Gauge Transformation of Double Field Theory for Open String

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Abstract

I combine the symmetry structure of both the normal and transverse coordinates to construct the Dirac-Born-Infeld (DBI) theory. Normal coordinates are dual to the Neumann boundary conditions and transverse coordinates dual to the Dirichlet boundary conditions. The gauge transformation of the generalized metric is also governed by the generalized Lie derivative as the massless closed string field theory. The massless closed string field theory gives the $C$-bracket, but the DBI theory gives the $F$-bracket from the closed algebra. The $F$-bracket can be a $\alpha'$ deformation of the $C$-bracket. From the symmetry point of view, we can deduce the suitable action with non-zero flux in open string. From the equations of motion of the scalar dilaton, it offers the generalized scalar curvature. Finally, I show the classical equivalence on the double sigma model and normal sigma model for open string.

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1 Introduction

The most interesting topic of the M-theory is duality. In particular, the T-duality shows the equivalence of the dual theories by exchanging radius and inversive radius. It extends the structure of the original theory and gives the equivalence among the different kind of theories. We have two typical examples, the string field theory \[1\] and DBI model \[2\] (the low energy effective theory of an open string ending on a single D-brane). T-duality exchanges winding and momentum modes in the closed string field theory. For the open string, it exchange the Dirichlet and Neumann boundary conditions by T-duality.

The recent interesting development of brane theory is to construct an action from the equivalence of commutative and non-commutative description \[3,4\] as the DBI, non-abelian DBI and Nambu-Poisson M5 (NP M5) \[5\]. This equivalence gives a very strong constraint to restrict the form of the action. This theory is the generalized DBI model with the description of a q-brane ending on a p-brane \[6\]. They also propose Nambu-Sigma model for the generalized DBI model. Their relation is similar to the open string sigma model and DBI model. If we set \(q = 1\), the generalized DBI can go back to the DBI theory, and Nambu-Sigma model can also go back to the Poisson-Sigma model \[7\]. The most interesting case in this theory is three-from background, expanding up to the second order, we can get the same theory for the M5-brane \[8\]. More references of the generalized DBI related to the generalized metric can be found in \[9\] and the study of the dimensional reduction is in \[10\]. Other related supersymmetric extension and different formulation are in \[11\].

The other main developments is the NP M5-brane, which is the low energy effective theory of a single M5-brane on flat spacetime with constant C-field background in three dimensional spacial directions. The relation between the original M5-brane and NP-M5-brane is similar to the relation between the non-commutative D-brane and DBI theory. The NS-NS Dp and R-R Dp-brane \[12\] can also be obtained from dimensional reduction of the NP M5-brane. D-brane and RR fields play an important role to promote T-duality to U-duality. The related approaches are in \[13\] and recent progress of the U-duality are in \[14\]. These D-brane theories also provide T-duality and S-duality these evidences to the NP-M5-brane theory. Especially, the S-duality is the most non-trivial consistent check for such kind of theories.

Other related developments is the generalized geometry \[15\]. Generalized geometry
combines tangent and cotangent bundle together to form a generalized tangent bundle. Then one can use the generalized Lie derivative to define the gauge transformation of the background and field strength in D-brane theory without restriction to the static gauge \[16\]. The authors also combine the non-commutative gauge theory with the generalized geometry \[17\], and use the \(\beta\)-transformation and replace tangent with cotangent bundle to study R-flux \[18\]. The other related formulation is double field theory (DFT) \[19–21\]. It doubles the coordinates with manifest \(O(D,D)\) structure, and deduces a well-defined theory for the massless closed string field theory with the strong constraint. The most interesting understanding of double field theory is to reformulate the standard ten dimensional supergravity \[22, 23\] with “stringy geometry” \[22, 24\] for understanding the non-geometric flux (Q-flux and R-flux) \[25\]. One interesting non-geometric solution is \(5\,5\,2\) \[26\]. The worldvolume theory for this exotic brane theory \(5\,5\,2\) is already constructed \[27\], which is based on \[28\]. These new structure of field theory is also expected to inspire new gravity model as the quantum correction of string worldsheet theory gives gravity model \[29\]. Double field theory can also be understood as the generalized geometry by the strong constraint. Other interesting directions of double field theory are \(\alpha'\) geometry \[30\], exceptional field theory \[31\], D-brane \[32\], relaxation of the constraint \[33\] and others \[34\].

We can also see \[35\] for a recent review. The other interested understanding of generalized geometry are curvature, torsion \[36\], Courant algebra \[37\], reduction \[38\], exceptional generalized geometry \[39\] and supergravity \[40\].

In this paper I build the double field theory of the DBI theory based on the generalized geometry \[16\]. I generalize the gauge transformation of the generalized geometry to the gauge transformation of the double field theory. I implement the boundary condition of the double coordinates as in the case for the massless modes of closed string field theory, i.e., coordinates dual to momentum and winding modes, and impose the strong constraint as the consistency relation, which is equivalent to use \(\bar{\partial}^M = 0\) to show gauge invariance. Without the gauge field, I can obtain the same gauge transformation as the massless closed string field theory and define the same generalized metric of the DBI theory in a similar way to the massless closed string field theory. The difference on the DBI theory and the massless closed string field theory is the closed algebra. For the massless closed string field theory it is based on the \(C\)-bracket, but we get the so-called \(F\)-bracket for our D-brane theory. The two brackets differs by an exact one-form, which is the \(\alpha'\) effect. If we perform the B-transformation on the Courant bracket, we need the condition \(dB=0\) to
make sure automorphism. For the $F$-bracket with the strong constraint, this should imply that double field formulation of the DBI theory does not have the $O(D, D)$ structure with including the gauge field. Based on the symmetry structure guided by the $F$-bracket, I write down the possible action for the D-brane. It offers a non-trivial flux in the D-brane theory. This action also provides the generalized scalar curvature from the symmetry and equations of motion of the scalar dilaton. Finally, I also construct the the double sigma model which is equivalent to open string theory at the on-shell level.

The plan of this paper is to first review the gauge transformation of the double field theory for the massless closed string field theory in Sec. 2. Then I construct the double field theory of the DBI theory in Sec. 3. It includes the gauge transformation, bracket, action and the discussion of the Ricci scalar. I discuss double sigma model for open string in Sec. 4. Finally, I conclude and discuss in Sec. 5.

2 Review of the Gauge Transformation of the Double Field Theory for the Massless Closed String Field Theory

I first review the gauge transformation of the double field theory for the massless closed field theory in this section. At first, I introduce some convenient notations for the DFT and then write down the gauge symmetry for the generalized metric formulation [21].

2.1 Basics

Double field theory is defined on a theory with doubled coordinates. The normal coordinates are associated with the momentum mode and the other one coordinates are associated with the winding mode. The field components are the metric field $(g_{MN})$, antisymmetric field $(b_{MN})$ and scalar dilaton $(d)$. We have two constraints

$$\partial_M \tilde{\partial}^M (\text{field}) = 0, \quad \partial_M \tilde{\partial}^M (AB) = 0,$$

where

$$\partial_M = \frac{\partial}{\partial x^M}, \quad \tilde{\partial}^M = \frac{\partial}{\partial \tilde{x}_M}.$$
The index $M = 0, 1 \cdots, D - 1$. The constraints also imply
\[ \partial_M A \tilde{\partial}^M B + \tilde{\partial}^M A \partial_M B = 0. \quad (3) \]
We need the above constraints (strong constraints) to show gauge invariance up to the cubic order. If we only consider the first constraint, the constraint is called the weak constraint. The reason why we need strong constraint is due to
\[ \partial_M \tilde{\partial}^M \delta \text{(field)} \neq 0, \quad (4) \]
where $\delta$ is the gauge transformation. When we use the strong constraint, it can be annihilated. According to the manifest $O(D, D)$ structure, we can rewrite the weak constraint as
\[ \partial^A \partial_A \text{(field)} = 0, \quad (5) \]
where $\partial_A$ is defined by
\[ \partial_A \equiv \begin{pmatrix} \tilde{\partial}^M \\ \partial_M \end{pmatrix} \quad (6) \]
and $\partial^A = \eta^{AB} \partial_B$. The index $A = 0, 1 \cdots, 2D - 1$. We use $\eta$ to raise and lower the indices for the arbitrary $O(D, D)$ tensors
\[ h = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad h^t \eta h = \eta, \quad \eta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad (7) \]
where $a, b, c$ and $d$ are $D$ by $D$ matrices. We can also define $X^A$ to combine normal coordinates and dual coordinates by
\[ X^A \equiv \begin{pmatrix} \tilde{x}_M \\ x^M \end{pmatrix}. \quad (8) \]

\section*{2.2 Gauge Transformation}

In this section, we review the gauge transformation and introduce the generalized Lie derivative, $C$-bracket, and $D$-bracket for the generalized metric formulation.\cite{[20],[21]}. Finally, we show the Courant and Dorfman bracket can be obtained from the $C$- and $D$-brackets by using constraint.
The gauge transformation is
\[
\delta E_{MN} \equiv \delta (g + b)_{MN} = D_M \xi_N - \tilde{D}_N \xi_M + \xi^P \partial_P E_{MN} + D_M \xi^P E_{PN} + \tilde{D}_N \xi^P E_{MP},
\]
\[
\delta d = -\frac{1}{2} \partial_P \xi^P + \xi^P \partial_P d,
\]
where
\[
e^{-2d} = \sqrt{-\det g e^{-2\phi}},
\]
\[
D_M = \partial_M - \mathcal{E}_{MN} \tilde{\partial}^N, \quad \tilde{D}_M = \partial_M + \mathcal{E}^{NM} \tilde{\partial}_N,
\]
and \(\phi\) is the dilaton. We can use the above gauge transformation to show gauge invariant up to cubic order with the strong constraint. Then we first introduce the generalized metric \(\mathcal{H}_{MN}\).
\[
\mathcal{H} \equiv \mathcal{H}^{**},
\]
\[
\mathcal{H} = \begin{pmatrix} g - bg^{-1}b & bg^{-1} \\ -g^{-1}b & g^{-1} \end{pmatrix}.
\]
This matrix is a symmetric matrix with the \(O(D, D)\) symmetry,
\[
\mathcal{H} \eta \mathcal{H} = \eta.
\]
The inverse of \(\mathcal{H}\) is
\[
\mathcal{H}^{-1} = \eta \mathcal{H} \eta,
\]
\[
\mathcal{H}^{-1} \equiv \mathcal{H}^{**} = (\mathcal{H}^{AB})^{-1} = \begin{pmatrix} g^{-1} & -g^{-1}b \\ bg^{-1} & g - bg^{-1}b \end{pmatrix}.
\]
The gauge transformation of the generalized metric is
\[
\delta \xi \mathcal{H}^{AB} = \xi^P \partial_P \mathcal{H}^{AB} + (\partial^A \xi_C - \partial_C \xi^A) \mathcal{H}^{CB} + (\partial^B \xi_C - \partial_P \xi^B) \mathcal{H}^{AC},
\]
where
\[
\xi^A \equiv \begin{pmatrix} \xi_M \\ \xi_M \end{pmatrix}.
\]
Then we can define the generalized Lie derivative from the gauge transformation of the generalized metric
\[
\hat{\mathcal{L}}_\xi \mathcal{H}^{AB} \equiv \delta \xi \mathcal{H}^{AB},
\]
which satisfies Leibniz rule. The generalized Lie derivative acting on the constant metric \((\eta)\) is zero, but the ordinary Lie derivative is not. The gauge algebra is closed by assuming strong constraint.

\[
[\hat{\mathcal{L}}_{\xi_1}, \hat{\mathcal{L}}_{\xi_2}] = \hat{\mathcal{L}}_{[\xi_1, \xi_2]_C},
\]

(20)

where the \(C\)-bracket is defined by

\[
[\xi_1, \xi_2]_C^A = \xi_1^C \partial_C \xi_2^A - \xi_2^C \partial_C \xi_1^A - \frac{1}{2} \eta^{AC} \eta_{DE} \xi_1^D \partial_C \xi_2^E + \frac{1}{2} \eta^{AC} \eta_{DE} \xi_2^D \partial_C \xi_1^E.
\]

(21)

We define the \(D\)-bracket for the generalized vector by

\[
[A, B]_D = \hat{\mathcal{L}}_A B.
\]

(22)

The difference on the \(D\)-bracket and \(C\)-bracket is a total derivative term

\[
[A, B]_D^A = [A, B]_C^A + \frac{1}{2} \partial^A (B^C A_C).
\]

(23)

Finally, we assume all parameters are independent of \(\tilde{x}\) to get the Courant bracket from \(C\)-bracket [20].

\[
[\xi_1, \xi_2]_C^M = \xi_1^P \partial_P \xi_2^M - \xi_2^P \partial_P \xi_1^M = ([\mathcal{L}_{\xi_1}, \xi_2]^M = (([\xi_1, \xi_2])^M,
\]

\[
[\xi_1, \xi_2]_{CM} = \xi_1^P \partial_P \tilde{\xi}_{2M} - \xi_2^P \partial_P \tilde{\xi}_{1M} - \frac{1}{2} (\xi_1^P \partial_M \tilde{\xi}_{2P} - \tilde{\xi}_{2P} \partial_M \xi_1^P) + \frac{1}{2} (\xi_2^P \partial_M \tilde{\xi}_{1P} - \tilde{\xi}_{1P} \partial_M \xi_2^P)
\]

\[
= \xi_1^P \partial_P \tilde{\xi}_{2M} - \xi_2^P \partial_P \tilde{\xi}_{1M} + (\partial_M \xi_1^P) \tilde{\xi}_{2P} - (\partial_M \xi_2^P) \tilde{\xi}_{1P}
\]

\[
\left(\mathcal{L}_{\xi_1} \tilde{\xi}_2 - \frac{1}{2} d(i_{\xi_1} \tilde{\xi}_2)\right)_M - \left(\mathcal{L}_{\xi_2} \tilde{\xi}_1 - \frac{1}{2} d(i_{\xi_2} \tilde{\xi}_1)\right)_M.
\]

(24)

It is exactly the same as

\[
[A + \alpha, B + \beta]_{\text{Cour}} = [A, B] + \mathcal{L}_A \beta - \mathcal{L}_B \alpha - \frac{1}{2} d(i_A \beta - i_B \alpha),
\]

(25)

where \(A, B\) are vectors, and \(\alpha, \beta\) are one-form. Similarly, we can also obtain the Dorfman bracket [15] from the \(D\)-bracket.

\[
[A + \alpha, B + \beta]_{\text{Dorf}} = [A, B] + \mathcal{L}_A \beta - i_B \alpha.
\]

(26)

For the consistent notation, we express the Dorfman bracket instead of the conventional way \((A + \alpha) \circ (B + \beta)\). The \(D\)-bracket has the Jacobi identity

\[
[A, [B, C]_D]_D = [[A, B]_D, C]_D + [B, [A, C]_D]_D,
\]

(27)

but it is not antisymmetric. For the \(C\)-bracket, it does not satisfy the Jacobi identity, but it is antisymmetric. In other words, the \(C\) and \(D\)-bracket are not Lie bracket.
3 Double Field Theory of the DBI Model

In this section, I first introduce our set up and notation for double field theory of the DBI model. Then I write down the gauge transformation without doubled coordinates. I also provide the proof to show the gauge invariance. Finally, I construct the gauge transformation in double field theory and define the $F$-bracket from the closed algebra. I also perform B-transformation on the $F$-bracket with strong constraint to compare with the Courant bracket.

3.1 Set Up and Notation

I define our notation for double field theory of the DBI model in this section. I construct double field theory of the DBI model by combining normal coordinates with transverse coordinates. Normal coordinates are associated with the Neumann boundary conditions. Transverse coordinates are associated with the Dirichlet boundary conditions. If I exchange coordinates, it is equivalent to exchange boundary conditions.

$$
\Lambda^M \equiv \left( \begin{array}{c} \epsilon^i \\ \Lambda_a \end{array} \right), \quad \epsilon^M \equiv \left( \begin{array}{c} \Lambda_i \\ \epsilon^a \end{array} \right), \quad A_M \equiv \left( \begin{array}{c} \phi^j \\ A_a \end{array} \right), \quad \partial_M \equiv \left( \begin{array}{c} \tilde{\partial}^i \\ \partial_a \end{array} \right), \quad \tilde{\partial}^M \equiv \left( \begin{array}{c} \partial_i \\ \tilde{\partial}^a \end{array} \right),
$$

(28)

where the index $a = 0, 1, \ldots, p$ and $i = (p + 1), (p + 2), \ldots, (D - 1)$ for D(D-1) brane theory. If I perform the T-duality on $i$ these directions, I need to use

$$
\tilde{\partial}^M(\text{field}) = 0, \quad \tilde{\partial}^i(\text{field}) = 0
$$

(29)

and

$$
\mathcal{E}'(X') = (a\mathcal{E}(X) + b)(c\mathcal{E}(X) + d)^{-1}, \quad d'(X') = d(X), \quad X' = h X, \quad \mathcal{E} \equiv g + B,
$$

(30)

where

$$
h = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad h^t \eta h = \eta, \quad \eta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix},
$$

(31)

and $B$ is the NS-NS two-form background. From $\mathcal{E}'(X') = (a\mathcal{E}(X) + b)(c\mathcal{E}(X) + d)^{-1}$ with the particular choice of $O(D, D)$ element $(h)$, I can get the Buscher’s rule. The
convention for $X$ is
\[ X^M = \left( \tilde{X}_i \right), \quad \tilde{X}_M = \left( X_i \right), \quad X^A = \left( \tilde{X}^M \right) \equiv X. \quad (32) \]
We use $\eta$ to define $X_A = \eta_{AB}X^B$. From the above set up, we can show that the exchange of coordinates is equivalent to perform the T-duality. I define the field strength
\[ F_{MN} \equiv \partial_M A_N - \partial_N A_M. \quad (33) \]
It combines the gauge fields with the scalar fields. If I exchange one normal coordinate and one transverse coordinate, it is analogous to perform dimensional reduction on one direction. I define a new element related to the $E$. I wish to define this new element based on the T-dual operation.
\[ t_{ab} \equiv E_{ab} - E_{ak}E^{kl}E_{lb}, \quad t^i_b \equiv E^{ik}E_{kb}, \quad t^i_a \equiv -E_{ak}E^{kj}, \quad t^{ij} \equiv E^{ij}. \quad (34) \]
We use
\[ t_{MN} = \begin{pmatrix} t^{ij} & t^i_b \\ t^i_a & t_{ab} \end{pmatrix} \quad (35) \]
to combine all new elements. If I consider the D($D$)-brane theory, we can obtain $t_{MN} = \mathcal{E}_{MN}$. For convenience, I also define $t_{MN} \equiv s_{MN} + a_{MN}$, where $s \equiv \frac{t_{MN} + t_{NP}}{2}$ and $a \equiv \frac{t_{MN} - t_{NP}}{2}$, and use
\[ t'(X') = (at(X) + b)(ct(X) + d)^{-1} \quad (36) \]
with a particular $O(D, D)$ element $(h)$ by replacing $t$ with $\mathcal{E}$ to obtain the Buscher’s rule.

### 3.2 Gauge Transformation

I first write the gauge transformation of the DBI theory.
\[
\begin{align*}
\delta t_{MN} & \equiv \partial_M \Lambda_N - \partial_N \Lambda_M + \mathcal{L}_t t_{MN} + \mathcal{L}_\Lambda t_{MN} + t_{MQ}(\tilde{\partial}^P \epsilon^Q - \tilde{\partial}^Q \epsilon^P)t_{PN}, \\
\delta s_{MN} & \equiv \mathcal{L}_s s_{MN} + \mathcal{L}_\Lambda s_{MN} + (s_{MQ}a_{NP} + s_{NQ}a_{MP})(\tilde{\partial}^Q \epsilon^P - \tilde{\partial}^P \epsilon^Q), \\
\delta a_{MN} & \equiv \partial_M \Lambda_N - \partial_N \Lambda_M + \mathcal{L}_a a_{MN} + \mathcal{L}_\Lambda a_{MN} - s_{MP}(\tilde{\partial}^P \epsilon^Q - \tilde{\partial}^Q \epsilon^P)s_{QN} - a_{MP}(\tilde{\partial}^P \epsilon^Q - \tilde{\partial}^Q \epsilon^P)a_{QN}.
\end{align*}
\quad (37)
\]
where
\[
\mathcal{L}_c t_{MN} = \epsilon^Q \partial_Q t_{MN} + (\partial_M \epsilon^Q) t_{QN} + t_{MQ} \partial_N \epsilon^Q, \quad \mathcal{L}_\Lambda t_{MN} = \Lambda_Q \bar{\partial}^Q t_{MN} + (\bar{\partial}^Q \Lambda_M) t_{NQ} - \bar{\partial}^Q \Lambda_N t_{MQ}.
\] (38)

I define the gauge transformation of the field strength
\[
\delta F_{MN} = \partial_M \Lambda_N - \partial_N \Lambda_M + \mathcal{L}_c F_{MN}.
\] (39)

From the gauge transformation of the field strength, I can also define the gauge transformation of the one-form gauge field. I will discuss this gauge transformation when we show the closed algebra. Now I use the above gauge transformation with \(\bar{\partial}^M = 0\) to show the DBI theory is gauge invariant. At first, I use some useful matrix identity to rewrite the DBI action for convenience to show gauge invariance. Now I decompose \(t\) as
\[
t = \begin{pmatrix}
delta_{ac} & -\epsilon_{ak} \epsilon^{kl} \\
0 & \epsilon^{il} \\
\end{pmatrix} \begin{pmatrix}
\epsilon_{cb} & 0 \\
\epsilon_{ib} & \delta_{lj} \\
\end{pmatrix} = \begin{pmatrix}
delta_{ac} & \epsilon_{ak} \\
0 & \epsilon_{ik} \\
\end{pmatrix}^{-1} \begin{pmatrix}
\epsilon_{cb} & 0 \\
\epsilon_{kb} & \delta_{kj} \\
\end{pmatrix}.
\] (40)

I define \(m \equiv \begin{pmatrix}
delta_{ac} & \epsilon_{ak} \\
0 & \epsilon_{ik} \\
\end{pmatrix}\) and \(n \equiv \begin{pmatrix}
\epsilon_{cb} & 0 \\
\epsilon_{kb} & \delta_{kj} \\
\end{pmatrix}\). Then I can obtain
\[
s = \frac{1}{2} \left( m^{-1} n + n^t (m^{-1})^t \right) = \frac{1}{2} m^{-1} (nm^t + mn^t) (m^{-1})^t.\] (41)

I also find
\[
mn^t = \begin{pmatrix}
\epsilon_{ac} & 2g_{ak} \\
0 & \epsilon_{ik} \\
\end{pmatrix}, \quad nm^t = \begin{pmatrix}
\epsilon_{ca} & 0 \\
2g_{ka} & \epsilon_{ki} \\
\end{pmatrix}.
\] (42)

Then I can show
\[
s = m^{-1} g (m^{-1})^t.\] (43)

It simultaneously implies
\[
det s = (\det m)^{-2} \det g = (\det t^{ij})^2 \det g.\] (44)
Now I decompose $\det t$ as

$$
\det t = \left( \det(s + a) \det(s + a) \right)^{\frac{1}{2}}
$$

$$
= \left( \det s \det(1 + s^{-1}a) \det(1 + as^{-1}) \det s \right)^{\frac{1}{2}}
$$

$$
= \det s \left( \det(1 + s^{-1}a) \det(1 - s^{-1}a) \right)^{\frac{1}{2}}
$$

$$
= \det s \left( \det(1 - s^{-1}as^{-1}a) \right)^{\frac{1}{2}}
$$

$$
= \left( \det s \right)^{\frac{1}{2}} \left( \det(s - as^{-1}a) \right)^{\frac{1}{2}}.
$$

(45)

For $t_F = t - F$, I can get the similar result.

$$
\det t_F = \left( \det s \right)^{\frac{1}{2}} \left[ \det \left( s - (a - F)s^{-1}(a - F) \right) \right]^{\frac{1}{2}}.
$$

(46)

By using

$$
\det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det \left( A - BD^{-1}C \right) \det D,
$$

we obtain

$$
\det t_F = \det \mathcal{E}^{ij} \det \left( P(\mathcal{E}) - F \right) = \det t^{ij} \det \left( P(\mathcal{E}) - F \right),
$$

(48)

where

$$
P(\mathcal{E})_{ab} \equiv \mathcal{E}_{ab} + \partial_a \phi^k \mathcal{E}_{kb} + \mathcal{E}_{ak} \partial_b \phi^k + \partial_a \phi^i \mathcal{E}_{ij} \partial_b \phi^j.
$$

(49)

Then I can get

$$
\left[ - \det \left( P(\mathcal{E}) - F \right) \right]^{\frac{1}{2}} = \left( - \det t_F \right)^{\frac{1}{2}} \frac{1}{\left( \det t^{ij} \right)^{\frac{1}{2}}}
$$

$$
= \left( - \det s \right)^{\frac{1}{2}} \frac{1}{\left( \det t^{ij} \right)^{\frac{1}{2}}} \left[ \det \left( s - (a - F)s^{-1}(a - F) \right) \right]^{\frac{1}{2}}
$$

$$
= \left( - \det g \right)^{\frac{1}{2}} \left[ \det \left( s - (a - F)s^{-1}(a - F) \right) \right]^{\frac{1}{2}}.
$$

(50)
Then I calculate the gauge transformation of det $g$.

$$\delta (\text{det} \, g) = (\text{det} \, g) g^{-1} \delta g = (\text{det} \, g) g^{ab} \left( \epsilon^c \partial_c g_{ab} + \partial_a \epsilon^c g_{cb} + \partial_b \epsilon^c g_{ca} \right) = \epsilon^c \partial_c \text{det} \, g + 2 \partial_c \epsilon^c \text{det} \, g. \quad (51)$$

Similarly I can also get

$$\delta \left[ \text{det} \left( s - (a - F) s^{-1} (a - F) \right) \right] = \epsilon^c \partial_c \left[ \text{det} \left( s - (a - F) s^{-1} (a - F) \right) \right]$$

$$+ 2 \partial_c \epsilon^c \left[ \text{det} \left( s - (a - F) s^{-1} (a - F) \right) \right]. \quad (52)$$

It is easy to imply

$$\delta \left[ \left( - \text{det} \, g \right)^{\frac{1}{2}} \right] = \epsilon^c \partial_c \left( - \text{det} \, g \right)^{\frac{1}{2}} + \frac{1}{2} \partial_c \epsilon^c \left( - \text{det} \, g \right)^{\frac{1}{2}},$$

$$\delta \left[ \text{det} \left( s - (a - F) s^{-1} (a - F) \right)^{\frac{1}{2}} \right] = \epsilon^c \partial_c \text{det} \left( s - (a - F) s^{-1} (a - F) \right)^{\frac{1}{2}}$$

$$+ 1 \partial_c \epsilon^c \text{det} \left( s - (a - F) s^{-1} (a - F) \right)^{\frac{1}{2}}. \quad (53)$$

Then I can use the above gauge transformation to show

$$\delta \left[ - \text{det} \left( P(E) - F \right) \right]^{\frac{1}{2}} = \partial_c \left\{ \epsilon^c \left[ - \text{det} \left( P(E) - F \right) \right]^{\frac{1}{2}} \right\}. \quad (54)$$

Because the gauge transformation of the dilation is $\delta \phi = \epsilon^c \partial_c \phi$, I only have a boundary term that we can integrate out to show gauge invariance on the DBI action. Double field theory needs strong constraint to show gauge invariance so I already show this gauge transformation is also a symmetry for the double field theory. It is also easy to deduce the gauge transformation of the scalar dilaton

$$\delta d = \epsilon^M \partial_M d - \frac{1}{2} \partial_M \epsilon^M. \quad (55)$$
I can also rewrite this theory by using \( d \), \( t \) and \( F \). Then the Lagrange is

\[
e^{-2d} \left( - \det(t - F) \right)^{\frac{1}{2}} \left( - \det t^{ij} \right)^{\frac{1}{2}} \frac{1}{(- \det \frac{t^{ij} + t_{ij}}{2})^{\frac{1}{2}}}.
\] (56)

But this Lagrange does not have the \( O(D, D) \) structure. Later I will discuss more about it when I check closed algebra. However this setup is based on the generalized geometry [16]. I provide a way to extend the generalized geometry to the double field theory.

### 3.3 Bracket

In this section, I discuss what kind bracket that appears in the double field of the D-brane. Before we can calculate the closed algebra to obtain the Courant bracket. Then it can be generalized to the \( C \)-bracket. Now we use the same logic to show this thing. Because the gauge transformation of D-brane without the one-form gauge field is the same as the gauge transformation of the closed string theory, we still have the the Courant bracket in this theory. If I include gauge field, I will obtain the \( F \)-bracket from the closed algebra. Let me to show the calculation now.

\[
\delta_2 A_M = \Lambda_{2M} + \epsilon_2^N F_{NM},
\]

\[
\delta_1 \delta_2 A_M = \epsilon_2^N \delta_1 F_{NM}
\]

\[
= \epsilon_2^N \left( \partial_N \Lambda_{1M} - \partial_M \Lambda_{1N} + \epsilon_1^P \partial_P F_{NM} + (\partial_N \epsilon_1^P) F_{PM} + (\partial_M \epsilon_1^P) F_{NP} \right),
\]

\[
[\delta_1, \delta_2] A_M = \epsilon_2^N \left( \partial_N \Lambda_{1M} - \partial_M \Lambda_{1N} \right) - \epsilon_1^N \left( \partial_N \Lambda_{2M} - \partial_M \Lambda_{2N} \right)
\]

\[
+ \epsilon_2^N \epsilon_1^P \partial_P F_{NM} + \epsilon_2^N (\partial_N \epsilon_1^P) F_{PM} + \epsilon_2^N (\partial_M \epsilon_1^P) F_{NP}
\]

\[
- \epsilon_1^N \epsilon_2^P \partial_P F_{NM} - \epsilon_1^N (\partial_N \epsilon_2^P) F_{PM} - \epsilon_1^N (\partial_M \epsilon_2^P) F_{NP},
\]

\[
\epsilon_2^N \epsilon_1^P \partial_P F_{NM} - \epsilon_1^N \epsilon_2^P \partial_P F_{NM} = \epsilon_2^N \epsilon_1^P \left( \partial_P F_{NM} - \partial_N F_{PM} \right)
\]

\[
= \epsilon_2^N \epsilon_1^P \left( - \partial_P \partial_M A_N + \partial_N \partial_M A_P \right)
\]

\[
= \epsilon_2^N \epsilon_1^P \partial_M F_{NP}.
\] (57)
It shows that I can have
\[ \epsilon' M = \epsilon_1^N \partial_N \epsilon_2^M - \epsilon_2^N \partial_N \epsilon_1^M, \]
\[ A'_M = \epsilon_1^N \left( \partial_N \Lambda_{2M} - \partial_M \Lambda_{2N} \right) - \epsilon_2^N \left( \partial_N \Lambda_{1M} - \partial_M \Lambda_{1N} \right) - \partial_M \left( \epsilon_2^N \epsilon_1^P F_{NP} \right), \]
\[ [\delta_1, \delta_2] A_M = -\delta' A_M. \] (58)

This calculation shows that the closed algebra parameters contain gauge field. We usually do not wish this situation to occur because I also need to doubled gauge fields. If we doubled gauge fields, it is not the double field theory that we consider. We only doubled coordinates in the double field theory. For removing this field dependence, we need to redefine the gauge transformation of the gauge field without changing the gauge transformation of the field strength.

\[ \delta A_M = \Lambda_M + \partial_M (\epsilon^N A_N) + \epsilon^N F_{NM} \]
\[ = \Lambda_M + \mathcal{L}_\epsilon A_M. \] (59)

Then I can get
\[ \epsilon'^M = \epsilon_1^N \partial_N \epsilon_2^M - \epsilon_2^N \partial_N \epsilon_1^M, \]
\[ A'_M = \epsilon_1^N \partial_N \Lambda_{2M} + (\partial_M \epsilon_1^N) \Lambda_{2N} - \epsilon_2^N \partial_N \Lambda_{1M} - (\partial_M \epsilon_2^N) \Lambda_{1N} \]
\[ = \mathcal{L}_\epsilon \Lambda_{2M} - \mathcal{L}_\epsilon \Lambda_{1M}. \] (60)

I also define a new bracket from this closed algebra.
\[ [\xi_1, \xi_2]_F^A = \left( \xi_1^D \partial_D \xi_2^A - \xi_2^D \partial_D \xi_1^A \right) - \frac{1}{2} \left( \xi_1^D \partial^A \xi_2^D - \xi_2^D \partial^A \xi_1^D \right) - \frac{1}{2} \partial^A \left( \xi_2^D Z^D E \xi_1^E \right), \] (61)
where
\[ Z = Z^A_B = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \] (62)

I note that \( Z \) is not an \( O(D, D) \) matrix. But I still use \( \eta \) to raise or lower index. It is easy to deduce
\[ [\delta_1, \delta_2] = -\delta_{[\xi_1, \xi_2]} F. \] (63)
If I do not consider double field theory, the gauge transformation of the gauge field has ambiguity. For considering double field theory, this ambiguity will be removed. It also implies that double field theory has more constraints to restrict a theory to construct the action and find the gauge transformation easily. Let us to know more properties of this new bracket. I want to know that the property of automorphism exists whether or not after I perform B-transformation. B-transformation is
\[
e^B \equiv \begin{pmatrix} 1 & 0 \\ B & 1 \end{pmatrix}, \quad e^B \left( \begin{pmatrix} X \\ \xi \end{pmatrix} \right) = \begin{pmatrix} X \\ \xi + BX \end{pmatrix} = \begin{pmatrix} X \\ \xi + iX B \end{pmatrix}.
\] (64)

This transformation is also a symmetry of the sigma model. I first calculate the Courant bracket
\[
[e^B(X + \xi), e^B(Y + \eta)]_{\text{Cour}} = [X + \xi + iXB, Y + \eta + iYB]_{\text{Cour}}
\]
\[
= [X + \xi, Y + \eta]_{\text{Cour}} + [X, iYB]_{\text{Cour}} + [iXB, Y]_{\text{Cour}}
\]
\[
= [X + \xi, Y + \eta]_{\text{Cour}} + \mathcal{L}_{X}iYB - \frac{1}{2} diXiYB - \mathcal{L}_{Y}iXB + \frac{1}{2} diYiXB
\]
\[
= [X + \xi, Y + \eta]_{\text{Cour}} + i[X,Y]B + iYiXB + iYiXdB
\]
\[
= e^B \left( [X + \xi, Y + \eta]_{\text{Cour}} \right) + iYiXdB.
\] (65)

If \( dB = 0 \), I can get automorphism after I use the B-transformation. It shows that this theory can define a \( H \)-flux (\( dH = 0 \)) and possibly be extended to the \( O(D, D) \). For the closed string theory, we already use the \( O(D, D) \) to represent this theory with the \( H \)-flux. For the D-brane theory without the gauge field, we should have the same understanding. However we include the gauge field to obtain the \( F \)-bracket. Before we check automorphism, we define the notation for the \( F \)-bracket with strong constraint
\[
[X + \xi, Y + \eta]_F = [X, Y] + \mathcal{L}_X\eta - \mathcal{L}_Y\xi.
\] (66)

Then I examine automorphism.
\[
[e^B(X + \xi), e^B(Y + \eta)]_F = [X + \xi + iXB, Y + \eta + iYB]_F
\]
\[
= [X + \xi, Y + \eta]_F + [X, iYB]_F + [iXB, Y]_F
\]
\[
= [X + \xi, Y + \eta]_F + \mathcal{L}_X iYB - \mathcal{L}_Y iXB
\]
\[
= [X + \xi, Y + \eta]_F + i[X,Y]B + iYiXB + diYiXB
\]
\[
= e^B \left( [X + \xi, Y + \eta]_F \right) + iYiXdB - diYiXB.
\] (67)
Then we cannot only use $dB = 0$ to show automorphism. It indicates that we lose the $O(D, D)$ structure if we still wish to include the one-form gauge field in this theory. I summarize the observation. If I include the gauge field in our theory, the action cannot be represented by the $O(D, D)$ structure. If I only consider background, I can still define a theory with the $O(D, D)$ structure. It should be a reason to explain that I cannot write DBI part in terms of the pure $O(D, D)$ element similar to the generalized metric formulation of the double field theory \cite{21}. I separate two parts to discuss the action. The first part is the DBI part that I already mentioned. The other one part is that I only discuss the background fields without involving gauge field up to two derivative. Now I discuss how to formulate this part. It is also interesting to find the form of the action with two derivative terms are uniquely determined. I write this part based on the $O(D, D)$ structure, $\mathbb{Z}_2$ symmetry, gauge symmetry with the strong constraint and two derivative. I first consider the $\mathbb{Z}_2$ symmetry.

$$B_{MN} \to -B_{MN}, \quad \tilde{\partial}^M \to -\tilde{\partial}^M.$$  
(68)

This implies

$$E_{MN} \to E_{NM}.$$  
(69)

We can rewrite $\tilde{\partial}^M \to -\tilde{\partial}^M$ as

$$\partial_A \to Z \partial_A.$$  
(70)

The off-diagonal matrices of the $H^{AB}$ change sign under the transformation $B_{MN} \to -B_{MN}$. It shows

$$H^{AB} \to Z H^{AB} Z, \quad H_{AB} \to Z H_{AB} Z.$$  
(71)

Then I can construct the action with respect to the gauge symmetry (with the strong constraint) from the all possible $O(D, D)$ elements ($\partial_A$, $H^{AB}$, $H_{AB}$ and $d$) within the third order up to a boundary term. The action is

$$S_2 = \int dx \, d\bar{x} \, e^{-2d} \left( \frac{1}{8} H^{AB} \partial_A H^{CD} \partial_B H_{CD} - \frac{1}{2} H^{AB} \partial_B H^{CD} \partial_D H_{AC} ight. \
- 2 \partial_A d \partial_B H^{AB} + 4 H^{AB} \partial_A d \partial_B d \bigg).$$  
(72)

This action is uniquely determined from the above symmetry. The action of the DBI part is

$$S_1 = \int dx \, d\bar{x} \, e^{-2d} \left( - \det(t - F) \right)^{1/2} \left( - \det t^{ij} \right)^{1/2} \frac{1}{(- \det \frac{t^{ij} + t_{ij}}{2})^{1/2}}.$$  
(73)
The total action is

\[ S = S_1 + \alpha S_2 \]

\[ = \int dx \, d\tilde{x} \, e^{-2\phi} \left[ \left( -\det(t - F) \right)^{\frac{1}{2}} \left( -\det t^{ij} \right)^{\frac{1}{2}} \frac{1}{(-\det \frac{t + t^T}{2})^{\frac{1}{2}}} \right. \]

\[ + \alpha \left( \frac{1}{8} \mathcal{H}^{AB} \partial_A \mathcal{H}^{CD} \partial_B \mathcal{H}_{CD} - \frac{1}{2} \mathcal{H}^{AB} \partial_B \mathcal{H}^{CD} \partial_D \mathcal{H}_{AC} \right. \]

\[ - 2 \partial_A d \partial_B H^{AB} + 4 \mathcal{H}^{AB} \partial_A d \partial_B d \right), \]

where \( \alpha \) is an arbitrary constant. If I use \( \tilde{\partial}^M = 0 \), I can obtain

\[ \int dx \, \sqrt{-\det g} e^{-2\phi} \left[ \left( -\det(g + B - F) \right)^{\frac{1}{2}} \left( -\det g \right)^{-\frac{1}{2}} + \alpha \left( R + 4(\partial \phi)^2 - \frac{1}{12} H^2 \right) \right] \]

(74)

for D\((D-1)\)-brane theory with the nontrivial background, \( R \) is the Ricci scalar and \( H = dB \) is the three form field strength. Now I define the generalized scalar curvature in this double field theory based on the symmetry and equations of motion of the scalar dilaton.

\[ \mathcal{R} \equiv \left( -\det(t - F) \right)^{\frac{1}{2}} \left( -\det t^{ij} \right)^{\frac{1}{2}} \frac{1}{(-\det \frac{t + t^T}{2})^{\frac{1}{2}}} \]

\[ + \alpha \left( 4 \mathcal{H}^{AB} \partial_A \partial_B d - \partial_A \partial_B H^{AB} - 4 \mathcal{H}^{AB} \partial_A d \partial_B d + 4 \partial_A H^{AB} \partial_B d 

\[ + \frac{1}{8} \mathcal{H}^{AB} \partial_A \mathcal{H}^{CD} \partial_B \mathcal{H}_{CD} - \frac{1}{2} \mathcal{H}^{AB} \partial_C \partial_D \mathcal{H}_{BD} \right). \]

(76)

It satisfies

\[ \delta_\xi \mathcal{R} = \xi^A \partial_A \mathcal{R} \]

(77)

with \( \tilde{\partial}^M = 0 \). In the final, I discuss about the \( \alpha' \) correction of the bracket. If I do not include the one-form gauge field, the \( C \)-bracket appears in the closed algebra. The \( \alpha' \) correction of the flat D-brane only comes from the gauge field. I can reinterpret our bracket by

\[ [\xi_1, \xi_2]^A_F = [\xi_1, \xi_2]^A_C - \frac{1}{2} \partial^A \left( \xi_2 D Z^D e \xi_1^E \right). \]

(78)
The difference on the $F$-bracket and $C$-bracket is $-\frac{1}{2} \partial^A \left( \xi^2 D^E Z^F E \xi^1 \right)$. If we use $\tilde{\partial} M = 0$, the difference is an exact one form. From this construction, we also easily understand the candidate of the suitable gauge transformation should be different from the Courant bracket by an exact one form. It also explains why we never use the twisted Courant bracket $([X + \xi, Y + \eta]_{\text{twist}} \equiv [X + \xi, Y + \eta]_{\text{Cour}} + i_Y i_X H)$ to be the gauge transformation of the massless closed string theory.

## 4 Double Sigma Model of Open String

In this section, I show the discussion of the sigma model for open string. I start from

$$S_{\text{bulk}} = -\frac{1}{2} \int d^2 \sigma \left( - \partial_1 X^A \mathcal{H}_{AB} \partial_1 X^B + \partial_1 X^A \eta_{AB} \partial_0 X^B \right).$$

(79)

I will show that (79) gives the same equations of motion as the sigma model of open string for the flat worldsheet metric with $(-, +)$ signature on the bulk. The equations of motion of (79) is

$$\partial_1 \left( \mathcal{H}_{AB} \partial_1 X^B - \eta_{AB} \partial_0 X^B \right) = \frac{1}{2} \partial_1 X^B \partial_A \mathcal{H}_{BC} \partial_1 X^C.$$  

(80)

For showing the equivalence, we use strong constraint $\tilde{\partial} M = 0$. Then I use

$$\mathcal{H}^M B \partial_1 X^B - \eta^M B \partial_0 X^B = 0$$  

(81)

to remove the half degrees of freedom. It is equivalent to

$$g^{-1} \partial_1 \tilde{X} - g^{-1} B \partial_1 X - \partial_0 X = 0.$$  

(82)

For convenience, I rewrite it as

$$\partial_1 \tilde{X} = B \partial_1 X + g \partial_0 X.$$  

(83)

I discuss the gauge transformation of this double sigma model. The gauge transformation of $X$ is governed by the generalized Lie derivative. The generalized Lie derivative is

$$\mathcal{L}_\xi V^A = \xi^C \partial_C V^A + (\partial^A \xi_C - \partial_C \xi^A) V^C.$$  

(84)
The gauge transformation of the background is
\[ \delta g_{MN} = \mathcal{L}_{\epsilon} g_{MN}, \]
\[ \delta B_{MN} = \partial_M \Lambda_N - \partial_N \Lambda_M + \mathcal{L}_{\epsilon} B_{MN} \] (85)
with \( \tilde{\partial}^M = 0 \). The gauge parameters do not depend on worldsheet coordinates. Then I can find (81) is covariant under the gauge transformation with \( \tilde{\partial}^M = 0 \). It implies that we do not need to modify (81) if I include the one-form gauge field. Then I substitute (81) to the other one equations of motion.

\[ \partial_1 \left( \mathcal{H}_{MB} \partial_1 X^B - \eta_{MB} \partial_0 X^B \right) \]
\[ = \partial_1 \left( B g^{-1} \partial_1 \tilde{X} + (g - B g^{-1} B) \partial_1 X - \partial_0 \tilde{X} \right)_M \]
\[ = \partial_1 \left( B g^{-1} \partial_1 \tilde{X} + (g - B g^{-1} B) \partial_1 X \right)_M - \partial_0 (g \partial_0 X + B \partial_1 X)_M \]
\[ = \partial_1 (g \partial_1 X + B \partial_0 X)_M - \partial_0 (g \partial_0 X + B \partial_1 X)_M. \] (86)

\[ \frac{1}{2} \partial_1 X^B \partial_M \mathcal{H}_{BC} \partial_1 X^C \]
\[ = \frac{1}{2} \partial_1 \tilde{X} \partial_M g^{-1} \partial_1 X + \partial_1 X \partial_M (B g^{-1}) \partial_1 X + \frac{1}{2} \partial_1 X \partial_M (g - B g^{-1} B) \partial_1 X \]
\[ = -\frac{1}{2} \partial_0 X \partial_M g \partial_0 X + \frac{1}{2} \partial_1 X \partial_M g \partial_1 X + \partial_1 X \partial_M B \partial_0 X. \] (87)

Combining (86) and (87) to show the same equations of motion as
\[ \frac{1}{2} \int d^2 \sigma \left( \partial_\alpha X^M g_{MN} \partial_\beta X^N - \epsilon^{\alpha \beta} \partial_\alpha X^M B_{MN} \partial_\beta X^N \right). \] (88)

If I impose the Neumann boundary condition on \( \alpha = 1 \) this direction and consider the constant background on the double sigma model, I should put the boundary term
\[ S_{\text{boundary}} = -\int d\sigma^0 A_M \partial_0 X^M \] (89)
to guarantee the gauge invariance and boundary condition. This boundary term breaks \( O(D, D) \) structure. It is consistent with my previous statement. Because the DBI theory on the constant background comes from the fluctuation of the gauge field, we cannot write the DBI theory in terms of the \( O(D, D) \) elements. However the above double sigma model already has the classical equivalence. It should be interesting to calculate the one-loop \( \beta \) function to get the DBI theory from quantum fluctuation.
5 Conclusion

I construct the double field theory of the DBI theory and sigma model of open string. The construction of the DBI theory is based on the generalized geometry. It is interesting to understand the difference on $\alpha'$ or the effect of the one-form gauge field. It is equivalent to compare the $C$-bracket with $F$-bracket. Then the $F$-bracket implies the DBI action cannot be written down by the $O(D, D)$ elements. From the symmetry, I also write down the suitable form for the action with nontrivial flux. It should be interesting to study the non-geometric flux in D-brane theory. The study of non-geometric flux should be similar to the closed string theory. It should also be interesting to consider the non-commutative geometry on this new action. It should not be the Moyal product for the non-constant background. Finding a non-commutative structure on this D-brane should inspire us a new physics to us. For the double sigma model, I find a boundary term or the gauge field that breaks the $O(D, D)$ structure. All of these give a consistent understanding. I also show the generalized scalar curvature based on symmetry and the equations of motion of the scalar dilaton. This generalized scalar curvature contains the DBI part and the non-trivial background part. Double sigma model provides a strong evidence on the bulk with non-constant background by the classical equivalence. I only need to put the boundary term to get Neumann boundary condition and gauge invariance without modifying the other relations.

The study of the D-brane should inspire us to construct the M5-brane theory. Double field theory provides more constraints for us to construct the action. For the D-brane theory, I use the gauge symmetry with the strong constraint, $Z_2$ symmetry and manifest $O(D, D)$ structure to find the non-constant background term. Then it is the unique action. For the M5-brane theory, we should also have the similar conditions to construct this theory. It is also interesting to understand the non-geometric flux on the M5-brane.

From the study of the double sigma model of the closed string, we already understood the equivalence from the one-loop $\beta$ function, which is consistent at the level of the quantum fluctuation. However, the classical equivalence does not imply that we still have the quantum equivalence. This implies that the double sigma model of open string should be important in studying the one-loop $\beta$ function to understand the quantum equivalence in a consistent manner. It is also a nontrivial consistent check on this theory and should give more interesting new physics for the double field theory.
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