Exceptional Coset Spaces and Unification in Six Dimensions

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Abstract

The coset spaces $E_8/\text{SO}(10) \times H_F$ allow complex structures which can account for three quark-lepton generations including right-handed neutrinos. We show that in the context of supersymmetric SO(10) gauge theories in 6 dimensions they also provide the Higgs fields which are needed to break the electroweak and $B - L$ gauge symmetries, and to generate small neutrino masses via the seesaw mechanism.
The standard model gauge groups of electroweak and strong interactions are naturally unified in the simple gauge group SU(5) \[1\]. With the increasing experimental evidence for neutrino masses and mixings the larger gauge group SO(10) \[2\] with the Pati-Salam subgroup SU(4) \times SU(2) \times SU(2) \[3\] appears particularly attractive, since all quarks and leptons of a single generation, including the right-handed neutrino, are then also unified in a single multiplet.

The unification groups SU(5) = E_6, SO(10) = E_7 and E_8 \[4\] belong to the sequence of exceptional groups E_n which terminates at E_8. This largest exceptional group is attractive for unification \[5, 6\] since its smallest multiplet, the 248-dimensional adjoint representation, is large enough to accommodate all three generations of quarks and leptons. The theory is naturally supersymmetric. However, in addition to the three known quark-lepton generations the theory also predicts three light mirror generations, contrary to observation.

The group \( E_8 \) also appears in the ten-dimensional Yang-Mills supergravity theory where a cancellation of all gauge and gravitational anomalies is obtained by means of the Green-Schwarz mechanism for the group \( E_8 \times E_8 \) \[7\]. Compactification to four dimensions \[8, 9\] can yield low energy effective theories with unbroken \( N = 1 \) supersymmetry and chiral fermions, similar to the structure of the standard model. The number of families is determined by the Euler characteristic of the compact manifold.

Further, the group \( E_8 \) has been considered in attempts to relate quarks and leptons to a coset space \( G/H \), where \( G \) is an appropriate simple group and \( H \) contains the standard model gauge group \[10\]. By pairing the scalar degrees of freedom of \( G/H \) into complex fields, which become the superpartner of quarks and leptons, the problem of mirror families can be avoided \[11\]. Particularly attractive are coset spaces \( E_8/(SO(10) \times H_F) \) where \( H_F \) is a subgroup of \( SU(3) \times U(1) \) \[11\]-\[14\]. In the case \( H_F = SU(3) \times U(1) \) the representation of chiral multiplets is unique,

\[
\Omega = (16, 3)_1 + (16^*, 1)_3 + (10, 3^*)_2 + (1, 3)_4 .
\]

Hence, in addition to three quark-lepton generations contained in the three 16’s of SO(10), one mirror generation, 16*, occurs, again in contrast to observation. For \( H_F = SU(2) \times U(1)^2 \) and \( H_F = U(1)^3 \) various complex structures are possible \[13\], which contain either three 16’s and one 16*, or two 16’s and two 16*’s in addition to three 10’s and SO(10) singlets.

Orbifold compactifications \[9\] have recently been applied to GUT field theories. The breaking of the GUT symmetry then automatically yields the required doublet-triplet splitting of Higgs fields \[15\]. Several SU(5) models have been constructed in 5 dimensions \[14\]-\[18\], whereas 6 dimensions are required for the breaking of SO(10) \[13, 20\]. The \( N=2 \)
supersymmetric SO(10) theory in 6 dimensions can also be used as a starting point to obtain a SU(5) GUT with three generations [21].

In the following we want to point out that the complex structure (1), when combined with the orbifold compactification of the 6d SO(10) theory, yields in a simple way the supersymmetric standard model with right-handed neutrinos. The three 16’s, which contain quarks and leptons, are brane fields; the 16* and the three 10’s are bulk fields. Vacuum expectation values of bulk fields break the symmetries SU(2) × U(1)Y and U(1)X, which are left unbroken by the orbifold compactification.

Consider the SO(10) gauge theory in 6d with N=1 supersymmetry. The gauge fields $V_M(x,y,z)$, with $M = \mu, 5, 6, x^5 = y, x^6 = z$, and the gauginos $\lambda_1, \lambda_2$ are conveniently grouped into vector and chiral multiplets of the unbroken N=1 supersymmetry in 4d,

$$V = (V_\mu, \lambda_1) , \quad \Sigma = (V_{5,6}, \lambda_2) .$$

Here $V$ and $\Sigma$ are matrices in the adjoint representation of SO(10). Symmetry breaking is achieved by compactification on the orbifold $T^2/(\mathbb{Z}_2^I \times \mathbb{Z}_2^{PS} \times \mathbb{Z}_2^{GG})$. The discrete symmetries $\mathbb{Z}_2$ break the extended supersymmetry in 4d; they also break the SO(10) gauge group down to the subgroups $SO(10), G_{PS} = SU(4) \times SU(2) \times SU(2)$ and $G_{GG} = SU(5) \times U(1)_X$, respectively, at three different fixpoints.
\[ P_I V(x, -y, -z) P^{-1}_I = \eta_I V(x, y, z) , \]
\[ P_{PS} V(x, -y + \pi R_5/2, -z) P^{-1}_{PS} = \eta_{PS} V(x, y + \pi R_5/2, z) , \]
\[ P_{GG} V(x, -y, -z + \pi R_6/2) P^{-1}_{GG} = \eta_{GG} V(x, y, z + \pi R_6/2) . \]

Here \( P_I = I \), the matrices \( P_{PS} \) and \( P_{GG} \) are given in ref. [19], and the parities are chosen as \( \eta_I = \eta_{PS} = \eta_{GG} = +1 \). The extended supersymmetry is broken by choosing in the corresponding equations for \( \Sigma \) all parities \( \eta_i = -1 \). At the fixpoints in the extra dimensions, \( O = (0, 0) \), \( O_{PS} = (\pi R_5/2, 0) \) and \( O_{GG} = (0, \pi R_6/2) \) the unbroken subgroups are \( \text{SO}(10) \), \( G_{PS} \) and \( G_{GG} \), respectively. In addition, there is a fourth fixpoint at \( O_{fl} = (\pi R_5/2, \pi R_6/2) \) [21], which is obtained by combining the three discrete symmetries \( Z_2 \), \( Z_2^{PS} \) and \( Z_2^{GG} \) defined above,

\[ P_{fl} V(x, -y + \pi R_5/2, -z + \pi R_6/2) P^{-1}_{fl} = + V(x, y + \pi R_5/2, z + \pi R_6/2) . \]

The unbroken subgroup at the fixpoint \( O_{fl} \) is flipped \( \text{SU}(5) \), i.e. \( G_{fl} = \text{SU}(5)' \times \text{U}(1)' \).

The physical region is obtained by folding the shaded regions in fig. 1 along the dotted line and gluing the edges. The result is a ‘pillow’ with the four fixpoints as corners. The unbroken gauge group of the effective 4d theory is given by the intersection of the \( \text{SO}(10) \) subgroups at the fixpoints. In this way one obtains the standard model group with an additional \( \text{U}(1) \) factor, \( G_{SM} = \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)_Y \times \text{U}(1)_X \). The difference of baryon and lepton number is the linear combination \( B - L = \sqrt{\frac{16}{15}} Y - \sqrt{\frac{5}{3}} X \). The zero modes of the vector multiplet \( V \) form the gauge fields of \( G_{SM} \).

The vector multiplet \( V \) is a \( 45 \)-plet of \( \text{SO}(10) \) which has an irreducible anomaly in 6 dimensions. It is related to the irreducible anomalies of hypermultiplets in the fundamental and the spinor representations by (cf. [23]),

\[ a(45) = -2a(10) , \quad a(16) = a(16^*) = -a(10) . \]

Hence, the anomaly of the vector multiplet can be cancelled by adding two \( 10 \) hypermultiplets, \( H_1 \) and \( H_2 \). The cancellation of the reducible anomalies can be achieved by means of the Green-Schwarz mechanism [7].

For these hypermultiplets we have to define parities with respect to the discrete symmetries,

\[ P_I H(x, -y, -z) = \eta_I H(x, y, z) , \]
\[ P_{PS} H(x, -y + \pi R_5/2, -z) = \eta_{PS} H(x, y + \pi R_5/2, z) , \]
\[ P_{GG} H(x, -y, -z + \pi R_6/2) = \eta_{GG} H(x, y, z + \pi R_6/2) . \]
Table 1: Parity assignment for the bulk 10 hypermultiplets. $H_1^c = H_d$ and $H_2 = H_u$.

with $\eta_i = \pm 1 \ (i = I, PS, GG)$. All hypermultiplets split under the extended 6d supersymmetry into two N=1 4d chiral multiplets, $H = (H, H')$. They have the 6d superpotential interactions [24],

$$S_6 = \int d^6x \left( \int d^2\Theta H'(\partial + \sqrt{2}g\Sigma)H + h.c. \right),$$

where $\partial = \partial_5 - i\partial_6$ and $g$ is the 6d gauge coupling. Invariance of the action then requires that the parities of $H$ and $H'$ are opposite. In the following we denote by $\eta_i$ the parities of the first 4d chiral multiplet, and we choose $\eta_I = +1$.

The discrete symmetry $Z_{PS}$ implies automatically a splitting between the SU(2) doublets and the SU(3) triplets contained in the 10-plets. The choice $\eta_{PS} = +1$ leads to massless SU(2) doublets and massive colour triplets (cf. table 1). Choosing further $\eta_{GG} = +1$ for $H_1$ and $\eta_{GG} = -1$ for $H_2$, selects the doublet $H^c$ from the SU(5) 5*-plet contained in $H_1$, and the doublet $H$ from the SU(5) 5-plet of $H_2$ (cf. table 1). The doublets $H^c$ and $H$ have the quantum numbers of the Higgs fields $H_d$ and $H_u$ in the supersymmetric standard model.

Quarks and leptons can be incorporated by adding 16-plets, additional 10-plets etc. in the bulk and on the fixpoints. Without any constraint on the multiplicity of these fields there are many possibilities [21, 22]. It is remarkable that for the complex structure $\Omega$ given in eq. (1), the requirement of SO(10) anomaly cancellations in the bulk determines the distribution of multiplets uniquely. As already mentioned, two 10’s are required to cancel the anomaly of the 45 vector multiplet. Hence, the (10,3*) have to be bulk fields. The anomaly of the third 10-plet can only be cancelled by the (16*,1), which leaves (16,3) as brane fields. Note, that in general a bulk field contains two 4-dimensional $N = 1$ chiral multiplets which transform as complex conjugates of each other with respect to the gauge group. Hence, in our SO(10) model in 6 dimensions the content of
We now have to choose the parities for $H_3$, the third 10-plet, and for $\Phi^c$, the $16^*$-plet. The presence of $\Phi^c$ offers the possibility to break $U(1)_{B-L}$ spontaneously by a vacuum expectation value of its SU(5) singlet component $N$ (table 2). To have $N$ as zero mode fixes the parities of $\Phi^c$ to be $\eta_{PS} = \eta_{GG} = -1$. Then an additional colour triplet, $D$, appears as zero mode. As we shall see, $D$ can acquire a Dirac mass together with another colour triplet, $G^c$, which can be chosen as zero mode of the third 10-plet. The corresponding parities of $H_3$ are $\eta_{PS} = -1$, $\eta_{GG} = +1$. The parities of all components of $H_i$ and $\Phi^c$ are listed in tables 1 and 2.

Knowing the parities of all fields we can now discuss the superpotential. We consider the three brane fields $16$, $\psi_i$, on the SO(10) symmetric fixpoint $O$. To restrict the number of terms we require R-invariance and an additional global $U(1)_{\tilde{X}}$ symmetry (cf. table 3). The most general superpotential up to quartic terms is then given by,

$$ W_4 = h_d \psi \psi H_1 + h_u \psi \psi H_2 + \frac{h_N}{M_*} \psi \psi \Phi^c \Phi^c + M_1 H_1 H_3 + M_2 H_2 H_3 + \lambda_1 \Phi^c \Phi^c H_3 , $$

where we choose $M_* > 1/R_{5,6}$ to be the cutoff of the 6d theory, and the bulk fields have been properly normalised. The first three terms are familiar from ordinary SO(10) GUTs whereas the last three terms are additional couplings among bulk fields.

It is instructive to consider in eq. (12) just the zero mode interactions and their

| $H_1$ | $H_2$ | $\psi_i$ | $\Phi^c$ | $H_3$ |
|-------|-------|----------|----------|-------|
| 0     | 0     | 1        | 0        | 2     |

Table 3: Charge assignments for the symmetries $U(1)_R$ and $U(1)_{\tilde{X}}$.  

4d chiral multiplets is larger than the complex structure (1).

| SO(10) | 16* |
|--------|-----|
| $G_{PS}$ | $(4^*, 2, 1)$ | $(4^*, 2, 1)$ | $(4, 1, 2)$ | $(4, 1, 2)$ |
| $G_{GG}$ | $10^*+1$ | $5_{-3}$ | $10^*+1$ | $5_{-3}, 1_{+5}$ |
| $Q^c$ | $Z_{2}^{PS}$ | $Z_{2}^{GG}$ | $L^c$ | $Z_{2}^{PS}$ | $Z_{2}^{GG}$ | $U^c, E^c$ | $Z_{2}^{PS}$ | $Z_{2}^{GG}$ | $D, N$ |
| $\Phi^c$ | $-$ | $-$ | $-$ | $+$ | $+$ | $-$ | $+$ | $+$ |

Table 2: Parity assignment for the bulk $16^*$ hypermultiplet.
couplings to a single heavy field in the bilinear and cubic terms. In standard notation, with \( \psi = (q, u^c, e^c, l, d^c, n^c) \), one obtains,

\[
W_4 = h_d (q d^c + l e^c) H_1^c + h_u (q u^c + l n^c) H_2^c \\
+ \frac{h_N}{M_\ast} (n^c N)^2 + \frac{h_N^2}{M_\ast} (d^c D)^2 + \frac{h_N^2}{M_\ast} d^c D n^c N \\
+ \lambda_1 D N G_3^c + M_1 H_1^c H_3 + M_2 H_2 H_3^c \\
+ h_d (q u^c + l n^c) H_1 + h_d (q q + u^c e^c + d^c n^c) G_1 + h_d (q l + u^c d^c) G_1^c \\
+ h_u (q d^c + l e^c) H_2^c + h_u (q q + u^c e^c + d^c n^c) G_2 + h_u (q l + u^c d^c) G_2^c ; \quad (13)
\]

here the three couplings \( h_N, h_N^c \) and \( h_N^c \) correspond to the three SO(10) invariants which can be formed from \( \psi \psi \Phi^c \Phi^c \).

Vacuum expectation values \( v_1 = \langle H_1^c \rangle, v_2 = \langle H_2 \rangle \) and \( v_N = \langle N \rangle \) yield the mass terms,

\[
W_m = m_u u u^c + m_\nu n n^c + m_d d d^c + m_e e e^c + m_N n n^c , \quad (14)
\]

with mass matrices \( m_u = m_\nu = h_u v_2, m_d = m_e = h_d v_1 \) and \( m_N = h_N v_N^2 / M_\ast \). With \( v_{1,2} \ll v_N \) this gives, to first approximation, a good description of the observed properties of quarks and leptons. Since there are two Higgs doublets, \( H_1 \) and \( H_2 \), the unwanted relation of minimal SO(10) models, \( m_u = m_d \), is avoided. The evidence for small neutrino masses, together with the seesaw mechanism \[27\], suggests that U(1)\(_{B-L}\) is broken near the unification scale, i.e. \( v_N \sim 1/R_{5,6} \), which implies large Majorana masses \[26\]. This also leads to a large mass term, \( m_T D G_3^c \), for the colour triplet zero modes of \( \Phi^c \) and \( H_3 \), with \( m_T = \lambda_1 v_N \). The vacuum expectation value \( \langle N \rangle = v_N \) breaks U(1)\(_X\) × U(1)\(_{X}\) to the diagonal global subgroup, leaving U(1)\(_R\) unbroken. As a consequence, integrating out the heavy fields \( G_{1,2}, G_{1,2}^c \) does not lead to a dimension-5 operator for proton decay. For the same reason, integrating out \( H_3 \) and \( H_3^c \) does not generate a \( \mu \)-term \( H_3^c H_2 \). Finally, the electroweak scale \( v_{1,2} \) may be induced together with the \( \mu \)-term by supersymmetry breaking. The resulting low energy effective theory is just the supersymmetric standard model with right-handed neutrinos.

What determines the vacuum expectation value \( \langle N \rangle \) which breaks U(1)\(_X\) and therefore U(1)\(_{B-L}\)? Note, that these U(1) symmetries are anomalous, since the zero modes of \( \Phi^c \) and \( H_3 \) lead to 4d anomalies SU(3)\(^2\)×U(1)\(_X\) and U(1)\(^3\)\(_X\). In general, these can be compensated by a Chern-Simons term\[\footnote{For recent discussions in 5-dimensional theories and references, see \[28, 29\].}]\. One may then hope to break U(1)\(_X\) spontaneously by means of a Fayet-Iliopoulos term on the Georgi-Glashow fixpoint \( O_{GG} \). This will be discussed in more detail elsewhere \[27\].

Alternatively, one can avoid the occurrence of anomalies by ‘partial doubling’, familiar from supersymmetric \( \sigma \)-models where one associates an entire chiral multiplet with
Table 4: Parity assignment for the bulk 10 and 16 hypermultiplets.

some degrees of freedom of the coset space $G/H$ (cf. [30]). Let us then add two bulk fields, a 16-plet, $\Phi$, and a fourth 10-plet, $H_4$, which have no irreducible 6d anomaly. The choice of parities $\eta_{PS} = -1$ and $\eta_{GG} = -1$ for both fields leads to the zero modes $D^c, N^c$ and $G$ (cf. table 4). The zero modes of all bulk fields now form a real, and therefore anomaly free, representation of $G_{SM'}$. The additional superpotential terms on the SO(10) brane are,

$$W' = M^2 S + \lambda_2 S \Phi \Phi^c + \lambda_3 \Phi H_4$$

$$+ \frac{\lambda_4}{M_*} S \Phi^c H_1 + \frac{\lambda_5}{M_*} S \Phi H_2 + \frac{\lambda_6}{M_*} \Phi^c H_1 H_3 + \frac{\lambda_7}{M_*} \Phi \Phi^c H_2 H_3,$$

where we have also included one of the SO(10) singlets, $S$. The $R$- and $\tilde{X}$-charges of the additional fields are $R_\Phi = 0, R_{H_4} = R_S = 2, \tilde{X}_\Phi = a, \tilde{X}_{H_4} = -2a, \tilde{X}_S = 0$. Without the singlet field $S$, there is a bulk D- and F-flat direction, $\langle N \rangle = \langle N^c \rangle = v_N$ [27]. The couplings of $S$ on the brane lift this degeneracy, and one has $v_N = \langle N \rangle = \langle N^c \rangle = \sqrt{-M^2/\lambda_2}$. The cubic term in (15) gives mass to two zero modes $G$ and $D^c$, $N - N^c$ makes the U(1)$_X$ vector multiplet massive, and $N + N^c$ has a common mass term with $S$. Hence, one obtains the same low energy theory as in the anomalous model discussed above.

We have shown that complex structures of coset spaces $E_8/\text{SO}(10) \times H_F$ can provide the starting point of a full grand unified model in the context of supersymmetric SO(10)
theories in 6 dimensions with orbifold compactification. Open questions concern the
breaking of supersymmetry, the origin of the brane superpotential and, in particular,
the possible connection to theories in 10 dimensions with \( E_8 \) symmetry.

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