Structure of Proton

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Abstract

Electron–proton scattering in elastic and highly inelastic region is reviewed in a unified approach. The importance of parity–violating scattering due to electro–weak interference in probing the structure of proton is emphasized. The importance of longitudinal spin–spin asymmetry as well as parity violating longitudinal asymmetry to extract the structure functions of proton in both regions are discussed. The recoil polarization of proton in the elastic scattering is also discussed.

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I. INTRODUCTION

Lepton–proton (nucleon) scattering is a very direct means of probing nucleon structure. In particular electron–nucleon scattering serves to produce a virtual photon of space like four momentum which probes nucleon structure in a very clean way. Elastic scattering experiments have been carried out extensively and we now have a fairly detailed knowledge of nucleon form factors as functions of virtual photon mass ($Q^2$). In these experiments, the nucleons recoils elastically, the photon interacts with the nucleon constituents in a coherent manner and these form factors are related roughly to average shape of a nucleon. On the other hand, the inelastic total scattering cross-section is described by the structure functions $W_2$ and $W_1$ that depend upon both the photon energy ($\nu$) and the photon mass ($Q^2$). In the inelastic processes the photon interacts in an incoherent manner and it probes, roughly, the instantaneous construction of the nucleon rather than the average shape found in the elastic scattering experiments. Thus the study of inelastic lepton–hadron scattering at high energies and large momentum transfers (deep inelastic scattering) may give information about the structure pertaining to any fundamental constituents of a nucleon. In the high energy inelastic experiments, the quantities measured are the momenta and scattering angles of electrons. The cross-section thus measured represents a sum over all the final hadronic states possible for a definite laboratory energy and hence for an invariant mass of the hadrons. The elastic scattering can be regarded as a special case of inelastic scattering in which the invariant hadronic mass is replaced by the mass ($m$) of the nucleon and the structure functions $W_2 (\nu, q^2)$ and $W_1 (\nu, q^2)$ are then given by the two form factors $F_2 (q^2)$ and $F_1 (q^2)$ multiplied by the $\delta$–function $\delta (\nu - q^2/2m)$.

To further probe the structure of the proton, it is important to investigate the spin dependent structure functions. In fact, the European Muon Collaboration (EMC) measurement of polarized muon-proton deep inelastic scattering gave the first indication that
strange content of the proton is not zero. Apart from this direct evidence; there were also indications (see below) that simple picture of a proton having only non-strange valence \(u\) and \(d\) quarks is not tenable. Instead of using both polarized lepton beams and polarized proton targets; one can dispense with either polarized proton targets or polarized lepton beams. The interference between the photon and \(Z\)-boson exchange can supply the missing polarization content. Thus the parity violating scattering of polarized (unpolarized) electron beam on unpolarized (polarized) proton target can give information about the matrix elements \(\langle p | \bar{q} \Gamma_{\mu} q | p \rangle\), \(\Gamma_{\mu} = i \gamma_{\mu}, \gamma_{\mu} \gamma_{5}\); \(q = u, d, s\). In particular parity violating elastic scattering would give direct information about the axial and vector form factors \(G^Z_A\), \(F^Z_1\) and \(F^Z_2\) due to \(Z\)-exchange. These form factors involve both isosinglet and octet parts. While the octet form factors can be extracted from the \(\beta\)-decay of hyperon and electromagnetic form factors; the singlet form factors have to be extracted from the direct measurement of \(G^Z_A\), and \(F^Z_2\) from the parity violating electron scattering.

The purpose of this article is to review the structure of proton. We follow a general approach which is applicable to both elastic and deep inelastic scattering.

II. ELECTRON SCATTERING ON UNPOLARIZED NUCLEON

Lord Rutherford was the first physicist to use scattering experiments to probe the structure of the matter. He used scattering of \(\alpha\)–particles on atoms. He found that his experimental results are compatible with a point like positive charge inside the atom which he called atomic nucleus—a discovery of tremendous importance.

We now consider the scattering of electrons on nucleons (see Fig. 1)

\[
k \equiv (k, iE), \quad k' \equiv (k', iE')
\]

\[
p \equiv (0, im)
\]

\[
q = k - k', \quad \nu = E - E'
\]

\[
q^2 \approx 2EE' (1 - \cos \theta), \quad m_e \approx 0, \quad Q^2 = -q^2
\]
For elastic scattering $p_X = p'$

$$p'^2 = -m^2$$

The differential cross-section is proportional [1] to

$$L_{\mu\nu} J_{\mu\nu}$$

where $L_{\mu\nu}$, $J_{\mu\nu}$ are given by

$$L_{\mu\nu} = \frac{1}{2m^2_c} [k_\mu k'_\nu + k_\mu k'_\nu - \delta_{\mu\nu} k \cdot k']$$

$$J_{\mu\nu} = 2\pi \left[ \frac{1}{m} \left( p_\mu - \frac{p \cdot q}{q^2} q_\mu \right) \left( p_\nu - \frac{p \cdot q}{q^2} q_\nu \right) W_2 (\nu, q^2) \right.$$ 
\[+ \left( \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) W_1 (\nu, q^2) \]  

Note that $L_{\mu\nu}$ is the leptonic part which we get after summing over the final electron spin and taking the spin average of initial electron. Leptonic part is completely known. The hadronic part is described in terms of two structure functions $W_2$ and $W_1$ after taking the spin average of the target nucleon and summing over all the quantum numbers of $p_X$.

The differential cross-section is given by

$$\frac{d^2 \sigma}{dq^2 d\nu} = \left( \frac{d\sigma}{dq^2} \right)_{\text{Mott}} \left[ W_2 (\nu, q^2) + 2 \tan^2 \theta \frac{q^2}{2} W_1 (\nu, q^2) \right]$$

where

$$\left( \frac{d\sigma}{dq^2} \right)_{\text{Mott}} = \frac{4\pi\alpha^2 E'}{q^4} \frac{E}{\cos^2 \theta}$$

give the differential cross-section for the scattering of electron on spinless structureless proton. The presence of the structure functions in Eq. (4) indicates that proton is not a point particle.

For elastic scattering $\nu = q^2/2m$. For this case the structure functions are given by

$$W_2 = \left[ F_1^2 (q^2) + \tau F_2^2 (q^2) \right] \delta \left( \nu - \frac{q^2}{2m} \right)$$

$$W_1 = \left[ F_1 (q^2) + F_2 (q^2) \right]^2 \delta \left( \nu - \frac{q^2}{2m} \right)$$

(6)
where $\tau = q^2/4m^2$. Thus from (4), we get [2]

$$\frac{d\sigma}{dq^2} = \left(\frac{d\sigma}{dq^2}\right)_{\text{Mott}} \left\{ \frac{G^2_E (q^2) + \tau G^2_M (q^2)}{1 + \tau} + 2 \tan^2 \frac{\theta}{2} G^2_M (q^2) \right\}$$

(7)

where

$$G_E (q^2) = F_1 (q^2) - \tau F_2 (q^2)$$

$$G_M (q^2) = F_1 (q^2) + F_2 (q^2)$$

(8)

The Pauli form factors $F_1$ and $F_2$ are normalized as follows:

$$F_1^p (0) = 1, \quad F_2^p (0) = \kappa_p \quad \kappa_p = 1.792$$

$$F_1^n (0) = 0, \quad F_2^n (0) = \kappa_n \quad \kappa_n = -1.913$$

(9)

$\kappa_p$ and $\kappa_n$ are the anomalous magnetic moments of the proton and the neutron respectively.

Experimental data is analysed in terms of Sachs form factors $G_E (q^2)$ and $G_M (q^2)$ which are normalized as follows:

$$G^p_E (0) = 1, \quad G^p_M = \mu_p = 2.792$$

$$G^n_E (0) = 0, \quad G^n_M = \mu_n = -1.913$$

(10)

From the fig. 1 and Eq. (4), it is easy to see that the structure functions $W_1$ and $W_2$ are related to the absorptive part for the forward Compton scattering for virtual photon. In fact using optical theorem one finds

$$\left(1 + \frac{\nu^2}{q^2}\right) \frac{W_2}{W_1} - 1 = \frac{\sigma_L}{\sigma_T},$$

(11)

where $\sigma_L$ and $\sigma_T$ are the longitudinal and transverse total Compton scattering cross-section respectively. For the elastic scattering, Eq. (11) gives

$$\frac{\sigma_L}{\sigma_T} = \frac{G^2_E}{\tau G^2_M}$$

(12)

The virtual longitudinal polarization $\epsilon$ is given by
\[ \epsilon = \frac{1}{1 + 2 \left(1 + \frac{\nu^2}{q^2}\right) \tan^2 \frac{\theta}{2}} = \frac{1}{1 + 2 (1 + \tau) \tan^2 \frac{\theta}{2}} \]  

(13)

The Rosenbluth scattering cross-section given in Eq. (7) can be written as follows [2]:

\[ \frac{d\sigma}{dq^2} = \left( \frac{d\sigma}{dq^2} \right)_{\text{Mott}} \frac{1}{\epsilon (1 + \tau)} \left[ \tau G_M^2 + \epsilon G_E^2 \right] \]  

(14)

It is clear from Eq. (14) that \( G_M \) can be extracted from the scattering cross-section at \( \epsilon = 0 \), while \( G_E \) is extracted from the \( \epsilon \)-dependance. \( \epsilon \) can be varied at fixed photon energy and momentum \( (q, i\nu) \) by varying the electron energy and scattering angle. A global analysis of data indicated that \( G_E \) and \( G_M \) follows the dipole form, \( G_D = (1 + q^2 / 0.71)^{-2} \); \( G_M \) with great accuracy while \( G_E \) with less precisely as to extract \( G_E \) at high \( q^2 \) is less accurate; because of \( \epsilon/\tau \) weighting of the electric term relative to the magnetic term. For, \( q^2 < 1 \) GeV\(^2\), one finds

\[ \frac{G_M}{\mu_p G_D} \approx \frac{G_E}{G_D} \approx 1 \]

However, recent measurements of recoil polarization of proton in [3,4] elastic scattering show that ratio \( G_M/G_E \) systematically decreases as \( q^2 \) increases from 0.5 to 3.5 GeV\(^2\), indicating for the first time a definite difference in the spatial distribution of charge and magnetization currents in the proton. If confirmed, it will indicate the breakdown of universality of electric and magnetic distribution at high \( q^2 \) in a proton.

**III. STRANGENESS IN THE PROTON**

The simple picture of the proton having only non-strange valence u and d quarks has been questioned In fact it was shown for quite some time that the \( \Sigma \) term in pion-nucleon scattering [5] which is given by

\[
\sum_{\pi N}(0) = \frac{1}{2} \langle p \left| \left[ F_{1-i2}^5, F_{1+i2}^5, H_M \right] \right| p \rangle \]

(15)

where \( F_i^5 \) are the axial-vector charges and \( H_M \) is the chiral-symmetry breaking Hamiltonian in QCD:
\[ H_M = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s \]
\[ = \sqrt{6}m_0S_0 + \frac{2}{\sqrt{3}}(\bar{m} - m_s)S_8 + (m_u - m_d)S_3 \]  

(16)

where

\[ m_0 = \frac{m_u + m_d + m_s}{\sqrt{3}} , \quad \bar{m} = \frac{m_u + m_d}{2} \]
\[ S_0 = \frac{\bar{u}u + \bar{d}d + \bar{s}s}{\sqrt{6}} , \quad S_8 = \frac{\bar{u}u + \bar{d}d - 2\bar{s}s}{2\sqrt{3}} , \quad S_3 = \frac{\bar{u}u - \bar{d}d}{2} \]  

(17)

\[ \text{From above equations, one can write} \]
\[ \sum_{\pi N}(0) = 2\bar{m}\left( \frac{1}{\sqrt{3}} \left( S_8 + \sqrt{2}S_0 \right) \right) \]
\[ = \bar{m}\left( \bar{u}u + \bar{d}d \right) \]  

(18)

Now \( \langle p | S_8 | p \rangle \) is entirely determined from the Gell–Mann–Okubo mass splitting:
\[ \langle p | S_8 | p \rangle = \frac{\sqrt{3}m_\Xi - m_\Lambda}{2 \left( m_s - \bar{m} \right)} \]  

(19)

Above pattern of symmetry breaking implies \( (\lambda_0 = \sqrt{2}\lambda_8) \)
\[ \langle p | S_0 | p \rangle = \sqrt{2} \langle p | S_8 | p \rangle \]  

(20)

Thus one gets
\[ \sum_{\pi N}(0) = \frac{3(m_\Xi - m_\Lambda) \bar{m}}{m_s - \bar{m}} \]  

(21)

Using \( \bar{m}/(m_s - \bar{m}) = 0.4 \) [6], and experimental masses for \( m_\Xi \) and \( m_\Lambda \), one obtains
\[ \sum_{\pi N}(0) \approx 25 \text{ MeV} \]  

(22)

which is about half the value of \( \sum_{\pi N}(0) \) extracted from low-energy pion–nucleon scattering: namely [5]
\[ \sum_{\pi N}(0) = 51 \pm 5 \text{ MeV} \]  

(23)

Experimental value of \( \sum_{\pi N}(0) \) implies that Eq.(20) is not valid and has to be modified. Let us write
\[
\sqrt{2} \langle p|S_0|p\rangle = 2(1 + C_s) \langle p|S_8|p\rangle
\]  
(24a)

so that

\[
\langle p|\bar{s}s|p\rangle = \frac{2}{3} C_s \frac{1}{1 + \frac{8}{3} C_s} \left\langle p \left| \frac{1}{2} \left( \bar{u}u + \bar{d}d \right) \right| p \right\rangle
\]  
(24b)

Then Eq. (18) is modified as follows

\[
\sum_{\pi N} (0) = \frac{2}{\sqrt{3}} \bar{m} (3 + 2C_s) \langle p|S_8|p\rangle
\]

\[
= \frac{(m_\Xi - m_\Lambda) \bar{m}}{m_s - \bar{m}} (3 + 2C_s)
\]  
(25)

Comparing it with the experimental value (Eq. 23), one gets

\[
C_s = \frac{3}{2}
\]  
(26)

Now in the valence quark model if the proton primarily consists u and d quarks one must have [7]

\[
\langle p|\bar{s}s|p\rangle \ll \left\langle p \left| \frac{1}{2} \left( \bar{u}u + \bar{d}d \right) \right| p \right\rangle
\]  
(27)

But Eq. (24) and (26) implies that

\[
\langle p|\bar{s}s|p\rangle = \frac{1}{2} \left\langle p \left| \frac{1}{2} \left( \bar{u}u + \bar{d}d \right) \right| p \right\rangle
\]  
(28)

that is proton has about 50% probability of containing \(\bar{s}s\) pairs. The same conclusion viz that strange content of a proton is not negligible was derived in Ref. [8]. We considered the effect of isospin violating part (i.e. the third term) of the Hamiltonian (16) on the pion nucleon vertex function viz

\[
\Gamma = \left\langle p \pi^0 \left| H_{\Delta I=1}^{\Delta S=0} \right| p \right\rangle
\]  
(29)

In the soft pion limit, one gets [8]

\[
\Gamma = -\frac{m_d - m_u}{F_\pi^2} \frac{2}{\sqrt{3}} \left\langle p \left| \left( P_8 + \sqrt{2} P_0 \right) \right| p \right\rangle
\]  
(30)
Note that the same combination of the pseudoscalar densities enters as that for the scalar densities in the $\sum_{\pi N}$. Using PCAC and flavor SU(3), we get

$$\frac{\delta g}{g_\pi} = (m_d - m_u) \frac{3F/D - 1}{F/D + 1} \frac{1 + \frac{2}{3}C_p}{(\bar{m} + m_s) + 2 (1 + C_p) (\bar{m} - m_s)}$$

(31)

Now since $\delta g/g_\pi$ has no pion pole, it should vanish in the chiral limit ($m_d, m_u \to 0$). However, for $C_p = 0$, Eq. (31) reduces to

$$\frac{\delta g}{g_\pi} = \sqrt{3} \frac{m_d - m_u}{m_d + m_u} \frac{1}{\sqrt{3}} \frac{3F/D - 1}{F/D + 1}$$

(32)

which does not vanish in the chiral limit. Hence $C_P$ cannot be zero. i.e. strange content of the proton is not negligible. This conclusion derived without any experimental input but purely from the validity of chiral SU(2)$\times$SU(2) symmetry. It is intersting to see [8] that in the chiral SU(3)$\times$SU(3) limit

$$C_P \approx -1$$

(33)

Finally, since, in the non–relativistic quark model

$$\bar{q}i\gamma_5 q \sim \bar{q} \frac{\sigma \cdot (p' - p)}{2m_q} q$$

(34)

where $m_q$ is the constituent–quark mass, the value $C_P \approx -1$ implies

$$\langle p | -\bar{s}i\gamma_5 s | p \rangle \approx 2 \left( \frac{1}{2} \left( \bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d \right) | p \right)$$

(35)

This indicates that the strange quarks are polarized opposite to the proton’s spin while the up and down quarks are polarized along the proton spin.

Additional piece of evidence that proton has strange–quark content comes from the European Muon Collaboration (EMC) data [9] for the polarized electroproduction structure function. These experiments involve the scattering of polarized electron beam of the polarized proton target in the deep inelastic region. Since in the deep inelastic region, one gets information about the elementary constituents of the proton; these experiments directly probe the spin content of protons in terms of quark spin. We define the quark contribution to proton spin: [1,11]
\[ \langle p | -\bar{q} i \gamma_\mu \gamma_5 q | p \rangle = \Delta \bar{q} (-S_\mu), \quad q = u, d, s \] (36)

where \( S_\mu = \bar{\psi} i \gamma_\mu \gamma_5 \psi \) is the spin of the proton. Taking into account, the gluon contribution to the proton spin.

\[ \Delta \bar{q} = \Delta q - \frac{\alpha_s}{2\pi} \Delta G_q \] (37)

However, this separation is not unambiguous. \( \Delta \bar{q} \) are related to the axial vector coupling constants \( g_A, g_A^8 \) and \( g_A^0 \) as follows

\[ \Delta \bar{u} - \Delta \bar{d} = g_A = 1.2670 \pm 0.0035 \]
\[ \Delta \bar{u} + \Delta \bar{d} - 2\Delta \bar{s} = g_A^8 = 3F - D \]
\[ F = 0.463 \pm 0.023 \]
\[ D = 0.803 \pm 0.040 \] (38)

These experimental values are obtained from axial vector coupling in \( \beta \)-decay of hyperons. However for the singlet constant

\[ g_A^0 = \Delta \bar{u} + \Delta \bar{d} + \Delta \bar{s} \]

one cannot get information from \( \beta \)-decay; because the known hyperons belong to Octet of SU(3). EMC data give [9,10]:

\[ \Delta \bar{u} = 0.78 \pm 0.07 \]
\[ \Delta \bar{d} = -0.48 \pm 0.08 \]
\[ \Delta \bar{s} = -0.14 \pm 0.07 \] (39)

so that

\[ \Delta \bar{u} + \Delta \bar{d} + \Delta \bar{s} = 0.16 \pm 0.22 \] (40)

which is consistent with zero. In other words

\[ g_A^0 \equiv \Delta \bar{\Sigma} = \Delta \Sigma - \frac{3\alpha_s}{2\pi} \Delta \bar{G} = 0.16 \pm 0.22 \] (41)
where $\Delta \Sigma = \Delta u + \Delta d + \Delta s$ is the quark contribution to the spin of the proton and $\Delta \tilde{G}$ is the singlet part of $\Delta G$. Various estimates of $\Delta \Sigma$ indicate that $\Delta \Sigma = 0$, which implies that $\frac{\alpha_s}{2\pi}(-\Delta \tilde{G}) = 0.05 \pm 0.07$. Thus one can say that the quarks do not contribute to the spin of the proton (this is known as spin crises for the proton) implying in view of angular momentum sum rule

$$\frac{1}{2} = \Delta \Sigma + \Delta \tilde{G} + L_z \quad (42)$$

that its spin is carried out by gluons and/or orbital angular momentum of its constituents. $\Delta \Sigma \simeq 0$ is in complete disagreement with the naive quark model (NQM) result which predicts $\Delta \Sigma = 1$. Thus it is very important to measure both $g_A^0$ and $F_2^0(0)$ [the SU(3), singlet anomalous magnetic moment of the proton] experimentally in order to determine the flavor and spin of content of the proton.

Since straightforward interpretation of $g_A^0$ in terms of the quark contribution to the proton spin is not possible due to the anomaly in the iso–singlet axial vector current; (cf. Eq. (41)); it is important to determine both $g_A^0$ and $F_2^0(0)$ directly from experiment. While $g_A^0$ can be directly determined by neutrino–proton elastic scattering [12], but experimentally $F_2^0(0)$ can only be determined by electron proton scattering. In reference [13], it was suggested that parity violating polarized electron scattering on unpolarized protons (nucleons) can give information about $g_A^0$ and $F_2^0(0)$ and hence about the matrix elements $\langle p | s \Gamma_{\mu} s | p \rangle$, $\Gamma_{\mu} = i\gamma_{\mu}$, $i\gamma_{\mu} \gamma_5$. In 1990, it was suggested in Ref. [14] that elastic parity violating unpolarized electron scattering on polarized protons can give information about iso-singlet axial–vector ($g_A^0$) and vector ($F_2^0$) form factors. These experiments thus can throw some light on the strange content of the proton. For a review and recent work see Ref. [15].

**IV. PARITY–VIOLATING ELECTRON SCATTERING AND PROTON FORM FACTORS**

It is well known that to get information for spin–dependent structure functions [16,17]; one needs polarized electron beam and polarized proton target. However, for the electron–
proton scattering, the interference between photon and Z–boson exchange diagrams (see Fig. 1; for Z–boson exchange replace $\gamma \rightarrow Z$) can also give us information by scattering of polarized (unpolarized) electrons on unpolarized (polarized) protons.

In electroweak theory (see for example Refs: [1] and [10]), the electromagnetic and weak neutral currents coupled to photon and Z–boson respectively are given by for the quarks

$$J_{\mu}^{\text{e.m.}} = i \bar{q} \gamma_{\mu} Q q = i \bar{q} \gamma_{\mu} \frac{1}{2} \left[ \lambda_3 + \frac{1}{\sqrt{3}} \lambda_8 \right] q$$

$$J_{\mu}^Z = i \bar{q} \gamma_{\mu} (1 + \gamma_5) \frac{1}{2} \left[ \lambda_3 + \frac{1}{\sqrt{3}} \lambda_8 - \frac{1}{\sqrt{6}} \lambda_0 \right] q - 2 x_W J_{\mu}^{\text{e.m.}}$$ (43)

For electron, we have

$$J_{\mu}^{\text{e.m.}} = i \bar{e} \gamma_{\mu} e$$

$$J_{\mu}^Z = \frac{i}{2} \left[ g_V \bar{e} \gamma_{\mu} e + g_A \bar{e} \gamma_{\mu} \gamma_5 e \right]$$ (44)

$$g_V = -\frac{1}{2} + 2 x_W = -\frac{1}{2} \left( 1 - 4 x_W \right) = \frac{1}{2} v_e$$

$$g_A = -\frac{1}{2} - \frac{1}{2} a_e$$

$$x_W = \sin^2 \theta_W$$ (45)

The vector and axial–vector form factors of the proton are defined as

$$(2\pi)^3 \left( \frac{p_0 p'_0}{m^2} \right)^{1/2} \langle p' J_{\mu}^Z p \rangle = \bar{u} (p') i \left[ F_1^Z \gamma_{\mu} - \frac{F_2^Z}{2m} \sigma_{\mu\lambda} q^\lambda + G_A^Z \gamma_{\mu} \gamma_5 \right]$$ (46)

For $J_{\mu}^{\text{e.m.}}$: $F_1^Z \rightarrow F_1^\gamma$, $F_2^Z \rightarrow F_2^\gamma$, $G_A^Z = 0$. Note form factors are functions of $q^2$

Thus we have

$$F_{1,2}^{\gamma p} (0) = F_{1,2}^{3} (0) + \frac{1}{\sqrt{3}} F_{1,2}^{8} (0)$$

$$F_{1,2}^{Z p} (0) = (1 - 2 x_W) \left[ F_{1,2}^{3} (0) + \frac{1}{\sqrt{3}} F_{1,2}^{8} (0) \right] - \frac{1}{\sqrt{6}} F_{1,2}^{0} (0)$$

$$G_A^{Z p} (0) = G_A^{3} (0) + \frac{1}{\sqrt{3}} G_A^{8} (0) - \frac{1}{\sqrt{6}} G_A^{0} (0)$$ (47)

Since

$$F_1^{\gamma p} (0) = 1, \quad F_2^{\gamma p} (0) = \kappa_p, \quad F_1^{\gamma n} (0) = 0, \quad F_2^{\gamma n} (0) = \kappa_n$$ (48)
we get

\[ F_3^3 (0) = \frac{1}{2}, \quad F_8^3 (0) = \sqrt{3} \frac{1}{2}, \quad F_0^3 (0) = \sqrt{3} \frac{1}{2} \]

\[ F_2^3 (0) = \frac{1}{2} (\kappa_p - \kappa_n), \quad F_8^8 (0) = \sqrt{3} \frac{1}{2} (\kappa_p + \kappa_n), \quad F_0^0 (0) = \sqrt{3} \frac{1}{2} \kappa_0 \]  

(49)

Note that \( F_0^0 (0) \) is not fixed i.e. isosinglet anomalous magnetic moment is not known experimentally. In terms of \( u, d, \) and \( s \) quarks:

\[ F_{1,2}^3 (0) = \frac{1}{2} \left( F_{1,2}^u (0) - F_{1,2}^d (0) \right) \]

\[ F_{1,2}^8 (0) = \frac{1}{2} \frac{1}{\sqrt{3}} \left( F_{1,2}^u (0) + F_{1,2}^d (0) - 2F_{1,2}^s (0) \right) \]

\[ F_{1,2}^0 (0) = \frac{1}{2} \frac{\sqrt{2}}{3} \left( F_{1,2}^u (0) + F_{1,2}^d (0) + F_{1,2}^s (0) \right) \]  

(50)

Thus we get

\[ F_1^u (0) = 2, \quad F_1^d (0) = 1, \quad F_1^s (0) = 0 \]

\[ F_2^u (0) = \kappa_p + \kappa_0, \quad F_2^d (0) = \kappa_n + \kappa_0, \quad F_2^s (0) = \kappa_0 - (\kappa_p + \kappa_n) \]  

(51)

For \( Z \)-exchange we have

\[ F_{1,2}^{Z_p} (0) = \frac{1}{2} (1 - 4x_w), \quad F_{1,2}^{Z_p} (0) = (1 - 2x_w) \kappa_p - \frac{1}{2} \kappa_0 \]  

(52)

\[ G_A^{Z_p} (0) = \frac{1}{2} \left[ (F - D) + \frac{1}{3} (3F - D) - \frac{1}{3} g_A^0 \right] \]  

(53)

where in writing Eq. (53), we have used Eq. (47)

\[ G_A^3 (0) = \frac{1}{2} \left( G_A^u (0) - G_A^d (0) \right) = \frac{1}{2} g_A = \frac{1}{2} (F + D) \]

\[ G_A^8 (0) = \frac{1}{2} \sqrt{3} \left( G_A^u (0) + G_A^d (0) - 2G_A^s (0) \right) = \frac{1}{2} \sqrt{3} (3F - D) \]

\[ G_A^0 (0) = \frac{1}{2} \frac{\sqrt{2}}{3} \left( G_A^u (0) + G_A^d (0) + G_A^s (0) \right) = \frac{1}{2} \frac{\sqrt{2}}{3} g_A^0 \]  

(54)

Further we note that

\[ G_A^u (0) = F + \frac{1}{3} D + \frac{1}{3} g_A^0 \]

\[ G_A^d (0) = -\frac{2}{3} D + \frac{1}{3} g_A^0 \]

\[ G_A^s (0) = -\frac{1}{3} (3F - D) + \frac{1}{3} g_A^0 \]  

(55)
The simple picture of a proton having only non-strange valence $u$ and $d$ quarks require that

\[
F_s^u (0) = 0 \implies \kappa_0 = \kappa_p + \kappa_n \\
G_A^u (0) = 0 \implies g_A^0 = 3F - D
\]  

(56)

Thus strangeness in proton means that the value of $\kappa_0$ and $g_A^0$ are different from those given in Eq. (56). To answer this question experimentally, the parity violating elastic scattering of polarized (unpolarized) electrons on unpolarized (polarized) protons are of importance. The interference between the photon exchange and $Z$-exchange result in the form factors $F_2^Z$ and $G_A^Z$ which depend on the singlet anomalous magnetic moment $\kappa_0$ and the isosinglet axial vector constant $g_A^0$ [cf. Eqs. (52) and (53)]. It may be noted that electromagnetic form factors $F_2^{\gamma p}$ is independent of $\kappa_0$. The experimental results both in polarized deep inelastic lepton–nucleon scattering and for elastic neutrino proton scattering are consistent with $g_A^0 = 0$. In fact, the latter experiments [12] gives $g_A^0 = 0.12 \pm 0.23$. This trend that singlet form factors is zero is consistent with our discussion in section III. $g_A^0 = 0$ and $\kappa_0 = 0$, implies [Eqs. (55) and (51)]

\[
G_A^s (0) = - \left( G_A^u (0) + G_A^d (0) \right) \\
F_s^u (0) = - \left( F_2^u (0) + F_2^d (0) \right) = - (\kappa_p + \kappa_n)
\]  

(57)

V. PARITY-VIOLATING UNPOLARIZED SCATTERING ON POLARIZED PROTONS

In parity violating electron scattering, the weak and electromagnetic interactions interfere. The unpolarized electron scattering on polarized nucleons can be described in terms of six spin-dependent structure functions (if we neglect the contribution to the scattering cross-section which goes to zero as $m_e \to 0$). These structure function [18] are given by

\[
\tilde{S}_{\mu\nu} = (2\pi)^3 \frac{P_0}{m} \int d^4 z \; e^{-iq \cdot z} \langle p, n \left| J_{\mu}^{\text{em}} (z), J_{\nu}^{Z} (0) \right| p, n \rangle \\
= \tilde{S}_{\mu\nu}^{(-)} + \tilde{S}_{\mu\nu}^{(+)}
\]  

(58a)
where

\[
\tilde{S}^\nu_{\mu \nu} = -\frac{2\pi}{m} \tilde{G}_1^e \epsilon_{\mu \nu \rho \sigma} q_\rho n_\sigma + \frac{2\pi}{m^3} \tilde{G}_2^e \epsilon_{\mu \nu \rho \sigma} q_\rho (p \cdot q n_\sigma - n \cdot q p_\sigma) - \frac{2\pi}{m} \tilde{G}_3^e \epsilon_{\mu \nu \rho \sigma} p_\rho n_\sigma
\]  

\[(58b)\]

\[
\tilde{S}^\nu_{\mu \nu} = -\frac{2\pi}{m} \tilde{H}_2^e (p_\mu n_\nu + p_\nu n_\mu) - \frac{2\pi}{m} \tilde{H}_3^e \delta_{\mu \nu} n \cdot q - \frac{2\pi}{m^3} \tilde{H}_4^e p_\mu p_\nu n \cdot q
\]  

\[(58c)\]

\(n\) is the polarization vector of nucleon (\(n^2 = n_\mu n_\mu = 1, p_\mu n_\mu = p \cdot n = 0\)). Since we are considering unpolarized electron, the lepton part \(\tilde{L}_{\mu \nu}\) is given by (cf. Eq. (44))

\[
\tilde{L}_{\mu \nu} = \frac{v_e}{8m_e^2} \left[ k'_\mu k'_\nu + k' k_\mu - \delta_{\mu \nu} k \cdot k' \right] + \frac{a_e}{8m_e^2} \epsilon_{\mu \nu \alpha \beta} k_\alpha k'_\beta
\]  

\[
= \tilde{L}^{(+)}_{\mu \nu} + \tilde{L}^{(-)}_{\mu \nu}
\]  

\[(59)\]

The scattering cross-section is given by

\[
\frac{d^2\sigma}{dq^2 d\nu} = \frac{4\pi\alpha}{q^2} \frac{8G_F}{\sqrt{2}E^2} \frac{m_e^2}{8\pi^2} \left[ \tilde{L}^{(-)\nu}_{\mu \nu} \tilde{S}^{(-)}_{\mu \nu} + \tilde{L}^{(+)}_{\nu \mu} \tilde{S}^{(+)}_{\mu \nu} \right]
\]  

\[(60)\]

\(\)From Eqs. (55) and (59), we get

\[
\tilde{L}^{(-)}_{\nu \mu} \tilde{S}^{(-)}_{\mu \nu} = \frac{a_e}{8m_e^2} \frac{4\pi}{m} \left\{ \tilde{G}_1^e (k \cdot q k' \cdot n - k' \cdot q k \cdot n) \\
- \tilde{G}_2^e \left[ p \cdot q (k \cdot q k' \cdot n - k' \cdot q k \cdot n) + q \cdot n (k \cdot p k' \cdot q - k \cdot q k' \cdot p) \right] \\
+ \tilde{G}_3^e (k \cdot p k' \cdot n - k' \cdot p k \cdot n) \right\}
\]  

\[(61)\]

\[
\tilde{L}^{(+)}_{\nu \mu} \tilde{S}^{(+)}_{\mu \nu} = -\frac{v_e}{8m_e^2} \frac{4\pi}{m} \left\{ \tilde{H}_2^e (p \cdot k k' \cdot n + p \cdot k k' \cdot n) - \tilde{H}_3^e k \cdot k' q \cdot n \\
+ \tilde{H}_4^e \left( p \cdot k p \cdot k' - \frac{1}{2} p^2 k \cdot k' \right) q \cdot n \right\}
\]  

\[(62)\]

In the laboratory frame, it is convenient to take \(\vec{k}\) along \(z\)-axis. Thus in the Lab frame (for \(E \gg m_e\)):

\[
k \equiv (0, 0, E, iE),
\]

\[
k' = E' \left( \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta, i \right),
\]

\[
p \equiv (0, im)
\]  

\[(63)\]

We will confine ourself to longitudinal polarization of proton:
\[ n = (0, 0, \lambda, 0); \quad \lambda = \pm 1 \]  

Then from Eq. (60), using Eqs. (61), (62) and (63), we get [18]

\[
\frac{d^2\sigma}{dq^2 d\nu} = \lambda \frac{\alpha G_F}{\sqrt{2} q^2 m E^2} \left\{ v_e \left[ 2m \tilde{H}_5^e EE' (1 + \cos \theta) - \tilde{H}_5^e q^2 (E - E' \cos \theta) - \tilde{H}_4^e \left( 2EE' - \frac{q^2}{2} \right) (E - E' \cos \theta) \right] + a_e q^2 \left[ \tilde{G}_1^e (E + E' \cos \theta) - \tilde{G}_2^e \frac{q^2}{m} + m \tilde{G}_3^e \right] \right\}
\]

(64)

For elastic scattering the structure functions are given by\(^1\) [14]

\[
\tilde{H}_2^e = G_A^Z (F_1^\gamma - \tau F_2^\gamma) \delta \left( \nu - \frac{q^2}{2m} \right),
\]

\[
\tilde{H}_3^e = -G_A^Z (F_1^\gamma + F_2^\gamma) \delta \left( \nu - \frac{q^2}{2m} \right),
\]

\[
\tilde{H}_4^e = -G_A F_2^\gamma \delta \left( \nu - \frac{q^2}{2m} \right)
\]

(65)

(66)

Using Eqs. (66) and (67), we get the scattering cross-section [14,15]

\[
\frac{d\sigma}{dq^2} = \lambda \frac{G_F q^2}{\sqrt{2} \pi \alpha} \left\{ v_e G_A^Z \left[ (F_1^\gamma + \tau \frac{m}{E} F_2^\gamma) + 2\tau \left( 1 + \frac{m}{E} \right) \tan^2 \frac{\theta}{2} (F_1^\gamma + F_2^\gamma) \right] + 2a_e \tan^2 \frac{\theta}{2} \frac{1}{2} \left( \frac{E}{m} - \tau \frac{m}{E} \right) \left( F_1^\gamma (F_1^Z + F_2^Z) + F_2^Z (F_1^\gamma + F_2^\gamma) \right) - \tau (F_1^\gamma F_1^Z - F_2^Z F_2^\gamma) \right\}
\]

(67)

(68)

Now using Eqs. (13) and (14), the longitudinal asymmetry for the scattering of unpolarized electrons on polarized protons can be written

\[
\mathcal{A}_L^p = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-}
\]

\[
= -\frac{G_F q^2}{\sqrt{2} \pi \alpha \Sigma_\gamma} \left\{ v_e G_A^Z \left[ \epsilon \left( G_E \left( 1 - \tau \frac{m}{E} \right) + \tau G_M \left( 1 + \frac{m}{E} \right) \right) + (1 - \epsilon) \left( 1 + \frac{m}{E} \right) G_M \right] \right\}
\]

\(1\)There were some errors in this paper; these have been corrected here
\[ + \frac{a_e}{2} \frac{1 - \epsilon}{1 + \tau} \left[ \left( \frac{E}{m} - \frac{m}{E} \right) \left( G_E G_M^Z + G_E^Z G_M + 2\tau G_M G_M^Z \right) \right. \\
\left. - 2\tau \left( G_E G_M^Z + G_E^Z G_M - (1 - \tau) G_M G_M^Z \right) \right] \]  

where we have used the Sachs form factors defined in Eq. (8) and

\[ \Sigma_\gamma = \tau G_M^2 + \epsilon G_E^2 \]

Recently it has been pointed out that internal structure of the proton can be investigated with polarization transfer. For one photon exchange, the scattering of longitudinally polarized electrons results in a transfer of polarization to the recoil proton (see Sec. VIII).

Let us now discuss the polarization of recoil proton for our case. For this case, the four vector \( n_\mu \) refers to the recoil proton, i.e.

\[ n \cdot p' = 0, \]
\[ p' = k - k' + p, \]
\[ E_{p'} = E - E' + m \]

\[ p' = k - k' \]
\[ \equiv (-E' \cos \phi \sin \theta, -E' \sin \phi \sin \theta, E - E' \cos \theta) \]
\[ = |p'| (\cos \phi \sin \beta, \sin \phi \sin \beta, \cos \beta) \]

Hence we have

\[ |p'| = 2m \sqrt{\tau (1 + \tau)} \]
\[ E_{p'} = m (1 + 2\tau) \]
\[ -E' \sin \theta = |p'| \sin \beta = 2m \sqrt{\tau (1 + \tau)} \sin \beta \]
\[ E - E' \cos \theta = m \tau \left( 1 + \frac{m}{E} \right) = 2m \sqrt{\tau (1 + \tau)} \cos \beta \]

For transverse polarization of recoil proton,

\[ p' \cdot n = 0 = q \cdot n, \quad n_0 = 0 \]
\[ n \equiv (\cos \phi \cos \beta, \sin \phi \sin \beta, -\sin \beta) \]
Then from Eqs. (60–62, 66, 67), using Eqs. (73,74,76), we get

\[
\left( \frac{d\sigma}{dq^2} \right)_{\text{T recoil}} = G_F \frac{q^2}{\sqrt{2} \pi \alpha} \left( \frac{d\sigma}{dq^2} \right)_{\text{Mott}} \frac{\tan (\theta/2)}{\sqrt{\tau} (1 + \tau)} \times \left\{ v_e \left( \frac{E_m}{E} \right) \left( 1 - \frac{m}{E} \right) G_A^Z G_E + a_e \tau \left( G_M G_A^Z + G_E G_M \right) \right\} \quad (77)
\]

For longitudinal polarization of recoil proton,

\[
n \equiv - (\cos \phi \sin \beta, \sin \phi \sin \beta, \cos \beta) \quad n_0 = \frac{p' \cdot n}{E_p'} \quad (78)
\]

Again, from Eqs. (60–62, 66, 67), using Eqs. (73–76), and retaining only the contribution of \( n \), we get

\[
\left( \frac{d\sigma}{dq^2} \right)_{\text{L recoil}} = - G_F \frac{q^2}{\sqrt{2} \pi \alpha} \left( \frac{d\sigma}{dq^2} \right)_{\text{Mott}} \frac{1}{\epsilon (1 + \tau)} \sqrt{\frac{\tau}{1 + \tau}} \times \left\{ v_e G_A^Z \left[ \frac{\epsilon}{1 + \tau} (G_M + \tau G_E) - (1 - \epsilon) G_M \right] + a_e (1 - \epsilon) \left( \frac{E + E'}{2m} \right) G_M G_A^Z \right\} \quad (79)
\]

Hence for the longitudinal polarization of the recoil proton

\[
I_L \equiv \frac{\left( \frac{d\sigma}{dq^2} \right)_{\text{L recoil}}}{\left( \frac{d\sigma}{dq^2} \right)_{\text{em}}} = - G_F \frac{q^2}{\sqrt{2} \pi \alpha} \frac{1}{\tau G_M^2 + \epsilon G_E^2} \frac{\sqrt{\tau}}{1 + \tau} \times \left\{ v_e G_A^Z \left[ \frac{\epsilon}{1 + \tau} (G_M + \tau G_E) - (1 - \epsilon) G_M \right] + a_e (1 - \epsilon) \left( \frac{E + E'}{2m} \right) G_M G_A^Z \right\} \quad (80)
\]

It may be noted that electroweak interference due to one-photon exchange and Z-exchange can induce polarization in the recoil proton, although the lepton beam is unpolarized. The V–A term in the lepton sector i.e. the term \( \epsilon_{\mu \nu \alpha \beta} k_{\alpha} k'_{\beta} \) which is similar in content to \( \epsilon_{\mu \nu \alpha \beta} q_\alpha s_\beta \) (\( s_\mu \), polarization of lepton) contracted with the antisymmetric part of hadronic sector [cf. Eq. (58b)] gives rise to the term with coefficient \( a_e \) in Eq. (77) and (79). The first term with \( v_e \) coefficient in Eqs. (77) and (79) arises due to the contraction of symmetric part of leptonic tensor with the symmetric part of hadronic tensor in Eq. (58c). The symmetric part of hadronic tensor arises due to V–A term in hadronic sector which would vanish in
the absence of electro-weak interference. Experimentally it may be possible to detect the polarization of recoil proton in the unpolarized electron proton scattering. The second term in Eq. (80) is similar in character to that of polarization transfer from lepton to proton (Sec. VIII).

VI. PARITY–VIOLATING POLARIZED ELECTRONS SCATTERING ON UNPOLARIZED PROTONS

For this case, the relevant structure functions are given by

\begin{equation}
\tilde{J}_{\mu\nu} = (2\pi)^3 \frac{p_0}{m} \int e^{-iq\cdot z} \left\langle p \left[ J^\text{em}_\mu(z), J^Z_\nu(0) \right] | p \right\rangle
\end{equation}

\begin{equation}
= 2\pi \left\{ \tilde{W}_2 \frac{m^2}{q^2} \left[ p_\mu p_\nu - p_\cdot q \left( p_\mu q_\nu + p_\nu q_\mu \right) + \frac{(p \cdot q)^2}{q^4} q_\mu q_\nu \right] \\
+ \left( \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \tilde{W}_1 + \frac{1}{2m^2} \varepsilon_{\mu\nu\rho\sigma} p_\rho q_\sigma \tilde{W}_3 \right\}
\end{equation}

(81)

For polarized electrons, the lepton part containing the electron polarization part is given by

\begin{equation}
\left( \tilde{L}_{\mu\nu} \right)_s = -\frac{1}{4m_e} \varepsilon_{\mu\nu\alpha\beta} q_\alpha s_\beta - \frac{a_e}{4m_e} \left[ k'_\mu s_\nu + k'_\nu s_\mu - \delta_{\mu\nu} k' \cdot s + k' \rightarrow k \right]
\end{equation}

(82)

where \( s_\mu \) is the polarization vector of electron. The scattering cross-section containing only polarized part is given by

\begin{equation}
\frac{d^2\sigma}{dq^2 dv} = \frac{4\pi\alpha}{q^2} \frac{8G_F m_e^2}{\sqrt{2} E^2} \tilde{L}_{\nu\mu} \tilde{J}_{\mu\nu}
\end{equation}

\begin{equation}
= -\frac{\alpha}{q^2} G_F m_e \left\{ v_e \left( q^2 p \cdot s + p \cdot q k' \cdot s \right) \tilde{W}_3 \frac{m^2}{m^2} \\
+ a_e \left[ \tilde{W}_2 \left( 2p \cdot k' p \cdot n + m^2 k' \cdot n \right) - 2\tilde{W}_1 k' \cdot n \right] \right\}
\end{equation}

(83)

we note

\begin{equation}
k \cdot s = 0
\end{equation}

\begin{equation}
k = (0, 0, 1, i)
\end{equation}

(84)

For longitudinally polarized electron beam
\[ s = \lambda(0, 0; \frac{E}{m_e}, \frac{iE}{m_e}) \]

Hence for this case

\[
\frac{d^2\sigma}{dq^2 d\nu} = \frac{\lambda}{q^2 \sqrt{2} E^2} \left( v_e \tilde{W}_3 \frac{E + E'}{2m} q^2 - a_e \left[ \tilde{W}_2 E E' (1 + \cos \theta) + \tilde{W}_1 E E' (1 - \cos \theta) \right] \right)
\]

\[
= \frac{\lambda G_E q^2}{2\pi \sqrt{2} \alpha} \left( \frac{d\sigma}{dq^2} \right)_{\text{Mott}} \left\{ v_e \tilde{W}_3 \frac{E + E'}{m} \tan^2 \frac{\theta}{2} - a_e \left[ \tilde{W}_2 + 2\tilde{W}_1 \tan^2 \frac{\theta}{2} \right] \right\}
\]

(85)

(86)

For elastic scattering

\[
\tilde{W}_1 = 2\tau (F_1^\gamma + F_2^\gamma) \left( F_1^Z + F_2^Z \right) \delta \left( \nu - \frac{q^2}{2m} \right)
\]

\[
\tilde{W}_2 = 2 \left( F_1^\gamma F_1^Z + \tau F_2^\gamma F_2^Z \right) \delta \left( \nu - \frac{q^2}{2m} \right)
\]

\[
\tilde{W}_3 = -2G_A^Z (F_1^\gamma + F_2^\gamma)
\]

(87)

Hence the elastic scattering cross-section for longitudinally polarized electrons on unpolarized protons is given by [13]

\[
\frac{d\sigma^\pm}{dq^2} = \pm G_E q^2 \left( \frac{d\sigma}{dq^2} \right)_{\text{Mott}} \left\{ v_e \left[ -2G_A^Z (F_1^\gamma + F_2^\gamma) \frac{E - m\tau}{m} \tan^2 \frac{\theta}{2} \right] - a_e \left[ (F_1^\gamma F_1^Z + \tau F_2^\gamma F_2^Z) + 2\tau \tan^2 \frac{\theta}{2} (F_1^\gamma + F_2^\gamma) (F_1^Z + F_2^Z) \right] \right\}
\]

(88)

For this case the asymmetry is given by

\[
\mathcal{A}^e = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-}
\]

\[
= -\frac{G_E q^2}{\sqrt{2} \pi \alpha} \frac{1}{\Sigma_{\gamma}} \left\{ v_e (1 - \epsilon) \left( \frac{E}{m} - \tau \right) G_A G_M \right. \]

\[
\left. + a_e \left[ \epsilon \left( G_E^Z G_E + \tau G_M^Z G_M \right) + \tau (1 - \epsilon) G_M^Z G_M \right] \right\}
\]

(89)

It may be noted that asymmetry \( \mathcal{A}^e \) given in Eq. (89) and longitudinal asymmetry \( \mathcal{A}_L^e \) given in Eq. (69) complement each other.

**VII. MEASUREMENT OF THE ELASTIC FORM FACTORS \( G_A^Z, G_M^Z \)**

The elastic scattering of electrons on proton target involves the form factors \( G_E, G_M, G_E^Z, G_M^Z \) and \( G_A^Z \). These form factors characterize the internal structure of proton. In particular
$G_Z^A$ and $G_Z^M$ contain both octet and singlet part. Thus the measurement of $G_Z^A$ and $G_Z^M$ would give the information about the singlet form factors which in turn determines the strange content of the proton. The electromagnetic form factors follow the dipole form

$$G_E = \frac{1}{\mu_p} G_M = G_D = \frac{1}{[1 + q^2/0.71 \text{ GeV}^2]^2}$$

We take the dipole form for $G_Z^A$, $G_Z^E$ and $G_Z^M$:

$$G_Z^E = \frac{G_E^Z(0)}{[1 + q^2/m_V^2]^2}$$
$$G_Z^M = \frac{G_M^Z(0)}{[1 + q^2/m_V^2]^2}$$
$$G_Z^A = \frac{G_A^Z(0)}{[1 + q^2/m_A^2]^2}$$

We will assume that

$$m_V \equiv m_V^3 = m_V^8 = m_V^0 = 0.84 \text{ GeV}, \quad m_V^2 = 0.71 \text{ GeV}^2$$
$$m_A \equiv m_A^3 = m_A^8 = m_A^0 = 1.03 \text{ GeV}, \quad m_A^2 = 1.06 \text{ GeV}^2$$

The equality of $m_V^3 = m_V^8$ follows from the electromagnetic form factors; similarly $m_A^3 = m_A^8$ follows from $\beta$-decay. The particular value for $m_A$ is taken from Ref. [12]. Now

$$G_A^Z(0) = \frac{1}{2} \left[ (F + D) + \left( F - \frac{1}{3} D \right) - \frac{1}{3} g_A^0 \right],$$

where experimentally

$$F + D = 1.2670 \pm 0.0035$$
$$F - \frac{1}{3} D = 0.25 \pm 0.05$$

EMC and elastic neutrino scattering data are consistent with $g_A^0 \approx 0$. For $g_A^0 = 0$,

$$G_A^s(0) = - \left( G_A^u(0) + G_A^d(0) \right)$$

where as $g_A^0 = 3F - D$, gives

$$G_A^s(0) = 0$$
For the vector form factors

\[ G_E^Z(0) = \frac{1}{2} (1 - 4x_W) = 0.036; \quad \text{for} \ x_W = 0.2322 \]

\[ G_M^Z(0) = (1 - 2x_W) \mu_p - \frac{1}{2} (1 + \kappa_0) \]

Again the singlet anomalous magnetic moment \( \kappa_0 \) is unknown. For \( \kappa_0 = 0 \):

\[ G_M^S(0) = - \left( G_M^a(0) + G_D^M(0) \right) \]

where as for \( \kappa_0 = \kappa_p + \kappa_n \)

\[ G_M^S(0) = 0 \]

In our analysis of parity violating scattering, we will use two sets of values

set (i) .

\[ G_A^Z(0) = \frac{1}{2} \left[ (F + D) + \left( F - \frac{1}{3} D \right) \right] = 0.76 \]

\[ G_M^Z(0) = (1 - 2x_W) \mu_p - \frac{1}{2} = 0.99 \]

set (ii) .

\[ G_A^Z(0) = \frac{1}{2} \left[ (F + D) + \left( F - \frac{1}{3} D \right) - \left( F - \frac{1}{3} D \right) \right] = 0.63 \]

\[ G_M^Z(0) = (1 - 2x_W) \mu_p - \frac{1}{2} (1 + \kappa_p + \kappa_n) = 1.05 \]

We note that since \( G_E^Z(0) / G_M^Z(0) \approx 0.036 \), it is a good approximation to neglect the contribution of \( G_E^Z \) as compared with \( G_M^Z \).

Using Rosenbluth method, it is possible to extract the form factors by measuring the longitudinal proton and electron asymmetries as well as recoil proton polarizations at a fixed \( q^2 \) over the range of \( \epsilon \) values that are obtained by changing the beam energy and scattering angle. In order to use this technique, we re-express the asymmetries and recoil polarization given in Eqs. (69), (89), (80) and (79) in terms of \( \epsilon \) and \( \tau \). For this purpose we note
Hence from Eqs. (69), (89), (80) and (79) we get

\[ \tan^2 \frac{\theta}{2} = \frac{1 - \epsilon}{2\epsilon (1 + \tau)} \]  
(100)

\[ \epsilon = \frac{E^2 - 2mE\tau - m^2\tau}{E^2 - 2mE\tau + m^2\tau + 2m^2\tau^2} \]  
(101)

\[ \frac{E}{m - \tau} = \sqrt{\tau (1 - \tau)} \sqrt{1 + \epsilon \over 1 - \epsilon} \]  
(102)

First we note that since the form factor \( G \) its contribution is suppressed compared to the terms multiplied by \( G \), we get from these equations

\[ \mathcal{A}_L^p = - \left( \frac{G_F q^2}{\sqrt{2\pi} \alpha} \right) \left( \frac{1}{\tau G_M^2 + \epsilon G_E^2} \right) \left( \frac{1}{1 + \epsilon + 2\epsilon\tau} \right) \times \left\{ v_e G_A^Z \left[ 2\epsilon^2 (1 + \tau) G_E + 2\tau G_M (1 - \epsilon + \epsilon\tau) + \sqrt{\tau (1 + \tau) (1 - \epsilon^2) (-\epsilon G_E + (1 - \epsilon + \epsilon\tau) G_M) \right] \right. \\
+ a_n \left[ 2\epsilon \left( \tau G_M G_E + G_E^2 G_M \right) (1 - \epsilon G_E + \tau G_M^2) \right] \right\} \]  
(103)

\[ \mathcal{A}_e = - \left( \frac{G_F q^2}{\sqrt{2\pi} \alpha} \right) \left( \frac{1}{\tau G_M^2 + \epsilon G_E^2} \right) \times \left\{ v_e \sqrt{\tau (1 + \tau) (1 - \epsilon) G_M} + a_n \left[ \epsilon G_E^2 G_M + \tau G_M^2 \right] \right\} \]  
(104)

\[ \mathcal{I}_L = - \left( \frac{G_F q^2}{\sqrt{2\pi} \alpha} \right) \left( \frac{1}{\tau G_M^2 + \epsilon G_E^2} \right) \sqrt{\tau \over 1 + \tau} \times \left\{ v_e G_A^Z \left[ (G_M + \tau G_E) - (1 - \epsilon) G_M \right] + a_n \sqrt{\tau (1 + \tau) (1 - \epsilon) G_M^2 \right\} \]  
(105)

\[ \mathcal{I}_T = - \left( \frac{G_F q^2}{\sqrt{2\pi} \alpha} \right) \left( \frac{1}{\tau G_M^2 + \epsilon G_E^2} \right) \sqrt{1 \over 2\tau} \times \left\{ v_e G_A^Z G_E^2 \left[ 2\epsilon (1 + \tau) \over 1 + \epsilon + 2\epsilon\tau \right] \left[ \sqrt{\tau (1 + \tau) (1 + \epsilon) + \sqrt{\tau (1 - \epsilon)} \right] + a_n \sqrt{\epsilon (1 - \epsilon) (G_E G_M^2 + G_E^2 G_M) \right\} \]  
(106)

First we note that since the form factor \( G_A^Z \) is multiplied by \( v_e = - \left( 1 - 4 \sin^2 \theta_W \right) = -0.07 \), its contribution is suppressed compared to the terms multiplied by \( a_n = -1 \). Thus overall contribution of \( G_A^Z \) is suppressed except for \( \epsilon = 1 \) in Eqs. (103), (105) and (106). For \( \epsilon = 1 \), we get from these equations

\[ \mathcal{A}_L^p (\epsilon = 1) = - \left( \frac{G_F q^2}{\sqrt{2\pi} \alpha} \right) \left( \frac{G_M^2}{\tau G_M^2 + \epsilon G_E^2} \right) \left( \frac{1}{2 (1 + \tau)} \right) \left\{ v_e G_A^Z \left[ 2 (1 + \tau) \frac{G_E}{G_M} + 2\tau^2 \right] \right\} \]  
(107)

\[ \mathcal{I}_L (\epsilon = 1) = - \left( \frac{G_F q^2}{\sqrt{2\pi} \alpha} \right) \left( \frac{G_M^2}{\tau G_M^2 + \epsilon G_E^2} \right) \sqrt{\tau \over 1 + \tau} \left\{ v_e G_A^Z \left[ (1 + \tau) \frac{G_E}{G_M} \right] \left( 1 + \tau \right) \right\} \]  
(108)

\[ \mathcal{I}_T (\epsilon = 1) = \left( \frac{G_F q^2}{\sqrt{2\pi} \alpha} \right) \left( \frac{G_M^2}{\tau G_M^2 + \epsilon G_E^2} \right) \left\{ v_e G_A^Z G_E \left( \frac{1}{1 + \tau} \right) \right\} \]  
(109)
On the other hand from Eq. (104), we get at \( \epsilon = 1 \)

\[
\mathcal{A}_e (\epsilon = 1) = - \left( \frac{G_F q^2}{\sqrt{2} \pi \alpha} \right) \left( \frac{G_M^2}{\tau G_M^2 + G_E^2} \right) a_e G_M^2
\]  

(110)

It is thus clear from Eqs. (107–110), that it is possible to extract \( G_A^Z / G_M \) and \( G_M^Z / G_M \) using Rosenbluth method. In any case we have plotted \( \mathcal{A}_p, \mathcal{A}_e, I_L, \) and \( I_T, \) versus \( \epsilon \) for \( q^2 = 0.5 \) GeV, 1 GeV, 2.64 GeV, 3.2 GeV, 5.6 GeV, and 10 GeV. In these plots we have taken

\[
\frac{G_A^Z}{G_M} = \frac{G_A^Z(0)}{\mu_p} (1 + q^2/0.71 \text{GeV}^2)^2
\]

(111)

\[
\frac{G_M^Z}{G_M} = \frac{G_M^Z(0)}{\mu_p}, \quad \frac{G_E}{G_M} = \frac{1}{\mu_p}
\]

(112)

For \( G_A^Z(0) \) and \( G_M^Z(0), \) we have used two sets of values given in Eqs. (98) and (99). These plots are shown in Figures 2, 3, 4 and 5. These figures can be used to extract the form factors in future experiments.

**VIII. POLARIZED ELECTRONS SCATTERING ON POLARIZED PROTONS**

For this case, we replace \( \tilde{S}_{\mu \nu} \to S_{\mu \nu} \) and \( \tilde{G}_{1,2} \to (1/2) G_{1,2} \) in Eq. (58c); the other structure functions are not relevant for this case. Then the scattering cross-section for polarized electrons on polarized protons is given by

\[
\frac{d^2 \sigma}{dq^2 dv} = \frac{1}{\pi} \left( \frac{d\sigma}{dq^2} \right)_{\text{Mott}} \frac{4 m_e^2 q^2}{q^2} \tan^2 \frac{\theta}{2} \left[ (L_{\nu \mu})_s S_{\mu \nu} \right],
\]

(113)

where [cf. Eq. (83)]

\[
(L_{\nu \mu})_s = - \frac{1}{m_e} \epsilon_{\nu \mu \alpha \beta} q_\alpha s_\beta
\]

(114)

Thus

\[
(L_{\nu \mu})_s S_{\mu \nu} = \frac{1}{2 m m_e} \frac{2 \pi}{2 m m_e} \left[ -G_1 \left( q^2 n \cdot s - n \cdot q s \cdot q \right) + \frac{G_2}{m^2} \left( q^2 p \cdot q n \cdot s - q^2 n \cdot q p \cdot s \right) \right]
\]

(115)

Hence for longitudinally polarized electrons, the scattering cross-section is given by
\[
\frac{d^2\sigma^\rightarrow}{dq^2d\nu} = \left( \frac{d\sigma}{dq^2} \right)_{\text{Mott}} \frac{4E}{mq^2} \tan^2 \frac{\theta}{2} \times \left\{ -G_1 \left( q^2 (n_z - n_0) - n \cdot qE' \left( 1 - \cos \theta \right) \right) + \frac{G_2}{m} \left( -q^2 \nu (n_z - n_0) + q^2 n \cdot q \right) \right\} 
\]

(116)

For longitudinally polarized protons, \( \vec{n} = \lambda_n \vec{e}_z \), \( n_0 = 0 \), \( \lambda_n = \pm 1 \), we get

\[
\frac{d^2\sigma^\rightarrow}{dq^2d\nu} = -2 \left( \frac{d\sigma}{dq^2} \right) \frac{1}{m} \tan^2 \frac{\theta}{2} \left[ G_1 \left( E + E' \cos \theta \right) - \frac{q^2}{m} G_2 \right] 
\]

(117)

Now longitudinal spin–spin asymmetry is defined as:

\[
A_\parallel = \frac{\frac{d\sigma^\rightarrow}{dq^2} - \frac{d\sigma^\rightarrow}{dq^2}}{\frac{d\sigma^\rightarrow}{dq^2} + \frac{d\sigma^\rightarrow}{dq^2}} = 2 \left( \frac{d\sigma}{dq^2} \right) \frac{1}{m} \tan^2 \frac{\theta}{2} \left[ G_1 \left( E + E' \cos \theta \right) - \frac{q^2}{m} G_2 \right] / \frac{d^2\sigma}{dq^2d\nu} 
\]

(118)

where [cf. Eq. (4)]

\[
\frac{d^2\sigma}{dq^2d\nu} = \left( \frac{d\sigma}{dq^2} \right)_{\text{Mott}} W_1 \left[ \frac{W_2}{W_1} + 2 \tan^2 \frac{\theta}{2} \right] = \left( \frac{d\sigma}{dq^2} \right)_{\text{Mott}} \frac{W_1}{\epsilon (1 + \nu^2/q^2)} [1 + \epsilon R] = \left( \frac{d\sigma}{dq^2} \right) \frac{2 \tan^2 \frac{\theta}{2}}{1 - \epsilon} W_1 [1 + \epsilon R] 
\]

(119)

where we have used

\[
\left( 1 + \frac{\nu^2}{q^2} \right) \frac{W_2}{W_1} - 1 = \frac{\sigma_L}{\sigma_T} = R 
\]

(120)

\[
\epsilon = \frac{1}{1 + 2 (1 + \nu^2/q^2) \tan^2 \frac{\theta}{2}} 
\]

(121)

Hence longitudinal spin–spin asymmetry is given by [19]

\[
A_\parallel = \frac{1}{m} (1 - \epsilon) \left[ G_1 \left( E + E' \cos \theta \right) - \frac{q^2}{m} G_2 \right] \frac{1}{W_1 [1 + \epsilon R]} 
\]

(122)

For elastic scattering

\[
\frac{1}{m} \left( E + E' \cos \theta \right) = 2 \left[ \frac{E}{m} - \tau - \frac{m \tau}{E} \right] W_1 = G_M^2 \delta \left( \nu - \frac{q^2}{2m} \right) 
\]

(123)
\[ R = \frac{\sigma_L}{\sigma_T} = \frac{G_E^2}{\tau G_M^2} \]

\[ G_1 = (G_E + \tau G_M) \frac{G_M}{1 + \tau} \delta \left( \nu - \frac{q^2}{2m} \right) \]

\[ G_2 = -\frac{1}{2} (G_M - G_E) \frac{G_M}{1 + \tau} \delta \left( \nu - \frac{q^2}{2m} \right) \]

(124)

Hence we get from Eq. (123)

\[ A_\parallel = 2 (1 - \epsilon) \frac{\tau}{1 + \tau} \left( \frac{G_M}{\tau G_M^2 + G_E^2} \right) \left\{ \left( \frac{E}{m} - \frac{m}{E} \right) (G_E + \tau G_M) - \tau (2G_E - (1 - \tau) G_M) \right\} \]

(125)

Finally we discuss the polarization of the recoil proton. For this case \( n_\mu \) refers to recoil proton. Then, from Eq. (93), using Eqs. (71–76) and (125), we get [3,4]

\[ \left( \frac{d\sigma}{dq^2} \right)_T = \left( \frac{d\sigma}{dq^2} \right)_{\text{Mott}} \left[ -\frac{8\tau}{\sqrt{\tau (1 + \tau)}} \tan \frac{\theta}{2} \right] G_M G_E \]

(126)

\[ \left( \frac{d\sigma}{dq^2} \right)_L = \left( \frac{d\sigma}{dq^2} \right)_{\text{Mott}} \left[ \frac{4\tau}{\sqrt{\tau (1 + \tau)}} \frac{E + E'}{m} \tan^2 \frac{\theta}{2} \right] G_M^2 \]

(127)

These equations can be put in the form

\[ \mathcal{I}_T = \left( \frac{d\sigma}{dq^2} \right)_T / \left( \frac{d\sigma}{dq^2} \right) = \frac{\epsilon (1 + \tau)}{1 + \frac{\epsilon G_E^2}{\tau G_M^2}} \left[ -8 \sqrt{\frac{\tau}{1 + \tau}} \tan \frac{\theta}{2} \right] \frac{G_E}{G_M} \]

(128)

\[ \mathcal{I}_L = \left( \frac{d\sigma}{dq^2} \right)_L / \left( \frac{d\sigma}{dq^2} \right) = \frac{\epsilon (1 + \tau)}{G_M^2 \left( 1 + \frac{\epsilon G_E^2}{\tau G_M^2} \right)} \left[ 4 \sqrt{\frac{\tau}{1 + \tau}} \frac{E + E'}{m} \tan^2 \frac{\theta}{2} \right] G_M^2 \]

(129)

\[ \frac{G_E}{G_M} = - \left( \frac{\mathcal{I}_T}{\mathcal{I}_L} \right) \left( \frac{E + E'}{2m} \right) \tan \frac{\theta}{2} \]

(130)

Equation (130) has been used in Refs. [3,4] to experimentally extract the \( G_E/G_M \) by measuring the transverse and longitudinal recoil proton polarization. Their result show a systematic decrease of ratio \( G_E/G_M \) as \( q^2 \) increases from 0.5 to 5.6 GeV\(^2\), indicating for the first time a definite difference in the spatial distribution of charge and magnetization currents in the proton.
However, it is possible to extract the form factors $G_E$ and $G_M$ from the longitudinal spin–spin asymmetry given in Eq. (125). From Eq. (125), we can write

$$A_\parallel = \frac{G_M}{\tau G_M^2 + \epsilon G_E^2} \frac{1}{1 + \epsilon + 2\epsilon \tau} \frac{2\tau}{1 + \tau} \left\{ 2\epsilon (1 + \tau) (G_E + \tau G_M) \sqrt{\tau (1 + \tau) (1 - \epsilon^2)} + (1 - \epsilon) \left[ 2G_E (\epsilon - \tau + 2\epsilon \tau^2) + \tau G_M (2\epsilon (1 + \tau) + (1 - \tau) (1 + \epsilon + 2\epsilon \tau)) \right] \right\}$$

Hence if we plot $A_\parallel$ versus $\epsilon$ for fixed $\tau$, we can get information about the form factors $G_E$ and $G_M$. In particular we note that for $\epsilon = 0$

$$A_\parallel (\epsilon = 0) = \frac{2\tau}{G_M 1 + \tau} \left[ -2G_E + G_M - \tau G_M \right] = \frac{2\tau}{1 + \tau} \left[ 1 - \tau - 2\frac{G_E}{G_M} \right] = 2\tau \left[ \frac{F_2 - F_1}{F_2 + F_1} \right] = -2\tau \left[ \frac{1 - F_2/F_1}{1 + F_2/F_1} \right]$$

This is an interesting result; it would supplement the result obtained from the recoil proton polarization discussed above.

**IX. DEEP INELASTIC SCATTERING**

We now briefly discuss the deep inelastic scattering [1,9,10,19–21]. In this region the structure functions are found to be independent of four momentum transfer $q^2$ at fixed Bjorken variable $x = q^2/2m\nu$. In the scaling region both $q^2$ and $\nu$ are large but $x$ remains fixed. The deep inelastic scattering is analysed in terms of Bjorken variable $x$ and inelasticity $y$.

$$x = \frac{q^2}{2m\nu}, \quad y = \frac{\nu}{E}$$

It is convenient to introduce another variable

$$\gamma^2 = \frac{q^2}{\nu^2} = \frac{4m^2x^2}{q^2}$$

In terms of these variables we can write
$$\tan^2 \frac{\theta}{2} = \frac{\gamma^2 (1 - \epsilon)}{2 \epsilon (1 + \gamma^2)} = \frac{\gamma^2 y^2}{4 (1 - y) - \gamma^2 y^2}$$  \hfill (135)$$

$$\epsilon = \frac{4 (1 - y) - \gamma^2 y^2}{4 (1 - y) + 2 y^2 + \gamma^2 y^2}$$ \hfill (136)$$

We can express the electron–proton scattering cross-section given in Eq. (4) in deep inelastic region as

$$\frac{d^2 \sigma}{dq^2 d\nu} = \left( \frac{d\sigma}{dq^2} \right)_{\text{Mott}} \frac{\gamma^2 F_1}{m (1 + \gamma^2) \epsilon} [1 + \epsilon R]$$

$$\frac{d^2 \sigma}{dxdy} = \left( \frac{d\sigma}{dq^2} \right)_{\text{Mott}} E \frac{Am x F_1}{(1 + \gamma^2) \epsilon} [1 + \epsilon R]$$ \hfill (137)$$

In deriving this result we have used

$$\nu W_2 \left( \nu, q^2 \right) = F_2 \left( x, q^2 \right)$$

$$m W_1 \left( \nu, q^2 \right) = F_1 \left( x, q^2 \right)$$ \hfill (138)$$

$$\left( 1 + \frac{\nu^2}{q^2} \right) \frac{W_2}{W_1} = 1 + R$$

$$\frac{F_2 (1 + \gamma^2)}{2 x F_1} = 1 + R$$ \hfill (139)$$

$$2 \left( 1 + \frac{\nu^2}{q^2} \right) \tan^2 \frac{\theta}{2} = \frac{1 - \epsilon}{2 \epsilon}$$ \hfill (140)$$

The scattering cross-section for polarized longitudinal electrons on longitudinally polarized protons given in Eq. (117) can be expressed in the deep inelastic region as

$$\frac{d^2 \sigma}{dxdy} = \left( \frac{d\sigma}{dq^2} \right)_{\text{Mott}} \frac{4 m x (1 - \epsilon)}{(1 + \gamma^2) \epsilon} \left[ g_1 \left( 2 - y - \frac{1}{2} \gamma^2 y^2 \right) \frac{1}{y} - \gamma^2 g_2 \right]$$ \hfill (141)$$

where we have used $\nu G_1 = g_1$, $\nu^2 G_2 = mg_2$.

The longitudinal spin-spin asymmetry defined in Eq. (122) can be written as

$$A_\parallel = (1 - \epsilon) \frac{1}{F_1 (1 + \epsilon R)} \left[ g_1 \left( 2 - y - \frac{1}{2} \gamma^2 y^2 \right) \frac{1}{y} - \gamma^2 g_2 \right]$$ \hfill (142)$$

Since $\gamma^2 \to 0$ as $q^2 \to \infty$; for large $q^2$, it is a good approximation to put $\gamma^2 = 0$. In what follows we will make this approximation. Thus we take
\[ \epsilon \approx \frac{4 (1 - y)}{4 (1 - y) + 2 y^2} = \frac{2 (1 - y)}{1 + (1 - y)^2} \equiv \frac{2 (1 - y)}{Y_+} \]

\[ 1 - \epsilon \approx \frac{y^2}{Y_+} \]

\[ 1 - \epsilon (1 - y) = \frac{1 - (1 - y)^2}{1 + (1 - y)^2} = \frac{Y_-}{Y_+} = (1 - \epsilon) \left( \frac{2 - y}{y} \right) \]  \hspace{1cm} (143)

Thus we get a simple expression for the longitudinal spin-spin asymmetry [19]

\[ \mathcal{A}_\parallel = \frac{g_1 [1 - \epsilon (1 - y)]}{F_1 (1 + \epsilon R)} \]  \hspace{1cm} (144)

This gives

\[ g_1 = \frac{\mathcal{A}_\parallel (1 + \gamma^2) F_2}{D} \frac{2x (1 + R)}{1 + \epsilon R} \]  \hspace{1cm} (145)

Similarly the parity violating scattering cross-section of electrons on longitudinal polarized protons given in Eq. (65) can be expressed in deep inelastic region [18]

\[ \frac{d^2 \sigma^-}{dq^2 d\nu} = \frac{G_F q^2}{\sqrt{2\pi \alpha}} 4mE \frac{1 - \epsilon}{4\epsilon} \left( \frac{d\sigma}{dq^2} \right)_{\text{Mott}} \times \left\{ v_e \left\{ \frac{2 (1 - y)}{y^2} \tilde{h}_2^e - x \tilde{h}_3^e - (1 - y) \frac{1}{y^2} \tilde{h}_4^e \right\} + a_e \left[ \tilde{g}_1^e 2 - \frac{y}{y} \right] \right\} \]  \hspace{1cm} (146)

where we have used,

\[ \nu \tilde{G}_1^e = \tilde{g}_1^e, \hspace{0.5cm} \nu^2 \tilde{G}_2^e = m \tilde{g}_2^e, \hspace{0.5cm} \nu \tilde{H}^e_3 = \tilde{h}_3^e, \hspace{0.5cm} \nu \tilde{H}^e_4 = m \tilde{h}_4^e. \]  \hspace{1cm} (147)

Now using the light cone algebra sum rules [18]

\[ \tilde{h}_3^e (x) = -\frac{1}{x} \tilde{h}_2^e (x) \]

\[ \tilde{h}_4^e (x) = 0 \]  \hspace{1cm} (148)

we get

\[ \frac{d^2 \sigma^-}{dx dy} = \frac{G_F q^2}{\sqrt{2\pi \alpha}} 4mE \frac{1 - \epsilon}{4\epsilon} \left( \frac{d\sigma}{dq^2} \right)_{\text{Mott}} \frac{1}{y^2} \times \left\{ v_e \left\{ 2 (1 - y) + y^2 \right\} \tilde{h}_2^e + a_e \left[ \tilde{g}_1^e xy (2 - y) \right] \right\} \]  \hspace{1cm} (149)

Hence the longitudinal proton asymmetry is given by
\[ \mathcal{A}_L^p = \frac{d^2 \sigma^-}{dx dy} - \frac{d^2 \sigma^+}{dx dy} \]
\[ = \frac{-G_F q^2}{\sqrt{2} \pi \alpha} \left( \frac{1}{4} \right) \frac{v_e Y_+ \frac{1}{2} h_2^e (x, q^2) + a_e Y_- \tilde{g}_1^e (x, q^2)}{F_1 (1 + \epsilon) y^2} \]
\[ = \frac{-G_F q^2}{\sqrt{2} \pi \alpha} \frac{v_e \frac{1}{2} h_2^e (x, q^2) + a_e [1 - \epsilon (1 - y)] \tilde{g}_1^e (x, q^2)}{4 F_1 (1 + \epsilon) R} \] (150)

Finally for the parity violating polarized electron scattering on unpolarized proton, we get [cf. Eq. (86)]

\[ \frac{d^2 \sigma^-}{dx dy} = \frac{G_F q^2}{\sqrt{2} \pi \alpha} \left( \frac{d \sigma}{dq^2} \right)_{\text{Mott}} \times \left\{ v_e \frac{1}{Y_+} \bar{x} \bar{F}_3 - a_e \left[ \bar{F}_2 - \frac{y^2}{Y_+} \bar{F}_L \right] \right\} \] (151)

where we have used the relation

\[ 2x \bar{F}_1 = \bar{F}_2 - \bar{F}_L \] (152)

Hence the longitudinal electron asymmetry

\[ \mathcal{A}_L^e = \frac{G_F q^2}{\sqrt{2} \pi \alpha} \frac{v_e \frac{1}{Y_+} \bar{x} \bar{F}_3 - a_e \left[ \bar{F}_2 - \frac{y^2}{Y_+} \bar{F}_L \right]}{2x F_1 (1 + \epsilon) R} \]
\[ = \frac{G_F q^2}{\sqrt{2} \pi \alpha} \frac{v_e [1 - \epsilon (1 - y)] x \bar{F}_3 - a_e \left[ \bar{F}_2 - (1 - \epsilon) \bar{F}_L \right]}{2x F_1 (1 + \epsilon) R} \] (153)

From the experimental measurements of asymmetries \( \mathcal{A}_\parallel \) and \( \mathcal{A}_L^p \), the structure functions \( g_1, \tilde{g}_1^e \) and \( \tilde{h}_2^e \) can be extracted. Note that they are functions of \( x \); their dependence on \( q^2 \) is weak (logarithmic) and can be taken care of by QCD corrections. These structure functions in turn give information about the fundamental constituents of the proton, particularly their spin content. The asymmetry \( \mathcal{A}_L^p \) involve two functions \( \tilde{g}_1^e \) and \( \tilde{h}_2^e \). The structure function \( \tilde{g}_1^e \) is multiplied by a \( y \)-dependent term, whereas \( \tilde{h}_2^e \) is just multiplied by a constant. Hence by measuring \( \mathcal{A}_L^p \) at various values of \( y \), it is possible to extract both \( \tilde{h}_2^e \) and \( \tilde{g}_1^e \).

The spin-dependent structure functions \( g_1 \) and \( \tilde{g}_c \) satisfy the sum rules given below; the right-hand-side of these sum rules is given in terms of the spin-content of elementary constituents of proton.

30
For the structure function $g_1(x)$, light cone algebra gives the sum rule

$$\int_0^1 g_{1}^{p,n}(x) \, dx = \left[ \pm \frac{1}{6} a_3 + \frac{1}{6\sqrt{3}} a_8 + \frac{1}{3\sqrt{3}} a_0 \right]$$  \hspace{1cm} (154)$$

$a_3$ and $a_8$ are given as follows

$$a_3 = \frac{1}{2} g_A = \frac{1}{2} (F + D)$$
$$a_8 = \frac{1}{2\sqrt{3}} g_A = \frac{1}{2\sqrt{3}} (3F - D)$$  \hspace{1cm} (155)$$

The $a_0$ being singlet cannot be written in terms of $F$ and $D$; but we put

$$a_0 = \frac{1}{2} \sqrt{\frac{2}{3}} g^0_A$$  \hspace{1cm} (156)$$

Hence, from Eq. (154), using Eqs. (155) and (156), we get the Bjorken sum rule [16]

$$\int_0^1 [g_{1}^{p}(x) - g_{1}^{n}(x)] \, dx = \frac{1}{6} g_A \left( 1 - \frac{\alpha_s(q^2)}{\pi} \right)$$  \hspace{1cm} (157)$$

The factor multiplying $g_A$ is the QCD correction to the sum rule. Bjorken sum rule is well satisfied experimentally. Also for proton only, the sum rule can be written as follows

$$\int_0^1 g_{1}^{p}(x) \, dx = \frac{1}{12} \left[ (F + D) + \frac{1}{3} (3F - D) + \frac{4}{3} g^0_A \right]$$  \hspace{1cm} (158)$$

However, if we take

$$g^0_A = g_8 = 3F - D,$$  \hspace{1cm} (159)$$

we get the Ellis–Jafee sum rule [17]

$$\int_0^1 g_{1}^{p}(x) \, dx = \frac{1}{12} \left[ 1 + \frac{5}{3} \frac{3F - D}{F + D} \right]$$  \hspace{1cm} (160)$$

This sum rule is in disagreement with the data. This shows that Eq. (159) does not hold experimentally viz the strange content of the proton is not zero.

In the naive quark–parton model, the sum rule (159) can be written as

$$\int_0^1 g_{1}^{p,n}(x) \, dx = \frac{1}{2} \sum_q e^2_q \Delta q$$  \hspace{1cm} (161)$$
where
\[ \Delta q = \int_0^1 \left\{ \left[ q^\uparrow (x) + \bar{q}^\uparrow (x) \right] - \left[ q^\downarrow (x) + \bar{q}^\downarrow (x) \right] \right\} \]  
(162)

Here \( \Delta q \) is the quark contribution to the first moment of \( g_1 (x) \). There is also gluon contribution to it. To include this we replace \( \Delta q \) by \( \Delta \tilde{q} \) defined in Eqs. (37) and (38). Then using Eqs. (38), we can write the sum rules (157) and (158) as (same results follow from Eq. (161))

\[ \int_0^1 [g_1^p (x) - g_1^n (x)] dx = \frac{1}{6} \left( \Delta \tilde{u} - \Delta \tilde{d} \right) \left( 1 - \frac{\alpha_s (q^2)}{\pi} \right) \]  
(163)

\[ \int_0^1 g_1^p (x) dx = \frac{1}{12} \left[ \left( \Delta \tilde{u} - \Delta \tilde{d} \right) + \frac{1}{3} \left( \Delta \tilde{u} - \Delta \tilde{d} - 2 \Delta \tilde{s} \right) + \frac{4}{3} \left( \Delta \tilde{u} + \Delta \tilde{d} + \Delta \tilde{s} \right) \right] \]
\[ = \frac{1}{2} \left[ \frac{4}{9} \Delta \tilde{u} + \frac{1}{9} \Delta \tilde{d} + \frac{1}{9} \Delta \tilde{s} \right] \]  
(164)

Now we discuss the sum rule for \( \tilde{g}_1^{ep} \). Light cone algebra gives the sum rule

\[ \int_0^1 [\tilde{g}_1^{ep} (x) + 2xW g_1^p (x)] dx = \left[ \frac{1}{12} a_3 + \frac{1}{4\sqrt{3}} a_8 + \frac{1}{4} \sqrt{\frac{2}{3}} a_0 \right] \]
\[ = \frac{1}{24} \left[ (F + D) + (3F - D) + 2g_0^a \right] \]
\[ = \frac{1}{12} \left[ 2 \Delta \tilde{u} + \Delta \tilde{d} \right] \]  
(165)

However, from Eqs. (165) and (158), eliminating the singlet contribution, we get the sum rule

\[ \int_0^1 [4\tilde{g}_1^{ep} (x) - (3 - 8xW) g_1^p (x)] dx = \frac{1}{6} (F - D) \left( 1 - \frac{\alpha_s (q^2)}{\pi} \right) \]
\[ = \frac{1}{6} \left( \Delta \tilde{d} - \Delta \tilde{s} \right) \left( 1 - \frac{\alpha_s (q^2)}{\pi} \right) \]  
(166)

Note that this sum rule involves only the proton target.

We conclude that the sum rules given in Eqs. (163) and (166) are independent of unknown signet axial vector constant \( g_0^a \).

The sum rules (163), (164), and (165) provide means to extract \( \Delta \tilde{u} \), \( \Delta \tilde{d} \), and \( \Delta \tilde{s} \) from deep inelastic scattering without any put from the hypron \( \beta \)-decay. This would test the
consistency of electro-weak theory at low energy with that in the deep inelastic region. This will be possible only in the future experiments with the measurement of $A_L^p$.

Finally it is convenient to write the asymmetry $A_e$ given in Eq. (153) in terms of the following structure functions.

\[
F_2 = \frac{1}{2} (F_2^{ep} + F_2^{en})
\]

\[
\tilde{F}_2 = \frac{1}{2} (\tilde{F}_2^{ep} + \tilde{F}_2^{en})
\]

\[
\tilde{F}_3 = \frac{1}{2} (\tilde{F}_3^{ep} + \tilde{F}_3^{en})
\]

\[
F_{cc}^2 = \frac{1}{2} (F_{cc}^{ep} + F_{cc}^{en})
\]

\[
F_{cc}^3 = \frac{1}{2} (F_{cc}^{ep} + F_{cc}^{en})
\]

(167)

The following relations hold between these structure functions

\[
\tilde{F}_2 = 4 \frac{1}{4} F_{cc}^{2} - 2x_W F_2
\]

\[
\tilde{F}_3 = 4 \frac{1}{4} F_{cc}^{3}
\]

\[
F_{cc}^{2} - \frac{18}{5} F_2 = 4x \frac{1}{5} \left[ \frac{2}{\sqrt{3}} v_8 - \sqrt{\frac{2}{3}} v_0 \right]
\]

\[
\tilde{F}_2 = 4x \frac{1}{12} \left[ (3 - 4x_W) \frac{1}{\sqrt{3}} v_8 + (3 - 8x_W) \sqrt{\frac{2}{3}} v_0 \right]
\]

(168)

If

\[
v_0 = \sqrt{2} v_8,
\]

(169)

then

\[
\frac{F_2}{18 F_{cc}^{2}} = 1 \ (1.007 \pm 0.063)
\]

(170)

\[
\frac{\tilde{F}_2}{F_2} = \frac{9}{10} \left( 1 - \frac{20}{9} x_W \right)
\]

(171)

The experimental value given in Eq. (170) shows that the condition (169) is well satisfied experimentally. It verifies that charges of $u$ and $d$ valence quarks as their mean charges is $5/18$ i.e. there is no sea of strange quarks. From the experimental measurement of
asymmetry $A_e$ given in Eq. (153) at various values of $y$, $\bar{F}_2$ can be extracted as the leading contribution comes from the term containing $\bar{F}_2$ which is multiplied by a constant $a_e = -1$ independent of $y$. It would test Eq. (171) experimentally.

The electron proton scattering is a very direct means of probing the structure of proton. We have given a unified approach for the elastic and the highly inelastic scattering of electrons of proton target. In the case of elastic scattering, the structure functions reduce to the form factors which are functions of $q^2$. These form factors give the spatial distribution of charge and magnetization currents in the proton. The Rosenbluth extraction of the electromagnetic form factors $G_E$ and $G_M$ has been supplemented recently by measurements of recoil polarization of proton. These experiments indicate break down of electric and magnetic distribution at high $q^2$ in a proton. In this respect, we have pointed out in this article that the longitudinal spin-spin asymmetry $A_\parallel$ in the elastic scattering of polarized electrons of polarized protons may provide an additional probe of these form factors. In particular the asymmetry $A_\parallel$ at $\epsilon = 0$, would directly give the ratio $F_2/F_1$. The parity violating probe of the proton by scattering of polarized (unpolarized) electrons on unpolarized (polarized) protons would give additional information about the structure of proton both in the elastic and highly inelastic region. In this respect the measurements of the asymmetries $A_L^p$ and $A_e$ are of particular importance for the strange content of the proton. Moreover we have pointed out that the observation of recoil polarization of proton induced by electro-weak interference in the elastic scattering of electrons on proton target will be of interest for the structure of proton.

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Figure Captions

1. Scattering of electrons on nucleons

2. Plot of the longitudinal asymmetry $A_L^p$ versus $\epsilon$ [see Eq. (103)] for various values of $q^2$; Upper and lower figures correspond to the sets of parameters (i) and (ii) given in Eq. (98) and (99) respectively. Solid line for $q^2 = 0.5$, dashed line $q^2 = 1$, dotted line $q^2 = 2.64$, dash-dotted line $q^2 = 3.2$, dash-dot-dot line $q^2 = 5.6$, dash-dot-dot-dot line $q^2 = 10$.

3. Plot of the longitudinal asymmetry $A_e$ versus $\epsilon$ [see Eq. (104)], line description is same as Fig. 2.
4. Plot of the longitudinal recoil polarization of proton $I_L$ versus $\epsilon$ [see Eq. (105)], line description is same as Fig. 2.

5. Plot of the transverse recoil polarization of proton $I_T$ versus $\epsilon$ [see Eq. (106)], line description is same as Fig. 2.
Fig. 1 Electron–photon scattering
Fig. 2
Fig. 3
Fig. 4
Fig. 5