Universal Extra Dimensions and the Muon Magnetic Moment

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Abstract

We analyze the muon anomalous magnetic moment in the context of universal extra dimensions. Our computation shows that the bound from electroweak data on the size of these dimensions allows only a small shift in the muon magnetic moment given by Kaluza-Klein modes of standard model fields. In the well-motivated case of two universal extra dimensions, additional contributions arising from physics at scales where the effective 6-dimensional standard model breaks down, given by dimension-ten operators, have a natural size comparable to the sensitivity of the muon $(g-2)$ experiment at BNL.

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1 Introduction

There are good reasons to imagine that all the standard model fields propagate in a larger number of spatial dimensions compactified at a scale \(1/R \lesssim 1\) TeV. This framework could provide a mechanism for electroweak symmetry breaking \(^1\) and supersymmetry breaking \(^2\), and it relates the number of fermion generations to the requirement of anomaly cancellation \(^3\). Recently, it was pointed out that the compactification radius \(R\) of these universal extra dimensions, can be surprisingly large, as large as \(1/(300\) GeV). The reason is that momentum conservation in extra dimensions leads to Kaluza-Klein (KK) number conservation and therefore to the absence of vertices with a single nonzero KK mode. There are thus no tree-level contributions to the electroweak observables, and no single KK mode production at colliders. Interestingly, the tightest bounds on \(R\), derived from the experimental constraints on the \(\rho\) parameter \(^4\) and on the \(b \to s\gamma\) process \(^5\), leave room for a discovery of KK modes in Run II at the Tevatron.

The higher dimensional standard model is an effective field theory, valid below some scale \(M_s\) in the multi-TeV range. Its lowest dimension operators correspond directly to the familiar terms of the 4-dimensional standard model. Corrections to the leading low energy theory are encoded in a tower of operators of increasing dimension allowed by the field content and the symmetries. This effective higher dimensional theory, after compactification to four dimensions, leads to the standard model together with two classes of corrections. The first arises from physics above \(1/R\) but below the cutoff \(M_s\), and corresponds to virtual KK modes of the standard model particles. The second arises from the unknown physics at scales \(M_s\) and above, and is parametrized by the coefficients of the tower of higher-dimension operators.

In this paper, we extend the considerations of Ref. \(^4\) by discussing the muon anomalous magnetic moment in the context of universal extra dimensions. We compute the one-loop contribution from standard model KK modes in universal extra dimensions, and find that it is too small to be seen by the Muon \(g-2\) Collaboration \(^6\). This computation confirms the estimate of Agashe, Deshpande and Wu \(^7\).

We then consider the effects of the higher-dimension operators. Concentrating on the well motivated case of the (chiral) 6-dimensional standard model, we find that the contributions to the muon magnetic moment from dimension-ten operators could naturally be large enough to be experimentally measurable for typical values of \(M_s\), of a few TeV.

In Section 2, we discuss universal extra dimensions in the framework of effective field
theory, and display the leptonic sector of this theory, appropriate for the computation of the muon anomalous magnetic moment. In Section 3, we compute at one-loop level the muon magnetic moment induced by the KK modes of standard model fields, and in Section 4 we analyze higher-dimension operators within the chiral 6-dimensional standard model. Conclusions and a comparison with the $\rho$ parameter computation are contained in Section 5.

2 Universal extra dimensions

The idea of universal extra dimensions is very simple: all standard model fields propagate in some extra spatial dimensions. These universal dimensions are taken to be flat, and must be compactified on an orbifold such that the zero-mode fermions are endowed with 4-dimensional chirality. The simplest $\delta$-dimensional orbifold of this kind is obtained by compactifying each pair of extra dimensions on $T^2/\mathbb{Z}_2$ and, for odd $\delta$, the remaining extra dimension on $S^1/\mathbb{Z}_2$, as explicitly shown in [4]. The $(4+\delta)$-dimensional quarks, $Q_i, U_i, D_i$, and leptons, $L_i, E_i, (i = 1, 2, 3$ is a generational index) are decomposed in KK modes such that only one left- (right-) handed component of each weak doublet (singlet) is even under the orbifold projection. For example, the second generation lepton fields have zero-modes $L_2^{(0)} = (\nu_{\mu L}, \mu L)$ and $E_2^{(0)} = \mu R$.

The $(4 + \delta)$-dimensional Lagrangian looks very similar to that of the 4-dimensional standard model. There are kinetic terms for the $(4 + \delta)$-dimensional $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge fields, a kinetic term and potential for the $(4+\delta)$-dimensional Higgs doublet, $H$, as well as the following terms involving quarks and leptons:

$$\left( \mathcal{Q}, \mathcal{U}, \mathcal{D}, \mathcal{L}, \mathcal{E} \right) i\Gamma^\alpha D_\alpha \left( \mathcal{Q}, \mathcal{U}, \mathcal{D}, \mathcal{L}, \mathcal{E} \right) - \left[ \mathcal{Q} \left( \hat{\lambda}_U M i \sigma_2 H^* + \hat{\lambda}_D D H \right) + \mathcal{L} \hat{\lambda}_E \mathcal{E} H + \text{h.c.} \right] \right) \left( \mathcal{Q}, \mathcal{U}, \mathcal{D}, \mathcal{L}, \mathcal{E} \right)$$

We use the notation $x^\alpha$ or $x^\beta$ for all space-time coordinates ($\alpha, \beta = 0, 1, ..., 4 + \delta$), and $x^\rho$ or $x^\tau$ for the non-compact coordinates ($\rho, \tau = 0, 1, 2, 3$). $D_\alpha$ are the covariant derivatives associated with the $SU(3)_C \times SU(2)_W \times U(1)_Y$ group. $\Gamma^\alpha$ are anti-commuting $2k+2 \times 2k+2$ matrices, where $k$ is an integer such that $\delta = 2k$ or $\delta = 2k+1$. When $\delta$ is even, the quark and lepton fields may have $4 + \delta$ chirality, defined by the eigenvalues $\pm 1$ of $\Gamma_{4+\delta}$, the analogue of $\gamma_5$ in four dimensions. Anomaly considerations are discussed in Refs. [4, 3]. A summation over a generational index is implicit in Eq. (2.1). The $(4 + \delta)$-dimensional Yukawa couplings, $\hat{\lambda}_U, \hat{\lambda}_D, \hat{\lambda}_E$, are $3 \times 3$ matrices and have mass dimension $-\delta/2$.

The standard model operators listed above have mass dimension ranging up to $4 + 2\delta$. 2
They are the lowest dimension operators allowed by gauge symmetry. Corrections are described by a tower of higher-dimension operators, each suppressed by inverse powers of the multi-TeV scale $M_s$ at which the effective theory breaks down.

In Ref. [4], this effective theory was used to show that, because the KK number is conserved and thus the contributions to experimental observables arise only from loops, the bound from the electroweak data on the size of universal extra dimensions is rather loose. The main constraint comes from weak-isospin violating effects, encoded in the $\Delta \rho = \alpha T$ parameter. In the case of a single extra dimension the electroweak parameters may be computed reliably at one-loop level using the Lagrangian (2.1), revealing that the compactification scale $1/R$ could be as low as 300 GeV. Higher order corrections, suppressed by inverse powers of $RM_s$, are small. In the case of two universal extra dimensions, the contributions of the KK modes to electroweak observables become logarithmically sensitive to $M_s$, meaning that they are not reliably computable by relying only on physics below $M_s$. A rough estimate can be made, however, indicating that in this case $1/R$ could be as low as roughly 500 GeV. We return to this discussion in section 5, after describing our results for the muon $g - 2$.

The 4-dimensional Lagrangian is obtained by dimensional reduction from the $(4 + \delta)$-dimensional theory. The decomposition of the standard model fields in KK modes leads to a variety of trilinear and quartic interactions. Contributions to the one-loop muon anomalous magnetic moment arise from the leptonic part of Eq. (2.1). The vector-like KK modes associated with the weak-doublet, $L^j = (L^j_\mu, L^j_\nu)$, and -singlet, $E^j$, muon fields have electroweak symmetric masses $M_j$, with $j \geq 1$. In the case of one universal extra dimension $M_j = j/R$, while for more dimensions the KK spectrum is denser. The zero-mode Higgs doublet acquires a VEV, breaking the electroweak symmetry, and leading to mass mixing between the $L^j_\mu$ and $E^j_2$ KK modes, level by level. The mixing angle is suppressed by the ratio of the muon mass to KK mass,

$$\sin \alpha_j \approx \frac{m_\mu}{2M_j} + O\left(\frac{m_\mu^3}{M_j^3}\right).$$

The trilinear interaction of the zero-mode muon mass eigenstate, $\mu'$, with the higher KK modes of the $Z$ boson, $Z^j_a$, and the muon KK mass eigenstates, $L^j_\mu$ and $E^j_2$, is described by the following terms in the 4-dimensional Lagrangian:

$$\frac{g}{\cos \theta_W} \left\{ Z^j_a \left[ \tilde{L}^j_\mu \gamma^\rho (g_{\mu L} \cos \alpha_j P_L + g_{\mu R} \sin \alpha_j P_R) - \tilde{E}^j_2 \gamma^\rho (g_{\mu R} \cos \alpha_j P_R + g_{\mu L} \sin \alpha_j P_L) \right] \mu' \right\} - iZ^j_4 \left[ \tilde{L}^j_\mu (g_{\mu L} \cos \alpha_j P_L - g_{\mu R} \sin \alpha_j P_R) - \tilde{E}^j_2 (g_{\mu R} \cos \alpha_j P_R - g_{\mu L} \sin \alpha_j P_L) \right] \mu' \right\} + \text{h.c.}$$

(2.3)
As usual, $g$ is the $SU(2)_W$ gauge coupling, $P_{L,R} = (1 \mp \gamma_5)/2$ and

$$g_{\mu L} = -\frac{1}{2} + \sin^2 \theta_W, \quad g_{\mu R} = \sin^2 \theta_W. \quad (2.4)$$

In Eq. (2.3) we have displayed the interactions involving the KK modes of only one scalar component of the $Z$. For $\delta$ extra dimensions there are $\delta$ scalars associated with a gauge boson, at each KK level. For the one-loop computations that we perform in the next section each scalar KK mode contributes by the same amount, so it is sufficient to consider the exact form of the interactions involving $Z^j_4$.

Another interaction entering the muon $g - 2$ computation is that of a photon KK mode, $A^j_\rho$ and $A^j_4$, with a muon zero-mode and a muon j-mode. It may be obtained from Eq. (2.3) by substituting $Z^j_{\rho,4}$ with $-A^j_{\rho,4} \sin \theta_W \cos \theta_W$ and setting $g_{\mu L} = g_{\mu R} = 1$. It is also straightforward to write the interactions of the $W$ scalar KK modes with a muon zero-mode and a muon-neutrino KK mode in the weak eigenbasis:

$$-\frac{ig}{\sqrt{2}} W^{+j}_4 \tau^j_{\mu R} \mu_L^L + \text{h.c.} \quad (2.5)$$

The $W^{+j}_\rho \tau^j_{\mu L} \gamma^\rho \mu_L$ vertex, involving the $W$ boson KK modes, as well as the $A_\rho W^{+j}_4 G^{-j}$ and $G^{+j} \tau^j_{\mu L} \mu_R$ vertices, involving the KK modes of the charged Goldstone boson eaten by the $W$, are identical with the standard model ones for the corresponding zero-modes. Finally, the interactions of the photon zero-mode with the muon or $W$ boson KK modes are diagonal and determined by the corresponding electric charge.

3 $g_\mu - 2$ from KK modes of standard model fields

The anomalous magnetic moment of the muon is the coefficient $a_\mu \equiv g_\mu - 2$ in the 4-dimensional momentum space operator

$$-a_\mu \frac{e}{2m_\mu} A_{\rho} (p_{\text{out}} - p_{\text{in}}) \not{p} (p_{\text{out}}') i\sigma^{\rho\tau} (p_{\tau}^\text{out} - p_{\tau}^\text{in}) \mu' (p_{\text{in}}'). \quad (3.1)$$

In this section, we compute the contribution $a_\mu^{\text{KK}}$ arising from KK modes associated with universal extra dimensions.

The standard model in universal extra dimensions leads to the one-loop corrections to $a_\mu$ shown in Figs. 1 and 2. Each diagram gives a contribution of order $(\alpha/\pi)m_\mu^2/M_j^2$, reflecting the decoupling of the KK modes\footnote{The result is proportional to $(m_\mu R)^2$, unlike the linear dependence on $m_\mu R$ in Ref. [8], due to the chiral couplings of the muon to the gauge boson KK modes.}. An important feature of these diagrams is
that the contributions from individual KK levels are independent, so that the result is simply a sum over KK levels:

\[
a_{\mu}^{KK} = \frac{\alpha}{8\pi} \sum_j D_j \frac{m_j^2}{M_j^2} \left\{ c(Z) + c(A) + c(W) + c(G^\pm) + \left[ c(A^4) + c(Z^4) + c(W^4) \right] \delta \right\},
\]

(3.2)

where \( D_j \) is the degeneracy of the \( j \)th KK level. The coefficients \( c(Z) \), \( c(A) \), \( c(W) \) and \( c(G^\pm) \) correspond to the diagrams shown in Fig. 1. The coefficients \( c(A^4) \), \( c(Z^4) \) and \( c(W^4) \), given by diagrams which involve scalar KK modes of the photon, \( Z \) and \( W \) (see Fig. 2), are multiplied by the number of extra dimensions because the higher dimensional gauge fields have \( 4 + \delta \) components. Using the interactions displayed in Section 2, we compute the diagrams of Figs. 1 and 2 in the Feynman gauge, and obtain the following values for each of the coefficients:

\[
c(A) = -\frac{2}{3} c(A^4) = \frac{2}{3}, \tag{3.3}
\]

\[
c(Z) = -\frac{3 + 4 \sin^2 \theta_W \cos 2\theta_W}{3 \sin^2 2\theta_W}, \tag{3.4}
\]

\[
c(Z^4) = \frac{1 + 12 \sin^2 \theta_W \cos 2\theta_W}{6 \sin^2 2\theta_W}, \tag{3.5}
\]

\[
c(W) = 2 c(W^4) = -c(G^\pm) = -\frac{1}{3 \sin^2 \theta_W}. \tag{3.6}
\]

The sum of all the diagrams takes the form

\[
a_{\mu}^{KK} = \frac{\alpha}{24\pi \sin^2 2\theta_W} \left[ -3 + 4 \sin^2 \theta_W - \frac{\delta}{2} \left( 3 + 8 \sin^2 \theta_W \right) \right] \sum_j D_j \frac{m_j^2}{M_j^2}, \tag{3.7}
\]

5
and using $\sin^2 \theta_W \approx 0.231$, we find

$$a_{\mu}^{\text{KK}} \approx -5.8 \times 10^{-11} \ (1 + 1.2\delta) \, S_{\text{KK}},$$  \hspace{1cm} (3.8)

where we defined

$$S_{\text{KK}} \equiv \sum_j \frac{6D_j}{\pi^2} \left[ \frac{300 \text{ GeV}}{M_j} \right]^2. \hspace{1cm} (3.9)$$

With a single universal extra dimension, the degeneracy factor $D_j$ is unity and the sum is convergent as in the case of precision electroweak observables. The smallest value for $1/R$ allowed by the electroweak data is approximately 300 GeV, giving $S_{\text{KK}} \approx 1$, and thus leading to a negative value for $a_{\mu}^{\text{KK}}$ of order $10^{-10}$. This is smaller than the final expected $1\sigma$ sensitivity of the muon $g - 2$ experiment at BNL [6]. The negative sign of this small contribution increases slightly the discrepancy between the standard model prediction [9] and experiment.

For the more interesting case of two universal extra dimensions, the KK sum diverges logarithmically, indicating that as in the case of the electroweak observables, important contributions can arise from physics at scales above $M_s$ as well as below. The contribution from physics below $M_s$ can be estimated by cutting off the KK mode sum at an $M_j$ of order $M_s$. As noted in Ref. [4], this procedure for the electroweak observables leads to $1/R \gtrsim 500$ GeV and $M_s R \lessgtr 5$. This leads to $S_{\text{KK}} \lessgtr 1$, and therefore $a_{\mu}^{\text{KK}}$ is of order $10^{-10}$ or smaller in the 6-dimensional standard model.
4 The 6-dimensional standard model and short distance effects on $g_\mu - 2$

In the previous section we showed that, with one or two universal extra dimensions, the value of $g_\mu - 2 \equiv a_\mu$ induced by loops with standard model KK fields below the effective theory cutoff $M_s$ is smaller than the final expected $1\sigma$ sensitivity of the muon $g - 2$ experiment at BNL [6]. However, physics above $M_s$ also contributes to $a_\mu$, and these effects could be large given that the $SU(3)_C \times SU(2)_W \times U(1)_Y$ interactions are strongly coupled at these scales. From a low-energy effective theory point of view, the effect of physics above $M_s$ is parametrized by higher-dimension operators suppressed by powers of $M_s$. In the case of one universal extra dimension, the effective 5-dimensional theory breaks down at $M_s \gtrsim 10$ TeV, so the operators suppressed by powers of $M_s$ are not likely to induce a large $a_\mu$. With more dimensions the cut-off $M_s$ is lower. We concentrate in what follows on the case of two universal extra dimensions.

In six dimensions, the standard model is chiral as in four dimensions and is highly constrained by anomaly cancellation and Lorentz invariance. The quarks and leptons are 4-component Weyl fermions of definite chirality which we label by $+$ and $−$. The cancellation of local anomalies imposes one of the following two chirality assignments: $Q_+, U_-, D_-, L_\mp, E_\pm$. Each of these 6-dimensional chiral fermions leads in the effective 4-dimensional theory to either a left- or right-handed zero-mode fermion depending on the orbifold boundary conditions. The 6-dimensional standard model is the only known theory that constrains the number of fermion generations to be $n_g = 3 \text{mod } 3$, based on the global anomaly cancellation condition [3].

The gravitational anomaly cancels only if within each generation there is a gauge singlet fermion with 6-dimensional chirality opposite to that of the lepton doublet [4, 3]. These gauge singlet fermions can have Yukawa couplings to the Higgs and lepton doublet fields, which at one loop give rise to a negative shift in $a_\mu$. However, the Yukawa couplings of the zero modes have to be smaller than $\sim 10^{-10}$ in order to avoid too large Dirac neutrino masses. There are mechanisms to explain this small parameter, involving additional dimensions accessible only to gravity and the singlet fermions [10]. The large number of singlet fermion KK modes associated with these additional dimensions enhance the contribution to the muon anomalous magnetic moment, but even then there is no reason to expect a sizable $a_\mu$ from the neutrino sector.

Here we point out that the chiral 6-dimensional standard model includes higher-
dimension operators suppressed by powers of $M_s$ that can naturally have a substantial contribution to $a_\mu$. In the 6-dimensional Lagrangian these appear as dimension-ten operators:

$$L + i 2 \left[ \mathcal{W}^\alpha_{\alpha\beta}, \mathcal{B}^\beta_{\alpha\beta} \right] \frac{\hat{\lambda}_E}{M_s^2} \left( C_B \hat{g}^\prime + C_W \hat{g} \right) \mathcal{E} \mathcal{H} + \text{h.c.},$$

where $\mathcal{W}_{\alpha\beta}, \mathcal{B}_{\alpha\beta}$ are the 6-dimensional $SU(2)_W \times U(1)_Y$ field strengths, and $C_W, C_B$ are dimensionless parameters determined by the unknown physics above $M_s$. We have defined them by extracting the 6-dimensional $SU(2)_W \times U(1)_Y$ gauge couplings $\hat{g}, \hat{g}^\prime$, and charged lepton Yukawa coupling matrix, $\hat{\lambda}_E$. These have inverse mass dimension, and are related to the corresponding 4-dimensional couplings by

$$\left\{ \hat{g}, \hat{g}^\prime, \hat{\lambda}_E \right\} = \sqrt{2} \pi R \left\{ g, g^\prime, \lambda_E \right\}.$$

After the two extra dimensions are integrated out, the operator (4.1) gives rise to a number of terms in the 4-dimensional Lagrangian. Only the Higgs doublet zero-mode acquires a VEV, leading to interactions of the leptons with gauge bosons described by dimension-five operators. Among those that involve only zero-modes, the following operator contributes to $a_\mu$:

$$\frac{e m_\mu}{2 M_s^2} \mathcal{F} \left[ U^\dagger (C_B + C_W) U \right]_{22} \frac{i}{2} \left[ \gamma^\rho, \gamma^\tau \right] \hat{\mu} \mathcal{F}_{\rho\tau},$$

where $F_{\rho\tau}$ is the electromagnetic field strength, $\hat{\mu}$ is the muon mass eigenstate, and $U$ is the unitary matrix that relates the mass eigenstate charged leptons to the weak eigenstates. In general, $C_W$ and $C_B$ are $3 \times 3$ matrices in flavor space. However, the gauge fields have generational-independent couplings, so we expect that the flavor-dependence of the operator (4.1) is due only to the presence of the Higgs field and shows up predominantly through the Yukawa coupling matrix $\hat{\lambda}_E$. In other words, we expect $C_W$ and $C_B$ to be approximately flavor independent:

$$C^{ii'}_{W,B} = c_{W,B} \left( \delta_{ii'} + e^{ii'}_{W,B} \right),$$

with $e^{ii'}_{W,B} \ll 1$ ($i, i' = 1, 2, 3$), on the order of the squared lepton Yukawa couplings. Therefore, the muon anomalous magnetic moment induced by the operator (4.3) is given by

$$a_\mu^{\text{op}} \approx \frac{2 m_\mu^2}{M_s^2} (c_B + c_W).$$

If $M_s$ is taken to be the scale where the standard model gauge interactions become non-perturbative, then $RM_s \approx 5$. The bound on the size of two universal extra dimensions
imposed by the electroweak data is $1/R \gtrsim 500$ GeV \cite{4}. Since anomaly cancellation in six dimensions does not allow a straightforward supersymmetric extension of the standard model \cite{3}, the scale where the 6-dimensional standard model breaks down should be not much higher than a few TeV in order to avoid fine-tuning in the Higgs sector. The result for $a_\mu^{\text{op}}$ can be written as

$$a_\mu^{\text{op}} \approx 3.6 \times 10^{-9} (c_B + c_W) \left( \frac{2.5 \text{ TeV}}{M_s} \right)^2. \quad (4.6)$$

The operators (4.1) arise at scales of order $M_s$, and because they involve gauge fields, their coefficients are expected to be proportional to the 6-dimensional gauge couplings. At the same time, these operators break the chiral symmetry of the leptons, and from the Yukawa terms in the 6-dimensional Lagrangian we know that such breaking is accompanied by Yukawa couplings. Hence, it is natural to expect the coefficients $C_B$ and $C_W$, defined in Eq. (4.1) by extracting the 6-dimensional gauge and Yukawa couplings, to be of order unity at the scale $M_s$. Furthermore, upon dimensional reduction the volume suppression is entirely absorbed in the gauge and Yukawa couplings\footnote{Consequently, the estimate (4.3) is similar to that arising from higher-dimension operators associated with muon substructure in a 4-dimensional context \cite{15}.}. As a result, the values of $c_B$ and $c_W$ at scales comparable to the muon mass differ from those at the scale $M_s$ by factors of order one, mostly due to the one-loop running. Notice that the theory is perturbative at scales below $M_s$.

We conclude that physics from above the cutoff $M_s$ of the effective, 6-dimensional standard model naturally gives a contribution to $a_\mu$ comparable to the current sensitivity of the Muon $g - 2$ experiment at BNL. The sign of this contribution cannot be determined within the effective 6-dimensional standard model. Future reductions in the experimental uncertainty and improvements in the estimate of hadronic contributions to $a_\mu$ would allow a measurement of $(c_B + c_W)/M_s^2$ in the context of universal extra dimensions.

Although the operators (4.1) are expected to be approximately flavor diagonal, the constraints on flavor-changing neutral currents are severe enough to warrant attention. The process $\mu^- \to e^- \gamma$ is the most constraining in this context. The tree level decay width for this process is

$$\Gamma(\mu^- \to e^- \gamma) \approx \frac{\alpha m_\mu^5}{2 M_s^4} \left\{ |U^\dagger (C_B + C_W) U|_{12} \right\}^2. \quad (4.7)$$

The experimental limit of $\Gamma(\mu^- \to e^- \gamma) < 3.6 \times 10^{-27}$ MeV \cite{12} imposes a bound on the
off-diagonal \((i \neq i')\) elements of \(C_{W,B}\),

\[
c_{W,B}^{ii'} \lesssim \frac{10^{-4}}{c_{W,B}} \left( \frac{2.5 \text{ TeV}}{M_s} \right)^2.
\]

## 5 Conclusions

In Ref. [4], it was pointed out that all the standard model fields could propagate in a larger number of spatial dimensions, compactified at a scale \(1/R\) as small as 300 GeV. In this paper, we addressed the implications of this idea of “universal extra dimensions” for the muon anomalous magnetic moment. For one or two extra dimensions, we computed the one-loop contribution of the KK modes of the standard model fields and found that it is too small to be detected by the Muon \((g - 2)\) experiment at BNL [6]. We then analyzed higher-dimension operators in the context of the 6-dimensional standard model. For the cut-off \(M_s\) in a range such that fine-tuning in the Higgs sector is eschewed (a few TeV), the contribution to the muon anomalous magnetic moment is naturally as large as the currently quoted discrepancy [6]. The sign of this contribution, however, is determined by the unknown physics above \(M_s\).

It is interesting that the natural expectation of the contribution to \(g_\mu - 2\) from physics above \(M_s\) is an order of magnitude larger than the contribution from physics below \(M_s\), arising from the KK modes of standard model fields. This is in contrast to the case of the weak-isospin violating \(\rho\) parameter discussed in [4]. The dimension-ten weak-isospin violating operator \((c_T \hat{\lambda}_\mathcal{H}/2M_s^2) \left( \mathcal{H}^\dagger D_\alpha \mathcal{H} \right)^2\) has a coefficient \(c_T\) (defined by extracting the 6-dimensional quartic Higgs coupling \(\hat{\lambda}_\mathcal{H}\)) of order unity if the weak-isospin is maximally violated by physics above \(M_s\). The volume suppression resulting from integration over the two extra dimensions is absorbed in the Higgs and gauge couplings, so this operator gives \(\Delta \rho \sim \frac{M_s^2}{M_h^2}\) (\(M_h\) is the Higgs boson mass), comparable to the one-loop KK contribution. The reason for this difference is partly that the one-loop contributions to \(g_\mu - 2\) involve only lepton KK modes, while \(\Delta \rho\) is enhanced by a color factor and the largeness of the top Yukawa coupling.

We emphasize that the low scale of new physics, of a few TeV, where the 6-dimensional standard model is expected to break down, is an opportunity, allowing phenomenologically interesting higher-dimension operators, but also a challenge requiring further study of mechanisms that suppress dangerous operators [13].

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