The impact of primordial black holes on the 21-cm angular-power spectrum in the dark ages

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We investigate the impact of radiation from primordial black holes (PBHs), in the mass range of $10^{15} \lesssim M_{\text{PBH}} \lesssim 10^{17}$ g and $10^{2} \lesssim M_{\text{PBH}} \lesssim 10^{4} M_{\odot}$, on the 21-cm angular-power spectrum in the dark ages. PBHs in the former mass range effect the 21-cm angular-power spectrum through the evaporation known as Hawking radiation, while the radiation from the accretion process in the latter mass range. In the dark ages, radiation from PBHs can increase the ionization fraction and temperature of the intergalactic medium, change the global 21-cm differential brightness temperature and then effect the 21-cm angular-power spectrum. Taking into account the effects of PBHs, we find that in the dark ages, $30 \lesssim z \lesssim 100$, the amplitude of the 21-cm angular-power spectrum is decreased depending on the mass and mass fraction of PBHs. We also investigate the potential constraints on the mass fraction of PBHs in the form of dark matter for the future radio telescope in lunar orbit or on the farside surface of the Moon.

I. INTRODUCTION

In the standard cosmological model, dark matter makes up about 27% of the Universe. Although many astronomical observations have confirmed the existence of dark matter, its nature has so far been unknown. Among the many dark matter models, weakly interacting massive particles (WIMPs) is the most important one. However, so far, all relevant experiments to detect WIMPs have not found any signs of them. Other dark matter models, such as primordial black holes (PBHs), have attracted extensive attention again. Recently, the gravitational waves generated by the merger of black holes detected by LIGO/Virgo may be partly caused by PBHs.

PBHs can be formed by the collapse of large density perturbation existing in the early Universe and their masses spread a wide range (see, e.g., Refs. [5, 20]). A PBH smaller than $M_{\text{PBH}} \sim 10^{17}$ g loses mass through evaporation due to Hawking radiation. A massive PBH with mass $M_{\text{PBH}} \gtrsim 10^{2} M_{\odot}$ radiates energy in the process of accretion. The extra energy injection from PBHs can effect the evolution of the intergalactic medium (IGM). The changes in the thermal history of IGM will be reflected in, e.g., the anisotropy of the cosmic microwave background (CMB) and the global 21-cm signal.

Recently, the Experiment to Detect the Global Epoch of Reionization Signature reported the detection of the global 21-cm signal centred at redshift $z \sim 17$ with an amplitude twice as large as expected. Although this result needs to be further verified by other experiments, the observation can be used to study the related properties of PBHs, such as limiting their mass fraction. According to the theory, there are also 21-cm absorption signals in the dark ages of the Universe (30 $\lesssim z \lesssim 100$) and these radio signals have been redshifted to the low frequency range ($14 \lesssim \nu_{21} \lesssim 46$ MHz). The Earth’s ionosphere makes it impossible to detect these low-frequency signals from the Earth. Radio telescopes in orbit around the moon or on the farside of the moon have been proposed to avoid the influence of the ionosphere. In Ref. [31], the authors have investigated the effect of PBHs accretion radiation on the global 21-cm signal in the dark ages, and explored the ability of future radio telescopes to limit the mass fraction of PBHs for the mass range of $10^{15} \lesssim M_{\text{PBH}} \lesssim 10^{4} M_{\odot}$. Although the resulting constraints are not the strongest, they are still competitive with that of the lower redshift period because the dark ages are less affected by the formation of cosmic structures. Similar to the anisotropy of the cosmic microwave background, the 21-cm signals can also be studied using angular-power spectrum.

In this paper, we focus on the influence of PBHs on the 21-cm angular-power spectrum in the dark ages. We mainly investigate the radiation from evaporation and accretion process of PBHs, corresponding to the mass range of $10^{15} \lesssim M_{\text{PBH}} \lesssim 10^{17}$ g and $10^{2} \lesssim M_{\text{PBH}} \lesssim 10^{4} M_{\odot}$, respectively. In view of future extraterrestrial radio telescope, we investigate the ability of future detection of the 21-cm angular-power spectrum to limit the abundance of PBHs.

This paper is organized as follows. In Sec. II we investigate the influence of PBHs on the thermal history of the IGM and the global 21-cm signal in the dark ages. The 21-cm angular-power spectrum including PBHs and the future potential upper limits on the abundance of PBHs are discussed in Sec. III. The conclusions are given in Sec. IV. Throughout the paper we will use the cosmological parameters from Planck-2018 results.

1 A PBH with mass $M_{\text{PBH}} \lesssim 10^{15}$ g has a shorter lifetime than the age of the Universe. Here we only consider PBH with mass greater than $10^{15}$ g. It should be pointed out that the lower mass PBH can still affect the 21-cm angular-power spectrum in the dark ages.
II. THE GLOBAL 21-CM SIGNAL IN THE DARK AGES INCLUDING PBHS

A. The thermal history of the IGM including PBHS

The changes in the thermal history of the Universe due to the injection of extra energy have been investigated by previous works (see, e.g., Refs. [59–61]). Here we review the main points and one can refer to, e.g., Refs. [59–61] for more details.

The interactions between the particles emitted from PBHs with that existing in the Universe, result in the changes of the thermal history of the IGM. Taking into account the effects of heating, ionization, and excitation, the changes of the degree of ionization ($x_e$) and the temperature of IGM ($T_k$) with the redshift are governed by the following equations [59, 62]

\begin{align}
(1 + z) \frac{dx_e}{dz} &= \frac{1}{H(z)} [R_e(z) - I_e(z) - I_{PBH}(z)], \quad (1) \\
(1 + z) \frac{dT_k}{dz} &= \frac{8\sigma_T a_B T^4_{CMB} x_e (T_k - T_{CMB})}{3n_e c H(z)} + f_{He} x_e \\
&- \frac{2}{3k_B H(z)} [K_{PBH} + 2T_k], \quad (2)
\end{align}

where $R_e(z)$ and $I_e(z)$ are the recombination and ionization rate for the case of with no PBHs, respectively. The ionization and heating rate caused by PBHs can be written as follows [36, 63, 65]:

\begin{align}
I_{PBH} &= f_i(z) \frac{1}{n_b} E_0 \left[ \frac{dE}{dt} \right]_{PBH} \quad (3) \\
K_{PBH} &= f_h(z) \frac{1}{n_b} \left[ \frac{dE}{dt} \right]_{PBH} \quad (4)
\end{align}

where $n_b$ is the number density of baryon. $E_0$ stands for the ground state energy of hydrogen atom. $f(z)$ corresponds to the energy fraction injected into the IGM for ionization, heating and exciting, respectively. It has been studied detailly in, e.g., Refs. [29, 61, 66], and we use the public code ExoCLASS [67, 68] to calculate $f(z)$ numerically.

For evaporating PBHs, the energy injection rate per unit volume is given by [64, 69]:

\begin{equation}
\left. \frac{dE}{dVdt} \right|_{PBH,eva} = f_{pbh} \rho_{DM} \frac{dM_{PBH}}{dt}, \quad (5)
\end{equation}

where $f_{pbh} = \rho_{PBH}/\rho_{DM}$. Throughout this paper we have adopted a monochromatic PBH mass function for our calculations. The mass-loss rate of a black hole is [3, 21]

\begin{equation}
\frac{dM_{BH}}{dt} = -5.34 \times 10^{25} f(M_{BH}) \left( \frac{M_{BH}}{g} \right)^{-2} \text{ g s}^{-1} \quad (6)
\end{equation}

where $f(M_{BH})$ is the number of particle species emitted directly and we have used the formula given in Ref. [22].

For accreting PBHs, the energy injection rate per unit volume can be written as [29, 60]

\begin{equation}
\left. \frac{dE}{dVdt} \right|_{PBH,acc} = f_{pbh} \rho_{DM} \frac{L_{acc,PBH}}{M_{PBH}}, \quad (7)
\end{equation}

where $L_{acc,PBH}$ is the accretion luminosity, which is proportional to the Bondi-Hoyle rate $M_{BH}$ [29]:

\begin{equation}
L_{acc,PBH} = \epsilon \dot{M}_{BH} c^2, \quad (8)
\end{equation}

where $\epsilon$ is the radiative efficiency depending on the accretion details. The authors of [30] made a detailed analysis of the accretion process of PBHs, finding $\epsilon = 10^{-5}$ ($10^{-3}$) $\dot{m}$ for collisional ionization (photoionization). Here we use $\epsilon = 10^{-5}\dot{m}$ for our calculations, corresponding to the conservative case. $\dot{m}$ is the dimensionless Bondi-Hoyle accretion rate, which is in the form of the Eddington luminosity $L_{Edd}$ as $\dot{m} = M_{BH} c^2 / L_{Edd}$.

![FIG. 1. The changes of the IGM temperature ($T_k$) and the spin temperature ($T_s$) of $z$. The redshifted right: evaporating PBH with mass $M_{PBH} = 10^{-13}$ g and mass fraction $f_{PBH} = 10^{-4}$. Right: accreting PBH with mass $M_{PBH} = 10^4 M_\odot$ and mass fraction $f_{PBH} = 10^{-3}$. For comparison, we also show the plots for the case of with no PBH $f_{PBH} = 0$ (red lines). The CMB temperature ($T_c$) is also shown (brown dashed line).](https://camb.info/)
B. The global 21-cm signal including PBHs

Here we review the main issues about the global 21-cm signal. For more details and in-depth discussion, one can refer to, e.g., Refs. [45, 46] and references therein.

The global 21-cm signal is usually described by the differential brightness temperature \( \delta T_{21} \). Relative to the CMB background, \( \delta T_{21} \) can be written as follows [70, 72]:

\[
\delta T_{21} = 26(1 - x_c) \left( \frac{\Omega_b h^2}{0.02} \right) \left[ 1 + \frac{0.3}{10^{-10} \Omega_m} \right]^{1/2} \times \left( 1 - \frac{T_{\text{CMB}}}{T_s} \right) \text{mK},
\]

(9)

where \( \Omega_b \) and \( \Omega_m \) are the density parameters of baryonic matter and dark matter, respectively. \( h \) is the reduced Hubble constant. \( T_s \) is the spin temperature defined as [45, 46]

\[
\frac{n_b}{n_o} = 3 \exp \left( -\frac{0.068 K}{T_s} \right),
\]

(10)

where \( n_0 \) and \( n_1 \) are the number densities of hydrogen atoms in triplet and singlet states, respectively. Specifically, the spin temperature can be written in the form of a weighted mean of the CMB temperature \( T_{\text{CMB}} \) and the IGM temperature \( T_k \) [70, 73]

\[
T_s = T_{\text{CMB}} + \left( y_a + y_c \right) T_k, \quad (11)
\]

where \( y_o \) corresponds to the Wouthuysen-Field effect and we use the formula given in, e.g., Refs. [63, 73, 74]:

\[
y_o = \frac{P_{10}}{A_{10}} \frac{0.068}{T_k} \frac{e^{-0.3 + \frac{1}{2kT_k}}}{e^{-0.3 + \frac{1}{2kT_k}}} \left( 1 + \frac{0.4}{T_k} \right)^{-1}, \quad (12)
\]

where \( A_{10} = 2.85 \times 10^{-15} \text{ s}^{-1} \) is the Einstein coefficient of hyperfine spontaneous transition. \( P_{10} \) is the radiative de-excitation rate due to Ly\( \alpha \) photons [45, 46]. Taking into account the collisions between hydrogen atoms and other particles, \( y_c \) can be written as [72, 70]

\[
y_c = \frac{0.068(C_{HH} + C_{He} + C_{PH})}{A_{10}T_k}, \quad (13)
\]

where \( C_{HH,He,PH} \) are the deexcitation rates of collisions [72, 74, 70].

The changes of the spin temperature \( T_s \) with redshift are shown in Fig. 1. It can be seen that \( T_s \) becomes larger than that of with no PBH, depending on the mass and mass fraction of PBH. The changes of the differential brightness temperature \( \delta T_{21} \) with redshift are shown in Fig. 2. The amplitude of the 21-cm absorption signal is decreased due to the influence of PBH. For larger mass fraction of PBH with a fixed mass, emission signal appears as shown in Fig. 2 for \( f_{PBH} = 10^{-3} \) with \( M_{PBH} = 10^{16} \text{ g} \).

FIG. 2. The changes of the differential brightness temperature \( \delta T_{21} \) with redshift for mass fraction \( f_{PBH} = 10^{-4} \) (blue solid line) and \( f_{PBH} = 10^{-3} \) (green solid line). Left: evaporating PBH with mass \( M_{PBH} = 10^{16} \text{ g} \). Right: accreting PBH with mass \( M_{PBH} = 10^{4} M_\odot \). The case of with no PBH \( f_{PBH} = 0 \) is also shown (red dashed line).

III. THE 21-CM ANGULAR-POWER SPECTRUM AND UPPER LIMITS ON THE MASS FRACTION OF PBHS

Similar to the CMB anisotropy, the fluctuations of \( \delta T_{21} \) can also be described by the 21-cm angular-power spectrum, which can be calculated by using a standard Boltzmann code. The calculation details of 21-cm angular-power spectrum can be found in Ref. [44] and the numerical code is available in CAMB. Here we have used the public code CAMB for our calculations, which has been used in previous section to investigate the thermal history of the IGM including the effects of PBHs. The 21-cm angular-power spectrum at redshift \( z = 50 \) is shown in Fig. 3.

FIG. 3. The 21-cm angular power spectrum at redshift \( z = 50 \) including PBH with mass \( M_{PBH} = 10^{4} M_\odot \) and \( f_{PBH} = 10^{-3} \) (green solid line). Left: evaporating PBH with mass \( M_{PBH} = 10^{4} M_\odot \). Right: accreting PBH with mass \( M_{PBH} = 10^{4} M_\odot \).

For the case of with no PBH, the amplitude of the 21-cm angular-power spectrum is about \( 1 \sim 3 \text{ mK} \) for \( l \sim 10^3 - 10^5 \). Since the angular-power spectrum is roughly proportional to \( |\delta T_{21}| \) [44, 54], therefore, for the case of with PBH, the angular-power spectrum is decreased depending on the mass and mass fraction of PBH. In Fig. 4 we also show the 21-cm angular-power spectrum for a scale \( l = 1400 \) in the redshift range \( 30 \lesssim z \lesssim 100 \). For the case of with no PBH, the largest amplitude of the angular-power spectrum appears at red-
shift $z \sim 50$ \footnote{Including the effects of PBH, the largest amplitude shifts to the lower redshifts.} Including the effects of PBH, the largest amplitude shifts to the lower redshifts.

The 21-cm signal ($\nu_{21} = 1421$ MHz) from the redshift range $30 \lesssim z \lesssim 100$ has been redshifted into the frequency range $14 \lesssim \nu_{21} \lesssim 46$ MHz. Due to the influence of the Earth’s ionosphere, it is difficult to detect these low frequency signals from the Earth. A radio telescope, either in lunar orbit or on the farside surface of the Moon, has been proposed to detect these radio signals \cite{17, 52}. For a radio telescope, the uncertainty of the $C_l$ at a multipole $l$ is \cite{57, 78, 79}.

$$
\sigma_{C_l} = \sqrt{\frac{2(C_l + C_l^N)^2}{f_{\text{sky}}(2l + 1)}},
$$

where $C_l^N$ is the noise power spectrum \cite{57, 78}.

$$
\ell^2 C_l^N = \frac{(2\pi)^2 T_{\text{sky}}^2}{\Delta \nu t_{\text{obs}} f_{\text{cover}}^2} \left( \frac{l}{l_{\text{max}}} \right)^2,
$$

where $t_{\text{obs}}$ is the observation time, $l_{\text{max}}$ is the maximum multipole observable, and $f_{\text{cover}}$ is the array covering factor. $T_{\text{sky}}$ is the sky temperature \cite{46, 50, 81}.

$$
T_{\text{sky}} = 180 \left( \frac{\nu}{180 \text{ MHz}} \right)^{-2.6} \text{ K}.
$$

For a future radio telescope, e.g., on the lunar surface \cite{78, 82}, with a array size $D \sim 300$ km, the maximum multipole could reach $l_{\text{max}} \sim 10^3$ at redshift $z \sim 50$. Therefore, for the array covering factor $f_{\text{cover}} \sim 0.75$ and bandwidth $\Delta \nu \sim 50$ MHz, the uncertainty of the $C_l$ at $z = 50$ for $l = 1400$ could be $\sigma_{C_l} \sim 0.02$ mK for 1000 hours observation time and $f_{\text{sky}} \sim 1$. Therefore, a large deviation of the 21-cm angular-power spectrum from the default case could be detected for the future radio telescope. On the other hand, future observations of the 21-cm angular-power spectrum can be used to put limits on the abundance of PBHs. Here we will make a simple study of the abundance of PBHs for the future detection, and more detailed studies are left for future work.

Instead of focusing on the sensitivity of a specific radio telescope, we have set $\sigma_{C_l} = 0.1$ and 0.01 mK for our calculations, which could be achieved in the future. Moreover, for simply, we have focused on the maximum sensitivity at a specific scale $l$ instead of all scales \cite{57}. By requiring the deviation of the 21-cm angular-power spectrum less than $\sigma_{C_l}$ for $l = 1400$ at redshift $z = 50$, we find the upper limits on the mass fraction of PBHs $f_{\text{PBH}}$, which are shown in Fig. \ref{fig:5}. For evaporating PBHs, the strongest limit is $f_{\text{PBH}} \sim 10^{-8}$ ($10^{-9}$) for $\sigma_{C_l} = 0.1$ (0.01) mK for $M_{\text{PBH}} \sim 10^{15}$ g. For accreting PBHs, the strongest limit is $f_{\text{PBH}} \sim 6 \times 10^{-5}$ ($10^{-6}$) for $\sigma_{C_l} = 0.1$ (0.01) mK for $M_{\text{PBH}} \sim 10^4 M_\odot$. Note that these constraints are comparable to the existing ones \cite{5}.

Since these limits are from the dark ages, where the influence of astrophysical factors are smaller than that in later period, therefore, future extraterrestrial detection of the radio signal can give very competitive results for limiting the mass fraction of PBHs.

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Note that in obtaining the above limits, we have fixed other cosmological parameters except $f_{\text{PBH}}$. Due to degeneracy, the deviation of the 21-cm angular-power spectrum from the default case can also be caused by the changes in other cosmological parameters \cite{57}. A complete multi-parameter analysis should be carried out by the statistical method such as Markov Chain Monte Carlo, and we will perform these analyses in future work.

IV. CONCLUSIONS

We have investigated the impact of PBHs on the thermal history of IGM and the 21-cm angular-power spectrum in the dark ages. We have focused on the evaporating and accreting PBHs in the mass range of $10^{15} \lesssim M_{\text{PBH}} \lesssim 10^{16}$ g and $10^2 \lesssim M_{\text{PBH}} \lesssim 10^4 M_\odot$, respectively. The radiation from PBHs results in increasing the gas and spin temperature compared with the case of...
with no PBHs. The amplitude of the 21-cm absorption signal is decreased, and emission signal appears for larger mass fraction of PBHs. The fluctuations of the 21-cm differential brightness temperature can be described by the 21-cm angular-power spectrum. Taking into account the effects of PBHs, the 21-cm angular-power spectrum is decreased in the dark ages depending on the mass and mass fraction of PBHs. The peak value of the 21-cm angular-power spectrum appears at redshift $z \sim 50$ for the case of with no PBHs, and shifts to the lower redshifts including PBHs.

The 21-cm signals from the dark ages have been redshifted into the lower frequency $\nu_2 \lesssim 46$ MHz ($30 \lesssim z \lesssim 100$). It is difficult to detect these radio signals from the Earth due to the influence of Earth’s ionosphere. Extraterrestrial radio telescopes, such as in lunar orbit or on the lunar surface, have been proposed. In view of the future radio telescopes, we have estimated the upper limits on the mass fraction of PBHs. Instead of focusing on a specific radio telescope, we have set the uncertainty $\sigma_l = 0.1$ and 0.01 mK, which can be achieved in the future, for our calculations. For a evaporating PBH with mass $M_{PBH} \sim 10^{18}$g, the upper limit is $f_{PBH} \sim 10^{-8}$ ($10^{-9}$) for $\sigma_l = 0.1$ (0.01) mK. For a accreting PBH with mass $M_{PBH} \sim 10^4$ $M_\odot$, the upper limit is $f_{PBH} \sim 6 \times 10^{-5}$ ($10^{-6}$) for $\sigma_l = 0.1$ (0.01) mK. Compared with the cosmic dawn, the dark ages is less effected by the astrophysical factors. Therefore, the detection of the 21-cm signal or angular-power spectrum in the dark ages will be of great significance to reveal the related properties of PBHs.

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