Analysis of laser wakefield dynamics in capillary tubes

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Abstract. A general approach to the modifications of the spectrum of a laser pulse interacting with matter is elaborated and used for spectral diagnostics of laser wakefield generation in guiding structures. Analytical predictions of the laser frequency red shift due to the wakefield excited in a capillary waveguide are confirmed by self-consistent modeling results. The role of ionization blue shift, and nonlinear laser pulse and wakefield dynamics on the spectrum modification, is analyzed for recent experiments on plasma wave excitation by an intense laser pulse guided in hydrogen-filled glass capillary tubes up to 8 cm long. The dependence of the spectral frequency shift, measured as a function of filling pressure, capillary tube length and incident laser energy, is in excellent agreement with the simulation results, and the associated longitudinal accelerating field is in the range 1–10 GV m\textsuperscript{-1}.

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1. Introduction

The interaction of short, intense laser pulses with plasmas produces large amplitude wakes. The high-field amplitude associated with these wake waves can be used to accelerate particles to high energies over very short lengths compared with the conventional accelerator technology [1]–[3]. In linear or moderately nonlinear regimes, these fields are of the order of $1–10 \text{ GV m}^{-1}$, and relativistic electrons injected into the wave can acquire an energy of the order of $1 \text{ GeV}$ over a length of the order of a few centimeters. The control of the characteristics of the electron beam as it is accelerated is crucial for achieving a usable laser–plasma accelerator unit. It is linked to the control of the accelerating electric field structure over several centimeters in a plasma. Diagnostics providing a detailed knowledge of the field structure and time evolution are therefore important for the progress of accelerator development.

Several diagnostics methods have been used to measure plasma waves created by the laser wakefield, in particular multi-shot or single-shot frequency domain interferometry [4, 5]. They rely on the interference in the frequency domain of two separate probe pulses and have been used to measure density perturbations over lengths of the order of a few millimeters. The dispersion effects on the probe pulse and the variations of the phase velocity of the excited plasma wave make these diagnostic techniques difficult to use over longer propagation distances [6]. Modifications of the spectrum of a single probe pulse propagating in a time- and space-varying plasma density [7] can be used to diagnose the laser wakefields [6, 8] without restrictions to the wake wave amplitude and dispersion effects.

The amplitude of the plasma wave excited by the laser pulse over a large length can be obtained in a single shot by the analysis of the modifications of the spectrum of the driving pulse. This method, proposed in [9, 10] and compared with the experimental measurements for a supersonic helium gas jet target of sub-millimeter length in [11], relies on the modifications of the spectrum induced by the time-varying plasma density within the pulse duration. Recently, it has been used successfully to measure the plasma wave excited in the wake of an intense laser guided in a gas-filled capillary tube over 8 cm [12].

In this paper, analytical expressions for the frequency shift due to the laser wakefield excited in a capillary tube are derived and compared with self-consistent modeling including gas ionization, nonlinear laser pulse and wakefield dynamics. A detailed comparison with experimental data shows that excellent agreement is obtained in a broad range of plasma densities, which allows us to determine the value of the accelerating field amplitude.
2. Analytical theory (general approach)

The general frequency momentum theory [13] describes the relation between the frequencies, \( \omega \), averaged over the spectrum of incident and output radiation of intensity \( I_\alpha(\omega, R) \) from the interaction volume \( V \):

\[
\langle \omega^2 \rangle_\alpha = \oint_S \int_0^{+\infty} \omega^2 I_\alpha(\omega, R) d\omega \, dS \left[ \oint_S \int_0^{+\infty} I_\alpha(\omega, R) d\omega \, dS \right]^{-1}, \quad \alpha = \text{in, out},
\]

where \( dS \) is the vector element normal to the surface \( S \) and \( n \) is the unit Poynting vector at a given point on the surface \( S \). It can be shown without any restrictions on the radiation intensity and geometry that the frequency shift of the output radiation \( \langle \omega^2 \rangle_{\text{out}} \) relative to the incident one, \( \langle \omega^2 \rangle_{\text{in}} = \omega_0^2 \), has the form

\[
\langle \omega^2 \rangle_{\text{out}} - \omega_0^2 = \frac{1}{\varepsilon_{\text{out}}} \left\{ \int_V d^3r \int_{-\infty}^{+\infty} dt \left[ \omega_0^2 E \cdot j - \frac{\partial E}{\partial t} \frac{\partial j}{\partial t} \right] + \frac{1}{8\pi} \int_V d^3r \right\},
\]

where

\[
f(t=+\infty) = f(t=+\infty) - f(t=-\infty), \quad E (B) \text{ and } j \text{ are the electric (magnetic) field and the current in the medium of volume } V, \text{ respectively},
\]

\( \varepsilon_{\text{in}} \) is the energy of incident radiation and \( \varepsilon_{\text{out}} \) is the energy of radiation coming out of the volume \( V \). The zero-frequency moment of Maxwell equations gives the energy conservation law for \( \varepsilon_{\text{in}} \) and \( \varepsilon_{\text{out}} \):

\[
\varepsilon_{\text{out}} = \varepsilon_{\text{in}} - \int_V d^3r \int_{-\infty}^{+\infty} dt \left[ E \cdot j - \frac{1}{8\pi} \int_V d^3r (E^2 + B^2) \right]_{t=+\infty}^{t=-\infty}.
\]

The first integral term in (2) describes in particular a blue frequency shift due to optical field ionization (OFI), while the last summands in (2) and (3) account for the plasma wave existing after the laser pulse. When the peak pulse intensity is much higher than the one corresponding to the optical ionization threshold, ionization and wakefield generation are separated in space and time. In this case, it can be shown that the total frequency shift due to both processes is the sum of the shift \( \delta\omega_{\text{ion}} \) due to ionization and the shift \( \delta\omega_{\text{wf}} \) associated with wakefield generation:

\[
\frac{\delta\omega}{\omega_0} \equiv \frac{\langle \omega^2 \rangle_{\text{out}} - \omega_0^2}{2\omega_0^2} \approx \frac{\delta\omega_{\text{ion}}}{\omega_0} + \frac{\delta\omega_{\text{wf}}}{\omega_0},
\]

where, in accordance with (2), the shift due to plasma wake generation is determined from the energy of the wake electric field, \( E_p \), excited in the plasma [10]:

\[
\frac{\delta\omega_{\text{wf}}}{\omega_0} \approx -\frac{1}{\varepsilon_{\text{out}}} \frac{1}{16\pi} \int_V E_p^2 \, dV.
\]

Assuming that the pulse intensity distribution is cylindrically symmetric with a characteristic radius larger than the plasma skin depth \( k_p^{-1} = c/\omega_p \) (where \( c \) is the speed of light and \( \omega_p = \sqrt{4\pi e^2 n_0/m} \) is the electron plasma frequency, where \( e, m \) and \( n_0 \) are the electron charge,
mass and background plasma density), the plasma wakefields can be found with the help of the equation for the wakefield potential, $\Phi$, [14]

$$\left\{ (\Delta_{\perp} - k_p^2) \frac{\partial^2}{\partial \xi^2} - \frac{\partial \ln n_0}{\partial r} \frac{\partial^3}{\partial r \partial \xi^2} + k_p^2 \Delta_{\perp} \right\} \Phi - \frac{k_p^4}{2} \left[ 1 - \frac{1 + |a|^2/2}{(\Phi + \delta \Phi_\delta)^2} \right] = \frac{k_p^2}{4} \Delta_{\perp} |a|^2, \quad (6)$$

where $\Delta_{\perp} = (1/r)\partial/\partial r(r\partial/\partial r)$ is the transverse part of the Laplace operator, $\xi = z - ct$, $a(\xi, z, r) = eE_L/(mcn_0)$ is the normalized complex envelope of the laser field, related to the high-frequency laser field by

$$\tilde{E}_L(r, t) = \text{Re} \{E_L(\xi, z, r)\exp(ik_0\xi)\},$$

where $k_0 = \omega_0/c = 2\pi/\lambda_0$ is the vacuum laser wave vector and $\delta \Phi_\delta$ describes the effect of OFI on wakefield generation [14]. In the weakly relativistic approximation ($|a| \ll 1, |\delta \Phi| \equiv |\Phi - 1| \ll 1$), assuming that the contribution due to ionization is negligible, and for the background plasma density $n_0$ uniform in space (or weakly inhomogeneous over lengths of the order of $k_p^{-1}$), the equation for the potential derived from (6) is

$$\left( \frac{\partial^2}{\partial \xi^2} + k_p^2 \right) \delta \Phi = \frac{1}{4} |a|^2. \quad (8)$$

With the help of $\partial \Phi/\partial \xi = eE_pz/m_ec^2$ and $\partial \Phi/\partial r = eE_pr/m_ec^2$, equation (8) yields the wakefield amplitude $E_p$ [10]:

$$E_p^{2,\text{max}} = \frac{me^2c^2 \omega_p^2}{e^2} \frac{1}{16} \left\{ k_p^2 \left[ \int_{-\infty}^{\infty} d\xi e^{-ik_p\xi} |a|^2 \right]^2 + \left| \frac{\partial}{\partial r} \right| \int_{-\infty}^{\infty} d\xi e^{-ik_p\xi} |a|^2 \right\} \right\}. \quad (9)$$

An expression similar to (9) was obtained for the electron density perturbations in [15] (see also [2]).

In order to obtain an analytical expression for the frequency shift of (5), a simplified expression for the laser pulse envelope is used in (9), where the reciprocal effect of the wakefield plasma wave on pulse propagation is neglected. Let us consider a Gaussian laser pulse focused at the capillary entrance, $z = 0$, as

$$a(\xi, r, z = 0) = a_0 \exp \left[ -\frac{r^2}{r_0^2} - \frac{1}{c^2} \frac{(\xi - \xi_0)^2}{\delta^2} \right], \quad (10)$$

where $a_0 \equiv 0.86 \times 10^{-9} (I_0[W \, cm^{-2}])^{1/2} \lambda_0[\mu m]$ is the normalized laser pulse amplitude ($I_0$ is the laser peak intensity). When the focal spot size of the laser pulse $r_0$ is matched to the inner radius of the capillary tube, $R_{\text{cap}}$, i.e. when $r_0 \equiv 0.65R_{\text{cap}}$, about 98% of the incident energy is transferred to the fundamental capillary mode, which can be described as [8, 14, 16,]

$$|a(\xi, z, r)|^2 = |a_0|^2 \exp \left( -2\delta k'' \xi - 2\frac{(\xi - \xi_0)^2}{\delta k''^2} \right) J_0^2 \left( b_1 \frac{r}{R_{\text{cap}}} \right), \quad (11)$$

where $b_1 \equiv 2.405$ is the first root of $J_0(x) = 0$, $J_0$ is the Bessel function of integer order 0 and $\delta k''$ is the damping coefficient associated to the loss of energy by refraction through the capillary wall with dielectric constant $\varepsilon_w$:

$$\delta k'' = \frac{b_1^2}{2k_p^2 R_{\text{cap}}^3 \sqrt{\varepsilon_w - 1}}.$$

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Using (11) and (9) in (5), the frequency shift as a function of the laser pulse propagation distance $z$ can be written as

$$
\frac{\delta \omega_{\text{ef}}(z)}{\omega_0} = -\frac{1}{64} \sqrt{\frac{\pi}{2}} \left(C_1 + \frac{C_2}{k_p^2 R_{\text{cap}}^2} \right) \left(\frac{\omega_p}{\omega_0}\right)^2 |a_0|^2 D(\Omega) \frac{k_p}{\delta k_{z_1}^2} \left[1 - \exp\left(-4\delta k_{z_1}^2 z\right)\right], \tag{12}
$$

where the constants $C_1$ and $C_2$ are determined by the radial laser intensity distribution for the fundamental mode

$$
C_1 \equiv \int_0^{b_1} x J_0^4(x) \left(\int_0^{b_1} x J_0^2(x) \right)^{-1} \approx 0.5655,
$$

$$
C_2 \equiv 4b_1^2 \int_0^{b_1} x J_1^2(x) J_0^2(x) \left(\int_0^{b_1} x J_0^2(x) \right)^{-1} \approx 4.361,
$$

and the function $D(\Omega) = \Omega \exp(-\Omega^2/4)$, with $\Omega = \omega_p \tau$, reflects the resonant behavior of wakefield generation, depending on the laser pulse duration with Gaussian time envelope in (11). Note that for a small energy loss by refraction through the capillary wall, when $\delta k_{z_1}^2 z \ll 1$, the frequency shift (12) grows linearly with the capillary length $z$.

In the following sections, the analytical prediction for the frequency shift due to wakefield generation of (12) will be compared with the results of fully self-consistent modeling, including gas ionization and nonlinear multi-mode laser pulse propagation in a capillary tube.

### 3. Modeling results and comparison with analytical predictions

To describe nonlinear laser pulse propagation in a gas-filled capillary, the following wave equation for the pulse envelope was used [14]

$$
\left\{ 2i k_0 \frac{\partial}{\partial z} + 2 \frac{\partial^2}{\partial z \partial \xi} + \Delta_\perp \right\} a = k_0^2 \left( \frac{n}{n_c \gamma} a - iG_{\text{ion}} \right), \tag{13}
$$

where $n$ is the slowly varying electron density, $n_c = m \omega_0^2/(4\pi e^2)$ is the critical density for the laser frequency $\omega_0$, $\gamma = [1 + (p/mc^2 + |a|^2/2)^{1/2}]$ is the relativistic factor of plasma electrons with momentum $p$ slowly varying in time and the last term on the right-hand side of (13) is the normalized ionization current, which represents the laser energy losses in the process of OFI [17]. In the quasi-static approximation [18], the relativistic plasma response $n/\gamma$ can be expressed through a single scalar function, the potential $\Phi$ determined by (6) [14]:

$$
\frac{n}{\gamma} = n_0 \frac{1 + k_p^{-2} \Delta_\perp \Phi}{\Phi + \delta \Phi_S}. \tag{14}
$$

When the laser pulse propagates in ionizing gas, equations (13), (14) and (6) should be supplemented with the equations for ionization kinetics, which determine the electron density $n_0(r, \xi)$ produced by OFI [14, 17, 19]

$$
\frac{\partial n_0}{\partial \xi} = -\frac{1}{c} \sum_{k=0}^{Z_n-1} \tilde{W}_k N_k, \tag{15}
$$

References not shown here for brevity.
where \( N_k \) and \( \bar{W}_k \) are the averages over the laser period of the ion densities and probabilities of tunneling ionization of ions [20] with charge state \( k, k = 0, 1, \ldots, Z_n \) and \( Z_n \) is the atomic charge number.

For the numerical modeling of laser pulse propagation and wakefield generation inside a gas-filled capillary tube by (6) and (13), boundary conditions have to be supplied. For linearly polarized laser pulses, the following boundary condition at the capillary tube wall, \( r = R_{\text{cap}} \), is used:

\[
\frac{\partial a}{\partial r} = 2i k_0 \frac{(\varepsilon_w - 1)^{1/2}}{\varepsilon_w + 1} \left( 1 - \frac{i}{k_0} \frac{\partial}{\partial \xi} \right) a,
\]

which describes correctly the structure of eigenmodes and their damping due to the energy loss through the capillary walls (see [14, 16]). The boundary conditions for the wakefield potential are \( \Phi = 1 \) in the unperturbed gas in front of the laser pulse, \( \xi \to +\infty \), and at the capillary wall \( r = R_{\text{cap}} \).

Taking into account (7), the mean-square frequency, defined by (1), at a distance \( z \) in the capillary is expressed using the laser envelope as follows:

\[
\langle \omega^2 \rangle(z) \equiv \left( \int_{-\infty}^{\infty} \int_{0}^{R_{\text{cap}}} |a(\omega, r, z)|^2 r \, dr \, d\omega \right)^{-1} \int_{-\infty}^{\infty} (\omega + \omega_0)^2 \int_{0}^{R_{\text{cap}}} |a(\omega, r, z)|^2 r \, dr \, d\omega,
\]

where \( a(\omega, r, z) \) is the dimensionless laser pulse envelope in Fourier space. The corresponding frequency shift is defined as

\[
\delta \omega(z) = \frac{\langle \omega^2 \rangle(z) - \omega_0^2}{2 \omega_0}.
\]

The solution to the system of equations (13)–(16) and (6) permits us to also obtain the normalized spectrum of the propagating laser field, centered at the carrier frequency \( \omega_0 \), integrated over the transverse cross section:

\[
I(\omega + \omega_0, z) \equiv \left( \max_{\omega} \int_{0}^{R_{\text{cap}}} |a(\omega, r, z)|^2 r \, dr \right)^{-1} \int_{0}^{R_{\text{cap}}} |a(\omega, r, z)|^2 r \, dr.
\]

This solution and spectrum (19) will be analyzed below and the resulting frequency shift given by (18) will be compared with the analytical expression (12), and to the experimental results in section 5. In order to compare with the experimental results, it is useful to define the relative wavelength shift. For a mean-square frequency shift \( \delta \omega \) small compared with \( \omega_0 \), determined by (1), (4) and (17), (18), the relative wavelength shift and the relative frequency shift are of opposite signs:

\[
\frac{\Delta \lambda z}{\lambda_0} = \left[ 1 - \frac{1}{\Delta \omega(z)/\omega_0 + 1} \right] \approx - \frac{\Delta \omega(z)}{\omega_0} \approx - \frac{\delta \omega(z)}{\omega_0},
\]

where

\[
\Delta \lambda(z) \equiv \left( \int_{0}^{\infty} I(\lambda, z) \, d\lambda \right)^{-1} \int_{0}^{\infty} \lambda I(\lambda, z) \, d\lambda - \lambda_0
\]

and \( \Delta \omega(z) \equiv \left( \int_{0}^{\infty} I(\omega, z) \, d\omega \right)^{-1} \int_{0}^{\infty} \omega I(\omega, z) \, d\omega - \omega_0 \) are the averaged wavelength and frequency shifts and \( I(\lambda, z) \) and \( I(\omega, z) \) are the spectral intensity, averaged over the radius, as functions of wavelength and frequency, respectively.

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Figure 1. Wavelength shift calculated with (12), the solid line (not marked), and obtained in simulations as functions of the laser propagation distance in a capillary filled with 20 mbar of hydrogen; lines with markers: OFI included (dots) and pre-ionized plasma (open squares). The inset shows the wavelength shift for the same parameters but in a capillary filled with helium. Parameters are indicated in the text.

The results of self-consistent modeling by (13)–(16) and (6) are illustrated in figures 1–4 for quasi-monomode laser pulse propagation, when the Gaussian laser pulse of (10) is focused at the entrance of a capillary of radius \( R_{\text{cap}} = 50 \mu m \), with the matched spot size \( r_0 = 0.67R_{\text{cap}} = 33.5 \mu m \) and \( a_0 = 0.24 \); for a full-width at half-maximum (FWHM) pulse duration \( \tau_{\text{FWHM}} = \sqrt{2\ln 2} \tau = 51 \text{ fs} \) and \( \lambda_0 = 0.8 \mu m \), it corresponds to a laser pulse energy of 0.12 J and an intensity \( I_0 = 1.3 \times 10^{17} \text{ W cm}^{-2} \).

The red wavelength shift (20) due to wakefield generation in a capillary filled with hydrogen at a pressure of 20 mbar is shown in figure 1 by lines with markers for OFI included (marked by dots) and for a pre-ionized plasma (marked by open squares). The corresponding spectra are shown in figure 2. They are in good agreement with the analytical prediction from (12) shown in figure 1 by a solid line (not marked). At this laser pulse intensity, well above the threshold value for the OFI of hydrogen gas, the laser pulse propagates in the capillary in the quasi-monomode regime over a few centimeters without substantial distortions and visible influence of ionization to the laser spectrum.

For the same parameters, when the capillary is filled with helium (with an atomic density half that of hydrogen in order to get the same free electron density at full ionization), the wavelength shift is determined completely by OFI: it is blue (negative) and of much larger amplitude than the red shift in hydrogen, as seen in the inset of figure 1 and in figure 2 for the lines marked by solid squares. Note that the integral wavelength shift (see (17), (18) and (20)) is determined not only by the shift of the maximum of the spectrum, but also by the ‘blue tail’ of the spectrum, not seen completely in figure 2 and extending in this case up to 400 nm. The laser pulse and the wakefield generated in the capillary filled with helium reveal in this case
Figure 2. Spectra of transmitted laser pulses integrated over the radius obtained in simulations after propagation over a length of 70 mm in a capillary filled with 20 mbar of hydrogen: OFI included (marked by dots) and pre-ionized plasma (open squares); the same for capillary filled with helium, the line is marked by solid squares. The dashed line shows the spectrum at the capillary entrance. Parameters are the same as in figure 1.

Figure 3. (a) Wavelength shift calculated with (12), unmarked solid line, and obtained in simulations as functions of the laser propagation distance in a capillary filled with 40 mbar of hydrogen, for $\tilde{e}_L = 50$ mJ: OFI included (marked by dots) and pre-ionized plasma (open squares). (b) Normalized laser pulse envelope $|a(r, \xi)|$ after propagating in a capillary of 50 mm long filled with 40 mbar of hydrogen (OFI included). All parameters are the same as in (a). (c) The same as in (b), but in pre-ionized plasma.

A pronounced longitudinal and radial modulation caused by the variation of electron density produced self-consistently by the OFI of helium [14, 21].

For higher gas pressure and lower laser pulse intensities, gas ionization starts to play a substantial role in the laser pulse spectrum modification even for capillaries filled with hydrogen. As an example, figure 3(a) shows the wavelength shifts for a capillary filled with...
Figures 3(b) and (c) exhibit the amplitude of the normalized laser pulse envelope $|a(r, \xi)|$ at a distance $z = 50\,\text{mm}$ for the same parameters as figure 3(a). In figure 3(b), where ionization is included, the front of the pulse at $k_p\xi \simeq 17$ is eroded due to refraction-induced ionization [21]. At the same time, there is no significant influence of the nonlinearity of pre-ionized plasma on the pulse propagation, as seen in figures 3(a) and (c).

For the same pressure of hydrogen (40 mbar), but higher laser intensity, $I_0 = 1.3 \times 10^{17}\,\text{W}\,\text{cm}^{-2}$, the plasma nonlinearity starts to play a substantial role in the pulse dynamics and transmitted laser spectrum, as illustrated in figure 4.

Comparison of figures 4(b) and (c) shows that the OFI of gas leads to the shortening and sharpening of the front of the pulse [21]; in addition, the self-phase modulation and nonlinear group velocity dispersion in the plasma causes a sharpening of the pulse tail just after the intensity maximum [22] (see figure 4(b)). Both these effects lead to pulse shortening that increases the wakefield generation [14, 23] and the subsequent red wavelength shift, as seen in figure 4(a), as the initial pulse length for this pressure was about twice as long as the linear resonant one.

4. Experimental arrangement

An experiment was performed using the multi-TW Ti:sapphire laser system at the Lund Laser Centre, which provided up to 0.5 J energy pulses onto the target, with an FWHM pulse duration of $40 \pm 5\,\text{fs}$ at a central wavelength of $\lambda_0 = 786\,\text{nm}$. The experimental setup scheme is presented in figure 5. The 30 mm-diameter laser beam was focused with an $f = 1.5\,\text{m}$ spherical mirror.
Figure 5. Schematic view of the experimental setup. All the elements up to the vacuum chamber window were under vacuum.

at the entrance of a capillary tube. In order to correct the aberrations of the phase front, a deformable mirror was used after the compressor. The laser beam transmitted through the capillary tube was attenuated by reflecting it from a glass wedge and then collimated by an achromatic $f_1 = 1$ m lens ($L_1$), which could be translated in vacuum along the beam axis to collect light either from the focal plane (i.e. with the capillary tube removed) or from the exit plane of the tube. The beam was then focused by an $f_2 = 1$ m achromatic lens ($L_2$) and magnified 10× by a microscope objective. The beam was split into two parts. Focal spot or capillary tube output images were recorded by a 16-bit charged-couple device (CCD 1 in figure 5). The transmitted part was sent to a visible-to-near-infrared imaging spectrometer equipped with a 16-bit CCD camera (CCD 2 in figure 5). The spectral resolution was 0.1 nm.

The laser beam path up to the vacuum chamber window was under vacuum. Glass capillary tubes with inner radius $R_{\text{cap}} = 50 \pm 1 \mu$m and length varying between 1.2 and 8.1 cm were used. Hydrogen gas ($H_2$) was injected into the tubes through two thin (≈ 100 $\mu$m) slits located between 2.5 and 5 mm from each end of the tube. The filling pressure was varied between 0 and 80 mbar. Each capillary tube could be used for at least a hundred laser shots, when the laser beam remained well centered at the capillary entrance. Pointing variations due to thermal drifts and mechanical vibrations were therefore minimized or compensated for. Laser guiding at input intensities up to $10^{18}$ W cm$^{-2}$ was achieved with more than 90% energy transmission in evacuated or hydrogen-filled gas tubes up to 8 cm long.

For the data presented here, in order to investigate the moderately nonlinear regime, the input intensity was kept lower than $3 \times 10^{17}$ W cm$^{-2}$. The laser pulse duration was $\tau_0 = 45 \pm 5$ fs and the associated bandwidth was approximately 25 nm (FWHM); each pulse had a small negative linear chirp, i.e. short wavelengths preceded longer wavelengths. The energy distribution in the focal plane exhibited an Airy-like pattern with a radius at first minimum of $r_0 = 40 \pm 5$ $\mu$m.
5. Comparison between modeling results and experimental data

To model wakefield generation and laser pulse spectrum modifications with initial conditions as close as possible to the experimental ones, the measured radial laser intensity distribution was approximated by a radial profile averaged over the azimuthal angle as shown in figure 6. This approximation was used for all the modeling results shown below. In the simulations, the FWHM laser pulse duration was 51 fs with a linear negative chirp of $-550 \, \text{fs}^2$, which corresponds to the experimentally measured laser spectrum width at the focal plane in vacuum.

Figure 7 shows the wavelength shift (21) measured in the experiment (squares with error bars) and obtained in the modeling (line with circles) for the experimentally measured radial distribution of laser intensity at the capillary entrance as a function of the laser pulse energy for a 7 cm-long capillary filled with hydrogen molecular gas at a pressure of 40 mbar. The dashed line is the analytical prediction (12) with the on-axis laser pulse amplitude adjusted so that the laser pulse energy in the main mode (11) equals the experimental measurement. The measured and modeled red wavelength shifts are in a good agreement in all the energy range shown in figure 7 and they are larger than the linear analytical prediction (12). At this gas pressure, above the linear resonant one equal to 25 mbar, and for laser pulse energies higher than 100 mJ, both ionization and nonlinear group velocity dispersion are responsible for the laser pulse shortening and consequently are more effective for wakefield generation as shown in figure 4. For energies lower than 100 mJ, the sharpening of the front edge of the pulse due to ionization is combined with the transverse modification of the structure of the laser pulse guided in the capillary; the use of the non-Gaussian distribution of figure 6 at the entrance thus induces a larger wavelength shift. This can be seen by comparison with figure 3(a) where the wavelength shift obtained for an initially Gaussian pulse in transverse direction is almost zero at 70 mm, whereas in figure 7 it is of the order of 7 nm for 50 mJ.
Figure 7. Wavelength shift measured in the experiment (squares with error bars) and obtained in the modeling (line with circles) as functions of the laser pulse energy for a 70 mm-long capillary filled with hydrogen gas at a pressure of 40 mbar. The dashed line is the analytical prediction (12).

The agreement between modeled and measured red shifts observed in figure 7 is supported by the agreement achieved for the transmitted laser pulse spectra shown in figure 8 for laser pulse energies 50, 120 and 150 mJ measured for different shots (black and gray solid lines in figures 8(a) and (c)) and obtained in full-scale modeling (dashed lines). The blue solid line in figure 8(b) shows a spectrum averaged over four shots. This shows that not only the pulse broadening but also the growth of the amplitude of the modulations can be reproduced by the simulations when the energy is increased.

The wavelength shifts as functions of pressure for different lengths of a capillary are shown in figure 9 at a laser pulse energy 120 mJ. For short, 12 mm-long capillaries the measured, modeled and analytically predicted shifts are close to zero as the ionization blue shift roughly cancels the wakefield red shift as can be seen for 20 mbar in figure 1(a). For longer capillaries, when wakefield generation has a substantial effect on the transmitted laser spectra, the measured and modeled dependences shown in figures 9(b)–(d) closely follow analytical predictions up to a pressure of the order of 25 mbar, which corresponds to the value determined by the linear resonant condition for the wakefield generation, $\omega_p \tau = 2$, for the Gaussian time envelope (10).

For higher pressures, in excess of 40 mbar and for capillaries longer than 50 mm, the measured and modeled red shifts grow with increasing pressure as opposed to the drop in the analytical behavior caused by the departure from the resonance between the laser pulse length and plasma wavelength (see figure 9, dashed lines). An increase of the wakefield generation and corresponding red wavelength shift for pressures higher than that determined by the linear resonant condition is caused by nonlinear laser pulse shortening. For monomode laser pulse propagation and 70 mm capillary length, this process is illustrated in figure 4(a) for a hydrogen pressure of 40 mbar. For the radial distribution of figure 6 and for higher pressure, the propagation in the capillary is multi-mode, leading to a more important pulse shortening as shown in figure 10 for the capillary length 71 mm and pressure 60 mbar (see also [24]). However, it should be noted that even for this strong nonlinear modification of the laser pulse,
Figure 8. Laser pulse spectra for pulse energies 50 (a), 120 (b) and 150 mJ (c) measured in the experiment (solid lines are for different shots in (a) and (c) and the average of 4 shots is shown in (b)) and obtained in simulations (dashed lines). Parameters are indicated in the text.
Figure 9. Wavelength shifts as functions of pressure at a laser pulse energy of 120 mJ for different lengths of capillary $L_{\text{cap}} = 12$ (a); 50 (b); 71 (c) and 81 mm (d). Squares with error bars are experimentally measured values, solid lines are the results of modeling and dashed lines are for analytical prediction (12).

the wakefield generated along the capillary tube is very regular as can be seen in figure 10(c), where the radial distribution of the wakefield potential as a function of the local time in the co-moving frame (moving with the speed of light) is shown by the color map. The zero of the local time corresponds to the time when the laser pulse maximum reaches the capillary entrance, see figure 10(a). The increasing deviations of the modeled red shift from the one measured as the pressure and the capillary length are increased (see figure 9) are attributed to the asymmetry of the focal spot in the experiment, whereas cylindrical symmetry, assumed in the model, leads to more pronounced nonlinear effects.

Figure 11 shows the wavelength shifts as functions of the length of the capillary filled with hydrogen gas, measured in the experiment and obtained in the modeling for pressures of 20 and 40 mbar and a laser pulse energy of 120 mJ. The wavelength shift grows linear as a function of length for the 20 mbar case, in agreement with analytical prediction (12). The fit of experimental data by simulation results demonstrates that the plasma wave is excited over a length as long as 8 cm. As the pressure is increased, the nonlinear laser pulse evolution is amplified with the propagation length leading to larger plasma wave amplitude.
Figure 10. Normalized laser pulse intensity (the hatched area) and wakefield potential (the solid line) on the axis at the entrance (a) and after propagation in a capillary of length 71 mm (b), as functions of the local time; radial distribution of the wakefield potential in time (c). Parameters of modeling correspond to figure 9(c) at pressure 60 mbar.

Figure 11. Wavelength shifts as functions of the length of capillary filled with 20 mbar of hydrogen gas measured in the experiment (solid squares with error bars) and obtained in the modelling (solid line); open circles with error bars and dashed line are for a pressure of 40 mbar. Laser pulse energy is 120 mJ.

6. Conclusion

Analytical predictions of the laser pulse frequency red shift based on the general approach of the frequency momentum theory formulated without any restrictions on the radiation intensity and geometry were used to analyze the wakefield generated by a short intense laser pulse propagating in a gas-filled capillary tube. The full-scale self-consistent modeling completely confirms the results obtained analytically in the following conditions: monomode propagation of the laser pulse, in a moderately nonlinear regime, with intensities much higher than the threshold value for the OFI of hydrogen filling the capillary, and gas pressures lower than the value determined by the linear resonant condition for wakefield generation. For higher pressures and laser pulse energies, modeling results exhibit nonlinear pulse shortening, enhanced wakefield...
generation and frequency red shift closely fitting experimental results. Even for intensities much higher than the threshold value for the OFI of filling gas, ionization-induced refraction can substantially modify the laser pulse, and the influence of ionization is more pronounced for higher pressures and lower pulse energies.

The agreement between modeled and measured red shifts observed as functions of pulse energy, pressure of hydrogen gas filling the capillary and capillary length is supported by the agreement achieved for the transmitted laser pulse spectra obtained in simulations and experiment. The value of the longitudinal accelerating field in the plasma obtained from the simulation is in the range 1–10 GV m\(^{-1}\). The average product of gradient and length achieved in this experiment is of the order of 0.4 GV at a pressure of 50 mbar; it could be increased to several GV by extending the length and diameter of the capillary tube with higher laser energy.

In conclusion, the outgoing spectra of driving laser pulses measured after propagation in gas-filled capillaries supported by relevant modeling can provide detailed information on laser pulse dynamics and on the main characteristics of the accelerating fields excited in the wake of the laser pulses over the long distances necessary for efficient acceleration of electrons to high energies.

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**References**

[1] Tajima T and Dawson J 1979 *Phys. Rev. Lett.* **43** 267
[2] Gorbunov L M and Kirsanov V I 1987 *Sov. Phys.—JETP* **66** 290
[3] Esarey E, Sprangle P, Krall J and Ting A 1996 *IEEE Trans. Plasma Sci.* **24** 252
[4] Marques J R *et al* 1996 *Phys. Rev. Lett.* **76** 3566
  Siders C W *et al* 1996 *Phys. Rev. Lett.* **76** 3570
[5] Le Blanc S P, Gaul E W, Matlis N H, Rundquist A and Downer M C 2000 *Opt. Lett.* **25** 764
[6] Dias J M, Silva L O and Mendonca J T 1998 *Phys. Rev. ST Accel. Beams* **1** 031301
[7] Wilks S C, Dawson J M, Mori W B, Katsouleas T and Jones M E 1989 *Phys. Rev. Lett.* **62** 2600
  Esarey E, Ting A and Sprangle P 1990 *Phys. Rev.* **42** 3526–31
[8] Andreev N E, Chegotov M V, Cros B, Mora P and Vieux G 2006 *Phys. Plasmas* **13** 053109
[9] Andreev N E, Fortov V E and Chegotov M V 2003 Spectral analysis of the ultrafast processes by the interaction of short laser pulses with matter *Physics of Extreme States of Matter* ed V E Fortov *et al* (Chernogolovka: Institute of Problems of Chemical Physics RAS) p 11
  Chetogov M V 2004 *Proc. Scientific Session of Moscow Institute of Engineering Physics* MIFI-2004 p 163
[10] Andreev N E and Chegotov M V 2005 *J. Exp. Theor. Phys.* **101** 56–63
[11] Murphy C D *et al* 2006 *Phys. Plasmas* **13** 033108
[12] Wojda F *et al* 2009 *Phys. Rev.* **80** 066403

*New Journal of Physics* **12** (2010) 045024 (http://www.njp.org/)
[13] Chegotov M V 2002 Tech. Phys. 47 1002
Chegotov M V 2003 Quantum Electron. 33 370–6

[14] Andreev N E, Nishida Y and Yugami N 2002 Phys. Rev. E 65 056407
Andreev N E, Cros B, Gorbunov L M, Matthieussent G, Mora P and Ramazashvili R R 2002 Phys. Plasmas 9 3999–4009

[15] Leemans W, Siders C W, Esarey E, Andreev N, Shvets G and Mori W B 1996 IEEE Trans. Plasma Sci. 24 331

[16] Dorchies F et al 1999 Phys. Rev. Lett. 82 4655
Courtois C, Couairon A, Cros B, Marque’s J R and Matthieussent G 2001 Phys. Plasmas 8 3445
Cros B, Courtois C, Matthieussent G, Di Bernardo A, Batani D, Andreev N and Kuznetsov S 2002 Phys. Rev. E 65 026405

[17] Andreev N E, Chegotov M V and Veisman M E 2000 IEEE Trans. Plasma Sci 28 1098
Andreev N E, Veisman M E, Cadjan M G and Chegotov M V 2000 Plasma Phys. Rep. 26 947

[18] Sprangle P, Esarey E and Ting A 1990 Phys. Rev. Lett. 64 2011

[19] Kandidov V P, Kosareva O G and Shlenov S A 1994 Quantum Electron. 24 971

[20] Ammosov M V, Delone N B and Krainov V P 1986 Sov. Phys.—JETP 64 1191
Delone N B and Krainov V P 1998 Phys.—Usp. 41 469
Delone N B and Krainov V P 1998 Usp. Fiz. Nauk 168 531

[21] Andreev N E, Chegotov M V and Pogosova A A 2003 J. Exp. Theor. Phys. 96 885

[22] Esarey E, Schroeder C B, Shadwick B A, Wurtele J S and Leemans W P 2000 Phys. Rev. Lett 84 3081
Hubbard R F, Sprangle P and Hafizi B 2000 IEEE Trans. Plasma Sci. 28 1122

[23] Andreev N E, Kuznetsov S V, Pogosova A A, Steinhauer L C and Kimura W D 2003 Phys. Rev. ST Accel. Beams 6 041301
Kimura W D et al 2005 IEEE Trans. Plasma Sci. 33 3–7

[24] Skobelev S A, Kulagin D I, Stepanov A N, Kim A V, Sergeev A M and Andreev N E 2009 JETP Lett. 89 540–6