STUDY OF BARYON NUMBER AND LEPTON FLAVOUR VIOLATION IN THE NEW MINIMAL SUPERSYMMETRIC SO(10)GUT

A THESIS
Submitted to the
FACULTY OF SCIENCE
PANJAB UNIVERSITY, CHANDIGARH
for the degree of
DOCTOR OF PHILOSOPHY

2014

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Dedicated To My Parents
Acknowledgements

First and foremost I would like to express my sincere gratitude to my advisor Prof. C. S. Aulakh for his guidance, consistent support, patience and encouragement during the period of my research work. His tireless efforts to do research acted as an inspiration for me.

I am indebted to all my teachers who always persuaded me to acquire higher education. I owe my thanks to my family for their unconditional support and encouragement throughout my Ph.D. I express my special thanks to my brother Gurpal Singh Khosa for bearing the brunt of my frustration and rages at times, for always understanding my work priorities, for maintaining my link with basic physics and for providing me homely environment.

I am thankful to my fellow labmate Ila Garg for various stimulating discussions, constructive criticism, companionship at the oddest of times.

I am grateful to the chairperson of Department of Physics, P.U. Chandigarh for providing me required facilities to work. I would like to acknowledge University Grants Commission, India for financial support during my Ph.D.

I am thankful to my senior Dr. Rama Gupta for guiding me time to time. There are countless contributors to whom I am indebted but few whom I could not skip to mention who anticipated my work and were always there for me whenever I needed them are Siman Dhillon, Amanpreet Chahal, Khushwant Chahal, Manbir Kaur, Jagdish Kaur, Harleen Asees Gill, Rajni Bansal, Arshdeep and Jashanpreet Kaur Bath.

Above all I am grateful to Mighty God for blessing me with great opportunities and keeping me motivated to have the enthusiastic attitude towards research.
Abstract

We study baryon number (B) and lepton flavour violation (LFV) in a supersymmetric model based on SO(10) gauge group called New Minimal Supersymmetric SO(10) Grand Unified Theory (NMSGUT).

We calculated one loop GUT scale threshold corrections to the relation between the NMSGUT and effective minimal supersymmetric standard model (MSSM) Yukawa couplings. Strong renormalization of the Higgs line entering these vertices, due to the large number of GUT fields coupled to them allows lowering of the SO(10) couplings required for fitting MSSM couplings. Since the same SO(10) Yukawas are responsible for B-violation, proton decay lifetimes compatible with experimental limits are generically achievable. We successfully searched the NMSGUT parameter space for values that allowed accurate fits of known MSSM couplings and have acceptable dimension five operator mediated B violation rates.

The spectra of sparticles used in the B violation calculations were improved by including one loop corrections. Searches including these corrections require careful control to avoid instability. We found fits compatible with the MSSM data and B violation limits even after inclusion of loop corrections.

The effective theory of the NMSGUT includes lepton number violating couplings and thus our fits also imply predictions for LFV rates. We computed NMSGUT estimates- based on successful fits- for important observables in the lepton sector such as lepton flavour violating processes (e.g. $l_i \rightarrow l_j \gamma, l_i \rightarrow 3l_j$), the muon g-2 anomaly ($a_\mu$) and the CP violation parameter ($\epsilon_{CP}$) relevant for high scale leptogenesis scenarios. For LFV estimation we have included heavy right handed neutrino thresholds.

We computed the two loop renormalization group evolution equations of the NMSGUT hard and soft supersymmetry breaking parameters. These equations are useful for running the parameters from Planck scale ($M_P$) to the unification scale ($M_X^0$). With a randomly chosen set of couplings at $M_P$, we run down them to $M_X^0$ and find a significant variation in the soft parameters. These changes could explain crucial features such as negative non universal Higgs mass squared values needed by the NMSGUT for successful fitting of fermion Yukawas.
We propose generation of the Standard Model fermion hierarchy by extension of the NMSGUT with $O(N_g)$ family gauge symmetry. In this scenario Higgs representations of SO(10) also carry family indices and are called Yukawons. VEVs of these Yukawon fields break GUT and family symmetry and generate MSSM Yukawa couplings dynamically. As in the NMSGUT, the effective MSSM matter fermion couplings to the light Higgs pair are determined by the null eigenvectors of the MSSM type Higgs doublet superfield mass matrix $\mathcal{H}$. A consistency condition on the doublet ([1, 2, ±1]) mass matrix ($\text{Det}(\mathcal{H}) = 0$) is required to keep one pair of Higgs doublets light in the effective MSSM. We show that the Yukawa structure generated by null eigenvectors of $\mathcal{H}$ are of generic kind required by the MSSM. We studied a toy model with two generations as well as the realistic three generation ($N_g = 3$) case. We considered a number of generic possibilities, with random GUT scale parameters, which produce acceptable Yukawa eigenvalues and lepton and quark mixing angles, but small neutrino masses. This justifies searches for realistic dynamical fermion fits in the future by generalizing the programs and techniques used for the NMSGUT.
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Chapter 1

Introduction

1.1 Standard Model and Beyond

The Standard Model (SM) supplemented by effective operators describing small neutrino masses is the established model of particle physics whose predictions have been tested experimentally up to high accuracy. It is a renormalizable, spontaneously broken chiral Yang Mills quantum field theory describing strong and electroweak interactions, based upon the principle of local gauge invariance with the gauge group $G_{SM} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$ and 3 families of 15 chiral fermion fields describing the known matter particles and antiparticles. Corresponding to this gauge group there are twelve gauge bosons $(G^a, W^i, B)$ out of which three become massive after spontaneous symmetry breaking of the gauge group

$$SU(3)_C \times SU(2)_L \times U(1)_Y \longrightarrow SU(3)_C \times U(1)_{em}$$

due to the vacuum expectation value (VEV) of Higgs field $\Phi(1,2,1)$

$$\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} ; \quad \langle \Phi \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$ \hspace{1cm} (1.1)

where $v=246$ GeV. Three linear combinations of the four $SU(2)_L \times U(1)_Y$ gauge bosons $(W^i, B)$, $W^\pm \left( \frac{W_1 \pm iW_2}{\sqrt{2}} \right)$ and $Z \left( c_W W^3 - s_W B \right)$ acquire mass while the other
orthogonal combination of $W_3$ and $B$- i.e. $s_W W^3 + c_W B$ remains massless and is identified as the photon (the gauge boson of the electromagnetic interactions). Here $W^i (i = 1, 2, 3)$ and $B$ denote the gauge boson of $SU(2)_L$ and $U(1)_Y$ respectively, $c_W(s_W) = \cos \theta_W (\sin \theta_W)$, $\theta_W$ is Weinberg angle, defined as $\tan \theta_W = \frac{g'}{g} (g'$ and $g$ are gauge couplings of $U(1)_Y$ and $SU(2)_L$ respectively). 8 gluons ($G^a, a = 1...8$) corresponding to unbroken $SU(3)_C$ remain massless. Fermionic matter consists of three generations, embedded in the SM gauge group with following quantum numbers:

\[
Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad ; \quad L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}
\]

\[
Q_{LA}(3, 2, \frac{1}{3}), u_{RA}(3, 1, \frac{4}{3}), d_{RA}(3, 1, -\frac{2}{3}), L_{LA}(1, 2, -1), e_{RA}(1, 1, -2) \quad (1.2)
\]

where A= 1, 2, 3 is the generation index. Clearly, all the fermion fields are present in chiral pairs (i.e. both $\Psi_L$ and $\Psi_R$) except the neutral fermion $\nu_L$. Dirac mass terms like $m_\psi \bar{\psi} \psi$ are not allowed by gauge invariance. However, charged fermion masses are generated via the Yukawa couplings of the fermions with the Higgs doublet since the Higgs acquire a VEV. Visible, stable, matter content of the universe is thought to be made of first generation fermions (except for $\nu_\mu, \nu_\tau$ which persist) since the heavier generations decay to them with lifetimes shorter than $10^{-10}$s.

Discovery of Higgs boson at Large Hadron Collider (LHC) \cite{1, 2} confirmed the existence of all the SM particles. Although the SM seems to be a very successful theory but there seems to be no justification for its basic assumptions like the existence of arbitrary Yukawa couplings. There is no profound explanation for the origin of families and their observed mixing structure. Furthermore, charge quantization remains unexplained. The fourth fundamental interaction- gravity- is not included. Besides these structural defects other flaws of the SM are the following unexplained experimental observations:

1. Neutrino oscillations indicate non zero neutrino masses \cite{3, 4, 5, 6, 7, 8}. Although neutrino masses can be included via (non-renormalizable) dimension $\geq 5$ operators \cite{9} in the effective Lagrangian, there is no way to generate neutrino masses in the renormalizable SM without any extension. This indicates SM is incomplete.
2. SM does not explain observed baryon asymmetry of the universe: $n_B/n_{\gamma} \sim 10^{-10}$.

3. Only 4% of the universe is visible matter, while the remaining 96% is dark matter (DM) and dark energy. SM has no DM candidate. Dark energy is an even greater mystery.

Answers to these questions require physics beyond SM. Therefore in spite of its experimental successes, the SM suffers from a number of limitations, and can’t be an ultimate theory of nature. It is only an effective theory at low energies ($\leq 200$ GeV) of some more profound and complete theory. For instance one can generate tiny neutrino masses through seesaw mechanism by extending the particle content of SM [10]. Depending upon the nature of additional particle one can have Type I [10], II [11] or III [12] seesaw contribution. Type I seesaw requires addition of a gauge singlet right handed (“sterile”) neutrinos. Type II and III need Higgs and fermion which are triplet irreducible representations (irreps) (of $SU(2)_L$) respectively. Several beyond SM approaches like Supersymmetry (Susy), Grand Unified Theories (GUT), Extra Dimension, String Theory, Technicolor Model and 4th generation model etc. have been considered. String theory is a theoretical tool which replaces point particle by one dimensional string whose excitations are the usual point particle fields. Extra dimensions models are based on the assumption that real world is higher dimensional and its extra spatial dimensions are compact. Susy and GUT together provide an attractive framework which has explanation of most of the above mentioned open questions. From now onwards we will focus on this approach.

1.2 Supersymmetric Grand Unification

1.2.1 Supersymmetry

Supersymmetry is the most appealing extension of the SM. It relates the existence of the fermions with bosons and vice versa. Each SM particle has its partner called superpartner which has the same quantum numbers as the particle except the spin which differs by half. Certain relations among the allowed coupling constants ensure
the invariance of the actions under transformations with anti-commuting Lorentz spinor parameter of Susy [13]. This leads to conserved supercurrents and Noether supercurrents that are constants of motion. Higgs mass is sensitive to radiative corrections from new physics which vary quadratically with the scale of heavy particles associated with the new physics. Stabilization of Higgs mass was the main motivation to introduce Susy. It stabilizes Higgs mass against radiative corrections by cancelling the loop contribution of a particle with the contribution from its superpartner [14]. Supersymmetric version of the SM is called Minimal Supersymmetric Standard Model (MSSM). Superpartners of the SM fermions are scalars, called s-particles while that of gauge and Higgs bosons are fermions called gaugino and higgsino respectively. These gauginos and higgsinos mix to form neutralino and chargino states. The field content of the MSSM is given in Table 1.1. Renormalization Group (RG) evolved gauge couplings of the MSSM accurately unify at GUT scale of order of $10^{16}$GeV which has been a strong motivational hint to study Susy.

Both fermions and bosons are placed into an irreducible representation of Susy algebra called supermultiplet. Supersymmetric gauge theories typically use chiral and vector supermultiplets. Interaction and mass terms for the various superfields are described by analytic function (of chiral superfields $\Phi$), called superpotential:

$$W = \frac{M^{ij}}{2} \Phi_i \Phi_j + \frac{y^{ijk}}{6} \Phi_i \Phi_j \Phi_k \quad (1.3)$$

where $M^{ij}$ is the mass matrix and $y^{ijk}$ is the Yukawa coupling. Scalar potential is computed from the superpotential:

$$V = F^{*i}F_i + \frac{1}{2} \sum_a D^a D^a; \quad F^i = \frac{\partial W}{\partial \phi_i}; \quad D^a = g^a \phi^j T^a \phi \quad (1.4)$$

Index $a$ run over the adjoint representation of the group so the corresponding term exhibit gauge interactions.

Clearly, Susy is broken in nature because none of the Susy particles has been observed till date. Even without being exact symmetry, Susy can solve the hierarchy problem in a elegant way provided Susy is softly broken. ‘Soft breaking’ means that the symmetry breaking terms are super-renormalizable (i.e. mass or scalar...
trilinear terms). Soft mass terms are introduced by hand in the Lagrangian ($L_{\text{soft}}$) to distinguish the mass of Susy particles from their SM partners. Supersymmetric Lagrangian is given by

$$L = L_{\text{Susy}} + L_{\text{soft}}$$ (1.5)

Here $L_{\text{Susy}}(\phi, \psi, A_\mu, \lambda, F, D)$ is globally supersymmetric action coupling matter and gauge fields to each other and their superpartners. The generic form is:

$$L_{\text{Susy}} = D^\mu \phi^* D_\mu \phi + \bar{\psi} i \gamma^\mu D_\mu \psi + i g_a \sqrt{2} \phi^* T^a \lambda^a \psi$$

$$+ \frac{g_a^2}{2} D^2_a - \left\lvert \frac{\partial W}{\partial \phi} \right\rvert^2 - \frac{1}{2} \frac{\partial^2 W}{\partial \phi^2} \psi \psi + h.c.$$ (1.6)

We follow notation of [15, 16] (see these Refs. for reviews). $L_{\text{soft}}(\phi, \lambda)$ is the Susy violating part having the generic form of gaugino mass, scalar bilinear and trilinear terms:

$$L_{\text{soft}} = M \lambda \lambda + m_\phi^2 |\phi|^2 + (A_0 W(\phi) + h.c.)$$ (1.7)

Note that all the parameters ($M, m_\phi, A_0$) in $L_{\text{soft}}$ have mass dimensions. If one consider the different soft masses and trilinear couplings for the MSSM scalars and gaugino masses then one would end up with 105 free parameters [17] and this scenario is called unconstrained MSSM. However it is known that flavour violation in the soft breaking terms must be nearly absent to avoid disastrous levels of flavour changing interactions not observed at low energy [18]. There are various theoretical mechanisms that may explain Susy breaking like gravity [19, 20], gauge [21] and anomaly [22, 23] mediation scenario which provide some kind of flavour blind boundary conditions at GUT scale. As MSSM is an effective theory of the GUT, soft parameters are evolved from the GUT scale down to the electroweak scale. Model with universal boundary conditions for soft sector parameters is called constrained MSSM. Besides constrained and unconstrained model, phenomenologically more predictive model based upon the assumptions that the CKM matrix is the only source of CP violation and flavour mixing, diagonal soft masses and trilinear couplings with degenerate first two generation, has been studied extensively. These assumptions reduce the parameters to 19 and the model is called phenomenological MSSM [24]. The superpotential of the MSSM is:
1.2 Supersymmetric Grand Unification

| Particle | Quarks | L = \left( \begin{array}{c} u_L \cr d_L \end{array} \right) |
|----------|--------|-------------|
|          | Squarks| \tilde{Q} = \left( \begin{array}{c} \tilde{u}_L \\
|          |        | \tilde{d}_L \end{array} \right) |
|          | Scalars| \tilde{L} = \left( \begin{array}{c} \tilde{\nu}_L \\
|          |        | \tilde{e}_L \end{array} \right) |
|          | Scalars| \tilde{S} = \left( \begin{array}{c} \tilde{\nu}_L \\
|          |        | \tilde{e}_L \end{array} \right) |
| Gauge Boson | Gleno | \tilde{W}^+_L, \tilde{W}_3 |
| Higgs Boson | Higgs | \tilde{H}_u, \tilde{H}_d |

Table 1.1: MSSM fields

\[ W_{MSSM} = \bar{u}Y_uQH_u + \bar{d}Y_dQH_d + \bar{e}Y_eLH_d + \mu H_uH_d \tag{1.8} \]

Here \( Q, L, \bar{u}, \bar{d}, \bar{e} \) are the chiral superfields introduced in the Table 1.1. The parameters \( Y_u, Y_d \) and \( Y_e \) are Yukawa couplings of up type quark, down type quark and charge leptons respectively. \( H_u \) and \( H_d \) are the Higgs doublets whose VEVs \((v_u, v_d, \tan \beta = \frac{v_u}{v_d})\) generate mass for the up and down type fermions. The first three terms are the familiar SM Yukawa interactions while the last term is the Higgs mixing or \( \mu \) term. Besides these terms in the superpotential the following baryon number (B) and lepton number (L) violating terms are also allowed by gauge invariance (but are excluded from the MSSM on phenomenological grounds)

\[ W_{\Delta B/\Delta L = 1} = \frac{\lambda_{ijk}}{2} L_iL_j\bar{e}_k \] \[ + \frac{\lambda'_{ijk}}{2} Q_iQ_j\bar{d}_k \] \[ + \mu' L_iH_u \] \[ + \frac{\lambda''_{ijk}}{2} \bar{u}_i\bar{d}_j\bar{d}_k \] \[ \tag{1.9} \]

These operators imply fast proton decay rate unless the couplings are suppressed (e.g. \(|\lambda'\lambda''| \leq 10^{-24}\left(\frac{M_{\text{Susy}}}{100 \text{ GeV}}\right)^2\)). To forbid these \( d = 4 \) operators, a discrete symmetry known as R-parity is introduced:

\[ R = (-1)^{3(B-L)+2s} \tag{1.10} \]
Here \( s \) is the spin of the particle. SM particles have \( R=1 \) and Susy particles have -1 value. Exact conservation of R-parity require each vertex must have even number of \( R = -1 \) particle, so \( W_{\Delta B/\Delta L=1} \) is not allowed. Further, the “lightest supersymmetric particle (LSP)” is stable by R-parity conservation and proves a suitable “weakly interacting massive particle (WIMP)” dark matter candidate. This is a crucial phenomenological virtue of R-parity conserving MSSM.

1.2.2 Grand Unified Theories

The quest for unification started back in the 19\(^{th}\) century with the unification of electric and magnetic forces as electromagnetic force by Clerk Maxwell. Unification of weak and electromagnetic interaction is successfully achieved in the SM but leaves many unanswered questions. Unification of all the known interactions except gravity is known as Grand Unified Theory. It offers a framework to solve many of the shortcomings of the SM. The first evidence for physics beyond SM came with discovery of neutrino masses implied by neutrino oscillation data [3, 4, 5, 6, 7, 8] which require theoretical explanation. The seesaw mechanism [10] is the most appealing mechanism for explaining smallness of neutrino masses although simply tuning of Yukawa couplings is also still viable. It relates smallness of neutrino masses with the existence of heavy particles. It provides some hint of high scale physics. Essentially the d=5 neutrino mass or Weinberg operator [9] and its higher dimension generalization are generated when heavy right handed neutrino (\( \bar{\nu}(1,1,0) \)) are integrated out (Type I) or a heavy triplet scalar mass suppress a neutrino mass inducing VEV (Type II) and so on. Nearly exact unification of the RG evolved three MSSM gauge couplings at high scale \( \sim 10^{16} \text{ GeV} \) [25, 26] is strongest motivation to study GUTs. All these clues reinforced the proposal [27, 28] to search for the larger gauge symmetry of the nature represented by some higher group whose effective theory is the SM. The basic idea is to unify gauge as well as matter content to reduce arbitrariness of SM. \( G_{SM} \subset G_{GUT} \), so GUT group should be of rank \( \geq 4 \).

In 1974, Pati-Salam proposed the first-ever GUT based upon \( G_{PS} \equiv SU(4) \times SU(2)_L \times SU(2)_R \) gauge group [27]. \( G_{PS} \) contains the left-right symmetric gauge
Supersymmetric Grand Unification

The group $G_{LR} \equiv SU(3) \times U(1)_{B-L} \times SU(2)_L \times SU(2)_R$ which breaks to SM. Left-right (LR) symmetric models offer an appealing understanding of the origin of parity violation in the SM. LR symmetry requires existence of the right handed neutrino. If the Majorana mass of $\bar{\nu}$ is large a small neutrino mass is generated through seesaw mechanism \[10\]. SM fermion and $\bar{\nu}$ are embedded in the LR models as

\[
\psi_{\mu \alpha}(4, 2, 1) = Q(3, 2, 1/3) + L(1, 2, -1) \\
\hat{\psi}_\alpha^\mu(\bar{4}, 1, \bar{2}) = \bar{d}(3, 1, 2/3) + \bar{u}(3, 1, -4/3) + \bar{e}(1, 1, 2) + \bar{\nu}(1, 1, 0)
\]

where L, Q, $\bar{e}$, $\bar{d}$, $\bar{u}$ are written with their SM quantum numbers and the indices $\mu$, $\alpha$, $\dot{\alpha}$ refer to $SU(4)$, $SU(2)_L$ and $SU(2)_R$ respectively. 4-plet of SU(4) treats lepton as a fourth color. Higgs triplets ($\Delta_L(1, 3, 1)$ and $\Delta_R(1, 1, 3)$) or doublets ($\chi_L(1, 2, 1) \oplus \chi_R(1, 1, 2)$) and bidoublet ($\phi(1, 2, 2)$) can implement symmetry breaking

\[
G_{LR} \xrightarrow{(\Delta_R) \text{ or } (\chi_R)} G_{SM} \xrightarrow{(\phi) \, (\chi_L)} SU(3)_c \times U(1)_{em}
\]

$W_L$ and $W_R$ are gauge bosons corresponding to $SU(2)_L$ and $SU(2)_R$ respectively with $m_{W_R} >> m_{W_L}$ as these are missing experimental signature. Electric charge is given by

\[
Q_{em} = T_{3L} + T_{3R} + \frac{B - L}{2}
\]

Although $G_{PS}$ unified matter content but still we have 3 gauge couplings, therefore no reduction in number of gauge parameters.

Soon after Pati-Salam, Georgi and Glashow proposed single gauge group GUT $SU(5)$ \[28\] which can embed SM gauge group. This is the smallest gauge group (rank 4) which provides unification of gauge interactions. SM fermions are embedded in the 5 dim and 10-plet (2 index antisymmetric) of SU(5) as

\[
5 = \{L, \bar{d}\} \quad ; \quad 10 = \{Q, \bar{e}, \bar{u}\}
\]

SU(5) group is simple, so it explains charge quantization. It has 24 gauge bosons
which transform under the maximal subgroup $SU(3) \times SU(2) \times U(1)$ as :

$$24 = (8,1,0) + (1,3,0) + (1,1,0) + (3,2,-\frac{5}{3}) + (3^*,2,\frac{5}{3})$$  \hspace{1cm} \text{(1.14)}$$

Here $(8,1)$ are the SU(3) gluons, $(1,3)$ are SU(2) gauge bosons, $(1,1)$ is gauge boson of U(1) group. Remaining 12 ($(3,2)+(3^*,2)$) are new heavy $X$ and $Y$ gauge bosons (leptoquark) which are responsible for proton decay. The experimental lower limit on the life time of proton is more than $10^{33}$ yrs. This implies the mass of carriers $X, Y$ should be greater than $10^{15}$ GeV. Exchange of $X, Y$ or other Higgs leptoquark leads to violation of baryon and lepton number but B-L is conserved. SU(5) symmetry spontaneously breaks to SM by 24-plet Higgs field ($\Sigma$), while the SM doublets lie in a 5-plet ($H$) of SU(5) :

$$SU(5) \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_c \times U(1)_{em}$$

SU(5) accommodates SM fermions of the same family in different representations ($\{ L, \bar{d} \} \in 5; \{ \bar{Q}, \bar{e}, \bar{u} \} \in 10$). It predicts equal Yukawa couplings for down quark and charged leptons which is not true for first two generations. The original model of Georgi-Glashow fails to produce neutrino masses. Realistic SU(5) models can be build by addition of $24_F$ [29, 30] or $15_H$ [31, 32] multiplets as well as a (singlet) right handed neutrino. Since it is a SU(5) singlet the gauge symmetry does not offer any relation between neutrino and charged lepton masses. Then one can produce neutrino masses through seesaw mechanism (Type I and III in case of $24_F$, Type II in $15_H$ and Type I with $\bar{\nu}$).

SO(10) is a rank 5 GUT [33] candidate which offers unification of matter as well as of gauge interactions. The embedding chain $SU(5) \times U(1) \subset SO(10) \rightarrow SO(10) \times U(1) \subset E_6$ shows that $E_6$ is a “maximal” GUT gauge group [34]. This thesis is based upon a successful supersymmetric SO(10) model so we will elaborate SO(10) properties in detail in the following section.
1.3 SO(10) GUT

1.3.1 Group Theory Essentials

SO(10) is a special orthogonal group of rank 5 with 45 parameters. Group elements are generated from generators \((J)\)

\[ O = \exp \left( \frac{i}{2} \theta^{ij} J_{ij} \right) ; \quad i, j = 1...10 \]  

(1.15)

Antisymmetric generators in the fundamental 10-plet representation are:

\[ (J_{ij})_{kl} = -i \delta_{i|k} \delta_{j|l} \]  

(1.16)

here square bracket represents antisymmetrization and these generators obey algebra

\[ [J_{ij}, J_{kl}] = i \delta_{k|i} J_{j|l} - i \delta_{l|i} J_{j|k} \]  

(1.17)

The fundamental (10-plet) representation \(H_i\) transforms as :

\[ H_i' = O_{ij} H_j \]  

(1.18)

Taking tensor product of the fundamental representation one can form higher dimensional symmetric or anti-symmetric representation. For example 45, 120, 210, 54 are \((2, 3, 4)\) index antisymmetric and two index symmetric traceless vector representations respectively. 126 \((\Sigma)\) is self-dual 5 index anti-symmetric representation which requires special attention because it plays a crucial role in SO(10) model building specially for neutrino masses.

\[ \tilde{\Sigma}_{i_1...i_5} = -\frac{i}{5!} \epsilon_{i_1...i_{10}} \Sigma_{i_6...i_{10}} ; \quad \tilde{\Sigma} = \Sigma \]  

(1.19)

Similarly one can project out the anti self-dual \(\overline{126}\) :

\[ \tilde{\Sigma} = - \Sigma \]  

(1.20)
Apart from tensor representations, orthogonal groups have spinor representations generated using Clifford algebra of the $2^N$ ($N=5$ for SO(10)) dimensional $\Gamma_i$ matrices:

$$\{\Gamma_i, \Gamma_j\} = 2\delta_{ij} \quad (1.21)$$

From these generators one constructs:

$$\Sigma_{ij} = \frac{[\Gamma_i, \Gamma_j]}{4i} \quad (1.22)$$

The generators $\Sigma_{ij}$ obey the SO(10) commutation algebra (see Eq. (1.17)). The explicit form of $\Gamma$ and hence $\Sigma$ matrices can be found in [35, 36]. Spinor representation of SO(2N) is $2^N$ dimensional (32 dim for SO(10)) which transform as

$$\Psi' = \exp(-i\theta_{ij}\Sigma_{ij})\Psi \quad (1.23)$$

Irreducible 16($\psi$) dimensional spinor representation is constructed using projectors:

$$\psi = \frac{1+\Gamma_{FIVE}}{2}\Psi \quad ; \quad \bar{\psi} = \frac{1-\Gamma_{FIVE}}{2}\Psi \quad ; \quad \Gamma_{FIVE} = i\Gamma_2\Gamma_4...\Gamma_{10} \quad (1.24)$$

By taking direct products with tensors, spinor representation lead to an additional class of (“double valued”) representations.

We strictly follow the notations of Ref. [36] throughout the thesis. For completeness we mention the most frequently used ones: $a,b,c(1..6)$ and $\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}(1..4)$ are SO(6) and SO(4) indices respectively, $\mu, \nu, \lambda...$ represent SU(4) indices and run from 1 to 4; $\bar{\mu}, \bar{\nu}, \bar{\lambda}$ run over the color subgroup (1 to 3) of SU(4); $\alpha, \beta...(\check{\alpha}, \check{\beta}...)$ denote $SU(2)_L(SU(2)_R)$ doublet indices and vary from 1 to 2; A, B... and i, j, k.. are SO(10) spinor and vector indices and run from 1 to 16 and 1 to 10 respectively; A, B, C=1...3 are also used for family indices.

1.3.2 Virtues of Supersymmetric SO(10) GUT

- RG evolved gauge couplings of MSSM accurately unify at $M_{GUT} \sim 10^{16.25}$ GeV.
• It is a natural home for Type I and Type II seesaw mechanism which generate neutrino masses in the milli-eV range, via high scale B-L breaking, without any tuning of Yukawa couplings as is required when the B-L breaking scale is small. This follows since minimal Susy SO(10) embeds the minimal Susy LR models \cite{37, 38, 39, 40, 41} which have high scale breaking of B-L symmetry.

• Large tan $\beta$ SO(10) models provide third generation Yukawa unification \cite{42}. Type II seesaw dominated models relate atmospheric neutrino mixing angle with $b - \tau$ unification \cite{43}.

• Another important aspect of SO(10) gauge group is that M-parity \((-1)^{3(B-L)}\) is effectively part of SO(10) gauge symmetry since $U(1)_{B-L} \subset SO(10)$. It can be preserved \cite{39, 40, 44} till low energy with suitable choice of VEV of Higgs field. Using only B-L even VEVs, R/M- parity preservation ensures stable LSP which can act as a cold dark matter candidate.

• Observed baryon asymmetry of the universe can be understood via leptogenesis which explains baryogenesis through sphaleron processing of a lepton asymmetry created in L and CP violating decays of heavy neutrino.

### 1.3.3 Model Building

As mentioned spinor representation of an orthogonal group provides special motivation to study SO(10) GUT. 16-plet of SO(10) can accommodate exactly 15 fermions of one SM generation along with right handed neutrino. 16-plet decompose under two maximal subgroups of SO(10) : $G_{PS}$ and $SU(5) \times U(1)$ as :

$$16 = \psi_{\mu\alpha}(4, 2, 1) + \psi_{\dot{\alpha}}^\mu(\bar{4}, 1, 2) = 10 + \bar{5} + 1$$ (1.25)

From Eq. 1.11 we know how SM fermion and right handed neutrino are embedded in $\psi_{\mu\alpha}$ and $\psi_{\dot{\alpha}}^\mu$. As all the matter fields are present in a single irreducible representation thus gauge interactions in SO(10) conserve parity. SO(10) has 45 gauge bosons which decompose under the Pati-Salam group as

$$45 = (15, 1, 1) + (1, 3, 1) + (1, 1, 3) + (6, 2, 2)$$ (1.26)
1.3 SO(10) GUT

An additional 33 gauge bosons (besides the 12 present in the SM) called leptoquarks, mediate B and L violating interactions. One can have different symmetry breaking chains via the two maximal subgroups:

\[
SO(10) \rightarrow \begin{cases} 
SU(5) \times U(1) \rightarrow SU(5) \rightarrow G_{SM} \\
G_{PS} \rightarrow G_{LR} \rightarrow G_{SM}
\end{cases}
\] (1.27)

These breaking chains proceed via different Higgs sectors. As clear from the \(G_{PS}\) and SU(5) GUT, spontaneous symmetry breaking of the larger unified group to SM gauge group requires different higher Higgs representation depending upon the group under consideration. Choice of different combination of Higgs irreps give different SO(10) models. Possible choices are 45, 54, 126, 210 Higgs irreps which contain MSSM singlet. Further, Higgs content of the model is chosen not just to break the GUT symmetry but it should also able to produce realistic fermion mass mixing data. The tensor product of two 16-plets is

\[
16 \otimes 16 = 10 \oplus 126 \oplus 120
\] (1.28)

Since 10, 120 irreps are real and 126 is complex, the above tensor decomposition suggests that only 10, \(\overline{126}\) and 120 Higgs irreps can couple to matter bilinear at SO(10) Yukawa vertex.

There are two main classes of SO(10) GUTs distinguished by whether they use doublets or triplets to break the right handed gauge group \(SU(2)_R\) and whether the seesaw is renormalizable or not. Model builders considered small representations like 10, 16, \(\overline{16}\), 45 [45], out of these 45, 16 and \(\overline{16}\) break SO(10) symmetry to MSSM and 10-plet is required for electroweak symmetry breaking. This model can not produce realistic fermion masses without using non-renormalizable operators. In the renormalizable regime one needs to use large representations like \(\overline{126}\). Further other higher Higgs irreps like 210 or 54 are required for gauge symmetry breaking. In this scenario two possible Higgs sets sufficient for spontaneous symmetry breaking to MSSM are 210 \(\oplus\) 126 \(\oplus\) \(\overline{126}\) and 54 \(\oplus\) 45 \(\oplus\) 126 \(\oplus\) \(\overline{126}\). In particular, the model based upon 10, 126, \(\overline{126}\), 210 Higgs representations has minimum number of parameters and is under development since 1982 [46, 47], was named
as Minimal Supersymmetric Grand Unified Theory (MSGUT)\footnote{48}. $126, \overline{126}, 210$ Higgs irreps break symmetry to the MSSM and $10, \overline{126}$ generate fermion masses.

Since 2000\footnote{36, 49} a conversion of SO(10) tensor, spinor representation and their invariants in terms of unitary subgroups $G_{PS}$ and $SU(5) \times U(1)$ has facilitated model building in SO(10). Recently a SO(10) model using only single pair of irreducible Higgs representation ($144 + 144$) is proposed\footnote{50, 51}. $144$ irrep has both vector and spinor index. Adjoint and 5-plet of SU(5) contained in $144$ break the gauge symmetry to $SU(3)_c \times U(1)_{em}$. Quartic coupling of Higgs and matter ($16.16.144.144$) generates fermion masses.

Babu-Mohapatra proposal that $10, \overline{126}$ Higgs can completely determine the fermion Yukawa couplings\footnote{52}, triggered intense interest in fermion fitting in SO(10) models\footnote{43, 53}. MSGUT, a fully specified theory with only 26 real (hard) parameter failed\footnote{54, 55, 56} to fit realistic fermion mass mixing data because Type I seesaw contribution which dominates over Type II seesaw yields too small neutrino masses. Faced with this impasse, Aulakh and Garg\footnote{55, 57} investigated the role of $120$ plet (which can couple to matter bilinear) in the context of its direct contribution to fermion masses. Earlier the $120$-plet was considered\footnote{56, 58, 59} mostly as a perturbation to $10, \overline{126}$, to suppress proton decay or to explain different quark and lepton mixing with arbitrary assumptions. The observation that MSGUT accompanied by the $120$-plet Higgs (which is the next to minimal candidate) where the $120$ and $10$-plet fit the charged fermion masses and the $126$ is freed to fit neutrino masses succeeded in achieving a realistic fit. Since the Type I seesaw neutrino masses are inversely proportional to the $126$ Yukawa coupling, the freed (to be small) $126$-plet coupling enhances the Type I seesaw masses to viable values (Type II contribution gets further suppressed) allowing enough freedom to fit all the fermion mass and mixing data (the d, s quark Yukawa couplings require special treatment and this yields important information on sparticle spectra). The small $126$ coupling provide right handed neutrino masses in a leptogenesis\footnote{60} compatible range ($10^8 - 10^{12}$ GeV). In this way, the GUT based upon the $210 \oplus 10 \oplus 120 \oplus 126 \oplus 126$ Higgs irreps, known as a New Minimal Supersymmetric SO(10) GUT (NMSGUT), emerged as a realistic GUT.
1.4 Thesis Outline

This thesis is a report on development of a realistic Susy SO(10) model called NMSGUT. The principal new contributions have made are the following:

- The NMSGUT \cite{57} while able to fit the MSSM fermion hierarchy and predict specific testable super spectra, yields a proton decay life time $\sim 10^{27}$ yrs if threshold corrections due to $\sim 700$ superheavy fields are ignored. We calculated one loop corrections to the effective MSSM Yukawa vertices. We found that the tree level relation between MSSM and Yukawa coupling is strongly renormalized due to the large number of fields and couplings renormalizing the MSSM Higgs field. This allows natural suppression to $\tau_p > 10^{34}$ yrs on the “Higgs dissolution edge” ($Z_{H,R} \approx 0$) in GUT parameter space.

- In Ref. \cite{57} tree level sparticle spectrum were used. One loop corrections to sparticle masses can be large specially for the minisplit Susy spectra with large $A_0$, $\mu$ and $M_A$ parameters found in \cite{57}. We incorporate these corrections in the search program.

- Since the NMSGUT generates neutrino masses from B-L violating VEVs and the GUT scale slepton soft masses and trilinear couplings are renormalized by loop corrections and right handed neutrino thresholds one expects \cite{61, 62, 63} significant lepton flavour violation (LFV) in the benchmark observable like $l_i \rightarrow l_j \gamma$ rates. We calculate these and other lepton sector predictions of the NMSGUT.

- One expects the soft Susy parameters to obey GUT relations at some high scale (e.g. Planck scale or string scale) which need not coincide with the MSSM coupling unification scale $M_X^0 = 10^{16.33}$ GeV. Thus the RG equations predicting flow of NMSGUT couplings between these scales should be calculated and used to improve the estimate of plausible soft parameter values at $M_X^0$. We calculated the complete two loop NMSGUT RG equations for soft and hard parameters and quote the hard parameter equations in the thesis. The soft parameter RG equations are available in \cite{64, 65}.
1.4 Thesis Outline

- The successful fitting of the fermion hierarchy in the NMSGUT naturally motivates attempts at flavour unification which generate the successful NMSGUT and ultimately MSSM fermion couplings dynamically. We have proposed a novel dynamical scenario implementing this idea using the experience gained from unifying the MSSM fermion hierarchy in the NMSGUT.

Our aim is to check the model compatibility with experimental data. First of all successful GUT should be able to fit the SM fermion mass mixing data \( (m_{q,i}, \theta_{12,23,13}^{\text{CKM}}, \delta_{12,23,13}^{\text{CKM}}, \theta_{12,23,13}^{\text{PMNS}}, \delta_{12,23,13}^{\text{PMNS}}, \Delta m_{s}^{2}) \). Proton decay is a peculiarity of GUT so it becomes a fundamental test. In the lepton sector the experimental upper bound \([66, 67, 68]\) for the branching ratio of lepton flavour violating decays \( \mu \rightarrow e\gamma, \tau \rightarrow \mu\gamma \), etc. and the muon anomaly \([69]\) require thorough investigation of how these limits constrain the parameter space of the model. We will discuss all these issues chapterwise.

In Chapter 2 we review the structure of the model, in particular, fermion mass generation, effect of superheavy thresholds on gauge couplings and FORTRAN fitting program algorithm. Tree level Susy spectrum is presented in the Appendix. In Chapter 3 a generic mechanism is introduced to suppress fast \( d=5 \), baryon decay in SO(10) GUT with an example solution. Appendix contains detailed formulae for Higgs renormalization factors. Loop corrected Susy spectrum is presented in the Chapter 4 with approximate formulae which clarify the dominant contributions and fits incorporating these loop corrections. Chapter 5 is devoted to the lepton sector phenomenological implications of the model which includes \( \Delta F=1 \) LFV processes, \( \Delta F=0 \) \( (a_{\mu}) \) and calculation of leptogenesis parameters. In Chapter 6, we present the SO(10) renormalization group equations for the NMSGUT soft and hard parameters. In Chapter 7, dynamical Yukawa generation in SO(10) GUTs extended with the family group \( O(N_{g}) \) is discussed. In Chapter 8, we summarize our work and conclusions. We also indicate avenues for further research.
Chapter 2

New Minimal Supersymmetric SO(10) Grand Unified Theory

2.1 Introduction

The so called “NMSGUT” is a renormalizable SO(10) supersymmetric grand unified theory based upon $10(H) \oplus 120(\Theta) \oplus 126(\Sigma) \oplus \overline{126}(\overline{\Sigma}) \oplus 210(\Phi)$ Higgs irreps. All the SM fermions along with the right handed neutrino are accommodated in three copies ($\psi_A$) of $16$-plet. $126$, $\overline{126}$, $210$ Higgs irreps participate in spontaneous symmetry breaking (SSB) from Susy SO(10) to MSSM in steps or at once and are therefore called adjoint type Higgs multiplets (AM). $10$, $\overline{126}$, $120$ Higgs irreps couple to matter bilinear to generate fermion masses and are hence called fermion mass (FM) type Higgs multiplets. $120$-plet has no MSSM singlet so it does not participate in symmetry breaking. Use of $126$ irrep in the GUT scale SSB offers automatic implementation of high scale Type I and Type II seesaw. $126$ is introduced to preserve Susy in the GUT scale SSB which exhibits crucial R-parity conservation (only B-L=2 even fields have VEVs). $10$ and $120$-plet are mainly responsible for generating charged fermion masses and small Yukawa coupling of $126$ is crucial for viable neutrino masses. The heavy right handed neutrino in range $10^{8-12}$ GeV (compatible with leptogenesis) and milli-eV neutrino masses as required by neutrino oscillations, through seesaw mechanism are achievable. Thus this model
is capable of fitting known SM fermion mass mixing data.

The structure of the theory includes its mass spectra, RG evolution, effective MSSM, B violation effective superpotential, threshold effects, fermion fitting etc. and has already been elucidated \[70\], \[48\], \[36\], \[54\], \[55\], \[57\]. Extensive computer codes were developed, incorporating the NMSGUT formulae to search the GUT parameter space for viable parameter sets \[71\]. In this chapter we review the structural features and predictions of the model. We include the description of how the GUT threshold corrections modify MSSM Yukawas that are incorporated in the NMSGUT code. The actual threshold corrections are given in the next chapter.

\section*{2.2 Structure}

\subsection*{2.2.1 Spontaneous Symmetry Breaking}

The superpotential of theory which involves Yukawa couplings for Higgs and matter fermions is given by

$$W_{\text{NMSGUT}} = \frac{1}{2} M_H H_i^2 + \frac{m}{4!} \Phi_{ijkl} \Phi_{ijkl} + \frac{\lambda}{4!} \Phi_{ijkl} \Phi_{klnm} \Phi_{mijn} + \frac{\eta}{4!} \Phi_{ijkl} \Sigma_{ijmno} \Sigma_{klmno}$$

$$+ \frac{M}{5!} \Sigma_{ijklm} \Sigma_{ijklm} + \frac{1}{4!} H_i \Phi_{jklm}(\gamma \Sigma_{ijklm} + \gamma \Sigma_{ijklm})$$

$$+ \frac{m_{\Theta}}{2(3!)} \Theta_{ijkl} \Theta_{ijkl} + \frac{k}{3!} \Theta_{ijkl} H_m \Phi_{mijn} + \frac{\rho}{4!} \Theta_{ijkl} \Theta_{mnk} \Phi_{ijmn}$$

$$+ \frac{1}{2(3!)} \Theta_{ijkl} \Phi_{klnm}(\zeta \Sigma_{lmnij} + \zeta \Sigma_{lmnij}) + h_{AB} \psi_A^T \psi_B^T \zeta_{ij} \gamma_i \psi_B H_i$$

$$+ \frac{1}{5!} \tilde{f}_{AB} \psi_A^T \psi_B \zeta_{ij} \gamma_i \psi_B \Sigma_{i1...i5} + \frac{1}{3!} \tilde{g}_{AB} \psi_A^T \psi_B \zeta_{ij} \gamma_i \gamma_j \psi_B \Theta_{ijkl} \psi_B \Theta_{ijkl}$$ \hspace{1cm} (2.1)

Here $h$, $f$ and $g$ are Yukawa couplings of $10$, $126$, $120$ Higgs. These are complex symmetric ($h$, $f$) and anti-symmetric ($g$) matrices in flavour space. We can diagonalize one out of these by performing U(3) rotations in the flavour space since the kinetic terms of the three $16$-plets are invariants under U(3). These Yukawas contribute 21 real parameters (real diagonal $h(3)+$ complex symmetric $f(12)+$ complex antisymmetric $g(6)$). In addition to these, superpotential has trilinear couplings ($\lambda, \eta, \gamma, \bar{\gamma}, k, \rho, \zeta, \bar{\zeta}$) and masses ($M_H, M, m_{\Theta}, m$) which contribute 24 parameters. In
total model has 45 real parameters out of which 2 can be fixed by using fine tuning condition for Higgs mass (which we will explain later) and 5 phases can be removed by redefining Higgs fields (therefore we choose real \((\gamma, \eta, m, M)\)). M is determined from \(\lambda, \eta, m, x\) \((M = \frac{\xi \eta m}{\lambda})\) and further \(\xi\) parameter is determined from \(x\) (solution of Eq. 2.5). We are left with 37 parameters. Although this seems a lot, it is minimal in comparison to any other SO(10) GUT which provides realistic fermion mass mixing data and experiment compatible B-decay rates. The GUT scale (SM neutral) VEVs that break the gauge symmetry down to the SM symmetry (in the notation of [36]) are

\[
\langle (15, 1, 1) \rangle_{210}, \quad \langle (15, 1, 3) \rangle_{210}, \quad \langle (1, 1, 1) \rangle_{210}, \quad \langle (10, 1, 3) \rangle_{126}, \quad \langle \Sigma \rangle_{126}, \quad \langle \Sigma \rangle_{126}.
\]

As a function of these VEVs the superpotential becomes

\[
W = m(p^2 + 3a^2 + 6\omega^2) + 2\lambda(a^3 + 3p\omega^2 + 6a\omega^2) + (M + \eta(p + 3a - 6\omega))\sigma\bar{\sigma}
\]

(2.3)

It is sufficient to calculate F terms from the above superpotential and investigate the conditions for them to vanish. The VEVs of 210 do not contribute to any D term leaving only \(D_{B-L} \sim (|\sigma|^2 - |\bar{\sigma}|^2)\) contribution to the D-term potential. Thus vanishing of F and D terms determine MSSM vacuum. Dimensionless VEVs (in units of \(m/\lambda\)) can be ensured by writing all VEVs in terms of single complex parameter \(x(= - \frac{\lambda \omega}{m} = -\bar{\omega})\):

\[
\tilde{p} = \frac{x(5x^2 - 1)}{(1 - x)^2}; \quad \tilde{a} = \frac{x^2 + 2x - 1}{1 - x}; \quad \bar{\sigma}\bar{\bar{\sigma}} = \frac{2\lambda x(1 + x^2)(1 - 3x)}{\eta(1 - x)^2}
\]

(2.4)

Note that \(\text{Arg}(\sigma) + \text{Arg}(\bar{\sigma})\) is B-L invariants while \(\text{Arg}(\sigma) - \text{Arg}(\bar{\sigma})\) can be set to zero by a B-L transformation. Thus effectively \(\sigma = \bar{\sigma}\). Then \(F_{p, a, \omega} = 0\) and vanishing of \(F_{\sigma, \bar{\sigma}}\) requires [70, 48, 72]:

\[
8x^3 - 15x^2 + 14x - 3 + \xi(1 - x)^2 = 0
\]

(2.5)
where $\xi = \frac{\lambda M}{\eta m}$. For each value of $\xi$, three solutions of $x$ are available. The complex parameter $x$ is used for systematic survey of parameter space of the model \cite{70, 54, 55} since its variation directly affects the VEVs and thus the masses in the theory and each value of $x$ fixes a unique $\xi$, whereas solving (Eq. 2.5) for $x$ given $\xi$ requires checking three solutions separately.

### 2.2.2 Superheavy Spectrum

SO(10) Higgs representations are decomposed into the SM gauge group representations by first decomposing into Pati-Salam (PS) labels. As an example we will discuss splitting of 10-plet. We first need to decompose 10-plet under $SO(6) \times SO(4)$ as:

$$10(H_i) = 6(H_a) + 4(H_{\tilde{\alpha}}) \quad (2.6)$$

Here $a, \tilde{\alpha}$ are SO(6) and SO(4) indices respectively. $SO(6) \sim SU(4)$ and $SO(4) \sim \{SU(2), SU(2)\}$ facilitate PS decomposition. Complete technology of orthogonal to unitary conversion is presented in \cite{36}. PS decomposition of 10-plet is:

$$H_i(10) = H_{\mu\nu}(6, 1, 1) + H_{\alpha\dot{\alpha}}(1, 2, 2) \quad (2.7)$$

6-plet ([$\mu\nu$]) is two index antisymmetric representation of SU(4). Second multiplet is SU(4) singlet and doublet of both $SU(2)_L$ and $SU(2)_R$, represented by $\alpha$ and $\dot{\alpha}$ respectively. From the breaking chain

$$SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y$$

Hypercharge is given by

$$Y = 2T_{3R} + (B - L) \quad (2.8)$$

so that one finds the beautiful LR symmetric electromagnetic charge formula:

$$Q = T_{3L} + T_{3R} + \frac{B - L}{2} \quad (2.9)$$
PS to the MSSM decomposition can be easily achieved using SU(4) decomposition ($\mu = \bar{\mu} + 4, \bar{\mu} = 1, 2, 3$) and hypercharge formulae given above. Under the SM gauge group ($SU(3)_C \times SU(2)_L \times U(1)_Y$) the 10 plet decomposes as:

$$10 = H_\alpha(1, 2, 1) + \bar{H}_\alpha(1, 2, -1) + t_\bar{\mu}(3, 1, -\frac{2}{3}) + \bar{t}_\bar{\mu}(\bar{3}, 1, \frac{2}{3})$$  \hspace{1cm} (2.10)

The $H_\alpha$, $\bar{H}_\alpha$ contribute to the MSSM doublets while $t$, $\bar{t}$ exchange contribute to proton decay. Similarly other Higgs irreps are decomposed into the PS labels:

$$\Sigma(126) = \Sigma^{(s)}(10, 1, 3) + \Sigma^{(s)}(\bar{10}, 3, 1) + \Sigma^{(s)}(15, 2, 2) + \Sigma^{(s)}(6, 1, 1)$$  \hspace{1cm} (2.11)

$$\Sigma(126) = \Sigma^{(s)}(10, 3, 1) + \Sigma^{(s)}(\bar{10}, 1, 3) + \Sigma^{(s)}(15, 2, 2) + \Sigma^{(s)}(6, 1, 1)$$  \hspace{1cm} (2.12)

$$\Phi(210) = \Phi^{(s)}(15, 1, 1) + \Phi^{(s)}(1, 1, 1) + \Phi^{(s)}(15, 1, 3) + \Phi^{(s)}(15, 3, 1) + \Phi^{(s)}(6, 2, 2) + \Phi^{(s)}(10, 2, 2) + \Phi^{(s)}(\bar{10}, 2, 2)$$  \hspace{1cm} (2.13)

$$\Theta_{ijk}(120) = \Theta^{(s)}(10, 1, 1) + \Theta^{(s)}(1, 1, 1) + \Theta^{(s)}(\bar{10}, 1, 1) + \Theta^{(s)}(15, 2, 2) + \Theta^{(s)}(6, 3, 1) + \Theta^{(s)}(1, 2, 2)$$  \hspace{1cm} (2.14)

As discussed for 10-plet, all Higgs irreps are decomposed into SM labels. The 592 ($10 + 120 + 126 + 126 + 210$) fields in the Higgs sector fall precisely into 26 different types of SM gauge representations which are labelled by the 26 letters of the English alphabet [70, 57]. The decomposition of SO(10) in terms of its “Pati-Salam” labels (i.e. the maximal subgroup $SU(4) \times SU(2)_R \times SU(2)_L$) provided a translation manual [36] from SO(10) to unitary group labels. Using this technology all the invariants of the superpotential are decomposed into PS labels. Decomposition of invariants corresponding to $10 + 126 + 126 + 210$ are given in [70, 36] and the ones involving 120-plet are given in [57]. To illustrate we give decomposition of one invariant $\eta \phi \Sigma \Sigma$. 

The supermultiplet masses are determined from this decomposition by using symmetry breaking VEVs. Mass terms are divided into 3 types: unmixed chiral, mixed pure chiral and mixed chiral-gauge \[70\]:

1. Unmixed chiral are those chiral fermions which transform as SM conjugate pairs and form Dirac fermions. In total there are 13 multiplets of this type: \(A[1, 1, \pm 4], B[6, 2, \pm \frac{5}{3}], I[3, 1, \pm \frac{6}{3}], M[6, 1, \pm \frac{5}{3}], N[6, 1, \mp \frac{1}{3}], O[1, 3, \mp 2], Q[8, 3, 0], S[1, 3, 0], U[3, 3, \pm \frac{4}{3}], V[1, 2, \mp 3], W[6, 3, \pm \frac{2}{3}], Y[6, 2, \mp \frac{1}{3}], Z[8, 1, \pm 2]\). For example \(A[1, 1, 4]\) and \(\tilde{A}[1, 1, -4]\) form a Dirac fermion and its mass originate from \(M\Sigma\Sigma\) and using \(210\) VEV in \(\eta\phi\Sigma\Sigma\) superpotential invariants.

\[
\tilde{A}[1, 1, -4] = \frac{\Sigma_{44}(R-)}{\sqrt{2}} ; \quad A[1, 1, 4] = \frac{\Sigma_{44}(R+)}{\sqrt{2}} \quad (2.28)
\]

First term of Eqns. \(2.16\), \(2.17\) and \(2.29\) will contribute to \(m_A\):

\[
m_A = 2(M + \eta(p + 3a + 6\omega)) \quad (2.29)
\]
2. Mixed pure chiral scenario correspond to when we have more than one multiplet from different SO(10) Higgs having same SM quantum numbers. The model has $C[8, 2, \pm 1]$, $D[3, 2, \pm \frac{2}{3}]$, $K[3, 1, \mp \frac{5}{3}]$, $L[6, 1, \pm \frac{2}{3}]$, $P[3, 3, \mp \frac{2}{3}]$, $R[8, 1, 0]$, $h[1, 2, \pm 1]$ and $t[3, 1, \mp \frac{2}{3}]$ multiplets that belong to this category. We will give explicit form of $h$ mass matrix when we will discuss the emergence of effective MSSM Higgs.

3. Mixed chiral gauge are the multiplet which mixes among themselves and also with gauge particles. These multiplets are named as $E[3, 2, \pm \frac{1}{3}]$, $J[3, 1, \pm \frac{4}{3}]$, $X[3, 2, \mp \frac{5}{3}]$, $F[1, 1, \pm 2]$, $G[1, 1, 0]$. Mass matrix for $E$ is given by:

\[
\begin{pmatrix}
-2(M + \eta(a - \omega)) & 0 & 0 & 0 & \text{(i} \omega - \text{i} \rho + 2a)\zeta \\
0 & -2(M + \eta(a - 3\omega)) & -2\sqrt{2}i\eta\sigma & 2i\eta\sigma & ig\sqrt{2}\sigma^* \text{ (i} 3i\omega + \text{i} 2a)\zeta \\
0 & 2i\sqrt{2}\eta\sigma & -2(m + \lambda(a - \omega)) & -2\sqrt{2}\lambda\omega & 2g(a^* - \omega^*) & -\sqrt{2}\zeta\sigma \\
0 & -2i\eta\sigma & -2\sqrt{2}\lambda\omega & -2(m - \lambda\omega) & \sqrt{2}g(\omega^* - p^*) & \sigma\zeta \\
(i \omega - \text{ip} - 2ia)\zeta & 0 & \text{(i} \omega - \text{ip} - 2ia)\zeta & -\sqrt{2}\zeta\sigma & \sigma\zeta & 0 & -(\text{mea} + \frac{6}{5}a - \frac{2}{5}\rho\omega) \\
\end{pmatrix}
\]

Rows and columns are labelled by $(\bar{E}_1, \bar{E}_2, E_3, E_4, E_5, E_6) [3, 2, -\frac{1}{3}] \oplus (E_1, E_2, E_3, E_4, E_5, E_6)[3, 2, \frac{1}{3}] \oplus (\Sigma_{\mu \alpha}, \Sigma_{\mu \alpha}^{\alpha}, \phi^{(a)}\mu_{\alpha \beta}, \phi^{(s)}\mu_{\alpha \beta}, \phi^{(a)}\mu_{\alpha \beta}, \phi^{(s)}\mu_{\alpha \beta}, \phi^{(a)}\mu_{\alpha \beta}, \phi^{(s)}\mu_{\alpha \beta}).$ 5th row and column are gaugino contributions.

As a check, SU(5) irreps mass spectra is generated from above spectra using special direction

\[ p = a = \pm \omega \quad (2.30) \]

of VEVs [57]. The superheavy fields play a crucial role as they provide threshold corrections to the unification scale, gauge couplings and Higgs fields etc. that we will explain in the subsequent sections and chapters. The complete GUT scale spectrum and couplings of NMSGUT have been given in [70] [36] [57] [72] [73].

### 2.2.3 RG Analysis

After symmetry breaking large number of fields get mass of order of GUT scale which give threshold correction to the gauge couplings [75] [74]. In [36] effect of superheavy thresholds on $\alpha_G(M_X)$, $\sin^2 \theta_W$ and $M_X$ is investigated using Weinberg and Hall approach. Alternatively [57] one can predict $\alpha_3(M_Z)$ instead of $\sin^2 \theta_W$. Effect
of these superheavy thresholds on $\alpha_3(M_Z)$, $\alpha^{-1}_G(M_X)$ and $\log_{10} M_X$ ($\Delta_G, \Delta_3, \Delta_X$) is calculated in [76] using precisely measured value of $\sin^2 \theta_W(M_Z)$. It was shown that although there are large number of superheavy fields their spectrum spread around $M^0_X$ gives contribution of both signs so that the sums can be reasonable modification of the tree level results. This analysis has disproved the conjecture that large number of fields will give huge corrections to the untameable observables making Susy SO(10) unification meaningless [77]. In [36, 57, 76] mass of the lightest vector particle mediating proton decay ($X[3, 2, \pm \frac{5}{3}]$) is chosen as the matching scale ($M_X$) between the effective MSSM and GUT scale. Relation between the gauge couplings of the effective MSSM ($\alpha_i^2 = \frac{g^2_i}{4\pi}$) and GUT ($\alpha_G$) is given by [75, 74]

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_G(M_X)} + 8\pi b_i \ln \frac{M_X}{M_Z} + 4\pi \sum_j \frac{b_{ij}}{b_j} \ln X_j - 4\pi \lambda_i(M_X) + \ldots \ (2.31)$$

here second and third term represent one-loop and two-loop gauge running

$$X_j = 1 + 8\pi b_j \alpha_G(M^0_X) \ln \frac{M_X}{M_Z} \ (2.32)$$

$b_i, b_{ij}$ (i, j=1, 2, 3) are the one-loop and two-loop gauge beta function coefficients:

$$\{b_1, b_2, b_3\} = \frac{1}{16\pi^2} \{33/5, 1, -3\}$$

$$b_{ij} = \frac{1}{(16\pi^2)^2} \begin{pmatrix} 199/25 & 27/5 & 88/5 \\ 9/5 & 25 & 24 \\ 11/5 & 9 & 14 \end{pmatrix}$$

(2.33)

Last term of the Eq. (2.31) represent the leading order effects of superheavy thresholds. In the $\overline{MS}$ scheme one has:

$$\lambda_i(\mu) = -\frac{2}{21}(b_{iv} + b_{iga}) + 2(b_{iv} + b_{iga}) \ln \frac{M_V}{\mu} + 2b_{is} \ln \frac{M_V}{\mu} + 2b_{if} \ln \frac{M_F}{\mu} \ (2.34)$$

where $b_v, b_s, b_f, b_{ga}$ denote one-loop beta functions of vectors, scalars, fermions and goldstone bosons respectively with a sum over heavy mass eigenstates. Corrections depend upon the ratio of masses so they are independent of $m$ (mass of 210-plet), the
overall mass scale parameter. Dots (in Eq. 2.31) represent the two loop contribution of matter Yukawa couplings. Three equations (Eq. 2.31) are used to determine \( \alpha_3(M_Z) \), \( M_X \) and \( \alpha_G(M_X) \). The threshold correction formulae are:

\[
\Delta^{(th)}(\ln M_X) = \frac{\lambda_1(M_X) - \lambda_2(M_X)}{2(b_1 - b_2)}
\]

\[
\Delta_X \equiv \Delta^{(th)}(\text{Log}_{10} \frac{M_X}{1 \text{ GeV}}) + \Delta^{(2\text{-loop})}(\text{Log}_{10} \frac{M_X}{1 \text{ GeV}})
\]

\[
= 0.222 + \frac{5(\bar{b}'_1 - \bar{b}'_2)}{56\pi} \text{Log}_{10} \frac{M'}{M_X}
\]

\[
\Delta_3 \equiv \Delta^{(th)}(\alpha_3(M_Z))
\]

\[
= \frac{100\pi(b_1 - b_2)\alpha(M_Z)^2}{[(5b_1 + 3b_2 - 8b_3)\sin^2 \theta_W(M_Z) - 3(b_2 - b_3)]^2} \sum_{ijk} \epsilon_{ijk}(b_i - b_j)\lambda_k(M_X)
\]

\[
= 0.00311667 \sum_{M'} (5\bar{b}'_1 - 12\bar{b}'_2 + 7\bar{b}'_3) \ln \frac{M'}{M_X}
\]

\[
\Delta_G \equiv \Delta^{(th)}(\alpha^{-1}_G(M_X)) + \Delta^{(2\text{-loop})}(\alpha^{-1}_G(M_X)) = \frac{4\pi(b_1\lambda_2(M_X) - b_2\lambda_1(M_X))}{b_1 - b_2}
\]

\[
= -1.27 + \frac{1}{56\pi} \sum_{M'} (33\bar{b}'_2 - 5\bar{b}'_1) \ln \frac{M'}{M_X}
\] (2.35)

Here \( \bar{b}'_i = 16\pi^2b'_i \) are one-loop \( \beta \) function coefficients \( (\beta_i = b_ig_i^3) \) for multiplets with mass \( M' \) and \( \lambda_i \) are the leading contributions of the superheavy thresholds [74, 70]. Using the experimental values

\[
M_Z = 91.1876 \pm 0.0021 \text{ GeV} \quad \text{; } \quad (\alpha(M_Z))^{-1} = 127.918 \pm 0.18
\]

\[
\sin^2 \theta_W = 0.23122 \pm 0.00015 \quad \text{; } \quad m^t_{\text{pole}} = 172.7 \pm 2.9 \text{ GeV} \quad (2.36)
\]

threshold corrections are estimated. Two expressions of \( M_X = M_X^010^{\Delta_X} \) and \( M_X = m_{\lambda_X} = |m/\lambda|g\sqrt{4|\tilde{a} + \tilde{w}|^2 + 2|\tilde{p} + \tilde{\omega}|^2} \) determine \( m \) parameter:

\[
\Delta_X = \Delta(\text{Log}_{10} \frac{M_X}{1\text{ GeV}})
\]

\[
|m| = M_X^010^{\Delta_X} \frac{|\lambda|}{g\sqrt{4|\tilde{a} + \tilde{w}|^2 + 2|\tilde{p} + \tilde{\omega}|^2}} \text{ GeV} \quad (2.37)
\]

where
2.2 Structure

\[ g = \sqrt{4\pi(25.6 + \Delta_G)^{-1}} \]  \hspace{1cm} (2.38)

is the threshold corrected SO(10) gauge coupling. Theory should remain perturbative after including threshold effects and mass of the proton decay mediating gauge boson should not be lowered too much so as not to violate experimental bounds. These requirements constrain the corrections as:

\[ -20.0 \leq \Delta_G \equiv \Delta(\alpha_G^{-1}(M_X)) \leq 25 \]
\[ 3.0 \geq \Delta_X \equiv \Delta(\log_{10}M_X) \geq -0.3 \]
\[ -0.017 < \Delta_3 \equiv \hat{\alpha}_3(M_Z) < -0.004 \] \hspace{1cm} (2.39)

All the superheavy VEVs and hence masses are determined in terms of parameter $x$. Solution of this variable depend upon the superpotential parameters through $\xi = \frac{\lambda M}{\eta m}$ parameter. Threshold corrections are not very sensitive to $\lambda, \eta, \gamma, \bar{\gamma}$ as shown by scanning the parameter space. Systematic survey of behavior of these unification stability monitoring parameters versus $x$ and $\xi$ is shown in [70, 54, 55].

2.2.4 MSSM Higgs

VEVs $p, a, \omega, \sigma, \bar{\sigma}$ of the multiplets of $\mathbf{210}, \mathbf{216}, \mathbf{T_{26}}$ Higgs irreps break the gauge symmetry to the SM gauge group. $\mathbf{10}$-plet (see Eq. 2.10) has $h[1, 2, 1]$ multiplet having MSSM Higgs quantum numbers. Similarly other Higgs irreps $\mathbf{210}, \mathbf{126}, \mathbf{T_{26}}, \mathbf{120}$ also have these multiplets which participate in electroweak symmetry breaking. Six such doublets are

\[ h^{(1)} = H_{a1} \hspace{1cm} ; \hspace{1cm} h^{(2)} = \Sigma_{a1}^{(15)} \hspace{1cm} ; \hspace{1cm} h^{(3)} = \Sigma_{a1}^{(15)} \]
\[ h^{(4)} = \frac{\phi^{44}_{a}}{\sqrt{2}} \hspace{1cm} ; \hspace{1cm} h^{(5)} = \Theta_{a1} \hspace{1cm} ; \hspace{1cm} h^{(6)} = \Theta_{a1}^{(15)} \] \hspace{1cm} (2.40)

where $\Sigma_{a1}^{(15)}, \Sigma_{a1}^{(15)}$ refer to singlet inside $(15, 2, 2)$ submultiplet of the $\mathbf{126}, \mathbf{T_{26}}$. $h^{(4)}$ comes from $(10, 2, 2)$ of the $\mathbf{210}$, $h^{(5)}$ and $h^{(6)}$ refer to the singlet inside the $(1, 2, 2)$ and $(15, 2, 2)$ of $\mathbf{120}$ submultiplet. Similarly the six doublets which transform as
2.2 Structure

The doublets mix via a $6 \times 6$ mass matrix $\mathcal{H}(W = \bar{h}HH + \ldots)$

\[
\mathcal{H} = \begin{pmatrix}
-M_H & \gamma \sqrt{3}(\omega - a) & -\gamma \sqrt{3}(\omega + a) & -\gamma \sigma & kp & -\sqrt{3}ik\omega \\
-\gamma \sqrt{3}(\omega + a) & 0 & -(2M + 4\eta(a + \omega)) & 0 & -\sqrt{3}i\omega & i(p + 2\omega)\zeta \\
\gamma \sqrt{3}(\omega - a) & -(2M + 4\eta(a - \omega)) & 0 & -2\eta \sqrt{3} & \sqrt{3}\zeta\omega & -i(p - 2\omega)\zeta \\
-\sigma \gamma & -2\eta \sigma \sqrt{3} & 0 & -2m + 6\lambda(\omega - a) & \zeta \sigma & \sqrt{3}\zeta \sigma \\
kp & \sqrt{3}i\omega & -\sqrt{3}\omega \zeta & \zeta \sigma & -m\zeta & \frac{m}{\sqrt{3}}i\omega \\
\sqrt{3}ik\omega & i(p - 2\omega)\zeta & -i(p + 2\omega)\zeta & -\sqrt{3}i\zeta \sigma & -\sqrt{3}\zeta \sigma & -m\zeta - \frac{m}{\sqrt{3}}i\omega \\
\end{pmatrix}
\]

Rows and columns of mass matrix are labelled by $(\bar{h}_1, \bar{h}_2, \bar{h}_3, \bar{h}_4, \bar{h}_5, \bar{h}_6)[1, 2, -1] \oplus (h_1, h_2, h_3, h_4, h_5, h_6)[1, 2, 1]$. As MSSM is effective theory of the model so one Higgs should be light. By tuning the parameters so that $\text{Det}\mathcal{H} = 0$ one can keep one pair of Higgs doublets $H_{(1)}, \bar{H}_{(1)}$ (defined by left and right null eigenstates of the mass matrix $\mathcal{H}$) light. We denote the components of the right (left) null eigenvectors as $\alpha_i(\bar{\alpha}_i), i = 1\ldots6$, normalized to one and real first component. $U$ and $\bar{U}$ are the unitary transformations

\[
h = UH \quad ; \quad \bar{h} = \bar{U}\bar{H}
\]

which diagonalize $\mathcal{H}^\dagger\mathcal{H}$ and $\mathcal{H}\mathcal{H}^\dagger$ so that $\bar{U}^\dagger\mathcal{H}U$ is diagonal and positive. Since $U_{i1} = \alpha_i, \bar{U}_{i1} = \bar{\alpha}_i$, in the Dirac mass matrices of the effective MSSM we can replace $\langle h_i \rangle \rightarrow \alpha_i \nu_u, \langle \bar{h}_i \rangle \rightarrow \bar{\alpha}_i \nu_d$. Thus the “Higgs fractions($\alpha_i, \bar{\alpha}_i$)” (analytical expressions can be found in [57]) specify how much the different GUT scale doublets $h_i, \bar{h}_i$ contribute to the electroweak (EW) symmetry breaking by $H = H_{(1)}, \bar{H} = \bar{H}_{(1)}$.

2.2.5 Fermion Masses

As mentioned the Yukawa couplings of the pairs of bidoublets contained in the Higgs set $10 \oplus 126 \oplus 120$ give rise to charged fermion masses. Therefore Yukawa coupling matrices $y_l, y_u, y_d$ and $y_\nu$ at high scale ($M_X$) are predicted in terms of the $Y_{AB}^{10} = h_{AB}, Y_{AB}^{126} = f_{AB}$ and $Y_{AB}^{120} = g_{AB}$ which specify the Yukawa couplings
of the $10, 126, 120$ to $16.16$. For explicit relations of the fermion Dirac masses decomposition of $16 \cdot 16 \cdot (10 \oplus 120 \oplus 126)$ is required, which can be found in [70, 57]. For illustration we present decomposition of $16 \cdot 16 \cdot 10$ in terms of the PS and the SM labels:

\[
W_{FM}^H = h_{AB}^T C_2^{(5)} \gamma_i^{(5)} \psi_B H_i
\]

\[
= \sqrt{2} h_{AB} \left[ H_{\mu\nu} \hat{\psi}_A^\mu \hat{\psi}_B^\nu + \tilde{H}^{\mu\nu} \psi_\mu A \psi_\nu B - H^{\alpha\dot{\alpha}} (\hat{\psi}_A^\mu \psi_{\alpha B} + \psi_{\alpha A} \hat{\psi}_B^\mu) \right]
\]

\[
= 2\sqrt{2} h_{AB} [\bar{t}_1 (\epsilon \bar{u}_A \bar{d}_B + Q_A L_B) + t_1 (\frac{\epsilon}{2} Q_A Q_B + \bar{u}_A \bar{e}_B - \bar{d}_A \bar{\nu}_B)]
\]

\[
- 2\sqrt{2} h_{AB} \bar{h}_1 [\bar{d}_A Q_B + \bar{e}_A L_B] + 2\sqrt{2} h_{AB} h_1 [\bar{u}_A Q_B + \bar{\nu}_A L_B]
\]

(2.43)

From these invariants one obtains the Yukawa couplings just by replacing the MSSM Higgs by corresponding Higgs fractions as

\[
y^u = (\hat{h} + \hat{f} + \hat{g}) \quad r_1 = \frac{\bar{\alpha}_1}{\alpha_1} \quad r_2 = \frac{\bar{\alpha}_2}{\alpha_2}
\]

\[
y^d = (r_1 \hat{h} + r_2 \hat{f} + r_6 \hat{g}) \quad r_6 = \frac{\bar{\alpha}_6 + i\sqrt{3}\bar{\alpha}_5}{\alpha_6 + i\sqrt{3}\alpha_5}
\]

\[
y^\nu = (\hat{h} - 3\hat{f} + (r_5 - 3)\hat{g}) \quad r_5 = \frac{4i\sqrt{3}\alpha_5}{\alpha_6 + i\sqrt{3}\alpha_5}
\]

\[
y^l = (r_1 \hat{h} - 3r_2 \hat{f} + (\bar{r}_5 - 3\bar{r}_6)\hat{g}) \quad \bar{r}_5 = \frac{4i\sqrt{3}\alpha_5}{\alpha_6 + i\sqrt{3}\alpha_5}
\]

\[
\hat{g} = 2ig \sqrt{\frac{2}{3}} (\alpha_6 + i\sqrt{3}\alpha_5) \quad \hat{h} = 2\sqrt{2} h_1 \alpha_1 \quad \hat{f} = -4\sqrt{\frac{2}{3}} if_2
\]

(2.44)

By multiplying these Yukawas with electroweak VEVs ($v_u, v_d, \tan \beta = \frac{v_u}{v_d}$) one can get fermion masses. Higgs fractions and SO(10) Yukawas determine MSSM matter fermion Yukawas which produce the experimental fermion mass mixing data. To generate Majorana masses ($M_{\psi\psi}$) for the left and right handed neutrino we need SU(4) 10-plet which contains $L = \pm 2$ components, since the lepton number of the Majorana mass term is 2. The multiplets having 10 of SU(4) from $210$ and $120$ do not have VEVs, thus the only option remaining is $(10, 1, 3)_{126}$ and $(\bar{10}, 3, 1)_{126}$ that can couple to matter bilinear. Majorana mass of the right handed neutrinos is determined by the coupling of the neutrino to the $126$: 
2.3 Viable Parameter Space Search

\[ M_{AB}^\rho = 8\sqrt{2} f_{AB} \tilde{\sigma} \]  

(2.45)

Majorana mass term and Dirac mass term (which mixes the left and right neutrinos) give rise to Type I seesaw contribution by eliminating \( \bar{\nu}_A \)

\[ W = \frac{1}{2} M_{AB}^\rho \bar{\nu}_A \bar{\nu}_B + \bar{\nu}_A m_{\nu AB} \nu_B + ..... \]

\[ W_{\text{eff}} = \frac{1}{2} M_{AB}^{\nu(I)} \nu_A \nu_B + ..... \]

\[ M_{AB}^{\nu(I)} = -((m_{\nu}^\nu)^T (M_{\nu}^{-1}) m_{\nu})_{AB} \]  

(2.46)

In addition to this there is another contribution to the neutrino mass known as Type II seesaw. The Type II neutrino mass is

\[ M_{AB}^\nu = 16i f_{AB} \langle \bar{O}_- \rangle \]  

(2.47)

where \( \langle \bar{O}_- \rangle \) is \( SU(2)_L \) triplet VEV (\( \in (10, 3, 1) \)) of \( 126 \) whose computation requires inspection of relevant terms in the superpotential:

\[ \langle \bar{O}_- \rangle = (i\gamma\sqrt{2}\alpha_1 + 2i\sqrt{6}\eta\alpha_2 - \sqrt{6}\zeta\alpha_6 + i\sqrt{2}\zeta\alpha_5)\alpha_4 \frac{v_u^2}{M_O} \]  

(2.48)

and \( M_O = 2(M + \eta(3a - p)) \) is mass of superheavy \( O \) multiplet \( [57] \).

2.3 Viable Parameter Space Search

With appropriate formulae in hand, next task is to check the compatibility of the model with experimental data. FORTRAN and Mathematica codes were developed \([71]\) for fermion fitting along with viable unification, electroweak symmetry breaking, including Susy threshold corrections and for B-decay calculations. \( \chi^2 \) analysis is performed to fit SM mass-mixing data at two scales- GUT scale (\( M_X^0 \)) and electroweak scale (\( M_Z \)). GUT scale fitting is based upon the random searches of 37 model parameters (listed in Section \( [2.2.1] \) and \( x \) parameter (which is chosen complex and later the phase of \( \lambda \) is fixed to remove this freedom). \( M_Z \) scale calculations involve Susy threshold corrections which require estimation of Susy spectra. Soft
Susy breaking Lagrangian is:

\[-\mathcal{L}_{\text{soft}} = m_{\tilde{Q}}^2 \tilde{Q} \tilde{Q} + m_{\tilde{u}}^2 \tilde{u} \tilde{u} + m_{\tilde{d}}^2 \tilde{d} \tilde{d} + m_{\tilde{L}}^2 \tilde{L} \tilde{L} + m_{\tilde{e}}^2 \tilde{e} \tilde{e} + M_1^2 \tilde{B} \tilde{B} + \frac{M_2}{2} \tilde{W} \tilde{W} + \frac{M_3}{2} \tilde{g} \tilde{g} + \text{h.c.}\]

\[+ M_H^2 H_d H_d + M_H^2 H_u H_u - (BH_d H_u + \text{h.c.}) + \left( A_u \tilde{Q} \tilde{u} H_u + A_d \tilde{Q} \tilde{d} H_d + A_l \tilde{L} \tilde{H} H_d + \text{h.c.} \right) + \left( \frac{M_1}{2} \tilde{B} \tilde{B} + \frac{M_2}{2} \tilde{W} \tilde{W} + \frac{M_3}{2} \tilde{g} \tilde{g} + \text{h.c.} \right). \quad (2.49)\]

where \(\tilde{Q}, \tilde{d}, \tilde{u}, \tilde{e}\) and \(\tilde{L}\) are scalar components of \(Q, d, u, e\) and \(L\) respectively. \(\tilde{g}, \tilde{W}\) and \(\tilde{B}\) are gaugino’s of SU(3), SU(2) and U(1) respectively. SU(2) and generation indices are suppressed. Tree level Susy spectrum is presented in the Appendix. The soft Susy breaking parameters at the GUT scale \((M_X)\) are described by universal soft Susy breaking parameters- \(m_0\) (universal scalar mass), \(m_\frac{1}{2}\) (universal gaugino mass), and \(A_0\) (dimensionless universal scalar trilinear coupling) and Higgs mass parameters \((M_H^2, M_{\tilde{H}}^2)\):

\[
m_{\tilde{Q}}^2 = m_{\tilde{u}}^2 = m_{\tilde{d}}^2 = m_{\tilde{L}}^2 = m_{\tilde{e}}^2 = m_0^2
\]

\[
M_1 = M_2 = M_3 = m_\frac{1}{2}
\]

\[
A_u = A_d = A_l = A_{\nu} = A_0 \quad ; \quad M_H^2, M_{\tilde{H}}^2
\]

(2.50)

This scenario is called supergravity non universal Higgs mass (SUGRY-NUHM) parameters at GUT scale. Use of non-universal Higgs masses is justified as the light Higgs of MSSM is a combination of six doublets from \(10, 210, 120, 126, \bar{126}\) Higgs irreps. In Chapter 6 we will see how RG flow of the soft parameters between \(M_P\) and \(M_X^0 = 10^{16.33}\) GeV can support the NUHM assumptions. Low scale fitting is based upon these parameters. Algorithm of the program is represented by a flowchart 2.1. Task of various functions/subroutine and variables used in the flowchart is discussed below:

1. Parameter Masteriter(miter) represents the number of loop iterations from one scale to another. \(\text{iter1}\) and \(\text{iter2}\) are high scale and low scale iteration parameters of the search engine.
2.3 Viable Parameter Space Search

Figure 2.1: Flowchart of FORTRAN search program.
2. Subroutine AMOEBA1/AMOEBA2- There are various public available packages to find minima of non-linear functions. Downhill simplex method of Nelder and Mead \cite{78} has been used to find the minima of highly non-linear function in a multi-dimensional space because it involves function evaluations only. It is based upon the n-simplex having n+1 vertices in n-dimensional space. The AMOEBA subroutine \cite{78} contracts, expands and reflects the simplex so as to converge upon (local) minima. AMOEBA1 and AMOEBA2 are modified versions of general subroutine AMOEBA which perform search in different dimensional space and call separate appropriate functions. AMOEBA1 is used for GUT scale parameter search while AMOEBA2 is used for low scale searches.

3. Subroutine GUTTHRESH does the calculations discussed in the Section \ref{2.2.2} and \ref{2.2.3}. First of all it calculates the superheavy spectrum. Then effect of threshold correction to unification stability monitoring parameter is checked and overall mass scale $m$ parameter is fixed. Then coefficients of d=5 proton decay LLLL and RRRR operator are calculated \cite{70,57}. It contains penalties to get $\Delta_G$, $\Delta_3$ and $\Delta_X$ within the required range (see Eq. \ref{2.39}). It provides $\Delta_G$, $\Delta_3$, $\Delta_X$ and $m$ parameters as output.

4. Function FUNKFERM- It calculates $\chi^2_X$ value by comparing model predicted values with target (which is run up SM experimental mass mixing data). For a given set of values of superpotential parameters and Yukawa couplings, fermion Yukawas are calculated using Eq. \ref{2.44}. Then it calculates the eigenvalues and mixing angles in quark and lepton sector. After that

$$\chi^2_X = \sum_i \left( \frac{(O - \bar{O}_i)^2}{\delta O_i} \right)$$

(2.51)

fitting of SM mass mixing data (18 parameters) is done. $\delta O_i$ is an estimate of the uncertainty in the GUT scale value based upon extrapolation of uncertainty at the measured scale (see e.g. \cite{79}). $\bar{O}$ and $O$ are experimental and model predicted values respectively. Here the sum $(i)$ run over the Yukawa couplings $(9)$, quark and lepton mixing angle, CKM phase and neutrino mass square differences. This function returns argument $\text{funk}\left(\chi^2_X\right)$.
5. Subroutine PMX2PS uses two-loop MSSM renormalization group equations [80] [81] to run the hard (Yukawa and gauge coupling) and soft parameters from GUT scale to electroweak scale and vice versa.

6. Function FUNKTUNE does $M_Z$ scale calculations. Main task of this function is to perform low scale fermion fitting. Fitting of $y_d$ and $y_s$ require inclusion of Susy threshold corrections and which further require Susy spectrum estimation. Yukawa couplings, gauge couplings, scalar masses and gaugino masses at $M_Z$ are input of this subroutine. Tree level Susy spectrum is calculated using SPheno subroutine (Susy spectrum code) [82] TreeMassesMSSM. Penalties are imposed to get positive (and above some lower limit) squark and slepton mass square parameter. Higgs mass is computed using one loop effective potential. $\mu$ and $B$ parameters are calculated using one loop electroweak symmetry breaking conditions and the run down values of the Higgs mass parameters at $M_Z$.

\[
\mu^2 = \frac{1}{2} \left[ \tan 2\beta \left( (M_H^2 - t_2) \tan \beta - (M_H^2 - t_1) \cot \beta \right) - M_Z^2 \right] \\
B = \frac{1}{2} \left[ \tan 2\beta \left( (M_H^2 - t_2) - (M_H^2 - t_1) \right) - M_Z^2 \sin 2\beta \right] \tag{2.52}
\]

where $t_{1,2}$ are tadpoles of the effective potential, calculated using a SPheno subroutine based on the formulae of [83]. These can be extrapolated back to $M_X^0$ to find $\mu$ and $B$ at $M_X^0$ since the RGEs of the other soft masses are independent of these. Both $\mu$ and $B$ are assumed real and positive. $M_A$ (pseudo-scalar mass) is calculated using the above one-loop corrected value of $B$ parameter

\[
M_A^2 = \frac{2B}{\sin 2\beta} \tag{2.53}
\]

Tree level spectrum is again calculated using updated value of $\mu$ and $B$ parameter. Susy threshold corrections to the Yukawa couplings are calculated. Then using off-diagonal run down values of Yukawas, tree level spectra is calculated to verify that ignoring generation mixing does not make much difference. Aim is to find the suitable set of soft parameters which give appropriate corrections
2.3 Viable Parameter Space Search

to the NMSGUT run down Yukawas to fit them with the SM data. Following penalties are imposed upon soft parameters [57]:

\[
|\tilde{m}_{i,L}/M_1| \geq 0.9 ; \\
|\tilde{m}_{u,d,\tilde{D}}/M_3| \geq 0.75 ; \\
\tilde{M}_{u,d} > 500 \text{ GeV} \\
\mu, |A_0| < 150 \text{ TeV} ; \\
\tilde{M}_{i,H^\pm} > 200 \text{ GeV} \\
\tilde{W} : 200 \text{ GeV} < M_2(M_Z) < 1000.0 \text{ GeV} \\
\tilde{g} : 500 \text{ GeV} < M_3(M_Z) < 1000.0 \text{ GeV}
\]

(2.54)

Higgs mass measurements are available since December 2011, in [57] SM Higgs was required to be heavier than 114 GeV (LEP limit) and the Bino lighter than the lightest sfermion. Susy threshold corrected run down Yukawas values are compared with SM Yukawa via a \( \chi^2_Z \).

\[
\chi^2_Z = \sum_i \left(1 - \frac{y_{i,\text{MSSM}}}{y_{i,\text{SM}}} \right)^2
\]

(2.55)

Here \( y_{i,\text{MSSM}} \) are threshold corrected Yukawas. This function returns \( \chi^2_Z \). AMOEBEA2 subroutine uses this function and calculates its value at each vertex of the simplex.

Program starts with the GUT scale searches. It requires target extrapolated SM data. Using two loop RGEs of MSSM [80, 81], central fermion experimental data (Yukawa + neutrino mass difference and (quark, lepton sector) mixing data) is extrapolated to one loop unification scale \( M_X^0=10^{16.33} \text{ GeV} \) ignoring right handed neutrino thresholds and assuming normal hierarchy for left handed neutrino masses (neutrino mass splitting is calculated from extrapolated coefficient of d=5 operator [9, 84]). At \( M_X^0 \) canonical parameters are extracted which serves as target for model calculations. This target file and two other input files having random set of NMSGUT superpotential couplings and SUGRY-NUHM parameters are provided. To start with, the value of variable \textit{bestfunk} (whose role will be explained later) is fixed along with the initialization of many other parameters. In a downhill simplex method one needs to provide the initial vertex of simplex around which
it starts searching in the parameter space. Random input provided is that initial vertex. With slight random changes from this point all the vertices of the simplex are generated. At each vertex of simplex GUTTHRESH calculations and function FUNKFERM is calculated. AMOEBA1 subroutine also calls GUTTHRESH and function FUNKFERM for viable unification and function evaluations and it compares the value of \( \text{funk} \) at each vertex and select the one with minimum \( \text{funk} \). If calculated \( \text{funk} \) at that point is less than or equal to initial \( \text{bestfunk} \) value then it replaces that parameter set in the input file. In the next iteration it start searching around that point. If the lowest funk value is more than the initial \( \text{bestfunk} \) then in the next iteration program starts searching around the old point, but as the vertices of simplex are randomly generated so simplex will be different from the earlier one. Once all the iterations at high scale are completed set of hard parameters (Yukawa and gauge couplings) at GUT scale is obtained.

Then program starts searching for soft parameters. It reads initially provided random SUGRY-NUHM parameters and generates simplex as discussed for GUT scale searches. Notice that now simplex dimensionality is different. Using MSSM RGEs, the diagonal Yukawa couplings and scalar masses are run down to \( M_Z \) scale. With the fixed value of hard couplings and random soft couplings, program calls FUNKTUNE at each vertex of simplex and calculates \( \chi^2_Z \). Again like \( M_X \) scale, AMOEBA2 select the point with minimum value of \( \chi^2_Z \) and compare that \( \chi^2_Z \) with the initially chosen \( \text{besttune} \) value. Soft parameter input is replaced if the selected point is better than initially provided. After completing all the low scale iterations, program provides a set of SUGRY-NUHM parameters. Then new target set of (Susy threshold) corrected MSSM couplings is provided for next iteration high scale calculations. Procedure is repeated \( \text{Masteriter} \) number of times to get reasonable fitting of MSSM data at high and low scale. At the end the program prints \( \text{bestfunk} \) and \( \text{besttune} \) value representing \( \chi_X, \chi_Z \) and stores corresponding parameters in the input/output files.
2.4 Distinct Predictions

2.4.1 Normal $s$-hierarchy

NMSGUT fits \cite{57} prefer large negative values of Higgs mass squared soft parameters $M_{H,\tilde{H}}^2 \simeq (100 \text{ TeV})^2$. One loop $\beta$ function of scalar’s soft mass RGEs contains terms proportional to $M_{H,\tilde{H}}Y_f^*Y_f$. Due to the large third generation Yukawa couplings, this term dominates for the third generation evolution and their masses evolve to large values compared to the first and second generation. So the model predicts normal $s$-hierarchy opposite to the common wisdom. The RG flow of Yukawa coupling and scalar masses exhibiting this behaviour is given in \cite{85}.

2.4.2 Large $A_0$ and $\mu$ Parameter

If only $10$ and $120$-plet fit charged fermion masses with $126$ Yukawa coupling ($f_{AB}$) chosen tiny the NMSGUT can only generate $y_{d,s}(M_X^0)$ values which are smaller by a factor of 3-5 than the extrapolated values of SM Yukawas at $M_X^0$. Therefore SM down and strange quark Yukawa require lowering by a factor of 5 to fit these with run down values of Yukawas. This lowering is achieved by large $\tan \beta$ (preferred by SO(10) GUTs for third generation Yukawa unification) driven threshold corrections which require specified soft Susy breaking parameters \cite{57}. Gluino contribution is a dominant one loop correction which is proportional to Susy breaking parameter $\mu$, so large $\mu$ parameter provides significant Susy threshold corrections to $y_d$ and $y_s$ to match it with GUT renormalized value down to $M_Z$ scale. For third generation, bottom quark, slight raising is required, so Susy threshold corrections should not change it too much. This cancellation can be achieved by large $A_0$ (soft trilinear couplings) parameter. After the Higgs discovery large $A_0$ is favoured in Susy-GUTs for Higgs mass of 126 GeV \cite{86}. Heavy third $s$-generation and large $A_0$ raise the tree level Higgs mass from 91 GeV to 126 GeV.
2.4.3 Bino LSP and Light Smuon

Solutions found have pure Bino LSP and the light charginos are pure Winos. A striking feature is that there are solutions with next to LSP (NLSP) as light smuon which can generate a significant corrections to the muon g-2 and thus remove the observed anomaly $a_\mu \sim 10^{-9}$. Moreover a light smuon provides DM co-annihilation channel to get the acceptable relic density.

2.5 Discussion

NMSGUT superpotential parameter and SUGRY-NUHM type soft supersymmetry breaking parameters \( \{m_0, m_{1/2}, A_0, B, M_{H,H}^2\} \) along with \( \mu \) specified at the MSSM one loop unification scale \( M_X^0 = 10^{16.33} \) GeV can fit the fermion mass mixing data as shown in [70, 57]. The parameter \( \{m_0, m_{1/2}, A_0, M_{H,H}^2\} \) are randomly chosen by the search program while \( \mu \) and \( B \) are fixed from electroweak symmetry breaking conditions. Moreover the constraints from fermion fitting are combined with the requirements of unification as well as electroweak symmetry breaking conditions. Solution sets presented in [57] have large proton decay rates of order of \( 10^{-27} \) yrs\(^{-1} \). However optimized search with respect to baryon decay, including GUT scale threshold correction will be discussed in the next chapter.

Among the superpotential parameters \( h_{33} \) (largest of all elements of Yukawas \( h, f, g \) ) is crucial for fitting fermion mass-mixing data, it alone can fit third generation within 5% error. Next relevant parameter is \( g_{23} \). \( f_{AB} \) is irrelevant for charge fermion masses and its small value is crucial for neutrino masses as it enhances Type I and suppress Type II seesaw contribution. Fits yield right handed neutrino mass in \( 10^8 - 10^{13} \) GeV range which is compatible with leptogenesis. Heavy right handed neutrino and small \( f_{AB} \) generates neutrino masses of order of meV. Fermion Yukawas obey \( b - \tau \) unification (\( |\frac{y_b - y_\tau}{y_\beta - y_\mu}| \approx 1 \)) noted in [87, 88, 89, 90, 91] based upon the \( 10 - 120 \)-plet FM Higgs system. In most of solutions, superheavy thresholds raise unification scale \( M_X \) closer to \( M_P \). Superheavy spectrum varies from \( 10^{15} \) GeV to \( 10^{19} \) GeV.

In addition to the superpotential couplings, \( \tan \beta \) is also a crucial parameter
for realistic fermion mass generation by the GUT Yukawas. For third generation Yukawa unification \((y_t \approx y_b \approx y_\tau)\), NMSGUT prefer large \(\tan \beta \ (\sim 50)\). As discussed in \cite{57} large \(\tan \beta\) driven Susy threshold corrections provides a route for successful fitting of fermion masses at \(M_Z\). Fermion fitting and experiment compatible MSSM Higgs mass require decoupled/mini-split Susy spectrum \cite{86,92} : \(|M|_{H,\tilde{H}} \sim 100 \mathrm{TeV}\), heavy CP-odd Higgs \(M_A\), large \(\mu\) and trilinear coupling \(A_0\), light gaugino \((< 1 \mathrm{TeV})\) with pure Bino as LSP, normal s-hierarchy with \(m_{\tilde{f}_i} \sim 50 \mathrm{TeV}\) and degenerate first two generations. With this kind of soft spectrum gaugino mass deviate from the Susy-GUT ratio 1:2:7 operative at one loop level. Fits prefer large \(A_0\) and \(\mu\) parameter \(\sim 100 \mathrm{TeV}\) which suppress flavour changing neutral current processes and avoid problem with charge and color breaking/unbounded from below vacua \cite{93}.

Besides realistic B-decay rates, other improvements in the fitting programs which are part of the thesis are loop corrected Susy spectrum, inclusion of right handed neutrino thresholds for LFV estimation and to consider the effect of soft parameter RG running from \(M_P\) to \(M_X^0\). This will be discussed in Chapter 4, 5 and 6 respectively.
Appendix: MSSM Tree Level Spectrum

The MSSM is an effective theory of NMSGUT. Its particle content is given in the Table 1.1 and corresponding superpotential and soft Lagrangian ($\mathcal{L}_{\text{soft}}$) are given by Eq. (1.8) and (2.49) respectively. Here we discuss tree level spectrum of the MSSM.

Sparticle Masses

Squark and slepton mass term in the Lagrangian is given by

$$\mathcal{L}_{\text{soft}} = \frac{1}{2} \left( \tilde{f}_L \quad \tilde{f}_R \right) M_f^2 \left( \begin{array}{c} \tilde{f}_L \\ \tilde{f}_R \end{array} \right)$$  \hspace{1cm} (2.56)

where $\tilde{f}$ represents $\tilde{u}$, $\tilde{d}$, $\tilde{l}$ and $\tilde{v}$. Mass matrices are:

$$M_{\tilde{e}}^2 = \begin{pmatrix} m_{\tilde{e}}^2 + m_{\tilde{e}}^2 - (\frac{1}{2} - S_W^2)M_Z^2C_{2\beta} & \frac{1}{\sqrt{2}}(v_1A_e - \mu Y_e v_2) \\ \frac{1}{\sqrt{2}}(v_1A_e - (\mu Y_e)^* v_2) & m_{\tilde{e}}^2 + m_{\tilde{e}}^2 - S_W^2M_Z^2C_{2\beta} \end{pmatrix}$$ \hspace{1cm} (2.57)

$$M_{\tilde{e}}^2 = \begin{pmatrix} m_{\tilde{e}}^2 + \frac{1}{2}M_Z^2C_{2\beta} & 0 \\ 0 & 0 \end{pmatrix}$$  \hspace{1cm} (2.58)

$$M_{\tilde{u}}^2 = \begin{pmatrix} m_{\tilde{u}}^2 + m_{\tilde{u}}^2 - (\frac{1}{2} - \frac{2}{3}S_W^2)M_Z^2C_{2\beta} & \frac{1}{\sqrt{2}}(v_2A_u - \mu Y_u v_1) \\ \frac{1}{\sqrt{2}}(v_2A_u - (\mu Y_u)^* v_1) & m_{\tilde{u}}^2 + m_{\tilde{u}}^2 + \frac{2}{3}S_W^2M_Z^2C_{2\beta} \end{pmatrix}$$ \hspace{1cm} (2.59)

$$M_{\tilde{d}}^2 = \begin{pmatrix} m_{\tilde{d}}^2 + m_{\tilde{d}}^2 - (\frac{1}{2} - \frac{1}{3}S_W^2)M_Z^2C_{2\beta} & \frac{1}{\sqrt{2}}(v_1A_d - \mu Y_d v_2) \\ \frac{1}{\sqrt{2}}(v_1A_d - (\mu Y_d)^* v_2) & m_{\tilde{d}}^2 + m_{\tilde{d}}^2 - \frac{1}{3}S_W^2M_Z^2C_{2\beta} \end{pmatrix}$$ \hspace{1cm} (2.60)

$$S_W^2 = \sin^2 \theta_W \quad ; \quad C_{2\beta} = \cos 2\beta$$

Here $m_{\tilde{L}}^2$, $m_{\tilde{Q}}^2$, $m_{\tilde{u}}^2$, $m_{\tilde{d}}^2$ and $m_{\tilde{e}}^2$ are soft mass parameters. $A_f$, $Y_f$ and $m_f$ are the soft trilinear couplings, fermion Yukawa couplings and masses. Tree level sparticle masses are obtained by diagonalizing above (Hermitian) mass matrices via unitary transformations:-

$$\tilde{U}_u M_u^2 \tilde{U}_u^\dagger = \Lambda_u^2 \quad ; \quad \tilde{U}_d M_d^2 \tilde{U}_d^\dagger = \Lambda_d^2$$ \hspace{1cm} (2.61)
\[ \tilde{U}_e M_\tilde{e}^2 \tilde{U}_e^\dagger = \Lambda_\tilde{e}^2 \quad ; \quad \tilde{U}_\nu M_\tilde{\nu}^2 \tilde{U}_{\nu}^\dagger = \Lambda_\tilde{\nu}^2 \] (2.62)

where \( \Lambda_\tilde{u}^2, \Lambda_\tilde{d}^2, \Lambda_\tilde{e}^2 \) and \( \Lambda_\tilde{\nu}^2 \) are positive definite mass square parameter.

### Higgs Masses

Physical Higgs particles of the MSSM are: CP-odd neutral \( A \), CP-even \( h \) and \( H \), and charged Higgs \( H^\pm \). Masses of these particles can be computed from \( M_A \) (and \( \tan \beta \)) which itself is determined from \( B \) parameter:

\[ M_A = B(\tan \beta + \cot \beta) \] (2.63)

\( B \) is calculated using EW symmetry breaking conditions.

\[
M_{H,h}^2 = \frac{1}{2}(M_A^2 + M_Z^2) \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2M_Z^2\cos^2\beta} \] (2.64)

\[
M_{H^\pm}^2 = M_A^2 + M_W^2 \] (2.65)

### Gaugino and Higgsino Masses

Gauginos and Higgsino mix to form chargino and neutralino eigenstates.

\[
\mathcal{L}_{\text{chargino}} = -\tilde{\chi}^-^T \mathcal{M}_{\tilde{\chi}^+} \tilde{\chi}^+ + h.c. \] (2.66)

\[
\tilde{\chi}^+ = (-i\tilde{W}^+, \tilde{H}_u^+)^T \quad \tilde{\chi}^- = (-i\tilde{W}^-, \tilde{H}_d^-)^T
\]

\[
\mathcal{L}_{\text{neutralino}} = -\frac{1}{2} \tilde{\chi}^0^T \mathcal{M}_{\tilde{\chi}^0} \tilde{\chi}^0 + h.c. \] (2.67)

\[
\tilde{\chi}^0 = (-i\tilde{B}, -i\tilde{W}_3, \tilde{H}_d, \tilde{H}_u)^T
\]

Here \( \tilde{B}, \tilde{W}_3, \tilde{H}_d \) and \( \tilde{H}_u \) are Bino, Wino and Higgs components. Neutralino and chargino mass matrices are obtained from \( \mathcal{L}_{\text{soft}}, \mathcal{L}_{\text{int}} \) (matter-gauge-Higgs) and superpotential:
\[ M_{\tilde{\chi}^+} = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin \beta \\ \sqrt{2} M_W \cos \beta & \mu \end{pmatrix} \]  

(2.68)

\[ M_{\tilde{\chi}} = \begin{pmatrix} M_1 & 0 & -M_Z \cos \beta S_W & M_Z \sin \beta S_W \\ 0 & M_2 & M_Z \cos \beta C_W & -M_Z \sin \beta C_W \\ -M_Z \cos \beta S_W & M_Z \cos \beta C_W & 0 & \mu \\ M_Z \sin \beta S_W & -M_Z \sin \beta C_W & -\mu & 0 \end{pmatrix} \]  

(2.69)

Matrices \( M_{\tilde{\chi}^\pm} \) and \( M_{\tilde{\chi}^0} \) are diagonalized to get masses:

\[ U^\dagger_{\pm} M_{\tilde{\chi}^\pm} U_+ = \Lambda_C \quad ; \quad N^\dagger M_{\tilde{\chi}^0} N = \Lambda_N \]  

(2.70)

Here \( U_- \), \( U_+ \) and \( N \) are unitary matrices and \( \Lambda_C \), \( \Lambda_N \) are positive definite masses.
Chapter 3

Baryon Decay and GUT Scale
Threshold Corrections

3.1 Introduction

GUTs place quarks and leptons in common irreducible representations. Quarks can transform into leptons by exchanging gauge and Higgs leptoquarks, thus GUTs predict baryon violating processes such as proton decay e.g. \((p \rightarrow \pi^0 e^+)\). However non observation of proton decay has put a stringent lower limit \([94]\) on its life time

\[
\tau(p \rightarrow e^+\pi^0) > 8.2 \times 10^{33}\text{yrs}
\]

\[
\tau(p \rightarrow K^+\bar{\nu}) > 2.3 \times 10^{33}\text{yrs}
\]

and this contradicts the simplest models. Hence one must investigate more refined models, among which the most appealing are supersymmetric GUTs. In Susy GUTs B and L are violated by the exchange of superheavy color triplets. In SO(10) GUTs, B-L is preserved by all the vertices since it is part of gauge symmetry. Gauge mediated dimension 6 operator proton decay rate is estimated as

\[
\Gamma_p \approx \frac{\alpha_{\text{GUT}}^2 m_p^5}{M_G^4}
\]  \hspace{1cm} (3.1)
Here \( M_G \) is mass of superheavy gauge boson. Even with \( M_G \sim 10^{16.25} \) GeV this gives \( \tau_p \sim 10^{36} \) yrs. Threshold corrections can raise the unification scale near to the Planck scale so this contribution can be even more strongly suppressed. R-parity forbids fast d=4 baryon number violating operators. The remaining contribution is d=5 operators (involving two fermion and two scalars exchanging triplet Higgsino). Scalars are converted into fermions via gaugino or higgsino dressing [95, 96, 97]. In Susy GUTs dimension 5 operators thus give leading contribution to proton decay [98, 99] as these are suppressed only by \( \frac{1}{M_H M_{\text{Susy}}} \), where \( M_H \) is the mass of triplet Higgsino. Experimental limits put the stringent constraint on the model parameters [100]. In this chapter we will investigate d=5 operator baryon decay and uncover a generic and natural mechanism to suppress these.

### 3.2 Dimension 5, Baryon Decay Operators

SO(10) Yukawa interaction include many superheavy Higgs-fermion interactions. By using the superpotential equations of motion for the heavy fields (just as we eliminate \( W^\pm, Z \) to get the Fermi effective theory of weak interactions from the SM), we obtain two types of d=5 operator which lead to proton decay. The effective superpotential has generic form:

\[
W_{\text{eff}}^{\Delta B \neq 0} = -L_{ABCD}\left(\frac{1}{2}\epsilon Q_A Q_B Q_C L_D\right) - R_{ABCD}\left(\epsilon \bar{e}_A \bar{u}_B \bar{u}_C \bar{d}_D\right)
\]  \hspace{1cm} (3.2)

where the first and second term represent contribution of \( SU(2)_L \) doublets and singlets, therefore are called LLLL and RRRR operator respectively. In NMSGUT 10, 126 and 120 irrep have \( t[3,1,\pm\frac{2}{3}] \oplus P[3,3,\pm\frac{2}{3}] \oplus K[3,1,\pm\frac{8}{3}] \) multiplets that couple to fermion and violate B+L. From PS decomposition of the relevant superpotential invariants \( L_{ABCD} \) and \( R_{ABCD} \) are obtained [36, 70, 57]:

\[
L_{ABCD} = S_1^1\tilde{h}_{AB}\tilde{h}_{CD} + S_1^2\tilde{h}_{AB}\tilde{f}_{CD} + S_2^1\tilde{f}_{AB}\tilde{h}_{CD} + S_2^2\tilde{f}_{AB}\tilde{f}_{CD} - S_1^6\tilde{h}_{AB}\tilde{g}_{CD} \\
- S_2^6\tilde{f}_{AB}\tilde{g}_{CD} + \sqrt{2}(P^{-1})_2^1\tilde{g}_{AC}\tilde{f}_{BD} - (P^{-1})_2^2\tilde{g}_{AC}\tilde{g}_{BD}
\]  \hspace{1cm} (3.3)
3.3 Baryon Decay Rate

\[ R_{ABCD} = S_1 h_{AB} h_{CD} - S_1 f_{AB} f_{CD} + S_2 f_{AB} f_{CD} - i \sqrt{2} S_4 f_{AB} f_{CD} \]
\[ + i \sqrt{2} S_4 f_{AB} f_{CD} + S_6 g_{AB} h_{CD} - i S_7 g_{AB} h_{CD} + S_7 g_{AB} f_{CD} \]
\[ + i S_1 h_{AB} g_{CD} - i S_2 g_{AB} g_{CD} + \sqrt{2} S_4 g_{AB} g_{CD} + i S_7 g_{AB} g_{CD} \]
\[ - \sqrt{2} (K^{-1}) h_{AD} g_{BC} - (K^{-1}) g_{AD} g_{BC} \] (3.4)

where \( S = T^{-1}, T \) is \( t[3, 1, \pm 2/3] \) multiplets mass matrix and

\[ h_{AB} = 2 \sqrt{2} h_{AB}; \quad f_{AB} = 4 \sqrt{2} f_{AB}; \quad g_{AB} = 4 g_{AB} \] (3.5)

where \( \alpha, \beta \) and \( \gamma \) are the colour indices and \( SU(2) \) indices are suppressed. Different Susy GUTs will furnish different coefficient arrays \( L_{ABCD}, R_{ABCD} \) and the task is to convert this information together with the assumptions for the soft Susy breaking terms (till the superpartners are discovered) into predictions for the baryon decay rate into different channels.

3.3 Baryon Decay Rate

We calculate proton decay rates due to \( d=5 (\Delta B = \pm 1) \) operators using formulas of [95]. As a check we compare the result calculated by using the formalism of [96] separately and verify they are the same. The calculation of baryon decay rates is done in steps as follows.

- Firstly one has to renormalize the \( 2 \times 3^4 = 162 \) component arrays \( L_{ABCD} \) and \( R_{ABCD} \) from \( M_X \) down to \( M_S \sim M_Z \) using the MSSM RG equations supplemented by the RGEs for the coefficients \( L_{ABCD}, R_{ABCD} \). The RGE for \( L_{ABCD} \) [95] is

\[ (4\pi)^2 \frac{d}{dt} L_{ABCD} = \left( -8g_3^2 - 6g_2^2 - \frac{2}{3} g_1^2 \right) L_{ABCD} \]
\[ + (Y^T Y)^*_{AA'} L_{A'B'CD} (Y^T Y)^*_{AB} L_{A'B'CD} \]
\[ + (Y^T Y)^*_{AA'} L_{A'B'CD} (Y^T Y)^*_{CD} L_{A'B'CD} + (Y^T Y)^*_{DD'} L_{ABC'D} \] (3.6)
3.3 Baryon Decay Rate

It is easy to see that last four terms correspond to $SU(2)$ invariant corrections to the three $Q_L$ and one $L_L$ external lines of the LLLL operator by a loop involving Higgs exchange while the first term counts the dressing by gauge particle exchange on an external line. A similar equation governs the RRRR evolution \[95\]. In the combined system of the MSSM, soft Susy, LLLL and RRRR- 447 RGEs must be integrated down from the scale $M_X$ to $M_Z$. Below that scale one has to treat fermions and bosons differently and thus one must pass to a component field description (instead of superfield).

- Next the $d=4$ superpotential is converted into the $d=5$ effective Lagrangian at $M_Z$ involving 2 scalar and 2 fermion fields. In order to determine the effective Lagrangian that governs the nucleon decay we must evaluate the dressing diagrams that convert the two scalar (A) fields into the corresponding fermi fields by exchange of a gluino, chargino or neutralino field. For example gluino exchange is governed by the diagrams shown in Fig 3.1. Calculation of a 1-loop diagram is simplified by assuming that the momenta of the external fermion lines are negligible compared to superpartner masses. It is clear that in order to calculate the loop we must write the Lagrangian in the mass diagonal basis so that the propagators can be easily inserted. Thus we not only diagonalize the $3 \times 3$ Yukawa coupling matrices by the usual biunitary transformations.

$$\hat{f}_{\text{weak}} = U^\dagger f_{\text{mass}} \quad , \quad \hat{f}_{\text{weak}} = V^\dagger \tilde{f}_{\text{mass}} \quad (3.7)$$
but also diagonalize the $6 \times 6$ charged sfermion mass squared matrices given in the Appendix of Chapter 2. These are written in the weak basis (denoted by hat on $\tilde{F}$)
\[(\tilde{f}, \tilde{f}^\dagger)^T = \hat{F} \quad (3.8)\]
or the mass diagonal basis
\[\mathcal{L} = -\hat{F}^\dagger M_\tilde{F}^2 \hat{F} = -\tilde{F}^\dagger \Lambda_\tilde{F}^2 \tilde{F} \quad ; \quad \tilde{F} = \tilde{U}_\tilde{F} \hat{F} \quad (3.9)\]
where $\Lambda_\tilde{F}^2$ is diagonal positive definite. Similarly one must also diagonalize the $4 \times 4$ neutralino and $2 \times 2$ chargino mass matrix by means of a symmetric unitary ($N$) and a biunitary ($U^+, U^-$) transformations respectively. Due to the diagonalization the Yukawa couplings in the theory when rewritten in the mass diagonal basis become quite complicated.

- The superpotential
\[W = \phi_1 \phi_2 \phi_3 \phi_4 \quad (3.10)\]
will yield 2 fermion, 2 scalar terms in the Lagrangian :
\[\mathcal{L} = -\frac{1}{2}(\psi_1 \psi_2)A_3 A_4 + \text{permutations} \quad (3.11)\]

So from $W_{\Delta B=\pm 1}$, one obtain the following $d=5$ terms:
\[\mathcal{L}_5 = \epsilon_{abc} \{ C(\tilde{u}dul_L)^{MNij} \tilde{u}^a_M \tilde{d}^b_N (u^c_L l_{Lj}) + C(\tilde{u}dul_R)^{MNij} \tilde{u}^a_M \tilde{d}^b_N (u^c_R l_{Rj}) + C(\tilde{u}dul_L)^{MNij} \tilde{u}^a_M \tilde{d}^b_N (u^c_L d^e_{Lj}) + C(\tilde{u}dul_R)^{MNij} \tilde{u}^a_M \tilde{d}^b_N (u^c_R d^e_{Rj}) + C(\tilde{d}\nu ud_L)^{Mijk} \tilde{d}^a_M \tilde{\nu} (u^b_L d^c_{Lj}) \} \quad (3.12)\]

We have mentioned the coefficients corresponding to channel: proton decays into the charged lepton. Contribution for the other decay channels can be found in [95]. Here $a$, $b$ and $c$ are SU(3) fundamental indices. Subscript L and R represent fermion chirality. The coefficients $C(\tilde{f} \tilde{f} f'' f'''$) are defined in terms of $L_{ABCD}$ and $R_{ABCD}$ as follows:
3.3 Baryon Decay Rate

\( C(\tilde{u}\tilde{d}u_L)^{MNij} = - \sum_{A,B,C,D=1}^3 (L^{ABCD} - L^{CBAD})(\tilde{U}^\dagger_d)_A^M (\tilde{U}^\dagger_u)_B^N (U^\dagger_u)_C^i (U^\dagger_e)_D^j \)

\( C(\tilde{e}\tilde{\nu}u_L)^{Mij} = - \sum_{A,B,C,D=1}^3 (L^{ABCD} - L^{CBAD})(\tilde{U}^\dagger_d)_C^M (\tilde{U}^\dagger_\nu)_D^i (U^\dagger_u)_B^j (U^\dagger_d)_A^k \)

\( C(\tilde{u}\tilde{c}d_L)^{MNij} = - \sum_{A,B,C,D=1}^3 (L^{ABCD} - L^{CBAD})(\tilde{U}^\dagger_u)_A^M (\tilde{U}^\dagger_d)_B^N (U^\dagger_d)_C^i (U^\dagger_u)_D^j \)

\( C(\tilde{u}\tilde{d}u_R)^{MNij} = -2 \sum_{A,B,C,D=1}^3 R^{ABCD}(\tilde{U}^\dagger_u)_C^{M+3} (\tilde{U}^\dagger_d)_D^{N+3} (V^\dagger_u)_B^i (V^\dagger_e)_A^j \) \hspace{1cm} (3.13)\)

Here the indices M, N, i, j, k run from 1 to 3 and \( \tilde{U}_f, U_f, V_f \) are the unitary matrices which diagonalize scalars and fermions respectively.

- After redefining the fields to diagonalize the mass matrices, we can write the interaction Lagrangian in mass basis. The quark (lepton)-squark (slepton)-gaugino/higgsino (gluino, chargino and neutralino) interaction terms are given by:

\[
\mathcal{L}_{\text{gauge-Yukawa}} = \mathcal{L}_{\text{int}}(\tilde{g}) + \mathcal{L}_{\text{int}}(\tilde{g}^+) + \mathcal{L}_{\text{int}}(\tilde{g}^0) \quad (3.14)
\]

\[
\mathcal{L}_{\text{int}}(\tilde{g}) = -i\sqrt{2}g_3 \bar{\tilde{d}}^I \bar{\tilde{g}} \left[ \left( \Gamma^{(d)}_{gL} \right)_I \tilde{d}^j + \left( \Gamma^{(d)}_{gR} \right)_I \tilde{d}^j \right] d_j \\
- i\sqrt{2}g_3 \bar{\tilde{u}}^I \bar{\tilde{g}} \left[ \left( \Gamma^{(u)}_{gL} \right)_I \tilde{u}^j + \left( \Gamma^{(u)}_{gR} \right)_I \tilde{u}^j \right] u_j + \text{h.c.} \quad (3.15)
\]

\[
\mathcal{L}_{\text{int}}(\tilde{\chi}^\pm) = g_2 \chi^\pm \left[ \left( \Gamma^{(d)}_{CL} \right)_I ^{\alpha j} P_L + \left( \Gamma^{(d)}_{CR} \right)_I ^{\alpha j} P_R \right] d_j \bar{\tilde{u}}^I \\
+ g_2 \chi^\pm \left[ \left( \Gamma^{(u)}_{CL} \right)_I ^{\alpha j} P_L + \left( \Gamma^{(u)}_{CR} \right)_I ^{\alpha j} P_R \right] u_j \bar{\tilde{d}}^I \\
+ g_2 \chi^\pm \left[ \left( \Gamma^{(l)}_{CL} \right)_I ^{\alpha j} P_L + \left( \Gamma^{(l)}_{CR} \right)_I ^{\alpha j} P_R \right] l_j \bar{\tilde{e}}^I \\
+ g_2 \chi^\pm \left( \Gamma^{(e)}_{CL} \right)_I ^{\alpha j} P_L V_j \bar{\tilde{e}}^I + \text{h.c.} \quad (3.16)
\]
3.3 Baryon Decay Rate

\[ \mathcal{L}_{\text{int}}(\chi^0) = g_2 \chi \left[ \left( \Gamma^{(d)}_{NL} \right)^{\pi j}_I P_L + \left( \Gamma^{(d)}_{NR} \right)^{\pi j}_I P_R \right] d_j \tilde{d}^i + g_2 \chi \left[ \left( \Gamma^{(u)}_{NL} \right)^{\pi j}_I P_L + \left( \Gamma^{(u)}_{NR} \right)^{\pi j}_I P_R \right] u_j \tilde{u}^i + g_2 \chi \left[ \left( \Gamma^{(l)}_{NL} \right)^{\pi j}_I P_L + \left( \Gamma^{(l)}_{NR} \right)^{\pi j}_I P_R \right] l_j \tilde{l}^i + g_2 \chi \left( \Gamma^{(\nu)}_{NL} \right)^{\pi j}_I P_L \nu_j \tilde{\nu}^i + \text{h. c.} \] (3.17)

where \( P_{L/R} = \frac{1}{2}(1 \pm \gamma_5) \), \( g_2 \) and \( g_3 \) are gauge couplings of SU(2) and SU(3) respectively, \( I = 1, 2, \cdots 6 \) represents squarks and charged sleptons, \( i, j, k = 1, 2, 3 \) refer to fermions and sneutrinos, \( \alpha = 1, 2 \) and \( \alpha \) (\( = 1, 2, 3, 4 \)) denote chargino and neutralino. Mixing factors (\( \Gamma \)s) involve unitary matrices \( \tilde{U}_j \), \( U_f \), \( V_f \), \( U_{\pm} \) and \( N \) which diagonalize scalars, fermion (bi-unitary transformations), chargino and neutralino mass matrices.

\[
\begin{align*}
\left( \Gamma^{(d)}_{gL} \right)^{\alpha j}_I &= \sum_{k=1}^3 \left( \tilde{U}_d \right)^{k}_I \left( \tilde{U}_d \right)^{j}_k + \sum_{k=1}^3 \left( \tilde{U}_d \right)^{k+3}_I \left( V_d^{T} \right)^{j}_k \\
\left( \Gamma^{(u)}_{gL} \right)^{\alpha j}_I &= \sum_{k=1}^3 \left( \tilde{U}_u \right)^{k}_I \left( \tilde{U}_u \right)^{j}_k + \sum_{k=1}^3 \left( \tilde{U}_u \right)^{k+3}_I \left( V_u^{T} \right)^{j}_k \\
\left( \Gamma^{(d)}_{cL} \right)^{\alpha j}_I &= \sum_{k=1}^3 \left\{ - \left( \tilde{U}_u \right)^{k}_I \left( U_+ \right)^{\alpha}_1 \left( U_d \right)^{j}_k + \frac{1}{g_2} \sum_{l=1}^3 \left( \tilde{U}_u \right)^{k+3}_I \left( U_+ \right)^{\alpha}_2 \left( Y_d \right)^{j}_l \left( U_d \right)^{j}_k \right\} \\
\left( \Gamma^{(d)}_{cR} \right)^{\alpha j}_I &= \frac{1}{g_2} \sum_{k,l=1}^3 \left( \tilde{U}_d \right)^{k}_I \left( Y_u \right)^{\alpha}_2 \left( Y_d \right)^{j}_k \left( U_- \right)^{\alpha}_l \\
\left( \Gamma^{(u)}_{cL} \right)^{\alpha j}_I &= \sum_{l=1}^3 \left\{ - \left( \tilde{U}_d \right)^{l}_I \left( U_u \right)^{\alpha}_1 \left( U_d \right)^{j}_k + \frac{1}{g_2} \sum_{k=1}^3 \left( \tilde{U}_d \right)^{k+3}_I \left( Y_d \right)^{j}_k \left( U_u \right)^{\alpha}_1 \left( U_d \right)^{j}_k \right\} \\
\left( \Gamma^{(u)}_{cR} \right)^{\alpha j}_I &= -\frac{1}{g_2} \sum_{k,l=1}^3 \left( \tilde{U}_d \right)^{k}_I \left( Y_u \right)^{\alpha}_2 \left( U_u \right)^{j}_k \left( U_+ \right)^{\alpha}_2 \\
\end{align*}
\]
\[
\begin{align*}
(\Gamma_{CL}^{(l)})_{I}^{\alpha j} & = \sum_{k=1}^{3} (\bar{U}_d)_{I}^{k} (U_l)_{j}^{k} (U_+)^{\alpha}_{I} \\
(\Gamma_{CR}^{(l)})_{I}^{\alpha j} & = \frac{1}{g_2} \sum_{k,l=1}^{3} (\bar{U}_e)_{I}^{k} (Y_e)_{l}^{k} (U_+)^{\alpha}_{2} \\
(\Gamma_{NL}^{(d)})_{M}^{\eta B} & = \sum_{A=1}^{3} \left\{ \frac{1}{\sqrt{2}} (\bar{U}_d)_{M}^{A} (U_d)^{B}_{A} \left( (N_+)^{\eta}_{2} - \frac{g_1}{3\sqrt{2}g_2} (N_+)^{\eta}_{1} \right) \\
& - \frac{1}{g_2} \sum_{C=1}^{3} (\bar{U}_d)_{M}^{C} (Y_d)_{A}^{C} (U_d)^{B}_{A} (N_+)^{\eta}_{3} \right\} \\
(\Gamma_{NR}^{(d)})_{M}^{\eta B} & = \sum_{A=1}^{3} \left\{ -\frac{\sqrt{2}g_1}{3g_2} (\bar{U}_d)_{M}^{A+3} (V_d)^{B}_{A} (N_+)^{\eta}_{1} \\
& - \frac{1}{g_2} \sum_{C=1}^{3} (\bar{U}_d)_{M}^{C} (Y_d)_{A}^{C} (V_d)^{B}_{A} (N_+)^{\eta}_{3} \right\} \\
(\Gamma_{NL}^{(u)})_{M}^{\eta B} & = \sum_{A=1}^{3} \left\{ \frac{1}{\sqrt{2}} (\bar{U}_u)_{M}^{A} (U_u)^{B}_{A} \left( - (N_+)^{\eta}_{2} - \frac{g_1}{3\sqrt{2}g_2} (N_+)^{\eta}_{1} \right) \\
& - \frac{1}{g_2} \sum_{C=1}^{3} (\bar{U}_u)_{M}^{A+3} (Y_u)_{A}^{C} (U_u)^{B}_{C} (N_+)^{\eta}_{4} \right\} \\
(\Gamma_{NR}^{(u)})_{M}^{\eta B} & = \sum_{A=1}^{3} \left\{ -\frac{2\sqrt{2}g_1}{3g_2} (\bar{U}_u)_{M}^{A+3} (V_u)^{B}_{A} (N_+)^{\eta}_{1} \\
& - \frac{1}{g_2} \sum_{C=1}^{3} (\bar{U}_u)_{M}^{A} (Y_u)_{A}^{C} (V_u)^{B}_{C} (N_+)^{\eta}_{4} \right\} \\
(\Gamma_{NL}^{(e)})_{M}^{\eta B} & = \sum_{A=1}^{3} \left\{ \frac{1}{\sqrt{2}} (\bar{U}_e)_{M}^{A} (U_e)^{B}_{A} \left( (N_+)^{\eta}_{2} + \frac{g_1}{\sqrt{2}g_2} (N_+)^{\eta}_{1} \right) \\
& - \frac{1}{g_2} \sum_{C=1}^{3} (\bar{U}_e)_{M}^{A+3} (Y_e)_{A}^{C} (U_e)^{B}_{C} (N_+)^{\eta}_{3} \right\} \\
(\Gamma_{NR}^{(e)})_{M}^{\eta B} & = -\sum_{A=1}^{3} \left\{ \frac{g_1}{\sqrt{2}g_2} (\bar{U}_e)_{M}^{A+3} (V_e)^{B}_{A} (N_+)^{\eta}_{1} \\
& + \frac{1}{g_2} \sum_{C=1}^{3} (\bar{U}_e)_{M}^{C} (Y_e)_{C}^{A} (V_e)^{B}_{A} (N_+)^{\eta}_{3} \right\} \quad (3.20)
\end{align*}
\]

The gluino mass matrix does not need diagonalization so one can evaluate the
3.3 Baryon Decay Rate

diagram in Fig 3.1 to give a contribution to $\mathcal{L}_{\text{eff}}$

$$
\mathcal{L}_{\text{eff}} = \frac{4}{3i} \frac{g_3^2}{M_\tilde{g}} \frac{1}{16\pi^2} (-2i L_{miljk})(\tilde{U}_u^\dagger)_{mN}(\tilde{U}_d^\dagger)_{lM} \left( \Gamma_{gL}^{(u)} \right)_N \left( \Gamma_{gL}^{(d)} \right)_M \\
\times H(\tilde{u}_N, \tilde{d}_M) \epsilon_{abc} (u_{\ell L}^a d_{LL}^b) (u_{\ell L}^c l_{Lk})
$$

(3.21)

Here the function $H(\tilde{u}_N, \tilde{d}_M)$ arises from the standard loop integration, having form :

$$
H(x, y) = \frac{1}{x-y} \left( \frac{x \log x}{x-1} - \frac{y \log y}{y-1} \right)
$$

(3.22)

and the arguments of loop function $H$ are Susy particle mass squared ratios:

$$
\tilde{d}_M = \frac{m_{\tilde{d}_M}^2}{M_\tilde{g}^2}, \quad \tilde{u}_M = \frac{m_{\tilde{u}_M}^2}{M_\tilde{g}^2}
$$

(3.23)

- After calculating one-loop (gluino, neutralino and chargino) dressing diagrams, effective Lagrangian containing four-fermi interaction terms relevant to the proton decay into charged lepton channels is given by:

$$
\mathcal{L}_{\Delta B \neq 0} = \frac{1}{(4\pi)^2} \epsilon_{abc} \left\{ \tilde{C}_{LL}(udul)^{ik} (u_{\ell L}^a d_{LL}^b) (u_{\ell L}^c l_{Lk}) + \tilde{C}_{RL}(udul)^{ik} (u_{\ell R}^a d_{Ri}^b) (u_{\ell L}^c l_{Lk}) \\
+ \tilde{C}_{LR}(udul)^{ik} (u_{\ell L}^a d_{LL}^b) (u_{\ell R}^c l_{Rk}) + \tilde{C}_{RR}(udul)^{ik} (u_{\ell R}^a d_{Ri}^b) (u_{\ell R}^c l_{Rk}) \right\}
$$

(3.24)

$\tilde{C}$ coefficients can be found in [95].

- Finally matrix elements of the four Fermi operators involving quark and lepton fields must be evaluated between the baryon and meson initial and final states to obtain the amplitude for baryon decay in any channel (e.g. $p \rightarrow e^+ \pi^0$). Chiral Lagrangian technique [101] is used to convert the effective quark Lagrangian to the effective hadronic Lagrangian. Then partial decay widths of the nucleon are given as

$$
\Gamma(B_i \rightarrow M_j l_k) = \frac{m_i}{32\pi} \left( 1 - \frac{m_j^2}{m_i^2} \right)^2 \frac{1}{f_\pi^2} (A_{\text{Long}}^2) \left( |A_{L}^{ijk}|^2 + |A_{R}^{ijk}|^2 \right)
$$

(3.25)
where $m_i$ and $m_j$ are the masses of baryon and meson respectively. $f_\pi$ is the pion decay constant having value 139 MeV. A factor of $A_{Long} \approx .22$ is used to take into account the renormalization from $M_Z$ to 1 GeV. The explicit expressions for $A_{L,R}^{ijk}$ (defined in terms of $\tilde{C}$ coefficients) can be found in [95].

As shown in [57] using the tree level Yukawas successful fitting of fermion mass mixing data is obtained in the NMSGUT but proton life time is 6-7 order of magnitude smaller than the experimental limit. In the literature particular textures of Yukawa couplings and discrete symmetries are considered to suppress fast B-decay rates [58]. In the next section we will discuss a generic mechanism to suppress fast B-decay rates in Susy-GUTs.

### 3.4 GUT Scale Threshold Corrections

We have computed one-loop GUT scale threshold correction to a Yukawa coupling of matter field due to heavy fields running in self energy loops on lines leading into the Yukawa vertex when the external light Higgs comes from any of the 6 possible components [57] using technique of [102]. These threshold corrections are very significant due to the large Higgs representation used and also play a crucial role in obtaining parameter sets compatible with constraints on B violation.

#### 3.4.1 Formalism

In supersymmetric theories, non-renormalization theorem [103] implies that superpotential couplings are modified only by wave function renormalization. We have calculated the large number of the NMSGUT Yukawa vertices that couple light fermions and Higgs field. To calculate corrections we need to move into the basis where mass matrices of heavy fields are diagonal. We can redefine heavy field $\Phi$ to diagonalize the mass term in the superpotential

$$\Phi = U^\phi \Phi' ; \quad \Phi = V^\phi \Phi' \Rightarrow \Phi^T M \Phi = \Phi'^T M_{Diag} \Phi'$$

(3.26)
Matter Yukawa vertices:

\[ \mathcal{L} = \left[ f^T_c Y_f f H_f \right]_F + h.c. + ... \]  (3.27)

As shown in Fig. 3.2, heavy superfields can circulate on any of the three chiral superfields which give wave function renormalization in the kinetic terms:

\[ \mathcal{L} = \left[ \sum_{A,B} (\bar{f}^A_f (Z_f)^B_f \bar{f}^B_f + f^A_f (Z_f)^B_f f^B_f) + H^\dagger Z_H H + \bar{H}^\dagger Z_{\bar{H}} \bar{H} \right]_D + .. \]  (3.28)

Here A, B = 1, 2, 3 are the generation indices and \( H \) and \( \bar{H} \) are the MSSM light Higgs doublets. Light Higgs fields are the combinations of all the heavy Higgs \( h_i \), \( i = 1...6 \) fields of GUT

\[ H = \sum_i \alpha^*_i h_i \quad ; \quad \bar{H} = \sum_i \bar{\alpha}^*_i \bar{h}_i \]  (3.29)

Here \( \alpha_i \) and \( \bar{\alpha}_i \) are the Higgs fractions which describe the contribution of different Higgs fields to light Higgs and are first columns of the unitary matrices which diagonalize the Higgs mass matrix. One needs to define a new basis to write kinetic terms of light matter and Higgs fields in canonical form as:

\[ f = U_{Z_f} \Lambda_{Z_f}^{-1} \bar{f} = \bar{U}_{Z_f} \bar{f} \quad ; \quad \bar{f} = \bar{U}_{Z_f} \Lambda_{Z_f}^{-1} \bar{f} = U_{Z_f} \bar{f} \]
\[ H = \frac{\tilde{H}}{\sqrt{Z_H}} \quad ; \quad \bar{H} = \frac{\tilde{H}}{\sqrt{Z_{\bar{H}}}} \quad (3.30) \]

Here \( U_{Z_f}, \bar{U}_{Z_f} \) are the unitary matrices that diagonalize \( Z_{f,\bar{f}} \) to positive definite form \( \Lambda_{f,\bar{f}} \). Then

\[ \mathcal{L} = \left[ \sum_{A} (\tilde{\bar{f}}_{A}^{\dagger} \tilde{f}_{A} + \tilde{f}_{A}^{\dagger} \tilde{f}_{A}) + \bar{H}^{\dagger} \bar{H} + \tilde{H}^{\dagger} \tilde{H} \right]_{D} + \left[ \tilde{\bar{f}}^{T} \bar{Y}_{f} \tilde{f} \bar{H}_{f} \right]_{F} + h.c. + .. \quad (3.31) \]

As a result the MSSM Yukawa couplings in terms of the tree level (SO(10) determined) Yukawas change as

\[ \tilde{Y}_{f} = \Lambda_{Z_{f}}^{-\frac{1}{2}} U_{Z_{f}}^{T} \frac{Y_{f}}{\sqrt{Z_{H_{f}}}} U_{Z_{f}} \Lambda_{Z_{f}}^{-\frac{1}{2}} = \tilde{Y}_{f}^{T} U_{Z_{f}}^{T} \bar{U}_{Z_{f}} \quad (3.32) \]

Generic form of correction factor for any chiral field \( \Phi_i \) is \((Z = 1 - \mathcal{K}) : \)

\[ \mathcal{K}^j_i = -\frac{g_{10}^2}{8\pi^2} \sum_{\alpha} Q_{ik}^{\alpha} \psi_{i}^{\dagger} \gamma^{\mu} A_{\mu}^{\alpha} \psi_{k} + \frac{1}{32\pi^2} \sum_{kl} Y_{ikl}^{*} Y_{jkl} F(m_k, m_l) \quad (3.33) \]

Here first term is the contribution of coupling of \( \Phi_i \) to gauge field \( (A_{\mu}) \) in \( \mathcal{L} = g_{10} Q_{ik}^{\alpha} \psi_{i}^{\dagger} \gamma^{\mu} A_{\mu}^{\alpha} \psi_{k} \) \((g_{10} \text{ is SO(10) gauge coupling}) \) and second term is Yukawa contribution \((W = \frac{1}{6} Y_{ijk} \Phi_i \Phi_j \Phi_k) \). \( F \) is symmetric Passarino-Veltman loop function.

When both the fields running in the loop are heavy fields then \( F(m_1, m_2) \) has the form

\[ F_{12}(M_A, M_B, Q) = \frac{1}{(M_A^2 - M_B^2)} (M_A^2 \ln \frac{M_A^2}{Q^2} - M_B^2 \ln \frac{M_B^2}{Q^2}) - 1 \quad (3.34) \]

which reduces to just

\[ F_{11}(M_A, Q) = F_{12}(M_A, 0, Q) = \ln \frac{M_A^2}{Q^2} - 1 \quad (3.35) \]

when one field is light \((M_B \to 0)\). One should avoid the sum over light index when both the fields running in the loop are Higgs fields.
3.4 GUT Scale Threshold Corrections

3.4.2 Explicit Form of Correction Factors

In the NMSGUT, right handed neutrino Majorana masses are 3-4 order of magnitude smaller than the GUT scale. Therefore while calculating GUT scale threshold correction to the Yukawa coupling we treat right handed neutrino as light particle like other SM fermions. The calculation for the corrections to the light Higgs doublet lines $H, \overline{H}$ is much more complicated than the matter lines since these are mixtures of pairs of doublets from the $10, 120$ (2 pairs), $126, 126, 210$ SO(10) Higgs multiplets:

$$H = (V^H)^\dagger h \quad ; \quad \overline{H} = (U^H)^\dagger \overline{h}$$ (3.36)

Here $V^H$ and $U^H$ are the unitary matrices which diagonalize Higgs mass matrix ($\mathcal{H}$). The couplings of the GUT field doublets $h_a, \overline{h}_a, = 1, 2...6$ to various pairs of the 26 different MSSM irrep-types (labelled conveniently by the letters of the alphabet : see \[70, 57\]) that occur in this theory is worked out using the technology \[36\] of SO(10) decomposition via the PS group. There are again precisely 26 different combinations of GUT multiplets (labelled by the letter pairs for irreps) which can combine to give operators that can form singlets with the MSSM $H[1, 2, 1]$ (their conjugates gave singlets with $\overline{H}[1, 2, -1]$). Then we get

$$(16\pi^2)K_H = 3K_{JD} + 8K_{RC} + 9K_{XP} + K_{VF} + 3K_{EJ} + 9K_{PE} + 6K_{BM} + 3K_{XT}$$
$$+ 3K_{Df} + 24K_{QC} + 3K_{TE} + 6K_{YL} + 18K_{WB} + 8K_{CZ} + 9K_{EU}$$
$$+ 9K_{UD} + 3K_{HO} + K_{V\tilde{A}} + 3K_{K\tilde{X}} + K_{HF} + 6K_{NY} + 18K_{YW}$$
$$+ 3K_{V\tilde{O}} + 6K_{LB} + 3K_{S\tilde{H}} + K_{G\tilde{H}}$$ (3.37)

To illustrate the correction factor from the $JD$ channel on Higgs line is given by:

$$K_{JD} = \sum_a \sum_{a'} \left( \gamma V_{2a}^J U_{1a'}^D \right. - \frac{\gamma}{\sqrt{2}} V_{3a}^J U_{1a'}^D + \bar{\gamma} V_{2a}^J U_{2a'}^D + \bar{\gamma} \frac{\bar{\gamma}}{\sqrt{2}} V_{3a}^J U_{2a'}^D - \frac{ik}{\sqrt{2}} V_{3a}^J U_{3a'}^D \right) V_{11}^H$$
$$+ \left( \frac{2\eta}{\sqrt{3}} V_{2a}^J U_{1a'}^D \right. - \sqrt{6} \eta V_{3a}^J U_{1a'}^D - \frac{2i\bar{\eta}}{\sqrt{3}} V_{2a}^J U_{3a'}^D + \sqrt{3} \frac{3i\bar{\xi}}{2} V_{3a}^J U_{3a'}^D \right) V_{21}^H + \left( \frac{-i}{\sqrt{6}} \xi V_{3a}^J U_{3a'}^D \right.$$
3.5 Threshold Effects on $\Gamma_{d=5}^{\Delta B \neq 0}$

As discussed SO(10) Yukawa couplings ($h, f, g$) and heavy masses $M_i$ determine both fermion masses and coefficient of $d=5$ baryon decay operator

$$L_{ABCD} \langle \{h, g, f\}_{AB}, M_i \rangle, R_{ABCD} \langle \{h, g, f\}_{AB}, M_i \rangle$$

Canonical kinetic terms after including wavefunction renormalization factor require transformation of fields to the tilde basis (Eq. 3.30) using bi-unitary transformation. Then fermion Yukawas ($\tilde{Y}_f$) are diagonalized to mass basis (denoted by primes) via the unitary matrices ($U_f^{L,R}$) made up of the left and right eigenvectors of $\tilde{Y}_f$. Phases of unitary matrices are fixed by the requirement that $(U_f^L)^T \tilde{Y}_f U_f^R = \Lambda_f$ should yield positive definite $\Lambda_f$:

$$W = (\tilde{f}^T)^T \Lambda_f \tilde{f}^T \tilde{H}_f$$

$$f = \tilde{U}_f U_f^R f' = \tilde{U}_f^T f' \quad ; \quad \tilde{f} = \tilde{U}_f U_f^L \tilde{f} = \tilde{U}_f^T \tilde{f}$$

These calculations were done in collaboration with Prof. C. S. Aulakh and Ila Garg.
3.6 Fits Including Threshold Corrections

\[ \Lambda_f = \left( U^T_f \right)^T \tilde{U}_Z \frac{Y_f}{\sqrt{Z_f}} \tilde{U}_Z U^R_f \]  \hspace{1cm} (3.39)

As a result the coefficient \( L_{ABCD} \), \( R_{ABCD} \) of \( d=5, \Delta B = \pm 1 \) decay operator in terms of the Yukawa eigenstate basis transform as

\[ L'_{ABCD} = \sum_{a,b,c,d} L_{abcd} (\tilde{U}'_Q)_a (\tilde{U}'_Q)_b (\tilde{U}'_L)_c (\tilde{U}'_L)_d \]

\[ R'_{ABCD} = \sum_{a,b,c,d} R_{abcd} (\tilde{U}'_Q)_a (\tilde{U}'_Q)_b (\tilde{U}'_L)_c (\tilde{U}'_L)_d \]  \hspace{1cm} (3.40)

MSSM Higgs \((H, \bar{H})\) are mixtures of 6 pairs of doublets from NMSGUT Higgs irreps so Higgs lines have contribution from all the invariants (couplings) of superpotential. Although these couplings are small but large number of terms add up to an appreciable correction. **Imposing unitarity and perturbativity** \( Z > 0 \) **one can find the regions of the parameter space where couplings are small but \( |Z_H, \bar{H}| \approx 0 \). Thus the factor \( 1/\sqrt{Z_H, \bar{H}} \) (Eq. 3.39) will lower the magnitude of the SO(10) Yukawas required to match MSSM data by a factor of \( 10^{-1} \) to \( 10^{-2} \) and still maintain perturbativity. \( d=5 \) operators have no external Higgs line so lowered SO(10) couplings will suppress \( d=5 \) operators by a factor of \( 10^{-4} \) to \( 10^{-8} \).

### 3.6 Fits Including Threshold Corrections

Besides \( d=5 \), B decay operator coefficients, wavefunction renormalization also modifies the relation between other GUT and MSSM parameters. MSSM \( \mu \) and \( B \) parameters are larger than the same GUT parameters by the factor of \( (Z_H Z_{\bar{H}})^{-1/2} \). Scalar soft masses and soft Higgs masses will be modified by a factor of \( Z_f^{-1} \) and \( Z_{H/\bar{H}}^{-1} \) respectively. Trilinear soft parameters will remain same as wavefunction renormalization factors are absorbed by the Yukawa couplings \((A = A_0 \tilde{Y})\). These thresholds redefine \( f_{AB} \) as

\[ \tilde{f}_{AB} = (\tilde{U}^T f \tilde{U})_{AB} \]  \hspace{1cm} (3.41)

This changes the right handed neutrino Majorana masses \(((M_\nu)_{AB} \sim f_{AB} \tilde{\sigma})\). \( Y_\nu \) and Higgs field redefinition modify the Type I seesaw formula.

If we use the NMSGUT parameters for the solutions presented in [57] and
3.6 Fits Including Threshold Corrections

Table 3.1: Eigenvalues of the wavefunction renormalization matrices $Z_f$ for fermion lines and for MSSM Higgs ($Z_{H,H}$) for solutions presented in [57].

| Solution 1 | Solution 2 |
|------------|------------|
| $Z_{u}$   | 0.928326   | $Z_{u}$   | $-7.526729$ |
| $Z_{d}$   | 0.915317   | $Z_{d}$   | $-7.845885$ |
| $Z_{e}$   | 0.9004179  | $Z_{e}$   | $-8.830309$ |
| $Z_{Q}$   | 0.942772   | $Z_{Q}$   | $-7.880892$ |
| $Z_{L}$   | 0.911375   | $Z_{L}$   | $-9.203739$ |
| $Z_{H}$   | $-109.367$ | $Z_{H}$   | $-264.776$  |

$Z_{H, H} = -109.367$, $-193.755$  

$Z_{H, H} = -264.776$, $-386.534$

calculate threshold correction factors for fermion and Higgs lines ($Z_{H, H}, Z_{f, f}$) we notice that both the solutions have large negative value of $Z_{H, H}$ and second one even has negative eigenvalues for $Z_{f, f}$ as shown in Table 3.1. So we performed a fresh search including the GUT scale threshold corrections to Yukawas. Our basic search criteria is same as described in the previous chapter but now include an additional subroutine implementing threshold corrections to fermion and Higgs fields. Other improvements we implemented relative to [57] are:

1. We imposed the strict unitarity and perturbativity

$$Z_{f, f, H, H} > 0$$

so searches now prefer smaller values of superpotential parameters as compared to the case without GUT scale threshold corrections (see Table 2 [57] for details).

2. Including GUT scale threshold corrections we searched for fits of fermion mass-mixing data in terms of NMSGUT superpotential parameters that are com-
3.6 Fits Including Threshold Corrections

compatible with B decay limits. We have constrained the B decay rates while searching:

$$\text{Max}(L'_{ABCD}, R'_{ABCD}) < 10^{-22} \text{ GeV}^{-1}$$

to get proton lifetime above $10^{34}$ Yrs. This constraint forces the search towards the regions of parameter space which produce $Z_{H,H} \ll 1$

3. Range of soft Susy parameters is almost same as [57] except gluino mass ($M_{\tilde{g}}$) which is kept greater than 1000 GeV in accordance with the latest LHC results. Loop corrected Higgs mass is required to be in the experimentally indicated range

$$124 \text{ GeV} < M_h < 126 \text{ GeV}$$

4. Another improvement is inclusion of Susy threshold effects on gauge unification parameters $\alpha_3(M_Z), M_X, \alpha(M_X)$ to take into account the spread out spectrum of supersymmetric masses. A weighted sum over all the Susy particles ($M_{\text{Susy}}$) is used in $\Delta_{\alpha_s}^{\text{Susy}}$ as given in [105].

$$\Delta_{\alpha_s}^{\text{Susy}} \approx -\frac{19\alpha_s^2}{28\pi} \ln \frac{M_{\text{Susy}}}{M_Z}$$

$$M_{\text{Susy}} = \prod_i m_i \frac{1}{5} \left(4b_1^2 - 9.6b_1^2 + 5.6b_3^2\right)$$ (3.42)

$$\Delta_X^{\text{Susy}} = \frac{1}{11.2\pi} \sum_i (b_1 - b_2) \log_{10} \frac{m_i}{M_Z}$$ (3.43)

$$\Delta_G^{\text{Susy}} = \frac{1}{11.2\pi} \sum_i (6.6b_2 - b_1) \ln \frac{m_i}{M_Z}$$ (3.44)

Here $b_1, b_2, b_3$ are the 1-loop $\beta$ function coefficient of U(1), SU(2), SU(3) in the MSSM respectively. $\Delta_{\alpha_s}^{\text{Susy}}$ can be significant so it changes the allowed range at GUT scale. We considered the following limits for $\Delta_{\alpha_s}^{\text{Susy}}$ in the search program.

$$-0.0146 < \Delta_{\alpha_s}^{\text{Susy}} < -0.0102$$ (3.45)

Typically we find $M_{\text{Susy}} \approx 10$ TeV. Our constraints on the gauge unification parameters are thus:
3.6 Fits Including Threshold Corrections

\[-22.0 \leq \Delta_G \equiv \Delta(\alpha^{-1}_G(M_X)) \leq 25\]
\[3.0 \geq \Delta_X \equiv \Delta(\log_{10}M_X) \geq -0.03\]
\[-0.0126 < \Delta_3 \equiv \Delta\alpha_3(M_Z) < -0.0122\]  \hspace{1cm} (3.46)

3.6.1 Example Fit

1. In Table 3.2 we give the values of the NMSGUT superpotential parameters, changes in gauge unification parameters- \(\Delta_x, \Delta_\alpha, \Delta_a\) (from GUT and Susy), \(x\) parameter, the superheavy spectrum and the mSUGRY-NUHM parameters preferred by the fitting search program. We also give heavy right handed neutrino masses along with Type I and II contribution to light neutrino masses. All parameters are modified (by GUT scale thresholds) parameters. Tree level relation \(\frac{y_d - y_e}{y_s - y_{\mu}} \sim 1\) \cite{87, 88, 89, 90, 91} is no longer applicable.

2. Table 3.3 shows the successful fitting of extrapolated fermion mass mixing data at GUT scale. Column 2 contains the values achieved by the model. Column 3 shows experimental error. In the central block eigenvalues of the fermion correction factors \((Z_f, \bar{Z}_f)\), \(Z_{H,\bar{H}}\) are given and in the lower block Higgs fractions \(\alpha_i, \bar{\alpha}_i\) are presented (which along with the SO(10) Yukawas determine fermion masses). These parameters are determined by the fine tuning condition to keep one pair of Higgs doublets light. As discussed \(Z_{H,\bar{H}} \ll 1\) lowers the SO(10) Yukawa couplings therefore \(Z_f, \bar{Z}_f\) are close to unity since these are determined by the lowered Yukawas. In Table 3.4 (column 2) we show, run down to \(M_Z\), values of the fermion masses generated by GUT. Notice that \(y_d\) and \(y_s\) are smaller by a factor of 3 as compared to their SM values. Fermion masses including large \(\tan\beta\) driven Susy threshold corrections are given in 4th column.

3. In Table 3.5 we have run down values of soft Susy masses (including \(M^2_{H,\bar{H}}\)) and trilinear couplings. These parameters determine Susy threshold corrections to fermion Yukawas. \(\mu\) and \(B\) are determined by electroweak symmetry breaking conditions (their values at \(M_X\) were obtained by running backup to \(M_X\)). We used the tree level formulae given in the Appendix (of Chapter 2), to calculate
Susy spectra. Tables 3.6 and 3.7 show Susy spectra ignoring and including generation mixing. Off-diagonal running changes the spectra marginally.

4. Here the spectrum has same characteristic as shown in [57] and mentioned in the previous chapter: $\mu, A_0, B > 100$ TeV and normal s-hierarchy. Sometimes we get light smuon having mass 100-200 GeV along with Bino (LSP). These special solutions are crucial for model phenomenology as they offer a co-annihilation channel to LSP and can predict appropriate contribution to $(g - 2)_\mu$. Unoptimized values of important beyond SM (BSM) observables are presented in Table 3.8.

5. Using the formalism of [95, 96] we have calculated the decay rate of proton and neutron to the different channel as shown in Table 3.9. Clearly, B-decay rates are compatible with the experiment. In Table 3.10 we consider the chargino and gluino contribution separately and it shows chargino dominance.

6. Other solution with large value of $M_\mu$ are also obtained. They may be found in [65, 104].

### 3.7 Conclusions and Outlook

We have computed the one-loop GUT scale threshold corrections [104] to the tree level Yukawas at an SO(10) Yukawa vertex. Threshold corrections at $M_\chi$ to the Higgs lines are very significant due to the large Higgs representation used. There exist regions in parameter space where the effective MSSM Higgs renormalization factor can have very small value ($Z_{H,\bar{H}} \approx 0$). These corrections lower the SO(10) Yukawas required to match MSSM fermion data. The same Yukawas determine coefficients of d=5 baryon violation operators. The lowered SO(10) Yukawas solved the problem of fast B decay in NMSGUT [57]. We have shown example solutions of NMSGUT parameters and soft Susy breaking parameters which accurately fit fermion mass mixing data and are compatible with B decay rates. Solutions found have not only the Yukawa couplings but also the superpotential parameters significantly lowered in magnitude as compared to the tree level solutions. We have not
Table 3.2: NMSGUT superpotential couplings and SUGRY-NUHM soft parameters at $M_X$ which accurately fit fermion mass-mixing data respecting RG constraints. Unification parameters and mass spectrum of superheavy and superlight fields are also given.
### 3.7 Conclusions and Outlook

| Parameter | Target $= \bar{O}_i$ | Uncert. $= \delta_i$ | Achieved $= O_i$ | Pull $= \frac{(O_i - \bar{O}_i)}{\delta_i}$ |
|-----------|-----------------------|---------------------|------------------|-----------------------------------|
| $y_u/10^{-6}$ | 2.062837 | 0.788004 | 2.066323 | 0.004424 |
| $y_e/10^{-3}$ | 1.005548 | 0.165915 | 1.010599 | 0.030440 |
| $y_t$ | 0.369885 | 0.014795 | 0.369792 | −0.006256 |
| $y_d/10^{-5}$ | 11.438266 | 6.668509 | 12.421488 | 0.147443 |
| $y_s/10^{-3}$ | 2.169195 | 1.023860 | 2.189195 | 0.019534 |
| $y_b$ | 0.456797 | 0.237078 | 0.527664 | 0.298917 |
| $y_e/10^{-4}$ | 1.240696 | 0.186104 | 1.224753 | −0.085665 |
| $y_{\mu}/10^{-2}$ | 2.589364 | 0.388405 | 2.603313 | 0.035911 |
| $y_{\tau}$ | 0.543441 | 0.103254 | 0.532427 | −0.106669 |
| $\sin \theta_{12}^\mu$ | 0.2210 | 0.001600 | 0.2210 | −0.0003 |
| $\sin \theta_{13}^q/10^{-4}$ | 29.1907 | 5.000000 | 29.0755 | −0.0230 |
| $\sin \theta_{23}^q/10^{-3}$ | 34.3461 | 1.300000 | 34.3574 | 0.0087 |
| $\delta^\nu$ | 60.0212 | 14.000000 | 59.7774 | −0.0174 |
| $(m_{12}^2)/10^{-5}$(eV)$^2$ | 5.2115 | 0.552419 | 5.2189 | 0.0133 |
| $(m_{23}^2)/10^{-3}$(eV)$^2$ | 1.6647 | 0.332930 | 1.6650 | 0.0011 |
| $\sin^2 \theta_{12}^\mu$ | 0.2935 | 0.058706 | 0.2926 | −0.0152 |
| $\sin^2 \theta_{13}^q$ | 0.4594 | 0.137809 | 0.4412 | −0.1317 |
| $\sin^2 \theta_{23}^q$ | 0.0250 | 0.019000 | 0.0267 | 0.0892 |
| $(Z_\mu)$ | 0.957467 | 0.957908 | 0.957908 |
| $(Z_d)$ | 0.950892 | 0.951332 | 0.951332 |
| $(Z_b)$ | 0.925116 | 0.925579 | 0.925580 |
| $(Z_e)$ | 0.944853 | 0.945306 | 0.945308 |
| $(Z_Q)$ | 0.968740 | 0.969189 | 0.969190 |
| $(Z_L)$ | 0.949564 | 0.950011 | 0.950013 |
| $Z_R, Z_H$ | 0.000273 | 0.001151 |
| $\alpha_1$ | 0.1609 − 0.0000i | $\bar{\alpha}_1$ | 0.1188 − 0.0000i |
| $\alpha_2$ | −0.3140 − 0.6026i | $\bar{\alpha}_2$ | −0.4802 − 0.2961i |
| $\alpha_3$ | −0.0477 − 0.4786i | $\bar{\alpha}_3$ | −0.4842 − 0.2469i |
| $\alpha_4$ | 0.3903 − 0.1942i | $\bar{\alpha}_4$ | 0.5795 + 0.0171i |
| $\alpha_5$ | −0.0449 + 0.0061i | $\bar{\alpha}_5$ | −0.0415 − 0.1241i |
| $\alpha_6$ | −0.0071 − 0.2982i | $\bar{\alpha}_6$ | 0.0274 − 0.1349i |

Table 3.3: Fit with $\chi_X = \sqrt{\sum_{i=1}^{17} \frac{(O_i - \bar{O}_i)^2}{\delta_i^2}} = 0.3988$. Target values, at $M_X$ of the fermion Yukawa couplings and mixing parameters, together with the estimated uncertainties, achieved values and pulls. The eigenvalues of the wavefunction renormalization for fermion and Higgs lines are given with Higgs fractions $\alpha_i, \bar{\alpha}_i$ which control the MSSM fermion Yukawa couplings.
3.7 Conclusions and Outlook

Parameter \( \text{SM}(M_Z) \) \( n^{\text{GUT}}(M_Z) \) \( n^{\text{MSSM}} = (m + \Delta m)^{\text{GUT}}(M_Z) \)

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| \( m_d/10^{-3} \) | 2.90000 | 1.08183 | 3.01515 |
| \( m_s/10^{-3} \) | 55.00000 | 19.06631 | 53.14737 |
| \( m_b \) | 2.90000 | 3.17508 | 3.05602 |
| \( m_e/10^{-3} \) | 0.48657 | 0.45157 | 0.45925 |
| \( m_\mu \) | 0.10272 | 0.09594 | 0.09902 |
| \( m_\tau \) | 1.74624 | 1.65725 | 1.65734 |
| \( m_u/10^{-3} \) | 1.27000 | 1.10509 | 1.27687 |
| \( m_c \) | 0.61900 | 0.54048 | 0.62449 |
| \( m_t \) | 172.50000 | 145.99987 | 170.88573 |

Table 3.4: Values of the SM fermion masses in GeV at \( M_Z \) compared with the masses obtained from values of GUT derived Yukawa couplings run down from \( M_X \) to \( M_Z \) both before and after threshold corrections. Fit with \( \chi_Z = \sqrt{\sum_{i=1}^{9} \frac{(m^i_{\text{MSSM}} - m^i_{\text{SM}})^2}{(m^i_{\text{MSSM}})^2}} = 0.1153 \).

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| \( M_1 \) | 210.10 | \( m_{\tilde{Q}_1} \) | 14446.81 |
| \( M_2 \) | 569.81 | \( m_{\tilde{Q}_2} \) | 14445.85 |
| \( M_3 \) | 1000.14 | \( m_{\tilde{Q}_3} \) | 24609.79 |
| \( m_{\tilde{t}_1} \) | 1761.31 | \( A_{11}^{(l)} \) | -121907.75 |
| \( m_{\tilde{t}_2} \) | 210.71 | \( A_{22}^{(l)} \) | -121757.58 |
| \( m_{\tilde{t}_3} \) | 20777.09 | \( A_{33}^{(l)} \) | -77289.04 |
| \( m_{\tilde{L}_1} \) | 15308.21 | \( A_{11}^{(u)} \) | -148456.63 |
| \( m_{\tilde{L}_2} \) | 15258.47 | \( A_{22}^{(u)} \) | -148455.19 |
| \( m_{\tilde{L}_3} \) | 21320.16 | \( A_{33}^{(u)} \) | -76985.25 |
| \( m_{\tilde{d}_1} \) | 8402.95 | \( A_{11}^{(d)} \) | -122521.00 |
| \( m_{\tilde{d}_2} \) | 8401.45 | \( A_{22}^{(d)} \) | -122518.53 |
| \( m_{\tilde{d}_3} \) | 51842.14 | \( A_{33}^{(d)} \) | -44046.92 |
| \( m_{\tilde{Q}_4} \) | 11271.93 | \( \tan \beta \) | 51.00 |
| \( m_{\tilde{Q}_5} \) | 11270.77 | \( \mu(M_Z) \) | 125591.16 |
| \( m_{\tilde{Q}_6} \) | 40274.01 | \( B(M_Z) \) | 2.7861 \times 10^9 |
| \( M_H^2 \) | -1.6336 \times 10^{10} | \( M_H^2 \) | -1.7391 \times 10^{10} |

Table 3.5: Values (in GeV) of the soft Susy parameters at \( M_Z \) (evolved from the soft SUGRY-NUHM parameters at \( M_X \)) (\( M_{\text{Susy}} = 12.6 \text{ TeV} \)).
### 3.7 Conclusions and Outlook

#### Field Mass (GeV)

| Field | Mass (GeV) |
|---|---|
| $M_{\tilde{g}}$ | 1000.14 |
| $M_{\tilde{\chi}^\pm}$ | 569.81, 125591.22 |
| $M_{\tilde{\chi}^0}$ | 210.10, 569.81, 125591.20, 125591.20 |
| $M_\rho$ | 15308.069, 15258.322, 21320.059 |
| $M_{\tilde{e}}$ | 1761.89, 15308.29, 211.57, 15258.60, 20674.72, 21419.56 |
| $M_{\tilde{\nu}}$ | 11271.80, 14466.76, 11270.63, 14445.80, 24607.51, 40275.87 |
| $M_{\tilde{d}}$ | 8402.99, 11272.10, 8401.48, 11270.95, 40269.19, 51845.93 |
| $M_A$ | 377025.29 |
| $M_{H^\pm}$ | 377025.30 |
| $M_H$ | 377025.28 |
| $M_h$ | 124.00 |

Table 3.6: Susy spectrum calculated ignoring generation mixing effects.

| Field | Mass (GeV) |
|---|---|
| $M_{\tilde{g}}$ | 1000.72 |
| $M_{\tilde{\chi}^\pm}$ | 570.11, 125537.00 |
| $M_{\tilde{\chi}^0}$ | 210.22, 570.11, 125536.98, 125536.98 |
| $M_\rho$ | 15257.98, 15307.71, 21350.169 |
| $M_{\tilde{e}}$ | 242.61, 1765.59, 15258.25, 15307.93, 20733.03, 21453.81 |
| $M_{\tilde{\nu}}$ | 11258.18, 11270.54, 14444.57, 14445.53, 24609.90, 40301.29 |
| $M_{\tilde{d}}$ | 8400.19, 8401.71, 11258.52, 11270.84, 40294.63, 51879.28 |
| $M_A$ | 377430.83 |
| $M_{H^\pm}$ | 377430.84 |
| $M_H$ | 377430.82 |
| $M_h$ | 124.13 |

Table 3.7: Susy spectrum calculated including generation mixing effects.

| Parameter | Value |
|---|---|
| $\text{BR}(b \to s\gamma)$ | $3.294 \times 10^{-4}$ |
| $\Delta a_\mu$ | $1.06 \times 10^{-9}$ |
| $\Delta \rho$ | $6.03 \times 10^{-7}$ |
| $\epsilon / 10^{-7}$ | 0.12 |
| $\delta_{PMNS}$ | 6.21$^\circ$ |

Table 3.8: Unoptimized values for the solution presented.
3.7 Conclusions and Outlook

Table 3.9: d=5 operator mediated nucleon lifetimes $\tau_{p,n}$ (yrs), decay rates $\Gamma$ (yr$^{-1}$) and branching ratios in the different channels.

| Parameter | Value |
|-----------|-------|
| $\tau_p(M^+\bar{\nu})$ | $9.63 \times 10^{34}$ |
| $\Gamma(p \to \pi^+\bar{\nu})$ | $4.32 \times 10^{-37}$ |
| $\Gamma(p \to K^+\bar{\nu})$ | $3.63 \times 10^{-34}$ |
| $\Gamma(p \to K^+\bar{\nu}_e,\mu,\tau)$ | $4.32 \times 10^{-37}$ |
| $\Gamma(p \to K^+\bar{\nu})$ | $9.50 \times 10^{-36}$ |
| $\Gamma(p \to K^0\bar{l}^+)$ | $4.239 \times 10^{-37}$ |
| $\Gamma(p \to K^0\bar{l}^+)$ | $1.742 \times 10^{-37}$ |
| $\Gamma(p \to K^0\bar{l}^+)$ | $2.232 \times 10^{-4}$ |
| $\Gamma(p \to \eta^0\bar{l}^+)$ | $5.544 \times 10^{-37}$ |
| $\Gamma(p \to \eta^0\bar{l}^+)$ | $1.33 \times 10^{-3}$ |
| $\Gamma(p \to \eta^0\bar{l}^+)$ | $1.33 \times 10^{-3}$ |
| $\Gamma(p \to \eta^0\bar{l}^+)$ | $2.32 \times 10^{-4}$ |
| $\Gamma(n \to \pi^0\nu)$ | $1.084 \times 10^{35}$ |
| $\Gamma(n \to \pi^0\nu)$ | $2.170 \times 10^{-37}$ |
| $\Gamma(n \to \pi^0\nu)$ | $1.321 \times 10^{-3}$ |
| $\Gamma(n \to \pi^0\nu)$ | $1.321 \times 10^{-3}$ |
| $\Gamma(n \to \eta^0\nu)$ | $8.79 \times 10^{-36}$ |
| $\Gamma(n \to \eta^0\nu)$ | $8.79 \times 10^{-36}$ |
| $\Gamma(n \to \eta^0\nu)$ | $2.177 \times 10^{-37}$ |
| $\Gamma(n \to \eta^0\nu)$ | $2.177 \times 10^{-37}$ |
| $\Gamma(n \to \eta^0\nu)$ | $5.39 \times 10^{-4}$ |

Table 3.10: First and second column contain ratio of nucleon decay considering only chargino and gluino contribution to the total decay rate.

| Parameter | $\Gamma_{\text{chargino}}/\Gamma_{\text{total}}$ | $\Gamma_{\text{gluino}}/\Gamma_{\text{total}}$ |
|-----------|---------------------------------------------|---------------------------------------------|
| $p \to K^+\bar{\nu}$ | 0.795 | 0.029 |
| $p \to \pi^+\bar{\nu}$ | 1.123 | 0.011 |
| $p \to K^0\bar{l}^+$ | 0.794 | 0.029 |
| $p \to \pi^0\bar{l}^+$ | 1.255 | 0.023 |
| $p \to \eta^0\bar{l}^+$ | 0.61 | 0.015 |
| $n \to \pi^0\nu$ | 1.255 | 0.023 |
| $n \to K^0\nu$ | 1.111 | 0.015 |
| $n \to \eta^0\nu$ | 0.969 | 0.009 |
optimized our fits for different phenomenological constraints from quark and lepton sector. However we have calculated some important beyond SM (BSM) parameters such as $a_\mu$, $\rho$, $\text{BR}(b \rightarrow s\gamma)$ which respect the experiment limits. One of the example fits with a light smuon ($\tilde{\mu}_R$) shows significant Susy contribution to $a_\mu$.

We have also implemented the effects of Susy spectra on gauge unification parameters $\alpha_3(M_Z)$, $M_X$ and $\alpha(M_X)$. Susy spectra of the fits including super-heavy threshold corrections have same characteristics as the one without GUT scale threshold corrections (exhibited in [57]). Thus they have large $A_0$, $\mu \sim 100$ TeV (for realistic fermion mass and mixing data and $M_h \sim 125$ GeV) parameters and heavy s-particle spectra which seems to be a likely scenario after Higgs discovery. Fits obtained deviate significantly from $\frac{y_b-y_\tau}{y_b-y_\mu} \approx 1$ which is characteristic of $10-120$-plet generated fermion fits [87, 88, 89, 90, 91].

This mechanism of suppressing fast dimension 5 proton decay rates including GUT scale threshold to light fields, is generic for all realistic Susy GUTs in which the light MSSM Higgs arise from a mixture of GUT Higgs doublets coupled to a large number of superheavy fields. The effect of d=6 B violation operator with one external Higgs line remains to be checked.
Appendix: Higgs Field Correction Factors

In this Appendix, we give correction factors from all type of GUT multiplets to the $H$ and $\bar{H}$ (see Eq. [3.37]):

$$K_{R\bar{C}} = \sum_{a=1}^{d(R)} \sum_{a'=1}^{d(C)} \left( \frac{i k}{\sqrt{2}} V_{2a} R U_{3a}^C - \gamma V_{1a} U_{2a}^C + \frac{\gamma}{\sqrt{2}} V_{2a} R U_{1a}^C - \frac{\bar{\gamma}}{\sqrt{2}} V_{2a} R U_{1a}^C \right) V_{11}^H$$

$$+ \left( \frac{2 \eta}{\sqrt{3}} V_{1a} R U_{2a}^C - \frac{2}{3} \eta V_{2a} R U_{2a}^C + \frac{i \bar{\gamma}}{\sqrt{6}} V_{2a} R U_{3a}^C \right) V_{21}^H$$

$$+ \left( \frac{2 \eta}{\sqrt{3}} V_{1a} R U_{1a}^C + \frac{2}{3} \eta V_{2a} R U_{1a}^C + \frac{i \bar{\gamma}}{\sqrt{6}} V_{2a} R U_{3a}^C \right) V_{31}^H$$

$$+ \left( \frac{\zeta}{\sqrt{2}} V_{2a} R U_{2a}^C - \frac{i \rho}{3 \sqrt{6}} V_{2a} R U_{3a}^C + \frac{\bar{\zeta}}{\sqrt{2}} V_{2a} R U_{1a}^C \right) V_{51}^H$$

$$- \left( \frac{i \bar{\gamma}}{\sqrt{6}} V_{2a} R U_{2a}^C + \frac{\bar{\gamma}}{\sqrt{6}} V_{2a} U_{1a}^C - \frac{\rho}{3 \sqrt{3}} V_{2a} R U_{3a}^C \right) V_{61}^H \left| F_{12}(m^R_a, m^C_a, Q) \right|^2 (3.47)$$

$$K_{XP} = \sum_{a=1}^{d(X)} \sum_{a'=1}^{d(P)} \left( \frac{\gamma V_{1a} X U_{2a}^P - k}{\sqrt{2}} V_{2a} X U_{2a}^P \right) V_{11}^H - \left( \frac{2 \bar{\zeta}}{\sqrt{3}} V_{1a} X U_{2a}^P + \frac{\bar{\gamma}}{\sqrt{6}} V_{2a} X U_{2a}^P \right) V_{21}^H$$

$$+ \left( \frac{\zeta}{\sqrt{6}} V_{2a} X U_{2a}^P + \frac{2 \eta}{\sqrt{3}} V_{1a} X U_{1a}^P - \frac{2 \sqrt{2} \eta}{\sqrt{3}} V_{2a} X U_{1a}^P \right) V_{31}^H$$

$$+ \left( \frac{\rho}{3 \sqrt{2}} V_{2a} X U_{2a}^P + \bar{\zeta} V_{1a} X U_{1a}^P \right) V_{51}^H + \frac{i}{\sqrt{3}} \left( \sqrt{2} \zeta V_{2a} X U_{1a}^P \right) V_{61}^H \left| F_{12}(m^X_a, m^P_a, Q) \right|^2 (3.48)$$

$$K_{VF} = \sum_{a=1}^{d(F)} \left( k V_{4a}^F - i \gamma V_{1a}^F \right) V_{11}^H - \left( 2 \sqrt{3} i \eta V_{1a}^F + \sqrt{3} \zeta V_{4a}^F \right) V_{21}^H - \sqrt{3} \zeta V_{4a}^F V_{31}^H$$

$$- 2 \sqrt{3} \lambda V_{4a}^F V_{41}^H + \zeta V_{1a}^F V_{51}^H - \left( \sqrt{3} \zeta V_{1a}^F - \frac{i \rho}{\sqrt{3}} V_{4a}^F \right) V_{61}^H \left| F_{12}(m^F, m_a^F, Q) \right|^2 (3.49)$$
3.7 Conclusions and Outlook

\[ K_{EJ} = \sum_{a=1} \sum_{a'=1} \left( \gamma V_{2a}^E U_{3a'}^J + \sqrt{2} \gamma i V_{2a}^E U_{3a'}^J - \frac{\gamma}{\sqrt{2}} V_{2a}^E U_{3a'}^J + \gamma V_{1a}^E U_{3a'}^J + \frac{\bar{\gamma}}{\sqrt{2}} V_{1a}^E U_{3a'}^J \right) \]

\[ + ik V_{4a}^E U_{5a'}^J - \frac{ik}{\sqrt{2}} V_{6a}^E U_{3a'}^J \right) V_{11}^H + \left( \frac{2\eta}{\sqrt{3}} V_{2a}^E U_{2a'}^J + 2 \sqrt{\frac{2}{3}} \eta V_{3a}^E U_{1a'}^J + \sqrt{\frac{2}{3}} \eta V_{2a}^E U_{3a'}^J \right) V_1^H \]

\[ + 4i \delta \left( \frac{V_{4a}^E U_{4a'}^J - \frac{i \delta}{\sqrt{6}} V_{6a}^E U_{3a'}^J + \frac{2i \delta}{\sqrt{3}} V_{4a}^E U_{2a'}^J - \frac{i \delta}{\sqrt{3}} V_{6a}^E U_{5a'}^J \right) V_{21}^H + \left( \frac{2i \delta}{\sqrt{3}} V_{6a}^E U_{2a'}^J \right) V_3^H \]

\[ + \sqrt{\frac{3}{2}} i \zeta V_{6a}^E U_{3a'}^J - \frac{2 \sqrt{\frac{3}{2}} i \zeta V_{6a}^E U_{3a'}^J + i \zeta V_{4a}^E U_{5a'}^J + \sqrt{6} \eta V_{1a}^E U_{3a'}^J + \frac{2 \eta}{\sqrt{3}} V_{2a}^E U_{3a'}^J \right) V_3^H \]

\[ - \zeta \frac{V_{2a}^E U_{3a'}^J - \sqrt{2} \zeta V_{3a}^E U_{1a'}^J - \frac{\zeta}{\sqrt{2}} V_{2a}^E U_{3a'}^J \right) V_{41}^H + \left( \frac{\sqrt{2} \zeta}{\sqrt{3}} \right) V_{6a}^E U_{5a'}^J - \frac{\zeta}{\sqrt{3}} V_{3a}^E U_{1a'}^J \right) V_5^H \]

\[ - \frac{\rho}{3 \sqrt{3}} V_{4a}^E U_{5a'}^J + \frac{\rho}{3 \sqrt{3}} V_{6a}^E U_{3a'}^J + \frac{2 \zeta}{\sqrt{3}} V_{4a}^E U_{1a'}^J - \frac{2 \zeta}{\sqrt{3}} V_{6a}^E U_{2a'}^J + \frac{\sqrt{2} \zeta}{\sqrt{3}} V_{3a}^E U_{3a'}^J \]

\[ + \frac{i \zeta}{\sqrt{6}} V_{6a}^E U_{3a'}^J - \frac{\sqrt{3}}{2} i \zeta V_{6a}^E U_{3a'}^J - \frac{2 i \zeta}{\sqrt{3}} V_{6a}^E U_{5a'}^J \right) V_{61}^H \]

\[ F_{12}(m_a^E, m_{a'}^J, Q) \]

\[ -2 g_{10}^2 \left| \frac{2 i}{\sqrt{3}} U_{2a}^E V_{21}^H + \frac{2 i}{\sqrt{3}} U_{1a}^E V_{31}^H - \sqrt{2} U_{3a}^E V_{41}^H + \frac{2 i}{\sqrt{3}} (U_{6a}^E V_{61}^H) \right|^2 F_{12}(m_a^E, m_{a'}^J, Q) \]

\[ -2 g_{10}^2 \left| \frac{2 i}{\sqrt{3}} V_{5a}^J V_{21}^H - i V_{5a}^J V_{41}^H + \frac{i}{\sqrt{2}} V_{3a}^J V_{41}^H + i V_{5a}^J V_{51}^H - \frac{1}{\sqrt{3}} V_{5a}^J V_{61}^H \right|^2 \]

[3.50]

\[ K_{PE} = \sum_{a=1} \sum_{a'=1} \left( \gamma V_{1a}^P U_{3a'}^E - \frac{k}{\sqrt{2}} V_{2a}^P U_{4a'}^E \right) V_{11}^H \]

\[ + \left( \frac{2\eta}{\sqrt{3}} V_{1a}^P (U_{3a'}^E - \frac{k}{\sqrt{2}} U_{4a'}^E) + \frac{\bar{\zeta}}{\sqrt{6}} V_{2a}^P U_{4a'}^E \right) V_{21}^H \]

\[ - \frac{\zeta}{\sqrt{3}} \left( 2 V_{2a}^P U_{3a'}^E + V_{2a}^P \right) V_{31}^H + \left( \frac{2 \sqrt{2} \eta i V_{1a}^P U_{2a'}^E - \frac{\rho}{\sqrt{3}} V_{2a}^P \right) V_{41}^H \]

\[ + \sqrt{\frac{2}{3}} i \zeta V_{2a}^P \left( V_{3a}^P U_{4a'}^E \right) V_{41}^H + \left( \frac{\rho}{\sqrt{3}} V_{2a}^P \right) V_{51}^H \]

\[ - \frac{\sqrt{2}}{\sqrt{3}} V_{1a}^P U_{4a'}^E - \frac{i \rho}{3 \sqrt{3}} V_{2a}^P U_{3a'}^E + \frac{i \rho}{3 \sqrt{6}} V_{2a}^P U_{4a'}^E \right) V_{61}^H \]

\[ F_{12}(m_a^P, m_{a'}^E, Q) \]

[3.51]
3.7 Conclusions and Outlook

\[
K_{BM} = \left| \sqrt{2} i \gamma V_{11}^H - 2 \sqrt{\frac{2}{3}} i \eta V_{21}^H - \sqrt{2} i \zeta V_{51}^H - \sqrt{\frac{2}{3}} \zeta V_{61}^H \right|^2 F_{12}(m^B, m^M, Q) \quad (3.52)
\]

\[
K_{XT} = \sum_{a=1}^{d(D)} \sum_{a'=1}^{d(T)} \left( (k V_{1a}^T U_{6a}^T - \gamma V_{2a}^T U_{3a}^T - i \gamma V_{1a}^T U_{4a}^T - \zeta V_{2a}^T U_{2a}^T - \frac{i k}{\sqrt{2}} V_{2a}^T U_{7a}^T) V_{11}^H \\
+ \left( \sqrt{\frac{2}{3}} \gamma V_{1a}^T U_{6a}^T - \frac{\gamma}{\sqrt{3}} V_{2a}^T U_{6a}^T - 2 \sqrt{\frac{2}{3}} \eta V_{1a}^T U_{3a}^T - \frac{2 i \eta}{\sqrt{3}} V_{1a}^T U_{4a}^T - 2 \sqrt{\frac{2}{3}} i \eta V_{2a}^T U_{4a}^T \\
- \frac{\zeta}{\sqrt{3}} V_{1a}^T U_{6a}^T - \sqrt{\frac{2}{3}} V_{2a}^T U_{6a}^T + \frac{i \zeta}{\sqrt{3}} V_{1a}^T U_{7a}^T - \frac{\zeta}{\sqrt{3}} V_{2a}^T U_{7a}^T - \frac{2 i \zeta}{\sqrt{3}} V_{1a}^T U_{2a}^T \\
+ \frac{i \gamma}{\sqrt{3}} V_{1a}^T U_{3a}^T - \frac{\gamma}{\sqrt{3}} V_{2a}^T U_{3a}^T + \frac{i \zeta}{\sqrt{3}} V_{1a}^T U_{4a}^T - \frac{\zeta}{\sqrt{3}} V_{2a}^T U_{4a}^T - \frac{2 i \zeta}{\sqrt{3}} V_{1a}^T U_{2a}^T \\
+ \frac{i \rho}{\sqrt{3} \gamma} V_{1a}^T U_{6a}^T - \frac{i \rho}{\sqrt{3}} V_{2a}^T U_{6a}^T + \frac{\rho}{3 \sqrt{3}} V_{1a}^T U_{7a}^T + \frac{\rho}{3 \sqrt{6}} \zeta V_{2a}^T U_{7a}^T - \frac{2 \zeta}{\sqrt{3}} V_{1a}^T U_{2a}^T \\
\right) V_{11}^H \right| F_{12}(m^X_a, m^T_a, Q) - 2 g_1^2 - V_{1a}^T V_{11}^H - \left( \frac{i \gamma}{3 \sqrt{3}} V_{2a}^T U_{6a}^T + \frac{i \zeta}{3 \sqrt{3}} V_{2a}^T U_{7a}^T - \frac{i \gamma}{3} V_{2a}^T U_{2a}^T \right) V_{11}^H - \left( \frac{i \zeta}{3 \sqrt{3}} V_{2a}^T U_{6a}^T + \frac{i \zeta}{3 \sqrt{3}} V_{2a}^T U_{7a}^T - \frac{i \gamma}{3} V_{2a}^T U_{2a}^T \right) V_{11}^H \\
+i V_{5a}^T V_{41}^H - \left( \frac{i \zeta}{3 \sqrt{3}} V_{2a}^T U_{6a}^T + \frac{i \zeta}{3 \sqrt{3}} V_{2a}^T U_{7a}^T - \frac{i \gamma}{3} V_{2a}^T U_{2a}^T \right) V_{11}^H \right| F_{12}(m^X_a, m^T_a, Q) \quad (3.53)
\]

\[
K_{DI} = \sum_{a=1}^{d(D)} \left( \gamma V_{2a}^D - \gamma V_{1a}^D + i k V_{2a}^D \right) V_{11}^H + \left( \frac{i \zeta}{\sqrt{3}} V_{3a}^D - \frac{2 \eta}{\sqrt{3}} V_{2a}^D \right) V_{21}^H \\
+ \left( - i \zeta \sqrt{\frac{2}{3}} V_{2a}^D - 2 \sqrt{\frac{2}{3}} \eta V_{1a}^D \right) V_{31}^H + \left( \zeta V_{2a}^D - \frac{i \rho}{3} V_{3a}^D + \zeta V_{1a}^D \right) V_{51}^H \\
- \frac{1}{\sqrt{3}} \left( i \zeta V_{2a}^D - 3 i \zeta V_{1a}^D + \frac{2 \rho}{3} V_{3a}^D \right) V_{61}^H \right|^2 F_{12}(m^I, m^D_a, Q) \quad (3.54)
\]

\[
K_{QC} = \sum_{a=1}^{d(C)} \left( \frac{i \gamma}{\sqrt{2}} U_{1a}^C - \frac{i \gamma}{\sqrt{2}} U_{2a}^C - \frac{k}{\sqrt{2}} U_{3a}^C \right) V_{11}^H + \left( \sqrt{\frac{2}{3}} i \eta U_{2a}^C - \frac{\zeta}{\sqrt{6}} U_{3a}^C \right) V_{21}^H \\
- \left( \frac{\zeta}{\sqrt{6}} U_{2a}^C + \frac{2 \sqrt{\frac{2}{3}} \eta U_{1a}^C}{} \right) V_{31}^H + \left( \frac{i \zeta}{\sqrt{2}} U_{2a}^C - \frac{\rho}{3 \sqrt{2}} U_{3a}^C + \frac{i \zeta}{\sqrt{2}} U_{1a}^C \right) V_{51}^H \\
+ \left( \frac{\zeta}{\sqrt{6}} U_{2a}^C + \frac{\zeta}{\sqrt{6}} U_{1a}^C \right) V_{61}^H \right|^2 F_{12}(m^Q, m^C_a, Q) \quad (3.55)
\]
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\[ K_{TE} = \sum_{a=1} \sum_{a'=1} \left| \left( \gamma V_{5a}^T U_{1a'}^E - \gamma V_{5a}^T U_{4a'}^E - \gamma V_{2a}^T U_{1a'}^E - \gamma V_{2a}^T U_{4a'}^E - i \gamma V_{4a}^T U_{3a'}^E + k V_{6a}^T U_{3a'}^E \right) \right|^2 \]

\[ + ik V_{5a}^T U_{6a'}^E - \frac{ik}{\sqrt{2}} V_{7a}^T U_{4a'}^E \right) V_{11}^H + \left( 2 \sqrt{\frac{2}{3}} \eta V_{3a}^T U_{3a'}^E + 2 \sqrt{\frac{2}{3}} \eta V_{5a}^T U_{3a'}^E + i \gamma \sqrt{\frac{2}{3}} V_{1a}^T U_{3a'}^E \right) \]

\[ + \frac{\gamma}{\sqrt{3}} V_{1a}^T U_{4a'}^E - \frac{\eta}{\sqrt{3}} V_{6a}^T U_{6a'}^E - i \sqrt{3} \zeta V_{5a}^T U_{6a'}^E + \frac{2 i \zeta}{\sqrt{3}} V_{7a}^T U_{4a'}^E - \frac{\bar{\zeta}}{\sqrt{6}} V_{7a}^T U_{4a'}^E \]

\[ + \left( \frac{2}{3} \zeta V_{6a}^T U_{4a'}^E \right) V_{21}^H + \left( \frac{i \zeta}{\sqrt{6}} V_{7a}^T U_{4a'}^E - \frac{\zeta}{\sqrt{3}} V_{6a}^T U_{4a'}^E - \frac{2}{3} \zeta V_{5a}^T U_{6a'}^E \right) \]

\[ + \sqrt{\frac{2}{3} \zeta V_{7a}^T U_{4a'}^E} - \frac{\gamma}{\sqrt{3}} V_{1a}^T U_{3a'}^E - 2 \sqrt{\frac{2}{3} \eta V_{2a}^T U_{3a'}^E} - \frac{2 i \eta}{\sqrt{3}} V_{4a}^T U_{3a'}^E + \frac{2 \eta}{\sqrt{3}} V_{5a}^T U_{2a'}^E \]

\[ -2 \sqrt{\frac{2}{3} \eta V_{3a}^T U_{6a'}^E + 2 \eta V_{5a}^T U_{1a'}^E - 2 \gamma V_{1a}^T U_{1a'}^E \right) \]

\[ - \gamma V_{1a}^T U_{2a'}^E - k V_{1a}^T U_{6a'}^E - \frac{\sqrt{2} \rho}{3} V_{5a}^T U_{6a'}^E - \frac{i \rho}{3 \sqrt{2}} V_{7a}^T U_{6a'}^E + \sqrt{\frac{2}{3}} \zeta V_{7a}^T U_{6a'}^E - \zeta V_{5a}^T U_{6a'}^E \]

\[ + \sqrt{\frac{2}{3} i \zeta V_{7a}^T U_{6a'}^E} + \sqrt{\frac{2}{3}} i \zeta V_{7a}^T U_{6a'}^E - \sqrt{\frac{2}{3}} i \zeta V_{7a}^T U_{6a'}^E \right) \]

\[ + \frac{\rho}{3 \sqrt{3}} V_{7a}^T U_{6a'}^E + \frac{\rho}{3 \sqrt{3}} V_{7a}^T U_{6a'}^E + \frac{2 \rho}{3 \sqrt{3}} V_{5a}^T U_{6a'}^E \right) V_{51}^H \]

\[ + \left( \sqrt{\frac{2}{3}} \zeta V_{2a}^T U_{3a'}^E \right) \]

\[ + \left( \frac{2}{3} i \zeta V_{3a}^T U_{3a'}^E + \sqrt{3} i \zeta V_{5a}^T U_{4a'}^E - \frac{\zeta}{\sqrt{3}} V_{6a}^T U_{4a'}^E - \frac{2}{3} \zeta V_{2a}^T U_{3a'}^E \right) \]

\[ - \frac{\zeta}{\sqrt{3}} V_{5a}^T U_{2a'}^E + \frac{\zeta}{\sqrt{3}} V_{5a}^T U_{3a'}^E + \frac{\zeta}{\sqrt{3}} V_{2a}^T U_{3a'}^E - \frac{\rho}{3 \sqrt{3}} V_{5a}^T U_{3a'}^E \]

\[ - \frac{\rho}{3 \sqrt{3}} V_{7a}^T U_{4a'}^E + \frac{\rho}{3 \sqrt{3}} V_{7a}^T U_{4a'}^E + \frac{2 \rho}{3 \sqrt{3}} V_{5a}^T U_{6a'}^E \right) V_{51}^H \]

\[ - 2 g_{10} - U_{1a}^T V_{11}^H + \frac{i}{\sqrt{3}} U_{2a}^T V_{21}^H + \left( \frac{2}{3} U_{3a}^T \right) V_{31}^H - \frac{i}{\sqrt{2}} U_{7a}^T V_{51}^H \]

\[ - \left( \frac{U_{7a}^T}{\sqrt{6}} + i \frac{g_{10}}{\sqrt{3}} U_{6a}^T \right) V_{51}^H \]
3.7 Conclusions and Outlook

\[ K_{C\bar{Z}} = \sum_{a=1}^{d(C)} \left[ \left( \gamma V_{2a}^C - \gamma V_{1a}^C - ikV_{3a}^C \right) V_{11}^H + \left( \frac{2\eta}{\sqrt{3}} V_{1a}^C - \frac{i\zeta}{\sqrt{3}} V_{3a}^C \right) V_{21}^H 
- \left( \frac{i\zeta}{\sqrt{3}} V_{3a}^C + \frac{2\eta}{\sqrt{3}} V_{2a}^C \right) V_{31}^H + \left( \frac{i\rho}{3} V_{3a}^C \right) \right] \right] \right|^2 F_{12}(m^Z, m_a^C, Q) \] (3.59)

\[ K_{E\bar{U}} = \sum_{a=1}^{d(E)} \left[ \left( \frac{\sqrt{2}}{3} i\eta V_{1a}^E + \frac{\zeta}{\sqrt{6}} V_{6a}^E \right) V_{31}^H + \left( \frac{2\sqrt{2}i\lambda V_{4a}^E - 2\lambda V_{3a}^E \right) V_{41}^H \right] \right] \right|^2 \left( \sum_{a=1}^{d(E)} \right) \right] \right]^2 F_{12}(m^U, m_a^E, Q) \] (3.60)

\[ K_{U\bar{D}} = \sum_{a=1}^{d(D)} \left[ \left( \frac{\sqrt{2}}{3} i\eta V_{1a}^D - \frac{i\zeta}{\sqrt{2}} V_{2a}^D + \frac{k}{\sqrt{2}} V_{3a}^D \right) V_{11}^H + \left( \frac{\zeta}{\sqrt{6}} U_{3a}^D - \frac{2\sqrt{2}}{3} i\eta U_{1a}^D \right) V_{21}^H \right] \right] \right|^2 F_{12}(m^U, m_a^D, Q) \] (3.61)

\[ K_{HO} = \sum_{a=2}^{d(H)} \left[ \gamma V_{4a}^H V_{11}^H + 2\sqrt{3}\eta V_{4a}^H V_{21}^H + \left( \gamma V_{1a}^H + 2\sqrt{3}\eta V_{2a}^H + \zeta V_{5a}^H + \sqrt{3}i\zeta V_{6a}^H \right) V_{41}^H \right] \right] \right|^2 F_{12}(m^O, m_a^H, Q) \] (3.62)

\[ K_{V\bar{A}} = \left| \sqrt{2}i\gamma V_{11}^H + 2\sqrt{2}\eta V_{31}^H - \sqrt{2}i\zeta V_{51}^H - \sqrt{6}\zeta V_{61}^H \right|^2 F_{12}(m^V, m^A, Q) \] (3.63)
\[ K_{K\bar{X}} = \sum_{a=1}^{d(K)} \sum_{a'=1}^{d(X)} \left( \sqrt{2i\gamma} V_{1a}^{K} U_{1a'}^{X} + i k V_{2a}^{K} U_{2a'}^{X} \right) V_{11}^{H} \]
\[ + \left( \frac{i\bar{\zeta}}{\sqrt{2}} V_{2a}^{K} U_{2a'}^{X} - 2\sqrt{3} \xi V_{2a}^{K} U_{1a'}^{X} \right) V_{21}^{H} \]
\[ + \left( 2\sqrt{3} i\eta V_{1a}^{K} U_{1a'}^{X} - \frac{i\zeta}{\sqrt{3}} V_{2a}^{K} U_{2a'}^{X} + 2\sqrt{3} \xi V_{1a}^{K} U_{1a}^{X} \right) V_{31}^{H} \]
\[ + \left( \frac{i\rho}{3} V_{2a}^{K} U_{1a}^{X} - \sqrt{2} i\bar{\zeta} V_{1a}^{K} U_{1a}^{X} \right) V_{31}^{H} + \left( \frac{\rho}{3} \sqrt{2} V_{2a}^{K} U_{1a}^{X} \right) \]
\[ + \left( \frac{\rho}{3} V_{2a}^{K} U_{2a}^{X} - \sqrt{2} i\bar{\zeta} V_{1a}^{K} U_{1a}^{X} \right) V_{31}^{H} + \left( \frac{\rho}{3} \sqrt{2} V_{2a}^{K} U_{1a}^{X} \right) \]
\[ + \frac{2 U_{2a}^{K} V_{1a}^{H}}{3} + i U_{2a}^{K} V_{1a}^{H} + \frac{U_{2a}^{K} V_{1a}^{H}}{3} + 2 g_{10}^{2} \left( m_{a}^{K}, m_{a'}^{X}, Q \right) \]  
(3.64)

\[ K_{HF} = \sum_{a=2}^{d(H)} \sum_{a'=1}^{d(F)} \left( \gamma U_{2a}^{F} V_{2a}^{H} - i\bar{\zeta} U_{2a}^{F} V_{2a}^{H} - \gamma U_{2a}^{F} V_{2a}^{H} + k U_{2a}^{F} V_{2a}^{H} - i k U_{2a}^{F} V_{2a}^{H} \right) V_{11}^{H} \]
\[ + \left( \zeta U_{2a}^{F} V_{2a}^{H} - \frac{2i\bar{\zeta}}{\sqrt{3}} U_{2a}^{F} V_{2a}^{H} - \frac{4\eta}{3} U_{2a}^{F} V_{2a}^{H} - \zeta U_{2a}^{F} V_{2a}^{H} \right) V_{21}^{H} \]
\[ + \left( \frac{4\eta}{3} U_{2a}^{F} V_{2a}^{H} - \frac{2i\bar{\zeta}}{\sqrt{3}} U_{2a}^{F} V_{2a}^{H} - \frac{4\eta}{3} U_{2a}^{F} V_{2a}^{H} - \zeta U_{2a}^{F} V_{2a}^{H} - 2\sqrt{3} i\eta U_{2a}^{F} V_{2a}^{H} \right) \]
\[ + \gamma U_{2a}^{F} V_{2a}^{H} + \left( 2\sqrt{3} i\eta U_{2a}^{F} V_{2a}^{H} + \frac{i\rho}{3} U_{2a}^{F} V_{2a}^{H} - \zeta U_{2a}^{F} V_{2a}^{H} - \sqrt{3} \zeta U_{2a}^{F} V_{2a}^{H} \right) V_{31}^{H} \]
\[ + \left( i\gamma U_{2a}^{F} V_{1a}^{H} + 3 \zeta U_{2a}^{F} V_{1a}^{H} - k U_{2a}^{F} V_{1a}^{H} + \sqrt{3} \zeta U_{2a}^{F} V_{2a}^{H} \right) V_{31}^{H} \]
\[ + \left( i \gamma U_{2a}^{F} V_{1a}^{H} + \zeta U_{2a}^{F} V_{1a}^{H} - \zeta U_{2a}^{F} V_{1a}^{H} + \frac{i\rho}{3} U_{2a}^{F} V_{2a}^{H} \right) V_{31}^{H} \]
\[ + \left( i k U_{2a}^{F} V_{1a}^{H} - \frac{2i\bar{\zeta}}{\sqrt{3}} U_{2a}^{F} V_{1a}^{H} - \frac{4\eta}{3} U_{2a}^{F} V_{1a}^{H} - \zeta U_{2a}^{F} V_{1a}^{H} + \sqrt{3} \zeta U_{2a}^{F} V_{1a}^{H} \right) \]
\[ - \left( \frac{i\rho}{3} U_{2a}^{F} V_{2a}^{H} \right) V_{31}^{H} + \left( \frac{2 U_{2a}^{H} V_{2a}^{H}}{3} + U_{2a}^{H} V_{31}^{H} + U_{3a}^{H} V_{31}^{H} + U_{3a}^{H} V_{31}^{H} + U_{5a}^{H} V_{31}^{H} + U_{6a}^{H} V_{31}^{H} \right) \]
\[ + U_{61}^{H} V_{61}^{H} \]  
(3.65)
\[ K_{NY} = \left| \sqrt{2i\gamma} V_{11}^H - 2\sqrt{\frac{2}{3}} i\eta V_{31}^H - \sqrt{2i\zeta} V_{51}^H + \sqrt{\frac{2}{3}} \zeta V_{61}^H \right|^2 F_{12}(m^N, m^Y, Q) \] (3.66)

\[ K_{YW} = \left| \gamma V_{11}^H - \frac{2\eta}{\sqrt{3}} V_{31}^H + \zeta V_{51}^H + \frac{i\zeta}{\sqrt{3}} V_{61}^H \right|^2 F_{12}(m^Y, m^W, Q) \] (3.67)

\[ K_{VO} = \left| \gamma V_{11}^H + 2\sqrt{3\eta} V_{31}^H + \zeta V_{51}^H - \sqrt{3i\zeta} V_{61}^H \right|^2 F_{12}(m^Y, m^O, Q) \] (3.68)

\[ K_{LB} = \sum_{a=1}^{d(L)} \left| \left( kV_{2a}^L - i\gamma V_{1a}^L \right) V_{11}^H + \frac{\zeta}{\sqrt{3}} V_{2a}^L V_{21}^H \right|^2 F_{12}(m^B, m_a^L, Q) \] (3.69)

\[ K_{SH} = \sum_{a=2}^{d(H)} \left| \left( \frac{i\gamma}{\sqrt{2}} U_{2a}^H - \frac{i\gamma}{\sqrt{2}} U_{3a}^H - \frac{k}{\sqrt{2}} U_{6a}^H \right) V_{11}^H - \left( 2\sqrt{\frac{2}{3}} i\eta U_{3a}^H - \sqrt{\frac{2}{3}} \zeta U_{6a}^H + \frac{i\zeta}{\sqrt{2}} U_{5a}^H \right) \right|^2 F_{12}(m^S, m_a^H, Q) \] (3.70)
3.7 Conclusions and Outlook

\[ K_{GH} = \sum_{a=2}^{d(H)} \sum_{a'=1}^{d(G)} \left( \frac{\gamma}{\sqrt{2}} V_{3a}^G U_{3a}^H - \gamma V_{2a}^G U_{2a}^H - \sqrt{2} i \gamma V_{4a}^G U_{4a}^H - \gamma V_{2a}^G U_{2a}^H - \frac{\gamma}{\sqrt{2}} V_{3a}^G U_{3a}^H \right. \\
+ \frac{i k}{\sqrt{2}} V_{3a}^G U_{6a}^H + k V_{1a}^G U_{5a}^H) V_{11}^H + \left( \frac{2\sqrt{2}}{2} \eta V_{3a}^G U_{3a}^H - \frac{4\eta}{\sqrt{3}} V_{2a}^G U_{3a}^H - 2\sqrt{6}i\eta V_{4a}^G U_{4a}^H \right. \\
- \frac{2}{3} \bar{\kappa} V_{3a}^G U_{6a}^H + i \bar{\kappa} V_{1a}^G U_{5a}^H - \frac{\bar{\kappa}}{\sqrt{2}} V_{3a}^G U_{5a}^H - \gamma V_{2a}^G U_{2a}^H + \frac{\bar{\kappa}}{\sqrt{2}} V_{3a}^G U_{1a}^H \right) V_{21}^H \\
- \left( \frac{2}{3} \bar{\kappa} V_{3a}^G U_{6a}^H + i \bar{\kappa} V_{1a}^G U_{5a}^H + \frac{\bar{\kappa}}{\sqrt{2}} V_{3a}^G U_{5a}^H + \gamma V_{2a}^G U_{2a}^H + \frac{\bar{\kappa}}{\sqrt{2}} V_{3a}^G U_{1a}^H \right) V_{21}^H \\
+ \frac{4\eta}{\sqrt{3}} V_{3a}^G U_{3a}^H - 2\sqrt{2} i \gamma V_{4a}^G U_{4a}^H - 2\sqrt{3} \lambda V_{2a}^G U_{4a}^H + \bar{\eta} V_{3a}^G U_{5a}^H + \left( \frac{2}{3} \bar{\kappa} V_{3a}^G U_{6a}^H + \frac{\bar{\kappa}}{\sqrt{2}} V_{3a}^G U_{5a}^H + \gamma V_{2a}^G U_{2a}^H + \frac{\bar{\kappa}}{\sqrt{2}} V_{3a}^G U_{1a}^H \right) V_{11}^H \\
\left. + \left( k V_{1a}^G U_{1a}^H - \frac{i \rho}{3\sqrt{2}} V_{3a}^G U_{2a}^H + \frac{\bar{\kappa}}{\sqrt{2}} V_{3a}^G U_{2a}^H + \frac{\bar{\kappa}}{\sqrt{2}} V_{3a}^G U_{3a}^H + 2 i \gamma V_{4a}^G U_{4a}^H \right) V_{51}^H \right) \\
+ \left( \frac{2}{3} \bar{\zeta} V_{3a}^G U_{2a}^H + \frac{\bar{\zeta}}{\sqrt{2}} V_{3a}^G U_{2a}^H + \frac{\bar{\zeta}}{\sqrt{2}} V_{3a}^G U_{3a}^H + \gamma V_{2a}^G U_{2a}^H + \frac{\bar{\zeta}}{\sqrt{2}} V_{3a}^G U_{1a}^H \right) V_{51}^H \\
\left. - \sqrt{2} i \gamma V_{5a}^G U_{5a}^H - \sqrt{2} \bar{\zeta} V_{5a}^G U_{6a}^H + \sqrt{2} i \gamma V_{5a}^G U_{5a}^H + 2 i \gamma V_{5a}^G U_{5a}^H \right) V_{61}^H \right)^2 F_{12}(m_a^H, m_{a'}, Q) \\
\left. - \sqrt{5} \left( V_{1a}^H V_{11}^H + V_{2a}^H V_{21}^H + V_{3a}^H V_{31}^H - 4 V_{4a}^H V_{41}^H + V_{5a}^H V_{51}^H \right) \right)^2 F_{12}(m_a^H, m_{a'}, Q) \\
\left. + \gamma V_{2a}^G U_{2a}^H - \frac{\gamma}{\sqrt{2}} V_{3a}^G U_{3a}^H + \frac{i k}{\sqrt{2}} V_{3a}^G U_{2a}^H + k V_{1a}^G U_{5a}^H \right) V_{11}^H + \left( \frac{2}{2} \eta V_{3a}^G U_{3a}^H \right. \\
- \frac{4\eta}{\sqrt{3}} V_{2a}^G U_{3a}^H - 2\sqrt{6} i \gamma V_{4a}^G U_{4a}^H - \frac{2}{3} \bar{\zeta} V_{3a}^G U_{5a}^H + i \bar{\zeta} V_{1a}^G U_{5a}^H = \frac{\bar{\zeta}}{\sqrt{2}} V_{3a}^G U_{5a}^H \\
\left. - \gamma V_{2a}^G U_{11}^H + \frac{\bar{\zeta}}{\sqrt{2}} V_{3a}^G U_{11}^H \right) V_{21}^H - \left( \frac{2}{3} \bar{\zeta} V_{3a}^G U_{6a}^H + i \bar{\zeta} V_{1a}^G U_{6a}^H + \frac{\bar{\zeta}}{\sqrt{2}} V_{3a}^G U_{6a}^H \right. \\
\left. + \gamma V_{2a}^G U_{11}^H + \frac{\bar{\zeta}}{\sqrt{2}} V_{3a}^G U_{11}^H + \frac{4\eta}{\sqrt{3}} V_{2a}^G U_{2a}^H + 2 \sqrt{2} \eta V_{3a}^G U_{2a}^H \right) V_{31}^H + \left( \sqrt{6} \gamma V_{3a}^G U_{4a}^H \right. \\
\left. - 2\sqrt{3} \lambda V_{2a}^G U_{4a}^H - \sqrt{2} i \gamma V_{5a}^G U_{5a}^H - \sqrt{6} \gamma V_{5a}^G U_{6a}^H + \sqrt{2} i \gamma V_{5a}^G U_{5a}^H \right) V_{41}^H \\
\left. + 2\sqrt{6} i \gamma V_{5a}^G U_{5a}^H \right) V_{41}^H + \left( k V_{1a}^G U_{11}^H - \frac{i \rho}{3\sqrt{2}} V_{3a}^G U_{6a}^H + \frac{\bar{\zeta}}{\sqrt{2}} V_{3a}^G U_{2a}^H \right) \right) \right) \right) \\
\left. + \frac{\zeta}{\sqrt{2}} V_{3a}^G U_{3a}^H + \sqrt{2} i \gamma V_{4a}^G U_{4a}^H \right) V_{51}^H + \left( \frac{2}{3} \bar{\zeta} V_{3a}^G U_{5a}^H + i \bar{\zeta} V_{1a}^G U_{5a}^H = \frac{\bar{\zeta}}{\sqrt{2}} V_{3a}^G U_{5a}^H \right) \\
2 \frac{\rho}{\sqrt{3}} V_{2a}^G U_{6a}^H"}
3.7 Conclusions and Outlook

\[-\frac{ik}{\sqrt{2}}V_{3a}^G U_{11}^H + \sqrt{\frac{2}{3}}i\zeta V_{3a}^G U_{31}^H - i\zeta V_{1a'}^G U_{31}^H - \sqrt{6}\zeta V_{4a}^G U_{41}^H\]

\[+ \frac{i\rho}{3\sqrt{2}}V_{3a}^G U_{51}^H \right| V_{61}^H \right|^2 F_{11}(m_{\lambda C}, Q) - 2g_{10}^2 \left| \frac{i}{\sqrt{2}} \right| \left( V_{11}^H V_{11}^H + V_{21}^H V_{21}^H + V_{31}^H V_{31}^H \right) \right|^2 F_{11}(m_{\lambda C}, Q) \]  

(3.71)

For the \( \mathcal{H}[1, 2, -1] \) line we have

\[(16\pi^2)K_{\mathcal{H}} = 8K_{RC} + +9K_{UE} + 9K_{PX} + 3K_{TX} + 6K_{MB}3K_{DJ} + 9K_{EP} + K_{VF} \]

\[+ 3K_{JE} + 6K_{LY} + 8K_{ZC} + 3K_{ET} + 3K_{ID} + 3K_{SH} + 24K_{QC} + 9K_{D\bar{U}} \]

\[+ 6K_{Y\bar{N}} + K_{F\bar{H}} + 3K_{X\bar{K}} + K_{Y A} + 6K_{BL} + 18K_{B\bar{W}} + 3K_{H\bar{O}} \]

\[+ 18K_{W\bar{Y}} + 3K_{YO} + K_{GH} \]  

(3.72)

\[K_{RC} = \sum_{a=1}^d (R) \sum_{a'=1}^d \left| \left( \gamma V_{1a'}^RV_{1a'}^C + \gamma V_{2a}^RV_{2a'}^C + \gamma V_{1a}^RV_{2a'}^C - \frac{\bar{\zeta}}{\sqrt{2}} V_{2a}^RV_{3a'}^C + \frac{ik}{\sqrt{2}} V_{2a}^R V_{3a'}^C \right) U_{11}^H \right|^2 \]

\[+ \left( \frac{i\zeta}{\sqrt{6}} \right) \left( V_{2a}^R V_{3a'}^C - \frac{2\eta}{3} \eta V_{1a}^R V_{1a'}^C - \sqrt{\frac{2}{3}} \eta V_{2a}^R V_{1a'}^C \right) U_{21}^H \]

\[+ \left( \frac{2}{3} \eta V_{2a}^R V_{1a'}^C - \frac{2\eta}{\sqrt{3}} \eta V_{2a}^R V_{1a'}^C + \frac{3\zeta}{\sqrt{6}} \eta V_{2a}^R V_{3a'}^C \right) U_{31}^H \]

\[+ \left( \frac{\zeta}{\sqrt{2}} \right) \left( V_{2a}^R V_{1a'}^C + 2\eta V_{2a}^R V_{1a'}^C - \frac{i\zeta}{3\sqrt{3}} V_{2a}^R V_{3a'}^C \right) U_{51}^H \]

\[- \left( \frac{\rho}{3} \right) \left( V_{1a}^R V_{3a'}^C + \frac{i\zeta}{\sqrt{6}} V_{2a}^R V_{1a'}^C + \frac{i\zeta}{\sqrt{6}} V_{2a}^R V_{3a'}^C \right) U_{61}^H \right|^2 F_{12}(m^R, m^C_{\lambda}, Q) \]  

(3.73)

\[K_{UE} = \sum_{a=1}^d \left| \left( \rho \zeta U_{2a}^E - k U_{6a}^E \right) U_{11}^H \right|^2 \left( 2i\eta U_{1a}^E - \bar{\zeta} U_{6a}^E \right) U_{21}^H \]

\[+ \left( 6i\eta U_{2a}^E + 3\zeta U_{6a}^E \right) U_{31}^H + \left( 2\lambda U_{3a}^E - \sqrt{2} \lambda U_{4a}^E \right) U_{41}^H \]

\[+ \left( \rho U_{2a}^E + i\zeta U_{6a}^E \right) U_{51}^H + \left( \zeta U_{1a}^E - 3\zeta U_{2a}^E + \frac{2i\rho}{3} U_{6a}^E \right) U_{61}^H \right|^2 \]

\[\times F_{12}(m^U, m^U_{\lambda}, Q) - 2g_{10}^2 \left| \frac{U_{41}^H}{\sqrt{2}} \right|^2 F_{12}(m^U, m^U_{\lambda}, Q) \]  

(3.74)


\[ K_{P\bar{X}} = \sum_{a=1}^{3} \sum_{a'=1}^{3} \left( \frac{k}{\sqrt{2}} V_{2a}^P U_{2a'}^X - \gamma V_{1a}^P U_{1a'} \right) U_{11}^H + \left( \frac{2\sqrt{2} \eta}{\sqrt{3}} V_{1a}^P U_{2a} - \frac{2\eta}{\sqrt{3}} V_{1a}^P U_{1a'} \right) \\
- \frac{\zeta}{\sqrt{6}} V_{2a}^P U_{2a'} \right) U_{21}^H + \frac{\zeta}{\sqrt{3}} V_{2a}^P \left( 2U_{1a}^X + \frac{U_{2a}}{\sqrt{2}} \right) U_{31}^H \\
- \left( \frac{\rho}{3\sqrt{2}} V_{2a}^P U_{2a'} + \zeta V_{1a}^P U_{1a'} \right) U_{51}^H + \left( \frac{i\sqrt{2} \zeta}{\sqrt{3}} V_{1a}^P U_{2a} - \frac{i\zeta}{\sqrt{3}} V_{1a}^P U_{1a'} \right) \\
+ \frac{i\rho}{3\sqrt{6}} V_{2a}^P U_{2a'} - \frac{i\rho}{3\sqrt{6}} V_{2a}^P U_{2a'} \right) U_{61}^H \bigg| \bigg| F_{12}(m_a^P, m_{\lambda X}, Q) \\
- 2g_{10}^2 \bigg| \bigg| i \sqrt{2} \gamma U_{1a}^H U_{2a}^H + \frac{U_{2a}^P}{\sqrt{2}} U_{51}^H + \frac{i}{\sqrt{6}} \frac{U_{2a}^P U_{61}^H}{U_{61}^H} \bigg| \bigg| F_{12}(m_a^P, m_{\lambda X}, Q) \] (3.75)

\[ K_{T\bar{X}} = \sum_{a=1}^{3} \sum_{a'=1}^{3} \left( \gamma U_{2a}^X V_{3a}^T - i\gamma U_{1a}^X V_{4a} + \gamma U_{2a'}^X V_{2a}^T - \frac{i\kappa}{\sqrt{2}} U_{2a}^X V_{7a}^T - kU_{1a}^X V_{6a}^T \right) U_{11}^H \\
+ \left( \frac{\zeta}{\sqrt{3}} U_{1a}^X V_{6a}^T + \frac{2i\zeta}{\sqrt{3}} U_{1a}^X V_{7a}^T - \frac{i\zeta}{\sqrt{6}} V_{7a}^X U_{2a}^T - \sqrt{2} \frac{\zeta}{3} U_{2a}^X V_{6a}^T - \frac{2i\gamma}{\sqrt{3}} U_{1a}^X V_{1a'}^T \right) U_{21}^H \\
- \frac{\gamma}{\sqrt{3}} U_{2a}^X V_{1a'}^T - \frac{4\eta}{\sqrt{6}} U_{1a}^X V_{3a}^T \right) U_{21}^H + \left( \frac{\gamma}{\sqrt{3}} U_{2a}^X V_{1a'}^T - \frac{i\sqrt{2} \gamma}{\sqrt{3}} U_{1a}^X V_{1a'}^T + 2\sqrt{2} \frac{\eta}{3} U_{1a}^X V_{2a}^T \\
- 2\sqrt{2} \frac{3}{\sqrt{3}} \eta U_{2a}^X V_{4a}^T - \frac{2i\eta}{\sqrt{3}} U_{1a}^X V_{4a} + \frac{\zeta}{\sqrt{3}} U_{1a}^X V_{6a} + \frac{i\zeta}{\sqrt{6}} V_{2a}^X U_{7a}^T + \frac{\sqrt{2} \zeta}{3} U_{2a}^X V_{6a}^T \right) U_{31}^H \\
+ 2i\lambda \left( U_{2a'}^X + \sqrt{2} U_{1a'}^X \right) V_{5a}^T U_{41}^H + \left( kU_{2a}^X V_{1a'}^T - \frac{i\rho}{3\sqrt{2}} U_{2a}^X V_{7a}^T + i\zeta U_{1a}^X V_{4a} \right) U_{51}^H \\
+ \left( \frac{i\sqrt{2} \zeta}{\sqrt{3}} U_{2a}^X V_{2a} + \frac{\zeta}{\sqrt{3}} U_{1a}^X V_{4a} + \sqrt{2} \frac{\zeta}{3} U_{2a}^X V_{4a} + \frac{i\zeta}{\sqrt{3}} U_{2a}^X V_{2a} - \frac{2}{\sqrt{3}} i\kappa U_{1a}^X V_{1a} \right) U_{51}^H \\
+ \frac{i\zeta}{\sqrt{3}} U_{2a}^X V_{3a}^T - \frac{2i\zeta}{\sqrt{3}} U_{1a}^X V_{3a} - \frac{\rho}{3\sqrt{3}} U_{1a}^X V_{7a} - \frac{\rho}{3\sqrt{6}} U_{2a}^X V_{7a} \\
+ \frac{i\rho}{3\sqrt{3}} U_{1a}^X V_{6a}^T \right) U_{61}^H \bigg| \bigg| F_{12}(m_a^X, m_{\lambda t}, Q) \\
- 2g_{10}^2 \left| U_{1a}^T U_{11}^H - \frac{i}{\sqrt{3}} U_{2a}^T U_{21}^H + \left( \frac{\sqrt{2}}{3} U_{4a}^T - \frac{iU_{3a}^T}{\sqrt{3}} \right) U_{31}^H \\
- iU_{5a}^T U_{41}^H - \frac{1}{\sqrt{2}} U_{7a}^T U_{51}^H + \left( \frac{\sqrt{2}}{3} U_{6a}^T - \frac{U_{7a}^T}{\sqrt{6}} \right) U_{61}^H \bigg| \bigg| F_{12}(m_{\lambda X}, m_a^T, Q) \] (3.76)

\[ K_{MB} = - \sqrt{2} \gamma U_{11}^H + 2 \frac{\sqrt{2}}{3} i\eta U_{31}^H + \frac{2i\zeta U_{51}^H}{\sqrt{6}} - \frac{\sqrt{2}}{3} \zeta U_{61}^H \bigg| \bigg| F_{12}(m^M, m^B, Q) \] (3.77)
3.7 Conclusions and Outlook

\[ K_{DI} = \sum_{a=1}^{d(D)} \sum_{a'=1}^{d(J)} \left( \frac{\bar{\eta}}{\sqrt{2}} V_{1a}^D U_{3a'}^J - \bar{\gamma} V_{1a}^D U_{2a'}^J - \gamma V_{2a}^D U_{2a'}^J - \frac{\bar{\gamma}}{\sqrt{2}} V_{2a}^D U_{3a'}^J - \frac{i k}{\sqrt{2}} V_{3a}^D U_{3a'}^J \right) U_{11}^H \]

\[ + \left( \sqrt{\frac{2}{3}} \eta V_{2a}^D U_{3a'}^J - \frac{2 \eta}{\sqrt{3}} V_{2a}^D U_{2a'}^J + \frac{2 i \bar{\eta}}{\sqrt{3}} V_{3a}^D U_{2a}^J - \frac{i \bar{\eta}}{\sqrt{6}} V_{3a}^D U_{3a'}^J \right) U_{21}^H \]

\[ + \left( \sqrt{6} \eta V_{1a}^D U_{3a'}^J - \frac{2 \eta}{\sqrt{3}} V_{1a}^D U_{2a'}^J - \frac{2 i \bar{\eta}}{\sqrt{3}} V_{3a}^D U_{2a}^J + \sqrt{\frac{3}{2}} i \bar{\eta} V_{3a}^D U_{3a'}^J \right) U_{31}^H \]

\[ + \left( \frac{i \rho}{3} V_{3a}^D U_{5a'}^J - \frac{4 \rho}{3} V_{3a}^D U_{1a'}^J - \frac{2 i \bar{\rho}}{\sqrt{3}} V_{3a}^D U_{3a'}^J \right) U_{41}^H \]

\[ - \frac{2 g_{10}^2}{\sqrt{3}} \left( \frac{2 i}{\sqrt{3}} \left( |U_{2a}^D U_{21}^H + U_{1a}^D U_{31}^H + U_{3a}^D U_{31}^H |^2 \right) F_{12}(m_a^D, m_{a'}^D, Q) \right) \] (3.78)

\[ K_{EP} = \sum_{a=1}^{d(E)} \sum_{a'=1}^{d(P)} \left( \frac{k}{\sqrt{2}} V_{4a}^E U_{2a'}^P - \bar{\eta} V_{3a}^E U_{1a'}^P \right) U_{11}^H + \left( \frac{2 \bar{\eta}}{\sqrt{3}} V_{2a}^E U_{2a'}^P + \frac{i \bar{\eta}}{\sqrt{6}} V_{4a}^E U_{2a'}^P \right) U_{21}^H \]

\[ + \left( \frac{2 \sqrt{2} \eta}{\sqrt{3}} V_{4a}^E U_{1a'}^P - \frac{2 \eta}{\sqrt{3}} V_{3a}^E U_{1a'}^P - \frac{\bar{\epsilon}}{\sqrt{6}} V_{4a}^E U_{2a'}^P \right) U_{31}^H \]

\[ + \left( 2 \sqrt{2} \eta V_{3a}^E U_{1a'}^P + \frac{\rho}{3 \sqrt{2}} V_{6a}^E U_{2a'}^P + i \sqrt{2} \zeta V_{3a}^E U_{1a'}^P + \sqrt{2} \zeta V_{6a}^E U_{1a'}^P \right) U_{41}^H \]

\[ - \left( \frac{\rho}{3 \sqrt{2}} V_{4a}^E U_{2a'}^P + \bar{\zeta} V_{3a}^E U_{1a'}^P \right) U_{51}^H + \left( \frac{i \bar{\epsilon}}{\sqrt{3}} V_{3a}^E U_{1a'}^P - \frac{i 2 \bar{\zeta}}{\sqrt{3}} V_{4a}^E U_{2a'}^P \right) U_{51}^H \]

\[ - \frac{i \rho}{3 \sqrt{3}} V_{3a}^E U_{2a'}^P + \frac{i \rho}{3 \sqrt{6}} V_{4a}^E U_{2a'}^P \right) U_{51}^H + \left( \frac{2 \sqrt{2} \eta}{\sqrt{3}} V_{4a}^E U_{1a'}^P + \bar{\eta} V_{3a}^E U_{1a'}^P \right) U_{51}^H \]

\[ - \frac{2 g_{10}^2}{\sqrt{3}} \left( \left| \frac{2 i}{\sqrt{3}} \left( \frac{U_{2a}^P U_{21}^H + U_{1a}^P U_{31}^H + U_{3a}^P U_{31}^H \right) \right| F_{12}(m_a^E, m_{a'}^E, Q) \right) \] (3.79)

\[ K_{VF} = \sum_{a=1}^{d(F)} \left( \frac{i \bar{\eta} U_{1a}^F + k U_{4a}^F} \right) U_{11}^H + \sqrt{3} \bar{\zeta} U_{4a}^F U_{21}^H \right) U_{31}^H \]

\[ + 2 \sqrt{3} \lambda U_{2a}^F U_{41}^H + i \zeta U_{1a}^F U_{51}^H \right) \right) \right) U_{51}^H \]

\[ - \frac{2 g_{10}^2}{\sqrt{3}} \left( \frac{U_{4a}^F U_{21}^H + U_{1a}^F U_{31}^H + \frac{i \rho}{3 \sqrt{3}} U_{4a}^F + \sqrt{3} \bar{\zeta} U_{1a}^F \right) U_{51}^H \]

\[ - \frac{2 g_{10}^2}{\sqrt{3}} \left( \left| \frac{2 i}{\sqrt{3}} \left( \frac{U_{2a}^P U_{21}^H + U_{1a}^P U_{31}^H + U_{3a}^P U_{31}^H \right) \right| F_{12}(m_V^F, m_{a'}^F, Q) \right) \] (3.80)
3.7 Conclusions and Outlook

\[ K_{JE} = \sum_{a=1}^{d(J)} \sum_{a'=1}^{d(E)} \left| \left( \frac{\gamma}{\sqrt{2}} V_{3a} U_{2a}^E - i \sqrt{2 \gamma} U_{3a} U_{2a}^J V_{1a}^J - \gamma U_{2a}^E V_{3a} - \gamma V_{2a} U_{1a}^E - \frac{\gamma}{\sqrt{2}} V_{3a} U_{1a}^E \right) \right|^2 \]

\[ - \frac{i k}{\sqrt{2}} V_{3a} U_{6a'}^E - i k V_{5a} U_{4a'}^E \right) U_{11}^H + \left( \sqrt{\frac{3}{2}} i \zeta V_{3a} U_{6a'}^E + \frac{2i k \zeta}{\sqrt{3}} V_{2a} U_{6a'}^E + \frac{2 \sqrt{2 \gamma} i \zeta}{\sqrt{3}} V_{3a} U_{1a}^E \right) \]

\[ - \frac{i \zeta}{\sqrt{3}} V_{5a} U_{4a'}^E - \frac{2 \eta}{\sqrt{3}} V_{2a} U_{1a}^E - \sqrt{6 \eta} V_{3a} U_{1a}^E \right) U_{21}^H + \left( - \frac{i \zeta}{\sqrt{6}} V_{3a} U_{6a'}^E + \frac{2i \zeta}{\sqrt{3}} V_{2a} U_{6a'}^E \right) \]

\[ + \frac{i \zeta}{\sqrt{3}} V_{5a} U_{4a'}^E - \frac{2 \eta}{\sqrt{3}} V_{2a} U_{2a}^E - \sqrt{\frac{2}{3}} \eta V_{3a} U_{2a}^E - \frac{4i \eta}{\sqrt{3}} V_{1a} U_{E}^E - \sqrt{\frac{8}{3} i \eta} V_{1a} U_{3a}^E \right) \]

\[ + \left( 2 \sqrt{2} i \lambda V_{1a} U_{3a}^E - \frac{4i}{\sqrt{3}} V_{2a} U_{6a'}^E - \frac{2 i \lambda V_{3a} U_{4a'}^E - 2 i \lambda V_{1a} U_{3a}^E} \right) U_{41}^H \]

\[ + \left( \frac{i \rho}{3 \sqrt{2}} V_{3a} U_{6a'}^E - \frac{i \rho}{3 \sqrt{2}} V_{5a} U_{4a'}^E - \frac{\zeta}{\sqrt{3}} V_{3a} U_{2a}^E - \frac{\sqrt{2 i \zeta} V_{3a} U_{2a}^E + \sqrt{\frac{2}{3} i \zeta} V_{3a} U_{2a}^E} \right) \]

\[ + \left( \frac{2}{\sqrt{3}} \zeta V_{1a} U_{4a'}^E + \frac{2 i \zeta}{\sqrt{3}} V_{1a} U_{3a}^E - \frac{2 i \zeta}{\sqrt{3}} V_{2a} U_{2a}^E - \frac{\sqrt{3} i \zeta V_{1a} U_{3a}^E} \right) \]

\[ - \frac{2 i}{3 \sqrt{3}} \zeta V_{2a} U_{6a'}^E - \frac{\rho}{3 \sqrt{3}} V_{5a} U_{4a'}^E - \frac{2 \rho}{3 \sqrt{3}} V_{3a} U_{3a}^E - \frac{2 \rho}{3 \sqrt{3}} V_{3a} U_{6a'}^E \]

\[ - \frac{\rho}{3 \sqrt{3}} V_{2a} U_{6a'}^E \right) U_{61}^H \right|^2 \]

\[ - 2 g_{y_0}^2 \left| - \frac{2 i}{\sqrt{3}} \left( V_{1a} U_{3a}^E + U_{2a} U_{4a'}^E \right) U_{11}^H - \frac{2 i}{\sqrt{3}} \left( V_{2a} U_{4a'}^E \right) U_{31}^H - \frac{2 i}{\sqrt{3}} \left( U_{3a} U_{6a'}^E \right) U_{41}^H \right|^2 \]

\[ - 2 g_{y_0}^2 \left| - \frac{2 i}{\sqrt{3}} \left( V_{1a} U_{3a}^E + U_{2a} U_{4a'}^E \right) U_{11}^H - \frac{2 i}{\sqrt{3}} \left( V_{2a} U_{4a'}^E \right) U_{31}^H - \frac{2 i}{\sqrt{3}} \left( U_{3a} U_{6a'}^E \right) U_{41}^H \right|^2 \]

\[ - \frac{1}{\sqrt{3}} \left| U_{5a} U_{6a'}^E \right|^2 \]

\[ K_{LY} = \sum_{a=1}^{d(L)} \left| \left( i \gamma V_{1a}^L + k V_{2a}^L \right) U_{11}^H - \frac{\zeta}{\sqrt{3}} V_{2a} U_{21}^H + \left( 2 i \gamma V_{1a}^L - \zeta V_{2a}^L \right) \frac{U_{31}^H}{\sqrt{3}} \right|^2 \]

\[ - \left( \frac{i \rho}{3 \sqrt{3}} V_{2a}^L + \frac{\zeta}{\sqrt{3}} V_{1a}^L \right) U_{61}^H \right|^2 \]

\[ K_{ZC} = \sum_{a=1}^{d(C)} \left| \left( i k U_{3a}^C + \gamma U_{2a}^C - \gamma U_{1a}^C \right) U_{11}^H + \left( i \zeta U_{3a}^C - \gamma U_{2a}^C \right) \frac{U_{21}^H}{\sqrt{3}} + \left( i \zeta U_{3a}^C + \gamma U_{1a}^C \right) \frac{U_{31}^H}{\sqrt{3}} \right|^2 \]

\[ + \left( U_{2a}^C + \frac{i \rho}{3} U_{3a}^C + \zeta U_{1a}^C \right) U_{51}^H - \frac{i}{\sqrt{3}} \left( U_{2a}^C + \zeta U_{1a}^C \right) U_{61}^H \right|^2 \]

(3.81)

(3.82)

(3.83)
\[ K_{ET} = \sum_{a'=1} d(T) d(E) \left( \frac{d(T) d(E)}{d(T) d(E)} \right) \]

\[ + \left( \frac{\gamma U_{3a}^T V_{4a}^E - \gamma U_{5a}^T V_{2a}^E - i\gamma U_{4a}^T V_{3a}^E + 2\gamma U_{5a}^T V_{1a}^E - kU_{6a}^T V_{3a}^E}{3} \right) \]

\[ - i\eta U_{5a}^T V_{6a}^E + \frac{ik}{\sqrt{2}} U_{7a}^T V_{4a}^E \right) U_{11}^H + \left( \frac{\eta U_{3a}^T V_{4a}^E - \sqrt{2i} U_{1a}^T V_{3a}^E + 2\sqrt{2i} U_{3a}^T V_{3a}^E}{\sqrt{3}} \right) \]

\[ - 2\eta U_{4a}^T V_{3a}^E + \frac{2\eta}{\sqrt{3}} U_{7a}^T V_{2a}^E - \frac{2\sqrt{2\eta}}{\sqrt{3}} U_{6a}^T V_{6a}^E + \frac{2\sqrt{2\eta}}{\sqrt{3}} U_{6a}^T V_{6a}^E \]

\[ + \frac{i\zeta}{\sqrt{6}} U_{7a}^T V_{4a}^E - \frac{i\zeta}{\sqrt{6}} U_{7a}^T V_{6a}^E \right) U_{12}^H + \left( 2\sqrt{2i} U_{2a}^T V_{3a}^E - \frac{2\sqrt{2\eta}}{\sqrt{3}} U_{3a}^T V_{3a}^E - 2\sqrt{2i} U_{1a}^T V_{1a}^E \right) \]

\[ - \frac{i}{\sqrt{3}} U_{1a}^T V_{6a}^E + \frac{\zeta}{\sqrt{3}} U_{5a}^T V_{3a}^E + i\zeta U_{5a}^T V_{6a}^E + \frac{2i\zeta}{\sqrt{3}} U_{7a}^T V_{3a}^E \]

\[ - \frac{i}{\sqrt{6}} U_{7a}^T V_{4a}^E + \frac{i}{\sqrt{6}} U_{7a}^T V_{6a}^E \right) U_{31}^H + \left( 2\sqrt{2i} U_{2a}^T V_{2a}^E - 2i\eta U_{5a}^T V_{3a}^E + 2\sqrt{2\eta} U_{4a}^T V_{1a}^E - \sqrt{2i} U_{1a}^T V_{1a}^E \right) \]

\[ - \frac{\eta}{3} U_{4a}^T V_{6a}^E + \frac{\sqrt{2\rho}}{3} U_{4a}^T V_{6a}^E - \frac{i}{3\sqrt{2}} U_{7a}^T V_{6a}^E + \frac{2i\zeta}{\sqrt{3}} U_{6a}^T V_{6a}^E \]

\[ - \frac{\eta}{3} U_{4a}^T V_{6a}^E + \frac{\sqrt{2\rho}}{3} U_{4a}^T V_{6a}^E - \frac{i}{3\sqrt{2}} U_{7a}^T V_{6a}^E + \frac{2i\zeta}{\sqrt{3}} U_{6a}^T V_{6a}^E \]

\[ + \left( \frac{2i\zeta}{\sqrt{3}} U_{7a}^T V_{3a}^E + \frac{2i\zeta}{\sqrt{3}} U_{7a}^T V_{6a}^E \right) U_{31}^H + \left( 2\sqrt{2i} U_{2a}^T V_{3a}^E - \frac{2\sqrt{2\eta}}{\sqrt{3}} U_{3a}^T V_{3a}^E - 2\sqrt{2i} U_{1a}^T V_{1a}^E \right) \]

\[ + \frac{\rho}{3\sqrt{3}} U_{7a}^T V_{3a}^E + \frac{U_{7a}^T V_{4a}^E}{\sqrt{2}} + 2U_{5a}^T V_{6a}^E - iU_{6a}^T V_{3a}^E \right) U_{61}^H \mid F_{12}(m'_{a}, m_{E}, Q) \]

\[ - 2g_2^{11} \left( \frac{\eta}{\sqrt{3}} V_{1a}^T U_{11}^H + \left( \frac{i}{\sqrt{3}} V_{2a}^T - \frac{V_{7a}^T}{\sqrt{3}} \right) U_{21}^H + \frac{i}{\sqrt{3}} V_{3a}^T U_{31}^H - \frac{i}{\sqrt{3}} V_{7a}^T U_{51}^H \right) \]

\[ + \left( \frac{V_{7a}^T}{\sqrt{6}} - i\sqrt{2} V_{6a}^T \right) U_{61}^H \mid F_{12}(m'_{a}, m_{\lambda_{E}}, Q) \]  

\[ K_{ID} = \sum_{a=1} d(D) \left( \frac{d(D)}{d(D)} \right) \left( \frac{d(D)}{d(D)} \right) \]

\[ + \left( - i\zeta U_{3a}^D - \gamma U_{1a}^D \right) U_{11}^H + \frac{\rho}{\sqrt{3}} U_{1a}^D \mid F_{12}(m'_{a}, m_{a}^D, Q) \]  

\[ \text{(3.84)} \]

\[ \text{(3.85)} \]
\[ K_{SH} = \sum_{a=2}^d \left( i\gamma V_{3a}^H - i\gamma V_{2a}^H + kV_{6a}^H \right) U_{11}^H \frac{1}{\sqrt{2}} + \left( 2 \sqrt{\frac{2}{3} i\eta V_{3a}^H} - \sqrt{\frac{2}{3} \zeta V_{6a}^H} + \frac{i\tilde{\zeta}}{\sqrt{2}} V_{5a}^H \right) \nonumber \\
+ \frac{i\gamma}{\sqrt{2}} V_{1a}^H U_{21} + \left( \frac{i\zeta}{\sqrt{2}} V_{5a}^H - \sqrt{\frac{2}{3} \zeta V_{6a}^H} - \frac{i\gamma}{\sqrt{2}} V_{1a}^H - 2 \frac{2}{3} i\eta V_{2a}^H \right) U_{31}^H \nonumber \\
- \sqrt{6} i\lambda V_{4a} U_{11}^H \left( \frac{\rho V_{6a}^H - i\zeta V_{3a}^H - i\zeta V_{2a}^H}{\sqrt{2}} \right) \nonumber \\
+ \left( \sqrt{\frac{2}{3} \zeta V_{1a}^H - k \frac{V_{3a}^H}{\sqrt{2}} + \sqrt{\frac{2}{3} \zeta V_{3a}^H} - \frac{\rho}{3} \sqrt{V_{5a}^H} \right) U_{21}^H \nonumber \\
\left( i\gamma V_{31}^H - i\gamma V_{21}^H + kV_{61}^H \right) U_{11}^H \frac{1}{\sqrt{2}} + \left( 2 \sqrt{\frac{2}{3} i\eta V_{31}^H} - \sqrt{\frac{2}{3} \zeta V_{61}^H} + \frac{i\tilde{\zeta}}{\sqrt{2}} V_{51}^H \right) \nonumber \\
+ \frac{i\gamma}{\sqrt{2}} V_{11}^H U_{21} + \left( \frac{i\zeta}{\sqrt{2}} V_{51}^H - \sqrt{\frac{2}{3} \zeta V_{61}^H} - \frac{i\gamma}{\sqrt{2}} V_{11}^H - 2 \frac{2}{3} i\eta V_{21}^H \right) U_{31}^H \nonumber \\
- \sqrt{6} i\lambda V_{41} U_{11}^H \left( \frac{\rho V_{61}^H - i\zeta V_{31}^H - i\zeta V_{21}^H}{\sqrt{2}} \right) \nonumber \\
+ \left( \sqrt{\frac{2}{3} \zeta V_{11}^H - k \frac{V_{31}^H}{\sqrt{2}} + \sqrt{\frac{2}{3} \zeta V_{31}^H} - \frac{\rho}{3} \sqrt{V_{51}^H} \right) U_{21}^H \nonumber \\
\left( i\gamma V_{31}^H - i\gamma V_{21}^H + kV_{61}^H \right) U_{11}^H \frac{1}{\sqrt{2}} + \left( 2 \sqrt{\frac{2}{3} i\eta V_{31}^H} - \sqrt{\frac{2}{3} \zeta V_{61}^H} + \frac{i\tilde{\zeta}}{\sqrt{2}} V_{51}^H \right) \nonumber \\
\nonumber \right)^2 \nonumber \\
F_{12}(m^H, m^S, Q) \tag{3.86} \]

\[ K_{QC} = \sum_{a=1} \left( i\gamma V_{1a}^C + kV_{3a}^C - i\gamma V_{2a}^C \right) U_{11}^H \frac{1}{\sqrt{2}} + \left( 2 i\eta V_{1a}^C + \zeta V_{3a}^C \right) U_{21}^H \nonumber \\
\nonumber \\
+ \left( \zeta V_{3a}^C + 2 i\eta V_{2a}^C \right) U_{11}^H \frac{1}{\sqrt{2}} + \left( \frac{\rho}{3} V_{3a}^C - i\zeta V_{1a}^C - i\zeta V_{2a}^C \right) U_{21}^H \nonumber \\
\nonumber \\
- \left( \zeta V_{1a}^C + \zeta V_{2a}^C \right) U_{21}^H \frac{1}{\sqrt{2}} \nonumber \\
\nonumber \right)^2 \nonumber \\
F_{12}(m^Q, m_a^C, Q) \tag{3.87} \]

\[ K_{DO} = \sum_{a=1} \left( i\gamma V_{1a}^D - i\gamma V_{2a}^D - kV_{3a}^D \right) U_{11}^H \frac{1}{\sqrt{2}} + \left( \sqrt{\frac{2}{3} \zeta V_{3a}^D} - \sqrt{6} i\eta V_{2a}^D \right) U_{21}^H \nonumber \\
\nonumber \\
- \left( \frac{\zeta}{\sqrt{6}} V_{3a}^D + \sqrt{\frac{2}{3} i\eta V_{1a}^D} \right) U_{11}^H \frac{1}{\sqrt{2}} + \left( i\zeta V_{2a}^D - \frac{\rho}{3} V_{3a}^D + i\zeta V_{1a}^D \right) U_{21}^H \nonumber \\
\nonumber \\
+ \left( \zeta V_{1a}^D - 3 \zeta V_{2a}^D - \frac{2 i\rho}{3} V_{3a}^D \right) U_{21}^H \frac{1}{\sqrt{2}} \nonumber \\
\nonumber \right)^2 \nonumber \\
F_{12}(m^U, m_a^D, Q) \tag{3.88} \]

\[ \nonumber \nonumber \\
K_{Y\bar{N}} = -\sqrt{2} i\gamma U_{11}^H + 2 \sqrt{\frac{2}{3} i\eta U_{21}^H} + \sqrt{2} i\zeta U_{51}^H \right)^2 \nonumber \\
\nonumber \nonumber \nonumber \\
F_{12}(m^M, m^B, Q) \tag{3.89} \]
3.7 Conclusions and Outlook

\[ K_{FH} = \sum_{a=1}^{d(F)} \sum_{a'=2}^{d(H)} \left[ i\gamma V_{1a}^F U_{4a'}^H + \gamma U_{3a}^H V_{2a}^F - \bar{\gamma} V_{2a}^F U_{2a}^H + kV_{4a}^F U_{4a'}^H + ikV_{2a}^F U_{6a'}^H \right] U_{11}^H \\
+ \left( \frac{4\eta}{\sqrt{3}} V_{2a}^F U_{3a}^H + 2\sqrt{3}i\eta V_{1a}^F U_{4a}^H - \sqrt{3}X V_{4a}^F U_{4a'}^H - \frac{2i\bar{\gamma}}{\sqrt{3}} V_{2a}^F U_{6a'}^H - \bar{\gamma} V_{2a}^F U_{5a'}^H \right) U_{11}^H \\
+ \left( \frac{4}{\sqrt{3}} \eta V_{2a}^F U_{2a}^H + \frac{2i\bar{\gamma}}{\sqrt{3}} V_{2a}^F U_{6a}^H + \bar{\gamma} V_{2a}^F U_{5a}^H + \sqrt{3}X V_{4a}^F U_{4a'}^H \right) U_{31}^H \\
+ \left( i\gamma V_{1a}^F U_{5a}^H - \bar{\gamma} V_{2a}^F U_{6a}^H - \frac{i\rho}{\sqrt{3}} V_{4a}^F U_{6a'}^H \right) U_{41}^H \\
+ \left( \frac{4i}{3} i\gamma V_{2a}^F U_{5a}^H + \frac{4i}{3} \bar{\gamma} V_{2a}^F U_{6a}^H + \sqrt{3}X V_{4a}^F U_{4a'}^H + \frac{i\rho}{\sqrt{3}} V_{4a}^F U_{6a'}^H \right) U_{51}^H \\
+ \left( i\gamma V_{1a}^F U_{6a}^H - \frac{i\rho}{3} V_{4a}^F U_{6a'}^H - i\gamma V_{1a}^F U_{4a'}^H + \frac{i\rho}{\sqrt{3}} V_{4a}^F U_{6a'}^H \right) U_{61}^H \bigg| F_{12}(m_a^F, m_{a'}^F, Q) \]

\[ = \sum_{a'=2}^{d(H)} 2g_{10}^2 \left| i \left( V_{1a}^H U_{11}^H + V_{2a}^H U_{21}^H + V_{3a} U_{31} + V_{5a}^H U_{51} \right) \right|^2 F_{12}(m_a^F, m_{a'}^F, Q) - 2g_{10}^2 \left| i \left( V_{11} U_{11}^H + V_{21} U_{21}^H \right) \right|^2 F_{11}(m_{a'}^F, Q) \]  

\[ K_{KK} = \sum_{a=1}^{d(X)} \sum_{a'=1}^{d(K)} \left[ i\gamma \sqrt{2} V_{1a}^X U_{1a'}^K + ik V_{2a}^X U_{2a'}^K \right] U_{11}^H \left( \frac{2\sqrt{2}i\eta}{\sqrt{3}} V_{1a}^X U_{1a'}^K + \frac{4\eta}{\sqrt{3}} V_{2a}^X U_{2a'}^K \right) \\
- \frac{i\bar{\gamma}}{\sqrt{3}} V_{2a}^X U_{2a'}^K \bigg| U_{1a}^H + \frac{i\bar{\gamma}}{\sqrt{3}} \left( 2\sqrt{2} V_{1a}^X - V_{2a}^X \right) \bigg| U_{2a}^K U_{31} \bigg( \sqrt{2}i V_{1a}^X U_{1a'}^K \right) \\
- \frac{i\rho}{3} \left( V_{2a}^X U_{2a'}^K \right) \bigg| U_{51}^H + \left( \frac{\rho}{3\sqrt{3}} V_{2a}^X U_{2a'}^K + \frac{2\rho}{3\sqrt{3}} V_{1a} U_{2a'}^K \right) - \frac{\gamma}{\sqrt{3}} V_{1a}^X U_{1a'}^K \bigg| U_{61}^H \bigg| F_{12}(m_a^X, m_{a'}^K, Q) \]  

\[ = \frac{-2g_{10}^2}{\sqrt{3}} \left| V_{1a}^K U_{2a}^H - iV_{2a} K_{5a}^K U_{51} + \frac{V_{2a} K_{5a}^K U_{61}^H}{\sqrt{3}} \right|^2 F_{12}(m_a^X, m_{a'}^K, Q) \]  

\[ K_{VA} = -\sqrt{2}i\gamma U_{11}^H - 2\sqrt{6} i\eta U_{21}^H + \sqrt{2}i\zeta U_{51}^H - \sqrt{6} \zeta U_{61}^H \bigg| F_{12}(m^V, m^A, Q) \]
3.7 Conclusions and Outlook

\[ K_{BL} = \sum_{a=1} \left[ -\left(i\gamma U_{1a} + ku_{2a}\right) u_{11}^H + \left(2i\eta u_{1a} - \bar{\zeta} u_{2a}\right) \sum_{a} u_{11}^{H} - \zeta u_{2a}^{H} \right] + i\bar{\zeta} u_{1a}^{L} u_{11}^{H} + \left(\zeta u_{1a} + \frac{i\rho}{3} u_{2a}^{L}\right) \sum_{a} u_{11}^{H} \right|^{2} F_{12}(m^{B}, m_{a}^{L}, Q) \] (3.93)

\[ K_{BW} = \left| -\gamma u_{11}^{H} + \frac{2\eta}{\sqrt{3}} u_{21}^{H} - \zeta u_{51}^{H} - \frac{i\zeta}{\sqrt{3}} u_{61}^{H} \right|^{2} F_{12}(m^{B}, m^{W}, Q) \] (3.94)

\[ K_{OH} = \sum_{a=2} \left| \bar{\gamma} u_{4a}^{H} u_{11}^{H} + 2\sqrt{3}\eta u_{4a}^{H} u_{31}^{H} + \left(2\sqrt{3}\eta u_{3a}^{H} + \zeta u_{5a}^{H} - \sqrt{3}i\zeta u_{6a}^{H} + \gamma u_{1a}^{H}\right) u_{41}^{H} \right|^{2} F_{12}(m^{H}, m^{O}, Q) \] (3.95)

\[ K_{WY} = \left| -\gamma u_{11}^{H} + \frac{2\eta}{\sqrt{3}} u_{21}^{H} - \zeta u_{51}^{H} + \frac{i\zeta}{\sqrt{3}} u_{61}^{H} \right|^{2} F_{12}(m^{W}, m^{Y}, Q) \] (3.96)

\[ K_{VO} = \left| -\gamma u_{11}^{H} - 2\sqrt{3}\eta u_{21}^{H} - \zeta u_{51}^{H} - \sqrt{3}i\zeta u_{61}^{H} \right|^{2} F_{12}(m^{V}, m^{O}, Q) \] (3.97)

\[ K_{GH} = \sum_{a=1} \sum_{a'=2} \left( \gamma v_{2a}^{G} v_{3a}^{H} + \frac{\gamma}{\sqrt{2}} v_{3a}^{G} v_{3a}^{H} - \sqrt{2}i\gamma v_{5a}^{G} v_{4a}^{H} + \gamma v_{2a}^{G} v_{2a}^{H} - \frac{\gamma}{\sqrt{2}} v_{3a}^{G} v_{2a}^{H} \right) \right] \left| v_{11}^{H} + \left(2\sqrt{3}\eta v_{3a}^{G} v_{3a}^{H} + \frac{4\eta}{\sqrt{3}} v_{2a}^{G} v_{3a}^{H} - \sqrt{3} \frac{2}{3} i\zeta v_{3a}^{G} v_{6a}^{H} \right) v_{11}^{H} + \left(\frac{4\eta}{\sqrt{3}} v_{2a}^{G} v_{2a}^{H} \right) \right|^{2} F_{12}(m^{G}, m^{V}, Q) \] (3.98)
3.7 Conclusions and Outlook

\[
+ \left( \sqrt{\frac{2}{3}} i \zeta V_{3a}^6 + i \zeta V_{1a}^6 \right) + \sqrt{\frac{2}{3}} i \zeta V_{3a}^6 + i \zeta V_{1a}^6 - \frac{2}{3} \sqrt{3} V_{3a}^6 V_{2a}^6
\]

\[
+ \frac{i \rho}{3\sqrt{2}} V_{3a}^6 V_{3a}^6 - \frac{i k}{\sqrt{2}} V_{3a}^6 V_{3a}^6 + \sqrt{6} \xi V_{4a}^6 V_{5a}^6 \right) u^H_{61} \right| F_{12}(m_a, m_{a'}, Q)
\]

\[
d(H)\left( \sum_{a'=2} 2 \gamma_{10} \right)^2 F_{12}(m_a, m_{a'}, Q)
\]

\[
d(G)\left( \sum_{a=1} \right) \left| \left( \omega V_{2a}^6 V_{3a}^6 - \sqrt{2} i \omega V_{2a}^6 V_{3a}^6 + \omega V_{2a}^6 V_{3a}^6 - \frac{\omega}{\sqrt{2}} V_{3a}^6 V_{3a}^6 \right) u^H_{21} + \left( i \zeta V_{1a}^6 V_{61} - \sqrt{2} i \zeta V_{3a}^6 V_{61} \right)
\]

\[
- \frac{2}{3} \xi V_{3a}^6 V_{3a}^6 + \sqrt{2} i \zeta V_{4a}^6 V_{4a}^6 + \sqrt{2} i \zeta V_{4a}^6 V_{4a}^6 + \sqrt{2} i \zeta V_{4a}^6 V_{4a}^6 + \frac{\omega}{\sqrt{2}} V_{3a}^6 V_{3a}^6
\]

\[
+ \frac{i k}{\sqrt{2}} V_{3a}^6 V_{3a}^6 + \frac{i k}{\sqrt{2}} V_{3a}^6 V_{3a}^6 + \sqrt{6} \xi V_{5a}^6 V_{4a}^6 \right) u^H_{61} \right| F_{11}(m_a, Q)
\]

\[
- 2 \gamma_{10} \left( \frac{i}{\sqrt{5}} \right) \left( u_{51}^H \right) \left( u_{51}^H \right) \left( u_{61}^H \right) ^2 F_{11}(m_{\lambda_G}, Q)
\]

\[
(3.98)
\]
Chapter 4

Loop Corrected Susy Spectra

4.1 Introduction

The discovery that the Higgs mass is, at around 126 GeV, almost 28% larger than the tree level upper limit in the MSSM [1, 2], has emphasized the crucial role of loop corrections in the MSSM [106]. For the precise estimation of Susy particle masses, it is essential to consider the loop effects. Susy threshold corrections to the SM Yukawas are already incorporated in the NMSGUT [57]. The NMSGUT requires GUT scale [104] and Susy threshold corrections [57] to suppress fast B-decay rates and for fermion fitting respectively. In the previous chapter tree level Susy spectrum is presented (however the Higgs mass is one-loop corrected). The next step regarding inclusion of quantum effects is to calculate one loop corrections to the Susy spectrum. NMSGUT fits [57, 104] prefer mini-split Susy spectrum: gaugino/higgsino masses $\sim 10^2$ TeV, heavy third s-generation $\sim 10$ TeV, $\mu, A_0, M_H, \bar{H} \sim 10^2$ TeV and first and second generation lie in between neutralino/chargino and third generation sparticle masses. In this chapter we present a one-loop corrected effective MSSM spectrum in the context of NMSGUT. We will investigate the significant corrections received from the NMSGUT kind of low energy spectrum. Formulae for the one loop corrections to the entire Susy spectrum are well known [107, 108, 109, 110, 111, 83]. Particularly Ref. [83] is a pedagogical manual for one loop corrections to fermion, sparticle and gauge boson spectra. The modified dimensional reduction (DR) renormalization scheme [112] is convenient to use in Susy theories. The poles of the loop corrected
propagators determine the physical masses. Thus the physical mass \( m_{\text{phys}} \) of a boson is given by the solution of

\[
M^2 = \hat{M}^2(Q) - \Re(\Pi(M^2)) \tag{4.1}
\]

Here \( \hat{M}^2(Q) \) is tree level mass parameter at scale \( Q \) and \( \Pi(M^2) \) is self energy contribution (the sensitivity to \( Q \) of \( m_{\text{phys}} \) should decrease as one increases the order of perturbation since it should be a RG invariant). To avoid negative pole mass self energy is iteratively calculated. One needs to calculate self energy of \( W \) and \( Z \) boson to calculate \( \overline{\text{DR}} \) EW symmetry breaking VEV. Fermion masses are also calculated from the pole of corresponding propagator. One-loop self energy of Susy particles affects:

- Gluino, Chargino and Neutralino Masses
- Higgs sector :- heavy Higgs boson masses \( (M_A, M_H, M_{H^+}) \) and light Higgs boson \( (M_h) \)
- Squarks and Sleptons

Complete formulae of these self energy calculations are presented in [83]. In the next section we will discuss dominant corrections with explicit expressions.

4.2 Dominant Corrections

FORTRAN subroutines implementing formulae of [83] are also available [82]. We have interfaced these subroutines with our FORTRAN code. We calculated the loop corrections to the tree level spectrum presented in the previous chapter (see Tables 3.2-3.6) and [104]. Including loop corrections (some of) the first and second generation squark and slepton masses can turn negative as shown in the example Tables 4.1 and 4.2. We identified the dominant dangerous corrections- i.e. those which can drive some sparticle masses to negative values- using the solutions presented in the previous chapters. The significant corrections are discussed below :-

4.2 Dominant Corrections

4.2.1 Gluino Mass

Tree level gluino mass is

\[ M_{\tilde{g}} = M_3(Q) \] (4.2)

The physical gluino mass is given by the solution of

\[ M_{\tilde{g}} = M_3(Q) - \Re \Sigma_{\tilde{g}}(M_{\tilde{g}}^2) \] (4.3)

where \( \Sigma_{\tilde{g}}(M_{\tilde{g}}^2) \) is gluino self-energy:

\[
\Sigma_{\tilde{g}}(p^2) = \frac{g_3^2}{16\pi^2} \left\{ -M_{\tilde{g}} \left( 15 + 9 \ln \frac{Q^2}{M_{\tilde{g}}^2} \right) - \sum_q \sum_{i=1}^2 M_{\tilde{g}} B_1(p, m_q, m_{\tilde{q}_i}) \right. \\
\left. + \sum_q m_q \sin 2\theta_q \left[ B_0(p, m_q, m_{\tilde{q}_1}) - B_0(p, m_q, m_{\tilde{q}_2}) \right] \right\} \] (4.4)

where \( Q \) is the renormalization scale (which we take to be \( M_Z \)), \( g_3 \) and \( \theta_q \) are SU(3) gauge coupling constant and squark mixing angle respectively. One loop correction to gluino mass comes from gluon/gluino and quark(m_q)/squark(m_{\tilde{q}}) loops [83]. The loop-function \( B_0 \) has the form

\[
B_0(p, m_1, m_2) = \frac{1}{\epsilon} - \ln \left( \frac{p^2}{Q^2} \right) - f_B(x_+) - f_B(x_-) \] (4.5)

where

\[
x_{\pm} = \frac{s \pm \sqrt{s^2 - 4p^2(m_1^2 - \epsilon)}}{2p^2} ; \quad s = p^2 - m_2^2 + m_1^2 \\
f_B(x) = \ln(1 - x) - x \ln(1 - x^{-1}) - 1 \] (4.6)

and \( 1/\epsilon \) represents infinite contribution. The function \( B_1 \) is defined in terms of loop-functions \( B_0 \) and \( A_0 \) :

\[
A_0(m) = m^2 \left( \frac{1}{\epsilon} + 1 - \ln \frac{m^2}{Q^2} \right) \] (4.7)

\[
B_1(p, m_1, m_2) = \frac{1}{2p^2} \left[ A_0(m_2) - A_0(m_1) + (p^2 + m_1^2 - m_2^2)B_0(p, m_1, m_2) \right] \] (4.8)
4.2 Dominant Corrections

First term of Eq. (4.4) represents the gluon/gluino contribution. Quark/squark contribution is given by second and third terms. Out of these the third term has negligible contribution since it is proportional to quark masses. First and second term have opposite contribution. Second term involves the loop function $A_0$ which is proportional to $m^2 \tilde{q}$, therefore it provides the dominant contribution. Gluino mass gets approximately 30% corrections (see Tables 4.1 and 4.2).

4.2.2 Neutralino and Chargino Masses

MSSM gauginos and Higgsinos form chargino and neutralino eigenstates. Neutralino mass matrix including radiative corrections is given by:

$$M_{\tilde{\chi}^0 \text{1-loop}} = M_{\tilde{\chi}^0} + \frac{1}{2} \left( \delta M_{\tilde{\chi}^0} (p^2) + \delta M_T (p^2) \right)$$

where

$$\delta M_{\tilde{\chi}^0} (p^2) = - \Sigma^0_R (p^2) M_{\tilde{\chi}^0} - M_{\tilde{\chi}^0} \Sigma^0_L (p^2) - \Sigma^0_S (p^2)$$

Here $M_{\tilde{\chi}^0}$ is the tree-level neutralino mass matrix (Eq. 2.69), and the factors $\Sigma^0_{L,R,S} (p^2)$ are matrix corrections. Self energy has contributions from (quark/squark, lepton/slepton, neutrino/sneutrino), chargino/W-boson, neutralino/Z-boson and gaugino/Higgs loops. Dominant contribution comes from quark/squark, lepton/slepton, neutrino/sneutrino loops. Third generation quark/squark provide maximum corrections:

$$(\Sigma^0_L (p^2))_{ij} \tilde{q} \tilde{q} \approx \sum_{k=1}^{2} a_{\tilde{\chi}^0_{ij} q\tilde{q}k} a_{\tilde{\chi}^0_{ij} q\tilde{q}k} \text{Re} \ B_1 (p, m_q, m_{\tilde{q}k})$$

$$(\Sigma^0_S (p^2))_{ij} \tilde{q} \tilde{q} \approx \sum_{k=1}^{2} b_{\tilde{\chi}^0_{ij} q\tilde{q}k} a_{\tilde{\chi}^0_{ij} q\tilde{q}k} m_t \text{Re} \ B_0 (p, m_q, m_{\tilde{q}k})$$

Here $q$ represents top/bottom quark and $\tilde{q}$ refer to the corresponding scalar. $a_{\tilde{\chi}^0_{ij} q\tilde{q}k}$ are neutralino-fermion-sfermion couplings and sum over $k$ includes contribution of scalar partners of both left and right handed quarks. Similarly, one-loop chargino
mass matrix is as follows:

\[ M^{1\text{-loop}}_\tilde{\chi} = M_\tilde{\chi} - \Sigma_R^+(p^2)M_\tilde{\chi} - M_\tilde{\chi} \Sigma_L^+(p^2) - \Sigma_S^+(p^2) \]  

(4.13)

where \( M_\tilde{\chi} \) is the tree-level chargino mass matrix (Eq 2.68). Quark/squark corrections have form:

\[ (\Sigma^+_L(p^2))_{ij} \approx \sum_{k=1}^{2} \alpha_{q_i q'_{k}}^+ a_{q'_{k} q}^+ \cdot \text{Re} B_1(p,m_q,m_{\tilde{q}'_k}) \]  

(4.14)

\[ (\Sigma^+_S(p^2))_{ij} \approx \sum_{k=1}^{2} \beta_{q_i q'_{k}}^+ a_{q'_{k} q}^+ m_q \cdot \text{Re} B_0(p,m_q,m_{\tilde{q}'_k}) \]  

(4.15)

Here \( q' \) denotes bottom (top) when \( q \) is top (bottom). \( \Sigma^0_R^+ \) can be obtained from \( \Sigma^+_L \) by replacing the couplings. Neutralino/chargino corrections are approximately 7%.

### 4.2.3 Higgs Mass

The MSSM has two Higgs doublets \( H_1 \) and \( H_2 \) whose VEVs generate fermion masses, so to start with we have 8 degrees of freedom. \( W^\pm \) and \( Z \) bosons eat 3 degrees of freedom and become massive so we are left with 5 degrees of freedom i.e. the neutral Higgs \( (h,H) \), charged Higgs \( H^\pm \), CP odd Higgs \( A \). Higgs soft mass parameters \( m_{H_{1,2}}^2 \) at \( M_Z \) determine \( \mu \) and \( B \):

\[ \mu^2 = \frac{1}{2} \left[ \tan 2\beta \left( m_{H_2}^2 \tan \beta - m_{H_1}^2 \cot \beta \right) - M_Z^2 \right] \]

\[ B = \frac{1}{2} \left[ \tan 2\beta \left( m_{H_2}^2 - m_{H_1}^2 \right) - M_Z^2 \sin 2\beta \right] \]  

(4.16)

\( M_A \) (\( B (\tan \beta + \cot \beta) \)) is computed from \( B \) and then tree level Higgs masses are calculated. Tadpoles need to be calculated to take into account the one loop radiative...
corrections to the Higgs masses. Vanishing of tadpoles provides:

\[ \mu^2 = \frac{1}{2} \left[ \tan 2\beta \left( m_{H_2}^2 \tan \beta - m_{H_1}^2 \cot \beta \right) - M_Z^2 - \mathcal{Re} \Pi_{ZZ}^T(M_Z^2) \right] \]

\[ M_A^2 = \frac{1}{c_{2\beta}} \left( m_{H_2}^2 - m_{H_1}^2 \right) - M_Z^2 - \mathcal{Re} \Pi_{ZZ}^T(M_Z^2) - \mathcal{Re} \Pi_{AA}(m_A^2) + b_A \] (4.17)

\[ \bar{m}_{H_1}^2 = m_{H_1}^2 - \frac{t_1}{v_1} \quad \bar{m}_{H_2}^2 = m_{H_2}^2 - \frac{t_2}{v_2} \] (4.18)

\[ b_A = s^2_\beta \frac{t_1}{v_1} + c^2_\beta \frac{t_2}{v_2} \] (4.19)

Here \( \Pi_{ZZ} \) and \( \Pi_{AA} \) are self energies of Z-boson and CP odd pseudo-pseudoscalar \( A \).

Charged Higgs mass including loop corrections is given by:

\[ M_{H^+}^2 = M_A^2 + M_W^2 + \mathcal{Re} \left[ \Pi_{AA}(M_A^2) - \Pi_{H^+H^-}(m_{H^+}^2) + \Pi_{WW}^T(M_W^2) \right] \] (4.20)

Remaining two CP-even Higgs mass are determined from the pole of matrix:

\[ \mathcal{M}_s^2(p^2) = \begin{pmatrix} \hat{M}_Z^2 c^2_\beta + \hat{M}_A^2 s^2_\beta - \Pi_{s_1 s_1}(p^2) + \frac{t_1}{v_1} & -(\hat{M}_Z^2 + \hat{M}_A^2) s_\beta c_\beta - \Pi_{s_1 s_2}(p^2) \\ -(\hat{M}_Z^2 + \hat{M}_A^2) s_\beta c_\beta - \Pi_{s_2 s_1}(p^2) & \hat{M}_Z^2 s^2_\beta + \hat{M}_A^2 c^2_\beta - \Pi_{s_2 s_2}(p^2) + \frac{t_2}{v_2} \end{pmatrix} . \] (4.21)

Here \( \hat{M}_Z^2 \) and \( \hat{M}_A^2 \) are \( \overline{\text{DR}} \) masses. Explicit form of self-energies (\( \Pi_{ss'} \)) can be found in [33].

### 4.2.4 Squark and Slepton Masses

Tree level squark and slepton masses are calculated from \( 6 \times 6 \) mass matrices. It is convenient to breakup self energy corrections as \( 3 \times 3 \) blocks (\( \Pi_{iLjL}, \Pi_{iLjR}, \Pi_{iRjR} \)):

\[ \mathcal{M}_f^2(p^2) = \begin{pmatrix} M_{iLjL}^2 - \Pi_{iLjL}(p^2) & M_{iLjR}^2 - \Pi_{iLjR}(p^2) \\ M_{iRjL}^2 - \Pi_{iRjL}(p^2) & M_{iRjR}^2 - \Pi_{iRjR}(p^2) \end{pmatrix} . \] (4.21)
4.2 Dominant Corrections

| Parameter | Tree level Masses (GeV) | Loop Corrected Masses (GeV) |
|-----------|-------------------------|-----------------------------|
| $M_0$     | 1000.14                 | 1297.10                     |
| $M_{\chi^\pm}$ | 569.81, 109858.12   | 628.52, 123993.68          |
| $M_{\tilde{\chi}^0}$ | 210.1, 569.8, 109858.1, 109858.1 | 215.9, 628.5, 123993.7, 123993.7 |
| $M_0^2$   | $\{2.34, 2.33, 4.55\} \times 10^8$ | $\{2.0, 2.02, 8.42\} \times 10^9$ |
| $M_\tilde{\ell}^2$ | $\{3.10, 234.34\} \times 10^6$, 45071.29 | $\{-3.50, 1.99, -3.46, 2.01\} \times 10^9$ |
| $M_{\tilde{u}}^2$ | $\{2.33, 4.29, 4.57\} \times 10^8$ | $\{8.42, 9.38\} \times 10^9$ |
| $M_{\tilde{d}}^2$ | $\{1.27, 2.09, 1.27, 2.09\} \times 10^8$ | $\{-0.45, 2.56, -0.45, 2.56\} \times 10^9$ |
| $M_A$     | 517662.74               | 40556.46                    |
| $M_{H^\pm}$ | 517662.75               | 40590.03                    |
| $M_H$     | 517662.74               | 40624.33                    |
| $M_H$     | 89.09                   | 493.46                      |

Table 4.1: Tree level and loop corrected Susy spectra corresponding to the soft parameter presented in the previous chapter. The loop corrected squark and slepton masses turn negative.

| Parameter | Tree level Masses (GeV) | Loop Corrected Masses (GeV) |
|-----------|-------------------------|-----------------------------|
| $M_0$     | 1200.01                 | 1542.55                     |
| $M_{\chi^\pm}$ | 590.18, 155715.46   | 646.67, 160020.65          |
| $M_{\tilde{\chi}^0}$ | 246.4, 590.2, 155715.4, 155715.4 | 255.8, 646.7, 160020.6, 160020.6 |
| $M_0^2$   | $\{2.35, 2.35, 9.08\} \times 10^8$ | $\{25.43, 25.71, 115.44\} \times 10^9$ |
| $M_\tilde{\ell}^2$ | $\{1.43, 2.35, 1.43, 2.35\} \times 10^8$ | $\{-4.42, 2.52, -4.35, 2.55\} \times 10^9$ |
| $M_{\tilde{u}}^2$ | $\{9.08, 14.87\} \times 10^8$ | $\{14.22, 19.38\} \times 10^9$ |
| $M_{\tilde{d}}^2$ | $\{1.64, 1.81, 1.64, 1.81\} \times 10^8$ | $\{-0.57, 3.21, -0.57, 3.21\} \times 10^9$ |
| $M_A$     | 584560.76               | 25390.88                    |
| $M_{H^\pm}$ | 584560.76               | 25398.06                    |
| $M_H$     | 584560.76               | 25409.02                    |
| $M_h$     | 89.35                   | 411.44                      |

Table 4.2: Tree level and loop corrected Susy spectra corresponding to solution 2 of [104]. The loop corrected squark and slepton masses turn negative.
4.2 Dominant Corrections

Self energies are calculated at the tree mass scale for each particle. Sfermion masses are fixed as poles of propagator

\[ \text{Det}[p_i^2 - M_f^2(p_i^2)] = 0 \] (4.22)

Squark and slepton receive corrections from electroweak gauge bosons, neutralinos, charginos, sfermion quartic interactions (up squark, down squark, charged slepton, sneutrinos), pseudoscalar Higgs, neutral Higgs and charged Higgs. Out of these Higgs sector corrections are dominant because of decoupled Susy spectra. For up type squarks, these are:

\[
\Pi_{Higgs-sector}^{\tilde{u}_L \tilde{u}_R}(p^2) \approx \frac{1}{2} \sum_{n=1}^{4} \left( Y_u^2 D_{nu} - \frac{g^2 g_{ul}}{2 \cos^2 \theta_W} C_n \right) A_0(m_{H_n^0}) \\
+ \sum_{n=3}^{4} \left( Y_u^2 D_{nu} + g^2 \left( \frac{g_{ul}}{2 \cos^2 \theta_W} - I_3^u \right) C_n \right) A_0(m_{H_{n-2}^+}) \\
+ \sum_{n=1}^{2} \sum_{i=1}^{2} \left( \lambda_{H_n^0 \tilde{u}_L u_i} \right)^2 B_0(p, m_{H_n^0}, m_{\tilde{u}_i}) \\
+ \sum_{i,n=1}^{2} \left( \lambda_{H_n^+ \tilde{u}_L \tilde{d}_i} \right)^2 B_0(p, m_{\tilde{d}_i}, m_{H_n^+}) \] (4.23)

Here \( H_n^0 \) denotes \( H, h, G^0 \) and \( A^0, H_1^+ (H_2^+) \) refer to \( H^+ (G^+) \), \( Y_u / Y_d \) is the Yukawa coupling of up/down quarks, the factors \( D_{nu} \) and \( C_n \) are sine/cosine functions of Higgs mixing angles \( \alpha, \beta \) [33]. The parameters \( I_3^u, g \) and \( g_f (I_3^f - Q_f \sin^2 \theta_W) \) represent SU(2) quantum number, SU(2) gauge coupling and weak neutral current coupling. Self energy \( \Pi_{Higgs-sector}^{f_R f_R}(p^2) \) can be obtained by replacing \( g_{ul} \) by \( g_{ur} \) and \( \lambda_{H_n^0 \tilde{u}_L \tilde{f}_i} / \lambda_{H_n^+ \tilde{u}_L \tilde{b}_i} \) by \( \lambda_{H_n^0 \tilde{u}_R \tilde{f}_i} / \lambda_{H_n^+ \tilde{u}_R \tilde{b}_i} \) in \( \Pi_{Higgs-sector}^{f_L f_L}(p^2) \).

\[
\Pi_{Higgs-sector}^{f_L f_L}(p^2) \approx \sum_{n=1}^{4} \sum_{i=1}^{2} \lambda_{H_n^0 \tilde{u}_L \tilde{u}_i} \lambda_{H_n^0 \tilde{u}_R \tilde{u}_i} B_0(p, m_{H_n^0}, m_{\tilde{u}_i}) \\
+ \sum_{i,n=1}^{2} \lambda_{H_n^+ \tilde{u}_L \tilde{d}_i} \lambda_{H_n^+ \tilde{u}_R \tilde{d}_i} B_0(p, m_{\tilde{d}_i}, m_{H_n^+}) \] (4.24)

Corrections due to quartic Higgs couplings involve loop function \( A_0 \) which is proportional to \( m^2 \), so these terms dominate for the heavy \( M_A \), as predicted by the
NMSGUT. If we switch off the terms containing $A_0$ in the Higgs contribution then we get positive loop corrected masses. But if we ignore all the Higgs corrections then third generation masses become negative as shown in Tables 4.3 and 4.4. Chargino/neutralino contribution is appreciable for third generation:

\[
\Pi_{\tilde{q}l\tilde{q}_L}^{C/N}(p^2) \approx \sum_{i=1}^{4} \left[ f_{iq\tilde{q}_L} G(p, m_{\tilde{\chi}_{i}^0}, m_q) - 2 g_{iq\tilde{q}_L} m_{\tilde{\chi}_{i}^0} m_t B_0(p, m_{\tilde{\chi}_{i}^0}, m_q) \right] \\
+ \sum_{i=1}^{2} \left[ f_{iq'\tilde{q}_L} G(p, m_{\tilde{\chi}_{i}^+}, m_{q'}) - 2 g_{iq'\tilde{q}_L} m_{\tilde{\chi}_{i}^+} m_{q'} B_0(p, m_{\tilde{\chi}_{i}^+}, m_{q'}) \right] \quad (4.25)
\]

\[
\Pi_{\tilde{q}l\tilde{q}_R}^{C/N}(p^2) \approx \sum_{i=1}^{2} \left[ f_{iq\tilde{q}_R} G(p, m_{\tilde{\chi}_{i}^+}, m_q) - 2 g_{iq\tilde{q}_R} m_{\tilde{\chi}_{i}^+} m_q B_0(p, m_{\tilde{\chi}_{i}^+}, m_q) \right] \quad (4.26)
\]

Couplings $f$ and $g$ are defined as

\[
f_{iff_j} = |a_{\tilde{\chi}_{i}ff_j}|^2 + |b_{\tilde{\chi}_{i}ff_j}|^2 \\
g_{iiff_j} = 2 \Re \left( b_{\tilde{\chi}_{i}ff_j}^* a_{\tilde{\chi}_{i}ff_j} \right) \quad (4.27)
\]

Loop function $G$ is defined in terms of functions $B_0$ and $A_0$.

\[
G(p, m_1, m_2) = (p^2 - m_1^2 - m_2^2) B_0(p, m_1, m_2) - A_0(m_1) - A_0(m_2) \quad (4.28)
\]

Chargino/neutralino and Higgs corrections have opposite contribution. One needs to decrease the magnitude of Higgs corrections to get positive first and second generation sparticle masses. The chargino and neutralino masses are proportional to $\mu$ parameter. We have softened this contribution via penalty on $\mu$ and $M_A$ which we will discuss in the next section. In the tree level example spectra presented in the previous chapter $M_A \sim 3\mu$.

### 4.3 Numerical Procedure

Our fitting criteria are the same as discussed in the previous chapters. To calculate the one loop Susy spectrum, we have interfaced the LoopmassesMSSM subroutine.
4.3 Numerical Procedure

Input
\[ g_i, \tan \beta, Y_u, Y_d, Y_t, M_i, A_u, A_d, A_t \]
\[ M_H^2, M_H^2, m^2_l, m^2_u, m^2_d, m_{\tilde{Q}}^2 \]

Calculate \( \mu \) and B using EWSB conditions

Calculate tree level MSSM spectrum

Check precision

Yes

Calculate tree level spectrum

Calculate tadpole

No

Self energy pseudo scalar Higgs

Check precision

Yes

Calculate \( \Pi_{ZZ} \), \( \overline{DR} \) VEV and two loop tadpole

No

Self energy charged Higgs

Check precision

Yes

Self energy neutral Higgs

No

Self energy of chargino and neutralino

Yes

Self energy squark, slepton

OUTPUT
\[ M_{\tilde{\chi}^\pm}, M_{\tilde{\chi}_0^\pm}, M_{H,h}, M_{A}, M_{H^\pm} \]
\[ M_{\tilde{t}_1}^2, M_{\tilde{t}_2}^2, M_{\tilde{b}_1}^2, M_{\tilde{b}_2}^2, M_{\tilde{g}}, \mu, B \]

Figure 4.1: Flowchart of SPheno subroutine LoopMassesMSSM.
4.3 Numerical Procedure

| Parameter | Loop Corrected Masses (GeV) |
|-----------|-----------------------------|
| $M_{\tilde{e}}^2$ | $\{2.61, 22.35, 2.2205\} \times 10^7, -4.41 \times 10^8, 305993.39\}$ |
| $M_{\tilde{u}}^2$ | $\{1.36, 1.96, 1.36, -21.61, -12.93\} \times 10^8$ |
| $M_{\tilde{d}}^2$ | $\{0.81, 1.36, 1.36, -12.93, -4.46\} \times 10^8$ |

Table 4.3: Charged slepton and squark masses ignoring Higgs sector corrections for the solution presented in the previous chapter. Third generation squark and slepton masses turn negative.

| Parameter | Loop Corrected Masses (GeV) |
|-----------|-----------------------------|
| $M_{\tilde{e}}^2$ | $\{1.488, 2.33, 1.431, 2.301, -1.777, 0.82\} \times 10^8$ |
| $M_{\tilde{u}}^2$ | $\{1.64, 1.87, 1.64, 1.87, -24.52, -20.98\} \times 10^8$ |
| $M_{\tilde{d}}^2$ | $\{1.32, 1.87, 1.31, 1.87, -20.10, -18.07\} \times 10^8$ |

Table 4.4: Charged slepton and squark masses ignoring Higgs sector corrections corresponding to solution 2 of [104]. Third generation squark and slepton masses turn negative.

of SPheno [82] with our low scale calculation subroutine FUNKTUNE (see flowchart 2.1). We will discuss only the extra penalties imposed and structure of the additional subroutine. Algorithm of LoopmassesMSSM is represented by a flowchart 4.1. It calculates one-loop radiative corrections to the Susy particles but for the Higgs masses two-loop corrections [113, 114, 115] are also included. We run the MSSM RGEs from $M_{GUT}$ to $M_Z$ to get hard and soft parameters at $M_Z$ (considered the renormalization matching scale), which are the inputs to LoopmassesMSSM. First of all the subroutine calculates $\mu$ and $B$ parameter using electroweak symmetry breaking conditions (EWSB) (Eq. 4.16). Using calculated $\mu$ and $B$ tree-level Susy spectra is calculated. $Z$-boson self energy ($\Pi_{ZZ}$) is used to compute $\overline{\text{DR}}$ VEV

$$v^2(Q) = 4M_Z^2 + R e \Pi_{ZZ}(M_Z^2) \overline{g^2(Q)} + \overline{g^2(Q)}$$ (4.29)

This VEV is used to calculate one-loop and two-loop tadpoles further in the spectrum calculations. $\mu$ and $B$ are calculated using two-loop effective potential. This process is repeated until consistent values are achieved. Then the tree level sparticle
4.3 Numerical Procedure

spectrum is calculated using updated $\mu$ and $B$ value.

As the Higgs sector masses are parameterized in terms of CP- odd pseudo-scalar Higgs mass ($M_A$) and $\tan \beta$, so first of all two loop corrections to pseudo-scalar Higgs are calculated and the process is repeated to get the convergent value. To calculate one-loop corrected charged Higgs masses (Eq. 4.20), self energy of charged Higgs and W-boson is calculated. Then neutral Higgs masses are calculated taking into account the two loop corrections [113, 114, 115, 116]. After this one loop correction to gaugino-higgsino sector are calculated. At the end loop corrections to the squarks and sleptons are calculated. Subroutine \textit{LoopmassesMSSM} calls several subroutines from SPheno modules - LoopMasses, Couplings and TwoLoopHiggs to compute loop corrections to Susy particles. Each subroutine has \textit{in/out} argument \textit{kont} which traces the occurrence of negative mass square parameter. We have modified the subroutine to accumulate \textit{kont} from all the called subroutines to the end so that program moves forward using absolute value of the negative quantity. Output \textit{kont} of \textit{LoopmassesMSSM} is added to $\chi^2_Z$. In other words penalties are imposed to get positive masses.

NMSGUT solutions [57, 104] have $A_0, \mu |M_{H,R}|$ parameter of order of $10^5$ GeV. Large value of $M_A$ gives huge corrections to sleptons and squarks masses and can turn these negative. To overcome these obstacles one way is to decouple each Susy particle at its mass threshold [117]. But implementation of this procedure is not trivial in multiple iteration search code. Other possibility is to limit the size of loop correction factor which is controlled by $\mu$ and $M_A$. While finding the solution we restricted the (\textit{LoopmassesMSSM} output) parameters such that $.3 < \left( \frac{\mu}{M_{A_{\text{loop}}}} \right)^2 < 2.7$ by imposing the penalty in the program. Lower limit is decided from the tree level fits which have $M_A \approx 3 \mu$. Two successful solution sets are given in Tables 4.5, 4.17 which have $\mu > M_A$. We also give the tree level spectrum for both the solutions in Table 4.9 and 4.16. Spectrum calculated using one loop EWSB conditions and including only one-loop corrections to CP-odd Higgs boson and neutral Higgs, is given in Tables 4.11 and 4.18. Since the two-loop corrections are small the spectrum is slightly modified. The values of $\mu$ and $B$ calculated using one-loop EWSB conditions are also provided. Results of nucleon decay rate calculations corresponding to the solution found are
Table 4.5: Fit 1: Values of the NMSGUT-SUGRY-NUHM parameters at $M_X$ derived from an accurate fit to all 18 fermion data and compatible with RG constraints. Unification parameters and mass spectrum of superheavy and superlight fields are also given.
given in Table 4.19. These fits have $A_0$, $\mu$, $M_H^2$ and $M_{\tilde{H}}^2$ parameter lesser by a factor of 10 as compared to the tree level fits. Parameter $B$ is decreased by a factor of 100. Loop corrected spectrum have $M_A^{\text{Tree}}$ slightly greater than $\mu$ parameter but earlier (tree level fits) we had $M_A \approx 3\mu$ where as now $M_A^{\text{loop}} \approx .5\mu$.

4.4 Discussion

We have found the NMSGUT superpotential couplings and mSUGRY-NUHM parameter sets which provide realistic fermion mass-mixing data, B-decay rates respecting experimental limits and consistent loop corrected Susy spectra as shown in Table 4.15. Computation of sfermion self enerlies require special treatment as heavy CP-pseudo scalar $A$ and chargino/neutralino give huge corrections. To control these corrections the $(\frac{\mu}{M_A})^2$ ratio is kept within the range : 0.3-2.7. Consequently the heavy chargino/neutralino masses and heavy Higgs masses ($M_A, H, H^\pm$) are smaller (O(80) TeV, 40-50 TeV in our example fits). Loop corrected spectra retain the distinctive feature (of NMSGUT): normal s-hierarchy. The reason is the preference for huge negative soft Higgs masses ($M_{H,\tilde{H}}^2$) $\approx -10^9$ GeV$^2$ in the fits. LSP is pure Bino as before. We shall see (in Chapter 6) that the NMSGUT provides a natural reason why the soft Higgs masses become negative already at the GUT scale. Loop corrected spectrum have right handed up squarks as the lightest sfermion instead of light smuon. The proton decay rates are still suppressed as explained in Chapter 3.

As discussed this ratio $\frac{\mu}{M_A}$ is crucial for the Higgs sector and Higgsino loop correction to the scalars. Values of this ratio found in the fits approach the upper limit applied. We have not yet found light smuon solution after including loop corrections which require parameter space scan with different ranges of $\frac{\mu}{M_A}$. As a check we calculated the one-loop spectrum using package SuSpect [118] by providing it soft masses (at $M_Z$, given in Tables 4.8 and 4.15) along with $\mu$ and $M_A$. We found acceptable agreement between the two packages except for sfermion masses which are not at all accurate in SuSpect when $M_A$ is very large since no contribution at
\[ \chi_X = \sqrt{\sum_{i=1}^{17} \frac{(\bar{O}_i - O_i)^2}{\delta_i}} = 0.0794 \]

Table 4.6: Fit 1 with \( \chi_X \). Target values, at \( M_X \) of the fermion Yukawa couplings and mixing parameters, together with the estimated uncertainties, achieved values and pulls. The eigenvalues of the wavefunction renormalization for fermion and Higgs lines are given with Higgs fractions \( \alpha_i, \bar{\alpha}_i \) which control the MSSM fermion Yukawa couplings.
### 4.4 Discussion

| Parameter | SM($M_Z$) | $m^{GUT}(M_Z)$ | $m^{MSSM}$ = $(m + \Delta m)^{GUT}(M_Z)$ |
|-----------|-----------|----------------|------------------------------------------|
| $m_d/10^{-3}$ | 2.900000 | 0.96198 | 2.90145 |
| $m_s/10^{-3}$ | 55.00000 | 34.51146 | 55.27487 |
| $m_b$ | 2.90000 | 2.46374 | 2.95452 |
| $m_e/10^{-3}$ | 0.48657 | 0.48374 | 0.48402 |
| $m_\mu$ | 0.10272 | 0.10222 | 0.10228 |
| $m_\tau$ | 1.74624 | 1.72797 | 1.72682 |
| $m_u/10^{-3}$ | 1.27000 | 1.12553 | 1.27181 |
| $m_c$ | 0.61900 | 0.54834 | 0.61960 |
| $m_t$ | 172.50000 | 147.29259 | 172.36306 |

Table 4.7: Fit 1: Values of the SM fermion masses in GeV at $M_Z$ compared with the masses obtained from values of GUT derived Yukawa couplings run down from $M_X$ to $M_Z$ both before and after threshold corrections. Fit with $\chi_Z = \sqrt{\sum_{i=1}^{9} \frac{(m_{i}^{MSSM} - m_{i}^{SM})^2}{(m_{i}^{MSSM})^2}} = .0234$.

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| $M_1$ | 132.41 | $m_{\tilde{u}_1}$ | 5433.34 |
| $M_2$ | 260.08 | $m_{\tilde{u}_2}$ | 5435.12 |
| $M_3$ | 900.00 | $m_{\tilde{e}_1}$ | 38053.13 |
| $m_{\tilde{t}_1}$ | 17466.95 | $A_{11}^{0(u)}$ | -8987.41 |
| $m_{\tilde{t}_2}$ | 17545.48 | $A_{22}^{0(u)}$ | -8977.63 |
| $m_{\tilde{t}_3}$ | 33074.48 | $A_{33}^{0(u)}$ | -6202.67 |
| $m_{\tilde{L}_1}$ | 9270.57 | $A_{11}^{0(u)}$ | -9976.33 |
| $m_{\tilde{L}_2}$ | 9344.03 | $A_{22}^{0(u)}$ | -9976.12 |
| $m_{\tilde{L}_3}$ | 21841.66 | $A_{33}^{0(u)}$ | -5835.50 |
| $m_{\tilde{d}_1}$ | 13413.69 | $A_{11}^{0(d)}$ | -9998.92 |
| $m_{\tilde{d}_2}$ | 13418.47 | $A_{22}^{0(d)}$ | -9998.43 |
| $m_{\tilde{d}_3}$ | 28449.38 | $A_{33}^{0(d)}$ | -6484.53 |
| $m_{\tilde{Q}_1}$ | 12775.38 | $\tan \beta$ | 51.00 |
| $m_{\tilde{Q}_2}$ | 12778.24 | $\mu(M_Z)$ | 76197.57 |
| $m_{\tilde{Q}_3}$ | 34335.50 | $B(M_Z)$ | $1.8027 \times 10^8$ |
| $M_H^2$ | $-4.2990 \times 10^9$ | $M_{H}^2$ | $-6.1500 \times 10^9$ |

Table 4.8: Fit 1: Values (in GeV) of the soft SUSY parameters at $M_Z$ (evolved from the soft SUGRY-NUHM parameters at $M_X$) which determine the SUSY threshold corrections to the fermion Yukawas ($M_{\text{SUSY}} = 2.89$ TeV).
## 4.4 Discussion

Table 4.9: Fit 1 : Tree level spectra of supersymmetric partners calculated ignoring generation mixing effects.

| Field | Mass (GeV) |
|-------|------------|
| $M_{\tilde{g}}$ | 900.00 |
| $M_{\tilde{\chi}^\pm}$ | 260.08, 76197.65 |
| $M_{\tilde{\chi}^0}$ | 132.41, 260.08, 76197.62, 76197.62 |
| $M_{\tilde{\nu}}$ | 9270.348, 9343.811, 21841.570 |
| $M_{\tilde{e}}$ | 9270.69, 17467.00, 9344.11, 17545.56, 21840.17, 33075.53 |
| $M_{\tilde{u}}$ | 5433.23, 12775.27, 5435.00, 12778.13, 34335.69, 38053.43 |
| $M_{\tilde{d}}$ | 12775.51, 13413.72, 12778.34, 13418.53, 28445.36, 34338.89 |
| $M_A$ | 95903.61 |
| $M_{H^\pm}$ | 95903.65 |
| $M_H$ | 95903.61 |
| $M_h$ | 90.40 |

Table 4.10: Fit 1 : Loop corrected spectra of supersymmetric partners calculated ignoring generation mixing effects.

| Field | Mass (GeV) |
|-------|------------|
| $M_{\tilde{g}}$ | 1174.85 |
| $M_{\tilde{\chi}^\pm}$ | 297.42, 82465.86 |
| $M_{\tilde{\chi}^0}$ | 141.08, 297.27, 82465.83, 82465.83 |
| $M_{\tilde{\nu}}$ | 11506.049, 11573.595, 24714.071 |
| $M_{\tilde{e}}$ | 11506.29, 14622.41, 11573.76, 14725.34, 24712.40, 33944.89 |
| $M_{\tilde{u}}$ | 9584.69, 12249.73, 9585.36, 12253.39, 34217.01, 34917.59 |
| $M_{\tilde{d}}$ | 12248.89, 12251.97, 12250.38, 12260.36, 34536.37, 34896.71 |
| $M_A$ | 46913.36 |
| $M_{H^\pm}$ | 46913.38 |
| $M_H$ | 46913.56 |
| $M_h$ | 127.00 |
4.4 Discussion

| Field  | Mass(GeV)                  |
|--------|----------------------------|
| $M_{\tilde{g}}$ | 1174.85                  |
| $M_{\tilde{\chi}^\pm}$ | 297.55, 81878.03          |
| $M_{\tilde{\chi}^0}$ | 141.11, 297.40, 81878.00, 81878.00 |
| $M_{\tilde{\nu}}$ | 11443.477, 11510.892, 24614.190 |
| $M_{\tilde{e}}$ | 11443.72, 14720.30, 11511.06, 14821.88, 24612.54, 33887.74 |
| $M_{\tilde{u}}$ | 9484.66, 12272.04, 9485.34, 12275.64, 34327.99, 34878.81 |
| $M_{\tilde{d}}$ | 12271.74, 12292.45, 12274.80, 12299.65, 34375.94, 34845.54 |
| $M_A$ | 44572.62                  |
| $M_{H^\pm}$ | 44570.38                  |
| $M_H$ | 44570.54                  |
| $M_h$ | 124.90                    |

Table 4.11: Loop corrected spectra of supersymmetric partners calculated ignoring two loop tadpoles and two loop corrections to CP-odd pseudo scalar and neutral Higgs. $\mu=75664.64$ and $B=1.7528 \times 10^8$.

all from $M_A$ is included on the grounds that large $M_A(>>500$ GeV) corresponds to high fine tuning (as noted on the SuSpect webpage [119]: comment on bug in Version 2.43). However the actual corrections [83] are proportional to $M_A^2$ and can be as large as 100%! Fortunately the subroutines from SPheno used by us include the complete corrections of [83].

We are still searching for fits with light smuons so as to keep open the possibility of muon g-2 anomaly resolution and as DM co-annihilation channel.
### 4.4 Discussion

Parameters and mass spectrum of superheavy and superlight fields are also given. Susy contribution to \( \alpha_1 \) and \( \Delta m/\mu \) are calculated. 

| Parameter | Value | Field \([SU(3), SU(2), Y]\) | Masses (Units of \(10^{16}\text{GeV}\)) |
|-----------|-------|----------------------------|-------------------------------------|
| \(\chi_x\) | 0.0986 | \(A[1, 1, 4]\) | 1.53 |
| \(\chi_z\) | 0.0206 | \(B[6, 2, 5/3]\) | 0.0762 |
| \(h_{11}/10^{-6}\) | 5.5774 | \(C[8, 2, 1]\) | 0.99, 2.47, 5.06 |
| \(h_{22}/10^{-4}\) | 4.6169 | \(D[3, 2, 7/3]\) | 0.11, 3.48, 5.93 |
| \(h_{33}\) | 0.0240 | \(E[3, 2, 1/3]\) | 0.10, 0.72, 1.90 |
| \(f_{11}/10^{-6}\) | \(-0.0179 + 0.2696i\) | \(F[1, 1, 2]\) | 1.898, 2.78, 5.25 |
| \(f_{12}/10^{-6}\) | \(-1.3919 - 4.0550i\) | \(F[1, 1, 2]\) | 0.23, 0.57 |
| \(f_{13}/10^{-5}\) | 0.2657 + 0.0487i | \(F[1, 1, 2]\) | 0.57, 3.26 |
| \(f_{22}/10^{-4}\) | 8.7306 + 6.8083i | \(G[1, 1, 0]\) | 0.016, 0.15, 0.54 |
| \(f_{23}/10^{-4}\) | \(-0.0063 + 2.3851i\) | \(G[1, 1, 0]\) | 0.542, 0.65, 0.69 |
| \(f_{33}/10^{-3}\) | \(-1.0545 + 0.7715i\) | \(h[1, 2, 1]\) | 0.299, 2.25, 3.46 |
| \(g_{12}/10^{-4}\) | 0.2210 + 0.1157 | \(I[3, 1, 10/3]\) | 4.82, 23.74 |
| \(g_{13}/10^{-5}\) | \(-7.3381 + 1.6935i\) | \(J[3, 1, 4/3]\) | 0.24 |
| \(g_{23}/10^{-4}\) | \(-0.3245 - 1.9495i\) | \(J[3, 1, 4/3]\) | 0.216, 0.55, 1.22 |
| \(\lambda/10^{-2}\) | \(-4.2087 + 0.6911\) | \(K[3, 1, 8/3]\) | 1.22, 3.63 |
| \(\eta\) | \(-0.2959 + 0.0877i\) | \(K[3, 1, 8/3]\) | 1.84, 3.54 |
| \(\rho\) | 0.5579 - 0.2820i | \(L[6, 1, 2/3]\) | 1.80, 2.41 |
| \(\kappa\) | 0.1661 + 0.1028i | \(M[6, 1, 8/3]\) | 2.00 |
| \(\zeta\) | 0.9140 + 0.7588i | \(N[6, 1, 4/3]\) | 1.89 |
| \(\bar{\zeta}\) | 0.2446 + 0.6802i | \(O[1, 3, 2]\) | 2.61 |
| \(m/10^{16}\text{GeV}\) | 0.0091 | \(P[3, 3, 2/3]\) | 0.65, 3.53 |
| \(m_{\Theta}/10^{16}\text{GeV}\) | \(-2.280e^{-i\text{Arg}(\lambda)}\) | \(Q[8, 3, 0]\) | 0.198 |
| \(\gamma\) | 0.2136 | \(R[8, 1, 0]\) | 0.08, 0.26 |
| \(\bar{\gamma}\) | \(-3.6511\) | \(S[1, 3, 0]\) | 0.3049 |
| \(x\) | 0.7881 + 0.5686i | \(t[3, 1, 2/3]\) | 0.20, 0.48, 0.90, 2.5 |
| \(\Delta_X\) | 0.68 | \(U[3, 3, 4/3]\) | 3.90, 4.31, 26.3 |
| \(\Delta_G^{	ext{Tot}}, \Delta_G^{	ext{GUT}}\) | \(-20.5212, -23.2360\) | \(U[3, 3, 4/3]\) | 0.257 |
| \({\Delta \alpha}_3^{	ext{Tot}}, \Delta \alpha_3^{	ext{GUT}}\)(\(M_Z\)) | \(-0.0126, -0.0024\) | \(V[1, 2, 3]\) | 0.199 |
| \(M^{\nu}/10^{12}\text{GeV}\) | 0.001057, 2.68, 55.62 | \(W[6, 3, 2/3]\) | 1.89 |
| \(M_{\nu}^{\mu}/10^{-10}\text{eV}\) | 3.2, 8108.4, 168413.3 | \(X[3, 2, 5/3]\) | 0.067, 2.112, 2.112 |
| \(M_{\nu}(\text{meV})\) | 1.551304, 7.26, 40.66 | \(Y[6, 2, 1/3]\) | 0.09 |
| \{Evals[\bar{f}]\}/10^{-6}\) | 0.02572, 65.14, 1352.43 | \(Z[8, 1, 2]\) | 0.26 |

| Soft parameters | \(|m_{1/2}| = 269.969\) | \(m_0 = 10781.649\) | \(A_0 = -1.1 \times 10^4\) |
| \(\mu\) | 7.49 \times 10^4 | \(-3.04 \times 10^8\) | \(\tan \beta = 52.00\) |
| \(M_R^2\) | \(-4.79 \times 10^9\) | \(-7.12 \times 10^9\) | \(R_{\beta\mu} = 8.3509\) |

Max(\(|L_{ABCD}|, |R_{ABCD}|\) \(6.08 \times 10^{-22}\text{GeV}^{-1}\)

| Susy contribution to | \(M_{\text{Susy}} = 2.92\text{TeV}\) |
| \(\Delta_{X,G,3}\) | \(\Delta_{\text{Susy}} = 0.236\) |
| \(\Delta_{G} = 2.715\) |
| \(\Delta_{\text{Susy}} = -0.010\) |

Table 4.12: Fit 2: Values of the NMSGUT-SUGRY-NUHM parameters at \(M_X\) derived from an accurate fit to all 18 fermion data and compatible with RG constraints. Unification parameters and mass spectrum of superheavy and superlight fields are also given.
Table 4.13: Fit 2 with $\chi_X = \sqrt{\sum_{i=1}^{17} \frac{(O_i - \bar{O}_i)^2}{\delta_i^2}} = 0.0986$. Target values, at $M_X$ of the fermion Yukawa couplings and mixing parameters, together with the estimated uncertainties, achieved values and pulls. The eigenvalues of the wavefunction renormalization for fermion and Higgs lines are given with Higgs fractions $\alpha_i, \bar{\alpha}_i$ which control the MSSM fermion Yukawa couplings.
4.4 Discussion

Table 4.14: Fit 2: Values of the SM fermion masses in GeV at $M_Z$ compared with the masses obtained from values of GUT derived Yukawa couplings run down from $M_X$ to $M_Z$ both before and after threshold corrections. Fit with $\chi_Z = \sqrt{\sum_{i=1}^{9} \frac{(m_{\text{MSSM}} - m_{\text{SM}})^2}{(m_{\text{MSSM}})^2}} = .0206$.

| Parameter | SM($M_Z$) | $m^{\text{GUT}}(M_Z)$ | $m^{\text{MSSM}} = (m + \Delta m)^{\text{GUT}}(M_Z)$ |
|-----------|-----------|-----------------|-----------------|
| $m_d/10^{-3}$ | 2.900000 | 1.09359 | 2.90288 |
| $m_s/10^{-3}$ | 55.000000 | 33.848800 | 55.128400 |
| $m_b$ | 2.900000 | 2.46581 | 2.94708 |
| $m_c/10^{-3}$ | 0.48657 | 0.48450 | 0.48488 |
| $m_\mu$ | 0.10272 | 0.10320 | 0.10327 |
| $m_\tau$ | 1.74624 | 1.72938 | 1.72822 |
| $m_u/10^{-3}$ | 1.27000 | 1.13124 | 1.27398 |
| $m_c$ | 0.61900 | 0.54992 | 0.61931 |
| $m_t$ | 172.50000 | 147.89942 | 172.73656 |

Table 4.15: Fit 2: Values (in GeV) of the soft Susy parameters at $M_Z$ (evolved from the soft SUGRY-NUHM parameters at $M_X$) which determine the Susy threshold corrections to the fermion Yukawas ($M_{\text{Suy}} = 2.92$ TeV).

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| $M_1$ | 117.91 | $m_{\tilde{\nu}_1}$ | 3840.82 |
| $M_2$ | 231.79 | $m_{\tilde{\nu}_2}$ | 3842.97 |
| $M_3$ | 800.05 | $m_{\tilde{\nu}_3}$ | 35037.40 |
| $m_{\tilde{t}_1}$ | 15569.94 | $A_{11}^{(l)}$ | -7954.28 |
| $m_{\tilde{t}_2}$ | 15650.58 | $A_{22}^{(l)}$ | -7944.94 |
| $m_{\tilde{t}_3}$ | 30854.82 | $A_{33}^{(l)}$ | -5342.45 |
| $m_{\tilde{L}_1}$ | 7850.18 | $A_{11}^{(u)}$ | -8933.50 |
| $m_{\tilde{L}_2}$ | 7929.53 | $A_{22}^{(u)}$ | -8933.32 |
| $m_{\tilde{L}_3}$ | 20331.96 | $A_{33}^{(u)}$ | -5164.69 |
| $m_{\tilde{d}_1}$ | 11787.72 | $A_{11}^{(d)}$ | -8852.66 |
| $m_{\tilde{d}_2}$ | 11792.39 | $A_{22}^{(d)}$ | -8852.22 |
| $m_{\tilde{d}_3}$ | 26506.85 | $A_{33}^{(d)}$ | -5605.50 |
| $m_{\tilde{Q}_1}$ | 11197.93 | $\tan \beta$ | 52.00 |
| $m_{\tilde{Q}_2}$ | 11200.73 | $\mu(M_Z)$ | 69679.92 |
| $m_{\tilde{Q}_3}$ | 31725.86 | $B(M_Z)$ | $1.4655 \times 10^8$ |
| $M_H^2$ | $-3.6171 \times 10^9$ | $M_{\tilde{H}}^2$ | $-5.1774 \times 10^9$ |
4.4 Discussion

| Field Mass(GeV) |
|----------------|
| $M_{\tilde{g}}$ 800.05 |
| $M_{\chi^\pm}$ 231.78, 69680.01 |
| $M_{\chi^0}$ 117.91, 231.78, 69679.97, 69679.98 |
| $M_{\tilde{\nu}}$ 7849.919, 7929.269, 20331.856 |
| $M_{\tilde{e}}$ 7850.32, 15570.00, 7929.62, 15650.67, 20330.35, 30855.95 |
| $M_{\tilde{u}}$ 87313.43 |
| $M_{\tilde{d}}$ 90.47 |

Table 4.16: Fit 2: Tree level spectra of supersymmetric partners calculated ignoring generation mixing effects.

| Field Mass(GeV) |
|----------------|
| $M_{\tilde{g}}$ 1046.78 |
| $M_{\chi^\pm}$ 264.60, 75524.08 |
| $M_{\chi^0}$ 125.28, 264.44, 75524.04, 75524.04 |
| $M_{\tilde{\nu}}$ 9970.212, 10040.719, 22929.231 |
| $M_{\tilde{e}}$ 9970.50, 12962.67, 10040.90, 13067.75, 22927.44, 31698.40 |
| $M_{\tilde{u}}$ 8060.81, 10699.37, 8061.49, 10702.88, 31650.65, 32370.72 |
| $M_{\tilde{d}}$ 10698.71, 10701.36, 10699.79, 10709.91, 32068.70, 32371.52 |
| $M_A$ 42900.74 |
| $M_{H^\pm}$ 42900.76 |
| $M_H$ 42900.90 |
| $M_h$ 127.00 |

Table 4.17: Fit 2: Loop corrected spectra of supersymmetric partners calculated ignoring generation mixing effects.
Table 4.18: Loop corrected spectra of supersymmetric partners calculated ignoring two loop tadpoles and two loop corrections to CP-odd pseudo scalar and neutral Higgs. $\mu=69242.22$ and $B=1.4198 \times 10^8$.

| Field                  | Mass(GeV) |
|------------------------|-----------|
| $M_g$                  | 1046.78   |
| $M_{\chi^\pm}$        | 264.73, 75039.61 |
| $M_{\chi^0}$          | 125.32, 264.56, 75039.57, 75039.58 |
| $M_{\tilde{g}}$       | 9903.666, 9974.010, 22818.584 |
| $M_{\tilde{t}}$       | 9903.95, 13064.36, 9974.20, 13167.78, 22816.81, 31624.16 |
| $M_{\tilde{b}}$       | 7951.10, 10722.79, 7951.80, 10726.22, 31729.56, 32294.13 |
| $M_{\tilde{d}}$       | 10722.63, 10744.40, 10725.47, 10751.32, 31874.24, 32275.46 |
| $M_A$                  | 40376.80 |
| $M_{H^\pm}$           | 40373.97 |
| $M_H$                  | 40374.08 |
| $M_h$                  | 124.85   |

Table 4.19: Table of d=5 operator mediated proton and neutron lifetimes $\tau_{p,n}$ (yrs), decay rates $\Gamma$(yr$^{-1}$) and branching ratios in the different channels.
Chapter 5

Lepton Flavour Violation

5.1 Introduction

The SM renormalizable Lagrangian respects four global U(1) symmetries (individual family lepton numbers $L_e$, $L_\mu$, $L_\tau$ and B). Right handed neutrinos are absent by choice in the SM. Without including non-renormalizable operators, neutrinos remain massless in the SM. However it is clear from the neutrino oscillation data that neutrinos are massive and their flavour states mix with each other. Therefore it is confirmed that lepton flavour is violated in nature. Most mechanisms for generating neutrino masses inevitably lead to lepton number violation and the simplest one is the so called ‘seesaw mechanism’. SO(10) GUTs can naturally accommodate seesaw and Susy together since they embed the minimal Susy LR models which have high scale breaking of B-L symmetry [37, 38, 39, 40, 41]. In Susy GUTs lepton flavour violation (LFV) in the neutrino sector generates LFV in the charged lepton sector.

Besides explaining smallness of neutrino masses, another attractive feature of the seesaw mechanism is that it provides a natural mechanism for generating the observed baryon asymmetry of the universe through leptogenesis. Once the MSSM is extended with three right handed neutrinos, the lepton number asymmetry can arise via a lepton number and CP violating out of equilibrium decay of the right handed neutrino. This can be followed by processing of the net lepton number into baryon asymmetry via sphaleron processes at scales near to but higher than $M_Z$ [60]. The E821 experiment [120] at Brookhaven National Laboratory observed
5.2 Standard Model Muon Decay

Muon decay ($\mu \to e\nu_e\bar{\nu}_\mu$) in the SM is mediated by $W^\pm$ gauge bosons. Due to suppression of heavy gauge boson propagation this decay is represented as point like 4-fermion interaction governed by the interaction Lagrangian

$$\mathcal{L}_{\text{int}} = -\frac{G_F}{\sqrt{2}}[\bar{e}\gamma^\mu(1-\gamma_5)\nu_e\bar{\nu}_\mu\gamma^\mu(1-\gamma_5)\mu]$$  \hspace{1cm} (5.1)

Here $G_F(=\frac{\alpha^2}{4\sqrt{2}M_W^2})$ is Fermi constant.

$$\Gamma(\mu \to e\nu_e\bar{\nu}_\mu) = \frac{G_F^2 m_\mu^5}{192\pi^3}$$  \hspace{1cm} (5.2)

The $\mu \to e\nu_e\bar{\nu}_\mu$ channel accounts for nearly 100% of the muon decay width. However many other channels for muon decay width are conceivable in the SM and beyond. For example the diagram 5.1 accounts for the lepton flavour violating decay $\mu \to e\gamma$ in the SM supplemented by Majorana mass term for neutrino mass (coded in the Weinberg operator $(M^c)_{ij}L_iHL_jH$). Decay rate and branching ratio (BR) are given by:

$$\Gamma(\mu \to e\gamma) = \frac{G_F^2 m_\mu^5}{192\pi^3} \frac{3\alpha}{32\pi} \left[ \sum_i U^*_{ei} U_{e\mu} m_\nu_i^2 \right]^2$$  \hspace{1cm} (5.3)

$$B(\mu \to e\gamma) = \frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\nu_e\bar{\nu}_\mu)} \approx \frac{\alpha}{2\pi} \left( \frac{\Delta m^2}{M_W^2} \right)^2 \approx 10^{-55}$$  \hspace{1cm} (5.4)

Here $U$ is the neutrino mixing matrix, $\Delta m^2$ is neutrino mass square difference parameter and $M_W$ is mass of W-boson. SM predicts unmeasurable BR for $\mu \to e\gamma$ even after the introduction of neutrino masses and mixing. MEG experiment [66](and several others [67, 68]) have provided an improved upper bound on BR for rare decays.
5.3 LFV in Supersymmetric GUTs

The SM extensions predict lepton flavour violation. In the unified scenario LFV has been studied by Barbieri et al. considering large top Yukawa couplings [61, 62].

| Process      | Present Upper Bound |
|--------------|---------------------|
| $\mu \to e\gamma$ | $5.7 \times 10^{-13}$ |
| $\tau \to \mu\gamma$ | $4.4 \times 10^{-8}$ |
| $\tau \to e\gamma$ | $3.3 \times 10^{-8}$ |
| $\mu \to eee$ | $1.0 \times 10^{-12}$ |
| $\tau \to eee$ | $2.7 \times 10^{-8}$ |
| $\tau \to \mu\mu\mu$ | $2.1 \times 10^{-8}$ |

Table 5.1: Experimental upper bound [66, 67, 68] for the BR of LFV processes.
The main consequence of the seesaw mechanism in supersymmetric theories is the violation of lepton number in the scalar sector leading to rare lepton flavour violating decays. Many groups have discussed this issue considering different frameworks [63, 121, 122, 123, 124, 125, 126]. However NMSGUT also predicts neutrino Yukawa coupling like the Yukawa couplings of other SM fermions. LFV ultimately originates from the neutrino Yukawa coupling, because neutrino Yukawa couplings in general are not diagonal in the basis in which charged lepton Yukawa coupling and right-handed neutrino mass matrix are diagonal. The off-diagonal neutrino Yukawa couplings will give rise to LFV in the soft slepton masses through the RGEs [63, 121].

Lepton flavour violation will be generated through the RGE even if the supersymmetry breaking mechanism at the high scale conserves flavour as it does in mSUGRY. Leptonic superpotential of the MSSM + three right handed neutrino is given by:

\[ W_{lep} = e_i^c(Y_e)_{ij}L_jH_d + \nu_i^c(Y_\nu)_{ij}L_jH_u - \frac{1}{2}\nu_i^cM_{\nu i}^{\nu_j}\nu_j^c \quad (5.5) \]

Here \( H_u \) and \( H_d \) are MSSM Higgs doublets whose VEVs generate mass of the up-type and down-type fermions, \( (Y_e, Y_\nu) \) are charged lepton and neutrino Yukawa couplings and \( M_\nu^R \) is right handed neutrino Majorana mass. In Susy-GUTs, right-handed neutrino Yukawa couplings generate lepton flavour violation in the soft Susy breaking lagrangian

\[ -L_{lep}^{soft} = (m_L^2)_{ij}\bar{L}_i^*\tilde{\bar{L}}_j + (m_{\bar{\nu}}^2)_{ij}\bar{\tilde{\nu}}_{R i}^*\bar{\tilde{\nu}}_{R j} + (A_e^e)_{ij}\bar{\tilde{\nu}}_{R i}H_d\tilde{\bar{L}}_j + (A_\nu^\nu)_{ij}\bar{\tilde{\nu}}_{R i}H_u\tilde{\bar{L}}_j + h.c. \quad (5.6) \]

where the \( \tilde{L} \), \( \tilde{\nu}_R \) and \( \tilde{\bar{\nu}}_R \) are the leptons Susy partners (sleptons). The parameters \( (m_L^2, m_{\nu}^2) \) and \( (A_e, A_\nu) \) are slepton’s soft mass matrices squared and trilinear couplings. Additional contribution by neutrino Yukawa couplings to the one loop RGE of the slepton masses \( (m_L^2) \) is given by [63]

\[ \frac{1}{16\pi^2}[(m_L^2Y_\nu^\dagger Y_\nu + Y_\nu^\dagger Y_\nu m_L^2)_{ij} + 2(Y_\nu^\dagger m_\tilde{\nu}^2 Y_\nu + M_H^2 Y_\nu^\dagger Y_\nu + A_\nu^\dagger A_\nu)_{ij}] \quad (5.7) \]
The leading contribution to the off-diagonal entries of the slepton mass squared matrix can be estimated as [63]

\[
\Delta(m_{\tilde{L}}^2)_{ij} \approx \frac{2m_0^2 + M_H^2 + A_0^2}{8\pi^2} \sum_k (Y_\nu^*)_{ik} (Y_\nu)_{jk} \log \left( \frac{M_X}{M_k} \right)
\]

(5.8)

The flavour violation in the slepton sector contributes through the one loop diagrams to charged flavour violating processes such as rare muon and tau decay ($\mu \to e\gamma, \tau \to \mu\gamma, \tau \to e, e, e$ etc.) and yields BR:

\[
BR(l_i \to l_j \gamma) \approx \frac{\alpha^3 ((m_{\tilde{L}}^2)_{ij})^2}{G_F^2 M_S^8}
\]

(5.9)

Where $\alpha$ is fine structure constant and $M_S$ is Susy particles mass scale. Feynman diagrams contributing to these processes are shown in Fig. 5.2. The upper bounds on BR of these processes may constrain very strictly the elements of soft mass matrices. It is important to estimate these LFV rates from NMSGUT on the basis of fits of the fermion data. To estimate LFV, one needs to consider the effect of right handed neutrino evolution upto their mass scale which we will discuss in the following section.
5.4 NMSGUT LFV Predictions

In NMSGUT using the formulae of Eq. (2.44) Yukawa couplings of SM fermion and neutrino are calculated. Then GUT scale threshold corrections (discussed in Chapter 3) are applied to the tree level Yukawas (see Eq. 3.32). At GUT scale right handed neutrino are present in the theory (along with the other fermions) which need to be integrated out below their mass scale. We shift to the right handed neutrino and charged lepton diagonal basis by redefining the fields:

\[ Y'_{\nu} = U^T_\nu \tilde{Y}_\nu ; \quad Y'_{e} = V^T_e \tilde{Y}_e U_e ; \quad M_{\nu}^D = U^T_\nu M_\nu U_\nu \]  

(5.10)

Here \( \tilde{Y}_f \) (\( f = \nu, e \)) is threshold corrected Yukawa coupling, redefined in a basis where kinetic terms have canonical form.

5.4.1 Right Handed Neutrino Thresholds

We incorporated the three thresholds associated with the heavy right handed neutrino masses in the theory. For this neutrino Dirac coupling is progressively introduced [127] on passing these thresholds when going higher in energy and right handed neutrino are decoupled one by one going the other way.

\[
\begin{align*}
&MSSM \quad EFT1 \\
&\kappa^{(1)} \quad M_{\nu_1} \quad \xrightarrow{M_{\nu_2}} \quad MSSM + \bar{\nu}_1 \quad EFT2 \\
&\kappa^{(2)} \quad M_{\nu_2} \quad \xrightarrow{M_{\nu_3}} \quad MSSM + \bar{\nu}_1, \bar{\nu}_2 \quad EFT3 \\
&\kappa^{(3)} \quad M_{\nu_3} \quad \xrightarrow{\text{Full Theory}} \quad MSSM + \bar{\nu}_1, \bar{\nu}_2, \bar{\nu}_3
\end{align*}
\]

To do this one needs to consider different effective theories (EFTs) in each of the energy ranges defined by right handed neutrino masses. We use the RGEs of the MSSM parameters extended with three right handed neutrino superfields [128] upto the energy scale of the lightest right handed neutrino by using the matching condition at the thresholds of right handed neutrino as discussed below. Along with these parameters we also consider the running [129] of dimension five (Weinberg [9]) operators which provide effective neutrino mass matrices below successive heavy mass thresholds. In the full theory there is no \( \kappa \) (coefficient of dimension five operator). Once third generation right handed neutrino is eliminated, a dimension five
Weinberg operator will appear in the theory. Leptonic superpotential in the above discussed basis is:

\[ W_{lep} = \bar{e}^T_A Y^e_{AB} e_B + \bar{\nu}^T_A Y^\nu_{AB} \nu_B - \frac{1}{2} \nu^T_A M^\nu_{AA} \nu_A \]  

(5.11)

here \( A, B = 1, 2, 3 \) are generation indices. Split \( A, B \) into \( a, 3 \) and \( b, 3 \) where \( a, b \) run from 1 to 2. At scale \( M^\nu_3 \) we diagonalize the mass matrix of right handed neutrinos. Writing only last 2 terms of the superpotential (Eq. 5.11) with indices \( a, b \) and 3 in right handed diagonal basis:

\[ W^{FT}_{lep} = \bar{\nu}^T_a Y^\nu_{ab} \nu_b + \bar{\nu}^T_3 Y^\nu_{33} \nu_3 + \bar{\nu}^T_3 Y^\nu_{3a} \nu_a + \bar{\nu}^T_3 Y^\nu_{33} \nu_3 \]

\[ -\frac{1}{2} \bar{\nu}^T_a M^\nu_{aa} \nu_a - \frac{1}{2} \bar{\nu}^T_3 M^\nu_{33} \nu_3 \]  

(5.12)

By solving the superpotential equation of motion for the heavy singlet \( \bar{\nu}_3 \) to leading order in \( (M^\nu_3)^{(−1)} \) we get:

\[ \bar{\nu}_3 = \frac{Y^\nu_{33} \nu_3 + Y^\nu_{3a} \nu_a}{M^\nu_{33}} = \frac{Y^\nu_{3A} \nu_A}{M^\nu_{33}} \]  

(5.13)

Substituting back in (5.12) the effective superpotential is given by:

\[ W_{eff} = \bar{\nu}^T_a Y^\nu_{ab} \nu_B - \frac{1}{2} \bar{\nu}^T_a M^\nu_{aa} \nu_a + \nu^T_A \left( 1 \frac{Y^\nu_{3A}^T Y^\nu_{3B}}{M^\nu_{33}} \right) \nu_B \]  

(5.14)

From third term we identify the coupling in EFT3 \( \kappa^{(3)} \):

\[ \kappa^{(3)}_{AB} = \frac{1}{2} (Y^\nu)^T_{A3} M^{-1}_{33} (Y^\nu)_{3B} \]  

(5.15)

This condition should be imposed at \( \mu = M^\nu_3 \) (largest eigenvalue of \( M^\nu \)). Now the Yukawa matrix \( Y^{(3)}_{aB} \) is \( 2 \times 3 \) and \( M^{(3)}_{ab} \) is \( 2 \times 2 \).

\[ W^{EFT3}_{eff} = \bar{\nu}^T_a Y^{(3)}_{aB} \nu_B - \frac{1}{2} \bar{\nu}^T_a M^{(3)}_{ab} \nu_b + \nu^T_A \kappa^{(3)}_{AB} \nu_B \]  

(5.16)

In the full theory light neutrino mass matrix is given by
\[ m_{\nu}^{\text{FT}} = \frac{v^2}{2} Y_\nu^{\top} M_{\bar{\nu}}^{-1} Y_\nu \]  
(5.17)

After integrating out \( \bar{\nu}_3 \)

\[ m_{\nu}^{\text{EFT3}} = v^2 \kappa^{(3)} + \frac{v^2}{2} (Y^{(3)}_\nu)^{\top} (M^{(3)}_{\nu})^{-1} Y^{(3)}_\nu \]  
(5.18)

Another way of calculating \( \kappa^{(3)} \) is to compare \( m_\nu \) in both the theories at \( \mu = M_3^\nu \) because light neutrino mass matrix in both the theory should match (\( m_\nu^{\text{FT}} = m_\nu^{\text{EFT3}} \)) at right handed neutrino threshold. By evolving the RGEs down to the scale \( M_2^\nu \), one has to repeat the same procedure. After integrating out \( \bar{\nu}_2 \) at \( \mu = M_2^\nu \), the Yukawa matrix and the effective neutrino mass operator are further modified. At Scale \( M_2^\nu \) superpotential is given by Eq. (5.16). Again using the superpotential equation of motion for \( \bar{\nu}_2 \) effective superpotential is given by

\[ W_{\text{EFT2}}^{\text{eff}} = \bar{\nu}_1^T Y^{(3)}_1 \nu_A - \frac{1}{2} \bar{\nu}_1^T M_{11}^{(3)} \bar{\nu}_1 + \nu_A \left( \frac{(Y^{(3)}_\nu)^{T} Y^{(3)}_{A2} Y^{(3)}_{2B}}{2 M_{22}^{(3)}} + \kappa^{(3)}_{AB} \right) \nu_B \]  
(5.19)

\[ \kappa^{(2)}_{AB} = \frac{(Y^{(3)}_\nu)^{T} Y^{(3)}_{A2} Y^{(3)}_{2B}}{2 M_{22}^{(3)}} + \kappa^{(3)}_{AB} \]  
(5.20)

\[ m_{\nu}^{\text{EFT2}} = v^2 \kappa^{(2)} + \frac{v^2}{2} (Y^{(2)}_\nu)^{\top} (M^{(2)}_{\nu})^{-1} Y^{(2)}_\nu \]  
(5.21)

Again \( \kappa^{(2)} \) can be obtained from matching of \( m_{\nu}^{\text{EFT3}} \) and \( m_{\nu}^{\text{EFT2}} \) at \( \mu = M_2 \). After a further RGE running to \( \mu = M_1^\nu \), the above steps are repeated for \( \bar{\nu}_1 \), so that \( \kappa^{(1)} \) is given by

\[ \kappa^{(1)}_{AB} = \frac{(Y^{(2)}_\nu)^{T} Y^{(2)}_{A1} Y^{(2)}_{1B}}{2 M_{11}^{(2)}} + \kappa^{(2)}_{AB} \]  
(5.22)

\[ W_{\text{eff}}^{\text{EFT1}} = \nu_A^T \kappa^{(1)}_{AB} \nu_B \]  
(5.23)

After this the effective theory is the MSSM \[81\] with Weinberg operators giving all three light neutrino a mass so we use the RGEs of MSSM and \( \kappa = \kappa^{(1)} \) to run the parameters from \( M_1^\nu \) to \( M_Z \). Left handed neutrino masses at electroweak scale are given by

\[ m_\nu = v^2 \kappa(M_Z) \]
5.4 NMSGUT LFV Predictions

| Parameter | Value (at $M_X$) Without RHN | Value (at $M_X$) With RHN |
|-----------|-----------------------------|---------------------------|
| $Y_u$     | $\{2.082 \times 10^{-6}, 1.014 \times 10^{-3}, 0.348\}$ | $\{2.076 \times 10^{-6}, 1.002 \times 10^{-3}, 0.346\}$ |
| $Y_d$     | $\{9.269 \times 10^{-5}, 3.325 \times 10^{-3}, 0.291\}$ | $\{8.303 \times 10^{-5}, 3.516 \times 10^{-3}, 0.286\}$ |
| $Y_l$     | $\{1.101 \times 10^{-4}, 2.328 \times 10^{-2}, 0.456\}$ | $\{1.225 \times 10^{-4}, 2.294 \times 10^{-2}, 0.452\}$ |
| $g_1, g_2, g_3$ | $\{0.728, 0.734, 0.728\}$ | $\{0.728, 0.734, 0.728\}$ |

Table 5.2: Effect of right handed neutrino (RHN) thresholds on gauge and Yukawa couplings for the fit presented in Tables 5.10-5.15.

**Soft Sector**

In the soft sector we have terms involving right handed sneutrino in addition to the MSSM soft

$$
- \mathcal{L}_{\text{soft}}^{\text{MSSM}} = - \mathcal{L}_{\text{soft}}^{\text{MSSM}} + \bar{\nu}_A^* (m_{\tilde{\nu}}^2)_{AB} \tilde{\nu}_B + (\bar{\nu}_A^T A'_\nu_{AB} \tilde{L}_B H + \frac{1}{2} \bar{\nu}_A^T (b_M)_{AB} \tilde{\nu}_B + h.c) \quad (5.24)
$$

We treat heavy right handed sneutrino in the same way as their fermion partner. At each threshold we move to diagonal basis of the right handed neutrino. We apply the same rotation to the right handed sneutrino, because sparticle also follow superpotential equation of motion. In order to integrate out right handed sneutrino, we simply remove the last row and column from soft mass matrix in right handed neutrino diagonal basis. Similarly our trilinear coupling $A_\nu$ is modified at each threshold. These thresholds have very small effect on gauge and Yukawa unification as shown in Table 5.2. But these thresholds are important for lepton flavour violation phenomenology which is our next task.

5.4.2 LFV Decay Rate

The decay rates for these processes are calculated using the amplitudes depicted by Fig. 5.2 and take the form

$$
\Gamma(l_i \to l_j \gamma) = \frac{e^2}{16\pi} m_{l_i}^5 (|A^L|^2 + |A^R|^2) \quad ; \quad A^{L,R} = A^{(n)L,R} + A^{(c)L,R} \quad (5.25)
$$
where $A^{(n)L,R}$ and $A^{(c)L,R}$ are the contributions from the neutralino and chargino loops respectively. In order to calculate these loop contributions one must write the interaction Lagrangian (fermion-sfermion-neutralino, fermion-sfermion-chargino) in the mass diagonal basis. To do this one needs to diagonalize the slepton mass matrices and to consider the mixing in the neutralino and chargino sectors. Fermion-sfermion-gaugino/higgsino interaction Lagrangian relevant to the $\mu \to e\gamma$, $\tau \to e\gamma$ and $\tau \to \mu\gamma$ processes is
\[ L^{int} = \bar{l}_i (N^R_{iAX} P_R + N^L_{iAX} P_L) \tilde{\chi}_0 \bar{l}_X \]
\[ + \bar{l}_i (C^R_{iAX} P_R + C^L_{iAX} P_L) \tilde{\chi}_X \bar{\nu}_X + h.c. \] (5.26)

One obtains \cite{63}
\[ A^{(n)L} = \frac{1}{32\pi^2} \frac{1}{m_{l_X}^2} \left[ N^L_{jAX} N^L_{iAX} \frac{(1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x)}{6(1 - x)^4} \right. \]
\[ + N^L_{jAX} N^{R*}_{iAX} \frac{M_{\tilde{\chi}^0} (1 - x^2 + 2x \ln x)}{m_{\nu}} \left. \right] \] (5.27)
\[ A^{(c)L} = \frac{1}{32\pi^2} \frac{1}{m_{l_X}^2} \left[ C^L_{jAX} C^L_{iAX} \frac{(2 + 3y - 6y^2 + 6y \ln y)}{6(1 - y)^4} \right. \]
\[ + C^L_{jAX} C^{R*}_{iAX} \frac{M_{\tilde{\chi}^-} (-3 + 4y - y^2 - 2 \ln y)}{m_{\mu}} \left. \right] \] (5.28)
\[ A^{(n)R} = A^{(n)L}|_{L \leftrightarrow R} \quad ; \quad A^{(c)R} = A^{(c)L}|_{L \leftrightarrow R} \] (5.29)

Here \( x = M_{\tilde{\chi}^0}^2 / m_{l_X}^2 \), \( y = M_{\tilde{\chi}^-}^2 / m_{\nu}^2 \) are ratios of neutralino mass squared to the charged slepton mass square and chargino mass squared to the sneutrino mass square respectively. The neutralino-slepton vertices used in the Eqns. 5.27-5.29 have form
\[ N^R_{iAX} = -\frac{g_2}{\sqrt{2}} \left\{ (-O^N_{A2} - O^N_{A1} \tan \theta_W) (U^*)_{i,i+3} + \frac{m_{\nu}}{M_W \cos \beta} O^N_{A3} (U^*)_{i,i+3} \right\} \] (5.30)
\[ N^L_{iAX} = -\frac{g_2}{\sqrt{2}} \left\{ \frac{m_{\nu}}{M_W \cos \beta} O^N_{A3} (U^*)_{i,i+3} + 2O^N_{A1} \tan \theta_W (U^*)_{i,i+3} \right\} \] (5.31)

Similarly chargino-sneutrino vertices are :-
\[ C^R_{iAX} = -g_2 O^C_{A1} (U^*)_{i,i} \] (5.32)
\[ C^L_{iAX} = -g_2 \frac{m_{\nu}}{\sqrt{2}M_W \cos \beta} O^C_{A2} (U^*)_{i,i} \] (5.33)

The matrices \( O^c, U^l, U^\nu \) and \( O^N \) are the unitary matrices which diagonalize chargino, slepton and neutralino mass matrices respectively. Notice before diagonalization slepton mass matrices are rotated to the diagonal fermion basis. The processes \( l_i \to 3l_j \) also involve same vertices. These have contributions from both
5.5 Numerical Fit

We used the solution sets presented in the previous chapter to calculate LFV. If we directly use these fits (which are found integrating out all the heavy right handed neutrinos at GUT scale) and run down hard and soft parameter using neutrino thresholds then soft Susy parameters at $M_Z$ are slightly changed as shown in Tables 5.3 and 5.6 but this give rise to different sparticle LR mixing. Low scale fermion masses for both the fits are given in Tables 5.5 and 5.8. Susy threshold corrections are sensitive to LR mixing, so down and strange quark masses (Susy threshold corrections modify them by factor of 3) do not match with MSSM data. Therefore we rerun multiple iteration search program using these solutions to get appropriate Susy threshold corrections to down type quarks. As shown RHN thresholds do not change Yukawa unification, so while running experimental data to get a target for GUT scale fitting we used the MSSM RGEs. We assume the normal hierarchy for the left handed neutrino. After some iteration, program found a reasonable fermion fitting at the low scale. The fitting criteria is same as discussed in previous chapters. We need off-diagonal running from GUT scale to EW scale to calculate BR for LFV processes. Since the loop corrections including generation mixing are not available, we use tree level spectrum to calculate BR for LFV processes. We calculate the loop corrected Susy spectrum using diagonal running corresponding to the same solutions to avoid possibility of tachyons after inclusion of one loop corrections. Loop corrected Susy spectrum for previous chapter fits taking into account the effect of heavy neutrino thresholds is presented in Tables 5.4 and 5.7. BR for the LFV processes for these fits is given in Table 5.9. Complete solution with acceptable down type quark masses is presented in Table 5.10, 5.15 and LFV BR is given in Table 5.16. For fitting purpose we have calculated the PMNS mixing angles at GUT scale which is permissible only if we integrate out all the heavy right handed neutrino at that scale. Since with the fixed hierarchy of left handed neutrino mixing angles do not change dramatically with RG evolution.
### Table 5.3: Values (in GeV) of the soft Susy parameters at $M_Z$ (evolved from the soft SUGRY-NUHM parameters at $M_X$) which determine the Susy threshold corrections to the fermion Yukawas for the Fit 1 of Chapter 4 including heavy neutrino thresholds.

| Field       | Mass (GeV)       |
|-------------|-----------------|
| $\tilde{g}$ | 1174.36         |
| $M_{\tilde{g}}^{\pm}$ | 298.03, 82096.67 |
| $M_{\tilde{g}}^\theta$ | 141.16, 297.89, 82096.64, 82096.64 |
| $M_{\tilde{g}}^\rho$ | 11481.787, 11580.152, 26082.412 |
| $M_{\tilde{\epsilon}}$ | 11482.03, 14652.15, 11580.32, 14755.14, 26080.54, 33699.23 |
| $M_{\tilde{m}}$ | 9559.21, 12256.54, 9559.87, 12260.19, 34000.41, 34793.04 |
| $M_{\tilde{d}}$ | 12256.10, 12267.83, 12258.83, 12275.49, 34484.99, 34785.22 |
| $M_A$ | 46231.20 |
| $M_{H^{\pm}}$ | 46231.22 |
| $M_H$ | 46231.39 |
| $M_h$ | 126.76 |

Table 5.4: Loop corrected spectra of supersymmetric partners calculated ignoring generation mixing effects for the Fit 1 of Chapter 4 including heavy neutrino thresholds.
### 5.5 Numerical Fit

Parameter $SM(M_Z)$  $m^{GUT}(M_Z)$  $m^{MSSM} = (m + \Delta m)^{GUT}(M_Z)$
---
$m_d/10^{-3}$  2.90000  0.96362  1.48676
$m_s/10^{-3}$  55.00000  34.56512  53.08815
$m_b$  2.90000  2.46805  2.96674
$m_c/10^{-3}$  0.48657  0.48450  0.48478
$m_\mu$  0.10272  0.10240  0.10245
$m_\tau$  1.74624  1.72406  1.72292
$m_u/10^{-3}$  1.27000  1.12199  1.26776
$m_c$  0.61900  0.54662  0.61764
$m_t$  172.50000  147.05640  172.08579

Table 5.5: Values of the SM fermion masses in GeV at $M_Z$ compared with the masses obtained from values of GUT derived Yukawa couplings run down from $M_X$ to $M_Z$ (for the Fit 1 of Chapter 4 including heavy neutrino thresholds) both before and after threshold corrections. Fit with $\chi^2 = \sqrt{\sum_{i=1}^{9} \frac{(m^{MSSM}_i - m^{SM}_i)^2}{(m^{MSSM}_i)^2}} = 0.9516$.

Parameter  Value  Parameter  Value
---
$M_1$  117.93  $m_{\tilde{b}_1}$  3864.14
$M_2$  232.11  $m_{\tilde{u}_2}$  3866.24
$M_3$  799.50  $m_{\tilde{b}_3}$  34774.44
$m_{\tilde{t}_1}$  15566.41  $A_{11}^{u(l)}$ -7964.82
$m_{\tilde{t}_2}$  15647.11  $A_{22}^{u(l)}$ -7954.65
$m_{\tilde{t}_3}$  30584.36  $A_{33}^{u(l)}$ -5286.11
$m_{\tilde{L}_1}$  7847.40  $A_{11}^{u(a)}$ -8865.38
$m_{\tilde{L}_2}$  7959.05  $A_{22}^{u(a)}$ -8865.38
$m_{\tilde{L}_3}$  21805.42  $A_{33}^{u(a)}$ -5140.46
$m_{\tilde{d}_1}$  11791.57  $A_{11}^{d(d)}$ -8862.73
$m_{\tilde{d}_2}$  11796.24  $A_{22}^{d(d)}$ -8862.29
$m_{\tilde{d}_3}$  26543.70  $A_{33}^{d(d)}$ -5625.41
$m_{\tilde{Q}_1}$  11198.87  $\tan \beta$  52.00
$m_{\tilde{Q}_2}$  11201.66  $\mu(M_Z)$  69411.39
$m_{\tilde{Q}_3}$  31595.82  $B(M_Z)$  1.4439 $\times 10^8$
$M_H^2$  $-3.6225 \times 10^9$  $M_H^2$  $-5.1348 \times 10^9$

Table 5.6: Values (in GeV) of the soft Susy parameters at $M_Z$ (evolved from the soft SUGRY-NUHM parameters at $M_X$) which determine the Susy threshold corrections to the fermion Yukawas. The matching of run down fermion Yukawas in the MSSM to the SM parameters determines soft SUGRY parameters at $M_X$ for the Fit 2 of Chapter 4 including heavy neutrino thresholds.
5.5 Numerical Fit

| Field | Mass(GeV) |
|-------|-----------|
| $\tilde{g}$ | 1046.16 |
| $M_{\chi}^\pm$ | 265.08, 75198.58 |
| $M_{\chi}^0$ | 125.32, 264.92, 75198.54, 75198.55 |
| $M_{\tilde{\nu}}$ | 9947.132, 10043.320, 24208.033 |
| $M_{\tilde{e}}$ | 9947.42, 12990.49, 10043.51, 13095.20, 24206.02, 31400.95 |
| $M_{\tilde{d}}$ | 8038.40, 10707.75, 8039.08, 10711.23, 31389.36, 32198.29 |
| $M_A$ | 10706.71, 10717.49, 10709.12, 10724.95, 31979.96, 32224.47 |
| $M_{H^\pm}$ | 42237.34 |
| $M_H$ | 42237.36 |
| $M_h$ | 42237.49 |

Table 5.7: Loop corrected spectra of supersymmetric partners calculated ignoring generation mixing effects for the Fit 2 of Chapter 4 including heavy neutrino thresholds.

| Parameter | SM(MZ) | $m_{\text{GUT}}$(MZ) | $m_{\text{MSSM}}$ = ($m + \Delta m$)$_{\text{GUT}}$(MZ) |
|-----------|--------|----------------------|--------------------------------------------------|
| $m_d/10^{-3}$ | 2.90000 | 1.09275 | 1.69684 |
| $m_s/10^{-3}$ | 55.00000 | 33.82299 | 52.55803 |
| $m_b$ | 2.90000 | 2.46505 | 2.92709 |
| $m_e/10^{-3}$ | 0.48657 | 0.48414 | 0.48450 |
| $m_{\mu}$ | 0.10272 | 0.10312 | 0.10319 |
| $m_{\tau}$ | 1.74624 | 1.72120 | 1.72004 |
| $m_u/10^{-3}$ | 1.27000 | 1.12569 | 1.26768 |
| $m_c$ | 0.61900 | 0.54722 | 0.61625 |
| $m_t$ | 172.50000 | 147.39488 | 172.15940 |

Table 5.8: Values of the SM fermion masses in GeV at $M_Z$ compared with the masses obtained from values of GUT derived Yukawa couplings run down from $M_X$ to $M_Z$ (for the Fit 2 of Chapter 4 including heavy neutrino thresholds) both before and after threshold corrections. Fit with $\chi_Z = \sqrt{\sum_{i=1}^{9} \frac{(m_{\text{MSSM}} - m_{\text{SM}})^2}{(m_{\text{MSSM}})^2}} = 0.7108$. 

5.6 Anomalous Magnetic Moment of Muon

Dirac magnetic moment of the muon corresponding to the tree level Feynman diagram is equal to 2. Difference between the classical results and observed value is called anomalous magnetic moment denoted by $a_\mu$. So, anomalous magnetic moment of muon in the MSSM is a contribution of loops involving sparticles to the magnetic moment of the muon. The magnetic moment $\vec{\mu}$ of a muon is related to gyromagnetic ratio as

$$\vec{\mu} = g \left( \frac{e}{2m_\mu} \right) \vec{S} = 2(1 + a_\mu) \left( \frac{e}{2m_\mu} \right) \vec{S} ; \quad a_\mu = \frac{1}{2}(g_\mu - 2) \quad (5.34)$$

Its SM prediction consists of three type of contribution from QED, hadronic loops and weak interactions

$$a_\mu(\text{SM}) = a_\mu(\text{QED}) + a_\mu(\text{Had}) + a_\mu(\text{Weak}) \quad (5.35)$$

The precisely measured magnetic moment of muon has a significant deviation from the theoretical prediction:

$$\Delta a_\mu = a_\mu(\text{exp}) - a_\mu(\text{SM}) = 287(63)(49) \times 10^{-11} \quad (5.36)$$

The $\Delta a_\mu$ may thus represent the contribution of new physics beyond SM. Supersymmetry is one of the leading candidate for new physics. The deviation $a_\mu$ may be due to heavy sparticle contributions [69].

| Process     | Fit - 1               | Fit - 2               |
|-------------|-----------------------|-----------------------|
| $\mu \to e\gamma$ | $1.1906 \times 10^{-18}$ | $1.3544 \times 10^{-18}$ |
| $\tau \to \mu\gamma$ | $1.2475 \times 10^{-17}$ | $9.1312 \times 10^{-18}$ |
| $\tau \to e\gamma$ | $1.8641 \times 10^{-19}$ | $2.1977 \times 10^{-19}$ |
| $\mu \to eee$ | $1.8303 \times 10^{-18}$ | $3.2487 \times 10^{-18}$ |
| $\tau \to \mu\mu\mu$ | $3.5397 \times 10^{-19}$ | $2.6097 \times 10^{-19}$ |
| $\tau \to eee$ | $6.8617 \times 10^{-21}$ | $8.1293 \times 10^{-21}$ |

Table 5.9: BR of LFV processes.
### 5.6 Anomalous Magnetic Moment of Muon

| Parameter | Value | Field | Masses (Units of $10^{16}$GeV) |
|-----------|-------|-------|-------------------------------|
| $\chi_x$  | 0.3907| $A[1, 1, 4]$ | 1.48 |
| $\chi_z$  | 0.1495| $B[6, 2, 5/3]$ | 0.1074 |
| $h_{11}/10^{-6}$ | 6.7245 | $C[8, 2, 1]$ | 0.92, 2.49, 5.58 |
| $h_{22}/10^{-4}$ | 5.2504 | $D[3, 2, 7/3]$ | 0.06, 3.64, 6.61 |
| $h_{33}$  | 0.0254| $E[3, 2, 1/3]$ | 0.14, 0.75, 2.01 |
| $f_{11}/10^{-6}$ | -0.0922 + 0.2132i | $F[1, 1, 2]$ | 2.007, 3.00, 5.84 |
| $f_{12}/10^{-6}$ | -2.2696 - 3.3813i | $K[1, 1, 0]$ | 0.22, 0.64 |
| $f_{13}/10^{-5}$ | -0.0869 - 0.2560i | $L[6, 1, 2/3]$ | 0.64, 3.91 |
| $f_{22}/10^{-5}$ | 11.4395 + 7.9139i | $M[1, 1]$ | 0.022, 0.20, 0.69 |
| $f_{23}/10^{-4}$ | -0.7711 + 3.7117i | $N[6, 1, 4/3]$ | 0.693, 0.74, 0.79 |
| $f_{33}/10^{-3}$ | -1.4865 + 0.7342i | $O[1, 3, 2]$ | 0.346, 2.59, 3.67 |
| $g_{12}/10^{-4}$ | 0.2283 + 0.1776i | $P[3, 3, 2/3]$ | 5.47, 25.08 |
| $g_{13}/10^{-5}$ | -12.714 - 5.964i | $Q[3, 1, 10/3]$ | 0.34 |
| $g_{23}/10^{-4}$ | -0.6201 - 1.0201i | $R[3, 1, 4/3]$ | 0.294, 0.52, 1.30 |
| $\lambda/10^{-2}$ | -5.6613 + 0.3565i | $S[1, 3, 0]$ | 1.30, 4.20 |
| $\eta$  | -0.2629 + 0.0792i | $T[3, 1, 8/3]$ | 1.90, 4.19 |
| $\rho$  | 0.7241 - 0.3943i | $U[6, 1, 2/3]$ | 1.75, 2.82 |
| $k$  | 0.5952 - 0.0681i | $V[6, 1, 8/3]$ | 1.86 |
| $\zeta$  | 0.9355 + 0.6921i | $W[6, 1, 1/3]$ | 1.76 |
| $\bar{\zeta}$  | 0.3158 + 0.7630i | $X[6, 1, 4/3]$ | 1.76 |
| $m/10^{16}$GeV | 0.0127 | $Y[8, 3, 0]$ | 1.90, 4.20 |
| $m_{\Theta}/10^{16}$GeV | -2.785e^{-iArg(\lambda)} | $Z[8, 3, 0]$ | 0.268 |
| $\gamma$  | -0.1239 | $\omega[8, 1, 0]$ | 0.12, 0.37 |
| $\tilde{\gamma}$  | -3.5165 | $\omega[1, 3, 0]$ | 0.4309 |
| $x$  | 0.7949 + 0.6027i | $t[3, 1, 2/3]$ | 0.25, 0.52, 0.87, 2.67 |
| $\Delta_X$  | 0.54 | & & 4.28, 4.56, 27.50 |
| $\Delta_G^{Tot}, \Delta_G^{GUT}$  | -19.131, -22.039 | $U[3, 3, 4/3]$ | 0.861 |
| $\{\Delta\alpha_3^{Tot}, \Delta\alpha_3^{GUT}\}(M_Z)$  | -0.0126, -0.0022 | $V[1, 2, 3]$ | 0.282 |
| $M^{\nu}/10^{12}$GeV  | 0.001112, 2.97, 91.23 | $W[6, 3, 2/3]$ | 1.64 |
| $M^{\nu}_{Hu}/10^{-10}$eV  | 1.8, 4679.6, 143542.9 | $X[3, 2, 5/3]$ | 0.10, 2.22, 2.22 |
| $M_{\nu}$ (meV)  | 1.153351, 7.08, 40.00 | $Y[6, 2, 1/3]$ | 0.12 |
| $\{\text{Eval}_3[f]\}/10^{-6}$  | 0.0213, 56.99, 1747.14 | $Z[8, 1, 2]$ | 0.37 |

| Soft parameters at $M_X$ | $m_{3/2} = 317.022$ | $m_0 = 17260.604$ | $A_0 = -1.39 \times 10^4$ |
| at $M_X$ | $\mu = 1.05 \times 10^5$ | $B = -4.86 \times 10^8$ | $\tan\beta = 51.0000$ |
| $M_{H_2} = -9.8 \times 10^9$ | $M_{H_1}^2 = -1.4 \times 10^{10}$ | $R_{H_u} = 8.5386$ |

Max($||L_{ABCD}||, ||R_{ABCD}||$) $= 7.65 \times 10^{-22}$GeV$^{-1}$

|Susy contribution to $M_{SUSY}$ = 3.25 TeV | $\Delta_X^{SUSY} = -0.241$ | $\Delta_G^{SUSY} = 2.908$ | $\Delta\alpha_3^{SUSY} = -0.010$ |

Table 5.10: Values of the NMSGUT-SUGRY-NUHM parameters at $M_X$ derived from an accurate fit to all 18 fermion data and compatible with RG constraints. Unification parameters and mass spectrum of superheavy and superlight fields are also given.
5.6 Anomalous Magnetic Moment of Muon

| Parameter     | Target = $\hat{O}_i$ | Uncert. = $\delta_i$ | Achieved = $O_i$ | Pull = $(O_i - \hat{O}_i)/\delta_i$ |
|---------------|-----------------------|-----------------------|------------------|-------------------------------------|
| $y_u/10^{-6}$ | 2.070378              | 0.790884              | 2.075721         | 0.006756                             |
| $y_c/10^{-3}$ | 1.009175              | 0.166514              | 1.001722         | -0.044756                            |
| $y_t$         | 0.345391              | 0.013816              | 0.345835         | 0.032130                             |
| $y_d/10^{-5}$ | 8.392364              | 4.892748              | 8.303114         | -0.018241                            |
| $y_s/10^{-3}$ | 3.492497              | 1.648459              | 3.515603         | 0.014017                             |
| $y_b$         | 0.281969              | 0.146342              | 0.286114         | 0.028327                             |
| $y_e/10^{-4}$ | 1.100058              | 0.165009              | 1.125226         | 0.152528                             |
| $y_\mu/10^{-2}$ | 2.323407         | 0.348511              | 2.293512         | -0.085780                            |
| $y_\tau$     | 0.461515              | 0.087688              | 0.451930         | -0.109316                            |
| $\sin \theta_{12}$ | 0.2210           | 0.001600              | 0.2210           | -0.0075                              |
| $\sin \theta_{13}/10^{-4}$ | 30.5099          | 5.000000              | 30.4663          | -0.0087                              |
| $\sin \theta_{23}/10^{-3}$ | 35.8968          | 1.300000              | 35.8938          | -0.0023                              |
| $\delta^9$ | 60.0238              | 14.000000             | 60.5468          | 0.0374                               |
| $(m_{12}^2)/10^{-5}(eV)^2$ | 4.8930         | 0.518662              | 4.8798           | -0.0255                              |
| $(m_{23}^2)/10^{-3}(eV)^2$ | 1.5558         | 0.311161              | 1.5497           | -0.0195                              |
| $\sin^2 \theta_{12}^L$ | 0.2945         | 0.058890              | 0.2886           | -0.0996                              |
| $\sin^2 \theta_{23}^L$ | 0.4656         | 0.139687              | 0.4314           | -0.2451                              |
| $\sin^2 \theta_{13}^L$ | 0.0255         | 0.019000              | 0.0290           | 0.1819                                |
| $Z_u$         | 0.961662             | 0.962167              | 0.962171         |                                    |
| $Z_d$         | 0.956080             | 0.956593              | 0.956596         |                                    |
| $Z_{\bar{d}}$ | 0.933182             | 0.933695              | 0.933702         |                                    |
| $Z_{\rho}$    | 0.949936             | 0.950417              | 0.950426         |                                    |
| $Z_{\bar{\rho}}$ | 0.972184         | 0.972726              | 0.972730         |                                    |
| $Z_{\bar{\nu}}$ | 0.954853         | 0.955402              | 0.955406         |                                    |
| $Z_R, Z_H$    | 0.001171             | 0.001631              |                  |                                    |
| $\alpha_1$   | 0.166 - 0.00i         | $\bar{\alpha}_1$    | 0.131 - 0.00i    |                                    |
| $\alpha_2$   | -0.492 - 0.462i      | $\bar{\alpha}_2$    | -0.551 - 0.156i  |                                    |
| $\alpha_3$   | -0.122 - 0.592i      | $\bar{\alpha}_3$    | -0.603 - 0.222i  |                                    |
| $\alpha_4$   | 0.145 - 0.26i        | $\bar{\alpha}_4$    | 0.466 - 0.088i   |                                    |
| $\alpha_5$   | -0.014 + 0.065i      | $\bar{\alpha}_5$    | -0.036 - 0.058i  |                                    |
| $\alpha_6$   | -0.039 - 0.239i      | $\bar{\alpha}_6$    | 0.005 - 0.111i   |                                    |

Table 5.11: Fit with $\chi_X = \sqrt{\frac{\sum_{i=1}^{17} (O_i - \hat{O}_i)^2}{\delta_i^2}} = 0.3907$. Target values, at $M_X$ of the fermion Yukawa couplings and mixing parameters, together with the estimated uncertainties, achieved values and pulls. The eigenvalues of the wavefunction renormalization for fermion and Higgs lines are given with Higgs fractions $\alpha_i, \bar{\alpha}_i$ which control the MSSM fermion Yukawa couplings.
5.6 Anomalous Magnetic Moment of Muon

Parameter | SM($M_Z$) | $m^{GUT}(M_Z)$ | $m^{MSSM} = (m + \Delta m)^{GUT}(M_Z)$
--- | --- | --- | ---
$m_d/10^{-3}$ | 2.90000 | 0.86783 | 3.39332
$m_s/10^{-3}$ | 55.00000 | 36.74367 | 55.05208
$m_b$ | 2.90000 | 2.45010 | 2.92919
$m_c/10^{-3}$ | 0.48657 | 0.49774 | 0.49787
$m_\mu$ | 0.10272 | 0.10140 | 0.10143
$m_\tau$ | 1.74624 | 1.71479 | 1.71386
$m_u/10^{-3}$ | 1.27000 | 1.11735 | 1.27186
$m_c$ | 0.61900 | 0.53922 | 0.61378
$m_t$ | 172.50000 | 146.60676 | 172.52755

Table 5.12: Values of the SM fermion masses in GeV at $M_Z$ compared with the masses obtained from values of GUT derived Yukawa couplings run down from $M_X$ to $M_Z$ both before and after threshold corrections. Fit with $\chi_Z = \sqrt{\sum_{i=1}^{9} \frac{(m_i^{MSSM} - m_i^{SM})^2}{(m_i^{MSSM})^2}} = 0.1495$.

Parameter | Value | Parameter | Value
--- | --- | --- | ---
$M_1$ | 139.08 | $m_{\tilde{u}_1}$ | 9611.03
$M_2$ | 273.94 | $m_{\tilde{u}_2}$ | 9612.67
$M_3$ | 941.89 | $m_{\tilde{u}_3}$ | 49603.05
$m_{\tilde{t}_1}$ | 23415.65 | $A_{11}^{(t)}$ | $-10267.66$
$m_{\tilde{t}_2}$ | 23515.87 | $A_{22}^{(t)}$ | $-10255.53$
$m_{\tilde{t}_3}$ | 43323.72 | $A_{33}^{(t)}$ | $-7063.33$
$m_{L_1}$ | 13753.64 | $A_{11}^{(u)}$ | $-11188.30$
$m_{L_2}$ | 13882.30 | $A_{22}^{(u)}$ | $-11188.05$
$m_{L_3}$ | 31213.03 | $A_{33}^{(u)}$ | $-6589.55$
$m_{\tilde{d}_1}$ | 18412.68 | $A_{11}^{(d)}$ | $-11323.83$
$m_{\tilde{d}_2}$ | 18419.55 | $A_{22}^{(d)}$ | $-11323.20$
$m_{\tilde{d}_3}$ | 37754.82 | $A_{33}^{(d)}$ | $-7403.99$
$m_{\tilde{Q}_1}$ | 17659.29 | $\tan\beta$ | 51.00
$m_{\tilde{Q}_2}$ | 17663.29 | $\mu(M_Z)$ | 99537.13
$m_{\tilde{Q}_3}$ | 45002.01 | $B(M_Z)$ | $2.7908 \times 10^8$
$M_H^2$ | $-7.5398 \times 10^9$ | $M_H^2$ | $-1.0487 \times 10^{10}$

Table 5.13: Values (in GeV) of the soft Susy parameters at $M_Z$ (evolved from the soft SUGRY-NUHM parameters at $M_X$) which determine the Susy threshold corrections to the fermion Yukawas. The matching of run down fermion Yukawas in the MSSM to the SM parameters determines soft SUGRY parameters at $M_X$ ($M_{SUSY} = 3.25$TeV).
5.6 Anomalous Magnetic Moment of Muon

| Field   | Mass (GeV) |
|---------|------------|
| $M_{\tilde{g}}$ | 941.89 |
| $M_{\tilde{\chi}^\pm}$ | 273.93, 99537.20 |
| $M_{\tilde{\chi}^0}$ | 139.08, 273.93, 99537.17, 99537.18 |
| $M_{\tilde{\nu}}$ | 13753.492, 13882.149, 31212.961 |
| $M_{\tilde{\chi}}$ | 13753.72, 23415.69, 13882.35, 23515.93, 31211.82, 43324.63 |
| $M_{\tilde{\eta}}$ | 9610.96, 17659.21, 9612.61, 17663.21, 45002.16, 49603.27 |
| $M_{\tilde{d}}$ | 17659.39, 18412.70, 17663.35, 18419.60, 37751.68, 45004.69 |
| $M_A$ | 119324.64 |
| $M_{H^\pm}$ | 119324.67 |
| $M_H$ | 119324.64 |
| $M_h$ | 90.31 |

Table 5.14: Tree level spectra of supersymmetric partners calculated ignoring generation mixing effects.

| Field   | Mass (GeV) |
|---------|------------|
| $M_{\tilde{g}}$ | 1259.36 |
| $M_{\tilde{\chi}^\pm}$ | 314.27, 107937.67 |
| $M_{\tilde{\chi}^0}$ | 148.50, 314.14, 107937.64, 107937.64 |
| $M_{\tilde{\nu}}$ | 16243.435, 16359.851, 34250.256 |
| $M_{\tilde{\chi}}$ | 16243.61, 20059.00, 16359.96, 20183.25, 34248.81, 44010.02 |
| $M_{\tilde{\eta}}$ | 13963.35, 17100.49, 13964.06, 17105.21, 44115.72, 44720.70 |
| $M_{\tilde{d}}$ | 17100.52, 17105.69, 17103.78, 17114.48, 44482.14, 44736.79 |
| $M_A$ | 59582.86 |
| $M_{H^\pm}$ | 59582.88 |
| $M_H$ | 59583.05 |
| $M_h$ | 127.17 |

Table 5.15: Loop corrected spectra of supersymmetric partners calculated ignoring generation mixing effects.

| Process | Branching Ratio from NMSGUT |
|---------|----------------------------|
| $\mu \to e \gamma$ | $1.2740 \times 10^{-19}$ |
| $\tau \to \mu \gamma$ | $1.9050 \times 10^{-18}$ |
| $\tau \to e \gamma$ | $4.7212 \times 10^{-20}$ |
| $\mu \to e e e$ | $6.8500 \times 10^{-19}$ |
| $\tau \to \mu \mu \mu$ | $7.0904 \times 10^{-20}$ |
| $\tau \to e e e$ | $2.1471 \times 10^{-21}$ |

Table 5.16: BR of LFV processes.
5.6.1 Analytic Formulae

The lowest order supersymmetric contribution \( a_{\mu}^{\text{SUSY}} \) to \( a_\mu \) from sneutrino-chargino and smuon-neutralino loops [63, 69] is shown in Figure 5.5

\[
a_{\mu}^{\text{SUSY,1L}} = a_{\mu}^{\tilde{\chi}^\pm} + a_{\mu}^{\tilde{\chi}^0}
\]

where \( a_{\mu}^{\tilde{\chi}^\pm} \) and \( a_{\mu}^{\tilde{\chi}^0} \) denote chargino and neutralino contribution, given as

\[
a_{\mu}^{\tilde{\chi}^\pm} = \frac{m_\mu^2}{48\pi^2 m_{\tilde{\chi}^\pm}^2} (|C_{2AX}^L|^2 + |C_{2AX}^R|^2) F_1^c(x_{AX}) + \frac{m_\mu m_{\tilde{\chi}^\pm}}{8\pi^2 m_{\tilde{\nu}}^2} \text{Re}[C_{2AX}^L C_{2AX}^R] F_2^c(x_{AX})
\]

\[
a_{\mu}^{\tilde{\chi}^0} = -\frac{m_\mu^2}{48\pi^2 m_{\tilde{\chi}^0}^2} (|N_{2AX}^L|^2 + |N_{2AX}^R|^2) F_1^n(x_{AX}) - \frac{m_\mu m_{\tilde{\chi}^0}}{8\pi^2 m_{\tilde{\nu}}^2} \text{Re}[N_{2AX}^L N_{2AX}^R] F_2^n(x_{AX})
\]

Here \( A=1...4 \) (1, 2) and \( X=1, 2 \) denote the neutralino(chargino) and smuon indices respectively. Variables of loop functions are defined as ratios of mass squares \( x_{AX} = m_{\tilde{\chi}_A}^2/m_{\tilde{\mu}_m}^2 \) (\( m_{\tilde{\chi}_A}^2/m_{\tilde{\nu}_X}^2 \)) for chargino (neutralino) contribution and loop functions have form :

\[
F_1^c(x) = \frac{2}{(1 - x)^4} (2 + 3x - 6x^2 + x^3 + 6x \log x)
\]

\[
F_2^c(x) = \frac{3}{(1 - x)^3} (-3 + 4x - x^2 + -2 \log x)
\]
Table 5.17: Susy contribution to muon g-2.

| Fit                  | $\Delta a_\mu/10^{-10}$ |
|----------------------|-------------------------|
| Fit1(4th Chapter)    | 0.0084                  |
| Fit2(4th Chapter)    | 0.0111                  |
| Fit1(5th Chapter)    | 0.0033                  |
| Fit1(3rd Chapter)    | 10.6                    |

It is clear from Eq. (5.38) that $a_\mu^{\text{susy}}$ is sensitive to smuon mass and $\tan \beta$ (terms linear in $m_{\tilde{\chi}^{0,\pm}}$ in Eqns. (5.37) and (5.38) are proportional to it). Susy contribution to g-2 is completely independent of color sparticles at leading order. It requires sleptons as light as $O(10^2)$ GeV and $(g-2)_\mu$ motivated LHC Susy searches have been discussed in [131, 132]. To understand the behavior of $a_\mu^{\text{susy}}$ one needs to investigate approximate relations for the different diagram contributions. Mass insertion method is used to calculate different diagrams [133, 132, 69, 131, 134]. MSSM prediction of $\Delta a_\mu$ depend upon left and right handed smuon masses ($m_{\tilde{\mu}_L}$, $m_{\tilde{\mu}_R}$), gaugino mass parameters ($M_1$, $M_2$), higgsino mass ($\mu$) and ratio of VEVs (tan $\beta$). Fits presented have heavy smuon so we use exact formulae of Eq. (5.37) and (5.38) for Susy contribution to $a_\mu$. There are tan $\beta$ enhanced two-loop diagrams also [69] which can modify the leading order results by 10% but we calculated only one loop diagrams. NMSGUT fits prediction for $\Delta a_\mu$ is given in Table 5.17. We see that except for the light smuon solution (presented in Chapter 3), the $\Delta a_\mu$ values are too small to resolve the muon g-2 anomaly.

5.7 Leptogenesis

The baryon asymmetry of the universe is defined as:

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = 6.1 \pm 0.3 \times 10^{-10}$$ (5.43)
This can be generated via:- B violation, C and CP violation and departure from thermal equilibrium as pointed out by Sakharov [135]. Leptogenesis [60, 136] is most promising mechanism to explain observed baryon asymmetry of the universe as CP violating decay of right handed neutrino (shown in Fig. 5.6) can fulfill these conditions. Leptonic CP asymmetry can be converted to baron asymmetry through sphaleron processes. Non-thermal leptogenesis involves right handed neutrinos generation through inflaton decay whose further decay can generate lepton and (after sphaleron processes) baryon asymmetry. CP asymmetry parameter relevant for leptogenesis is given by

\[
\epsilon_{\text{CP}} = \frac{\Gamma(N \rightarrow l_L + H) - \Gamma(N \rightarrow \bar{l}_L + \bar{H})}{\Gamma(N \rightarrow l_L + H) + \Gamma(N \rightarrow \bar{l}_L + \bar{H})} 
\]

(5.44)

In case of hierarchical spectrum:

\[
\epsilon_{\text{CP}} \simeq -\frac{3}{8\pi} \frac{\text{Im}\{Y_{\nu}Y_{\nu}^\dagger\}^2}{[Y_{\nu}Y_{\nu}^\dagger]_{12} M_{1L}^0} \frac{M_{1L}^0}{M_{2L}^0} 
\]

(5.45)
5.8 Conclusion

| Fit                          | $\epsilon_{CP}/10^{-8}$ |
|------------------------------|-------------------------|
| Fit1 (4th Chapter)           | 0.18                    |
| Fit2 (4th Chapter)           | 0.10                    |
| Fit1 (5th Chapter)           | 0.24                    |

Table 5.18: Lepton sector parameter.

Model independent upper bound on CP asymmetry [137]:

$$|\epsilon_{CP}| \leq \frac{3}{8\pi} \frac{M_1^0 (m_3 - m_1)}{\langle H_u^0 \rangle}$$

(5.46)

Here $m_{1,3}$ are left handed neutrino masses. The $\epsilon_{CP}$ parameter for the three discussed fits is given in Table 5.18. The desirable range of values for successful leptogenesis [137] is $\epsilon_{CP} \sim 10^{-7}$. Although the generic values we obtain are somewhat small, it should be noted that we have not optimized our fits to improve $\epsilon_{CP}$ yet we are not too far off. Since it depends sensitively on the off diagonal structure of $Y_\nu^\dagger Y_\nu$ and linearly on $M_1^0/M_2^0$, optimization could easily yields more satisfactory values. We will return to these questions in future research.

5.8 Conclusion

Neutrino Dirac Yukawa coupling is crucial for LFV predictions which require inclusion of heavy right handed neutrino thresholds. We found a reasonable fit implementing these thresholds and calculated BR for $l_i \rightarrow l_j \gamma$ and $l_i \rightarrow 3l_j$ LFV processes. NMSGUT predict BR for the these processes several order of magnitude smaller than the upper bound from experiments because the fits have negative soft Higgs masses ($M^2_H$) and heavy Susy spectrum. Negative soft Higgs masses provide cancellation in the off-diagonal entries of slepton mass matrices [138] which estimate the LFV. Solution presented has all the NMSGUT superpotential and soft parameters in the range corresponding to the previous chapter solutions. The smuon is heavy in all the solutions with loop corrected sfermion masses that we have so far found. Hence Susy contribution to muon $g-2$ is very small $\sim 10^{-12}$. $\Delta a_\mu$ is not sensitive to neutrino Yukawa coupling but depend upon the soft Susy spectrum which further
depend upon mSUGRY parameters at GUT scale. The value of the leptogenesis CP violation parameter is roughly in the desired ball park even without optimization.

This study shows that NMSGUT fits found are compatible with LFV constraints but further searches are needed to explore whether light smuon and adequate $\epsilon_{\text{CP}}$ are achievable.
Chapter 6

Renormalization Group Evolution
Equations of the NMSGUT

6.1 Introduction

Renormalization group equations (RGEs) are used to evolve the gauge couplings, superpotential parameters and soft terms from UV scales into physically meaningful quantities that describe physics near the electroweak scale. The scale of generation of soft Susy breaking parameters in mSUGRY and string motivated mechanisms is above the GUT scale, typically $M_P$. Thus a complete RG study of a Susy GUT requires evolution of GUT parameters from $M_P$ to GUT scale and then of the effective MSSM to electroweak/Susy breaking scale. Evolution between $M_P$ and $M_{GUT}$ can be (and is) very important for both the hard and soft Susy breaking parameters of the NMSGUT due to the large RG $\beta$ functions in SO(10). In this chapter we give formulae for NMSGUT $\beta$ function upto two loops for the first time and examine their effect. The form of the RGEs for supersymmetric theories is governed by the supersymmetric non-renormalization theorem [103]. According to this theorem the logarithmically divergent contributions to a particular process arise only from wave-function renormalisation, without any superpotential coupling renormalization. Variation of parameter $X$ with energy scale is given by
where \( t = \log(Q/Q_0) \), \( Q \) and \( Q_0 \) are the renormalization and reference scale respectively. \( \beta_X^{(1)} \) and \( \beta_X^{(2)} \) are one-loop and two-loop \( \beta \) functions. The 2-loop RGEs for the MSSM (effective theory of Susy-GUTs) and the soft Susy breaking parameters are well known \[81\], and are useful from low scale to GUT scale. However, to consider the effect of renormalization from Planck scale to GUT scale, one needs the explicit form of GUT dependent RGEs. We have computed the two-loop RGEs for gauge coupling constant, superpotential and soft Susy breaking parameters for the New Minimal Supersymmetric SO(10) Grand Unified Theory. General formulae for the evolution functions for any softly broken Susy simple gauge group theory are available \[81\] to compute SO(10) two-loop \( \beta \) function. A Mathematica package immediately gets stuck \[139\] on combinatorial complexity while performing the sums over irrep indices required to obtain RGE coefficients for SO(10) irreps. However special tricks using the properties of the model and SO(10) irrep index contraction make the sums over the components of the large irreps (\( 210, \ 126 \), \( 126 \), and \( 120 \)) used in the NMSGUT tractable. We got explicit results for all RGEs up to second order using gauge invariance as a guiding principle. This work is done in collaboration with Prof. C.S. Aulakh and Ila Garg\[64\].

### 6.2 Formalism

The one-loop \( \beta \)-function for the gauge coupling \[81\] is:

\[
\beta_g^{(1)} = g^3[S(R) - 3C(G)]
\]  

where

\( S(R) \) and \( C(G) \) are Dynkin index (including contribution of all superfields) and Casimir invariant respectively. Note that \( C(R) \ d(R) = S(R) \ d(G) \). Since the values are \( S(45,10,16,120,126,210) = (8,1,2,28,35,56) \) and \( C(R) = 45(8/45,1/10,2/16,28/120,35/126,56/210) \) we get one-loop \( \beta \) function for the SO(10) gauge coupling to be:
\[ \beta_{g_{10}}^{(1)} = 137 g_{10}^3 \] (6.3)

Notice that this implies very rapid change of \( g_{10} \) and hence require great care to avoid nonsensical results. The generic form of one-loop \( \beta \)-function for the superpotential parameters is \( (W = \frac{1}{6}Y^{ijk} \phi_i \phi_j \phi_k + ...) \):

\[ [\beta_Y^{(1)}]^{ij} = Y^{ijp} \gamma_p^{(1)} + (k \leftrightarrow i) + (k \leftrightarrow j) \] (6.4)

where \( i, j, k \) are the indices running over all the chiral fields in the theory and \( \gamma^{(1)} \) is the one loop anomalous dimension matrix. SO(10) gauge invariance implies that \( \gamma^i_j \) must be fieldwise and componentwise diagonal: thus simplifies their computation enormously. The NMSGUT has \( (\lambda, k, \rho, \gamma, \bar{\gamma}, \eta, \zeta, \bar{\zeta}) \) superpotential couplings representing the following interactions:

\[
\begin{align*}
\lambda : & 210^3 ; \\
\eta : & 210 \cdot 126 \cdot \overline{126} ; \\
\rho : & 120 \cdot 120 \cdot 210 \\
k : & 10 \cdot 120 \cdot 210 ; \\
\gamma \oplus \bar{\gamma} : & 10 \cdot 210 \cdot (126 \oplus \overline{126}) \\
\zeta \oplus \bar{\zeta} : & 120 \cdot 210 \cdot (126 \oplus \overline{126})
\end{align*}
\] (6.5)

and mass parameters:

\[
\begin{align*}
\mu_\Phi : & 210^2 ; \\
\mu_\Sigma : & 126 \cdot \overline{126} ; \\
\mu_H : & 10^2 ; \\
\mu_\Theta : & 120^2
\end{align*}
\] (6.6)

The generic one-loop anomalous dimension parameters associated with superfields carry the crucial structure governing the NMSGUT RGEs and are given by

\[
\gamma_i^{(1)} = \frac{1}{2} Y_{ipq} Y^{jpq} - 2 g^2 \delta^i_j C(i) \] (6.7)

Now we discuss the contribution of superpotential invariant \( \rho \Phi_{ijkl} \Theta_{ijm} \Theta_{klm} \) to \( \gamma_{\Phi}^{(1)} \). Let us focus on what couples to a given component of the \( 210 \)

\[
\rho \frac{1}{4!} \Phi_{ijkl} \Theta_{ijm} \Theta_{klm} = \sum_m \rho \frac{1}{4!} 4.2 \Phi_{1234} (\Theta_{12m} \Theta_{34m} - \Theta_{13m} \Theta_{24m} + \Theta_{14m} \Theta_{23m})
\]
Here \( m \) runs over remaining 6 values since the 120 plet is totally antisymmetric. We see that since the SO(10) symmetry will require \( \gamma_j \) diagonality within an irrep one can obtain the \( \gamma_j \) by counting the possibilities for any representative irrep element on the external lines and SO(10) allowed index combinations on the summed indices. In this example we can have 18 possible combinations that couple to \( \Phi_{1234} \). Therefore

\[
\frac{1}{2} \left| Y_{\{\Phi_{1234}, \Theta, \Sigma\}} \right|^2 = \frac{18 |\rho|^2}{9} = 2 |\rho|^2
\]

(6.8)

Similarly

\[
\frac{\gamma}{4!} \Phi_{ijkl} H_{m} \Sigma_{ijklm} = \gamma \Phi_{1234} (H_{5} \Sigma_{12345} + H_{6} \Sigma_{12346} + ....)
\]

(6.9)

The six allowed index values for \( H \) (i.e. 5-10) give

\[
\sum_{H, \Sigma} Y_{\{\Phi_{1234}, H, \Sigma\}} Y_{\{\Phi_{1234}, H, \Sigma\}}^* = 6 |\gamma|^2
\]

(6.10)

The invariant \( k H_{i} \Theta_{jkl} \Phi_{ijkl} \) will contribute to \( \gamma_{\Phi}^{(1)} \)

\[
k \frac{3!}{3!} H_{i} \Theta_{jkl} \Phi_{ijkl} = k \Phi_{1234} (H_{1} \Theta_{234} - H_{2} \Theta_{134} + H_{3} \Theta_{124} - H_{4} \Theta_{312}) + ....
\]

(6.11)

\[
\sum_{H, \Theta} Y_{\{\Phi_{1234}, H, \Theta\}} Y_{\{\Phi_{1234}, H, \Theta\}}^* = 4 |k|^2
\]

(6.12)

Thus the anomalous dimension matrix reduces to a common anomalous dimension for each independent component of each field.

\[
\gamma_{\Phi}^{(1)} = 240 |\eta|^2 + 4 |k|^2 + 180 |\lambda|^2 + 2 |\rho|^2 + 6 (|\gamma|^2 + |\bar{\gamma}|^2) + 60 (|\zeta|^2 + |\bar{\zeta}|^2) - 24 g_{10}^2
\]

(6.13)

\[
\gamma_{\Sigma}^{(1)} = 200 |\eta|^2 + 10 |\gamma|^2 + 100 |\zeta|^2 - 25 g_{10}^2
\]

(6.14)

\[
\gamma_{H}^{(1)} = 84 |k|^2 + 126 (|\gamma|^2 + |\bar{\gamma}|^2) + 32 \text{Tr} [\bar{f} \cdot f] - 25 g_{10}^2
\]

(6.15)

\[
\gamma_{\Theta}^{(1)} = 7 (|k|^2 + |\rho|^2) + 105 (|\zeta|^2 + |\bar{\zeta}|^2) + 8 \text{Tr} [\bar{g} \cdot g] - 21 g_{10}^2
\]

(6.16)

\[
(\gamma_{\Phi}^{(1)})^B_A = (\gamma_{\Phi}^{(1)})_{AB} = 252 f^\dagger \cdot f + 120 g^\dagger \cdot g + 10 h^\dagger \cdot h - \frac{45 g_{10}^2}{4}
\]

(6.17)
Here $h$, $f$ and $g$ are Yukawa couplings of $10$, $126$ and $120$ Higgs irreps. Using the $\gamma$’s one can compute RGEs for all the superpotential parameters. For example one loop $\beta$ function for $\lambda$ is:

$$\beta^{(1)}_\lambda = 3\gamma^{(1)}_\Phi \lambda$$ (6.19)

In addition to superpotential parameters and gauge coupling, we need to compute $\mathcal{L}_{\text{soft}}$ parameters RGEs:

$$\mathcal{L}_{\text{soft}} = -\frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k - \frac{1}{2} h^{ij} \phi_i \phi_j - \frac{1}{2} (m^2)^{ij}_j \phi_i \phi_j - \frac{1}{2} M \lambda \lambda + \text{h.c.}$$ (6.20)

In total we have \{\hat{\lambda}, \hat{k}, \hat{\rho}, \hat{\gamma}, \hat{\eta}, \hat{\zeta}, \hat{h}, \hat{f}, \hat{g}\}, \{b_\Phi, b_\Sigma, b_H, b_\Theta\} and \{m^2_\Phi, m^2_\Sigma, m^2_\Sigma, m^2_\Theta, m^2_H, m^2_{\tilde{\Psi}}\} parameters in the NMSGUT soft Lagrangian. One loop $\beta$- function for $h^{ijk}$ is \[81\] :

$$[\beta^{(1)}_h]_{ijk} = \frac{1}{2} h^{ij} Y_{lmn} Y^{mnk} + Y^{ijk} Y_{l}^{\alpha\beta\gamma} h^{\alpha\beta\gamma} - 2(h^{ijk} - 2M Y^{ijk})g^2 C(k)$$

\[ (k \leftrightarrow i) + (k \leftrightarrow j) \] (6.21)

We define :

$$\tilde{\gamma}^{(1)}_i = \frac{1}{2} Y_{ipq} Y^{ipq} ; \hat{\gamma}^{(1)}_i = \frac{1}{2} Y_{ipq} h^{jipq} ; \hat{\gamma}^{(1)}_i = \frac{1}{2} h_{ipq} h^{jipq}$$ (6.22)

Then arguments similar to those given above yield :

$$\tilde{\gamma}^{(1)}_\Phi = 240|\eta|^2 + 4|k|^2 + 180|\lambda|^2 + 2|\rho|^2 + 6(|\gamma|^2 + |\bar{\gamma}|^2) + 60(|\zeta|^2 + |\bar{\zeta}|^2)$$ (6.23)

$$\hat{\gamma}^{(1)}_\Phi = 240|\bar{\eta}|^2 + 4|\bar{k}|^2 + 180|\bar{\lambda}|^2 + 2|\bar{\rho}|^2 + 6(|\bar{\gamma}|^2 + |\bar{\bar{\gamma}}|^2) + 60(|\bar{\zeta}|^2 + |\bar{\bar{\zeta}}|^2)$$ (6.24)

$$\hat{\gamma}^{(1)}_\Phi = 240|\bar{\eta}|^2 + 4|\bar{k}|^2 + 180|\bar{\lambda}|^2 + 2|\bar{\rho}|^2 + 6(|\bar{\gamma}|^2 + |\bar{\bar{\gamma}}|^2) + 60(|\bar{\zeta}|^2 + |\bar{\bar{\zeta}}|^2)$$ (6.25)

$$\beta^{(1)}_\lambda = 3\tilde{\lambda} \hat{\gamma}^{(1)}_\Phi + 6\lambda \hat{\gamma}^{(1)}_\Phi - 72g^2 (\bar{\lambda} - 2M \lambda)$$ (6.26)
\[ [\beta_b^{(1)}]^{ij} = \frac{1}{2} b^i Y_{lmn} Y^{mnj} + \frac{1}{2} Y^{ijl} Y_{lmn} b^{mn} + \mu^i Y_{lmn} h^{mnj} \]
\[-2(b^{ij} - 2M\mu^{ij})g^2 C(i) + (i \leftrightarrow j) \] (6.27)
\[ \beta_{b^\Phi}^{(1)} = 2b_\Phi \bar{\gamma}_\Phi^{(1)} + 4\mu_\Phi \bar{\gamma}_\Phi^{(1)} - 48g_{10}^2 (b_\Phi - 2M\mu_\Phi) \] (6.28)
\[ [\beta_{m^2}^{(1)}]^{ij} = \frac{1}{2} Y_{ipq} Y^{pqn} (m^2)^{\frac{n}{2}} + \frac{1}{2} Y^{ijp} Y_{pqn} (m^2)^{\frac{n}{2}} + 2Y_{ipq} Y^{jpr} (m^2)^{\frac{p}{2}} \]
\[ + h_{ipq} h^{jpr} - 8\delta_i^{jm} M M^\dagger g^2 C(i) + 2g^2 t_i^{A^j} \text{Tr}[t^A m^2] \] (6.29)
\[ \beta_{m^2}^{(1)} = 2\bar{\gamma}_{\Phi}^{(1)} m_\Phi^2 + 720 m_\Phi^2 |\lambda|^2 + m_\Phi^2 (12|\gamma|^2 + 12|\bar{\gamma}|^2 + 8|k|^2) \]
\[ + m_0^2 (8|p|^2 + 12(|\zeta|^2 + |\tilde{\zeta}|^2) + 8|k|^2) + m_2^2 (480|\eta|^2 + 12|\gamma|^2 + 120|\zeta|^2) \]
\[ + m_2^2 (480|\eta|^2 + 12|\bar{\gamma}|^2 + 120|\bar{\zeta}|^2) + 2\bar{\gamma}_{\Phi}^{(1)} - 96|\lambda|^2 g_{10}^2 \] (6.30)

The two loop anomalous dimensions \( \gamma^{(2)} \) are the main building blocks of two loop \( \beta \) functions, having generic form:
\[ \gamma_i^{(2)} j = -\frac{1}{2} Y_{imn} Y^{npq} Y_{pqr} Y^{mrj} + g_{10}^2 Y_{ipq} Y^{jpr} [2C(p) - C(i)] \]
\[ + 2\delta_i^{Aj} g_{10}^2 [C(i) S(R) + 2C(i)^2 - 3C(G) C(i)] \] (6.31)

Again they are fieldwise and independent component wise diagonal. Only the first term require attention. The intermediate sums over \( n, r \) can be broken field wise and thereafter using diagonality of the one loop anomalous dimensions (with respect to independent irrep components) already computed:
\[ Y_{imn} Y^{npq} Y_{pqr} Y^{mrj} = Y_{imnH} \bar{\gamma}_H^{(1)} Y^{mnHj} + Y_{imn\Phi} \bar{\gamma}_\Phi^{(1)} Y^{mn\Phi j} + ... \] (6.32)

As discussed for the one loop, SO(10) gauge invariance provide \( \gamma^{(2)} \) and constraint \( n=r \) (in the first term), so we have (as already the sum is over independent field components)
\[ Y_{imn} Y^{npq} Y_{pqr} Y^{mrj} = Y_{imn} (\bar{\gamma})_n^{\gamma mnj} \] (6.33)
6.3 Numerical Analysis

One needs to examine the superpotential invariant involving two fields carrying field component indices \( mn \). Thus the total contribution can be written with the help of one loop anomalous dimension parameters. For example:

\[
\gamma^{(2)}_{\Phi} = -(240|\eta|^2(\bar{\gamma}^{(1)}_{\Sigma} + \bar{\gamma}^{(1)}_{\bar{\Sigma}}) + 4|k|^2(\bar{\gamma}^{(1)}_{H} + \bar{\gamma}^{(1)}_{\bar{\Theta}}) + 6|\gamma|^2(\bar{\gamma}^{(1)}_{H} + \bar{\gamma}^{(1)}_{\bar{\Sigma}})
+ 360|\lambda|^2\bar{\gamma}^{(1)}_{\bar{\Phi}} + 4|\rho|^2\bar{\gamma}^{(1)}_{\bar{\Theta}} + 6|\bar{\gamma}|^2(\bar{\gamma}^{(1)}_{H} + \bar{\gamma}^{(1)}_{\bar{\Sigma}}) + 60|\zeta|^2(\bar{\gamma}^{(1)}_{\bar{\Theta}} + \bar{\gamma}^{(1)}_{\bar{\Sigma}})
+ 60|\bar{\gamma}|^2 + 60|\bar{\gamma}|^2 + 1320|\zeta|^2 + 1320|\bar{\zeta}|^2) + 3864g_{10}^4
\]

Two-loop \( \beta \) functions for other superpotential parameters are given in the Appendix and for the soft couplings can be found in [64, 65]. We will use these RGEs to estimate the variation of the soft parameters between \( M_P \) and \( M_X^0 \). NMSGUT fits discussed in the previous chapters have large negative soft Higgs masses (\( M_{H,\bar{H}}^2 \)). SO(10) RGEs can explain origin of these kind of couplings.

6.3 Numerical Analysis

We throw SO(10) gauge and Yukawa couplings and soft parameters randomly in the perturbative range. Along with this we choose soft breaking parameters according to SUGRY soft term form (assuming canonical soft terms). Susy breaking i.e. with all gaugino masses zero, all soft scalar masses equal, \( A_0=2m_{3/2} \), \( b_i=(A_0-m_{3/2})\mu_i \) at the Planck scale. We chose \( m_{3/2}=20 \) TeV and renormalize them from \( M_P \) and \( M_X^0 \). Large coefficient of the trilinear couplings in the anomalous dimension make these RGEs to evolve fast between \( M_P \) to \( M_X^0 \). The values of hard and soft parameters at two scales (\( M_P \) and \( M_X^0 \)) are given in Tables 6.1 and 6.2 respectively. This shows that the evolution can be very significant and in particular the soft masses change rapidly. The large value of \( \beta_g \) makes a UV fixed point impracticable. Soft mass evolution is shown in Fig. 6.1.
| Parameter | Value at $M_P$ | Value at $M_R^{0}(10^{16.33}$ GeV) |
|-----------|----------------|----------------------------------|
| $\lambda$ | $-0.0434 + 0.0078i$ | $-0.0098 + 0.0018i$ |
| $\eta$   | $-0.3127 + 0.0798i$ | $-0.0969 + 0.0247i$ |
| $\rho$   | $0.9544 - 0.2698i$ | $0.141 - 0.0399i$ |
| $k$      | $0.0273 + 0.0991i$ | $0.0015 + 0.0053i$ |
| $\gamma$ | $0.4711$ | $0.0318$ |
| $\bar{\gamma}$ | $-3.2719$ | $-0.2922$ |
| $\zeta$  | $1.0091 + 0.8305i$ | $0.1876 + 0.1544i$ |
| $\bar{\zeta}$ | $0.3596 + 0.5898i$ | $0.0885 + 0.1452i$ |
| $h_{11}/10^{-6}$ | $4.4602$ | $1.0843$ |
| $h_{22}/10^{-4}$ | $4.1031$ | $0.9971$ |
| $h_{33}$ | $0.0244$ | $0.0059$ |
| $h_{12}/10^{-12}$ | $0.0$ | $-2.9819 + 4.8131i$ |
| $h_{13}/10^{-11}$ | $0.0$ | $-2.3318 + 4.0693i$ |
| $h_{23}/10^{-9}$ | $0.0$ | $-6.1280 + 11.4938i$ |
| $f_{11}/10^{-6}$ | $-0.0044 + .16207i$ | $-0.0049 + .1811i$ |
| $f_{22}/10^{-5}$ | $6.675 + 4.8457i$ | $7.4587 + 5.4144i$ |
| $f_{33}/10^{-4}$ | $-9.264 + 2.7876i$ | $-10.3507 + 3.1146i$ |
| $f_{12}/10^{-6}$ | $-8.4951 - 1.7825i$ | $-0.9492 - 1.9917i$ |
| $f_{13}/10^{-6}$ | $5.4964 + 1.1479i$ | $0.6141 + 1.2826i$ |
| $f_{23}/10^{-4}$ | $-4.266 + 2.231i$ | $-0.4767 + 2.4927i$ |
| $g_{12}/10^{-5}$ | $1.4552 + 1.599i$ | $0.9755 + 1.0718i$ |
| $g_{13}/10^{-5}$ | $-1.1784 + .49613i$ | $-7.8988 + 3.3255i$ |
| $g_{23}/10^{-4}$ | $-1.6648 - 1.18436i$ | $-1.1159 - 0.7939i$ |
| $\mu_\Phi$ | $10^{15}$ GeV | $3.71 \times 10^{13}$ GeV |
| $\mu_H$ | $10^{15}$ GeV | $3.19 \times 10^{13}$ GeV |
| $\mu_\Sigma$ | $10^{15}$ GeV | $50.87 \times 10^{13}$ GeV |
| $\mu_\Theta$ | $10^{15}$ GeV | $24.25 \times 10^{13}$ GeV |
| $g$ | $2.2519$ | $0.3445$ |

Table 6.1: Values of NMSGUT parameters at two different scales evolved by using one-loop SO(10) RGEs.
6.3 Numerical Analysis

| Parameter | Value at $M_P$ | Value at $M_X^{0\{10^{16.33}\}}$ GeV |
|-----------|---------------|--------------------------------------|
| $\lambda$ | $-43.4231 + 7.7508i$ | $-0.3413 + 0.0613i$ |
| $\eta$   | $-312.69 + 79.7879i$ | $-450.6968 + 115.0163i$ |
| $\tilde{\rho}$ | $954.387 - 269.837i$ | $-556.0236 + 157.1827i$ |
| $\tilde{k}$ | $27.3142 + 99.0676i$ | $-12.3372 - 44.7845i$ |
| $\tilde{\gamma}$ | $471.122$ | $-185.8855$ |
| $\tilde{\gamma}$ | $-3271.91$ | $707.8617$ |
| $\tilde{\zeta}$ | $1009.13 + 830.517i$ | $-255.3635 - 210.1669i$ |
| $\tilde{\eta}$ | $359.587 + 589.788i$ | $183.1857 + 300.4532i$ |
| $\tilde{h}_{11}$ | 0.1784 | 0.0219 |
| $\tilde{h}_{22}$ | 16.4052 | 2.014 |
| $\tilde{h}_{33}$ | 976.7480 | 119.8772 |
| $\tilde{h}_{12}/10^{-7}$ | 0.0 | $-2.1449$ |
| $\tilde{h}_{13}/10^{-6}$ | 0.0 | $-1.3053$ |
| $\tilde{h}_{23}/10^{-4}$ | 0.0 | $-5.1246$ |
| $\tilde{f}_{11}/10^{-3}$ | $-0.1744 + 6.483i$ | $-0.1496 + 5.56i$ |
| $\tilde{f}_{22}$ | $2.6701 + 1.9383i$ | $2.2899 + 1.6623i$ |
| $\tilde{f}_{33}$ | $-37.0558 + 11.1505$ | $-31.7754 + 9.5616i$ |
| $\tilde{f}_{12}/10^{-2}$ | $-3.3980 - 7.1301i$ | $-2.9143 - 6.1149i$ |
| $\tilde{f}_{13}/10^{-2}$ | $2.1986 + 4.5916i$ | $1.8853 + 3.937i$ |
| $\tilde{g}_{12}$ | $-1.7064 + 8.9336$ | $-1.4633 + 7.6523i$ |
| $\tilde{g}_{13}$ | $0.5821 + 0.6396i$ | $0.2408 + 0.2646i$ |
| $\tilde{g}_{23}$ | $-6.6592 - 4.7374i$ | $-2.7549 - 1.9599i$ |

| $M$ | 0 | 0 |
|-----|----|----|
| $b_\Phi$ | $2.0 \times 10^{19}\text{GeV}^2$ | $-2.465 \times 10^{18}\text{GeV}^2$ |
| $b_H$ | $2.0 \times 10^{19}\text{GeV}^2$ | $-6.2536 \times 10^{17}\text{GeV}^2$ |
| $b_\Sigma$ | $2.0 \times 10^{19}\text{GeV}^2$ | $-1.0311 \times 10^{18}\text{GeV}^2$ |
| $b_{\Theta}$ | $2.0 \times 10^{19}\text{GeV}^2$ | $-2.576 \times 10^{18}\text{GeV}^2$ |

| $m_\phi^2$ | $m_{3/2}$ | $-12577864.9856\text{GeV}^2$ |
| $m_H^2$ | $m_{3/2}$ | $-151814083.220\text{GeV}^2$ |
| $m_{\Theta}^2$ | $m_{3/2}$ | $-55926753.8102\text{GeV}^2$ |
| $m_\Sigma^2$ | $m_{3/2}$ | $16687663.4739\text{GeV}^2$ |
| $m_{\Sigma}^2$ | $m_{3/2}$ | $100530987.171\text{GeV}^2$ |

$\text{Eval } m_\Phi^2 \{m_{3/2}, m_{3/2}, m_{3/2}\} \{3.9999, 3.9999, 3.9993\} \times 10^{8}\text{GeV}^2$

Table 6.2: Values of NMSGUT soft parameters at two different scales evolved by using one-loop SO(10) RGEs. $\{\tilde{\lambda}, \tilde{k}, \tilde{\rho}, \tilde{\gamma}, \tilde{\eta}, \tilde{\zeta}, \tilde{\bar{h}}, \tilde{f}, \tilde{g}\}=A_0(\lambda, k, \rho, \gamma, \eta, \zeta, \bar{h}, f, g)$, $A_0 = 40\text{ TeV}$, $m_{3/2} = 20\text{ TeV}$. 
6.4 Discussion and Outlook

We derived the NMSGUT RG equations to determine the RG evolution of couplings between $M_P$ and $M_X^0$ (the matching scale between GUT and effective theory) assuming pure supergravity canonical scenario for the starting parameter ansatz. Evaluating the effects of the evolution on randomly chosen sets of parameter values we see that the RG evolution has dramatic effects on the soft susy breaking parameters. Firstly most of the soft Susy squared masses of the SO(10) Higgs irreps become negative. It provides a potentially robust justification of the negative values of $M^2_{H,\bar{H}}$, the NMSGUT fits already needed. Note that the distinctive normal s-hierarchy at low scale is strongly correlated with the large negative $M^2_{H,\bar{H}}$ we use in the fits. Gaugino masses will be generated by two loop RG evolution, however even $M=0$ at the GUT scale yielded adequate gaugino masses at the electroweak scale. The other dramatic effect is the intermediate scale ($O(m_{3/2}^2 M_X)$) values of the soft parameters $b_{\Phi,\Sigma,\Theta,\bar{H}}$ required by the canonical SUGRY ansatz and induced by the dependence $\frac{db}{dt} \sim \mu m_{3/2}$. Actually the running values of $b_{\Phi,\Sigma,\Theta,\bar{H}}$ turn negative after starting positive so it is possible that they run to smaller and more acceptable values $O(m^2_{3/2})$ for a suitable set of starting parameters. We are currently studying the detailed implications.
Appendix

One-loop RGEs

One-loop beta functions for the SO(10) superpotential parameters and Yukawa couplings of \(10, \overline{126}\) and \(120\):

\[
\beta_{\lambda}^{(1)} = 3\gamma_{\Phi}^{(1)} \lambda ; \; \beta_{\eta}^{(1)} = \eta(\gamma_{\Sigma}^{(1)} + \gamma_{\Phi}^{(1)})
\]

\[
\beta_{\gamma}^{(1)} = \gamma(\gamma_{H}^{(1)} + \gamma_{\Sigma}^{(1)} + \gamma_{\Phi}^{(1)}) ; \; \beta_{\eta}^{(1)} = \eta(\gamma_{H}^{(1)} + \gamma_{\Sigma}^{(1)} + \gamma_{\Phi}^{(1)})
\]

\[
\beta_{k}^{(1)} = k(\gamma_{H}^{(1)} + \gamma_{\Theta}^{(1)} + \gamma_{\Phi}^{(1)}) ; \; \beta_{\zeta}^{(1)} = \zeta(\gamma_{\Theta}^{(1)} + \gamma_{\Sigma}^{(1)} + \gamma_{\Phi}^{(1)})
\]

\[
\beta_{f}^{(1)} = f(\gamma_{\Psi}^{(1)}) + (\gamma_{\Psi}^{(1)})^{T} \cdot f + f \cdot \gamma_{\Psi}^{(1)}
\]

\[
\beta_{\mu_{\Phi}}^{(1)} = 2\gamma_{\Phi}^{(1)} \mu_{\Phi} ; \; \beta_{\mu_{H}}^{(1)} = 2\gamma_{H}^{(1)} \mu_{H}
\]

\[
\beta_{\mu_{\Sigma}}^{(1)} = (\gamma_{\Sigma}^{(1)} + \gamma_{\Phi}^{(1)}) \mu_{\Sigma} ; \; \beta_{\mu_{\Theta}}^{(1)} = 2\gamma_{\Theta}^{(1)} \mu_{\Theta}
\]

Two-loop RGEs

\[
\tilde{\gamma}_{\Sigma}^{(1)} = 200|\eta|^{2} + 10|\gamma|^{2} + 100|\zeta|^{2}
\]

\[
\tilde{\gamma}_{\Sigma}^{(1)} = 200|\eta|^{2} + 10|\gamma|^{2} + 100|\zeta|^{2} + 32\text{Tr}[f^{\dagger} \cdot f]
\]

\[
\tilde{\gamma}_{H}^{(1)} = 84|k|^{2} + 126(|\gamma|^{2} + |\bar{\gamma}|^{2}) + 8\text{Tr}[h^{\dagger} \cdot h]
\]

\[
\tilde{\gamma}_{\Theta}^{(1)} = 7(|k|^{2} + |\rho|^{2}) + 105(|\zeta|^{2} + |\bar{\zeta}|^{2}) + 8\text{Tr}[g^{\dagger} \cdot g]
\]
\[\gamma^{(1)}_\psi = 252 f^\dagger f + 120 g^\dagger g + 10 h^\dagger h\] (6.47)

\[\beta^{(2)}_{g_{10}} = \frac{9709 g_{10}^5}{45} - \frac{g_{10}^3}{45} (9 \gamma^{(1)}_H + 21 \gamma^{(1)}_\Theta + 25 \gamma^{(1)}_\Sigma + 25 \gamma^{(1)}_\Phi) + \frac{135}{4} (8 \text{Tr}[h^\dagger h] + 8 \text{Tr}[g^\dagger g] + 32 \text{Tr}[f^\dagger f])\] (6.48)

\[\gamma^{(2)}_\Sigma = -\left(200|\eta|^2(\gamma^{(1)}_\Phi + \gamma^{(1)}_\Sigma) + 10|\gamma|^2(\gamma^{(1)}_H + \gamma^{(1)}_\Phi) + 100|\zeta|^2(\gamma^{(1)}_\Theta + \gamma^{(1)}_\Phi)\right) + g_{10}^2 (4800|\eta|^2 + 80|\gamma|^2 + 2000|\zeta|^2) + 4050 g_{10}^4 \] (6.49)

\[\gamma^{(2)}_\Sigma = -\left(200|\eta|^2(\gamma^{(1)}_\Phi + \gamma^{(1)}_\Sigma) + 10|\gamma|^2(\gamma^{(1)}_H + \gamma^{(1)}_\Phi) + 100|\zeta|^2(\gamma^{(1)}_\Theta + \gamma^{(1)}_\Phi)\right) + \text{Tr}[f^\dagger \gamma^{(1)}_\psi . f]] + g_{10}^2 (4800|\eta|^2 + 80|\gamma|^2 + 2000|\zeta|^2) - 80 \text{Tr}[f^\dagger f] + 4050 g_{10}^4 \] (6.50)

\[\gamma^{(2)}_H = -\left(84|k|^2(\gamma^{(1)}_\Phi + \gamma^{(1)}_\Sigma) + 126|\gamma|^2(\gamma^{(1)}_\Phi + \gamma^{(1)}_\Sigma) + 126|\gamma|^2(\gamma^{(1)}_\Phi + \gamma^{(1)}_\Sigma)\right) + \text{Tr}[h^\dagger \gamma^{(1)}_\psi h]] + g_{10}^2 (3024|k|^2 + 5040|\gamma|^2 + 5040|\zeta|^2) + 108 \text{Tr}[h^\dagger h] + 1314 g_{10}^4 \] (6.51)

\[\gamma^{(2)}_\Theta = -\left(7|\rho|^2(\gamma^{(1)}_\Phi + \gamma^{(1)}_H) + 7|\rho|^2(\gamma^{(1)}_\Phi + \gamma^{(1)}_H)\right) + \text{Tr}[g^\dagger \gamma^{(1)}_\psi g] + 105|\zeta|^2(\gamma^{(1)}_\Sigma + \gamma^{(1)}_\Phi) + g_{10}^2 (84|k|^2 + 168|\rho|^2) + 2940|\zeta|^2 + 2940|\zeta|^2 + 12 \text{Tr}[g^\dagger g]) + 3318 g_{10}^4 \] (6.52)

\[\gamma^{(2)}_\psi = -\left(h^\dagger \gamma^{(1)}_\psi^T h - g^\dagger \gamma^{(1)}_\psi^T g + f^\dagger \gamma^{(1)}_\psi^T f + h^\dagger h \gamma^{(1)}_H - g^\dagger g \gamma^{(1)}_\Theta + f^\dagger f \gamma^{(1)}_\Sigma\right) + g_{10}^2 (90 h^\dagger h + 2520 g^\dagger g + 6300 f^\dagger f) + \frac{26685 g_{10}^4}{16} I \] (6.53)

\(\beta^{(2)}\) can be obtained from Eqns. 6.35-6.42 replacing \(\gamma^{(1)}\) by \(\gamma^{(2)}\).
Chapter 7

Dynamical Yukawa Couplings

7.1 Introduction

The SM fermion mass-mixing data poses several questions. Fermion masses vary from milli-eV from neutrino to between 0.5 MeV to 174 GeV for charged fermions. Leptonic mixing is large as compared to quark sector mixing. Why do we have three fermion generations? Do they follow some flavour symmetry? The mass hierarchy is different for up type quarks, down type quarks and for leptons. All these questions constitute the flavour puzzle posed by the SM and neutrino oscillations data. To understand the origin of observed flavour structure of the SM data is most basic problem of flavour physics. Introduction of family symmetry and generation of flavour structure by Yukawa couplings arising as VEVs of “spurion” fields offers an attractive alternative prospect for understanding flavour structure [140]. Model builders have considered various possibilities like discrete (tetrahedral group $A_4$ and permutation group $S_3$), abelian/non-abelian (global or local) symmetries. The establishment of the lepton mixing pattern triggered great interest in the discrete family symmetry approach (for reviews see [141, 142, 143]). Mostly SU(5) GUT and discrete family symmetry combination is considered. In the so called Yukawa-on models [144] different symmetry is considered for each type of fermion. The dimension-1 Yukawa-on field ($\mathcal{Y}$) makes the Higgs vertex non-renormalizable ($\mathcal{L} = f^c \mathcal{Y} f H/\Lambda_\mathcal{Y} + ...$) and Yukawa-on dynamics is controlled by a high-scale $\Lambda_\mathcal{Y}$.

In our view the strongest motivation and hint for the flavour symmetry comes
from third generation Yukawa unification in Susy SO(10) GUTs at large \( \tan \beta \). It indicates that the GUT gauge symmetry breaking may generate the fermion hierarchy. Combining this hint with the successful fitting of the fermion data in the NMSGUT motivated us to extend the minimal renormalizable supersymmetric SO(10) GUT with \( O(N_g) \) family group \([135]\). In minimal Susy SO(10)GUT \([46, 47, 48, 57]\), MSSM Higgs pair emerges from the large number of MSSM type doublets of UV theory and fermion hierarchy is generated by the SO(10) matter Yukawa couplings. In this extended scenario Higgs multiplets of SO(10) also carry family index ("Yukawons") and their VEVs generate Yukawa couplings of SM fermion and neutrino. In our study Yukawons also carry representation of the gauge (SM/GUT) dynamics. As explained in the previous chapters MSGUT completed with \( 120 \)-plet called NMSGUT \([57]\), can generate realistic fermion mass mixing data and experiment compatible B-decay rates after the inclusion of superheavy thresholds. Therefore, from our viewpoint of combined family and GUT unification, it is the logical base for a dynamical theory of flavour. To start with, we study extension of MSGUT based upon the \( 10 \oplus 210 \oplus \bar{126} \oplus 126 \) Higgs irrep. We will comment on the minor changes required to include the \( 120 \)-plet: which may ultimately be necessary.

### 7.2 Yukawon Ultra Minimal GUTs

Yukawon Ultra Minimal GUTs are an extension of minimal supersymmetric SO(10) model by \( O(N_g) \) family gauge group. The \( 10(H) \oplus 210(\Phi) \oplus \bar{126}(\Sigma) \oplus 126(\Sigma) \) Higgs irreps become symmetric representations of \( O(N_g) \) family group. Matter fermions are present in the form of three copies of \( 16(\Psi) \)-plet. Superpotential of the model has same form as of MSGUT (with sum over flavour indices):

\[
W_{GUT} = \text{Tr}(m\Phi^2 + \lambda \Phi^3 + M\bar{\Sigma}.\Sigma + \eta \Phi.\bar{\Sigma}.\Sigma + \Phi.H.(\gamma\Sigma + \bar{\gamma}.\bar{\Sigma}) + M_H.H.H)
\]

\[
W_F = \Psi_A.(hH_{AB} + f\Sigma_{AB} + g\Theta_{AB})\Psi_B
\]  

(7.1)

Here A and B are the family indices. Now SO(10) Yukawa couplings \( h, f, g \) are complex number because flavour indices are carried by MSGUT Higgs irreps them-
selves. Here we have included 120-plet in \( W_F \) but for simplicity we study only MSGUTs. However addition of 120-plet does not effect GUT SSB since it does not contain any MSSM singlet. Notice that 120-plet carry antisymmetric representation of family group. In this scenario matter fermion Yukawa couplings are reduced from 15(21) to just 3(5) parameters in MSGUT(NMSGUT) with 3 generations so we call it \textbf{Yukawon Ultra Minimal Grand Unified Theory (YUMGUTs)}. Each Higgs irrep contains one MSSM Higgs type multiplet \([1, 2, \pm 1]\). Mass matrix is given as

\[
\mathcal{H} = \begin{pmatrix}
-M_H & \frac{\gamma}{\sqrt{3}} \Omega(\omega - a) & -\frac{\gamma}{\sqrt{3}} \Omega(\omega + a) & -\frac{\gamma}{\sqrt{3}} \Omega(\bar{\sigma}) \\
\frac{\gamma}{\sqrt{3}} \Omega(\omega - a) & -(2M + 4\eta \Omega(a - \omega)) & \varnothing_d & -2\eta \frac{\gamma}{\sqrt{3}} \Omega(\bar{\sigma}) \\
-\frac{\gamma}{\sqrt{3}} \Omega(\omega + a) & \varnothing_d & -(2M + 4\eta \Omega(\omega + a)) & \varnothing_d \\
-\frac{\gamma}{\sqrt{3}} \Omega(\bar{\sigma}) & -2\eta \frac{\gamma}{\sqrt{3}} \Omega(\sigma) & \varnothing_d & 6\lambda \Omega(\omega - a) - 2m
\end{pmatrix}
\]

The rows are labelled by the \( N_g(N_g + 1)/2 \)-tuples (ordered and normalized, for a symmetric \( \phi_{AB}, A, B = 1..N_g \), as \( \{\phi_{11}, \phi_{22}, ... \phi_{N_gN_g}, \sqrt{2}\phi_{12}, \sqrt{2}\phi_{13}, ..., \sqrt{2}\phi_{N_g-1N_g}\} \)) containing MSSM type \( \mathcal{H}[1, 2, -1] \) doublets from 10, 126, 210. The columns represent \( \mathcal{H}[1, 2, 1] \) doublets in the order 10, 126, 210. \( \varnothing_d \) is the d dimensional null square matrix. The matrix function \( \Omega \left( \frac{N_g(N_g+1)}{2} \right) \) dimensional) is introduced to write \( \mathcal{H} \) in compact notations and its form is determined by symmetric invariant \( \phi_{AB}\phi_{BC}\phi_{CA} \) (here one field can have VEV and other two should contain \( H \) and \( \bar{H} \)). For \( N_g = 2 \) it is

\[
\Omega[V] = \begin{pmatrix}
V_{11} & 0 & V_{12}/\sqrt{2} \\
0 & V_{22} & V_{12}/\sqrt{2} \\
V_{12}/\sqrt{2} & V_{12}/\sqrt{2} & (V_{11} + V_{22})/2
\end{pmatrix}
\]

with labels \{\( \mathcal{H}_{11}, \mathcal{H}_{22}, \sqrt{2}\mathcal{H}_{12} \) \( \oplus \) \{\( H_{11}, H_{22}, \sqrt{2}H_{12} \)\}. Higgs mass matrix is now 2\( N_g(N_g+1) \) dimensional which would become \( N_g(3N_g+1) \) dimensional if we include 120-plet. MSSM being a effective theory requires one light Higgs pair out of these large number of Higgs multiplets. Consistency condition of the light Higgs pair assumption (fine tuning \( \text{Det}\mathcal{H} = 0 \)) ensures this. From left \( (\hat{W}) \) and right \( (\hat{V}) \) null eigenvectors we can determine MSSM Yukawa couplings. For \( N_g = 2 \) Yukawas of up
and down type quarks we get

\[ Y_u = \left( \begin{array}{c} \hat{h} + \hat{f}V_4 \\sqrt{2} \\ \hat{h} + \hat{f}V_6 \\sqrt{2} \end{array} \right) \left( \begin{array}{c} \hat{h} \hat{V}_3 + \hat{f}V_6 \\sqrt{2} \\ \hat{h} \hat{V}_2 + \hat{f}V_3 \end{array} \right) \right) ; \quad \hat{h} = 2\sqrt{2}h \]

\[ Y_d = \left( \begin{array}{c} \hat{h}W_1 + \hat{f}W_7 \\sqrt{2} \\ \hat{h}W_3 + \hat{f}W_9 \\sqrt{2} \end{array} \right) \left( \begin{array}{c} \hat{h}W_2 + \hat{f}W_8 \\sqrt{2} \\ \hat{h}W_3 + \hat{f}W_9 \\sqrt{2} \end{array} \right) \right) ; \quad \hat{f} = -4i\sqrt{\frac{2}{3}}f \] (7.3)

By replacing \( \hat{f} \rightarrow -3\hat{f} \) in \( Y_u, Y_d \) we can get \( Y_\nu, Y_l \). Clearly for \( f \sim h \) one can get \( Y_\nu, Y_l \) different from \( Y_u, Y_d \) as \( f \ll h \) implies \( Y_u \approx Y_\nu \) and \( Y_d \approx Y_l \). Higgs mass matrix consequently \( \hat{V}, \hat{W} \) are determined in terms of symmetry breaking VEVs \( (p, a, \omega, \sigma, \bar{\sigma}) \). Next step is to calculate these VEVs.

### 7.3 Spontaneous Symmetry Breaking

The multiplets \( \mathbf{210}, \mathbf{126, 126} \) break the GUT and flavour symmetry to MSSM. YUMGUT superpotential written in terms of VEVs \( (p, a, \omega, \sigma, \bar{\sigma}) \) of SM singlets:

\[ W = \text{Tr}\left[ m(p^2 + 3a^2 + 6\omega^2) + 2\lambda(a^3 + 3p\omega^2 + 6a\omega^2) \right] \]

\[ + \text{Tr}\left[ M\sigma\bar{\sigma} + \eta(p + 3a - 6\omega)(\sigma\bar{\sigma})/2 \right] \] (7.4)

Susy vacuum is determined by the vanishing of F and D terms. The F-term vanishing equations can be written as:

\[ 2m(p - a) - 2\lambda a^2 + 2\lambda \omega^2 = 0 \] (7.5)

\[ 2m(p + \omega) + \lambda(p + 2a + 3\omega)\omega + \lambda\omega(p + 2a + 3\omega) = 0 \] (7.6)

\[ M\sigma + \eta(\chi\sigma + \sigma\chi)/2 = 0 \] (7.7)

\[ M\bar{\sigma} + \eta(\chi\bar{\sigma} + \bar{\sigma}\chi)/2 = 0 \] (7.8)

\[ \sigma\sigma + \bar{\sigma}\bar{\sigma} = -\frac{4}{\eta}(mp + 3\lambda\omega^2) \equiv F \] (7.9)
where $\chi \equiv (p + 3a - 6\omega)$. D-terms include SO(10) and family D-terms. SO(10) has only one non-trivial D-term: $D_{B-L}$:

$$\sigma_{AB} - \bar{\sigma}_{AB} = 0$$  \hspace{1cm} (7.10)

The set of homogenous Eqns. (7.7,7.8) can be written in a more transparent form as

$$\Xi \cdot \hat{\Sigma} = \Xi \cdot \hat{\bar{\Sigma}} = 0$$  \hspace{1cm} (7.11)

where $\hat{\Sigma}, \hat{\bar{\Sigma}}$ are $(N_g(N_g + 1)/2)$-plet of $\sigma, \bar{\sigma}$ VEVs. Nontrivial solutions of Eqns. (7.7,7.8) for $\sigma, \bar{\sigma}$ exist only if $Det[\Xi] = 0$. In the MSGUT($N_g=1$) the linear condition ($\chi = -M/\eta$) supplements the Eqns. (7.5,7.6) and allows determination of $p, a, \omega$ via a cubic equation for $\omega$. After solving F-term conditions (actual procedure will be discussed in the next section), the D-term conditions ($D_{B-L} = 0$ from SO(10) and $D^A = 0$ from $O(N_g)$) need to be solved. In $N_g=1$ case

$$D_{B-L} = |\sigma|^2 - |\bar{\sigma}|^2 = 0$$  \hspace{1cm} (7.12)

Since $\text{Arg}[\sigma] - \text{Arg}[\bar{\sigma}]$ can be removed by $U(1)_{B-L}$ transformations, we choose $\sigma = \bar{\sigma}$. Here also we only consider the cases corresponding to $\sigma_{AB} = \bar{\sigma}_{AB}$, so that $D_{B-L}$ is automatically zero.

The D-terms of the family group vanish automatically only for trivial solutions of the F-terms conditions. We are interested in non-trivial solutions because only these can generate generation mixing. One needs to introduce additional fields to cancel GUT sector contribution to the family D-terms. F-terms corresponding to extra fields should not interfere with the GUT F terms so as not to disturb the MSGUT SSB. The best possible choice is to locate these fields in the hidden sector. In [146] it has been shown that Bajc-Melfo (BM) two field superpotential is an appropriate candidate. In the next section we will discuss how BM superpotential enables YUMGUTs.
7.4 Bajc-Melfo Superpotential

Two field \((S_s, \phi_s)\) BM superpotential reads:

\[ W_H = S_s(\mu_B \phi_s + \lambda_B \phi_s^2) \] (7.13)

It has a Susy preserving global minima at \(S_s = \phi_s = 0\) and \(S_s = 0, \phi_s = -\frac{\mu_B}{\lambda_B} \) and Susy breaking local minima at \(\langle \phi_s \rangle = -\frac{\mu_B}{2\lambda_B}\) where \(S_s\) remains undetermined with a condition \(|\langle S_s \rangle| \geq |\langle \phi_s \rangle|\). \(\langle S \rangle\) can be fixed either by radiative corrections \([147]\) or by couplings to N=1 supergravity \([148, 149]\). In \([146]\) \(\langle S \rangle\) is determined by coupling \(S, \phi\) fields to N=1 supergravity as reviewed below.

7.4.1 Coupling to Supergravity

Supergravity potential \([150, 151, 152, 153]\) for the scalar field \(Z_I\) is

\[ V = E(|F^I + Z^* Z \kappa^2 W|^2 - 3\kappa^2 |W|^2) \quad ; \quad E \equiv e^{\sum_i |Z_i|^2} \] (7.14)

Visible sector VEVs \((z_i)\) and \(\phi_s\) preserve global Susy \((\frac{\partial W}{\partial Z_i} = 0 = D(Z^I))\)

\[ V(S_s) = e^{\kappa^2 |S_s|^2 + \delta \{ (\delta \kappa^2 |\hat{W}_0 + S_s\theta|^2 + |\theta + \kappa^2 S_s(\hat{W}_0 + S_s\theta)|^2 - 3\kappa^2 |\hat{W}_0 + S_s\theta|^2 \}} \] (7.15)

Where \(\delta = \kappa^2 (|\tilde{\phi}_s|^2 + \sum_i |\tilde{z}_i|^2)\) denotes Susy preserving VEVs contribution and the contribution of \(S_s\) has been separated. The potential written in terms of dimensionless variables

\[ x = \frac{\hat{W}_0}{\theta} \quad ; \quad y = \kappa S_s \quad ; \quad \varphi_x = \text{Arg}[x] \quad ; \quad \varphi_y = \text{Arg}[y] \] (7.16)

\[ \tilde{V} \equiv \frac{V}{|\theta|^2} = \left\{ (|x|^2 + |y|^2)(\delta - 3) + (1 + |y|^2)^2 + |x|^2 |y|^2 + 2 \cos(\varphi_y - \varphi_x)|x||y|(\delta - \varphi_y - \varphi_x) \right\} \] (7.17)
Minimum is achieved if

\[ V = V_{[y]} = V_{\varphi_y} = 0 = V_{[y]|\varphi_y} ; \quad V_{[y]|y}, V_{\varphi_y} > 0 \]  

(7.18)

The solution is

\[ \varphi_y = \varphi_x ; \quad x = 2 - \sqrt{3} - \delta \]

\[ y = y_0 = \sqrt{3 - \delta} - 10\delta \bar{y} \delta V = 4\sqrt{3 - \delta} \]

\[ \frac{\partial \varphi_y}{\partial \varphi_y} V = 4\delta \sqrt{3 - \delta} - 16\delta - 32\sqrt{3 - \delta} + 56 \]

(7.19)

provided

\[ \delta < 3 - \left(1 + \sqrt{\frac{\kappa^2 \theta}{2\lambda_B}}\right)^2 \approx 2 \]

(7.20)

The globally undetermined VEV \( \langle S_s \rangle \sim M_\rho \) is now fixed. In gravity mediated scenario, gravitino mass is given by

\[ m^2_3 = \kappa^2 |\sqrt{E(W_0 + \overline{W_H})}| \]

(7.21)

Typical range of gravitino mass require cancellation among \( \bar{W} \) and \( W_{GUT} \) such that

\[ |\overline{W_0 + W_H}| = M_\rho \theta | < 10^{39} - 10^{41} \text{GeV}^3 \]

(7.22)

BM superpotential parameters determine Susy breaking scale :

\[ \sqrt{|F_S|} = |\theta| = \left|\begin{array}{c} \frac{\mu_B}{2\sqrt{\lambda_B}} \end{array}\right| \sim 10^{10.5} - 10^{11.5} \text{GeV} \]

(7.23)

### 7.4.2 Gauged \( O(N_g) \) with Hidden Sector Superpotential

Considering BM superpotential as a hidden sector of supergravity, total Superpotential is given by

\[ W = W_H + W_{GUT}(z_i) \]

(7.24)
7.4 Bajc-Melfo Superpotential

where \( W_H(S_s, \phi_s) = W_0 + S_s(\mu_B \phi_s + \lambda_B \phi_s^2) \), \( z_i \) represents GUT chiral and symmetric multiplets of \( O(N_g) \) whose VEVs determine supersymmetric vacuum from vanishing of F and D terms

\[
F^i = \frac{\partial W}{\partial z_i} = 0
\]

\[
D^a_{GUT}(\bar{z}_i, \bar{z}_i^*) = 0 \tag{7.25}
\]

Supergravity potential representing D-term contribution is

\[
V_D = \frac{g_\alpha^2}{4} \left[ (z_i^* + \frac{F^i}{\kappa^2 W}) (T^\alpha)_i^j z_j + h.c. \right] \tag{7.26}
\]

Since F terms vanish for all \( z_I \) except \( S_s \) (which is gauge singlet) we are left with just global supersymmetric D-terms. The \( O(N_g) \) D terms are given as :

\[
D^a_{O(N_g)} = \text{Tr} (\hat{\phi}^I [T^a, \hat{\phi}] + \hat{S}^I [T^a, \hat{S}]) + \bar{D}_X^a
\]

\[
\bar{D}_X^a = \sum_i \bar{z}_i^I T^a \bar{z}_i \tag{7.27}
\]

where \( \bar{D}_X^a \) is the visible sector contribution, and \( T^a, T^a \) are \( O(N_g) \) generators in the fundamental and generic representations. We can consider \( S, \phi \) as traceful multiplets of \( O(N_g) \), traceless part still remains undetermined, to fix family D-terms :

\[
W_H = \text{Tr} S (\mu_B \phi + \sqrt{N_g} \lambda_B \phi^2) \tag{7.28}
\]

\[
S = \hat{S} + \frac{1}{\sqrt{N_g}} S_s I_{N_g} \quad ; \quad \text{Tr} \hat{S} = 0 \tag{7.29}
\]

here \( I_{N_g} \) is the unit matrix of order \( N_g \). All \( O(N_g) \) non-singlet fields : \( \hat{S}_{ab}, \hat{\phi}_{ab} \) and visible sector Higgs fields enter into the family D-terms. As discussed in the previous section flavour singlet VEV (\( S_s \)) is determened by supergravity effects and non-singlet part is fixed by the \( O(N_g) \) D-terms and supergravity soft mass terms.
7.4.3 \( \langle S \rangle \) fixation

A. \( N_g = 2 \)

Using \( O(2) \approx U(1) \) isomorphism, \( S \) (similarly \( \phi \)) is defined as

\[
S = \frac{1}{2} \begin{pmatrix}
\sqrt{2}S_+ + S_- + S_s + S_- - i(S_+ - S_-) \\
i(S_- - S_+) - i(S_- - S_+) - \sqrt{2}S_s - (S_+ + S_-)
\end{pmatrix}
\] (7.30)

where \( S_\pm, S_s \) are properly normalized fields so that

\[
\text{Tr} S^\dagger S = S^\dagger S + S^\dagger S_- + S^\dagger S_s
\]

and

\[
\text{Tr} S\phi = S_s\phi_s + S_{(\phi)}
\]

The superpotential becomes

\[
W_H = \mu_B(S_s\phi_s + S_{(\phi)}) + \lambda_B(S_s\phi_s^2 + 2S_s\phi_s\phi_- + 2\phi_sS_{(\phi)})
\] (7.31)

\( F_s \) is non zero(\( \theta \)) and all other F-term vanish for \( \bar{\phi}_s = -\frac{\mu_B}{2\lambda_B}, \bar{\phi}_\pm = 0 \) and \( S_{s,\pm} \) remain undetermined. Using the values of fields :

\[
V(S_+, S_-) = \frac{m_3^2}{2} \left(|S_+|^2 + |S_-|^2\right) + \frac{g_f^2}{2} \left(|S_+|^2 - |S_-|^2 + \bar{D}_X\right)^2
\] (7.32)

here \( \bar{D}_X = \sum q_i|\bar{z}_i|^2 \), \( q_i \) is the family symmetry charge of the visible sector VEV \( Z_i \).

The minimum will occur when

\[
S_{-x} = \sqrt{|\bar{D}_X| - x \frac{m_3^2}{g^2}} \; ; \; \; S_x = 0 \; \; (x = \text{Sign}[\bar{D}_X])
\] (7.33)

Detailed mass spectrum of \( S, \phi \) fields can be found in [146].

B. \( N_g = 3 \)

\( S_{ab} \) (\( \phi_{ab} \)) in terms of \( T_3 \) eigenfields :
\[
\begin{pmatrix}
\frac{1}{6}(\sqrt{6} S_0 + 3i S_{-2} - 3i S_{+2} + 2\sqrt{3} S_s) & \frac{1}{2}(S_{+2} + S_{-2}) & \frac{1}{2}(S_+ + S_-) \\
\frac{1}{2}(S_{-2} + S_{+2}) & \frac{1}{6}(\sqrt{6} S_0 - 3i S_{-2} + 3i S_{+2} + 2\sqrt{3} S_s) & \frac{1}{2}(S_+ - S_-) \\
\frac{1}{2}(S_+ + S_-) & \frac{1}{2}i(S_+ - S_-) & \frac{1}{\sqrt{3}}(S_s - \sqrt{2} S_0)
\end{pmatrix}
\]

where \( S_{s,0,\pm,\pm2} \) are properly normalized fields so that

\[
\text{Tr} S^\dagger S = S^\dagger_{+2} S_{+2} + S^\dagger_{-2} S_{-2} + S^\dagger_+ S_+ + S^\dagger_- S_- + S^\dagger_s S_s \\
\text{Tr} S \phi = S_s \phi_s + S_{(+\phi_-)} + S_{(+2\phi_2)}
\]

Now D-terms form a \( O(3) \) vector. It is convenient to use a basis where D-terms point in the third direction \( \vec{D}_X^\alpha = \delta_3^\alpha |\vec{D}_X| \). This can be achieved by performing the following rotations:

\[
O = R_{23}[\theta_X].R_{12}\left[\frac{\pi}{2} - \varphi_X\right]
\]

\[
\theta_X = \text{ArcTan} \left[ \sqrt{\frac{V_2^2 + V_3^2}{V_1^2}} \right] ; \quad \varphi_X = \text{ArcTan} \left[ \frac{V_2}{V_1} \right]
\]

where \( V^\alpha = \vec{D}_X^\alpha \). The potential for the flat directions from \( \hat{S}' \) is now

\[
V[\hat{S}'] = m_{3/2}^2(|S'_0|^2 + |S'_+|^2 + |S'_{-2}|^2 + |S'_{+2}|^2 + |S'_{-2}|^2) \\
+ \frac{g_f^2}{2} \{( |S'_+|^2 + 2|S'_{+2}|^2 - |S'_-|^2 - 2|S'_{-2}|^2 + (\vec{D}_X')^2 \} \\
+ 2\text{Tr}(S'^\dagger[T_+, S'])\text{Tr}(S'^\dagger[T_-, S'])
\]

Solution found is

\[
|\vec{S}_{-2}| = \sqrt{\frac{|\vec{D}_X|^2}{2} - \frac{m_{3/2}^2}{4g_f^2}} ; \quad |\vec{S}_{-2,+,+2}| = 0
\]

In [146] it is shown that BM type hidden sector necessarily imply a number of light SM singlet scalars \( (O(m_{3/2})) \) and even lighter fermions that get mass only from radiative effects. These modes are reminiscent of the light moduli in string theory. Note that these light modes supplement the singlet \( (G[1,1,0] \) sector) pseudo-Goldstones from the visible sector yielding a very rich set of possible DM candidates. Light modes of the BM superpotential may provide light DM candidates (< 50) GeV.
as indicated by the DAMA/LIBRA [154] experiments.

7.5 Analytical and Numerical Analysis

Writing VEVs \( \{ p, \omega, a, \sigma, \dot{\sigma}, \chi \} = \frac{m}{\lambda} \{ P, W, A, \acute{\sigma}, \acute{\dot{\sigma}}, \acute{\chi} \} \), \( \dot{\chi}_A = \dot{\chi}_{AA} + \xi \) in units of \( m/\lambda \), we can eliminate all the parameters in F-term equations (Eq. 7.5-7.9) except two ratios \( \xi = \frac{M}{m} \) and \( \frac{\lambda}{\eta} \):

\[
2 \left( \frac{m}{\lambda} \right)^2 (P - A - A^2 + W^2) = 0 \tag{7.38}
\]

\[
\left( \frac{m}{\lambda} \right)^2 (P + W + (P + 2A + 3W)W + W(P + 2A + 3W)) = 0 \tag{7.39}
\]

\[
\left( \frac{m}{\lambda} \right)^2 (\xi \sigma + (\dot{\chi} \sigma + \dot{\sigma} \dot{\chi})/2) = 0 \tag{7.40}
\]

\[
\left( \frac{m}{\lambda} \right)^2 (\xi \tilde{\sigma} + (\tilde{\chi} \tilde{\sigma} + \tilde{\sigma} \tilde{\chi})/2) = 0 \tag{7.41}
\]

\[
\tilde{\sigma} \sigma + \tilde{\dot{\sigma}} \dot{\sigma} = -\frac{4\lambda}{\eta} (P + 3W^2) = \frac{\lambda^2 F}{m^2} = \tilde{F} \tag{7.42}
\]

It is convenient to use dimensionless form of equations for SSB analysis because we can get most of the VEVs independent of model parameters. Before analyzing realistic SM case (\( N_g = 3 \)) we will study the simplest toy model (\( N_g = 2 \)).

7.5.1 Toy Model (\( N_g = 2 \))

For \( N_g = 2 \), \( \hat{\Sigma} = \{ \tilde{\sigma}_{11}, \tilde{\sigma}_{22}, \tilde{\sigma}_{12} \} \), the matrix \( \Xi \) involves the combinations \( \tilde{\chi}_A = \tilde{\chi}_{AA} + \xi \):

\[
\Xi = \begin{pmatrix}
\tilde{\chi}_1 & 0 & \tilde{\chi}_{12} \\
0 & \tilde{\chi}_2 & \tilde{\chi}_{12} \\
\tilde{\chi}_{12} & \tilde{\chi}_{12} & \tilde{\chi}_1 + \tilde{\chi}_2
\end{pmatrix} \tag{7.43}
\]
\[ \text{Det}[\Xi] = (\tilde{\chi}_1 + \tilde{\chi}_2)(\tilde{\chi}_{12}^2 - \tilde{\chi}_1 \tilde{\chi}_2) = 0 \]

\[ \Rightarrow \quad \tilde{\chi}_1 = -\tilde{\chi}_2 \quad \text{or} \quad \tilde{\chi}_{12} = \pm \sqrt{\tilde{\chi}_1 \tilde{\chi}_2} \quad (7.44) \]

\text{Det}[\Xi] should vanish for non-trivial solutions. Null 2 \times 2 minors provide \( \tilde{\chi}_1 = \tilde{\chi}_2 = \tilde{\chi}_{12} = 0 \) (\( \Xi \equiv 0 \)). Thus \( \text{Rank}[\Xi] < 2 \) implies \( \text{Rank}[\Xi] = 0 \) so that all the six \( \sigma, \bar{\sigma} \) remain undetermined. However we find that \( \text{Rank}[\Xi] = 0 \) is a degenerate case implying large colored and charged pseudo-Goldstone multiplets so we consider only \( \text{Rank}[\Xi] = 2 \) case. For a non-trivial solution, one out of two factors \((\tilde{\chi}_1 + \tilde{\chi}_2)\) and \((\tilde{\chi}_{12}^2 - \tilde{\chi}_1 \tilde{\chi}_2)\) of \( \text{Det}[\Xi] \) should vanish. One can calculate \( \bar{\sigma}_{11} \) and \( \bar{\sigma}_{22} \) in terms of \( \bar{\sigma}_{12} \) from \( \Xi \cdot \hat{\Sigma} = 0 \) as

\[ \bar{\sigma}_{11} = -\frac{\tilde{\chi}_{12}}{\tilde{\chi}_1} \bar{\sigma}_{12} \quad ; \quad \bar{\sigma}_{22} = -\frac{\tilde{\chi}_{12}}{\tilde{\chi}_2} \bar{\sigma}_{12} \quad (7.45) \]

\[ \text{Det}[\bar{\sigma}] = \frac{(\tilde{\chi}_{12}^2 - \tilde{\chi}_1 \tilde{\chi}_2)}{\tilde{\chi}_1 \tilde{\chi}_2} \bar{\sigma}_{12}^2 \quad (7.46) \]

\text{Det}[\bar{\sigma}] has a factor \((\tilde{\chi}_{12}^2 - \tilde{\chi}_1 \tilde{\chi}_2)\) in common with \( \text{Det}[\Xi] \) which will cause \( \text{Det}[\bar{\sigma}] \) to also vanish if we choose this factor to be zero to make \( \text{Det}[\Xi] \) vanish. In MSGUTs (also in YUMGUTs), Majorana mass of the right handed neutrinos is determined by \( (\bar{\Sigma}) = \bar{\sigma} \), which requires invertible VEV for Type I seesaw contribution. Therefore we analyze only the branch \((\tilde{\chi}_1 + \tilde{\chi}_2) = 0 \) for vanishing \( \text{Det}[\Xi] \). Eq. \( (7.42) \) then implies

\[ \bar{\sigma}_{11}^2 = \frac{\bar{F}_{11} \tilde{\chi}_{12}^2}{2(\tilde{\chi}_{12}^2 + \tilde{\chi}_1^2)} \quad ; \quad \bar{F}_{11} = \bar{F}_{22} \quad ; \quad \bar{F}_{12} = 0 \quad (7.47) \]

We solve Eq. \( (7.39) \) (linear in \( P \)) for all the components of \( P \). Using calculated \( P \) values, solve \( \tilde{\chi}_1 = -\tilde{\chi}_2 \), \( \bar{F}_{12} = 0 \) and \( \bar{F}_{11} = \bar{F}_{22} = 0 \) for \( A_{11}, A_{12} \) and \( A_{22} \). The remaining equations (Eq. \( (7.38) \)) can be completely expressed in terms of \( W \) and \( \xi \). We used a minimization method for a numerical solution of \( W \) for a convenient \( \xi \). Using these numerical values of \( P, A, W, \bar{\sigma} \) (given in Appendix A) and randomly chosen YUMGUT parameters \( (\lambda, \eta, \gamma, \bar{\gamma}, h, f) \), we find \( M_H \) values from \( \text{Det}[H] = 0 \). Then Yukawas corresponding to all allowed value of \( M_H \) are determined. Yukawa eigenvalues, mixing angles and neutrino masses are presented in Table \( (7.1) \) for \( f \sim h \) and in Table \( (7.2) \) when \( f \) is smaller by a factor of \( 10^{-3} \). In \( f \sim h \) case we have acceptable fermion hierarchy and mixing but too small neutrino masses which is the main failure of MSGUT. One can boost Type I seesaw contribution by suppressing
### Table 7.1: Yukawa eigenvalues and mixing angles for $N_g=2, f=-0.13$.

| Parameter | Value                  |
|-----------|------------------------|
| $M_H$     | I     | II     | III    |
|           | 0.049 + 0.190i         | 0.599 + 0.791i | 1.39 + 0.80i |
| $Y_u$     | 0.1537, 0.0080         | 0.1293, 0.0118  | 0.0685, 0.0214 |
| $Y_d$     | 0.0537, 0.0043         | 0.0562, 0.0051  | 0.0359, 0.0052 |
| $Y_l$     | 0.0424, 0.0027         | 0.0712, 0.0065  | 0.0147, 0.0063 |
| $Y_\nu$   | 0.2515, 0.0233         | 0.0576, 0.0053  | 0.0911, 0.0028 |
| $\theta_{\text{CKM}}$(deg.) | 5.15          | 2.27 $\times$ 10$^{-6}$ | 7.41          |
| $\theta_{\text{PMNS}}$(deg.) | 14.5         | 2.32 $\times$ 10$^{-5}$ | 33.7          |
| $m_\nu$(meV) | 0.0255, 0.2791 | 0.0013, 0.0144  | 0.0011, 0.0121 |
| $\Delta m_\nu^2$(eV$^2$) | 7.73 $\times$ 10$^{-8}$ | 2.06 $\times$ 10$^{-10}$ | 1.45 $\times$ 10$^{-10}$ |

$\tilde{M}_\nu^c \equiv \lambda M_{\nu^c}/m = \{0.6969, 0.0636\}$. $m/\lambda$ is taken to be 10$^{16}$ GeV to estimate $\Delta m_\nu^2$. $\lambda = -0.038 + .005 i, \eta = 0.4, \gamma = 0.32, \bar{\gamma} = -1.6, h = .34, \xi = 0.8719 + .5474i$.

### Table 7.2: Effect of reducing $f$ : Yukawa eigenvalues and mixing angles for $N_g=2, f=-0.00013$ and other parameters same as in Table [7.1]. Notice that the light neutrino masses are in an acceptable range but $Y_u=Y_\nu$, $Y_d=Y_l$ and the quark mixing is negligible.

| Parameter | Value                  |
|-----------|------------------------|
| $M_H$     | I     | II     | III    |
|           | 0.049 + 0.190i         | 0.599 + 0.791i | 1.39 + 0.80i |
| $Y_u$     | 0.1761, 0.0131         | 0.1108, 0.0101  | 0.0721, 0.0140 |
| $Y_d$     | 0.0507, 0.0038         | 0.0569, 0.0052  | 0.0283, 0.00552 |
| $Y_l$     | 0.0507, 0.0038         | 0.0569, 0.0052  | 0.0283, 0.0055 |
| $Y_\nu$   | 0.1762, 0.0131         | 0.1108, 0.0101  | 0.0721, 0.0140 |
| $\theta_{\text{CKM}}$(deg.) | 0.00486          | 2.47 $\times$ 10$^{-9}$ | 0.00767          |
| $\theta_{\text{PMNS}}$(deg.) | 8.7             | 2.79 $\times$ 10$^{-6}$ | 26.8            |
| $m_\nu$(meV) | 10.05, 110.19 | 4.86, 53.29     | 4.39, 48.12     |
| $\Delta m_\nu^2$(eV$^2$) | 0.01204             | 0.00282         | 0.00230         |

$\tilde{M}_\nu^c \equiv \lambda M_{\nu^c}/m = \{0.000697, 0.0000636\}$. 

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f which implies \( Y_u = Y_\nu \) and \( Y_d = Y_\ell \) (see Table 7.2). Type II seesaw contribution is generated by VEV of O[1,3,-2] multiplet \([36, 70, 55, 57]\) which is now family group triplet. In the MSGUT(NMSGUT) Type I dominates over the Type II. Type II contribution needs to be re-examined in the YUMGUT. Complete superheavy spectra (in units of \( m/\lambda \)) for the solution found is presented in Table 7.7 in Appendix B. Only the SM singlet sector \( G[1,1,0] \) has pseudo-Goldstones (which can act as DM candidates).

7.5.2 Realistic Case \((N_g = 3)\)

Symmetry breaking equations in this case are more complex and offer a number of phenomenologically interesting possibilities like light sterile neutrino and novel DM candidate from MSSM singlet sector \( G[1,1,0] \). Like \( N_g = 2 \) case, \( \tilde{\sigma} \) equations can be written as

\[
 \Xi \cdot \hat{\Sigma} = 0 \quad (7.48)
\]

Now

\[
\Xi = \begin{pmatrix}
\tilde{x}_1 & 0 & 0 & \tilde{x}_{12} & \tilde{x}_{13} & 0 \\
0 & \tilde{x}_2 & 0 & \tilde{x}_{12} & 0 & \tilde{x}_{23} \\
0 & 0 & \tilde{x}_3 & 0 & \tilde{x}_{13} & \tilde{x}_{23} \\
\tilde{x}_{12} & \tilde{x}_{12} & 0 & \tilde{x}_{11} + \tilde{x}_2 & \tilde{x}_{23} & \tilde{x}_{13} \\
\tilde{x}_{13} & 0 & \tilde{x}_{13} & \tilde{x}_{23} & \tilde{x}_{11} + \tilde{x}_3 & \tilde{x}_{12} \\
0 & \tilde{x}_{23} & \tilde{x}_{23} & \tilde{x}_{13} & \tilde{x}_{12} & \tilde{x}_{2} + \tilde{x}_3
\end{pmatrix} \quad (7.49)
\]

and \( \hat{\Sigma} = \{\tilde{\sigma}_{11}, \tilde{\sigma}_{22}, \tilde{\sigma}_{33}, \tilde{\sigma}_{12}, \tilde{\sigma}_{13}, \tilde{\sigma}_{23}\} \)

\[
\text{Det}[\Xi] = (\tilde{x}_1 \tilde{x}_2 \tilde{x}_3 - \tilde{x}_1 \tilde{x}_2^2 - \tilde{x}_{12} \tilde{x}_3 + 2 \tilde{x}_{12} \tilde{x}_{13} \tilde{x}_{23} - \tilde{x}_{13}^2 \tilde{x}_2) \\
(\tilde{x}_1^2 \tilde{x}_2 + \tilde{x}_1 \tilde{x}_3 - \tilde{x}_1 \tilde{x}_{12}^2 - \tilde{x}_1 \tilde{x}_{13}^2 + \tilde{x}_1 \tilde{x}_2^2 + 2 \tilde{x}_1 \tilde{x}_2 \tilde{x}_3 + \tilde{x}_1 \tilde{x}_3^2 \\
- \tilde{x}_{12} \tilde{x}_2 - 2 \tilde{x}_{12} \tilde{x}_{13} \tilde{x}_{23} - \tilde{x}_{13} \tilde{x}_3 + \tilde{x}_2 \tilde{x}_3 - \tilde{x}_2 \tilde{x}_{23}^2 + \tilde{x}_2 \tilde{x}_3^2 - \tilde{x}_{23} \tilde{x}_3)(7.50)
\]
$\text{Det} [\Xi]$ should vanish for the non-trivial solution. Order of mass matrices will be double that of $N_g = 2$ case and can be written using $\Omega_3[V]$ matrix function defined as:

$$
\Omega_3[V] \equiv \begin{pmatrix}
V_{11} & 0 & 0 & \frac{V_{12}}{\sqrt{2}} & \frac{V_{13}}{\sqrt{2}} & 0 \\
0 & V_{22} & 0 & \frac{V_{12}}{\sqrt{2}} & 0 & \frac{V_{23}}{\sqrt{2}} \\
0 & 0 & V_{33} & 0 & \frac{V_{13}}{\sqrt{2}} & \frac{V_{23}}{\sqrt{2}} \\
\frac{V_{12}}{\sqrt{2}} & \frac{V_{13}}{\sqrt{2}} & 0 & \frac{V_{11} + V_{22}}{2} & \frac{V_{23}}{2} & \frac{V_{13}}{2} \\
\frac{V_{12}}{\sqrt{2}} & 0 & \frac{V_{23}}{2} & 0 & \frac{V_{11} + V_{22}}{2} & \frac{V_{12}}{2} \\
0 & \frac{V_{23}}{2} & \frac{V_{13}}{2} & \frac{V_{12}}{2} & \frac{V_{23} + V_{23}}{2} & \frac{V_{12}}{2}
\end{pmatrix}
$$

(7.51)

Matter Yukawas can be written by the same procedure as in the $N_g=2$ case:

$$
Y_u = \begin{pmatrix}
\hat{h}V_1 + \hat{f}V_7 & (\hat{h}V_4 + \hat{f}V_{10})/\sqrt{2} & (\hat{h}V_5 + \hat{f}V_{11})/\sqrt{2} \\
(\hat{h}V_4 + \hat{f}V_{10})/\sqrt{2} & \hat{h}V_2 + \hat{f}V_8 & (\hat{h}V_6 + \hat{f}V_{12})/\sqrt{2} \\
(\hat{h}V_5 + \hat{f}V_{11})/\sqrt{2} & (\hat{h}V_6 + \hat{f}V_{12})/\sqrt{2} & \hat{h}V_3 + \hat{f}V_9
\end{pmatrix}
$$

(7.52)

$$
Y_d = \begin{pmatrix}
\hat{h}W_1 + \hat{f}W_{13} & (\hat{h}W_4 + \hat{f}W_{16})/\sqrt{2} & (\hat{h}W_5 + \hat{f}W_{17})/\sqrt{2} \\
(\hat{h}W_4 + \hat{f}W_{16})/\sqrt{2} & \hat{h}W_2 + \hat{f}W_{14} & (\hat{h}W_6 + \hat{f}W_{18})/\sqrt{2} \\
(\hat{h}W_5 + \hat{f}W_{17})/\sqrt{2} & (\hat{h}W_6 + \hat{f}W_{18})/\sqrt{2} & \hat{h}W_3 + \hat{f}W_{15}
\end{pmatrix}
$$

To avoid pseudo-Goldstones, we start with the non-degenerate case $\text{Rank}[\Xi] = 5$.

**A.** $\text{Rank}[\Xi] = 5$

Using Cramer’s rule, we can solve $\Xi \cdot \hat{\Sigma} = 0$ for five $\hat{\sigma}$ variables in terms of undermined one (say $\hat{\sigma}_{23}$)

$$
\begin{pmatrix}
\Xi_5 & v \\
v^T & \bar{\chi}_2 + \bar{\chi}_3
\end{pmatrix}
\begin{pmatrix}
\hat{\sigma} \\
\hat{\sigma}_{23}
\end{pmatrix} = 0 \quad \Rightarrow \quad \hat{\sigma} = -(\Xi_5^{-1}v)\hat{\sigma}_{23}
$$

(7.53)

Here $\hat{\sigma} = (\hat{\sigma}_{11}, \hat{\sigma}_{22}, \hat{\sigma}_{33}, \hat{\sigma}_{12}, \hat{\sigma}_{13}), \Xi_5$ and $v$ are upper left 5 × 5 block and 6th column (deleting the last element) of $\Xi$ respectively. We can construct $\hat{\sigma}$ from $\hat{v} = -(\Xi_5^{-1}v)$.
\[
\tilde{\sigma} = \begin{pmatrix}
\tilde{v}_1 & \tilde{v}_4 & \tilde{v}_5 \\
\tilde{v}_4 & \tilde{v}_2 & 1 \\
\tilde{v}_5 & 1 & \tilde{v}_3
\end{pmatrix} \tilde{\sigma}_{23}
\] (7.54)

Then
\[
\text{Det}[\tilde{\sigma}] = \frac{\text{Det}[\Xi]N_5(\tilde{\chi})}{D_5(\tilde{\chi})} \tilde{\sigma}_{23}^3
\]

where
\[
N_5(\tilde{\chi}) = (\tilde{\chi}_{13}^2 \tilde{\chi}_2 - 2\tilde{\chi}_{12} \tilde{\chi}_{13} \tilde{\chi}_{23} + \tilde{\chi}_1 \tilde{\chi}_{23} + \tilde{\chi}_{12}^2 \tilde{\chi}_3 - \tilde{\chi}_1 \tilde{\chi}_2 \tilde{\chi}_3)(\tilde{\chi}_{12} \tilde{\chi}_{13}^2 + \tilde{\chi}_{13} \tilde{\chi}_2 \tilde{\chi}_{23} - \tilde{\chi}_1 \tilde{\chi}_2 \tilde{\chi}_3 - \tilde{\chi}_{12} \tilde{\chi}_{23} \tilde{\chi}_3)
\]
\[
D_5(\tilde{\chi}) = (-\tilde{\chi}_1 \tilde{\chi}_{12} \tilde{\chi}_{13}^2 + \tilde{\chi}_1^2 \tilde{\chi}_{13}^2 \tilde{\chi}_2 - \tilde{\chi}_{12}^2 \tilde{\chi}_{13}^2 \tilde{\chi}_2 + \tilde{\chi}_1 \tilde{\chi}_{13} \tilde{\chi}_2^2 + \tilde{\chi}_1^2 \tilde{\chi}_{12} \tilde{\chi}_3 \\
-\tilde{\chi}_{12}^2 \tilde{\chi}_{13} \tilde{\chi}_3 - \tilde{\chi}_1^2 \tilde{\chi}_2 \tilde{\chi}_3 + \tilde{\chi}_1 \tilde{\chi}_{12} \tilde{\chi}_{23} \tilde{\chi}_3 + \tilde{\chi}_1 \tilde{\chi}_{13} \tilde{\chi}_2 \tilde{\chi}_3 - \tilde{\chi}_1^2 \tilde{\chi}_2 \tilde{\chi}_3 + \tilde{\chi}_1^2 \tilde{\chi}_2 \tilde{\chi}_3 \\
-2\tilde{\chi}_{12} \tilde{\chi}_{13} \tilde{\chi}_2 \tilde{\chi}_{23} \tilde{\chi}_3 + \tilde{\chi}_1 \tilde{\chi}_{12} \tilde{\chi}_{23} \tilde{\chi}_3 + \tilde{\chi}_1 \tilde{\chi}_{12} \tilde{\chi}_3^2 + \tilde{\chi}_1 \tilde{\chi}_{12}^2 \tilde{\chi}_3^2 - \tilde{\chi}_1^2 \tilde{\chi}_2 \tilde{\chi}_3^2 + \tilde{\chi}_1^2 \tilde{\chi}_2 \tilde{\chi}_3^2 \\
-\tilde{\chi}_1 \tilde{\chi}_{12}^2 \tilde{\chi}_3^2)^3
\] (7.55)

Thus
\[
\text{Det}[\tilde{\sigma}] \sim \text{Det}[\Xi] \Rightarrow \text{Det}[\tilde{\sigma}] = 0 = \text{Det}[M_{\nu}]
\] (7.56)

It implies the existence of one or more light sterile neutrino depending upon the zero eigenvalues of \(\tilde{\sigma}\) VEV. We proceed by solving the \(\text{Det}[\Xi] = 0\) condition for \(\tilde{\chi}_1\):
\[
\tilde{\chi}_1 = \frac{(\tilde{\chi}_{13}^2 \tilde{\chi}_2 - 2\tilde{\chi}_{12} \tilde{\chi}_{13} \tilde{\chi}_{23} + \tilde{\chi}_1 \tilde{\chi}_{23} \tilde{\chi}_3)}{(\tilde{\chi}_1 \tilde{\chi}_2) - \tilde{\chi}_{23}^2}
\] (7.57)

Like \(N_g=2\) case, we solve for the undetermined variable \(\tilde{\sigma}_{23}\) from one of the equations of Eq. (7.42) and P using Eq. (7.39). In the search program the remaining equations are used to solve for A and W. Notice the factor
\[
(\tilde{\chi}_1 \tilde{\chi}_2 \tilde{\chi}_3 - \tilde{\chi}_1 \tilde{\chi}_{23}^2 - \tilde{\chi}_{12} \tilde{\chi}_3 + 2\tilde{\chi}_{12} \tilde{\chi}_{13} \tilde{\chi}_{23} - \tilde{\chi}_{13} \tilde{\chi}_2)
\]
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| S.No. | $M_H$            | $Y_u$          | $Y_d$          |
|-------|------------------|----------------|----------------|
| 1.    | $2.55 + 0.13i$   | $0.007, 0.019, 0.368$ | $0.007, 0.014, 0.306$ |
| 2.    | $1.44 - 0.61i$   | $0.027, 0.13, 0.409$ | $0.009, 0.083, 0.242$ |
| 3.    | $1.28 + 0.75i$   | $0.063, 0.228, 0.424$ | $0.019, 0.083, 0.186$ |
| 4.    | $1.16 + 0.67i$   | $0.062, 0.193, 0.439$ | $0.02, 0.099, 0.188$ |
| 5.    | $1.06 - 0.73i$   | $0.009, 0.076, 0.458$ | $0.008, 0.078, 0.321$ |
| 6.    | $0.02 - 0.03i$   | $0.022, 0.254, 0.604$ | $0.009, 0.104, 0.289$ |

| S.No. | $Y_1$          | $Y_\nu$          | $\{\theta_{13}, \theta_{12}, \theta_{23}\}^Q$ |
|-------|----------------|------------------|-----------------------------------------------|
| 1.    | $0.007, 0.026, 0.421$ | $0.014, 0.032, 0.533$ | $0.56, 13.18, 1.58$ |
| 2.    | $0.023, 0.094, 0.314$ | $0.018, 0.213, 0.566$ | $3.42, 8.71, 3.87$ |
| 3.    | $0.031, 0.103, 0.212$ | $0.015, 0.187, 0.624$ | $6.65, 6.59, 1.11$ |
| 4.    | $0.029, 0.094, 0.259$ | $0.018, 0.283, 0.42$  | $2.6, 5.18, 1.96$  |
| 5.    | $0.009, 0.073, 0.4$  | $0.008, 0.214, 0.558$ | $1.51, 11.19, 4.61$ |
| 6.    | $0.007, 0.148, 0.338$ | $0.01, 0.159, 0.608$  | $1.04, 1.57, 6.03$  |

| S.No. | $m_\nu$(eV) | $M_\nu$ |
|-------|-------------|---------|
| 1.    | $0.16, 0.1597, 0.0104, 0.0104, 1.7 \times 10^{-6}$  | $365.07, 0, 0$ |
| 2.    | $0.2056, 0.2054, 0.05, 0.0499, 3.8 \times 10^{-6}$  | $365.07, 0, 0$ |
| 3.    | $0.3021, 0.302, 0.0781, 0.0781, 4.4 \times 10^{-7}$  | $365.07, 0, 0$ |
| 4.    | $0.1806, 0.1805, 0.1254, 0.1254, 7.8 \times 10^{-7}$  | $365.07, 0, 0$ |
| 5.    | $0.1946, 0.1945, 0.0533, 0.0532, 7.2 \times 10^{-7}$  | $365.07, 0, 0$ |
| 6.    | $0.2837, 0.2836, 0.0129, 0.0128, 6.4 \times 10^{-6}$  | $365.07, 0, 0$ |

Table 7.3: Yukawa eigenvalues and mixing angles for $N_g=3$ ($\text{Rank}[ar{\Xi}] = 5$), $f=0.9+0.7i$, $\lambda = 0.48+0.3i$, $\eta = 0.25$, $h = 1.3$, $\gamma = 0.05$, $\bar{\gamma} = -1.2$, $\xi = 3.645+0.363i$. $M_\nu$ is independent of $M_H$ value chosen.

of $\text{Det}[ar{\Xi}]$ occurs twice in $\text{Det}[\bar{\sigma}]$. We used this factor to achieve vanishing $\text{Det}[ar{\Xi}]$ in our numerical search program. So $\bar{\sigma}$ VEVs (see Appendix A) have two zero eigenvalues. We therefore need to integrate out only one heavy right handed neutrino. Leptonic superpotential is:

$$W_{\text{lep}} = \tilde{\nu}_A^T Y_{AB}^\nu \nu_B + \frac{1}{2} \tilde{\nu}_A^T M_{AB}^\nu \nu_B$$  \hspace{1cm} (7.58)

Using superpotential equation of motion:

$$\tilde{\nu}_3 = -\frac{Y_{34}^\nu \nu_A}{M_{33}^\nu}$$  \hspace{1cm} (7.59)
In a right handed neutrino diagonal basis, the effective superpotential reads:

$$W_{\text{eff}} = \bar{\nu}^T a Y_{\nu a B} \nu a^B + \frac{1}{2} \bar{\nu}^T M_{\nu a a} \nu a - \nu A^T \left( \frac{1}{2} Y^{\nu}_{3 A} Y^{\nu}_{3 B} M_{33}^\nu \right) \nu B$$  \hspace{1cm} (7.60)

We can write a dimension five ($\kappa$) operator for three left handed neutrinos:

$$\kappa_{AB} = -(Y^{\nu})^T_{A3} M_{33}^{-1}(Y^{\nu})_{3B}$$  \hspace{1cm} (7.61)

The light sterile neutrino will get Dirac mass only, so the mass matrix is given by:

$$M_{\text{light}} = \frac{1}{2} \begin{pmatrix} \kappa_{11} & \kappa_{12} & \kappa_{13} & Y^{\nu}_{11} & Y^{\nu}_{21} \\ \kappa_{21} & \kappa_{22} & \kappa_{23} & Y^{\nu}_{12} & Y^{\nu}_{22} \\ \kappa_{31} & \kappa_{32} & \kappa_{33} & Y^{\nu}_{13} & Y^{\nu}_{23} \\ Y^{\nu}_{11} & Y^{\nu}_{12} & Y^{\nu}_{13} & 0 & 0 \\ Y^{\nu}_{21} & Y^{\nu}_{22} & Y^{\nu}_{23} & 0 & 0 \end{pmatrix}$$  \hspace{1cm} (7.62)

Using the above solution and random superpotential parameters, we have calculated the Yukawa structure and neutrino masses for all the $M_H$ values as shown in Table 7.3. Notice that in this case neutrino masses are larger comparative to the earlier case due to the mixing of Dirac coupling. Superheavy spectrum is shown in Tables 7.8 and 7.9, it also exhibits pseudo-Goldstones in the $G[1,1,0]$ sector.

**B. Rank[Ξ] = 4**

In this case, one can determine 4 $\tilde{\sigma}$ variables, out of a total of six. We have additional conditions for vanishing 5 $\times$ 5 minors of $\Xi$ along with $Det[\Xi]$. By calculating $\tilde{\sigma}_{11}, \tilde{\sigma}_{22}, \tilde{\sigma}_{33}, \tilde{\sigma}_{12}$ in terms of ($\tilde{\sigma}_{13}, \tilde{\sigma}_{23}$), we can construct $\tilde{\sigma}$ as we have discussed earlier for Rank[Ξ] = 5:

$$\tilde{\sigma} = A\tilde{\sigma}_{13} + B\tilde{\sigma}_{23}$$  \hspace{1cm} (7.63)

where the matrices A and B are functions of the $\tilde{\chi}$ elements. Now we can’t factorize $Det[\tilde{\sigma}]$ separating $\tilde{\chi}$ elements and $\tilde{\sigma}_{13}, \tilde{\sigma}_{23}$ as in the previous case. So none of the $Det[\tilde{\sigma}]$ factors is common with $Det[\Xi]$. Even in the special case: $\tilde{\sigma}_{13} = \tilde{\sigma}_{23}$ $Det[\tilde{\sigma}]$ factors are different from that of $Det[\Xi]$. Therefore, Rank[Ξ] = 4 could be
a workable scenario with Type I seesaw neutrinos and without light sterile neutrinos. Vanishing of the common factor of $5 \times 5$ minors results in a complicated system. For convenience, we choose two factors to vanish which results in null dimension 5 minors. Thus, we get three conditions, one from $Det(\Xi)=0$ and following two from $5 \times 5$ minors:

$$\tilde{x}_{13} \tilde{x}_{23} - \tilde{x}_{12} \tilde{x}_3 = 0$$

$$\tilde{x}_{12}^2 \tilde{x}_{13} + \tilde{x}_{12} \tilde{x}_{23} \tilde{x}_{23} + \tilde{x}_{13} \tilde{x}_{23}^2 = 0$$ (7.64)
We solve these equations for \( \tilde{\chi}_2 \) and \( \tilde{\chi}_3 \)

\[
\tilde{\chi}_1 = \frac{\tilde{\chi}_{12} \tilde{\chi}_{13}}{\tilde{\chi}_{23}} ; \quad \tilde{\chi}_2 = -\frac{\tilde{\chi}_{13} (\tilde{\chi}_{12}^2 + \tilde{\chi}_{23}^2)}{\tilde{\chi}_{12} \tilde{\chi}_{23}} ; \quad \tilde{\chi}_3 = \frac{\tilde{\chi}_{13} \tilde{\chi}_{23}}{\tilde{\chi}_{12}} \quad (7.65)
\]

Besides Eqns. \( 7.39 \), \( 7.40 \), \( 7.42 \) we have seven equations (4 \( \tilde{\sigma} \) equations and 3 above conditions). For consistency we fix \( \xi \) parameter using one extra condition.

We calculate Yukawa eigenvalues (Eq. \( 7.52 \)) along with quark and lepton mixing angles for large and small values of \( f \) and a random illustrative set of superpotential parameters. The results are given as Tables \( 7.4 \) and \( 7.5 \) and superheavy...
spectrum in Tables 7.10 and 7.11.

C. Using symmetric irrep of \( O(3) \)

Another alternative that we have investigated is considering only the 5-dimensional irrep of \( O(3) \). We write the traceless symmetric \( 3 \times 3 \) representation as

\[
\hat{\phi}_{AB} = \phi_{11} \frac{(\lambda_3)_{AB}}{\sqrt{2}} + \phi_{22} \frac{(\lambda_8)_{AB}}{\sqrt{2}} + \frac{\phi_{KL} \delta_{(A \theta_B)}}{\sqrt{2}}
\]

Here \( \lambda_3 \) and \( \lambda_8 \) are the usual diagonal \( 3 \times 3 \) Gell-Mann matrices. Matrix \( \Xi \) is given by

\[
\Xi = \left(\begin{array}{cccc}
-\tilde{\chi}_{22} + 2\xi & -\tilde{\chi}_{11} - \tilde{\chi}_{22} + \xi & \tilde{\chi}_{12} & 0 & -\tilde{\chi}_{23} \\
-\tilde{\chi}_{11} - \tilde{\chi}_{22} + \xi & -\tilde{\chi}_{11} + 2\xi & \tilde{\chi}_{12} & -\tilde{\chi}_{13} & 0 \\
\tilde{\chi}_{12} & \tilde{\chi}_{12} & (\tilde{\chi}_{11}+\tilde{\chi}_{22}+2\xi) & \tilde{\chi}_{23} & \tilde{\chi}_{11} \\
0 & -\tilde{\chi}_{13} & \tilde{\chi}_{23} & -\tilde{\chi}_{22}+2\xi & \tilde{\chi}_{12} \\
-\tilde{\chi}_{23} & 0 & \tilde{\chi}_{13} & \tilde{\chi}_{12} & -\tilde{\chi}_{11}+2\xi
\end{array}\right)
\] (7.66)

\( \hat{\Sigma} = \{\hat{\sigma}_{11}, \hat{\sigma}_{22}, \hat{\sigma}_{12}, \hat{\sigma}_{13}, \hat{\sigma}_{23}\} \). In the present scenario matrix function \( \Omega'_3[V] \) has the following form:

\[
\Omega'_3[V] = \left(\begin{array}{cccc}
\frac{V_{11}+V_{22}}{2} & \frac{V_{11}-V_{22}}{2\sqrt{3}} & 0 & \frac{V_{13}}{2} & -\frac{V_{23}}{2} \\
\frac{V_{11}-V_{22}}{2\sqrt{3}} & \frac{V_{11}+V_{22}}{2} & \frac{V_{13}}{2} & -\frac{V_{23}}{2} & \frac{V_{23}}{2} \\
0 & \frac{V_{13}}{2\sqrt{3}} & \frac{V_{11}+V_{22}}{2} & \frac{V_{23}}{2} & \frac{V_{13}}{2} \\
-\frac{V_{23}}{2\sqrt{3}} & \frac{V_{13}}{2} & \frac{V_{11}+V_{22}}{2} & \frac{V_{13}}{2} & \frac{V_{13}}{2}
\end{array}\right)
\] (7.67)

Higgs mass matrix can be obtained by using \( \Omega'_3 \) and rows and columns are labelled by \( \{\sqrt{\frac{H_{11}+H_{22}}{2}}, \sqrt{\frac{3}{2}}(H_{11}+H_{22}), \sqrt{2}H_{12}, \sqrt{2}H_{13}, \sqrt{2}H_{23}\} \) and \( \{\sqrt{\frac{H_{11}+H_{22}}{2}}, \sqrt{\frac{3}{2}}(H_{11}+H_{22}), \sqrt{2}H_{12}, \sqrt{2}H_{13}, \sqrt{2}H_{23}\} \). Up and down quark Yukawas are given as:

\[
Y_u = \left(\begin{array}{cccc}
\hat{h}(\frac{V_1}{\sqrt{2}} + \frac{V_8}{\sqrt{6}}) + \hat{f}(\frac{V_1}{\sqrt{2}} + \frac{V_8}{\sqrt{6}}) & \hat{h}\frac{V_1}{\sqrt{2}} + \hat{f}\frac{V_1}{\sqrt{2}} & \hat{h}(\frac{V_1}{\sqrt{2}} + \frac{V_8}{\sqrt{6}}) + \hat{f}(\frac{V_1}{\sqrt{2}} + \frac{V_8}{\sqrt{6}}) & \hat{h}\frac{V_1}{\sqrt{2}} + \hat{f}\frac{V_1}{\sqrt{2}} \\
\hat{h}\frac{V_2}{\sqrt{2}} + \hat{f}\frac{V_2}{\sqrt{2}} & \hat{h}\frac{V_2}{\sqrt{2}} + \hat{f}\frac{V_2}{\sqrt{2}} & \hat{h}\frac{V_2}{\sqrt{2}} + \hat{f}\frac{V_2}{\sqrt{2}} & \hat{h}\frac{V_2}{\sqrt{2}} + \hat{f}\frac{V_2}{\sqrt{2}} \\
\hat{h}\frac{V_3}{\sqrt{2}} + \hat{f}\frac{V_3}{\sqrt{2}} & \hat{h}\frac{V_3}{\sqrt{2}} + \hat{f}\frac{V_3}{\sqrt{2}} & \hat{h}\frac{V_3}{\sqrt{2}} + \hat{f}\frac{V_3}{\sqrt{2}} & \hat{h}\frac{V_3}{\sqrt{2}} + \hat{f}\frac{V_3}{\sqrt{2}} \\
-2\hat{h}\frac{V_6}{\sqrt{6}} - 2\hat{f}\frac{V_6}{\sqrt{6}} & -2\hat{h}\frac{V_6}{\sqrt{6}} - 2\hat{f}\frac{V_6}{\sqrt{6}} & -2\hat{h}\frac{V_6}{\sqrt{6}} - 2\hat{f}\frac{V_6}{\sqrt{6}} & -2\hat{h}\frac{V_6}{\sqrt{6}} - 2\hat{f}\frac{V_6}{\sqrt{6}}
\end{array}\right)
\]
### 7.5 Analytical and Numerical Analysis

| S.No. | $M_H$        | $Y_u$       | $Y_d$       | $\{\theta_{13}, \theta_{12}, \theta_{23}\}^Q (deg.)$ |
|-------|--------------|-------------|-------------|-------------------------------------------------|
| 1.    | $135.29 + 11.98i$ | .054, .068, .1718 | .0001, .0012, .0019 | 8.23, 34.35, 32.04 |
| 2.    | $25.4 + 1.72i$   | 0.0, .0588, .0938 | 0.0, .0021, .0034 | $(1, 1.7, 1.2) \times 10^{-6}$ |
| 3.    | $24.6 + 1.16i$   | 0.0, .0614, .1325 | 0.0, .0016, .0036 | $8.12 \times 10^{-8}$, 0, 0 |
| 4.    | $18.41 + 1.4i$   | .001, .0555, .125  | .0035, .011, .0264 | 3.48, 6.01, 8.67 |
| 5.    | $18.32 + 1.23i$  | 0.0, .0584, .1322  | 0.0, .0112, .0255 | $4.86 \times 10^{-9}$, 0, 0 |

| S.No. | $Y_1$      | $Y_\nu$  | $\{\theta_{13}, \theta_{12}, \theta_{23}\}^L (deg.)$ | $\bar{M}_{\nu}$ |
|-------|------------|----------|-------------------------------------------------|-----------------|
| 1.    | .0029, .0066, .013 | .174, .2183, .551 | 5.92, 32.15, 9.6 | 22.39, 8.89, 7.05 |
| 2.    | 0.0, .0177, .0283 | 0.0, .2272, .3629 | $(1.1, 8.5) \times 10^{-7}$, 28.7 | 22.39, 8.89, 7.05 |
| 3.    | 0.0, .0068, .0147 | 0.0, .236, .5094 | $(0.3, 1.2) \times 10^{-6}$, 19.4 | 22.39, 8.89, 7.05 |
| 4.    | .0098, .0136, .038  | .0028, .122, .275  | 14.33, 20.53, 35.13 | 22.39, 8.89, 7.05 |
| 5.    | 0.0, .015, .034   | 0.0, .1284, .291   | $5.7 \times 10^{-9}$, 0, 42.03 | 22.39, 8.89, 7.05 |

| S.No. | $\mu_{\nu}(\text{meV})$ | $\Delta m_{\nu}^2(\text{eV}^2)$ |
|-------|--------------------------|----------------------------------|
| 1.    | $(1.29, 1.62, 4.1) \times 10^{-2}$ | $(.096, 1.42) \times 10^{-9}$ |
| 2.    | $6.5 \times 10^{-11}$, .012, .028 | $(1.49, 6.18) \times 10^{-10}$ |
| 3.    | $2.0 \times 10^{-10}$, .017, .045 | $(2.75, 17.9) \times 10^{-10}$ |
| 4.    | $1.5 \times 10^{-6}$, .010, .011 | $(1.08, .23) \times 10^{-10}$ |
| 5.    | $1.1 \times 10^{-10}$, .012, .013 | $(1.33, .29) \times 10^{-10}$ |

Table 7.6: Yukawa eigenvalues and mixing angles for traceless case $N_f=3$ ($\text{Rank} \Xi = 4$), $f = 0.23 + .04 i$. $\bar{M}_{\nu} \equiv \lambda M_{\nu} / m$. $m/\lambda$ is taken to be $10^{16}$ GeV to estimate $\Delta m_{\nu}^2$. $\lambda = 0.18 - 0.03i$, $\eta = 0.34$, $\gamma = -0.53$, $\tilde{\gamma} = -2.60$, $h = .14$, $\xi = 7.677 + 0.15772i$. $M_{\nu}$ is independent of $M_H$ value chosen.

\[
Y_d = \begin{pmatrix}
\hat{h}(\frac{W_{\nu}}{\sqrt{2}}) + \hat{f}(\frac{W_{\mu}}{\sqrt{2}}) + \hat{f}(\frac{W_{\mu}}{\sqrt{2}}) \\
\hat{h}(\frac{W_{\nu}}{\sqrt{2}}) + \hat{f}(\frac{W_{\mu}}{\sqrt{2}}) \\
\hat{h}(\frac{W_{\nu}}{\sqrt{2}}) + \hat{f}(\frac{W_{\mu}}{\sqrt{2}})
\end{pmatrix}
\]

(7.68)

We have solved the least degenerate ($\text{Rank} \Xi = 4$) case. This option has relatively fewer parameters so it is easier to perform numerical searches. We solved some equations analytically and remaining numerically to find solution. Yukawa eigenvalues and mixing angles are given in Table 7.6, superheavy spectrum in Tables 7.12 and 7.13.
7.6 Discussion and Outlook

We proposed\textsuperscript{[145]} dynamical generation of flavour based upon the Susy SO(10) and family gauge group. In literature O(3) family symmetry with traceful representation is considered for non-renormalizable and non-GUT Yukawa-on models\textsuperscript{[144]}, however our model is renormalizable and GUT based. Yukawon fields break flavour and GUT symmetry spontaneously. Emergence of light Higgs of the effective MSSM among the large number of YUMGUT MSSM type Higgs multiplets is ensured by the consistency condition $\text{Det}(\mathcal{H})=0$. SM fermion and neutrino Yukawa couplings are generated by the VEV of the Yukawon field. SO(10) Yukawa couplings are just single complex number thus parameter reduction is one of the main virtue of YUMGUTs. Consistent SSB is achieved with the introduction of $(O(N_g)$ symmetric- two field $S, \phi)$ BM (hidden sector) superpotential. $O(N_g)$ singlet $(S_s)$ breaks Susy and traceless part $\hat{S}$ is fixed against visible sector fields contribution to $O(N_g)$ D terms and thus facilitates YUMGUTs.

We have analyzed the toy model ($N_g=2$) and realistic case ($N_g=3$) without any optimization. As explained earlier the rank of the coefficient matrix $\Xi$ of the $F_{\sigma,\bar{\sigma}}=0$ equation is crucial for determining the SSB. For $N_g=2$ $\text{Rank}[\Xi] < 2 \Rightarrow \text{Rank}[\Xi] = 0$, so to avoid problematic pseudo-Goldstone non-degenerate rank reduction of homogeneous system provides a possible route to find Yukawa eigenvalues different by a factor of about 10, small quark and large lepton mixing angle.

In the realistic case ($N_g = 3$) we have considered several possibilities like $\text{Rank}[\Xi] = 5$, $\text{Rank}[\Xi] = 4$. If we consider the reducible 6-dimensional symmetric representation of O(3) with equal superpotential couplings for traceless and singlet part then non-degenerate case ($\text{Rank}[\Xi] = 5$) give rise to light sterile neutrino. This motivates reconsideration of the no-go \textsuperscript{[55]} in the MSGUT using light sterile neutrinos. Rank reduction of homogeneous system provides a possible route to find non-zero eigenvalues of $\sigma(\bar{\sigma})$ VEV. We have also studied the case using traceless-dimensional representation. We can’t use traceless representation for $N_g=2$ because cubic invariants in the superpotential do not contribute. We have calculated the complete superheavy spectrum for all the cases considered (given in Appendix B) to check the existence of pseudo-Goldstones which may be present when there is
Higgs duplication. Spectra do not contain pseudo-Goldstone except the SM singlet G[1,1,0] sector which does not affect unification.

In all the cases studied acceptable Yukawa hierarchy and mixing is achieved but neutrino masses generated are too small. Type I contribution can be raised by suppressing $f$ which provides unacceptable Yukawa structure. We have not considered the contribution of Type II seesaw generated by the VEV of symmetric multiplet $O^-$. Although we expect it to be small as compared to Type I as in MSGUTs, but some special points may yield significant contribution. Addition of 120-plet, which along with 10-plet is mainly responsible for generating charged fermion masses, is another way to get neutrino masses in experimentally measured range. The $\text{Rank}[\Xi] = 5$ case (considering 6 dim symmetric representation) phenomenology needs to be investigated because this provides sterile neutrino. With optimization, one can expect to find the flavour blind parameters of YUMGUT which can produce actual MSSM Yukawas. To completely demonstrate this idea we need to produce realistic SM mass mixing data respecting NMSGUT fitting features which will require a huge computational effort.

A number of experimental signals such as light moduli fields and singlet pseudo-Goldstones $[146]$ which can also be DM candidates are associated with our proposal. These fields also cause cosmological problems. Thus our work has laid the basis for an extensive program of future studies in unification and cosmology.
Appendix A : YUMGUT VEVs

The values of the VEVs of the YUMGUT Higgs fields responsible for breaking $SO(10) \rightarrow$ MSSM in units of $m/\lambda \sim 10^{16}$ GeV are:

1. $N_g=2$, $\text{Rank}[\Xi]=2$

\[
W = \begin{pmatrix}
0.141 - 0.203i & 0.3168 + 0.189i \\
0.3168 + 0.189i & -0.2667 + 0.3075i
\end{pmatrix}
\] (7.69)

\[
P = \begin{pmatrix}
-0.236 - 0.2001i & 0.1787 - 0.028i \\
0.1787 - 0.028i & -0.2297 + 0.1202i
\end{pmatrix}
\] (7.70)

\[
A = \begin{pmatrix}
-0.23 - 0.3521i & 0.3382 + 0.0777i \\
0.3382 + 0.0777i & -0.4475 + 0.2227i
\end{pmatrix}
\] (7.71)

\[
\tilde{\sigma} = \tilde{\sigma} = \begin{pmatrix}
0.0863 - 0.2366i & 0.1973 + 0.1041i \\
0.1973 + 0.1041i & -0.0863 + 0.2366i
\end{pmatrix}
\] (7.72)

\[
D_X = 2(|p^+|^2 - |p^-|^2 + 3(|a^+|^2 - |a^-|^2) + 6(|w^+|^2 - |w^-|^2) + \frac{1}{2}|\sigma_+|^2 - |\sigma_-|^2 + \frac{1}{2}|\tilde{\sigma}_+|^2 - |\tilde{\sigma}_-|^2) = -8.94
\] (7.73)

2. $N_g=3$

VEVs are written in prime basis where D-terms point in third direction.

A. $\text{Rank}[\Xi]=5$ (tracefull symmetric representation)

\[
\tilde{\sigma}' = \tilde{\sigma}' = \begin{pmatrix}
-8.1532 + 14.4793i & -11.7404 - 7.4196i & 0.194 - 1.1043i \\
-11.7404 - 7.4196i & 6.6912 - 9.4853i & 0.9135 + 0.2088i \\
0.194 - 1.1043i & 0.9135 + 0.2088i & 0.0124 + 0.0746i
\end{pmatrix}
\] (7.74)
7.6 Discussion and Outlook

\[ A' = \begin{pmatrix} -2.7937 - 0.4459i & -0.0298 - 1.8182i & 2.573 - 0.2572i \\ -0.0298 - 1.8182i & -2.6651 + 0.5903i & -0.4297 - 1.5744i \\ 2.573 - 0.2572i & -0.4297 - 1.5744i & -0.8325 - 0.1193i \end{pmatrix} \] (7.75)

\[ P' = \begin{pmatrix} 2.6802 - 0.9855i & 0.589 - 1.518i & -13.9797 + 0.005i \\ 0.589 - 1.518i & 4.1178 + 0.5751i & 1.9362 - 0.8664i \\ -13.9797 + 0.005i & 1.9362 - 0.8664i & 6.1195 + 0.1825i \end{pmatrix} \] (7.76)

\[ W' = \begin{pmatrix} 1.0342 + 0.1527i & -0.4614 + 0.2617i & 2.0891 + 0.4169i \\ -0.4614 + 0.2617i & -1.4776 - 0.1941i & -0.3529 + 2.8359i \\ 2.0891 + 0.4169i & -0.3529 + 2.8359i & 1.3112 + 0.0711i \end{pmatrix} \] (7.77)

\[ D'^a_X = 3319.0 \delta^a_3 \] (7.78)

B. \textit{Rank}[\Xi] = 4 (tracefull symmetric representation)

\[ \tilde{\sigma}' = \tilde{\sigma}' = \begin{pmatrix} -0.0515 - 2.2441i & 1.6389 + 0.9556i & 0.8735 + 1.689i \\ 1.6389 + 0.9556i & 0.4307 + 0.2916i & -1.8341 - 2.1534i \\ 0.8735 + 1.689i & -1.8341 - 2.1534i & 0.926 + 1.8694i \end{pmatrix} \] (7.79)

\[ A' = \begin{pmatrix} -1.1162 + 0.1493i & -0.6261 - 0.4133i & -0.1611 + 0.4422i \\ -0.6261 - 0.4133i & 0.3311 + 0.7292i & -0.3128 - 0.1764i \\ -0.1611 + 0.4422i & -0.3128 - 0.1764i & -0.7771 - 0.6807i \end{pmatrix} \] (7.80)

\[ P' = \begin{pmatrix} 0.7956 - 0.2474i & 0.0406 - 0.4419i & 0.6712 + 0.2745i \\ 0.0406 - 0.4419i & 0.3486 + 1.7078i & -0.0373 - 0.1994i \\ 0.6712 + 0.2745i & -0.0373 - 0.1994i & -0.4144 - 0.7517i \end{pmatrix} \] (7.81)

\[ W' = \begin{pmatrix} -0.0025 - 0.4259i & 0.0644 + 0.4904i & -0.3268 - 0.5708i \\ 0.0644 + 0.4904i & -0.3575 - 0.0647i & 0.0554 + 0.2052i \\ -0.3268 - 0.5708i & 0.0554 + 0.2052i & 0.5644 + 0.6209i \end{pmatrix} \] (7.82)
\[ D^a_X = 47.04 \delta_3^a \] (7.83)

C. \( \text{Rank}[\Xi] = 4 \) (traceless symmetric representation)

\[ \bar{\sigma}' = \tilde{\sigma}' = \begin{pmatrix} 1.5506 - 5.4394i & 1.7398 + 4.1109i & 0.0011 - 0.0811i \\ 1.7398 + 4.1109i & -1.6868 + 2.0758i & 0.0202 + 0.042i \\ 0.0011 - 0.0811i & 0.0202 + 0.042i & 0.1361 + 3.3636i \end{pmatrix} \] (7.84)

\[ A' = \begin{pmatrix} -1.8304 - 0.7199i & 1.5213 - 0.445i & -0.0854 + 0.0056i \\ 1.5213 - 0.445i & 0.6854 + 0.7288i & -0.0906 - 0.0529i \\ -0.0854 + 0.0056i & -0.0906 - 0.0529i & 1.145 - 0.009i \end{pmatrix} \] (7.85)

\[ P' = \begin{pmatrix} -0.2976 + 0.0331i & 0.3609 - 0.0673i & -0.2904 + 0.0343i \\ 0.3609 - 0.0673i & 0.2394 + 0.4181i & -0.5147 - 0.2562i \\ -0.2904 + 0.0343i & -0.5147 - 0.2562i & 0.0581 - 0.4512i \end{pmatrix} \] (7.86)

\[ W' = \begin{pmatrix} 1.0622 + 0.3626i & -0.7826 + 0.3279i & -0.0567 + 0.0102i \\ -0.7826 + 0.3279i & -0.3667 - 0.2597i & -0.1406 - 0.0597i \\ -0.0567 + 0.0102i & -0.1406 - 0.0597i & -0.6955 - 0.1029i \end{pmatrix} \] (7.87)

\[ D^a_X = 275.15 \delta_3^a \] (7.88)

**Appendix B : Superheavy Spectra**

We present here superheavy spectra (in units of the MSGUT scale parameter \( m/\lambda \)) in the toy model \( (N_g = 2) \) and realistic case \( (N_g = 3) \):
### 7.6 Discussion and Outlook

| Field  | Masses                      |
|--------|-----------------------------|
| $[SU(3), SU(2), Y]$ |                     |
| $A[1, 1, 4]$ | 4.093, 3.321, 0.137         |
| $B[6, 2, 5/3]$ | 0.106, 0.099, 0.091         |
| $C[8, 2, 1]$ | 1.727, 1.727, 1.224, 1.224, 0.614, 0.614 |
| $D[3, 2, 7/3]$ | 1.919, 1.433, 1.191, 0.810, 0.205, 0.134 |
| $E[3, 2, 1/3]$ | 1.475, 1.043, 0.716, 0.716, 0.677, 0.594, 0.506 |
|                   | 0.404, 0.277, 0.087, 0.073, 0.050, 0.004 |
| $F[1, 1, 2]$ | 1.794, 1.794, 1.681, 1.317, 0.289, 0.228, 0.018 |
| $G[1, 1, 0]$ | 1.672, 1.665, 1.248, 1.248, 0.766, 0.766, 0.504, 0.469, 0.208 |
|                   | 0.201, 0.079, 0.068, 0.055, 0.011, 0.009, 0.0 |
| $h^{(1)}[1, 2, 1]$ | 3.799, 2.812, 1.398, 1.182, 0.983, 0.74, 0.588, 0.511, 0.159, 0.024, 0.013 |
| $h^{(2)}[1, 2, 1]$ | 3.947, 2.961, 1.623, 1.247, 1.009, 0.726, 0.556, 0.51, 0.14, 0.044, 0.005 |
| $h^{(3)}[1, 2, 1]$ | 4.161, 3.196, 2.049, 1.289, 0.979, 0.710, 0.540, 0.520, 0.152, 0.029, 0.010 |
| $I[3, 1, 10/3]$ | 0.210, 0.192, 0.003         |
| $J[3, 1, 4/3]$ | 1.889, 1.889, 0.946, 0.740, 0.453, 0.278, 0.119, 0.086, 0.021, 0.006 |
| $K[3, 1, -8/3]$ | 1.591, 1.237, 0.116         |
| $L[6, 1, 2/3]$ | 1.066, 0.916, 0.757         |
| $M[6, 1, 8/3]$ | 1.340, 0.958, 0.493         |
| $N[6, 1, -4/3]$ | 1.795, 1.178, 0.345         |
| $O[1, 3, -2]$ | 1.127, 0.886, 0.084         |
| $P[3, 3, -2/3]$ | 0.902, 0.754, 0.595         |
| $Q[8, 3, 0]$ | 0.163, 0.126, 0.083         |
| $R[8, 1, 0]$ | 0.170, 0.119, 0.107, 0.086, 0.066, 0.047 |
| $S[1, 3, 0]$ | 0.090, 0.058, 0.011         |
| $t^{(1)}[3, 1, -2/3]$ | 3.264, 2.802, 1.824, 1.496, 1.175, 1.019, 0.89 |
|             | 0.824, 0.598, 0.495, 0.343, 0.202, 0.055, 0.026, 0.007 |
| $t^{(2)}[3, 1, -2/3]$ | 3.418, 2.936, 1.873, 1.636, 1.2, 1.053, 0.909, 0.824 |
|             | 0.692, 0.532, 0.454, 0.211, 0.077, 0.018, 0.001 |
| $t^{(3)}[3, 1, -2/3]$ | 3.650, 3.156, 2.097, 1.747, 1.273, 1.116, 0.926, 0.824 |
|             | 0.779, 0.541, 0.466, 0.223, 0.116, 0.023, 0.002 |
| $U[3, 3, 4/3]$ | 0.084, 0.070, 0.054         |
| $V[1, 2, -3]$ | 0.227, 0.208, 0.003         |
| $W[6, 3, 2/3]$ | 1.693, 1.324, 0.902         |
| $X[3, 2, -5/3]$ | 1.666, 1.666, 0.149, 0.102, 0.072, 0.070, 0.066 |
| $Y[6, 2, -1/3]$ | 0.167, 0.118, 0.058         |
| $Z[8, 1, 2]$ | 0.100, 0.086, 0.070         |

Table 7.7: Mass spectrum of superheavy fields in units of $m/\lambda \sim 10^{16}$ GeV for $N_g=2$. Only the spectra of $h[1, 2, \pm 1]$, $t[3, 1, \mp 2/3]$ (given in colors S.No. 1 (Blue), S.No. 2 (Green), S.No. 3 (Red)) depend on the value of $M_H$ chosen (see Table 7.1 for the values of $M_H$).
### Table 7.8: Mass spectrum of superheavy fields in units of $m/\lambda \sim 10^{16}$ GeV in 6-dim symmetric tracefull ($\text{Rank}[\Sigma] = 5$) scenario with $N_g = 3$. Only the spectra of $h[1, 2, \pm 1]$, $t[3, 1, \mp 2/3]$ depend on the value of $M_H$ chosen (see Table 7.9 for the spectra for each of the 6 values of $M_H$).

| Field $[\text{SU}(3), \text{SU}(2), Y]$ | Masses |
|--------------------------------------|--------|
| $A[1, 1, 4]$                        | 9.93, 8.08, 7.34, 4.06, 2.53, 0.84 |
| $B[6, 2, 5/3]$                      | 6.46, 6.38, 5.8, 5.18, 4.83, 2.88 |
| $C[8, 2, 1]$                        | 4.62, 4.62, 4.36, 4.36, 4., 4., 2.2, 2.2, 1.88, 1.88, 0.35, 0.35 |
| $D[3, 2, 7/3]$                      | 7.55, 4.64, 4.25, 3.95, 2.71, 2., 1.92, 1.61, 1.39, 1.1, 1.08, 0.53 |
| $E[3, 2, 1/3]$                      | 28.48, 28.48, 20.62, 19.48, 18.45, 18.19, 14.85, 13.38 |
|                                      | 5.94, 4.47, 3.32, 2.87, 2.54, 2.28, 2.2, 1.54, 1.36 |
|                                      | 1.28, 1.16, 1.06, 0.97, 0.8, 0.4, 0.25, 0.24 |
| $F[1, 1, 2]$                        | 24.66, 24.66, 21.78, 20.8, 19.4, 18.4 |
|                                      | 14.65, 13.88, 4.6, 3.65, 1.9, 1.7, 0.14 |
| $G[1, 1, 0]$                        | 45.55, 45.55, 39.11, 37.34, 36.03, 35.82, 27.46, 26.75, 15.96 |
|                                      | 14.2, 14.0, 13.83, 11.92, 11.41, 10.75, 8.62, 7.15, 6.35, 6.03 |
|                                      | 5.27, 4.47, 3.4, 2.15, 1.77, 0.76, 0.66, 0.11, 0.05, 0, 0, 0 |
| $I[3, 1, 10/3]$                     | 17.23, 13.25, 11.82, 4.61, 2.61, 1.17 |
| $J[3, 1, 4/3]$                      | 25.09, 25.09, 23.7, 23.02, 20.65, 19.5, 16.49, 15.45, 7.68 |
|                                      | 7.07, 5.18, 4.66, 4.51, 3.25, 3.07, 1.94, 0.86, 0.68, 0.23 |
| $K[3, 1, -8/3]$                     | 8.41, 5.21, 4.54, 1.46, 1.01, 0.44 |
| $L[6, 1, 2/3]$                      | 13.27, 9.28, 5.3, 4.92, 3.49, 0.91 |
| $M[6, 1, 8/3]$                      | 12.48, 8.46, 6.42, 4.37, 3.29, 0.71 |
| $N[6, 1, -4/3]$                     | 14.23, 10.71, 7.19, 7.07, 3.87, 0.22 |
| $O[1, 3, -2]$                       | 14.05, 9.35, 8.34, 5.48, 3.22, 1.05 |
| $P[3, 3, -2/3]$                     | 9.63, 7.14, 5.65, 2.8, 1.6, 1.27 |
| $Q[8, 3, 0]$                        | 15.14, 12.38, 8.53, 7.75, 2.31, 0.93 |
| $R[8, 1, 0]$                        | 27.29, 18.82, 13.31, 10.7, 9.3, 6.36 |
|                                      | 6.12, 6.07, 5.45, 3.68, 2.58, 0.15 |
| $S[1, 3, 0]$                        | 29.76, 19.87, 14.44, 10.53, 7.96, 1.74 |
| $U[3, 3, 4/3]$                      | 24.78, 15.76, 13.17, 6.8, 5.83, 3.13 |
| $V[1, 2, -3]$                       | 18.19, 16.8, 15.59, 13.91, 4.48, 2.43 |
| $W[6, 3, 2/3]$                      | 6.72, 5.37, 4.41, 2.72, 1.44, 0.15 |
| $X[3, 2, -5/3]$                     | 16.48, 16.48, 11.8, 9.05, 8.37, 6.22, 3.69 |
|                                      | 3.5, 2.79, 1.91, 1.31, 0.77, 0.49 |
| $Y[6, 2, -1/3]$                     | 7.51, 7., 6.1, 3.69, 2.2, 1.52 |
| $Z[8, 1, 2]$                        | 27.05, 18.02, 10.94, 9.02, 8.06, 0.93 |
### Table 7.9: Mass spectrum of superheavy fields $h[1, 2, \pm 1]$, $t[3, 1, \mp 2/3]$ which depend on the value of $M_H$ chosen in units of $m/\lambda \sim 10^{16}$ GeV in 6-dim symmetric tracefull ($Rank(\Xi) = 5$) scenario with $N_g = 3$ for all values of $M_H$.

| $M_H$ | $h[1, 2, 1]$ | $t[3, 1, -2/3]$ |
|-------|--------------|------------------|
| 2.55 + 0.13i | 34.81, 32.15, 27.12, 21.08 | 39.36, 35.17, 27.86, 26.76, 24.06 |
|         | 20.53, 18.99, 12.6, 12.06 | 21.67, 18.22, 13.95, 12.81, 10.72, 8.94 |
|         | 11.62, 10.96, 10.38, 9.51, 7.29 | 8.28, 7.46, 6.5, 5.07, 4.49, 4.09 |
|         | 6.87, 3.57, 2.7, 2.28, 2.06 | 3.44, 3.08, 2.23, 1.52, 1.36, 0.78 |
|         | 0.74, 0.59, 0.32, 0.13, 0.05 | 0.49, 0.43, 0.26, 0.21, 0.11, 0.04 |
| 1.44 − 0.61i | 34.77, 32.11, 27.08, 21.06 | 38.97, 36.25, 35.14, 27.88, 27.49, 24.01 |
|         | 20.5, 18.95, 12.5, 11.96 | 21.68, 18.09, 13.86, 12.87, 10.71, 8.73 |
|         | 11.55, 10.81, 10.28, 9.54, 7.3 | 8.17, 7.54, 6.53, 5.05, 4.49, 4.07 |
|         | 6.87, 3.5, 2.59, 2.31, 1.97 | 3.45, 2.92, 2.24, 1.49, 1.29, 0.61 |
|         | 0.5, 0.38, 0.23, 0.11, 0.04 | 0.44, 0.37, 0.23, 0.18, 0.1, 0.05 |
| 1.28 + 0.75i | 34.77, 32.1, 27.08, 21.05 | 38.97, 36.25, 35.13, 27.88, 27.48, 24.0 |
|         | 20.5, 18.95, 12.49, 11.95 | 21.69, 18.08, 13.86, 12.87, 10.7, 8.72 |
|         | 11.55, 10.81, 10.27, 9.54, 7.3 | 8.16, 7.55, 6.53, 5.05, 4.49, 4.07 |
|         | 6.86, 3.5, 2.59, 2.32, 1.96 | 3.45, 2.91, 2.24, 1.49, 1.29, 0.58 |
|         | 0.49, 0.37, 0.22, 0.11, 0.01 | 0.44, 0.35, 0.21, 0.17, 0.1, 0.06 |
| 1.16 + 0.67i | 34.77, 32.1, 27.07, 21.05 | 38.96, 36.24, 35.13, 27.88, 27.47, 24.0 |
|         | 20.49, 18.94, 12.48, 11.94 | 21.69, 18.07, 13.85, 12.88, 10.7, 8.7 |
|         | 11.54, 10.79, 10.26, 9.55, 7.3 | 8.15, 7.56, 6.54, 5.05, 4.49, 4.07 |
|         | 6.86, 3.49, 2.58, 2.32, 1.95 | 3.46, 2.89, 2.24, 1.49, 1.28, 0.55 |
|         | 0.450, 0.34, 0.21, 0.12, 0.01 | 0.43, 0.33, 0.21, 0.16, 0.1, 0.06 |
| 1.06 − 0.73i | 34.77, 32.1, 27.07, 21.05 | 38.96, 36.24, 35.13, 27.89, 27.46, 24.0 |
|         | 20.49, 18.94, 12.48, 11.94 | 21.69, 18.07, 13.85, 12.88, 10.7, 8.69 |
|         | 11.54, 10.79, 10.25, 9.55, 7.31 | 8.15, 7.56, 6.54, 5.05, 4.49, 4.07 |
|         | 6.86, 3.49, 2.58, 2.32, 1.95 | 3.45, 2.89, 2.24, 1.49, 1.28, 0.54 |
|         | 0.46, 0.34, 0.22, 0.14, 0.03 | 0.43, 0.34, 0.21, 0.16, 0.1, 0.06 |
| 0.02 − 0.03i | 34.75, 32.08, 27.05, 21.04 | 38.95, 36.24, 35.12, 27.91, 27.4, 23.97 |
|         | 20.47, 18.92, 12.44, 11.9 | 21.69, 18., 13.82, 12.92, 10.68, 8.58 |
|         | 11.51, 10.74, 10.19, 9.57, 7.33 | 8.1, 7.6, 6.56, 5.05, 4.5, 4.06 |
|         | 6.86, 3.48, 2.57, 2.33, 1.9 | 3.46, 2.84, 2.24, 1.48, 1.25, 0.43 |
|         | 0.36, 0.28, 0.23, 0.21, 0.09 | 0.31, 0.22, 0.19, 0.13, 0.11, 0.06 |
### 7.6 Discussion and Outlook

| Field            | Masses                                                                 |
|------------------|------------------------------------------------------------------------|
| $A[1, 1, 4]$     | 1.531, 1.462, 1.282, 1.265, 0.379, 0.072                               |
| $B[6, 2, 5/3]$   | 2.885, 2.445, 2.409, 1.901, 1.219, 1.001                              |
| $C[8, 2, 1]$     | 1.195, 1.195, 0.911, 0.911, 0.889, 0.889                              |
|                  | 0.625, 0.625, 0.604, 0.604, 0.594, 0.594                              |
| $D[3, 2, 7/3]$   | 1.269, 0.896, 0.862, 0.719, 0.667, 0.625                              |
|                  | 0.612, 0.511, 0.412, 0.354, 0.191, 0.131                              |
| $E[3, 2, 1/3]$   | 6.259, 6.259, 4.381, 3.93, 3.543, 2.986, 2.936, 2.512, 1.437, 1.123 |
|                  | 1.092, 1.016, 0.965, 0.915, 0.775, 0.733, 0.701, 0.677               |
|                  | 0.598, 0.425, 0.322, 0.241, 0.218, 0.189, 0.12                       |
| $F[1, 1, 2]$     | 6.007, 6.007, 5.122, 3.971, 3.444, 3.184, 2.948                      |
|                  | 2.772, 1.425, 1.044, 0.462, 0.194, 0.119                            |
| $G[1, 1, 0]$     | 10.192, 10.192, 8.226, 7.903, 6.583, 6.275, 5.522, 4.998, 3.994     |
|                  | 3.749, 2.885, 2.787, 2.528, 2.217, 2.148, 2.029, 1.792, 1.711, 1.673|
|                  | 1.333, 0.973, 0.948, 0.877, 0.79, 0.616, 0.42, 0.062, 0, 0, 0, 0    |
| $I[3, 1, 10/3]$  | 2.885, 2.736, 2.417, 2.372, 1.463, 0.301                           |
| $J[3, 1, 4/3]$   | 6.398, 6.398, 4.68, 3.929, 3.452, 3.086, 2.974, 2.642, 1.678, 1.272 |
|                  | 1.193, 1.144, 0.965, 0.904, 0.542, 0.371, 0.161, 0.106, 0.032        |
| $K[3, 1, −8/3]$  | 1.009, 0.766, 0.757, 0.625, 0.559, 0.468                           |
| $L[6, 1, 2/3]$   | 1.533, 1.08, 1.044, 0.626, 0.613, 0.596                           |
| $M[6, 1, 8/3]$   | 1.580, 1.328, 1.268, 1.048, 0.785, 0.388                           |
| $N[6, 1, −4/3]$  | 1.628, 1.311, 0.983, 0.933, 0.7, 0.487                           |
| $O[1, 3, −2]$    | 1.359, 1.219, 0.813, 0.699, 0.494, 0.218                           |
| $P[3, 3, −2/3]$  | 0.8, 0.698, 0.625, 0.357, 0.192, 0.179                           |
| $Q[8, 3, 0]$     | 2.887, 2.692, 2.093, 1.587, 1.258, 1.238                           |
| $R[8, 1, 0]$     | 3.538, 2.886, 2.656, 2.515, 2.429, 1.9                      |
|                  | 1.779, 1.528, 1.318, 0.889, 0.875, 0.774                           |
| $S[1, 3, 0]$     | 2.92, 1.885, 1.511, 0.892, 0.843, 0.507                           |
| $U[3, 3, 4/3]$   | 1.518, 1.479, 1.172, 0.965, 0.278, 0.256                           |
| $V[1, 2, −3]$    | 3.775, 2.588, 1.961, 0.965, 0.857, 0.767                           |
| $W[6, 3, 2/3]$   | 1.27, 1.098, 1.005, 0.826, 0.727, 0.696                           |
| $X[3, 2, −5/3]$  | 2.885, 2.33, 2.33, 2.085, 1.796, 1.746, 1.367                     |
|                  | 1.063, 0.963, 0.92, 0.876, 0.871, 0.339                           |
| $Y[6, 2, −1/3]$  | 2.51, 1.743, 1.682, 0.966, 0.906, 0.893                           |
| $Z[8, 1, 2]$     | 3.394, 2.183, 2.124, 1.157, 0.984, 0.96                           |

Table 7.10: Mass spectrum of superheavy fields in units of $m/\lambda \sim 10^{16}$ GeV in 6-dim symmetric tracefull scenario ($\text{Rank}[\Xi] = 4$).
## 7.6 Discussion and Outlook

| $M_H$          | $h[1, 2, 1]$ | $t[3, 1, -2/3]$ |
|---------------|-------------|-----------------|
| $-4.323 + 1.47i$ | 10.09, 9.86, 8.22, 7.93, 7.31 | 9.96, 9.82, 7.84, 7.49, 7.12, 6.57 |
| 4.6, 5.85, 3.74, 3.49, 3.26 | 5.94, 3.43, 3.29, 3.05, 2.22, 1.84 |
| 2.39, 2.04, 1.76, 1.41, 0.89 | 1.72, 1.49, 1.32, 1.16, 1.05, 1.0 |
| 0.75, 0.63, 0.61, 0.48 | 0.99, 0.79, 0.74, 0.65, 0.63, 0.61 |
| 0.33, 0.27, 0.1, 0.03 | 0.52, 0.4, 0.32, 0.28, 0.2, 0.12 |
| $0.465 + 3.382i$ | 9.71, 9.48, 7.76, 7.42, 6.71 | 9.63, 9.39, 7.35, 7.03, 6.59, 6.28 |
| 6.27, 5.16, 3.58, 3.17, 2.64 | 5.11, 3.18, 2.91, 2.7, 2.1, 1.83 |
| 2.4, 1.96, 1.72, 1.55, 0.92 | 1.68, 1.46, 1.36, 1.21, 1.11, 1.04 |
| 0.66, 0.63, 0.61, 0.52 | 1.02, 0.8, 0.74, 0.66, 0.63, 0.61 |
| 0.32, 0.26, 0.13, 0.03 | 0.49, 0.37, 0.25, 0.22, 0.17, 0.13 |
| $0.76 - 2.193i$ | 9.52, 9.22, 7.52, 7.1, 6.35 | 9.46, 9.09, 7.14, 6.74, 6.23, 5.87 |
| 5.94, 4.61, 3.29, 2.87, 2.52 | 4.46, 3.01, 2.51, 2.14, 2.09, 1.91 |
| 2.09, 1.92, 1.76, 1.17, 0.97 | 1.72, 1.43, 1.28, 1.2, 1.12, 1.07 |
| 0.72, 0.66, 0.62, 0.52 | 1.03, 0.79, 0.77, 0.63, 0.61, 0.59 |
| 0.31, 0.22, 0.17, 0.04 | 0.47, 0.35, 0.27, 0.18, 0.14, 0.1 |
| $-0.002 + 0.968i$ | 9.4, 9.01, 7.38, 6.9, 6.16 | 9.34, 8.87, 7.04, 6.54, 5.98, 5.61 |
| 5.7, 4.19, 3.18, 2.61, 2.36 | 3.84, 2.83, 2.25, 1.99, 1.83, 1.69 |
| 2.11, 1.38, 1.15, 1.07, 1.01 | 1.64, 1.38, 1.32, 1.29, 1.17, 1.06 |
| 0.73, 0.66, 0.63, 0.36 | 0.95, 0.82, 0.79, 0.63, 0.61, 0.57 |
| 0.2, 0.13, 0.06, 0.02 | 0.38, 0.2, 0.13, 0.1, 0.08, 0.03 |
| $-0.508 - 0.209i$ | 9.39, 8.99, 7.37, 6.86, 6.15 | 9.33, 8.84, 7.03, 6.5, 5.96, 5.54 |
| 5.64, 4.12, 3.16, 2.45, 2.41 | 3.76, 2.84, 2.1, 1.99, 1.79, 1.7 |
| 2.13, 1.35, 1.24, 0.95, 0.82 | 1.62, 1.46, 1.35, 1.27, 1.14, 0.98 |
| 0.71, 0.67, 0.63, 0.39 | 0.91, 0.82, 0.81, 0.63, 0.59, 0.55 |
| 0.17, 0.11, 0.05, 0.02 | 0.28, 0.14, 0.12, 0.08, 0.06, 0.03 |
| $-0.092 - 0.032i$ | 9.37, 8.98, 7.35, 6.84, 6.13 | 9.32, 8.82, 7.03, 6.48, 5.95, 5.52 |
| 5.61, 4.1, 3.17, 2.45, 2.37 | 3.71, 2.83, 2.07, 1.99, 1.82, 1.7 |
| 2.17, 1.32, 1.2, 0.88, 0.85 | 1.62, 1.45, 1.33, 1.27, 1.14, 1. |
| 0.72, 0.68, 0.63, 0.37 | 0.89, 0.83, 0.81, 0.63, 0.59, 0.55 |
| 0.12, 0.08, 0.05, 0.02 | 0.31, 0.1, 0.07, 0.06, 0.04, 0.02 |

Table 7.11: Mass spectrum of superheavy fields $h[1, 2, \pm 1]$, $t[3, 1, \mp 2/3]$ which depend on the value of $M_H$ chosen in units of $m/\lambda \sim 10^{16}$ GeV in 6-dimensional symmetric tracefull ($\text{Rank}(\Xi) = 4$) scenario with $N_y = 3$ for all values of $M_H$. 


### 7.6 Discussion and Outlook

| Field | Masses |
|-------|--------|
| $A[1, 1, 4]$ | $0.602, 0.596, 0.515, 0.461, 0.446$ |
| $B[6, 2, 5/3]$ | $1.099, 0.861, 0.826, 0.248, 0.244$ |
| $C[8, 2, 1]$ | $0.563, 0.563, 0.552, 0.552, 0.532$ |
| $D[3, 2, 7/3]$ | $0.584, 0.571, 0.563, 0.561, 0.531$ |
| $E[3, 2, 1/3]$ | $8.843, 8.843, 1.48, 1.47, 0.92, 0.858, 0.847$ |
| $F[1, 1, 2]$ | $8.262, 8.262, 1.603, 1.591, 0.997, 0.901$ |
| $G[1, 1, 0]$ | $15.3, 15.3, 2.62, 2.601, 1.866, 1.317, 1.315, 1.111, 1.11$ |
| $I[3, 1, 10/3]$ | $0.974, 0.898, 0.462, 0.149, 0.06$ |
| $J[3, 1, 4/3]$ | $9.188, 9.188, 1.438, 1.428, 0.908, 0.83, 0.821, 0.573$ |
| $K[3, 1, −8/3]$ | $0.565, 0.555, 0.53, 0.483, 0.48$ |
| $L[6, 1, 2/3]$ | $0.596, 0.574, 0.542, 0.452, 0.45$ |
| $M[6, 1, 8/3]$ | $0.692, 0.635, 0.582, 0.359, 0.356$ |
| $N[6, 1, −4/3]$ | $0.549, 0.548, 0.516, 0.503, 0.496$ |
| $O[1, 3, −2]$ | $0.772, 0.771, 0.419, 0.396, 0.282$ |
| $P[3, 3, −2/3]$ | $0.596, 0.593, 0.504, 0.471, 0.45$ |
| $Q[8, 3, 0]$ | $0.967, 0.846, 0.595, 0.14, 0.112$ |
| $R[8, 1, 0]$ | $1.18, 0.959, 0.856, 0.441, 0.406$ |
| $S[1, 3, 0]$ | $0.394, 0.319, 0.31, 0.274, 0.272$ |
| $U[3, 3, 4/3]$ | $1.279, 1.269, 0.537, 0.098, 0.024$ |
| $V[1, 2, −3]$ | $1.065, 1.05, 0.422, 0.167, 0.076$ |
| $W[6, 3, 2/3]$ | $0.63, 0.602, 0.549, 0.421, 0.416$ |
| $X[3, 2, −5/3]$ | $2.084, 2.084, 0.857, 0.791, 0.772, 0.699$ |
| $Y[6, 2, −1/3]$ | $0.577, 0.238, 0.161, 0.07, 0.053$ |
| $Z[8, 1, 2]$ | $0.768, 0.669, 0.501, 0.063, 0.045$ |

Table 7.12: Mass spectrum of superheavy fields in units of $m/\lambda \sim 10^{16}$ GeV in 5-dimensional symmetric traceless case ($\text{Rank}[\Xi] = 4$).
7.6 Discussion and Outlook

| $M_H$       | $h[1, 2, 1]$ | $t[3, 1, -2/3]$ |
|-------------|--------------|-----------------|
| $135.29 + 11.98i$ | 136.697, 136.68, 136.528 | 136.517, 136.501, 136.392, 135.942 |
|             | 135.959, 135.936, 2.939 | 135.923, 2.293, 2.282, 1.407, 0.697 |
|             | 2.925, 1.921, .78, .77, 0.655 | 0.687, 0.652, 0.609, 0.593, 0.573, .559 |
|             | 0.55, 0.62, 0.557, 0.505 | 0.527, 0.522, 0.52, 0.483, 0.477 |
|             | 0.481, 0.466, 0.266, 0.217 | 0.391, 0.387, 0.169, 0.139, 0.072 |
| $25.4 + 1.72i$ | 29.744, 29.664, 28.968 | 28.908, 28.83, 28.323, 26.094 |
|             | 26.179, 26.063, 3.287, 3.252 | 25.996, 2.715, 2.688, 1.506, 1.195 |
|             | 1.821, 1.52, 0.992, 0.927 | 0.86, 0.809, 0.679, 0.633, 0.599, .572 |
|             | 0.658, 0.605, 0.579, 0.517 | 0.556, 0.534, 0.526, 0.398, 0.392 |
|             | 0.284, 0.115, 0.103, 0.008 | 0.258, 0.177, 0.161, 0.026, 0.007 |
| $24.6 + 1.16i$ | 29.034, 28.952, 28.237 | 28.176, 28.096, 27.575, 25.282 |
|             | 25.37, 25.25, 3.299, 3.263 | 25.181, 2.732, 2.703, 1.533, 1.202 |
|             | 1.828, 1.55, 1.003, 0.937 | 0.868, 0.816, 0.682, 0.635, 0.599, .572 |
|             | 0.659, 0.606, 0.585, 0.518 | 0.556, 0.534, 0.526, 0.398, 0.392 |
|             | 0.285, 0.104, 0.092, 0.008 | 0.259, 0.17, 0.153, 0.031, 0.012 |
| $18.41 + 1.4i$ | 24.01, 23.909, 23.03 | 22.965, 22.865, 22.217, 19.321 |
|             | 19.435, 19.281, 3.413, 3.353 | 19.19, 2.849, 2.841, 1.831, 1.228 |
|             | 2.055, 1.669, 1.115, 1.043 | 0.942, 0.885, 0.734, 0.644, 0.604, .577 |
|             | 0.697, 0.633, 0.602, 0.528 | 0.562, 0.537, 0.528, 0.398, 0.392 |
|             | 0.291, 0.076, 0.061, 0.004 | 0.276, 0.109, 0.095, 0.069, 0.061 |
| $18.32 + 1.23i$ | 23.926, 23.825, 22.943 | 22.878, 22.777, 22.127, 19.218 |
|             | 19.332, 19.177, 3.416, 3.355 | 19.086, 2.893, 2.844, 1.838, 1.228 |
|             | 2.062, 1.669, 1.118, 1.046 | 0.943, 0.887, 0.735, 0.645, 0.604, .577 |
|             | 0.699, 0.633, 0.602, 0.528 | 0.562, 0.537, 0.528, 0.398, 0.392 |
|             | 0.291, 0.078, 0.062, 0.004 | 0.276, 0.108, 0.095, 0.069, 0.061 |

Table 7.13: Mass spectrum of superheavy fields $h[1, 2, \pm 1]$, $t[3, 1, \mp 2/3]$ which depend on the value of $M_H$ chosen in units of $m/\lambda \sim 10^{16}$ GeV in 5-dim symmetric traceless ($\text{Rank}[\Xi] = 4$) scenario with $N_g=3$ for all values of $M_H$. 

Chapter 8

Summary and Outlook

This thesis is based upon a particular GUT, called New Minimal Supersymmetric SO(10) Grand Unified Theory (NMSGUT) [57], which is capable of producing realistic fits of basic fermion mass mixing data. Baryon decay is a peculiarity of GUTs and various extensions of SM predict lepton flavour violation. The main motive of our study is to check the compatibility of the model (NMSGUT) with experimental data, particularly constraints from baryon number and lepton flavour violation, and on the basis of realistic NMSGUT parameter sets to further refine the NMSGUT predictions and thus subject it to stringent falsification tests. Apart from this we aimed to improve the NMSGUT fitting process by inclusion of loop effects on Susy spectrum, consideration of heavy right handed neutrino thresholds and RG improvements in the large NMSGUT FORTRAN code.

This work emphasizes the importance of GUT scale threshold corrections for baryon number violation rates. MSSM is the effective theory of the GUT and its light Higgs is a combination of different Higgs multiplets from all the Higgs irrep of the NMSGUT. So, light Higgs can have wave function corrections from all the heavy fields at SO(10) Yukawa vertex. Wavefunction renormalizion constant of Higgs line can have very small value $Z_{H,\bar{H}} \approx 10^{-2}$. This lowers the tree level SO(10) Yukawas required to match the GUT derived effective MSSM fermion Yukawas with MSSM data. Since the same Yukawas determine the $d = 5 \Delta B \neq 0$ operator we get suppressed B violation rates. We have shown that instead of being problematic the large number of superheavy particles at GUT scale can cure the long standing problem of
fast baryon decay rates in Susy GUTs. We have extended the already available FOR-
TRAN and Mathematica codes of NMSGUT calculations \[57\]. Including GUT scale
threshold corrections, we have searched for the set of NMSGUT superpotential pa-
rameters and mSUGRY NUHM soft parameters, respecting RG constraints, which
accurately fit the fermion mass mixing data and experiment compatible B-decay
rates (i.e. \(< 10^{-34} \text{ yrs}^{-1}\)). These fits have smaller values of all the superpotential
parameters as compared to the tree level fits. Soft parameters prefer the same range
as found before and provide mini-split Susy spectrum with heavy third s-generation.

The other pressing issue on which we have focussed is computation of one loop
corrected Susy spectrum. Direct inclusion of one loop self energies to the Susy spec-
trum of NMSGUT solution (fits which produce realistic fermion data and acceptable
B-decay rates) drives slepton and squark masses to negative values. Heavy CP odd
pseudoscalar Higgs provides huge corrections. Fresh searches were performed to
get positive loop corrected sfermion masses by implementing a penalty on the ratio
\((\frac{\mu}{M_A})^2(0.3-2.7)\). This ratio is crucial for the Higgs sector and Higgsino loop correction
to the scalars and solutions found have this ratio close to the upper limit applied.
We have not yet found light smuon solution (which is very desirable for dark matter
phenomenology and to resolve muon g-2 anomaly ) after including loop corrections.
We perhaps require either more searches or deeper RG analysis of the soft param-
ters. The refined NMSGUT fits have large \(A_0, M_{H,\tilde{H}}, \mu\) and \(B\) parameters with
heavy third s-generation like the tree level fits. These distinct predictions of the
model will be tested at LHC with the discovery of Susy particles.

Branching ratio for different lepton flavour violating \((l_i \rightarrow l_j \gamma\) and \(l_i \rightarrow 3 l_j \gamma\)
processes is calculated. These processes do not provide additional constraints on the
soft mass matrices since the calculated BR is much smaller than the experimental
upper bound because of the large sfermion masses and negative soft Higgs mass
square parameters. We have calculated the \(\Delta a_\mu\) for the loop corrected fits presented
in Chapter 4 and 5. Since smuon is not light, this contribution is also very small.
We have not yet estimated charged lepton electric dipole moments but can and will
do so in upcoming studies.

Another puzzle for the NMSGUT has been the necessity of using non universal
Higgs doublets mass squared values that are negative : which is difficult when the
soft terms come from supergravity. We calculated the NMSGUT RG equations and used them to study parameter evolution between $M_{\text{Planck}}$ and $M_{\text{GUT}}$. We found that the universal positive scalar mass squared parameters provided by SUGRY easily become negative due to the RG flow thus removing the problem in principle. Future fits will thus include this third stage of RG evolution.

Finally, we considered a new scenario based upon the SO(10) and $O(\mathcal{N}_g)$(family) gauge symmetry. In this framework, Higgs irreps of SO(10) also carry family indices whose VEVs break GUT and family symmetry thus generating matter Yukawas with enough structure to account for the observed hierarchy. So the number of SO(10) Yukawa couplings reduce dramatically. Therefore this scenario is called Yukawon ultra minimal grand unified theory (YUMGUT). Consistent SSB requires introduction of a special type (`Bajc-Melfo`) of superpotential. Study of toy model ($\mathcal{N}_g=2$) and realistic three generation case show that the realistic MSSM data can be produced dynamically by the VEV of Yukawon field. Consideration of $\mathbf{126}-\mathbf{126}$ VEV homogeneous equations of different rank provides new directions for model phenomenology such as existence of sterile neutrinos. YUMGUT also offers novel dark matter candidates from a SM singlet sector as well as from the hidden sector fields.

This study shows that NMSGUT is quite compatible with B and L violation experimental data. All these observations make NMSGUT a leading candidate for physics beyond SM and a mature theory of particle physics.
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