Hydraulic investigation of finite crested stepped spillways
Abbas Parsaie and AmirHamzeh Haghiabi

ABSTRACT
In this paper, the hydraulic properties of the finite-crested stepped spillway (FCSS) including discharge coefficient ($C_d$) and the ratio of energy dissipation (EDR) were experimentally investigated. Results indicated that the $C_d$ of the FCSS changes between 0.9 and 1.2, while the ratio of the upstream head to the length of the crest ($h_{up}/L_c$) changes between 0.25 and 1.8. The $h_{up}/L_c$ is the main parameter affecting the $C_d$. The value $h_{up}/L_c$ equal to 0.6 is a good criterion for designing the crest of the FCSS. At this point, the $C_d$ of FCSS is about 1.0. The performance of FCSS regarding the EDR changes between 95 and 40 percent. By increasing the discharge of flow and skimming flow formation, the performance of the FCSS related to energy dissipation is dramatically decreased.

Key words | cascade weir, energy dissipation, flow measurement, short crested weir

HIGHLIGHTS
- Based on the mentioned reports, the $C_d$ of finite-crest weirs is more than the board and sharp-crested weirs, hence in this study, the use of it as the crest of a stepped spillway is suggested.
  In other words, finite-crested stepped spillways are proposed and their hydraulic properties including the $C_d$ and their ability in energy dissipation are investigated.

INTRODUCTION
Water resources management, especially flood control, is one of the most important actions for the development of human societies. River engineering projects and dam construction projects are among these measures (Patel et al. 2018; Mehta & Yadav 2020). Spillways are one of the most costly and important components of the flood evacuation systems of dams. A spillway usually consists of guide walls, approach channel, crest, chute, and an energy dissipator placed at the toe of the chute. The proper performance of the spillway depends on the optimal design of all its components. For example, the proper design of the guide walls could remove cross waves and helps the smooth transfer of flow from the dam’s reservoir to the approach channel (Parsaie et al. 2018). The crest is the main part of the spillway in determining its discharge capacity (Parsaie 2016). As the flow passes over the crest and flows into the chute, its velocity increases rapidly, which in turn leads to a decrease in pressure. As a result, the potential for occurrences of cavitation is increased significantly (Parsaie et al. 2016). On the other hand, the high kinetic energy of the flow may cause scouring downstream of chutes (Rajaratnam & Chamani 1995; Chamani & Rajaratnam 1999; Rashki Ghaleh Nou et al. 2019, 2020). The best approach to remove the cavitation is aeration and the reduction of flow velocity. One of the best ideas to reduce or eliminate the cavitation is to step up the chute. Besides, in some cases, it has been observed that steps have been used as aerated structures that eliminate cavitation with more confidence (Pfister et al. 2006; Pfister & Hager 2011). A typically stepped spillway is made of a flat crest and a stepped chute (Roushangar et al. 2014). The use of this
structure, due to its hydraulic properties, has been welcomed by many hydraulic engineers. Investigators have divided the flow regime on stepped spillways into three classes as nappe, transition, and skimming flow. Gonzalez et al. (2008) examined the effects of rough steps on the flow pattern over stepped spillways. Felder & Chanson (2011) studied the impact of non-uniformity in height of steps on energy dissipation and flow regime. They stated that the effect of non-uniformity of the step’s height on increasing energy dissipation is negligible and makes the flow pattern unstable. They found that the roughness of steps has no significant effect on changing the flow regime in all three known flow regimes. Felder & Chanson (2012) studied the distribution of velocity and pressure on the crest of the Broad crest stepped spillway. Felder & Chanson (2014) investigated the influence of pooled steps on energy dissipation of flow on stepped spillways. They demonstrated that the formation of the pool on steps increases energy dissipation by about 30 percent. Roushangar et al. (2018) using laboratory studies investigated the discharge coefficient of the Broad crest stepped spillway. They reported the experimental $C_d$ values vary from 0.58 to 0.64 for nappe flow, and from 0.59 to 0.7 for the skimming flow regime. Reviewing the literature shows that most researches have focused on the mechanism of energy dissipation.

Recently finite-crested weirs have been investigated by few numbers of investigators. The finite-crest weirs are categorized as the short crested weirs that their crest length is more than the sharp-crest weir and is less than the broad-crest weirs. Based on the reports, the $C_d$ of finite-crest weirs is more than the broad and sharp-crested weirs (Azimi et al. 2012; Azimi Amir et al. 2014; Mohammadzadeh-Habili et al. 2016). Hence, in this study, the finite-crest is suggested for applying as the crest of the stepped spillway. In other words, in this study, the finite-crested stepped spillways are proposed and their hydraulic properties including the $C_d$ and its ability in energy dissipation is investigated.

**MATERIAL AND METHODS**

**Dimensional analysis**

The sketch of FCSS is shown in Figure 1. As presented in this figure, $y_{up}$ and $h_{up}$ are the depth and head of flow over the crest, $H_{up}$ and $H_{down}$ are the totals of the head upstream and downstream, respectively. $P$ is the height of FCSS, $h_s$ and $L_s$ are the height and lengths of steps, respectively. $y_1$ and $y_2$ are the conjugated depths of the hydraulic jump. $L_c$ is the length of the crest. The discharge capacity per channel width ($q$) of FCSS as presented in Equation (1) (Parsaie & Haghiaibi 2019; Shamsi et al. 2019) is proportional to the $h_{up}$ and $C_d$. The hydraulic and geometric parameters affecting the $C_d$ are given in Equation (2), respectively.

$$q = C_d \sqrt{g \left( \frac{2}{3} \right)} h_{up}^{1.5}$$

$$C_d = f(L_c, P, V, h_{up}, g, \rho, \mu, \sigma)$$

As presented in Equation (2), the $C_d$ is proportional to the velocity of approached flow ($V$), water density ($\rho$), acceleration due to gravity ($g$), dynamic viscosity ($\mu$), surface tension ($\sigma$), $P$, $L_c$, and $h_{up}$. Using the Buckingham Pi theorem as the dimensional analysis technique, the involved dimensionless parameters in $C_d$ are derived and given in Equation (3). Notably, the $\rho$, $V$, and $h_{up}$ are considered as repetitive parameters.

$$\Pi(P) = \rho h_{up}(P)$$

$$\Pi(P) = (ML^{-3})^{a_1} (LT^{-1})^{b_1} (L)^{c_1} (L) = M^0 L^0 T^0$$

$$\Pi(P) = \frac{P}{h_{up}}$$

$$\Pi(L_c) = \frac{L_c}{h_{up}}$$
\[ \Pi (g) = \rho v h_{up}(g) = \frac{h_{up}^2}{V^2}, \sqrt{\frac{1}{\Pi (g)}} = \frac{V}{\sqrt{h_{up} g}} = Fr \Rightarrow \Pi (g) = Fr \]

\[ \Pi (\mu) = Re \]

\[ \Pi (\sigma) = We \]

\[ C_d = f\left(\frac{h_{up}}{L_c}, \frac{P}{h_{up}}, Fr, We, Re\right) \]  (3)

where \( F_r \), \( W_e \) and \( R_e \) are the Froude, Weber, and Reynolds numbers, \( h_{up}/L_c \) is the ratio of the head of the flow over the crest (at enough distance from the crest) to the length of the crest named the relative upstream head, and \( P/h_{up} \) is the ratio of the height of FCSS to the flow head over the crest. For evaluation of the performance of FCSS in terms of energy dissipation of flow, the Bernoulli equation is applied at its upstream (Equation (4)) and downstream (Equation (5)). To determine the performance of the FCSS concerning energy dissipation, it is enough to minus the \( H_{down} \) from the \( H_{up} \) (Equation (6)).

\[ H_{up} = P + y_{up} + \frac{V^2}{2g} = P + y_{up} + \frac{q^2}{2g(P + y_{up})^2} \]  (4)

\[ H_{down} = \frac{y_1 + \frac{q^2}{2g\eta_1^2}}{1} \]  (5)

\[ EDR = \frac{\Delta H}{H_{up}} = \frac{H_{up} - H_{down}}{H_{up}} = \left(1 - \frac{H_{down}}{H_{up}}\right) \times 100 \]  (6)

where \( EDR \) is the energy dissipation ratio. The factors involved in the energy dissipation of the flow passing through the FCSS are seen in Equation (7). Using the Buckingham \( \Pi \) theorem, the dimensionless parameters affecting the energy dissipation of flow on the FCSS are derived as

\[ \frac{\Delta H}{H_{up}} = f(L_c, h_s, \eta_s = h_s \cos(\theta), L_s, P, h_{up}, g, \rho) \]  (7)

\[ \Pi (P) = \rho g h_{up}(P) = \frac{P}{h_{up}^2} \frac{P - Nh}{h_{up}} \prod (P) = N \frac{h_s}{h_{up}} \]

\[ \Pi (L_c) = \frac{L_c}{h_{up}} \]

\[ \Pi (L_s) = \frac{L_s}{h_{up}} \]

\[ \Pi (K_s) = \frac{h_{up}}{K_s} \frac{h_{up} - 1.5y_c}{h_s} \prod (h_{up}) = \frac{h_s}{h_{up}} \]

\[ \Pi (P) \times \frac{1}{\prod (L_s)} = \frac{h_s}{h_{up}} \cdot \frac{h_{up}}{L_s} = S \]

\[ EDR = \frac{\Delta H}{H_{up}} = f\left(\frac{y_c}{h_s}, S, N\right) \]  (8)

In this equation, the \( h_s \) is the roughness caused by the size of steps and longitudinal slope of the chute. \( y_c \) is the critical depth of flow. The \( N \) is the number of steps.

### Experimental setups

Physical laboratory models of FCSS were constructed from galvanized iron covered by blue epoxy. The main channel is 12 m long with a rectangular cross-section that is 0.9 m deep and 0.50 m wide. The left side wall of the channel is made of Plexiglas and its right wall is made from smooth concrete covered by blue epoxy. The bed of the channel is made of a well-pointed steel sheet. To control the formation of the hydraulic jump, a sluice gate is installed downstream. The longitudinal slope of the main channel is equal to 0.0001. The depths of flow upstream and downstream of the structure were measured by a point gage with ±0.1 mm sensitivity. The discharge of flow was measured with a triangular weir that was installed downstream for this purpose. The properties of the models are given in Table 1.
RESULTS AND DISCUSSION

In this section, the results obtained in this study are presented. As presented in the dimensional analysis, the hydraulic properties of the FCSS including $C_d$ and EDR is the function of the relative upstream head ($h_{up}/L_c$), the slope of the downstream stepped chute (ratio of the height of steps to their lengths: ($S$)), the number of steps ($N$), the Froude number ($Fr$), Weber number ($We$) and Reynolds number ($Re$). According to Reclamation (1987) experiences, when the $P = h_{up}$ is more than 3 ($P = h_{up} \gg 3$), the height of the spillway does not affect the $C_d$. In this study, the maximum value of $h_{up}$ is about 0.18 m, and spillway height is 0.8, hence the minimum value of $P = h_{up}$ is about 4.44. Therefore, the effect of this parameter has not been investigated. It should be noted that spillways are usually constructed perpendicular to the flow path; therefore, they are always designed for the subcritical flow. Hence, the effect of the $Fr$ on $C_d$ is negligible. The flow over the FCSS is fully turbulent and the minimum value of $y_{up}$ is 5 cm; therefore, the effects of $Re$ and $We$ are negligible. The stage-discharge relations of the FCSS are shown in Figure 2. It is noteworthy that since the velocity of the flow approaching the spillway is very low, the head of flow over the crest of the FCSS is approximately equal to the flow depth over the crest ($y_{up} = h_{up}$). According to Equation (1), a mathematical formula of $1.138 \sqrt{(2/3) g(h_{up})^{1.5}}$ can describe the stage-discharge relation of the FCSS. From this equation, it is understood that the average value of the $C_d$ of this structure (crest length = 0.1 m) is equal to 1.138. With a 50 percent increase in the length of the crest (0.15 m), the mathematical formula $1.028 \sqrt{(2/3) g(h_{up})^{1.5}}$ can model the stage-discharge relation. In this case, the average value of $C_d$ is about 1.028. A comparison of the average value of the $C_d$ with the previous one shows that a 50% increase in crest length reduces the average value of $C_d$ by about 10%. By doubling the length of the crest ($L_c = 0.20 m$) compared to the first one ($L_c = 0.10 m$), the $0.952 \sqrt{(2/3) g(h_{up})^{1.5}}$ can model the stage-discharge relation. In this case, the average value of $C_d$ is about 0.95. A comparison of this case with the first one shows that the average $C_d$ decreases by about 17%.

The distribution of the $C_d$ against the $h_{up}$ is presented in Figure 3. As shown in this figure, the range of the $C_d$ for the model with a crest length of 0.1 m varies from 0.8 to 1.2. It is clear from Figure 3 that the flow head of about 0.08 m is a turning point for the curve of the $C_d$. The curve trend to this value is increased dramatically and then for more than this, its increase is not significant. By increasing the crest length by 50%, the $C_d$, similar to the stage-discharge curve, shows a significant decrease. On average, with a 50 percent

Table 1 | The summary of properties of rough stepped spillway

| $P$(m) | $L_c$(m) | $h_s$(m) | $L_s$(m) | $N$ | Q(l/s) |
|-------|---------|---------|---------|-----|-------|
| 0.8 m | 0.10, 0.15, 0.20 | 0.10 | 0.10 | 8 | 10–90 |
|       | 0.10 | 0.15 | 8 |
|       | 0.05 | 0.05 | 16 |
|       | 0.05 | 0.10 | 16 |
increase in crest length, the $C_d$ decreases by about 10%. In general, by increasing the crest length by about 100% ($L_c = 0.20m$), the $C_d$ decreases by approximately 17%.

The examination of Figure 3 shows that the geometrical properties of the downstream stepped chute such as the slope and the size of the steps do not affect the $C_d$. This observation, of course, confirms supercritical flow theory. According to this theory, the downstream part of the supercritical flow (chute) does not affect the upstream (crest). The values of the $C_d$ versus the $h_{up}/L_c$, as a dimensionless parameter for design purposes, are shown in Figure 4. As shown in this figure, the $h_{up}/L_c$ of 0.6 can be the appropriate point for the crest design of this structure.

Sensitivity analysis of the parameters involved in the $C_d$ shows that the geometric properties of the downstream part of the crest such as the slope of the stepped chute as well as the size of the steps do not affect the $C_d$. But the most important factor affecting the $C_d$ is the $h_{up}/L_c$. The rate of increase of this parameter from 0.2 to 0.6 is very effective in increasing the $C_d$ so that with increasing the $h_{up}/L_c$ from 0.2 to 0.6, the $C_d$ increases from 0.6 to about 1.0. Thus, with a relative increase of about 200%, the $C_d$ increases by about 66%. However, for values greater than 0.6 for the $h_{up}/L_c$, the changes in the $C_d$ are not significant because, with the increase of the $h_{up}/L_c$ from 0.6 to 1.8 (an increase of about 300%), the $C_d$ will increase by about 20% (the $C_d$ changes from 1.0 to 1.2).

As mentioned in the introduction section, a few studies have been conducted on finite-crested weirs; hence, in this section, the $C_d$ obtained in this study are compared with their results. To this end, three of these studies, Shahheydari et al. (2014), Mohammadzadeh-Habili et al. (2016), and Sargison Jane & Percy (2009), were selected and the results of the comparison are given in Figure 5. As shown in this figure, the $C_d$ of FCSS obtained in this study is close to the results of the previous study.

The performances of FCSS in terms of energy dissipation are shown in Figure 6. In this figure, the $EDR$ versus the $y_c/h_s$ has been plotted. As shown in this figure, the FCSS can dissipate the energy of flow between 95 and 40% while the relative critical depth ($y_c/h_s$) changes between 0.2 and 1.4. As can be seen from this figure, as the flow discharge ($y_c/h_s$) increases, the performance of the FCSS concerning $EDR$ decreases sharply. In other words, the performance of the FCSS is very sensitive to increasing the flow discharge, because with increasing flow discharge, the skimming flow regime is formed and the effect of the dimensions of the steps on the flow energy will be very low, which will
greatly reduce energy consumption. Examination of this figure shows that the performance of FCSS starts at very high values (95%) and decreases with increasing discharge (40%). This is because as the flow discharge increases, the flow regime changes from nappe to skimming flow. In the nappe flow regime, the energy loss is mainly due to the collision of the nappe flow with the floor of the down step. In fact, in the nappe flow regime, the steps have a performance like a drop structure installed in tandem together. The formation of the skimming flow regime creates a pseudo layer between the steps and the main body of the jet stream, and in this case, the effect of the step’s size is greatly reduced. As the steps become larger, the formation of the skimming flow is delayed. For a given height of steps, increasing the length of steps has a significant effect on increasing the FCSS performance concerning EDR. It is noteworthy that this increase in step length affects the rate of energy dissipation if it results in a change in the flow regime from skimming to nappe or transition. According to this, for models with a step height equal to 10 cm, the nappe flow or transient flow regimes were observed, and increasing the length of steps from 10 to 15 cm did not change the flow regime, but reinforced the nappe flow pattern. Hence, in this case, increasing the length of steps does not have a significant effect on energy dissipation. But in the case of the FCSS with a height of 5 cm, doubling the length of the steps will delay the flow pattern formation. According to Figure 6 and laboratory observations, for values of $y_c/h_s \leq 1.5$, the nappe flow and transient flow regimes were observed for all models (with step height of 5 cm), and for values more than 1.5 ($y_c/h_s > 1.5$), doubles the length of the steps, delaying the formation of the skimming flow regime. However, increasing the length of the steps also creates more roughness in the flow path, which ultimately reduces the energy of flow (increases the performance of FCSS concerning energy dissipation).

CONCLUSION

Stepped spillways are one of the most common hydraulic structures used in hydraulic engineering and watershed management projects. In this study, the hydraulic properties of the finite-crested stepped spillway including discharge coefficient and energy dissipation were investigated. The $C_d$ of FCSS changes between 0.8 and 1.2 while the $h_{up}/L_c$ ranges between 0.2 and 1.7. A ratio of upstream head to the length of the crest $(h_{up}/L_c)$ equal to 0.6 is the suitable point for the design of the crest of the stepped spillway. At this point, the $C_d$ of FCSS is about 1.0. By increasing the $h_{up}/L_c$ more than this, the $C_d$ is increased slightly. The geometric properties of the downstream chute including the size of steps and slope of chute do not affect the $C_d$. The performance of the FCSS concerning energy dissipation was changed between 95 and 40%. Increasing the size of steps has a significant effect on increasing energy dissipation, but it should be noted that increasing the step’s size should not cause instability of the flow pattern.

To continue this research, it is suggested that researchers determine the optimal size of the steps according to the conditions of the skimming flow and the maximum amount of energy dissipation.

Due to the use of stepped spillways (stepped weirs) in watershed management projects, it is suggested to investigate the effect of porosity and roughness of the surfaces of crest and steps on the discharge coefficient and the amount of energy dissipation.

CONFLICT OF INTEREST

None.

DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

REFERENCES

Azimi, A. H., Rajaratnam, N. & Zhu, D. Z. 2012 Discharge characteristics of weirs of finite crest length with upstream and downstream ramps. Journal of Irrigation and Drainage Engineering 139 (1), 75–83.
Azimi Amir, H., Rajaratnam, N. & Zhu David, Z. 2014 Submerged flows over rectangular weirs of finite crest length. Journal of Irrigation and Drainage Engineering 140 (5), 06014001. doi:10.1061/(ASCE)IR.1943-4774.0000728.
Chamani, M. R. & Rajaratnam, N. 1999 Characteristics of skimming flow over stepped spillways. *Journal of Hydraulic Engineering* **125** (4), 361–368. doi:10.1061/(asce)0733-9429(1999)125:4(361).

Felder, S. & Chanson, H. 2011 Energy dissipation down a stepped spillway with nonuniform step heights. *Journal of Hydraulic Engineering* **137** (11), 1543–1548. doi:10.1061/(asce)hy.1943-7900.0000455.

Felder, S. & Chanson, H. 2012 Free-surface profiles, velocity and pressure distributions on a broad-crested weir: a physical study. *Journal of Irrigation and Drainage Engineering* **138** (12), 1068–1074. doi:10.1061/(ASCE)IR.1943-4774.0000515.

Felder, S. & Chanson, H. 2014 Effects of step pool porosity upon flow aeration and energy dissipation on pooled stepped spillways. *Journal of Hydraulic Engineering* **140** (4), 04014002. doi:10.1061/(asce)hy.1943-7900.0000858.

Gonzalez, C. A., Takehashi, M. & Chanson, H. 2008 An experimental study of effects of step roughness in skimming flows on stepped chutes. *Journal of Hydraulic Research* **46** (sup1), 24–35. doi:10.1080/00221686.2008.9521937.

Mehta, D. J. & Yadav, S. M. 2020 Hydrodynamic simulation of river Ambica for riverbed assessment: a case study of Navsari region. In: *Advances in Water Resources Engineering and Management* (R. AlKhaddar, R. Singh, S. Dutta & M. Kumari, eds), Springer, Singapore, pp. 127–140.

Mohammadzadeh-Habili, J., Heidarpour, M. & Haghiabi, A. 2016 Comparison the hydraulic characteristics of finite crest length weir with quarter-circular crested weir. *Flow Measurement and Instrumentation* **52**, 77–82. https://doi.org/10.1016/j.flowmeasinst.2016.09.009.

Parsaie, A. 2016 Predictive modeling the side weir discharge coefficient using neural network. *Modeling Earth Systems and Environment* **2** (2), 1–11. doi:10.1007/s40808-016-0123-9.

Parsaie, A. & Haghiabi, A. H. 2019 The hydraulic investigation of circular crested stepped spillway. *Flow Measurement and Instrumentation* **70**, 101624. https://doi.org/10.1016/j.flowmeasinst.2019.101624.

Parsaie, A., Dehdar-Bebahani, S. & Haghiabi, A. H. 2016 Numerical modeling of cavitation on spillway’s flip bucket. *Frontiers of Structural and Civil Engineering* **10** (4), 438–444. doi:10.1007/s11709-016-0357-y.

Parsaie, A., Moradinejad, A. & Haghiabi, A. H. 2018 Numerical modeling of flow pattern in spillway approach channel. *Jordan Journal of Civil Engineering* **12** (1), 1–9.

Patel, S. B., Mehta, D. J. & Yadav, S. M. 2018 One dimensional hydrodynamic flood modeling for Ambica River, South Gujarat. *Journal of Emerging Technologies and Innovative Research* **5** (4), 595–601.

Pfister, M. & Hager, W. H. 2011 Self-entrainment of air on stepped spillways. *International Journal of Multiphase Flow* **37** (2), 99–107. https://doi.org/10.1016/j.ijmultiphaseflow.2010.10.007.

Pfister, M., Hager, W. H. & Minor, H.-E. 2006 Bottom aeration of stepped spillways. *Journal of Hydraulic Engineering* **132** (8), 850–853.

Rajaratnam, N. & Chamani, M. R. 1995 Energy loss at drops. *Journal of Hydraulic Research* **33** (3), 373–384. doi:10.1080/00221689509498578.

Rashki Ghaleh Nou, M., Azhdary Moghaddam, M., Shafai Bajestan, M. & Azamatkhull, H. M. 2019 Estimation of scour depth around submerged weirs using self-adaptive extreme learning machine. *Journal of Hydroinformatics* **21** (6), 1082–1101. doi:10.2166/hydro.2019.070.

Rashki Ghaleh Nou, M., Azhdary Moghaddam, M., Shafai Bajestan, M. & Mohammad Azamatkhull, H. 2020 Control of bed scour downstream of ski-jump spillway by combination of six-legged concrete elements and riprap. *Ain Shams Engineering Journal*. https://doi.org/10.1016/j.asej.2020.01.009.

Roushangar, K., Akghar, S., Salmisi, F. & Shiri, J. 2014 Modeling energy dissipation over stepped spillways using machine learning approaches. *Journal of Hydrology* **508**, 254–265. https://doi.org/10.1016/j.jhydrol.2013.10.053.

Roushangar, K., Akghar, S. & Salmisi, F. 2018 Estimating discharge coefficient of stepped spillways under nappe and skimming flow regime using data driven approaches. *Flow Measurement and Instrumentation* **59**, 79–87. https://doi.org/10.1016/j.flowmeasinst.2017.12.006.

Sargison Jane, E. & Percy, A. 2009 Hydraulics of broad-crested weirs with varying side slopes. *Journal of Irrigation and Drainage Engineering* **135** (1), 115–118. doi:10.1061/(ASCE)0733-9437(2009)135:1(115).

Shahheydari, H., Nodoshan, E. J., Barati, R. & Moghadam, M. A. 2017 Energy dissipation down a stepped spillway with nonuniform step heights. *International Journal of Multiphase Flow* **37** (4), 368. doi:10.1016/j.ijmultiphaseflow.2010.11.007.

Shamsi, Z., Parsaie, A. & Haghiabi, A. H. 2015 The hydraulic investigation of stepped spillway with varying side slopes. *Journal of Irrigation and Drainage Engineering* **131** (9), 1154–1158. doi:10.1061/(asce)ir.1943-7900.0000515.

Shamsi, Z., Parsaie, A. & Haghiabi, A. H. 2019 Optimum hydraulic design of cylindrical weirs. *ISH Journal of Hydraulic Engineering* **1–5**, doi:10.1080/09715010.2019.1683474.

US Bureau of Reclamation 1987 *Design of Small Dams*. U.S. Department of the Interior, Bureau of Reclamation.

First received 13 November 2020; accepted in revised form 10 March 2021. Available online 22 March 2021.