Finite-time synchronization and identification of the Markovian switching delayed network with multiple weights

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Abstract
This paper focuses on finite-time synchronization and parameters identification in the Markovian switching complex delayed network with multiple weights. Considering the control cost required for network synchronization, finite-time control technique and pinning control strategy are adopted. Based on these methods, when finite-time synchronization of the network is achieved, the unknown parameters of the system can also be identified in finite time. In addition, in order to solve the problem of the network performance change caused by network topology switching, an Optimal Nodes Selection Control Strategy (ONSCS) is proposed. On the one hand, finite-time synchronization and identification of the network can be realized faster; on the other hand, it also further reduces the energy consumption and control cost of the network. Finally, two sets of comparative numerical simulations are given to prove the superiority and applicability of the proposed ONSCS.

1 | INTRODUCTION

Nowadays, as a comprehensive discipline, complex network has received extensive attention from experts and scholars in different disciplines. In the past 10 years, the research of complex networks has made great achievements. The research of complex networks can not only better understand complex systems in the real world, but also provides a theoretical basis for the design of networks with good performance. These theoretical results will be widely used in various disciplines such as biology, communication and computer [1–4]. However, researchers have found that real networks or systems are often affected by some uncertain factors, such as: random changes in weather and environment, random maintenance of the system, random changes in connections between systems and these factors will cause random switching of the network topology. This random switching may depend on Markovian processes [5, 6]. Therefore, it is of practical significance to study the complex network models with Markovian switching [7, 8].

Synchronization is very common as an important non-linear phenomenon, it is to study the behaviour propagation mechanism of complex networks from a dynamic perspective. Synchronization is also one of the important research aspects in the Markovian switching complex network [9–11]. Generally speaking, from the perspective of synchronization objects, synchronization can be divided into outer synchronization and inner synchronization [12, 13]. The outer synchronization means that the behaviour of corresponding nodes between two or more networks tends to be consistent. In [14], in order to study the outer synchronization problem of a class of Markovian switched neural networks, a unified theoretical model framework is proposed. The inner synchronization means that the behaviour of each node in a network tends to be consistent. In [15], a universal pinning controller with different power parameter ranges is designed to achieve inner synchronization of the system. At present, various control methods have been proposed to achieve inner synchronization of systems. Among them, pinning control strategy is one of the most effective methods to achieve synchronization of the Markovian switching networks, but pinning control strategy also faces two problems; one is the selection of controlled nodes; the other is the design of the controller [16, 17]. Compared with the research on the latter, the research on the former is ignored by scholars. However, the problem of controlled nodes selection faced by pinning control strategy will be studied in this paper, and an Optimal Nodes Selection Control Strategy (ONSCS) is proposed to achieve synchronization of Markovian switching complex network.

The above discussed are basically single weight complex network models, but single weight Markovian switching network models can no longer meet the requirements of real networks.
In many practical engineering fields, it is often more practical to realize the synchronization of systems in a finite time rather than an infinite time [31, 32]. This is because finite time control technology not only allows engineers to know the specific time of system synchronization, but also saves synchronization time and reduces control costs. In addition, in the synchronization process of the system, the finite-time control method has also shown better robustness [33, 34]. Based on finite-time technology and intermittent control, synchronization of complex dynamical networks with Markovian switching topologies is realized in [35]. In [36], synchronization of two classes of Markovian jump complex networks is achieved in a finite time via feedback control. Thus, the finite-time synchronization of multi-weighted Markovian switching complex delayed network model with stochastic perturbations will be investigated in this paper [37, 38].

Based on the above discussion, the main contributions of this paper are as follows. In order to make the studied network model closer to the actual system, the multi-weighted Markovian switching complex delayed network model with stochastic perturbations and unknown system parameters is established. Then, considering the control cost and synchronization time problems, the pinning control method and finite time control techniques are adopted in the synchronization process. Finally, based on the controlled nodes selection problem faced by pinning control strategy, the ONSCS is proposed to achieve network synchronization. To the best of our knowledge, there are rarely studies on the multi-weighted Markovian switching complex delayed network model with stochastic perturbations; On the other hand, the selection of controlled nodes in the synchronization of Markovian switching networks is always ignored. Thus, the research has value in terms of theory and practice.

The rest of the paper is arranged as follows. In Section 2, some important conditions and network models are given. The sufficient conditions and theoretical analysis process for finite-time synchronization and identification via pinning control are given in Section 3. The sufficient conditions and theoretical analysis process for finite-time synchronization and identification via the ONSCS are given in Section 4. In Section 5, two sets of simulations are given to prove the validity of the above analysis. Finally, Section 6 concludes the paper.

## 2 | MODELS AND PRELIMINARIES

In order to complete the theoretical analysis and proof in Sections 3 and 4, the mathematical model of the network is established, and some necessary assumptions and lemmas are also given in this section.

Consider the following uncertain dynamical system:

$$\dot{x}(t) = f(x(t), \xi),$$  \hspace{1cm} (1)
where $\mathbf{x}(t) \in \mathbb{R}^n$ is the state vector; $\zeta \in \mathbb{R}^m$ is the system parameter vector; $f : \mathbb{R}^n \to \mathbb{R}^n$ is a non-linear vector function on $\mathbf{x}(t)$. Uncertain dynamical system (1) can also be rewritten as

$$\dot{\mathbf{x}}(t) = f_1(\mathbf{x}(t)) + f_2(\mathbf{x}(t))\zeta,$$

(2)

where $f_1 : \mathbb{R}^n \to \mathbb{R}^n$ and $f_2 : \mathbb{R}^n \to \mathbb{R}^m$ are continuous vector function and matrix function on $\mathbf{x}(t)$, respectively.

Define a right-continuous Markovian process $\{r(t), t \geq 0\}$ in the complete probability space $(\Omega, \mathcal{F}, \{P_t\}_{t \geq 0}, P)$, which takes values in the finite state space $\mathcal{C} = \{1, 2, \ldots, m\}$ with generator $\Pi = (\pi_{pq})_{m \times m} (p, q \in \mathcal{C})$. Define the transition probability (from the $p$th mode at time $t$ to the $q$th mode at time $t + \Delta t$) in the following form:

$$P\{r(t + \Delta t) = q | r(t) = p\} = \begin{cases} \pi_{pq}\Delta t + o(\Delta t) & \text{if } q \neq p, \\ 1 + \pi_{pp}\Delta t + o(\Delta t) & \text{if } q = p, \end{cases}$$

(3)

where $\lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t} = 0 (\Delta t > 0)$, and $\pi_{pq} \geq 0$ is the transition rate from mode $p$ at time $t$ to mode $q$ at time $t + \Delta t$ that satisfies

$$\pi_{pp} = - \sum_{q=1, q \neq p}^{m} \pi_{pq}. 

(4)

Consider the Markovian switching dynamical delayed network model with multiple weights and stochastic perturbations, if the network has $N$ nodes, then it can be described as

$$\dot{x}_i(t) = f_{i1}(x_i(t)) + f_{i2}(x_i(t))\Delta x_i + \varepsilon \sum_{j=1}^{N} a_{ij}(r(t))\Gamma_1x_j(t - \tau) + \varepsilon \sum_{j=1}^{N} b_{ij}(r(t))\Gamma_2x_j(t - \tau) + \sigma_i(t, x_i(t), r(t))\omega(t),$$

(5)

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{im}(t))^T \in \mathbb{R}^n$ is the state vector of node $i$; $f_{i1}$ and $f_{i2}$ have the same meanings as $f_1$ and $f_2$ in Equation (2), respectively; $\Delta x_i \in \mathbb{R}^n$ is an unknown parameter vector of system; time delays $\tau$ satisfy condition $\tau > 0$; $\Gamma_1 \in \mathbb{R}^{m \times m}$ and $\Gamma_2 \in \mathbb{R}^{m \times m}$ are inner coupling matrices; $\varepsilon > 0$ is the coupling strength. $A(r(t)) = a_{ij}(r(t)) \in \mathbb{R}^{N \times N}$ and $B(r(t)) = b_{ij}(r(t)) \in \mathbb{R}^{N \times N}$ are the coupling configuration matrices, which represent the topological structure of the network mode $r(t)$ at time $t - \tau$. The matrices $A(r(t))$ and $B(r(t))$ can be defined as follows: if there exists a connection between node $i$ and node $j$ ($i \neq j$), then $a_{ij}(r(t)) \neq 0$ ($b_{ij}(r(t)) \neq 0$); otherwise $a_{ij}(r(t)) = 0$ ($b_{ij}(r(t)) = 0$). The diagonal elements of matrices $A$ and $B$ are defined by $a_{ii}(r(t)) = - \sum_{j=1, j \neq i}^{N} a_{ij}(r(t)); b_{ii}(r(t)) = - \sum_{j=1, j \neq i}^{N} b_{ij}(r(t))$. Moreover, the noise intensity function is expressed as $\sigma_i(t, x_i(t), r(t))$.

The isolated node of the network (5) is given by

$$\dot{s}(t) = f(s(t), t).$$

(6)

Here, $s(t)$ can be regarded as a particular solution of the formula (6).

As mentioned in Section 1, the pinning control strategy is adopted to achieve system (3) synchronization. Under this control method, a small fraction $\varpi = (0, 1)$ of the nodes in the network (5) are selected as controlled nodes, where $[\varpi N]$ represents to keep the integer part. Then, the pinning controlled network can be described by

$$\dot{x}_i(t) = f_{i1}(x_i(t)) + f_{i2}(x_i(t))\Delta x_i + \varepsilon \sum_{j=1}^{N} a_{ij}(r(t))\Gamma_1x_j(t - \tau) + \varepsilon \sum_{j=1}^{N} b_{ij}(r(t))\Gamma_2x_j(t - \tau) + \sigma_i(t, x_i(t), r(t))\hat{\omega}(t) + u_i,$n

(7)

where $u_i(t) \in \mathbb{R}^n$ is the controller to be designed.

**Definition 1.** The synchronization of multi-weighted Markovian switching delayed network (5) is realized via an effective controller, if there exists a constant $t^* > 0$, such that

$$\lim_{t \to t^*} E\|e_i(t)\| = \lim_{t \to t^*} E\|x_i(t) - s(t)\| = 0. 

(8)

if for arbitrary $t \geq t^*$, we have $\|e_i(t)\|_2 = \|x_i(t) - s(t)\|_2 \equiv 0$, then the synchronization of complex dynamical network (5) will be realized in the finite time $t^*$. Here, $e_i(t) = (e_{i1}(t), e_{i2}(t), \ldots, e_{in}(t))^T$ represents state error of $i$th node in the system (5); $E(\cdot)$ represents the mathematical expectation.

**Remark 1.** The unknown parameter vector $\Delta x_i$ of the system is identified, if there exists a constant $t^* > 0$, such that

$$\lim_{t \to t^*} \|\Delta x_i - \alpha_i\| = 0 

(9)

and for any $t \geq t^*$, if we have $\|\Delta x_i\| = \|\Delta x_i - \alpha_i\| \equiv 0$, then the system parameter vector is achieved in the finite time $t^*$. 

In the next follow, some assumptions and lemmas needed for the proof will be given.

**Assumption 1.** For the non-linear function \( f(\cdot) : \mathbb{R}^e \to \mathbb{R}^e \) and \( \forall \xi(t), y(t) \in \mathbb{R}^e \), there exists a non-negative constant \( \nu \) and a symmetric positive matrix \( P \) such that

\[
(x(t) - y(t))^T f(t, x(t), \xi) - f(t, y(t), \xi) \leq \nu(x(t) - y(t))
\]

where \( \xi \) is the system parameter vector.

**Assumption 2.** The noise intensity function \( \sigma_j(t, \epsilon_j(t), r(t)) \) satisfies the uniform Lipschitz condition and there exists a normal number \( \rho_j \), such that

\[
\text{trace}\left[ \sigma_i^T(t, \epsilon_j(t), r(t)) \sigma_j(t, \epsilon_j(t), r(t)) \right] \leq \rho_j(r(t)) \epsilon_i^T(t) \epsilon_j(t).
\]

**Assumption 3.** The uncertain vector parameter \( \tilde{\alpha}_i \) is norm bounded and satisfies

\[
\left\{ \begin{array}{l}
\|\tilde{\alpha}_i\|_2 \leq \kappa, \\
\|b_i\| \tilde{\alpha}_i\|_2^{-1} \leq \Phi,
\end{array} \right.
\]

where \( 0 < \beta < 1; \kappa, \Phi \) and \( b_i \) are known positive constants, denote \( \Lambda = (\kappa, ..., \kappa)^T \in \mathbb{R}^e \).

**Assumption 4.** \((39)\) Let \( 0 < \epsilon < 1 \) and \( \lambda > 0 \). For any \( 0 \leq n \leq t \), if there exists a continuous function \( g(g(0) > 0) \) such that

\[
g(t) - g(n) \leq -\lambda \int_n^t (g(s))^\epsilon ds.
\]

**Lemma 1.** \((40)\) Suppose that \( d_1, d_2, ..., d_n \) are positive numbers, \( c \) \( (0 < c < 1) \) is a positive constant, then we have

\[
(d_1 + d_2 + \cdots + d_n)^\epsilon \leq d_1^\epsilon + d_2^\epsilon + \cdots + d_n^\epsilon.
\]

**Lemma 2.** For any two vectors of the same dimension \( \xi \in \mathbb{R}^e \) and \( \tilde{\xi} \in \mathbb{R}^e \), if there is a positive constant \( a \) and satisfies \( \|\xi\| \leq a \), then the following inequality holds:

\[
(\xi - \tilde{\xi})^T (\xi - a) \leq a(a + 1) (a + \|\tilde{\xi}\|)\]

where \( A \) is an \( n \)-dimension vector composed of \( a \), i.e. \( A = (a, ..., a)^T \in \mathbb{R}^e \).

**Proof.** Because \( a > 0 \), then \( \|A\|_2 \leq na \) holds.

Let \( \|\xi\| \leq a \) hold, then we have

\[
(\xi - \tilde{\xi})^T (A - \tilde{\xi}) \leq \|\xi - \tilde{\xi}\| \cdot \|A - \tilde{\xi}\|_2
\]

\[
\leq (\|\xi\|_2 + \|\tilde{\xi}\|_2) (\|A\|_2 + \|\tilde{\xi}\|_2)
\]

\[
= \|A\|_2^2 + \|\xi\|_2 \|\tilde{\xi}\|_2 + \|A\|_2 \|\tilde{\xi}\|_2 + \|\tilde{\xi}\|_2 \|A\|_2
\]

\[
\leq a^2 + a \|\tilde{\xi}\|_2 + a \cdot na + \|\tilde{\xi}\|_2 \cdot na
\]

\[
= a(a + 1)(a + \|\tilde{\xi}\|_2).
\]

\[\square\]

### 3 Finite-Time Synchronization and Identification of Markovian Switching Network Via Pinning Control

As mentioned in Section 1, the pinning control is one of the most effective methods to achieve inner synchronization of network. This is mainly because the pinning control does not need to control all the nodes of the system, which reduces the control cost of the actual engineering. In this section, the sufficient conditions for network synchronization and identification are first given. Next, on the basis of pinning control strategy, the theoretical analysis process of system synchronization and identification will be given.

According to \((6)\) and \((5)\), the following synchronization error system can be obtained:

\[
\begin{align*}
\dot{e}_i(t) &= f(x_i(t)) - f(x_i(t)) + \varepsilon \sum_{j=1}^{N} a_{ij}(r(t)) \Gamma_1 \epsilon_j(t - \tau) \\
&\quad + \varepsilon \sum_{j=1}^{N} b_{ij}(r(t)) \Gamma_2 \epsilon_j(t - \tau) + \sigma_i(t, \epsilon_i(t), r(t)) \omega(t) + \nu_i, \\
&\quad i = 1, 2, ..., l,
\end{align*}
\]

\[
\begin{align*}
\dot{e}_i(t) &= f(x_i(t)) - f(x_i(t)) + \varepsilon \sum_{j=1}^{N} a_{ij}(r(t)) \Gamma_1 \epsilon_j(t - \tau) \\
&\quad + \varepsilon \sum_{j=1}^{N} b_{ij}(r(t)) \Gamma_2 \epsilon_j(t - \tau) + \sigma_i(t, \epsilon_i(t), r(t)) \omega(t), \\
&\quad i = l + 1, l + 2, ..., N.
\end{align*}
\]

(17)

In the succeeding theorem, an effective controller and updated law will be designed, so that the errors \( \epsilon_i(t) \) can gradually converge to zero and identify the unknown parameter vector.
Theorem 1. Let Assumptions 1–4 hold. Based on the following designed controller and updated law, the synchronization error system (17) will tend to zero in the finite time.

\[
\begin{align*}
\dot{u}_i(t) &= -k(r(t))\text{sign}(e_i(t))|e_i(t)|^\beta - c \sum_{j=1}^{N} a_{ij}(r(t))\Gamma_1 e_j(t - \tau) \\
&\quad - \eta_i(r(t))e_i(t) - \frac{e_i(t)}{\|e_i(t)\|_2} \Phi \kappa(u + 1)(\kappa + \|\dot{\alpha}_i(t)\|_2) \\
&\quad - \sum_{j=1}^{N} b_{ij}(r(t))\Gamma_2 e_j(t - \tau), \quad \text{if } e_i(t) \neq 0, \\
\dot{u}_i(t) &= 0, \quad \text{if } e_i(t) = 0, \\
\dot{\alpha}_i(t) &= -b_i(r(t))f_1^T(\xi_i(t))e_i + \Phi(I - \dot{\alpha}_i(t)), \\
\end{align*}
\]

where the control parameters \(\eta_i(r(t)), k(r(t)), \beta,\) and \(b_i(r(t))\) are positive constants; \(b_i^{1+\beta/2}(r(t)) \geq 1/b_i^0, r \in C,\) and if the following inequality can be satisfied:

\[
\begin{align*}
\tilde{\Omega} + \tilde{K} + \left(\frac{1}{2}\Theta(r(t)) - \Xi(r(t))\right) \otimes I_N \leq 0, \\
\sum_{q = 1}^{N} \pi_{pq}(O(q) - D(p)) \leq 0,
\end{align*}
\]

(18)

where \(\Theta(r(t)) = \text{diag}(\rho_1(r(t)), ... , \rho_N(r(t)), 0, ..., 0); \tilde{\Omega} = \sqrt{N}I_N \otimes P; \tilde{K} = \sum_{i=1}^{N} \pi_{ii}(O(q) - D(p)); O(q)\) is a positive definite matrix of proper dimension; \(D(p)\) is arbitrary symmetrical matrix; \(I\) is an identity matrix, \(I_N = \text{diag}(1, ..., 1, 0, ..., 0).\) Then the synchronization and parameters identification of the system (5) will be realized in a finite time

\[
t^* \leq \tau + \frac{V(0, e(0), r(0))^{1-\gamma}}{\mu 2^\gamma (1 - \gamma)},
\]

(20)

where \(\gamma = (1 + \beta)/2; \sigma = \min(k(r(t))); \hbar\) is an appropriate positive constant, \(\mu = \hbar \min(\sigma, \lambda_f^2/(1 + \beta)/2, 1); V(0, e(0), r(0)) = \frac{1}{2} \sum_{i=1}^{N} e_i^T(0) e_i(0) + \frac{1}{2} \sum_{i=1}^{N} \frac{1}{b_i(0)} \dot{\alpha}_i^T(0) \dot{\alpha}_i(0); e_i(0) \) and \(\dot{\alpha}_i(0)\) are the initial conditions.

Proof. We design the Lyapunov–Krasovskii functional as

\[
\begin{align*}
V'(t, e(t), p) &= \frac{1}{2} \sum_{i=1}^{N} e_i^T(t) e_i(t) + \frac{1}{2} \sum_{i=1}^{N} \frac{1}{b_i(p)} \dot{\alpha}_i^T(t) \dot{\alpha}_i(t) \\
&\quad + \sum_{i=1}^{N} \int_{t-\tau}^{t} e_i^T(\Theta)e_i(\Theta)d\Theta.
\end{align*}
\]

(21)

According to the differential operator \(\mathcal{L}\) ([41]), we can get

\[
\begin{align*}
\mathcal{L}V'(t, e(t), p) &= \sum_{i=1}^{N} e_i^T(t) \left\{ f_1(\xi_i(t)) + f_2(\xi_i(t))\dot{\alpha}_i(t) \\
&\quad - f_2(x(t))\alpha_i(t) + \frac{1}{2} \sum_{i=1}^{N} a_{ij}(p) e_j(t - \tau) \right\} \\
&\quad - \sum_{i=1}^{N} e_i^T(t - \tau) e_i(t - \tau) \\
&\quad - f_1(\xi(t)) + \sum_{i=1}^{N} b_i(p) e_i(t - \tau) + u_i(t) \\
&\quad + \frac{1}{2} \sum_{i=1}^{N} \text{trace}[\sigma_i^T(\Theta)e_i(\Theta)e_i(\Theta)d\Theta] \\
&\quad + \sum_{i=1}^{N} e_i^T(t)e_i(t) + \sum_{q=1}^{M} \pi_{pq} \frac{1}{2} \sum_{i=1}^{N} e_i^T(t)e_i(t) \\
&\quad + \sum_{q=1}^{M} \pi_{pq} \sum_{i=1}^{N} \int_{t-\tau}^{t} e_i^T(\Theta)e_i(\Theta)d\Theta.
\end{align*}
\]

(22)

Let Assumptions 1 and 2 hold, then we can get
\[ \mathcal{L} V (t, e(t), p) \leq \sum_{i=1}^{N} e_i^T(t) \left\{ \nu P e_i(t) + e \sum_{j=1}^{N} a_{ij}(p) \Gamma_j e_j(t - \tau) \right. \\
+ e \sum_{j=1}^{N} b_{ij}(p) \Gamma_2 e_j(t - \tau) + f_{j2}(x_i(t)) \bar{\xi}_i(t) \\
+ \eta_i(t) \right\} + \frac{1}{2} \sum_{p=1}^{w} \pi_{pq} \sum_{i=1}^{N} e_i^T(t) O(q) e_i(t) \\
+ \frac{1}{2} \rho_i(p) \sum_{j=1}^{N} e_j^T(t) e_j(t) + \sum_{i=1}^{N} \Phi \bar{\xi}_i^T(t)(\Lambda - \alpha_i(t)) + \frac{1}{2} \rho_i(p) \sum_{i=1}^{N} e_i^T(t) e_i(t) \\
+ \sum_{i=1}^{N} \Phi \bar{\xi}_i^T(t)(\Lambda - \alpha_i(t)) + \frac{1}{2} \rho_i(p) \sum_{i=1}^{N} e_i^T(t) e_i(t) \]
According to Lemma 1, Assumption 3, and $h_j^{1+\beta/2}(r(t)) \geq 1/b'$, we can get
\[
\sum_{i=1}^{N} \Phi \tilde{\alpha}^T_j(r_i(t) - \alpha_i(t)) \geq \sum_{i=1}^{N} h'\|\tilde{\alpha}_i\|^2_2 - \sum_{i=1}^{N} \Phi \tilde{\alpha}^T_j(r_i(t) - \alpha_i(t)) \geq \left( \sum_{i=1}^{N} \frac{1}{h_i(p)} \tilde{\alpha}^T_j(r_i(t) - \alpha_i(t)) \right)^{1+\beta/2}. \tag{29}
\]

According to Lemma 1, we can get
\[
\mathcal{L}V(t, e(t), \mathfrak{p}) \leq \sum_{i=1}^{N} e_j^T(t)\nu P e_j(t) + \frac{1}{2} \rho_j(p) \sum_{i=1}^{N} e_j^T(t) e_j(t)
+ \frac{1}{2} \sum_{j=1}^{m} \pi_{pq} \sum_{i=1}^{N} e_j^T(t) O(q) e_i(t)
- \sum_{i=1}^{N} e_j^T(t) \eta_j e_i(t) - k(p) \left( \sum_{i=1}^{N} e_j^T(t) e_i(t) \right)^{1+\beta/2}
+ \sum_{j=1}^{m} \pi_{pq} \sum_{i=1}^{N} e_j^T(t) (O(q) - D(p)) e_i(t) \frac{1+\beta/2}{2}
- \lambda \left( \sum_{i=1}^{N} e_j^T(t) e_i(t) \right) \frac{1+\beta/2}{2}
- \left( \sum_{i=1}^{N} \frac{1}{h_i(p)} \tilde{\alpha}^T_j(r_i(t)) \tilde{\alpha}_i(t) \right)^{1+\beta/2}
\]

where $D(p) = D^T(p) (p \in \mathcal{C})$ is a symmetric matrix of suitable dimension, and has $\sum_{q=1}^{m} \pi_{pq} D(p) = 0$; $e(t) = (e_1(t), \ldots, e_N(t))^T \in \mathbb{R}^{mN}$.

According to the sufficient conditions (19) of Theorem 1 and denote $\gamma = (1+\beta)/2$, then we take the expectation on both sides of Equation (30)
\[
\mathbb{E} \left[ \mathcal{L}V(t, e(t), \mathfrak{p}) \right] \leq -\mu' 2^\gamma \mathbb{E} \left[ V^\gamma(t, e(t), \mathfrak{p}) \right]. \tag{31}
\]

Assume that there exist a positive constant $\mathfrak{h}$ satisfies $E[V^\gamma(t_0, e(t_0), \mathfrak{p})] \geq \mathfrak{h} E[V^\gamma(t_0, e(t_0), \mathfrak{p})]^\gamma$ and denote $\mu = \mu' \mathfrak{h}$. Thus
\[
\mathbb{E} \left[ \mathcal{L}V(t, e(t), \mathfrak{p}) \right] \leq -\mu 2^\gamma \mathbb{E} \left[ V(t, e(t), \mathfrak{p}) \right]^\gamma. \tag{32}
\]

Integrate both sides of inequality (32), and by solving the integral equation we have
\[
E \left[ \left[ V(t, e(t), \mathfrak{p}) \right]^{1-\gamma} \right] \leq E \left[ \left[ V(t_0, e(t_0), r(t_0)) \right]^{1-\gamma} \right]
- \mu 2^\gamma (1 - \gamma) (t - t_0).
\tag{33}
\]
Then we can get
\[
\ell - \ell_0 \leq \frac{E\left(\left|V(t, e(t), r(t))\right|\right)^{1-\gamma} - E\left(\left|V\left(t, e_0, r(t)\right)\right|\right)^{1-\gamma}}{\mu^{\gamma}(1 - \gamma)}.
\]
(34)

If there exists \( V'(t, e(t), r(t)) \equiv 0 \) for any \( t_0 \), then \( V'(t) \) will converge to zero in a finite time \( t^* \). Therefore, when \( t_0 = 0 \), the finite time \( t^* \) can be estimated by
\[
t^* \leq t + \frac{V(0, r(0))^{1-\gamma}}{\mu^{\gamma}(1 - \gamma)},
\]
(35)
where \( V(0, r(0)) = \frac{1}{2} \sum_{i=1}^{N} e_i^r(0)e_j(0) + \frac{1}{2} \sum_{i,j=1}^{N} k_i(r(0))\tilde{\beta}_{ij}(0)\tilde{\alpha}_{ij}(0). \]

Thus, based on the traditional pinning control method and some inequality techniques, the synchronization and parameters identification of the network model (5) established in Section 2 can be achieved in a finite time \( t^* \).

Remark 2. In Theorem 1, the controlled nodes of the Markovian switching network are the same in the network synchronization and identification process, these controlled nodes may consume more control costs and energy consumption. On the basis of Theorem 1, a new controlled nodes selection strategy is proposed in Section 4 to reduce control costs.

Remark 3. When the finite-time pinning synchronization multi-weighted Markovian switching complex dynamical network (5) is realized, the unknown parameter vector \( \tilde{\alpha}_i \) of the network (5) will gradually converge to the true value in a finite time \( t^* \).

Remark 4. The speed of network synchronization and parameters identification depends on the selection of control parameters \( \eta_i, k_i, b_i(r(t)) \) in the controller (18). In addition, the inequalities (19) are sufficient conditions rather than necessary conditions to realize system synchronization and identification.

4 | FINITE-TIME SYNCHRONIZATION AND IDENTIFICATION OF MARKOVIAN SWITCHING NETWORK VIA THE ONSCS

In the previous section, through theoretical analysis and proof, we verified that the finite-time pinning synchronization and parameters identification of multi-weighted Markovian switching complex delayed network with stochastic disturbances (5) can be realized under Theorem 1. However, compared with general complex systems, the parameters and topological structure of Markovian switching complex networks will change randomly, which will affect the performance and dynamic behaviour of the system more seriously. Therefore, in order to solve these problems, we need to adopt the fast and stable pinning control strategy, which can achieve network synchronization in a shorter time by controlling fewer nodes. Below, the new control strategy will be introduced to achieve this goal.

Step 1: Divide \( t \) into \( \Delta \) segments, then we have
\[
t = (t_{0}, t_1] \cup (t_1, t_2] \cup \cdots \cup (t_{\Delta-2}, t_{\Delta-1}] \cup (t_{\Delta-1}, t_{\Delta}].
\]
(36)

where \( \Delta \geq 1 \); define \( (t_{\Delta-1}, t_\Delta) \) as any time interval, \( \delta = 1, 2, \ldots, \Delta. \)

Step 2: In Markovian switching networks, the selection of the optimal controlled nodes is affected by the changes of system parameters and topology. Therefore, a selection method of optimal controlled nodes is introduced. In any time interval \((t_{\Delta-1}, t_{\Delta})\), the set \( \mathcal{I}_t^2 \) can be defined as follows:
\[
\mathcal{I}_t^2 = Y_1 = \left\{ a_{ij}(r(t_{\Delta-1})) = a_{ij}(r(t_{\Delta})) \quad i \neq j \right\}, \xi^2_1 \geq \chi \xi^1_1,
\]
\[
\mathcal{I}_t^2 = Y_2 = \left\{ a_{ij}(r(t_{\Delta-1})) \neq a_{ij}(r(t_{\Delta})) \quad i \neq j \right\}, \xi^2_1 < \chi \xi^1_1,
\]
(37)

where \( \mathcal{I}_t^2 \) is a set of control nodes selected when the network topology changes; \( Y_1 \) and \( Y_2 \) represent the sets of nodes with fixed and variable connection modes, respectively; \( \chi \) is defined as the screening factor and satisfies \( \chi \geq 0.7 \); The definitions of \( \xi^1_1(t) \) and \( \xi^2_1(t) \) are as follows:
\[
\xi^1_1(t) = \frac{1}{\psi} \sum_{j \in Y_1} \left\| x_i(t) - s(t) \right\|,
\]
\[
\xi^2_1(t) = \frac{1}{\psi} \sum_{j \in Y_2} \left\| x_i(t) - s(t) \right\|,
\]
(38)
where \( \psi \) and \( \psi' \) represent the number of elements in the \( Y_1 \) and \( Y_2 \) sets, respectively, and satisfy \( \psi + \psi' = N \).

Step 3: In any time interval \((t_{\Delta-1}, t_\Delta)\), the network (5) average synchronization error is described as
\[
\xi^\Delta_i(t) = \frac{1}{t_{\Delta} - t_{\Delta-1}} \int_{t_{\Delta-1}}^{t_{\Delta}} \left\| x_i(t) - s(t) \right\| dt, \quad i = 1, 2, \ldots, N,
\]
(39)
\[\text{where } \mathcal{I}_t^\Delta(t) \text{ represents the average synchronization error of node } i \text{ in interval } (t_{\Delta-1}, t_\Delta). \]

Step 4: In each time interval, the optimal controlled nodes are obtained by comparing the average synchronization error of each node, and the selection strategy of controlled nodes in each time interval can be described as follows: if the number of controlled nodes is recorded as \( \mathcal{I}^\Delta_1 \) at time \( t_{\Delta-1} \), then the first \( \mathcal{I}^\Delta_1 = \lceil \chi N \rceil \) nodes with the largest network average error \( \xi^\Delta_i(t) \) in the set of \( \mathcal{I}^\Delta_1 \) are the controlled nodes at time \( t_{\Delta} \) in the time interval \((t_{\Delta-1}, t_{\Delta})\), and the numbers of these nodes are written as \( \delta_{\max}^\Delta(t), t = 1, 2, \ldots, \mathcal{I}^\Delta_1. \)
The error equation of the $i$th node at time $t_5$ is described as
\[ e^δ_i(t) = \dot{x}^δ_i(t) - x_i, \quad i = 1, 2, ..., N. \] (41)

Then error system in the time interval $(t_{k-1}, t_k]$ can be obtained
\[
\begin{align*}
\dot{e}^δ_i(t) &= f(x^δ_i(t)) - f(x(t)) + \sum_{j=1}^{N} a_{ij}(r(t)) \Gamma \dot{e}^δ_j(t - \tau) \\
&\quad + \epsilon \sum_{j=1}^{N} b_{ij}(r(t)) \Gamma_2 \dot{e}^δ_j(t - \tau) + \sigma^δ_i(t, x^δ_i(t), r(t)) \dot{\omega}(t) \\
&\quad + \delta^δ_i(t), \quad i = \delta_{\max}(1), \delta_{\max}(2), ..., \delta_{\max}(N),
\end{align*}
\] (42)

Obviously, the sum of the synchronization error system (42) $\delta = 1, 2, ..., \Delta$ ($\Delta \to \infty$) converges to zero in a finite time, then the network (5) can achieve the pinning synchronization via the ONSCS. Therefore, under the action of an effective controller and adaptive laws, the finite-time synchronization and parameters identification of Markovian switching delayed network with multiple weights via the ONSCS can be solved.

**Theorem 2.** Design the controller and update laws as follows:
\[
\begin{align*}
\nu^δ_i(t) &= -\eta^δ_i(r(t))e^δ_i(t) - k(r(t)) \text{sign}(\dot{e}^δ_i(t)) \||\dot{\phi}^δ_i(t)||^\phi \\
&\quad - \epsilon \sum_{j=1}^{N} (a_{ij}(r(t)) \Gamma_1 + b_{ij}(r(t)) \Gamma_2) \dot{e}^δ_j(t - \tau) \\
&\quad - \frac{\dot{\phi}^δ_i(t)}{\||\dot{\phi}^δ_i(t)||_2^2} \Phi \varsigma + \frac{\dot{\phi}^δ_i(t)}{\||\dot{\phi}^δ_i(t)||_2^2}, \dot{e}^δ_i(t) \neq 0 \quad (43)
\end{align*}
\]
\[
\begin{align*}
\nu_i^0(t) &= 0, \quad \dot{\phi}^δ_i(t) = 0, \\
\dot{\phi}^δ_i(t) &= -\theta^δ_i(r(t))f^δ_i(x^δ_i(t))e^δ_i(t) + \Phi(\Delta - \dot{\phi}^δ_i(t)) \\
i &= \delta_{\max}(1), \delta_{\max}(2), ..., \delta_{\max}(N), \delta = 1, 2, ..., \Delta,
\end{align*}
\] (44)

where $b_i^{(1+\beta)/2}(r(t)) \geq 1/b', \text{control parameters } \eta^δ_i(r(t)), k(r(t)) \text{ and } \theta^δ_i(r(t)) \text{ are positive constants, } r \in C$, and if the following inequality can
be satisfied:

\[
\begin{cases}
\hat{\Omega} + \hat{K} + \left(\frac{1}{2} \Theta^\Delta(r(t)) - \Xi^\Delta(r(t))\right) \otimes I_n \leq 0,

\sum_{q=1}^{N} \pi_{pq} (O(q) - D(p)) \leq 0,
\end{cases}
\]

(44)

where \( \Theta^\Delta(r(t)) = \text{diag}\{\rho^\Delta_1(r(t)), \ldots, \rho^\Delta_{\max}(r(t)), 0, \ldots, 0\}; \Xi^\Delta(r(t)) = \left\{\delta, \ldots, 0\right\}; \hat{\Omega} = \upsilon I_n \otimes P; \hat{K} = I_N \otimes \upsilon_{pq}; \gamma^\Delta_1(t) = \text{diag}\{\text{sign}(\delta_1(t)), \text{sign}(\delta_2(t)), \ldots, \text{sign}(\delta_m(t))\}; \upsilon_{pq} = \sum_{q=1}^{N} \pi_{pq} (D(q) - Q(p)); D(q) \) is an appropriate positive definite matrix; \( Q(p) \) is any symmetrical matrix; \( I \) is an identity matrix, \( I_N = \text{diag}\{1, 1, 0, \ldots, 0\}. \) Then the synchroniztion and identification of the Markovian switching complex network (1) can be achieved in the finite time \( t^* \)

\[
t^* \leq \tau + \frac{V(0, e(0), r(0))^{-1} - \gamma}{\mu 2Y(1 - \gamma)},
\]

(45)

where other parameters and \( V(0, e(0), r(0)) \) have the same meaning as the parameters in formula (20).

**Proof.** The Lyapunov–Krasovskii function in the entire time interval is described as (21), then the Lyapunov–Krasovskii function constructed in the interval \( [t_2 - 1, t_2] \) is as follows:

\[
V^\Delta(t, \delta(t), p) = \frac{1}{2} \sum_{i=1}^{N} (\delta_i(t))^T \delta_i(t)
\]

\[
+ \frac{1}{2} \sum_{i=1}^{N} \frac{1}{b_i^\Delta(p)} (\mathbf{\alpha}_i^\Delta(t))^T \mathbf{\alpha}_i^\Delta(t)
\]

\[
+ \sum_{i=1}^{m} \int_{t - \tau}^{t} (\delta_i(\Theta))^T \delta_i(\Theta) d\Theta.
\]

(46)

According to the differential operator \( \mathcal{L} \) ([41]), we can get

\[
\mathcal{L} V^\Delta(t, \delta(t), p) = \sum_{i=1}^{N} (\delta_i(t))^T \dot{\delta}_i(t) + \sum_{i=1}^{N} (\dot{\delta}_i(t))^T \delta_i(t)
\]

\[
+ \sum_{i=1}^{m} \frac{1}{b_i^\Delta(p)} (\mathbf{\alpha}_i^\Delta(t))^T \mathbf{\alpha}_i^\Delta(t)
\]

\[
- \sum_{i=1}^{N} (\delta_i(t - \tau))^T \delta_i(t - \tau)
\]

\[
+ \sum_{i=1}^{N} \pi_{pq} \frac{1}{2} \sum_{j=1}^{N} (\delta_j(\Theta))^T \delta_j(\Theta) d\Theta.
\]

(47)

In the view of controller (43), the proof process is similar to the proof of Theorem 1, so we can get

\[
\mathcal{L} V^\Delta(t, \delta(t), p) \leq \sum_{i=1}^{N} (\delta_i(t))^T \left(\nu P - \eta^\Delta_i(p)\right) \dot{\delta}_i(t)
\]

\[
- \frac{k(p) \left(\sum_{i=1}^{N} (\delta_i(t))^T \delta_i(t)\right)^{\frac{1 + \beta}{2}}}{2}
\]

\[
- \sum_{i=1}^{N} \Phi (\mathbf{\alpha}_i^\Delta(t))^T (\mathbf{\alpha}_i^\Delta(t) - \mathbf{\alpha}_i^\Delta(t))
\]

\[
+ \frac{1}{2} \rho^\Delta_1(p) \sum_{i=1}^{N} (\dot{\delta}_i(t))^T \dot{\delta}_i(t)
\]

\[
- \lambda \sum_{i=1}^{N} \int_{t - \tau}^{t} (\delta_i(\Theta))^T \delta_i(\Theta) d\Theta.
\]
where $O(q)$ is a positive definite matrix of suitable dimension. According to Lemma 1, we can get

$$\mathcal{L}V^\delta(t, \hat{\delta}(t), p) \leq \sum_{i=1}^{N} (e_i^\delta(t))^T (v_p - \eta^\delta_p(p)) e_i^\delta(t)$$

$$+ \sum_{\ell=1}^{m} \sum_{i=1}^{N} \int_{t-\tau}^{t} \left( \sum_{i=1}^{N} (e_i^\delta(t))^T O(q) e_i^\delta(t) \right) d\theta$$

$$+ \sum_{\ell=1}^{m} \sum_{i=1}^{N} \int_{t-\tau}^{t} \left( \sum_{i=1}^{N} (e_i^\delta(t))^T O(q) e_i^\delta(t) \right) d\theta$$

$$\leq \frac{1}{2} (e^\delta(t))^T (\Theta^\delta_p(p) \otimes I_N) e^\delta(t)$$

$$+ \frac{1}{2} (e^\delta(t))^T (\Theta^\delta_p(p) \otimes I_N) e^\delta(t)$$

$$+ \frac{1}{2} \left( \Sigma_{i=1}^{N} (e_i^\delta(t))^T O(q) e_i^\delta(t) \right)$$

$$\leq (e^\delta(t))^T \left( \tilde{\Omega} + \tilde{K} + \frac{1}{2} \Theta^\delta_p(p) - \Sigma^\delta \right)$$

$$\times I_N e^\delta(t) - \mu' 2^{1+\beta} \left( V^\delta(t, \hat{\delta}(t), p) \right)^{1+\beta}.$$  (49)

where $D(p) = D^T(p) (p \in C)$ is a symmetric matrix of suitable dimension, and has $\sum_{\ell=1}^{m} \pi_{\ell} D(p) = 0$; $\hat{\delta}(t) = (\hat{\delta}_1(t), \ldots, \hat{\delta}_m(t))^T \in \mathbb{R}^{mN}$.

According to the sufficient conditions (43) of Theorem 2, and denote $\gamma = (1 + \beta)/2$, then we take the expectation on both sides of Equation (49)

$$E[\mathcal{L}V^\delta(t, \hat{\delta}(t), p)] \leq -\mu' 2^{1+\beta} E[V^\delta(t, \hat{\delta}(t), p)]^{1+\beta}. \quad (50)$$

Assume that there exist a positive constant $h$ satisfies $E[V^\delta(t_0, \hat{\delta}(t_0), p)] \geq h (E[V^\delta(t_0, \hat{\delta}(t_0), p)]^{1+\beta}$ and denote $\mu = \mu' h$. Then

$$E[\mathcal{L}V^\delta(t, \hat{\delta}(t), p)] \leq -\mu V^\delta(t, \hat{\delta}(t), p)]^{1+\beta}. \quad (51)$$

Integrate both sides of inequality (51) in the time interval $(t_{-1}, t_{\delta})$, and by solving the integral equation we have

$$E \left[ \left. \left( V^\delta(t_{-1}, \hat{\delta}(t_{-1}), r(t_{-1})) \right)^{1+\beta} \right| t_{-1} \right] \leq -\mu V^\delta(t_{-1}, \hat{\delta}(t_{-1}), r(t_{-1}))^{1+\beta}$$

$$+ E \left[ \left. \left( V^\delta(t_{-1}, \hat{\delta}(t_{-1}), r(t_{-1})) \right)^{1+\beta} \right| t_{-1} \right]. \quad (52)$$

Then, in the time interval $(t_{-1}, t_{\delta})$, we have

$$t_{\delta} - t_{-1} \leq \frac{E \left[ \left. \left( V^\delta(t_{-1}, \hat{\delta}(t_{-1}), r(t_{-1})) \right)^{1+\beta} \right| t_{-1} \right]}{\mu 2^{1+\beta} (1 - \gamma)}$$

$$- \frac{E \left[ \left. \left( V^\delta(t_{-1}, \hat{\delta}(t_{-1}), r(t_{-1})) \right)^{1+\beta} \right| t_{-1} \right]}{\mu 2^{1+\beta} (1 - \gamma)}. \quad (53)$$

Similar to inequality (53), in the time interval $(t_{\delta-1}, t_{\delta-1})$ we have

$$t_{\delta-1} - t_{\delta-2} \leq \frac{E \left[ \left. \left( V^\delta(t_{\delta-2}, \hat{\delta}(t_{\delta-2}), r(t_{\delta-2})) \right)^{1+\beta} \right| t_{\delta-2} \right]}{\mu 2^{1+\beta} (1 - \gamma)}$$

$$- \frac{E \left[ \left. \left( V^\delta(t_{\delta-2}, \hat{\delta}(t_{\delta-2}), r(t_{\delta-2})) \right)^{1+\beta} \right| t_{\delta-2} \right]}{\mu 2^{1+\beta} (1 - \gamma)}. \quad (54)$$

Similarly, the corresponding inequalities in the time interval $(t_{\delta-3}, t_{\delta-2}), (t_{\delta-4}, t_{\delta-3}), \ldots, (t_1, t_\delta), (t_\delta, t_1)$ can also be obtained, and the $\delta + 1$ inequalities can be added to get

$$t_{\delta} - t_0 \leq \frac{E \left[ \left. \left( V^\delta(t_{\delta}, \hat{\delta}(t_{\delta}), r(t_{\delta})) \right)^{1+\beta} \right| t_{\delta} \right]}{\mu 2^{1+\beta} (1 - \gamma)}$$

$$- \frac{E \left[ \left. \left( V^\delta(t_{\delta}, \hat{\delta}(t_{\delta}), r(t_{\delta})) \right)^{1+\beta} \right| t_{\delta} \right]}{\mu 2^{1+\beta} (1 - \gamma)}. \quad (55)$$
Then, when $\delta = 1, 2, \ldots, \Delta$ satisfies $\Delta \to \infty$, the following inequality holds:
\[
t - t_0 \leq \frac{E\left(\|V(t, e(t), r(t))\|^{1-\gamma}\right) - E\left(\|V(t_0, e(t_0), r(t_0))\|^{1-\gamma}\right)}{\mu Z_1(1-\gamma)}.
\] (56)

If there exists $V(t, e(t), r(t)) \equiv 0$ for any $t_0$, then $V(t)$ will converge to zero in a finite time $t^\ast$. Therefore, when $t_0 = 0$, the finite time $t^\ast$ can be estimated by
\[
t^\ast \leq t + \frac{V(0, r(0))^{1-\gamma}}{\mu Z_1(1-\gamma)}.
\] (57)

Hence, the synchronization and parameters identification of Markovian switching delayed network (5) with multiple weights can be achieved via the ONSCS in a finite time $t^\ast$.

Remark 6. In Theorem 2, the synchronization process is divided into $\Delta$ time intervals, and $l$ controlled nodes are selected for each time interval based on the ONSCS; The proof process of each time interval is the same as that of Theorem 1, finally each time interval is added up to obtain the whole synchronization process. Therefore, Theorem 1 is the basis of Theorem 2, and Theorem 2 is an extension of Theorem 1.

Remark 7. Under the ONSCS in this section, the finite-time synchronization of the multi-weighted Markovian switching delayed network (5) can be realized, and at the same time the unknown parameter vector $\hat{\alpha}_i(t)$ of the network is also identified in the finite time $t^\ast$.

Remark 8. The inequalities (44) in Theorem 2 are sufficient conditions rather than necessary conditions to realize system synchronization and parameters identification via the ONSCS. In addition, the ONSCS mentioned can also be applied to other more general Markovian switching complex network models.

Remark 9. As we all know, when the network coupling strength $\varepsilon$ is sufficiently large, the pinning synchronization of the complex network can be achieved with a small number of controlled nodes. However, through the ONSCS in this section, not only the control requirements of the synchronization are improved, but also the synchronization control and parameters identification of the multi-weighted Markovian switching delayed network is achieved with weak coupling strength and fewer controlled nodes.

## 5 ILLUSTRATIVE EXAMPLES

In this section, two sets of comparative simulations will be given to prove the superiority and wide applicability of the ONSCS. First, the 30-node chaotic system is used to illustrate the superiority of the proposed ONSCS; Secondly, in order to prove the wide applicability of the proposed ONSCS, the 100-node chaotic system is given; Finally, some important simulation results are obtained.

**Example 1.** In this simulation, the node dynamics of complex network is described by the chaotic system, and Figure 3 shows the trajectory of the chaotic system.

\[
\begin{aligned}
\dot{x}_1(t) &= -\hat{\alpha}_{11} x_1(t) - x_2(t)x_3(t) + \hat{\alpha}_{22} x_3(t) \\
\dot{x}_2(t) &= -\hat{\alpha}_{12} x_2(t) + x_1(t)x_3(t) \\
\dot{x}_3(t) &= \hat{\alpha}_{13} (x_1(t) - x_3(t)) \\
\end{aligned}
\] (58)

The system (58) can also be rewritten as:

\[
\begin{bmatrix}
\dot{x}_{11}(t) \\
\dot{x}_{12}(t) \\
\dot{x}_{13}(t)
\end{bmatrix} =
\begin{bmatrix}
-x_{11}(t) & x_{12}(t) & 0 \\
-x_{12}(t) & 0 & 0 \\
0 & 0 & x_{11}(t) - x_{13}(t)
\end{bmatrix}
\begin{bmatrix}
\hat{\alpha}_{11} \\
\hat{\alpha}_{12} \\
\hat{\alpha}_{13}
\end{bmatrix}
+ \begin{bmatrix}
-x_{12}(t)x_{13}(t) \\
x_{11}(t)x_{13}(t) \\
0
\end{bmatrix} (f_{12}(x(t)))
\] (59)

where $\hat{\alpha}_i$ is the system unknown parameter vector, and the true value is $\hat{\alpha}_i = (\hat{\alpha}_{11}, \hat{\alpha}_{12}, \hat{\alpha}_{13})^T = (1, 20, 5.46)^T$.

In order to prove the superiority of the ONSCS mentioned in Section 4, the network model, the topology switching mode and all parameters are kept consistent in the two simulations. The multi-weighted Markovian switching complex network is composed of 30 nodes, and the coupling configuration matrices of the network in two modes are given as Figure 4. Moreover, Figure 5 shows the system mode switching process.

**Example 1.1.** In this simulation, the synchronization and parameters identification of the multi-weighted Markovian switching complex network with stochastic disturbances will be realized via traditional pinning control method in a finite time. According to Theorem 1, the first three nodes of the network (5) are selected as the controlled nodes, i.e.

\[u_i(t) = (u_{i1}(t), u_{i2}(t), u_{i3}(t), 0, \ldots, 0),\]

where $u_{i1}(t), u_{i2}(t), u_{i3}(t)$ are the control input for the first three nodes, respectively.
The other network parameters will be given below:

\[
\Gamma_1 = \begin{bmatrix} 0.7 & 0 & 0 \\ 0 & 0.7 & 0 \\ 0 & 0 & 0.7 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \Pi = \begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix},
\]

\[
\sigma_j(t, e_j(t), 1) = \sqrt{\frac{2}{3}} e_1(t), \quad \sigma_j(t, e_j(t), 2) = \sqrt{\frac{2}{3}} e_2(t), \quad \sigma_j(t, e_j(t), 3) = \sqrt{\frac{2}{3}} e_3(t).
\]

In addition, the simulation start time is determined to be \( t_0 = 0 \), and time delay \( \tau = 0.02 \). The coupling strength of the network is taken as \( c = 10 \). The other control parameters in Theorem 1 are:

\[
\eta(1) = 50; \quad \eta(2) = 40; \quad k(1) = 1; \quad k(2) = 2; \quad h(1) = 40; \quad h(2) = 32; \quad \beta = 0.6; \quad \Xi(1) = \text{diag}(90, 75, 82, 0, ..., 0); \quad \Xi(2) = \text{diag}(103, 80, 98, 0, ..., 0); \quad \rho(1) = 2; \quad \rho(2) = 1.25.
\]

In Figure 6, the curves of network synchronization errors \( e_j(t) \) versus time are shown, from which one can see that \( e_j(t) \) have been converged to zero at about \( t \approx 4 \). The identification process curves of system unknown parameter vector \( \hat{\alpha}_i \) are given in Figure 7, we can clearly observe that the unknown parameters \( \hat{\alpha}_i (i = 1, 2, 3) \) converge to their truth values \((1; 20; 5.46)^T\) at about \( t \approx 10 \). In summary, the synchronization and identification of Markovian switching network can be achieved at about \( t \approx 10 \), which further proved that synchronization and identification of the system can be achieved through traditional pinning control strategy.

**Example 1.2.** In this simulation, the finite-time synchronization and parameters identification of the multi-weighted Markovian switching complex delayed network with stochastic disturbances will be realized via the ONSCS. According to ONSCS in Section 4, the controller \((43)\) is applied at time \( t_\delta \) of each time interval \([t_{\delta-1}, t_\delta]\), and the node with the largest average synchronization error in Equation \((39)\) is selected as the controlled node, that is to say, only one controlled node is selected in the example.

The value of the control parameters in Theorem 2 is the same as that in Theorem 1, especially, \( \Xi(1) = \text{diag}(90, 0, ..., 0); \Xi(2) = \text{diag}(103, 0, ..., 0) \). Moreover, the coupling strength of
the network is taken as $\varepsilon = 0.01$. According to (45), we can get $t^* \leq 9.4283$ by simple calculation.

In Figure 8, the curves of network synchronization errors $e_i(t)$ versus time are shown, from which one can see that $e_i(t)$ have been converged to zero at about $t \approx 3$. The identification process curves of system unknown parameter vector $\hat{\alpha}_i$ are given in Figure 9, we can clearly observe that the unknown parameters $\hat{\alpha}_i$ ($i = 1, 2, 3$) converge to their truth values $(1; 20; 5.46)^T$ at about $t \approx 7$. Figure 10 shows the number of controlled nodes at time $t_\delta$ of the time interval $[t_{\delta - 1}, t_\delta]$. In summary, the synchronization and identification of Markovian switching network can be achieved at about $t \approx 7$, which further proved that synchronization and identification of the system can be achieved through traditional pinning control strategy.

By comparing the results of Examples 1.1 and 1.2, the following conclusions can be drawn. On the one hand, under the traditional pinning control strategy and the proposed ONSCS, the synchronization and parameters identification of the network can be realized in a finite time. On the other hand, finite-time synchronization and identification of multi-weighted Markovian switching complex delayed network with stochastic perturbations can be achieved more quickly and stably under the ONSCS. In addition, based on the proposed ONSCS, the synchronization and identification of the network can be achieved under the condition of weak coupling strength and one controlled node, the energy consumption and control cost of the system are reduced. This also further illustrates the superiority of the proposed method.

Example 2. In this simulation, the node dynamics of complex network is described by the Lorenz system, and Figure 11 shows the trajectory of the Lorenz system.

\[
\begin{align*}
\dot{x}_1(t) &= \hat{\alpha}_1 (x_2(t) - x_1(t)) \\
\dot{x}_2(t) &= \hat{\alpha}_2 x_1(t) x_3(t) - x_2(t) \quad i = 1, 2, \ldots, N. \\
\dot{x}_3(t) &= x_1(t) x_2(t) - \hat{\alpha}_3 x_3(t)
\end{align*}
\]

The system (60) can also be rewritten as

\[
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t) \\
\dot{x}_3(t)
\end{bmatrix} =
\begin{bmatrix}
x_2(t) - x_1(t) & 0 & 0 \\
0 & 0 & x_1(t) \\
0 & -x_3(t) & 0
\end{bmatrix}
\begin{bmatrix}
\hat{\alpha}_1 \\
\hat{\alpha}_2 \\
\hat{\alpha}_3
\end{bmatrix}
\]
where $\hat{\alpha}_i$ is the system unknown parameter vector, and the true value is $\hat{\alpha}_i = (\hat{\alpha}_{i1}, \hat{\alpha}_{i2}, \hat{\alpha}_{i3})^T = (10, 8/3, 28)^T$.

In order to prove the broad applicability of the ONSCS mentioned in Section 4, the above network model was adopted in this simulation. The multi-weighted Markovian switching complex network is composed of 100 nodes, and the network topology is switched in two modes, Figure 12 shows the system mode switching process.

The other system parameters are given as below:

$$
\Gamma_1 = \begin{bmatrix}
0.8 & 0 & 0 \\
0 & 0.8 & 0 \\
0 & 0 & 0.8 \\
\end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}, \quad \Pi = \begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix},
$$

$$
\sigma_i(t, \epsilon_i(t), 1) = \text{diag}\left\{ \frac{\sqrt{2}}{2} \epsilon_{i1}(t), \frac{\sqrt{2}}{2} \epsilon_{i2}(t), \frac{\sqrt{2}}{2} \epsilon_{i3}(t) \right\},
$$

$$
\sigma_i(t, \epsilon_i(t), 2) = \text{diag}\left\{ \sqrt{2} \epsilon_{i1}(t), \sqrt{2} \epsilon_{i2}(t), \sqrt{2} \epsilon_{i3}(t) \right\}.
$$

In addition, the simulation start time is determined to be $t_0 = 0$, and time delay $\tau = 0.02$. The coupling strength of the network is taken as $\epsilon = 10$. The other control parameters in Theorem 1 are: $\eta(1) = 5, \eta(2) = 8; k(1) = 10, k(2) = 14; h(1) = 80, h(2) = 71; \beta = 0.6; \Sigma(1) = \text{diag}(90, 75, 82, 0, \ldots, 0); \Sigma(2) = \text{diag}(103, 80, 98, 0, \ldots, 0); \rho(1) = 2, \rho(2) = 1.25$. According to (35), we can get $r^* \leq 28.3764$ by simple calculation.

In Figure 13, the curves of network synchronization errors $\epsilon_i(t)$ versus time are shown, from which one can see that $\epsilon_i(t)$ have been converged to zero at about $t \approx 16$. The identification process curves of system unknown parameter vector $\hat{\alpha}_i$ are given in Figure 14, we can clearly observe that the unknown parameters $\hat{\alpha}_i(i = 1, 2, 3)$ converge to their truth values $(10; 8/3; 28)^T$ at about $t \approx 25$. In summary, the synchronization and identification of Markovian switching network can be achieved at about $t \approx 25$, which proved once again that synchronization and identification of the system can be achieved through traditional pinning control strategy.

Example 2.2. In this simulation, the finite-time synchronization and parameters identification of multi-weighted Markovian switching complex delayed network with stochastic disturbances will be realized via the ONSCS. According to the ONSCS mentioned in Section 4, the controller (43) is applied at time $k$ of each time interval $[t_{k-1}, t_k]$, and the node with the largest average synchronization error in Equation (39) is selected as the controlled node, that is to say, only one controlled node is selected in the example.

The value of the control parameters in Theorem 2 is the same as that in Theorem 1, especially, $\Sigma(1) = \text{diag}(90, 0, \ldots, 0)$, $\Sigma(2) = \text{diag}(103, 0, \ldots, 0)$. Moreover, the coupling strength of
the network is taken as \( \epsilon = 0.01 \). According to (45), we can get \( t^* \leq 8.2217 \) by simple calculation.

In Figure 15, the curves of network synchronization errors \( e_i(t) \) versus time are shown, from which one can see that \( e_i(t) \) have been converged to zero at about \( t \approx 5 \). The identification process curves of system unknown parameter vector \( \hat{\alpha}_i \) are given in Figure 16, we can clearly observe that the unknown parameters \( \hat{\alpha}_i (i = 1, 2, 3) \) converge to their truth values \((10; 8/3; 28)^T\) at about \( t \approx 6 \). In summary, the synchronization and identification of Markovian switching network can be achieved at about \( t \approx 6 \), which proved once again that synchronization and identification of the system can be achieved through traditional pinning control strategy.

By comparing the results of Examples 2.1 and 2.2, the following conclusions can be drawn. On the one hand, the conclusion in Examples 1 is verified again. On the other hand, it also shows that the proposed ONSCS can achieve faster synchronization and parameters identification under larger networks scale. In addition, an interesting thing has also been discovered, in the 100-node network, comparing the actual simulation time for network synchronization and identification by the traditional pinning control method and the proposed ONSCS, the actual simulation time required by the latter is much shorter than that of the former. This also further illustrates the wide applicability and superiority of the proposed method.

Except for the above conclusions, by comparing Examples 1 and 2, the following conclusions can be drawn. On the one hand, according to the results of Examples 1.1 and 2.1, we can find that the synchronization and parameters identification time of the network gradually increases with the increase of network nodes; On the other hand, according to the results of Examples 1.2 and 2.2, we can find that the synchronization and parameters identification time of the network will not change significantly with the increase of network nodes. According to the simulation results in this section, the correctness of the theoretical analysis in Sections 3 and 4 is proved, and the superiority and wide applicability of the proposed ONSCS are also better proved.

6 CONCLUSIONS

This paper proposed a new node selection method to improve the efficiency of synchronization and identification of multi-weighted Markovian switching complex delayed network. First, by designing a finite-time adaptive pinning controller, unknown parameters are also identified while network synchronization is achieved. Secondly, considering that the existence of Markovian switching will change the performance of the network, and in order to reduce energy consumption and control costs, the ONSCS is proposed, which can select more important nodes for control in Markovian switching network. Finally, in order to prove the superiority and applicability of the proposed method, two sets of comparative simulation experiments are given. Moreover, by the ONSCS, the synchronization and parameters identification of the system can be realized more quickly and stably under the condition of weak coupling strength \( \epsilon = 0.01 \) and one controlled node, and the synchronization and parameters identification time of the network will not increase significantly with the increase of the network scale. In the future, we will continue to study the following aspects. Based on the model proposed in this manuscript, it is very interesting to study the non-linear coupled Markovian switching complex network models with multiple weights and time-varying delays. On the other hand, it is also challenging to extend the ONSCS proposed in this manuscript to the other complex networks synchronization.

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REFERENCES

1. Zhang, C., et al.: Synchronization in nonlinear complex networks with multiple time-varying delays via adaptive periodically intermittent control. Int. J. Adapt. Control Signal Process. 33(1), 39–51 (2019)
2. Yang, C., et al.: Synchronization for nonlinear complex spatio-temporal networks with multiple time-invariant delays and multiple time-varying delays. Neural Process. Lett. 50(2), 1051–1064 (2019)
3. Sakthivel, R., et al.: Observer-based robust synchronization of fractional-order multi-weighted complex dynamical networks. Nonlinear Dyn. 98(2), 1231–1246 (2019)
4. Wang, Z., Liu, X.: Synchronization of interconnected discontinuous neural networks with nonlinear coupling functions. IEEE Access 7, 25,804–25,814 (2019)
5. Shen, H., et al.: Generalized state estimation for Markovian coupled networks under round-robin protocol and redundant channels. IEEE Trans. Cybern. 49(4), 1292–1301 (2019)
6. Shen, H., et al.: Slow state variables feedback stabilization for semi-Markov jump systems with singular perturbations. IEEE Trans. Auton. Control 63(8), 2709–2714 (2018)
7. Ren, H., Deng, F., Peng, Y.: Finite time synchronization of Markovian jumping stochastic complex dynamical systems with mix delays via hybrid control strategy. Neurocomputing 272, 683–693 (2018)
8. Ma, Y., Ma, N., Chen, L.: Synchronization criteria for singular complex networks with Markovian jump and time-varying delays via pinning control. Nonlinear Anal. Hybrid Syst. 29, 85–99 (2018)
9. Wang, X., et al.: Finite-time synchronization control relationship analysis of two classes of Markovian switched complex networks. Int. J. Control Autom. Syst. 16(6), 2845–2858 (2018)
10. Sakthivel, R., et al.: Synchronization of complex dynamical networks with random coupling delay and actuator faults. ISA Trans. 94, 57–69 (2019)
11. Selvaraj, P., Sakthivel, R., Kwon, O.M.: Finite-time synchronization of stochastic coupled neural networks subject to Markovian switching and input saturation. Neural Networks 105, 154–165 (2018)
12. Chen, W.H., Liu, Y., Zheng, W.X.: Synchronization analysis of two-time-scale nonlinear complex networks with time-scale-dependent coupling. IEEE Trans. Cybern. 49(9), 3255–3267 (2019)
13. Li, W., Gao, X., Li, R.: Stability and synchronization control of inertial neural networks with mixed delays. Appl. Math. Comput. 367, 124779 (2020)
14. Wang, P., Sun, D., Zhao, M.: Finite-time and fixed-time anti-synchronization of Markovian neural networks with stochastic disturbances via switching control. Neural Networks 123, 1–11 (2020)
15. Liu, X., et al.: Finite-fixed-time pinning synchronization of complex networks with stochastic disturbances. IEEE Trans. Cybern. 49(6), 2398–2403 (2019)
16. Wang, D., Huang, L.: Robust synchronization of discontinuous Cohen-Grossberg neural networks: Pinning control approach. J. Franklin Inst. 355(13), 5866–5892 (2019)
17. Lu, W., Han, Y., Chen, T.: Pinning networks of coupled dynamical systems with Markovian switching couplings and event-triggered diffusions. J. Franklin Inst.-Eng. Appl. Math. 352(9), 3526–3545 (2015)
18. Jia, Y., Wu, H., Cao, J.: Non-fragile robust finite-time synchronization for fractional-order discontinuous complex networks with multi-weights and uncertain couplings under asynchronous switching. Appl. Math. Comput. 370, 124929 (2020)
19. Xiao, F., Gan, Q., Huang, X.: Research on fixed-time synchronization of delayed complex dynamical networks with multi-weights. J. Syst. Sci. Math. Sci. 40(1), 15–28 (2020)
20. An, X.-J., Zhang, L., Zhuang, J.-G.: Research on urban public traffic network with multi-weights based on single bus transfer junction. Physica A 436, 748–755 (2015)
21. Yao, X., Zhang, C., Xia, D.: Synchronization of stochastic multiple weighted coupled networks with Markovian switching. Adv. Differ. Equations 2020(1), 159 (2020)
22. Liu, Y., et al.: Finite-time synchronization for a class of multi-weighted complex networks with Markovian switching and time-varying delay. Complexity 2020, 1–25 (2020)
23. Li, H.-L., et al.: Finite-time synchronization and parameter identification of uncertain fractional-order complex networks. Physica A 533, 122027 (2019)
24. Li, L., et al.: Parameter identification and synchronization between uncertain delay networks based on the coupling technology. Physica A 534, 120713 (2019)
25. Wu, K., et al.: Synchronization for impulsive hybrid-coupled reaction-diffusion neural networks with time-varying delays. Commun. Nonlinear Sci. Numer. Simul. 82, 105031 (2020)
26. Liu, Y., et al.: Finite-time synchronization of complex-valued neural networks with multiple time-varying delays and infinite distributed delays. Neural Process. Lett. 50(2), 1773–1787 (2018)
27. Liu, Q., et al.: Cluster synchronization of Markovian switching complex networks with hybrid couplings and stochastic perturbations. Physica A 526, 120937 (2019)
28. Du, H.: Modified function projective synchronization between two fractional-order complex dynamical networks with unknown parameters and unknown bounded external disturbances. Physica A 526, 120997 (2019)
29. Wang, X., et al.: An improved fuzzy event-triggered asynchronous dissipative control to T-S FMSs with nonperiodic sampled data. IEEE Trans. Fuzzy Syst. (2020)
30. Mathiyalagan, K., Park, J.H., Sakthivel, R.: Synchronization for delayed memristive BAM neural networks using impulsive control with random nonlinearities. Appl. Math. Comput. 259, 967–979 (2015)
31. Shen, H., et al.: Finite-time event-triggered $H_{\infty}$ control for T-S fuzzy Markov jump systems. IEEE Trans. Fuzzy Syst. 26(5), 3122–3135 (2018)
32. Xie, W., et al.: Finite-time asynchronous $H_{\infty}$ resilient filtering for switched delayed neural networks with memory unideal measurements. Inf. Sci. 487, 156–175 (2019)
33. Chen, T., Peng, S., Zhang, Z.: Finite-time synchronization of Markovian jumping complex networks with non-identical nodes and impulsive effects. Entropy 21(8), 779 (2019)
34. Wang, X., et al.: A new settling-time estimation protocol to finite-time synchronization of impulsive memristor-based neural networks. IEEE Trans. Cybern. 1–11 (2020)
35. Guo, Y., Chen, B., Wu, Y.: Finite-time synchronization of stochastic multi-links dynamical networks with Markovian switching topologies. J. Franklin Inst.-Eng. Appl. Math. 357(1), 359–384 (2020)
36. Wang, X., et al.: Finite-time synchronization control relationship analysis for two classes of Markovian jump complex networks under feedback control. Adv. Differ. Equations 2018(1), 3821–18 (2018)
37. Xie, Q., et al.: Finite-time synchronization and identification of complex delayed networks with Markovian jumping parameters and stochastic perturbations. Chaos Solitons Fractals 86, 35–49 (2016)
38. Sakthivel, R., et al.: Reliable anti-synchronization conditions for BAM memristive neural networks with different memductance functions. Appl. Math. Comput. 275, 213–228 (2016)
39. Yin, J., et al.: Finite-time stability and instability of stochastic nonlinear systems. Automatica 47(12), 2671–2677 (2011)
40. Wang, H., et al.: Finite time chaos control for a class of chaotic systems with input nonlinearities via TSM scheme. Nonlinear Dyn. 69(4), 1941–1947 (2012)
41. Yuan, C., Mao, X.: Robust stability and controllability of stochastic differential delay equations with Markovian switching. Automatica 40(3), 343–354 (2004)

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