Non-Markovian effects on the geometric phase

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Abstract – The geometric phases of a two-level atom interacting with non-Markovian environments are calculated and the non-Markovian effects on the geometric phases are discussed in this paper. Three kinds of methods that describe the non-Markovian process, projection superoperator technique, memory kernel master equation and post-Markovian master equation, are used in the discussions. The results show that when the dissipation rate is large, the non-Markovian effects change the geometric phase more strikingly than the small one.

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Introduction. – The so-called Berry phase or geometric phase was first pointed out by Berry [1] when he studied a pure quantum system that undergoes an adiabatic cyclic evolution. Simon gave an interpretation of this phase in the language of differential geometry and fibre bundles [2]. Since then, large numbers of work have been done to generalize the conception of Berry phase. For example, Aharonov and Anandan extended the Berry phase to the case of non-adiabatic evolution [3], Samuel and Bhandari [4] generalized the Berry phase in cyclic evolution to the case of non-cyclic evolution.

Another direction of the generalization is to study the geometric phase for mixed states and nonunitary evolution. This work was first done by Uhlmann [5] via the mathematical context of purification. Sjöqvist et al. [6] gave an alternate definition of geometric phase for nondegenerate density operators undergoing unitary evolution based on quantum interferometry [6]. This study has been extended to the case of degenerate density operators by Singh et al. [7]. Geometric phase in nonunitary evolution have been addressed in refs. [8,9]. Recently a kinematic approach to the geometric phase for mixed quantal states in nonunitary evolution is proposed by Tong et al. [10].

The conception of off-diagonal geometric phase for pure states was proposed in ref. [11] and has been extended to mixed states in ref. [12]. The relation of geometric phase between entangled system and its subsystem has been studied in ref. [13].

The intrinsic nature of geometric phase provides us an impelling tool for fault tolerance quantum computation [14]. However, the unavoided interaction between system and environment would destroy the coherence of the quantum system and hence limit the implementation of quantum computation. So the study of environment effects on the geometric phase is highly required. Geometric phases in the dephasing system and spin environment were studied in ref. [15], which used the exact solution of the system dynamics and then the dynamics of the system is fully non-Markovian. However the exactly solvable models for an open system are only few, most problems are treated with certain approximations. For instance, the finite temperature effect of the environment on the mixed state geometric phases [16] and the effects of a squeezed vacuum reservoir on geometric phases [17] were studied within the Markovian approximation. It is well known that when the dissipation is large or the reservoir is finite and the initial state of the composite system (system and environment) is entangled, the process exhibits strong non-Markovian effects. In this paper we shall study the non-Markovian effects on the geometric phase of a two-level atom interacting with a non-Markovian environment. Three kinds of methods which describe the non-Markovian process, projection superoperator technique, memory kernel master equation and post-Markovian master equation, are considered. We calculate the geometric phase for the system governed by these equations and discuss the results obtained with different dissipation rates and different memory effects.

Throughout this paper, the geometric phases are calculated according to the formula of Tong [10], see eq. (1) on the next page.
where \( \lambda_i(t) \) are instantaneous eigenvalues of the density matrix \( \rho(t) \) and \( |\varphi_i(t)\rangle \) are the corresponding eigenstates. The dot on \( \varphi_i(t) \) denotes the derivative with respect to time \( t \), Arg expresses the argument of a complex number.

The structure of this paper is organized as follows. In the second section we discuss a two-level atom interacting with a two-band environment. The evolution equation is obtained through the projection superoperator methods. In the third section, we calculate the geometric phase in an open system governed by the memory kernel master equation and in the fourth section governed by the post-Markovian master equation. Conclusions and discussions are presented in the last section.

**Non-Markovian effects with generalized Lindblad master equation.** In this section, we will calculate the geometric phase of a two-level atom (system) coupled to a two-band environment in both Markovian and non-Markovian cases. The evolutions of the reduced density matrix for the atom in both cases are obtained through correlated projection superoperators method [18–22], different chosen projection operators would differentiate the Markovian and non-Markovian cases. To calculate the geometric phase we solve the time evolution of the system in the interaction picture [35] back to the Schrödinger picture according to the free Hamiltonian of the system \( H_S = \frac{1}{2} \omega \sigma_z \).

\[
\Psi(t) = \cos \frac{\theta}{2} |1\rangle + \sin \frac{\theta}{2} e^{i\phi} |0\rangle,
\]

where \( |1\rangle \) and \( |0\rangle \) are the excited state and ground state of the atom, respectively. This state corresponds to a state vector in Bloch sphere with polar angle \( \theta \) and azimuthal angle \( \phi \).

**Model.** Consider a two-level atom coupled to a two-band environment \( E \). The environment consists of a large number of energy levels which are arranged in two energy bands with the same width (see fig. 1). The levels of each band are equidistant. The lower energy band contains \( N_1 \) levels while the upper band \( N_2 \) levels. The transition of the atom is in resonance with distance \( \omega \) between the bands (we set \( \hbar = 1 \)). The total Hamiltonian for such a system is [22] \( H = H_S + H_E + V \), with

\[
H_S = \frac{1}{2} \omega \sigma_z,
H_E = \sum_{n_1} \frac{\delta \epsilon}{N_1} |n_1 \rangle \langle n_1 | + \sum_{n_2} \left( \omega + \frac{\delta \epsilon}{N_2} \right) |n_2 \rangle \langle n_2 |,
V = \lambda \sum_{n_1 n_2} c(n_1, n_2) \sigma_+ |n_1 \rangle \langle n_2 | + H.c.,
\]

where the index \( n_1 \) denotes the levels of the lower energy band and \( n_2 \) the levels of the upper band, \( \sigma_+ \) and \( \sigma^- \) are Pauli operators, \( \lambda \) is the overall strength of the interaction, \( c(n_1, n_2) \) are coupling constants which are independent of each other and satisfy

\[
\langle c(n_1, n_2) \rangle = 0,
\]

\[
\langle c(n_1, n_2)c(n_1', n_2') \rangle = 0,
\]

\[
\langle c(n_1, n_2)c^*(n_1', n_2') \rangle = \delta_{n_1,n_1'} \delta_{n_2,n_2'}. 
\]

Setting \( H_0 = H_S + H_E \), we transform our discussion to the interaction picture that,

\[
V(t) = \sigma^+ B(t) + \sigma^- B^\dagger(t),
\]

where

\[
B(t) = \lambda \sum_{n_1 n_2} c(n_1, n_2) e^{-i \omega (n_1, n_2) t} |n_1 \rangle \langle n_2 |,
\]

and

\[
\omega(n_1, n_2) = \delta \epsilon \left( \frac{n_2}{N_2} - \frac{n_1}{N_1} \right).
\]

**Markovian case.** When the standard projection is used, the evolution of the reduce system must be Markovian [18–20]. If we project the composite system’s state into the form \( \rho = (T_E \rho_{EP}) \otimes \rho_{ES} \), where \( \rho \) is the projection superoperator and \( \Pi_1 = \sum_n |n_1 \rangle \langle n_1 | \) the second-order Markovian master equation in the interaction picture is obtained [22]:

\[
\frac{d}{dt} \rho_S(t) = \gamma_2 \left( \sigma^- \rho_S(t) \sigma^- - \frac{1}{2} \{ \sigma^+ \sigma^-, \rho_S(t) \} \right),
\]

where

\[
\gamma_2 = \frac{2 \pi \lambda^2 N_2}{\delta \epsilon}.
\]

Fig. 1: A two-level atom coupled to an environment consisting of two energy bands with a finite number of levels.
\[
\Phi_{GP} = \text{Arg} \left[ \left( \cos \frac{\theta}{2} \sin \frac{\theta_t}{2} + \sin \frac{\theta}{2} \cos \frac{\theta_t}{2} e^{-i\omega t} \right) \exp \left( -i\omega \int_0^t \cos^2 \frac{\theta_t}{2} \, dt \right) \right].
\]

This equation is the same as that which describes a two-level atom spontaneously decaying in vacuum under the Markovian approximation. The time dependence of the reduced system in the Schrödinger picture may be obtained easily by a simple calculation. With the initial state eq. (2), the final state follows:

\[
\rho(t) = \left( \begin{array}{cc}
\cos^2 \frac{\theta}{2} e^{-\gamma t} & \frac{1}{2} \sin \theta e^{-i(\phi + \omega t)} e^{-\frac{1}{2} \gamma t} \\
\frac{1}{2} \sin \theta e^{i(\phi + \omega t)} e^{-\frac{1}{2} \gamma t} & 1 - \cos^2 \frac{\theta}{2} e^{-\gamma t}
\end{array} \right).
\]

(9)

The two eigenvalues of \(\rho(t)\) are

\[
\lambda_\pm = \frac{1}{2}(1 \pm \eta),
\]

(10)

where

\[
\eta = \sqrt{\left(1 - 2 \cos^2 \frac{\theta}{2} e^{-\gamma t}\right)^2 + \sin^2 \theta e^{-\gamma t}}.
\]

(11)

It is obvious that the eigenvalue \(\lambda_+ = 0\) at \(t = 0\). From the formation of eq. (1), we can see that the eigenvalue \(\lambda_-\) and its corresponding eigenstate \(|-\rangle\) contribute zero to the geometric phase. This simplifies the calculation. The eigenstate corresponding to \(\lambda_-\) can be written as

\[
|\rangle = \sin \frac{\theta_t}{2} |1\rangle + \cos \frac{\theta_t}{2} e^{i(\phi + \omega t)} |0\rangle,
\]

(12)

where

\[
\tan \frac{\theta_t}{2} = \frac{\sin \theta e^{-\frac{1}{2} \gamma t}}{1 + \eta - 2 \cos^2 \frac{\theta}{2} e^{-\gamma t}}.
\]

(13)

Obviously, for \(t = 0\), \(\tan \frac{\theta_t}{2} = \cot \frac{\theta}{2}\), as expected. Now we substitute eqs. (10) and (12) into eq. (1) to obtain the geometric phase at time \(t\),

\[
\text{see eq. (14) above}
\]

We consider the geometric phase acquired after a quasi-period \(T = \frac{2\pi}{\omega}\). In this case, we can rewrite the geometric phase as

\[
\Phi_{GP} = -\omega \int_0^T \cos^2 \frac{\theta_t}{2} \, dt,
\]

(15)

where \(\cos^2 \frac{\theta_t}{2}\) can be obtained according to eq. (13)

\[
\cos^2 \frac{\theta_t}{2} = \frac{(1 + \eta - 2 \cos^2 \frac{\theta}{2} e^{-\gamma t})^2}{(1 + \eta - 2 \cos^2 \frac{\theta}{2} e^{-\gamma t})^2 + \sin^2 \theta e^{-\gamma t}}.
\]

(16)

Non-Markovian case. If we choose a correlated projection

\[
P \rho = \text{Tr}_E(\Pi_1 \rho) \otimes \frac{1}{N_1} \Pi_1 + \text{Tr}_E(\Pi_2 \rho) \otimes \frac{1}{N_2} \Pi_2
\]

\[
= \rho_S^{(1)} \otimes \frac{1}{N_1} \Pi_1 + \rho_S^{(2)} \otimes \frac{1}{N_2} \Pi_2,
\]

where \(\Pi_2 = \sum_n |n\rangle \langle n|\), the evolution of the reduced system is said to be non-Markovian [18–20]. At this stage the master equation in the interaction picture is [22]

\[
\frac{d}{dt} \rho_S^{(1)}(t) = \gamma_1 \sigma^+ \rho_S^{(2)}(t) \sigma^- - \frac{\gamma_2}{2} \left\{ \sigma^+ \sigma^- - \rho_S^{(1)}(t) \right\},
\]

(17)

\[
\frac{d}{dt} \rho_S^{(2)}(t) = \gamma_2 \sigma^- \rho_S^{(1)}(t) \sigma^+ - \frac{\gamma_1}{2} \left\{ \sigma^- \sigma^+ - \rho_S^{(2)}(t) \right\},
\]

where \(\gamma_1\) has a similar definition to \(\gamma_2\) and the state of atom is given by \(\rho_S = \rho_S^{(1)} + \rho_S^{(2)}\). This equation is of a generalized Lindblad form [19] and gives an excellent approximation of the reduced system’s dynamics [22].

We assume that initially only the lower band is populated. This means \(\rho_S^{(2)}(0) = 0\). We also set \(\gamma_1 = \gamma_2 = \gamma\). The evolution of the atom with the initial condition (2) in the Schrödinger picture is

\[
\rho(t) = \left( \begin{array}{cc}
\frac{1}{2} \left(1 + e^{-2\gamma t}\right) \cos^2 \frac{\theta}{2} & \frac{1}{2} \sin \theta e^{-i(\phi + \omega t)} e^{-\frac{1}{2} \gamma t} \\
\frac{1}{2} \sin \theta e^{i(\phi + \omega t)} e^{-\frac{1}{2} \gamma t} & 1 - \frac{1}{2} \left(1 + e^{-2\gamma t}\right) \cos^2 \frac{\theta}{2}
\end{array} \right).
\]

(18)

Following the same procedure mentioned in the Markovian process, the geometric phase acquired after a quasi-periods \(T = \frac{2\pi}{\omega}\) takes the same form as eq. (15), where \(\theta_t\) and \(\eta\) are defined by

\[
\cos^2 \frac{\theta_t}{2} = \frac{(\sin^2 \frac{\theta}{2} + \eta - \cos^2 \frac{\theta}{2} e^{-2\gamma t})^2}{(\sin^2 \frac{\theta}{2} + \eta - \cos^2 \frac{\theta}{2} e^{-2\gamma t})^2 + \sin^2 \theta e^{-\gamma t}},
\]

(19)

\[
\eta = \sqrt{(1 + e^{-2\gamma t}) \cos^2 \frac{\theta}{2} - 1} + \sin^2 \theta e^{-\gamma t}.
\]

(20)

The geometric phases as a function of the initial polar angle \(\theta\) in both the Markovian and non-Markovian cases are shown in fig. 2. In this figure, we have set the transition frequency \(\omega = 1\) and \(\gamma_1 = \gamma_2 = \gamma\). The geometric phase \(\Phi_{GP}\) and the polar angle \(\theta\) are plotted in units of \(\pi\). We see from this figure that when the dissipation is weak \((\gamma = 0.1)\), the geometric phases for both processes are similar especially when \(\theta > \frac{\pi}{2}\), whereas when the dissipation is large \((\gamma = 1)\), the non-Markovian effect changes the geometric phase drastically, especially when \(\theta \rightarrow 0\). Because the time dependences for the off-diagonal elements \(\rho_{12}\) and \(\rho_{21}\) are the same, we shall study the difference of the diagonal elements \(\rho_{11}\) in both cases. The decay functions are \(A_M = e^{-\gamma t}\) and \(A_N = \frac{1}{2}(1 + e^{-2\gamma t})\) for the Markovian and non-Markovian equation, respectively.

When \(\gamma t\) is small enough, we can make a Taylor expansion and ignore all items higher than the second order. Then \(A_M = A_N = 1 - \gamma t\), i.e. the dynamics are the same for weak dissipation and short time, so it can be easily understood that the geometric phases are nearly the same. When the dissipation is large, items higher than the
\[ \xi(R, t) = \exp \left( \frac{\tau}{2} \right) \left\{ \frac{1}{\sqrt{1 - 4R^2}} \sinh \left[ \frac{\tau}{2} \sqrt{1 - 4R^2} \right] + \cosh \left[ \frac{\tau}{2} \sqrt{1 - 4R^2} \right] \right\}, \]  
(24)

Fig. 2: Geometric phase in a system governed by the generalized Lindblad master equation. The geometric phase \( \Phi_{GP} \) and the polar angle \( \theta \) are plotted in units of \( \pi \). We set the transition frequency \( \omega = 1 \) and time \( \tau = \frac{\pi}{2} \). The parameter \( \gamma \) is chosen as: top: \( \gamma = 0.1 \), bottom: \( \gamma = 1 \). The solid line indicates the Markovian process while the dash-dotted line denotes the non-Markovian one.

second order cannot be ignored for a quasi-period. In this case, we set \( \theta \to 0 \), i.e., the initial state vector in Bloch sphere is near the \( z \)-axis. After a quasi-period, the vector turns to the hemisphere containing the \( -z \)-axis in the Markovian case while it remains in the initial hemisphere in the non-Markovian one. This is the reason why the difference is so large between both processes with large dissipation rate at \( \theta \to 0 \), with which the geometric phase is usually interpreted as the solid angle of the evolution track in the Bloch sphere. We also note that in figs. 2, 3 and 4 in this paper, the left boundaries of \( \theta \) are all set \( \theta \to 0 \). If \( \theta = 0 \), the geometric phase in all these processes are zero. The qualitative analysis (see the solid line in the bottom figure) is the following. In the Markovian case, because of the semi-positivity of \( \eta \), the two branches of the eigenvalues \( \lambda_\pm \) maintain the relation \( \lambda_+ > \lambda_- \) all the time for \( \theta \neq 0 \). The calculation of the geometric phase given above is available. When \( \theta \to 0 \) and \( \gamma t = \ln 2 \) the two eigenvalues tend to be equal, i.e. the eigenstates of the density matrix are approximately degenerate. After this critical point, the relation \( \lambda_+ > \lambda_- \) holds again in the above calculation. However, as a matter of fact, in the case of \( \theta = 0 \), the two branches should be crossing, and the relation \( \lambda_+ > \lambda_- \) should be changed after the critical point. Moreover, when the degeneration occurs, the geometric phase should be calculated as discussed at the end of ref. [10], which in our model shows that when \( \theta = 0 \), the matrix elements of \( \rho(t) \) and the eigenvalues and as well as the coefficients of the eigenstates are all real. Those facts make the geometric phase zero all the time. This results in the difference in the two cases \( \theta = 0 \) and \( \theta \to 0 \).

Non-Markovian effects with exponential memory. — In this section, we will calculate the geometric phase of a two-level atom by using a memory kernel master equation with exponential memory phenomenologically. This equation may lead to a non-positive reduced density matrix [23] and the positivity for a qubit has been discussed in ref. [24]. In this paper, we focus on the geometric phase only and do not take the positivity into account.

We consider a two-level atom interacting with a vacuum. An integrodifferential master equation containing the memory kernel in the interaction picture is [23,24]

\[ \frac{d\rho}{dt} = \int_0^t K(t-t') \mathcal{L} \rho(t') dt' = \int_0^t K(t') \mathcal{L} \rho(t-t') dt', \]

(21)

where \( \mathcal{L} \) is the Liouvillian superoperator that may take the form

\[ \mathcal{L} \rho = \frac{1}{2} \gamma_0 (2\sigma^- \rho \sigma^++\sigma^+ \sigma^- + \rho \sigma^- \sigma^+), \]

(22)

and \( K(t) \) is the memory kernel and here we choose an exponential memory phenomenologically

\[ K(t) = \gamma e^{-\gamma t}. \]

(23)

We call \( \tau_R = \frac{\tau}{2} \) the memory time. \( \gamma_0 \) is the dissipation constant. One can solve this integrodifferential equation by taking its Laplace transform, determining the poles and inverting the solution in the standard way. The analytic solution of this equation [25] with initial condition eq. (2) in Schrödinger representation is

\[ \rho(t) = \left( \begin{array}{cc} \xi(R, \tau) \cos^2 \frac{\theta}{2} & \frac{1}{\sqrt{2}} \sin \theta e^{-i(\phi+\omega) t} \xi(R, \tau) \\ \frac{1}{\sqrt{2}} \sin \theta e^{i(\phi+\omega) t} \xi(R, \tau) & 1 - \xi(R, \tau) \cos^2 \frac{\theta}{2} \end{array} \right), \]

where \( \xi(R, \tau) \) is given by

see eq. (24) above

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memory time $\gamma$. The geometric phase $\Phi_{GP}$ and the polar angle $\theta$ are plotted in units of $\pi$. We see from the figure that for weak dissipation (small dissipation constant, $\gamma_0 = 0.1$), the three curves stand much close, even when the memory time is long ($\gamma = 0.1$). For large dissipation rate, long time memory lead to a large departure from the Markovian process. When the memory time is shortened ($\gamma = 10$), the result tends to the Markovian.

**Geometric phase with post-Markovian master equation.** – In ref. [26], a post-Markovian master equation including bath memory effects via a phenomenological introduced memory kernel $K(t)$ is derived. Compared with the memory kernel master equation, the advantage of the post-Markovian master equation is that for a qubit it keeps the positivity of the density matrix for an exponential memory [25]. The general form of the post-Markovian master equation for a two-level atom in the zero-temperature reservoir is

$$\frac{d}{dt}\rho = \mathcal{L} \int_0^t K(t') e^{\mathcal{L}t'} \rho(t-t') dt',$$  \hspace{1cm} (27)

where $\mathcal{L}$ is the Liouvillian superoperator and $K(t)$ is the exponential memory kernel. It has been proved that the memory kernel master equation is a special case of post-Markovian master equation and one may derive the memory kernel master equation from the post-Markovian equation in the limit $\gamma_0 \ll \gamma$ [25].

The time dependent of the state for atom and the geometric phase after a quasi-periods have the same form as that from the memory kernel master equation, the only difference is that the quantity $\xi(R, \tau)$ is replaced by [25]

$$\xi(R, \tau) = \frac{e^{-R\tau} - R e^{-\tau}}{1 - R}. \hspace{1cm} (28)$$

The geometric phase for the post-Markovian master equation is shown in fig. 4. A similar feature as that from the memory kernel master equation can be seen in the figure. Furthermore when the dissipation rate is small, the figure is nearly the same as that obtained from the memory kernel master equation. The difference between the two cases can be found in the large dissipation case, e.g., the geometric phase departure from the Markovian process more largely with $\gamma = 0.1$ than that in the memory kernel master equation.

**Conclusion.** – In summary, we have studied the geometric phases of a two-level system coupled to a non-Markovian environment. The non-Markovian effects on the geometric phase are presented and discussed. We have chosen three different methods to describe the non-Markovian process. The common feature is that the geometric phase tends to zero with $\theta$ approaching $\pi$. As the memory kernel master equation can be reproduced from the post-Markovian master equation in the limit of $\gamma_0 \ll \gamma$, the geometric phase calculated confirmed this point in this limit. Our results also show that for all the
methods the non-Markovian effects change the geometric phase more strikingly when the dissipation rate and the population of the excited state in the initial state is large. This can be understood as the competition between the decoherence and the non-Markovian effects in the open system.

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