Polarization properties of 'slow' light

V. A. Reshetov, I. V. Meleshko

Department of General and Theoretical Physics, Tolyatti State University, 14 Belorousskaya Street, 446667 Tolyatti, Russia

Abstract

The propagation of the arbitrarily polarized pulse of the weak probe field through the resonant medium of Λ-type three-level atoms with degenerate levels adiabatically driven by the coherent coupling field is considered. It is shown that such pulse is decomposed in the medium into two orthogonally polarized dark-state polaritons propagating with different group velocities. The expressions for the polarizations and group velocities of these two polaritons are obtained. The dependence of these polarizations and group velocities on the values of the angular momenta of resonant levels, on the polarization of the coupling field and on the initial atomic state is studied.

1 Introduction

The remarkable reduction of the group velocity of light pulses [1, 2] based on the phenomenon of the electromagnetically induced transparency (EIT) [3, 4] provided a number of applications, the most promising among them being the implementation of quantum memory [5, 6, 7]. The recent experiments [8, 9, 10] on EIT-based quantum memory demonstrate the continuously enhancing memory efficiency and fidelity, bringing it close to practical applications. Such memory is based on the concept of dark-state polaritons in the three-level Λ-type systems, proposed in [11, 12] and soon realized in rubidium vapor in the experiment [13]. The group velocity of such polaritons may be controlled by the adiabatically varying intensity of the driving field to store single-photon pulses in the resonant media or to retrieve them. The most natural way for q-bit encoding is provided by the photon two polarization degrees of freedom, as it was implemented in the experiments [9, 10]. However the group velocity of the dark-state polariton may depend essentially on its polarization due to the optical anisotropy induced by the polarization of the driving field, while for the effective storage of the photon polarization q-bit its both polarization components must be stopped in the medium simultaneously. The objective of the present paper is to study the polarization properties of the dark-state polaritons formed in the three-level Λ-type systems with degenerate levels, which are in many experiments the hyperfine structure components of alkali atoms degenerate in the projections of the atomic total angular momentum on the quantization axis.
2 Basic equations and relations

We consider the pulse of the weak probe field propagating along the sample axis \(Z\) with the carrier frequency \(\omega\), which is in resonance with the frequency \(\omega_0\) of an optically allowed transition \(J_a \rightarrow J_c\) between the ground state \(J_a\) and the excited state \(J_c\), while the strong coherent coupling field propagates in the same direction with the carrier frequency \(\omega_c\), which is in resonance with the frequency \(\omega_{c0}\) of an optically allowed transition \(J_b \rightarrow J_c\) between the long-lived state \(J_b\) and the same excited state \(J_c\) (Fig. 1). Here \(J_a, J_b\) and \(J_c\) are the values of the angular momenta of the levels. The electric field strength of the coupling field and that of the probe field may be put down as follows:

\[
E_c = e_c(t-z/c)l_c e^{-i\omega_c (t-z/c)} + c.c.,
\]

\[
E = e(t, z)e^{-i\omega (t-z/c)} + c.c.,
\]

where \(e_c\) is the slowly-varying amplitude of the coupling field and \(l_c\) is its constant unit polarization vector, while \(e\) is the slowly-varying vector amplitude of the probe pulse, which satisfies the equation:

\[
\left(\frac{\partial}{\partial t} + c \frac{\partial}{\partial z}\right) e = \frac{i\omega n_0 |d|}{2\varepsilon_0} tr \{\hat{g} \hat{\rho}\},
\]

as it follows from Maxwell equations, while the evolution of the atomic slowly-varying density matrix \(\hat{\rho}\) in the rotating-wave approximation is described by the equation:

\[
\frac{\partial \hat{\rho}}{\partial t} = i\frac{1}{2} \left[ \hat{V}, \hat{\rho} \right] + \left( \frac{d \hat{\rho}}{dt} \right)_{rel},
\]

\[
\hat{V} = 2(\Delta \hat{P} + \delta \hat{P}_b) + \hat{D} + \hat{D}^\dagger, \quad \hat{D} = \Omega_c \hat{\rho}_c + \hat{G},
\]

\[
\hat{g} = \hat{g}_c l_c, \quad \hat{G} = (2|d|/\hbar) \hat{g} e^*.
\]

Here \(n_0\) is the concentration of resonant atoms, \(\hat{g}\) and \(\hat{\rho}_c\) are the dimensionless electric dipole moment operators for the transitions \(J_a \rightarrow J_c\) and \(J_b \rightarrow J_c\), \(d = d(J_a J_c)\) and \(d_c = d(J_b J_c)\) being the reduced matrix elements of the electric dipole moment operators for these transitions, \(\Delta = \omega - \omega_0\) and \(\Delta_c = \omega_c - \omega_{c0}\) are the frequency detunings from resonance of the probe and of the coupling fields, while \(\delta = \Delta - \Delta_c\), \(\hat{P}_a\) is the projector on the subspace of the atomic level \(J_a\) (\(a = a, b, c\)), \(\Omega_c = 2|d_c|\hbar|e_c|/h\) is the reduced Rabi frequency for the coupling field. The matrix elements of the circular components \(\hat{g}_q\) and \(\hat{g}_{cq}\) (\(q = 0, \pm 1\)) of vector operators \(\hat{g}\) and \(\hat{g}_c\) are expressed through Wigner 3J-symbols [14]:

\[
(\hat{g}_q)^{ac}_{m_a m_c} = (-1)^{J_a - m_a} \begin{pmatrix} J_a & 1 & J_c \\ -m_a & q & m_c \end{pmatrix},
\]

\[
(\hat{g}_{cq})^{bc}_{m_b m_c} = (-1)^{J_b - m_b} \begin{pmatrix} J_b & 1 & J_c \\ -m_b & q & m_c \end{pmatrix}.
\]
Finally, the term \((d\hat{\rho}/dt)_{rel}\) in the equation (3) describes the irreversible relaxation. Initially the atoms are at the ground state \(a\) the atomic density matrix being
\[
\hat{\rho}(0) = \hat{\rho}_a.
\]
In the linear approximation for the probe field we obtain from the equations (4)-(5) for the elements of the atomic density matrix the following equations:

\[
\begin{align*}
\left( \frac{\partial}{\partial t} + \gamma - i\Delta \right) \hat{\rho}^{ca} &= \frac{i}{2} \left( \Omega_c \hat{g}_c \hat{\rho}^{ba} + \hat{G}^\dagger \hat{\rho}_a \right), \\
\left( \frac{\partial}{\partial t} + \Gamma - i\delta \right) \hat{\rho}^{ba} &= \frac{i}{2} \Omega_c \hat{g}_c \hat{\rho}^{ca},
\end{align*}
\]

where
\[
\hat{\rho}^{\alpha\beta} = \hat{P}_\alpha \hat{\rho} \hat{P}_\beta, \quad \alpha, \beta = a, b, c,
\]
while the irreversible relaxation is simply characterized by the two real relaxation rates – \(\gamma\) for the optically allowed transition \(J_a \rightarrow J_c\) and \(\Gamma\) for the optically forbidden transition \(J_a \rightarrow J_b\):

\[
\begin{align*}
\left( \frac{d\hat{\rho}}{dt} \right)^{ca}_{rel} &= -\gamma \hat{\rho}^{ca}, \\
\left( \frac{d\hat{\rho}}{dt} \right)^{ba}_{rel} &= -\Gamma \hat{\rho}^{ba}.
\end{align*}
\]

In the adiabatic approximation, when the coupling field varies slowly:

\[
\gamma T \gg 1, \quad \Omega_c^2 T \gg \gamma,
\]

\(T\) being the characteristic time scale, while the relaxation at the forbidden transition remains negligible

\[
\Gamma T \ll 1,
\]
in the case of single-photon and two-photon resonances
\[ \Delta \lesssim \gamma, \quad \delta T \ll 1, \]
from (9)-(10) it follows:
\begin{align*}
\hat{\Omega}_c \hat{g}_c \hat{\rho} + \hat{G}_c \hat{\rho}_a &= 0, \\
\frac{\partial \hat{\rho}^{ba}}{\partial t} &= i \frac{1}{2} \hat{\Omega}_c \hat{g}_c \hat{\rho}^{ca}.
\end{align*}
(11)
(12)
The equation (11), which is the approximation of the relation
\[ \hat{D}_c \hat{g}_c \hat{\rho}^{ba} = 0, \]
linear in the probe field, means that only the dark states contribute to the solution of the equation (12). In order to express \( \hat{\rho}^{ca} \) through \( \hat{\rho}^{ba} \) we multiply both parts of the equation (12) by the matrix \( \hat{g}_c \) from the left and consider the orthonormal set of eigenvectors \( |c_n> \) of the hermitian operator \( \hat{g}_c \) acting at the subspace of the excited level \( c \):
\[ \hat{g}_c |c_n> = c_n^2 |c_n>, \quad n = 1, ..., 2J_c + 1, \]
(13)
the corresponding eigenvalues being non-negative \( c_n^2 \geq 0 \). Then, after multiplying both parts of the equation (12) from the left by the matrix
\[ \hat{P}_b^{c} = \sum_n |c_n><c_n|, \]
(14)
we obtain
\[ \hat{P}_b^{c} \hat{\rho}^{ca} = - \frac{2i}{\Omega_c} \frac{\partial}{\partial t} \left( \hat{D}_c \hat{g}_c \hat{\rho}^{ba} \right), \]
(15)
where
\[ \hat{P}_b^{c} = \sum_n |c_n><c_n|, \]
(16)
while the summation in the equations (14) and (16) is carried out only over eigenvectors \( |c_n> \) with non-zero eigenvalues \( c_n^2 > 0 \), \( \hat{P}_b^{c} \) being the projector on the subspace formed by such eigenvectors. The eigenvectors \( |c_n> \) with zero eigenvalues \( c_n^2 = 0 \) are not affected by the coupling field and may be neglected under the assumed approximation. Now let us introduce the vector field
\[ p = tr \left\{ \hat{g}_c \hat{D}_c \hat{g}_c \hat{\rho}^{ba} \right\}, \]
(17)
describing the induced coherence at the forbidden transition \( J_a \to J_b \). With the two orthonormal vectors \( \mathbf{l}_i \) in the polarization plane \( XY \) \( 1 \mathbf{l}_k = \delta_{jk}, \quad j, k = 1, 2 \) we obtain from (3), (15) and (11) for the components \( e_k = i \mathbf{l}_k \) and \( p_k = \mathbf{p} \mathbf{l}_k \) the following equations:
\[ \left( \frac{\partial}{\partial t} + c \frac{\partial}{\partial z} \right) e_k = \frac{\omega n_0 |d|}{c_0 \Omega_c} \frac{\partial p_k}{\partial t}, \]
(18)
\[ p_k = -\frac{2|d|}{\hbar \Omega_c} \sum_j a_{kj} e_j, \]  
\[ a_{kj} = \text{tr} \left\{ \hat{\rho}_a \hat{g}_k \hat{D}_c \hat{g}_j^\dagger \right\}, \quad \hat{g}_k = \hat{g}_k^\dagger. \]

Now let us choose the two orthonormal vectors \( \mathbf{l}_i \) as the two eigenvectors of the hermitian \( 2 \times 2 \) matrix \( a_{jk} \), defined by (20). Then
\[ a_{kj} = a_k \delta_{kj}, \]
where \( a_k \) (\( k = 1, 2 \)) are the two real eigenvalues of this matrix. By introducing via canonical transformation the field of the dark-state polariton [12]:
\[ \Psi_k = \cos \theta_k e_k - \sin \theta_k \lambda_k p_k, \]
where
\[ \lambda_k = \sqrt{\hbar \omega n_0 \over 2 \varepsilon_0 a_k}, \]
and the angle \( \theta_k \) is determined by the equation
\[ \tan \theta_k = \frac{2|d|a_k \lambda_k}{\hbar \Omega_c}, \]

neglecting the retardation of the coupling field \( \Omega_c(t - z/c) \sim \Omega_c(t) \), we obtain from (18)-(24) the following equation:
\[ \frac{\partial \Psi_k}{\partial t} + c \cos^2 \theta_k \frac{\partial \Psi_k}{\partial z} = 0, \]
which describes the propagation of the dark-state polariton with the group velocity
\[ V_{gr}^k = c \cos^2 \theta_k = \frac{c}{1 + n_{gr}^2}, \]
\[ n_{gr}^2 = \tan^2 \theta_k = \frac{2|d|^2 n_0 \omega a_k}{\hbar \Omega_c^2 \varepsilon_0}. \]

3 Discussion

As it follows from (22)-(27), the arbitrarily polarized pulse of the weak probe field is decomposed under the action of the strong coupling field into two components polarized along the two eigenvectors of tensor (20) propagating with different group velocities, which difference is determined by the difference in the two eigenvalues of tensor (20). So the polarization properties of the dark-state polaritons are totally determined by the hermitian \( 2 \times 2 \) tensor \( a_{jk} \) defined by the equation (20). This tensor in its turn is determined by the values of the angular momenta of resonant levels, by the polarization of the coupling field and by the initial atomic state. Let us now calculate the eigenvalues \( a_1 \) and \( a_2 \).
and the corresponding eigenvectors $l_1$ and $l_2$ of tensor $a_{ik}$ for some transitions $J_a \rightarrow J_c \rightarrow J_b$ with the angular momenta characteristic for the experiments on the hyperfine structure components of atomic levels. In the case of equilibrium initial atomic state and linearly polarized coupling field 

$$\hat{\rho}_a = \frac{\hat{P}_a}{2J_a + 1}, \quad l_c = l_x,$$

we obtain:

$$a_1 = 2, \ a_2 = 0, \ l_1 = l_y, \ l_2 = l_x,$$

for transitions $J_a = 0 \rightarrow J_c = 1 \rightarrow J_b = 1$, 

$$a_1 = 1.111, \ a_2 = 0.972, \ l_1 = l_x, \ l_2 = l_y,$$

for transitions $J_a = 1 \rightarrow J_c = 1 \rightarrow J_b = 2$, 

$$a_1 = 2, \ a_2 = 1.5, \ l_1 = l_x, \ l_2 = l_y,$$

for transitions $J_a = 1 \rightarrow J_c = 2 \rightarrow J_b = 2$, 

$$a_1 = 1.295, \ a_2 = 0.907, \ l_1 = l_x, \ l_2 = l_y,$$

for transitions $J_a = 2 \rightarrow J_c = 2 \rightarrow J_b = 3$, 

$$a_1 = 2.96, \ a_2 = 1.787, \ l_1 = l_x, \ l_2 = l_y,$$

for transitions $J_a = 2 \rightarrow J_c = 3 \rightarrow J_b = 3$, 

$$a_1 = 1.445, \ a_2 = 0.944, \ l_1 = l_x, \ l_2 = l_y,$$

for transitions $J_a = 3 \rightarrow J_c = 3 \rightarrow J_b = 4$, 

$$a_1 = 3.832, \ a_2 = 2.151, \ l_1 = l_x, \ l_2 = l_y,$$

for transitions $J_a = 3 \rightarrow J_c = 4 \rightarrow J_b = 4$.

In the case of equilibrium initial atomic state and circularly polarized coupling field 

$$\hat{\rho}_a = \frac{\hat{P}_a}{2J_a + 1}, \quad l_c = l_{+1},$$

we obtain:

$$a_1 = 2, \ a_2 = 0, \ l_1 = l_{+1}, \ l_2 = l_{-1},$$

for transitions $J_a = 0 \rightarrow J_c = 1 \rightarrow J_b = 1$, 

$$a_1 = 2.222, \ a_2 = 0.833, \ l_1 = l_{+1}, \ l_2 = l_{-1},$$

for transitions $J_a = 1 \rightarrow J_c = 1 \rightarrow J_b = 2$, 

$$a_1 = 1.444, \ a_2 = 0.611, \ l_1 = l_{+1}, \ l_2 = l_{-1},$$

$$a_1 = 1.444, \ a_2 = 0.611, \ l_1 = l_{+1}, \ l_2 = l_{-1},$$

$$a_1 = 1.444, \ a_2 = 0.611, \ l_1 = l_{+1}, \ l_2 = l_{-1},$$

$$a_1 = 1.444, \ a_2 = 0.611, \ l_1 = l_{+1}, \ l_2 = l_{-1},$$

$$a_1 = 1.444, \ a_2 = 0.611, \ l_1 = l_{+1}, \ l_2 = l_{-1},$$

$$a_1 = 1.444, \ a_2 = 0.611, \ l_1 = l_{+1}, \ l_2 = l_{-1},$$

$$a_1 = 1.444, \ a_2 = 0.611, \ l_1 = l_{+1}, \ l_2 = l_{-1},$$

$$a_1 = 1.444, \ a_2 = 0.611, \ l_1 = l_{+1}, \ l_2 = l_{-1},$$

$$a_1 = 1.444, \ a_2 = 0.611, \ l_1 = l_{+1}, \ l_2 = l_{-1},$$

$$a_1 = 1.444, \ a_2 = 0.611, \ l_1 = l_{+1}, \ l_2 = l_{-1},$$

$$a_1 = 1.444, \ a_2 = 0.611, \ l_1 = l_{+1}, \ l_2 = l_{-1},$$

$$a_1 = 1.444, \ a_2 = 0.611, \ l_1 = l_{+1}, \ l_2 = l_{-1},$$

$$a_1 = 1.444, \ a_2 = 0.611, \ l_1 = l_{+1}, \ l_2 = l_{-1},$$

$$a_1 = 1.444, \ a_2 = 0.611, \ l_1 = l_{+1}, \ l_2 = l_{-1},$$

$$a_1 = 1.444, \ a_2 = 0.611, \ l_1 = l_{+1}, \ l_2 = l_{-1},$$

$$a_1 = 1.444, \ a_2 = 0.611, \ l_1 = l_{+1}, \ l_2 = l_{-1},$$

$$a_1 = 1.444, \ a_2 = 0.611, \ l_1 = l_{+1}, \ l_2 = l_{-1},$$

$$a_1 = 1.444, \ a_2 = 0.611, \ l_1 = l_{+1}, \ l_2 = l_{-1},$$

$$a_1 = 1.444, \ a_2 = 0.611, \ l_1 = l_{+1}, \ l_2 = l_{-1},$$

$$a_1 = 1.444, \ a_2 = 0.611, \ l_1 = l_{+1}, \ l_2 = l_{-1},$$

$$a_1 = 1.444, \ a_2 = 0.611, \ l_1 = l_{+1}, \ l_2 = l_{-1},$$

$$a_1 = 1.444, \ a_2 = 0.611, \ l_1 = l_{+1}, \ l_2 = l_{-1},
for transitions $J_a = 1 \rightarrow J_c = 2 \rightarrow J_b = 2$,

$$a_1 = 2.59, \quad a_2 = 1.12, \quad l_1 = l_{+1}, \quad l_2 = l_{-1},$$

for transitions $J_a = 2 \rightarrow J_c = 2 \rightarrow J_b = 3$,

$$a_1 = 1.392, \quad a_2 = 0.832, \quad l_1 = l_{+1}, \quad l_2 = l_{-1},$$

for transitions $J_a = 2 \rightarrow J_c = 3 \rightarrow J_b = 3$,

$$a_1 = 2.89, \quad a_2 = 1.362, \quad l_1 = l_{+1}, \quad l_2 = l_{-1},$$

for transitions $J_a = 3 \rightarrow J_c = 3 \rightarrow J_b = 4$,

$$a_1 = 1.402, \quad a_2 = 0.971, \quad l_1 = l_{+1}, \quad l_2 = l_{-1},$$

for transitions $J_a = 3 \rightarrow J_c = 4 \rightarrow J_b = 4$.

Here $l_x$ and $l_y$ denote the unit vectors of the Cartesian axes, while

$$l_{\pm 1} = \mp \frac{l_x \pm il_y}{\sqrt{2}}$$

are the two circular vectors.

In the experiment [9] the coupling field was linearly polarized in the direction of propagation of the probe field: $l_c = l_z$ ($\pi$-polarized), while it propagated in the perpendicular direction, and the atoms were prepared at the pure Zeeman state with the zero projection on the quantization axis. Under such conditions the group velocity of the probe pulse does not depend on its polarization for the arbitrary value of angular momenta, as it follows from (20). The same remains true for the unprepared atoms, which are initially in the equilibrium state with equally populated Zeeman sublevels, as it also follows from (20).

## 4 Conclusions

In the present article we consider the propagation of the arbitrarily polarized pulse of the weak probe field through the resonant medium of $\Lambda$-type three-level atoms with degenerate levels adiabatically driven by the coherent coupling field. It is shown that such pulse is decomposed in the medium into two orthogonally polarized dark-state polaritons propagating with different group velocities. The polarizations of these two polaritons and the difference in their group velocities are determined by the values of the angular momenta of resonant levels, by the polarization of the coupling field and by the initial atomic state.

### Acknowledgements

Authors are indebted for financial support of this work to Russian Ministry of Science and Education (grant 2.2407.2011).
References

[1] M.M. Kash, V.A. Sautenkov, A.S. Zibrov, L. Hollberg, G.R. Welch, M.D. Lukin, Y. Rostovtsev, E.S. Fry, M.O. Scully, Physical Review Letters 82 (1999) 5229.

[2] D. Budker, D.F. Kimball, S.M. Rochester, V.V. Yashchuk, Physical Review Letters 83 (1999) 1767.

[3] M.O. Scully, M.S. Zubairy, Quantum Optics, Cambridge University Press, Cambridge, 1997.

[4] M. Fleischhauer, A. Imamoglu, J.P. Marangos, Reviews of Modern Physics 77 (2005) 633.

[5] A.I. Lvovsky, B.C. Sanders, W. Tittel, Nature Photonics 3 (2009) 706.

[6] C. Simon, M. Afzelius, J. Appel, A. Boyer de la Giroday, S.J. Dewhurst, N. Gisin, C.Y. Hu, F. Jelezko, S. Kroll, J.H. Muller, J. Nunn, E. Polzik, J. Rarity, H. de Riedmatten, W. Rosenfeld, A.J. Shields, N. Skold, R.M. Stevenson, R. Thew, I. Walmsley, M. Weber, H. Weinfurter, J. Wrachtrup, R.J. Young, European Physical Journal D 58 (2010) 1.

[7] M. Himsworth, P. Nisbet, J. Dilley, G. Langfahl-Klabes, A. Kuhn, Applied Physics B 103 (2011) 579.

[8] Y.-W. Cho, Y.-H. Kim, Optics Express 18 (2010) 25786.

[9] S. Riedl, M. Lettner, C. Vo, S. Baur, G. Rempe, S. Durr, Physical Review A 85 (2012) 022318.

[10] S. Zhou, S. Zhang, C. Liu, J. F. Chen, J. Wen, M. M. T. Loy, G. K. L. Wong, S. Du, Optics Express 20 (2012) 24124.

[11] M.D. Lukin, S.F. Yelin, M. Fleischhauer, Physical Review Letters 84 (2000) 4232.

[12] M. Fleischhauer, M.D. Lukin, Physical Review Letters 84 (2000) 5094.

[13] D.F. Phillips, A. Fleischhauer, A. Mair, R.L. Walsworth, M.D. Lukin, Physical Review Letters 86 (2001) 783.

[14] I.I. Sobelman, Introduction to the Theory of Atomic Spectra, Pergamon, New York, 1972.