Hydrodynamics and Nonlocal Conductivities in Vortex States

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A hydrodynamical description for vortex states in type II superconductors with no pinning is presented based on the time-dependent Ginzburg Landau equation (TDGL). In contrast to the familiar extension of a single vortex dynamics based on the force balance, our description is consistent with the known rotating superfluid hydrodynamics and, at long distance limit, with that following from the Landau level approach for high field case, and enables one to study nonlocal conductivities perpendicular to the field in terms of Kubo formula. Typically, the nonlocal conductivities deviate from the usual vortex flow expressions as the nonlocality parallel to the field becomes weaker than that perpendicular to the field measuring a degree of positional correlations, and, for instance, the dc Hall conductivity nonlocal only in directions perpendicular to the field becomes zero in the vortex lattice with infinite shear viscosity. Various situations are discussed based on the resulting expressions.

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1. Introduction

At present, understanding theoretically the nonlocality \([1,2]\) of linear resistivity in a clean system (with no pinning) seems to be one of central problems on the vortex states of a type II superconductor. By analogy to the usual viscous fluid, Marchetti and Nelson \([1]\) have previously proposed within a linear hydrodynamics that, deep in the liquid regime, a shear-viscous force proportional to \(-\partial_\perp^2 \partial_t s\) will take the place of the shear term in the elastic force \(f_{\text{el}}\) in the following force-balance equation for the vortex lattice

\[
\partial_t (-\Gamma_1 s + \Gamma_2 (\hat{z} \times s)) + J \times B = f_{\text{el}},
\]

and that consequently, a positive nonlocal term proportional to \(q_\perp^2\) should exist in the conductivity perpendicular to \(B\). Here \(s\) denotes the displacement field of vortex positions, \(J\) the external current, \(B\) the uniform flux density parallel to \(z\) axis, \(q_\perp\) the wavevector perpendicular to \(B\), and \(\Gamma_1\) and \(\Gamma_2\) will be given later. This equation is essentially an extension \([3]\) of the single vortex dynamics to interacting vortex states. Subsequently, the presence of other nonlocal terms was proposed \([2]\) even in the case with no sample disorder on the basis of experiments in heavily twinned samples \([4]\), and, deep in the liquid regime, the presence of a large conductivity nonlocal in the field direction was argued. It is important to \textit{theoretically} clarify to what degree these proposals can be justified beyond the phenomenology \([1,2,5]\) and within a basic dynamical model for type II superconductors such as the time dependent Ginzburg-Landau equation (TDGL) which reasonably describes the \textit{uniform} linear dissipation in terms of Kubo formula \([6]\). Actually, the eq. \((1)\) is not correct in the following senses. Firstly, it cannot be used in calculations of conductivity based on the use of Kubo formula \([5,6]\). Secondly, it is not clear from \((1)\), in which \(f_{\text{el}}\) is independent of \(J\) and of the time derivative \(\partial_t\), how the possible viscous (nonlocal) terms in the dc conductivity are changed as the vortex lattice is formed. Further, the dispersion \(\sim q_\perp^4\) of the shear mode found as a phase fluctuation formally and generally \([7,8]\) cannot be seen in eq. \((1)\).

In the present paper, a correct hydrodynamical approach, or equivalently, a dynamical harmonic analysis on the transport in a vortex state, particularly the vortex lattice, is presented on the basis of TDGL. Our approach has no difficulties mentioned above and makes it possible to study nonlocal conductivities above and below the melting transition with no pinning, and several consequences on the resulting conductivities are discussed. As has been understood through studies \([9]\) on nonlocal linear and static responses, the use of the harmonic analysis for 3D and layered systems often lead to misunderstandings on physics in the liquid regime, and hence we will not consider situations with negligibly small positional correlations of vortices.

2. TDGL hydrodynamics

Our starting point is TDGL describing dynamics of the order parameter \(\psi\)

\[
a(\gamma + i\gamma')\partial_t \psi + [b(|\psi|^2 - \rho_0) + a \xi_0^2 (-i\partial - \frac{2\pi}{\phi_0} A)^2] \psi = 0,
\]

where \(\text{curl}A = B \hat{z}\), \(\gamma\), \(a\), and \(b\) are positive, \(\phi_0\) is the flux quantum, \(\xi_0\) the coherence length, and \(\rho_0\) is the spatial average of the mean squared order parameter. Since the linearized
version of (2) is considered throughout the paper, the noise term introducing fluctuations in equilibrium was neglected, and growths of time scales will be phenomenologically included later as the only fluctuation effects. Within the harmonic analysis given below, spatial variations of the gauge field have to be neglected in a consistent sense with the corresponding derivation of the infinite diamagnetic susceptibility in the Meissner state.

Particularly upon calculations of linear responses, it is convenient to work rather within the harmonic approximation of the corresponding ‘quantum’ action with the gauge field disturbance $\delta A$ (and with $\hbar = 1$)

$$S_{\text{har}} = \int_\tau \left[ \beta \sum_\Omega a(\gamma|\Omega| + i\gamma'|\Omega)|\psi_\Omega|^2 + \int_\tau \left[ a\xi_0^2(-i\partial - \frac{2\pi}{\phi_0}(A + \delta A(\tau)))\psi(\tau)\right]^2 \right.$$

$$+ b(2|\psi_0|^2|\psi(\tau)|^2 + \frac{1}{2}(\psi_0^*\psi^2(\tau) + \text{c.c.}))\right].$$

(3)

Since we are interested only in frequencies and wavevectors accompanying $\delta A$ and summations with respect them are not performed, examinations based on (3) are equivalent to the linearized analysis of (1) and formally independent of temperature. Here $\tau$ is the imaginary time, $\psi_\Omega$ is the temporal Fourier transform of $\psi(\tau)$, $\Omega$ is Matsubara frequency, $\beta$ the inverse temperature, and $\psi_0$ the mean field solution of $\psi$. The superconducting (and real) part $\sigma_{ij}$ of dc linear conductivity tensor should be always calculated in terms of Kubo formula

$$\sigma_{ij}(q) = \left. \frac{\partial \rho_s(q, i\Omega_0)}{\partial \Omega_0} \right|_{\Omega_0 \to +0} = \left. \frac{\partial}{\partial \Omega_0} \frac{\delta^2(-\beta^{-1}\ln \text{Tr}_\psi \exp(-S_{\text{har}}))}{\delta A_i(q, \Omega_0) \delta A_j(-q, -\Omega_0)} \right|_{\Omega_0 \to +0},$$

(4)

where $\Omega_0$ is the external (Matsubara) frequency, $i, j = x$ or $y$, and the absence of dc uniform ($q = 0$) superfluid density was used (see below). Since the ac and uniform case of (4) always becomes the usual vortex flow expression in the present formulation, the dc conductivity will be only considered hereafter. Since the harmonic modes appearing in (4) are spatial and temporal variations around the mean field solution linearly excited by the gauge field disturbance, the linear response quantities in vortex states resulting from such a harmonic analysis [10] are not accompanied by the temperature $\beta^{-1}$, implying that this approach is not applicable to the linear dissipation parallel to $B$, which is a consequence of critical fluctuations [6]. Therefore, considerations are largely focused on the vortex flow configuration.

Before presenting general results, it is useful to first see results on the linear conductivity perpendicular to $B$ in the Landau level (LL) approach within the lowest and next-lowest LLs (see sec.4 in Ref.7). In such a high field approximation the mean field solution and the shear elastic mode are found within the lowest LL, and the next-lowest LL, which does not participate in constructing the mean field solution [7, 9], provides the uniform displacement of vortices. Since, in constructing basis functions of eigenmodes, Matsubara frequencies in the ‘quantum’ action play similar roles to the wavevector parallel to $B$, including dynamical terms is easily performed following Ref.7. For later convenience an amplitude-dominated mode in the lowest LL, denoted by $\delta \rho$, will also be included. Then,
to the lowest order in \( q_\perp \), the action (3) becomes \( S_{\text{LL}} = S_0 + S_1 \), where

\[
S_0 \approx \beta \alpha \sum_{r} \int \left( \gamma \rho_0 |\Omega||\chi_\Omega|^2 + \gamma' \Omega \delta \rho_{\Omega}^* \chi_\Omega + \frac{b}{2a} |\delta \rho_\Omega|^2 \right) + \rho_0 \xi_0^2 |\partial_\perp \chi_\Omega| \left( 2 |\partial_\perp \chi_\Omega|^2 + C_{66} \frac{4}{2a} r_B^2 |\partial_\perp \chi_\Omega|^2 \right),
\]

\[
S_1 \approx \frac{\beta \rho_0 a}{2r_B^2} \sum_{r} \int \left( \gamma |\Omega||s_0|^2 + \gamma' \Omega (s_0^* x s_0) z + \xi_0^2 |\partial_\perp s_0|^2 \right) + \frac{2 \xi_0^2}{r_B^2} |s_0| + B^{-1} (\delta A_{\perp 0} \times \hat{z}) (\hat{z}), \tag{5'}
\]

where \( s^T \) (the transverse component of \( s \)) is given by \( r_B^2 (\partial_\chi \times \hat{z}) \) with vortex spacing \( r_B = \sqrt{\phi_0/2\pi B} \), \( s_0 \) the uniform displacement with vanishing \( q_\perp \), \( \chi \) a longitudinal phase variable, and \( C_{66} \) the resulting shear modulus. Variational equations with replacement \( i \Omega \rightarrow \omega \) give eigenmodes of the vortex lattice. Note that the resulting dispersion \( \omega(\text{real frequency}) = -i (a \gamma \rho_0)^{-1} C_{66} (q_\perp r_B)^4 \) for the shear mode[7,8] is different from that following from (1), and hence that the extrapolation [1,3] of the uniform (or equivalently, the single-) vortex dynamics to the shear mode is not justified within TDGL. Through the London limit (7) given below, the action (5) with the constraint \( s^T = r_B^2 (\partial_\chi \times \hat{z}) \) is found to correctly give hydrodynamical results at longer distances than

\[
l = r_B \sqrt{C_{66} r_B^2 / 2a \rho_0 \xi_0^2},
\]

which is typically of the order \( r_B \).

The absence of the dc uniform superfluid density (helicity modulus) is obvious from (4) and (5'), implying that a constant twist pitch \( \delta A_0 \) is transmuted into an uniform displacement of vortices keeping the free energy invariant. That is, situation is different from an elastic matter with free energy invariant with respect to uniform displacements. From (4) and (5'), we easily obtain the mean field expressions on diagonal \((i = j)\) and Hall \((i \neq j)\) vortex flow conductivities

\[
\sigma_{ij} (q_\perp = 0, q_\parallel) = \rho_{s0} \frac{r_B^2}{2 \xi_0^2} \left( \frac{\gamma \delta_{ij} + \gamma' \varepsilon_{ij}}{1 + r_B^2 q_\parallel^2 / 2} \right), \tag{6}
\]

where \( \rho_{s0} = 2(2\pi \xi_0 / \phi_0)^2 \rho_0 a \) is the mean field superfluid density in zero field. Due to the constraint mentioned above, (5) does not give any nonlocal (and longitudinal) corrections to (6) associated with the freezing to the vortex lattice. Consistently with (6), the nonlocal superfluid density, namely the real part of the linear response function \( \rho_s(q, \omega + i0^+) \), becomes \( \rho_s(q_\perp = 0, q_\parallel) \simeq \rho_{s0} r_B^2 q_\parallel^2 / 2 \) in dc limit. As pointed out in Ref.9, this \( q_\parallel^2 \) behavior does not change through a 3D melting transition [11].

Next let us here comment on eigenmodes of (3) in 2D and nondissipative \((\gamma = 0)\) case, corresponding to the rotating superfluid \( ^4\text{He} \) at zero temperature described according to
the Gross-Pitaevskii equation. Consistently with Ref.12 and 13, the order parameter in (3) will be divided into the amplitude $\rho$ and phase $\varphi$; $\psi = \sqrt{\rho} \exp(i\varphi)$ and, just as in the usual London approximation, the presence of the field-induced vortices will be taken into account through the topological condition[14] on $\partial_\mu \varphi = (\partial_\varphi, \partial_\tau \varphi)$ by neglecting any fluctuation-induced vortices. As a result, we obtain the following harmonic action

$$S_{\text{ph}} = a \int_{r, \tau} (i\gamma' \delta \rho \partial_\tau \chi + \rho_0 \xi_0^2 (\partial \chi - r_B^{-2} (\dot{z} \times s))^2 + \frac{b}{2a} \delta \rho^2 + \frac{C_{66}}{2a} (\partial_\tau s^T)^2).$$

(7)

The only difference of this action from (5) is that the longitudinal current

$$\mathbf{v}_s = 2a \xi_0^2 (\partial \chi - r_B^{-2} (\dot{z} \times s^T)) \equiv 2a \xi_0^2 \mathbf{\dot{v}}$$

is nonzero in (7), which is necessary in considering longitudinal linear responses nonlocal in directions perpendicular to the field. When $q/\ell > 1$, the minimal coupling in (8) between $\chi$ and $s^T$ is removed, and in principle the phase fluctuation behaves like in zero field case at such short lengths. Variational equations resulting from (7) agree with those following from an ‘em’ analogy in Ref.12, where an importance of $v_s$ at nonzero frequency was stressed in a context of superfluid $^4$He. The action (7) does not include the term

$$S_{\text{inc}} = i \int_{r, \tau} \gamma'' \rho a \frac{\rho a}{2 r_B^2} (s \times \partial_\tau s)_z,$$

(9)

which is necessary in recovering the limit of incompressible fluid (i.e., $\delta \rho = 0$ and $v_s = 0$). This additional term with the coefficient $\gamma''$, which should become $\gamma'$, cannot be detected in this phase-only analysis for (3) (Hereafter, $\gamma''$ will be assumed to be equivalent to $\gamma'$). The eigenvalues following from $S_{\text{ph}} + S_{\text{inc}}$ precisely coincide, when $\gamma' = 1$, with those derived by Sonin[13] and give dispersions $\omega_{sh}^2 \simeq (a \gamma')^{-2} b C_{66} (q \perp r_B)^4 / (1 + (q \perp l_{\text{inc}})^2)$ and $\omega_s^2 \simeq (2 \xi_0^2 / \gamma' r_B^2)^2 (1 + (q \perp l_{\text{inc}})^2)$ of two modes corresponding to the shear (massless) and compression (massive) elastic modes, respectively, where $l_{\text{inc}} = \sqrt{b \rho_0 / 2a r_B^2} / \xi_0$. The quadratic dispersion of $\omega_{sh}$ at low $q \perp$ is an origin of the destruction [7, 8] of the long-ranged phase coherence in the vortex lattice. The variational equation, with replacement $-i\tau \rightarrow t$(real time), with respect to the shear displacement of $S_{\text{ph}} + S_{\text{inc}}$ becomes the transverse part of (1) with no external current if $\Gamma_1 = 0$, $\Gamma_2 = a \rho_0 \gamma'' 2\pi B / \phi_0$, and $v_s = 0$. In this case the constraint $v_s = 0$ does not imply that in (5) but rather corresponds to the limit of incompressible fluid. Following [13], this limit is valid only at shorter lengths than $l_{\text{inc}}$, which is at most of the order of several vortex spacings in a type II superconductor near the melting line in a moderate field.

The above agreement with the well-known rotating superfluid hydrodynamics justifies the presence of the minimal-coupling in $v_s$ between the phase field $\chi$ and shear displacement $s^T$. Based on this, we assume below that the $v_s^2$ term appearing in London limit(7) will be found at the static level by summing up many higher LLs in GL harmonic analysis around the mean field state. When combining this with (5) and (5)', we are naturally led to invoking the following action [15] appropriate to examining nonlocal conductivities:

$$S_{\text{nl}} = \beta \sum_q \int_G \rho_0 a |\Omega| (\gamma |\chi_q|² + \tilde{\gamma}_q r_B^{-2} |s_q|²) + \frac{C_{66} (q, |\Omega|)}{2 q}\frac{r_B^{-2} |s_q|²}{|s_q|²}$$

5
\[ + a \xi^2 \rho_0 \int_r \left[ |\tilde{\Omega} - \frac{2\pi}{\phi_0} \delta A_{\perp,\perp}|^2 + r_B^{-2} |s_{\perp,\perp} + B^{-1}(\delta A_{\perp,\perp} \times \tilde{z})|^2 \right] \\
+ \frac{r_B^{-2}}{2} |\partial_s \Omega|^2 + |\partial_s \chi - \frac{2\pi}{\phi_0} \delta A_{\perp,\perp}|^2, \]  

(10)

where \( \delta A_{\perp} (\delta A_{\perp}^T) \) denotes the longitudinal (transverse) part of the gauge disturbance defined within \( x-y \) plane. For later convenience the shear modulus is assumed to have possible frequency and wavevector dependences, and the time scale of \( s \) was phenomenologically changed taking account of a possibility that it may have nonlocal corrections \( \tilde{\gamma}_q - \gamma > 0 \), due to an origin other than the freezing to the vortex lattice \([1,2]\) and irrelevant to statics of vortex states at long distances. The amplitude mode leading to a \( \Omega^2 |\chi|^2 \) term was neglected. The term \( (9) \) has to be included when deriving a nonlocality of Hall conductivity. Again, the variational equation with respect to \( s^T \) of (10) is different from (1) with \( \Gamma_1 = a \rho_0 \gamma 2\pi B/\phi_0 \) due to the presence of nonzero \( v_s \). Further, by neglecting the gauge disturbance \( \delta A \) and examining the dispersion of the shear mode in the vortex lattice, it is found that, due to the presence of the first term in (10), setting \( v_s = 0 \) from the outset, as in (1), always leads to an erroneous result on the dispersion. Rather, at low enough \( q \), the constraint \( v_s \simeq 0 \) is established, and consistently, the second term in (10) becomes unimportant compared to the first term. The first term in (10) having the same form as in the lowest LL case (5) is required within TDGL formalism and actually is justified because the \( \chi \) variable to be related to the shear displacement at small but nonzero \( q \) was shown\([7]\) to, up to the lowest order in \( q_\perp \), become a phase change around \( \psi_0 \) even if higher LLs are \textit{fully} included.

### 3. Nonlocal Conductivities

It is straightforward to, using (4) and (10), find the nonlocal conductivities. First, let us discuss the vortex lattice where \( C_{66}(q = 0, \Omega = 0) \neq 0 \). In this case the longitudinal (\( \parallel q_\parallel \)) part of the diagonal conductivity and the Hall conductivity are given by

\[
\sigma_{xx}^{(s)}(q) = \rho_s \frac{r_B^2}{2\xi_0^2} \frac{\tilde{\gamma}_q q_\parallel^4 + \gamma q_\perp^4 l^2(q_\perp^2 + 2(lq_\perp/r_B)^2)}{(q_\parallel^2(1 + q_\parallel^2 r_B^2/2) + l^2q_\perp^2)^2}, \\
\sigma_{xy}^{(s)}(q) = \rho_s \frac{r_B^2}{2\xi_0^2} \frac{\gamma' q_\parallel^2}{(1 + q_\parallel^2 r_B^2/2)(q_\parallel^2(1 + q_\parallel^2 r_B^2/2) + q_\perp^2 l^2)}.
\]

(11)

using the length \( l \propto \sqrt{C_{66}} \) defined previously. The transverse part \( \sigma_{xx}^{(s)T} \) of the diagonal conductivity merely becomes, even in the liquid regime, \( \tilde{\gamma}_q \sigma_{xx}(q_\parallel = 0, q_\parallel)/\gamma \) (see (6)), and hence, in contrast to (11), is not affected by the positional correlation \([16]\). We note that the term \( \sim q_\parallel^2 + q_\perp^4 l^2 \) in denominators is the dispersion of the 3D shear (Goldstone) mode destroying the true off-diagonal long ranged order \([7,8]\). In general, the magnitudes of these conductivities significantly depend much on the \textit{relative} size of \( |q_\parallel| \) and \( |q_\perp| \). For instance, when \( |q_\parallel| > l|q_\perp|/r_B \sim |q_\perp| \), both \( \sigma_{xx}^{(s)L} \) and \( \sigma_{xy}^{(s)} \) are well approximated by \( \sigma_{ij} \) given in (6) with replacement \( \gamma \to \tilde{\gamma}_q \). On the contrary, when \( |q_\parallel| < lq_\perp^2 \), \( \sigma_{xx}^{(s)L} \) typically becomes the value \( \gamma \rho_s/2q_\perp^4 \xi_0^2 \) at zero \( q_\parallel \) (or in 2D case). This expression is the same as the
2D result in zero field \[17\](see also \[5\]), reflecting the fact that it arises entirely from the phase variation \(\chi\). Correspondingly, \(\sigma_{xy}^{(s)}\) decreases like \(\sim (q_{||}/q_{\perp}^2)^2\) and vanishes at zero \(q_{||}\) or in 2D case. In other words, as the nonlocality parallel to \(B\) becomes negligible compared to that perpendicular to \(B\), contributions of a finite \(\tilde{\gamma}_q\) are covered and overcome by the perfect positional correlation, i.e., the presence of the Goldstone mode. This feature is lost, at fixed \(q_{\perp}\) and \(q_{||}\), as \(l\) becomes shorter, i.e., with increasing field, which is consistent with the absence of the nonlocal conductivity induced by the positional correlation within the lowest LL (see a sentence following \((6)\)). In addition, note that, as seen in \(q_{||} \rightarrow 0\) limit, the infinite shear viscosity of the vortex lattice does not mean a zero value of the nonlocal resistivity. We emphasize that the remarkable difference between the uniform limit, the infinite shear viscosity of the vortex lattice does not mean a zero value of the nonlocal resistivity. 

When trying to understand the liquid regime (disordered state) in the present approach, some comments are necessary. As pointed out in Ref.9, the harmonic analysis cannot be used in the situation where the contributions with nonzero reciprocal lattice vectors are negligible even if the amplitude fluctuation of the order parameter is negligible. In such situations the nonvanishing transverse diamagnetic susceptibility results from the static superconducting fluctuation and, in layered systems, shows a dimensional crossover due to a competition between the layer spacing and a finite phase coherence length which cannot be found in the harmonic analysis. Consistently, this dimensional crossover will be seen also in the nonlocal conductivity. On the other hand, it is practically difficult at present to understand possible nonlocal corrections, just above the freezing point to the vortex lattice, to the vortex flow expression \((6)\) according to the fluctuation theory \([6]\). For these reasons, we will only consider the case just above the freezing point to the vortex lattice and with vanishing \(q_{||}\) (see, however, the last section).

In this case, the corresponding results to \((11)\) are given by

\[
\sigma_{xx}^{(l)}(q_{\perp},q_{||} = 0) = \rho s_0 \frac{r_B^2}{2 \xi_0} \frac{\tilde{\gamma}_q + \eta q_{\perp}^2 q_B^2 / \rho_0 a}{1 + q_{\perp}^2 q_B^2 (\tilde{\gamma}_q + \eta q_{\perp}^2 q_B^2 / \rho_0 a) / 2 \gamma} \gamma',
\]

\[
\sigma_{xy}^{(l)}(q_{\perp},q_{||} = 0) = \rho s_0 \frac{r_B^2}{2 \xi_0} \frac{\tilde{\gamma}_q + \eta q_{\perp}^2 q_B^2 / \rho_0 a}{1 + q_{\perp}^2 q_B^2 (\tilde{\gamma}_q + \eta q_{\perp}^2 q_B^2 / \rho_0 a) / 2 \gamma} \gamma',
\]

where the Maxwell form \[18\] \(C_{66}(q,\Omega) \sim \eta |\Omega|\) with shear viscosity \(\eta\) for the dynamical shear modulus was assumed. Interestingly, the expressions \((12)\) smoothly lead to \((11)\) in zero \(q_{||}\) for the vortex lattice by taking \(\eta\) to be infinity. The terms \(1 + \eta(q_{\perp} r_B)^4 / 2 \gamma \rho_0 a\)
in the denominators of (12) again originates from the spectrum \( \omega \sim -iC_{66}(q_{\perp}r_B)^4 \) of the shear mode, which cannot be found from (1). Consequently, at high \( q_{\perp} \) of the order \( r_B^{-1} \), both (11) and (12) are well approximated by (6), suggesting that, for such a rapid variation of the external current, not only collective effects but also the sample disorder is irrelevant because the time scale to be affected by the sample disorder (pinning) will be \( \tilde{\gamma}_q \) and not be \( \gamma \). In general, in the case with a pinning effect where \( \tilde{\gamma}_q \) will grow, the viscous effect accompanying the freezing transition becomes relatively negligible. Further, the expressions (12) suggest that the low temperature limits of the nonlocal conductivities for small \( q_{\perp} \) in 2D systems with pinnings, where we have no transition at nonzero temperatures, again become the corresponding (and above-mentioned) results in the vortex lattice independent of \( \tilde{\gamma}_q \).

4. Comments and Conclusion

Finally, we will discuss consequences of possible viscous effects [1, 2] in pinning-free systems, other than \( \eta \), which should appear in nonlocalities of \( \tilde{\gamma}_q \). According to (11) and (12), when \( |q_{\parallel}| \ll |q_{\perp}| \), it is difficult to practically divide contributions of \( \tilde{\gamma}_q \) from the positional correlation, and hence, we will only consider the case with vanishing \( q_{\perp} \) but nonzero \( q_{\parallel} \), where the diagonal conductivities can be always expressed by (6) with replacement \( \gamma \to \tilde{\gamma}_q \). Further we will focus on the 3D region below a dimensional crossover suggested in sec.3 (We note that this dimensional crossover is quite different from that argued through experiments in Ref.4). For this case, it is often argued [19] that thermally activated cutting (and reconnection) processes among vortices induced by a nonuniform \( (q_{\parallel} \neq 0) \) current will lead to a very long length scale and significantly increase \( \tilde{\gamma}_q \) even if the (thermally-induced) entanglement is absent [2]. This picture suggests that the time scale grows unlimtedly even in the vortex lattice under cooling, and hence that the bent vortices cannot move at low enough temperatures. It should be noted that such an argument is also applicable to a nondissipative case (with zero \( \gamma \) but nonzero \( \gamma' \)) by imagining a nonlocal growth of \( \gamma' \). However, it is even unclear to us if this is a correct argument in the context of the linear hydrodynamics. In superfluid \( ^4 \)He[20] the reconnection process does not seem to need a remarkable slow dynamics. In addition, the system-size dependence of a temperature [4, 21] characteristic of apparently nonlocal vortex motions is inconsistent with the argument based on the thermally activated vortex cutting processes, because the measured size dependence inevitably means an algebraic decrease of the cutting barrier with increasing the system-size, although intuitively such a decrease of the barrier should not be expected. In relation to this issue, we emphasize that viscous terms appearing in \( \tilde{\gamma}_q \) cannot reduce to elastic (and static) terms in the vortex lattice and that the (if any) entanglement [1] making a growth of \( \tilde{\gamma}_q \) possible must disappear deep in the vortex lattice [22]. In Ref.9, the presence of a large and positive nonlocal (\( \sim q_{\parallel}^2 \)) contribution in \( \tilde{\gamma}_q \) was questioned on the basis of the observation that, in contrast to the pinning-induced activation form of the time scale, any nonlocal (i.e., wavevector-dependent) growth of the time scale may be incompatible with the uniform vortex flow due to the nonlinearity in Kubo formula in the nonGaussian fluctuation theory [6]. Nevertheless, a possibility of a large \( \tilde{\gamma}_q \) for nonzero \( q \) should be further searched in an extension of the fluctuation theory [6] which is not available at present. Experimentally, measurements in twin-free samples corresponding to those in [4, 21] should be performed. Actually, the data on the resistivity
parallel to $B$ in [21] seem to intuitively contradict the data in a more 3D-like situation [23].

In conclusion, a hydrodynamics for vortex states in type II superconductors consistent with the rotating superfluid hydrodynamics has been presented in order to study the nonlocal conductivities perpendicular to the applied field. In particular, the vanishing Hall conductivity and divergent diagonal conductivity in the case with nonlocality in perpendicular directions to the field will be applicable to (if any) 2D hexatic state and useful in experimentally judging its existence or absence in type II superconductors.

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[11] This means that the elastic terms associated with the compression mode are not affected by the 3D melting transition (see [9]). It also invalidates a corresponding result $\sim |q_0|$ in a putative liquid phase [Feigel’man and Ioffe, preprint] even if the fluctuating magnetic field is neglected, because it would mean that such an intermediate phase has a stronger ordering than in the vortex lattice.
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[13] See (4.71) to (4.73) in E.B.Sonin, Rev.Mod.Phys.59, 87(1987). Parameters $c_T$, $c$, and $\Omega$ appearing in this reference are given using our notation by $c_T^2 \equiv 2\xi_0^2C_{06}/a\rho_0\gamma'^2$, $\Omega \equiv \xi_0^2/r_B^2\gamma'$, and $c^2 \equiv 2b\rho_0\xi_0^2/a\gamma'^2$.
[14] See Appendix D in [7], and V.N.Popov, Functional integrals in Quantum Field Theory and Statistical Physics(Reidel, Dordrecht, 1983).
[15] See also Appendix D in [7], and A.I.Larkin and Y.N.Ovchinikov, J.Low Temp.Phys. 34, 409(1979).
[16] Correspondingly, we have two kinds of the transverse parts of diamagnetic susceptibility due to the presence of two (elastic) modes, and they are given by $\chi_{\perp}^{(s)} = r_B^2\rho_s|q_0|^2 +$
\[ 2(lq_B + rB)^2/[q_B^2(2 + (qr_B)^2) + 2l^2q_B^4] \] and \( \chi^{(c)}_B \approx r_B^2 \rho \rho_{\parallel}/2 \) for the shear and compression modes, respectively. Note that the divergent behavior of \( \chi^{(s)}_B(q_B = 0, q_B \rightarrow 0) \) does not contradict the vanishing superfluid density perpendicular to \( B \). Since we have neglected in (5) and (10) an \( O(q_B^2 s L^2) \) term playing no roles in nonlocal linear dissipations, the longitudinal susceptibility is zero in the present approach.

[17] H.J. Mikeska and H. Schmidt, J. Low Temp. Phys. 2, 371 (1970).
[18] For instance, V.M. Vinokur et al., JETP 73, 610 (1991).
[19] For instance, C. Carrano and D. S. Fisher, Phys. Rev. B 51, 534 (1995).
[20] K.W. Schwartz, Phys. Rev. B 38, 2398 (1988).
[21] D. Lopez et al., Phys. Rev. B 50, 7219 (1994).
[22] In addition, the picture in [1] that the thermally-induced entanglement will become more remarkable with increasing field is questionable, because a situation dominated by the lowest LL mode, in which the entanglement is absent (see [9] and M.J.W. Dodgson and M.A. Moore, preprint), must become valid with increasing field.
[23] W.K. Kwok et al., Phys. Rev. Lett. 64, 966 (1990).