NONPERTURBATIVE SUSY GAUGE THEORIES
WITH SOFT MASSES

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ABSTRACT

After briefly reviewing the nonperturbative dynamics of $N = 1$ supersymmetric field theories, soft SUSY breaking mass terms are introduced into the SUSY gauge theories and their effects on gauge symmetry breaking pattern are studied. For $N_f < N_c$, we include the dynamics of the non-perturbative superpotential and use the original (s)quark and gauge fields. For $N_f > N_c + 1$, we formulate the dynamics in terms of dual (s)quarks and a dual gauge group $SU(N_f - N_c)$. The mass squared of squarks can be negative triggering the spontaneous breakdown of flavor and color symmetry. The general condition for the stability of the vacuum is derived. We determine the breaking pattern, derive the spectrum, and argue that the masses vary smoothly as one crosses from the Higgs phase into the confining phase exhibiting the complementarity.

1. Introduction

The presence of very small mass scale $m_W$ compared to the fundamental scale of unified theories can be explained by symmetry reasons using supersymmetry (SUSY). Moreover supersymmetry naturally incorporates gravity which has a naturally large mass scale $M_{\text{Planck}}$. Now supersymmetric theory has become a standard theory to solve this gauge hierarchy problem and to unify all the forces in nature. Recent advances to understand the dynamics of supersymmetric gauge theories has provided a rich and concrete structure of nonperturbative effects [1-5]. Many exact results are found for low energy effective field theories in $N = 2$ SUSY gauge theories. There are also certain results for $N = 1$ SUSY field theories.

Particularly interesting aspects of SUSY field theories are related to SUSY breaking. SUSY breaking may be classified into three classes:
1. Soft breaking

2. Spontaneous breaking

3. Dynamical breaking

The soft breaking of supersymmetry was used in the original proposal of supersymmetric grand unified theories and provides a general framework for the usual formulation of the Minimal Supersymmetric Standard Model. In the context of the new results on nonperturbative dynamics, it is worthwhile to reconsider the SUSY breaking.

In a series of papers, it was argued that the addition of perturbative, soft supersymmetry breaking mass terms, with \( m^2 \geq 0 \), essentially preserves the qualitative picture of the dynamics derived for \( N = 1 \) supersymmetric QCD (SQCD). An effective low energy theory is used in terms of color singlet meson and (for \( N_f \geq N_c \)) baryon fields, appropriate for the confining phase, and the effects of the non-perturbative superpotential of Affleck, Dine and Seiberg are included.

More recently we have investigated \( N = 1 \) supersymmetric QCD, again with soft supersymmetry breaking mass terms added, but this time with \( m^2 < 0 \) for at least some of the squark fields. In the present paper, we will first review briefly the nonperturbative dynamics of \( N = 1 \) SUSY gauge theories, and then report our findings.

2. Nonperturbative Superpotential

The analysis of \( N = 1 \) SUSY gauge theories is based on the following two fundamental ingredients:

1. Holomorphy

2. Duality

These principle allow a determination of exact superpotential in low energy effective field theories with \( N = 1 \) Supersymmetry.

On the other hand, the Kähler potential can only be determined in the case of \( N = 2 \) supersymmetry.

Let us consider supersymmetric Yang-Mills theory with gauge group \( SU(N_c) \) and \( N_f \) flavors of squarks and quarks (with \( N_f < N_c \)), transforming under the representation \( N_c \oplus \bar{N}_c \) of \( SU(N_c) \). This theory is the natural supersymmetric extension of QCD, and will be referred to as SQCD. The corresponding chiral superfields

\[
\hat{Q}_{a}^i, \quad \bar{\hat{Q}}_{i}^a \quad a = 1, \cdots, N_c; \quad i = 1, \cdots, N_f,
\]

contain the squark fields \( Q \) and \( \bar{Q} \) and the left-handed quark fields \( \psi_Q \) and \( \psi_{\bar{Q}} \) respectively. There is a natural color singlet meson chiral superfield \( \hat{T} \), defined by

\[
\hat{T}_{i}^{j} = \hat{Q}_{i}^{a} \bar{Q}_{a}^{j}
\]

with scalar components \( T_{ij} \). Superfields are denoted by a cap on the scalar components.
As a starting point we consider classical massless SQCD whose Lagrangian $\mathcal{L}_0$ is determined by $SU(N_c)$ gauge invariance, by requiring that the superpotential for the quark superfields vanish identically:

$$
\mathcal{L}_0 = \int d^4\theta \text{tr}\{\hat{Q} e^{2g\hat{V}} \hat{Q} + \hat{\bar{Q}} e^{-2g\hat{V}} \hat{\bar{Q}}\} + \frac{1}{2} \int d^2\theta \text{tr} WW + \frac{1}{2} \int d^2\theta \text{tr} \bar{W} \bar{W} \tag{3}
$$

This theory has a global symmetry, $G_f = SU(N_f)_Q \times SU(N_f)_{\bar{Q}} \times U(1)_B \times U(1)_R$, with $R$-charges (baryon number) are given by $1 - N_c/N_f$ (1) for $Q$, and $1 - N_c/N_f$ (−1) for $\bar{Q}$.

Exact nonperturbative results in supersymmetric gauge theories can be given for the $F$-type term which is a chiral superspace integral of a superpotential $W_{NP}$ of the quark superfields $\hat{Q}$ and $\hat{\bar{Q}}$ given as follows:

$$
\int d^2\theta W_{NP}(\hat{Q}, \hat{\bar{Q}}) = (N_c - N_f)\Lambda^{3+2N_f/(N_c-N_f)} \int d^2\theta (\det \hat{Q} \hat{\bar{Q}})^{-1/(N_c-N_f)} \tag{4}
$$

3. Dynamics for $N_f < N_c$

### 3.1. Soft Supersymmetry Breaking Mass Terms

We choose to break supersymmetry explicitly, by adding to the Lagrangian $\mathcal{L}_0$ soft supersymmetry breaking terms for the quark supermultiplet.

For the sake of simplicity, we shall add to the Lagrangian only soft supersymmetry breaking squark mass terms and neglect effects due to gaugino masses and supersymmetric flavor masses. Generic mass squared for squark and antisuquarks are given by matrices $M^2_Q$ and $M^2_{\bar{Q}}$

$$
\mathcal{L}_{sb} = -\{\text{tr}QM^2_Q Q^\dagger + \text{tr} \bar{Q}^\dagger M^2_{\bar{Q}} \bar{Q}\} \tag{5}
$$

As we remarked above, when $M^2_Q$ and $M^2_{\bar{Q}}$ are proportional to the identity matrix, the global flavor symmetry is unchanged: $SU(N_f)_Q \times SU(N_f)_{\bar{Q}} \times U(1)_B \times U(1)_R$.

When either $M^2_Q$ or $M^2_{\bar{Q}}$ is not positive definite, we expect the pattern of symmetry breaking to be substantially different. Global flavor symmetry should be spontaneously broken, and $Q$ and/or $\bar{Q}$ should acquire non-vanishing vacuum expectation values. These non-zero vacuum expectation values, in turn, are expected to break color $SU(N_c)$ and give mass to some of the gauge particles through the Higgs mechanism. This is the so-called Higgs phase.

According to standard lore, (originally derived from lattice gauge theory) the confining and Higgs phases are smoothly connected to one another in at least some region of parameter space. There should be a one to one correspondence between the observables in both phases, suggesting that – in principle – color singlet meson fields could still be used to describe the dynamics of the Higgs phase. In practice, however, a formulation in terms of colored fields appears more suitable instead. Indeed, physical free quarks and certain massive gauge bosons are expected to appear in the low energy spectrum, and it is unclear how to represent these degrees of freedom in terms of meson
variables. Thus, we shall use the original squark $Q$, $\bar{Q}$, quark $\psi_Q$, $\psi_{\bar{Q}}$, and gauge boson and fermion fields as physical variables at low energy.

3.2. Vacuum Stability

Without SUSY breaking soft masses, the vacuum is known to run away. For generic matrices $M_Q^2$ and $M_{\bar{Q}}^2$, we find the stability condition for the vacuum as

$$m_Q^2 + m_{\bar{Q}}^2 \geq 0 \quad (6)$$

for any pair of $i, j = 1, \cdots N_f$.

For simplicity, we explicitly analyze only the case where the mass squared for all $Q$'s and $\bar{Q}$'s are equal to $-m_Q^2$ and $m_{\bar{Q}}^2$ respectively, thus preserving the entire global symmetry $SU(N_f)_Q \times SU(N_f)_{\bar{Q}} \times U(1)_B \times U(1)_R$. The vacuum configuration is assumed to be Poincaré invariant and so that the values of $Q$ and $\bar{Q}$ in the vacuum are space-time independent. We shall determine these expectation values at the semi-classical level.

By making a global $SU(N_c) \times SU(N_f)_Q \times U(1)_B$ transformation on $Q$, and the remaining $SU(N_f)_{\bar{Q}} \times U(1)_R$ transformation on $\bar{Q}$, we can always rotate the vacuum expectation values of $Q$ and $\bar{Q}$ to the following arrangement

After a somewhat lengthy analysis, we find that there is a unique minimum of the potential at which the first $N_f \times N_f$ block of $Q$ and $\bar{Q}$ are nonvanishing and are proportional to the identity

$$\langle 0|Q|0 \rangle = \begin{pmatrix} Q_0I_{N_f} \\ 0 \end{pmatrix} \quad \langle 0|\bar{Q}|0 \rangle = \begin{pmatrix} Q_0I_{N_f} & 0 \end{pmatrix}. \quad (7)$$

The minimum conditions are

$$0 = (\gamma - 1)Q_0^{-2\gamma}Q_0^{-2\gamma} + \gamma Q_0^{-2-2\gamma}Q_0^{-2-2\gamma} - \frac{g^2}{2\gamma}(Q_0^2 - \bar{Q}_0^2) + m_Q^2 \quad (8)$$

3.3. Spectrum and Unbroken Symmetries

We have obtained the spectra in this vacuum.

The properties of the vector bosons are summarized in the table 1.

A summary of all spin 1/2 fields and masses is given in the table 2.

The results on spin 0 boson masses are summarized in the table 3. The masses of the would-be-Goldstone bosons are referred to by the gauge fields into which they have combined, so the corresponding masses can be found in the table 1.

As anticipated, the scalar particles contain massless Nambu-Goldstone boson corresponding to the spontaneous breakdown of $U(1)_R$ and $SU(N_f)$ global symmetry.

3.4. Complementarity between Confining and Higgs Phase
Table 1: The spin 1 fields and their masses ($N_f < N_c$)

| Spin | Masses | Multiplicity | $SU(N_f) \times SU(N_c - N_f)$ |
|------|--------|--------------|---------------------------------|
| $A_{\mu}(0)$ | $g v \sqrt{2/\gamma}$ | 1 | $1 \otimes 1$ |
| $A_{\mu}(1)$ | $g v \sqrt{2}$ | $N_f^2 - 1$ | adjoint $\otimes 1$ |
| $A_{\mu}(2)$ | $g v$ | $2 N_f (N_c - N_f)$ | $N_f^2 \otimes (N_c - N_f) \oplus N_f \otimes (N_c - N_f)^*$ |
| $A_{\mu}(3)$ | 0 | $(N_c - N_f)^2 - 1$ | $1 \otimes$ adjoint |

Table 2: The spin 1/2 fields and their masses ($N_f < N_c$)

| Spin 1/2 | Masses | Multiplicity | $SU(N_f) \times SU(N_c - N_f)$ |
|----------|--------|--------------|---------------------------------|
| $\psi(0), \bar{\psi}(0), \lambda(0)$ | $\mathcal{M}^0_{(0)}, \mathcal{M}^0_{(0)}$ | 3 | $1 \otimes 1$ |
| $\psi(1), \bar{\psi}(1), \lambda(1)$ | $\mathcal{M}^0_{(1)}, \mathcal{M}^0_{(1)}$ | $3(N_f^2 - 1)$ | adjoint $\otimes 1$ |
| $\psi(2), \lambda(2), \bar{\psi}(2), \lambda(2)$ | $\pm \mathcal{M}_{(2)}$ | $2 N_f (N_c - N_f)$ | $2 N_f \otimes (N_c - N_f)$ |
| | $\pm \mathcal{M}_{(2)}$ | $2 N_f (N_c - N_f)$ | $2 N_f \otimes (N_c - N_f)^*$ |

According to the complementarity argument, one should be able to relate our mass spectra calculated in terms of colored elementary fields with the result in ref. 12 where the meson fields $T$ are used as fundamental variables in the low energy effective theory. They have assumed a standard minimal kinetic term for the meson fields, because the Kähler potential can not be constrained by holomorphy. Consequently their results on mass spectra differed from ours, although overall qualitative picture was the same.

On the other hand, if we take large VEV for squark fields, we should be in a nearly perturbational region. Therefore there should be the correct Kähler potential for the color singlet meson which correspond to our choice of perturbative kinetic term for squarks. We indeed find that a similar analysis as Aharony et al 12 with the Kähler potential for meson

$$K[T] = 2 \text{tr} \left( T^* T \right)^{\frac{1}{2}}$$

reproduces our mass spectra correctly.

This shows that the complementarity is also valid in supersymmetric situation, and the higher order terms in Kähler potential are important when some of the fields acquire VEV.

4. Dynamics for $N_f > N_c + 1$

4.1. Dual Description

We have also analyzed the case of $N_f > N_c + 1$ using the dual description. Qualitative picture is similar to the previous case of $N_f < N_c + 1$.

The dual description for the gauge group $SU(N_c)$ and $N_f$ flavors of quarks and
Table 3: The spin 0 fields and their masses ($N_f < N_c$)

| Spin 0 | Masses | Multiplicity | $SU(N_f) \times SU(N_c - N_f)$ |
|--------|--------|--------------|-------------------------------|
| $Q^{(0)}$, $\bar{Q}^{(0)}$ | $[A_{\mu(0)}]$, 0, $\mathcal{M}^z_{(0)}$ | 4 | $1 \otimes 1$ |
| $Q^{(1)}$, $\bar{Q}^{(1)}$ | $[A_{\mu(1)}]$, 0, $\mathcal{M}^z_{(1)}$ | $4(N_f^2 - 1)$ | adjoint $\otimes 1$ |
| $Q^{(2)}$, $\bar{Q}^{(2)}$ | $[A_{\mu(2)}]$, $\mathcal{M}_{(2)}$ | $4N_f(N_c - N_f)$ | $N_f^* \otimes (N_c - N_f) \oplus c.c.$ |

antiquarks (with $N_f > N_c + 1$) has a gauge group $SU(\tilde{N}_c)$ with $N_f$ flavors, where $\tilde{N}_c = N_f - N_c$. The elementary chiral superfields in the dual theory are dual quark $\tilde{q}$ and meson $\tilde{T}$ superfields,

$$\tilde{q}^a \hat{q}^i \hat{T}^i a = 1, \cdots, \tilde{N}_c; \quad i, j = 1, \cdots N_f. \quad (10)$$

Since $\tilde{q}$, $\hat{q}$ and $\hat{T}$ are effective fields, their kinetic terms need not have canonical normalizations; in particular, they can receive nonperturbative quantum corrections. Thus, we introduce into the (gauged) Kähler potential for $\tilde{q}$, $\hat{q}$ and $\hat{T}$ normalization parameters $k_\tilde{q}$ and $k_\hat{T}$ as follows

$$K[\tilde{q}, \hat{q}, \hat{T}, \hat{v}] = k_\tilde{q} tr(\tilde{q}^\dagger e^{2\tilde{g}\phi} \tilde{q} + \hat{q} e^{-2\tilde{g}\phi} \hat{q}^\dagger) + k_\hat{T} tr\hat{T}^\dagger \hat{T}. \quad (11)$$

Here, we denote by $\hat{v}$ the $SU(\tilde{N}_c)$ color gauge superfield, and by $\tilde{g}$ the associated coupling constant. (Pure gauge terms will not be exhibited explicitly.) In principle, these normalization parameters are determined by the dynamics of the underlying microscopic theory. Furthermore, it has been pointed out that a superpotential coupling $q$, $\tilde{q}$ and $T$ should be added as follows

$$W = \tilde{q}^a \hat{T}^i \hat{q}^j a. \quad (12)$$

4.2. Soft Breaking Terms and Stability

We add soft supersymmetry breaking terms to the Lagrangian for the dual quark and meson supermultiplets. For simplicity we shall assume that $R$-symmetry is maintained so that neither $A$-terms nor gaugino masses are present in the Lagrangian.

When the eigenvalues of $M_q^2$, $M_{\tilde{q}}^2$ and $M_T^2$ can take generic positive or negative values, the scalar potential may be unbounded from below. A necessary condition for which the potential is bounded from below is that $M_T^2$ be a positive definite matrix. This is because there is no quartic term of $T$.

The D terms vanish when the vacuum expectation values are given by:

$$\langle 0 | q | 0 \rangle = \begin{pmatrix} q_1 & \cdots & 0 \\ \ddots & \ddots & \ddots \\ q_{\tilde{N}_c} & \cdots & 0 \end{pmatrix}, \quad \langle 0 | \tilde{q} | 0 \rangle = \begin{pmatrix} \tilde{q}_1 & \cdots & \tilde{q}_{\tilde{N}_c} \\ \ddots & \ddots & \ddots \\ 0 & \cdots & 0 \end{pmatrix}, \quad (13)$$

with the combinations $|q_i|^2 - |\tilde{q}_i|^2$ independent of $i$. 
If we set the squark masses to be zero, the space where $|q_i|^2$ is independent of $i$ and $\bar{q} = 0$ is a subspace of the moduli space of vacua. If we insist on flavor symmetric mass squared matrix and on having a negative eigenvalue, we are forced to have a potential unbounded from below. In fact, in the next subsection, we shall establish more generally that to have a potential bounded from below, we must have

$$m_1^2 + \cdots + m_{N_c}^2 \geq 0,$$

(14)

where $m_i^2$ are eigenvalues of the matrix $M_\bar{q}^2$ or $M_q^2$, and they are set to be $m_1^2 \leq m_2^2 \leq \cdots \leq m_{N_f}^2$.

Therefore we consider the simplest stable situation, where the $n$ eigenvalues of $M_\bar{q}^2$ are negative and same, while all the others are positive or zero. The $n$ should be smaller than $N_c$. For simplicity we shall also assume that the soft supersymmetry breaking positive mass squared terms for squarks have a flavor symmetry $SU(N_f - n)Q \times SU(N_f)_Q$. As a result, the $N_f - n$ positive eigenvalue of $M_q^2$ are all the same, while the $N_f$ eigenvalues of $M_\bar{q}^2$ are the same : $M_\bar{q}^2 = m_\bar{q}^2 I_{N_f}$. We also assume $M_T^{2i} = m_T^2 \delta^i_\bar{j} \delta^k_i$.

4.3. Vacuum Configurations

After all, we find that there are only two solutions for possible minimum, described as follows.

1. Only $q_1, \cdots, q_n \neq 0$, while $q_i = 0$, $i > n$ and $\bar{q}_i = 0$ for all $i$. The values of $q_1, \cdots, q_n$ are the same. We call the common value as $q_0$. The value of $q_0$ and the potential in this configuration are given by

$$q_0^2 = \frac{2}{\bar{g}^2 \bar{\gamma}} m^2_{q_1}, \quad V = -\frac{n}{\bar{g}^2 \bar{\gamma}} m^4_{q_1}.$$

(15)

where

$$\bar{\gamma} = \frac{\bar{N}_c - n}{N_c}.$$

(16)

2. Only $q_1, \cdots, q_n \neq 0$ and $\bar{q}_1, \cdots, \bar{q}_n \neq 0$, while $q_i = \bar{q}_i = 0$, $i > n$. The values of $q_0 = q_1 = \cdots = q_n$ and $\bar{q}_0 = \bar{q}_1 = \cdots = \bar{q}_n$ are then given by

$$
\begin{pmatrix}
q_0^2 \\
\bar{q}_0^2
\end{pmatrix} = \frac{1}{\bar{\gamma} \bar{g}^2 - \frac{1}{k_T}} \left( \frac{1}{2} \bar{\gamma} \bar{g}^2 k_T (m_{q_1}^2 - m_{\bar{q}}^2) + m_{\bar{q}}^2 \right)
\begin{pmatrix}
m_{q_1}^2 - \frac{1}{2} \bar{\gamma} \bar{g}^2 k_T (m_{q_1}^2 - m_{\bar{q}}^2) - m_{q_1}^2
\end{pmatrix}
$$

(17)

Given the fact that $q_1^2$ and $\bar{q}_1^2$ must be positive, this expression yields a solution only when the following condition is satisfied

$$\frac{1}{2} \bar{\gamma} \bar{g}^2 k_T (m_{q_1}^2 - m_{\bar{q}}^2) \geq m_{q_1}^2.$$

(18)

The value of the potential at the stationary point is given by

$$V = -\frac{n}{4 \bar{\gamma} \bar{g}^2 - \frac{1}{k_T}} \left( m_{\bar{q}}^2 - \frac{1}{2} \bar{\gamma} \bar{g}^2 - \frac{1}{k_T} m_{q_1}^2 \right)^2 - \frac{n}{\bar{\gamma} \bar{g}^2} m_{q_1}^4.$$

(19)
Therefore, whenever conditions (18) is satisfied, solution 2 is the absolute minimum of the potential and describes the true ground state. If condition (18) is not satisfied, solution 1 is the absolute minimum, and flavor symmetry is spontaneously broken.

In our model, the Lagrangian has a global $SU(N_f - n)_Q \times SU(n)_Q \times U(1)_Q \times SU(N_f)_Q \times U(1)_B \times U(1)_R$ symmetry. When the coupling $\tilde{g}$ is too weak, the condition (18) is not satisfied, solution 1 is the absolute minimum, and flavor symmetry is not broken.

On the other hand, when the gauge coupling is strong, the condition (18) is satisfied. Therefore solution 2 is the absolute minimum, and flavor symmetry is spontaneously broken to $SU(N_f - n)_Q \times SU(N_f - n)_Q \times SU(n)_V \times U(1)_V \times U(1)_{B'} \times U(1)_{R'}$, where $SU(n)_V$ is the diagonal subgroup of $SU(n) \subset SU(\tilde{N}_c)$ and $SU(n)_Q \times SU(n)_Q$. The spontaneous breaking of the global symmetry induces spontaneous breaking of color gauge symmetry $SU(\tilde{N}_c) \rightarrow SU(\tilde{N}_c - n)$.

4.4. Spectrum and Unbroken Symmetries

Here we examine the mass spectrum after this spontaneous gauge symmetry breaking, $SU(\tilde{N}_c) \rightarrow SU(n) \times SU(\tilde{N}_c - n)$. The properties of the vector bosons are summarized in the table 4.

A summary of all spin 1/2 fields and masses is given in the table. One interesting feature of the present case is that the gauge symmetry is broken without chiral symmetry breaking. We find Nambu-Goldstone bosons for spontaneous
breakdown of $SU(N) / SU(N-f - n)$.

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