Time drift of subtended angles as a new cosmological probe

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Abstract: We here propose the time drift of subtended angles as a new possible cosmological probe. In particular, with the coming era of microarcsecond astrometry, our proposal can be used to measure the Hubble expansion rate of our universe in a direct way.
1. Introduction

Research in cosmology has become extraordinarily lively in the past quarter century. In particular, the use of Type Ia supernovae as standard candles lead to the discovery that the expansion of our universe is accelerating, implying that the energy of the universe may be dominated by some sort of exotic matter, with a ratio of pressure to density less than one third, dubbed dark energy\[1\]. However, besides the cosmological constant, there are various other dark energy models devised to explain the mysterious accelerating expansion. On the other hand, the current accelerating expansion of our universe can also be accounted for by either the modification of Einstein gravity or the violation of Copernican principle. With the other cosmological probes, some of them has been ruled out, but there still remain a number of theoretical models surviving such observational tests\[2\].

In this era of empirical cosmology, it is thus significant to propose some new observational programs that may shed light on our understanding of the universe in a way independent from other known cosmological probes. By comparing and combining results from very different methods of determining cosmological parameters, we may both obtain stronger constraints than any method alone would impose, and test techniques against one another to identify signatures of systematic effects. In particular, in a field so afflicted by systematic errors as cosmology, having many independent but complementary techniques is the best way to ensure that we are on the right track to explore the genuine mechanism underlying the evolution of our universe. We here contribute to such a theme by proposing the time drift of subtended angles as a new cosmological probe. As depicted in Fig[1], the ordinary cosmological probes are usually related to the sky survey along the past light cone of today. However, we have another
way to acquire the evolution information of our universe by measuring the time drift of cosmological objects. The classical exemplification is Sandage-Loeb test\cite{3, 4}.

In next section we shall popularize the spirit of time drift by confining ourselves onto the case of subtended angles and argue that the time drift of subtended angle may provide us with a new cosmological probe, just like Sandage-Loeb test. Some discussions will be presented in the last section.

2. Time drift of subtended angles

In what follows we assume that on large scales our universe is described by the FLRW metric

\[
ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2d\Omega^2\right),
\]

where \(k = 1, 0, -1\) correspond to closed, flat, and open universes, respectively.

Imagine a luminous object that extends a proper distance \(D_{\perp}\) perpendicular to the line of sight. Suppose the light was emitted from the object at the time \(t_1\) and is

\[\text{Figure 1: Time drift versus sky survey in the redshift space.}\]
observed by us at \( t_0 \). By the definition of the angular diameter distance, the object will subtend an angle as follows

\[
\theta = \frac{D_\perp}{d_A(z)},
\]

where

\[
d_A(z) = a(t_1)S\int_{t_1}^{t_0} \frac{dt}{a(t)} = \frac{a(t_0)}{1 + z} S[\frac{1}{a(t_0)} \int_0^z \frac{dz'}{H(z')}],
\]

with \( S[x] = \sin x, x, \sinh x \) for closed, flat, and open universes, respectively.

Then after a time integral \( \delta t_0 \), the variation of the subtended angle gives

\[
\delta \theta = -\frac{D_\perp}{d_A^2(z)} \delta d_A(z) + \frac{\delta D_\perp}{d_A(z)}
\]

\[
= -\frac{D_\perp}{d_A^2(z)} \{ \dot{a}(t_1) \delta t_1 S[\int_{t_1}^{t_0} \frac{dt}{a(t)}] + a(t_1) S'[\int_{t_1}^{t_0} \frac{dt}{a(t)}] (\delta t_0 - \frac{\delta t_1}{a(t_1)}) \} + \frac{\dot{D}_\perp \delta t_1}{d_A(z)}
\]

\[
= -\frac{D_\perp}{d_A(z)} [H(z) - \frac{\dot{D}_\perp}{D_\perp}] \delta t_1 = -\frac{H(z) - \frac{\dot{D}_\perp}{D_\perp}}{1 + z} \theta \delta t_0,
\]

where \( S' \) denotes the differentiation with respect to its argument, and \( \frac{\delta a}{\delta t_1} = \frac{\dot{a}(t_0)}{a(t_1)} = 1 + z \) has been used. If the size evolution rate of the luminous object can be ignored compared with the Hubble flow, then the time drift of the subtended angle can be expressed as

\[
\delta v \equiv \left| \frac{\delta \theta}{\theta} \right| = \frac{H(z)}{1 + z} \delta t_0,
\]

which implies that in principle we can determine \( H(z) \) by measurement of the subtended angle at the redshift \( z \) and its variation over the time interval \( \delta t_0 \).

Now let us make a rough estimate of the feasibility of such a proposal by specializing to the fiducial concordance \( \Lambda \)CDM model, where the Hubble expansion rate is given by

\[
H(z) = H_0 \sqrt{\Omega_m^0 (1 + z)^3 + 1 - \Omega_m^0}
\]

with the Hubble constant \( H_0 = 70 \text{km}^{-1}\text{Mpc}^{-1} \) and \( \Omega_m^0 = 0.3 \).

By Eq. (2.6), the Hubble expansion rate increases with redshift, in particular, as shown in Fig 2, goes up from \( 70 \text{km}^{-1}\text{Mpc}^{-1} \) at the present to \( 700 \text{km}^{-1}\text{Mpc}^{-1} \) or so at the redshift \( z = 6 \). It is difficult to imagine that the magnitude of \( \frac{\dot{D}_\perp}{D_\perp} \) can be achieved to the same order as the Hubble expansion rate for those virialized galaxies or clusters of galaxies. Thus at least such virialized systems can serve as those cosmological objects for us to apply our proposal to.

Before proceeding, let us recall the fact that we have been getting ready for an era in which the microarcsecond astrometry will become norm [5]. Thus if we assume
a ten year baseline for $\delta t_0$, then as shown in Fig.3, it follows from Eq.(2.5) that the subtended angle of the observed object should be at least one tenth degree such that its variation can be detected in the microarcsecond telescopes. If we start from the nearby object with the angle diameter distance $100\, Mpc$, its size requires to be of order $0.1\, Mpc$, which can actually be satisfied by some galaxies, not to mention the typical clusters of galaxies. Furthermore, as illustrated in Fig.4, contrary to our naive intuition, the angular diameter distance decreases at high redshifts. In particular, it arrive at its maximum of around $2000\, Mpc$ at the redshift $z = 1.5$ or so. This implies that the maximal size of the required luminous object is of order $1\, Mpc$, which is the typical size of clusters of galaxies. Therefore, the time drift of subtended angle seems to be a very promising cosmological probe.

3. Discussions

We have argued that the time drift of subtended angles may be used as a new promising cosmological probe to measure the Hubble expansion rate of our universe in a direct way.

However, so far the analysis of its feasibility rests on an order of magnitude estimate. It is thus important to see whether the realistic data will distort such an estimate dramatically. On the other hand, although it is hard to imagine that the virialized systems will undertake a size evolution to the same order as the Hubble expansion, it may be expected that at least a few of them will violate this general belief. Nevertheless we
may employ the other side of the same coin by instead using Eq.\ref{Eq:2.4} to determine the size evolution rate for those exceptional objects and non-virialized ones if we know the evolution law of our universe from other cosmological probes, and vice versa, since the subtended angle by definition entangles the whole universe with those luminous objects in it.
It is interesting to ask whether we can disentangle them intrinsically. One way is to combine it with the time drift of the redshift difference from the far and near points of the object to obtain an evolution free cosmological probe, reminiscent of Alcock-Paczynski test[6]. Another way out is to go directly for the time drift of angle diameter distance, which by expression is immune to the size evolution effect totally.

All of these issues are worthy of further investigation but beyond the scope of this paper, we expect to report them elsewhere in the near future.

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