A COMPTON UPSCATTERING MODEL FOR SOFT LAGS IN THE LOWER KILOHERTZ QUASI-PERIODIC OSCILLATION IN 4U 1608–52

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ABSTRACT

An empirical Compton upscattering model is described that reproduces both the fractional amplitude (rms) versus energy and the soft time lags in the ≈830 Hz quasi-periodic oscillation (QPO) observed in 4U 1608–52 on 1996 March 3. A combination of two coherent variations in the coronal and soft photon temperatures (with their relative contributions determined by enforcing energy conservation) gives rise to the QPO’s energy-dependent characteristics. All input parameters to the model, save a characteristic plasma size and the fraction of Comptonized photons impinging on the soft photon source, are derived from the time-averaged photon energy spectrum of the same observation. Fits to the fractional rms and phase lag data for this kilohertz QPO imply that the spatial extent of the plasma is in the range from ~4 to 15 km.

Subject headings: accretion, accretion disks — radiation mechanisms: thermal — stars: neutron — X-rays: stars

1. INTRODUCTION

Soon after the launch of the Rossi X-Ray Timing Explorer (RXTE), rapid (300–1300 Hz), nearly periodic variability in the X-ray light curves of low-mass X-ray binary (LMXB) systems was discovered (Strohmayer et al. 1996; van der Klis et al. 1996). These oscillations, referred to as kilohertz quasi-periodic oscillations (QPOs), have now been observed in over two dozen neutron star–bearing LMXBs (see van der Klis 2000 for a review). They are distinguished by high frequencies and high quality factors (Q = FWHM/frequency) and tend to be seen in pairs, with nearly constant frequency separation between the lower and higher frequency peaks (also called the lower and upper QPO). The high frequencies of kilohertz QPO are thought to tie them to phenomena taking place in the inner regions of accretion disks surrounding neutron stars, making them potentially valuable probes of strong gravity.

A broad range of theoretical ideas have been proposed to explain the phenomenology of kilohertz QPO. In these models, one of the frequencies is generally identified as the Keplerian frequency of the innermost orbit of an accretion disk. The sonic point model (Miller, Lamb, & Psaltis 1998) identifies the second frequency as the beat of the primary QPO with the spin of the neutron star. Stella & Vietri (1999) have proposed a general relativistic precession/apsidal motion model wherein the primary frequency is the Keplerian frequency of a slightly eccentric orbit and the secondary is due to the relativistic apsidal motion of this orbit. On the other hand, in the two-oscillator model (Osherovich & Titarchuk 1999, applied to 4U 1608–52; Titarchuk & Osherovich 1999), the secondary frequency is due to the transformation of the primary (Keplerian) frequency in the rotating frame of the neutron star magnetosphere. The strength (and weakness) of these models lie in their ability (or inability) to predict the variation of the frequency separation with the kilohertz QPO frequency and the variation of their frequency with other low-frequency QPOs observed in the source (see van der Klis 2000 for a comparison of recent observations with model predictions). However, these dynamic models generally do not address how these dynamic oscillations affect, or couple to, the radiative processes that finally give rise to the oscillations seen in the X-ray spectra. The observed energy-dependent features of the QPO, namely, the strength of the QPO versus photon energy (fractional rms vs. energy) and the phase lag of fluctuations at different photon energies relative to variations at a reference energy, are expected to depend primarily on the radiative mechanism and its coupling to the dynamic behavior of the system.

While a unified model for kilohertz QPO, which self-consistently incorporates both the dynamical and radiative mechanisms, remains illusive, significant progress can be achieved by studying the radiative response of a system to an oscillating perturbation. Such an analysis would make definite predictions about the energy-dependent features of the QPO, thereby placing constraints on the radiative mechanism, and perhaps shed light on the nature of the coupling between the dynamical and radiative processes.

The power-law photon energy spectra often observed in LMXBs suggests that the dominant radiative mechanism in these systems is Compton upscattering of soft photons in a hot plasma (Sunyaev & Titarchuk 1980; Podznyakov, Sobol, & Sunyaev 1983). The temporal behavior of such models has been studied in a variety of circumstances (e.g., Wijers, van Paradijs, & Lewin 1987; Hua, Kazanas, & Titarchuk 1997; Böttcher & Liang 1999). For QPO in particular, both the fractional amplitude versus energy and the relative time lag versus energy can be predicted based on models where a hot, uniform plasma is illuminated by soft blackbody photons, accounting for the system’s response to variations in either the soft photons or the plasma (Wijers et al. 1987; Lee & Miller 1998).

In Compton upscattering models, a photon’s mean residence time in the hot plasma increases with the energy of the escaping photon since, on average, higher energies require a larger number of scatters. Thus, it would seem that, for these models, variability in high-energy photons should be delayed relative to their lower energy counterparts (i.e., a hard time lag is expected). In fact, analysis of simple Comptonization models, where the variability is caused only by oscillations in the intensity of the soft input spectrum, reveal that the time lag between the high-energy, $E_\gamma$, and low-energy, $E_\nu$, photons should roughly scale as $\log (E_\gamma/E_\nu)$ (Nowak & Vaughan 1996). It is therefore rather surprising that soft lags (i.e., variability in soft photons delayed with respect to hard photons) seem to be the norm in kilohertz QPO (Vaughan et al. 1998; Kaaret et al. 1999; Markwardt, Lee, & Swank 1999). This inconsistency...
has led to the suggestion that Compton downscattering could be the dominant radiative process in these systems. However, it is difficult to reconcile the observed increase in the amplitude of the oscillations with energy with such models. Moreover, this would point to the unlikely scenario in which the radiative process for the time-averaged spectrum (Compton upscattering) differs from that for the QPOs (Compton downscattering).

This Letter describes a Compton upscattering model that gives a consistent interpretation of the time-averaged photon spectra as well as the energy dependence of both the fractional rms amplitude and time lag for the 830 Hz QPO detected in 4U 1608-52 on 1996 March 3 (Berger et al. 1996). The observed soft lag is a natural consequence of the model provided that the temperatures of the plasma and of the soft photon source oscillate coherently at the QPO frequency. It is further shown that coherent oscillation of the temperatures is expected when energy transfer between the plasma and the soft photon source is taken into account. The model imposes limits on the size of the Comptonizing region and places constraints on any future model invoked to explain how the dynamical origin of size of the Comptonizing region and places constraints on any

2. THE COMPTON UPSCATTERING MODEL

Consider a uniform hot plasma characterized by its temperature \( T_p \), size \( l \), and optical depth \( \tau = n_e \sigma_T l \), where \( n_e \) is the electron density and \( \sigma_T \) is the Thomson cross section. The hot plasma upscatters soft photons from an external source. Unlike previous work, the soft photons are assumed to be described by a Wien spectrum (at a temperature \( T_s \)) instead of a blackbody. Thus, the soft photon source lies in a Comptonizing region with an optical depth large enough that the emergent spectrum saturates to a Wien peak. The choice of a Wien spectrum rather than a blackbody spectrum is motivated by the fact that for \( T_s \approx 1 \) keV and for reasonable electron densities \(<10^{25} \text{ cm}^{-3}\) the photon density inside the source will not be in thermal equilibrium. It should be noted that broadband observations of the black hole system Cygnus X-1 reveal that the soft photon component (which is assumed to provide the seed photons for Comptonization) is better described as a Wien peak rather than a blackbody or a sum of blackbodies (Di Salvo et al. 2001). Since the soft photon source could itself be heated by the Comptonized photons from the hot plasma, it is assumed that a fraction \( f \) of the Comptonized photons is incident on the soft photon source. In the framework of this model, oscillations in the photon escape rate from the hot plasma are due to corresponding coherent oscillations in the plasma temperature \( T_p \) and the soft photon source temperature \( T_s \), while the size \( l \), the electron density \( n_e \), and the total rate of photons emerging from the plasma remain constant.

For nonrelativistic temperatures \( kT_s \ll m_e c^2 \) and low photon energies \( E \ll m_e c^2 \), the evolution of the photon density inside the hot plasma is governed by the simplified Kompaneets equation (with the induced scattering term neglected; Kompaneets 1957):

\[
n_s \frac{dn_s}{dt} = \frac{1}{m_e c^2} \frac{d}{dE} \left[ -4kT_s E n_s + E^2 n_s + kT_s \frac{d}{dE} (E^2 n_s) \right] + t_s n_s - t_s n_{esc}.
\]

Here \( n_s \) is the photon density inside the plasma, \( E \) is the energy, and \( t_s \) = \( l/c \tau \) is the Thomson collision timescale. The number of photons per second per unit volume injected into the plasma from the soft photon source is \( n_s = C T_s^{-1} E^2 \exp(-E/kT_s) \), where \( C \) is a normalization constant. This is balanced by the corresponding rate of photons escaping from the plasma, \( n_{esc} \approx n_e (N_{esc}) \), where \( N_{esc} \approx \tau (\tau + 1) \) is the average number of scatterings a photon undergoes before escaping.

The time-averaged spectrum arising from the plasma can be estimated using \( n_{esc} \approx n_e (N_{esc}) \), where \( n_e \) is the steady state photon number density corresponding to time-averaged values of plasma temperature \( T_{p0} \) and soft photon Wien temperature \( T_{s0} \). For \( kT_{p0} \ll E \ll kT_{s0} \), it can be shown from equation (1) that the spectrum is a power law with photon index \( (\Gamma) \) related to \( N_{esc} \) by

\[
N_{esc} = \frac{(m_e c^2/kT_s)}{\Gamma^2 + \Gamma - 2}.
\]

Lee & Miller (1998) studied the temporal behavior of a Comptonizing medium by computing the linear response of the photon spectrum to a sinusoidally varying physical parameter at a given angular frequency \( \omega \). Following their formulation, the plasma temperature is approximated as \( T_p = T_{p0}(1 + \delta T_p e^{-it}) \) and the soft photon temperature as \( T_s = T_{s0}(1 + \delta T_s e^{-it}) \), where \( \delta T_p \) and \( \delta T_s \) are, in general, small complex quantities. The resulting time-varying photon number density is taken to be \( n_s = n_{0s}(1 + \delta n_s e^{-it}) \) with the output spectrum given by \( n_{esc} = n_e/N_{esc} \), \( \delta n_s \) is related to \( \delta T_p \) and \( \delta T_s \) by the linearized Kompaneets equation (Lee & Miller 1998):

\[
\left( \frac{1}{N_{esc}} - i\omega t \right) n_{0s} \delta n_s = -\frac{1}{m_e c^2} \frac{d}{dE} \left[ 4E kT_{s0} n_{0s} (\delta T_s + \delta n_s) \right]
\]

\[
- E^2 n_{0s} \delta n_s
\]

\[
- \frac{d}{dE} E^2 kT_{s0} n_{0s} (\delta T_s + \delta n_s)
\]

\[
- t_s n_{0s} \left( \frac{E}{kT_{s0}} - 3 \right) \delta T_s.
\]

The above equation implicitly assumes that the soft photon source spectrum \( n_s \) is a Wien peak with a constant photon number flux. It is useful to define \( \delta n_s \) as the photon variation \( \delta n_s \) corresponding to a unit variation in the plasma temperature \( (\delta T_p = 1) \) and similarly \( \delta n_s \) corresponding to a unit variation in the soft photon temperature \( (\delta T_s = 1) \). Thus, \( \delta n_s \) \((\delta n_s)\) can be computed from equation (3) by setting \( \delta T_p = 1 \) \((\delta T_p = 1) \) and \( \delta T_s = 0 \) \((\delta T_s = 0) \). Since only the linear response is being considered, the variation in total photon number density can be written as

\[
\delta n_s = \delta T_p \delta n_{sc} + \delta T_s \delta n_{sc}.
\]

Extending this method to include higher order terms leads to harmonics of the fundamental represented by the linear response. In the absence of any observed kilohertz QPO harmonics and in view of the simple assumptions made in this model, only the linear response will be considered.

Variations in the plasma temperature will induce a variation in the plasma luminosity. Since a fraction \( f \) of the Comptonized photons reenter and heat up the soft photon source, a variation in the plasma luminosity causes a variation in the soft
For a given variation in plasma temperature ($\delta T_p$) and time-averaged model parameters ($T_s, T_c, N_{esc}$, $l$, $A$, and $f$), the fluctuations in the photon number density ($\delta n_p$) can be obtained from equations (3), (4), and (6). The complex quantity $\delta n_p(E)$ leads to the time lag versus energy and amplitude versus energy for the QPO. Since the disk parameters ($T_s, T_c, N_{esc}$, and $A$) can, in principle, be constrained by fitting the time-averaged spectrum of the source, $\delta n_p(E)$ depends only on the amplitude of the plasma temperature variation, the plasma size, and the fraction of photons incident on the soft photon source.

3. RESULTS

The predictions of the Comptonization model described in § 2 are compared with the RXTE Proportional Counter Array (PCA) observations of the LMXB 4U 1608−52 on 1996 March 3 (Berger et al. 1996), where a 830 Hz QPO (the lower peak; see Mendez et al. 1998) was discovered. The time-averaged spectrum from this observation is well described by the model with a reduced $\chi^2 = 1.03$ in the energy range 3–20 keV. The soft photon Wien temperature and the photon index are well constrained to be $kT_s = 1.11 \pm 0.04$ keV and $\Gamma = 3.55 \pm 0.05$, respectively. However, since no cutoff in the spectrum was observed, spectral fitting imposed only a lower limit on the plasma temperature $kT_p > 20$ keV. Since the Kompaneets equation (1) is valid only for $kT_p < m_c^2$, $kT_p = 25$ keV has been chosen for the analysis (choosing $kT_p = 50$ keV makes no significant qualitative difference in the results). These values of $kT_s$ and $\Gamma$ give $N_{esc} = 1.44$ (eq. [2]) and a Compton amplification factor $A \approx 1.33$.

The model predictions and RXTE observations of the amplitude (rms fraction) as a function of energy for the 830 Hz QPO (Berger et al. 1996) are compared in Figure 1 for different values of $l$ and $f$. The overall normalization of the curve is fixed by choosing a best-fit value of $|\delta T_p|$. The predicted amplitude is zero at $E \approx 4$ keV (comparable to the peak of the input spectrum) since the emergent spectrum pivots about this energy. This behavior is expected, as oscillations in the soft photon Wien temperature and in the plasma temperature both preserve photon number. The lag is illustrated as a function of energy in Figure 2 (Vaughan et al. 1998) compared to the model prediction for the same values of $l$ and $f$ used in Figure 1. Reasonable fits to both variations are obtained for $4 \text{ km sec}^{-1} < l < 15 \text{ km sec}^{-1}$ and $0.5 < f < 0.85$.

Since the amplitudes of plasma and Wien temperature oscillations are nearly equal (i.e., $|\delta T_s| \approx |\delta T_p|$), the photon fluctuations at high energies, $\delta n_{ph} = \delta n_p(E \approx 15 \text{ keV})$, are predominately caused by $\delta T_s$, while the photon fluctuations at low energy, $\delta n_{ph} = \delta n_p(E \approx 3 \text{ keV})$, are predominately caused by $\delta T_p$. Thus, the time lag between $\delta T_s$ and $\delta n_{ph}$ is $\Delta t_{\delta n} \approx N_{ph} t_s$, where $N_{ph} \approx 9$ is the average number of scatters a high-energy photon undergoes. The time lag between $\delta T_p$ and $\delta n_{ph}$ can be expressed as $\Delta t_{\delta n} \approx t_p + \Delta t_{\delta n}$, where $t_p$ is the time lag between
Thus, the time lag between high- and low-energy photons is \( \Delta t_{\text{H}} \approx \Delta t_{\text{L}} \), so that when \( \Delta t_{\text{L}} > \Delta t_{\text{H}} \), the system will show soft lags.

If the soft photon source is assumed to be a blackbody instead of a Wien peak, an additional ad hoc oscillation in the soft photon number flux is required for the predicted rms fraction versus energy to agree with observations. Thus, a Wien spectrum is favored for the soft photons, since it minimizes the number of oscillating components needed.

In our model, the primary oscillation takes place in the plasma with the soft photon source responding to the variation. Alternatively, it is possible that the soft photon Wien temperature oscillates and, in order to keep the total luminosity of the plasma constant, induces variability in the plasma temperature. However, contrary to observations, such a model does not predict soft lags. Thus, it seems that the QPO manifests itself only as an oscillation in the plasma temperature rather than variations in other parameters of the Comptonization model.

The large value for the fraction of Comptonized photons incident on the soft photon source, \( f \approx 0.5 \), suggests that the geometry of the hot plasma can be described as a corona on top of a cold accretion disk. The size of the corona (\( l \approx 5 \) km) indicates that it is a significant fraction of the radius of the star. This result implies that the nearly coherent variability making up the kilohertz QPO is not limited to a narrow region and hints at the existence of a global mode in the corona. In such an interpretation, the high \( Q \)-values (~100) characteristic of the high-frequency oscillations would provide a strong constraint on the excitation and damping of the mode (Nowak et al. 1997). We point out that the Comptonization model does not provide insight into the detailed geometry of the source. An alternative geometry would be an X-ray source (i.e., the neutron star) surrounded by a flaring accretion disk. This geometry introduces an additional time lag due to the light travel time between the Comptonizing plasma and soft photon source.

Since the energy spectra did not exhibit an observable cutoff, the plasma temperature could be significantly larger than the assumed 25 keV. In that case, the average energy transfer per scatter would become a significant fraction of the photon energy, and Kompaneets’s equation, and the model predictions with it, would no longer be accurate (Katz 1987). Hence, simple extensions of our model to higher plasma temperatures are not useful, and detailed fitting to the observations will be warranted only when models incorporating nonstandard geometries in sophisticated time-dependent Monte Carlo methods (incorporating energy balance) are developed. In this light, the ability of the simple model presented here to explain the main features of a kilohertz QPO’s energy-dependent properties is highly encouraging.

In the future, this work will be extended to other systems exhibiting QPOs, particularly to the upper kilohertz QPO of the doublet observed in neutron star systems. Such an endeavor will not only check the robustness of the results presented here but also provide further constraints on theoretical models of the QPO phenomenon.

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