COSMIC SHEAR WITHOUT SHAPE NOISE

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ABSTRACT

We describe a new method for reducing the shape noise in weak lensing measurements by an order of magnitude. Our method relies on spectroscopic measurements of disk galaxy rotation and makes use of the Tully-Fisher (TF) relation in order to control for the intrinsic orientations of galaxy disks. For this new proposed experiment, the shape noise ceases to be an important source of statistical error.

Using COSMOLIKE, a new cosmological analysis software package, we simulate likelihood analyses for two spectroscopic weak lensing survey concepts (roughly similar in scale to Dark Energy Task Force Stage III and Stage IV missions) and compare their constraining power to a cosmic shear survey from the Large Synoptic Survey Telescope (LSST). Our forecasts in seven-dimensional cosmological parameter space include statistical uncertainties resulting from shape noise, cosmic variance, halo sample variance, and higher-order moments of the density field. We marginalize over systematic uncertainties arising from photometric redshift errors and shear calibration biases considering both optimistic and conservative assumptions about LSST systematic errors.

We find that even the TF-Stage III is highly competitive with the optimistic LSST scenario, while evading the most important sources of theoretical and observational systematic error inherent in traditional weak lensing techniques. Furthermore, the TF technique enables a narrow-bin cosmic shear tomography approach to tightly constrain time-dependent signatures in the dark energy phenomenon.

Subject headings: cosmology: observations — gravitational lensing: weak — methods: observational

1. INTRODUCTION

Weak gravitational lensing has been advertised as a powerful probe of cosmology (e.g. Albrecht et al. 2006; Hoekstra & Jain 2008), and is a major science driver for several ongoing and future surveys, such as the Dark Energy Survey, HyperSuprime-Cam, the Large Synoptic Survey Telescope, Euclid, and the Wide-Field Infrared Survey Telescope. It is the least indirect method available for constraining the distributions of both dark and luminous matter in the universe. Weak lensing by large-scale structure — termed cosmic shear — promises powerful constraints on both the growth of structure and the expansion history of the Universe. For cosmic shear, typical fluctuations in the matter density field projected over cosmological distances produce lensing distortions to galaxy ellipticities of order $10^{-3}$. The noise (per ellipticity component) resulting from the random intrinsic orientations and ellipticities of shapes, by contrast, is $\sigma_\epsilon \sim 0.26$ (Chang et al. 2013). In order to detect the cosmic shear signal at high significance, lensing analyses must use every available galaxy. This comes at a high cost in increased systematic error, as shear measurements using faint and poorly-resolved galaxy images are especially susceptible to calibration biases (c.f. Hirata & Seljak 2003; Massey et al. 2013). For all of these reasons, it is highly desirable to control for sources of intrinsic scatter in lensing observables.

Several methods have been proposed for reducing the shape noise using additional observables to infer the unlensed properties of galaxies. Polarization in radio observations provides an estimate of the unlensed position angle (e.g., Brown & Battye 2011). Spatially-resolved kinematic maps carry information about the intrinsic orientation (Blain 2002; Morales 2006). In the context of weak lensing magnification, a scaling relation can be used to predict the unlensed size of a galaxy from other photometric quantities (Bertin & Lombardi 2002; Huff & Graves 2011). This paper presents a novel combination of the latter two approaches, employing minimally-resolved disk galaxy kinematics and the Tully-Fisher scaling relation to estimate both components of shear while suppressing shape noise.

Independent of these developments, the coming decade is likely to see a considerable increase in the capacity of massively multi-object spectroscopy of galaxies at moderate redshifts. Two such instruments currently under development at the time of this writing include the Prime Focus Spectrograph for the Subaru telescope (Takada et al. 2012) and the DESI spectrograph (Levi et al. 2013). The primary science surveys anticipated for these instruments require spectroscopic target densities of $1 \text{arcmin}^{-2}$, which is nearly an order of magnitude above the previous generation of spectroscopic surveys.

It is the coincidence between this surge in spectroscopic capacity and the widespread scientific interest in weak lensing that motivates the present work. We forecast
the cosmological constraining power of a cosmic shear measurement using a large spectroscopic data set (in combination with high-quality imaging) comparable in size to those expected from the aforementioned multi-object spectrographs. We show that, by using a combination of minimally-resolved disk galaxy kinematics and the Tully-Fisher scaling relation, a spectroscopic weak lensing experiment has the potential to greatly improve on the statistical and systematic errors of conventional lensing measurements.

1.1. The Tully-Fisher Relation

The Tully-Fisher (TF) relation (Tully & Fisher 1977) is a well-studied scaling relation between the rotation velocity and luminosity (or stellar mass) (Reyes et al. 2012) of disk galaxies, such that

\[ v_{\text{circ}} = \left( \frac{L}{L_0} \right)^n \] (1)

Here \( L \) is a measure of (variously) the total stellar mass or luminosity of the galaxy, \( v_{\text{circ}} \) is the rotation speed of the disk, and \( L_0 \) normalizes the relation. The power law \( n \) is constrained empirically to be \( \sim 1/3 \), with little redshift evolution (Miller et al. 2011).

The TFR was originally proposed, and has been frequently employed, as a cosmological distance indicator (Strauss & Willick 1995; Giovanelli et al. 1997; Courteau et al. 2000; Springob et al. 2007). It remains an important constraint on models of the formation of disk galaxies (Governato et al. 2007; Piontek & Steinmetz 2011; Portinari & Sommer-Larsen 2007).

Disk galaxies are inclined at random with respect to the line of sight to the observer, so the measured rotation velocity is related to the true circular velocity of the disk by \( v_{\text{obs}} = v_{\text{circ}} \sin i \). Correcting for the effects of inclination has been an important observational complication inherent in TF studies to date. Typically, the observer estimates \( \sin i \) by modeling the galaxy as an axisymmetric disk within finite thickness (typically parameterized by the edge-on aspect ratio \( q_j \)). In this case, the inclination angle \( i \) can be estimated from the ratio of the galaxy’s image \( q \) by

\[ \sin i = \left( \frac{1 - q^2}{1 - q_j^2} \right)^{1/2}. \] (2)

After inclination correction, estimates of the intrinsic fractional scatter in \( v_{\text{circ}} \) at fixed luminosity or stellar mass are typically 0.06 dex or less (Miller et al. 2011; Reyes et al. 2012).

In this work, we suggest a reversal of the normal measurement process. Rather than applying an inclination correction to individual galaxies, we propose to use each galaxy’s offset from the TFR to estimate its true, unlensed shape. This procedure does not require that the TFR be known in advance – in narrow slices in redshift, it can be estimated from the upper envelope of the scatter in the \( L - v_{\text{circ}} \sin i \) plane (Obreschkow & Meyer 2013). The degree to which TFR offset can control for the scatter in galaxy shapes is set by the intrinsic scatter in the scaling relation: if face-on disk galaxies are not round, noise will be introduced into Eq. 2, which will in turn increase the intrinsic TFR scatter, \( \sigma_{\text{int}} \). For the analysis that follows, the aforementioned constraints on \( \sigma_{\text{int}} \) set the intrinsic scatter in galaxy ellipticities at fixed \((L, v_{\text{circ}} \sin i)\).

1.2. The Effect of Shear on Tully-Fisher Observables

The effects of an arbitrary small shear on an elliptical curve – tracing, say, an isophotal contour in the image of a disk galaxy – are well-documented elsewhere, though we include them in the discussion below in abbreviated form for the sake of convenience and completeness. The effects on the line-of-sight velocity field of the disk are not so widely discussed (though see Morales (2006)), and the difference between the two is key for this measurement.

Shear is a spin-2 field, parameterized by two numbers. In a Cartesian coordinate system, these parameters are the magnitude of the distortion projected along the major coordinate axes (\( \gamma_+ \)) and at odd multiples of \( \pi/4 \) with respect to the major axes (\( \gamma_\times \)).

For a sufficiently small distortion, the effect of the shear can be written as a linear transformation \( A \), which maps the unsheared onto the sheared coordinates as \( x \rightarrow Ax \), where

\[ A = \begin{pmatrix} 1 + \gamma_+ & \gamma_\times \\ \gamma_\times & 1 - \gamma_+ \end{pmatrix}. \] (3)

In polar coordinates \((r, \theta)\), the shear is \((\gamma, \phi)\) and the effect of the distortion is

\[ r \rightarrow r (1 + \gamma \cos(2(\theta - \phi))) \]
\[ \theta \rightarrow \theta - \gamma \sin(2(\theta - \phi)). \] (4)

These expressions describe the remapping of the line-of-sight velocity field of the galaxy; in particular, the shift in \( \theta \) describes the angular shift in the kinematic major and minor axes of the rotating disk.

The effects of a shear on an elliptical isophote are twofold: the position angle of the major axis shifts, and the axis ratio is changed. If we choose the coordinate system to be aligned with the unlensed isophotal major axis (i.e., \( \theta = 0 \)), then the effect of a shear distortion on these two quantities is

\[ q \rightarrow q (1 + 2\gamma_+) \]
\[ \theta \rightarrow \theta + \frac{1 + q^2}{1 - q^2} \gamma_\times. \] (7)

With this choice of coordinates, the two shear components are

\[ \gamma_+ = \gamma \cos(2\phi), \]
\[ \gamma_\times = \gamma \sin(2\phi), \] (9)

so the shift in \( \theta \) in Eq. 5 at the position of the minor photometric axis becomes \( \gamma_\times \). The angle between the photometric and kinematic major axes resulting from the shear is the difference between this and Eq. 7

\[ \delta\theta_{\text{major}} = \frac{2q^2}{1 - q^2} \gamma_\times. \] (10)

The corresponding shift along the photometric minor axis is larger, as here the shift in position angle is \(-\gamma_\times\):

\[ \delta\theta_{\text{minor}} = \frac{2}{1 - q^2} \gamma_\times. \] (11)
For a rotating disk, where the velocity amplitude is proportional to \( \cos(\theta) \), the shear will produce a non-zero line-of-sight velocity along the photometric minor axis. The ratio between this quantity and the velocity along the photometric major axis is

\[
\frac{v_\|}{v_\perp} = \frac{\sin(\delta \theta_{\text{minor}})}{\cos(\delta \theta_{\text{major}})} \approx \delta \theta_{\text{minor}},
\]

from which the \( \gamma_x \) shear component can be straightforwardly extracted.

In the absence of measurement noise and intrinsic scatter in the TFR, the \( \gamma_+ \) component of the shear is just the fractional difference between the measured isophotal axis ratio and that predicted from the TFR. In summary, the effect of a small shear on the Tully-Fisher observables \((v_\perp, v_\|, L, q)\) is

\[
v_\| = \left( \frac{L}{L_0} \right)^n \left( \frac{1 - q^2}{1 + q^2} \right)^{\frac{1}{2}} \left( 1 - \frac{2q^2}{1 - q^2} \gamma_+ \right),
\]

\[
v_\perp = v_\| \frac{2}{1 - q^2} \gamma_x.
\]

In terms of the standard expressions for TFR observables, and letting absolute magnitude be \( M_B = -2.5 \log_{10} \left( \frac{L}{L_0} \right) \), the TFR is written as

\[
\log v_\| = a + b(M_B - M_0),
\]

and our estimators for shear become

\[
\gamma_+ = \frac{1 - q^2}{2q^2} \left[ \log v_\| - \frac{1}{2} \log \left( \frac{1 - q^2}{1 + q^2} \right) - (a + b(M_B - M_0)) \right]
\]

\[
\gamma_x = \frac{1 - q^2}{2} \frac{v_\perp}{v_\|}.
\]

It should be noted that magnification will also affect the TFR, by modifying the measured \( L \); we will defer consideration of this effect to a future work.

2. A TULLY-FISHER WEAK LENSING SURVEY

2.1. Effective Shape Noise

We estimate the effective shape noise that would arise from a hypothetical TF lensing experiment by generating catalogs of mock observables with appropriate noise properties. Each quantity in Eq. 16 is generated according to the following procedure:

1. An absolute magnitude \( M_B \) for each mock catalog entry is drawn from a normal distribution with mean -20.5 and standard deviation of unity.

2. For each mock catalog entry, \( \log_{10} v_{\text{circ}} \) is drawn from a Gaussian with mean 2.142 - 0.128(M\( _B + 20.558 \)), and a standard deviation (modeling the intrinsic TFR scatter) of \( \sigma_{\text{int}} = .05 \). The TFR coefficients here are taken from Reyes et al. (2012).

3. The cosine of the inclination angle \( i \) is drawn uniformly from \([0, 1]\), and the image axis ratio \( q \) is assigned as per Eq. 2.

4. The measured velocity \( v_{\text{obs}} \) is drawn from a normal distribution with mean \( v_{\text{circ}} \sin i \) and standard deviation equal to the characteristic velocity error \( \sigma_v \).

With this mock TF shape catalog, we estimate the noise in \( \gamma = \sqrt{\gamma_+^2 + \gamma_x^2} \) per ellipticity component \( \sigma_{\epsilon,\text{TF}} \) using an Expectation-Maximization (EM) algorithm. We note that the distribution of residuals in \( \gamma_+ \) and \( \gamma_x \) are well-fit by a Moffat distribution with \( \beta = 0.5 \). For this distribution, the maximum-likelihood estimator for the effective shape noise \( \sigma_{\epsilon,\text{TF}} \) is arrived at iteratively (Dempster et al. 1977), where for each iteration the mean, standard deviation, and set of weights are recalculated separately for each shear component from those of the previous iteration as

\[
\mu = \frac{\sum w_i \gamma_i}{\sum w_i},
\]

\[
\sigma_{\epsilon,\text{TF}}^2 = \frac{\sum w_i (\gamma_i - \mu)^2}{\sum w_i},
\]

\[
w_i = \frac{\beta \sigma_{\epsilon,\text{TF}}^2 + (\gamma_i - \mu)^2}{\beta \sigma_{\epsilon,\text{TF}}^2 + 1}.
\]

The sums above are over the input simulated data vector. We iterate until \( \sigma_{\epsilon,\text{TF}} \) converges.

The resulting effective shape noise is shown as a function of the velocity error in Fig. 1. For 17 km/s velocity errors, the effective shape noise improves by an order of magnitude over traditional methods, to \( \sigma_{\epsilon,\text{TF}} = 0.021 \).

With this improvement, LSST-equivalent levels of shape noise should be achievable with spectra for 0.4 galaxies per square arcminute. This is comparable to the target densities planned for the next generation of large spectroscopic surveys, and while the instruments currently under construction for these surveys have not been designed to obtain the spatially-resolved spectroscopy necessary for a spectroscopic lensing survey, they may be capable of the measurements discussed here. We discuss this point in general terms in Sect. 2.2.1, but defer instrument-specific survey considerations to a later analysis.

2.2. Designing a Tully-Fisher Lensing Survey

Here we describe two TF survey concepts. The first (hereafter TF-Stage III) is intended to be representative of an experiment that could be performed with instruments similar to those currently under development, relies on optical spectroscopy to measure rotation curves, and covers 5,000 square degrees. The second (hereafter TF-Stage IV) is intended to represent a more optimistic future survey, and assumes a greater redshift reach (which will require an infrared spectrograph) and a survey area of 15,000 square degrees, which is similar to the planned LSST footprint after masking (Chang et al. 2013).

We estimate the number and redshift distribution of viable targets for each of these two surveys with the Cosmos Mock Catalog (CMC) (Jouvel et al. 2009). The CMC, created using data from COSMOS\textsuperscript{11}, zCOSMOS (Lilly et al. 2007), and GOODS-N\textsuperscript{12}, was designed specific-
Figure 1. Effective shape noise $\sigma_{TF}$ (calculated as in Sect. 2.1 for a sample of disk galaxies as a function of the measurement error on the disk circular velocity.

Figure 2. Redshift distributions from the CMC (solid, histogram) for the TF-Stage III experiment. The smooth fit to this is the short-dashed green line, and the redshift distribution used to construct the TF-Stage III covariances is shown in the long-dashed blue line. When the emission line is detected out to this distance from the galaxy center. The actual line emission detection threshold will of course depend on the exposure time. Achieving this signal-to-noise ratio should be possible on an 8–m telescope with a PFS-like spectrograph in 30-minute exposures\textsuperscript{14}, which for this program would entail approximately 4–5 years of dedicated observations.

The available galaxy density set by applying these constraints to the CMC is $2.9/\text{arcmin}^2$. This number is most sensitive to the emission line strength requirement; halving the emission line detection threshold approximately doubles the available target density.

We do not expect a feasible spectroscopic lensing survey to realistically exceed a target density of one galaxy per square arcminute. To construct the redshift distributions for both TF surveys, we first fit a smooth distribution of the usual form:

$$p(z) \propto z^\alpha e^{-\left(\frac{z}{z_0}\right)^\beta}$$

(20)

to the redshift distribution of CMC sources that meet the selection criteria described above. We then subsample this to our fiducial target density assuming that the high-redshift tail is left in place, smoothly reducing the number density at lower redshift in a manner proportional to the comoving volume. The resulting redshift distributions for the CMC selection, its smoothed fit, and the fiducial survey redshift distributions for the TF-Stage

\textsuperscript{13} http://lamwws.oamp.fr/cosmowiki/RealisticSpectroPhotCat

\textsuperscript{14} as we are targeting larger, brighter galaxies than the PFS and DESI surveys, the fractional contribution of sky flux to the total flux in each fiber is substantially smaller than for the redshift survey components of those programs.
The primary obstacle is the collection of order $10^7$ resolved spectra. Several wide-field imaging surveys (HSC, LSST, DES) with a weak lensing focus are already planned or underway; we assume that any of these might be used for target selection and shape measurement for a TF survey. The primary obstacle is the collection of order $10^7$ resolved spectra.

Two massively multi-object fiber-fed spectroscopic instruments are currently in the advanced planning stage: the Prime Focus Spectrograph for the Subaru telescope (Takada et al. 2012) and the DESI spectrograph (Schlegel et al. 2009). Each is capable of producing target densities in a single exposure of 0.5 per square arcminute. Spatially resolved spectroscopy can in principle be obtained with multiple pointings. DESI, in particular, is planning to collect 50 million galaxy spectra, the majority of which are at $z > 1$. While we defer a more detailed, instrument-specific feasibility study to a future paper, it seems clear that a TF lensing survey is not drastically more challenging than currently planned projects.

3. Modeling Cosmological Quantities

We present a side-by-side comparison of a Stage IV Dark Energy experiment (pseudo-LSST) and the proposed Tully-Fisher measurements. In this section, we present a calculation of the expected cosmological constraints from each of these two surveys, including both statistical and systematic error contributions. The following sections describe the prediction code, the systematic errors we consider here, and our method for incorporating the systematics into our model.

3.1. Prediction Code

The simulated likelihood analysis in this paper is computed using COSMOlike (see Eifler et al. 2013, for an early version; the official release will be presented in Krause et al. 2013 in prep). We compute the linear power spectrum using the Eisenstein & Hu (1999) transfer function and models the non-linear evolution of the density field as described in Takahashi et al. (2012). We compute time-dependent dark energy models ($w = w_0 + (1-a)w_a$) following the recipe of iCOSMO (Refregier et al. 2011), which in the non-linear regime interpolates Halofit between flat and open cosmological models (please also see Schrabback et al. 2010, for more details).

From the density power spectrum we compute the shear power spectrum as

$$C^{ij}(l) = \frac{9H_0^4G^2}{4c^4} \int_0^{\chi_h} d\chi \frac{g^i(\chi)g^j(\chi)}{a^2(\chi)} P_\delta \left( \frac{l}{f_K(\chi)} , \chi \right) ,$$

with $l$ being the 2D wave vector perpendicular to the line of sight, $\chi$ denoting the comoving coordinate, $\chi_h$ is the comoving coordinate of the horizon, $a(\chi)$ is the scale factor, and $f_K(\chi)$ the comoving angular diameter distance. The lens efficiency $g^i$ is defined as an integral over the redshift distribution of source galaxies $n(\chi(z))$ in the $i$th tomographic interval

$$g^i(\chi) = \int_\chi^{\chi_h} d\chi' n^i(\chi') \frac{f_K(\chi' - \chi)}{f_K(\chi')} .$$

Since we chose five tomographic bins, the resulting data vector which enters the likelihood analysis consists of 15 tomographic shear power spectra, each with 20 logarithmically spaced bins ($l \in [30; 5000]$), hence 300 data points overall. In the following analysis we assume different redshift distributions (depending on the probe/survey considered), however we always choose five tomography bins with equal number densities in each $z$-bin.

3.2. Statistical Covariances

Under the assumption that the shear field is Gaussian (which means that the shear 4pt-function can be expressed in terms of 2pt-functions) the covariance of projected shear power spectra can be expressed as (Hu & Jain 2004)

$$\text{Cov}_{ij} \left( C^{ij}(l_1)C^{kl}(l_2) \right) = \langle \Delta C^{ij}(l_1) \Delta C^{kl}(l_2) \rangle = \frac{2\pi \delta_{l_1l_2}}{A_1A_1} \left[ C^{ik}(l_1)C^{jl}(l_1) + C^{il}(l_1)C^{jk}(l_1) \right] ,$$
with
\[ \bar{C}^{ij}(l_1) = C^{ij}(l_1) + \delta_i \delta_j \frac{\sigma^2}{n^2}, \] (24)
where the superscripts indicate the redshift bin and \( n^i \) is the density of source galaxies in the \( i^{th} \) redshift bin.

Since non-linear structure growth at late time induces significant non-Gaussianities in the shear field Eq. 23

\[
\text{Cov}_{\text{NG}}(C^{ij}(l_1), C^{kl}(l_2)) = \int_{|l_1| \leq l_1} \frac{d^2 l_1}{A(l_1)} \int_{|l_2| \leq l_2} \frac{d^2 l_2}{A(l_2)} \left[ \frac{1}{\Omega_m} T^{ijkl}_{\kappa,0}(1, -1, 1', -1') + T^{ijkl}_{\kappa,\text{HSV}}(1, -1, 1', -1') \right]. \tag{25}
\]
The convergence trispectrum \( T^{ijkl}_{\kappa,0} \) is, in the absence of finite volume effects, defined as
\[
T^{ijkl}_{\kappa,0}(1, l_2, l_3, l_4) = \left( \frac{3 H_0^2}{2 c^2 \omega_m} \right)^4 \int_0^{\chi_h} d\chi \left( \frac{\chi}{\bar{\rho}}(\chi) \right)^4 g^I g^J g^K g^L \times \chi^{-6} T_{\delta,0} \left( \frac{l_1}{\chi}, \frac{l_2}{\chi}, \frac{l_3}{\chi}, \frac{l_4}{\chi}, z(\chi) \right), \tag{26}
\]
with \( T_{\delta,0} \) the matter trispectrum (again, not including finite volume effects), and where we abbreviated \( g^I = g^I(\chi) \).

We model the matter trispectrum using the halo model (Seljak 2000; Cooray & Sheth 2002), which assumes that all matter is bound in virialized structures that are modeled as biased tracers of the density field. Within this model the statistics of the density field can be described by the dark matter distribution within halos on small scales, and is dominated by the clustering properties of halos and their abundance on large scales. In this model, the trispectrum splits into five terms describing the 4-point correlation within one halo (the one-halo term \( T^{1h} \)), between 2 to 4 halos (two-, three-, four-halo term), and a so-called halo sample variance term \( T_{\text{HSV}} \), caused by fluctuations in the number of massive halos within the survey area,
\[
T = T_0 + T_{\text{HSV}} = [T_{1h} + T_{2h} + T_{3h} + T_{4h}] + T_{\text{HSV}}. \tag{27}
\]
The two-halo term is split into two parts, representing correlations between two or three points in the first halo and two or one point in the second halo. As halos are the building blocks of the density field in the halo approach, we need to choose models for their internal structure, abundance and clustering in order to build a model for the trispectrum. Our implementation of the one-, two- and four-halo term contributions to the matter trispectrum follows Cooray & Hu (2001), and we neglect the three-halo term as it is subdominant compared to the other terms at the scales of interest for this analysis. Specifically, we assume NFW halo profiles (Navarro et al. 1997) with the Bullock et al. (2001) fitting formula for the halo mass–concentration relation \( c(M, z) \), and the Sheth & Tormen (1999) fit functions for the halo mass function \( \frac{dn}{dM} \) and linear halo bias \( b(M) \), neglecting terms involving higher order halo biasing.

Within the halo model framework, the halo sample variance term is described by the change of the number of massive halos within the survey area due to survey-scale density modes; following Sato et al. (2009) it is calculated as

\[
T^{ijkl}_{\kappa,\text{HSV}}(1, -1, 1', -1') = \left( \frac{3 H_0^2}{2 c^2 \omega_m} \right)^4 \int_0^{\chi_h} d\chi \frac{dV}{d\Omega}(\chi)^2 \left( \frac{\chi}{\bar{\rho}}(\chi) \right)^4 g^I g^J g^K g^L \times \chi^{-6} T_{\delta,0} \left( \frac{l_1}{\chi}, \frac{l_2}{\chi}, \frac{l_3}{\chi}, \frac{l_4}{\chi}, z(\chi) \right), \tag{28}
\]

4. SIMULATED LIKELIHOOD ANALYSES

\textsc{cosmolike} computes the analytic covariance and the data vector from a fiducial cosmology (see Table 1) as described in Sect. 3. We assume the covariance to be known, implying that it is fixed with respect to cosmological parameters. This choice can influence cosmological constraints (Eifler et al. 2009); however given that we sample a relatively limited parameter space, especially for our most important comparison (LSST optimistic vs. TF-Stage IV), we believe that it will not change our results qualitatively. We point out that data analyses from
the high precision Stage IV surveys require an improved handling of theoretical uncertainties (e.g., Krause & Hirata 2010); however since the data vector is created internally in COSMOlike we can exclude these terms in the data and model vector.

In the simulated analysis we sample a seven dimensional cosmological parameter space with flat priors at the boundaries of the parameter range (see Table 1). We compare four different surveys (see Table 2 for the exact parameters): two purely photometric surveys mimicking DES and LSST and two versions of the Tully Fisher Lensing surveys, TF-Stage III and TF-Stage IV. For LSST we additionally consider an optimistic and a conservative systematics scenario.

The design of the Tully Fisher Lensing surveys is detailed in Sect. 2; the number density of galaxies for TF-Stage III and TF-Stage IV (1.1/arcmin²) is limited by the number of spectra that can be acquired. The survey parameters that we assume in the analyses are summarized in Table 2. Please note that throughout

### Table 1

Fiducial cosmology and range of cosmological parameters used in the likelihood analyses

| Survey        | Ωₘ | σ₈ | nₐ | w₀ | wₐ | h₀ |
|---------------|----|----|----|----|----|----|
| Fiducial      | 0.315 | 0.829 | 0.9603 | -1.0 | 0.0 | 0.049 | 0.673 |
| Min           | 0.1 | 0.6 | 0.85 | -2.0 | -2.5 | 0.04 | 0.6 |
| Max           | 0.6 | 0.95 | 1.06 | 0.0 | 2.5 | 0.055 | 0.76 |

In a photometric survey, galaxies are grouped into tomographic bins by their photometric redshifts zₕ. To account for the degradation due to uncertainties in the photometric redshift estimates, we compute the true underlying redshift distribution nₜ(z) of galaxies in tomography bin zₜ₋₁ < z < zₜ₊₁ as

$$nₜ(z) = \int_{zₜ₋₁}^{zₜ₊₁} dzₕ nₕ(z)p(zₕ|z)$$

using a simple parameterization from Ma et al. (2006) to model p(zₕ|z), the distribution of photometric redshifts given true redshift,

$$p(zₕ|z) = \frac{1}{\sqrt{2\pi \sigma_z(z)}} \exp \left[ \frac{-(z - zₕ - zₜ + 1)}{2\sigma_z(z)^2} \right]$$

i.e. a Gaussian distribution with rms σₖ and offset zₕ from the true redshift (but see Hearin et al. (2010) for discussion of critical outliers). We assume photometric redshift estimates to be unbiased on average (⟨zₕ⟩ = z) and marginalize over the uncertainty of the width of the distribution Δσₜ and the uncertainty of the redshift bias Δzţ assuming Gaussian distributions with parameter values listed in Table 3.

Since the TF surveys require spectra from each galaxy we assume no error from redshift uncertainty for these.

#### 4.1.2. Shear Calibration Biases

In addition to photo-z uncertainties we consider multiplicative shear calibration bias in the analyses, which we implement as prefactors of the modeled shear power spectra, i.e. MᵢMᵢCᵢ(ℓ). The superscripts i, j correspond to the tomography bins. We model shear calibration uncertainties as a Gaussian PDF around a fiducial value of 1 and we further assume that the PDFs vary independently in each tomography bin; hence we use five additional parameters to model shear calibration (see Table 3 for parameter ranges). Shear calibration uncertainty affects

### Table 2

Survey parameters

| Survey        | area [deg²] | σₚ | nₐ | zₚmax | zₚmean | zₚmed |
|---------------|-------------|----|----|--------|---------|--------|
| TF-Stage III  | 5,000       | 0.021 | 1.1 | 1.68   | 0.90    | 0.73   |
| TF-Stage IV   | 15,000      | 0.021 | 1.1 | 3.85   | 1.09    | 0.84   |
| DES           | 5,000       | 0.26  | 10  | 2.0    | 0.84    | 0.63   |
| LSST          | 15,000      | 0.26  | 31  | 3.5    | 1.37    | 0.93   |

*Values taken from DES documents and internal communication within the DES collaboration.

bValues match specifications outlined in Chang et al. (2013).

The paper σₚ refers to the shape noise per component of the ellipticity.

### 4.1. Systematic Uncertainties

In addition to the seven cosmological parameters we consider up to seven parameters for photo-z and shear calibration uncertainties. Note that for LSST we consider two different scenarios, termed conservative and optimistic, which differ in the range of photo-z and shear calibration uncertainty prior. The LSST optimistic scenario assumes major breakthroughs in photo-z and shape measurement methods compared to the current state of the art, while the conservative scenario only assumes modest progress.

#### 4.1.1. Photometric Redshifts

In a photometric survey, galaxies are grouped into tomographic bins by their photometric redshifts zₕ. To account for the degradation due to uncertainties in the photometric redshift estimates, we compute the true underlying redshift distribution nₜ(z) of galaxies in tomography bin zₜ₋₁ < z < zₜ₊₁ as

$$nₜ(z) = \int_{zₜ₋₁}^{zₜ₊₁} dzₕ nₕ(z)p(zₕ|z)$$

using a simple parameterization from Ma et al. (2006) to model p(zₕ|z), the distribution of photometric redshifts given true redshift,

$$p(zₕ|z) = \frac{1}{\sqrt{2\pi \sigma_z(z)}} \exp \left[ \frac{-(z - zₕ - zₜ + 1)}{2\sigma_z(z)^2} \right]$$

i.e. a Gaussian distribution with rms σₖ and offset zₕ from the true redshift (but see Hearin et al. (2010) for discussion of critical outliers). We assume photometric redshift estimates to be unbiased on average (⟨zₕ⟩ = z) and marginalize over the uncertainty of the width of the distribution Δσₜ and the uncertainty of the redshift bias Δzₜ assuming Gaussian distributions with parameter values listed in Table 3.

Since the TF surveys require spectra from each galaxy we assume no error from redshift uncertainty for these.

#### 4.1.2. Shear Calibration Biases

In addition to photo-z uncertainties we consider multiplicative shear calibration bias in the analyses, which we implement as prefactors of the modeled shear power spectra, i.e. MᵢMᵢCᵢ(ℓ). The superscripts i, j correspond to the tomography bins. We model shear calibration uncertainties as a Gaussian PDF around a fiducial value of 1 and we further assume that the PDFs vary independently in each tomography bin; hence we use five additional parameters to model shear calibration (see Table 3 for parameter ranges). Shear calibration uncertainty affects

### Table 3

Systematic error uncertainty parameters

| Survey        | σₚ | Δσₚ | Δzₜ | ΔM |
|---------------|----|-----|-----|----|
| TF-Stage III  | -  | -   | -   | 0.0032 |
| TF-Stage IV   | -  | -   | -   | 0.0016 |
| DES           | 0.1(1 + z) | 0.1 × σₚ | 0.01 | 0.02 |
| LSST, conservative | 0.05(1 + z) | 0.01 | 0.01 | 0.01 |
| LSST, optimistic | 0.05(1 + z) | 0.002 | 0.003 | 0.002 |

*Values taken from DES documents and internal communication within the DES collaboration.

bValues match specifications outlined in Chang et al. (2013).

the paper σₚ refers to the shape noise per component of the ellipticity.

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both photometric and Tully Fisher Lensing surveys; however in the latter case our galaxy sample generally has significantly higher S/N (S/N ≥ 50). Predicting future progress in shear calibration performance is of course difficult; for current measurements, however, the dominant systematic calibration errors appear to arise from noise rectification bias, which scales as (S/N)² (Refregier et al. 2012). It seems safe to assume that calibration biases will
be reduced by limiting the measurement to bright, well-resolved galaxies, and so we adopt the aforementioned S/N scaling and assume a reduction in $\Delta M$ by a factor of 6.25 when going from DES/LSST to the TF experiments. We note that for TF-Stage IV we rescale the conservative LSST shear calibration uncertainty, not the optimistic one (see Table 3).

4.2 Details of the Analyses and Results

Given the data vector and the covariance COSMO-LIKE samples the parameter space using parallel MCMC (Goodman & Weare 2010) implemented through the EMCEE python package\(^{15}\). The computing time for the 300-dimensional model vector (including photo-z and multiplicative shear calibration) at each point in parameter space is \(\sim 1s\), which in combination with the parallel MCMC technique allows for an extremely fast sampling of the considered parameter space.

We assume a Multivariate Gaussian being the functional form of the likelihood \(L\); its width being solely determined by the covariance matrix

\[
L(D|p_{co}, p_{nu}) \sim \exp\left(-\frac{1}{2} \left[(D-M)^T C^{-1} (D-M)\right]\right),
\]

where \(p_{co}\) denotes the cosmological parameter vector, \(p_{nu}\) the nuisance parameter vector, \(D = D(p_{fid}, p_{nu})\) is the data vector consisting of the 300 $C(l)$ that are computed from the fiducial model, \(M = M(p_{co}, p_{nu})\) is the corresponding model vector at a given point in cosmological and nuisance parameter space, and \(C\) is the covariance described in Sect. 3.2.

We use Bayes theorem to compute the posterior probability

\[
P(p_{co}, p_{nu}|D) = \frac{P_{r}(p_{co}, p_{nu}) L(D|p_{co}, p_{nu})}{E(D)}
\]

with \(E\) being a normalization called evidence. We assuming flat prior probability \(P\), in the cosmological parameter space (see Table 1 for details) and Gaussian priors for our nuisance parameters (see Sect. 4.1). For the LSST and the Tully Fisher analyses the priors do not impact the contours at all; for the DES analysis the prior on \(u_0\) cuts off outer regions of the corresponding parameter space.

Constraints that are marginalized over nuisance parameters (or cosmological parameters that are not of interest) are calculated as

\[
L(D|p_{co}) = \int dp_{nu} \exp\left(-\frac{1}{2} \chi^2(p_{co}, p_{nu})\right).
\]

For the final runs of the simulated likelihood analyses we compute 420,000 steps in the MCMC and reject the first 10,000 steps as a burn-in phase. We also run several shorter chains to check for convergence.

We produce a compressed summary of the different experiments by computing a measure of the cosmological information content equal to \(|\Xi|^{-\frac{1}{2}},\) where \(n\) is the number of cosmological parameters of interest, and \(\Xi\) is the covariance matrix of the MCMC outputs

\[
\Xi_{ij} = \text{cov}(p_{coi}, p_{coj})\,.
\]

This information measure corresponds roughly to the geometric average of the constraints on the \(p_{co}\), or the square of the size of the ball in parameter hyper-space enclosing the \(1 - \sigma\) likelihood surface; it is worth noting that this particular measure of experimental merit is insensitive to the number of parameters. Table 5 shows the ratio of this quantity for each of the four surveys considered here to that of the Dark Energy Survey.

For better illustration we also show two-dimensional contour plots for the most interesting cases, i.e. TF-Stage III and TF-Stage IV vs. LSST conservative and optimistic (Fig. 4). These 95% confidence regions are marginalized over all other cosmological parameters (five) and nuisance parameters (five and seven for TF and LSST, respectively).

4.3 Discussion

This analysis makes a number of conservative assumptions which favor photometric weak lensing measurements. We do not include any intrinsic alignment contamination in the LSST or DES cosmic shear forecasts, nor do we allow for the possibility of catastrophic photometric redshift errors. Neither of these effects are present for the TF survey concepts, though both are important limitations of traditional methods (Hirata & Seljak 2004; Hearn et al. 2010). We also assume that the shear calibration biases scale as \((S/N)^2\), despite claims that noise rectification bias, which appear to be the dominant source of shear calibration problems for many existing shape measurement methods, can be removed at this order (Kacprzak et al. 2012). Allowing for a higher order calibration-S/N scaling would further enhance the cosmological information content of the TF surveys. Finally, we have made no attempt to optimize the extraction of 3D lensing information. It is likely that a tomographic analysis using additional redshift bins would further improve the power of the TF-Stage III and TF-Stage IV analyses.

Nevertheless, Fig. 4 and Table 5 show that the TF-Stage III experiment – which would require only an overlapping DESI-like spectrograph and a DES-like imaging survey – is comparable in constraining power to our optimistic LSST forecasts. The TF-Stage IV experiment provides constraining power well in excess of any other optical ground-based lensing measurement and offers a way to break through the information ceiling set for traditional lensing experiments by the surface density of galaxies suitable for shape measurement.

For a better understanding of the individual error contributions we show the correlation matrices of the LSST survey (left) and the TF-Stage III survey (right) in Fig. 5. As described in Sect. 3.2 our covariance consists of shape noise, cosmic variance (including higher order terms), and halo sample variance. Shape noise and second order cosmic variance act on the diagonal and secondary diagonal only, while halo sample variance and higher order cosmic variance act on all elements of the covariance.

For the LSST survey one can see that the larger shape

\(^{15}\) http://dan.iel.fm/emcee/
Figure 4. Results of the simulated likelihood analyses. We show the 95% confidence regions for the TF-Stage III survey (black, solid), the TF-Stage IV survey (red, dashed) in comparison with the LSST-optimistic (green, dotted) and pessimistic scenario (blue, dotted-dashed). We marginalize over shear calibration and (for LSST only) photometric redshift systematic errors.

This dominance permits an analysis with narrower tomographic bins, which is an extremely powerful tool to explore time-dependent signatures in the dark energy phenomenon and separately constrain expansion history and structure growth.

Regarding the robustness of our TF constraints we point out that an error of the disk circular velocity of 13 km/s is likely a conservative assumption. If instead we assume an error of 10 km/s, which is reasonably achievable with today’s instruments already, we can tolerate a ~35% percent failure rate in obtaining the required galaxy spectra while still achieving the constraints shown in Fig. 4.

5. CONCLUSIONS

In this paper we have presented a new method to extract cosmological information from Weak Gravitational Lensing. Using the well-established Tully-Fisher scal-
Figure 5. Correlation plots of the LSST covariance (left) and the TF covariance (right). The data vector consists of 15 tomography power spectra, each with 20 l-bins, i.e. 300 data points altogether. Shape noise only acts on the main and secondary diagonals; see text for further explanation.

Table 4
Marginalized One-Parameter Constraints

| Survey               | \( \Omega_m \) | \( \sigma_8 \) | \( n_s \) | \( w_0 \) | \( \omega_a \) | \( H_0 \) |
|---------------------|----------------|---------------|----------|----------|---------------|---------|
| LSST-optimistic     | 0.016          | 0.015         | 0.0097   | 0.14     | 0.52          | 0.030   |
| LSST-conservative   | 0.020          | 0.018         | 0.012    | 0.25     | 0.90          | 0.036   |
| TF-Stage III        | 0.0011         | 0.0097        | 0.012    | 0.14     | 0.50          | 0.037   |
| TF-Stage IV         | 0.0064         | 0.0056        | 0.0065   | 0.073    | 0.25          | 0.026   |

Table 5
Cosmological Information Content relative to DES

| Survey      | Information Content (relative to DES) |
|-------------|---------------------------------------|
| LSST-pessimistic | 2.39                                  |
| LSST-optimistic    | 3.31                                  |
| TF-Stage III      | 3.60                                  |
| TF-Stage IV       | 7.10                                  |

The limitation of our method clearly is the need for spectroscopic information and hence the limited number of galaxies that can be observed spectroscopically within a given time interval. As a result the model surveys we present in this paper (TF-Stage III and TF-Stage IV) have an average number density of galaxies of 1.1/arcmin², however they are not affected by photo-z uncertainty and only little by shear calibration errors. For our model surveys we adopt the DETF “Stage X” terminology in the sense that TF-Stage III covers 5,000deg² (similar to DES), and TF-Stage IV covers 15,000deg² (similar to LSST); also we assume a similar improvement in survey depth when going from Stage III to Stage IV.

Using the cosmic shear module of CosmoLike we run various simulated cosmic shear tomography likelihood analyses, in a multi-dimensional parameter space (seven cosmological parameters, and five and two parameters for shear calibration and photo-z errors, respectively). These simulated analyses include full Non-Gaussian covariances and a realistic sampling of the parameter space which is a major improvement over Fisher forecasts.

Our main findings are that already TF-Stage III is competitive with LSST, depending on the assumptions of how strongly nuisance parameters impact the LSST constraints; TF-Stage IV clearly outperforms even the optimistic scenarios for LSST. In this context we mention all LSST analyses assume zero intrinsic alignment contamination, which potentially is of similar importance as photo-z and shear calibration uncertainties.

It is however important to note that any TF-survey obviously relies on overlapping photometric and spectroscopic data, hence the main intention of this comparison is to strongly advocate a spectroscopic survey.
Cosmic Shear Without Shape Noise

overlapping with LSST. The interesting prospect of this overlap is not just a TF-Stage IV survey but a combination of TF-Stage IV and LSST. Galaxies without spectra will substantially contribute to the constraints, especially since the overlap with spectroscopic data allows for improved photo-z and shear calibration and IA mitigation schemes.

Another interesting prospect is the design of an optimal TF-lensing tomographic survey. The small shape noise and the accurate redshift information allows for substantially more tomographic bins and hence for a precise measurement of expansion history vs structure growth. We point out that the TF-lensing method presented in this paper can be applied to cluster lensing, galaxy-galaxy lensing and other cross-lensing probes, thereby overcoming possible limitations of these probes due to shape noise.

Using the TF method presented in this paper cosmic shear for the first time is no longer fundamentally limited by shape noise errors and systematics associated with it but by the instrumental capabilities of multi-object spectrographs.

ACKNOWLEDGMENTS

We thank Klaus Honscheid, David Weinberg, Rachel Mandelbaum, Chris Hirata, Bhuvnesh Jain, and Peter Schneider for very useful discussions and advice. We also thank the Center for Cosmology and Astroparticle Physics at the Ohio State University and the Center for Particle Cosmology at the University of Pennsylvania for hosting us during critical phases of the paper. This paper is based upon work supported in part by the National Science Foundation under Grant No. 1066293 and the hospitality of the Aspen Center for Physics. The research of TE and EK was funded in part by NSF grant AST 0908027 and U. S. Department of Energy grant DE-FG02-95ER40893.

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