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They're back! Post-financialization diversification benefits of commodities

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A B S T R A C T

Do alternative assets such as commodities improve portfolio diversification? The empirical evidence is generally positive but mixed, and almost exclusively focuses on U.S. data. Using several distinct commodity indexes over the period 1993–2019, we investigate the case of an investor in Canada, a commodity-currency country where equities are already exposed to commodity beta. We use spanning tests and several out-of-sample performance measures for both risk-averse and disappointment-averse investors. Overall, we find that while the diversification potential of commodities was limited in Canada before and during financialization, the post-financialization period offers new opportunities. The evidence suggests that portfolio performance is significantly improved using some, but not all, commodity indexes. Thus, the choice of a relevant commodity index matters as a vehicle for diversification. Finally, compounding an international component to the sectorial diversification of the portfolio can significantly improve its performance.

1. Introduction

There is a well-documented negative correlation between commodity returns and equity assets (Bodie & Rosansky, 1980; Fortenbery & Hauser, 1990; Erb and Harvey, 2006, 2016; Gorton and Rouwenhorst, 2006; Ankrum & Hensel, 1993; Abanomey & Mathur, 1999). This feature of commodity markets has prompted financial institutions, institutional investors, and traders to consider commodities as a new asset class with useful diversification potential. Whether commodities offer good diversification potential continues to be debated in the literature. For instance, while Gorton and Rouwenhorst (2006) and Bhardwaj, Gorton and Rouwenhorst (2016) are bullish about the benefits of including commodities in an asset portfolio, Erb and Harvey (1996, 2016) are skeptical. This interest has stimulated growth in commodity index investments during the 2000s and accelerated the financialization of these markets, which has contributed to its expansion (Tang & Xiong, 2010). The financialization process is important for markets and investors, as the significant influx of capital into commodities has affected risk sharing, by integrating these previously segmented markets with other financial markets (Basak & Pavlova, 2016; Henderson, Pearson, & Wang, 2014).

The financialization of commodities has further implications for firms and investors in smaller open economies characterized as “commodity currency” markets. Commodity-currency countries are relatively small, open economies whose exports consist largely of primary commodities (e.g., Chen & Rogoff, 2003; Ready, Roussanov, & Ward, 2017). Examples are Canada, Australia, Norway and New Zealand. For such countries, fluctuations in international commodity prices are important to explain their terms of trade and exchange rates, and have financial implications for carry trades. Indeed, changes in commodity prices are linked to changes in exchange rates (Ferraro, Rogoff, & Rossi, 2015). For example, the U.S. dollar negatively covaries with commodity prices, while currencies such as the Canadian dollar positively covary with commodity prices (Clements & Fry, 2008).

This paper focuses on the strategic value of commodities in asset allocation for investors in a commodity-currency market such as Canada over the 1993–2019 period. In this setting, we investigate the value of adding this asset class to a portfolio using spanning tests as well as out-of-sample techniques for a wide range of assumptions regarding investor preferences. Moreover, we revisit the diversification potential of commodities with a decade of post-2008 financial crisis data coupled with strategic sub-period analysis.

The Canadian case is particularly relevant for our purposes due to five main reasons. First, Canada is the only commodity-currency market included in the G7 and is therefore of strategic importance. Second, its proximity and similarity to the U.S. allows for a comparison with the

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existing U.S. evidence not unlike a quasi-natural experiment. Third, its trade openness, as Canada is one of the most opened country both in the G7 (Kia, 2013) and when compared to other commodity currencies. Fourth, commodity trade accounts for 43% of Canada’s nominal exports in 2015 (Bank of Canada, 2016). These exports are dominated by crude oil, but are well diversified across a wide range of commodities.¹ The potential for an investor to diversify through commodities in commodity-currency economies such a Canada is unresolved and has important implications in terms of asset allocation. Fifth, the existence of a diversified Canadian commodity index with known weights allows for real-life replication.

We investigate the evolution in the attractiveness of commodities as vehicles for diversification for Canadian investors in order to bridge two gaps in the recent empirical evidence: i) the post-financialization period and ii) the case of non-U.S. investors. First, the recent post-financialization period may have changed the potential for commodities to improve portfolio performance. Indeed, increased participation by institutional investors in commodity markets was an important driver of financialization. Inflows of funds increased from the mid-2000s until the financial crisis, when institutional investors retreated. After the financial crisis and recession, investment in commodities by institutions including investment banks increased once again until 2014, when prices of commodities, and crude oil in particular, tumbled.² For instance, the Financial Times (2015) reports that commodity assets under management held by investors fell from $150bn pre-crisis to $67bn in 2015. Indeed, in 2014 alone, investors pulled $24.2bn in commodity assets.

One expected outcome of large investor outflows is lower correlations between commodities and equities, which should give back commodities their diversification potential. This hypothesis is suggested by Bhardwaj, Gorton, and Rouwenhorst (2016), but to our knowledge has not been formally tested. Moreover, research suggests that the diversification effect is typically reduced during periods of market turbulence (Butler & Joaquin, 2002; King & Wadhwani, 1990), so the last sub-period we examine allows for an updated assessment of this claim. In particular, empirical evidence with respect to the diversification benefits of commodities during volatile periods is mixed. On the one hand, many empirical studies were conducted before 2008, and their results imply that diversification benefits are amplified during periods of market turbulence (Gorton & Rouwenhorst, 2006). On the other hand, Tang and Xiong (2010) show that correlations between commodity and equity returns increased substantially during the recent subprime crisis.

In addition, we perform this analysis for different investor levels of relative risk aversion and disappointment aversion (DA) (Gul, 1991). In this framework, Canadian investors can be disappointment-averse as well as more or less risk-averse (e.g., Routledge & Zin, 2010). Our methodology allows for an out-of-sample analysis that goes beyond mean-variance or traditional power utility analysis. Thus, we investigate possible differences in the diversification gains associated with disappointment aversion.

Our second contribution is to study asset allocation outcomes for commodity-currency economies. Indeed, most research uses U.S. data, yet optimal investment decisions for U.S. investors may not be appropriate for investors in other markets. Research has found that commodity price volatility has large effects on such economies (e.g., Groen & Pesenti, 2010; Jordan, Vivian, & Wohar, 2016; Ready et al., 2017). Consider, for example, the effect of a commodities boom on Canadian markets. The appreciation of the Canadian currency has the effect of mitigating the impact of the boom. Prices in the domestic currency increase less than do world prices, so the profitability of the export sector increases less than expected. Domestic consumers benefit from this appreciation through inexpensive imports. This automatic stabilizer reduces the cyclical volatility of the Canadian economy (Büyükşahin, Haigh, & Robe, 2010). Thus, the positive correlation between changes in commodity prices and the domestic currency may reduce diversification gains. The strategic value of adding commodities to a portfolio for such investors is therefore unclear and warrants an empirical assessment.

Our results highlight three main findings. First, the potential for diversification for commodities has changed in Canada. While it was limited prior to and during financialization, in recent years Canadian investors can use commodities to improve their portfolio performance, as measured in terms of spanning tests and out-of-sample outcomes. Second, the choice of a particular commodity index is important. Our study includes several commodity indexes—both local and global alternatives—including or excluding the energy sector. While all indexes considered in this study have a similar diversification potential as measured by spanning tests, our results suggest that the Bloomberg commodity index is superior to the others out-of-sample. Finally, our results suggest that it is beneficial to add an international component to the sectorial diversification provided by commodities. We find that not only do U.S. commodity indexes provide better diversification potential compared to a Canadian commodity index, but also that using a commodity-currency index is superior to commodity indexes, based on spanning tests.

2. Literature review

The significance of our contribution is highlighted by the fact that most of the literature concerns U.S. investors and sample periods ending before or during the 2008–09 financial crisis. We thus review three main strands of the literature on portfolio diversification using commodities: (i) evidence based on pre-financial crisis sample data vs. evidence including the crisis period and later years, where no consensus has yet been reached; (ii) comparing the large evidence for U.S. investors with the more limited international evidence and (iii) performance differences between commodity indexes and individual commodities.

2.1. Overview of the literature on the diversification properties of commodities

The empirical evidence is generally supportive of the diversification role of commodities (see e.g., Abanomy & Mathur, 1999; Ankrim & Hensel, 1993; Anson, 1999; Belousova & Dorfleitner, 2012; Bessler & Wolff, 2015; Cheung & Miu, 2010; Daskalaki, Skiadopoulos, & Topaloglou, 2017; Forstenberg & Hauser, 1990; Willenbrock, 2011). The early literature in particular has found clear benefits to adding commodities to portfolios of stocks and bonds (Bodie & Rosansky, 1980; Forstenberg & Hauser, 1990; Ankrim & Hensel, 1993; Satyanarayan & Varangis, 1996; Abanomy & Mathur, 1999; Georgiev, 2001; Conover, Jensen, Johnson, & Mercer, 2010). More recent literature, however, has found that commodity diversification has weakened as a result of financialization and the financial crisis. For instance, Daskalaki and Skiadopoulos (2011) argue that in-sample diversification performance does not translate into out-of-sample success.

Since the early 2000s, investors, regulators, and academics have paid increasing attention to commodity futures markets and other commodity-linked investments (e.g., Skiadopoulos, 2013). This renewed interest is explained by the prospect of improving the risk-return profile of portfolios. Interest in commodities is also linked to periodic
bull cycles such as the commodity boom of 2004–2008. There is no consensus, however, on whether commodity booms and busts are due to speculative forces (e.g., Singleton, 2011; Tang & Xiong, 2010) or changes in fundamentals (e.g., Hamilton & Wu, 2015; Rouwenhorst & Tang, 2012; Stoll & Whaley, 2011). The source of these fluctuations matters for the diversification potential of commodities (Skia dopoulos, 2013). Before the Commodity Futures Modernization Act was passed in 2004, commodity prices included a premium for idiosyncratic risk (DeRoon, Nijman, & Veld, 2000) and commodities were weakly correlated with equities (Gorton & Rouwenhorst, 2006) as well as within their commodity category (Erb & Harvey, 2006). Commodity-equity correlations increased during the 2008–09 financial crisis (Tang & Xiong, 2010), but they remained lower their peak of the previous decade (Büyüksahin et al., 2010). For the period leading up to 2008 (i.e., the commodity boom and financialization period), research finds that diversification benefits are amplified during volatile periods (Gorton & Rouwenhorst, 2006).

2.2. Investors in the U.S. and other countries

Although most of the literature concerns the U.S., optimal portfolios for U.S. investors may not be the best guide for investors in other countries, especially commodity-currency economies. Research on European representative investors has found that adding commodities lowers portfolio risk and can increase returns (Belousova & Dorfleitner, 2012). However, for UK investors this may be true for only certain weighted portfolios or indexes of commodities (Giamouridis, Sakkas, & Tesseromatis, 2014). Commodities seem to help European investors even during bear markets (Delatte and Lopez, 2013), in particular gold and silver (Sarafrazi, Hammoudah, & Araújo Santos, 2014). On the other hand, research on commodity-currency countries is very limited. Cheung and Miu (2010) study commodity diversification for U.S. and Canadian portfolios over 1970–2005. They find that commodities help, but only during bull markets, and that benefits are greatest when risk-free rates are low.

The literature has expanded to look at emerging markets, in particular China. This research finds that commodities are beneficial except during crisis periods, when they do not in general provide a safe haven, whether for BRICS investors (Bekiros, Boubaker, Nguyen, & Uddin, 2017) or China (Hammoudah, Nguyen, Reboredo, & Wen, 2014). In China, only soybeans and soybean meal futures seem to be consistently useful (Liu, Tse, & Zhang, 2018) while gold provides a safe haven for energy stocks (Wen & Nguyen, 2017). Commodities also help investors in Asia by indirectly gaining international risk exposure (Batten, Szilagyi, & Wagner, 2015).

2.3. Diversification using commodity indexes and individual commodities

Gaining commodity exposure can involve positions in specific commodity indexes, or in individual commodity futures or portfolios of futures. In general, research finds that individual commodities are highly correlated within groups, so it is sufficient to examine one representative commodity per group (see e.g., Batten et al., 2015; Gao & Liu, 2014; Silvennoinen and Thorp, 2013). In some cases, however, specific commodities perform better than others, which justifies reviewing such evidence.

Overall, the recent literature finds that for U.S. equity investors, newer, third-generation indexes perform better than early-generation indexes over different sample periods (Daigler, Dupoyet, & You, 2017; Henriksen, Pichler, Westgaard, & Frydenberg, 2019; Miffre, 2016; Yan and Garcia, 2017). Daskalaki et al. (2017) confirm this finding for both second- and third-generation commodity indexes. Fethke and Prokopczuk (2018) agree that third-generation commodity indexes perform better, but find that their performance is not consistent. Henriksen (2018) concludes that diversifying using older commodity indexes was only beneficial prior to 2011.

Some of the literature explicitly considers dynamic models, assuming that investors can generate and exploit short-term forecasts. The findings are consistent with the rest of the literature. Commodities are generally beneficial, but help less during crisis periods (e.g., Fousekis & Grigoriadis, 2019; Öztek & Öcal, 2017). While dynamic models using forecasts increase portfolio returns, economic gains for investors may be limited due to higher volatility and turnover costs (e.g., Bernardi, Leippold, & Lohre, 2018; Lombardi & Ravazzolo, 2016; Pouliias & Papapostolou, 2018). Lastly, benefits vary by investor risk aversion (Cotter, Eyiah-Dokor, & Poti, 2017; Zhang and Lin, 2019) and investment horizon (Cai, Fang, Chang, Tian, & Hamori, 2020).

Research on the benefits of individual commodities is not conclusive, although the value of adding precious metals such as gold seems to be a consistent finding (e.g., Baur & Lucey, 2010). Demiralay, Bayraci, and Gencer (2019) find that lean hogs, feeder cattle, natural gas, orange juice, and gold are beneficial to add to a portfolio, while Bekiros, Nguyen, Uddin, and Sjö (2016) find that gold, platinum and heating oil are most helpful. Adding energy commodities may lower risk for a commodity portfolio in the post-financial crisis period (Rehman, Bouri, Eraslan, & Kumar, 2019). However, indexes that are energy-light or focused on metals perform better for U.S. investors over 1983–2013 than do indexes that load more on energy or agricultural commodities (Bessler and Wolff, 2015). While precious metals reduce downside risk, they may also lower the portfolio’s Sharpe ratios (Bredin, Conlon, & Poti, 2017). Gold is usually considered the best safe haven commodity, but other metals like palladium may perform as well (Agyei-Ampomah, Gounopoulos, & Mazouz, 2014).

3. Methodology

We use a portfolio optimization methodology based on the approach of Daskalaki and Skia dopoulos (2011) (thereafter, DS (2011)). The methodology allows us to move beyond the mean-variance framework and include higher-order moments for more general investor preferences. In this section, we present empirical evidence on the diversification potential of commodities, using both spanning tests and out-of-sample tests.

3.1. Spanning tests

Regression-based spanning techniques work by adding a risky asset to the initial MV efficient frontier used as a reference point (DeRoon & Nijman, 2001). This spanning test has the following null hypothesis: the mean-variance frontier of the reference assets increased by the test asset coincides with the reference asset boundary only. Then, no mean-variance investor will benefit from adding the new asset to their optimal portfolio constructed only from reference assets.

The technique used here is based on the stochastic discount factor (SDF) framework (DeRoon, Nijman, & Werker, 1996, 2003). An SDF $M_{t+1}$ is such that

$$\mathbb{E} [M_{t+1} R_{t+1} | \mathcal{F}_t] = l_t$$

where $R_{t+1}$ is the vector of gross returns of all $k$ assets forming the traditional portfolio (equities, bonds, and risk-free assets), $l_t$ denotes the information available at time $t$, and $\mathcal{F}_t$ is a unit vector of dimension $k$. The SDF is derived from first-order conditions of a portfolio optimization problem where the investor maximizes the expected utility of his or her terminal wealth (DeRoon & Nijman, 2001). In this case, the SDF is proportional to the first derivative of the utility function of wealth, given the investor’s optimal portfolio choice $w$.

$$M_{t+1} = \lambda U'(w R_{t+1})$$

where $\lambda$ is a constant corresponding to the subjective discount factor specific to the individual and $w$ is the optimal portfolio weighting vector. $M$ is a set of SDFs allowing the recovery of the price of the $k$ assets in the reference portfolio. It shows that returns $R_{t+1,\text{test}}$ of the
Table 1

Summary statistics.

|                          | Avg return | Std   | SR   | Skewness | Kurtosis | JB p value |
|--------------------------|------------|-------|------|----------|----------|------------|
| Bank of Canada 1 Month Commercial Paper Rates | 3%         | 0.6%  | 0.43 | 2.2      | 0.002    |
| S&P/TSX Composite Index  | 9%         | 14.1% | 0.42 | −0.97    | 6.36     | 0.001      |
| S&P Canada Aggregate Bond Index Total Return | 6%         | 4.2%  | 0.72 | −0.01    | 5.28     | 0.001      |
| Bank of Canada Commodity Avg Price      | 4.2%       | 18.5% | 0.07 | −0.5     | 4.73     | 0.001      |
| Bank of Canada Commodity Energy Avg Price | 9.2%       | 32.8% | 0.19 | −0.1     | 4.55     | 0.001      |
| Bank of Canada Commodity Ex Energy Avg Price | 2.6%       | 11%   | −0.04| −0.01    | 3.43     | 0.306      |
| Bloomberg Commodity Index Total Return  | 3.1%       | 12.6% | 0.01 | −0.15    | 3.6      | 0.056      |
| Bloomberg Energy Subindex Total Return  | 3%         | 27.2% | 0.01 | 0.17     | 4.08     | 0.004      |
| Bloomberg Exenergy Subindex Total Return | 3%         | 11.4% | 0.00 | 0.05     | 3.71     | 0.044      |
| Morningstar Commodity Currency Index TR | 6.8%       | 12.1% | 0.31 | 1.35     | 14.94    | 0.001      |

This table presents the descriptive statistics for the various asset classes used in this study. Results are based on monthly observations from February 1993 to April 2019. The summary statistics reported being the annualized mean returns, standard deviations, Sharpe Ratio (SR), skewness, kurtosis and the p-value Jarque-Bera normality test.

3.1.2. Non-mean-variance spanning tests

If the set of returns \( R_{t+1} \) also spans \( R_{t+1}^{new} \) for investors with a non-mean-variance utility function \( U(\mathbf{w}^\top R_{t+1}) \), then the error term \( \epsilon_{t+1} \) in Eq. (5) must be orthogonal to the derivative of the marginal utility \( U'(\mathbf{w}^\top R_{t+1}) \). Therefore, the set \( M \) considered includes MV linear SDFs as well as non-MV utility function SDFs that correspond to different degrees of risk aversion. Eq. (2) implies that any value given for the risk aversion coefficient imposes a different SDF that should be included in the set \( M \). Therefore, the spanning test must be performed by examining whether the relative restrictions are valid for each specific risk aversion coefficient value. Following DeRoon et al. (1996, 2003), DS (2011) estimate Eq. (3) by projecting the returns of the tested asset on the set \( M \) of SDFs:

\[
R_{t+1}^{new} - R_{t+1}^i = \alpha + \beta(R_{t+1} - R_{t+1}^i) + \sum_{i=1}^{n} \chi_i (\mathbf{w}^\top R_{t+1}) \epsilon_{t+1} \tag{7}
\]

where \( \epsilon_{t+1} \) is orthogonal to the set of SDFs \( M \) considered.

The null hypothesis for the spanning test is

\[
H_0: \alpha = 0 \tag{6}
\]

To account for the presence of autocorrelation and heteroskedasticity in the residual term, we use Newey-West (1987) standard errors. Moreover, before running the regression for Eq. (7), the unobserved regressors (i.e., marginal utilities) must be estimated. To this end, and to obtain the optimal portfolio weights, we assume the investor's preferences are described by a power utility function. This function implies decreasing absolute and constant relative risk aversions, which are desirable properties. The power utility function is defined as:

\[
U(w) = w^\gamma - 1, \quad \gamma > 0
\]

where \( w \) is the portfolio weight and \( \gamma \) is the risk aversion coefficient.
The optimal portfolio weights are estimated using the generalized method of moments (GMM) (see e.g., Cochrane, 2005). To implement this approach, the moment conditions generated by our SDFs must be defined. Given the non-MV utility function, Eqs. (1) and (2) imply that the returns of the \( K \) reference assets must satisfy the following conditions:

\[
E(U'(w_i^* R_{i+1}) R_{i+1} \mid l_i) = l_i \quad \forall i
\]

(10)

where \( i \) corresponds to the \( i \)th risk aversion value. \( \lambda U'(w_i^* R_{i+1}) \) corresponds to the SDF \( M_{i+1} \) which prices each return \( R_{i+1} \). The GMM approach estimates the parameters by making sample averages as close as possible to one another. For a sample of size \( T \), the moment conditions \( g_T(w_i^*) \) are defined as the sample mean of the errors \( u_{i+1}(w_i^*) \), such that

\[
g_T(w_i^*) = \frac{1}{T} \sum_{t=1}^{T} u_t(w_i^*) = E_T [u_t(w_i^*)] = E_T [\varepsilon U'(w_i^* R_t) R_t - l_i].
\]

(11)

By definition, for each risk aversion value \( i \), the SDF should fix the price of the three reference assets. Therefore, this approach provides three moment conditions for estimating \( w_i^* \). Finally, we obtain the GMM estimator of \( w_i^* \), that is, \( J_T(w_i^*) \), minimizing the following quadratic function:

\[
J_T(w_i^*) = g_T(w_i^*)' W g_T(w_i^*)
\]

(12)

where \( W \) is a positive definite weighting matrix. Since the number of unknowns is equal to the number of moment conditions, \( W \) is equal to the identity matrix.

### 3.2. Out-of-sample analysis: tests on expected utility maximization

In this section, we describe how we perform out-of-sample portfolio optimization by directly maximizing the expected utility of investors. We assume that investors' preferences are described by the power utility function (Eq. (9)). To ensure the robustness of our results, we consider several levels of relative risk aversion (RRA = 2, 4, 6, 8, 10). In addition, to account for more general behavioral characteristics in investor preferences we use the disappointment aversion framework (DA) introduced by Gul (1991). This model allows for investors to be loss-averse in addition to risk-averse. It means that at a certain reference point, investors are more sensitive to wealth losses than to gains. That is, they react asymmetrically. Our DA value function is based on the power utility function as in DS (2011), that is:

For brevity, only results for 2, 6 and 10 are reported.
Fig. 3. Cross correlations between Market Index and Commodity Indices.

This figure shows, from a Canadian and American perspective, the time-varying correlation between monthly rates of return on the market and commodities indices from February 1993 to April 2019. The correlations are computed with 72 months rolling window. Panel A uses the S&P/TSX Composite Index for representing the Canadian stock market and all prices are in Canadian dollars. Panel B uses the S&P 500 Total Return Index as proxy for the US stock market and all prices are in US dollars.

Table 2
Spanning tests.

| Test asset | 1993–2019 | 1993–2003 | 2004–2008 | 2008–2019 | 2007–2009 |
|------------|-----------|-----------|-----------|-----------|-----------|
|            | Full period | Pre-fin. Period 1 | Finan. Period 2 | Period 3 | Global fin Crisis |
| BCPI       | 0.01       | 3.13      | 0          | 2.13      | 0.2       |
|            | (0.92)     | (0.79)    | (0.99)     | (0.91)    | (0.65)    |
| BCPXN      | 0.6        | 5.53      | 0.36       | 4.27      | 0.4       |
|            | (0.44)     | (0.48)    | (0.55)     | (0.64)    | (0.53)    |
| BCPXIE     | 0.09       | 5.91      | 0.52       | 4.87      | 0.3       |
|            | (0.76)     | (0.43)    | (0.47)     | (0.56)    | (0.87)    |
| BCOM       | 0.19       | 12.1*     | 1.09       | 9.67      | 0         |
|            | (0.66)     | (0.06)    | (0.3)      | (0.14)    | (0.98)    |
| BCOMN      | 0.14       | 8.3       | 1.34       | 9.75      | 0.1       |
|            | (0.71)     | (0.22)    | (0.25)     | (0.14)    | (0.69)    |
| BCOMXE     | 0.12       | 2.83      | 0          | 4.07      | 0.37      |
|            | (0.73)     | (0.83)    | (0.95)     | (0.67)    | (0.54)    |
| MSCC       | 6.84***    | 11.16*    | 4.06**     | 28.1***   | 0         |
|            | (0.01)     | (0.08)    | (0.04)     | (0.00)    | (0.99)    |

This table presents the Wald test statistics and respective values for the null hypothesis: the mean-variance frontier of the reference assets increased by the test asset coincides with the reference asset boundary only. The results are based on monthly observations from February 1993 to April 2019. In each period, the MV spanning hypothesis (Eq. (5)) and the non-MV spanning hypothesis (Eq. (7)) are tested separately. The non-MV spanning test is done with a power utility function and use different levels of risk aversion coefficients (RRA = 2, 4, 6, 8, 10) and a subjective annualized discount factor of 0.95. The reference assets consist of the S&P/TSX Composite Index, the S&P Canada Aggregate Bond Total Return Index and the Bank of Canada’s 1-month commercial paper rate. All test statistics are based on a Newey-West covariance matrix with five offsets. One, two and three stars indicate significance at the level of 10%, 5% or 1% respectively.
| Table 3 | Out-of-sample performance: Bank of Canada Commodity Avg Price Indices. |
|---------|---------------------------------------------------------------|
| RRA     | K = 36     | K = 60     | K = 72     |
| ...     | 2  | 6   | 10  | 2  | 6  | 10   | 2  | 6  | 10   |
| Panel A: February 1993 to April 2019 |
| A = 1.0 |                    |                    |                    |
| Trad.   | 0.17 | 0.28 | 0.37 | 0.14 | 0.37 | 0.46 | 0.17 | 0.36 | 0.54 |
| Port. turn. | 63.8% | 50.7% | 40.5% | 31.3% | 19.9% | 21.3% | 27.2% | 18.0% | 18.0% |
| BCPI    | 0.10** | 0.17*** | 0.17*** | 0.16** | 0.28*** | 0.28*** | −0.01* | 0.23** | 0.41** |
| (p-Value) | 0.05 | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.07 | 0.04 | 0.02 |
| Port. turn. | 94.4% | 65.2% | 52.4% | 55.8% | 31.0% | 27.6% | 47.7% | 25.7% | 22.2% |
| Ret. loss | −4.7% | −2.8% | −2.4% | −3.8% | −2.8% | −1.9% | −3.8% | −1.7% | −1.2% |
| BCPEN   | 0.17** | 0.17** | 0.17** | 0.17** | 0.17** | 0.17** | −0.05* | 0.24* | 0.43* |
| (p-Value) | 0.12 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.06 | 0.08 | 0.06 |
| Port. turn. | 90.7% | 56.6% | 46.2% | 50.1% | 25.8% | 24.1% | 42.1% | 21.5% | 20.2% |
| Ret. loss | −4.4% | −2.7% | −2.2% | −4.2% | −2.3% | −1.6% | −4.3% | −1.5% | −1.0% |
| BCPXPE  | 0.14 | 0.23 | 0.31 | 0.18 | 0.31 | 0.38 | 0.25 | 0.32 | 0.44* |
| (p-Value) | 0.38 | 0.29 | 0.26 | 0.35 | 0.25 | 0.16 | 0.25 | 0.31 | 0.09 |
| Port. turn. | 101.3% | 70.4% | 56.1% | 57.6% | 38.0% | 31.9% | 45.8% | 31.1% | 27.3% |
| Ret. loss | −1.6% | −1.3% | −1.2% | −0.3% | −1.5% | −1.3% | 0.6% | −1.1% | −1.3% |
| Panel B: February 1993 to December 2003 |
| A = 1.0 |                    |                    |                    |
| Trad.   | 0.36 | 0.39 | 0.43 | 0.18 | 0.26 | 0.27 | 0.23 | 0.23 | 0.19 |
| Port. turn. | 41.0% | 35.0% | 34.6% | 37.9% | 26.6% | 24.8% | 42.2% | 29.4% | 28.4% |
| BCPI    | 0.25** | 0.30** | 0.08** | 0.15** | 0.18*** | 0.21 | 0.19 | 0.16 | 0.11 |
| (p-Value) | 0.06 | 0.02 | 0.03 | 0.03 | 0.01 | 0.01 | 0.30 | 0.19 | 0.11 |
| Port. turn. | 58.7% | 42.7% | 39.4% | 46.0% | 29.8% | 26.2% | 44.2% | 30.5% | 25.5% |
| Ret. loss | −2.5% | −1.5% | −1.0% | −1.3% | −0.9% | −0.6% | −0.4% | −0.3% | −0.2% |
| BCPXPE  | 0.22*** | 0.26*** | 0.07* | 0.18** | 0.20** | 0.18 | 0.20 | 0.17 | 0.28 |
| (p-Value) | 0.01 | 0.01 | 0.07 | 0.05 | 0.05 | 0.18 | 0.30 | 0.28 | 0.28 |
| Port. turn. | 59.2% | 41.5% | 38.3% | 44.0% | 28.9% | 25.7% | 44.1% | 30.3% | 25.2% |
| Ret. loss | −3.9% | −1.7% | −1.2% | −1.4% | −0.7% | −0.5% | −0.6% | −0.5% | −0.4% |
| BCPXPE  | 0.34 | 0.37 | 0.16 | 0.20 | 0.23 | 0.20 | 0.20 | 0.17 | 0.32 |
| (p-Value) | 0.16 | 0.29 | 0.23 | 0.42 | 0.19 | 0.21 | 0.36 | 0.34 | 0.32 |
| Port. turn. | 63.6% | 50.3% | 43.6% | 49.3% | 32.7% | 28.8% | 51.7% | 34.2% | 27.6% |
| Ret. loss | −2.0% | −1.1% | −0.9% | −0.7% | −0.8% | −0.5% | −0.8% | −0.5% | −0.3% |
| Panel C: January 2004 to March 2008 |
| A = 1.0 |                    |                    |                    |
| Trad.   | 0.10 | 0.16 | 0.16 | 0.12 | 0.23 | 0.23 | 0.09 | 0.15 | 0.15 |
| Port. turn. | 29.0% | 27.0% | 28.8% | 34.9% | 21.2% | 13.2% | 68.5% | 29.5% | 18.3% |
| BCPI    | 0.06 | 0.02 | 0.05 | 0.07* | 0.38* | 0.38* | 0.04 | 0.04 | 0.03 |
| (p-Value) | 0.21 | 0.24 | 0.08 | 0.08 | 0.08 | 0.08 | 0.34 | 0.22 | 0.21 |
| Port. turn. | 51.9% | 34.6% | 34.4% | 44.5% | 24.1% | 14.9% | 70.4% | 30.9% | 19.1% |
| Ret. loss | −2.7% | −1.6% | −1.1% | −1.5% | −0.9% | −0.7% | −0.7% | −0.7% | −0.7% |
| BCPXPE  | 0.10* | 0.10* | 0.17* | 0.14 | 0.10 | 0.05 | 0.08 | 0.10 | 0.10 |
| (p-Value) | 0.12 | 0.09 | 0.42 | 0.42 | 0.42 | 0.20 | 0.19 | 0.19 | 0.19 |
| Port. turn. | 51.9% | 34.5% | 34.4% | 41.4% | 25.2% | 15.6% | 69.0% | 31.9% | 19.6% |
| Ret. loss | −4.4% | −2.2% | −1.6% | −1.7% | −1.3% | −1.4% | −1.5% | −1.8% | −1.7% |
| BCPXPE  | 0.14 | 0.10 | 0.05 | 0.26 | 0.26 | 0.08 | 0.10 | 0.10 | 0.10 |
| (p-Value) | 0.34 | 0.45 | 0.37 | 0.39 | 0.45 | 0.45 | 0.40 | 0.40 | 0.40 |
| Port. turn. | 57.1% | 41.6% | 36.6% | 51.0% | 27.3% | 16.8% | 78.4% | 34.4% | 21.2% |
| Ret. loss | −2.5% | −1.4% | −1.3% | −1.4% | −1.7% | −1.7% | −1.3% | −1.7% | −1.7% |
| (continued on next page)
Table 3 (continued)

| RRA | K = 36 | K = 60 | K = 72 |
|-----|--------|--------|--------|
|     |        |        |        |
| **SR** |        |        |        |
| BCPHEN | 0.74  | 0.86  | 0.77   |
| (p-Value) | (0.21) | (0.30) | (0.16) |
| Port. turn. | 60.8% | 35.3% | 33.7% |
| Ret.-loss | −6.6% | −10.7% | −1.5% |
| BCPXEN | 0.99  | 0.98  | 0.89   |
| (p-Value) | (0.46) | (0.38) | (0.46) |
| Port. turn. | 74.3% | 48.4% | 45.2% |
| Ret.-loss | −2.2% | −4.4% | −0.9% |
| **SR** |        |        |        |
| Panel D: August 2007 to December 2009 |

| **SR** |        |        |        |
| Panel E: April 2008 to April 2019 |

| **SR** |        |        |        |
| A = 0.6 |

| **SR** |        |        |        |
| A = 0.6 |

(continued on next page)
This table presents performance measures (annualized Sharpe Ratio (SR), portfolio turnover, and annualized return-loss) in the case where the expected utility is maximized as part of a power utility function (\(A = 1\)) and with a disappointment utility \(A = 0.6\). The values of the Memmel’s (2003) SR test are indicated in parentheses below the test. The null hypothesis is that the SR obtained from the set of traditional investment opportunities is equal to that derived from the augmented set of commodities. The results are reported for different rolling window sizes (\(K = 36, 60, 72\) observations) and different relative risk aversion degrees (\(RRA = 2, 6, 10\)). Investors have access to commodity investments through the Bank of Canada Commodity Avg Price (BCPI), the Bank of Canada Commodity Energy Avg Price (BCPIEN) or the Bank of Canada Commodity Excluding Energy Avg Price (BCPIXE). Results are based on monthly observations from February 1993 to April 2019 and prices are in Canadian dollars. One, two and three stars indicate that annualized SR is statistically significantly different at the level of 10%, 5% or 1% respectively. Results are in bold when the commodity-augmented portfolio improves performance compared to the traditional portfolio.

| RRA  | K = 36 |  | K = 60 |  | K = 72 |  |
|------|--------|---|--------|---|--------|---|
|      | 2      | 6 | 10     | 2 | 6      | 10 | 2 | 6 | 10 |
| Trad. SR | 0.20  | 0.32 | 0.41 | 0.59 | 0.48 | 0.51 | 0.46 | 0.56 | 0.59 |
| Port. turn. | 57.8% | 44.6% | 39.5% | 32.5% | 32.3% | 34.4% | 33.3% | 26.6% | 24.8% |
| BCPI SR | 0.11  | 0.20 | 0.29 | 0.43 | 0.35 | 0.38* | 0.34 | 0.46 | 0.48* |
| (p-Value) | (0.35) | (0.17) | (0.14) | (0.17) | (0.12) | (0.06) | (0.22) | (0.15) | (0.09) |
| Port. turn. | 73.2% | 52.7% | 45.2% | 42.1% | 36.1% | 35.6% | 40.7% | 30.4% | 26.4% |
| Ret.-loss | 1.0%  | 1.3% | 0.9%  | 1.2% | 0.9% | 0.9%  | 1.3% | 0.9% | 0.7%  |
| BCPIEN SR | 0.07* | 0.17* | 0.26* | 0.43* | 0.37* | 0.42** | 0.32* | 0.48* | 0.50* |
| (p-Value) | (0.09) | (0.08) | (0.05) | (0.06) | (0.05) | (0.05) | (0.06) | (0.06) | (0.06) |
| Port. turn. | 71.8% | 49.4% | 41.3% | 39.6% | 33.2% | 33.8% | 39.9% | 27.8% | 24.7% |
| Ret.-loss | 2.7%  | 1.4% | 1.0%  | 1.5% | 0.7% | 0.5%  | 1.4% | 0.6% | 0.5%  |
| BCPIXE SR | 0.01  | 0.18* | 0.27* | 0.41* | 0.38 | 0.44 | 0.30* | 0.41** | 0.48* |
| (p-Value) | (0.11) | (0.09) | (0.06) | (0.08) | (0.18) | (0.22) | (0.08) | (0.05) | (0.06) |
| Port. turn. | 69.1% | 58.3% | 50.6% | 45.5% | 39.3% | 42.0% | 45.1% | 35.3% | 30.2% |
| Ret.-loss | 1.9%  | 1.5% | 1.3%  | 1.8% | 0.9% | 0.7%  | 1.8% | 1.4% | 0.9%  |

Optimization by direct maximization of the utility function involves solving the following problem:

\[
(W_t^*) = \begin{cases} 
\frac{W_t^{1-\gamma} - 1}{1 - \gamma} - \frac{1}{A} & \text{if } W_t > \mu_W \\
\frac{W_t^{1-\gamma} - 1}{1 - \gamma} - \frac{1}{A} & \text{if } W_t < \mu_W
\end{cases}
\]

where \(\gamma\) corresponds to the level of relative risk aversion, \(A \leq 1\) is the DA coefficient which controls the curvature of the utility function in the loss region, and \(\mu_W\) is the reference point used to differentiate gains from losses. Note that loss aversion decreases as \(A\) increases, and that when \(A = 1\) we have the standard power utility function with no disappointment aversion. Consistent with the literature (e.g., Driessen & Maenhout, 2007), we use \(A = 0.6, 0.8\) and we set the reference point \(\mu_W\) as the initial wealth invested at the risk-free rate. This modelling choice implies that investors use the risk-free rate as a reference point to differentiate between gains and losses. We further impose general constraints on the optimization process that the weights must be within the interval \([-100\%, +200\%]\). The constraint has been shown to work well (e.g., Daskalaki, Kostakis, & Skladopoulos, 2014).

Optimization by direct maximization of the utility function involves solving the following problem:

\[
\max_{\mathbf{W}_t} \left[ E[U(W_{t+1})] = \int \ldots \int U \left[ \sum_{i=1}^{N} \theta_i r_i \right] dF(r_1 \ldots r_N), \text{out } \sum_{i=1}^{N} \theta_i = 1 \right] \quad (14)
\]

where \(F(r_1 \ldots r_N)\) is the joint allocation function of \(N\) returns at time \(t + 1\), \(W_t\) is the investor’s wealth level at time \(t\), \(r_i\) is the return of asset \(i\) during the period and \(\theta_i\) is the weighting invested in the asset \(i\). The joint allocation function must be estimated and requires assumptions about the estimators or the parametric form of the distribution. This optimization is therefore subject to estimation errors. To circumvent this problem, we use the full-scale optimization (FSO) method proposed by Cremers, Kritzman, and Page (2005) and Adler and Kritzman (2007), as in DS (2011).

With this method, each of the periodic \(T\) sets of \(N\) empirical returns is treated as a future scenario with probability \(T^{-1}\). Utility is computed for each possible vector \(\theta_i\), as well as each scenario in the sample. Vector \(\theta_i\), having the highest average utility among all the scenarios, is the combination of optimal allocations \(\theta_{FSO}\). Full-scale optimization therefore involves solving the following problem:

\[
\theta_{FSO} = \arg \max_{\theta} \left( -T^{-1} \sum_{i=1}^{T} U(\theta^T R_t) \right), \text{where } \theta \in \Omega \quad (15)
\]

\[
\Omega = \{ -1 \leq \theta \leq 2, \theta^T = 1 \} \quad (16)
\]

where \(\Omega\) contains a budget constraint (\(\theta^T = 1\), where \(i\) is a unit vector) and a constraint on weights. These constraints ensure that all funds have been invested and allow short-sales up to a certain level.

The formation of the optimal portfolio is dynamic with a rolling-window approach of \(K\) months. At each point \(t\) in time, the last \(K\) observations are used to estimate the asset allocation that maximizes the expected utility function. The weights estimated at time \(t\) are used to calculate the out-of-sample return achieved during the period \(t, t + 1\). This process is repeated until the end of the sample is reached, and a series of \(T\) monthly out-of-sample returns is obtained. The series of time series returns obtained is then used to compute the out-of-sample performance of the two optimal portfolios. As a robustness exercise, we also perform additional out-of-sample analysis accounting for higher order moments by estimating the expected utility function using a Taylor second-order series expansion.5

3.3. Performance measurement

We use three commonly used performance measures: Sharpe ratio (SR), portfolio turnover, and a return/loss measure of risk-adjusted

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5 For brevity, only results for DA = 0.6 are reported since both cases yields similar results.

5 Robustness checks have been performed with this constraint. We have also tried successively the following constraints: a) the weights are within \(-100\%, +100\%\) interval and b) the weights are within the \(-100\% +200\%\) interval and we had a fee of 5 basis point is applied when an asset is sold (except RF). Results are similar to the one reported here and omitted for brevity.
### Table 4
Out-of-sample performance: Bloomberg Commodity Index.

| RRA     | K = 36 | K = 60 | K = 72 |
|---------|--------|--------|--------|
|         |        |        |        |
| A = 1.0 |        |        |        |
| Trad.   |        |        |        |
| SR      | 0.17   | 0.28   | 0.37   |
| Port.   | 63.8%  | 50.7%  | 40.5%  |
| turn.   | 31.3%  | 19.9%  | 21.3%  |
| (p-Value)| (0.25) | (0.24) | (0.29) |
| Ret.-loss| 1.0%   | -0.4%  | -0.4%  |
| BCOMM   | 0.26   | 0.47   | 0.60   |
| SR      | 0.46   | 0.75   | 0.54   |
| Port.   | 44.7%  | 25.7%  | 19.0%  |
| turn.   | 4.4%   | 2.4%   | 1.1%   |
| (p-Value)| (0.06) | (0.03) | (0.08) |
| Ret.-loss| -0.5%  | -0.4%  | -0.5%  |
| BCOME   | 0.20   | 0.31   | 0.29   |
| SR      | 0.16   | 0.28   | 0.34   |
| Port.   | 42.5%  | 23.6%  | 19.4%  |
| turn.   | 25.6%  | 1.9%   | 0.9%   |
| (p-Value)| (0.08) | (0.05) | (0.10) |
| Ret.-loss| -0.5%  | -0.4%  | -0.5%  |
| BCOMEX  | 0.20   | 0.31   | 0.29   |
| SR      | 0.16   | 0.28   | 0.34   |
| Port.   | 42.5%  | 23.6%  | 19.4%  |
| turn.   | 25.6%  | 1.9%   | 0.9%   |
| (p-Value)| (0.08) | (0.05) | (0.10) |
| Ret.-loss| -0.5%  | -0.4%  | -0.5%  |

| Panel B: February 1993 to December 2003 |
|-----------------------------------------|
| A = 1.0                                 |
| Trad.                                   |
| SR -0.07                                 |
| Port. turn. 92.5%                       |
| (p-Value) (0.41) (0.29) (0.30) (0.01) (0.00) (0.01) (0.05) (0.45) (0.39) |
| Ret.-loss -0.2%                         |
| BCOMM -0.21                             |
| Port. turn. 103.3%                      |
| (p-Value) (0.35) (0.33) (0.20) (0.24) (0.49) (0.44) (0.48) (0.34) (0.38) |
| Ret.-loss -0.3%                         |
| BCOME -0.13                             |
| Port. turn. 114.7%                      |
| (p-Value) (0.17) (0.28) (0.39) (0.34) (0.38) (0.42) (0.26) (0.38) (0.26) |
| Ret.-loss 3.4%                          |

| Panel C: January 2004 to March 2008 |
|-------------------------------------|
| A = 1.0                             |
| Trad.                               |
| SR 0.97                               |
| Port. turn. 32.0%                     |
| (p-Value) (0.21) (0.38) (0.41) (0.20) (0.12) (0.07) (0.26) (0.32) (0.36) |
| Ret.-loss -0.6%                      |

(continued on next page)
| RRA | K = 36 |       | K = 60 |       | K = 72 |       |
|-----|--------|-------|--------|-------|--------|-------|
|     |        | 2     | 6     | 10    | 2     | 6     | 10    | 2     | 6     | 10    |
| BCOMEN | SR  | 0.42 | 0.80 | 0.78  | 0.29 | 0.27 | 0.36  | 0.18 | 0.42 | 0.60  |
| (p-Value) | (0.10) | (0.30) | (0.32) | (0.31) | (0.33) | (0.28) | (0.28) | (0.28) | (0.26) |
| Port. turn. | 72.5% | 34.5% | 35.1% | 48.5% | 25.2% | 23.6% | 50.8% | 21.7% | 19.7% |
| Ret.-loss | −11.5% | −12.1% | −1.9% | −11.9% | −3.5% | −1.9% | −3.5% | −1.9% | −1.9% |
| BCOMEX | SR  | 0.80 | 0.81 | 0.77  | 0.17* | 0.12** | 0.25** | 0.52 | 0.44 | 0.52  |
| (p-Value) | (0.30) | (0.33) | (0.30) | (0.08) | (0.03) | (0.02) | (0.45) | (0.24) | (0.14) |
| Port. turn. | 76.0% | 48.6% | 38.0% | 63.5% | 37.8% | 29.6% | 43.1% | 27.6% | 24.7% |
| Ret.-loss | −2.6% | −0.5% | −0.4% | −5.5% | −3.7% | −2.7% | 0.7% | −2.2% | −2.3% |
| A = 0.6 | Trad. | 0.91 | 0.79 | 0.83  | 0.22 | 0.45 | 0.52  | 0.31 | 0.24 | 0.19  |
| Port. turn. | 29.3% | 33.0% | 42.2% | 40.9% | 27.0% | 31.1% | 29.0% | 30.0% | 29.9% |
| Ret.-loss | −11.5% | −1.2% | −1.0% | −5.2% | −3.5% | −2.2% | 0.7% | −2.2% | −3.2% |
| BCOM | SR  | 0.83 | 0.76 | 0.76  | −0.03* | 0.01** | 0.25** | 0.58* | 0.48* | 0.41*  |
| (p-Value) | (0.35) | (0.43) | (0.39) | (0.10) | (0.02) | (0.04) | (0.10) | (0.07) | (0.10) |
| Port. turn. | 76.0% | 48.6% | 42.8% | 62.7% | 27.4% | 23.6% | 43.1% | 27.6% | 24.7% |
| Ret.-loss | −1.6% | −1.5% | −0.6% | 1.1% | −1.1% | −0.6% | −1.6% | −0.1% | −0.2% |
| Panel D: August 2007 to December 2009 | A = 1.0 | Trad. | 0.69 | −0.57 | −0.46 | −0.63 | −0.40 | −0.26 | −0.54 | −0.43  |
| Port. turn. | 68.0% | 68.6% | 55.8% | 42.1% | 41.7% | 31.7% | 38.4% | 39.7% | 26.8% |
| Ret.-loss | −11.5% | −0.5% | −0.4% | −5.5% | −3.7% | −2.7% | 0.7% | −2.2% | −2.3% |
| BCOM | SR  | −0.43 | −0.50 | −0.32 | −0.42 | −0.33 | −0.24 | −0.15* | −0.13* | 0.02*  |
| (p-Value) | (0.18) | (0.38) | (0.28) | (0.22) | (0.37) | (0.45) | (0.07) | (0.06) | (0.08) |
| Port. turn. | 115.6% | 85.1% | 66.9% | 78.6% | 48.3% | 43.1% | 75.7% | 39.4% | 28.0% |
| Ret.-loss | −1.6% | −1.5% | −0.6% | 1.1% | −1.1% | −0.6% | −2.2% | 0.1% | −0.2% |
| Panel E: April 2008 to April 2019 | A = 1.0 | Trad. | −0.34 | −0.15 | −0.10 | −0.56 | −0.49 | −0.44 | −0.57 | −0.52  |
| Port. turn. | 45.9% | 31.3% | 27.7% | 67.6% | 38.3% | 32.6% | 64.1% | 36.9% | 38.4% |
| Ret.-loss | −11.5% | −0.5% | −0.4% | −5.5% | −3.7% | −2.7% | 0.7% | −2.2% | −2.3% |
| BCOM | SR  | −0.51 | −0.22 | −0.18 | −0.61 | −0.51 | −0.46 | −0.20** | −0.21** | −0.26*  |
| (p-Value) | (0.31) | (0.40) | (0.38) | (0.40) | (0.46) | (0.40) | (0.04) | (0.03) | (0.06) |
| Port. turn. | 78.7% | 61.4% | 49.6% | 79.5% | 63.1% | 45.7% | 76.8% | 69.4% | 37.5% |
| Ret.-loss | −2.5% | −1.5% | −1.2% | 1.1% | −1.3% | −1.2% | 1.2% | 2.9% | 2.9% |
| BCOMEN | SR  | −0.45 | −0.38 | −0.33 | −0.55 | −0.51 | −0.44 | −0.14* | −0.19** | −0.27**  |
| (p-Value) | (0.43) | (0.29) | (0.29) | (0.48) | (0.46) | (0.49) | (0.02) | (0.01) | (0.01) |
| Port. turn. | 100.5% | 75.2% | 58.6% | 67.5% | 55.6% | 40.7% | 36.9% | 34.8% | 38.5% |
| Ret.-loss | −2.4% | −3.0% | −2.2% | 0.5% | −0.6% | −0.2% | 6.7% | 3.3% | 1.7% |
| BCOMEX | SR  | −0.63 | −0.30 | −0.19 | −0.65 | −0.55 | −0.51 | −0.68 | −0.53 | −0.40  |
| (p-Value) | (0.30) | (0.43) | (0.43) | (0.31) | (0.33) | (0.15) | (0.49) | (0.42) | (0.29) |
| Port. turn. | 98.5% | 36.9% | 31.5% | 94.3% | 47.7% | 39.0% | 72.6% | 38.7% | 36.0% |
| Ret.-loss | −5.1% | −1.5% | −0.8% | −3.0% | −1.4% | −1.1% | −2.1% | −0.1% | 0.1% |

(continued on next page)
returns that are net of transaction costs (see e.g., DeMiguel, Garlappi, & Uppal, 2009; Kostakis et al., 2011, and Daskalaki & Skiadopoulos, 2011).

The first measure compares the risk-adjusted performance of our investment alternatives. A Memmel (2003) statistic is used to test whether the Sharpe ratios (SR) of the two strategies are statistically different. The estimator $\overline{SR}_c$ of strategy $c$ is defined as the average of excess out-of-sample returns, $\bar{R}_c$, divided by the standard deviation $\hat{\sigma}_c$: $\overline{SR}_c = \frac{\bar{R}_c}{\hat{\sigma}_c}$ (17)

Second, we compute the portfolio turnover rate to quantify the degree of rebalancing required to implement each of the two strategies. The portfolio turnover rate $PT_c$ of strategy $c$ is defined as the average absolute change in the weights on the T-K rebalancing periods and on all N available assets:

$PT_c = \frac{1}{T-K} \sum_{t=1}^{T-K} \sum_{j=1}^{N} \left( |w_{c,j,t+1} - w_{c,j,t}| \right)$

(18)

where $w_{c,j,t}$ is the optimal weighting of the portfolio in asset $j$ under strategy $c$ at time $t$, $w_{c,j,t+1}$ is the weighting before rebalancing at time $t + 1$, and $w_{c,j,t+1}$ is the desired weighting after rebalancing at time $t + 1$. The turnover rate of the portfolio can be interpreted as the average percentage of the portfolio value that is redistributed over the period.

Third, we use a measure of risk-adjusted returns that is net of transaction costs (DeMiguel et al., 2009). Consistent with prior research, we set the transaction cost per share $pc$ to 50 basis points for equities and bonds, 35 basis points for commodities futures indexes, and 0 for risk-free assets. The net-of-transaction-costs wealth $NW_e$ for strategy $c$ is given by:

$NW_{c,i+1} = NW_{c,i+1}(1 + r_{c,p,i}) \left( 1 - pc \times \sum_{j=1}^{N} |w_{c,j,i+1} - w_{c,j,i}| \right)$

(19)

where $r_{c,p,i}$ is the portfolio return achieved prior to rebalancing. Therefore, the net-of-transaction-costs return is defined by:

$\text{RNTC}_{c,i+1} = \frac{NW_{c,i+1}}{NW_{c,i}} - 1$

(20)

Therefore, using this metric of returns that are net of transaction costs, the return-loss measure is interpreted as the additional return required for the traditional investment strategy to perform as well as the commodity-augmented strategy, in terms of the Sharpe ratio. Let $\mu_{nc}$, $\mu_c$ be the monthly average of the RNTC sample of the strategy with the expanded and restricted investment opportunity, respectively, and $\sigma_{nc}$, $\sigma_c$ be the corresponding standard deviations. Then, the return-loss measure is given by:

$\text{return} - \text{loss} = \frac{\mu_{nc} - \sigma_{nc} - \mu_c}{\sigma_c}$.  

(21)

### 4. Data

#### 4.1. Periods studied

Most previous research has investigated sample periods ending during or prior to the financial crisis. This study covers a longer period, from February 1993 to April 2019 as well as strategic sub-periods. The first sub-period studies the pre-financialization period from January 1993 to December 2003 during which investor interest in commodities was renewed. The second period studied, January 2004 to March 2008, covers the financialization period during which prices of most commodities increased significantly and some reached historic highs. The last sub-period, April 2008 to April 2019, is the post-financialization period during which institutional investor interest in commodities waned and outflows of funds were large.

#### 4.2. Data

This study adopts the perspective of a Canadian investor and all monthly returns are expressed in Canadian dollars. The reference portfolio consists of three assets: the S&P/TSX Composite Index, the S&P Canada Aggregate Bond Total Return Index, and the Bank of Canada 1-month commercial paper rate. They represent the equity market, the bond market, and the risk-free rate, respectively. To gain exposure to the commodities asset class, the following commodity indices are considered:

a) The Bank of Canada Commodity Price Index (BCPI) and the Bank of...
Table 5

| RRA | K = 36 | K = 60 | K = 72 |
|-----|--------|--------|--------|
|     | 2      | 6      | 10     | 2      | 6      | 10     | 2      | 6      | 10     |

### Panel A: February 1993 to April 2019

| A = 1.0 |       |       |       |
|---------|-------|-------|-------|
| Trad. SR | 0.17 | 0.28 | 0.37 |
| Port. turn. | 63.8% | 50.7% | 40.5% |
| MCC SR | 0.12 | 0.29 | 0.35 |
| (p-Value) | (0.39) | (0.49) | (0.45) |
| Port. turn. | 93.8% | 68.8% | 58.0% |
| Ret.-loss | -2.0% | -0.9% | -1.1% |

### Panel B: February 1993 to December 2003

| A = 1.0 |       |       |       |
|---------|-------|-------|-------|
| Trad. SR | 0.18 | 0.34 | 0.38 |
| Port. turn. | 92.5% | 72.4% | 49.8% |
| MCC SR | 0.11 | 0.25 | 0.38 |
| (p-Value) | (0.03) | (0.01) | (0.01) |

### Panel C: January 2004 to March 2008

| A = 1.0 |       |       |       |
|---------|-------|-------|-------|
| Trad. SR | 0.91 | 0.79 | 0.83 |
| Port. turn. | 92.3% | 83.0% | 69.6% |
| MCC SR | 0.67** | 0.62** | 0.62** |
| (p-Value) | (0.03) | (0.01) | (0.01) |

### Panel D: August 2007 to December 2009

| A = 1.0 |       |       |       |
|---------|-------|-------|-------|
| Trad. SR | -0.69 | -0.57 | -0.46 |
| Port. turn. | 68.0% | 68.6% | 55.8% |
| MCC SR | -0.15 | -0.26 | -0.15 |
| (p-Value) | (0.19) | (0.22) | (0.21) |

### Panel E: April 2008 to April 2019

| A = 1.0 |       |       |       |
|---------|-------|-------|-------|
| Trad. SR | 0.33 | 0.29 | 0.38 |
| Port. turn. | 54.5% | 43.6% | 38.5% |

(continued on next page)
4.3. Descriptive statistics

Table 5 (continued)

| RRA | \(K=36\) | \(K=60\) | \(K=72\) |
|-----|-----|-----|-----|
|     | \(2\) | \(6\) | \(10\) | \(2\) | \(6\) | \(10\) | \(2\) | \(6\) | \(10\) |
| Trad. SR | 0.20 | 0.32 | 0.41 | 0.59 | 0.48 | 0.51 | 0.46 | 0.56 | 0.59 |
| Port. turn. | 57.8% | 44.6% | 39.5% | 32.5% | 32.3% | 34.4% | 33.3% | 26.6% | 24.8% |
| MCC SR | 0.21 | 0.33 | 0.45 | 0.54 | 0.51 | 0.56 | 0.39 | 0.52 | 0.60 |
| (p-Value) | (0.48) | (0.48) | (0.40) | (0.41) | (0.42) | (0.38) | (0.38) | (0.40) | (0.48) |
| Port. turn. | 84.8% | 59.2% | 51.2% | 37.9% | 30.4% | 33.0% | 40.1% | 27.5% | 25.0% |
| Ret.-loss | −0.8% | −0.6% | −0.3% | −0.7% | 0.0% | 0.2% | −1.3% | −0.6% | −1.1% |

This table presents performance measures (annualized Sharpe Ratio (SR), portfolio turnover, and annualized return-loss) in the case where the expected utility is maximized as part of a power utility function \((A = 1)\) and with a disappointment utility \(A = 0.6\). The values of the Memmel’s (2003) SR test are indicated in parentheses below the test. The null hypothesis is that the SR obtained from the set of traditional investment opportunities is equal to that derived from the augmented set of commodities. The results are reported for different rolling window sizes \((K = 36, 60, 72\) observations\) and different relative risk aversion degrees \((\text{RRA} = 2, 6, 10)\). Investors have access to commodity investments through the Morningstar Commodity Currency Index TR (MCC). Results are based on monthly observations from February 1993 to April 2019 and prices are in Canadian dollars. One, two and three stars indicate that annualized SR is statistically significantly different at the level of 10%, 5% or 1% respectively. Results are in bold when the commodity-augmented portfolio improves performance compared to the traditional portfolio.

Comparing the U.S. and Canadian trends in indexes underscores the differences between the two markets and further justifies our analysis. Fig. 2 further shows, from a Canadian and U.S. perspective, the evolution of the main indices during two sub-periods of interest. Indeed, for the period from January 2005 to June 2008, Daskalaki and Skiadopoulos (2011) conclude that adding commodities to a U.S. investor’s portfolio is profitable. This result is not surprising since this period is the longest and the strongest commodity bull cycle since the Second World War (Conceição and Marone, 2008). This is clearly seen in Panel B. Turning to the Canadian market, the S&P TSX has generally outperformed commodity indices. But given the significant presence of the commodities sector in Canada, the impact of a commodities boom is not the same for the Canadian and U.S. economies. As the Canadian dollar appreciates, this mitigates the impact of the boom. Indeed, prices in the domestic currency increase less than do world prices, the profitability of the export sector increases less than expected, and thus domestic consumers benefit from the appreciation in the form of cheap imports.

4.3. Descriptive statistics

Table 1 presents descriptive statistics of the monthly returns for the various asset classes over the period February 1993 to April 2019. In general, commodity indices have a lower average monthly return and a higher standard deviation than do equities and bonds. As a result, bonds and equities typically have higher Sharpe ratios than do commodity indices. This is also true for the Commodity Currency Index. The evidence we report is therefore consistent with previous studies, and confirms that commodity indices taken alone under-perform other asset classes (e.g., Jensen, Johnson, & Mercer, 2000). With the exception of the Bloomberg Commodity Index and the Bank of Canada Commodity Ex Energy Index, we reject at the 5% level the null hypothesis that returns are normally distributed (using the Jarque-Bera test).

Fig. 1 shows the evolution of prices over the full period. It suggests a strong positive correlation between the two commodity indices. Correlation between commodities and the stock market index is weak particularly from February 1993 to December 2004, but it was positive in 2008. Among the indices selected to represent commodities, the Morningstar Commodity Currency Index has more stable returns. In addition, it is the only index that had positive returns during the 2008–2009 financial crisis.

Table 1 presents descriptive statistics of the monthly returns for the various asset classes over the period February 1993 to April 2019. In general, commodity indices have a lower average monthly return and a higher standard deviation than do equities and bonds. As a result, bonds and equities typically have higher Sharpe ratios than do commodity indices. This is also true for the Commodity Currency Index. The evidence we report is therefore consistent with previous studies, and confirms that commodity indices taken alone under-perform other asset classes (e.g., Jensen, Johnson, & Mercer, 2000). With the exception of the Bloomberg Commodity Index and the Bank of Canada Commodity Ex Energy Index, we reject at the 5% level the null hypothesis that returns are normally distributed (using the Jarque-Bera test).

5. Results

5.1. In-sample analysis: spanning tests

This section presents and discusses the results of spanning tests where a commodity index is added to the conventional mix of equities, bonds, and risk-free assets. If the spanning condition holds, the asset being tested (here, commodities) does not provide any significant improvement to the tangency portfolio or global minimum-variance portfolio. Thus, it does not improve upon the efficient frontier. The MV spanning hypothesis (Eq. (5)) and the non-MV spanning hypothesis (Eq. (7)) are tested separately with a power utility function. The non-MV
| Panel A: February 1993 to April 2019 |
|------------------------------------|
| **A = 1.0**                        |
| **BCOM**                           |
| SR 0.26 0.38 0.44 0.16 0.38 0.40 0.36 0.58 0.70 |
| Port. turn. 90.5% 62.4% 52.2% 50.5% 29.9% 31.3% 44.7% 25.7% 24.4% |
| (p-Value) (0.49) (0.19) (0.41) (0.02) (0.05) (0.04) (0.26) (0.28) (0.33) |
| Ret.-loss 0.7% 2.1% 0.5% 6.6% 2.8% 2.2% 2.2% 0.9% 0.4% |
| **BCOMEN**                         |
| SR 0.26 0.50 0.47 0.46** 0.59** 0.60** 0.46 0.66 0.76 |
| Port. turn. 80.6% 55.8% 50.0% 45.0% 29.7% 30.2% 42.5% 25.6% 24.8% |
| (p-Value) (0.32) (0.16) (0.50) (0.21) (0.31) (0.18) (0.03) (0.03) |
| Ret.-loss 0.7% 2.1% 0.5% 6.6% 2.8% 2.2% 2.2% 0.9% 0.4% |
| **BCOMEX**                         |
| SR 0.20 0.26 0.31 0.16 0.28 0.34 0.25 0.34** 0.45** |
| Port. turn. 96.1% 67.5% 54.5% 52.8% 34.3% 31.6% 44.3% 29.5% 26.8% |
| (p-Value) (0.32) (0.16) (0.05) (0.21) (0.31) (0.18) (0.03) (0.03) |
| Ret.-loss 0.7% 2.1% 0.5% 6.6% 2.8% 2.2% 2.2% 0.9% 0.4% |
| **Panel B: February 1993 to December 2003** |
| **A = 1.0**                        |
| **BCOM**                           |
| SR −0.14 −0.17 −0.01 −0.55 −0.32 −0.36 −0.12 0.37 0.51 |
| Port. turn. 125.6% 81.8% 57.0% 67.8% 30.5% 41.5% 48.4% 26.9% 26.8% |
| (p-Value) (0.39) (0.45) (0.38) (0.02) (0.01) (0.03) (0.33) (0.21) (0.17) |
| Ret.-loss −1.8% 0.7% −0.8% 13.7% 5.8% 4.8% 3.7% 0.3% 0.4% |
| **BCOMEN**                         |
| SR −0.21 −0.14 −0.09 0.12** 0.23*** 0.17*** 0.12 0.34 0.62 |
| Port. turn. 103.3% 66.7% 51.2% 42.0% 33.6% 28.2% 39.2% 29.7% 28.6% |
| (p-Value) (0.05) (0.05) (0.08) (0.22) (0.34) (0.27) (0.02) (0.04) (0.03) |
| Ret.-loss −1.8% 0.7% −0.8% 13.7% 5.8% 4.8% 3.7% 0.3% 0.4% |
| **BCOMEX**                         |
| SR 0.13 0.17 0.24 −0.02** 0.14*** 0.06* 0.17 0.25 0.27 |
| Port. turn. 84.3% 71.0% 60.5% 54.0% 28.1% 36.2% 41.8% 23.5% 26.8% |
| (p-Value) (0.18) (0.14) (0.22) (0.00) (0.05) (0.08) (0.08) (0.45) (0.31) |
| Ret.-loss −4.0% −2.7% −1.7% −0.4% −0.4% −0.4% −3.3% −1.8% −1.3% |
| **Panel C: January 2004 to March 2008** |
| **A = 1.0**                        |
| **BCOM**                           |
| SR 0.76 0.86 0.83 0.24 0.15 0.24 0.26 0.54 0.75 |
| Port. turn. 90.2% 46.4% 42.7% 61.0% 32.6% 26.2% 61.3% 25.2% 19.4% |
| (p-Value) (0.42) (0.80) (0.78) (0.29) (0.27) (0.36) (0.18) (0.42) (0.60) |
| Ret.-loss −5.1% 0.8% 0.5% 3.6% 2.7% 2.4% 1.1% −0.7% −0.9% |
| **BCOMEN**                         |
| SR −0.34 −0.24* −0.19* 0.07*** −0.01** −0.01** 0.12 0.16 0.16 |
| Port. turn. 68.6% 46.4% 41.3% 54.0% 28.1% 36.2% 41.8% 23.5% 34.8% |
| (p-Value) (0.14) (0.17) (0.23) (0.03) (0.02) (0.03) (0.33) (0.41) (0.41) |
| Ret.-loss −4.7% −3.9% −2.6% 3.6% 2.7% 2.4% 1.4% −0.7% −0.9% |
| **BCOMEX**                         |
| SR 0.14 0.12 0.15 0.15** 0.09** 0.07** 0.30 0.27 0.26 |
| Port. turn. 84.3% 71.0% 60.5% 54.0% 28.1% 36.2% 41.8% 23.5% 26.6% |
| (p-Value) (0.36) (0.44) (0.46) (0.03) (0.04) (0.05) (0.45) (0.45) (0.31) |
| Ret.-loss −0.0% −1.6% −1.0% 4.0% 0.9% −0.2% 0.8% −2.0% −2.5% |
| **Panel D: August 2007 to December 2009** |
| **A = 1.0**                        |
| **BCOM**                           |
| SR 0.83 0.76 0.76 −0.03 0.09 0.25 0.58 0.48 0.41 |
| Port. turn. 43.3% 46.3% 52.0% 32.7% 36.5% 35.3% 31.1% 31.5% 33.3% |
| (p-Value) (0.34) (0.32) (0.49) (0.15) (0.13) (0.13) (0.26) (0.36) (0.36) |
| Ret.-loss 0.7% −0.2% 0.9% 4.6% 3.5% 2.9% 1.8% −0.8% −0.7% |
| **BCOMEN**                         |
| SR 0.74 0.73 0.70 0.04 0.28 0.38 0.29 0.31 0.30 |
| Port. turn. 54.0% 37.3% 43.4% 56.7% 33.3% 30.0% 42.4% 33.5% 33.3% |
| (p-Value) (0.40) (0.46) (0.42) (0.36) (0.13) (0.24) (0.15) (0.20) (0.28) |
| Ret.-loss 0.6% 1.4% 0.6% 1.0% 3.0% 1.5% −2.3% −0.7% −0.2% |

(continued on next page)
spanning tests use risk aversion coefficients of different levels (RRA = 2, 4, 6, 8, 10) and a subjective annualized discount factor of 0.95. We consider the full sample (1993–2019) and four sub-periods, namely before, during, and after the financialization period as well as during the global financial crisis.

The results are summarized in Table 2, which reports Wald test statistics, with p-values in parentheses. This test has the null hypothesis that there is spanning, meaning that the portfolio frontier augmented with the tested asset (i.e., commodities) coincides with the frontier from the reference portfolio. First, we consider the full sample period. We fail to reject the null hypothesis of MV spanning at the 5% level for all indexes considered except the Morningstar commodity currency index. In the non-MV case, we fail to reject the null for all indexes except Morningstar CC (p < 0.10). However, we find important differences when we look at each sub-period. In period 1 (before financialization), we generally fail to reject the spanning null hypothesis. However, we reject the null with the BCOMXE in period 2 (during financialization) and in all cases in period 3 (after financialization) for power utility (at the 1% level). These results suggest that period 1 results are driving the full sample findings. For the global financial crisis period, we fail to reject the null hypothesis of MV spanning at the 5% level for all indexes considered, while we reject the null of non-MV at 5% for every index except the MSC. Thus, the benefits of commodities vary by investor type and by commodity index type, and in addition are time-varying. Note that the full-sample performance of the Morningstar CC index suggests that investors should consider combining international diversification with commodity diversification.

During the third sub-period (where commodities perform best), institutional investors retreated from commodity markets, as prices—which partly recovered after the 2008–09 financial crisis and institutional diversification with commodity diversification.
recession—dropped even further in 2014. For example, WTI crude oil prices fell by about 73% between July 2014 and February 2016. As institutional investors usually take long-only positions, falling commodity prices encouraged an outflow of funds. Moreover, commodity index investment returns historically benefited from positive roll yield returns, as commodity futures curves were usually in backwardation. Over the period 2005–2015, however, curves were more often in contango, implying a negative futures roll return (Erb & Harvey, 2016). Given the outflows of institutional investor positions in commodities, the evidence suggests that correlations with financial instruments have decreased, and therefore that benefits from commodity diversification have increased.

The finding that we reject more often for power utility than MV is consistent with DS (2011), and suggests that gains from commodity diversification are greater when we consider the impact of higher moments on expected utility (which by definition MV ignores). This result also corroborates the broader literature’s emphasis on risk measures related to skewness and kurtosis (crash risk, asymmetry, jump risk, tail risk), whose importance for investors is now well established. However, the results in period 1 suggest that adding commodities to the investment portfolio may not always help against crash and tail risk. The reason for our findings differ somewhat from DS (2011) is likely the nature of the Canadian stock market index. This index loads more heavily on commodity-related industries and firms, which are exposed to commodity beta (e.g., Boons, Roon, & Szymanska, 2014). This result suggests that the investor should carefully choose the commodity index to yield benefits, particularly if the equity market is already exposed to commodity beta.

5.2. Out-of-sample analysis of performance: direct maximization of expected utility

5.2.1. Sharpe ratios

This section presents results for the out-of-sample (OOS) performance of traditional portfolios and those augmented by a commodity index. All optimal portfolios are constructed to directly maximize investor expected utility (for a range of risk aversion and disappointment aversion specifications). Results are presented in Tables 3 to 5 and include the Memmel test on Sharpe ratios (SRs). The null hypothesis is equality of the SRs for traditional investment opportunities and the commodity-increased portfolio.

First, we discuss the results regarding the Bank of Canada (BCIP) indexes (with and without energy), and present these in Table 3. Optimal portfolios formed using the BCIP index generate Sharpe ratios that are lower than those excluding commodities. Nearly all differences are statistically significant at the 5% level using the Memmel test. This finding is confirmed in sub-periods. The results improve if we use the index excluding energy (which is a particularly important industry in Canada, especially since the shale gas boom). Overall, however, the investor’s portfolio performance is not improved by adding the BCIP index.

Next, we consider the Bloomberg commodity index in Table 4. Sharpe ratios increase and the improvements are generally significant based on the Memmel test (at least at the 10% level and often 5%). In the full sample, improvements are significant for the 72-month window, both for the risk-averse and disappointment-averse investor. However, there are important differences across sub-periods, consistent with the evidence from spanning tests. Performance is significantly better in sub-period 3 for the 36- and 72-month windows, and to a lesser extent also in the 60-mo window. However, performance in periods 1 and 2 is either similar or worse (in the 60-month window).

The gains from the Bloomberg index are consistent with the spanning tests in terms of sub-period analysis, as they are clustered in the full period and the last sub-period of the sample. Excluding energy has a marginal impact here, as the results are similar. However, the comparison between diversification opportunities with and without energy in the index presented in Table 6 shows the possibility of some gains over the BCOM when including only the energy sector for 60- and 72-month windows for the full sample. During the global financial crisis, excluding the energy sector results in significantly smaller Sharpe ratios than the full index.

The main point we make is that there are economically and statistically significant improvements in OOS performance (across measures and utility specifications) at the 60- and 72-month horizons when investors use the Bloomberg index. However, performance is usually worse when the Bank of Canada commodity index is used. Thus, consistent with our evidence from spanning tests, we find that commodity diversification tends to work best when combined with international diversification. Indeed while the Bloomberg index is foreign (since U.S.-based), the BoC index is domestic. Indeed, the results suggest that it is better to turn to international diversification at the same time as adding commodities, and that is what the Bloomberg (BCOM) index offers. This finding is related to evidence from Carrieri, Errunza, and Sarkissian (2004) who find that investors should use international and sector diversification to improve portfolio performance.

Next, we discuss the commodity related foreign exchange indexes. Table 6 shows that the OOS performance of the optimal portfolios formed with the Morningstar Commodity Currency Index (MCC) does not improve on the traditional investment portfolio. This result is consistent across utility specifications, windows, and sub-periods. Together with the spanning tests, the results imply that while MCC is not spanned by the traditional portfolio, adding it does not generate statistically significant improvements out of sample. The two approaches tackle different questions and are two useful pieces of evidence to understand what exactly are the diversification benefits of commodities for investors in our sample. A commodity currency index can improve on the efficient frontier but not yield significant OOS performance, while a foreign commodity index, while possibly spanned, can nonetheless yield economically and statistically significant OOS improvements.

5.2.2. Alternative performance measures

Tables 3, 4, and 6 also present OOS results using alternative performance measures. These results are mostly consistent with the Sharpe ratios/Memmel test results.

First, we consider the return-loss measure, where a positive value implies that despite a portfolio turnover that is greater than that of a traditional investment portfolio, investors can still earn a higher return by adding commodities to his or her portfolio. We find an improvement for the Bloomberg index for the full sample (36 and 72 mo) and for period 3 (all windows), but performance is worse in periods 1 and 2. The global financial crisis period also shows positive return-loss measures, but gains disappear with disappointment aversion $A = 0.6$. These findings lend support to the main results, as investors benefit more from commodities in the most recent sub-period, post financialisation (2008–2019P), and for longer windows. Using the Bank of Canada index yields consistently worse performance, except during the global financial crisis for the index excluding the energy sector. This is another indication that excluding energy in the case context can lead to interesting diversification. The commodity-currency index shows mixed but typically weakly negative results, irrespective of the value for disappointment aversion.

Finally, portfolio turnover is similar across commodity indices. It is always higher when we add a commodity index, and it is nearly always decreasing in risk aversion and decreasing in the length of the window. However, this relationship is less clearly monotonic under disappointment aversion, as turnover is sometimes first decreasing and then increasing in risk aversion or window length or both. Overall, the results...

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8 This information comes from data provided by the St Louis FRED and Energy Information Administration.
for alternative performance measures are robust and lend strong support to our main findings using Sharpe ratios for risk-averse or appointment-averse investors.

6. Conclusion

This paper investigates the diversification potential of several commodity indexes over a long sample period (1993–2019) that includes the pre-, during, and post-financialization phases in commodity markets as well as the global financial crisis. We present evidence using spanning tests and out-of-sample performance measures. Our study focuses on Canada, a commodity-currency market the most opened country in the G-7 for this period and an ideal benchmark for this question because of its closeness with the USA, for which evidence is widely studied in the literature. Thus, this analysis examines the diversification potential of commodities for countries whose economies and currencies are strongly affected by commodities.

Our results highlight three main findings. First, while the potential for diversification using commodities had diminished prior to and during financialization, it is now back. We argue that the retreat of institutional investors from commodity markets market—associated with prices falling and volatility increasing—is one of the reasons why correlations are now lower than before. This translates into better diversification potential in the third sub-period we study (2008–2019). This improved diversification in the post-financialization sup-period also translates into a reasonable diversification potential using commodities during the global financial crisis. Therefore, not only are commodities back in style, but our evidence suggest that they may offer some diversification against economic turmoil, suggesting a safe-haven characteristic sometimes associated with commodities (see e.g., Miffre & Fernandez-Perez, 2015). Our results also suggest that, in general, commodity-linked improvements in terms of Sharpe ratios decrease with the investor’s level of risk aversion. This finding is in line with recent evidence presented in Lin and Zhang (2019) and Cai et al. (2020). Thus, our results are most relevant for investors with a lower relative risk aversion, who have the most to gain.

Second, we find that the choice of a particular diversification vehicle matters, as the indexes we consider do not contribute the same diversification profile. How much an index loads on the energy sector and how liquid it is appear to be important factors. Our results suggest that the Bloomberg index is superior to the others in terms of diversification for investors in commodity-currency economies. In terms of country diversification, our results suggest that a lighter loading in the energy sector (which tends to be pro-cyclical) provides some diversification during the global financial crisis.

Third and last, our paper provides a strong case for coupling international diversification along with sectoral (commodity) diversification. In our results, we find that the Morningstar commodity currency index improves on the efficient frontier, while regular commodity indexes do not. Furthermore, the U.S. commodity index has better out-of-sample performance than the Canadian commodity index. These results are especially relevant for more sophisticated investors, who are not constrained by the same investment set as institutional investors, and who can profit from the current diversification potential of commodities despite their volatility and strong cycles.

The results suggest that investment managers in commodity-currency economies such as Canada should pay particular attention to macroeconomic forces. Indeed, despite an ostensibly stronger link with the real economy, financial markets do not reflect only the export-based economy, because if it were the case there would be no diversification gains—which is not what we find. As for policymakers, our results suggest paying particular attention to interest rates, as diversification benefits increase in low rate environments. In the current context of near record low rates in 2020 following the Covid–19 crisis, diversification using commodities may be even more useful for investors. Future research could investigate the diversification potential of single and basket commodities internationally, especially in emerging markets.

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