Gauge Invariance and Anomalous Dimensions of a Light-Cone Wilson Loop in Light-Like Axial Gauge

A. Bassetto

Dipartimento di Fisica, Università di Padova and INFN, Sezione di Padova, 35100 Padova, Italy

I. A. Korchemskaya, G. P. Korchemsky

Dipartimento di Fisica, Università di Parma and INFN, Gruppo Collegato di Parma, 43100 Parma, Italy

and

G. Nardelli

Dipartimento di Fisica, Università di Trento and INFN, Gruppo Collegato di Trento, 38050 Povo (Trento), Italy

Abstract

Complete two-loop calculation of a dimensionally regularized Wilson loop with light-like segments is performed in the light-like axial gauge with the Mandelstam-Leibbrandt prescription for the gluon propagator. We find an expression which exactly coincides with the one previously obtained for the same Wilson loop in covariant Feynman gauge. The renormalization of Wilson loop is performed in the $\overline{MS}$–scheme using a general procedure tailored to the light-like axial gauge. We find that the renormalized Wilson loop obeys a renormalization group equation with the same anomalous dimensions as in covariant gauges. Physical implications of our result for investigation of infrared asymptotics of perturbative QCD are pointed out.

*On leave from the Moscow Energetic Institute, Moscow, Russia
†On leave from the Laboratory of Theoretical Physics, JINR, Dubna, Russia
1. Introduction

There exists a unique possibility in non-Abelian gauge theories to formulate the nontrivial dynamics of gauge fields in terms of the gauge invariant collective variables $W_C$

\[ W_C = \frac{1}{N} \langle 0 | \text{tr} \mathcal{T} \mathcal{P} \exp \left( ig \int_C dx^\mu A^a_\mu(x) t_a \right) | 0 \rangle, \]

called the Wilson loop expectation values. Here, $A^a_\mu(x)$ is the gauge field, $\mathcal{T}$ orders gauge field operators in time and $\mathcal{P}$ orders generators $t_a$ of the $SU(N)$ gauge group along a closed integration path $C$. With this definition, $W_C$ is a gauge invariant non local functional of a gauge field depending on the integration path and satisfying nontrivial loop equations.

However, a closer consideration shows that $W_C$ is not a well defined quantity for some paths $C$ lying in 4–dimensional Minkowski space-time. The problem is that for a closed path $C$ having either cusps, or self-intersections, or segments lying on the light-cone, $W_C$, when calculated in perturbation theory, is divergent even after renormalization of gauge field and coupling constant.

Perturbation theory expansion of $W_C$ looks like

\[ W_C = 1 + \sum_{n=2}^{\infty} (ig)^n \int_C dx_1^{\mu_1} \ldots \int_C dx_n^{\mu_n} \theta(x_1 > \ldots > x_n) \frac{1}{N} \text{tr} G_{\mu_1 \ldots \mu_n}(x_1, \ldots, x_n), \quad (1.1) \]

where $G_{\mu_1 \ldots \mu_n}(x_1, \ldots, x_n)$ is the $n$–point Green function and the $\theta$–function orders points $x_1, \ldots, x_n$ along the integration path $C$.

Let us consider the simplest closed path in Minkowski space-time shown in fig. 1 with parallel segments lying on the light-cone. Under integration over, say, $x_1$ and $x_2$ along the path $C$, the renormalized Green functions $G_{\mu_1 \ldots \mu_n}(x_1, x_2, \ldots, x_n)$ may have singularities at coinciding points ($x_1 = x_2$) and light-cone singularities ($(x_1 - x_2)^2 = 0$), which give rise to very specific divergences of $W_C$.

These divergences were considered for a long time as an undesirable byproduct in dealing with the loop equations. This opinion radically changed after it was found that those specific divergences are of the utmost importance in perturbative QCD. It turned out indeed that there is an intimate relation between their renormalization and infrared asymptotics of hard processes in perturbative QCD. For instance, the Wilson loop expectation values calculated along paths partially lying on the light-cone obey renormalization group equations (RG) which coincide with the Altarelli-Parisi-Lipatov and the Brodsky-Lepage evolution equations.

The “bremsstrahlung” function, well known in QED for a long time, is nothing but the cusp anomalous dimension of the Wilson loop. The same function, called velocity dependent anomalous dimension, was rediscovered recently within the heavy quark effective field theory. That is why the investigation of the renormalization properties of Wilson loops has not a pure academic interest.

In a previous paper the calculation of the Wilson loop along the light-like path of fig. 1 was performed in the second order of perturbation theory in the Feynman gauge. The dimensional regularization was used and all divergences were subtracted in the $\bar{MS}$–scheme. It was shown that the renormalized Wilson loop, being an even dimensionless function of the scalar product $(n \cdot n^*)$, of the renormalization point $\mu$ and of the coupling constant, has a form

\[ W^R_C = W^R_C(\rho, g), \quad \rho^2 = (n \cdot n^*) \mu^2, \]

and obeys the following RG equation:

\[ \left( \frac{\partial}{\partial \rho} + \beta(g) \frac{\partial}{\partial g} \right) \log W^R_C(\rho, g) = -2\Gamma_{\text{cusp}}(g)(\log(\rho^2 + i0) + \log(-\rho^2 + i0)) - \Gamma(g), \quad (1.2) \]

$n$ and $n^*$ being the two light-like vectors

\[ n_\mu = (T, 0, 0, -T), \quad n^*_\mu = (L, 0, 0, L), \quad (n \cdot n) = (n^* \cdot n^*) = 0, \quad (n \cdot n^*) = 2LT. \]

with $L, T > 0$. The unusual property here is that the r.h.s. being considered as anomalous dimension depends on the renormalization point. It implies that $W_C$ does not renormalize multiplicatively.

Equation (1.2) contains two gauge invariant anomalous dimensions: $\Gamma_{\text{cusp}}(g)$ and $\Gamma(g)$. The first one is related to the asymptotic behaviour of the so-called cusp anomalous dimension and has very interesting properties. In
particular, $\Gamma_{\text{cusp}}(g)$ does not depend on the form of the integration path $C$ and is equal at two-loop order to

$$\Gamma_{\text{cusp}}(g) = \frac{\alpha_s}{\pi} C_F + \left( \frac{\alpha_s}{\pi} \right)^2 C_F \left( C_A \left( \frac{67}{36} - \frac{\pi^2}{12} \right) - N_f \frac{5}{18} \right). \quad (1.3)$$

with $\alpha_s = \frac{g^2}{\pi}$. Here, $C_F = \frac{N_c^2 - 1}{2N_c}$ and $C_A = N$ are color factors of the gauge group $SU(N)$. The anomalous dimension $\Gamma(g)$ does depend on $C$ and it was found at two-loop order for the path in fig. 1 to be

$$\Gamma(g) = -\left( \frac{\alpha_s}{\pi} \right)^2 C_F \left( \left( 7\zeta(3) - \frac{202}{27} + \frac{11}{36}\pi^2 \right) C_A + \left( \frac{28}{27} - \frac{\pi^2}{18} \right) N_f \right). \quad (1.4)$$

The RG equation (1.2) was checked in Feynman gauge and gauge invariance implies that it should be fulfilled in any gauge with the same anomalous dimensions $\Gamma_{\text{cusp}}(g)$ and $\Gamma(g)$. This is one of the statements we are going to verify.

In the present paper we calculate the Wilson loop expectation value along path of fig. 1 in the light-like axial (LLA) gauge:

$$n^\mu A^a_\mu(x) = 0 \quad (1.5)$$

with the gauge fixing vector along one of the sides of the integration path at the second order of perturbation theory using dimensional regularization with $D = 4 - 2\epsilon$.

It is well-known that the free propagator of the gauge field is more singular in axial gauges owing to the presence of additional “spurious” singularities. In the LLA gauge we are considering, a prescription, the Mandelstam-Leibbrandt (ML) causal prescription [1], has been proposed to consistently define it as

$$G_{\mu\nu}(k) = -\frac{i}{k^2 + i\epsilon} \left( g_{\mu\nu} - \frac{k_\mu n_\nu + k_\nu n_\mu}{(kn) + i\epsilon \cdot (kn^*)} \right). \quad (1.6)$$

We stress that the $+i\epsilon$ prescription only specifies how the integration contour is to be distorted near the pole. In particular it is irrelevant in the expression

$$\frac{(nk)}{(nk) + i\epsilon \cdot (n^*k)} = 1,$$

always in the sense of the theory of distributions; one can also prove that Mandelstam’s proposal does indeed coincide with Leibbrandt’s one, namely

$$\frac{1}{(nk) + i\epsilon \cdot (n^*k)} = \frac{(n^*k)}{(nk)(n^*k) + i\epsilon}.$$ 

The vector propagator contains the famous $(kn) = 0$ singularity and, as a consequence, individual Feynman diagrams contributing to $W_C$ in LLA gauge may contain additional poles at $D - 4$. These additional poles should be compensated in the sum of all diagrams contributing to $W_C$, in order to ensure its gauge invariance. The mechanism of this compensation is not trivial and very sensitive to the prescription one uses to define the propagator [4]. Our purpose it to show that we shall indeed find full consistency using the ML prescription, at least up to the second order in the loop expansion.

Calculating Wilson loop along the path of fig. 1 in the LLA gauge, we will meet three different kinds of divergences: specific light-cone singularities of Wilson loops, singularities which are peculiar of the light-cone gauge we are using and conventional divergences of the Yang-Mills theory. Since the first two of them originate from the same phase space region of gluon momenta, i.e. gluons propagating along the light-cone, an interplay is $a priori$ possible destroying their correct compensation. It would mean that the LLA gauge with the ML prescription is sick.

This doubt was raised in a recent paper [11]. The leading singularities of the Wilson loop of fig. 1 were calculated using the LLA gauge with the ML prescription in the second order of perturbation theory and the following expression was obtained

$$W_C \sim \exp \{ B\alpha_s^2 C_F C_A (D - 4)^{-4} + \mathcal{O}((D - 4)^{-3}) \},$$

$B$ being a suitable constant, which turned out to be different from the Feynman gauge result: $B = 0$. 

3
In what follows we present a complete two-loop calculation of the same dimensionally regularized Wilson loop. As dimensional regularization does not spoil gauge invariance, we shall verify that the expression we obtain in the LLA gauge for the unrenormalized (but dimensionally regularized) Wilson loop exactly coincides with the analogous quantity evaluated in Feynman gauge up to terms vanishing when \( D = 4 \), i.e. not only leading and non-leading logarithmic terms but up to finite ones. This is a complete check of gauge invariance and thereby of the correctness of the ML prescription, which is crucial to this goal.

Then, as a second step, we shall renormalize Wilson loop expectation value in the \( \overline{MS} \)-scheme according to the theory of renormalization in LLA gauge with ML prescription developed in refs.\cite{12}. We show that it indeed obeys the RG equation (1.2) with the same anomalous dimensions (1.3) and (1.4) as in covariant gauges.

The paper is organized as follows. In Sect. 2 the properties of free gluon propagator are described and the full set of non vanishing Feynman diagrams contributing to \( W_C \) is identified. In Sect. 3 we determine \( W_C \) at the first order of perturbation theory. Second order Feynman diagrams are evaluated in Sect. 4 and summed to get the final expression for the dimensionally regularized Wilson loop, recovering the Feynman gauge result. In Sect. 5 the Wilson loop renormalization, which presents non trivial features in LLA gauge, is performed in the \( \overline{MS} \)-scheme and a check is made of the anomalous dimensions against the corresponding ones in Feynman gauge, thereby supporting the renormalization procedure in LLA gauge developed in refs.\cite{12}. Sect. 6 contains concluding remarks. Some technical details of our calculations are presented in the Appendices.

2. Free gluon propagator

To calculate the vacuum average Wilson loop we use the definition (1.1). The integration path \( C \) is shown in fig. 1. It lies on the light-cone and is formed by two light-like vectors \( n \) and \( n^\ast \) we have already introduced. As a consequence, in the LLA gauge (1.5) gluons cannot be attached to any segment of the path \( C \) going along the gauge fixing vector. We parameterize the two remaining segments of the path \( C \) as follows:

\[
x_1(t) = n^\ast t, \quad t \in [0, 1]; \quad x_2(s) = n + n^\ast s, \quad s \in [1, 0].
\]  

(2.1)

Note that parameter \( s \) runs from 1 to 0 in order to provide the correct orientation of the integration path.

There are a lot of Feynman diagrams contributing to the Wilson loop expectation value at the second order of perturbation theory. However, the number of diagrams one has to evaluate can be drastically reduced using the non-Abelian exponentiation theorem \cite{13}. According to this theorem

\[
W_C = 1 + \sum_{n=1}^{\infty} W^{(n)} = \exp \sum_{n=1}^{\infty} w^{(n)},
\]  

(2.2)

where \( w^{(n)} \) is given by the contribution of \( W^{(n)} \) with the maximal non-Abelian color factor to the \( n \)-th order of perturbation theory which is equal to \( C_F \) for \( n = 1 \) and \( C_F C_A \) for \( n = 2 \). The two-loop diagrams containing color factors \( C_F \) and \( C_F C_A \) are shown in figs. 2, 3 and 4 where we omitted symmetric diagrams. It turned out that, owing to the properties of the gluon propagator with the ML prescription, many of these diagrams give vanishing contributions.

The peculiar features of the free gluon propagator, when using the ML prescription for the “spurious” singularity, have been discussed in \cite{14}. In particular it has been shown that the propagator behaves in the coordinate representation as a tempered distribution at variance with the expression one would obtain adopting the Cauchy principal value prescription. In this last case additional infrared singularities would plague the component \( n^\mu n^\nu G^{\mu\nu}(x) \).

The explicit expressions for the propagator with the ML prescription in the coordinate representation are

\[
G_{\alpha\beta}(x) = \delta_{\alpha\beta}G(x), \quad \alpha = 1, 2,
\]

with the causal scalar distribution

\[
G(x) = \frac{1}{4\pi^2} \frac{1}{x^2 - i\delta},
\]

and

\[
G_{\alpha\beta}(x) = \frac{\partial}{\partial x^\alpha} \int_0^{x^+} d\xi G(\xi, x^-), \quad G_{\alpha\mu}(x) = \frac{\partial}{\partial x^\mu} \int_0^{x^+} d\xi G(\xi, x^-), \quad G_{+\mu}(x) = 0.
\]
We have used here the light-cone variables \( x_-, x_+ \) and \( x_T \) for the components of vector \( x \). Their definitions and some useful identities can be found in Appendix A.

The origin of this reasonable behavior from a mathematical point of view can be traced back to the causal nature of the ML prescription in which the “spurious” pole complies with the Feynman poles position in the complex energy plane so that no pinches occur under Wick’s rotation of the integration contour. From the physical point of view it has been shown [13] that a “longitudinal” ghost enter the theory. This ghost, while decoupling in all physical quantities, is responsible for the mild infrared behavior of the gluon propagator. Nevertheless, since the propagator is a generalized function, local limits may not exist. Indeed, using the expression for the \( G_{-+}(x) \) component one immediately finds that the function

\[
G(x) = n^*_\mu n^*_\nu G^{\mu\nu}(x)
\]

is divergent for \( x_T = 0 \).

Then one uses the dimensional regularization to define it at \( x_T = 0 \), as

\[
G(x_+, x_T = 0, x_-) = n^*_\mu n^*_\nu G^{\mu\nu}(x_+, x_T = 0, x_-) = \frac{2\pi^{-D/2}\Gamma(D/2)}{4-D} \frac{(xn^*)^2}{(-x^2 + i0)^{D/2}}. \tag{2.3}
\]

Of course, it is the “spurious” \((kn) = 0\) singularity which is responsible for the appearance of a pole in (2.3) for \( D = 4 \).

It is now straightforward to realize that one- and two-loop diagrams with all gluons attached to the same contour side (see e.g. figs. 2a and 3a,b) give zero. Consider first the contribution of the diagram of fig. 2a:

\[
(i\gamma)^2 C_F \mu^{4-D} \int_0^1 ds \int_0^s dt n^*_\mu n^*_\nu G^{\mu\nu}(n^*(s-t)).
\]

Using the definition (2.3) we find that it vanishes as

\[
n^*_\mu n^*_\nu G^{\mu\nu}(n^*(s-t)) \sim (n^*n^*)^{2-D/2} = 0 \tag{2.4}
\]

for \( \text{Re}D < 4 \). The diagrams of fig. 3a and 3c also vanish since the same function enters the corresponding path integrals. We stress that the property (2.4) is a peculiar consequence of the ML prescription. To show this, let us perform again the evaluation of the diagram of fig. 2a in the momentum representation:

\[
(i\gamma)^2 C_F \mu^{4-D} \int_0^1 ds \int_0^s dt \frac{dk_- dk_+}{(2\pi)^D} e^{-ik_- L \sqrt{2(s-t)}} \frac{2i}{(2k_+ k_- - k^2_T + i0)} \frac{k_-}{(k_+ + i0 k_-)}.
\]

It is important to note that \( k_+ \) does not enter the exponent when the gluon is attached to the same contour side. As a consequence, the integral over the \( k_+ \) component is zero because with the ML prescription poles of Feynman and “spurious” denominators lie on the same side of the real \( k_+ \) axis. The same arguments can be applied to show that the diagram of fig. 3b gives zero. In this case we have an integral over two momenta which vanishes after one closes the integration contours over “+” components of both momenta without encountering any pole.

3. One-loop calculation

In the lowest order of perturbation theory, Wilson loop (1.1) has a single nonvanishing contribution coming from the diagram of fig. 2b

\[
W^{(1)}_{C} = (ig)^2 C_F \mu^{4-D} \int_0^1 ds \int_1^0 dt n^*_\mu n^*_\nu G^{\mu\nu}(x_2(t) - x_1(s)),
\]

where the functions \( x_1(s) \) and \( x_2(t) \) are defined in (2.1). Notice, that the vector \( x_1(s) - x_2(t) \) lies in the plane of the Wilson loop and one can use expression (2.3) for the propagator in Minkowski space-time. After substitutions of (2.3) and (2.4) one gets

\[
W^{(1)}_{C} = g^2 C_F \mu^{4-D} \frac{2\Gamma(D/2)}{4-D} (nn^*)^2 \int_0^1 ds \int_0^1 dt |2(nn^*)(s-t) + i0|^{-D/2}.
\]

\(^1\text{In the Feynman gauge, the propagator of the gauge field is equal to} \ -\frac{g^2}{4\pi^2} \Gamma(D/2 - 1)(-x^2 + i0)^{1-D/2} \text{and is regular for} \ D = 4.\)
The singularity of the integrand for \( s = t \) comes from the propagation of the gluon in fig. 2b along the light-cone from point \( n^*s \) to \( n + n^*t \). After an integration which is trivial in the region \( \text{Re}D < 2 \) and a subsequent analytic continuation up to \( \text{Re}D < 4 \), one finds

\[
W^{(1)}_C = - \frac{g^2}{(2\pi)^{D/2}} C_F \frac{4\Gamma(D/2 - 1)}{(4 - D)^2} \left[ (-\rho^2 + i0)^{2-D/2} + (\rho^2 + i0)^{2-D/2} \right]. \tag{3.1}
\]

We have obtained the same result in the momentum representation, i.e. integrating first over the contour and performing the Fourier transform afterwards. Although calculations are more involved in this way, they can be done directly in the strip \( 3 < \text{Re}D < 4 \).

We conclude that in the first order of perturbation theory the Wilson loop expectation value contains a double pole. This expression coincides exactly with the analogous expression in the Feynman gauge.

4. Two-loop calculation

Calculating the Wilson loop in the second order of perturbation theory we use non-Abelian exponentiation and restrict ourself only to diagrams containing the non-Abelian color factor \( C_FC_A \). As was shown in Sect. 2, among these diagrams only those shown in fig. 4 give non vanishing contributions whereas diagrams of figs. 3a,b,c do vanish.

4.1. Crossed diagram

The contribution of the “crossed” diagram of fig. 4a to \( W_C \) contains the integration of two propagators over the path:

\[
W^{(2)}_{\text{crossed}} = (ig)^4 C_F(C_F - C_A/2) \int_0^1 ds_1 \int_0^{s_1} ds_2 \int_0^1 dt_1 \int_{t_1}^{t_1} dt_2 G(n + n^*(t_1 - s_1))G(n + n^*(t_2 - s_2)).
\]

After substitution of (2.3), \( W^{(2)}_{\text{crossed}} \) acquires a double spurious pole before the integration over \( s^- \) and \( t^- \) parameters which are ordered along the path as shown in fig. 4a.

\[
W^{(2)}_{\text{crossed}} = \frac{g^4}{\pi^D} C_F(C_F - C_A/2) \mathcal{M}^{8-2D} \frac{4\Gamma^2(D/2)}{(4 - D)^2} (nn^*)^4
\]

\[
\times \int_0^1 ds_1 \int_0^{s_1} ds_2 \int_0^1 dt_1 \int_{t_1}^{t_1} dt_2 [(2(nn^*)(s_1 - t_1) + i0)(2(nn^*)(s_2 - t_2) + i0)]^{D/2}.
\]

As in the previous case, we expect to get additional singularities from the propagation of both gluons along the light-cone which correspond to the following values of parameters: \( s_1 = s_2 = t_1 = t_2 \). Indeed, a careful integration for \( \text{Re}D < 2 \) and therefrom an analytic continuation up to \( \text{Re}D < 4 \) leads to

\[
W^{(2)}_{\text{crossed}} = \frac{g^4}{(2\pi)^{D}} C_F(C_F - C_A/2) \frac{4\Gamma^2(D/2 - 1)}{(4 - D)^4} \left\{ \frac{D - 2}{D - 3} \left[ (-\rho^2 + i0)^{4-D} + (\rho^2 + i0)^{4-D} \right] \right. \\
\left. + 4 \left( 1 - \frac{1}{D - 3} \right) \frac{\Gamma^2(3 - D/2)}{\Gamma(5 - D)} \left[ (-\rho^2 + i0)(\rho^2 + i0) \right]^{2-D/2} \right\}. \tag{4.1}
\]

Again the result has been checked in the momentum representation, where the calculation can be directly performed in the strip \( 3 < \text{Re}D < 4 \).

\( W^{(2)}_{\text{crossed}} \) has a third pole at \( D = 4 \). Thus, the leading singularity of \( W^{(2)}_{\text{crossed}} \) is formed by a double pole arising from the “spurious” contributions and only one single light-cone singularity. At the same time an analogous crossed diagram in the Feynman gauge gives rise to a fourth-order pole which however cancels in the sum of all diagrams. It means that the Feynman gauge is more singular than LLA gauge as far as light-cone singularities are concerned. This property is well known in perturbative QCD. As we shall show below, all the diagrams of fig. 4 have no fourth pole at all.
4.2. Self-energy diagram

The contribution of the diagram of fig. 4b to the Wilson loop reads:

\[ W_{\text{self}}^{(2)} = (ig)^2 C_F \mu^{4-D} \int_0^1 ds \int_1^0 dt G_1(n + n^*(t-s)), \]

where in the momentum representation the function \( G_1(k) \) is defined as

\[ G_1(k) = \frac{1}{2} n^*_\rho G^{\mu\nu}(k) \Pi_{\nu\rho}(k) G^{\rho\lambda}(k) n_\lambda. \]

Here \( G_{\mu\nu}(k) \) is the free gluon propagator (1.6) and \( \Pi_{\nu\rho}(k) \) is the one-loop gluon self-energy operator in the LLA gauge. There are no Faddeev-Popov ghosts and \( \Pi_{\nu\rho}(k) \) gets contribution only from the gluon loop.

The calculation of the one-loop self-energy \( \Pi_{\nu\rho}(k) \) in the LLA gauge is a very cumbersome problem. As was shown in [16], the self-energy \( \Pi_{\nu\rho}(k) \) is transverse and it is decomposed into seven independent tensor structures some of which depending on the gauge fixing vectors \( n \) and \( n^* \). Moreover the contributions to the self-energy of the tensors involving \( n_\rho \) and \( n^*_\rho \) exhibit non polynomial residues at \( D = 4 \), whereas the residue of the contribution of the “physical” tensor \( (g_{\nu\rho} k^2 - k_\nu k_\rho) \) gives rise in the LLA gauge to the one-loop \( \beta \)–function, since \( Z_1 = Z_3 \) in this gauge as is well known.

Notice that, after substitution of the self-energy into (4.3), all tensors do contribute to the function \( G_1(k) \) and the residue at the pole differs from the \( \beta \)–function.

The one-loop self-energy was calculated in [16] up to \( O((D - 4)^3) \)–terms. It is important to recognize that these terms, which vanish formally for \( D = 4 \), might become divergent after integration over the path in (4.2) due to light-cone singularities. This is the reason why one is obliged to first calculate the self-energy for an arbitrary \( D \), then to integrate over the path and only eventually perform a Laurent expansion at \( D = 4 \). We have performed this calculation in the momentum representation and some details are given in Appendix B.

We present here only the final expression for the function (4.3)

\[ G_1(k) = -\frac{4(n^*)^2}{k^2 [nk]} I_1 + 4 \left\{ \frac{(n^*)^2}{[nk]} - 4 \frac{(n^*_k)(nn^*)}{k^2} \right\} I_2 - 2 \frac{3D - 2}{D - 1} \frac{(n^*_k)(nn^*)}{k^2 [nk]} I_3 - 8 \frac{(nn^*)^2}{k^2} I_4 \]

where the integrals \( I_1 \) ... \( I_4 \) are listed in Appendix B. We have checked that \( G_1(k) \), after Laurent expansion around \( D = 4 \), coincides with the expression (4.3) calculated starting from the self-energy \( \Pi_{\nu\rho}(k) \) found in [16].

By performing the Fourier transform, we obtain the function \( G_1 \) in the coordinate representation

\[ G_1(x) = -\frac{g^2}{8\pi^D} C_A \mu^{4-D} \left\{ \frac{(nn^*^2)}{(-x^2 + i0)^{D-2}} \frac{8}{(4-D)^3} \left\{ \frac{\Gamma(3-D/2)\Gamma(D-3)}{\Gamma(5-D)} - \Gamma^2(D/2 - 1) \right\} \right\}, \]

which is valid for \( xT = 0 \).

Inserting (4.4) into (4.2) and integrating over the path, in a region \( \text{Re}D < 2 \), we get the following contribution from the diagram of fig. 4b to the Wilson loop

\[ W_{\text{self}}^{(2)} = \frac{g^4}{(2\pi)^D} C_F C_A \left[ (-\rho^2 + i0)^{4-D} + (\rho^2 + i0)^{4-D} \right] \times \frac{4}{(4-D)^4(4-D)} \left\{ \frac{\Gamma(3-D/2)\Gamma(D-3)}{\Gamma(5-D)} - \Gamma^2(D/2 - 1) \right\}, \]

which can be continued to the “natural” strip \( 3 < \text{Re}D < 4 \). Again the same result can be recovered directly in this strip by performing the calculation in the momentum representation.

The expression for \( W_{\text{self}}^{(2)} \) exhibits the third pole coming from the double pole of the function (4.3) and the single light-cone singularity of the path integral in (4.2) for \( s = t \).

\[ \text{We do not explicitly consider the contribution of the quark loop, which does not entail any new feature with respect to the one in Feynman gauge [12].} \]
4.3. Three gluon diagram

The calculation of the diagram with the three-gluon vertex is the most cumbersome one. At variance with the previous diagrams where we have integrated free gluon propagators in the coordinate representation along the path, here we have to deal with the additional vertex of a self-gluon interaction. In the coordinate representation it implies integration over the intermediate point \( z \). Then the contribution of the diagram of fig. 4c is given by

\[
W_{3-\text{gluon}}^{(2)} = (ig)^{4} \frac{1}{2} C F C_{\mu} \Lambda^{8-2D} \int_{0}^{1} ds_{1} \int_{0}^{1} ds_{2} \int_{0}^{s_{2}} ds_{3} \times \int d^{D}z \int \frac{d^{D}k}{(2\pi)^{D}} e^{ik_{1}(x_{2}(s_{1})-z)} e^{ik_{2}(x_{1}(s_{2})-z)} e^{ik_{3}(x_{1}(s_{3})-z)} V(k_{1}, k_{2}, k_{3}),
\]

where the parameters \( s_{i} \) are ordered along the path and gluons are attached to the path at points \( x_{1}(s_{1}), x_{2}(s_{2}) \) and \( x_{2}(s_{3}) \) defined in (4.3). Here \( \frac{d^{D}k}{(2\pi)^{D}} \) denotes integration over gluon momentum and

\[
V(k_{1}, k_{2}, k_{3}) = -i\Gamma_{\mu\nu\rho}(k_{1}, k_{2}, k_{3}) G_{\mu\nu'}(k_{1}) G_{\nu\rho'}(k_{2}) G_{\rho\mu'}(k_{3}) n^{\mu}_{i} n^{\nu}_{r} n^{\rho}_{s},
\]

\( \Gamma_{\mu\nu\rho}(k_{1}, k_{2}, k_{3}) \) being the standard three-gluon vertex expression. Note that integration over the intermediate point \( z \) ensures momentum conservation \( k_{1} + k_{2} + k_{3} = 0 \).

Our strategy for calculating \( W_{3-\text{gluon}}^{(2)} \) is the following. Instead of integrating over \( z \), we first work out the expression for \( V(k_{1}, k_{2}, k_{3}) \) in the momentum representation, then perform independent integrations over the three gluon momenta in (4.6) and integrate over \( z \) at the very end.

After some algebra we get from (4.7) an expression for \( V(k_{1}, k_{2}, k_{3}) \) involving four addenda

\[
V(k_{1}, k_{2}, k_{3}) = V_{1} + V_{2} + V_{3} + V_{4},
\]

where the expression for \( V_{1} \) contains only one single “spurious” denominator \( [kn] \equiv (kn) + i0 \cdot (kn*) \):

\[
V_{1} = \frac{2(mn*)}{k_{1}^{2} k_{2}^{2} k_{3}^{2}} \left\{ \left( (k_{1}n^{*}) \left( \frac{k_{3}n^{*}}{k_{3}n} \right) - \left( \frac{k_{2}n^{*}}{k_{2}n} \right) \right) + \left( k_{2}n^{*}\right) (k_{3}n^{*}) \left( \frac{1}{k_{2}n} - \frac{1}{k_{3}n} \right) + \left( k_{2} - k_{3}\right) n^{*} \left( \frac{k_{1}n^{*}}{k_{1}n} \right) \right\},
\]

the expression for \( V_{2} \) has two “spurious” denominators

\[
V_{2} = \frac{mn}{k_{1}n[k_{3}n]} \left( \frac{1}{k_{2}^{2}} - \frac{1}{k_{3}^{2}} \right) + \frac{(mn*)^{2}}{k_{1}n[k_{1}^{2}k_{3}^{2}]} \left( \frac{k_{1}n^{*}}{k_{1}n} - \frac{k_{3}n^{*}}{k_{3}n} \right) + \frac{(mn*)^{2}}{k_{1}n[k_{1}^{2}k_{3}^{2}]} \left( \frac{k_{2}n^{*}}{k_{2}n} - \frac{k_{3}n^{*}}{k_{3}n} \right)
\]

and \( V_{3} \) and \( V_{4} \) depend only on two momenta

\[
V_{3} = \frac{(mn*)^{2}}{k_{2}n}[k_{3}n]^{2} k_{3}^{2}, \quad V_{4} = \frac{(mn*)^{2}}{k_{1}n}[k_{1}n][k_{2}n][k_{3}n]^{2} k_{2}^{2}
\]

It turns out that, after substitution of \( V_{4} \) into (4.6) and integration over momenta \( k_{2} \) and \( k_{3} \), we get a vanishing result even before integration over the parameters \( s_{i} \).

Consider the first term in the expression \( V_{4} \). Its integration over \( k_{3} \) leads to \( z = x_{1}(s_{3}) \) and the resulting expression, being considered as a function of \( (k_{2}n) \), has singularities coming from two propagators \( \frac{1}{k_{2}n} \) and \( \frac{1}{k_{2}n+0} \) which lie on the same side of real axis in the complex \( (k_{2}n) \)-plane. Hence the integral over the \( (k_{2}n) \) component of momentum \( k_{2} \) vanishes. The same argument applies to the second term in \( V_{4} \) and therefore one concludes that \( V_{4} \) does not contribute to (4.8).

Substituting expressions for \( V_{1}, V_{2} \) and \( V_{3} \) into (4.6) one finds that the result of the integration over momenta \( k_{1}, k_{2} \) and \( k_{3} \) can be expressed in terms of the following three basic functions:

\[
F_{1}(x) = \int \frac{d^{D}k}{(2\pi)^{D}} e^{ikx} \frac{1}{(k^{2} + i0)[kn]},
\]

\[
F_{2}(x) = \int \frac{d^{D}k}{(2\pi)^{D}} e^{ikx} \frac{1}{(k^{2} + i0)},
\]

\[
F_{3}(x) = \int \frac{d^{D}k}{(2\pi)^{D}} e^{ikx} \frac{1}{[kn]}.
\]

\[\text{We have used here the identity } k^{2} = 2(kn)(kn*)/(mn*) - k^{2}_{T} \text{ with } k_{T} = (k_{1}, k_{2}).\]
Let us first consider the contribution of the expression \( V_3 \) into \( W^{(2)}_{3-\text{gluon}} \). Since \( V_3 \) does not depend on \( k_1 \), integration over this momentum leads to a \( \delta \)-function implying \( z = x_2(s_1) = n + n^* s_1 \) and we eventually get

\[
\frac{1}{2} g^4 C_F C_A \delta^{8-2D}(nn^*)^2 \int_0^1 ds_1 \int_0^1 ds_2 \int_0^{s_2} ds_3 \left( \frac{\partial}{\partial s_2} - \frac{\partial}{\partial s_3} \right) F_1(-n - n^*(s_1 - s_2)) F_1(-n - n^*(s_1 - s_3)).
\]

Using the expression (C.2) for the function \( F_1(x) \) with the vector \( x \) lying in the plane of the path \( C \) given in Appendix C, we integrate over the parameters \( s_i \) and obtain the contribution to \( W^{(2)}_{3-\text{gluon}} \) arising from \( V_3 \)

\[
\begin{align*}
- \frac{g^4}{(2\pi)^D} C_F C_A \frac{\Gamma^2(D/2 - 1)}{(4 - D)^4} \left\{ \frac{8 \Gamma^2(3 - D/2)}{\Gamma(5 - D)} \left[ (-\rho^2 + i0)(\rho^2 + i0) \right]^{2-D/2} \\
- \frac{1}{D - 3} \left[ (-\rho^2 + i0)^{4-D} + (\rho^2 + i0)^{4-D} \right] \right\}.
\end{align*}
\]

This expression has a fourth order pole in the dimensional regularization parameter and contains two different structures as an even function of \((nn^*)\).

Calculation of the integrals coming from \( V_1 \) and \( V_2 \) is more complicated. We shall only sketch here the evaluation of one of them taking only two terms from \( V_1 \) as an example:

\[
\frac{2(k_2 n^*)(k_3 n^*)(nn^*)}{k_1^2 k_2^2 k_3^2} \left( \frac{1}{|k_2 n|} - \frac{1}{|k_3 n|} \right).
\]

Replacing the vertex \( V \) by this expression in (4.10), one gets

\[
J = \int d^D z F_2(n + n^* s_1 - z) \frac{\partial^2}{\partial s_2 \partial s_3} (F_1(n^* s_2 - z) F_2(n^* s_3 - z) - F_2(n^* s_2 - z) F_1(n^* s_3 - z)),
\]

where the functions \( F_i \) are defined in (4.9) and their explicit expressions can be found in Appendix C. At variance with the previous case, the integral over the intermediate point \( z \) is not trivial and is given by (C.3).

After substitution of (C.3) into \( J \) we integrate over the parameters \( s_i \) and eventually get:

\[
J = \left[ (-2(nn^*) + i0)^{4-D} + (2(nn^*) + i0)^{4-D} \right]
\]

\[
\times \int_0^1 d\beta_1 \beta_1^{2-D/2} \beta_2^{D/2-2} \beta_3^{D/2-2} \frac{M(\beta_2, \beta_3)}{1 - \beta_2}.
\]

The integral over the parameters \( \beta_i \) for an arbitrary \( D \) can be expressed in the form of a convergent series, which however does not sum to elementary functions. For our purposes it is enough to expand it in powers of \((4-D)\) with the following result:

\[
J = \frac{g^4}{(2\pi)^D} C_F C_A \frac{\Gamma(D - 3)}{4(4 - D)^2} \left[ (-\rho^2 + i0)^{4-D} + (\rho^2 + i0)^{4-D} \right]
\]

\[
\times \left\{ (1 - \zeta(3))(4 - D) + \left( \frac{5}{2} - 2\zeta(3) + \frac{\pi^4}{120} \right)(4 - D)^2 + O((4 - D)^3) \right\},
\]

where \( \zeta(z) \) is the Riemann function.

The calculation of the remaining terms entering the expressions for \( V_1 \) and \( V_2 \) is analogous. For all of them we get the final expressions given in (C.4), which contain similar integrals over the parameters \( \beta_i \) with the same function \( M(\beta_2, \beta_3) \) defined in (4.12).
Summing all expressions \(\text{(4.10)}\) and \(\text{(C.4)}\) we obtain the contribution of the diagram of fig. 4c to the Wilson loop, we multiply it by factor 2 to take into account the symmetric diagram with two gluons attached to the lower contour side and get

\[
W_{\text{4-gluon}}^{(2)} = \frac{g^4}{(2\pi)^D} C_F C_A \left\{ - \left[ (-\rho^2 + i0)(\rho^2 + i0) \right]^{2-D/2} \frac{8\Gamma^2(3-D/2)\Gamma^2(D/2-1)}{(4-D)^2\Gamma(5-D)} 
+ \left[ (-\rho^2 + i0)^{4-D} + (\rho^2 + i0)^{4-D} \right] \Gamma(D-3) 
\right.
\]
\[
\left. \frac{2}{(4-D)^4} \left( 2\Gamma(D/2-1)\Gamma(3-D/2) + \frac{1}{D-3} \frac{\Gamma^2(D/2-1)}{\Gamma(D-3)} - \frac{6-D}{D-2} \frac{\Gamma^2(3-D/2)}{\Gamma(5-D)} \right) 
- \frac{\pi^2}{4-D} \frac{1}{12} + \frac{1}{4-D} \frac{2-5\zeta(3)}{4} + \frac{\pi^4}{80} + \frac{5}{4} - \zeta(3) \right\} . \tag{4.14}
\]

This complicated expression is valid up to terms vanishing for \(D = 4\).

The contribution coming from diagrams with three gluon vertex has been computed also in momentum representation, namely integrating first over the contour and then over momenta. All integrals in this case can be directly performed in the strip \(3 < \text{Re}D < 4\). Details of the calculation are reported in Appendix D. The final result of course fully coincides with expression \(\text{(4.14)}\) and provides an independent check.

We have calculated in this section all two-loop diagrams contributing to the Wilson loop expectation value with non-Abelian color factor \(C_F C_A\). To find two-loop unrenormalized Wilson loop using non-Abelian exponentiation theorem we sum expressions \(\text{(4.1)}, \text{(4.3)}\) and \(\text{(4.14)}\), skip terms with color factor \(C_F^2\) in front and identify the resulting expression with the exponent \(w^{(2)}\) in \(\text{(2.2)}\).

Notice that unrenormalized Wilson loop is a finite function of \(D\) as long as \(D \neq 4\). Trying to give a physical meaning to this quantity, we have to take care of poles at \(D \neq 4\) and to perform the renormalization procedure, which eventually will provide us with the two-loop renormalized Wilson loop for \(D = 4\). Renormalization will be performed in the next section.

At this stage we can already settle our first problem, namely whether the LLA gauge with the ML prescription is sick in the calculation of the Wilson loop under consideration. We stress that unrenormalized Wilson loop for \(D \neq 4\) is a well defined gauge invariant quantity. The same unrenormalized Wilson loop was calculated in \(\text{(8)}\) using Feynman gauge. We take here the opportunity of correcting a misprint in the expression for \(W_F\) in ref.\(\text{(8)}\), where the constant \(\frac{2}{\pi^2}\) should be replaced by \(\frac{1}{\pi^2}\).

Combining two-loop expressions for unrenormalized Wilson loop in the axial and Feynman gauges and performing their Laurent expansions near \(D = 4\) we find that Wilson loops are identical in both gauges up to terms vanishing for \(D = 4\). It means that LLA gauge with the ML prescription works perfectly, at least up to two-loop order.

### 5. Wilson loop in the light-like axial gauge

In the previous section we found the two-loop unrenormalized expressions for all independent diagrams contributing to the light-like Wilson loop. To give a meaning to these expressions for \(D = 4\) we have to define the renormalization procedure which, when applied to unrenormalized Wilson loop, removes all divergences including overlapping divergences like the one of the gluon self-energy in fig. 4b.

Calculating diagrams we have met three different kinds of divergences: specific light-cone divergences of the Wilson loops, spurious divergences of the LLA gauge and conventional divergences of the Yang-Mills theory. All of them have manifested themselves as poles in the dimensional regularization parameter.

The problem with the renormalization of the first kind of singularities has to do with to the fact that Wilson loop is a non-local functional of the gauge potentials. As was shown in ref.\(\text{(2)}\), had the integration path neither cusps nor light-like segments, it would be enough to renormalize Green functions and coupling constant to remove all divergences from \(W_C\). But since the integration path under consideration has both kinds of peculiarities, one should first renormalize the gauge potentials and coupling constant and then subtract the remaining specific light-cone and cusp divergences.

The general theory of renormalization in LLA gauge has been developed in refs.\(\text{(12)}\). In particular it has been shown there that the vector potential \(A_{\mu}(x)\) renormalizes at all orders in the loop expansion with two
The operator \((nD)^{-1}\) is understood as a series expansion in the coupling constant with boundary conditions giving rise to the ML prescription. It is immediate to realize that renormalization does not change the gauge condition.

To our subsequent calculations, only the effective part of the zero-th order expansion in the coupling constant \(g\) is renormalized as \(Z_3^{1/2} g^R\) as \(Z_1 = Z_3\) in this gauge. As a consequence the Wilson loop gets the expression in terms of renormalized potentials

\[
W_C = \frac{1}{N} \text{tr} \mathcal{T} \mathcal{P} \exp \left( i g^R \int_C dx^\mu [A_\mu^R(x) - (1 - \bar{Z}_3^{-1}) n_\mu \Omega^R(x)] \right) |0\rangle.
\]

The renormalization constants \(Z_3\) and \(\bar{Z}_3\) at order \(g^2\) are given by [17]

\[
Z_3 = 1 + \frac{11 g^2}{24 \pi^2} \frac{C_A}{4-D}, \quad \bar{Z}_3 = 1 + \frac{g^2}{4 \pi^2} \frac{C_A}{4-D}.
\]

The extra-term in the Wilson loop (5.3) gives rise at \(O(g^4)\) only to a contribution coming from the cross-product, which turns out to be equivalent to the following correction to the gluon propagator

\[
\delta G_{\mu\nu}(x) = \frac{g^2}{2 \pi^2} \frac{C_A}{4-D} G_{\mu\nu}(x).
\]

In turn, the divergent part at \(D = 4\) of the gluon propagator due to the self-energy insertion of fig.4b can easily be extracted from [16]. It originates from the following two contributions of the self-energy tensor

\[
\Pi_{\mu\nu}(k) = i \frac{g^2}{8 \pi^2} C_A \left\{ \frac{11}{3} \frac{1}{4-D} (k^2 g_{\mu\nu} - k_\mu k_\nu) + \frac{4}{4-D} \frac{(nk)}{(nn^*)} (k_\nu n_\rho + k_\rho n_\nu) \right\}.
\]

One can easily check that the contribution from the second tensor is exactly compensated by the correction in the expression of the Wilson loop due to the presence of \(\Omega\). Therefore only the term which is proportional to the \(\beta\)-function is to be subtracted and the resulting path integral coincides (up to factor \(-\frac{11 g^2}{24 \pi^2} C_A \frac{1}{4-D}\)) with one-loop expression [13]

\[
W_{c.t.}^{(2)} = \frac{g^4}{8 \pi^2} \frac{11}{3} C_F C_A \left\{ \frac{\Gamma(D/2 - 1)}{(D - 4)^3} \left[ (-2(nn^*) + i0)^{2-D/2} + (2(nn^*) + i0)^{2-D/2} \right] \right\}.
\]

Once this expression has been subtracted from the sum of contributions to the Wilson loop, turning the coupling constant \(g\) into \(g^R\), not all divergent parts at \(D = 4\) are eliminated. The remaining divergences are just light-cone divergences of the Wilson loop, which we also have to subtract taking care of overlapping singularities. Here we meet some simplifications in the structure of light-cone singularities in LLA gauge. It is indeed well known from perturbative QCD that diagrams like the one of fig. 4, having non-Abelian color factor \(C_F C_A\) and either being nonplanar (see fig. 4a) or containing gluon self-interaction, do not exhibit overlapping light-cone (or collinear) divergences. It means that, in order to renormalize those diagrams, one can simply subtract all poles remaining after renormalization of Green functions. Then, using the non-Abelian exponentiation theorem, one
realizes that, in order to renormalize all light-cone singularities of the Wilson loop $W$, it is enough to subtract poles from the exponent $w^{(n)}$, which does not contain any overlapping light-cone singularity.

After having subtracted those remaining poles in the $\overline{\text{MS}}$-scheme according to this procedure, we verify that the resulting renormalized expression satisfies the RG equation (1.2) with the same anomalous dimensions of eqs. (1.3) and (1.4), providing a further test of gauge invariance and, at the same time, of the way in which renormalization operates in the LLA gauge.

6. Conclusions

We end by summarizing what we have achieved in previous sections. We have first evaluated in perturbation theory at $\mathcal{O}(g^4)$ a light-like Wilson loop for the Yang-Mills theory in LLA gauge with the ML prescription for the vector propagator. All singularities, the ones related to the ultraviolet behaviour of the theory, the ones peculiar of the contour we have chosen (light-like lines and cusps) and the ones related to the gauge choice, have been dimensionally regularized and manifest themselves as poles at $D = 4$.

As dimensional regularization does not spoil gauge invariance, we have checked that our result does indeed completely coincide with the analogous one obtained in ref.[3] for the same contour in Feynman gauge. In this way a complete test at $\mathcal{O}(g^4)$ of the correctness of the ML prescription in LLA gauge has been achieved.

Then we have renormalized our result in the $\overline{\text{MS}}$-scheme. In so doing we explicitly extend the general prescriptions given in refs.[12] to a non-local operator (the Wilson loop). We again recover in a quite non-trivial way the Feynman gauge findings: in particular both cusp and anomalous dimensions, which obey the RG equation and are relevant to the physics of soft radiation, do indeed respectively coincide with their Feynman gauge counterparts, providing a further test in favour of general procedures concerning canonical quantization and renormalization in LLA gauge as well as of its usefulness in practical calculations.

Acknowledgements

Two of us (I.A.K. and G.P.K.) would like to thank G.Marchesini for numerous and useful discussions.

7. Appendices

Appendix A. Light-cone variables

For an arbitrary vector $x = (x_0, x_1, x_2, x_3)$ light-cone variables are defined as

$$x_+ = \frac{1}{\sqrt{2}}(x_0 + x_3), \quad x_- = \frac{1}{\sqrt{2}}(x_0 - x_3), \quad x_T = (x_1, x_2)$$

The following identities are fulfilled for two 4-dimensional vectors $x$ and $y$:

$$x_\pm = x^\mp, \quad x^2 = 2x_+x_- - x_T^2, \quad (xy) = x_+y_- + x_-y_+ - (x_T \cdot y_T), \quad d^4x = dx_+dx_-d^2x_T$$

For gauge fixing vectors $n$ and $n^*$ we have

$$n_+ = 0, \quad n_- = \sqrt{2}T, \quad n_T = 0, \quad n^*_+ = \sqrt{2}L, \quad n^*_- = 0, \quad n^*_T = 0.$$

Appendix B. Useful integrals in the evaluation of the self-energy diagram

We list here the integrals used in the expression $G_1(p)$. We follow the conventions given in Appendix A for light-cone coordinates. In the defining integrals, ML prescription for the spurious poles will always be understood.

$$I_1(p) = \int \frac{d^Dk}{(2\pi)^D} \frac{1}{k^2[p_+ + k_+]}$$
In Eqs. (B.2) and (B.4), 

\[ \eta \]

\[ \mathrm{V} \to x. \]

be done here we remain them undone in order to simplify further integration of the product of these functions (4.9). Their explicit expressions are

Performing calculation of the diagram with a three-gluon vertex we have introduced three functions defined in Appendix C. Diagram with three-gluon vertex (coordinate representation)

Using (C.2) we have calculated in (4.10) the contribution of

\( F_1 \) \( F_2 \) \( F_3 \)

\[ (B.3) \]

In Eqs. (B.2) and (B.4), \( \eta \in (0,1) \).

**Appendix C. Diagram with three-gluon vertex (coordinate representation)**

Performing calculation of the diagram with a three-gluon vertex we have introduced three functions defined in [3]. Their explicit expressions are

\[ F_1(x) = \frac{\Gamma(1-D/2)}{(4\pi)^{D/2} (nn^*)} \int_0^\infty d\alpha \alpha^{D/2-2} \int_0^1 d\xi \xi^{1-D/2} e^{-i\alpha(2x_x+x_z^2/\xi)} \]

\[ F_2(x) = \frac{\Gamma(3-D/2)}{(4\pi)^{D/2} (nn^*)} \int_0^\infty d\alpha \alpha^{D/2-2} e^{-i\alpha x_z^2} = -\frac{i}{4\pi D/2} (-x^2 + i0)^{1-D/2}, \]

\[ F_3(x) = \frac{\Gamma(3-D/2)}{(4\pi)^{D/2} (nn^*)} \delta(x_T) \int_0^\infty d\alpha e^{-i\alpha x_z^2} = \frac{1}{\pi (nn^*)} \delta^{(D-2)}(x_T) \]

for arbitrary \( D \)-dimensional vector \( x \) in the coordinate space. Although some integrals over \( \alpha \)-parameters can be done here we remain them undone in order to simplify further integration of the product of these functions over \( x \). Expression for \( F_1(x) \) is greatly simplified as the vector \( x \) lies in the plane of the vectors \( n \) and \( n^* \):

\[ F_1(x_+, x_T = 0, x_-) = \frac{\Gamma(D/2 - 1)}{2\pi D/2} \frac{(nn^*)}{4 - D} (-x^2 + i0)^{1-D/2}, \]

Using (C.2) we have calculated in (4.11) the contribution of \( V_3 \)-term. Calculation of the contribution due to \( V_1 \) and \( V_2 \) terms involves integration of products of functions (C.1) over intermediate point \( z \). One of these
integrals entering into (4.11) can be calculated using (C.1) as
\[
\int d^Dz F_2(n^*s_1 - z)(F_1(n^*s_2 - z)F_2(n^*s_3 - z) - F_2(n^*s_2 - z)F_1(n^*s_3 - z))
\]
\[
= \frac{i}{32\pi^D} \frac{\Gamma(D-3)}{4-D} \int_0^1 d\beta \beta_1^{2-D/2} \beta_2^{D/2-2} \frac{1 - \beta_2^{D/2-2}}{1 - \beta_2} (\Phi(\beta_1, \beta_2, \beta_3) - \Phi(\beta_1, \beta_3, \beta_2))
\]  
(C.3)

where \(d\beta \equiv d\beta_1 d\beta_2 d\beta_3 \delta(1 - \beta_1 - \beta_2 - \beta_3),\ 0 \leq \beta_i \leq 1\) and the notation was used for the function \(\Phi(\beta_1, \beta_2, \beta_3) = [2(n^*)^3(\sum_{i=1}^3 \delta s_i) + i0]^{3-D}\) The \(\beta\) parameters in (C.3) have appeared after one has performed the following change of the integration variables: \(\alpha_i = \lambda \beta_i\) \((i = 1, 2, 3)\) and integrated over \(\lambda\) from 0 to \(\infty\). Substituting (C.3) into (4.11) and integrating over \(s_i\)-parameters we get (4.12).

Different terms entering into expressions for \(V_1\) and \(V_2\) are grouped into 6 items. Contribution of one of them is given by (4.13). The calculation of the remaining 5 terms is analogous. One first rewrites them as integrals of a product of \(F\)-functions over \(z\), uses relations (C.1) to express the result of \(z\)-integration as integral over \(\alpha\)-parameters and then integrates over \(s\)-parameters taking into account their ordering along the path. At the end of this procedure substituting \(\alpha_i = \lambda \beta_i\) we get integrals over \(\beta\)-parameters analogous to (4.12). Summarizing, we can write the contribution of 6 items from \(V_1\) and \(V_2\) to the diagram of fig. 4c as
\[
\frac{g^4}{64\pi^D} C_F C_A \left[ (-2(nn^*) + i0)^4 - (2(nn^*) + i0)^4 - D \right] \frac{\Gamma(D-3)}{(4-D)^2} (I_1 + I_2 + I_3 + I_4 + I_5 + I_6)
\]  
(C.4)

where \(I_1, I_2, I_3\) correspond to three items in \(V_1\) and \(I_4, I_5, I_6\) to three items in \(V_2\):

\[
I_1 = - \int_0^1 d\beta \beta_1^{2-D/2} \beta_2^{D/2-2} \frac{1 - \beta_2^{D/2-2}}{1 - \beta_2} M(\beta_2, \beta_3)
\]
\[
= (1 - \zeta(3))(4 - D) + \left( \frac{5}{2} - 2\zeta(3) + \frac{4\pi^4}{144} \right)(4 - D)^2 + O((4 - D)^3)
\]

\[
I_2 = \int_0^1 d\beta \beta_1^{1-D/2} \beta_2^{D/2-2} \beta_3^{D/2-3} (1 - \beta_2^{D/2-2}) M(\beta_2, \beta_3)
\]
\[
= \frac{4\Gamma(D/2 - 1)}{(4-D)^2} \left( \frac{\Gamma(3 - D/2) - \Gamma(D/2 - 1)}{2\Gamma(3 - D/2)} \right)
\]

\[
I_3 = - \int_0^1 d\beta \beta_1^{2-D/2} \beta_3^{D/2-3} \frac{1 - \beta_2^{D/2-2}}{1 - \beta_2} M(\beta_2, \beta_3)
\]
\[
= \frac{\pi^2}{6} - \frac{5}{27}(4 - D) + \frac{21}{120}\pi^4(4 - D)^2 + O((4 - D)^3)
\]

\[
I_4 = \int_0^1 d\beta \beta_1^{3-D/2} \beta_2^{D/2-3} \beta_3^{D/2-2} M(\beta_2, \beta_3) = - \frac{4\Gamma(D/2 - 1)}{(4-D)^2} \left( \frac{\Gamma(3 - D/2) - \Gamma(D/2 - 1)}{2\Gamma(3 - D/2)} \right)
\]

\[
I_5 = - \int_0^1 d\beta \beta_1^{2-D/2} \beta_2^{D/2-3} \beta_3^{1} M(\beta_2, \beta_3)
\]
\[
= \frac{2\Gamma(D/2 - 1)\Gamma(3 - D/2)}{(4-D)^2} - \frac{2\Gamma^2(3 - D/2)}{(4-D)\Gamma(5-D)} \left[ \psi(3 - D/2) - \psi(D/2 - 1) \right]
\]

\[
I_6 = \int_0^1 d\beta \beta_1^{3-D/2} \beta_2^{D/2-2} \beta_3^{1} M(\beta_2, \beta_3)
\]
\[
= \frac{2\Gamma(D/2 - 1)\Gamma(3 - D/2)}{(4-D)^2} + \frac{\Gamma(3 - D/2)\Gamma(2-D/2)}{(5-D)} \left[ \psi(3 - D/2) - \psi(D/2) - \frac{1}{D/2 - 1} \right]
\]

where \(\psi(z) = d \log \Gamma(z)/dz\). Note that \(I_1\) was used in (4.12).

Appendix D. Diagram with three-gluon vertex
(momentum representation)

Among the various diagrams, the evaluation of the three-gluon diagram given in Fig. (4.c) is the most delicate. In the main text and in the previous Appendix we have shown how the calculation can be performed in the coordinate representation. Since this is the only diagram in which the difference between the computation in the momentum and coordinate representation is appreciable, it is instructive to sketch the main lines of calculation in the momentum representation.

We start from Eq. (4.4). The trivial integration over \(dz\) reproduces the momentum conservation delta distribution. Then, we can write the three-gluon diagram as

\[
W_{3\text{-gluon}} = \alpha \sum_{i=1}^{3} \frac{d^{3D}p d^{3D}q d^{3D}r}{(2\pi)^{2D}} F_i(p, q, r) \mathcal{G}(p, q, r) \delta(p + q + r) \equiv \sum_{i=1}^{3} W^{(i)} \quad (D.1)
\]

where \(\alpha = 2ig^4C_A C_F \mu^{8-2D}\),

\[
F_1(p, q, r) = \frac{2}{p_+} \left\{ (pq)^r_{[r_+] - (pr)^q_{[q_+]}} \right\} + \left\{ \text{cyc. perm. of } (p, q, r) \right\} \quad (D.2)
\]

\[
F_2(p, q, r) = \frac{2p_+}{q_+} (q_- - r_-) \quad (D.3)
\]

\[
F_3(p, q, r) = \frac{2q_+}{r_+} (r_- - p_-) + \frac{2r_+}{q_+} (p_- - q_-) \quad (D.4)
\]

and \(\mathcal{G}(p, q, r)\) denotes the so called “geometrical factor”, encoding all the local geometrical properties of the loop contour

\[
\mathcal{G}(p, q, r) = e^{2ip_+ T} \int_0^{2L} dx^- e^{ip-x^L} \int_0^{2L} dy^- e^{i q-y^L} \int_0^{2L} dz^- e^{i r-z^L}
\]

\[
= \frac{ie^{2ip_+ T}}{p_-} (e^{2ip-L} - 1) \left\{ \frac{1}{p_- r_-} (e^{-2iL p_+} - 1) + \frac{e^{2iL r_-}}{q_- r_-} (e^{2iL q_-} - 1) \right\} \quad (D.5)
\]

In the amplitude factors \(F_i\), all the spurious singularities are defined through ML distributions. \(\mathcal{G}(p, q, r)\), instead, does not contain any pole, and all the singularities in the \(q_-\), \(p_-\) and \(r_-\) variables are fictitious. Since the \(F_i(p, q, r)\) are antisymmetric under the exchange \((q, r) \rightarrow (r, q)\), the geometrical factor \(G\) in the integrand (D.1) can be more conveniently rewritten by performing the change of variables \((q, r) \rightarrow (r, q)\) in the second term of the curly bracket, eq. (D.5). Thus, a successive integration over \(d^3 r\) leads to

\[
W_{3\text{-gluon}} = -i\alpha \sum_{i=1}^{3} \frac{d^{3D}p d^{3D}q e^{2ip_+ T}}{(2\pi)^{2D}} F_i(p, q) \frac{(e^{2ip-L} - 1)(e^{2iq-L} - 1)}{p_-^2 q_-^2 q_+^2 (p + q)^2} \left[ 1 + e^{-2iL(p_- + q_-)} \right]
\]

\[
\equiv \sum_{i=1}^{3} W^{(i)} \quad (D.6)
\]

where the amplitudes \(\tilde{F}_1(p, q)\) are now defined by

\[
\tilde{F}_1(p, q) = \frac{2}{p_+} \left\{ \frac{1}{q_+} (q_- (2p_+^2 + pq) - p_- (q_+^2 + 2pq)) \right\} + \left\{ \frac{1}{p_+ + q_+} (q_- (pq - p^2) + p_- (pq - q^2)) \right\}, \quad (D.7)
\]

\[
\tilde{F}_2(p, q) = \frac{2p_-}{|p_+|} (p_- + 2q_-), \quad (D.8)
\]

\[
\tilde{F}_3(p, q) = \frac{2}{|p_+ + q_+|} (p_+^2 - q_-^2) - \frac{2}{|q_+|} (q_- (q_- + 2q_-)). \quad (D.9)
\]
To cast $\tilde{F}_1$ in the form (D.7), we repeatedly used the so called “splitting formula”

\[
\frac{1}{[q_+]p_+ + q_+] = \frac{1}{[p_+]} \left( \frac{1}{[q_+]} - \frac{1}{[p_+ + q_+]} \right),
\]

which is an identity, in the sense of the theory of distributions, when the spurious singularities are defined by ML prescription. Eq. (D.6) is particularly convenient for the computation of the three contributions $W^{(i)}$ in momentum representation. In fact, for each of the terms $W^{(i)}$, it is necessary to compute only the term with the “1” factor in the last square bracket of eq. (D.6), as the second one, proportional to $\exp[-2iL(p_- + q_-)]$, can be obtained from the first by the replacement $L \to -L$. Following these suggestions and using (for instance) Schwinger parameterization for Feynman denominators and ML-distributions, one can get the following results for $W^{(1)}$ and $W^{(2)}$

\[
W^{(1)} = -i\alpha \frac{(4LT)^{4-D}e^{-i\pi D/2}\cos(\pi D/2)}{(2\pi)^D} \frac{\Gamma(D - 4)}{(D - 4)^2} \times \left\{ \frac{3}{\Gamma(D/2 - 1)} \frac{\Gamma(D/2 - 2)}{\Gamma(D - 2)} + 4 \frac{\Gamma^2(3 - D/2)}{(D/2 - 1)\Gamma(5 - D)} \right\} + 4\Gamma(2 - D/2)\Gamma(D/2 - 1) \right\},
\]

\[
W^{(2)} = i\alpha \frac{(4LT)^{4-D}e^{-i\pi D/2}\cos(\pi D/2)}{(2\pi)^D} \frac{\Gamma(D - 4)}{(D - 4)^2} \times \left\{ \frac{\Gamma(D/2 - 1)}{\Gamma(D/2)} \frac{\Gamma(D/2 - 2)}{\Gamma(D - 2)} - 2\Gamma(D/2 - 2)\Gamma(3 - D/2) \right\}.
\]

It should be stressed that, contrary to what happens in the coordinate representation, the expressions $W^{(1)}$ and $W^{(2)}$ have been obtained keeping the dimension $D$ in the natural strip $3 < \text{Re}D < 4$, and therefore they do not entail any analytical continuation.

The computation of $W^{(3)}$ is more delicate and require additional care. Following the same instructions used for $W^{(1)}$ and $W^{(2)}$, one can get the following integral form for $W^{(3)}$

\[
W^{(3)} = i\alpha \frac{(4LT)^{4-D}e^{-i\pi D/2}\cos(\pi D/2)}{(2\pi)^D} \frac{\Gamma(D - 4)}{(D - 4)^2} \int_0^1 d\zeta \zeta^{D/2 - 2} \frac{2 - \zeta}{1 - \zeta} \varphi(\zeta) \times \int_0^\infty \frac{d\eta}{1 + \eta} [\eta + 1 - \zeta]^{D/2 - 5} [\eta + 1 - \zeta]^{2 - D/2} - (\eta\zeta)^{2 - D/2},
\]

where $\varphi(\zeta) = |\zeta^{4-D} - (1 - \zeta)^{4-D} + 1 - 2\zeta|$. The integral in (D.13) presents nasty singularities in $\zeta = 1$ if $D$ lies in the natural strip. To overcome this problem yet remaining in the $3 < \text{Re}D < 4$ region, we can for instance perform first an integration by parts in $d\eta$ and write

\[
\int_0^\infty \frac{d\eta}{1 + \eta} (\eta + 1 - \zeta)^{D/2 - 3} = \frac{(1 - \zeta)^{D/2 - 2}}{2 - D/2} - \frac{2F_1(2 - D/2, 3 - D/2; 4 - D/2; \zeta)}{(2 - D/2)(3 - D/2)},
\]

\[
\int_0^\infty \frac{d\eta}{1 + \eta} (\eta + 1 - \zeta)^{D/2 - 3} = \frac{\Gamma^2(3 - D/2)}{\Gamma(6 - D)} (1 - \zeta)^{D/2 - 2} \times 2F_1(1, 3 - D/2; 6 - D; \zeta),
\]

Notice that the two hypergeometric series in the r.h.s. are analytic functions in $|\zeta| \leq 1$ if $D$ lies in its natural strip. Therefore, the remaining integral in $d\zeta$ can be evaluated integrating term by term in the series expansion of the hypergeometrics. The final result is

\[
W^{(3)} = -i\alpha \frac{(4LT)^{4-D}e^{-i\pi D/2}\cos(\pi D/2)}{(2\pi)^D} \frac{\Gamma(D - 4)}{(D - 4)^2} \left\{ \frac{1}{D/2 - 2} \frac{\Gamma^2(3 - D/2)}{\Gamma(5 - D)} \right\}.
\]
\[
-3\Gamma(3-D/2)\Gamma(D/2-1) + 2\frac{\Gamma^2(D/2 - 1)}{\Gamma(D - 2)} + \frac{2}{\Gamma(2-D/2)} \sum_{n=0}^{\infty} \frac{\Gamma(n + 2 - D/2)}{(n + 3 - D/2)n!} \\
\times \left[ \psi(n + D/2) - \psi(n + 3 - D/2) + \frac{2}{(n + D/2 - 1)(n + D/2)} \right] + (4 - D)\Gamma(3-D/2) \\
+ \frac{1}{n + 3 - D/2} \frac{\Gamma(n + D/2 - 1)\Gamma(5-D)}{\Gamma(4 + n - D/2)} + \frac{2}{\Gamma(2-D/2)} \sum_{n=0}^{\infty} \frac{\Gamma(n + 3 - D/2)}{\Gamma(n + 6 - D)} \\
\times \left[ \Gamma(D/2 - 2) \left( \frac{\Gamma(n + 5 - D)}{\Gamma(n + 3 - D/2)} - \frac{\Gamma(n + 2)}{\Gamma(n + D/2)} \right) \right] \\
+ 2\frac{\Gamma(n + 1)\Gamma(D/2)}{\Gamma(n + 1 + D/2)} + \frac{\Gamma(n + 5 - D)\Gamma(D/2 - 1)}{\Gamma(n + 4 - D/2)} - \frac{\Gamma(n + 1)\Gamma(3-D/2)}{\Gamma(n + 4 - D/2)} \right] 
\]

Eq. (D.16) concludes the calculation of \( W^{(3)} \) and therefore of the whole three-gluon graph. Notice that all the series in eq. (D.16) are convergent for \( 3 < \text{Re} D < 4 \).

Although not straightforward, it can be checked that collecting all the expressions for \( W^{(i)} \) and performing the Laurent expansion around \( D = 4 \), the residues at the \( D = 4 \) poles as well as the finite term exactly coincide with the corresponding quantities evaluated in the coordinate representation, eq. (4.12).
References

[1] A.A.Migdal, Phys. Rep. 102 (1983) 316.

[2] A.M.Polyakov, Nucl. Phys. B164 (1980) 171;
I.Ya.Aref’eva, Phys. Lett. B93 (1980) 347;
V.S.Dotsenko and S.N.Vergeles, Nucl. Phys. B169 (1980) 527;
R.A.Brandt, F.Neri and M.-A.Sato, Phys. Rev. D24 (1981) 879.

[3] I.A.Korchemskaya and G.P.Korchemsky, Phys. Lett. 287B (1992) 169.

[4] S.V.Ivanov, G.P.Korchemsky and A.V.Radyushkin, Sov. J. Nucl. Phys. 44 (1986) 145; G.P.Korchemsky
and A.V.Radyushkin, Phys. Lett. 171B (1986) 459; Nucl. Phys. 283B (1987) 342.

[5] G.P.Korchemsky and G.Marchesini, Parma Univ. preprint UPRF–92–354 [hep-ph/9210281].

[6] G.P.Korchemsky and A.V.Radyushkin, Phys. Lett. 279B (1992) 359.

[7] For a review see: H.Georgi, “Heavy Quark Effective Field Theory,” preprint HUTP–91–A039 (1991).

[8] G.P.Korchemsky, Mod. Phys. Lett. A4 (1989) 1257.

[9] S.Mandelstam, Nucl. Phys. B213 (1983) 149; G.Leibbrandt, Phys. Rev., D29 (1984) 1699.

[10] A.Bassetto, M.Dalbosco, I.Lazzizzera and R.Soldati, Phys. Rev., D31 (1985) 2012.

[11] A.Andrași and J.C.Taylor, Nucl. Phys. B375 (1992) 341.

[12] A.Bassetto, M.Dalbosco and R.Soldati, Phys. Rev., D36 (1987) 3138; see also A.Bassetto, G.Nardelli and
R.Soldati, “Yang–Mills Theories in Algebraic Non-Covariant Gauges”, World Scientific, Singapore, 1991.

[13] J.G.M.Gatheral, Phys. Lett. 133B (1984) 90;
J.Frenkel and J.C.Taylor, Nucl. Phys. B246 (1984) 231.

[14] A.Bassetto, Phys. Rev. D46 (1992) 3676.

[15] A.Bassetto, in Proceedings of the 8th Warsaw Symposium on Elementary Particle Physics, Kazimierz
(Poland) 1985.

[16] M.Dalbosco, Phys. Lett. 180B (1986) 121.

[17] A.Bassetto, M.Dalbosco and R.Soldati, Phys. Rev., D33 (1986) 617.
Figure captions:

**Fig. 1:** Integration path for Wilson loop. Parallel segments belong on the light-cone.

**Fig. 2:** One-loop contributions to the Wilson loop expectation value. The graph (a) vanishes due to causal properties of ML prescription.

**Fig. 3:** Diagrams containing the following subgraphs give vanishing contributions to Wilson loop expectation value.

**Fig.4:** Relevant diagrams contributing to the Wilson loop to the second order: “crossed” diagram (a), “self-energy” diagram (b) and “three-gluon” diagram (c).