Report on Thermal Neutron Diffusion Length Measurement in Reactor Grade Graphite Using MCNP and COMSOL Multiphysics

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Abstract. Neutron diffusion length in reactor grade graphite is measured both experimentally and theoretically. The experimental work includes Monte Carlo (MC) coding using ‘MCNP’ and Finite Element Analysis (FEA) coding suing ‘COMSOL Multiphysics’ and Matlab. The MCNP code is adopted to simulate the thermal neutron diffusion length in a reactor moderator of 2m x 2m with slightly enriched uranium ($^{235}U$), accompanied with a model designed for thermal hydraulic analysis using point kinetic equations, based on partial and ordinary differential equation. The theoretical work includes numerical approximation methods including transcendental technique to illustrate the iteration process with the FEA method. Finally collision density of thermal neutron in graphite is measured, also specific heat relation dependability of collision density is also calculated theoretically, the thermal neutron diffusion length in graphite is evaluated at 50.85±0.3cm using COMSOL Multiphysics and 50.95±0.5cm using MCNP. Finally the total neutron cross-section is derived using FEA in an inverse iteration form.

1. Introduction
This work demonstrates an analytic approach accompanied with models of Finite Element Analysis (FEA) and Monte Carlo (MC) with an experimental measure on neutron cross-section and slowing down process. In MC approach Monte Carlo N-Particle Transport Code (MCNP) is used to simulate the simplified version of reactor moderation process. Similarly in FEA the moderator modelled (Assuming a symmetrical distribution) using point kinetic equations, based on partial and ordinary differential equation in software package.

2. Theoretical Calculations
Having the number of particles found in a volume element $dr$ where $dr = dx dy dz$ at $r$ with a vector with solid angle $dΩ$ at $Ω$ be donated by [1]:

$$N(r, Ω, t)drdΩ$$

(1)

Therefore can have:
\[
\frac{dN}{dt} = -NV\sigma + \int N(r, \Omega', t)V\sigma_f(\Omega, \Omega')d\Omega' + S(r, \Omega, t)
\]  

(2)

Where the first term \(dN/dt\) donates the number of particles present in given volume (particle density) and second term \(-NV\sigma\) represents the total number of particles removed from the given volume by scattering and capture. \(\sigma\) is representing the total cross-section. The third term represents the total number of particles scattered into the given volume, and \(f(\Omega, \Omega')\) represents the relative probability of scattering through an angle whose cosine is \(\Omega, \Omega'\), where \(\Omega'\) is a unit vector in the direction of the initial velocity and \(\Omega\) is unit vector in the final direction. Finally \(S(r, \Omega, t)\) is the external source term available in the system and is given by:

\[
N(r, \Omega, t)drd\Omega = N(Z, \varphi)dZd\varphi
\]  

(3)

Where \(\varphi\) is the cosine of the velocity vector in the \(Z\) direction and \(\phi\) is the longitude of velocity vector.

Figure 1. The Velocity Vector: Where \(\varphi\) is the cosine of the velocity vector in the \(Z\) direction and \(\phi\) is the longitude of velocity vector and \(\Omega\) is unit vector in the final direction.

Now an assumption can be made such:

\[
N_0(Z) = 2\pi \int_{-1}^{+1} N(Z, \varphi)d\varphi
\]

(4)

Rewrite the equation 4 as:

\[
V\varphi \frac{\delta N(Z, \varphi)}{dZ} = -N(Z, \varphi)V\sigma + \int N(Z, \varphi')V\sigma_f(\varphi_0)d\Omega + S(z)
\]

(5)

Where \(\varphi_0\) is the cosine of the angle between initial and final velocities and it can be found by:

\[
cos\theta cos\theta' + sin\theta sin\theta' cos(\phi - \phi')
\]

(6)
Or using:

\[ \varphi_0 = \varphi \varphi' + \sqrt{1 - \varphi^2} \sqrt{1 - \varphi'^2} \cos(\phi - \phi') \] (7)

Now if the collision function of \( F(\varphi_0) \) expanded in spherical harmonics:

\[ F(\varphi_0) = \sum_{l=0}^{\infty} \frac{2l + 1}{4\pi} F_l(\varphi_0) \] (8)

With \( F_1 = \int f(\varphi_0) P_1(\varphi_0) d\Omega \). Using similar expansion the phase density function will be:

\[ N(Z, \varphi) = \sum_{l=0}^{\infty} \frac{2l + 1}{4\pi} N_l(Z) P_1(\varphi) \] (9)

Where \( N(Z, \varphi) = \int N(Z, \varphi) P_1(\varphi) d\Omega \), with assumption that \( N(Z, \varphi) \) is isotropic, three conditions must be satisfied all the times: first, it is far from the source (equal to Mean Free Path (MFP)), second, it is far from the boundaries; third, the probability of capture is small compared to probability of scattering. Having all the conditions satisfied, the following can be assumed:

\[ N(Z, \varphi) \approx \frac{1}{4\pi} (N_0(Z) + 3\varphi N_1(Z)) \] (10)

Where the second term in the bracket donates the particle flux \( (J) \). Here \( N_1 = \int \varphi N(Z, \varphi) d\Omega = J/V \). For simplicity we choose our unit such that \( V = 1 \) and \( \sigma = 1 \), hence:

\[ 1 - f = \frac{\sigma_s}{\sigma} \] (11)

Now the Boltzmann equation takes the form of:

\[ N \frac{dN}{dZ} = -N + (1 - f) \int N(Z, \varphi') f(N_0) d\Omega' + S(Z) \] (12)

By integrating the equation over all possible angles \( (d\Omega) \) we have:

\[ \frac{dN_1}{dZ} = -N_0 + (1 - f) N_0 + 4\pi S(Z) \] (13)

\( f \) is normalized in such a way that \( \int f(\Omega, \Omega') d\Omega' = \int f(\Omega, \Omega) d\Omega = 1 \), hence going back to Eq. \( \int f(\Omega, \Omega) d\Omega = 1 \), for the case \( l = 0 \) we have:

\[ F_0 = \int f(\varphi_0) d\Omega = 1 \] (14)

Hence by integrating over all angles and Multiplying by \( \varphi \) we have:

\[ \frac{1}{3} \frac{dN_0}{dZ} = -N_1 + (1 - f) F_1 N_1 \] (15)

3
Now the second order differential equation gives:

\[- \frac{1}{3(1 - f_1)} \frac{d^2 \phi_0}{dZ^2} = -fN_0 + S_0(Z) \]  \hspace{1cm} (16)

This also can be written as:

\[ \nabla^2 N_0 - \frac{1}{L^2} N_0 + \frac{1}{D} S_0 = 0 \]  \hspace{1cm} (17)

Eqs. (16) and (17) are known as diffusion equation. Here $L$ is diffusion length and $D$ is diffusion coefficient and it is equal to $\frac{1}{3} \frac{\lambda_s}{1 - f_1} = \frac{\lambda_{tr}}{3} V$, where $\lambda_s$ and $\lambda_{tr}$ are the scattering and transport mean free path. $\lambda_{tr}$ can be calculated from:

\[ \lambda_{tr} = \frac{\lambda_s}{1 - \langle \cos \theta \rangle_{av}} \]  \hspace{1cm} (18)

and $\langle \cos \theta \rangle_{av}$ is equal to $\int f(\phi_0) \phi_0 d\Omega = f_1$. Also $L^2$ can be measured using following relation:

\[ L^2 = \frac{\lambda_c \lambda_{tr}}{3} \]  \hspace{1cm} (19)

Here $\lambda_c$ is the capture mean free path.

**MAXIMUM ENERGY LOSS**

If a neutron with initial velocity $V_0$ collides with a nucleus of mass $M$ (at rest), then in the Centre of Mass (CoM) system, the initial velocity is $MV_0/M + 1$ after collision. The momentum of of neutron and the nucleus will be equal to magnitude oppositely directed vector. Figure 2 demonstrates the collision in CoM system.

As demonstrated in Fig. 2 the $\theta$ is the deflection angle and $\Theta$ is angle on the final velocity $v$.

The $v^2$ in this case is given by:

\[ \frac{Mv_0}{M + 1} \cos \theta + \frac{v_0}{M + 1} = v\cos \Theta \]  \hspace{1cm} (20)

\[ \left( \frac{Mv_0}{M + 1} \right)^2 + \left( \frac{v_0}{M + 1} \right)^2 - \frac{2Mv_0^2}{M + 1} \cos \theta = v^2 \]  \hspace{1cm} (21)

so

\[ \cos \theta = 1 - \frac{(M + 1)^2}{2M} \left( 1 - \frac{v}{v_0} \right)^2 \]  \hspace{1cm} (22)

since $u = \log \frac{E_0}{E}$ then:

\[ \cos \theta = 1 - \frac{(M + 1)^2}{2M} \left( 1 - e^{-u} \right) \]  \hspace{1cm} (23)
Figure 2. The Collision in Centre-of-Mass System: If a neutron with initial velocity $V_0$ collides with a nucleus of mass $M$ at rest, then in the Centre of Mass (CoM) system the initial velocity is $MV_0/M + 1$ after collision. The momentum of of neutron and the nucleus will be equal to magnitude oppositely directed vector. Here $\theta$ is the deflection angle and $\Theta$ is angle on the final velocity $v$.

Now differential cross-section gives:

$$
\frac{d\cos\theta}{du} = -\frac{(M + 1)^2}{2M} e^{-u}
$$

Hence:

$$
\cos\Theta = -\frac{(M + 1)^2}{2} e^{-\frac{u}{2}} - \frac{M - 1}{2} e^{\frac{u}{2}}
$$

Therefore the maximum logarithmic energy loss can be calculated from:

$$
q_M = \log\left(\frac{M + 1}{M - 1}\right)^2
$$

The $q_M$ is at most when $\Theta = \pi$. Now going back to the problem we can redefine the collision density function as:

$$
F(\phi_0, u) = \frac{(M + 1)^2}{8\pi M} e^{-u} \times \delta(\phi_0 - \left(\frac{M + 1}{2}\right) e^{-\frac{u}{2}} - \frac{M - 1}{2} e^{\frac{u}{2}})
$$

The term $\frac{(M+1)^2}{8\pi M} e^{-u}$ is the normalization constant chosen to satisfy $\int d\Omega \int du f(\phi_0, u) = 1$ and
δ is the Dirac δ function. So that δ(x − a) = 0 when x ≠ 0 and \( \int d(x - a)F(x)dx = F(a) \). Now the average logarithmic loss (ξ) can be calculated from:

\[
\xi = 1 - \frac{(M + 1)^2}{4M} q_m e^{-q_m}
\]

(28)

and (cos)\( v \) is 2/3M.

Energy Distribution of Slowed Down Neutrons
I. Stationary Case

The average collision density per unit time, with logarithmic energy intervals is given by:

\[
\psi_0(u) = \int_0^u du' \psi_0(u') h(u') f_0(u - u') + \delta(u)
\]

(29)

where \( f_0(u) = (M + 1)^2 e^{-u}/4m \) for \( u \leq q_m \) and it is zero otherwise. In stationary case the total number of neutron produced is unity per unit in this case, i.e. for \( M = 12 \), \( f + 0(u) \) becomes 3.5e\( -u \). Hence the equation (29) becomes:

\[
\psi_0(u) = \int_{u-q_m}^u du' \psi_0(u') h(u') 3.5e^{-(u-u')} + \delta(u)
\]

(30)

where \( q_m = 0.72 \)

II. Time-dependent Case

The time dependent when there is no absorption in the system and source strength is unity and is given by:

\[
\frac{l(u)}{v} \frac{d\psi_0}{dt} + \psi_0(u, t) = \int_0^u du' \psi_0(u', t) e^{-(u-u')} + \delta(u)\delta(t)
\]

(31)

where \( l(u) \) is the mean free path and if the mean free path is constant, the Laplacian form of the equation for \( M \neq 1 \) becomes:

\[
1 + \frac{s l_0}{v} \phi_0(u, s) = \int_0^u du' \phi_0(u', s) f_0(u - u') + \delta(u)
\]

(32)

now:

\[
\phi(w, s) = \frac{2}{(1 - r^2)w^2} \int_{ru}^w dw' \frac{w' \phi(w', s)}{(1 + w')}
\]

(33)

Eq. (33) applies for \( u > q_m \) where \( w = l_0 s/v, r = M - 1/M + 1 \) and \( \phi(w, s) = (1 + w) \phi_0(u, s) \), so that the mean free path is proportional to velocity.

III. Rigorous Numerical Solution

The slowing down process is not an easy approach, therefore a more discrete form of solution also could be defined using:

\[
F(E) = \int_{E'}^\infty \sum_s (E' \rightarrow E) \phi(E') dE' + \delta(u)
\]

(34)
Where $\sum_{S}(E' \rightarrow E)$ is the scattering term between energies $E'$ and $E$. Recalling $\sum_{S}$:

$$
\sum_{S}(E' \rightarrow E) = \sum_{S}(E')P(E' \rightarrow E)
$$

(35)

Where $P(E' \rightarrow E)$ is the probability of collision happens between $E'$ and $E$. It can be defined by:

$$
P(E' \rightarrow E).E'(1 - \alpha) = 1
$$

(36)

Hence:

$$
P(E' \rightarrow E) = \frac{1}{(1 - \alpha)E'}
$$

(37)

Now:

$$
\sum_{S}(E' \rightarrow E) = \frac{\sum_{S}(E')}{(1 - \alpha)E'}
$$

(38)

For $M = 1$ the collision is defined as:

$$
F(E) = \int_{E}^{E_0} \sum_{S}(E') \frac{\phi(E')}{E'} dE' + \delta(E)
$$

(39)

Also the solution with capture process:

$$
F_c(E) = \frac{\sum_{S}(E_0) S_0}{\sum_{S}(E_0) \phi(E_0)} - \int_{E}^{E_0} \frac{\sum_{S}(E')}{\sum_{S}(E')} dE' + \delta(E)
$$

(40)

Also For $M \neq 1$ the collision density can be found:

$$
F(E) = \int_{E/\alpha}^{E} \frac{\sum_{S}(E')}{(1 - \alpha)E'} \phi(E') dE' \frac{\delta(E)}{(1 - \alpha)E_0} + \frac{\delta(E)}{(1 - \alpha)E_0}
$$

(41)

This only applicable if $\alpha E_0 < E < E_0$. A theoretical calculation is performed for an arbitrary system and Fig 3 is derived. For graphite the collision density is also measured for different neutron energy range as demonstrated by Fig. 4.

and for graphite:

The oscillations are due to Plaezack Oscillations which is the fundamental phenomenon associated with the neutron slowing-down [2]. And finally for the $M \neq 1$ with capture:

$$
F(E) = (\sum_{S}(E) + \sum_{\alpha}(E))\phi(E)
$$

$$
= \left(\int_{E/\alpha}^{E} \frac{\sum_{S}(E')}{(1 - \alpha)E'} \phi(E') dE' + \frac{S_0}{(1 - \alpha)E_0}\right)
$$

$$
= \int_{E/\alpha}^{E} \frac{\sum_{S}(E')}{\sum_{S}(E')} \frac{F(E')}{(1 - \alpha)E'} dE' + \frac{S_0}{(1 - \alpha)E_0}
$$

(42)
3. Model Set-up in MCNP

The MCNP code is developed in Los Alamos National Laboratory and it is well-known for analysing the transport of neutron and γ-rays in matter. MCNP is a continuous energy modeller with generalized geometry time dependent code that implements data from nuclear libraries such as, Evaluated Nuclear Data File (ENDF), Evaluated Nuclear Data Library (ENDL), Activation Library (ACTL).

The code structure is divided into four main sections, geometry definitions, surface definitions, material cards, and tallies. Geometry of MCNP is a three-dimensional form defined using cell and surface cards. For instance, Fig. 5 demonstrates the geometry setup in this system. Figure 5 illustrates the reactor moderator, where each cylinder represents the fuel rod containing slightly enriched $^{235}$U. The moderator is reactor grade graphite.

The user can instruct the code to make various analysis with tally cards. The tallies are to measure the particle current on the surface, particle flux and energy deposition. In fact any
Figure 5. The MCNP Geometry Set up: Figure demonstrates the reactor moderator module. The dimension was set 2m x 2m.

Quantity in form of Eq. 43 can be tallied \[ C = \int \phi(E) f(E) dE \] (43).

Here \( \phi(E) \) represent the particle flux and \( f(E) \) is the cross-section quantities given in the libraries. Table 1 demonstrates the six MCNP standard tallies.

In MCNP when neutron collides with a nucleus: the nuclide will be identified depends on the preferences of target, that is either the \( S(\alpha, \beta) \) treatment or velocity of target; therefore the nucleus will be sampled for low energy neutrons; neutron capture or absorption will be modelled.

| Property     | Data                                      |
|--------------|-------------------------------------------|
| F1:N         | Surface Current                           |
| F2:N         | Surface Flux                              |
| F4:N         | Track Length Estimate of Cell Flux        |
| F5a:N        | Flux at a Point                            |
| F6:N         | Track Length Estimate of Energy Dependence |
| F7:N         | Track Length Estimate of Fission Energy Dependence |
and either elastic or inelastic reaction depend on the model performance. However sometimes different nuclide form a material, (where the collision occurs) therefore we can have:

\[
\sum_{i=1}^{k-1} \sum_{i=1}^{n} \sum_{i=1}^{n} < \xi \sum_{i=1}^{k} \sum_{i=1}^{n} (44)
\]

where \( \sum_{i=1}^{n} \) is the microscopic total cross-section of nuclide \( i \). The total cross-section is sum of the capture cross-sections in the cross-section reference table.

The collision between thermal neutrons and the target will be effected by thermal motion of the atoms, chemical binding and lattice structure of the target. This is called Free Gas Thermal Treatment. Hence the effective scattering cross-section in laboratory system is given by \([4]\):

\[
\sigma_{eff}^{s}(E) = \frac{1}{v_n} \int \int \sigma_{s}(v_{ref})v_{rel}P(v)dv \frac{d\varphi}{2} (45)
\]

Here \( v_n \) is particle velocity, \( v_{rel} \) is the relative velocity, \( P(v) \) is the probability density function and \( \varphi \) as explained before is the cosine angle of velocity vector. The relative velocity can therefore is given by:

\[
v_{rel} = (v_n^2 + v^2 - 2v_nv_{\varphi})^{1/2} (46)
\]

The density function is also given by:

\[
P(v) = \frac{4}{\pi^{1/2}} \beta^2 v^2 e^{-\beta^2 v^2} (47)
\]

where \( \beta = (\frac{AM_n}{2kT})^{1/2} \). However most of the time in equation 45 the \( \sigma_s \) can be ignored for heavy nuclei, where \( \sigma_{rel} \) can have moderating effect and is given by \([4]\):

\[
P(v, \varphi) \propto \sqrt{v^2 + v^2 - 2v_nv_{\varphi}}v^2 e^{-\beta^2 v^2} (48)
\]

In MCNP there are also two types of capture, analogue and implicit. Analogue occurs when the particle is killed with probability of \( \frac{\sigma_a}{\sigma_t} \). Where \( \sigma_a \) and \( \sigma_t \) is the absorption and total cross-section respectively. Implicit capture happens when neutron weight \( (W_n) \) is reduced by number of collisions and is given by:

\[
W_{n+1} = (1 - \frac{\sigma_a}{\sigma_t})W_n (49)
\]

The elastic scattering directed by two body kinematics:

\[
E_{out} = \frac{1}{2}E_{in}((1 - \alpha)\Theta_c m + 1 + \alpha) (50)
\]

Where \( \Theta_c m \) is the center of mass cosine of angle between incident and existing path direction. Where in inelastic an scatter the particle reaction is chosen such as \( (n,n') \), \( (n,2n) \), \( (n,f) \), and \( (n,n'\alpha) \), and is given by \([4]\):

\[
E' = E'_{cm} + \frac{E + 2\Theta(A + 1)\sqrt{EE'_{cm}}}{10 (A + 1)^2} (51)
\]
\[ \varphi_{\text{lab}} = \Theta_{\text{cm}} \sqrt{\frac{E'_{\text{cm}}}{E}} + \frac{1}{A+1} \sqrt{\frac{E}{E'}} \]  

(52)

Here \( \varphi_{\text{lab}} \) is cosine of laboratory scattering angle. However for thermal energy neutron, \( S(\alpha, \beta) \) treatment is needed. For inelastic treatment the secondary particle distribution will be represented by set of discrete energies between \( 4eV \) to \( 10^{-5}eV \).

4. Model Set-up in COMSOL

The Partial Differential Equation (PDE) module of COMSOL package supports three types of formation: coefficient form, general form, weak form. The coefficient form is a linear system where as the general and weak form supports non-linear, and more flexible form of definitions is supported by weak form. In this report one study is performed for thermal group transport using equation based general form of the system.

In equation based system the independent variable \( u_1, u_2 \) will be defined in following equation:

\[
e_a \frac{\delta^2 u}{\delta t^2} + d_a \frac{\delta u}{\delta t} - \nabla \cdot (c \nabla u + \alpha u - \gamma) + \beta \nabla u + au = f
\]  

(53)

Where \( e_a \) is the mass matrix, and \( e_a \frac{\delta^2 u}{\delta t^2} \) is called mass term. \( d_a \frac{\delta u}{\delta t} \) is called damping term, \( \nabla \cdot (c \nabla u + \alpha u - \gamma) \) is called diffusive flux, \( \beta \) is convection flux, \( a \) is the absorption coefficient, and \( f \) is the source term.

Environmental factors are defined by enforcing boundary conditions using Dirichlet equation. Dirichlet imposes Laplace equation (our transport equation) to the system domain. It is therefore more convenient to have the numerical Laplace such that:

\[
\nabla = \frac{\delta^2 U}{\delta x^2} + \frac{\delta U}{\delta y^2}
\]  

(54)

Now that Laplace equation is defined we need to numerically define the flux and multiply the two values, hence:

\[
\nabla \cdot (-c \nabla u - \alpha u + \gamma)
\]  

(55)

Equation 55 is called flux vector. Here \( \alpha \) is the velocity term, \( \gamma \) is the source term. \( c \) can be also indirectly calculated for an anisotropic material.

Equation 55 is in computational domain \( (\omega) \), thus the calculations need to satisfy all conditions in boundary domain. This is called Neumann-Dirichlet where the boundary will be transformed from \( \omega \) to \( d\omega \) (from computational boundary to domain boundary). This transformation is also described as domain decomposition preconditioner [5]. Thus the partial differential equation is given by \( \nabla^2 u + u = 0 \) where \( \nabla \) donated as Laplacian therefore we will have:

\[
\frac{du}{dn}(x) = \nabla^2 u(x).n(x)
\]  

(56)

Where \( n \) refers to a normal vector, thus we can rewrite the equation 55 as:

\[
n.(c \nabla u + \alpha u - \gamma) = g - h_N
\]  

(57)
where \( g \) and \( h^t_N \) donate the boundary source term and the Lagrange multiplier factor. \( h^t_N \) is needed in a mixed field situation as it corresponds to local maxima and minima. In some respect \( h^t_N \) can also refers to the velocity [6].

Now by taking the energy dependent diffusion equations we have:

\[
\frac{1}{v} \frac{\delta \phi}{\delta t} \cdot \nabla D \cdot \nabla \phi + \sum_t \phi = \int_0^\infty \sum_s (E \rightarrow E') \phi(E')dE' + \chi(E) \int \bar{v}(E') \sum_f (E') \phi(E)d(\bar{E}') + S(r, E, t)
\]

(58)

Where in multi-group theory discrete energies varies with \( G \) discrete group as:

| Group ‘1’ | Group ‘2’ | Group ‘3’ |
|-----------|-----------|-----------|
| Thermal   | Epithermal| Fast      |
| \( E_1 \) | \( E_2 \)  | \( E_3 \) |

**Figure 6.** Discrete Group Relation, Thermal, Epithermal, Fast Region

The group flux can be obtained by integrating total fluxes across the group energy range. Hence the parameters can be defined as below:

I. Total Cross Section:

\[
\sum_t \phi_g = \int_{E_g}^{E_g-1} \sum_t (E) \phi(E)dE = \int_{E_g}^{E_g-1} \sum_t (E) \phi(E) \frac{dE}{\phi_g}
\]

(59)

II. Diffusion Length:

\[
D_g = \frac{\int_{E_g}^{E_g-1} E_g D(E) \nabla \phi(E)dE}{\int_{E_g}^{E_g-1} E_g \nabla \phi(E)dE}
\]

(60)

III. Inverse Velocity:

\[
\frac{1}{v_g} = \frac{\int_{E_g}^{E_g-1} E_g v_{g[E]} \frac{1}{\phi(E)}dE}{\phi_g}
\]

(61)

IV. Fissile Spectrum Term

\[
\chi_g = \int_{E_g}^{E_g-1} E_g \chi(E)dE
\]

(62)

Therefore the stationary solution for many group equation can be given by(in this work the equation is only solved for thermal group spectrum):

\[
\frac{1}{v_g} \frac{\delta \phi_g}{\delta t} \cdot \nabla D_g \cdot \nabla \phi_g + \sum_t \phi_g = \sum_{g'} \sum_s g'^{-\rightarrow g} \phi_{g'}' + \chi_g \sum_{g'} \frac{(G)\bar{v}_{g'} \sum_f \phi_{g'} + S_g}{g'}
\]

(63)
Where the right and left hand side of the equation present the loss and production term respectively, $v$ is the average neutron speed $\chi_g$ is the fraction of prompt neutrons. For the simulation the Arbitrary Lagrangian Eulerian (ALE) mapping mesh analysis is used.

5. Discussion and Results
This report has reviewed the neutron diffusion length both using COMSOL Multiphysics and MCNP. The total number of 5,000,000 meshes used for the iteration process in COMSOL. Both the thermal neutron flux and absorption property of graphite with respect to its cross-section features have been evaluated. The thermal diffusion length therefore calculated was $50.85 \pm 0.3cm$ in COMSOL and $50.95 \pm 0.5cm$ in MCNP. Figure 7 demonstrates the distribution of thermal neutron increases as they penetrate deeper into graphite compared both in MCNP and COMSOL. The red line in the figure is also illustrates the experiment done in the lab on graphite using $Am – Be$ source. The $Am – Be$ was canned on top of an aluminium cylindrical tube. Two set-up is used in this experiment, a cadmium cover with nominal thickness of 0.1 cm (As explained previously the cadmium has cut-off of 0.55eV) and were constructed to fully overlap the detector edges to avoid leakages. The flux distribution is measured by putting the source at a fixed location and relocating the detector at 25 cm distance intervals in horizontal and vertical directions.
Figure 7. Figure demonstrates the neutron diffusion length using COMSOL and MCNP, the circles illustrate the COMSOL results whereas the stars demonstrate MCNP. The red line as well shows the experiment was done using BF3 tube.
As shown in figure 7 the distribution calculated using COMSOL is less than half order of magnitude higher than MCNP. For completion the absorption probability cross-section in FEA is also evaluated using inverse iteration technique as demonstrated by figure 9. It is clearly shown as the neutron travels deeper into graphite they probability of absorption in graphite is also increases.

To understand the respond of absorption cross-section to different thermal neutron energies, the evaluated values are compared with with [7], [8], [9], [10], [11], [12] and [13] as demonstrated in figure 9.

**Figure 8.** The Total Neutron Absorption in Graphite: Derived Using COMSOL. It shows the probability of absorption increases as the neutrons travel deeper in to the graphite, the areas red shows the highest and blue the lowest.

6. Acknowledgement
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Figure 9. The Total Neutron Absorption Cross-section in Graphite Compared with [7], [8], [9], [10], [11], [12] and [13]. Fittings are extracted by permission from [14].

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