Abstract – In this paper we study the dynamical generation of mass in the Lorentz-violating \(CP(N-1)\) model defined in two- and three-dimensional aether-superspace. We show that even though the model presents a phase structure similar to the usual Lorentz-invariant case, the mass dynamically generated by quantum corrections has a dependence on the Lorentz-violating background properties, except for space-like LV vector parameter. This is to be contrasted with the behavior of the quantum electrodynamics in the two-dimensional aether-superspace, where the dynamical generation of mass was shown to exhibit an explicit dependence on the aether parameters in every possible case.

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Introduction. – The possibility of small deviations of the Lorentz symmetry has been much studied in the last years, from the theoretical and the experimental viewpoint. One might imagine a more fundamental theory at the Planck scale involving tensorial fields which acquire nonvanishing vacuum expectation values, appearing at low energy as the coupling of Standard Model fields to constant background tensors: in this way, the Standard Model could be understood as an effective field theory receiving small Lorentz-violating corrections. This approach was systematized in [1], leading to a rich set of stringent constraints on the Lorentz violation (LV) coefficients [2].

The same idea can also be implemented in the context of supersymmetric models. Supersymmetry, at the more fundamental level, is understood as a generalization of Poincaré symmetry, yet one may incorporate in the supersymmetric algebra translation invariant deformations of spacetime, including Lorentz violation, since the supercharges and the translation generators form a closed sub-algebra [3]. As an example, in the context of models with spacetime noncommutativity, where LV appears naturally from the string background that originates the noncommutativity [4], supersymmetry was argued to be a natural way to avoid difficulties in the perturbative consistency of such models [5–7]. Supersymmetry has also been claimed to avoid some naturalness problems [8] in Lorentz-violating theories, but not in the case of noncommutative models as discussed in [9]. This discussion of naturalness in the context of LV in (commutative) supersymmetric models was developed on more general grounds in [10,11], with the conclusion that supersymmetry forbids large Lorentz-violating quantum corrections that could become phenomenologically problematic. As a matter of fact, the supersymmetry algebra is highly constrained, and, as a consequence, supersymmetric models themselves are very constrained regarding possible terms in the classical Lagrangian and the quantum corrections, which is the main reason why some problematic contributions are avoided in many of the contexts mentioned above. For the same reason, consistent deformations of the supersymmetry algebra are not trivial to obtain, some noteworthy examples involving the use of Hopf algebras [12–17].

Another possibility, presented in [18], is to deform supersymmetry by the introduction of a constant background tensor \(k_{\mu\nu}\), by modifying the usual supercharge (in four spacetime dimensions)

\[
\mathcal{Q}_\alpha = \partial_\alpha - i\tilde{\theta}^\alpha \bar{\sigma}_\alpha^\mu \partial_\mu,
\]

by means of the substitution

\[
\partial_\mu \rightarrow \partial_\mu + k_{\mu\nu} \partial_\nu,
\]
where, hereafter, Greek and Latin indices are used to represent, respectively, spacetime and spinor indices. In this way, it is possible to introduce LV in supersymmetric models preserving most of the supersymmetric structure. The superalgebra is deformed to

$$\left\{ Q_\alpha, \tilde{Q}_\beta \right\} = \sigma^{\mu\nu}_\alpha \beta 2i (\partial_\mu + k_{\nu} \partial^\nu), \quad (3)$$

and one may still define superfields, and calculate quantum corrections using supergraphs. For the specific choice

$$k_{\mu\nu} = \alpha k_\mu k_\nu, \quad (4)$$

the resulting deformed models are known as “aether-like”, where now the constant vector $k_\mu$ defines a preferred direction in spacetime. Some quantum properties of these models in three and four spacetime dimensions were studied in [19,20], both for scalar and gauge theories, by using the fact that the essential structure of supergraph calculations is preserved by the deformation (1).

The study of lower-dimensional theories is motivated by the fact that many models can be constructed presenting interesting properties, sometimes resembling those of more complicated four-dimensional models, but in a simpler setting. On the other hand, unique results can also be obtained, such as the existence of a topological mass for the photon is also affected by the constant background $k_\mu$, where the aether-like Schwinger model was studied, showing that the background $k_\mu$ affects the pole structure of the propagator, and that the perturbative mass generation for the photon is also affected by the constant background vector, since the generated mass is of the form

$$M = e^2 \Delta / (8\pi^2), \quad (5)$$

where $\Delta = (1 + \alpha k_\mu k_\mu)^{-1}$. It is noteworthy that the dependence of $M$ on the LV parameter $k_\mu$ is such that the mass does not depend on the direction of $k_\mu$, in the sense that $k_\mu k_\mu$ is a scalar from the observer point of view (and a set of independent scalars from the particle point of view). A similar result was found when looking from LV contributions to axions physics in [28].

In this work we study the incorporation of aether-like Lorentz violation in the $CP^{(N-1)}$ model. Essentially, a $CP^{(N-1)}$ model contains a set of $N$ scalars $\varphi_i$, on which it is imposed a constraint of the form $\sum_i \varphi_i^2 = \text{const}$. On the classical level, the model is globally $SU(N)$ invariant, and a local $U(1)$ gauge symmetry can be incorporated. Several interesting properties of such models have been reported in the literature, including a nontrivial phase structure [29], instantons solutions and confinement [30,31]. The coupling with fermions preserves the phase structure but the long-range force is obstructed by the fermionic fields [32]. More recently, the $CP^{(N-1)}$ model, both with and without supersymmetry, was studied in the context of spacetime noncommutativity, and questions like renormalizability [33–35], and the structure of BPS and non-BPS solitons [36–42] were addressed.

Our work will proceed as follows. First, we will present the aether-superfield formulation of the model and study its phase structure, which we will see is not modified by the Lorentz violation. Then, leading-order corrections to the effective action of the auxiliary and gauge superfields will be calculated, and the corresponding dispersion relations discussed, showing that in general the dynamics is consistent if $\alpha$ is assumed to be small enough. Afterwards, the quadratic part of the effective action of the scalar superfield at the subleading order will also be calculated. We will close with some final remarks about our results.

**$CP^{(N-1)}$ model in aether-superspace and its phase structure.** – The two-dimensional aether-superspace was presented in [27], revealing straight similarity with the three-dimensional one: the conventions of three-dimensional superspace can be directly applied to the two-dimensional one [43], therefore we are adopting the superspace conventions as described in [44] for the three-dimensional case, and we will be able to develop a discussion that encompasses both two and three spacetime dimensions.

The $CP^{(N-1)}$ model defined in the aether-superspace is a model $SU(N)$ globally and $U(1)$ supergauge invariant, involving an $N$-uple of complex scalar aether-superfields $\Phi_\alpha$, subject to the constraint $\Phi_\alpha^* \Phi_\alpha = N/g$, where $g$ is a constant. We refer the reader to [19,27] for more details on the modification in the superfield component structure induced by the presence of the aether. The constraint can be implemented by the use of a Lagrange multiplier $\Sigma$, which is a real scalar aether-superfield. As a consequence, the Lorentz-violating supersymmetric $CP^{(N-1)}$ model in the aether-superspace can be defined by the action

$$S = -\int d^D x d^2 \theta \left\{ \frac{1}{2} \tilde{\nabla}^2 \Phi_\alpha \Phi_\alpha + \Sigma \left( \Phi_\alpha \Phi_\alpha - \frac{N}{g} \right) \right\}, \quad (6)$$

where $D$ is the dimension of spacetime, which in this work will be taken as $D = 2$ or $D = 3$, $\tilde{\nabla}_\alpha = \tilde{D}_\alpha = D_\alpha - i \bar{\theta}^\beta \gamma^\mu_{\alpha\beta} \partial_\mu$, with $\partial_\mu = \partial_\mu + k_\mu \partial^\mu$. Lorentz violation is described by the tensor $k_{\mu\nu}$, where $k_\mu$ is a constant vector with $k_\mu k_\mu$ being equal either to 1, −1 or 0, and $\alpha$ is small [45,46]. The gauge
aether-superfield $\Gamma_\alpha$ has no kinetic term in the classical Lagrangian so it has no propagating degrees of freedom at the classical level, in the same way as the Lagrange multiplier $\Sigma$, but both acquire nontrivial dynamics at the quantum level.

The usual $CP^{(N-1)}$ model presents a rich phase structure [29]: in one phase the symmetry $SU(N)$ is broken down to $SU(N-1)$, whereas in the other the model remains $SU(N)$ symmetric while mass generation is observed for the fundamental bosonic fields. We will verify the possibility of symmetry breaking in our case by assuming that $\Sigma$ and the $a = N$ component of $\Phi_a$ acquire nonvanishing vacuum expectation values (VEVs), $(\Sigma) = m$ and $\langle \Phi_N \rangle = \sqrt{N} v$. By redefining the aether-superfields in terms of new ones with vanishing VEV, $\Phi_a \rightarrow \Phi_a$ (for $a = 1, \ldots, N-1$), $\Phi_N \rightarrow \Phi_N + v \sqrt{N}$, $\Sigma \rightarrow \Sigma + m$, the action in eq. (6) can be cast as

$$S = \int d^4x d^2q \left\{ \Phi_a (\bar{\cal D}^2 - m) \Phi_a - \Sigma (\bar{\cal D} \Phi_a - \frac{N}{g} \Phi_a) \right\} - \frac{1}{2} \bar{\Phi}_a \Phi_a \Gamma^\alpha \Gamma_\alpha$$

$$+ \frac{i}{2} \left( \bar{\cal D}^\alpha \Phi_a \Gamma_\alpha - \bar{\Phi}_a \bar{\cal D}^\alpha \Gamma_\alpha \right)$$

$$+ \bar{\Phi} \sqrt{N} \bar{\cal D}^\alpha \Phi_N \Gamma_\alpha - \bar{\Phi} v \sqrt{N} \Gamma_\alpha \Delta_\alpha \Phi_N$$

$$- \frac{\sqrt{N}}{2} \bar{\Phi} \Gamma^\alpha \Gamma_\alpha$$

$$+ \frac{N}{2} \bar{\Phi} \Gamma_\alpha$$

$$- \Sigma \Phi_N \bar{\Phi} v \sqrt{N}$$

$$- \Sigma \Phi_N v \sqrt{N} + \Sigma \Phi_N v \bar{\Phi} \sqrt{N} \right\}. \quad (7)$$

The propagator for the first $(N-1)$ components of $\Phi_a$ is readily seen to be given by

$$\langle T \Phi_a(p, \theta_1) \bar{\Phi}_b(-p, \theta_2) \rangle = -i \delta_{ab} \left( \frac{D^2 + m^2}{p^2 + m^2} \right) \delta^2(\theta_1 - \theta_2). \quad (8)$$

The redefined aether-superfields must have vanishing vacuum expectation values, $(\Sigma) = 0$ and $\langle \Phi_N \rangle = 0$, implying in the equations

$$i \int \frac{d^Dq}{(2\pi)^D} \left\{ \frac{1}{q^2 + m^2} + \frac{1}{g} + \delta_g - \bar{v} \bar{v} = 0, \quad (9) \right.$$  

$$m \bar{v} = 0. \quad (10)$$

From eq. (10), we see that for $v = 0$ and $m \neq 0$ a nonvanishing mass is generated for the fundamental aether-superfields $\Phi_a$, in the gauge symmetric phase. In such a case, eq. (9) reads

$$\delta_g = -\frac{1}{g} - i \int \frac{d^Dq}{(2\pi)^D} \left\{ \frac{1}{q^2 + m^2} \right\} \quad (11)$$

corresponding to the renormalization of the coupling constant $g$. Notice that in two dimensions the above integral is logarithmic divergent, while in three dimensions it is finite because the integral (linear UV divergent) is completely regularized by dimensional reduction. For the other phase, we have $v \neq 0$ and $m = 0$, implying that the global symmetry is reduced to $SU(N-1)$, while the scalar aether-superfields remain massless. We therefore conclude that the presence of the aether-like Lorentz violation does not change the phase structure of the $CP^{(N-1)}$ model.

**Effective action at leading order.** We shall be interested in the generation of mass in the model, so we will work in the symmetric phase from now on. Moreover, the renormalization of the model is not affected by the choice of phase because both have the same ultraviolet behavior [47]. Therefore, in the symmetric phase ($v = 0$ and $m \neq 0$) the action reads

$$S = \int d^4x d^2q \left\{ \Phi_a (\bar{\cal D}^2 - m) \Phi_a - \Sigma \left( \bar{\Phi} \Phi_a - \frac{N}{g} \right) \right\}$$

$$- \frac{1}{2} \bar{\Phi}_a \Phi_a \Gamma^\alpha \Gamma_\alpha + \frac{i}{2} \left[ \bar{\cal D}^\alpha \Phi_a \Gamma_\alpha - \bar{\Phi}_a \bar{\cal D}^\alpha \Gamma_\alpha \right], \quad (12)$$

where $a = 1, \ldots, N$; in this case, the propagator of the $\Phi_N$ aether-superfield is also given by eq. (8).

**Effective action of the $\Sigma$ aether-superfield.** At the classical level, the $\Sigma$ aether-superfield is a constraint without dynamics, however it acquires a nonlocal kinetic term becoming a propagating dynamical superfield by means of quantum corrections. In the leading order of the $1/N$ expansion, the radiative correction to its quadratic effective action, shown in fig. 1, is given by

$$\Gamma^{(2)}_{\Sigma} = \frac{1}{2} \int \frac{d^Dp}{(2\pi)^D} d^2\theta \Sigma(-\hat{\bar{\theta}}, \hat{\bar{\theta}})$$

$$\times \left[ N f(\hat{\bar{\theta}})(\bar{\cal D}^2 + 2m) \right] \Sigma(\hat{\bar{\theta}}, \hat{\bar{\theta}}), \quad (13)$$

where

$$f(\hat{\bar{\theta}}) = \int \frac{d^2q}{(2\pi)^2} \left( \frac{1}{(\hat{\bar{\theta}}^2 + m^2)} \right), \quad (14)$$

and the computer package presented in [48] was used for the manipulations of covariant superderivatives. In order to evaluate the integral, we use the Feynman trick to
From eq. (13), we obtain the radiative induced $\Sigma$ in $D$ dimensions obtaining

$$f(\vec{p}) = \int \frac{d^D q}{(2\pi)^D} \frac{1}{[(\vec{q} + \vec{p})^2 + m^2][(\vec{q}^2 + m^2)]} = \Delta \int_0^1 dz \frac{i}{(4\pi)^{D/2}} \frac{\Gamma(2 - D/2)}{\Gamma(2)} \frac{1}{M^{2-D/2}}, \quad (15)$$

where $M = m^2 + \vec{p}^2 z(1 - z)$. The factor

$$\Delta = \det \left( \frac{\partial q^m}{\partial \eta^i} \right) = \det^{-1}(\delta_n^m + k_n^m) \quad (16)$$

is the Jacobian of the change of variables $q \rightarrow \eta$ used to bring the integral to its final form.

In particular, for $D = 2$, $f(\vec{p})$ reads

$$f(\vec{p}) = \Delta \int_0^1 dz \left\{ \frac{1}{4\pi |m^2 + \vec{p}^2 (1 - z)|} + O(D - 2) \right\}$$

$$= -\frac{\Delta \arctan \left( \frac{|\vec{p}|}{\sqrt{-4m^2 - \vec{p}^2}} \right)}{\pi |\vec{p}| \sqrt{-4m^2 - \vec{p}^2}}$$

$$= \left\{ \begin{array}{ll}
\Delta/(8\pi |m|), & \text{for } \vec{p} \rightarrow \infty,
\Delta/(8\pi |m|), & \text{for } \vec{p} \rightarrow 0.
\end{array} \right. \quad (17)$$

while in $D = 3$,

$$f(\vec{p}) = \Delta \int_0^1 dz \left\{ \frac{1}{8\pi \sqrt{m^2 + \vec{p}^2 (1 - z)}} + O(D - 3) \right\}$$

$$= \frac{\Delta}{8\pi |\vec{p}|} \ln \left( \frac{2|m| + |i|\vec{p}|}{2|m| + i|\vec{p}|} \right)$$

$$= \left\{ \begin{array}{ll}
\Delta/(8|m|), & \text{for } \vec{p} \rightarrow \infty,
\Delta/(8\pi |m|), & \text{for } \vec{p} \rightarrow 0.
\end{array} \right. \quad (18)$$

From eq. (13), we obtain the radiative induced $\Sigma$ propagator,

$$\langle T \Sigma(\vec{p}, \theta_1) \Sigma(-\vec{p}, \theta_2) \rangle = \frac{i}{N} \frac{(\vec{D}^2 - 2m)}{f(\vec{p})(\vec{p}^2 + 4m^2)} \delta(\theta_1 - \theta_2). \quad (19)$$

This propagator has a regular infrared behavior ($\vec{p}^2 \rightarrow 0$) in both dimensions, while it decreases as $1/|\vec{p}|$ for $D = 3$ and $1/\ln |\vec{p}|$ for $D = 2$ in the ultraviolet limit ($\vec{p}^2 \rightarrow \infty$). We notice that the nonlocal $f(\vec{p})$ factor in the denominator does not introduce additional poles, so the dispersion relation for the aether-superfield $\Sigma$ is given by

$$\vec{p}^2 + 4m^2 = p^2 + 2k_m p^m p^0 + k^m k_m p_0 + 4m^2 = 0, \quad (20)$$

or

$$p^2 + \alpha \left[ 2 + \alpha k^2 \right] (k^0 p_0)^2 + 4m^2 = 0, \quad (21)$$

in terms of the aether LV vector $k^0$.

It is interesting to analyze the consequences of eq. (21) separately for $k^0 k_0$ being $\pm 1$ or zero, as was done in [19]. Starting with space-like $k^0$, i.e., $k^0 k_0 = -1$, we choose coordinates such that $k^\mu = (0, \vec{k})$, $\vec{k}$ being a unitary space vector. With this, eq. (21) reduces to

$$E^2 = p^2 + 4m^2 + \alpha (2 + \alpha)(\vec{k} \cdot \vec{p})^2, \quad (22)$$

and, by taking $\vec{p} = 0$, the rest mass of the particle if found to be $2m$, independently of $\alpha$ and $\vec{k}$ and, therefore, of the LV background. Generally, however, the particle energy depends on the orientation of its momentum with respect to the LV vector $k$. We may also notice that if $\alpha = -1$ and $\vec{p}$ is collinear to $\vec{k}$, we obtain $E^2 = 4m^2$, independently of $\vec{p}$, therefore representing a degenerate, inconsistent dynamics. Otherwise, for any small value of $\alpha$, the dynamics is consistent. Now considering time-like $k^0$, i.e., $k^0 k_0 = -1$, we choose coordinates such that $k^\mu = (1, 0)$, thus obtaining

$$E^2 = \vec{p}^2 + 4m^2 \quad (1 - \alpha)^2, \quad (23)$$

corresponding to a rest mass given by $2m/(1 - |\alpha|)$. In this case, we see an explicit dependence of the dynamics (which is consistent in principle as far as $|\alpha| < 1$) on the LV properties, as given by the value of $\alpha$. Finally, for the light-like case, we have a more complicated dispersion relation,

$$E^2 \left[ 1 - 2\alpha (k^0)^2 \right] + 4\alpha k^0 (\vec{k} \cdot \vec{p}) E$$

$$= -2\alpha(\vec{k} \cdot \vec{p})^2 - \vec{p}^2 - 4m^2 = 0. \quad (24)$$

To simplify matters, the reference frame is chosen so that $\vec{k} = (k^0, k^0, 0, 0)$. We consider two particular cases: first, $\vec{p}$ parallel to $\vec{k}$, leading to

$$E = (1 - 2\alpha)^{-1} \left[ 2\alpha |\vec{p}| \pm \sqrt{\vec{p}^2 + 4m^2 (1 - 2\alpha)} \right], \quad (25)$$

and also $\vec{p}$ perpendicular to $\vec{k}$, in which case we obtain

$$E^2 = \vec{p}^2 + 4m^2 \quad (1 - 2\alpha). \quad (26)$$

We see that the result for $\vec{p} \perp \vec{k}$ is similar to the time-like case studied before, as far as $\alpha < 1/2$ to avoid imaginary energies. For $\vec{p} \parallel \vec{k}$, given a sufficiently small value of $|\alpha|$, we have one positive and one negative energy state, which are to be reinterpreted, in the quantum theory, as the energies for particles and antiparticles. In this case, it is noteworthy that the energies of particles and their corresponding antiparticles will be slightly different. Finally, both for parallel and perpendicular cases, the rest mass is given by $2m/\sqrt{1 - 2\alpha}$.

**Effective action of the gauge aether-superfield.** The gauge aether-superfield $\Gamma_\alpha$ is also nondynamical at classical level, but similarly to $\Sigma$ it will have a nontrivial kinetic term generated by quantum corrections. The leading $1/N$
radiative corrections to the quadratic part of the $\Gamma_\alpha$ effective action are represented in fig. 2. The first contribution, corresponding to the left diagram in fig. 2, is given by

$$
\Gamma^{(2)}_{1a} = -\frac{N}{2} \int \frac{d^D p}{(2\pi)^D} \int d^D \theta \Gamma_\beta(-\tilde{p}, \theta) \times \int \frac{d^D q \Delta}{(2\pi)^D} \frac{C^{\alpha \beta}}{q^2 + m^2} \Gamma_\alpha(\tilde{p}, \theta), 
$$

(27)

while for the right one,

$$
\Gamma^{(2)}_{1b} = \frac{N}{2} \int \frac{d^D p}{(2\pi)^D} \int d^D \theta \Gamma_\beta(-\tilde{p}, \theta) \times \int \frac{d^D q \Delta}{(2\pi)^D} \frac{1}{(q^2 + m^2)(q^2 + m^2)} \times \left[ \frac{q^{\alpha \beta} - mC^{\alpha \beta}}{[(q + \tilde{p})^2 + m^2][(q^2 + m^2)]} W_\alpha(\tilde{p}, \theta), \right)
$$

(28)

Summing up the two contributions, and using the relation $\hat{D}_\mu \hat{D}_\nu = i\partial_\mu + C_{\nu \mu} \hat{D}^2$, we obtain

$$
\Gamma^{(2)}_1 = \frac{N}{2} \int \frac{d^D p}{(2\pi)^D} \int d^D \theta \Gamma_\beta(-\tilde{p}, \theta) \times \int \frac{d^D q \Delta}{(2\pi)^D} \frac{1}{(q^2 + m^2)[(q + \tilde{p})^2 + m^2]} W_\alpha(\tilde{p}, \theta),
$$

(29)

where $W_\alpha = \frac{1}{2} \hat{D}^2 \hat{D}^\alpha \hat{\Gamma}_\beta$ is the Maxwell aether-superfield strength.

Using the identity

$$
\int \frac{d^D q}{(2\pi)^D} \frac{q^{\alpha \beta}}{[(q + \tilde{p})^2 + m^2][q^2 + m^2]} = -\frac{p^{\alpha \beta}}{2} \int \frac{d^D q}{(2\pi)^D} \frac{1}{[(q + \tilde{p})^2 + m^2][q^2 + m^2]},
$$

(30)
eq (29) can be cast as

$$
\Gamma^{(2)}_1 = -\frac{N}{2} \int \frac{d^D p}{(2\pi)^D} \int d^D \theta f(\tilde{p}) \times [W_\alpha(-\tilde{p}, \theta) W_\alpha(\tilde{p}, \theta) + 2m \Gamma_\alpha(-\tilde{p}, \theta) W_\alpha(\tilde{p}, \theta)].
$$

(31)

In $D = 3$, the previous effective action describes the dynamics of a (nonlocal) Maxwell-Chern-Simons aether-superfield. It is well known that the presence of the Chern-Simons (CS) term $\Gamma_\alpha W_\alpha$ generates a topological massive pole for the $\Gamma_\alpha$ two-point superpropagator. In $D = 2$, the effective action we obtained represents the dynamics of a massive gauge-invariant aether-superfield, as discussed in the ordinary two-dimensional superspace in [49,50].

The propagator of the $\Gamma_\alpha$ aether-superfield at leading order can be obtained after a gauge fixing. For convenience, we use a covariant nonlocal gauge fixing, together with the corresponding Faddeev-Popov terms, given by

$$
S_{GF} = \frac{N}{4} \int \frac{d^D p}{(2\pi)^D} d^D \theta f(\tilde{p}) \left\{ \frac{1}{2 \xi} \hat{D}^\beta \Gamma_\beta \hat{D}^\alpha \Gamma_\alpha - c' \hat{D}^2 c \right\}.
$$

(32)

Notice that the ghosts aether-superfields decouple from $\Gamma_\alpha$ because we are working in an Abelian gauge theory. With this gauge choice, we obtain the following gauge aether-superfield propagator:

$$
\left< T \Gamma^\alpha(\tilde{p}, \theta) \Gamma^\beta(\tilde{p}, \theta) \right> = \frac{i}{\sqrt{N f(\tilde{p})}} \times \left[ \hat{D}^2 (\tilde{p}^2 + 4m^2) \delta(\theta_1 - \theta_2) \right].
$$

(33)

The pole at $\tilde{p}^2 = 0$ is not physical, being dependent on the gauge parameter $\xi$. To verify this, the standard procedure is to project the superpropagator given in eq. (33) for the physical component fields. For example, for $2 + 1$ spacetime dimensions, we use the decomposition

$$
\Gamma_\alpha(\tilde{p}, \theta_1) = \chi_\alpha - \theta_\alpha B + i\theta^\beta V_{\alpha \beta} - \theta^2 (\lambda_\alpha + i\theta_{\alpha \beta} \chi^\beta),
$$

(34)
together with the selection of the Wess-Zumino gauge, $B = \chi = 0$, to notice that

$$
\left< T \Gamma^\alpha(\tilde{p}, \theta_1) \Gamma^\beta(\tilde{p}, \theta_2) \right> = -\theta^\alpha \theta^\beta \hat{D}^\alpha \hat{D}^\beta \hat{V}_{\alpha \beta}(\tilde{p}) V_{\alpha \beta}(\tilde{p}) \cdots,
$$

(35)
where $\hat{V}_{\alpha \beta}(\tilde{p}) V_{\alpha \beta}(\tilde{p})$ is the desired propagator for the gauge field $V_{\mu \nu}$. To obtain its explicit form, one starts by calculating explicitly $D^2 \delta(\theta_1 - \theta_2)$ and $(D^2)^2 \delta(\theta_1 - \theta_2)$,

$$
D^2 \delta(\theta_1 - \theta_2) = -1 + \theta^\alpha \theta^\beta \tilde{p}_{\alpha \beta} + \frac{1}{4} \theta^\alpha \theta^\beta \theta^\gamma \theta^\delta \tilde{p}_{\alpha \beta \gamma \delta},
$$

(36)
and

$$
(D^2)^2 \delta(\theta_1 - \theta_2) = \frac{1}{2} \theta^\alpha \theta^\beta \theta^\gamma \theta^\delta \tilde{p}_{\alpha \beta \gamma \delta} + \theta^\alpha \theta^\gamma \theta^\delta \tilde{p}_{\alpha \beta \gamma \delta} + \frac{1}{4} \theta^\alpha \theta^\beta \theta^\gamma \theta^\delta \tilde{p}_{\alpha \beta \gamma \delta}.
$$

(37)
The identity $\hat{D}^\alpha \hat{D}^\beta \hat{D}^\alpha = \tilde{p}_{\alpha \beta} + C_{\alpha \beta} \hat{D}^2$ can be used to rewrite eq. (33) in terms of $D^2$ and $D^4$ only, and using the results of the last equation to isolate the required part, i.e.,

$$
\left< T V_{\alpha \beta}(\tilde{p}) V_{\alpha \beta}(\tilde{p}) \right> = \left< T \Gamma_\alpha(\tilde{p}, \theta_1) \Gamma_\beta(-\tilde{p}, \theta_2) \right>_{\theta_1 = \theta_2}.
$$

(38)
In $D = 2$ one obtains the final result as

$$
\left< T V_{\alpha \beta}(\tilde{p}) V_{\alpha \beta}(\tilde{p}) \right> = \frac{F_{\mu \nu \alpha \beta}(\tilde{p})}{\tilde{p}^2 (\tilde{p}^2 + 4m^2)} + \frac{\xi G_{\mu \nu \alpha \beta}(\tilde{p})}{(\tilde{p}^2)^2},
$$

(39)
Effective action at subleading order. – We are also able to evaluate the quadratic part of the effective action of the matter aether-superspace \( \Phi \) at sub-leading order, whose contributions arise from the diagrams depicted in fig. 3. The corresponding amplitude is given by

\[
\Gamma^{(2)}_\Delta = \frac{1}{N} \int \frac{d^D p}{(2\pi)^D} d^2 \theta \phi(-\bar{p}, \theta) (D^2 - m) \phi(p, \theta) 
\times \int \frac{d^D q \Delta}{(2\pi)^D f(\bar{q})} \times \left\{ \frac{1}{2(\bar{q}^2 + m^2)} \right\},
\]

which after some algebraic manipulations, and choosing the gauge \( \xi = 0 \), leads to

\[
\Gamma^{(2)}_\Delta = \frac{1}{N} \int \frac{d^D p}{(2\pi)^D} d^2 \theta \phi(-\bar{p}, \theta) (D^2 - m) \phi(p, \theta) 
\times \int \frac{d^D q \Delta}{(2\pi)^D f(\bar{q})} \times \left\{ \frac{1}{2(\bar{q}^2 + m^2)}[\bar{q}^2 (q + \bar{p})^2 + m^2] \right\},
\]

Final remarks. – In this paper we have studied the dynamical generation of mass in the two- and three-dimensional Lorentz-violating supersymmetric \( CP^{(N-1)} \) model defined in the aether-superspace. Even though the phase structure of the model is not affected by the Lorentz violation, we showed that in the \( CP^{(N-1)} \) model the dynamically generated mass has an explicit dependence on the aether properties, except for space-like LV vector. The aether properties dependence on the physical dynamically generated mass has also been noticed in the quantum electrodynamics in the two-dimensional aether-superspace [27]. We have also calculated the leading quantum corrections, in the large-\( N \) approximation, to the two-point vertex functions of the scalar, gauge and auxiliary aether-supersfields, showing that the latter two acquire dynamics at the quantum level, and that the generated propagators are sensitive to the LV. The dispersion relation governing their dynamics was studied, and shown to be consistent for small \( \alpha \). Finally, subleading corrections to the scalar propagators were also obtained, also exhibiting a dependence on the aether parameter.

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