Lepton Flavor Violating Decays as Probes of Neutrino Mass Spectra and Heavy Majorana Neutrino Masses

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We investigate the lepton flavor violating (LFV) rare decays in the supersymmetric minimal seesaw model in which the Frampton-Glashow-Yanagida ansatz is incorporated. The branching ratio of $\mu \rightarrow e\gamma$ is calculated in terms of the Snowmass Points and Slopes (SPS). We find that the inverted mass hierarchy is disfavored by all SPS points. In addition, once the ratio of $\text{BR}(\tau \rightarrow \mu\gamma)$ to $\text{BR}(\mu \rightarrow e\gamma)$ is measured, one may distinguish the normal and inverted neutrino mass hierarchies, and confirm the masses of heavy right-handed Majorana neutrinos by means of the LFV processes and the thermal leptogenesis mechanism. It is worthwhile to stress that this conclusion is independent of the supersymmetric parameters.

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I. INTRODUCTION

Recent solar [1], atmospheric[2], reactor[3] and accelerator[4] neutrino oscillation experiments have provided us with very robust evidence that neutrinos are massive and lepton flavors are mixed. The canonical seesaw mechanism [5] gives a very simple and appealing explanation of the smallness of left-handed neutrino masses – it is attributed to the largeness of right-handed neutrino masses. The existence of neutrino oscillations implies the violation of lepton flavors. Hence the lepton flavor violating (LFV) decays in the charged-lepton sector, such as $\mu \rightarrow e+\gamma$, should also take place. They are unobservable in the Standard Model (SM), because their decay amplitudes are expected to be highly suppressed by the ratios of neutrino masses ($m_i \lesssim 1$ eV) to the $W$-boson mass ($M_W \approx 80$ GeV). In the supersymmetric extension of the SM, however, the branching ratios of such rare processes can be enormously enlarged. Current experimental bounds on the LFV decays $\mu \rightarrow e\gamma$, $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ are\cite{6}

$$\text{BR}(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11},$$
$$\text{BR}(\tau \rightarrow e\gamma) < 1.1 \times 10^{-7},$$
$$\text{BR}(\tau \rightarrow \mu\gamma) < 6.8 \times 10^{-8}. \quad (1)$$

The sensitivities of a few planned experiments\cite{7} may reach $\text{BR}(\mu \rightarrow e\gamma) \lesssim 1.3 \times 10^{-13}$, $\text{BR}(\tau \rightarrow e\gamma) \sim \mathcal{O}(10^{-8})$ and $\text{BR}(\tau \rightarrow \mu\gamma) \sim \mathcal{O}(10^{-8})$.

For simplicity, here we work in the framework of the minimal supergravity (mSUGRA) extended with two heavy right-handed Majorana neutrinos. Then all the soft breaking terms are diagonal at high energy scales, and the only source of lepton flavor violation in the charged-lepton sector is the radiative correction to the soft terms through the neutrino Yukawa couplings. In other words, the low-energy LFV processes $l_i \rightarrow l_j + \gamma$ are induced by the RGE effects of the slepton mixing. The branching ratios of $l_j \rightarrow l_i + \gamma$ are given by\cite{8, 9}

$$\text{BR}(l_j \rightarrow l_i \gamma) \approx \frac{\alpha^3}{G_F m_S^2} \left[ \frac{3m_0^2 + A_0^2}{8\pi^2 v^2 \sin^2 \beta} \right]^2 |C_{ij}|^2 \tan^2 \beta, \quad (2)$$

where $m_0$ and $A_0$ denote the universal scalar soft mass and the trilinear term at $\Lambda_{\text{GUT}}$, respectively. In addition, $m_S$ is a typical mass of superparticles, can be approximately written as $\mathcal{O}(10)$

$$m_S^2 \approx 0.5m_0^2 M_{1/2}^2 (m_0^2 + 0.6M_{1/2}^2)^2 \quad (3)$$

with $M_{1/2}$ being the gaugino mass; and

$$C_{ij} = \sum_k (M_D)_{ik} (M_D)^*_{jk} \ln \frac{\Lambda_{\text{GUT}}}{M_k} \quad (4)$$

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with $\Lambda_{\text{GUT}} = 2.0 \times 10^{16}$ GeV to be fixed in our calculations. $M_D$ and $M_i$ (for $i = 1, 2$) represent the Dirac neutrino mass matrix and the masses of the heavy right-handed Majorana neutrinos, respectively.

To calculate the branching ratios of $l_j \to l_i + \gamma$, we need to know the following parameters in the framework of the mSUGRA: $M_{1/2}$, $m_0$, $A_0$, $\tan \beta$ and $\tan(\mu)$. These parameters can be constrained from cosmology (by demanding that the proper supersymmetric particles should give rise to an acceptable dark matter density) and low-energy measurements (such as the process $b \to s + \gamma$ and the anomalous magnetic moment of muon $g_\mu - 2$). Here we adopt the Snowmass Points and Slopes (SPS) \cite{11} listed in Table 1. These points and slopes are a set of benchmark points and parameter lines in the mSUGRA parameter space corresponding to different scenarios in the search for supersymmetry at present and future experiments.

Since the canonical seesaw models are usually pestered with too many parameters, specific assumptions have to be made for $M_D$ or $M_i$ so as to calculate the LFV rare decays. In addition, the LFV processes have also been discussed in the supersymmetric minimal seesaw model (SM) \cite{13, 14, 15, 16}, in which only two heavy right-handed Majorana neutrinos are introduced \cite{17, 18}. In the MSM, all model parameters can in principle be fixed by use of the LFV rare decays and electric dipole moment of the electron \cite{13}. Ibarra and Ross discuss that the LFV processes may constrain the masses of right-handed heavy majorana neutrinos \cite{14}. In the Frampton-Glashow-Yanagida (FGY) ansatz, Raidal and Strumia have given analytic approximations to the LFV rare decays for the normal hierarchy case \cite{15}. As shown in the literature \cite{19}, the FGY ansatz only includes three unknown parameters: $M_1$, $M_2$ and the smallest mixing angle $\theta_z$. In this paper, we are going to numerically compute the LFV processes in terms of $\theta_z$ for both normal and inverted hierarchies in the FGY ansatz. In the following parts, we shall show that the branching ratio of $\mu \to e\gamma$ may distinguish the neutrino mass hierarchies \cite{12}. In addition, once the ratio of BR($\tau \to \mu\gamma$) to BR($\mu \to e\gamma$) is measured, one may distinguish the normal and inverted neutrino mass hierarchies, and confirm the masses of two heavy right-handed Majorana neutrinos by means of the LFV processes and the thermal leptogenesis mechanism \cite{20}. It is worthwhile to stress that this conclusion is independent of the mSUGRA parameters. The remaining parts of this paper are organized as follows. In Section II, we briefly describe the main features of the FGY ansatz and the thermal leptogenesis in the minimal supersymmetric standard model (MSSM). In Section III, the branching ratio of $\mu \to e\gamma$ is computed in terms of the SPS. In Section IV, we numerically calculated the ratio of BR($\tau \to \mu\gamma$) to BR($\mu \to e\gamma$). Finally the summary are given in Section V.

II. THE FGY ANSATZ AND THE THERMAL LEPTOGENESIS

In the supersymmetric extension of the MSM, two heavy right-handed Majorana neutrinos $N_{iR} \ (i = 1, 2)$ are introduced as the $SU(2)_L$ singlets. The Lagrangian relevant for lepton masses can be written as \cite{21}

$$-\mathcal{L}_{\text{lepton}} = \overline{\ell_L Y_i} E_R H_1 + \overline{\ell_L Y_{\nu}} N_R H_2 + \frac{1}{2} N_R^c N_R + h.c.,$$

(5)

where $\ell_L$ denotes the left-handed lepton doublet, while $E_R$ and $N_R$ stand respectively for the right-handed charged-lepton and neutrino singlets. $H_1$ and $H_2$ (with hypercharges $\pm 1/2$) are the MSSM Higgs doublet superfields. After the spontaneous gauge symmetry breaking, one obtains the charged-lepton mass matrix $M_l = v_1 Y_l$ and the Dirac neutrino mass matrix $M_D = v_2 Y_{\nu}$. Here $v_i$ is the vev of the Higgs doublet $H_i \ (i = 1, 2)$. An important parameter $\beta$ is defined by $\tan \beta \equiv v_2/v_1$ or $\sin \beta \equiv v_2/v$ with $v \approx 174$ GeV. The heavy right-handed Majorana neutrino mass matrix $M_R$ is a $2 \times 2$ symmetric matrix and $M_D$ is a $3 \times 2$ matrix. Without loss of generality, we work in the flavor basis where $M_l$ and $M_R$ are both diagonal, real and positive; i.e., $M_l = \text{Diag}\{m_1, m_2, m_\tau\}$ and $M_R = \text{Diag}\{M_1, M_2\}$. The seesaw relation \cite{3}

$$M_\nu = -M_D M_R^{-1} M_D^T$$

(6)

remains valid. Note that this canonical seesaw relation holds up to the accuracy of $\mathcal{O}(M_D^2/M_R^2) \approx 22$. Since $M_R$ is of rank 2, $M_\nu$ is also a rank-2 matrix with $|\det(M_\nu)| = m_1 m_2 m_3 = 0$, where $m_i \ (i = 1, 2, 3)$ are the masses of three light neutrinos. As for the three neutrino masses $m_i$, the solar neutrino oscillation data have set $m_2 > m_1 \geq m_3$. Now that the lightest neutrino in the MSM must be massless, we are then left with either $m_1 = 0$ (normal mass hierarchy) or $m_3 = 0$ (inverted mass hierarchy). With the best-fit values $\Delta m^2_{\text{sun}} \approx m_2^2 - m_1^2 = 8.0 \times 10^{-5}$ eV$^2$ and $\Delta m^2_{\text{atm}} \equiv |m_3^2 - m_2^2| = 2.5 \times 10^{-3}$ eV$^2$, one can numerically calculate the neutrino masses \cite{24}.

In the FGY ansatz \cite{17}, $M_D$ is taken to be of the form

$$M_D = \begin{pmatrix} a_1 & 0 \\ a_2 & b_2 \\ 0 & b_3 \end{pmatrix}.$$

(7)
With the help of Eqs.(6) and (7), one may straightforwardly arrive at

$$M_\nu = -\left( \begin{array}{ccc} \frac{a_1^2}{M_1} & \frac{a_1 a_2}{M_1} & 0 \\ \frac{a_1 a_2}{M_1} & \frac{a_2^2}{M_1} + \frac{b_2^2}{M_2} & \frac{b_1 b_3}{M_2} \\ 0 & \frac{b_2 b_3}{M_2} & \frac{b_3^2}{M_2} \end{array} \right). \quad (8)$$

Without loss of generality, one can always redefine the phases of left-handed lepton fields to make $a_1$, $b_2$ and $b_3$ real and positive. In this basis, only $a_2$ is complex and its phase $\phi \equiv \arg(a_2)$ is the sole source of CP violation in the model under consideration. Because $a_1$, $b_2$ and $b_3$ of $M_D$ have been taken to be real and positive, $M_\nu$ may be diagonalized in a more general way

$$M_\nu = (P_i V) \left( \begin{array}{ccc} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{array} \right) (P_i V)^T, \quad (9)$$

where $P_i = \text{Diag}\{e^{i\alpha}, e^{i\beta}, e^{i\gamma}\}$ and $V$ is lepton flavor mixing matrix \cite{25} parameterized as

$$V = \left( \begin{array}{ccc} c_x c_z & s_x c_z & s_z \\ -c_x s_y s_z - s_x c_y e^{-i\delta} & -s_x s_y s_z + c_x c_y e^{-i\delta} & s_y c_x \\ -c_x c_y s_z + s_x s_y e^{-i\delta} & -s_x c_y s_z - c_x s_y e^{-i\delta} & c_y c_x \end{array} \right) \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (10)$$

with $s_x \equiv \sin \theta_x$, $c_x \equiv \cos \theta_x$ and so on. $\theta_x \approx \theta_{\text{sun}}$, $\theta_y \approx \theta_{\text{atm}}$ and $\theta_z \approx \theta_{\text{diz}}$ hold as a good approximation \cite{20}. In view of the current experimental data, we have $\theta_x = 34^\circ$ and $\theta_y = 45^\circ$ (best-fit values) as well as $\theta_z < 10^\circ$ at the 99% confidence level \cite{23}. It is worth remarking that there is only a single nontrivial Majorana CP-violating phase ($\sigma$) in the MSM, as a straightforward consequence of $m_1 = 0$ or $m_3 = 0$.

Note that all six phase parameters $(\delta, \sigma, \phi, \alpha, \beta$ and $\gamma$) have been determined in terms of $r_{23} = m_2/m_3 \approx 0.18$, $\theta_x, \theta_y$ and $\theta_z$ \cite{19}:

$$\delta = \pm \arccos \left[ \frac{c_x^2 s_y^2 - c_y^2 s_x^2 (c_x^2 s_y^2 + s_x^2 s_y^2)}{2 c_x s_y c_x s_y s_x s_z} \right],$$

$$\sigma = \frac{1}{2} \arctan \left[ \frac{c_x s_y \sin \delta}{c_x^2 s_y s_z + c_x s_y \cos \delta} \right],$$

$$\alpha = \frac{1}{2} \arctan \left[ \frac{r_{23}^2 s_z^2 s_x^2 \sin 2\sigma}{s_z^2 + r_{23}^2 s_x^2 s_z^2 \cos 2\sigma} \right],$$

$$\beta = -\gamma - \arctan \left[ \frac{c_x s_y s_z \sin \delta}{s_x s_y - c_x c_y s_z \cos \delta} \right],$$

$$\gamma = \frac{1}{2} \arctan \left[ \frac{s_x^2 \sin 2\sigma}{r_{23}^2 s_z^2 s_x^2 + s_x^2 \cos 2\sigma} \right],$$

$$\phi = \alpha + \beta - \arctan \left[ \frac{s_x c_y s_z \sin \delta}{c_x s_y + s_x s_y \cos \delta} \right] \quad (11)$$

for the normal mass hierarchy ($m_1 = 0$). Similar results can also be obtained for the inverted mass hierarchy ($m_3 = 0$) \cite{19}, but we do not elaborate on them here. Because $|\cos \delta| \leq 1$ must hold, we find $0.077 \leq s_z \leq 0.086$ ($m_1 = 0$) and $0.0075 \leq s_z \leq 0.174$ ($m_3 = 0$). Hence, a measurement of the unknown angle $\theta_z$ becomes crucial to test the model.

In the flavor basis where the mass matrices of charged leptons and right-handed neutrinos are diagonal, one can calculate the CP asymmetry in the decays of the lighter right-handed neutrino and obtain \cite{27}

$$\varepsilon_1 \approx \frac{3}{8 \pi v^2 \sin^2 \beta} \frac{|M_1|(M_\nu)_{12}^2 |(M_\nu)_{23}|^2 2^2 \sin 2\phi}{|(M_\nu)_{11}|^2 + |(M_\nu)_{12}|^2 |(M_\nu)_{33}|} \quad (12)$$

when $M_1 \ll M_2$, where we have used

$$a_1^2 = M_{11}(M_\nu)_{11}^2, \quad a_2^2 = M_{12}(M_\nu)_{12}^2/(M_\nu)_{11}, \quad b_3^2 = M_{33}(M_\nu)_{33}^2/(M_\nu)_{33}^2 \quad (13)$$
Then $\varepsilon_1$ can result in a net lepton number asymmetry $Y_L$. The lepton number asymmetry $Y_L$ is eventually converted into a net baryon number asymmetry $Y_B$ via the nonperturbative sphaleron processes \cite{28,29}, which is given by

$$Y_B = \frac{n_B - n_{\bar{B}}}{s} = -c \ Y_L = -c \ \frac{\kappa}{g_*} \ \varepsilon_1,$$

(14)

where $c = 28/79 \approx 0.35$, $g_* = 228.75$ is an effective number characterizing the relativistic degrees of freedom which contribute to the entropy $s$ of the early Universe, and $\kappa$ accounts for the dilution effects induced by the lepton-number-violating wash-out processes. The dilution factor $\kappa$ can be figured out by solving the full Boltzmann equations \cite{29}, however, we take the following approximate formula \cite{30}:

$$\kappa \approx 0.3 \left( \frac{10^{-3} \text{ eV}}{\bar{m}_1} \right) \left( \ln \left( \frac{\bar{m}_1}{10^{-9} \text{ eV}} \right) \right)^{-0.6}$$

(15)

with $\bar{m}_1 = (M_D^4 M_D^4)_{11}/M_1$. It is clear that $\varepsilon_1$ and $Y_B$ only involve two unknown parameters: $M_1$ and $\theta_z$. A generous range $8.5 \times 10^{-11} \lesssim Y_B \lesssim 9.4 \times 10^{-11}$ has been drawn from the recent Wilkinson Microwave Anisotropy Probe (WMAP) observational data \cite{32}. It has been shown that $M_1 > 3.4 \times 10^{10}$ GeV ($m_3 = 0$) are required by the current observational data of $Y_B$ \cite{21}. Flavor effects in the mechanism of thermal leptogenesis have recently attracted a lot of attention \cite{33}. Because the $\tau$ Yukawa coupling are in thermal equilibrium for $10^9 \text{ GeV} \lesssim M_1 \lesssim 10^{12}$ GeV, the flavor issue should be taken into account in our model for the $m_1 = 1$ case. Using the analytic approximate formulas in the Ref. \cite{34}, one may calculate the CP-violating asymmetry $\varepsilon_{\tau\tau}$ and the corresponding wash-out parameter $K_{\tau\tau}$ for the $\tau$ lepton doublet in the final states of $N_1$ decays. We find $\varepsilon_{\tau\tau} = 0$ and $K_{\tau\tau} = 0$ because of $M_{D31} = 0$, namely the $N_1$ decays involving the $\tau$ lepton doublet don’t contribute to the baryon asymmetry $Y_B$. On the other hand, the CP-violating asymmetry $\varepsilon_{\tau\tau}$, which is produced by the out-of-equilibrium decays of $N_2$, vanishes. Therefore, such flavor effects may be negligible in our paper.

Note that there is in general a potential conflict between achieving successful thermal leptogenesis and avoiding overproduction of gravitinos in the MSM with supersymmetry \cite{35} unless the gravitinos are heavier than $\sim 10 \text{ TeV}$ \cite{36}. If the mass scale of gravitinos is of $O(1)$ TeV, one must have $M_1 \lesssim 10^6$ GeV. This limit is completely disfavored in the FGY ansatz. Such a problem could be circumvented in other supersymmetric breaking mediation scenarios (e.g., gauge mediation \cite{14}) or in a class of supersymmetric axion models \cite{37}, where the gravitino mass can be much lighter in spite of the very high reheating temperature. For simplicity, we choose $Y_B = 9.0 \times 10^{-11}$ as an input parameter in this paper. Hence, one may analyze the dependence of $M_1$ on $\theta_z$ from the successful leptogenesis.

### III. DISTINGUISH THE NEUTRINO MASS HIERARCHY

With the help of Eqs. (4) and (7), $|C_{ij}|^2$ can explicitly be written as

$$|C_{12}|^2 = |a_1|^2 |a_2|^2 \left( \ln \frac{A_{\text{GUT}}}{M_1} \right)^2,$$

$$|C_{13}|^2 = 0,$$

$$|C_{23}|^2 = |b_2|^2 |b_3|^2 \left( \ln \frac{A_{\text{GUT}}}{M_2} \right)^2.$$  

(16)

Because of $|C_{13}|^2 = 0$, we are left with $\text{BR}(\tau \rightarrow e\gamma) = 0$. If $\text{BR}(\tau \rightarrow e\gamma) \neq 0$ is established from the future experiments, it will be possible to exclude the FGY ansatz. Using Eq. (13), we reexpress Eq. (16) as

$$|C_{12}|^2 = M_1^2 \ |(M_\nu)_{12}|^2 \left( \ln \frac{A_{\text{GUT}}}{M_1} \right)^2,$$

$$|C_{23}|^2 = M_2^2 \ |(M_\nu)_{23}|^2 \left( \ln \frac{A_{\text{GUT}}}{M_2} \right)^2.$$ 

(17)

As shown in Section II, $M_1$ may in principle be confirmed by leptogenesis for given values of $\sin \theta_z$, but $M_2$ is entirely unrestricted from the successful leptogenesis with $M_2 \gg M_1$.

We numerically calculate $\text{BR}(\mu \rightarrow e\gamma)$ for different values of $\sin \theta_z$ by using the SPS points \cite{44}. The results are shown in Fig. 1. Since the SPS points 1a and 1b (or Points 2 and 3) almost have the same consequence in our scenario, we only focus on Point 1a (or Point 3). When $\sin \theta_z \rightarrow 0.077$ or $\sin \theta_z \rightarrow 0.086$, the future experiment
is likely to probe the branching ratio of $\mu \to e + \gamma$ in the $m_3 = 0$ case. The reason is that $\sin \theta_2 \to 0.077$ (or $\sin \theta_2 \to 0.086$) implies $\phi \to -\pi/2$ (or $\phi \to 0$). Furthermore, the successful leptogenesis requires a very large $M_1$ due to $\varepsilon_1 \propto \sin 2\phi$. It is clear that all SPS points are all unable to satisfy $BR(\mu \to e\gamma) \leq 1.2 \times 10^{-11}$ in the $m_3 = 0$ case. Therefore, we can exclude the $m_3 = 0$ case when the SPS points are taken as the mSUGRA parameters. When $\sin \theta_2 \approx 0.014$, $BR(\mu \to e\gamma)$ arrives at its minimal value in the $m_3 = 0$ case. For the SPS slopes, larger $M_{1/2}$ yields smaller $BR(\mu \to e\gamma)$. We plot the numerical dependence of $BR(\mu \to e\gamma)$ on $M_{1/2}$ in Fig. 2, where we have adopted the SPS slope 3 and taken $300 \text{ GeV} \leq M_{1/2} \leq 1000 \text{ GeV}$. We find that $M_{1/2} \geq 474 \text{ GeV}$ (or $M_{1/2} \geq 556 \text{ GeV}$) can result in $BR(\mu \to e\gamma) \leq 1.2 \times 10^{-11}$ for $\sin \theta_2 = 0.014$ (or $\sin \theta_2 = 0.1$). For all values of $M_{1/2}$ between $300 \text{ GeV}$ and $1000 \text{ GeV}$, $BR(\mu \to e\gamma)$ is larger than the sensitivity of some planned experiments, which ought to examine the $m_3 = 0$ case when the SPS slope 3 is adopted. The same conclusion can be drawn for the SPS slopes 1a and 2. In view of the present experimental results on muon $g - 2$, one may get $M_{1/2} \lesssim 430 \text{ GeV}$ for $\tan \beta = 10$ and $A_0 = 0$ [39], implying that the $m_3 = 0$ case should be disfavored.

IV. DETERMINE THE HEAVY MAJORANA NEUTRINO MASSES

In this paper, an important conclusion is the masses of two heavy right-handed Majorana neutrinos ($M_1$ and $M_2$) can be derived through the leptogenesis and the LFV rare decays. With the help of Eqs.(2) and (17), one can obtain

$$R = \frac{BR(\tau \to \mu\gamma)}{BR(\mu \to e\gamma)} = M_2^2 \left| (M_\nu)_{23} \right|^2 \frac{\ln(\Lambda_{\text{GUT}}/M_2)^2}{M_2^2 \left| (M_\nu)_{12} \right|^2 \ln(\Lambda_{\text{GUT}}/M_1)^2}. \tag{18}$$

Since the successful leptogenesis can be used to fix $M_1$, a measurement of the above ratio will allow us to determine or constrain $M_2$. On the other hand, we may calculate the ratio $R$ by inputting the appropriate $M_2$. It is worthwhile to remark that the ratio $R$ is independent of the mSUGRA parameters. To illustration, we show the numerical results of $R$ as functions of $\sin \theta_2$ in Fig. 3(a) for the $m_1 = 0$ case and Fig. 3(b) for the $m_3 = 0$ case, respectively. In the $m_3 = 0$ case, when $\sin \theta_2 \to 0.0075$, the phase $\phi \to 0$ which implies $M_1$ and $M_2$ may be larger than the GUT scale $\Lambda_{\text{GUT}} = 2.0 \times 10^{16} \text{ GeV}$. So we demand that $M_1$ and $M_2$ must be less than the GUT scale. Below the scale $\Lambda_{\text{GUT}}$, $M_2^2(\ln(\Lambda_{\text{GUT}}/M_2)^2$ obtain its maximum value at $M_2 = \Lambda_{\text{GUT}}/e = 7.4 \times 10^{15} \text{ GeV}$. \tag{19} \vspace{5mm}

Consequently, we can derive the maximum value of $R$ with $M_2 = 7.4 \times 10^{15} \text{ GeV}$. In Fig. 3(b), the numerical results at the left side of the red (green) dash-dot line are “nonphysical”, because these results are corresponding to $M_2 > 7.4 \times 10^{15} \text{ GeV}$ for the $M_2 = 10M_1$ ($M_2 = 50M_1$) case. We can obtain the ratio $M_2/M_1$ once $R$ is measured, furthermore calculate the heavy neutrino mass $M_2$. From Fig. 3, one may derive $R < 2 \times 10^3$ for the $m_1 = 0$ case and $R < 8 \times 10^3$ for the $m_3 = 0$ case, respectively. If the future experiments prove $R \geq 8 \times 10^3$, the inverted mass hierarchy case can be excluded. It is worthwhile to stress that the above conclusions are independent of the mSUGRA parameters.

Finally, let us comment on the $M_1 \gg M_2$ case [13]. In this case, the CP-violating asymmetry $\varepsilon_2$, which is produced by the out-of-equilibrium decay of $N_2$, may finally survive. In order to producing the positive cosmological baryon number asymmetry ($Y_B > 0$), $\delta$ of Eq.(11) must take - sign. The flavor effects may be neglected since the successful thermal leptogenesis requires $M_2 \geq 10^{12} \text{ GeV}$ ($m_1 = 0$) and $M_2 \geq 4.4 \times 10^{13} \text{ GeV}$ ($m_3 = 0$). For $M_1 \geq 5M_2$, we calculate the branching ratio of $\mu \to e\gamma$, find the inverted hierarchy case is disfavored for all SPS points and slopes. For the normal hierarchy case, we may distinguish the $M_1 \ll M_2$ case ($R \geq 14$) and the $M_1 \gg M_2$ case ($R \lesssim 24$) in terms of the ratio $R$. In addition, the masses of heavy neutrinos may be quasi-degenerate [40]. In the $M_1 \simeq M_2$ case, the so-called resonant leptogenesis may occur [41]. Such a scenario could allow us to relax the lower bound on the lighter right-handed Majorana neutrino mass. Therefore, the branching ratios of $\mu \to e\gamma$ and $\tau \to \mu\gamma$ may be much less than the sensitivities of a few planned experiments. $R \simeq \left| (M_\nu)_{23} \right|^2/(\left| (M_\nu)_{12} \right|^2$ can be derived from Eq.(18). Furthermore, one may obtain $0.04 \lesssim R^{-1} \lesssim 0.07$ (normal hierarchy) and $0.00045 \lesssim R^{-1} \lesssim 0.22$ (inverted hierarchy).

V. SUMMARY

We have analyzed the LFV rare decays in the supersymmetric version of the MSM in which the FGY ansatz is incorporated. In this scenario, there are only three unknown parameters $\theta_2$, $M_1$ and $M_2$. The successful leptogenesis may fix $M_1$ for given values of $\theta_2$. We have numerically calculated the branching ratio of $\mu \to e\gamma$ for different values of $\sin \theta_2$ by using the SPS Points. Then one can find the inverted mass hierarchy case is disfavored by all SPS points.
For the SPS slopes, the planned experiments may measure $\mu \rightarrow e\gamma$ in the $m_3 = 0$ case. In addition, once the ratio of $\text{BR}(\tau \rightarrow \mu \gamma)$ to $\text{BR}(\mu \rightarrow e\gamma)$ is measured, we may also distinguish the neutrino mass hierarchies and fix the masses of two heavy right-handed Majorana neutrinos by means of the LFV processes and the leptogenesis mechanism. It is worthwhile to stress that this conclusion is independent of the mSUGRA parameters.

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[42] We may also distinguish the normal and inverted neutrino mass hierarchies through measuring $\theta_z$, the Jarlskog parameter of CP violation ($J_{CP}$) and the effective mass of neutrinoless double beta decay ($\langle m_{ee} \rangle$)[19].
[43] Note that $\varepsilon_1$ is inversely proportional to the mSUGRA parameter $\sin^2 \beta$. Because $\tan \beta \lesssim 3$ is disfavored (as indicated by the Higgs exclusion bounds[31]), here we focus on $\tan \beta \geq 5$ or equivalently $\sin^2 \beta \geq 0.96$. Hence $\sin^2 \beta \approx 1$ is a reliable approximation in our discussion.
[44] For the SPS point 5, Antusch et al. [12] have pointed out that the full RGE running results differ by several orders of magnitude from the leading Log approximation results. In addition, we don’t consider a suppression factor of $\sim 15\%$ from the QED correction [35].
TABLE I: Some parameters for the Snowmass Points and Slopes (SPS) in the mSUGRA. The masses are given in GeV. $\mu$ appeared in the Higgs mass term has been taken as $\mu > 0$ for all SPS.

| Point | $M_{1/2}$ | $m_0$ | $A_0$ | $\tan \beta$ | Slope |
|-------|-----------|-------|-------|-------------|--------|
| 1 a   | 250       | 100   | -100  | 10          | $m_0 = -A_0 = 0.4M_{1/2}$, $M_{1/2}$ varies |
| 1 b   | 400       | 200   | 0     | 30          |        |
| 2     | 300       | 1450  | 0     | 10          | $m_0 = 2M_{1/2} + 850$ GeV, $M_{1/2}$ varies |
| 3     | 400       | 90    | 0     | 10          | $m_0 = 0.25M_{1/2} - 10$ GeV, $M_{1/2}$ varies |
| 4     | 300       | 400   | 0     | 50          |        |
| 5     | 300       | 150   | -1000 | 5           |        |
FIG. 1: Numerical illustration of the dependence of $\text{BR}(\mu \rightarrow e\gamma)$ on $\sin \theta_z$: (a) in the $m_1 = 0$ case; and (b) in the $m_3 = 0$ case. The black solid line and black dash-dot line denote the present experimental upper bound on and the future experimental sensitivity to $\text{BR}(\mu \rightarrow e\gamma)$, respectively.
FIG. 2: Numerical illustration of the dependence of $\text{BR}(\mu \rightarrow e\gamma)$ on $M_{1/2}$ for SPS slope 3 in the $m_3 = 0$ case. The black solid line and black dash-dot line denote the present experimental upper bound on and the future experimental sensitivity to $\text{BR}(\mu \rightarrow e\gamma)$, respectively.
FIG. 3: Numerical illustration of the dependence of $\text{BR}(\tau \rightarrow \mu \gamma)/\text{BR}(\mu \rightarrow e\gamma)$ on $\sin \theta_z$: (a) in the $m_1 = 0$ case; and (b) in the $m_3 = 0$ case.