Energy Resolution Improvement of Scintielectron detectors: Priorities and Prospects

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Abstract

The development prospects of a scintillator-photodiode type detector with an improved energy resolution attaining few per cent, \( R \) from 1 to 2\%, are considered. The main resolution components have been analyzed theoretically, their theoretical and physical limits have been established. Empirical data on the properties of novel scintillators have been presented confirming the possibility of the \( R \) improvement. New optimization methods have been proposed to optimize the detector statistical fluctuations and the scintillator intrinsic resolution. A specific importance of the intrinsic resolution is shown as a limiting threshold factor at the ionizing radiation energy values from 662 keV to 10 MeV and over.

I. INTRODUCTION

Among the solid detectors, the scintillator-PMT (SC-PMT) and scintillator-photodiode (SC-PD) type detectors are widely used along with the semiconductor (SCD) ones. In the SCD, the ionizing radiation is transformed directly into the charge carriers. In contrast, in the SC-PMT and SC-PD ones, a two-stage transformation takes place, first into optical photons (in the scintillator) and then into the charge carriers (in the photoreceiving device). The double transformation results in the energy losses and redistribution (dispersion). Therefore, the sensitivity and energy resolution of the SCD are one decimal order higher. It is of a principal importance that, as the SCD volume increases from 1 mm\(^3\) to 1 or 10 cm\(^3\), these advantage become lost due to the carrier re-capturing in the traps. Therefore, the parameters of modern large-volume SCD are not so high as those of scintielectron detectors at room temperatures. The energy resolution of a SC-PD pair including the traditional CsI(Tl) scintillator of a volume up to 1 cm\(^3\) and a silicon [1] or HgI\(_2\) [2] photodiode at room temperature is about 5% to 6% on the 662keV line. The advance in the development of large wide-band SCD is only insignificant during lasr few years. An alternative way is to develop a novel high-efficiency scintillator matched spectrally to a solid photoreceiver. This will provide a detector with a volume of at least 10cm\(^3\) having the energy resolution of few per cent.

The theory of energy transformation process in scintillators is developed well. There are several models to describe that process and to calculate the energy resolution, e.g., [3–6] and others. The possible correlation between important characteristics of the process, such as the conversion efficiency, the light collection coefficient, the self-absorption, etc., on the one hand, and the energy resolution of the scintillation and of the entire system, on the other one, is, however, still insufficiently studied. The theoretical threshold of the energy resolution was not considered. It is just the above-mentioned problems that are the consideration objects in this work.

II. ENERGY RESOLUTION COMPONENTS.

STATISTICAL CONTRIBUTIONS

Let the energy resolution of a scintillator-photodiode couple be considered. Under approximation of the Gaussian shape of the electron signal output line, the resolution \( R \) takes the form

\[
R = \frac{FWHM}{MAX} = G \frac{\Delta E}{\langle E \rangle}; \quad G = \sqrt{8 \ln 2} \approx 2.355,
\]  

(1)
where $\langle E \rangle$ is the spectral line average energy; $\Delta E = \left[ \langle E^2 \rangle - \langle E \rangle^2 \right]^{1/2}$, its dispersion. The dependence on the radiation energy $E_\gamma$ is defined by the system efficiency in the photoabsorption peak, $\eta_p$:

$$E = \eta_p E_\gamma; \langle E \rangle = \langle \eta_p \rangle E_\gamma; \Delta E = f(E_\gamma; \langle \eta_p \rangle; \Delta \eta_p; ...).$$  \hfill (2)

The conversion efficiencies of the system, namely, the total, $\eta$ (the electron yield of the PD per 1 MeV of the ionizing radiation), scintillation, $\eta_{sc}$ (the light yield per 1 MeV), and peak, $\eta_p$ (the fraction of the total efficiency $\eta$ falling on the photoabsorption peak), ones, depend on the radiation energy as well as on a series of physical and geometric parameters denoted in (2) by dots (...). These efficiencies are fluctuating quantities. That is why there is no direct (monotonous) dependence of the line width, $\Delta E$, and thus, of the detector resolution, $R = R(\eta)$, on its conversion efficiency $\eta$ measured in the current regime. However, the correlations associated with the light yield improvement and its more efficient use are conserved. These correlations result as a rule in a resolution improvement that is confirmed by experimental data.

The fluctuation unmonotony, $\Delta E = \Delta E(\eta)$, is due to that the main parameters of the Gaussian distribution, namely, the mean value and dispersion, are independent of each other. Physically, this fact is due to that the energy transformation and transfer processes are multifactorial and indefinite. The spectral line widening does not result from the statistical contributions only. The latter are related directly to the conversion efficiency $\eta$. Non-statistical fluctuations are of no less importance. The latter include the fluctuations of various geometric-dynamic and spatially non-uniform factors influencing the detector conversion output. Moreover, fluctuations due to that the light yield is not in proportion with the energy $E_\gamma$ and those of the mutually dependent (competing) quantities are associated with the above-mentioned ones. Due to that complication, the most efficient method to determine the resolution consists in that the most important noise channels are separated in such a way that the corresponding fluctuations may be assumed to be independent. Then, each of those contributions is estimated. The independent fluctuation are added vectorially [7]. Therefore, we have for the total resolution (the index $k$ is the noise channel number):

$$R^2 = \sum_k R^2_k.$$  \hfill (3)

It is naturally that the resolution of a scintilelectron detector can be determined as

$$R^2 = R^2_{sc} + R^2_{st} + R^2_{pd},$$  \hfill (4)

where $R_{sc}$ is the scintillator intrinsic resolution; $R_{st}$, the statistical fluctuations of the energy carriers (photons and electrons); $R_{pd}$, the noise of the photodiode and electronic devices. Note that each contribution in the formula (4) may contain in its turn partial components characterizing the specific widening mechanisms. Since $R_k \leq R$ for each partial channel, it is necessary to attain the noise level of at least 1 to 2% for each component to provide the same value of the total resolution.

Let the conditions be determined allowing us to neglect the photodiode noise. This contribution depends mainly on the total number of noising electrons $N_{noise}$ relative to the useful electronic signal

$$R_{pd} \propto R_{noise} \propto \frac{N_{noise}}{N_e} \approx \frac{(\Delta E)_{pd}}{\eta E_\gamma},$$  \hfill (5)

where $\Delta E_{pd}$ is the corresponding energy spread; $\eta$ means the mean value of the conversion efficiency, if not otherwise specified. It will be shown below that a sufficiently high $\eta$ (of the order of 10%) is required to compensate the statistical detector noise. Then at the mean energy values of 0.5 to 1 MeV, the useful signal $N_e$ should be at least 30000 electrons. It follows from the expression (5) that the photodiode resolution will be smaller than 1 to 2% on condition that the spread $(\Delta E)_{pd}$
The theoretical limit is attained: the Poisson one becomes substantial. The fluctuations of the former drop steeper as those of the latter. This points directly that an increase in $\eta$ yields, in the limiting case $\eta \leq 1$. Thus, the electronic noises may be assumed to be suppressed strongly, at least under forced cooling. Their level $R_{pd}$ is lower than 1%. Then it is just the statistical noises that are of decisive importance in the moderate energy spectroscopy.

The statistical resolution $R_{st}$ consists of independent contributions of quantum fluctuations of the number (or energy) of the scintillation photons $R_{st,ph}$ and the photodiode photoelectrons $R_{st,el}$:

$$R_{st}^2 = R_{st,ph}^2 + R_{st,el}^2$$

(6)

In contrast to the cascade theory [9] for the SC-PMT assembly, the statistical fluctuations of photons cannot be neglected in this case. For SC-PD with a small $R_{st}$, a nearly ideal energy transport from the scintillation quanta to the photodiode photons is required. Therefore, the statistical noises of the photons and electrons will be of the same order of magnitude. The $R_{st,ph}$ contribution is often neglected also in SC-PD detectors, but at worse resolution values exceeding 4 $- 5\%$.

Let a formula be derived for $R_{st}$ in a detector with a high conversion efficiency. The $\gamma$-quantum energy transformation into a scintillation photon and then into an electronic signal will be considered to that end as independent events within a Bernoulli’s test series [7]. The statistical fluctuations of such a test series are described by a binomial distribution with the mean value $x = N_p$ and the dispersion $Dx = N_p(1 - p)$. Here $N$ it the number of tests, i.e., the number of converted particles at 100% efficiency. The event probability, $p$, means the scintillation efficiency $\eta_{sc}$ and the total one $\eta$, respectively. Thus, the partial statistical contributions will be expressed as

$$R_{st,j} = 2.355 \sqrt{\frac{1 - \eta_j}{\eta_j N_j}} \times N_j = \frac{E_j}{\varepsilon_j}; \ j = (sc; el),$$

(7)

where $\varepsilon_{sc}$ and $\varepsilon_{el}$ are the mean energy of the scintillation quantum and the mean energy of an electron-hole pair formation in the semiconductor, respectively. The total statistical contribution is

$$R_{st} = \frac{2.355}{\sqrt{\eta \left(E_\gamma / \varepsilon_{el}\right)}} \left[1 + K_Y \left(1 - \eta_{sc} \frac{\varepsilon_{sc}}{\varepsilon_{el}} + \frac{\varepsilon_{sc}}{\varepsilon_{el}}\right)\right]^{1/2},$$

(8)

where the coefficient $K_Y = (\eta/\eta_{sc}) (\varepsilon_{sc}/\varepsilon_{el})$ is introduced. Usually, $\varepsilon_{sc} = hc/\lambda = 1239/\lambda(nm) \approx 2 \div 3eV$ and, for example, $\varepsilon_{el}(Si) = 3.6eV$ in a silicon SCD. In detectors with low conversion yields, in the limiting case $\eta \leq \eta_{sc} < 1$, the usual Poisson noise of the charge carriers is obtained:

$$R_{st} \approx R_{st,el}^{poisson} = R_{st,el} \approx \frac{2.355}{\sqrt{\eta \left(E_\gamma / \varepsilon_{el}\right)}}.$$

(9)

At $\eta_{sc}$ exceeding 20% or $\eta$ not less than 10%, the distinctions of the binomial distribution from the Poisson one become substantial. The fluctuations of the former drop steeper as those of the latter. This points directly that an increase in $\eta_{sc}$ is of prospects for the maximum attenuation of the threshold statistical fluctuations. Under ideal energy transformation when $\eta = \eta_{sc} = 1$, the theoretical limit is attained:

$$R_{st}^{ideal} = R_{st} (\eta = 1) = 0.$$

(10)
An important feature of the statistical resolution follows from the expression (8). This resolution is characterized by a monotonous dependence on both conversion efficiencies. The other contributions to the total resolution do not exhibit that property. It is useful to transform the Eq. (8) into the form

$$R_{st} = \frac{2.355}{\sqrt{\eta_{sc}} \left( E_\gamma / \varepsilon_{el} \right)} \sqrt{K_Y^{-1} + \left( 1 - \eta_{sc} \frac{\varepsilon_{el} + \varepsilon_{sc}}{\varepsilon_{sc}} \right)}. \quad (11)$$

Solving this equation for \( \eta_{sc} \), we obtain

$$\eta_{sc} = \left( \frac{6.91}{\lambda E_\gamma} \right) \left[ \frac{1 + K_Y^{-1}}{R_{st}^2 + \left( \frac{6.91}{\lambda E_\gamma} \right) \frac{\varepsilon_{el} + \varepsilon_{sc}}{\varepsilon_{sc}}} \right], \quad (12)$$

where the energy \( E_\gamma \) should be expressed in (keV) and the scintillation wavelength \( \lambda \), in (nm). This expression defines the scintillation efficiency necessary to provide the preset statistical noise level in a detector with the coefficient \( K_Y \) defining the matching between the scintillator and the photodiode, \( 0 < K_Y < 1 \). At the ideal spectral and optical matching, the \( K_Y \) value is defined mainly by the light collection coefficient \( K_c \), so that \( (\varepsilon_{el}/\varepsilon_{sc})K_Y \approx K_c \). For a “red” scintillator with \( \lambda = 640\,\text{nm} \), at a good light collection with \( K_c = 0.6 \) to 0.8, the statistical resolution at the 662keV energy is small on the condition following from the Eq. (12),

$$R_{st}(E_\gamma \propto 1\,\text{MeV}) \leq 1\% \iff \eta_{sc} \geq 25\%. \quad (13)$$

For the total conversion efficiency \( \eta \), that level will amount 10 to 15%. In the energy region up to 10\,MeV and over, the statistical resolution drops dramatically, since \( R_{st} \propto 1/\sqrt{E_\gamma} \). In the best scintillation assemblies, the statistical noise at the 662keV line is from 2% to 4%, see., e.g., [10] and other publications. At high \( E_\gamma \) from 10\,MeV to 1\,GeV, this contribution will decrease by a factor from 3 − 4 to several decimal orders and amounts several fractions of a percent.

Thus, the statistical contribution can be minimized substantially in a detector with a high scintillation efficiency and well-matched SC-PD couple. The development of new scintillators based on semiconductors (e.g., ZnSe(\text{Te}) [10]) or rare-earth elements (e.g., LaCl3(\text{Ce}) [11]) evidences the real attaining of extremely small \( R_{st} \) values even in the moderate energy region. The high light yield is necessary also to optimize the intrinsic energy resolution of the scintillator, \( R_{sc} \). It includes usually non-statistical fluctuations. This quantity is a natural threshold of the limiting improved detector resolution. This is associated with that \( R_{sc} \) includes residual (slightly dependent on or independent at all of \( E_\gamma \)) contributions at any radiation energy, including the high one. Its part will be decisive in the improved-spectrometry detectors.

### III. THE SCINTILLATOR INTRINSIC RESOLUTION

The decisive role of the intrinsic resolution noted in [3–6], [8–11] and other works is clearly pronounced in alkali halide scintillators where it amounts from 4% to 5%. In heavy oxides BGO and CWO the internal resolution of sufficiently small samples (less than 10\,cm³ in volume) is negligible, since their light yield is in proportion to the ionizing radiation energy. Note that recent studies, in particular, in [12–15], confirm convincingly the dominating dependence between the light yield non-proportionality and the deterioration of the detector intrinsic resolution. The non-proportionality is due mainly to the conversion efficiency non-linearity with respect to the secondary electron formation at the radiation absorption by the scintillator and to uncorrelated processes of the multiple Compton’s scattering. The proportionality effect is manifested in numerous experimental data. For example, RbGd2Br7(\text{Ce}) seems to exhibit a rather good intrinsic resolution at the proportional light yield [13]; its total resolution at the 662keV line is 4.1%, the statistical contribution of the PMT statistical noise of 3.5% being the main component in this case. The intrinsic resolution of ZnSe(\text{Te}) scintillator amounts 3.26% at the same line of \(^{37}\text{Cs}\) while the total resolution is 5.37% [14].

The intrinsic resolution includes several components. Some of those drop monotonously depending on \( E_\gamma \). There are components, however, independent of or slightly depending on the ionizing
radiation energy provided that it is absorbed completely in the crystal (the escape resolution and edge effects being neglected). Among those threshold contributions, the substantional resolution of the scintillator $R_{sc}$ and the light collection resolution $R_{lc}$ are most substantial, so that

$$R_{sc}^2 = R_{sub}^2 + R_{rc}^2 + R_r^2,$$

where the insubstantial rest of the intrinsic resolution is denoted as $R_r$. To neglect the escape resolution, it is necessary to use the crystals having the characteristic dimension $L$ not less than

$$L \geq L_c \approx \frac{0.45 E_{sc}(MeV)}{\rho (g/cm^3)} ,$$

where $L_c$ is the radiative $\delta$-electron free path in the radiation energy region under consideration. The edge effects can be neglected in “heavy” crystals of a volume of several tens of $mm^3$ or more. To retain a small crystal volume at very high energy levels, metallized reflectors are to be used. Other advantages offered by the mirror scintillator boundary will be considered below. The substantional resolution $R_{sub}$ is due mainly to the light yield non-proportionality, $E_{sc} = \eta_{sc} E_\gamma$; $\eta_{sc} = \eta_{sc}(E_\gamma) \neq \text{const.}$, and by its spatial inhomogeneity. The light collection contribution $R_{lc}$ is defined by the geometric-dynamic fluctuations in the crystal of a preset shape at fixed optical parameters in the crystal volume and at its boundary.

The spatial inhomogeneity of scintillations, $\eta_{sc} = \eta_{sc}(\vec{r})$, is of great importance, in particular, in activated compounds. The resolution of inhomogeneities is defined by the factor $R_{inhom} = 2.36(\Delta \eta_{sc})/\langle \eta_{sc} \rangle$ with the spatial averaging of the corresponding fluctuations. To suppress the statistical noise, a high mean scintillation efficiency $\langle \eta_{sc} \rangle$ of about 25% is necessary at moderate energies. To attain the low macroinhomogeneity resolution in this case, the dispersion $\Delta \eta_{sc}$ should not exceed 0.1%. It is just a superhigh homogeneity of the activator distribution that answers to this requirement.

There are scintillators with extra small $R_{sub}$ values. Their specific feature is the light yield proportionality. Those include the above-mentioned tungstates. For example, CdWO4 of 200 $cm^3$ volume has $R_{sub}$ less than 0.3% (after this contribution is isolated from the total resolution) and 0.03% to 0.08% at the crystal volume from 3 $cm^3$ to 20 $cm^3$ (data of [16] were used for the estimations). ZnSe(Se) shows a rather good linearity with $\eta_{sc}(5.9keV)/\eta_{sc}(662keV) = 85\%$ and $\eta_{sc}(16.6keV)/\eta_{sc}(662keV) = 90\%$ [14], the physical light yield being 28000ph/MeV. Some complex oxides behave somewhat worse. So, Lu3Al5O12(Ce) has the light yield 13000 ph/MeV and $\eta_{sc}(16.6keV)/\eta_{sc}(662keV) = 76\%$, while LuAlO3(Ce) where the light yield is decreased down to 11000 ph/MeV shows $\eta_{sc}(16.6keV)/\eta_{sc}(662keV) = 71\%$ [15]. To compare, the non-linearity of NaCl(Tl) amounts about 80% at the light yield of 40000 ph/MeV. Nevertheless, the above-mentioned modern scintillators and other ones offer good prospects in the attaining of the high scintillator energy resolution (both intrinsic and total one) as their spectrometric characteristics will be further improved.

To minimize the substantional contribution, it is necessary to develop a material with a high scintillation efficiency as well as high light yield proportionality and homogeneity. The scintillators exhibiting the intrinsic (or nearly intrinsic) luminescence type seem to be of priority. There is no strict theory explaining in terms of physical phenomena why unactivated scintillators or those activated with isovalent impurities show substantially improved substantional resolutions. This regularity, however, is observed in experiments. This may be due to the following physical reasons. The absence of a non-isovalent activator, on the one hand, provides the uniform and homogeneous distribution of the emission centers over the sites of the ideal crystal lattice. On the other hand, the absence of the direct energy transformation where its intermediate transport from the matrix to the emission centers is required is accompanied by losses and results in the detected pulse widening.

The scintillators containing non-isovalent impurities (emission centers) show a considerable non-proportionality of the light yield, in particular, near the K-absorption threshold. For example, among yttrium garnets, it is just crystals containing the isovalent cerium admixture, YAlO3(Ce) (YAP) and Y3Al5O12(Ce) (YAG) that exhibit the best proportionality while Y2SiO5(Ce) (YSO) with non-isovalent Si and Ce the worse one, cf. [13]. The proportionality of Lu2SiO5(Ce) (LSO) and Gd2SiO5(Ce) (GSO) is rather poor. In the same time, it is considerably better at zinc
selenide with isovalent tellurium or at lanthanum chloride with isovalent cerium. Furthermore, tungstates and germanates with their intrinsic emission centers in the crystal matrix show an ideal proportionality. Non-activated NaI and CsI also exhibit a smaller intrinsic resolution than those activated with sodium or thallium. Moreover, at low temperatures (liquid nitrogen) pure NaI and CsI have considerably higher light yields relative to NaI(Tl) amounting 217% and 190%, respectively [16]. The light yield is increased when the interaction with the lattice phonons is suppressed. The considerable increase of the scintillation efficiency can be associated with the more pronounced definiteness of the energy transformation in the absence of activator. This should be accompanied by an improvement in the substantial resolution. It is just what is observed even at room temperatures when the light yield of activated compounds is enhanced. This is due to the same cause, a very small intrinsic resolution of non-activated scintillators. Finally, it is to note that it is just the PbWO4 (PWO) scintillator that has been chosen as the main component of high-energy calorimeters for the CERN accelerator and other ones. This is connected, not in the last place, with its extremely low substantial resolution \( R_{\text{sub}} \). At high energies, 1GeV to 100GeV and over, a resolution smaller than 1% has been already attained for such detectors.

The geometric fluctuations of the light collection, \( R_{lc} \), under homogeneous scintillation distribution are defined only by the light collection coefficient dispersion. This resolution component, in spite of its non-statistical character, has the following form in the Gaussian approximation:

\[
R_{lc} = 2.355 \sqrt{\frac{\langle K_c^2 \rangle - \langle K_c \rangle^2}{\langle K_c \rangle}}. \tag{16}
\]

The light collection resolution and the coefficient \( K_c \) itself depend on the scintillator geometric shape and optical properties, namely, the light reflection, refraction and absorption. The resolution attains its minimum under ideal light collection with mirror reflection at the boundary and without light absorption in the scintillator.

The reason for the ideality of the mirror light collection is established in the frame of the stochastic (geometric-dynamic) light collection theory [17]. The picture of the geometric (light) rays distribution in a detector of macroscale size can be substituted by a dynamic model, a billiard with elastic reflections from the boundary. The billiards are described by special dynamic systems, the reversible maps in a symmetric phase space with coordinates \((\varphi_1, \varphi_2)\). A couple of such coordinates defines a ray with two successive reflection points at the billiard (detector) boundary. The light collection parameters are expressed in terms of the invariant distribution function \( f \) for the corresponding dynamic flow. The latter defines the total set of all possible light rays being reflected from the billiard boundary. It is just the possibility to consider the light collection picture not for individual trajectories but in its entirety that makes a substantial distinction of this approach from widespread numerical models. The non-averaged quantities \( K_c \) and \( K_c^2 \) are expressed as

\[
K_c = \iint f(\varphi_1, \varphi_2) \chi_c(\varphi_1, \varphi_2) A(\varphi_1, \varphi_2) \, d\varphi_1 \, d\varphi_2; \tag{17}
\]

\[
K_c^2 = \iint f(\varphi_1, \varphi_2) \chi_c^2(\varphi_1, \varphi_2) A^2(\varphi_1, \varphi_2) \, d\varphi_1 \, d\varphi_2, \tag{18}
\]

where \( \chi_M \) is the characteristic function of the set \( M \) (equal to unity in the points of the set and zero outside of it); \( \chi_c \) corresponds to the light pick-off in the phase space that does not include the captured light region. The factor \( A \propto \exp(-\mu_{sc} L) \) describes the light losses; \( \mu_{sc} \) is the optical absorption coefficient of the scintillator; \( L \) – the length of the latter. The expressions (17) and (18) should be averaged over the flash distribution in the scintillator, depending on the corresponding distribution (that was assumed above to be homogeneous). For the mirror light collection with mirror boundaries, \( r \approx 1 \); neglecting the absorption in an optically transparent crystal, we have \( A \approx 1 \). Using the projection property \( \chi^2 = \chi \), we obtain the equality \( \langle K_c^2 \rangle = \langle K_c \rangle \). Then, starting from (17)–(18), we obtain the relationship

\[
R_{lc}^{\text{mirror}} = 2.355 \sqrt{\frac{1 - \langle K_c \rangle}{\langle K_c \rangle}}. \tag{19}
\]
Mathematically, the obtained fluctuation character corresponds to the binomial distribution. At the ideal light collection, when \( K_c = 1 \) and \( r = 1 \), the theoretical limit is attained:

\[
R_{lc}(K_c = 1; r = 1) = 0 ,
\]

that is confirmed by numerical and experimental data (see e.g. \cite{10}, \cite{16}). The limiting high \( K_c \) values are attainable in detectors where the dynamics of light rays is completely chaotic. Due to the “disjoining of correlations”, the captured light is absent therein and the technical light yield is limited by the absorption factor only. The mentioned detector shape answers to the chaotic billiard geometry. The known geometries of those type include the “stadium”, the cube with an internal void, etc. As an exception, it is just sphere corresponding to the integrable billiard that is characterized by the ideal light collection.

The light collection becomes changed considerably if the absorption takes place (\( A \neq 1 \)). This results not only in a reduced intensity of the scintillation propagation but causes the light collection inhomogeneity. The effective light collection with a high resolution is attained in small-size scintillators and/or in those having a high optical transparency. It follows from experimental data, numerical calculations and theoretical estimations that a scintillator of regular geometry with a volume of about 10cm\(^3\) (e.g., a cylinder of commensurable height and diameter values about 3cm) and a high-quality mirror boundary (\( r \) from 0.8 to 0.9) will exhibit a sufficiently low light collection resolution on condition that

\[
R_{lc} \leq 1\% \Leftrightarrow \mu_{sc} L \leq 0.1 \quad (10 L \leq l_{sc}) ,
\]

where \( l_{sc} = \mu_{sc}^{-1} \) is the light ray free path in the scintillator. On the same conditions, a good light collection \( K_c \), up to 60\%, is attained. It is just the alkali halide crystals that exhibit a high transparency, \( l_{sc}[\text{NaI(Tl)}] \) attaining 2m; it is somewhat lower in tungstates, \( l_{sc} \) being from 10cm to 50cm. Nevertheless, the \( l_{sc} \) values of about 30cm that are required for a scintillator of about 10cm\(^3\) volume are quite attainable. Note that the optical transparency increase improves, in an indirect manner, the substantial resolution \( R_{sub} \) that depends also on the energy loss in the scintillator.

The increase of the scintillator volume results as a rule in a worse intrinsic energy resolution due to the optical absorption. But exceptions are possible when a special geometry is chosen that optimizes the light collection. The search for scintillators with high atomic numbers, \( Z_{eff} \), and density values, \( \rho \), becomes very actual. This approach allows to avoid the sharp increase of the scintillator volume in the moderate and high energy spectrometry. The improvement of the light collection and its resolution is a geometric-dynamic problems to a considerable extent and does not require any substantial changes in the scintillator production technology. Basing on the modern advances in the non-linear physics, some untraditional scintillator geometries could be developed, such as asymmetrical shapes, varying curvature boundaries, “stadium” type defocusing billiards, systems containing topological defects, e.g., holes. The light collection inhomogeneities must be optimized in such scintillators due to strong stochastic mixing of the light rays.

Thus, the light collection intrinsic resolution is minimized in mirror-reflecting scintillators of appropriate geometry and high optical transparency. Alternatively, volume-diffuse systems consisting of small crystalline grains in an optically conductive medium can be used for scintillators with high optical losses. The intrinsic material resolution is expected to be small in non-activated scintillators or “combined” ones (where the emission of an isovalent activator and the intrinsic one of the scintillator are combined) at a high conversion efficiency, homogeneity and the complete absorption of the ionizing radiation. Scintillators with a high \( Z_{eff} \) and meeting the condition \((2 \div 3) l_r \leq L\), where \( l_r \) is the radiation attenuation length. The intrinsic scintillator resolution can be lowered down to a level of 1 – 2\% by satisfying the above-mentioned conditions in combination.

IV. CONCLUSIONS

It follows from the above theoretical analysis that there are no physical reasons hindering to attain sufficiently small values of the main energy resolution components in a SC-PD detector. Only in the low energy range (several hundreds keV) where it is just the statistical contribution that predominates, the latter increases sharply as the radiation energy drops even in scintillators with a limiting scintillation quantum yield. In contrast, the \( R \) values from 15 to 2\% are quite attainable in the range of 662keV to 10MeV and over. To that end, it is necessary to increase
substantially the total conversion efficiency (1.5 to 2 times) and the scintillation efficiency (up to 25 to 30%) as compared to the available SC-PD detectors having $R$ of 3 to 5%. Moreover, optically transparent, homogeneous scintillators exhibiting a high light yield proportionality, in particular those close to the proper emission type, are to be searched for. To improve the light collection, the search for untraditional scintillator geometry (e.g., the “stadium” type one) providing a strong chaotization of light beams therein (at the mirror surface) should be preferred. The last advances in the field of “heavy” crystals with light yields exceeding that of CsI(Tl) (see [10],[11], [18-20], etc.) evidence clearly that these problems are quite realizable and offer good prospects.

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