Direct detection of singlet dark matter in classically scale-invariant standard model

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Abstract

Classical scale invariance is one of the possible solutions to explain the origin of the electroweak scale. The simplest extension is the classically scale-invariant standard model augmented by a multiplet of gauge singlet real scalar. In the previous study it was shown that the properties of the Higgs potential deviate substantially, which can be observed in the International Linear Collider. On the other hand, since the multiplet does not acquire vacuum expectation value, the singlet components are stable and can be dark matter. In this letter we study the detectability of the real singlet scalar bosons in the experiment of the direct detection of dark matter. It is shown that a part of this model has already been excluded and the rest of the parameter space is within the reach of the future experiment.
1 Introduction

Higgs boson was discovered in 2012 at the CERN Large Hadron Collider [1,2]. Since then, its properties, such as spin, parity and couplings to the standard model fermions and gauge bosons, have been measured and it turned out that they are consistent with the standard model prediction. In spite of the success of the standard model up to now, however, it is commonly believed that the standard model is not the ultimate theory of particle physics. In fact there are lots of unsolved problems in the field of particle physics as well as cosmology.

One of them is the origin of the spontaneous symmetry breakdown of the electroweak gauge group. In the standard model, the electroweak symmetry is broken by Higgs field that has an ad hoc tachyonic mass term. One explanation for the tachyonic mass is supersymmetry. In supersymmetric extension of the standard model, the negative mass term is induced radiatively. On the other hand, radiative symmetry breaking is possible in non-supersymmetric theory, which is known as Coleman-Weinberg (CW) mechanism [3]. In the CW mechanism spontaneous symmetry breaking is induced at quantum level from classically scale-invariant scalar potential. Although it turned out that the CW mechanism with a Higgs does not work for the electroweak symmetry breaking, simple extensions of the Higgs sector are known to be phenomenologically viable (see, e.g., [4–29]).

Recently Higgs properties were studied in a classically scale-invariant standard model augmented by an electroweak singlet scalars that form a multiplet of global $O(N)$ symmetry [30]. It was shown that the Higgs self-couplings deviate significantly from the standard model prediction. Such feature can be observed as a prominent signal of this model at the next-generation lepton collider experiment, such as the International Linear Collider (ILC) [31, 32]. On the other hand, it was also shown that the singlet field does not get a vacuum expectation value (VEV). Then, other Higgs properties are unaffected since there is no mixing between the singlet and Higgs. Another important consequence is the stability of the singlet field due to unbroken $O(N)$ symmetry. If the reheating temperature of the universe is higher than the mass of the singlet, the singlet field is thermalized. Then non-vanishing thermal relic of the singlet remains, which can play a role of dark matter.

In this letter we study direct detection of the real singlet dark matter with $O(N)$ symmetry. It was pointed out in a similar framework where the thermal relic abundance of the singlet dark matter is too small to explain the present energy density of
dark matter by taking into account the 125 GeV Higgs \[7,10,12\]. This is due to an enhanced annihilation cross section caused by a large singlet-Higgs coupling. On the contrary, however, the large singlet-Higgs coupling may result in a large scattering cross section of the singlet with nucleon. According to the study of Ref. \[30\], the couplings of the singlet with the standard model particles are fixed for the successful electroweak symmetry breaking via the CW mechanism, which makes it possible to determine the relic abundance and the scattering cross section of the singlet with nucleon at a high precision. The scattering cross section of singlet scalar dark matter is also discussed in several literature mentioned above. We revise the calculation of the spin-independent cross section of singlet scalar particle by adopting the formalism given in Ref. \[34\] where next-to-leading order QCD effect is properly taken into account. It will be shown that part of the model has already been excluded by recent LUX result \[35\] and the future experiments will be able to probe almost the entire parameter space of the model.

Here is the organization of this letter. In Sec. 2 we briefly explain the model, including the prescription how to determine model parameters. Then the thermal relic and the scattering cross section of the singlet are calculated, and the detection of the singlet scalar is discussed in Sec. 3. Sec. 4 is dedicated to conclusion.

2 The Model

In the framework with classical scale invariance it is known that the standard model without the Higgs mass term has already been excluded. In order to construct phenomenologically viable model, therefore, it is necessary to extend the model, e.g., by adding a new particle to the model. The simplest extension is to introduce a gauge singlet real scalar field. Such a singlet scalar can couple to the Higgs in general, then the singlet contributes to the CW potential. The effect strongly depends on the degree of freedom of the singlet field. To see the impact we introduce a fundamental representation of a global \( O(N) \) symmetry, \( S = (S_1, \ldots, S_N)^T \). Consequently, the tree-level scalar potential which is allowed under the symmetry is

\[
V = \lambda_H (H^\dagger H)^2 + \lambda_H S H^\dagger H S_i S_i + \frac{\lambda_S}{4} (S_i S_i)^2,
\]

where \( H \) is the Higgs doublet field \( H = (H^+, H^0)^T \), and summed over \( i = 1, \ldots, N \) for \( N \geq 2 \). \( Z_2 \) symmetry is assumed for \( N = 1 \) case, whereas it is also a subgroup
of $O(N)$ symmetry and always survives in the scale-invariant tree-level potential for $N \geq 2$.

The electroweak symmetry breaking is induced via the CW mechanism. To see this, the scalar fields can be taken without loss of generality as $H = (1/\sqrt{2})(0, \phi)^T$ and $S = (\varphi, 0, \cdots, 0)^T$ where $\phi$ and $\varphi$ are classical fields of the real scalars. Then the effective potential at one-loop level is given by

$$V_{\text{eff}}(\phi, \varphi) = V_{\text{tree}}(\phi, \varphi) + V_{1\text{-loop}}(\phi, \varphi),$$

(2.2)

with

$$V_{\text{tree}}(\phi, \varphi) = \frac{\lambda_H}{4}\phi^4 + \frac{\lambda_{HS}}{2}\phi^2\varphi^2 + \frac{\lambda_S}{4}\varphi^4,$$

(2.3)

$$V_{1\text{-loop}}(\phi, \varphi) = \frac{1}{4(4\pi)^2} \sum_i n_i M_i^4(\phi, \varphi) \left[ \ln \frac{M_i^2(\phi, \varphi)}{\mu^2} - c_i \right],$$

(2.4)

in $\overline{\text{MS}}$-scheme with renormalization scale $\mu$. Index $i$ denotes the fields which run in the loop diagrams. ($n_i$, $M_i^2$ and $c_i$ are given in Appendix A). The electroweak symmetry is spontaneously broken if $\partial V_{\text{eff}}/\partial \phi|_{\phi = \langle \phi \rangle} = 0$ with $\langle \phi \rangle \neq 0$, which implies $\lambda_H \sim \frac{N\lambda_{HS}^2}{16\pi^2} - \frac{3y_t^4}{16\pi^2}$ as a necessary condition. Then $\lambda_H$ should be regarded as the next-to-leading order in terms of the order counting of the dimensionless couplings. Consequently we rewrite the effective potential as

$$V_{\text{eff}} = V_{\text{LO}} + V_{\text{NLO}},$$

(2.5)

with $V_{\text{LO}}$ and $V_{\text{NLO}}$ being regarded as leading order (LO) and next-to-leading order (NLO) of the scalar potential;

$$V_{\text{LO}} = \frac{\lambda_{HS}}{2}\phi^2\varphi^2 + \frac{\lambda_S}{4}\varphi^4,$$

(2.6)

$$V_{\text{NLO}} = \frac{\lambda_H}{4}\phi^4 + \frac{F_{\text{app}}^2(\phi, \varphi)}{64\pi^2} \left[ \ln \frac{F_{\text{app}}(\phi, \varphi)}{\mu^2} - \frac{3}{2} \right] + \frac{3}{64\pi^2} (\lambda_{HS}\varphi^2)^2 \left[ \ln \frac{\lambda_{HS}\varphi^2}{\mu^2} - \frac{3}{2} \right]$$

$$+ \frac{N + 1}{64\pi^2} (\lambda_{HS}\phi^2 + \lambda_S\varphi^2)^2 \left[ \ln \frac{(\lambda_{HS}\phi^2 + \lambda_S\varphi^2)^2}{(\lambda_{HS}\phi^2 + \lambda_S\varphi^2)^2} - \frac{3}{2} \right]$$

$$- \frac{12}{64\pi^2} M_i^4(\phi) \left[ \ln \frac{M_i^2(\phi)}{\mu^2} - \frac{3}{2} \right]$$

$$+ \frac{6}{64\pi^2} M_W^4(\phi) \left[ \ln \frac{M_W^2(\phi)}{\mu^2} - \frac{5}{6} \right] + \frac{3}{64\pi^2} M_Z^4(\phi) \left[ \ln \frac{M_Z^2(\phi)}{\mu^2} - \frac{5}{6} \right],$$

(2.7)
where $F_{+\text{app}}$ is given in Appendix A. In Ref. [30] it is shown that the successful electroweak symmetry breaking with the Higgs mass $m_h \simeq 125$ GeV can be realized in a given number of $N$. Table I shows the results. Roughly speaking, the Higgs mass is expected to be $m_h \sim (\sqrt{N} \lambda_{HS}/4\pi)v_H$ with $v_H = 246$ GeV, which is consistent with the numerical results in Table I. With the proper order counting, the effective potential around the VEV is obtained by replacing the scalar fields as \( \phi \to v_H + h, \varphi^2 \to s_i s_i \) and expanding by powers of $h$ and $s_i s_i$;

\[
V_{\text{eff}} = \text{const.} + \frac{1}{2} m_h^2 h^2 + \frac{1}{2} m_s^2 s_i s_i + \frac{\lambda_{hhh}}{3!} v_H h^3 + \frac{\lambda_{hhhh}}{4!} h^4 \\
+ \frac{\lambda_{hss}}{2} v_H h s_i s_i + \frac{\lambda_{hhss}}{4} h^2 s_i s_i + \frac{\lambda_{ssss}}{4!} (s_i s_i)^2 + \cdots ,
\]

where $m_s$ is the mass of singlet. We have taken $\mu = v_H$ and omitted irrelevant terms in our later discussion. The results for $\lambda_{hhhh}$, $\lambda_{hhh}$, $\lambda_{hss}$, $\lambda_{ssss}$ and $m_s$ are summarized in Table 2 [30].

Basically $\lambda_H$ and $\lambda_{HS}$ are chosen to give rise to $v_H = 246$ GeV and $m_h = 125$ GeV, which determine the couplings (except for the singlet self-coupling) and the singlet mass. $\lambda_S$, on the other hand, has little impact on these results. Since the Higgs self-couplings $\lambda_{hhhh}$ and $\lambda_{hhhh}$ significantly deviate from the SM prediction, the precise measurement of the Higgs self-couplings is a viable way to test this model.

Another important fact shown in Ref. [30] is that the singlet does not get VEV. Without a VEV of the singlet, Higgs properties, such as Higgs production or decay rates, are unaffected. On the other hand, unbroken $O(N)$ symmetry forbids $s_i$ to decay. Such stable particles can change the thermal history of the universe. If the reheating temperature in the early universe is higher than the singlet mass, the singlet particles are thermalized and their number densities freeze out eventually. Then the thermal relics can be components of dark matter. In this model the parameters which determine the interaction of $s_i$ with the standard model particles (the Higgs field in our case) are completely fixed as discussed above. Therefore its nature is

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#1 This is the results derived from the potential referred as (I) in Ref. [30].

#2 The couplings are obtained from the parameters shown in Table 1 which corresponds to case (I) in Ref. [30]. Since the couplings change by a few % in cases (II) or (III), we will use the couplings from case (I) in our later calculation.

#3 This fact is guaranteed to all orders in perturbative expansion. Strictly speaking, non-perturbative effect might break $O(N)$, which could allow non-zero VEV for the singlet. Possible (or known) non-perturbative effect is anomaly. In our model, however, $O(N)$ multiplet is scalar, thus it is anomaly free. Though one may concern another unknown non-perturbative effect, the situation is the same for the standard model, i.e. unbroken $U(1)_{em}$ symmetry.
Table 1: The input parameters of the analysis in Ref. [30]. The values $\lambda_S = 0.10$ are benchmarks, but there are very few $\lambda_S$-dependences for $\lambda_{hhh}$, $\lambda_{hhhh}$, $\lambda_{hss}$.

|   | 1   | 4   | 12  |
|---|-----|-----|-----|
| $\mu$ | $v_H = 246 \text{ GeV}$ |     |     |
| $y_t$ | 0.919 |     |     |
| $g$   | 0.644 |     |     |
| $g'$  | 0.359 |     |     |
| $\lambda_H$ | -0.11 | -0.0045 | 0.075 |
| $\lambda_{HS}$ | 4.8 | 2.4 | 1.4 |
| $\lambda_S$ | 0.10 | 0.10 | 0.10 |

highly predictable, and in fact we will see in the next section that the experiments of direct detection of dark matter provide the powerful tool to probe the model.

Before closing this section it is worth noting that $N = 1$ case is favored by two reasons [30]. In terms of Veltman’s condition [36] the level of fine-tuning for the Higgs mass at the electroweak scale gets milder compared to larger $N$ cases. Second, the fine-tuning is relaxed compared to the standard model and larger $N$ case in a sense that the cutoff scale due to Landau pole is predicted to be around TeV. We will come back to this point later.

### 3 Detection of Singlet Scalars

As we discussed the singlet scalars are stable and they can play a role of dark matter. Since the singlet scalars interact with Higgs boson, they are thermalized in the early universe if the reheating temperature is higher than the singlet mass.\(^4\) Then the relic abundance is determined by the conventional freeze-out scenario. What we need is the annihilation cross section of $s_i$. Relevant annihilation processes are $s_i s_i \rightarrow W^+ W^-$, $ZZ$, $hh$ and $tt$. The cross sections for the processes are given by

$$
\sigma_{s_i s_i \rightarrow W^+ W^-} = \frac{\beta_f(s, m_W^2)}{4 \pi s \beta_i} \left( \frac{\lambda_{hss} m_W^2}{s - m_h^2} \right)^2 \left[ 2 + \frac{1}{4} \left( \frac{s - 2 m_W^2}{m_W^2} \right)^2 \right],
$$

\(^{#4}\)To be specific, the reheating temperature should be larger than 300–600 GeV (see Table 2), which is a canonical case in the early universe.
Table 2: The results about the interactions among the Higgs boson and singlet scalar bosons derived in Ref. [30]. They are defined in Eq. (2.8). Some of them are also input parameters of the calculations in Section 3. These values are those in case (I), except for $\lambda_{ssss}$ which is in (II) of Table 4 in Ref. [30]. In our notation of the SM Higgs potential the predicted self-couplings are $\lambda^{(SM)}_{hhh} = \lambda^{(SM)}_{hhhh} = 6\lambda_H = 3m_h^2/v_H^2 \simeq 0.78$.

| $N$ | 1   | 4   | 12  |
|-----|-----|-----|-----|
| $\lambda_{hhh}$ | 1.32 | 1.32 | 1.32 |
| $\lambda_{hhhh}$ | 2.9  | 2.9  | 2.9  |
| $\lambda_{hss}$  | 11.4 | 5.02 | 2.80 |
| $\lambda_{hsss}$ | 14   | 5.6  | 3.0  |
| $\lambda_{ssss}$ | 6.5  | 1.9  | 0.9  |
| $m_s [\text{GeV}]$ | 556  | 378  | 285  |

Numerically it is found that the annihilation mode to $t\bar{t}$ is subdominant.\(^5\) It is straightforward to compute the thermal-averaged cross section from above expression. We use the formula in Ref. [37] to get the relic abundance. (We have checked that the density parameter by solving the Boltzmann equation agrees with the approximated result within a few %.) The results are shown in Table 3. (We note

\[\sigma_{s_i s_i \to ZZ} = \frac{\beta_f(s, m_Z^2)}{8\pi s\beta_i} \left( \frac{\lambda_{hss} m_Z^2}{s - m_h^2} \right)^2 \left[ 2 + \frac{1}{4} \left( \frac{s - 2m_Z^2}{m_Z^2} \right)^2 \right], \quad (3.2)\]

\[\sigma_{s_i s_i \to hh} = \frac{1}{16\pi s\beta_i} \left[ \beta_f(s, m_h^2)^2 + 4\lambda_{hss}^2 v_H^2 \log \frac{t_+}{t_-} + \frac{2}{s\beta_i} \left( \frac{s\beta_i \beta_f(s, m_h^2)}{t_+ t_-} + \frac{2}{2m_h^2 - s} \log \frac{t_+}{t_-} \right) \right], \quad (3.3)\]

\[\sigma_{s_i s_i \to t\bar{t}} = \frac{3\beta_f^2(s, m_t^2)}{8\pi \beta_i} \left( \frac{\lambda_{hss} m_t^2}{s - m_h^2} \right)^2, \quad (3.4)\]

where $m_W$, $m_Z$ and $m_t$ are the masses of $W$, $Z$ and $t$, respectively. $s$ is the center-of-mass energy in the initial state, $\beta_i = (1 - 4m_s^2/s)^{1/2}$, $\beta_f(s, m^2) = (1 - 4m^2/s)^{1/2}$ and

\[\tilde{\lambda} = \lambda_{hss} + \frac{\lambda_{hhh} \lambda_{hss} v_H^2}{s - m_h^2}, \quad t_\pm = m_h^2 - \frac{s}{2} \left[ 1 \mp \beta_i \beta_f(s, m_h^2) \right]. \quad (3.5)\]

\(^5\)This is expected since the amplitude of fermion pair final state is chirality suppressed, \textit{i.e.}, the cross section is proportional to $m_t^2/m_s^4$ instead of $1/m_s^2$. 

6
that when \(N \geq 2\) all scalars \(s_1, \ldots, s_N\) become dark matter.) It has figured out that the relic abundance \(\Omega_{s_i}\) of \(s_i\) is much smaller than that of dark matter, which means that \(s_i\) cannot be the main component of dark matter. This is due to the large annihilation cross section enhanced by the large couplings (mainly \(\lambda_{hss}\)). On the other hand, however, \(s_i\) can be detected in the experiment of direct detection of dark matter, which we will discuss below.

For evaluation of the spin-independent cross section of \(s_i\) with nucleon, we adopt the formalism given in Ref. [34]. (See also Refs. [38–41] for earlier works.) In the present case only scalar-type operators are induced by Higgs-exchange diagram, then the effective Lagrangian for the scattering process is

\[
\mathcal{L}_{\text{eff}} = \sum_{i=q,G} C^q_S \mathcal{O}^q_S + C^G_S \mathcal{O}^G_S,
\]

where

\[
\mathcal{O}^q_S = m_q s_i^2 \bar{q}q, \quad \mathcal{O}^G_S = \frac{\alpha_s}{\pi} s_i^2 G^a_{\mu\nu} G^{a\mu\nu}.
\]

\(m_q\) is quark mass, \(G^a_{\mu\nu}\) is the gluon field strength and \(\alpha_s\) is the strong coupling constant. By integrating out the Higgs boson (and top quark), the Wilson coefficients at the electroweak scale \(\mu_W \simeq m_Z\) at the next-to-leading order in \(\alpha_s\) are given by

\[
C^q_S(\mu_W) = \frac{\lambda_{hss}}{2m_h^2},
\]

\[
C^G_S(\mu_W) = -\frac{\lambda_{hss}}{24m_h^2} \left[ 1 + \frac{11\alpha_s}{4\pi} \right].
\]

The amplitude is given by the hadronic matrix elements, i.e.

\[
\langle N|m_q\bar{q}q|N \rangle, \langle N|\frac{\alpha_s}{\pi} s_i^2 G^a_{\mu\nu} G^{a\mu\nu}|N \rangle \quad (N = p, n),
\]

which are obtained from lattice simulations [42,43] and the QCD trace anomaly [14,46].

\[
\langle N|m_q\bar{q}q|N \rangle = m_N f_{t_q}^{(N)},
\]

\[
\langle N|\frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu}|N \rangle = m_N \frac{4\alpha_s^2}{\beta_s(N_f=3)} \left[ 1 - (1 - \gamma_m) \sum_{q=u,d,s} f_{t_q}^{(N)} \right].
\]

Here \(m_N\) is nucleon mass, \(f_{t_q}^{(N)}\) is mass fractions, e.g., \(f_{t_q}^{(p)} = 0.019(5), f_{t_q}^{(n)} = 0.027(6)\) and \(f_{t_q}^{(p)} = 0.009(22)\), which are evaluated in Ref. [11] based on Refs. [42,43]. \(\beta_s\) and \(\gamma_m\) are the beta function of \(\alpha_s\) and the anomalous dimension of quark mass defined

\[\Theta^\mu_\mu = \frac{3\alpha_s}{4\pi} G^a_{\mu\nu} G^{a\mu\nu} + (1 - \gamma_m) \sum_q m_q \bar{q}q,\]

and \(m_N = \langle N|\Theta^\mu_\mu|N \rangle\) to derive Eq. (3.11).
Table 3: Fractions of the energy density of $s_i$ in the observed dark matter density, and spin-independent cross sections of the scalars with proton multiplied by their fractions in the present dark matter density. The cross sections are computed with the effective couplings $f_S^{(p)}|_{\text{NLO}}$ shown in Table 4.

| $N$ | $\Omega_{s_i}/\Omega_{\text{DM}}$ | $\sigma_{SI}^{(p)} [10^{-46} \text{ cm}^2]$ |
|-----|-------------------------------|--------------------------------|
| 1   | $2.01 \times 10^{-4}$         | 6.77                          |
| 4   | $4.54 \times 10^{-4}$         | 25.6                          |
| 12  | $8.07 \times 10^{-4}$         | 74.5                          |

by $\beta_s = \mu \frac{d\alpha_s}{d\mu}$ and $\gamma_m m_q = \mu \frac{dm_q}{d\mu}$, respectively. In Eq. (3.11) the number of flavors $N_f = 3$ is taken since we only know the mass fractions for the light quarks, which means that the Wilson coefficients $C_S^q$ and $C_S^G$ should be evaluated at the hadronic scale $\mu_{\text{had}} \simeq 1 \text{ GeV}$. This can be done by the matching procedure at each quark threshold (bottom and charm quarks) and by solving the renormalization group equation for the Wilson coefficients. (For the details, such as the matching and renormalization group evolution at the next-to-leading order in $\alpha_s$, see Ref. [34]). Finally the spin-independent cross section is given by

$$\sigma_{SI}^{(N)} = \frac{1}{\pi} \left( \frac{m_N^2}{m_s + m_N^2} \right)^2 |f_S^{(N)}|^2,$$

with

$$f_S^{(N)} = \sum_{q=u,d,s} C_S^q(\mu_{\text{had}}) \langle N | m_q \bar{q} q | N \rangle + C_S^G(\mu_{\text{had}}) \langle N | \frac{\alpha_s}{\pi} G^{a\mu\nu} G_{a\mu\nu} | N \rangle.$$  (3.13)

If renormalization group evolution is ignored as well as taking the leading order threshold matching, then the effective scattering amplitude is simply given by

$$\frac{f_S^{(N)}}{m_N}|_{\text{est}} \approx \frac{\lambda_{hss}}{2m_h^2} \left[ \frac{2}{9} + \frac{7}{9} \sum_{q=u,d,s} f_{Tq}^{(N)} \right],$$

which is often used in the literature. To see the impact of the proper matching procedure to the effective coupling $f_S^{(N)}$, we show the numerical values in Table 4. The difference between the LO and the NLO results is about 4%, while the one between the rough estimation and the NLO is about 7%, which gives rise to about 14% deviation in the spin-independent cross section.

Now we are ready to see the experimental consequence of the model. Since the scalar fields are not the main component of the present DM, the “effective” cross
Table 4: Amplitude for $s_i$-proton scattering. $f_S^{(p)}|_{\text{est}}$ is given by Eq. (3.14) and $f_S^{(p)}|_{\text{LO}}, f_S^{(p)}|_{\text{NLO}}$ are the results obtained by appropriate matching at the leading order and the next-to-leading order, respectively.

| $N$ | 1  | 4  | 12 |
|-----|----|----|----|
| $f_S^{(p)}|_{\text{est}}[10^{-5}\text{ GeV}^{-1}]$ | 9.07 | 3.99 | 2.23 |
| $f_S^{(p)}|_{\text{LO}}[10^{-5}\text{ GeV}^{-1}]$ | 10.2 | 4.49 | 2.50 |
| $f_S^{(p)}|_{\text{NLO}}[10^{-5}\text{ GeV}^{-1}]$ | 9.77 | 4.30 | 2.40 |

The cross section of the singlet with nucleon is obtained by multiplying the fraction of the total abundance of the scalars in the present DM density,

$$\tilde{\sigma}_SI^{(N)} = \sigma_{SI}^{(N)} \sum_{i=1,\cdots,N} \frac{\Omega_{s_i}}{\Omega_{\text{DM}}} , \quad (3.15)$$

where $\Omega_{\text{DM}} = 0.264$ [45]. The results are given in Table 3 and Fig. 1. In Table 3 we have used the NLO result. Compared with the most stringent bound for the cross section [35], $N = 12$ case has already been excluded at 90% C.L. The others, i.e. $N = 1$ and 4 cases, are still viable, but $N = 4$ case is very close to the present bound. The cross section is far below the present bound for $N = 1$ case. However, it is much larger than the neutrino background in the direct detection experiments [46]. Therefore, our model (with any number of $N$) can be tested in ton-scale future experiments, such as LZ program [47]. Recall that $N = 1$ is favored in terms of Veltman’s condition as well as the fine-tuning of the Higgs mass, which is discussed in the previous section. Thus the result shows that the most well-motivated case will be able to be examined in the future experiments.

## 4 Conclusion

In this letter we have studied direct detection of singlet scalar dark matter in a classically scale-invariant extension of the standard model. The model extends the Higgs sector to have an additional electroweak singlet scalars that form a multiplet of global $O(N)$ symmetry, and the electroweak symmetry is broken via Coleman-Weinberg mechanism. Recently the Higgs self-couplings as well as new couplings and the singlet mass were precisely computed in Ref. [30]. In the work it was shown the Higgs self-couplings deviate from the standard model prediction significantly,
Figure 1: Spin-independent cross sections of the singlet scalars compared with the LUX 90\% C.L. bound [35]. Orange, green and purple points represent $N = 1$, 4 and 12 cases, respectively. Triangle and circle points represent rough estimation and the next-to-leading order calculation, respectively.

which can be observed at the next-generation collider experiments such as the ILC. Another important outcome of their analysis is unbroken $O(N)$. Consequently the singlet scalars are cosmologically stable and can play a role of dark matter. Since all couplings and mass parameters are fixed for given number of $N$ [30], it is possible to precisely predict the nature of the singlet scalars. Therefore detection of the singlet scalars is complementary for the test of the model.

Assuming that the reheating temperature is above the singlet mass, we have computed the thermal relic abundance of the singlet scalars, and the scattering amplitude of the scalars with nucleon. For the precise determination of the scattering cross section we have used the formalism [34] which takes into account the next-to-leading order QCD effect in consistent way. We have focused on three benchmarks, \textit{i.e.} $N = 1$, 4 and 12 (the singlet masses are predicted as 556, 378 and 285 GeV, respectively). Then it has been found that although the relic abundance is much
smaller than the present dark matter \( (\Omega_s/\Omega_{\text{DM}} \sim \mathcal{O}(10^{-4})) \), the scattering rate is enhanced due to the large Higgs-singlet coupling. To be concrete, \( N = 12 \) case has already been excluded by the LUX experiments, meanwhile \( N = 4 \) case is near the bound. In \( N = 1 \) case which is favored in terms of the fine-tuning regarding the Higgs mass, the effective spin-independent cross section \( (\simeq 6.8 \times 10^{-46} \text{ cm}^2) \) is far below the current bound. It is, however, much larger than the neutrino background. Thus it is concluded that the whole parameter space of this scenario is testable in the future ton-scale detector of dark matter direct detection.

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**A Parameters in the effective potential**

Here is the list of parameters of the one-loop potential in Eq. (2.2):

\[
\begin{align*}
n_W &= 6, \quad M_W^2 = \frac{1}{4}g^2\phi^2, \quad c_W = \frac{5}{6}; \\
n_Z &= 3, \quad M_Z^2 = \frac{1}{4}(g^2 + g'^2)\phi^2, \quad c_Z = \frac{5}{6}; \\
n_t &= -12, \quad M_t^2 = \frac{1}{2}y_t^2\phi^2, \quad c_t = \frac{3}{2}; \\
n_{\pm} &= 1, \quad M_{\pm}^2 = F_{\pm}, \quad c_\pm = \frac{3}{2}; \\
n_{H_{\text{NG}}} &= 3, \quad M_{H_{\text{NG}}}^2 = \lambda_H\phi^2 + \lambda_H S\varphi^2, \quad c_{H_{\text{NG}}} = \frac{3}{2}; \\
n_{S_{\text{NG}}} &= N - 1, \quad M_{S_{\text{NG}}}^2 = \lambda_H S\phi^2 + \lambda_S \varphi^2, \quad c_{S_{\text{NG}}} = \frac{3}{2},
\end{align*}
\]

(A.1)

where \( i = W, Z, t \) show \( W, Z, t \) in the loop, respectively. \( i = \pm \) indicates \( \varphi, \phi \), while \( i = H_{\text{NG}}, S_{\text{NG}} \) stand for the degrees of freedom which are orthogonal to \( \phi \) and \( \varphi \), respectively. In the effective potential with the precise order counting \((i.e.\)
in Eq. (2.5), we use

\[
F_{\pm \text{app}}(\phi, \varphi) = \frac{\lambda_{HS}}{2} \phi^2 + \frac{\lambda_{HS} + 3\lambda_S}{2} \varphi^2 \\
\pm \sqrt{\left(-\frac{\lambda_{HS}}{2} \phi^2 + \frac{\lambda_{HS} - 3\lambda_S}{2} \varphi^2\right)^2 + 4\lambda_{HS}^2 \phi^2 \varphi^2},
\]

(A.2)

for \( F_\pm \). The reason for dropping \( F_- \) in Eq. (2.5) is explained in Ref. [30] (see also Ref. [48]).

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