with some variability. Moreover, the probability parameter $r$ allows to drive the density
of the network between the empty \((r = 0)\) and complete \((r = 1)\) networks as special cases.

We use simulations to generate several different realizations of the market interaction network and then compute the first three moments of the resulting equilibrium price vectors: average price, the relative standard deviation of prices and the skewness. While an average price allows to contrast the average markup in the incomplete networks to the complete network case, relative standard deviation shows the extent to which prices are dispersed. Finally, skewness allows to test whether the price distribution shows heavy tails to either sides, i.e. to reveal asymmetry in the pricing behavior, resulting from the underlying network structure of market interactions.

We set the model parameters for the simulation as follows. Without being restrictive, we set the four model parameters \(\alpha, \gamma, \epsilon\) and \(\omega\) identically to one (as it is visible from (18), these are just scale prices). Under this parametrization, \(\hat{p} = 1 + M/(M - 1)\) in the complete reference network. It follows that if \(M \to \infty\) then \(\hat{p} \to 2\) from above, so we expect average prices not to be lower than 2. Also, if \(M > 51\), the difference between the limit price 2 and the actual price becomes less than 1\%. Following from this observation, we limit the number of producers in the simulations at \(M = 100\) as we do not expect significant changes in the resulting prices above this threshold (this assumption is reinforced by the results in Fig. 1). Also, it follows from (20) that the number of consumers does not have a considerable effect on prices, so we set the number of consumers to be equal to the number of producers in the simulations: \(N = 2M\).

The Erdős–Rényi random graph model has two parameters: the size of the network and the probability of link formation. The size of the network in our case is given by the number of producers and consumers \((M)\). The probability parameter is the expected value of network density. In this respect, we have to obey condition (15) under which the equilibrium price vector exists. If all consumers have at least two product varieties to choose from \((d_j \geq 2\) for all \(j)\) and all producers have at least one consumer, then this condition holds as well. Given the uniform probability of link formation in the Erdős–Rényi network, we expect \(d_j \geq 2\) to hold if the link formation probability \(r\) is at least \(2/M\). In other terms, if the link formation probability is below this threshold, we expect many generated networks to remain infeasible, so results are less reliable in this range.

Given that the model parameters are fixed at unity, we simulate the optimal price vectors for different combinations of \(M\) and \(r\). For all combinations, 1000 independent interaction networks and the resulting price vectors are calculated. In what follows, we present the averages over these independent simulations in order to estimate the expected values of the three moments under consideration.

Figure 1 contains the results of these simulations in a comprehensive format. The three panels reflect the three moments of the optimal price vector (from top to bottom: mean, relative standard deviation and skewness) under different network structures. The corresponding value of the moments is represented by coloring: blue shades refer to lower, while red shades show higher values. The horizontal axis of all panels show the probability parameter \((r)\) of the network generating algorithm, which practically coincides with the (expected) density of the resulting networks.
The vertical axes measure network size, i.e. the number of producers/consumers. Axes are shown in logarithmic scale for the sake of easy reading.

First, we should take a look at average prices (panel A of Fig. 1). As already noted, price level 2 serves as a lower limit for average prices. Also, the right edge of the panels serve as the homogeneous reference point: in this case network density is 100%, the network is complete. Prices are homogeneous in this case, but depend on the number of producers on the market \((M)\). If the network density decreases, i.e., the network becomes incomplete, the average price increases. Moreover, at lower network densities, prices tend to be higher even for larger networks. Overall, we observe a higher average price if the network becomes sparser or there are less producers. Both tendencies point towards more market power on the side of the producers which naturally rises markups, resulting in higher prices. It is important to note that significant deviations from the lower limit price 2 (which arises under many producers and a complete interaction network) are constrained to extreme network structures where density and size are very low.
Then, panel B of Fig. 1 indicates how the variability of individual prices (as measured by the relative standard deviation of the price distribution) is shaped by network structure. The picture is quite similar to average prices in panel A: there is no significant price dispersion for most of the network structures analyzed here, but visible heterogeneity arises in sparse and small networks.

Finally, panel C of Fig. 1 reflects the extent to which the price distribution under optimal price setting is asymmetric. Again, a more skewed price distribution is found for low densities, but also for large networks. This skew is positive, which means that the distribution has a heavier right tail: relatively more producers set low prices while relatively few producers set higher prices. This result is somewhat contradictory as in this type of network nodes are very similar with respect to their positions, so one expects the prices to be symmetric as well, even if they are heterogeneous. To resolve this contradiction recall that the relatively lower skewness at the bottom-left part of panel C in Fig. 1 can be due to a measurement bias as the distribution from which skewness is calculated has a very small size. Taking this into account, the reliable range of the picture shows higher skewness for lower densities which is in line with expectations. The denser a network is, the more local differences in degrees (positions) are eliminated by indirect connections. So the less dense a network becomes, the stronger local differences become in the sense that indirect connections are not fully able to eliminate them.

To sum up our results with the symmetric Erdős–Rényi type network structures, we found that a significant departure from the homogeneous setup (complete network) arises if the market interaction network is strongly selective (density is low, \( r < 0.2 \)), and/or the number of competitors is relatively low (\( M < 20 \)). At lower densities (strong incompleteness) of the market interaction network, we observe higher average prices due to the stronger market power and higher price dispersion. On the other hand, the price distribution tends to show a heavy right tail (positive skew) for low densities but larger network sizes.

### 4.3 Price heterogeneity in asymmetric network structures

The Erdős–Rényi type random networks were used in the previous section to generate the market interaction structure. This network has a strong symmetry with a Poisson degree distribution where large deviations from a typical connectedness (degree) is very unlikely. As a result, nodes fulfill the same role in the network and we expect them to behave similarly. This was reinforced by the findings so far which indicated that significant price dispersion arises under a small network and/or a sparse one.

In this section, we change the underlying structure of the market interaction network and check whether inherent asymmetry in this network has an effect on the distribution of optimal prices. In order to do this, we employ a specific network generating algorithm proposed by [26] and [15]. The algorithm takes the size of the network (\( N, M \)) and its density (\( \delta \)) as given, alongside with two scale parameters \( k^1 \) and \( k^2 \). The latter are used to set the extent of asymmetry in the degree distribution.
of producers \((k^1)\) and consumers \((k^2)\) separately. All indices \(z\) in the rows and columns of matrix \(A\) get a weight according to \(w_z = z^{-k_l} / \sum_z z^{-k_l}\), where \(l \in \{1, 2\}\). Then the cell in row \(i\) and column \(j\) is weighted as \(\hat{w}_{ij} = w_i^1 w_j^2\). As \(\sum_{ij} \hat{w}_{ij} = 1\) holds, we can consider these weights as probabilities and then all cells in \(A\) are set to one with the respective probability \(\hat{w}_{ij}\). More details on the algorithm can be found in Appendix D.

If \(k^l = 0 (l \in \{1, 2\})\), the respective dimension of the resulting adjacency matrix has a Poisson degree distribution resembling the Erdős–Rényi graph while if \(k^l = 1\) the degree distribution follows a power law, resembling a scale-free network structure with strong asymmetry in network positions (see the illustrative histograms in Fig. 3). To sum up, this algorithm for setting the network structure in \(A\) allows us to see how positional heterogeneity in the market interaction network shapes price dispersion. By varying \(k^1\) and \(k^2\) we can simulate networks where some of the producers, consumers or both have a central position in the network with many connections while others are more peripheral with only a few connections.

In what follows, we present the results of a similar simulation exercise as in the previous section. We set \(\alpha = \gamma = \epsilon = \omega = 1\) and \(M = 100\). The choice of network size (number of producers) is driven by the previous observation that at this size there is no significant deviation from the homogeneous case neither in average prices nor in price dispersion. The scale parameters \(k^l\) are then used to check whether asymmetry in the underlying network structure can lead to such deviations. We let \(k^1\) and \(k^2\) go from 0 to 1, while network density is also set at a given level. In the simulations we use two densities: a relatively sparse network at \(\delta = 0.2\) and a relatively dense one at \(\delta = 0.6\). It is important to note that the more dense the network becomes, the less room remains for the asymmetry to unfold, so the specific features resulting from this asymmetry become less relevant.

In Fig. 2 we display the results in a similar fashion as for the Erdős–Rényi networks. However, instead of plotting density and network size on the axes, now the two scale parameters \(k^1\) and \(k^2\) are used. Network size is fixed at \(M = 100\), while the figure represents two specific densities: 20% on the left-hand side and 60% on the right-hand side. The panels from the top to the bottom still show the three moments. The bottom-left corners of all panels correspond to \(k^1 = k^2 = 0\), which is the symmetric case analyzed in the previous section.

The pattern in average prices (panels A and D of Fig. 2) is similar for both densities. While the degree asymmetry of producers \((k^1)\) positively affects average price, the degree asymmetry of consumers \((k^2)\) slightly decreases average price – however, the former effect is much stronger. This means that market concentration, when some producers have many consumers while most of the producers only have a few, increases markups as a result of increasing market power. At the same time, a similar concentration on the consumer side lowers this market power and lowers prices. On the other hand, the effect of network asymmetry on average price remains quite limited even at a large \(k^1\): in the case of strong asymmetry in the network structure the average price rises above the limiting value of 2 by roughly 0.5% (when the density is 20%). This can be contrasted to the results in Fig. 1, where we observed more a roughly two-fold difference between average prices between high and low network.
densities. This shows that sparsity and network size are much stronger determinants of the average price than network asymmetry.

With respect to price dispersion, the picture is similar to average prices at low density (panel B of Fig. 2). Strong asymmetry on the side of producers increases price heterogeneity, but the extent of this deviation from the homogeneous case is only 1.2% at most. This shows again that an asymmetric network structure at this network size does not have a huge effect on price dispersion. Moreover, at a higher density (panel E of Fig. 2) the maximum dispersion is even lower.

An interesting picture emerges for skewness. In the relatively sparse network (panel C of Fig. 2) the pattern follows that of an average price and price dispersion. This is quite straightforward as a higher $k^1$ means stronger positive skew in producer degrees which is projected then into prices. If the network density is higher (panel F of Fig. 2), we see an overall negative skew. This is in line with the finding in panel C of Fig. 1, which shows a slightly negative skew in this range. However, increasing
producer asymmetry ($k^1$) typically pushes skewness towards the positive range here as well as in panel C. On the other hand, the role of consumer degree distribution becomes stronger in this case. For symmetric producer degrees (around $k^1 = 0$), more asymmetry on the consumer side results in stronger negative skew with a few producers setting lower prices while the majority is setting higher ones. For asymmetric producer degrees (around $k^1 = 1$) increasing consumer asymmetry slightly increases the skewness of prices.

5 Summary

In this paper we examined how the structure of market interaction networks shape price-setting behavior, most importantly price dispersion. Through this attempt, we contribute to the literature on incomplete information and price heterogeneity on competitive markets. We introduced explicit supplier-buyer interaction networks into a standard monopolistic competition model with finite producers/varieties and examined (i) whether it is possible for producers to optimally set their prices under incomplete connectedness, i.e. incomplete information and (ii) how specific network structures (local interactions) shape the equilibrium price distribution, especially price dispersion. The latter question was analyzed in three setups by considering the complete network, a symmetric network structure of the Erdős–Rényi graph and an asymmetric network structure exhibiting scale-free properties.

Our results prove that producers are able to simultaneously set optimal prices in this setup under realistic conditions. It is only required that all producers are connected to at least one such consumer which has more than one variety to choose from. On the other hand, it is not required that the market interaction network is connected, so segmented markets can be described by this model as well.

We found that under a complete interaction network, producers set identical prices which is in line with the standard monopolistic competition model. However, even a slight departure from the complete network brings heterogeneity to the prices.

We used simulations to examine whether price heterogeneity under incomplete market interaction networks is economically significant or not. The results in this respect show that it is primarily the sparsity of the network structure which significantly increases price dispersion and the low number of producers. An interesting finding is that sparsity and network size dominate heterogeneity in the sense that it is not really relevant whether the interaction network is symmetric or asymmetric. Even for asymmetric network structures, where the degree distributions of producers or consumers follow a power-law, price dispersion vanishes for dense networks.

There are several logical routes along which this analysis can be extended in the future. First, we excluded a strategic price setting in the sense that producers take competitors’ prices as given. It would be interesting to see the effects of the network structure on pricing if producers take into account how their pricing decision changes that of others. Second, network structure was assumed exogenous in
this paper. However, whether buying from a given producer or not may change over time and as a response to prices which calls for an endogenous approach in network formation. Third, overcoming the limitations of simulations, an analytic approach could derive general results about the specific network structures.

Appendix A: Proof of positive prices

Proof Let us define matrix $V$ as

$$V = I_M - \kappa Q \iff Q = \frac{1}{\kappa}(I_M - V)$$

(A.1)

where $0 < \kappa \leq 2/(N - M)$ is a constant and $I_M$ is the identity matrix of size $M$. If $\|V\|_\infty < 1$ holds, then we can write $Q^{-1} = \kappa(I_M - V)^{-1} = \kappa\sum_{n=0}^{\infty} V^n$. Moreover, if all elements in $V$ are non-negative then the elements of $Q^{-1}$ are surely non-negative so $p > 0$. More specifically, because $Q^{-1}$ can not have rows full of zeros, we have the strict condition $p > 0$.

In the first step, we examine the absolute value of the diagonal elements of $V$ and the sum of absolute values of off-diagonal elements in the same row. It is true for the absolute value of the diagonal element in row $i$ that

$$\left| \kappa \left( \sum_j b_{ij}^2 + \sum_j b_{ij}(b_{ij} - 1) - \sum_j b_{ij} \right) + 1 \right| \leq 1 - 2\kappa \sum_j b_{ij}(1 - b_{ij}),$$

(A.2)

as $-1/4 \leq b_{ij}(b_{ij} - 1) \leq 0$ holds for all $i$. For the off-diagonal elements of row $i$ we have

$$\kappa \sum_{k \neq i} \sum_j b_{ij}b_{kj} = \kappa \sum_j b_{ij}(1 - b_{ij})$$

(A.3)

From all this we have $\|V\|_\infty < 1$, moreover, choosing an appropriate $\kappa$ ensures that the elements of $V$ are non-negative and at least the elements on the main diagonal are positive. With this we proved that the elements of the optimal price vector are non-negative, moreover, neither of them are zero.

Another way to prove the non-negativity of the elements of $Q^{-1}$ is through the binomial inverse theorem (see e.g. [35]). According to the binomial inverse theorem:

$$(\tilde{B} + C - BB^T)^{-1} = \sum_{n=0}^{\infty} \left( (\tilde{B} + C)^{-1}BB^T \right)^n (\tilde{B} + C)^{-1}$$

(A.4)

Because the inverse of $\tilde{B} + C$ is a diagonal matrix the $i$th diagonal element of which is the reciprocal of the $i$th diagonal element of $\tilde{B} + C$, the infinite sum defined previously consists of matrices with exclusively non-negative elements.
Appendix B: Proof of positive output allocation

Proof Let us start from (1). For the allocation of the output to be economically feasible, we need $x_{ij} \geq 0$ to hold. This means that

$$b_{ij} y_j \geq e b_{ij} (p_i - \overline{p_j}) \quad \text{(B.1)}$$

Using the definition of $\overline{p_j}$ and after some rearrangements we get

$$\frac{b_{ij} y_j}{e} + b_{ij} \sum_{k \neq i} b_{kj} p_k \geq b_{ij} (1 - b_{ij}) p_i \quad \text{(B.2)}$$

Modifying further:

$$\frac{b_{ij} y_j}{2e} + \frac{b_{ij} \sum_{k \neq i} b_{kj} p_k}{2} \geq \frac{b_{ij} (1 - b_{ij}) p_i}{2} \quad \text{(B.3)}$$

Summing up over all consumers:

$$\frac{\sum_j b_{ij} y_j}{2e} + \frac{\sum_j \sum_{k \neq i} b_{ij} b_{kj} p_k}{2} \geq \frac{p_i}{2} \sum_j b_{ij} (1 - b_{ij}) \quad \text{(B.4)}$$

then dividing both sides with $\sum_{j=1}^{N-M} b_{ij} (1 - b_{ij})$:

$$\frac{\sum_j b_{ij} y_j}{2e \sum_j b_{ij} (1 - b_{ij})} + \frac{\sum_j \sum_{k \neq i} b_{ij} b_{kj} p_k}{2 \sum_j b_{ij} (1 - b_{ij})} \geq \frac{p_i}{2} \quad \text{(B.5)}$$

Using the (17) price equation:

$$p_i - \frac{\omega}{2\alpha} \geq \frac{p_i}{2} \quad \text{(B.6)}$$

which means that $p_i \geq \omega / \alpha$.

Appendix C: Proof of positive profits

Proof Let us substitute the optimal price $p_i$ from (17) into the profit function (6). After rearrangement we get

$$\pi_i = y_i \left[ \frac{\sum_j b_{ij} y_j}{2e \sum_j b_{ij} (1 - b_{ij})} + \frac{\sum_j \sum_{k \neq i} b_{ij} b_{kj} p_k}{2 \sum_j b_{ij} (1 - b_{ij})} - \frac{\omega}{2\alpha} \right] \quad \text{(C.1)}$$

We know from the previous discussion that prices are positive, but if prices are positive, then (12) ensures that $y_i > 0$ for all $i$. As a result, it is enough to show that the expression in the parentheses in (C.1) is positive. This requires
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\[
\frac{\sum_j b_{ij} r_j}{\epsilon \sum_j b_{ij} (1 - b_{ij})} + \frac{\sum_j \sum_{k \neq i} b_{ij} b_{kj} p_k}{\sum_j b_{ij} (1 - b_{ij})} > \frac{\omega}{\alpha} \quad (C.2)
\]
to hold. (C.2) is true if

\[
\frac{\sum_j b_{ij} r_j}{\epsilon \sum_j b_{ij} (1 - b_{ij})} + \frac{\omega}{\alpha} \quad (C.3)
\]
is also true, where \( \rho = \min\{p_1, p_2, \ldots, p_M\} \). From (B.6) we know that

\[
\rho > \frac{\rho}{2} + \frac{\omega}{2\alpha} \quad (C.4)
\]
This means that all producers have positive profits, so from Lemma 1 it also follows that the allocation under optimal pricing is economically feasible.

Appendix D: Generating an asymmetric degree distribution

The task is to fill up an \( M \times N \) matrix \( A \) with zeros and ones so that (i) the number (density) of the ones is given and (ii) the degree distribution along both dimensions (rows and columns) can vary between the Poisson distribution of the Erdős–Rényi network and the power law of the scale-free network. In order to accomplish this task, we used the following algorithm, based on [26] and [15].

1. Set the density \( \delta \) and two parameters describing the asymmetry of the degree distributions row- and column-wise, separately. Let us denote the former by \( k^1 \) and the latter by \( k^2 \). If \( k^l = 0 \), then we have a Poisson degree distribution while for \( k^l = 1 \) we have a power law degree distribution along the respective dimensions \( (l \in \{1, 2\}) \). Also, set the dimensions of the matrix \( M \) and \( N \).
2. Determine the number of edges/ones to be distributed in \( A \) as \( \delta MN \), rounded to the nearest integer.
3. Calculate weights for all indices in the rows \( (l = 1) \) and columns \( (l = 2) \) as follows: \( w_i^l = z^{-k^l} / \sum z^{-k^l} \). It is clear that for \( k^l = 0 \) we get identical weights for all rows/columns and for \( k^l = 1 \) we have a power law with exponent \(-1\), where the first indices get higher weights and the consecutive indices get lower weights according to the power law.
4. Calculate weights for all entries in matrix \( A \) as \( \hat{w}_{ij} = w_i^1 w_j^2 \). It is true that \( \sum_{i,j} \hat{w}_{ij} = 1 \).
5. Consider the cells of matrix \( A \) as a population and draw a random sample out of this population according to the respective weights \( \hat{w}_{ij} \).
6. Set the cells of \( A \) corresponding to the drawn sample to one and the rest of the cells to zero.
After the procedure above, one may check the feasibility of the generated adjacency matrix so that it fits the condition (15). Figure 3 shows that the algorithm nicely resembles reference distributions in extreme cases: the Poisson distribution for \( k_1 = 0 \) and the power law for \( k_1 = 1 \).

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**Declarations**

**Conflicts of interest** The authors declare that they have no conflict of interest.

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