Non-Hermitian Majorana Modes Protect Degenerate Steady States

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Majorana zero modes (MZMs) ensure robust spectral degeneracies in topological phases of superconductors. We demonstrate that non-Hermitian generalizations of MZMs are responsible for a degeneracy in the complex spectrum which describes dynamics of a continuously-observed many-body system, implying a degenerate steady state. Specifically, we study coherent, measurement-induced, non-unitary evolution of a one-dimensional (1D) transverse field Ising model (TFIM) where spins are allowed to decay to a third level by emitting a photon into a cavity. Within a Lindblad approach, the effective Hamiltonian governing the two-level system in the absence of photonic emission can be mapped to a non-Hermitian topological superconductor (TSC). Using a generalization of particle-hole symmetry, we argue for the stability of MZMs which results in a robust two-fold spectral degeneracy. Increasing the decay rate above a critical amount can close a bulk gap, and we uncover the resulting topological phase diagram. Qualitative features of our study generalize to two-dimensional (2D) chiral TSCs. Our work provides an example of a dynamical quantum system which hosts symmetry-protected topological order that lies beyond the Hermitian tenfold way.

Introduction. Recent experimental \cite{1,2} and theoretical \cite{2,21} efforts have aimed at generalizing topological band theory to non-Hermitian operators which generate the dynamics of open systems. While origins of this field primarily began with motivations from classical topological photonics in the presence of gain or loss, more recent studies suggest that the formalism is very relevant for quantum many-body physics \cite{21,29}.

In this Letter, we use the framework of non-Hermitian topology to demonstrate that a dissipative quantum many-body system can possess symmetry-protected topologically-degenerate steady states, in analogy with the equilibrium paradigm of topologically-degenerate fermionic ground states. The zero-temperature ground state of fermions can exhibit a “topological degeneracy,” e.g. orthogonal ground states are related to each other by a nonlocal zero-energy Majorana excitation: $|\mathrm{gnd}_1\rangle = \alpha_{\mathrm{gnd}_1} |\mathrm{gnd}_2\rangle$ \cite{32}. In open quantum systems, there is no notion of a ground state, but systems generally evolve into a steady state. We provide an example where a coherent, partially-projected, many-body system possesses topologically-degenerate steady states.

The tenfold way \cite{31} provides a useful starting point to discuss symmetry-protected topological phases in Hermitian systems. In a non-Hermitian setting there are more than ten random matrix ensembles, called the Bernard- LeClair (BL) classes \cite{32} which generalize the Altland-Zirnbauer (AZ) classes \cite{33} in the absence of Hermiticity. The enlarged set of unique BL classes are based on four symmetries, described below. Any Hamiltonian matrix $H$ can possess $K, C, Q, P$ symmetries, defined as

\begin{align*}
K: \quad H &= \epsilon_k k H^s k^\dagger, \quad kk^* = \pm I \quad (1) \\
C: \quad H &= \epsilon_c c H^T c^\dagger, \quad cc^* = \pm I \quad (2) \\
Q: \quad H &= \epsilon_q q H^1 q^\dagger, \quad q^2 = I \quad (3) \\
P: \quad H &= -p H p^\dagger, \quad p^2 = I \quad (4)
\end{align*}

where $\epsilon_{k,c,q} = \pm 1$, $I$ is the identity, and the transformation matrices $k, c, q, p$ must be unitary. $K, C$ symmetries are reminiscent of time-reversal ($\epsilon_k = \epsilon_k = (+1)$) and particle-hole ($\epsilon_c = \epsilon_k = (-1)$) symmetry of the AZ classification with the observation that $H^T$ is no longer equal to $H^T$ for non-Hermitian models. $P$ symmetry is guaranteed for any model with both $K$ and $C$ symmetry; $P$ symmetry is equivalent to the AZ chiral symmetry. To obtain a proper understanding of symmetry-protected topological phases in the absence of Hermiticity, it is useful to consider the relationships given above.

Very recent studies have constructed a topological periodic table using the BL classes as a basis, dubbed the 38-fold way \cite{34,36}. In this work, we discuss a physical system which hosts edge modes which are protected by BL symmetries. We study a non-Hermitian superconductor with particle-hole symmetry which obeys:

$H_k = s H^T_k s^1$ but explicitly breaks the conventional expression: $H_k \neq s H^T_k s^1$ where $s$ transforms particles to holes. In the language of the BL classification, the model possesses a $C$ symmetry but breaks $K$. In fact, the $C$ symmetry (not $K$) is fundamental for all fermionic Bogoliubov-de Gennes (BdG) models due to the inherent particle-hole structure \cite{34}. We show that this non-Hermitian particle-hole symmetry can be used to protect topological Majorana modes in both 1D and 2D, which are responsible for robustly degenerate steady states.

Setup, model, and particle-hole symmetry. Before specializing to the many-body case, we first discuss a single atomic three-level system, with states: $|\uparrow\rangle, |\downarrow\rangle, |c\rangle$. We consider a scenario similar to Ref. \cite{36}: The two-level system $|\uparrow\rangle, |\downarrow\rangle$ undergoes unitary time evolution according to some Hermitian Hamiltonian $\mathcal{H}$. The only way for an atom to end up in the state $|c\rangle$ is by starting in the state $|\uparrow\rangle$ and spontaneously emitting a photon into a cavity mode. (See Fig. 1) In the Lindblad formalism \cite{37}, the quantum jump operator of this dissipative three-level system
Then in the absence of photonic emission, the system is projected onto a two-level subspace which undergoes coherent, non-unitary dynamics. The resulting steady state can be degenerate. (Emission from up to down can be disallowed by selection rules.)

In this study, we suppose that we can constantly check for the absence of photonic emission, the three-level system is a “dark mode” of the Lindbladian. By checking for the absence of photonic emission, the three-level system is projected onto a two-level subspace which undergoes coherent, non-unitary dynamics. The resulting steady state can be degenerate. (Emission from $\uparrow$ to $\downarrow$ can be disallowed by selection rules.)

The non-equilibrium dynamics is described by the master equation:

$$\frac{d\rho}{dt} = -i\left(\mathcal{H}_\text{eff}\rho - \rho\mathcal{H}_\text{eff}^\dagger\right) + 2\gamma L\rho L^\dagger$$

where $\rho$ is the density matrix of the three-level system, $\mathcal{H}_\text{eff} = \mathcal{H} - i\gamma L^\dagger L = \mathcal{H} - i\gamma |\uparrow\rangle\langle\uparrow|$, and $\gamma$ is the emission rate. This has a convenient physical interpretation known as the quantum stochastic wavefunction: In a time step $dt$, a system prepared in a pure state will either evolve coherently according to a non-Hermitian effective Hamiltonian $\mathcal{H}_\text{eff}$, or a “quantum jump event” will decouple the system by moving a pure state from $|\psi\rangle$ to $L|\psi\rangle$ [38, 39]. Averaging over all such trajectories will produce the same expectation values as formally solving the Lindblad master equation for the evolution of the density matrix.

In this study, we suppose that we can constantly check if a photon has been emitted. (In practice this can be achieved via postselection by measuring the occupation of the $c$ state [35].) Partially-projected spin systems have been experimentally probed for a single atom [10, 11], and many-body systems have been investigated theoretically [10, 12]. Lack of emission leads to coherent, non-unitary dynamics generated by $\mathcal{H}_\text{eff}$ which represents exponential decay in the probability of finding the system in the $|\uparrow\rangle$ state as a function of time. This agrees with our intuition: Conditioning on the fact that no photons are emitted, the probability of finding the spin system in the up state decreases over time. In what follows, we will focus on an $N$-particle generalization of this setup. The many-body steady state can exhibit a robust degeneracy, and we uncover a phase diagram for this interacting, dissipative spin system by mapping the problem to a non-Hermitian topological superconductor.

Consider $N$ of such three-level systems, which interact coherently according to a transverse-field Ising model. Then in the absence of photonic emission, the $\uparrow, \downarrow$ subsystem evolves via the effective Hamiltonian:

$$\mathcal{H} = -J \sum_n \sigma_n^x \sigma_{n+1}^x + u \sum_n \sigma_n^x - i\gamma \sum_n (\sigma_n^+ + 1)$$

where $\sigma_n^x$ represents the $n$th Pauli spin operator on the lattice site $n$ of $N$. (In the following analysis, we drop the term proportional to the identity since this does not affect dynamics.) The first two terms represent the standard Hermitian TFIM, while the last term represents the non-Hermitian contribution responsible for non-unitary dynamics of the eigenstates. The spectrum of (6) is generically complex, and an arbitrary initial state will tend to the steady state, which is defined as the state with the largest imaginary part of its eigenvalue. The many-body spectrum can be found exactly by performing a Jordan-Wigner transformation [33] to fermionic degrees of freedom

$$\sigma_j^+ = \exp\left(-i\pi \sum_{k=1}^{j-1} c_k^\dagger c_k\right) c_j^\dagger, \quad \sigma_j^- = 2c_j^\dagger c_j - 1.$$  

The Hamiltonian reads:

$$\mathcal{H} = 2\bar{u} \sum_n c_n^\dagger c_n - J \sum_n \left( c_n^\dagger c_{n+1} + c_n c_{n+1}^\dagger + h.c. \right)$$

where $\bar{u} = u - i\gamma$, and $c_n$ represents a complex spinless fermion on site $n$. The Hamiltonian is rewritten as

$$\mathcal{H} = \frac{1}{2} \left( c^\dagger c \right) H \left( c c^\dagger \right)$$

where $c = (c_1, \ldots, c_N)$ and $H$ is a $2N \times 2N$ non-Hermitian matrix.

In the language of the Bernard-LeClair scheme, the particle-hole structure of the BdG formalism imposes a $C$ symmetry on the Hamiltonian, but explicitly breaks $K$ symmetry:

$$C : \quad H = -\Sigma_x H^T \Sigma_x \quad (10)$$

$$K : \quad H \neq -\Sigma_x H^* \Sigma_x \quad (11)$$

where $\Sigma_x = \sigma_x \otimes \mathbb{I}_N$. We emphasize that in the Hermitian limit, $H^* = H^T$ such that $C$ and $K$ symmetries are redundant. Once non-Hermitian terms are added, $K$ symmetry is violated while $C$ remains. In contrast to previous studies [10, 11], the model studied above does not possess $PT$ symmetry and hence energies will generically be complex. It also does not possess time-reversal symmetry due to the decaying nature of the eigenstates. Nevertheless, we will demonstrate that only $C$ symmetry is needed to protect Majorana zero modes in 1D and 2D.

To diagonalize the Hamiltonian, we make a transformation to fermionic quasiparticles:

$$\left( \begin{array}{c} q \\ p^\dagger \end{array} \right) = \left( \begin{array}{c} c^\dagger \\ c \end{array} \right) V, \quad \left( \begin{array}{c} p^\dagger \\ q \end{array} \right) = V^{-1} \left( \begin{array}{c} c^\dagger \\ c \end{array} \right).$$

The two flavors of quasiparticles $q, p^\dagger$ arise due to the right and left eigenvectors of non-Hermitian matrices [13]. The $C$ symmetry of the BdG Hamiltonian imposes a structure on the transformation matrix:

$$V = \Sigma_x \left( V^{-1} \right)^T \Sigma_x.$$  

FIG. 1. A two-level system $\uparrow, \downarrow$ can decay to a third level $c$ by starting in $\uparrow$ and spontaneously emitting a photon into a cavity. (The $c$ state is a “dark mode” of the Lindbladian.) By checking for the absence of photonic emission, the three-level system is projected onto a two-level subspace which undergoes coherent, non-unitary dynamics. The resulting steady state can be degenerate. (Emission from $\uparrow$ to $\downarrow$ can be disallowed by selection rules.)
Remarkably, this expression guarantees that quasiparticles obey generalized fermionic statistics:

$$\{p_i^\dagger, q_j\} = \delta_{i,j}, \ \{p_i, p_j^\dagger\} = \{q_i, q_j^\dagger\} = 0.$$  \hspace{1cm} (14)

The diagonalized second-quantized Hamiltonian reads:

$$\mathcal{H} = \frac{1}{2} (q \ p^\dagger) \Lambda \left( q^\dagger \right) = \sum_i E_i \left( p_i^\dagger q_i + \frac{1}{2} \right)$$  \hspace{1cm} (15)

where $\Lambda = \text{Diag}[-E_1, \ldots, -E_N, E_1, \ldots, E_N]$, $\text{Re} \ E_i > 0$ is a diagonal matrix whose entries correspond to the energies of the system. The quasiparticle vacuum state is defined as: $q_i |\text{vac}\rangle = 0$, and an excited state with energy $E_i$ is $p_i^\dagger |\text{vac}\rangle$.

**Stability of Majorana modes.** Before calculating the spectrum, we generalize the stability of Majorana zero modes to include robustness against non-Hermitian terms in the Hamiltonian. In a 1D Hermitian TSC (e.g. the Kitaev chain [30]), the MZM is protected at zero energy due to its particle-hole symmetry:

$$\psi_0 \propto \Sigma \psi_0^* \implies E_0 = 0 \hspace{1cm} (16)$$

where $H \psi_0 = 0$. Any term entering the Hamiltonian which preserves the bandgap cannot perturb the MZM away from zero energy.

In the non-Hermitian case, each eigenvalue $E$ has an associated right and left eigenvector, defined as

$$H \psi = E \psi$$  \hspace{1cm} (17)

$$H^\dagger \lambda = E^* \lambda.$$  \hspace{1cm} (18)

The non-Hermitian MZM satisfies the condition

$$\psi_0 \propto \Sigma \lambda_0^* \implies E_0 = 0. \hspace{1cm} (19)$$

We find that the $C$ symmetry protects the MZM at zero energy in direct analogy with the Hermitian case. This ensures a two-fold degeneracy in the many-body spectrum of the dynamical system. Our analysis generalizes the protection of MZMs in a TSC with respect to Hermiticity-breaking terms in the Hamiltonian.

**Many-body spectrum.** The spectrum of the system for weak decay is given in Fig. 2. We find that the energy of the MZM remains unchanged upon inclusion of decay, in agreement with the analysis from the previous section. The same cannot be said about bulk modes, all of which acquire a non-zero imaginary component to their energy. Interestingly, we find that some quasiparticles get amplified while others decay. (In order to properly calculate observables we must renormalize the wavefunction after time evolving $\exp(-i\mathcal{H}t)$.) We can understand this behavior by examining the composition of the quasiparticles in terms of electrons and holes. Quasiparticles which are mostly composed of electrons ($c$ terms) acquire a negative imaginary energy indicating growth, while modes composed of holes ($c$ terms) acquire a positive imaginary energy indicating decay. Intuitively this agrees with the idea that electronic dissipation leads to the proliferation of holes. MZMs are equally composed of electrons and holes, hence their imaginary component is zero.

What does a complex energy spectrum imply for the dynamics of the system? An arbitrary initial state can be rewritten as a superposition of quasiparticles $p_i^\dagger$ acting on the vacuum. Upon evolving the state in time via $\exp(-i\mathcal{H}t)$, the terms with the most hole-like quasiparticles will start to dominate the wavefunction, since these are the modes which get amplified in time. The eigenstate with the largest imaginary energy will dominate the steady-state behavior. Each many-body eigenstate has a two-fold degeneracy due to the MZMs which couple to form a nonlocal zero mode.

**Phase diagram in the presence of decay.** We have seen that the presence of weak decay (small $\gamma$) leaves the Majorana mode pinned at zero energy while bulk modes generically pick up a complex dispersion. As the decay rate is further increased, it is possible to induce a topological phase transition via closing a band gap. In order to uncover the topological phase boundary, it is easiest to examine the Bloch Hamiltonian by transforming to mo-
momentum coordinates \( k \). The bulk dispersion is found to be
\[
E_k = \pm 2\sqrt{(u - i\gamma - J \cos k)^2 + J^2 \sin^2 k}.
\]  
We easily identify the band closing points which occur at the critical values: \( u^2 + \gamma^2 = J^2 \). The phase diagram is depicted in Fig. 3. This agrees with our intuition: If we consider the non-Hermitian TFIM in Eq. (6), this phase boundary corresponds to the statement that spontaneous symmetry breaking will occur whenever the magnitude of the strength of the transverse field is less than that of the nearest-neighbor interaction. Indeed our analysis suggests that the \( \mathbb{Z}_2 \) eigenstate degeneracy of the symmetry-broken TFIM persists in the presence of a complex transverse field.

Two-dimensional TSC. We demonstrate that the qualitative results from our study generalize to two-dimensional models. Consider the simplest TSC in 2D in the presence of uniform electronic loss \( \gamma \):
\[
\mathcal{H}_{2D} = \sum_{m,n} -t \left( c_{m+1,n}^\dagger c_{m,n} + c_{m,n+1}^\dagger c_{m,n} + h.c. \right) \\
+ \left( \Delta c_{m+1,n}^\dagger c_{m,n}^\dagger + h.c. \right) + \left( i\Delta c_{m,n+1}^\dagger c_{m,n}^\dagger + h.c. \right) \\
- (\mu + i\gamma - 4t) c_{m,n}^\dagger c_{m,n} \tag{21}
\]
where we assume a square lattice geometry and \( c_{m,n} \) annihilates a spinless fermion on lattice site \((m, n)\) of an \( N \times N \) square lattice [44]. The terms represent hopping \( t \), pairing \( \Delta \), and the chemical potential \( \mu \). To observe edge modes we impose periodic boundary conditions in the \( y \) direction while maintaining a finite slab in \( x \). We rewrite operators in terms of their Fourier transform:
\[
c_{m,n} = \frac{1}{\sqrt{N}} \sum_{k_y} e^{ik_y n} c_{m,k_y}.
\]  
The Hamiltonian takes the form:
\[
\mathcal{H}_{2D} = \sum_{k_y} c_{k_y}^\dagger H(k_y) c_{k_y} \tag{23}
\]
where \( c_{k_y} = (c_{m=1,k_y}^\dagger, \ldots, c_{m=1,-k_y}^\dagger, \ldots)^T \). The spectrum is found by diagonalizing \( H_{k_y} \) and is given in Fig. 3 for a weakly decaying model. (Henceforth, we use the scalar \( k \) to represent \( k_y \).) Notice that a single edge mode is localized on each side of the chain with opposite group velocity which results in a net chirality.

The MZM is protected at the high-symmetry point in the Brillouin zone due to particle-hole symmetry. The Bloch Hamiltonian satisfies
\[
H(k) = -\Sigma_x H^T(-k) \Sigma_x.
\]  
Non-Hermitian Majorana modes at a high-symmetry point in the Brillouin zone (e.g. \( k = 0 \)), are related via
\[
\psi_{edge}(k = 0) \propto \Sigma_x \lambda_{edge}^*(k = 0). \tag{25}
\]

This again suggests that \( E(k = 0) = 0 \) due to \( C \) symmetry, in agreement with the numerics given in Fig. 4.

Experimental protocol and signatures. The non-Hermitian coherent time evolution described in this study is expected to occur in a continuously-monitored three level system in the absence of a “quantum jump event,” i.e. the observation of a photon in the cavity. We briefly outline the envisioned experimental setup, along with expected signatures.

Consider a chain of spins which is initially prepared in a symmetry-broken state in the \( x \) direction, i.e. \(|\rightarrow \rightarrow \rightarrow \rangle \). We then quench the system by allowing the spin-up state in \( z \) to decay to a third level by emitting a photon into a cavity mode with rate \( \gamma \), and potentially applying a transverse field \( u \). After propagating the system in time, but before the first photonic emission into the cavity, we turn off the evolving Hamiltonian and make measurements in the resulting steady state. We expect this steady state to remain symmetry-broken when \( \gamma^2 + u^2 < J^2 \), i.e. \(|\sigma_x \rangle \neq 0 \), whilst a unique steady-state is chosen in the opposite limit, i.e. \(|\sigma_x \rangle = 0 \). Our simple setup predicts a quantum phase transition for a non-Hermitian extension of the famous transverse-field Ising model.

Conclusions and outlook. In this Letter, we have studied the coherent, non-unitary dynamics of a three-level system coupled to a photonic cavity mode. In the absence of photonic emission into the cavity, the two-level subspace evolves according to a non-Hermitian extension of the transverse-field Ising model. We solved for the spectrum exactly by mapping the system to a dissipative topological superconductor via a Jordan-Wigner transformation. Due to its inherent particle-hole structure, the model possesses a \( C \) symmetry (in the Bernard-LeClair classification) which protects Majorana zero modes at the edge of the sample against non-Hermitian terms in both 1D and 2D. We uncovered a topological phase diagram in the decay rate. In the language of the spin model, we demonstrated that symmetry-breaking phases are expected to occur in com-
plex extensions of the transverse field, so long as the magnitude of the field is less than that of the coupling. Finally, we demonstrated that these ideas generalize in a straightforward fashion to 2D, and proposed a simple experimental protocol to observe this uniquely non-Hermitian phase transition. Our work extends the ideas of topological phase transitions to non-equilibrium many-body setups.

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Note added. Upon completion of this work, a preprint appeared which discusses non-Hermitian Majorana modes in a different physical context [45].

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