Revisiting the Soltan Argument Based on a Semianalytical Model for Galaxy and Black Hole Evolution

Hikari Shirakata$^{1,2}$, Toshihiro Kawaguchi$^3$, Takashi Okamoto$^1$, Masahiro Nagashima$^4$, and Taira Oogi$^5$

$^1$Department of Cosmuscences, Graduate School of Science, Hokkaido University, N10 W8, Kitaku, Sapporo, 060-0810, Japan; shirakata@astro1.sci.hokudai.ac.jp
$^2$The Technical Research Center, Tadano Ltd., 2217-13, Hayashi-machi, Takamatsu, Kagawa, 761-0301, Japan
$^3$Department of Economics, Management and Information Science, Onomichi City University, 1600-2, Hisayamada, Onomichi, Hiroshima, 722-8506, Japan
$^4$Faculty of Education, Bunkyo University, 3337, Minami-ogishima, Koshigaya, Saitama 343-8511, Japan
$^5$Kavli Institute for the Physics and Mathematics of the Universe, Todai Institutes for Advanced Study, the University of Tokyo, 5-1-5, Kashiwanoha, Kashiwa, 277-8583 Japan

Received 2019 November 8; revised 2020 May 30; accepted 2020 June 2; published 2020 July 23

Abstract

We show the significance of the super-Eddington accretion for the cosmic growth of supermassive black holes (SMBHs) with a semianalytical model for galaxy and black hole evolution. The model explains various observed properties of galaxies and active galactic nuclei at a wide redshift range. By tracing the growth history of individual SMBHs, we find that the fraction of the SMBH mass acquired during the super-Eddington accretion phases to the total SMBH mass becomes larger for less massive black holes and at higher redshift. Even at $z \sim 0$, SMBHs with $M_{\rm BH} > 10^7 M_{\odot}$ have acquired more than 50% of their mass by super-Eddington accretions, which is apparently inconsistent with the classical Soltan argument. However, the mass-weighted radiation efficiency of SMBHs with $M_{\rm BH} > 10^7 M_{\odot}$ obtained with our model, is about 0.08 at $z \sim 0$, which is consistent with Soltan’s argument within the observational uncertainties. We, therefore, conclude that Soltan’s argument cannot reject the possibility that SMBHs are grown mainly by super-Eddington accretions.

Unified Astronomy Thesaurus concepts: Active galactic nuclei (16); Supermassive black holes (1663); Galaxy formation (595)

1. Introduction

Almost all galaxies at $z \sim 0$ have a supermassive black hole (SMBH) at their center (e.g., Magorrian et al. 1998). SMBHs are considered to have grown by gas accretion (Salpeter 1964; Lynden-Bell 1969) and BH–BH coalescence (e.g., Kauffmann & Haehnelt 2000). When the gas accretion occurs, the SMBH can be observed as an active galactic nucleus (AGN), which emits vast radiation when the material gets accreted onto the SMBH. The radiative energy per unit time, $L_{\rm bol}$ (i.e., the bolometric luminosity of an AGN), can be described by the gas accretion rate, $\dot{M}$, as

$$L_{\rm bol} = \epsilon \dot{M} c^2,$$ (1)

where $c$ and $\epsilon$ are the speed of light and the radiation efficiency, respectively. The mass increment per unit time for a black hole (BH), $\dot{M}_{\rm BH}$, is described as $\dot{M}_{\rm BH} = (1 - \epsilon) \dot{M}$.

As an indicator for how rapid an SMBH grows, the luminosity and accretion rate normalized by the Eddington limit have been employed. The Eddington luminosity and Eddington accretion rate are defined as

$$L_{\rm Edd} = \frac{4 \pi c G M_p m_p \sigma_T}{\sigma_T} M_{\rm BH},$$ (2)

$$\dot{M}_{\rm Edd} = L_{\rm Edd}/c^2,$$ (3)

where $G$, $m_p$, $\sigma_T$, and $M_{\rm BH}$ are the gravitational constant, proton mass, cross-section for the Thomson scattering, and the mass of a BH, respectively. Assuming $\epsilon \sim 0.1$ for the sub-Eddington accretion rate, the gravitational force balances radiative pressure on the accreted gas in a spherical accretion and illumination case at $\dot{M} \sim 10 \dot{M}_{\rm Edd}$ (i.e., $L_{\rm bol} \sim L_{\rm Edd}$).

The radiation efficiency, $\epsilon$, depends on the SMBH spin, defined as $a = c J / M_{\rm BH}^2$ ($J$ is the angular momentum of the BH), the Eddington ratio defined as $L_{\rm bol} / L_{\rm Edd}$, and the innermost radius of the accretion disk. Assuming that the disk extends down to the innermost stable circular orbit, $\epsilon$ is $\sim 0.06$ with $a = 0$ (i.e., the Schwarzschild BH) and $\sim 0.43$ with $a \rightarrow 1$ (i.e., the Kerr BH; Bardeen 1970). The properties of the accretion disk also depend on the Eddington ratio. The efficiency becomes maximum at $L_{\rm bol} / L_{\rm Edd} \sim 0.01$–1 and decreases at lower and higher $L_{\rm bol} / L_{\rm Edd}$ regimes (Abramowicz et al. 1988) due to the effects of photon trapping (at $L_{\rm bol} / L_{\rm Edd} \gtrsim 1$) and advection cooling (Begelman 1978). The dependence of the radiation efficiency on the Eddington ratio has been investigated by several authors (e.g., Mineshige et al. 2000; Watarai et al. 2000; Kawaguchi 2003).

The contribution of the super-Eddington accretion to the cosmic growth of SMBHs is also important for understanding the coevolution of SMBHs and galaxies via outflow (Zamanov et al. 2002; Aoki et al. 2005; Komossa et al. 2008) and for constraining the mass of seed black holes. Observations have found luminous quasars at $z > 6$, whose SMBH masses are estimated as $>10^7 M_{\odot}$ (e.g., Mortlock et al. 2011; Wu et al. 2015; Bañados et al. 2018). Such SMBHs at high redshift tend to have shorter timescales from their birth to the observed time than local SMBHs. To explain the existence of such SMBHs at $z \sim 6$, SMBHs should have grown with a higher Eddington ratio or should form in a very early epoch of the universe, or their seed black hole mass should be large (namely, $>10^7 M_{\odot}$).

As a case study with a semianalytic model of galaxy formation (hereafter SA model), Pezzulli et al. (2016) suggest that a luminous QSO at $z \sim 6$, SDSS J1148 + 5251, obtains $\sim 80\%$ of its mass at a super-Eddington accretion rate owing to its dense and gas-rich environment. Also, various theoretical
studies investigate the environment of the seed BHs of $z \sim 6$ QSOs with analytical methods (e.g., Madau & Rees 2001; Omukai et al. 2008; Tanaka & Haiman 2009), hydrodynamical simulations (e.g., Hirano et al. 2014; Chon et al. 2016; Regan et al. 2019), and SA models (e.g., Valiante et al. 2016). The environment and the effect of high radiative pressure of BHs with super-Eddington accretions have been investigated (e.g., Inayoshi et al. 2016).

In our previous paper (Shirakata et al. 2019a), we presented the theoretical predictions of Eddington ratio distribution functions (ERDFs) of AGNs by using an SA model “New Numerical Galaxy Catalogue ($i$1GCG)” (Makiya et al. 2016; Shirakata et al. 2019b). In Shirakata et al. (2019a), we found that SMBH growths via super-Eddington accretions become more significant at higher redshift and for less massive SMBHs. In this paper, we analyze our model data in the same way as Soltan’s argument and conclude that our results of the significance of the super-Eddington accretion is consistent with Soltan’s argument. Before showing our main results, we review Soltan’s argument (Section 2) and briefly describe the growth model of SMBHs and the analysis method (Section 3). In Section 4, we show how significant the super-Eddington accretion is, by addressing the mass fraction of SMBHs acquired through super-Eddington growth, and compare the model results with observational data in the same manner as Soltan’s argument. Finally, we discuss the consistency between our model results and Soltan’s argument and summarize our results in Section 5. Unless otherwise stated, we employ the N-body simulation (Ishiyama et al. 2015) with the box size 560$h^{-1}$ Mpc and 4096$^3$ particles (the smallest halo mass is $8.79 \times 10^8 M_\odot$ with 40 dark matter particles). The parameters used in this paper are the same as those in Shirakata et al. (2019b) and Shirakata et al. (2019a).

2. Points to Review in Soltan’s Argument

Soltan’s argument (Soltan 1982) is one of the well-known discussions on the cosmic growth of SMBHs. To understand the significance of the gas accretion for the cosmic growth of SMBHs, the following two values have been compared:

$$\rho_{BH}(z=0) = \int_{\log(M_{BH,min})}^{\infty} \rho_{BH}(M_{BH}, z=0) d\log M_{BH},$$

$$\rho_{AGN}^{acc}(z=0) = \int_{\log(L_{bol,min})}^{\infty} d\log L_{bol} \int_{0}^{\infty} (1-\epsilon)L_{bol} \Phi_{AGN}(L, z) \frac{dt}{dz},$$

where $\rho_{BH}(z)$ and $\rho_{AGN}^{acc}(z)$ are the SMBH mass density at redshift $z$, and the accreted gas mass density from $z = \infty$ to $z$, respectively. The AGN luminosity function (LF), $\Phi_{AGN}$, and SMBH mass function (MF), $\Phi_{BH}$, should be observables. Yu & Tremaine (2002, hereafter YT02) compared $\rho_{BH}$ and $\rho_{AGN}^{acc}$, which are obtained from type-1 QSO LFs, and found that they become comparable to each other when $\epsilon \sim 0.1 - 0.3$ is assumed. Therefore, SMBHs are considered to have grown mainly by the gas accretion, not BH–BH coalescence.

Soltan’s argument also constrains how rapid the SMBH growth is. Since $\epsilon \sim 0.1 - 0.3$ is consistent with the standard accretion disk (Shakura & Sunyaev 1973), it is often interpreted that SMBHs would have grown mainly by sub-Eddington accretions. This scenario is also supported by observational studies of the ERDF at $z \sim 0$ (e.g., Schulze & Wisotzki 2010). One might conclude that the super-Eddington accretion is rare, and it is unimportant for the cosmic growth of the SMBHs.

However, other observational studies (e.g., McLure & Dunlop 2004; Nobuta et al. 2012; Kelly & Shen 2013) suggest that the super-Eddington accretion becomes more common at higher redshift. Also, several authors (e.g., Mortlock et al. 2011; Wu et al. 2015; Bañados et al. 2018) have found that QSOs (i.e., the brightest class of AGNs) at $z \gtrsim 6$ with $M_{BH} > 10^8 M_\odot$ are growing at $\lambda_{Edd} \gtrsim 1$. On the theoretical side, some studies have found that the super-Eddington accretions should play a role in the cosmic growth of SMBHs by using hydrodynamic simulations (e.g., Anglés-Alcázar et al. 2017) and SA models (e.g., Shirakata et al. 2019a). These recent findings may conflict with Soltan’s argument.

One can argue that $\epsilon$, based on various observations, does not necessarily indicate that its value lies tightly between 0.1 and 0.3 and that super-Eddington accretion could be the dominant mode for the SMBH growth as follows. First, as shown in Kawaguchi et al. (2004) in detail, $\rho_{BH}(z=0)$ and $\rho_{AGN}^{acc}(z=0)$, described in Equations (4) and (5), are governed by SMBHs with $\sim 10^{8-9} M_\odot$. As shown below, properties and amounts of BHs with $< 10^8 M_\odot$ are neglected. LFs and MFs are fitted by double power-law functions. Each function has a steep slope at the luminous/massive end and a flat slope at the other end. Such a function has a point (hereafter “knee”) at which the slope becomes $-1$. Assuming such functions, the integration values of LFs and MFs are mostly determined by the values around the knees, with a weak dependence on the slopes of the LFs/MFs below the knees. In Soltan’s argument, $\rho_{BH}(z=0)$ is obtained by the SMBH MFs, whose knee is located at $M_{BH} \sim 1.4 - 3.5 \times 10^9 M_\odot$ (Shankar et al. 2004). Therefore, details (census and activity) on the SMBHs with $M_{BH} < 10^9 M_\odot$ have little influence on the $\rho_{AGN}^{acc}(z=0)$ and $\rho_{BH}(z=0)$.

Second, $\rho_{BH}(z=0)$ and $\rho_{AGN}^{acc}$ still have large uncertainties to put a constraint on the value of $\epsilon$ (see Novak 2013 for more details). As for $\rho_{BH}(z=0)$, several empirical scaling relations between the SMBH mass and host bulge properties such as the luminosity, velocity dispersion, and stellar mass have been employed for obtaining the local SMBH MF. YT02 adopted $\rho_{BH}(z=0) = (2.5 \pm 0.4) \times 10^3 M_\odot$ Mpc$^{-3}$, assuming a relation between the SMBH mass and velocity dispersion. Vika et al. (2009) derived $\rho_{BH}(z=0) \sim 4.9 \times 10^3 M_\odot$ Mpc$^{-3}$, using a relation between the SMBH mass and bulge luminosity. Tucci & Volonteri (2017) assumed an analytic expression for the local SMBH MF with a Schechter shape and a Gaussian scatter. They suggested that $\rho_{BH}(z=0) = 4.3(6.6) \times 10^3 M_\odot$ Mpc$^{-3}$ for a Gaussian scatter of 0.3 (0.5) dex. Considering the relation between SMBH mass and the Sérsic index for bulge surface density profile and its error, Mutlu-Pakdil et al. (2016) estimated $\rho_{BH}(z=0) = 2.04^{+1.16}_{-0.75} \times 10^3 M_\odot$ Mpc$^{-3}$, besides, the value of $\rho_{BH}(z=0)$ suffers another uncertainty if $M_{BH}$ inferred from the emission line width is underestimated (Kormendy & Ho 2013). As for $\rho_{AGN}^{acc}(z=0)$, the bolometric correction from the AGN B-band or X-ray luminosity has large uncertainties. For example, the bolometric correction from the $B$-band luminosity, $C_{B}$, adopted in YT02 is 11.8, while, according to a later study (Marconi et al. 2004), the value is about 7 for
QSOs. Even if these uncertainties of the bolometric correction and $\rho_{BH}(z = 0)$ are less than 1 dex, the value of $\epsilon$ could change drastically.

The evolution of AGN LFs also involves large uncertainties. In Soltan (1982) and YTO2, $\rho_{AGN}(z = 0)$ was estimated from optical AGN LFs. This means that only the contribution of “type-1” QSOs was considered. The values are $4.7 \times 10^{4}(0.1/\epsilon) M_{\odot} \text{Mpc}^{-3}$ (Soltan 1982) and $2.1 \times 10^{5}(C_{0}/11.8)[0.1(1 - \epsilon)/\epsilon] M_{\odot} \text{Mpc}^{-3}$ (YTO2). The evolution of the shape of optical AGN LFs has been under discussion, which largely affects the value of $\rho_{AGN}^{acc}(z = 0)$. Optical AGN LFs have been assumed as a double power-law shape:

$$\Phi_{AGN}(M_{B}, z) = \Phi_{AGN}^{\star} \left( \frac{M_{B} - m_{crit}}{10^{0.4/[0.6 + (z - z_{0})]} + 0.6} \right)^{-1},$$

where $M_{B}$ is the B-band magnitude, $\Phi_{AGN}^{\star}$, $\beta_{1}$, $\beta_{2}$, $M_{B}^{\star}(z)$ are adjustable parameters. In YTO2, the characteristic magnitude, $M_{B}^{\star}(z)$, was assumed to evolve as:

$$M_{B}^{\star}(z) = -21.14 + 5 \log h - 2.5(1.36z - 0.27z^{2}),$$

although recent observations find no such strong evolution of $M_{B}^{\star}$ especially at $z > 3$ (e.g., Akiyama et al. 2018). If there is no strong evolution of $M_{B}^{\star}$, $\rho_{AGN}^{acc}(z = 0)$ obtained with the evolution of $M_{B}^{\star}$ (Equation 6) is overestimated. Recent analysis, on the other hand, obtains $\rho_{AGN}^{acc}(z = 0)$ from the integration of X-ray AGN LFs, that is, the contribution of both type-1 and type-2 objects is considered. Shankar et al. (2009) found $\rho_{AGN}^{acc}(z = 0)$ should be $\sim 5 \times 10^{3} M_{\odot} \text{Mpc}^{-3}$ with $\epsilon \sim 0.075$ so that $\rho_{AGN}^{acc}(z = 0)$ becomes $\sim \rho_{BH}(z = 0)$.

Given the various uncertainties above, the dominance of the sub-Eddington accretion suggested by Soltan’s argument is worth reassessing. In other words, the predominance of the super-Eddington accretion should be carefully considered.

3. Methods

3.1. SMBH Growth in the Semi-analytic Model

We briefly describe the modeling of the SMBH growth (see Shirakata et al. 2019b for more details). The seed BH mass is $10^{3} M_{\odot}$ for all seed BHs, which are placed when galaxies newly form. We assume that an SMBH grows with its host bulge via starbursts induced by galaxy mergers and/or disk instabilities. The accreted gas mass onto the SMBH, $\Delta M_{acc}$, and stellar mass formed by a starburst, $\Delta M_{\text{star, burst}}$, have the following relation:

$$\Delta M_{acc} = f_{BH} \Delta M_{\text{star, burst}},$$

where $f_{BH} = 0.02$ is chosen to reproduce the local BH mass—bulge mass relation.

The gas accretion rate is described as follows:

$$\dot{M}(t) = \frac{\Delta M_{acc}}{t_{acc}} \exp\left( -\frac{t - t_{\text{start}}}{t_{acc}} \right),$$

where $t_{\text{start}}$ and $t_{acc}$ are the starting time of the accretion and the accretion timescale per one accretion event, respectively.\(^{7}\)

\(^{5}\) The seed BH mass does not largely affect the properties of AGNs and SMBHs at $z \lesssim 6$, since the seed mass is negligible compared with the total amount of the accreted gas onto a BH (see Shirakata et al. 2016).

\(^{7}\) $M$ was described as $M_{BH}$ in Shirakata et al. (2019b).

We use the same model of the accretion timescale as Shirakata et al. (2019b), $t_{acc} = \tau_{\text{bulge}} f_{\text{dyn, bulge}} + \tau_{\text{loss}}$. The first term of the right-hand side is proportional to the dynamical time of their host bulges, $f_{\text{dyn, bulge}}$, where the value of the free parameter, $\tau_{\text{bulge}}$, is 0.58. The second term represents the timescale for the angular momentum loss in the “gas reservoir” (e.g., circumnuclear disks) and accretion disk. We define $t_{\text{loss}}$ as $t_{\text{loss,0}} \left( M_{BH}/M_{\odot} \right)^{\beta} \left( \Delta M_{acc}/M_{\odot} \right)^{\gamma}$, where the values of the free parameters, $t_{\text{loss,0}}$, $\beta$, and $\gamma$, are 1.0 Gyr, 3.5, and -4.0, respectively (see Shirakata et al. 2019b).

The AGN bolometric luminosity is described with $\dot{m} = M/\dot{M}_{\text{Edd}}$ (with $\dot{M}_{\text{Edd}} = L_{\text{Edd}}/c^{2}$) as

$$\lambda_{\text{Edd}} = \frac{L_{\text{bol}}}{L_{\text{Edd}}} = \left[ 1 + 3.5 \{ 1 + \tanh(\log(\dot{m}/\dot{m}_{\text{crit}})) \} + \frac{\dot{m}_{\text{crit}}}{\dot{m}} \right]^{-1}. \quad (9)$$

We employ the formula, Equation (9), based on Kawaguchi (2003), which takes into account various corrections (e.g., gravitational redshift, transverse Doppler effect). In this paper, $\dot{m}_{\text{crit}} = 10$ is assumed. The radiation efficiency, $\epsilon = (L_{\text{bol}}/M^{2})$, is defined as $\lambda_{\text{Edd}}/\dot{m}$. In the super-Eddington regime (i.e., $\dot{m} > \dot{m}_{\text{crit}}$), $\epsilon$ gradually decreases from 0.1 at $\dot{m} = \dot{m}_{\text{crit}}$ to $\epsilon \sim 0.2$ at $\dot{m} \sim 30$, and to $\epsilon \sim 0.01$ at $\dot{m} \sim 600$ (see Figure 1 in Shirakata et al. 2019a). For obtaining $B$-band luminosity of AGNs, we employ the bolometric correction obtained by Marconi et al. (2004).

3.2. Analysis for the Significance of the Super-Eddington Growth

To evaluate the significance of the super-Eddington accretion, we calculate the accreted masses, $\Delta M_{se}(z), \Delta M_{QSO}(z)$, and $\Delta M_{se,QSO}(z)$, as follows:

$$\Delta M_{se}(z) = \sum_{z} \int_{z}^{\infty} M_{BH, i}(z') \frac{dt}{dz'} dz',$$

$$\text{for } \dot{m} > \dot{m}_{\text{crit}}, \quad (10)$$

$$\Delta M_{QSO}(z) = \sum_{z} \int_{z}^{\infty} M_{BH, i}(z') \frac{dt}{dz'} dz',$$

$$\text{for } L_{\text{bol}} > 1.44 \left[ 10^{12} L_{\odot} \right], \quad (11)$$

$$\Delta M_{se,QSO}(z) = \sum_{z} \int_{z}^{\infty} M_{BH, i}(z') \frac{dt}{dz'} dz',$$

$$\text{for } \dot{m} > \dot{m}_{\text{crit}} \text{ and } L_{\text{bol}} > 1.44 \left[ 10^{12} L_{\odot} \right], \quad (12)$$

respectively. The summation $(i)$ is taken for the SMBH subsample determined by the SMBH mass at a redshift $z'$, $M_{BH, i}(z')$. We define QSOs as AGNs with $L_{\text{bol}} > 1.44 \left[ 10^{12} L_{\odot} \right]$, which corresponds to the absolute B-band magnitude, $M_{B, i} \sim -23.7$. By taking the ratio between $\Delta M_{se}$ (or $\Delta M_{QSO}$ or $\Delta M_{se,QSO}$) and the sum of the total SMBH mass, $\sum_{z} M_{BH, i}(z)$, we can estimate the importance of super-Eddington accretions (or of QSO phases or of super-Eddington accretions in QSO phases).
We also estimate the mass-weighted mean radiation efficiency, $\bar{\epsilon}(z)$, as

$$\bar{\epsilon}(z) = \frac{\sum_i \int_z^\infty \epsilon_i \dot{M}_{BH,i}(z') \frac{dz'}{dt} \, dz'}{\sum_i \dot{M}_{BH,i}(z)},$$

where $\epsilon_i$ is the radiation efficiency obtained from $\lambda_{Edd}/\dot{m}$ of the $i$-th SMBH.

4. Results

4.1. Comparison with Soltan’s Argument

We make a straightforward comparison with Soltan’s argument. Since our model reproduces observed AGN LFs at $0 < z < 6$ and local SMBH MF (Shirakata et al. 2019b), the model should return the consistent result with Soltan’s argument, i.e., $\rho_{\text{AGN}}^{\text{acc}}(z = 0)$ obtained from the QSO luminosity functions (with $\epsilon \sim 0.1$–0.3) becomes $\sim \rho_{\text{BH}}(z = 0)$.

First, we obtain $\rho_{\text{AGN}}^{\text{acc}}(z = 0)$ from the QSO luminosity functions at $0 \leq z \leq 5$ by our model. Following the conventional procedure, we fix $\epsilon = 0.1$ in converting the AGN luminosity into the mass accretion rate, while we have calculated the AGN luminosity in the model from Equations (8) and (9). The resultant $\rho_{\text{AGN}}^{\text{acc}}(z = 0)$ is $3.6 \times 10^5 M_\odot \text{Mpc}^{-3}$, including all type-1 and type-2 AGNs with $M_B < -23$, which is only 1.7 times larger than the $\rho_{\text{AGN}}^{\text{acc}}(z = 0)$ only for type-1 AGNs obtained by YT02 since the evolution of QSO LFs assumed in YT02 seems to be inconsistent with recent observational results (see also Section 2). The value of $\rho_{\text{AGN}}^{\text{acc}}(z = 0)$ obtained by our model becomes $9.4 \times 10^4 M_\odot \text{Mpc}^{-3}$, assuming that type-1 QSOs account for 16% of total AGNs with $M_B < -23$ (Shirakata et al. 2019b).

Second, we determine the lower limit of the integration, $M_{\text{BH,min}}$, in Equation (4). By integrating the model SMBH MF at $z \sim 0$ obtained by our model, we find that $\rho_{\text{AGN}}^{\text{acc}}(z = 0)$ becomes $\sim \rho_{\text{BH}}(z = 0)$, when $M_{\text{BH,min}}$ is $\sim 1.7 \times 10^4 M_\odot$ (under the assumption that all AGNs are type-1) and $\sim 1.1 \times 10^5 M_\odot$ (under the assumption that type-1 QSOs account for 16% of total AGNs with $M_B < -23$, Shirakata et al. 2019b). Given that the knee of the SMBH MF at $z \sim 0$ places at $1.4 - 3.5 \times 10^5 M_\odot$ (Shankar et al. 2004), the model result without obscuration (i.e., the same assumption as YT02) is consistent with Soltan’s argument since the value of $\rho_{\text{BH}}$ is determined by the value around the knee, as described in Section 2. Therefore, we conclude that the SMBHs with $M_{\text{BH}} \gtrsim 1.7 \times 10^5 M_\odot$ have grown mainly by gas accretions during QSO phases.

4.2. Growth History of Individual SMBHs

In order to show the redshift evolution in another way, we trace the evolution of individual SMBHs with $\log(M_{\text{BH}}(z')/M_\odot) = [7, 8], [8, 9]$, and $>9$ in Figure 1. Each panel shows the distribution of $M_{\text{BH}}/M_{\text{BH},i}$ at $z = 0, 2$, and 4 for each SMBH mass bin. By investigating the distribution, we assess how typical AGNs at each redshift- and mass-bins behave. The peak of the distribution moves toward the higher $M_{\text{BH}}/M_{\text{BH},i}$ at higher redshift, meaning that higher-$z$ SMBHs have greater contribution of super-Eddington accretion. However, even SMBHs with $M_{\text{BH}}(z = 0) > 10^5 M_\odot$ have more than 50% of their mass by super-Eddington accretions. The same suggestion is obtained in Shirakata et al. (2019a).

Figure 2 shows the evolution of $\Delta M_{\text{BH}}(z)/\sum_i M_{\text{BH},i}(z)$, $\Delta M_{\text{QSO}}(z)/\sum_i M_{\text{BH},i}(z)$, $\Delta M_{\text{QSO},\text{QSO}}(z)/\sum_i M_{\text{BH},i}(z)$, $\Delta M_{\text{QSO}}(z)/\Delta M_{\text{QSO}}(z)$, respectively. The vertical axis in the first panel (top left panel) corresponds to the horizontal axis of Figure 1. From this figure, we find that

1. for SMBHs with $10^3 M_\odot < M_{\text{BH}}(z = 0) < 10^5 M_\odot$, about half of the super-Eddington growth occurs at less luminous AGN phases, not QSO phases (by comparing panels (1) and (2));
2. even SMBHs with $10^5 M_\odot < M_{\text{BH}}(z = 0)$, SMBHs do not acquire their whole mass in QSO phases (panel (3)); and
3. typical QSOs at any redshift and mass bins have acquired their masses mostly at super-Eddington phases (panel (4)). In other words, a significant fraction of QSOs at any redshift are expected to show $\lambda_{\text{Edd}} \gtrsim 1$. This is consistent with the result by Collin et al. (2002) who found that about half of nearby bright QSOs (PG QSOs) are accreting close to or exceeding the Eddington rate.
Even super-Eddington growth plays a role in the cosmic growth of SMBHs, the probability with which we can observe super-Eddington accreting SMBHs is low. We investigate the duration of each super-Eddington accretion episode in Figure 3. The median value of the duration obtained from our model becomes shorter at higher redshift; 12 Myr at $z \sim 0$ and 4 Myr at $z \sim 6$, although super-Eddington accretion becomes more common at higher redshift. The decreasing trend with redshift results from shorter $t_{\text{acc}}$ at higher redshift. Due to the short duration of the super-Eddington phase, the fraction of super-Eddington accreting SMBHs with $10^{7.8}$--$M_\odot$ at an output time in our model, for example, is only $\sim 4 \times 10^{-3}$% among all SMBHs (and $\sim 6.6$% in all $\lambda_{\text{Edd}} > 0.01$ AGNs) at $z \sim 0$, and $\sim 1$% (and $\sim 36.3$%) at $z \sim 6$ (the bottom panel of Figure 3).

Examples of actual $\lambda_\text{sd}$ history of each SMBH is shown in Figure 4. We choose three SMBHs with $M_{\text{BH}} = 10^{8.0}$--$M_\odot$ at $z \sim 0$, which have acquired $\sim 20$, 50, and 80% of their mass with super-Eddington accretions. As shown in Figure 3, super-Eddington accretions do not last long and accretion rates stay at $\dot{m} < 10$ for most of their lives.

In the panel (1) of Figure 2, we also show that the less massive SMBHs have a higher value of $\Delta M_{\text{ac}}/M_{\text{BH}}$, which was also mentioned in detail in Shirakata et al. (2019a) with the same model as this paper. Recent observations show that AGNs with less massive BHs tend to have higher values of $\lambda_{\text{Edd}}$ at $z \sim 0$, which is qualitatively consistent with our model prediction. For example, observational samples of Dong et al. (2012) and Liu et al. (2018) with $10^{5.5}$--$M_\odot$ of SMBH mass at $z \sim 0$ have the $\lambda_{\text{Edd}}$ distribution peaking at $\log(\lambda_{\text{Edd}}) \sim -0.4$. For more massive SMBHs, Figure 3 of Schulze & Wisotzki (2010) shows the number distribution of $\lambda_{\text{Edd}}$ for AGNs with $M_{\text{BH}} \sim 10^{7.8}$--$M_\odot$ from the Hamburg/ESO Survey. The distribution peaks at $\log(\lambda_{\text{Edd}}) \sim -1$, which is smaller than for the less massive SMBHs. The data of SDSS QSO shows the similar distribution at $z = 0.4$, peaking at $\log(\lambda_{\text{Edd}}) \sim -0.8$, with $M_{\text{BH}} \sim 10^7M_\odot$ (Kelly & Shen 2013).

5. Discussion and Conclusions

We have investigated the growth history of SMBHs by using an SA model, which explains various observed properties of galaxies and AGNs at a wide redshift range. Since Soltan’s argument is based on the AGN LFs at $z < 6$ and the local SMBH MF, the growth processes of SMBHs in our model should satisfy the constraints by the argument, which imply that the SMBHs have grown mainly via sub-Eddington accretion events. When we adopt the radiation efficiency of $\sim 0.1$ in the QSOs in our model AGNs and estimate the accreted mass during the QSO phase from $z = 6$ to 0, we obtain the mass density of SMBHs with $M_{\text{BH}} > 1.7 \times 10^9M_\odot$ at $z = 0$. This is in line with Soltan’s argument. We, however, find that even SMBHs with $M_{\text{BH}} > 10^9M_\odot$ at $z \sim 0$ have acquired more than 50% of their mass by the super-Eddington accretion and a significant fraction of QSOs at any redshift are expected to have undergone the super-Eddington accretion.

We have noted that super-Eddington accretions are difficult to observe since the durations of each super-Eddington accretion event is short; 12 Myr at $z \sim 0$ and 4 Myr at $z \sim 6$. Due to the short duration of the super-Eddington phase, the fraction of super-Eddington accreting SMBHs at an output time in our model is only $\sim 4 \times 10^{-3}$% among all SMBHs at $z \sim 0$, and $\sim 1$% at $z \sim 6$ (Figure 3). In other words, our model predicts that only $\sim 6.6$% of SMBHs in observed AGNs with $\lambda_{\text{Edd}} > 0.01$ are $\lambda_{\text{Edd}} > 1$ at $z \sim 0$. Therefore, just from observational data, we underestimate the importance of super-Eddington accretions for cosmic growth of SMBHs.

One might think that if SMBHs acquire their mass mainly by super-Eddington accretions, then the SMBH mass at $z \sim 0$ cannot be provided by observed QSOs with sub-Eddington accretions. Our model predictions, however, show no contradiction with the observations on which Soltan’s argument is based as we discussed above.

To understand well these apparently contradictory results, we investigate the evolution of the mass-weighted radiation efficiency, $\bar{\varepsilon}$ (Equation (13)), for different SMBH mass bins in Figure 5. Since super-Eddington accretion is more common at
higher redshift (see also the top panel of Figure 5), $\bar{\varepsilon}$ is small at high redshift (e.g., $\sim0.04$ at $z \sim 4$ with $\log(M_{\text{BH}}(z=4)/M_\odot) = [8, 9]$). In contrast, $\bar{\varepsilon}$ becomes larger, $\sim0.08$, at $z \sim 0$ for SMBHs with $\log(M_{\text{BH}}(z=0)/M_\odot) > 8$. This value of $\bar{\varepsilon} \sim 0.08$ is not rejected by Soltan’s argument given various uncertainties discussed in Section 2, although more than half of SMBHs with $M_{\text{BH}}(z=0) > 10^8 M_\odot$ in our model acquire their mass mainly by super-Eddington accretions as shown in Figure 2. This is because, (1) $\varepsilon$ decreases slowly toward higher $m$, and (2) the accretion with higher $m$ ($\dot{m} \gg \dot{m}_{\text{crit}}$) is rarer, considering the shape of ERDFs (Shirakata et al. 2019a).

Conclusions of this paper are as follows:

1. The Soltan argument does not reject the possibility that SMBHs are grown mainly by super-Eddington accretions, because assumptions employed in the classical Soltan argument (e.g., $z$-evolution of AGN LFs) do not match the current observational data.
2. When we take current statistical data (estimated values and their uncertainties) of galaxies and AGNs at face value, our semianalytical model suggests that SMBHs have grown mainly by super-Eddington accretions.
3. Further observations with smaller uncertainties will judge the conclusion of this paper.

As we discuss in Section 2, uncertainties in observational estimates for $\rho_{\text{BH}}$ and $\rho_{\text{AGN}}^{\text{acc}}$ are crucial in Soltan’s argument. For example, relations between SMBH mass and properties of the host galaxies are important. SMBHs also reside in the disk-dominated galaxies. In such cases, SMBH mass is difficult to estimate from the host galaxy’s properties and the uncertainty of $\rho_{\text{BH}}$ will become larger. The difference of AGN SEDs among individual AGNs and SMBHs and an increase of the number of AGNs with $M_{\text{BH}} < 10^7 M_\odot$ are also important. In this paper, we employ the bolometric correction independently of the SMBH mass or Eddington ratio, which is the same treatment as different semianalytic models (e.g., Fanidakis et al. 2012; Hirschmann et al. 2012; Menci et al. 2014). Theoretical and observational understandings of AGN SEDs will help to improve the discussion. When the uncertainties of observational estimates for $\rho_{\text{BH}}$ and $\rho_{\text{AGN}}^{\text{acc}}$ are reduced and/or when the shape of observed AGN LFs and SMBH MFs are determined with smaller errors, our conclusion in this paper will be reevaluated. However, the main conclusion of this paper that SMBHs predominantly acquire their mass through super-Eddington accretion remains unchanged (see Appendix). Also, if the averaged radiation efficiency at higher redshift is estimated, it will be helpful to judge the importance of super-Eddington accretions on the cosmic growth of SMBHs. As an example, Davies et al. (2019) estimate the averaged radiation efficiency of two $z > 7$ SMBHs from their mass and ionized region size. The estimated values are 0.08 and 0.1. The increase of the sample size is needed for statistical discussions.

We appreciate the detailed review and useful suggestions of the anonymous referee, which have improved our paper. H.S. has been supported by the Sasakawa Scientific Research Grant from the Japan Science Society (29-214), JSPS KAKENHI (18J12081), and a grant from the Hayakawa Satio Fund awarded by the Astronomical Society of Japan. T.K. has been supported by JSPS KAKENHI (17K05389) and the grant from the Urakami Scholarship Foundation. T.O. is financially supported by MEXT KAKENHI Grant (18H04333) and the Grant-in-Aid (19H01931). M.N. has been supported by the Grant-in-Aid (17H02867 and 18H05437) from the MEXT of Japan. This work was also supported in part by the World Premier International Research Center Initiative (WPI), MEXT, Japan, by MEXT Priority Issue 9 on Post-K Computer (Elucidation of the Fundamental Laws and Evolution of the Universe), and by JICFuS.

**Appendix**

**The Effect of the Parameter Choice**

Here we show the robustness of our main result, the predominance of the super-Eddington accretion for the cosmic growth of SMBHs. Our model reproduces current observational AGN LFs and SMBH MFs. If observational AGN LFs and SMBH MFs largely changed their shapes by future observations, the model parameters should take different values and the model prediction about the dominance of the super-Eddington accretion might change. We present the results with different parameter choices below and show that the main conclusion of this paper is quite general and does not change even with other extreme parameter choices.

As described in Section 3, the gas accretion rate is modeled as $\dot{M}(t) = \frac{\Delta M_{\text{acc}}}{t_{\text{acc}}} \exp\left(-\frac{t-t_{\text{coll}}}{t_{\text{acc}}}\right)$, where $\Delta M_{\text{acc}}$ is the total accreted gas mass. In principle, the Eddington ratio for an SMBH with a given mass becomes smaller with the smaller value of $\Delta M_{\text{acc}}$ or larger value of $t_{\text{acc}}$. We note that if we make $t_{\text{acc}}$ longer, bright AGNs become difficult to emerge since we assume that the maximum accretion rate is given as $\Delta M_{\text{acc}}/t_{\text{acc}}$.

We test several combinations of parameters, in which one parameter has a value different from the default one with the remaining parameters having the default values. The parameter values we test here are $\gamma_{\text{BH}} = 3, \gamma_{\text{BH}} = 4, t_{\text{coll}} = 0$, and $\log(\epsilon_{\text{SMBH}}) = -1.66$ (corresponding to a stronger AGN feedback that quenches the formation of massive galaxies at $z < 1$; Maiya et al. 2016 and Shirakata et al. 2019b). When we choose $\gamma_{\text{BH}} = 3, 4$ or $t_{\text{coll}} = 0$, we cannot reproduce the shape of AGN LFs especially at $z < 1$. When we choose $\log(\epsilon_{\text{SMBH}}) = -1.66$, massive galaxies at $z < 1$ cannot form because of the strong AGN feedback. The case with $\gamma_{\text{BH}} = 4,$
the accretion timescale of SMBHs becomes longer and possibly exceeds the cosmic age (depending on the SMBH mass). Figure A1 shows the results with different combinations of parameters, which is the same figure as Figure 2 (the fraction of mass acquired through super-Eddington accretion). Even the most extreme case with $\gamma_{BH} = 4$, $\Delta M_{se}/\Delta M_{BH}$ is reduced only $\sim 30\%$. As shown here, drastic changes of free parameters related with the SMBH growth do not have large impacts on the main results of this work.

**ORCID iDs**

Masahiro Nagashima https://orcid.org/0000-0003-2938-7096

**References**

Abramowicz, M. A., Czerny, B., Lasota, J. P., & Szuszkiewicz, E. 1988, ApJ, 332, 646

Akiyama, M., He, W., Ikeda, H., et al. 2018, PASJ, 70, S34

Anglés-Alcázar, D., Faucher-Giguère, C.-A., Quataert, E., et al. 2017, MNRAS, 472, L109

Aoki, K., Kawaguchi, T., & Ohta, K. 2005, ApJ, 618, 601

Baltados, E., Venemans, B. P., Mazzucchelli, C., et al. 2018, Natur, 553, 473

Bardeen, J. M. 1970, Natur, 226, 64

Begelman, M. C. 1978, MNRAS, 184, 53

Chon, S., Hirano, S., Hosokawa, T., & Yoshida, N. 2016, ApJ, 832, 134

Collin, S., Boisson, C., Mouchet, M., et al. 2002, A&A, 388, 771

Davies, F. B., Hennawi, J. F., & Eilers, A.-C. 2019, ApJL, 884, L19

Dong, R., Greene, J. E., & Ho, L. C. 2012, ApJ, 761, 73

Fanidakis, N., Baugh, C. M., Benson, A. J., et al. 2012, MNRAS, 419, 2797

Hirano, S., Hosokawa, T., Yoshida, N., et al. 2014, ApJ, 781, 60

Hirschmann, M., Somerville, R. S., Naab, T., & Burkert, A. 2012, MNRAS, 426, 237

Inayoshi, K., Haiman, Z., & Ostriker, J. P. 2016, MNRAS, 459, 3738

Ishiyama, T., Enoki, M., Kobayashi, M. A. R., et al. 2015, PASJ, 67, 61

Kauffmann, G., & Haehnelt, M. 2000, MNRAS, 311, 576

Kawaguchi, T. 2003, ApJ, 593, 69

Kawaguchi, T., Aoki, K., Ohta, K., & Collin, S. 2004, A&A, 420, L23

Kelly, B. C., & Shen, Y. 2013, ApJ, 764, 45

Komossa, S., Xu, D., Zhou, H., Storchi-Bergmann, T., & Binette, L. 2008, ApJ, 680, 926

Kormendy, J., & Ho, L. C. 2013, ARA&A, 51, 511

Liu, H.-Y., Yuan, W., Dong, X.-B., Zhou, H., & Liu, W.-J. 2018, ApJS, 235, 40

Lynden-Bell, D. 1969, Natur, 223, 690

Madau, P., & Rees, M. J. 2001, ApJL, 551, L27

Magorrian, J., Tremaine, S., Richstone, D., et al. 1998, AJ, 115, 2285

Makiya, R., Enoki, M., Ishiyama, T., et al. 2016, PASI, 68, 25

Marconi, A., Risaliti, G., Gilli, R., et al. 2004, MNRAS, 351, 169

McLure, R. J., & Dunlop, J. S. 2004, MNRAS, 352, 1390

Menci, N., Gatti, M., Fiore, F., & Lamastra, A. 2014, A&A, 569, A37

Mineshige, S., Kawaguchi, T., Takeuchi, M., & Hayashida, K. 2000, PASI, 52, 499

Mortlock, D. J., Warren, S. J., Venemans, B. P., et al. 2011, Natur, 474, 616

Mutlu-Pakdil, B., Seigar, M. S., & Davis, B. L. 2016, ApJ, 830, 117

Nobuta, K., Aoki, M., Ueda, Y., et al. 2012, ApJ, 761, 143

Novak, G. S. 2013, arXiv:1310.3833

Omomukai, K., Schneider, R., & Haiman, Z. 2008, ApJ, 686, 801

Pezzulli, E., Valiante, R., & Schneider, R. 2016, MNRAS, 458, 3047

Regan, J. A., Downes, T. P., Volonteri, M., et al. 2019, MNRAS, 486, 3892

Salpeter, E. E. 1964, ApJ, 146, 811

Schulze, A., & Wisotzki, L. 2010, A&A, 516, A87

Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337

The Astrophysical Journal, 898:63 (8pp), 2020 July 20 Shirakata et al.
Shankar, F., Salucci, P., Granato, G. L., de Zotti, G., & Danese, L. 2004, MNRAS, 354, 1020
Shankar, F., Weinberg, D. H., & Miralda-Escudé, J. 2009, ApJ, 690, 20
Shirakata, H., Kawaguchi, T., Okamoto, T., et al. 2016, MNRAS, 461, 4389
Shirakata, H., Kawaguchi, T., Oogi, T., Okamoto, T., & Nagashima, M. 2019a, MNRAS, 487, 409
Shirakata, H., Okamoto, T., Kawaguchi, T., et al. 2019b, MNRAS, 482, 4846
Soltan, A. 1982, MNRAS, 200, 115
Tanaka, T., & Haiman, Z. 2009, ApJ, 696, 1798
Tucci, M., & Volonteri, M. 2017, A&A, 600, A64
Valiante, R., Schneider, R., Volonteri, M., & Omukai, K. 2016, MNRAS, 457, 3356
Vika, M., Driver, S. P., Graham, A. W., & Liske, J. 2009, MNRAS, 400, 1451
Watarai, K.-y., Fukue, J., Takeuchi, M., & Mineshige, S. 2000, PASJ, 52, 133
Wu, X.-B., Wang, F., Fan, X., et al. 2015, Natur, 518, 512
Yu, Q., & Tremaine, S. 2002, MNRAS, 335, 965
Zamanov, R., Marziani, P., Sulentic, J. W., et al. 2002, ApJL, 576, L9