Ground state of three qubits coupled to a harmonic oscillator with ultrastrong coupling

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We study the Rabi model composed of three qubits coupled to a harmonic oscillator without involving the rotating-wave approximation. We show that the ground state of the three-qubit Rabi model can be analytically treated by using the transformation method, and the transformed ground state agrees well with the exactly numerical simulation under a wide range of qubit-oscillator coupling strengths for different detunings. We use the pairwise entanglement to characterize the ground state entanglement between any two qubits and show that it has an approximately quadratic dependence on the qubit-oscillator coupling strength. Interestingly, we find that there is no qubit-qubit entanglement for the ground state if the qubit-oscillator coupling strength is large enough.

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I. INTRODUCTION

Recent experimental progresses related to the qubit-oscillator system in ultrastrong coupling regime have been reported in different light-matter interaction systems [1–10], where the coupling strength between a single qubit and a single oscillator reaches a significant fraction of the oscillator and qubit frequencies. In this ultrastrong coupling regime, the ubiquitous Jaynes-Cummings model [11] under the rotating-wave approximation (RWA) is expected to break down leading to a mass of unexplored physics and giving rise to unconventional quantum phenomena [12–24]. For example, superradiance transition [14], vacuum Rabi-splitting [1, 12], photon blockade [13], Bloch-Siegert shift [5], and plasmonic effect [10].

Since the Hamiltonian of a qubit-oscillator system contains counter-rotating terms that make the computational subspace unclosed, the fully analytical solution to the ground state of this Hamiltonian in the ultrastrong coupling limit is still not found. Although the spectrum and eigenfunction of the Rabi model beyond the RWA are known by numerical diagonalization in a truncated finite-dimensional Hilbert space [25, 26], the analytical solution to the qubit-oscillator system beyond the RWA is quite necessary for capturing the fundamental physics clearly. Such an analytical treatment has the potential to be extended to more complicated models for the implementation of the quantum information processing (QIP) [27]. Therefore, various mathematics approaches have been proposed to analytically obtain the ground state properties of the single-qubit Rabi model in the ultrastrong coupling regime [28, 30, 31]. For example, the generalized-RWA method [30, 31] functions well when the qubit frequency is smaller than the oscillator frequency, the variational treatment for the ground state [28, 29, 32] captures reasonably in the single-qubit Rabi system which is very hard to be generalized to multi-qubit Rabi systems, and the transformation method is a perturbation expansion and has been successfully applied to the single-qubit Rabi system [37, 40, 42].

Recently, the Tavis-Cummings model beyond the RWA has been extended to the multi-qubit case using an adiabatic approximation method when the qubit frequency is far larger than the oscillator frequency [38], and the ground state of the nearly resonant Rabi model of two qubits coupled to a harmonic oscillator has been analytically treated by using both the variational and the transformation methods [43]. The Rabi model of three and more qubits coupled to a common oscillator in the ultrastrong coupling regime has more potential applications in QIP [21, 22] than the single-qubit Rabi model, such as protected quantum computation [21], which is expected to be very promising with the circuit QED architecture.

However, the ground states of the three- and more-qubit Rabi models in the ultrastrong coupling regime have not been extensively studied. Braak generalized the method based on the $Z_2$ symmetry [35] to the three-qubit Dicke model [44] to analytically determine the system’s spectrum, which is dependent on the composite transcendental function defined through its power series. However, this method can not be extended to determine the concrete form of the ground state.

Different from the Ref. [44], we focus here on the analytic ground state of the three-qubit Rabi model in the ultrastrong coupling regime by the transformation method. By mapping the three-qubit Rabi model into a solvable Jaynes-Cummings-like model, we show that the ground state energy and the ground state of this three-qubit Rabi model can be approximately determined by the analytic expression based on the transformation method, which agrees well with the exactly numerical simulation in the ultrastrong coupling regime under different detunings. The ground state entanglement between any two qubits is characterized by using the pairwise entanglement and has a quadratic dependence on the qubit-oscillator coupling strength, which is able to be analytically determined within a wide range of parameters. The interesting feature in the ground state entanglement exists

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in its maximum value, which decreases quickly to zero and never increases again as the qubit-oscillator coupling strength is large enough.

II. TRANSFORMED GROUND STATE

The Hamiltonian of three identical qubits coupled to a harmonic oscillator without the rotating-wave approximation is \( \hbar = 1 \)

\[
H = \frac{1}{2}w_a(J_+^2 + J_-^2) + w_c a^\dagger a + g(a^\dagger + a)J_z,
\]

where \( a \) and \( a^\dagger \) are respectively the annihilation and creation operators of the harmonic oscillator with frequency \( w_c \). \( J_\| \{ \| = \pm, \pm \} \) describes the collective atomic operator of a spin-\( \frac{2}{3} \) system, satisfying the angular momentum commutation relations \( [J_z, J_\pm] = \pm J_\pm \) and \( [J_+, J_-] = 2J_z \). Physically, the spin-\( \frac{2}{3} \) system is non-trivial and the states are entangled in terms of individual qubit configurations. \( w_a \) is the transition frequency of each qubit. \( g \) represents the collective qubit-oscillator coupling strength.

The key point in this paper is to determine the ground state energy \( E_g \) and the ground state \( |\phi_g\rangle \) for the three-qubit Rabi system in the ultrastrong coupling regime, where \( H|\phi_g\rangle = E_g|\phi_g\rangle \). To derive the analytic ground state, we define \( |m\rangle \) to be an eigenvector of \( J_z \), i.e., \( J_z|m\rangle = m|m\rangle \) \( (m = -\frac{2}{3}, -\frac{2}{3}, \frac{2}{3}) \). Besides, we will respectively use \( |X\rangle_f \) and \( |0\rangle_f \) to represent the coherent field state with the real amplitude \( X \) and the vacuum field state.

In what follows we extend the transformation method used in the single- and two-qubit Rabi models \([42, 43]\) to the three-qubit Rabi model. To transform the Hamiltonian \( H \) into a mathematical form without the counterrotating wave terms, we apply a unitary transformation to the Hamiltonian \( H \):

\[
H' = e^SHe^{-S},
\]

with

\[
S = \chi(a^\dagger - a)J_z,
\]

where \( \chi \) is a variable to be determined. Then the transformed Hamiltonian \( H' \) is decomposed into three parts \([43]\):

\[
H' = H'_0 + H'_1 + H'_2,
\]

with

\[
H'_0 = \eta w_a J_x - (2g\chi - wc^2)J_z^2 + wc a^\dagger a,
\]

\[
H'_1 = (g - wc\chi)(a^\dagger + a)J_z + i\eta w_a \chi(a^\dagger - a)J_y,
\]

\[
H'_2 = w_a J_x \left\{ \cosh \left[ \chi(a^\dagger - a) \right] - \eta \right\} + i w_a J_y \left\{ \sinh \left[ \chi(a^\dagger - a) \right] - \eta \chi(a^\dagger - a) \right\},
\]

where \( \eta = \frac{1}{2} |0\rangle \langle \chi(a^\dagger - a)|0\rangle = \exp[-\frac{\chi^2}{2}] \). The terms \( \cosh \left[ \chi(a^\dagger - a) \right] \) and \( \sinh \left[ \chi(a^\dagger - a) \right] \) in \( H'_2 \) have the dominating expansions:

\[
cosh\left[ \chi(a^\dagger - a) \right] = \eta + O(\chi^2),
\]

\[
\sinh\left[ \chi(a^\dagger - a) \right] = \chi \eta(a^\dagger - a) + O(\chi^3),
\]

here \( O(\chi^2) \) and \( O(\chi^3) \) represent the double- and multi-photon transition processes containing higher-order operators for \( a^\dagger \) and \( a \), which can be neglected as an approximation when \( \chi \) is small. Thus, \( H' \approx H'_0 + H'_1 \). By now, our approximation procedure is the same as that in Ref. \([43]\). However, the main difference exists in the diagonalization for \( H'_0 \) with the collective qubit vectors. Such an operator \( H'_f = \eta w_a J_x - (2g\chi - wc^2)J_z^2 \) appearing in the qubit basis \( \Gamma = \{ |\frac{1}{2}\rangle, |\frac{1}{2}\rangle, |\frac{1}{2}\rangle, |\frac{1}{2}\rangle \} \) represents a renormalized four-level qubit system. This is different from the two-qubit Rabi system \([43]\) which is a renormalized three-level atomic system corresponding to the diagonalization for \( H_0 \). Through diagonalizing the operator \( H'_f \) in the qubit basis \( \Gamma \), we obtain the renormalized qubit eigenvector \( |\varphi_k\rangle \) with the eigenvalue \( \lambda_k \) \( (k = 1, 2, 3, 4) \) as following:

\[
\lambda_1 = 5A - \frac{1}{2}B - 2\sqrt{4A^2 + AB + \frac{1}{4}B^2},
\]

\[
|\varphi_1\rangle = \frac{1}{N_1} \left( |\frac{1}{2}\rangle - \frac{3}{2}\rangle + K_1|\frac{1}{2}\rangle + \frac{1}{2}\rangle + \frac{3}{2}\rangle \right),
\]

\[
\lambda_2 = 5A + \frac{1}{2}B - 2\sqrt{4A^2 + AB + \frac{1}{4}B^2},
\]

\[
|\varphi_2\rangle = \frac{1}{N_2} \left( |\frac{1}{2}\rangle - \frac{3}{2}\rangle + K_2|\frac{1}{2}\rangle + \frac{1}{2}\rangle + \frac{3}{2}\rangle \right),
\]

\[
\lambda_3 = 5A - \frac{1}{2}B + 2\sqrt{4A^2 + AB + \frac{1}{4}B^2},
\]

\[
|\varphi_3\rangle = \frac{1}{N_3} \left( |\frac{1}{2}\rangle - \frac{3}{2}\rangle + K_3|\frac{1}{2}\rangle + \frac{1}{2}\rangle + \frac{3}{2}\rangle \right),
\]

\[
\lambda_4 = 5A + \frac{1}{2}B + 2\sqrt{4A^2 + AB + \frac{1}{4}B^2},
\]

\[
|\varphi_4\rangle = \frac{1}{N_4} \left( |\frac{1}{2}\rangle - \frac{3}{2}\rangle + K_4|\frac{1}{2}\rangle + \frac{1}{2}\rangle + \frac{3}{2}\rangle \right),
\]

and

\[
A = -\frac{1}{4}(2g\chi - wc^2),
\]

\[
B = \eta w_a,
\]

\[
K_1 = \frac{1}{\sqrt{3B}} \left( 8A + B + 4\sqrt{4A^2 + AB + \frac{1}{4}B^2} \right),
\]

\[
K_2 = \frac{1}{\sqrt{3B}} \left( 8A - B + 4\sqrt{4A^2 - AB + \frac{1}{4}B^2} \right),
\]

\[
K_3 = \frac{1}{\sqrt{3B}} \left( 8A + B - 4\sqrt{4A^2 + AB + \frac{1}{4}B^2} \right),
\]

\[
K_4 = \frac{1}{\sqrt{3B}} \left( 8A - B - 4\sqrt{4A^2 - AB + \frac{1}{4}B^2} \right),
\]

where \( N_k = \sqrt{2 + 2K_k^2} \) \( (k = 1, 2, 3, 4) \) is the normalization factor for the eigenvector \( |\varphi_k\rangle \). For \( \chi \approx g \approx w_a \),
the eigenvalues here are arranged in the decreasing order through the numerical simulation, i.e., \( \lambda_1 < \lambda_2 < \lambda_3 < \lambda_4 \). Therefore, \( H' \) can be expanded with the above renormalized eigenvectors:

\[
H' = \sum_{k=1}^{4} \lambda_k |\varphi_k\rangle_a \langle \varphi_k| + \left[ (C_1 a + C_2 a^\dagger) |\varphi_1\rangle_a \langle \varphi_2| + (C_3 a + C_4 a^\dagger) |\varphi_3\rangle_a \langle \varphi_4| + (C_7 a + C_8 a^\dagger) |\varphi_3\rangle_a \langle \varphi_4| + H.c. \right] + w_c a^\dagger a,
\]

where \( C_x (x = 1, 2, 3, \ldots, 8) \) is the coefficient depending on the variable \( \chi \). It is obvious to see that \( C_1, C_3, C_5 \) and \( C_7 \) represent the coupling strengths of the corresponding counter-rotating terms with respect to the renormalized eigenvectors in Eq. \( \text{(12)} \).

Similar to the single- and two-qubit Rabi systems \([12, 13]\), the main task after transforming the Hamiltonian \( H \) into \( H' \) is to eliminate the counter-rotating terms for the eigenvector with the lowest eigenenergy. The major obstacle here is to remove the different coupling coefficients \( C_1 \) and \( C_3 \) of two counter-rotating terms for the eigenvector \( |\varphi_1\rangle_a \) simultaneously. This is very different from the single-qubit \([12] \) and the two-qubit Rabi models \([13]\), which just have one counter-rotating term for the approximate ground state vector. Although it is not possible to simultaneously remove the coefficients \( C_1 \) and \( C_3 \) for all the values of \( \chi \), we find that the conditions \( C_1 = 0 \) and \( C_3 \approx 0 \) can be both satisfied when \( 0 \leq \chi \leq 0.5 \), meaning two counter-rotating terms for the approximate ground state vector \( |\varphi_1\rangle_a \) can both be eliminated if the qubit-oscillator interaction is not too strong. The coefficients \( C_1 \) and \( C_3 \) have the following analytical forms:

\[
C_1 = (3 + K_1 K_2) (g - w_c \chi)
\]

\[
C_3 = (3 + K_1 K_4) (g - w_c \chi)
\]

\[
-\eta w_c \chi \left( \sqrt{3} K_1 + 2 K_1 K_2 - \sqrt{3} K_2 \right),
\]

\[
-\eta w_c \chi \left( 3 K_1 + 2 K_1 K_4 - \sqrt{3} K_4 \right).
\]

Therefore, \( |\varphi_1\rangle_a \) is expected to be the approximate ground state vector if the conditions \( C_1 = 0 \) and \( C_3 \approx 0 \) are both satisfied, then the ground state \( |\phi_\chi\rangle \) of this three-qubit Rabi system approximates the transformed ground state \( |\phi_\chi'\rangle \):

\[
|\phi_\chi'\rangle = e^{-S} |\varphi_1\rangle_a |0\rangle_f
\]

\[
= \frac{1}{N_1} \left[ - | - \frac{3}{2} a | \frac{3}{2} \lambda f + K_1 | - \frac{1}{2} a | \frac{1}{2} \lambda f - K_1 | \frac{1}{2} a - \frac{1}{2} \lambda f + | \frac{3}{2} a - \frac{3}{2} \lambda f \right],
\]

and the transformed ground state energy \( E_\chi' \) is:

\[
E_\chi' \simeq \lambda_1
\]

\[
= \frac{5}{4} w_c \chi^2 - \frac{5}{2} g \chi - \frac{1}{2} w_a e^{-\frac{\chi^2}{2}} - \left[ (2 g - w_c \chi)^2 \right]^{\frac{1}{2}}
\]

\[-w_a (2 g - w_c \chi)^2 e^{-\frac{\chi^2}{2}} + w_a^2 e^{-\chi^2}. \]

According to the condition \( C_1 = 0 \), the numerical solution of \( \chi \) is plotted as a function of the coupling strength \( g \) for different qubit-oscillator detunings in Fig. 1(a). We find \( \chi \) has a proportional relation with \( g \): \( \chi \simeq \frac{g}{w_c + w_a} \). By substituting the result \( \chi \) from Fig. 1(a) into Eq. (14), we obtain the corresponding solution for \( C_3 \) in Fig. 1(b), which shows the conditions \( C_1 = 0 \) and \( C_3 \approx 0 \) can be both satisfied when \( 0 \leq g \leq 0.5 w_a \). This guarantees two counter-rotating terms in the eigenvector \( |\varphi_1\rangle_a \) are both eliminated if the qubit-oscillator coupling is not too strong. In Fig. 1(c), we have estimated the accuracy between the transformed ground state energy \( E_\chi' \) and the exact ground state energy \( E_\chi \) under different qubit-oscillator detunings, in which \( E_\chi' \) has an approximately quadratic dependence on \( g \):

\[
E_\chi' \simeq \frac{3}{2} w_a - \frac{3}{2} w_c + 3 w_a g^2.
\]

We see that the transformed ground state energy achieves the nearly perfect matching with the exactly numerical value within the ultrastrong coupling regime \( g \leq 0.5 w_a \). When \( g = 0.5 w_a \), the errors for the transformed ground state energy at \( w_c = 0.8 w_a \) and \( w_c = w_a \), and \( w_c = 1.2 w_a \) are 0.49%, 0.19%, and 0.07%, respectively. Especially, when there is a positive detuning \( w_c - w_a > 0 \), the transformed ground state energy fits much better with the exact value for a wide range of \( g \) than that with the negative qubit-oscillator detuning or the exact qubit-oscillator resonance, and its error is only 0.9% even when \( g = 0.8 w_a \) for \( w_c = 1.2 w_a \). This result coincides with the variational behavior of \( C_3 \) in Fig. 1(b), in which \( C_3 \) grows much smaller than \( C_1 \).
FIG. 2. (Color online) The pairwise entanglement $N_{ρ_{ps}}$ as a function of the coupling strength $g$ under different qubit-oscillator detunings: (a) $w_l = 0.8w_a$; (b) $w_l = w_a$; (c) $w_l = 1.2w_a$. The solid line (dashed line) corresponds to the pairwise entanglement of the transformed (exact) ground state.

slower with $w_c = 1.2w_a$ than the case with $w_c = 0.8w_a$ or $w_c = w_a$ when the coupling strength satisfies $g > 0.5w_a$.

To examine the reliability of the transformed ground state $|φ_g′⟩$, we use the fidelity $F$, which is defined as $F = ⟨φ_g′|φ_g⟩$, as a measurement for the transformation method. From the result of Fig. 1(d), we find that the exact ground state for the three-qubit Rabi model can be approximately represented by $|φ_g⟩$ within the ultrastrong coupling regime $0 ≤ g ≤ 0.5w_a$. For example, we obtain the fidelity with high value $F > 99\%$ for $g ≤ 0.5w_a$ under different qubit-oscillator detunings.

III. GROUND STATE ENTANGLEMENT

To investigate the qubit-qubit entanglement for the present three-qubit Rabi model in the ground state, in which the prescription set out for symmetricDicke states is used, we proceed to consider the pairwise entanglement [13, 14] between any two qubits.

Taking the transformed ground state $|φ_g⟩$ in Eq. (15), the reduced density matrix $ρ_{ps}$ of any two qubits can be written as:

$$ρ_{ps} = \begin{pmatrix} ρ_{11} & 0 & ρ_{14} \\ 0 & ρ_{22} & ρ_{23} \\ 0 & ρ_{32} & ρ_{33} \end{pmatrix},$$

(18)

where

$$ρ_{11} = ρ_{44} = \frac{N^2 - 2N + 4J^2_1}{4N(N - 1)} = \frac{1}{6} + \frac{1}{3(1 + K_1^2)}.$$

$$ρ_{14} = ρ_{41} = \frac{(J_1^2)}{N(N - 1)} = \frac{\sqrt{3}K_1}{3(1 + K_1^2)}e^{-2x^2},$$

$$ρ_{22} = ρ_{23} = ρ_{32} = ρ_{33} = \frac{N^2 - 4J^2_1}{4N(N - 1)} = \frac{K_1}{3(1 + K_1^2)},$$

(19)

and the standard basis in $ρ_{ps}$ is $\{|e_i⟩|e_m⟩⟩, |e_i⟩|s_m⟩⟩, |s_i⟩|e_m⟩⟩, |s_i⟩|s_m⟩⟩$, with $|e_i⟩$ (|$e_m⟩$) and $|s_i⟩$ ($|s_m⟩$) ($l, m = 1, 2, 3$; and $l ≠ m$) denoting the excited and ground state of the $l$th ($m$th) qubit, respectively. Therefore, the pairwise entanglement $N_{ρ_{ps}}$ can be expressed as:

$$N_{ρ_{ps}} = 2\max\left\{0, |ρ_{14}|, |ρ_{14}| - |ρ_{23}|, |ρ_{14}| - |ρ_{32}|, |ρ_{14}| - |ρ_{23}|, |ρ_{14}| - |ρ_{32}|, |ρ_{14}| - |ρ_{23}|, |ρ_{14}| - |ρ_{32}|\right\}.$$  

(20)

In the ultrastrong coupling regime $g ≤ 0.5w_a$, we can numerically verify:

$$N_{ρ_{ps}} = \frac{2(\sqrt{3}K_1e^{-2x^2} - K_1^2)}{3(1 + K_1^2)} ≃ \frac{1}{4(w_a + 9w_c)^2}g^2.$$  

(21)

Fig. 2 illustrates the pairwise entanglement $N_{ρ_{ps}}$ obtained from the transformed and the exact ground states versus the coupling strength $g$ under different detunings. We see that the pairwise entanglement has a quadratic dependence on $g$ at small coupling strength, which is mathematically captured by the approximate power law between $N_{ρ_{ps}}$ and $g$ in Eq. (20). If $g > 0.5w_a$, discrepancies for the numerical results between the transformed and exact ground states become bigger as the coupling strength increases further. The maximal pairwise entanglement between any two qubits is determined by the detuning $Δ = w_c - w_a$. For the positive detuning $Δ > 0$, the increase of $Δ$ leads to the increase of the maximal entanglement in the qubit system, whereas it is opposite for the negative detuning case. Interestingly, the pairwise entanglement $N_{ρ_{ps}}$ decreases quickly to zero after reaching its maximum, and remains at zero even when the coupling strength $g$ increases further, which means that there is no qubit-qubit entanglement in the ground state of such a model any more if the coupling strength $g$ is large enough. For example, the pairwise entanglement decreases to zero for $g = 1.5w_a$ at the exact resonant case and never increases again even when $g$ increases further. This feature is distinguished from the result of Ref. [13] and can be explained as follows. When the field and one qubit is traced out, the first and fourth terms of Eq. (15) do not result in the entanglement of the other two qubits. In other words, the pairwise entanglement is contributed by the $W$-state components $|− \frac{1}{2}⟩_a$ and $|\frac{1}{2}⟩_a$. The coefficient $K_1$ of the terms involving $|− \frac{1}{2}⟩_a$ and $|\frac{1}{2}⟩_a$ quickly decreases to zero when $g$ is large enough resulting in the vanishing of pairwise entanglement.

IV. CONCLUSION

In summary, we have shown that the ground state of the three-qubit Rabi model in the ultrastrong coupling regime can be analytically treated by the transformation method. The transformed ground state fits very well with the exact ground state for different detunings even when the coupling strength $g$ increases to $0.5w_a$. When $g = 0.5w_a$, the error of the transformed ground state energy is only $0.19\%$ at $w_a = w_c$, and the fidelity for the transformed ground state keeps higher than
99% when $g \leq 0.5\omega_w$ under different qubit-oscillator detunings. Finally, we use the pairwise entanglement to analytically examine the qubit-qubit entanglement, and the result shows that the ground state entanglement has an approximately quadratic dependence on the qubit-oscillator coupling. Interestingly, we find that there is no ground state entanglement if the qubit-oscillator coupling strength is large enough.

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