Projecting Cosmological Problems on the Story of Gravitinos

Gongjun Choi,1* and Tsutomu T. Yanagida,1,2†

1 Tsung-Dao Lee Institute, Shanghai Jiao Tong University, Shanghai 200240, China
2 Kavli IPMU (WPI), UTIAS, The University of Tokyo, 5-1-5 Kashiwanoha, Kashiwa, Chiba 277-8583, Japan
(Dated: April 8, 2021)

In this paper, we discuss interesting potential implications for the supersymmetric (SUSY) universe in light of cosmological problems on (1) the number of the satellite galaxies of the Milky Way (missing satellite problem) and (2) a value of the matter density fluctuation at the scale around $8h^{-1}$Mpc ($S_8$ tension). The implications are extracted by assuming that the gravitino of a particular mass can be of help to alleviate the cosmological tension. We consider two gravitino mass regimes vastly separated, that is, $m_{3/2} \simeq 100$eV and $m_{3/2} \simeq 100$GeV. We discuss non-trivial features of each supersymmetric universe associated with a specific gravitino mass by projecting potential resolutions of the cosmological problems on each of associated SUSY models.

I. INTRODUCTION

In spite of the null observation of the sparticles (supersymmetric partners) in the LHC to date, the minimal supersymmetric Standard Model (MSSM) is still considered to be the most promising one among various proposals for a new physics beyond the Standard Model (BSM). Its capability to explain intermediate scalar masses in a natural manner as well as to achieve improved unification of gauge couplings of the Standard Model and the radiatively induced electroweak symmetry breaking (EWSB) should be still fully appreciated. In the particle physics side, as an alternative to the LHC probe, the long standing puzzle of the muon $g-2$ anomaly may provide us with a chance of probing supersymmetry (SUSY) (for example, see Ref. [1]). Then in cosmology side what observational and experimental results can serve as the playground in which indirect ways of probing SUSY can be discussed?

With the curiosity followed by the last question above, in this work we attend to the two problems encountered in $\Lambda$CDM cosmology: missing satellite problem [2, 3] and $S_8$ tension.1 For the galactic scales ($\lesssim 1$Mpc), the observed number of the satellite galaxies of the Milky Way is given by $N_{\text{sat}} \lesssim 60$ whereas $\Lambda$CDM model predicts $N_{\text{sat}} \sim 160$ [4, 5]. Thus this disagreement in $N_{\text{sat}}$ may imply a non-cold dark matter (DM) candidate accompanied by a suppressed matter power spectrum although it still remains possible to find more satellite galaxies in future. On the other hand, when compared to what results from a $\Lambda$CDM model fit to cosmic microwave background (CMB) data, $S_8$ values obtained from weak lensing surveys are smaller by $2-3\sigma$ level [6–9] (see, for example, Ref. [10] for the current status of the problem and plausible resolutions). Although the discrepancy doesn’t seem vastly large, the problem itself might be regarded as a firm one since it concerns the length scale where the linear perturbation theory is applicable. Namely, there seems no other factors within $\Lambda$CDM model on which we blame for the problem as far as the problem cannot be resolved even when systematic errors in local direct measurement of $S_8$ are fully improved and taken into account. More interestingly, $S_8$ appears to be consistent with other sub-galactic small scale issues typified by core-cusp problem [11] and too-big-to-fail problem [12] in that both are about mass deficit at small scales as compared to $\Lambda$CDM model prediction.2

Inspired by theoretical merits of the SUSY and the aforesaid cosmological issues challenging for $\Lambda$CDM model, in this work we pose a question about what we could learn about the supersymmetric universe provided we take the cosmological problems as important clues. It would be remarkable if a certain particle whose presence is inevitable in SUSY models can naturally induce the mass deficit at either the galactic scale or the length scales near $8h^{-1}$Mpc thanks to its properties that SUSY imposes. In this context, we give our special attention to the gravitino that arises in the supergravity (SUGRA) models. As will be shown, we find that gravitinos with masses $m_{3/2} \simeq 100$eV and $m_{3/2} \simeq 100$GeV can address the missing satellite problem and the $S_8$ tension respectively in accordance with two distinct mechanisms when the MSSM is extended by the additional gauge group $U(1)_{B-L}$ and three right-handed neutrinos. Here $B$ and $L$ stand for the baryon and lepton number respectively. Irrelevant to the seasaw mechanism and leptogenesis, one of three right-handed neutrinos will be shown essential for the gravitino to alleviate the cosmological problems.

---

1 The amplitude of matter fluctuation is quantified by $S_8 \equiv \sigma_8^2(\Omega_m/0.3)^{4/3}$ where $\Omega_m$ is the fraction of the total energy density of the universe contributed by DM and baryon, and $\sigma_8$ is the root mean square of matter fluctuations at $8h^{-1}$Mpc.

2 These two problems might be able to be addressed within the stories of the gravitinos we discuss in this work since suppression of the matter power spectrum and decaying nature may be of help to resolve too-big-to-fail problem and core-cusp problem respectively. Furthermore, 100eV gravitino case can be combined with other DM scenario to treat the two issues. Bearing this point in mind, we do not focus on the two problems in the work.
| $U(1)_{B-L}$ | $Z_4$ | $N_u$ | $N_{i=2,3}$ | $N_1$ | $\Phi$ | $\Phi$ | $X$ |
|---|---|---|---|---|---|---|---|
| 10 | 0 | 5 | $H_u$ | $H_d$ | $N_{i=2,3}$ | $N_1$ | $\Phi$ | $\Phi$ | $X$ |
| +1 | 1 | -3 | -2 | +2 | +5 | $+5-10$ | +10 | $+10$ | 0 |

TABLE I. Quantum numbers of chiral superfields for particle contents of the model under $U(1)_{B-L}$ and $Z_4$. For denoting the MSSM matter fields, we borrow the notation of representations of $SU(5)_{GUT}$.

The outline of this paper is as follows. In Sec. II, we discuss the underlying extended MSSM model. This setup is to serve as the common framework in which we discuss two cases differentiated by the distinct gravitino masses. In Sec. III, we discuss the phenomenological strategy for the gravitino with $m_{3/2} \simeq 100$ eV to address the missing satellite problem and resulting properties of the associated SUSY model (case I). In Sec. IV, the parallel discussion for the gravitino with $m_{3/2} \simeq 100$ GeV and the $S_8$ tension is made (case II). Our conclusion is given in Sec. V. In what follows, we would use the notation $N_i$ for both a chiral superfield of the right-handed neutrino and its fermion component, $\tilde{N}_i$ for the right-handed sneutrino (scalar component of $N_i$).

II. BASIC SET-UP

For both of the case I ($m_{3/2} \simeq 100$ eV) in Sec. III and the case II ($m_{3/2} \simeq 100$ GeV) in Sec. IV, we assume an extension of the MSSM as the underlying particle physics model. For the extension, on top of the MSSM gauge symmetry group, we further assume $U(1)_{B-L}$ gauge symmetry and the discrete $Z_4$ symmetry. The later will be assumed global and local in the case I and the case II respectively. As for the particle content of the model, we extend that of the MSSM by introducing three chiral supermultiplets $N_i$ ($i = 1, 2, 3$) for the right-handed neutrinos, two chiral supermultiplets $\Phi$ and $\Phi$ of which condensation of scalar components spontaneously breaks $U(1)_{B-L}$ symmetry. These additional fields are taken to be singlets under the MSSM gauge symmetry group. Especially the three right-handed neutrinos are required to cancel the mixed anomalies of $U(1)_{B-L}$ and $U(1)_{B-L} \times [\text{gravity}]^2$. We show the quantum number assignment of the particle content of the model under $U(1)_{B-L}$ and $Z_4$ in Table I. For denoting the MSSM matter fields, we borrow the notation of representations of $SU(5)_{GUT}$. Only one of the right-handed neutrinos is assumed charged under $Z_4$, which is motivated by phenomenological reasons as discussed in Sec. III and Sec. IV.

With the symmetry and particle contents specified above, the superpotential of the model extending that of the MSSM ($W_{\text{MSSM}}$) is given by

$$W = W_{\text{MSSM}} + \sum_{i=2}^{3} \kappa_{i\alpha} N_i L_{i\alpha} H_u + \sum_{i=2}^{3} \frac{y_i}{2} \Phi N_i + \lambda X (2\Phi - V^2_{B-L}),$$

where $L_\alpha$ and $H_u$ are the chiral superfields for the MSSM lepton and the up-type Higgs $SU(2)_L$ doublets ($SU(2)_L$ indices are contracted via $\epsilon$-tensor), $\kappa_{i\alpha}$, $y_i$, and $\lambda$ are dimensionless coupling constants, $V_{B-L}$ is a $U(1)_{B-L}$ symmetry breaking scale, $\alpha$ is the flavor index ($\alpha = e, \mu, \tau$) and $i$ is the index for the mass eigenstate of the right-handed neutrino. The two right-handed neutrinos $N_2$ and $N_3$ are taken to be responsible for the active neutrino masses via the seesaw mechanism [13–15] and the baryon asymmetry via the primordial leptogenesis [17]. For these two mechanisms to work properly, two right-handed-neutrinos suffice [18].

We end this section by commenting on the mass term for $N_1$ and the discrete $Z_4$ symmetry. After $U(1)_{B-L}$ breaking, the right-handed neutrino $N_1$ can have different masses in forms depending on whether $Z_4$ is assumed to be global or gauged. For the former case, a spurion with an appropriate $Z_4$ charge can be introduced so that $N_1$ may obtain a suppressed mass smaller than those of $N_2$ and $N_3$. The later case, however, never allows for the mass term of $N_1$ and hence $N_1$ remains massless. In the coming sections, for the phenomenological purpose, we shall assume that $Z_4$ is the global symmetry in the case I (Sec. III) so that the right-handed neutrino $N_1$ can have a suppressed mass compared to $N_2$ and $N_3$. In contrast, we assume $Z_4$ is the gauged symmetry in the case II (Sec. IV) and thus $N_1$ becomes massless.

III. 100eV GRAVITINO (CASE I)

A. Phenomenology

In the case where the current DM population is explained as an admixture of cold DM (CDM) and warm DM (WDM), one of sub-galactic small scale problems that $\Lambda$CDM model confronts can be alleviated. To put it concretely, this picture dubbed the mixed DM (MDM) model is expected to result in the smaller number of predicted dwarf satellites ($N_{\text{sat}}$) of the Milky Way in comparison with what $\Lambda$CDM model predicts [19, 20]. Thereby as far as the missing satellite problem is concerned, a MDM model can provide us with theoretical predictions more compatible with experimental observations. The underlying physics is that the presence of WDM component induces suppression of the matter power spectrum on scales below the free-streaming length of WDM. With the mass of WDM ($m_{\text{wdm}}$) and the fraction of DM contributed by WDM ($f_{\text{wdm}} \equiv \rho_{\text{wdm}}/\rho_{\text{dm}}$) related to the free-streaming length and the amount of mat-

3 The importance of $U(1)_{B-L}$ gauge symmetry was clearly explained and stressed in [16].
matter fluctuation suppression respectively, the two quantities can be constrained by requiring that the MDM model’s estimate for $N_{\text{sat}}$ be greater than (or close to) an observed value.

Along this line of reasoning, the constraints on $m_{\text{wdm}}$ and the present temperature ratio between a WDM component and the active neutrino, $T_{\text{wdm},0}/T_{\nu,0}$, were obtained in Ref. [19] based on weak gravitational lensing survey data, CMB data and the observed number of Milky way satellites. We notice that values of $(m_{\text{wdm}}, T_{\text{wdm},0}/T_{\nu,0})$ obeying $m_{\text{wdm}} > 90\text{eV}$ and $T_{\text{wdm},0}/T_{\nu,0} < 0.19$ are not only consistent with weak gravitational lensing survey data and CMB data, but also satisfying the conservative lower bound of $N_{\text{sat}}$ at $2\sigma$ level.\footnote{As for $N_{\text{sat}}$, two different approaches are used in Ref. [19]. The observed value $N_{\text{sat}} = 58 \pm 11$ serves as the lower bound $N_{\text{sat}}$ in a rather conservative approach while a MDM model is strictly required to produce $N_{\text{sat}} = 58 \pm 11$ in more aggressive one.} Furthermore, for values of $(m_{\text{wdm}}, T_{\text{wdm},0}/T_{\nu,0})$ close to $(90\text{eV},0.19)$, it becomes more probable for a MDM model to produce $N_{\text{sat}}$ not far from the observed value.

In accordance with this observation, we consider the MDM scenario where the warm and sub-dominant component is attributed to the gravitino with the mass near $m_{3/2} \simeq 100\text{eV}$ with the purpose of alleviating the missing satellite problem within a SUSY model. This small mass renders the gravitino the lightest supersymmetric particle (LSP) in the model. The corresponding SUSY-breaking scale reads

$$m_{3/2} = \frac{|F|}{\sqrt{3}M_P} \simeq 100\text{eV} \quad \rightarrow |F| \simeq O(10^{11})\text{GeV}^2,$$  

(2)

where $F$ is the order parameter for the SUSY breaking and $M_P \simeq 2.4 \times 10^{18}\text{GeV}$ is the reduced Planck mass. As a thermal relic of the primordial MSSM thermal bath, the gravitino (longitudinal component of the goldstino) becomes free particle when the temperature of the MSSM thermal bath reaches [21]

$$T_d \sim \max \left[ m_{\tilde{g}} , 26\text{GeV} \left( \frac{g_*(T_d)}{100} \right) \frac{1}{1\text{keV}} \left( \frac{m_{3/2}}{100\text{GeV}} \right)^2 \left( \frac{500\text{GeV}}{m_{\tilde{g}}} \right)^2 \right]$$  

(3)

where $m_{\tilde{g}}$ is a gluino mass, $g_*(T_d)$ is the effective number of relativistic degrees of freedom at $T_{\text{MSSM}} = T_d$. With the SUSY-breaking scale specified in Eq. (2), we may assume a low scale gauge-mediated SUSY breaking (GMSB) scenario based on, for example, strongly interacting conformal gauge mediation discussed in [22–24]. Then TeV-scale gluino mass is expected to be dominantly generated by the gauge mediation, which implies TeV scale $T_d$ in accordance with Eq. (3). On account of this, taking into account the MSSM particle contents in the thermal bath, we shall take the approximated value $g_*(T_d) \simeq 230$ hereafter.

In terms of the decoupling temperature, $T_d$, the relic abundance of the thermal gravitino WDM is given by

$$\Omega_{3/2} h^2 = \left( \frac{T_{3/2,0}}{T_{\nu,0}} \right)^3 \frac{m_{3/2}}{94\text{eV}} = \left( \frac{10.75}{g_*(T_d)} \right) \frac{100\text{GeV}}{94\text{eV}},$$  

(4)

where $\Omega_{3/2}$ is the fraction of the critical energy density contributed by the present gravitino energy density, $h$ is a dimensionless present Hubble expansion rate defined via $H_0 = 100h\text{km/Mpc/sec}$ and $T_{3/2,0}$ is the present gravitino temperature. The second equality is obtained by considering the MSSM thermal bath entropy conservation between the times of the gravitino decoupling and the active neutrino decoupling in the absence of any entropy production. Given Eq. (4) and $g_*(T_d) \simeq 230$, it is immediately realized that the gravitino with $m_{3/2} \simeq 100\text{eV}$ makes contribution as large as $\sim 40\%$ to the current DM energy without an entropy production. $g_*(T_d) \simeq 230$ is translated to $T_{3/2,0}/T_{\nu,0} \simeq 0.36$ which is too large to produce satellite dwarfs of the Milky Way as many as the observed value $N_{\text{sat}} = 50 - 60$. Therefore, it is unavoidable to require an early time entropy production provided that the gravitino is to serve as the warm minor component DM in the supersymmetric MDM scenario. We notice that the fraction $f_{3/2} \equiv \rho_{3/2}/\rho_{\text{DM}}$ becomes $2 - 3\%$ for $T_{3/2,0}/T_{\nu,0} \simeq 0.13 - 0.15$ which is not only consistent with the constraint $T_{\text{wdm},0}/T_{\nu,0} < 0.19$ quoted above but satisfying the constraint on $f_{3/2}$ given in Ref. [20] based on CMB+BAO+SAT data sets.\footnote{In Ref. [20], the constraint on the fraction $f_{\text{ncdm}} \equiv \rho_{\text{ncdm}}/\rho_{\text{DM}}$ in the MDM model is presented for the case where $T_{\text{ncdm},0}/T_{\nu,0} = 1$ holds. Thus when reading the constraint on $f_{\text{ncdm}}$ for a given $m_{3/2}$, one needs to refer to the constraint on the fraction $f_{\text{wdm}}$ at $m_{\text{ncdm}} = (T_{3/2,0}/T_{\nu,0})^{-1} m_{3/2}$.} Therefore, we shall take

\begin{figure}[h]
\centering
\includegraphics[width=\linewidth]{PLOT1}
\caption{The ratio of the matter power spectra in the MDM (ACWDW) model with the gravitino and that in the $\Lambda$CDM model obtained based on the analytic formula given in Ref. [25]. The assumed parameters for the plot are $m_{3/2} = 100\text{eV}$ and $T_{3/2,0}/T_{\nu,0} \simeq 0.15$.}
\end{figure}
\(T_{3/2,0}/T_{\nu,0} \simeq 0.13 - 0.15\) (\(f_{3/2} = 2\% - 3\%\)) with a mechanism of the entropy production which is to be discussed in Sec. III B. This choice is expected to yield \(N_{\text{sat}}\) close to \(\sim 60\).

In Fig. 1, we show the ratio of the matter power spectra resulting from the MDM (ACWDM) model and the \(\Lambda\)CDM model. The plot is obtained based on the analytic formula given in Ref. [25] for the parameters \(m_{3/2} = 100\text{eV}\) and \(T_{3/2,0}/T_{\nu,0} \simeq 0.15\) (\(f_{3/2} = 3\%\)). The largest scale at which the matter power spectrum starts to distinguish the MDM (ACWDM) model from \(\Lambda\)CDM model corresponds to the comoving wavenumber [26]

\[
k \simeq 0.5 h\text{Mpc}^{-1} \left(\frac{T_{\nu,0}}{T_{3/2,0}}\right)^{m_{3/2}/1\text{keV}} \left(\frac{0.7}{h}\right) \times \left(1 + 0.085 \ln \left(\frac{T_{\nu,0}}{T_{3/2,0}}\right) \left(\frac{0.1}{\Omega_{m} h^2}\right)^{m_{3/2}/1\text{keV}}\right)^{-1}.
\]

Note that the comoving length scale \(\lambda = 2\pi/k\) is nothing but the present value of the particle horizon. For \(m_{3/2} \simeq 100\text{eV}\) and \(T_{3/2,0}/T_{\nu,0} \simeq 0.13 - 0.15\), we see that the comoving number in Eq. (5) becomes \(k \sim 0.4 h\text{Mpc}^{-1}\) which can be seen in Fig. 1. This suppression of \(P_{\text{ACDM}}\) as compared to \(P_{\Lambda\text{CDM}}\) for \(k \gtrsim 0.4 h\text{Mpc}^{-1}\) is the essential reason why \(N_{\text{sat}}\) produced by the model is close to \(\sim 60\), but far from \(\Lambda\)CDM prediction of \(N_{\text{sat}} \sim 160\).

**B. Model I**

In Sec. III A, we discussed how the gravitino with \(m_{3/2} \simeq 100\text{eV}\) and \(T_{3/2,0}/T_{\nu,0} \simeq 0.13 - 0.15\) can help us grasp a better understanding for the observed value of \(N_{\text{sat}} \simeq 50 - 60\) than that based on the \(\Lambda\)CDM model. This intriguing scenario, however, turns out to necessarily ask for a certain mechanism to dilute relic abundance of the thermal gravitino since the expected value of \(T_{3/2,0}/T_{\nu,0}\) in the standard cosmological history (without any entropy production) is equal to \((10.75/g_{*}(T_{\nu})^{1/3} \simeq 0.36\) with \(g_{*}(T_{\nu}) \approx 230\). Hence, it becomes an important issue whether there can be a natural way within a SUGRA model to achieve a desired amount of the entropy production. In this section, we present a mechanism in which the requisite entropy production can occur via the decay of the lightest right-handed neutrino \(N_1\) and its superpartner sneutrino \(\tilde{N}_1\). The basic set-up of the model is presented in Sec. II.

As was mentioned in the last paragraph of Sec. II, for the case I, we assume a discrete \(Z_4\) global symmetry and a spurion \(\epsilon\) with \(Z_4\) charge \(-1\). Then aside from the terms given in Eq. (1) we have the following new terms added to the superpotential of the model

\[
W \supset \kappa_{1a} \epsilon N_1 L_a H_u + \frac{y_1}{2} e^2 \Phi N_1 N_1.
\]

On the spontaneous breaking of \(U(1)_B-L\) by the acquisition of a vacuum expectation value (VEV) of \(\Phi\) and \(\overline{\Phi} (\langle \Phi \rangle = \langle \overline{\Phi} \rangle \equiv V_{B-L}/\sqrt{2})\), \(N_i (i = 1 - 3)\) obtain the masses \(m_1 = (y_{11}^2 V_{B-L})/\sqrt{2}\) and \(m_{i=2,3} = (y_{i=2,3} V_{B-L})/\sqrt{2}\). This shows that \(m_1 \ll m_2, m_3\) can be the case for \(\epsilon \ll 1\). As an exemplary value of \((U(1)_B-L)\) breaking scale leading to a successful explanation of the active neutrino masses via the seesaw mechanism, we take \(V_{B-L} \sim 10^{15}\text{GeV}\) close to the GUT scale.

For a reheating temperature satisfying \(T_{\text{RH}} > m_2, m_3\), the out-of-equilibrium decay of \(N_2\) and \(N_3\) (and their superpartners \(\tilde{N}_2\) and \(\tilde{N}_3\)) produces a primordial lepton asymmetry (thermal leptogenesis). On the other hand, both of \(N_1\) and \(\tilde{N}_1\) are produced mainly via the \(s\)-channel MSSM particle scattering mediated by the gauge boson and the gaugino of \((U(1)_B-L)\). The five relevant dominant scattering events are (1) the scattering among the SM fermions to produce either a pair of \(N_1\)s or a pair of \(\tilde{N}_1\)s (\(f_{\text{SM}} + f_{\text{SM}} \rightarrow N_1(N_1) + N_1(N_1)^\ast\)) (2) the scattering among the MSSM sfermions to produce either a pair of \(N_1\)s or a pair of \(\tilde{N}_1\)s (\(f_{\text{SM}} + f_{\text{SM}} \rightarrow N_1(N_1) + N_1(N_1)^\ast\)) and (3) the scattering between a SM fermion and its superpartner to produce \(N_1\) and \(\tilde{N}_1\) (\(f_{\text{SM}} + f_{\text{SM}} \rightarrow N_1 + \tilde{N}_1\)). After the gauge boson and the gaugino of \((U(1)_B-L)\) are integrated-out, the scattering rates for these five processes are all given by \(\Gamma \propto T^5/V_{B-L}^2\). Thus, these scattering events are expected to generate equal primordial relic abundances of \(N_1\) and \(\tilde{N}_1\). The estimate of the comoving number density of \(N_1\) generated by the first process is given by [27]

\[
Y_s \equiv \frac{n_1}{s} \sim \frac{n_{\text{SM}} \Gamma(f_{\text{SM}} + f_{\text{SM}}^\ast \rightarrow 2N_1s)/H}{T_{\text{RH}}^3} \cdot (3 \times 10^{-3}) \left(\frac{g_*(T_{\text{RH}})}{230}\right)^{-2} \times \left(\frac{V_{B-L}}{10^{13}\text{GeV}}\right)^{-4} \left(\frac{T_{\text{RH}}}{5 \times 10^{13}\text{GeV}}\right)^3,
\]

where \(n_1 (n_{\text{SM}})\) is the number density of \(N_1 (\text{SM fermions})\), \(H\) is the Hubble expansion rate, and \(s\) is the entropy density of the MSSM thermal bath. Because of \(s \propto T^3, n_{\text{SM}} \propto T^3\) and \(H \propto T^2\), it can be easily seen that \(N_1\) production is most efficient at the reheating era and hence we evaluate \(Y_s\) at \(T = T_{\text{RH}}\) in Eq. (7). Therefore when taken into account together, the five scattering processes eventually give rise to the following comoving number densities \(Y_1\) and \(\tilde{Y}_1\) of \(N_1\) and \(\tilde{N}_1\) respectively in terms of \(Y_s\)

\[
Y_1 \simeq \frac{5}{2} Y_s, \quad \tilde{Y}_1 \simeq \frac{5}{2} Y_s.
\]

After the non-thermal production of \(N_1\) and \(\tilde{N}_1\), the above comoving number densities in Eq. (8) are conserved until the time is reached when the decay rates \(\Gamma(N_1 \rightarrow H_u + L)\) and \(\Gamma(\tilde{N}_1 \rightarrow L(\tilde{L}) + \tilde{H}_u(H_u))\) become comparable.
to the Hubble expansion rate. Note that both $N_1$ and $\tilde{N}_1$ can decay thanks to the first term in Eq. (6). Neglecting the small mass difference between $N_1$ and $\tilde{N}_1$ due to the SUSY-breaking, we expect the time when the decay of $N_1$ and $\tilde{N}_1$ takes place to be close to one another, which is estimated by

$$T_{\text{decay}} \simeq 3 \left( \frac{\kappa_{10} \epsilon}{10^{-16}} \right) \left( \frac{m_1}{10^{14} \text{GeV}} \right)^{1/2} \text{GeV}$$

$$\simeq 0.8 \left( \frac{\kappa_{10} \epsilon}{10^{-16}} \right) \left( \frac{y_{11}^2}{10^{-12}} \right)^{1/2} \left( \frac{V_{B-L}}{10^{15} \text{GeV}} \right)^{1/2} \text{GeV},$$

where $\kappa_{10}$ denotes the largest one of the three Yukawa coupling constants.

In the case where the amount of the additionally produced entropy through the decay of $N_1$ and $\tilde{N}_1$ is so large as to be greater than the entropy of the existing MSSM thermal bath, the relic abundance of the gravitino is subject to the dilution.\(^7\) We may quantify the amount of the dilution by $\Delta \equiv s'/s$ where $s'$ ($s$) is the entropy density of the universe after (prior to) $N_1$ and $\tilde{N}_1$ decay. Accordingly, the primordial abundance in Eq. (4) is subject to the modification to become

$$\Omega_{3/2}^{(\text{after})} h^2 = \frac{\Omega_{3/2} h^2}{\Delta} = \frac{1}{\Delta} \left( \frac{T_{3/2,0}}{T_{c,0}} \right)^3 \left( \frac{m_{3/2}}{94 \text{eV}} \right).$$

This means the temperature ratio decreases by $\Delta^{1/3}$. So in order to have the temperature ratio $\sim 0.13 - 0.15$ today, $\Delta$ is required to be $\sim 10 - 20$.\(^8\)

Now by using $\Delta = s'/s \simeq (4/3)(m_1(Y_1 + \tilde{Y}_1)/T_{\text{decay}})$ and $m_1 = (y_{11}^2 V_{B-L})/\sqrt{2}$, and referring to Eq. (7), Eq. (8) and Eq. (9) we can estimate $\Delta$ as\(^9\)

$$\Delta \simeq 19 \left( \frac{y_{11}^2}{10^{-12}} \right)^{1/2} \left( \frac{\kappa_{10} \epsilon}{10^{-16}} \right)^{-1} \left( \frac{g_*(T_{\text{RH}})}{230} \right)^{-2} \times \left( \frac{V_{B-L}}{10^{15} \text{GeV}} \right)^{1/2} \left( \frac{T_{\text{RH}}}{5 \times 10^{13} \text{GeV}} \right)^3.$$  

(11)

Observing Eq. (11), it is realized that the required $\Delta \sim 10 - 20$ can be readily accomplished indeed for the values of quantities used for the normalization.\(^10\)

Therefore, we showed that the desired entropy production can be successfully accommodated within the model thanks to the decay of the lightest right-handed neutrino which is necessary for gauging $U(1)_{B-L}$, but not necessary for the successful operation of the seesaw mechanism and the (thermal) leptogenesis. As an example of a parameter set yielding $\Delta = 10 - 20$, we may take $(m_1, T_{\text{RH}}, V_{B-L}, \kappa_{10} \epsilon) = (10^3 \text{GeV}, 5 \times 10^{13} \text{GeV}, 10^{15} \text{GeV}, 10^{-10})$ with $g_*(T_{\text{RH}}) = 230 - 300$.

For consistency of the scenario, we notice that it remains to be checked that the gravitino decouples from the MSSM thermal bath before the summed energy density of the non-relativistic $N_1$ and $\tilde{N}_1$ dominates the energy budget of the universe, which was the assumption of deriving Eq. (3). By comparing the energy density of the MSSM thermal bath and that of $N_1$ and $\tilde{N}_1$, we obtain the following temperature below which the energy of the universe is dominated by that of $N_1$ and $\tilde{N}_1$

$$T_c = \frac{4}{3} m_1(Y_1 + \tilde{Y}_1)$$

$$\simeq 15 \text{GeV} \left( \frac{y_{11}^2}{10^{-12}} \right) \left( \frac{g_*(T_{\text{RH}})}{230} \right)^{-2} \left( \frac{V_{B-L}}{10^{15} \text{GeV}} \right)^3 \times \left( \frac{T_{\text{RH}}}{5 \times 10^{13} \text{GeV}} \right)^3.$$  

(12)

Thus for the exemplary parameter values we are targeting, we see that $T_c \simeq 15 \text{GeV}$ is obtained and thus confirmed to be smaller than $T_d \sim m_3$ in Eq. (3).

Now for consistency of the scenario, we finalize the analysis by imposing the following generic condition

$$T_d > T_c > T_{\text{decay}} > 10 \text{MeV},$$  

(13)

where the last inequality was introduced to guarantee the successful big bang nucleosynthesis (BBN). Note that the second inequality was originally intended to demand that $T_c$ be greater than the MSSM thermal bath when $N_1$ and $\tilde{N}_1$ decay. But, since $T_{\text{MSSM}}$ differs from a temperature of the thermal bath created from the $N_1$ and $\tilde{N}_1$ decay by an $O(1)$ factor, it suffices to require $T_c > T_{\text{MSSM}}$. In Fig. 2, we show the parameter space of $(y_{11}^2, \kappa_{10} \epsilon)$ satisfying Eq. (13) as the green shaded region. Shown together as the red (blue) dashed line is the set of the points in the parameter space yielding $\Delta = 10$ ($\Delta = 20$). For $V_{B-L} = 10^{15} \text{GeV}$, the comparison of the panel (a) and (b) shows that $T_{\text{RH}}$ as large as $\sim 5 \times 10^{13} \text{GeV}$ is preferred from the model building point of view given the different suppression of $y_{11}^2$ and $\kappa_{10} \epsilon$ by different powers of $\epsilon < 1$.

On the other hand, in comparison of the panels (a), (c) and (d), we see that the consistent parameter space exists for each different $V_{B-L}$ as far as the ratio $V_{B-L}/T_{\text{RH}}$ is similar to the case of the panel (a).

We end this section by commenting on the suppression of the couplings in Eq. (6) made by the spurion $\epsilon$. Observing that each $N_1$ is accompanied by a single $\epsilon$, one may regard $\epsilon$ as the analog of the normal-
FIG. 2. Parameter space of $(y_1\epsilon^2, \kappa_1\alpha \epsilon)$ for a fixed $V_{B-L}$ and a fixed $T_{RH}$. The left and right panels show how the consistent parameter space changes when $T_{RH}$ changes. The green shaded region corresponds to the parameter space which satisfies the condition in Eq. (13). Each of the dashed red and blue is the collection of the points yielding $\Delta = 10$ and 20 respectively.

...
IV. 100GeV GRAVITINO (CASE II)

A. Phenomenology

For the case where a CDM candidate is able to decay to a massless and a massive decay product, such a decay has been employed to address various cosmological issues including small scale problems for the sub-galactic scale [30, 31], $S_8$ tension [32, 33] and $H_0$ tension [34–38]. The framework of this so-called “decaying dark matter (DDM)” is to assume a mother CDM which decays to a massless radiation and a massive daughter WDM. Given this set-up, it is unavoidable for the decay products to obtain a non-vanishing three momentum. The decay process is parametrized by a life time ($\Gamma^{-1}_{\text{cdm}}$) of the mother CDM and a fraction ($\xi$) of the rest mass of the CDM transferred to the massless decay product. As a result, the four momenta of the massless and the massive decay products are given by $p_n = (\xi m_{\text{cdm}}, \vec{p}_n)$ and $p'_n = ((1 - \xi)m_{\text{cdm}}, -\vec{p}_n)$ respectively where $m_{\text{cdm}}$ is a mass of the mother CDM. In addition, the dispersion relation of the massive decay product leads to $(m_{\text{wdm}}/m_{\text{cdm}}) = \sqrt{1 - 2\xi}$ with $m_{\text{wdm}}$ a mass of the massive decay product.

Recently, a study invoking the DDM framework to address $S_8$-tension was conducted in Refs. [33, 42] through a Monte Carlo Markov Chain (MCMC) against up-to-date CMB, BAO and uncalibrated SNIa data. Interestingly, it was argued that with the inclusion of the prior on $S_8$ value, $S_8 \simeq 0.77$ was obtained within the DDM scenario with ($\Gamma^{-1}_{\text{cdm}}, \xi) = (56\text{Gyrs}, 7 \times 10^{-3})$ (best-fit), resolving the tension. A compelling point in the result of the analysis is that the decrease in $S_8 \simeq 0.77$ as compared to what is inferred from $\Lambda$CDM model is mostly induced by decrease in $\sigma_8$ with $\Omega_m$ unaffected, which is better in making agreement with the BOSS galaxy clustering constraint on ($\Omega_m, \sigma_8$) where the degeneracy between the two gets broken [43].

The physics behind this resolution is nothing but the suppression of the growth of the matter fluctuation at $k \sim 0.1-1h\text{Mpc}^{-1}$ caused by the free-streaming of the massive decay product when compared to the $\Lambda$CDM model prediction. Namely, the way for causing the suppression is identical to that of a usual WDM scenario. Yet, the DDM scenario is distinguished from the typical WDM scenario in that the time-dependency is found in both of the amount of the suppression and the cut-off scale in the matter power spectrum [33]. For that reason the suppression of the linear matter power spectrum can be delayed until the time as late as $z \sim 2 - 3$ for a large enough $\Gamma^{-1}_{\text{cdm}}$ and a small enough $\xi$. In that case, the DDM scenario becomes closer to $\Lambda$CDM model in its effect on CMB power spectrum with $\ell \gtrsim O(10)$ by avoiding to cause a significant early integrated Sachs-Wolfe effect and in its prediction for the growth rate ($f\sigma_8 \equiv (d\ln \delta_m/d\ln a)\sigma_8$) for the time $z \gtrsim 1$. With these distinctions between the DDM and the WDM scenarios, the potential significant late integrated Sachs-Wolfe effect on the CMB anisotropy power spectrum for low $\ell$ regime ($\ell \sim 10$) is expected to be caused by the late time decay of the DDM and it further distinguishes the DDM and the usual WDM scenarios.

Now inspired by the above phenomenologically compelling resolution to $S_8$, a natural question from the particle physics side is whether there exists a well-motivated BSM model accommodating the DDM scenario addressing the $S_8$ tension. Obviously one challenging point in answering the question concerns the non-trivial mass spectrum of the three ingredients. We notice that the best-fit value of $\xi \simeq 7 \times 10^{-3}$ is converted to 1% mass difference between the mother CDM and the massive decay product. In the next section, we demonstrate that the model presented in Sec. II can naturally realize the phenomenological DDM scenario alleviating the $S_8$ tension when the assumed $Z_4$ symmetry is the gauge symmetry.

B. Model II

With the basic set-up presented in Sec. II and the discrete symmetry $Z_4$ specified as the gauged one, the model II presented in this section does not have the additional superpotential terms given in Eq. (6). Thus, even after the spontaneous breaking of $U(1)_B-L$ at the energy scale around $V_{B-L} \sim 10^{10}\text{GeV}$, the right-handed neutrino $N_1$ still remains massless. Because of this, sneutrino $\tilde{N}_1$ also remains massless until the SUSY-breaking takes place.

Given this set-up, we find that this situation is well suited for realizing the DDM scenario discussed in Sec. IV A. $N_1$ is expected to obtain a mass once the SUSY-breaking takes place and is mediated to $\tilde{N}_1$. Now the soft mass of $N_1$ can be very close to a mass of gravitino provided it is dominantly generated by the gravity mediation contribution. Remarkably since $\tilde{N}_1$ is the SM gauge singlet, even if we are to explain heavy enough mass spectrum for the sparticles in the MSSM consistent with the null observation of SUSY particles in the LHC by relying on a gauge mediation, the soft-mass of $\tilde{N}_1$ can be easily dominated by the gravity mediation contribution provided we assume messengers are singlets under $U(1)_{B-L}$. Along this line of reasoning, we see that the coupling between the gravitino, $N_1$ and $N_1$ in our model can be an excellent candidate of the concrete particle physics example realizing the DDM scenario resolving the $S_8$ tension.\[^{14}\]

\[^{11}\] See also Refs. [39–41] for the criticism on the DDM solution to the $H_0$ tension.

\[^{12}\] This model, however, cannot resolve the Hubble tension [33].

\[^{13}\] $Z_4$ in our model does not suffer from any gauge anomaly.

\[^{14}\] Another possibility is to consider the MSSM extended by an
For mapping to our model the phenomenological best-fit values of \((\Gamma^{-1}_{\text{obs}}, \xi) = (56\text{Gyrs}, 7 \times 10^{-3})\), we refer to the rate of the gravitino decay to \(\tilde{N}_1\) and \(N_1\) [44]

\[
\Gamma(\tilde{G}_\mu \to \tilde{N}_1 + N_1) = \frac{m_{3/2}^3}{102\pi M_P^2} \times \left[ 1 - \left(\frac{m_1}{m_{3/2}}\right)^2 \right] \left[ 1 - \left(\frac{m_1}{m_{3/2}}\right)^2 \right]^3 \ .
\]  
(14)

where \(m_1\) is the soft mass of \(\tilde{N}_1\). The replacement of the mass ratio \(m_1/m_{3/2}\) with \(\sqrt{1-2\xi}\) and the substituting the best-fit value in Eq. (14) yield \(m_{3/2} \approx 216\text{GeV}\). The SUSY-breaking scale read from this \(m_{3/2}\) amounts to

\[
m_{3/2} = \frac{|F|}{\sqrt{3}M_P} \approx 216\text{GeV} \ ightarrow \ |F| \approx \mathcal{O}(10^{21})\text{GeV}^2 \ .
\]  
(15)

For the gravitino with \(m_{3/2} \approx 216\text{GeV}\) to explain the current DM relic density, we notice that the high enough reheating temperature is necessarily required as shown below. In Ref. [45], the careful analysis of the collision term of the Boltzmann equation for the gravitino was made, which gives rise to the thermal gravitino relic abundance

\[
\Omega_{3/2}h^2 = 0.21 \left(\frac{T_{\text{RH}}}{10^{10}\text{GeV}}\right) \left(\frac{100\text{GeV}}{m_{3/2}}\right) \left(\frac{m_\tilde{g}(\mu)}{1\text{TeV}}\right)^2 \ ,
\]  
(16)

where \(m_\tilde{g}(\mu)\) is the running gluino mass. For an exemplary gluino mass \(m_\tilde{g}(\mu) \approx 3\text{TeV}\) and \(\Omega_{\text{DM}}h^2 \approx 0.12\), we see from Eq. (16) that \(T_{\text{RH}} \approx 10^9\text{GeV}\) is required for the gravitino DM with \(m_{3/2} \approx 216\text{GeV}\). The fact that this possibility with \(T_{\text{RH}} \approx 10^9\text{GeV}\) and \(m_{3/2} \approx 100\text{GeV}\) is consistent with the thermal leptogenesis was pointed out in Ref. [48]. Having \(T_{\text{RH}} \approx 10^9\text{GeV}\) and \(V_{3/2-1} \sim 10^{15}\text{GeV}\) in mind, it can be easily seen that production of both \(N_1\) and \(\tilde{N}_1\) is suppressed since the interaction rate of the main production channels (see discussion in Sec. III) \(\Gamma \sim T^5/V_{3/2-1}^4\) never has a chance to exceed the Hubble expansion rate during the radiation-dominated era. Therefore, the gravitino being the sole DM candidate cannot be spoiled by \(\tilde{N}_1\).

On the other hand, the gravitino DM with \(m_{3/2} \approx 216\text{GeV}\) is subject to two potentially problematic issues: the model might cause deviation of the light element amount and formation history from the standard BBN scenario and additional unwanted relic abundance of the non-thermal gravitino DM [21]. To avoid these dangers, we focus on the relatively simple case where the next-to-lightest SUSY particle (NLSP) is the gluino \((\tilde{g})\).\(^{17}\) To this end, we may consider the gauge mediation model with a pair of messengers \((\Psi, \overline{\Psi})\) transforming as \((5, \overline{5})\) under \(SU(5)\) with their mass satisfying \(M_{\text{mess}}^2 \gg F\).\(^ {18}\) We assume that the colored \(SU(2)_{L}\) singlet messengers \((D, \overline{D})\) are heavier than \(SU(2)_{L}\) doublet messengers \((L, \overline{L})\). Then, thanks to the annihilation driven by the strong interaction, the gluino relic comoving number density \((n_\tilde{g}/s)\) resulting from the freeze-out process is expected to be much smaller than \(\mathcal{O}(10^{-12})\) which is the comoving number density of 100GeV gravitino DM. Note that even for the case where the neutralino is the DM candidate as LSP and coannihilation with the gluino and gluino-gravitino bound state formation are taken into account, the comoving number density is still given as \(\mathcal{O}(10^{-14})\) [50]. Therefore, the amount of the potential non-thermal gravitinos produced from the gluino decay is negligible.

Regarding the successful BBN, the gluino mass should be constrained from the requirement that its decay takes place before \(T_{\text{MSSM}} \approx 10\text{MeV}\) is reached. From the comparison of the decay rate of gluino to the Hubble expansion rate below,

\[
\Gamma(\tilde{g} \to g + \tilde{G}_\mu) \approx \frac{1}{48\pi} \frac{m_\tilde{g}^5}{m_{3/2}^2 M_P^2} \approx \frac{T^2}{M_P} \approx \mathcal{H} \Rightarrow m_\tilde{g} = 10 - 20\text{TeV} \quad \text{for} \quad T_{\text{MSSM}} = 10\text{MeV},
\]  
(17)

we see that the mass of messengers \((D, \overline{D})\) is required to satisfy \(M_{\text{mess}} \lesssim 10^{15}\text{GeV}\). Once this condition is met, the potential problematic hadro-dissociation effects caused by the decay products of \(\tilde{g}\) could be avoided and the success of the BBN procedure can be guaranteed. In sum, the model can successfully avoid the so-called cosmological gravitino problem as far as we adopt the gauge mediation model with the gluino NLSP to explain SUSY-breaking soft masses and assume \(m_\tilde{g} \gtrsim 10\text{TeV}\).

Next, with the successful identification of the vertex for explaining the phenomenologically required specific mass spectra, we are still left with the question for the tiny difference between \(m_{3/2}\) and \(m_1\). In other words, we wonder if there is a theoretically plausible way to account for \(m_1^2 \approx (0.98 - 0.99) \times m_{3/2}\). To this end, we consider the possibility where there exists a single UV

\(^1\) Note that more precisely speaking, of course, \(\tilde{g}\) is the next-to-NLSP (NNLSP) because \(\tilde{N}_1\) and the gravitino are lighter than \(\tilde{g}\). Nevertheless, since a very tiny amount of \(N_1\) is produced at the late universe after recombination, we treat \(\tilde{g}\) as the NLS in our discussion.

\(^2\) The inequality \(M_{\text{mess}}^2 \gg F\) is required for the stability of the SUSY-breaking vacuum to last for a time longer than the age of the universe [49] in a perturbative gauge mediation model.
cutoff in the theory and it is not $M_P$, but $\Lambda_{UV} \simeq 4\pi M_P$ (higher cutoff) [51, 52]. As an example, this could be the case if we assume a hidden interaction under which the SUSY-breaking field $Z$ is strongly coupled ($g \sim 4\pi$), but the MSSM fields are weakly-coupled ($h \sim 1$) at the energy scale near $M_P$. Then in accordance with the Naive dimensional analysis (NDA) [53, 54], the effective Kähler potential for the energy scale below $\Lambda_{UV} \simeq 4\pi M_P$ can be written as\(^{19}\)

$$K_{\text{eff}} = \frac{\Lambda_{UV}^2}{g^2} K^{(Z,Y)}_{\text{eff}} \left[ \left( \frac{gZ}{\Lambda_{UV}} \right) \left( \frac{gZ}{\Lambda_{UV}} \right)^\dagger \left( \frac{hY}{\Lambda_{UV}} \right) \left( \frac{hY}{\Lambda_{UV}} \right)^\dagger \right] + \frac{\Lambda_{UV}^2}{h^2} K^{(Y)}_{\text{eff}} \left[ \left( \frac{hY}{\Lambda_{UV}} \right) \left( \frac{hY}{\Lambda_{UV}} \right)^\dagger \right],$$

(19)

where $Y$ collectively denotes chiral superfields of the MSSM sector, and couplings in $K^{(Z,Y)}_{\text{eff}}$ and $K^{(Y)}_{\text{eff}}$ are order 1.

Now from the Eq. (19), it can be seen that for those sfermions whose masses are dominantly generated by the gravity mediation, the universal soft mass up to a leading correction by higher dimensional operators can be given by

$$m_f^2 \simeq (1 - \frac{1}{g^2}) \times m_{3/2}^2,$$

(20)

if an $O(1)$ coefficient of the operator $O \sim (Z^4 Y^4)/(\Lambda_{UV}^2)$ is negative. Now that the sneutrino mass $\tilde{N}_1$ is dominantly generated by the gravity mediation in our model, its required mass $m_{\tilde{N}_1}^2 \simeq (0.98 - 0.99) \times m_{3/2}^2$ could be understood to be stemming from the large UV-cutoff $\Lambda_{UV} \simeq 4\pi M_P$.

We finalize this section by commenting on a feature of the higher cutoff hypothesis based on Refs. [51, 52]. As shown in the above, in the model building the higher cutoff hypothesis can be invoked when one needs a justification for suppression of higher dimensional operators including the SUSY-breaking field $Z$. These operators might result in operators multiplied by $m_{3/2}$ or $m_{3/2}^2$. Then the higher cutoff hypothesis can naturally explain a dimensionless coefficient of order $(4\pi)^{-1}$ or $(4\pi)^{-2}$. As an example of the application of this hypothesis, for a SUSY-model where soft masses are dominantly generated by the gravity-mediation, the problematic potential

\[^{19}\text{Graviton being assumed to be weakly coupled under the hidden interaction, the NDA enables us to write down the Einstein-Hilbert action as}\]

$$S = \frac{\Lambda_{UV}^4}{g^4} \int d^4x \left( \frac{h^2 R}{2\Lambda_{UV}^2} \right) = M_P^2 \int d^4x \left( \frac{R}{2} \right),$$

(18)

where $R$ is the Ricci scalar. We may recognize this point as the origin of $\Lambda_{UV} \simeq 4\pi M_P$ in the higher cutoff hypothesis.

V. CONCLUSION

In this paper, we studied several implications on SUGRA models with the assumption that the missing satellite problem and $S_8$ tension can be alleviated by presence of gravitinos. To this end, we took as the basic common set-up the MSSM extended by $U(1)_{B-L}$ gauge symmetry, the discrete $Z_4$ symmetry and chiral supermultiplets of $\Phi, \Psi$ and three right-handed neutrinos $N_i$ ($i = 1 - 3$) as shown in Table. I. We considered the case where only two $N_i$ ($i = 2, 3$) are responsible for the seesaw mechanism and the leptogenesis. The remaining lightest right-handed neutrino $N_1$, as the only field charged under $Z_4$, was invoked for different purposes depending on the cosmological problems at hand. This varied uses of $N_1$ were achieved by relying on the different nature of $Z_4$, i.e. global or gauged.

For the missing satellite problem, we attend to the mixed DM scenario where DM population consists of two components and the minor component is a WDM with 100eV scale mass and the fraction $2 - 3\%$. Projecting the possibility on the low SUSY-breaking scenario with 100eV gravitino LSP, we figured out that it is necessary to have a mechanism to dilute the gravitino relic abundance. We demonstrated that the decay of the lightest right-handed neutrino $N_1$ and sneutrino $\tilde{N}_1$ can induce the required dilution provided $Z_4$ is chosen to be the global symmetry. For the scenario to have a consistent sparticle mass spectrum, a low scale gauge mediation is demanded for the SUSY-breaking mediation to the visible sector. Since the entropy production is the requisite for this possibility, it might be interesting to ask what cosmological (or astrophysical) probes can investigate whether there has ever been any time of an entropy production causing a temporary increase in the evolution of the Hubble expansion rate. If there is, such a probe can be a way to test the 100eV gravitino scenario we propose in this work. In addition, the early matter-dominated era before BBN in our model may enhance the production of the primordial black holes if there are.\(^{20}\)

For the $S_8$ tension, we attended to the decaying DM solution with the parameters $(\Gamma^{-1}_{\text{cdm}}, \xi) = (56\text{Gyr}, 7 \times 10^{-3})$. We find that our model offers the 100GeV gravitino as the excellent candidate for the DDM if we take $Z_4$ as the gauge symmetry and the higher cutoff hypothesis. We pointed out that if the mass of $N_1$ can be dominantly generated by the gravity mediation, $\tilde{N}_1$ can serve as the excellent candidate for the massive warm decay product. Combined with the question about a consistent sparticle spectrum, this possibility requires a gauge

\[^{20}\text{We thank K. Inomata for useful discussion for this point.}\]
mediation model incorporating messengers as heavy as \( \sim 10^{15}\text{GeV} \) and charged only under the MSSM gauge symmetry group. Notably we proposed the theoretically justified reasoning for the phenomenologically required fine-tuning for the mass difference between the gravitino and \( \tilde{N}_1 \) by relying on the higher cutoff hypothesis.

In sum, we projected the known resolutions to two cosmological problems on SUSY models to obtain potential interesting properties of the universe with SUSY. Both of the cases were shown to be consistent with the thermal leptogenesis with \( T_{\text{RH}} \gtrsim 10^6\text{GeV} \). Although each mass was associated with each individual problem, it does not necessarily exclude the possibility that all the aforementioned problems can be addressed with a single \( m_{3/2} \); for 100eV gravitino case, it remains unclear that physics other than DM (e.g., baryonic effect) can alleviate another galactic scale problems. And also for 100GeV case, the decaying DM nature may be of help to alleviate the core-cusp problem and the induced suppression in the matter power spectrum is expected to yield \( N_{\text{aux}} \) definitely smaller than \( \Lambda_{\text{CDM}}'s \) prediction. Another interesting possibility could be the presence of “multi effective gravitinos” [55] and ours are two of them such that DM population may consist of \( 97 \sim 98\% \) of decaying cold 100GeV gravitinos and 2 \sim 3\% of 100eV gravitinos.

**ACKNOWLEDGMENTS**

T. T. Y. is supported in part by the China Grant for Talent Scientific Start-Up Project and the JSPS Grant-in-Aid for Scientific Research No. 16H02176, No. 17H02878, and No. 19H05810 and by World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan.

[1] T. Moroi, Phys. Rev. D 53, 6565 (1996). [Erratum: Phys.Rev.D 56, 4424 (1997)], arXiv:hep-ph/9512396.
[2] B. Moore, S. Ghigna, F. Governato, G. Lake, T. R. Quinn, J. Stadel, and P. Tozzi, Astrophys. J. Lett. 524, L19 (1999), arXiv:astro-ph/9907411.
[3] S. Y. Kim, A. H. G. Peter, and J. R. Hargis, Phys. Rev. Lett. 121, 211302 (2018), arXiv:1711.06267 [astro-ph.CO].
[4] A. Schneider, JCAP 04, 059 (2016), arXiv:1601.07553 [astro-ph.CO].
[5] E. Polisensky and M. Ricotti, Phys. Rev. D 83, 043506 (2011), arXiv:1004.1459 [astro-ph.CO].
[6] C. Heymans et al., Mon. Not. Roy. Astron. Soc. 432, 2433 (2013), arXiv:1303.1808 [astro-ph.CO].
[7] T. Abbott et al. (DES), Phys. Rev. D 98, 043526 (2018), arXiv:1708.01530 [astro-ph.CO].
[8] C. Hikage et al. (HSC), Publ. Astron. Soc. Jap. 71, Publications of the Astronomical Society of Japan, Volume 71, Issue 2, April 2019, 43, https://doi.org/10.1093/pasj/psz010 (2019), arXiv:1809.09148 [astro-ph.CO].
[9] H. Hildebrandt et al., Astron. Astrophys. 633, A60 (2020), arXiv:1812.06076 [astro-ph.CO].
[10] E. Di Valentino et al., (2020), arXiv:2008.11285 [astro-ph.CO].
[11] B. Moore, T. R. Quinn, F. Governato, J. Stadel, and G. Lake, Mon. Not. Roy. Astron. Soc. 310, 1147 (1999), arXiv:astro-ph/9903164.
[12] M. Boylan-Kolchin, J. S. Bullock, and M. Kaplinghat, Monthly Notices of the Royal Astronomical Society: Letters 415, L40–L44 (2011).
[13] T. Yanagida, Proceedings: Workshop on the Unified Theories and the Baryon Number in the Universe: Tsukuba, Japan, February 13-14, 1979, Conf. Proc. C7902131, 95 (1979).
[14] M. Gell-Mann, P. Ramond, and R. Slansky, Supergravity Workshop Stony Brook, New York, September 27-28, 1979, Conf. Proc. C790927, 315 (1979), arXiv:1306.4669 [hep-th].
[15] P. Minkowski, Phys. Lett. 67B, 421 (1977).
