Analytical study on impact strength of steel fibre reinforced concrete

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Abstract. In recent years, there has been an increasing tendency to build resilience to the impact of civilian and military infrastructure due to the increasing number of attacks or calamities. An alternative solution is to improve the impact resistance of such structures by incorporating different types and dosages of fibres into the concrete that can greatly reduce the damage of concrete structures due to the impact load. This research work focuses on the experimental setup based on Drop Weight Test-ACI 544. The objective is to build a model for impact energy using variation in plain concrete and fibre reinforced concrete. Furthermore, this research work investigates and describes the impact resistance of fibre reinforced concrete to calculate the reliability of the model. Lastly, this analytical study is based on testing an experimental model using various mathematical and statistical methods.

Keywords. Fibre reinforced concrete, Reliability function, Statistical methods.

1. Introduction
The impact strength of concrete can be measured by the drop-weight test due to its simplicity and attractive method, as recommended by the ACI Committee 544. The real facts resemble a huge deviation under this test. Some of the reasons for this are-

- First crack are put under visual recognition and accordingly the test results are interpreted. One fact which can be observed is that the crack can occur in any direction which can affect the end result.
- The manual approach of dropping the hammer from an exact height is beyond control.
- Tough crude aggregate, fibre or matrix play an important role in determining the single point impact for resistance of concrete.
- A heterogeneous mix material is used for making the concrete can also cause the variation for the drop-weight test result.

The amount of deviation in impact resistance affect the shape of aggregate, distribution of fibres, its geometry etc. that may be due to the variation in the mix design. The end results of the impact experimental test using steel fibre in concrete can be analyzed using the proper statistical tests.
2. Literature Survey

Natraja., M.C., et al., have examined the effect of plain concrete as well as effect of steel fibre-reinforced concrete. The variability in the impact resistance has been determined using drop weight test. The results due to the coefficient of variation calculated in percentage increase the quantity of blows beyond first crack for both type of concretes used in their study [1]. Murali G., et al., studied the statistical variations in impact strength of steel fibre reinforced concrete. Their Normality test results suggested that if three samples are used to determine \(N_1\) at 95% confidence level then the error developed is about 50% [2]. Murali G., et al., had tried to establish an empirical relationship between compressive strength vs. impact energy of steel-fibre reinforced concrete [3]. Song. P.S., et al., investigate the strength reliability of polypropylene hybrid fibre reinforced concrete with reinforced concrete using steel fibre. They found that the hybrid fibre reinforced concrete showed smaller variation than steel fibre reinforced concrete [4]. BahadorAbolpour, et al., applied statistical methods in their experimental study based on the raw materials as well as concrete mix [5]. Chirag R. Parmar et. al. study on ready mixed concrete and has interpreted their result by using SPSS software in charotar region of Gujarat state [6].

3. Experimental Setup

The Numerical study is based on strength of steel fibre reinforced concrete. The experimental setup includes Drop Weight Test-Ac1 544 where steel fibre having low carbon and it's both end were hooked are used. This test gives the quantity of blows needed to induce the required distress levels in the test sample. This figure is used as a qualitative estimate of the energy consumed by the sample at the specified distress levels.

The quantity of blows for the first visible crack (\(N_1\)) and final failure (\(N_2\)) of concrete specimens are indexed using sample size of four rounds given in Table no.1. In addition, Table no.2 provides the Average results of impact energy for Initial crack and final crack.

| Sample Number | 1  | 2  | 3  | 4  |
|---------------|----|----|----|----|
| Mix I/d       | 5/8| 6/8| 7/9| 7/10|
| 0%            |    |    |    |     |
| 0.5%A         | 6/9| 7/9| 8/10| 8/11|
| 0.5%B         | 7/10| 8/11| 9/13| 9/15|
| 0.5%C         | 10/13| 12/14| 11/16| 14/16|
| 0.5%D         | 11/16| 12/16| 14/17| 15/18|
| 0.5%E         | 12/19| 14/20| 16/21| 17/22|
| 1%A           | 9/12| 10/13| 11/13| 11/14|
| 1%B           | 10/14| 11/15| 12/16| 13/18|
| 1%C           | 15/21| 16/23| 18/23| 19/25|
| 1%D           | 22/32| 23/34| 24/35| 25/36|
| 1%E           | 28/35| 29/36| 30/38| 32/39|
| 1.5%A         | 9/13| 10/14| 11/15| 12/16|
| 1.5%B         | 12/18| 13/19| 14/20| 15/21|
| 1.5%C         | 14/26| 14/28| 16/28| 17/30|
| 1.5%D         | 26/32| 28/35| 29/37| 30/39|
| 1.5%E         | 30/38| 33/44| 34/49| 37/54|

\(N_1/N_2\)

It can be observed that, by adding 0.5%, 1.0% and 1.5% dosage of steel fibre the number of blows is increasing. This indicates that the increase in percentages of fibre in concrete provides more strength.
Table 2. Average results of experimental impact energy for Initial crack and final crack

| I/d   | N₁  | N₂  | N₂ - N₁ | Impact Energy (N₁) | Impact Energy (N₂) |
|-------|-----|-----|---------|---------------------|---------------------|
| Control | 6   | 9   | 3       | 127.1875            | 178.0625            |
| 0.5A   | 7   | 10  | 3       | 147.5375            | 198.4125            |
| 0.5B   | 8   | 12  | 4       | 167.8875            | 249.2875            |
| 0.5C   | 12  | 15  | 3       | 239.1125            | 300.1625            |
| 0.5D   | 13  | 17  | 4       | 264.55              | 340.8625            |
| 0.5E   | 15  | 21  | 6       | 300.1625            | 417.175             |
| 1.0A   | 10  | 13  | 3       | 208.5875            | 264.55              |
| 1.0B   | 12  | 16  | 4       | 234.025             | 320.5125            |
| 1.0C   | 17  | 23  | 6       | 345.95              | 468.05              |
| 1.0D   | 24  | 34  | 11      | 478.225             | 696.9875            |
| 1.0E   | 30  | 37  | 7       | 605.4125            | 752.95              |
| 1.5A   | 11  | 15  | 4       | 213.675             | 295.075             |
| 1.5B   | 14  | 20  | 6       | 274.725             | 396.825             |
| 1.5C   | 15  | 28  | 13      | 310.3375            | 569.8               |
| 1.5D   | 28  | 36  | 8       | 574.8875            | 727.5125            |
| 1.5E   | 34  | 46  | 13      | 681.725             | 941.1875            |

The above table shows the average reading of four specimens. By adding dosage of steel fibre (0.5%, 1.0% and 1.5%) the energy intake required to cause the visibility of the first crack was increased by 136%, 376% and 435%, respectively. The energy necessary to cause failure of concrete specimen was increased by 134%, 322% and 428% over the plain concrete specimen (control - 0%). Hence it was observed that, increasing the volume fraction of steel fibre increases the impact energy of concrete significantly, in both the first crack stage as well as failure stage.

This demonstrates that steel fibres act as an effective crack barrier in case of FRC, when encountered with an impact load. Consequently, crude concrete has an early brittle failure relative to the FRC, which has quite better ductile properties.

Based on N₁ and N₂, the calculated impact energy values can be determined by using the formulae given below.

\[ \text{Impact energy} \ U = \left( \frac{n \cdot m \cdot V^2}{2} \right) \]  \hspace{1cm} (1)

Where, V stands for velocity of the hammer at impact, n is the number of blows and m is the drop mass. Substituting these values in Equation (1), impact energy is-

\[ U = \frac{44.3 \times 2994.01^2}{2 \times 9810} = 20.345 \text{ KN mm} \]

\[ U_1 = U \times N_1 \]

\[ U_1 = 20.345 \times N_1 \text{ ................. (For first crack)} \]

\[ U_2 = U \times N_2 \]

\[ U_2 = 20.345 \times N_2 \text{ ................. (For Failure crack)} \]
Based on the experimental setup, this study aims to model the impact energy variation in plain concrete and fiber reinforced concrete. The three data sets collected for 4 different types of concrete mix, in terms of % as well as in Kg are given below-

Table 3. Total Volume Fraction of steel fibre (Vf) in concrete

| Sr No | Designation of Concrete Mix | Total Volume Fraction of steel fibre (V_f) in concrete | Volume Fraction of steel fibre SF (V_f) in concrete | Volume Fraction of steel fibre LF (V_f) in concrete |
|-------|-----------------------------|-----------------------------------------------------|--------------------------------------------------|--------------------------------------------------|
|       |                             | in %       | in Kg                           | in %       | in Kg                           |
| 1     | I Control                   | No fibre   | No fibre                        | 0%         | No fibre                        | 0% |
|       | A                           | 100%       | 39.25                           | 0%         | 0%                              |
|       | B                           | 75%        | 29.44                           | 25%        | 9.81                           |
| 2     | II A                        | 0.50%      | 39.25                           | 50%        | 19.62                          | 19.62 |
|       | D                           | 25%        | 9.81                            | 75%        | 29.44                          |
|       | E                           | 0%         | 0%                              | 100%       | 39.25                          |
|       | A                           | 100%       | 78.5                            | 0%         | 0%                             |
|       | B                           | 75%        | 58.87                           | 25%        | 19.63                          |
| 3     | III A                       | 1%         | 78.5                            | 50%        | 39.25                          | 39.25 |
|       | D                           | 25%        | 19.63                           | 75%        | 58.87                          |
|       | E                           | 0%         | 0%                              | 100%       | 78.5                           |
|       | A                           | 100%       | 117.75                          | 5%         | 0%                             |
| 4     | IV B                        | 1.50%      | 117.75                          | 75%        | 88.31                          | 25%  | 29.43 |
|       | C                           | 50%        | 58.87                           | 50%        | 58.87                          |
|       | D                           | 25%        | 29.43                           | 75%        | 88.31                          |
|       | E                           | 0%         | 0%                              | 100%       | 117.75                         |

Where, 0.50%, 1% and 1.5% deals with
SF = Short Fibre Aspect Ratio – 30 (1mm Dia & 30mm Length)
LF= Long Fibre Aspect Ratio – 60 (1mm Dia & 60mm Length)

4. Data Analysis
This section include three major analytics which determine the infer underlying the data sets. The details of the analytics and the respective interpretation are given below-

4.1. Normality Test
Statistically, normality test check whether the experimental data set follows normal distribution. Using Table no.1, the data following table provides the outcomes for the Impact strength.

Table 4. Descriptive Statistics for Impact Strength

| Crack | Minimum | Maximum | Average (blows) | Std.Dev. | Coeff. of Var. | 95% Conf. Int. (blows) |
|-------|---------|---------|----------------|---------|---------------|------------------------|
| N_1   | 5       | 37      | 16             | 8       | 52            | 18                     |
|       |         |         |                |         |               | 14                     |
| N_2   | 8       | 54      | 22             | 11      | 50            | 25                     |
|       |         |         |                |         |               | 19                     |
Table 4 shows the descriptive statistical parameters for N2 are much better as compared to N1 type.

![Figure 1. Histogram of Normality for N1](image1)

![Figure 2. Histogram of Normality for N2](image2)

The frequency histograms presented in Figure No. 1 and 2, interprets that the N1 and N2 distribution does not coincide with the bell-shaped curve and hence fails to follows the normal distribution. This may be due the variation in the mix concrete dosages.

4.2. Mathematical Modeling

To establish the relationship between input variable (concentration of short fibre and long fibre) and output variable (impact energy based on number of blows). We are using regression analysis method.

Let us consider the non linear regression model as follows,

\[ u = k (Vf)^a (Sf)^b (Lf)^c \]

Where, 
- \( u \) - impact energy
- \( Sf \) – Content of Short fibre
- \( Lf \) – Content of Long fibre
- \( Vf \) – Total volume of fibre content

Taking Log on both sides

\[ \log(u) = \log(k) + a \log(Vf) + b \log(Sf) + c \log(Lf) \]

\[ U = K + aVf + bSf + cLf \]

The normal equations are given by-

\[ U = nK + a \sum Vf + b \sum Sf + c \sum Lf \]  \hspace{1cm} (2)

\[ UCo = K \sum Vf + a \sum Vf^2 + b \sum VfSf + c \sum VfLf \]  \hspace{1cm} (3)

\[ USf = K \sum Sf + a \sum Sf^2 + b \sum SfSf + c \sum LfSf \]  \hspace{1cm} (4)

\[ ULf = K \sum Lf + a \sum VfLf + b \sum LfSf + c \sum Lf^2 \]  \hspace{1cm} (5)

on solving normal equation (2), (3), (4) & (5) by matrix method we get

\[ U_1 = 313.48 \ (Vf)^{0.52} \ (Sf)^{-0.08} \ (Lf)^{0.07} \] \hspace{1cm} (6)

\[ U_2 = 452.995 \ (Vf)^{0.56} \ (Sf)^{-0.07} \ (Lf)^{0.08} \] \hspace{1cm} (7)

From the equation we have observed that the index of short fibre (Sf) is negative that means the impact strength goes on decreasing with increase in short fibre concentration.
whereas the index of long fibre (Lf) is positive which means impact strength will increase with increase in concentration of long fibre.

4.3. Reliability of Model

Based on the 15 sets of different mix, following table provides the experimental and calculated impact energy (U) values. Using the coefficient of determination R² are calculated for N₁ and N₂ types.

| Experimental U₁ | Calculated U₁ | Experimental U₂ | Calculated U₂ |
|-----------------|---------------|-----------------|---------------|
| 147.50          | 138.58        | 198.36          | 179.54        |
| 167.85          | 202.93        | 249.23          | 276.05        |
| 239.05          | 220.83        | 300.09          | 301.19        |
| 264.49          | 240.83        | 340.78          | 327.19        |
| 300.09          | 365.04        | 417.07          | 469.54        |
| 208.54          | 187.87        | 264.49          | 251.92        |
| 233.97          | 289.61        | 320.43          | 410.83        |
| 345.87          | 315.14        | 467.94          | 448.25        |
| 478.11          | 343.69        | 696.82          | 486.94        |
| 605.26          | 551.31        | 752.77          | 733.39        |
| 213.62          | 224.47        | 295.00          | 307.12        |
| 274.66          | 356.58        | 396.73          | 518.41        |
| 310.26          | 388.02        | 569.66          | 565.62        |
| 574.75          | 423.17        | 727.33          | 614.44        |
| 681.56          | 701.67        | 940.96          | 951.97        |

In terms of statistics, the COD is denoted by R² and is basically used to get the error between the experimental and numerical values. The COD should always be less than 1 for its best fit using prediction concept. Here R² values are 0.82 for N₁ and 0.91 for N₂ respectively. This indicated model is very much fit to predict any future set of values.

5. Conclusion

The findings based on the above analytical study can be concluded as follows:

- The descriptive statistics defines the characteristics of the data set used in this study. The N₁ and N₂ distribution do not coincide with the normal curve and hence are not normally distributed. This may be due to the impact of ground reality of the outcomes of the experiment that voiles any ideal assumptions.
- The mathematical model based on a non-linear regression provides the valuable information regarding the impact strength based on the negative effect of short fibre concentration and the positive effect of long fibre concentration.
- The robustness of the model is determined by the coefficient of determination for both N₁ and N₂ type. Any value near to 1.0 exhibits a perfect fit, and hence implies a highly reliable model for future forecasts.

The future scope for studying the impact failure strength variation in the various concrete can be modeled using Weibull distribution and hence to indicate the reliability function. Weibull distribution with two parameters can provide the base for the statistical inferences for impact failure strength of the FRC.

6. References

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