Pair correlations of a spin-imbalanced Fermi gas on two-leg ladders

A. E. Feiguin and F. Heidrich-Meisner

1Microsoft Project Q, University of California, Santa Barbara, CA 93106, USA
2Condensed Matter Theory Center, Department of Physics, University of Maryland, College Park, MD 20742, USA
3Institut für Theoretische Physik C, RWTH Aachen University, 52056 Aachen

(Dated: Feb. 4, 2009)

We study the pair correlations of a spin-imbalanced two-leg ladder with attractive interactions, using the density matrix renormalization group method (DMRG). We identify regions in the phase diagram spanned by the chemical potential and the magnetic field that can harbor Fulde-Ferrell-Larkin-Ovchinnikov (FFLO)-like physics. Results for the pair structure factor, exhibiting multiple pairing wave-vectors, substantiate the presence of FFLO-like correlations. We further discuss phase separation scenarios induced by a harmonic trap, which differ from the case of isolated chains.

The experimental realization of fermionic superfluids in ultracold atom gases under clean conditions and with a great control over interactions has paved the way toward a detailed understanding of the BEC-BCS crossover of spin-balanced, ultracold Fermi gases [1]. Now, the case of a polarized two-component Fermi gas, realized by unequally populating the two lowest hyperfine states, has moved into the focus of current experimental work [2].

Intriguing properties such as phase separation in a trap and the transition from superfluidity to a normal state driven by the population imbalance have attracted a great deal of attention, but it is, in particular, the search for exotic superfluids such as the FFLO one [3, 4] that drives the current interest in imbalanced Fermi gases. In an FFLO state, the order parameter is spatially inhomogeneous with Cooper pairs with a finite center-of-mass momentum. In recent experiments on three dimensional (3D) ultracold gases, this state remains elusive, and theoretical work indicates that in 3D, the phase space volume of this phase in the interaction-polarization plane is small [5]. Yet, reducing the spatial dimension renders this pairing mechanism more effective as a larger portion of the Fermi surfaces of minority and majority spins can be matched [6]. In particular, in one dimension (1D), where a true condensation is prohibited, the existence of an FFLO-type state with quasi-long range order has been proven by means of analytical [7, 8] as well as numerically exact approaches [9–12]. This also pertains to the experimentally relevant case of a harmonic trap [10–12].

It is then natural to ask whether such quasi-FFLO states seen in 1D are stable against coupling chains to 2D or 3D ensembles, in order to connect the aforementioned theoretical results for 1D to those available for 2D [6, 13–16]. For 1D chains weakly coupled to a 3D array, the polarization-interaction phase diagram has recently been derived in Refs. [6, 7, 12, 17]. Here we present a rigorous and quasi-exact numerical analysis of pairing correlations in two coupled chains, using DMRG [18].

The physics of spin [19] and Hubbard ladders [20] has proven to be unique and interesting in itself, due to the emergence of exotic quantum phases – such as spin liquids – driven by strong correlations [19]. The experimentally realization of ladders in optical lattices as arrays of double wells could be an important step toward understanding the experimental and theoretical challenges that we may face before scaling to 2D systems. Double wells have already been studied in recent experiments [21] and Danshita et al. [22] have given a prescription of how the parameters of a double well potential need to be tuned to create a ladder system in an optical lattice. Methods to detect the FFLO state in experiments using noise correlations or interferometry are discussed in Refs. [12, 23].

We study the Hubbard model on a $2 \times L$ ladder:

$$H = -t_\perp \sum_{i=1,\sigma}^{L} (c_{i+1,\sigma}^{\dagger} c_{i,\sigma} + h.c.) + U \sum_{l=1,\sigma}^{L} n_{i,l,\uparrow} n_{i,l,\downarrow} - t_\parallel \sum_{l,i=1,\sigma}^{L-1} (c_{i,l,\sigma}^{\dagger} c_{i+1,l,\sigma} + h.c.) - \mu N - \hbar N p / 2 - V_{\text{trap}} \sum_{l,i=1,\sigma}^{L} [i - (L + 1)/2]^2 n_{i,l,\sigma}, \quad (1)$$

where $c_{i,l,\sigma}^{\dagger}$ is a fermionic annihilation (creation) operator acting on a site on rung $i$ and leg $l$ ($l = 1, 2$). The hopping matrix elements along rungs and legs are denoted by $t_\perp$ and $t_\parallel$, respectively. We study the case of strongly attractive onsite interactions $U = -8t_\parallel$ (see Ref. [22] for a discussion of how $U$, $t_\parallel$, and $t_\perp$ are related to experimentally controllable parameters in an optical lattice). Further, $n_{i,l,\sigma} = c_{i,l,\sigma}^{\dagger} c_{i,l,\sigma}$, yielding the number of fermions of each species as $N_{\sigma} = \sum_{i,l}(n_{i,l,\sigma})$, with $N = N_{\uparrow} + N_{\downarrow}$ and the pseudo-spin index $\sigma = \uparrow, \downarrow$. As customary, $h$ is the magnetic field, $\mu$ the chemical potential, $n = N/(2L)$ the filling factor, and $p = (N_{\uparrow} - N_{\downarrow})/N$ measures the polarization (we use $N_{\uparrow} > N_{\downarrow}$). A harmonic trapping potential is introduced by the last term in Eq. (1).

Our analysis is mostly concerned with the case of $t_\perp = t_\parallel$ and thus not restricted to weakly coupled chains. We first study the emergence of FFLO-like correlations as a function of the ratio $t_\perp/t_\parallel$ and filling $n$ for $V_{\text{trap}} = 0$. 

arXiv:0809.1539v2 [cond-mat.str-el] 4 Feb 2009.
The bulk phase diagram is displayed in Figs. 1 (a) and (b), showing contour plots of $n$ and $p$ in the $h$ vs. $\mu$ plane, respectively. We infer the presence of five phases (for the single chain case, see Ref. [25]): I: the empty phase ($n = 0, p = 0$); II: a spin gapped phase ($p = 0, n \leq 1$); III: fully polarized, but less than half filled ($p = n, n < 1$); IV: partially polarized ($n < 1$); V: fully polarized ($n = 1$). Phase IV is the candidate for FFLO-like physics and we will show that a rich structure in the real-space modulation of the pair correlation function emerges.

As a guidance for the interpretation of our DMRG results, we next consider the noninteracting limit (see, e.g., Ref. [9]). The dispersion relation at $U = 0$ with periodic boundary conditions along the legs of the ladder is: $\epsilon(k) = -2t_\parallel \cos(k) - t_\perp \cos(k)$. In the imbalanced case, the Fermi surfaces are mismatched, and we compute the pairing momenta $Q = (Q_\parallel, Q_\perp)$ of possible FFLO-like states from the difference of the Fermi-wave vectors for majority and minority spins, i.e., $Q = k_{F_\parallel} - k_{F_\perp}$, $Q_\parallel = k_{F_\parallel} - k_{F_\perp}$, and $Q_\perp = k_{F_\parallel} - k_{F_\perp}$. Depending on $p, n$, and $t_\perp/t_\parallel$, the band structure can allow for more than one pairing wave vector $Q$, yielding a quasi-condensate with multiple contributing modes. This may also be thought of as coexisting quasi-condensates with $Q_\perp = 0$ and $Q_\parallel = \pi$. Examples for the expected polarization dependence of the pairing momenta $Q$ are displayed with lines in Fig. 2 for generic fillings of $n < 1$ and $t_\perp/t_\parallel = 1$ and 2. To label these branches, we use a symbol $Q_{\perp}(k_{\parallel}, k_{\perp})$, which specifies $Q_\perp$ as well as the $k_{\parallel}$ of the majority and the minority spins. For instance, $\pi(\pi, 0)$ represents $Q_\perp = \pi$ pairs, formed from majority spins with $k_{\parallel} = \pi$ and minority spins with $k_{\perp} = 0$.

The disappearance of certain branches is due to a band getting completely filled or depleted, causing kinks in the other branches at the same $p$. In the case of the ladder, there are two constraints on the four Fermi momenta, $Q_{\parallel}$, $Q_{\perp}$, and $\mu$.
from Fig. 2(b). Yet, at $U < 0$, we find at maximum one peak in the pair structure factor in each channel. We compare the position of these peaks as extracted from the DMRG data to the $Q_{\parallel}$ computed in the $U = 0$ limit, for the parameters of Fig. 2. In the case of $t_{\perp} = t_{\parallel}$, we choose the fillings such that several cuts through phase IV of Fig. 1 can be followed (as indicated by the labels IVa-IVd in Fig. 2). We subdivide phase IV according to how many bands are partially filled ($i.e.$, $0 < n < 1$): IVa: four bands; IVb: $(0, \sigma = \uparrow, \downarrow)$ partially filled; IVc: three bands; IVd: two bands ($\pi, \uparrow$) and $(0, \downarrow)$ full.

At $n = 0.7$ [Fig. 2(b)], the $Q_{\parallel} = 0$ quasi-condensate at small $p$ starts on the $0^{\uparrow}\pi^{\downarrow}$ branch and then continuously moves to the $0^{(0,0)}$ branch. At larger $p$, a second condensate with $Q_{\parallel} = \pi$ builds up (circles), namely one of the $\pi^{(0,0)}$ type. Again, the positions $Q_{\parallel}$ of the quasi-condensates well agree with the $U = 0$ predictions (lines in Fig. 2) at polarizations $p \gtrsim 0.3$. In the case of a large filling $n = 0.9$ and $t_{\perp} = t_{\parallel}$ [see Fig. 2(c)], the picture is very similar to the $n = 0.7$ case, with the difference that at large $p$, one enters into region IVd, where the $(0, \uparrow)$ band is filled, and hence only the $\pi^{(0,0)}$ branch survives.

A simpler structure for the quasi-condensate is expected for the parameters of both Figs. 2(a) and (d), where at maximum, two modes may exist. In these two cases all possible $(Q_{\parallel}, Q_{\perp})$ indeed contribute to the quasi-condensate at finite $U$ and, moreover, their momenta $Q_{\parallel}$ mostly agree with the $U = 0$ predictions. An interesting effect occurs at polarizations $p \gtrsim 0.35$ in Fig. 2(d), where at $U = 0$, no phase IVc is present. The interactions renormalize the dispersion such that the $(0, \uparrow)$ band remains partially filled up to $p \lesssim 0.7$, allowing the $(0^{(0,0)})$ branch to survive up to that value of polarization. Therefore, due to the interactions, a small window for the emergence of IVc opens up. For $t_{\perp} > 2t_{\parallel}$ and $n = 0.7$, only one quasi-condensate is realized (not shown in the figures). We emphasize that at any polarization and in all cases studied, at least one mode with $Q_{\parallel} > 0$ is present.

At small $p$, the decay of pair correlations is consistent with $|\rho_{i,r,s}^{\text{pair}}| \sim |\cos(Q_{\parallel} |r - s|)/|r - s|^n| \lesssim 2/7$, as expected for a single-$Q$ quasi-condensate [7]. The same applies to all cases in which a single branch is present in Fig. 2. In all other cases, our data suggest that the envelope of $|\rho_{i,r,s}^{\text{pair}}|$ follows a power-law. Moreover, we find that pair correlations decay slower than those of competing instabilities such as density-density correlations [12] at small $p$, while at present, we cannot make a definite statement about larger polarizations. Out of the possible quasi-condensates, typically the ones with the smallest $Q_{\parallel}$ win and show up with a significant weight in the pair structure factor. The most 1D-like behavior with a single $(Q_{\parallel}, Q_{\perp} = 0)$ mode is encountered in phase IVb, which for $t_{\perp}/t_{\parallel} = 1.2$ extends up to full polarization for $n \lesssim 0.4$. In IVb, $Q_{\parallel} = 2\pi n p$, as in 1D [10].

We now turn to the effect of a harmonic trapping potential on the pairing correlations and the density profile by setting $V_{\text{trap}} = 0.002t_{\parallel}$ with $N = 40$ fermions and $t_{\perp} = t_{\parallel}$. As for phase separation, our results displayed in Fig. 4 indicate that at small polarizations ($p < 0.2$), there are three shells: an inner core with a vanishing polarization, a thin shell that is partially polarized with increased spin fluctuations, and fully polarized wings [see Fig. 4(b), $p = 0.1$ there]. This is distinctly different from the case of uncoupled chains with the same $U$ and filling $\lfloor 8, 10 \rfloor$, where FFLO-like correlations develop in the core as soon as $p > 0$. By invoking the local density approximation, we trace this back to the slope of the boundary between phases II and IV that has a different sign in the case of ladders (see Fig. 1) as compared to chains [25]; thus the system goes through IV in the center of the trap before entering III toward the edges. At $U = -8t_{\parallel}$, the formation of fully polarized wings sets in as soon as $p > 0$.

At larger polarizations, there are only two phases: in the core, the local polarization $p_{i,t} = (n_{i,t,\uparrow} - n_{i,t,\downarrow})/(n_{i,t,\uparrow} + n_{i,t,\downarrow})$ increases and the unpolarized phase gives room to a partially polarized one, while the wings remain fully polarized [see Fig. 4(c) for the example of $p = 0.6$]. We relate the phases emerging in the trapped situation to those of the bulk system shown in Fig. 1: the unpolarized phase is a $Q_{\parallel} = 0$ superfluid – phase II – while the partially polarized one has, similar to phase IV, FFLO-like features. Our results for the pair structure factor, presented in Fig. 4(a), confirm this picture: For $p < 0.2$, a coherence peak at $Q_{\parallel} = 0$ dominates in the $k_{\perp} = 0$ channel, while, as soon as the core assumes a finite polarization, develops into a $Q_{\parallel} > 0$ peak, characteristic of an FFLO state. The evolution of the coherence peaks for both $k_{\perp} = 0$ and $\pi$ resembles that of the untrapped system [Figs. 2(a) and (b)]. The coherence peaks at both $Q_{\parallel} = 0$ and $\pi$ survive up to saturation. At high
densities (not shown here), a band-insulating Fock state with uncorrelated tightly bound pairs appears in the center of the trap, displacing the other phases toward the edges, similar to the 1D [10] and the 2D cases [13].

Discussion – The physics of ladders already features some characteristics of higher dimensional systems. In particular, the positive slope of the boundary of phase II (see Fig. 1) is a remarkable effect of the dimensionality that changes the order of the concentric phase-separated shells in a trap, in agreement with mean-field calculations in 2D and 3D. To learn about the incipient 2D physics, it is edifying to look at a ladder with $t_L = 2t_\parallel$. In this case, the IVb and IVd regions occupy most of phase IV, with only a small sliver of IVc at high densities due to the renormalization of the bandwidth [see Fig. 2(d)]. In the isotropic 2D system, region IVb would correspond to two partially filled bands, while IVd has a band of majority spins above half filling, touching the boundaries of the Brillouin zone. At a low density (phase IVb), the physics fits within a single-band picture [see, e.g., Fig. 2(a)], similar to a chain. At larger densities, the pairing acquires contributions from multiple Fermi points. Similarly, in higher dimensions and at low density, the problem has rotational symmetry, and the FFLO order parameter can be faithfully described by the modulus of a single wave vector $|Q|$. However, at higher densities the Fermi surface(s) acquires a diamond-like shape, and a multi-modal description is expected to be more accurate [26]. This would translate into complex real-space modulations of the order parameter, such as those observed in Ref. [13]. Beyond 1D, nesting between bands with opposite spin is weak [5], particularly at high densities and high magnetic fields, when the Fermi surfaces have very different shapes, with a large mismatch in Fermi velocities. Thus, the FFLO state becomes unfavorable compared to a normal polarized state. The ladder system, however, is strongly nested to the effect that we find a pairing instability of the FFLO-type at all $p$ and $n$ studied, with a much richer structure than in the single-chain case.

We thank D. Huse and G. Roux for fruitful discussions.

[1] I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. 80, 885 (2008); S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. 80, 1215 (2008).
[2] M. W. Zwierlein et al. Science 311, 492 (2006); G. B. Partridge et al., ibid. 311, 503 (2006); Y. Shin et al., Phys. Rev. Lett. 97, 030401 (2006); G. B. Partridge et al., ibid 97, 190407 (2006).
[3] P. Fulde and A. Ferrell, Phys. Rev. 135, A550 (1964).
[4] A. Larkin and Y. Ovchinnikov, Zh. Eksp. Teor. Fiz 47, 1136 (1964).
[5] D. E. Sheehy and L. Radzihovsky, Phys. Rev. Lett. 96, 060401 (2006); J. Kinnunen, L. M. Jensen, and P. Törmä, ibid. 96, 110403 (2006); K. Machida, T. Mizushima, and M. Ichikawa, ibid 97, 120407 (2006); S. Pilati and S. Giorgini, ibid. 100, 030401 (2008).
[6] M. M. Parish et al., Phys. Rev. Lett. 99, 250403 (2007).
[7] K. Yang, Phys. Rev. B 63, 140511(R) (2001).
[8] H. Hu, X.-J. Liu, and P. D. Drummond, Phys. Rev. Lett. 98, 070403 (2007); G. Orso, ibid. 98, 070402 (2007).
[9] G. Roux et al., Phys. Rev. Lett. 97, 087207 (2006); G. Roux et al., Phys. Rev. B 75, 245119 (2007).
[10] A. E. Feiguin and F. Heidrich-Meisner, Phys. Rev. B 76, 220508(R) (2007).
[11] M. Tezuka and M. Ueda, Phys. Rev. Lett. 100, 110403 (2008); G. Batrouni et al., ibid 100, 116405 (2008); M. Rizzi et al., Phys. Rev. B 77, 245105 (2008); G. Xi-anlong and R. Asgari, Phys. Rev. A 77, 033604 (2008); M. Machida et al., ibid. 77, 053614 (2008); M. Casula, D. M. Ceperley, and E. J. Mueller, ibid. 78, 033607 (2008).
[12] A. Lüscher, R. M. Noack, and A.M. Läuchli, Phys. Rev. A 78, 013637 (2008).
[13] M. Iskin and C.J. Williams, Phys. Rev. A 78, 011603(R) (2008); Y. Chen et al., arXiv:0710.5484.
[14] A. Moreo and D. J. Scalapino, Phys. Rev. Lett. 98, 216402 (2007).
[15] T. Koponen et al., New. J. Phys. 10, 045104 (2008).
[16] L. He and P. Zhuang, Phys. Rev. A 78, 033613 (2008).
[17] E. Zhao and W. V. Liu, Phys. Rev. A 78, 063605 (2008).
[18] S. R. White, Phys. Rev. Lett. 69, 2863 (1992); U. Schollwöck, Rev. Mod. Phys. 77, 259 (2005).
[19] E. Dagotto and T. M. Rice, Science 271, 618 (1996).
[20] T. Giamarchi, Quantum Physics in One Dimension, Clarendon Press, Oxford, 2004.
[21] P. Lee et al., Phys. Rev. Lett. 99, 020402 (2007), S. Trotzky et al., Science 319, 295 (2008).
[22] I. Danshita, C.A.R. Sa de Melo, and C. W. Clark, Phys. Rev. A 77, 063609 (2008).
[23] V. Gritsev, E. Demler, and A. Polkovnikov, Phys. Rev. A 78, 063624 (2008).
[24] S. Sachdev and K. Yang, Phys. Rev. B 73, 174504 (2006).
[25] F. Essler et al., The one-dimensional Hubbard model, Cambridge University Press, 2005.
[26] J.A. Bowers and K. Rajagopal, Phys. Rev. D 66, 065002 (2002); R. Combescot and C. Mora, Europhys. Lett. 68, 79 (2004).