We present two efficient schemes for the deterministic generation and the complete nondestructive analysis of hyperentangled Bell states in both the polarization and spatial-mode degrees of freedom (DOF) of two-photon systems, assisted by the nitrogen-vacancy (NV) centers in diamonds coupled to microtoroidal resonators as a result of cavity quantum electrodynamics. With the input-output process of photons, two-photon polarization-spatial hyperentangled Bell states can be generated in a deterministic way and their complete nondestructive analysis can be achieved. These schemes can be generalized to generate and analyze hyperentangled Greenberger-Horne-Zeilinger states of multiphoton systems as well. Compared with previous works, these two schemes relax the difficulty of their implementation in experiment as it is not difficult to obtain the $\pi$ phase shift in single-sided NV-cavity systems. Moreover, our schemes do not require that the transmission for the uncoupled cavity is balanceable with the reflectance for the coupled cavity. Our calculations show that these schemes can reach a high fidelity and efficiency with current technology, which may be a benefit to long-distance high-capacity quantum communication with two DOF of photon systems.

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I. INTRODUCTION

Recently, hyperentanglement, which is defined as the entanglement in multiple degrees of freedom (DOF) of a quantum system [1–3], has attracted much attention as it has some important applications in quantum information processing. It can speed up quantum computation (e.g., hyperparallel photonic quantum computation [4,5]) and it can be used to assist the complete Bell-state analysis and entanglement purification. In 2003, Walborn et al. [6] presented a simple linear-optical scheme for the complete Bell-state analysis of photons with hyperentanglement. In 2006, Schuck et al. [7] demonstrated the complete deterministic analysis of polarization Bell states with only linear optics assisted by polarization—time-bin hyperentanglement. In 2007, Barbieri et al. [8] demonstrated the complete and deterministic discrimination of polarization Bell states assisted by momentum entanglement. In 2009, Wilde and Uskov [9] proposed a quantum-error-correcting-code scheme assisted by linear-optical hyperentanglement. In 2010, some deterministic entanglement purification protocols (EPPs) [10–12] were proposed with the polarization-spatial hyperentanglement of photon systems, which can solve the troublesome problem that a large amount of quantum resources would be sacrificed in the conventional EPPs, and they are very useful in practical quantum repeaters. In 2012, Wang et al. [13] proposed an important quantum repeater protocol with polarization-spatial hyperentanglement.

The hyperentanglement of photon systems can also be used to largely increase the channel capacity of quantum communication. For example, Barreiro et al. [14] beat the channel capacity limit of photonic superdense coding with polarization-orbital–angular-momentum hyperentanglement in linear optics in 2008. In 2010, Sheng et al. [15] gave the first scheme for the complete hyperentangled–Bell-state analysis (HBSA) for quantum communication, and designed the pioneering model for the quantum teleportation with two DOF of photon pairs, resorting to cross-Kerr nonlinearity. In 2011, Pisenti et al. [16] pointed out the limitations for manipulation and measurement of entangled systems with inherently linear unentangling devices. In 2012, Ren et al. [17] proposed another interesting scheme for the complete HBSA for photon systems by using the giant nonlinear optics in quantum-dot–cavity systems and presented the entanglement swapping with photonic polarization-spatial hyperentanglement. In 2012, Wang et al. [18] introduced an interesting scheme for the complete HBSA for photon systems by the giant circular birefringence induced by double-sided quantum-dot–cavity systems. In 2013, Ren et al. [19] proposed the parameter-splitting method to extract the polarization-spatial maximally hyperentangled photons when the coefficients of the initial partially hyperentangled states are known, and this fascinating method can be achieved with the maximum success probability by performing the protocol only once, resorting to linear-optical elements only. They [19] also gave the first hyperentanglement concentration protocol (hyper-ECP) for unknown polarization-spatial less-hyperentangled states with linear-optical elements only [19]. Ren and Deng [20] presented the first hyperentanglement purification protocol (hyper-EPP) for two-photon systems in polarization-spatial hyperentangled states, and it is very useful in the high-capacity quantum repeaters with hyperentanglement. In 2014, Ren et al. [21] gave a two-step hyper-EPP for polarization-spatial hyperentangled states with the quantum-state–joining method, and it has a far higher efficiency. At the same time, Ren and Long [22] proposed a general hyper-ECP for photon systems assisted by quantum-dot spins inside optical microcavities. Recently, Li and Ghose presented an interesting hyper-ECP resorting to linear optics [23] and another efficient hyper-ECP for the multipartite hyperentangled state via the cross-Kerr nonlinearity [24].

Another attractive candidate for solid-state quantum information processing is the nitrogen-vacancy (NV) center in a diamond, owing to its long decoherence time even at room
temperature [25] and the fact that its spin can be initialized and read out via a highly stable optical transition [26,27]. By using the coherent manipulation of an electron spin and nearby individual nuclear spins, Dutt et al. [28] demonstrated a controllable quantum register in NV centers in 2007. Decoherence-protected quantum gates for a hybrid solid-state register [29] were also experimentally demonstrated on a single NV center. As this system allows for high-fidelity polarization and detection of single electron- and nuclear-spin states even under ambient conditions [27,30–32], the multipartite entanglement among single spins in diamond was demonstrated by Neumann et al. [33] in 2008. In 2010, Togan et al. [34] realized the quantum entanglement generation of an optical photon and an NV center. Photon Fock states on demand can be implemented in a low-temperature solid-state quantum system with an NV center in a nanodiamond coupled to a nearby high-\(Q\) optical cavity [35].

Recently, a combination of NV centers and microcavities, i.e., a promising solid-state cavity quantum electrodynamics (QED) system, has gained widespread attention [36–44]. One of the microcavities is called microtoroidal resonator (MTR) with a quantized whispering-gallery mode (WGM) and is required to be of a high-\(Q\) factor and a small mode volume [45,46]. However, when a MTR couples to a fiber, its \(Q\) factor is surely degraded [38]. The single-photon input-output process from a MTR in experiment also has been demonstrated [38]. In 2009, the quantum nondemolition measurement on a single spin of an NV center was proposed with a low error rate [39] and was experimentally demonstrated through Faraday rotation [40] in 2010. In 2011, Chen et al. [42] proposed an efficient scheme to entangle separate NV centers by coupling to MTRs. In 2013, Wei and Deng [43] proposed some interesting schemes for compact quantum gates on electron-spin qubits assisted by diamond NV centers inside cavities.

In this paper, we present two efficient schemes to generate deterministically hyperentangled states, i.e., hyperentangled Bell states and hyperentangled Greenberger-Horne-Zeilinger (GHZ) states, in which photons are entangled in both the polarization and spatial-mode DOF, assisted by the NV centers in diamonds coupled to MTRs and the input-output process of photons as a result of cavity QED. We also propose a scheme to completely distinguish the 16 polarization-spatial DOF of photons with the identical transition frequency, respectively. Considering the structure in Fig. 1, which can be modeled as a hybrid solid-state system as by an external magnetic field or crystal strain, is labeled as [34] \(A_2\) = \(|E_-\rangle|+\rangle + |E_+\rangle|−\rangle\), where \(|E_-\rangle\) and \(|E_+\rangle\) are the orbital states with the angular momentum projections \(+1\) and \(-1\) along the NV axis, respectively. The optical transition is allowed between the levels \(|m = 0\rangle\) and the excited state \(|A_2\rangle\) owing to the total angular-momentum conservation [34,47].

An NV center can be modeled as a \(Λ\)-type three-level structure with the ground state \(|\rangle = |m = −1\rangle\) and \(|+\rangle = |m = +1\rangle\), and the excited state is \(|1\rangle = |A_2\rangle\). The transitions \(|−\rangle \leftrightarrow |1\rangle\) and \(|+\rangle \leftrightarrow |1\rangle\) in the NV center are resonantly coupled to the right \((R)\) and the left \((L)\) circularly polarized photons with the identical transition frequency, respectively. Considering the structure in Fig. 1, which can be modeled as a single-sided cavity, one can write the Heisenberg equations of motion for this system as follows:

\[
\frac{da}{dt} = -\left[ i(\omega_e - \omega_a) + \frac{\kappa}{2} + \frac{\kappa}{2} \right] a - g \sigma_- - \sqrt{\kappa} a_{\text{in}},
\]

\[
\frac{d\sigma_-}{dt} = -\left[ i(\omega_0 - \omega_a) + \gamma \right] \sigma_- - g \sigma_a,
\]

\[
a_{\text{out}} = a_{\text{in}} + \sqrt{\kappa} a,
\]

FIG. 1. (Color online) (a) Schematic diagram for an NV center inside a MTR. (b) The electron energy-level configuration of an NV center in a MTR with the relevant transitions driven by different polarized photons. \(L\) \((R)\) represents the left \((right)\) circularly polarized photon.

This article is organized as follows. In Sec. II, we will introduce the diamond-NV-center system and its single-photon input-output process. The generation of hyperentangled Bell states and hyperentangled GHZ states, and the complete nondestructive HBBA, are presented in Secs. III and IV. A discussion and a summary are given in Sec. V.
where \( \omega, \omega_c, \) and \( \omega_0 \) are the frequencies of the photon, cavity mode, and the atomic-level transition, respectively. \( g \) is the coupling strength between the NV center and the cavity mode, \( \gamma \) and \( \gamma_c \) are the decay rates of the NV center and the cavity field, respectively. \( \gamma_k \) is the side leakage rate of the cavity. \( a_{in} \) and \( a_{out} \) are the input and output field operators, respectively.

In the weak-excitation approximation, the reflection coefficient in the steady state can be obtained,

\[
r(\omega) = 1 - \frac{\kappa \left[ i (\omega_0 - \omega) + \frac{\gamma_k}{2} \right]}{i (\omega - \omega_0) + \frac{\gamma_k}{2} \left[ i (\omega - \omega_0) + \frac{\gamma_k}{2} + \frac{\gamma_c}{2} \right] + g^2}.
\]

(2)

For \( g = 0 \), the reflection coefficient \( r_0(\omega) \) is

\[
r_0(\omega) = \frac{i (\omega - \omega_0) - \frac{\gamma_k}{2}}{i (\omega_0 - \omega) + \frac{\gamma_k}{2}}.
\]

(3)

From Eqs. (2) and (3), one can see that if \( \omega_0 = \omega_c = \omega \), then

\[
r(\omega) = \frac{(\kappa_+ - \kappa) \gamma + 4g^2}{(\kappa_+ + \kappa) \gamma + 4g^2}, \quad r_0(\omega) = \frac{\kappa_+ - \kappa}{\kappa_+ + \kappa}.
\]

(4)

If the NV center is in the initial state \(|-\rangle \) (\(|+\rangle \)) and a single polarized photon \(|L\rangle \) (\(|R\rangle \)) is input, then the photon will experience a phase shift \( e^{i \phi} \) owing to the Faraday rotation. However, if the initial state of the NV center is \(|-\rangle \) (\(|+\rangle \)), then the input photon with \(|R\rangle \) (\(|L\rangle \)) polarization will get a phase shift \( e^{i \phi_0} \). In the resonant condition \( \omega_0 = \omega_c = \omega \), when \( \kappa_+ \ll \kappa \) and \( 4g^2 \gg \gamma \gamma_c \), we approximately have \( \phi = 0 \) and \( \phi_0 = \pi \) from Eq. (2). The change of the input photon can be summarized as follows [43]:

\[
|L\rangle |+\rangle \rightarrow |L\rangle |+, \quad |R\rangle |-\rangle \rightarrow -|R\rangle |-\rangle,
\]

\[
|L\rangle |+\rangle \rightarrow -|L\rangle |+, \quad |L\rangle |-\rangle \rightarrow |L\rangle |-\rangle.
\]

(5)

III. PHOTONIC HYPERENTANGLEMENT GENERATION

We first describe how to generate two-photon polarization-spatial hyperentangled Bell states assisted by NV centers coupled to MTRs as a result of cavity QED, and then extend this approach for the generation of three-photon polarization-spatial hyperentangled GHZ states.

A. Generation of two-photon hyperentangled Bell states

A two-photon hyperentangled Bell state in both the polarization and the spatial-mode DOF can be expressed as

\[
|\Phi_+^{1+}\rangle_R = \frac{1}{\sqrt{2}} (|RR\rangle |LL\rangle)_{ab} (|a_1 b_1\rangle + |a_2 b_2\rangle)_{ab}.
\]

(6)

Here, \(|R\rangle \) and \(|L\rangle \) denote the right-circular polarization and the left-circular polarization of photons, respectively. \( a_1 (b_1) \) and \( a_2 (b_2) \) are the different spatial modes for photon \( a \) (\( b \)). The subscripts \( P \) and \( S \) denote the polarization and the spatial-mode DOF, respectively. \( a \) and \( b \) represent the two photons in the hyperentangled state. The four Bell states in the polarization DOF can be expressed as

\[
|\Phi_+^{1+}\rangle_P = \frac{1}{\sqrt{2}} (|RR\rangle \pm |LL\rangle),
\]

(7)

\[
|\Phi_+^{1\pm}\rangle_P = \frac{1}{\sqrt{2}} (|LR\rangle \pm |RL\rangle),
\]

and those in the spatial-mode DOF can be written as

\[
|\Phi_+^{s+}\rangle_S = \frac{1}{\sqrt{2}} (|a_1 b_1\rangle \pm |a_2 b_2\rangle),
\]

(8)

\[
|\Phi_+^{s\pm}\rangle_S = \frac{1}{\sqrt{2}} (|a_2 b_2\rangle \pm |a_1 b_1\rangle).
\]

The principle of our scheme for the two-photon polarization-spatial hyperentangled Bell states generation (HBSG) assisted by NV centers coupled to MTRs as a result of cavity QED is shown in Fig. 2. Here SW is an optical switch and BS represents a 50:50 beam splitter which can accomplish the following transformation in the spatial-mode DOF of the photons:

\[
K_{a_1(b_2)}^{\dagger} \rightarrow \frac{1}{\sqrt{2}} (K_{c_1(d_1)}^{\dagger} + K_{c_2(d_2)}^{\dagger}),
\]

(9)

\[
K_{a_2(b_1)}^{\dagger} \rightarrow \frac{1}{\sqrt{2}} (K_{c_2(d_1)}^{\dagger} - K_{c_1(d_2)}^{\dagger}).
\]

Suppose that two NV centers, NV1 and NV2, are initialized to the superposition states \(|\varphi^+\rangle = |\varphi^+\rangle = \frac{1}{\sqrt{2}} (|-\rangle + |+\rangle), \) and the two photons \( a \) and \( b \) with the same frequency are prepared in the same initial state, \(|\phi\rangle_a = |\phi\rangle_b = \frac{1}{\sqrt{2}} (|R\rangle + |L\rangle)\).

FIG. 2. (Color online) Schematic diagram for two-photon polarization-spatial HBSG. PBS represents a circularly polarized beam splitter which is used to transmit the \( L \) polarized photon and reflect the \( R \) polarized photon, respectively. SW represents an optical switch and BS is a 50:50 beam splitter. HWP represents a half-wave plate which is used to perform a bit-flip operation \( X = \{ |R\rangle \langle L| + |L\rangle \langle R| \} \) on the photon in the polarization DOF. NV1 and NV2 represent two NV centers. \( k_i \) and \( k_j \) (\( k = a,b,c,d \)) represent different spatial modes.
Photon $a$ and $b$ are successively sent into the device shown in Fig. 2. A time interval $\Delta t$ exists between two photons, and $\Delta t$ should be less than the spin coherence time $T$. When photon $a$ passes through the cavity, the optical switch (SW) is switched to await photon $b$. After passing through the two NV centers, the two photons $a$ and $b$ can be entangled with the electron spins in the NV centers in the two cavities. The corresponding transformations on the states can be described as follows:

$$|\Phi\rangle_0 = |\phi\rangle_a|\phi\rangle_b|\psi^+\rangle_1|\psi^+\rangle_2$$

$$\rightarrow \quad |\Phi\rangle_{PBS} = \frac{1}{\sqrt{2}}(|L\rangle|a_1\rangle + |R\rangle|a_2\rangle)(|L\rangle|b_1\rangle + |R\rangle|b_2\rangle)|\psi^+\rangle_1|\psi^+\rangle_2$$

$$\rightarrow \quad |\Phi\rangle_{NV_1} = \frac{1}{\sqrt{2}}|(LL\rangle|a_1b_1\rangle + |RR\rangle|a_2b_2\rangle)(|\psi^+\rangle_1$$

$$\rightarrow \quad |\Phi\rangle_{BS} = \frac{1}{\sqrt{2}}|(|\Phi^+_1\rangle_p|\Phi^-_1\rangle_s|\psi^+\rangle_1 + |\Phi^-_1\rangle_p|\Phi^+_1\rangle_s|\psi^-\rangle_1|\psi^+\rangle_2$$

$$\rightarrow \quad |\Phi\rangle_{NV_2} = \frac{1}{\sqrt{2}}\left(|\Phi^+_2\rangle_p|\Phi^-_2\rangle_s|\psi^+\rangle_1|\psi^+\rangle_2$$

$$+ |\Phi^-_2\rangle_p|\Phi^+_2\rangle_s|\psi^-\rangle_1|\psi^+\rangle_2$$

$$+ |\Phi^+_2\rangle_p|\Phi^-_2\rangle_s|\psi^-\rangle_1|\psi^+\rangle_2$$

$$+ |\Phi^-_2\rangle_p|\Phi^+_2\rangle_s|\psi^+\rangle_1|\psi^-\rangle_2\right).$$

(10)

Here, $|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|-\rangle \pm |+\rangle)$.

From Eq. (10), one can see that the injecting photons $a$ and $b$ pass through the circularly polarizing beam splitter (PBS) in sequence. Each of them is split into two wave packets. The part of photon $a$ ($b$) in the state $|L\rangle$ is transmitted to path $a_1$ ($b_1$), while the part in the state $|R\rangle$ is reflected by PBS to path $a_2$ ($b_2$), and subsequently it interacts with the first NV center (named NV$_1$). The wave packets from the spatial modes $a_1$ and $a_2$ ($b_1$ and $b_2$) are mixed at the beam splitter (BS). The half-wave plate (HWP) is used to keep the polarization state of the photon unchanged. After BSs, the two-photon systems are divided into two groups, according to the state of NV$_2$ when it is measured with the basis $\{|\psi^+\rangle, |\psi^-\rangle\}$; so are the outcomes of the measurement on NV$_1$. The relationship between the measurement outcomes of these two NV centers and the polarization-spatial hyperentangled Bell states of the two photons is shown in Table I.

From Table I, one can see that if NV$_1$ is in the state $|\psi^+\rangle_1$ and NV$_2$ is in the state $|\psi^-\rangle_2$, then the two-photon system $ab$ is in the hyperentangled Bell state $|\Phi^-_1\rangle_p|\Phi^+_1\rangle_s$. When the two NV centers are in the states $|\psi^+\rangle_1$ and $|\psi^+\rangle_2$, respectively, the two-photon system is in the hyperentangled Bell state $|\Phi^+_2\rangle_p|\Phi^-_2\rangle_s$. The hyperentangled Bell states $|\Phi^-_1\rangle_p|\Phi^+_1\rangle_s$ and $|\Phi^+_2\rangle_p|\Phi^-_2\rangle_s$ also correspond to different combinations of the states of the two NV centers. Therefore, one can generate the polarization-spatial hyperentangled Bell states of the two-photon system by measuring the states of the two NV centers.

B. Generation of three-photon hyperentangled GHZ states

Similar to the case for two-photon polarization-spatial hyperentangled Bell states, we denote a three-photon polarization-spatial hyperentangled GHZ state as

$$|\xi^+_1\rangle_{PS} = \frac{1}{\sqrt{2}}(|RRR\rangle + |LLL\rangle)_{abc}(|a_1b_1c_1\rangle + |a_2b_2c_2\rangle)_{abc}. $$

(11)

Here $a$, $b$, and $c$ represent the three photons in the hyperentangled state. The eight three-photon GHZ states in the polarization DOF can be expressed as

$$|\Psi^+_1\rangle_p = \frac{1}{\sqrt{2}}(|RRR\rangle \pm |LLL\rangle),$$

$$|\Psi^+_2\rangle_p = \frac{1}{\sqrt{2}}(|LRR\rangle \pm |RLR\rangle),$$

$$|\Psi^+_3\rangle_p = \frac{1}{\sqrt{2}}(|RLR\rangle \pm |LRL\rangle),$$

$$|\Psi^+_4\rangle_p = \frac{1}{\sqrt{2}}(|RRL\rangle \pm |LRR\rangle),$$

and the eight GHZ states in the spatial-mode DOF are

$$|\Psi^+_1\rangle_s = \frac{1}{\sqrt{2}}(|a_1b_1c_1\rangle \pm |a_2b_2c_2\rangle),$$

$$|\Psi^+_2\rangle_s = \frac{1}{\sqrt{2}}(|a_2b_1c_1\rangle \pm |a_1b_2c_2\rangle),$$

$$|\Psi^+_3\rangle_s = \frac{1}{\sqrt{2}}(|a_1b_2c_1\rangle \pm |a_2b_1c_2\rangle),$$

$$|\Psi^+_4\rangle_s = \frac{1}{\sqrt{2}}(|a_2b_2c_1\rangle \pm |a_1b_1c_2\rangle).$$

(13)

The principle of our scheme for the generation of a three-photon polarization-spatial hyperentangled GHZ state is shown in Fig. 3, considering all three NV centers are in the initial state $|\psi^+\rangle_1 = |\psi^+\rangle_2 = |\psi^+\rangle_3 = \frac{1}{\sqrt{2}}(|-\rangle + |+\rangle)$ and the flying photons $a$, $b$, and $c$ are in the superposition state $|\phi\rangle_a = |\phi\rangle_b = |\phi\rangle_c = \frac{1}{\sqrt{2}}(|a\rangle + |b\rangle)$. A brief description of our scheme for hyperentangled–GHZ-state generation can be written as follows.

Each of the three photons $a$, $b$, and $c$ is split into two wave packets by PBS. The photon in the state $|L\rangle$ is injected into the paths $a_1$, $b_1$, and $c_1$, while the photon in the state $|R\rangle$ is sent into the paths $a_2$, $b_2$, and $c_2$. The photons in the paths $a_2$ and

|Γ  | |Γ  | |Γ  | |Γ  |
|---|---|---|---|
|a  |b  |c  |d  |
|e  |f  |g  |h  |
|i  |j  |k  |l  |
|m  |n  |o  |p  |
|q  |r  |s  |t  |
|u  |v  |w  |x  |
|y  |z  |Γ  |Γ  |

Table I. The relation between the outcomes of the two NV centers and the final polarization-spatial hyperentangled Bell states.
b_2 interact with NV_1, and those in the paths b_1 and c_2 interact with NV_2. BSs mix the spatial modes a_1 and a_2, b_1 and b_2, and c_1 and c_2. The states of the three photons are divided into four groups, according to the states of NV_1 and NV_2. Under the condition that NV_3 is imported, the hyperentangled states of the photons can be determined by measuring the states of the NV centers. The evolution of the whole system can be described as

$$\Psi_0 = |\phi\rangle_a |\phi\rangle_b |\phi\rangle_c |\varphi^+_1\rangle_1 |\varphi^+_2\rangle_2 |\varphi^-_3\rangle_3$$

$$\longrightarrow \frac{1}{2\sqrt{2}} (|\Psi^{\Phi_1}_1\rangle_P |\Psi^{\Phi_2}_2\rangle_P |\varphi^+_1\rangle_1 |\varphi^-_2\rangle_2 |\varphi^-_3\rangle_3$$

- $$|\Psi^{\Phi_4}_4\rangle_P |\Psi^{\Phi_5}_5\rangle_P |\varphi^+_1\rangle_1 |\varphi^-_2\rangle_2 |\varphi^-_3\rangle_3$$

+ $$|\Psi^{\Phi_6}_6\rangle_P |\Psi^{\Phi_7}_7\rangle_P |\varphi^-_1\rangle_1 |\varphi^-_2\rangle_2 |\varphi^-_3\rangle_3$$

$$|\Psi^{\Phi_8}_8\rangle_P |\Psi^{\Phi_9}_9\rangle_P |\varphi^-_1\rangle_1 |\varphi^-_2\rangle_2 |\varphi^-_3\rangle_3$$)

Here, U_T represents the total operation by PBS, NV_1, NV_2, BS, and NV_3 in sequence; also, $$|\varphi_{\pm}\rangle = \frac{1}{\sqrt{2}} (|\varphi_1\rangle \pm |\varphi_2\rangle)$$. The relation between the outcomes of the measurements on the three NV centers and the obtained final polarization-spatial hyperentangled GHZ states is shown in Table II.

Table II shows that if the three NV centers are in the states $$|\varphi^+_1\rangle_1$$, $$|\varphi^-_2\rangle_2$$, and $$|\varphi^-_3\rangle_3$$, respectively, then the three-photon system abc is in the polarization-spatial hyperentangled GHZ state $$|\Phi^{\Phi}_1\rangle_P |\Phi^{\Phi}_2\rangle_P |\Phi^{\Phi}_3\rangle_P$$. When the NV centers are in the state $$|\varphi^+_1\rangle_1$$, $$|\varphi^-_2\rangle_2$$, and $$|\varphi^-_3\rangle_3$$, the final hyperentangled GHZ state of the three photons is $$|\Phi^{\Phi}_1\rangle_P |\Phi^{\Phi}_2\rangle_P |\Phi^{\Phi}_3\rangle_P$$. Our scheme can, in principle, generate the eight deterministic hyperentangled three-photon GHZ states by measuring the states of the NV centers.

Table II. The relation between the outcomes of the three NV centers and the final hyperentangled GHZ states.

| NV_1     | NV_2     | NV_3     | Hyperentangled GHZ states  |
|----------|----------|----------|-----------------------------|
| $$|\varphi^+_1\rangle_1$$ | $$|\varphi^-_2\rangle_2$$ | $$|\varphi^-_3\rangle_3$$ | $$|\Psi^{\Phi}_1\rangle_P |\Psi^{\Phi}_2\rangle_P |\Psi^{\Phi}_3\rangle_P$$  |
| $$|\varphi^+_1\rangle_1$$ | $$|\varphi^-_2\rangle_2$$ | $$|\varphi^-_3\rangle_3$$ | $$|\Psi^{\Phi}_1\rangle_P |\Psi^{\Phi}_2\rangle_P |\Psi^{\Phi}_3\rangle_P$$  |
| $$|\varphi^+_1\rangle_1$$ | $$|\varphi^-_2\rangle_2$$ | $$|\varphi^-_3\rangle_3$$ | $$|\Psi^{\Phi}_1\rangle_P |\Psi^{\Phi}_2\rangle_P |\Psi^{\Phi}_3\rangle_P$$  |
| $$|\varphi^+_1\rangle_1$$ | $$|\varphi^-_2\rangle_2$$ | $$|\varphi^-_3\rangle_3$$ | $$|\Psi^{\Phi}_1\rangle_P |\Psi^{\Phi}_2\rangle_P |\Psi^{\Phi}_3\rangle_P$$  |
| $$|\varphi^+_1\rangle_1$$ | $$|\varphi^-_2\rangle_2$$ | $$|\varphi^-_3\rangle_3$$ | $$|\Psi^{\Phi}_1\rangle_P |\Psi^{\Phi}_2\rangle_P |\Psi^{\Phi}_3\rangle_P$$  |

IV. COMPLETE NONDESTRUCTIVE POLARIZATION-SPATIAL HYPERENTANGLED–BELL-STATE ANALYSIS

The principle of our scheme for distinguishing the 16 hyperentangled Bell states in both the polarization and the spatial-mode DOF of entangled photon pairs is shown in Fig. 4. Here, we use four NV centers coupled to four MTRs and a few linear-optical elements to achieve the complete nondestructive analysis of hyperentangled Bell states, considering that two NV centers are both initialized in the superposition state $$|\varphi^+_1\rangle = |\varphi^+_2\rangle = \frac{1}{\sqrt{2}} (|\varphi^+\rangle + |\varphi^-\rangle)$$, and a hyperentangled photon pair is in one of the 16 hyperentangled Bell states. As shown in Fig. 4, one can let photon a pass through the cavities first and then photon b. SW is used for the arrival of photon b after photon a passes through the cavity. Δt is the time interval between photon a and photon b, which is smaller than the spin-coherence time of the NV center.

According to Eq. (5), after the photons pass through NV_1 and NV_2, the evolution of the system composed of the two photons and the two NV centers is shown in Table III. One can see that the 16 polarization-spatial hyperentangled states can be divided into four groups. If both the spins of NV_1 and NV_2 are changed to be $$|\varphi^-\rangle_1 |\varphi^-\rangle_2$$, then the photon pair ab is in one of the four hyperentangled Bell states $$|\Phi^{\Phi}_1\rangle_P |\Phi^{\Phi}_2\rangle_P$$.
and $|\Phi^+_{2}\rangle_P|\Phi^-_{1}\rangle_S$. If both NV1 and NV2 stay in the initial state $|\psi^+\rangle_1|\psi^+\rangle_2$, then the photon pair $ab$ is in one of the four hyperentangled Bell states $|\Phi^+_{1}\rangle_P|\Phi^-_{1}\rangle_S$ and $|\Phi^-_{2}\rangle_P|\Phi^-_{2}\rangle_S$. Otherwise, when NV1 and NV2 are in the state of $|\psi^-\rangle_1|\psi^+\rangle_2$ (or $|\psi^-\rangle_1|\psi^-\rangle_2$), the photon pair $ab$ is in one of the four hyperentangled Bell states $|\Phi^-_{1}\rangle_P|\Phi^+_{1}\rangle_S$ and $|\Phi^+_{2}\rangle_P|\Phi^+_{2}\rangle_S$ or $|\Phi^-_{2}\rangle_P|\Phi^+_{2}\rangle_S$ and $|\Phi^+_{1}\rangle_P|\Phi^+_{1}\rangle_S$.

Subsequently, one can let the photons pass through the BS and quarter-wave plate (QWP) which are used to transform the phase information discrimination to parity information discrimination, both for the polarization DOF and the spatial-mode DOF. According to Table III, after the photons pass through BS and QWP, the initial hyperentangled Bell state $|\Phi^+_{2}\rangle_P|\Phi^-_{1}\rangle_S$ and $|\Phi^-_{2}\rangle_P|\Phi^-_{1}\rangle_S$ for the same group will become

$$\begin{align}
|\Phi^+_{1}\rangle_P|\Phi^-_{1}\rangle_S & \xrightarrow{NV_1, NV_2, BS, QWP} |\Phi^+_{1}\rangle_P|\Phi^-_{1}\rangle_S, \\
|\Phi^-_{1}\rangle_P|\Phi^+_{1}\rangle_S & \xrightarrow{NV_1, NV_2, BS, QWP} |\Phi^+_{2}\rangle_P|\Phi^-_{1}\rangle_S, \\
|\Phi^+_{2}\rangle_P|\Phi^-_{1}\rangle_S & \xrightarrow{NV_1, NV_2, BS, QWP} |\Phi^+_{1}\rangle_P|\Phi^-_{1}\rangle_S, \\
|\Phi^-_{2}\rangle_P|\Phi^-_{1}\rangle_S & \xrightarrow{NV_1, NV_2, BS, QWP} |\Phi^+_{2}\rangle_P|\Phi^-_{1}\rangle_S. 
\end{align}$$

(15)

From Table III, one can see that the final hyperentangled Bell states of two-photon systems in the same group will be changed into four different groups. By letting the photons pass through NV3 and NV4, which have the same syndetic connection with NV1 and NV2, one can distinguish those hyperentangled Bell states. The hyperentangled Bell states in the other three groups will have the same conditions. After passing through all the elements shown in Fig. 4, the final states of the two photons become those shown in Table IV.

**TABLE III.** The relation between the initial states and the final states of the system after the photons pass through NV1 and NV2.

| Initial states | Final states |
|----------------|-------------|
| $|\Phi^+_{1}\rangle_P|\Phi^+_{1}\rangle_S|\psi^+\rangle_1|\psi^+\rangle_2$ | $|\Phi^+_{1}\rangle_P|\Phi^-_{1}\rangle_S|\psi^-\rangle_1|\psi^-\rangle_2$ |
| $|\Phi^-_{1}\rangle_P|\Phi^+_{1}\rangle_S|\psi^+\rangle_1|\psi^+\rangle_2$ | $|\Phi^+_{2}\rangle_P|\Phi^+_{2}\rangle_S|\psi^-\rangle_1|\psi^-\rangle_2$ |
| $|\Phi^-_{1}\rangle_P|\Phi^-_{1}\rangle_S|\psi^+\rangle_1|\psi^+\rangle_2$ | $|\Phi^-_{2}\rangle_P|\Phi^-_{2}\rangle_S|\psi^-\rangle_1|\psi^-\rangle_2$ |
| $|\Phi^+_{2}\rangle_P|\Phi^-_{2}\rangle_S|\psi^+\rangle_1|\psi^+\rangle_2$ | $|\Phi^-_{2}\rangle_P|\Phi^+_{2}\rangle_S|\psi^-\rangle_1|\psi^-\rangle_2$ |
| $|\Phi^-_{2}\rangle_P|\Phi^-_{2}\rangle_S|\psi^+\rangle_1|\psi^+\rangle_2$ | $|\Phi^+_{1}\rangle_P|\Phi^+_{1}\rangle_S|\psi^-\rangle_1|\psi^-\rangle_2$ |
| $|\Phi^+_{2}\rangle_P|\Phi^-_{1}\rangle_S|\psi^+\rangle_1|\psi^+\rangle_2$ | $|\Phi^-_{1}\rangle_P|\Phi^+_{2}\rangle_S|\psi^-\rangle_1|\psi^-\rangle_2$ |
| $|\Phi^-_{2}\rangle_P|\Phi^+_{1}\rangle_S|\psi^+\rangle_1|\psi^+\rangle_2$ | $|\Phi^+_{1}\rangle_P|\Phi^-_{2}\rangle_S|\psi^-\rangle_1|\psi^-\rangle_2$ |

This time, by using linear-optical elements, one can transform the state of the two-photon system into its initial hyperentangled Bell state. That is, one can completely distinguish the 16 hyperentangled Bell states with the measurement outcomes of the states of the four NV centers rather than using single-photon detectors to proceed destructive measurement. The relation between the initial hyperentangled Bell states and the measurement outcomes of the states of the NV centers is shown in Table V.

From the analysis above, one can see that the hyperentangled Bell states in both the polarization and the spatial-mode DOF can be completely distinguished assisted by NV centers in diamonds confined in MTRs, and that our analysis is nondestructive. Our scheme can be generalized to the complete analysis of multiphoton polarization-spatial hyperentangled GHZ states by importing more NV-center–cavity systems.

**V. DISCUSSION AND SUMMARY**

In our schemes, the reflection coefficient of input photon pulse and the phase shift induced on the output photon play a crucial role. Under the resonant condition $\omega_0 = \omega_0 = \omega$, if the cavity side leakage is neglected, the fidelities of our schemes can reach 100% in the strong-coupling regime with $r(\omega) \equiv 1$ and $r(\omega) \equiv -1$. If the cavity leakage is taken into account, the spin-selective optical transition rules employed in our work become

$$\begin{align}
|R>|+> & \rightarrow |R>|+>, \quad |R>|-> \rightarrow r_0|R>|->, \\
|L>|+> & \rightarrow r_0|L>|+>, \quad |L>|-> \rightarrow r|L>|->. 
\end{align}$$

(16)

**TABLE IV.** The relation between the initial hyperentangled Bell states and the final states of the two photons after passing through all the elements.

| Initial states | Final states | Initial states | Final states |
|----------------|-------------|----------------|-------------|
| $|\Phi^+_{1}\rangle_P|\Phi^+_{1}\rangle_S$ | $|\Phi^+_{1}\rangle_P|\Phi^-_{1}\rangle_S$ | $|\Phi^-_{2}\rangle_P|\Phi^-_{2}\rangle_S$ | $|\Phi^+_{2}\rangle_P|\Phi^-_{2}\rangle_S$ |
| $|\Phi^-_{1}\rangle_P|\Phi^-_{1}\rangle_S$ | $|\Phi^+_{2}\rangle_P|\Phi^+_{2}\rangle_S$ | $|\Phi^-_{2}\rangle_P|\Phi^-_{2}\rangle_S$ | $|\Phi^+_{1}\rangle_P|\Phi^-_{1}\rangle_S$ |
| $|\Phi^+_{2}\rangle_P|\Phi^-_{1}\rangle_S$ | $|\Phi^-_{2}\rangle_P|\Phi^+_{2}\rangle_S$ | $|\Phi^-_{2}\rangle_P|\Phi^-_{1}\rangle_S$ | $|\Phi^+_{1}\rangle_P|\Phi^+_{2}\rangle_S$ |
| $|\Phi^-_{2}\rangle_P|\Phi^+_{1}\rangle_S$ | $|\Phi^-_{1}\rangle_P|\Phi^+_{2}\rangle_S$ | $|\Phi^-_{2}\rangle_P|\Phi^+_{1}\rangle_S$ | $|\Phi^-_{1}\rangle_P|\Phi^+_{1}\rangle_S$ |

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TABLE V. The relation between the initial hyperentangled Bell states and the measurement outcomes of the states of the NV centers.

| Bell states   | NV₁ | NV₂ | NV₃ | NV₄ |
|--------------|-----|-----|-----|-----|
| | | | | |

Considering the practical implementation of the system, we numerically simulate the relation between the fidelities (the efficiencies) and the coupling strength \( g \), the cavity decay rate \( \kappa \), and the NV-center decay rate \( \gamma \). We define the fidelity of the process for generating or completely analyzing the \( \eta \) and the coupling strength \( \kappa \) numerically simulate the relation between the fidelities (the efficiencies) and the coupling strength \( \kappa \).

**Efficiency of HBSG scheme**

\[
F_i = \frac{(r - r_0)^2(r^2 + r_0^2 + 2)^2}{8(r^2 + r_0^2)(r^2 + r_0^2 + 2)}.
\]  

(17)

By computation, it is found that since the hyperentangled Bell states \( |\Phi^+_i\rangle_P|\Phi^+_i\rangle_S \), \( |\Phi^+_i\rangle_P|\Phi^-_i\rangle_S \), and \( |\Phi^-_i\rangle_P|\Phi^-_i\rangle_S \) have the same parity conditions with the hyperentangled Bell state \( |\Phi^+_i\rangle_P|\Phi^-_i\rangle_S \), the fidelities of our scheme for their generations are also \( F_1 \), \( F_2 \), \( F_3 \), and \( F_4 \) correspond to the fidelities for generating the hyperentangled Bell states \( |\Phi^+_i\rangle_P|\Phi^+_i\rangle_S \), \( |\Phi^+_i\rangle_P|\Phi^-_i\rangle_S \), and \( |\Phi^-_i\rangle_P|\Phi^-_i\rangle_S \), respectively. Here,

\[
F_2 = \frac{(r^2 + r_0^2 + 2)^4}{16(r^4 + r_0^4 + 2)}, \quad F_3 = \frac{(r - r_0)^4}{4(r^2 + r_0^2)^2},
\]  

(18)

\[
F_4 = \frac{(1 - r_0)^2(r - r_0)^2}{4(1 + r_2)(r^2 + r_0^2)}.
\]

The fidelities of our HBSG scheme vary with the parameter \( g/\sqrt{\kappa \gamma} \), shown in Fig. 5(a). For our HBSG scheme, the efficiency is defined as the ratio of the number of the output photons to the input photons, which can be written as

\[
\eta_1 = \frac{1}{2} \left( r^2 + r_0^2 + 2 \right)^4.
\]  

(19)

The efficiency of our HBSG scheme varies with the parameter \( g/\sqrt{\kappa \gamma} \), shown in Fig. 5(b).
for the hyperentangled Bell state $|\Phi\rangle$ vs the parameter $\kappa_s/\kappa$. Here, the efficiency of our HBSA protocol is

$$\eta = \frac{1}{2^6}(r^2 + r_0^2 + 2)^8.$$  

Both the fidelity and efficiency of our HBSA scheme vary with the parameter $g/\sqrt{\kappa'\gamma}$, shown in Fig. 6. The plots indicate that for $g/\sqrt{\kappa'\gamma} \geq 0.5$, the higher fidelities and efficiencies of our HBSG scheme depend on the higher NV-cavity coupling strength. When the parameter $g/\sqrt{\kappa'\gamma}$ is larger than 3, the fidelity and the efficiency will be higher than $F = 99.58\%$ and $\eta = 51.48\%$ for the leakage rates $\kappa_s/\kappa = 0.06$.

The fidelity of our HBSG and HBSA schemes can be reduced a few percent because of the imperfection of electron-spin population, the imperfection of the NV electron-spin readout, and the imperfection of frequency-selective microwave manipulation. For $m_s = 0$, the preparation fidelity is $99.7 \pm 0.1\%$, and for $m_s = \pm 1$, it is $99.2 \pm 0.1\%$ [49]. The average readout fidelity of the NV electron spins is $93.2 \pm 0.5\%$ [49]. By using isotopically purified diamonds and polarizing the nitrogen nuclear spin, we can reduce the microwave manipulation imperfection. The spin decoherence may also reduce the fidelity of our schemes. In our schemes, the single photons are successively sent into the devices, and the time interval $\Delta t$ between two photons should be less than the electron-spin coherence time $T$. In experimental cases, the spin-relaxation time $T_1$ of NV centers in diamond scales from microseconds to seconds at low temperature and the dephasing time $T_2$ is about 2 ms in an isotopically pure diamond [33,50]. The electron-spin coherence time $T > 10$ ms [51], which is much longer than the photon coherence time $\sim 10$ ns and the subnanosecond electron-spin manipulation control [52].

For practical operations, the photon loss due to absorption and scattering, the mismatch between the input pulse mode and MTR, and the inefficiency of the detectors will bring ineffectiveness to our schemes. Interestingly, the photon loss will just affect the success efficiency, rather than the fidelity. The present experimental technology can generate 300 000 high-quality single photons within 30 s [53]. Another key ingredient of our protocols is the coupling between NV centers and MTRs. In realistic experiments, the strong coupling between the NV centers and the WGM has been demonstrated in different kinds of microcavities [37,54–58]. The coupling strength between NV centers and the WGM can reach $g/2\pi \sim 0.3−1$ GHz [37,55–57]. The $Q$ factor of the chip-based microcavity is higher than 25 000. Considering the parameters $[g_{ZPL},\kappa,\gamma_{total},\gamma_{ZPL}]/2\pi = [0.30,26,0.013,0.0004]$ GHz (where ZPL is the zero phonon line) of an NV center coupled to a microdisk [55], we have $g \approx 3/\sqrt{\kappa'\gamma}$ and the fidelities of our HBSG and HBSA schemes can exceed 99%. Therefore, our protocols are feasible in experiment.

Compared with previous works [15,17,18], these two schemes relax the difficulty of their implementation in experiment as it is not difficult to generate the $\pi$ phase shift in single-sided NV-cavity systems. Moreover, single-sided NV-cavity systems have a long coherence time even at room temperature (1.8 ms) [50], different from quantum-dot–cavity systems. The first HBSA scheme by Sheng et al. [15] was achieved with cross-Kerr nonlinearity. It is perfect in theory [59,60]. The second HBSA scheme by Ren et al. [17] with single-sided quantum-dot–cavity systems requires the $\pi$ phase difference of the Faraday rotation between a hot cavity and an empty cavity, and it can be achieved with the nonlinear interaction between a photon and a quantum dot twice. Interestingly, our schemes do not require that the transmission for the uncoupled cavity is balanceable with the reflectance for the coupled cavity. Moreover, the coherent manipulation of the spin of a single NV center to accomplish quantum information and computation tasks at room temperature has been presented [61,62], which provides the basis for the current schemes.

In summary, we have proposed two efficient schemes for the deterministic generation and the complete nondestructive analysis of hyperentangled Bell states in both polarization.
and spatial-mode DOF assisted by NV centers in MTRs. The HBSG protocol can also be extended to achieve the generation of multiphoton hyperentangled GHZ states efficiently. Compared with previous works [15,17,18], our schemes relax the difficulty of their implementation in experiment. Our calculations show that the proposed schemes can work with a high fidelity and efficiency under the current experimental techniques, which may be a benefit to long-distance high-capacity quantum communication, such as quantum teleportation, quantum dense coding, and quantum superdense coding with two DOF of photon systems.

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