Prediction of the received global solar irradiance for clear days – simple approach

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Abstract. The prediction of the global solar radiation through a distribution function $q(t)$, W/m\textsuperscript{2} for clear days is given where “$t$” is the local day time.: The distribution is based on a simple model and is expressed through well-established parameters such as the length of the solar day “$t_d$” and the maximum value of the received solar irradiance $q_{\text{max}}$, W/m\textsuperscript{2} at time instant “$t_0$”. $q_{\text{max}}$ is expressed in terms of the solar constant. Comparison between the computed values of $q(t)$ and the corresponding published experimental data for Hong Kong (China), Valencia (Spain) and Makah (Saudi Arabia) are given as illustrative examples. A test for the degree of fitting is also clarified.

1. Introduction.
The prediction of the global solar irradiance received on a horizontal surface is important for solar technological applications. For example, the evaluation of the performance and efficiency of the solar photovoltaic cells, and flat plate solar collectors and concentric solar collectors depend on the received solar radiation. Different authors measured the incident global radiations $q(t)$ W/m\textsuperscript{2} using different recording instruments. For example our data for Makah \[1\] was measured using Eppley precision pyranometer model (PSP). Several attempts \[1-8\] have been made to predict such a function. Such attempts have different degrees of fitting accuracy. The analysis of the published measured long term meteorological data is important to test the validity of any of the predicting formula.

2. The suggested model.
The experimental measurements for the considered daily solar irradiance $q(t)$, W/m\textsuperscript{2} for clear days \[9-12\] show a symmetrical distribution, that passes through a maximum value $q_{\text{max}}$, W/m\textsuperscript{2} at the midday time “$t_0$” between sunrise time $t_s$ and sunset time $t_s$. Trials to estimate the total solar energy received per unit area of a horizontal surface along the day time is done through the integration process.

$$I = \int_{t_s}^{t_d} q(t) \, dt$$ (1)

Equation (1) represents the area confined under the curve representing the function $q(t)$. Figure (1-b).

It is interested to notice that the value of this integration for different suggested models of $q(t)$ equals

$$\approx \frac{1}{2} q_{\text{max}} t_d.$$ (2)

For example, it is equal to 0.533 $q_{\text{max}}$ $t_d$ \[2\], 0.5567 $q_{\text{max}}$ $t_d$ \[4\] and 0.564 $q_{\text{max}}$, $t_d$ \[5, 9, 10\].
This situation encouraged the authors of the present trial to suggest the required distribution as a triangle as shown in figure (1).

![Figure 1. Schematic Diagram for the suggested model](image)

The area of such triangle is given as a second order determinant [13]. This gives the ordinary formula:

\[ \text{The area of the triangle} = \frac{1}{2} t_d q_{\text{max}}. \]  

(2)

Thus the present trial represents a simple approach to evaluate the required function based on simple geometrical considerations. This facilitates the technical computations for solar energy exploitations.

The introduced formula contains well defined physical parameters, namely:

1. The length of the solar day "t_d" expressed in terms of the latitude \( \varphi \), which is the angle made by the radial line joining the given location to the center of the earth with its projection on the equatorial plane, and in terms of the solar declination angle "\( \delta \)" which is the angle between the line joining the centers of the sun and the earth and its projection on the equatorial plane.

2. The midday time "\( t_o \)" at which the solar irradiance received by a unit area of a horizontal plane attains maximum value \( q_{\text{max}} \).

Moreover \( q_{\text{max}} \) is suggested in terms of the extraterrestrial solar constant adjusted for the variation of the distance between the sun and the earth and along the time of the year [1,7,8]. This makes it possible to have a mathematical closed system to find the required function on pure theoretical basis. This means that the suggested trial is not a semi-empirical formula. Applying the similarity condition for the triangle one can write the relation \( \frac{q(t)}{q_{\text{max}}} = \frac{t}{t_o} \), from which the distribution for the first half of the day time is given in the form:

\[ q(t) = \frac{q_{\text{max}}}{t_o} t, \quad 0 < t \leq t_o. \]  

(3)

For the second half, the relation is written in the form :

\[ \frac{q(t)}{q_o} = \frac{t_d - t}{t_d - t_o}, \quad t_o \leq t \leq t_d. \]  

(4)
From which, one can write the relation:

\[ q(t) = q_{\text{max}} \left( \frac{t_s - t}{t_d - t_s} \right), \quad t_s \leq t \leq t_d \]  

(5)

This distribution suffers from discontinuity at \( t = t_o \) and satisfies for shifted time for which \( t_r = 0 \), the following conditions:

At \( t = t_r = 0 \),

\[ q(t_o) = 0 \]  

(6)

At \( t = t_d \)

\[ q(t_d) = 0 \]  

(7)

At \( t = t_o \)

\[ q(t_o) = q_{\text{max}} \]  

(8)

The length of the solar day is given by the relation: [14]

\[ t_d = \frac{12}{15} \cos^{-1} \left( -\tan \varphi \tan \delta \right) \]  

(9)

Where, \( \varphi \) is the latitude, it is the angle made by the radial line joining the given location to the Center of the earth with its projection on the equatorial plane.

\( \delta \) is the solar declination angle, which is the angle between the line joining the centers of the sun and the earth and its projection on the equatorial plane and is given as:

\[ \delta = 23.45 \sin 360 \left( \frac{284 + n}{365} \right) \]  

(10)

\( (1 \leq n \leq 365) \), \( n \) is the number of the day of the year starting from 1 January.

Following El-Adawi [1], one can suggest the estimation of \( q_{\text{max}} \) in terms of the solar constant to be in the form:

\[ q_{\text{max}} = \alpha \overline{S} \]  

(11)

where \( \overline{S} \), is the extraterrestrial solar constant adjusted for the variation of the distance between the sun and the earth and along the time of the year given as [1,7]:

\[ \overline{S} = S \left( 1 + 0.033 \cos \left( \frac{360 + n}{365} \right) \right) \]  

(12)

Where, \( S = 1353 \text{ W/m}^2 \) [15] is the solar constant, and

\[ \alpha \leq 1 \quad [1] \quad \text{is a correction factor} \]

\[ \alpha = 0.94 \quad \text{For Makkah [6,16]} \]
3. Computations.
In the following we considered the meteorological experimental data published for the function q(t) for three locations. These are:

1- Hong Kong, China $[114^\circ 10' E, 22^\circ 19' N]$ December, 1979 [10].

2- Valencia, Spain, $[39^\circ 44' N]$ December, 1992 [11] and

3- Makah – Saudi Arabia $[38^\circ 30' E, 21^\circ 30' N]$ - 18/3/1983 [16].

The function q(t) is computed according to equations (3) and (4) and compared with the corresponding published experimental data. The obtained results are given in tables 1, 2, 3 and are illustrated in Figures 2, 3 and 4, respectively.

Moreover, the value of $q_{max}$ is computed for Makah according to equation (11). The obtained value of $q_{max} = 878 W/m^2$ [6] While the experimental value is 938 W/m^2 with relative error 6%. The computed values for q(t) using the obtained value 878 W/m^2 are also given in table 3 and are compared with the corresponding experimental values of q(t).

### Table 1: Hong Kong, China

| t | $t_o$ | $q_{exp}$ | $q_{cat}$ | $\epsilon$ |
|---|---|---|---|---|
| 5.5 | 0 | 0 | 0 | 0 |
| 6.5 | 1 | 100 | 108.0909 | 8.1% |
| 7.5 | 2 | 255.58 | 216.1818 | 15.4% |
| 8.5 | 3 | 422.26 | 324.2727 | 23.2% |
| 9.5 | 4 | 536.15 | 432.3636 | 19.4% |
| 11 | 5.5 | 594.5 | 594.5 | 0 |
| 11.5 | 6 | 583.36 | 540.4545 | 7.4% |
| 12.5 | 7 | 541.71 | 432.3636 | 20.2% |
| 13.5 | 8 | 425.03 | 324.2727 | 23.7% |
| 14.5 | 9 | 277.8 | 216.1818 | 22.2% |
| 15.5 | 10 | 122.23 | 108.0909 | 11.6% |
| 16.5 | 11 | 0 | 0 | 0 |

**Figure 2.** Hong Kong, China, $[114^\circ 10' E, 22^\circ 19' N]$[10].

### Table 2: Valencia Spain
\( t_d = 15 \), \( t_o = 7.5 \), \( q_{\text{max}} = 947.36 \text{ w/m}^2 \)

| \( t \) | \( t^\text{\$} \) | \( q_{\text{exp}} \) | \( q_{\text{cat}} \) | \( \epsilon = \frac{q_{\text{cat}} - q_{\text{exp}}}{q_{\text{exp}}} \) |
|-------|-------|--------|--------|------------------|
| 7     | 0     | 0      | 0      | 0                |
| 8.5   | 1.5   | 135.338| 189.472| 28.6%            |
| 9.5   | 2.5   | 306.76 | 315.7867| 2.9%            |
| 10.5  | 3.5   | 496.24 | 442.1013| 12.2%           |
| 11.5  | 4.5   | 676.69 | 568.416 | 19.0%           |
| 12.5  | 5.5   | 812.03 | 694.7307| 16.9%           |
| 13.5  | 6.5   | 915.78 | 821.0453| 11.5%           |
| 14.5  | 7.5   | 947.36 | 947.36  | 0               |
| 15.5  | 8.5   | 924.812| 821.0453| 11.2%           |
| 16.5  | 9.5   | 834.586| 694.7307| 16.8%           |
| 17.5  | 10.5  | 703.759| 568.416 | 19.2%           |
| 18.5  | 11.5  | 541.35 | 442.1013| 18.3%           |
| 19.5  | 12.5  | 342.85 | 315.7867| 7.9%            |
| 20.5  | 13.5  | 171.429| 189.472 | 10.5%           |
| 22    | 15    | 0      | 0      | 0               |

**Figure 3.** Valencia, Spain, [39\(^\circ\) 44\(^\prime\) N] December (1992) [11].

**Figure 4.** Makah, Saudi, [38\(^\circ\) 5\(^E\), 21\(^\circ\) 5\(^N\)][16]

**Table 3:** Makah, Saudi Arabia\((q_{\text{max}} \text{ (Eq.11)}) = 878 \text{W/m}^2\)
\[ t_d = 13, t_e = 6.5, q_{\text{max}} = 938 \text{ W/m}^2 \]

| \( t \) | \( t_e \) | \( q_{\text{exp}} \) | \( q_{\text{cal}} \) | \( \epsilon_{\text{cal}} \) | \( \epsilon_{\text{exp}} \) | \( \epsilon \) |
|---|---|---|---|---|---|---|
| 6 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7.5 | 1.5 | 168 | 216.4615 | 28.8 \% | 202.60 | 20 |
| 8.5 | 2.5 | 393 | 360.7692 | 8.2 \% | 337.70 | 14 |
| 9.5 | 3.5 | 600 | 505.0769 | 15.8 \% | 472.78 | 21 |
| 10.5 | 4.5 | 767 | 649.3846 | 15.3 \% | 607.86 | 21 |
| 11.5 | 5.5 | 890 | 793.6923 | 10.8 \% | 742.94 | 16 |
| 12.5 | 6.5 | 938 | 938 | 0 | 878 | 6 |
| 13.5 | 7.5 | 902 | 793.6923 | 12.0 \% | 742.94 | 17.6 |
| 14.5 | 8.5 | 760 | 649.3846 | 14.6 \% | 607.86 | 20 |
| 15.5 | 9.5 | 586 | 505.0769 | 13.8 \% | 472.78 | 19 |
| 16.5 | 10.5 | 367 | 360.7692 | 1.7 \% | 337.70 | 7 |
| 17.5 | 11.5 | 133 | 216.4615 | 62.7 \% | 202.60 | 52 |
| 19 | 13 | 0 | 0 | 0 | 0 | 0 |

As a measure to the degree of fitting the relative percentage errors

\[
\frac{q_{\text{exp}} - q_{\text{cal}}}{q_{\text{exp}}} = 100 \%
\]

are computed and are given in the corresponding tables.

4. Discussion.

The results reveal that the degree of fitting differs from point to point along the curve \( q(t) \) with maximum relative error 20\% except for few extreme points of the distribution near sunrise and sunset hours. At these points, low level of solar insulation is received. Thus such points are not effective for practical exploitation of solar energy. Moreover, the scatter of the fitting values is attributed to the fact that the model is devoted for days with clear sky. There are several parameters that affect such distribution for ordinary days such as the relative humidity, the extent of cloud cover and its nature; the aerosol content of the atmosphere [9, 10]. These must be taken into consideration in forthcoming trials.

5. Conclusion

1. The introduced model to predict \( q(t) \), W/m\(^2\) for global solar irradiance is not the best. The advantage of the introduced model is that it is based on simple geometrical considerations. This is useful for practical applications.
2. The obtained fitting degree is acceptable for points other than the extreme points.
3. The model for \( q(t) \) represents a closed system, in which \( q_{\text{max}} \) in the expression for \( q(t) \) can be estimated in terms of the solar constant.
4. The model is based on well-established and well defined parameters such as the length of the solar day \( t_d \). And \( (t_e) \) at which \( q(t) \) attains its maximum value.

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