Research Article

Blowup of Solutions to the Compressible Euler-Poisson and Ideal MHD Systems

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Received 22 November 2019; Revised 13 January 2020; Accepted 30 January 2020; Published 21 February 2020

Academic Editor: Antonio Scarfone

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In the present paper, we study the blowup of the solutions to the full compressible Euler system and pressureless Euler-Poisson system with time-dependent damping. By some delicate analysis, some Riccati-type equations are achieved, and then, the finite time blowup results can be derived.

1. Introduction

In the present paper, we are concerned with the blowup of the solutions to three models of compressible fluids, namely, the full Euler system, pressureless Euler-Poisson system, and ideal MHD system. In addition, there can also be time-dependent velocity damping in these three systems.

At first, we consider the Cauchy problem to the full compressible Euler system with time-dependent damping.

\[
\begin{align*}
\partial_t \rho + \text{div} (\rho u) &= 0 \quad \text{(conservation of mass)}, \\
\partial_t (\rho u) + \text{div} (\rho u \otimes u) + \nabla p &= -\alpha(t) \rho u \quad \text{(conservation of momentum)}, \\
\partial_t \left( \rho c_v + \frac{1}{2} \rho |u|^2 \right) + \text{div} \left( u \left( \rho c_v + \frac{1}{2} \rho |u|^2 + p \right) \right) &= -\alpha(t) \rho |u|^2 \quad \text{(conservation of energy)},
\end{align*}
\]

where \( x \in \mathbb{R}^n (n \geq 1), \ V = (\partial_1, \cdots, \partial_n), \text{ and } \rho, u = (u^1, \cdots, u^n), \ p = p(\rho, S), \ e = e(\rho, S), \text{ and } S \) stand for the density, velocity, pressure, inner energy, and entropy, respectively. The coefficient of the damping term \(-\alpha(t) \rho u\) in the second equation of (1) verifies \( \alpha(t) \geq 0 \) (when \( \alpha(t) \equiv 0 \), it means that there is no damping in the system (1)). The inner energy \( e(\rho, S) \) satisfies

\[
de = TdS + \frac{p}{\rho^2} d\rho,
\]

where \( T > 0 \) is the temperature. The equation of state is given by

\[
p(\rho, S) = P_0 \rho^\gamma e^{\delta c_v},
\]

with \( P_0 > 0 \) and the adiabatic exponent \( \gamma = c_p/c_v > 1 \) \((\gamma \approx 1.4 \text{ for the air})\). The positive constants \( c_p \) and \( c_v \) are the specific heat at constant pressure and volume, respectively. For convenience, let \( c_v = 1 \).

The global existence theories for the Euler equations with damping can be found in [1–6] and the references therein. In [7, 8], the behavior of solutions of the system (1) was considered. For the compressible Euler equations in 3D, the finite time blowup of classical solutions was proved by Sideris in [9]. Recently, the authors in [10] have proved the blowup result of the solutions to (1) in finite time with \( \alpha(t) \equiv 1 \) and
γ ∈ {2} ∪ [3,∞). In this paper, by more careful analysis on the pressure and some Riccati-type equations, we can achieve a similar blowup result of system (1) with the adiabatic exponent γ > 1 and the general damping a(t) ≥ 0.

Next, we study the pressureless Euler-Poisson system with time-dependent damping in \( \mathbb{R}^n (n ≥ 2) \).

\[
\begin{align*}
\partial_t \rho + \text{div} (\rho u) &= 0, \\
\partial_t (\rho u) + \text{div} (\rho u \otimes u) + \nabla \Phi - \alpha(t) \rho u &= 0, \\
\Delta \Phi &= -\rho - m_{\text{total}}, \\
\rho(0, x) &= \rho_0(0, x), \quad u(0, x) = u_0(x),
\end{align*}
\]

where \( \Phi(t, x) \) and \( m_{\text{total}} = \int \rho_0(x) dx \) are the electrostatic potential and total mass, respectively. \( \alpha = 0 \) coincides with the pressureless Euler system. \( \alpha = 1 \) stands for repulsive forces and attractive forces with \( \alpha = -1 \).

The results for existence theories can be found in [11–14]. For the blowup results for compressible Euler-Poisson equations without time-dependent damping \( \alpha(t) = 0 \), see [15–18] with repulsive forces and [17, 19, 20] with attractive forces. In this paper, we established the finite time blowup result for the Euler-Poisson equations with time-dependent damping \( \alpha(t) ≥ 0 \) and attractive forces, under suitable initial conditions.

At last, we are concerned with the classical solutions to the 2D ideal compressible transverse MHD flow. Let \( x ∈ \mathbb{R}^3 \), and the 3D compressible isentropic MHD system reads

\[
\begin{align*}
\partial_t \rho + \text{div} (\rho u) &= 0, \\
\partial_t (\rho u) + \text{div} (\rho u \otimes u) + \nabla \cdot (\mu \nabla \rho) &= 0, \\
\partial_t B &= \text{curl} (u \times B), \\
\text{div} B &= 0, \\
\rho(0, x) &= \rho_0(0, x), \quad u(0, x) = u_0(x), \quad B(0, x) = B_0(x),
\end{align*}
\]

where \( \rho, u = (u_1, u_2, u_3) \), \( p = p(\rho) = \rho_0 \rho^\gamma \), and \( B = (B_1, B_2, B_3) \) denote the density, velocity, pressure, and magnetic field. The “ideal” means that there is no viscosity or resistivity in the system (5). In this paper, we consider the 2D transverse flow as follows:

\[
\begin{align*}
\rho(t, x) &= \rho(t, x_1, x_2), \\
u(t, x) &= (u^1(t, x_1, x_2), u^2(t, x_1, x_2), 0), \\
B(t, x) &= (0, 0, B(t, x_1, x_2)).
\end{align*}
\]

The divergence-free condition of the magnetic field \( B \) is naturally fulfilled. Let \( x = (x_1, x_2) \), \( \nabla = (\partial_1, \partial_2) \), and \( u = (u^1, u^2) \); then, (5) with the structural assumption (6) can be simplified as the following 2D hyperbolic system:

\[
\begin{align*}
\partial_t \rho + \nabla \cdot (\rho u) &= 0, \\
\partial_t (\rho u) + \nabla \cdot (\rho u \otimes u) + \nabla \left( p(\rho) + \frac{1}{2} B^2 \right) &= 0, \\
(\partial_t + u \cdot \nabla) B + B \cdot \nabla u &= 0, \\
\rho(0, x) &= \rho_0(x), \quad u(0, x) = u_0(x), \quad B(0, x) = B_0(x).
\end{align*}
\]

In [21], under similar initial condition to that of [22], the author obtained the global existence of classical solution of the transverse MHD flow. For the large initial data in 3D and small data in 1D, Rammaha [23] proved the finite time blowup of solutions to the MHD system. In this article, under some suitable conditions, we proved the blowup results for the 2D transverse flow.

The paper is organized as follows. In Section 2, we will present our main results. In Sections 3–5, by using the spectral dynamics method as in [10, 19, 20, 24], some Riccati-type equations are achieved, and then, we will give the proof of the main results.

2. Main Results

In this section, we will introduce our main results in the paper. Now, we give the first result. To this purpose, some conditions on the initial data of the Euler system (1) are stated as follows.

**Hypothesis HE.** There exists \( a_1 ∈ \mathbb{R}^n \) such that

\[
\begin{align*}
\sum_{0 ≤ t ≤ 2} \left| \nabla \rho_0^{\gamma - 1}(a_1) \right| + \sum_{|t| = 1, 2} \left| \nabla \rho_0 S_0(a_1) \right| &= 0, \\
|S_0(a_1)| &< ∞,
\end{align*}
\]

\[
\begin{align*}
\text{div} u_0(a_1) &≤ -n \left( \int_0^t e^{-\int_0^s a(t) \, dt} \, ds \right)^{1-1}, \\
\text{for some } t^* > 0, \\
(\partial_i u_0^j - \partial_j u_0^i)(a_1) &= 0, \quad i, j = 1, \ldots, n.
\end{align*}
\]

Define the first blowup time \( t^*_E \) by

\[
\int_0^{t^*_E} e^{-\int_0^s a(t) \, dt} \, ds = \inf_{a_1} \frac{-n}{\text{div} u_0(a_1)},
\]

where the infimum taking over all the \( a_1 \) verifies the Hypothesis HE. We assume that the infimum is reached at \( a_1 \), and for convenience, we omit the tilde above \( a_1 \). The aforementioned two conventions also work in the following part. It is easy to check that \( t^*_E \) is well defined, since the term in the integration on the left side of (9) is positive. On the other hand, the Hypothesis HE implies that \( t^*_E > 0 \).

**Theorem 1.** Let \( (\rho, u, S) \) be the \( C^1 \)-smooth solution to the system (1) with \( γ > 1, a(t) ≥ 0 \), and the initial data \( (\rho_0, u_0, S_0) \).

Suppose that \((\rho_0, u_0, S_0)\) satisfies the Hypothesis HE; then, the classical solution will blow up before or at \( t^*_E \).
Remark 2. Now, we give the conditions on $\text{div}\ u_0$ in Hypothesis HH and the blowup time $t^*_E$ for some special choices of $\alpha(t)$. It is not hard to check that

$$
\alpha(t) \equiv 0, \quad \text{div}\ u_0(a_t) < 0, \quad t^*_E = \inf_{a_t} \frac{-n}{\text{div}\ u_0(a_t)},
$$

$$
\alpha(t) \equiv 1, \quad \text{div}\ u_0(a_t) < -n, \quad t^*_E = -\sup_{a_t} \ln \left(1 + \frac{n}{\text{div}\ u_0(a_t)}\right).
$$

(10)

The condition on $\text{div}\ u_0$ and the blowup time in [10] are $\text{div}\ u_0(a_t) < -n - 1$ and $T^* \leq \inf (n(n + 1)/|\text{div}\ u_0(a_t)|)$. Choose $\inf\text{div}\ u_0(a_t) = -2n$; then, our bound is strictly less than their bound $t^*_E = \log 2 < ((n + 1)/2)$. In addition, for $\alpha(t) = (\alpha/(1 + t))$ with nonnegative constant $\alpha$, we achieve

$$
\begin{align*}
\alpha < 1, & \quad \text{div}\ u_0(a_t) < 0, \quad t^*_E = \inf_{a_t} \left(1 + \frac{n(1-\alpha)}{\text{div}\ u_0(a_t)}\right)^{(1-\alpha)/\alpha} - 1, \\
\alpha = 1, & \quad \text{div}\ u_0(a_t) < 0, \quad t^*_E = \inf_{a_t} e^{-\alpha\text{div}\ u_0(a_t)} - 1, \\
\alpha > 1, & \quad \text{div}\ u_0(a_t) < -n(\alpha - 1), \quad t^*_E = \inf_{a_t} \left(1 + \frac{n(\alpha - 1)}{\text{div}\ u_0(a_t)}\right)^{-1/(\alpha-1)} - 1.
\end{align*}
$$

(11)

We also make some assumptions on the initial data of the Euler-Poisson system (4).

**Hypothesis $H_{EP}$**. There exists $a_2 \in \mathbb{R}^n$ such that

$$
\rho_0(a_2) > 0, \quad \text{div}\ u_0(a_2) < -n \left(\frac{1}{2} \alpha(0) + m_0\right),
$$

$$
\left(\partial_t u_0^i - \partial_j u_0^j\right)_{(a_2)} = 0, \quad i, j = 1, \ldots, n,
$$

$$
t^*_{EP} = \frac{1}{2m_0} \inf_{a_t} \ln \frac{nm_0 - (n/2)\alpha(t) - \text{div}\ u_0(a_t)}{-nm_0 - (n/2)\alpha(t) - \text{div}\ u_0(a_t)} > 0,
$$

(12)

where the constant $m_0$ is defined by

$$
m_0 = \sqrt{\frac{1}{n} m_{\text{total}} + \frac{1}{4} \alpha^2(0)} > 0.
$$

(13)

The second result in the present paper is stated in the following.

**Theorem 3.** Let $(\rho, u, \Phi)$ be the smooth solution to the system (4) with $\delta = -1, \alpha(t) \geq 0, \alpha'(t) \leq 0$, and the initial data $(\rho_0, u_0)$. Suppose that $(\rho_0, u_0)$ satisfies the Hypothesis HEP; then, singularity will be developed before or at $t^*_{EP}$.

At last, we turn to our last result. The initial data of the MHD system (7) is assumed to verify.

**Hypothesis $H_{MHD}$**. There exists $a_3 \in \mathbb{R}^2$ such that

$$
\left|\frac{B_0(x)}{\rho_0(x)}\right| < \infty, \quad \forall x \in \mathbb{R}^2,
$$

$$
\sum_{\varepsilon \in \varepsilon_2} \left|\nabla t^\varepsilon \rho_0(a)\right| + \sum_{\varepsilon \in \varepsilon_2} \left[\nabla \left(\frac{\partial t^\varepsilon}{\rho_0} \rho_0(a)\right)\right] + \left|\nabla \left(\frac{B_0(a)}{\rho_0}\right)\right| = 0,
$$

$$
\left(\partial_t u^0_0 - \partial_t u^0_1\right)_{(a_t)} = 0, \quad \text{div}\ u_0(a_t) < 0,
$$

$$
t^*_{MHD} = \inf_{a_t} \frac{-n}{\text{div}\ u_0(a_t)} > 0.
$$

(14)

**Theorem 4.** Let $(\rho, u, B)$ be the smooth solution to the system (7) with the initial data $(\rho_0, u_0, B_0)$. Suppose that $(\rho_0, u_0, B_0)$ satisfies the Hypothesis $H_{MHD}$ with $\gamma > 1$; then, the solution will blowup before or at $t^*_{MHD}$.

**Remark 5.** The special case $B \equiv 0$ in (5) coincides with the isentropic Euler system (1).

**Remark 6.** If $\rho(t, x) > 0$, the first and third equations in (7) derive the following well-known “frozen” law:

$$
\left(\partial_t + u \cdot \nabla\right) \frac{B}{\rho} = 0.
$$

(15)

Under the assumption $|B_0(x)/\rho_0(x)|<\infty$, in the Hypothesis $H_{MHD}$, it is natural to define $B(t, x) = 0$ when $\rho(t, x) = 0$. For the special case $B_0(x)/\rho_0(x) \equiv 1, B(t, x) \equiv \rho(t, x)$ holds for any $t > 0$ when the smooth solution exists.

**Remark 7.** We give some examples of the Hypothesis $H_{MHD}$. Near the point $a_3$

$$
\rho_0(x) = \begin{cases} 
|x - a_3|^{3(\gamma - 1)/2}, & \gamma \geq 3, \\
|x - a_3|^{6(\gamma - 1)/2}, & 1 < \gamma < 3.
\end{cases}
$$

(16)

**Remark 8.** The sign conditions of $\text{div}\ u_0$ in (8) and (14) are necessary. If the spectrum of $\nabla u_0$ is nonnegative, global existence was given in [21, 22].

### 3. Blowup of the Euler System with Time-Dependent Damping

In this section, we deal with the Proof of Theorem 1. Before the proof, a new reformulation of (1) is achieved as follows.
Lemma 9. Under the assumption that \((\rho, u, S) \in C^1\), the equations in (1) can be rewritten as

\[
\begin{align*}
D_t \rho + \rho \, \text{div} \, u & = 0, \\
D_t u + \frac{1}{\rho} \nabla p & = -\alpha(t) u, \\
D_t S & = 0,
\end{align*}
\]

where \(D_t = \partial_t + u \cdot \nabla\) denotes the material derivative.

Proof. The first two equations in (17) are trivial, and we focus on the last equation of the entropy \(S\). Multiplying the second equation in (17) by \(\rho u\), it implies that

\[
\partial_t \left( \frac{1}{2} \rho |u|^2 \right) + \text{div} \left( \frac{1}{2} \rho u |u|^2 \right) + u \cdot \nabla p = -\alpha(t) |u|^2. \tag{18}
\]

Subtracting the third equation in (1) by the above identity, then

\[
\partial_t (\rho e) + \text{div} \, (\rho u e) + \rho \, \text{div} \, u = 0. \tag{19}
\]

This, together with (2), derives

\[
\rho (\partial_t + u \cdot \nabla) S + \frac{P}{\rho} (\partial_t + u \cdot \nabla) \rho + \rho \, \text{div} \, u = \rho (\partial_t + u \cdot \nabla) S = 0, \tag{20}
\]

which implies \(D_t S = 0\) in \(\{(t, x)|\rho(t, x) > 0\}\). Noting the \(C^1\)-smoothness of \(S\), it is natural to define \(D_t S = 0\) in \(\{(t, x)|\rho(t, x) = 0\}\). This completes the proof of Lemma 9.

Proof of Theorem 1. Denote \(U := \nabla u\) and apply \(V\) to the second equation in (17); then, we achieve

\[
D_t U = -UU - \nabla \left( \frac{1}{\rho} \nabla p \right) - \alpha(t) U. \tag{21}
\]

Let \(U_+ = (1/2)(U + U^T)\), and we deduce from (21) that

\[
D_t U_+ = -U_+ U_+ - U_- U_- - \nabla \left( \frac{1}{\rho} \nabla p \right) - \alpha(t) U_+, \tag{22}
\]

\[
D_t U_- = -U_+ U_+ - U_- U_- - \nabla \left( \frac{1}{\rho} \nabla p \right) - \alpha(t) U_. \tag{23}
\]

where \(\nabla((1/\rho)\nabla p)_+ = (1/2)[\nabla((1/\rho)\nabla p) + (\nabla((1/\rho)\nabla p))^T]\).

Now, we turn to the gradient of the pressure. According to (3) with \(\gamma > 1\), we find that

\[
\nabla \left( \frac{1}{\rho} \nabla p \right) = \nabla \left( \frac{1}{\rho} (p_0 \rho^\gamma e) \right) = p_0 \nabla \left( e \frac{1}{\rho} \nabla \rho^\gamma \right) + p_0 \nabla \left( \rho^\gamma - 1 \nabla e \right) = \frac{p_0 e}{\gamma - 1} \nabla \rho^\gamma - 1 + p_0 \nabla \left( \rho^\gamma - 1 \nabla e \right) + p_0 \rho^\gamma - 1 \nabla e. \tag{24}
\]

Define the flow line \(X(t, a)\) starting from \(a\) by

\[
\frac{d}{dt} X(t, a) = u(t, X(t, a)), \quad X(0, a) = a. \tag{25}
\]

Then, we conclude from the first and third equations in (17) that

\[
\sum_{b \in \Sigma |e \leq a} |\nabla \rho^\gamma - 1(t, X(t, a_t))| \leq \sum_{\alpha \in \Sigma} |\nabla \rho^\gamma - 1(t, X(t, a_t))|, \tag{26}
\]

where \(c_0\) is a positive constant. On the other hand, due to the Hypothesis \(\text{HE}\) on \(U_0(0, a_1)\) and the fact that equation (23) is homogenous in \(U_\omega\) we obtain \(U_\omega(t, X(t, a_1)) = 0\).

Thereafter, we have proved that

\[
\frac{d}{dt} U_+(t, X(t, a_1)) \leq -(U_+, U_+)(t, X(t, a_1)) - \alpha(t) U_+(t, X(t, a_1)). \tag{27}
\]

This, together with

\[
\text{trace}(U_+, U_+) \geq \frac{1}{n} \text{trace}(U_+)^2 = \frac{1}{n} \left( \text{div} \, u \right)^2, \tag{28}
\]

implies

\[
\frac{d}{dt} \left( \frac{1}{n} \text{div} \, u(t, X(t, a_1)) \right) \leq -\frac{1}{n} \left( \text{div} \, u \right)^2(t, X(t, a_1)) - \alpha(t) \text{div} \, u(t, X(t, a_1)). \tag{29}
\]

Let \(y(t) = -(1/n)e^{\int_0^t \alpha(t) dt} \text{div} \, u(t, X(t, a_1));\) then, we find that

\[
y'(t) \geq e^{\int_0^t \alpha(t) dt} y_1(t) \geq 0. \tag{30}
\]
This means \( y_1(t) \geq y_1(0) > 0 \) and

\[
y_1(t) \geq \frac{y_1(0)}{1 - y_1(0)\int_0^t e^{-\int_0^s a(r) ds} dr} \longrightarrow +\infty, \quad t \longrightarrow t_E^*(t < t_*^E). \tag{31}
\]

Note that there exists \( C(t_E^*) > 0 \) such that for any \( t \in [0, t_E^*) \)

\[
\frac{1}{C(t_E^*)} \leq e^{\int_0^t a(r) dr} \leq C(t_E^*). \tag{32}
\]

Then, we infer that \( \text{div } u(t, X(t, a_t)) \) blows up before or at \( t_E^* \). This completes the Proof of Theorem 1.

### 4. Blowup of the Euler-Poisson System with Time-Dependent Damping

In the present section, we turn our attention to the Proof of Theorem 3.

**Proof of Theorem 3.** Similar to (22) and (23), we can obtain

\[
D_t U_+ = -U_+ U_+ - U_- U_- - \nabla^2 \Phi - \alpha(t) U_+, \tag{33}
\]

\[
D_t U_- = -U_+ U_- - U_- U_+ - \alpha(t) U_-. \tag{34}
\]

By taking the trace of the two sides of equation (33), we find that

\[
\begin{align*}
D_t \text{div } u &= -\text{trace}(U_+ U_+) - \text{trace}(U_- U_-) \\
&- \Delta \Phi - \alpha(t) \text{div } u \leq -\frac{1}{n}(\text{div } u)^2 \\
&- \text{trace}(U_+ U_-) - (\rho - m_{\text{total}}) - \alpha(t) \text{ div } u,
\end{align*}
\tag{35}
\]

where we have used (28) again. We conclude from the Hypothesis HEP and (34) that \( U_-(t, X(t, a_t)) = 0 \).

Let \( y_2(t) := (1/n) \text{ div } u(t, X(t, a_t)) + (1/2)\alpha(t); \text{ then, it is easy to check that} \)

\[
y'_2(t) \leq -y_2^2(t) + \frac{m_{\text{total}}}{n} + \frac{1}{4} \alpha^2(t) + \frac{1}{2} \alpha'(t) \\
\leq -y_2^2(t) + \frac{1}{n} m_{\text{total}} + \frac{1}{4} \alpha^2(0) = -y_2^2(t) + m_0^2, \tag{36}
\]

where we have used the fact \( \alpha'(t) \leq 0, \) and \( m_0 \) is defined in (13). Now, we make the ansatz that before the formation of singularity

\[
y_2(t) < -m_0 < 0, \tag{37}
\]

which derives \( y_2^2(t) > m_0^2. \) Thereafter, we find that

\[
\frac{d}{dt} \ln \frac{m_0 - y_2(t)}{-m_0 - y_2(t)} = \frac{2m_0 y'_2(t)}{y_2^2(t) - m_0^2} \leq -2m_0, \tag{38}
\]

Integrating the above differential inequality yields

\[
1 + \frac{2m_0}{-m_0 - y_2(t)} = \frac{m_0 - y_2(t)}{-m_0 - y_2(t)} \leq e^{-2m_0 t} \frac{m_0 - y_2(0)}{-m_0 - y_2(0)}.
\tag{39}
\]

This, together with \( (m_0 - y_2(0))/(m_0 - y_2(0)) = 1 + (2nm_0/\langle -nm_0 - (n/2)\alpha(0) - \text{ div } u_0(a_2) \rangle) > 1 \) which follows from the Hypothesis HEP, implies

\[
-m_0 - y_2(t) \leq e^{-2m_0 t} (m_0 y_2(t))/(m_0 y_2(t)) - 1 \longrightarrow +\infty, \\
\quad t \longrightarrow t_E^*(t < t_*^E). \tag{40}
\]

According to the boundedness of \( \sup_{t \in [0, t_0]} |\alpha(t)| \), we achieve that \( \text{div } u(t, X(t, a_t)) \) blows up before or at \( t_E^* \). On the other hand, we also conclude from (40) that \( -m_0 - y_2(t) > 0 \).

Consequently, the ansatz (37) makes sense. This completes the proof of Theorem 3.

### 5. Blowup of 2D Ideal Compressible Transverse MHD Flow

In the last section, we finish the proof of Theorem 4.

**Proof of Theorem 4.** The Proof of Theorem 4 is analogous with that of Theorem 1. Instead of (22) and (23), we achieve

\[
D_t U_+ = -U_+ U_+ - U_- U_- - \nabla^2 \Phi - \alpha(t) U_+, \tag{41}
\]

\[
D_t U_- = -U_+ U_- - U_- U_+ - \alpha(t) U_.
\]

According to the “frozen” law, it is convenient to regard \( B/\rho \) as a new unknown. Consequently, similar to (24), we find that

\[
\frac{\text{div } u(t, X(t, a_t))}{\rho} = -\frac{1}{n}(\text{div } u)^2(t, X(t, a_t)),
\tag{43}
\]

By an analogous argument as in Section 2 and the Hypothesis HMHD, we finally derive

\[
\frac{d}{dt} \text{div } u(t, X(t, a_t)) \leq -\frac{1}{n}(\text{div } u)^2(t, X(t, a_t)),
\tag{43}
\]
which yields
\[
\text{div } u(t, X(t, a_3)) \leq \frac{n \text{ div } u_0(a_3)}{n + t \text{ div } u_0(a_3)} \longrightarrow -\infty, \\
t \longrightarrow t^*_\text{MHD} = \inf_{a_3} \frac{-n}{\text{ div } u_0(a_3)}.
\] (44)

**Data Availability**

No data were used to support this study.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

**Acknowledgments**

Wenming Hu was supported by the NSFC (Nos. 11601236 and 11971237) and by the Natural Science Foundation of the Jiangsu Higher Education Institutions of China (No. 19KJA320001).

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