Application of mathematical modeling to solve problems of optimization of lattice structures

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Abstract. The paper deals with the application of mathematical modeling for solution of problems of optimum design of mesh structures. We will consider the method of analysis of power circuits for solving problems of structural optimization of such structures. A system for automating a computational experiment, modeling, and numerical calculation is proposed in the paper. The authors consider a mathematical model of joint deformation of the rib structure and skin, presented algorithms for solving the problem of optimal design. The problem of choosing the optimal variant of the arrangement of annular ribs in a cylindrical structure is considered as an example

1. Introduction
Lattice structures are often used in engineering [1-4] and constructing. VG Shukhov began to use lattice structures in the industry for the first time. The towers of the Shukhov system are widely spread due to their efficiency, lightness, stability [5]. Lattice shell structures, arbitrary shapes [6] and other structures are often used in the design of buildings and other building structures today. The use of mesh structures is widespread in aircraft [7] and missilery [1]. Lattice anisogrid composite shells are the constituent elements of the aggregates of spacecraft. They are power structures. For such structures, a set of structural elements must ensure the strength and rigidity of the shell under load. Lattice structures have a distinctive feature - a system of intersecting helical and circular ribs (Figure 1).

Figure 1. The lattice structure:
a_h – the distance between pairs of spiral ribs;
a_c – the distance between annular ribs;
φ – the angle of the spiral edge to the generatrix;
b_h – helical rib thickness;
b_c – circular rib thickness;
H – thickness of rib structure (height of rib section).

The use of lattice structures in mechanical engineering allows us to reduce the constructions’ mass with constant loads. This fact allows for increasing the mass of the payload. Therefore, the economic efficiency of the design increases. The optimum design of mesh structures improves efficiency and economy. One of the reasons is to reduce the excess safety margin. Such tasks have a large number of restrictions. Restrictions must be met when operating the mesh design. The optimality criterion is either the minimum mass of the constructions or the minimum cost of the constructions.
2. Tasks of optimal design of lattice structures

The optimization problem for the mass of a multi-element statically indefinable construction is one-criterion. Strength, rigidity, and stability of structures are used as constraints (state variables of the structure) when solving problems of optimization of lattice anisogride structures. Strength conditions are that the maximum normal stresses do not exceed the tensile strength of the material (compression), and the maximum transverse shear stresses do not exceed the shear strength for all sections of the structural edges. Conditions for overall stability are that the specified load does not exceed the critical one, resulting in a loss of stability. Since the condition on general stability does not take into account the axisymmetric form of loss of stability, the additional condition on axisymmetric stability is applied. The condition for local loss of stability is to limit the compressive forces in the edge segments between the points of intersection by the Euler critical force.

Restrictions on the strength, stiffness, and stability of such structures are expressed through the parameters of the stress-strain state at design loads. These parameters depend on the variable design parameters of the structure. The parameters of the stress-strain state are fields of displacements, stresses, and strains, that is, the value of each of these parameters depends on the position of the point (elements or node) of the structure. Therefore, constraints on strength and stiffness should be formulated for a sufficiently large number of characteristic points of the structure.

Thus, the formulation of the problem may include a large number of restrictions, which are several orders of magnitude greater than the number of variable parameters.

We consider a mathematical model of static deformation of the power structure at the “black box” level. The internal structure of the object being modeled is not disclosed. The model captures only the response of the object to the applied effects [8]. Thus, the model has the form

\[ q = K(p)r, \]  

where \( q \) denotes the vector of state variables (displacements, stresses, and deformations), \( r \) denotes the vector of the variable effects (applied forces), \( K \) denotes the model operator that maps the elements of the action space to the state space elements, \( p \) denotes a vector of model parameters that includes the "internal" characteristics of the modeled object (physical and mechanical constants of materials, geometric dimensions of the structure, dimensions of sections of structural elements, etc.).

The task of optimizing the structure by mass is formulated as follows: find the values of the structural parameters \( p \) that provides a minimum of the quality criterion (mass), provided that the state parameters \( q \) satisfy the system of specified constraints (strength, rigidity, and stability, etc.). Formally, the problem of optimizing the construction by mass based on the model (4) can be put in the following form.

It is known:
- initial values of the structural parameters of the model \( X_0 \),
- a vector of variable effects \( r \),
- model of the reaction of the structure to the effects \( q = K(p)r \).

It is required to determine: the structural parameters of the model \( X \in p \), for which the constraints of the structural parameters \( F(p) \geq 0 \) and the constraints of the state parameters \( \Phi(q) \geq 0 \) are satisfied, which ensure the minimum of the objective function \( Z(p) \rightarrow \min \). The mass of the structure is chosen as the objective function \( Z(p) \).

The structural parameters \( p \) are heterogeneously. Designable optimized parameters can have both qualitative and quantitative scale. The quantitative scale has such parameters as the number of spiral or annular ribs, the angle of inclination of the rib to the generator and the geometric parameters of the cross-section of the rib structure. Such parameters as a variant of connecting the ribs in the structure, the material of execution, the absence or presence of structural elements refer to parameters with a quality scale.

The solution of the optimal design problem can be based on the analysis of power circuits and typical solutions or the methods of topological and parametric optimization. When choosing a method for solving an optimization problem, the designer must take into account the features and type of
construction. It is obvious that the use of analysis of force schemes and standard solutions requires a great deal of experience and theoretical knowledge from the designer. Also, the set of solutions considered is limited. Thus, the result of such an analysis depends on a large number of factors. Nevertheless, the analysis of power circuits is the only use case for the optimization of parameters with a qualitative scale.

Topological and parametric optimization methods have limitations on applicability. The number of restrictions, the number of parameters and the structure of the area of restrictions affect the choice of numerical methods [9]. For example, topological optimization determines the structure of the structure. Therefore, it cannot be applied to structures with a certain rigid structure.

Since the optimization problem has heterogeneous parameters, the designer can use both methods based on the analysis of power circuits and numerical methods based on optimization algorithms to solve it.

Methods of optimal design of mesh structures of regular rib structure without skin are currently well developed and known [1, 10]. When designing complex engineering and building structures that have structural and technological cuts, reinforcement or plating, the use of such methods is difficult. One of the reasons is the use of the continual approach in these methods [11]. It is known that continual design does not provide sufficient accuracy for irregular structures.

3. Modeling of the stress-strain state of mesh structures
The mathematical model [12] of the deformation of the anisogrid lattice structure is based on the idea of joint deformation of the rib structure and skin if any. For this, the following kinematic hypotheses were used.

1. The deformation of the skin corresponds to the classical Kirchhoff-Love hypothesis: the material normal coincides with the geometric one, does not warp and does not change its length, that is, the deformations of the normal and transverse shifts are absent.

2. Each edge is represented as a set of short beams. The beams have a section height commensurate with the length of the beam. The deformation of the beam corresponds to the hypothesis of Timoshenko: the cross-sections of the beam do not change the shape and dimensions but do not remain perpendicular to the curved axis of the beam.

3. The displacements of the points of the beams of different kinds (spiral, annular) at their intersections are continuous and identical.

4. The movements of the ribs and the skin coincide on the lines of their contact surface, which run parallel to the axes of the ribs.

We consider the mutual arrangement of the geometric normal to the reduction surface and the material normals for the skin and the rib structure to represent the joint deformation of the shell and the rib structure.

The scheme of joint deformation of the shell and rib structure is shown in Fig. 2. The material normal in the shell is the geometric one: it rotates from the initial position to an angle $R$ at deformation. In the edge, the material normal rotates in space by an angle $R$. The angle $R$ between the geometric and material normal is equal to the strain of the transverse shear.

The kinematic hypothesis for beams ignores the deformation section. Therefore, the rotation of the beam relative to its longitudinal axis must be zero when there is equality of movement of the rib and beam on the contact surface. This is unacceptable since the height of the cross-section exceeds the width by a factor of 10 or more. Thus, the weaker condition of equality of displacements on the contact line can be considered as the assumption of "in stock" in hardness. It is more rational for the description of kinematics than the assumption of the coincidence of displacements on the entire contact surface (it leads to an overestimation of hardness).

Works [12, 13] describe in detail the acquisition of functions that describe the field of displacements of a mesh structure with and without skin. These functions are basic and ensure the consistency of the movements of the ribs and the skin in the nodes on the surface of the cast.

Each constructive element is given by a corresponding set of finite elements for constructing a discrete mesh shell model. Spiral, annular ribs and frames are represented in the form of beams –
The movements are approximated by a one-dimensional Hermitian polynomial of the third order. The skin was represented in the form of triangular plates – Zenkevich-Argiris plate. The displacements in the skin are approximated by an incomplete cubic polynomial.

Calculation of the stress-strain state of an anisogrid lattice-shell structure is performed by the finite element method in the variational formulation according to the Lagrange principle. Thus, the discrete model of the static deformation of the reticulate shell of a regular and irregular structure allows ensuring the compatibility of deformation of the skin and rib elements on their reduction lines. The model describes the static deformation of a net design without skin as a special case.

The discretization of the model of the mesh structure without plating is carried out using only one-dimensional two-node finite elements. The stiffness parameters of the beam (cross-section of the beam) are taken into account in the elastic matrix in the variational formulation of the finite element method:

$$
\begin{bmatrix}
EF & ES_t & ES_n & 0 \\
ES_t & EI_t & EI_m & 0 \\
ES_n & EI_m & EI_n & 0 \\
0 & 0 & 0 & GI_{np}
\end{bmatrix}
$$

(2)

where $E$ denotes the modulus of elasticity of the material, $G$ denotes the shear modulus of the material, $S_t$, $S_n$ denotes static moments, $I_t$, $I_m$, $I_n$ denotes moments of inertia, $I_{np}$ denotes torsional stiffness, $F$ denotes the cross-sectional area.

4. Algorithm for solving the optimization problem and the tool for numerical calculation

Figure 3 is an algorithm for solving an optimization problem based on the analysis of power circuits. The algorithm will work correctly if all the considered variants of computational models correspond to the geometry and behavior of real structures. The fulfillment of this condition requires that the model takes into account a variety of parameters, including the boundary conditions [14]. Software packages with open source can allow variations in the assignment of various conditions and simulation parameters. Later in the paper, the authors consider the implementation of the mathematical model in the software package "Composite NC Anisogrid" [15]. This package has open-source, the ability to modify and customize solutions. The "Composite NK Anizogrid" program package was developed at the Novokuznetsk branch of Kemerovo State University.

A software package is a tool for constructing application programs using visual programming technology. The algorithm for calculating an application program is presented as a sequence of calculations of the value of functional objects. In this case, the arguments of functional objects are other functional objects. Each functional object implements a specific subroutine.
The oriented graph describes the composition of functional objects, where the vertices are functional objects. Arcs of the graph appear when the argument of the object is another functional object.

Thus, the designer models the calculation process as a network of visual images of software objects. Objects have two types - executable and structure-forming. Structure-forming objects are responsible for implementing loops, branching, and getting references to other blocks. The translation of the functional objects graphical representation scheme forms a sequence of the algorithm’s commands.

The software package "Composite NK Anisogrid" is based on the use of the finite element method. The software package includes computational experiment modules that calculate approximation functions. This allows you to effectively solve the problem of analysis and synthesis for structures of complex geometry and irregular structure. One of the modules is intended for the automated construction of approximation functions of responses according to the results of computational experiments. The core data of the module are a text file containing a plan-matrix of the computational experiment and files with responses.

His calculation of the approximation function consists of the following steps:

- loading the planning matrix and the responses of the factor experiment;
- checking the completeness of loading the input data for constructing approximating functions;
- determination of the required type of basic functions and the formation of a set of basic functions for the response functions;
- calculation of the coefficients of the approximating function.

This software package was tested on the calculation of the stress-strain state and stability of mesh anisogrid structures. The results of the numerical calculation are compared with analytical calculations and known CAD systems. Currently, the software package is used in modeling and design of grid anisogrid structures “Central Scientific Research Institute of Special Mechanical Engineering” (Hotkovo, Moscow region).

5. Solution of the optimal design problem

Engineers can vary the relative position of the helical and circular ribs in the design of mesh structures. In paper [16], the author lists the classification of lattice structures according to the location of circular ribs (Figure 4). The classification is based on two factors: number of circular ribs per helical cross and distance from the axis of the circular edge to the nearest cross of the helical ribs. Strength characteristics and purpose of the construction affect the choice of the location of the edges. For example, double annular (Figure 4) ribs are convenient when attaching equipment to a lattice structure.

The works of Vasiliev, Nikitin, Razin [17] show that in practice there are two main forms of the mutual arrangement of the ribs, which are more technologically rational than the triangular cell (Figure 5-7). In the case of a uniform stress state of a product, the application of continual modeling to such structures gives equivalent results for all the considered cell shapes. This is due to the averaging of the lattice structure in the simulation.

When the regularity of the rib structure is violated, there are reinforcements, and when the structure is locally loaded, stresses arise in its elements, which vary considerably within a triangular cell. Designers use discrete modeling based on the finite element method to obtain results consistent with the results of field experiments in such cases. The choice of the type of intersection of the helical and circular ribs, the cross-sectional type of circular ribs can influence the results of numerical calculations.

The methods described in this work can be used to determine the relative position of the edges. In this case, the modeling of lattice structures will be carried out on the basis of discrete modeling by the finite element method. This task has two variation factors. The factors correspond to the classification factors - the distance to the intersection of the spiral ribs and the number of annular ribs between the intersections. Since one of the factors has a qualitative scale, the methods of numerical optimization cannot be applied.
**Figure 3.** The block diagram of the algorithm for solving the problem of optimizing the design based on the analysis of power circuits.

- Specifying design parameters
- Construction of the matrix of the computational experiment
- Building discrete models
- Calculation of the stress-strain state for the stress-strain state for a series of models
- Comparison of stress-strain state calculations for a series of models
- Determination of the optimal value of the optimization criterion and the corresponding values of the design parameters

**Figure 4.** Classification of lattice structures.
The software package "Composite NC Anisogrid" allows you to simulate double annular ribs using the task of the cross section (Fig. 8, 9). This is possible with a small distance between the annular ribs. Thus, the mesh structure shown in Figure 7 can be modeled as a triangular cell with a cross section of in Fig. 9.

The design model is a cylindrical lattice structure without skin. The design has 32 pairs of spiral ribs and a slope of the spiral rib to the generatrix of 27.3 degrees. The height of the structure is 3 radii of the lower edge of the structure. The upper edge of the structures was loaded through a rigid node, located in the plane of the upper section, with a compressive force of 1000 kgf with a moment of 1000 kgf. Figure10 shows discrete models of design with varying factors.

The authors analyzed the values of the calculated stress in the helical edges of the structures between the nodes of the intersection of the edges. Comparative analysis shows that the differences between the axial stresses in the spiral edges of the structure for different values of variable parameters reach 1-3%, and in shear stresses - 19-30%. The definition of the optimal variant when reaching the minimum values of normal and tangential stresses corresponds to the conclusions given in [17].

The bending of the ribs in the structure shown in Figure 6c is larger than in the structure with triangular cells (Figure 5). Analysis of the longitudinal stresses showed that in the presence of concentrated loads, the structure shown in Fig. 7 is more efficient than the structure shown in Figures 6. Table 1 presents the absolute deviations of the stresses in the grid structure relative to the structure with a triangular cell.
Figure 10. Discrete models with a triangular cell.

Figure 11. Discrete models with the location of the annular ribs at 50% of the cell height.

6. Conclusions
The paper shows an approach to the construction of a mathematical model of joint deformation of a mesh structure formed by two families of ribs and skin; approach to the construction of a discrete model of the mesh design, taking into account the specification of the cross-sectional type of the edges of the mesh structure and the relative position of the edges.

These approaches are applied to the analysis of power structures in determining the optimal type of mesh structure. The results of numerical simulation and analysis show that the mesh structure presented in Figure 7 is optimal.

Table 1. The absolute deviations of the stresses in the grid structure relative to the structure with a triangular cell.

| distance from the axis of the annular edge | number of annular ribs per spiral cross | Longitudinal Stresses $\sigma_c$ | Flexural Stresses $\tau_{st}$ | Flexural Stresses $\tau_{sn}$ |
|-------------------------------------------|----------------------------------------|-------------------------------|-----------------------------|-----------------------------|
| minimum technological                    |                                        | -0.1% – 1.69%                 | 25.16% – 25.17%             | -2.03% – 6.9%              |
| middle of the cell                        |                                        | -0.001% – -0.36%              | 25.044% – 25.1%             | 0.58 – 2.25%               |
| number of annular ribs per spiral cross   |                                        |                               |                             |                             |
| single edge                               |                                        |                               |                             |                             |
| double edge                               |                                        |                               |                             |                             |

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