Gflash as a Parameterized Calorimeter Simulation for the CMS Experiment

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Abstract. Gflash is a parameterized simulation of electromagnetic and hadronic showers. It uses parameterizations of longitudinal and lateral shower profiles in homogeneous or sampling calorimeters taking into account an individual shower fluctuation and correlation. Gflash can take over the Geant4 based tracking at the first inelastic interaction inside calorimeters. It can be tuned flexibly to data and speed up the calorimeter simulation significantly for high energy particle showers. In this paper, the Gflash implementation developed for the CMS calorimeter and tuned to the CMS test beam data is described. The performance of Gflash is also presented.

1. Introduction
The CMS experiment primarily uses the GEANT4 toolkit (Geant4) to simulate the passage of particles through the CMS detector. Geant4 provides various physics models that describe interactions of particles with all detector components. Physics processes included in a specific physics list govern to track all secondaries as well as the primary particle down to a certain minimum energy. In general, it requires much efforts to tune simulation of a general purposed high energy detector due to its complexity as well as various choices of options applied to a certain physics process. In addition, the computing time to simulate an event for the CMS experiment may takes several minutes due to energetic particles copiously produced by the high energy collision at LHC as well as multiple underlying interactions. Usually the calorimeter simulation is the primary contributor of the CPU time per event due to the large decay volume designed to produce particle showers by electromagnetic cascades or hadronic interactions.

Gflash is a parameterized simulation package for electromagnetic (EM) and hadronic showers. It was originally developed for the calorimeter simulation of the H1 experiment at HERA. Gflash uses parameterizations of longitudinal and lateral shower profiles in homogeneous or sampling calorimeters taking into account an individual shower fluctuation and correlations between parameters. The main advantage of using Gflash is to speed up the calorimeter simulation while it can be tuned flexibly to data. Details of the original program Gflash can be found elsewhere [1] [2].

The EM shower parameterization of Gflash can be applied to any homogeneous detector if one can use the energy in the unit of critical energy \( E_c \) and the length in the unit of the radiation length \( X_\text{o} \) according to the Rossi’s approximation B [3]. Since the CMS EM calorimeter (ECAL) is homogeneous \( PbWO_4 \) and relatively long \( 25X_\text{o} \), the CMS EM shower parameterization adapt the original Gflash program without much modifications. However, both longitudinal and lateral parameterizations for the hadron shower are newly developed to take
into account for the CMS specific detector structure, especially the presence of passive materials between ECAL and HCAL.

2. Particle Shower Parameterizations of Gflash

The simulation of particle showers in Gflash is divided into the spatial distribution of the deposited energy \( E_{dp} \) for a shower and the energy fraction of the deposited energy which is visible in the active medium. The spatial distribution of \( E_{dp} \) within the azimuthal symmetric calorimeter volume containing showers can be expressed as

\[
dE_{dp}(\vec{r}) = \frac{E_{dp} L(z) dz R(r)}{2\pi} dr.
\]  

where \( L(z) \) describes the longitudinal energy sub-profile along the shower depth \( z \) and \( R(r) \) the lateral sub-profile to the radial direction from the shower axis \( r \). The energy fraction of the deposited energy which is visible in the active medium is determined by sampling fractions (or calibration scales) and their fluctuations.

2.1. Electromagnetic Shower Parameterization

The mean longitudinal energy profile of electromagnetic showers in Gflash is assumed to take a \( \Gamma \)-distribution,

\[
L(x) = \frac{x^{\alpha-1}e^{-x}}{\Gamma(\alpha)}
\]

where \( x = \beta z \) and \( z \) is the shower depth in the unit of radiation length. The shape of showers is characterized by a correlated pair of parameter \( \alpha \) and \( \beta \) which are corresponding to the first and second moments of the \( \Gamma \)-distribution. The distribution has a maximum at \( z_{max} = (\alpha - 1)/\beta \) along the longitudinal axis \( \hat{z} \).

The lateral profile of energy distribution for both electromagnetic and hadronic showers is described by a simple ansatz

\[
R(r) = \frac{2r R_0^2}{(r^2 + R_0^2)^2}
\]

where \( r \) in units of Molière radius is the radial distance perpendicular to the shower direction and with origin in the center of the shower, and \( R_0 \) is a free parameter that is a function of the shower energy \( E \) and the shower depth \( z \). The mean of \( R_0(E, z) \) is parameterized as

\[
\langle R_0 \rangle(E, z) = [R_c + Q(E)z]^n
\]

where \( n = 1(2) \) for electromagnetic (hadronic) shower.

2.2. Hadronic Shower Parameterization Specialized for CMS

The longitudinal profile of hadron showers developed for the CMS calorimeter simulation is written as a combination of sub-profiles in two detector regions,

\[
F = f_{ecal} F_{ecal} + f_{hcal} F_{hcal},
\]

where \( F_{ecal} \) and \( F_{hcal} \) represent the hadronic shower profile inside electromagnetic calorimeters (ECAL) and hadronic calorimeters (HCAL), respectively. In the CMS Gflash model for hadronic showers, both \( F_{ecal} \) and \( F_{hcal} \) taking the same functional hypothesis are the superposition of two Gamma distributions:

\[
F_{ecal}(or F_{hcal}) = [cL(x_e; \alpha_e, \beta_e)dx_e + (1-c)L(x_h; \alpha_h, \beta_h)dx_h],
\]
where $c$ is the fraction of the deposited energy by any $\pi^0$ produced from the primary or secondary interactions, $L(x_e; \alpha_e, \beta_e)$ and $L(x_h; \alpha_h, \beta_h)$ are sub-profiles of the spatial energy distribution by $\pi^0$ and by any charged hadron tracks in the units of $d_i$ ($d_e = X_o$, $d_h = \lambda_o$), respectively. We then parameterize the set of parameters describing the longitudinal profile. In particular, $\alpha$ and $\beta$ of each $\Gamma$ distribution can be semi-analytically estimated by using its mean ($\mu/\beta$) and variance ($\alpha/\beta^2$) of the distribution. To build an initial set of parameters, $\tilde{x} = (c, \alpha_e, \beta_e, \alpha_h, \beta_h)$, we use the CMS detector simulation with Geant4 to construct the statistical ansatz of the deposited hit energy distribution. The correlations between them are then parameterized as follow,

$$\tilde{x} = \tilde{\mu} + \tilde{\sigma}C\tilde{z} \quad \text{with} \quad \rho = C C^T$$

where $\tilde{\mu}$ is the vector of the means of $x_i$, $\tilde{\sigma}$ is the vector of the standard deviation of $x_i$, $C$ is the Cholesky decomposed matrix of the correlation matrix ($\rho$), and $\tilde{z}$ is a vector of normal distributed random numbers. For the lateral profile, the mean and the variance of $R_o$ are also newly parameterized depending on the position of the shower start to take into account the specific detector structure of the CMS calorimeters.

3. Tuning Shower Parameterizations of Gflash to the Test Beam Data

In order to tune shower shapes and responses of Gflash, we use CMS test beam data taken in 2006 for electrons and hadrons ($\pi^\pm$, $K^\pm$, $p$, $\bar{p}$). The version 9.2.p01 of Geant4 with the CMS test beam geometry is used to tune Gflash parameterizations. The default physics list of Geant4 used up to the first inelastic interaction includes the standard EM physics list and QGSP_BERT. The simulated energy response for the pure hadronic showers is calibrated with 50 GeV/$c$ electron beams.

3.1. Tuning the EM parameterization

To tune the EM response of Gflash, the reference response of electron test beam data is used. For each electron beam energy ($E_{\text{beam}} = 20, 30, 50, 80, 120, 150$ GeV), the nominal energy response in $N \times N$ ($N = 1, 3, 5$) crystals is obtained from energy scan data. Optimized weights are used for the amplitude reconstruction and the crystals are inter-calibrated. The reference response of the detector is obtained by selecting only the electrons impacting close to the point of maximum containment which is determined for each crystal in order to disentangle the effects due to not fully containing the shower. Simulated data using Gflash are generated with the same set of configuration for simulation and reconstruction and they are then analyzed using the same analysis software as for the test beam data.

Since the absolute response may be sensitive to the calibration scale and the containment correct, the ratio of $N \times N$ crystal response (relative response) is a scale-independent measure of the energy response. The relative response is also a good measure for the lateral spread of the shower. Figure 1 shows an example of energy shape comparisons between Gflash and the test beam data for the 50 GeV electron beam. Both the absolute response ($N \times N$ crystal response) and the relative response (ratio of $N \times N$ responses) are in good agreement. Figure 2 shows the mean energy response and its resolution as the beam energy. Again, comparisons between Gflash and results from test beam data are in good agreement.

3.2. Tuning the Hadronic Parameterization

To tune hadronic response of Gflash, the reference response of the hadron test beam data is used. For the nominal energy response, the energy response of Gflash is compared to the sum
Figure 1. The energy response of 50 GeV $e^-$ of Gflash (red) and the 2006 test beam data (black); the nominal energy response in $N \times N$ ($N = 1, 3, 5$) crystals (top) and the ratio of $N \times N$ crystal response (bottom).

Figure 2. The mean energy response (top) and energy resolution (bottom) of $e^-$ as the beam energy of Gflash (red) and the 2006 test beam data (black).

of energy in ECAL (7 × 7 matrix) and HCAL (4 × 3), and their correlations. For the lateral shower, the energy in each $\eta$-tower or $\phi$-tower is compared using tower-by-tower energy matrix of ECAL (9 × 9) and HCAL (4 × 3).
For each energy of each particle type, the energy shape is tuned for 1) pure hadronic response (hadronic energy for $E_{ECAL} < 0.8$ GeV), 2) EM energy (ECAL) response 3) hadronic energy (HCAL) response. The total energy response is not attempted to be tuned, but used only as the final measure of tuning. Figure 3 shows examples of the energy shape tuning for 100 GeV $\pi^-$ beam and 20 GeV $p$ beam, respectively.

**Figure 3.** The energy shapes of 100 GeV $\pi^-$ (left) and 20 GeV $p$ (right) of Gflash (blue) compared to the 2006 test beam (red).

The measured mean energy as the beam energy ($\langle E \rangle / P$) for all available $\pi^-$ and $p$ beams are shown in Figure 4 and Figure 5, respectively. Compared to results from test beam data, both the average response and the resolution of Gflash are in good agreement for $\pi^-$ and $p$.

**Figure 4.** The energy response and resolution ($\pi^-$) as the beam energy of Gflash (blue) compared to those of the test beam data (red).
Figure 5. The energy response and resolution (proton) as the beam energy of Gflash (blue) compared to those from the test beam data (red).

4. Performance

Figure 6 shows the CPU time per event and the memory usage for first 100 $t\bar{t}$ events from Gflash and Geant4. The CPU gain using Gflash with respect to Geant4 is around factor 2 at low energy (2 GeV) and factor 10 at high energy (300 GeV). However, there is only marginal increment in the memory usage compared to Geant4.

![Time per Event for Single Pions](image)

![Memory Usage: RSS](image)

Figure 6. The CPU performance and the usage of memory for the first 100 $t\bar{t}$ events.

References

[1] Grindhammer G, Rudowicz M, and Peters S 1990 Nucl. Instrum. and Methods A290 469
[2] Grindhammer G and Peters S 2000 Preprint hep-ex/0001020
[3] Rossi B 1952 High-Energy Particles (New York: Prentice Hall)