Searching for Cold Dark Matter. A case of coexistence of Supersymmetry and Nuclear Physics.

J.D. VERGADOS and T.S. KOSMAS

Theoretical Physics Section, University of Ioannina, GR-45110, Greece.

Abstract

The direct detection rate for supersymmetric cold dark matter (CDM) particles is calculated for a number of suitable nuclear targets. Both the coherent and spin contributions are considered. By considering representative phenomenologically acceptable input in the restricted SUSY parameter space, detectable rates are predicted for some choices of the parameters. The modulation effect, due to the Earth’s annual motion, has also been considered and found to be $\leq 4\%$. Its precise value depends on the mass of CDM particles (LSP) and the structure of the target.

Presented by J.D. Vergados.
1 Introduction

There are many arguments supporting the fact that, the cold dark matter of the universe, i.e. its component which is composed of particles which were non-relativistic at the time of structure formation, is at least 60%. There are two interesting cold dark matter candidates: i) Massive Compact Halo Objects (MACHO’s) and ii) Weak Interacting Massive Particles (WIMP’s). The MACHO’s cannot exceed 40% of the CDM component. In the present work we discuss a special WIMP candidate connected with the supersymmetry, i.e. the lightest supersymmetric particle (LSP).

We examine the possibility to directly detect the LSP via the recoiling of a nucleus \((A,Z)\) in the elastic scattering process:

\[ \chi + (A, Z) \rightarrow \chi + (A, Z)^* \]  

(\(\chi\) denotes the LSP). In this investigation, we proceed with the following steps:

1) We write down the effective Lagrangian at the elementary particle (quark) level obtained in the framework of supersymmetry as described in Refs. [3]-[11]

2) We go from the quark to the nucleon level using an appropriate quark model for the nucleon. Special attention in this step is paid to the scalar couplings, which dominate the coherent part of the cross section and the isoscalar axial current, which, as we will see, strongly depend on the assumed quark model. [10]

3) We compute the relevant nuclear matrix elements [18]-[25] using as reliable as possible many body nuclear wave functions hoping that, by putting as accurate nuclear physics input as possible, one will be able to constrain the SUSY parameters as much as possible.

4) We calculate the modulation of the cross sections due to the earth’s revolution around the sun by a folding procedure assuming a Maxwell Boltzmann distribution [3] of velocities for LSP.

There are many popular targets [26]-[30] for LSP detection as e.g. \(^{19}F, ^{23}Na, ^{27}Al, ^{29}Si, ^{40}Ca, ^{73,74}Ge, ^{127}I, ^{207}Pb\), etc. Among them \(^{207}Pb\) has been recently proposed [3] as a theoretical laboratory. Furthermore, it can be an important detector, since its spin matrix element, especially the isoscalar one, does not exhibit large quenching as that of the light and up to now much studied \(^{29}Si\) and \(^{73}Ge\) nuclei. [18]

Our purpose is to calculate LSP-nucleus scattering cross section using some representative input in the restricted SUSY parameter space, [11]-[14, 30, 31] to compute the coherent LSP-nucleus scattering cross sections throughout the periodic table and study the spin matrix elements of \(^{207}Pb\), since this target, in addition to its experimental qualifications, has the advantage of a rather simple nuclear structure. We compare our results to those obtained [18] for other proposed cold dark matter detection targets. We finally present results obtained by using new input SUSY parameters [31] obtained in a phenomenologically allowed parameter space.

2 Effective Lagrangian

Before proceeding with the construction of the effective Lagrangian we will briefly discuss the nature of the lightest supersymmetric particle (LSP) focusing on those ingredients which are of interest to dark matter.
2.1 The nature of LSP

In currently favorable supergravity models the LSP is a linear combination \[ \tilde{\chi} \] of the neutral four fermions \( \tilde{B}, \tilde{W}_3, \tilde{H}_1 \) and \( \tilde{H}_2 \) which are the supersymmetric partners of the gauge bosons \( B_\mu \) and \( W^\mu_\mu \) and the Higgs scalars \( H_1 \) and \( H_2 \). Admixtures of s-neutrinos are expected to be negligible.

In the above basis the mass-matrix takes the form \[ \text{Eq. C86} \]

\[
\begin{pmatrix}
M_1 & 0 & -m_2 c_\beta s_w & m_2 s_\beta s_w \\
0 & M_2 & m_2 c_\beta c_w & -m_2 s_\beta c_w \\
-m_2 c_\beta s_w & m_2 c_\beta c_w & 0 & -\mu \\
m_2 s_\beta s_w & -m_2 c_\beta c_w & -\mu & 0
\end{pmatrix}
\] (2)

In the above expressions \( c_W = \cos \theta_W, \ s_W = \sin \theta_W, \ c_\beta = \cos \beta, \ s_\beta = \sin \beta \), where \( \tan \beta = \langle v_2 \rangle / \langle v_1 \rangle \) is the ratio of the vacuum expectation values of the Higgs scalars \( H_2 \) and \( H_1 \). \( \mu \) is a dimensionful coupling constant which is not specified by the theory (not even its sign). The parameters \( \tan \beta, M_1, M_2, \mu \) are determined by the procedure of Refs. \[ 30, 31 \] using universal masses of the GUT scale.

By diagonalizing the above matrix we obtain a set of eigenvalues \( m_j \) and the diagonalizing matrix \( C_{ij} \) as follows

\[
\begin{pmatrix}
\tilde{B}_R \\
\tilde{W}_{3R} \\
\tilde{H}_{1R} \\
\tilde{H}_{2R}
\end{pmatrix}
= (C_{ij})
\begin{pmatrix}
\chi_{1R} \\
\chi_{2R} \\
\chi_{3R} \\
\chi_{4R}
\end{pmatrix}
= (C_{ij}^*)
\begin{pmatrix}
\chi_{1L} \\
\chi_{2L} \\
\chi_{3L} \\
\chi_{4L}
\end{pmatrix}
\]

(3)

Another possibility to express the above results in photino-zino basis \( \tilde{\gamma}, \tilde{Z} \) via

\[
\begin{align*}
\tilde{W}_3 &= \sin \theta_W \tilde{\gamma} - \cos \theta_W \tilde{Z} \\
\tilde{B}_0 &= \cos \theta_W \tilde{\gamma} + \sin \theta_W \tilde{Z}
\end{align*}
\] (4)

In the absence of supersymmetry breaking \( (M_1 = M_2 = M \text{ and } \mu = 0) \) the photino is one of the eigenstates with mass \( M \). One of the remaining eigenstates has a zero eigenvalue and is a linear combination of \( \tilde{H}_1 \) and \( \tilde{H}_2 \) with mixing angle \( \sin \beta \). In the presence of SUSY breaking terms the \( \tilde{B}, \tilde{W}_3 \) basis is superior since the lowest eigenstate \( \chi_1 \) or LSP is primarily \( \tilde{B} \). From our point of view the most important parameters are the mass \( m_x \) of LSP and the mixings \( C_j, \ j = 1, 2, 3, 4 \) which yield the \( \chi_1 \) content of the initial basis states. These parameters which are relevant here are shown in Table 1.

We are now in a position to find the interaction of \( \chi_1 \) with matter. We distinguish three possibilities involving Z-exchange, s-quark exchange and Higgs exchange.

2.2 The relevant Feynman diagrams

2.2.1 The Z-exchange contribution

This can arise from the interaction of Higgsinos with Z which can be read from Eq. C86 of Ref. \[ 32 \]

\[
L = \frac{g}{\cos \theta_W} \frac{1}{4} \left[ \tilde{H}_{1R} \gamma_\mu \tilde{H}_{1R} - \tilde{H}_{1L} \gamma_\mu \tilde{H}_{1L} - \tilde{H}_{2R} \gamma_\mu \tilde{H}_{2R} - \tilde{H}_{2L} \gamma_\mu \tilde{H}_{2L} \right] Z^\mu
\] (5)
Using Eq. (3) and the fact that for Majorana particles $\bar{\chi}\gamma_{\mu}\chi = 0$, we obtain

$$L = \frac{g}{\cos\theta_W} \frac{1}{4} (|C_{31}|^2 - |C_{41}|^2) \bar{\chi}_1 \gamma_{\mu} \gamma_5 \chi_1 Z^\mu$$

which leads to the effective 4-fermion interaction (see Fig. 1)

$$L_{\text{eff}} = \frac{g}{\cos\theta_W} \frac{1}{4} 2(|C_{31}|^2 - |C_{41}|^2)(-\frac{g}{2\cos\theta_W} \frac{1}{q^2 - m_Z^2} \bar{\chi}_1 \gamma_{\mu} \gamma_5 \chi_1) J^Z_{\mu}$$

where the extra factor of 2 comes from the Majorana nature of $\chi_1$. The neutral hadronic current $J^Z_\lambda$ is given by

$$J^Z_\lambda = -\bar{q}_\lambda \{ \frac{1}{3} \sin^2\theta_W - \left[ \frac{1}{2}(1 - \gamma_5) - \sin^2\theta_W \right] \tau_3 \} q$$

at the nucleon level it can be written as

$$J^Z_\lambda = -\bar{N}_\gamma \{ \sin^2\theta_W - g_V(\frac{1}{2} - \sin^2\theta_W)\tau_3 + \frac{1}{2} g_A \gamma_5 \tau_3 \} N$$

Thus we can write

$$L_{\text{eff}} = -\frac{G_F}{\sqrt{2}} (\bar{\chi}_1 \gamma^\lambda \gamma_5 \chi_1) J_\lambda (Z)$$

where

$$J_\lambda (Z) = N \gamma_\lambda [f^0_V (Z) + f^1_V (Z) \tau_3 + f^0_A (Z) \gamma_5 + f^1_A (Z) \gamma_5 \tau_3] N$$

and

$$f^0_V (Z) = 2(|C_{31}|^2 - |C_{41}|^2) \frac{m^2_Z}{m^2_Z - q^2} \sin^2\theta_W$$

$$f^1_V (Z) = -2(|C_{31}|^2 - |C_{41}|^2) \frac{m^2_Z}{m^2_Z - q^2} g_V(\frac{1}{2} - \sin^2\theta_W)$$

$$f^0_A (Z) = 0$$

$$f^1_A (Z) = 2(|C_{31}|^2 - |C_{41}|^2) \frac{m^2_Z}{m^2_Z - q^2} \frac{1}{2} g_A$$

with $g_V = 1.0, g_A = 1.24$. We can easily see that

$$f^1_V (Z)/f^0_V (Z) = -g_V(\frac{1}{2\sin^2\theta_W} - 1) \simeq -1.15$$

Note that the suppression of this $Z$-exchange interaction compared to the ordinary neutral current interactions arises from the smallness of the mixings $C_{31}$ and $C_{41}$, a consequence of the fact that the Higgsinos are normally quite a bit heavier than the gauginos. Furthermore, the two Higgsinos tend to cancel each other.

2.2.2 The $s$-quark mediated interaction

The other interesting possibility arises from the other two components of $\chi_1$, namely $\bar{B}$ and $\bar{W}_3$. Their corresponding couplings to $s$-quarks can be read from the appendix C4 of Ref. [32]. They are

$$L_{\text{eff}} = -g\sqrt{2} \{ \bar{q}_L [T_3 \bar{W}_3 R - \tan\theta_W (T_3 - Q) \bar{B}_R] \bar{q}_L - \tan\theta_W \bar{q}_R Q \bar{B}_L \bar{q}_R \} + HC$$

(17)
where \( \tilde{q} \) are the scalar quarks (SUSY partners of quarks). A summation over all quark flavors is understood. Using Eq. (3) we can write the above equation in the \( \chi_i \) basis. Of interest to us here is the part

\[
L_{\text{eff}} = g\sqrt{2}\left\{ (\tan\theta_W)(T_3 - Q)C_{11} - T_3C_{21})\tilde{q}_L\chi_{1R}\tilde{q}_L \\
+ \tan\theta_W C_{11} Q\tilde{q}_R\chi_{1L}\tilde{q}_R \right\}
\] (18)

The above interaction is almost diagonal in the quark flavor. There exists, however, mixing between the s-quarks \( \tilde{q}_L \) and \( \tilde{q}_R \) (of the same flavor) i.e.

\[
\begin{align*}
\tilde{q}_L &= \cos\theta_{\tilde{q}}\tilde{q}_1 + \sin\theta_{\tilde{q}}\tilde{q}_2 \\
\tilde{q}_R &= -\sin\theta_{\tilde{q}}\tilde{q}_1 + \cos\theta_{\tilde{q}}\tilde{q}_2
\end{align*}
\] (19)

with

\[
\begin{align*}
tan2\theta_{\tilde{u}} &= \frac{m_u(A + \mu \cot\beta)}{m^2_u - m^2_{\tilde{u}_R} + m^2_Z \cos 2\beta/2} \\
tan2\theta_{\tilde{d}} &= \frac{m_d(A + \mu \tan\beta)}{m^2_d - m^2_{\tilde{d}_R} + m^2_Z \cos 2\beta/2}
\end{align*}
\] (21)

The above effective interaction can be written as

\[
L_{\text{eff}} = (g\sqrt{2})^2\left\{ [B_L \cos \theta_{\tilde{q}} \tilde{q}_L\chi_{1R} - B_R \sin \theta_{\tilde{q}}\tilde{q}_R\chi_{1L}]\tilde{q}_1 \\
+ [B_L \sin \theta_{\tilde{q}}\tilde{q}_L\chi_{1R} + B_R \cos \theta_{\tilde{q}}\tilde{q}_R\chi_{1L}]\tilde{q}_2 \right\}
\] (23)

The above effective interaction can be written as

\[
L_{\text{eff}} = L^{LL+RR}_{\text{eff}} + L^{LR}_{\text{eff}}
\] (24)
The first term involves quarks of the same chirality and is not much affected by the mixing (provided that it is small). The second term involves quarks of opposite chirality and is proportional to the s-quark mixing.

i) The part \( L_{\text{eff}}^{LL+RR} \)

Employing a Fierz transformation \( L_{\text{eff}}^{LL+RR} \) can be cast in the more convenient form

\[
L_{\text{eff}}^{LL+RR} = (g\sqrt{2})^2(\frac{1}{2})(|B_L|^2 \\
\left(\frac{\cos^2\theta_q}{q^2 - m_{q_1}^2} + \frac{\sin^2\theta_q}{q^2 - m_{q_2}^2}\right)\bar{q}_L \gamma^\lambda q_L \chi_{1R}^\gamma \chi_{1L}^\gamma \\
+ |B_R|^2(\frac{\sin^2\theta_q}{q^2 - m_{q_1}^2} + \frac{\cos^2\theta_q}{q^2 - m_{q_2}^2})\bar{q}_R \gamma^\lambda q_R \chi_{1L}^\gamma \chi_{1L}^\gamma \right) \tag{25}
\]

The factor of 2 comes from the majorana nature of LSP and the (-1/2) comes from the Fierz transformation. Equation (25) can be written more compactly as

\[
L_{\text{eff}} = -\frac{G_F}{\sqrt{2}}2\{\bar{q}\gamma_\lambda(\beta_{0R} + \beta_{3R}\tau_3)(1 + \gamma_5)q \\
- \bar{q}\gamma_\lambda(\beta_{0L} + \beta_{3L}\tau_3)(1 - \gamma_5)q\}(\chi_1\gamma^\gamma\chi_1) \tag{26}
\]

with

\[
\beta_{0R} = \left(\frac{4}{9}\chi^{2}_{u_R} + \frac{1}{9}\chi^{2}_{d_R}\right)|C_{11}\tan\theta_W|^2 \\
\beta_{3R} = \left(\frac{4}{9}\chi^{2}_{u_R} - \frac{1}{9}\chi^{2}_{d_R}\right)|C_{11}\tan\theta_W|^2 \tag{27}
\]

\[
\beta_{0L} = \frac{1}{6}C_{11}\tan\theta_W + \frac{1}{2}C_{21}|^2\chi^{2}_{u_L} + \frac{1}{6}C_{11}\tan\theta_W - \frac{1}{2}C_{21}|^2\chi^{2}_{d_L} \\
\beta_{3L} = \frac{1}{6}C_{11}\tan\theta_W + \frac{1}{2}C_{21}|^2\chi^{2}_{u_L} - \frac{1}{6}C_{11}\tan\theta_W - \frac{1}{2}C_{21}|^2\chi^{2}_{d_L}
\]

with

\[
\chi^{2}_{q_L} = c_q\frac{m_q^2}{m_{q_i}^2 - q^2} + s_q\frac{m_q^2}{m_{q_i}^2 - q^2} \\
\chi^{2}_{q_R} = s_q\frac{m_q^2}{m_{q_i}^2 - q^2} + c_q\frac{m_q^2}{m_{q_i}^2 - q^2} \\
c_q = \cos\theta_q, \ s_q = \sin\theta_q \tag{28}
\]

The above parameters are functions of the four-momentum transfer which in our case is negligible. Proceeding as in Sec. 2.2.1 we can obtain the effective Lagrangian at the nucleon level as

\[
L_{\text{eff}}^{LL+RR} = -\frac{G_F}{\sqrt{2}}(\bar{\chi}_1\gamma^\gamma\chi_1)J_\lambda(\bar{q}) \tag{29}
\]

\[
J_\lambda(\bar{q}) = \bar{N}\gamma_\lambda\{f_V^0(\bar{q})\gamma_5 + f_V^1(\bar{q})\gamma_3 + f_A^0(\bar{q})\gamma_5 + f_A^1(\bar{q})\gamma_5\gamma_3\}N \tag{30}
\]

with

\[
f_V^0 = 6(\beta_{0R} - \beta_{0L}), \quad f_V^1 = 2(\beta_{3R} - \beta_{3L}) \\
f_A^0 = 2g_V(\beta_{0R} + \beta_{0L}), \quad f_A^1 = 2g_A(\beta_{3R} + \beta_{3L}) \tag{31}
\]
We should note that this interaction is more suppressed than the ordinary weak interaction by the fact that the masses of the s-quarks are usually larger than that of the gauge boson $Z^0$. In the limit in which the LSP is a pure bino ($C_{11} = 1, C_{21} = 0$) we obtain

\[
\beta_{0R} = \tan^2\theta_W \left( \frac{4}{9} \chi_{uR}^2 + \frac{1}{9} \chi_{dR}^2 \right)
\]

\[
\beta_{3R} = \tan^2\theta_W \left( \frac{4}{9} \chi_{uR}^2 - \frac{1}{9} \chi_{dR}^2 \right)
\]

\[
\beta_{0L} = \frac{\tan^2\theta_W}{36} (\chi_{uL}^2 + \chi_{dL}^2)
\]

\[
\beta_{3L} = \frac{\tan^2\theta_W}{36} (\chi_{uL}^2 - \chi_{dL}^2)
\]

(32)

Assuming further that $\chi_{uR} = \chi_{dR} = \chi_{uL} = \chi_{dL}$ we obtain

\[
f_1^V(\bar{q})/f_0^V(\bar{q}) \simeq +2 \frac{9}{14}
\]

\[
f_3^V(\bar{q})/f_0^V(\bar{q}) \simeq + 6 \frac{11}{22}
\]

(33)

If, on the other hand, the LSP were the photino ($C_{11} = \cos\theta_W, C_{21} = \sin\theta_W, C_{31} = C_{41} = 0$) and the s-quarks were degenerate there would be no coherent contribution ($f_0^V = 0$ if $\beta_{0L} = \beta_{0R}$).

ii) $L_{eff}^{LR}$

From Eq. (23) we obtain

\[
L_{eff}^{LR} = -(g\sqrt{2})^2 \sin 2\theta_W B_L(q) B_R(q) \frac{1}{2} \left[ \frac{1}{q^2 - m_{\tilde{q_1}}^2} - \frac{1}{q^2 - m_{\tilde{q_2}}^2} \right]
\]

\[
(q_L \chi_{1R} \chi_{1L} q_R + \bar{q}_R \chi_{1L} \bar{\chi}_{1R} q_L)
\]

Employing a Fierz transformation we can cast it in the form

\[
L_{eff} = -\frac{G_F}{\sqrt{2}} \left[ \beta_+(\bar{q}q \chi_{11} + \bar{q}\gamma_5 q \chi_1 \gamma_5 \chi_1 - (\bar{q}\sigma_{\mu\nu} q)(\chi_1 \sigma^{\mu\nu} \chi_1)) + \beta_-(\bar{q}\gamma_5 q \chi_{11} - \bar{q}\gamma_5 q \chi_1 \gamma_5 \chi_1 - (\bar{q}\sigma_{\mu\nu} q)(\chi_1 \sigma^{\mu\nu} \chi_1)) \right]
\]

where

\[
\beta_+ = \frac{1}{3} \tan \theta_W C_{11} \left[ 2 \sin 2\theta_W \frac{1}{6} C_{11} \tan \theta_W + \frac{1}{2} C_{21} \right] \Delta_{\bar{u}}
\]

\[
\beta_- = \sin 2\theta_W \frac{1}{6} C_{11} \tan \theta_W - \frac{1}{2} C_{21} \Delta_{\bar{d}}
\]

with

\[
\Delta_{\bar{u}} = \frac{(m_{\tilde{u}_1}^2 - m_{\tilde{u}_2}^2) M_W^2}{(m_{\tilde{u}_1}^2 - q^2)(m_{\tilde{u}_2}^2 - q^2)}
\]

and an analogous equation for $\Delta_{\bar{d}}$. Here $u$ indicate quarks with charge 2/3 and d quarks with charge -1/3.
In going to the nucleon level and ignoring the negligible pseudoscalar and tensor components in the spirit of Ref. [33] we obtain

\[ L_{\text{eff}} = \frac{G_F}{\sqrt{2}} [f_s^0(\bar{q})\bar{N}N + f_s^1(\bar{q})\bar{N}\tau_3 N]\bar{\chi}_1 \chi_1 \]  

(34)

with

\[ f_s^0(\bar{q}) = 1.86\beta_+ \quad \text{and} \quad f_s^1(\bar{q}) = 0.48\beta_- \quad (\text{Model A}) \]  

(35)

(see sect. 2.2.3) The appearance of scalar terms in s-quark exchange has been first noticed in Ref. [7]. It has also been noticed there that one should consider explicitly the effects of quarks other than u and d [10] in going from the quark to the nucleon level. We first notice that with the exception of t s-quark the \( \bar{q}_L - \bar{q}_R \) mixing small. Thus

\[
\sin^2 \theta_{\bar{u}} \Delta \bar{u} \simeq \frac{2m_u(A + \mu \cot \beta)m_W^2}{(m_{\bar{u}_L}^2 - q^2)(m_{\bar{u}_R}^2 - q^2)}
\]

\[
\sin^2 \theta_{\bar{d}} \Delta \bar{d} \simeq \frac{2m_d(A + \mu \tan \beta)m_W^2}{(m_{\bar{d}_L}^2 - q^2)(m_{\bar{d}_R}^2 - q^2)}
\]

Then the amplitude for this s-quark contribution is proportional to the quark mass (à la Higgs). Thus the amplitude for finding such quarks in the nucleon can be computed in a way which is similar to that of the Higgs coupling (see Sec. 2.2.3). For the t s-quark the mixing is complete, which implies that the amplitude is independent of the top quark mass. Hence in the case of the top quark we do not get an extra enhancement in going from the quark to the nucleon level. As we will see in the next section we get an enhancement due to quarks other than u and d (see model B in the next section). This is not enough, however, to dominate even over \( \beta f_0^l \) in the SUSY parameter space considered here. Thus, \( f_s^0(\bar{q}) \) can be neglected in front of the isoscalar scalar coupling coming from Higgs exchange (see sect. 2.2.3).

2.2.3 The intermediate Higgs contribution

The coherent scattering can be mediated via the intermediate Higgs particles which survive as physical particles (see Fig. 2). The relevant interaction can arise out of the Higgs-Higgsino-gaugino interaction which takes the form

\[
L_{H\chi\chi} = \frac{g}{\sqrt{2}} \left( \bar{W}_R^3 \bar{H}_{2L}^* H_2^0 - \bar{W}_R^3 \bar{H}_{1L}^* H_1^0 \right) - \tan \theta_w (\bar{B}_R^* \bar{H}_{2L}^* H_2^0 - \bar{B}_R \bar{H}_{1L}^* H_1^0) + H.C.
\]

(36)

Proceeding as above we can express \( \bar{W} \) an \( \bar{B} \) in terms of the appropriate eigenstates and retain the LSP to obtain

\[
L = \frac{g}{\sqrt{2}} \left( (C_{21} - \tan \theta_w C_{11}) C_{41} \bar{\chi}_{1R} \chi_{1L} H_2^{0*} \right) - \left( C_{21} - \tan \theta_w C_{11} \right) C_{31} \bar{\chi}_{1R} \chi_{1L} H_1^{0*} + H.C.
\]

(37)

We can now proceed further and express the fields \( H_1^{0*} \), \( H_2^{0*} \) in terms of the physical fields \( h \), \( H \) and \( A \). The term which contains \( A \) will be neglected, since it yields only a pseudoscalar coupling which does not lead to coherence.
Thus we can write

\[ \mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \bar{\chi} \chi \bar{N}[f_s^0(H) + f_s^1(H)\tau_3]N \]  

(38)

where

\[ f_s^0(H) = \frac{1}{2}(g_u + g_d) + g_s + g_c + g_b + g_t \]  

(39)

\[ f_s^1(H) = \frac{1}{2}(g_u - g_d) \]  

(40)

with

\[ g_{a_i} = \left[ g_1(h) \frac{\cos \alpha}{\sin \beta} + g_2(H) \frac{\sin \alpha}{\sin \beta} \right] m_{a_i}, \quad a_i = u, c, t \]  

(41)

\[ g_{\kappa_i} = \left[ - g_1(h) \frac{\sin \alpha}{\cos \beta} + g_2(H) \frac{\cos \alpha}{\cos \beta} \right] m_{\kappa_i}, \quad \kappa_i = d, s, b \]  

(42)

\[ g_1(h) = 4(C_{11}\tan \theta_W - C_{21})(C_{41}\cos \alpha - C_{31}\sin \alpha) \frac{m_N m_W}{m_h^2 - q^2} \]  

(43)

\[ g_2(H) = 4(C_{11}\tan \theta_W - C_{21})(C_{41}\sin \alpha - C_{31}\cos \alpha) \frac{m_N m_W}{m_H^2 - q^2} \]  

(44)

where \( m_N \) is the nucleon mass, and the parameters \( m_h, m_H \) and \( \alpha \) depend on the SUSY parameter space (see Table 1). If one ignores quarks other than \( u \) and \( d \) (model A) and uses \( m_u = 5 MeV = m_d/2 \) finds [33]

\[ f_s^0 = 1.86(g_u + g_d)/2, \quad f_s^1 = 0.49(g_u - g_d)/2, \]  

(45)

As we have already mentioned, one has to be a bit more careful in handling quarks other than \( u \) and \( d \) since their couplings are proportional to their mass. [10, 11] One encounters in the nucleon not only sea quarks (\( u \bar{u}, d \bar{d} \) and \( s \bar{s} \)) but the heavier quarks also due to QCD effects, which were estimated at the one loop level in Ref. [34, 35] This way one obtains the pseudoscalar Higgs-nucleon coupling \( f_s^0(H) \) by using effective quark masses as follows

\[ m_u \rightarrow f_u m_N, \quad m_d \rightarrow f_d m_N, \quad m_s \rightarrow f_s m_N \]

\[ m_Q \rightarrow f_Q m_N, \quad (\text{heavy quarks } c, b, t) \]

where \( m_N \) is the nucleon mass. The isovector contribution is now negligible. The parameters \( f \) can be obtained by chiral symmetry breaking terms in relation to phase shift and dispersion analysis. [10, 11] Following Cheng [34, 35] we obtain

\[ f_u = 0.021, \quad f_d = 0.037, \quad f_s = 0.140, \quad \sum_Q f_Q = 0.240 \quad \text{(model B)} \]

Another possible solution is

\[ f_u = 0.023, \quad f_d = 0.034, \quad f_s = 0.400, \quad \sum_Q f_Q = 0.120 \quad \text{(model C)} \]

In the present work we will consider these solutions (models B and C) and compare them with the solution obtained in the spirit of Addler et al. [33] (Model A above). For a more detailed discussion we refer the reader to Refs. [10, 11].
2.3 Expressions for the nuclear matrix elements

Combining for results of the previous section we can write

\[ L_{eff} = -\frac{G_F}{\sqrt{2}} \left\{ (\bar{\chi}_1 \gamma^\lambda \gamma_5 \chi_1) J_\lambda + (\bar{\chi}_1 \chi_1) J \right\} \]  

(46)

where

\[ J_\lambda = \bar{N} \gamma_\lambda (f^0_\nu + f^1_\nu \tau_3 + f^0_A \gamma_5 + f^1_A \gamma_5 \tau_3) N \]  

(47)

with

\[ f^0_\nu = f^0_\nu (Z) + f^0_\nu (\bar{q}), \quad f^1_\nu = f^1_\nu (Z) + f^1_\nu (\bar{q}) \]
\[ f^0_A = f^0_A (Z) + f^0_A (\bar{q}), \quad f^1_A = f^1_A (Z) + f^1_A (\bar{q}) \]  

(48)

and

\[ J = \bar{N} (f^0_s + f^1_s \tau_3) N \]  

(49)

We have neglected the uninteresting pseudoscalar and tensor currents. Note that, due to the Majorana nature of the LSP, \( \bar{\chi}_1 \gamma^\lambda \chi_1 = 0 \) (identically). We have seen that, the vector and axial vector form factors can arise out of Z-exchange and s-quark exchange. \[6\]–\[4\] They have uncertainties in them. Here we consider the three choices in the allowed parameter space of Ref. [30] and the eight parameter choices of Ref. [31] These involve universal soft breaking masses at the scale. Non-universal masses have also recently been employed. [11]–[13] In our choice of the parameters the LSP is mostly a gaugino. Thus, the Z- contribution is small. It may become dominant in models in which the LSP happens to be primarily a Higgsino. [36] The transition from the quark to the nucleon level is pretty straightforward in the case of vector current contribution. We will see later that, due to the Majorana nature of the LSP, the contribution of the vector current, which can lead to a coherent effect of all nucleons, is suppressed. [3] The vector current is effectively multiplied by a factor of \( \beta = v/c \), \( v \) is the velocity of LSP (see Tables 2(a),(b)). Thus, the axial current, especially in the case of light and medium mass nuclei, cannot be ignored.

For the isovector axial current one is pretty confident about how to go from the quark to the nucleon level. We know from ordinary weak decays that the coupling merely gets renormalized from \( g_A = 1 \) to \( g_A = 1.24 \). For the isoscalar axial current the situation is not completely clear. The naive quark model (NQM) would give a renormalization parameter of unity (the same as the isovector vector current). This point of view has, however, changed in recent years due to the so-called spin crisis, [37]–[39] i.e. the fact that in the EMC data [37] it appears that only a small fraction of the proton spin arises from the quarks. Thus, one may have to renormalize \( f_A^0 \) by \( g_A^0 = 0.28 \), for u and d quarks, and \( g_A^0 = -0.16 \) for the strange quarks, [38, 39] i.e. a total factor of 0.12. These two possibilities, labeled as NQM and EMC, are listed in Tables 2(a),(b). One cannot completely rule out the possibility that the actual value maybe anywhere in the above mentioned region. [39]

The scalar form factors arise out of the Higgs exchange or via s-quark exchange when there is mixing [40] between s-quarks \( \bar{q}_L \) and \( \bar{q}_R \) (the partners of the left-handed and right-handed quarks). We have seen in Ref. [4] that they have two types of uncertainties in them. One, which is the most important, at the quark level due to the uncertainties in the Higgs sector. The actual values of the parameters \( f_S^0 \) and \( f_S^1 \) used here, arising mainly from Higgs exchange, were obtained by considering 1-loop corrections in the Higgs sector. As a result, the lightest Higgs mass is now a bit higher, i.e. more massive than the value of the Z-boson. [16, 17]

The other type of uncertainty is related to the step going from the quark to the nucleon level [41] (see sect. 2.2.3). Such couplings are proportional to the quark masses, and hence sensitive to the
small admixtures of $qar{q}$ (q other than u and d) present in the nucleon. Again values of $f_S^0$ and $f_S^1$ in the allowed SUSY parameter space are considered (see Tables 2(a),(b)).

The invariant amplitude in the case of non-relativistic LSP can now be cast in the form

$$|\mathcal{M}|^2 = \frac{E_j E_i - m_j^2 + \mathbf{p}_i \cdot \mathbf{p}_f}{m_x^2} |J_0|^2 + |J|^2$$

$$\approx \beta^2 |J_0|^2 + |J|^2$$  \hspace{1cm} (50)

where $m_x$ is the LSP mass, $|J_0|$ and $|J|$ indicate the matrix elements of the time and space components of the current $J_\lambda$ of Eq. (49), respectively, and $J$ represents the matrix element of the scalar current $J$ of Eq. (50). Notice that $|J_0|^2$ is multiplied by $\beta^2$ (the suppression due to the Majorana nature of LSP mentioned above). It is straightforward to show that

$$|J_0|^2 = A^2 |F(q^2)|^2 \left( f_V^0 - f_V^1 \frac{A - 2Z}{A} \right)^2$$

$$J^2 = A^2 |F(q^2)|^2 \left( f_S^0 - f_S^1 \frac{A - 2Z}{A} \right)^2$$

$$|J|^2 = \frac{1}{2J_i + 1} |\langle J_i || J_0 \rangle |^2 \left[ f_A^0 \Omega_0(q) + f_A^1 \Omega_1(q) \right] |J_i|^2$$  \hspace{1cm} (53)

with $F(q^2)$ the nuclear form factor and

$$\Omega_0(q) = \sum_{j=1}^{A} \sigma(j) e^{-i\mathbf{q} \cdot \mathbf{x}_j}, \hspace{1cm} \Omega_1(q) = \sum_{j=1}^{A} \sigma(j) \tau_3(j) e^{-i\mathbf{q} \cdot \mathbf{x}_j}$$  \hspace{1cm} (54)

where $\sigma(j)$, $\tau_3(j)$, $\mathbf{x}_j$ are the spin, third component of isospin ($\tau_3(p) = |p|$) and coordinate of the j-th nucleon and $\mathbf{q}$ is the momentum transferred to the nucleus.

The differential cross section in the laboratory frame takes the form

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_0}{\pi} \left( \frac{m_x}{m_N} \right)^2 \frac{1}{(1 + \eta)^2} \xi \left\{ \beta^2 |J_0|^2 \left[ 1 - \frac{2\eta + 1}{(1 + \eta)^2} \xi^2 \right] + |J|^2 + |J|^2 \right\}$$  \hspace{1cm} (55)

where $m_N$ is the proton mass, $\eta = m_x/m_N A$, $\xi = \mathbf{p}_f \cdot \mathbf{q} \geq 0$ (forward scattering) and

$$\sigma_0 = \frac{1}{2\pi} (G_F m_N)^2 \simeq 0.77 \times 10^{-38} cm^2$$

The momentum transfer $\mathbf{q}$ is given by

$$|\mathbf{q}| = q_0 \xi, \hspace{1cm} q_0 = \frac{\beta m_x c}{1 + \eta}$$  \hspace{1cm} (57)

Some values of $q_0$ (forward momentum transfer) for some characteristic values of $m_x$ and representative nuclear systems (light, medium and heavy) are given in Table 3. It is clear from Eq. (57) that the momentum transfer can be sizable for large $m_x$ and heavy nuclei ($\eta$ small).

The total cross section can be cast in the form

$$\sigma = \sigma_0 \left( \frac{m_x}{m_N} \right)^2 \frac{1}{(1 + \eta)^2} \left\{ A^2 \left[ \beta^2 (f_V^0 - f_V^1 \frac{A - 2Z}{A})^2 \right] + \left( f_S^0 - f_S^1 \frac{A - 2Z}{A} \right)^2 |I_0(u_0)| \right\}$$

$$+ \left( f_A^0 \Omega_0(0) \right)^2 I_{00}(u_0) + 2 f_A^0 f_A^1 \Omega_0(0) \Omega_1(0) I_{01}(u_0)$$

$$+ \left( f_A^1 \Omega_1(0) \right)^2 I_{11}(u_0) \}$$  \hspace{1cm} (58)
The quantities \( I_\rho \) entering Eq. (58) are defined as (see Fig. 3)

\[
I_\rho(u_0) = (1 + \rho)u_0^{(1+\rho)} \int_0^{u_0} x^{1+\rho} |F(x)|^2 \, dx, \quad \rho = 0, 1
\]  

(59)

where \( F(q) \) the nuclear form factor and

\[
u_0 = q_0^2 b^2 / 2
\]

(60)

Using appropriate expressions for the form factors (in a harmonic oscillator basis with size parameter \( b \)) we obtain [40]-[42]

\[
I_\rho(u_0) = 1 \frac{\Omega_\rho}{A^2} \{ Z^2 I_\rho^p(u_0) + 2 NZ I_\rho^p(u_0) + N^2 I_\rho^p(u_0) \}
\]

(61)

where

\[
I_\rho^p(u_0) = (1 + \rho) \sum_{\lambda=0}^{N_{\text{max}}(\alpha)} \sum_{\nu=0}^{N_{\text{max}}(\beta)} \frac{\theta_\alpha^{(\lambda)}}{\lambda} \frac{\theta_\beta^{(\nu)}}{\nu} 2^{\lambda+\nu+\rho} (\lambda + \nu + \rho)! \frac{u_0^{1+\rho}}{u_0^{1+\rho}} \times [1 - e^{-u_0^{1+\rho}} \sum_{\kappa=0}^{\lambda+\nu+\rho} u_0^\kappa / \kappa!]
\]

(62)

\((\alpha, \beta = N, Z)\). The coefficients \( \theta_\alpha^{(\lambda)} \) are given in Ref., [40] for light and medium nuclei, and in Ref. [41] for heavy nuclei.

The integrals \( I_{\rho\rho'} \), with \( \rho, \rho' = 0, 1 \), (see Fig. 4) result by following the standard procedure of the multipole expansion of the \( e^{-i q \cdot r} \) in Eq. (54). One finds

\[
I_{\rho\rho'}(u_0) = 2 \int_0^1 \xi d\xi \sum_{\lambda, \kappa} \frac{\Omega^{(\lambda, \kappa)}_\rho(u_0 \xi^2)}{\Omega_\rho(0)} \frac{\Omega^{(\lambda, \kappa)}_{\rho'}(u_0 \xi^2)}{\Omega_{\rho'}(0)}
\]

(63)

where, in the special case in which the ground state of \( ^{207}\text{Pb} \) is approximated by a \( 2p_{1/2} \) neutron-hole, one finds

\[
I_{00} = I_{01} = I_{11} = 2 \int_0^1 \xi [F_{2p}(u_0 \xi^2)]^2 \, d\xi
\]

(64)

Even though the probability of finding a pure \( 2p_{1/2} \) neutron hole in the \( \frac{1}{2}^- \) ground state of \( ^{207}\text{Pb} \) is greater than 95\%, the ground state magnetic moment is quenched due to the \( 1^+ \) p-h excitation involving the spin orbit partners. Hence, we expect a similar suppression of the isovector spin matrix elements [43]-[44] (see Table 4(a),(b)). For comparison, we present our results for \( A = 207 \) together with those of \( A = 29 \) and \( A = 73 \) (Ref. [18]) in Table 4(a).

3 Convolution of the cross section with the velocity distribution

The cross sections which would be given from an LSP-detector participating in the revolution of the earth around the sun would appear retarded. In this section we are going to study this effect by using the method of folding. To this aim let us assume that the LSP is moving with velocity \( v_z \) with respect to the detecting apparatus. Then the detection rate for a target with mass \( m \) is given by

11
\[
\frac{dN}{dt} = \frac{\rho(0)}{m} \frac{m}{Am_N} |v_z| \sigma(v) \quad (65)
\]

where \(\rho(0) = 0.3\text{GeV/cm}^3\) is the LSP density in our vicinity. This density has to be consistent with the LSP velocity distribution. Such a consistent choice can be a Maxwell distribution

\[
f(v') = (\sqrt{\pi}v_0)^{-3}e^{-(v' / v_0)^2} \quad (66)
\]

provided that

\[
v_0 = \sqrt{(2/3)\langle v^2 \rangle} = 220\text{Km/s} \quad (67)
\]

For our purposes it is convenient to express the above distribution in the laboratory frame, i.e.

\[
f(v, v_E) = (\sqrt{\pi}v_0)^{-3}e^{-(v + v_E)^2 / v_0^2} \quad (68)
\]

where \(v_E\) is the velocity of the earth with respect to the center of the distribution. Choosing a coordinate system in which \(\hat{x}_2\) is the axis of the galaxy, \(\hat{x}_3\) is along the sun’s direction of motion \((v_0)\) and \(\hat{x}_1 = \hat{x}_2 \times \hat{x}_3\), we find that the position of the axis of the ecliptic is determined by the angle \(\gamma \approx 29.80\) (galactic latitude) and the azimuthal angle \(\omega = 186.3^0\) measured on the galactic plane from the \(\hat{x}_3\) axis. [45]

Thus, the axis of the ecliptic lies very close to the \(x_2x_3\) plane and the velocity of the earth is

\[
v_E = v_0 + v_1 = v_0 + v_1 (\sin \alpha \hat{x}_1 - \cos \alpha \cos \gamma \hat{x}_2 + \cos \alpha \sin \gamma \hat{x}_3) \quad (69)
\]

and

\[
v_0 \cdot v_1 = v_0v_1 \frac{\cos \alpha}{\sqrt{1 + \cot^2 \gamma \cos^2 \omega}} \approx v_0v_1 \sin \gamma \cos \alpha \quad (70)
\]

where \(v_0\) is the velocity of the sun around the center of the galaxy, \(v_1\) is the speed of the earth’s revolution around the sun, \(\alpha\) is the phase of the earth orbital motion, \(\alpha = 2\pi(t - t_1)/T_E\), where \(t_1\) is around second of June and \(T_E = 1\text{year}\).

The mean value of the event rate of Eq. (65), is defined by

\[
\langle \frac{dN}{dt} \rangle = \frac{\rho(0)}{m} \frac{m}{Am_N} \int f(v, v_E) \mid v_z \mid \sigma(|v|)d^3v \quad (71)
\]

Then we can write the counting rate as

\[
\langle \frac{dN}{dt} \rangle = \frac{\rho(0)}{m} \frac{m}{Am_N} \sqrt{\langle v^2 \rangle \langle \Sigma \rangle} \quad (72)
\]

where

\[
\langle \Sigma \rangle = \int \frac{|v_z|}{\sqrt{\langle v^2 \rangle}} f(v, v_E)\sigma(|v|)d^3v \quad (73)
\]

Thus, taking the polar axis in the direction \(v_E\), we get

\[
\langle \Sigma \rangle = \frac{4}{\sqrt{6\pi}v_0^3} \int_0^\infty v^3dv \int_{-1}^1 |\xi|d\xi e^{-(v_0^2 + v_E^2 + 2v_0v_E\xi)/v_0^2}\sigma(v) \quad (74)
\]

or

\[
\langle \Sigma \rangle = \frac{2}{\sqrt{6\pi}v_E} \int_0^\infty vdv F_0\left(\frac{2v_E}{v_0^2}\right) e^{-(v_0^2 + v_E^2)/v_0^2}\sigma(v) \quad (75)
\]
with
\[ F_0(\chi) = \chi \sinh \chi - \cosh \chi + 1 \] (76)

One can also write Eq. (75) as follows
\[ \langle \Sigma \rangle = \left( \frac{2}{3} \right)^{\frac{1}{2}} \int_0^{\infty} \frac{v}{v_0} f_1(v) \sigma(v) dv \] (77)

with
\[ f_1(v) = \frac{1}{\sqrt{\pi}} \frac{v_0}{v_E^2} F_0(\frac{2v_E}{v_0}) e^{-\frac{(v^2 + v_E^2)}{v_0^2}} \] (78)

In the case in which the first term in Eq. (76) becomes dominant, we get
\[ f_1(v) = \frac{1}{\sqrt{\pi}} \frac{v}{v_0v_E} \left\{ e^{\text{exp} \left[ - \frac{(v - v_E)^2}{v_0^2} \right]} - e^{\text{exp} \left[ - \frac{(v + v_E)^2}{v_0^2} \right]} \right\} \] (79)
in agreement with Eq. (8.15) of Ref. [3]. In Eq. (75) the nuclear parameters are implicit in the cross section \( \sigma(v) \) given from Eq. (58). The nuclear physics dependence of \( \langle \Sigma \rangle \) could be disentangled by taking note of the extra velocity dependence of the coherent vector contribution in \( \sigma(v) \) and introducing the parameters
\[ \delta = \frac{2v_E}{v_0} = 0.27, \quad \psi = \frac{v}{v_0}, \quad u = u_0\psi^2 \] (80)

where the quantity \( u_0 \) is the one entering the nuclear form factors of Eq. (59) for \( v = v_0 \), which in this case is given by
\[ u_0 = \frac{1}{2} \left( \frac{2\beta_0 m_x c^2 b}{\sqrt{(1 + \eta) \hbar c}} \right)^2, \quad \beta_0 = \frac{v_0}{c} \] (81)

Afterwards, we can write Eq. (73) as
\[
\langle \Sigma \rangle = \left( \frac{m_x}{m_N} \right)^2 \frac{\sigma_0}{(1 + \eta)^2} \left\{ A^2 \left[ (\beta^2) \left( f_V^0 - f_V^1 A - 2Z \right)^2 \left( J_0 - \frac{2\eta + 1}{2(1 + \eta)^2} J_1 \right) \right. \right.
\] \[ \left. + \left( f_S^0 - f_S^1 A - 2Z \right)^2 \tilde{J}_0 \right) \right. \]
\[ + \left( f_A^0 \Omega_0(0) \right)^2 J_{00} + 2f_A^0 f_A^1 \Omega_0(0) \Omega_1(0) J_{01} + \left( \frac{f_A^1}{2}\Omega_1(0) \right)^2 J_{11} \right\} \] (82)

If we assume that \( J_{00} = J_{01} = J_{11} \), as seems to be the case for \( ^{207}\text{Pb} \), the spin dependent part of Eq. (78) is reduced to the familiar expression \( \left[ f_A^0 \Omega_0(0) + f_A^1 \Omega_1(0) \right]^2 J_{11} \), where the quantity in the bracket represents the spin matrix element at \( q = 0 \).

The parameters \( \tilde{J}_0, J_\rho, J_\rho^\sigma \) describe the scalar, vector and spin part of the counting rate, respectively, and they are given by
\[ \tilde{J}_0(\lambda, u_0) = \frac{2}{\sqrt{6\pi}} \frac{e^{-\lambda^2}}{\lambda^2} \int_0^{\infty} \psi e^{-\psi^2} F_0(2\lambda\psi) I_0(u_0\psi^2) d\psi \] (83)
\[ J_\rho(\lambda, u_0) = \frac{2}{\sqrt{6\pi}} \frac{e^{-\lambda^2}}{\lambda^2} \int_0^{\infty} \psi^3 e^{-\psi^2} F_0(2\lambda\psi) I_\rho(u_0\psi^2) d\psi \] (84)
\[ J_{\rho\sigma}(\lambda, u_0) = \frac{2}{\sqrt{6\pi}} \frac{e^{-\lambda^2}}{\lambda^2} \int_0^\infty \psi e^{-\psi^2} F_0(2\lambda \psi) I_{\rho\sigma}(u_0 \psi^2) d\psi \]  
\[ \lambda = \frac{v_E}{v_0} = \left[ 1 + \delta \cos \alpha \sin \gamma + (\delta/2)^2 \right]^{1/2} \]  

The parameters \( I_\rho, I_{\rho\sigma} \) have been discussed in the previous section (see Figs. 3 and 4). The above integrals are functions of \( \lambda \) and \( u_0 \). The latter depends on \( v_0 \), the nuclear parameters and the LSP mass. These integrals can only be done numerically. Since, however, \( \lambda \) is close to unity, we can expand in powers of \( \delta \) and make explicit the dependence of these integrals on the earth’s motion. Thus,

\[ \tilde{J}_0(\lambda, u_0) = \frac{2}{\sqrt{6\pi}} B_1 \left[ \tilde{K}_0^{(0)}(u_0) + \delta \sin \gamma \cos \alpha \tilde{K}_0^{(1)}(u_0) \right] \]  
\[ J_\rho(\lambda, u_0) = \frac{2}{\sqrt{6\pi}} B_2 \left[ K_\rho^{(0)}(u_0) + \delta \sin \gamma \cos \alpha K_\rho^{(1)}(u_0) \right] \]  
\[ J_{\rho\sigma}(\lambda, u_0) = \frac{2}{\sqrt{6\pi}} B_1 \left[ K_{\rho\sigma}^{(0)}(u_0) + \delta \sin \gamma \cos \alpha K_{\rho\sigma}^{(1)}(u_0) \right] \]  

The integrals \( \tilde{K}_0^0, K_\rho^0 \) and \( K_{\rho\sigma}^0 \) are normalized so that they become unity at \( u_0 = 0 \) (negligible momentum transfer). We find

\[ B_1 = \frac{1}{e} \int_0^\infty \psi e^{-\psi^2} F_0(2\psi) d\psi = \frac{1}{e} + 2\nu \approx 1.860 \]  
\[ B_2 = \frac{2}{3e} \int_0^\infty \psi^3 e^{-\psi^2} F_0(2\psi) d\psi = \frac{2}{3} \left( \frac{3}{e} + 7\nu \right) \approx 4.220 \]  

with

\[ \nu = \int_0^1 e^{-t^2} dt \approx 0.747 \]  

Furthermore,

\[ \tilde{K}_0^l = \frac{1}{eB_1} \int_0^\infty \psi e^{-\psi^2} F_l(2\psi) I_0(u_0 \psi^2) d\psi, \quad l = 0, 1 \]  
\[ K_\rho^l = \frac{2}{3eB_2} \int_0^\infty \psi^3 e^{-\psi^2} F_l(2\psi) I_\rho(u_0 \psi^2) d\psi, \quad l = 0, 1 \]  
\[ K_{\rho\sigma}^l = \frac{1}{eB_1} \int_0^\infty \psi e^{-\psi^2} F_l(2\psi) I_{\rho\sigma}(u_0 \psi^2) d\psi, \quad l = 0, 1 \]  

with \( F_0(\chi) \) given in Eq. (76) and

\[ F_1(\chi) = 2 \left[ \left( \frac{\chi^2}{4} + 1 \right) \cosh \chi - \chi \sinh \chi - 1 \right] \]  

The counting rate can thus be cast in the form

\[ \left\langle \frac{dN}{dt} \right\rangle = \left\langle \frac{dN}{dt} \right\rangle_0 (1 + h \cos \alpha) \]  

where \( \left\langle \frac{dN}{dt} \right\rangle_0 \) is the rate obtained from the \( l = 0 \) multipole and \( h \) the amplitude of the oscillation, i.e. the ratio of the component of the multipole \( l = 1 \) to that of the multipole \( l = 0 \). Below (see also Tables 5(a),(b)) we compute separately the amplitude of oscillation for the scalar, vector and spin parts of the event rate i.e. the quantity \( h = \delta \sin \gamma K^1(u_0)/K^0(u_0) \). Note the presence of the geometric factor \( \sin \gamma = 1/2 \), which reduces the modulation effect.
In order to get some idea of the dependence of the counting rate on the earth’s motion, we will evaluate the above expressions at $u_0 = 0$. We get

$$\tilde{K}_0^0 = K_{\rho\sigma}^0 \approx K_{\rho}^0 = 1$$ (98)
$$\tilde{K}_0^1 = K_{\rho\sigma}^1 = \frac{\nu}{1/e + 2\nu} \approx 0.402$$ (99)
$$K_{\rho}^1 = \frac{3/(2e) + (11/2)\nu/2}{3/e + 7\nu} \approx 0.736$$ (100)

Thus, for $\sin \gamma \approx 0.5$

$$\tilde{J}_0 \approx J_{\rho\sigma} = \frac{2}{\sqrt{6\pi}} 1.860(1 + 0.054 \cos \alpha) = 0.857(1 + 0.054 \cos \alpha)$$ (101)
$$J_{\rho} = \frac{2}{\sqrt{6\pi}} 4.220(1 + 0.099 \cos \alpha) = 1.944(1 + 0.099 \cos \alpha)$$ (102)

We see that, the modulation of the detection rate due to the earth’s motion is quite small ($h \approx 0.05$). The corresponding amplitude of oscillation in the coherent vector contribution, Eq. (101), is a bit bigger ($h \approx 0.10$). However, this contribution is suppressed due to the Majorana nature of LSP (through the factor $\beta^2$). The modulation due to the Earth’s rotation is expected to be even smaller.

The exact $K^l$ integrals, for the $l=0$ and $l=1$, are shown in Figs. 5(a),(b), (c). The most important of these integrals, those of Eq. (93) associated with the scalar interaction, are shown in Fig. 5(a). In Fig. 5(b) we present the integrals of Eq. (94) for $\rho = 0$ associated with the vector interaction (the integral for $\rho = 1$ is analogous but it is less important). Finally in Fig. 5(c) the integrals of Eq. (93) for $\rho = 1$ and $\sigma = 1$ are shown. The others are practically indistinguishable from these and are not shown (see Ref. [41]).

Before closing this section we should mention that, the folding procedure can also be applied in the differential rate in order to obtain the corresponding convoluted expression for $d\sigma/d\Omega$, i.e. before doing the angular integration in Eq. (53) and obtain the total cross section Eq. (58). In fact this may be important since the modulation effect in the total cross section is small due to cancellations. In fact preliminary results indicate that the modulation effect can get as high as 20%. The high value occurs, unfortunately, in the regions where the total differential rate becomes too small.

4 Results and discussion

The three basic ingredients of our calculation were the input SUSY parameters, a quark model for the nucleon and the structure of the nuclei involved. The input SUSY parameters used for the results presented in Tables 1 and 2 have been calculated in a phenomenologically allowed parameter space (cases #1, #2, #3 of Ref. 30 and cases #4-9 of Ref. 31).

For the coherent part (scalar and vector) we used realistic nuclear form factors and studied three nuclei, representatives of the light, medium and heavy nuclear isotopes ($Ca$, $Ge$ and $Pb$). In Tables 5(a),(b) and 6 we show the results obtained for three different quark models denoted by A (only quarks u and d) and B, C (heavy quarks in the nucleon). We see that the results vary substantially and are very sensitive to the presence of quarks other than u and d into the nucleon.
The spin contribution, arising from the axial current, was computed in the case of $^{207}$Pb system. For the isovector axial coupling the transition from the quark to the nucleon level is trivial (a factor of $g_A = 1.25$). For the isoscalar axial current we considered two possibilities depending on the portion of the nucleon spin which is attributed to the quarks, indicated by EMC and NQM. The ground state wave function of $^{208}$Pb was obtained by diagonalizing the nuclear Hamiltonian in a 2h-1p space which is standard for this doubly magic nucleus. The momentum dependence of the matrix elements was taken into account and all relevant multipoles were retained (here only $\lambda = 0$ and $\lambda = 2$).

In Table 4(a), we compare the spin matrix elements at $q = 0$ for the most popular targets considered for LSP detection $^{207}$Pb, $^{73}$Ge and $^{29}$Si. We see that, even though the spin matrix elements $\Omega^2$ are even a factor of three smaller than those for $^{73}$Ge obtained in Ref. [18] (see Table 4(a)), their contribution to the total cross section is almost the same (see Table 4(b)) for LSP masses around 100 GeV. Our final results for the quark models (A, B, C, NQM, EMC) are presented in Tables 5(a),(b) for SUSY models #1-#3 [30] and Table 6 for SUSY models #4-#9 [31].

5 Conclusions

In the present study we found that for heavy LSP and heavy nuclei the results are sensitive to the momentum transfer as well as to the LSP mass and other SUSY parameters. From the Tables 5(a),(b) and 6 we see that, the results are also sensitive to the quark structure of the nucleon. We can, however, draw the following general conclusions.

(i) The coherent scalar (associated with Higgs exchange) for model A (u and d quarks only) is comparable to the vector coherent contribution. Both are at present undetectable. For models B and C (heavy quarks in the nucleon) the coherent scalar contribution is dominant. Detectable rates $\langle dN/dt \rangle_0 \geq 100 \, y^{-1} Kg^{-1}$ are possible in a number of models with light LSP.

(ii) The folding of the total event rate with the velocity distribution provides the total modulation effect $h$. In all cases it is small, less than ±5%.

(iii) The spin contribution is sensitive to the nuclear structure. It is undetectable if the LSP is primarily a gaugino.

Acknowledgements: One of the authors (JDV) would like to acknowledge partial support of this work by ΠΝΕΔ 1895/95 of the Greek secretariat for Research, by TMR No ERB FMAX-CT96-0090 of the European Union, the INTAS 93-1648 program for travel support to attend the Workshop and the workshop organizers for hospitality.

References

[1] G.F. Smoot et al., (COBE data), Astrophys. J. 396 (1992) L1.

[2] D.P. Bennett et al., (MACHO collaboration), A binary lensing event toward the LMC: Observations and Dark Matter implications, Proc. 5th Annual Maryland Conference, edited by S. Holt (1995); Preprint astro-ph/9606012.
C. Alcock et al., (MACHO collaboration), Phys. Rev. Lett. 74 (1995) 2967; Preprint astro-ph/9606165.

[3] G. Jungman et al., Phys. Rep. 267 (1996) 195.

[4] J.D. Vergados, J. of Phys. G 22 (1996) 253.

[5] T.S. Kosmas and J.D. Vergados, Phys. Rev. D 55 (1997) 1752.

[6] M.W. Goodman and E. Witten, Phys. Rev. D 31 (1985) 3059.

[7] K. Griest, Phys. Rev. Lett. 62 (1988) 666; Phys. Rev. D 38 (1988) 2357; D 39 (1989) 3802.

[8] J. Ellis, and R.A. Flores, Phys. Lett. B 263 (1991) 259; Phys. Lett B 300 (1993) 175; Nucl. Phys. B 400 (1993) 25; J. Ellis and L. Roszkowski, Phys. Lett. B 283 (1992) 252.

[9] V.A. Bednyakov, H.V. Klapdor-Kleingrothaus and S.G. Kovalenko, Phys. Lett. B 329 (1994) 5.

[10] M. Drees and M.M. Nojiri, Phys. Rev. D 48 (1993) 3483; Phys. Rev. D 47 (1993) 4226.

[11] A. Bottino et al., Mod. Phys. Lett. A 7 (1992) 733; Phys. Lett. B 265 (1991) 57; Z. Berezhinsky et al., Astroparticle Phys. 5 (1996) 1.

[12] P. Gondolo, these proceedings.

[13] L. Arnowitt, Dark matter predictions in non-universal soft breaking masses, these proceedings.

[14] L. Roszkowski, neutralino dark matter, these proceedings.

[15] Z. Berezhinsky, Dark matter Candidates, these proceedings.

[16] J. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. B 257 (1991) 83.

[17] H.E. Haber and R. Hempfling, Phys. Rev. Lett. 66 (1991) 1815.

[18] M.T. Ressell et al., Phys. Rev. D 48 (1993) 5519.

[19] V.I. Dimitrov, J. Engel and S. Pittel, Phys. Rev. D 51 (1995) R291.

[20] J. Engel, Phys. Lett. B 264 (1991) 114.

[21] K. Freese, J. Frieman and A. Gould, Phys. Rev. D 37 (1988) 3388.

[22] J. Engel and P. Vogel, Phys. Rev. D 40 (1989) 3132. A.F. Pacheco and D. Strottman, Phys. Rev. D 40 (1989) 2131.

[23] J. Engel, S. Pittel and P. Vogel, Int. J. Mod. Phys. E (1992) 1.

[24] F. Iachello, L.M. Krauss and G. Maino, Phys. Lett. B 254 (1991) 220.

[25] M.A. Nikolaev and H.V. Klapdor-Kleingrothaus, Z. Phys. A 345 (1993) 373.

[26] P.F. Smith and J.D. Lewin, Phys. Rep. 187 (1990) 203.

[27] M. Roman-Robinson, Evidence for Dark Matter, Proc. Int. School on Cosmological Dark Matter, Valencia, Spain, 1993 p.7. (ed. J.W.F. Valle and A. Perez).
[28] C.S. Frenk, The large Scale Structure of the Universe, in Ref. [27] p. 65; J.R. Primack, Structure Formation in CDM and HDM Cosmologies, ibid p. 81.

[29] J.R. Primack, D. Seckel and B. Sadoulet, Ann. Rev. Nucl. Part. Sci. 38 (1988) 751; F. von Feilitzch, Detectors for Dark Matter Interactions Operated at Low Temperatures, Int. Workshop on Neutrino Telescopes, Venezia Feb. 13-15, 1990 (ed. Milla Baldo Ceolin) p. 257.

[30] G.L. Kane et al., Phys. Rev. D 49 (1994) 6173.

[31] D.J. Castaño, E.J. Piard and P. Ramond, Phys. Rev. D 49 (1994) 4882; D.J. Castaño, Private Communication.

[32] H.E. Haber and G.L. Kane, Phys. Rep. 117 (1985) 75.

[33] S.L. Addler, Phys. Rev. D 11 (1975) 3309.

[34] T.P. Cheng, Phys. Rev. D 38 (1988) 2869.

[35] H-Y. Cheng, Phys. Lett. B 219 (1989) 347.

[36] G.L. Kane and J.D. Wells, Higgsino Cold Dark Matter Motivated by Collider Data, UM-TH-9604, SLAC-PUB-7131, hep-ph/9603330; M. Drees, M.M. Noijiri, D.P. Roy and Y. Yamada, Light Higgsino Dark matter, KEK-Th505, Lep-ph 970 1219.

[37] J. Ashman et al., EMC collaboration, Nucl. Phys. B 328 (1989) 1.

[38] R.L. Jaffe and A. Manohar, Nucl. Phys. B 337 (1990) 509; J. Ellis and M. Carliuer, The Strange Spin of the nucleon, CERN-Th/95334, Hep-ph 960 1280.

[39] P.M. Gensini, In Search of the Quark Spins in the Nucleon: A next-to-Next-to Leading Order in QCD Analysis of the Ellis-Jaffe Sum Rule, preprint DFUPG-95-GEN-01, hep-ph/9512440.

[40] T.S. Kosmas and J.D. Vergados, Nucl. Phys. A 536 (1992) 72.

[41] T.S. Kosmas and J.D. Vergados, Proc. 6th Hellenic Symp. on Nucl. Phys., Athens, 25-26 May, 1995, in press.

[42] T.S. Kosmas and J.D. Vergados, Nucl. Phys. A 510 (1990) 641.

[43] J.D. Vergados, Phys. Lett. B 36 (1971) 12; 34 B (1971) 121.

[44] T.T.S. Kuo and G.E. Brown, Nucl. Phys. 85 (1966) 40.

[45] G.H. Herling and T.T.S. Kuo, Nucl. Phys. A 181 (1972) 113.

[46] K. Alissandrakis, Private Communication (1996).
Table 1. The essential parameters describing the LSP and Higgs. For the definitions see the text.

| Solution | #1    | #2    | #3    | #4    | #5    | #6    | #7    | #8    | #9    |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $m_x$ (GeV) | 126   | 27    | 102   | 80    | 124   | 58    | 34    | 35    | 50    |
| $m_h$     | 116.0 | 110.2 | 113.2 | 124.0 | 121.0 | 105.0 | 103.0 | 92.0  | 111.0 |
| $m_H$     | 345.6 | 327.0 | 326.6 | 595.0 | 567.0 | 501.0 | 184.0 | 228.0 | 234.0 |
| $m_A$     | 345.0 | 305.0 | 324.0 | 594.0 | 563.0 | 497.0 | 179.0 | 207.0 | 230.0 |
| tan2$\alpha$ | 0.245 | 6.265 | 0.525 | 0.410 | 0.929 | 0.935 | 0.843 | 1.549 | 0.612 |
| tan$\beta$ | 10.0  | 1.5   | 5.0   | 5.4   | 2.7   | 2.7   | 5.2   | 2.6   | 5.3   |

Table 2(a). The coupling constants entering $L_{eff}$, eqs. (46), (47) and (49) of the text, for solutions #1 - #3.

| Quantity                        | Solution #1     | Solution #2     | Solution #3     |
|---------------------------------|-----------------|-----------------|-----------------|
| $\beta f_V^0$                   | $1.746 \times 10^{-5}$ | $2.617 \times 10^{-5}$ | $2.864 \times 10^{-5}$ |
| $f_V^0 / f^0$                   | -0.153          | -0.113          | -0.251          |
| $f_S^0 (H)$ (model A)            | $1.31 \times 10^{-5}$ | $1.30 \times 10^{-4}$ | $1.38 \times 10^{-5}$ |
| $f_S^0 (B)$ (mode A)             | -0.275          | -0.107          | -0.246          |
| $f_S^0 (H)$ (model B)            | $5.29 \times 10^{-4}$ | $7.84 \times 10^{-3}$ | $6.28 \times 10^{-4}$ |
| $f_S^0 (H)$ (model C)            | $7.57 \times 10^{-4}$ | $7.44 \times 10^{-3}$ | $7.94 \times 10^{-4}$ |
| $f_A^0 (NQM)$                    | $0.510 \times 10^{-2}$ | $3.55 \times 10^{-2}$ | $0.704 \times 10^{-2}$ |
| $f_A^0 (EMC)$                    | $0.612 \times 10^{-3}$ | $0.426 \times 10^{-2}$ | $0.844 \times 10^{-3}$ |
| $f_A^1$                          | $1.55 \times 10^{-2}$ | $5.31 \times 10^{-2}$ | $3.00 \times 10^{-2}$ |

Table 2(b). The same as in Table 1(a) for solutions #4 - #9.

| Solution | #4        | #5        | #6        | #7        | #8        | #9        |
|----------|-----------|-----------|-----------|-----------|-----------|-----------|
| $\langle \beta^2 \rangle^{1/2} f_V^0$ | $0.225 \times 10^{-4}$ | $0.190 \times 10^{-4}$ | $0.358 \times 10^{-4}$ | $0.108 \times 10^{-4}$ | $0.694 \times 10^{-4}$ | $0.864 \times 10^{-4}$ |
| $f_V^0 / f^0$              | -0.0809   | -0.0050   | -0.0320   | -0.0538   | -0.0464   | -0.0369   |
| $f_S^0 (A)$                 | -0.179 $10^{-4}$ | -0.236 $10^{-4}$ | -0.453 $10^{-4}$ | -0.266 $10^{-4}$ | -0.210 $10^{-3}$ | -0.131 $10^{-3}$ |
| $f_S^0 (B)$                 | -0.531 $10^{-2}$ | -0.145 $10^{-2}$ | -0.281 $10^{-2}$ | -0.132 $10^{-1}$ | -0.117 $10^{-1}$ | -0.490 $10^{-2}$ |
| $f_S^0 (C)$                 | -0.315 $10^{-2}$ | -0.134 $10^{-2}$ | -0.261 $10^{-2}$ | -0.153 $10^{-1}$ | -0.118 $10^{-1}$ | -0.159 $10^{-2}$ |
| $f_A^0 (A)$                 | -0.207 $10^{-5}$ | -0.407 $10^{-5}$ | 0.116 $10^{-4}$ | 0.550 $10^{-4}$ | 0.307 $10^{-4}$ | 0.365 $10^{-4}$ |
| $f_A^0 (NQM)$               | 6.950 $10^{-3}$ | 5.800 $10^{-3}$ | 1.220 $10^{-2}$ | 3.760 $10^{-2}$ | 3.410 $10^{-2}$ | 2.360 $10^{-2}$ |
| $f_A^0 (EMC)$               | 0.834 $10^{-3}$ | 0.696 $10^{-3}$ | 0.146 $10^{-2}$ | 0.451 $10^{-2}$ | 0.409 $10^{-2}$ | 0.283 $10^{-2}$ |
| $f_A^1$                      | 2.490 $10^{-2}$ | 1.700 $10^{-2}$ | 3.440 $10^{-2}$ | 2.790 $10^{-1}$ | 1.800 $10^{-1}$ | 2.100 $10^{-1}$ |
Table 3. The quantity $q_0$ (forward momentum transfer) in units of $fm^{-1}$ for three values of $m_1$ and three typical nuclei. In determining $q_0$ the value $\langle \beta^2 \rangle^{1/2} = 10^{-3}$ was employed.

| Nucleus | $m_1 = 30.0 \, GeV$ | $m_1 = 100.0 \, GeV$ | $m_1 = 150.0 \, GeV$ |
|---------|---------------------|---------------------|---------------------|
| Ca      | 0.174               | 0.290               | 0.321               |
| Ge      | 0.215               | 0.425               | 0.494               |
| Pb      | 0.267               | 0.685               | 0.885               |

Table 4(a). Comparison of the static spin matrix elements for three typical nuclei, $Pb$ (present calculation) and $^{73}Ge, ^{29}Si$ (see Ref. [18]).

| Component | $^{207}Pb_{1/2}$ | $^{73}Ge_{9/2^+}$ | $^{29}Si_{1/2^+}$ |
|-----------|------------------|-------------------|-------------------|
| $\Omega^2_1(0)$ | 0.231           | 1.005             | 0.204             |
| $\Omega_1(0)\Omega_0(0)$ | -0.266          | -1.078            | -0.202            |
| $\Omega^2_0(0)$ | 0.305           | 1.157             | 0.201             |

Table 4(b). Ratio of spin contribution ($^{207}Pb/^{73}Ge$) at the relevant momentum transfer with the kinematical factor $1/(1 + \eta)^2$, $\eta = m/Am_N$.

| Solution | #1 | #2 | #3 | #4 | #5 | #6 | #7 | #8 | #9 |
|----------|----|----|----|----|----|----|----|----|----|
| $m_x \,(GeV)$ | 126 | 27 | 102 | 80 | 124 | 58 | 34 | 35 | 50 |
| NQM      | 0.834 | 0.335 | 0.589 | 0.394 | 0.537 | 0.365 | 0.346 | 0.337 | 0.417 |
| EMC      | 0.645 | 0.345 | 0.602 | 0.499 | 0.602 | 0.263 | 0.341 | 0.383 | 0.479 |
Table 5(a). The quantity $\langle dN/dt \rangle_0$ in $y^{-1} Kg^{-1}$ and the modulation parameter $h$ for the coherent vector and scalar contributions in the cases #1 - #3 and for three typical nuclei.

| Case | Vector Contribution | Scalar Contribution |
|------|---------------------|---------------------|
|      | $(dN/dt)_0$       | $h$                | $(dN/dt)_0$       | $h$                |
| Pb   | (×10⁻³)            | Model A          | Model B          | Model C           |
| #1   | 0.264 0.029        | 0.151 × 10⁻³     | 0.220            | 0.450 -0.002      |
| #2   | 0.162 0.039        | 0.410 × 10⁻¹     | 142.860          | 128.660 0.026     |
| #3   | 0.895 0.038        | 0.200 × 10⁻³     | 0.377            | 0.602 -0.001      |
| Ge   | #1 0.151 0.043     | 0.779 × 10⁻⁴     | 0.120            | 0.245 0.017       |
|      | #2 0.053 0.057     | 0.146 × 10⁻¹     | 51.724           | 46.580 0.041      |
|      | #3 0.481 0.045     | 0.101 × 10⁻³     | 0.198            | 0.316 0.020       |
| Ca   | #1 0.079 0.053     | 0.340 × 10⁻⁴     | 0.055            | 0.114 0.037       |
|      | #2 0.264 0.060     | 0.612 × 10⁻²     | 22.271           | 20.056 0.048      |
|      | #3 0.241 0.053     | 0.435 × 10⁻⁴     | 0.090            | 0.144 0.038       |

Table 5(b). The spin contribution in the $LSP - ^{207}Pb$ scattering for two cases: EMC data and NQM Model for solutions #1, #2, #3.

| Solution | EMC DATA | NQM MODEL |
|----------|----------|-----------|
|          | $(dN/dt)_0$ $(y^{-1} Kg^{-1})$ | $h$ | $(dN/dt)_0$ $(y^{-1} Kg^{-1})$ | $h$ |
| #1       | 0.285 × 10⁻² | 0.014 | 0.137 × 10⁻² | 0.015 |
| #2       | 0.041 | 0.046 | 0.384 × 10⁻² | 0.056 |
| #3       | 0.012 | 0.016 | 0.764 × 10⁻² | 0.017 |
Table 6. The same parameters as in Tables 5(a) for $Pb$ for the solutions #4 – #9. Cases #8, #9 are no-scale models. The values of $\langle dN/dt\rangle_0$ for Model A and the Vector part must be multiplied by $\times10^{-2}$.

| Case | A   | B   | C   | Scalar Part | Vector Part | Spin Part |
|------|-----|-----|-----|-------------|-------------|-----------|
|      |     |     |     | $\langle dN/dt \rangle_0$ | $h$          | $\langle dN/dt \rangle_0$ | $h$     |
| #4   | 0.03| 22.9| 8.5 | 0.003       | 0.04        | 0.04      | $0.8010^{-3}$ | $0.1610^{-2}$ | 0.015 |
| #5   | 0.46| 1.8 | 1.4 | -0.003      | 0.03        | 0.03      | $0.3710^{-3}$ | $0.9110^{-3}$ | 0.014 |
| #6   | 0.16| 5.7 | 4.8 | 0.007       | 0.11        | 0.11      | $0.4410^{-3}$ | $0.1110^{-2}$ | 0.033 |
| #7   | 4.30| 110.0 | 135.0 | 0.020 | 0.94 | 0.065 | 0.67 | 0.87 | 0.055 |
| #8   | 2.90| 73.1 | 79.8 | 0.020 | 0.40 | 0.065 | 0.22 | 0.35 | 0.055 |
| #9   | 2.90| 1.6 | 1.7 | 0.009       | 0.95        | 0.059    | 0.29 | 0.37 | 0.035 |
FIGURE CAPTIONS

Fig. 1. Two diagrams which contribute to the elastic scattering of LSP with nuclei: Z-exchange in Fig. 1(a) and s-quark exchange in Fig. 1(b). Due to the Majorana nature of LSP only its pseudovector coupling contributes. \( J_\lambda \) can be parametrized in terms of four form factors \( f_0^V, f_1^V, f_0^A, f_1^A \). The scalar terms arising from s-quark mixing are negligible in the SUSY parameter space considered here.

Fig. 2. The same as in Fig. 1, except that the intermediate Higgs exchange is considered. This leads to an effective scalar interaction with two form factors \( f_0^S \) (isoscalar) and \( f_1^S \) (isovector).

Fig. 3. The integrals \( I_0 \) which describe the dominant scalar contribution (coherent part) of the total cross section as a function of the LSP mass \( m_x \equiv m_1 \), for three typical nuclei: Ca, Ge and Pb. The value \( \langle \beta^2 \rangle^{1/2} = 10^{-3} \) was used.

Fig. 4. (a) Plot of the integrals \( I_{11} \) as a function of the LSP mass \( m_x \equiv m_1 \). This integral gives the spin contribution to the LSP-nucleus total cross section for \(^{207}\)Pb. The integrals \( I_{00} \) and \( I_{01} \) are similar. (b) Plot of the integrals \( I_{11}(u) \) and \( I_0(u) \) for Pb. Note that \( I_{11} \) is quite a bit less retarded compared to \( I_0 \). For definitions see the text.

Fig. 5. Contributions of K integrals (for \( l=0 \) and \( l=1 \)) entering the event rate due to earth’s revolution around the sun: \( \tilde{K}_{l0} \) in Fig. 5(a), \( K_{l0} \) in Fig. 5(b) and \( K_{l1} \) in Fig. 5(c). The other integrals \( K_{00} \) and \( K_{01} \) are similar to \( K_{11} \).
Fig. 1
\begin{figure}
\centering
\begin{subfigure}{\textwidth}
\centering
\includegraphics[width=\textwidth]{a.png}
\caption{}
\end{subfigure}
\begin{subfigure}{\textwidth}
\centering
\includegraphics[width=\textwidth]{b.png}
\caption{}
\end{subfigure}
\end{figure}
\[ \tilde{K}_0^l / \equiv / 0 \]
\[ K_0^l / \equiv / 1 \]
\[ K_0^l / \equiv / 1 \]

\( P b \)

\( m_1 \) (GeV)

\( l = 0 \)
\( l = 1 \)

\( \text{Pb} \)

\( 207\text{Pb} \)

\( m_1 \) (GeV)