Integrated petrographical and petrophysical studies of sandstones from the Araba Formation for reservoir characterization

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Abstract
The present study aims at an integration of both petrographic and petrophysical data of 45 sandstone samples collected from Araba Formation (Lower Paleozoic) for reservoir characterization. The studied Araba Formation is classified into three types of facies: (i) quartz arenite, (ii) sublithic quartz arenite, and (iii) calcareous quartz arenite. The mineralogical composition of the studied sandstone samples includes mainly quartz and detrital clays, some pyrite, and iron oxides as cement. The petrographic investigation of the studied Araba Formation reveals various diagenetic processes including compaction and pressure solution, cementation with iron oxides, carbonates, or clays. The porosity values for investigated samples range from 14.5 to 27.8% and permeability values from 2.4 mD to 8.4 D, the surface area per unit pore volume \( S_{\text{por}} \) from 0.011 to 17 \( \mu \text{m}^{-1} \), the formation factor from 6 to 13, and the dimensionless volumetric magnetic susceptibility from 13 \( 10^{-6} \) to 126 \( 10^{-6} \). The samples of facies (i) and (ii) are characterized by higher values of porosity and permeability. The samples of facies (iii) indicate increasing permeability with rising porosity. The relationship between permeability and formation factor reflects the effect of porosity changes. The specific surface \( S_{\text{por}} \) shows an inverse relationship to permeability. The petrophysical investigations provide relationships between permeability and other reservoir parameters such as porosity, formation factor, specific surface, and longitudinal and transverse relaxation time resulting from nuclear magnetic resonance (NMR) relaxometry. The permeability prediction model based on NMR data proves to be applicable for samples from facies (i) and (ii), whereas the predictive quality is considerably lower for facies (iii).

Keywords Permeability · Magnetic susceptibility · Formation factor · Specific surface · Nuclear magnetic resonance

Introduction
Petrophysical properties such as permeability and porosity of reservoir rocks control the storage and flow of fluids in the pore space and therefore the production of producible fluids in oil, gas, or water reservoirs. Both porosity and permeability can be easily determined in the laboratory. Permeability is a physical quantity that is described by Darcy’s law for the flow of viscous fluids in porous media. Permeability is controlled by many factors such as pore space geometry, connectivity, cementation, rock texture, grain size, and grain shape. A positive correlation is mostly observed between porosity and permeability. However, high rock porosity does not guarantee a measurable permeability. Pore throats that enable the flow of fluids must interconnect the individual pores. Whereas pumice stone and shale are characterized by a very high porosity but very low permeability, micro-fractured carbonates show low porosity but high permeability. A pore network consists of larger spaces, which are referred to as pore bodies, and smaller pore spaces, which are referred to as pore throats. In other words, the volume fraction of pore space is reflected by the measured porosity, while the measured permeability is related to the size of pore throats. The geometric relationship between pore spaces and pore throats controls the relationship between porosity and permeability. Many authors (e.g., Kozeny 1927; Carman 1937; Timur 1968; Scheidegger 1974; El Sayed 1981;
Herron 1987; Adler et al. 1990; Schön 1996; Tiab and Donaldson 2015) have studied the relationship between porosity and permeability.

Many authors used hydraulic flow units (HFUs) for permeability modeling and rock typing (e.g., Farouk et al. 2021, McDougall and Teama 2018). In the present study, we apply the concept of HFU for reservoir quality classification. This concept, which was proposed by Amaefule et al. (1993), is based on the parameters reservoir quality index (RQI) and flow zone indicator (FZI).

Coates et al. (1999) published a relationship between permeability and nuclear magnetic resonance (NMR) longitudinal relaxation time $T_1$ and porosity. A similar equation was proposed by Straley et al. (1997) using the transverse relaxation time $T_2$. The predictive quality of different models is evaluated by a comparison between measured and predicted permeability.

The present work aims at a detailed investigation of the reservoir properties of three sandstone facies of the Lower Paleozoic Araba Formation. The petrographical investigations provide a detailed description of the different facies with special emphasis on mineralogical composition, rock texture, and cementing materials. The findings enable an interpretation of the paleoenvironmental conditions that prevailed during the deposition of these sediments. The petrophysical study is mainly devoted to analyzing core samples collected from sandstones of the Araba Formation for studying the effects of petrophysical properties on the permeability. We apply petrophysical models to predict permeability from nuclear magnetic resonance (NMR) data. The study improves the prediction of permeability, which is considered to be very important in reservoir evaluation, production, and field development.

**Geological situation in the area of investigation**

The Sinai Peninsula (Egypt) covers an area of about 61,000 km². Highly dissected mountains consisting of igneous and metamorphic rocks rise to a height of 2675 m (Gebel Mussa) and form the high relief mountainous tip of the peninsula at its southern portion as a part of the Arabo-Nubian massif. The central part of Sinai consists of sub-horizontally layered Mesozoic and Tertiary sediments, which form the plateau of Gebel El-Tih and Gebel Ekma. The northerly flowing affluent of Wadi El-Arish drains the plateau. Alternating Mesozoic faulted domes, anticlines, and synclines, which are known as the Syrian Arc System, form a contrasting topography of low alluvial plains and high hill masses north of latitude 30° N. These Syrian Arc structures sink seaward in the north. The structures are hidden under the Quaternary coastal plain and continental shelf deposits (Said 1990).

The study area Wadi Saal lies in east-central Sinai, as shown in Fig. 1.

The Paleozoic section in central Sinai has been subdivided into four formations from basement upward starting with the Araba Formation. This formation with a thickness of up to 170 m overlies the basement. It is assigned to Carboniferous age or older. The overlying Naqus Formation, which reaches a thickness of 462 m, is assigned to the Carboniferous era. However, Issawi et al. (1981) assigned it to Early-Cambrian. The Abu Durba Formation, which attains 166 m, follows the Naqus Formation and is assigned to Carboniferous age. Finally, the Ataqa Formation is assigned to the Early-Carboniferous.

Many authors have studied the sedimentary succession in southwestern Sinai (e.g., Abdel-Hameed et al. 1986; Allam 1989; Allam and Khalil 1987; Beleity et al. 1986; Brenklen and Marchant 1982; El-Mansey et al. 1988; Issawi et al. 1981; Salem et al. 1998; Hassanaia et al. 2007). In Egypt, the Lower Paleozoic rocks are exposed in various regions and include surface rock units (Taba, Araba, and Naqus formations) and subsurface rock units (Shifa, Kohla, and Basur formations) (Wanas 2011). The Cambrian to Silurian sequences form one of the world’s greatest concentrations of Lower Paleozoic reservoirs in Algeria, Libya, and Tunisia. Equivalent sequences in Egypt, Morocco, Niger, and Chad might also have hydrocarbon potential (Crossley and McDougall 1998).

A total of 45 samples were collected from the outcrops of Araba Formation in Wadi Saal, east central Sinai between latitude N 29° and N 30° and longitude E 34° and E 35° (Fig. 1).

**Methods**

**Mineralogical investigations**

The sandstone samples selected for microscopy were saturated by a blue dye resin to enable a better differentiation between pore spaces types (Williams and Patzl 2016). Careful investigation and description of thin sections was carried out using a polarizing microscope. Special attention is paid to types of porosity due to their significance in reservoir quality description and diagenesis. The classification of the studied sandstones is done according to Williams et al. (1982). Sandstones containing 10% or more argillaceous material (such as quartz wacke, arkosic wacke, and lithic wacke) are regarded as impure, while arenites that contain little or no argillaceous matrix (such as quartz arenite, arkosic arenite, and lithic arenite) are classified as pure sandstones. According to this classification, maturity of the sandstone increases with decreasing the percentage of argillaceous material. The
variation between arkosic and lithic arenites is related to differences in the source material. The feldspathic arenites and quartz-rich lithic arenites indicate a higher maturity. Fully mature types consist of quartz (quartz arenites) or quartz and abundant grains of pure chert. Most of these mature sandstones are characterized by a longer sedimentary history than the less mature rocks. Usually, the mature sandstones were involved in more than one cycle of erosion and deposition.

Petrophysical methods

Petrophysical investigations were carried out to determine parameters for reservoir characterization, such as porosity, density, permeability, magnetic susceptibility, and formation factor. The special core analysis includes the determination of specific surface and measurements of nuclear magnetic resonance (NMR) relaxometry. The 45 sandstone samples were analyzed in the Egyptian Petroleum Research Institute and at the Institute of Geophysics at Clausthal University of Technology, Germany. Using a diamond-drilling machine, cylindrical plugs of 2.54 cm diameter and varying lengths (2 to 4 cm) have been prepared. The samples were cleaned in a Soxhlet extractor. Methanol was used to remove water and residual salts. The petrophysical measurements were performed either in dry state or with full water saturation.

Density and porosity

The volumetric mass density of a substance is defined by the ratio of mass and volume. In the case of rock samples, different density values are determined: natural raw density, dry density, and grain density. The natural raw density $d_n$ considers the ratio between total mass ($m_{tot}$) and the total volume ($V$) of the sample:

$$d_n = \frac{m_{tot}}{V}. \quad (1)$$

For a dry sample, the mass of the sample corresponds to the mass of the solid constituents ($m_g$). The equation determines the dry density:

$$d_{dry} = \frac{m_g}{V}. \quad (2)$$
The grain density, which considers the mass of the grains and only the volume of the solid grains \( V_g \), is determined by:

\[
d_g = \frac{m_g}{V_g}.
\] (3)

The volume of the pore space \( V_p \) corresponds to the difference between the total volume \( V \) and the volume of the solid phase \( V_g \).

Porosity \( \Phi \) describes the volume of the voids (pores, cracks, fissures, fractures, etc.) related to the considered rock volume. It is defined as the ratio of the volume of pore space \( V_p \) to the total volume \( V \) of the rock sample:

\[
\Phi = \frac{V_p}{V}.
\] (4)

The dimensionless parameter \( \Phi \) is provided as fraction \((<1)\) or in percent. Porosity is the most important parameter for evaluating the storage capacity of reservoir rocks.

Permeability

The gas permeability \( k \) is determined in a steady state flow experiment that determines the flow rate \( q \) as a function of the pressure difference between inlet pressure \( P_1 \) and outlet pressure \( P_2 \):

\[
k = \frac{\mu q L P_1}{A(P_1^2 - P_2^2)}.
\] (5)

with \( \mu \) being the viscosity of gas, \( A \) being the cross-sectional area, and \( L \) being the length of the cylindrical sample.

Hydraulic flow unit concept

We use the hydraulic flow unit (HFU) concept of Amaefule et al. (1993), which is based on a modified Kozeny-Carman equation (Kozeny 1927; Carman 1937) and the mean hydraulic radius. The concept introduces the reservoir quality index \( RQI \) and the flow zone indicator \( FZI \) as defined in the following equations:

\[
RQI = 0.0314 \sqrt{k/\Phi},
\] (6)

\[
FZI = \frac{RQI(1 - \Phi)}{\Phi}.
\] (7)

If permeability \( (k) \) is provided in Milli-Darcy (mD) and the porosity \( (\Phi) \) is defined as a fraction then \( RQI \) and \( FZI \) are determined in micrometer (\( \mu \m \)). Considering Eq. 7, a log–log plot of \( RQI \) versus normalized porosity index \( NPI = \frac{\Phi}{1 - \Phi} \).

yields a straight line with a unit slope for a constant value of \( FZI \) that characterizes an HFU. The parameter \( FZI \) depends on the grain size and the hydraulic tortuosity. Assuming that these microscopic parameters do not change while diagenetic processes, the Kozeny-Carman equation formulates a specific porosity — permeability relationship for each HFU. Modifications of HFU concepts try to overcome simplified assumptions. Nooruddin and Hossain (2012) propose a modified \( FZI \) that is based on a relationship between tortuosity and the cementation factor. The modified flow zone index \( FZI^* \) suggested by Mirzaei-Paiaman et al. (2018) is equivalent with the definition of \( RQI \) in Eq. 6. It considers pore size instead of grain size.

Specific surface area

The specific surface describes the surface area of the pore space normalized to the total rock volume \( (S_{tot}) \), to the pore volume \( (S_{por}) \), or to the mass of the dry sample \( (S_m) \). The quantities mentioned above are related by the following equations:

\[
S_{tot} = d_g (1 - \Phi) S_m,
\] (9)

\[
S_{por} = d_g \left( \frac{1 - \Phi}{\Phi} \right) S_m.
\] (10)

The specific surface depends mainly on the shape and size of pores and the microstructure and morphology of the interface between minerals and pore space. The specific surface varies over a wide range. The “micromorphology” of the pore surface influences the value of the specific surface. The specific surface \( S_{por} \) increases with the decrease of the mean pore or grain size as a general tendency. The presence of clay particles and the growth of various types of surface structures in the pore space increase the surface.

Formation factor

Archie (1942) defined the property of a porous medium known as formation factor \( F \) as:

\[
F = \frac{\sigma_w}{\sigma_0},
\] (11)

with \( \sigma_0 \) being the conductivity of a sample that is completely saturated with brine of conductivity \( \sigma_w \).

Wyllie and Gregory (1953) investigated the influence of particle size and cementation on the formation factor of various materials. The observed formation factor for artificially cemented aggregates showed that the cemented aggregates
exhibit a greater variation in porosity than the unconsolidated aggregates.

The general form of the relation between formation factor and porosity reads:

\[ F = a \Phi^{-m}, \] (12)

with \( m \) being the cementation exponent and \( a \) being another free parameter. We prefer using the fixed parameter \( a = 1 \), because this value results for a porosity \( \Phi \) approaching the upper theoretical limit of one. In this case, \( F \) becomes equal one as expected from Eq. 11. The inverse value of the formation factor estimates the effective interconnected porosity (Revil and Cathles 1999).

**Magnetic susceptibility**

Mineral constituents with stronger magnetization control the magnetic properties of rocks even if the fraction of these minerals is small. Magnetic properties describe the behavior of any substance under the influence of an external magnetic field. Minerals are classified according to their magnetic properties in diamagnetic, paramagnetic, and ferro-, antiferro-, and ferrimagnetic behavior. For diamagnetic and paramagnetic substances, the strength of the induced magnetization \( M \) (magnetic dipole moment per unit volume) is directly related to the strength of the external magnetic field \( H \):

\[ M = \kappa H, \] (13)

with the factor of proportionality \( \kappa \) being the magnetic susceptibility.

The phenomenon of ferro-, antiferro-, and ferrimagnetism is related to a spontaneous magnetization without an external magnetic field.

**Nuclear magnetic resonance**

We refer to the textbooks of Kenyon (1997) or Coates et al. (1999) for an overview on the fundamentals of the nuclear magnetic resonance (NMR) method. NMR relaxometry investigates the behavior of nuclear spins of hydrogen nuclei in a magnetic field after perturbation by an energizing pulse. The decaying magnetization signal is transformed into a relaxation time distribution (RTD) that is related to the pore size distribution. The resulting characteristic pore sizes have proved to be key parameters for permeability prediction. Two different measurement modes provide a distribution either of longitudinal relaxation time (\( T_1 \)) or of transverse relaxation time (\( T_2 \)). The individual relaxation time (\( T_i \) with \( i = 1 \) or \( i = 2 \)) is attributed to a pore space with a certain ratio between pore surface (\( A_{\text{por}} \)) and pore volume (\( V_{\text{por}} \)) by:

\[
\frac{1}{T_i} = \rho_i \frac{A_{\text{por}}}{V_{\text{por}}} + \frac{1}{T_{ib}}.
\] (14)

with \( \rho_i \) being the surface relaxivity and \( T_{ib} \) being the bulk relaxation time of the water in the pore space. For sandstones with small pores, the contribution of \( 1/T_{ib} \) becomes negligible. Therefore, we ignore the second term at the right side of Eq. 14 in further calculations. In free fluids, the relaxation time depends mainly on fluid viscosity. Therefore, only small differences between \( T_1 \) and \( T_2 \) are observed in free fluids. In natural rocks, the transverse relaxation time \( T_2 \) is shortened by dephasing effects generated by the diffusion of the hydrogen nuclei in an inhomogeneous magnetic field that is caused by varying susceptibility of minerals and pore fluid.

We determined the longitudinal and transverse RTD of 18 sandstone samples of the Araba Formation with a MARAN 7 equipment that is operating at a Larmor frequency of 7 MHz. The samples were saturated with a low salinity brine (~0.5 g/l NaCl) before the measurement.

**Results**

**Petrographic investigation**

The petrographic investigation is focused on the texture and composition of the analyzed samples. The petrographic classification and diagenetic history are discussed. The petrography and mineralogy of the available sandstone samples were examined by polarizing microscopy of thin sections. Petrographically, the sandstone samples of the studied Araba Formation can be classified into three facies associations as proposed in an earlier study by El Sayed et al. (2008): (i) quartz arenite, (ii) sublithic quartz arenite, and (iii) calcareous quartz arenite. We assign 20 samples to facies (i), 14 samples to facies (ii), and 11 samples to facies (iii).

The quartz arenite facies (i) is mainly composed of fine, angular to sub-rounded, moderately well sorted monocrystalline quartz grains. This facies is characterized by the presence of some detrital clays, some pyrite, and iron oxides as cement, which reduces, to some extent, the rock porosity and permeability. Regarding texture and mineral composition, facies (i) represents a mature sandstone. The presence of iron oxides indicates its formation under oxidizing conditions. Iron oxides growing around quartz grains form post depositional cement or rim cement. Primary intergranular porosity becomes visible in Fig. 2A (yellow arrow). Detrital clays (blue arrow) and pyrite (red arrow) are marked in Fig. 2B. Iron oxide (red arrow) is indicated in Fig. 2C, and iron oxide surrounding the quartz grains (red arrow) is shown in Fig. 2D.
The sublithic quartz arenite facies (ii) is mainly composed of fine, angular to sub-rounded, moderately well sorted monocrystalline quartz grains. This facies is characterized by the presence of some lithics, detrital clays, pyrite, and iron oxides. The facies is characterized by mature texture and intergranular porosity as shown in Fig. 2E (yellow arrow). Pyrite (red arrow) and iron oxide (yellow arrow) are marked in Fig. 2F.

The calcareous quartz arenite facies (iii) is mainly composed of fine, angular to sub-rounded, moderately well sorted, monocrystalline quartz grains. It is characterized by carbonates and some iron oxides, which reduce the rock porosity and permeability. This facies indicates a mature texture. The carbonates form a post depositional cement between the quartz grains. Intergranular porosity becomes visible in Fig. 2G (yellow arrow). Carbonate cement (red arrow) and iron oxide (yellow arrow) are marked in Fig. 2H.

The petrographic investigation of the studied Araba Formation reveals various diagenetic processes that include compaction and pressure solution, cementation with iron oxides, and cementation with carbonates and clays. Several features confirm that the investigated sandstones have been subjected to considerable compaction. Among these features are the pressure solution effects, which result in the alteration of the original grain shapes (see Fig. 2E and F). Morris et al. (1979) stated that simultaneous cementation and pressure solution obscure the original grain shapes and result in concave-convex boundaries. Cementation with iron oxides is a common diagenetic feature of the investigated sandstones. The cementation may alternate with carbonates (see Fig. 2H) and/or clays (Fig. 2F). Iron oxides occur as pore-lining (Fig. 2D) or developed as pore-filling between quartz grains (Fig. 2H). The habit and orientation of the iron oxides indicate that they are authigenic in origin and have been precipitated in the pore spaces by intruding fluids.

The environment of deposition of Paleozoic sandstones in the central Sinai has been a matter of dispute. Issawi et al. (1981) stated that fluvial processes deposited the lower part of the Paleozoic sequence in the Wadi Firan area (Araba and Naqus formations) while the upper part (Abu Durba and Ataqa formations) was generated under alternative marine and fluvial conditions or deltaic mode of deposition. According to Wanas (2011), the sedimentological analysis confirms that the Araba Formation was deposited in a marginal-marine environment.

**Petrophysical relationships**

The minima, maxima, average values, and standard deviations of all measured parameters, which were calculated for the three facies, are compiled in Table 1.

**Dry density — porosity**

As expected from theory, an inverse relationship between dry density and porosity is shown for different sandstone facies in Fig. 3. The following equations control the relationship:

(a) for the facies (i):

\[ \Phi = 1.09 - 0.43d_{\text{dry}}, \]  

and (c) for the facies (iii):

\[ \Phi = 0.82 - 0.29d_{\text{dry}}, \]
with the dry density $d_{\text{dry}}$ given in g/cm$^3$ and the porosity $\Phi$ as a fraction. The facies of quartz arenite is characterized by the highest porosity and the lowest density, whereas the other two facies indicate higher dry density and lower porosity. The samples of facies (iii) indicate the widest variation of porosity and dry density.

$$\Phi = 1.00 - 0.38d_{\text{dry}},$$  \hspace{1cm} (17)

Porosity — permeability

Figure 4 shows the relationship between porosity and the logarithm of permeability. The data points follow the expected positive trend between porosity and permeability. Considering the small variation in porosity, the facies (i) and (ii) do not provide any reliable equation for the relationship. The following equation describes the relationship between porosity and permeability for the facies (iii):

$$k = [10]^{(11.17\Phi - 0.71)},$$ \hspace{1cm} (18)
Porosity — reservoir quality index

Figure 5 displays the relationship between the porosity and the logarithm of the reservoir quality index (RQI) for all samples. The facies (i) and (ii) are characterized by \( RQI > 1 \) µm. These values indicate a good reservoir quality. However, the facies (iii), which is characterized by \( RQI < 1 \) µm, is indicative of poor to fair reservoir quality. The porosity — RQI relationship for the facies (iii) reads:

\[
RQI = 10^{(4.47\Phi - 1.28)}.
\]  

It is characterized by a weak coefficient of determination \( (R^2 = 0.33) \). The other two facies do not provide any usable equation.

Permeability — reservoir quality index

Considering the definition of the \( RQI \) in Eq. (6), a strong correlation is expected between permeability and \( RQI \). Figure 6 displays the nearly linear relationship between the logarithm of \( k \) and the logarithm of \( RQI \). We get the following equations:

(a) for the facies (i):

\[
RQI = 10^{(-1.19) * [k]^{0.49}}.
\]

(b) for the facies (ii):

\[
RQI = 10^{(-1.19) * [k]^{0.50}}.
\]

and (c) for the facies (iii):

\[
RQI = 10^{(-1.08) * [k]^{0.45}}.
\]

The exponents of \( k \) are close to 0.5 as suggested by Eq. (6). Considering the weak variation in porosity within the studied facies, it becomes obvious that the \( RQI \) reflects mainly changes in permeability.
Hydraulic flow units

The core data provides the basis for the analysis of hydraulic flow units (HFU). Core porosity and permeability were used to calculate \( RQI \) and \( NPI \) values for each core sample using Eqs. (6) and (8) respectively. Figure 7 displays the log–log plot of \( RQI \) versus \( NPI \) as proposed by Amaefule et al. (1993). All samples following a line with a unit slope correspond to a single value of \( FZI \) and belong to the same HFU. According to the \( FZI \) values, the reservoir is classified into the following hydraulic flow units: impervious from 0 to 1 \( \mu \)m, poor from 1 to 2.5 \( \mu \)m, fair from 2.5 to 5 \( \mu \)m, and good for more than 5 \( \mu \)m. We observe a wide variation of \( FZI \) with the lowest values for facies (iii). It becomes obvious that the samples of each facies do not belong to a single HFU. Therefore, a simple porosity — permeability relationship cannot be formulated neither for the formation nor for each facies.

Figure 8 indicates a linear relationship between \( RQI \) and \( FZI \) with a high coefficient of determination \( (R^2 = 0.992) \). The slope corresponds to the mean value of the \( NPI \). The modified HFU concepts proposed by Nooruddin and Hosain (2012) and Mirzaei-Paiaman et al. (2018) with alternative formulations of \( FZI \) do not reduce the general variability of \( FZI \).

As shown in Fig. 4, a simple permeability prediction based exclusively on a porosity–permeability relationship does not exist for the investigated Araba Formation.

Specific surface — permeability

The specific surface \( (S_{sp}) \) — permeability relationship for the studied samples is shown in Fig. 9. We observe a decrease in permeability with increasing specific surface. The expected trend results from the inverse relationship between \( S_{sp} \) and effective pore radius. Obviously, the calcite cement in facies (iii) causes a strong increase in specific surface and a decrease in effective pore size.

Porosity — formation factor

The cross plot between the logarithm of porosity and logarithm of formation factor is shown in Fig. 10. The data points follow the expected trend with a decrease of formation factor with increasing porosity. The fitting equation:

\[
F = \frac{1}{\Phi^{1.43}}
\]

with a cementation exponent \( m = 1.43 \) is characterized by a low coefficient of determination \( (R^2 = 0.389) \). The low
cementation exponent suggests only a moderate cementation for the investigated sandstone facies.

**Formation factor — permeability**

The cross plot between the logarithm of formation factor and logarithm of permeability, which is shown in Fig. 11, yields the following fitting equation for the samples of this study:

\[
\log(k) = [10^{9.09} \times (\log F)^{-7.16}]
\]

with a coefficient of determination \(R^2 = 0.54\).

**Dry density — magnetic susceptibility**

The cross plot between dry density and magnetic susceptibility, which is displayed in Fig. 12, shows highly scatted...
data points. The plot indicates a positive trend with a slight increase in magnetic susceptibility for increasing dry density. The samples of facies (i) and (ii) are characterized by a low magnetic susceptibility and low variation in dry density. The samples of facies (iii) indicate higher values of dry density. However, the susceptibility values are comparable with the larger values of the other two facies.

**Ratio between longitudinal and transverse relaxation time from NMR**

The weighted logarithmic mean values of the distribution of longitudinal ($T_{lm1}$) and transverse ($T_{lm2}$) relaxation times are compared in the graph of Fig. 13. The general trend reflects a linear relationship between the mean relaxation times $T_{lm1}$ and $T_{lm2}$. As predicted by the theory, the weighted mean values of longitudinal relaxation time are higher than the transverse ones. The ratio $T_{lm1}/T_{lm2}$ varies between 1.21 and 3.24.

**Petrophysical model for permeability prediction**

A petrophysical model is usually designed to estimate an important reservoir parameter that is difficult to measure or estimate either in the laboratory or in the field. The model should be designed to derive this parameter from a conventional or easily measurable physical parameter.

The empirical Schlumberger Doll Research (SDR) equation (Kenyon 1997) is commonly used for estimating permeability from the weighted geometric mean relaxation time ($T_{lm1}$ or $T_{lm2}$) and porosity by:

$$k^{*}_{NMR} = a_i T_{lm i}^{2} \Phi^4. \quad (25)$$

The prefactors $a_i$ have to be adjusted for a certain formation by calibrating with core data. The exponent of two for the relaxation time results from dimensional considerations when considering the proportionality between the pore radius $r$ and relaxation time. Empirical studies suggested a porosity exponent of four (e.g., Coates et al. 1999; Dunn et al. 2002).

We use the factors $a_1 = 0.744$ and $a_2 = 1.933$ (for $k$ in mD and $T$ in ms), which have been determined for the sample set by Weller et al. (2014). The relationship between the measured and predicted permeability is shown in Figs. 14 and 15. The average logarithmic deviation between predicted $k^*$ (Eq. 25) and measured permeability $k$ is calculated by the formula (e.g., Weller et al. 2014):

$$d = \frac{1}{n} \sum_{j=1}^{n} [\log_{10}(k_j) - \log_{10}(k^*_j)]. \quad (26)$$

A value of $d = 1$ corresponds to an average deviation of one order of magnitude (or a factor of 10). Robinson et al. (2018) rate one order of magnitude deviation as an acceptable estimation if samples from different lithologies are regarded. The use of the longitudinal relaxation time provides with $d = 0.37$ a slightly lower value in comparison with

![Fig. 13](image) Mean relaxation time $T_{lm1}$ versus mean relaxation time $T_{lm2}$ derived from NMR for all samples.

![Fig. 14](image) Comparison of measured and predicted permeability using the weighted logarithmic longitudinal relaxation time ($T_{lm1}$) and porosity for all samples. The two dashed lines on either side of the diagonal indicate a deviation of one order of magnitude ($d = 1$).
the use of transverse relaxation time \( (d = 0.39) \). The values \( d < 1 \) indicate a reliable permeability prediction. However, it should be mentioned that the lower permeability values of the samples of facies (iii) show a higher deviation between measured and predicted permeability. The deviations for individual samples reach more than one order of magnitude.

The Schlumberger Doll Research equation, which uses porosity and mean relaxation times from NMR, proves to be applicable for permeability prediction for sandstones of facies (i) and (ii). The predicting quality is lower for the low-permeability samples of facies (iii).

A narrow range of high porosity values characterizes the samples of facies (i). Although a slight reduction is observed for the porosity of facies (ii), the permeability values of the two facies are similar. A remarkable decrease in porosity and permeability is observed for the calcareous quartz arenite facies (iii).

The pore volume-related surface shows an inverse relationship to permeability. This behavior may be explained by the inverse relationship between \( S_{por} \) and the effective pore radius. The formation factor proves to be another parameter that is closely related to permeability. The resulting exponent \((-7.16)\) in the power law equation between permeability and formation factor indicates the strong effect of small variations in the effective porosity.

The values of reservoir quality index \( (RQI) \) and flow zone indicator \( (FZI) \) are dominated by the permeability of the individual samples. Changes in porosity indicate only a minor influence on these parameters. Considering the \( FZI \) for classification, the samples of facies (i) and (ii) are characterized as fair to good hydraulic flow units, whereas most samples of facies (iii) originate from impervious or poor hydraulic flow units. Considering the wide range of \( FZI \), a simple porosity — permeability relationships cannot be formulated neither for the Araba Formation nor for each facies.

The Schlumberger Doll Research equation, which uses porosity and mean relaxation times from NMR, proves to be applicable for permeability prediction for sandstones of facies (i) and (ii). The predicting quality is lower for the low-permeability samples of facies (iii).

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Declarations

Conflict of interest The authors declare no competing interests.

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