Study on External Limit Load and Instability Characteristics of C-shaped Bellows

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Abstract. The metal bellow is the main flexible component in the pipeline compensator, which is used to compensate for the change of pipeline or structural displacement caused by thermal expansion and contraction, mechanical displacement or vibration, and is widely used in various fields of industrial production. In this paper, the critical load of the external pressure instability of the C-shaped bellow is theoretically deduced, and the corresponding finite element method is verified by this theory. On this basis, the finite element analysis is carried out for C-shaped bellows of different sizes. The influences of parameters such as diameter, wall thickness, wave radius, wave pitch and wave height on the limit load and critical instability load are clarified. The present study could provide an effective basis for the mechanical analysis and design of bellows, and also could provide a reference for the installation and use of bellows.

1. Introduction

The metal bellow is the main flexible component in the pipeline compensator, which is used to compensate for the change of pipeline or structural displacement caused by thermal expansion and contraction, mechanical displacement or vibration, and is widely used in various fields of industrial production. For different types of bellows, some research and explorations had been conducted by scholars from both domestic and foreign countries. Zhang [1] used ANSYS to carry out finite element analysis and calculation of single-layer and multi-layer corrugated pipes subjected to external pressure and tension load, and the deformation and stress distribution of unstable bellows were obtained. The deformation of multi-layer corrugated pipe under the action of external pressure tension load was also studied by experiments. Li [2] employed the finite element method to conduct linear and nonlinear buckling analysis on the bellow subjected to external pressure. The results show that the linear buckling load of the bellow increases first and then decreases with increasing tensile displacement, and increases with increasing waveform parameters. Osweiller [3] analyzed the stress distribution of U-shaped bellows under pressure and displacement load using linear elastic-based finite element software, and obtained similar results with EJMA [4] equation. Li [5] carried out finite element analysis for the bellow of the shut-off valve under certain working conditions. The research shows that the structural size has a great influence on the stress distribution under the action of pressure load and axial displacement. The size has different effects on the stress distribution of the bellow. Tan [6] took
two kinds of S-shaped bellows with a diameter of 70 mm as the research object, established a single-wave finite element model of the bellow, to study the influence of geometrical dimensions of the straight line segment and the circular arc segment of the wave plate on the stiffness of bellows. The waveform structure is obtained by optimization analysis to obtain a waveform, which has a small deviation from the design formula. The results show that the wave plate waveform size has a significant influence on the stiffness of the bellow. The wave plate stiffness of the inner circle bevel waveform is larger than the inner circle straight edge waveform. The linear segment waveform has a greater influence on the stiffness than the arc segment waveform. Scheibner [7] studied the calculation process of the geometric characteristics of the cylindrical compensator, and had developed an automatic calculation program for columnar compensation of pipeline heating systems based on the basic concepts of mechanical calculation of pipeline and column compensator. The program was utilized to determine the total deformation and compensation times of the pipe expansion joint. To study the stress distribution of U-shaped bellows under the separate and combined action of axial displacement and internal pressure, Becht [8] used finite element software MARC to perform an elastoplastic analysis. The analysis results show that when the bellows is in the elastic state, the stress generated by the separate action of axial displacement and internal pressure can be superimposed, and it is pointed out that the rigidity and stress of the bellows are sensitive to the dimensional deviation. Tsukimori [9] studied the elastic instability of the bellow and the instability of elastoplastic columns by the combination of finite element analysis and experimental method. Skoczen [10] studied the influence of the change of support conditions of the pipes connecting with the two ends of the expansion joint on the instability of the elastic column of this expansion joint, the instability safety checking of the bellow is enhanced through the above research. Thin-walled C-shaped bellows with large diameter are prone to suffer instability under the action of external pressure, and the stress when they are instable is often far less than the yield limit of the material. The instability of the bellow manifests as a peak collapse, and the failure mode is similar to the instability of the external pressure cylinder. The stability checking of the bellow subjected to external pressure is conducted by an equivalent cylinder method. For the instability, the bellow is taken as a rigid body in theoretical equation, and the equivalent diameter is obtained, and then instability calculation and checking is performed according to the external pressure cylinder theory. However, the inertia moment of the undeformed waveform is used in the calculation of the equivalent cylinder, and the influence of plasticity is not considered. Therefore, the obtained critical instability under external pressure is much higher than the actual instability pressure of the bellow.

In this paper, the finite element method is used to analyze the external limit load and instability of bellows under external pressure, which would have an important influence on the design and application of C-shaped bellows.

2. Theoretical Calculation of the Instability Load of C-shaped Bellows under External Pressure

The wave structure of C-shaped bellows is shown in Fig.1, and the meaning of each symbol is as follows:

- $\alpha$ —— Equivalent half fillet of waveform;
- $b$ —— Wave pitch;
- $h$ —— Wave height;
- $r$ —— Inside radius of wave;
- $r_1$ —— Radius of the transition fillet;
- $t$ —— Thickness;
- $t_f$ —— The minimum thickness of wave;
- $P_{cr}$ —— The theoretical instability load;
- $R$ —— Radius of pipe;

The average stiffness method is used to calculate the instability load of C-shaped bellows under external pressure.

(1) Assume $t=t_f$. 

...
(2) Calculate equivalent half fillet of waveform.

\[ c = r + t_1 - (h + t) \]  

\[ \alpha_1 = \arcsin \frac{r + t + c}{n_1 + t_1 + r} \]  

\[ a = (n_1 + t_1 + r) \cos \alpha_1 \]  

\[ \alpha_2 = \arctan \frac{c}{a} \]  

Thus,

\[ \alpha = 90 - \alpha_2 \]  

(3) Calculate the average bending stiffness of the pipe wall.

\[ D = \frac{E_h I_3}{12(1 - \mu^2)} + \frac{E_t J}{b} \]  

(4) Calculate the instability load of the bellow under external pressure, \( P_{cr} \).

\[ P_{cr} = \frac{3D}{\left( R - \frac{t}{2} \right)^3} \]  

3. Finite Element Analysis

3.1. Geometric Model

C-shaped bellows studied in this paper are made of 6061 high-strength aluminum alloy by hydroformed. Figure 2 shows the geometric model of C-shaped bellows, and the dimensions are listed in Table 1.

![Figure 1. Wave structure of C-shaped bellows.](image1)

![Figure 2. Geometric model of C-shaped bellows.](image2)
3.2. Material Model
The material parameters of 6061 high-strength aluminum alloy are shown in Table 2.

| Table 1. Parameters of the bellow/mm. |
|--------------------------------------|
| \( r_1 \) | \( R \) | \( t \) | \( r \) | \( b \) | \( h \) | \( t_1 \) |
| 20     | 310  | 3.5 | 35   | 200  | 34   | 3.5    |

| Table 2. Material properties of 6061. |
|--------------------------------------|
| Young’s Modulus/MPa | 68646 |
| Poisson’s Ratio      | 0.3   |
| Yield Stress/MPa     | 100   |

Local plastic deformation may occur when the bellows is in their working conditions. The instability load calculated with power hardening law model is larger than that calculated with elastic-perfectly plastic model. In order to keep a suitable margin of safety in engineering application, the elastic perfectly-plastic model is used to consider the influence of the plastic behavior. The material model is presented in Fig.3.

3.3. Material Model
On the base of model and load symmetry, and in order to improve the calculation efficiency, 1/4 symmetry model is employed for analysis. The symmetry model is shown in Fig.4.

The Solid185 element is used for meshing. To check mesh sensitivity, the model with parameters shown in Table 1 is created first. Then the model is divided into 2 layers, 3 layers and 4 layers along the thickness direction. The limit load analysis, eigenvalue buckling analysis and nonlinear buckling analysis are carried out respectively, and the results are listed in Table 3. As shown in Table 3, the deviations are all less than 3%. It is believed that the model with 2 or more layers can satisfy the
analysis requirements. And in this paper, the model with 3 layers along the thickness direction is selected to conduct the following analysis, and the global element size is 0.5mm. The number of nodes is 29,5320, and the number of elements is 22,0800. A local mesh is demonstrated in Fig.5.

| Layers | Limit Load Value | Eigenvalue Instability Load Value | Nonlinear Instability Load Value |
|--------|------------------|----------------------------------|----------------------------------|
|        | Deviation        | Deviation                        | Deviation                        |
| 2      | 1.2094           | --                               | 1.5259                           |
|        |                  |                                  | --                               |
| 3      | 1.2133 0.32%     | 1.5321 0.41%                     | 1.1255 0.64%                     |
| 4      | 1.2336 1.67%     | 1.5564 1.59%                     | 1.1257 0.01%                     |

According to the loading conditions of the bellow, a fixed constraint is applied to the left end of the model, a symmetric constraint is applied to the symmetry plane, and an external pressure load is applied to the outside surface. The loading and boundary condition is shown in Fig.6.

3.4. Analysis Type
The analysis types used in this paper conclude limit load analysis, eigenvalue buckling analysis and nonlinear buckling analysis.

The control variable method is used in this paper. Only one of the structural parameters is changed each time. And then the limit load analysis, eigenvalue buckling analysis and nonlinear buckling analysis are performed for the bellow to obtain the limit load and the instability load under external pressure.

4. RESULTS ANALYSIS AND DISCUSSION
According to the influence of different structural parameters, combining theoretical and numerical results, the variation of limit load, theoretical instability load and the instability load of eigenvalue buckling analysis and nonlinear buckling analysis is given.

The control parameter is r1, and the values of the other parameters are shown in Table 1. The variation of the limit load and the instability load of the bellow under external pressure with r1 is shown in Fig.7. It can be seen that as the r1 increases, the limit load and the nonlinear instability load
almost remain unchanged, and the eigenvalue instability load decreases slightly. The eigenvalue instability load is the theoretical solution of bellows under ideal conditions, which is consistent with the theoretical instability load. The eigenvalue instability load is always higher than the nonlinear instability load. This is because the influence of geometric nonlinearity on the structure is ignored in the eigenvalue load calculation process.

The control parameter is $R$, and the values of the other parameters are shown in Table 1. The variation of the limit load and the instability load with $R$ is shown in Fig.8. It can be seen that as $R$ increases, the theoretical instability load, eigenvalue instability load and the limit load decrease, which indicates that the external pressure bearing capacity of the bellow decreases. The eigenvalue instability load is higher than the nonlinear instability load, and with smaller diameter, the difference becomes larger, which means that the smaller pipe radius results in greater influence of geometric nonlinearity on the critical instability load of structural under external pressure.

![Figure 7. The variation of limit load and instability load with $r_1$.](image1)

![Figure 8. The variation of limit load and instability load with $R$.](image2)

The control parameter is $t$, and the values of the other parameters are shown in Table 1. The variation of the limit load and the instability load with $t$ is shown in Fig.9. It is noted that as $t$ increases, the theoretical instability load, eigenvalue instability load, nonlinear instability load and limit load increase, indicating the external pressure bearing capacity of the bellow increases. The eigenvalue instability load is higher than the nonlinear instability load. This is because the influence of geometric nonlinearity on the structure is ignored in the eigenvalue load calculation process, and the eigenvalue instability load is not conservative.
The control parameter is \( r \), and the values of the other parameters are shown in Table 1. The variation of the limit load and the instability load with \( r \) is shown in Fig.10. One note that as \( r \) increases, the limit load almost remain unchanged, and the eigenvalue instability load and the nonlinear instability load increase. The eigenvalue instability load is consistent with the theoretical instability load. When \( r \) is in the range of 25-30mm, the eigenvalue instability load is less than the limit load, and the nonlinear instability load is limited by the eigenvalue instability load, indicating that the instability has occurred before the strength reaches its limit. When \( r \) is greater than 30mm, the limit load is less than the eigenvalue instability load, and the nonlinear instability load is limited by the limit load, indicating that the external pressure bearing capacity is limited by the structural strength. And it is found that when \( r \) is greater than 30 mm, the nonlinear instability load gradually approaches the limit load, meaning that the structural strength and stiffness are fully utilized.

![Figure 9. The variation of limit load and instability load with t.](image)

![Figure 10. The variation of limit load and instability load with r.](image)

The control parameter is \( b \), and the values of the other parameters are shown in Table 1. The variation of the limit load and the instability load with \( b \) is shown in Fig.11. It is observed that as \( b \) increases, the theoretical instability load and eigenvalue instability load decrease significantly, but the limit load and the nonlinear instability load almost remain unchanged. The eigenvalue instability load is consistent with the theoretical instability load. The nonlinear instability load is consistent with the limit load, indicating that the nonlinear instability load is limited by its limit load, and the structural strength is insufficient under this circumstance.
The control parameter is $h$, and the values of the other parameters are shown in Table 1. The variation of the limit load and the instability load with $h$ is shown in Fig.12. One observes that as $h$ increases, the theoretical instability load and eigenvalue instability load increase, and the nonlinear instability load and limit load almost remain unchanged. The eigenvalue instability load is consistent with the change of theoretical instability load. The nonlinear load is significantly less than the eigenvalue instability load, indicating that the geometric defect has a significant influence on the nonlinear instability load.

![Figure 11. The variation of limit load and instability load with b.](image)

The control parameter is $t_1$, and the values of the other parameters are shown in Table 1. The variation of the limit load and the instability load with $t_1$ is shown in Fig.13. It can be seen that as $t_1$ increases, the eigenvalue instability load and theoretical instability load increase slightly. The increasing of the nonlinear instability load and the limit load value is not obvious. The nonlinear instability load is less than the limit load value, indicating that the bearing capacity of the structural under external pressure is limited by the property of material stiffness.

![Figure 12. The variation of limit load and instability load with h.](image)
5. Conclusion

(1) The transition fillet $r_1$, the wave height $h$ and the minimum thickness $t_1$ of the waveform do not much affect the limit load and the instability load of C-shaped bellows.

(2) The limit load and the instability load of C-shaped bellows decrease with the increasing of the radius $R$ and the wave pitch $b$, and increase with the increasing of the thickness $t$ and the inside radius of wave $r$.

(3) The nonlinear instability load is limited by the limit load and the eigenvalue instability load, and it is always less than the minimum value of the two. If the nonlinear instability load is limited by the limit load, the structural strength is insufficient; if the nonlinear instability load is limited by the eigenvalue instability load, the structural stiffness is insufficient.

Acknowledgments

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