A Search for Lorentz Invariance and CPT Violation with the MINOS Far Detector

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We searched for a sidereal modulation in the MINOS far detector neutrino rate. Such a signal would be a consequence of Lorentz and CPT violation as described by the Standard-Model Extension framework. It also would be the first detection of a perturbative effect to conventional neutrino mass oscillations. We found no evidence for this sidereal signature and the upper limits placed on the magnitudes of the Lorentz and CPT violating coefficients describing the theory are an improvement by factors of $20 - 510$ over the current best limits found using the MINOS near detector.

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Neutrinos have provided many crucial insights into particle physics, including the existence of physics beyond the minimal Standard Model (SM) with the detection of neutrino oscillations [1, 2]. Because oscillations are interferometric in nature, they are sensitive to other indicators of new physics. Such indicators include potential small amplitude signals persisting to the current epoch whose origin is a fundamental theory that unifies quantum physics and gravity at the Planck scale, $m_p \approx 10^{19}$ GeV. One promising category of Planck-scale signals is the violation of the Lorentz and CPT symmetries that are central to the SM and General Relativity. The Standard Model Extension (SME) is the comprehensive effective field theory that describes Lorentz (LV) and CPT violation (CPTV) at attainable energies [2].

The SME predicts behaviors for neutrino flavor change that are different from conventional neutrino oscillation theory. The probability of flavor change in the SME depends on combinations of $L$, the distance traveled by the neutrino, and the product of distance and the neutrino energy, $L \times E_\nu$. For conventional oscillation theory the transition probability depends only on $L/E_\nu$. The SME also predicts that the neutrino flavor change probability depends on the angle between the direction of the neutrino and the LV/CPTV field in the sun-centered inertial frame in which the SME is formulated [4]. Experiments like MINOS [3], whose neutrino beam is fixed on the Earth, are well-suited to search for this behavior, which would appear as a periodic variation in the detected neutrino rate as the beam swings around the field with the sidereal frequency $\omega_\oplus = 2\pi/(365.246\,\text{day})$.

MINOS has a near detector (ND) located 1 km from the neutrino beam source and a far detector (FD) located 735 km from the neutrino source. Because of their different baselines, the ND and FD are sensitive to two separate limits of the general SME formulated for the neutrino sector. The predicted SME effects for baselines less than 1 km are independent of neutrino mass [3], and both MINOS [3] and LSND [7] reported searches for these effects. Recent theoretical work has shown that SME effects are a perturbation to the dominant mass oscillations for neutrinos having the appropriate $L/E_\nu$ to experience oscillations [8]. Since the probability for transitions due to LV increases with baseline, experiments with baselines greater than $\approx 100$ km are especially sensitive to LV and CPTV. The following analysis using MINOS FD data is the first search for perturbative LV and CPTV effects in admixture with neutrino oscillations.

According to the SME, the transition probability for $\nu_\mu \rightarrow \nu_\tau$ transitions over long baselines is $P_{\mu\tau} \approx P_{\mu\tau}^{(0)} + P_{\mu\tau}^{(1)}$, where $P_{\mu\tau}^{(0)}$ is the conventional mass oscillation probability for transitions between two flavors and $P_{\mu\tau}^{(1)}$ is the perturbation due to LV and CPTV, with $P_{\mu\tau}^{(1)}/P_{\mu\tau}^{(0)} < 1$. In the SME, $P_{\mu\tau}^{(1)}$ is given by

$$P_{\mu\tau}^{(1)} = 2L \left( P_C^{(1)} \right)_{\mu\tau} + \left( P_A^{(1)} \right)_{\mu\tau} \sin \omega_\oplus T_\oplus + \left( P_A^{(1)} \right)_{\mu\tau} \cos \omega_\oplus T_\oplus \sin 2\omega_\oplus T_\oplus + \left( P_B^{(1)} \right)_{\mu\tau} \cos \omega_\oplus T_\oplus \cos 2\omega_\oplus T_\oplus \right),$$

where $L = 735$ km is the distance from neutrino production in the NuMI beam to the MINOS FD [2], $T_\oplus$ is the local sidereal time (LST) at neutrino detection, and the coefficients $(P_C^{(1)})_{\mu\tau}$, $(P_A^{(1)})_{\mu\tau}$, $(P_B^{(1)})_{\mu\tau}$, $(P_{c_B}^{(1)})_{\mu\tau}$, and $(P_{c_B}^{(1)})_{\mu\tau}$ contain the LV and CPTV information. These coefficients depend on the SME coefficients that explicitly describe LV and CPTV, $(a_L)_{\mu\tau}$, $(b_L)_{\mu\tau}$, as well as the neutrino mass-squared splitting, $\Delta m_{32}^2$ [8]. For two-flavor transitions, only the real components of $(a_L)_{\mu\tau}$ and $(b_L)_{\mu\tau}$ contribute to the transition probability.

The magnitudes of the functions in eq. (1) depend on the direction of the neutrino propagation in a fixed coordinate system on the rotating Earth. The direction vectors are defined by the cotangents of the NuMI beam line $\chi = (90^\circ - \text{latitude}) = 42.1793347^\circ$, the beam zenith angle $\theta = 86.7255^\circ$ defined from the $z$-axis which points up toward the local zenith, and the beam azimuthal angle $\phi = 203.909^\circ$ measured counterclockwise from the $x$-axis chosen to lie along the detector’s long axis.

This analysis selected data using standard MINOS beam quality requirements and data quality selections [2]. The neutrino events used must interact in the 4.0 kiloton FD fiducial volume [8] and be charged-current (CC) in nature. The selection method described in [8] allowed the identification of the outgoing muon in a CC interaction. As in [8], we focused on these events to maximize the $\nu_\mu$ disappearance signal.

The data used come from the run periods listed in Table I; also shown are the number of protons incident on the neutrino production target (POT) for each period and the total number of events observed. To avoid biases, we performed the analysis blindly with the procedures de-
determined using only the Runs I and II data. The Run III data, comprising more than 50% of the total, were included only after finalizing the analysis procedures.

| Run Dates       | POT     | CC Events |
|-----------------|---------|-----------|
| May05 – Feb06   | 1.24 × 10^{20} | 281 |
| Sep06 – Jul07   | 1.94 × 10^{20}  | 453 |
| Sep07 – Jun09   | 3.88 × 10^{20}  | 954 |

We tagged each neutrino event with the time determined by the Global Positioning System (GPS) receiver located at the FD site that reads out absolute Universal Coordinated Time (UTC) and is accurate to 200 ns [10]. The GPS time of the accelerator extraction magnet signal defined the time of each 10 μs beam spill. We converted the time of each neutrino event and spill to local sidereal time $t_{\text{SST}}$ (LST) in standard ways [11]. The uncertainty in the GPS time stamps introduced no significant systematic error into the analysis [6]. We placed each detected CC event into a histogram that ranged from 0-1 in local sidereal phase (LSP), the LST of the event divided by the length of a sidereal day. We used the LSP for each spill to place the number of POT for that spill, whether or not there was a neutrino event associated with it, into a second histogram. By dividing these two histograms, we obtained the normalized neutrino event rate as a function of LSP, in which we searched for sidereal variations.

Since the search for sidereal variations used a Fast Fourier Transform (FFT) algorithm that works most efficiently for $N = 2^n$ bins [12] and eq. (1) only puts power into the Fourier terms $\omega n T_{\text{SST}}$ and $2\omega n T_{\text{SST}}$, we chose $N = 2^{14} = 16$ bins to retain these harmonic terms while still providing sufficient resolution in sidereal phase to detect a sidereal signal. With this choice, each bin spanned 0.063 in LSP or 1.5 hours in sidereal time.

The statistical similarity of the event rates for all runs was tested by comparing the rate for run $i$ in LSP phase bin $j$, $\tilde{R}_{ij}$, with the weighted mean rate for that bin, $\bar{R}_j$. The distribution of $\tilde{r} = (\tilde{R}_{ij} - \bar{R}_j)/\sigma_{ij}$, where $\sigma_{ij}$ is the statistical uncertainty in $R_{ij}$ for all $i$, $j$, is Gaussian with $\tilde{r} = 0$ and $\sigma = 1$, as expected for statistically consistent runs. Given this result, we combined the runs into a single data set whose rate as a function of LSP is shown in Fig. 1. The mean rate is $2.36 \pm 0.06$ events per $10^{18}$ POT and the uncertainties shown in the figure are statistical.

We searched for a sidereal signal by looking for excess power in the FFT of the data in Fig. 1 at the frequency corresponding to exactly 1 sidereal day. We used two statistics in our search, $p_1 = \sqrt{S_1^2 + C_1^2}$ and $p_2 = \sqrt{S_2^2 + C_2^2}$, where $S_1^2$ is the power returned by the FFT for $\sin(\omega_0 T_{\text{SST}})$, $C_1^2$ is the power returned for $\cos(\omega_0 T_{\text{SST}})$, and $S_2^2$ and $C_2^2$ are the analogous powers for the second harmonics. We used the quadratic sum of powers to minimize the effect of the arbitrary choice of zero point in phase at $0^\circ$ LST. Table II gives the $p_1$ and $p_2$ values returned by the FFT for the total data set.

![Fig. 1: Event rate as a function of LSP for the total data set.](image)

| statistic  | $p(\text{FFT})$ | $p_F$ |
|------------|-----------------|-------|
| $p_1$      | 1.09            | 0.26  |
| $p_2$      | 1.13            | 0.24  |

We determined the significance of our measurements of $p_1$ and $p_2$ by simulating $10^4$ experiments without a sidereal signal. To construct these experiments we used the data themselves by randomizing the LSP of each CC event $10^4$ times and placing each instance into a different phase histogram. We next randomized the LSP of each spill $10^4$ times and placed the POT for each instance into another set of histograms. For the POT histograms we drew the phases randomly from the LSP histogram of the start times for all spills. Dividing an event histogram by a POT histogram produced one simulated experiment. The randomization of both the spill and event LSP removed any potential sidereal variation from the data.

We performed the FFT on the simulated experiments and computed the $p_1$ and $p_2$ statistics for each. The resulting distributions of $p_1$ and $p_2$ are nearly identical as shown in Fig. 2. The solid line at $p(\text{FFT}) = 2.26$ divides the $p_2$ histogram at the point where 99.7% of the entries have lower values of the statistic. Consequently we took $p(\text{FFT}) = 2.26$ as the 99.7% confidence limit that a measured $p(\text{FFT})$ for either harmonic indicates the distribution had no sidereal signal. That is, we adopted $p(\text{FFT}) \geq 2.26$ as our signal detection threshold.

Based on this threshold, the $p_1$ and $p_2$ statistics in Table III show no evidence for a sidereal signal. Thus, the normalized neutrino event rate exhibits no statistically significant variation that depends on the direction of the earth-based neutrino beam in the sun-centered inertial frame. In the context of the SME, this result is inconsistent with the detection of LV and CPTV. The third column of Table III gives the probability, $p_F$, that the
Since we found no sidereal signal, we determined upper limits on the $(\alpha_L)^{\nu}\nu$ and $(\nu_L)^{\nu}\nu$ coefficients that describe LV and CPTV in the SME using the standard MINOS Monte Carlo simulation [2]. Neutrinos are simulated by modeling the NuMI beam line, including the hadron production and subsequent propagation through the focusing elements, hadron decay in the 675 m decay pipe, and the probability that any neutrinos produced will intersect the FD 735 km away. These neutrinos along with weights determined by decay kinematics, are used in the detailed simulation of the FD.

To find the limits, we chose $|\Delta m^2_{32}| = 2.43 \times 10^{-3} \text{ eV}^2$ and $\sin^2(2\theta_{23}) = 1$, the values measured by MINOS [2]. Our tests show that changing these values within the allowed uncertainty does not alter the limits we found.

We determined the limits for each SME coefficient individually. We constructed a set of experiments in which one coefficient was set to be small but non-zero and the remaining coefficients were set to zero. We simulated a high statistics event histogram by picking events with a random sidereal phase drawn from the distribution of start times for the data spills and weighted these simulated events by both their survival probability and a factor to account for the different exposures between the data and the simulation. Simultaneously we simulated a spill histogram by entering the average number of POT from the data, at the sidereal phase of each simulated spill histogram by picking events with a random sidereal phase drawn from the distribution of start times for the data spills and weighted these simulated events by both their survival probability and a factor to account for the different exposures between the data and the simulation. Simultaneously we simulated a spill histogram by entering the average number of POT from the data, at the sidereal phase of each simulated spill. The division of these two histograms resulted in the LSP histogram we used to compute the $p_1$ and $p_2$ statistics. We then increased the magnitude of the non-zero SME coefficient and repeated the process until either $p_1$ or $p_2$ was greater than the 2.26 detection threshold. To reduce fluctuations we computed the limit 100 times and averaged the results. Table III gives the magnitude of the coefficient required to produce a signal above threshold. This procedure could miss fortuitous cancelations of SME coefficients. We did not consider these cases. We cross-checked these limits by simulating 750 low statistics experiments for each coefficient limit given in Table III. Each experiment had the same total number of neutrinos as the data. The distributions of the $p_1$ and $p_2$ statistics for all the experiments were used to determine the confidence level at which the measured values in Table III are excluded by each limit. The exclusion using this method is > 99.7% C.L. for all coefficients.

In summary we found no evidence for sidereal variations in the neutrino rate in the MINOS FD. This result, when framed in the SME [4, 5], leads to the conclusion that we have detected no evidence for the violation of Lorentz or CPT invariance described by this framework for neutrinos traveling over the 735 km baseline from their production in the NuMI beam to the MINOS FD. The limits on the SME coefficients in Table III for the FD that come from this null result improve the limits we found for the ND by factors of order 20 - 510 [6]. This

harmonic powers we found were due to statistical fluctuations. $P_F$ is the probability of drawing a value of $p_1$ or $p_2$ from the parent distribution in Fig. 2 at least as large as found in the data.

We next determined the minimum detectable sidereal signal strength we could have found in this experiment by injecting a sidereal signal of the form $A \sin(\omega_{\odot} T_{\odot})$, where $A$ is a fraction of the mean event rate, into a new set of $10^4$ simulated experiments and repeating the FFT analysis. We found 68% of the experiments gave $p_1 \geq 2.26$ for $A = 9\%$. These simulations indicated there was a 68% probability of detecting a sidereal modulation if the signal amplitude had been 9% of the mean event rate.

We tested the sensitivity of our results to several sources of systematic uncertainty. First we confirmed through simulated experiments that the analysis was insensitive to our choice of zero point in phase, $0^\circ$ LST, due to the definition of $p_1$ and $p_2$. Next we tested sources that could introduce false sidereal signals into the data and mask an LV detection. Degradation of the NuMI target caused secular drifts of ~5% in the neutrino production rate on a time scale of ~6 months. Doubling this trend introduced no detectable signal for either secular decreases or increases. The known $\pm1.0\%$ uncertainty in the number of POT per spill [2] was too small to affect this analysis. Because of the non-uniformity of the data taking throughout the solar year, diurnal effects, like temperature variations on the POT counting devices, could have introduced a false signal. However, systematic differences between the day and night event rates were smaller than the statistical errors in the rates themselves and could not introduce a false signal. Atmospheric effects could also have imparted a sidereal signal on the data if there were a solar diurnal modulation in the event rate that beats with a yearly modulation [13]. Using methods described in [14], we found that this false sidereal signal is < 0.2% of the mean event rate and well below the detection threshold.

FIG. 2: Distributions of the $p_1$ and $p_2$ statistics from the FFT analysis of $10^4$ simulated experiments without a sidereal signal. The adopted signal detection limit is $p(FFT) = 2.26$. 

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TABLE III: 99.7% C.L. limits on SME coefficients for $\nu_\mu \to \nu_\tau$; $(aL)_{\mu\tau}^N$ have units [GeV]; $(cL)_{\mu\tau}^{0,3}$ are unitless. The columns labeled $I$ show the improvement from the near detector limits.

| Coeff. | Limit | $I$ |
|--------|-------|-----|
| $(aL)_{\mu\tau}^X$ | $5.9 \times 10^{-23}$ | 510 |
| $(aL)_{\mu\tau}^N$ | $6.1 \times 10^{-23}$ | 490 |
| $(cL)_{\mu\tau}^X$ | $0.5 \times 10^{-23}$ | 20 |
| $(cL)_{\mu\tau}^N$ | $0.5 \times 10^{-23}$ | 20 |
| $(cL)_{\mu\tau}^{YY}$ | $2.4 \times 10^{-23}$ | 230 |
| $(cL)_{\mu\tau}^{ZZ}$ | $0.7 \times 10^{-23}$ | 170 |

The improvement is due to the different behavior of the oscillation probability in the short and long baseline approximations coupled with the significantly increased baseline to the FD. These improvements more than offset the significant decrease in statistics in the FD. They are the first limits to be determined for the neutrino sector in which LV and CPTV are assumed to be a perturbation on the conventional neutrino mass oscillations.

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