Lagrangian formulation of massive fermionic totally antisymmetric tensor field theory in $AdS_d$ space

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Abstract

We apply the BRST approach, developed for higher spin field theories, to Lagrangian construction for totally antisymmetric massive fermionic fields in $AdS_d$ space. As well as generic higher spin massive theories, the obtained Lagrangian theory is a reducible gauge model containing, besides the basic field, a number of auxiliary (Stückelberg) fields and the order of reducibility grows with the value of the rank of the antisymmetric field. However, unlike the generic higher spin theory, for the special case under consideration we show that one can get rid of all the auxiliary fields and the final Lagrangian for fermionic antisymmetric field is formulated only in terms of basic field.

1 Introduction

One of the interesting aspects of higher spin field theory (see e.g. the recent reviews [1]) in various dimensions is a possibility to construct the models using the fields with mixed symmetry of tensor indices (see the recent papers [2, 3, 4, 5] and references therein). Since the totally antisymmetric fields are particular cases of generic mixed symmetry fields, the methods developed in higher spin field theory can be applied for Lagrangian formulation of totally antisymmetric fields.

In our recent paper [6] we constructed the Lagrangians for totally antisymmetric massive and massless bosonic fields in curved space-time using BRST approach which was earlier applied for description of totally symmetric higher spin field models [7, 8, 9, 10, 11, 12, 13, 14, 15] and mixed symmetry higher spin fields [16]. In this paper we find the Lagrangians for massive fermionic totally antisymmetric tensor fields in AdS space using the BRST approach.

In principle, the Lagrangian for totally antisymmetric fermionic field can be derived using the generic method developed in [16] for mixed symmetry fields. This method uses the bosonic creation
and annihilation operators which automatically take into account symmetry of the group of the indices. All the other symmetry conditions on the indices are formulated in the form of constraints. Such a procedure possesses a definite advantage, however demands including a number of auxiliary fields and gauge symmetries in the Lagrangian. In particular, for totally antisymmetric fields it means that the antisymmetry index condition is absent from the very beginning and should be a consequence of the equations of motion and gauge fixing. Therefore a straightforward application of the generic method to totally antisymmetric tensor fields being absolutely correct, yields complicated enough Lagrangian formulation. In the paper under consideration we use fermionic creation and annihilation operators thus taking into account the antisymmetry index condition from the very beginning. As a result the Lagrangian will contain less number of the auxiliary fields and will be more simpler in comparison with a result of straightforward application of the generic method [16].

The paper is organized as follows. In the next section we show that the equations of motion for antisymmetric fermionic field are noncontradictory for arbitrary dimensions only in spaces of constant curvature if one supposes the absence of the terms with the inverse powers of the mass. The rest part of the paper deals with the fermionic fields on AdS background. In section 3 we rewrite the equations of motion for antisymmetric fermionic field in the operator form and find the closed algebra generated by such operators. Then according to the generic procedure of Lagrangian construction [12] we derive in section 4 the additional parts and in section 5 we first find the extended expressions for the operators and then on the base of their algebra we construct BRST operator. Finally in section 6 we determine the Lagrangian for the antisymmetric fermionic field. In the Appendices we give some details of calculations missed in the main part of the paper. In appendix A we describe the calculations of the additional parts. In appendix B we show that the obtained Lagrangian indeed reproduces the true equations determining the irreducible representation of the $AdS_d$ group on massive fermionic antisymmetric fields and in appendix C we simplify the Lagrangian by removing all the auxiliary fields and get the final Lagrangian in terms of physical field only. It should be noted that such Lagrangian has not been previously presented in the literature.

2 Consistency of fermionic field dynamics in curved space

In this section we show that unlike the bosonic case [6] there are no consistent equations of motion for fermionic totally antisymmetric fields minimally coupled to arbitrary curved space-time.

It is well known that a rank-$n$ totally antisymmetrical tensor-spinor field $\psi_{\mu_1\ldots\mu_n}$ (the Dirac index is suppressed) will describe the irreducible massive representation of the Poincare group if the following conditions are satisfied

$$
(i\gamma^\nu \partial_\nu - m)\psi_{\mu_1\ldots\mu_n} = 0, \quad \gamma^\mu \psi_{\mu_2\ldots\mu_n} = 0, \quad \partial^\mu \psi_{\mu_2\ldots\mu_n} = 0,
$$

(1)

with $\{\gamma^\mu, \gamma^\nu\} = -2g^{\mu\nu}$. When we put these equations on an arbitrary curved spacetime we see that if we do not include the terms with the inverse powers of the mass then there is no freedom to add any terms with the curvature and it unambiguously follows that in curved space equations (1) take the form

$$
(i\gamma^\nu \nabla_\nu - m)\psi_{\mu_1\ldots\mu_n} = 0, \quad \gamma^\mu \psi_{\mu_2\ldots\mu_n} = 0, \quad \nabla^\mu \psi_{\mu_2\ldots\mu_n} = 0.
$$

(2)

Let us show that the mass-shell and divergence-free equations are inconsistent in arbitrary curved space. For this purpose we take the divergence of the mass-shell equation and suppose that equations

Of course, finally both such ways lead to the same Lagrangian. Here we point out a possibility to obtain the final result more simple way.
Riemann tensor of the Ricci tensor is zero. Let us try to find from (3) what spaces do not give any condition on $\psi_{\mu_1...\mu_n}$. As is known an antisymmetric tensor rank-3 antisymmetric tensor rank-3 of the AdS group \[17\] if the following conditions are satisfied

\[ R_{\mu\nu\alpha\beta} = C_{\mu\nu\alpha\beta} + \frac{1}{d-2}(R_{\mu\alpha}g_{\nu\beta} + R_{\nu\beta}g_{\mu\alpha} - R_{\mu\beta}g_{\nu\alpha} - R_{\nu\alpha}g_{\mu\beta}) \]

where $C_{\mu\nu\alpha\beta}$ is the Weyl tensor, and substitute this decomposition into (3)

\[ i\gamma^\nu \left\{ C_{\mu_2} \alpha_1^\mu_1 \nu \psi_{\mu_1\mu_3...\mu_n} + \cdots + C_{\mu_n} \alpha_1^\mu_1 \nu \psi_{\mu_1...\mu_{n-1}\alpha} - \frac{1}{4} C_{\alpha_3} \beta_1^\mu_1 \nu \gamma_{\alpha_3} \beta_1 \psi_{\mu_1...\mu_n} \right\} + \frac{i}{2} \frac{d - 2n}{d - 2} R^{\alpha_1 \gamma_1} \psi_{\mu_1...\mu_n} = 0, \]

where we have used $\gamma^\mu_1 \psi_{\mu_1...\mu_n} = 0$. From (5) we see that for compatibility of (2) one should suppose that $C_{\mu\nu\alpha\beta} = 0$. After this condition (5) is reduced to

\[ \frac{i}{2} \frac{d - 2n}{d - 2} R^{\alpha_1 \gamma_1} \psi_{\mu_1...\mu_n} = 0. \]

One can see that the last expression in (3) assumes if the space is arbitrary curved then $\psi_{\mu_1...\mu_n} = 0$. Let us try to find from (3) what spaces do not give any condition on $\psi_{\mu_1...\mu_n}$. Let us decompose the Riemann tensor

\[ R_{\mu\nu\alpha\beta} = \frac{1}{4} \gamma^\mu \nabla_\mu - m \right) \psi_{\mu_1...\mu_n} = \frac{1}{2} \frac{d - 2n}{d - 2} R^{\alpha_1 \gamma_1} \psi_{\mu_1...\mu_n} = 0 \]

First we see that in even dimensional spaces when $d = 2n$, with $n$ being the tensor rank of the field there is no restriction on the Ricci tensor. This special case will be studied elsewhere. Now we consider arbitrary values of $d$ and $n$. In this general case one must suppose that the traceless part of the Ricci tensor is zero $R_{\mu\nu} = R_{\mu\nu} = \frac{1}{d} g_{\mu\nu} R = 0$, what means that $R_{\mu\nu} = \frac{1}{d} g_{\mu\nu} R$. Next from the corollary of the Bianchi identity $2\nabla^\nu R_{\mu\nu} = \nabla_\mu R$ one finds that $R = \text{const.}$ That is equations (2) are compatible with each other only on the spaces with constant curvature. Therefore the rest part of the paper will be devoted to Lagrangian construction for the fields in AdS space. Note that in spaces of constant curvature there is a possibility to modify equations of motion (2), since a parameter (the radius of the curvature) with dimension of length appears.

### 3 Algebra of constraints for fermionic fields in $AdS_d$

As is known an antisymmetric tensor rank-$n$ fermionic field will realize irreducible massive representation of the AdS group \[17\] if the following conditions are satisfied

\[ [\hat{\gamma}^\mu \nabla_\mu - m + r^\frac{1}{2} (n - \frac{d}{2})] \psi_{\mu_1...\mu_n} = 0, \quad \gamma^\mu_1 \psi_{\mu_1...\mu_n} = 0, \quad \nabla^\mu_1 \psi_{\mu_1...\mu_n} = 0. \]

Here $r$ is defined from $R_{\mu\alpha\beta\gamma} = r(g_{\mu\beta}g_{\alpha\gamma} - g_{\mu\alpha}g_{\beta\gamma})$. Analogously to the bosonic case \[6\] in order to avoid manipulations with a number of indices we introduce auxiliary Fock space generated by fermionic creation and annihilation operators $a^+_a$, $a_a$ satisfying the anticommutation relations

\[ \{a^+_a, a_b\} = \eta_{ab}, \quad \eta_{ab} = \text{diag}(-, +, +, \cdots, +). \]
As usual the tangent space indices and the curved indices are converted one into another with the help of vielbein $e^a_\mu$ which is assumed to satisfy the relation $\nabla_\nu e^a_\mu = 0$. Then in addition to the conventional gamma-matrices

$$\{\gamma_a, \gamma_b\} = -2\eta_{ab},$$  

(9)

we introduce a set of $d + 1$ Grassmann odd objects \[10, 11\] which obey the following gamma-matrix-like conditions

$$\{\tilde{\gamma}^a, \tilde{\gamma}^b\} = -2\eta^{ab}, \quad \{\tilde{\gamma}^a, \tilde{\gamma}\} = 0, \quad \tilde{\gamma}^2 = -1$$  

(10)

and connected with the “true” gamma-matrices by the relation

$$\gamma^a = \tilde{\gamma}^a \tilde{\gamma} = -\tilde{\gamma} \tilde{\gamma}^a.$$  

(11)

After this we define derivative operator

$$D_\mu = \partial_\mu + \omega_\mu^{\ ab} M_{ab}, \quad M_{ab} = \frac{1}{2}(a^+_a a_b - a^+_b a_a) - \frac{1}{8} (\tilde{\gamma} a \tilde{\gamma}_b - \tilde{\gamma}_b \tilde{\gamma} a),$$  

(12)

which acts on an arbitrary state vector in the Fock space

$$|\psi\rangle = \sum_{n=0} \ a^{+\mu_1} \cdots a^{+\mu_n} \psi_{\mu_1 \cdots \mu_n} (x) |0\rangle$$  

(13)

as the covariant derivative\[2\]

$$D_\mu |\psi\rangle = \sum_{n=0} \ a^{+\mu_1} \cdots a^{+\mu_n} \ (\nabla_\mu \psi_{\mu_1 \cdots \mu_n}) |0\rangle.$$  

(14)

As a result equations \[17\] can be realize in the operator form

$$\tilde{t}_0 |\psi\rangle = 0, \quad t_1 |\psi\rangle = 0, \quad l_1 |\psi\rangle = 0,$$  

(15)

where

$$\tilde{t}_0 = i\tilde{\gamma}^\mu D_\mu + \tilde{\gamma} (r^{\frac{1}{2}} g_0 - m), \quad g_0 = a^+_a a^\mu - \frac{d}{2}, \quad t_1 = \tilde{\gamma}^\mu a^\mu, \quad l_1 = -i a^\mu D_\mu.$$  

(16)

Lagrangian construction within the BRST approach \[12\] demands that we must have at hand a set of operators which is invariant under Hermitian conjugation and which forms an algebra \[12, 13\]. In order to determine the Hermitian conjugation properties of the constraints we define the following scalar product

$$\langle \tilde{\Phi} | \Phi \rangle = \int d^d x \sqrt{-g} \sum_{n,k=0} \langle 0 | \Psi_{\nu^1 \cdots \nu_k}^+ (x) \tilde{\gamma}^0 a^{\nu_k} \cdots a^{\nu_1} a^{+\mu_1} \cdots a^{+\mu_n} \Phi_{\mu_1 \cdots \mu_n} (x) |0\rangle.$$  

(17)

As a result we see that constraint $\tilde{t}_0$ is Hermitian and the two other are non-Hermitian\[3\]

$$t_1^+ = a^+_\mu \tilde{\gamma}^\mu, \quad l_1^+ = -i a^{+\mu} D_\mu.$$  

(18)

Now in order to have an algebra we add to the set of operators all the operators generated by the (anti)commutators of $\tilde{t}_0$, $t_1$, $l_1$, $t_1^+$, $l_1^+$. Therefore we have to add the following three operators

$$\tilde{t}_0 = D^2 - m^2 + r \left( -g_0^{2} + g_0 + t_1^+ t_1 + \frac{d(d+1)}{4} \right),$$  

$$g_0 = a^+_a a^\mu - \frac{d}{2}, \quad g_m = m,$$  

(19)

(20)

\[2\]we assume that $\partial_\mu a^+_a = \partial_a a^\mu - \partial_\mu |0\rangle = 0$.

\[3\]we assume that $(\tilde{\gamma}^\mu)^+ = \tilde{\gamma}^0 \tilde{\gamma}^\mu \tilde{\gamma}^0$, $(\tilde{\gamma})^+ = \tilde{\gamma}^0 \tilde{\gamma} \tilde{\gamma}^0 = -\tilde{\gamma}$. 

where $D^2 = g^{\mu\nu} \left( D_\mu D_\nu - \Gamma^a_{\mu\nu} D_a \right)$. As a result set of operators $\tilde{t}_0, \tilde{l}_0, t_1, l_1, t_1^+, l_1^+$, $g_0, g_m$ is invariant under Hermitian conjugation and form an algebra.

The method of Lagrangian construction within the BRST approach [12] requires enlarging of the initial operators $\tilde{o}_i = (\tilde{t}_0, \tilde{l}_0, t_1, l_1, t_1^+, l_1^+, g_0, g_m)$ so that the enlarged Hermitian operators contain arbitrary parameters and the set of enlarged operators form an algebra. A procedure of constructing of these enlarged operators $\tilde{O}_i = \tilde{o}_i + \tilde{o}'_i$ for the operators $\tilde{o}_i$ is considerably simplified if the initial operators $\tilde{o}_i$ (super)commute with their additional parts [4]. This happens because the additional parts must contain $\tilde{\gamma}$ which is also present in $\tilde{t}_0$. Therefore in order that initial operators (super)commute with additional parts we make a non-degenerate linear transformation $o_1 = U^\dagger_j \tilde{o}_j$ and remove $\tilde{\gamma}$ from $\tilde{t}_0$. Thus we modify $\tilde{t}_0$ and $\tilde{l}_0$

\begin{align*}
    t_0 &= \tilde{t}_0 + \tilde{\gamma}(g_m - r\frac{2}{3}g_0), \\
    l_0 &= \tilde{l}_0 + g_m^2 + r g_0^2,
\end{align*}

\begin{align*}
    t_0 &= i\tilde{\gamma}^\mu D_\mu, \\
    l_0 &= D^2 + r(g_0 + t_1^+ t_1 + \frac{d(d+1)}{4})
\end{align*}

with the other operators being unchanged. Due to this transformation of the initial operators we can apply the method of constructing of additional parts elaborated in [12]. Algebra of new initial operators is given in Table 1 with

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
$[\downarrow, \rightarrow]$ & $t_0$ & $t_1$ & $t_1^+$ & $l_0$ & $l_1$ & $l_1^+$ & $g_0$ & $g_m$ \\
\hline
$t_0$ & $2l_0$ & $2l_1$ & $-2l_1^+$ & 0 & (23) & (24) & 0 & 0 \\
$t_1$ & $-2l_1$ & 0 & $2g_0$ & (25) & 0 & $-t_0$ & $t_1$ & 0 \\
$t_1^+$ & $2l_1^+$ & $-2g_0$ & 0 & (26) & $t_0$ & 0 & $-t_1^+$ & 0 \\
l_0 & 0 & $-t_0$ & (27) & (28) & (29) & 0 & 0 & 0 \\
l_1 & (23) & 0 & $-t_0$ & (27) & $\frac{1}{2}rt_{1}^{+2}$ & (29) & $l_1$ & 0 \\
l_1^+ & (24) & $t_0$ & 0 & (28) & (29) & $\frac{1}{2}rt_{1}^{+2}$ & $-l_1^+$ & 0 \\
g_0 & 0 & $-t_1$ & $t_1^+$ & 0 & $-l_1$ & $l_1^+$ & 0 & 0 \\
g_m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
\end{tabular}
\caption{Algebra of the initial operators}
\end{table}

Next step in the procedure of Lagrangian construction is finding the additional parts for the initial operators given in Table 1.

\footnote{We suppose that the additional parts are constructed from new (additional) creation and annihilation operators and from the constants of the theory. See e.g. [12].}
Table 2: Algebra of the additional parts

| \{ l_i \rightarrow \} | t_0' | t_1' | t_1'^{+} | l_0' | l_1' | l_1'^{+} | g_0' | g_m' |
|-------------------|-------|-------|-----------|------|------|-----------|------|------|
| t_0'             | 2t_0' | 2t_1' | -2t_1'^{+} | 0    | (34) | (35)      | 0    | 0    |
| t_1'             | -2t_1' | 0    | 2g_0'      | (36) | 0    | -t_0'     | t_1' | 0    |
| t_1'^{+}         | 2t_1'^{+} | -2g_0' | 0       | (37) | t_0' | 0         | -t_1'^{+} | 0    |
| l_0'             | 0     | - (36) | - (37)    | 0    | (38) | (39)      | 0    | 0    |
| t_1'             | (34)  | 0    | -t_0'      | - (38)| - \frac{1}{2} t_1'^{+} + (32) | l_1' | 0    |
| l_1'^{+}         | (35)  | t_0' | 0         | (36) | (32) | - \frac{1}{2} t_1'^{+} + l_1'^{+} | 0    | 0    |
| g_0'             | 0     | -t_1' | t_1'^{+}   | 0    | -l_1' | l_1'^{+}  | 0    | 0    |
| g_m'             | 0     | 0    | 0         | 0    | 0    | 0         | 0    | 0    |

4 The additional parts

In this section we are going to find explicit expressions for the additional parts in terms of new (additional) creation and annihilation operators and from the constants of the theory. The requirements the additional parts \( o_i' \) must satisfy is as follows: 1) The enlarged operators \( O_i = o_i + o_i' \) are in involution relation \([O_i, O_j] \sim O_k\); 2) each Hermitian operator must contain an arbitrary parameter linearly which values shall be defined later from the condition of reproducing the equation of motion (7).

To find explicit expression for the additional parts we must first determine their algebra. The procedure of finding the algebra of the additional parts for nonlinear algebras was elaborated in [12]. To be complete we explain this procedure using anticommutator \( \{ L_1, L_1^{+} \} \) as an example. Supposing that the initial operators \( o_i \) (super)commute with the additional parts \( o_i' \) one finds

\[
\{ L_1, L_1^{+} \} = \{ l_1, l_1^{+} \} + \{ l_1', l_1'^{+} \} = -l_0 + r(g_0^2 + \frac{1}{2} g_0 + \frac{1}{2} t_1^{+} t_1) + \{ l_1', l_1'^{+} \}.
\]

(30)

Then we express all the initial operators through the enlarged and the additional ones \( O_i = O_i - o_i' \) and order the operators so that the enlarged operators stand on the right side

\[
\{ L_1, L_1^{+} \} = -L_0 + rG_0^2 - 2rg_0G_0 + \frac{1}{2} rG_0 + \frac{1}{2} rT_1^{+} T_1 - \frac{1}{2} rt_1^{+} T_1 - \frac{1}{2} rt_1' T_1^{+} + l_0' + r(g_0^2 + \frac{1}{2} g_0 + \frac{1}{2} t_1^{+} t_1) + \{ l_1', l_1'^{+} \}.
\]

(31)

In order to satisfy the first requirement \([O_i, O_j] \sim Q_k\) we put

\[
\{ l_1', l_1'^{+} \} = -l_0' - r(g_0^2 + \frac{1}{2} g_0 + \frac{1}{2} t_1^{+} t_1)
\]

(32)

and as a consequence we get

\[
\{ L_1, L_1^{+} \} = -L_0 + rG_0^2 - 2rg_0G_0 + \frac{1}{2} rG_0 + \frac{1}{2} rT_1^{+} T_1 - \frac{1}{2} rt_1^{+} T_1 - \frac{1}{2} rt_1' T_1^{+}.
\]

(33)

Thus we have found anticommutator for the additional parts \( \{ l_1', l_1'^{+} \} \) (32) and simultaneously anticommutator for the enlarged operators \( \{ L_1, L_1^{+} \} \) (33). Repeating the same procedure for the other (anti)commutators we find the algebra of the additional parts and the algebra of the extended operators. The algebra of the additional parts is given \( ^5 \) in Table 2 with

\[ ^5 \] The algebra of the extended operators will be discussed later. It is given in Table 3 at page 9.
\[ \{ t'_0, l'_1 \} = r(g'_0 + \frac{1}{2})t'_1, \]  
\[ \{ t'_0, l'_1^+ \} = r t'_1^+ (g'_0 + \frac{1}{2}), \]  
\[ [t'_1, l'_0] = -r(2g'_0 + 1)t'_1, \]  
\[ [t'_1^+, l'_0] = r t'_1^+ (2g'_0 + 1), \]  
\[ [l'_0, l'_1] = r(2g'_0 + 1)l'_1, \]  
\[ [l'_0, l'_1^+] = -r l'_1^+ (2g'_0 + 1). \]  

Using this algebra one can find explicit expressions for the additional parts in terms of new (additional) creation and annihilation operators. The method which allows us to do this is described in appendix A. The result takes the form

\[
\begin{align*}
t'_1^+ &= b^+, \quad l'_1^+ = m_1 f^+ - \frac{r}{4m_1} b^{+2} f, \quad g'_0 = b^+ b + f^+ + h, \quad g_m = h_m, \\
t'_0 &= -\gamma m_0 - 2m_1 f^+ b + \frac{r}{2m_1} (b^+ b + 2h) b^+ f, \\
l'_0 &= -m_0^2 - r(b^+ b + 2h) b^+ b - 2r(b^+ b + h + \frac{1}{2}) f^+ f, \\
t'_1 &= \gamma (m_0^2 + m_1) f + (2f^+ f + b^+ b + 2h)b, \\
l'_1 &= \gamma m_0 b + m_1 f^+ b^2 + \frac{m_0^2}{m_1} f - \frac{r}{m_1} (h + \frac{1}{2})(b^+ b + h) f - \frac{r}{4m_1} b^{+2} b^2 f,
\end{align*}
\]

where we have introduced one pair of fermionic \( f^+, f \) and one pair of bosonic \( b^+, b \) creation and annihilation operators with the standard (anti)commutation relations

\[
\{ f^+, f \} = 1, \quad [b^+, b] = 1.
\]

According to the second requirement the found additional parts for Hermitian initial operators contain arbitrary parameters linearly: operators \( t'_0, g'_0, g'_m \) contain parameters \( m_0, h, h_m \) respectively. Operator \( l'_0 \) cannot contain independent arbitrary parameter since \( l'_0 = (t'_0)^2 \). Parameters \( m_0 \) and \( h_m \) have dimension of mass, and parameter \( h \) is dimensionless. The values of these parameters will be defined later from the condition of reproducing equations of motion \( (7) \). Also expressions for additional parts \( (40) \)–\( (44) \) contain arbitrary (nonzero) parameter \( m_1 \) with dimension of mass. Its value remains arbitrary and it can be expressed from the other parameters of the theory \( m_1 = f(m, r) \neq 0 \). The arbitrariness of this parameter does not influence on the reproducing of the equations of motion for the physical field \( (7) \).

Note that the additional parts do not obey the usual properties

\[
(t'_0)^+ \neq t'_0 \quad (l'_0)^+ \neq l'_0, \quad (t'_1)^+ \neq t'_1^+, \quad (l'_1)^+ \neq l'_1^+
\]

if one use the standard rules of Hermitian conjugation for the new creation and annihilation operators

\[
(b)^+ = b^+, \quad (f)^+ = f^+.
\]

To restore the proper Hermitian conjugation properties for the additional parts we change the scalar product in the Fock space generated by the new creation and annihilation operators as follows:

\[
\langle \tilde{\Psi}_1 | \tilde{\Psi}_2 \rangle_{\text{new}} = \langle \tilde{\Psi}_1 | K | \Psi_2 \rangle,
\]

for any vectors \( |\Psi_1\rangle, |\Psi_2\rangle \) with some yet unknown operator \( K \). This operator is determined by the condition that all the operators of the algebra must have the proper Hermitian properties with respect
to the new scalar product:

\[ \langle \tilde{\Psi}_1 | K t'_0 | \Psi_2 \rangle = \langle \tilde{\Psi}_2 | K t'_0 | \Psi_1 \rangle^*, \quad \langle \tilde{\Psi}_1 | K t'_1 | \Psi_2 \rangle = \langle \tilde{\Psi}_2 | K t'_1 | \Psi_1 \rangle^*, \]
\[ \langle \tilde{\Psi}_1 | K t'_0 | \Psi_2 \rangle = \langle \tilde{\Psi}_2 | K t'_0 | \Psi_1 \rangle^*, \quad \langle \tilde{\Psi}_1 | K t'_1 | \Psi_2 \rangle = \langle \tilde{\Psi}_2 | K t'_1 | \Psi_1 \rangle^*, \]
\[ \langle \tilde{\Psi}_1 | K g'_0 | \Psi_2 \rangle = \langle \tilde{\Psi}_2 | K g'_0 | \Psi_1 \rangle^*, \quad \langle \tilde{\Psi}_1 | K g'_m | \Psi_2 \rangle = \langle \tilde{\Psi}_2 | K g'_m | \Psi_1 \rangle^*. \] (51)

Since the problem with the proper Hermitian conjugation of the operators are in \((b^+, f^+)-sector\) of the Fock space then the modification of the scalar product concerns only this sector. Therefore operator \(K\) acts as a unit operator in the entire Fock space, but for the \((b^+, f^+)-sector\) where the operator has the form

\[
K = \sum_{k=0}^{\infty} \frac{C_k(k)}{k!} \left[ |0, k\rangle \frac{1}{2h+k} \langle 0, k | + |1, k\rangle \frac{2m_0^2 - rh(2h+k+1)}{4h m_1^2} \langle 1, k | \\
+ |1, k\rangle \frac{\tilde{\gamma}m_0}{2h m_1} \langle 0, k+1 | + |0, k+1\rangle \frac{\tilde{\gamma}m_0}{2h m_1} \langle 1, k | \right],
\] (52)

where

\[
C_k(k) = 2h (2h+1) \ldots \ldots (2h+k-2)(2h+k-1)(2h+k),
\] (53)
\[
|0, k\rangle = (b^+)^k |0\rangle, \quad |1, k\rangle = f^+(b^+)^k |0\rangle. \] (54)

Thus in this section we have constructed the additional parts \((40-44)\) for the operators which obey all the requirements. In the next section we determine the algebra of the enlarged operators and construct BRST operator corresponding to this algebra.

5 The deformed algebra and BRST operator

The algebra of the enlarged operators can be determined by the method described in the previous section where we obtained anticommutator \(\{L_1, L_1^+\}\) in the explicit form \((43)\). Looking at this anticommutator we see that its r.h.s. are quadratic and therefore there is different possibilities to order operators

\[
\{L_1, L_1^+\} = -L_0 + rG_0^2 - 2rg'_0G_0 - r(\frac{1}{2} - \xi)G_0 + \frac{1}{2}r\xi T_1^+ T_1 + \frac{1}{2}r(1 - \xi)T_1 T_1^+ - \frac{1}{2}rT_1^+ T_1 - \frac{1}{2}rt_1T_1^+, \]
(55)

where we have introduced parameter \(\xi\) responsible for the operator ordering. The same is valid for the other (anti)commutators. Each ordering leads to different forms of BRST operator. We will not investigate all the possibilities of ordering here and choose only one of them which corresponds to the supersymmetric ordering of the enlarged operators in the rhs. The algebra\(^6\) of the enlarged operators corresponding to the supersymmetric ordering is presented in Table 3, where

\[^6\text{We put arbitrary constant } h_m \text{ to } -m \text{ and get that enlarged operator } G_m = g_m + g'_m = 0. \text{ In what follows we forget about } G_m.\]
Here, $q$, $q_i$, $q_i^+$ and $\eta_0$, $\eta_i^+$, $\eta_i$, $\eta_G$ are, respectively, the bosonic and fermionic ghost “coordinates” corresponding to their canonically conjugate ghost “momenta” $p_0$, $p_i^+$, $p_i$, $P_0$, $P_i$, $P_i^+$, $P_G$. They obey the (anti)commutation relations

$$\{q_0, P_0\} = \{q_i, P_{i^+}\} = \{\eta_i^+, P_1\} = 1, \quad \text{and} \quad [q_0, p_0] = [q_i, p_i^+] = [q_i^+, p_i] = i.$$ (66)

Table 3: Algebra of the enlarged operators

| $\{1, \rightarrow\}$ | $T_0$ | $T_1$ | $T_1^+$ | $L_0$ | $L_1$ | $L_1^+$ | $G_0$ |
|----------------------|-------|-------|---------|-------|-------|---------|-------|
| $T_0$                | $2L_0$| $2L_1$| $-2L_1^+$| 0     | (56)  | (57)    | 0     |
| $T_1$                | $-2L_1$| 0     | $2G_0$  | (58)  | 0     | $-T_0$ | $T_1$ |
| $T_1^+$              | $2L_1^+$| $-2G_0$| 0     | (59)  | $T_0$ | 0       | $-T_1^+$|
| $L_0$                | 0     | $-\frac{56}{58}$| $-\frac{59}{59}$| 0     | (60)  | (61)    | 0     |
| $L_1$                | (56)  | 0     | $-T_0$ | (60)  | (63)  | (62)    | $L_1$ |
| $L_1^+$              | (57)  | $T_0$ | 0     | (61)  | (62)  | (61)    | $-L_1^+$|
| $G_0$                | 0     | $-T_1$| $T_1^+$| 0     | $-L_1$| $L_1^+$ | 0     |

The construction of a nilpotent fermionic BRST operator for a nonlinear superalgebra is based on the same principles as those developed in [3, 12] (for a general consideration of operator BFV quantization, see the reviews [19]). The BRST operator constructed on a basis of the algebra given by Table 3 is

$$Q' = q_0T_0 + \eta_1^+T_1 + \eta_1T_1^+ + \eta_0L_0 + q_1^+L_1 + q_1L_1^+ + \eta_GG_0 + (q_i^+q_i - q_i^2)P_0 + 2i\eta_1^+q_0p_1 - 2iq_0\eta_1^+p_1 - 2i\eta_1^+\eta_GP_0 + i(\eta_1^+q_1 - q_i^+\eta_i)\rho_0 + \eta_G(\eta_i^+P_1 - \eta_iP_{i^+} + iq_i^+p_i - iq_1^+\eta_i) - rq_i^+q_1(G_0 - 2g_0)P_G - r(\eta_1^+\eta_0 - \frac{1}{2}q_i^+q_0)[(G_0 - 2g_0)P_1 + (T_1 - 2t_1^+)P_G] - r(\eta_0\eta_1 - \frac{1}{2}q_0q_1)[(G_0 - 2g_0)P_{i^+} + (T_1^+ - 2t_1'^+)P_G] + rq_i^+\eta_0[(G_0 - 2g_0)i\eta_1 + (L_1 - 2t_1^+)P_G] - r\eta_0q_1[(G_0 - 2g_0)i\eta_1^+ + (L_1^+ - 2t_1'^+)P_G] - \frac{r}{4}[q_1(T_1^+ - 2t_1') + q_i^+(T_1 - 2t_1')]\eta_1^+P_1 + q_i^+P_{i^+} + \frac{r^2}{4}\eta_0(q_iT_1^+ - q_i^+T_1)(q_iP_{i^+} + q_i^+P_1)P_G.$$ (65)
and possess the standard ghost number distribution, $gh(C^i) = -gh(P_i) = 1$, providing $gh(Q') = 1$. The resulting BRST operator $Q'$ is Hermitian with respect to the new scalar product $(18)$. Let us turn to Lagrangian construction on the base of BRST operator $Q'$ $(65)$.

# 6 Construction of Lagrangians

In this section we construct Lagrangians of antisymmetric fermionic massive fields in the AdS space. This construction goes along the line of $(14)$. First we extract the dependence of the BRST operator $Q'$ $(65)$ on the ghosts $\eta_G, P_G$

$$Q' = Q + \eta_G (\sigma + h) + A P_G$$

$$Q = q_0 \left[ T_0 + 2i(\eta_1^p p_1 - \eta_1 p_1^+) + \frac{r}{2}(G_0 - 2g_0)(q_1^+ P_1^+ + q_1^+ P_1) \right] + i(\eta_1^p q_1 - q_1^p \eta_1)p_0$$

$$+ \eta_1 \left[ L_0 + r(G_0 - 2g_0')(q_1^+ P_1 - \eta_1 P_1^+ + iq_1^+ p_1 - iq_1^p) \right] + (q_1^+ q_1 - q_0^2)P_0$$

$$+ \eta_1^k T_1 + \eta_1 T_1^+ + q_1^p L_1 + q_1 L_1^+$$

$$- \frac{r}{4} \left[ q_1(T_1^+ - 2t_1') + q_1^p (T_1 - 2t_1') \right] (q_1 P_1^+ + q_1^p P_1)$$

$$\sigma + h = G_0 + \eta_1^k P_1 - \eta_1 P_1^+ + iq_1^+ p_1 - iq_1^p$$

where explicit expression for the operator $A$ is not essential. Then we choose the following representation of the Hilbert space:

$$(p_0, q_1, p_1, P_0, P_G, \eta_1, P_1)|0\rangle = 0$$

and suppose that the vectors and gauge parameters do not depend on $\eta_G$ $(14)$

$$|\chi\rangle = \sum_{k_i} (q_0)^{k_1} (q_1^+)^{k_2} (p_1^+)^{k_3} (\eta)^{k_4} (\eta_1^+)^{k_5} (P_1^+)^{k_6} (b^+)^{k_7} (f^+)^{k_8} a^{+\mu_1} \cdots a^{+\mu_k} \chi_{\mu_1 \cdots \mu_k} (x)|0\rangle.$$

The sum in $(71)$ is taken over $k_0, k_1, k_2, k_3, k_7$ running from 0 to infinity and over $k_4, k_5, k_6, k_8$ running from 0 to 1. Then we derive from the equation on the physical vector $Q'|\chi\rangle = 0$ and from the reducible gauge transformations $\delta|\chi\rangle = Q'|\Lambda\rangle$ a sequence of relations:

$$Q|\chi\rangle = 0,$$

$$\delta|\chi\rangle = Q|\Lambda\rangle,$$

$$\delta|\Lambda\rangle = Q|\Lambda^{(1)}\rangle,$$

$$\delta|\Lambda^{(i-1)}\rangle = Q|\Lambda^{(i)}\rangle,$$

$$\delta|\Lambda^{(i)}\rangle = (\sigma + h)|\Lambda^{(i)}\rangle = 0,$$

$$gh(|\chi\rangle) = 0,$$

$$gh(|\Lambda\rangle) = -1,$$

$$gh(|\Lambda^{(1)}\rangle) = -2,$$

$$gh(|\Lambda^{(i)}\rangle) = -(i + 1).$$

The middle equation in $(72)$ presents the equations for the possible values of $h$

$$h = \frac{d}{2} - n,$$

with $n$ being related to the tensor rank of antisymmetric tensor-spinor. By fixing the tensor rank of the antisymmetric field we also fix the parameter $h$ according to $(76)$. Having fixed a value of $h$ we should substitute it into each of the expressions $(72)$ - $(75)$, see $(14)$ for more details.

Next step is to extract the zero ghost mode from the operator $Q$. This operator has the structure

$$Q = \eta_0 \bar{L}_0 + (q_1^+ q_1 - q_0^2)P_0 + q_0 \bar{T}_0 + i(\eta_1^+ q_1 - \eta_1 q_1^+)p_0 + \Delta Q,$$
where $\tilde{T}_0$, $\tilde{L}_0$, $\Delta Q$ is independent of $\eta_0$, $P_0$, $q_0$, $p_0$

\[
\tilde{T}_0 = T_0 + 2i(\eta_1^p p_1 - \eta_1 p_1^*) + \frac{r}{2}(G_0 - 2g_0^q)(q_1^p P_1^* + q_1^* P_1) \tag{78}
\]

\[
\tilde{L}_0 = L_0 + r(G_0 - 2g_0^q)(\eta_1^p P_1 - \eta_1 P_1^* + iq_1^p p_1 - iq_1 p_1^*) \tag{79}
\]

\[
\Delta Q = \eta_1^T T_1 + \eta_1 T_1^* + q_1^T L_1 + q_1 L_1^* - \frac{r}{4}[q_1(T_1^* - 2T_1^\prime) + q_1^* (T_1 - 2T_1^\prime)](q_1^p P_1^* + q_1^* P_1) \tag{80}
\]

Also we decompose the state vector and the gauge parameters as

\[
|\chi\rangle = \sum_{k=0}^{\infty} g_0^k (|\chi_0^k\rangle + \eta_0 |\chi_1^k\rangle), \quad g_h(|\chi_m^k\rangle) = -(m + k), \tag{81}
\]

\[
|A^{(i)}\rangle = \sum_{k=0}^{\infty} g_0^k (|A^{(i)k}_0\rangle + \eta_0 |A^{(i)k}_1\rangle), \quad g_h(|A^{(i)k}_m\rangle) = -(i + k + m + 1). \tag{82}
\]

Then following the procedure described in [14] we get rid of all the fields except two $|\chi_0^0\rangle$, $|\chi_0^1\rangle$ and the leftmost equation in (72) is reduced to

\[
\Delta Q|\chi_0^0\rangle + \frac{1}{2}\{\tilde{T}_0, q_1^* q_1\}|\chi_0^1\rangle = 0, \tag{83}
\]

\[
\tilde{T}_0|\chi_0^0\rangle + \Delta Q|\chi_0^1\rangle = 0, \tag{84}
\]

where $\{A, B\} = AB + BA$. State vector (71) and as a consequence $|\chi_0^0\rangle$, $|\chi_0^1\rangle$ (81) contain physical fields of all ranks. Due to the fact that the operators $\Delta Q$, $\tilde{T}_0$, $q_1^* q_1$ commute with $\sigma$ we derive from (83), (84) the equations of motion corresponding to the physical field of tensor rank-$n$

\[
\Delta Q|\chi_0^0|^n + \frac{1}{2}\{\tilde{T}_0, q_1^* q_1\}|\chi_0^1|^n = 0, \tag{85}
\]

\[
\tilde{T}_0|\chi_0^0|^n + \Delta Q|\chi_0^1|^n = 0, \tag{86}
\]

where the $|\chi_0^0|^n$, $|\chi_0^1|^n$ are assumed to obey the relations

\[
\sigma|\chi_0^0|^n = (n - d/2)|\chi_0^0|^n, \quad \sigma|\chi_0^1|^n = (n - d/2)|\chi_0^1|^n. \tag{87}
\]

The field equations (85), (86) are Lagrangian ones and can be deduced from the following Lagrangian

\[
L_n = n|\tilde{\chi}_0^0|^2 + n|\tilde{\chi}_0^1|^2 - \frac{1}{2}(\tilde{T}_0, q_1^* q_1)|\chi_0^1|^n + \frac{1}{2}(\tilde{T}_0, q_1^* q_1)|\chi_0^1|^n + n|\chi_0^0| K_n \Delta Q|\chi_0^1|^n + n|\chi_0^1| K_n \Delta Q|\chi_0^0|^n, \tag{88}
\]

where the standard scalar product for the creation and annihilation operators is assumed, and the operator $K_n$ is the operator $K$ (52), where the following substitution is made $h \to d/2 - n$.

The equations of motion (85), (86) and the action (88) are invariant with respect to the gauge transformations

\[
\delta|\chi_0^0|^n = \Delta Q|\Lambda_0^0|^n + \frac{1}{2}\{\tilde{T}_0, q_1^* q_1\}|\Lambda_0^1|^n, \tag{89}
\]

\[
\delta|\chi_0^1|^n = \tilde{T}_0|\Lambda_0^0|^n + \Delta Q|\Lambda_0^1|^n, \tag{90}
\]

\[\text{The physical fields in (71) are those which correspond to } k_1 = k_2 = k_3 = k_4 = k_5 = k_6 = k_7 = k_8 = 0 \text{ and arbitrary } k_0 \text{ which is equal to } n \text{ being the tensor rank of tensor-spinor. The other fields in decomposition (71) are St"uckelberg (} k_8 = 1 \text{) or auxiliary (} k_8 = 0 \text{).}
\]

\[\text{The Lagrangian is defined as usual up to an overall factor.}\]
which are reducible, with the gauge parameters $|\Lambda^{(i,j)}_{0/n}|, j = 0, 1$ subject to the same conditions as those for $|\chi^0_{0/n}|$ in (87),

$$
\delta|\Lambda^{(i,0)}_{0/n}| = \Delta Q|\Lambda^{(i+1,0)}_{0/n}| + \frac{1}{2}\{\tilde{T}_0, q^+_i q_i\}|\Lambda^{(i+1,1)}_{0/n},
$$

$$
|\Lambda^{(0,0)}_{0/n}| = |\Lambda^0_{0/n}|,\quad (91)
$$

$$
\delta|\Lambda^{(i,1)}_{0/n}| = \tilde{T}_0|\Lambda^{(i+1,0)}_{0/n}| + \Delta Q|\Lambda^{(i+1,1)}_{0/n},
$$

$$
|\Lambda^{(0,1)}_{0/n}| = |\Lambda^1_{0/n}|,\quad (92)
$$

with finite number of reducibility stages $i_{\text{max}} = n - 1$.

We now determine the value of the arbitrary parameter $m_0$ using the condition that the equations (7) [or in operator form (15)] for the basic vector $|\psi\rangle$ (13) be reproduced. To this end it is necessary that conditions (7) be implied by Eqs. (85), (86). Note that the general vector $|\chi^0_{0/n}|$ includes the basic vector $|\psi\rangle$ (13)

$$
|\chi^0_{0/n}| = |\psi\rangle + |\psi_A\rangle_n, \quad |\psi_A\rangle_n\bigg|_{\text{ghosts}=b^+=f^+}=0. \quad (93)
$$

In appendix [13] we shall demonstrate that due to the gauge fixing and a part of the equations of motion the vector $|\psi_A\rangle_n$ can be completely removed and the resulting equations of motion have the form

$$
T_0|\psi\rangle_n = (t_0 - \gamma m_0)|\psi\rangle_n = 0, \quad T_1|\psi\rangle_n = t_1|\psi\rangle_n = 0, \quad L_1|\psi\rangle_n = l_1|\psi\rangle_n = 0, \quad (94)
$$

so the action actually reproduces the correct equations of motion (7). The above relations permit one to determine the parameter $m_0$ in a unique way as follows

$$
m_0 = m + r^\frac{1}{2}(d/2 - n) = m + r^\frac{1}{2}h. \quad (95)
$$

Thus we have constructed Lagrangians for antisymmetric fermionic fields of any tensor rank using the BRST approach.

Finally we note that Lagrangian (88) can be simplified. In particular one can remove all the auxiliary field and write Lagrangian $\mathcal{L}_n$ for rank-$n$ antisymmetric fermionic field in terms of basic field $\psi_{\mu_1...\mu_n}$ only (see details in appendix [C])

$$
\mathcal{L}_n = \sum_{k=0}^{n} \frac{1}{(n-k)!} \bar{\psi}^{\mu_1...\mu_{n-k}} [(-1)^k i \gamma^\sigma \nabla_\sigma - m_0] \psi^{\mu_1...\mu_{n-k}}
- i \sum_{k=0}^{n-1} \frac{(-1)^k}{(n-k-1)!} \left( \bar{\psi}^{\mu_1...\mu_{n-k}} \nabla_{\mu_1} \psi^{\mu_2...\mu_{n-k}} + \bar{\psi}^{\mu_2...\mu_{n-k}} \nabla^{\mu_1} \psi^{\mu_1...\mu_{n-k}} \right), \quad (96)
$$

where $m_0 = m + r^\frac{1}{2}(d/2 - n)$ with $m$ being the mass and we have denoted

$$
\psi_{\mu_{k+1}...\mu_n} = \frac{1}{k!} \gamma^{\mu_k} \ldots \gamma^{\mu_1} \psi_{\mu_1...\mu_n}, \quad \bar{\psi}_{\mu_{k+1}...\mu_n} = \frac{1}{k!} \bar{\psi}_{\mu_1...\mu_n} \gamma^{\mu_1} \ldots \gamma^{\mu_k}. \quad (97)
$$

If we put the mass $m = 0$ in (96) (which means $m_0 = r^\frac{1}{2}(d/2 - n)$) then one should expect that Lagrangian (96) becomes gauge invariant Lagrangian for the rank-$n$ massless antisymmetric fermionic field.

---

9Lagrangian (96) is Lagrangian (88) multiplied by $(-1)^n$. See (C.21).
7 Summary

We have constructed the Lagrangian formulation for massive fermionic antisymmetric tensor field theory in $AdS_d$ space and found the various equivalent forms of the Lagrangian. In general, the Lagrangian contains basic field together with a number of auxiliary and St"uckelberg fields determining the reducible gauge model. Such a situation is a standard for massive higher spin field theories. However, the specific features namely fermionic antisymmetric field allowed to eliminate completely all the auxiliary and St"uckelberg fields from the action and obtain the Lagrangian only in terms of basic field. As far as we know, such a Lagrangian has never been presented before in the literature.

We have demonstrated that if we don’t include in the equations of motion for antisymmetric field the terms with the inverse powers of the mass then the equations of motion in curved space of arbitrary dimension are consistent only in space of constant curvature. Then we have shown that the BRST approach which was earlier applied for totally symmetric or mixed symmetry higher spin fields perfectly works for massive fermionic antisymmetric fields in $AdS_d$ space.

The initial point of Lagrangian construction is reformulating the massive irreducible representation of the $AdS_d$ on fermionic antisymmetric tensor fields as operator constraints in auxiliary Fock space. Then we found the closed algebra generated by these operators and applied the BRST construction \cite{12}. As a result we obtained the reducible gauge Lagrangian theory, the corresponding Lagrangian and (St"uckelberg) gauge transformations are given by (88), (89)-(92) and the order of reducibility grows with the value of the rank of the antisymmetric field. Like all the Lagrangians constructed on the base of the BRST approach, the Lagrangian in the case under consideration possess rich gauge symmetry and contain many auxiliary fields. Partially fixing some of the symmetries or/and eliminating some auxiliary fields it is possible to derive the various intermediate Lagrangian formulations. In particular one can write Lagrangian with some number of auxiliary fields without gauge symmetry (C.18) or with gauge symmetry (C.14). In particular, the Lagrangian in terms of basic field only (i.e. without any auxiliary fields and gauge symmetries) is also obtained (96).

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A Calculation of the additional parts

In this Appendix we show how the representation of the algebra given in Table 2 can be constructed in terms of some creation and annihilation operators.

Let us consider a representation of this algebra with the vector $|0\rangle_V$ annihilated by the operators $l'_1$ and $t'_1$

$$l'_1|0\rangle_V = t'_1|0\rangle_V = 0,$$  \hspace{1cm} (A.1)

One points out that the standard (local) Lagrangian of totally symmetric massive higher spin field theory with auxiliary fields can be transformed to equivalent but nonlocal Lagrangian without auxiliary fields \cite{20}. In the given paper, speaking about Lagrangian without auxiliary fields we mean a local Lagrangian without auxiliary fields. Also we emphasize that the Lagrangian formulations for totally antisymmetric tensor-spinor fields, which were studied in the given paper, have never been constructed earlier at all.
and being the eigenvector of the operators $t'_0$, $l'_0$, $g'_0$ and $g'_m$

\[ t'_0|0\rangle_V = -\tilde{\gamma} m_0|0\rangle_V, \quad l'_0|0\rangle_V = -m_0^2|0\rangle_V, \quad g'_0|0\rangle_V = h|0\rangle_V, \quad g'_m|0\rangle_V = h_m|0\rangle_V, \quad (A.2) \]

where $m_0$, $h_m$ are arbitrary constants with dimension of mass and $h$ is an arbitrary dimensionless constant. They are the arbitrary constants which must be contained in the additional parts of Hermitian operators. Next we choose the basis vectors of this representation as follows

\[ |0, n\rangle_V = (t'^+_1)^n|0\rangle_V, \quad |1, n\rangle_V = \frac{l'^+_1}{m_1} (t'^+_1)^n|0\rangle_V, \quad (A.3) \]

where $m_1$ is an arbitrary nonzero constant with dimension of mass. It may be constructed from the parameters of the theory $m_1 = f(m, r) \neq 0$.

Now using commutators given in Table 2 and (A.1)–(A.3) one finds

\[ t'^+|0, n\rangle_V = |0, n + 1\rangle_V, \quad t'^+|1, n\rangle_V = |1, n + 1\rangle_V, \quad (A.4) \]

\[ l'^+|0, n\rangle_V = m_1|1, n\rangle_V, \quad l'^+|1, n\rangle_V = -\frac{r}{4m_1}|0, n + 2\rangle_V, \quad (A.5) \]

\[ g'_0|0, n\rangle_V = (n + h)|0, n\rangle_V, \quad g'_0|1, n\rangle_V = (n + 1 + h)|1, n\rangle_V, \quad (A.6) \]

\[ g'_m|0, n\rangle_V = h_m|0, n\rangle_V, \quad g'_m|1, n\rangle_V = h_m|1, n\rangle_V, \quad (A.7) \]

\[ t'_0|0, n\rangle_V = -2nm_1|1, n - 1\rangle_V - \tilde{\gamma} m_0|0, n\rangle_V, \quad (A.8) \]

\[ t'_0|1, n\rangle_V = \frac{r}{2m_1} \left(n + 1 + 2h\right)|0, n + 1\rangle_V - \tilde{\gamma} m_0|1, n\rangle_V, \quad (A.9) \]

\[ l'_0|0, n\rangle_V = -\left[rn(n + 2h) + m_0^2\right]|0, n\rangle_V, \quad (A.10) \]

\[ l'_0|1, n\rangle_V = -\left[rn(n + 2h) + m_0^2\right]|1, n\rangle_V - 2r(n + h + \frac{1}{2})|1, n\rangle_V, \quad (A.11) \]

\[ l'_1|0, n\rangle_V = n(n - 1 + 2h)|0, n - 1\rangle_V, \quad (A.12) \]

\[ l'_1|1, n\rangle_V = \tilde{\gamma} \frac{m_0}{m_1}|0, n\rangle_V + n(n + 1 + 2h)|1, n - 1\rangle_V \quad (A.13) \]

\[ l'_1|0, n\rangle_V = n\tilde{\gamma} m_0|0, n - 1\rangle_V + n(n - 1)m_1|1, n - 2\rangle_V \quad (A.14) \]

\[ l'_1|1, n\rangle_V = n\tilde{\gamma} m_0|1, n - 1\rangle_V - \frac{r}{4m_1} n(n - 1)|0, n\rangle_V + \frac{m_0^2 - r(n + h + \frac{1}{2})(n + h)}{m_1}|0, n\rangle_V. \quad (A.15) \]

Now let us turn to construction of a representation of the operator algebra given in Table 2 in terms of creation and annihilation operators. The number of pairs of these operators and their statistics is defined by the number and the statistics of the operators used in the definition of the basis vectors (A.3). Thus we introduce one pair of the bosonic and one pair of fermionic creation and annihilation operators with the standard commutation relations

\[ [b, b^+] = 1, \quad \{f, f^+\} = 1, \quad (A.16) \]

corresponding to $t'_1$, $t'^+_1$ and $l'_1$, $l'^+_1$ respectively. After this we map the basis vectors (A.3) and the basis vectors of the Fock space generated by $b^+$, $f^+$

\[ |0, n\rangle_V \longleftrightarrow (b^+)^n|0\rangle = |0, n\rangle = |n\rangle, \quad (A.17) \]

\[ |1, n\rangle_V \longleftrightarrow f^+(b^+)^n|0\rangle = |1, n\rangle, \quad (A.18) \]

and find from (A.4)–(A.15) form of the operators in terms of the creation and annihilation operators $b$, $b^+$, $f$, $f^+$. They are given by relations (A.10)–(A.14).
Let us consider antisymmetric fermionic field of tensor rank $n$. Then we have a reducible gauge theory with $n - 1$ reducibility stages. Due to restriction [S7] and the ghost number restriction [see the right-hand formulae in (82)], the lowest-stage gauge parameters have the form

$$|\Lambda^{(n-1)0}_{0/n} = (p_1^+)^{n-1} \{ P_1^+ \lambda \} _0 + p_1^+ |\lambda_1 \rangle_0 \} , \quad (B.19)$$

$$|\Lambda^{(n-1)1}_{0/n} \equiv 0, \quad (B.20)$$

with the subscripts of the state vectors being associated with the eigenvalues of the corresponding state vectors (S7). In what follows we shall omit these subscripts. We see that gauge parameter $|\Lambda^{(n-1)0}_{0} \rangle$ cannot depend (in particular) on $f^+$. It can be verified directly that one can eliminate the dependence on $f^+$ from the gauge function $|\Lambda^{(n-2)0}_{0} \rangle$ of the $(n - 2)$-th stage. The gauge function $|\Lambda^{(n-2)1}_{0} \rangle$ has no $f^+$ dependence due to the same reason as $|\Lambda^{(n-1)0}_{0} \rangle$. It is then possible to verify that one can remove the dependence of $|\Lambda^{(n-3)0}_{0} \rangle$, $|\Lambda^{(n-3)1}_{0} \rangle$ on $f^+$ with the help of the remaining gauge parameters $|\Lambda^{(n-2)0}_{0} \rangle$, $|\Lambda^{(n-2)1}_{0} \rangle$ which do not depend on $f^+$.

We now suppose that we have removed the dependence on $f^+$ from the gauge functions of the $i$-th stage $|\Lambda^{(i)j}_{0} \rangle$, $j = 0, 1$, i.e., we have $f|\Lambda^{(i)0}_{0} \rangle = 0$. Let us consider the gauge transformation for $|\Lambda^{(i-1)j}_{0} \rangle$. It has the following structure

$$\delta|\Lambda^{(i-1)j}_{0} \rangle = q_1 L_1^+ |\Lambda^{(i)j}_{0} \rangle + \ldots = m_1 q_1 f^+ |\Lambda^{(i)j}_{0} \rangle + \ldots \quad (B.21)$$

The $f^+$ dependent part of $|\Lambda^{(i-1)j}_{0} \rangle$ and $q_1 f^+ |\Lambda^{(i)j}_{0} \rangle$ have the same decomposition on creation operators (71). Therefore we can eliminate the $f^+$ dependent part of $|\Lambda^{(i-1)j}_{0} \rangle$ having used all the restricted gauge parameters $|\Lambda^{(i)j}_{0} \rangle$.

The same argumentation is valid for the gauge transformations of fields $|\chi^j_{0} \rangle$. But in this case we do not use all the gauge parameters since in $|\Lambda^0_0 \rangle$ there are terms independent of $p_1^+$ and they are annihilated by $q_1 f^+$. These terms have the following ghost structure

$$|\Lambda^0_0 \rangle = P_1^+ |\lambda \rangle + \ldots \quad (B.22)$$

where $|\lambda \rangle$ depends on $a^{+\mu}$ and $b^{+}$ and independent of $f^{+}$ due to the condition $f|\Lambda^0_0 \rangle = 0$. We can use the remaining gauge parameter $|\lambda \rangle$ to eliminate the $b^{+}$ dependence in the ghost independent part of $|\chi^0_0 \rangle$ with the help of the transformation

$$\delta|\chi^0_0 \rangle = \eta_1 T_1^+ P_1^+ |\lambda \rangle + \ldots = b^+ |\lambda \rangle + \ldots \quad (B.23)$$

Now we have used all the gauge parameters. Thus the gauge conditions on the fields are

$$f|\chi^0_0 \rangle = f|\chi^1_0 \rangle = 0, \quad p_1 P_1 b|\chi^0_0 \rangle = 0. \quad (B.24)$$

Let us turn to the elimination of the rest auxiliary fields with the help of the equations of motion.
B.2 Removing of the auxiliary fields with the equations of motion

Let us decompose the equations of motion \( (85), (86) \) on \( f^+ \). The equations of motion at \( f^+ \) are

\[
(q_1^+ b^2 + q_1)|\chi_0^0\rangle = 2b|\chi_0^0\rangle, \quad \quad \quad \quad \quad \quad (q_1^+ b^2 + q_1)|\chi_0^0\rangle = 2q_1^+ q_1 b|\chi_0^0\rangle.
\]

(B.25)

Then we decompose fields \( |\chi_0^0\rangle \) and \( |\chi_1^0\rangle \) in ghosts \( \eta_1^+, \mathcal{P}_1^+ \)

\[
|\chi_0^j\rangle = |\chi_0^0\rangle + \eta_1^+ |\chi_0^1\rangle + \mathcal{P}_1^+ |\chi_0^0\rangle + \eta_1^+ \mathcal{P}_1^+ |\chi_0^1\rangle, \quad \quad j = 0, 1
\]

(B.26)

and substitute this decomposition into (B.25). First we consider the following pair of equations corresponding to (B.26)

\[
(q_1^+ b^2 + q_1)|\chi_0^0\rangle = 2b|\chi_0^0\rangle, \quad \quad \quad \quad \quad \quad (q_1^+ b^2 + q_1)|\chi_0^0\rangle = 2q_1^+ q_1 b|\chi_0^0\rangle.
\]

Decomposing fields \( |\chi_0^0\rangle \) in power series of bosonic ghosts \( q_1^+, p_1^+ \)

\[
|\chi_0^0\rangle = \sum_{k=0}^{[n/2]} \frac{(-iq_1^+ p_1^+)^k}{k!} |\chi_{00k}\rangle, \quad \quad |\chi_0^0\rangle = \sum_{k=1}^{[(n+1)/2]} (q_1^+)^{k-1} \frac{(-iq_1^+ p_1^+)^k}{k!} |\chi_{00k}\rangle
\]

(B.28)

where fields \( |\chi_{00k}\rangle \) have ghost number equal to zero \( gh(|\chi_{00k}\rangle) = 0 \). Substituting (B.28) into (B.27) and considering the obtained equations from the lowest power of \( p_1^+ \) (and taking into account the gauge \( b|\chi_0^0\rangle = 0 \)) we get that all \( |\chi_{00k}\rangle = 0, k \geq 1 \). That is we have \( |\chi_{010}\rangle = 0 \) and \( |\chi_{000}\rangle = |\psi\rangle \), with \( |\psi\rangle \) being the physical field (13).

Next we consider one more pair of equations (B.25) corresponding to \( (\eta_1^+)^0 (\mathcal{P}_1^+)^0 \) coefficient of decomposition (B.26)

\[
(q_1^+ b^2 + q_1)|\chi_1^1\rangle = 2b|\chi_0^0\rangle, \quad \quad \quad \quad \quad \quad (q_1^+ b^2 + q_1)|\chi_0^1\rangle = 2q_1^+ q_1 b|\chi_0^1\rangle.
\]

(B.29)

Doing decomposition of the fields in power series of ghosts \( q_1^+, p_1^+ \) analogous to (B.28) and considering equations from the lowest powers of \( p_1^+ \) one concludes that \( |\chi_{010}\rangle = |\chi_{111}\rangle = 0 \).

Let us now turn to the equations which are coefficients of equations \( (85), (86) \) at \( (f^+)^0 (\eta_1^+)^0 (\mathcal{P}_1^+)^0 \)

\[
T_0|\psi\rangle - 2ip_1^+ |\chi_{01}\rangle + T_1^+ |\chi_{010}\rangle = 0,
\]

(B.30)

\[
q_1^+ L_1|\psi\rangle + T_1^+ |\chi_{010}\rangle + q_1^+ (1 - 2ip_1^+ q_1)|\chi_{010}\rangle = 0
\]

(B.31)

and at \( (f^+)^0 (\eta_1^+)^1 (\mathcal{P}_1^+)^0 \)

\[
2ip_1^+ |\chi_{01}\rangle - T_1^+ |\chi_{011}\rangle = 0,
\]

(B.32)

\[
T_1|\psi\rangle - T_1^+ |\chi_{011}\rangle - q_1^+ (1 - 2ip_1^+ q_1)|\chi_{011}\rangle = 0,
\]

(B.33)

where we have taken into account that \( |\chi_{000}\rangle = |\psi\rangle \) and \( |\chi_{010}\rangle = |\chi_{010}\rangle = |\chi_{111}\rangle = 0 \).

Let us consider the first pair of the equations. We decompose fields \( |\chi_{01}^0\rangle \) in bosonic ghosts \( q_1^+, p_1^+ \)

\[
|\chi_{01}^0\rangle = q_1^+ \sum_{k=0}^{[n/2]} \frac{(-iq_1^+ p_1^+)^k}{k!} |\chi_{01k}\rangle, \quad \quad |\chi_{01}^0\rangle = \sum_{k=0}^{[(n-1)/2]} \frac{(-iq_1^+ p_1^+)^k}{k!} |\chi_{01k}\rangle,
\]

(B.34)

where all fields \( |\chi_{01k}\rangle \) have ghost number equal to zero. Starting from the highest power of \( p_1^+ \) we conclude that all the \( |\chi_{01k}\rangle \) are \( 0 \) except \( |\chi_{010}\rangle \). Now equation (B.30) reduce to

\[
T_0|\psi\rangle + T_1^+ |\chi_{010}\rangle = 0.
\]

(B.35)
Decomposing field $|\chi_{010}\rangle$ in power series of creation operator $b^+$ and substituting this decomposition into (B.35) we find that as a result $|\chi_{010}\rangle = 0$. That is we get $|\chi_{00}\rangle = |\chi_{01}\rangle = 0$.

Similar consideration of equations (B.32), (B.33) leads us to conclusion that $|\chi_{10}\rangle = |\chi_{11}\rangle = 0$.

Thus we have shown that all the auxiliary fields are equal to zero due to the gauge condition (B.24) or as a solution to the equations of motion. The equations of motion on the physical field $|\psi\rangle$ followed from (B.30), (B.33), (B.31) are

$$T_0|\psi\rangle = \langle t_0 - \gamma m_0|\psi\rangle = 0, \quad T_1|\psi\rangle = t_1|\psi\rangle = 0, \quad L_1|\psi\rangle = l_1|\psi\rangle = 0$$

which coincide with (94) and with (15) or in component form with (7).

C Simplified Lagrangians

Let us try to simplify Lagrangian (88) and write it in terms of the physical field only. For this purpose we decompose fields $|\chi_{00}\rangle$, $|\chi_{01}\rangle$ in power series of fermionic ghost fields $\eta_1^+$, $\mathcal{P}_1^+$ (B.26) and substitute into (88). One has

$$\mathcal{L}_n = \langle \chi_{00}\rangle K_n \left\{ T_0|\chi_{00}\rangle - 2ip_1^+|\chi_{00}\rangle + \frac{r}{2}(G_0 - 2g_0)q_1^+|\chi_{00}\rangle \right\}$$

$$+ \langle \chi_{01}\rangle K_n \left\{ -T_0|\chi_{01}\rangle + 2ip_1|\chi_{00}\rangle + 2i\eta_1^+|\chi_{01}\rangle + T_1|\chi_{00}\rangle - \left( \langle q_1^+L_1 + q_1L_1^+ \rangle |\chi_{01}\rangle \right) \right\}$$

$$+ \langle \chi_{10}\rangle K_n \left\{ -T_0|\chi_{10}\rangle + \frac{r}{2}(G_0 - 2g_0)q_1|\chi_{00}\rangle + \frac{r}{2}(G_0 - 2g_0)q_1^+|\chi_{01}\rangle \right\}$$

$$- \langle \chi_{11}\rangle K_n \left\{ -T_0|\chi_{11}\rangle + 2ip_1^+q_1p_1|\chi_{00}\rangle + q_1^+|\chi_{01}\rangle + \frac{r}{2}(G_0 - 2g_0)q_1^+|\chi_{10}\rangle \right\}$$

Then we partially fix the gauge analogously to as in Appendix B so that parameters $|\Lambda_0^0\rangle$ and $|\Lambda_0^1\rangle$ do not depend on $f^+: \langle f|\Lambda_0^0\rangle = \langle f|\Lambda_0^1\rangle = 0$. After the partial gauge fixing the gauge transformations of the fields (89) and (90) become irreducible. Decomposed the gauge parameters $|\Lambda_0^0\rangle$ and $|\Lambda_0^1\rangle$
analogously to (B.26) we substitute them into (89) and (90). The result is

\[
\delta |\chi_{00}\rangle = (q_+^+ L_1 + q_1 L_1^+) |\Lambda_{00}^{00}\rangle + T_1^+ |\Lambda^{00}_{01}\rangle - \frac{r}{4} \left[ q_1 (T_1^+ - 2t_1^+) + q_1^+ (T_1 - 2t_1') \right] q_1^+ |\Lambda_{00}^{10}\rangle
\]

\[
+ T_0 q_1^+ q_1 |\Lambda_{00}^{00}\rangle - 2i q_1^+ p_1 q_1 |\Lambda_{01}^{10}\rangle + q_1^+ |\Lambda_{01}^{00}\rangle + \frac{r}{2} (G_0 - 2g_0) q_1^+ q_1 |\Lambda_{10}^{11}\rangle
\]  
\[
(C.2)
\]

\[
\delta |\chi_{00}\rangle = T_1 |\Lambda_{00}^{00}\rangle - (q_1^+ L_1 + q_1 L_1^+) |\Lambda_{00}^{10}\rangle - T_1^+ |\Lambda_{01}^{00}\rangle
\]

\[
- T_0 q_1^+ q_1 |\Lambda_{00}^{00}\rangle - 2i q_1^+ p_1 q_1 |\Lambda_{01}^{10}\rangle + q_1^+ |\Lambda_{01}^{00}\rangle - q_1^+ |\Lambda_{01}^{11}\rangle + 2i q_1^+ p_1 q_1 |\Lambda_{11}^{11}\rangle
\]  
\[
(C.3)
\]

\[
\delta |\chi_{01}\rangle = -(q_+^+ L_1 + q_1 L_1^+) |\Lambda_{01}^{00}\rangle - \frac{r}{4} \left[ q_1 (T_1^+ - 2t_1^+) + q_1^+ (T_1 - 2t_1') \right] q_1 |\Lambda_{00}^{00}\rangle
\]

\[
+ T_0 q_1^+ q_1 |\Lambda_{01}^{00}\rangle + 2i q_1^+ q_1 p_1 |\Lambda_{01}^{10}\rangle + q_1^+ |\Lambda_{01}^{00}\rangle - \frac{r}{2} (G_0 - 2g_0) q_1^+ q_1^2 |\Lambda_{01}^{11}\rangle
\]  
\[
(C.4)
\]

\[
\delta |\chi_{10}\rangle = T_0 |\Lambda_{00}^{00}\rangle - 2i p_1^+ |\Lambda_{00}^{10}\rangle + \frac{r}{2} (G_0 - 2g_0) q_1 |\Lambda_{01}^{00}\rangle
\]

\[
+ (q_+^+ L_1 + q_1 L_1^+) |\Lambda_{01}^{10}\rangle + T_1^+ |\Lambda_{01}^{00}\rangle - \frac{r}{4} \left[ q_1 (T_1^+ - 2t_1^+) + q_1^+ (T_1 - 2t_1') \right] q_1^+ |\Lambda_{01}^{11}\rangle
\]  
\[
(C.5)
\]

\[
\delta |\chi_{11}\rangle = -T_0 |\Lambda_{00}^{00}\rangle + 2i p_1 |\Lambda_{00}^{10}\rangle + 2i p_1^+ |\Lambda_{01}^{00}\rangle + T_1 |\Lambda_{01}^{10}\rangle - (q_1^+ L_1 + q_1 L_1^+) |\Lambda_{11}^{10}\rangle - T_1^+ |\Lambda_{11}^{11}\rangle
\]  
\[
(C.6)
\]

\[
\delta |\chi_{10k}\rangle = -T_0 |\Lambda_{01}^{00}\rangle + \frac{r}{2} (G_0 - 2g_0) q_1 |\Lambda_{01}^{00}\rangle
\]

\[
- (q_+^+ L_1 + q_1 L_1^+) |\Lambda_{01}^{10}\rangle - \frac{r}{4} \left[ q_1 (T_1^+ - 2t_1^+) + q_1^+ (T_1 - 2t_1') \right] \left( q_1^+ |\Lambda_{01}^{10}\rangle + q_1^+ |\Lambda_{11}^{10}\rangle \right)
\]  
\[
(C.7)
\]

\[
\delta |\chi_{11k}\rangle = T_0 |\Lambda_{01}^{00}\rangle + 2ip_1 |\Lambda_{01}^{10}\rangle - \frac{r}{2} (G_0 - 2g_0) q_1 |\Lambda_{01}^{00}\rangle
\]

\[
+ (q_+^+ L_1 + q_1 L_1^+) |\Lambda_{11}^{10}\rangle + T_1 |\Lambda_{11}^{10}\rangle + \frac{r}{4} \left[ q_1 (T_1^+ - 2t_1^+) + q_1^+ (T_1 - 2t_1') \right] q_1^+ |\Lambda_{11}^{11}\rangle
\]  
\[
(C.8)
\]

\[
\delta |\chi_{11k}\rangle = T_0 |\Lambda_{01}^{00}\rangle + 2i p_1 |\Lambda_{01}^{10}\rangle - \frac{r}{2} (G_0 - 2g_0) q_1 |\Lambda_{01}^{00}\rangle
\]

\[
+ (q_+^+ L_1 + q_1 L_1^+) |\Lambda_{11}^{10}\rangle + T_1 |\Lambda_{11}^{10}\rangle + \frac{r}{4} \left[ q_1 (T_1^+ - 2t_1^+) + q_1^+ (T_1 - 2t_1') \right] q_1^+ |\Lambda_{11}^{11}\rangle
\]  
\[
(C.9)
\]

Let us proceed the gauge fixing. Now we remove the fields depending on ghost $q_1^+$. That is we get rid of fields $|\chi_{01}^{01}\rangle$ (using $|\Lambda_{00}^{00}\rangle$ and $|\Lambda_{00}^{10}\rangle$ completely), $|\chi_{01}^{10}\rangle$ (using $|\Lambda_{01}^{00}\rangle$ and $|\Lambda_{01}^{10}\rangle$ completely), $|\chi_{01}^{11}\rangle$ (using $|\Lambda_{01}^{00}\rangle$ partially and $|\Lambda_{01}^{11}\rangle$ completely), $|\chi_{01}^{11}\rangle$ (using $|\Lambda_{01}^{00}\rangle$ partially and $|\Lambda_{01}^{11}\rangle$ completely). A part of parameters $|\Lambda_{00}^{00}\rangle$ and $|\Lambda_{01}^{01}\rangle$ remains unused. These unused gauge parameters we denote as $|\Lambda_{00}^{00}\rangle$ and $|\Lambda_{01}^{01}\rangle$ and they are defined from the following decomposition of $|\Lambda_{00}^{00}\rangle$ and $|\Lambda_{01}^{01}\rangle$ in power series of bosonic ghosts $q_1^+, p_1^+$

\[
|\Lambda_{00}^{00}\rangle = \sum_{k=0}^{(n-1)/2} (q_1^+)^k \left( \frac{-ip_1^+}{(k+1)!} \right) |\Lambda_{00k}^{00}\rangle.
\]

\[
|\Lambda_{01}^{01}\rangle = \sum_{k=0}^{(n-1)/2} \left( \frac{-iq_1^+ p_1^+}{k!} \right) |\Lambda_{01k}^{01}\rangle.
\]  
\[
(C.10)
\]

Here $|\Lambda_{00k}^{00}\rangle, |\Lambda_{01k}^{01}\rangle$ depend on $a^{\mu +}, b^+$ only (they are independent of $f^+$ due to the gauge fixing $f|\Lambda_0^+ = 0$). After the last partial gauge fixing Lagrangian (C.1) for the residuary fields are

\[
\mathcal{L}_n = \langle \chi_{00}^{00} K_n \left\{ T_0 |\chi_{00}^{00}\rangle - 2i p_1^+ |\chi_{01}^{00}\rangle + (q_1^+ L_1 + q_1 L_1^+) |\chi_{00}^{10}\rangle + T_1^+ |\chi_{01}^{10}\rangle \right\}
\]

\[
+ \langle \chi_{01}^{10} K_n \left\{ T_0 q_1^+ q_1 |\chi_{00}^{10}\rangle + q_1^+ (1 - 2i p_1^+ q_1) |\chi_{01}^{10}\rangle + (q_1^+ L_1 + q_1 L_1^+) |\chi_{00}^{10}\rangle + T_1^+ |\chi_{01}^{11}\rangle \right\}
\]

\[
+ \langle \chi_{01}^{11} K_n \left\{ (1 + 2i q_1^+ p_1) q_1 |\chi_{00}^{10}\rangle + T_1 |\chi_{00}^{10}\rangle \right\} + \langle \chi_{01}^{10} K_n \left\{ 2i p_1 |\chi_{00}^{10}\rangle + T_1 |\chi_{01}^{10}\rangle \right\},
\]  
\[
(C.11)
\]
Let us decompose the fields entering in Lagrangian (C.11) in power series of bosonic ghosts $q_i^+ p_i^+$

$$
|\chi_{00}^{00}\rangle = \sum_{k=0}^{[n/2]} \frac{(-iq_i^+ p_i^+)^k}{k!}|\Psi_{n-2k}\rangle,
|\chi_{00}^{10}\rangle = \sum_{k=0}^{[n-1]/2} \frac{(q_i^+)^k (-ip_i^+)k+1}{(k+1)!}|\Psi_{n-2k-1}\rangle,
$$

$$
|\chi_{01}^{00}\rangle = \sum_{k=0}^{[(n-2)/2]} (q_i^+)^{k+1} \frac{(-ip_i^+)^k}{k!}|A_{n-2k-2}\rangle,
|\chi_{01}^{10}\rangle = \sum_{k=0}^{[n-1]/2} \frac{(-iq_i^+ p_i^+)^k}{k!}|A_{n-2k-1}\rangle,
$$

(12)

(13)

where all $|\Psi_k\rangle$ and $|A_k\rangle$ depend on $a^{\mu+}, b^{+}, f^{+}$ only and their subindices coincide with the eigenvalues of operator $\sigma \equiv (87)$. Substituting these decompositions of the fields into (C.11) one finds

$$
\mathcal{L}_n = \langle \tilde{\Psi}_n|K_n\{T_0|\Psi_n] + L_1^+ |\Psi_{n-1}\rangle + T_1^+ |A_{n-1}\rangle \}
$$

$$
+ \sum_{k=1}^{n-1} \langle \tilde{\Psi}_k|K_n\{T_0|\Psi_k] + L_1|\Psi_{k+1}\rangle + L_1^+ |\Psi_{k-1}\rangle + (n - k)|A_k\rangle + T_1^+ |A_{k-1}\rangle \}
$$

$$
+ \langle \tilde{\Psi}_0|K_n\{T_0|\Psi_0\] + L_1|\Psi_1\rangle + n|A_0\rangle \}
+ \sum_{k=0}^{n-1} \langle \tilde{\Psi}_k|K_n\{T_0|\Psi_k\] + L_1|\Psi_{k+1}\rangle + (n - k)|A_k\rangle + T_1^+ |A_{k+1}\rangle \}
$$

(14)

Solving the equation of motion of $\langle \tilde{A} \rangle$ we can express all $|\Psi_k\rangle$ in terms of $|\Psi_n\rangle$

$$
|\Psi_{n-k}\rangle = \frac{(-1)^k}{k!}\langle T_1\rangle^k|\Psi_n\rangle.
$$

(15)

Now let us fix the gauge completely using the residual gauge parameters $|\Lambda_{000}^{00}\rangle$ and $|\Lambda_{0010}\rangle$. With their help we get rid of the dependence of the field $|\Psi_n\rangle$ on $f^+$ and $b^+$ respectively. That is the gauge condition is

$$
f|\Psi_n\rangle = b|\Psi_n\rangle = 0.
$$

(16)

Let us denote the part of fields $|\Psi_k\rangle$ and $|A_k\rangle$ which are independent of $f^+$ and $b^+$ as $|\psi_k\rangle$ and $|\alpha_k\rangle$ respectively. Then due to (C.16) we have that $|\Psi_n\rangle = |\psi_n\rangle$ and $|\psi_n\rangle$ is the physical field and due to (C.15) we get that all other $|\Psi_k\rangle$ are also independent of $f^+$, $b^+$

$$
|\Psi_{n-k}\rangle = \frac{(-1)^k}{k!}\langle T_1\rangle^k|\Psi_n\rangle = \frac{(-1)^k}{k!}\langle T_1\rangle^k|\psi_n\rangle = \frac{(-1)^k}{k!}\langle T_1\rangle^k|\psi_{n-k}\rangle = |\psi_{n-k}\rangle.
$$

(17)

Thus after the gauge fixing (C.16) Lagrangian (C.14) become

$$
\mathcal{L}_n = \langle \tilde{\psi}_n|\{(t_0 - \tilde{\gamma}m_0)|\psi_n\} + l_1^+|\psi_{n-1}\rangle + t_1^+|\alpha_{n-1}\rangle \}
$$

$$
+ \sum_{k=1}^{n-1} \langle \tilde{\psi}_{n-k}|\{(t_0 - \tilde{\gamma}m_0)|\psi_{n-k}\} + l_1|\psi_{n-k+1}\rangle + l_1^+|\psi_{n-k-1}\rangle + k|\alpha_{n-k}\rangle + t_1^+|\alpha_{n-k-1}\rangle \}
$$

$$
+ \langle \tilde{\psi}_0|\{(t_0 - \tilde{\gamma}m_0)|\psi_0\} + l_1|\psi_1\rangle + n|\alpha_0\rangle \}
+ \sum_{k=1}^{n} \langle \tilde{\alpha}_{n-k}|\{k|\psi_{n-k}\} + l_1|\psi_{n-k+1}\rangle \}
$$

(18)

and it has no gauge symmetry. Finally we can express all $|\psi_k\rangle$ through $|\psi_n\rangle$ using (C.17) and write Lagrangian in terms of the physical field only

$$
\mathcal{L}_n = \sum_{k=0}^{n} \frac{1}{(k!)^2} \langle \tilde{\psi}_n|(t_1^+)^k(t_0 - \tilde{\gamma}m_0)(t_1)^k|\psi_n\rangle
- \sum_{k=0}^{n-1} \frac{1}{k!(k+1)!} \langle \tilde{\psi}_n|(t_1^+)^k(l_1^+ t_1 + t_1^+ l_1)(t_1)^k|\psi_n\rangle.
$$

(19)
Let us rewrite Lagrangian (C.19) in the component form. Using the explicit expressions of the operators and

\[ |\psi_n\rangle = \frac{(-i)^n}{n!} a^+_{\mu_1} \cdots a^+_{\mu_n} \psi(x)_{\mu_1 \ldots \mu_n} |0\rangle \quad \langle \bar{\psi}_n| = \langle 0| \psi^+_{\mu_1 \ldots \mu_n} \gamma^0 a_{\mu_1} \cdots a_{\mu_n} \frac{i^n}{n!} \]  

we find

\[ (-1)^n L_n = \sum_{k=0}^{n} \frac{1}{(n-k)!} \bar{\psi}^{\mu_1 \ldots \mu_{n-k}} \gamma^\sigma \nabla_\sigma m_0 \psi_{\mu_1 \ldots \mu_{n-k}} \]
\[ - i \sum_{k=0}^{n-1} \frac{(-1)^k}{(n-k-1)!} \left( \bar{\psi}^{\mu_1 \ldots \mu_{n-k}} \nabla_{\mu_1} \psi_{\mu_2 \ldots \mu_{n-k}} + \bar{\psi}^{\mu_2 \ldots \mu_{n-k}} \nabla_{\mu_2} \psi_{\mu_1 \ldots \mu_{n-k}} \right), \]  

where we have denoted

\[ \psi^{\mu_{k+1} \ldots \mu_n} = \frac{1}{k!} \gamma^{\mu_k} \cdots \gamma^{\mu_1} \psi_{\mu_1 \ldots \mu_n}, \quad \bar{\psi}_{\mu_1 \ldots \mu_n} = \frac{1}{k!} \bar{\psi}^{\mu_1 \ldots \mu_n} \gamma^{\mu_1} \cdots \gamma^{\mu_k}, \quad \psi^{\mu_1 \ldots \mu_n} = \psi^{+}_{\mu_1 \ldots \mu_n} \gamma^0. \]  

Thus we have constructed Lagrangian for antisymmetric massive tensor-spinor field in terms of the basic field only.

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