Freak Wave Formation from Phase Coherence

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Abstract. This paper describes freak waves by a (pseudo-)maximal wave proposed in [1]. The freak wave is a consequence of a group event that is present in a time signal at some position and contains successive high amplitudes with different frequencies. The linear theory predicts the position and time of the maximal amplitude wave quite well by minimizing the variance of the total wave phase of the given initial signal. The formation of the freak wave is shown to be mainly triggered by the local interaction of waves evolving from the group event that already contained large local energy. In the evolution, the phases become more coherent and the local energy is focussed to develop a larger amplitude. We investigate two laboratory experimental signals, a dispersive focussing wave with harmonic background and a scaled New Year wave. Both signals generate a freak wave at the predicted position and time and the freak wave can be described by a pseudo-maximal wave with specific parameters.

1. Introduction
Freak waves were first recognized since four and a half decades ago [2]. In general, it is unexpectedly high waves that suddenly appear in a relatively mild background wave field. Many definitions of freak wave have been proposed by researchers [3, 4, 5] and the most common definition is a wave for which the ratio of the maximum crest height and the significant wave height (Hs) is greater than 1.25. Freak waves are also known as extreme wave, rogue wave, giant wave, or monster wave [6]. Understanding the mechanism of freak wave appearance is important as it may cause damages in the ocean or shore. Indonesian region has the largest number of fatalities caused by freak waves [7, 8]. On 1 January 2016, local media reported a freak wave up to 9m high came to the shore, specifically at Bantul Jati and Perawan Desa beach in Malang regency and dragged five people [9, 10].

In the past two decades much research about freak waves have been studied, but the mechanisms of the freak wave appearance are still under discussion. A freak wave can be generated by a simple consequence of linear superposition of waves [11]. Another mechanism was proposed that the interactions between envelope solitons provide an explanation for the formation of extreme waves [12]. Study of [13] gives an overview that the occurrence of extreme waves is highlighted into two scenarios: nonlinear mechanism of second order bound waves and third order four-wave resonance interaction of free waves. In [14] a number of physical mechanisms of extreme waves are described: spatial focusing, dispersive focusing, and nonlinear focusing. Besides these three mechanisms, reference [15] discussed other possible mechanisms, such as nonlinear modulational Benjamin-Feir (BF) instability, wave-current interaction, nonlinear wave group dynamics, and wave-coast interaction. While the
BF instability develops, the population of extreme waves increases, but this only occurs for very long-crested waves [16]. Hence, for waves with broad spectra the linear interaction might be the major cause. Dispersive focusing is the linear mechanism and occurs even in linear Gaussian sea. It was proposed in the eighties [17] that the mechanism of the extreme waves in a Gaussian sea are particular realizations of the evolution of well defined wave groups. The waves are dispersive with phase and group velocity inversely proportional to the frequency at deep water. The dispersive focusing can be explained by creating a long wave group with decreasing frequencies, then the dispersion forces the group to contract to a few wavelengths at a given position. This mechanism is also suggested in [18, 7, 11, 19]. Reference [13] confirmed that extreme waves most likely occur due to the dynamics of a single wave group in agreement with the dynamics imposed by the Zakharov equation [20].

This paper contributes in understanding the appearance of freak waves. We propose one mechanism of freak waves occurrence. We discuss extreme waves that are generated by the linear mechanism without confronting that the nonlinear term will also add some contributions in the wave amplitude. The extreme wave is mainly developed by a group event, part of the initial signal with successive high amplitudes. We give the prediction and description of the extreme waves by a concept of (pseudo-)maximal wave [1], that is based on the phase coherence. We implement it in two different experimental signals, a dispersive focusing wave with harmonic background and an experimental version of the well-known New Year wave. We investigate the dynamics of the group event in both signals in forward time by numerical simulations of the AB-equation [21, 22, 23, 24].

2. Description of a freak wave
One design wave describing a freak wave was proposed by Walker et al. [25]. The design wave has become known as New Wave, which is shown to be an acceptable local linear model for large waves. It is based on the average shape of an extreme wave in a linear random Gaussian process. The full model of New Wave uses Stokes regular wave expansion up to nonlinear fifth order. Another design wave was proposed for a linear description of a freak wave based on the phase coherence by a so-called (pseudo-)maximal wave [1]. In the exceptional case of a fully coherent wave phase, a freak wave can be described well by a so-called maximal wave. The difference with the concept of the New Wave is that the pmw can be designed completely by knowledge of the spectrum, without the necessity as for the New Wave to determine the amplitude based on the probability of appearance.

Suppose \( \eta_0(t) \) is a given signal at \( x = X_0 \) and let \( \hat{\eta}_0(\omega) = |\hat{\eta}_0(\omega)|e^{i\theta_0(\omega)} \) be the Fourier transform of the signal. A maximal wave corresponding to \( \eta_0(t) \) is the linear wave that is defined as follows:

\[
\eta_{\text{max}}(x,t) = \int |\hat{\eta}_0(\omega)| \cos (k(x - X_0) - \omega t) \, d\omega, \tag{1}
\]

in which \( k \) and \( \omega \) are related by the exact dispersion relation, \( \omega = \Omega(k) = \text{sign}(k) \sqrt{gk \tanh(kD)} \), with water depth \( D \) and gravitational acceleration \( g \). In this maximal wave definition, the maximum amplitude is assumed to be at \( t = 0 \). An irregular wave is obtained in case the phases are uniformly distributed in \( (-\pi, \pi) \). Then, a freak wave can be described in between a completely random wave and a fully coherent wave. For a given random phase \( \theta(\omega) \in (-\pi, \pi] \) as a function of wave frequencies with \( \theta(\omega) = -\theta(-\omega) \), we consider a pseudo-maximal wave (pmw) corresponding to \( \eta_0(t) \), for which the phases are restricted for certain \( \alpha \in (0,1) \) to the phases \( \theta_\alpha(\omega) = \alpha \theta(\omega) \), as follows:

\[
[\eta_{pm}(x,t)]_\alpha = \int |\hat{\eta}_0(\omega)| \cos (k(x - X_0) - \omega t + \theta_\alpha(\omega)) \, d\omega. \tag{2}
\]

For given fraction \( \alpha \), the phases of all frequencies are constrained by \( |\theta(\omega)| \leq \alpha \pi \). Therefore the maximal amplitude decreases for increasing \( \alpha \). Based on [26], we can also implement a weak
pmw to describe a freak wave by restricting the phases of only the frequencies of large energy carrying modes. The restriction is typically for frequencies within one (or a half) standard deviation $\sigma_\omega$, around the mean frequency $\omega_m$, that is $\theta_\alpha(\omega) = \alpha \theta(\omega)$ for $|\omega - \omega_m| < \sigma_\omega$.

3. Coherence of a freak wave

3.1. Coherence and prediction of a freak wave

For a given initial signal $\eta_0(t)$ at a position $X_0$, the uni-directional linear wave evolution is

$$
\eta(x, t) = \int |\eta_0(\omega)| \cos(k(x - X_0) - \omega t + \theta_0(\omega)) \, d\omega.
$$

The phase information in the frequency frame is defined by the total wave phase, $\Phi(x, t, \omega) = k(x - X_0) - \omega t + \theta_0(\omega)$ with $\theta_0(\omega)$ is the initial phase. Then we estimate the focussing position and time, $(X_{foc}, T_{foc})$, at which the variance of the total wave phase is minimal, $PV_{foc} = \text{Var}(\Phi(T_{foc}, X_{foc}, \omega))$. We also measure a degree of coherence, $\Gamma \in (0, 1)$ by

$$
\Gamma_{foc} = 1 - \frac{3}{\pi^2} PV_{foc}.
$$

For random phase $\theta_\alpha = \alpha \theta$, the phase variance and the parameter $\alpha$ is related by $PV(\theta_\alpha) = \frac{(\alpha \pi)^2}{3}$. Therefore, using the spectrum of $\eta_0(t)$, and calculating $\Gamma_{foc}$, we obtain the pmw with the phase band $\alpha_{foc}$. Instead of the pmw that estimates the freak wave in the neighbourhood of $(X_{foc}, T_{foc})$ only, we can reconstruct a predictive signal which is the signal of the linear wave evolution at position $X_{foc}$:

$$
\eta_{foc}(t) = \int |\eta_0(\omega)| \cos(\Phi(X_{foc}, t - T_{foc}, \omega)) \, d\omega
$$

To see the performance of the linear-based prediction, we will compare the pseudo-maximal signal (pms) and the signal prediction with the numerical simulation results by the nonlinear AB-equation in Section 4.

3.2. Local coherence

Subsection 3.1 discusses that a freak wave occurs when the total wave phase computed from the given signal has minimal variance. This gives the impression that a freak wave is developed by the whole interval from a given signal. However, most waves will not contribute to develop the freak wave. According to [27, 28, 29, 30, 31], a freak wave can be generated locally from clustered waves. Given a time signal, we investigate parts of the signal that may generate a freak wave based on the amount of the local energy, so-called group events [26]. The local energy is computed using wavelet transform. For a given signal, $\eta_0(t)$, the wavelet transform gives a complex valued function, $W\eta(u, \omega) = |W\eta(u, \omega)| e^{-i\theta(u, \omega)}$. The absolute value represents the distributed local energy and the angle represents the phase in the frame of time and frequency. We introduce a dominating group, that is chosen as the group event containing local energy larger than a certain threshold. Thus we investigate the local behaviour of the dominating group that will generate a freak wave. We compute the local time of the maximal energy in the time interval $u \in [t_1, t_2]$, by

$$
\tau(\omega) = \left\{ u \left| |W\eta(u, \omega)| = \max_u |W\eta(u, \omega)| \right. \right\}.
$$

Then, we define a local time spreading of waves as a function of frequencies, $\phi(\omega)$.

$$
\phi(\omega) = \left\{ \varphi(v) \left| \varphi(v) = \min_v \phi(v) \right. \right\} \text{ with } \varphi(v) = (\tau(\omega) - v) \text{ mod } 2\pi \text{ and } v \in [t_1, t_2]
$$
At the focussing time, $T_{\text{foc}}$, the mean of the local time spreading is then minimal, $\varphi(\omega) = \varphi_{T_{\text{foc}}}(\omega)$. If the dominating group gives a constant $\tau(\omega)$, all frequencies contribute at the same time which leads to a coherent state. We measure the appearance of a freak wave by local coherence, that is computed from the maximum ($M$), mean ($\mu$), or standard deviation ($\sigma$) of the time spreading. For a dominating group at $x$ position, the local coherence is defined as $\Gamma_{M,\mu,\sigma}(x)$, depending on the choice of the parameters, $M, \mu, \sigma$:

$$
\Gamma_M(x) = 1 - \frac{M}{\pi}, \quad \Gamma_\mu(x) = 1 - \frac{2\mu}{\pi}, \quad \Gamma_\sigma(x) = 1 - \frac{\sqrt{3}\sigma}{\pi}.
$$

We will show that the evolution of the highest amplitude of the dominating wave group is highly correlated with the local coherence in the restricted frequency interval.

4. Results

This section presents experimental cases, a dispersive focussing wave with harmonic background and the scaled New Year wave. From the measurements, we use the measured elevation data from the first gauge after the wave flap as the influx signal for the numerical simulations. We investigate the coherence of the freak waves and show the evolution of the dominating group.

4.1. Dispersive focussing wave with harmonic background

The experiment was executed at the water depth of 2.12m in the Towing tank 1 at Ship Hydromechanics Laboratory, Delft University of Technology. The performance of the linear and nonlinear AB-simulations is very convincing. Compared to the measurements at seven gauges, the linear and nonlinear AB-simulations give a high correlation, 0.97 and 0.98 respectively. This shows that in this case, the waves are mostly linear and there is only little change by the nonlinear contribution.

Figure 1 shows that the freak wave is mainly developed by the dominating group event from the influx signal. The linear-based prediction gives a minimum phase variance 0.45, related to a pmw with $\alpha = 0.37$ and a degree of coherence 0.86. It occurs at $t = 116.35s$ and $x = 70.2m$ from the wave flap, while the nonlinear AB-simulation results a maximal amplitude at $t = 116.72s$ and $x = 72m$. The maximum amplitude from the linear prediction reaches $1.27Hs$ and the nonlinear AB-simulation reaches $1.53Hs$. Both can be seen in Figure 2. We also describe the freak wave by the pms with parameter $\alpha = 0.37$. From Figure 2 we can see that the pms can describe the freak wave quite well. The evolution of the local energy of the dominating group is shown in Figure 3; at $x = 40m$ the energy is distributed in 10s and at $x = 70m$ it is squeezed in approximately 5s. The correlation between the maximal amplitude and the local coherences can be seen in Figure 4, showing that $\Gamma_M$ and $\Gamma_\sigma$ seem to be better than $\Gamma_\mu$ to measure the appearance of the freak wave.
4.2. Experimental New Year wave

The experiment was conducted at the water depth of 1m, approximately 1:70 of the original New Year wave [3] by MARIN hydrodynamic laboratory with code 204001. The numerical simulations by the linear and nonlinear AB-equation perform well. The mean correlations with the signal measurements at five gauges are 0.78 for the linear and 0.9024 for the nonlinear. This shows that there is significant improvement by adding the nonlinear contribution.

Based on the linear prediction, the maximal amplitude reaches $1.58Hs$ at $x = 48.15m$ and $t = 162s$, while the nonlinear AB-simulation gives $1.9Hs$ at $x = 49.6m$ and $t = 164.5s$. The pmw with parameter $\alpha = 0.56$ describes the freak wave quite well around the focussing time, even though the trough of the freak wave is deeper. Figure 7 shows the local energy at various
positions. The main wave frequency becomes higher: at $x = 20m$ the main wave frequency is about 3.5 and at $x = 50m$ it is about 4. This gives an indication that the waves become steeper. The local phase coherences $\Gamma_M$ and $\Gamma_\sigma$ give quite low correlation and $\Gamma_\mu$ seems to be the best indicator to measure the appearance of the freak wave in this case.

5. Conclusion
Freak wave appears at time and position of high phase coherence. It can be estimated by linear prediction, through minimizing the variance of the total wave phase of a given initial signal. It turns out that the freak wave can be described by a pseudo-maximal wave with a specific parameter that is related to the minimum phase variance. In the two examples presented in this paper, both freak wave positions and times are well estimated by the linear prediction. We remark that the prediction is not a statistical approach, but deterministically based on the phase coherence. The freak wave from the dispersive focussing wave with harmonic background is more coherent than the experimental version of the New Year wave. Both are well described by the pseudo-maximal wave with $\alpha = 0.37$ and $\alpha = 0.56$ respectively. We showed that the freak waves are developed by local waves interactions from a dominating group event that initially contained large energy.

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