Fatigue Topology Optimization Based on Global Stress Constraint Method

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Abstract. In topology optimization design for continuous structure, common mechanical properties considered in optimization model focus on structural strength, stiffness and stability. However, fatigue characteristic of structure is easily ignored, which is one of the main causes of engineering failure. Therefore, it is necessary to consider the fatigue characteristic for the optimization design of the structure. In order to investigate the influence of fatigue on structural topology in optimization design, fatigue life filter function is introduced and then the model and solution method of fatigue topology optimization for continuous structures are proposed based on independent continuous mapping method (ICM). The fatigue topological optimization model is established with structure minimum weight as objective and fatigue life as constraint, and the model is solved by global stress constraint method. The results extend the ICM method and this basic theory provides a new idea for fatigue topology optimization.

Keywords: Topology optimization; Fatigue; ICM; Global stress constraint method

1. Introduction

Topology optimization of continuum structure is a challenging research direction in the field of structural optimization after the size optimization and shape optimization. The purpose of topology optimization is to find the best distribution form of structural stiffness in the design space, or search for the best transmission path form in the design domain, so as to optimize some performance of structure or reduce the weight of structure. Topological optimization methods for continuum structure include homogenization method [1], variable density method [2], variable thickness method [3], evolutionary structural optimization method [4], level set method [5], moving morphable component (MMC) [6], and independent continuous mapping (ICM) [7].

Fatigue life is the time or cycle times of the structure used from the beginning to the damage of the cyclic load. Fatigue failure occurs under the action of alternating stress which is far below the strength limit or even the yield limit of material. Fatigue failure occurs with no obvious portent, and has huge potential risks. However, there are few studies on the topology optimization of fatigue now, and most foreign scholars use SIMP method. Holmberg [8] introduced the critical fatigue stress to the constraint, which is determined through the damage factor in the fatigue analysis and so as to form a constraint condition together with the static stress. Jacob [9] used fatigue damage factor to describe the fatigue response, then a fatigue topological optimization model was established with the minimum cost as objective and the fatigue life as constraint, where all constraints were grouped into a global constraint with P-norm function. Lee [10] presented a new topology optimization method that consider fatigue
life in the frequency domain, and this work established a model with the minimum volume as objective and the accumulated damage as constraint.

In this paper, combined ICM method and fatigue analysis method, a fatigue topological optimization model is established with the minimum structure weight as objective and the fatigue life as constraint. Fatigue constraint is transformed into the stress constraint by using the S-N curve. And the global stress constraint method is used to solve the optimal model. The results of the study provide new ideas and new methods for conceptual design of structures considering the fatigue characteristics.

2. Fatigue analysis method

The earliest anti-fatigue design method is the nominal stress method. It is based on the S-N curve of materials, combined with fatigue damage accumulation theory, to check fatigue strength or to calculate fatigue life. In this paper, the nominal stress method used to predict the life of structural [11]. S-N curve shows relationship between peak stress of cyclic load and fatigue life is described. The most common form is power function, as follows

\[
\sigma^\beta \cdot L = C
\]

where \( \beta \) and \( C \) are material constant.

In addition, the structural damage is based on the fatigue accumulated damage criterion, according to the Miner rule, that the fatigue damage can be linearly accumulated, as follows

\[
D = \sum D_i = \sum_{i=1}^{n} \frac{m_i}{M_i}
\]

where \( M_i \) represent the fatigue life of \( i \)-th stress level, \( m_i \) represent the cycle times of \( i \)-th stress level. D is equal to 1, indicating material damage.

3. ICM method

The ICM method means "independent, continuous, mapping ". "Independence" means that the topological variables are independent of the variables of the physical parameters to express the solid or void of an element. New functions are introduced to realize "continuous" and "mapping". The continuous approximation property of function successfully converts the 0/1 integer programming problem into a continuous mathematical model, and this is the independent continuous topological variables defined in the ICM method, which are independent of any element physical properties. The filtering function and the polishing function are the results of the continuous infinite approximation of the hurdle function and the step function [7]. In this paper, three filter functions are used as follows

\[
w_i = f_w(t_i)w_i^0, \quad k_i = f_k(t_i)k_i^0, \quad L_i = \frac{L_i^0}{f_k(t_i)}
\]

where \( w_i, k_i, L_i \) are the \( i \)-th element weight, stiffness matrix and allowable fatigue life respectively. \( w_i^0, k_i^0, L_i^0 \) represent the inherent element weight, stiffness matrix, and allowable fatigue life.

In this paper, the global stress constraint method is used to solve fatigue topology optimization. The global stress constraint method comes from the forth strength theory, which is established from the distortional strain energy density that is a part of strain energy density. Therefore, the peak stress of cyclic load can be transformed into strain energy related to fatigue, which is called dynamic strain energy.

4. The establishment and solution of fatigue topography optimization model

At first, lightweight topological optimization design considering fatigue characteristics is described as a mathematical programming model based on ICM. The objective is to find the optimal layout of structure with minimum material under the constraints of fatigue life.
where $t \in (t_1, \ldots, t_N)^T$ is the topological variable vector, $W$ is the total weight of structure. $L_i$ is the $i$-th element fatigue life.

Then, the constraint does a transformation using Eq. (1), and put weight filter function $f_w(t_i)$, fatigue life filter function $f_L(t_i)$ into it. Eq. (5) can be obtained

$$\begin{align*}
&\text{find } t \in (t_1, \ldots, t_N)^T \\
&\text{make } W = \sum_{i=1}^N w_i \rightarrow \min \\
&\text{s.t. } L_i \geq L_i^0 \\
&\quad (i = 1, \ldots, N)
\end{align*}$$

where, $\sigma_d(L_i)$ is $i$-th element’s fatigue life corresponding to the peak stress of cyclic load, $\sigma_{ul}(L_i^0)$ is $i$-th element’s allowable fatigue life corresponding to the peak stress of cyclic load.

In order to describe the energy related to fatigue, we introduce coefficient of association $\xi_i$,

$$\begin{align*}
e_{Li} = \xi_i e_i
\end{align*}$$

according to the global stress constraint method, Eq. (7) is written as

$$\begin{align*}
\frac{(1+\mu)\sigma_d^2(L_i^0)W}{3E} < e_{Li}
\end{align*}$$

the model can be written as

$$\begin{align*}
&\text{find } t \in (t_1, \ldots, t_N)^T \\
&\text{make } W = \sum_{i=1}^N f_w(t_i)w_i^0 \rightarrow \min \\
&\text{s.t. } e_{Li} < f_L(t_i)^{2\beta_i}(t_i) \cdot e_{Li} \\
&\quad (0 < t_i \leq t_i \leq 1; i = 1, \ldots, N)
\end{align*}$$

Assuming $f_w(t_i) = t_i^{\alpha_w}$, $f_L(t_i) = t_i^{\alpha_L}$, let $x_i = \frac{1}{t_i^{2\alpha_L-\alpha_w}}$, the constraint of Eq. (8) can be written as

$$\begin{align*}
\sum_{i=1}^n f_w(t_i(x_i)) \cdot e_{Li}(x_i) \cdot x_i < e_{Li}
\end{align*}$$

For the sake of presentation, let $B_i = f_w(t_i(x_i)) \cdot e_{Li}(x_i)$, therefore

$$\begin{align*}
\sum_{i=1}^n B_i \cdot x_i < e_{Li}
\end{align*}$$

The next work is to approximate the objective function by second-order Taylor expansion and ignore the constant terms.

$$f_w(t_i) = t_i^{\alpha_w} = x_i^{-\frac{\alpha_w}{2\alpha_L\beta_i-\alpha_w}}$$

then we can set $A = -\frac{\alpha_w}{2\alpha_L\beta_i+\alpha_k}$, therefore, the objective can be expressed as follows

$$W = ax_i + bx_i^2 \rightarrow \min$$
where \( a_i = \sum_{i=1}^{N} A(A+1)(x_i^0)^{d-1} \cdot w_i^0 \), \( b_i = \frac{1}{2} \sum_{i=1}^{N} A(A-1)(x_i^0)^{d-2} \cdot w_i^0 \).

Finally, a quadratic programming model is shown in Eq. (13)

\[
\begin{align*}
\text{find} & \quad \mathbf{x} \in (x_1, \cdots, x_N)^T \\
\text{make} & \quad W = a_i x_i + b_i x_i^2 \rightarrow \min \\
\text{s.t.} & \quad \sum_{i=1}^{n} B_i \cdot x_i < e_k \\
& \quad (0 < t_i \leq t \leq 1; i = 1, \cdots, N)
\end{align*}
\] (13)

where \( a_i = \sum_{i=1}^{N} A(A+1)(x_i^0)^{d-1} \cdot w_i^0 \), \( b_i = \frac{1}{2} \sum_{i=1}^{N} A(A-1)(x_i^0)^{d-2} \cdot w_i^0 \), \( A = -\frac{\alpha_u}{2\alpha_i \beta^{-1} + \alpha_k} \), \( B_i = f_{ul}(t_i^{(v)}) \cdot e_{k(i)}^{(v)} \).

5. Numerical example

In this paper, example proves the feasibility and effectiveness of the fatigue topology optimization based on the ICM method. In the numerical example, the concentrated dynamic load is the form of Sine function. as shown in Figure 1.

**Figure 1.** Cyclic loading form

The design domain and size are shown in Figure 2, a central cyclic external load is applied to the upper end of the right side of the structure. To avoid the influence of stress concentration, the force is spread on the three nodes of the structure. Young’s modulus \( E = 210\text{GPa} \), Poisson’s ratio \( \mu = 0.25 \). The original structure weight is 144 kg. In the process of fatigue topology optimization, the structure fatigue life constraint is 1500 cycles.

**Figure 2.** The basic structure

Figure 3 is an iterative history of the weight and fatigue life.
Figure 3. The iteration curve of history

Table 1 gives the optimal topology diagram and the corresponding stress nephogram and fatigue life nephogram.

Table 1: The result of topology optimization

| Optimal structure | Stress nephogram | Fatigue life nephogram |
|-------------------|------------------|------------------------|
| Image             | Image            | Image                  |

After optimization, the structure weight decreases from 144 kg to 61.71 kg, and the fatigue life is 1562. From the iteration curve of history, we can see that the fatigue life of the structure converges steadily and the structure weight converges steadily with the number of iterations when the structure satisfies the fatigue. From table 1, the structure of the optimal topology is clear and meets the requirement of force transmission, and the distribution of the stress nephogram and fatigue life nephogram are more uniform.

6. Conclusion
Based on ICM (Independent Continuous Mapping) method, a fatigue topological optimization model is established with the minimum structure weight as objective and the fatigue life as constraint. The topology optimization considering structural fatigue characteristics is realized. Numerical example verifies the effectiveness and feasibility of fatigue topology optimization based on the global stress constraint method.

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