Research on Constant Current Control of Regenerative Braking in Hybrid Energy Storage Electric Bicycle

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Abstract. Electric vehicle has gradually become a hot research topic due to the energy crisis and environmental pollution problems. But the traffic is becoming much more crowded because of the sharp increase in the number of private cars, for this reason, the electric bicycle has been more and more popular owing to the convenience of it, especially the electric bicycle is easier than vehicle to park, but the mileage short problem of electric bicycle is always the main factor to influence the development of electric bicycle, moreover, electric bicycles are chiefly used under urban conditions which have frequent start and stop process, so single energy source electric bicycle can not be very good to adapt to these driving conditions. So far, there is little research about the application of the super capacitor and regenerative braking system of the electric bicycle. In this paper, regenerative braking system of hybrid energy storage electric bicycle is the research object, the mathematical model of regenerative braking system of hybrid energy storage electric bicycle has been established, regenerative braking system of hybrid energy storage consists of the brushless dc motor, Buck - Boost DC-DC converter, super capacitor and controller. In order to validate the regenerative braking system model, the regenerative braking simulation test system has been designed and developed, the equivalent small power brushless dc wheel motor drive system is used to simulate the drive system of electric bicycle. The mathematical model of regenerative braking system is calculated and experimental verified, the results show that the established mathematical model is accurate and effective. In order to maintain small braking torque fluctuation range during the braking process, constant current control strategy has been employed to control the armature current. As simulation result and experiment data shows, the hybrid energy storage system can effectively absorb the regenerative braking energy. Stable braking torque can be implemented and higher energy recovery efficiency has been realized in the process by adopting the constant current control strategy.

1. Introduction
With the increasingly serious problems of pollution and energy crisis, the development of new energy vehicles is valued\cite{1}, including electric vehicles, electric bicycles, etc. In recent years, the global electric vehicle market has been growing at a faster pace, as a result that the production and sales of electric vehicles are significantly improved\cite{2}. In China, the traffic problems of most working class have been solved because of the emergence of electric bicycle, what’s more, the electric bicycle is featured by economical and convenient, and the most important is that electric bicycles are in
consumption capacity of most people, so electric bicycles can increasingly catch more people's attention. The driving conditions of electric bicycle are mainly urban roads, according to the analysis and research of vehicle driving cycle, the average speed of vehicle is relatively slow which is basically less than 20km/h due to traffic congestion[3], under this condition, braking energy recovery has great potential, according to the experimental results of some typical driving conditions, such as, under the driving conditions in China, braking energy recovery efficiency is up to 25.7%, under the European ECE conditions, braking energy recovery efficiency is up to 22.9%, under Japan 10.15 driving condition, braking energy recovery efficiency is up to 19.0%. Based on these data, the regenerative braking of electric bicycle is with great potential.

There are still technical defect in the power battery which limit the driving range of electric vehicles to a great extent, causing the promotion of electric vehicles blocked[4]. In order to overcome these problems, the electric bicycle is equipped with a hybrid energy storage regenerative braking system which is composed of a battery and super capacitors. As an auxiliary energy source, the super capacitor is featured by low resistance, high work efficiency, long cycle life and wide operating temperature range, what is the most important is that the super capacitors are suitable for frequent high current charge and discharge field[5]. So the energy of the regenerative braking system of the hybrid energy storage regenerative braking system is better adapted to the driving of the electric bicycle in the city condition. The regenerative braking system can recycle some of the energy in the braking process to extend the driving range[6]. There are many different structures of regenerative braking system of hybrid energy storage, most structures can be divided into three systems: the super capacitor and the battery are connected directly to the motor controller, the super capacitor is connected with the motor controller through the DC/DC converter, the super capacitor and the battery are connected with the motor controller through the DC/DC converter. In this paper, the second one is considered. At the time of braking, the recovered energy is stored in the DC/DC converter by the motor and through the DC/DC converter.

There are two ways to realize the electric vehicle electro-mechanical braking, one is in series, one is in parallel[7]. The parallel braking is that the motor regenerative braking is superimposed on the conventional hydraulic brake or pneumatic braking system, what should be only done is to control the motor braking torque, there few hybrid form of control parameters, the control accuracy requirement is low and very easy to realize. Series brake can be divided into two kinds, one is the combination of regenerative braking technology and brake by wire technology, the other is to transform the traditional hydraulic brake, the pressure source is provided by the brake master cylinder or the energy storage device, to achieve the real-time control of hydraulic brake by adjusting the various types of valves on the active line. Series control is more complex, the accuracy of the requirements are relatively high[8]. Because the speed of the electric vehicle is relatively high, the requirement for stability is relatively high when braking. As a result, the electric vehicle needs to use some high control precision and good stability of the regenerative braking control method. Compared with electric vehicle, the speed of electric bicycle is relatively low and the front and back wheels of the electric bicycle are controlled by two brakes, so the brake requirements are not as high as the electric vehicle, only the easy regenerative braking control technology can achieve good braking energy recovery efficiency and good braking stability in electric bicycle.

After using the regenerative braking technology, under the circumstance of meeting the braking performance and braking stability, the purpose of this paper is to maximize the energy recovery efficiency of regenerative braking system. So the studies on the regenerative braking control strategy should be reasonable and efficient. A.M.Walker analysis the requirement of regenerative braking and mechanical braking, the mechanical brake is reduced to obtain a higher energy recovery efficiency by improving the energy storage system, but on the motor braking torque and mechanical braking distribution of the optimal problem is only a qualitative analysis because there is no detailed theoretical analysis[9]. LIU Zhiqiang designs a boost chopper circuit to recover energy for battery energy storage equipment[10]. There are three regenerative braking control strategy: the maximum regenerative energy, the maximum regenerative efficiency, and constant braking torque based on
storage battery. The maximum regenerative energy and the maximum regenerative efficiency control modes ignore battery charge current limit and energy absorption efficiency. At the same time the maximum efficiency control mode requires to detect real-time resistance moment which is difficult in practice. The control object of constant braking torque control mode is the braking current, assuming that the motor speed and the rolling resistance torque remain constant in its mathematical model. But the actual motor power voltage and rolling resistance bureau are changing, and its initial value are no more than allow current of battery.

In the first of this paper, the force analysis is carried out on the regenerative braking of the electric bicycle, then the circuit diagram of regenerative braking is designed, finally, The regenerative braking control strategy is used for the experiment of regenerative braking system and the analysis of the experimental data. In this paper, considering the energy efficiency of a storage device and the control strategy, choosing the ultra-capacitor to recycle braking energy, ultra-capacitor can be used to high power, high charging and discharging current. This paper uses the ultra-capacitor as the auxiliary power to overcome the disadvantage of battery for the characteristics of high energy absorption efficiency of ultra-capacitor. This paper chooses the braking current as the control object and makes the braking current changes with the instructions in the braking process, making the electric braking torque and pedal opening the approximate linear ratio with the method of feedback control.

2. Force analysis of the front and rear wheel of electric bicycle in the braking process

![Fig.1 The force of the wheel during braking](image)

Fig.1 draw the force condition of the front and back wheels of the electric bicycle during braking time on a good road, in the braking process, rolling resistance moment and deceleration of the inertial force and inertial moment are ignored.

From Fig.1, when the front wheel is braking, the front wheel load is as follows: \( W_1 \) \((N)\) is the front wheel vertical load, \( F_3 \) \((N)\) is the normal reaction force of ground to wheel, in terms of size, \( W_2 \) is equal to \( F_3 \), but they're in the opposite direction.

\[
W_2 = F_3 = \left( \frac{G}{L} \right) \left( b - \varphi h \right)
\]

(1)

Where \( G \) \((N)\) is the gravity of electric bicycle, \( L \) \((m)\) is the distance from the center of the front and back of the electric bicycle, \( b \) \((m)\) is the horizontal distance from the center of mass of the
electric bicycle to the center of the rear wheel, \( \varphi \) is the ground adhesion coefficient, \( h_g (m) \) is the vertical height of the center of mass of the electric bicycle from the ground.

\[ T_3 (N \cdot m) \] is the friction torque of the friction plate in the rear wheel brake and the relative slip of the brake drum or the brake disc, \( F_6 (N) \) is the thrust of Axle to wheel, \( F_5 (N) \) is the the is the ground braking force.

\[ F_5 = T_2 / r \]  

Where \( T_2 (N \cdot m) \) is the friction torque of the friction plate in the front wheel brake and the relative slip of the brake drum or the brake disc, \( r \ (m) \) is the wheel radius.

From Fig.1, when the rear wheel is braking, the rear wheel load is as follows: \( T_1 (N \cdot m) \) is the friction torque of the friction plate in the rear wheel brake and the relative slip of the brake drum or the brake disc, \( M (N \cdot m) \) is the motor braking torque, \( F_3 (N) \) is the thrust of Axle to wheel, \( F_2 (N) \) is the ground braking force which is made up of mechanical braking force and electric force.

\[ F_2 = T_1 / r + M / r \]  

Where \( T_1 (N \cdot m) \) is the friction torque of the friction plate in the rear wheel brake and the relative slip of the brake drum or the brake disc, \( M (N \cdot m) \) is the motor braking torque, \( r \ (m) \) is the wheel radius.

\( W_1 (N) \) is the wheel vertical load, \( F_1 (N) \) is the normal reaction force of ground to wheel. In terms of size, \( W1 \) is equal to \( F_1 \), but they're in the opposite direction.

\[ W_1 = F_1 = (G / L) * (a - \varphi h_g) \]  

Where \( G (N) \) is the gravity of electric bicycle, \( L (m) \) is the distance from the center of the front and back of the electric bicycle, \( a (m) \) is the horizontal distance from the center of mass of the electric bicycle to the center of the front wheel, \( \varphi \) is the ground adhesion coefficient, \( h_g (m) \) is the vertical height of the center of mass of the electric bicycle from the ground.

From the above we can know, vehicle braking force is made up of two parts, one is mechanical braking force, the other is electric force. In order to allow the electric bicycle to recycle as much energy as possible, so the resistance of the motor to the wheel is less than the ground adhesive force, that is to say:

\[ M / r <= F_1 * \varphi \]  

If the electric force is greater than the ground adhesion, the rear wheel locking phenomenon will appear, this will not receive the energy.

2.1 The structure of the system circuit

Regenerative braking system consists of a brushless DC motor, a variable structure Bi Buck-Boost converter, the super capacitor group and a controller. The composition principle of Regenerative braking system is shown in figure 1, where, \( V_b \) : Storage battery; SC: super capacitor; A, B, C: three-phase winding brushless DC motor; \( L_0 \) and \( R_0 \) are the motor's winding resistance and self-induction respectively; \( K_1 \) : two-way control relay; K: the main relay; \( L_1 \) : energy storage inductor (DC-DC inductance); \( q_1, q_2 \), \( Q_1 - Q_6 \) : field effect tube; The \( K_1, K, \) Bi Buck - Boost Buck circuit, \( Q_1 - Q_6 \) of motor drive controller, \( e_x, e_y, e_c \) for motor winding induction electromotive force.
There are two main types of regenerative braking state, the first one is the booster state. In this state, the motor switches to the generator state, main relay switch $K$ switches to the positive pole of the super capacitor, two-way control relay switch $K_1$ switches to static contact 1, PWM wave makes field effect tube $q_2$ chopper, the motor can booster charge the ultra-capacitor through the energy storage and release of the energy storage inductance. The second one is the buck state. In this state, the motor switches to the generator state, main relay switch $K$ switches to the positive pole of the super capacitor, two-way control relay switch $K_1$ to static contact 2, PWM wave makes field effect tube $q_1$ chopper, the motor can buck charge the ultra-capacitor through the energy storage and release of the energy storage inductance.

2.2 System boost circuit

The circuit structure of booster regenerative braking process is shown in figure 3:

During high-level of power tube $q_2$, the circuit constitutes a loop through $q_2$, inductor $L_1$ stores energy, which is equal to the motor power charging the inductance $L_1$, the circuit model is:
\[
\frac{d i_{L1}}{dt} + \frac{R'}{L} i_{L1} = \frac{V_{m0}}{L}
\]  
(6)

Supposing \( i_{L1} = i_0 \) at the initial time of the PWM high level of \( q_2 \), the expression of the inductor current when \( q_2 \) is turn-on can be obtained by the equation (6):

\[
i_{L1} = \frac{V_{m0}}{R'} + (i_0 - \frac{V_{m0}}{R'}) e^{-\frac{R'}{L} t}
\]  
(7)

\( i_{L1} \) is the current in the rising stage and the initial value for the next phase current. \( V_{m0} = K_n \eta \) is the induced electromotive force of the motor, \( K_n \) is the coefficient of the induced electromotive force of the motor, \( t \) is the elapsed time of this stage; \( L \) (H) is the inductance of the circuit, \( L = 2L_0 + L_1 \), where \( L_0 \) (H) is the single-phase winding inductance of the motor (H), \( L_1 \) (H) is the inductance of DC-DC; \( R' \) (\( \Omega \)) is the resistance of the circuit, \( R' = 2R_0 + R_L + R_f \), where \( R_0 \) (\( \Omega \)) is the single-phase winding resistance of the motor, \( R_L \) (\( \Omega \)) is the resistance DC-DC inductance, \( R_f \) (\( \Omega \)) is the resistance of the switch tube.

During the period of the PWM low level of \( q_2 \), \( q_2 \) is shut off. Since the inductor current cannot mutate, the current flows though the reverse parallel diode of \( q_1 \) to the ultra-capacitor. At this time, the motor and the inductance charges the ultra-capacitor mutually. Circuit model is:

\[
\frac{d^2 V_c}{dt^2} + \frac{R^*}{L} \frac{d V_c}{dt} + \frac{V_c}{LC} = \frac{V_{m1}}{LC}
\]  
(8)

Supposing the super capacitor voltage is \( V_{c0} \) at the initial time of the PWM low level of \( q_2 \), the inductor current is the final value of the last stage. The initial current value is \( i_L = C \frac{dV_c}{dt} = i_{L1} \), \( V_c \) (V) is the terminal voltage of the ultra-capacitor, \( V_{m1} \) is the motor electromotive force, \( R^* \) (\( \Omega \)) is the circuit resistance, \( R^* = 2R_0 + R_L + R_c \), where \( R_c \) is the ultra-capacitor resistance, \( C \) (F) is the capacitance of the ultra-capacitor; the expression of the inductor current and the capacitor voltage can be gained from formula (8):

\[
i_{L2} = C(A_1 p_1 e^{p_1 t} + A_2 p_2 e^{p_2 t})
\]  
(9)

\[
V_c = A_1 e^{p_1 t} + A_2 e^{p_2 t} + V_{m1}
\]  
(10)

\( A_1, A_2 \) is the general solution coefficients of the homogeneous equation; \( p_1, p_2 \) is the root of the characteristic equation.

2.3 System buck circuit

The circuit structure of buck regenerative breaking is shown in figure 4:
The buck process can also be divided into two stages, chopping \( q_1 \) can achieve the buck process. Assuming that the generated voltage of motor is constant during this period, take the conduction duty ratio of power tube \( q_1 \) as \( D \), gaining the following circuit mathematical model. (PWM signal period \( T \) is far less than the system time constant).

The first stage of buck regenerative braking is the rising stage of the inductance current, in this stage, \( q_1 \) turns on, which is controlling the PWM wave of \( q_1 \) on high level. This process is the same as the second phase of the booster braking process. Supposing the ultra-capacitor voltage is \( V_{c0} \) at the initial time of the PWM high level of \( q_1 \), the initial value of the inductor current is \( i_L = C \frac{dV_c}{dt} = i_0 \), the expression of the inductor current and the capacitor voltage can be gained:

\[
i_{L1} = C(B_1 p_1 e^{p_1 t} + B_2 p_2 e^{p_2 t})
\]

\[
V_{c1} = B_1 e^{p_1 t} + B_2 e^{p_2 t} + V_{m0}
\]

(11) \( B_2 \) is the general solution coefficients of the homogeneous equation.

The second stage of buck regenerative braking is the decline stage of the inductance current, \( q_1 \) shutting down, which is controlling the PWM wave of \( q_1 \) on low level. The circuit model is as below:

\[
\frac{d^2V_c}{dt^2} + \frac{R''}{L} \frac{dV_c}{dt} + \frac{V_c}{LC} = 0
\]

(13)

Where \( R'' (\Omega) \) is the circuit resistance, \( R'' = R_e + R_L \), \( L (H) \) is the circuit inductance, \( L = L_1 \). Supposing the ultra-capacitor voltage is \( V_{c1} \) at the initial time of the PWM low level of \( q_1 \), the initial value of the inductor current is \( i_L = C \frac{dV_c}{dt} = i_{L1} \), the expression of the inductor current and the capacitor voltage of this process can be gained from formula (13):

\[
i_{L2} = C(B_3 p_3 e^{p_3 t} + B_4 p_4 e^{p_4 t})
\]

\[
V_{c2} = B_3 p_3 e^{p_3 t} + B_4 p_4 e^{p_4 t}
\]

(14) \( B_3, B_4 \) is the general solution coefficients of the homogeneous equation; \( p_3, p_4 \) is root of the characteristic equation.

3. Constant braking torque control strategy

The braking torque control is the same as the braking current control. In this paper, the constant
torque refers to the fixed constant braking current under the fixed pedal opening extent. The advantage of this control strategy is that the strategy is simple and its braking feeling is approximate to the mechanical brake. According to the rear wheel brake instructions and the super capacitor state of the electric bicycle and the operation information of the motor, the regenerative braking controller can control the motor to realize the regenerative braking.

The constant torque control is the same as the constant current control. The control object is the difference between the given current and the actual current. By the motor torque closed loop PI control, this strategy outputs the duty ratio D of the DC-DC converter. According to the driver braking torque, this strategy keeps the braking current changing with the instruction current by controlling the braking current, recovering the braking energy of the motor.

In order to ensure the normal operation of the system, there must be a limit constraint in this control strategy:

$$T_{p_{\text{max}}} = \min(T_{m_{\text{max}}} \frac{P_{m_{\text{max}}}}{\eta_m} n)$$

where $T_{p_{\text{max}}}$ is the current maximum allowable torque (Nm); $T_{m_{\text{max}}}$ is the maximum allowable braking torque of the motor (Nm); $P_{m_{\text{max}}}$ is the current maximum allowed power (W); $\eta_m$ is the generating efficiency of the motor (%); $n$ is the motor speed (rpm). At the same time the following formula can be obtained:

$$P_{c_{\text{max}}} = \min(P_{m_{\text{max}}}, I_{c_{\text{max}}}, V_c)$$

Where $P_{c_{\text{max}}}$ is the maximum allowable charging power of the ultra-capacitor (W); $I_{c_{\text{max}}}$ is the maximum allowable current of the ultra-capacitor (A); $V_c$ is the current terminal voltage of the ultra-capacitor (V). The two limiting conditions of the motor is constant, given by the motor performance parameter; The two limiting conditions and the current terminal voltage of the ultra-capacitor are variable, taken from their transient values. The motor speed is taken from its running transient.

4.Equivalent test bench of regenerative braking system

In order to verify the model of the regenerative braking system, the equivalent test bench of regenerative braking system is established, its structure is shown in Fig.5:
The load of the test bench is in coaxial symmetric arrangement with two identical flywheels. The flywheel is fixed on both ends of the flywheel shaft by means of a flat key connection, the flywheel is axially compressed with a bolt through the sleeve; The flywheel shaft is parallel to the ground and is connected with the base of the test bench through two sets of bearings to ensure the stability of the rotation of the flywheel. The friction wheel is connected with a flat key in the middle of the flywheel shaft, wheel motor for testing is connected with the friction wheel through the wheel to ensure that the wheels and the friction wheels are pressed, the change of ground adhesion coefficient can be simulated by adjusting the force of compaction when no slip in motion.

The flywheel shaft, the friction wheel and the flywheel and other rotating parts together constitute the flywheel mechanism which is used to model the kinetic energy of an electric vehicle, taking the actual parameters of electric vehicle as the condition to design the parameters of test bench, the parameters are shown in Table 1:

| Test bench parameters              | Table 1 |
|-----------------------------------|---------|
| Wheel radius $R$ (m)              | $0.21$  |
| Maximum speed $v_{\text{max}}$ (km/h) | $27.7$  |
| Equivalent full load mass $m$ (kg) | $155$   |
| The minimum transmission ratio of the wheel to the friction wheel $i$ | $2$     |

Notes: The maximum speed of the motor is 350r/min, that is, the maximum speed is 27.7 km/h.

From Fig.5, the wheel motor is contacted with the friction wheel through the wheel. When the wheel hub motor is used as the driving motor, the wheel is driven by the friction wheel to drive the flywheel mechanism to rotate, all driven components provide load for the motor to simulate the resistance moment of the normal running of the electric vehicle, when the regenerative braking is regenerated, the wheel hub motor becomes a generator, the flywheel mechanism provides the driving torque for the wheel motor through the friction wheel to simulate the moment of inertia of the electric vehicle in the braking progress. Therefore, according to the principle of conservation of energy, when the line speed of the friction wheel is equal to the speed of the wheel line, the kinetic energy of the rotation of the flywheel mechanism and other rotating parts should be equal to the kinetic energy of the electric vehicle when driving. If take the electric translational energy as the kinetic energy, the above energy relation can be expressed as:

$$
\frac{1}{2}mv^2 = \frac{1}{2}J\omega_f^2
$$

(18)

Where $m$ is the full load quality of electric vehicle, unit is kg; $v$ is the speed of electric vehicle, unit is m/s, $J$ is the moment of inertia of the flywheel mechanism, unit is kg·m$^2$, $\omega_f$ is the angular velocity of flywheel.

$$
v = \omega_R R = \frac{\omega_f}{i} R
$$

(19) Where

$\omega_R$ is the wheel angular velocity, unit is rad/s, $i$ is the transmission ratio of wheel to friction wheel.

According to formula 16 and 17, we can conclude that:

$$
J = \frac{mR^2}{i^2}
$$

(20)

According to the parameters of Table 1, we can conclude that $J = 1.7$ kg·m$^2$.

Taking the rotary inertia $J = 1.7$ kg·m$^2$ of rotary inertia as the target to design test bench can ensure the requirements of the test. Using finished flywheel of diesel engine, finally, to ensure that the total moment of inertia of the flywheel mechanism is 1.7 kg·m$^2$ by adjusting the moment of inertia.
of the flywheel shaft and the friction wheel.

5. Validation of regenerative braking control strategy

The numerical simulation of regenerative braking process is carried out according to the established mathematical model of regenerative braking of electric bicycle, test and validation of regenerative braking process using constructed equivalent simulation test system. In regenerative braking system, the motor rated power is 350W, rated voltage is 48V. The rated voltage of the ultra-capacitor is 60V, rated capacity is $C = 4.7F$, $L = 0.3H$, $K_n = 0.1056$, $R_c = 0.95\, \Omega$, $L_n = 0.01H$, $R_n = 0.1\, \Omega$, $L_t = 0.2H$, $R_L = 0.08\, \Omega$, $R_f = 0.01\, \Omega$.

Fig. 6 shows the hardware circuit and the regenerative braking controller:

Fig. 6 Hardware circuit and the regenerative braking controller

The initial braking speed of the motor in simulation test is 350 n/min, the control current is 10A, the initial voltage of the ultra-capacitor is $V_{c0} = 30V$. The rolling resistance moment was obtained by fitting the measured rotating speed data of the flywheel system. The curve of the simulation and the test is shown in figure 7 a, b, c.
Fig. 7 (a) is the test and simulation motor current under the constant current control strategy, and the two are very similar as shown in Fig. 7 (a). The first half of the currents keep in a steady level of about 10A. With the decrease of the rotating speed, the EMF levels down, leading the decrease of the inductance current. Fig. 7(b) is the test and simulation terminal voltage curve of the ultra-capacitor. There is a little difference between simulation and test curve, but the terminal data is almost the same, under the number of 0.7V. At the beginning, because of the voltage drop of the internal resistance of the ultra-capacitor, the terminal voltage of the ultra-capacitor increases rapidly. By the end of the barking process, the motor is in the stage of plug barking, the ultra-capacitor does not charge any more, voltage keeping steady. Fig. 7(c) is the test and simulation motor rotating speed curve under the constant current control strategy. The changing tendency of the test and simulation curve is similar, as well as the terminal time, about 17s. At the stage of steady inductance current, the rotating speed of the motor decreases linearly, and during the final barking stage, the rotating speed decreases nonlinearly for the failure of motor current keeping steady.

The analysis above have proved the correctness of the model and the accessibility of control strategy in the aspect of theory and experiment.

6. The analysis of the experimental data

In the initial condition of $n=350\text{rpm}$ and $V_c = 30V$, the regenerative braking experiment is done under different control. The experiment data is shown in table 2.

| Control current $curent$ (A) | Duration of Constant current braking (s) | Angular speed of the motor braking in constant current phase (rad/s²) | Rate of energy recovery (%) | Total braking time (s) |
|-----------------------------|----------------------------------------|---------------------------------|---------------------------|------------------------|
| 20                          | 7                                      | 3.47                            | 28.4                      | 13.1                   |
| 15                          | 9.6                                    | 2.88                            | 33.7                      | 13.6                   |
| 10                          | 13                                     | 2.32                            | 40.5                      | 15.7                   |
| 5                           | 20.3                                   | 1.6                             | 36.6                      | 23                     |

From table 2, regenerative braking system in this paper is at least able to recycle 28.4% energy, electric bicycle which is full of electricity in general can ride around 45 km, according to the statistics, China's conservative estimate number of the electric bicycle is about three hundred and seventy million, assuming that all the electric bikes are taken regenerative braking system used in this paper, then a total of the driving range can be extended to about Four billion and eight hundred million kilometers, now a gasoline car is about to cost five wool a kilometer, according to above calculation that will save two billion four hundred million yuan, now the price of gasoline is about 5.7 yuan a liter, then will save more than four hundred million liters of gasoline.

According to the data in the table, we can calculate the size of the electric force and the ground
adhesion. Depending on the speed of the motor, the current and the radius of the wheel, we can calculate the braking torque on the size of the braking force of the wheel during braking process. Assuming $F_r$ as the electric force,

$$F_r = \frac{(9550 \times P)}{r \times n}$$

(21)

Where $n$ is the motor speed, unit is $r/min$, $P$ is the motor power, unit is $kw$, $r$ is the wheel radius, unit is $m$.

Assuming $F_g$ as the ground adhesion,

$$F_g = F_1 \times \phi$$

(22)

Where calculation of $F_1$ refer to formula (4) and $\phi$ is the ground adhesion coefficient.

According to the data in the table and calculation of formula (21) and (22), we draw a conclusion that the electric force is less than the ground adhesion. Next, on this basis, we can draw the following conclusions:

The experiment data from table1 shows that under the same initial conditions, the bigger the braking current is, the bigger the braking torque in condition of boost regenerative brake is. The total braking time under the control current of 20A decreases about 50% than the condition of 5A. The braking angular speed of the motor can reaches to 3.47 rad/s\(^2\) in the range of the maximum allowable braking current. Combining with the Fig.10, the rotating speed of the motor is decreasing in a linear tendency during the stage of steady braking constant braking current, reaching the goal of the close similarity between the regenerative braking feeling and the mechanical braking, as well as a good braking stability.

As the table1 shows, under the same initial conditions, according to the total braking time and electric braking time, the braking of the vehicle is made up of mechanical braking and electric braking, in addition, with the decrease of braking current, the proportion of electric braking time in the total braking time is more and more large. For example, when the braking current is 20A, the constant current braking time is 6.1 seconds while the mechanical braking time is 7 seconds, the proportion of constant current braking time was 41.6%; when the braking current is 5A, the constant current braking time is 7 seconds while the mechanical braking time is 20.3 seconds, the proportion of constant current braking time was 11.7%.

The efficiency of energy recovery is basically in a tendency of the less the current, the higher the recovery rate. This is mainly because the motor generator voltage cannot meet the braking needs because of the big braking instructions, leading to the increase of the PWM duty ratio of DC-DC, increasing conduction losses, decreasing energy recovery rate. The recovery rate is over 40% when the control current is 10A, and also can reach 28% when the control current is at the top of 20A. When the current is reach the extent of lower limit, the energy recovery rate will go down slightly due to the systematic internal resistance and the action of rolling resistance moment. The result of experiment is similar to the simulation.

7. Conclusion
Firstly, this paper has analyzed the force of the front and back wheels of the electric bicycle during braking time and established dynamic modeling of electric bicycle.

Secondly, this paper studies on the ultra-capacitor stored energy regenerative braking system, and divides the braking process into two parts of boost and buck process through the theory analysis on the braking process.

Thirdly, this paper has develop the control circuit of regenerative braking of electric bicycle, including boost control circuit and buck control circuit, at the same time, the circuit is modeled and analyzed.

Fourthly, this paper has established equivalent test bench of regenerative braking system, constant current braking control strategy has been validated by equivalent test bench. Experiment has proved that the control strategy is able to effectively recycle energy.
The result of the experiment shows that the smaller the brake current is, the smaller the proportion of mechanical braking is, the higher the energy recovery efficiency is. That is to say, only the proportion of electric braking is more, the energy recovery efficiency may be higher. The maximum energy recovery efficiency of the ultra-capacitor energy stored regenerative braking system can be up to 40%, as well as a good braking performance.

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