Determining the Neutrino Mass Hierarchy with Cosmology

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The combination of current large scale structure and cosmic microwave background (CMB) anisotropies data can place strong constraints on the sum of the neutrino masses. Here we show that future cosmic shear experiments, in combination with CMB constraints, can provide the statistical accuracy required to answer questions about differences in the mass of individual neutrino species. Allowing for the possibility that masses are non-degenerate we combine Fisher matrix forecasts for a weak lensing survey like Euclid with those for the forthcoming Planck experiment. Under the assumption that neutrino mass splitting is described by a normal hierarchy we find that the combination Planck and Euclid will possibly reach enough sensitivity to put a constraint on the mass of a single species. Using a Bayesian evidence calculation we find that such future experiments could provide strong evidence for either a normal or an inverted neutrino hierarchy. Finally we show that if a particular neutrino hierarchy is assumed then this could bias cosmological parameter constraints, for example the dark energy equation of state parameter, by $\gtrsim 1\sigma$, and the sum of masses by 2.3$\sigma$.

I. INTRODUCTION

Accurately determining the absolute value of the neutrino mass is one of the main goals of particle physics. However since the effect of individual neutrinos is small it is cosmological observations, that observe the cumulative effect of neutrinos on large scales, that present the most powerful way to bound the absolute neutrino mass scale. Albeit indirect and model dependent, cosmological constraints are currently stronger than those coming from beta-decay experiments (for recent reviews see $^1$ and $^2$). For example, the Mainz $^3$ and Troitsk $^4$ Tritium decay experiments give upper limits on the single electron neutrino mass of $m < 2.05\text{eV}$ and $m < 2.3\text{eV}$ respectively, at 95\%c.l.. Cosmological data coming from Cosmic Microwave Background (CMB) measurements from WMAP $^5$ experiment combined with baryonic oscillation data $^6$ and various Supernovae observations ($^7$, $^8$) lower this limit to $m \lesssim 0.22\text{eV}$ $^9$. On the other hand observations of flavour oscillations in atmospheric and solar neutrinos provide evidence not only for a non-zero neutrino mass, but also for a difference between masses, measuring squared mass differences of $^{10}$:

$$|\Delta m^2_{31}| = |m^2_3 - m^2_1| = 2.2^{+1.1}_{-0.8} \cdot 10^{-3}\text{eV}^2$$
$$\Delta m^2_{21} = m^2_2 - m^2_1 = 7.9^{+1.0}_{-0.8} \cdot 10^{-5}\text{eV}^2$$

where ranges indicated are 3$\sigma$ confidence level and $m_1$, $m_2$ and $m_3$ are the three mass eigenstates. The ambiguity in the sign of $\Delta m^2_{31}$ leads to an uncertainty about the neutrino mass scheme, allowing for two possible hierarchies: the normal hierarchy, given by the scheme $m_3 \gg m_2 > m_1$, or the inverted hierarchy $m_2 > m_1 \gg m_3$. Note that given equation $^{11}$ an inverted hierarchy scenario would be automatically excluded by measuring a total mass $\sum m_\nu \lesssim 0.1\text{eV}$.

It is commonly perceived that cosmology is able to constrain the total neutrino mass $\sum m_\nu$, while mass differences between eigenstates can be neglected. This is an excellent approximation as shown in $^{11}$ at least for the CMB anisotropies power spectrum, since the effect of neutrino mass on the CMB is related to the physical density of massive neutrinos $\omega_\nu$, i.e. to their total mass $\sum m_\nu$. Individual neutrino masses do have an effect on the matter power spectrum, due to the different transition redshifts from relativistic to non-relativistic behaviour. This effect is still much smaller than that due to the total mass and can be safely neglected in analysing current cosmological data. Nevertheless in the near future various experiments will reach a much higher accuracy in reconstructing the matter power spectrum. It is therefore timely to consider the possibility that these surveys will be sensitive to single neutrino masses. In recent papers (see for example $^{12}$ and $^{13}$) a forecast has been made considering future observations of CMB anisotropies, CMB lensing and galaxy distribution finding that this kind of data doesn’t seem able to reach enough accuracy to discriminate between the two hierarchies. In $^{14}$ has been shown that future accurate measurements of the redshifted 21 cm signal from the epoch of reionization can in principle measure individual masses but will be very difficult to achieve the precision required to distinguish normal and inverted hierarchies.

In $^{15}$ an explicit and more general parameterization of neutrino mass splitting was introduced, representing various possible hierarchies; with the only simplifying approximation that two neutrinos are of the same mass $m_1 = m_2$. This approximation is well justified by equation $^{11}$. In $^{15}$ it was shown that even a cosmic variance limited CMB experiment would not be able to detect a difference in individual neutrino masses. In addition CMB lensing, even if limited by cosmic variance only, is strongly inhibited.
in measuring the mass hierarchy by degeneracies with other cosmological parameters.

In this article we use a Fisher matrix formalism applied to the same parameterization of \[15\] to assess the ability of future cosmic shear measurements, like those achievable with Euclid/DUNE \[16\] experiment, combined with Planck \[17\] CMB data to place constraints on single neutrino masses. It is almost a decade that cosmic shear has been recognized as one of the most powerful tools to constrain the total neutrino mass (see for example \[18\], \[19\] and \[20\]) and hence an investigation into how far these experiments can proceed in the exploration of the neutrino properties is well justified.

The article is organized as follows: in section \[II\] we review in more detail the effects of the total neutrino mass and of individual neutrino masses on cosmological observables. We also describe the parameterization of \[15\] which will be used throughout the rest of the article. In section \[III\] we describe the implementation of our cosmic shear and CMB Fisher matrices. Section \[IV\] shows results from our forecasts as function of various assumptions about the neutrino mass hierarchy, and for different parameter sets. We also discuss results in the light of our parameterization of the neutrino mass hierarchy. Our forecasts show that accurate measurements of the matter power spectrum from Euclid, combined with CMB data from Planck, can reach the accuracy required to constrain single neutrino masses. In section \[V\] we conduct a more accurate analysis of our results through a Bayesian evidence investigation. We also show that assuming a wrong hierarchy can lead to a bias in the recovered value of other cosmological parameters, in particular for the dark energy equation of state, generally comparable with the 1\(\sigma\) sensitivity. The largest bias is found in the total neutrino mass, due to the degeneracy involving \(\sum m_\nu\) and the hierarchy parameter, at 2.3\(\sigma\). Finally, in section \[VI\] we summarize our conclusions.

II. EFFECTS OF NEUTRINO MASS SPLITTING

The main effect of a non-zero neutrino mass on cosmology is through the collisionless fluid behaviour that causes neutrinos to free-stream over a typical length which is known as free-streaming length \(\lambda_{FS}\). The consequence of this free propagation is a cancellation of neutrino density fluctuations on scales smaller than \(\lambda_{FS}\) and a slow-down in the growth of perturbation on these scales. The matter power spectrum results are then damped for wave vectors \(k > k_{FS} \approx 2\pi/\lambda_{FS}\). The free streaming wave vector of a single species \(k_{FS}\) depends on the mass of that species \(22\):

\[
k_{FS}(z) = \frac{\sqrt{\frac{3}{4} |\Omega_m(1 + z)^3 + \Omega_\Lambda|}}{(1 + z)^2} \left(\frac{m}{1\text{eV}}\right) h\text{Mpc}^{-1},
\]

where \(h = H_0/(100\text{km}\text{s}^{-1}\text{Mpc}^{-1})\). On these small scales the matter power spectrum is suppressed with respect to the power spectrum of a cosmology with massless neutrinos by an amount that depends mainly on the fraction \(f_\nu\) of matter density in the form of massive neutrinos \((f_\nu = \Omega_\nu/\Omega_m)\). In the case of degenerate masses \(\Omega_\nu\) can be expressed as a function of the total neutrino mass:

\[
\Omega_\nu \approx \frac{\sum m_\nu}{93.14\text{h}^2\text{eV}}.
\]

However, as pointed out in \(22\), even in the case of non-degenerate masses this relation remains a good approximation.

Massive neutrinos become non-relativistic at a redshift given by \(z_{nr} \approx \frac{2 \cdot 10^3 m_\nu}{1\text{eV}}\), so that neutrinos with masses up to \(\sim 0.5\text{eV}\) are still relativistic at time of recombination. As a result the effect of neutrino free streaming on the CMB power spectrum is negligible for small neutrino masses. In this case the main effect of neutrino masses on the CMB is indirect, related to the delay of matter-radiation equality. This causes a small shift in the peaks of the power spectrum and a slight increase of their heights due to a longer duration of the Sachs-Wolfe effect.

From this discussion is clear that neutrino mass affects growth of structure in two ways: the matter power spectrum is suppressed by an amount that depends mainly on the total mass \(\sum m_\nu\) but also, even if for a minor amount, on single masses, because the time of transition to non-relativistic regime depends on single masses. The typical wave vector over which this suppression can be observed also depends on the mass of single neutrino species as shown by equation \(22\). Therefore a reconstruction of the matter power spectrum can in principle give information both on \(\sum m_\nu\) and on single masses (through the suppression of the power spectrum and \(k_{FS}\)), even if this second effect is generally much smaller than that due to the total mass. In the following sections we investigate the ability of future cosmological experiments to reach the sensitivity required to detect differences in neutrino masses. Following \[15\] we parameterize mass splitting by introducing the parameter \(\alpha\) defined as the fraction of the total mass in the third neutrino mass eigenstate:

\[
m_3 = \alpha \sum m_\nu.
\]

The other two eigenstates are assumed to share the same mass \(m_1 = m_2\). This approximation is supported by the observed differences in the squares of neutrino masses as measured by oscillations of atmospheric neutrinos, \(|\Delta m_{32}^2/\Delta m_{12}^2| \approx 0.5\cdot 10^{-2}\) \(10\). The advantage of this parameterization is that it allows us...
to represent in a simple way various mass hierarchies such as total degeneracy ($\alpha = 1/3$) and normal or inverted hierarchy given respectively by $\alpha \sim 1$ and $\alpha \ll 1$.

### III. FORECAST FOR WEAK LENSING TOMOGRAPHY

Weak lensing (see [27] for a recent review or http://www.gravitationallensing.net) is a particularly powerful probe of cosmology since it simultaneously measures the growth of structure through the matter power spectrum, and the geometry of the Universe through the lensing effect. Since weak lensing probes the dark matter power spectrum directly it is not limited by assumptions about galaxy bias (how galaxies are clustered with respect to the dark matter). Future weak lensing surveys will measure photometric redshifts of billions of galaxies allowing the possibility of 3D weak lensing analysis (e.g. [32, 33, 34, 35]) or a tomographic reconstruction of growth of structures as a function of time through a binning of the redshift distribution of galaxies ([32, 33, 34, 35]) or a tomographic reconstruction of the Universe through the lensing effect. Since weak lensing simultaneously measures the growth of structure through an angular distortion parameter (with target value $\alpha = 0.03(1+z)$), we then repeated the analysis for a fiducial normal hierarchy scheme for neutrino mass. We then repeated the analysis for a fiducial normal hierarchy scheme for neutrino masses. The usual definition of the Fisher matrix [30] is:

$$ F_{\alpha\beta} \equiv - \frac{\partial^2 \ln L}{\partial p_\alpha \partial p_\beta} $$

where $L$ is the likelihood function for a set of parameters $p_i$. When the derivatives of $F_{\alpha\beta}$ are evaluated at the fiducial model the Fisher matrix gives an estimate of the best statistical error achievable on the parameters (via the Cramer-Rao inequality) for the method and survey design considered

$$ \sigma(p_i) \geq \sqrt{(F^{-1})_{ii}}. $$

The Fisher matrix for weak lensing is given by (e.g. [24])

$$ P_{\alpha\beta}^{WL} = f_{sky} \sum_\ell \frac{(2\ell + 1)\Delta \ell}{2} \frac{\partial P_{ij}}{\partial p_\alpha} C_{\mu
u} \frac{\partial P_{km}}{\partial p_\beta} C_{\mu
u}^{-1} (5) $$

where $P_{ij}(\ell)$ is the convergence weak lensing power spectrum that depends on the non-linear matter power spectrum at redshift $z$, $P_{nl}(k, z)$, obtained by correcting the linear matter power spectrum $P(k, z)$ using the option halofit of CAMB [25]. $\Delta \ell$ is the step used for $\ell$ and

$$ C_{jk} = P_{jk} + \delta_{jk}(\gamma_{8}^2) n_j^{-1}. (6) $$

In the last expression $\gamma_{8}^2$ is the rms intrinsic galaxy ellipticity (and we assume $\langle \gamma_{8}^2 \rangle = 0.22$) and $n_j$ is the number of galaxies per steradian belonging to the $j$th bin

$$ n_j = 3600 d \left( \frac{180}{\pi} \right)^2 \hat{n}_j, \quad (7) $$

where $d$ is the number of galaxies per square arctminute and $\hat{n}_j$ is the fraction of sources belonging to the $j$th bin. For the Euclid experiment we take $d = 35$ and $f_{sky} = 0.5$. The galaxy redshift distribution is assumed to have the form $D(z) \propto z^2 \exp(-z/z_0)^{1.5}$ with $z_0 = 0.9$. For this experiment photometric redshift uncertainties are assumed to be $\sigma_z = 0.03(1+z)$. We treat this uncertainty following the approach of [26] where a galaxy with redshift $z$ could be wrongly classified at a redshift $z_{ph}$. Letting $p(z|z_{ph})$ be the probability that this happens the distribution of galaxies in the $i$th bin is modified to take into account this 'leakage' between bins $D_i(z) = \int_{z_{ph}}^{z_{max}} d z_{ph} D(z)p(z|z_{ph})$. We choose a simple Gaussian form for $p(z|z_{ph})$

$$ p(z|z_{ph}) = \frac{(2\pi\sigma_z^2)^{-1/2}}{\exp \left[ -\frac{(z_{ph} - z)^2}{2\sigma_z^2} \right]}. $$

The Fisher matrix for weak lensing is then added to that of CMB to obtain constraints from the combination Planck+Euclid

$$ F_{\alpha\beta}^{tot} = F_{\alpha\beta}^{WL} + F_{\alpha\beta}^{CMB}. $$

For a CMB experiment the Fisher matrix is given by

$$ F_{\alpha\beta}^{CMB} = \sum_{\ell=2}^{\ell_{max}} P_{TT} \sum_{PP',QQ'} \frac{\partial \Delta C_{\ell}^{PP'}}{\partial p_\alpha} (Cov_{\ell}^{-1})_{PP',QQ'} \frac{\partial \Delta C_{\ell}^{QQ'}}{\partial p_\beta} (8) $$

where the couples $PP'$ and $QQ'$ mean in our case $TT$, $TE$ or $EE$ (temperature and E-mode polarisation). $Cov_{\ell}$ is the power spectrum covariance matrix at the $\ell$th multipole and $\ell_{max}$ is the maximum multipole available given the angular resolution of the considered experiment, for Planck we use $\ell_{max} = 2000.$
The other specifications we used are listed in Table I. The total Fisher matrix $F_{\text{tot}}$ is then inverted to obtain uncertainties on cosmological parameters of our model.

| PLANCK Channel/GHz | FWHM $f_{\text{sky}}$ | $\Delta T/T$ | $\Delta P/T$ |
|--------------------|------------------------|--------------|--------------|
| 100                | 9.5′                   | 2.5          | 4.0          |
| 143                | 7.1′                   | 2.2          | 4.2          |
| 217                | 5.0′                   | 4.8          | 9.8          |

TABLE I: specifications for Planck experiment used in the Fisher matrix calculation. $\Delta T/T$ and $\Delta P/T$ are sensitivities ($\mu K/K$) for temperature and polarization respectively.

IV. RESULTS

In this section we show results of our forecasts on neutrino mass parameters $\sum m_\nu$ and $\alpha$.

A. Parameter Constraints

The constraints of Table I and Fig. 1 show that for $\sum m_\nu = 0.055$eV, if the neutrino mass hierarchy is described by a normal hierarchy ($\alpha = 0.95$) then the combination of weak lensing data from an experiment like Euclid and accurate measurements of the CMB power spectrum, achievable with Planck, can become sensitive to the mass of single species. The 1σ uncertainties on the sum of masses is $\sigma_{\sum m_\nu} = 0.037$eV, which is in agreement with other forecasts (see for example [20]) and confirms the ability of these surveys to detect neutrino mass. In particular this work has a resonance with Debono et al. (2009) (in preparation) in which the effect of parameter sets on neutrino mass constraints are investigated. We find that for the parameters that are common between the two articles there is agreement between the predicted errors, despite the slightly different parameter sets, power spectrum estimation approaches (CAMB vs Eisenstein & Hu code [21]) and assumptions.

As one can see for a typical normal hierarchy scenario with $\alpha = 0.95$ the combination Euclid+Planck can reach a $\sim 20\%$ sensitivity on $\alpha$ with an error $\sigma_\alpha = 0.19$. For this normal hierarchy scenario we have repeated the Fisher matrix calculation using a larger parameter space, including running $dn_\alpha/d\ln k$ of the spectral index and the dark energy equation of state parameter $w$, to check the weakening in the constraints on $\alpha$ induced by degeneracies among these parameters. One may expect that running and $w$ would have a large degeneracy with neutrino mass since they both can add an effective damping on small scale. However, as shown in in Table II and in Fig. 1 constraints on $\alpha$ are not seriously weakened – for this second 10-parameter space $\sigma_\alpha = 0.22$.

| experiment   | $\sigma_{\sum m_\nu}$ | $\sigma_\alpha$ |
|--------------|------------------------|-----------------|
| Planck       | 0.49 eV                | $> 1$           |
| Planck+Euclid| 0.037 eV               | 0.22            |

TABLE II: constraints from Planck+Euclid Fisher matrix on neutrino mass parameters for the 10-parameter space described in the text.

FIG. 1: 68% and 95% probability contours (two-parameter) in the plane $\alpha - \sum m_\nu$ for Planck+Euclid from our Fisher matrix calculation for the 10-parameter space described in the text.

B. Parameterisation Investigation

For the smaller 8-parameter set we repeated the Fisher matrix calculation for different fiducial values for $\alpha$. In Fig. 2 we show the relative uncertainties in $\alpha$ from the combination Euclid+Planck as a function of the target model. The figure is a combination of a general trend, that causes the uncertainty on $\alpha$ to decrease for $\alpha$ increasing, and the way in which constraints from CMB combine with those from weak lensing. The loss of sensitivity around $\alpha \approx 0.88$ is in fact due to a strong rotation of the degeneracy in the plane $\alpha - \sum m_\nu$ in the Euclid Fisher matrix, as shown in Fig. 3 from an anti-correlation to a positive correlation. For the Planck Fisher matrix instead the degeneracy between these two parameters is less dependent on the fiducial value of $\alpha$. As one can see for $\alpha \approx 0.88$ the constraints from Euclid are almost independent of $\alpha$, and the combination of the two experiments lose the ability to break the degeneracy between these parameters. This is confirmed also by the fact that the only other parameter...
that shows a significant peak of uncertainty in correspondence of $\alpha \approx 0.88$ is the total mass $\sum m_\nu$ itself that increase from $\sigma_{\sum m_\nu} \approx 0.025$eV for $\alpha = 0.86$ to $\sigma_{\sum m_\nu} \approx 0.034$eV for $\alpha = 0.88$.

We emphasise that this is a peculiar effect of this parameterisation, we have tested the derivative numerically for convergence. Therefore we advocate this parameterisation but with a strong warning that results are highly dependant on the fiducial value of $\alpha$, particularly around $\alpha \approx 0.88$.

We note however that the general decrease in $\sigma_\alpha/\alpha$ is quite intuitive since for $m_3 \approx \sum m_\nu$, a variation in $\alpha$ means a variation only in the mass that can have an effect on the growth of structure. In this limit the others two eigenstates have a very low mass and are relativistic up to a very low redshift. For example for our target model ($\sum m_\nu \approx 0.055$eV and $\alpha = 0.95$) the two eigenstates have $m_1 = m_2 \approx 1.4 \cdot 10^{-3}$eV, this results in the neutrinos being relativistic up to a redshift $z \sim 2$. Hence for $\alpha \rightarrow 1$ the sensitivity to this parameter increases. Conversely when $\alpha$ is significantly different from 1, for example in the case of total degeneracy, when $\alpha = 1/3$, a variation in $\alpha$ and hence in $m_3$ implies an opposite variation in $m_1$ and $m_2$ which now have a non-negligible mass. These variations partially compensate reducing the sensitivity of cosmology to $\alpha$.

This reasoning is confirmed by the comparison between derivatives of the matter power spectrum $P(k)$ (that enters in the calculation of the weak lensing convergence power spectrum) with respect to $\alpha$ calculated for different values of $\alpha$. In Fig. 4 are shown derivatives $dP/d\alpha$ for the case $\alpha = 0.95$ and $\alpha = 1/3$. The latter is about two orders of magnitude smaller than the first. As one can see the two derivatives have opposite signs due to the different change in $k_{FS}$ induced by a change in $\alpha$ in the two target models. When $\alpha \rightarrow 1$ an increase in $\alpha$ causes an increase in $k_{FS}$ according to equation (4) while, as we have said above, $m_1$ and $m_2$ are not important. Instead, for $\alpha = 1/3$ an increase in this parameter causes a decrease of $m_1$ and $m_2$ (which now cannot be neglected) and the overall effect is a decrease in $k_{FS}$.

V. BAYESIAN ANALYSIS

To assess better the power of these cosmological probes to detect a neutrino mass difference we present a Bayesian evidence forecast. Calculation of Bayesian evidence allows one to make a comparison of different models, as opposed to parameter estimation within a model. According to Bayes theorem the (posterior) probability of a set of parameters $\theta$ describing a model $\mu$ given the data $d$ is

$$p(\theta|d, \mu) = \frac{L(\theta)p(\theta|\mu)}{p(d|\mu)},$$

(9)
where $\mathcal{L}(\theta) = p(d|\theta, \mu)$ is the likelihood function, $p(\theta|\mu)$ is the prior probability on the parameters of the model $\mu$ and $p(d|\mu)$ is the Bayesian evidence; that has the role of a normalization constant, being $p(d|\mu) = \int d\theta p(d|\theta, \mu)p(\theta|\mu)$.

The following is a summary of the technique describe in [29]. Given a cosmological model $M'$ described by a number of parameters $n'$, a common problem is to verify whether data require the inclusion of some new parameters in the model so as creating a new more complicated model $M$ with a number of parameter $n > n'$. In this case one has to take the ratio of the posterior probabilities of the two models $p(M'|d)/p(M|d)$. This ratio can be obtained from Bayes theorem:

$$\frac{p(M'|d)}{p(M|d)} = \frac{B_p(M')}{p(M)}$$  \hspace{1cm} (10)

where $p(M')$ and $p(M)$ are the prior probabilities of the two models and $B$ is the Bayes factor given by the ratio between Bayesian evidences:

$$B = \frac{\int d\theta' p(d|\theta', M')p(\theta'|M')}{\int d\theta p(d|\theta, M)p(\theta|M)}$$ \hspace{1cm} (11)

where $\theta$ and $\theta'$ are the set of parameters of $M'$ and $M$. In the case we are considering the two models are nested, in the sense that they share the same $n'$ parameters. For nested models and for Gaussian likelihoods approximates to

$$B = (2\pi)^{-p/2} \frac{\sqrt{\det F}}{\sqrt{\det F'}} \exp\left( -\frac{1}{2} \delta\theta_\alpha F_{\alpha\beta} \delta\theta_\beta \right) \prod_{q=1}^{p} \Delta\theta_{n'+q},$$ \hspace{1cm} (12)

where the index $q$ runs over the $p = n - n'$ additional parameters of $M$ with respect to $M'$ and $\Delta\theta$ are the prior ranges on the parameters. Note that the Fisher matrix $F$ is $n \times n$ while $F'$ is $n' \times n'$.

The $p$ parameters are assumed to be fixed at a certain fiducial values in the model $M'$ and are shifted by an amount $\delta\psi$ with respect to their fiducial value in $M$. For $\alpha, \beta = 1, \ldots, n'$ then this shift $\delta\theta_\alpha$ is given by

$$\delta\theta_\alpha = -(F'^{-1})_{\alpha\beta}G_{\beta\gamma} \delta\psi_\gamma \hspace{1cm} \gamma = 1, \ldots, p,$$ \hspace{1cm} (13)

where for $\alpha, \beta = 1, \ldots, p$ we have $\delta\theta_\alpha = \delta\psi_\alpha$. The quantity $G$ that appears in (13) is a block $n' \times p$ of the full Fisher matrix $F$.

The calculation of Bayes factor through Fisher matrices helps to clarify whether future experimental data will be sensitive to a wrong assumption about some parameters (for example fixing $p$ parameters to wrong fiducial values). Bayesian analysis is known to be conservative in the sense that models with a smaller number of parameters are favoured until data strongly require the introduction of new parameters. Hence, for the case of nested models only very sensitive experiments will have the power to discern that a certain parameter is kept fixed to an incorrect value.

In what follows we apply equation (12) to our neutrino Fisher matrices so that we can understand if future weak lensing and CMB data will achieve enough sensitivity to require a parameterization of the mass hierarchy. All results showed below are for the combination of the Euclid and Planck experiments and hence the Fisher matrix used in the calculation of Bayes factor is the total Fisher matrix $F_{\alpha\beta}^{tot} = F_{\alpha\beta}^{WL} + F_{\alpha\beta}^{CMB}$. We assume that the true model $M$ is represented by the 10-parameter model described in the previous section with a normal hierarchy scheme for neutrinos ($\alpha = 0.95$) and $\sum m_\nu = 0.055\text{eV}$. We next consider a simpler 9-parameter model $M'$ in which the parameter $\alpha$ is fixed to a certain value, shifted of an amount $\delta\alpha$ with respect to $\alpha = 0.95$ and calculate the Bayes factor for these two competing models.

Results of the evidence calculation are shown in Fig. 5 where we plot $|\ln B|$ as a function of $|\delta\alpha|$. The horizontal lines corresponds to the values of the Jeffreys scale [30]: $|\ln B| < 1$ means inconclusive evidence, $1 < |\ln B| < 2.5$ is a substantial evidence, $2.5 < |\ln B| < 5$ is a strong evidence and $|\ln B| > 5$ is considered decisive evidence. The area leftward of the cusp corresponds to a Bayes factor $B > 1$ which in our case means evidence for the simpler model $M'$. For $|\delta\alpha| \gtrsim 0.24$, $B$ becomes smaller than 1 indicating that data would require the introduction of $\alpha$ in the analysis, and will be sensitive to differences between neutrino masses. In particular for $\delta\alpha \approx -0.62$, that represents the common (and under our hypothesis wrong) assumption of total degeneracy between neutrino masses, the data would give a strong evidence for model $M$ requiring a parameterization of neutrino mass differences.

If data will be able to give evidence for a hierarchy of neutrino masses then it is proper to verify what the effect of assuming degeneracy of masses, or a wrong hierarchy, on other cosmological parameters will be. As shown in [29] fixing one parameter to a wrong value causes a shift in the best fit value of other parameters according to [13]. We calculate the bias in cosmological parameters due to a wrong assumption for $\alpha$ and assuming normal hierarchy ($\alpha = 0.95$) as true model. The results are shown in Table [11] and show that assuming a degenerate hierarchy ($\delta\alpha = -0.62$) or an inverted hierarchy ($\delta\alpha = -0.9$) would cause a shift in other cosmological parameters comparable with the $1\sigma$ statistical error. Only the shift in $\sum m_\nu$ is significantly greater than the $1\sigma$ error due to the high degeneracy between $\alpha$ and $\sum m_\nu$.

These results confirm those of Fig. 5 the data will be accurate enough to require a parameterization of the mass splitting, this causes a non-negligible bias in other parameters in the case of a wrong assumption. However the bias on cosmological parameters is generally smaller than the $1\sigma$ uncertainties. No-
shown in Fig. 6 where we have plotted the contour values of $|\ln B|$ as a function of $[\delta \alpha]$ and $\delta \Sigma m_\nu$. The inner contours from 1 to 5 correspond to values $B > 1$, and so to an evidence which favours the simpler model, while the outer contours are relative to values $B < 1$ and hence to evidence for the more complicated model $M$. The star in the plot refers to the wrong assumption of total degeneracy (but fixing the total mass to the true value $\delta \Sigma m_\nu = 0$). As one can see, under these assumptions the data would favour the simpler model giving a substantial evidence for $M''$.

The evidence for the simpler model is due to the smaller number of parameters of $M''$ with respect to $M$ (the Occams razor term, see [20]) and also to the negative degeneracy between $\alpha$ and $\Sigma m_\nu$ (given by the off-diagonal term of the inverted full Fisher matrix $(F^{-1})_{\alpha \Sigma m_\nu} \sqrt{(F^{-1})_{\alpha \alpha}(F^{-1})_{\Sigma m_\nu \Sigma m_\nu}} \simeq -0.44$). Because of this degeneracy there is a region of confusion in the plane of Fig. 6 in which a wrong assumption in $\alpha$ is compensated by a wrong, and opposite in sign, assumption for $\Sigma m_\nu$, leading to evidence in favour of the simpler model $M''$. This explains why in Fig. 6 for a fixed $\delta \alpha$ the the Bayes factor initially increases becoming greater than 1 for $\delta \Sigma m_\nu$ increasing. Of course when $\delta \Sigma m_\nu$ becomes large enough the Bayes factor decreases because the data start to favour the true model $M$.

If one assumes an inverted hierarchy (represented by the square in the plot), for example fixing the total mass to the minimum value allowed for this hierarchy ($\Sigma m_\nu \simeq 0.11$eV) and $\alpha$ to a low value (we take $\alpha = 0.05$), one obtains a strong evidence for the more complicated model $M$.

These evidence calculations indicate that a future weak lensing survey could become sensitive to the hierarchy of neutrino masses requiring a suitable parameterization of mass splitting. We note however that even if a model assuming total degeneracy could be still favoured. This indicates that Planck and Euclid data could give strong evidence for the existence of a hierarchy among neutrino masses.

| Parameter | Inverted | Degenerate |
|-----------|----------|------------|
| $w$       | 0.041    | -0.047     | -0.033     |
| $\Omega_b h^2$ | $10^{-4}$ | -0.4 $\cdot 10^{-4}$ | -2.7 $\cdot 10^{-5}$ |
| $\Omega_c h^2$ | 0.00065  | 0.0013     | 0.00090    |
| $h$       | 0.013    | 0.0049     | 0.0036     |
| $\tau$    | 0.0028   | -0.0012    | -0.00082   |
| $n_s$     | 0.0022   | -0.0036    | -0.0024    |
| $A_s$     | $1.44 \cdot 10^{-11}$ | $5.75 \cdot 10^{-12}$ | $3.94 \cdot 10^{-12}$ |
| $\Sigma m_\nu$(eV) | 0.037     | 0.086      | 0.060       |
| $dn_\nu/d\ln k$ | 0.0031     | -0.0019    | -0.0012    |

TABLE III: 1σ errors on cosmological parameters and the bias ($\delta$) due to a wrong assumption in the neutrino mass hierarchy.

FIG. 5: absolute value of $\ln B$ as a function of $[\delta \alpha]$. The lines indicates the limits of the Jeffreys scale. On the right of the cusp is $B < 1$, meaning evidence favours a more general parameterization of neutrino mass hierarchy.

FIG. 6: Jeffreys scale contours of $\ln B$ as a function of $\delta \alpha$ and $\delta \Sigma m_\nu$. The inner part of the plot (from the innermost $\ln B = 1$ contour) corresponds to values $B > 1$ and hence evidence for the simpler model (see text). The cross indicates the assumption of degenerate masses fixing the total mass to the correct value; the square indicates a typical inverted hierarchy scenario.

Table exceptions include the dark energy equation of state parameter. Assuming total degeneracy of neutrino masses, the shift on $w$ is smaller than 1σ but becomes slightly greater assuming an inverted hierarchy. Our Fisher matrix analysis indicates that this is due essentially to the shift in $\Sigma m_\nu$, and hence to the degeneracy $w-\Sigma m_\nu$ because there is no significant correlation between $w$ and $\alpha$.

We also show the results of an evidence calculation for an even simpler model $M''$, in which both the total mass $\Sigma m_\nu$ and $\alpha$ are fixed, over the model $M$. Note that now we are comparing an 8-parameter model with a 10-parameter model. The results are
VI. CONCLUSIONS

In this paper we have investigated the ability of future cosmic shear measurements, like those achievable with the proposed Euclid mission, to constrain differences in the mass of individual neutrino species. Using an explicit parameterization of neutrino mass splitting and a Fisher matrix formalism we have found that the combination Euclid+Planck will reach enough sensitivity to put constraints on the fraction of mass in the third neutrino mass eigenstate ($\alpha$) under the assumption that the neutrino mass scheme is described by a normal hierarchy, with fiducial value $\alpha = 0.95$: $\sigma_\alpha = 0.22$. We have also investigated the parameterization dependence of our results, repeating our forecasts calculation for different fiducial values of $\alpha$ and finding that constraints on $\alpha$ are generally decreasing for increasing $\alpha$. We have found a loss of sensitivity around $\alpha \approx 0.88$ due to a strong rotation of the degeneracy in the plane $\sum m_\nu - \alpha$ in the Euclid Fisher matrix, suggesting a considerable dependence of the results on the fiducial value around this point. We have then studied more deeply the power of detecting neutrino mass splitting considering Bayesian degeneracies with $\alpha$, also shown that assuming a wrong hierarchy can introduce bias constraints on other cosmological parameters, in particular with those parameters involved in degeneracies with $\sum m_\nu$ and $\alpha$. For the dark energy equation of state we found a bias greater than the 1$\sigma$ statistical error ($\sigma_w = 0.041$ from Euclid+Planck) assuming an inverted hierarchy $d_w = -0.047$ and comparable with 1$\sigma$ assuming degenerate masses $\delta_w = -0.033$.

In conclusion we emphasize that, even if these constraints are strongly dependent on the parameterization used, the possibility of having a splitting in neutrino masses cannot be neglected in analysing future data from Euclid-like experiments. A wrong assumption about neutrino mass hierarchies can indeed cause non-negligible bias on other cosmological parameters and in particular on the dark energy equation of state.

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