Modeling and analyzing of nonlinear dynamics for linear guide slide platform considering assembly error

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Abstract It is industry tendency to accurately predict the dynamics of the mechanical systems with full consideration of errors. Based on the Hertz contact theory and general bearing modeling methods, this study proposed a more practical model using numerical method to investigate the influence of assembly error on the dynamics of linear guide slide platform. First, the modeling methods of five types of assembly errors are established, based on which, a nonlinear dynamic model is developed to investigate the influence of assembly error. In consideration of assembly error, the modeling method enables the sum of restoring forces and restoring moments equal to zero when no external load applied to the platform. Second, the simulation results indicate that the assembly error can cause uneven load distribution, change the dynamics of the system. In addition, different from previous research results, the stability of the system cannot be improved by simply increasing the preload. Last, in order to validate the proposed method, the proposed model is compared with previous fewer degrees-of-freedom model, and a series of experiments are conducted on a specialized platform to estimate the parameters of the system and verify the proposed model.

Keywords Linear guideway · Assembly error · Nonlinear dynamics · Dynamics modeling

Abbreviations

$\Delta \delta^x_i$, $\Delta \delta^y_i$ Straightness assembly errors of rail II in $x$ and $y$ directions

$\Delta \epsilon_x$, $\Delta \epsilon_y$, $\Delta \epsilon_z$ Rotation assembly errors of rail II about $x$, $y$ and $z$ axis

$\Delta x^a_{ijk}$, $\Delta y^a_{ijk}$ Type $a$ assembly error induced contact deformation along $x$ and $y$ axis

$\Delta x^b_{ijk}$, $\Delta y^b_{ijk}$ Type $b$ assembly error induced contact deformation along $x$ and $y$ axis

$\Delta x^c_{ijk}$, $\Delta y^c_{ijk}$ Type $c$ assembly error induced contact deformation along $x$ and $y$ axis

$\Delta x^d_{ijk}$, $\Delta y^d_{ijk}$ Type $d$ assembly error induced contact deformation along $x$ and $y$ axis

$\Delta x^e_{ijk}$, $\Delta y^e_{ijk}$ Type $e$ assembly error induced contact deformation along $x$ and $y$ axis

$l_k$ Distance between the $k^{th}$ and the first loaded ball in the grooves of each carriage

$S^c_0$ Initial distance between the groove curvature centers with consideration of assembly error

$r_{cg}$, $r_{rg}$ The groove radius of carriage and rail
The initial contact angle with consideration of assembly error

The contact deformation of the kth ball in the jth groove of the ith carriage

The contact force of the kth ball in the jth groove of the ith carriage

The contact angle of the kth ball in the jth groove of the ith carriage

The contact deformation of the kth ball along x and y directions under external load

The displacement of the platform along x and y axis under external load

The distance between grooves along x and y directions

The distance between carriages along x and y directions

The angular displacement about x, y and z axis

Total restoring force of platform along x, and y axis

Total restoring moment of platform about x, y and z axis

Moment of inertia of platform about x, y and z axis

Width, length, and height of platform

Subscripts

i  ith rail
j  jth carriage
k  kth ball
x  x Axis
y  y Axis
z  z Axis
a  Type a assembly error
b  Type b assembly error
c  Type c assembly error
d  Type d assembly error
e  Type e assembly error

1 Introduction

Linear guide slide platform can be used in machine tool system such as 3D printers, sliding doors and automation settings; the dynamic behavior of linear guide slide platform can seriously influence the manufacturing performance of machine tool. As a vital part of machine tools system, due to low friction, high stiffness, long service life, high precision, excellent dynamic response, run smoothly and easy to install, the linear guide can be used with high vibration requirement [1–3]. The positioning performance of machine tool can be greatly constrained by the errors caused by cutting force and temperature changes [4]. Not just temperature and cutting force, assembly error can also cause geometric deviations. It is well known that the linear guide platform assembly error can lead groove wear, increase clearances, and reduce the positioning accuracy of the system [5]. In addition, due to the factors such as human error and routine maintenance, the assembly error is inevitable. However, the influence of the linear guide platform assembly error on the dynamic characteristic is not well understood, and the factors considered in the previous studies are not comprehensive enough. In order to fill this gap, this study focuses on modeling of linear guide slide platform with assembly error and presents discussion results.

Lots of researches focus on the formulation of force–deflection relationship of linear guideway to investigate the static and dynamic characteristics, and some aspects have been well studied. Xu et al. [6] developed a five degrees-of-freedom dynamic model to analyze the dynamics of linear guideway. It found that super harmonic resonance can be easier induced by larger excitation amplitude, and the dynamics of the linear guide are more sensitive to the corresponding mass. The modeling method was comprehensive, but the assembly error was not mentioned in this literature. To accurately investigate the dynamic characteristics of linear guide, Kong et al. [7] proposed a piecewise-nonlinear model, studied the simulation results based on the multi-term incremental harmonic balance method, it revealed that higher order nonlinearities were more responsive to the dynamic behavior than lower-order nonlinearities. Nevertheless, only the force–deflection relationship in normal direction was presented. Wu et al. [8] studied the dynamic characteristics of rolling guideway when there exist external moment applied to the carriage block, the study revealed that the modal frequencies were related to the corresponding motion of freedom. Ohta et al. [9] studied the influence of grease characteristics on vibration and sound characteristics, and they found that the absolute viscosity of grease was highly relevant to the amplitude of sound and vibration and decreased with them. Liu et al. [10] proposed an
analytical model of ball screw feed system with consideration of assembly error; the study found that the assembly error could have impacts on the dynamic responses of the system. However, this study did not take the sum of restoring force $\sum F = 0$ and the sum of restoring moment $\sum M = 0$ into consideration when no external load applied to the system. Wang et al. [11] developed a time-varying model to study the dynamic behavior of machine tool feed system. The phenomenon of coupling effect such as jump discontinuity, hardening and softening type nonlinearity had been observed. Beyond that, the coupling vibrations could not only influence the vibration amplitude but also the motion state of the system. However, the proposed model neglects the yawing motion of the platform. Jiang et al. [12] derived a five degrees-of-freedom deformation equation, based on which, the vertical, rolling, pitching, and yawing stiffness of linear guide could be obtained. However, the method was presented without experimental validation. Xu et al. [13] developed a 18 degrees-of-freedom dynamic model of ball screw feed system. The model considered the load deformation relationship of each component as a series of mass–spring–damper systems. Parivash et al. [14] proposed a coupled tribology-dynamic model of linear guideways with consideration of the influence of the lubricant and the coupling between vertical degrees and horizontal directions, and the results showed that it was necessary to consider the coupled solution of vertical and horizontal directions into the lubricated contacts. Notwithstanding, only normal force of linear guide was considered. Pawelko et al. [15] proposed a new methodology and method to model the preloaded rolling guides, and various degrees of simplification of the modeling methods were studied. Yang et al. [16] developed a hybrid model of linear rolling guideway with considering the flexibility of the block, and the complex loading conditions in the vertical and horizontal directions are considered. Yasunori and Tanakab [17] studied the relationship between nonlinear friction and vibration of linear rolling guideway, and the influence of lubricant specifications on the damping ratio was investigated. Li et al. [18] proposed a time-varying model with consideration of crowning of rolling linear guide, and the ball passing frequency was predicted. In addition, the proposed dynamic model was validated by an experiment. Wang et al. [19] developed a three degrees-of-freedom nonlinear dynamic model to investigate the vibration characteristic of machine tool platform mounted on linear guideways. The model considered the mechanics and contact characteristic of the guideways, but only three degrees-of-freedom were considered. Wang et al. [20] investigated the relationship between vibration and groove wear of linear guideways and analyzed the failure mechanism of rolling linear guide, but the rolling and yawing motion of the table were neglected. Ohta and Tanaka [21] developed a static model of linear guideway with consideration of the flexibility of the carriage and rail, and the calculated vertical stiffness obtained from the theoretical formulations was compared with the measured vertical results. By using experiment and simulation method, researchers focus on the dynamic response of the carriage block under various factors. Hung et al. [22] developed a finite element model to study the dynamic behaviors and cutting stability of a vertical milling system. The model used Hertzian contact theory for coupling the linear components and the machine structures.

To reduce the ball passage vibration of a linear guideway, Ohta et al. [23] proposed a method of designing crowning, the method used approximate curve to control point. It found that the vibration of ball passage could be decreased by changing the tilting and normal stiffness. Sun et al. [24] presented a 5-degrees-of-freedom static model of linear roller bearing, the structure deformation of the carriage was calculated by finite element method, and the developed model was verified by comparing with the calculation results obtained by commercial software. Sun et al. [25] proposed an analytical model using hertz contact theory, on the base of which, a full finite element model was developed, and the influence of external load and preload on the static characteristics was analyzed. Considering the contact stiffness of the interface between ball and groove, Lin et al. [26] proposed a finite element model to study the dynamic stiffness of the spindle head. The results from simulation and experimental showed that the dynamic stiffness increased with the preload of the linear guide. Zou and Wang [27] proposed a contact stiffness model which took wear and thermal deformation of linear guideway into account, and an experiment on a specialized linear guide was conducted to validate the model. Ohta [28] investigated the relationship between sound pressure level and linear velocity and found that the main frequency of vibration response...
was caused by the rolling vibration, pitching vibration and vertical vibration of the carriage. Tao et al. [29] proposed a calculation model of stiffness of carriage based on Archard wear theory, the amount of wear of the grooves of linear guideway was predicted, and a series of experiments were conducted to verify the simulation results.

The above-mentioned studies focused on the modeling and understanding the dynamic behaviors with different system parameters through experiments and model-based methods. Besides, the modeling and analysis of mechanical systems errors had been also reported by researchers. Liu et al. [30] using homogeneous coordinate transformation to model the position deviation of machine tool table system, and the dynamic response of the system was obtained by the integration of Newmark and Monte Carlo method. However, the test did not consider the effect of position deviations. Pawel [31] proposed a new method to model the geometric deviations of machine tool table, and the influence of the geometric errors on joint kinematic errors was discussed. Ma et al. [32] developed an analytical model with consideration of the assembly errors of linear axis of CNC machine tool; the straightness and rotation deviations were calculated under four different conditions. Sun et al. [33] proposed a model to establish the relationship between assembly error and positioning accuracy, and the influence of volumetric errors of linear guideways on repeatability of positioning accuracy based on genetic algorithm was studied. To enhance the assembly performance of machine center, Sun [34] proposed a method based on computer aid engineering analysis. Mayer and Cloutier [35] proposed an analytical model and conducted a series of experiments to study the relations between the geometric error of linear guideway and the motion error of carriage.

Concluding from the studies, previous researches concerning the modeling of linear guide and discussing the influence of variety of parameters, fewer studies propose a comprehensive model and give modeling methods of assembly errors. On one hand, since human error and routine maintenance can lead geometric error, which is usually inevitable. On the other hand, it is tendency to accurate predict the vibration response of the system with consideration of the variation of working parameters. This study depicts the five types of assembly errors as the changes between curvature centers in position, based on which, the corresponding expression of initial contact angular and initial contact deformation of each ball can be obtained. Since the system can be stationary in the absence of external load, the modeling guideline of this study is that the sum of the restoring force and the sum of the restoring moment equals to zero on the initial condition. Based on the mentioned modeling method and guideline, the influence of the assembly errors has been studied in the following section. The outline of this paper is as follows: after the introduction in Sect. 1, Sect. 2 modeled five types of assembly errors and proposed a 5-degrees-of-freedom dynamic model of linear guide slide platform. Section 3 revealed the combined influence of assembly error and system parameters on the dynamic and static characteristic of the system. In addition, a series of experiments had been conducted to validate the proposed model and estimate the parameter of system.

2 Dynamic model and governing equations of motion

Figure 1 shows the schematic of linear slide guide platform capable of vibrations in five directions; a typical linear guide slide platform consists of four carriages, a platform and two rails. O represents the geometric center of the platform when there exist no assembly error, and O’ is the new geometric center of the platform under the action of assembly error. The time-varying external force \( F_x \) and \( F_y \) are applied to the geometric center of the platform along x and y axis. Since the platform can move along z axis, the external force along z axis is neglect. Besides, there exist three external moments (\( M_x, M_y, M_z \)) acting on the platform about x, y and z axis, and the assembly errors of the platform are caused by the relative displacement between bases due to the rails are mounted on the bases. With the aim of predicting the dynamic response of linear guideway slide platform with assembly error, a practical model is developed. As shown in Fig. 4, five modes of vibrations including \( x, y, \varphi_x, \varphi_y, \varphi_z \) are considered. The axial stiffness (stiffness of z axis) of the platform is usually provided by ball screw feed system; thus, the axial modes of the platform is neglect. In order to investigate the effect of assembly error of linear guide slide platform, a
5-degrees-of-freedom dynamic model is proposed in this study. The assembly errors are divided into five types, and the modeling methods of each type of assembly error are established.

2.1 Modeling of assembly error of linear guide slide platform

In this subsection, the assembly errors of linear guide platform are modeled as five types, namely, translation displacements along $x$ and $y$ axis and angular displacement about $x$, $y$ and $z$ axis. As can be shown in Fig. 2, Rail I is fixed by default, the assembly error of the platform is caused by the deviation of Rail II. The schematic of error-deformation relationship of balls for each type of assembly error can be shown in Figure. Due to the assembly error induced movement of the base where the rails mounted on, the assembly errors of rails are modeled as five types including: straightness error along $x$ and $y$ axis (Type $a$ and Type $b$), and rotation error about $x$, $y$ and $z$ axis (Type $c$, Type $d$ and Type $e$). It is noteworthy that the deviation of Rail II along $z$ axis is neglect in Fig. 2, this is because the dynamic and static simulations are carried out based on the state of stress equilibrium of the system. In other words, under the action of any types of assembly error and no external applied on the platform, the sum of the restoring force and the sum of the restoring moment of the system are equal to zero. Since the main goal of this study is to investigate the dynamics of linear guide slide platform, both the rails and the carriages are considered to be rigid. The method of modeling five types of assembly error is achieved by calculating the distance between the curvature center of grooves.

2.1.1 Modeling of type $a$ error

As shown in Fig. 2a, type $a$ assembly error is caused by the rotation of rails around $x$ axis. To facilitate modeling, the contact deformation of balls are decomposed into $x$ and $y$ axis. It can be observed in Fig. 2a, type $a$ assembly error only influences the deformation along $y$ direction. Therefore, the assembly error-induced contact deformation along $x$ axis satisfies

$$
\Delta x_{ijk} = 0
$$

Since the contact deformations of balls of different carriages are not equal to each other. According to the position of each ball, and the contact deformation along $y$ axis of the $k$th ball in $j$th groove of $i$th carriage can be calculated as follows

$$
\Delta y_{ijk} = (-1)^j \left\{ \begin{array}{ll}
\Delta \phi_x (0.5l_z + l_k) & j = 1, 2 \\
-\Delta \phi_x (0.5l_z + l_k) & j = 3, 4 
\end{array} \right.
$$

where $\Delta \phi_x$ represents the rotational error of rail II around $x$ axis. To describe the position of each ball, the notation $l_k$ is introduced to represent the distance between the $k$th ball and the geometric center along the $x$ axis in a carriage, and the expression can be given by

$$
l_k = -0.5N_br + r_b(k - 1)
$$
2.1.2 Modeling of type b error

As shown in Fig. 2b, the type b assembly error is caused by the rails rotating around y axis. Different from type a, this kind of assembly error cannot influence the contact deformation of balls around y axis. Therefore, for type b, the assembly error-induced contact deformation along y axis is equal to zero

$$\Delta y_{ijk}^b = 0$$

and the position related and assembly error-induced contact deformation along x axis of the kth ball in jth groove of ith carriage can be calculated by

$$\Delta x_{ijk}^a = (-1)^{i+j+1} \left\{ \begin{array}{ll} \Delta \varepsilon_{ij}(0.5l_z + l_k) & j = 1, 3 \\ -\Delta \varepsilon_{ij}(0.5l_z + l_k) & j = 2, 4 \end{array} \right.$$  (5)

where $\Delta \varepsilon_{ij}$ represents the rotational assembly error of rail II around y axis.

2.1.3 Modeling of type c error

Type c assembly error is caused by simple static translation displacement of rails, and the schematic can be shown in Fig. 2c. The contact deformation of balls along x axis cannot be influenced by type c assembly error, and the assembly error-induced contact deformation along y axis is zero

$$\Delta y_{ijk}^c = 0$$  (6)
For carriage 1 and carriage 4, the contact deformation of the kth ball in jth groove caused by type c assembly error along x axis can be given by

\[
\Delta x_{ijk} = \begin{cases} 
\Delta \delta_{i}^c & i = 1, 4, j = 2, 4 \\
-\Delta \delta_{i}^c & i = 1, 4, j = 1, 3 
\end{cases}
\]

(7)

and for carriage 2 and carriage 3, the deformation is

\[
\Delta x_{ijk} = \begin{cases} 
-\Delta \delta_{i}^c & i = 2, 3, j = 2, 4 \\
\Delta \delta_{i}^c & i = 2, 3, j = 1, 3 
\end{cases}
\]

(8)

where \( \Delta \delta_{i}^c \) represents the straightness error of rail II along x axis.

### 2.1.4 Modeling of type d error

Type d assembly error is attribution to the rotation of rails around its central axis z. The schematic of type d assembly error is shown in Fig. 2d. It can be observed in the figure, and the distance \( l_x \) between rails remains unchanged. Unlike previous cases, this kind of assembly error can both change each ball preload along x and y axis. The error-deformation relationship of balls can be shown in Fig. 3, and the modeling methods of type d can be obtained. According to the geometric relationship shown in Fig. 2e, the incline angle \( \theta \) can be calculated as follows:

\[
\theta = \arctan \frac{\Delta \delta_{i}^c}{l_x}
\]

(13)

As can be seen in Fig. 2e, the assembly error-induced deformation along y axis is

\[
\Delta y_{ijk} = 0
\]

(14)
In carriage 1 and carriage 4, the contact deformation of the \(k\)th ball caused by type \(e\) assembly error can be given by
\[
\Delta x_{ijk} = \begin{cases} 
-\Delta \delta_y^e \sin \theta & i = 1, 4, j = 1, 3 \\
\Delta \delta_y^e \sin \theta & i = 1, 4, j = 2, 4 
\end{cases}
\]
(15)
in carriage 2 and carriage 3, the contact deformation of the \(k\)th ball caused by type \(e\) assembly error can be given by
\[
\Delta x_{ijk} = \begin{cases} 
\Delta \delta_y^e \sin \theta & i = 2, 3, j = 1, 3 \\
-\Delta \delta_y^e \sin \theta & i = 2, 3, j = 2, 4 
\end{cases}
\]
(16)
where \(\Delta \delta_y^e\) represents the straightness error of rail II along \(y\) axis. Therefore, five types of assembly errors have been modeled in above, and the contact deformation along \(x\) and \(y\) axis caused by any mentioned type of assembly error can be expressed by
\[
\begin{align*}
\Delta x_{ijk} &= \Delta x_{ijk}^e + \Delta x_{ijk}^b + \Delta x_{ijk}^c + \Delta x_{ijk}^d + \Delta x_{ijk}^e \\
\Delta y_{ijk} &= \Delta y_{ijk}^e + \Delta y_{ijk}^b + \Delta y_{ijk}^c + \Delta y_{ijk}^d + \Delta y_{ijk}^e
\end{align*}
\]
(17)
In order to calculate the initial contact deformation, the effect of assembly error is considered into pre-deformation of each ball. According to the classic bearing theory [13], assembly error induced pre-deformation of the \(k\)th ball in \(j\)th groove of \(i\)th carriage can be given by
\[
\delta_0^e = 0.5 \left( \sqrt{\Delta x_{ijk}^e^2 + \Delta y_{ijk}^e^2} - S_0 + 2\delta_0 \right)
\]
(18)
where \(\delta_0\) represents the contact deformation caused by initial preload. For the \(k\)th ball in \(j\)th groove of \(i\)th carriage, the initial distance between the groove curvature centers of carriage and rail can be expressed by
\[
S_0^e = r_b + r_g - 2r_b + 2\delta_0
\]
(19)
where \(r_b\) is the diameter of ball, \(r_g\) and \(r_r\) represent the groove curvature diameter of carriage and rail, and the expression of initial contact angle is
\[
x_0^e = \arctan \frac{\Delta y_{ijk}}{\Delta x_{ijk}}
\]
(20)
\[\text{2.2 Restoring force of linear guide slide platform}\]
In Sect. 2.1, the expression for the pre-deformation of each ball was given, and the initial distance between groove curvature centers and the contact angle of each ball is derived. In this section, the expressions of assembly errors will be used to calculate the restoring force and restoring moment of the linear guide slide platform.

In this study, external loads in five directions lead five corresponding displacements: namely, \(x, y, \varphi_x, \varphi_y\), and \(\varphi_z\). The spatial contact deformation can be decomposed into \(x\) and \(y\) directions. According to geometric relationship shown in Fig. 1, among them, translational displacement \(x\) and angular displacement \(\varphi_x\), have influence on the deformation along \(x\) direction, translational displacement \(y\) and angular displacement \(\varphi_y\), have influence on the deformation along \(y\) direction. The size of deformations is determined by the spatial position of ball. In addition, the effect of assembly errors is considered into \(S_0^e\) and unloaded initial contact angle \(x_0^e\); here, \(S_0^e\) is the unloaded initial distance between the rail groove and carriage groove curvature centers. As shown in Eq. 21–36, due to the difference in ball spatial position, the sign of some part of the formula can be different. The contact deformation of the \(k\)th ball in each groove of Carriage 1 along \(x\) and \(y\) axis can be given by
\[
\begin{align*}
\Delta x_{11k} &= x + \varphi_x (0.5l_c + l_k) + S_0^e \cos x_0^e \\
\Delta y_{11k} &= y - \varphi_y (0.5l_c + l_k) + 0.5\varphi_z (l_e - L_e) + S_0^e \sin x_0^e
\end{align*}
\]
(21)
\[
\begin{align*}
\Delta x_{12k} &= -x - \varphi_x (0.5l_c + l_k) + S_0^e \cos x_0^e \\
\Delta y_{12k} &= y - \varphi_y (0.5l_c + l_k) + 0.5\varphi_z (l_e + L_e) + S_0^e \sin x_0^e
\end{align*}
\]
(22)
\[
\begin{align*}
\Delta x_{13k} &= x + \varphi_x (0.5l_c + l_k) + S_0^e \cos x_0^e \\
\Delta y_{13k} &= -y + \varphi_y (0.5l_c + l_k) - 0.5\varphi_z (l_e - L_e) + S_0^e \sin x_0^e
\end{align*}
\]
(23)
\[
\begin{align*}
\Delta x_{14k} &= -x - \varphi_x (0.5l_c + l_k) + S_0^e \cos x_0^e \\
\Delta y_{14k} &= -y + \varphi_y (0.5l_c + l_k) - 0.5\varphi_z (l_e + L_e) + S_0^e \sin x_0^e
\end{align*}
\]
(24)
For Carriage 2 is

\[
\begin{align*}
\Delta x_{1k} &= x - \varphi_x(0.5l + l_k) + S_0^x \cos \varphi_x \\
\Delta y_{1k} &= y - \varphi_y(0.5l + l_k) - 0.5\varphi_z(l_k + L_a) + S_0^y \sin \varphi_x
\end{align*}
\]  

(25)

\[
\begin{align*}
\Delta x_{2k} &= -x - \varphi_x(0.5l + l_k) + S_0^x \cos \varphi_x \\
\Delta y_{2k} &= y - \varphi_y(0.5l + l_k) - 0.5\varphi_z(l_k - L_a) + S_0^y \sin \varphi_x
\end{align*}
\]  

(26)

\[
\begin{align*}
\Delta x_{3k} &= x + \varphi_x(0.5l + l_k) + S_0^x \cos \varphi_x \\
\Delta y_{3k} &= y + \varphi_y(0.5l + l_k) + 0.5\varphi_z(l_k + L_a) + S_0^y \sin \varphi_x
\end{align*}
\]  

(27)

\[
\begin{align*}
\Delta x_{4k} &= -x - \varphi_x(0.5l + l_k) + S_0^x \cos \varphi_x \\
\Delta y_{4k} &= y + \varphi_y(0.5l + l_k) + 0.5\varphi_z(l_k - L_a) + S_0^y \sin \varphi_x
\end{align*}
\]  

(28)

For Carriage 3, the contact deformation of the kth ball along x and y axis can be calculated, respectively

\[
\begin{align*}
\Delta x_{1k} &= x - \varphi_x(0.5l + l_k) + S_0^x \cos \varphi_x \\
\Delta y_{1k} &= y + \varphi_y(0.5l + l_k) - 0.5\varphi_z(l_k + L_a) + S_0^y \sin \varphi_x
\end{align*}
\]  

(29)

\[
\begin{align*}
\Delta x_{2k} &= -x - \varphi_x(0.5l + l_k) + S_0^x \cos \varphi_x \\
\Delta y_{2k} &= y + \varphi_y(0.5l + l_k) - 0.5\varphi_z(l_k - L_a) + S_0^y \sin \varphi_x
\end{align*}
\]  

(30)

\[
\begin{align*}
\Delta x_{3k} &= x + \varphi_x(0.5l + l_k) + S_0^x \cos \varphi_x \\
\Delta y_{3k} &= y + \varphi_y(0.5l + l_k) + 0.5\varphi_z(l_k + L_a) + S_0^y \sin \varphi_x
\end{align*}
\]  

(31)

\[
\begin{align*}
\Delta x_{4k} &= -x - \varphi_x(0.5l + l_k) + S_0^x \cos \varphi_x \\
\Delta y_{4k} &= y + \varphi_y(0.5l + l_k) + 0.5\varphi_z(l_k - L_a) + S_0^y \sin \varphi_x
\end{align*}
\]  

(32)

And for Carriage 4, the expression of the kth ball along x and y axis is as follows

\[
\begin{align*}
\Delta x_{1k} &= x - \varphi_x(0.5l + l_k) + S_0^x \cos \varphi_x \\
\Delta y_{1k} &= y + \varphi_y(0.5l + l_k) + 0.5\varphi_z(l_k + L_a) + S_0^y \sin \varphi_x
\end{align*}
\]  

(33)

\[
\begin{align*}
\Delta x_{2k} &= -x - \varphi_x(0.5l + l_k) + S_0^x \cos \varphi_x \\
\Delta y_{2k} &= y + \varphi_y(0.5l + l_k) + 0.5\varphi_z(l_k - L_a) + S_0^y \sin \varphi_x
\end{align*}
\]  

(34)

\[
\begin{align*}
\Delta x_{3k} &= x + \varphi_x(0.5l + l_k) + S_0^x \cos \varphi_x \\
\Delta y_{3k} &= y + \varphi_y(0.5l + l_k) - 0.5\varphi_z(l_k + L_a) + S_0^y \sin \varphi_x
\end{align*}
\]  

(35)

\[
\begin{align*}
\Delta x_{4k} &= -x - \varphi_x(0.5l + l_k) + S_0^x \cos \varphi_x \\
\Delta y_{4k} &= y + \varphi_y(0.5l + l_k) - 0.5\varphi_z(l_k - L_a) + S_0^y \sin \varphi_x
\end{align*}
\]  

(36)

where \(x, y\) represent the displacement of the geometric center along x and y axis, and \(\varphi_x, \varphi_y\) and \(\varphi_z\) represent the angular displacement about x, y and z axis. \(L_a\) is the distance between grooves in a carriage along x axis, which can be shown in Fig. 1. In order to obtain external load related restoring force and restoring moment, the contact deformation of the kth ball in jth groove of ith carriage under external load can be calculated by

\[
\delta_{ijk} = 0.5 \left( \sqrt{\Delta x_{ijk}^2 + \Delta y_{ijk}^2 - S_0^\varphi + 2\delta_{ijk}^\varphi} \right)
\]  

(37)

According to the Hertz contact theory [36, 37], the contact force of rolling ball can be given by

\[
Q_{ijk} = K \delta_{ijk}^{3/2}
\]  

(38)

where \(K\) is the Hertz contact stiffness, and the contact angle is

\[
x_{ijk} = \arcsin \frac{\Delta y_{ijk}}{\Delta x_{ijk}}
\]  

(39)

The total restoring force along x axis of the vibration system can be given by

\[
\text{Fig. 4 Model schematic of linear guide slide platform}
\]
The total restoring force along $y$ axis of the vibration system can be given by

$$F_y = \sum_i^4 \sum_i^4 \sum_k^N \begin{cases} Q_{ijk} \cos \alpha & j = 1, 3 \\ -Q_{ijk} \cos \alpha & j = 2, 4 \end{cases}$$

The total restoring moment about $x$ axis of the vibration system can be given by

$$F_x = \sum_i^4 \sum_i^4 \sum_k^N \begin{cases} Q_{ijk} \sin \alpha j = 1, 2 \\ -Q_{ijk} \sin \alpha j = 3, 4 \end{cases}$$

Table 1 [41] Specifications of the linear guide

| Specification | Value |
|---------------|-------|
| Number of raceways of each carriage | 4 |
| Initial contact angle ($^\circ$) | 45 |
| Number of loaded balls in a groove | 12 |
| Radius of ball $r_b$ (mm) | 6.35 |
| Radius of groove $r_g$ (mm) | 3.2 |
| Elasticity modulus $E$ (GPa) | 206 |
| Preload $F_p$ (N) | 623 |
| Thickness of worktable (mm) | 30 |
| Distance between rails in $x$ direction $l_x$ (mm) | 350 |
| Distance between carriages in $z$ direction $l_z$ (mm) | 300 |
| Distance between grooves in $x$ direction in carriage $L_x$ (mm) | 32 |
| Distance between grooves in $y$ direction in carriage $L_y$ (mm) | 10 |
| Moment of inertia of the platform $I_x, I_y, I_z$ (kg m$^2$) | 0.2419, 0.5491, 0.3259 |
| Mass of platform $m$ (kg) | 33 |
| Hertz contact stiffness (N/m$^{3/2}$) | $8.07171 \times 10^9$ |

Table 2 Detailed procedure of the numerical calculation

1: given the initial parameters of the system
2: calculate the initial contact deformation of each ball according to the proposed method
3: ensure that the sum of the restoring force and restoring moment is equal to zero when there is no external load applied on the platform
4: repeat
5: calculate the dynamic response under harmonic excitation using multiple threads
6: until stopping criterion is met
7: return solution

Fig. 5 A comparison of different modeling methods between the traditional and the proposed approaches
The total restoring moment about y axis of the vibration system can be given by

\[
M_y = \sum_{j=1}^{4} \sum_{i=1}^{2} \sum_{k=1}^{N_k} Q_{ijk} \cos \alpha_{lz} + \zeta_{lk}(i = 1, 2)
\]

\[
+ \sum_{j=1}^{4} \sum_{i=3}^{4} \sum_{k=1}^{N_k} Q_{ijk} \cos \alpha_{lz} - \zeta_{lk}(i = 3, 4)
\]

\[
- \sum_{j=1}^{4} \sum_{i=1}^{2} \sum_{k=1}^{N_k} Q_{ijk} \sin \alpha_{lz} + \zeta_{lk}(i = 1, 2)
\]

\[
- \sum_{j=1}^{4} \sum_{i=3}^{4} \sum_{k=1}^{N_k} Q_{ijk} \sin \alpha_{lz} - \zeta_{lk}(i = 3, 4)
\]

(42)

The total restoring moment about y axis of the vibration system can be given by

\[
M_y = \sum_{j=1}^{4} \sum_{i=1}^{2} \sum_{k=1}^{N_k} Q_{ijk} \cos \alpha_{lz} + \zeta_{lk}(i = 1, 2)
\]

\[
- \sum_{j=1}^{4} \sum_{i=3}^{4} \sum_{k=1}^{N_k} Q_{ijk} \cos \alpha_{lz} - \zeta_{lk}(i = 3, 4)
\]

\[
+ \sum_{j=1}^{4} \sum_{i=1}^{2} \sum_{k=1}^{N_k} Q_{ijk} \sin \alpha_{lz} + \zeta_{lk}(i = 1, 2)
\]

\[
+ \sum_{j=1}^{4} \sum_{i=3}^{4} \sum_{k=1}^{N_k} Q_{ijk} \sin \alpha_{lz} - \zeta_{lk}(i = 3, 4)
\]

(43)
The total restoring moment about \( z \) axis of the vibration system can be given by

\[
M_z = \sum_{j=1}^{4} \sum_{k=1}^{4} \sum_{l=1}^{N_b} Q_{ijkl} \cos (l_z - (1)l_k) + \sum_{j=3}^{4} \sum_{k=1}^{4} \sum_{l=1}^{N_b} Q_{ijkl} \cos (l_z - (1)l_k) \\
- \sum_{j=1}^{4} \sum_{k=1}^{4} \sum_{l=1}^{N_b} Q_{ijkl} \cos (l_z + (1)l_k) + \sum_{j=3}^{4} \sum_{k=1}^{4} \sum_{l=1}^{N_b} Q_{ijkl} \cos (l_z + (1)l_k) \quad i = 2, 3
\]

(44)

In this study, for any given type and value of assembly error and no external load applied on the system, the restoring force and restoring moment \( F_x, F_y, M_x, M_y, \) and \( M_z \) satisfies

\[
F_x \left( x = 0, y = 0, \varphi_x = 0, \varphi_y = 0, \varphi_z = 0, \Delta \delta_x^c, \Delta \delta_y^c, \Delta \epsilon_x^c, \Delta \epsilon_y^c, \Delta \epsilon_z^c \right) = 0 \quad (45)
\]

\[
F_y \left( x = 0, y = 0, \varphi_x = 0, \varphi_y = 0, \varphi_z = 0, \Delta \delta_x^c, \Delta \delta_y^c, \Delta \epsilon_x^c, \Delta \epsilon_y^c, \Delta \epsilon_z^c \right) = 0 \quad (46)
\]

\[
M_x \left( x = 0, y = 0, \varphi_x = 0, \varphi_y = 0, \varphi_z = 0, \Delta \delta_x^c, \Delta \delta_y^c, \Delta \epsilon_x^c, \Delta \epsilon_y^c, \Delta \epsilon_z^c \right) = 0 \quad (47)
\]

\[
M_y \left( x = 0, y = 0, \varphi_x = 0, \varphi_y = 0, \varphi_z = 0, \Delta \delta_x^c, \Delta \delta_y^c, \Delta \epsilon_x^c, \Delta \epsilon_y^c, \Delta \epsilon_z^c \right) = 0 \quad (48)
\]

\[
M_z \left( x = 0, y = 0, \varphi_x = 0, \varphi_y = 0, \varphi_z = 0, \Delta \delta_x^c, \Delta \delta_y^c, \Delta \epsilon_x^c, \Delta \epsilon_y^c, \Delta \epsilon_z^c \right) = 0 \quad (49)
\]
\[ I_x = \frac{1}{12}m(w^2 + d^2) \]
\[ I_y = \frac{1}{12}m(w^2 + h^2) \]
\[ I_z = \frac{1}{12}m(h^2 + d^2) \]

2.2.1 Equations of motions

In this section, considering the influence of different types of assembly error and external load, an equivalent nonlinear dynamic model of the linear guide slide platform can be simplified as a 5-degrees-of-freedom mass-spring-damper model. According to the schematic which is shown in Fig. 4, the governing equations of motion can be expressed as follows

\[ m\ddot{x} + c_x\dot{x} + F_x = F_x(t) \]  
(50)

\[ m\ddot{y} + c_y\dot{y} + F_y = F_y(t) \]  
(51)

\[ I_x\ddot{\phi}_x + c_xI_x\dot{\phi}_x + M_x = M_x(t) \]  
(52)

\[ I_y\ddot{\phi}_y + c_yI_y\dot{\phi}_y + M_y = M_y(t) \]  
(53)

\[ I_z\ddot{\phi}_z + c_zI_z\dot{\phi}_z + M_z = M_z(t) \]  
(54)

where \( c_x \) and \( c_y \) represent the damping coefficient, and the value can be determined by the test in Sect. 3.2, \( x \) and \( y \) represent the displacement of the geometric center of the platform along \( x \) and \( y \) axis, \( \phi_x \), \( \phi_y \), and \( \phi_z \) represent the angular displacement of the geometric center of the platform about \( x \), \( y \), and \( z \) axis. \( F_x(t) \) and \( F_y(t) \) represent the time-varying exciting force and satisfy \( F_x(t) = F_y(t) = F_0\sin(\omega t) \), \( \omega \) represents the excitation frequency, \( M_x(t) \), \( M_y(t) \), and \( M_z(t) \) are the external moment load about \( x \), \( y \) and \( z \) axis and satisfy \( M_x(t) = M_y(t) = M_z(t) = MF_0\sin(\omega t) \).
3 Simulation and discussion

In this section, the fourth-order Runge–Kutta method with fixed time step is used to analyze the governing equations of motion. The studied linear guide slide platform is defined in Table 1. To accelerate the calculation progress, the initial condition for differential equation is set to \( [x, \dot{x}, y, \dot{y}, \phi_x, \dot{\phi}_x, \phi_y, \dot{\phi}_y, \phi_z, \dot{\phi}_z] = [0,0,0,0,0,0,0,0,0,0] \), and the value is determined by the mean of the solution when the steady state solution is obtained. In this study, steady state solution means the properties of the solution does not change over time. To obtain high precision solution, the absolute tolerance and relative tolerance are set to \( 10^{-4} \) and \( 10^{-4} \). To accelerate convergence, the time step in numerical simulation is set as \( \Delta t = 10^{-6} \) \([38]\). The detailed procedure of the calculation is listed in Table 2. According to Ref.[39], the positioning accuracy of machine tool is less than 5 \( \mu \)m/30cmm. In this study, in order to meet the industrial requirements and analyze the influence of assembly error qualitatively, the max translation error of platform along \( x \) and \( y \) axis is 5\( \mu \)m, which means \( \text{max}(\Delta x_{ijk}) < 5 \mu \text{m} \) and \( \text{max}(\Delta y_{ijk}) < 5 \mu \text{m} \). In the simulation, the default system parameters are as follows: \( F_p = 623 \) N \([40]\), \( F_0 = 100000 \) N, and \( \omega = 4350 \text{ rad/} \)
The model number of linear guide is THK HSR 35C, and the geometric parameters are listed in Table 1.

The comparison between the traditional modeling method and the proposed modeling method is shown in Fig. 5. Furthermore, the detailed calculation steps can be shown in Fig. 6.

3.1 Static analysis of linear guide slide platform

Assembly error can influence the load distribution of linear guide; according to Archard wear theory [42], the ball with greater contact force is more likely to cause groove wear. In addition, the assembly error can affect the positioning accuracy of the platform. Therefore, the investigation of the influence of assembly error on the static characteristics is crucial.

Due to the force applied on each carriage is symmetric, taking carriage 1 as an example, Fig. 7 shows the load distribution of balls in carriage 1. As shown in Fig. 7a, with the increasing straightness assembly error $\Delta \delta_x$, under the combined effect of preload and assembly error, the load of balls in groove 2 and groove 4 decreases to zero first, increases the assembly error further, and the value of each ball remains unchanged due to the contact deformation is equal to zero. Furthermore, the balls in a groove are subjected to the same load.

As can be seen in Fig. 7b, for straightness assembly error $\Delta \delta_y$, the contact deformation of balls in groove 1 and groove 3 decreases with $\Delta \delta_y$, but the degree of linearity of the curve is less than Fig. 7a. As shown in Fig. 7c and d, for rotation assembly error $\Delta \delta_x$ and $\Delta \delta_y$, the balls in different positions in a groove are subjected to different loads, the further distance between the ball and the geometric center of carriage,
the larger load it is subjected to. As shown in Fig. 7e, for rotation assembly error $\Delta e$, the loads of the balls in groove 1 and groove 4 increase with $\Delta e$ more linearly.

The load distribution between balls in a specific groove influenced by assembly error has been discussed above. To further investigate the effect of assembly error on load distribution in a carriage, the comparison of $\Sigma Q_k$ between different grooves can be shown in Fig. 8. As shown in Fig. 8a, the value of $\Sigma Q_k$ increase with $\Delta e$ in groove 2 and groove 4, and the value of $\Sigma Q_k$ in groove 1 and groove 3 decrease to zero and do not change with assembly error. This is because
the value of preload decrease to zero and the ball is not deformed. In addition, the difference between the loads of left grooves (groove 1 and groove 3) and right grooves (groove 2 and groove 4) increases with assembly error \( \Delta \delta_y \). This means the balls in groove 1 and groove 3 are prone to cause wear under the action of assembly error \( \Delta \delta_y \). Similarly, left grooves (groove 1 and groove 3) shown in Fig. 8b, upper grooves (groove 1 and groove 3) shown in Fig. 8c, right grooves shown in Fig. 8d, groove 1 and groove 4 shown in Fig. 8e, these cases are prone to cause groove uneven wear.

The assembly error forms one of the biggest source of inaccuracy in machine tool [4]. Figure 9 shows the relationship between the external static load and the positioning accuracy of the linear guide platform under the action of different assembly errors. It can be shown in the figure, due to the existence of initial error, the system with assembly error has lower positioning accuracy.
3.2 Nonlinear dynamic analysis of linear guide slide platform

In this section, to investigate the influence of different types of assembly errors on dynamic responses of linear guide slide platform, the bifurcation diagrams, the largest Lyapunov exponent (LLE), and 3D frequency spectrums are presented with excitation frequency, excitation amplitude, assembly error, and preload as control parameters.

3.2.1 The influence of excitation frequency

Figure 10 presents the corresponding bifurcation diagram of the linear guide slide platform vibration system with respect to excitation frequency under different types of assembly errors $\Delta \delta x, \Delta \delta y, \Delta \delta x, \Delta \delta y,$ and $\Delta \delta z$. As shown in Fig. 10 (a), b, e and f, as the increase in excitation frequency, the motion of the system is periodic-1 motion before resonance frequency, the values of LLE are less than zero which can be shown in Fig. 10 (a), b, e and f. With the increase in excitation frequency, the system enters to quasi-periodic motion and chaotic motion, the values of LLE alternate among negative, zero, and positive, this indicates that the excitation frequency is close to the resonance frequency and resonance phenomenon occurs. By increasing excitation frequency further, the motion of the system return to periodic-1 motion, and the values of LLE are less than zero. By comparing the bifurcation diagram shown in Fig. 10(a), b, e and f, there exist small differences between them, including the length of the interval of quasi-periodic motion. For rotation assembly error $\Delta \delta x$ and $\Delta \delta y$ which can be shown in Fig. 10c and d, unlike with the bifurcation diagrams shown in Fig. 10b, e and f, in the interval close to resonance frequency, the system exhibits periodic-1 motion with a relatively short interval. In addition, compared to normal system, the jump frequency increases under the effect of assembly error except assembly error $\Delta \delta z$. The 3D-lyapunov exponents diagram corresponding to Fig. 10 can be shown in Fig. 11.

In order to illustrate the dynamic characteristics of linear guide slide platform in detail, the bifurcation

![Fig. 17 3D LLE diagram for different types of assembly error. a Normal system, b straightness assembly error $\Delta \delta x$, c straightness assembly error $\Delta \delta y$, d rotation assembly error $\Delta \delta x$, e rotation assembly error $\Delta \delta y$, f rotation assembly error $\Delta \delta z$.](image)

![Fig. 18 Vibration responses at $F_0 = 95800$ N. a Waveform diagram, b Spectrum, c Phase diagram and Poincaré section.](image)

\[ \frac{1.18}{25}f \quad \frac{2.19}{25}f \quad \frac{3.22}{25}f \quad \frac{4.24}{25}f \quad \frac{5.26}{25}f \quad \frac{6.27}{25}f \]
diagram, LLE diagram, and 3D frequency spectrum of the system without assembly error are shown in Fig. 12. In the second interval which is close to resonance frequency, the vibration system shows quasi-periodic and chaotic motions, frequency demultiplication appears \( \frac{23}{25} f, \frac{24}{25} f, \frac{26}{25} f \). Furthermore, the main frequency of the system is \( f \). Figure 13 displays the vibration response of the system without assembly error at \( \omega = 4280 \text{ rad/s} \). As shown in Figure, only the main frequency component appears, and the waveform and phase diagram are regular, which illustrates the motion of the system exhibits periodic-1 motion. As can be seen in Fig. 14, frequency demultiplication components appear in Fig. 14b, \( f \) is the dominant frequency component, \( 19/25 f \) is the second largest one, the Poincaré section and phase diagram is regular which further illustrate the motion of the system is quasi-periodic motion at this point. Figure 15 shows the vibration response at \( \omega = 4408 \text{ rad/s} \); as can be seen in the figure, the Poincaré section and phase diagram are disorder, the continuous frequency component appears in the spectrum, which means the system exhibits chaotic motion at this point.

It can be concluded from the bifurcation diagram shown in Fig. 10a–e. Under the action of different types of assembly error, the system can exhibit different nonlinear dynamic behaviors with respect to excitation frequency \( \omega \). In addition, the assembly error can influence the frequency of jump discontinuity. By comparing the subfigures shown in Fig. 10, straightness assembly error \( \Delta \delta_x \), straightness assembly error \( \Delta \delta_y \), and rotation error assembly \( \Delta \varepsilon_x \) can increase the jump frequency of the system. However, rotation error assembly \( \Delta \varepsilon_y \) and rotation error assembly \( \Delta \varepsilon_z \) decrease it.

3.2.2 The influence of excitation amplitude

Excitation amplitude is one of the most important factors that influence the dynamic behaviors of linear guide slide platform. In order to investigate the combined influence of assembly error and excitation amplitude, the bifurcation diagrams of the system with excitation amplitude \( F_0 \) as control parameter for different assembly errors are shown in Fig. 16. As shown in Fig. 16a, b, d, e and f, as the increase in excitation amplitude, the motion of the system from simply periodic-1 motion in the first interval, to quasi-periodic motion in the second interval. The corresponding LLE diagram can be shown in Fig. 16a, b, d, e and f, the value of LLE first fluctuate intensively less than zero and then, increase up beyond zero. In Fig. 16c, it is noteworthy that the system exhibits simply periodic-1 motion in the whole range of \( F_0 \in [60000, 100000] \text{N} \). In addition, compare with normal system, jump discontinuity and quasi-periodic disappear (Fig. 17).

In order to illustrate the influence of excitation amplitude on the dynamics of the system, Figs. 18 and 19 show the waveform, frequency, phase diagram and Poincaré section at \( F_0 = 95800 \text{ N} \) and \( F_0 = 97400 \text{ N} \) for assembly error \( \Delta \varepsilon_z \), it can be observed that the frequency demultiplication components appear in the spectrum and the Poincaré section and Phase diagram
are regular, which indicates that the system exhibits quasi-periodic motion at these two points.

In conclusion, the excitation amplitude is a sensitive parameter, which influence the motion state of the system and lead the system to take on relatively strong nonlinearity and instability. Furthermore, the combined effect of the assembly error and excitation amplitude makes the motion of the system complex.

3.2.3 The influence of assembly error

In this section, the dynamic characteristics of the system are investigated with five types of assembly error as control parameter. As can be seen in Fig. 20 (a) and (b), there exist two intervals, according to the corresponding LLE diagram shown in Fig. 21, and the system exhibits quasi-periodic motion and chaotic motion in the first interval. As $\Delta \delta_x$ and $\Delta \delta_y$ are increased, the values of LLE are less than zero, and therefore, the nature of the motions of the system is periodic-1. As it can be seen in Fig. 20c), d and e, the system exhibits predominantly chaotic and quasi-periodic motion in the whole assembly error range, and the value of LLE fluctuates intensively in Fig. 21.

For assembly error $\Delta \delta_z$, with the increase in $\Delta \delta_z$ from low to high, the nature of the motion are chaotic and...
quasi-periodic motion. In Fig. 20e, it is noteworthy that the nature of the system does not change with assembly error $D_{e_z}$.

To investigate the dynamic characteristic of the system, Fig. 22 shows the bifurcation diagram, LLE diagram, and 3D spectrum with assembly error $D_{e_y}$ as control parameter. As shown in the 3D spectrum, in the first interval, the main frequency component is $f$, the demultiplication frequency component appears, as the increase in assembly error $D_{e_y}$, the demultiplication frequency component disappears in the second interval. In addition, as shown in Fig. 23, the waveform and Poincaré section are irregular, which illustrate that the system displays chaotic motion at this point.

3.2.4 The influence of preload

In order to investigate the combined influence of preload and assembly error on the dynamic characteristics of the system, Figs. 24 and 25 present the bifurcation diagram and LLE diagram with preload as control parameter in the interval $[0.9, 1.15] F_p$. It can be seen in Fig. 24 (a), b, c, and f, there exist three intervals in the bifurcation diagrams, by increasing the preload, the motion of the system from simple periodic-1 motion in the first interval, through chaotic and quasi-periodic motion in the second interval, the jump discontinuity phenomenon can be observed in this interval, at last the system enters to periodic-1 and quasi-periodic motion in the third interval. It can be observed that the stability of the system does not improve with the increase in the preload of the system. As shown in Fig. 24d and e, the system experienced a short range of quasi-periodic motion in the third interval. In addition, in Fig. 24c, compared with other bifurcations, the preload entering into chaotic motion is less than other cases. The dynamic response of the system with assembly error $D_{e_z}$ can be seen in Fig. 24; the continuous frequency components appear in the 3D frequency spectrum, which indicates the system

![Diagram](image-url)
Fig. 23 Vibration responses with rotation assembly error $\Delta \delta_x$ at $\omega = 4408$ rad/s. 

- a Waveform diagram,
- b Spectrum, c Phase diagram and Poincaré

Sect. 3.2.4 The influence of preload

Fig. 24 Bifurcation diagram with preload as control parameter. 

- a Normal system, b straightness assembly error $\Delta \delta_y$, c straightness assembly error $\Delta \delta_y$, d rotation assembly error $\Delta \delta_x$, e rotation assembly error $\Delta \delta_x$, f rotation assembly error $\Delta \delta_z$.

A$_s$: periodic motion
B: Quasi-periodic motion
C: Chaotic motion
exhibits chaotic motion in the corresponding interval (Fig. 26).

The vibration response of the system at preload = 0.9832Fp can be shown in Fig. 27; there exist numerous points in Poincaré section, continuous frequency components appear, waveform diagram and phase diagram are disorder, which illustrates that the system exhibits chaotic motion at this point. The vibration response of the system at preload = 0.9555Fp is shown in Fig. 28; frequency demultiplication (39f/54, 41f/54, 52f/54, 56f/54, 58f/54) components appears, and Poincaré section and phase diagram are regular, which indicates that the system displays quasi-periodic motion.

4 Model verification and system parameter estimation

In this section, a series of tests are conducted on a specialized linear guideway slide platform to validate the proposed dynamic model and estimate the system parameters.

4.1 Model validation

In order to validate the proposed model, experiment results, Wang’s model (4 DOF) [40] and the proposed model(5 DOF) are compared with the same parameters in Table 1. As shown in Figs. 29 and 30, the comparison of vibration response can be obtained. It can be found that the vibration response and the spectrum for 5 DOF model is larger than 4 DOF model. This is because the number of degrees of freedom, which leads the additional coupled vibration.

4.2 System parameter estimation

Damping ratio determines the amplitude of vibration response at resonance frequency, and therefore, the estimation of damping ratio is critical for the design and investigation of the system [43]. In order to estimate the damping ratio of the system, an impact
A hammer test is conducted, and the test setup is shown in Fig. 31. As shown in the figure, an impact hammer (Sinocera LC-01A) is used to excite a range of frequencies above the noise floor of the instrument, the output of the system is measured by an accelerometer (Piezotronics 352C04) and processed by a data collection system (DongHua5956) and a PC. The frequency response function (FRF) can be obtained by:

\[
FRF = \frac{\text{output}}{\text{input}}
\] (55)

According to the equation in Ref. [44], the expression of damping ratio \(\zeta\) can be given by:

\[
\zeta = \frac{\omega_2 - \omega_1}{2\omega_m}
\] (56)

where \(\omega_1\) and \(\omega_2\) represent the half-power frequency when the corresponding amplitude \(A_1\) and \(A_2\) are equal to \(A_{max}/\sqrt{2}\), and \(\omega_m\) is the resonance frequency.

The FRF of the system along x and y axis can be shown in Fig. 32, \(\omega_{x1} = 1683 \text{ Hz}, \ \omega_{x2} = 1784 \text{ Hz}, \ \omega_{y1} = 687 \text{ Hz,} \ \omega_{y2} = 706.4 \text{ Hz,}\) and the damping ratio along x and y axis can be estimated \(\zeta_x = 0.0297\) and \(\zeta_y = 0.0139\). Therefore, the viscous damping coefficient along x and y directions can be estimated by \(c = 4\pi\zeta\omega_pm\), respectively.

4.3 Experimental verification

In this section, a test is conducted on a specialized linear guide worktable to measure the vibration response of the system, which is used to compare the vibration response obtained by the model. The schematic of the test system can be shown in Fig. 33, the test system is composed of two subsystem, namely, data collection system and excitation system. The descriptions of the two system are as follows, the exciting force applied on the worktable is generated by
**Fig. 29 a** Experimental results. **B** The comparison between 5-DOF model and 4-DOF model at $F_0 = 75$ N, $f = 280.4$ Hz along x direction

**Fig. 30 a** Experimental results. **B** The comparison between 5-DOF model and 4-DOF model at $F_0 = 75$ N, $f = 239.6$ Hz along y direction
an electromagnetic shaker (Sinocera JZK-50), the excitation amplitude and excitation frequency of the force are controlled by a signal generator (Sinocera YE1311) and a power amplifier (YE5874A), and the excitation amplitude is measured by a piezoelectric force sensor (Sinocera CL-YD-331A). The vibration response of the system is measured by an accelerometer (Piezotronics 352C04) from five directions; a data collection system (DH5956) and a PC are used to process and save data. In this experiment, using a gasket to simulate rotation error $D_{ex}$. The thickness of the gasket is 4 mm, $L = 150$ cm, and therefore, the rotation error of Rail II is $D_{ex} = 0.0053$. The comparisons of dynamic response between simulation and experiment are shown in Fig. 34.

5 Conclusion

In this study, a practical 5-degrees-of-freedom nonlinear dynamic model of linear guide slide platform is developed, and five types of assembly errors including two straightness errors and three rotational errors are introduced into the nonlinear dynamic model. The influence of the assembly error on load distribution and positioning accuracy is analyzed. The combined effect of system parameters and each type of assembly error on the dynamic characteristic of the system is studied. A series of experiment are conducted to validate the proposed method. Some conclusions of the study can be obtained as follows:

(1) All five types of assembly errors can cause uneven load distribution between grooves, and rotation error $\Delta_{ex}$ and $\Delta_{ey}$ can lead to uneven load distribution in a groove.

(2) Assembly error can influence the dynamic characteristic of the linear guide slide platform, change the frequency of jump discontinuity and resonance frequency.

(3) Excitation amplitude can have impact to the system, but the system with straightness
assembly error $\Delta c_2$ is less sensitive to excitation amplitude.

(4) Different from the previous study, the nonlinear dynamics of the system cannot be improved by simply increasing the preload.

In reality, the assembly error is not the only factor which can influence the dynamic characteristics of the system. Beyond that, thermal deformation-induced error, wear, pitting and other kinds of errors can also influence the dynamics of the system. To deeply study the vibration response of linear guide slide platform, a more reasonable and practical model of linear guide slide platform will be established in further study.

Authors’ contributions ZL contributed to methodology, investigation, experimental, writing—Original Draft, writing—review & editing. MX contributed to resources and supervision. HZ contributed to resources, writing—reviewing and editing, supervision, writing—review & editing. CL conceived the presented idea. GY conceived the presented idea. ZL carried out the experiment. HM contributed to coding. CW carried out the experiment. YZ contributed to resources and supervision.

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Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability The data sets supporting the results of this article are included within the article and its additional files.
Ethical approval This chapter does not contain any studies with human participants or animals performed by any of the authors.

Consent to participate Not applicable. The article involves no studies on humans.

Consent for publication TAII authors have read and agreed to the published version of the manuscript.

Appendix

According to [45], the moment of inertia of the platform about x, y and z axis can be calculated by

\[ I_x = \frac{1}{12} m(w^2 + d^2) \]

\[ I_y = \frac{1}{12} m(w^2 + h^2) \]

\[ I_z = \frac{1}{12} m(h^2 + d^2) \]

where \( m \) is the mass of the platform, \( h, w, \) and \( d \) represent the height, width, and length, respectively.

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