CMB component separation in the pixel domain

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We show that the popular ILC approach is unstable in respect to the division of the sample of map pixels to the set of “homogeneous” subsamples. For suitable choice of such subsamples we can obtain the restored CMB signal with amplitudes ranged from zero to the amplitudes of the observed signal. We propose approach which allows us to obtain reasonable estimates of $C_\ell$ at $\ell \leq 30$ and similar to WMAP $C_\ell$ for larger $\ell$. With this approach we reduce some anomalies of the WMAP results.

In particular, our estimate of the quadrupole is well consistent to theoretical one, the effect of the “axis of evil” is suppressed and the symmetry of the north and south galactic hemispheres increases. This results can change estimates of quadrupole polarization and the redshift of reionization of the Universe. We propose also new simple approach which can improve WMAP estimates of high $\ell$ power spectrum.

\textbf{INTRODUCTION}

During last years fundamental results are obtained with the analysis of fluctuations of relic radiation \cite{1, 9} observed by WMAP mission. Key problem of such analysis is the cosmic microwave background (CMB) component separation from the Galactic foregrounds in the pixel domain. Several approaches were used to separate CMB from the observed signal. They are internal linear combination (ILC) and maxima entropy methods \cite{10}, the blind and Wiener filtering methods \cite{11, 12}, harmonic ILC \cite{13}, fast independent component analysis (FASTICA) \cite{14} etc. Among these approaches the ILC method is very convenient because in fact it requires minimal additional assumptions in respect to the separated signals. Detailed discussion of the ILC approach with many corrections can be found in \cite{4, 10}. The instability of the low multipoles reconstruction with the ILC method owing to the correlation between the CMB and foregrounds was discussed in \cite{15}. Recently some problems arising with the ILC method were discussed...
In Planck review [17], there is considered the final component separation pipeline for the Planck mission, which involves a combination of methods and iterations between processing steps targeted at different objectives such as diffuse component separation, spectral estimation, and compact source extraction.

At the same time some anomalies in results of WMAP team are widely discussed. Among other these are the small amplitude of quadrupole component, unexpected correlations between components with \( \ell = 2 \& 3 \) ("axis of evil"), noticeable asymmetry between north and south galactic hemispheres, existence of few deep walls in the CMB map etc. Final step with these discussions is paper [18] where all these anomalies are explained as random fluctuations.

In this paper we show that the ILC method is unstable in respect to the definition of "homogeneous" regions. As is shown below different criteria of homogeneity and corresponding division of the full sample of map pixels to set of "homogeneous" subsamples leads to different CMB maps and even different \( C_\ell \). Thus, for suitable procedure we can obtain the CMB signal in wide range of its amplitude. In fact these amplitudes can vary from zero to the amplitude of observed signal.

In Section 2 we represent four different procedure which can be used for the division of the map pixels in the set of 'homogeneous' subsamples with analytical and numerical estimates of efficiency CMB component separation. In Section 3 we apply our "best" approach to the observed Q and V channels of WMAP and show that we can suppress some of the anomalies noted above. Sec. 4 includes the summary of our results and discussion of methodical problems. In particular, we propose new approach for the analysis of high \( \ell \) power spectrum which can improve now available results.
SEPARATION OF THE CMB SIGNAL WITH ILC APPROACH

The ILC approach

The observed map is built as a set of pixels each of which contains combination $S(\theta_i)$ of the CMB signal $C(\theta_i)$ and the foreground $F(\theta_i)$. If we have maps at two different frequencies then we can write

$$S_1(\theta_i) = C(\theta_i) + F_1(\theta_i) ,$$

$$S_2(\theta_i) = C(\theta_i) + F_2(\theta_i) ,$$

and we like to perform the linear extraction of the CMB signal as follows

$$C(\theta_i) = \alpha S_1(\theta_i) + (1 - \alpha) S_2(\theta_i)$$

$$= S_2(\theta_i) + \alpha [S_1(\theta_i) - S_2(\theta_i)] ,$$

The general expression for $\alpha$ determined by the condition of minimal dispersion of cleaned map is

$$\alpha = -\langle Q_2 Q_{12} \rangle / \langle Q_{12}^2 \rangle ,$$

$$\sigma_C^2 = \langle C^2 \rangle - \langle C \rangle^2 = \langle Q_2^2 \rangle - \langle Q_2 Q_{12} \rangle^2 / \langle Q_{12}^2 \rangle .$$

Here

$$Q_1(\theta_i) = S_1(\theta_i) - \langle S_1 \rangle , \quad Q_2(\theta_i) = S_2(\theta_i) - \langle S_2 \rangle ,$$

$$Q_{12}(\theta_i) = Q_1(\theta_i) - Q_2(\theta_i) , \quad \langle Q_1 \rangle = \langle Q_2 \rangle = 0 ,$$

and $\langle \rangle$ means the averaging over the considered subsample of pixels.

However, as is seen from (1 & 2),

$$\alpha \langle (1 - F_1/F_2) \rangle = 1 , \quad \alpha = \alpha_f = -(1 - \langle F_1/F_2 \rangle) \quad (4)$$

where in accordance with the main ideas of the approach we consider $\alpha$ as a constant.

Relation (4) points out the best value of the parameter of separation $\alpha = \alpha_f$. This value depends upon the ratio $F_1/F_2$ and the scatter of $\alpha$ is determined by the scatter of this ratio for the subsample used. Moreover, two values of the parameter of separation, $\alpha$ (3) and $\alpha_f$ (4), are different and this difference decreases for decreased scatter of ratio $F_1/F_2$. This means that in order to improve the separation we must divide the full sample of pixels into set of more homogeneous subsamples using the distribution of ratios $F_1(\theta_i)/F_2(\theta_i)$. After the component separation within these subsamples we get set of cleaned pixels sum of which forms the cleaned map and allows to perform further analysis of this map with better precision. Example of such component separation is considered below (model 1).

However, such approach cannot be used in practice when the foregrounds are a priory unknown and for the component separation we would have to use criteria expressed through the observed signals. As we show below the cleaned map strongly depends upon these criteria.

In the further analysis we consider the pix-
els as independent ones and ignore the possible correlations of the signal amplitude in the neighboring pixels. The inclusion of such correlations allows to improve the component separation but makes the procedure of separation more complex.

As demonstration of these statements we consider below both analytically and numerically four models of map division on ”homogeneous” subsamples prepared with various definitions of ”homogeneity”. We determine the “homogeneous” subsamples in respect to the function $G$ of amplitudes of signals

$$G_i = G(\theta_i) = G(S_1(\theta_i), S_2(\theta_i))$$

The $i^{th}$ bin contains $K_i$ pixels for which we have

$$i \leq G_i/\Delta \leq i + 1 \quad (5)$$

where $\Delta$ is a given common width of the bins. The bin center is the mean amplitude of the function $G_{ik}$

$$\langle G_i \rangle = \sum_{k=1}^{K_i} G_{ik}/K_i, \quad (6)$$

By the way for all bins we have the symmetric distribution of functions $G_{ik}$ with

$$|\delta_k| = |G_{ik} - \langle G_i \rangle| \leq \Delta, \quad \langle \delta_i \rangle = \langle G_i - \langle G_i \rangle \rangle \equiv 0.$$  

For each subsample we obtain $\alpha_i$ according to the standard relation (3) and get the CMB signal, $C(\theta_{ik})$ for each pixel of considered subsample with relation (2).

In main this approach is similar to that used in [4] in order to take into account the inhomogeneities of the foreground. However, their selection of 12 pixel subsamples differs from ones discussed below. Our analysis confirms that the correct result can be obtained only for the known a priori foregrounds. In all other cases we can obtain the approximate estimate of the CMB signal only. But deviations between the input and restored CMB signals depend upon the criteria homogeneity and decreases for less $\Delta$. For larger $\Delta$ all approaches give comparable results.

**Four models of separation of the CMB signal**

The theoretical consideration reveals the main influences of the selection criteria but real estimates of quality of separation can be found with simulations only. To test the various methods of component separation we generate the CMB signals with the standard power spectrum and Gaussian distribution of amplitudes, using the foregrounds from WMAP [19] we transform the generated CMB signals to observed ones and separate the CMB signals with various approaches. The final estimates of precision achieved for the full map relate to the comparison of introduced and restored $C_\ell$. 

model 1

Let us consider the set of subsamples with
\[
G(\theta_i) = F_1(\theta_i)/F_2(\theta_i) = 1 + \beta + \delta(\theta_i), \quad (7)
\]
\[
\langle G \rangle = 1 + \beta, \quad |\delta| \leq \Delta,
\]
\[
F_1 = F_2(1 + \beta + \delta_i), \quad \langle Q_{12} \rangle = \beta\langle F_2 \rangle + \langle F_2\delta \rangle,
\]
Here \(1 + \beta\) is the center of the subsample and \(\delta_i = \delta(\theta_i)\) characterizes the (small) random scatter of the pixel amplitude in respect of the central point \((\langle \delta \rangle = 0)\).

For such subsample we get
\[
\alpha = -\frac{1 + o(\delta)}{\beta + o(\delta)}, \quad \alpha_f = -\frac{1}{\beta},
\]
and for \(\delta \to 0\) we have \(\alpha \to \alpha_f = -1/\beta\),

\[
C(\theta_i) = C(\theta_i) + \Delta_C(\theta_i), \quad (8)
\]
\[
\Delta_C(\theta_i) = F_2(\theta_i) \frac{\langle F_2\delta \rangle/\langle F_2 \rangle + \delta(\theta_i)}{\beta + \delta(\theta_i)} \propto \delta.
\]
As is seen from this relation
\[
\Delta_C(\theta_i) \to 0 \quad \text{for} \quad \delta \to 0 \quad (9)
\]

For such choice of the pixel subsamples we get accurate component separation precision of which depends upon the bin size, \(\Delta\), and increases for smaller \(\Delta\). Numerical simulations confirm this conclusion.

model 2

Let us consider the set of the pixel subsamples with
\[
G_i = S_1(\theta_i)/S_2(\theta_i) = 1 + \beta + \delta(\theta_i) \quad (10)
\]
where again \(1 + \beta\) is the center of the subsample and \(\delta(\theta_i)\) characterizes the (small) random scatter of the pixel amplitude in respect of the central point \((\langle \delta \rangle = 0, |\delta| \leq \Delta)\). In the case
\[
S_1(\theta_i) - S_2(\theta_i) = S_2(\theta_i)(\beta + \delta_i), \quad Q_2(\theta_i) = S_2(\theta_i) - \langle S_2 \rangle,
\]
\[
Q_{12}(\theta_i) = \beta Q_2(\theta_i) + S_2(\theta_i)\delta(\theta_i) - \langle S_2\delta \rangle, \quad (11)
\]
\[
\langle Q_{12}^2 \rangle = \beta^2 \langle Q_2^2 \rangle + 2\beta \langle \delta S_2 Q_2 \rangle + o(\delta^2),
\]
\[
\langle Q_2 Q_{12} \rangle = \beta \langle Q_2^2 \rangle + \langle \delta S_2 Q_2 \rangle, \quad \alpha \approx -1/\beta + o(\delta).
\]

Therefore,
\[
C(\theta_i) = S_2(\theta_i) \frac{\langle \delta S_2 Q_2 \rangle + o(\delta^2)}{\beta \langle Q_2^2 \rangle + o(\delta)} \propto o(\delta), \quad (12)
\]
\[
\sigma_C^2 = \langle Q_2^2 \rangle \left[ 1 - \frac{1 + o_1(\delta)}{1 + o_2(\delta)} \right] \propto o(\delta).
\]
Thus, we see that \(C(\theta_i) \propto \Delta, \sigma^2_C \propto \Delta\), and for \(\Delta \to 0\) we have \(C(\theta_i) \to 0, \sigma^2_C \to 0\). For such pixel subsamples we get the extremal result – the signal CMB equal zero. The same result can be obtained for an arbitrary function \(G = G(S_1/S_2)\). Numerical models confirm this tendencies.

model 3

Let us consider the set of pixel subsamples with
\[
G_i = S_1(\theta_i) = S_0[1 + \delta(\theta_i)], \quad (13)
\]
\[
Q_{12} = S_0\delta(\theta_i) - Q_2.
\]
Here $S_0$ is the center of the subsample and $\delta(\theta_i)$ characterizes the (small) random scatter of the pixel amplitude in respect to the central point ($\langle \delta \rangle = 0$, $|\delta| \leq \Delta/S_0$). In the case

$$\langle Q_{12}^2 \rangle = \langle Q_2^2 \rangle - 2S_0 \langle \delta S_2 \rangle + S_0^2 \langle \delta^2 \rangle,$$

$$\langle Q_2 Q_{12} \rangle = -\langle Q_2^2 \rangle + S_0 \langle \delta Q_2 \rangle,$$

$$C(\theta_i) = S_2(\theta_i)(1 - \alpha) + \alpha S_0[1 + \delta(\theta_i)].(14)$$

Thus, for $\delta \to 0$ we get

$$\alpha \to 1 \quad C(\theta_i) \to S_0, \quad \sigma^2_C \to 0. \quad (15)$$

For such choice of the function $G$, we get unexpected result - for small $\Delta \to 0$ the signal CMB is equal to $S_1 = S_0$. Numerical simulations confirm these tendencies and as is seen from the Table 1 for small $\Delta$ the selected signal $C$ is quite close to the input one $S_1$ and strongly differs from $S_2$. For larger $\Delta$ this difference disappears.

TABLE I. Two examples of the reconstruction of the CMB signal with the model 3 (arbitrary units)

| $\Delta$      | Npixels | $\langle S_1 \rangle$ | $\langle S_2 \rangle$ | $\langle C \rangle$ |
|---------------|---------|-----------------------|-----------------------|---------------------|
| 0.2mK         | 256129  | 8.7 ± 5.4             | 5.6 ± 5.3             | 3.3 ± 6.3           |
| 0.002mK       | 2835    | 1.0 ± 0.57            | -20. ± 17             | 0.4 ± 0.8           |
| 0.002mK       | 2922    | 3.0 ± 0.6             | -18. ± 18             | 1.3 ± 1.6           |

Let us consider the set of pixel subsamples with

$$G_i = S_1(\theta_i) - S_2(\theta_i) = F_1(\theta_i) - F_2(\theta_i) = \beta(1 + \delta_i)(16)$$

$$Q_{12} = \beta \delta(\theta_i) \quad \langle \delta \rangle = 0, \quad |\delta_i| \leq \Delta/\beta.$$ 

It is interesting that the best reconstruction is obtained for the larger $\Delta$ and for restored and input signals the ratio $C_\ell/C_{in}$ decreases with $\Delta$.

For $\Delta = 2, 0.2$ & $0.002mK$ reconstruction of the modeling CMB signal with foregrounds in Q and V bands are presented in Fig. 1. It is interesting that the best reconstruction is obtained for the larger $\Delta$ and for restored and input signals the ratio $C_\ell/C_{in}$ decreases with $\Delta$. 

![Fig. 1](image-url)
grounds only what is some advantage of this approach. In the case
\[ \langle Q_2 Q_{12} \rangle = \beta \langle S_2 \delta \rangle, \quad \langle Q_{12}^2 \rangle = \beta^2 \langle \delta^2 \rangle, \quad \alpha = -\frac{\langle S_2 \delta \rangle}{\langle \delta^2 \rangle}, \]
\[ C(\theta_i) = S_2(\theta_i) - [1 + \delta(\theta_i)] \langle S_2 \delta \rangle / \langle \delta^2 \rangle \quad (17) \]
\[ \sigma_C^2 = \langle Q_2^2 \rangle - \langle S_2 \delta \rangle^2 / \langle \delta^2 \rangle \]

For such choice of the function \( G_i \) results depend upon the bin size but even for \( \delta \to 0 \) they are not tend to real CMB signal. In the case the choice of optimal \( \Delta \) can be done with simulations.

Examples of such reconstruction of the input CMB signal with \( \Delta = 2, 0.2, \& 0.002mK \) are presented in Fig. 2. As is seen from this figure reconstructed signal is weakly sensitive to used small \( \Delta \) and is oscillated around the level \( C_\ell / C_{in} \sim 1. - 1.1 \).

The difference between models 3 and 4 is illustrated by Fig. 3 where we see the probability distribution function \( P(\alpha) \) for fraction of pixels versus the separation coefficient \( \alpha \).

**POWER SPECTRUM FROM Q AND V BANDS OF WMAP**

As was found in previous Section the best reconstruction of \( C_\ell \) is possible with approach used in the model 4. Applying this approach with \( \Delta \leq 0.2mK \) for Q and V bands of the WMAP maps we get \( C_\ell \) which significantly differ from ones presented in WMAP publications. In these cases we have from several tens to several thousands of ‘homogeneous’ regions instead of 12 regions used in WMAP analysis. These \( \Delta T^2_\ell \) are plotted in Fig. 3 and \( a_{2m} \) are listed in Table I. However, for
broad bins with $\Delta \geq 10mK$ our results become quite similar to the WMAP ones.

In contrast, for our parameters of quadrupole we get

$$\lambda_1 = 68.3 \mu K, \quad (l, b) = (-75^\circ, \ 9.1^\circ),$$

and

$$\lambda_2 = 12.0 \mu K, \quad (l, b) = (13.1^\circ, \ -8.7^\circ),$$

with

$$\Delta T^2 = -\frac{3}{5\pi} (\lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3) = 1070 \mu K^2.$$ 

The orientations (22) differ from both the dipole direction

$$(l, b)_D = (-96^\circ, 48^\circ),$$

and from orientations (21).

As is well known, the five quadrupole coefficients are equivalent to the components of a symmetric traceless tensor. For the principle values and orientation of tensor axes for the 3 years WMAP quadrupole we have

$$\lambda_1 = 27.1 \mu K, \quad (l, b) = (-0.8^\circ\pm13^\circ, \ 63.3^\circ\pm1^\circ),$$

$$\lambda_2 = 12.9 \mu K, \quad (l, b) = (15.5^\circ \pm 3^\circ, \ 25.8^\circ \pm 1^\circ),$$

$$\lambda_3 = -40 \mu K, \quad (l, b) = (-77.6^\circ\pm5^\circ, \ 6.5^\circ\pm4^\circ),$$

As is seen from Fig. 4 the most serious differences are found for $\ell = 2, 4,$ and for even $\ell \leq 30$. For these even $\ell$ our estimates $C_\ell$ exceed ones obtained by WMAP by a

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**TABLE II. Amplitudes of quadrupole components in $\mu K$ for $\Delta = 2\mu K$**

|      | WMAP Model 4 |
|------|--------------|
| $a_{2,0}$ | 11.48 -65.2 |
| $a_{2,1}$ | -0.05 -13.8 |
| $a_{2,-1}$ | 4.86 9.0 |
| $a_{2,2}$ | -14.41 -17.3 |
| $a_{2,-2}$ | -18.80 -11.0 |

With $a_{2m}$ listed in Table II we get for the quadrupole

$$\Delta T_Q^2 \approx 1070 \mu K^2,$$ (18)

what is close to theoretical expectations

$$\Delta T_{th}^2 \approx 1250 \mu K^2.$$ (19)

and exceeds estimate obtained by WMAP

$$\Delta T_Q^2 \approx 249 \mu K^2.$$ (20)

![FIG. 4. The $10^{-3}\Delta T^2$ for the WMAP data (points) and obtained according to the method used for the model 4 (stars) with $\Delta = 20\mu K$. Solid and dashed lines show the theoretically expected values and their scatter.](image)
factor of $\sim 1.5$ what emphasizes the symmetry of the CMB signal in north and south hemispheres. For $\ell \geq 30$ the difference becomes small. It is interesting that for odd $\ell$ deviations from WMAP results regularly do not exceed 10%. The random scatter of the method depends upon the bin size used, $\Delta$, but not exceed $\sim 10 - 15\%$ what does not distort essentially our estimates of the power spectrum.

These results noticeably change low $\ell$ part of the power spectrum and significantly suppress the effect of “axis of evil”. However, they do not distort strongly main conclusions of WMAP which are weakly depend upon this part of the power spectrum.

Let us emphasis only that new estimate of $C_2$ can noticeably changes the estimates of the quadrupole polarization and, therefore, the redshift of reionization.

**SUMMARY AND DISCUSSION**

In this paper we show that the separation of foregrounds and the CMB signal with the ILC method strongly depends upon the choice of 'homogeneous' subsamples of pixels. For foregrounds presented in WMAP papers our more stable estimates of the CMB fluctuations are obtained for the selection criteria used in the model 4. Theoretical consideration [16] shows that with this approach we cannot perform the very high precision cleaning. However, numerical analysis demonstrates that for suitable choice of the bin size, $\Delta$, the precision $\sigma \approx 10\%$ can be achieved. It can be expected that the application of refined technique developed by WMAP team will allow to decrease the errors up to values presented in [4].

**Main results**

The best results are obtained for the frequency channels Q & V and are presented in Fig. 4. Main results of our analysis can be summarized as follow:

1. The measured amplitude of quarupole is more than that given by WMAP by a factor of 2.1 what eliminates disagreement between the theoretically expected and measured values.

2. The coordinates of the quadrupole are changed while our estimates of the octupole remain the same as in WMAP. This fact substantially reduces the effect of "axes of evil".

3. All even $C_\ell$ with $4 \leq \ell \leq 20$ are more then those given by WMAP by a factor of $\approx 1.5 - 2$ what emphasizes the symmetry of the CMB signal in north and south hemispheres.
4. Deviations of odd $C_\ell$ from that given by WMAP do not exceed a factor of 1.2 - 1.3.

5. At $\ell \geq 30$ deviations of our estimates from the WMAP data do not exceed $\approx 5\%$.

6. At $\ell \leq 20$ the expected error of measured $C_\ell$ is $\sim 10\%$.

These results indicate that the main conclusions of the WMAP team remain correct. However, the change of the large scale characteristics leads to the moderate change of estimates of $\sigma_8$ and especially the estimates of low $\ell$ polarization and, therefore, the redshift of reionization of the Universe. These corrections could be important for analysis of the epoch of reionization and formation of earlier galaxies.

The further more detailed analysis of possible divisions of the full sample of pixels to the 'homogeneous' subsamples can find more effective methods of subsample selection than that used in the paper. In particular, the account of correlation of the signal amplitude in neighboring pixels can improve the quality of the cleaned map of the CMB signal.

Of course, this approach can be extended for the three and more frequency channels.

Methodical comments

The considered models allow us to obtain some inferences related to the method of linear component separation. Thus, we see that:

1. The method of linear component separation is unstable and the resulting CMB map strongly depends upon criteria homogeneity used for the selection of the set of subsample under consideration.

2. The best separation is possible with using the foreground measurements (model 1). However, such approach is of no concern for a practice as we do not know a priori the foregrounds.

3. Models 2 and 3 demonstrate that with a suitable choice of the selection criteria we can obtain arbitrary estimates for the CMB signal.

4. Reasonable estimates of the CMB signal can be obtained with the selection criteria used in the model 4. However even in this model the CMB signal can be found with errors which depend upon the bin size $\Delta$ used for the subsample selection.

5. Comparison of theoretical estimates of $\sigma_C$ for models 3 and 4 with numerical estimates of $C_\ell$ shows that sometime
the former ones do not characterize adequately the final precision achieved.

It can be expected that final results depend upon the actual foreground. This inference is confirmed by comparison results obtained for various pairs of frequency channels.

Let us note that further cleaning can be performed by recurrent comparison of the cleaned maps obtained for two pairs of frequencies. With the WMAP data we cannot test this approach as the quality of maps obtained for QV channels significantly exceed the quality of maps found for other pairs of frequency channels. However, for many channels of the PLANCK mission such approach becomes useful.

Estimates of the high $\ell$ power spectrum

As is well known for the real maps of the CMB with the finite number of pixels the determination of the power spectrum for larger $\ell$ is complex because the polar regions with relatively small number of pixels along the azimuthal coordinate cannot be used. By the way at high $\ell$ we would have to analyze the noisy regions in the vicinity of equator what decreases the precision achieved.

To decrease the influence of the noisy galactic equator we can use the simple procedure what is change of the map orientation. Indeed, if we will build the map in coordinate system with the galactic equator situated along some map meridian then we will have less noisy pixels situated along the map equator while some of the noisy pixels will be shifted to polar regions. Example of such map is presented in Fig. 5.

Of course, such approach requires preparation of two different maps one of which have the ordinary orientation and is used for the analysis of the low $\ell$ part of power spectrum while second one with the orthogonal orientation can be used for analysis of high $\ell$ components of the power spectrum.

This approach seems to be quite effective but it must be tested with real repixelized maps.

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\begin{figure}[h]
\centering
\includegraphics[width=0.4\columnwidth]{fig5.png}
\caption{The CMB map for the Q channel after rotation of the coordinate system.}
\end{figure}

\textsuperscript{1} \url{http://healpix.jpl.nasa.gov/}
age, which we used to transform the WMAP7 maps into the coefficients $a_{\ell m}$. This work made use of the GLESP\footnote{http://www.glesp.nbi.dk} package for the further analysis of the CMB data on the sphere. This paper was supported in part by Russian Foundation for Basic research grant Nr. 08-02-00159 and Nr. 09-026-12163, and Ministry of education Nr. 1336. O.V.V. also acknowledges partial support from the "Dynasty" Foundation.

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