Classification from Ambiguity Comparisons

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Abstract

Labeling data is an unavoidable pre-processing procedure for most machine learning tasks. However, it takes a considerable amount of time, money, and labor to collect accurate explicit class labels for a large dataset. A positivity comparison oracle has been adapted to relieve this burden, where two data points are received as input and the oracle answers which one is more likely to be positive. However, when information about the classification threshold is lacking, this oracle alone can at most rank all data points on the basis of their relative positivity; thus, it still needs to access explicit class labels. In order to harness pairwise comparisons in a more effective way, we propose an ambiguity comparison oracle. This oracle also receives two data points as input, and it answers which one is more ambiguous, or more difficult to assign a label to. We then propose an efficient adaptive labeling algorithm that can actively query only pairwise comparison oracles without accessing the explicit labeling oracle. We also address the situation where the labeling budget is insufficient compared to the dataset size, which can be dealt with by plugging the proposed algorithm into an active learning algorithm. Furthermore, we confirm the feasibility of the proposed oracle and the performance of the proposed labeling algorithms theoretically and empirically.

1 Introduction

The performance of a classifier heavily relies on the quality of its training data, especially the quality of labels. The annotation step for labels is usually expensive and time-consuming. Crowdsourcing reduces the annotation budget but the accuracy can be hardly guaranteed. This paper focuses on improving the quality of acquired labels while keeping the annotation budget low.

In order to address the instability of explicit class label annotation, collecting pairwise comparison information is a promising alternative. It is believed that pairwise comparisons show a more reliable performance\textsuperscript{29}. Previously, the positivity comparison oracle was used to collect information concerning which
one of two unlabeled data points is more likely to be positive. This oracle has been extensively used in interactive classification \[10, 32\] and preference learning \[7, 11\]. Especially in binary classification, high accuracy label assignment can be achieved with a lower budget using both the positivity comparison oracle and the usual explicit labeling oracle \[32\]. However, in certain application scenarios, achieving explicit class labels for even a small dataset could be difficult or impossible. For example, in order to correctly diagnose whether a patient is infected with a specific disease, it is essential to test their gene expression profiles. When the cost of conducting the tests is not affordable or facing a sudden explosion of a totally unknown virus, it is difficult to confirm the RNA sequence and mass-produce test kits in a very short time. Thus, doctors tend to diagnose using implicit information related to the disease, such as microscope images or CT images. On the other hand, when data are accidentally or intentionally made to be deceptive, such as cursively written characters \[8\], fake news articles and Deepfake videos \[20\], it is difficult for common annotators to assign accuracy labels. In these situations, we face a lack of trustable explicit class labels, and with positivity comparisons alone, we cannot recover explicit class labels.

In this paper, we tackle this problem by proposing a new pairwise comparison oracle that compares the ambiguities or classification difficulties. This oracle is weak enough that we believe it is easier to answer than explicit class labels and explicit class labels cannot be recovered from only ambiguity comparisons. This oracle is also strong enough that explicit class labels can be elicited when both this oracle and the positivity comparison oracle are available, as we will show later by an interactive labeling algorithm which enjoys an efficient query complexity. The output labels of this algorithm can be fed into various downstream tasks. Specifically, we consider the k-NN algorithm \[1\], which is a commonly used non-parametric classification algorithm that is easy to implement and enjoys theoretical guarantees. Moreover, for cases where the dataset is too large compared to an insufficient query budget, we show that plugging the proposed algorithm into an existing active framework also results in a principled learning algorithm.

In summary, for the problem of interactive labeling with access to only pairwise comparison oracles, our contributions are three-fold:

- We propose a novel pairwise comparison oracle that compares the ambiguity, or classification difficulty, of two unlabeled data points.
- We propose a feasible labeling algorithm accessing only the aforementioned two kinds of pairwise comparison oracles without accessing any explicit class labels, as well as its applications under different query budgets.
- We establish the error rate bound for the proposed algorithm and generalization error bounds for its applications, and confirm their empirical performance.
2 Labeling with only pairwise comparisons

In this section, we introduce the two comparison oracles and the proposed labeling algorithm.

2.1 Preliminaries

We consider the binary classification problem. Let $\mathcal{X} \subset \mathbb{R}^d$ denote the $d$-dimensional sample space and $\mathcal{Y} = \{+1, -1\}$ denote the binary label space. Let $\mathcal{P}_{XY}$ denote the underlying data distribution over $\mathcal{X} \times \mathcal{Y}$ and $\eta(x) \triangleq p(Y = +1 | X = x)$ denote the underlying conditional probability for a data point $x$ being positive. Then $h^* \triangleq \text{sign}(\eta(x) - 0.5)$ is the Bayes classifier minimizing the classification risk $R(f) \triangleq \mathbb{E}_{(x, y) \sim \mathcal{P}_{XY}}[\mathbb{I}_{f(x) \neq y}]$ for a classifier $f: \mathcal{X} \to \mathcal{Y}$. In this problem setting, we are given only data points drawn from $\mathcal{P}_X$, the marginal distribution over $\mathcal{X}$. Without accessing class labels $\{h^*(x) | x \in \mathcal{X}\}$, we query the following two oracles for essential information.

2.2 Two pairwise comparison oracles

**Positivity comparison oracle** This oracle receives two data points as input and answers whether the first data point has a higher probability of being positive. The answer “$+1$” means “yes” and “$-1$” means “no”. This is a common oracle that has been used in many different fields such as interactive classification \[\text{[16, 32]}\] and preference learning \[\text{[7, 11]}\]. We denote this oracle by $O_1: \mathcal{X} \times \mathcal{X} \to \{+1, -1\}$ and define it with the following noise condition.

**Condition 1.** The distribution $\mathcal{P}_{XY}$ and oracle $O_1$ satisfies the condition with noise parameter $\epsilon_1 \geq 0$ if $\mathbb{E}_{x_1, x_2 \sim \mathcal{P}_X}[\mathbb{I}_{O_1(x_1, x_2)(\eta(x_1) - \eta(x_2)) < 0}] = \epsilon_1$.

**Ambiguity comparison oracle** This is our proposed oracle that receives two data points as input and answers whether the first one is more ambiguous or more difficult to classify. The answer “$+1$” means “yes” and “$-1$” means “no”. We define the ambiguity of a data point $x \in \mathcal{X}$ as the difference between $\eta(x)$ and the classification threshold $0.5$. The smaller this difference $|\eta(x) - 0.5|$ is, the more ambiguous $x$ to be classified. We denote this oracle by $O_2: \mathcal{X} \times \mathcal{X} \to \{+1, -1\}$ and define it with the following noise condition.

**Condition 2.** The distribution $\mathcal{P}_{XY}$ satisfies this condition with noise parameter $\epsilon_2 \geq 0$ if $\mathbb{E}_{x_1, x_2 \sim \mathcal{P}_X}[\mathbb{I}_{O_2(x_1, x_2)(|\eta(x_2) - 0.5| - |\eta(x_1) - 0.5|) < 0}] = \epsilon_2$.

Note that the above conditions only assume the error rates. Thus, answers may not hold for a proper order. Namely, it is possible to collect positive answers from $O_1(x_1, x_2)$ and $O_1(x_2, x_1)$ (asymmetricity), or $O_1(x_1, x_2)$, $O_1(x_2, x_3)$ and $O_1(x_3, x_1)$ (intransitivity) for $x_1, x_2, x_3 \sim \mathcal{P}_X$. The same holds for $O_2$. Therefore our assumptions are relatively weak compared to parametric models, such as the Bradley-Terry-Luce (BTL) model \[\text{[5, 22]}\].
2.3 Proposed labeling algorithm

We propose a labeling algorithm that does not access the explicit labeling oracle and can still output accurate labels. Given a set of unlabeled data points $D$ sampled from $P_X$ with size $n$, the idea is to first select a subset of $t$ data points $D' \subset D$ as a proxy or delegation for the classification threshold with $t \ll n$. Note that we do not need to know the ranking order of either $D'$ or $D \setminus D'$, and we want to find the subset by accessing the oracles as few times as possible. This can be formulated as a top-$t$ items selection problem from noisy comparisons. To this end, we choose the theoretical-guaranteed and practically promising algorithm proposed by Mohajer et al. [24] as the first step of our algorithm. Then we use the selected delegation subset and the positivity comparison oracle to decide labels of $D \setminus D'$. Our algorithm is described in following three steps.

1. We use $O_2$ to find $D'$, a subset of $t$ most ambiguous data points.
2. For each data point $x \in D \setminus D'$, we use $O_1$ to compare it with all (or part of) data points in $D'$ to infer its label by majority votes.
3. Since we do not assume $D$ is i.i.d. sampled to adapt this algorithm to more general situations, the worst case could happen for any labeling of $D'$. Therefore, we can just assign random labels to data points in $D'$, or repeat the whole algorithm using $D'$ as input.

This algorithm can efficiently infer labels without requiring unnecessary information such as the ranking order based on posterior probabilities. The algorithm is formally described in Algorithm 1. An error rate bound for inferred labels under noise conditions is established in Section 3.1.

Algorithm 1 Proposed Labeling Algorithm

Input: Positive integer $t$, dataset $D$ with size $n$.
1: Select $t$ most ambiguous data points from $D$ using the algorithm of Mohajer et al. [24] and $O_2$. Denote the selected set as $D'$.
2: for $x_i \in D \setminus D'$ do
3: If $\sum_{x_j \in D'} O_1(x_i, x_j) \geq \frac{1}{2}$, then let $\hat{y}_i \leftarrow 1$, else let $\hat{y}_i \leftarrow 0$.
4: end for
5: Assign random $\hat{y}_i$ for $x_i \in D'$.
Output: Inferred labels $\hat{Y} \doteq \{\hat{y}_i\}_{i=1}^n$.

2.4 Learning classifiers under different budgets

For down-stream tasks, we can feed $D$ and $\hat{Y}$ into any algorithms that rely on samples from $P_{X|Y}$. In this paper, we consider the general application of learning a binary classifier.
Sufficient budget case In this case, we assume enough budget for running Algorithm 1 on the whole dataset. Then, we can obtain the inferred labels and feed them into any classification algorithms. In this paper, we consider the simplest non-parametric $k$-NN algorithm [1], which is easy to implement and enjoys good theoretical guarantees. A generalization bound for classifiers obtained in this case is established in Section 3.2.

Insufficient budget case In this case, we consider a more practical situation where the dataset is too large compared to the budget; thus, we cannot afford to run Algorithm 1 on the whole dataset. We resort to using active learning with Algorithm 1 as a subroutine for the selected batch at each step. The same as Algorithm 3 of Xu et al. [32], we consider a disagreement-based active learning algorithm calling the proposed Algorithm 1 at each step. Algorithm 2 describes the detailed algorithm.

Algorithm 2 Disagreement-based active learning algorithm (Algorithm 3 of Xu et al. [32]).

Input: $\epsilon$, a sequence of $n_i$, hypothesis set $H$

1: $H_1 \gets H$
2: for $i = 1, 2, \ldots, \lceil \log \frac{1}{\epsilon_1} \rceil$ do
3: $S_i \gets \text{i.i.d. sample from } P_X$ with size $n_i$.
4: $D_i \gets \text{DIS}(S_i, H_i)$.
5: Run Algorithm 1 with $\epsilon_i = \frac{1}{2^{i+2}}$ and $D_i$, obtain $\{\hat{y}_j\}_{j=1}^{D_i}$.
6: $H_{i+1} \gets \{h \in H_i : \sum_{j=1}^{n_i} 1_{h(x_j) \neq \hat{y}_j} \leq \epsilon_i n_i\}$
7: end for

Output: Any Classifier in $H_{i+1}$

3 Theoretical analysis

In this section, we establish the error rate bound for Algorithm 1 and generalization error bounds for the $k$-NN algorithm and Algorithm 2.

3.1 Analysis of the proposed labeling algorithm

Theorem 1 (Error rate bound). Suppose Condition 1 and Condition 2 hold for $\epsilon_1, \epsilon_2 \in [0, 0.5)$. Let $t = \Omega \left( \frac{\log^2 2}{2(0.5 - \epsilon_2)^2} \right)$. Fix $\epsilon > 0$ and assume $D$ to be a set of size $n > \frac{t}{\epsilon}$ that contains data points $x \in X$. Then, there exist constants $C_1$ and $C_2$ such that for an execution of Algorithm 1 on $D$ with parameters $t$ and $m \geq C_1 \max(\log \log n, \log t)$, with probability at least $1 - \delta$ where we denote $\delta \triangleq \delta(C_2, n, t, \epsilon_1)$ for simplicity, the error rate of inferred labels is bounded as $\frac{|\{i \in [n] : h(x_i) \neq \hat{h}(x_i)\}|}{n} \leq \epsilon$. The query complexity is $O \left( \frac{n}{\epsilon_1^2} \right)$ for $O_1$ and $O \left( \frac{n \log \log n}{\epsilon_2^2} \right)$ for $O_2$. 

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Proof can be found in Appendix A. Note that there are two hyper-parameters for the algorithm: the size of the delegation subset \( t \) and the repetition number for each comparison \( m \). The above theory shows a principled way to select the hyper-parameter \( t \), which only depends on the error condition \( \epsilon_1 \). For a reasonable range of \( \epsilon_1 \leq 0.4 \), Algorithm 1 only requires \( t \) to be at most 35, which is relatively small compared to the size of a modern dataset. For the other hyper-parameter \( m \), we empirically observe that a surprisingly small value, even 1, shows promising performance. For the query complexities, the \( O(n) \) factor should be required by at least one oracle, since we cannot decide the label of a data point without accessing it at least once.

3.2 Analysis of nearest neighbors classifiers

We establish a generalization error bound for classifiers obtained by combining Algorithm 1 and \( k \)-NN. We want to estimate the function \( \eta(x) \) from the inferred labels by Algorithm 1. For \( x \in \mathcal{X} \), we denote indices of other points in a descending distance order by \( \{ \tau_q(x) \}_{q=1}^{n-1} \). This means that for a metric \( \rho \), it holds \( \rho(x, x_{\tau_q(x)}) \leq \rho(x, x_{\tau_{q+1}(x)}) \) for \( q \in [1, n-2] \). Thus, we can denote the resulting \( k \)-NN classifier as \( \hat{f}(x; k) = \frac{1}{k} \sum_{q=1}^{k} \hat{y}_{\tau_q(x)} \).

We then introduce two essential assumptions. First, we need a general assumption for achieving fast convergence rates for \( k \)-NN classifiers.

**Assumption 1** (Measure smoothness). With \( \lambda > 0 \) and \( \omega > 0 \), for all \( x_1, x_2 \in \mathcal{X} \), it satisfies

\[
|\eta(x_1) - \eta(x_2)| \leq \omega \mu (B_{\rho_{(x_1, x_2)}}(x_0))^\lambda,
\]

where \( B_{\rho_{(x_1, x_2)}}(x_0) \) denotes a ball with center \( x_0 \) and radius \( \rho_{(x_1, x_2)} \).

Then, we need the following Tsybakov’s margin condition [23], which is a common assumption for establishing fast convergence rates.

**Assumption 2** (Tsybakov’s margin condition). There exist \( \alpha \geq 0 \) and \( C_\alpha \geq 1 \) such that for all \( \xi > 0 \) we have

\[
\mu \left( \left\{ x \in \mathcal{X} : 0 < \left| \eta(x) - \frac{1}{2} \right| < \xi \right\} \right) \leq C_\alpha \xi^\alpha.
\]

Finally, we establish the generalization error bound.

**Theorem 2** (Generalization error bound for \( k \)-NN). Suppose the conditions for Theorem 1 hold. Let the input and the output of Algorithm 1 be \( D = \{ x_i \}_{i=1}^n \) and \( \hat{Y} = \{ \hat{y}_i \}_{i=1}^n \). Let \( \hat{f}(x; k) \) be the \( k \)-NN classifier obtained and \( f^*(x) = \mathbb{1}_{\eta(x) \geq \frac{1}{2}} \) be the Bayes classifier. Then, using the same notations as Theorem 1, supposing that Assumption 1 holds with \( \lambda > 0 \) and \( \omega > 0 \), and Assumption 2 holds with \( \alpha \geq 0 \) and \( C_\alpha \geq 1 \), for \( \delta' \in (0, 1) \), \( 4 \log(\frac{1}{\delta'}) + 1 \leq k \leq \frac{n}{2} \), with probability at least \( (1 - \delta)(1 - \delta') \), we have

\[
R(\hat{f}) \leq R(f^*) + C_\alpha \left( \frac{2\epsilon}{k} + \omega \left( \frac{2k}{n} \right)^\lambda \right)^{\alpha+1}.
\]
Proof can be found in Appendix B. The difference between the above bound and other generalization bounds under unknown asymmetric noise [12, 24] is that Theorem 2 does not require the labels to be an i.i.d. sample from an underlying distribution, as they are instead inferred by Algorithm 1.

3.3 Analysis of disagreement-based active learning

We establish the generalization error bound by the following corollary to justify Algorithm 2. Its proof can be found in Appendix C.

**Corollary 3** (Generalization error bound for active learning). Suppose conditions for Theorem 1 hold. Then, for an execution of Algorithm 2 with \( \epsilon \in (0, 1) \), \( \epsilon_i = \frac{1}{2^i+2} \), with probability at least \( 1 - \delta \), the output \( \hat{h} \) satisfies \( P_{x \sim P_X} [\hat{h}(x) \neq h^*(x)] \leq \epsilon \).

4 Related work

**Weakly-supervised learning** Learning classifiers from passively obtained comparisons without explicit class labels have been studied, such as learning from similarity comparisons [3, 28] and learning from triplet comparisons [9]. However, the feasibility of learning in such cases relies on various inevitable assumptions. Bao et al. [3] assumes the group with more data to be the positive class. The other two methods [28, 9] assume specific data generation processes, which may not always hold in some applications. Moreover, none of these methods have theoretical guarantees for noisy comparisons. On the other hand, learning from totally unlabeled data has also been studied [10, 21]. However, these methods require at least two datasets with different class priors \( p(Y = +1) \), and they also need to know these class priors exactly, which can be impossible in some cases. The proposed labeling algorithm is transductive and can be combined with non-parametric classifiers, while above existing methods mainly rewrite the classification risk and require a differentiable model. Furthermore, the proposed algorithm does not require above assumptions and additional information such as exact class priors.

**Preference learning** Results of \( O_1 \) are mainly used in this learning paradigm to learn a (partial) ranking over data points. However, ranking cannot induce labels without further information as the class prior is needed to decide the classification threshold. At the same time, labeling cannot induce ranking as there is no information on the ranking order of data with the same label. Similar arguments also hold for bipartite ranking [24].

**Active learning** Interactive classification with oracles that do not answer the explicit class labels has been studied [4, 16, 32, 30]. Beygelzimer et al. [4] uses a search oracle that receives a function set as input and outputs a data point with its explicit class label. Other two methods use the same oracle as \( O_1 \). However,
they all need to access the explicit labeling oracle. On the other hand, Balcan et al. \cite{2} uses only the class conditional queries (CCQ) without accessing the explicit labeling oracle. However, labels can be inferred from a single CCQ query. Although we cannot directly compare, we claim that $O_2$ is weaker than CCQ because labels cannot be inferred from the query results of $O_2$.

5 Experiments

In this section, we confirmed the feasibility and performance of the proposed algorithm using both simulation and crowdsourcing data.

5.1 Simulation study

All experiments were repeated ten times on a server with an Intel(R) Xeon(R) CPU E5-2698 v4 @ 2.20GHz CPU and a Tesla V100 GPU. Their mean values and standard deviations are reported.

5.1.1 Sufficient budget case

When considering constructing binary datasets from multi-class datasets, experiments in existing work usually split the whole dataset into two parts, such as separating odd numbers and even numbers for handwritten digits. However, as the focus of the proposed oracle is the ambiguity, it is important to simulate experiments that have some kind of ambiguities in its expression. For image datasets, the ambiguity can be expressed as visual similarity between two classes. Therefore, we constructed following eight binary classification datasets that have visual similarity to some extent.

- **MNIST-a** denotes the MNIST \cite{19} data with label 1 (7877 data) and 7 (7293 data).
- **MNIST-b** denotes the MNIST data with label 3 (7141 data) and 5 (6313 data).
- **FMNIST-a** denotes the Fashion-MNIST \cite{31} data with label T-shirt/top (7000 data) and shirt (7000 data).
- **FMNIST-b** denotes the Fashion-MNIST data with label pullover (7000 data) and coat (7000 data).
- **KMNIST-a** denotes the Kuzushiji-MNIST \cite{8} data with the second label (7000 data) and eighth label (7000 data).
- **KMNIST-b** denotes the Kuzushiji-MNIST data with the third label (7000 data) and seventh label (7000 data).
- **CIFAR10-a** denotes the CIFAR-10 \cite{18} data with label automobile (5000 data) and truck (5000 data).
• **CIFAR10-b** denotes the CIFAR-10 data with label deer (5000 data) and horse (5000 data).

For all datasets except CIFAR-10, a logistic regression classifier was first learned on all selected data with one hundred thousand maximum iteration. The oracles were then simulated using the output conditional probabilities of this logistic regression classifier. For CIFAR-10, a ResNet152 [15] classifier was first trained on the whole dataset (10 classes) for 100 epochs. Then, the 2048-dimension expressions before the last fully connected layer were used as low dimensional features, which were then used to train a logistic regression classifier. The logistic regression classifier and the k-NN classifiers are trained on these 2048-dimension features instead of the original input. We set $k = 5$ for $k$-NN classifiers throughout all experiments. We randomly split training and test set according to the $4:1$ ratio for every repetition of the algorithms. We do not have sensitive hyper-parameters to tune, thus we did not separated a validation set. For the Co-teaching experiments, we set batchsize as 1024 and epochs as 100. We adopted the public codes provided by the authors, thus followed all other default settings therein, such as learning rate schedule.

We considered the conservative case where the noise rates are high and the repetition number $m$ is small. Theorem 1 indicates that the size of the delegation subset $t$ usually has a maximum of 35, thus we set $t$ to be 10 or 35. Table 1 shows that a larger set of delegation set (corresponding to a higher $t$) contributes to a better label accuracy, thus a better generalization ability. This behavior matches the expectation as the inferred label for each non-delegation data point becomes more accurate. We also observed that even with a small $t$, $k$-NN shows promising generalization ability.

Table 1: Performance when the repetition number $m = 1$, noise rates $\epsilon_1 = \epsilon_2 = 0.4$.

| Dataset   | Label Accuracy ($t=10$) | $k$-NN Test Accuracy ($t=10$) | Label Accuracy ($t=35$) | $k$-NN Test Accuracy ($t=35$) |
|-----------|-------------------------|-------------------------------|--------------------------|-------------------------------|
| MNIST-a   | 67.89 (0.37)            | 77.63 (0.83)                  | 80.94 (0.47)             | 92.36 (0.60)                 |
| MNIST-b   | 67.10 (0.52)            | 76.11 (0.79)                  | 80.46 (0.37)             | 92.93 (0.37)                 |
| FMNIST-a  | 65.78 (0.26)            | 70.96 (0.45)                  | 76.38 (0.20)             | 81.40 (0.19)                 |
| FMNIST-b  | 66.25 (0.34)            | 72.28 (0.50)                  | 77.25 (0.24)             | 83.36 (0.20)                 |
| KMNIST-a  | 68.69 (0.56)            | 78.90 (1.07)                  | 81.64 (0.62)             | 94.30 (0.58)                 |
| KMNIST-b  | 67.99 (0.26)            | 77.45 (0.45)                  | 78.88 (0.36)             | 90.16 (0.33)                 |
| CIFAR10-a | 69.34 (0.44)            | 80.09 (0.82)                  | 82.07 (0.41)             | 94.28 (0.31)                 |
| CIFAR10-b | 68.67 (0.20)            | 78.47 (0.59)                  | 81.83 (0.50)             | 93.95 (0.42)                 |

Table 2 shows the results of the optimism situation when the noise rates were low and sufficient budget for a larger $m$ was available.

We next confirmed the quality of inferred labels using a more powerful model. Co-teaching [13] is a recently proposed training method for extremely noisy
Table 2: Performance when repetition $m = 10$, noise rate $\epsilon_1 = \epsilon_2 = 0.1$. 

| Dataset    | Label Accuracy ($t=10$) | Test Accuracy ($t=10$) | Label Accuracy ($t=35$) | Test Accuracy ($t=35$) |
|------------|--------------------------|-------------------------|--------------------------|-------------------------|
| MNIST-a    | 99.74 (0.01)             | 99.39 (0.03)            | 99.84 (0.01)             | 99.35 (0.03)            |
| MNIST-b    | 97.12 (0.03)             | 98.36 (0.09)            | 97.22 (0.02)             | 98.36 (0.06)            |
| FMNIST-a   | 87.19 (0.06)             | 83.95 (0.18)            | 87.38 (0.06)             | 84.14 (0.16)            |
| FMNIST-b   | 88.84 (0.04)             | 86.26 (0.20)            | 88.86 (0.04)             | 86.67 (0.18)            |
| KMNIST-a   | 98.78 (0.01)             | 99.12 (0.05)            | 98.90 (0.01)             | 99.00 (0.02)            |
| KMNIST-b   | 92.33 (0.03)             | 94.53 (0.14)            | 92.36 (0.03)             | 94.85 (0.09)            |
| CIFAR10-a  | 99.87 (0.02)             | 99.92 (0.02)            | 99.97 (0.01)             | 99.95 (0.01)            |
| CIFAR10-b  | 99.86 (0.01)             | 99.98 (0.01)            | 99.94 (0.01)             | 99.98 (0.01)            |

labels. It holds two classifiers which feed their small loss data points to the other classifier for training. Although lacking theoretically guarantees, it is reported promising performance [13]. We used relatively small ResNet18 [15] models and restrain from tuning any hyper-parameters for Co-teaching.

Figure 1 shows results with same size of delegation set in the same color, and uses dot lines to show results with fewer repetition numbers. We observe that setting $m = 1$ already shows promising accuracy, with $t$ set to be the theoretical maximum 35. For the same value of $t$, increasing $m$ from 1 to 10 can offer only little improvement on the accuracy. Setting $m$ to 1 means we only query each pair once and proceed the algorithm believing the answer is correct. This shows that the proposed algorithm is highly robust to query noise, as it shows promising performance using the single noisy result without repeating the same query many times. Moreover, the low noise rate regime shows comparable performance under different settings, which means the proposed algorithm can generally achieve high performance with low budget.

![Figure 1: Generalization performance of co-teaching classifiers.](image)

Figure 2 shows the detailed investigation on the Fashion MNIST datasets. It shows similar tendency as the previous Co-teaching results on CIFAR-10.
datasets.

Figure 2: Generalization performance of $k$-NN classifiers for Fashion-MNIST datasets.

### 5.1.2 Insufficient budget case

In this case, because Algorithm 2 needs to loop over every available hypothesis at each step, it is infeasible to start with a large hypotheses set. Note that even for the MNIST dataset with 784 features and the simplest linear models, using a discrete exploring space of size 10 for the parameter corresponding to each feature creates a huge hypotheses set of size $10^{784}$. Therefore, in order to illustrate the feasibility of the algorithm, we used 2-dimensional toy data generated from two Gaussian distributions that are symmetric to the origin point. Specifically, we used two Gaussian distributions with mean value of $(2, 2)$ and $(-2, -2)$ and the identical matrix as both covariances. From these distributions, we drew ten thousand data points in total, with each data point having an equal probability to be generated from either distribution. Then a logistic regression classifier is trained with one hundred thousand maximum iteration to simulate the oracles. For the hypothesis set, we used 1000 equally separated linear classifiers passing through the origin point. Setting the desiring precision $\epsilon = 0.1$ resulted three steps based on Algorithm 2. Table 3 shows the number of left candidate hypotheses and their test accuracy at each step.

| Step  | Number of Left Hypotheses | Test Accuracy of Left Hypotheses |
|-------|---------------------------|---------------------------------|
| Step 1| 674.10 (4.97)              | 96.98 (0.44)                    |
| Step 2| 525.60 (7.34)              | 99.29 (0.19)                    |
| Step 3| 196.90 (71.85)             | 99.78 (0.11)                    |
5.2 User study

The previous section investigated the proposed algorithm using artificial oracles, and the feasibility in real-world situations remains untouched. Therefore, we conducted user study using crowdsourcing in this section.

5.2.1 Character recognition task

In this task, we focused on the classification of Kuzushiji (cursive Japanese) [8], which is important for advocating research on Japanese historical books and documents.

Goals We want to justify the proposed oracle and confirm whether the proposed algorithm can work on results collected through user study without simulation. Specifically, we want to (1) confirm whether data pairs selected by the proposed algorithm are easier for ambiguity comparison than explicit labeling, and (2) confirm whether the proposed algorithm can work on only crowdsourcing results. We will introduce the data and the general interface we used in user study, followed by detailed description of each user study setting in the following paragraphs.

Data From the Kuzushiji-MNIST dataset [8], we selected the 5-th and the 10-th characters to form the binary classification task. The reading alphabet is ‘NA’ for the 5-th character and ‘WO’ for the 10-th character. Figure 3 shows them in a standard font. Albeit the visual similarity, these two characters are important auxiliary words with distinct meanings. Thus, wrongly recognizing the two characters can harm the understanding of the sentence. This recognition task has a natural affinity with ambiguity comparison, as in daily writing, the difficulty of recognizing a hand written character is easier to interpret, rather than recognizing the exact character.

Methods We prepared three types of questions: explicit labeling, pairwise positivity comparison, and pairwise ambiguity comparison. We also asked annotators for the difficulty of each question when necessary. The interfaces are shown by the following list.

- Figure 4 shows how we ask annotators for explicit labels.
- Figure 5 shows how we ask annotators for pairwise positivity comparisons. If we fix one label such as ‘NA’ and ask which one is more likely to be ‘NA’, there are cases that both images in a pair look similar to ‘WO’, thus it’s difficult to answer. Therefore, we also ask annotators to choose either ‘NA’ or ‘WO’ that is used as the criterion of positivity.
Figure 6 shows how we ask annotators for pairwise ambiguity comparisons. As this is a newly proposed comparison question and annotators may be not used to answer it, we give an explanatory example on how to select.

Figure 7 shows how we ask annotators for difficulty evaluation of ambiguity comparisons compared to explicit labeling. We asked annotators to answer both queries first to familiarize them with the problem.

Task Description

Hand written character images of 'NA' or 'WO' are to be shown. Please answer questions on their recognition.

Question 1: Please answer the exact character.

○ 'NA'  ○ 'WO'

Figure 4: Questionnaire of explicit labeling.

**Justification for ambiguity comparisons**  In this user study, we confirmed whether the data pairs selected by the algorithm for $O_2$ are difficult for explicit labeling. We first greedily selected 25 medoids from all data points. Then, we ran the proposed algorithm on these 25 data points using artificial oracles, and collected the 42 pairs that were selected for $O_2$. Finally, we conducted user study from 50 annotators on explicit labeling and ambiguity comparison on these 42 pairs. For each, we also asked the difficulty of ambiguous comparisons compared to explicit labeling using scores from one to five, with a smaller score indicating an easier question. Furthermore, we collected difficulty evaluation of explicit labeling for each image from 10 annotators.

In order to investigate the difficulty evaluation on pair attributes, we introduce the *individual difficulty* for each single image. Another difficulty will be introduced in the following paragraph. Then, based on the user evaluation of *individual difficulties*, we classified data pairs into three types: (1) the ‘E’ type containing two easy data points, (2) the ‘&’ type containing one easy and one difficult data point, and (3) the ‘D’ type containing two difficult data points.

We then aggregated the user evaluations based on pair types. Table 4 shows the mean and standard deviation of the difficulty evaluations for each type, as well as t statistics and p values when conducted one sample t test.
Task Description

Hand written character images of 'NA' or 'WO' are to be shown. Please answer questions on their recognition. Please annotate the one that looks more like 'NA', or the one that looks more like 'WO'. Please also answer the criterion of the selection.

Question1:

The one looks more like ○ 'NA' ○ 'WO' is ○ image A ○ image B.

Figure 5: Questionnaire of pairwise positivity comparisons.

Task Description

Hand written character images of 'NA' or 'WO' are to be shown. Please answer questions on selecting the one easier to recognize. For example, given the following two images,

The left one if more cursive, and the right one is more clear to be recognized as 'WO'. Thus the right one turns out to be the image that is more easier to recognize.

Question1: Please select the image that is more easier to recognize.

○ Image A ○ Image B

Figure 6: Questionnaire of pairwise ambiguity comparisons.
against value 3, which means two types of query have equal difficulty. From the results, we can conclude that as pair type changes from ‘E’ to ‘D’, ambiguity comparison becomes less favored against explicit labeling. For type ‘D’, the mean of difficulty evaluations is not significantly different from 3, as the p value $0.06 > 0.05$. However, as the proposed algorithm focuses on separating difficult images, random decisions on images with similar difficulty do not harm the performance.

### Table 4: Statistics of difficulty evaluation of ambiguity comparisons.

|                | Type ‘E’ | Type ‘&’ | Type ‘D’ | Total |
|----------------|----------|----------|----------|-------|
| Number of Pairs| 12       | 25       | 5        | 42    |
| Number of Total Evaluations | 600     | 1250     | 250      | 2100  |
| Mean           | 2.57     | 2.82     | 2.84     | 2.75  |
| Standard Deviation | 1.28     | 1.38     | 1.35     | 0.34  |
| t statistic    | -8.23    | -4.68    | -1.91    | -5.15 |
| p value        | $1.19 \times 10^{-15}$ | $3.22 \times 10^{-6}$ | 0.06 | $4.69 \times 10^{-6}$ |

**Algorithm feasibility using simulated pairwise comparisons** In this user study, we first greedily selected 50 medoids as training data. We then collected explicit label feedback from 20 annotators. For a single image in these 50 medoids, we simulated its class probability by the proportion of class assignments in the 20 evaluations. For example, if 15 annotators assigned positive label to an image, we defined its probability to be positive as $\frac{15}{20} = 0.75$. These probabilities were then used to simulate both kinds of pairwise comparisons. Using inferred labels as input, the last layer of a pre-trained neural network was
Figure 8 shows the label accuracy and the test accuracy when using different numbers of medoids as training data. The test accuracy measures the performance of each classifier learnt from inferred labels on a test dataset of size 100, which is uniformly selected without replacement excluding the training data points. It can be clearly observed for the full supervision case that more training data contribute higher accuracies. It is not clear for the other two methods, because they rely on not only the number of training data, but also the quality of their pairwise comparison feedback. Although there were 64\% ties among all ambiguity pairwise comparisons, the proposed method showed consistent performance. However, with 24\% ties among all positivity pairwise comparisons, the existing method failed to perform consistently, even with parameter tuning.

Algorithm feasibility using user feedback on pairwise comparisons
In this user study, we confirmed the performance of each algorithm on only crowdsourcing results. We greedily selected 25 medoids \cite{27}, collected answers for all possible combinations among these medoids from 10 annotators, and used aggregated majority as input to the existing algorithm \cite{32} using both positivity comparisons and explicit class labels and the proposed algorithm. We adopted a pre-trained neural network and fine-tuned its last layer considering the small number of training data.

Figure 9 and Figure 10 show the label accuracies and test accuracies for results of 25 greedily selected medoids and 25 uniformly selected data points, respectively. The test accuracy measures the performance of each classifier learnt from inferred labels on a test dataset of size 100, which is uniformly selected without replacement after the selection of training data points. For label accuracies, we calculated the scores for each trial. For test accuracies, we uses aggregated results and calculated only once. The mean value from results of 10 annotators are shown in dashed lines and the standard deviation are shown by the shadow. The value from aggregated results are shown in solid lines. The
The proposed algorithm showed competitive performance to fully supervised learning without accessing explicit class labels at all.

![Figure 9: Performance on medoids.](image)

![Figure 10: Performance on uniformly selected data points.](image)

When increasing the number of training data, we observed the proposed algorithm could also show stable and promising generalization ability competitive with full supervision. However, the performance of the existing algorithm was not stable, because it separated data points into small bags, and queried a random subset of each bag for explicit class labels. With fewer training data, the size of each bag was small and it could query most of a bag for explicit class labels, thus achieved high labeling accuracy. However, with more training data, a reasonable budget restrained the size of the subset from each bag for querying explicit class labels, thus resulting the drop in performance.

Then we analysed the properties of pairs selected for $O_2$. Different from last paragraph, these pairs were selected by the algorithm ran on crowdsourcing results. We introduce another type of difficulty: pair difficulty for a pair of data points. We investigated the relationship between pair types and pair difficulties. The user evaluation of pair difficulties were 0.16 ($\pm$0.33), 0.17 ($\pm$0.10)
and 0.56 (±0.09), respectively. It matches the intuition that annotators confused when both images were difficult to classify.

Figure 11 shows the trajectories of actually queried ambiguity comparisons of 10 trials, indicating easy pairs by white and difficult pairs by orange. Note that each query consisted of a pair of images. Taking Trial 02 as an example, we observe that for the first query, an annotator found it easy to assign the explicit class label to one image and difficult for the other. This also holds for the second query. The same annotator then found it difficult to assign explicit class labels to both images in the third query. Another annotator found it easy to compare ambiguities than explicit labeling images in the first and second queries, and difficult for the third query. We can observe that more difficult pairs are queried on the latter half of the executions. This can be interpreted that the algorithm successfully separated difficult data from easy data at an early stage. Note that for the purpose of separating data by different difficulties, the results of ‘E’ pairs and ‘D’ pairs do not effect too much as the data points in these pairs have similar difficulty.

Figure 12 shows the corresponding histogram. As pair type becoming difficult, the proportion of pairs evaluated as difficult for ambiguity comparison increased as expected. Although blue areas are more preferable than orange areas, the proposed algorithm is not significantly influenced by the orange proportion of ‘D’ pairs.

User comments At the end of each questionnaire, we also asked annotators to answer their opinions on these tasks in free text. We select some of representative opinions and list their English translation.

The following list shows advantages of positivity comparisons over explicit labeling.

- It is easy to choose between “NA” or “WO” even if you can’t read the word.
- You can choose the one you can easily recognize.
- You can choose the letters by your feeling.

1The translation is based on the results of DeepL (https://www.deepl.com/translator).
Figure 12: Histogram of ambiguity comparison query types and difficulties.

- Unlike direct judgments, there is no clear correct answer, so it is possible to create questions that are easy for anyone to answer.
- When it’s not too curled up, it’s easy to choose.

The following list shows disadvantages of positivity comparisons over explicit labeling.
- If you cannot read either of them, your selection criteria will be blurred.
- It is hard to judge a flaw when it’s curled up.
- It is not sure if the decision is accurate.
- You need to stop and compare both images carefully, and may feel a great sense of hesitation before making a decision.
- Unlike direct judgement, there is no clear correct answer, and if neither letter is difficult to judge, you don’t have to think about the answer. You can make a good choice.

The following list shows advantages of ambiguity comparisons over explicit labeling.
- It’s easy to choose if you can read one or the other somehow.
- It’s quick and intuitive and I understand it quickly.
- Can be narrowed down if both are recognized as “NA” or “WO”.
It’s easy to imagine how easy it is to read by just the simple criterion of being able to read, and how easy it is to read by pronouncing it in your head.

It is highly flexible and does not have any restrictions.

The following list shows disadvantages of ambiguity comparisons over explicit labeling.

- You can only seem to read them, but you can’t tell whether you actually chose the correct answer or not.
- I don’t know if other people can quickly recognize.
- If the words are not read as “NA” or “WO”, I use the elimination method to select.
- When neither of them is likely to be readable, I tend to choose them at random.
- Unlike direct judgments, there is no clear correct answer, which makes it difficult to evaluate the competence of the annotator.

As we can see from above lists, it is difficult to choose when both images in a pair are not recognizable. This may affect the accuracy of the existing method, as it is required to sort the whole dataset. However, this does not significantly downgrade the performance of the proposed algorithm, as either one in the pair satisfying the desired ambiguity. Moreover, it is interesting to see the various criterion used by annotators.

5.3 Car preference task

The pairwise positivity oracle $O_1$ is extensively used in preference learning. Thus in this study, we used a car dataset [17] to simulate a binary classification using user preference, denoting car images a user likes as positive and those a user dislikes as negative. Note the true labels differ for each user, as different users may have different preferences for cars.

Goals We want to verify if the proposed comparison oracle is useful for binary classification of individual user preference.

Method In this user study, we conducted crowdsourcing in two ways.

- First, we collected user preference by five-stage evaluation. Stage one indices the user likes the car very much and stage five indices the opposite. This can be seen as different ranges of $p(y = 1|x)$ for a given image $x$, thus can be used to simulate both pairwise comparison oracles. For eliciting explicit labels, we considered the first two stages as positive.
• Second, we directly collected user feedback of two kinds of pairwise comparisons on all possible pairs for a fixed set of training data.

We used an interface that is similar to the one used in the first user study.

Data  The original dataset \cite{17} consists of 196 categories. We trained a ResNet18 \cite{15} model for classifying car categories to extract useful features. Based on extracted features, we greedily selected a single medoid for each class to collect 196 images. For the first crowdsourcing task on five-stage evaluation, we then uniformly selected 150 images. For the second task, we greedily selected 25 medoids based on extracted features for training and used the left 125 images for testing. We collected user feedback of all possible 300 pairs for both kinds of pairwise comparisons. All tasks are answered by four users. After inferring labels, we trained both a neural network classifier and a \(k\)-NN classifier for each setting.

Algorithm feasibility using simulated pairwise comparisons  Using five-stage evaluation to simulate pairwise comparisons, we had the freedom of choosing various sets of training data points. Thus, we conducted experiments with different sizes of training data points that are selected either uniformly or greedily as medoids using extracted features. The simulated feedback was noisy in the sense that when two images have the same stage evaluation, we can only randomly answer one with equal probability.

As shown in Figure 13, the proposed method using only simulated pairwise comparisons showed competitive performance to fully supervision. The performance of the existing method was not stable, because the quick sort subroutine is very sensitive to the results of pairwise comparison, which could be random in this case. However, the proposed algorithm showed consistent performance under the same situation.

User Comments  After a user finished answering all questions, we asked comments on the following open questions. The answers below are summaries of comments from four users.

Question 1: What are the characteristics of pairs that are easy for preference ambiguity comparison.

• When one of the car falls in the middle of like and dislike, or falls in a preferred category.

• When two cars are completely different from each other.

Question 1: What are the characteristics of pairs that are difficult for preference ambiguity comparison.

• When two cars have similar appearance or preference.

Question 3: What factors decide the difficulty of preference ambiguity comparison.
Figure 13: Test accuracies using simulated pairwise comparisons.
Question 4: What other items that preference ambiguity comparison may work?

- Food; plants; shoes; cloth; things that are unusual in daily life.

Question 5: What other measures other than preference ambiguity comparison may work?

- Fairness; measures that everyone is familiar with; measures that based on experience.

6 Conclusion

In this paper, we address the problem of interactive labeling and propose a novel ambiguity comparison oracle, followed by a noise-tolerant theoretical-guaranteed labeling algorithm without accessing explicit class labels at all. We then confirm the performance of the algorithm theoretically and empirically. For future work, eliminating $O(n)$ from one of the query complexity can improve the efficiency. On the other hand, extending the ambiguity comparison oracle to multiple data points and multiple classes is a promising direction.

Broader Impact

We believe this research will benefit researchers in all fields who are seeking for a more effective and less laborious annotation method for their unlabeled datasets. It can foster applications of machine learning by lowering the annotation barrier for people without specific professional knowledge. It can also benefit domain experts with professional knowledge by saving their time for more important tasks. Furthermore, collecting comparison information can potentially mitigate annotation biases of explicit labeling. It can also serve the aim of protecting privacy by not querying the explicit class labels in some cases.

For the negative side, it may harm the performance of downstream classification models when the comparison annotation is mostly incorrect. However, there would be no consequential ethical issues of failure of the method.

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A Proof of Theorem 1

Proof. The Algorithm 1 consists of two steps: selection of relatively ambiguous points and assigning labels by majority vote.

For the first step, the algorithm of Mohajer et al. [24] is executed using parameters $K = t$ and $m$. By adapting Theorem 1 of Mohajer et al. [24], we know that if $m \geq C_1 \max((\log \log n \log t) / (0.5 - \epsilon_2)^2)$, then the correct top-$t$ points can be identified with probability at least $1 - \log n^{-C_2}$.

For the second step, we analyze the probability that a point $x \in D \setminus D'$ is correctly inferred. Without loss of generality, we assume the correct label for $x$ is 1 and we calculate the probability that $\sum_{x_j \in D'} O_1(x, x_j) \geq 1 / 2$.

Let $Z_j \equiv O_1(x, x_j)$ denotes the random variable representing the outcome of every call to oracle $O_1$. Because $D'$ is assumed to be correctly identified, so $p(y|x) \geq p(y|x_j)$ for every $x_j \in D'$, thus the expectation of $Z_j$ is $1 - \epsilon_1$. Also note that $Z_j$ only takes a value of either 0 or 1, thus by applying Hoeffding’s inequality to $Z_1, Z_2, \ldots, Z_t$, we have

$$\Pr \left[ \frac{1}{t} \sum_{j=1}^{t} Z_j - (1 - \epsilon_1) \leq -(0.5 - \epsilon_1) \right] \leq \exp \left( -2t(0.5 - \epsilon_1)^2 \right). \quad (1)$$

This actually expresses the probability that $\frac{1}{t} \sum_{j=1}^{t} Z_j$ is smaller than $0.5$.

Let $a \equiv \exp \left( -2t(0.5 - \epsilon_1)^2 \right)$. Because $t$ is selected so that $a \leq \frac{1}{2}$ and $\frac{1}{t} \sum_{j=1}^{t} Z_j$ is bounded within $[0, 1]$, therefore for a single $x \in D \setminus D'$ it holds that

$$\Pr \left[ \frac{1}{t} \sum_{j=1}^{t} Z_j \geq \frac{1}{2} \right] \geq 1 - a$$

$$\geq \exp(-a(a + 1)). \quad (2)$$

For points in $D'$, because we assign random labels, there is positive probability that all assigned labels are wrong.

In conclusion, for all points in $D \setminus D'$ correctly labeled, the error rate $\epsilon = \frac{1}{n}$ can be achieved with probability at least $1 - \delta$ where $\delta \equiv 1 - (1 - \log n^{-C_2}) \exp(-a(a + 1)(n - t))$.

For query complexities, as $O_1$ is queried $t(n - t)$ times, the query complexity of $O_1$ is $O \left( \frac{n}{t^2} \right)$. Moreover, as indicated by Eq. (17) of [24], the query complexity of $O_2$ is $O \left( \frac{n \log \log n}{\epsilon^2} \right)$. \hfill \Box

B Proof of Theorem 2

Proof. First, we bound the difference between $\hat{f}(x; k)$ and $f(x)$. Similar to Reeve et al. [26], we define $\hat{f}(x; k) = \mathbb{E}_{p(y|x)} = \frac{1}{k} \sum_{q=1}^{k} y_{\tau_q(x)}$. 

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Then we have
\[ |\hat{f}(x; k) - f(x)| \leq |\hat{f}(x; k) - \hat{f}(x; k)| + |\hat{f}(x; k) - f(x)|. \] (4)

For the first term in RHS, from Theorem 1 we know it is bounded by \( \frac{2}{\epsilon} \) with probability at least \( 1 - \delta \). For the second term in right hand side, from Lemma 4.1 in [26], we have it is bounded by \( \omega \left( \frac{2k}{n} \right)^{\lambda} \) with probability at least \( 1 - \delta' \) for \( \delta' > 0 \) and \( \frac{n}{2} \geq k \geq 4 \log \left( \frac{1}{\delta'} \right) + 1 \). Thus combing the two inequalities, we have the left hand side is bounded by \( \Delta \triangleq \frac{2}{\epsilon} + \omega \left( \frac{2k}{n} \right)^{\lambda} \) with probability at least \( (1 - \delta)(1 - \delta') \). This means with at least the same probability, a randomly drawn point from \( \mathcal{X} \) will fall in the set
\[ \mathcal{X}' \triangleq \{ x \in \mathcal{X} : |\hat{y}(x) - \eta(x)| \leq \Delta \}. \]

Thus
\[
\begin{align*}
R(\hat{f}) - R(f^*) & = \int_{\mathcal{X}} |\eta(x) - \frac{1}{2} f(x)_{\neq f^*}(x) d\mu(x) \\
& = \int_{\mathcal{X}'} |\eta(x) - \frac{1}{2} f(x)_{\neq f^*}(x) d\mu(x) \text{ (with probability at least}(1 - \delta)(1 - \delta')) \\
& \leq \int_{\mathcal{X}} |\eta(x) - \frac{1}{2}| |\eta(x) - \frac{1}{2}| \leq \Delta d\mu(x) \\
& \leq C \Delta^{\alpha+1}.
\end{align*}
\]

\[ \Box \]

C Proof of Corollary 3

Proof. Similar to the approach in Xu et al. [32], we use induction to show that at the end of every step \( i \), \( \mathbb{E}_{P_x}[h(x) \neq h^*(x)] \leq 4\epsilon_i \) always holds with probability at least \( (1 - \delta)^{\log(\frac{1}{\epsilon})} \) for a universal \( \delta \), which is obvious for \( i = 0 \).

Then, with a little abusing of notations, we have
\[
| x \in S_i : h(x) \neq h^*(x) | = | x \in D_i : h(x) \neq h^*(x) | \\
\leq | x \in D_i : h(x) \neq \hat{y} | + | x \in D_i : h^*(x) \neq \hat{y} | \\
= 2\epsilon_i |S_i|.
\]

Thus \( \mathbb{P}_{x \sim S_i}[h(x) \neq h^*(x)] = \frac{|x \in S_i : h(x) \neq h^*(x)|}{|S_i|} \leq 2\epsilon_i \). Having \( c_0 \in (1, \infty) \) and \( \gamma \in (0, 1) \), using Lemma 3.1 from Hanneke et al. [14], we have \( \mathbb{P}_{x \sim P_x}[h(x) \neq h^*(x)] \leq 4\epsilon_i \) with probability at least \( 1 - \gamma \), providing \( c_0^{d \log \left( \frac{\log(\frac{1}{\epsilon})}{|S_i|} \right) + \log(\frac{1}{\epsilon})} \leq \epsilon_i \).

Setting \( \gamma = 1 - (1 - \delta)^{\log(2\epsilon)} \). We have \( \mathbb{P}_{P_x}[\hat{h}(x) \neq h^*(x)] \leq \epsilon \) with probability at least \( (1 - \delta)^{\log(\frac{1}{\epsilon})}(1 - \delta)^{\log(2\epsilon)} = 1 - \delta \) at the end of the algorithm. \[ \Box \]