Construction of 1-Bit Transmit Signal Vectors for Downlink MU-MISO Systems: QAM constellations

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Abstract—In this paper, we investigate the construction of transmit signal for a base station (BS) with a massive number of antenna arrays under cost-effective 1-bit digital-to-analog converters (DACs). Because of the coarse nonlinear property, conventional precoding methods could not yield satisfactory performances. Moreover, finding an optimal transmit signal is computationally implausible due to its combinatorial nature. Thus, it is still an open problem to construct a 1-bit transmit signal efficiently. We first derive a feasibility condition which ensures that each user’s noiseless observation belongs to a desired decision region, and then formulate it as linear constraints. Taking into account the robustness to a noise, we develop a mixed-integer-linear-programming (MILP) problem. Also, we propose an efficient algorithm to solve it (equivalently, to generate a 1-bit transmit vector). We further compare the computational complexities of the proposed and existing methods. Simulation results validate the computation complexity and the detection performance of the proposed method.

Index Terms—Massive MISO, 1-bit DAC, Downlink, precoding, Linear programming.

I. INTRODUCTION

In recent years, massive multiple-input single-output (MISO) has been actively investigated for fifth-generation (5G) and future wireless communication systems due to its significant gain in spectral efficiency [1]. In contrast, because of the large number of antennas, dealing with a high hardware cost and considerable power consumption become one of the key challenges. In massive MISO systems, the use of cheap and efficient building block, e.g., digital-to-analog converters (DACs), has attracted the most interest as a promising low-power solution [2], [3]. Considering the same clock frequency and resolution, it is known that DACs have lower power consumption than analog to digital converters, therefore research on low-resolution DACs are often ignored for this reason. However, in downlink multiuser massive MISO systems, the number of transmit antenna at base station (BS) is much larger than the number of receive antennas. In this context, we should consider DACs’ power consumption, cost, and computation complexity. In downlink systems, conventional precoding method such as zero-forcing (ZF) and regularized ZF (RZF) achieve almost optimal performance effectively [4]. These linear precoding schemes have a low complexity and widely used in wireless communication with high resolution DACs (e.g., 12 bits). But in reality, massive MISO must be built with low cost DACs. This is because power consumption due to quantization increases exponentially as resolution increases. Many non-linear precoding methods with phase-shift-keying (PSK) constellation have been studied actively in the system, which achieve good performances with low complexities [5]–[8]. Especially, near-optimal performance with PSK is demonstrated using branch and bound method (B&B) in [8]. However, the above methods cannot be applied to more practical quadrature-amplitude-modulation (QAM) constellations, due to the property of QAM as boundness of decision regions. Recently, some precoding methods with QAM has been investigated in [9], [10]. Exploiting a superposition coding of two QPSK symbols to generalize a 16-QAM symbol, the authors in [9] formulated an optimization problem using gradient projection to obtain 1-bit transmit vectors, and stored them in a look-up-table per coherent channel. In [10], non-linear 1-bit precoding schemes for Massive MIMO with high-order QAM were proposed which are enabled by semidefinite relaxation and \(\ell_\infty\)-norm relaxation. These methods aim at reducing the performance-loss compared with conventional cases with infinite-resolution DACs. In the aggregate, we need to study an effective precoding method resulting in good performance and low complexity at QAM constellation in the downlink MU-MISO system with 1-bit DACs.

Unlike PSK constellations, the boundness of a decision region in QAM constellations should be carefully considered to design a transmit signal vector. Although the 1-bit precoding constructions under QAM constellations have been studied in various perspectives, they do not provide an elegant complexity-performance trade-off. In this paper, we suggest a novel direction to construct a 1-bit transmit signal vector in the system. The first key contribution is to derive a simple feasibility condition which ensures that each user’s noiseless received signal is located in a desired decision region. Also, we present an optimization problem as mixed integer linear programming (MILP), by incorporating the robustness to a noise into the feasibility criterion. Unfortunately, it is too complex to solve the MILP optimally. We thus propose an efficient algorithm to solve the MILP, in which it is first solved with a LP relaxation and then the resulting solution is refined to satisfy the 1-bit constraint. Via simulation results, the proposed method shows better performances than the existing benchmark methods. In addition, the complexity comparisons of the proposed and existing methods demonstrate the potential...
of the proposed direction and algorithm.

This paper is organized as follows. In Section II we provide useful notations and definitions, and describe a system model. In Section III we propose an efficient method to construct a transmit signal vector for downlink MU-MISO systems with 1-bit DACs. Section IV provides simulation results. Conclusions are provided in V.

II. PRELIMINARIES

In this section, we provide useful notations which will be used throughout the paper, and then describe the system model.

A. Notation

The lowercase and uppercase bold letters represent column vectors and matrices, respectively. The symbol $(\cdot)^T$ denotes the transpose of a vector or a matrix. For any vector $x$, $x_i$ represents the $i$-th component of $x$. Let $\{a : b\} \triangleq \{a, a + 1, \ldots, b\}$ for any integer $a$ and $b$ with $a < b$. $\text{Re}(a)$ and $\text{Im}(a)$ represent the real and complex parts of a complex vector $a \in \mathbb{C}$, respectively. Given a $x \in \mathbb{C}$, we let
\[
g(x) = [\text{Re}(x), \text{Im}(x)]^T, \tag{1}
\]
and the inverse mapping of $g$ is denoted as $g^{-1}$. Also, $g$ and $g^{-1}$ are the component-wise operations, i.e.,
\[
g([x_1, x_2]^T) = [\text{Re}(x_1), \text{Im}(x_1), \text{Re}(x_2), \text{Im}(x_2)]^T.
\]
For a complex-value $x$, its real-valued matrix expansion $\phi(x)$ is defined as
\[
\phi(x) = \begin{bmatrix} \text{Re}(x) - \text{Im}(x) \\ \text{Im}(x) \text{ Re}(x) \end{bmatrix}.
\tag{2}
\]
As an extension into a vector, we have $\phi([x_1, x_2]^T) = [\phi(x_1)^T, \phi(x_2)^T]^T$.

B. System Model

We consider a downlink MU-MISO system in which BS equipped with $N_t \gg K$ transmits antennas serves $K$ users, each of which has a single antenna. As the natural extension of our earlier work in [5], where PSK constellations were only considered, this paper focuses on $4^n$-QAM with $n \geq 2$. Let $\mathcal{C}$ denote the set of constellation points of $4^n$-QAM. Also, let $x = [x_1, \ldots, x_N]^T$ be a transmit vector at the BS. Then, the received signal vector $y \in \mathbb{C}^K$ at the $K$ users is given as
\[
y = \sqrt{\rho}Hx + z, \tag{3}
\]
where $H \in \mathbb{C}^{K \times N_t}$ denotes the frequency-flat Rayleigh fading channel, each of which component follows a complex Gaussian distribution with zero mean and unit variance, and $z \in \mathbb{C}^{K \times 1}$ denotes the additive Gaussian noise vector whose each element are distributed as complex Gaussian random variables with zero mean and unit variance, i.e., $z \sim \mathcal{CN}(0, \sigma^2 = 1)$. The signal-to-noise ratio (SNR) is defined as $\text{SNR} = \rho/\sigma^2$, where $\rho$ denotes the per-antenna power constraint. Throughout the paper, it is assumed that the channel matrix $H$ is perfectly known at the BS.

Given a message vector $s \in \mathbb{C}^K$, BS needs to construct a transmit vector $x$ such that each user $k$ can recover the desired message $s_k$ successfully. Toward this, our goal is to construct a precoding function $P$:
\[
x = P(H, s), \tag{4}
\]
which produces a transmit vector $x$ from the channel matrix $H$ and the message vector $s$. Focusing on the impact of 1-bit DACs on the downlink precoding, we assume that BS is equipped with 1-bit DACs while all $K$ users are equipped with infinite-resolution ADCs. Accordingly, each component $x_i$ of the transmit vector $x$ is restricted as
\[
\text{Re}(x_i) \text{ and } \text{Im}(x_i) \in \{-1, 1\}. \tag{5}
\]
Since this restriction causes a severe non-linearity, conventional precoding methods, developed by exploiting the linearity, cannot ensure an attractive performance. The goal of this paper is to construct a precoding function $P(H, s)$ having a manageable complexity and suitable for the considered non-linear MISO channels.

III. THE PROPOSED TRANSMIT-SIGNAL VECTORS

We formulate an optimization problem to construct a transmit-vector $x$ under $4^n$-QAM. Especially, this problem can be represented as a manageable mixed integer linear programming (MILP). We remark that our earlier work on PSK [5] cannot be employed as the decision regions of $4^n$-QAM are bounded (see Fig. 1). For the ease of exploration, an equivalent real-valued expression will be used in the following:
\[
y = \sqrt{\rho}Hx + z, \tag{6}
\]
where $\tilde{y} = g(y)$, $\tilde{x} = g(x)$, $\tilde{z} = g(z)$, and $\tilde{H} = \phi(H) \in \mathbb{R}^{2K \times 2N_t}$ denotes the real-value expansion matrix of $H$.

Before explaining the main result, we provide the useful definitions which will be used throughout the paper.

Definition 1: (Decision region) For any constellation point $s \in \mathcal{C}$, the decision region of $s$ is defined as
\[
\mathcal{R}(s) \triangleq \left\{ y \in \mathbb{C} : |y - s| \leq \min_{c \in \mathcal{C} : c \neq s} |y - c| \right\}. \tag{7}
\]
This region implies that any received signal $y \in \mathcal{R}(s)$ is detected as $s$. Also, the corresponding real-valued decision region is given as
\[
\tilde{\mathcal{R}}(s) = g(\mathcal{R}(s)). \tag{8}
\]

Definition 2: (Base region) A base region $\mathcal{B}_i \subseteq \mathbb{R}^2, \forall i \in [1 : n]$, is defined as
\[
\mathcal{B}_i \triangleq \{ \alpha_1^i m_1^i + \alpha_2^i m_2^i : \alpha_1^i, \alpha_2^i > 0 \}, \tag{9}
\]
where $m_\ell^i$ represents a basis vector with
\[
m_\ell^i = \begin{cases} g(\sqrt{2} \cos(\frac{\pi}{4}(1 + 2i))) & \text{if } \ell = 1 \\ g(\sqrt{2} \sin(\frac{\pi}{4}(1 + 2i))) & \text{if } \ell = 2. \end{cases} \tag{10}
\]

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In the sequel, the decision region in Definition 1 will be represented by the intersections of the $n$ base regions in Definition 2 with proper shift values. This representation makes it easier to formulate an optimization problem.

First of all, we need to decide the size of bounded decision regions, i.e., the parameter $\tau$ in Fig. 1 should be determined. Note that $2\tau = d_{\min}$ denotes the minimum Euclidean distance of the given constellation points. In PSK, $\tau$ is always infinite regardless of a channel matrix, whereas in $4^n$-QAM, it should be well-optimized. Specifically, $\tau$ should be chosen as large as possible to ensure a reliable performance, provided that a noiseless received signal belongs to the corresponding decision regions at all the $K$ users. Unfortunately, it is not tractable to find an optimal $\tau$ according to a given channel matrix. In this paper, we follow the asymptotic result in [11], where $\tau$ is fully determined as a function of $N_t$ and $K$:

$$
\tau = \frac{\sqrt{2\pi}}{\sqrt{\hat{f}(K, n)}} - \frac{2\rho N_t^2}{6},
$$

where

$$
\hat{f}(K, n) = K^{\frac{2n}{3}} + \frac{1}{3} \left( 2^{n-1} - 1 \right) + 2 \sqrt{K^{\frac{2n}{3}} + \left( 2^{2n-4} - 4 \right)} \cdot \frac{3}{22.5 \left( 2^{n-1} \right)^3}.
$$

We will explain how to construct a transmit-signal vector for a given decision size $\tau$. Given $4^n$-QAM, each symbol is indexed by a length-$n$ quaternary vector $(i_1, \ldots, i_n)$ with $i_j \in \{0 : 3\}$, i.e.,

$$
\mathcal{C} = \left\{ s^{(n)}_{(0, \ldots, 0)}, s^{(n)}_{(0, \ldots, 1)}, \ldots, s^{(n)}_{(3, \ldots, 3)} \right\}.
$$

Each constellation point can be represented as a linear combination of the $n$ basis symbols $c_i$’s such as

$$
s^{(n)}_{(i_1, \ldots, i_n)} = \tau \sum_{l=1}^{n} 2^{n-l} c_{i_l},
$$

Here, the basis symbols are defined as

$$
c_i = \sqrt{2} \left\{ \cos \left( \frac{\pi}{4} (1 + 2i) \right) + j \sin \left( \frac{\pi}{4} (1 + 2i) \right) \right\},
$$

for $i \in \{0 : 3\}$. For the ease of expression, we represent the the constellation $\mathcal{C}$ and the corresponding decision regions $\mathcal{R}(s^{(n)}_{(i_1, \ldots, i_n)})$ in the corresponding real-valued forms:

$$
\tilde{\mathcal{C}} = \left\{ g(s^{(n)}_{(0, \ldots, 0)}), g(s^{(n)}_{(0, \ldots, 1)}), \ldots, g(s^{(n)}_{(3, \ldots, 3)}) \right\},
$$

and

$$
\tilde{\mathcal{R}} \left( s^{(n)}_{(i_1, \ldots, i_n)} \right) = g \left( \mathcal{R} \left( s^{(n)}_{(i_1, \ldots, i_n)} \right) \right).
$$

A transmit vector $x$ should ensure that a noiseless received signal at the $k$-th user (i.e., $r_k = h_k^T x$) should be placed in the corresponding decision regions for all users $k \in \{1 : K\}$. This necessity condition implies that $x$ should satisfy the following condition:

$$
g(r_k) \in \tilde{\mathcal{R}} \left( s^{(n)}_{(\mu_{k,1}, \ldots, \mu_{k,n})} \right), k \in \{1 : K\}.
$$

for $k \in \{1 : K\}$, where $r_k = h_k^T x$ denotes a noiseless received signal (i.e., $y_k = h_k^T x + z_k$).

**Feasibility condition:** The condition in (18) will be rewritten in a way that the optimization problem can be interpreted as an LP problem. The decision region in (18) can be expressed as the intersections of the $n$ shifted base regions in Definition 2

$$
\tilde{\mathcal{R}} \left( s^{(n)}_{(i_1, \ldots, i_n)} \right) \triangleq \tilde{\mathcal{B}}_1 \bigcap_{l=2}^{n} \left\{ \tilde{\mathcal{B}}_l + 2^n -(l-1) g \left( s^{(l-1)}_{(i_1, \ldots, i_{l-1})} \right) \right\},
$$

where the shifted base region with a bias $c$ is defined as

$$
\tilde{\mathcal{B}}_l + c \triangleq \{ \alpha_1 m_1^1 + \alpha_2 m_2^2 + c : \alpha_1, \alpha_2 > 0 \}.
$$

Then, the condition in (18) holds if $g(r_k)$ can be represented by the following $n$ linear equations with some positive coefficients, i.e.,

$$
g(r_k) = \alpha_{k,1} m_1^1 + \alpha_{k,2} m_2^2 + 2^n g(0)
$$

$$
= \alpha_{k,1} m_1^1 + \alpha_{k,2} m_2^2 + 2^n g(s^{(1)}_{(\mu_{k,1})})
$$

$$
= \alpha_{k,1} m_1^1 + \alpha_{k,2} m_2^2 + 2^n g(s^{(n-1)}_{(\mu_{k,1}, \ldots, \mu_{k,n-1})}),
$$

for some $\alpha_{k,1}$, $\alpha_{k,2}$, $\alpha_{k,1} > 0$. The condition in (21) is called a feasibility condition as it can guarantee that $r_k \in \tilde{\mathcal{R}} \left( s^{(n)}_{(\mu_{k,1}, \ldots, \mu_{k,n})} \right)$ for $k \in \{1 : K\}$. In other words, if this condition is satisfied, all $K$ users can reliably detect the desired messages in higher SNRs.

**Example 1:** Assuming 16-QAM, we will explain how to obtain the feasibility condition in (19). Consider the decision region $\mathcal{R}(s^{(2)}_{(0,2)})$. From Fig. 1, the decision region is represented by the intersection of the two base regions $B_0$ (i.e., the infinite region with blue basis in Fig. 1) and $B_2 + s^{(0)}_{(0,2)}$ (i.e., the infinite region with red basis in Fig. 1). Thus, the decision region (i.e., the gray region in Fig. 1) is represented as

$$
\mathcal{R} \left( s^{(2)}_{(0,2)} \right) \triangleq \{ B_0 + 2^2 g(0) \} \cap \left\{ B_2 + 2^1 s^{(1)}_{(0,2)} \right\}.
$$

Also, from Definition 2, the above condition can be repre-
sent by the following two linear equations:

\[ g(r_k) = \alpha_{k,1}^2 m_k^1 + \alpha_{k,2}^2 m_k^2 + 2^2 g(0), \]
\[ = \alpha_{k,2}^2 m_k^2 + 2^2 g \left( \frac{1}{\alpha_{k,2}^2} \right), \]  
(23)

for some positive coefficients \( \alpha_{k,1}^2, \alpha_{k,1}, \alpha_{k,2}, \alpha_{k,2}^2 > 0 \). This is equivalent to the condition in \( (22) \). In the same way, we can verify the feasibility condition in \( (19) \).

We are now ready to derive MILP problem which can generate an optimal transmit vector \( x \) under 1-bt DAC constraints. We first represent the feasibility condition in a matrix form. Define the \( n \) copies of the channel vector \( h_k \) as

\[ \tilde{H}^k = [h_k^1, \ldots, h_k^n]^T, \]  
(24)

where \( h_k \) denotes the \( k \)-th row of \( H \), \( 1_n \) denotes the length-\( n \) all-1 vector, and \( \otimes \) indicates Kronecker product operator. Also, the corresponding real-valued expression is denoted as

\[ \tilde{H}^k = \phi(H^k). \]  
(25)

Accordingly, the \( n \)-extended received vector at \( k \)-th user is defined as

\[ r^k = g(H^k x) = \tilde{H}^k \tilde{x} = 1_n \otimes g(r_k). \]  
(26)

We next express the right-hand side of \( (21) \) (i.e., linear constraints) in a matrix form. From Definition 2, we let:

\[ M_i = [m_i^1, m_i^2]^T = \begin{bmatrix} \text{Re}(c_i) & 0 \\ 0 & \text{Im}(c_i) \end{bmatrix}. \]  
(27)

We remark that \( M_i \) is symmetric and orthogonal matrices, i.e.,

\[ M_i M_j = I. \]  
(28)

Since the decision region of a constellation point \( 4^n \)-QAM is formed as the conjunction of \( n \) shifted base regions, we need to establish a tightly packed format that can cope with both base regions and shifts (biases). The former is addressed by the basis matrix \( M^{\mu} \) and coefficient vector \( \alpha^k \), which are respectively written as

\[ M^{\mu} = \text{diag}(M_{\mu,k,1}, \ldots, M_{\mu,k,n}), \]
\[ \alpha^k = [\alpha_{k,1}^1, \alpha_{k,1}^2, \ldots, \alpha_{k,n}^1, \alpha_{k,n}^2]^T. \]  
(29)

(30)

Lastly, the whole series of the biases are formed as the bias vector \( b \), defined as

\[ b^{\mu} = g(2n \cdot 0, 2n-1 \cdot z_{(\mu,1,j)}^{(1)}, \ldots, 2^1 \cdot z_{(\mu,1,j-1,n-1)}^{(n-1)}). \]  
(31)

From \( (29) - (31) \), the matrix form of \( k \)-th user’s feasibility conditions \( (21) \) is given as

\[ r^k = M^{\mu} \alpha^k + b^{\mu}. \]  
(32)

Leveraging the expression designed for each user, we construct the cascaded matrix form of feasibility condition on all \( K \) users as

\[ r = \tilde{H} \tilde{x} = \tilde{M} \alpha + \tilde{b}, \]  
(33)

where

\[ \tilde{M} \triangleq \text{diag}(M^{\mu_1}, \ldots, M^{\mu_K}), \]
\[ \tilde{H} \triangleq [(\tilde{H}^1)^T, \ldots, (\tilde{H}^K)^T]^T, \]
\[ r \triangleq [(r^1)^T, \ldots, (r^K)^T]^T, \]
\[ \tilde{b} \triangleq [(b^{\mu_1})^T, \ldots, (b^{\mu_K})^T]^T. \]

Thus, the feasibility condition in \( (33) \) is rewritten as

\[ \alpha = \tilde{M} \tilde{H} \tilde{x} - \tilde{M} \tilde{b} \triangleq \tilde{\alpha} \triangleq \Lambda \tilde{x} - \Lambda \tilde{b}, \]  
(34)

where we used the fact that \( \tilde{M} = \tilde{M} \) from \( (28) \). We remark that \( \Lambda \in \mathbb{R}^{2nK \times 2N_t} \) and \( \Lambda_{\tilde{b}} \in \mathbb{R}^{2nK \times 1} \) are fully determined by the channel matrix \( \tilde{H} \) and users’ messages \( \{\mu_k : k \in [1 : K]\} \).

**Robustness:** A feasible transmit vector can provide an attractive performance in higher SNR regimes. Whereas, it could not guarantee the robustness to an additive Gaussian noise. To enhance the robustness, one reasonable way is to make a noiseless received signal to be placed in the center of the decision region. By taking this goal into account, we formulate the following optimization problem:

\[ \mathcal{P}_1 : \max \min_{\tilde{x}} \{a_{i,j,k} : i = 1, 2, j = 1, 2, k \in [1 : K]\} \]  
(35)

s.t. \[ \tilde{\alpha} = \Lambda \tilde{x} - \Lambda \tilde{b}, \]
\[ a_{1,k,j}^1 \geq 0, k \in [1 : K], j \in [1 : n], \]
\[ \tilde{x} \in \{-1, 0, 1\}^{2^N_t}. \]

To be specific, we aim at moving away the noiseless received signal from the boundaries of the detection lines. Fig. 2 verifies the proposed approach, where \( 10^4 \) normalized noiseless signals \( Hx \) are plotted with \( N_t = 8, K = 2, \) and \( n = 2 \). The blue points depict the noiseless received signals when ZF precoding in \( (4) \) is used with the assumption of infinite resolution. In contrast, the red points show the noiseless received signals.
LP problem by relaxing the integer constraint in algorithm to solve MILP problem \( P \) performance. However, its computation complexity is quite respectively. The MILP problem can be solved via branch-and-
denoted as \( \text{method} \) \cite{12}, and the corresponding relaxed LP solution is \( \tilde{x} \). We artificially change candidates

We first evaluate the complexity of the optimal method (i.e.,
by the total number of the required real-valued multiplications.
In the remaining part of this section, we present an efficient
algorithm which consists of LP solver and greedy
algorithm. To solve the LP problem in \( P_3 \), we use the interior point method \cite{12}. The corresponding complexity (denoted as \( \mathcal{X}_L \)) is given in \( \mathcal{X}_L \) such as \( \mathcal{X}_L \), where \( \| \cdot \|_1 \) and \( \varepsilon \) denote the induced \( \ell_1 \)-norm of matrix and the accuracy, respectively.

In this section, we compare symbol error rate (SER) per-
In the remaining part of this section, we present an efficient
algorithm which consists of LP solver and greedy
algorithm. To solve the LP problem in \( P_3 \), we use the interior point method \cite{12}. The corresponding complexity (denoted as \( \mathcal{X}_L \)) is given in \( \mathcal{X}_L \) such as \( \mathcal{X}_L \), where \( \| \cdot \|_1 \) and \( \varepsilon \) denote the induced \( \ell_1 \)-norm of matrix and the accuracy, respectively.

The complexity of optimization-based method \cite{42} cannot be directly compared with algorithm-based methods, since the complexity of optimization-based is obtained by analytic upper bound in worst case, not required number of real multiplications. For fair comparisons, thus, the complexities of all methods are compared via execution time (see Table I).

**Algorithm 1** Greedy Algorithm

**Input:** \( x_{\text{LP}} \in \mathbb{R}^{2nK \times 1}, \Lambda \in \mathbb{R}^{2nK \times 2N_i} \) and \( \Lambda_b \in \mathbb{R}^{2nK \times 1} \).

**Initialization:** \( \bar{x} = x_{\text{LP}} \)

for \( i = 1:2N_i \) do
for \( j \in \{-1,1\} \) do
\( x_i = j \) and \( \alpha^{(j)} = \Lambda \bar{x} - \Lambda_b \)
end for
end for
Update \( x_i \leftarrow \text{argmax}_{j \in \{-1,1\}} \{ \min(\alpha^{(j)}) \} \)

**Output:** \( \bar{x} \in \mathbb{R}^{2N_i \times 1} \)

when the proposed 1-bit transmit vectors, obtained from the solutions of \( P_2 \), are used. Fig. 2 clearly shows that the red points can provide more robustness than the blue points.

Furthermore, \( P_1 \) is transformed to MILP:

\[
P_2 : \quad \arg\max_{\bar{x}, t} \quad t \\
\text{s.t.} \quad \Lambda \bar{x} - \Lambda b, t \geq t, \quad i \in [1:2nK] \\
\quad x_i \in \{-1,1\}^{2N_i},
\]

where \( \Lambda_i \) and \( \Lambda_{b,i} \) denote the \( i \)-th row of \( \Lambda \) and \( \Lambda_b \), respectively. The MILP problem can be solved via branch-and-bound (B&B) method \cite{11}, which can achieve a near optimal performance. However, its computation complexity is quite expensive for a realistic implement \cite{8}.

In the remaining part of this section, we present an efficient algorithm to solve MILP problem \( P_2 \). We first solve the LP problem by relaxing the integer constraint in \( P_2 \) as the bounded interval:

\[
P_3 : \quad \arg\max_{\bar{x}, t} \quad t \\
\text{s.t.} \quad \Lambda \bar{x} - \Lambda b, t \geq t, \quad i \in [1:2nK] \\
\quad -1 \leq \bar{x}_j \leq 1, \quad j \in [1:2N_i].
\]

This problem can be efficiently solved via interior point method \cite{12}, and the corresponding relaxed LP solution is denoted as \( x_{\text{LP}} \). Then, we refine the solution of \( P_3 \) via a greedy algorithm (see Algorithm 1) so that it fulfills the desired one-bit constraints. Starting from the solution of \( P_3 \), i.e., \( x_{\text{LP}} \), the main idea behind the second stage is to choose an antenna index \( i \), to test the possible values of the antennas, that is \( x_i \in \{-1,1\} \), to calculate the set of scaling coefficients when we artificially change \( x_i \), and finally to set \( x_i = j \) where the substitution of \( j \in \{-1,1\} \) for \( x_i = j \) insists the maximization of the minimum element in the coefficients.

**A. Computation complexity**

We compare the proposed algorithm with the existing methods in terms of a computational complexity. Following the related works \cite{5}, \cite{6}, the computational complexity is measured by the total number of the required real-valued multiplications.

We first evaluate the complexity of the optimal method (i.e.,
an exhaustive search) which explores all the possible signal candidates \( \bar{x} \in \{-1,1\}^{2N_i} \). Since each candidate requires \( 2nK \cdot 2N_i \) operations to generate the magnitude of coefficients in the feasibility conditions in (34), the total complexity of the exhaustive search is computed as

\[
\mathcal{X}_e = 4nK N_i \cdot 2^{2N_i}.
\]

Also, as a low-complexity method, we consider the symbol-scaling method proposed in \cite{6}, where the total computation complexity is given as

\[
\mathcal{X}_{SS} = 4N_i^2 + 24nkN_i - 2nk.
\]

We next focus on the computational complexity of the proposed algorithm which consists of LP solver and greedy algorithm. For the LP solver, the interior point method in \cite{12} is assumed.

The proposed method is divided into LP solver and greedy algorithm. To solve the LP problem in \( P_3 \), we use the interior point method \cite{12}. The corresponding complexity (denoted as \( \mathcal{X}_{LP} \)) is given in \( \mathcal{X}_{LP} \) such as \( \mathcal{X}_{LP} \), where \( \| \cdot \|_1 \) and \( \varepsilon \) denote the induced \( \ell_1 \)-norm of matrix and the accuracy, respectively.

Also, the quantized LP represents the algorithm that directly quantizes the solution of \( P_3 \) to generate 1-bit transmit vector using sign function, given as

\[
x_q = \text{sign}(x_{\text{LP}}).
\]

Thus, the corresponding complexity is the same as that of LP as

\[
\mathcal{X}_q = \mathcal{X}_{LP}.
\]

Also, the complexity of the greedy algorithm is obtained as

\[
\mathcal{X}_{\text{greedy}} = 2 \cdot 2nK \cdot 2N_i = 8nK N_i.
\]

Thus, the total computation complexity of proposed method is computed as

\[
\mathcal{X}_{\text{pro}} = \mathcal{X}_{LP} + \mathcal{X}_{\text{greedy}} = \mathcal{X}_{LP} + 8nK N_i.
\]

**IV. SIMULATION RESULTS**

In this section, we compare symbol error rate (SER) performances of various methods such as symbol scaling (SS), quantized LP (i.e., solving \( P_3 \)) and the proposed algorithm. In addition, we evaluate the computational complexities with the simulation time (i.e., execution time) of the above methods and MILP-based method (i.e., solving \( P_2 \)). Recall that SNR is defined as per-antenna signal power to noise, i.e., \( \rho/\sigma^2 \).

Fig. 3 shows the SER performance comparisons of the above algorithms for downlink MU-MISO systems with 1-bit DACs, where \( N_i = 128, \quad K = 16, \quad 64\text{-QAM} \). Without 1-bit constraints, LP method (i.e., solving \( P_3 \)) provides an optimal performance with infinite-resolution DACs. This can be interpreted as the lower-bound of the above 1-bit constraint
\( x_{LP} = (6N_t + 2nK + 1)^{1.5}(2N_t + 1)^2 \cdot \log \left( \frac{8N_t^2 + (20 + 4nK)N_t + 2\sqrt{2N_t} + 8nK + \|M\|_1 + 6 + \varepsilon^2}{\varepsilon} \right). \) (42)

Fig. 3. Performance comparisons of LP, Quantized LP, Symbol-scaling and the proposed methods for the downlink MU-MISO systems with 1-Bit DACs, where \( N_t=128, \ K=16, \) and 4\(^2\)-QAM.

TABLE I

| Precoding Methods | Time (seconds) | MILP (B&B) | SS | Proposed Method |
|-------------------|----------------|------------|----|-----------------|
|                   |                | 922        | 0.0022 | 0.0087 |

methods. Note that, in this setting, we cannot evaluate the performance of MILP due to its unmanageable complexity. At high SNR, we observe that quantized LP suffers from a severe error-floor. Thus, it is required to consider the other LP-based methods instead of using the direct quantization of LP. For this reason, we apply the greedy algorithm which determines the entries of \( x_{LP} \) such that they belong to \( \{-1, 1\} \) while keeping the feasibility and robustness. In Fig.3 we can see that the proposed method achieves an attractive performance close to a lower bound, with low-complexity, while SS method which yielded a good performance in PSK constellations, suffers from an error-floor in QAM constellations.

In comparisons of computational complexities, we consider the average running time in executing \( 10^4 \) realizations of each algorithm, and the corresponding results are summarized in Table I. For simulations, we consider the downlink MU-MISO systems, where \( N_t = 32, \ K = 4, \) 4\(^2\)-QAM, and SNR = 5dB. It is shown that MILP has an infeasible complexity in realistic implementation while the proposed method has extremely lower complexity than MILP. Moreover, the proposed method has a similar order of complexity with the symbol scaling method. In other words, these methods provide a lower complexity. Combining the results of Fig. 3 and Table I we can conclude that the proposed method achieves an elegant complexity-performance tradeoff.

V. CONCLUSION

We proposed the construction of 1-bit transmit signal vector for downlink MU-MISO systems with QAM constellations. In this regard, we derived the linear feasibility constraints which ensures that each user can recover the desired message successfully, and transformed them into the cascaded matrix form. From this, we constructed mixed integer linear programming (MILP) problem whose solution generates a 1-bit transmit vector to satisfy the feasibility and guarantee the robustness to a noise. To address the computational complexity of MILP, we proposed the LP-relaxed algorithm consisting of two steps: i) solve the relaxed LP; ii) refine the LP solution to fit into the 1-bit constraint. Via simulation results, we demonstrated that the proposed method shows better performances than the benchmark methods with low-complexity. One promising future work is to further reduce the complexity of the proposed method without the cost of the performance loss.

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