Evaporating Black Holes And An Entropic Scale-Hierarchy

Hadi Salehi

Institute for Studies in Theoretical Physics and Mathematics,
P.O.Box 19395-1795, Niavaran-Tehran, Iran

and

Arnold Sommerfeld Institute for Mathematical Physics, TU Clausthal,
Leibnizstr. 10, D-38678 Clausthal-Zellerfeld, Federal Republic of Germany

Abstract

It is argued that a characteristic length may be associated with the entropic state of a spherically symmetric black hole in the cosmological context. This length is much smaller than the Schwarzschild-radius of a black hole and may act as a regulator of arbitrarily high frequencies apparently entering the usual derivation of Hawking’s radiation.

1 Introduction

One of the most impressive predictions of general relativity is that the process of gravitational contraction of a ‘sufficiently’ large mass at some stages leads to the formation of a black hole. For an outside observer a black hole appears to be a state in which the maximal achievable limit of entropy production by gravitational contraction is achieved. The form of entropy function was suggested by Bekenstein [1] by means of information theoretic arguments. His result was greatly strengthened by Hawking’s remarkable discovery [2] that quantum effects lead to the association of a radiation with the black hole mass at a temperature which corresponds to the assumed form of information theoretic Bekenstein entropy, namely (c resp. $\hbar$ is the velocity of light resp. the Planck constant)

$$T_H = \frac{\hbar c^3}{4\pi MG},$$

(1)
where $M$ denotes the black hole mass. Perhaps it is one of the most important characteristics of this formula that it interconnects all the fundamental constants of the nature in a single thermodynamic relation and it is widely believed that a generic understanding of this interconnection will lead to new fundamental insights about the unification of physical concepts in general. The present paper, in essence, is the outgrowth of attempts to understand the origin of this interconnection from a quite universal point of view. It should, however, be remarked at the outset that many aspects of the ideas to be reported are far from being dynamically justified and the problem of a strict dynamical approach is not addressed. Accordingly, the paper ought to be understood as an extremely modest contribution towards a strict dynamical elaboration of the results.

2 Cosmic thermalization of gravitational constant

The idea as to whether the gravitational constant might fundamentally be related to an entropic property of the universe as a whole to my knowledge has never been worked out in any details. A strong indication suggesting an entropic approach is provided by the relation (2). When written in the alternative form

$$G^{-1} = 4\pi \frac{MT_H}{\hbar c^3},$$

(2)

it suggests that in the presence of a black hole an outside asymptotic observer may interpret the gravitational constant as an emergent feature of an equilibrium type process having its origin in the gravitational trend of a ‘sufficiently’ large mass towards a maximal entropic state. There is, of course, an inherent ambiguity in a statement of this type, for it is impossible to give an account of the entropic state of a sufficiently large mass without an extrinsic knowledge of the numerical value of $G$.

One may consider two different feasible ways of establishing the value of gravitational constant extrinsically. In fact, one might either simply identify the ‘extrinsic’ value of $G$, denoted in the following by $G_{ex}$, as a constant of nature, or else consider it as fundamentally related to $\hbar$, $c$, the temperature of the microwave background radiation $T_{mic}$, and the entire visible mass of the universe $M_{uni}$. Although these are two alternative ways of establishing the value of $G_{ex}$, in the presence of a black hole the physical implications in the second case will be very different. In the rest of this paper an attempt is made to explore the physical implications of the second alternative.

First, it should be realized that in a universe which is thermodynamically unique there must be only one feasible way of relating the extrinsic value of $G$ to $\hbar$, $c$, $T_{mic}$, and $M_{uni}$. Quite in the sense of (2) this relation may be assumed to be linear with respect to the equilibrium parameter (temperature), namely

$$G^{-1}_{ex} = f(\hbar, c, M_{uni}, \alpha) T_{mic},$$

(3)

For this mass I shall take in the following the value $10^{53}kg$ with the realization that there may exist possible uncertainties in the estimate of this value.
where $\alpha$ is imagined to represent a still unspecified quantity needed to establish the value of $G_{ex}$. The explicit form of the relation (3) may be suggested by a general consideration. First, note that (3) would act as a stringent constraint imposed by the state of the universe as a whole on the locally observed value of physical constants about an arbitrary point of observation. Since such a constraint would affect rules of local measurements, so by necessity $\alpha$ must be treated as a 'cosmic' field $\alpha(x)$.

It should, however, be realized that, not knowing absolut standards of units in the process of local measurements, the conceivability of a cosmic field $\alpha(x)$ regulating the thermodynamic connection between all physical constants about an arbitrary point of observation is meaningless unless $\alpha(x)$ is invariant under a position dependent transformation of units.

The need for this limitation is apparent as the value of $\alpha(x)$ at each space time point is to reflect an absolute property of the universe as a whole and, hence, must remain unaffected by a change of the particular standards of units used in local observation.

It is clear that the last arbitrary way of achieving the kind of invariance mentioned is to require $\alpha(x)$ to be dimensionless. Now, dimensional arguments may be used to restrict the form of functional connection in (3) to

$$G_{ex}^{-1} = \alpha(x) \frac{T_{mic} M_{uni}}{\hbar c^3},$$

The order of magnitude of $\alpha(x)$ may be estimated by the requirement that the intersection of $\alpha$ in the present epoch of the universe would adjust $G_{ex}^{-1}$ to a value close to the reciprocal of the observed value of gravitational constant. This is a statement of numerical coincidence of the thermodynamically predicted value of $G$ with its observed value. It gives the estimate

$$\alpha \sim 10^{-28}$$

This relation has a significance in a rough-order-of-magnitude manner only, but indicates the impossibility of achieving a cosmic thermodynamic connection of $G$ with $\hbar$ and $c$ without producing an immensely small dimensionless number.

Physically, there is one way of interpreting this observation\textsuperscript{2}: To an observer restricted to observations in the present epoch of the universe the allowed contribution of the entire visible mass of the universe to the 'thermal' source of gravitational entropy appears to be remarkably fractional, that is, $\alpha$ may have the physical significance of an absolute scale factor which, if applied to the entire visible mass of the universe, would provide a measure of the 'entropic' mass scale of the universe, $M_{ent}$\textsuperscript{3}. Of course, much work is needed to

\textsuperscript{2}The reader should notice the correspondence between the next statement and the appreciation that the gravitational entropy of the present universe is remarkably low [3].

\textsuperscript{3}If we take the gravitational entropy $s$ of the expanding, time-asymmetric universe, considered as a thermodynamic system, as somehow related to $M_{uni}$, we may associate with the universe an entropic mass scale $M_{ent}$ by means of the relation (we are using here the Planck units)

$$\frac{ds}{dM_{uni}} \sim M_{ent}.$$
understand the dynamical origin of the entropic mass scale of the universe\footnote{In a dynamical approach it seems reasonable to suspect a connection between $\alpha(x)$ and the expectation value $< \phi^2 >$ of a quantized scalar field $\phi$. Such a theory, although conceptually different \cite{4}, would have common features with the Brance-Dicke theory \cite{5}.} but in line with interpretation given above, that mass scale in a rough order of magnitude estimate should be related to $M_{\text{uni}}$ by a scale factor transformation applied to $M_{\text{uni}}$, namely

$$M_{\text{ent.}} \sim \alpha M_{\text{uni}}.$$ \hspace{1cm} (6)

Since such a scale transformation applied to the entire visible mass of the universe may affect the dynamically ascertainable value of mass for an arbitrarily localized and quasi isolated gravitating system (considered as thermodynamic system) in the cosmological context, the appearance of the entropic mass scale $M_{\text{ent.}}$ may prove to have essential consequences. First note that due to the very presence of $M_{\text{ent.}}$ it seems rather natural to suspect that the universe may be in a state in which each localized and quasi isolated gravitating system 'irreversibly' contributes to $M_{\text{ent.}}$ by a fraction $\sim \alpha$ of its mass. In this way $\alpha$ may regulate a sort of universal transfer of mass into the thermal source of gravitational entropy.

If a statement of this sort is admitted then, due to the very existence of $M_{\text{ent.}}$, the dynamically ascertainable value of mass for a localized and quasi isolated gravitating system in the cosmological context ought to exceed its 'bare' gravitational mass $M$ in the 'idealized' classical picture (the mass dynamically determined by the static Schwarzschild-metric at large distances from the system, see \cite{6}) by a term of the order of $\alpha M$.

For a black hole in the cosmological context\footnote{There may be problems of principle in the definition of a black hole in the cosmological context, for the discussion of some aspects the reader is referred to \cite{7}.} formed by an actual process of gravitational contraction this has the remarkable consequence that, asymptotically, the temperature of the radiation would differ from Hawking's temperature by a term of the order of $\alpha$, namely

$$T = T_H + o(\alpha), \quad \alpha \sim 10^{-28}.$$ \hspace{1cm} (7)

### 3 Scale-hierarchy

There is now an important question regarding the effect of cosmic thermalization of gravitational constant. Could it not be that this thermalization could define an additional scale other than the linear dimension of a black hole, i.e. its Schwarzschild-radius, for black hole radiation?

Unavoidably, a new scale may be defined if we look for a characteristic length scale, say $l$, for which the ratio of $l$ and the linear dimension of a black hole, i.e. its Schwarzschild-radius $r_g$, becomes of the same order of magnitude as the small number $\alpha$, namely

$$\frac{l}{r_g} \sim \alpha.$$ \hspace{1cm} (8)
Now the relation (7) may be written as

\[ T = T_H + o \left( \frac{l}{r_g} \right), \]  

which indicates that the real entropic state of a black hole must unavoidably exhibit the scale hierarchy

\[ l \ll r_g. \]  

Note that for a black hole of a typical astrophysical dimension \( \sim 10^5 \text{cm} \) the length \( l \) will define a short distance scale which in term of the Planck length \( l_P \) would have the order of magnitude \( \sim 10^{10} l_P \).

4 Short distance cut-off

Recently there have been efforts in understanding the role played by arbitrarily high frequencies apparently entering the usual derivation of Hawking radiation [8][9][10]. The difficulty of the usual derivation is that, given a static detector placed far away from a spherically symmetric black hole, the long time response of the detector to outgoing modes of a quantized field becomes (as a consequence of the 'infinite' gravitational redshift effect) causally correlated to the behaviour of incoming modes with arbitrarily high frequencies in the past, the relevant frequencies increasing as (units are used in which \( c = \hbar = G = 1 \))

\[ \omega \sim \frac{1}{|v|} \]  

as the advanced time \( v \) approaches the long time limit \( v \to 0 \) (\( v = 0 \) corresponds to the formation of the horizon), details may be found in [9]. It is now evident that in the long time one comes very soon into the real difficulties concerning the interpretation of 'infinitely' magnified frequencies. I would like to argue that the difficulty might have its origin in an improper separation of the entropic state of a black hole from the effect of scale-hierarchy (10). I am very well aware of the difficulty of the problem, still the following extremely heuristic remark appears to be worth mentioning.

The point is that the horizon, which in the usual analysis forms at \( v = 0 \), in the presence of \( l \) is expected to form at a slightly smaller value of the advanced time (since the scale \( l \) has its origin in an increment of the black hole mass). This 'entropic' shifting of horizon may have the following consequence. First, note that in the frame of a free falling observer beginning his journey from the rest at infinity the advanced time near the horizon would change oppositely at a rate which is of the order of the rate at which the Schwarzschild-radial coordinate \( r \) would change, see appendix. Thus, in the presence of \( l \) the entropic shifting of horizon may give rise to the possibility of the existence of an effective cut-off frequency \( \sim l^{-1} \) for incoming modes in that frame. The heuristic nature of this argument, however, should once again be stressed.
5 Summary and Outlook

The essence of the investigations made above is that the real entropic state of a black hole cannot be separated from the state of the universe as a whole. This aspect is clearly reflected in the presence of the short distance scale $l$ in the scale hierarchy (10). This scale has its origin in a remarkably fractional contribution of the mass of a black hole (in the moment of its formation) to the entropic mass scale of the universe and may have the effect of prohibiting the unphysical ultrahigh frequency modes apparently entering the usual derivation of Hawking’s radiation.

There are many open problems to be resolved. For example, due to the ‘infinite’ gravitational redshift effect, the causal structure of black hole space-time in the semiclassical picture appears to be very sensitive to an (even) small entropic shifting in the location of horizon, leading, therefore, to the problem of how the appearance of $l$ may limit the validity of the conclusions drawn in the semiclassical picture. In addition there is the problem of properly understanding the physical status of the length scale $l$. Regarding this last problem it would seem, first, that the actual process of gravitational contraction of a sufficiently large mass in the present epoch of the universe may define a minimal bound for physical length scales as $l$ in (10) appears to be ‘entropically’ bounded from below by the limit mass of the hydrostatic stability for a super dense star. It is quite conceivable that the appearance of this ‘entropic’ minimal length may act as a possible reason for the postulate of a fundamental irreversibility inherent in ultrashort distance physics and for the break down of the universal requirement of Lorentz symmetry.

Appendix

Let $p$ be an affin parameter along a free falling geodesics of the Schwarzschild-metric. Since this metric does not depend on the Schwarzschild-time $t$ the quantity $p_t = g_{t\mu} dx^\mu / dp = (1 - 2M/r) dt / dp$ is conserved. Thus, the affin parameter may normalized so that

\[
\frac{dt}{dp} = (1 - \frac{2M}{r}).
\]

(12)

In this normalization the equation governing the radial motion of the geodesics takes the form

\[
\left(\frac{dr}{dp}\right)^2 + \left(1 - \frac{2M}{r}\right)\left(\frac{J^2}{r^2} + E\right) - 1 = 0,
\]

(13)

where $E$ and $J$ are two integration constants, the first one being related to the proper time by, see [11]

\[
d\tau^2 = E dp^2.
\]

(14)

If the geodesics starts from the rest at infinity, it follows from (13) $E = 1$, and consequently (13) with respect to $\tau$ can be written as

\[
\left(\frac{dr}{d\tau}\right)^2 + \left(1 - \frac{2M}{r}\right)\left(\frac{J^2}{r^2} + 1\right) - 1 = 0.
\]

(15)
Near the horizon $r \approx 2M$, we may write it in the form

$$\left(\frac{dr}{d\tau}\right)^2 - \frac{2M}{r} \approx 0,$$

(16)

Thus, the rate of the change of $r$ with respect to the proper time near the horizon must be $dr/d\tau \approx -(2M/r)^{1/2}$. Now, it is easy to determine the rate at which advanced time $v$ would change near the horizon along the geodesics. Using

$$v = t + \dot{r}^*$$

$$\dot{r}^* = r + 2M \ln\left(\frac{r}{2M} - 1\right),$$

(17)

one finds $dv/d\tau \approx 1/2$.

References

1. Bekenstein J D 1973 Phys. Rev. D7,2333
2. Hawking S W 1975 Commun. Math. Phys. 43
3. Penrose R 1979 General Relativity, an Einstein Centenary Survay ed. by Hawking S W and Israel W, p.581 (Cambridge university Press)
4. Salehi H Proceeding of international symposium of generalized symmetries, Germany, World Scientific (1994)
5. Brance C and Dicke R 1961 Phys. Rev. D 124, 3, 925
6. Tolman Relativity Thermodynamics and Cosmology Oxford (1966)
7. Qadir A Proc. of the fifth. Marcell Grossman Meeting on general relativity, Part A. World Sceitific (1989)
8. Jacobson Th 1991 Phys. Rev. D 44,1731
9. Salehi H 1992 Class. Quantum Grav.9 2557-2571
10 Jacobson Th 1993 Phys. Rev. D 48,728
11. Weinberg S Gravitation and Cosmology, John Wiley and Sons (1972)