CP VIOLATION IN HIGGS DECAYS
DUE TO MAJORANA NEUTRINOS

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Abstract

The possibility of observing CP violation in the decays of Higgs particles into top-quark, W- and Z-boson pairs induced by heavy Majorana neutrinos is discussed. In the context of minimal “see-saw” models with interfamili mixings, we find that Majorana neutrinos may give rise to sizable CP-odd effects at the one-loop electroweak order. We present numerical estimates of typical CP-odd observables that might be triggered at high-energy e+e− colliders.

There has been growing effort in the literature to analyze gauge models and possible mechanisms of producing sizable CP-violating effects at high-energy colliders. Several scenarios that could account for CP-violating effects at supercollider energies were considered. Most of them were concerned with extensions of the Higgs sector of the Standard Model (SM). Here, we shall present the attractive possibility that heavy Majorana neutrinos could play a crucial role in accounting for sizable CP-odd effects at future e+e− linear colliders.

In the following, we shall address the question whether loop graphs mediated by heavy Majorana neutrinos can induce measurable CP-odd signals in the decays H0 → tt, H0 → W+W−, and H0 → ZZ. In particular, we shall consider the following CP-odd asymmetry parameters:

\[ A_{CP}^{(t)} = \frac{\Gamma(H^0 \rightarrow t_L\bar{t}_L) - \Gamma(H^0 \rightarrow t_R\bar{t}_R)}{\Gamma(H^0 \rightarrow tt)}, \]

\[ A_{CP}^{(W)} = \frac{\Gamma(H^0 \rightarrow W^+_{(1)}W^-_{(1)}) - \Gamma(H^0 \rightarrow W^+_{(-1)}W^-_{(-1)})}{\Gamma(H^0 \rightarrow W^+_{(1)}W^-_{(1)}) + \Gamma(H^0 \rightarrow W^+_{(-1)}W^-_{(-1)})}, \]

where the subscripts L, R, and (±1) denote the top-quark helicity and the W-boson transverse polarization, respectively. Similarly to Eq. (2), a CP asymmetry, \( A_{CP}^{(Z)} \), for the decay

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channel $H^0 \to ZZ$ can be constructed. The states $t_L$ and $W^-_{(1)}$ are connected with the states $t_R$ and $W^+_{(-1)}$ by $CP$ conjugation. Therefore, $A^{(t)}_{CP}$ and $A^{(W)}_{CP}$ represent genuine $CP$-violating parameters that can be determined experimentally. One should also remark that the $CP$ asymmetries $A^{(t)}_{CP}$, $A^{(W)}_{CP}$, and $A^{(Z)}_{CP}$ resulting from the Feynman graphs depicted in Fig. 1 cannot be induced if the heavy neutrinos are of the standard Dirac type.

**Fig. 1:** Feynman graphs giving rise to a $CP$-odd part in the $H^0 t\bar{t}$, $H^0 W^+ W^-$, and $H^0 ZZ$ couplings.

The minimal class of models that predict heavy Majorana neutrinos can be obtained by simply introducing $n_G$ right-handed neutrinos, $\nu^0_{Ri}$, to the SM, where $n_G$ denotes the number of generations. The relevant interactions of the Majorana neutrinos with the $W^-$, $Z^-$, and Higgs bosons in such minimal models are given by [6]

$$
\mathcal{L}_{int}^W = - \frac{g_W}{2\sqrt{2}} W^{-\mu} \bar{\nu}_i B_{l1,j} \gamma^\mu (1 - \gamma_5) n_j + \text{H.c.},
$$

(3)

$$
\mathcal{L}_{int}^Z = - \frac{g_W}{4\cos\theta_W} Z^\mu \bar{\nu}_i \gamma^\mu \left[ i\text{Im}C_{ij} - \gamma_5 \text{Re}C_{ij} \right] n_j,
$$

(4)

$$
\mathcal{L}_{int}^H = - \frac{g_W}{4M_W} H^0 \bar{\nu}_i \left[ (m_{n_i} + m_{n_j}) \text{Re}C_{ij} + i\gamma_5 (m_{n_j} - m_{n_i}) \text{Im}C_{ij} \right] n_j,
$$

(5)

where

$$
C_{ij} = \sum_{k=1}^{n_G} U_{ki}^\nu \ell_{kj}^\nu,
$$

(6)

$$
B_{l1,j} = \sum_{k=1}^{n_G} V_{l1,k} U_{kj}^\nu.
$$

(7)

The matrices $B$ and $C$ satisfy a great number of identities that will also help us to estimate our $CP$ asymmetries. These identities are written down as [1]

$$
\sum_{i=1}^{2n_G} B_{l1,i} B_{l2,i}^* = \delta_{l1l2},
$$

(8)
The mixings $B_{lN}$ and $C_{\nu N}$ are constrained by a global analysis of low-energy and LEP data \[8\]. Notice that Eqs. (8)–(12) are not accidental, but represent generalized GIM-type unitary relations and guarantee the renormalizability of our model.

Note that the Lagrangians of Eqs. (3)–(6) violate the $CP$ symmetry of the model. Apart from the $CP$ violation introduced by the known complex Cabbibo-Kobayashi-Maskawa-type matrix, $B_{l_i n_j}$, the neutral-current interactions described by Eqs. (4) and (5) violate the $CP$ symmetry of the model, as the neutral particles, $H^0$ and $Z$, couple simultaneously to $CP$-even ($\bar{n}_i n_j$ or $\bar{n}_i \gamma_5 n_j$) and $CP$-odd ($\bar{n}_i \gamma_\mu n_j$ or $\bar{n}_i \gamma_\mu \gamma_5 n_j$) operators with two different Majorana neutrinos (i.e., $n_i \neq n_j$). As a result, the $H^0 t \bar{t}$, $H^0 W^+ W^-$, and $H^0 Z Z$ couplings contain non-zero $CP$-odd parts which are generated radiatively (see also Fig. 1) \[5\].

We shall now look for $CP$-odd correlations of the type $\langle (\vec{s}_t - \vec{s}_{\bar{t}}) \cdot \vec{k}_t \rangle$ in the decay $H^0 \rightarrow t \bar{t}$. The important ingredient for a non-zero $A^{(t)}_{CP}$ are Eqs. (4) and (5), which violate the $CP$ symmetry of the model, as mentioned above. Thus, vacuum-polarization transitions between the $CP$-even Higgs and the $CP$-odd $Z$ boson, which are not allowed in the $SM$, can now give rise to a pseudoscalar part in the $H^0 t \bar{t}$ coupling (see also Fig. 1(a)) and hence lead to the non-vanishing $CP$ asymmetry

$$A_{CP}^{(t)} = \frac{\alpha_w}{4} \text{Im} C_{ij}^2 \sqrt{\lambda_i \lambda_j} \frac{\lambda_i - \lambda_j}{\lambda_H} \frac{\lambda_H^{1/2}(\lambda_H, \lambda_i, \lambda_j)}{\lambda_H^{1/2}(\lambda_H, \lambda_t, \lambda_t)} ,$$

where

$$\lambda_i = \frac{m^2_{n_i}}{M^2_W}, \quad \lambda_H = \frac{M^2_H}{M^2_W}, \quad \lambda_t = \frac{m^2_t}{M^2_W}, \quad \lambda(x, y, z) = (x - y - z)^2 - 4yz. \tag{14}$$

To get $A_{CP}^{(t)} \neq 0$, two non-degenerate heavy Majorana neutrinos are required. On the other hand, a counting of the $CP$-odd phases existing generally in the $SM$ with right-handed neutrinos gives \[7\]

$$N_{CP} = N_L(N_R - 1), \tag{15}$$

where $N_L$ and $N_R$ are the numbers of left-handed and right-handed neutrinos, respectively. In the three-generation model, one has $\text{Im} C_{ij N_1 N_2}^2 \leq 10^{-2}$ \[8\]. However, the situation changes drastically if one introduces an additional left-handed neutrino field, $\nu^0_{L_4}$, in the
Lagrangian of the SM and assumes a non-trivial mixing between the two right-handed neutrinos. Using Eq. (14), one can find the helpful relations

\begin{align}
\text{Im}C_{\nu_4 N_1}^2 &= \sin \delta_{CP} |C_{\nu_4 N_1}|^2, \\
\text{Im}C_{\nu_4 N_1}^2 &= -\frac{m_{N_1}}{m_{N_2}} \sin \delta_{CP} |C_{\nu_4 N_1}|^2, \\
\text{Im}C_{N_1 N_2}^2 &= \frac{m_{N_4}}{m_{N_2}} \sin \delta_{CP} |C_{\nu_4 N_1}|^2.
\end{align}

In these models, the mixing $|C_{\nu_4 N_1}|^2$ can be of order one. Detailed investigation reveals that, for $M_H \approx 400$ GeV, $CP$ asymmetries $A_{CP}^{(0)} \approx 1 \times 10^{-2}$ are possible in four-generation scenarios with Majorana neutrinos \cite{5,9}. We again emphasize the fact that high-mass Dirac neutrinos with standard diagonal couplings cannot give rise to a non-zero $CP$ asymmetry.

Heavy Majorana neutrinos can also generate a $CP$-odd part in the $H^0 W^+ W^-$ coupling through the triangle graph shown in Fig. 1(b). Actually, one has to look for $CP$-violating correlations of the form

\[
\epsilon_{\mu\nu\rho\sigma} \epsilon_{(+1)}^{\mu}(\hat{\varepsilon}^{(+)\nu}(\hat{\varepsilon}^{(-)}\kappa_4 \kappa_6 = M_H \hat{k}_+ (\hat{\varepsilon}^{(+)\nu}(\hat{\varepsilon}^{(-)}\kappa_4 \kappa_6,
\]

where the polarization vectors, $\hat{\varepsilon}_{(+)}^{\mu}(\hat{\varepsilon}^{(-)}$, describe two transverse $W$ bosons with helicity $+1$. The presence of $CP$-odd terms given in Eq. (19) leads to the $CP$ asymmetry

\[
A_{CP}^{(W)} = \frac{\alpha_W}{4} \left[ \text{Im}(B_{t_{k_4}} B_{t_{k_4}}^* C_{ij}) \sqrt{\lambda_i \lambda_j} (F_+ + F_-) + \text{Im}(B_{t_{k_i}} B_{t_{k_j}}^* C_{ij}) (\lambda_i F_+ + \lambda_j F_-) \right],
\]

where $\alpha_W = (g_W^2 / 4\pi)$. The functions $F_{\pm}$ in Eq. (24) describe the absorptive contributions from three different kinematic configurations of the intermediate states that can become on-shell. A straightforward computation gives

\[
F_{\pm}(\lambda_i, \lambda_j, \lambda_{l_k}) = \theta(M_H - m_{n_i} - m_{n_j}) \left[ \pm \frac{\lambda_{1/2}(\lambda_H, \lambda_i, \lambda_j)}{2\lambda^{1/2}(\lambda_H, \lambda_W, \lambda_W)} + G_{\pm}(\lambda_i, \lambda_j, \lambda_{l_k}) \right]
\]

\[
\times \ln \left( \frac{t^+(\lambda_i, \lambda_j) - \lambda_{l_k}}{t^-(\lambda_i, \lambda_j) - \lambda_{l_k}} \right) + \theta(M_W - m_{n_i} - m_{l_k}) \left[ \left( -\frac{\lambda_H - 4\lambda_W}{\lambda_W} \right) \right]
\]

\[
\pm \frac{\lambda_H}{\lambda_W} \frac{\lambda_{1/2}(\lambda_W, \lambda_i, \lambda_{l_k})}{4\lambda^{1/2}(\lambda_H, \lambda_W, \lambda_W)} + G_{\pm}(\lambda_i, \lambda_j, \lambda_{l_k}) \ln \left( \frac{t^+(\lambda_i, \lambda_{l_k}) - \lambda_j}{t^-(\lambda_i, \lambda_{l_k}) - \lambda_j} \right) \]

\[
+ \theta(M_W - m_{n_j} - m_{l_k}) \left[ \left( \frac{\lambda_H - 4\lambda_W}{\lambda_W} \right) + \frac{\lambda_H}{\lambda_W} \right] \frac{\lambda_{1/2}(\lambda_W, \lambda_j, \lambda_{l_k})}{4\lambda^{1/2}(\lambda_H, \lambda_W, \lambda_W)}
\]

\[
+ G_{\pm}(\lambda_i, \lambda_j, \lambda_{l_k}) \ln \left( \frac{t^+(\lambda_j, \lambda_{l_k}) - \lambda_i}{t^-(\lambda_j, \lambda_{l_k}) - \lambda_i} \right),
\]

where

\[
\lambda_{l_k} = \frac{m_{l_k}^2}{M_W^2}, \quad \lambda_W = 1, \quad \lambda_Z = \frac{M_Z^2}{M_W^2},
\]

\[
(22)
\]
\[ t^\pm(x, y) = -\frac{1}{2} \left[ \lambda_H - x - y - 2\lambda_W \mp \lambda^{1/2}(\lambda_H, x, y)\lambda^{1/2}(\lambda_H, \lambda_W, \lambda_W) \right], \]  
\[ t^\pm(x, y) = \lambda_H + x - \frac{1}{2}\lambda_W(\lambda_W + x - y) \pm \frac{1}{2}\lambda^{1/2}(\lambda_H, x, y)\lambda^{1/2}(\lambda_H, \lambda_W, \lambda_W), \]  
\[ G_\pm(x, y, z) = \frac{x - y}{4\lambda_H} \pm \frac{2(\lambda_W + z) - x - y}{4(\lambda_H - 4\lambda_W)}. \]  

The dominant contribution to \( A_{CP}^{(W)} \) comes from the \( n_i n_j \) on-shell states. To measure \( A_{CP}^{(W)} \), a discrimination between the events \( W_{(+1)}^+ W_{(+1)}^- \) and \( W_{(-1)}^+ W_{(-1)}^- \) from the total number of \( W \) bosons produced by Higgs-boson decays is needed. A branching-ratio estimate of these specific decay mode can be obtained from the following ratio:

\[ R^{(W)} = \frac{\Gamma(H^0 \rightarrow W_{(+1)}^+ W_{(+1)}^-) + \Gamma(H^0 \rightarrow W_{(-1)}^+ W_{(-1)}^-)}{\Gamma(H^0 \rightarrow W^+ W^-)} \approx \frac{8M_W^4}{M_H^4}. \]  

Since one expects to produce 100–1000 Higgs bosons, with \( M_H < 400 \) GeV, at a 500-GeV \( e^+e^- \) collider with an integrated luminosity of 20 fb\(^{-1}\), it should, in principle, be possible to see \( R^{(W)} \) values of order 1 and \( CP \) asymmetries of \( A_{CP}^{(W)} \approx 10\% \) in the SM with four generations.

Table 1: Numerical results of \( A_{CP}^{(Z)}/\text{Im} C_{N_1 N_2}^2 \) and \( A_{CP}^{(Z)}/\text{Im} C_{N_1 N_3}^2 \) for “see-saw” models with three and four generations, respectively.

| \( m_{N_2} [\text{GeV}] \) | \( m_H \) | \( m_{N_1}(m_{\nu_2}) \) | \( m_{N_2} \) | \( 3\text{Gens} \) | \( 4\text{Gens} \) |
|--------------------------|---------|----------------|---------|------------|------------|
| 300                      | 400 GeV | -1.60 \( 10^{-2} \) | -1.41 \( 10^{-2} \) |
| 400                      | 250 GeV | -3.70 \( 10^{-2} \) | -3.23 \( 10^{-2} \) |
| 500                      |         | -4.95 \( 10^{-2} \) | -4.29 \( 10^{-2} \) |
| 600                      |         | -5.74 \( 10^{-2} \) | -4.94 \( 10^{-2} \) |
| 1000                     |         | -7.09 \( 10^{-2} \) | -6.02 \( 10^{-2} \) |

Finally, heavy Majorana neutrinos induce, through the triangle graph of Fig. 1(c), a non-zero \( CP \) asymmetry in the decay \( H^0 \rightarrow ZZ \), which is computed to be

\[ A_{CP}^{(Z)} = \frac{\alpha_W}{4} \left[ \text{Im}(C_{ij} C_{jk} C_{ki}) (\lambda_i F_1 + \lambda_j F_2) + \text{Im}(C_{ij}^* C_{jk} C_{ki}) \sqrt{\lambda_i \lambda_j} (F_1 + F_2) \right. \\
- \left. \left( \text{Im}(C_{ij}^* C_{jk} C_{ki}) \sqrt{\lambda_i \lambda_k} + \text{Im}(C_{ij} C_{jk} C_{ki}) \sqrt{\lambda_j \lambda_k} \right) (R + F_1 - F_2) \right], \]  

where

\[ R = \theta(M_H - m_{\nu_i} - m_{\nu_j}) \left[ \frac{1}{2} \ln \left( \frac{t^+(\lambda_i, \lambda_j) - \lambda_k}{t^-(\lambda_i, \lambda_j) - \lambda_k} \right) \right]. \]
\[ + \theta(M_W - m_{n_i} - m_{n_j}) \frac{1}{2} \ln \left( \frac{\bar{t}^+(\lambda_i, \lambda_k) - \lambda_j}{\bar{t}^-(\lambda_i, \lambda_k) - \lambda_j} \right) \]
\[ + \theta(M_W - m_{n_j} - m_{n_k}) \frac{1}{2} \ln \left( \frac{\bar{t}^+(\lambda_j, \lambda_k) - \lambda_i}{\bar{t}^-(\lambda_j, \lambda_k) - \lambda_i} \right). \]  

(28)

The functions $F_1$, $F_2$, $t^\pm$, and $\bar{t}^\pm$ emerge from Eqs. (21)–(25) by substituting $\lambda_W \rightarrow \lambda_Z$ and $\lambda_{l_k} \rightarrow \lambda_k$. From Table 1 we see that $A_{CP}^{(Z)}$ can be of the order of 10%. An effect of this size should be observable at the next $e^+e^-$ linear collider.

In conclusion, we have seen that heavy Majorana neutrinos represent an attractive alternative to complicated multi-Higgs-boson scenarios when it comes to accounting for possible CP-violating phenomena in the decays of Higgs bosons into $t\bar{t}$, $W^+W^-$, and $ZZ$ pairs. The minimal extension of the SM by right-handed neutrinos (assuming the Majorana mass matrix, $m_M$, to be bare and, hence, the absence of Majoron fields) can naturally describe sizable CP-violating effects, of the order of 10%, at high-energy $e^+e^-$ linear colliders. Although the CP asymmetries $A_{CP}^{(t)}$, $A_{CP}^{(W)}$, and $A_{CP}^{(Z)}$ may not be directly accessible by experiment, the CP-violating signals originating from such Higgs-boson decays will, however, be transcribed to the decay products of the top-quark, $W^-$, and $Z$-boson pairs. More realistic $CP$-odd projectors may be constructed by considering, for instance, angular-momentum distributions or energy asymmetries of the produced charged leptons and jets.

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