On the Measurement of Fundamental Parameters of White Dwarfs in the Gaia Era

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Abstract

We present a critical review of the determination of fundamental parameters of white dwarfs discovered by the Gaia mission. We first reinterpret color–magnitude and color–color diagrams using photometric and spectroscopic information contained in the Montreal White Dwarf Database (MWDD), combined with synthetic magnitudes calculated from a self-consistent set of model atmospheres with various atmospheric compositions. The same models are then applied to measure the fundamental parameters of white dwarfs using the so-called photometric technique, which relies on the exquisite Gaia parallaxes and photometric data from Panoramic Survey Telescope And Rapid Response System, Sloan Digital Sky Survey, and Gaia. In particular, we discuss at length the systematic effects induced by these various photometric systems. We then study in great detail the mass distribution as a function of effective temperature for the white dwarfs spectroscopically identified in the MWDD, as well as for the white dwarf candidates discovered by Gaia. We pay particular attention to the assumed atmospheric chemical composition of cool, non-DA stars. We also briefly revisit the validity of the mass–radius relation for white dwarfs and the recent discovery of the signature of crystallization in the Gaia color–magnitude diagram for DA white dwarfs. We finally present evidence that the core composition of most of these white dwarfs is, in bulk, a mixture of carbon and oxygen, an expected result from stellar evolution theory, but never empirically well established before.

Key words: stars: fundamental parameters – techniques: photometric – techniques: spectroscopic – white dwarfs

1. Introduction

The Sloan Digital Sky Survey (SDSS) has certainly been among the most important developments in the last few years in terms of observational data of white dwarf stars since the Palomar-Green survey (Green et al. 1986). The SDSS has been looking at 10,000 deg² of high-latitude sky in five bandpasses (ugriz) and producing images in these five bandpasses from which galaxies, quasars, and stars were selected for follow-up spectroscopy. The selection effects in this survey are important, as discussed in Kleinman et al. (2004, see also Eisenstein et al. 2006a), and therefore, it cannot be considered as a complete survey in any sense. The number of white dwarf stars discovered in the SDSS has rapidly grown from the first Data Release (2551 white dwarfs, Kleinman et al. 2004) to well over 30,000 white dwarfs in Data Release 12 (Kepler et al. 2016). Not only has the SDSS significantly increased the number of spectroscopically confirmed white dwarfs since the last published version of the McCook & Sion catalog (McCook & Sion 1999), but it has provided a phenomenal source of homogeneous photometric observations in the ugriz system as well as optical spectroscopy for most objects. The SDSS has led to numerous detailed spectroscopic analyses of white dwarfs, in particular for DA (hydrogen-line) and DB (helium-line) stars (Eisenstein et al. 2006b; Kepler et al. 2007; Tremblay et al. 2011; Koester & Kepler 2015; Genest-Beaulieu & Bergeron 2019, hereafter GBB19).

Another ongoing, and perhaps more important, revolution in the white dwarf field is the Gaia mission, which will probably discover about 400,000 white dwarfs when the mission is completed, with a detection probability of almost 100% up to a distance of 100 pc (Jordan 2007). The Gaia Data Release 2 (Gaia Collaboration et al. 2018, hereafter GaiaHRD) has already provided precise astrometric and photometric data for ~260,000 high-confidence white dwarf candidates (Gentile Fusillo et al. 2019). These combined astrometric and photometric data sets allow, for the first time, the measurements of fundamental parameters (effective temperature, radii, and mass) of large samples of white dwarf stars, using the so-called photometric technique (Hollands et al. 2018; Tremblay et al. 2019a, GBB19). Most white dwarf candidates identified by Gaia still require spectroscopic observations and confirmations, however. Furthermore, Gaia provides photometry only in wide passbands—G, GP, and GR—which are not the most optimal set to be used for model atmosphere modeling. Fortunately, there are now a growing number of all-sky (or almost) surveys with narrower bands, such as the Panoramic Survey Telescope And Rapid Response System (Pan-STARRS; Chambers et al. 2016; Tonry et al. 2012).

Given that such large ensembles of combined astrometric (trigonometric parallaxes in particular) and photometric data have become available only recently, the fundamental parameters derived from the photometric technique are not as mature and well understood as those obtained from spectroscopy. To overcome this situation, we present in this paper a comprehensive investigation of the physical parameters of white dwarfs derived from the photometric technique. In Section 2, we first investigate the classical color–magnitude and color–color diagrams for white dwarfs, both observationally and theoretically. In Section 3, we discuss the determination of physical parameters of white dwarfs using the photometric technique, with a particular emphasis on the use of various photometric systems. Finally, we explore in Section 4 various implications of our results for white dwarf

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1 Actually, the broadband filter G is almost the sum of the GP and GR bandpasses, and in that sense, it does not represent an independent data point in the sampling of the spectral energy distribution.
physics, including a detailed study of the mass distributions under various assumptions about the chemical composition, and a closer look at the crystallization process. Our discussion and conclusions follow in Section 5.

2. Color–Magnitude and Color–Color Diagrams

The first Hertzsprung–Russell diagram for white dwarfs using data from the Gaia mission has been presented in Figure 13 of GaiaHRD—in this case, $M_G$ versus $(G_{BP}-G_{RP})$. We show in Figure 1 our own version of this color–magnitude diagram, but only for objects with a distance $D < 100$ pc, based on Gaia photometric and parallax data extracted from the Montreal White Dwarf Database (MWDD)\(^2\) (Dufour et al. 2017). We use different color symbols to distinguish white dwarf candidates from Gaia, as well as spectroscopically identified DA and non-DA (He II-line DO, DB, carbon-line DQ, metal-line DZ, featureless DC, and all subtypes) stars. Composite systems with a known M-dwarf companion are also excluded from this diagram. For the selection of white dwarf candidates, instead of using the extensive catalog of Gentile Fusillo et al. (2019), we use the same cuts as in GaiaHRD (see their Section 2.1), to which we add $G - 10 + 5\log_{10} \pi > 10 + 2.6 \ (G_{BP}-G_{RP})$ to select only white dwarfs, a cut also suggested in GaiaHRD. We also restrict our sample to $\sigma_{\pi}/\pi < 0.1$.

Also reproduced in this color–magnitude diagram are theoretical color sequences for 0.6 $M_\odot$ white dwarfs with pure hydrogen and pure helium atmospheric compositions, calculated using state-of-the-art model atmospheres described in Section 3. As discussed in GaiaHRD, the most striking feature in this diagram is a clear bifurcation between the DA and non-DA stars in the range $0.0 < (G_{BP}-G_{RP}) < 0.8$. Although the pure hydrogen models follow nicely the observed DA sequence, the pure helium models actually go through the gap where the bifurcation between the DA and the DB stars occurs. There are at least two interpretations that have been proposed for these results, also discussed in GaiaHRD. The first obvious interpretation of this bifurcation is atmospheric composition, which differentiates the hydrogen-atmosphere DA white dwarfs from the helium-atmosphere non-DA stars. In this case, it has been suggested that perhaps the helium models simply fail to go through the observed non-DA sequence at 0.6 $M_\odot$ because of physical inaccuracies in the model atmospheres. An alternative explanation that has been proposed is related to differences in stellar mass, which affect the radius of

\(^2\) http://montrealwhitedwarfdatabase.org/
the star and, thus, the absolute magnitude in such color–magnitude diagrams. In this other case, the observed non-DA sequence in Figure 1 could be interpreted as a higher-than-average mass for these objects. We now explore these two interpretations further using our full set of synthetic colors for white dwarfs.

As can be seen in Figure 1, pure hydrogen and pure helium models overlap almost completely in color–magnitude diagrams ($\Delta M_G \lesssim 0.2$), except at low effective temperature where collision-induced absorption (CIA) by molecular hydrogen becomes important. However, the differences in colors between hydrogen- and helium-atmosphere white dwarfs become much more important when the Balmer jump is considered, which can be measured by observing the flux on each side of the hydrogen photoionization threshold (from the $n = 2$ level), located at $\lambda \sim 3640$ Å. One of the best examples is the color–color diagram built from SDSS magnitudes, $(u - g)$ versus $(g - r)$, displayed in Figure 2 (see also the $M_g$ versus $(u - g)$ color–magnitude diagram shown in Figure 15 of GaiaHRD). Because of the relatively small number of white dwarfs within 100 pc with $ugriz$ data available, we include in this figure objects at all distances (still with $\sigma_\pi/\pi < 0.1$), and the colors are therefore dereddened following the procedure described in Harris et al. (2006) and discussed in Section 3.1. Also reproduced in Figure 2 are our theoretical colors for pure hydrogen and pure helium model atmospheres at various temperatures and stellar masses. The important effect produced by the Balmer jump, as measured by the $(u - g)$ color-index, is clearly visible between $T_{\text{eff}} \sim 8000$ and 20,000 K (see Shipman & Sass 1980 for a detailed physical explanation of the log $g$ dependence of this jump). More importantly, the theoretical colors reproduce perfectly the observed sequences for both the DA and non-DA white dwarfs in this diagram, and there is a sensitivity on the mass for DA stars leading to inferred masses around 0.6 $\epsilon M_\odot$, as expected. We conclude, at least on the basis of this color–color diagram, that there appears to be no major problems with the color predictions from our model atmospheres.

In Figure 3, we show the same $M_g$ versus $(G_{\text{BP}} - G_{\text{RP}})$ color–magnitude diagram as before (with $D < 100$ pc), but this time by splitting the DA and non-DA stars into two panels, excluding the Gaia white dwarf candidates. We also superpose
in each panel the theoretical colors for the corresponding pure hydrogen or pure helium atmospheric composition, and for various values of effective temperature and stellar mass. For the DA stars (left panel), the observed sequence follows the 0.6 $M_\odot$ model sequence almost perfectly (slightly lower than 0.6 $M_\odot$ actually), with a small departure toward lower masses below $T_{\text{eff}} \sim 5500$ K. We can also identify a large population of extreme low-mass DA white dwarfs ($M < 0.4$ $M_\odot$)—most likely unresolved double degenerate binaries—as well as a nearly horizontal sequence of massive DA stars, which has recently been interpreted as evidence for crystallization (Tremblay et al. 2019b).

In the case of non-DA white dwarfs (right panel), the observed sequence follows very nicely the 0.6 $M_\odot$ model sequence from $T_{\text{eff}} \sim 30,000$ K down to 12,000 K, which corresponds to the region where DB white dwarfs are located. We also find little evidence for any large population of low- or high-mass white dwarfs in this temperature range. Below 12,000 K, however, a significant departure from the 0.6 $M_\odot$ sequence can be observed, with a sudden and nearly constant shift toward larger masses by an amount of $\sim 0.1 M_\odot$, down to $T_{\text{eff}} \sim 6000$ K. Below $\sim 6000$ K, the scatter in mass for the non-DA stars increases considerably, with the average mass of the bulk of objects decreasing steadily, even reaching values well below 0.4 $M_\odot$ at $T_{\text{eff}} \lesssim 4500$ K. We also find evidence again for overluminous, unresolved double degenerates in this temperature range. The most plausible explanation for these strange features is that most cool non-DA white dwarfs in this diagram probably have hydrogen-dominated atmospheres. Indeed, a closer inspection of the results displayed in Figure 1 reveals that the coolest non-DA white dwarfs represent a natural extension of the cool DA sequence. It is thus possible that most of the coolest non-DA stars in the right panel of Figure 3 have hydrogen-dominated atmospheres and belong instead in the left panel with other hydrogen-rich objects. We revisit this point further in Section 4.2.2.

Color–magnitude and color–color diagrams, despite their general interest in terms of the global properties of the sample, contain limited information in terms of interpreting the physical parameters of white dwarfs, most importantly effective temperatures, stellar masses, and atmospheric compositions. We now turn our attention to the more physical approach of fitting spectral energy distributions with the predictions of detailed model atmospheres.

3. Photometric Determinations of Physical Parameters

3.1. The Photometric Technique

The photometric technique has been developed and first applied to large ensembles of white dwarfs by Bergeron et al. (1997). Briefly, a set of magnitudes is converted into average fluxes using appropriate zero-points, and fitted—using a least-squares method—with average synthetic fluxes calculated from model atmospheres with the appropriate chemical composition. In this case, the fitted parameters are the effective temperature, $T_{\text{eff}}$, and the solid angle, $\pi (R/D)^2$, where $R$ is the radius of the star, and $D$ its distance from Earth. If the distance is known from the parallax measurement, the radius can be measured directly and converted into stellar mass ($M$) using evolutionary models, which provide the required temperature-dependent mass–radius relation. Further details about this fitting method are given in GBB19 and will not be repeated here. In particular, we use the same atmosphere and evolutionary models as in GBB19, with the following exception. Here we rely on a new grid of pure helium models computed using the atmosphere code described in Blouin et al. (2018). This code includes numerous high-density effects relevant for the modeling of cool helium-rich white dwarfs. Among the most
important effects considered, continuum opacities are corrected for collective interactions (Iglesias et al. 2002; Rohrmann 2018), CIA from the He—He—He interaction is included (Kowalski 2014), an ab initio equation of state is used (Becker et al. 2014), and the ionization equilibrium of helium is assessed from the ab initio calculations of Kowalski et al. (2007).

GBB19 adopted the dereddening procedure outlined in Harris et al. (2006) where the extinction is assumed to be negligible for stars with distances less than 100 pc, to be maximum for those located at $|z| > 250$ pc from the galactic plane, and to vary linearly along the line of sight between these two regimes. Although this approach is probably valid for stars well above the galactic plane, as is the case for the SDSS white dwarfs analyzed by GBB19, it is certainly not appropriate for stars very close to the galactic plane ($z \sim 0$), for which no reddening is predicted, even if the object lies at extremely large distances from Earth. A more reasonable dereddening procedure has been proposed by Gentile Fusillo et al. (2019, see their Section 4), which is applicable to all-sky surveys, such as the Gaia sample. Note that in the case of white dwarfs in the SDSS, GBB19 demonstrated that both dereddening procedures yield similar results (see their Figure 15). However, we found that, in some cases, the procedure proposed by Gentile Fusillo et al. yields spurious results when the star is nearby and that the maximum extinction along the line of sight is large. One example is GD 50 (WD 0346—011) at a distance of only 31.2 pc, and a maximum extinction of $E(B-V) = 0.1601$ along the line of sight (Schlafly & Finkbeiner 2011). For this hot DA star, we obtain a spectroscopic temperature of 42,670 K and a photometric temperature of 40,845 K on the basis of Pan-STARRS grizy photometry, assuming the dereddening procedure of Harris et al.; there is no reddening in this case because $D < 100$ pc. However, if we use the procedure proposed by Gentile Fusillo et al., this photometric temperature jumps to 59,090 K, i.e., more than 16,000 K above the spectroscopic value. We find similar problems with other nearby objects in our sample. As discussed by the authors, improved Gaia DR2 reddening maps in 3D will eventually supersede these simple parameterizations, but in the meantime, we will consider both dereddening procedures, depending on the white dwarf sample analyzed.

A few additional details are also worth mentioning. Even though modern magnitude measurements, such as SDSS or Pan-STARRS, are quoted with extremely small uncertainties—sometimes as small as millimagnitudes—the conversion to average fluxes remains the largest source of uncertainty when using the photometric technique (Holberg & Bergeron 2006). For example, while the Pan-STARRS photometric system attempts to be as close as possible to the AB magnitude system, the Pan-STARRS implementation has an accuracy of only $\sim 0.02$ mag according to Tonry et al. (2012). Also, the SDSS magnitude system is not exactly on the AB magnitude system either, and corrections must be added to the $uiz$ bandpasses: $\mu_{AB} = \mu_{SDSS} - 0.040$, $i_{AB} = i_{SDSS} + 0.015$, and $z_{AB} = z_{SDSS} + 0.030$ (Eisenstein et al. 2006a). Vega-based magnitudes also suffer from similar problems because, more often than not, the magnitude of Vega is not even known on the given system. For these reasons, we always adopt in our fitting procedure a lower limit of 0.03 mag uncertainty in all bandpasses. This ensures that all bandpasses have more equal weights, and that one magnitude with an extremely small error bar does not drive the overall photometric solution.

### 3.2. Adopted Photometric System

One of the critical issues when using the photometric technique is to identify which is the most optimal and reliable photometric system. Obviously, the SDSS $ugriz$ photometry has proven to be very useful and reliable (see GBB19 and references therein), but it is available for only a portion of the sky, while magnitudes measured by Gaia and Pan-STARRS are almost all-sky surveys (three-quarters of the entire sky for Pan-STARRS). Gentile Fusillo et al. (2019) have investigated the use of several photometric systems, including $Gaia (G, G_{BP},$ and $G_{RP})$, and found a generally good agreement between the derived atmospheric parameters. In what follows, we present our own assessment of the internal consistency between effective temperature and mass values obtained from photometric fits based on various photometric systems (point-spread function magnitudes). To do so, we first rely on the spectroscopic sample of relatively bright DA stars of Gianninas et al. (2011). More specifically, we compare spectroscopic temperatures and masses with those obtained from photometry, using a procedure identical to that described in detail by GBB19 for DA stars in the SDSS. We exclude from our sample all composite systems containing an M-dwarf companion, as well as bright objects for which the magnitudes are saturated (the SDSS photometry in particular). Because most of the DA stars in the Gianninas et al. sample are nearby objects, we adopt the dereddening procedure of Harris et al. (2006) to avoid the problems discussed above. We point out, however, that the dereddening procedure of Gentile Fusillo et al. (2019) yields similar results, but many spurious results are observed, such as the case of GD 50 mentioned in the previous section.

We show in the upper panel of Figure 4 the differences between spectroscopic and photometric temperatures measured from fits to Pan-STARRS grizy photometry. The spectroscopic solutions have been obtained using our grid of model atmospheres for DA stars with the ML2/$\alpha = 0.7$ version of the mixing-length theory, and the 3D corrections$^3$ from Tremblay et al. (2013) have been applied to both $T_{\text{eff}}$ and $\log g$ values. Our results can be contrasted with those obtained for DA stars in the SDSS by GBB19 (see their Figures 13 and 17). While comparisons between spectroscopic and photometric temperatures both show a systematic offset—with the spectroscopic temperatures being larger than the photometric values—the differences observed here are much larger (10%—20%) than those reported by GBB19 ($\leq 10\%$). Note that we exclude from this discussion the largest discrepant cases (up to 40%), which occur for unresolved double degenerate binaries (or binary candidates), shown by red symbols in Figure 4. More important, the comparison between spectroscopic and photometric masses (bottom panel of Figure 4) shows a systematic offset of about 0.1 $M_\odot$, with spectroscopic masses being larger, while GBB19 (see their Figure 17) find instead a much better agreement, with no apparent systematic offset.

Because the results of GBB19 are based on both a different white dwarf sample (SDSS) and a different set of photometric measurements ($ugriz$), we explore here the same sample of DA stars drawn from the SDSS, but we restrict our analysis to white dwarfs with $D < 100$ pc to minimize the effects of

$^3$ Note that these 3D corrections do not apply to the photometric solutions.
interstellar reddening. We also exclude all objects with $M < 0.48 \, M_\odot$, which are most likely unresolved double degenerate binaries. Instead of using the spectroscopic temperatures as a reference, we rely on photometric temperatures based on SDSS $ugriz$ photometry ($T_{ugriz}$). The temperatures obtained from various photometric systems and filter sets are compared with $T_{ugriz}$ in Figure 5. The top panel shows the comparison with Pan-STARRS $grizy$ photometry, also used in Figure 4. Although the temperatures are in good agreement below $T_{\text{eff}} \sim 15,000$ K, an increasing shift appears above this temperature, with the SDSS $ugriz$ temperatures exceeding those derived from Pan-STARRS $grizy$ ($\Delta T_{\text{phot}} = T_{grizy} - T_{ugriz}$ in the upper panel of Figure 5). Hence, had GBB19 used Pan-STARRS instead of SDSS photometry, they would have found discrepancies between spectroscopic and photometric temperatures that are much larger than those reported in their Figures 13 and 17, and consistent with our results displayed in Figure 4 using the Gianninas et al. sample.

Because the discrepancy between photometric temperatures becomes more important at high temperatures, we first believed that the specific use of the $u$ filter in the model fits based on SDSS $ugriz$ photometry was responsible for the observed shift in temperature. As discussed by GBB19 (see in particular their Figure 4), the $u$ magnitude becomes important when fitting hot white dwarfs that are in the Rayleigh–Jeans regime. This is also illustrated in Figure 6 of Genest-Beaulieu & Bergeron (2014), where photometric temperatures obtained using the full $ugriz$ photometric set are compared with those derived only from $griz$. To test our hypothesis, we fitted the same sample of DA white dwarfs by excluding the $u$ bandpass, thus fitting only the SDSS $griz$ photometry, the results of which are displayed in the second panel of Figure 5. We can see that even though the scatter at high temperatures has increased significantly, no systematic shift appears, in sharp contrast with the results based on Pan-STARRS photometry. Genest-Beaulieu & Bergeron (2014) reached a similar conclusion (see their Section 2.4.2).

To better understand these systematics differences, we performed the same experiment as in Figure 8 of Genest-Beaulieu & Bergeron (2014), which shows the histogram distributions between observed (obs) and predicted theoretical (th) magnitudes for each individual bandpass of the $ugriz$ system. As discussed by the authors, if the photometric system
is well calibrated—at least in a relative sense—all histograms should appear symmetrical and centered on $m_{\nu,\text{obs}} - m_{\nu,\text{th}} = 0$. We present in Figure 6 such histograms based on our own photometric fits of the SDSS DA sample using various photometric systems and filter sets. The top row in this figure is with the Pan-STARRS $grizy$ photometry. Although the $gri$ histograms appear well centered, the $y$ histogram in particular shows a long, extended tail, indicating a potential problem with this bandpass. By removing the $y$ magnitude from our fits, we obtain the results displayed in the second row of Figure 6 and in the third panel of Figure 5. We notice that the histograms using only the Pan-STARRS $griz$ photometry have not changed significantly, with the exception of the $z$ histogram, which now appears more centered. More importantly, however, the discrepancy between the photometric temperatures in Figure 5 has remained essentially unchanged.

The third and fourth rows in Figure 6 show our results using SDSS $griz$ and $ugriz$ photometry, respectively. The results using the full $ugriz$ photometric set are qualitatively consistent with those shown in Figure 8 of Genest-Beaulieu & Bergeron (2014), although the dispersion in their case is much smaller than ours, because they restricted their DA sample to the best photometric fits ($T_{\text{eff}} < 20,000$ K), while ours is based on distance ($D < 100$ pc). A more interesting result is the comparison between Pan-STARRS and SDSS for the same $griz$ filter sets (second and third row), which reveals a much smaller dispersion with the Pan-STARRS photometry. This demonstrates that the Pan-STARRS photometric system is superior to the SDSS photometric system, at least in a relative sense, a conclusion we also reached on the basis of our inspection of the individual photometric fits. Hence, despite the larger discrepancies between the spectroscopic and photometric temperatures obtained from Pan-STARRS $grizy$ photometry (as observed in Figure 4), it would be a shame not to take advantage of its superior photometric quality.

Figure 5. Differences between photometric temperatures as a function of $T_{ugriz}$—the temperature obtained from SDSS $ugriz$ photometry—for DA stars drawn from the SDSS with $D < 100$ pc. $\Delta T_{\text{phot}}$ represents the difference between the temperature obtained using the photometric data set indicated in each panel, and $T_{ugriz}$. The dashed lines indicate equal temperatures.
As a final experiment, we attempted to combine the SDSS $u$ bandpass with the Pan-STARRS $grizy$ photometry, the results of which are displayed in the bottom panels of Figures 5 and 6. Surprisingly, the resulting photometric temperatures are now in excellent agreement with those obtained using SDSS $ugriz$ photometry, which implies that the substitution of the SDSS $griz$ with Pan-STARRS $grizy$ has no significant effect on the measured photometric temperature, although the quality of our fits has increased significantly. We adopted the same strategy for the DA stars in the Gianninas et al. sample, and included the SDSS $u$ magnitude, whenever possible. The results are displayed in Figure 7. We see that the temperature discrepancy has been significantly reduced and that the photometric and spectroscopic masses are now in much better agreement. These results are now entirely consistent with those reported by GBB19 for the DA stars in the SDSS. We mention again that we achieve results that are virtually identical to those shown in Figure 7 if we substitute the Pan-STARRS $grizy$ photometry with SDSS $griz$ data, although individual fits are often superior with Pan-STARRS. Unfortunately, SDSS $u$ magnitudes are available for only a fraction of objects in our sample, a situation that will likely change, thanks to the ongoing Canada–France Imaging Survey (Ibata et al. 2017), which will map 10,000 square-degrees of the northern high Galactic latitude sky in the $u$-band (CFIS-$u$).

Finally, we also obtained results using Gaia photometry that are similar to those shown in Figure 4, in particular the large offsets in temperature and mass. Instead of repeating the same figure, we summarize our results in Figure 8, where the cumulative mass distributions are displayed using four different sets of photometry: Pan-STARRS grizy, SDSS $u$ + Pan-STARRS grizy (corresponding to Figures 4 and 7, respectively), SDSS ugriz, and Gaia G, G_BP, and G_RP photometry. Note that the number of stars in each panel is different because of the availability of the data in a given photometric system. Also given in each panel are the average masses and 1$\sigma$ dispersion values. As already mentioned above, the results obtained with SDSS $u$ + Pan-STARRS grizy and with SDSS ugriz are virtually identical, although the peak of the mass distribution is more sharply defined using the Pan-STARRS photometry, perhaps an indication of the superior quality of this particular data set. If we omit the $u$ bandpass, however, we obtain average masses that are lower, no matter whether we use Pan-STARRS grizy alone or Gaia photometry. Even though both of these subsamples are significantly larger, nearly identical results are obtained if we restrict our analysis to a common subsample.
The results displayed in Figure 8 can probably explain the differences, of the same order (∼0.03 $M_\odot$), between the average mass of DA white dwarfs in the SDSS reported by Tremblay et al. (2019a, see their Figure 13) on the basis of Gaia photometry, $\langle M \rangle = 0.586 M_\odot$, and that found by GBB19 (see their Figure 20) on the basis of SDSS ugriz photometry, $\langle M \rangle = 0.617 M_\odot$. We also note that the mean mass obtained by Tremblay et al. for DA stars in the Gianninas et al. sample using Gaia photometry, $\langle M \rangle = 0.599 M_\odot$, is comparable to the value reported in Figure 8, $\langle M \rangle = 0.587 M_\odot$, even though there are several differences between both analyses, the first of which is the different procedure for taking into account interstellar reddening. Also, our zero-points for the Gaia photometry are calculated under the assumption of zero magnitude for Vega, which differ from their zero-points listed in Table 3 of Gentile Fusillo et al. (2019).

Despite these differences, our conclusion remains the same. The photometric temperatures obtained by neglecting the $u$ bandpass may be significantly underestimated, especially for hotter stars in the Rayleigh–Jeans regime. For a given luminosity, the photometric fit will yield a larger radius, and thus a smaller mass, given the mass–radius relation for white dwarfs. Consequently, the mean mass for a given white dwarf sample will be smaller than the mean value obtained from fits where the $u$ magnitude is included, by ∼0.03 $M_\odot$ according to our results.

4. Implications for White Dwarf Physics

4.1. The Mass–Radius Relation Revisited

In this section we revisit some of the results of Bédard et al. (2017), who presented a detailed spectroscopic and photometric analysis of 219 DA and DB white dwarfs for which trigonometric parallax measurements were available at that time, with the aim of testing the mass–radius relation for white dwarfs. In order to compare physical quantities on an equal
footing, Bédard et al. first compared the parallactic distance, $D_\pi$, obtained directly from the trigonometric parallax, to the distance inferred from the mass–radius relation, $D_{MR}$. The latter is calculated by first using evolutionary models to convert the spectroscopic log $g$ into radius, which is then combined with the photometric value of $(R/D)^2$ to obtain the desired distance $D_{MR}$. The 1σ confidence level between these two distance estimates can then be calculated, including all sources of

Figure 8. Cumulative mass distributions measured using the photometric technique for DA white dwarfs drawn from the sample of Gianninas et al. (2011). Different photometric sets are used, as labeled in each panel, and the corresponding mean masses and dispersions are also given. Note that the number of stars, indicated in each panel, differs because of the availability of the photometry.
uncertainties associated with both the spectroscopic and photometric techniques.

The original results from Bédard et al. are reproduced in the left panel of Figure 9, where all white dwarfs are plotted in the $R$ versus $M$ diagram, together with various mass–radius relations for different effective temperatures and core compositions. Here, the radius $R$ is obtained directly from the photometric technique, while the mass is derived by combining this photometric radius with the spectroscopic log $g$ measurements. The right panel of Figure 9 shows the same results but with the updated trigonometric parallaxes from Gaia. With the exception of the unresolved double degenerate binaries, or binary candidates (shown as filled and dotted circles, respectively), we can see that the white dwarfs are more closely packed near the theoretical mass–radius relations with the Gaia parallaxes than with the older measurements. However, we also note that the number of objects with distances that differ by more than the 1σ confidence level (shown in red) has not changed significantly. This somewhat surprising result is due to the fact that the errors on the Gaia parallaxes are now extremely small—as can be estimated by the average 1σ uncertainty on the radius $R$ displayed in both panels of Figure 9—while the errors on the mass are still dominated by the large uncertainties of the spectroscopic log $g$ determinations.

Several puzzling discrepancies reported by Bédard et al. are worth discussing here. The first case is G87-7, whose distance obtained from C/O-core models, $D_{MR} = 17.5$ pc, was found to be significantly different from the parallactic distance, $D_\pi = 15.7$ pc, measured by Hipparcos. Bédard et al. suggested that G87-7 could have instead an iron core, because the distance inferred from Fe-core models, $D_{MR} = 15.9$ pc, was in much better agreement with the parallactic distance. This discrepancy has been resolved with the Gaia parallax, $D_\pi = 17.07$ pc, which is now in excellent agreement with the distance inferred from C/O-core models. Two additional objects of interest are the ZZ Ceti white dwarfs Ross 548 and GD 1212, for which Bédard et al. obtained from C/O-core models, $D_{MR} = 32.6$ pc and $18.7$ pc, respectively, in strong disagreement with the parallactic distances of $D_\pi = 63.3$ pc and $15.9$ pc, respectively. In both cases, no physical model could explain the observed discrepancies. Fortunately, the improved parallax measurements for the two ZZ Ceti stars, respectively, $D_\pi = 32.76$ pc and $18.66$ pc, are now in perfect agreement with the distances inferred from C/O-core models.

We end this section by discussing another puzzling result. In the analysis of Bédard et al. (2017), the agreement between their spectroscopic and photometric parameters was found to be excellent, especially for effective temperatures (see their Figure 4), in sharp contrast with the results discussed in GBB19 and those reported here, where differences of the order of 10% are found between the two temperature estimates. However, if we plot differences between temperatures rather than comparing values against each other, we do find a small systematic offset. Also, the sample of Bédard et al. contains very few hot stars, and if we superpose their results with those shown in Figure 7 for the complete Gianninas et al. sample, the temperature discrepancies observed in both samples are entirely consistent.

4.2. Mass–Temperature Distributions

4.2.1. The Sample of Spectroscopically Identified White Dwarfs

We now investigate further the physical parameters obtained from photometric fits by first looking at the white dwarfs identified spectroscopically in the MWDD. Given our results above, we combine the SDSS $u$ magnitude with the Pan-STARRS grizy photometry because this sample is completely dominated by objects identified in the SDSS (over 95% of the objects in our sample include the SDSS $u$ magnitude). We rely solely on Pan-STARRS photometry if the SDSS $u$ bandpass is not available. Again, we restrict our analysis to Gaia parallaxes with uncertainties smaller than 10%.
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Figure 10. Stellar masses as a function of effective temperature for all spectroscopically identified white dwarfs in the MWDD. The parameters have been determined using photometric fits to SDSS $u$ and Pan-STARRS $g$-band photometry, mostly, and $Gaia$ parallaxes with $\sigma_p/\pi < 0.1$. DA (white symbols, 7340 objects) and non-DA (red symbols, 2348 objects) white dwarfs have been fitted under the assumption of pure hydrogen and pure helium models, respectively. Also shown as solid curves are theoretical isochrones, labeled in units of $10^9$ yr, obtained from cooling sequences with C/O-curves correspond to isochrones with the main-sequence lifetime taken into account.

Figure 10 shows the stellar masses as a function of $T_{\text{eff}}$ for all DA and non-DA white dwarfs identified spectroscopically in the MWDD. Here we simply assume pure hydrogen and pure helium compositions for the DA and non-DA stars, respectively, and we restrict our analysis to a range of effective temperatures where the photometric technique is the most reliable ($T_{\text{eff}} < 30,000$ K, GBB19). The stellar masses have been derived from the measured stellar radii using evolutionary models$^4$ similar to those described in Fontaine et al. (2001) with (50/50) C/O-core compositions, $q(\text{He}) = 0.1$, $q(\text{H}) = 10^{-2}$, and $q(\text{He}) = 10^{-4}$ or $10^{-10}$ for DA and non-DA stars, respectively. Also shown are theoretical isochrones, with and without the main-sequence lifetime taken into account, as described in Bergeron et al. (2001, see their Section 5.5).

As mentioned above, the white dwarfs displayed in Figure 10 are completely dominated by those identified in the SDSS, and, consequently, there is a strong bias toward hotter objects given the color cuts inherent to this survey. If we first consider the DA stars, we can see that the mass distribution is well centered around $0.6 M_\odot$, with a significant number of high-mass and low-mass white dwarfs. The low-mass DA stars ($M \lesssim 0.5 M_\odot$) in this diagram correspond most certainly to unresolved double degenerate binaries whose photometric masses are meaningless because these have been obtained under the assumption of a single star. The high-mass DA stars, on the other hand, are believed to be the end result of stellar mergers (Kilic et al. 2018), or alternatively, they can also be explained as the outcome of the initial-to-final mass relation (El-Badry et al. 2018). Note that the results presented here for the DA stars are entirely consistent with those displayed in Figure 7 of GBB19.

The situation is relatively more complex for the non-DA white dwarfs shown in Figure 10. As a reminder, these non-DA white dwarfs include DB, DQ, DZ, DC, and all other subtypes (DO stars are outside the temperature range displayed here). Below $T_{\text{eff}} \approx 5000$ K, the DC spectral type gives no indication about the atmospheric composition because both hydrogen and helium lines become invisible at these temperatures (see, e.g., Bergeron et al. 1997). The DC stars in this temperature range may as well have hydrogen-dominated compositions. We defer our discussion of these objects further below. Above $T_{\text{eff}} \approx 11,000$ K, the non-DA population is dominated by DB/DBA white dwarfs, whose masses are well centered around $0.6 M_\odot$, in agreement with the results of GBB19 (see their Figure 8). We also see that the number of extremely high-mass and extremely low-mass non-DA white dwarfs is relatively small compared to DA stars. Actually, the high-mass, non-DA stars above $\sim 10,000$ K observed here correspond in majority to warm DQ white dwarfs (S. Coutu et al. 2019, in preparation).

The most striking feature in Figure 10, by far, is the large number of non-DA white dwarfs below $T_{\text{eff}} \approx 11,000$ K with masses significantly above $0.6 M_\odot$. The number of non-DA stars in this temperature range with masses around $0.6 M_\odot$ is actually quite small. The “step function” in mass observed here corresponds exactly to the discontinuity in the color–magnitude diagram described in Figure 3. Such high masses inferred from photometric fits is reminiscent of a similar problem observed in the context of cool DQ stars analyzed with pure helium models. For instance, Dufour et al. (2005, see their Figure 8) compared the effective temperatures and masses derived from pure helium models with those obtained with models including

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$^4$ See [http://www.astro.umontreal.ca/~bergeron/CoolingModels](http://www.astro.umontreal.ca/~bergeron/CoolingModels).
carbon. The temperatures measured from models including carbon are found to be significantly lower than the pure helium solutions. In this case, the inclusion of carbon in the equation of state increases the number of free electrons and, thus, the contribution of the He\(^+\) free–free opacity, which, in turn, affects the atmospheric structure, in particular in the continuum-forming region. Because the derived temperatures are significantly reduced, larger stellar radii—and thus smaller masses—are required to match the observed stellar flux. In some cases, the masses are decreased by as much as 0.2 \(M_\odot\) when carbon is included.

The key ingredient in the above argumentation is the presence of additional free electrons in otherwise pure helium atmospheres, whether these are coming from carbon, other metals, or even hydrogen. Note that in this context, GBB19 have demonstrated (see their Figure 6) that the presence of hydrogen in DBA white dwarfs does not affect the masses inferred from photometry in the temperature range where DB stars are found (\(T_{\text{eff}} \gtrsim 11,000\) K). Hence, their measured masses in Figure 10 are free of this uncertainty and completely reliable. In fact, they overlap perfectly with those of DA stars in the same temperature range, but the almost complete absence of normal-mass (\(M \sim 0.6\) \(M_\odot\)) non-DA stars below 11,000 K suggests that pure helium white dwarfs are extremely rare, if they exist at all. In the range 11,000 K \(\lesssim T_{\text{eff}} \lesssim 6000\) K, we actually find that about 25% of the objects are DQ stars, while another 25% are DZ stars, but most are DC white dwarfs.

It is a well-known fact that the ratio of non-DA to DA white dwarfs increases dramatically below \(T_{\text{eff}} \sim 10,000\) K (see, e.g., Fontaine & Wesemael 1987). The sudden increase in the number of non-DA stars in this temperature range has been interpreted as the result of the mixing of the superficial convective hydrogen layer with the deeper and much more massive convective helium envelope (see Rolland et al. 2018 and references therein). In this mixing scenario, the relatively small amount of hydrogen in the upper layers is being thoroughly mixed within the underlying helium convection zone, resulting in photospheric hydrogen abundances that are extremely small, but not zero. Rolland et al. (2018) have explored this scenario more quantitatively by using state-of-the-art envelope models to predict the hydrogen-to-helium abundance ratio as a function of the temperature at which convective mixing occurs (see their Figure 16). Qualitatively, the thicker the hydrogen layer, the cooler the mixing temperature, and the larger the predicted photospheric hydrogen abundance upon mixing. After mixing occurs, the hydrogen abundance remains almost constant with time. In some cases, the presence of hydrogen might be revealed by the detection of a weak H\(\alpha\) absorption feature, heavily broadened by van der Waals interactions, as observed for instance in L745-46A and Ross 640 (see Figure 14 of Giammichele et al. 2012); other examples from the SDSS can be found in Rolland et al. (2018), but as the star cools off, the weak H\(\alpha\) feature rapidly falls below the limit of visibility. Hence, in more representative cases, the object becomes a DC white dwarf, or a DZ star if the atmosphere was already contaminated with metals.

With this idea in mind, we performed the following experiment. Instead of fitting the non-DA stars in our sample with pure helium models, we used mixed H/He models where the hydrogen abundance is adjusted as a function of effective temperature. In the range of DB stars (\(T_{\text{eff}} > 11,000\) K), we adopt a typical value of \(\log N(H)/N(\text{He}) = -5\), while below this temperature, we gradually increase the hydrogen abundance following the predictions of the convective mixing scenario displayed in Figure 16 of Rolland et al. (2018). Below 6000 K, where these simulations stop and where we have very few objects in our sample, we assume the same composition as in DB white dwarfs. As mentioned above, the cool non-DA stars in this temperature range may have hydrogen-dominated compositions, and these will be discussed separately below. The results of our experiment are displayed in Figure 11. As
anticipated, the masses for the DB stars have not changed because, in this temperature range, helium remains the main electron donor. At lower temperatures, however, the large masses observed in Figure 10 have been significantly reduced and now overlap perfectly with those of DA stars. Our results clearly indicate that the photometric parameters obtained from pure helium models may be unreliable, and that more reasonable masses are derived when additional electron donors are included, whether they are in the form of metals (including carbon) or hydrogen. We stress here that this is only an experiment, and that stellar parameters of individual white dwarfs can be obtained only through a tailored analysis of each object with an appropriate model atmosphere grid. In that sense, accurate parameters for DC stars may be impossible to achieve because only upper limits on the hydrogen abundance can be measured. One thing our results indicate, however, is that this limit is certainly not zero.

4.2.2. The Sample of White Dwarfs in Gaia

We now turn our attention to white dwarfs found in Gaia, which include both spectroscopically identified white dwarfs as well as white dwarf candidates. We restrict our analysis to objects with a distance $D < 100$ pc for which interstellar reddening is assumed to be negligible. If we had access to the $u$ bandpass for all objects, we could, in principle, differentiate hydrogen-rich from helium-rich objects in the temperature range where the Balmer jump is important (see Figure 2). Because $u$ magnitudes are available for only a limited number of objects in our sample, we adopt a simpler approach and fit only the Pan-STARRS $grizy$ photometry by assuming that all white dwarfs have either pure hydrogen, pure helium, or mixed H/He atmospheric compositions (as defined above). Also, because we do not include the $u$ bandpass, our stellar masses for hot stars might be underestimated by $\sim 0.03 M_\odot$ on average. The stellar masses obtained under these assumptions are displayed in Figure 12 as a function of effective temperature.

The distribution of Gaia white dwarfs within 100 pc shown in Figure 12 is markedly different from that displayed in Figure 10 (or 11), regardless of the assumed atmospheric composition, because the latter is mostly based on UV-excess selection criteria rather than distances. Consequently, the number of cool white dwarfs in our distance-limited sample is significantly larger, while the number at higher temperatures is much smaller. At high temperatures ($T_{\text{eff}} > 11,000$ K), the differences in mass inferred from pure H or pure He models—applicable to DA and DB stars, respectively—are of the order of $\sim 0.1 M_\odot$, but the differences between the pure He and mixed H/He solutions in the same temperature range are completely negligible, in agreement with our discussion above. At lower temperatures, however, the effects due to atmospheric composition are much more pronounced. Somewhat unexpectedly, the average mass drops as we go from pure H to pure He, and then to mixed H/He models. In particular, the bulk of massive white dwarfs in the 6000–10,000 K temperature range has an average mass above $0.7 M_\odot$ under the assumption of pure hydrogen atmospheres, and of $\sim 0.6 M_\odot$ with mixed H/He models.

We can explore these results more quantitatively by looking at the cumulative mass distributions derived with our various assumptions about the atmospheric composition, the results of which are displayed in Figure 13. The pure hydrogen models yield the sharpest peak around $0.6 M_\odot$, but also a well-developed high-mass bump with a peak around $0.8 M_\odot$. This mass distribution is completely analogous to that reported by Kilic et al. (2018, see their Figure 5); in this case, the high-mass bump has been interpreted by the authors as evidence of a large population of merged white dwarfs. We can see, however, that this high-mass bump virtually vanishes if we use mixed H/He models. Pure helium models yield results that are in between. Obviously, the true mass distribution requires the knowledge of the atmospheric composition of each individual white dwarf in our sample. Nevertheless, our results indicate that the number of massive ($M \sim 0.8 M_\odot$) white dwarfs is certainly much smaller than previously reported by Kilic et al.

Another feature worth discussing in Figure 12 is the behavior at the cool end of the distribution ($T_{\text{eff}} \lesssim 5000$ K). Both the pure helium and the mixed H/He models give rise to a large number of extremely low-mass white dwarfs—$M < 0.5 M_\odot$, and even $M < 0.4 M_\odot$ with mixed models. On the other hand, pure hydrogen models bring the masses of most cool white dwarfs comfortably above $0.5 M_\odot$. Because pure hydrogen white dwarfs in this temperature range are featureless, we predict that most, but not all, cool white dwarfs identified by Gaia are H-rich DC stars.

4.3. The Crystallization Sequence

It is instructive to reconsider the mass-effective temperature distribution of Gaia DA white dwarfs in the light of the recent discovery of the signature of crystallization—a characteristic pile-up forming a sequence going across evolutionary tracks—in the Gaia color–magnitude diagram for such stars (Tremblay et al. 2019b). To this end, we plotted in Figure 14 our most reliable estimates of mass and effective temperature for spectroscopically identified DA white dwarfs having Gaia parallaxes with $\sigma_\pi/\pi < 0.1$. Next, using the DA subset of the standard evolutionary models, which we briefly described above in Section 4.1 (these are the same models used also in Tremblay et al. 2019b), we plotted a series of tightly spaced cooling isochrones in the mass-effective temperature plane. The (variable) density of these many isochrones indicates graphically phases of slowing down and of accelerated cooling. Note that our (50/50) C/O-core models include the release of latent heat upon crystallization, but no additional source of energy associated with possible phase separation between C and O. Further details concerning our approach to the crystallization process can be found in Tremblay et al. (2019b).

The lower solid curve in Figure 14 corresponds to the onset of crystallization at the center of an evolving model (at constant mass, from left to right). From that point on, with further cooling, the solidification front progresses upward in the star from the center, and latent heat is progressively released. By the time some 80% of the total mass of the star has solidified—this is indicated by the upper solid curve—most of the latent heat has been spent. The release of latent heat corresponds to a slowdown in the evolution of a white dwarf (specifically, the cooling rate decreases), and this is well illustrated in the tightening up of the isochrones in between the two solid curves. Note that the DA white dwarfs found in between the two solid curves in Figure 14 correspond to a subset of the stars populating the “crystallization sequence” reported by Tremblay et al. (2019b) in their observational Gaia color–magnitude diagram (see their Figure 2). They are less numerous, but they do define a pile-up structure that is clearly associated with the release of latent heat.
The most significant effect of crystallization on the evolution of white dwarfs, however, is not the slowdown associated with the release of latent heat, but rather the so-called Debye cooling phase, i.e., the subsequent transition, in the solid phase, from the classical regime where the specific heat of a solid is independent of temperature to the quantum regime where it goes down from that constant value with decreasing temperature. In the quantum regime, the specific heat decreases quickly with cooling, which rapidly depletes the reservoir of thermal energy and produces a spectacular increase of the cooling rate, leading to the concomitant rapid shift to the black dwarf phase. This is well illustrated in Figure 14, through the turnover of the isochrones toward low effective temperatures, most obviously seen in the more massive models. For the present purposes, we loosely defined the transition from the classical to the quantum regime through the dotted curve. The latter has been obtained by isolating the evolving model where the central temperature becomes, from above, equal to the central Debye temperature (function of density and composition, but not of temperature). According to our models, a few massive DA white dwarfs in our sample—those above the dotted curve—are likely crystallized white dwarfs having

**Figure 12.** Stellar masses as a function of effective temperature for all white dwarfs found in *Gaia* within a distance of 100 pc, including both spectroscopically identified white dwarfs as well as white dwarf candidates. The parameters have been determined using photometric fits to Pan-STARRS grizy photometry and *Gaia* parallaxes with $\sigma_p/p < 0.1$. The white dwarfs in each panel have been fitted under the assumption of pure hydrogen, pure helium, or mixed H/He models, as indicated in the figure. Also shown are the same isochrones as in Figure 10. The horizontal lines represent constant masses of $M = 0.6 \pm 0.1 M_\odot$. 
reached the quantum regime and undergone rapid Debye cooling. They must be still quite young (τ_{cool} \leq 4.5\,\text{Gyr}; see Figure 10), and their predecessors must already have faded away to the black dwarf state.

Beyond crystallization, another phenomenon complicates things in the late cooling history of white dwarfs, and that is convective coupling (see, e.g., Tremblay et al. 2015). The onset of convective coupling, when superficial convection first reaches into the degenerate core (where essentially all of the thermal energy resides), is indicated by the dashed curve in Figure 14. The phenomenon is weakly mass dependent and clearly interacts with the manifestations of crystallization, as can be observed through the changes of slope in the solid and dotted curves to the right of the convective coupling boundary. For a middle-of-the-road DA white dwarf with \( M_\odot \sim 0.6 \, M_\odot \), crystallization and convective coupling occur at nearly the same time, so it is almost impossible to untangle their effects. Only for the larger masses are the two mechanisms occurring during different phases of their evolution, and this is well depicted in Figure 14. For example, picking the case of a 1.0\,M_\odot model, it is clear from the figure that the star is already highly solidified and in its Debye phase by the time convective coupling turns on. When it does, it acts in two phases (like crystallization), and this is well illustrated by the behavior of the isochrones to the right of the dashed curve for that particular mass. First, there is an initial release of thermal energy as the outer envelope becomes much more transparent to photons through convective transport (the isochrones bunch together), and, second, this is followed by a phase of accelerated cooling, convective cooling (the isochrones separate from each other), compared to the purely radiative case (see Tremblay et al. 2015 for details). Note that convective cooling, in the presence of Debye cooling as is the case in this example, accelerates considerably the passage to the black dwarf state.

For more representative masses, the effects of crystallization and convective coupling manifest themselves initially in their phases of slowing down, which add up together (release of latent heat superposed on the extra thermal energy liberated when the envelope becomes suddenly more transparent through convection). This is best seen in the behavior of the isochrones, which come very close to each other, just to the right of the dashed curves and in between the two solid curves in the range 0.5 to 0.6\,M_\odot. This is where a maximum pile-up of stars is expected. Unfortunately here, our sample of DA white dwarfs is limited by the availability of suitable photometric data, and one cannot test for this possibility. However, we consider a larger sample in what follows.

**4.4. Constraint on the Core Composition of Gaia White Dwarfs**

We reconsider the sample of all white dwarfs found in Gaia within a distance of 100 pc, including both spectroscopically identified white dwarfs as well as white dwarf candidates. Both panels of Figure 15 are a zoomed-in view of the mass-effective temperature distribution depicted in the upper panel of Figure 12, the latter obtained under the specific assumption that the atmospheres of all these stars are made of pure hydrogen. We emphasize, once again, that detailed atmospheric analyses are required on a case-by-case basis, but our approach has merits from a statistical standpoint, especially in view of our suggestion above that the majority of the coolest white dwarfs in the Gaia sample likely have hydrogen-dominated atmospheres.

In the lower panel of Figure 15, one’s attention is attracted by the strong correlation that exists between the behavior of the isochrones that bunch closest together and the maximum density of white dwarfs observed in an area roughly centered around \( T_{\text{eff}} \sim 5100\,\text{K} \) and \( M \sim 0.56\,M_\odot \). This is the expected behavior discussed at the end of the previous subsection. As well, the theoretical crystallization sequence (the two solid curves) sandwiches remarkably well the observed distribution of stars to the right of the convective coupling boundary in this lower panel. Hence, we find that our C/O-core models provide a natural explanation for the observed clump of low-mass, cool Gaia white dwarfs in the mass-effective temperature diagram. These objects are likely currently undergoing a significant slowdown in their cooling history due to the superposition of latent heat and of extra thermal energy associated with the first phase of convective coupling.

We note, in the present context, that there is a strong dependency between the locations of isochrones and of the theoretical crystallization sequence in a mass-effective temperature diagram and the core composition of the models. This is illustrated by comparing the lower with the upper panel of Figure 15, where, in the latter case, isochrones and characteristic curves have been plotted for models with pure C cores instead of C/O cores. Note that these two sets of models are quite similar (same envelope stratification, same physical assumptions), except for the core composition. Given that carbon ions are less charged than their oxygen counterparts, the
former solidify at lower temperature in a dense Coulomb plasma; given a density (equivalently, a total mass), it follows that the theoretical crystallization sequence is shifted to lower effective temperatures in the upper panel. This dependency on the core composition of models is most interesting and directly implies that the core composition of Gaia white dwarfs can be estimated, at least in bulk.

The top panel of Figure 15 indicates that the two solid curves, the crystallization sequence, do a significantly poorer job of containing the high-density clump of stars than what is depicted in the lower panel. This suggests that, as a whole, the Gaia white dwarfs are probably not made up of pure C. A similar and complementary result would be obtained with pure O-core models, which would predict, this time, a crystallization sequence too far to the left of the observed clump. Of course, a proper statistical analysis, well beyond the scope of this paper, is needed to infer correctly the most probable bulk core composition for white dwarfs in the Gaia sample. Nevertheless, we wish to attract the attention of the reader to that very real possibility. Currently, we find that C/O-core models perform better than pure C (and very likely pure O) models. The statement that the core composition of Gaia white dwarfs, in bulk, is more likely a mixture of C and O than pure C or pure O is hardly a surprise. However, and to our knowledge, this is the first time that such inference can be made on the basis of a very large number of stars.

Note that, in this context, changing the core composition of evolutionary models of the kind has little influence on the locations of the classical-to-quantum boundary and none on the location of the convective coupling boundary.

5. Discussion

Despite its overall simplicity and straightforward approach, the photometric technique is certainly not devoid of uncertainties, most likely related to photometric calibration issues. For instance, the question of whether Pan-STARRS and SDSS magnitudes are precisely on the AB magnitude system remains. As shown in our analysis, these two photometric systems yield different effective temperatures, even when we consider only the griz filters common to both sets. To overcome these calibration problems, Holberg & Bergeron (2006) proposed a procedure whereby empirical corrections are applied to various photometric systems, which are calculated by comparing the observed magnitudes with those predicted from spectroscopic determinations of Teff and log g for DA stars. However, this procedure assumes that the spectroscopic solutions are accurate, which may not be the case according to the analysis of GBB19. Nevertheless, our experiments with various photometric systems indicate that the SDSS ugriz photometric system, or a combination of SDSS u with Pan-STARRS grizy photometry, probably yield the most accurate Teff measurements. When the u bandpass is omitted, however, the effective temperatures are often underestimated (with the exception of SDSS griz), and the corresponding stellar masses are underestimated as well, but according to our results displayed in Figure 8, these mass differences remain small, ñ0.03 M⊙ on average, which is about the size of the mass uncertainty of the spectroscopic method (Liebert et al. 2005).

The Hertzsprung–Russell diagram for white dwarfs presented in Figure 13 of GaiaHRD suggested that perhaps there was a problem with the model atmospheres for DB white dwarfs, because the 0.6M⊙ theoretical sequence failed to reproduce the observed sequence in the MG versus (G BP−G RP) diagram. However, our results displayed in Figure 3 show that

\[
\begin{align*}
&MWDD DA stars + Gaia (<10\%) \\
&M_{\odot} \\
&T_{\text{eff}} (K)
\end{align*}
\]
the theoretical models for the DB stars ($T_{\text{eff}} \gtrsim 11,000$ K) overlap perfectly with the observed sequence. The observed shift actually occurs at lower temperatures where non-DA stars exist in the form of DZ, DQ, or DC white dwarfs. By assuming pure helium compositions for these objects, we found that the observed shift in absolute magnitude corresponds to a shift in mass in a $M$ versus $T_{\text{eff}}$ diagram (see Figure 10). However, we also found that these high masses could be significantly reduced to more normal masses around 0.6 $M_\odot$ if we include a small amount of hydrogen in the atmosphere, the presence of which has been interpreted as the result from the convective mixing of DA into non-DA white dwarfs at low temperatures. Alternatively, the presence of other electron donors, such as carbon or metals, would have the same effect. Consequently, the population of massive white dwarfs interpreted as stellar mergers by Kilic et al. (2018) is probably less important than previously established.

According to our results displayed in Figure 10, the number of cool (11,000 K > $T_{\text{eff}}$ > 6000 K), pure helium-atmosphere white dwarfs with normal masses must be extremely small. Now this is an odd result when considering the chemical evolution of DB white dwarfs. Figure 14 of Rolland et al. (2018)—reproduced here in Figure 16 and extended to much lower effective temperatures—shows the hydrogen-to-helium abundance ratio as a function of effective temperature for several DB, DBA, and cool, He-rich DA white dwarfs, together with the predictions from envelope calculations for homogeneously mixed models at 0.6 $M_\odot$. By following a curve with a constant value of total hydrogen mass (labeled as $\log M_H/M_\odot$ in Figure 16), one can predict the evolution of the photospheric

Figure 15. Magnified view of the low-mass, low-effective-temperature corner of the upper panel of Figure 12 showing the highest-density clump of stars observed in the empirical data. The curves are similar to those found in Figure 14. The lower (upper) panel refers to C/O-core (pure C-core) models. Note that, unlike the curves defining the convective coupling and the classical-to-quantum boundaries, the locations of the isochrones and of the theoretical crystallization sequence (the two solid curves) depend strongly on the core composition of the models.
hydrogen abundance as a function of time, or cooling temperature. The simulations indicate that a significant fraction of DB white dwarfs should have hydrogen abundances well below $\log N(\text{H})/N(\text{He}) = -6$ once they reach temperatures below $T_{\text{eff}} = 10,000$ K or so, and in particular for the “pure” DB stars with no detectable traces of hydrogen (shown as white circles in Figure 16). Additional calculations, not discussed in our paper, indicate that hydrogen abundances larger than $\log N(\text{H})/N(\text{He}) > -6$ are required to affect the photometric masses above $\sim 8000$ K. Because DB white dwarfs have normal masses around $0.6 \, M_\odot$ (see, e.g., Bergeron et al. 2011), they should appear as $\sim 0.6 \, M_\odot$, pure helium white dwarfs in Figure 10. Yet, there are none. We thus propose that another electron donor makes the photometric masses of cooled-off DB stars appear normal, namely carbon. This implies that the progenitors of normal-mass DQ stars are DB white dwarfs. This proposition is perfectly in line with the well-known carbon dredge-up scenario, which suggests a natural connection between PG1159, DO, DB, and normal-mass DQ white dwarfs (see, e.g., Pelletier et al. 1986; Dufour et al. 2005; Fontaine & Brassard 2005). Brassard et al. (2007) last reviewed the question of the carbon pollution observed in the atmospheres of DQ stars and in some DB stars. They proposed the existence of a convectively driven cold wind in DB stars to slow down the separation of C and O from He and, thus, maintain some observable amounts of primordial C in their atmospheres. Alternatively, the presence of radiative turbulence (of still unknown origin, however) could be the source of competition against gravitational settling. Current thinking, based on the hydrodynamic simulations of Tremblay et al. (2015), rather suggests that overshooting at the base of the superficial He convection zone in DO and DB white dwarfs could be the actual competing mechanism. In any case, carbon does appear as a natural electron donor in cooled-off DB white dwarfs, which produce normal-mass DQ stars.

One potential problem with the interpretation of the presence of trace amounts of hydrogen in cool, helium-rich atmospheres is the behavior at extremely low effective temperatures. Indeed, a typical hydrogen abundance of $\log N(\text{H})/N(\text{He}) = -5$ should produce an extremely strong infrared flux deficiency at $T_{\text{eff}} < 4000$ K, resulting from CIAs by molecular hydrogen due to collisions with helium, as shown for instance in Figure 8 of Bergeron & Leggett (2002). This absorption is even more important than in pure hydrogen atmospheres and also manifests itself at much higher temperatures and higher luminosities. However, the color–magnitude diagram displayed in Figure 1 shows no evidence for such a large population of cool white dwarfs. We turn again to Figure 16 for a possible explanation of this lack of mixed H/He white dwarfs. The simulations show that for most values of the total hydrogen mass, the predicted hydrogen-to-helium abundance ratio remains nearly constant, below $T_{\text{eff}} \sim 11,000$ K, until the star reaches a temperature of $\sim 6000$ K, at which point the bottom of the mixed H/He convection zone plunges deep into the star, resulting in photospheric hydrogen abundances that are at least two orders of magnitudes smaller. Hence, we suggest that most non-DA stars below $10,000$ K must have nearly pure helium atmospheres when they reach the end of the cooling sequence.

Figure 16. Hydrogen-to-helium abundance ratio as a function of effective temperature for all DB, DBA, and cool, He-rich DA white dwarfs taken from the analysis of Rolland et al. (2018); the DZA stars from Dufour et al. (2007) and Giammichele et al. (2012) are also displayed. The hydrogen detection limits at Hα are indicated by blue lines. Also reproduced are the predictions from our envelope calculations for homogeneously mixed models at $0.6 \, M_\odot$ for the ML2/$\alpha = 0.6$ version of the mixing-length theory. Each curve is labeled with the corresponding value of $\log N(\text{H})/N(\text{He})$. The Astrophysical Journal, 876:67 (20pp), 2019 May 1 Bergeron et al.
An alternative and more exotic explanation proposed by Bergeron et al. (1997, see their Section 6.3.2) is that some non-DA stars could experience a sudden transition, where all the hydrogen thoroughly diluted within the stellar envelope somehow makes it back to the surface, thus transforming a mixed H/He white dwarf into a hydrogen-dominated atmosphere white dwarf. This scenario would be consistent with the fact that the coolest white dwarfs in the Gaia sample displayed in Figure 12 appear to have hydrogen-dominated atmospheres. More definitive conclusions will have to wait until better photometric analyses of the coolest white dwarfs become available, including photometric measurements in the near-infrared.

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