Properties of a magnetic superconductor with weak magnetization - application to $ErNi_2B_2C$

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Abstract

Using a Ginsburg-Landau free energy functional, we study the $H - T$ phase diagram of a weak magnetic superconductor, where the magnetization from the magnetic component is marginal in supporting a spontaneous vortex phase in absence of external magnetic field. In particular, the competition between the spiral state and spontaneous vortex phase is analysed. Our theory is applied to understand the magnetic properties of $ErNi_2B_2C$.

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Recently, there has been a renewed interest in the study of magnetic superconductors [1–4], following the discoveries of the magnetic superconductor \( \text{ErNi}_2\text{B}_2\text{C} \), which, in the superconducting phase, has a vortex line-lattice with very unusual properties [2]. Ng and Varma [5] proposed that many of the unusual properties of \( \text{ErNi}_2\text{B}_2\text{C} \) can be understood by assuming that the system has an instability towards forming a *spontaneous vortex phase* at low temperatures below \( T_{WF} \sim 2.3K \). The coexistence of weak ferromagnetism and superconductivity in the system in the presence of external magnetic field was confirmed in neutron diffraction measurement [3] and in magnetic hysteresis measurement [4] recently.

The studies of Ng and Varma [5] focused at the temperature range \( T > T_{WF} \), when long-ranged magnetic ordering is absent. In this paper we shall study numerically the \( H-T \) phase diagram of a weak magnetic superconductor, where the magnetization from the magnetic moment is not strong enough to support a spontaneous vortex phase in absence of magnetic field. The behaviour of the system at both \( T > T_{WF} \) and \( T < T_{WF} \) will be studied. In particular, we shall investigate the competition between the spiral phase [6–8] and spontaneous vortex phase.

We start with the Ginsburg-Landau Gibb’s free energy functional \( G \) describing a magnetic superconductor system [7] in the presence of external field \( \vec{H} \),

\[
G = \int d^3x \left[ \frac{1}{2}a|\psi|^2 + \frac{1}{4}b|\psi|^4 + \frac{\hbar^2}{2m}(\nabla - \frac{i2e}{\hbar c} \vec{A})|\psi|^2 + \frac{\vec{B}^2}{8\pi} \right.
\]

\[
+ \frac{1}{2}\alpha|\vec{M}|^2 + \frac{1}{4}\beta|\vec{M}|^4 + \frac{1}{2}\gamma^2|\nabla \vec{M}|^2 - \vec{B}.\vec{M} - \frac{\vec{B}.\vec{H}}{4\pi},
\]

(1)

where \( \vec{B} = \nabla \times \vec{A} \) is the total magnetic field, \( \vec{M} \) is magnetization and \( \psi \) is the superconducting order parameter. We assume here that the superconducting component is isotropic in space but the magnetic component is restricted to lie only on the \( x-z \) plane, as is believed to be the case of the \( \text{ErNi}_2\text{B}_2\text{C} \) compound [1]. The anisotropy within the \( x-z \) plane is neglected in this study. The effect of in-plane anisotropy will be discussed at the end of the paper. We shall work in the London limit where contributions from vortex cores will be neglected, and shall consider only fluctuations in the *phase* of the superconducting order parameter \( \psi \). In this limit, we obtain an effective GL functional \( G_{\text{eff}} \) in terms of the magnetization
component and the vorticity (vortex density) field $\kappa(\vec{x})$, where

$$
G_{eff}[\vec{M}, \kappa] = \int d^3x \left\{ -\frac{a}{4} \rho + \frac{1}{4\pi} (\kappa)^2 - \frac{1}{4\pi} \kappa(\vec{H} + 4\pi \vec{M}) + \frac{1}{2} \alpha |\vec{M}|^2 + \frac{1}{4} \beta |\vec{M}|^4 + \frac{1}{2} \gamma |\nabla \vec{M}|^2 \right\} \tag{2a}
$$

$$
- \frac{1}{4} \int d^3x d^3x' \kappa(\vec{x} - \vec{x}') (\kappa - H - 4\pi \vec{M})_{\mu}(\vec{x}) (\Pi^{-1})_{\mu\nu}(\vec{x} - \vec{x}') (\kappa - H - 4\pi \vec{M})_{\nu}(\vec{x}'),
$$

with Fourier transform of $(\Pi^{-1})_{\mu\nu}(\vec{x})$ given by

$$
(\Pi^{-1})_{\mu\nu}(\vec{q}) = \frac{1}{2\pi} \left( \frac{\lambda_0^2 q^2}{(1 + \lambda_0 q^2)} \left[ \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right] \right). \tag{2b}
$$

$\rho = |\psi|^2$ and $\lambda_0$ are the superfluid density and London penetration depth, respectively, in GL theory in the absence of the magnetic component. Notice that only transverse magnetization $(\nabla \times \vec{M} \neq 0)$ couples to vortices. We shall study the spontaneous vortex phase and spiral phase of the system in this paper using approximate, variational solutions of $G_{eff}$. We first consider the case when the external magnetic field $H$ is along $z$-axis, where the magnetization couples directly to $H$.

We consider trial solutions of $G_{eff}$ of form

$$
\vec{M}(\vec{r}, z) = M_o \hat{z} + M_1 \cos(Qy) \hat{z} + M_2 \sin(Qy) \hat{x},
$$

$$
\kappa(\vec{r}, z) = \Phi_o \sum_i \hat{z} \delta(\vec{r} - \vec{R}_i),
$$

where $\vec{r} = (x, y)$. The solution represents an ideal flux-line lattice in $\hat{z}$ direction with straight vortex lines, where $\vec{R}_i = (X_i, Y_i)$ represents the position of vortices in the $x - y$ plane which form a regular lattice. The magnetic component may form a spiral state with magnetization lying on the $x - z$ plane, where $\vec{Q} = Q \hat{y}$ is a spiral wave vector yet to be determined, or form a spontaneous vortex phase with $M_o \neq 0$. Notice that we have also included the possibilities of co-existence phases $(M_o, M_1, M_2 \neq 0)$ in our trial solution.

Substituting Eq. (3) into Eq. (2) we obtain after some algebra

$$
G = \frac{\alpha}{2} \left( M_o^2 + \frac{M_1^2 + M_2^2}{2} \right) + \frac{\beta}{4} \left( M_o^4 + M_o^2 \frac{M_1^2 + M_2^2}{2} + 2 M_o^2 M_1^2 + \frac{(M_1^4 + M_2^4)}{2} + \frac{M_1^2 M_2^2}{4} \right) \tag{4}
$$

$$
+ \frac{\gamma}{2} Q^2 \frac{(M_1^2 + M_2^2)}{2} - \frac{1}{4\pi} B(H + 4\pi M_o) + \frac{B^2}{8\pi} - \frac{2\pi \lambda_0^2 Q^2}{1 + \lambda_0^2 Q^2} \frac{(M_1^2 + M_2^2)}{2}
$$

$$
+ \frac{1}{8\pi} \sum_{\vec{q}_N} \frac{B^2}{(1 + \lambda_0^2 q_N^2)} - \delta_{\vec{q}, \vec{q}_N} \frac{BM_1}{1 + \lambda_0^2 Q^2}.
$$
where $V$ is the volume of the system. The fourier transform of vortex density field $\vec{\kappa}(\vec{q}, q_z)$ is

$$\vec{\kappa}^{(0)}(\vec{q}, q_z) = \delta(q_z) \hat{z} B \delta^{(2)}(\vec{q} - \vec{q}_N) V,$$

where $\vec{q}_N$’s are reciprocal lattice vectors and $B = n \Phi_0$ is the average magnetic field trapped in the flux-line lattice, $n$ is the density of vortex lines. Notice that $M_1$ couples to the flux-line lattice when $\vec{Q} = \vec{q}_N$ for some $\vec{q}_N$, reflecting the fact that the existence of a flux-line lattice induces fluctuations in magnetization which is commensurate with the lattice.

To simplify our calculation we replace the sum $S = \sum_{q_N} \frac{8\pi^2 q_N^2}{8\pi(1 + \lambda_0 q_N^2)}$ by the approximate value $S = BH_{c1}/4\pi$ \[10\] which is valid at $H \sim H_{c1}$, where $H_{c1}$ is the lower critical field in the absence of the magnetic component. Logarithmic correction to $S$ arises at intermediate value of $H_{c2} > H > H_{c1}$ which we shall neglect in the following since we shall be interested mainly at qualitative features at regions around $H \sim H_{c1}$. Minimizing the energy with respect to $Q$ and assuming that $\vec{Q} \neq \vec{q}_N$ we obtain $Q^{-2} = Q_s^{-2} \sim \delta^2 \lambda_0^2$, where $Q_s$ is the wave vector characterizing spiral instability [6,7] in magnetic superconductors, $\delta = \gamma/4\pi \lambda_0^2$.

To examine the importance of magnetization fluctuations commensurate with the flux-line lattice, we have also consider $Q = q_1 \sim (B ln(\kappa)/H_{c1})^{1/4} \lambda_0^{-1}$ in our study, where $q_1$ is the smallest reciprocal lattice vector of the flux-line lattice. The corresponding free energies are minimized with respect to $B, M_o, M_1, M_2$ at various temperatures and external magnetic fields $H$ to determine the phase diagram. We have chosen parameters such that temperatures are measured relative to superconducting transition temperature $T_c$, which is set to be $T_c = 10$. We have chosen also $\kappa = \lambda_0/\xi = 5$, and $\alpha = \alpha_o(T/t_m - 1)$, where $\alpha_o = 10$ and $\beta = 20$. $t_m$ is chosen such that $T_M = 3$ (or $0.3 T_c$), where $T_M$ is the ferromagnetic transition temperature in the absence of superconducting component. Notice that in the presence of the superconducting component, the actual magnetic transition temperature $T_{WF}$ is below $T_M$ because the Meissner effect forbids appearance of uniform magnetization in a superconductor [8,9]. Moreover, the zero field magnetization $M_{sat}$ at $T = 0$ in absence of superconducting component is chosen to be $M_{sat} = H_{c1}/4\pi$, i.e. the system is marginal.
in supporting a spontaneous vortex phase.

Minimizing the energy of the system we find that magnetic fluctuations commensurate with flux-line lattice is always unimportant compared with the spiral instability and the system is in a spiral state with $M_1 = M_2$ and $M_o = 0$ at zero magnetic field at temperatures $T < T_{WF} \sim 2.74$. The system goes through a second order phase transition to a non-magnetic superconducting state at $T = T_{WF}$. At $T < T_{WF}$, the system stays at the spiral state when external magnetic field is applied, until $H \to H_{cs} < H_{c1}$, where the system undergoes a first order phase transition into a magnetic-field assisted spontaneous vortex phase with finite $M_o$, and with $M_1 = 0$ and small $M_2$. The presence of $M_2 \neq 0$ is a residue effect of spiral instability in the spontaneous vortex state. We find numerically that $M_2$ reduces rapidly to zero as external field $H$ increases further. The $H - T$ phase diagram is shown in Fig.1, where (VP), (SP), (MP) denotes the magnetic field-assisted spontaneous vortex phase, spiral phase and Meissner phase, respectively. $H^*_{c1}$ is the zero temperature lower critical field in the absence of the magnetic component. The dotted and dashed lines give $H_{c2}$ and $H_{c1}$ of the superconducting component, respectively, in the absence of the magnetic component. The solid line gives $H_{cs}$ and represents a first order transition line from the spiral phase to the vortex phase at $T < T_{WF}$, and represents the usual transition from Meissner phase to vortex phase at $T > T_{WF}$. The dot-dashed line denotes the second order transition from Meissner phase to spiral phase at $H < H_{cs}$. 

\[\text{Figure 1: $H - T$ phase diagram.}\]
FIG. 1. H-T phase diagram of the magnetic superconductor

To make comparisons with experiments we show in Fig. 2 the total magnetization $M = (B - H)/4\pi$ versus external magnetic field $H/H_{c1}$ at three different temperatures $T = 0.0, 2.0, 4.0$. 
The qualitative behaviour of the $M - H$ curve is similar to those observed experimentally in the $ErNi_2B_2C$ compound at low temperature [3]. Notice a sudden jump in magnetization at $H = H_{cs}$ appears when $T < T_{WF}$, signaling the first-order transition to magnetic field assisted spontaneous vortex phase. This sudden jump is absent at $T \geq T_{WF}$, justifying the identification of $T_{WF}$ as the spiral state transition temperature. The sudden jump in magnetization will probably be smeared out by disorder in realistic samples, and $H_{cs}$ can be considered as an effective lower critical field observed in magnetic superconductors at $T < T_{WF}$. Notice that in previous analysis of Ng and Varma [4], the lower critical field $H_{c1}$ for magnetic superconductors was computed from single vortex line energy and it was found that the value of $H_{c1}$ is not much affected by the magnetic component in the spiral state. However, we find here that $H_{cs}$ is smaller than $H_{c1}$ at $T < T_{WF}$, indicating that an effective reduction in $H_{c1}$ can occur because of the first order transition to the magnetic field assisted spontaneous vortex phase. As $M_{sat}$ increases further, we find numerically that $H_{cs}$ starts to decrease as temperature decreases until when $4\pi M_{sat} \sim 4H_{c1}^*$ where $H_{cs} \to 0$ at $T \to 0$, indicating a zero temperature phase transition of the system to the spontaneous vortex phase at zero magnetic field. As $M_{sat}$ increases further, the region where the spontaneous
vortex state is stable at zero field expands further and the spiral state becomes stable only at a finite temperature range $0 < T_{sp} < T < T_{WF}$, in agreement with results from previous studies [7,8].

Recently it was observed in magnetization studies of the $ErNi_2B_2C$ compound that substantial hysteresis developed in the $M - H$ loop at temperature $T \leq T_{WF}$ [4], and the result was interpreted as coming from a $\sim$ three fold increase of the pinning force of the flux line lattice at $T < T_{WF}$. Here we shall consider the effect of spiral phase in magnetic hysteresis. We note that the spiral phase and spontaneous vortex phase are locally stable phases at $T \leq T_{WF}$ over a wide range of magnetic fields. The spiral phase is (locally) stable at $H < H_{l1}^1$, and the spontaneous vortex state is (locally) stable at $H > H_{l2}^1$, where $H_{l1}^1(H_{l2}^1)$ depends on the parameters in the GL functional, and is a decreasing (increasing) function of temperature, with $H_{l1}^1 > H_{l2}^1$ in general. We propose that the huge magnetic hysteresis observed at $T < T_{WF}$ is a result of the system being trapped in the two meta-stable states when magnetic field increases (decreases) at temperature $T < T_{WF}$. When magnetic field increases from zero, the system is trapped in the spiral state until $H \geq H_{l1}^1$, where the spiral state becomes locally unstable and the system goes to the spontaneous vortex state. Similarly the system is trapped in the spontaneous vortex state when magnetic field decreases, until $H \leq H_{l2}^1$, where the system goes back to the spiral state. As a result, huge magnetic hysteresis can be observed at magnetic field range $H_{l2}^1 < H < H_{l1}^1$. 

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In figure 3 we show the hysteresis magnetization $\Delta M = |M_{up} - M_{down}|$ as function of external magnetic field $H/H_{c1}^*$ at three different temperatures $T = 0.0, 1.1, 2.2$, assuming the above picture. The narrowing of the hysteresis region and reduction in $\Delta M$ as temperature increases is in qualitative agreement with what is observed experimentally [4]. Notice that first order jumps of $\Delta M$ is found at $H = H_1^l$ and $H = H_2^l$ in our theory. These jumps should be smeared our by defects in real samples. Notice also that additional features are observed experimentally at $H \sim H_1^l$ at low temperatures [4], suggesting that there exist additional structures in the spiral-spontaneous-vortex state phase diagram not included in our simple variational solution.

Next we consider the case when the applied magnetic field is almost perpendicular to the magnetic plane and makes a small angle $\theta$ with the $\hat{y}$-axis. In this case, the coupling between magnetization and external field is weak. However, it was observed experimentally that the flux-line lattice seen to rotate away from the applied field direction at temperature $T \to T_{WF}$ [2]. This observation can be explained qualitatively by development of in-plane magnetization as $T \to T_{WF}$ [3]. Here we shall study numerically this tilting effect.
at temperatures both above and below $T_{WF}$.

The trial solutions of $G_{eff}$ we consider is similar to Eq.(3), except that $\kappa(\vec{r}, z)$ is in Eq.(3) is replaced by

$$\kappa(\vec{r}, z) = \Phi_o \sum_i (\cos(\theta_t)\hat{y} + \sin(\theta_t)\hat{z})\delta^{(2)}(\vec{r} - \vec{R}_i),$$

where $\theta_t$ is the angle the flux-line lattice made with the $\hat{y}$-axis. Correspondingly, the term $-B(H + 4\pi M_o)/4\pi$ in Eq.(4) is replaced by

$$- \frac{B(H + 4\pi M_o)}{4\pi} \rightarrow - \frac{1}{4\pi} B(H\cos(\theta - \theta_t) + 4\pi M_o \sin(\theta_t)).$$

Minimizing the energy of the system we obtain $\tan(\theta_t) = (H\sin(\theta) + 4\pi M_o)/H\cos(\theta)$.

![FIG. 4. Angle of the flux-line lattice with the $\hat{y}$-axia as a function of magnetic field at three different temperatures $T = 0.0, 1.4, 2.4$. The angle between applied magnetic field and $\hat{y}$-axia is 1.8°.](image)

The resulting angle $\theta_t$ is shown in figure 4 as a function of external magnetic field $H/H^*_{c1}$ at three different temperatures $T = 0.0, 1.4, 2.8$ with $\theta = 0.01\pi(1.8^\circ)$. At $T = 0$, $\theta_t$ remains at 1.8° at low field, indicating that the system is in the spiral state. Flux-line lattice with $\theta_t = \theta (M_o = 0)$ begins to form at $H \sim H_{c1}$, because the coupling of the magnetization to
the applied field is very weak at small $\theta$. A first order phase transition to the (magnetic-field assisted) spontaneous vortex phase occurs at $H \sim 1.6H_{c1}^*$, where in-plane magnetization $M_o$ becomes nonzero ($M_1 = 0, M_2$ small) and $\theta_t$ jumps to a value $\sim 25^\circ$. As magnetic field increases further, $\theta_t$ decreases because the in-plane magnetization is already near saturation and the rate of increase in $M_o$ is slower than the rate of increase in $H$. Similar result is also found at $T = 1.4$, except that the first order transition occurs at a higher field and $\theta_t$ is smaller. We find numerically that there exists also a small region between the spiral and spontaneous vortex phase where the two states are co-existing in the system with a small value of $M_o$ but large $M_1 \sim M_2$, with $\theta_t \sim 4^\circ$. This state is stable only at intermediate temperatures below $T_W$ and at a narrow range of magnetic field $H > H_{c1}$. At temperatures above but close to $T_W$, we find a continous increase of $\theta_t$ as function of $H$ at $H > H_{c1}$, in agreement with previous experimental observation \cite{2}. When $\theta = 0$, direct first order transitions to spontaneously-tilted flux-line lattice phase \cite{5} is also found at $T < T_W$, when magnetic field is large enough.

Summarizing, using simple variational solutions of the GL free energy functional, we have investigated numerically the magnetic behaviours of a magnetic superconductor where the magnetic component is weak and marginal in supporting a spontaneous vortex phase. We have studied both the cases when magnetic field is parallel to the magnetic plane, and when it is almost perpendicular to the magnetic plane. Our results are in general agreement with experimental observations. Furthermore, we have made several theoretical predictions in this paper, including the appearance of spiral phase at zero field at $T < T_W$, which can be tested by neutron scattering experiment. Notice that we have neglected in-plane magnetic anisotropy in our study. In the case of $ErNi_2B_2C$ where magnetization are strongly confined to point at either $\hat{x}$ or $\hat{z}$ directions, a smooth spiral phase is impossible and will be replaced by sharp domain structures \cite{5} that can be disordered easily. In this case the sharp elastic neutron peaks at spiral wavevector $\vec{Q}_s$ will be replaced by a broadened peak centered at wavevector $\vec{q} = 0$ with width of order $Q_s$. We predict also a first order transition to spontaneous vortex phase when external field is strong enough, both when the
field direction is in the magnetic plane, and when it is almost perpendicular to the plane. In general a stronger magnetic field is needed for this transition to occur when the angle between the applied field and the magnetic plane is larger. When the magnetic field is almost perpendicular to the plane, the first order transition to spontaneous vortex phase appears as a spontaneous tilting transition of the flux-line lattice, which can be tested experimentally.

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