Hybrid Rotation Averaging: A Fast and Robust Rotation Averaging Approach

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Abstract

We address rotation averaging (RA) and its application to real-world 3D reconstruction. Local optimisation based approaches are the defacto choice, though they only guarantee a local optimum. Global optimisers ensure global optimality in low noise conditions, but they are inefficient and may easily deviate under the influence of outliers or elevated noise levels. We push the envelope of rotation averaging by leveraging the advantages of global RA method and local RA method. Combined with a fast view graph filtering as preprocessing, the proposed hybrid approach is robust to outliers. We apply the proposed hybrid rotation averaging approach to incremental Structure from Motion (SfM) by adding the resulting global rotations as regularisers to bundle adjustment. Overall, we demonstrate high practicality of the proposed method as bad camera poses are effectively corrected and drift is reduced.

1. Introduction

Rotation averaging is a problem that consists of estimating absolute camera orientations that agree as well as possible with a set of pairwise relative orientations. Errors expressing disagreements between estimated absolute orientations and the measured relative orientations are hereby distributed over each pairwise constraint. Rotation averaging is essential in global or hierarchical Structure from Motion (SfM) [29, 15, 36, 49, 50], as well as Simultaneous Localization and Mapping (SLAM) [8] where it can accelerate camera pose estimation and reduce drift accumulation.

In global SfM, we typically start by constructing a view graph $G$ that encodes all connections between pairs of views $i$ and $j$ by an edge $(i,j)$, each one including the relative motion between image $i$ and image $j$. Rotation averaging then gives us the absolute orientation of each view, and it is typically followed by a translation averaging step [26, 31, 22, 51] to also obtain absolute positions. Triangulation of 3D points and joint optimisation over all parameters (i.e. bundle adjustment [39]) completes the reconstruction. In SLAM, rotation averaging has been used in the back-end pose graph optimisation [33, 8] to flexibly encounter large drift accumulations or—more generally—replace the time-consuming bundle adjustment step.

Rotation averaging was first proposed using the quaternion representation [23]. Later solutions can be categorised into approaches based on either local and global optimisation. Local optimisation approaches such as the one presented by Chatterjee and Govindu [9] are well studied and practical. However, these methods only return the nearest local minimum. To overcome this limitation, the community has also proposed global optimisation approaches [40, 33, 19, 18, 16]. Though the retrieval of global optima can be guaranteed, they have large computational cost and high sensitivity against outliers, and thus are impractical when applied to large-scale SfM problems.

In this paper, we focus on improving the efficiency and robustness of rotation averaging method, and push its application to challenging scenes. We make a combination of global solver and local solver to solve rotation averaging, with global optimality is guaranteed and outliers insensitive. Rotation averaging based on chordal distances can be reformulated as a semi-definite program (SDP) with a low-rank constraint. Taking advantage of low-rank factorisation of original SDP, we can apply the Riemannian-Staircase-based [4] methods, with global optimum guaranteed. In
principle, any global solvers [33, 37, 17] can be used in our hybrid framework, we adopt the block coordinate minimisation method [37] to better leverage the view graph sparsity. By preprocessing the graph with fast view graph filtering, graph sparsity can be further exploited to accelerate the optimisation.

Previous works mainly apply rotation averaging to global SfM. Though global SfM is efficient, translation averaging is often complicated by the unknown scale of relative translations and the difficulty of identifying outliers. In this work, we embed the proposed approach into an incremental SfM pipeline. The strategy is inspired by the work of Cui et al. [13], who propose a hybrid SfM scheme where camera rotations are estimated globally, and camera centers are estimated incrementally by a perspective-2-point (P2P) method. Though the approach is efficient, some globally estimated rotations are not correct preventing camera centers from proper registration and scene reconstructions from completeness. Inspired by Cui’s approach [13], we apply rotation averaging in a traditional incremental SfM pipeline, however use the perspective-3-point (P3P) to register camera rotations and centers.

We further propose a novel cost function to optimise camera poses and landmarks that alleviates drift accumulation, in which camera rotations obtained from rotation averaging are used as regularisers. The resulting SfM approach, named RA-SfM (Rotation Averaged Structure from Motion), shows high practicality and at least comparable performance to other methods. Though the approach is efficient, some globally estimated rotations are not correct preventing camera centers from proper registration and scene reconstructions from completeness. Inspired by Cui’s approach [13], we apply rotation averaging in a traditional incremental SfM pipeline, however use the perspective-3-point (P3P) to register camera rotations and centers.

We further propose a hybrid rotation averaging scheme, which combines the global optimiser with fast view graph filtering and the local optimiser, that is able to robustly handle outliers.

• We refine the traditional incremental bundle adjustment cost function by adding the obtained global rotations as a regularisation term, which significantly alleviates drift accumulation in incremental SfM.

The practicality and superiority of the proposed scheme is demonstrated by extensive experiments on large-scale real-world datasets.

2. Related Work

Motion averaging [23, 24] is widely used in global SfM pipelines [29, 15, 36, 49, 50] as an answer to the drift problem occurring in incremental SfM [3, 46, 34, 14]. The first solution to rotation averaging goes back to Govindu [23], who uses the quaternion representation and solves the problem by linear least-squares fitting. More reliable results were later on gained by optimising over a Lie algebra [24]. In practice, the problem is complicated by the existence of outliers. To enhance the robustness of rotation averaging, absolute rotations may first be initialised under the $L_1$-norm, and then refined by Iteratively Reweighted Least Squares (IRLS) [9, 10]. Despite great progress, all aforementioned approaches can only guarantee a locally optimal solution. Another local approach was proposed by Crandall et al. [11, 12], who couple the cost function with regularisation terms to enhance robustness. However, the method is computationally demanding as it relies on discrete belief propagation over a Markov random field.

Fredriksson and Olsson [21] exploit Lagrangian duality to become the first to find a globally optimal solution to the rotation averaging problem. In a similar approach, Eriksson et al. [18] perform the optimisation directly on the rotation matrix by minimising chordal distances. By removing the determinant constraint on the rotation from the original SDP, they elegantly prove that there is no duality gap between the primal problem and its dual when residual errors are bounded below an angular residual threshold.

Rotation averaging can be converted into an SDP optimisation problem [6]. Wang and Singer [41] solve it by the Alternating Direction Method of Multipliers (ADMM) [5, 44]. Eriksson et al. [18] use a row-by-row block coordinate descent method (BCM) [43]. However, due to the slow convergence of ADMM and the repetitive fill-in procedures of BCM, neither approach proves to be practical when applied to large-scale datasets. A seminal work on the solution of SDP problems is presented by Burer and Monteiro [7], where the positive semi-definite variable is replaced by an appropriate factorisation, and the minimal rank variable is chosen to enhance computational speed. The Burer-Monteiro factorisation later inspired Boumal [4], who proposes a general optimisation technique named the Riemannian staircase algorithm, where the rank variable is augmented until the KKT condition is met, thus guaranteeing global optimality. Rosen et al. [33] address the SDP problem of pose graph optimisation in the Special Euclidean space (SE(n)). When translation variables are decoupled from rotations, they first find the second-order critical point by the second-order Riemannian trust-region method, and then adopt the low-rank optimisation framework of [4] to guarantee global optimality [33]. Inspired by Wang’s work et al. [42], which solves the low-rank SDP problem by block coordinate descent method, Tian et al. [37] extends this work to Steifel manifold, and further applied a Riemannian BCM method to pose graph optimisation in distributed settings [38]. Building on SE-Sync [33], Dellaert et al. [16] propose Shonan rotation averaging, a method in which the rotation matrix is vectorized, thus permitting the use of existing gradient-based optimisation methods (e.g. Gauss-Newton or Levenberg-Marquardt) on the manifold of rotation matrices. However, both SE-Sync and Shonan ro-
tation averaging need to evaluate and maintain the Jacobian matrix, which slows down both approaches.

3. Notations and Preliminaries

Let $\mathcal{G} = \{V,E\}$ be an undirected graph, where $V$ represents the collection of nodes and $E$ the set of edges. Let $m = |E|$ be the number of edges and $n = |V|$ be the number of nodes. Let $\text{tr}(\cdot)$ denote the trace of a square matrix. Given two matrices $A \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{m \times n}$, let $\langle A, B \rangle = \sum_i \sum_j A_{ij} B_{ij}$. We therefore have $\text{tr}(A^T B) = \langle A, B \rangle$. Let $\text{blockdiag}(A)$ represent the block diagonal matrix of $A$, and $\text{symblockdiag}(A) = \frac{1}{2} \text{blockdiag}(A + A^T)$.

The set of rotations in 3D forms the Special Orthogonal Group $\text{SO}(3)$, i.e.,

$$\text{SO}(3) = \{ R \in \mathbb{R}^{3 \times 3} \mid R^T R = I, \det(R) = 1 \}. \quad (1)$$

Since $\text{SO}(3)$ is a Lie group, there exists an exponential mapping between a rotation $R$ and its Lie algebra $\mathfrak{so}(3)$ representation $w$ [28]:

$$R = \exp([w]_\times). \quad (2)$$

The absolute rotations are grouped in $\mathcal{R} = \{ R_1, R_2, \ldots, R_n \}$, where $R_i \in \text{SO}(3)$, $i \in [n]$. Relative rotations are represented by $\mathcal{R}_{\text{rel}} = \{ R_{ij} \}$, where $R_{ij} \in \text{SO}(3)$, $i, j \in [n]$, $i < j$ is the rotation from $R_i$ to $R_j$. The chordal distance between two rotations is measured by [25]

$$d_{\text{chord}}(R_1, R_2) = \| R_1 - R_2 \|_F, \quad (3)$$

where $\| \cdot \|_F$ represents the Frobenius norm of a matrix.

4. Hybrid Rotation Averaging

Globally optimal rotation averaging is sensitive to outliers, thus requiring an additional step to clean the view graph. In this section, we first present an efficient preprocessing step to filter outliers in the view graph. We then apply a block coordinate descent (BCD) method [37] to optimise the low-rank formulation of rotation averaging. Its global optimality can be guaranteed theoretically. Finally, we apply a local optimisation step to further refine the result in the case of scenes that have many erroneous edges.

4.1. Fast View Graph Filtering

The view graph plays an important role in our SfM pipeline. We clean the view graph for two main reasons: (1) Solutions of global rotation averaging algorithms can be biased by outliers. In addition, global optimality is only guaranteed when the residuals for each edge are bounded below a certain threshold [18]. (2) Some view pairs are redundant and even have negative impact on the quality of SfM results. Zach et al. [47] proposed a view graph filtering (VGF) technique to obtain a high quality initial view graph, where loop constraints of rotation triplets are utilised to detect outliers. Specifically, edge $(i, j)$ is an outlier if its angular error satisfies

$$d(R_{ij} R_{jk} R_{ki}, I) > \epsilon. \quad (4)$$

Despite its effectiveness, [47] needs to validate all triplets, which is impractical for large-scale datasets. However, [35] suggests that it is not necessary to check all triplets to distinguish inliers from outliers, and that an increased number of valid 2D-2D image correspondences usually suggests more reliable two-view geometries. We propose an efficient view graph filtering method that relies on this observation. In the following, we denote a group of 3 nodes as a triplet, and a triplet with two valid edges and one unverified edge as a weak triplet.

Given an initial view graph $\mathcal{G}$, we start by constructing a maximum spanning tree (MST), where the weight of an edge is the number of valid 2D-2D correspondences. The relative rotations from this MST are all treated as valid. We then check the triplets along with the MST. That is, all adjacent edges that share a common node in the MST are used to build triplets. Next, we generate many weak triplets. Now supposing that edges $(i, j)$ and $(j, k)$ are valid and edge $(i, k)$ exists, we use criterion (4) to verify the validity of edge $(i, k)$. An iteration is completed once all such weak triplets have been verified. After the first iteration, new weak triplets are generated based on which we can perform another iteration. We empirically found that 3 iterations are sufficient for successful rotation averaging.

4.2. Global Rotation Averaging Review

In this section, we first review a globally optimal guaranteed rotation averaging method [37], then the sparsity pattern of the view graph is further exploited to accelerate the algorithm.

Given a set of relative rotations $\{ R_{ij} \}$, where $i, j \in [n]$, the aim of rotation averaging is to obtain the absolute rotations $\{ R_i \}$ that satisfy the constraints

$$R_{ij} = R_j R_i^{-1} = R_j R_i^T \quad (5)$$

between absolute and relative rotations. Usually there are more edges than nodes in an undirected graph, so there are more constraints than unknowns. Rotation averaging can be formulated as

$$\min_{R_1, \ldots, R_n} \sum_{(i,j) \in E} d^p(R_{ij}, R_j R_i^T), \quad (6)$$

where $d^p(\cdot)$ represents a distance measure under a $p$-norm. While there are a lot of local methods [23, 24, 9, 10] giving a least-squares solution to problem (6), here we exploit a global optimisation approach that can obtain the global optimum.
Chordal distances is popular as the distance measure in (6). Each residual along an edge of $G$ will hence read
\begin{align}
\|R_j - R_{ij} R_i\|^2_F &= ||R_j||^2_F - 2 \text{tr}(R_j^T R_{ij} R_i) + ||R_i||^2_F \\
&= 6 - 2 \text{tr}(R_j^T R_{ij} R_i).
\end{align}

The set of absolute rotations can be represented by
\[ R = [R_1, R_2, \ldots, R_n], \] (8)
and the graph $G$ can be represented by
\[ G = \begin{bmatrix}
0 & a_{12}R_{12} & \cdots & a_{1n}R_{1n} \\
a_{21}R_{21} & 0 & \cdots & a_{2n}R_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{n1}R_{n1} & a_{n2}R_{n2} & \cdots & 0
\end{bmatrix}, \] (9)
where $a_{ij} = 1$ if the edge between views $i$ and $j$ exists, and $a_{ij} = 0$ otherwise. By combining Eqs. (7), (8) and (9), problem (6) can be rewritten as
\[ \min_R (6 - \text{tr}(R^T G R)) \Leftrightarrow \min_R - \text{tr}(R^T G R). \] (10)

The primal problem of rotation averaging is finally given by
\[ \min_R - \text{tr}(R^T G R) \quad \text{s.t.} \quad R \in \text{SO}(3)^n. \] (11)

Eriksson et al. [18] solve the dual problem with determinant constraint relaxation
\[ \min_X - \text{tr}(G X) \quad \text{s.t.} \quad X_{ii} = I_3, \ i = 1, \ldots, n, \ X \succeq 0, \] (12)
where $X$ is partitioned as
\[ X = \begin{bmatrix}
X_{11} & X_{12} & \cdots & X_{1n} \\
X_{21} & X_{22} & \cdots & X_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
X_{n1} & X_{n2} & \cdots & X_{nn}
\end{bmatrix}. \] (13)

They furthermore prove that there is no duality gap between the primal problem (11) and its dual problem (12) if the maximum residuals stay below a certain threshold. If $R^*$ is the global optimum of problem (11), the solution of problem (12) satisfies
\[ X^* = R^*^T R^*. \] (14)

For readers who are interested in the duality gap and global optimality proof, we kindly refer them to [18] for more details.

Problem (12) is an SDP problem [6]. Since every $X \succeq 0$ can be factored as $Y^T Y$ for some $Y$ [7], and—in the case of rotation averaging—the optimal value of $X$ satisfies $X^* = R^*^T R^*$ [18], there is an implicit constraint on $X$ such that rank($X$) = 3. Thus, $X$ can be reformulated as
\[ X = Y^T Y, \] (15)
where $Y = [Y_1, Y_2, \ldots, Y_n], Y_i^T Y_i = I, \forall i \in [n]$. By substituting (15) into problem (12), a new problem is obtained
\[ \min_Y - \text{tr}(G Y^T Y) \quad \text{s.t.} \quad Y = [Y_1, Y_2, \ldots, Y_n], \ Y_i^T Y_i = I, \forall i \in [n]. \] (16)

Tian et al. [37] proposed a block coordinate minimisation (BCM) method to solve the low-rank SDP problem (16). The update of $Y_j$ in problem (16) is determined by [37]
\[ Y_j^* = U_j I_{3 \times 3} V_j^T = U_j V_j^T, \] (17)
where $U_j \Sigma V_j$ is the singular value decomposition of $-Q_j$. In Algorithm 1, we outline the BCM with graph sparsity.

In steps 6~7 of Algorithm 1, the time complexity of $O(n)$ is only an upper bound occurring for general cases. In practice, due to the commonly sparse structure of SfM problems, time complexity can be further reduced to $O(d)$, where $d$ is the degree of the nodes. This property is important for accelerating the optimisation. Notice that, with our fast view graph filtering, the graph can be more sparse and we can gain more acceleration.

**Algorithm 1 BCM for SDP [37] with Graph Sparsity**

**Input:** relative rotations $\mathcal{R}_{rel}$, maxIterNum, $Y^0$.

**Output:** First-order critical point $Y^*$

1: $k \leftarrow 0$; $Q_j^0 \leftarrow \sum_{i \neq j} Y_i G_{ij}, \forall j \in [n]$.
2: while $k < \text{maxIterNum}$ and not converge do
3:   for $i < n$ do
4:     $j_k \leftarrow i$
5:     Update $Y_j^{k+1}$ by Eq. (17)
6:   for $\forall j \neq j_k, G_{jk} \neq 0$ do
7:     $Q_j^{k+1} \leftarrow Q_j^k + (Y_j^{k+1} - Y_j^{k}) G_{jk}$
8:   end for
9:   $k \leftarrow k + 1$
10: end while

**Discussion of Global Optimality:** Problem (16) is non-convex, and there is no guarantee that we can obtain the global optimum. In Wang’s method [42], the global optimum is guaranteed by selecting an appropriate step size and random initialisation (Theorem 3.4). However, the theorem is only hold for variables that are scalar. For RA problem, global optimality is not hold since we optimised on manifold. Boumal [4] proposes a general framework
named the Riemannian Staircase algorithm (RS), which can find the global optimum. As previous work has applied a Riemannian based method to ensure the global optimum [33, 38, 17], we recommend interested readers to refer to them.

4.3. Local Refinement

The global optimisation presented in Sec. 4.2 assumes that the input relative rotations do not contain any outliers. As a result, it is sensitive to outliers. To further improve the robustness and accuracy of the low-rank BCM method, we follow the suggest-and-improve framework of [32]. The global optimisation approaches can obtain a good solution close to the global optimum. Still, it could be further refined by a gradient descent algorithm. We adopt this framework and use the method of Chatterjee and Govindu [9] as a local optimiser. This method performs an Iteratively Reweighted Least Square (IRLS) under Lie algebra, which leads to an efficient and robust optimiser. The rotation $R_{ij}, R_i, R_j$ can be represented by the corresponding Lie algebra $\omega_{ij}, \omega_i, \omega_j$, respectively. Using the Baker-Campbell-Hausdorff (BCH) equation, the relationship of Eq. (5) can be converted to

$$\omega_{ij} = \omega_j - \omega_i. \quad (18)$$

By collecting the relative constraints, we obtain

$$A\omega_{\text{global}} = \omega_{\text{rel}}. \quad (19)$$

Here $A$ is a sparse matrix, in which all consecutive $3 \times 3$ blocks are zeros except two matrices $I$ and $-I$. Encapsulating Eq. (19) with a Huber loss $\rho(x) = \frac{x^2}{2} + \frac{\sigma^2}{2}$, we can optimise a robust cost function under least square meaning

$$\arg\min_{\omega_{\text{global}}} \sum \rho(\|A\omega_{\text{global}} - \omega_{\text{rel}}\|). \quad (20)$$

4.4. Hybrid of Global Rotation Averaging and Local Refinement

We outline our hybrid rotation averaging algorithm in Algorithm 2. An ablation study of robustness against outliers is shown in Fig. 2. The outlier ratio ranges from 0 to 50% and is incremented in steps of 5%. We display the mean rotation error over 30 experiments. As can be observed, both VGF and local refinement improve the robustness of the global rotation averaging approach.

5. Application to Structure-from-Motion

In this section, we apply our rotation averaging method to an incremental SFM pipeline, which is known to suffer from the drift problem. Our hybrid SFM pipeline can be summarised as follows: We first construct the view-graph and obtain global rotations from our proposed fast and robust rotation averaging approach. Next, we create a seed reconstruction by selecting two appropriate images. We then continue by incrementally registering adjacent camera poses using a RANSAC-based [20] P3P [27] algorithm, and triangulate landmarks. To reduce the accumulation of errors in our incremental SFM pipeline, we perform local bundle adjustment after each successful registration of an image, and global bundle adjustment whenever the number of recently added views surpasses a certain threshold.

The drift problem is not solved, as each newly computed camera pose is affected by a small error, and these errors accumulate along the graph. Traditional incremental SFM pipelines have no way to rectify these errors. To tackle this problem, we introduce a novel cost function with averaged rotations as regularisers for bundle adjustment. Let $I_i$ denote the measurements of image $i$. Each image observes 3D landmarks given by the set $\mathcal{P}_i$. Let $u_{il} \in I_i$ furthermore denote the image keypoint measurement of landmark $l$ in frame $i$. Grouping image measurements with the precomputed known rotations, we have

$$Z = \{I_i, \hat{R}_i, \hat{R}_j | (i, j) \in \mathcal{E}\} \quad (21)$$

In SFM, camera poses are estimated as well as 3D points observed by cameras. We have the following unknowns:

$$\mathcal{X}_i = \{R_i, C_i, \mathcal{P}_i\}, \; \mathcal{X} = \{\mathcal{X}_i | i \in [0, N]\}, \quad (22)$$

![Algorithm 2 Hybrid Rotation Averaging Algorithm](image)

**Figure 2.** Ablation study of robustness. Outliers are generated by perturbing ground truth by rotation between $60^\circ - 90^\circ$. **global** represents the proposed low-rank BCM method, and **hybrid** represents the proposed hybrid method with VGF and local refinement.
where $R_i$ and $C_i$ are the estimated camera rotation and center, respectively. Note that the sets $P_i$ are intersection, which—in a slight abuse of notation—is ignored for the sake of simplicity.

We first give the proposed cost function as below:

$$\sum_{i \in N} \sum_{l \in \mathcal{L}_i} \rho_v \left( \| r_{Z_{il}} \|^2 \right) + \sum_{(i,j) \in \mathcal{E}} w_{ij} \left\| r_{R_{ij}} \right\|^2,$$

where $\rho_v(\cdot)$ is a robust loss function, and $w_{ij}$ is an individual weighting function for each known rotation term. In this paper, we fix $w_{ij}$ as a constant. This objective divides into two terms which are explained as follows.

**Visual Term:** We adopt traditional re-projection error in bundle adjustment as our visual term

$$r_{Z_{il}} = u_{il} - \Pi(R_i, C_i, P_i, l),$$

where $\Pi(\cdot)$ is the back-projection function that projects landmarks into the image plane. Note that the latter also depends on camera intrinsics, which—for the sake of a simplified and general notation—are not specified.

**Known Rotation Term:** The added known rotation term is

$$r_{R_{ij}} = \log(\hat{R}_{ij}^T \hat{R}_i R_j^T R_j),$$

where log is the logarithm map $SO(3) \rightarrow \mathfrak{so}(3)$. This known rotation term is used as a regulariser in the complete cost function.

We further make an explanation for our cost function: the first one corresponds to the reprojection error, the second one is used to penalize the large RPE that is caused by poses drift. Since the reprojection error might be small even when camera poses drift (Because triangulated 3D points are geometrically coherent with camera poses), however, the known rotation term can measure the discrepancy between the averaged rotations and incrementally recovered rotations in this situation. Thus the optimiser tries to minimize this discrepancy, and poses drift is alleviated.

### 6. Experimental Results

Our experiments aim at demonstrating the accuracy, efficiency, and robustness of the proposed methods. We implement Levenberg-Marquardt (LM) [30], row-by-row block coordinate descent (RBR-BCD) [18], and our hybrid rotation averaging in C++. Besides, the implementation of SE-Sync [33] and Shonan [16] are obtained from the given websites of their papers. For HSfM [13] and LUD [31], we use [10] as the rotation averaging solver, and the Ceres solver [2] for bundle adjustment. All approaches are tested on a laptop with a 2.7 GHz CPU and 8GB RAM.

#### 6.1. Evaluation of Hybrid Rotation Averaging on Synthetic Datasets

We designed 7 synthetic datasets to evaluate the performance of our rotation averaging approach. The view and relative rotation numbers are shown in Table 1, and denoted by $n$ and $\#$ edges respectively. The ground truth absolute rotations are initialised randomly. The relative rotations are constructed by a spanning tree expanded by random edges until the given number of relative poses is reached. All relative rotations are derived from ground truth, and perturbed by random angular rotations about randomly selected axes. The perturbation angles are normally distributed with 0 mean and a variance of either $\sigma = 0.2$ rad or $\sigma = 0.5$ rad. Initial absolute rotations are chosen randomly.

The evaluation results are shown in Table 1, where we compare our method against LM [30], RBR-BCD [18], Shonan [16] and SE-Sync [33]. In terms of efficiency, RBR-BCD is the slowest and almost 1000 times slower than others when $n = 1000$. SE-Sync is faster than LM but stays within the same order of magnitude. While SE-Sync is slightly faster than ours when cameras number is below 500, the hybrid rotation averaging approach is $1 \sim 2$ orders of magnitude faster than SE-Sync when the number of views grows beyond 1000. While Shonan is also a low-rank method, as well as SE-Sync and ours, it is almost 2 magnitude slower than SE-Sync, and 3 magnitude slower than ours. In terms of the scale of the solved problems, RBR-BCD failed when the camera number increased to 5000, 10000, or 50000. This is primarily due to insufficient memory for optimisation, and we marked the corresponding cells in the table by “-“.

LM, Shonan and SE-Sync failed when the camera number reaches 50000, as there is insufficient memory to hold the Jacobian matrix. Besides, Shonan is not converged when camera number is 10000 with variance 0.5, and we marked the corresponding cells by “NC”. As BCM approach only needs to compute the SVD of a small block matrix and evaluate $d$ matrix operations of $3 \times 3$ matrices in each iteration, we are able to solve all the evaluated large-scale datasets.

In terms of accuracy, LM achieves the global optimum with certain probability (30% - 70% as reported in [18]), and Table 1 only shows the best results. While all the evaluated globally optimal methods have the same accuracy and can both obtain the global optimum for successful cases.

#### 6.2. Evaluation of RA-SfM on Real-World Datasets

We evaluate the performance of our RA-SfM on large scale real-world datasets and compare it against state-of-the-art incremental [34], global [31] and hybrid [13] SfM approaches. Since the quasi-convex SiM approach [48] is sensitive to outliers and extremely slow in such datasets, we did not evaluate it in our experiment.

Figure 3 shows the reconstruction results of COLMAP and our RA-SfM on the Campus [15] dataset. This dataset, which has a loop, mainly contains plants that can produce lots of wrong matching results. COLMAP [34] fails to reconstruct this dataset, as the camera poses drift and the loop...
Table 1. Comparison of runtime on synthetic datasets. $n$ is the number of rotations, $\bar{R}$ represents the average rotation error (unit: degree).

| $n$ | #edges | $R$ | $n$ | $R$ | $n$ | $R$ | $n$ | $R$ | $n$ | $R$ |
|-----|--------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 20  | 30     | 0.2 | 1.800e-03 | 0.001 | 1.813e-06 | 0.007 | 1.719e-06 | 0.089 | 1.499e-06 | $< 1e-06$ | 1.502e-06 | $< 1e-06$ |
| 100 | 300    | 0.2 | 5.328e-03 | 0.007 | 2.607e-06 | 0.526 | 2.596e-06 | 0.787 | 2.589e-06 | 0.003 | 2.588e-06 | 0.003 |
| 500 | 1000   | 0.2 | 1.483e-03 | 0.264 | 3.006e-06 | 176.316 | 2.995e-06 | 18.47 | 2.901e-06 | 0.079 | 2.994e-06 | 0.006 |
| 1000| 4000   | 0.2 | 1.800e-01 | 0.321 | 2.810e-06 | 2.362 | 2.731e-06 | 22.730 | 2.641e-06 | 0.372 | 2.490e-06 | 0.008 |
| 5000| 20000  | 0.2 | 2.210e-01 | 0.237 | 3.790e-02 | 2.579 | 3.988e-02 | 29.57 | 3.930e-02 | 0.343 | 4.100e-02 | 0.105 |
| 10000| 40000  | 0.2 | 1.320e-01 | 24.256 | - | - | 2.375e-06 | 1108.435 | 2.012e-06 | 13.398 | 2.012e-06 | 0.092 |
| 50000| 200000 | 0.2 | - | - | - | - | NC | NC | 3.612e-02 | 15.205 | 3.600e-02 | 1.042 |
| 50000| 200000 | 0.2 | - | - | - | - | - | - | 2.941e-06 | 6.193 |

Figure 3. Reconstruction results for the Campus dataset [15]. Left: COLMAP [34], Right: Our RA-SfM.

is not closed. Our approach closes the loop successfully, as the known rotation optimisation is able to further constrain camera poses after the initial registration.

We also evaluated our approach on the online datasets from [45], which are collections of challenging unordered images. The datasets contains many wrong epipolar geometries due to extreme viewpoint, scale and illumination changes. The runtime and accuracy results are shown in Table 2. As can be observed, our approach recovers the most camera poses in all online datasets, and has the lowest mean reprojection error (MRE) in most of the online datasets. For our RA-SfM, the time for rotation averaging is separately given. COLMAP [34] is the second best approach in terms of recovered camera poses and MRE, which shows the robustness of incremental SfM approaches. While LUD [31] is the most efficient among the evaluated approaches, it has large MRE, and the recovered camera poses is less than ours or even COLMAP. HSfM [13] is faster than COLMAP and our approach, because it only samples 2 correspondences to compute the camera center in each RANSAC iteration. HSfM [13] recovers the fewest camera poses, and fails to recover the correct camera centers.

Some visual results for online datasets are shown in Fig. 4. For each sub-figure, the top and bottom images are the results obtained by COLMAP [34] and our RA-SfM, respectively. For Ellis Island dataset, we showed two different parts in the first two columns of Fig. 4, where the red rectangle area shows the comparison result. For Gendarmen-

market dataset, the reconstruction result of COLMAP is bad on the left part, which indicates wrong camera poses. For Vienna Cathedral dataset, our approach can recover more scene details than COLMAP, as is again indicated by the red rectangle.

7. Conclusion

This paper presents a hybrid rotation averaging method that is robust to outliers. We combine the existing global and local RA methods, with fast view graph filtering is applied at first to increase graph sparsity, so as to accelerate optimisation. The exposition is rounded off by a soft embedding into an incremental SfM pipeline leading to accurate, reliable, and highly efficient results.

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Table 2. Comparison of runtime and accuracy on online datasets [45]. \(N_i\) denotes the number of images, \(N_v\) denotes the number of registered cameras, \(T_i\) denotes the reconstructed 3D landmarks, MRE means mean reprojection error in pixel.

| Dataset               | \(N_i\) | \(N_v\) | MRE (px) | \(T_i\) (s) | \(N_i\) | \(N_v\) | MRE (px) | \(T_i\) (s) | \(N_i\) | \(N_v\) | MRE (px) | \(R_0\) (s) | \(T_i\) (s) |
|-----------------------|---------|---------|----------|------------|---------|---------|----------|------------|---------|---------|----------|------------|------------|
| Alamo                 | 2,915   | 660     | 94K      | 682        | 578     | 140K    | 1.28     | 260        | 522     | 149K    | 1.62     | 1,079      | 895        | 141K      | 3.84     | 2,784      |
| Ellis Island          | 2,587   | 315     | 64K      | 732        | 234     | 16K     | 1.54     | 24         | 208     | 34K     | 2.53     | 169        | 727        | 146K      | 0.72     | 6.4        | 3,954      |
| Gendarmenmarkt        | 1,463   | 861     | 123K     | 627        | 705     | 87K     | 1.51     | 104        | 542     | 74K     | 1.94     | 377        | 1,023      | 202K      | 0.70     | 6.2        | 3,001      |
| Madrid Metropolis     | 1,344   | 368     | 43K      | 251        | 350     | 51K     | 1.18     | 56         | 292     | 51K     | 1.48     | 221        | 438        | 66K       | 0.59     | 2.2        | 1,440      |
| Montmarte            | 2,298   | 506     | 98K      | 723        | 462     | 166K    | 1.64     | 194        | 418     | 155K    | 1.95     | 1,041      | 528        | 105K      | 0.67     | 1.7        | 1,446      |
| Notre Dame            | 1,431   | 1408    | 349K     | 22,788     | 550     | 262K    | 2.06     | 259        | 526     | 281K    | 2.30     | 2,375      | 1,409      | 353K      | 0.75     | 1.7        | 3,323      |
| NYC Library           | 2,550   | 453     | 77K      | 420        | 336     | 70K     | 1.52     | 75         | 282     | 74K     | 1.99     | 356        | 519        | 100K      | 0.65     | 2.1        | 1,734      |
| Piazza del Popolo     | 2,251   | 437     | 47K      | 380        | 329     | 38K     | 1.65     | 62         | 286     | 35K     | 1.92     | 212        | 966        | 122K      | 0.66     | 4.3        | 5,228      |
| Piccadilly            | 7,351   | 2,336   | 260K     | 0.75       | 1,961   | 2,301   | 202K     | 1.83       | 262     | 1,665   | 185K     | 2.169      | 3,041      | 363K      | 0.80     | 23.8       | 15,324     |
| Roman Forum           | 2,364   | 1,409   | 222K     | 0.70       | 1,041   | 1,045   | 256K     | 1.71       | 182     | 1,071   | 262K     | 2.237      | 1,460      | 267K      | 0.77     | 4.6        | 5,002      |
| Tower of London       | 1,576   | 578     | 109K     | 0.61       | 678     | 485     | 140K     | 1.65       | 95      | 398     | 149K     | 1.816      | 672        | 139K      | 0.58     | 2.2        | 2,040      |
| Trafalgar             | 15,685  | 5,211   | 450K     | 0.74       | 5,122   | 5,044   | 378K     | 1.56       | 713     | 3,446   | 318K     | 1.95       | 5,761      | 7,085      | 597K     | 0.72     | 58.5       | 15,048     |
| Union Square          | 5,961   | 763     | 53K      | 0.67       | 693     | 803     | 41K      | 1.65       | 107     | 769     | 38K      | 1.88       | 1,763      | 809        | 57K      | 0.52     | 4.5        | 1,962      |
| Vienna Cathedral      | 6,288   | 933     | 190K     | 0.74       | 1,244   | 849     | 203K     | 1.91       | 173     | 662     | 252K     | 2.36       | 2,807      | 1,173      | 903K     | 0.71     | 9.4        | 1,611      |
| Yorkminster           | 3,368   | 456     | 105K     | 0.70       | 997     | 421     | 132K     | 1.75       | 135     | 417     | 129K     | 1.93       | 1,487      | 614        | 183K     | 0.64     | 5.8        | 926        |

Figure 4. Visual reconstruction results for some of the online datasets [45]. For each sub-figure, the top and bottom images are respectively the results obtained from COLMAP [34] and our RA-SIM. (The first two columns are results of parts of the Ellis Island datasets.)

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