1. Introduction

The continuous casting of alloy metals is a very valid and commonly known technology, which is described in detail in both the scientific and the professional literature for example in [1-7]. These papers present numerical methods for the calculation of a thick slab produced in a continuous casting process. For the continuous casting of thin plates specific conditions are present. The relatively low thermal resistance due to its small thickness compared to the thermal resistance contact layer allows it to use a simple analytical model to describe the phenomenon.

A special technology is applied for the continuous casting of thin metal rods. In [8] the slow solidification of a cylinder with a constant heat flux is analyzed. However, the development of the shape of the solidification front has not been investigated. In [9] the shape of the solidification crust has been investigated for thin metal rods. The shape and
development of the solidification front is very important in the continuous casting of thin plates, because an appropriate shape of the solidification front in the casting process guarantees its continuity. The second problem is the shape of the solidification front for the different forms of thin castings. This problem is the main subject of this paper.

Another problem is the role of the contact layer for the flow of heat in continuous casting processes. This effect is very large and thus dominants over other factors. The contact layer is an additional resistance of the flow of heat from the liquid metal to the cold surface. The analysis of the role of the contact layer in a solidification process was recently studied experimentally and theoretically by Loulou et al. [10-12], by Lipnicki [13], by Lipnicki et al. [14] and by Lipnicki and Weigand [15, 16]. Beside other things the impact of the contact layer on continuous casting process for thin metal plates is examined in this paper.

2. Problem formulation and solution

It is assumed that the liquid metal flow with the heat of fusion $\lambda$, the density $\rho$, the kinematic viscosity $\nu$ and the diffusivity of heat $\alpha$, has the mean velocity $u$ and the temperature $T_L$, which is larger than the solidification temperature $T_F$, inside the thin plate channel (see Fig.1). The frozen layer with thickness $H-\delta$ is formed on the cold surface of the channel, which has both the width $2H$ and the length $l$. The channel is assumed to be very narrow, $H/b<<1$ where $b$ the depth of the channel is. The temperature of the cold surface is equal to $T_W$, which is lower than the fusion temperature $T_F$ of the liquid metal. Thus, a contact layer is established between the flowing liquid metal and the cold surface of the channel.

There is a contact layer (small gap), which causes an additional resistance of the heat flow $\lambda(\alpha \cos H)$. Across the contact layer, there is the temperature difference $(\bar{T}-T_w)$; where $\bar{T}$ is the temperature of the inner surface of the frozen layer (see Fig. 2). Between the channel wall and the cooling water with the temperature $T_0$ the resistance $\lambda(\alpha \cos H)$ is present for transferring the heat flux to the water.

Very important among other things is the solidification front, where $2\delta$ denotes the free channel width.

For the development of the solidification front on the cold surface the velocity of the thickness of the frozen layer is given by $-d\delta/dt$. The main goal of this publication is to determine the thickness of the frozen layer depending on the cooling conditions, which are determined by the properties of both the cooling and the liquid metal flow.

3. Theoretical analytical model

The law of conservation of energy with equates the instantaneous latent heat and the heat from the liquid flowing metal by convection to the frozen layer can be described as

$$\alpha (T_e - T_s) - \rho L \frac{d\delta}{dt} \frac{\lambda}{H-\delta} (T_e - \bar{T}) = \alpha_c (\bar{T} - T_w) = \alpha_e (T_w - T_s)$$

The term on the left side of equation (2) denotes the heat from the liquid metal and the heat for the phase change from the solidified metal. The subsequent expressions show the heat flowing through the contact layer and the heat flow to the cooling water.

For the outer surface of the contact layer one obtains from equation (2)
After substituting the temperature $T_w$ and $T_f$ given by equations (3) and (4) into equation (2) the following equation is obtained for the time depending thickness $\delta^{-1}$, describing the position of the solidification front. Hereby the introducing the following dimensionless variables

$$\delta = \frac{\delta}{H}, \quad \tau = \frac{\tau}{H^2}; \quad \beta = \frac{\Delta T}{\beta H}$$

which are respectively the Stephan number, a dimensionless time, the Fourier number, a dimensionless free channel width, the dimensionless thermal resistance for heat transfer to the coolant, the dimensionless thermal resistance of the contact layer and the parameter overheating have been introduced.

Equation (2) can be reduced as follows, after introducing the non-dimensional quantities

$$\frac{\partial}{\partial \tau} \frac{1}{\tau} = \frac{1}{\beta + 1 - \delta}, \quad (6)$$

where the parameter $\beta$ is equal to

$$\beta = \frac{1}{B_i_{\text{CON}}} + \frac{1}{B_i_{\text{L}}}.$$ 

This equation can be integrated after separation of variables to give the solidification time

$$\tau = \int \frac{\beta + 1 - \delta}{\beta + 1 - \delta} d\delta,$$

and if the parameters $\beta$ and $\beta$ are constants the above equation may be integrated

$$\tau = -\frac{1 - \delta}{\beta} - \frac{1}{\beta \delta^2} \ln \frac{\beta(1 - \delta) - 1}{\beta \delta - 1}.$$

The above simple equations fulfill the initial condition: $\tau = 0, \delta = 1$.

The solidification time depends on the position of the liquid metal flow relative to the inlet of the crystallizer (see Figure 2). At the inlet of the crystallizer ($x = 0$), the solidification time is equal to zero ($\tau = 0$) and increases with distance $x$ according to equation (1). However, the solidification front is steady with respect to the crystallizer, because this situation results from the superposition of both motions: Liquid metal flow in the axial direction $x$ with the mean velocity $u$ and the solidifying metal in the perpendicular direction. The solidification time is described by equation (1).

The above analysis shows that the steady state interface can be expressed by the following equations. From equations (1), (7) and (8) it follows

$$\hat{x} = \frac{\text{Re} \cdot \text{Pr} \cdot a}{\text{Ste}} \int \left[ \beta(1 - \delta) - 1 \right] d\delta$$

and

$$\hat{x} = \frac{\text{Re} \cdot \text{Pr} \cdot \bar{a}}{\text{Ste}} \left[ \frac{1 - \delta^2}{\beta \delta - 1} \right].$$

In these equations, the dimensionless quantities are defined by

$$\hat{x} = \frac{x}{H}, \quad \text{Re} = \frac{uH}{\nu}, \quad \text{Pr} = \nu \cdot a \cdot \frac{a}{a}.$$ 

If liquid metal is not overheated $\beta = 0$ the equation (6) takes the form

$$\frac{d\hat{\delta}}{d\tau} = \frac{1}{\beta + 1 - \hat{\delta}}.$$

The solution of the above equation with the initial condition $\tau = 0, \hat{\delta} = 1$ can be obtained to be

$$\tau = \left( \beta + 1 \right) \left[ 1 - \hat{\delta}^2 \right] - \frac{1}{2} \left( \frac{1 - \hat{\delta}^2}{\beta \hat{\delta} - 1} \right), \quad (12)$$

4. Results and discussion

The shapes of the solidified crusts for different casting forms (plate and rods), external conditions and different parameters of the liquid metal are given in Figures 3 and 4. Shapes of the solidification fronts for flat thin plates and thin rods are very different. This difference explains the rapidly decreasing heat flux for bars as compared to flat plates as you get closer to the axis of symmetry. In order to show the quantitative impact of the thermodynamic parameters on the solidification process the corresponding graphs are prepared.

Fig. 3. Solidification front for a liquid metal for thin plat, $\text{Pr} = 0.01, \text{Ste} = 4, \beta = 0.2, \bar{a} = 1.0$
Figures 3-6 show shapes of the solidification front for different Reynolds numbers and given values of Pr, Ste, for different overheat parameters $\beta$ and different Biot numbers expressed by the parameters $\beta$. As it can easily be seen, the shape and size of the solidification front depend on the resistance of the contact layer $\lambda/(\alpha c \cos H)$, the parameter $\beta$, the Prandtl number Pr and the Reynolds number Re. From the obtained solution it can be seen that the size of the solidification front grows with increasing values of the Reynolds number, the Prandtl number, the resistance of the contact layer and the parameter $\beta$.

As can be seen the maximum length of the liquid metal flow corresponds to the thickness $\delta = 0$ and is equal to

$$\bar{x}_{\text{max}} = \frac{-\text{Re} \cdot \text{Pr} \cdot \bar{a}}{\text{Ste}} \left[ \frac{1}{\beta} + \frac{1}{\beta^2} \ln \left( \frac{\beta + 1 - 1}{\beta \beta - 1} \right) \right].$$  \hspace{1cm} (17)

The elevation of the interface should not exceed the height of the mold. This may result in an interruption of the continuous casting process. In addition, the maximum length should satisfy the additional condition

$$\bar{x}_{\text{max}}H \rho g < p_b,$$  \hspace{1cm} (18)

which represents the supply of liquid metal to the mold and guarantees the continuity of the stream.

Figure 6 shows the shapes of the solidification fronts of the liquid metals for similar external conditions, that is, both at the same flow rates of the liquid metal and the same cooling conditions for comparison. It can be seen that the liquid silver solidifies faster than the liquid copper.

Tin is freezing slowly, so the solidification front is more elongated. Knowledge of the shape of the solidification front,
especially the maximum height of the solidification front $x_{\text{max}}$, is very important and is needed for determining the technological parameters of the continuous process.

5. Conclusions

The paper is focused on the process of continuous casting of a thin metal plate in comparison with the continuous casting of thin metal rods. The solidification of pure liquid metal in an internal crystallizer flow is investigated and a simplified analytical model is presented. The role of the contact layer between the metal and the surface of the cooling channel is very important and might dominate the continuous casting of a thin metal plate. A simple analytical model is used to predict the freezing fronts for different metal flows. It is seen that these fronts might differ drastically, depending on the type of metal used. The shape of the solidified front depend on the shape of the casting form. Also the influence of the boundary conditions on the development of the frozen crust can nicely been studied with the simple model presented here.

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