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Spin-one color superconductivity in compact stars? - an analysis within NJL-type models

Abstract We present results of a microscopic calculation using NJL-type model of possible spin-one pairings in two flavor quark matter for applications in compact star phenomenology. We focus on the color-spin locking phase (CSL) in which all quarks pair in a symmetric way, in which color and spin rotations are locked. The CSL condensate is particularly interesting for compact star applications since it is flavor symmetric and could easily satisfy charge neutrality. Moreover, the fact that in this phase all quarks are gapped might help to suppress the direct Urca process, consistent with cooling models. The order of magnitude of these small gaps ($\simeq 1$ MeV) will not influence the EoS, but their also small critical temperatures ($T_c \simeq 800$ keV) could be relevant in the late stages neutron star evolution, when the temperature falls below this value and a CSL quark core could form.

Keywords spin-one color superconductors · compact star interiors · neutron star cooling

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1 Introduction

The most favorable places in nature where color superconducting states of matter are expected to occur are the interiors of compact stars, with temperatures well below 1 MeV and central densities exceeding the nuclear saturation density $\rho_0$ by several times.

Color superconductivity has been widely studied from non-perturbative low-energy QCD models where gaps of the order of magnitude of $\simeq 100$ MeV has been calculated (Rapp et al. (1998); Alford et al. (1998)). One of the effective models most used is the Nambu Jona-Lasinio (NJL) model that considers that the quarks interact locally by a 4-point vertex effective force and disregards the gluon degrees of freedom. The model uses an attractive interaction in the scalar meson channel that causes spontaneous chiral symmetry breaking if the coupling is strong enough. At the mean field level, the model shows how quarks acquire a dynamical constituent mass, which is proportional to the vacuum expectation value of the scalar field. The NJL model has then been extended and widely used to described successfully chiral restoration at finite temperatures and color superconductivity at finite density (for a review see Buballa (2005)).

For compact star applications, color superconducting quark matter phases enforcing color and charge neutrality has been widely studied (Alford et al. (2001); Steiner et al. (2002)). It has been shown that local charge neutrality may disfavor the occurrence of phases with large gaps where quarks with different flavor pair in a spin-0 condensate, like the 2SC phase (Alford & Rajagopal (2002)). On the other hand, NJL-type model calculations show that the intermediate density region of the neutral QCD phase diagram, where the quark chemical potential is not sufficiently large to have the strange quark deconfined ($\mu \geq 430 - 500$ MeV, see Buballa & Oertel (2002); Neumann et al. (2002)), might be dominated by $u, d$ quarks (Ruster et al. (2005); Blaschke et al. (2005)). If this is the case, 2-flavor quark matter phases may occupy a large volume in the core of compact stars (Grigorian et al. (2004); Shovkovy et al. (2003)).

While the stability of neutral 2SC pure phase is rather model dependent and might be unlikely for moderate coupling constants (Aguilera et al. (2005); Gomez Dumm et al. (2006)), phases with other pairing patterns (or none at all) become important for the phenomenology of neutron stars. Then, besides other possibilities, quarks could pair in spin-one condensates (Schafer (2000); Alford et al. (2000)).

1 no pairing (normal quark matter), pairing with displacement of the Fermi surfaces (LOFF or crystalline structure) or deformation of the Fermi surfaces, interior gap structure, gapless 2SC or gCFL, etc.
These condensates with small pairing gaps ($\Delta \simeq 1$ MeV), are not expected to have influence on the equation of state but could strongly affect the transport and thermal properties of quark matter and therefore leads to consequences for the phenomenology of compact stars.

An energy gap in the quasiparticle excitation spectrum introduces a suppression of the neutrino emissivity and the specific heat of the paired fermions by a Boltzmann factor $\exp(-\Delta/kT)$ for $T$ smaller than the critical temperature $T_c$ for pair formation. Thus, since the neutrino luminosity is strongly dominated by the neutrino emission from the core, the cooling of a neutron star during its early life will be affected by the dense matter pairing pattern. On the other hand, most of the specific heat of the star is provided by a Boltzmann factor $\exp(-\Delta/kT)$ which we will made explicit in Sec. 3.2 and Fig 3.

To study this qualitatively, we present in this work a summary of the results obtained in previous works of NJL model calculations of two spin-one pairing patterns, focusing in the CSL phase and a new analysis of features that are relevant to the crust confinement of the magnetic field (see discussion in Absence of ungapped modes in Sec. 3.2 and Fig 3).

After performing a linearization of $S_{\text{eff}}$ in the presence of the condensates and providing the inverse of the fermion propagator in Nambu-Gorkov space

$$S^{-1}(p) = \left( \begin{array}{c|c} \hat{p} + \mu_f & \hat{M} \\ \hline -\hat{M}^+ & \hat{p} - \mu_f \end{array} \right),$$

performing usual techniques of thermal field theory, the thermodynamical potential $\Omega(T, \mu_f)$ can be derived. At the mean-field level, i.e. the stationary points

$$\frac{\delta \Omega}{\delta \Delta^i} = 0, \quad \frac{\delta \Omega}{\delta M_f} = 0,$$

define a set of gap equations for $\Delta^i$ and $M_f$. Among the solutions, the stable one is the solution which corresponds to the absolute minimum of $\Omega$.

Thus, phases that present no gapless modes with small pair-breaking energies (e.g. $\Delta \simeq 1$ MeV) prevent the direct Urca to work uncontrolled and might help to give a consistent picture of the observed data (see Page et al. (2000, 2004); Yakovlev & Pethick (2004); Grigorian et al. (2005)).

On the other hand, it has been demonstrated that the CSL phase exhibit a Meissner effect and the magnetic field of a neutron star would be expelled from a CSL quark core if it does not exceed the critical magnetic field $B_c$ that destroy the superconducting phase. Schmitt et al. (2003). This hypothesis might be consistent with recent investigations that indicate the crust confinement of the magnetic field (Geppert et al. 2004; Pérez-Azorín et al. 2004).

To study this qualitatively, we present in this work a summary of the results obtained in previous works of NJL model calculations of two spin-one pairing patterns, focusing in the CSL phase, and a new analysis of features that are relevant to apply the CSL in phenomenological applications, like compact star cooling or protoneutron star evolution.

2 Brief on the model

We consider a two-flavor system of quarks, $q = (u, d)^T$, with an NJL-Lagrangian $\mathcal{L}_{\text{eff}} = \mathcal{L}_0 + \mathcal{L}_{\bar{q}q} + \mathcal{L}_{q\bar{q}}$, which contains a free part $\mathcal{L}_0 = \bar{q}(i\not \! \! \partial - m)q$, a quark-antiquark interaction channel $\mathcal{L}_{q\bar{q}}$ that corresponds to the condensate for each flavor $f$

$$\sigma_f = \langle \bar{q} f q \rangle, \quad (1)$$

and a diquark channel $\mathcal{L}_{q\bar{q}}$ with color superconducting condensate matrix $\hat{\Lambda}$ (components $\Lambda^i$) which we will made explicit in Sec. 3. The condensates $\sigma_f$ are responsible for dynamical chiral symmetry breaking in vacuum and define the constituent quark masses as

$$M_f = m_f - 4G\sigma_f \quad (2)$$

being $G$ the coupling constant in the meson scalar channel. In the density regime investigated, these condensates are relatively small but their finite size have consequences in the dispersion relations (see discussion in Absence of ungapped modes in Sec. 3.2 and Fig 3).

3 Spin-one pairing for compact stars applications

3.1 Unlikely: 2SCb phase

Besides the spin-0 isospin singlet condensate (2SC) of two colors (e.g. red and green), the remaining unpaired color (say blue) could pair in an anisotropic spin-one channel with an expectation value of $\hat{\xi} = \langle \hat{q}^T C\sigma^0 \tau_2 \bar{p}^0 \hat{q} \rangle$ (see Buballa et al. (2003) for NJL and Aguilera & Blaschke (2005) for a non-local extension). Then, in the 2SCb phase we assume

$$\hat{\Delta}^{2SCb} = \Lambda (\gamma_5 \tau_2 \lambda_2) (\delta_{+,+} + \delta_{+,u}) + \Lambda' (\sigma^0 \tau_2 \hat{P}_3^{(c)}) \delta_{+,b}$$

and the thermodynamical potential is

$$\Omega^{2SCb}(T, \mu) = \frac{(M_f - m)^2}{4G} + \frac{|\Delta|^2}{4G_1} + \frac{|\Delta'|^2}{16G_2}$$

$$-4 \sum_{\pm, j=1}^{3} \int \frac{d^3 p}{(2\pi)^3} \left[ E_{j}^\pm + T \ln(1 + e^{-E_{j}^\pm / T}) \right].$$

(6)
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The dispersion relations for the 2SC-paired quarks \((r, g)\)

\[
(E_{1,2}^-)^2 = (E^-)^2 = (\epsilon - \mu)^2 + |\Delta|^2,
\]

with the free particle energy \(\epsilon^2 = p^2 + M^2\) and for the third color \((b)\) quarks

\[
(E_3^-)^2 = (\epsilon_{\text{eff}} - \mu_{\text{eff}})^2 + |\Delta'_{\text{eff}}|^2
\]

where the effective variables depend on the angle \(\theta\), with \(\cos \theta = p_3/|p|\), and are defined as

\[
\epsilon_{\text{eff}}^2 = p^2 + M_{\text{eff}}^2, \quad M_{\text{eff}} = M + \frac{\mu}{\mu_{\text{eff}}}, \quad \mu_{\text{eff}}^2 = \mu^2 + |\Delta'_{\text{eff}}|^2 \sin^2 \theta, \quad |\Delta'_{\text{eff}}|^2 = |\Delta'|^2 (\cos^2 \theta + \frac{M^2}{\mu_{\text{eff}}^2} \sin^2 \theta).
\]

The antiparticles states \(E^+\) are obtained replacing \(\mu\) by \(-\mu\) and the coupling constants are taken as \(G_1 = 3/4G_2\) and \(G_2 = 3/16G_2\), from instanton induced interactions.

In Fig. 1 we show the results of solving the gap equations \((\ref{eq:gap_equations})\) for \(M_f, \Delta\) and \(\Delta'\). The gaps \(\Delta'\) are strongly \(\mu\)-dependent rising functions and typically of the order of magnitude of keV, at least two orders of magnitude smaller than the corresponding 2SC gaps. For the nonlocal extension, the gaps are even smaller, not larger than \(\Delta' \approx 0.05\) MeV in the \(\mu\)-range shown in Fig. 1. Such small gaps will have no influence on the equation of state and in this pattern the are no quarks that remain unpaired. Nevertheless, it is unlikely that the blue quark pairing could survive the constraint imposed by charge neutrality: the Fermi seas of the up and down quarks should differ by about 50-100 MeV and this is much larger than the magnitude of the gap in the symmetric case.

3.2 Likely: Color Spin Locking phase in compact stars

Single flavor spin-1 pairs are good candidates since they are inert against large splittings in the quark Fermi levels for different flavors caused by charge neutrality. They have been introduced first in \(\text{Schaefer (2004)}\) and \(\text{Alford et al. (2000)}\) and their properties have been investigated later more in detail, see \(\text{Schmitt (2004)}\) and more recently \(\text{Alford & Cowan (2006)}\). Among many possible pairing patterns (polar, planar, A, CSL), the transverse Color-Spin-Locking phase (CSL) has been demonstrated to be the ground state of a spin-1 color superconductor at \(T = 0\) having the largest pressure \(\text{Schmitt (2004)}\). Their non-relativistic limit reproduces the B-phase in \(^3\text{He}\), which locks angular momentum and spin and is also the most stable phase at \(T = 0\). We will show the features that make the CSL likely to occur in the interior of compact stars.

Single flavor and color neutral condensates. In the CSL condensates the color and spin are locked

\[
\langle q_f^2 \gamma^2 \lambda_{\alpha} q_f \rangle = \langle q_f^2 \gamma^2 \lambda_{\alpha} q_f \rangle = \langle q_f^2 \gamma^2 \lambda_{\alpha} q_f \rangle \equiv \eta_f (13)
\]

in an antisymmetric antitriplet in the color-space and an axial vector in the spin-space. A scheme of the pairing is shown in Fig 2 on the left. One can see immediately, that since

\[
\text{Cooper pairs in the CSL phase are single flavor the results will not be affected by charge neutrality and thus we have overcome one of the most restrictively constraints of quark matter in compact stars. Moreover, there are equal number of color antitriplets (\(\bar{r}, \bar{g}, \bar{b}\)) and color neutrality is automatically fulfilled. The CSL diquark gaps are}
\]

\[
\Delta_f = 4H_f, \eta_f
\]

Fig. 1 Spin-0 2SC + spin-1 of the blue quarks pairing pattern (left panel) and the corresponding dynamical mass and energy gaps (right panel) for symmetric two-flavor quark matter, \(\mu_u = \mu_d\). Parameters: \(A = 595.5\) MeV, \(G_A^2 = 3.37\), \(m = 5.56\) MeV. Fixed \(M = 380\) MeV.

Fig. 2 Spin-1 CSL pairing (left panel) and the corresponding dynamical mass and energy gaps (right panel as a function of a single flavor chemical potential \(\mu_f\), with \(f = u, d\)). For the NJL model (thick lines) the same parameterization as Fig. 1 is used. For the nonlocal extension (label L3), parameters are fixed to \(M = 380\) MeV, see \(\text{Aguilera et al. (2009)}\).
where \( H_v = 3/8G \) from Fierz-transformation of single gluon exchange.

One finds that the different flavors \((u, d)\) decouple \((\text{Aguilera et al. (2005)})\) and the thermodynamic potential is given by the sum

\[ \Omega_q(T, \mu) = \Omega_u(T, \mu_u) + \Omega_d(T, \mu_d) \]

where

\[ \Omega_f(T, \mu_f) = \frac{1}{8G}(M_f - m)^2 + \frac{3}{8G} |\Delta_f|^2 \]

\[ - \sum_{k=1}^{6} \frac{d^3p}{(2\pi)^3} \left( E_{f,k} + 2T \ln \left( 1 + e^{-E_{f,k}/T} \right) \right) \]

\[ (15) \]

**Absence of ungapped modes** Defining the effective variables as modified by the factor \( M_f/\mu_f \) that counts for finite mass effects

\[ \varepsilon_{f,\text{eff}}^2 = p^2 + M_{f,\text{eff}}^2 \]

\[ (16) \]

\[ M_{f,\text{eff}} = \frac{\mu_f}{\mu_{f,\text{eff}}} M_f \]

\[ (17) \]

\[ \mu_{f,\text{eff}}^2 = \mu_f^2 + |\Delta_f|^2 \]

\[ (18) \]

\[ \Delta_{f,\text{eff}} = \frac{M_{f,\text{eff}}}{\mu_{f,\text{eff}}} |\Delta_f| \]

\[ (19) \]

we can express the dispersion relation \( E_i \) for the first particle mode in the standard form

\[ E_{f,1}^2 = (\varepsilon_{f,\text{eff}} - \mu_{f,\text{eff}})^2 + \Delta_{f,\text{eff}}^2 \]

\[ (20) \]

while for the other particle modes the following approximation can be used

\[ E_{f,2:5}^2 \simeq (\varepsilon_{f} - \mu_f)^2 + c_{f,3:5} |\Delta_f|^2 \]

\[ (21) \]

\[ c_{f,3:5} \]

The dispersion relations for the antiparticles \((k = 2, 4, 6)\) can be obtained replacing \( \mu \) by \(-\mu\) and the corresponding coefficients \( c_{f,3:5} \) by \( c_{f,4:6} \) (see \text{Aguilera et al. (2005)} for a complete treatment).

The results of solving the gap equations \( (1) \) for the CSL phase are shown in Fig.\( \text{2} \) (right panel). The dynamical quark mass and the diquark gap are plotted as functions of \( \mu_f \). Their corresponding excitation energies \( E_i \), \( i = 1 - 6 \) for the critical chemical potential \( \mu_c \), the onset for the CSL phase, are shown in Fig.\( \text{3} \).

From Fig.\( \text{2} \) we see that the CSL gaps are strongly \( \mu_f \)-dependent functions in the considered domain. Since the constituent mass in vacuum \( M \) determines the \( \mu_f \) at which the chiral phase transition takes place (and thus the onset for the superconducting phase), the low density region is qualitatively determined by the parameterization. This is crucial for the later matching of the quark sector with a high density nuclear matter \( \text{EoS} \) and for the construction of stable configurations of hybrid stars. Models with an onset of the quark matter phase at high densities might not give stable quark cores \((\text{Buballa et al. (2004)})\).

From Fig.\( \text{3} \) we have learned that there are no gapless modes as a direct consequence of keeping the finite size of the mass of the \( u, d \) quarks in \( (19) \). This may play a crucial role suppressing the neutrino emissivities and preventing the direct Urca process to work. With unpaired quark species, the direct Urca is so efficient that cool down the star too fast, in disagreement with observational data\(^2\).

3.3 Neutral CSL and critical temperatures

It is straightforward now to consider charge neutral matter introducing an electron chemical potential \( \mu_e \), with \( \mu_u = \mu - \frac{3}{5} \mu_e \) and \( \mu_d = \mu + \frac{1}{5} \mu_e \), such that the total charge vanishes. Therefore, the effect of introducing charge neutrality is that the energy gap for the two flavors splits in two branches: \( \Delta_f, \Delta u \), where \( \Delta_u < \Delta_f \) for a given \( \mu \) as it shown in Fig.\( \text{4} \).

The difference between them could be as large as a factor \( \approx 10 \) for low densities. The onset for neutral CSL matter is displayed in both parameterizations considered.

We analyze the temperature dependence of the gaps in Fig.\( \text{5} \). They show a decreasing behavior that ends at the critical temperature \( T_c \), where a second order phase transition to normal (unpaired) quark matter occurs. It is worth to notice that the critical temperature \( T_c \) for the CSL phase deviates from the BCS relation \( T_c \approx 0.57 \Delta (T = 0) \), as it has been demonstrated for weak coupling in \((\text{Schmitt (2004)})\), and follows

\[ T_{c,f} \approx 0.82 \Delta_f (T = 0) \]

\[ (22) \]

Our calculations confirm this result and are shown in the phase diagram of Fig.\( \text{6} \). After the critical potential \( \mu_c \) for the occurrence of CSL phase, \( \mu_c = 382.1 \text{ MeV} \) for the \( M = 380 \text{ MeV} \) parameterization, two lines labeling \( T_c \) for the two flavor species appear: as quark matter cool down, first the \( d \)

\[^2\text{ although data might be also consistent with models that allow direct Urca in quark matter, lowering the core temperature but retain the surface temperature high enough due to e.g. Joule heating in the crust (\text{Pons et al. (2006)})} \]
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Fig. 4 Neutral matter: $\Delta_u, \Delta_d$ as a function of $\mu$. Two different parameterizations are considered: $M = 300$ MeV on the left and $380$ MeV on the right. Note the different scales for $\Delta$ in both cases showing that $\Delta$ is very sensitive to the variation of the constituent mass $M$ and the corresponding parameterization.

Fig. 5 Neutral matter: Temperature dependence of the CSL gaps.

Fig. 6 Two flavor neutral matter at intermediate densities: critical temperatures are shown for the $d$ (solid line) and for $u$ (dashed line) quarks as a function of $\mu$. The thick lines correspond to the parameterization of NJL with $M = 300$ MeV and the thin lines to $M = 380$ MeV.

model calculations performed with massless $u-d$ quarks and the mass of the $s$ quark taken through an effective chemical potential, show that for large strange quark mass, CSL dominates the phase diagram at low temperature and a second order phase transition to unpaired quark matter occurs as the temperature increases. So, we expect that when the densities in the interior of a compact star are high enough to allow for two-flavor quark matter but not so high to have the strange quark deconfined, and the temperature has fallen below $T_c$, a CSL quark core might develop. Moreover, for such small gaps the pressure is expected to be approximately the one of the normal $u-d$ quark matter and it has been obtained that hybrid stars with a relatively large normal quark matter core can be stable (Grigorian et al. (2004)).

3.4 Neutral CSL and magnetic field

Let’s suppose that in the core of a neutron star the density is high enough that a quark core could be formed and has already condensed to the CSL phase. The final question we try to address is the nature of the interaction between the magnetic field present in a neutron star and the CSL quark core. Although this is a question that requires a careful analysis, the aim of this section is to review the work done on this matter and present estimations derived from the CSL gaps calculated before.

For the interaction of the CSL phase with an external magnetic field $B$ it has been stated that the CSL phase exhibits an electromagnetic Meissner effect (Schmitt et al. (2003)) since all the gluons and the photon acquires a non-vanishing Meissner mass proportional to the quark chemical potential (multiplied by the appropriate gauge coupling) (Schmitt et al. (2004)). The formation of the Cooper pairs breaks the symmetry (color-spin) $SU(3)_c \times SO(3)_J$ to $SO(3)_{c+J}$, that is a global symmetry. Concerning electromagnetism, it was shown
that there is no symmetry subgroup left of $SU(3)_c \times U(1)_{em}$ different from the trivial and thus there is no combination of color and electric charge for which the Cooper pairs are invariant.

This is in contrast to the spin–0 2SC and CFL phases that although the condensate has non-zero electric charge, there is a residual local symmetry $\hat{U}(1)$ with an associated linear combination of the photon and a gluon (“rotated photon”) that remains massless. The “rotated” $\hat{B}$ field can penetrate and propagate in the 2SC and in the CFL phase (Alford et al. 2000). Therefore, 2SC and CFL phases are not superconductors from the electromagnetic point of view and the magnetic field $\hat{B}$ will penetrate without the restriction to be quantized to flux tubes and is stable over very long time scales. On the other hand, $B$ cannot penetrate in the CSL phase unless it exceeds the critical magnetic field $B_c$ that destroy the superconducting phase.

Similarly to the ordinary superconductors, we can estimate the penetration length as the inverse of the Meissner mass of the photon (γ) and/or the gluon (a) that would penetrate the matter following (Schmitt et al. 2004)

$$\lambda_{\gamma,a} \simeq \frac{1}{m_{\gamma,a}} \simeq \frac{1}{(e,g)\mu}$$  \hspace{1cm} (23)

where $e, g$ are their corresponding coupling constants in dense matter. At densities in neutron stars, photons are weakly coupled $e^2/4\pi \simeq 1/137$ while gluons are strongly coupled $g^2/4\pi \simeq 1$ and taking a typical value of $\mu \simeq 400$ MeV we obtain that the penetration length is $\lambda \simeq 1 - 10$ fm.

The coherence length is proportional to the inverse of the energy gap $\xi \simeq 1/\Delta$ and from our previous results we obtained that being $\Delta \simeq 1$ MeV $\approx 1/200$ fm. Then, the ratio

$$\frac{\lambda}{\xi} \simeq \frac{1 - 10}{200} \ll 1/\sqrt{2},$$  \hspace{1cm} (24)

confirms that CSL is a Type-I superconductor and the magnetic field will be expelled from macroscopic regions if it does not exceed the critical field to destroy the condensate. This may be consistent with recent investigations that state that surface temperature anisotropies inferred from the observations are due to the crust conflation of the magnetic field (Geppert et al. 2004; Pérez-Azorín et al. 2006). Moreover, studies of the light curves of neutron stars have shown evidence of precession and if this is confirmed, this might be inconsistent with Type II superfluidity. Because of the strong interaction between the rotational vortices and the flux tubes in the latter, Type I superconductivity may be required (Link 2003).

We can estimate the critical field $B_c$ using the Ginzburg-Landau approach following Bailin & Love (1983).

$$B_c^2 = \frac{8\mu_P}{7\xi(3)} \left(\frac{k_B T}{T_c}\right)^2$$  \hspace{1cm} (25)

Assuming that due to the low mass of $u,d$, the Fermi momentum $p_F \simeq \mu$ and if the $T << T_c$ then the parameter

4 Heaviside-Lorentz units are used: $\mu_0 = e_0 = 1$, $\hbar = c = k = 1$. $e = \sqrt{4\pi/137} \approx 0.3$, $g \approx 3.5$ and thus $\text{GeV}^2 = 5.10^{19}$ G

$t = (T - T_c)/T_c \simeq 1$. We can also express $T_c \simeq 0.82 \Delta (T = 0)$ and consider the $\mu$-dependence of the gaps to obtain $B_c(\mu) \simeq 0.8 \mu \Delta(\mu)$ and in relevant units

$$B_c(\mu) = 1.6 \left( \frac{\mu}{400\text{MeV}} \right) \left( \frac{\Delta(\mu)}{\text{MeV}} \right) 10^{16} G.$$  \hspace{1cm} (26)

Therefore for typical values of $\mu = 400$ MeV and CSL gaps not exceeding $\Delta = 1$ MeV, we have that $B_c \simeq 10^{16}$ G. In Fig. 7 we display $B_c(\mu)$ taking into account the density dependence of our previous NJL model results. As the figures show, $B_c$ is highly density dependent, and due to the different $\mu_u, \mu_d$ there is a splitting in the critical field for the two flavors: $B_u^c \leq B_d^c$.

What are the consequences for compact stars? First, due to the strong density dependence, we expect that if a CSL quark core forms, it begins to grow from the center of the star. Since it is a Type I superconductor, the magnetic field will be expelled from it. If $B$ is not so large that the CSL phase persist against the magnetic field ($B \leq B_u^c$), then, the characteristic times for the expulsion of the field over macroscopic regions and for the quark core grow start to compete. According to Ouyed et al. (2004), the magnetic field expulsion time over a sizable region ($\simeq 10$ m) can be evaluated as proportional to the electrical conductivity in the normal phase $\sigma_{el} \simeq 10^{23 - 24}\text{s}^{-1}$ (Shovkovy & Ellis 2003),

$$\tau_{\exp} \simeq \frac{10^{23}\text{s}^{-1}T}{\left(\frac{\delta}{10m}\right)^2 \left(\frac{B}{10^{15}\text{G}}\right)^8}$$  \hspace{1cm} (27)

getting typical values of seconds. An estimation of how fast the quark core develops is unknown, but if it is much smaller than $\tau_{\exp}$ it might not have time to expel $B$. Since CSL and the magnetic field do not coexist, $B$ would be frozen in an mixed state composed of alternating regions of normal material with flux density $H_c$ and superconducting material exhibiting Meissner screening. This would be also the case for intermediate fields, comparable with $B_c$. For the case $B_u^c \leq B \leq B_d^c$, although $B$ will try to destroy the $u-$condensates,
$d$-currents will cancel the field inside, restoring the superconducting phase. As a result, we speculate that the field structure will be rearranged to give the mixed state described above.

The spin down estimates from highly magnetized neutron stars (magnetars) show that the magnetic field could be as large as $B \approx 10^{15}$ G (Woods & Thompson (2004)). If one consider that they could reach even higher values (one order of magnitude more?) as a result of flux conservation into confined regions, the scenario of a CSL quark core under the stress of magnetic fields that are of the order of $B_c$ seems not to be unrealistic.

4 Summary and outlook

We summarize here the interesting features that the CSL phase presents for compact star applications:

- since Cooper pairs are single flavor they are not affected by charge neutrality
- color neutrality is automatically fulfilled
- there are no gapless modes. This has important consequences e.g. in neutrino emissivities, suppressing the direct Urca process that leads to a rapid cooling
- their small gaps ($\Delta \approx 1$ MeV) will not influence on the EoS, and the pressure is expected to be approximately the one of the normal $ud$-quark matter without pairing. Similar to the latter, stable hybrid star configurations could be obtained with a relatively large quark matter core.
- it exhibits a Meissner effect and the magnetic field of a neutron star would be expelled from a CSL quark core if it does not exceed the critical magnetic field $B_c$ that destroy the superconducting phase. This hypothesis might be consistent with recent investigations that indicate the crust confinement of the magnetic field. Moreover, the hypothesis of a Type I superconductor is supported by the inferred precession of some compact objects.

This study presents a qualitative analysis of the spin-1 pairing and cannot be taken as conclusive to decide whether this phase could be realized in neutron star cores. However, the features listed show that CSL could be consistent with the phenomenology of compact stars in many aspects.

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