A probabilistic evaluation of the argument of correction in affirmative action

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Abstract

Objective: The issue of affirmative action is a major point of discussion in distributive justice and allocation of scarce resources such as job-offerings or college seats. Philosophical arguments around this focus on diversity, historical justice and correction. The objective of this study is to examine the argument of correction in relation to affirmative action, which states that affirmative action helps correct biases in metrics used to measure merit. Method: We develop a stochastic formulation of the argument of correction and analyse it using a probability-theory approach. Result: We find with the help of two counter-examples that the argument of correction is not generally valid, when we admit the stochastic nature of the problem. Conclusion: Though the argument of correction may be valid in some special cases, its general lack of validity presents a major challenge to this dominant argument widely used in the literature.

Keywords: Affirmative action; distributive Justice; allocation of resources

1 Introduction

Discrimination in the midst of diversity has been a serious concern across multi-racial, multi-ethnic and multi-religious countries and institutions. With the consistent and prolonged seclusion of deprived and socially stigmatised and stereotyped groups from institutes of quality education and fair employment, there appears to be validity in the cause of ‘affirmative action’—which Muller described as the practice of giving preferential treatment to racial minorities in educational and employment opportunities, based on the premise that there is a need to raise the socioeconomic status of these underprivileged races in spheres where they remain underrepresented, as opposed to privileged races. Regardless of the racial emphasis, we would hold this definition to be true for all forms of affirmative action, including those where the criterion is not racial, such as caste, gender or economic status. In India for instance, the primary criterion for ‘reservation’—an affirmative action policy that reserves a certain number of seats in educational institutes and certain forms of employment, is based on caste. The constitution of India, specifically mentions special arrangements for such castes. Inspired by the US policies of affirmative action, France too has initiates policies, but in a much smaller scale. Multi-criteria affirmative action also exists, as can be seen in some schools in Israel, where the residential area, parental education etc. were chosen as the basis for the reservation of seats. The private sector too can be
included in the discussion. For instance, in a recent study, Khora has studied its effects, in light of the argument that the "private sector cannot discriminate because if it does, it is at its own peril"[8]. There is of course, serious contestation regarding the issue too, a major concern being that the principle of merit is violated in the pursuit of equality.

The idea of affirmative action has drawn scholarly debate for long. Extensive studies have been conducted in both empirical and theoretical spheres, ranging from finding patterns and explicit recording of history to decision-theoretic modelling.

An impressive body of literature explores the difference in perspective of the privileged and the underprivileged regarding affirmative action, difference in abilities and productivity etc. Kluegel and Smith[7] for instance, empirically found that whites in the US systematically understate the discrimination and difficulties of the blacks. This we believe can even perhaps be made more serious by affirmative action. Coate and Loury[8], through the use of a model of statistical-discrimination concluded, that even if the productivity of two communities are identical, and even in the presence of affirmative action, employers may end up believing in a productivity differential. With regard to the actual effects on efficiency, Holzer and Neumark[9] and Estevan and Morin[10] found very little evidence for any fall in efficiency due to affirmative action. Recently, Li and Qui have also found evidence that the family background affects educational attainments in China[11]. Studying judicial rulings related to affirmative action from 1994 to 2004, Blume and Long concluded that affirmative action declined significantly after referenda or administrative decision in the US[12]. In 2016, Frisancho and Krishna made a study of the effects of affirmative action in one of the leading engineering institutions of India. They found that Scheduled Caste and Scheduled Tribe students tend to fall behind their same-major peers as the semesters progress. Interestingly, they also find that minority students earn less than what they would have earned had they chosen less selective majors[13]. A glance at such empirical literature reveals how opinions and perceptions diverge from empirical facts, which are often complicated, and that such divergence is contingent on the location of a person on the socio-economic map. It is pertinent to scrutinise theoretical arguments in a careful manner in this context, as bias may enter otherwise fine-looking interlocutions.

From a theoretical standpoint, scholars have argued the issue in the light of distributive justice and ethics, as well as decision-theoretic frameworks. The proponents of affirmative action build their arguments using three broad characterisations—

1. Diversification—Affirmative action allows students and employees of such communities to enter, who would have otherwise not been able to enter. This leads to the diversity of viewpoints, experiences and perspectives, leading to richer intellectual and work discourse, at the same time, exposing people to different communities and enhancing social harmony.
2. Compensation—Affirmative action compensates for historical injustice. Members of underprivileged communities may lack resources and perhaps even productivity, not because of inherent problems, but because of past exclusion from education and opportunities. The society should thus provide them better chance than prosperous communities, to allow them to reach the level of achievement of the prosperous communities.
3. Correction—This argument bears a relation to the meritocratic principle. Suppose we seek to reward those who have worked harder or more efficiently. A way to gauge this merit would be to use a metric, such as marks attained in a test. The test-scores are not affected merely by hard-work, but they are also affected by the opportunities received by the candidate. Thus, to attain the same scores as that of a privileged candidate, a candidate from an underprivileged community would probably have had to work harder. To correct for this bias in the measurement metric, special provisions favouring candidates from marginalised communities must be made.

These three classes of arguments dominate the philosophical discourse surrounding affirmative action. Each argument has been attacked and defended using various methods, but they stand to debate even today. The arguments have been documented and discussed in numerous works, such as in Tierney[14], Crosby[15] and Sandler[16]. A recent contribution to this debate can be found in Chukwuma[17], claiming that past wrongs cannot be corrected with present wrongs and that affirmative action leads to "mediocrity". Interestingly, it argues that affirmative action makes the oppressed perpetual victims of injustice by telling them that they are unable to succeed without help. Premdas also comments that affirmative action is “alien” to western democracy, as western society is based on individual identity and merit[18]. A recent contribution in defence of affirmative action is Brown, where the policies are defended both principally and consequentially as long as the victim is "educationally robbed”[19].

In this paper, we seek to examine the argument of correction in a mathematically rigorous manner, and comment on its validity. While the other classes of arguments may be debatable too, we abstain from commenting on them. Thus, the purpose of the current paper is neither to defend affirmative action nor to refute it, as not all aspects of it are debated. We solely seek to understand and scrutinize the argument of correction for a more rigorous and informed look into this very crucial issue of affirmative action.

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The argument of correction and a stochastic formulation

A proper understanding of the argument of correction requires at least a brief exploration of the principle of merit. A common conceptualisation of meritocracy includes the definition of merit in terms of tested competency and ability. It is often alleged by the opponents of affirmative action that it hampers meritocracy. The same argument is also found in the book “Liberal Education: The Politics of Race and Sex on Campus” by D’Souza and Goldner. It claims that affirmative action fails to achieve equality, and also dilutes merit in educational institutes. The argument of correction, in a form, is a way to answer this allegation. Proponents of the argument of correction, including Tierney and Walton et al. may agree that the more able or hard-working individual must be preferred, but argue that common methods of gauging ability, such as standardized tests are biased against the deprived people. It is because privileged people may score more not just because of working harder or due to higher ability, but simply because of having better resources and support, at least on average. This essentially entails that if two candidates receive exactly the same score, then the one with lesser resources has a higher probability of working harder or having more ability. This of course is not always the case, as there are a lot of other factors involved apart from the ones taken into account when affirmative action policies are devised, but this is thought to be true on average. For example, suppose candidate A comes from a rich household and B comes from a poor household and both receive the same scores. While candidate B is not guaranteed to have worked harder, the probability that B has worked harder, it is argued, must be higher than the probability that A has worked harder. And because these are complementary events, it simply means that the probability that candidate B has worked harder should be more than 0.5. The reason cited could be that because of better resources, rich people who work as hard as some poor people would receive a higher score on average. Thus, while it is impossible to always pick the candidate who has worked harder or is more able (i.e. more “meritorious”), if we always pick the one who had less resources, we would correctly choose the more meritorious student more than 50 per cent of the time, and less than 50 per cent of the time if choose the privileged student instead. Thus, affirmative action, according to this argument, does not hamper meritocracy, but facilitates it. Let us state this argument, first in verbal logical terms and then in formal mathematical terms to scrutinise it—

Argument

Premise 1: On average, candidates who are from well-off backgrounds score higher.

Premise 2: On average, candidates who work harder/ are efficient score higher.

Premise 3: Two candidates have attained the same score.

Conclusion: The probability that the candidate from the more well-off background has worked harder is less than the probability that the candidate from the less well-off background has worked harder.

To state the argument formally, let us introduce some notation. We consider two candidates, 1 and 2 (this is not an assumption and does not lead to any loss in generality as multiple binary comparisons may be made to account for the case with 2+ candidates). Let e1, r1 be the “effort” and “support from background” of candidate 1 and e2, r2 be the “effort” and “support from background” of candidate 2. Let m1, m2 be the scores of candidate 1 and 2 respectively. Both higher “effort” and higher “support from background” mean a higher score on average.

Let, the difference in efforts be e = e1 – e2 and the difference in support from backgrounds be r = r1 – r2. Let the difference in scores be m = m1 – m2. Because both higher efforts and higher support from background mean higher scores on average, a higher difference in efforts and a higher difference in supports from background essentially mean a higher difference in marks. Accordingly, we have the following relation, where f is an increasing function in both e and r, in Equation (1)—

\[ E(m) = f(e, r) \]

The correction argument can then be stated mathematically as (Equation (2))—

\[ E(m) = f(e, r) \Rightarrow P(\text{er} \geq 0 | m = 0) < 0.5 \]

Here, \( P(\text{er} \geq 0 | m = 0) \) is the probability if two candidates have scored the same, that the candidate who has a better background (or is privileged) is also the one who has worked harder. This is because, if candidate 1 is the more privileged student, then r > 0, in that case er ≥ 0 if and only if e ≥ 0 too. Similarly, if candidate 1 is the underprivileged one, then r < 0 and in that case, er ≥ 0 if and only if e ≤ 0 too. The correction argument entails that this probability should be less than 0.5, as expected difference in scores is an increasing function of both e and r.

Examining the argument

As we have mathematically stated the correction-argument now, let us proceed to an examination of it. In this examination, in line with the principle of justice, we presuppose no relation between e and r a-priori and also consider all possibilities to be equally likely in the beginning. Accordingly, let e1, e2, r1, r2 have independent uniform distributions from -1 (the lowest
possible) to 1 (the highest possibly). Because $f$ has not been specified, this does not lead to any loss in generality. Accordingly, a-priori, $e$ and $r$, as the differences of uniform and independent variables, have triangular distributions from -2 to 2 with a peak at 0. Because of this triangular distribution, we have,

$$ P(e \geq 0) = P(r \geq 0) = \frac{1}{2} $$

$$ P(e \geq 0, r \geq 0) = P(e < 0, r < 0) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} $$

Therefore, we have,

$$ P(er \geq 0) = P(e \geq 0, r \geq 0) + P(e < 0, r < 0) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} $$

Now, we have,

$$ P(er \geq 0 \mid m = 0) = \frac{p(m = 0 \mid er \geq 0)P(er \geq 0)}{p(m = 0)} $$

$$ \Rightarrow P(er \geq 0 \mid m = 0) = \frac{p(m = 0 \mid er \geq 0)}{2p(m = 0)} \quad (\text{Since, } P(er \geq 0) = \frac{1}{2}) $$

To extract further meaning from the expression, let us manipulate the numerator and the denominator of the expression above separately. For the numerator we have,

$$ p(m = 0 \mid er \geq 0) $$

$$ = \frac{p(m = 0 \text{ and } er \geq 0)}{P(er \geq 0)} $$

$$ = \frac{p(m = 0 \text{ and } [(e \geq 0, r \geq 0) \text{ or } (e < 0, r < 0)])}{1/2} $$

$$ = \frac{p(m = 0, e \geq 0, r \geq 0) + p(m = 0, e < 0, r < 0)}{1/2} $$

Again, we can further manipulate the two terms in the numerator to extract further meaning. For the first term—

$$ p(m = 0, e \geq 0, r \geq 0) $$

$$ = \int_{0}^{2} \int_{0}^{2} p(m = 0, e = x, r = y) dx dy $$

$$ = \int_{0}^{2} \int_{0}^{2} p(m = 0, e = x, r = y) p(e = x, r = y) dx dy $$
\[
= \frac{1}{4} \int_{-2}^{2} \int_{-2}^{2} p(m = 0) e = x, r = y) \, dx \, dy
\]

Similarly, for the second term, performing similar manipulations we have,
\[
p(m = 0, e < 0, r < 0) = \frac{1}{4} \int_{-2}^{2} \int_{-2}^{2} p(m = 0) e = x, r = y) \, dx \, dy
\]

Putting these expressions in the numerator, we get the value of \(p(m = 0) e = x, r = y)\) to be,
\[
\frac{1}{2} \left[ \int_{0}^{2} \int_{0}^{2} p(m = 0) e = x, r = y) \, dx \, dy + \int_{-2}^{0} \int_{-2}^{0} p(m = 0) e = x, r = y) \, dx \, dy \right]
\]

Similarly, we also manipulate the denominator as follows—
\[
2p(m = 0)
\]
\[
= 2 \int_{-2}^{2} \int_{-2}^{2} p(m = 0, e = x, r = y) \, dx \, dy
\]
\[
= 2 \int_{-2}^{2} \int_{-2}^{2} p(m = 0) e = x, r = y) \, dx \, dy
\]
\[
= \frac{1}{2} \int_{-2}^{2} \int_{-2}^{2} p(m = 0) e = x, r = y) \, dx \, dy
\]

Thus, we have attained elaborate expressions for both the numerator and the denominator. We can now use these expressions to get the following expression—
\[
P(\text{er} \geq 0 \mid m = 0) = \frac{\int_{0}^{2} \int_{0}^{2} p(m = 0) e = x, r = y) \, dx \, dy + \int_{-2}^{0} \int_{-2}^{0} p(m = 0) e = x, r = y) \, dx \, dy}{\int_{-2}^{2} \int_{-2}^{2} p(m = 0) e = x, r = y) \, dx \, dy}
\]

Informally, the equation simply states that if two candidates have scored the same, then the probability that the privileged candidate is also the one who worked harder is simply a measure of the probability density of both the candidates receiving the same score (indicated by \(m = 0\)), when either both \(e\) and \(r\) are negative (meaning candidate 1 is both the less privileged and less hard-working) or when both \(e\) and \(r\) are non-negative (meaning candidate 1 is both more privileged and more hardworking) compared to the total probability density of receiving the same score.

The argument of correction, as discussed earlier, claims that this expression always has a value less than 0.5. For the sake of clarity, we reiterate, that this does not mean that it is always the case that the underprivileged one has worked harder, the correction argument simply argues that more than half of the time (that is with a probability higher than 0.5) the underprivileged candidate is the one who has worked harder. The premise of this argument, as stated earlier, is that on average, the privileged candidates tend to score higher even when the level of hard-work is the same. While this verbally appears to be a sound argument, as we have derived an explicit mathematical expression for the probability that it is not the case, we find that the
argument is not true in general. We do this, simply though the means of two counter-examples. We use the expression above to calculate the probabilities and report the probability of the more privileged candidate also being the one who has worked harder is less than 0.5. Thus, our demonstration of correction claims that this value is always less than 0.5, that is, if two candidates have scored the same, then the probability that the more privileged candidate is also the one who has worked harder is still greater than 0.5, contradicting what the argument of correction claims.

**Counter-examples to the correction argument**

Since the argument of correction is an implication, to disprove it, we need to find cases where the premise is true but the conclusion is false, i.e. we need to find cases where the expected difference in scores is an increasing function of both the difference in efforts and the difference in backgrounds, but the probability that the more privileged candidate out of two equal scoring candidates has worked harder is still greater than 0.5, contradicting what the argument of correction claims.

**Example 1:** Let us consider the following possible distribution for $m$:

$$p(m = a | e = x, r = y) = \frac{3}{16} (a - e - r)^2 \text{ when } x + y - 2 \leq a \leq x + y + 2$$

$$p(m = a | e = x, r = y) = 0 \text{ otherwise}$$

It can be verified that the expected value of this distribution is $x + y$, and thus the expected difference in marks is a strictly increasing function of both efforts/hard work and institutional support. Thus, the premise of the correction argument is true in case of this distribution.

To check whether the conclusion is true, we need to evaluate $p(er \geq 0 | m \geq 0)$ using the expression derived in the previous section. The following is the form of the expression in case of this distribution—

$$p(er \geq 0 | m \geq 0) = \frac{\int_0^2 \int_0^{x+y} x^2 \, dx \, dy + \int_0^1 \int_0^{x+y} dx \, dy}{\int_2^3 \int_2^{x+y} x^2 \, dx \, dy}$$

Evaluating this, we get,

$$p(er \geq 0 | m \geq 0) = \frac{3.5 + 3.5}{8} = 0.875$$

The argument of correction claims that this value is always less than 0.5, that is, if two candidates have scored the same, then the probability that the more privileged candidate is also the one who has worked harder is less than 0.5. Thus, our demonstration that the probability is $0.875 > 0.5$ shows that the argument of correction is not true in general, as a true premise and false conclusion signifies that the implication is false. It may be correct for some special cases, but definitely not true in general.

**Example 2:** Let us consider another example where this is seen. Consider the following distribution for $m$—

$$m \sim N(e + r, (e + r)^4)$$

That is, the difference in marks is normally distributed with mean $e + r$ and variance $(e + r)^4$. The expected value is a strictly increasing function of both hard work and institutional support, therefore, the premise of the argument of correction is true.

To check whether the conclusion is correct, note that in this case—

$$p(er \geq 0 | m \geq 0) = \frac{\int_0^2 \int_0^{x+y} e^{-0.5(x+y)^2} \, dx \, dy + \int_0^1 \int_0^{x+y} e^{-0.5(x+y)^2} \, dx \, dy}{\int_2^3 \int_2^{x+y} e^{-0.5(x+y)^2} \, dx \, dy}$$

Evaluating this, we get,

$$p(er \geq 0 | m \geq 0) \approx \frac{3.21203 + 3.21203}{8.82034} \approx 0.723$$

Here again, the value is $0.723 > 0.5$ showing that the argument of correction does not hold in this case either.

**2 Conclusion**

The argument of correction is one of the major arguments in support of affirmative action, but as we have seen, it is untrue in general when we admit the stochastic nature of the problem. It of course is the case that under some special circumstances,
a positive relation between the expected difference in scores and the difference in hard work and difference in institutional support will indeed imply that the probability of the more privileged one also being the one who has worked harder would be less than 0.5, but this entails that in any argument involving the argument of correction, the conditions under which it holds true must be stated. The current study does not delve into the possible necessary conditions for the argument of correction to hold true, future research can focus on this problem. Do note, however, that in both the counter-examples we provided, one of the assumptions of the Classical Linear Regression Model were violated each— in the first example, the residuals are not normally distributed about the mean and in the second example, the variance of the residuals is dependent on the predictor variables, thus admitting heteroscedasticity. It hints that fulfilling the CLRM assumptions may suffice to grant validity to the argument of correction, but we do not test this claim here. If that is the case, then defending the argument of correction would be relatively easy, in fact, such an exposition, we believe, would lead to a much stronger presentation of the argument of correction than the currently offered verbal justification.

The merit of the current study is that we have shown that the argument of correction is not generally true even if we weaken its premises and conclusion. It is common-sense that in a deterministic sense, the argument of correction is not generally true. But the probabilistic version of the argument already admits that the less privileged cannot be guaranteed to have worked harder. It only claims that the probability that the lessprivileged has worked harder is more than the probability that the more privileged has worked harder, given two candidates receive the same score. Our result showing that even this weak and seemingly highly plausible version of the argument is not true in general thus raises non-trivial and highly serious questions on the argument of correction. Finally, for the benefit of the reader, we would reiterate that the mathematical treatment in this study is not a model of society, but a pure logical argument. Thus, it is not prone to errors that arise due to simplification, regardless of the complexity of the real world. The logical content of the argument established in this study stays valid as a universal truth. Despite being empirically unfalsifiable, such arguments do hold much value in normative decision-making and in a fundamental understanding of the philosophical questions that surround social issues, as demonstrated by Sen in his mathematical treatment of Liberalism and Pareto Optimality for instance.

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