A spatiotemporal structure: common to subatomic systems, biological processes, and economic cycles

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Abstract. A theoretical model derived based on a quasi-stability concept applied to momentum conservation (Naitoh, JJIAM, 2001, Artificial Life Robotics, 2008, 2010) has revealed the spatial structure of various systems. This model explains the reason why particles such as biological cells, nitrogenous bases, and liquid droplets have bimodal size ratios of about 2:3 and 1:1. This paper shows that the same theory holds true for several levels of parcels from baryons to stars in the cosmos: specifically, at the levels of nuclear force, van der Waals force, surface tension, and the force of gravity. A higher order of analysis clarifies other asymmetric ratios related to the halo structure seen in atoms and amino acids. We will also show that our minimum hypercycle theory for explaining the morphogenetic cycle (Naitoh, Artificial Life Robotics, 2008) reveals other temporal cycles such as those of economic systems and the circadian clock as well as the fundamental neural network pattern (topological pattern). Finally, a universal equation describing the spatiotemporal structure of several systems will be derived, which also leads to a general concept of quasi-stability.

1. Introduction
There are some mysterious fractal features common to the levels of various systems ranging from subatomic to cosmic.

First, let us consider a spatial aspect related to the golden and silver ratios. A neutron impacting uranium 235 produces smaller child atoms that often have an asymmetric weight ratio of about 2:3 between the golden and silver ratios. In contrast, varying the impact speed of neutrons results in a nearly symmetric division of uranium 235. [1] Purines and pyrimidines in biological base pairs, biological cells after divisions, liquid droplets, and stars in the cosmos also have a fusion of symmetry of 1:1 and asymmetry of around 2:3. [2-5] There are also mesons with “two” quarks and baryons with “three”. [6] A model we developed previously based on fluid dynamics has revealed the reason for the fusion of symmetry of 1:1 and asymmetry of around 2:3 in the fractals found in nature.[2-5] The fusion of symmetry and asymmetry appears in various systems from atoms to stars, because each system is commonly related to a “breakup of flexible particles deformed”. Then, a higher order of analysis clarifies the reason why size ratios over 2.0 are also seen in other biological molecules such as amino acids as well as liquid droplets.[7, 8]

Second is a rhythmical aspect in time. We can see several chemical oscillation phenomena in the biosystems related to the origin of life, bacteria, morphogenetic processes, the brain, and the economy of the human network. The Brusselator, circadian clock, and heartbeat are also well-known examples. [9, 10] Until recently, little was known about the relation between these oscillations.

We have revealed that the core engine for giving life to living beings is a system of four categories of biological molecules, consisting of two types of information molecules and two types of functional molecules and resembling a four-stroke engine. [4] We have previously showed how the origin of life can be explained by this four-stroke engine, representing a minimum hypercyclic chemical reaction system. [4] This four-stroke engine induces the exponential growth of bio-systems.
Subsequently, fifth and sixth categories of molecules for repressing DNA replication were added to
the four-stroke engine, resulting in several oscillating phenomena with chemical reactions. [11-13] A
mathematical model based on these six categories of molecular groups showed that the oscillation
cycle has a rhythm of about “seven” times the fundamental beat. [11-13]
This model clarifies the common principle underlying several levels of bio-chemical oscillations,
because new organs emerge at “sevenfold” divisions of stem cells, while the circadian clock of about
24 hours is seven times the fundamental clock of about 3.5 hours. It is also stressed that one week has
“seven” days. Several economic cycles are seven times the fundamental durable periods of certain
products.

This interdisciplinary research spanning several categories of natural systems is expected to be
effective, because we cannot see subatomic particles, the origin of the cosmos, or the origin of life, nor
do we possess sufficient data on living beings and economical process.

However, little is known about the relation between spatial and temporal features. Thus, in this paper
we will show a universal equation describing the spatiotemporal structure of various systems along
with a generalized concept of quasi-stability.

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2. Spatial structure

2.1. Model [2-5]

Here, we define a parcel as a flexible spheroid having two long and short radii of
\[ a(t) \] and \[ b(t) \] dependent on time \( t \), for the aggregation of neutrons and protons in each child atom
resulting from the fission of a uranium 235 atom, a nitrogenous base in biological base-pairs of nucleic
acids hydrated with a lot of water molecules, a biological cell just before division, and a star at
breakup in the cosmos. The parcel becomes a sphere of the radius \( \left[ \frac{ab^5}{12} \right]^{1/3} \) under an equilibrium
condition. The deformation rate \( \gamma(t) \) is defined as \( \frac{a(t)}{b(t)} \), while a sphere without deformation

Then, we consider the form of two spheroid parcels connected in line at the time of the breakup
processes of uranium 235, at the replication stage of biological base-pair, at cell division, and at
division of star. This is because atoms, nitrogenous bases, cells, and stars will have the shape of a
gourd with two kinks at the later stage of breakup or at the first stage of coalescence. We derive a
theory for describing the deformation and motions of the two connected spheroid parcels having two
radii of \( r_{d1} \) and \( r_{d2} \) under equilibrium conditions and two deformation rates of \( \gamma_k \) \( [k = 1, 2] \), while the
size ratio of the two parcels is defined by \( \varepsilon = \frac{r_{d1}}{r_{d2}} \).

We model the relative motion between the two parcels, nonlinear convections inside the parcels, and
the interfacial force at the parcel surface. The interfacial force is evaluated in the form of \( \frac{\sigma}{r^m} \)
where \( m \) and \( \sigma \) are constants and \( r \) is the curvature of parcel surface. Several types of forces such as
nuclear force, van der Waals force, surface tension, coulomb force, and gravity can be explained by
varying \( m \). The relation \( m = 1 \) implies the surface tension of liquid. The mean density of the parcels
is \( \rho_L \).

We assume that the convection flow inside parcel is irrotational, i.e., potential one. There are random
collisions of water molecules and electrons with the parcel such as biological molecules and cells. It is
stressed that these random collisions from the outer region induce potential flow inside the flexible
continuum particle, i.e., irrotational flow, because the fluctuations of impulsive starts and stops
generate potential flow. [5] This potential flow is also applicable, because fluctuations entering the
parcels such as bases, cells, and atoms will be close to those of thermal fluctuations, which are less dissipative. (Fluctuation dissipation theorem) It is well-known that the potential flow assumption is also applicable for liquid droplets. [2, 3] The potential assumption will be no problems also for stars, because of large size and high speed.

Moreover, we must consider that a parcel is not often a continuum, because the number of nucleons and water molecules inside the parcels for atom and nitrogenous base will be less than the order of 1,000. The scale for averaging, i.e., the minimum scale representing the phenomenon, will be smaller than that in continuum mechanics. Thus, this small averaging window leads to a weak indeterminacy of physical quantities such as deformation and density because of discontinuity of nucleons and molecules.

Here, we derive the relation between dimensionless deformation rate $\gamma_i'(\equiv a_k / b_k [k = 1, 2])$ of each parcel dependent on dimensionless time $\tau_k = \sqrt[2]{\frac{8\sigma}{\rho \mu r_{2w}^2}} [k = 1, 2]$ and the size ratio of the two parcels of $\varepsilon = r_{d1} / r_{d2}$.

The stochastic governing equation having indeterminacy can be described as

$$\frac{d^2}{dt^2} \gamma_i = \left\{ m_{ij} \left( \frac{d}{dt} \gamma_j \right)^2 + m_{ij} \left( \frac{d}{dt} \gamma_j \right)^2 + m_{ij} \gamma_i \gamma_j \right\} / \text{Det} + \xi_i$$

[for $i = 1, 2, j = 1, 2, i \neq j$]

with

$$m_{ij} = \left[ (-\varepsilon - \varepsilon^4 + \frac{2}{3} \epsilon \varepsilon_0 \gamma_i^{-1/3}) B_{ij} + \frac{2}{9} \varepsilon^2 \varepsilon_0 \gamma_i^{-4/3} \right]$$

$$m_{ij} = \left[ \frac{2}{3} \varepsilon^2 \varepsilon_0 \gamma_i^{-1/3} B_{ij} - \frac{2}{9} \varepsilon^2 \varepsilon_0 \gamma_i^{-4/3} \right]$$

$$m_{ij} = (-\varepsilon - \varepsilon^4 + \frac{2}{3} \epsilon \varepsilon_0 \gamma_j^{-1/3}) C_{ij}$$

$$m_{ij} = \frac{2}{3} \varepsilon^2 \varepsilon_0 \gamma_j^{-1/3} C_{ij}$$

$$\text{Det} = (-\varepsilon - \varepsilon^4 + \frac{2}{3} \epsilon \varepsilon_0 \gamma_i^{-1/3} + \frac{2}{3} \epsilon \varepsilon_0 \gamma_j^{-1/3}) B_{ok} = \frac{1}{3 \gamma_k^2} \left( \frac{\gamma_k^2 - 2}{\gamma_k^2 - 1/2} \right),$$

$$C_{ok} = \frac{3}{8} \frac{2 \gamma_k^m}{\gamma_k^2 - 1/2},$$

and $E_{ok} = 3 - \frac{\gamma_k^7/3}{\gamma_k^2 - 1/2}$ [for $k = 1, 2$]

where the parameter $\delta_i$ denotes random fluctuation. It is stressed that this system is not the simple two-body problem of rigid body, because of flexible nonlinear deformations of the parcels.

[Equation 1 is derived only by the above assumptions and also purely mathematical transformation. The long derivation of Eq. (1) is in Ref. 3 confirmed by the referees, although only the stochastic term $\delta_i$ is not in Ref. 3.]
We then define the deviation from a sphere as \( y_i \), which is equal to \( n_i - 1 \). Taking the first order of approximation in the Taylor series leads to

\[
\frac{d^2 y_i}{dt^2} = \left[ -\frac{2}{3} \left( 3 - \varepsilon^3 - 2\varepsilon^{3\text{m}} \right) \left( \frac{dy_i}{dt} \right)^2 + 3(3 - \varepsilon^3) m y_i - 4\varepsilon^{1\text{m}} \left( \frac{dy_i}{dt} \right)^2 + 12\varepsilon^{1\text{m}} m y_i \right] / [3(\varepsilon^3 + 1)]
\]

\[ + \delta_{\text{st}}, \]

(2)

where the parameter \( \delta_{\text{st}} \) denotes random fluctuation.

Existing experimental data on breakup processes of liquid droplets of water and oil and biological cells show spheroid shapes at the timing of breakup or the later stage of breakup, which is the rate-determining stage. This also leads to a possibility that, for several levels of parcel breakups from subatomic to cosmic ones, the parcels will take approximately spheroid shape at the later stage after deformation. Thus, the present model reveals the essential principle underlying the fission process, biological divisions, and stars, whereas multi-dimensional calculations using supercomputers shows the process in detail.

2.2. Size (representative length)

Equation 2 shows that a symmetric ratio of 1.0 (\( \varepsilon=1 \)) makes the first term on the right-hand side of the equation zero, while an asymmetric ratio of \( \sqrt[3]{3} \) around 1.5 (\( \varepsilon^3=3 \)) makes the second term zero for each \( m \). The size ratios of 1.00 and approximately 1.50 can be described by the unified number of the n-th root of n.

We define a system as being quasi-stable when only one term on the right-hand side of the differential equation system governing the phenomenon is zero. (The quasi-stable principle may also be defined by the condition that either of volume or surface forces is zero.) The system of two parcels connected is relatively quasi-stable because \( d^2 x / dt^2 \) becomes smaller when the size ratio of connected parcels takes the values of \( \varepsilon=1 \) or \( \varepsilon^3=3 \).

The concept of quasi-stability is weaker than neutral stability. [3, 4, 5] The quasi-stability concept is necessary for living beings, because stronger stability cannot bring variations, i.e., adaption for environmental change and evolution. This quasi-stability is also possible for nonliving systems such as atoms, because atomic systems also vary in a long time. (This is also because both atomic systems and living beings have dense interaction between particles in closed volumes.)

Next, let us look at biological systems containing water flows. We can classify the five bases of adenines (A), guanines (G), cytosines (C), thymines (T), and uracils (U) into two groups: purines and pyrimidines. Purines, i.e., A and G, have a relatively large size, while pyrimidines, i.e., C, T, and U, are small. Asymmetric base pairs such as the Watson-Crick type of about 1:1 are used in living beings. This grouping specifically refers to the asymmetric size ratio of purines and pyrimidines of around 1.50 in their hydrogen bonds within DNA and RNA, although a symmetric size ratio of 1.00 is often observed in RNA. [4, 5] Symmetric and asymmetric size ratios are also observed at the cell level of microorganisms such as yeast. [2, 3]

These ratios of 1:1 and about 1:1.5 also correspond to those of child atoms generated by the breakup of uranium 235. This means that the probabilities of the size ratios of 1:1 and about 2:3 are relatively high, because uranium will have the shape of a gourd with two kinks just before its breakup.

The quasi-stable ratios of 1:1 for n=1 and 3 appear for each \( m \) (see Eq. 2). This universality also leads to the possibility that the present model can be applied for several levels of parcels from baryons to stars in the cosmos [5, 6, 14]: specifically, at the level of nuclear force, coulomb force, van der Waals force, surface tension, and force of gravity.
As Eqs. 1 and 2 show a slightly vague solution for the phenomenon, this indeterminacy also implies that size variations of $\varepsilon$ are possible in a limited range. This indeterminacy permits the possibility of sizes around 1:1 and also around $\frac{\sqrt{3}}{3}$, i.e., between 1:1 and about 2:3, although 1:1 and about 2:3 are more.

Let us consider the reason why the energy conservation law and variation principle such as the Bohr model do not explain the fusion of symmetry of 1:1 and asymmetry of around 2:3 in size and density (number density). The reason is that the momentum conservation in Eq. 1 models all of the relative motion between two parcels, nonlinear convections inside the parcels, interfacial forces at the parcel surfaces, and collisions with smaller molecules, whereas previous models based on energy conservation have tended to eliminate the relative motion between parcels. A second reason is that the quasi-stability principle was proposed instead of the variation principle.

Another important point is that natural systems should be explained in terms of four conservation concepts regarding mass, momentum, moment, and energy, as is seen in the thermo-fluid dynamics. Conservative quantity is not only energy. The present new explanation for two particle systems is necessary, because explanations based on two conservativity principles such as momentum and energy should be done. The following section also shows the importance of mass conservation law.

[The effect of special theory of relativity between mass and energy will not influence for the size ratio of child atoms obtained after the fission process of uranium 235 very much, because the effect work on both two child atoms at an identical rate. Moreover, impact of only one neutron to uranium will not produce fast deformations of uranium at the later stage of fission.]

2.3. Number ratio

It is also well known that several atoms in nature have the number ratios of protons and neutrons between 1:1 and 2:3. Here, let us examine the reason why larger atoms have larger number ratios close to 2:3.

The first and second terms on the right-hand side of Eq. 2 also show that symmetric ($\varepsilon=1$) and asymmetric ($\varepsilon^3=3$) parcel divisions are more stable for small disturbances of motion ($dy/dt$) and parcel deformation $y$, respectively (Fig. 1).

Let us separate baryon aggregation into two parts: the internal side around the center of the aggregation and the external side close to the surface. External baryons close to the surface move relatively easily, because one part of the baryon is free without any connection to other baryons. However, internal baryons often receive forces from many directions due to the presence of other ones, making it relatively difficult for them to move relative to the origin on the earth. Thus, inner baryons deform relatively easily without any translational motion of the gravity center. As a result, the inner and outer baryons determine whether baryons are asymmetric or symmetric, respectively. [4, 5] An important point is that the concept of inner asymmetry and outer symmetry can be seen.

Larger aggregations of parcels such as Thorium (Th) including baryons more than Helium (He) have more inner baryons, because the surface/volume ratio of the aggregation becomes smaller as size increases. (Fig. 2) More inner baryons for larger atoms bring more asymmetric number ratio of protons and neutrons, as larger nucleic acids such as rRNA also have more asymmetric number ratios of purines and pyrimidines close to 2:3. (Fig. 3) This is because of mass conservation law, i.e., because heavier particles such as purines and protons can be generated with less numbers. [4, 5]

A mysterious thing is that the masses of stable proton and neutron are almost same, while child atoms generated by fission of uranium 235 and nitrogenous bases (pyrimidine and purine) have the different weight ratio around 2:3. Qualitatively, proton is a little heavier than neutron. Thus, relatively heavy protons can be with less number in atomic core, because of mass conservation law. We may quantitatively be able to explain this mystery by the fact that the mass ratio of proton and neutron are around 2:3 as the forms such as lambda particle before atomic cores are stabilized, because the up and down quarks have the weight ratio between 1:1 and 1:2. Thus, the later stage of fission process for
generating atomic cores is described by the present theory. Then, the final or post-fission stage generates the atomic cores of stable protons and neutrons. This explanation is possible by the analogy with biological system, because nitrogenous bases are also connected with heavy molecules such as ribose or deoxyribose, which lead to nucleic acids. The weight ratio of purine-deoxyribose and pyrimidine-deoxyribose is close to 1:1, i.e., symmetric value. (Fig. 4)

The present theory also indicates that the number ratios around 1:1.3, around the center between 1:1 and about 1:1.5, will be relatively unstable. (Fig. 3) This can be understood by the fact that both fluctuations of deformation and velocity enter into the parcels connected with the medium number ratios of about 1:1.3, whereas only velocity fluctuations enter into small parcels such as those of Helium (He) and only deformations occur for large atoms such as Thorium (Th). Actually, atoms, which are a little larger than lead (Pb), are unstable.

Atoms larger over 300 nucleons are unstable, which cannot have the size ratios such as 1:1 and about 2:3. This can be explained by the fact that larger deformations enter into the atoms, because larger atoms get larger deformations.

**Figure 1** Asymmetric division due to parcel deformation and symmetric division in the absence of deformation.

**Figure 2.** Increase in inner parcels according to the increasing number of parcels.
Thus, this theory explains the reason why relative heavy atoms have more asymmetric number ratios close to 2:3 and also why the atoms having the number ratios around 1:1.3 are relatively unstable, while the energy conservation law such as those of Bohr clarifies the inevitability that neutron-rich system is more stable than proton-rich one. [The quasi-stability concept applied to momentum conservation law and mass conservation law bring the result compatible with that the Bohr’s drop model based on the variation principle for energy conservation law and also gives a further physics underlying the Shell model related to the magic number.]

2.4. Taylor series
Next, we take a higher order of the Taylor series for Eq. 1. Even-numbered terms such as the second and fourth ones show no other quasi-stable size ratios. However, odd-numbered terms result in other quasi-stable ratios. The third term in the Taylor series results in a quasistable ratio of about 3.5, the fifth term in ratios of about 2.5 and 2.1, and the seventh in a ratio of about 1.78. An important point is
that the terms of orders higher than the ninth are absolutely unstable, which leads to the existence of the maximum size limit of atoms. (Table 1)

It is also stressed that liquid fuel droplets generated by injectors and child atoms broken up from the fission of uranium 235 also have a threefold variation of sizes at the maximum and also that the molecular weights of the twenty types of amino acids show a threefold variation between 240 of cysteine as the maximum and 75 of glycine as the minimum. [1, 4, 5] The higher-order of analysis clarifies the ratios over 2:3 in several systems. (Table 1)

Free atoms in atmosphere and molecules outside living beings are not with the special ratios such as 1:1, about 2:3, and 3.5, because free atoms in atmosphere and molecules outside cells are relatively difficult to interact each other, while subatomic particles such as baryons and biological molecules like bases, nucleic acids, amino acids, proteins are strongly interactional inside closed regions.

Table 1 Quasi-stable ratios observed in atoms and biological systems.

| n-th term in Taylor series | Quasi-stable ratio | Atoms | Biological molecules |
|---------------------------|-------------------|-------|----------------------|
| 1                         | 1:1 & about 1.1.5 | Main size ratios of child atoms broken from uranium 235. Number ratios of neutrons and protons in atoms | Nitrogenous bases in DNAs and RNAs (Purines:Pyrimidines), DNA polymerases I, II, and III (I:II, I:III) |
| 3                         | about 1:3.5       | Higher limit of size ratios of child atoms broken from uranium. Number ratio of protons and neutrons in He 10 | Amino acids |
| 5                         | about 1:2.5 & about 1:2.1 | The intermediate size ratios of child atoms broken from uranium. Number ratio of protons and neutrons in Mg 32 | rRNA (23S:16S) |
| 7                         | about 1:1.78      | The intermediate size ratios of child atoms broken from uranium. | Ribosome (50S:30S) |
| 9, 11                     | unstable         | -     | -                    |

2.5. Further possibilities

Our previous report based on the quasi-stability concept applied to momentum conservation [2-5, 7, 8] revealed the reason why several particles such as biological cells, nitrogenous bases, and liquid droplets have the bimodal size ratios of about 2:3 and 1:1. There are mesons with “two” quarks and baryons with “three”. Therefore, an analysis based on Eq. 1 may also clarify the ratios in the elemental particles such as quarks.

The present paper extended with stochastic mechanics and indeterminacy principle and the higher order of analyses also reveal the other ratios over 2:3. The halo structures such as H10 and M32 have the number ratios of neutrons and protons over 2:3. [16] These number ratios similar to those in largest and smallest child atoms broken from uranium 235 and biological molecules such as amino acids will also be explained by the present theory of the higher-order of accuracy.

3. Temporal structure

3.1. Minimum drive element for the origin of life (Fore-stroke engine of biomolecules) [4]
Both information strings such as DNA and catalysts such as enzymes are necessary for bio-systems, which can replicate to counter the natural degradation of biomolecules. At least two types of information strings for the origin of life are necessary, which code catalysts for generating information strings and those for generating catalysts. This also implies the inevitability of two catalysts. Thus, the minimum system for self-organization at the origin of life has four types of molecules: two information strings and two catalysts.

We call molecules having information such as DNA “infostrings”, while the catalysts such as enzymes are called “catalysts”. The core system shown in Fig. 5 includes four molecules consisting of Infostring 1 that codes the catalyst generating infostrings, Infostring 2 that codes the catalyst for generating catalysts, Catalyst 1 generated by Infostring 1, and Catalyst 2 produced by Infostring 2. This four-cycle system works as a closed loop, if elements such as nTP and amino acids are input from the outside with energy. This is the minimum 4-stroke engine (minimum hypercycle), which produces molecules exponentially in cases where there are no degradations.

We define the densities of Infostrings 1 and 2 as $D_1$ and $D_2$, respectively. Then, the densities of Catalysts 1 and 2 are defined as $R_1$ and $R_2$, respectively. This yields the following essential equation system.

\[
\begin{align*}
D_i^{N+1} - D_i^N &= \alpha_i D_i^N \otimes R_i^N \\
R_i^{N+1} - R_i^N &= \alpha_i D_i^N \otimes R_i^N
\end{align*}
\]  

(3a, 3b)

where $D_i \otimes R_j$ denotes the smaller value among $D_i$ and $R_j$ with $i=1, 2$ and $j=1, 2$. Then, $\alpha_i$ and $N$ imply an arbitrary constant and generation, respectively.

Equation 3 can also be written by the form of differential equation as

\[
\begin{align*}
\frac{d}{dt} D_i &= \alpha_i D_i \otimes R_1 \\
\frac{d}{dt} R_i &= \alpha_i D_i \otimes R_2
\end{align*}
\]  

(4a, 4b)

Results computed with this equation system show nearly exponential increases in $D_i$ and $R_i$. It is stressed that variable $R_1$ on the left-hand side of Eq. (3b) never appears on the right-hand side of the equation, although common variables can be observed on both sides of Eq. (3a). Equation (3b) essentially shows the asymmetry topologically.

The simplest hypercycle of $D_1$, $D_2$, $R_1$ and $R_2$ works as “the engine of life” for promoting more complex reaction networks.
3.2. Minimum oscillator for morphogenetic process (Six-stroke engine of bio-molecules) [11-13]

Let us consider bacteria, eukaryotes, and cells in higher-order living systems. Four types of fundamental molecules (D1, D2, R1, and R2) must be extended to four molecular groups, because life today uses numerous types of genes and proteins. We can then define four macroscopic groups as follows.

D1: Gene group 1 that codes enzyme system R1 for DNA production
D2: Gene group 2 that codes enzyme system R2 for enzyme production
R1: Enzyme system 1 for DNA production, including DNA polymerases, topoisomerases, helicases, ligases, and primases
R2: Enzyme system 2 for enzyme production, including ribosomes with rRNAs and ribosome proteins, RNA polymerases, mRNAs, and tRNAs

It is stressed that D2 and R2 are for activating reactions, as is shown in the previous section. Next, we define D3 and R3 as other molecular groups for repressing reactions as in the case of transcription factors. These two groups are incorporated in the four groups of D1, D2, R1, and R2, because today’s cells and the morphogenetic processes of multi-cellular systems use negative controllers such as Oct-4 and SOX2 for producing tissues and organs [17]. Accordingly, D3 and R3 can be defined as follows.

D3: Gene group 3 that codes protein system R3 for repressing Di gene expressions
R3: Protein system 3 for repressing Di gene expressions, including the proteins of the transcription factors such as those of Oct-4 and SOX2

This leads to a macroscopic model having six types of molecular groups, or in other words, a six-stroke engine. (Fig. 6) We can describe the densities of the six groups at generation \( N \) after the mother cell generation by the following equations.
\[ D_i^{N+1} - D_i^N = \alpha_i D_i^N \otimes R_i^N, \quad (i = 1, 2, 3) \]

\[ R_i^{N+1} - R_i^N = \alpha_3 \delta (D_i^N - \epsilon_i R_3^N) \otimes R_2^N, \quad (i = 1, 2, 3) \]

where \( D_i \otimes R_j \) denotes the smaller value among \( D_i \) and \( R_j \) with \( i = 1, 2, 3 \) and \( j = 1, 2, 3 \) and also where \( \delta (x) \) denotes the larger one of \( x \) or 0, i.e., \( \max (x, 0) \). Parameter \( \epsilon_i \) with \( i = 1, 2, 3 \) denotes an arbitrary constant for controlling repression rates. In this report, we set to be \( \epsilon_1 = \epsilon_3 = 0 \), while \( \epsilon_2 = 1.0 \), because R3 usually represses D2.

Figure 6 Six-stroke engine as a minimum clock.

Figure 7 obtained by solving Eq. 5 shows the density ratios of \( D_i \) and \( R_i \). The condition of D2/R3 > 1.0 means that a part of D2 is not covered by R3. The condition of D2/R3 < 1.0 means that D2 is completely blocked by redundant R3. When R3 is dense in stem or induced pluripotent stem (iPS) cells, it means that stem and iPS cells can be reprogrammed by the presence of much Oct-4. This oscillation of D2/R3 will lead to changes in the gene combination for expressions, i.e., the emergence of new organs. Figure 7 clearly shows that the antagonism between the negative controller R3 and the positive replication factors R1 and R2 induces bifurcation events at rhythmic intervals constituting about seven divisions, although the intervals are slightly chaotic and the vibrational amplitude is attenuated.

The mathematical model based on these six categories of molecular groups, Eq. 5, showed that the oscillation cycle has a rhythm of “seven” times the fundamental beat. [11-13] It is stressed that this cycle of about seven divisions, i.e., the branching time between periodic bifurcation events, corresponds to the emergence timing of blast cysts, embryos, germ layers, tissues, and organs, which can be observed for about every seven cell divisions. Emphasis is placed on the fact that three system parameters, \( \alpha_{12} \), have less influence on the time cycle. The cycle of about seven divisions is fairly
stable. (The time cycles are between four and nine beats, while varying $\alpha_{ij}$ between 0.0 and 5.0. The cycle is about sevenfold beat on the average.)

![Diagram of time histories of Di and Ri during 15 generations, with the initial conditions of Di = 1 and Ri = 1.](image)

Figure 7 Time histories of Di and Ri during 15 generations, with the initial conditions of Di = 1 and Ri = 1.

3.3. Economic cycles and human society

There are several types of boom-and-bust economic cycles: the Kondratiev cycle of about 50 years, the Juglar cycle of about 7 years, and the Kitchin cycle of about 3.5 years, which are observed in human network systems. [18, 19]

The shortest economic cycle is one week of “seven” days. The longest repetitive catastrophes occur about every 70 years, which is seven times the actual durable period of automobiles of about 10 years. Thus, we hypothesize that several types of economic cycles are about seven times the length of the fundamental production cycles (durable periods). New organs that emerge with a sevenfold rhythm may correspond mathematically to the rise of new production or new business categories. [We classify all things in our economic system into two major categories: information and functional objects. Computers and industrial robots are examples of functional objects. Functional objects can be further divided into two subgroups: those such as computers for generating information and those such as industrial robots for generating functional products. Information can also be classified into two subgroups: for designing information machines such as computer and for functional products. These four subgroups correspond to D1, D2, R1, and R2 in Eqs.(3) and (5). Then, fifth subgroup of R3 such as alcohol for depressing D2 and sixth subgroup of D3 for generating R3 lead to the cycles.]
3.4. Fundamental neural network pattern

Next, let us remind the circadian clock in the brain, having a cycle of 24-25 hours. We know that, when suprachiasmatic nuclei (SCN) in brain are removed, the clock shows a cycle of about 3-4 hours. [20] This means that the sevenfold-beat cycle based on the six-stroke engine of molecules will also be present in the brain. The fundamental topological pattern of neural network in brain may also be that in Fig. 6, which is similar to Fig. 8. Six variables of Di and Ri (i=1-3) are redefined as activation level of neurons.

Neurons D1, D2, D3: Provoked pulse induced by input from outer region to Di goes into R1, which activates Di cyclically.

Neuron R2: Provoked pulse induced by input from outer region to R2 goes into D2, which activates R2 as accelerator.

Neuron R1: Provoked pulse induced by input to from outer region to R2 goes into D1, which activates R1 as accelerator.

Neuron R3: Provoked pulse induced by input from outer region to R2 goes into D3, which activates R3 as depression neuron.

Several systems and functions inside brain such as memory, thinking, pyramidal area, emotion, language, visual and auditory senses may also be governed by this seven-beat cycle of Fig. 8. It is also
stressed that the fundamental neural network pattern inside brain in space (topological pattern) will also be dominated with a fusion of symmetric and asymmetric connections in Fig. 8.

4. Quasi-stability extended to spatiotemporal structure

Let us think about whether or not the quasi-stability concept can also be applied to the evolutionary process. The macroscopic kinetic model describing the time-dependent populations of the four main groups of species in the evolutionary process [21] shows that only one group of species does not increase at the time of a transition in evolution, while the other groups vary in time. [Figs. 5 and 17 in reference 21]

Moreover, Eq. 5 explaining the biological cycles occurring in the brain and morphogenetic processes and economic cycles [11-13] shows that, when only one variable among six is invalid, drastic transitions occur in morphogenetic and economic processes. (Fig. 7)

Thus, a quasi-stable condition, i.e., a weakly stable situation at the transitions such as fission, evolution, and the emergence of organs, can be redefined by the rule that only a part of the equation system, i.e., only one term or one variable, is invalid at the time of a transition.

5. Governing equation unified for both spatial and temporal structures

Equation 4 for $D_1$ can be transformed to the form of

$$\frac{d^2}{dt^2} D_1 = m_1 \left[ \frac{d}{dt} D_1 \right]^2 + m_1 D_1 + m_s.$$  \hspace{1cm} (6)

with the additional term of $m_s$. The result is essentially identical to Eq. 2 and also to the deterministic equation for evolution in Reference 21.

Here, we eliminate only the third term $m_s$ on the right hand side of Eq. 6 to obtain

$$\frac{d^2}{dt^2} D_1 = m_1 \left[ \frac{d}{dt} D_1 \right]^2 + m_1 D_1.$$  \hspace{1cm} (7)

Equation 7 is essentially nonlinear due to the first term on the right-hand side. However, this equation has an analytical solution. [22] Increasing the initial disturbance leads to a transition from a trajectory of oval oscillations to parabolic types. Then, variations of input energy bring two types of modes (oval or parabolic), which are stable sustainment such as information storage and drastic change typified by the emergence of new organs.

We can conclude that living and nonliving systems from atoms to stars employ two types of modes due to Eqs. 6 and 7, in order to use quasi-stable spatiotemporal structures between stable and reorganized conditions.

6. Four-dimensional scenario of the biological self-organizing processes: clarified by the fusion of the foregoing sections

6.1 Quasi-stability and neutral stability in Section 2

Let us think about the asymmetric pattern of the Karman vortex streets [23] generated downstream of a bluff body. Emphasis is placed on the fact that Karman’s theoretical analysis based on the neutral stability principle clarifies only the instantaneous stability of the vortex array in space, but it does not show the temporal development of the vortices. Many researchers have confirmed that the analysis based on the neutral stability at the instantaneous stage has a high probability of showing the specific pattern of vortices over a long period in the natural observation.

Thus, this precedent set by Karman is further evidence that the quasi-stability principle, which is a little weaker than neutral stability, can explain why parcels are connected by biological size ratios such as 1:1, about 2:3 or about 1:3.5.
Here, let us recall that the terms of orders higher than the ninth are absolutely unstable, which leads to the existence of the maximum size limit of parcels in Table 1. This also shows that, in order to execute four-dimensional numerical computations for the self-organizing process of living beings, the 9th order of accuracy is necessary. This is a very important guiding principle for near-future simulations on supercomputers.

6.2 Probability and population in Section 3
The 4- and 6-stroke engine models described in Section 3 are in the category of population dynamics, which reveal the probability of each molecular group. Various processes, including those of living beings, are stochastic because of the indeterminacy shown by Eq. 1 in Section 2. Thus, we should consider the probabilities at which bio-chemical reaction processes such as replication, breakup, and degradation occur with some mutations. The replication process of DNA and cell divisions are not deterministic.

A quasi-stability analysis tells us that, when a base pair of two parcels connected (a base pair hydrated) and a cell just before division have the shape of a gourd with two kinks such as what occur at the later stage of breakup or at the first stage of coalescence of two parcels, the probabilities are high for size ratios of 1:1 and about 2:3, while various size ratios of candidates of nitrogenous bases and amino acids are generated by coalescence and breakup with an energy input. (An example of breakup in living beings is the first stage of DNA replication, while an example of coalescence is the final stage of DNA replication.)

High probabilities of size ratios of 1:1 and about 2:3 stabilize the primitive information of base pairs, which induce the complex information of DNA. As a result, there is greater production of various enzymes with different functions, leading to the probabilities of stable cycles of 4- and 6-strokes shown above. The probability of the above-mentioned scenario for self-organization of today’s human beings and ancient lives is larger than that due to a fully random strategy, although the probability of this scenario based on quasi-stability will not be very high.

Some amino acids softer than the bases may deform easily. This also leads to quasi-stable size ratios over 1.5 (about 1:3.5, 1:2.5, 1:2.1, and 1:1.78) for large deformations, because of the higher order terms of the Taylor expansion from the 3rd to the 7th, while size ratios of 1:1 and about 2:3 are also possible. Some large proteins are also very soft and may be unstable.

Thus, biological processes are not deterministic, nor do they have a one-to-one correspondence because of several passages of chemical reaction networks, which have the diversity of redundancy (many-and-many), degeneracy (many-to-one), and versatility (one-to-many). [24]

The foregoing discussion has concerned parcels connected by hydrogen bonds. Next, let us think about an interesting example of parcels connected by covalent bonds. Our previous report shows that the size distribution of various amino acids is similar to that of human chromosomes. [14] This similarity may imply that the present theoretical model can also be applied to the disconnection of covalent connections.

6.3 Oval and parabola in Section 5
The analytical solution for Eq. 7 in Section 5 shows that increasing the initial disturbance leads to a transition from a trajectory of oval oscillations to parabolic types. This transition implies that variation of the input energy brings two types of modes (oval or parabolic), which make possible both stable sustenance such as information storage in DNA and fast replication of DNA. This tendency of stability against a small disturbance and fast breakup due to a large disturbance is also confirmed by a numerical analysis of Eq. 1, when the size ratios of parcels are 1:1 and about 2:3. [3] This corresponds to the fact that a large energy input in the mitotic phase leads to fast breakup of DNA with base pairs having a size ratio of 1:1 and about 2:3 and more cells, while the size ratios are relatively stable for small disturbances during the silent period without any large energy input.
6.4 Four-dimensional scenario for information, structures, and functions of various biological molecules and organs

There is also a 2:3 ratio in immune globulin (IgX). Two types of large immune globulines (IgM and IgE) and three small ones (IgG, IgD, and IgA) are naturally selected, while the weight ratio of the small and large heavy chains is also 2:3. [11]

The other large molecule, ribosome, includes ribosomal RNA (rRNA). The average frequency ratio of purines and pyrimidines in lots of types of rRNA sequences is actually about between 1.0 and 1.5.

Both hydrophilic and hydrophobic amino acids exist in proteins. If we employ the classification of R-side chain grouping, the frequency ratios of hydrophilic and hydrophobic amino acids in proteins are also asymmetric in many cases, between 1:1 and approximately 2:3. [4] This asymmetric symbiosis of hydrophilic and hydrophobic molecules also produces the complex convexoconcave shapes of proteins, as hydrophobic parts are often folded inside proteins.

How are biological functions generated from the fundamental information represented by the asymmetric and symmetric ratios of 2:3 and 1:1 described in the previous sections?

Purine and pyrimidine pairs do not form in the presence of only one type of base. An extremely large frequency ratio of purine and pyrimidine, say, far larger than 1.5, cannot produce the stem in tRNA. The fact that tRNA has the weakly-asymmetric density ratio of purines and pyrimidines between 1:1 and about 2:3 produces the clover structure with loops (leaves) and stems, because the loci inside the leaves (loops) are free from hydrogen-bonded connections. [4] This clover structure is constructed with three leaves similar to fingers, which can catch other molecules and also carry the other molecules to ribosome. The “catch” is an important function for living being. As larger molecules such as rRNA and some enzymes are also with repeats of the leaves such as those in tRNA, the fingers catch mRNAs.

The above-mentioned asymmetric ratio around 2:3 reveals the relation between the three-dimensional structure and one-dimensional information, i.e., the reason why RNA such as tRNA and rRNA have clover structures with fingers, while DNA has a simple spiral. An important point is that the combination of a string (stem) and a ring (loop) in RNA results in a function, although a string such as DNA with information alone cannot generate a function.

Another interesting point is that the anti-codon inside the leaf of tRNA induced by the asymmetric ratio essentially permits a maximum of sixty-four patterns, because A, U, G, and C are possible for each of the three loci (4 x 4 x 4 = 64). Then, because the third locus inside the anti-codon has a relatively weak connection with the codon, the minimum number of patterns is sixteen. This implies that the number of amino acids is between sixteen and sixty-four. The weak connection at the third locus permits twenty types of amino acids, a little more than sixteen. It is emphasized that clarification of the leaf size with an anti-codon of three loci related to the asymmetric ratio of 2:3 for densities also leads to the general inevitability of twenty amino acids.

There is another important mechanism of the self-organizing process in living beings. It is well known that the electron cloud is not spherical around an atom heavier than helium (He). A non-spherical electron cloud induces string-like and ring-like molecules in amino acids and nitrogenous bases, without any spherical connections of carbons. Nitrogenous bases without a spherical electron cloud generate string-like DNA and RNA. This is an essential principle of chemistry.

Now, let us consider a rotating spheroid that is generated by deformation of a parcel such as a cell. The spheroid is geometrically asymmetric due to its rotating axis. The discrepancy from a complete sphere, i.e., its scabrous shape, induces a string-like connection of deformed parcels, although parcels with less deformation are connected as a spherical aggregation or a spherical surface aggregation in two- or three-dimensional space. Repeats of strings and spheres can also be observed in larger scales. [5, 14] Although oospheres still maintain a three-dimensional spherical shape overall during several divisions after being fertilized by antherozoids, the sphere gradually comes closer to being a one-dimensional string due to asymmetric divisions. The string will be close to the next larger sphere like yarn waste because of its flexibility. [5]
In our previous reports, we also simulated the morphogenetic process of the complex cerebral shape in the brain, including its main blood vessels, as well as that of asymmetric inner organs such as the liver and symmetric outer organs like the arms and legs. [4, 5, 25] These results correspond to the principle of inner asymmetry and outer symmetry discussed in Section 2.

7. Conclusion
A quasi-stability analysis shows that, when a pair of two nitrogenous bases, a cell just before division, an atom at the breakup has the shape of a gourd with two kinks such as what occur at the later stage of breakup or at the first stage of coalescence of two parcels, the probabilities are high for size ratios of 1:1, about 2:3, and also 1:3.5, while various size ratios of candidates of nitrogenous bases and amino acids are generated by coalescence and breakup with an energy input.

The population model of four and six strokes clarifies the standard circuit controlling various stages of living beings.
Fusion research based on time and space demonstrates a possible four-dimensional scenario of biological self-organization, although there are still many mysteries in natural systems.

Appendix: Volume and weight ratios
The weight and size (representative length) ratios of purines and pyrimidines are about 1.5. The asymmetric pair of flexible spheroid parcels of “purines surrounded by water molecules” and “pyrimidines surrounded by water molecules” will also have the size (length) ratio between 1.40-1.50. The weight and size (length) ratios of nitrogenous bases are proportional to the parcel size ratio (parcel length ratio). However, the weight and volume ratios of the two spheroid parcels are 3.0. The difference between the weight ratios of parcels and nitrogenous bases, i.e., about 1.5 for the weight ratio of bases and 3.0 for the weight ratio of parcels, is possible, because the molecular weight ratio of the parcels including water is larger than that of the purine-pyrimidine bases and also because the electric density distribution of Pai electrons will not be in a two-dimensional distribution, while the nitrogenous bases have one-dimensional string (ring).

Emphasis is placed on the fact that the size (length) and weight ratios of child atoms broken up from uranium 235 will also be close to 2:3, because atomic cores are mathematically similar to one-dimensional strings distorted or rings.

What is the difference between the weight ratio of two child atoms separated from uranium 235 and the weight ratio of the parcels including child atoms? It will be possible that the weight ratio of parcels is larger than that of protons and neutrons in atom or child atoms split from uranium 235, because of immerse mass. Immerse mass proportional to the parcel weight only appears while deformation and breakup occur. The immerse mass can be understood analogically from the fluid dynamic effect in which impulsive deformation motion is accompanied with added mass [15]. This immerse mass produces a weight ratio of 3.0 for the parcels, although the weight ratio of child atoms is about 1.5. This mass may be related to gluon, quark condensation, or the relativity theory effect. Parcels smaller than cells may be with immerse mass, which may come from lack of observation techniques. The weight ratios of liquid droplets, biological cells, and stars will be equal to those of parcels.

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