Post necking evaluation of the tensile test using artificial neural networks

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Abstract. This paper introduces a new method for evaluating the tensile test using finite element simulations and data-driven artificial neural networks. For this purpose, a synthetic data set was generated by finite element simulations using LS-DYNA. Artificial neural networks of two different topologies were trained and tested on parts of this synthetic data set. The networks use geometry information of the necking area as input data to predict a correction factor to convert the stress obtained from the tensile test to the equivalent flow stress. The best models are evaluated on the test set and good results are achieved.

1. Strategies for evaluating the tensile test

The flow properties of a given material can be represented by means of a flow curve. A flow curve shows the relationship of the stress at which the material begins or continues to flow versus the degree of deformation. One method to determine the flow curve is the uniaxial tensile test, an inexpensive and relatively simple method. Up to the beginning of specimen necking, the stress is uniaxial. Afterwards, the stress state becomes multiaxial. If the stress state is uniaxial, the flow curve can be determined using the stress-strain curve. After the initiation of necking, the evaluation becomes much more complex and inaccurate. For many materials, the beginning of the necking begins at relatively small degrees of deformation. However, degrees of deformation for industrial forming processes are significantly greater. For this reason, the stress-strain curve is extrapolated with different methods which result in inaccurate stress-strain curves.

Tu et al. divide strategies for evaluating the uniaxial tensile test into three different classes: analytical methods, iterative experimental-numerical methods and inverse methods [1] which will only be discussed briefly here. For further information [1] can be reviewed.

Analytical methods determine a correction factor to convert the true stress into a uniaxial equivalent stress. These methods are easy to use and quite simple, however compared to other methods not as accurate [1] when the existing stress state diverges from the uniaxial assumption. One analytical method is the formula introduced by Bridgman [2]. A more recent method is introduced by Paul et al. which aims to use strain data measured with digital image correlation (DIC) to correct the local stress and strain states in the necking area [3].

Finite element method simulations are conducted for iterative experimental-numerical methods. Furthermore, real tensile test experiments are carried out. Afterwards the parameters of the simulation are iteratively adjusted to minimize the deviation between the results of the simulation and the
experimental test. Iterative experimental-numerical methods can produce very accurate results. However, the better accuracy comes with greater complexity as well as a higher demand on resources and expertise [1].

Dunand et al. use an iterative procedure to adjust local deformations [4]. Marth et al. use a full-field method to characterize strain hardening parameters even after the localized constriction has developed [5].

The third category encompasses the inverse methods. For these methods a tensile specimen with a predefined equivalent stress-strain curve is modelled. Afterwards the equivalent strain after diffuse necking is characterized. The relationship between the true stress from numerical modelling and the equivalent stress at the equivalent strain is investigated, which is afterwards put into a formula used to correct the stress [1].

Schneider et al. study the stress distribution in the necking using FE simulation and an alternative correction formula for rectangular cross-section specimens is developed [6]. Coppeters et al. use the balance between internal and external work and an optimization based on the Levenberg-Marquardt method to estimate the parameter values of an associated hardening law [7].

The existing methods for evaluating the tensile test after necking are either easy to apply but less accurate or complex to apply with higher possible accuracy. Currently, there seems to be no solution for a method that is as simple to use and fast as well as accurate at the same time.

The method presented in this paper is intended to fill the gap by using a data driven approach. Such approaches, including artificial intelligence, enable finding patterns in large amounts of data. As a result, less expertise regarding the evaluation of tensile tests is required. However, the successful application of data-driven methods essentially depends on the quality and quantity of a representative database. Furthermore, because simulations are used to generate the given database for the case at hand, it is necessary to validate methods developed solely on synthetic data on data from real experiments such as the tensile test or bulge test [1].

2. Artificial neural networks

For this paper two of the commonly used topologies for artificial neural networks, multilayer perceptrons (MLP) and convolutional neural networks (CNN), were used to train models for the prediction of a correction factor to accurately determine the flow stress after necking.

MLPs are the default type of artificial neural network consisting of multiple fully connected layers which results in perceptrons in each layer being fully connected to all perceptrons in the previous and following layer. Therefore, the amount of parameters in a MLP model grows exponentially with the amount of neurons in a model. A high amount of neurons in a model is necessary if the patterns hidden in the data are complex [8].

The other investigated topology are convolutional neural networks. Neurons of resulting models can consider data in a specified window [9]. For this reason, neurons are only sparsely connected which results in less complex models with similar performance compared to MLP models.

Jeong et al. train an artificial neural network on data from finite element simulations which is then used to convert indentation load-depth curves into hardening parameters required to describe uniaxial tensile flow [10]. Jeník et al. determine hardening curves using an artificial neural network. However, this solution does not take the influence of geometry on the tensile behavior of the tested material into account. Therefore the method is limited to materials with no strongly pronounced strain hardening and anisotropy [11].

3. New method for stress analysis

The aim of this paper is to use Artificial Intelligence and geometry information of a specimen during the tensile test to predict the quotient $\alpha_{corr}$ of flow stress $\sigma_y$ and the true stress in tensile direction $\sigma_{true}$. 
Flow stress and true stress are frequently used expressions. For every timestep of every simulation this factor is calculated independently.

\[ \alpha_{corr} = \frac{\sigma_y}{\sigma_{true}} \]  

The multiplication of this correction factor with the stress obtained from a tensile test or simulation the flow stress can be calculated. The task at hand is a regression for which the dataset must contain the input as well as the output data. The factor to be predicted contains information that cannot be extracted from experimental tensile tests, but from finite element simulations. For this reason, simulations of the uniaxial tensile test are carried out and afterwards the geometry information used as input and the correction factor used as output data are extracted. The principal progression of the correction factor over a simulation is shown in figure 1.

The input data for the artificial neural networks are geometrical information of the specimen for each timestep of each simulation. The used geometrical data are the width differences between the current timestep and the first timestep in relation to the distance to the narrowest point of the specimen. The narrowest cross-section of the specimen is identified and using interpolation, 32 points in a given distance from the narrowest cross-section are extracted from the simulations. From each of the points the specimen width at this point is extracted and compared to the width of the same point at the initial state of the simulation.

Within one given simulation, the input data was extracted for every 20th timestep and visualized as shown in figure 2a. Markers of the same color are points extracted from the same timestep. The earlier the timestep the smaller is the width difference compared to the initial state of the simulation. With an increasing time step the necking area forms. Figure 2b shows a timestep of a specimen when the necking already started. The extracted points at that timestep are comparable to the 280th timestep shown in figure 2a.

![Figure 1. Correction factor for one simulation](image-url)
4. Virtual Experiments

4.1. Database

The database used for the training and testing of the artificial neural network models are generated using finite element method simulation. The simulations are carried out using only the Swift hardening law to sufficiently represent the simulated materials. The flow stress can be calculated with the Swift hardening law as shown in formula (2) [12].

\[ \sigma_y(\varepsilon_p) = k(\varepsilon_0 + \varepsilon_p)^n \]  

\( \sigma_y \) and \( \varepsilon_p \) are the equivalent stress and strain and \( k, \varepsilon_0 \) and \( n \) are parameters that assume material-dependent values normally determined by uniaxial tensile tests [13]. Furthermore, only isotropic hardening is simulated. A two-dimensional finite element method of a tensile test specimen is created. Afterwards a pretest is conducted to reduce the necessary computing power for resources necessary to create the database is minimized while its accuracy is as high as necessary. The parameters under study for this pretest are the amount of time between two states of the simulation and the amplitude height of the perturbation shell thickness.

The limits of the parameters of the selected hardening laws for steel as shown in formula 2 are derived from literature and adapted to the given task [14][15][16]. The upper and lower parameter limits are shown in table 1. From the resulting parameter space, 500 samples are taken using Latin Hypercube Sampling, a sampling method optimal for numerical experiments. Compared to the random Monte Carlo method, the necessary effort can be reduced by up to 50 % without having to forego the advantages of the method [17].

These samples are used to create the necessary input files for the simulations. The simulations are then carried out and used as a database for training and testing the artificial intelligence models. For this purpose, for every timestep in every simulation the previously mentioned input and output data is extracted automatically using a Python script.

For reproducibility reasons, a random seed was determined to generate the same outputs for functions that use random elements with a pseudo number generator. The data set is divided into a training and a testing set in the ratio of 4:1. Subsequently, the training set is divided into five subsets for cross validation. Four of these subsets are used to form a slightly smaller training set. The last subset is used...
as validation set resulting in each model being trained five times on different data. The average performance of each model is calculated and the model with the best average performance of each topology is then trained on the entire training set and afterwards evaluated on the testing set. The metric to evaluate the performance of each model is the mean squared error often used for regression.

**Table 1.** Swift hardening law parameters

| Parameter | Minimum | Maximum |
|-----------|---------|---------|
| $k$ in MPa | 600 | 1200 |
| $\varepsilon_0$ | 0.02 | 0.05 |
| $n$ | 0.05 | 0.3 |

### 4.2. Artificial Intelligence

Before the hyperparameter optimization and cross validation are carried out, pretests were conducted to estimate the range of complexity in which the best model of each topology might be situated. Instead of training many models with a wide range of complexity on the entire dataset, few manually chosen models with different complexities were tested to estimate the range of complexity in which the best models might be situated. Once the range was determined many models with their complexity inside the given range were trained and tested. Without these pretests more models need to be trained and tested to get an accurate model. By manually reducing the range of model complexity beforehand, the necessary computing power for solving the task could be reduced.

In addition, various pretests were carried out for the CNN topology to determine a range in which the optimal structure of the identification part is located. The optimal structure for the identification part for the task at hand consist of five one dimensional convolutional layers. In each layer the product of the number of filters and kernel size is the same. From one layer to the next the number of filters is halved and the kernel size is doubled. Afterwards the influence of this identification part on the complexity of the detection part as well as the model’s performance is investigated. The identification part of the CNN are the convolutional layers and the detection part is the MLP part of the CNN.

The models are trained on a system with 128 GB RAM, an Intel® e5-2667 v4 and a NVIDIA® Quadro M5000. The training process takes several days. The studied hyperparameters for the MLP and the CNN topology with their researched limits are shown in table 2 and table 3.

**Table 2.** Studied hyperparameters of the MLP topology

| Hyperparameter | Minimum | Maximum |
|----------------|---------|---------|
| Learning rate  | 0.00001 | 0.1     |
| Amount of trainable parameters | 351 | 15,145 |

**Table 3.** Studied hyperparameters of the CNN topology

| Hyperparameter | Minimum | Maximum |
|----------------|---------|---------|
| Learning rate  | 0.0001  | 0.1     |
| Amount of trainable parameters | 2,143 | 13,444 |
5. Evaluation
In this section the results of the conducted experiments are presented. Figure 3 shows the deviations between the predictions of the best models for each topology and the true values for the correction factor. The performance of the best model of each topology is nearly identical. The predictions of the best MLP model have a 5\% relative deviation with a confidence interval of 95\% and the predictions of the best CNN model have a 6\% relative deviation with a confidence interval of 95\%. However, the best MLP model has roughly eight times the number of parameters of the best CNN model. Also the best MLP model is the most complex model possible within the examined hyperparameter space. In contrast the best CNN model is in terms of model complexity in the bottom 20\%.

If the expected yield stress is to be 350 MPa, then there is a 95\% probability that the final corrected stress predicted by the best CNN model will be between 329 MPa and 371 MPa.

![Boxplots of the relative deviance of the prediction of the best models for each topology from the true values](image)

**Figure 3.** Boxplots of the relative deviance of the prediction of the best models for each topology from the true values

The fact that the best MLP models is the most complex suggests that an even more complex model could achieve better performance. So it stands to reason that since this topology has achieved better results, it should be investigated further. But even though the best CNN model has a slightly worse performance than the best MLP model, further research should focus on this topology, as similar performance compared to the MLP topology could be achieved with 1/8 of the model complexity.

6. Discussion and further research
The proposed method shows accurate results on the synthetic data generated from finite element simulations. However, the resulting models need to be validated on real experiments like the tensile test or bulge test. Furthermore, the database used to train the models can be made more diverse by using different hardening laws, different approaches to determine the flow curve, different flow curve models and wider ranges of material parameters. This can result in more robust models for stress correction and further validate the accuracy of the method presented in this paper. However, the more diverse database could mean that models have to become more complex to still achieve the same performance.
For this publication only one-dimensional data (of the width of the specimen) were used as input data. Instead, the depth of the specimen could be used in combination with the width, or the cross-sectional area could be used. Using these kinds of input data could result in more robust but more complex models as well.

The deviations in the predictions can be reduced in multiple ways. Each prediction of a model is for one time step in a simulation or a real tensile test. One possibility to reduce the impact of outliers on one prediction is to use a Gaussian filter or another topology that regards the input and output data as time series.

The prototype presented in this paper shows promising accuracies for a possible application on data from real experimental tensile tests and bulge tests.

7. Summary
A new method for stress correction based on artificial intelligence and finite element simulations is proposed. An artificial intelligence model predicts a stress correction factor on the basis of geometric data of the necking area of a tensile specimen. 500 finite element simulations using the Swift hardening law were carried out and used as database for the artificial neural network models. For the network topologies multilayer perceptrons and convolutional neural networks a hyperparameter optimization was carried out. The performance of the best model of each topology was evaluated and compared. The proposed method shows promising results on the synthetic data and is also usable on data from real tensile tests.

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