Reducing vortex density in superconductors using the ratchet effect

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(March 24, 2022)

PACS numbers:

1. A serious obstacle that impedes the application of low and high temperature superconductor (SC) devices is the presence of trapped flux \[\Phi\]. Flux lines or vortices are induced by fields as small as the Earth’s magnetic field. Once present, vortices dissipate energy and generate internal noise, limiting the operation of numerous superconducting devices \[1,2\]. Methods used to overcome this difficulty include the pinning of vortices by the incorporation of impurities and defects \[4\], the construction of flux dams \[5\], slots and holes \[6\] and magnetic shields \[2,3\] which block the penetration of new flux lines in the bulk of the SC or reduce the magnetic field in the immediate vicinity of the superconducting device. Naturally, the most desirable would be to remove the vortices from the bulk of the SC. There is no known phenomenon, however, that could form the basis for such a process. Here we show that the application of an ac current to a SC that is patterned with an asymmetric pinning potential can induce vortex motion whose direction is determined only by the asymmetry of the pattern. The mechanism responsible for this phenomenon is the so-called ratchet effect \[7-11\], and its working principle applies to both low and high temperature SCs. As a first step here we demonstrate that with an appropriate choice of the pinning potential the ratchet effect can be used to remove vortices from low temperature SCs in the parameter range required for various applications.

Consider a type II superconductor film of the geometry shown in Fig. 1a, placed in an external magnetic field \(H\). The superconductor is patterned with a pinning potential \(U(x, y) = U(x)\) which is periodic with period \(\ell\) along the \(x\) direction, has an asymmetric shape within one period, and is translationally invariant along the \(y\) direction of the sample. The simplest example of an asymmetric periodic potential, obtained for example by varying the sample thickness, is the asymmetric sawtooth potential, shown in Fig. 1b. In the presence of a current with density \(J\) flowing along the \(y\) axis the vortices move with the velocity

\[
v = (f_i + f_v + f_a)/\eta, \tag{1}\]

where \(f_i = (\mathbf{J} \times \hat{\mathbf{i}})\Phi_0 d/c\) is the Lorentz force moving the vortices transverse to the current, \(\hat{\mathbf{i}}\) is the unit vector pointing in the direction of the external magnetic field \(\mathbf{H}\), \(f_v = -\frac{dU}{dx}\hat{\mathbf{x}}\) is the force generated by the periodic potential, \(f_v\) is the repulsive vortex-vortex interaction, \(\Phi_0 = 2.07 \times 10^{-7} \text{ G cm}^2\) is the flux quantum, \(\eta\) is the viscous drag coefficient, and \(d\) is the length of the vortices (i.e. the thickness of the sample).

When a dc current flows along the positive \(y\) direction, the Lorentz force moves the vortices along the positive \(x\) direction with velocity \(v_+\). Reversing the current reverses the direction of the vortex velocity, but its magnitude, \(|v_-|\), due to the asymmetry of the potential, is different from \(v_+\). For the sawtooth potential shown in Fig. 1b, the vortex velocity is higher when the vortex is driven to the right, than when it is driven to the left (\(v_+ > |v_-|\)). As a consequence the application of an ac current (which is the consecutive application of direct and reverse currents with density \(J\)) results in a net velocity \(v = (v_+ + v_-)/2\) to the right in Fig. 1b. This net velocity induced by the combination of an asymmetric potential and an ac driving force is called ratchet velocity \[7-11\]. The ratchet velocity for the case of low vortex density (when vortex-vortex interactions are neglected) can be calculated analytically. Denoting the period of the ac current by \(T\), the ratchet velocity of the vortices in the \(T \to \infty\) limit is given by the expression

\[
v = \left\{ \begin{array}{ll}
0 & \text{if } f_L < f_1 \\
\frac{1}{2} (f_1 + f_2)(f_1 - f_2) & \text{if } f_1 < f_L < f_2 \\
\frac{f_1 f_2}{f_1 + f_2} & \text{if } f_2 < f_L,
\end{array} \right. \tag{2}
\]

where \(f_1 = \Delta U/\ell_1\) and \(f_2 = \Delta U/\ell_2\) are the magnitudes of the forces generated by the ratchet potential on the facets of length \(\ell_1\) and \(\ell_2\), respectively (see Fig. 1c). \(\Delta U\) is the energy difference between the maximum and the minimum of the potential, and \(f_L = |f_1| = J\Phi_0 d/c\).

Since for high magnetic fields vortex-vortex interactions play an important role, we have performed molecular dynamics simulations to determine the ratchet velocity for a collection of vortices. As Fig. 2 demonstrates, we find that for low vortex densities the numerical results follow closely the analytical prediction \[12\], and the magnitude of the ratchet velocity decreases with increasing vortex density. The vortex densities used in the simulations correspond to an internal magnetic field of about 0.7, 35, and 70 G, covering a wide range of magnetic fields. A key question for applications is if the ratchet
velocity \(v_0\) is large enough to induce observable vortex motion at experimentally relevant time scales. To address this issue in Fig. 3 we plotted \(v\) for Nb, a typical low temperature SC used in a wide range of devices, for which the potential \(U(x)\) is induced by thickness variations of the SC. The details of the model are described in the caption of Fig. 3. As the figure indicates, the maximum ratchet velocity (5.2 m/s) is high enough to move a vortex across the typical few micrometer wide sample in a few microseconds. Furthermore, increasing the vortex density by two orders of magnitude decreases the vortex velocity only by a factor of three.

Next we discuss a potentially rather useful application of the ratchet effect by demonstrating that it can be used to drive vortices out of a SC. Consider a SC film that is patterned with two arrays of the ratchet potential oriented in opposite directions, as shown in Fig. 3. During the application of the ac current the asymmetry of the potential in the right half moves the vortices in that region to the right, while vortices in the left half move to the left. Thus the vortices drift towards the closest edge of the sample, decreasing the vortex density in the bulk of the SC. We performed numerical simulations to quantitatively characterize this effect, the details and the parameters being described in the caption of Fig. 3. In Fig. 3 we summarize the effectiveness of vortex removal by plotting the reduced vortex density inside the SC as a function of the Lorentz force \(f_l\) and the period \(T\) of the current. One can see that there is a well defined region where the vortex density drops to zero inside the SC, indicating that the vortices are completely removed from the bulk of the SC. Outside this region we observe either a partial removal of the vortices or the ac current has no effect on the vortex density.

The \((f_l, T)\) diagram shown in Fig. 3 has three major regimes separated by two boundaries. The \(T_1 = 2\eta f_l / (f_{\text{edge}} + f_{\text{ac}})\) phase boundary (here we assume \(d/2 < \ell_2\) and \(f_{\text{in}}(x)\) is defined in Fig. 3) provides the time needed to move the vortex all the way up on the \(\ell_1\) long facet of the ratchet potential at the edge of the SC, i.e., to remove the vortex from the SC. When \(T < T_1\) the vortices cannot exit the SC. The \(T_2 = 2/\eta f_l / (f_{\text{in}} + f_{\text{ac}})\) phase boundary (where \(f_{\text{edge}}\) is also defined in Fig. 3) is the time needed for a vortex to enter from the edge of the SC past the first potential maxima. Thus, when \(T < T_2\) the vortices cannot overcome the edge of the potential barrier. These phase boundaries, calculated for non-interacting vortices, effectively determine the vortex density in the three phases. Vortex removal is most effective in regime I, where the vortices cannot move past the first potential barrier when they try to enter the SC, but they get past the barriers opposing their exit from the SC. Thus the vortices are swept out of the SC by the ratchet effect, and no vortex can reenter, leading to a vortex density \(\rho = 0\). Indeed, we find that the numerical simulations indicate complete vortex removal in the majority of this phase (see the contour lines in Fig. 3b). An exception is the finger structure near the crossing of the \(T_1\) and \(T_2\) boundaries. For fields and periods within the first finger (lowest in Fig. 3b) the vortex follows a periodic orbit inside a single potential well. The subsequent fingers represent stable periodic orbits between two, three, or more wells, respectively. Since the vortices cannot escape from these orbits, they remain trapped inside the SC, increasing the vortex density within the fingers in the phase diagram. Fig. 3 shows the analytically calculated envelopes of the regions where such trapping occurs. An important feature of the finger structure is that stable periodic orbits, which prevent vortex removal, do not exist above the line \(T_{\text{tip}} = f_l / (f_{\text{in}} + f_{\text{ac}})\) connecting the finger tips. In regime II vortices can enter the SC, but the ratchet effect is still sweeping them out, thus here we expect partial removal of the vortices, the final vortex density inside the SC being determined by the balance of vortex nucleation rate at the edge of the sample (which depends on the surface properties of the SC) and the ratchet velocity moving them out. In regime III the vortices cannot leave the SC and new vortices cannot enter the system, thus the initial density inside the SC is unchanged throughout this phase (\(\rho = \rho_0\)).

Since the forces \(f_{\text{in}}(x)\) and \(f_{\text{edge}}\) depend on \(H\), the position of the phase boundaries \(T_1\) and \(T_2\) also depends on the external magnetic field. In particular, there exists a critical field \(H^*\), such that for \(H > H^*\) regime I, where vortex removal is complete, disappears, but regime II with partial vortex removal does survive. We find that for Nb films of geometry described in Fig. 3 we have \(H^* \approx 10\text{G}\). However, since \(H^*\) is a consequence of the geometric barrier, its value can be modified by changing the aspect ratio of the film. Furthermore, for superconductors with elliptic cross section the geometric barrier can be eliminated, thus phase I with complete vortex removal could be extended to high magnetic fields as well.

Vortex removal is important for numerous SC applications and can improve the functioning of several devices. An immediate application of the proposed method would be improving the operation of superconducting quantum interference devices (SQUIDs), used as sensors in a wide assortment of scientific instruments. A long-standing issue in the performance of SQUIDs is \(1/f\) noise, arising from the activated hopping of trapped vortices. Reducing the vortex density in these superconductors is expected to extend the operation regime of these devices to lower frequencies.

Although over the past few years several applications of the ratchet effect have been proposed, such as separating particles, designing molecular motors, smoothing surfaces, or rectifying the phase across a SQUID, our proposal solves a well known acute problem of condensed matter physics, by removing vortices from a SC. In contrast with most previous applications, which require the presence of thermal noise, our
model is completely deterministic. Indeed, in Nb the variations in the pinning potential is \( \Delta U \approx 25 \text{ eV} \), which is more than \( 10^4 \) times larger than \( k_B T \approx 0.8 \text{ meV} \) at \( T_c = 9.26 \text{ K} \), thus rendering thermal fluctuations irrelevant. On the practical side, a particularly attractive feature of the proposed method is that it does not require sophisticated material processing to make it work: First, it requires standard few-micron scale patterning techniques (the micrometer tooth size was chosen so that a few teeth fit on a typical SQUID), but larger feature size will also function if the period \( T \) is increased proportionally. Second, the application of an ac current with appropriate period and intensity is rather easy to achieve. For applications where an ac current is not desired, the vortices can be flushed out before the normal operation of the device. On the other hand, if the superconducting device is driven by an ac current (e.g. rf SQUIDs, ac magnets, or wires carrying ac current), the elimination of the vortices will take place continuously during the operation of the device. The analytically predicted phase boundaries, whose position is determined by the geometry of the patterning, provide a useful tool for designing the appropriate patterning to obtain the lowest possible vortex density for current and frequency ranges desired for specific applications. Finally, although here we limited ourselves to low temperature SCs, the working principle of the ratchet effect applies to high temperature superconductors as well.

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Acknowledgements. We wish to thank D. J. Bishop, S. N. Coppersmith, D. Grier, H. Jeong, A. Koshelev and S. T. Ruggiero for very useful discussions and help during the preparation of the manuscript. This research was supported by NSF Career Award DMR-9710998.

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FIG. 1. Patterning with an asymmetric potential of a SC. (a) Schematic illustration of a SC in the presence of an external magnetic field $H$. A dc current with density $J$ flowing along the $y$ direction (indicated by the large arrow) induces a Lorentz force $f_L$ that moves the vortex in the $x$ direction. The SC is patterned with a pinning potential $U(x, y) = U(x)$, whose shape is shown in the lower panel. The potential is periodic and asymmetric along the $x$ direction, and is translationally invariant along $y$. (b) The pinning potential $U(x)$ along the SC cross section. The solid arrows indicate the vortex velocity $v_+ (v_-)$ induced by a direct $+J$ (reversed $-J$) current. The average, $v = (v_+ + v_-)/2$, is the ratchet velocity of the vortex, obtained when an ac current is applied. (c) The parameters characterizing a single tooth of the asymmetric potential.
FIG. 2. Ratchet velocity of the vortices as a function of the amplitude of the driving force $f_L$. The thick solid line corresponds to the analytical result (3) for a single vortex line. The symbols are the result of the numerical simulations for multiple vortices. The simulations were done using the model developed by Nori and collaborators [20–22], assuming that the rigid vortices are pointlike objects moving in the $x$-$y$ plane. At time zero the vortices are positioned randomly in the SC with a density $\rho_0$ and they move with velocity given by (1). The vortex-vortex interaction between two vortices at position $r_i$ and $r_j$ is modeled using $f_{vv} = \frac{4\pi d}{8\pi^2\lambda^2} K_1((r_i - r_j)/\lambda) \hat{r}_{ij}$, where $\hat{r}_{ij} = (r_i - r_j)/|r_i - r_j|$. Here the modified Bessel function $K_1$ is cut off beyond the distance $r = 25\lambda$, where $\lambda$ is the penetration depth (for Nb $\lambda = 45$nm at $T = 0$). The force $f$ generated by the sawtooth pinning potential shown in Fig. 1 is equal to $f_1$ when $k\ell < x < k\ell + \ell_1$, and $f_2$ when $k\ell + \ell_1 < x < (k + 1)\ell$, where $k = 0, 1, ..., N - 1$. We choose $\ell_1 = 20\lambda = 0.9 \mu m$, $\ell_2 = 5\lambda = 0.225 \mu m$, $\ell = \ell_1 + \ell_2$ and $N = 10$, giving for the total width of the sample $w = 11.25 \mu m$. Its length (along the $y$ direction) is set to $12 \mu m$. The sample has periodic boundary conditions in both the $x$ and $y$ direction. The Lorentz force due to the ac current is equal to $+f_L$ for $T/2$ time, and $-f_L$ for $T/2$ using $T = 0.3 \mu s$. We considered the simplest case, in which the potential is induced by thickness variations of a Nb superconductor thin film of thickness $d$, i.e. the SC thickness, $d + h(x)$, changes along the $x$ direction, following a sawtooth pattern. The pinning energy acting on the vortices is given by $U(x) = (d + h(x))\epsilon_0$, where $\epsilon_0$ is the line energy of the vortex per unit length. Thus the magnitudes of the forces acting on the vortices are $f_1 = \epsilon_0 \Delta h/\ell_1$ and $f_2 = \epsilon_0 \Delta h/\ell_2$ for the two facets of the $\Delta h$ high teeth (shown in Fig. 1c), and we choose $\Delta h = \ell_2$. For Nb we have $\epsilon_0 = 1.7 \times 10^{-11} N$, the viscosity per unit length is $\eta_0 = 7 \times 10^{-6} Ns/m^2$, yielding $\eta = \eta_0 d = 1.4 \times 10^{-12} Ns/m$ for a $d = 2000 \AA$ thick film. The total number of vortices in the simulation were $n = 5(\bigtriangleup)$, $n = 250(\bigtriangledown)$, and $n = 500(\bigtriangleup)$ corresponding to $\approx 0.7G(\bigtriangleup)$, $35G(\bigtriangledown)$, and $70G(\bigtriangleup)$ magnetic fields in the sample.
FIG. 3. Removing vortices from a SC using an asymmetric potential. (a) The potential necessary to remove vortices from the SC. Using the simulation method described in Fig. 2, we investigated a system consisting of \( N = 5 \) teeth oriented to the left and the same number oriented to the right, as shown in the figure, the parameters of each tooth being identical to that described in Fig. 1c. To mimic the pressure generated by the external magnetic field, which acts to push vortices into the SC, on the two sides we attached two reservoirs, that have a constant vortex density \( \rho_0 \) at all times. Thus, vortices can leave the SC for the reservoir, or new vortices can enter from the reservoir. In thin SC films, due to the Meissner current, there is a geometrical barrier that acts to trap the vortices inside the SC. Since most applications of SC’s involve thin films, we included in the simulations this geometrical barrier, that creates a force \( f_{\text{in}}(x) = -\frac{\mu_0 H_0}{2\pi} \frac{x}{\sqrt{w^2 - x^2}} \) for \(-w + d/2 < x < w - d/2\), and \( f_{\text{edge}} = 2\epsilon_0 - \frac{\mu_0 H_0}{2\pi} \sqrt{4w/d - 1} \) for \( x > w - d/2 \), and \(-f_{\text{edge}} \) for \( x < -w + d/2 \). Thus the geometrical barrier opposes the entry of the vortices at the edge of the SC, but once they move inside, it moves them towards the center of the SC. For successful vortex removal the ratchet effect has to be strong enough to move the vortices against \( f_{\text{in}}(x) \). (b) The \((f_L, T)\) diagram describing the effectiveness of the ratchet effect as a function of the parameters characterizing the driving current, \( f_L \). The color code corresponds to the the relative vortex density \( \rho/\rho_0 \), where \( \rho_0 \) is the initial vortex density corresponding to \( H = 1G \) and \( \rho \) is the final vortex density after the application of the ac current. As the color code indicates, there is a region where vortex removal is complete, the vortex density being equal to zero. The dashed lines correspond to the \( T_1 \) and \( T_2 \) boundaries, that are calculated analytically (see text) and separate the three main regimes: I: complete vortex removal in the majority of the regime, \( \rho = 0 \); II: partial vortex removal, \( 0 \leq \rho < \rho_0 \); and III: no change in the vortex density, \( \rho = \rho_0 \). The thin white lines denote the boundaries of the regions where vortex trapping due to periodic orbits occurs. These boundaries correctly reflect the structure of the fingers, but slightly deviate from the results of the numerical simulation, because the analytical calculation assumed an array of identical teeth.