Minimal Flavour Violation with Strong Higgs Dynamics

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ABSTRACT: We develop a variant of the Minimal Flavour Violation ansatz for the case of a strongly interacting heavy-Higgs boson sector. The tower of effective operators differs from that for a Higgs system in the linear regime, and the new operators obtained at leading order include a CP-odd one. We investigate the impact of these operators on $\Delta F = 1$ and $\Delta F = 2$ observables, demonstrating that the non-linear scenario may have an interesting impact on the anomalies in present data.

KEYWORDS: Minimal Flavour Violation, Strong Interacting Higgs, Flavour Changing Neutral Currents
1 Introduction

Electroweak precision tests [1], when analyzed within the SM and assuming a linearly realized Higgs sector, clearly point since long to a light scalar degree of freedom. By light we mean here and below a Higgs mass close to its present lower bound, while by heavy we will refer to a Higgs mass in the TeV range or larger.

Latest LHC data could be hinting to a SM-like light Higgs particle with a mass around 125 GeV [2, 3]. Would this hint be substantiated by LHC data in the next months, with the resulting picture pointing towards a pure SM scenario at those energies, the need to find alternative solutions to the electroweak hierarchy puzzle would be more compelling...
than ever: the SM “desert” should flourish with new physics (NP) at the TeV scale, to solve the electroweak hierarchy puzzle.

Nevertheless, the observed excess of events in the 120-130 GeV region is still well compatible with a background fluctuation, in which case the two most obvious avenues will be: i) either the Higgs is there but hidden in the data, for instance because it mixes with singlet scalars, which is known to generically dilute the strength of the signals (see for example [4] and reference therein); ii) or the Higgs mass is around the TeV range or even larger (up to the limit of infinite mass).

Possibility ii) is a very interesting physics option, which may correspond for instance to a composite scalar, like in technicolor [5, 6], or simply to the absence of a physical Higgs excitation. Recall that in the Standard Model of particle physics (SM), the Higgs mass is given by $m_H^2 \sim \lambda/G_F$, where $G_F$ denotes the Fermi constant and $\lambda$ is the quartic self-coupling of the Higgs field. For values of the Higgs mass $m_H \geq 1$ TeV, it follows that $\lambda \geq 1$: the Higgs interactions enter a strong (non-linear) regime. A heavy Higgs scenario thus points to non-elementary Higgs quanta and a rich strong dynamics around the TeV scale, a range of energy fortunately at reach at the LHC in a few years.

Notice as well that, if a light Higgs and nothing else is confirmed to exist below TeV scales, the Higgs sector could still encode a strong TeV dynamics to account for the hierarchy problem or, in other words, a non-linear realization of the electroweak symmetry breaking mechanism. For instance, recent models (see Ref. [7] for an overview) of a light composite Higgs sector include in the spectrum a light Higgs which is a quasi-Goldstone boson of the theory.

In the case of a purely weakly interacting Higgs, that is, of a purely linearly realized SM, the impact on low-energy observables of new physics at a scale $\Lambda \gg v$, where $v$ is the vacuum expectation value (vev) of the Higgs field, can be parameterized in a model-independent way as a tower of effective operators organized as a Taylor series in $v^2/\Lambda^2$. In contrast to this, in non-linear realizations of the Higgs mechanism the characteristic scale $\Lambda$ of the models is of order $v$, and a power expansion in $v^2/\Lambda^2$ is not pertinent. A convenient model-independent way to explore then the low-energy consequences of a strong Higgs dynamics is to treat the massless Goldstone bosons, which are the longitudinal components of the $W$ and $Z$ bosons, in the same spirit that low-energy hadronic physics is described by phenomenological chiral Lagrangians [8, 9]. In the limit in which the Higgs-boson mass goes to infinity, a gauged non-linear sigma model is often used to order and catalog the low-energy effects. The relevant symmetry protecting the longitudinal degrees of freedom of the gauge bosons is a global $SU(2)_L \times SU(2)_R$, spontaneously broken to the diagonal “custodial” symmetry $SU(2)_C$ and explicitly broken by the $U(1)_Y$ gauge interaction and by the (different) masses of fermions in each $SU(2)_L$ fermion doublet. The longitudinal degrees of freedom of the gauge bosons are more efficiently described in a compact way by a unitary matrix $U$ transforming as a $(2, 2)$ of the global symmetry group with $U^\dagger U = UU^\dagger = 1$.

The tower of effective operators that parameterizes generically the low-energy impact of a non-linear Higgs sector is well known to differ [10–12] from that of a purely linear regime. Those operators containing a Higgs field $\Phi$ in the purely linear regime, and weighted in the Taylor expansion as powers of $\Phi/\Lambda$, may not be present. Instead, the Goldstone
boson contribution encoded in $U$ does not exhibit such a mass or scale suppression, as $U$ is dimensionless: much as the emission of multiple gluons is not suppressed in the non-perturbative regime of QCD because the interaction strength is of $\mathcal{O}(1)$, the emission of the fields encoded in $U$ is not suppressed either. As a result the “chiral” dimension of the effective operators describing the dynamics of the scalar sector differs from that in the purely linear regime. Typically, an operator containing the Higgs field in the linear expansion has its non-linear counterpart as a lower dimension operator, while the operators made out exclusively of gauge and/or fermion fields keep the same dimension. The consequence is that the leading terms of the two expansions may and will differ.

Assuming a strongly interacting heavy-Higgs-boson sector and armed with the tools mentioned above, we develop in this work a variant of the Minimal Flavour Violation (MFV) [13] ansatz. The latter is not a model of flavour, but rather a general framework, where the flavour violation in the SM and beyond is described in terms of the known fermion mass hierarchies and mixings. A relevant outcome is that, while in a completely general beyond SM set-up the scale of new physics should be heavier than hundred TeV [15], in the MFV framework this energy scale is lowered to few TeV.

The MFV ansatz builds upon the fact that, in the limit of zero fermion masses, the SM exhibits a global flavour symmetry

$$ G_f = SU(3)_Q \times SU(3)_{U_R} \times SU(3)_{D_R} \quad (1.1) $$

plus three extra $U(1)$ factors corresponding to baryon number, hypercharge and the Peccei-Quinn symmetry [16, 17]. The non-abelian subgroup $G_f$ controls the flavour structure of the Yukawa matrices, and we focus on it for the remainder of this paper. Under $G_f$, the $SU(2)_L$ quark doublet, $Q_L$, and the $SU(2)_L$ quark singlets, $U_R$ and $D_R$, transform as:

$$ Q_L \sim (3, 1, 1), \quad U_R \sim (1, 3, 1), \quad D_R \sim (1, 1, 3). \quad (1.2) $$

The MFV hypothesis is a highly predictive working frame in which: i) it is assumed that, at low energies, the Yukawa couplings are the only sources of flavour and CP violation both in the SM and in whatever may be the flavour theory beyond it, which is consistent with the non-observation of flavour effects other than those predicted in the SM; ii) it is exploited the flavour symmetry which the SM exhibits in the limit of vanishing Yukawa couplings, and which is implicitly assumed to be respected by the BSM theory of flavour.

The SM Yukawa interactions,

$$ \mathcal{L}_Y = \bar{Q}_L Y_D D_R H + \bar{Q}_L Y_U U_R \tilde{H} + \text{h.c.}, \quad (1.3) $$

break explicitly both the custodial symmetry $SU(2)_C$ and the flavour symmetry group $G_f$. The technical realization of the MFV ansatz promotes the up and down Yukawa couplings $Y_{U,D}$ to be spurion fields [18, 19] which transform under $G_f$ as

$$ Y_U \sim (3, \overline{3}, 1), \quad Y_D \sim (3, 1, \overline{3}). \quad (1.4) $$

1In the present proposal we do not promote the custodial symmetry to be a true symmetry of the theory. This is particularly relevant, because imposing it the flavour symmetry of the kinetic terms would be $SU(3)^2$ and not $SU(3)^3$, as we are considering.
recovering the invariance under $\mathcal{G}_f$ of the full SM Lagrangian and also under $SU(2)_L \times SU(2)_R$ except for the $U(1)$ gauge interactions. Using these transformation properties, it is possible to write an effective Lagrangian invariant under $\mathcal{G}_f$, in which the flavour transformation of each operator is compensated by that of its operator coefficient, which is constructed with this aim out of the Yukawa spurions. The description given so far applies both to the case of a weakly interacting Higgs field, as in Ref. [13], and to the case where a strong interacting sector is present at the TeV scale. At our knowledge, there is only one example [14] of strong dynamics theory at the 1 TeV scale that leads to exact MFV, even if under certain assumptions. We are not attempting, here, to work on this new physics scenario, but rather to study the results of applying the MFV ansatz on this context. In both the weakly and strongly interacting cases, the only requirement is that the low-energy effects undergo the MFV ansatz, as stated above. Within this framework, we determine below the tower of flavour-changing operators for a variant of the MFV ansatz corresponding to the case of a strong interacting Higgs regime. One of the novelties to be discussed is the appearance at leading order of a CP-odd operator, whose coefficient is necessarily complex. Notice that in the “minimal” version of MFV the Yukawa couplings are assumed to be the only source of the breaking of both $\mathcal{G}_f$ and the CP symmetry, but this does not need to be the case, as pointed out in Refs. [20–23], and flavor-diagonal CP-odd phases can be allowed in generic MFV models. This is precisely what happens naturally at leading order in the strongly interacting Higgs scenario considered here, and the operator found may have a strong impact on CP-odd observables for the $B$ system.

More in general, in addition to write the full MFV effective Lagrangian at leading order for the case of a strongly interacting Higgs, we explore the phenomenological limits and consequences of the MFV tower of operators that we derive, and compare with the analysis in the literature for the case of a Higgs in the linear regime. At this stage, our phenomenological analysis will concentrate on the following observables:

- $\Delta F = 1$ processes, which are sensitive to modifications of the fermion-Z couplings. $B \to \tau^+ \nu$ and the rare heavy meson decays $B \to \mu^+ \mu^-$, $B \to X_s \ell^+ \ell^-$, $K^+ \to \pi^+ \bar{\nu} \nu$ will be most relevant to the present analysis;

- $\Delta F = 2$ processes, which are very sensitive to modifications of the fermion-W couplings. Meson oscillations and semileptonic asymmetries will be discussed, and in particular the quantities $\Delta M_{B_{d,s}}$, the CP-asymmetry $S_{\Psi K_S}$ in the decay $B_d \to \Psi K_S$, the CP-asymmetry $S_{\Psi \phi}$ in the decay $B_s \to \Psi \phi$, $\epsilon_K$ and the like-sign dimuon charge asymmetry of semileptonic $B$ decays.

The operators considered below will produce additional flavour-conserving effects. In the spirit of MFV, we will not include such a discussion here, deferring the extended analysis to a future work [24].

It is worth mentioning that the results presented here, besides applying to heavy Higgs scenarios, could a priori be valid also in theories with both strong interacting Higgs dynamics and a light Higgs. For instance, in composite Higgs models all degrees of freedom of the Higgs doublet, rather than just the gauge boson longitudinal ones, are considered
Goldstone bosons and there is a light physical Higgs. In general, a physical Higgs boson in the spectrum would change our low-energy (\(E < M_W\)) description, due to the presence of flavour-violating sources in addition to the Yukawa couplings \(Y_{u,d}\) (see for example Ref. [7]). However, there are specific limiting cases in which these extra sources do not contribute and all the flavour violation is encoded in \(Y_{u,d}\). If for example the SM fermions couple bilinearly to the strong sector, these extra sources are universal parameters that can be simply re-absorbed. On the other side, if the SM fermions couple linearly to fermionic operators of the strong sector and therefore these couplings may provide extra flavour violation, our analysis applies only under the assumption of universal couplings. Nevertheless, the models of this kind constructed up to now are not formulated as MFV models, as the flavour for the SM fermions is described by a weakly interacting sector, beside the strong one [14]. To extend them in this direction is beyond the scope of the present paper.

Finally, notice that we do not take in consideration in our analysis the possibility that the flavour breaking is non-linearly realized, as it has already been discussed in Ref. [22, 25].

The structure of the manuscript is as follows. Sect. 2 describes the general formalism of the non-linear realization of the symmetry breaking mechanism and the effective MFV Lagrangian considered. In Sect. 3 unitarity and CP violation are thoroughly discussed. Sect. 4 and Sect. 5 deal with the impact on \(\Delta F = 1\) and \(\Delta F = 2\) processes, respectively. The phenomenological impact is discussed in Sect. 6. Finally in Sect. 7 we conclude. Technical details are deferred to the Appendices.

### 2 The MFV Lagrangian for a Heavy Higgs

The standard implementation of the spontaneous SM electroweak symmetry breaking uses a complex scalar field \(\Phi\), which is a doublet under the \(SU(2)_L\) gauge symmetry:

\[
\Phi(x) \equiv \begin{pmatrix} \varphi_+(x) \\ \varphi_0(x) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1(x) - i\varphi_2(x) \\ v + H(x) + i\varphi_3(x) \end{pmatrix},
\]

where \(v = 246\) GeV. A convenient way [10, 12] to face the large \(m_H\) limit is to represent the Higgs field through a field \(M\) transforming as a \((2, 2)\) of the global \(SU(2)_L \times SU(2)_R\) symmetry:

\[
M(x) \equiv \sqrt{2} \begin{pmatrix} \Phi(x) \Phi(x) \end{pmatrix} = \sqrt{2} \begin{pmatrix} \varphi_0^*(x) & \varphi_+(x) \\ -\varphi_1^*(x) & \varphi_0(x) \end{pmatrix},
\]

where \(\bar{\Phi} = i\tau_2\Phi^\ast\). Under the \(SU(2)_L \times U(1)_Y\) gauge group the field \(M\) transforms as:

\[
M(x) \rightarrow L(x) M(x) R^\dagger(x),
\]

with \(L\) and \(R\) the \(SU(2)_L\) and \(U(1)_Y\) gauge transformations, given respectively by

\[
L(x) = \exp \left\{ i\bar{\epsilon}_L(x) \cdot \frac{\vec{\tau}}{2} \right\}, \quad R(x) = \exp \left\{ i\epsilon_Y(x) \frac{\tau_3}{2} \right\},
\]

where the \(\tau_3\) dependence reflects the opposite hypercharges of \(\Phi\) and \(\bar{\Phi}\). Accordingly, the Higgs scalar potential can be rewritten as:

\[
V(M) = \frac{1}{4} \lambda \left( \frac{1}{2} \text{Tr} [M^\dagger M] + \frac{\mu^2}{\lambda} \right)^2.
\]
For $\mu^2 < 0$, $\mathbf{M}$ acquires a non-vanishing vev,
\[
\langle \mathbf{M}^\dagger \mathbf{M} \rangle = v^2 \mathbf{1}
\quad \text{with} \quad v^2 = -\frac{\mu^2}{\lambda},
\] (2.6)
and the physical Higgs field gets a mass $m_H = \sqrt{-2\mu^2}$ breaking the global $SU(2)_L \times SU(2)_R$ symmetry down to $SU(2)_C$ and the local $SU(2)_L \times U(1)_Y$ gauge symmetry down to $U(1)_{EM}$.

In the limit of large Higgs mass, the physical scalar degree of freedom can be formally decoupled sending $\mu, \lambda \to \infty$ while keeping $v$ finite. A non-linear realization of the gauge symmetry breaking is thus obtained subject to the constraint $\mathbf{M}^\dagger \mathbf{M} = \mathbf{M} \mathbf{M}^\dagger = v^2$. All remaining low-energy degrees of freedom of $\mathbf{M}$ can be described through an adimensional unitary matrix $\mathbf{U}$,
\[
\mathbf{U}(x) = \frac{\mathbf{M}(x)}{v}
\] (2.7)
and its covariant derivative,
\[
\mathcal{D}_\mu \mathbf{U} \equiv \partial_\mu \mathbf{U} + \frac{i}{2} g' \tau_3 W^\mu_3 \mathbf{U} - \frac{i}{2} g'' \mathbf{U} \tau_3 B^\mu.
\] (2.8)
It is customary [11, 26, 27] to use the following combinations of $\mathbf{U}$ and $\mathcal{D}_\mu \mathbf{U}$, which transform covariantly under the SM gauge group:
\[
\mathbf{T} = \mathbf{U} \tau_3 \mathbf{U}^\dagger, \quad \mathbf{T} \to L \mathbf{T} L^\dagger,
\]
\[
\mathbf{V}_\mu = (\mathcal{D}_\mu \mathbf{U}) \mathbf{U}^\dagger, \quad \mathbf{V}_\mu \to L \mathbf{V}_\mu L^\dagger.
\] (2.9)
The power series in chiral dimensions is then a Taylor expansion whose terms are operators constructed out of $\mathbf{T}$, $\mathbf{V}_\mu/v$, $W^\mu_{\nu}/v^2$ and $B^\mu_{\nu}/v^2$.

The Lagrangian describing the interaction between the gauge fields and the scalar sector reads:
\[
\mathcal{L} = -\frac{1}{4} W^\mu_{\nu} W^{\mu\nu}_i - \frac{1}{4} B^\mu_{\nu} B^{\mu\nu} + \frac{v^2}{4} \text{Tr}[\mathbf{V}_\mu \mathbf{V}^\mu] + \delta \mathcal{L},
\] (2.10)
in which the first two terms have chiral dimensions $d_\chi = 4$ and the third one has instead $d_\chi = 2$. Those three terms preserve the global $SU(2)_L \times SU(2)_R$ symmetry. $\delta \mathcal{L}$ contains other gauge invariant terms, but includes among them terms breaking the custodial symmetry that the quantum corrections inevitably induce. At lowest order $d_\chi = 2$ there is only one term,
\[
\delta \mathcal{L}_{d_\chi=2} = a_{WZ} \frac{v^2}{4} \text{Tr}[\mathbf{T} \mathbf{V}_\mu] \text{Tr}[\mathbf{T} \mathbf{V}^\mu],
\] (2.11)
which breaks the custodial symmetry inducing a shift in the $Z$-boson mass with respect to the $W$-boson mass. This coupling tends to be unacceptably large in naive models of a strong interacting Higgs sector, from the original technicolor formulation [29, 30] to its modern variants, if not opportunely protected by some additional custodial symmetry. Quantitatively, realistic models need to limit the intensity of this $\delta \mathcal{L}_{d_\chi=2}$ induced coupling to $a_{WZ} < 0.001$, as it is well-known.

A large number of operators appear at dimension $d_\chi = 4$. Thirteen of them contribute to the so called oblique sector [11]. None of them break the custodial symmetry, but are severely constrained by EW precision tests [1]. However they carry no flavour and in
consequence they will not be further discussed below. In the following, instead, we will explicitly concentrate on those operators -among the $d_\chi = 4$ leading ones- relevant for flavour changing transitions.

In the non-linear formalism, the Yukawa interactions in Eqs. (1.3) may be written as:

$$\mathcal{L}_Y = \frac{v}{\sqrt{2}} \overline{Q}_L Y U Q_R + \text{h.c.},$$

(2.12)

where $Q_R = (u_R, d_R)$ and $Y$ is a $2 \times 2$ block matrix built up from the up and down Yukawas matrices. We will work in the usual MFV basis in which the down-sector Yukawa couplings are taken to be diagonal in flavour, with all flavour violation parameterized via the up sector,

$$Y \equiv \begin{pmatrix} Y_U & 0 \\ 0 & Y_D \end{pmatrix} = \begin{pmatrix} V^\dagger y_U & 0 \\ 0 & y_D \end{pmatrix},$$

(2.13)

where $y_U$, $y_D$ are diagonal matrices whose elements are the Yukawa eigenvalues and $V$ denotes the CKM matrix. It is also convenient and customary to define the combination:

$$\lambda_F \equiv Y_U Y_U^\dagger + Y_D Y_D^\dagger = V^\dagger y_U^2 V + y_D^2,$$

(2.14)

which transforms as a $(8, 1, 1)$ under the flavour group $G_f$ and will determine the strength of new flavour effects. In practice, as the SM Yukawa couplings for all fermions except the top are small\(^2\), the only relevant non-diagonal flavour structure will be given by the contraction of two $Y_U$ with $(\lambda_F)_{ij} \equiv (Y_U Y_U^\dagger)_{ij} \approx y^2 t V^\ast t_i V t_j$, for $i \neq j$.

2.1 $d_\chi = 4$ Contributions

Consider the $d_\chi = 4$ part of the Lagrangian which contains only combinations of the field $U$ and fermions:

$$\delta \mathcal{L}_{d_\chi=4} = a_i \mathcal{O}_i.$$  

(2.15)

Operators containing two right-handed fields $Q_R$ will be flavour conserving at leading order in the spurion expansion\(^3\) and of no interest in what follows. Four independent $d_\chi = 4$ flavour violating operators involving left-handed quarks $Q_L$ and the $U$ field can, instead, be written [26–28]:

$$\mathcal{O}_1 = i \frac{1}{2} \overline{Q}_L \lambda F \gamma^\mu \{ T, V_\mu \} Q_L, \quad \mathcal{O}_2 = i \overline{Q}_L \lambda F \gamma^\mu V_\mu Q_L,$$

(2.16)

$$\mathcal{O}_3 = i \overline{Q}_L \lambda F \gamma^\mu T V_\mu Q_L, \quad \mathcal{O}_4 = \frac{1}{2} \overline{Q}_L \lambda F \gamma^\mu [ T, V_\mu ] Q_L.$$

(2.17)

The operator $\mathcal{O}_2$ has been previously singled out in the context of composite Higgs models, see for example Ref. [31]. Notice from Eqs. (2.16) and (2.17) the interesting fact that in the non-linear realization of MFV a CP-odd operator, $\mathcal{O}_4$, emerges at leading order. Much

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\(^2\)In scenarios in which the bottom Yukawa coupling may be of order one, this assumption has to be accordingly modified.

\(^3\)With more insertions of the spurions, it may also result in flavour non-diagonal operators, but they will be Yukawa suppressed and in this sense subdominant.
like in the strong CP problem, the **mere presence** of $\mathcal{O}_4$ will violate CP without the need of introducing complex coefficients, as done for example in Refs. [21, 23]. The presence of this operator is a slight modification of the MFV ansatz\(^4\) as defined in Ref. [13]. We will analyze in detail its implications in the following sections.

### 2.2 Relation to the Linear Expansion

In the linear realization of the Higgs mechanism, the leading MFV “flavour” operators appear at dimension $d = 6$. In that case, four operators involving the Higgs field and two fermions were found; two of those, dubbed $\mathcal{O}_{H_1}$ and $\mathcal{O}_{H_2}$ in Ref. [13], produce the same low-energy effects than our operators $\mathcal{O}_1$ and $\mathcal{O}_2$ in Eq. (2.16) as can be seen replacing in the former the Higgs field by its vev. The *siblings* of the other two (linear) operators, labeled as $\mathcal{O}_{G_1}$ and $\mathcal{O}_{F_1}$ in Ref. [13] do not appear at the leading order $d = 4$ in the non-linear expansion, but only at $d = 5$. Conversely, the linear siblings of our leading operators $\mathcal{O}_3$ and $\mathcal{O}_4$ in Eq. (2.17) have not been considered in Ref. [13]: they would have dimension $d = 8$ in the linear realization, as it can be easily checked with the correspondences in Eqs. (A.22)–(A.24) in Appendix 1. In consequence, the phenomenological signals of MFV is expected to exhibit notable differences between the two scenarios.

### 2.3 The Effective Low-Energy Lagrangian

Aside from fermion masses, three parameters are relevant to our discussions: the weak isospin coupling constant $g$, the hypercharge coupling constant $g'$, and EW symmetry breaking scale $v$. The impact on them of the operators introduced in Eq. (2.16) and (2.17) is best seen looking at the modification of the low-energy effective Lagrangian in the unitary gauge, i.e. $U = 1$:

\[
\delta L_{d=4} = -\frac{g}{\sqrt{2}} \left[ W^{\mu +} \bar{U}_L \gamma_\mu \left( a_W + ia_{\text{CP}} \right) \left( y^2_U V + V y^2_D \right) D_L + h.c. \right] + \\
-\frac{g}{2c_W} Z^\mu \left[ a_Z^2 \bar{U}_L \gamma_\mu \left( y^2_U + V y^2_D V^\dagger \right) U_L + a_Z^2 \bar{D}_L \gamma_\mu \left( y^2_D + V^\dagger y^2_U V \right) D_L \right]
\]

where $c_W$ and $s_W$ stand for the cosine and sine of the weak angle $\theta_W$. The new couplings are codified through

\[
\begin{align*}
    a_Z^2 &\equiv a_1 + a_2 + a_3, \\
    a_W &\equiv a_2 - a_3, \\
    a_{\text{CP}} &\equiv -a_4.
\end{align*}
\]

It is customary to fix the values of $g$, $g'$ and $v$ by taking the three best known experimental quantities [32]: the fine structure constant $\alpha$ - as determined from the quantum Hall effect, the Fermi constant $G_F$ - as extracted from the muon lifetime- and the $Z$-boson mass

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\(^4\)Our only requirement is the invariance under the flavour group $\mathcal{G}_f$ for all operators built out of the SM model fields (and $U$) and the spurions $Y_{U,D}$. In Ref. [13] the invariance under CP was additionally assumed by restraining all operator coefficients to be real; no genuine CP-odd operator stems at leading order of the linear expansion (the sibling of $O_4$ would appear in it only at higher order). In our approach we will keep the new source of CP violation naturally present at leading order only for the non-linear expansion, for its theoretical and phenomenological interest.
\( M_Z \) as determined from the Z line-shape scan at LEP 1. Eqs. (2.18) and (2.19) show that none of the operators in Eqs. (2.16) and (2.17) contribute to \( \alpha \) or \( M_Z \), while three of them a priori correct \( G_F \): that are \( \mathcal{O}_2 \), \( \mathcal{O}_3 \) and \( \mathcal{O}_4 \). Nevertheless, as the determination of \( G_F \) only involves the first family of fermions, the impact of the new couplings discussed here can be neglected within the MFV scheme, where the effects of new operators come with a proportionality to the Yukawa couplings of the fermions involved.

On the contrary, those four operators of Eqs. (2.16) and (2.17) will have important phenomenological consequences for transitions involving heavier quarks. The contributions of \( \mathcal{O}_2 \), \( \mathcal{O}_3 \) and \( \mathcal{O}_4 \) will induce non-unitarity quark mixing and new CP-odd effects to fermion-\( W \) couplings. \( \mathcal{O}_1 \) affects instead fermion-\( Z \) couplings, to which \( \mathcal{O}_2 \) and \( \mathcal{O}_3 \) also contribute, resulting in tree-level flavour changing neutral currents (FCNC).

3 Non Unitarity and CP Violation

The tree-level modifications of the \( W \)-fermion couplings from \( \mathcal{O}_2 \), \( \mathcal{O}_3 \) and \( \mathcal{O}_4 \) can be parameterized as a modified, non-unitary, mixing matrix:

\[
\tilde{V}_{ij} = V_{ij} \left[ 1 + (a_W + ia_{CP})(y_{ui}^2 + y_{dj}^2) \right].
\]

(3.1)

In the limit of vanishing Yukawa couplings but the top one\(^5\), a good estimation of the departures from unitarity can be read from:

\[
\sum_k \tilde{V}_{ik}^* \tilde{V}_{jk} \simeq \delta_{ij} + \left[ 2 a_W y_i^2 + (a_W^2 + a_{CP}^2) y_i^4 \right] \delta_{ij} \delta_{jl} ,
\]

(3.2)

\[
\sum_k \tilde{V}_{ki}^* \tilde{V}_{kj} \simeq \delta_{ij} + \left[ 2 a_W y_i^2 + (a_W^2 + a_{CP}^2) y_i^4 \right] V_{ii}^* V_{ij}.
\]

(3.3)

As expected from Eq. (2.14), sizable unitarity corrections are expected in transitions involving the third family of quarks, still not severely constrained by experiments. In contrast, the unitarity corrections induced on the first two family sectors, being suppressed by small Yukawa eigenvalues and small mixing angles, are \( \mathcal{O}(10^{-4}) \), perfectly in agreement with present bounds [32]. Eqs. (3.2) and (3.3) illustrate in addition that \( a_{CP} \)-dependent non-unitarity corrections appear only at second order.

Let us consider now CP violation. The first remarkable aspect is that the new source of CP-violation only requires two fermion families, contrary to the SM case, see Eq. (3.1). This is evident from the non-unitary character of the NP correction. Indeed, even for two families the \( a_{CP} \)-dependent contributions cannot be rotated away through fermionic-field re-definitions due to the non-trivial flavour structure of both the up and down quark sectors, when the mixing matrix is non-unitary. This translates, for instance, in that the usual parametrization-invariant definition of the angles of the unitarity triangles now reads:

\[
\arg \left( -\frac{\tilde{V}_{il}^* \tilde{V}_{ik}^*}{\tilde{V}_{jk} \tilde{V}_{jl}^*} \right) = \arg \left( -\frac{V_{ik}^* V_{il}}{V_{jk}^* V_{jl}} \right) + a_{CP} \left[ 2 a_W \left( y_{u_j}^2 - y_{u_i}^2 \right) \left( y_{d_k}^2 - y_{d_i}^2 \right) + \left( 3 a_W^2 - a_{CP}^2 \right) \left( y_{u_j}^2 - y_{u_i}^2 \right) \left( y_{d_k}^2 - y_{d_i}^2 \right) \left( y_{u_j}^2 + y_{u_i}^2 + y_{d_k}^2 + y_{d_i}^2 \right) \right] + \mathcal{O}(a^4) ,
\]

(3.4)

\(^5\)These relations should be modified if one works in a framework in which \( y_b \approx y_t \).
showing that:

- All corrections are proportional to $a_{CP}$, as expected from the fact that the SM source of CP-violation is the only one remaining in the absence of $O_4$;

- Two quark families, non-degenerate in both the up and down sectors, are necessary and sufficient to induce physical CP-odd effects proportional to $a_{CP}$. In particular, no CP-odd effects are present in the one-family case;

- New CP-odd contributions appear generically at quadratic order $O(a_{CP} a_W)$. In contrast, when the operator $O_4$ is considered by itself (e.g. $a_W = a_Z^2 = a_{Z}^d = 0$ in Eq. (2.18)) the leading correction is $O(a_{CP}^3)$ or higher.

Overall, the arguments above imply that the leading CP-odd effects should be $\sim a_{CP}^2 y_b^2 y_t^2$, when two operators are considered simultaneously. In particular, using Eq. (3.4), one can give an estimation of how the angles of the $B_d$ unitarity triangle, $(\alpha, \beta, \gamma)$ get modified at leading order:

$$
\begin{align*}
\arg \left( - \frac{V_{tb}^* V_{ld}}{V_{ub}^* V_{ud}} \right) &\simeq \alpha + 2 y_b^2 y_t^2 a_W \ a_{CP}, \\
\arg \left( - \frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}} \right) &\simeq \beta - 2 y_b^2 y_t^2 a_W \ a_{CP}, \\
\arg \left( - \frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} \right) &\simeq \gamma - 2 y_c^2 y_b^2 a_W \ a_{CP} \simeq \gamma .
\end{align*}
$$

(3.5)

Interestingly, the angle $\gamma$ remains free from new physics contamination, up to corrections of order $y_c^2/y_t^2$ relatively to those for $\alpha$ and $\beta$.

In the next sections we will discuss $\Delta F = 1$ and $\Delta F = 2$ transitions.

4 $\Delta F = 1$ Observables

FCNC

The $d_\chi = 4$ operators $O_1$, $O_2$, and $O_3$ induce tree-level FCNC processes, as can be seen from the $Z$ couplings of the effective Lagrangian of Eq. (2.18). This results in important constraints on their coefficients. The siblings of $O_1$ and $O_2$ for the linear regime (i.e. $O_{H1}$ and $O_{H2}$) have been thoroughly analyzed in [13]. The bounds obtained there can be straightforwardly applied also to our case to constrain the specific combinations of coefficients that modify the neutral current.

For the purposes of the present work, only $\Delta F = 1$ processes involving $K$ and $B$ mesons need to be considered. In fact, as can be seen in Eq. (2.18) the new up-type tree-level FCNC contributions are governed by the down-type spurion $V_{Y_D^*} V_D^\dagger$, subleading with respect to $\lambda_F$, by at least a factor $y_b^2/y_t^2$. Therefore only bounds on $a_{Z}^d$ are going to be discussed in the following.
The down-type low-energy FCNC effective Lagrangian is usually written as

\[
\frac{G_F}{2\sqrt{2}\pi s_W^2} V_{ti}^* V_{tj} \sum_n C_n Q_n + \text{h.c.},
\]

(4.1)

where the Wilson coefficient \( C_n \) contain both the SM and the NP contributions:

\[
C_n = C_n^{SM} + C_n^{NP}
\]

(4.2)

and \( Q_n \) stands for the FCNC operators constructed with SM fields, the basis of which is customarily taken as:

\[
\begin{align*}
Q_{9V} &= \bar{d}_i \gamma_{\mu}(1-\gamma_5)d_j \bar{\nu}_i \gamma_{\mu}(1-\gamma_5)\nu_j, \\
Q_{9V} &= \bar{d}_i \gamma_{\mu}(1-\gamma_5)d_j \bar{\ell}_i \gamma_{\mu}\ell_j, \\
Q_{10A} &= \bar{d}_i \gamma_{\mu}(1-\gamma_5)d_j \ell_i \gamma_{\mu}\gamma_5 \ell_j,
\end{align*}
\]

(4.3)

A sum over all quark species is understood in \( Q_{7,9} \), with \( e_q \) denoting the quark electric charge. The main SM contribution to the Wilson coefficients results from top quark loop, while the effects of our \( d_x = 4 \) operators appear already at tree-level. The leading NP contributions coming from the non-linear MFV operators read:

\[
\begin{align*}
C_{9V}^{NP} &= -\kappa y_t^2 a_Z^d, & C_{7}^{NP} &= +2\kappa s_W^2 y_t^2 a_Z^d, \\
C_{9V}^{NP} &= \kappa (1 - 4s_W^2) y_t^2 a_Z^d, & C_{9}^{NP} &= -2\kappa c_W^2 y_t^2 a_Z^d, \\
C_{10A}^{NP} &= -\kappa y_t^2 a_Z^d, & C_{7}^{NP} &= C_{10A}^{NP} = 0.
\end{align*}
\]

(4.4)

where \( \kappa \equiv \pi s_W^2/(2\alpha) \) reflects the relative strength of the NP tree-level contribution with respect to the loop-suppressed SM one. Note that the CP-odd operator \( \mathcal{O}_4 \) does not modify the \( Z \) couplings and in consequence it has no impact at this order.

Different rare decays of mesons can be analyzed \([33]\) in order to bound the \( a_Z^d \) coefficient. The cleanest channels -those with less hadronic uncertainties- are reported in Table 1 and the following overall constraint \(-0.044 < a_Z^d < 0.009\) can be extracted, at 95% of CL.

The bounds on \( \Delta F = 1 \) FCNC transitions among up-type quarks, \( a_Z^d \), can be easily predicted to be of \( \mathcal{O}(a_Z^d y_t^2/y_t^3) \) and consequently do not provide any interesting additional information.

\[ B^+ \rightarrow \tau^+\nu \]

The branching ratio for \( B^+ \rightarrow \tau^+\nu \) will be of particular relevance in the phenomenological analysis below, and therefore we consider its impact in detail. In the SM, this decay
occurs as a tree-level charged current process. The main NP correction enters through the modification of the CKM matrix element $\tilde{V}_{ub}$, see Eq. (3.1), resulting in:

$$BR(B^+ \to \tau^+ \nu) = \frac{G_F^2 m_{B^+} \tau^2}{8\pi^2} \left(1 - \frac{m_{\tau}^2}{m_{B^+}^2}\right)^2 F_{B^+}^2 |V_{ub}|^2 \left|1 + (a_W + i a_{CP}) \frac{y_t^2}{\tau_{B^+}}\right|^2 \tau_{B^+},$$

(4.5)

with $F_{B^+}$ the $B$ decay constant.\footnote{The SM lepton-$W$ couplings have been assumed in writing Eq. (4.5). Even if we are not considering the lepton sector in our scenario, those couplings are strongly constrained by the SM electroweak analysis and therefore any analogous NP modification in the lepton sector should be safely negligible.} FCNC $Z$-mediated contributions to this process appear at the one-loop level and can be safely neglected.

5 \ $\Delta F = 2$ Observables and $B$ Semileptonic CP-Asymmetry

In this section we provide analytical expressions for the NP contributions to $K$ and $B$ mesons oscillations and $B$ semileptonic CP-asymmetry in terms of the three parameters $a_W$, $a_{CP}$ and $a_Z^d$ as there is no dependence on $a_Z^u$ in the observables considered. The formula reported here, concerning the NP part, have been obtained keeping only the dominant term in $\lambda_F$, Eq. (2.14), neglecting the Yukawa couplings of the lightest quarks\footnote{When considering the $B$ system in the formulae below, only terms proportional to $y_t^2$ and $y_b^2$ will be described. For $K$ mixing instead, $y_c$-dependent terms will be also retained.} and retaining solely the leading contributions in each of the new parameters. In practice this means that the expressions below will only contain terms at most quadratic in $a_W$, $a_{CP}$, $a_Z^d$ or their combinations. A detailed discussion on the Wilson coefficients and the renormalization group QCD evolution is deferred to App. B.

5.1 \ $\Delta F = 2$ Observables

The modified $W$ and $Z$ couplings exhibited by the effective low-energy Lagrangian in Eq. (2.18) induce deviations from the SM predictions for $\Delta F = 2$ observables. The main corrections enter through the box diagrams with $W$-bosons exchange. However contributions from tree-level FCNC $Z$ diagrams can be relevant as well and will be considered, while $Z$-mediated boxes and weak penguin diagrams can be safely neglected being suppressed with respect to the tree-level $Z$ contributions.

The effective Hamiltonian for $\Delta F = 2$ transitions is usually written as:

$$\mathcal{H}^{\Delta F=2}_{\text{eff}} = \frac{G_F^2 M_W^2}{4\pi^2} C(\mu) Q,$$

(5.1)

where $Q$ is the operator describing the neutral meson mixing:

$$Q = (d_i^a \gamma_\mu P_L d_j^a)(d_i^b \gamma_\mu P_L d_j^b),$$

(5.2)

and $C(\mu)$ denote the Wilson coefficient evaluated at a scale $\mu$. The mixing amplitude $M_{ij}^i$ ($i = K, d, s$) is defined from the effective hamiltonian by:

$$M_{12}^K = \frac{\langle K^0 | \mathcal{H}^{\Delta F=2}_{\text{eff}} | K^0 \rangle^*}{2 m_K}, \quad M_{12}^q = \frac{\langle B^0_q | \mathcal{H}^{\Delta F=2}_{\text{eff}} | B^0_q \rangle^*}{2 m_{B_q}},$$

(5.3)
with \( q = d, s \). For the \( K \) system, the mixing amplitude can be written as the sum of the SM and the NP contributions\(^8\) \( M_{12}^K = (M_{12}^K)_{SM} + (M_{12}^K)_{NP} \). Neglecting all contributions proportional to \( y_{u,d,s} \) with \( n > 2 \) one has:

\[
(M_{12}^K)_{SM} = R_K \left[ \eta_2 \lambda_2^2 S_0(x_t) + \eta_1 \lambda_c^2 S_0(x_c) + 2 \eta_3 \lambda_t \lambda_c S_0(x_c, x_t) \right]^*,
\]

\[
(M_{12}^K)_{NP} = R_K \left[ \eta_2 \lambda_2^2 \left( y^2_t (2 a_W + \eta^2_t a_{CP}^2) G(x_t) + \frac{(4 \pi y^2_t a_{CP}^2)^2}{g^2} \right) + 2 \eta_1 \lambda_c^2 a_W y_c^2 G(x_c) + 2 \eta_3 \lambda_t \lambda_c \left( y^2_t (2 a_W + a_{CP}^2) H(x_t, x_c) + 2 a_W y_c^2 H(x_c, x_t) \right) \right]^*,
\]

(5.4)

where \( \eta_i \) denote terms due to QCD higher order effects, \( S_0 \), \( G \) and \( H \) are loop functions defined in App. B and

\[
R_K = \frac{G_F^2 M_W^2 F_K^2 m_K^2 \hat{B}_K}{12 \pi^2},
\]

(5.5)

with \( \hat{B}_K \) the scale-independent hadronic B-mixing matrix element \([34]\). Eq. (5.4) shows only the leading terms in each parameter: linear terms in \( a_W \) and quadratic terms in \( a_{CP}^2 \) and \( a_Z^d \). Indeed, the effects of non-linear terms in \( a_W \) turn out to be negligible in the numerical analysis of the next section.

The \( K_L - K_S \) mass difference and the CP-violating parameter \( \varepsilon_K \) are given by

\[
\Delta M_K = 2 \left[ \text{Re}(M_{12}^K)_{SM} + \text{Re}(M_{12}^K)_{NP} \right],
\]

\[
\varepsilon_K = \frac{\kappa_e e^{i \varphi_e}}{\sqrt{2} (\Delta M_K)_{\exp}} \left[ \text{Im}(M_{12}^K)_{SM} + \text{Im}(M_{12}^K)_{NP} \right],
\]

(5.6)

where \( \varphi_e \) and \( \kappa_e \) (see Table 2) account for \( \varphi_e \neq \pi/4 \) and include long-distance contributions to \( \text{Im}(\Gamma_{12}) \) and \( \text{Im}(M_{12}) \).

For the \( B_{d,s} \) systems, it is useful to move to a slightly different notation for the mixing amplitude \( M_{12}^q \)\(^9\):

\[
M_{12}^q = (M_{12}^q)_{SM} C_{B_q} e^{2i \varphi_{B_q}},
\]

(5.7)

where \( C_{B_{d,s}} \) and \( \varphi_{B_{d,s}} \) parametrize the NP effects, while the SM contribution is given by:

\[
M_{12}^q = R_{B_q} \left[ \lambda_2^2 S_0(x_t) \right]^*, \quad \text{with} \quad R_{B_q} = \frac{G_F^2 M_W^2 F_{B_q}^2 m_{B_q} \hat{B}_{B_q} \eta_{B_q}}{12 \pi^2},
\]

(5.8)

with \( F_{B_q} \) and \( \hat{B}_K \) denoting the neutral \( B \) decay constant and mixing hadronic matrix elements, respectively. The mass differences in the \( B_{d,s} \) systems are given by

\[
\Delta M_q = 2 |M_{12}^q| \equiv (\Delta M_q)_{SM} C_{B_q},
\]

(5.9)

---

\(^8\)The expression for \( M_{12}^K \) is phase-convention dependent. In the following we will give all the results in the convention in which the phase of the \( K \to \pi \pi \) decay amplitude is vanishing.

\(^9\)The expression for \( M_{12}^q \) is phase-convention dependent and we adopt the convention in which the decay amplitudes of the corresponding processes, \( B_d^0 \to \psi K_S \) and \( B_s^0 \to \psi \phi \), have a vanishing phase.
with
\[
C_{B_d} = C_{B_s} = \left| 1 + 2a_W \left( y_t^2 \frac{G(x_t)}{S_0(x_t)} + y_b^2 \right) + \frac{4\pi y_t^2 a_\tilde{Z}^2}{g^2 S_0(x_t)} + 2i a_W a_{CP} y_t^2 y_b^2 \frac{G(x_t)}{S_0(x_t)} \right|^2. \tag{5.10}
\]

The mixing-induced CP asymmetries \(S_{\psi K_S}\) and \(S_{\psi \phi}\) in the decays \(B^0_d \to \psi K_S\) and \(B^0_s \to \psi \phi\), respectively, are described by
\[
S_{\psi K_S} = \sin(2\beta + 2\varphi_{B_d}), \quad S_{\psi \phi} = \sin(2\beta_s - 2\varphi_{B_s}), \tag{5.11}
\]
where \(\beta\) and \(\beta_s\) are angles in the unitary triangles,
\[
\beta \equiv \arg \left( -\frac{V_{tb}^* V_{cd}}{V_{tb}^* V_{td}} \right), \quad \beta_s \equiv \arg \left( -\frac{V_{tb}^* V_{ts}}{V_{cb}^* V_{cs}} \right), \tag{5.12}
\]
and the new phases are given by
\[
\varphi_{B_d} = \varphi_{B_s} = 2a_W a_{CP} y_t^2 y_b^2 \frac{G(x_t)}{S_0(x_t)}. \tag{5.13}
\]

It is interesting to point out that, at this level of approximation, the NP contributions to \(\Delta M_{B_d}\) and \(\Delta M_{B_s}\) are equal (see Eq. (5.10)) and as a result the ratio \(R_{\Delta M} \equiv \Delta M_{B_d}/\Delta M_{B_s}\) turns out to be not affected by NP. Any deviation from the SM value of this observable is then negligible in our framework. Another very clean observable is the ratio between \(\Delta M_{B_d}\) and the \(B^+ \to \tau^+\nu\) branching ratio [35]:
\[
R_{BR/\Delta M} = \frac{3\pi \tau_{B^+}}{4\eta_B} \frac{m_{B^+}^2}{M_{B^+}^2} \frac{|V_{ub}|^2}{|V_{tb}^* V_{td}|^2} \left( 1 - \frac{m_{\tau}^2}{m_{B_d}^2} \right)^2 \left[ 1 + (a_W + i a_{CP}) \frac{y_b^2}{\eta_B} \right], \tag{5.14}
\]
where we took \(m_{B^+} \approx m_{B_d}\), well justified considering the errors in the other quantities in this formula.

Finally, notice that the NP contributions to \(\Delta M_q\) (\(q = K, d, s\)) and \(\epsilon_K\) are all proportional to \(y_t^2\) and therefore one may expect large effects on these observables, driven by \(a_W\), \(a_{CP}\), and \(a_\tilde{Z}^2\). In contrast, the new contributions to \(S_{\psi K_S}\) and \(S_{\psi \phi}\) appear to be proportional to \(y_b^2\) and only small deviations from their SM values are thus expected.

### 5.2 The \(B\) Semileptonic CP-Asymmetry

In the \(B_q\) systems, in addition to \(\Delta M_q\), \(S_{\psi K_S}\) and \(S_{\psi \phi}\), a fourth observable provides rich information on meson mixing: the like-sign dimuon charge asymmetry of semileptonic \(b\) decays \(A_{sl}^b\):
\[
A_{sl}^b \equiv \frac{N_b^{++} - N_b^{--}}{N_b^{++} + N_b^{--}}, \tag{5.15}
\]
where \(N_b^{++}\) and \(N_b^{--}\) denote the number of events containing two positively or negatively charged muons, respectively. In \(p\bar{p}\) colliders, such events can only arise through \(B^0_d - \bar{B}^0_d\) or \(B^0_s - \bar{B}^0_s\) mixings. Due to the intimate link with meson oscillations, \(A_{sl}^b\) is also called semileptonic CP-asymmetry and gets contributions from both \(B_d\) and \(B_s\) systems [36, 37]:
\[
A_{sl}^b = (0.594 \pm 0.022) a_\tilde{Z}^d + (0.406 \pm 0.022) a_\tilde{Z}^s, \tag{5.16}
\]
where
\[
\begin{align*}
    a_{d}^d & \equiv \left| \frac{\Gamma_{d}^{d}}{M_{d}^{d}} \right| \sin \phi_{d} = (5.4 \pm 1.0) \times 10^{-3} \sin \phi_{d}, \\
    a_{s}^s & \equiv \left| \frac{\Gamma_{s}^{s}}{M_{s}^{s}} \right| \sin \phi_{s} = (5.0 \pm 1.1) \times 10^{-3} \sin \phi_{s},
\end{align*}
\]  
(5.17)

with
\[
\begin{align*}
    \phi_{d} & \equiv \arg \left( -\frac{M_{d}^{d}}{\Gamma_{d}^{d}} \right) = -4.3^\circ \pm 1.4^\circ, \\
    \phi_{s} & \equiv \arg \left( -\frac{M_{s}^{s}}{\Gamma_{s}^{s}} \right) = 0.22^\circ \pm 0.06^\circ.
\end{align*}
\]  
(5.18)

In order to provide the expression for NP contributions to the semileptonic CP-asymmetry, it is useful to adopt a notation for \( \Gamma_{q}^{q} \) similar to that introduced for \( M_{q}^{q} \):
\[
\Gamma_{q}^{q} = (\Gamma_{q}^{q})_{SM} \tilde{C}_{Bq} \quad \text{with} \quad \tilde{C}_{Bq} = 1 + 2a_{W}y_{b}^{2},
\]  
(5.19)
in the approximation used here. With such a notation, it follows that
\[
a_{d}^{q} = \left| \frac{\Gamma_{d}^{q}}{M_{d}^{q}} \right| \tilde{C}_{Bq} \sin \left( \phi_{q} + 2\varphi_{Bq} \right),
\]  
(5.20)

with \( C_{Bq} \) given in Eq. (5.10).

6 Phenomenological Analysis

Given the small corrections to the unitarity of the CKM matrix and to the angles of the unitarity triangle described in Eq. (3.5), and in particular the tiny and subdominant corrections to \( \gamma \), it is reasonable to adopt the Wolfenstein parametrization to describe the CKM matrix, where the parameters are fixed considering the value of \( V_{us}, V_{cb}, \gamma \) and \( |V_{ub}| \), which are related to tree-level processes and therefore hardly affected by NP contributions.

6.1 The input parameters and the SM analysis

The physical parameters considered in our analysis and their present experimental values are summarized in Table 2. First of all, notice that \( V_{us} \) and \( V_{cb} \) appear to be relatively well measured, compared to \( V_{ub} \) which has an error still of the order of 10\%. Moreover there is a latent tension between the exclusive and the inclusive experimental determinations of \( |V_{ub}| \), which translates into the well known \( \varepsilon_{K} - S_{\psi K_{S}} \) and \( BR(B^{+} \rightarrow \tau^{+}\nu) \) anomalies. Finally the angle \( \gamma \) of the \( B_{d} \) unitarity triangle, despite of being a tree-level SM processes, still suffers from a large uncertainty. \( Rebus sic stantibus \), two different scenarios may be depicted, either the inclusive or the exclusive determination of \( |V_{ub}| \) is preferred:

i) Using the exclusive determination of \( |V_{ub}| \), \( S_{\psi K_{S}} \) is predicted to be very close to the experimental determination of \( \sin(2\beta)_{b\rightarrow c\bar{c}s} \), while \( \varepsilon_{K} \approx 1.8 \times 10^{-3} \) is clearly below the measured value. Furthermore, for such a value of \( |V_{ub}| \) the \( BR(B^{+} \rightarrow \tau^{+}\nu) \) is predicted to be \( 0.85 \times 10^{-4} \), smaller than the central experimental value shown in Table 2 by more than 2\( \sigma \). If NP is advocated in order to solve (or at least to soften) these anomalies it should enhance the value of \( \varepsilon_{K} \) and \( BR(B^{+} \rightarrow \tau^{+}\nu) \), while having negligible impact on \( S_{\psi K_{S}} \).
ii) Using the inclusive determination of $|V_{ub}|$, the SM prediction for $\varepsilon_K$ is closer to its experimental determination and $BR(B^+ \to \tau^+\nu) \approx 1.35 \times 10^{-4}$ agrees with the measured value within the $1\sigma$ level. However, $S_{\psi K_S} \approx 0.8$ is above the measured value. If NP is advocated in order to solve (or at least to soften) this anomaly, it should deplete $S_{\psi K_S}$, while leaving basically unchanged $\varepsilon_K$ and $BR(B^+ \to \tau^+\nu)$.

In the previous section, we have shown that NP contributions to $S_{\psi K_S}$, encoded in $\varphi_{Bd}$, are negligible, being suppressed by $y^3_b$, while NP contributions to $\varepsilon_K$ are sizable. As a result, our framework seems to fit better inside scenario i) and, consequently, in the following we will assume the exclusive determination of $|V_{ub}|$ to be the “correct” one.

Assuming the SM, it is possible to consider the constraints on the $|V_{ub}| - \gamma$ parameter space, deriving from the independent measurements of $R_{\Delta M_{B}}$ and $S_{\psi K_S}$. The present situation is depicted in Fig. 1, for $S_{\psi K_S}$ (orange-red tone) and $R_{\Delta M_{B}}$ (green-blue tone) inside their corresponding $3\sigma$ error ranges (see Table 2). The figure shows that $R_{\Delta M_{B}}$ strongly reduces the allowed parameter space for $\gamma$ to two intervals: the first one is $\sim [21.5^\circ, 26.3^\circ]$, which is around the experimental lower $2\sigma$ value; the second one is $\sim [63.7^\circ, 68.5^\circ]$, very

| $G_F = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2}$ | \[32\] |
| $M_W = 80.399(23) \text{ GeV}$ | \[32\] |
| $s_W^2 \equiv \sin^2 \theta_W = 0.23116(13)$ | \[32\] |
| $\alpha(M_Z) = 1/127.9$ | \[32\] |
| $\alpha_s(M_Z) = 0.1184(7)$ | \[32\] |
| $m_u(2 \text{ GeV}) = 1.7 \div 3.1 \text{ MeV}$ | \[32\] |
| $m_d(2 \text{ GeV}) = 4.1 \div 5.7 \text{ MeV}$ | \[32\] |
| $m_s(2 \text{ GeV}) = 100^{+30}_{-20} \text{ MeV}$ | \[32\] |
| $\kappa_s(m_c) = (1.279 \pm 0.013) \text{ GeV}$ | \[38\] |
| $m_b(m_b) = 4.19^{+0.48}_{-0.06} \text{ GeV}$ | \[32\] |
| $M_t = 172.9 \pm 0.6 \pm 0.9 \text{ GeV}$ | \[32\] |
| $m_K = 497.614(24) \text{ MeV}$ | \[32\] |
| $F_K = 156.0(11) \text{ MeV}$ | \[34\] |
| $\hat{B}_K = 0.737(20)$ | \[34\] |
| $\phi_{B_s}^{\psi\phi} = 0.55^{+0.38}_{-0.36}$ | \[41, 42\] |
| $\varphi_s = (43.51 \pm 0.05)^\circ$ | \[45\] |
| $\kappa = (18.76(7))$ | \[32\] |
| $\varepsilon = (0.5765(65))$ | \[39\] |
| $\eta_2 = 0.496(47)$ | \[47\] |
| $\Delta M_{K} = 0.5292(9) \times 10^{-2} \text{ ps}^{-1}$ | \[32\] |
| $|\varepsilon_K| = 2.228(11) \times 10^{-3}$ | \[32\] |
| $\tau_{B^\pm} = (1641 \pm 8) \times 10^{-3} \text{ ps}$ | \[32\] |
| $BR(B^+ \to \tau^+\nu) = (1.65 \pm 0.34) \times 10^{-4}$ | \[32\] |

Table 2: Values of the experimental quantities used as input parameters. Notice that $m_i(m_i)$ are the masses $m_i$ at the scale $m_i$ in the $\overline{MS}$ scheme while $M_t$ is the top-quark pole mass.
close to the central value. In the next years, LHCb will lower the uncertainty on $\gamma$ and hopefully one of these two regions will be excluded. The figure also illustrates that $S_{\psi K_S}$ constrains $|V_{ub}|$ only to relatively small values, $\sim [3.0 \times 10^{-3}, 3.8 \times 10^{-3}]$, a pattern already mentioned above when discussing the exclusive analysis of $|V_{ub}|$. In the following we will refer to allowed $|V_{ub}| - \gamma$ parameter space, once included the combined bounds from $R_{\Delta M_B}$ and $S_{\psi K_S}$, as the “reduced” $|V_{ub}| - \gamma$ parameter space.

It Fig. 2 the variation of the SM predictions for $\varepsilon_K$, $\Delta M_{B_d,s}$ and $R_{BR/\Delta M}$ in the “reduced” $|V_{ub}| - \gamma$ parameter space is studied. In each of the sub-plots of Fig. 2 the variation of the SM central values predictions is shown, with the colors corresponding to the legend on the right side of each subplot. While the SM prediction for $\varepsilon_K$, Fig. 2 (a), turns out to be always smaller than its experimental determination, the SM predictions for the mass differences $\Delta M_{B_d}$, Fig. 2 (b), and $\Delta M_{B_s}$, Fig. 2 (c) are above the corresponding data. However, given the large theoretical uncertainties on these quantities, this tendency should not be considered more than just a “slight indication”. The ratio $R_{BR/\Delta M} \equiv BR(B^+ \rightarrow \tau^+ \nu)/\Delta M_{B_d}$ can be useful in order to reduce most of the theoretical uncertainties on $\Delta M_{B_d}$. Its SM prediction can be obtained from Eq. (5.14) setting $a_W = a_{CP} = 0$. Fig. 2 (d) depicts the variation of the SM prediction in the “reduced” $|V_{ub}| - \gamma$ parameter space. Even if always inside the $3\sigma$ error range, it appears evident the SM preference for a small $R_{BR/\Delta M}$ ratio, close to the lower experimental limit:

$$\left( R_{BR/\Delta M} \right)_{exp} = (3.25 \pm 0.67) \times 10^{-4} \text{ ps}. \quad (6.1)$$

These particular patterns of the SM predictions (always either above or below or inside the $3\sigma$ error ranges) are going to be relevant when discussing NP effects, because it is a common feature of all the points in the “reduced” $|V_{ub}| - \gamma$ parameter space.
In order to illustrate the features of the MFV scenario with a strong interacting Higgs sector, the numerical analysis of the following sections will be presented choosing as reference point, \((|V_{ub}|, \gamma) = (3.5 \times 10^{-3}, 66^\circ)\), corresponding to \(S_{\psi K_S} \simeq 0.692\) and \(R_{\Delta M_B} \simeq 2.83 \times 10^{-2}\). For this point, \(\gamma, S_{\psi K_S}, R_{\Delta M_B}\) and \(|V_{ub}|\) are inside their own 1σ error determinations\(^{10}\).

### 6.2 FCNC Constraints on \(a_{CP}, a_W\) and \(a_Z^d\)

The analysis presented here mainly relies on two observables: \(\varepsilon_K\) and \(R_{BR/\Delta M}\). Indeed, as seen in the previous section, \(S_{\psi K_S}\) and \(R_{\Delta M_B}\) turn out to be only slightly affected by NP and very close to their SM predictions. On the other hand, \(\Delta M_K\) and \(\Delta M_{B_{d,s}}\) are affected by large theoretical errors related to the long distance contributions. Therefore the only requirement on the neutral meson mass differences will be that \((\Delta M_K)_{\exp}\) is reproduced within ±40%. Finally, the discussion on \(S_{\psi d}\) and \(A_{sl}^b\) is presented separately.

\(^{10}\)We are considering the exclusive determination of \(|V_{ub}|\) as commented in the previous section.
As none of the observables considered here get contributions from $a_Z^2$, the analysis will be restricted to the remaining three parameters $a_{CP}$, $a_W$ and $a_Z^d$. The analytic expressions for the NP contribution to the considered observables are deferred to App. B.

Before entering into the detailed determination of the bounds on $a_{CP}$, $a_W$ and $a_Z^d$ from FCNC constraints, it is pertinent to point out that further constraints on the $d_{\chi=4}$ operator coefficients of the non-linear expansion may result from data on flavour-conserving transitions, such as for instance from the constraints on the oblique parameters $S$, $T$, $U$ [48, 49]. A rough order-of-magnitude estimate of the NP effects under consideration is given by the correction to the $T$ parameter,

$$T^{NP} - T^{SM} \approx a_X y_t^2 T^{SM}$$

(6.2)

where $T^{NP}$ is the total SM contribution due to the top and the NP loop, with $a_X$ denoting a generic NP coefficient of the $d_{\chi=4}$ Lagrangian Eq. (2.18), and

$$T^{SM} \approx \frac{3G_F M_t^2}{8\pi^2 \alpha \sqrt{2}}.$$  

(6.3)

The NP effects may be traded by a variation on the value of the quark top mass in the SM contribution. A change of the top mass of order of 5% produces a change of the $T$ parameter of 0.1, well inside the 2$\sigma$ LEP result (see Fig. E.2 in Ref. [50]). This translates into a bound of order $5 - 10\%$ on the couplings $a_X$. A thorough and detailed analysis of the one-loop impact on EW precision measurements is beyond the scope of the present paper and will be presented elsewhere [24].

6.2.1 $\varepsilon_K$ and $R_{BR/\Delta M}$

For the chose reference point in the $(|V_{ub}|, \gamma)$ parameter space, the central values for the SM predictions are:

$$\varepsilon_K = 1.8 \times 10^{-3}, \quad R_{BR/\Delta M} = 1.6 \times 10^{-4} \, \text{ps}.$$  

(6.4)

As anticipated before in Fig. 2, the SM prediction for $\varepsilon_K$ is below the experimental value, while that for $R_{BR/\Delta M}$ is already inside but close to the 3$\sigma$ error bound. The main question is if the MFV scenario with strong Higgs dynamics discussed here is able to accommodate contemporaneously both observables: in other words, whether $\varepsilon_K$ can be enhances by NP while not spoiling $R_{BR/\Delta M}$.

The NP corrections to $\varepsilon_K$ and $R_{BR/\Delta M}$ turn out to be inversely correlated in our scenario: when $\varepsilon_K$ increases $R_{BR/\Delta M}$ decreases, and vice-versa. More quantitatively, considering only one non-vanishing NP parameter at time we find the following behavior:

$$a_{CP} = \pm 0.1 \quad \rightarrow \quad \delta \varepsilon_K \approx 1.1\%, \quad \delta R_{BR/\Delta M} \approx -1.4\%,$$
$$a_W = 0.1(-0.1) \quad \rightarrow \quad \delta \varepsilon_K \approx +26%(-19\%), \quad \delta R_{BR/\Delta M} \approx -25%(+30\%),$$
$$a_Z^d = \pm 0.1 \quad \rightarrow \quad \delta \varepsilon_K \approx 124\%, \quad \delta R_{BR/\Delta M} \approx -62\%.$$  

(6.5)

Notice that $\varepsilon_K$ can be suppressed only for negative values of $a_W$, while it is always enhanced for positive ones; exactly the opposite happens for $R_{BR/\Delta M}$. Furthermore, $\varepsilon_K$ and $R_{BR/\Delta M}$
are \textit{a priori} very sensitive to the tree level FCNC Z-exchange contributions, parametrized by $a_Z^d$. However, this parameter is also strongly constrained by $\Delta F = 1$ observables, as shown in sec. 4. Finally, we have also checked that this pattern is do not strongly rely on the particular point chosen in the $|V_{ub}| - \gamma$ parameter space.

In Fig. 3 we show the results considering the NP contributions and the input parameters (see Table 2). We consider three different cases:

1. Only $a_{CP}$ is taken to be non-vanishing, see plots (a) and (b) of Fig. 3. This case shows the impact of the CP odd operator by itself. Plot (a) illustrates the $\varepsilon_K - R_{BR/\Delta M}$ correlation: in orange (green) the 1$\sigma$, 2$\sigma$ and 3$\sigma$ (from the darker to the lighter) experimental error ranges for $\varepsilon_K (R_{BR/\Delta M})$, in blue the correlation between $\varepsilon_K$ and $R_{BR/\Delta M}$, while the red star represents the SM predictions in Eq. (6.4). Plot (b) depicts the $a_{CP}$ parameter space in terms of $\varepsilon_K$ for $R_{BR/\Delta M}$ inside its 3$\sigma$ error range: only $|a_{CP}|$ values in the range $\sim [0.3, 0.5]$ can accommodate both observables.

2. Only $a_W$ and $a_Z^d$ parameters are taken to be non-vanishing, see plots (c) and (d) of Fig. 3. This illustrates the case when only the CP even operators are taken into account. This case can be considered as the strict MFV limit [13] of our scenario: the CP-invariance of the Lagrangian is recovered and there are no new CP-violating sources other than the CKM one. Plot (c) is the analogous of plot (a) and the same description applies. Furthermore, those two plots are very similar and therefore the impact of $a_{CP}$ alone and of $a_W$ and $a_Z^d$ together is comparable. Plot (d) shows the $a_W - a_Z^d$ parameter space: the blue points refer to $\varepsilon_K$ and $R_{BR/\Delta M}$ inside their 3$\sigma$ error ranges; the gray regions represent the $\Delta F = 1$ bounds on $a_Z^d$ parameter, $a_Z^d \in [-0.044, 0.009]$, and as a result a large part of the $a_W - a_Z^d$ parameter space is excluded: this puts a strong bound on $a_W$, which can take values only inside a narrow range $[-0.1, 0.1]$.

3. All parameters are considered to be simultaneously non-vanishing, see plots (e) and (f) of Fig. 3. Plot (e) is very similar to plots (a) and (c) and therefore the same conclusions apply. Plot (f) shows the $a_W - a_{CP}$ parameter space, in which the blue points refer to $\varepsilon_K$ and $R_{BR/\Delta M}$ inside their 3$\sigma$ error ranges and $a_Z^d$ is inside the bound coming from the $\Delta F = 1$ observable analysis. Only a small part of the $a_W - a_{CP}$ parameter space survives: it is however interesting to notice that $|a_W|$ and $|a_{CP}|$ can take even large values and this is the result of a partial cancellation among the contributions of $a_{CP}$ and $a_W$, as we have explicitly verified.

Summarizing, our MFV scenario with a strong interacting Higgs sector is able to accommodate simultaneously $\varepsilon_K$, $R_{BR/\Delta M}$, $S_{\psi K_S}$, and $R_{\Delta M_B}$, only including the theoretical errors. The NP parameter space is strongly constrained and only for a small part of it all the observables take values inside their own experimental 3$\sigma$ errors ranges. Correlations between contributions from $a_W$ and $a_{CP}$, even large values for these two parameters are allowed. Conversely, $a_Z^d$ is strongly constrained, mainly from $\Delta F = 1$ observables.

For the parameter space region selected by all considered constraints, the non-linear realization of MFV is able to solve the $\varepsilon_K - S_{\psi K_S}$ anomaly, but it is unable to remove
the SM tension on $BR(B^+ \to \tau^+\nu)$. However, a better agreement with the data can be found selecting different values for $|V_{ub}|$ and $\gamma$: indeed, slightly larger values for $|V_{ub}|$, while keeping $\epsilon_K$, $R_{\Delta M_B}$ and $S_{\psi K_S}$ in agreement with data at 1$\sigma$, would enhance $BR(B^+ \to \tau^+\nu)$.
towards its experimental central value.

### 6.2.2 $S_{\psi\phi}$ and $A_{sl}^b$

The mixing-induced CP asymmetry in the $B^0 \rightarrow \psi\phi$ decay has been measured by D0, which recently updated its analysis with 8 fb$^{-1}$ of data [41, 42], and by LHCb, which has presented its preliminary measurement [44]. The numerical results have been included in Table 2 and translate into the $S_{\psi\phi}$ asymmetry values:

$$D0 : S_{\psi\phi} = 0.52^{+0.23}_{-0.33} \quad \leftrightarrow \quad LHCb : S_{\psi\phi} = 0.13 \pm 0.18 \pm 0.07,$$

(6.6)

which are in agreement, even if the two central values are relatively different. In particular, the LHCb result is closer to the SM value, that for our ($|V_{ub}|, \gamma$) reference point turns out to have the central value

$$S_{\psi\phi} = 0.036.$$

(6.7)

Alike to the $S_{\psi K_S}$ asymmetry previously analyzed, in our scenario the NP contributions to $S_{\psi\phi}$ are expected to be suppressed by $y_b^2$-dependent terms, as shown in Eq. (5.13), and indeed our numerical analysis predicts only negligible deviations from the SM value.

The situation is different when the $B$ semileptonic CP-asymmetry $A_{sl}^b$ is considered. The central value of the SM prediction, for our ($|V_{ub}|, \gamma$) reference point, is given by:

$$A_{sl}^b = -2.3 \times 10^{-4} \quad (a_{sl}^d = -4.0 \times 10^{-4}, \quad a_{sl}^s = 1.9 \times 10^{-5}),$$

(6.8)

to be compared with the experimental determination by D0 [36], $A_{sl}^b = (-78.7 \pm 17.2 \pm 9.3) \times 10^{-4}$, which deviates from the SM prediction by more than $3\sigma$. The question now is whether the scenario explored here may provide contributions to $A_{sl}^b$ of the correct sign and sufficiently large to soften the tension with respect to the experimental data.

It turns out that indeed $A_{sl}^b$ could receive sizable NP contributions. For instance, for our reference ($|V_{ub}|, \gamma$) values, it results:

$$a_{CP} = 0.1(-0.1) \quad \rightarrow \quad \delta A_{sl}^b \approx 1.1\%(1.6\%)$$

$$a_{W} = 0.1(-0.1) \quad \rightarrow \quad \delta A_{sl}^b \approx 33\%(-23\%)$$

$$a_{Z}^d = \pm 0.1 \quad \rightarrow \quad \delta A_{sl}^b \approx 160\%.$$

(6.9)

This NP sensitivity of $A_{sl}^b$ is not a common feature of NP models as it has been discussed in Ref. [51] and therefore it is worth to further investigate it numerically.

Fig. 4 illustrates the correlation between $A_{sl}^b$ and $S_{\psi\phi}$, always for the reference ($|V_{ub}|, \gamma$) value, and constraining $\varepsilon_K$ and $R_{BR/\Delta M}$ to remain inside their $3\sigma$ error ranges and $a_{Z}^d$ to satisfy the $\Delta F = 1$ bounds. Again it is worth to explore the three different cases:

1. Only $a_{CP}$ is considered to be non-vanishing, see plot (a) of Fig. 4, spanning the range $[-1, 1]$. The red star represents the SM prediction as in Eqs. (6.7) and (6.8), that is between the $2\sigma$ and $3\sigma$ levels. The figure shows that only small deviations from the SM value are allowed, corresponding to large values of $a_{CP}$: such deviations go in the right direction, but are too small to reach the experimental central value.
2. In plot (b) of Fig. 4, we show the MFV scenario in which $a_{CP} = 0$, and $a_W$ and $a_Z^d$ are non-vanishing. Only few points survive and they all are quite close to the SM value: the reason can be understood looking at Fig. 3(c), where indeed $a_W$ is allowed to span only a narrow range $\sim [-0.1, 0.1]$, corresponding to tiny contributions to $A_{bt}^b$.

3. Finally, in plot (c) of Fig. 4, $a_W$ and $a_{CP}$ are both allowed to take values in the $[-1, 1]$ range. Tiny, but visible, deviations of $S_{\psi\phi}$ from the SM value are now allowed, but this is not the case for $A_{bt}^b$. This can be understood from Eqs. (5.10) and (5.13): when $a_W$ and $a_{CP}$ take the same values but opposite in sign, as it is the case here (compare with Fig. 3(e)), then a cancellation occurs in $C_{B_d s}$, and therefore $\delta A_{bt}^b \approx 0$; such cancellation, however, does not take place in $\varphi_{B_d s}$, and then $\delta S_{\psi\phi} \neq 0$.

Summarizing, in the MFV scenario with a strongly interacting Higgs sector explored in this paper, only tiny corrections to the SM predictions for $S_{\psi\phi}$ and $A_{bt}^b$ occur. This outcome is still experimentally allowed, mainly due to the large uncertainties which affect present measurements. Hopefully, LHCb will (partially) lower such uncertainties in the next future and this will further test the parameter space of our scenario.
7 Conclusions

Naturalness arguments about the electroweak sector of the SM strongly point to new dynamics around the TeV scale. On the other side, the experimentally allowed gap gets narrowing down for BSM models which naturally propose new light states, such as for instance the simplest supersymmetric models of the electroweak scale. In this situation, the possibility of a strong-interacting dynamics associated to the electroweak breaking sector regains new interest, be it in the simplest case of a TeV or heavier Higgs particle -an option still allowed by present LHC data within $3\sigma$- or when the strong dynamics is accompanied of a light Higgs mode. In both cases, it may well be that the new physics will show up first through its impact on non-standard fermion couplings and flavour effects.

Much as it is happening in direct Higgs searches, departures from the SM are neither being found in flavour physics, which is an optimal tool to indirectly detect new physics. The latter fact led to the fructiferous MFV ansatz. In this work, we have identified the leading effective couplings which would signal MFV for the case of a strong-interacting dynamics associated to the Higgs sector. The analysis does not refer to any particular BSM theory: an effective Lagrangian approach has been used, with the strong-Higgs dynamics parametrized as usual through a generic non-linear sigma model. The leading operators appear at mass dimension four, and we have allowed in the analysis custodial-breaking couplings, within their strong phenomenological bounds.

In the linear realization of MFV there were four effective operators involving quarks and the Higgs field at leading order (i.e. dimension six). The non-linear realization considered here turns out to have as well four operators of that type at its leading order (i.e. dimension four). But only two among the latter are “siblings” of two of the former set, in the sense that their low-energy impact on flavour observables is the same. The nature of the other two leading couplings differs for both regimes. In particular, one outstanding difference of MFV in the strong interacting Higgs scenario, compared with the purely perturbative regime, is the natural presence of a new source of CP violation at leading order of the effective field theory.

We have analyzed in depth the phenomenological constraints and hypothetical future impact of the leading effective couplings. All their contributions to low-energy measurements depend only on three parameters: the modification of the Z-fermion couplings for down quarks\textsuperscript{11} $a_d^Z$, and of the W-fermion couplings, $a_W$ and $a_{\text{CP}}$ for their real and imaginary parts, respectively. Because the corrections are tantamount to a non-unitary component in flavour mixing, new CP-odd effects follow even when only two fermion families are present, in contrast to the SM case.

In the linear realizations of MFV, either the operator coefficients are assumed to be $O(1)$ and the new physics scale needs to be larger than 5 TeV, or the flavour scale is assumed to be $\sim 1$ TeV, in which case the fermion-boson couplings must be small. In contrast, in the case of strongly-interacting Higgs sector explored here, the scale of the dimension four operators analyzed is necessarily the mass of the W and Z gauge bosons: as a consequence,

\textsuperscript{11}Because of the Yukawa dependence dictated by the MFV hypothesis, the Z-fermion couplings for up quarks $a_u^Z$ can be neglected

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the experimental constraints on $\Delta F = 1$ processes force $a_Z^d$ to be small, as for linear MFV, while $a_W$ and $a_{CP}$ can be even large. This is a big difference with respect to the linear case, because it is now easier to have correlations and in particular cancellations among different contributions: this happens for instance for $\epsilon_K$ and other observables, see Fig. 3.

A plethora of $\Delta F = 1$ and $\Delta F = 2$ processes have been analyzed in much detail in this work, including the existing tension between the exclusive and the inclusive experimental determinations of $|V_{ub}|$, which within the SM translates into the $\epsilon_K - S_{\psi K_S}$ anomaly and also the $BR(B^+ \rightarrow \tau^+ \nu)$ anomaly. In particular, the non-linear realization of MFV is able to remove the $\epsilon_K - S_{\psi K_S}$ anomaly, while it does not soften the SM tension for $BR(B^+ \rightarrow \tau^+ \nu)$. For the reference values $(|V_{ub}|, \gamma) = (3.5 \times 10^{-3}, 66^\circ)$, the new physics parameter space for the ensemble of observables $\epsilon_K$, $S_{\psi K_S}$, the ratio $BR(B^+ \rightarrow \tau^+ \nu)/\Delta M_{B_d}$, and $\Delta M_{B_d}/\Delta M_{B_s}$ turns out to be strongly constrained. Only for a small part of it all the observables take values inside their own experimental 3$\sigma$ errors ranges; when the type of cancellations mentioned above occur, $a_W$ and $a_{CP}$ may be even large. Furthermore, tiny corrections to the SM predictions for the CP asymmetries $S_{\psi \phi}$ and $A_{b_s}$ may occur, and this is still experimentally allowed. Hopefully LHCB will (partially) lower these experimental uncertainties in the near future, further constraining the parameter space of our scenario or heralding a signal of new physics.

We are living exciting times in which new data appear galore from several fronts; flavour physics is offering an increasingly piercing window on new physics and it could even shed the first light on a possible dynamic nature of the electroweak symmetry breaking mechanism itself.

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A Useful Formulas for non-linear $d_\chi = 4$ basis

In this appendix the derivation of $d_\chi = 4$ operators is sketched. To make contact with all the strong Higgs effective Lagrangian literature the relation between the notation used in this paper and the one also used in the literature (see for example [11]) is shown. In this appendix, only operators involving fermions and the strong Higgs sector are analyzed. For
the complete basis of operators, including all the gauge-strong Higgs interactions, one can refer to [10–12, 26, 27, 52–54].

The main quantities needed in the construction are the basic “covariant” quantities under the SM gauge group:

\[
\begin{align*}
T &= U \tau_3 U^\dagger, & T &\to L T L^\dagger, \\
V_\mu &= (D_\mu U) U^\dagger, & V_\mu &\to L V_\mu L^\dagger,
\end{align*}
\]

(A.1)

where for historical reasons \(L \in SU(2)_L\) and \(R \in U(1)_Y\) are denoted respectively. It is straightforward to verify that all these quantities are traceless:

\[
\text{Tr} [T] = \text{Tr} [V_\mu] = 0,
\]

(A.2)

and consequently, using the well known decomposition properties of a generic \(2 \times 2\) matrix, one can therefore decompose them as:

\[
T = \frac{1}{2} \text{Tr} [T \tau_i] \tau_i, \quad V_\mu = \frac{1}{2} \text{Tr} [V_\mu \tau_i] \tau_i.
\]

(A.3)

All operators, at any order in the chiral expansion, can be expressed in terms of the vectors \(T^i, V_\mu^i\) or of the corresponding traces. For sake of completeness we report here the explicit expression of the relevant traces in the unitary gauge:

\[
\begin{align*}
\text{Tr} [T \tau_i]_U &= 2 \delta_{i3} \quad \text{for } i = 1, 2, 3 \\
\text{Tr} [V_\mu \tau^a]_U &= ig W^a_\mu \quad \text{for } a = 1, 2 \\
\text{Tr} [TV_\mu]_U &= i \frac{g}{c_W} Z_\mu.
\end{align*}
\]

(A.4)

\(d_\chi = 4\) Contributions

Being \(T\) an hermitian and unitary matrix, it is straightforward to show that all the Higgs “covariant” quantities with a single Lorentz index, that can be built starting from Eq. (A.3) are given by:

\[
\begin{align*}
V_\mu &= \frac{1}{2} \text{Tr} [V_\mu \tau_i] \tau_i, \\
(TV_\mu + V_\mu T) &= \text{Tr} [TV_\mu] \mathbb{1}, \\
(TV_\mu - V_\mu T) &= \frac{i}{2} \epsilon^{ijk} \text{Tr} [T \tau_i] \text{Tr} [V_\mu \tau_j] \tau_k, \\
TV_\mu T &= \frac{1}{2} \left( \text{Tr} [TV_\mu] \text{Tr} [T \tau^i] - \text{Tr} [V_\mu \tau^i] \right) \tau_i.
\end{align*}
\]

(A.5)

Using the relations in Eqs. (A.1)-(A.5) the operators defined in Eqs. (2.16)-(2.17) can be written, alternatively, as:

\[
\begin{align*}
O_1 &= \frac{1}{2} J^\mu \text{Tr}[TV_\mu], \\
O_2 &= \frac{1}{2} J^\mu_i \text{Tr}[V_\mu \tau^i], \\
O_3 &= \frac{1}{2} J^\mu_i \left( \text{Tr}[TV_\mu] \text{Tr}[T \tau^i] - \text{Tr}[V_\mu \tau^i] \right), \\
O_4 &= \frac{i}{4} \epsilon^{ijk} \text{Tr}[T \tau_i] \text{Tr}[V_\mu \tau_j] J^\mu_k.
\end{align*}
\]

(A.6)
with $J^\mu$ and $J_i^\mu$ the $SU(2)_L$ singlet and triplet currents, respectively:

$$J^\mu = i \bar{Q}_L \lambda_F \gamma^\mu Q_L, \quad J_i^\mu = i \bar{Q}_L \lambda_F \gamma^\mu \tau_i Q_L. \quad (A.10)$$

One of the most relevant differences of the strong interacting Higgs scenario, when compared with the linear case is the presence of a new source of CP violation at chiral dimension $d = 4$. To realize explicitly the CP character of non-linear operators in the strong Higgs case one has to remember the CP transformation properties of the traces:

$$\mathbf{T}(t, x) \xrightarrow{CP} - \tau_2 \mathbf{T}(t, -x) \tau_2, \quad (A.11)$$

$$\mathbf{V}_\mu(t, x) \xrightarrow{CP} \tau_2 \mathbf{V}_\mu(t, -x) \tau_2. \quad (A.12)$$

By means of Eqs. (A.11) and (A.12) it is straightforward to recover the transformation properties of the traces:

$$\text{Tr}[\mathbf{T}_i] \xrightarrow{CP} \text{Tr}[\mathbf{T}_i^*], \quad (A.13)$$

$$\text{Tr}[\mathbf{V}_\mu \tau_i] \xrightarrow{CP} - \text{Tr}[\mathbf{V}_\mu^* \tau_i^*], \quad (A.14)$$

$$\text{Tr}[\mathbf{T} \mathbf{V}_\mu] \xrightarrow{CP} - \text{Tr}[\mathbf{T} \mathbf{V}_\mu^*]. \quad (A.15)$$

Using in addition the transformation properties of the singlet and triplet $SU(2)_L$ fermionic currents:

$$\bar{Q}_L \gamma^\mu Q_L \xrightarrow{CP} - \bar{Q}_L \gamma^\mu Q_L, \quad (A.16)$$

$$\bar{Q}_L \gamma^\mu \tau_i Q_L \xrightarrow{CP} - \bar{Q}_L \gamma^\mu \tau_i^* Q_L, \quad (A.17)$$

one can easily verify that $O_{1,2,3}$ are CP-even, while $O_4$ is CP-odd.

**Relation With the Linear Representation**

Finally we want to connect the operators in the non-linear basis with those defined in the linear realization. One can define the following four flavour operators involving fermions and Higgs fields:

$$O_{H1} = i \left( \bar{Q}_L \lambda_F \gamma^\mu Q_L \right) \left( \Phi^\dagger (D_\mu \Phi) - (D_\mu \Phi)^\dagger \Phi \right), \quad (A.18)$$

$$O_{H2} = i \left( \bar{Q}_L \lambda_F \gamma^\mu \tau_i Q_L \right) \left( \Phi^\dagger \tau_i (D_\mu \Phi) - (D_\mu \Phi)^\dagger \tau_i \Phi \right), \quad (A.19)$$

$$O_{H3} = i \left( \bar{Q}_L \lambda_F \gamma^\mu \tau_i Q_L \right) \left( \Phi^\dagger \tau_i (D_\mu \Phi) - (D_\mu \Phi)^\dagger \tau_i \Phi \right), \quad (A.20)$$

$$O_{H4} = i \epsilon^{ijk} \left( \bar{Q}_L \lambda_F \gamma^\mu \tau_i Q_L \right) \left( \Phi^\dagger \tau_j (D_\mu \Phi) - (D_\mu \Phi)^\dagger \tau_j \Phi \right). \quad (A.21)$$

The first two operators [13] appear in the linear expansion at dimension $d = 6$, while the last two appear only at dimension $d = 8$. To match the operators in the linear and non linear expansion one has to express the traces appearing in Eqs. (A.6)-(A.9) with the corresponding (linear) Higgs contribution. In particular one finds:

$$\text{Tr}[\mathbf{T} \mathbf{V}_\mu] \rightarrow - \frac{2}{v^2} \left( \Phi^\dagger (D_\mu \Phi) - (D_\mu \Phi)^\dagger \Phi \right) \quad (A.22)$$

$$\text{Tr}[\mathbf{V}_\mu \tau_i] \rightarrow \frac{2}{v^2} \left( \Phi^\dagger \tau_i (D_\mu \Phi) - (D_\mu \Phi)^\dagger \tau_i \Phi \right) \quad (A.23)$$

$$\text{Tr}[\mathbf{T} \tau^i] \rightarrow - \frac{4}{v^2} (\Phi^\dagger \tau^i \Phi). \quad (A.24)$$
Here the correspondence between $\langle M^\dagger M \rangle = v^2$ and $\langle \Phi^\dagger \Phi \rangle = v^2/2$ has to be kept in mind. Inserting Eqs. (A.22)-(A.24) in Eqs. (A.6)-(A.9) one can easily recover the following correspondence between operators in the linear and non linear realization:

$$O_1 \leftrightarrow -\frac{1}{v^2} O_{H1},$$  \hspace{1cm} (A.25)

$$O_2 \leftrightarrow \frac{1}{v^2} O_{H2},$$  \hspace{1cm} (A.26)

$$O_3 \leftrightarrow \frac{4}{v^2} O_{H3} - \frac{1}{v^2} O_{H2},$$  \hspace{1cm} (A.27)

$$O_4 \leftrightarrow -\frac{2}{v^2} O_{H4}.$$  \hspace{1cm} (A.28)

In particular notice that $d_\chi = 4$ operators $O_{1,2}$ correspond to the $d = 6$ linear operators $O_{H1}$ and $O_{H2}$, whereas $O_{3,4}$ to operators of dimension up to $d = 8$ in the linear expansion.

### B Formulae for the Phenomenological Analysis

In this appendix we provide details on the results presented in sects. 5 and 6.2.

#### B.1 $\Delta F = 2$ Wilson Coefficients

The Wilson coefficients of the $\Delta F = 2$ observables in presence of NP can be written separating the contributions from the box diagrams and the tree-level $Z$ diagrams, so that

$$C^{(M)} = \Delta^{(M)}_{Box} C + \Delta^{(M)}_{Z} C,$$  \hspace{1cm} (B.1)

where $M = K, B_d, B_s$. Taking into account only the corrections to the $W$-quark vertices, we find the following contributions to the Wilson coefficient relevant for the $M_0 - \bar{M}_0$ system at the matching scale $\mu_t \approx m_t$ ($m_t$ is the top quark mass $m_t$ computed at the scale $m_t$ in the $\overline{\text{MS}}$ scheme):

$$\Delta^{(M)}_{Box}(\mu_t) = \sum_{i,j = u,c,t} \tilde{\lambda}_i \tilde{\lambda}_j F_{ij},$$  \hspace{1cm} (B.2)

where for the $K$ and $B_q$ systems we have respectively

$$\tilde{\lambda}_i = \hat{V}_{is} \hat{V}_{id}, \hspace{1cm} \tilde{\lambda}_i = \hat{V}_{ib} \hat{V}_{iq},$$  \hspace{1cm} (B.3)

with $\hat{V}$ the modified CKM matrix. The $F_{ij}$ are the usual box functions with the exchange of $W$ and up-type quarks defined by

$$F_{ij} \equiv F(x_i, x_j) = \frac{1}{4} \left[ (4 + x_i x_j) I_2(x_i, x_j) - 8 x_i x_j I_1(x_i, x_j) \right],$$

$$I_1(x_i, x_j) = \frac{1}{(1 - x_i)(1 - x_j)} + \left[ \frac{x_i \ln(x_i)}{(1 - x_i)^2(x_i - x_j)} + (i \leftrightarrow j) \right],$$

$$I_2(x_i, x_j) = \frac{1}{(1 - x_i)(1 - x_j)} + \left[ \frac{x_i^2 \ln(x_i)}{(1 - x_i)^2(x_i - x_j)} + (i \leftrightarrow j) \right],$$  \hspace{1cm} (B.4)
with \( x_i = (m_i/M_W)^2 \) (with \( m_i \) should be understood as \( m_i(m_i) \)).

In the SM limit, i.e. switching off the modifications of the \( W \)-quark couplings, \( \tilde{V} \to V \) and therefore \( \tilde{\lambda}_i \to \lambda \) and it is possible to rewrite the previous expression in eq. (B.2) in terms of the usual Inami-Lim functions \( S_0(x_i), S_0(x_c) \) and \( S_0(x_c, x_t) \), using the unitarity relations of the CKM matrix:

\[
S_0(x_t) \equiv \frac{4x_t - 11x_t^2 + x_t^3}{4(1 - x_t)^2} - \frac{3x_t^3 \log x_t}{2(1 - x_t)^3} \quad \text{(B.5)}
\]

\[
S_0(x_c) \equiv x_c
\]

\[
S_0(x_c, x_t) \equiv x_c \left[ \log \frac{x_t}{x_c} - \frac{3x_t^2 \log x_t}{4(1 - x_t)^2} \right].
\]

When the NP contributions are taken into account, it is useful to analyse the modification of the CKM factors \( \lambda_i \) entering the meson oscillation Wilson coefficients: for the \( K \) system we have

\[
\tilde{\lambda}_i \equiv \tilde{V}_{id_i}^* \tilde{V}_{idi} = \lambda_i \left[ 1 + i a_{CP} \Delta_{12}^d + a_W (\Sigma_{1i}^d + \Sigma_{2i}^d) + (a_W^2 + a_{CP}^2) \Sigma_{12}^d \right], \quad \text{(B.6)}
\]

\[
\tilde{\lambda}_i' \equiv \tilde{V}_{u2i}^* \tilde{V}_{ui1} = \lambda_i' \left[ 1 + i a_{CP} \Delta_{12}^u + a_W (\Sigma_{1i}^u + \Sigma_{2i}^u) + (a_W^2 + a_{CP}^2) \Sigma_{12}^u \right], \quad \text{(B.7)}
\]

with \( \lambda_i = V_{id_2}^* V_{idi}, \lambda_i' = V_{u2i}^* V_{ui1} \) and

\[
\Delta_{12}^d = y_{x_1}^2 - y_{x_2}^2, \quad \Sigma_{1i}^d = y_{x_1}^2 + y_{x_2}^2, \quad \Sigma_{2i}^d = y_{x_2}^2 + y_{x_1}^2. \quad \text{(B.8)}
\]

So respectively for the \( B_s, B_d \) and \( K \) system one has to replace accordingly the quark labels. If now we consider the products \( \tilde{\lambda}_i \tilde{\lambda}_j \), we can parametrise the deviations from the SM expression as follows:

\[
\tilde{\lambda}_i \tilde{\lambda}_j = \lambda_i \lambda_j (1 + \delta \lambda_{ij}) \quad \text{(B.9)}
\]

where for the \( K \) system

\[
\delta \lambda_{ij} = 2i a_{CP} \Delta_{12}^k + a_W A_{ij}^k + a_{CP}^2 B_{ij}^k + a_W^2 C_{ij}^k + i a_{CP} a_W \Delta_{12}^k B_{ij}^k + (a_{CP}^2 + a_W^2)(i a_{CP} D_{ij}^k + a_W^2 E_{ij}^k) + (a_{CP}^2 + a_W^2)^2 L_{ij}^k, \quad \text{(B.10)}
\]

with

\[
A_{ij}^k = \Sigma_{1i}^x + \Sigma_{1j}^x + \Sigma_{2i}^x + \Sigma_{2j}^x, \quad B_{ij}^k = \Sigma_{1i}^x \Sigma_{2j}^x - \Sigma_{1j}^x \Sigma_{2i}^x, \quad C_{ij}^k = \Sigma_{1i}^x \Sigma_{1j}^x + \Sigma_{2i}^x \Sigma_{2j}^x + \Sigma_{1i}^x (\Sigma_{2j}^x + \Sigma_{2j}^x) + \Sigma_{1j}^x (\Sigma_{2i}^x + \Sigma_{2i}^x),
\]

\[
D_{ij}^k = \Delta_{12}^k (\Sigma_{1i}^x \Sigma_{2j}^x + \Sigma_{1j}^x \Sigma_{2i}^x), \quad E_{ij}^k = (\Sigma_{1i}^x + \Sigma_{2i}^x) \Sigma_{1j}^x \Sigma_{2j}^x + (\Sigma_{1j}^x + \Sigma_{2j}^x) \Sigma_{1i}^x \Sigma_{2i}^x, \quad L_{ij}^k = \Sigma_{1i}^x \Sigma_{1j}^x \Sigma_{2i}^x \Sigma_{2j}^x. \quad \text{(B.11)}
\]

Notice that the previous expressions hold for both for the \( K \) and the for \( B_q \) systems: only \( \Delta_{12}^x, \Sigma_{1i}^x \) and \( \Sigma_{2i}^x \) distinguish the different systems. With such notation, we can write the expression in eq. (B.2) as follows:

\[
\Delta_{Box}^{(K)} C(\mu_t) = \lambda_t^2 S'_0(x_t) + \lambda_c^2 S'_0(x_c) + 2 \lambda_t \lambda_c S'_0(x_c, x_t), \quad \text{(B.12)}
\]

\[
\Delta_{Box}^{(B_q)} C(\mu_t) = \lambda_t^2 S'_0(x_t),
\]
where

\[
S'_0(x_t) \equiv (1 + \delta \lambda_{tt}) F_{tt} + (1 + \delta \lambda_{uu}) F_{uu} - 2 (1 + \delta \lambda_{ut}) F_{ut},
\]

\[
S'_0(x_c) \equiv (1 + \delta \lambda_{cc}) F_{cc} + (1 + \delta \lambda_{uu}) F_{uu} - 2 (1 + \delta \lambda_{uc}) F_{uc},
\]

\[
S'_0(x_c, x_t) \equiv (1 + \delta \lambda_{ct}) F_{ct} + (1 + \delta \lambda_{uu}) F_{uu} - (1 + \delta \lambda_{uc}) F_{uc} - (1 + \delta \lambda_{ut}) F_{ut}.
\]

When all the $\delta \lambda_{ij}$ are put to zero, these relations coincide with the SM ones.

Moving to the tree-level FCNC $Z$ diagrams and integrating the $Z$ boson at $\mu_t^{12}$, we find the following contributions to the Wilson coefficients:

\[
\Delta C^{(K)}_Z(\mu_t) = \frac{4 \pi^2}{G_F^2 M_W^2} \frac{1}{2 M_Z^2} (C^{d,s})^2, \quad \Delta C^{(R)}_Z(\mu_t) = \frac{4 \pi^2}{G_F^2 M_W^2} \frac{1}{2 M_Z^2} (C^{q,b})^2,
\]

where the coefficients $C^{d,s}$ and $C^{q,b}$ are the FCNC couplings of the $Z$ boson to the down-type quarks,

\[
C^{d,s} = \frac{g}{2 \cos \theta_W} a^d_Z (\lambda_{FC})_{12}, \quad C^{d,s} = \frac{g}{2 \cos \theta_W} a^d_Z (\lambda_{FC})_{13}, \quad C^{q,b} = \frac{g}{2 \cos \theta_W} a^d_Z (\lambda_{FC})_{23}.
\]

The Wilson coefficients given above are evaluated at the $\mu_t$ scale and therefore the complete analysis requires the inclusion of the renormalisation group (RG) QCD evolution down to low energy scales, at which the hadronic matrix elements are evaluated by lattice methods.

In our model we can apply the same RG QCD analysis as in the SM context: indeed, both the effective operator, that arises from the modified box diagrams and the tree-level $Z$ diagrams by integrating out the heavier degrees of freedom, and the matching scale are the same as in the SM. In particular no new effective operators with different chiral structure from that one in eq. \(5.2\) and no higher scales than $\mu_t$ are present. All the NP effects are encoded into the Wilson coefficients.

By the use of the Wilson coefficients reported in this appendix and having in mind the previous discussion on the QCD evolution, we find the following full expressions for the mixing amplitudes:

\[
M^K_{12} = R_K \left[ \eta_2 \lambda^2 \lambda'_{12} S'_0(x_t) + \eta_1 \lambda^2 \lambda'_{12} S'_0(x_c) + 2 \eta_3 \lambda_t \lambda_c S'_0(x_c, x_t) + \eta_2 \Delta C^{(K)}_Z(\mu_t) \right],
\]

\[
M^q_{12} = R_{B_q} \left[ \lambda^2 \lambda'_{12} S'_0(x_t) + \Delta C^{(B_q)}_Z(\mu_t) \right].
\]

Analogously, in the presence of NP also $\Gamma^q_{12}$ is modified. Following Ref. \[55\], we can write $\Gamma^q_{12}$ in our model as

\[
\Gamma^q_{12} = - \left[ \lambda^2 \Gamma^\ast_{12} + 2 \lambda_c \lambda'_{12} \Gamma^{\ast \ast}_{12} + \lambda'_{12} \Gamma^{\ast \ast \ast}_{12} \right],
\]

where $\lambda_i$ are the CKM factors defined in eq. \(B.3\) and $\Gamma^q_{12}$ can be found in Ref. \[55\]. By using the same notation as in eq. \(B.9\) and using the unitarity of $\lambda_i$ to eliminate $\lambda_c$, we get

\[
\Gamma^q_{12} = - \left[ \lambda^2 \Gamma^\ast_{12} + 2 \lambda_t \lambda'_{12} (\Gamma^{\ast \ast}_{12} - \Gamma^{\ast \ast \ast}_{12}) + \lambda^2 \lambda'_{12} (\Gamma^{\ast \ast \ast}_{12} - 2 \Gamma^{\ast \ast \ast \ast}_{12} + \Gamma^{\ast \ast \ast \ast \ast}_{12}) \right].
\]

\textsuperscript{12}Integrating out the $Z$ boson at $\mu_t$ or at $M_Z$ introduces a subleading error in our computation, that can be safely neglected.
where

\[ \Gamma_{ij}^{\prime} = (1 + \delta \lambda_{ij}) \Gamma_{ij}^{ij}. \]  

(B.19)

Notice that in the limit in which \( \delta \lambda_{ij} \to 0 \), the SM expression is recovered.

### B.2 Approximate Analytical Expressions

Once we consider only the first terms in the expansion in \( a_W \) and \( a_{CP} \), the relevant parameters \( \delta \lambda_{ij} \) are simplified as follows: for the \( K \) system they are

\[
\begin{align*}
\delta \lambda_{uu} &= 2 (a_W - i a_{CP}) y_s^2, \\
\delta \lambda_{cc} &= 4 a_W y_c^2 - 2 i a_{CP} y_s^2, \\
\delta \lambda_{tt} &= 4 a_W y_t^2 - 2 i a_{CP} y_s^2, \\
\delta \lambda_{uc} &= 2 a_W y_c^2 - 2 i a_{CP} y_s^2, \\
\delta \lambda_{ut} &= \delta \lambda_{ct} = 2 a_W y_t^2 - 2 i a_{CP} y_s^2,
\end{align*}
\]

(B.20)

while for the \( B_q \) systems they are

\[
\begin{align*}
\delta \lambda_{uu} &= \delta \lambda_{cc} = \delta \lambda_{uc} = 2 (a_W - i a_{CP}) y_b^2, \\
\delta \lambda_{tt} &= 2 a_W (y_t^2 + y_b^2) - 2 i a_{CP} y_b^2 (1 + 2 a_W y_t^2), \\
\delta \lambda_{ct} &= 2 a_W (y_b^2 + y_t^2) - 2 i a_{CP} y_b^2 (1 + a_W y_t^2),
\end{align*}
\]

(B.21)

where the subsequent terms in the expansion are of order \( O(a_W^2, a_{CP}^2, a_W a_{CP}) \). Similarly, we can report the approximated expressions for the \( C^{d,s} \), \( C^{d,b} \) and \( C^{s,b} \) couplings, that enter the tree level \( Z \) contributions:

\[
\begin{align*}
C^{d,s} &= \frac{g}{2 \cos \theta_W} \ a_d^2 \ y_t^2 \ V_{ts}^* \ V_{td}, \\
C^{d,b} &= \frac{g}{2 \cos \theta_W} \ a_d^2 \ y_t^2 \ V_{tb}^* \ V_{td}, \\
C^{s,b} &= \frac{g}{2 \cos \theta_W} \ a_d^2 \ y_t^2 \ V_{tb}^* \ V_{ts}.
\end{align*}
\]

(B.22)

Finally we report the explicit expressions for \( G(x_t) \) and \( H(x_t, x_c) \) appearing in the expressions in sect. 5:

\[
\begin{align*}
G(x_t) &= 2 (F_u - F_{ut}) = 4 x_t + 2 x_t \log x_t - \frac{7 x_t^2 - x_t^3}{2 (1 - x_t)^2} + \frac{2 x_t^2 - 5 x_t^3}{(1 - x_t)^3} \log x_t, \\
G(x_c) &= 2 (F_{cc} - F_{uc}) = 2 x_c (2 + \log x_c), \\
H(x_t, x_c) &= F_{tc} - F_{ut} = x_c \left[ \frac{4 - 11 x_t + 7 x_t^2}{4 (1 - x_t)^2} + \frac{4 - 8 x_t + x_t^2}{4 (1 - x_t)^2} \log x_t \right], \\
H(x_c, x_t) &= F_{ct} - F_{uc} = -x_c \log x_c + x_t \frac{4 - 3 x_c}{4 (1 - x_t)} + \frac{4 x_t + x_c (4 - 8 x_t + x_t^2)}{4 (1 - x_t)^2} \log x_t.
\end{align*}
\]

(B.23)
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