A Simplified Method for the Design of Steel Beam-to-column Connections

Imola Kristóf¹, Zsanett Novák¹, Dezső Hegyi²*

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Abstract
The moment resistance of beam-to-column connections is frequently utilised in steel structures. Eurocode 3 suggests the component method to analyse such connections, and it implements the equivalent T-stub method to determine the resistance of the end plate of the beam. The latter method requires tedious and concentrated work. A simplified method is suggested to reduce the number of calculations and enable the designer to focus on construction aspects in the pre-design phase, or in education.

The resistance of the T-stub covers three possible failure modes: the yield of the plate, the failure of the of the bolt and simultaneous yield. The yield of the plate and simultaneous yield depend on numerous parameters, and they are verified by multiple equations. The failure of the bolts are more easily checked. In the present paper, requirements for geometric ratios are defined for the widely used steel sections to assure failure of the bolts at a lower level of the load than the yield of the plate. These parameters facilitate the simple calculation of the resistance of the bolts instead of the tedious work needed for the end plate resistance.

The paper presents a proper explanation for the design rules and the effect of the geometric parameters on the resistance of the end plate. Geometric parameters are suggested for the widely used hot rolled and typical welded beam sections. All the parameters fulfil the requirements of the equivalent T-stub method of Eurocode 3.

Keywords
beam-to-column connections, end plate, simplified design method

1 Introduction
Beam-to-column connections are widely used for steel structures. They provide moment resistant connections between beams and columns at the corners of frames or a moment resistant connection to elongate beams.

The analysis of the beam-to-beam connection is complex: due to the geometrical arrangement, different failure modes are possible, and some of them cannot be presented by simple equations. Finite element analysis can be practical to verify the loadbearing capacity of the connection, or Eurocode 3 (EC3) suggests using the component method for the analysis (Ádány et al., 2014; Fernezelyi, 2013; EC3.). It verifies the connection based on the failure of its elements, represented in Fig. 1.

The considered failure modes are the following:

a) Tension on the girder of the column
b) Compression on the girder of the column
c) Bending on the flange of the column
d) Bending on the end plate
e) Tension on the bolts
f) Tension on the girder of the beam
g) Compression on the girder and the flange
h) Shear on the girder of the column

Most of the component parts can be described by simple failure modes. Failure of the end plate exhibits the most complicated behaviour in the connection (Fernezelyi, 2008).
The T-stub method is suggested to determine the proper thickness of the end plate and the proper diameter of the bolts. Our target is to simplify the T-stub method because the resistance of the T-stub depends on a large set of equations. Each equation belongs to a different failure mode of the end plate or the bolt. Generally speaking, it cannot be predicted what kind of yield pattern describes the breaking of the end plate a priori. These equations are not very complicated to apply, but it is a tedious work to evaluate each. It is accompanied with an elevated risk of mistakes and a lack of a picturesque workflow.

The end plate is a bended element under out of plane forces: the tension of the flange of the beam, the tension of the bolt and the compression on the touch point of the opposite element of the connection. While the geometrical arrangement draws a complex geometry for the plate, the elastic stress distribution cannot be reasonably simplified for the everyday applications in the analysis. Plastic moment resistance is taken into consideration with different yield patterns.

Basically, there are three possible failures for the T-stub: the yield of the end plate (1st case, there is yield on the plate at the flange and at the bolt); the simultaneous yield of the end plate and the bolt (2nd case, there is yield on the plate at the flange); and the failure of the bolt (3rd case, the plate is elastic) (Fig. 2).

Determination of the failure of the 1st and the 2nd cases are based on a complex yield pattern analysis (Ádány et al., 2014; Fernezelyi, 2013; EC3.). Different possible failures must be compared, and the minimum of the resistances must be selected. The analysis of the 3rd case is a simple tensile resistance calculation for the bolt. This study aims to suggest the minimum plate thickness according to a certain diameter of bolt to eliminate the analysis of the T-stub for selecting the correct bolt diameter. The +bolt diameter can be chosen from the force required to balance the moment acting on the connection (Fig. 3).

The minimal thickness of the plates and the geometrical constraints are determined for the most typical I and H sections and welded I sections in the same range. Systematic calculations were carried out on the range of IPE 200 to 600 and the range of HEA 100 to 1000, and similar shape welded I beams. Two combinations of material qualities are taken into account: S235 steel with 8.8 bolts and S355 steel with 10.9 bolts. In all rows, two bolts are used with an identical diameter. This paper focuses on the failure of the T-stub. The shear failure of the bolts and all other failure modes of the end plate and the connection are omitted here. All the other failure modes should be analysed, but they typically can be omitted by proper geometric design, or they are simple to analyse.

The T-stub method determines the partial height of the end plate that belongs to one row of bolts (Fig. 4). This part of the end plate represents the resistance of the plate in the 1st and 2nd cases of failure (Fig. 2). The tension resistance of the T-stub in the case of the yield of the plate, 1st case:

\[ F_{T,1,Rd} = \frac{4M_{pl,1,Rd}}{m}, \]  

for the simultaneous yield of the plate and the bolt, 2nd case:

\[ F_{T,2,Rd} = \frac{2M_{pl,2,Rd} + n\sum F_{t,Rd}}{m + n}, \]  

and for the yield of the bolt, 3rd case:

\[ F_{T,3,Rd} = \sum F_{t,Rd}, \]  

where \( F_{T,1,Rd}, F_{T,2,Rd}, F_{T,3,Rd} \) are the tensile resistances of the T-stub, which belong to the three cases of the failure; \( \sum F_{t,Rd} \) is the summary of the tension resistance of the bolts, respectively. \( M_{pl,1,Rd}, M_{pl,2,Rd} \) are the yield moments of the end plate, \( n \) is the number of the bolts in the T-stub and \( m \) is explained in Fig. 5.

\[ M_{pl,1,Rd}, M_{pl,2,Rd} \] are determined by the effective length of the T-stub (Fig. 4):

\[ M_{pl,1,Rd} = \frac{\sum I_{eff,1} \cdot f_y}{4Y_{MO}}, \]
where \( l_{\text{eff},1} \) and \( l_{\text{eff},2} \) are the effective lengths of the T-stub that belong to the 1st and 2nd cases of failure, respectively; \( t \) is the thickness of the plate, \( f_y \) is the yield stress of the plate and \( \gamma_{M0} \) is the partial factor.

\[
M_{pl,2,rd} = \frac{\sum l_{\text{eff},2} \cdot t^2 \cdot f_y}{4\gamma_{M0}},
\tag{5}
\]

Equations (4) and (5) represent the yield moment resistance of the plate. The effective lengths are determined according to EC3 for different yield line patterns. We do not provide further details of the analysis of the different yield patterns in this paper, as we used the EC3 formulas on the systematic analysis of the chosen geometric solutions to obtain relevant data for the simplified method.

### 2.2 Some simplification on the geometry

The left side of Fig. 5 shows the EC3 notation of the end plate:

- \( b_p \): the width of the end plate
- \( w \): the horizontal distance between the bolts
- \( e, e'_x \): the bolt-edge distance
- \( m, m_x, m_z \): the bolt-mid-support distance
- \( p \): the vertical distance between the bolts

It would be beneficial to simplify the geometry to get common parameters for all bolt positions (Fig. 5 right side): over the flange, 1st row under the flange, middle and last row positions. With these common parameters, the simplification can be more general with an acceptable safety margin. According to our test calculations, the difference between the yield resistance of the plates for different positions is negligible under these simplifications.

The effective length of the T-stub over the flange is derived from obvious geometric properties; it is maximal at the peak value of

\[
l_{\text{eff}} = \frac{b_p}{2}.
\tag{6}
\]

To have equal resistance for all bolt positions

\[
l_{\text{eff}} = p = \frac{b_p}{2}.
\tag{7}
\]

One can address the question: for the internal bolts, do we get a larger resistance for the connection with shorter \( l_{\text{eff}} \) accompanied with a larger arm for the moment, or a longer \( l_{\text{eff}} \) should be used with the shorter arm to balance the design moment (Fig. 6)? For shorter \( l_{\text{eff}} \), one obtains smaller tension resistance for the T-stub; longer \( l_{\text{eff}} \) has the opposite effect. According to our test calculations (based on further parameters), the suggestion embodied in Eq. (7) is a reasonable choice in practice; furthermore, it is almost optimal.

During the optimisation of the values \( w, m \) and \( e \), we use the equilibrium of Eq. (7) and all the other simplifications of Fig. 5. By the application of these parameters, the same method can be used for all the bolts of the connection: outer and inner bolts, and the inner bolts in the first, middle and last positions.

### 2.3 The optimisation of the bolt position in one row

The distance from the flange (for the outer bolts) and the distance from the web (for the inner bolts) has a significant effect on the tension capacity of the T-stub. For the simplified method, an optimal position is suggested to obtain the best performance for the connection. The main steps of this optimisation are listed here.
By the simplification of the geometrical parameters in Section 2.2, the following inequalities hold for $l_{\text{eff}}$ (based on the yield patterns in the EC3):

\begin{align*}
1.625e + 2m &\geq \frac{b_p}{2}, \quad (8) \\
0.5w + 2m + 0.625e &\geq \frac{b_p}{2}, \quad (9) \\
2\pi m &\geq \frac{b_p}{2}, \quad (10) \\
\pi m + 2e &\geq \frac{b_p}{2}, \quad (11) \\
\alpha m &\geq \frac{b_p}{2}, \quad (12) \\
0.5\alpha m - (2m + 0.625e) &\geq \frac{b_p}{2}, \quad (13) \\
4m + 1.25e &\geq \frac{b_p}{2}, \quad (14)
\end{align*}

where $\alpha$ is a modification factor based on $m$ and $e$ and can be determined from Figure 6.11 of EC3.

The effect of the horizontal position of the bolt (the value of $w$) to the effective length of the T-stub is visualised in Fig. 7. Observe, that there is a certain value for $w$ (32 mm for $b_p = 100$ mm), which limits the effect of $w$ on $l_{\text{eff}}$ if we use Eq. (7). Over this value, the $b_p/2$ limitation has priority.

The tension resistance of the T-stub in the 1st and 2nd cases of failure with respect to $w$ is depicted in Fig. 8 and 9, respectively. In these diagrams, the curves are drawn with the constraint $l_{\text{eff}} \leq b_p/2$; the minimum envelope curve is used for each position later. The constraint of Eq. (7) together with the Eqs. (8)-(14) form knees in each curve, and it is clear that the bolts over the flange and the middle positions limit the resistance of the plate. The diagrams determine an optimal position for each $b_p$. The smaller and the larger values of $w$ can give a smaller resistance for the plate. Consequently, the designers are constrained to a narrow range of the geometric arrangement to get the optimal resistance of the connection. The presented simplified method is addressed to use the optimal position with greater freedom described later (See Section 3.2).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig08.png}
\caption{The tension resistance of the T-stub by $w$ in the 1st case $b_p = 100$ mm, $t = 16$ mm, IPE, S235 with 8.8.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig09.png}
\caption{The tension resistance of the T-stub by $w$ in the 2nd case $b_p = 100$ mm, $t = 16$ mm, IPE, S235 with 8.8.}
\end{figure}

\subsection*{2.4 The effect of the web}

The yield of the end plate depends on the connection of the web to the plate, namely the radius of the hot roll or the size of the weld. The geometry of this connection determines the location of the yield point of the plate, and hence it affects the resistance of the connection. The geometrical features of Fig. 2 are affected by this parameter. We aim to generalise this effect. The support of the web is denoted by $c$ (Fig. 10). The value of $c$ establishes a connection between $w$ and $m$:

\begin{equation}
2m + c \geq w.
\end{equation}
The radius of the hot rolled sections can be found for the most popular IPE and HEA sections. The possible size of the weld was analysed based on the IPE and the HEA sections. The thickness of the web and the moment resistance of the weld determines a size range for the web. The thickness of the web was the upper limit for the weld size ($c_{max}$). The minimum size was selected based on 70% of the moment (or shear) resistance of the hot rolled section ($c_{min}$). 70% of the moment resistance of the section is a reasonable choice for the resistance of the connection because, usually, buckling limits the capacity of the structural elements. Nevertheless, this limit is just a practical value for the present analysis; for actual structures, any geometry can be considered. The real value of $c$ must be verified before the application of the simplified method.

In the further analysis, $w$ is calculated based on $c$ between $c_{max}$ and $c_{min}$ (the less safe for each $b_p$). Figs. 9 and 10 used $c$ from the radius of the hot rolled IPE section. The range of $c$ represented on Fig. 11 influences the value of $w$, so it yields to an updated function of the tension resistance of the T-stub and $w$. The recalculation of Figs. 8 and 9 are given in Figs. 12 and 13. The new tensile resistances of the T-stub can be seen here.

From the analysis of Figs. 12 and 13 (and the same diagrams for different $b_p$-s), we found that Eq. (13) gives the limitation for $w$ with all our constraints. Fig. 14 represents the optimal $w$ by the width of the end plate. $w'$ is a rounded value to have an integer [mm] value for everyday engineering usage.
above the flange was used for all sections, two rows of bolts between the flanges for IPE sections and 1 row between the flanges for HEA sections (the bolts closer to the bottom flange were not considered in the moment resistance) (Fig. 15).

The radius of the hot rolled sections and the weld size is taken into consideration to obtain a realistic result. A range for \( w \) is plotted in Fig. 16. The envelope of \( w_{\text{max}} \) and \( w_{\text{min}} \) are based on \( c_{\text{max}} \) and \( c_{\text{min}} \) and the possible sizes of the bolt, washer and wrench.

By the parameters generated above for \( c \) and \( w \), all the practical requirements are fulfilled for a bolt to beam connection.

3 The simplified method for the T-stub

In Section 2, the optimal and practical geometric parameters were investigated. All these parameters are based on a systematic set of calculations using the T-stub method. A minimum thickness of the plate was set for the possible diameters of the bolts. Tables 1 and 2 represent these minimum thicknesses, where the yield of the bolt is guaranteed below the yield of the plate.

These tables give the minimum thickness for certain widths of the plate (\( b_p \)) and certain diameters of the bolt (\( D \)), and the tables represent the geometric constraints for the bolt position

| \( b_p \) [mm] | \( w \) [mm] | \( c_{\text{min}} \) [mm] | \( c_{\text{max}} \) [mm] | \( F_{\text{nb,rd}} \) [kN] |
|---------------|---------------|--------------------------|--------------------------|--------------------------|
| 100           | 50            | 10                       | 20                       | 16                       | 19                       | 21                       | 23                       | 26                       |
| 120           | 55            | 10.5                     | 21                       | 15                       | 18                       | 21                       | 23                       | 26                       | 28                       | 31                       |
| 140           | 60            | 11                       | 22                       | 15                       | 17                       | 20                       | 22                       | 25                       | 28                       | 30                       | 34                       |
| 160           | 65            | 11.5                     | 23                       | 15                       | 17                       | 20                       | 22                       | 25                       | 27                       | 29                       | 33                       |
| 180           | 70            | 12                       | 24                       | 14                       | 17                       | 19                       | 21                       | 24                       | 27                       | 29                       | 33                       | 36                       |
| 200           | 75            | 12.5                     | 25                       | 14                       | 17                       | 19                       | 21                       | 24                       | 27                       | 29                       | 32                       | 36                       |
| 220           | 80            | 13                       | 26                       | 14                       | 16                       | 19                       | 21                       | 24                       | 26                       | 28                       | 32                       | 36                       |
| 240           | 85            | 13.5                     | 27                       | 14                       | 16                       | 19                       | 21                       | 23                       | 26                       | 28                       | 32                       | 35                       |
| 260           | 90            | 14                       | 28                       | 14                       | 16                       | 19                       | 21                       | 23                       | 26                       | 28                       | 32                       | 35                       |
| 280           | 95            | 14.5                     | 29                       | 14                       | 16                       | 19                       | 21                       | 23                       | 26                       | 28                       | 31                       | 35                       |
| 300           | 100           | 15                       | 30                       | 14                       | 16                       | 19                       | 20                       | 23                       | 26                       | 27                       | 31                       | 35                       |
| Coulour legend | \( \Delta t= \) | 1 mm                     | 2 mm                     | 3 mm                     | 4 mm                     | 5 mm                     |

Fig. 14 The value of \( w \) with respect to the width of the end plate, S235 with 8.8.

Fig. 15 Bolt positions for the test calculations in the case of IPE and HEA sections.

Fig. 16 The value of \( w \) modified with regard to the installation possibilities.
(w) and the possible range of the support of the flange (c). The colour legend of the table is described in Section 3.2.

### 3.1 Application of the simplified method

For a certain section, the position of the bolts can be chosen by Fig. 17 and Tables 1 or 2. The geometric parameters determine the arm of force (Fig. 3) to balance the moment on the connection. The minimum resistance of the T-stub can be calculated by the moment equation based on the position of the bolts. For this minimum resistance, a suitable bolt diameter can be selected based on the resistance of the bolt. The minimum thickness of the end plate belongs to the diameter of the bolt and the width of the end plate and can be selected from Tables 1 or 2. It is crucial that the determined thickness is a minimum value for the certain bolt diameter! Perhaps the utilisation of the bolt is over 100%, but the behaviour of the connection is changed according to Fig. 2, so the minimum thickness of the plate belongs to the bolt diameter and not to the effect on the T-stub.

By using a thinner plate, either the 1st or the 2nd case of failure determine the resistance of the T-stub, and the yield of the end plate causes extra tension on the bolt (Fig. 2) caused by the touch point force at the edge of the flange. In other words, a type of brittle failure is risked with a thinner end plate. The present simplified method can only be used to determine the resistance of the T-stub. All other elements of the connection needed to be analysed.

The analysis of the flange of the column is almost the same as the analysis of the end plate. The simplified method can be used to verify the flange of the column if the effect of this reinforcement is taken into account.

### 3.2 The variability of the geometry

For existing connections or for some constraints outside of the range of this study, the horizontal position (w) of the bolts are not optimal. By using a thicker plate, it is possible to deviate from the ranges of Tables 1 and 2.

The white field of the tables is the practical range of the bolts for certain widths of end plate. This range is based on the optimal w position, and the sizes of commercial bolts are also considered. A 3-mm deviation in w is possible in this strip.

If we can apply the given (optimal) value for w, then the suggested thickness can be used as well. If we depart from the ±3 mm range (closer or further from the web or the flange), a thicker end plate must be used. Observe that the difference needed for the new thickness is reasonably small, the method is not very sensitive to the exact value of w close to the optimal position.

### Table 2

| b_p [mm] | w [mm] | c_min [mm] | c_max [mm] | D [mm] | F_{u,d} [kN] |
|----------|--------|------------|------------|--------|-------------|
| 100      | 50     | 10         | 20         | 12     | 48.6        |
| 120      | 55     | 10.5       | 21         | 14     | 66.2        |
| 140      | 60     | 11         | 22         | 16     | 90.4        |
| 160      | 65     | 11.5       | 23         | 18     | 110.6       |
| 180      | 70     | 12         | 24         | 20     | 141.1       |
| 200      | 75     | 12.5       | 25         | 22     | 174.5       |
| 220      | 80     | 13         | 26         | 24     | 203.3       |
| 240      | 85     | 13.5       | 27         | 26     | 264.4       |
| 260      | 90     | 14         | 28         | 28     | 323.1       |
| 280      | 95     | 14.5       | 29         | 30     | 100         |
| 300      | 100    | 15         | 30         | 32     | 50          |
|          |        |            |            |        | 10           |

Coulour legend: ∆t = 1 mm, 2 mm, 3 mm, 4 mm, 5 mm

### Fig. 17

The geometrical parameters for the application of the simplified method.
3.3 Ductile connections

EC3 suggests using ductile connections under dynamic effects or vibrations. This is important in earthquake hazard areas, structures for transportation or under vibration from technological elements. The yield of the tensile bolt does not fulfil this requirement due to the smaller ultimate strain of the high strength steel of the bolts (smaller, than the lower quality structural steel materials). The 1st and 2nd cases of failure are preferable to a yielding failure. Tables 1 and 2 can be used to determine the tension resistance of the T-stub for certain end plate thickness: this resistance is equal to the tension resistance of the bolt because the minimum thicknesses were determined for a certain bolt diameter by the different failure modes of the end plate. Consequently, we can choose the correct thickness for the required tensile resistance, and with a larger bolt, the failure of the plate is expected at a lower level of the load than that needed for the bolt to fail. This failure is predicted as a ductile failure.

However, the static model changes if the yield of the plate happens at a smaller load than the yield of the bolt! According to Fig. 2 the force $Q$ appears at the outer edge of the plate. This force increases the tension in the bolt, and it may cause the failure of the bolt at a lower level of load than that requested.

The minimum diameter for the bolt for the 1st and 2nd cases of failure can be determined by the following steps:

1. Find the required resistance of T-stub from the effect acting on the connection.
2. Choose the correct thickness of the end plate from Table 1 or 2 by the $F_{n,Rd}$ values.
3. Calculate the minima of $F_{T,Rd}$ by Eqs. (1) or (2).
4. Calculate the force on the bolt by Fig. 2.
5. Choose a bolt diameter with a larger resistance than this force.

The bolt diameter selected by these steps will be larger by one or two steps than the original size of the bolt that belongs to the required T-stub resistance. As the practical position of the bolt (washer and wrench size requirements) was taken into account by the failure of the bolt, special attention is needed for the design of the geometry.

4 Summary

The present paper discusses the tension resistance of the T-stub of the beam-to-bolt connection. A simplified method is introduced to choose the proper thickness of the end plate for certain diameters of bolt. The bolt is selected to balance the design moment on the connection by a simple equilibrium equation based on the geometrical arrangement of the connection. The suggested thickness of the plate is minimal to obtain failure of the bolt with the certain diameter in any case. By using Tables 1 and 2, the capacity of the bolts for the T-stub is determined by the tensile resistance of the bolts.

This method is only for determining the proper thickness of the end plate; all the other components of the connection must be verified. The method can be used to verify the flange of the column by taking into account the reinforcement of the diaphragms.

Although this method only gives suggestions on how to analyse one part of the connection, this step is the most complicated of the overall analysis. Consequently, the suggested simplification is significant.

There are useful software for the analysis based on the finite element method or the T-stub method of the EC, but they do not provide a clear picture of the structural behaviour of the connection. The suggested simplification can easily be used for manual calculations. It can be used practically in predesign or in education. While the complex failure of the end-plate is summarised by a set of data in a table, the user can concentrate on the equilibrium of the connection. The behaviour of one connection can be readily understood.

Tables 1 and 2 are valid only for the $c$ and $w$ values at certain $b_{f}$. All these values were chosen based on the analysis of the most common hot rolled IPE and HEA sections, with the welded sections based on the geometry of the hot rolled types. The geometric parameters and the minimum thickness are based on systematic analysis. For the analysis, the T-stub method of EC3 was investigated on IPE, HEA and welded sections.

The constraints of the geometrical parameters are based on the optimal positioning of the bolt; however, the optimal solution sometimes does not meet other requirements, or the engineer must verify an existing connection. To solve this problem, Tables 1 and 2 can be disengaged in a range to allow more freedom in design.

In certain situations, such as earthquake hazard or notable vibration from the live load, it is disadvantageous to use brittle structural elements or brittle connections. While the typical high strength steel has limited ductility, the failure of the bolt of the T-stub is prohibited in such situations. To obtain more ductile connections, the yield of the end plate can be achieved by selecting a larger bolt diameter. In this case, special care is needed for the force $Q$ in Fig. 2 to avoid failure of the bolt due to an undesired change of the static model!

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