Coherent bremsstrahlung at the BEPC collider

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June 12, 1998

Abstract

Coherent bremsstrahlung (CBS) is a specified type of radiation at colliders with short bunches. In the present paper we calculate the main characteristics of CBS for the BEPC collider. At this collider $dN_\gamma \sim 3 \cdot 10^8 dE_\gamma/E_\gamma$ photons of CBS will be emitted for a single collision of the beams in the energy range $E_\gamma \approx 240$ eV.

It seems that CBS can be a potential tool for optimizing collisions and for measuring beam parameters. Indeed, the bunch length $\sigma_z$ can be found from the CBS spectrum because critical energy $E_c \propto 1/\sigma_z$; the horizontal transverse bunch size $\sigma_x$ is related to $dN_\gamma \propto 1/\sigma_x^2$. Besides, CBS may be very useful for a fast control over an impact parameter $R$ between the colliding bunch axes because a dependence of $dN_\gamma$ on $R$ has a very specific behavior.

It seems quite interesting to investigate this type of radiation at the BEPC collider (for example, in the range of a visible light with the rate about $3 \cdot 10^{14}$ photons per second) and to apply it for the fast beam control.

Keywords: colliding beams, bremsstrahlung, coherent bremsstrahlung, beam parameter measurements, BEPC

*This work was partly supported by the National Natural Science Foundation of China (NSFC) and by Russian Foundation for Basic Research (code 96-02-19114)
1 Three types of radiation at colliders

Let us speak, for definiteness, about emission by electrons moving through a positron bunch. If the photon energy is large enough, one deals with the ordinary (incoherent) bremsstrahlung.

If the photon energy becomes sufficiently small, the radiation is determined by the interaction of the electron with the collective electromagnetic field of the positron bunch. It is known (see, e.g. §77 in Ref. [1]) that the properties of this coherent radiation are quite different depending on whether the electron deflection angle $\theta_d$ is large enough or rather small as compared with the typical emission angle $\theta_r \sim 1/\gamma_e$.

It is easy to estimate the ratio of these angles. The electric $E$ and magnetic $B$ fields of the positron bunch are approximately equal in magnitude, $|E| \approx |B| \sim eN_p/(\sigma_z(\sigma_x + \sigma_y)) \sim 200 \text{ G}$. These fields are transverse and they deflect the electron into the same direction. In such fields the electron moves around a circumference of radius $\rho \sim \gamma_e m_e c^2/(eB)$ and bends on the angle $\theta_d \sim \sigma_z/\rho$. On the other hand, the radiation angle $\theta_r$ corresponds to a length $l_p = \rho/\gamma_e \sim m_e c^2/(eB)$. Therefore, the ratio of these angles is determined by the dimensionless parameter $\eta$

$$\eta = \frac{r_e N_p}{\sigma_x + \sigma_y} \sim \frac{\theta_d}{\theta_r} \sim \frac{\sigma_z}{l_p}. \quad (1)$$

We call a positron bunch long if $\eta \gg 1$. The radiation in this case is usually called beamstrahlung. Its properties are similar to those for the ordinary synchrotron radiation in an uniform magnetic field (see, e.g. review [2]).

We call a positron bunch short if $\eta \ll 1$. In this case the motion of the electron can be assumed to remain rectilinear over the course of the collision. The radiation in the field of a short bunch differs substantially from the synchrotron one. In some respect it is similar to the ordinary bremsstrahlung, which is why we called it coherent bremsstrahlung (CBS).

In most colliders the parameter $\eta$ is either much smaller then 1 (all the $pp$, $\bar{p}p$ and relativistic heavy-ion colliders, some $e^+e^-$ colliders and B-factories) or $\eta \sim 1$ (e.g., LEP). Only for linear $e^+e^-$ colliders $\eta \gg 1$. Therefore, the CBS has a very wide region of applicability.

Below we use the following parameters for the BEPC collider

$$N_e = N_p = 2 \cdot 10^{11}, \quad E_e = E_p = 2 \text{ GeV}, \quad \sigma_x = 890 \mu\text{m}, \quad \sigma_y = 37 \mu\text{m}, \quad \sigma_z = 5 \text{ cm}. \quad (2)$$

Therefore, for BEPC the parameter $\eta$ is equal to

$$\eta = 0.608, \quad (3)$$

so for the BEPC collider our calculation gives, strictly speaking, an estimate only (though we have some reason to believe that the real parameter is determined not by the relation $\eta \ll 1$, but by the relation $\eta \ll 10$ - see Ref. [3]).

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1 We use the following notation: $N_e$ and $N_p$ are the numbers of electrons and positrons in the bunches; $\sigma_z$ is the longitudinal, $\sigma_x$ and $\sigma_y$ are the horizontal and vertical transverse sizes of the positron bunch; $\gamma_e = E_e/(m_e c^2)$ is the electron Lorentz factor; $E_c = 4\gamma_e^2 \hbar c/\sigma_z$ is the characteristic (critical) energy for the coherent bremsstrahlung photons; $r_e = e^2/(m_e c^2)$. 

A classical approach to CBS was given in Ref. [4]. A quantum treatment of CBS based on the rigorous concept of colliding wave packets were considered in [5], some applications of CBS to modern colliders in [6, 3, 7, 8, 9]. A new method to calculate CBS based on the equivalent photon approximation for the collective electromagnetic field of the oncoming bunch is presented in [8]. This method is much more simple and transparent as that previously discussed. It allows to calculate not only the classical radiation but to take into account quantum effects in CBS as well [10].

2 Distinctions of the CBS from the usual bremsstrahlung and from the beamstrahlung

In the usual bremsstrahlung the number of photons emitted by electrons is proportional to the number of electrons and positrons:

$$dN_\gamma \propto N_e N_p \frac{dE_\gamma}{E_\gamma}.$$  \hspace{1cm} (4)

With decreasing photon energies the coherence length $\sim 4\gamma^2 e \hbar c/E_\gamma$ becomes comparable to the length of the positron bunch $\sigma_z$. At photon energies

$$E_\gamma \lesssim E_c = 4\gamma^2 e \hbar c/\sigma_z$$  \hspace{1cm} (5)

the radiation arises from the interaction of the electron with the positron bunch as a whole, but not with each positron separately. The quantity $E_c$ is called the critical photon energy. Therefore, the positron bunch is similar to a “particle” with the huge charge $e N_p$ and with an internal structure described by the form factor of the bunch. The radiation probability is proportional to the squared number of positrons $N_p^2$ and the number of the emitted photons is given by

$$dN_\gamma \propto N_e N_p^2 \frac{dE_\gamma}{E_\gamma}.$$  \hspace{1cm} (6)

The CBS differs strongly from the beamstrahlung in the soft part of its spectrum. As one can see from (4) the total number of CBS photons diverges in contrast to the beamstrahlung for which (as well as for the synchrotron radiation) the total number of photons is finite.

3 Experimental status

The ordinary bremsstrahlung was used for luminosity measuring (for example, at the VEPP-4, HERA and LEP colliders).

The beamstrahlung has been observed in a single experiment at SLC [11] in which it has been demonstrated that it can be used for measuring a transverse bunch size of the order of 5 $\mu$m.

The main characteristics of the CBS have been calculated only recently and an experiment for its observation is now under preparation at VEPP-2M (Novosibirsk).
4 Qualitative description of CBS

We start with the standard calculation of bremsstrahlung (see \[12\], §93 and §97) at $ep$ collisions. This process is defined by the block diagram of Fig. 1, which gives the radiation of the electron (we do not consider the similar block diagram which gives the radiation of positron, the interference of these two block diagrams is negligible). We denote the

\[
e^{-} \quad \frac{E_e}{E_{\gamma}} \quad q = (\omega, q) \quad + \quad \frac{E_p}{E_{\gamma}}
\]

Figure 1: Feynman diagrams for the radiation of $e^-$ in the $e^- e^+ \rightarrow e^- e^+ \gamma$ process.

4-momentum of the virtual photon by $\hbar q = (\hbar \omega/c, \hbar q)$. The main contribution to the cross section is given by the region of small values $(-q^2)$. In this region the given reaction can be represented as a Compton scattering of the equivalent photon (radiated by the positron) on the electron. Therefore, one obtains

\[
d\sigma_{e^- e^+ \rightarrow e^- e^+ \gamma} = dN_{EP}(\omega, E_p) d\sigma_{e\gamma}(\omega, E_e, E_{\gamma}). \tag{7}
\]

Here

\[
dN_{EP}(\omega, E_p) \approx \frac{\alpha d\omega}{\omega} \int^{(-q^2)_{\text{max}}}_{(-q^2)_{\text{min}}} \frac{d(-q^2)}{(-q^2)} = \frac{\alpha d\omega}{\pi} \ln \frac{m_e^2}{(m_p \hbar \omega/E_p)^2}. \tag{8}
\]

is the number of equivalent photons (EP) with the frequency $\omega$ generated by the positron.

For the cross section (7) we obtain (the case of the $e^-$ radiation only)

\[
d\sigma_{e^- e^+ \rightarrow e^- e^+ \gamma} \approx 16 \alpha r_e^2 \left(1 - y + \frac{3}{4} y^2\right) \ln \frac{4E_e E_p (1 - y)}{m_e m_p c^4 y} \frac{dE_{\gamma}}{E_{\gamma}}, \quad y = \frac{E_{\gamma}}{E_e}. \tag{9}
\]

Just as in the standard calculations we can estimate the number of CBS photons using equivalent photon approximation. Taking into account that the number of EP increases by a factor $\sim N_p$ compared to the ordinary bremsstrahlung we get (using $d(-q^2) \rightarrow d^2 q_\perp/\pi$)

\[
dN_{EP} \sim N_p \frac{\alpha d\omega}{\omega} \frac{d^2 q_\perp}{q_\perp^2}. \tag{10}
\]

Since the impact parameter $\rho \sim 1/q_\perp$, we can rewrite this expression in another form

\[
dN_{EP} \sim N_p \frac{\alpha}{\pi^2} \frac{d\omega}{\omega} \frac{d^2 \rho}{\rho^2}. \tag{11}
\]

It is not difficult to estimate the region which gives the main contribution $|\rho_x| \sim \sigma_x, |\rho_y| \sim \sigma_y$. Integrating over this region we obtain estimates for $dN_{EP}$ and for the “effective cross section”

\[
dN_{EP} \sim N_p \frac{\alpha}{\pi} \frac{d\omega}{\omega} \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}, \quad d\sigma_{\text{eff}} \sim N_p \frac{\alpha r_e^2}{\sigma_x^2 + \sigma_y^2} \frac{dE_{\gamma}}{E_{\gamma}}. \tag{12}
\]

To illustrate the transition from the ordinary bremsstrahlung to CBS we present in Fig. 2 the photon spectrum for the BEPC collider. In the region of $E_{\gamma} \sim 100$ eV the number of photons dramatically increases by about 8 orders of magnitude.
Figure 2: Effective cross section for the emission of bremsstrahlung photons at BEPC as function of the photon energy. The huge increase at low photon energies is due to the coherent bremsstrahlung effect, the high energy tail corresponds to ordinary (incoherent) bremsstrahlung.

5 Possible applications

Coherent bremsstrahlung was not observed yet. Therefore, one can speak about applications of CBS on the preliminary level only. Nevertheless, even now we can see such features of CBS which can be useful for applications. They are the following.

A huge number of the soft photons whose spectrum is determined by the length of the positron bunch are emitted. The number of CBS photons for a single collision of the beams is (see Refs. [3, 8] for details)

\[ dN_\gamma = N_0 \Phi(E_\gamma/E_c) \frac{dE_\gamma}{E_\gamma}, \]  

(13)

Here for the flat Gaussian bunches (e.g. at \( a_y^2 = \sigma_{ey}^2 + \sigma_{py}^2 \ll a_x^2 = \sigma_{ex}^2 + \sigma_{px}^2 \)) constant \( N_0 \) is equal to

\[ N_0 = \frac{8}{3\pi} \alpha N_e \left( \frac{r_e N_p}{a_x} \right)^2 \frac{\arcsin(\sigma_{ex}/a_x)^2 + \arcsin(\sigma_{ey}/a_y)^2}{[1 - (\sigma_{ex}/a_x)^4]^{1/2}}, \]  

(14)

and for the flat and identical Gaussian bunches it is

\[ N_0 = \frac{8}{9\sqrt{3}} \alpha N_e \left( \frac{r_e N_p}{\sigma_x} \right)^2 \approx 0.5 \alpha N_e \eta^2. \]  

(15)

The function

\[ \Phi(x) = \frac{3}{2} \int_0^\infty \frac{1 + z^2}{(1 + z)^4} \exp[-x^2(1 + z)^2] \, dz; \]
\[ \Phi(x) = 1 \text{ at } x \ll 1; \quad \Phi(x) = (0.75/x^2) \cdot e^{-x^2} \text{ at } x \gg 1; \quad (16) \]

Some values of this function are: \( \Phi(x) = 0.80, 0.65, 0.36, 0.10, 0.0023 \) for \( x = 0.1, 0.2, 0.5, 1, 2 \) (see Ref. [3]).

In Table 1 we give the parameters \( E_c \) and \( N_0 \) for the BEPC collider as well as for some colliders now under development for comparison.

|        | BEPC | KEKB [7] | LHC, \( pp \) [6] | LHC, \( Pb Pb \) [9] |
|--------|------|----------|-------------------|----------------------|
| \( E (\text{GeV}) \) | 2    | 8/3.5    | 7000              | 574000               |
| \( E_c (\text{eV}) \) | 240  | 40000/7400 | 590              | 90                   |
| \( N_0 \) | \( 2.7 \cdot 10^8 \) | \( 20 \cdot 10^6 / 8 \cdot 10^6 \) | 80             | 50                   |

Specific features of CBS — a sharp dependence of spectrum (13) on the positron bunch length, an unusual behavior of the CBS photon rate in dependence on the impact parameter between axes of the colliding bunches, an azimuthal asymmetry and polarization of photons — can be very useful for an operative control over collisions and for measuring bunch parameters.

It may be convenient for BEPC to use the CBS photons in the range of \( \text{visible light } E_\gamma \sim 2 - 3 \text{ eV} \ll E_c = 240 \text{ eV} \). In this region the rate of photons will be

\[ \frac{dN_\gamma}{\tau} \approx 3 \cdot 10^{14} \frac{dE_\gamma}{E_\gamma} \text{ photons per second} \quad (17) \]

(here \( \tau = 0.8 \mu s \) is time between collisions of bunches at a given interaction region), and it is possible to use a polarization measurement without difficulties.

### 6 Collisions with the nonzero impact parameter of bunches

If the electron bunch axis is shifted in the vertical direction by a distance \( R_y \) from the positron bunch axis, the luminosity \( L(R_y) \) (as well as the number of events for the usual reactions) decreases very quickly:

\[ L(R_y) = L(0) \exp \left( -\frac{R_y^2}{4\sigma_y^2} \right). \quad (18) \]

In contrast, for the BEPC collider the number of CBS photons increases almost two times. The increase reaches 75% at \( R_y \approx 3\sigma_y \). After that, the rate of photons decreases, but even at \( R_y = 15\sigma_y \) the ratio \( dN_\gamma(R_y)/dN_\gamma(0) = 1.01 \). The corresponding results are presented in Fig 3 (for the calculation we used formulae from Ref. [3]).

The effect does not depend on the photon energy. It can be explained in the following way.
The solid line (a) is the ratio of the number of CBS photons \( dN_\gamma(R_y)/dE_\gamma \) for a vertical distance between beam axes \( R_y \) to that, \( dN_\gamma(0)/dE_\gamma \), at \( R_y = 0 \) vs. \( R_y/\sigma_y \), where \( \sigma_y \) is the vertical size of the bunch. The dotted line (b) is the same for luminosity \( L \).

At \( R_y = 0 \) a considerable portion of the electrons moves in the region of small impact parameters where electric and magnetic fields of the positron bunch are small. For \( R_y \) such as \( \sigma_y^2 \ll R_y^2 \ll \sigma_x^2 \), these electrons are shifted into the region where the electromagnetic field of the positron bunch are larger, and, therefore, the number of emitted photons increases. For large \( R_y \) (at \( \sigma_x^2 \ll R_y^2 \ll \sigma_z^2 \)), fields of the positron bunch are \( |E| \approx |B| \propto 1/R_y \) and, therefore, \( dN_\gamma \propto 1/R_y^2 \), i.e. the number of emitted photons decreases but very slowly.

This feature of CBS can be used for a fast control over impact parameters between beams (especially at the beginning of every run) and over transverse beam sizes. For the case of long bunches, such an experiment has already been performed at the SLC (see Ref. [11]).

### 7 Azimuthal asymmetry and polarization

If the impact parameter between beams is nonzero, an azimuthal asymmetry of the CBS photons appears, which can also be used for operative control over beams. For definiteness, let the electron bunch axis be shifted in the vertical direction by the distance \( R_y \) from the positron bunch axis. When \( R_y \) increases, the electron bunch is shifted into the region where the electric field of the positron bunch is directed almost in a vertical line. As a result, the equivalent photons (produced by the positron bunch) obtain a linear polarization in the vertical direction. The mean degree of such a polarization \( l \) for the BEPC collider is shown in Fig 4:
Figure 4: Degree of the equivalent photon polarization $l$ vs. $R_y/\sigma_y$, where $R_y$ is a vertical shift of the electron bunch axis.

Let us define the azimuthal asymmetry of the emitted photons by the relation

$$A = \frac{dN_\gamma(\varphi = 0) - dN_\gamma(\varphi = \pi/2)}{dN_\gamma(\varphi = 0) + dN_\gamma(\varphi = \pi/2)},$$

where the azimuthal angle $\varphi$ is measured with respect to the horizontal plane. It is not difficult to obtain that this quantity does not depend on photon energy and is equal to:

$$A = \frac{2(\gamma_e \theta)^2}{1 + (\gamma_e \theta)^2} l,$$

where $\theta$ is the polar angle of the emitted photon. From Fig 3 one can see that when $R_y$ increases, the fraction of photons emitted in the horizontal direction becomes greater than the fraction of photons emitted in the vertical direction.

If the equivalent photons have the linear polarization (and $l$ is its mean degree), then the CBS photons get also the linear polarization in the same direction. Let $l^{(f)}$ be the mean degree of CBS photon polarization. The ratio $l^{(f)}/l$ varies in the interval from 0.5 to 1 when $E_\gamma$ increases (see Table 2).

| $E_\gamma/E_c$ | 0   | 0.2 | 0.4 | 0.6 | 0.8 | 1   | 1.5 | 2   |
|----------------|-----|-----|-----|-----|-----|-----|-----|-----|
| $l^{(f)}/l$   | 0.5 | 0.7 | 0.81| 0.86| 0.89| 0.94| 0.96| 0.97|

8 Discussion

Coherent bremsstrahlung is a new type of radiation at storage rings which was not observed yet. Therefore, if it will be observed at BEPC, it will be a pioneer work
in this field. Besides, there is another interesting coincidence. The Lorentz factor
\[ \gamma_e = \frac{E_e}{m_e c^2} = 4 \cdot 10^3 \]
at the BEPC accelerator is of the same order as the Lorentz factor at the LHC collider ( \( \gamma_p = 7 \cdot 10^3 \) for pp collisions and \( \gamma_{Pb} = 3 \cdot 10^3 \) for PbPb collisions). It means that CBS spectrum at BEPC will be similar to that at LHC. As a result, an experience in observation and application of CBS at BEPC may be important for Large Hadron Colliders as well.

A serious problem for the observation of CBS may be the background due to synchrotron radiation on the external magnetic field of the accelerator. This background depends strongly on the details of the magnetic layout of the collider and cannot be calculated in a general form. To distinguish CBS from synchrotron radiation (SR) one can use such tricks as:

(i) According to Eqs. (6, 13, 15), the number of the emitted CBS photons is proportional to the number of electrons \( N_e \) and to the squared number of positrons \( N_p^2 \), i.e.
\[ dN_{CBS}^\gamma \propto N_e N_p^2. \]
As for SR, the number of SR photons is proportional to the number of electrons only
\[ dN_{SR}^\gamma \propto N_e. \]
Therefore, if one will observe that \( dN_{\gamma} \) changes when one changes the positron current, it will be the sign that the observed radiation is caused by CBS but not SR.

(ii) If one has a possibility to shift position of positron bunch in the vertical direction, one can clearly distinguish CBS from SR. Indeed, SR of electrons does not change in this case, but the number of CBS photons changes considerably as it can be seen from Fig. 3.

Acknowledgements

We are very grateful to Jin Li for providing us the BEPC parameters and to Chuang Zhang for useful discussions. Y.B.D. acknowledges the fellowship by INFN and V.G.S. acknowledges the fellowship by the Italian Ministry of Forein Affairs.

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