Determining the Hubble Constant without the Sound Horizon: Measurements from Galaxy Surveys

Oliver H. E. Philcox, Blake D. Sherwin, Gerrit S. Farren and Eric J. Baxter

1Department of Astrophysical Sciences, Princeton University, Princeton, NJ 08540, USA
2Department of Astrophysical Sciences and Theoretical Physics, University of Cambridge, Cambridge CB3 0WA, UK
3Kavli Institute for Cosmology, Cambridge University, Kavli Institute for Cosmology, Cambridge, Cambridge CB3 0HA, UK
4Department of Physics and Astronomy, Haverford College, Haverford, PA 19041, USA
5Institute for Astronomy, University of Hawaii, 2680 Woodlawn Drive, Honolulu, HI 96822, USA

Two scales are encoded in the galaxy power spectrum: the sound horizon at recombination and the horizon at matter-radiation equality. By combining BOSS DR12 galaxy power spectra with the theoretical size of the sound horizon scale at angular equality, we derive information from the sound-horizon at recombination, including CMB and calibrated BAO, usually independent of early-universe physics.

How do galaxy surveys measure the Hubble constant? Recent analyses have determined $H_0$ by comparing the angular scale of Baryon Acoustic Oscillations (BAO) with the theoretical size of the sound horizon scale at decoupling, $r_d$. A second ‘standard ruler’ exists however; the equality scale, i.e., the horizon wavenumber at matter-radiation equality. In this Letter, we explore the extent to which galaxy surveys can use this scale to place constraints on $H_0$ that are independent of the sound horizon.

Until recently, precise $H_0$ constraints have stemmed from two sources: the Cepheid-calibrated local distance ladder [e.g., 1] and anisotropies in the cosmic microwave background (CMB) [e.g., 2]. Today, a host of additional constraints are available, arising from data-sets such as galaxy and Lyman-alpha (Lyα) BAO [e.g., 3–8], strong gravitational lensing [e.g., 9, 10], and gravitational wave observations [11, 12]. Broadly, these fall into two camps: ‘indirect’ measurements, which require a full cosmological model for interpretation; and ‘direct’ probes, independent of early-universe physics.1 Probes in the former category, including CMB and calibrated BAO, usually derive information from the sound-horizon at recombination,2 calculated assuming $\Lambda$CDM. Previously, a tension between direct and indirect measurements seemed apparent; however, with the latest results from the TRGB-calibrated distance ladder [13] and strong-lensing [10], this distinction has become less clear. Nevertheless, there remains significant disagreement between indirect probes and the SH0ES distance ladder measurements [1], reaching a significance of $\sim 5\sigma$ [14].

Two primary possibilities exist to resolve this: (a) unresolved systematics [e.g., 15]; or (b) incompleteness of the cosmological model. For the latter, a wide variety of new-physics models have been proposed; many of these resolve the tension by providing mechanisms to reduce the sound horizon at recombination. As yet, there is no generally accepted solution.

Ref. [16] proposed a new method to shed light on the discrepancy, using CMB lensing to measure $H_0$ without the sound horizon scale; constraints were instead derived from the angular equality scale, $L_{eq}$. Being simply the projected wavenumber of modes entering the horizon at $z_{eq} \sim 3400$, this produces a definitive feature in the convergence spectrum, and can be used as a standard ruler. Importantly, the equality scale is sensitive to different redshifts than those of CMB and BAO analyses ($z \sim 1100$). This yields an important test: inconsistency of equality- and recombination-based $H_0$ constraints would give evidence for physics beyond $\Lambda$CDM 'recombination' and 'decoupling'; we thus use the terms interchangeably.

---

1 These are sometimes classified as ‘early’ and ‘late’ measurements respectively, but the terminology can be misleading, since an ‘early’ measurement does not necessarily involve high-redshift data-sets.
2 For the purposes of this Letter, there is little difference between...
operating at $z \gtrsim 10^3$. Combining Planck lensing with cosmological priors on $\Omega_m$ and $A_s$, Ref.[16] obtained $H_0 = 73.8 \pm 5.1$ km s$^{-1}$Mpc$^{-1}$; unfortunately, the projected improvements from future surveys were modest owing to the intrinsically large cosmic variance. Since the number of modes available to a 3D galaxy survey is typically much greater than for CMB lensing, one might expect stronger constraints on $H_0$ from this avenue: indeed, this was the primary source of $H_0$ information from galaxy surveys two decades ago [e.g., 17, 18]. In this Letter, we perform such a measurement with modern surveys.

EQUALITY AND THE SOUND HORIZON

A glance at the matter power spectrum reveals two features: the broadband peak at wavenumber $k_{\text{eq}} \sim 10^{-2}h\text{Mpc}^{-1}$ and the oscillatory behavior with period $\Delta k \sim 0.05h\text{Mpc}^{-1}$. The behavior around $k_{\text{eq}}$ is well known, arising from the transition between modes that enter the horizon in radiation-dominated and matter-dominated epochs. In galaxy surveys, resolving the peak is difficult (though possible with experiments such as SPHEREx [19]), due to relativistic effects and integral constraints [e.g., 20], alongside cosmic variance. More generally, $k_{\text{eq}}$ information is encoded in the shape of the power spectrum, and can be inferred from smaller scales, facilitated in part by the addition of amplitude and bias information from redshift-space distortions or priors. In ΛCDM, the equality scale is simply related to cosmological parameters;

$$k_{\text{eq}} = (2\Omega_{cb} H_0^2 z_{\text{eq}})^{1/2}, \quad z_{\text{eq}} = 2.5 \times 10^4 \Omega_{cb} h^2 \Theta_{2.7}^{-4/3} \quad (1)$$

[21, 22], where $\Theta_{2.7} \equiv T_{\text{CMB}}/(2.7\text{K})$ is the temperature of the CMB monopole, $\Omega_{cb} \equiv \Omega_{cdm} + \Omega_b$ (assuming neutrinos to be relativistic at $z_{\text{eq}}$), and $h \equiv H_0/(100$ km s$^{-1}$ Mpc$^{-1}$). Measuring $k_{\text{eq}}$ in $h\text{Mpc}^{-1}$ units probes the combination $\Omega_{cb} h \equiv (\omega_{cdm} + \omega_b)/h$, or, marginalizing over $\omega_b, \omega_{cdm}/h$. Given $k_{\text{eq}}$ and a probe of $\Omega_{cb}$, we can thus solve for the Hubble constant.

Complicating this is the second scale: the sound horizon at $z_d$, the redshift of photon-baryon decoupling. This is given by

$$r_d \equiv r_s(z_d) = \int_{z_d}^{\infty} \frac{c_s(z)}{H(z)} dz$$

$$\approx 55.154h \exp \left[ -72.3(\omega_\nu + 0.0006)^2 \right] \frac{0.2535}{\omega_b^2} \frac{1.2807}{\omega_b} h^{-1}\text{Mpc} \quad (2)$$

[23], where $H(z)$ and $c_s(z)$ are the Hubble parameter and sound-speed. $r_d$ sources two main features: the BAO wiggles with $\Delta k \approx 0.05h$ Mpc$^{-1}$, and a small-scale suppression of power on the baryonic Jeans scale [24]. Both have amplitudes scaling as $\omega_b/\omega_{cb}$ and could be used to infer the physical scale of the sound horizon. In combination with the measured angular scale, this constrains $H_0$. From (2), such measurements carry the degeneracy $\omega_{cdm} \propto h^4$ (after $\omega_b$ marginalization), and a measurement using the Jeans suppression is degenerate with $n_s$.

An $H_0$ measurement from the full-shape (FS) of the galaxy power spectrum will include information from both $r_d$ and $k_{\text{eq}}^{-1}$ standard rulers, whilst BAO analyses are sensitive only to $r_d$. To extract only information deriving from equality, one may wish to ‘marginalize over the sound horizon’; this is non-trivial since $r_d$ is not a direct input to any Boltzmann code, emerging only following simplifications such as tight-coupling. Here, we limit the $H_0$ information arising from the sound horizon simply by removing the usual informative prior on $\omega_b$, and thus the external $r_d$ calibration. For future data this may be insufficient: the BAO features and Jeans suppression can, in principle, calibrate each other, sourcing an effective sound-horizon prior.

DATA-SETS AND ANALYSIS

Redshift-Space Power Spectrum

Our main observational data-set is the twelfth data release (DR12) [3] of the Baryon Oscillation Spectroscopic Survey (BOSS), part of SDSS-III [25, 26]. Split across two redshift bins in each of the Northern and Southern galactic caps, the survey contains $\sim 1.2 \times 10^6$ galaxy positions with a total volume of $5.8 h^{-3}\text{Gpc}^3$. Here, we use the (unreconstructed) power spectrum monopole and quadrupole, each in 48 $k$-bins for $k \in [0.01, 0.25]/h\text{Mpc}^{-1}$, with covariances generated from a suite of 2048 MultiDark-Patchy mocks [27, 28], using the cosmology $\{\Omega_m = 0.30715, \Omega_b = 0.048, \sigma_8 = 0.8288, h = 0.6777, \sum m_\nu = 0\text{eV}\}$.

To extract maximal shape information, we model $P_r(k)$ with the Effective Field Theory of Large Scale Structure, following Ref. [29] (see also Ref. [30]). This includes one-loop perturbation theory, infra-red resummation of long-wavelength modes, and counterterms parametrizing the impact of small-scale physics. The model is convolved with the survey window function, and incorporates Alcock-Paczynski (AP) effects [31]. The procedure has been used in a number of works [22, 32–35], including a rigorous test on huge volume simulations [36]. Here, we

---

3 An ad-hoc rescaling of $r_d$ would be dangerous, as it would not conserve the stress-energy $T_{\nu\nu}$; instead, one should self-consistently add any new physics model to the perturbation equations.

4 beburl.github.io/hub/boss_papers.html

5 We use a fiducial value of $\Omega_m = 0.31$ to apply the AP rescaling to the BOSS data.
utilize the CLASS-PT implementation [37], with MCMC performed using montepython v3.3 alongside heavily optimized public likelihoods.\footnote{\url{github.com/michalychforever/ls_monterthon}.}

We vary the parameter set
\begin{equation}
\{ h, \omega_{cdm}, \omega_b, A_s, A_{s, Planck}, n_s, \sum m_{\nu} \} \times \{ b_1, b_2, b_{G_2}, b_4, c_{s,0}, c_{s,2}, P_{shot} \}
\end{equation}

where $A_{s, Planck} = 2.0989 \times 10^{-9}$, and $\sum m_{\nu}$ is the summed mass of three degenerate neutrinos. To aid convergence, we add an uninformative Gaussian prior on $\omega_b$ of width 50\% and flat priors of $[0, 0.18]$ eV and $[0.87, 1.07]$ to $\sum m_{\nu}$ and $n_s$ respectively. The second line gives nuisance parameters of the EFT model, which are allowed to vary independently in each of the four data patches, subject to the weak Gaussian priors of Ref.\cite{32}. Those entering the likelihood linearly ($b_4$, $c_{s,0}$, $c_{s,2}$ and $P_{shot}$) are marginalized analytically\cite{35, 38, 39}, reducing the total number of sampled parameters to $6 + 3 \times 4 = 18$.

Later, we will require mock data. This is generated from the theory model using the baseline cosmology \{$\omega_{cdm} = 0.118, \omega_b = 0.022, A_s/A_{s, Planck} = 1.025, h = 0.6777, n_s = 0.9649, \sum m_{\nu} = 0.06 \text{ eV}$\}, similar to the MultiDark-Patchy parameters, but with massive neutrinos. Three samples are created: (1) fiducial; (2) with negligible BAO wiggles; and (3) with fewer baryons (to reduce both BAO and baryon damping effects). For each, we fit nuisance parameters to the observed $P_l(k)$ and do not include noise. Set (2) is generated by increasing the BAO damping scale by 1000$\times$, whilst (3) reduces $\omega_b$ by 10$\times$ relative to the fiducial value, keeping $\omega_{cdm}$ and $A_s$ fixed.

**Cosmological Priors**

Equality based measurements of $H_0$ are assisted by information on $A_s$ (to constrain $k_{eq}$) and $\Omega_b$ (to break the $H_0 - \Omega_b$ degeneracy).\footnote{Knowledge of $b_1$ is also useful; this is provided by redshift-space distortions.} For the former, we employ a weak Gaussian prior of $A_s = (2.11 \pm 0.36) \times 10^{-9}$, centered on the Planck best-fit\cite{2}. The $r_d$-dependence of this is minimal, since the CMB measurement is limited by the optical depth and hence derives from very large scales; however, to be maximally conservative, we choose the prior width to be 10$\times$ that of the Planck constraint.

For the $\Omega_m$ prior, we principally use the marginalized result from Pantheon supernovae: $\Omega_m = 0.298 \pm 0.022$\cite{40}. This cannot constrain $H_0$ directly, since the supernova absolute magnitudes are unknown. An alternative source of $\Omega_m$ information is given by uncalibrated BAO measurements. A standard BAO analysis proceeds by comparing the radial and angular oscillatory scales to the $\Lambda$CDM sound horizon, providing information on $\Omega_m$ and $H_0 r_d$ through the evolution of the angular diameter distance and Hubble parameter \cite{e.g., 4]. To remove the dependence on $r_d$, we rescale the sound horizon by a free parameter $\alpha_{r_d}$. In this formalism, no knowledge of recombination physics is required, just the existence of a time-independent correlation function peak. Here, we use a range of galaxy BAO measurements from BOSS DR7 6dF and main galaxy samples\cite{41, 42}, and eBOSS DR14 Lyman-alpha measurements, (including cross-correlations with quasars)\cite{43, 44}. We exclude the BOSS DR12 BAO measurements, since they are covariant with the FS data-set, which would cause additional complications. Alone, the uncalibrated BAO give the constraint $\Omega_m = 0.308^{+0.025}_{-0.035}$.

**Additional Data-sets**

$H_0$ constraints from CMB lensing were demonstrated in Ref.\cite{16}. Due to the presence of projection integrals, the measurements are relatively free from $r_d$-calibration, even with a restrictive prior on $\omega_b$. The lensing power spectrum measures the combination $\Omega_{m, h}^2$, with a different scaling than that of $k_{eq}$; thus we may expect some degeneracy breaking when this is combined with FS measurements. Here, we use the public Planck 2018 CMB-marginalized lensing likelihood\cite{45}, assuming zero covariance between this and the BOSS data.\footnote{Technically, some correlation will be present since the probes partially overlap. Since the lensing kernel is much broader in redshift space than the BOSS selection function, and CMB lensing is only sensitive to modes that are perpendicular to the line-of-sight, we expect this to be small.} For analyses including the lensing data-set, we impose a twice tighter prior on $A_s$ of $(2.11 \pm 0.18) \times 10^{-9}$ (as in Ref.\cite{16}), to break the significant $A_s - L_{eq}$ degeneracy.

**RESULTS**

**Constraints from current data-sets**

Fig. 1 shows the cosmological constraints obtained. Combining BOSS power spectra with Pantheon $\Omega_m$ priors, we obtain $H_0 = 65.1^{+3.0}_{-4.4}$ km s$^{-1}$ Mpc$^{-1}$, below the best-fit SH0ES value at a 95\% confidence level (including non-Gaussianity of the posterior), even though our analysis does not depend on sound-horizon physics.\footnote{This corresponds to $k_{eq} = (1.40^{+0.10}_{-0.14}) \times 10^{-2}$ h Mpc$^{-1}$.} As expected, the $\omega_b$ posterior is broad, since, unlike in previous analyses, we have not imposed a restrictive prior.
There is a strong $h - \omega_{cdm} - \omega_b$ degeneracy, close to the expected linear relationship, rather than the $\omega_{cdm} \propto h^4$ scaling of $r_d$-calibration. The $\omega_b - \omega_{cdm}$ degeneracy indicates that a small amount of $\omega_{cdm}$ information comes from the BAO wiggles, though, as argued below, we do not expect this to inform our $H_0$ constraints. We note little dependence on the $A_s$ prior, with $< 10\%$ degradation in $\sigma_{H_0}$ if this is removed; this is expected since $A_s$ can be measured from the power spectrum through the loop corrections.

Using uncalibrated BAO instead of the Pantheon sample gives a similar posterior; $H_0 = 65.6^{+3.4}_{-5.5}$ km s$^{-1}$Mpc$^{-1}$. This is unsurprising; the sound horizon rescaling parameter, $\alpha_{rd}$, removes the $H_0$ information, and the marginalized $\Omega_m$ constraint from BAO alone is similar to Pantheon. Interestingly, the $\alpha_{rd}$ posterior is $0.993 \pm 0.016$; the combination of equality-based power spectra and (independent) uncalibrated BAO prefer a sound horizon consistent with $\Lambda$CDM.

Combination with Planck lensing shifts the $H_0$ posterior to larger values, with a marginalized limit of $70.6^{+3.7}_{-5.0}$ km s$^{-1}$Mpc$^{-1}$. Due to the addition of galaxy information, this is somewhat tighter than the lensing-only constraints of Ref. [16], though there is no improvement relative to the $P(k)$ posteriors, due to the broad error bars on the CMB lensing measurements and similar degeneracy directions. Note that the lensing-only constraint ($H_0 = 73.4 \pm 6.1$ km s$^{-1}$Mpc$^{-1}$) is shifted somewhat from that of Ref. [16], due to slightly different prior choices.

**Dependence on $r_d$**

We now demonstrate that our $H_0$ constraints do not receive significant information from the sound horizon, using three tests: repeating the analysis on mock data-sets without baryon oscillations and damping; employing scale cuts; and performing Fisher forecasts, where we can explicitly marginalize over $r_d$.

First, we turn to the synthetic data-sets discussed above. As shown in Fig. 2, our $H_0$ constraints are negligibly impacted by removing BAO wiggles or reducing baryonic damping. Since the mock data are generated to match the BOSS spectra, this is a strong indication that our $H_0$ constraints are independent of sound horizon physics. Note that the best-fit values of $\omega_{cdm}$ and $h$ are shifted by $\sim 0.5\sigma$ from the truth; this indicates a
(modest) prior-volume effect, confirmed by its removal when reanalyzing the data with a covariance appropriate for a 10× larger survey. Whilst this could be ameliorated by stricter nuisance parameter priors, given that the offsets are small, we do not include these. The mocks also highlight the importance of $\Omega_m$ priors; since the FS likelihood sources $\omega_{adm}$ information from BAO wiggles, the no-wiggle constraints on $H_0$ would degrade if an external prior was not present.

Scale cuts provide further evidence to support our conclusions. Fig. 3 shows the effect of reducing $k_{\text{max}}$ from 0.25 $h$ Mpc$^{-1}$ to 0.10 $h$ Mpc$^{-1}$, which, if information were coming from BAO wiggles, would be expected to greatly inflate the $H_0$ posterior. Notably, the reduction in constraining power is slight ($\sim 10\%$), though the nuisance parameters of the one-loop model suffer significant posterior inflation. Again, this indicates that the primary information is sourced by $k_{eq}^{-1}$ rather than $r_d$.

Finally, we consider a simplified Fisher analysis in which $r_d$ can be marginalized over exactly. This is made possible by using the Eisenstein-Hu transfer function [21] in CLASS-PT, rather than the usual output from CLASS. In addition to the five cosmological parameters $\{h, \omega_{adm}, \omega_b, A_s/A_s(\text{Planck}), n_s\}$ (the Eisenstein-Hu approximation does not allow for massive neutrinos) we vary a sound horizon rescaling parameter $\beta_{r_d}$. For simplicity, a single redshift bin (centered at $z = 0.51$) with the total BOSS volume is used, and window function effects are ignored. As seen in Fig. 4, the $H_0$ posteriors are broader than those found in Fig. 1; this is due to the assumptions of an Eisenstein-Hu model and exclusion of redshift evolution, yet the model retains qualitative utility. Marginalization over $r_d$ has little effect, reducing $\sigma_{H_0}$ by < 10%, reinforcing our conclusions that the $H_0$ constraints are insensitive to $r_d$. Without an $\Omega_m$ prior the marginalization gives significant degradation, with $\sigma_{H_0} = 11$ km s$^{-1}$ Mpc$^{-1}$; in this case, $\Omega_m$ information enters from the BAO wiggle amplitudes which are washed-out by the marginalization.

### Forecasting for Future Surveys

To estimate the potential of future surveys to constrain $H_0$ without the sound horizon, we perform a simplistic Fisher analysis, similar to that presented above. In particular, we consider a Euclid-like survey in eight redshift bins, taking the volumes and fiducial bias parameters from the forecast of Ref. [46]. For consistency, we slightly expand our $k$-range up to $k_{\text{max}} = 0.3h$ Mpc$^{-1}$ and do not impose nuisance parameter priors. Adopt-
ing the $A_\alpha$ and $\Omega_m$ priors of this Letter, and marginalizing over $r_d$, we obtain $\sigma_{H_0} \sim 1.7 \text{ km s}^{-1}\text{Mpc}^{-1}$; this tightens to $\sim 1.6 \text{ km s}^{-1}\text{Mpc}^{-1}$ with the more optimistic $\sigma_{\Omega_m} = 0.012$ prior of Ref. [16]. For future surveys, it is unclear whether removing the $\omega$ prior will be sufficient to ensure $r_d$-independence; this will be discussed in future work alongside a more complete forecast.

**DISCUSSION**

In the past decade, galaxy surveys have focused on measuring BAO. In this Letter, we make use of the fact that an additional standard ruler is present; the horizon size at matter-radiation equality, $k_{\text{eq}}^{-1}$. Combining galaxy power spectra from BOSS with cosmological priors on $\Omega_m$ gives equality-based constraints of $H_0 = 65.1^{+3.0}_{-5.4} \text{ km s}^{-1}\text{Mpc}^{-1}$ (power spectrum only) and $70.6^{+3.7}_{-5.0} \text{ km s}^{-1}\text{Mpc}^{-1}$ (adding Planck lensing). For BOSS such a measurement can be obtained simply by analyzing the data without use of an informative $\omega$ prior; we demonstrate this using mock catalogs, scale cuts and Fisher forecasts. For the next generation of surveys, simple forecasts indicate that sound horizon independent constraints of $\sigma_{H_0} \sim 1.6 \text{ km s}^{-1}\text{Mpc}^{-1}$ should be possible; more sophisticated techniques may be required to remove $r_d$ information, however.

To close, we consider implications for the `Hubble tension’. Most proposed mechanisms for its resolution rely on modifying the sound horizon at recombination, and thus altering the BAO scale. Given that equality-based measurements are sensitive to higher redshifts than BAO measurements, $H_0$ constraints anchored at $z_d$ and $z_{\text{eq}}$ may differ if new physics is at work, making this a valuable test of new physics prior to recombination. Here, we find good agreement between galaxy-only $H_0$ measurements derived from the sound horizon and equality scales, both of which favor lower values than those of SH0ES. If this consistency holds to much higher precision, it will place strong bounds on many beyond-CDM resolutions of the Hubble tension.

We thank Mikhail Ivanov, Julien Lesgourgues and Marko Simonovi{\v{c}} for insightful discussions; we are additionally grateful to Jo Dunkley, Dragan Huterer, Eiichiro Komatsu and Matias Zaldarriaga for comments on a draft of this manuscript. OHEP would like to thank the Max Planck Institute for Astrophysics for hospitality when this work was being finalized. OHEP acknowledges funding from the WFIRST program through NNG06F33C and NNN12AA01C. GSF acknowledges support through the Isaac Newton Studentship and the Cambridge Trust Vice Chancellor’s Award. BDS acknowledges support from an Isaac Newton Trust Early Career Grant, from a European Research Council (ERC) Starting Grant under the European Unions Horizon 2020 research and innovation programme (Grant agreement No. 851274), and from an STFC Ernest Rutherford Fellowship.

* ohep2@cantab.ac.uk

[1] A. G. Riess, S. Casertano, W. Yuan, L. M. Macri and D. Scolnic, *Large Magellanic Cloud Cepheid Standards Provide a 1% Foundation for the Determination of the Hubble Constant and Stronger Evidence for Physics beyond CDM*, ApJ 876 (2019) 85 [1903.07603].
[2] Planck Collaboration, N. Aghanim, Y. Akrami, M. Ashdown, J. Aumont, C. Baccigalupi et al., *Planck 2018 results. VI. Cosmological parameters*, arXiv e-prints (2018) arXiv:1807.06209 [1807.06209].
[3] S. Alam, M. Ata, S. Bailey, F. Beutler, D. Bizyaev, J. A. Blazek et al., *The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey: cosmological analysis of the DR12 galaxy sample*, MNRAS 470 (2017) 2617 [1607.03155].
[4] A. Cuceu, J. Farr, P. Lemos and A. Font-Ribera, *Baryon Acoustic Oscillations and the Hubble constant: past, present and future*, JCAP 2019 (2019) 044 [1906.11628].
[5] T. M. C. Abbott, F. B. Abdalla, J. Annis, K. Bechtol, J. Blazek, B. A. Benson et al., *Dark Energy Survey Year 1 Results: A Precise $H_0$ Estimate from DES Y1, BAO, and D/H Data*, MNRAS 480 (2018) 3879 [1711.00403].
[6] K. Aylor, M. Joy, L. Knox, M. Millea, S. Raghu, M. Millon, D. Sluse et al., *Sounds Discordant: Classical Distance Ladder and CDM-based Determinations of the Cosmological Sound Horizon*, ApJ 874 (2019) 4 [1811.00537].
[7] N. Schöneberg, J. Lesgourgues and D. C. Hooper, *The BAO+BBN take on the Hubble tension*, JCAP 2019 (2019) 029 [1907.11594].
[8] G. E. Addison, D. J. Watts, C. L. Bennett, M. Halpern, G. Hinshaw and J. L. Weiland, *Elucidating CDM: Impact of Baryon Acoustic Oscillation Measurements on the Hubble Constant Discrepancy*, ApJ 853 (2018) 119 [1707.06547].
[9] K. C. Wong, S. H. Suyu, G. C. F. Chen, C. E. Rust, M. Millon, D. Sluse et al., *H0LiCOW XIII. A 2.4% measurement of $H_0$ from lensed quasars: 5.3σ tension between early and late-Universe probes*, MNRAS (2020) [1907.04869].
[10] S. Birrer, A. J. Shahij, A. Galan, M. Millon, T. Treu, A. Agnello et al., *TDcosmo IV: Hierarchical time-delay cosmography – joint inference of the Hubble constant and galaxy density profiles*, arXiv e-prints (2020) arXiv:2007.02941 [2007.02941].
[11] B. P. Abbott, R. Abbott, T. D. Abbott, F. Acernese, K. Ackley, C. Adams et al., *A gravitational-wave standard siren measurement of the Hubble constant*, Nature 551 (2017) 85 [1710.05835].
[12] A. G. Riess, *The expansion of the Universe is faster than expected*, Nature Reviews Physics 2 (2019) 10 [2001.03624].
[13] W. L. Freedman, B. F. Madore, D. Hatt, T. J. Hoyt, I. S. Jang, R. L. Beaton et al., *The Carnegie-Chicago Hubble Program. VIII. An Independent Determination of the Hubble Constant Based on the Tip of the Red Giant Branch*, ApJ 882 (2019) 34 [1907.05922].
[14] L. Verde, T. Treu and A. G. Riess, *Tensions between the early and late Universe*, Nature Astronomy 3 (2019) 891 [1907.10625].
G. d’Amico, J. Gleyzes, N. Kokron, K. Markovic, L. Senatore, P. Zhang et al., The cosmological analysis of the SDSS/BOSS data from the Effective Field Theory of Large-Scale Structure, JCAP 2020 (2020) 005 [1909.05271].

C. Alcock and B. Paczynski, An evolution free test for non-zero cosmological constant, Nature 281 (1979) 358.

M. M. Ivanov, M. Simonović and M. Zaldarriaga, Cosmological parameters and neutrino masses from the final P l a n k and full-shape BOSS data, Phys. Rev. D 101 (2020) 083504 [1912.08208].

O. H. E. Philcox, M. M. Ivanov, M. Simonović and M. Zaldarriaga, Combining full-shape and BAO analyses of galaxy power spectra: a 1.6% CMB-independent constraint on H0, JCAP 2020 (2020) 032 [2002.04035].

M. M. Ivanov, E. McDonough, M. Simonović, M. W. Toomey, S. Alexander et al., Constraining Early Dark Energy with Large-Scale Structure, arXiv e-prints (2020) arXiv:2006.11235 [2006.11235].

O. H. E. Philcox, M. M. Ivanov, M. Zaldarriaga, M. Simonović and M. Schmittfull, Fewer Mocks and Less Noise: Reducing the Dimensionality of Cosmological Observables with Subspace Projections, in prep. (2020).

T. Nishimichi, G. D’Amico, M. M. Ivanov, L. Senatore, M. Simonović, M. Takada et al., Blinded challenge for precision cosmology with large-scale structure: results from effective field theory for the redshift-space galaxy power spectrum, arXiv e-prints (2020) arXiv:2003.08277 [2003.08277].

A. Chudaykin, M. M. Ivanov and M. Simonović, CLASS-PT: non-linear perturbation theory extension of the Boltzmann code CLASS, arXiv e-prints (2020) arXiv:2004.10607 [2004.10607].

S. L. Bridle, R. Crittenden, A. Melchiorri, M. P. Hobson, R. Kneissl and A. N. Lasenby, Analytic marginalization over CMB calibration and beam uncertainty, MNRAS 335 (2002) 1193 [astro-ph/0112114].

A. N. Taylor and T. D. Kitching, Analytic methods for cosmological likelihoods, MNRAS 408 (2010) 865 [1003.1136].

D. M. Scolnic, D. O. Jones, A. Rest, Y. C. Pan, R. Chornock, R. J. Foley et al., The Complete Light-curve Sample of Spectroscopically Confirmed SNe Ia from Pan-STARRS1 and Cosmological Constraints from the Combined Pantheon Sample, ApJ 859 (2018) 101 [1710.00845].

F. Beutler, C. Blake, M. Colless, D. H. Jones, L. Staveley-Smith, L. Campbell et al., The 6dF Galaxy Survey: baryon acoustic oscillations and the local Hubble constant, MNRAS 416 (2011) 3017 [1106.3366].

A. J. Ross, L. Samushia, C. Howlett, W. J. Percival, A. Burden and M. M. Ivanov, The clustering of galaxies in the SDSS-III Baryon Oscillation Spectroscopic Survey: modelling the clustering, halo occupation distribution of BOSS CMASS galaxies in the Final Data Release, MNRAS 460 (2016) 1173 [1509.06404].

M. M. Ivanov, M. Simonović and M. Zaldarriaga, Cosmological parameters from the BOSS galaxy power spectrum, JCAP 2020 (2020) 042 [1909.05277].

G. d’Amico, J. Gleyzes, N. Kokron, K. Markovic, L. Senatore, P. Zhang et al., The cosmological analysis of the SDSS/BOSS data from the Effective Field Theory of Large-Scale Structure, JCAP 2020 (2020) 005 [1909.05271].

C. Alcock and B. Paczynski, An evolution free test for non-zero cosmological constant, Nature 281 (1979) 358.

M. M. Ivanov, M. Simonović and M. Zaldarriaga, Cosmological parameters and neutrino masses from the final P l a n k and full-shape BOSS data, Phys. Rev. D 101 (2020) 083504 [1912.08208].

O. H. E. Philcox, M. M. Ivanov, M. Simonović and M. Zaldarriaga, Combining full-shape and BAO analyses of galaxy power spectra: a 1.6% CMB-independent constraint on H0, JCAP 2020 (2020) 032 [2002.04035].

M. M. Ivanov, E. McDonough, M. Simonović, M. W. Toomey, S. Alexander et al., Constraining Early Dark Energy with Large-Scale Structure, arXiv e-prints (2020) arXiv:2006.11235 [2006.11235].

O. H. E. Philcox, M. M. Ivanov, M. Zaldarriaga, M. Simonović and M. Schmittfull, Fewer Mocks and Less Noise: Reducing the Dimensionality of Cosmological Observables with Subspace Projections, in prep. (2020).

T. Nishimichi, G. D’Amico, M. M. Ivanov, L. Senatore, M. Simonović, M. Takada et al., Blinded challenge for precision cosmology with large-scale structure: results from effective field theory for the redshift-space galaxy power spectrum, arXiv e-prints (2020) arXiv:2003.08277 [2003.08277].

A. Chudaykin, M. M. Ivanov and M. Simonović, CLASS-PT: non-linear perturbation theory extension of the Boltzmann code CLASS, arXiv e-prints (2020) arXiv:2004.10607 [2004.10607].

S. L. Bridle, R. Crittenden, A. Melchiorri, M. P. Hobson, R. Kneissl and A. N. Lasenby, Analytic marginalization over CMB calibration and beam uncertainty, MNRAS 335 (2002) 1193 [astro-ph/0112114].

A. N. Taylor and T. D. Kitching, Analytic methods for cosmological likelihoods, MNRAS 408 (2010) 865 [1003.1136].

D. M. Scolnic, D. O. Jones, A. Rest, Y. C. Pan, R. Chornock, R. J. Foley et al., The Complete Light-curve Sample of Spectroscopically Confirmed SNe Ia from Pan-STARRS1 and Cosmological Constraints from the Combined Pantheon Sample, ApJ 859 (2018) 101 [1710.00845].

F. Beutler, C. Blake, M. Colless, D. H. Jones, L. Staveley-Smith, L. Campbell et al., The 6dF Galaxy Survey: baryon acoustic oscillations and the local Hubble constant, MNRAS 416 (2011) 3017 [1106.3366].

A. J. Ross, L. Samushia, C. Howlett, W. J. Percival, A. Burden and M. M. Ivanov, The clustering of galaxies in the SDSS-III Baryon Oscillation Spectroscopic Survey: modelling the clustering, halo occupation distribution of BOSS CMASS galaxies in the Final Data Release, MNRAS 460 (2016) 1173 [1509.06404].

M. M. Ivanov, M. Simonović and M. Zaldarriaga, Cosmological parameters from the BOSS galaxy power spectrum, JCAP 2020 (2020) 042 [1909.05277].

G. d’Amico, J. Gleyzes, N. Kokron, K. Markovic, L. Senatore, P. Zhang et al., The cosmological analysis of the SDSS/BOSS data from the Effective Field Theory of Large-Scale Structure, JCAP 2020 (2020) 005 [1909.05271].
[45] Planck Collaboration, N. Aghanim, Y. Akrami, M. Ashdown, J. Aumont, C. Baccigalupi et al., *Planck 2018 results. V. CMB power spectra and likelihoods*, arXiv e-prints (2019) arXiv:1907.12875 [1907.12875].

[46] A. Chudaykin and M. M. Ivanov, *Measuring neutrino masses with large-scale structure: Euclid forecast with controlled theoretical error*, Journal of Cosmology and Astroparticle Physics (2019) [1907.06666].