Flutter speed estimation using presented differential quadrature method formulation

Mohammad Ghalandari, Shahaboddin Shamshirband, Amir Mosavi & Kwok-wing Chau

To cite this article: Mohammad Ghalandari, Shahaboddin Shamshirband, Amir Mosavi & Kwok-wing Chau (2019) Flutter speed estimation using presented differential quadrature method formulation, Engineering Applications of Computational Fluid Mechanics, 13:1, 804-810, DOI: 10.1080/19942060.2019.1627676

To link to this article: https://doi.org/10.1080/19942060.2019.1627676

© 2019 The Author(s). Published by Informa UK Limited, trading as Taylor & Francis Group

Published online: 29 Jul 2019.

Submit your article to this journal

Article views: 111

View related articles

View Crossmark data
Flutter speed estimation using presented differential quadrature method formulation

Mohammad Ghalandari\textsuperscript{a}, Shahaboddin Shamshirband\textsuperscript{b,c}, Amir Mosavi\textsuperscript{d,e} and Kwok-wing Chau\textsuperscript{f}

\textsuperscript{a}Department of Aerospace Engineering, Sharif University of Technology, Tehran, Iran; \textsuperscript{b}Department for Management of Science and Technology Development, Ton Duc Thang University, Ho Chi Minh City, Vietnam; \textsuperscript{c}Faculty of Information Technology, Ton Duc Thang University, Ho Chi Minh City, Vietnam; \textsuperscript{d}Institute of Automation, Kando Kalman Faculty of Electrical Engineering, Obuda University, Budapest, Hungary; \textsuperscript{e}School of the Built Environment, Oxford Brookes University, Oxford, UK; \textsuperscript{f}Department of Civil and Environmental Engineering, Hong Kong Polytechnic University, Hong Kong, China

\section*{ABSTRACT}
In this paper the flutter behavior of a typical wing is investigated. The study is performed by presented Deferential Quadrature Method (DQM). The aerodynamic part adopted Wagner functions to model subsonic regime. Quasi steady and unsteady aerodynamics are considered to estimate the instability speed of the structure. Based on the presented model, a code is developed, for an arbitrary typical section beam. The obtained results validated the existing methods in the literature. The proposed method provides the advantage of finding the modes of oscillation and other dynamic parameters with less than 0.2\% difference.

\section*{ARTICLE HISTORY}
Received 1 May 2019
Accepted 2 June 2019

\section*{KEYWORDS}
aero-elasticity; subsonic regime; deferential quadrature method (DQM); typical section beam; aerodynamic; wing design; airfoil; computational fluid dynamics (CFD); flutter speed prediction

\section*{1. Introduction}
The flutter behavior of coupled two- and three-dimensional systems have been investigated both numerically and analytically in a vast amount of research. Some simplifications have been conducted to approximate this type of instability condition. Estimation of aerodynamics behavior established steady, quasi-steady, and non-steady theories. Each of the aerodynamic theories has been presented in both time and frequency domains. In the time domain, Wagner (1925) introduced the indicial function to study the lift response of a 2D flat plate in incompressible flow. Jones (1960), using the Laplace transformation method, studied the dynamic behavior of an airplane. Sears (1940) applied this method to assess the non-uniform motion of airfoil. Later, perusing unsteady aerodynamics modeling was developed by such new techniques as Finite state induced flow models and reduced order model (ROM). The new type of finite state induced flow was suggested by Peters, Karunamoorthy, and Cao (1995). However, many authors (Behbahani-Nejad, Haddadpour, \& Esfahanian, 2005; Dowell, Hall, \& Romanowski, 1997; Garrick, 1939; Hall, 1994; Marzocca, Librescu, \& Chiochcia, 2001; Peters et al., 1995) introduced, developed, and used the time domain methods to describe flows about airfoils, cascades, and wings, but most of the aerodynamic theories were presented in the frequency domain. Theodorsen and Mutchler (1935) introduced this theory to describe the lift response of structures in unsteady flows. In this area, some semi-analytical schemes have been suggested to determine instability condition. The first method is called P method and is usually applied to the steady and quasi-steady models. Most of the unsteady aerodynamic theories could not be determined by this technique. The K method (Bisplinghoff, Ashley, \& Halfman, 2013; Hodges \& Pierce, 2011) is another approach to studying structural response in an unsteady flow. This method introduces artificial structural damping to the system and determines only the instability point. Another scheme in this region was introduced by Hassig (1971). His technique is based on the oscillatory aerodynamic force and generalized P method. This scheme, known as P-K method later, was modified by Rodden et al. (Rodden \& Johnson, 1994; Rodden, Harder, \& Bellinger, 1979). The P-P method (Haddadpour \& Firouz-Abadi, 2009) in the Laplace domain is another technique to investigate damping and frequency quantity of an aero-elastic system.

Among some numerical approaches (Ghalandari, Mirzadeh Koohshahi, Mohamadian, Shamshirband, &
of the system, $e$ is the elastic axis, $GJ$ is the torsional stiffness, $I_{ea}$ is the mass moment of inertia, and $L$ and $M$ are lift and moment of the system, respectively. So by consideration of the wings as the cantilever beam, the boundary condition can be presented as:

$$x = L, \left( \frac{\partial^2 w}{\partial x^2} = 0, \frac{\partial^3 w}{\partial x^3} = 0 \right), \left( \frac{\partial \theta}{\partial x} = 0, \frac{\partial^3 \theta}{\partial x^3} = 0 \right)$$

$$x = 0, \left( w = 0, \frac{\partial w}{\partial x} = 0 \right), \left( \theta = 0, \frac{\partial^2 \theta}{\partial x^2} = 0 \right)$$

### 2.2. Aerodynamic modeling

The aerodynamic forces in quasi-steady and non-steady subsonic flows modeled by Wagner function in Duhamel integral form. In this model, lift and moment distribution around the elastic axis can be represented as below:

$$L = \pi \rho_\infty b^2 (\dot{w} + U_\infty \dot{\theta} - ba \ddot{\theta}) + 2\pi \rho_\infty U_\infty b$$

$$\left[ (\dot{\theta} + U_\infty \theta + b \left( \frac{1}{2} - a \right) \dot{\theta} ) \varphi(t) + \int_0^t \varphi(t - \lambda) \left[ \dot{w} + U_\infty \dot{\theta} + b \left( \frac{1}{2} - a \right) \dot{\theta} \right] d\lambda \right]$$

$$M_{ea} = \pi \rho_\infty b^3 \left[ a \ddot{w} - U_\infty \left( \frac{1}{2} - a \right) \dot{\theta} + b \left( \frac{1}{8} + a^2 \right) \ddot{\theta} + 2\pi \rho_\infty U_\infty b^2 \left( \frac{1}{2} + a \right)$$

$$\left[ (\dot{\theta} + U_\infty \theta + b \left( \frac{1}{2} - a \right) \dot{\theta} ) \varphi(t) + \int_0^t \varphi(t - \lambda) \left[ \dot{w} + U_\infty \dot{\theta} + b \left( \frac{1}{2} - a \right) \dot{\theta} \right] d\lambda \right]$$

where $a$ is the elastic axis location, $b$ is the half cord of the wing, $\rho_\infty$ is the air density, $U_\infty$ is the air speed, and $\varphi(t)$ is the Wagner function. It is represented as:

$$\varphi(t) = 1 - \alpha_1 e^{\beta_1 t} - \alpha_2 e^{\beta_2 t}$$

The constants coefficients of the Wagner function have been approximated by Jones (1938) as follows:

$$\alpha_1 = 0.165, \alpha_2 = 0.335, \beta_1 = 0.0455(U_\infty/b), \beta_2 = 0.3(U_\infty/b)$$

Using the Laplace transformation method with zero initial condition and definition of the Wagner function, Equation (3) can be written as:

$$L = \pi \rho_\infty b^2 (S^2 \ddot{w} + U_\infty S \ddot{\theta} - ba S^2 \ddot{\theta})$$

$$+ 2\pi \rho_\infty U_\infty b [\tilde{W}_d - \tilde{C}_1(S) - \tilde{C}_2(S)]$$
\[ M_{ea} = \pi \rho_\infty b^3 \left( a^2 \omega - U_\infty \left( \frac{1}{2} - a \right) S\delta \right) \]
\[ + b \left( \frac{1}{8} + a^2 \right) S^2 \delta + 2\pi \rho_\infty U_\infty b^2 \left( \frac{1}{2} + a \right) \]
\[ \times \left[ \bar{W}_a - \bar{C}_1(S) - \bar{C}_2(S) \right] \quad (5) \]

where \( S \) is the non-dimensional Laplace variable, \( \bar{C}_1(S) = 0.165 S\bar{W}_a/S + 0.0455, \) \( \bar{C}_2(S) = 0.335 S\bar{W}_a/S + 0.35, \) and \( \bar{W}_a \) is the Laplace transformation of \( W_a = \dot{w} + U_\infty \theta - ba \dot{\theta} \). Using Laplace inverse transformation, Equation (5), \( \bar{C}_1(S) \) and \( \bar{C}_2(S) \) respectively can be obtained as:

\[ L = \pi \rho_\infty b^2 (\dot{w} + U_\infty \dot{\theta} - ba \dot{\theta}) + 2\pi \rho_\infty U_\infty \]
\[ b \left[ (\dot{w} + U_\infty \theta - ba \dot{\theta}) - C_1(t) - C_2(t) \right] \]
\[ M_{ea} = \pi \rho_\infty b^3 \left( a\omega - U_\infty \left( \frac{1}{2} - a \right) \dot{\theta} \right) \]
\[ + b \left( \frac{1}{8} + a^2 \right) \dot{\theta} + 2\pi \rho_\infty U_\infty b^2 \left( \frac{1}{2} + a \right) \]
\[ \times \left[ (\dot{w} + U_\infty \theta - ba \dot{\theta}) - C_1(t) - C_2(t) \right] \quad (6) \]

and

\[ \bar{C}_1(t) + 0.0455 C_1(t) = 0.165 W_a \]
\[ \bar{C}_2(t) + 0.35 C_2(t) = 0.335 W_a \quad (7) \]

So lift and moment of the coupled system can be represented in the matrix form as follows:

\[ \begin{bmatrix} L \\ M \end{bmatrix} = [M_a] \begin{bmatrix} \dot{w} \\ \dot{\theta} \end{bmatrix} + [C_a] \begin{bmatrix} w \\ \theta \end{bmatrix} + [K_a] \begin{bmatrix} \dot{w} \\ \dot{\theta} \end{bmatrix} + [A_a] \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \quad (8) \]

where \( M_a, C_a, \) and \( K_a \) are aerodynamic mass, damping and stiffness matrix and introduced as:

\[ M_a = \pi \rho_\infty b^2 \begin{bmatrix} 1 \\ ba \\ b^2 \left( \frac{1}{8} + a^2 \right) \end{bmatrix} \]
\[ K_a = 2\pi \rho_\infty U_\infty ^2 b \begin{bmatrix} 0 \\ 0 \\ b \left( \frac{1}{2} + a \right) \end{bmatrix} \]
\[ C_a = 2\pi \rho_\infty b U_\infty \begin{bmatrix} 1 \\ b \left( \frac{1}{2} + a \right) \\ -ba \left( \frac{1}{4} + a^2 \right) \end{bmatrix} \]
\[ A_a = -2\pi \rho_\infty b U_\infty \begin{bmatrix} 1 \\ b \left( \frac{1}{2} + a \right) \\ b \left( \frac{1}{2} + a \right) \end{bmatrix} \quad (9) \]

And also the matrix form of Equation (7) as:

\[ [I] \begin{bmatrix} \bar{C}_1(t) \\ \bar{C}_2(t) \end{bmatrix} + [B] \begin{bmatrix} C_1(t) \\ C_2(t) \end{bmatrix} = [D_1] \begin{bmatrix} \dot{w} \\ \dot{\theta} \end{bmatrix} + [D_2] \begin{bmatrix} w \\ \theta \end{bmatrix} \quad (10) \]

where \( B, D_1, \) and \( D_2 \) signified as:

\[ D_2 = U_\infty \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]
\[ D_1 = \begin{bmatrix} 0.165 & 0 \\ 0.335 & -0.165ba \end{bmatrix} \]
\[ B = \begin{bmatrix} 0.0455 & 0 \\ 0 & 0.335 \end{bmatrix} \]

### 2.3. DQM and aero-elastic equation

As mentioned earlier, aero-elastic formulation of the wing is estimated by the following equations:

\[ EL \frac{\partial^4 w}{\partial x^4} + m \frac{\partial^2 w}{\partial t^2} + me \frac{\partial^2 \theta}{\partial t^2} = L \]
\[ -GJ \frac{\partial^2 \theta}{\partial x^2} + me \frac{\partial^2 w}{\partial x^2} + I_{ea} \frac{\partial^2 \theta}{\partial t^2} = M_{ea} \quad (11) \]

By the implementing of a differential equation in the domain of the solution, Equation (12) is characterized as follows:

\[ EL[I] \dddot{w}_d + m[I] \ddot{w}_d + me [I] \dddot{\theta}_d = \dddot{\bar{L}} \]
\[ -GJ[I] \dddot{\theta}_d + me[I] \ddot{\theta}_d + I_{ea}[I] \dddot{\theta}_d = \dddot{\bar{M}}_{ea} \quad (12) \]

Using the DQM technique Equation (13) is represented as:

\[ EL[K_{b_d}] \dddot{w}_d + m[I] \ddot{w}_d + me [I] \dddot{\theta}_d = \dddot{\bar{L}} \]
\[ -GJ[K_{\theta_d}] \dddot{\theta}_d + me[I] \ddot{\theta}_d + I_{ea}[I] \dddot{\theta}_d = \dddot{\bar{M}}_{ea} \quad (13) \]

So the stiffness matrix can be divided into the boundary and internal domain:

\[ EL([K_{b_d}]) \dddot{w}_d + m[I] \ddot{w}_d + me [I] \dddot{\theta}_d = \dddot{\bar{L}} \]
\[ -GJ([K_{\theta_d}]) \dddot{\theta}_d + me[I] \ddot{\theta}_d + I_{ea}[I] \dddot{\theta}_d = \dddot{\bar{M}}_{ea} \quad (14) \]

By enforcing boundary condition on \( (1,2,\ldots,N-1,N) \) points, the stiffness matrix on the boundary takes the following form:

\[ [K_b] \dddot{w}_b = 0 \]
\[ [K_{\theta_b}] \dddot{\theta}_b = 0 \quad (15) \]

Using the stiffness matrix which related to the boundary points, the relation between the boundary and domain
The matrix form of aero-elastic equation can be shown as follows:

\[
\begin{bmatrix}
K_{BB} & K_{Ba} \\
K_{Ta} & K_{dd}
\end{bmatrix}
\begin{bmatrix}
\ddot{w}_b \\
\ddot{\theta}_d
\end{bmatrix} = 0 \Rightarrow \ddot{w}_b = -((K_{BB})^{-1}[K_{Ba}])\ddot{w}_d
\]

\[
\begin{bmatrix}
K_{TB} & K_{Tb} \\
K_{Tb} & KTbb
\end{bmatrix}
\begin{bmatrix}
\ddot{\theta}_b \\
\ddot{\theta}_d
\end{bmatrix} = 0 \Rightarrow \ddot{\theta}_b = -((K_{Tb})^{-1}[K_{Tbb}])\ddot{\theta}_d
\]

(16)

So the DQM form of aero-elastic equation is presented as:

\[
[K_B] \ddot{w}_d + m[I] \ddot{w}_d + me[I] \ddot{\theta}_d = \ddot{L}
\]

\[
[K_T] \ddot{\theta}_d + me[I] \ddot{w}_d + I_{ea} [I] \ddot{\theta}_d = \ddot{M}_e
\]

(17)

Where \(K_B\) and \(K_T\) are:

\[
[K_B] = EI([K_{BB}] - [K_{Ba}][K_{Ba}]^{-1}[K_{Ba}])
\]

\[
[K_T] = -GJ([K_{Tb}] - [K_{Tb}][K_{Tb}]^{-1}[K_{Tb}])
\]

So the structural section of the aero-elastic equation can be obtained as:

\[
M_s \ddot{q} + C_s q + K_s q = f
\]

(18)

Where

\[
\begin{bmatrix}
\ddot{w} \\
\ddot{\theta}
\end{bmatrix} = \begin{bmatrix} m[I] & me[I] \end{bmatrix}
\]

\[
C_s = \begin{bmatrix} [0] & [0] \\ [0] & [0] \end{bmatrix} ;
K_s = \begin{bmatrix} [K_{Ba}] & [0] \\ [0] & [K_{Tb}] \end{bmatrix}
\]

Also, the DQM form of aerodynamic model can be written as follows:

\[
[I] \ddot{B}_1 + \left( \beta_1 \frac{U_{\infty}}{b} \right) [I]B_1 = [I] \ddot{w} - U_{\infty} [I] \theta + ba[I] \dot{\theta}
\]

\[- \frac{b}{2} \left( \frac{C_{Ia}}{\pi} - 1 \right) [I] \ddot{\theta}
\]

\[
[I] \ddot{B}_2 + \left( \beta_2 \frac{U_{\infty}}{b} \right) [I]B_2 = [I] \ddot{w} - U_{\infty} [I] \theta + ba[I] \dot{\theta}
\]

\[- \frac{b}{2} \left( \frac{C_{Ia}}{\pi} - 1 \right) [I] \dot{\theta}
\]

(19)

The matrix form of aero-elastic equation can be shown as:

\[
I \ddot{d} + E \dot{d} = D_1 q + D_2 q
\]

(20)

where

\[
I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} E = \frac{U_{\infty}}{b} \begin{bmatrix} \beta_1, 1 \\ 0 & \beta_2, 1 \end{bmatrix}
\]

\[
D_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{ba - \frac{b}{2} \left( \frac{C_{Ia}}{\pi} - 1 \right)}{1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\]

\[
D_2 = -U_{\infty} \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}
\]

3. Result and discussion

In this section, the verifications of the introduced coupled formulation are carried out for two test cases. The first test case is a clamped free typical airfoil and the second

Figure 1. Two-dimensional typical section airfoil.

Table 1. Parameters of two-dimensional airfoils (Haddadpour & Firouz-Abadi, 2009).

| PARAMETER Description | Case 1 |
|----------------------|--------|
| \(\sigma = \omega_{bt}/\omega_0\) | Bending to torsion frequencies ratio | 0.4 |
| \(A\) | Elastic axis position on the airfoil | -0.2 |
| \(x_a\) | Center of mass | .1 |
| \(\mu = m/\rho b^2\) | Mass ratio | 20 |
| \(r_a = l_a/mb^2\) | Radius of gyration | 0.24 |

Figure 2. Real part variation at subsonic speed.

Table 2. The variation of flutter speed vs. number of nodes.

| Number of nodes | 8   | 20  | 25  |
|-----------------|-----|-----|-----|
| Non-dimensional flutter speed | 1.8 | 1.98 | 1.985 |


Table 3. Given wing model data.

| Parameters     | Unit and description          | Value     |
|----------------|------------------------------|-----------|
| \( \rho_{\text{Sea Level}} \) | Density (sea level)Slug/ft\(^3\) | 0.002378  |
| \( \rho_{20 \text{Kft}} \)    | Density (20Kft)Slug/ft\(^3\)  | 0.001267  |
| \( E \)                      | Flexural stiffness (lb.ft)    | 23.6 \times 10^6 |
| \( G \)                      | Torsional stiffness (lb.ft)   | 2.39 \times 10^6  |
| \( I_{\text{ea}} \)          | Slug/ft\(^4\)/ft             | 1.943     |
| \( L \)                      | Wing span(ft)                | 20        |
| \( c = 2b \)                 | Cord (ft)                    | 6         |
| \( a \)                      | Elastic axis location (ft)    | \(-1/3\) |
| \( x_0 \)                    | Center of Mass               | 0.1997    |
| \( m \)                      | Mass per length (Slug/ft)     | 0.746     |

The study of the DQM result which is extracted by different nodes for frequencies shows that the instability speed approximations are very close to the other models (Table 2).

Investigations of the slender body are also performed for quasi-steady and non-steady aerodynamic models by the DQM method with the following specification (Table 3). The Results are extracted like previous examples: by P method at both sea level and 20,000 ft (Figures 3–6).

The flutter speeds of the given airfoil are estimated by 20 nodes for two aerodynamic models (Table 4). The results show minimum slight differences compared to the literature. In the following figures, in which are illustrated the results of flutter speed estimation, the \( U \), \( \omega \), and \( \zeta \) are air speed, frequency, and damping respectively.

Figure 3. (a) Frequency; and (b) damping part of the aero-elastic system for quasi-steady aerodynamic model at sea level.

Figure 4. (a) Frequency; and (b) damping part of the aero-elastic system for quasi-steady aerodynamic model at 20,000 ft.
Figure 5. (a) Frequency; and (b) damping part of the aero-elastic system for non-steady aerodynamic model at sea level.

Figure 6. (a) Frequency; and (b) damping part of the aero-elastic system for non-steady aerodynamic model at 20,000 ft.

Table 4. Comparison between presented models for aerodynamic model vs. literature.

| Unsteady | Quasi-steady | Unsteady | Quasi-steady |
|----------|--------------|----------|--------------|
| At sea level | At 20 Kft | At sea level | At 20 Kft |
| Present | 446 | 446 | 573 | 573 |
| Haddadpour & Firouz-Abadi (2009) | 447 | 447 | 574 | 574 |

4. Conclusion

In this study, a DQM model for the flutter speed prediction of a simplified airfoil (typical section) was developed. Using unsteady and quasi-steady Wagner function the instability condition of airfoil was studied. Comparison between the presented model and PP, Theodorsen, and also Peter’s aerodynamic methods for both a typical section airfoil and Goland wing for sea level and 20,000 ft above was conducted. The good agreements between the results, which are extracted by only 20 nodes and simplification of governing equations, show the tools powerful, with minimum 0.17% difference in aero-elastic instability estimation. Future studies, besides overcoming challenges which emerged from instability predicting differences especially in quasi-steady models, could be focused on the nonlinearity effect of aerodynamics beside the large deflection of structures in flutter speed approximation using a DQM approach.

Disclosure statement

No potential conflict of interest was reported by the authors.

References

Behbahani-Nejad, M., Haddadpour, H., & Esfahanian, V. (2005). Reduced order modeling of unsteady flows without static correction requirement. Journal of Aircraft, 42(4), 882–886.

Bellman, R., & Casti, J. (1971). Differential quadrature and long-term integration. Journal of Mathematical Analysis and Applications, 34(2), 235–238.

Bisplinghoff, R. L., Ashley, H., & Halfman, R. L. (2013). Aerelasticity. Mineola, New York, USA: Courier Corporation.

Dowell, E. H., Hall, K. C., & Romanowski, M. C. (1997). Eigenmode analysis in unsteady aerodynamics: Reduced order models. Applied Mechanics Reviews, 50(6), 371–386.

Garrick, I. E. (1939). On some Fourier transforms in the theory of non-stationary flows. Proceedings of the Fifth International Congress for Applied Mechanics, 590–593.
Ghalandari, M., Mirzadeh Koohshahi, E., Mohamadian, F., Shamshirband, S., & Chau, K. W. (2019). Numerical simulation of nanofluid flow inside a root canal. *Engineering Applications of Computational Fluid Mechanics, 13*(1), 254–264. doi:10.1080/19942060.2019.1578696.

Haddadpour, H., & Firouz-Abadi, R. D. (2009). True damping and frequency prediction for aeroelastic systems: The PP method. *Journal of Fluids and Structures, 25*(7), 1177–1188.

Hall, K. C. (1994). Eigenanalysis of unsteady flows about airfoils, cascades, and wings. *AIAA Journal, 32*(12), 2426–2432.

Hassan, M. T., & Nassar, M. (2015). Analysis of stressed Timoshenko beams on two parameter foundations. *KSCE Journal of Civil Engineering, 19*(1), 173–179.

Hassig, H. J. (1971). An approximate true damping solution of the flutter equation by determinantal iteration. *Journal of Aircraft, 8*(11), 885–889.

Hodges, D. H., & Pierce, G. A. (2011). *Introduction to structural dynamics and aeroelasticity* (Vol. 15). Cambridge, UK: Cambridge University Press.

Jang, S. K., Bert, C. W., & Striz, A. G. (1989). Application of differential quadrature to static analysis of structural components. *International Journal for Numerical Methods in Engineering, 28*(3), 561–577.

Jones, R. T. (1938). *Operational treatment of the nonuniform-lift theory in airplane dynamics*. Langley Field, VA, United States: NASA.

Jones, W. P. (1960). The potential theory of unsteady supersonic flow. By JW M ILES. Cambridge University Press, 1959. 220 pp. 45s. *Journal of Fluid Mechanics, 7*(2), 319–320.

Kang, K.-J., & Kim, B.-S. (2002). In-plane extensional vibration analysis of curved beams using DQM. *Journal of the Korean Society of Safety, 17*(1), 99–104.

Malik, M., & Bert, C. W. (1998). Three-dimensional elasticity solutions for free vibrations of rectangular plates by the differential quadrature method. *International Journal of Solids and Structures, 35*(3–4), 299–318.

Marzocca, P., Librescu, L., & Chiocchia, G. (2001). Aeroelastic response of 2-D lifting surfaces to gust and arbitrary explosive loading signatures. *International Journal of Impact Engineering, 25*(1), 41–65.

Mou, B., He, B. J., Zhao, D. X., & Chau, K. W. (2017). Numerical simulation of the effects of building dimensional variation on wind pressure distribution. *Engineering Applications of Computational Fluid Mechanics, 11*(1), 293–309. doi:10.1080/19942060.2017.1281845.

Peters, D. A., Karunamoorthy, S., & Cao, W.-M. (1995). Finite state induced flow models. I – Two-dimensional thin airfoil. *Journal of Aircraft, 32*(2), 313–322.

Rodden, W. P., Harder, R. L., & Bellinger, E. D. (1979). *Aeroelastic addition to NASTRAN* (Vol. 3094). National Aeronautics and Space Administration, Scientific and Technical . . .

Rodden, W. P., & Johnson, E. H. (1994). User’s guide of MSC/NASTRAN aeroelastic analysis. MSC/NASTRAN V68.

Sears, W. R. (1940). Operational methods in the theory of airfoils in non-uniform motion. *Journal of the Franklin Institute, 230*(1), 95–111.

Striz, A. G., Jang, S. K., & Bert, C. W. (1988). Nonlinear bending analysis of thin circular plates by differential quadrature. *Thin-Walled Structures, 6*(1), 51–62.

Theodorsen, T., & Mutchler, W. H. (1935). *General theory of aerodynamic instability and the mechanism of flutter*. Washington, D.C, USA: National Advisory Committee for Aeronautics.

Wagner, H. (1925). Über die Entstehung des dynamischen Auftriebes von Tragflügeln. *ZAMM - Zeitschrift für Angewandte Mathematik und Mechanik, 5*(1), 17–35.