Rapid Growth of Dust Particles in the Vicinity of Local Gas Density Enhancements in a Solar Nebula

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ABSTRACT

The results of a numerical study of the growth of solid particles, ranging from 1 micron to 1 millimeter in size, in the vicinity of an azimuthally symmetric density enhancement of a protostellar disk are presented. It is shown that the combined effect of gas drag and pressure gradients, which causes solid objects to rapidly migrate toward the location of the local maximum density, can also enhance the rate of growth of dust particles to larger objects. The results of numerical simulations of such growth processes are presented and the effects of the changes of the physical properties of particles on the rates of their growth are also discussed.

Subject headings: solar system: formation, planetary systems: formation, planetary systems: protoplanetary disks

1. Introduction

The focus of this paper is on the study of the growth rate of dust grains in the vicinity of a local maximum density of a nebula. It has recently been shown that in a nebula with local gas density enhancements, the combined effect of gas drag and pressure gradients causes solid objects to rapidly migrate toward the location of maximum density (Haghighipour & Boss, 2003a&b, hereafter HB03a&b). In those papers, the authors have studied the dynamics of a single object in a non-uniform gaseous disk with no particulate background material. The only interaction considered in those papers between the object and the disk was through the gas drag. If the disk contains a mixture of gas and small solid particles, however, the motion of an object will also be affected by its interaction with this particulate material.
For instance, an object may sweep up smaller particles and grow larger during its gas-drag induced migration. In this paper, I consider particulate material coupled to a gaseous disk with a local density enhancement, and study the rate of growth of small particles while sweeping up the particles of the background material.

For the past four decades, the growth of dust particles in a solar nebula was studied by many authors. In 1969, Safronov studied the growth of dust particles during their vertical descent in a quiescent nebula and showed that, considering their sticking growth, settling of dust particles could be quite rapid. Safronov's formulation indicated that dust particles could grow to 1 centimeter in size at a 1 AU radial distance from the Sun, assuming a 100% sticking probability (Safronov 1969). Growth and sedimentation of dust particles have also been studied extensively by Weidenschilling (1980, 1984) and Nakagawa et al. (1981, 1986). Weidenschilling (1980, 1984) considered coagulation of dust grains due to van der Waals force in their settling toward the midplane and showed that, while such particles grow to 1 centimeter in size, they create a turbulent layer in the vicinity of the midplane which increases the rates of their collision and their coalescence to larger objects. Nakagawa et al. (1981) studied the growth and settling of dust grains at the location of Earth in a nebula and showed that, similar to the results presented by Safronov, the settling of particles and the formation of a layer of centimeter-sized objects in the vicinity of the midplane can occur in a relatively short time (≈300 years). This study was later extended by Nakagawa et al. (1986) to the regions of Jupiter and Neptune in which the authors studied the settling and growth of dust particles in laminar phase of a nebula and presented analytical expressions for the rates of the growth and also vertical and radial motions of solids. By treating a dust layer as a two-dimensional fluid, Nakagawa et al. (1986) showed that the total settling time of dust grains in the two stages of gas- and dust-dominant phases are smaller than those estimated by Weidenschilling (1980) and Nakagawa et al. (1981). More recently, Supulver & Lin (2000) have studied the coagulation of small solids in a turbulent protostellar disk with an application to the formation of icy planetesimals in the outer region of the nebula.

In this paper, I extend the previous studies presented in HB03a&b by considering the growth of a dust particle due to interaction with the particulate background of a nebula in the vicinity of its local density enhancement. The main purpose of this paper is to study how the combined effect of gas drag and pressure gradients can speed up the coagulation of dust particles and the formation of centimeter-sized objects.

The model nebula considered in this paper is presented in § 2. Section 3 presents the equations of motion of a solid particle and § 4 deals with the analysis of the numerical results. Section 5 concludes this study by reviewing the results and discussing their applications.
2. The Model Nebula

The model nebula in this study is an isothermal and turbulence-free protostellar disk with a solar-type star at its center. The nebula is assumed to be a mixture of pure molecular hydrogen at hydrostatic equilibrium, and small submicron-sized solid particles. It is also assumed that the density of the gas maximizes at certain locations in the nebula.

In a cylindrical coordinate system with its origin at the location of the central star and its polar plane on the midplane of the nebula, the gas distribution function at any point \( (r, z) \) is given by (HB03b)

\[
\rho_g(r, z) = \rho_g(r, 0) \exp\left\{ \frac{8GM}{\pi \bar{v}_{th}^2} \left[ \frac{1}{(r^2 + z^2)^{1/2}} - \frac{1}{r} \right] \right\}. \tag{1}
\]

In equation (1), \( M \) is the mass of the central star, \( G \) is the gravitational constant, and \( \rho_g(r, 0) \) represents the gas distribution function on the midplane of the nebula. For simplicity, the density of the gas is considered to have an azimuthally symmetric maximum on the midplane and is given by (HB03a&b)

\[
\rho_g(r, 0) = \rho_0 \exp\left[ -\beta \left( \frac{r}{r_m} - 1 \right)^2 \right], \tag{2}
\]

where \( \rho_0, r_m \) and \( \beta \) are constant quantities (Figure 1).

The quantity \( \bar{v}_{th}^2 = 8K_B T/\pi m_H \) in equation (1) is the mean thermal velocity of the gas molecules. In this equation, \( K_B \) is the Boltzmann constant, \( T \) is the temperature of the gas, and \( m_H \) is the molecular mass of hydrogen. The gas density function as given by equation (1) ensures that along the vertical axis, the gravitational attraction of the central star will be balanced by the vertical component of the pressure gradients (HB03b).

Because of their submicron sizes, the particles of the background material of the nebula are strongly coupled to the gas. The motions of these particles are only affected by the gas drag. On the other hand, since molecular hydrogen obeys the equation of state of an ideal gas, and because the nebula is isothermal, any local enhancement in the gas density will also enhance the pressure of the gas. In the vicinity of such pressure enhanced regions, any solid object undergoes radial migration toward the location of the maximum pressure while approaching the midplane (Whipple 1964, HB03a&b). The rates of such migrations are small for small particles. For the particles of the background, although radial migration is extremely slow, along with vertical descent, it results in accumulation of particulate material at the locations of density enhancements and also in the vicinity of the midplane. In this
study, it is assumed that at any position in the nebula, the gas and particle distribution functions are proportional. That is, 
\[ \rho_{\text{dust}}(r, z) = f \rho_g(r, z), \]
where \( \rho_{\text{dust}}(r, z) \) is the distribution function of the background material. The solid/gas ratio \( f \) can attain different values at different positions and can also vary with time. However, for simplicity, in this study, it is assumed to be a constant number much smaller than unity.

3. Equation of Motion

An object in the model nebula considered here is subject to the gravitational attraction of the central star and the drag force of the gas. Because of its small size, such an object, similar to the particles of the background material, shows the tendency of staying with the gas and its dynamics is mostly driven by gas drag. The tendency of solid particles in being coupled to the gas is weaker for larger objects. For a particle with a size within the range considered here, its coupling to the gas is less strong than the coupling of the submicron-sized particles of the background material. As a result, the particle moves faster than the background material and collides with them. Such collisions may result in adhesion of the background particles to the moving object and increase its mass. Such a change in the mass of the object causes a change in its momentum and subsequently affects its dynamics.

The rate of the change of the momentum of an object due to the sweeping of the background material is proportional to the rate of the collision of the moving object with those particles. In general, whether a collision occurs between two objects depends on their relative velocity. For an object with mass \( m \) and radius \( a \) at position \( (r, z, \varphi) \) moving with velocity \( \mathbf{V} \) with respect to the central star, the rate of change of its momentum due to the sweeping of the background particles is equal to

\[ \frac{dP}{dt} = \pi \rho_{\text{dust}}(r, z) a^2 \frac{d\ell}{dt} (\mathbf{V} - \mathbf{U}), \]

where the sticking coefficient is equal to unity. In equation (3), \( P \) is the portion of the total momentum of the object that changes by its coalescence with smaller background particles and \( d\ell = [(dr)^2 + (dz)^2 + (rd\varphi)^2]^{1/2} \) is a line element along the path of the object. The quantity \( \mathbf{U} \) in equation (3) represents the velocity of the particles of the medium along the line element \( d\ell \). In writing equation (3), it has been assumed that, while sweeping smaller particles, the density of the object remains unchanged and it stays perfectly spherical. It is important to emphasize that in this equation \( m \) and \( a \) are functions of time, and are related as

\[ \frac{dm}{dt} = \pi \rho_{\text{dust}}(r, z) a^2 \frac{d\ell}{dt}. \]
Equation (4) immediately implies
\[ \frac{da}{dt} = \frac{1}{4} \left( \frac{\rho_{\text{dust}}}{\rho} \right) \frac{d\ell}{dt}, \]
where \( \rho \) is the density of the object.

For small particles such as those studied here, the magnitude of the velocity \( V \) is mainly dominated by its components in the radial and vertical directions. As shown in HB03a&b, these components have larger values for larger objects. That implies, while an object grows by sweeping the particles of the background, it will move faster in the radial and vertical directions. However, due to the strong coupling to the gas, for the particles of interest in this paper, the rates of increase in radial and vertical motions are quite small. As a result, the magnitude of the velocity of the object relative to the background material (i.e., \( |V - U| \)), is also small. This implies that, when migrating toward the location of a maximum gas density, the object approaches the background particles very slowly. Such a slow approach suggests a gentle encounter between the moving object and the particle of the background followed by the sweeping up of the smaller particle by the larger object. Laboratory experiments have indicated that the collisional coagulation of millimeter-sized and smaller particles can well be approximated by such a sweeping process (Wurm & Blum 1998).

As mentioned above, the small velocities of the background particles of the medium are the result of a strong coupling between the gas and these particles. For these particles such a coupling is so strong that with a good approximation, one can replace \( U(r, z, \varphi) \) in equation (3) with the velocity of the gas at that position. As a result, equation (3) can be written as
\[ \frac{dP}{dt} = \pi \rho_{\text{dust}}(r, z) a^2 \frac{d\ell}{dt} V_{\text{rel}}, \]
where \( V_{\text{rel}} \) is the relative velocity of the object with respect to the gas. In the cylindrical coordinate system considered here, the radial, vertical and tangential components of this velocity are, respectively, given by \( \dot{r}, \dot{z}, \) and \( r(\dot{\varphi} - \omega_g) \), where the motions of gas molecules along the \( z \)-axis have been neglected. Because the gas is at hydrostatic equilibrium, its angular velocity, \( \omega_g \), is slightly different from its Keplerian value and is given by
\[ \omega_g^2 = \frac{GM}{(r^2 + z^2)^{3/2}} + \frac{1}{r \rho_g(r, z)} \frac{\partial P_g(r, z)}{\partial r}. \]
In this equation, \( P_g(r, z) \) is the pressure of the gas.

Considering the mass-growth equation (4), the equation of motion of an object in our model nebula is given by
\[ m \ddot{R} = -\frac{GMm}{(r^2 + z^2)^{3/2}} R - \pi \rho_{\text{dust}}(r, z) a^2 \frac{d\ell}{dt} V_{\text{rel}} - F_{\text{drag}}. \]
where \( \mathbf{R}(r, z) \) is the position vector of the particle and

\[
\mathbf{F}_{\text{drag}} = \frac{4}{3} \frac{a^2}{a + \lambda} \bar{v}_{\text{th}} \left[ \lambda \rho_g(r, z) + \frac{3m_0}{2\sigma} \right] \mathbf{V}_{\text{rel}},
\]

represents the drag force of the gas. In equation (9), \( \sigma \) is the collisional cross section between two hydrogen molecules and \( \lambda \) is their mean free path.

Equation (9) has been written following Supulver & Lin (2000) and HB03a&b whose authors present \( \mathbf{F}_{\text{drag}} \) in a form that combines Epstein and Stokes drags in one formula. Because the focus of this paper is on the growth of 1-1000 micron-sized objects to 1 centimeter in radius, as shown in HB03&b, the gas Reynolds number (Re) for these objects in the model nebula presented here stays well below unity. This implies that the drag coefficient \( C_D \) in \( \mathbf{F}_{\text{drag}} \) has to be taken to be equal to \( 24/\text{Re} \) (Supulver & Lin 2000; Weidenschilling 1977, HB03a&b).

4. Numerical Results

As mentioned in §3, the distribution of the background material is considered to be proportional to the gas distribution function. Podolak & Cameron (1974) considered \( f = 0.0034 \) for a solar nebula with fully formed silicates and metals (also see Weidenschilling 1988). Supulver & Lin (2000), however, have considered \( f = 0.0045 \) by assuming that the abundance of H\(_2\)O in the nebula is the same as its solar value. They call this value of \( f \) nominal and have also considered \( f = 0.045 \) and 0.45 in order to study the effect of an increase in the concentration of material on the midplane, on collision and coagulation of small solids. The value of \( f \) in this study is chosen to be 0.0034. From equation (4), it is evident that larger values of \( f \) will result in faster growth of particles.

The equation of motion of a particle [Eq.(8)] and the growth equation (5) were integrated, numerically, for initial radii ranging from 1 to 1000 microns, and for different values of the gas temperature. In all these computations, the mass of the central star was chosen to be equal to the mass of the Sun, \( \beta = 1, r_m = 1\) AU, and \( \rho_0 = 10^{-10}\) g cm\(^{-3}\). The collisional cross section of hydrogen molecules, \( \sigma \), was taken to be \( 2 \times 10^{-15}\) cm\(^{-2}\), and their mean free paths \( \lambda (\text{cm}) = 4 \times 10^{-9}/\rho_g(r, z) \) (g cm\(^{-3}\)). The initial value of the height of a particle above the midplane was set equal to 1/10 of its initial radial distance from the Sun. The initial velocity of the particle along the vertical axis was taken to be zero, and in the radial direction, the particle was given an initial Keplerian circular velocity.

The growth of an object was followed until it reached 1 cm in size. Although centimeter-sized objects can also grow larger by sweeping smaller particles, a 1 cm radius was chosen...
as the upper limit since the assumed growth process, that is, sweeping of smaller particles, is more effective for dust grains and micron-sized objects. In a nebula with centimeter-sized particles, in addition to sweeping of the submicron particles of the background, there is also collision among centimeter-sized particles which may result in their coagulation and/or fragmentation. Such collisions can also generate dust particles. The collisions of micron-sized particles, on the other hand, are gentle and result in coalescence of the involved objects. To avoid the complexities associated with treating centimeter-sized particles then, the numerical simulations were stopped at $a = 1$ cm. Figure 2 shows the growth of a 10 micron-sized grain with a density of $2$ g cm$^{-3}$, initially at $(r = 3, z = 0.3)$ AU. The gas temperature is $300$ K. As shown here, in a very short time, the particle grows to 1 centimeter in size. During this time, it undergoes radial migration and also descends toward the midplane (Fig. 3). For a comparison, the particle’s migrations without mass-growth have also been plotted. As expected, the rates of the radial migration and vertical descent of the particle increase by increasing its size. Figures 3 and 4 clearly show that the combined effect of gas drag and pressure gradients causes particles to rapidly grow during their gas-drag induced migrations.

A comparison between the two graphs of Figure 3 indicates that during the time that the particle grows to 1 cm, it descends toward the midplane faster than it migrates radially. The actual path of the particle during this time has been shown in Figure 4. Such a rapid vertical descent was also reported by Nakagawa et al. (1986), where the authors have shown that, for settling dust particles, the ratio of the vertical and radial components of velocity is greater than one and it increases by increasing the size of the particle. The effect of the radial component of the particle’s velocity becomes more pronounced when the particle reaches 1 cm in radius and grows larger. Nakagawa et al. (1986) call this phase of the growth process the “gas-dominated” phase. During this phase, the particle undergoes an over-damped oscillatory motion in the vicinity of the midplane (HB03b).

To better understand the relation between the growth of the particle and its vertical and radial motions, it is useful to calculate the time and the distance of collision between the object and a submicron grain of the background material. To define these quantities, recall that it has been assumed that all collisions between the object and particles of the background result in 100% sticking. The time of collision is therefore defined as the time during which the mass of an object, $m(t)$, is increased to $m(t) + m_0$, where $m_0$ is the mass of a background grain. Because it has been assumed that after a collision, the object maintains its spherical shape and will have the same density, assuming a spherical shape for a background particle, and also assuming that it has similar density as the moving object, after a collision the radius of the object changes from $a(t)$ to $[a^3(t) + a_0^3]^{1/3}$, where $a_0$ is the radius of a background grain. The rate of change of radius, on the other hand, is given by equation (5). From this equation and also considering that $a_0 \ll a(t)$, the time of collision can be
approximately written as

$$\tau_c(t) \simeq \frac{4a_0^3}{3a^2(t)} \left( \frac{\rho}{\rho_{\text{dust}}} \right) \left( \frac{d\ell}{dt} \right)^{-1}. \quad (10)$$

Equation (10) can also be obtained by introducing an equivalent of a mean free path for the object. The mean free path of the object, shown by $\lambda_d$, is defined as the distance that the object travels before it sweeps a background grain. Similar to the equation of the mean free path of a gas molecule, $\lambda_d$ can be written as

$$\lambda_d(t) = \frac{m_0}{\pi[a(t) + a_0]^2 \rho_{\text{dust}}}. \quad (11)$$

Equation (11) immediately results in equation (10), noting that $\lambda_d = (d\ell/dt)\tau_c$.

Figure 5 shows the time of collision and the mean free path of the 10 micron object of Figure 2 during its growth to 1 cm. As shown here, the magnitudes of $\tau_c$ and $\lambda_d$ rapidly decrease in time. To show that in more detail, the time of growth has been divided into 4 segments. As the object sweeps the background material and grows in size, the rate of its gas-drag induced migration becomes faster. As a result, it approaches a background particle in a shorter time. Also, as shown in Equation (11), the mean free path of the object is inversely proportional to the square of its instantaneous radius. Since $a(t)$ increases at all times, $\lambda_d$ is a monotonically decreasing function of time.

Numerical simulations were also carried out for different values of the gas temperature. Figure 6 shows the time of the growth of a 10 micron particle to 1 cm for three different values of the temperature. As shown here, the time of growth increases at higher temperatures. That can be attributed to the fact that the spatial distribution of the background material is directly proportional to the gas density function [Eq. (1)]. By increasing the gas temperature, the FWHM of $\rho_g$ and $\rho_{\text{dust}}$ increases (Fig. 7) indicating an increase in the spatial distances between the background particles. This causes the time of collision to increase and the object will take a longer time to grow larger at higher temperatures.

5. Conclusions

The results of the numerical study of the growth rates of small particles in the vicinity of a local density enhancement of a solar nebula were presented. It was shown that, as a consequence of migration toward the location of a local density enhancement, solid objects can rapidly grow in size by sweeping up the smaller particles of the background material. Particles of interest in this study had sizes between 1 to 1000 micron and their growth-rates were studied until they reached 1 cm in radius. The results presented here indicate that, compared to the time of settling and growth of dust particles in a nebula without local
gas density enhancements (Safronov 1969; Weidenschilling 1980; Nakagawa et al. 1981; Weidenschilling 1988), in the vicinity of a local maximum gas density, the combined effect of gas drag and pressure gradients can increase the growth rates of dust grains by one order of magnitude.

In this study the focus was on the growth of small grains to objects of 1 cm in size. Although centimeter-sized objects can still acquire mass and grow larger by sweeping up smaller particles, their dynamics is not solely driven by gas drag and the sweeping process. The velocities of such particles are large enough to cause them to fragment and produce dust when they collide. On the other hand, as noted by Safronov (1969), Weidenschilling (1980), Nakagawa et al. (1981), Nakagawa et al. (1986), and Weidenschilling (1988), the largest that dust particles can grow to while settling toward the midplane is centimeter-sized. The concentration of centimeter-sized particles in the vicinity of the midplane creates a turbulent layer (Weidenschilling 1980) which requires a more detailed analysis. All these introduce complexities in treating centimeter-sized objects that are unnecessary for the purpose of this study. Studies are, however, underway in which the mutual interactions of centimeter-sized particles and the effect of turbulence on their collision and coalescence have been considered.

The spatial density of the particulate background of the nebula was chosen to be proportional to the gas distribution function. The solid/gas ratio \( f \) was chosen to be constant and equal to 0.0034. This is a minimal value for \( f \) that corresponds to a dust composition of metals and silicates on the midplane of a standard nebula at 1 AU (Podolak & Cameron 1974; Weidenschilling 1980, 1988). In a more realistic scenario, the value of \( f \) is a function of position and time. For instance, in the model nebula considered here, as time passes, the background material of the nebula settles toward the midplane and also migrates toward \( r = 1 \) AU, where the gas distribution function maximizes. This increases the dust/gas ratio around \( r = 1 \) AU on the midplane and decreases \( f \) at other locations. In such cases, one has to take the time variation of \( f \) into consideration. However, because in this study, the rates of radial and vertical migration of the background material are smaller than those of the objects of interest, and also because it has been assumed that the sticking probability is 100%, which results in faster growth of the particle, growing to 1 cm in size occurs so rapidly that the particles of the background will not have enough time to undergo appreciable radial and vertical migrations.

As mentioned earlier, the focus of this study has been on the growth of particles to 1 cm in size. For such small objects, the assumption of growth merely due to sweeping up smaller particles is quite plausible since these objects are coupled to the gas and they approach one another slowly. Once the sizes of objects increase 1 cm, their collisions may result in adhesion and/or fragmentation. As a result, their growth will not be entirely due to
sweeping up of smaller particles and will be affected by their individual velocities. Replacing
the relative velocities of such objects with their velocities relative to the gas is no longer a
valid approximation. In such cases, one has to follow individual particles and consider their
velocities relative to one another. Such an extension of this work is currently in preparation
for publication.

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Fig. 1.— Graphs of the gas distribution function. The temperature of the gas is 300 K, \( r_m = 1 \) AU, \( \beta = 1 \), and \( \rho_0 = 10^{-10} \) g cm\(^{-3}\). The top graph depicts the radial distribution of the gas at different heights above the midplane. The middle graph shows a topview of the nebula at \( z = 0.3 \) AU and the bottom graph shows its topview at \( z = 0.15 \) AU.
Fig. 2.— Graph of the radius versus time for a particle with an initial radius of 10 micron. The initial position of the particle is (3,0.3) AU, and the physical properties of the gas are similar to those of Figure 1.
Fig. 3.— Graphs of the radial migration (top) and vertical descent (bottom) of the 10 micron-sized particle of Figure 2. The solid lines correspond to $f = 0.0034$, and the dashed lines represent the radial and vertical motions with no mass-growth ($f = 0$). The horizontal axis represents the time of the growth of the particle to 1 centimeter.
Fig. 4.— The path of the 10 micron-sized particle of Figure 3. As shown here, during its growth to 1 centimeter, the motion of the particle is mostly vertical.
Fig. 5.— Graphs of the time of collision between a growing object and a particle of the background (left), and the mean free path of the object (right). The horizontal axes represent the time of growth. The initial radius of the object was 10 micron, and it was placed at (3,0.3) AU. The physical properties of the nebula are similar to those of Figure 1. Note different scales on vertical axes.
Fig. 6.— Graph of the growth of a 10 micron-sized particle with a density of 2 g cm$^{-3}$ for different values of the gas temperature. The particle was initially at (3.0, 0.3) AU.
Fig. 7.— Graph of the vertical distribution of the background material at $r = 3$ AU for different values of the gas temperature.