On the regularization scheme and gauge choice ambiguities in topologically massive gauge theories

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Abstract

It is demonstrated that in the (2 + 1)-dimensional topologically massive gauge theories an agreement of the Pauli-Villars regularization scheme with the other schemes can be achieved by employing pairs of auxiliary fermions with the opposite sign masses. This approach does not introduce additional violation of discrete ($P$ and $T$) symmetries. Although it breaks the local gauge symmetry only in the regulator fields’ sector, its trace disappears completely after removing the regularization as a result of superrenormalizability of the model. It is shown also that analogous extension of the Pauli-Villars regularization in the vector particle sector can be used to agree the arbitrary covariant gauge results with the Landau ones. The source of ambiguities in the covariant gauges is studied in detail. It is demonstrated that in gauges that are softer in the infrared region (e.g. Coulomb or axial) nonphysical ambiguities inherent to the covariant gauges do not arise.

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1 Introduction

Since the extensive work after Deser, Jackiw and Templeton [1] it is well known that in the (2 + 1)-dimensional topologically massive gauge theories there arise several ambiguities at the one loop level. First of all we mean the regularization scheme and gauge parameter dependence of physical quantities and some other, related to these, problems. The most distinct among them are the regularization scheme ambiguities. The problem is that at one loop approximation the Pauli-Villars (PV)
method in contrast to the other regularization schemes produces different gauge field polarization operators \[2\]–\[8\]. Another problem is that in calculation of the fermion self-energy the Landau gauge plays an outstanding role among the other covariant gauges \[9\]. The fermion pole-mass in an arbitrary covariant gauge turns out to be gauge parameter dependent. This trouble can be avoided if we use the auxiliary vector fields as PV regulators, but even in this case the numerical result differs from that of the Landau gauge \[1\].

In \((2 + 1)\) dimensions the PV regularization scheme unlike dimensional (or the other) regularization schemes introduces parity violation at the intermediate stage, the finite trace of which survives after removing PV regularization. In our previous publication \[9\] we have demonstrated that in abelian case it is possible to agree polarization operator calculations in all schemes by using pairs of opposite sign mass fermions as regulators. Below we are going to show that this result can be generalized for the nonabelian case too. We are going to study also the consistency problem of different gauges with the help of pairs of regulator vector fields i.e. we will show that the fermion pole mass calculated in an arbitrary covariant gauge coincides with the Landau gauge result if the gauge parameters for the pairs of regulator vector fields with opposite sign masses are set the same as that of the initial gauge field. As long as the gauge propagator in the covariant gauge is more infrared (IR) singular than in the other ones, we will study the same problem in the IR softer gauges (Coulomb and axial). we will show that they produce the Landau gauge result. Moreover it turns out that the nonphysical cut present in the Landau gauge disappears in IR softer gauges.

2 Polarization operator of the gauge field and the three-gluon vertex at one loop

Consider QED (or QCD) with massive fermions plus Chern-Simons term in \((2 + 1)\)-dimensions. Below we will follow notations from \[1\]. Due to the superrenormalizability of these models only few one-loop diagrams are divergent. In this section we study the gauge field polarization operator:

\[
\Pi_{\mu\nu}(k, m) = -ig^2 \int \frac{d^3q}{(2\pi)^3} tr \left[ \gamma_\mu S(k+q)\gamma_\nu S(q) \right]
\] (2.1)
and the three-gluon vertex:

$$\Pi_{\mu\nu\alpha}(k, q, m) = -g^3 \int \frac{d^3 p}{(2\pi)^3} tr [\gamma_\mu S(p)\gamma_\nu S(p + k)\gamma_\alpha S(p + k + q)] .$$  \hspace{1cm} (2.2)

Here $S(q) = -i(\hat{q} - m)^{-1}$. Explicit calculation with the cutoff $\Lambda$ yields [1], [5]:

$$\Pi_{\mu\nu}(k, m) = -\frac{g^2}{3\pi} \Lambda g_{\mu\nu} + \left( g_{\mu\nu} - k_\mu k_\nu k^2 \right) \Pi^{(1)}(k^2, m) + i\epsilon_{\mu\nu\lambda} k^\lambda \Pi^{(2)}(k^2, m) \hspace{1cm} (2.3)$$

with

$$\Pi^{(1)}(k^2, m) = \frac{g^2}{2\pi} \left[ \frac{\sqrt{m^2}}{2} - \left( \frac{m^2}{2} + \frac{k^2}{8} \right) \frac{1}{\sqrt{k^2}} \ln \left( \frac{2\sqrt{m^2} + \sqrt{k^2}}{2\sqrt{m^2} - \sqrt{k^2}} \right) \right], \hspace{1cm} (2.4)$$

$$\Pi^{(2)}(k^2, m) = \frac{g^2}{4\pi} \frac{m}{\sqrt{k^2}} \ln \left( \frac{2\sqrt{m^2} + \sqrt{k^2}}{2\sqrt{m^2} - \sqrt{k^2}} \right). \hspace{1cm} (2.5)$$

If we use the dimensional regularization result will turn out finite and (2.3) without the first term and (2.4)–(2.5) will be final expressions. In the PV scheme we have to consider combination:

$$\Pi^{Reg}_{\mu\nu}(k) = \sum_{i=1}^{N} C_i \Pi_{\mu\nu}(k, M_i) \hspace{1cm} (2.6)$$

where $C_1 = 1$, $M_1 = m$ and the limit $|M_{i>1}| \to \infty$ is to be taken. From the above expressions it is clear that

$$\Pi^{(1)}(k^2, |M_i| \to \infty) = -\frac{g^2}{12\pi} \frac{k^2}{\sqrt{M_i^2}} \to 0, \hspace{1cm} (2.7)$$

$$\Pi^{(2)}(k^2, |M_i| \to \infty) = \frac{g^2}{4\pi} \frac{M_i}{\sqrt{M_i^2}} = \frac{g^2}{4\pi} \text{sgn}(M_i). \hspace{1cm} (2.8)$$

If only one regulator field is used, we obtain:

$$\Pi^{(1)}_{PV}(k^2, m) = \Pi^{(1)}(k^2, m), \hspace{1cm} (2.9)$$

$$\Pi^{(2)}_{PV}(k^2, m) = \Pi^{(1)}(k^2, m) - \frac{g^2}{4\pi} \text{sgn}(M). \hspace{1cm} (2.10)$$
For massless fermions we have:

\[
\Pi^{(1)}_{PV}(k^2, m = 0) = \frac{g^2 \sqrt{-k^2}}{16},
\]
\[
\Pi^{(2)}_{PV}(k^2, m = 0) = -\frac{g^2}{4\pi} \text{sgn}(M). \tag{2.11}
\]

While the dimensional regularization yields:

\[
\Pi^{(1)}_{Dim}(k^2, m = 0) = \frac{g^2 \sqrt{-k^2}}{16},
\]
\[
\Pi^{(2)}_{Dim}(k^2, m = 0) = 0. \tag{2.12}
\]

So radiative corrections to the Chern-Simons term for massless fermions are absent in the dimensional regularization while in the PV one they are nonzero and depend on the sign of the regulator fermion mass! The latter is evidently absurd result — for massless theory there is no source of contribution to the antisymmetric \(\epsilon_{\mu\nu\lambda}\) structure and its appearance is an artefact of the PV regularization itself. Of course in the other regularization schemes, which respect discrete \((P, T)\) symmetries at the intermediate stage, no such contribution arises \[5\]. Hence it is natural to modify the PV method in a way to cancel the radiative corrections to the antisymmetric structure. It can be achieved if, apart from the condition of cancellation of divergence

\[
1 + \sum_{i=2}^{N} C_i = 0, \tag{2.13}
\]

another condition is also demanded \[9\]:

\[
\sum_{i=2}^{N} C_i \text{sgn}(M_i) = 0. \tag{2.14}
\]

Note first of all that it is impossible to satisfy both (2.13) and (2.14) if all regulators have the same sign masses. It is evident also that some of the coefficients \(C_i\) have to be fractional. E.g. for two regulator fermions with opposite sign masses solution of (2.13) and (2.14) is

\[
C_2 = C_3 = -\frac{1}{2}. \tag{2.15}
\]

Fractional coefficients make difficult conventional counterterm interpretation of the PV procedure \[10\]. But alternatively we can interpret fractional coefficients as
being originated from fractional charges of the regulator fermions, i.e., their coupling constants to the gauge fields are $|C_i|^{\frac{3}{2}} g$ instead of $g$. This interpretation is consistent for the abelian theory [11], but for the nonabelian one it causes local gauge symmetry violation in the auxiliary fields’ sector — in counter terms. The question is whether it will affect the final (after removing regularization) results. To find answer let us examine the nonabelian case which, as a rule, can make things clear. Here the PV regularization modifies the three-gluon vertex (2.2) too and the extra term is again proportional to the sign of the mass of the regulator fermion [6]:

$$\lim_{|M| \to \infty} \Pi_{\mu\nu\alpha}(p, k; M) = -\epsilon_{\mu\nu\alpha} \frac{g^3}{4\pi} \text{sgn}(M).$$  \hspace{1cm} (2.16)

Hence using the counterterms described above we will get an extra term:

$$g^3 \sum_{i=2}^{N} |C_i|^{\frac{3}{2}} \text{sgn}(M_i).$$  \hspace{1cm} (2.17)

In general, it is not clear whether this term will vanish provided (2.13) and (2.14) are satisfied except when all of the coefficients $C_i$ are the same: $C_i = -1/(N-1), N \geq 2$. Then (2.14) takes the following form:

$$\sum_{i=2}^{N} \text{sgn}(M_i) = 0$$  \hspace{1cm} (2.18)

and (2.17) vanishes if (2.18) is satisfied. As for the higher orders or other Green’s functions, the terms generated by counterterms vanish after removing regularization and hence leave no trace in the final results.

Condition (2.18) is quite interesting from the physical point of view. Evidently, it can be satisfied only if we take equal number of the regulator fermions with both signs of masses. It means that in the limit of the infinite regulator masses the regulator Lagrangian would preserve parity. Of course it does not matter how many pairs with opposite mass signs are taken. Therefore more than one pair of the regulators is abundant.

Thus if we use the above described generalized PV scheme which preserves parity, it will produce results identical to the other parity preserving schemes. It seems quite natural for us because the only distinction of principle of the ordinary PV scheme with the odd number of fermions is its parity-violating nature. Note that in the massive model parity is not a symmetry of the theory. But as in many similar cases, the counterterms of a parity-preserving (symmetric) model are sufficient to renormalize a parity-violating (broken symmetry) massive model as well.
3 The fermion propagator

We will see below that in the case of the fermion propagator one faces the gauge choice problem. In an arbitrary covariant gauge the gauge field propagator has the form [1]:

\[
D_{\mu\nu} = -\frac{i}{p^2 - \mu^2 + i0} (g_{\mu\nu} - \frac{p_{\mu}p_{\nu}}{p^2} - \frac{i\mu}{p^2} \epsilon_{\mu\nu\lambda p}^*) - i\xi \frac{p_{\mu}p_{\nu}}{p^4}
\]  

(3.1)

where \(\xi\) is the gauge parameter and \(\mu\) is the Chern-Simons constant. The corresponding gauge fixing Lagrangian is

\[
\mathcal{L}_{gf} = -\frac{1}{2\xi} (\partial_{\mu} A_{\mu})^2 .
\]  

(3.2)

Below we will use the one-loop results from [1]. The fermion self-energy operator is made up from three terms

\[
\Sigma_I(p) = \frac{-g^2}{16\pi} \int_{-\infty}^{\infty} \frac{da}{\hat{p} - a} \left[ \left( \frac{\mu}{a^2} + \frac{4m}{a} \right) \theta(a^2 - M^2) +\right. \\
+ \left. \frac{1}{\mu^2 a^2} (a^2 - m^2)^2 \theta(M^2 - a^2) \theta(a^2 - m^2) \right] ,
\]  

(3.3)

\[
\Sigma_{II}(p) = \frac{-g^2}{8\pi} \int_{-\infty}^{\infty} \frac{da}{\hat{p} - a} \left[ (a + m) \frac{\mu}{a^2} \theta(a^2 - M^2) +\right. \\
+ \left. \frac{(a - m)(a^2 - m^2)}{\mu a^2} \theta(M^2 - a^2) \theta(a^2 - m^2) \right] ,
\]  

(3.4)

\[
\Sigma_{III}(p) = \frac{-g^2}{16\pi} \int_{-\infty}^{\infty} \frac{da}{\hat{p} - a} \frac{(a + m)^2}{a^2} \theta(a^2 - m^2) ,
\]  

(3.5)

\[
\mathcal{M} \equiv m + |\mu| .
\]

Here contributions of the antisymmetric (\(\Sigma_{II}\)) and the gauge fixing terms (\(\Sigma_{III}\)) are separated. Expressions (3.3)–(3.5) can be obtained by arbitrary Lorentz invariant integration and hence by the dimensional regularization too. Result is finite — expected logarithmic divergence has disappeared. As for the regulator fermions in the PV regularization mentioned in the previous section, they do not contribute here. Although the IR singularity of the massless longitudinal gauge particles is integrable, there arise some pathologies in the mass shell behaviour [1]. Namely
a) $\Sigma_{III}(m) \neq 0$ and hence the fermion renormalized (pole) mass is gauge parameter ($\xi$) dependent, b) $\Sigma'_{III}(m)$ diverges and the wave function renormalization can not be defined.

These pathologies can be avoided in the Landau ($\xi = 0$) gauge where $\Sigma_{III}$ is absent. Hence the Landau gauge plays distinguished role.

Can the above mentioned troubles be avoided for $\xi \neq 0$? For that purpose authors of [1] considered the PV regularization using regulator vector fields. Sources of troubles reside in the infrared region and it may seem strange that the PV regularization cures them. But $\Sigma_{III}(\tilde{p})$ does not depend on the vector particle mass. Therefore its contribution to the total fermion self energy will cancel in the PV scheme if we take the gauge parameter of the regulator vector particle to be equal to the original one. Unfortunately this is not the end of the story — after removing regularization there will arise additional contribution to the $\Sigma_{II}$ structure. Indeed

$$
\Sigma_{II}(\tilde{p})|_{|\tilde{\mu}|\rightarrow\infty} = -\frac{g^2}{2\pi} \frac{\tilde{\mu}}{|\tilde{\mu}|} = -\frac{g^2}{2\pi} sgn(\tilde{\mu}).
$$

Here $\tilde{\mu}$ denotes the mass of the regulator vector particle. So the PV regularization result (e.g. for pole-mass) will differ from dimensionally regularized Landau gauge result due to the additional contribution (3.6).

The way out can be found if we notice that the additional term (3.6) is proportional to the sign of regulator mass and therefore we can extend the PV procedure in full analogy to the previous section. Namely, we can introduce several regulator vector fields and require that the following conditions are satisfied:

$$
1 + \sum_{i=2}^{N} C_i' = 0, \quad \sum_{i=2}^{N} C_i' sgn(\tilde{\mu}_i) = 0.
$$

The meaning of the coefficients $C_i'$ is the same as in the previous section and $\tilde{\mu}_i$ are the masses of the regulator gauge fields. Again, the fact that these auxiliary gauge fields interact with matter fields (and in nonabelian case selfinteract too) with couplings $g | C_i | \frac{1}{4}$ does not violate the gauge invariance of final expressions. Apparently if the gauge parameters for these fields were chosen to be equal then any covariant gauge would reproduce the dimensionally regularized Landau gauge results.

It is worth noting that quite similarly to the conditions (2.13)–(2.14) conditions (3.7) have solutions that correspond to the parity preservation in the end — pairs
of regulators with opposite sign masses. The simplest choice is just one pair with
\[ C'_2 = C'_3 = -\frac{1}{2}. \] (3.8)

So, described generalization of the PV scheme leads to the agreement of arbitrary covariant gauges. But evidently, introduction of any additional procedure for agreeing the covariant gauges seems to be quite artificial and has no well argumented theoretical basis.

4 The fermion propagator in the axial gauge

From the discussion in the previous section it is clear that the Landau ($\xi = 0$) gauge is the only applicable one out of the covariant gauges because it is free of the infrared singularities. Still, even in the Landau gauge, there remains the following problem: after calculating the pole mass and defining the wave function renormalization constant $Z_F$, one finds that there is a nonphysical cut in the continuum contribution starting at $p^2 = m^2$, i.e. earlier than the two-particle threshold at $p^2 = (m + \mu)^2$. The reason is the survival of the contributions of virtual, nonphysical processes originated by $p_\mu p_\nu/p^2$ and $\epsilon_{\mu\nu\alpha\beta}p_\alpha/p^2$ terms of the propagator in the Landau gauge.

The question whether this superfluous contribution does not show up in the physical processes (e.g. the Compton scattering amplitude etc.) is not a priori clear. Below we will see that this problem is absent in the IR softer gauges.

First let us consider an arbitrary axial gauge, when $L_{af} = -\frac{1}{2\xi}(nA)^2$ with $n_\mu$ being some spacelike vector, $n^2 < 0$. The propagator of the vector field has the form
\[ D_{\mu\nu} = -\frac{i}{p^2 - \mu^2}(g_{\mu\nu} - \frac{p_\mu n_\nu + p_\nu n_\mu}{np}) + \frac{n^2 p_\mu p_\nu}{(np)^2} - \frac{i\mu}{np} \epsilon_{\mu\nu\lambda\gamma} n_\lambda - i\xi \frac{n^2 p_\mu p_\nu}{(np)^2}. \] (4.1)

If $\xi = 0$ the general axial gauge is reduced to the homogeneous $nA = 0$ gauge. The $\xi$-dependent part does not contribute to the fermion pole mass because the corresponding integral is less singular compared to the covariant gauge. Indeed, in the axial gauge we have:
\[ \delta m_\xi \sim g^2 \xi \frac{p^2 (p^2 - m^2)}{p^2 (p^2 - m^2)} \int d^4q \frac{1}{(nq)^2((p-q)^2 - m^2)} \bigg|_{p=m} = 0, \] (4.2)
while in the covariant gauge we have:

$$\delta m_\xi \sim g^2 \xi \frac{\hat{p}^2}{p^2} (p^2 - m^2)^2 \int d^3 q \frac{1}{q^4 ((p - q)^2 - m^2)} \bigg|_{\tilde{q} = m} = 2i\pi g^2 \xi . \quad (4.3)$$

Hence we can neglect the $\xi$-dependent term and check the gauge $n_\mu$-vector independence of physical quantities. At this stage there arise two problems inherent in axial gauge. The first one is the problem of prescription for singular $(np)^{-k}$ denominators. It was demonstrated in [14] that it is possible to use any prescription $[np]$ that in the sense of generalized functions [15] satisfies condition $np/[np] = 1$. Another problem is the form of the correct equation for the fermion pole mass. The usual equation

$$S_F^{-1}(p) U(p) \bigg|_{p^2 = m^2} = 0 \quad (4.4)$$

(with $S_F$ being the fermion propagator) is useless in the axial gauge except for the choice $n_\mu \sim p_\mu$. Quite often the following equation is used [16]:

$$\bar{U}(p) S_F^{-1}(p) \bigg|_{\tilde{q} = m} U(p) = 0 . \quad (4.5)$$

In [17] it was demonstrated that this equation produces correct results only at one loop. Beginning from the second loop its solution becomes $n_\mu$-vector dependent. The correct approach is to equate the denominator of the fermion propagator to zero only after rationalizing it (i.e. after eliminating the $\gamma$-matrices from the denominator). Anyway our analysis does not exceed one loop and we can use (4.5) as well.

The (4.1) propagator (with $\xi$ set to zero) results the following contributions to the fermion self energy operator:

$$\Sigma_{g_{\mu\nu}}(p) = - \frac{ig^2}{(2\pi)^3} \int d^3 q \left\{ 3m - \hat{p} \frac{3m - \hat{p}}{(q^2 - \mu^2)((p^2 - q^2) - m^2)} + \frac{\hat{p}}{2p^2 (q^2 - \mu^2)((p^2 - q^2) - m^2)} \right\} , \quad (4.6)$$

$$\Sigma_{(np)^{-1}}(p) = - \frac{ig^2}{(2\pi)^3} \int d^3 q \left\{ \frac{(m^2 - p^2) \hat{n}}{q^4 ((q^2 - \mu^2)((p^2 - q^2) - m^2)} + \frac{(p - m) \hat{q} \hat{n} + \hat{n} \hat{q} (p - m)}{q^4 ((q^2 - \mu^2)((p^2 - q^2) - m^2)} \right\} , \quad (4.7)$$
\[ \Sigma_{(np)-z}(p) = \frac{-ig^2n^2}{(2\pi)^3} \int d^3q \left\{ \frac{m-\hat{p}}{(q^2-\mu^2)(nq)^2} + \frac{(\hat{p} - m)(p^2 - m^2)}{(nq)^2(q^2 - \mu^2)((p^2 - q^2) - m^2)} \right\} + \right\} \cdot \frac{(\hat{p} - m)(\hat{p} - m)}{(nq)^2(q^2 - \mu^2)((p^2 - q^2) - m^2)} \right\}, \quad (4.8) \]

\[ \Sigma_\mu(p) = -\frac{i2\mu g^2}{(2\pi)^3} \int d^3q \left\{ \frac{1}{(q^2 - \mu^2)((p^2 - q^2) - m^2)} + \right\} + \right\} \cdot \frac{(np) - m\hat{n}}{(nq)(q^2 - \mu^2)((p^2 - q^2) - m^2)} \right\}. \quad (4.9) \]

For definiteness we choose \( m > 0 \). Examining expressions (4.6)–(4.9) it is easy to see that the second terms in \( \Sigma_{\mu}(p) \) and \( \Sigma_{(np)-1}(p) \) will cause problems due to the \( \hat{n} \) dependence if one tries to employ the standard equation (4.4). At the same time it is clear that in (4.3) these terms do not contribute and the nonvanishing contributions (resulting in by \( \Sigma_{g_{\mu\nu}} \) and the first term in \( \Sigma_{\mu} \)) are independent of \( n_\mu \).

Explicit calculation of the pole mass using dimensional regularization results:

\[ \delta m = \frac{g^2}{16\pi} \left[ (2 - \frac{\mu}{m})^2 \ln(1 + 2\frac{m}{\mu}) + 2(1 - \frac{|\mu|}{m}) \right]. \quad (4.10) \]

Note that in the \( |\mu| \to 0 \) limit this expression diverges. So the topological (originated by CS term) mass plays the role of infrared regulator. On the other hand if we use the PV regularization introducing a single auxiliary vector field, then in place of (4.10) we have:

\[ \delta m_{PV} = \delta m_{\text{dim}} - \delta m_{|\mu_{\text{reg}}|\to\infty} = \delta m_{\text{dim}} + \frac{g^2}{2\pi} \text{sign}(\mu_{\text{reg}}). \quad (4.11) \]

It coincides with (3.6), i.e. there is a similar discrepancy between dimensional and PV regularizations in the axial gauge too. Hence this discrepancy can be avoided in the same manner by introduction of pairs of opposite mass sign regulator vector particles.

Evidently \( \delta m_{m\to0} = -\frac{g^2}{2\pi} \text{sign}(\mu) \), i.e., even if we start from massless theory, then due to the CS term the fermion mass will be generated. But the vector PV regularization will predict different values for the fermion mass. Moreover, for the specific choice of number and mass signs of auxiliary vectors it is possible to avoid generation of the mass.
Note also that when \( n_\mu \) is a constant vector, the \( n_\mu \)-dependent terms of the gauge field propagator do not contribute to the physical quantities, while the \( n_\mu \) independent ones have the structure:

\[
\int \frac{d^3 q}{(q^2 - \mu^2) ((p - q)^2 - m^2)}.
\]

These terms will produce physical cut beginning at \( p^2 = (m+\mu)^2 \). So the nonphysical cut (starting from \( p^2 \geq m^2 \)) present in the Landau gauge is absent in the axial gauge.

At the end let us mention that we have calculated the fermion pole mass also in the “most physical” Coulomb gauge. We have made use of the propagator

\[
D_{\mu\nu} = -i \frac{1}{p^2 - \mu^2 + i0} \left( g_{\mu\nu} + \vec{\nu} \vec{p}_\nu - \frac{n_\mu n_\nu \vec{p}_0^2}{\vec{p}^2} + \frac{i \mu}{\vec{p}^2} \epsilon_{\mu\nu\lambda} \vec{p}_\lambda \right),
\]

\[\bar{p}^\mu = (0, \vec{p}), \quad n^\mu = (1, \vec{0})\],

Its time-time component is not integrable in two-dimensional space. In [1] it was admitted that the infrared safe version of (4.12) (i.e. without the second and third terms) produces the pole mass identical to the Landau gauge. We have checked by explicit calculation that the same result can be obtained using (4.12). Moreover, like the axial gauge, the nonphysical cut is absent in the Coulomb gauge too.

5 Conclusions

Above we have considered ways and means to avoid undesirable nonphysical ambiguities arising in \((2 + 1)\)-dimensional topologically massive gauge theories. The above described generalization of the Pauli-Villars regularization seems to us to be the most interesting. Quite nontrivial feature is that the local gauge symmetry violated by the counterterm sector is restored after removing regularization. The crucial role is played by superrenormalizability of the models under consideration — due to this feature only finite number of the one loop diagrams are affected by the regularization. The ambiguities in the ordinary PV regularization are just artefacts of the regularization itself and it seems quite natural to avoid these ambiguities by preserving discrete symmetries in the auxiliary sector. It is evident that the described PV regularization scheme excludes, for example, appearance of the parity anomaly [18] and of the other analogous effects as well. Another question is why
should we employ the modified PV regularization, i.e., why should we specify the regularization scheme together with quantization of the model? There is no answer to this question in general. We can only require that the regularization used should not produce physically meaningless results. The suggested modification of the PV scheme satisfies this requirement and is compatible with the other schemes which does not introduce additional violation of discrete symmetries. The same is true about the existence of preferred gauge, such as the Landau gauge out of covariant gauges though even in the Landau gauge there remains trace of nonphysical processes resulting nonphysical cut in the fermion propagator. We have demonstrated above that by the means of generalization of the PV regularization in the vector particles’ sector the compatibility of all covariant gauges (including the Landau gauge) can be achieved. The PV procedure in this case takes care of IR but not UV problem. The considered example shows that one must take special care while introducing gauge fixing terms into Langrangian especially when gauge variant quantities like the Green’s functions are considered. As for the noncovariant gauges (e.g. axial or Coulomb), they, being “softer” in infrared, do not imply nonphysical singularities.

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