Compressed domain image classification using a multi-rate neural network

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Abstract: Compressed domain image classification aims to directly perform classification on compressive measurements generated from the single-pixel camera. While neural network approaches have achieved state-of-the-art performance, previous methods require training a dedicated network for each different measurement rate which is computationally costly. In this work, we present a general approach that endows a single neural network with multi-rate property for compressed domain classification where a single network is capable of classifying over an arbitrary number of measurements using dataset-independent fixed binary sensing patterns. We demonstrate the multi-rate neural network performance on MNIST and grayscale CIFAR-10 datasets. We also show that using the Partial Complete binary sensing matrix, the multi-rate network outperforms previous methods especially in the case of very few measurements.

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1. Introduction

Compressive sensing (CS) [1]–[3] is a mathematical framework for efficient signal acquisition and robust recovery. According to CS theory, a real-world image that is sparse or compressible under certain transformation basis can be reconstructed almost exactly from much fewer number of linear measurements than required by the Nyquist sampling theorem. The CS framework has inspired incorporation into a variety of imaging systems. There have been comprehensive reviews of current state of the field of CS imaging [4, 5]. In particular, in a class of imagers known as spatial multiplexing cameras, where instead of acquiring pixel intensities as in traditional cameras, coded linear measurements of the pixels of the scene are obtained and then used to computationally reconstruct the scene. In the imaging system where the CS framework has pushed the detector design to its limit of employing a single-pixel photodetector, such an imaging platform is commonly referred to as the single-pixel camera (SPC) [6]. The SPC efficiently implements the process of acquiring random linear measurements in hardware through the use of a spatial light modulator. This approach features great savings in signal acquisition and detector cost, and especially useful in imaging outside of the visible regime, e.g. infrared imaging [7], terahertz imaging [8] and hyperspectral imaging [9] where detector arrays are extremely expensive or even nonexistent. In this paper, we focus on applying machine vision algorithms directly on the measurements from a SPC platform as a single grayscale system.

In many traditional and compressive imaging applications, the goal is not to reconstruct the full image, but instead to solve an inference problem like anomaly detection or classification. Traditionally, such applications rely on acquiring high quality images and extracting features from the images for inference [10]–[12]. However, in the CS framework, reconstruction of the image, which is either through iterative optimization algorithms [13] or most recently through deep neural networks [14], have drawbacks: iterative algorithms are computationally expensive and reconstructions are poor at low measurement rates, and deep learning approaches require intensive and prolonged training process and the trained networks are dataset-dependent.
Alternatively, there has been quite a lot of work solving these inference problems in the compressed domain directly on compressive measurements thus bypassing reconstruction [15]–[20]. This single detector-based inference has additional virtues of reducing acquisition hardware costs and requiring fewer measurements, therefore directly impacting storage, transmission and computation costs.

As a first demonstration of inference directly on compressed measurements, Davenport et al. [19] employed the matched filter of compressed sensing patterns applied to a library of images to create a ‘smashed’ filter and demonstrated the validity of the random projections-based approach to solve classification problem. Later, Li et al. employed the same SPC system but with learned sensing patterns through data-dependent “secant projections” to perform classification directly in the compressed domain [20]. Recently, as neural networks have prevailed as a powerful tool in traditional imaging applications, the convolutional neural network (CNN) has also been employed in data-driven framework for compressed domain classification. Recent results [16], [17] have produced much higher classification accuracy compared to the smashed-filtering approach as well as being computationally more efficient.

Despite their progress in compressed domain image classification, there are several limitations with the current neural network methods. To begin with, the ratio of the number of linear measurements to the number of pixels in the reconstructed image is called the measurement rate. Previous methods require training a separate network model for each measurement rate. There are many applications where the measurement rate keeps changing or is indefinite but especially in a compressive sensor or imager, therefore such methods inevitably lead to the need of training a large number of neural network models which is hugely inefficient in storage and computation. Secondly, both previous neural network methods [16, 17] first project the compressive measurements into the image space either by a fixed [16] or learned [17] projection matrix, which increases the dimensionality that subsequent layers need to deal with and therefore computationally expensive especially when image size is large. Thirdly, the classification accuracy remains low at very low measurement rate like 0.01, but the low measurement rate is a regime that is especially interesting and useful in a lot of application scenarios, e.g. real time compressive classification, and power-constraint environments when taking measurements is expensive. Lastly, the learned sensing matrix [17], though achieves better performance compared to fixed sensing matrix, is in full precision. On a SPC platform, however, displaying floating-point patterns leads to reduced pattern speed and large storage cost in memory compared to binary patterns. In addition, fixed binary sensing patterns are data-independent and do not require learning from full image datasets which might not exist or extremely expensive to acquire, like infrared or hyperspectral datasets.

In this paper, we directly address these limitations. The contributions of this paper are as follows.

To begin with, we develop a general and efficient multi-rate training scheme that allows a single neural network to perform compressed domain classification on a range of measurement rates directly on compressive measurements acquired by a SPC using fixed binary sensing patterns. The effectiveness of this approach is demonstrated on standard datasets and the multi-rate neural network produces approximate equal classification accuracy at each measurement rate as that of a network valid for only that particular measurement rate. In this multi-rate training scheme, the network only needs to be trained with measurements of a few different measurement rates within the range of interest instead of being trained with all different measurement rates, therefore leading to great savings in computation. Such scheme can also be regarded as opening up an extra dimension for performing data augmentation that uniquely exists in CS related tasks through neural networks to endow the network with multi-rate property. Next, in this paper we perform the multi-rate scheme on the 2-layer feedforward network with the following benefits: the success of the multi-rate training scheme with the 2-layer feedforward neural network serves as a foundation to demonstrate the effectiveness of the multi-rate training scheme for any deeper networks when more layers are added after the first
fully connected layer; also we show that this shallow and simple network has approximately equal performance as the deeper networks on MNIST [16] while being computationally more efficient. Lastly, different fixed binary sensing patterns are tested. In particular, the relatively new Partial Complete (PC) matrix [21, 22] for compressive sensing as the binary sensing matrix is compared with the standard permuted Walsh Hadamard (PWH) matrix. Results show that PC matrix significantly increases the classification accuracy at very low measurement rates like 0.01, which due to the low frequency patterns from PC matrix which pick up more energy of the image.

2. Background and related work

2.1 Compressive sensing

Compressive sensing seeks to minimize the number of measurements to be taken from a signal while still retaining the information necessary to produce a nearly complete recovery, exploiting the structure of the signal. A core result of CS states the following: a length $N$ signal (or image) that is $K$-sparse in an arbitrary basis can be recovered from $M = O(K \log(N/k))$ linear measurements. Formally, a signal $x = \Psi \alpha$, where $\Psi$ is an orthonormal basis, and $\alpha$ is $K$-sparse, can be exactly reconstructed from measurements

$$ y = \Phi x = \Phi \Psi \alpha $$

where $\Phi$ is an $M \times N$ matrix called the sensing matrix or sensing patterns, provided that the product matrix $\Phi \Psi$ satisfies a certain restricted isometry property (RIP) [21]. The ratio of $M/N$ is called measurement rate. The measurement $y$ is a vector of weighted sum of entries in signal $x$, representing a highly concise encoding of $x$, based on which the image inference can be made. In real world experiments, the measurement $y$ is usually corrupted by noise, so the measurement $y$ can be represented as $y = \Phi x + \epsilon = \Phi \Psi \alpha + \epsilon$, where $\epsilon$ is an upper bound on the noise magnitude.

To recover the signal $x$ from the random measurement $y$, it has been shown that $l_1$ optimization could be used [1, 2, 23]:

$$ \hat{\alpha} = \arg \min \| \alpha \|_1 \quad \text{such that} \quad \| y - \Phi \Psi \alpha \|_2 < \epsilon $$

Thus we can closely approximate $x$ stably which is a $K$-sparse or compressible vector with high probability. Total variation (TV) regularization [24] is another well-known method for its ability to recover the edges or boundaries more accurately than $l_1$ method:

$$ \hat{x} = \arg \min \sum_i \| D_i \| \quad \text{such that} \quad \| y - \Phi x \|_2 < \epsilon $$

The TV minimization suggests that the gradient of the 2D image signal $x$ is sparse, so it can be considered as a generalized $l_1$ minimization problem on the image gradient map.

However, signal reconstruction through iterative optimization algorithms is computationally expensive and time consuming due to the algorithm being iterative in nature and not suitable for parallelization. Also, the reconstruction quality is poor at very low measurement rates. Furthermore, there are parameters in the algorithm that needs to be tuned or defined by the user making the algorithm difficult for practical use. The deep learning approaches for CS signal reconstruction, though much faster than iterative optimization algorithms, require intensity and prolonged training process. Besides, the trained networks only perform well on datasets that are similar to the training dataset, and need to be retrained or fine-tuned if to be used with different types of images.

2.2 Single-pixel compressive imaging
While this paper is simulation only and the final result will be applicable to a variety of compressive imaging systems, here we focus on its implementation in the original single-pixel camera architecture (SPC) [6]. The SPC, as showed in Figure 1, is an imaging platform that implements the image encoding process specified by (1) in the optical domain. A short description of the optical setup is as follows. The scene, which is the original signal $x$, is focused by a biconvex lens onto a digital micromirror device (DMD) functioning as a spatial light modulator. The DMD consists of an array of micro-sized mirrors where each mirror can tilt at either angle of $+12^\circ$ or $-12^\circ$ about their diagonals. To encode the signal, the DMD is programmed to display a sequence of sensing patterns consisting of binary elements $\{0, 1\}$ to modulate the intensities of scene pixels. Each sensing pattern is one row of the sensing matrix reshaped into a 2D configuration. When the image of the scene is projected onto the DMD, in effect the DMD mirrors tilting $+12^\circ$ (ON state) encodes 1 and the mirrors tilting $-12^\circ$ (OFF state) encodes 0 on the image pixels because the light reflected by mirrors tilting $+12^\circ$ comes out in one direction and by mirrors tilting $-12^\circ$ comes in an opposing direction, and a second lens sums up the light coming in the $+12^\circ$ direction while the rest of the light is discarded. The intensity of summed up light is detected by a single photon detector as measurement data corresponding to the current sensing pattern displayed on DMD.

![Fig.1](image.png)

**Fig.1.** Schematic of performing compressed domain classification using compressive measurements acquired from the SPC through neural networks.

The standard type of sensing matrix used in the original SPC is random binary matrix such as permuted Walsh Hadamard (PWH) matrix. Although existing SPC architectures allow for displaying floating-point values by temporal modulation of the micromirror being in the ON state during each measurement period, floating-point sensing patterns take up much more memory and lead to reduced pattern speed compared to binary patterns and therefore not preferable in real applications.

**2.3 Compressed domain classification**

In regards to compressed domain inference, Calderbank et al. [25] provided the first theoretical results that learning directly in the compressed domain is feasible. In particular, they provided bounds demonstrating that the performance of a linear SVM in the compressed domain is close to the performance of the best linear classifier in the uncompressed domain and that classifiers can be learned directly in the compressive domain. Davenport et al. [19] employed the ‘smashed’ filter where the random sensing matrix and a 1-nearest-neighbor classifier were used. Later, the same SPC system was employed but with learned patterns through data-dependent “secant projections” to perform classification directly in the compressed domain [15]. Most recently, Lohit et al. [16] employed a convolutional neural network (CNN) for compressed domain classification, which produced much higher classification accuracy compared to the smashed-filtering approach as well as being computationally more efficient. Following this work, Adler et al. [17] developed an end-to-end deep learning solution for compressed domain classification where sensing matrix is jointly learned with the inference operator. In the two previous cases, CS measurements are always projected first to the image space, either by
multiplying the transpose of sensing matrix to get $\Phi^T y$, or by learning the projection matrix as weights in a fully connected layer. Then convolutional layers follow starting on image space. There are many limitations associated with the previous neural network methods. To begin with, they trained single-rate neural networks that work only with a particular measurement rate. Next, the estimate image $\Phi^T y$ is extremely noisy and it increases the dimensionality of the input to the neural network, and therefore not a good justification of such projection procedure. Lastly, learning the projection matrix will drastically increase the number of weights to learn especially when the image is of large sizes.

3. CS multi-rate neural networks

This section describes the multi-rate training scheme for compressed domain classification with fixed binary sensing patterns using the 2-layer feedforward neural network. Then we discuss the significance of using the 2-layer feedforward network in tackling the problem. Also, we point out the values of fixed binary patterns for spatial multiplexers and the advantages of using the PC sensing matrix to raise the classification accuracy at very low measurement rates.

3.1 CS multi-rate training scheme for neural networks

We develop a general and efficient CS multi-rate training scheme that enables a single neural network to perform classification over a range of measurement rates, from $m_{r_{\text{min}}}$ to $m_{r_{\text{max}}}$, corresponding to $m_{\text{min}}$ through $m_{\text{max}}$ number of measurements. The trained multi-rate network produces approximate equal classification accuracy at each measurement rate as that of a neural network valid for only that particular measurement rate. A main discovery is that training on a few different measurement rates as shown in Figure 2 is sufficient for the neural network to achieve such desired performance at the untrained measurement rates. We will use the term ‘multi-rate neural network’ to denote a network trained by our CS multi-rate training scheme, and refer to a neural network which is valid for one measurement rate as a single-rate neural network, as those in previous methods [16, 17].

![Fig.2. Schematic of the multi-rate neural network scheme compared to single-rate neural networks.](image)

The details of the training scheme are presented as follows. We first select a set of different measurement rates: \{\(m_{r_1}, m_{r_2}, \ldots, m_{r_k}\)\}, where each measurement rate is between $m_{r_{\text{min}}}$ and $m_{r_{\text{max}}}$, and $k$ is the number of measurement rates selected. Then for every training image, compressive measurements corresponding to each of the selected measurement rates are generated. Then these measurement vectors are zero-padded at the end of each vector to reach length $m_{\text{max}}$, which is also the dimension of the input layer of the network. All these zero-padded measurement vectors form the new final training set. Then the network is trained on this final training set with standard optimization algorithms. When preparing the final training set, it is crucial for the data points in the training set to be randomized. Sequentially training on a series
of data batches where each batch is of a single measurement rate will fail to work as we have learned through experiment.

A question of key importance in the above training scheme is that how large should $k$ be? Should we include measurements of all possible measurement rates for each training image in the final training set, considering the network will be used to predict on all these measurement rates? The naive approach where all measurement rates between $m_{\text{min}}$ and $m_{\text{max}}$, or equivalently all the different number of measurements from $m_{\text{min}}$ through $m_{\text{max}}$ are used for each training image, however, will cause the final training set to explode. That is, the final training set will have $k = (m_{\text{max}} - m_{\text{min}})$ times more data points than the original image training dataset, which could drastically increase the training time and complexity if not infeasible at all. But we have found that it is unnecessary. We have shown through MNIST and grayscale CIFAR-10 datasets that training the network on only a few of the measurement rates is sufficient to achieve expected test accuracies and that adding more measurement rates for training produces approximately equal performance.

The above training scheme is simple in procedure and efficient in computation. It involves minimal processing in preparing the new training set, and then uses standard neural network training algorithms without the need for any modification or fine-tuning. The scheme we developed for creating a new CS multi-rate training dataset bears similarities but also key differences compared to the conventional data augmentation methods, which are used for preprocessing full images such as cropping, flipping and rotation. For one obvious difference, when cropping images for data augmentation, the size of cropped image cannot be too small and is usually at least 50% or higher of the original image in order to retain enough information of the scene, while in our CS multi-rate training scheme, $m_{\text{min}}$ can be a small fraction of $m_{\text{max}}$, e.g. $m_{\text{min}} = 10$, $m_{\text{max}} = 256$ in our experiment and $m_{\text{min}}$ is approximately 4% of $m_{\text{max}}$. The CS multi-rate training scheme of creating a new training dataset can be regarded as opening up another dimension for performing data augmentation that uniquely exists in CS related tasks to endow the neural network with multi-rate property, and it is in parallel to the conventional image data augmentation method. We call this scheme CS multi-rate data augmentation or CS multi-rate training. In this sense, for compressed domain classification using neural networks, two types of data augmentation can be performed: the conventional image augmentation which allows the system be invariant for changes in object location, lighting, angle, etc., and CS multi-rate data augmentation which endows the system with CS multi-rate property.

3.2 2-Layer feedforward neural network

The network architecture we use in this paper for the multi-rate neural networks is the 2-layer feedforward neural network. It consists of the following layers in sequence: an input layer with $m_{\text{max}}$ neurons; a fully-connected layer which we call the hidden layer with tan-sigmoid transfer function, with $H$ hidden neurons; an output layer with softmax transfer function, with $c$ output neurons, where $c$ is the number of classes of the dataset.

The significance of using the 2-layer feedforward neural network in this paper is as follows. Firstly, the success of the multi-rate training scheme with the 2-layer network demonstrates that a single fully-connected layer after the input layer can extract discriminative nonlinear features with multi-rate property by training on a few of measurement rates. Based on the foundation of the 2-layer feedforward network, the effectiveness of the CS multi-rate training scheme is expected to generalize to deeper networks where more layers are added after the hidden layer. Secondly, we show that the simple 2-layer network with binary sensing patterns has approximately equal performance as convolutional networks with $\Phi^1y$ or $\Phi^2y$ as input on MNIST and grayscale CIFAR-10. The 2-layer network scheme is potentially much more efficient in computation and power consumption, which is crucial in resource-constrained applications. Lastly, when adding more layers, the number of neurons in the first fully-connected layer can be designed to be smaller than the image space dimension to avoid
exploding number of weights and overfitting while still learning a projection matrix as the weights of the first fully-connected layer.

3.3 Partial Complete sensing matrix

In this paper, we focus on using fixed binary sensing matrices for taking CS measurements, in contrast to random Gaussian [16] or learned sensing matrix [17] which are in full-precision. Fixed binary sensing patterns are more beneficial in practice than random Gaussian or learned patterns. This is because, even though displaying floating-point patterns on the DMD in a SPC platform is possible by temporal modulation of bitplanes of the sensing pattern, it leads to reduced pattern speed and larger storage cost in memory compared to binary patterns. Also, fixed binary sensing patterns are data-independent and do not require learning from full images datasets which might not exist or extremely expensive to acquire, like infrared or hyperspectral datasets.

We compare two types of binary sensing patterns: the randomly permuted Walsh-Hadamard (PWH) matrix, and the Partial Complete (PC) matrix [21, 22]. In the PC matrix, the rows are grouped into blocks where the same signature pattern is shared within a block of rows, and the measurement values for rows in each block tend to have similar intensities. The first block of rows in the PC matrix consists of low frequency patterns, and since most natural images have energy concentrated in low frequency region, we propose to use PC matrix and expect it to perform better at very low measurement rates than PWH matrix.

4. Experimental results

In this section, we present the experimental results for performing the CS multi-rate training scheme on the 2-layer feedforward neural networks. We also compare the PC matrix and PWH matrix for sensing and found that PC sensing matrix greatly enhances the classification accuracy compared to PWH matrix at very low measurement rates. In addition, we compare the 2-layer network scheme with convolutional network scheme using fixed projection matrix to show that they have approximate equal performance.

4.1 Datasets

We use two widely used datasets MNIST [26] and CIFAR-10 [27], and we slightly modified them to suit our problem setting. Examples images of each modified datasets are shown in Figure 3. The MNIST dataset consists of $28 \times 28$ grayscale images of size-normalized and centered handwritten digits. It contains 60000 training images and 10000 test images. Each image belongs to one of 10 classes corresponding to the digit “0” through “9”. Since Hadamard matrix and PC matrix don’t exist with dimension $784 \times 784$ for sensing $28 \times 28$ images, we pad zeros around four sides of every image evenly so that the final image size becomes $32 \times 32$, and then the $1024 \times 1024$ PWH and PC matrices can be used for sensing. The CIFAR-10 dataset contains $32 \times 32$ color images belonging to 10 classes with 50000 training images and 10000 test images. Since our problem setting of interest concerns a monochromatic single-pixel camera with a single photodetector for short-wave infrared (SWIR) where each measurement taken is by nature a grayscale measurement of the scene, we convert the color images in CIFAR-10 to grayscale images by only keeping the intensity channel for classification to take this into account, which could reduce classification accuracy but more accurately reflects the type of SWIR image data that would be acquired.
4.2 Training details

For the multi-rate neural networks in this paper, we choose: $mr_{\text{min}} = 0.01$, $mr_{\text{max}} = 0.25$, and accordingly, $m_{\text{min}} = 10$, $m_{\text{max}} = 256$. For simplicity, we use the terminology ‘$k$-point training’ to indicate that, for every image in the original training image dataset, the compressive measurements corresponding to $k$ different measurement rates are used for training the multi-rate network. We compare 4, 6, 10 and 50 point training in the multi-rate 2-layer feedforward neural network with 400 hidden neurons. The trained network is then tested on every different number of measurements between $m_{\text{min}}$ and $m_{\text{max}}$. The number of measurements selected for training for all experiments in this paper are: for 4-point training, $m = [10, 51, 102, 256]$, correspondingly $mr = [0.01, 0.05, 0.1, 0.25]$; for 6-point training, $m = [10, 20, 51, 102, 150, 256]$; for 10-point training, $m = [10, 18, 26, 34, 42, 51, 75, 102, 180, 256]$; for 50-point training, $m$ starts from 10 with an increment of 5 until 250, plus 256. We follow these general principles in selecting $m$: Firstly, $m = [10, 51, 102, 256]$ are always included; Secondly, we select more points in the low measurement rate region where the network classification performance changes rapidly, and less in high measurement rate region where performance changes slower. The loss function used is cross-entropy. Specifically, we use the Adam [28] optimizer for training with the following parameters: initialLearningRate = 5e-5, learningRateDropFactor = 0.9, learningRateDropPeriod = 4, total number of epochs = 100.

4.3 Results and discussion

Figure 4 shows the test accuracy results of compressed domain classification on zero-padded MNIST for the multi-rate 2-layer feedforward neural networks with 400 hidden neurons using PWH sensing matrix and PC sensing matrix. Comparison is shown among different multi-rate networks that are trained with 4, 6, 10 and 50 different measurement rates, respectively. The single-rate network is also a 2-layer feedforward neural networks with 400 hidden neurons, but is trained with only one measurement rate of 0.25 corresponding to 256 measurements and in principle is expected to be valid only for predicting 256 measurements. In Figure 4, each curve represents the test result for one neural network. The multi-rate and single-rate networks are tested in the same way where measurement vectors are zero-padded at the end to reach length 256. The markers on each accuracy curve indicate that the network training dataset includes the specific number of measurements represented by that marker.
Fig. 4. Test accuracy of compressed domain classification for different multi-rate and single-rate neural networks on zero-padded MNIST dataset. The network design is the 2-layer feedforward neural network with 400 hidden neurons. Comparison is shown among different multi-rate networks that are trained with 4, 6, 10 and 50 different measurement rates, respectively. The single-rate network is trained with only one measurement rate of 0.25 corresponding to 256 measurements. Each curve represents the test accuracy for one network. The markers indicate that the network training dataset includes the specific number of measurements represented by that marker. It is clearly shown that the multi-rate training scheme of 10-point training produces approximately equal performance as the 50-point training and performs better than 4-point and 6-point training. The single-rate network performs poorly compared to multi-rate networks. Also the PC matrix has much better accuracy than PWH matrix at low measurement rates.

It is clearly demonstrated that, for both PWH matrix and PC matrix, selecting 10 different measurement rates for training produces approximately equal performance as selecting 50 different measurement rates and that the PC matrix has much better accuracy than PWH matrix at low measurement rates. For either 4-point or 6-point training, though the test accuracies are the same at those measurement rates which are selected for training compared to 10-point training, the accuracy curves have significant drops for measurement rates that the network didn’t see during training especially at very low measurement rates. Also the single-rate network performs poorly compared to multi-rate networks.

Fig. 5. Test accuracy of compressed domain classification for different multi-rate and single-rate neural networks on grayscale CIFAR-10. The network design is the 2-layer feedforward neural network with 400 hidden neurons. Comparison is shown among multi-rate networks that are trained with 10 or 50 different measurement rate using PC or PWH sensing matrix. The single-rate network is trained with only one measurement rate of 0.25 corresponding to 256
measurements. Each curve represents the test accuracy for one network. The markers indicate that the network training dataset includes the specific number of measurements represented by that marker. It is clearly shown that the multi-rate training scheme of 10-point training produces approximately equal performance as the 50-point training and that PC sensing matrix performs better than PWH sensing matrix especially at low measurement rates. Also the single-rate network performs poorly compared to multi-rate networks.

Since the MNIST dataset is not very complex, we also applied our multi-rate training scheme to the more elaborate image set grayscale CIFAR-10. As shown in Figure 5, the 10-point training produces approximately equal performance as 50-point training and the single-rate model has poor performance compared to multi-rate models. We show only the 10-point and 50-point training results for grayscale CIFAR-10 since we found that the trend is exactly the same as with MNIST for 4-point and 6-point training.

It is clearly shown that PC sensing matrix has much higher test accuracy at very low measurement rates compared to PWH matrix for both MNIST and grayscale CIFAR-10. This could be because that the first 64 sensing patterns from the $1024 \times 1024$ PC matrix that we use are all low frequency patterns and therefore capture more energy and information of the image to facilitate classification. We have also noticed the following: while the accuracy curve is smooth for 10-point and 50-point training using the PWH matrix, the accuracy curve using PC matrix has step-like features. It could be due to that the PWH matrix is a randomized matrix and each measurement picks up about the same amount of information of the image to facilitate classification, while the measurement values of the PC matrix could vary a lot between different blocks of rows and therefore rows in different blocks contribute different amount of information of the image.

Given the validation and success of our multi-rate training scheme, we then also compare the performance between the multi-rate neural network at a given measurement rate with the performance of a single-rate neural network valid only for that particular measurement rate. For zero-padded MNIST, using 2-layer networks with 400 hidden neurons, we train single-rate neural networks valid for measurement rates 0.01, 0.05, 0.1 and 0.25 separately, with PWH and PC matrices and compare with 10-point multi-rate training scheme. We also test the method in [16] for single-rate networks where we used Gaussian random sensing matrix and the Lenet-like network with two convolutional layers with $\Phi^T y$ as input. In addition, test results are compared at measurement rates of 0.08 and 0.15 which none of the multi-rate or single-rate networks were trained on. Results are shown in Table 1. There are two accuracies in each table cell for measurement rates 0.15 and 0.08 for single-rate neural networks, where the first is predicted by the single-rate network below this cell, and the second is predicted by the single-rate network above this cell. In the case of single-rate 2-layer feedforward neural network, more measurements do not yield any improvements since the input layer has no place to input them, and less measurements decreases the performance much than our multi-rate network. In the case of Lenet-like network that receives $\Phi^T y$ which is of size $32 \times 32$ as input, the performance decreases severely with more measurements. Since with more measurements, $\Phi^T y$ is expected to be a better estimate of the signal $x$, this drastic performance decrease with more measurements shows that the stability and generalizability of the method where $\Phi^T y$ is used as input to a convolutional neural network [16] are not good and potentially suffer from a lot of problems.
produces proxy signal residual network, Similar network below this cell, and the second is predicted by the single cell of 0.15 and 0.8 for layers.

tested on a deep network with 3 convolutional layers and another deep network with 7 residual layers.

Table 1. Test accuracy comparison of compressed domain classification between multi-rate and single-rate neural networks on zero-padded MNIST

| Measurement Rate | Number of Compressive Measurements | Multi-rate, PC, 2-layer network, 400 hidden neurons, 10-point training | Multi-rate, PWH, 2-layer network, 400 hidden neurons, 10-point training | Single-rate, PC, 2-layer network, 400 hidden neurons | Single-rate, PWH, 2-layer network, 400 hidden neurons | Single-rate, Gaussian, transpose projection, Lenet-like network |
|------------------|------------------------------------|-----------------------------------------------------------------------|-----------------------------------------------------------------------|-----------------------------------------------|-------------------------------------------------|-----------------------------------------------------|
| 0.25             | 256                                | 98.05%                                                                | 98.08%                                                                | 96.93%                                        | 96.90%                                          | 98.00%                                               |
| 0.15             | 154                                | 97.53%                                                                | 97.71%                                                                | 96.58%/                                      | 96.45%/                                         | 68.13%/                                              |
| 0.1              | 102                                | 97.19%                                                                | 97.54%                                                                | 96.58%                                        | 96.45%                                          | 96.68%                                               |
| 0.08             | 82                                 | 96.93%                                                                | 96.73%                                                                | 95.46%/                                      | 92.34%/                                         | 50.14%/                                              |
| 0.05             | 51                                 | 96.31%                                                                | 95.41%                                                                | 95.46%                                        | 92.34%                                          | 94.08%                                               |
| 0.01             | 10                                 | 78.61%                                                                | 66.43%                                                                | 79.60%                                        | 69.31%                                          | 68.24%                                               |

For grayscale CIFAR-10, as a comparison with the multi-rate 2-layer feedforward network, we also tried single-rate networks using $\Phi^+ y$ as input where $\Phi^+$ is pseudo-inverse of $\Phi$, and tested on a deep network with 3 convolutional layers and another deep network with 7 residual layers. Results for grayscale CIFAR-10 are shown in Table 2. Same as in Table 1, in the table cell of 0.15 and 0.8 for single-rate models, the first accuracy is predicted by the single-rate network below this cell, and the second is predicted by the single-rate network above this cell. Similar to MNIST, here we also observe that when using $\Phi^+ y$ as input to the convolutional or residual network, with more measurements where $\Phi^+ y$ is expected to be a better estimate of the signal $x$, the network actually performance worse, which again shows that using an estimate or proxy image as input suffer from stability and generalizability problems.

Table 2. Test accuracy comparison of compressed domain classification between multi-rate and single-rate neural networks on grayscale CIFAR-10

| Measurement Rate | Number of Compressive Measurements | Multi-rate, PC, 2-layer network, 400 hidden neurons, 10-point training | Multi-rate, PWH, 2-layer network, 400 hidden neurons, 10-point training | Single-rate, PC, 2-layer network, 400 hidden neurons | Single-rate, PWH, 2-layer network, 400 hidden neurons | Single-rate, PWH, pseudo-inverse projection, 3 convolutional layers |
|------------------|------------------------------------|-----------------------------------------------------------------------|-----------------------------------------------------------------------|-----------------------------------------------|-------------------------------------------------|-----------------------------------------------------|
| 0.25             | 256                                | 45.08%                                                                | 43.89%                                                                | 41.38%                                        | 41.13%                                         | 44.95%                                               |
| 0.15             | 154                                | 45.26%                                                                | 41.46%                                                                | 42.62%/                                      | 39.69%                                         | 36.02%/                                              |
| 0.1              | 102                                | 44.93%                                                                | 41.36%                                                                | 42.62%                                        | 39.69%                                         | 41.40%                                               |
| 0.08             | 82                                 | 45.52%                                                                | 39.95%                                                                | 39.81%/                                      | 38.72%                                         | 29.60%/                                              |
| 0.05             | 51                                 | 42.31%                                                                | 37.28%                                                                | 39.81%                                        | 38.72%                                         | 37.27%                                               |
| 0.01             | 10                                 | 31.10%                                                                | 26.02%                                                                | 33.86%                                        | 26.87%                                         | 26.49%                                               |

For both datasets, it is shown that the multi-rate 2-layer feedforward neural network produces approximately equal performance as single-rate models of 2-layer feedforward...
networks for PC and PWH matrixes and deeper networks with convolutional and residual layers for PWH matrix at the trained measurement rates of 0.01, 0.05, 0.1 and 0.25. Also, PC matrix increases classification accuracy compared to PWH matrix at very low measurement rates of 0.01 and 0.05.

5. Conclusions

In this paper, we developed a general and efficient CS multi-rate training scheme that endows a neural network with multi-rate property for compressed domain classification using fixed binary sensing patterns such that it is capable for predicting over a range of measurement rates directly on compressive measurements acquired by a single-pixel camera. In this approach, the network only needs to be trained on a few measurement rates within the range of interest. The effectiveness of this approach is demonstrated on zero-padded MNIST and grayscale CIFAR-10 datasets for compressed domain image classification. We also showed that Partial Complete sensing matrix performs better than permuted Walsh Hadamard matrix at very low measurement rates. In addition, we demonstrated that the 2-layer feedforward network has approximately equal performance as the deeper networks in previous methods while being potentially efficient in computation, and serves as a foundation to generalize the effectiveness of the CS multi-rate training scheme for deeper networks when more layers are added. During the preparation of this manuscript we have learnt that Prof. Pavan Turaga and his research group have developed a different rate-adaptive neural network for compressed domain classification where the sensing matrix is learned through a fully-connected layer [29] as opposed to the concurrent CS multi-rate training scheme that uses fixed binary sensing matrixes as presented here. Lastly, we feel this approach is not only useful in the single pixel camera architecture but will likely also be beneficial in use with other compressed sensing camera systems.

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