The Use of Metacognitive Strategies in Solving Mathematical Problems

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Abstract. The ability to solve mathematical problems is useful not only for the absorption of knowledge, but also for the formation of thinking and attitude in solving the problems. This research was conducted to describe the metacognitive strategies that students undertake when solving mathematical problems. Metacognitive strategy is one of the methods used in problem solving that emphasizes to build the well-structured thinking, by deciding what to do and where to go in realizing and overcoming obstacles. Data obtained through problem solving by the subject and then analyzed qualitatively. Through the application of metacognitive strategies, a systematic and well-structured problem solving is implemented based on the logical thinking steps, and allows to the right solution. For students, metacognitive strategies are very important to build a strong foundation of thinking and to be applied in various situations and levels of complexity problems.

Keywords: Metacognitive, Mathematics, knowledge, attitude

1. Introduction

One of the rationale in implementation of mathematics learning on education is to prepare students to obtain knowledge, attitudes and skills to be able to solve various forms of mathematical problems (Anggo,). This ability is useful not only for understanding mathematical knowledge itself, but also for the formation of thinking and attitude in solving problems encountered. Various ways in solving problems will form a creative personality in choose different approaches of the problem according to the context.

Through mathematical problem solving, students are expected to develop their abilities including to build new mathematical knowledge, solving problems in various contexts, implementing various strategies needed, and reflecting the process of solving mathematical problems [2]. All of these capabilities can be obtained if the problem given is in well designed so that the scope of the benefits obtained is not only to one problem being solved, but becomes wider on various other problems and covers many aspects of mathematical knowledge.

The thinking process in problem solving is an important thing that needs to pay attention by the teacher, especially to help students in developing their ability to solve problems, both in the formal mathematics and in the real world context. This is in line with Lester [3] that the main purpose of teaching problem solving in mathematics is not only to equip students with a set of skills or processes, but rather to enable students to think for themselves.
Thinking for oneself is related to students’ awareness about their ability to develop various ways that might be taken in solving problems. The process of realizing and organizing students’ own thinking, is known as metacognition [3].

Metacognition is simply defined as thinking about what one thinks. Metacognitive strategies in problem solving are the implementation of abilities in planning, monitoring, and evaluating one’s own learning, which is manifested in the stages of solving that must be passed to be able to solve problems. The use of metacognitive strategies in problem solving is one interesting factor that is much considered by education researchers. This is due to the possibility to build the strong and comprehensive understanding of the problem with logical reasons. This kind of understanding is always emphasized when mathematics learning takes place at all levels of education.

1.1. Mathematical Problem Solving

Before discussing mathematical problems, let us understand what the problem is. Jonassen defines two main characteristics of the problem, namely: (1) a problem is an entity that is not known in a context, (2) finding or solving something unknown must have a social, cultural, or intellectual value [4].

As the key to determining an existing situation is a problem, is to see how someone reacts to the situation [5]. If no strategy is easily found, then the situation is a problem. Conversely, if a person's strategy can be applied normally to a situation that is similar to the given situation, then it is not a problem, but just an exercise.

Mathematical problems in this study are problems related to school mathematics and require students to deal with unfamiliar situations through thinking flexibly and creatively [6]. In the process of learning mathematics in school, teachers usually present mathematical problems to be solved by students in the form of problems or tasks that must be solved.

Mathematical problems given to students in school are intended specifically to train students to develop their intellectual abilities in understanding, planning, doing, and getting solutions for every problem. Thus, the need to improve students’ ability to solve problems and become a successful problem solvers is an important theme in the standard content of the mathematics education curriculum in Indonesia (Curriculum 2013) and educational standards in several countries [7].

Problem solving is an embodiment of a mental activity consisting of various skills and cognitive actions [7] which are intended to get the correct solution of the problem. The problem solving activity involves a person's cognitive activities, so the effect on the ability of each person's to solve the problem will be different. A problem that is challenging and quite difficult for someone, may be a simple problem for others.

1.2. Metacognitive Strategies in solving Problems

Metacognition generally relates to the two dimensions of thinking, namely (1) self-awareness of cognition, the knowledge one has about one's own thinking, and (2) self-regulation of cognition, one's ability to use his consciousness to regulate his own cognitive processes [8]. Both metacognitive dimensions have interdependent nature that is interdependent with each other. In relation to problem solving, Brown divides metacognition into two broad categories, namely: (1) knowledge of cognition, as an activity that includes conscious reflection on one's thinking ability and activity, (2) cognitive regulation, as an activity of paying attention to mechanisms self-regulation during an effort to learn or solve problems [9].

The ability to solve problems is seen as a state of mutual influence and complexity between cognition and metacognition. When students have difficulty in problem solving, then the difficulty can be sourced from the inability to actively monitor and regulate the cognitive processes involved in problem solving [10]. This indicates that to be able to carry out complex tasks in problem solving well, metacognitive strategies are needed.

Brown expressed the skills or metacognitive abilities that are essential for every efficient problem solver include the ability to: (1) planning, including estimating results, and scheduling strategies, (2) monitoring, including testing, revision, and rescheduling the strategy carried out, and (3) checking,
including evaluation the results of implementing a strategy based on efficiency and effectiveness criteria [10].

Thus, the use of metacognitive strategies in the problem-solving process becomes important, both to get the right solution of the problem being solved, and to build a well-structured mathematical thinking. In the problem solving process, students need metacognition when deciding what to do and where to go by realizing and overcoming their shortcomings [11].

2. Research methods

The research is exploratory research with qualitative approach. The research subjects were students of FKIP Universitas Halu Oleo who were in first semester. The subjects selected based on the tests and interviews. The test was conducted by think aloud method, and then interviewed based on the test results. In order to get the validity of the data, a triangulation method was carried out, through the use of tests, observations, and interviews. The results were analyzed qualitatively to reveal the implementation of metacognitive strategies in problem solving.

The metacognitive strategy used in this study is as proposed by De Corte [12] covering heuristics that are carried out at the stages of problem solving, as presented in the table 1.

| STAGES               | STRATEGIES                                      |
|----------------------|-------------------------------------------------|
| Stage 1              | Build Mental Representation of the Problem       |
| Heuristic            | - Draw pictures                               |
|                      | - Create a list, scheme or table               |
|                      | - Separate relevant data from non-relevant     |
|                      | - Use real knowledge                           |
| Stage 2              | Determine How to Solve the Problem             |
| Heuristic            | - Make a flow chart                            |
|                      | - Predict and check                            |
|                      | - Look at the pattern                          |
|                      | - Simplify the numbers                         |
| Stage 3              | Perform The Required Break-Up Steps            |
| Stage 4              | Interpretation of Results And Formulate Answers|
| Stage 5              | Evaluate the Solutions                         |

3. Result and Discussion

Metacognitive strategies are analyzed after the subject has resolved the following problems: A weaver will make Wolio’s traditional sarong and dress, with the supply of 84 rolls of gold yarn and 70 rolls of silver yarn. To make a sarong requires 2 rolls of gold yarn, and 4 rolls of silver, while one traditional dress requires 5 rolls of gold and 3 rolls of silver. If the profit that can be obtained from a traditional sarong is Rp. 25,000, and the profit from one traditional dress is Rp. 35,000, what is the maximum number of traditional sarong and dress that can be made so that the maximum profit will obtained?

3.1. Build mental representation of the problem

The activities performed by the subject shows the ability to regulate thinking when translating the problem into a table. This activity was followed by the activities relating to the structure of mathematical expressions / symbols. Then, subject determines the use of the x and y symbols to express the variables used in the mathematical model. The use of this variable is well recognized as confirmed in the interview.
3.2. Determine how to solve the problem

As has been done before, then the subject makes a plan to formulate the constraints functions that are carried out by reflecting on the required conditions. For this purpose, the subject plans a representation to state the conditions needed in production, in order to obtain the maximum value. Then the subject conducts reflection activities on the concept, followed by an analysis of the expressions, symbols, or numbers which were used for the formulation of the constraint function. In practice, the subject controls terminology / notation, related to the constraint function. The result is the formulation of the constraint functions which are the condition for achieving the maximum value of the objective function.

The next step is to draw a Cartesian diagram, which is intended to illustrate the results of the solution that has been done. The drawing is done by paying attention to one by one point between the line with the coordinate axis, then draw the line

3.3. Perform the required break-up steps

Then the subject conducts an evaluation by paying attention to the picture on the Cartesian diagram that was just made. At this stage the subject analyzes the settlement area, which is then shaded. Shading is made in areas that do not meet the requirements, with the aim to facilitate the next solution step.

Elimination and substitution methods are used in determining the intersection point, then implemented by determining the equation of the intersecting line which is based on the constraint function. The next activities are to do calculations to determine the intersection of the lines. The intersection coordinates obtained directly are written on the Cartesian diagram.
3.4. Interpretation of results and formulate answers
After the critical points are obtained, then subject determines the maximum value by substitution of the critical points to the objective function. Subjects appear to test the results obtained to ensure that the results obtained are correct.

3.5. Evaluate the solutions
Based on the calculation results, then the subject can set the maximum results that can be achieved. There are some results that are not written explicitly by the subject, including not writing the maximum value and not translating the results into contextual situations of the problem. However, when interviewing the subject can explain these results very well, which shows that the subject is aware of the achievement of the goals set.

4. Conclusion
The application of metacognitive strategies in problem solving, allows students to build appropriate ways of thinking in dealing with problems by arousing the knowledge they already have and managing it to gain new knowledge.

Through the application of metacognitive strategies, a systematic, well-structured problem solving process will be built on the basis of a logical frame of mind, and it is possible to obtain the right solution. From the student's own side, metacognitive strategies are very important in building a strong foundation of thinking to be applied in various conditions and levels of complexity of the problems.

Students who are accustomed to applying metacognitive strategies in solving mathematical problems will be greatly helped in every problem solving, because all the steps taken to solve are always based on an awareness of what they are doing, why they are done and how to make improvements when mistakes occur.

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