**Bianchi-I cosmology with generalised Chaplygin gas and periodic deceleration parameter**

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Received: 27 June 2021 / Accepted: 03 February 2022 / Published online: 11 March 2022

**Abstract:** We investigate the Bianchi-I cosmological model in presence of generalised Chaplygin gas, variable gravitational and cosmological constant. The exact solutions of Einstein field equations are obtained with time varying periodic deceleration parameter. The graphical representation method has been used to discuss the physical and dynamical behaviour of the model. Further, the stability and physical acceptability of the obtained solutions have been investigated. Most of the parameters show periodic behaviour in this study due to the presence of cosine function in the deceleration parameter. In all cases, pressure is negative, which leads us to late time expansion of the universe. The considered models are found to be stable.

**Keywords:** Bianchi-I; Periodic deceleration parameter; Dark energy; Cosmological constant

**1. Introduction**

Cosmological and astronomical data [1–7] reveal that the universe is currently undergoing accelerating expansion and it has originated with a bang from a phase of very high density and temperature. For a long time, it was believed that either the universe will expand eternally or the inward pull of gravity gradually slows down the expansion of the universe and would ultimately come to a halt and contract into a big crunch singularity. At the end of the twentieth century, it was discovered that the universe might be expanding with acceleration. The fact of accelerating universe surprised the cosmologist as the idea of cosmic acceleration was against the standard predictions of decelerating expansion caused by gravity. The universe expansion is accelerating due to some exotic stuff termed as ‘Dark energy’ (DE) having highly negative pressure. The successive disclosure in this direction gave more and more documentation for a flat, dark energy-dominated accelerating universe [8–26]. However, by modifying the theory of gravity one can find an alternative way to explain the acceleration of expanding universe [26].

In General theory of relativity (GTR), cosmological constant \( \Lambda \) has been introduced by Einstein, which now is one of the feasible candidates of dark energy [27–29]. The homogeneous and isotropic model with cosmological constant in general relativistic framework is also known as Lambda cold dark matter (ΛCDM) model. However, the ΛCDM model faces some serious issues [8, 30] such as the fine-tuning (a typical small value), cosmic coincidence problems (although the universe is in accelerated phase of expansion, why the dark matter and the dark energy are of the same order?). To resolve these issues, various DE models with quintessence, k-essence, phantom, tachyon and so on [26], were analysed in the literature. In spite of these attempts, the perfect DE model is still lacking [26].

Chaplygin gas (CG) [31] with equation of state (EoS) \( p = -\frac{\rho}{\alpha} \) [32] where, \( \rho \) and \( p \) are energy density and pressure, respectively, is one of the most explored candidates of DE. Among the various classes of dark energy models, CG is a simple characterisation in order to understand the cosmic acceleration and yields a unified scenario for dark energy and dark matter (DM) [9–13]. The remarkable
feature of this unified model is that, at early times the CG behaves as a dust-like matter and at later times it behaves like cosmological constant. CG also plays a fascinating part in holography [33] and string theory [34] inspired models. CG depicts a transition to the present cosmic acceleration from a decelerated cosmic expansion and conceivably submits a deformation of ΛCDM model. CG model fails the tests connected with structure formation and observed strong oscillations of matter power spectrum [35]. To overcome this failure of CG model, the generalised Chaplygin gas (GCG) with EoS \( p = -\frac{B}{\rho} \) with \( 0 \leq x \leq 1 \) has been proposed [9, 10]. GCG model also acts like CG model i.e. at early times GCG mimics a dust-like matter and at later times it mimics a cosmological constant. With the aim of describing the unification of DE and DM, Zhang et al. [11] have proposed another version of GCG with EoS \( p = -\frac{A(R)}{\rho} \) \((R) is a scale factor\) termed as new generalised Chaplygin gas (NGCG). By using various combinations of latest observational data sample including SNIa and CMB, Salahedin et al. [12] have analysed the constraints on free parameters of NGCG model by using the statistical Markov Chain Monte Carlo method. Mamon et al. [13] have studied an extended GCG model with EoS of the NGCG and found that the NGCG model is consistent with gravitational thermodynamics. In the literature, the GCG model is again modified as modified generalised Chaplygin gas (MGCG) model with EoS \( p = \mu \rho - \frac{B}{\rho^x} \), where \( \mu \) is a positive constant [11]. This modification of GCG to MGCG has been done because the inferences from GCG models are almost similar to the CDM models [14]. In general, above examples of cosmic fluid may be written as \( p = f(\rho) \) and we term these formulations as barotropic fluid. Variable Chaplygin gas (VCG) \( p = -\frac{B}{\rho^x} \) is one of the forms among the class of modified forms of Chaplygin gas. A flat Friedmann–Lemaître–Robertson–Walker (FLRW) cosmological model with perfect fluid comprising of VCG yields ΛCDM model-like behaviour at late-times [15]. In anisotropic Bianchi-I background, MCG equation of state and the barotropic fluid model satisfying general linear EoS may also characterise different phases of the universe and approach to ΛCDM model at late-times [16, 19]. Nonlinear electrodynamics models may induce barotropic fluid-like scenario and result in varying deceleration parameter which is decreasing function of time [20].

In cosmological modelling, explanation of current accelerating stage and transition from decelerating past to accelerating stage of universe evolution are important and interesting ingredients. In order to explain the phase transition of the universe from decelerating to accelerating phase, one may also use varying deceleration parameter for anisotropic or isotropic universe model [17, 18]. Deceleration parameter denotes the rate at which the universe’s expansion is slowing down. The model of oscillating universe with quintom matter in FRW framework illustrates that the universe undergoes decelerating and accelerating expansion alternately and Hubble parameter oscillates and keeps positive for the time periodic varying deceleration parameter (TPVDP) [21]. The field equations of Barber’s second self-creation theory with TPVDP yield the values of state finder parameters \( r \) and \( s \) into state-finder parameters of standard ΛCDM model \( r = 1 \) and \( s = 0 \) for parameter \( m = 1 \) [36]. TPVDP model with string cloud in context of \( f(R, T) \) gravity yields vanishing string tension density in the considered universe scenario [37]. In the \( f(R, T) \) gravity model, the cosmological solutions with TPVDP have been obtained for the flat homogeneous and isotropic space-time and these solutions are consistent with observations for different values of model parameters [38]. Extension of FRW results in LRS Bianchi-I space-time with \( f(R, T) \) gravity yields the evolution of universe from decelerated phase to super-exponential expansion [39]. The late time acceleration of the universe may be caused by the presence of negative cosmological constant as a non-conventional mechanism with TPVDP [40]. Deceleration parameter has been used in modified gravity theories to investigate the cosmological implications along with other measurements in isotropic as well as anisotropic backgrounds [41–45]. The current accelerating expansion phase of universe may be explained in GTR framework with positive cosmological constant. The presence of cosmological constant in models may also affect the violation of energy conditions [46, 47]. In GTR framework, the Einstein field equations with the cosmological constant using variable redshift and shape functions have been solved for wormhole solutions [46]. The spherical regions for the wormhole may also satisfy the energy conditions for positive cosmological constant [46]. Energy conditions are coordinate invariant restrictions on energy momentum tensor and force various linear combinations of energy density and pressure of model to be positive [46–51]. The energy conditions have been used in the literature to derive many theorems such as the singularity theorems, black hole area increase theorem as well as the positive mass theorem [48–50].

The introduction of anisotropies in the cosmological modelling may give rise to a richer dynamical structure, yet the model remains simple enough to provide numerical and/or analytical results. These anisotropic models allow us to analyse the problems regarding behaviour of models on the approach to space-time singularities like why do the present day universe appear highly isotropic, the effects of anisotropy on astronomical observables, etc. [16, 19, 26, 39, 51]. By considering the features of periodic
deceleration parameter, role of Chaplygin gas and its modifications in cosmological modelling, it is worthwhile to investigate the dark energy cosmological model with time periodic varying deceleration parameter in Bianchi-I geometrical background. The paper is organised as follows: In Sect. 2, we write the field equations of general relativity with Bianchi-I space-time metric. In Sect. 3, we present the cosmological solutions by assuming the time periodic varying deceleration parameter and generalised Chaplygin gas. In Sect. 4, we investigate the stability and acceptability of the solution using classical stability criterion, using the square sound speed and energy conditions. In Sect. 5, we summarise our obtained result and concluding remarks.

2. Field equations

We take Bianchi-I space-time metric as,
\[ ds^2 = dt^2 - (R_1^2(t)dx^2 + R_2^2(t)dy^2 + R_3^2(t)dz^2), \]
where \( R_1, R_2, R_3 \) are directional scale factors. Einstein’s field equations with gravitational and cosmological constant for perfect fluid distribution are given as
\[ R_{ij} - \frac{1}{2} R g_{ij} = -8\pi GT_{ij} + \Lambda g_{ij}, \]
where \( R_{ij}, g_{ij}, R, G(t) \) and \( \Lambda(t) \) are Ricci tensor, metric tensor, Ricci scalar, gravitational constant and cosmological constant, respectively. \( T_{ij} \) is energy momentum tensor and is given as
\[ T_{ij} = (\rho + p)u_iu_j - pg_{ij}, \]
where \( \rho \) is energy density, \( p \) is perfect fluid pressure and \( u^i \) represents the four velocity vector such that \( u_iu^i = -1 \).

For the Bianchi-I space time metric represented by Eq. (1), the Einstein’s field Eq. (2) yields the following equations
\[ \frac{\ddot{R}_2}{R_2} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_1\dot{R}_3}{R_1R_3} = -8\pi Gp + \Lambda, \]
\[ \frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_1\dot{R}_3}{R_1R_3} = -8\pi Gp + \Lambda, \]
\[ \frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\dot{R}_1\dot{R}_2}{R_1R_2} = -8\pi Gp + \Lambda, \]
\[ \frac{\dot{R}_1\dot{R}_2}{R_1R_2} + \frac{\dot{R}_2\dot{R}_3}{R_2R_3} + \frac{\dot{R}_3\dot{R}_1}{R_3R_1} = 8\pi G\rho + \Lambda, \]
where overhead dot denotes derivative with respect to cosmic time \( t \). From Eqs. (4)-(7) one can easily obtain
\[ \dot{\rho} + (\rho + p)\left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3}\right) + \rho \frac{\dot{G}}{G} + \frac{\dot{\Lambda}}{8\pi G} = 0. \]

The energy momentum conservation equation \( (T^0_j = 0) \) suggests
\[ \dot{\rho} + (\rho + p)\left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3}\right) = 0. \]

From Eqs. (8) and (9) we have
\[ \rho \frac{\dot{G}}{G} + \frac{\dot{\Lambda}}{8\pi G} = 0. \]

Further, Eqs. (4)-(6) yield the solutions
\[ R_i = c_iRe^{\frac{\dot{R}_i}{R}t}, \]
where \( k_i \) and \( c_i \) \( (i = 1, 2, 3) \) are constants which satisfies \( \sum_{i=1}^{3} k_i = 0 \) and \( \prod_{i=1}^{3} c_i = 1 \). From the above results, one can notice that the metric potential can be explicitly expressed in terms of scale factor \( R = (R_1R_2R_3)^{\frac{1}{3}} \) represented by Bianchi-I space-time.

3. Cosmological Solutions

There are four linearly independent Eqs. (4)-(7) with six unknowns in the form of \( R_1, R_2, R_3, \rho, G \) and \( \Lambda \). Hence to solve the system of equations completely, two additional physically plausible relations among these variables are required. To obtain the cosmological solution we considered the time periodically varying deceleration parameter (TPVDP) of the form \[ q = m \cos(nt) - 1, \]
where \( m \) and \( n \) are positive constants. This type of deceleration parameter is known as TPVDP. The deceleration parameter plays a crucial role in determining the nature of the constructed models of the universe i.e. decelerating or accelerating in nature. According to the value/ranges of \( q \) the universe exhibits the expansion in the following ways \[ 15, 53]:
- \( q > 0 \) : Decelerating expansion
- \( q = 0 \) : Expansion with constant rate
- \( -1 < q < 0 \) : Accelerating power law expansion
- \( q = -1 \) : Exponential expansion/de Sitter expansion
- \( q < -1 \) : Super-exponential expansion

From the considered form of \( q \) in Eq. (12), the deceleration parameter shows periodic nature due to the presence of \( \cos(nt) \). The deceleration parameter lies in the interval \( -(m + 1) \leq q \leq m - 1 \). Here, we observed that:
1. For $m = 0$, the deceleration parameter $q$ is equal to $-1$ and the universe exhibits exponential expansion/de Sitter expansion.
2. For $m \in (0, 1)$, the deceleration parameter $q$ becomes negative and leads to accelerated expansion in a periodic way.
3. For $m = 1$, $q$ lies in the interval $[-2, 0]$, this shows that the universe evolves from expansion with constant rate to super exponential expansion in a periodic way followed by accelerating power law expansion to de Sitter expansion.
4. For $m > 1$, phase transition takes place from decelerating phase to accelerating phase in a periodic way where the universe starts with a decelerating expansion and evolves to super exponential expansion.

The constraints on generalised deceleration parameter from cosmic chronometers were investigated and specify the range as $q_0 = -0.53^{+0.17}_{-0.13}$ [52]. In the study of particle creation mechanism in higher derivative theory, Singh et al. [53] examine the values of free parameters using the observational range of $q$. The present observational limit of the considered deceleration parameter $q_0 = -0.53^{+0.17}_{-0.13}$ [52], suggests the following values of $m$ and $n$, which is given in Table 1.

Figures 1 and 2 represent the behaviour of deceleration parameter for different values of $m$ and $n$. One can easily analyse the behaviour of $q$ from the plots. It can be seen that, (i) for $n = 0.01$ and $0.01 \leq m \leq 0.64$, considered models are accelerating in nature i.e. $q < 0$ and (ii) for $n = 0.10$ and $1.70 \leq m \leq 3.20$, models show phase transition from decelerating phase to accelerating phase. In this situation, $q$ takes values from positive to negative. We are mainly focused to investigate the phase transition scenario so in all discussed models, as a representative case we have considered the value of $m$ in the range $1.70 \leq m \leq 3.20$ and $n = 0.10$.

In order to obtain the Hubble parameter from Eq. (12), we used the relation between Hubble parameter and deceleration parameter as $q = \frac{4}{m} (\frac{H}{\dot{H}}) - 1$, which leads to

$$H = \frac{n}{m \sin(nt) + nc_4}, c_4$$ is the constant of integration.

(13)

The value of integration constant $c_4$ does not affect the qualitative behaviour of the Hubble parameter but it affect the scale factor. We classify the considered cosmological model in three different cases as per $c_4 = 0, c_4 > 0$ and $c_4 < 0$, i.e. $c_4 = -c_5, c_5 > 0$.

3.1. Case-I: $c_4 = 0$

In this case, the Hubble parameter in Eq. (13) leads to

![Fig. 1 Deceleration parameter $q$ against cosmic time for $0.34 \leq m \leq 0.64$ and $n = 0.01$](image1)

![Fig. 2 Deceleration parameter $q$ against cosmic time for $1.70 \leq m \leq 3.20$ and $n = 0.10$](image2)

### Table 1

Computational range of the parameter $m$ for fixed values of $n$, obtained from the present value of the deceleration parameter

| $n$  | Interval of $m$ |
|------|-----------------|
| 0.01 | $0.34 \leq m \leq 0.64$ |
| 0.02 | $0.35 \leq m \leq 0.66$ |
| 0.03 | $0.37 \leq m \leq 0.69$ |
| 0.04 | $0.39 \leq m \leq 0.74$ |
| 0.05 | $0.43 \leq m \leq 0.82$ |
| 0.06 | $0.49 \leq m \leq 0.94$ |
| 0.07 | $0.59 \leq m \leq 1.11$ |
| 0.08 | $0.74 \leq m \leq 1.39$ |
| 0.09 | $1.02 \leq m \leq 1.93$ |
| 0.10 | $1.70 \leq m \leq 3.20$ |
We know the relation between Hubble parameter and scale factor as $H = \frac{n}{m\sin(nt)}$, which along with Eq. (14) leads to the scale factor of the form

$$\mathcal{R} = c_6 \left[ \frac{1 - \cos(nt)}{\sin(nt)} \right]^{\frac{1}{2}} = c_6 \left[ \tan \left( \frac{nt}{2} \right) \right]^{\frac{1}{2}},$$  

(15)

where $c_6$ is the constant of integration.

Further, Eqs. (11) and (15) yield the following metric potentials as

$$R_i = c_{11} \left[ \tan \left( \frac{nt}{2} \right) \right]^{\frac{1}{2}} \exp \left[ c_{12} \int \left[ \tan \left( \frac{nt}{2} \right) \right]^{-\frac{1}{2}} dt \right],$$  

(16)

where $c_{11} = c_i c_{6}$ and $c_{12} = \frac{k_i}{3c_6}$ for $i = 1, 2, 3$. We take the generalised Chaplygin gas, described by EoS [9, 10]

$$\rho = -\frac{A}{\rho^2}, \quad A > 0 \text{ and } 0 \leq x \leq 1.$$  

(17)

The expression for the energy density can be obtained from Eqs. (9), (14) and (17) as

$$\rho = \left[ A + \frac{c_7}{R^{3(1+x)}} \right]^{1/2} = \left[ A + \rho_0 \sin(nt) \right]^{\frac{3(1+x)}{2}} (1 + \cos(nt))^{\frac{3(1+y)}{2}} \frac{1}{R^{3(1+x)}},$$  

(18)

where $\rho_0 = c_7 c_{6}^{-3(1+x)}$ and $c_7$ is the constant of integration.

The directional Hubble parameter $H_i$ are given by,

$$H_i = \frac{\dot{R_i}}{R_i} = \frac{3c_i^2 + (\sin(nt))^{\frac{3(1+y)}{2}} (1 + \cos(nt))^{\frac{3(1+y)}{2}}}{3c_i^2 m \sin(nt)}, \quad i = 1, 2, 3.$$  

(19)

From Eqs. (4), (7), (16), (17) and (18), one can get the expression for Gravitational constant as

$$G = \frac{1}{8\pi(\rho + p)} \left[ \frac{k_4 (\sin(nt))^{-\frac{2}{3}} (\cos(nt) - 1)}{(\cos(nt) + 1)^{\frac{2}{3}} - 18c_0^6 n^2 \cos(nt)} \right],$$  

(20)

where $k_4 = m(k_3 k_2 + k_1 k_3 - k_2^2 - k_3^2)$. With the help of Eqs. (4) and (7), one can obtain the expression for cosmological constant as

$$\Lambda = \frac{1}{\rho + p} \left[ \frac{m^2 (\sin(nt))^{-\frac{2}{3}} (\cos(nt) - 1) (\cos(nt) + 1)^{\frac{m^2}{3}}}{(k_3 \rho + k_6 \rho^p - 27c_0^6 n^2 (\rho + p) + 18c_0^6 n^2 m \cos(nt)\rho)} \right],$$  

(21)

where $k_5 = k_2 k_3 + k_2^2 + k_3^2$ and $k_6 = k_1 k_2 + k_1 k_3 + k_2 k_3$.

The physical quantities of the observational interest are expansion scalar ($\Theta$), shear scalar ($\sigma^2$) and the anisotropic parameter ($A_m$), which are defined as follows:

$$\Theta = 3H = H_1 + H_2 + H_3,$$  

(22)

$$\sigma^2 = \frac{1}{2} \left[ H_1^2 + H_2^2 + H_3^2 \right] - \frac{\Theta^2}{6},$$  

(23)

$$A_m = \frac{1}{3} \sum_{i=1}^{3} \left( \frac{H_i - H^2}{H} \right)^2.$$  

(24)

In this case, the physical quantities are obtained as

$$\Theta = \frac{3n}{m \sin(nt)},$$  

(25)

$$\sigma^2 = \frac{(k_1^2 + k_2^2 + k_3^2) [1 + \cos(nt)]^{\frac{3}{2}} [\sin(nt)]^{-\frac{3-2m}{2}}}{18c_0^6 (1 - \cos^2(nt))},$$  

(26)

$$A_m = \frac{(k_1^2 + k_2^2 + k_3^2) m^2 [1 + \cos(nt)]^{\frac{3}{2}} [\sin(nt)]^{-\frac{3+2m}{2}}}{27c_0^6 n^2}.$$  

(27)

In this case, the state finder parameters are defined and expressed as

$$\rho = \frac{\rho}{\rho H^3} = m^2 \cos^2(nt) - 3m \cos(nt) + 1 + m^2,$$  

(28)

$$s = \frac{r - 1}{3(q - 0.5)} = \frac{2m (m \cos^2(nt) - 3 \cos(nt) + m)}{3(2m \cos(nt) - 3)}.$$  

(29)

Relation between $r$ and $s$ is given by

$$r = 1 + \frac{9}{2} s^2 = \frac{3}{2} s \sqrt{9 + 9s^2 - 4m^2}.$$  

(30)

In terms of the deceleration parameter the state finder parameters are given as

$$r = -1 - q + q^2 + m^2,$$  

(31)

$$s = \frac{2(-2 - q + q^2 + m^2)}{3(-1 + 2q)}.$$  

(32)

The energy density, pressure, cosmological constant and gravitational constant are periodic in nature, which is noticed from Figs. 4, 5, 6, 7, respectively, due to the presence of $\cos(nt)$ and $\sin(nt)$ terms in the expressions of
these physical quantities. Here, one can note that, $\rho$, $p$, $\Lambda$, $G \to \infty$ at $t = \frac{n_2 \pi}{n}$, $\forall n_2 \in \mathbb{Z}$. The energy density is positive, whereas pressure is negative for different values of $m$ with evolvement of time. In this case, the qualitative behaviour of energy density follows the pattern of higher energy density value to lower energy density value (approaching to zero) to higher energy density value. This process will continue due to the periodic nature of the terms $\cos(nt)$ and $\sin(nt)$ involved in the expression of energy density (18). Again it is also pointed out that, cosmological constant is positive for some values of $m$ and positive to negative values for some $m$ (See Fig. 6). Gravitational constant takes values from negative to positive values and positive to negative values in a periodic way for different values of $m$ (See Fig. 7). Further, it is noticed that the expansion scalar, shear scalar and anisotropy parameter also have singularity at $t = \frac{n_2 \pi}{n}$, $\forall n_2 \in \mathbb{Z}$ and they behave periodically.

The profile of scale factor $\mathcal{R}$, energy density $\rho$, pressure $p$, cosmological constant $\Lambda$ and gravitational constant $G$ against time is presented in Figs. 3, 4, 5, 6, 7), respectively, for fix $n$ and different values of $m$ with suitable choice of arbitrary constant involved in the expressions of the physical quantities. The scale factor is increasing with the evolvement of time in the provided range of time, which can be seen from Fig. 3. However, the qualitative behaviour is similar to tan function due to the presence of $\tan\left(\frac{nt}{2}\right)$ in Eq. (15) and $\mathcal{R} \to \infty$ at $t = \frac{2n_1 + 11\pi}{n}$, $n_1 \in \mathbb{Z}$.

3.2. Case-II: $c_4 > 0$

In this case, the Hubble parameter in Eq. (13) takes the form

$$H = \frac{n}{m \sin(nt) + nc_4}. \quad (33)$$

The relation $H = \frac{\dot{\mathcal{R}}}{\mathcal{R}}$, which along with (33) leads to the scale factor of the form

$$\mathcal{R} = c_8 e^{\sqrt{\frac{n_4 \tan\left(\frac{nt}{2}\right) + m}{n_4 \tan\left(\frac{nt}{2}\right) + m}}}, \quad c_8 \text{ is the constant of integration}. \quad (34)$$

Further, Eqs. (11) and (34) yield the following metric potentials as

$$R_i = c_{13} e^{\frac{2 \arctan\left(\frac{n_4 \tan\left(\frac{nt}{2}\right) + m}{n_4 \tan\left(\frac{nt}{2}\right) + m}\right)}{\sqrt{\frac{n_4 \tan\left(\frac{nt}{2}\right) + m^2}}}} \exp\left[\frac{-6 \arctan\left(\frac{n_4 \tan\left(\frac{nt}{2}\right) + m}{n_4 \tan\left(\frac{nt}{2}\right) + m}\right)}{\sqrt{n_4 \tan\left(\frac{nt}{2}\right) + m^2}} dt\right], \quad i = 1, 2, 3, \quad (35)$$

where $c_{13} = c_i c_8$ and $c_{14} = \frac{c_i}{c_8}$ ($i = 1, 2, 3$). The directional Hubble parameters are expressed as
The expression for the energy density can be obtained from Eqs. (9) and (34) as

\[ \rho = \left[ A + \frac{c_7}{R^{3(1+\alpha)}} \right]^{\frac{1}{2}} = \left[ A + \rho_1 e^{-\frac{6(1+\alpha) \arctan \left( \frac{\bar{c}_4 n \bar{c}_8^{\alpha} c_7}{\sqrt{\bar{c}_4^2 - m^2}} \right)} \frac{1}{4} \right]^{\frac{1}{2}}, \]  

(37)

where \( \rho_1 = c_7 c_8^{3(1+\alpha)} \). The expression for gravitational and cosmological constant are obtained as

\[ G = \frac{\bar{c}_8}{\bar{c}_4^2} \left[ m^2 \cos^4 \left( \frac{nt}{2} \right) - m^2 \cos^2 \left( \frac{nt}{2} \right) \right], \]

\[ 72\pi \bar{c}_8^6 \left( \rho + p \right) \left( m^2 \cos^4 \left( \frac{nt}{2} \right) - m^2 \cos^2 \left( \frac{nt}{2} \right) \right) \]

\[ \left( -n m c_4 \sin \left( \frac{nt}{2} \right) \cos \left( \frac{nt}{2} \right) - \frac{m^2 c_8^2}{4} \right) k_7, \]

(38)

where \( k_7 = \frac{k_i}{m} \).
Gravitational constant $G$ against cosmic time for different $m$, $n = 0.1$, $\sigma = 0.5$ and $A = 1$

Fig. 7

In this case, the physical quantities are obtained as

$$
\Theta = \frac{3n}{m \sin(nt) + nc_4},
$$

$$
\sigma^2 = \frac{(k_1^2 + k_2^2 + k_3^2)}{18c_8^6} e^{-\frac{12 \arctan \left( \frac{nc_4 \sin\left(\frac{nt}{2}\right)}{\sqrt{m^2 - 3n^2c_4^2}} \right)}},
$$

$$
A_m = -\frac{2nm^2c_4 \sin(nt) - m^2 - n^2c_4^2}{27n^2c_8^6}
$$

$$
r = \frac{\mathcal{R}}{\mathcal{R}H^3} = -\frac{(m \sin(nt) + nc_4)^3 \left(1 + 3m + 2m^2 + nm^2c_4 \sin(nt) + 4m^2 \cos^4\left(\frac{nt}{2}\right) - (4m^2 + 6m) \cos^2\left(\frac{nt}{2}\right)\right)}{(12nm^2c_4 + 4m^3 \sin(nt)) \cos^4\left(\frac{nt}{2}\right) - (12nm^2c_4 + 4m^3 \sin(nt)) \cos^2\left(\frac{nt}{2}\right) - 3n^2mc_4^2 \sin(nt) - n^3c_4^3}. \tag{43}
$$

$$
\Lambda = \frac{9n^2c_8^6 \left( m \cos^2\left(\frac{nt}{2}\right) - \frac{1}{2} \right) - \frac{3}{4} (\rho + p)}{12 \arctan \left( \frac{nc_4 \sin\left(\frac{nt}{2}\right)}{\sqrt{m^2 - 3n^2c_4^2}} \right)},
$$

$$
\rho = \frac{e^{-\frac{12 \arctan \left( \frac{nc_4 \sin\left(\frac{nt}{2}\right)}{\sqrt{m^2 - 3n^2c_4^2}} \right)}}}{\sqrt{m^2 - 3n^2c_4^2}} + \frac{(m^2 \cos^2\left(\frac{nt}{2}\right) - \frac{n^2c_4^2}{4}) (k_3 \rho + k_6 p)}{nm \sin\left(\frac{nt}{2}\right) \cos\left(\frac{nt}{2}\right) - n^2c_4^2}. \tag{39}
$$

In terms of the deceleration parameter the state finder parameters are given as

$$
r = \frac{\left( \sqrt{m^2 - 1 - 2q - q^2} + nc_4 \right)^3}{\left( m^2 - 1 - q + q^2 + nc_4 \sqrt{m^2 - 1 - 2q - q^2} \right)} \left( m^2 - 1 - q + q^2 + 3n^2c_4^2 \right) \sqrt{m^2 - 1 - 2q - q^2 + 3nc_4}, \tag{45}
$$

$$
s = \frac{r - 1}{3(q - 0.5)} = -\frac{2}{3(m \cos(nt) - 3)} \left( \frac{m \sin(nt) + nc_4)^3}{\left(1 + 3m + 2m^2 + nm^2c_4 \sin(nt) + 4m^2 \cos^4\left(\frac{nt}{2}\right)\right)} \right) - \frac{3}{2} (2m \cos(nt) - 3)
$$

$$
\times \left[ \left( \frac{12nm^2c_4 + 4m^3 \sin(nt)) \cos^4\left(\frac{nt}{2}\right) - (12nm^2c_4 + 4m^3 \sin(nt)) \cos^2\left(\frac{nt}{2}\right) - 3n^2mc_4^2 \sin(nt) - n^3c_4^3}{12 \arctan \left( \frac{nc_4 \sin\left(\frac{nt}{2}\right)}{\sqrt{m^2 - 3n^2c_4^2}} \right)} \right) \right) \right] + 1. \tag{44}
$$
Fig. 8  Scale factor $\mathcal{R}$ against cosmic time for different $m$

Fig. 9  Pressure $p$ against cosmic time for $1.70 \leq m \leq 3.20$ and $n = 0.10$

Fig. 10  Energy density $\rho$ against cosmic time for $1.70 \leq m \leq 3.20$ and $n = 0.10$

Fig. 11  Gravitational constant $G$ against cosmic time for $1.70 \leq m \leq 3.20, c_T = c_8 = A = 1, x = 0.5, c_4 = 55$ and $n = 0.10$

Fig. 12  Cosmological constant $\Lambda$ against cosmic time for $1.70 \leq m \leq 3.20, c_T = c_8 = A = 1, x = 0.5, c_4 = 55$ and $n = 0.10$
respect to cosmic time. We noticed that, both behave in a periodic way with time and also $\Lambda > 0$ and $G < 0$ with the evolution of cosmic time. $\Lambda > 0$ and $G < 0$ are free from the initial singularity.

Figures 13 and 14 show the qualitative behaviour of shear scalar and anisotropic parameters with respect to cosmic time. Both the parameters behave in a periodic way with time and positive with the evolution of cosmic time.

3.3. Case-III: $c_4 < 0$ i.e. $c_4 = -c_5$, $c_5 > 0$

In this case, the Hubble parameter in Eq. (13) takes the form

$$H = \frac{n}{m \sin(nt) - nc_5}. \quad (47)$$

The law $H = \frac{\dot{a}}{a}$ along with Eq. (47) leads to the scale factor of the form

$$R = c_6 e^{\sqrt{\frac{1}{m^2 - q^2}} \arctan \left( \frac{\sqrt{\frac{1}{m^2 - q^2}} - n}{\sqrt{\frac{1}{m^2 - q^2}}} \right)}, \quad (48)$$

$c_9$ is the constant of integration.

Further, Eqs. (11) and (48) yield the following metric potentials as

$$R_i = c_{6i} e^{-\sqrt{\frac{1}{m^2 - q^2}} \arctan \left( \frac{\sqrt{\frac{1}{m^2 - q^2}} - n}{\sqrt{\frac{1}{m^2 - q^2}}} \right)} \left( 2 \arctan \left( \frac{\sqrt{\frac{1}{m^2 - q^2}} - n}{\sqrt{\frac{1}{m^2 - q^2}}} \right) - \frac{6 \arctan \left( \frac{\sqrt{\frac{1}{m^2 - q^2}} - n}{\sqrt{\frac{1}{m^2 - q^2}}} \right)}{\sqrt{\frac{1}{m^2 - q^2}}} \right) dt,$$

where $c_{6i} = c_{5i} c_9$ and $c_{id} = \frac{k_i}{\sqrt{c_9}}$ ($i = 1, 2, 3$). The directional Hubble parameters are expressed as

$$H_i = \frac{6 \arctan \left( \frac{\sqrt{\frac{1}{m^2 - q^2}} - n}{\sqrt{\frac{1}{m^2 - q^2}}} \right)}{3 \sqrt{\frac{1}{m^2 - q^2}} - 6 \arctan \left( \frac{\sqrt{\frac{1}{m^2 - q^2}} - n}{\sqrt{\frac{1}{m^2 - q^2}}} \right)} \frac{\sqrt{\frac{1}{m^2 - q^2}} - n}{\sqrt{\frac{1}{m^2 - q^2}}} - nc_5 k_i + mk_i \sin(nt) \right)}{3c_5 \left( m \sin(nt) - nc_5 \right)}.$$

Figures 9 and 10 are representing the profile of pressure and energy density. As per the evolution of time, energy density and pressure are positive and negative, respectively, in a periodic way for different values of $m$.

Figures 11 and 12 show the qualitative behaviour of gravitational constant and cosmological constant with...
The expression for the energy density can be obtained from Eqs. (9) and (21) as
\[
\rho = \left[ A + \frac{c_7}{R^{3(1+z)}} \right]^{\frac{1}{1+z}} = \left[ A + \rho_2 e^{\frac{m c_6}{R^{1/3} \sqrt{\frac{a}{2} - \frac{1}{2}}}} \right]^{\frac{1}{1+z}},
\]
(51)
where \( \rho_2 = c_7 c_9^{-3(1+z)} \). The expressions for gravitational and cosmological constants are obtained as
\[
G = \frac{-9n^2mc_6^5}{12} \left( \frac{\cos^2\left(\frac{mt}{2}\right) - \frac{1}{2}}{\sqrt{\frac{a}{2} - \frac{1}{2}}} \right) e^{\frac{12 \arctan \left( \frac{nmc_5 \sin \left( \frac{mt}{2} \right) \cos \left( \frac{mt}{2} \right) - n^2 c_5^2 \right)}}{ \sqrt{\frac{a}{2} - \frac{1}{2} \rho} + m^2 \cos^2 \left( \frac{mt}{2} \right) - m^2 \cos^2 \left( \frac{mt}{2} \right) + nmc_5 \sin \left( \frac{mt}{2} \right) \cos \left( \frac{mt}{2} \right) - n^2 c_5^2 \right) k_7},
\]
(52)
\[
A = \frac{9n^2c_6^5}{12} \left( \frac{\rho m^3 \cos^2 \left( \frac{mt}{2} \right) - \frac{1}{2}}{4} \right) \frac{3}{4} \left( \rho + p \right)
\]
\[
e^{\frac{12 \arctan \left( \frac{nmc_5 \sin \left( \frac{mt}{2} \right) \cos \left( \frac{mt}{2} \right) - n^2 c_5^2 \right)}}{ \sqrt{\frac{a}{2} - \frac{1}{2} \rho} + m^2 \cos^2 \left( \frac{mt}{2} \right) - m^2 \cos^2 \left( \frac{mt}{2} \right) + nmc_5 \sin \left( \frac{mt}{2} \right) \cos \left( \frac{mt}{2} \right) - n^2 c_5^2 \right)}
\]
(53)
In this case, the physical quantities are obtained as

\textbf{Fig. 15} Hubble parameter \( H \) against cosmic time for \( m = 2.2, n = 0.1 \) and different \( c_4 \)

\textbf{Fig. 14} Anisotropic parameter \( A_m \) against cosmic time for \( 1.70 \leq m \leq 3.20, c_7 = c_8 = A = 1, x = 0.5, c_4 = 55 \) and \( n = 0.10 \)

\textbf{Fig. 16} Hubble parameter \( H \) against cosmic time for \( m = 0.5, n = 0.01 \) and different \( c_4 \)
\[ \Theta = \frac{3n}{m \sin(nt) - nc_5}, \]  
\[ \sigma^2 = \frac{(k_1^2 + k_2^2 + k_3^2)}{18c^2_9} e^{\frac{12 \arctan \left( \frac{m - nc_5 \tan \left( \frac{nt}{2} \right)}{\sqrt{k_1^2 + k_2^2 + k_3^2}} \right)}}, \]  
\[ 27n^2m^2c_9^2 \left( -2 \tan \left( \frac{nt}{2} \right) + \tan^2 \left( \frac{nt}{2} \right) \sin(nt) + \sin(nt) \right)^2 \]  
\[ + 4 \left( -\frac{nc_5}{2} - \frac{nc_5 \tan \left( \frac{nt}{2} \right)}{2} + m \tan \left( \frac{nt}{2} \right) \right)^2 \]  
\[ A_m = \frac{(k_1^2 + k_2^2 + k_3^2)(m \sin(nt) - nc_5)^2 e^{\frac{12 \arctan \left( \frac{m - nc_5 \tan \left( \frac{nt}{2} \right)}{\sqrt{k_1^2 + k_2^2 + k_3^2}} \right)}}{27n^2c_9^2 \left( -nc_5 - nc_5 \tan^2 \left( \frac{nt}{2} \right) + 2m \tan \left( \frac{nt}{2} \right) \right)^2}. \]  

In this case, the state finder parameters are defined and expressed as

\[ r = \frac{\ddot{R}}{\ddot{R}H^2} = \frac{(m \sin(nt) - nc_5)^3 \left( -4m^2 \cos^4 \left( \frac{nt}{2} \right) + (6m + 4m^2) \cos^2 \left( \frac{nt}{2} \right) + nmc_5 \sin(nt) - 3m - 2m^2 - 1 \right)}{4m^2(m \sin(nt) - 3nc_5) \cos^2 \left( \frac{nt}{2} \right) \left( \cos^2 \left( \frac{nt}{2} \right) - 1 \right) + n^3c_5^3 - 3mn^2c_5^2 \sin(nt)}, \]  
\[ s = \frac{r - 1}{3(g - 0.5)} = \frac{2}{3(2m \cos(nt) - 3)} \left[ \left( m \sin(nt) - nc_5 \right)^3 \left( -4m^2 \cos^4 \left( \frac{nt}{2} \right) + (6m + 4m^2) \cos^2 \left( \frac{nt}{2} \right) + nmc_5 \sin(nt) - 3m - 2m^2 - 1 \right) \right] \times \left[ \frac{4m^2(m \sin(nt) - 3nc_5) \cos^2 \left( \frac{nt}{2} \right) \left( \cos^2 \left( \frac{nt}{2} \right) - 1 \right)}{4m^2(m \sin(nt) - 3nc_5) \cos^2 \left( \frac{nt}{2} \right) \left( \cos^2 \left( \frac{nt}{2} \right) - 1 \right) + n^3c_5^3 - 3mn^2c_5^2 \sin(nt)} \right] - 1. \]
In terms of the deceleration parameter the state finder parameters are given as

\[
 r = \frac{\left(\sqrt{m^2 - 1 - 2q - q^2} - nc_5\right)^3}{\left(m^2 - 1 - 2q - q^2 + 3n^2c_5^2\right)\sqrt{m^2 - 1 - 2q - q^2} - 3nc_5\left(m^2 - 1 - 2q - q^2 + \frac{n^2c_5^2}{3}\right)},
\]

(59)

Table 2 \(\{r,s\}= (1,0)\) corresponding to the different values of \(m\) and \(q\)

| \(q\)     | \(m\)   | \(\{r,s\}\) pair          |
|----------|---------|-----------------------------|
| -1.00    | 0.000   | (1.000,0)                   |
| -0.80    | 0.748   | (1.0,2.846726e-01)\(\approx\) (1.0) |
| -0.60    | 1.020   | (1.0,0)                     |
| -0.40    | 1.200   | (1.0,8.223874e-01)\(\approx\) (1.0) |
| -0.20    | 1.327   | (1.0,1.057355e-01)\(\approx\) (1.0) |
| 0.00     | 1.414   | (1.0,2.960595e-01)\(\approx\) (1.0) |
| 0.20     | 1.470   | (1.0,0)                     |
| 0.40     | 1.497   | (1.0,0)                     |
| 0.60     | 1.497   | (1.0,0)                     |
| 0.80     | 1.470   | (1.0,0)                     |
| 1.00     | 1.414   | (1.0,2.960595e-01)\(\approx\) (1.0) |
Tables 2, 3, 4, respectively, for three cases. Here one can notice that, model discussed in case-I approaches to ΛCDM model for different value of \( q \) and \( m \). At current value of \( q = -\frac{1}{2} \), the model in case-II approaches to ΛCDM model, whereas model in Case-III does not approach to ΛCDM model in view of the values of \( n \) and \( m \) presented in Table 1.

### 4. Stability and physical acceptability of the solutions

#### 4.1. The squared sound speed

We determine the classical stability of considered models on the basis of an adiabatic squared sound speed. It is one of the important quantities in cosmology. Adiabatic squared sound speed for system is defined \([54]\), as

\[
c_s^2 = \frac{\partial p}{\partial \rho}.
\]

Here, \( c_s^2 \) has three possibilities i.e. \( c_s^2 < 0 \) or \( c_s^2 = 0 \) or \( c_s^2 > 0 \). The sign of \( c_s^2 \) is very important to investigate as it leads to the instability of the cosmological models through which one can reject or accept the constructed cosmological models. The case when, \( c_s^2 < 0 \), leads to classical instability of the cosmological models due to the uncontrolled growth of the energy density perturbation. The case when, \( c_s^2 > 0 \) may lead to the issue of occurrence of causality. As a matter of fact, it is usually considered as \( c_s \leq 1 \) and the bound on \( c_s^2 \) is \( 0 \leq c_s^2 \leq 1 \). In addition to that, the complementary bound \( c_s > 1 \) is used as a condition for rejecting the theories. The details regarding \( c_s^2 \) can be found in \([21]\).

In present study, \( c_s^2 \) is obtained as

\[
s = \frac{2}{3(2q - 1)}
\]

\[
\frac{\left(\sqrt{m^2 - 1 - 2q - q^2} - nc_5\right)^3}{\left(m^2 - 1 - q + q^2 - nc_5\sqrt{m^2 - 1 - 2q - q^2}\right) - 1} \times \frac{\left(m^2 - 1 - q + q^2 - nc_5\sqrt{m^2 - 1 - 2q - q^2}\right)}{\left(m^2 - 1 - 2q - q^2 + 3n^2c_5^2\right)}
\]

\[
-3nc_5\left(m^2 - 1 - 2q - q^2 + \frac{n^2c_5^2}{3}\right)
\]

(60)

Figs. 15 and 16 indicate the profile of Hubble parameter against cosmic time for different values of \( c_4 \). Similar qualitative behaviour as that of case-II is observed for \( n \) and \( m \) (Table 1). From Eqs. (31),(32),(45), (46), (59) and (60), we have evaluated the \( \{r, s\} \) pair for the different model parameters, which are presented in Tables 2, 3, 4, respectively, for three cases. Here one can notice that, model discussed in case-I approaches to ΛCDM model for different value of \( q \) and \( m \). At current value of \( q = -\frac{1}{2} \), the model in case-I approaches to ΛCDM model for \( m = \frac{3}{5} \). The models in case-II and Case-III, the \( \{r, s\} \) pair depends on the model parameters \( n, m, c_4, c_5 \) and \( q \). At current value of \( q = -\frac{1}{2} \), the model in case-II approaches to ΛCDM model, whereas model in Case-III does not approach to ΛCDM model in view of the values of \( n \) and \( m \) presented in Table 1.

| \( q \) | \( n \) | \( m \) | \( c_4 \) | \( r \) | \( s \) |
|---|---|---|---|---|---|
| -0.5 | 0.02 | 0.5358059184 | 250 | 1.000000001 | 1 | -3.333333333×10^{-10} ≈ 0 |
| -0.5 | 0.04 | 0.5097105689 | 250 | 0.9999999951 | 1 | 1.633333333×10^{-9} ≈ 0 |
| -0.5 | 0.06 | 0.5043861337 | 250 | 1.0000000008 | 1 | -2.666666667×10^{-9} ≈ 0 |
| -0.5 | 0.08 | 0.8416493515 | 10 | 1.000000001 | 1 | -3.333333333×10^{-10} ≈ 0 |

Table 3 \( \{r, s\} \) pair for different model parameters with fixed \( q \)

| \( q \) | \( n \) | \( m \) | \( c_5 \) | \( r \) | \( s \) |
|---|---|---|---|---|---|
| -0.5 | 0.02 | 1.127031913 | 1 | 1.000000001 | 1 | -3.333333333×10^{-10} ≈ 0 |
| -0.5 | 0.04 | 1.136137315 | 1 | 1.020403999 | 1 | 1.000000000 |
| -0.5 | 0.06 | 1.144350162 | 1 | 1.041217996 | 1 | -0.0137393200 |
| -0.5 | 0.08 | 1.154670505 | 1 | 1.062447982 | 1 | -0.0137393200 |

Table 4 \( \{r, s\} \) pair for different model parameters with fixed \( q \)
In Fig. 23, we have presented the profile of squared sound speed against time for different values of \( m \). It is observed from Fig. 23 that, \( c_s^2 \) is periodic in nature and \( 0 \leq c_s^2 < 0.6 \) in all cases. Thus, in account of the prescribed range of \( c_s^2 \), the solution presented here are stable.

4.2. Energy conditions

The pointwise energy conditions (ECs) depend on energy momentum tensor at a given point in a space time. These conditions are contractions of time like or null vector with the Einstein’s tensor and energy momentum tensor coming from Einstein’s field equations. ECs can be imposed in order to investigate the constraints on the free parameters involved in the cosmological models. For example, the evolution of acceleration or deceleration of the universe and the emergence of Big Rip singularity, can be related to the constraints imposed by the ECs. For energy momentum tensor \( T_{ij} = (\rho + p)u_iu_j - pg_{ij} \), the standard pointwise energy conditions are defined as follows [48–50]

- Null energy condition (NEC): \( \rho + p \geq 0 \)
- Weak energy condition (WEC): \( \rho \geq 0 \) and \( \rho + p \geq 0 \)
- Dominant energy condition (DEC): \( \rho \geq 0 \) and \( \rho \pm p \geq 0 \)
Strong energy condition (SEC): $\rho + p \geq 0$ and $\rho + 3p \geq 0$

We use the ECs to investigate the stability and physical acceptability of the solutions in the present models. In general, the universe in cosmological model should satisfy WEC and DEC and violate SEC for late time accelerated expansion of the universe. For each of the three cases, Figs. 24, 25 and 26 represent the profile of WEC, DEC and SEC, respectively, versus cosmic time. From these figures it can be seen that profile of energy conditions follows periodic variation for each of the cases. Considered model satisfies WEC $(\rho + p)$ (except for $m=1.7$ in case $I$) and DEC $(\rho - p)$, whereas SEC is satisfied and violated periodically throughout the evolution of the universe.

5. Conclusions

In this work, we have considered Bianchi-I cosmological models with generalised Chaplygin gas represented by Eq. (17) and cosmological solutions are obtained by using time periodic deceleration parameter represented by Eq. (12). By fixing the value of constant parameter $n$, we have computed the range of parameter $m$ (see Table 1 for present observational value (range) of deceleration parameter). Figures (1) and (2) show the variation of deceleration parameter with respect to cosmic time for $n = 0.01$ and $n = 0.10$ and corresponding range of $m$. For $n = 0.01$ deceleration parameter is negative throughout the evolution of the universe, whereas for $n = 0.10$ it shows transition from decelerated phase to accelerated phase periodically throughout the evolution of the universe. Hubble parameter represented by (13) is computed by using it’s relation with deceleration parameter. The integration constant $c_4$ affects the scale factor so we have discussed three different cases for the considered model depending on the positive negative and neutral nature of $c_4$. In each of the cases, we have analysed the solutions and physical quantities of the observational interest (such as expansion scalar $(\Theta)$, shear scalar $(\sigma^2)$, anisotropy parameter $(A_m)$, state finder parameters $(r$ and $s)$).

Gravitational constant $G$ varies from positive to negative in the present cosmological framework. With positive and negative $G$, we will have attractive and repulsive nature of gravity, respectively. Positive and negative cosmological constants also yield repulsive and attractive gravity, respectively. The dark energy is responsible for accelerating universe and positive cosmological constant acts as a mechanism for dark energy. The periodic nature of $\Lambda$ and $G$ also highlights the attractive as well as repulsive gravity era in the model. For more details one can refer to Ayuso et al. [55]. In bouncing model, the scale factor is having its minima at the bounce instant. The universe should be contracting before the bounce and expanding after the bounce. The solutions given in Eq. (34) and (48) may lead to asymmetric bounce for different values of model parameters. On the other hand, solution (15) will not exhibit a bouncing scenario; however, the toy model will exhibit periodic behaviour of scale factor but the behaviour is not of bouncing type.

In recent, Sahoo et al. [38] have discussed the periodic varying deceleration parameter in $f(R,T) = R + 2\lambda T$ gravity. For $\lambda = 0$, this reduces to the result in general relativity. In our investigation, we noticed that for all the models SEC is violated but WEC and SEC are violated in their study. For stability of the solutions we both have different approaches, they have used linear homogeneous perturbations in the FRW background, whereas we used the square sound speed. Our solutions are stable but their stability of solution depends on the parameter $k$ and $k$. Further, all other physical parameters we both have the same periodic qualitative behaviour. In all the figures, we have taken the cosmic time $t$ along the horizontal axis which is measured in giga years ($1Gyr = 10^9 years$). The conclusion of our study is as follows:

- Almost all the parameters which we have discussed in our study show periodic behaviour due to the choice of deceleration parameter.
- For each of the cases energy $\rho$ is positive and $p$ is negative throughout the evolution of the universe and
this negative pressure guarantees the late time expansion of the universe.

- Models discussed in Case-I and Case-II tend to $\Lambda$CDM model, whereas model in Case-III fails for the value of $n$ and $m$ provided in Table 1. The expressions obtained for expansion scalar, shear scalar, and anisotropic parameter bear singularities in Case-I, whereas these are free from singularities in Case-II and Case-III.

- The square sound speed satisfies the bounds $0 \leq c_s^2 \leq 1$ for each of the cases. So the considered model is stable and physically acceptable.

- For each of the cases considered model satisfies WEC (except for $m=1.7$ in case $-I$) and DEC, whereas it violates SEC periodically throughout the evolution of the universe. So using the results of energy conditions one can conclude that the violation of SEC may lead to the accelerating universe.

Acknowledgements The author B. K. Bishi is thankful to research supported wholly/in part by the National Research Foundation of South Africa (Grant Numbers: 118511) in the form of a postdoctoral fellowship at the University of Zululand, South Africa.

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