On massive spin 2 electromagnetic interactions

Yu. M. Zinoviev

Institute for High Energy Physics, Protvino, Russia

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Outlook

1. Constructive approach to interactions

2. E/m interactions: $m = 0, \Lambda \neq 0 \iff m \neq 0, \Lambda = 0$

3. Massive spin 2 example
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3. Massive spin 2 example
Massless particles interactions

- Iterative construction of Lagrangian and gauge transformations

\[ \mathcal{L} \sim \mathcal{L}_2 + g\mathcal{L}_3 + g^2\mathcal{L}_4 + \ldots \]

\[ \delta \Phi \sim \delta_0 + g\delta_1 + g^2\delta_2 + \ldots \]

- Universality (model independence) of linear approximation

\[ \mathcal{L}_3 \sim \Phi^A \Phi^B \Phi^C \implies \delta_1 \Phi^B \sim \Phi^C \xi^A, \quad \delta_1 \Phi^C \sim \Phi^B \xi^A \]

\[ \implies \text{"structure constants"} \]

- Next to linear approximations

\[ \implies \text{closure of the algebra} \]

\[ \implies \text{complete set of fields} \]
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Constructive approach to interactions

Gauge invariant description of massive particles

- A set of massless fields $\Phi_s, \Phi_{s-1}, \ldots, \Phi_0$ with all their usual gauge transformations
- Lagrangian

$$\mathcal{L} \sim \sum_{m=0}^{s} \left[ \partial \Phi_m \partial \Phi_m + m \Phi_m \partial \Phi_{m-1} + m^2 (\Phi_m \Phi_m + \Phi_m \Phi_{m-2}) \right]$$

- Guage transformations

$$\delta \Phi_m \sim \partial \xi_m + m [\xi_{m+1} + \xi_{m-1}]$$

- Works well both in flat space as well as $(A)dS$ spaces

\[\rightarrow\] Constructive approach for any collection of massive and/or massless particles
Gauge invariant description of massive particles

- A set of massless fields $\Phi_s$, $\Phi_{s-1}$, ..., $\Phi_0$ with all their usual gauge transformations
- Lagrangian
  \[ \mathcal{L} \sim \sum_{m=0}^{s} \left[ \partial \Phi_m \partial \Phi_m + m \Phi_m \partial \Phi_{m-1} + m^2 (\Phi_m \Phi_m + \Phi_m \Phi_{m-2}) \right] \]
- Gauge transformations
  \[ \delta \Phi_m \sim \partial \xi_m + m [\xi_{m+1} + \xi_{m-1}] \]
- Works well both in flat space as well as (A)dS spaces
- $\Longrightarrow$ Constructive approach for any collection of massive and/or massless particles
E/m interactions (integer spin)

- Massless particle in flat space
  \[ \mathcal{L} \sim D\Phi D\Phi + e_0 \Phi \Phi F + ? \]

- Massless particle in AdS space
  \[ \mathcal{L} \sim D\Phi D\Phi + \Lambda \Phi \Phi + e_0 \Phi \Phi F + \frac{e_0}{\Lambda} D\Phi D\Phi F + \ldots \]

- Massive particle in flat space
  \[ \mathcal{L} \sim D\Phi D\Phi + m^2 \Phi \Phi + e_0 \Phi \Phi F + \frac{e_0}{m^2} D\Phi D\Phi F + \ldots \]
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Massless vs flat limit

$m = 0, \Lambda = 0, e_0 = 0 \quad \Rightarrow \quad e_0 \rightarrow 0, \Lambda \rightarrow 0 \quad \Rightarrow \quad m = 0, \Lambda \neq 0, e_0 \neq 0$
Massless vs flat limit

\[ m = 0, \Lambda = 0, e_0 = 0 \quad \text{\(\iff\)} \quad m \neq 0, \Lambda = 0, e_0 \neq 0 \]

\[ m \to 0 \quad e_0 \to 0 \]

\[ m \neq 0, \Lambda = 0, e_0 \neq 0 \]
Massless vs flat limit

\[ m = 0, \Lambda = 0, e_0 = 0 \quad \leftrightarrow \quad m \neq 0, \Lambda = 0 \]

\[ e_0 \to 0, \Lambda \to 0 \]

\[ m \to 0, e_0 \to 0 \]

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\[ \Lambda \to 0 \]

\[ m \neq 0, \Lambda \neq 0, e_0 \neq 0 \]
Non-minimal e/m interactions in flat space

- Boulanger-Leclercq-Sundell vertices for spin $s$

\[ \mathcal{L}_1 \sim a_0 D^{s-1} \bar{\Phi} D^{s-1} \Phi F, \quad a_0 \sim \frac{1}{M^{2s-2}} \]

\[ \delta_1 A \sim a_0 D^{s-1} \Phi D^{s-1} \xi, \quad \delta_1 \Phi \sim a_0 D^{s-1} [FD^{s-2} \xi] \]

- $s = 2$ case in frame-like formulation

\[ \mathcal{L}_1 = \frac{a_0}{8} \{ \mu \nu \}_{ab} \left[ \bar{\omega}^{ab} F^{cd} \omega_{cd}^{\mu \nu} + \bar{\omega}^{cd} F^{cd} \omega_{ab}^{\mu \nu} - 
- 4 \bar{\omega}^{ac} F^{cd} \omega_{bd}^{\mu \nu} - \bar{\omega}^{cd} F^{ab} \omega_{cd}^{\mu \nu} + h.c. \right] \]

\[ \delta_1 h_{\mu}^{a} = -\frac{a_0}{2} [F_{\mu}^{b \eta} \eta_{ba} + \eta_{\mu}^{b} F^{ba} + \frac{1}{d' - 2} e_{\mu}^{a} (F \eta)] \]

\[ \delta_1 A_{\mu} = \frac{a_0}{2} [\bar{\omega}_{\mu}^{ab} \eta_{ab} + h.c.] \]
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- Boulanger-Leclercq-Sundell vertices for spin $s$

\[
\mathcal{L}_1 \sim a_0 D^{s-1} \tilde{\Phi} D^{s-1} \Phi F,
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\delta_1 A \sim a_0 D^{s-1} \Phi D^{s-1} \xi,
\quad \delta_1 \Phi \sim a_0 D^{s-1} [FD^{s-2} \xi]
\]

- $s = 2$ case in frame-like formulation

\[
\mathcal{L}_1 = \frac{a_0}{8} \left\{ \begin{array}{c} \mu \nu \\
abla \end{array} \right\} \left[ \tilde{\omega}_\mu^{\ ab} F^{cd} \omega_\nu^{\ cd} + \omega_\mu^{\ cd} F^{\ cd} \omega_\nu^{\ ab} - \\
-4 \tilde{\omega}_\mu^{\ ac} F^{\ cd} \omega_\nu^{\ bd} - \omega_\mu^{\ cd} F^{\ ab} \omega_\nu^{\ cd} \right] + h.c.
\]

\[
\delta_1 h^a_\mu = -\frac{a_0}{2} \left[ F_\mu^{\ b} \eta^{ba} + \eta_\mu^{\ b} F^{ba} + \frac{1}{d-2} e_\mu^a (F \eta) \right]
\]

\[
\delta_1 A_\mu = \frac{a_0}{2} \left[ \tilde{\omega}_\mu^{\ ab} \eta^{ab} + h.c. \right]
\]
Frame-like gauge invariant formulation

- **Fields:** \((\omega_\mu^{\, ab}, \, h_\mu^\, a), (C^{ab}, B_\mu), (\pi^a, \varphi)\)
- **Lagrangian**

\[
L_0 = \frac{1}{2} \left\{ \omega^\mu_{\, ab} \right\} \omega_\mu^{\, ac} \omega_\nu^{\, bc} - \frac{1}{2} \left\{ \omega^\mu_{\, abc} \right\} \omega_\mu^{\, ab} D_\nu h_\alpha^\, c + \frac{1}{8} C^{ab}_\mu d - \\
- \frac{1}{4} \left\{ C^{ab}_\mu B_\nu \right\} - \frac{3}{4} \pi_a^2 + \frac{3}{2} \left\{ \mu^a_\mu \right\} \pi^a D_\mu \varphi + \frac{m}{2} \left\{ C^{ab}_\mu h_\mu^\, b \right\} - \frac{3}{2} M \left\{ \mu^a_\mu \right\} \pi^a B_\mu + \\
+ \frac{M^2}{2} \left\{ h_\mu^\, a \right\} h_\nu^\, b - \frac{3}{2} m M \left\{ \mu^a_\mu \right\} h_\mu^\, a \varphi + \frac{3}{2} m^2 \varphi^2
\]

- **Gauge transformations**

\[
\delta_0 h_\mu^\, a = D_\mu \xi^a + \frac{m}{d-2} e_\mu^\, a \xi, \quad \delta_0 B_\mu = D_\mu \xi + m \xi_\mu, \quad \delta_0 \varphi = M \xi
\]

where \(M^2 = m^2 - \kappa (d - 2), \quad \kappa = \frac{2\Lambda}{(d-1)(d-2)}\)
Linear approximation for massive spin 2

- Lagrangian \((d = 4)\)

\[
\mathcal{L}_1 = \frac{a_0}{8} \{\mu\nu\}_{ab} \left[ \bar{\omega}_\mu^{\ab} F^{\cd} \omega_\nu^{\cd} + \bar{\omega}_\mu^{\cd} F^{\cd} \omega_\nu^{\ab} - 4 \bar{\omega}_\mu^{\ac} F^{\cd} \omega_\nu^{\bd} - \bar{\omega}_\mu^{\cd} F^{\ab} \omega_\nu^{\cd} - M^2 F^{\ab} (\bar{h}_\mu^{\bb} h_\nu^{\bb} - \bar{B}_\mu B_\nu) + h.c. \right]
\]

- Gauge transformations

\[
\delta_1 h_\mu^a = -\frac{a_0}{2} \left[ F_\mu^b \eta^{ba} + \eta_\mu^b F^{ba} + \frac{1}{2} e_\mu^a (F \eta) \right]
\]

\[
\delta_1 A_\mu = \frac{a_0}{2} \left[ \bar{\omega}_\mu^{\ab} \eta^{ab} + M^2 (h_\mu^{\bb} \xi^a - B_\mu \xi) + h.c. \right]
\]

- Electric charge

\[
e_0 = -\frac{(d - 3)}{(d - 2)} a_0 [m^2 - \kappa (d - 2)]
\]
Electric charge $e_0$ vs $m^2$, $\kappa$

$$m^2 = \kappa(d - 2)$$

$$e_0 = \text{const}$$

$$e_0 \sim m^2 - \kappa(d - 2)$$
Conclusion

- Gauge invariant description of massive particles $\Rightarrow$ constructive aproach to interactions for any collections of massive and/or massless particles
- Universality of linear approximation $\Rightarrow$ model independent results
- $m = 0, \Lambda \neq 0 \iff m \neq 0, \Lambda = 0$
- Generalization on higher spins $\Rightarrow$ work in progress
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