Decoherence induced by inelastic scattering in Aharonov-Bohm ring with embedded quantum dot

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Abstract. We theoretically investigate the Kondo effect in a quantum dot embedded in an Aharonov-Bohm (AB) ring. We obtain an analytical expression for the Kondo temperature $T_K$ as a function of the AB phase $\phi$ for the magnetic flux penetrating the ring, by the scaling method. We also examine the decoherence due to the inelastic process in the quantum dot embedded in an AB ring. Solving the scattering problem, we obtain the elastic part of the conductance. The inelastic part of the conductance $G_{\text{inel}}$ is obtained by the subtraction of elastic part from the total conductance. We show that $G_{\text{inel}} = 0$ at $T = 0$ and becomes significant when $T \gg T_K$. We also show that the inelastic part of the conductance depends on the magnetic flux.

1. Introduction

A quantum dot embedded in an Aharonov-Bohm (AB) ring, the so-called AB interferometer, has been studied experimentally for various phenomena, e.g. transmission phase through a quantum dot [1], phase in the Kondo effect [2], and Fano resonance [3]. Regarding the Kondo effect, the Fano-Kondo effect has been proposed theoretically [4] and observed in an experiment [5].

The violation of the quantum interference (decoherence) is one of the major issues in mesoscopic physics. A dephasing time has been measured through weak localization experiments in a 1D Au wire and a saturation of it at low temperatures has been reported [6]. The saturation has been explained theoretically as the dephasing due to the inelastic scattering by the magnetic impurities [7]. The AB interferometer should reveal the nature of decoherence due to the inelastic process in the impurity.

This paper consists of two parts. First, we present a scaling analysis to calculate the Kondo temperature ($T_K$) for the AB ring with an embedded quantum dot. We obtain the analytical expression for $T_K$ and show that $T_K$ depends on the magnetic flux penetrating the ring. Second, we calculate the elastic part of the conductance ($G_{\text{el}}$). The inelastic part of the conductance is given by $G_{\text{inel}} = G_{\text{tot}} - G_{\text{el}}$, where $G_{\text{tot}}$ is the total conductance. We show that $G_{\text{inel}}$ changes from 0 at $T = 0$ to the same order with the elastic part at $T \gg T_K$ by increasing the temperature. We also show that $G_{\text{inel}}$ depends on the magnetic flux penetrating the ring.

2. Model

We examine an extended impurity Anderson model (Fig. 1). A quantum dot with single energy level is connected to two external leads by the tunnel coupling $V_L$ or $V_R$. The asymmetry of two
Figure 1. Our model for an AB ring with an embedded quantum dot of single energy level $\epsilon_0$. The magnetic flux penetrating the ring is represented by the AB phase $\phi$ on the tunnel coupling $W$. Two external leads are described by a one-dimensional tight-binding model.

tunnel couplings is characterized by $\alpha = 4V_L^2V_R^2/(V_L^2 + V_R^2)^2$. Another arm of the AB ring is represented by a direct tunnel coupling $We^{i\phi}$ between the leads. The magnetic flux penetrating the ring is taken into account as AB phase $\phi$. The two leads are described by a one-dimensional tight-binding model with transfer integral $-t$. The Hamiltonian is

$$H^{(0)} = H_{d\ot} + H_{\text{leads}} + H_T,$$

$$H_{d\ot} = \sum_\sigma \epsilon_0 d_\sigma^\dagger d_\sigma + U\hat{n}_1\hat{n}_1,$$

$$H_{\text{leads}} = \sum_{i\neq 0} \sum_\sigma (-t a_{i+1,\sigma}^\dagger a_{i,\sigma} + \text{h.c.}) + \sum_\sigma (W e^{i\phi} a_{1,\sigma}^\dagger a_{0,\sigma} + \text{h.c.}),$$

$$H_T = \sum_\sigma (V_L d_\sigma^\dagger a_{0,\sigma} + V_R d_\sigma^\dagger a_{1,\sigma} + \text{h.c.}),$$

where $d_\sigma^\dagger$ and $d_\sigma$ are creation and annihilation operators in the dot state with spin $\sigma$, respectively, $a_{i,\sigma}^\dagger$ and $a_{i,\sigma}$ are those at site $i$ with spin $\sigma$ in the leads ($i \leq 0$ for the left lead, $i \geq 1$ for the right lead). $\hat{n}_\sigma = d_\sigma^\dagger d_\sigma$. $U$ is the charging energy in the dot.

We are interested in the Coulomb blockade region with one electron in the quantum dot, where $-\epsilon_0, \epsilon_0 + U \gg \Gamma, k_B T$, to investigate the Kondo effect. $\Gamma = \Gamma_L + \Gamma_R$ is the level broadening, where $\Gamma_\sigma = \pi \nu V_\sigma^2$ with $\nu = 1/(\pi t)$ being the density of states in the lead. The background transmission probability through the upper arm of the ring is given by $T_b = 4x/(1 + x)^2$ with $x = (W/t)^2$, at the Fermi level $\epsilon_F = 0$. We assume $\epsilon_0 + U \simeq -\epsilon_0$.

3. Scaling analysis

In this section we examine the scaling analysis to obtain the Kondo temperature. A unitary transformation for the eigenstates of $H_{\text{leads}}$ yields the mode coupled to the dot via $H_T$ and the mode completely decoupled to the dot [8]. Neglecting the decoupled modes, we obtain an equivalent model to model in Eq. (1) in which a quantum dot is coupled to a single lead with tunnel coupling $V = \sqrt{V_L^2 + V_R^2}$. In the model, density of states in the lead is $\nu(\epsilon_k) = \tilde{\nu} [1 + P(\phi)\epsilon_k/D_0]$ for $-D_0 \leq \epsilon_k \leq D_0$, where $D_0 = 2t$ is the bandwidth. Here $\tilde{\nu} = \nu/(1 + x)$, $P(\phi) = \sqrt{\alpha T_b} \cos \phi$, and $\epsilon_k$ is the energy for the conducting electron in the lead with momentum $k$.

We analyze the reduced model using the scaling method. The scaling procedure consists of two stages. In the first stage, by changing the bandwidth $D$ to $D - |dD|$, we obtain the scaling equations of

$$\frac{d\epsilon_0}{d\ln D} \simeq \frac{d\epsilon_1}{d\ln D} = 2\nu V^2 P(\phi) \frac{D}{D_0},$$

within the second-order perturbation with respect to the tunnel coupling $V$. By integrating Eq. (5) from the bandwidth $D_0$ to $D_1$ where the charge fluctuation is quenched, we obtain $\tilde{\epsilon}_i = \epsilon_i - 2\tilde{\nu} V^2 P(\phi) (1 - D_i/D_0)$, for $i = 0, 1$ and $D_1 \simeq -\epsilon_0$.

In the second stage, we start from the reduced model with the renormalized energy levels $\tilde{\epsilon}_i$, the bandwidth $D_1$, and the $\phi$ dependent density of states. To describe the spin fluctuation at
energy scale of $D < D_1$, we make the Kondo Hamiltonian by the Schrieffer-Wolff transformation. Then we obtain the Kondo model which describes the exchange coupling between the localized spin and conduction electrons in the lead ($H_J$), and the potential scattering of the conduction electrons by the quantum dot ($H_K$). By changing the bandwidth, we renormalize the coupling constants $J$ and $K$ so as not to change the low-energy physics within the second-order perturbation with respect to $H_J$ and $H_K$. Here $J$ and $K$ are the coupling constant for spin exchange and that for potential scattering, respectively. The coupling constants follow the scaling equations

$$\frac{dJ}{d \ln D} = -2\nu \left[ J^2 - 2JK P(\phi) \frac{D}{D_0} \right],$$  

(6)

$$\frac{dK}{d \ln D} = 2\nu \left( \frac{3}{4} J^2 + 4K^2 \right) P(\phi) \frac{D}{D_0}. $$  

(7)

The energy scale $D$ at which $J$ diverges is the Kondo temperature and it becomes

$$T_K(\phi) = T_K\left( \frac{\pi}{2} \right) \exp \left[ -\frac{U}{2\nu} P(\phi) - \frac{2\nu V^2}{U} P(\phi)^2 \right].$$  

(8)

The Kondo temperature is modulated by the magnetic flux. The magnetic flux dependence is much smaller than that in case of $-\epsilon_0 \ll D_0$ and $U = \infty$ [8].

4. Elastic and Inelastic part of conductance

In this section, first we present a brief review of the method used in [9]. In the paper the interactions are switched on and off adiabatically as $H = H_0 + e^{-\delta}H_{\text{int}}$, where $H_0 = p^2/2m$ and $H_{\text{int}}$ is interacting part of the Hamiltonian. For the model, the $S$-matrix is defined as $\langle i|S|i\rangle_{\text{in}} = \langle i|S|i\rangle_{\text{out}}$. We use the notation of $|a(t = \pm \infty)\rangle \equiv |a\rangle_{\text{out/\text{in}}}$. All of the information for the scattering process is involved in $T$-matrix defined by $i\hat{T} = \hat{S} - \hat{I}$. From the $T$-matrix, we can define the on-shell $T$-matrix $\hat{T}$ as $\langle f|\hat{T}|i\rangle_{\text{in}} = 2\pi \delta(E_f - E_i)\langle f|\hat{T}|i\rangle_{\text{in}}$. Then the total scattering cross section of an electron of momentum $p$ and spin $\sigma$ is given by $\sigma_{\text{tot}} = 2\Im \langle p\sigma|\hat{T}|p\sigma\rangle_{\text{in}}/v_F$, where $v_F$ is the Fermi velocity and $|p\sigma\rangle$ is the state that an electron with momentum $p$ and spin $\sigma$ is created above the Fermi sea at $t = -\infty$. The scattering process without any spin flip nor electron-hole excitation yields the elastic scattering cross section as

$$\sigma_{\text{el}} = \frac{1}{v_F} \int \frac{dp'd}{(2\pi)^3} 2\pi \delta(\epsilon_p - \epsilon_{p'}) |\langle p'\sigma|\hat{T}|p\sigma\rangle_{\text{in}}|^2.$$  

(9)

Then the inelastic cross section $\sigma_{\text{inel}}$ is calculated as $\sigma_{\text{inel}} = \sigma_{\text{tot}} - \sigma_{\text{el}}$. They have shown that $\sigma_{\text{el}}(\omega)$ is dominant for $\omega \ll T_K$ and $\sigma_{\text{inel}}(\omega)$ is dominant for $\omega \gg T_K$.

Applying the method for our model, we obtain

$$G_{\text{el}} = \frac{2e^2}{h} \int d\omega [-f'(|\omega|)] \left[ T_b + 2T_b \pi \nu \Im \hat{T}(\omega) - 2\sqrt{\alpha T_b} R_b \cos \phi \nu \Re \hat{T}(\omega) \right. $$

$$\left. + (\alpha + T_b - T_b \alpha \cos^2 \phi) \pi \nu T(\omega) \right] + 2\sqrt{\alpha T_b} \sin \phi (\pi \nu T(\omega) + |\pi \nu T(\omega)|^2),$$  

(10)

where $R_b = 1 - T_b$. Thus the inelastic part of the conductance is given by

$$G_{\text{inel}} = \frac{2e^2}{h} (\alpha + T_b - \alpha T_b \cos^2 \phi + 2\sqrt{\alpha T_b} \sin \phi) \int d\omega [-f'(|\omega|)] \left( \Im \pi \nu T(\omega) + |\pi \nu T(\omega)|^2 \right),$$  

(11)

if we use the expression of the total conductance $G_{\text{tot}}$ obtained in [4]. When the optical theorem $\Im \pi \nu T(\omega) = -|\pi \nu T(\omega)|^2$ holds, $G_{\text{inel}} = 0$ and $G_{\text{el}} = G_{\text{tot}}$. For $T = 0$, optical theorem holds and thus all the scattering processes are elastic.
By using the T-matrix for $T \ll T_K$ [10], we obtain

$$G_{el} = \frac{2e^2}{h} \alpha (1 - T_b \cos^2 \phi) - \frac{1}{3} \frac{2e^2}{h} \left[ 5\alpha (1 - T_b \cos^2 \phi) - T_b + 4\sqrt{\alpha T_b} \sin \phi \right] \left( \frac{\pi T}{T_K} \right)^2,$$

(12)

and

$$G_{inel} = \frac{2e^2}{h} \frac{2}{3} \left[ \alpha (1 - T_b \cos^2 \phi) + T_b + 2\sqrt{\alpha T_b} \sin \phi \right] \left( \frac{\pi T}{T_K} \right)^2,$$

(13)

thus $G_{el}/G_{inel} = O(T/T_K)^2$. Thus the elastic process is dominant for $T \ll T_K$. It is shown that $G_{inel}$ depends on $\phi$. By using the T-matrix for $T \gg T_K$ [10], we obtain

$$G_{el} = \frac{2e^2}{h} T_b - \frac{2e^2}{h} \frac{3\pi^2}{16} [T_b + 2\sqrt{\alpha T_b} \sin \phi] \frac{1}{[\ln(T/T_K)]^2},$$

(14)

and

$$G_{inel} = \frac{2e^2}{h} \frac{3\pi^2}{16} \left[ \alpha (1 - T_b \cos^2 \phi) - 2T_b - 2\sqrt{\alpha T_b} \sin \phi \right] \frac{1}{[\ln(T/T_K)]^2}.$$

(15)

It is easily seen that the inelastic term is dominant when $T_b \ll 1$, however the elastic part is also significant due to the interference term when $T_b \approx 1$. We show that the inelastic part of the conductance depends on $\phi$.

5. Conclusion

We have examined the Kondo effect in a quantum dot embedded in an AB ring. The scaling analysis has yielded an analytical expression for the Kondo temperature as a function of $\phi$. We have also obtained the elastic part of the conductance and the inelastic part of the conductance. The scattering process is totally elastic when $T = 0$, and the inelastic part becomes larger as increasing the temperature. Then $G_{inel}$ becomes the same order as $G_{el}$ at high temperatures. We have shown that $G_{inel}$ significantly depends on $\phi$.

Acknowledgments

This work was partly supported by a Grant-in-Aid for Scientific Research from the Japan Society for the Promotion of Science, and by the Global COE Program “High-Level Global Cooperation for Leading-Edge Platform on Access Space (C12).”

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