Cooling of Hybrid Stars with Spin Down Compression

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Abstract We study the cooling of hybrid stars coupling with spin-down. Due to the spin-down of hybrid stars, the interior density continuously increases, different neutrino reactions may be triggered (from the modified Urca process to the quark and nucleon direct Urca process) at different stages of evolution. We calculate the rate of neutrino emissivity of different reactions and simulate the cooling curves of the rotational hybrid stars. The results show the cooling curves of hybrid stars clearly depend on magnetic field if the direct urca reactions occur during the spin-down. Comparing the results of the rotational star model with the transitional static model, we find the cooling behavior of rotational model is more complicated, the temperature of star is higher, especially when direct urca reactions appear in process of rotation. And then we find that the predicted temperatures of some rotating hybrid stars are compatible with the pulsar’s data which are contradiction with the results of transitional method.

Key words: dense matter— stars: rotation—equation of state

1 INTRODUCTION

The interior of neutron stars contain matter beyond the nuclear saturation density which we have not seen yet. The cooling of neutron stars give us an important tool to study the properties of such dense matter. The traditional investigations of cooling often adopt the static star model which is not connect with spin-down. It is well known that neutron stars would spin-down due to magnetic dipole radiation. The spin-down compression of stars may lead to the changes in chemical composition (from nucleon matter to deconfined quark matter) and structure (mixed phase and quark phase appearing). The coupling of cooling and spin-down correlates stellar surface temperatures with rotational state as well as time. The interlinked processes of spin-down and cooling present intriguing prospects of gain insight in the fundamental properties of dense matter in neutron stars by confrontation with thermal emission data from observations (Stejner et al (2008)), (Page et al (2004), Yakovlev & Pethick (2004)).

Neutron stars are born with temperatures above $10^{10}$K. The dominant cooling mechanism of stars is the neutrino emission from the interior for the first several thousand years after birth, which can be generated via numerous reactions (Yakovlev & Pethick (2004)). For the nucleon direct urca (NDU) reaction, the most efficient one, is only possible if the fraction of proton exceeds a certain threshold. It is impossible to satisfy conservation of momentum unless the proton fraction exceed the value where both charge neutrality and the triangle inequality can be observed (Lattimer et al (1991)). Hence the traditional investigations of neutron stars often divide the cooling process into two regimes that are slow
and fast cooling due to slow and fast neutrino emission respectively. Slow cooling occurs in the low mass stars via neutrino emission produced mainly by the nucleon modified Urca(NMU) process. The fast cooling occurs in stars with mass critical one \( M_D \) (it is of course model dependent) via the NDU process. Comparing with the observed data, we can see that the fast cooling process would result in contradiction between the predicted temperatures of stars and the observations, which gives the challenge to the traditional model (Page et al. (2006)).

The neutron stars containing quark matter are called hybrid stars. Hybrid stars have more complicated interior structure and matter composition than the purely neutron stars. Due to spin-down compression, the interior density gradually increases, some the thermodynamic quantities such as neutrino emission luminosity and total heat capacity of stars continuously change with rotational frequency. Especially, the rate of neutrino emission would have an abrupt rise because of direct urca reactions occurring which would induce the rapid fall of temperatures of hybrid stars. Appearing of different direct Urca processes can result in different cooling behavior in the stages of evolution of the stars. The main difference between our model and traditional model is that our model combine the equation of thermal balance with the rotational stars structure and take magnetic dipole radiation model to investigate the changes of thermodynamic quantities and the cooling behavior of hybrid stars with spin frequency as well as time. The simulation of cooling curves of rotational stars are more complex than the traditional cases because of the magnetic field dependence and changes of rotational state. Surface temperatures of some stars including fast cooling processes are compatible with the pulsar’s data due to spin-down.

We take Glendenning’s hybrid stars model (Glendenning (1997)) based on the perturbation theory (Hartle (1967), Chubarian et al. (2000)) to study the rotational structure of stars. In our calculation, we only choose the simplest nucleon matter composition, namely neutrons, protons, electrons, and muons, and ignore superfluidity and superconductivity.

2 HYBRID STARS

As the stars spin-down, the nuclear matter are continuously converted into quark matter by the exothermic reactions, i.e. \( n \rightarrow u + 2d, p \rightarrow 2u + d \), s quarks immediately appear after weak decay. The deconfinement phase transition brought forward by Glendenning (1997, 1992) who had realized firstly the possibility of the occurrence of a mixed phase (MP) of hadron matter and quark matter in a finite density range inside neutron stars. Global charge neutrality of MP can be achieved by a positively charged amount of hadronic matter and a negatively charged amount of quark matter. The Gibbs condition for mechanical and chemical equilibrium at zero temperature between the two phases reads

\[
p_{HP}(\mu_n, \mu_e) = p_{QP}(\mu_n, \mu_e) = p_{MP}.
\]

(1)

Here \( \mu_n \) is the chemical potential of neutron and \( \mu_e \) is the chemical potential of the electron. The condition of global charge neutrality in the MP is

\[
0 = \frac{Q}{V} = \chi q_{QP} + (1 - \chi)q_{HP}.
\]

(2)

Here \( \chi = V_Q/V \) is the quark fraction in the MP, thus the energy density \( \epsilon_{MP} \) of the MP follows as

\[
\epsilon_{MP} = \frac{E}{V} = \chi \epsilon_{QP} + (1 - \chi) \epsilon_{HP}.
\]

(3)

We can obtain the equation of state (EOS) of MP using above equations. For the hadron part of star, we adopt the Argonne \( V18 + \delta \nu + U1X^* \) model (Akmal et al. (1998)) which is based on the models for the nucleon interaction with the inclusion of a parameterized three-body force and relativistic boost corrections. For the quark matter, we use the EOS of a effective mass bag-model EOS (Schertle et al. (1997)). In Figure 1, we show the EOS for hybrid stars with deconfinement transition as described above. We choose the parameters for quark matter EOS with s quark mass \( m_s = 150 MeV \), coupling constant \( g = 3 \) and different bag constants \( B = 108 MeV f m^{-3} \) and \( B = 136 MeV f m^{-3} \).
The rotation of stars lead to the change in structure. In present work, we apply Hartle’s approach (Hartle (1967)) to investigate the rotational structure of the stars. It is based on the treatment of a rotating star as a perturbation of non-rotating star, which can be obtained by expanding the metric of an axially symmetric rotating star in even powers of the angular velocity $\Omega$. We assume the frequency of stars at birth close to the Kepler(mass-shedding limit) frequency(Hartle & thorne (1968), Lattimer et al (1990), De Araujo et al (1995). We have shown the central density of rotating stars of different gravitational mass, as a function of its rotational frequency in Figure 2. These sequences of star models have same total baryon number but different central density and angular velocity. It is well known the perturbative approach fails when the angular velocity approaches the mass-shedding limit. However, the rotational frequencies of all known pulsars are much lower than the mass-shedding limit. So we can use the perturbative theory to investigate the structure of rotating stars unless we are specifically interested in the mass-shedding problem(Benhar et al (2005)). In Figure 2, dotted horizontal lines indicate deconfined quark matter produced and dashed horizontal lines indicate NDU nucleon direct Urca processes triggered. The appearance of quark matter will result in the occurrence the quark direct Urca(QDU) processes. Such processes are also efficient, but somewhat weaker than the NDU ones. We observe spin-down of stars lead to the changes in chemical composition and structure in Figure 2. In the case of bag constant $B = 108 MeV fm^{-3}$(left penal), the quark matter is produced in the interior for the stars mass $M > 1.4M_\odot$(here denote the static mass) in process of rotation. For $M = 1.6M_\odot$ star, for example, a purely neutron star can be transformed into a hybrid star and NDU reactions are triggered during spin-down. It is known that the occurrence of QDU and NDU processes lead to the rapid fall of temperatures of stars. In the section four, we discuss in detail spin-down leads to changes of the temperatures and cooling curves.

3 NEUTRINO EMISSIVITIES

The cooling of stellar could be carried out via two channels - neutrinos emission from the entire star and thermal emission of photons through transport of heat from the internal layers to the surface. The emission of neutrinos could carry away energy which would provide efficient cooling for warm neutron stars.
Fig. 2 Central density as a function of rotation frequency for rotating hybrid stars with different gravitational mass at zero frequency. All sequences have constant total baryon number. Dotted horizontal lines indicate deconfined quark matter is produced and dashed horizontal lines indicate nucleon direct Urca process is triggered.

For the hadron matter, we mainly consider three kinds of processes which are NDU, NMU as well as nucleon bremsstrahlung(NB). For the quark matter, we take QDU processes of unpaired quarks, the quark modified Urca (QMU) processes and the quark bremsstrahlung(QB) are considered. The most powerful neutrino emission is provided by direct Urca process. The rate of neutrino emissivity of NDU processes $n \rightarrow pe\bar{\nu}, pe \rightarrow n\nu$ is given by

$$\varepsilon_{NDU} \simeq 4.0 \times 10^{27} (Y_e \rho_s)^{1/3} T_9^6 \Theta_t \text{ erg cm}^{-3} \text{s}^{-1}$$  \hspace{1cm} (4)$$

where $T_9$ is the temperature in units of $10^9$ K, $\rho_s = 0.16 fm^{-3}$ is the nuclear saturation density, $\Theta_t = \theta(p_{Fp} + p_{Fn} - p_{Fe})$ is the threshold factor, with $\theta(x)$ being 1 for $x > 0$ and zero otherwise(Lattimer et al.(1991)). NDU processes can occur when the fraction of proton exceeds 11%. The presence of muons would raise it to about 15%.

The neutrino emissivity of quark matter has been given firstly by Iwamoto(1982). The QDU processes have been estimated as

$$\varepsilon_{QDU} \simeq 8.8 \times 10^{26} \alpha_c \left(\frac{\rho}{\rho_s}\right) Y_e^{1/3} T_9^6 \text{ erg cm}^{-3} \text{s}^{-1}$$  \hspace{1cm} (5)$$

with the standard value of the QCD coupling constant $\alpha_c \simeq 0.1$.

The emissivity of NMU and NB processes in the non-superfluid $npe$ matter are usually taken from Friman & Maxwell(1979), in which the one-pion-exchange Born approximation with phenomenological corrections was used for consideration. For the emissivity of QMU and QB processes, we take the results of Iwamoto(1982).

4 COOLING CURVES WITH SPIN DOWN

The traditional standard cooling model which often based on the Tolman-Oppenheimer-Volkoff(TOV) equation of hydrostatic equilibrium(Page et al. (2006), Yakovlev & Pethick (2004)). All thermodynamic quantities (neutrino emission luminosity and total heat capacity etc) are calculated when rotational frequency of stars is zero.
In present work, we investigate the cooling of hybrid stars with spin-down. The equation of thermal balance has been assumed spherical symmetry although it may be broken in a rotating star. It is reasonable for slowly rotating stars which could be treated as a perturbation to change the structure and chemical composition (Stejner et al. 2008). We combine the equation of thermal balance with the rotating structure equations of the stars (Kang & Zheng 2007, Hartle 1967) and rewrite the energy equation in the approximation of isothermal interior (Glen & Sutherland 1980).

\[
C_V(T_i, v) \frac{dT_i}{dt} = -L^\infty_{\nu}(T_i, v) - L^\infty_\gamma(T_s, v)
\]

(6)

\[
C_V(T_i, v) = \int_0^{R(v)} c(r, T) (1 - \frac{2M(r)}{r})^{-1/2} 4\pi r^2 dr
\]

(7)

\[
L^\infty_{\nu}(T_i, v) = \int_0^{R(v)} \varepsilon(r, T) (1 - \frac{2M(r)}{r})^{-1/2} e^{2\Phi} 4\pi r^2 dr
\]

(8)

Where \(T_i\) is the effective surface temperature, \(T_i(t) = T(r, t)\) is the red-shifted internal temperature, \(T(r, t)\) is the local internal temperature of matter, and \(\Phi(r)\) is the metric function (describing gravitational red-shift) (Yakovlev & Haesel 2003). Furthermore \(L^\infty_{\nu}(T_i, v)\) and \(C_V(T_i, v)\) are the total red-shifted neutrino luminosity and the total stellar heat capacity respectively which are functions of rotation frequency \(v\) and temperature \(T\); \(c(r, T)\) is the heat capacity per unit volume. \(L^\infty_{\nu} = 4\pi R^2(v) \sigma T^4_{\nu}(1 - R_g/R)\) is the surface photon luminosity as detected by a distant observer \(R_g\) is the gravitational radius of stellar ). The effective surface temperature detected by a distant observer is \(T^\infty = T_s \sqrt{1 - R_g/R}\). \(T_s\) is obtained from the internal temperature by assuming an envelope model (Gudmundsson et al. 1983, Potekhin et al. 1997). The spin-down of stars is due to the magnetic dipole radiation. The evolution of rotation frequency is given by

\[
\frac{dv}{dt} = -\frac{16\pi^2}{3Ic^3} \mu^2 v^3 \sin^2 \theta
\]

(9)

where \(I\) is the stellar moment of inertia, \(\mu = \frac{1}{2}B_mR^3\) is the magnetic dipole moment, and \(\theta\) is the inclination angle between magnetic and rotational axes. According to the Eq.(6)-(9), we can simulate the cooling of hybrid stars during spin-down.

In Figure 3 we present the cooling curves of a 1.6\(M_\odot\) rotational hybrid star for different magnetic field strengths \((10^9 - 10^{13}\text{G})\) with bag constant \(B = 108\text{MeV fm}^{-3}\). We find the cooling curves of the star are different in different magnetic fields, especially for the strong magnetic fields cases. As we know, the stronger magnetic field is, the faster rotational frequency slows down due to the magnetic dipole radiation, the direct Urca processes are triggered at the earlier time, which result in rapidly cooling in a shorter time. In the cases of weaker fields \((B < 10^{10}\text{G})\), direct urca processes appear at about the stage of photons cooling, the effect of magnetic fields is not important at the era. Using the same model in Figure 3, we show the neutrino emissivity of different reactions as well as the photon luminosity as functions of time and rotational frequency with magnetic field \(B_m = 10^{12}\text{G}\) in Figure 4. As spin-down of the star, deconfined quark matter appear in the core of star and then QDU reactions are triggered at spin frequency \(v = 1123\text{Hz}\). From then on, the neutrino emissivity of direct urca processes dominate the cooling of star. For star in age \(10^{-1} < t < 10^{-3}\text{yr}\), the QDU reactions provide most efficient neutrino emissivity. We find NDU processes are triggered at spin frequency \(v = 492\text{Hz}\) which lead to the rapid increase of neutrino emissivity. The photon emission control the cooling curves from about \(t = 10^4\text{yr}\) after birth.

In Figure 5, we plot the cooling curves of hybrid stars for different mass. The results of the rotational model with magnetic field \(B_m = 10^{11}\text{G}\) and the traditional model are both presented in the picture (The per star model include two cooling curves.). The results show the temperatures of the rotating stars model are higher than the transitional static cases, especially for the star including NDU reactions (1.6 and 1.7 \(M_\odot\) stars for bag constant \(B = 108\text{MeV fm}^{-3}\), 1.8 and 1.9 \(M_\odot\) stars for bag constant \(B = 136\text{MeV fm}^{-3}\). As we know that the traditional model use the equation of hydrostatic
Fig. 3 Cooling curves of $1.6 \, M_\odot$ rotational hybrid star for various magnetic fields (curve a: $10^{13}\, G$, b: $10^{12}\, G$, c: $10^{11}\, G$, d: $10^{10}\, G$, e: $10^9\, G$).

Fig. 4 $1.6 \, M_\odot$ hybrid star with magnetic field $B_m = 10^{12}$ for bag constant equal $108 \, MeV \, fm^{-3}$, the neutrino emissivity of different reactions as well as the photon luminosity as functions of time(left panel) and rotation frequency(right panel).

... equilibrium to study the configuration of stars. The central density of stars and the thermodynamic quantities of neutrino emission luminosity and total heat capacity etc are calculated with rotational frequency of stars being zero. For static model, these hybrid stars are born when direct reactions occur. For rotational model, direct urca processes appear with the central density gradually increases during spin-down of stars, which induce the temperature of rotating model is higher than the static model at the stage of neutrino cooling. From Figure 5, we can observe the cooling curves of some rotating hybrid stars($1.6M_\odot$ and $1.7M_\odot$ in left panel, $1.8M_\odot$ and $1.9M_\odot$ in right panel) are compatible with the
Cooling curves of hybrid stars for different mass with magnetic field $B_m = 10^{11}$ G and the cases in absence of rotation. The curves of the higher temperature correspond to our model for different mass stars. The observational data 1 to 11 are taken from tables 1 and 2 of Page et al. (2004). These stars are: 1. RX J0822-4247, 2. 1E 1207.4-5209, 3. PSR 0538+2817, 4. RX J0002+6246, 5. PSR 1706-44, 6. PSR 0833-45, 7. PSR 1055-52, 8. PSR 0656+14, 9. PSR 0633+1748, 10. RX J1856.5-3754, and 11. RX J0720.4-3125. The next two stars, labeled as a and b: a. PSR B0531+21 (Weisskopf et al. (2004)), b. PSR J0205+6449 (Slane et al. (2004))

Fig. 5 Cooling curves of hybrid stars for different mass with magnetic field $B_m = 10^{11}$ G and the cases in absence of rotation. The curves of the higher temperature correspond to our model for different mass stars. The observational data 1 to 11 are taken from tables 1 and 2 of Page et al. (2004). These stars are: 1. RX J0822-4247, 2. 1E 1207.4-5209, 3. PSR 0538+2817, 4. RX J0002+6246, 5. PSR 1706-44, 6. PSR 0833-45, 7. PSR 1055-52, 8. PSR 0656+14, 9. PSR 0633+1748, 10. RX J1856.5-3754, and 11. RX J0720.4-3125. The next two stars, labeled as a and b: a. PSR B0531+21 (Weisskopf et al. (2004)), b. PSR J0205+6449 (Slane et al. (2004))

observed data (Page et al. (2004), Weisskopf et al. (2004), Slane et al. (2004)) which are contradiction with the cooling curves of the transitional model. We find hybrid stars have complicated cooling behavior during spin-down. For $M = 1.5 M_\odot$ hybrid star (left panel), the effect of spin-down is weaker than the cases of $M = 1.6 M_\odot$ and $1.7 M_\odot$ because the interior of $M = 1.5 M_\odot$ star only appear QDU reactions, but $M = 1.6 M_\odot$ and $1.7 M_\odot$ stars include QDU and NDU reactions in process of rotation. Comparing $M = 1.6 M_\odot$ with $1.7 M_\odot$ stars (left panel), we find earlier appearing of NDU processes lead to more rapid cooling for the rotating hybrid stars.

We take the value of bag constant of relatively large ($B = 108, 136 MeV fm^{-3}$) in the paper. Comparing the results for two parameters, we find that critical mass of hybrid stars is smaller, the effect of spin-down is more important for the smaller mass hybrid stars with the decrease of bag constant. In the case of the smaller bag constant, spin-down may lead to the larger changes for temperature of the low mass hybrid stars. However, cooling behavior of hybrid stars for the different bag constant are similar if the changes of the rotational state are similar during spin-down process.

5 CONCLUSIONS AND DISCUSSIONS

The cooling of hybrid stars with spin-down have been studied in the paper. We combine the cooling equation with the rotational structure equation of stars and simulate the cooling curves of the hybrid stars. The results show the cooling curves have a clear magnetic field dependence if QDU or/and NDU reactions are triggered during spin-down. The time of direct Urca reactions triggered controls the occur-
ring of fast cooling. Comparing the cooling curves of the rotational models with the static models, we find the cooling behavior of rotational models are more complicated, the temperature of stars are higher, especially when direct urca reactions appear in process of rotation. We also find that the cooling curves of some rotational hybrid stars are consistent with the observational data in case of direct Urca neutrino emission. Through considering the inclusions of superfluidity and superconductivity, we expect cooling curves of these rotational hybrid stars can match the pulsar data well in future study.

It has been studied by many investigators that different EOS of hadron phase and model parameters of quark phase (bag constant $B$, coupling constant $g$) would influence the phase transition densities, rotational structure of hybrid stars and corresponding internal structure etc (Schertler et al. (2000); Pan et al. (2006)). We will investigate the effect of these parameters on the cooling of rotating hybrid stars in detail.

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