Atmospheric, long baseline, and reactor neutrino data constraints on $\theta_{13}$

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An atmospheric neutrino oscillation tool that uses full three-neutrino oscillation probabilities and a full three-neutrino treatment of the MSW effect, together with an analysis of the K2K, MINOS, and CHOOZ data, is used to examine the bounds on $\theta_{13}$. The recent, more finely binned, Super-K atmospheric data is employed. For $L/E_\nu \geq 10^4$ km/GeV, we previously found significant linear in $\theta_{13}$ terms. This analysis finds $\theta_{13}$ bounded from above by the atmospheric data while bounded from below by CHOOZ. The origin of this result arises from data in the previously mentioned very long baseline region; here, matter effects conspire with terms linear in $\theta_{13}$ to produce asymmetric bounds on $\theta_{13}$. Assuming CP conservation, we find $\theta_{13} = -0.07_{-0.11}^{+0.15}$ (90% C.L.).

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The phenomenon of neutrino oscillations [1, 2, 3, 4] has been observed in a variety of experiments: solar, long baseline (LBL) reactor, atmospheric, and LBL accelerator experiments. Including the constraint imposed by the CHOOZ reactor experiment [5], one may quantitatively determine the three mixing angles and two mass-squared differences that parameterize three-neutrino phenomenology [6]. An outstanding question is the value of the mixing angle $\theta_{13}$. Present analyses [6] yield $|\theta_{13}| \leq 0.15$. We examine the impact of small effects, particularly those linear in $\theta_{13}$ [7, 8, 9, 10], on extracting this small parameter from the data. We find that including the full three-neutrino oscillation probabilities and a full three-neutrino MSW calculation are important for determining this mixing angle.

Knowledge of $\theta_{13}$ is a particularly important part of neutrino oscillation phenomenology because its value sets the magnitude of possible CP violating effects and the size of effects that might be used to determine the neutrino mass hierarchy. There are presently three new reactor experiments under development which are designed to measure $\theta_{13}$. Daya Bay [11], Double CHOOZ [12], and RENO [13], as well as two long baseline experiments, T2K [14] and NOvA [15]. The subsequent generation of experiments, e.g., those which will ascertain the level of CP violation, cannot proceed until the current generation better determines the value of $\theta_{13}$.

The standard model of neutrinos conserves flavor, as is required by the data. Neutrino oscillations require adding a posteriori a mass term to the standard model Lagrangian. The standard model Lagrangian is diagonal in flavor; the added mass term is diagonal in the basis which governs vacuum propagation. The relation between the two bases is given by a phenomenological unitary matrix $U_{at}$. In the absence of CP violation, it is real. We employ the standard representation [16] written in terms of the three mixing angles, $\theta_{12}$, $\theta_{23}$, and $\theta_{13}$. In vacuum, the probability that a neutrino of flavor $\alpha$ and energy $E_\nu$ will be detected a distance $L$ from the source as a neutrino of flavor $\beta$ is given by

$$P_{\alpha\beta}(L/E_\nu) = \delta_{\alpha\beta} - 4 \sum_{k,c=1}^{3} (U_{\alpha k} U_{\beta k}) \sin^2 \varphi_{jk}$$

with $\varphi_{jk} := \frac{1}{2}\Delta_{jk} \frac{L}{E_\nu}$ and $\Delta_{jk} = m_3^2 - m_k^2$, where $L$ is measured in km, $E_\nu$ in GeV, and the mass eigenvalues $m_i$ in eV. If (anti-)neutrinos travel an appreciable distance through matter of sufficient density, then one must add to the Hamiltonian an effective potential to account for the (anti-)neutrino–matter interactions [17]. This potential yields different effective mixing angles and neutrino masses. For neutrinos which propagate through the earth over long baselines, it is crucial to take such matter effects into account. We do so by using the approach developed in Ref. [18]. We employ a two density model of the earth: a mantle of density 4.5 gm/cm$^3$ and a core of density 11.5 gm/cm$^3$ with radius 3486 km. This approach incorporates the MSW effect into a three-neutrino framework without the use of approximate oscillation formulae. We note that it is possible for parametric resonances to occur when neutrinos pass between regions of differing densities [19]; our treatment of matter effects automatically insures that such resonances are fully incorporated.

Atmospheric neutrino experiments are unique in that the baselines span several orders of magnitude making them sensitive to an enormous region of relevant parameter space. However, the SK-atmospheric experiment is the most difficult to model as one must use the detected charged lepton to infer the direction and energy of the incident neutrino. A complete description of our analysis tool can be found in Ref. [20]. Statistical errors are included in the chi-square function whereas systematic errors are accounted for by using the pull method [21]. We include 43 pulls, the most important being the overall flux normalization. We also include [20] a simple model
of the multi-ring events. For CHOOZ, K2K, and MINOS, we utilize standard analysis techniques \[20\].

We introduce the commonly used “sub-dominant approximation” to provide a comparison for the full three-neutrino treatment. It arises from an expansion in the ratio of the mass-squared differences, $\Delta_{12}/\Delta_{32}$. The oscillation probabilities are then given by

$$\begin{align*}
P_{ee} &= 1 - \sin^2 2\theta_{13} \sin^2 (\varphi_{32}) \\
P_{e\mu} &= \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 (\varphi_{32}) \\
P_{\mu\mu} &= 1 - 4 \cos^2 \theta_{13} \sin^2 \theta_{23} (1 - \cos^2 \theta_{13} \sin^2 \theta_{23}) \\
&\quad \times \sin^2 (\varphi_{32}).
\end{align*}$$

(2)

Additional correction terms can be added \[6, 22\]. We effect this approximation by setting $\Delta_{21} = 0$ in our full three-neutrino code; in this treatment, matter effects will differ slightly from the approximations used by others.

We take the bounds on the mixing angles as $\theta_{13} \in [-\pi/2, \pi/2]$ and $\theta_{12}, \theta_{23} \in [0, \pi/2]$, as suggested in Ref. \[23\]. For no CP violation, this produces an allowed parameter space that is a single connected region. The (equivalent) often used bounds, $\theta_{jk} \in [0, \pi/2]$ with Dirac CP phase $\delta = 0, \pi$, produce two disconnected regions.

We begin utilizing the sub-dominant approximation. Ref. \[4\] showed that atmospheric data alone restrict the allowed value of $\theta_{13}$, although less so than does CHOOZ. We use the data from Ref. \[4\], which are binned more finely in energy than the original data \[3\]. In Fig. 1, we plot $\Delta \chi^2$ versus $\theta_{13}$, varying $\theta_{23}$ and $\Delta_{32}$. The solid [black] curve contains only atmospheric data. Our results quantitatively reproduce those of Ref. \[4\] which also used the sub-dominant approximation. Both give $\sin^2 \theta_{13} < 0.14$ (or $|\theta_{13}| < 0.38$) for $\Delta \chi^2 < 4.6$. Reproducing this result is a strong test for our analysis tool. The dashed [blue] curve in Fig. 1 represents $\Delta \chi^2$ for the data set which includes the atmospheric \[4\], LBL (K2K and MINOS \[2\]), and CHOOZ \[5\] experiments. It is CHOOZ which restricts $\theta_{13}$ much more so than the atmospheric data in the sub-dominant approximation.

We next examine the contribution of the atmospheric data alone to $\Delta \chi^2$, the [black] solid curve in Fig. 1. The dashed [blue] curve employs the full data set. For positive $\theta_{13}$, the atmospheric data are more restrictive than even CHOOZ. The restrictions for negative $\theta_{13}$ are set by the CHOOZ data. Overall, we find the allowed region for $\theta_{13}$ to be asymmetric about zero, bounded from above by atmospheric data and bounded from below by CHOOZ. The final value is $\theta_{13} = -0.07^{+0.18}_{-0.11}$ at 90% confidence level, corresponding to $\Delta \chi^2 = 6.25$ for a three parameter analysis of this data set. In Fig. 1, we, as have others \[23\], find a non-zero value for $\theta_{13}$; furthermore, we find a
Which subset of atmospheric data results in the strict upper bound on $\theta_{13}$ and the lack thereof from below? To answer this, we examine statistically insignificant preference for a negative value.

We find that the sub-GeV fully contained events are responsible for two-thirds of this $\Delta \chi^2$. Furthermore, one-half of the total change in chi-squared (4.5) comes from the single angular bin, $-0.8 < \cos \vartheta < -0.6$, bin II, for fully contained $e$-like events, and the two lowest energy bins in which the charged lepton momentum, $p_\ell$, is less than 400 MeV/c. This is well into the very long baseline region mentioned previously where we expect contributions from terms linear in $\theta_{13}$. Bin I, $-1.0 < \vartheta < -0.8$, contains neutrinos which traverse the core suppressing their amplitude of oscillation.

In Fig. 3 we plot oscillation probabilities $\mathcal{P}_{\alpha\beta}(E^{-1}_\nu)$ for angular bin II and the lowest energy bin, $p_\ell < 240$ MeV/c. The solid curves use the best fit parameters, the dash (dot-dash) curves are for angular bin I and the lowest energy bin, in which the charged lepton momentum, $p_\ell$, is less than 400 MeV/c. This is well into the very long baseline region mentioned previously where we expect contributions from terms linear in $\theta_{13}$. Bin I, $-1.0 < \vartheta < -0.8$, contains neutrinos which traverse the core suppressing their amplitude of oscillation.

In this new era of precision neutrino experiments, small effects, such as those arising from $\theta_{13}$, require a careful treatment. Future reactor experiments are sensitive to $\theta_{13}$ and thus can determine the magnitude of $\theta_{13}$, but not its sign. The long baseline experiments will contain small effects that are linear in $\theta_{13}$, while an upgraded Super-K will produce additional data in the region most sensitive to effects linear in $\theta_{13}$. How these different data interplay with each other in determining
this mixing angle will be most interesting.

We find that present atmospheric data restrict the value of $\theta_{13}$ from above, while the limit from below remains as determined by CHOOZ, and that $\theta_{13} = -0.07^{+0.18}_{-0.13}$, assuming no CP violation. It is important to realize first that $\theta_{13}$ can be negative [23] and, second, that linear effects lead to asymmetric errors. Our analysis requires the use of the more finely binned atmospheric data, Ref. [4], the use of the full three-neutrino oscillation probabilities, and inclusion of the full MSW effect.

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