Lorentzian’ analysis of the accuracy of modern catalogues of stellar positions

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Abstract. There is a new approach for the estimation of the position accuracy and proper motions of the stars in astrometric catalogues by comparison of the stars’ positions in the researched and Hipparcos catalogues in different periods, but under a standard equinox. To verify this method was carried out the analysis of the star positions and proper motions UCAC2, PPM, ACRS, Tycho-2, ACT, TRC, FON and Tycho catalogues. As a result of this study was obtained that the accuracy of positions and proper motions of the stars in Tycho-2 and UCAC2 catalogues are approximately equal. The results of the comparison are represented graphically.

1. Introduction

The Hipparcos catalogue provides highly accurate data on the positions (about 1 mas) and proper motions (about 1 mas/yr) of the stars [1]. According to the IAU resolution, this catalogue is accepted as the astrometrical standard. Therefore Hipparcos is used for comparison to estimate the accuracy of other astrometrical catalogues.

This work offers a new graphic method for estimation of the position accuracy and proper motions of the stars in astrometrical catalogues. This method is based on comparison of the stars’ positions in different periods and \(J.2000.0\) equinox with their positions as given in the Hipparcos catalogue in the same periods and \(J.2000.0\) equinox. The number \(n\) of the absolute value of coordinates’ differences is more than the given value \(\Delta\) concerning the general number \(N\) of stars, is determined for each period in the researched and Hipparcos catalogues and as a percentage. The sum of the absolute values of coordinates’ differences is taken as 100%, which is equal to the general number of stars compared in both catalogues. The years for which the differences of coordinates are calculated are put on the abscissa axis and the percentages are put on the \(y\)-axis.

The use of a new method can be justified by the expediency of comprehensive research of the catalogues’ accuracy and by the use of a visual representation of the results obtained by the given method. The method is illustrated by examples of research on the PPM [2, 3], ACRS [4], Tycho-2 [5], ACT [6], TRC [7], FON [8], Tycho [1], UCAC2 [9] catalogues.

2. Graphic method of comparison of positions’ accuracy and proper motions of stars in astrometrical catalogues

The epoch of the modern PPM and Tycho-2 catalogues is \(T(0) = J.2000.0\) and for the Hipparcos and Tychocatalogues is \(T(0) = J.1991.25\). Obviously, for an epoch \(T(j)\) the coordinates of stars of the Hipparcos catalogue will be

\[
\alpha(H) = \alpha([1991.25]/1991.25) + \mu \alpha \times (T(j) - 1991.25) \quad \text{and} \quad \delta(H) = \delta([1991.25]/1991.25) + \mu \delta \times (T(j) - 1991.25),
\]

and for the researched catalogue

\[
\alpha(cat) = \alpha([T(0)]) + \mu \alpha \times (T(j) - T(0)) \quad \text{and} \quad \delta(H) = \delta([1991.25]/1991.25) + \mu \delta \times (T(j) - 1991.25).\]
Take the absolute values of these differences and sum up both $\alpha$ and $\delta$:

$$\Delta' = |\Delta(\alpha)| \times \cos(\delta) + |\Delta(\delta)|.$$  

At the same time, reject those differences whose absolute values are less than a given value, for example $\Delta = 0.2''$. This value is selected taking into account the positions accuracy of stars in the ground catalogues. Consider only those differences which are more than $\Delta$. Let the number of such differences equal $n$. Divide $n$ by $N$ and then centuple, i.e., to evaluate the number of differences of large $\Delta$ in percent. Denote this as $L(j)$. Perform such calculations for a number of periods (usually more than ten). Further, take $L(j)$ as a function from $T(j)$. Construct the plot of dependence of $L(j)$ on $T(j)$. Curves $L(j)$ are well described by the Lorential function. This is used in astrophysics for the approximation of photometric structures of spectral lines. It has the analytical expression

$$L(x) = \Delta(x)/[(x-x_0)^2 + \{\Delta(x/2)\}^2],$$

where $x_0$ is the abscissa of the minimum of a Lorential, and $\Delta(x)$ is an average width of a structure Lorential, similar to the equivalent width of the photometric profile of a spectral line.

3. Analysis of modern catalogues

In figure 1, the schedules of dependence of $L(j)$ on $T(j)$ for the PPM catalogue are indicated at $\Delta = 0.2''$ separately for the northern zone on the declination PPM($n$) and the southern zone PPM($s$). It is known that the observations of the northern and southern stars PPM were made in different years.

![Figure 1](image)

**Figure 1.** The dependencies of $L(j)$ on $T(j)$ for the PPM catalogue separately for the northern zone on the declination PPM($n$) and the southern zone PPM($s$).

This is clearly illustrated in figure 1. Thus, the abscissa of a minimum curve $L(j)$ gives information about the epoch of the observations of the researched catalogues. It is obvious that the ordinate of minimum $L(j)$ gives information about the accuracy of the positions of stars in the researched catalogue. The lower the ordinate at a given $\Delta$, the more precise the stars’ positions are in
the catalogue. PPM(\(n\)) has a minimum at 30.6\%, and PPM(\(s\)) at 18.6\%. So the stars’ positions in PPM(\(s\)) are more exact than in PPM(\(n\)).

Figure 2 presents curves \(L(j)\) for the Tycho-2, ACT, TRC, FON and Tycho catalogues at \(\Delta=0.2''\). It is evident from a comparison of curves for the Tycho and Tycho-2 catalogues that Tycho-2 (0.4\%) has more exact star positions than Tycho (48.8\%). The accuracy of the ACT and TRC catalogues is almost identical at 1.48\% and 1.87\%. At 31.7\%, the accuracy of the FON catalogue is worse than ACT and TRC, but greater than the Tycho catalogues. The steepness of curve \(L(j)\) gives information about the accuracy of the stars’ proper motions in the catalogues researched. The less steep the curve, the more exact the proper motions in the given catalogue. This is for simple reasons. In figure 1 it is visible that the stars of the Tycho-2 catalogue have the most exact proper motions. The ACT and TRC catalogues have approximately identical proper motions. The FON catalogue gives proper motions which are close to the proper motions of the stars in the Tycho-2 catalogue. The Tycho catalogue has the least accurate proper motions.

**Figure 2.** The curves \(L(j)\) for the Tycho-2, ACT, TRC, FON and Tycho catalogues.

**Figure 3.** The curves \(L(j)\) for PPM, ACRS and FON catalogues.
In figure 3, the PPM, ACRS and FON catalogues are compared. The positions of stars in the ACRS catalogue are a little more exact than in PPM, and are the least exact in the FON catalogue: 22.4%, 26.1% and 30.5% respectively. The stars of the FON catalogue have the most exact proper motions. ACRS and PPM have proper motions which are close on accuracy. Evaluations of the accuracy of the ACRS and PPM catalogues correspond to the results of research on their accuracy conducted at the Pulkovo Observatory [10].

In figure 4, the curves $L(j)$ are indicated for the UCAC2 and Tycho-2 catalogues at the value $\Delta=0.2\arcsec$. The accuracy of the positions and proper motions of the stars in these catalogues are approximately equal.

![Figure 4. The curves $L(j)$ for the UCAC2 and Tycho-2 catalogues.](image)

4. Summary and conclusions

This method of evaluating the accuracy of the catalogues permits us to determine external global errors by a graphic method, including random and regional errors. The term "external" means those errors received by way of comparison with the Hipparcos catalogue. That is, these errors are received accurate within the Hipparcos catalogue accuracy. Common external errors of positions in the UCAC2 catalogue are on $\alpha$ and $\delta$: 20 mas and 19 mas, while errors of proper motions on $\alpha$ and $\delta$ make 2 mas/yr and 2 mas/yr.

As evaluated by the authors, errors of observations for positions in the UCAC2 catalogue for stars with brightness 10÷14 m are about 20 mas. For proper motions, errors of observation for stars which are brighter than 12.5 m are 4÷6 mas/yr, while for stars which are weaker than 12.5 m these errors are about 1÷3 mas/yr.

This problem is discussed in more detail in the following work [9, 11].

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References

[1] European Space Agency 1997 The Hipparcos and TYCO Catalogues (ESA SP-1200) (Noordwijk: ESA Publications Division)

[2] Bastian U, Röser S, Yagudin L and Nesterov V 1993 PPM Star Catalogue: Positions and Proper Motions of 197 179 Stars South of -2.5 Degrees Declination (Heidelberg; Berlin; Oxford: Spektrum Acad. Verl.)

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[3] Röser S and Bastian U 1991 *PPM Star Catalogue, Positions and Proper Motion of 181 731 Stars North of -2.5 Degrees Declination* (Heidelberg; Berlin; Oxford: Spektrum Acad. Verl.)

[4] Corbin T E and Urban S E 1991 *Astrographic Catalogue Reference Stars* (Washington: U.S. Naval Observatory)

[5] Høg E, Fabricius C, Makarov V V, Urban S, Corbin T, Wycoff G, Bastian U, Schwekendiek P and Wicenec A 2000 *A&A* **355** L27

[6] Urban S E, Corbin T E and Wycoff G L 1998 *Astron. J.* **115** 2161

[7] Høg E, Kuzmin A, Bastian U, Fabricius C, Kuimov K, Lindegren L, Makarov V V and S. Röser 1998 *A&A* **335** L65

[8] Kislyuk V, Yatsenko A, Ivanov G, Pakuliak L and Sergeeva T 2000 The FON astrographic catalogue (FONAC): version 1.0 *Motion of Celestial Bodies, Astrometry and Astronomical Reference Frames: Proc. of Journees 1999 & IX Lohrman-Kolloquium* (Drezden: Lohrman Observatory) pp 13–15

[9] Zacharias N, Urban S E, Zacharias M I, Wycoff G L, Hall D M, Monet D G and Rafferty T J 2004 *Astron. J.* **127** 3043

[10] Evdokimov A E, Pickin Y D and Potter H I 1995 *Comparison of union ACRS, PPM, HIP catalogues* (Saint Petersburg: GAO RAS)

[11] Zacharias N, Zacharias M I, Urban S E and Høg E 2000 *Astron. J.* **120** 1148