Observation of discrete time–crystalline order in a disordered dipolar many–body system

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Understanding quantum dynamics away from equilibrium is an outstanding challenge in the modern physical sciences. Out-of-equilibrium systems can display a rich variety of phenomena, including self-organized synchronization and dynamical phase transitions1–3. More recently, advances in the controlled manipulation of isolated many-body systems have enabled detailed studies of non-equilibrium phases in strongly interacting quantum matter4–6; for example, the interplay between periodic driving, disorder and strong interactions has been predicted to result in exotic ‘time–crystalline’ phases7, in which a system exhibits incommensurate temporal correlations at integer multiples of the fundamental driving period, breaking the discrete time–translational symmetry of the underlying drive8–12. Here we report the experimental observation of such discrete time–crystalline order in a driven, disordered ensemble of about one million dipolar spin impurities in diamond at room temperature13–15. We observe long-lived temporal correlations, experimentally identify the phase boundary and find that the temporal order is protected by strong interactions. This order is remarkably stable to perturbations, even in the presence of slow thermalization16,17. Our sample has a high concentration (45 p.p.m.) of nitrogen–vacancy centers, giving rise to strong long-range magnetic dipolar interactions17. The spins are also subject to multiple sources of disorder, owing to lattice strain, paramagnetic impurities and the random positioning of nitrogen–vacancy centers. A strong, resonant microwave field is used to control spin orientations, resulting in an effective Hamiltonian (in the rotating frame)17

\[ H(t) = \sum_i \Omega_i(t)S_i^x + \Omega_j(t)S_j^y + \Delta_i S_i^z + \sum_j \left( I_{ij}/r_{ij}^2 \right) \left( S_i^z S_j^z + S_i^y S_j^y - S_i^x S_j^x \right) \]

Here, \( S_i^\mu (\mu \in \{x, y, z\}) \) are Pauli spin-1/2 operators acting on the effective two-level system spanned by the spin states \( \{|m_i = 0\} \) and \( \{|m_i = -1\} \), \( \Omega_{st}(t) \) is the Rabi frequency of the microwave driving, \( \Delta_i \) is a disordered on-site field with approximate standard deviation \( W = 2\pi \times 4.0 \text{ MHz} \), \( r_{ij} \) is the distance between spins \( i \) and \( j \) (average nearest-neighbour separation \( r_0 \approx 8 \text{ nm} \)), and \( I_{ij} \) are the orientation-dependent coefficients of the dipolar interaction. The average interaction, \( I_{ij} r_{ij}^{-2} \approx 2 \pi \times 105 \text{ kHz} \) (ref. 17), is much faster than typical spin coherence times15.

To probe the existence of time–crystalline order, we monitor the spin dynamics of an initial state that is polarized along the \( +\hat{x} \) direction. We begin by applying continuous microwave driving (spin locking) along \( \hat{x} \) with Rabi frequency \( \Omega_x = 2\pi \times 54.6 \text{ MHz} \) for a duration \( \tau_1 \) (Fig. 1a). Next, we rotate the spin ensemble by an angle \( \theta \) around the \( \hat{y} \) axis using a strong microwave pulse with \( \Omega_y = 2\pi \times 41.7 \text{ MHz} \) for duration \( \tau_2 = \theta/\Omega_y \ll \tau_1 \). This two-step sequence defines a Floquet unitary with a total period \( T = \tau_1 + \tau_2 \), and is repeated \( n \) times before the polarization \( P(nT) \) along the \( \hat{x} \) axis is measured. The resulting polarization dynamics are analysed in both the time and frequency (\( \nu \)) domain. Repeating these measurements with various values of \( \tau_1 \) and \( \theta \) allows us to independently explore the effect of interactions and global rotations.

Figure 1b–d depicts representative time traces and the corresponding Fourier spectra, \( S(\nu) \equiv \sum_n P(nT)e^{i2\pi \nu n} \), for various values of \( \tau_1 \) and \( \theta \). For relatively short interaction times (\( \tau_1 = 92 \text{ ns} \)) and nearly perfect \( \pi \) pulses (\( \theta \approx \pi \)), we observe that the spin polarization \( P(nT) \) alternates between positive and negative values, resulting in a subharmonic peak at \( \nu = 1/2 \) (Fig. 1b). In our experiment, the microwave pulses have an intrinsic uncertainty of 0.9% stemming from a combination of spatial inhomogeneity in the microwave fields, on-site potential disorder and the effect of dipolar interactions (see Methods). These effects eventually
cause the oscillations to decay, after approximately 50 periods. Although such temporal oscillations nominally break discrete time-translation symmetry, their physical origin is trivial. To see this, we note that for sufficiently strong microwave driving, \( \Omega_2 \gg W, J_{z\theta}/r_{\text{th}} \), the dynamics during \( \tau_1 \) are governed by an effective polarization-conserving Hamiltonian \( V \), \( H_{\text{eff}} \approx \sum_i \Omega_i S_i^x + \sum_{ij} (I_{ij}/r_{\text{th}}) S_i^z S_j^z \). During \( \tau_2 \), the evolution can be approximated as a global spin rotation \( R^x_2 \approx \exp(-i \theta \sum_i S_i^z) \). When \( \theta = \pi \), this pulse simply flips the sign of the \( \hat{x} \) polarization during each Floquet cycle, resulting in the \( \nu = 1/2 \) peak. However, this 2\( T \)-periodic response originates from the fine tuning of \( \theta \) and should not be robust against perturbations. Indeed, a systematic change in the average rotation angle to \( \theta = 1.034 \pi \) causes the 2\( T \)-periodicity to completely disappear, resulting in a modulated, decaying signal with two incommensurate Fourier peaks at \( \nu = 1/2 \pm (\theta - \pi)/(2\pi) \) (Fig. 1c). Remarkably, we find that a rigid 2\( T \)-periodic response is restored when interactions are enhanced by increasing \( \tau_1 \) to 989 ns, suggesting that the \( \nu = 1/2 \) peak is stabilized by interactions. In this case, we observe a sharp peak in the spectrum at \( \nu = 1/2 \) peak (inset) is extracted from a short-time Fourier transform with a time window of length \( m = 20 \) shifted from the origin by \( n_{\text{sweep}} \).
As shown in Fig. 2c, for $\theta = 1.034\pi$, this lifetime increases with the interaction time ($\tau_i$) and eventually approaches the independently measured spin depolarization time $T_1^p \approx 60 \mu s$. This demonstrates that, for sufficiently long interaction times, the observed periodic order is limited by only coupling to the environment. We associate this with DTC order. Within the DTC phase, the lifetime is essentially independent of $\theta$, indicating exceptional robustness (Fig. 2d).

We examined whether the observed periodic order could arise from an accidental $XY$ sequence or from inhomogeneous dephasing resulting from the effective single-particle disorder in the dressed state basis. To avoid the former, $\tau_1$ is always chosen as an integer multiple of $2\pi/\Omega_y$. For the latter, although it has been shown that disorder alone is insufficient for stabilizing a DTC phase in the absence of interactions, we verified this experimentally; implementing a rotary echo sequence that reduces such dephasing, we find no change in the lifetime of the DTC order and an enhancement in the subharmonic response at late times (see Methods and Extended Data Fig. 1). In principle, fast Markovian dephasing could also lead to apparent periodic order at extremely small values of $\theta - \pi$ by eliminating coherences along both $\hat{y}$ and $\hat{z}$, leaving only $\hat{x}$ polarization dynamics. In such a case, the decay rate of periodic order should increase quadratically with $\theta - \pi$. However, this explanation is inconsistent with the observed robustness of the lifetime of DTC order for a range of $\theta - \pi$ values (Fig. 2d) and the independently measured dephasing rate (see Methods).

To experimentally determine the DTC phase boundary, we focus on the long-time behaviour of the polarization time traces (50 < $n$ < 100) and compute the ‘crystalline fraction’, which is defined as the ratio of the $\nu = 1/2$ peak intensity to the total spectral power, $f = |S(\nu = 1/2)|^2/|\sum S(\nu)|^2$ (see Methods). Figure 3a shows $f$ as a function of $\theta$ for two different interaction times. For weak interactions ($\tau_1 = 92$ ns), $f$ has a maximum at $\theta = \pi$ and rapidly decreases as $\theta$ deviates by approximately 0.02$\pi$. However, for stronger interactions ($\tau_1 = 275$ ns), we observe a robust DTC phase, which manifests as a large crystalline fraction over a wide range 0.86$\pi < \theta < 1.13\pi$. We associate a phenomenological phase boundary with $f = 10\%$ and observe that the boundary enlarges with $\tau_1$, eventually saturating at $\tau_1 \approx 400$ ns (Fig. 3b). The phase boundary can also be visualized as the vanishing of the $\nu = 1/2$ peak and the simultaneous emergence of two incommensurate peaks (Fig. 3c).

The rigidity of the $\nu = 1/2$ peak can be qualitatively understood by constructing effective eigenstates of 2T Floquet cycles, including spin–spin interaction. We approximate the unitary time evolution over a single period as $U_T = R^{aT}_\theta e^{-i\Delta_T a}$ and solve for a self-consistent evolution using product states as a variational ansatz. To this end, we consider the situation in which a typical spin returns to its initial state after 2T, $|\psi(0)\rangle = e^{-i\theta z_S} e^{i\phi_S} e^{-i\phi_S} e^{i\theta z_S} |\psi(0)\rangle$, and self-consistently determine the interaction-induced rotation angle $\phi_S \equiv \sum J_{ij} r^2_j (S^+_j)_{\tau_1} \sim \sum J_{ij} r^2_j |\psi(0)\rangle^{S^+_j} |\psi(0)\rangle$, where $|\psi(0)\rangle$ is the initial spin state and $J_\tau = \sum J_{ij} r^2_j$ (see Methods). We expect $\phi_S$ to change sign after each Floquet cycle, because the average polarization $\langle \psi(0) | S^+_j | \psi(0) \rangle$ should be flipped. Intuitively, the self-consistent solution can be visualized as a closed path on the Bloch sphere (Fig. 3d), where each of the four arcs corresponds to one portion of the 2T-periodic evolution. When $\theta = \pi$, such a solution always exists. More surprisingly, even for $\theta = \pi$ a closed path can still be found for sufficiently strong interactions, $|\Delta_T| > 2\pi - \pi$ in such cases, the deviation in $\theta$ away from $\pi$ is compensated by the dipolar interactions (Fig. 3d). We obtain a theoretical phase boundary by numerically averaging the self-consistent solution over both disordered spin positions and local fields. The resultant phase boundary is in reasonable agreement with the experimental observations for short to moderate interaction times $\tau_1$, but overestimates the boundary at large $\tau_1$ (dashed line, Fig. 3b; see Methods).

Finally, Fig. 4 demonstrates that the discrete time-translation symmetry can be further broken down to $Z_4$ (refs 10–12, 29), resulting in DTC order at $\nu = 1/3$. Here, we utilize all three spin states of the nitrogen–vacancy centre. We begin with all of the spins polarized in the nitrogen–vacancy centre and then on the transition $|m_z = 0\rangle \rightarrow |m_z = -1\rangle$. As before, we apply resonant microwave pulses, each of duration $\tau_2$, first on the transition $|m_z = 0\rangle \rightarrow |m_z = -1\rangle$ and then on the transition $|m_z = 0\rangle \rightarrow |m_z = +1\rangle$. In combination, this sequence of operations defines a single Floquet cycle with period $T = \tau_1 + 2\tau_2$. As before, we measure the polarization $P(nT)$, which is defined as the population difference between $|m_z = 0\rangle$ and $|m_z = -1\rangle$ states (Fig. 4a). When each of the applied microwave sequences corresponds to an ideal $\pi$ pulse, this sequence realizes a cyclic transition with $Z_4$ symmetry (Fig. 4b), which is explicitly broken by any change in the pulse duration. The Fourier spectra of $P(nT)$ for various pulse durations and for two different values of $\tau_1$ are shown in Fig. 4c. With
which show that the lifetime of the DTC order is limited by the depolarization time $T_\text{DTC}^\circ$, owing to coupling with the environment\(^{28}\) (Fig. 2c). We have explicitly verified that the DTC order is not greatly affected by varying the initial polarization (see Methods). One possible explanation is that, owing to slow critical thermalization\(^{17}\), the spins in our system do not reach even a pre-thermal state. Finally, the interplay between coherent interactions and dephasing in open systems at long times could also have a role. Detailed understanding of such mechanisms requires further theoretical investigation.

A number of remarkable phenomena in quantum dynamics have recently been observed in engineered many-body systems consisting of ten to a few hundred particles\(^{3–6}\). The observations that we have presented here indicate that robust DTC order can occur in large systems without fine-tuned interactions and disorder, even in the regime in which localization is nominally not expected to occur. Our work raises important questions about the role of localization, long-range interactions and coupling to the environment in driven systems, and opens up several new avenues for fundamental studies and potential applications. In particular, it should be possible to extend these studies to realize novel dynamical phases in more complex driven Hamiltonians, and to explore whether such phases can be used to create and stabilize coherent quantum superposition states for applications such as quantum metrology\(^{18–20}\).

Online Content Methods, along with any additional Data display items and Source data, are available in the online version of the paper; references unique to these sections appear only in the online paper.

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**Author Contributions**
S.C. and M.D.L. developed the idea for the study. J.C., R.L. and G.K. designed and conducted the experiment. H.S., S.O., J.I. and F.J. fabricated the sample. S.C., H.Z., V.K., C.v.K., N.Y.Y. and E.D. conducted the theoretical analysis. All authors discussed the results and contributed to the manuscript.

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Reprints and permissions information is available at www.nature.com/reprints. The authors declare no competing financial interests. Readers are welcome to comment on the online version of the paper. Correspondence and requests for materials should be addressed to M.D.L. (lukin@physics.harvard.edu).

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METHODS

Experimental details. Our sample and experimental set-up have been previously described. We utilize a diamond sample containing a high concentration (about 45 p.p.m.) of nitrogen–vacancy (NV) centres, corresponding to an average NV–NV separation of 5 nm. For a single crystalline orientation of NV centres, selected by applying an external magnetic field, this corresponds to an average separation of 8 nm, resulting in a typical dipolar interaction strength of $2\pi \times 105$ kHz. The system exhibits strong on-site energy disorder, owing to the effects of lattice strain, the random orientation of NV centres and the presence of scattered paramagnetic impurities (consisting mainly of P1 centres and $^{13}$C nuclear spins). For each NV, the effective random field $\Delta_i$ is therefore a function of its local environment, including interaction effects of neighbouring NV centres. This results in an approximately Gaussian distribution with standard deviation $W = 2\pi \times 4.0$ MHz. We extract $W$ by measuring the linewidth of an electron spin resonance (ESR) spectrum with sufficiently weak microwave driving strength to avoid power broadening. To control the experimental probe volume, we fabricate a diamond nanobeam structure (about 300 nm $\times$ 300 nm $\times$ 20 nm) and confocally address a region of approximately 300 nm diameter using a green laser (532 nm). This realizes an effective three-dimensional excitation volume containing about 10$^7$ NV centres. By applying an external magnetic field along one of the diamond crystal axes, we spectrally isolate one group of NV centres and selectively address an effective two-level system between the $|m_s = -1\rangle$ and $|m_s = 0\rangle$ spin states via coherent microwave radiation. The addition of a microwave in-phase/quadrature (IQ) mixer allows for arbitrary rotations around any linear combination $\hat{\chi}$ and $\hat{\psi}$. 

Experimental sequence. Initial polarization of NV centres into $|m_s = 0\rangle$ is performed via laser illumination at a wavelength of 532 nm, a power of 50$mW$ and a duration of 100 ms. Subsequent application of a microwave pulse, aiming at rotating the spin ensemble into $|1\rangle = (|m_s = 0\rangle + |m_s = -1\rangle)/\sqrt{2}$. This sequence is repeated 22 2 times for a duration $\tau$. This so-called spin-locking technique suppresses two-spin (flip-flop and flop-flop) excitations and leads to non-trivial quantum dynamics. Under such dynamics, spins get entangled, resulting in mixed-state density matrices. These effects cannot be ignored in the case of long interaction times, effectively limiting the present existence of such a closed path stabilizes the time-crystalline phase. We emphasize that such a 2T-periodic path is a consequence of interactions; without the change in $\phi_0$, the eigenstates of the unitary evolution over one or two periods coincide and, therefore, unless the rotation angle is fine-tuned, $T$-periodic motion cannot be broken into a 2T period. The eigenstates of unitary evolution over one period can be obtained even as odd linear combinations, $|\Psi\rangle = \pm U|\Psi\rangle$, $U = \exp(-i\epsilon\hat{H}T/2\hbar)$, and the quasi-energy eigenvalue is given by $\epsilon^2 = (U|\Psi\rangle U^\dagger|\Psi\rangle)^{1/2}$.

To estimate the phase boundary, we numerically solve the self-consistency equation. Here, we include the effects of on-site disorder potential $\Delta_i$ in all four rotations and the disorder in $\hat{I}$ that arises from the random positions of NV centres. The distribution of $\hat{I}$ is simulated for 1,000 spins, which are randomly distributed in three dimensions with an average separation $\rho_0$ and minimum cutoff distance $\rho_{\text{min}} = 3$ nm (limited by NV–NV electron tunnelling). Instead of cos$\theta_0$, we solve for a self-consistent distribution for cos$\theta_0$, where $|\Psi\rangle$ is defined as the mean of the distribution. The average order parameter $|\langle\Psi|U|\Psi\rangle\langle\Psi\rangle|^{1/2}$ is computed for various values of $\tau_i$ and $\theta$ and compared with a threshold value of 0.1 to identify the phase boundary. The experimental and numerical phase boundaries are asymptotic about $\theta = \pi$. We attribute this asymmetry to the inherently asymmetric distribution of the effective rotation angle $\theta_i = \frac{\tau_i}{\tau}(\alpha_j + \Delta_j + \hat{I})$, which causes the transition to occur earlier for positive deviations $\theta - \pi$.

Although we assumed $\phi_0$ to be a classical variable in this analysis, the interaction-induced rotation angle is an operator that exhibits quantum fluctuations and leads to non-trivial quantum dynamics. Under such dynamics, spins get entangled, resulting in mixed-state density matrices. These effects cannot be ignored in the case of long interaction times, effectively limiting the present description. We believe that the diminished range of $\phi_0$ in the experimentally obtained phase diagram (Fig. 3b) is related to this effect.
Although it has been shown theoretically that disorder alone is insufficient (in the absence of interactions) for stabilizing DTC order \(^8\)–\(^{12}\), to experimentally demonstrate that the accidental decoupling is not responsible for the observed DTC ordering, we implement a ‘rotary echo’ sequence, wherein after half the interaction time \(\tau_1\) the microwave driving is flipped from \(\Omega_1\) to \(-\Omega_1\) (Extended Data Fig. 1a). In the limit of strong driving, such a sequence eliminates the phase acquired between the two dressed states for each spin, regardless of the exact value of \(\Omega_1\). As shown in Extended Data Fig. 1b, the lifetimes of the DTC order are nearly identical between the cases of the rotary echo and continuous \(i+\hat{x}\) driving. Moreover, the rotary echo spin polarization maintains a larger amplitude at late times, excluding the possibility of self-correcting dynamical decoupling as the origin of the observed DTC order.

**Markovian dephasing effects on DTC order.** The presence of the subharmonic peak at \(\nu=1/2\) at small values of \(\theta - \pi\) can, in principle, also be explained on the basis of fast Markovian dephasing in the dressed state basis. Indeed, for sufficiently fast dephasing, coherences along both \(\hat{y}\) and \(\hat{z}\) will be eliminated after each rotation, \(\hat{R}^\nu_{xy}\). Therefore, the only evolution that remains is the population dynamics along \(\hat{x}\), which exhibits 2T-periodicity from the alternating sign. Microscopically, such strong dephasing could potentially originate from either dipolar interactions between the spins or from coupling to an external (Markovian) environment.

Intuitively, the result of such dephasing can be understood as an ‘effective’ projective measurement of polarization along \(\hat{x}\) in each Floquet cycle, reminiscent of the quantum Zeno effect. To quantify and distinguish such dephasing-induced subharmonic rigidity, we consider the dynamics (over one Floquet period) of a single spin undergoing Markovian dephasing, with super-operator \(\widehat{D}[\rho] = -\gamma (\rho - 4\gamma i \delta^x \rho / \hbar) / 2\) and dephasing rate \(\gamma\). Assuming \(\theta - \pi \ll 1\), evolution falls into two well-known limits. In the under-damped limit (weak dephasing), \(\mathcal{S}(\nu)\) has two Lorentzian peaks at \(\nu = \pm 1/\gamma\) with a linewidth set by \(\gamma\), where \(\tau_1\) is the spin-locking duration and \(\cos(2\theta \gamma) = \cos\left(\theta + \pi / 2\right) / 2\). In the over-damped limit (strong dephasing), \(\mathcal{S}(\nu)\) (at late times) has a peak at \(\nu = 1/2\) with a linewidth (in Floquet units) of

\[
\Gamma \approx \frac{(\theta - \pi)^2}{2\text{tanh}(\gamma/2)}
\]

These over-damped oscillations of the spin polarization exhibit sign flips between the even and odd cycles, leading to a subharmonic Fourier response that is reminiscent of DTC order.

Although strong Markovian dephasing can indeed result in a \(\nu=1/2\) subharmonic peak, we observe three distinct experimental signatures that clearly show that our observations are not governed by this effect. First, the linewidth \(\Gamma\) (equation (1)) of the subharmonic peak should be quadratically sensitive to the deviation of \(\theta\) from \(\pi\). This is in stark contrast with our experimental observations shown in Fig. 2d, wherein this linewidth \(\Gamma\) is essentially independent of \(\theta\) within the DTC phase. Second, according to the dephasing model (equation (1)), the lifetime of the 3T-periodic DTC order is expected to be longer than that of the 2T-periodic DTC order owing to enhanced dephasing (from a lack of spin-locking) in the bare basis\(^{28}\). However, we observe the exact opposite behaviour. Finally, Markovian dephasing requires an effective environment with a relatively fast, sub-microsecond correlation time. This is also inconsistent with our experimental observations. In particular, we performed Rabi oscillation decay measurements with a rotary echo sequence, resulting in a lower bound of 1.3\(\mu\)s on the Markovian dephasing time \(\tau_2\). This time scale includes contributions from static on-site disorder and interactions, so the Markovian dephasing rate is, in fact, much slower than this. Indeed, we independently extracted the typical timescales of disorder fluctuations in our system\(^7\), and found that they are similar (60\(\mu\)s) to the depolarization timescale under spin-locking dynamics. Effects resulting from such slow dephasing should be completely negligible within a typical Floquet period. Therefore, we conclude that fast dephasing alone does not explain the observed DTC order.

At the same time, in the time crystalline order description based on interacting spin models\(^8\)–\(^{12}\), the time-crystalline order is expected to be robust and is not expected to exhibit any functional dependence on the angle \(\theta\), in complete agreement with experimental observations. This is also the case for our self-consistent description. We finally note that the interplay between coherent interactions and dephasing could potentially have a role in stabilizing DTC order at longer interaction times. A detailed understanding of such mechanisms requires further theoretical investigation.

**Derivation of effective Hamiltonian for the \(Z_3\) symmetry breaking phase.** Using microwave driving resonant with two different transitions (Fig. 4a), we realize dynamics involving all three spin states and observe robust 3T-periodic time-crystalline order. The unitary matrix of the time evolution during the fundamental period \(T\) is

\[
U_3 = \exp \left[ -\frac{\theta}{2} \sum_i \left( \sigma^z_{i+1} + \sigma^z_{i-1} \right) \right] \exp \left[ -i \frac{\gamma}{2} \sum_i \left( \sigma^x_{i-1} + \sigma^x_{i+1} \right) \right] \exp(-iH_2\gamma)
\]

where \(\sigma^{x,y}_a \equiv (m_a = a)\langle m_b = b | \) for spin \(i\) and \(H_2 = H_{\text{dis}} + H_{\text{int}}\) is the effective Hamiltonian of NV centres for all three spin states, including on-site disorder potentials \(H_{\text{dis}} = \sum_i \Delta_i \sigma^z_{i+1,i} + \sum_i \sigma^z_{i-1,i} + \text{dipolar interactions for spin-1 particles}\(^\text{17}\).

This Hamiltonian is obtained in the rotating frame under the secular approximation. The Hamiltonian \(H_2\) conserves the total population in any of the three spin states: \(\mathcal{P}_a = \sum_i \sigma^a_{i,i}\) with \(a \in \{0, \pm 1\}\). If each microwave pulse realizes a \(\pi\) pulse \((\theta = \pi)\), their combination results in a cyclic transition \(R_{\nu}^3 : \{m_a = +1\} \rightarrow \{m_a = 0\} \rightarrow \{m_a = -1\} \rightarrow \{m_a = +1\}\) and the population \(\mathcal{P}_a\) becomes periodic over three periods. Under such evolution, the effective Hamiltonian over three periods is given by \(D_{\nu}^3 = (H_2 + \left[R_\nu\right]^3) H_2 R_\nu^3 + \left[R_\nu\right]^3 H_2 R_\nu^3 / 3\), in which on-site disorder terms average to zero, and the interactions are modified to

\[
D_{\nu}^3 = \sum_{\nu} \left[ \sum_{\mathcal{P}_a} \sum_{\mathcal{P}_b} \sigma^a_{\nu} \sigma^b_{\nu} - \frac{1}{3} \sum_{\mathcal{P}_a} \sigma^a_{\nu} \sigma^b_{\nu} \right]
\]

The first term describes Ising-like interactions that shift energy when any pair of spins are in the same state, and the second term corresponds to spin-exchange interactions that allow polarization transport. These additional exchange interactions may lead to a shorter lifetime of the DTC order compared to that of the \(\nu=1/2\) DTC order. For small perturbations in the microwave pulse angle \(\epsilon = \theta - \pi\), the effective dynamics, to leading order, are governed by

\[
D_{\nu}^{\epsilon,++} \approx D_{\nu}^3 + \frac{\epsilon}{3} \sum_{\nu} \left( \sigma^z_{i+1} + \sigma^z_{i-1} + i\sigma^y_{i+1,-1} + \text{h.c.} \right)
\]

which explicitly breaks the conservation laws for \(\mathcal{P}_a\).

**Data availability.** The data generated during this study are available from the corresponding author on reasonable request.
Extended Data Figure 1 | Effect of rotary echo sequence. a, Experimental sequence: during the interaction interval $\tau_1$, the phase of the microwave driving along $\hat{x}$ is inverted after $\tau_1/2$. b, Comparison of time traces of $P(nT)$, measured at even (green) and odd (blue) integer multiples of $T$, in the presence (left) and absence (right) of an $\hat{x}/-\hat{x}$ rotary echo sequence at similar $\tau_1$ and $\theta$ (left, $\tau_1 = 379$ ns, $\theta = 0.979\pi$; right, $\tau_1 = 384$ ns, $\theta = 0.974\pi$). The rotary echo leads to more pronounced $2T$-periodic oscillations at long time. The microwave frequencies used in the rotary echo sequence are $\Omega_x = 2\pi \times 52.9$ MHz and $\Omega_y = 2\pi \times 42.3$ MHz.