Transport Properties of an Aharonov-Bohm Interferometer with an In-line Quantum Dot

Do Ngoc Son and Hideaki Kasai*

Graduate School of Engineering, Osaka University
2-1 Yamadaoka, Suita, Osaka 565-0871, Japan

(Received 18 January 2007; Accepted 20 January 2007; Published 26 January 2007)

We investigate the transport properties of a single quantum dot Aharonov-Bohm (AB) interferometer by analyzing the total current of the system in and out of the Kondo regime. We then interpolate the transport behavior of the AB interferometer in the linking regime. We show the explicit expression of the Kondo temperature as a function of the AB phase together with its dependence on other characteristics such as the linewidth of the ring, the finite Coulomb interaction and energy levels of the quantum dot. The total current as a function of AB phase and the ratio of temperature to the Kondo temperature of the system is also presented in this work.

[DOI: 10.1380/ejssnt.2007.29]

Keywords: Green’s function methods; Electrical transport; Kondo effect; Quantum effects; Aharonov-Bohm phase; Quantum dot; Aharonov-Bohm interferometer

I. INTRODUCTION

Two main parts of mesoscopic physics are concerned with exploiting and analyzing strong correlations and quantum interference effects. Strong correlations are due to spin-spin and Coulomb interactions between electrons. Quantum interference effects are due to the superposition of electron wave functions. By modern techniques of mesoscopic system fabrication, one can now produce systems which can include both strong correlations and interference effects at once [1, 2]. One of the most interesting systems is the Aharonov-Bohm interferometer with a quantum dot inserted in one of its paths, which was setup by Yacoby et al., in 1995 [2]. The measurement of magnetic flux-periodic current oscillations through the device proved for the first time that the part of the tunneling current through the quantum dot is coherent. By attaching additional terminals to the system, one was able to modify this interferometer [2-13]. At sufficiently low temperature, the transport through the modified AB interferometer is in the Kondo regime. This improved version of the experiment allowed measuring the phase of the transmission amplitude through the dot, which sharply increases by $\pi$. The Kondo-enhanced valley conductance is observed over a finite dc voltage bias applied across the quantum dot [5, 6, 9].

Much attention has been devoted to study electronic transport properties of the interferometer, which can be used for quantum computation and controlling spintronics devices [14-43]. Most of these works consider the transport through the interferometer in the Kondo regime or out of the Kondo regime. A question is raised that how the transport in the linking regime between in and out of the Kondo regime behaves. This question will be answered in our present work.

How the intradot Coulomb interaction influences the phase coherence of electronic transport through the AB interferometer has been the subject of debate. Several theoretical papers concluded that the intradot Coulomb interaction induces partial dephasing from the spin-flip process [18-20, 35, 36], while others argued that the intradot Coulomb interaction does not induce dephasing effect at all and transport through the quantum dot is fully coherent [37, 38]. The works have been devoted to investigate properties of the system in the small ring limit [18-20, 35-38]. In this letter, we don’t intend to judge the previous works. But we will explore transport properties of the system in the large ring limit.

Transport properties of the AB interferometer have been also investigated in the works of Refs. [41-43] using the expression of the Kondo temperature of an impurity not in a magnetic field found in the works of Haldane [44], and Tsvelick and Wiegmann [45]. It is well known that electron waves come from the left lead travel through different paths of the ring to the right lead of the interferometer and interfere with each other (see Fig. 1). According to the AB effect, each electron should carry an AB phase factor $\exp(-i\phi)$ where $\phi$ is the AB phase [46]. Hence the Kondo temperature of the quantum dot in the AB interferometer should depend on the AB phase or the magnetic flux [47, 48].

Simon et al. had shown the expression for the Kondo temperature of the quantum dot as a function of an AB phase in the limit of infinite intradot Coulomb interaction in the tight-binding Hamiltonian [47]. Lewenkopf and Weidenmuller utilized poor man’s scaling and renormalization group arguments to investigate numerically the Kondo temperature of the quantum dot [48]. The precise form of the dependence of the Kondo temperature on an

FIG. 1: A schematic description of the AB interferometer.
AB phase cannot be predicted.

In this letter, we show the total current through the two-terminal AB interferometer with an AB ring and a quantum dot inserted in one of the ring’s paths [2-6, 8-11] (see also Fig. 1). We consider the transport through the system in or out of the Kondo regime and in the linking regime between these two. We interpolate the transport behavior of the AB interferometer in the linking regime. We also present the new expression for the Kondo temperature of the quantum dot as a function of the AB phase, together with its dependence on the intradot Coulomb interaction, the linewidth of the ring and levels of the quantum dot.

II. THEORY

A. Model Hamiltonian

In this AB interferometer, the quantum dot can be considered as an impurity [49] based on the Anderson model [50]. We can express the Hamiltonian for the present system as follows

$$H = H_0 + H_T + H_C.$$  \hspace{1cm} (1)

Here, $H_0$ describes the totally isolated subsystems of two leads, the AB ring, and the quantum dot, and is given by

$$H_0 = \sum_{\kappa, \sigma, \alpha = L, R} \epsilon_{\kappa \alpha} C_{\kappa \alpha}^+ C_{\kappa \alpha} + \sum_{p \sigma} \epsilon_p C_{p \sigma}^+ C_{p \sigma} + \sum_{\sigma} \epsilon_d d_{\sigma}^+ d_{\sigma} ,$$  \hspace{1cm} (2)

where $\alpha$ stands for the left ($L$) and the right ($R$) leads, while $k$ and $\epsilon_{k \alpha}$ are the longitudinal wave number and the corresponding energy of the electron. The energies of the single particle states within the ring and within the quantum dot are $\epsilon_p$ and $\epsilon_d$, respectively. $C_{\kappa \alpha}^+, C_{p \sigma}^+$, $d_{\sigma}^+$ ($C_{\kappa \alpha}$, $C_{p \sigma}$, $d_{\sigma}$) are the creation (annihilation) operators for the electron in the leads, the ring, and the dot, respectively, while $\sigma$ is the spin index. The tunneling part $H_T$ consists of the couplings between the subsystems, and is given by

$$H_T = \sum_{\kappa \sigma p \sigma \alpha = L, R} (W_{\kappa p}^0 C_{\kappa \alpha}^+ C_{p \sigma} + h.c)$$
$$+ \sum_{p \sigma} [ (V_{pd}^l + V_{pd}^r e^{-i\phi}) C_{p \sigma}^+ d_{\sigma} + h.c] ,$$  \hspace{1cm} (3)

where the tunneling matrix elements $W_{\kappa p}$ describe the coupling between the ring and the leads, while the tunneling matrix elements $V_{pd}^l$ ($V_{pd}^r$) describe the coupling between the left (right) side of the dot and the ring. According to the AB effect, when electron waves from two paths of the ring meet again the phase difference between electron waves will be an AB phase factor $\exp(-i\phi)$, where $\phi = 2\pi \Phi e / h$ and $\Phi$ is the magnetic flux enclosed by the ring formed by the paths. We attach the magnetic flux on the right hand side of the dot, hence $V_{pd}^r$ carries the AB phase factor $\exp(-i\phi)$. The intradot Coulomb interaction Hamiltonian is given by

$$H_C = U n_1 n_1 ,$$  \hspace{1cm} (4)

where $U$ is the charging energy of the quantum dot. $n_\sigma$ is the number operator, $n_\sigma = d_{\sigma}^+ d_{\sigma}$.

B. The Kondo Temperature of the Quantum Dot in the Aharonov-Bohm Interferometer

By means of a Schrieffer-Wolff transformation [51], for $\epsilon_p$ near the Fermi level $\epsilon_F$ and for $\epsilon_d < \epsilon_F < \epsilon_d + U$, we obtain the exchange interaction coupling $J$ between the localized moment of the quantum dot and the conduction electron band as follows

$$J = -2|V_d|^2 (1 + \cos \phi) \frac{U}{\epsilon_d (\epsilon_d + U)} .$$  \hspace{1cm} (5)

Here, the tunneling matrix elements $V_{pd}^l$ ($V_{pd}^r$) are assumed to be real and equal to $V_d$. To show the relationship between $J$ and the Kondo temperature, we must utilize the renormalization group.

The renormalization group enables us to write the renormalization flow equation up to the third order of the running coupling constant $J_\rho$ as follows

$$\frac{\partial (J_\rho)}{\partial \ln D} = -2(J_\rho)^2 + 2(J_\rho)^3 + O((J_\rho)^4) .$$  \hspace{1cm} (6)

Here, $D$ is the cutoff energy or the energy of the largest excitations, $\rho$ is the density of states of the ring. Integrating this equation for $D' << D$, we obtain

$$\ln \left( \frac{D'}{D} \right) = -\frac{1}{2 J_\rho} + \frac{1}{2} \ln(2 J_\rho) + O(1) .$$  \hspace{1cm} (7)

The Kondo temperature $T_K$, where the system scales to strong coupling (very large $J_\rho$), is obtained by setting $D' = T_K$ in Eq. (7), which gives

$$T_K = D \sqrt{2 J_\rho} \exp(-\frac{1}{2 J_\rho}) .$$  \hspace{1cm} (8)

Equation (8) expresses the relationship between $T_K$ and $J$. From Eq. (5) and Eq. (8), we can deduce the Kondo temperature of the quantum dot for the cutoff energy of $U$ as

$$T_K = U \sqrt{\frac{2 \Delta (1 + \cos \phi) U}{\epsilon_d (\epsilon_d + U)}} \exp(\frac{\pi \epsilon_d (\epsilon_d + U)}{2 \Delta (1 + \cos \phi) U}) .$$  \hspace{1cm} (9)

Here, $\Delta = 2 \pi \rho |V_d|^2$ is the linewidth of the ring and $\phi$ is the AB phase. This expression shows the dependence of the Kondo temperature of the quantum dot on the AB phase, which doesn’t appear in the works of Haldane [44] and Tsvelick and Wiegmann [45]. Also, the linewidth $\Delta$ of the ring appears instead of the leads’ [44, 45]. Taking the limit $\cos \phi \rightarrow -1$, we have $T_K \rightarrow 0$ K.

C. The Total Current through the Aharonov-Bohm Interferometer

The current through the system from the leads to the central region (the ring-dot) is calculated from the time evolution of the occupation numbers in the leads [52], $N_\alpha = \sum_k C_{k \sigma}^+ C_{k \sigma} (\alpha = L, R)$, by

$$I_\alpha(t) = -\frac{i e}{h} \langle [H, N_\alpha] \rangle .$$  \hspace{1cm} (10)
Applying the Keldysh Green function technique found in the work of Jauho et al. [52], after integrating over time $t$, we have

$$I = \frac{ie}{2\pi} \sum_{\alpha \sigma \sigma'} \int \frac{d\epsilon}{\Gamma^L(\epsilon) + \Gamma^R(\epsilon)} [G_{\alpha \sigma'}^+(\epsilon) - G_{\alpha \sigma'}^-(\epsilon)] \cdot \left\{ [f_L(\epsilon) - f_R(\epsilon)] [G_{\alpha \sigma'}^+(\epsilon) - G_{\alpha \sigma'}^-(\epsilon)] \right\}.$$  

(11)

Here, the linewidth of the leads is $\Gamma^\alpha(\epsilon) = 2\pi \sum_k W_{k \sigma}^\alpha W_{k \sigma}^\alpha \delta(\epsilon - \epsilon_k) (\alpha = L, R)$. The Fermi distribution function of the leads is $f_L(R)(\epsilon)$. The energy of incoming electrons is $\epsilon$. The retarded and advanced Green functions of the ring-dot are $G_{\alpha \sigma'}^R(\epsilon)$ and $G_{\alpha \sigma'}^A(\epsilon)$ of the ring-dot,

$$G_{\alpha \sigma'}^{R,A}(\epsilon) = \frac{V_d(1 + e^{-i\phi})}{\epsilon - \epsilon_p \pm i\delta} G_{\sigma \sigma'}^{R,A}(\epsilon) - \sum_{\alpha} \epsilon G_{\sigma \sigma'}^{R,A}(\epsilon).$$  

(12)

Using equations (11)-(14), we get

$$I = A \sum_{\epsilon_p} [f_L(\epsilon_p) - f_R(\epsilon_p)] \left( \epsilon_p - \epsilon_d + \Delta(1 + \cos \phi) \frac{\epsilon_p}{T_K} \right)^2 + \Delta^2(1 + \cos \phi)^2 \left( \frac{\epsilon_p}{T_K} \right)^2,$$

(15)

where $A = (e/h)\Gamma^L R V_d/(\Gamma^L + \Gamma^R)$.

Out of the Kondo regime, the temperature $T$ of the system is higher than the Kondo temperature $T > T_K$. Here, the self-energy at the Fermi level and $U > \Delta$ is determined by

$$\Sigma^{r,a}(\epsilon_F) \approx i\Delta(1 + \cos \phi) B,$$

(16)

where

$$B = \ln \frac{T/T_K + \sqrt{\ln^2 (T/T_K) + 3\pi^2/4}}{T/T_K - \sqrt{\ln^2 (T/T_K) + 3\pi^2/4}}.$$  

(17)

Using equations (16), (17), and (11)-(13), we get

$$I = A \sum_{\epsilon_p} \left( \frac{\epsilon_p}{T_K} \right)^2 \left( \frac{\epsilon_p - \epsilon_d}{(\epsilon_p - \epsilon_d)^2 + \Delta^2(1 + \cos \phi)^2 (1 - B)} \right)^2.$$  

(18)

### III. DISCUSSION

Figure 2 shows the total current through the interferometer as a function of ratio $T/T_K$ for different values of AB phase $\phi = [0, \pi/3, 2\pi/3, \pi/2, 3\pi/4, 5\pi/6]$. Other parameters are $U = 1 V$, $\epsilon_d = -U/2$, $\epsilon_p = [0, 0.1Uk_BT_K]$, $V$ ($k_B$ is the Boltzmann constant) and $\Delta = U/2$. From Fig. 2, we see that the magnitude of the current first increases and then decreases with increasing of AB phase (the magnetic flux). For the ratio of the system’s temperature to the Kondo temperature is much smaller than
1. $T/T_K < < 1$, transport through the interferometer is in the Kondo regime. The total current through the interferometer is calculated by Eq. (15). For $T/T_K \geq 1$, the transport is out of the Kondo regime. The total current obeys Eq. (18). Unfortunately there is no reliable theoretical expression for the total current interpolating between Eq. (15) and Eq. (18) so that the interpolation by the dashed lines is somewhat arbitrary.

Some remarks should be made on the results. The magnetic flux dependence of the current leads to vary the direction of the current magnitude. This change occurs at the AB phase about $\phi = \pi/2$. The current magnitude changes from proportion of to inversion with the AB phase. The interpolating lines may be interpreted as an indication of the gradual transition from the coherent to incoherent state of the system, or from in to out of the Kondo regime.

IV. CONCLUSION

In conclusion, we have presented the expression for the Kondo temperature of the quantum dot as a function of AB phase and a finite intradot Coulomb interaction. We have also analyzed the current behavior of the AB interferometer in and out of the Kondo regime. Our result goes beyond other works by interpolating the current in the linking regime.

This work was partially supported by the 21st Century COE program (G18) from the Japan Society for the Promotion of Science (JSPS), and a Grant-in-Aid for Scientific Research on Priority Areas (Developing Next Generation Quantum Simulators and Quantum-Based Design Techniques) by the Ministry of Education, Culture, Sports, Science and Technology (MEXT).