Detection of the onset of Dean instability and effects of the rheological behavior in non-Newtonian fluids

H Fellouah, C Castelain, A Ould El Moctar and H Peerhossaini
Thermofluids, Complex Flows and Energy Research Group, Laboratoire de Thermocinétique, UMR CNRS 6607, Polytech Nantes, rue Christian Pauc, B.P. 90604, F-44306 NANTES Cedex 3, France
E-mail: ahmed.ould-el-moctar@univ-nantes.fr

Abstract. The onset of Dean instability in laminar secondary flow in 180° curved rectangular cross section channel was studied experimentally and numerically in Newtonian and non-Newtonian fluids. The development of the instability was observed; we showed that the Dean vortices develop first in the near wall zone on the concave wall, where the viscosity is weak and the shear rate is high, and then they penetrate the cross-section center characterized by a high viscosity for the pseudoplastic fluids and a solid (unsheared) zone for the yield fluids. Based on the complete formation of the Dean vortices, the critical value of the Dean number decreases with the increase of the power law index and with the decrease of the Bingham number. Contrarily to what is reported in the literature where the instability threshold was usually obtained visually, in this study a new criterion based on the radial gradient of the axial velocity to detect the instability threshold was defined. Comparison was made between numerical and experimental results concerning the instability threshold, obtained with the new criterion. Also, a comparison between the instability threshold using this new criterion and using the visualization technique is presented. We show that the value of the Dean number using the new criterion is comparable for the two studies, numerical and experimental. These values are weaker than those obtained with the visualization technique for the same conditions.

1. Introduction
The secondary flow in a curved duct consists of a pair of counter-rotating symmetrical vortices called the corner vortices (Moffatt [1]), end cells or Ekman vortices (Finlay and Nandakumar [2]). Corner vortices are due to a non-equilibrium state caused by an imbalance between the centrifugal force and viscous force and appear on the channel end walls. Beyond a critical Dean number

\[ De = \frac{UmD_h}{\nu} \sqrt{\frac{D_h}{Rc}} \]

where \( Um \) is the mean velocity, \( \nu \) the kinematic viscosity, \( D_h \) the channel hydraulic diameter and \( Rc \) the mean radius of curvature, a definition of this parameter in the case of non-Newtonian fluids is given by Fellouah et al. [3]), another pair of counter-rotating vortices appears on the concave wall of the duct. These vortices are due to Dean instability, one of the large family of centrifugal instabilities of which Taylor-Couette and Görtler instability are also members. Dean instability is a centrifugal instability appearing in Poiseuille flow in curved channels.
Curved channels are widely used in technological applications: heat exchangers, static mixers, engines are some most commonly uses. They provide compactness, high heat and mass-transfer rates, transverse mixing and an extended laminar flow regime. Detailed knowledge of the secondary flow in curved channels is thus crucial for better understanding and the control of the flow in these industrial applications. Because of these utilities curved channels flow with square or rectangular cross-sections have been studied, both numerically and/or experimentally, by numerous authors. Numerically we can cite Cheng et al. [3], Ghia and Sokhey [4], Hwang and Chao [5], Cheng and Akiyama [6] and Chandratilleke and Nursubyakto [7]. Experimentally we can cite Bara et al. [8], Cheng et al. [9], Sugiyama et al. [10], and Ligrani and Niver [11]. These studies described the structure of the secondary flow occurring in the curved channels with and without Dean instability. The commonly used criterion for the detection of the instability onset in these work is visual, i.e. the visual appearance of the Dean vortices. However, visualization is only an approximate method for detection of instability. As explained in Fellouah et al. [12-13], visualization remains a qualitative criterion; numerically, the visualization technique depends strongly on the number of iso-value contour lines selected. In other words, one can encounter a situation with two corner vortices (flow without instability) for a given Dean number and a given number of iso-value contour lines, whereas for the same Dean number and under the same operating conditions, if one increases the number of iso-value contour lines, two more Dean vortices may appear near the concave wall (flow with instability). Experimentally, instability is signalled by the appearance of the Dean vortices, but the Dean vortices are visible only when they are highly amplified. Therefore, in both studies — numerical and experimental visualization — the critical Dean number tends to be systematically overestimated.

Bara et al. [8], Ghia and Sokhey [4], Hille et al. [14] have tried to detect the critical Dean number for ducts of square cross-section, finding critical Dean numbers respectively of 137, 143, and 150. These discrepancies seem to arise from the use in all these studies of a qualitative criterion for the onset of the instability (experimental visualization or numerical path lines). Fellouah et al. [13] gave response to the pertinent question concerning the Dean instability threshold: how to calculate the critical Dean number for the appearance of the new pair of vortices, or equivalently how to detect the passage from flow with two corner vortices to flow with four cells (two corner vortices and two Dean vortices). In there study, they gave a numerical accurate criterion, based on the radial gradient of the axial velocity on the concave wall, to detect this instability threshold. This criterion has a physical signification of the shear stresses. This shear stress is very sensible to the modification of the secondary flow caused by the formation of the Dean vortices.

2. Experimental apparatus and methods
The aim of this work is to extend the work of Fellouah et al. [13] on the comparison between the instability threshold, in curved channel flow of b/a=8 and Rc/Dh=10, obtained numerically and experimentally for both Newtonian and non-Newtonian fluids. The non-Newtonian fluid is a shear thinning one and its rheological behaviour is described by a power law model in the range of shear rate (0.1 < \dot{\gamma} < 1000 s^-1).

The experimental setup which is described in Fellouah et al. [15] consists of a water tunnel facility starting with a straight duct to ensure the development of a Poiseuille flow. The straight duct is followed by the test section composed of a curved channel of rectangular cross-section with an aspect ratio of b/a=8; the curvature angle is 180° and the curvature ratio is 10.

The flow was visualized by the laser-induced fluorescence (LIF) technique. Dye (Fluorescein solutions of mass concentration 1 mg/l) is injected with a syringe upstream of the settling chamber; the tracer is distributed in the whole channel cross-section. This kind of injection allows overall visualization. The secondary flow was visualized by illuminating the channel cross-section by a laser sheet. For this purpose, a traversing mechanism was constructed in which the relative position of the camera and the laser light sheet is fixed. The traversing mechanism rotates around the center of curvature of the bend under investigation and scans the flow at different angular positions. Images
viewed by the camera are transferred directly to a digital image-processing system where they can be recorded and analyzed with a personal computer.

| Newtonian fluid : water | Pseudoplastic fluid : CMC 0.25% |
|------------------------|---------------------------------|
| De=115                 | De=122                          |
| De=127                 | De=128                          |
| De=137                 | De=139                          |
| De=165                 | De=150                          |
| De=230                 | De=263                          |

**Table 1.** Comparison between the experimental flow visualization obtained at $\theta=180^\circ$ of the curved channel for the Newtonian fluid (water $n=1$, $\kappa=0.001$ Pa s$^n$) and the pseudoplastic fluid (CMC solution $n=0.75$, $\kappa=0.097$ Pa s$^n$), $b/a=8$, $Re/D_h=10$.

To show the effect of the power law index on Dean instability, we represent on Table 1 the experimental evolution of the flow at the exit section ($180^\circ$) of the rectangular curved channel for different Dean numbers in the Newtonian fluid ($n=1$) and in a pseudoplastic fluid ($n=0.75$). For the two fluids, the Dean corners exist for all Dean numbers and they are always located near the flat walls of the channel. In these visualisations, the Dean vortices appear rapidly and occupy all the width of the cross-section of the channel for the Newtonian fluid and they appear slowly in the case of pseudoplastic fluid, they appear first in the parietal region of the concave wall and than they develop to occupy all the width cross-section of the channel. This is due to the variation of viscosity in the case of the pseudoplastic fluid, which is weak near the wall and high in the section centre.

To determine the critical Dean number, two situations can be considered. The first one consists in considering that the critical Dean number which corresponds to the complete formation of the first Dean vortices (complete formation of the first mushrooms). Based on this method, these visualisations show a shift of the value of the critical Dean number in the Newtonian and the pseudoplastic fluids. The critical Dean number is 137 for Newtonian fluid which is 150 in the pseudoplastic fluid ($n=0.75$), this result is in good agreement with those exiting in the literature (Bara et al. [11] and Shanthini and Nadakumar [16]). The second method consists in considering the critical Dean number corresponding to the beginning of the tracer perturbation on the concave wall. In this case, the shift of the critical Dean number is less pronounced. In the case of high Dean number (about 250), the flow is similar for the two types of fluid.
In addition to this experimental study, numerical simulations are carried out with the Fluent CFD code. The model represents the same geometry studied in the experimental part. A subroutine, based on the Papanastasiou [17] model, was added to the CFD code to solve the equations of motion in yield-stress fluids.

![Plane in the curved channel diagram](image)

**Figure 1.** Method for measurement of the radial gradient of the axial velocity.

### 3. Criterion for detection of the instability onset

To calculate this criterion, the first step consists in measuring, experimentally (or computing numerically), the axial velocity along the \(x\)-coordinate (from the point \(A\) to \(A'\) in Figure 1). The distance from \(A\) to \(A'\) is 40 mm which is largely sufficient to detect the formation of the first pair of Dean vortices. These measurements are carried out for different Dean numbers. For each Dean number and in the same \(x\)-coordinates positions, we measure the axial velocity along the segment \([BB']\), which is 1 mm away from the segment \([AA']\) along the \(y\)-coordinate. Then, for each \(x\)-coordinate position, we measure the difference between the two axial velocities (the first determined on segment \([AA']\) and the second on segment \([BB']\)). This difference is then divided by \(\Delta y=1\) mm to obtain the axial velocity gradient \((dw/dy)\) at each position of \(x\)-coordinates.

To facilitate the detection of the appearance of the Dean vortices, Fellouah *et al.* [13] calculate the slope of the axial velocity gradient profile along the \(x\)-coordinates axis for different Dean numbers. This slope changes the form with the appearance of the Dean vortices.

Figure 2 shows the slope of the axial velocity gradient respectively for a Newtonian and a pseudoplastic \((n=0.75, k=0.097)\) fluid obtained numerically and experimentally with the laser Doppler anemometry (LDA) technique.

As it can be seen from the two figures, slopes of the velocity gradient profiles are uniform for weak Dean numbers and present a perturbations at high Dean numbers. These perturbations are due to a normal velocity gradient caused by the formation of the Dean vortices. In the numerical study (Figure 2a), these perturbations have a sinusoidal variation whereas, in the experimental study (Figure 2b), the perturbations are not symmetrical. These differences are due to the errors of measurements which are due primarily to the vibration of the curved channel at strong Dean numbers because of the high pressures.

The variation of the slope form, from uniform to perturbed, is considered as a signature of the Dean instability threshold. From these two figures, it is easy to determine critical Dean numbers, both numerical and experimental, for the Newtonian and the pseudoplastic \((n=0.75)\) fluids. These results are summarized in Table 2. In this table, we have added the results of critical Dean numbers obtained visually, both numerically and experimentally.

Table 2 shows that numerical or experimental critical Dean numbers obtained by measuring the axial velocity gradient are weaker than those obtained by numerical and experimental visualizations. This result confirms the satisfactory precision of the criterion, based on the axial velocity gradient adopted in the present study for the determination of the onset of the Dean vortices.
Figure 2. Slope of the axial velocity gradient profile in the centre of the mid-plane for different Dean numbers, pseudoplastic fluid \((n=0.75, k=0.097)\), \(b/a=8\) and \(Rc/Dh=10\), \(\theta=180^\circ\).

| Method                  | Newtonian fluid \((n=1, k=0.001)\) | Pseudoplastic fluid \((n=0.75, k=0.097)\) |
|-------------------------|---------------------------------|----------------------------------------|
| Numerical results       | Velocity gradient \((dw/dy)\) | 86                                      | 110                                    |
|                         | visualization                   | 122 \((1)\)                             | 165 \((2)\)                            |
| Experimental results    | Velocity gradient \((dw/dy)\)       | 115                                    | 130                                    |
|                         | visualization                   | 131 \((1)\)                             | 145 \((2)\)                            |

Table 2. Comparison between critical Dean numbers obtained numerically and experimentally. \((1)\) results form Fellouah et al. [13] and \((2)\) results form Fellouah et al. [15].

Figure 3. Effect of rheological parameters on the onset of Dean instability, numerical study.
4. Influence of the rheological parameters on Dean instability

Figure 3 shows the effect of the variation of two rheological parameters on Dean instability, the power-law index (Figure 3a) and the Bingham number (Figure 3b). The critical Dean number decreases with increasing power-law index \( n \). This is due to the local change of the apparent viscosity in the pseudoplastic fluid. It is weaker near the wall and higher at the center of the channel cross section, and thus it is difficult for unstable Dean vortices to form. Agrawal et al. [18] showed that pseudoplasticity decreases the intensity of the secondary flow. All these factors lead to a delay in the onset of Dean instability with increasing power-law index. In Figure 3b, increasing Bingham number delays the onset of Dean instability. Alexandrou et al. [19] observed that the size of the plug region increases with the Bingham number. In addition, when the Bingham number increases, the yield stress effect in the flow increases too. To overcome the effect of the yield stress increase, the inertial forces and hence the Dean number must be increased.

5. Conclusions

We studied numerically and experimentally the effects of the power law index and the yield stress on the Dean instability in curved channels of rectangular cross-section. The experimental study was carried out in a curved channel of aspect ratio 8 and curvature ratio 10. We defined a new criterion, based on the radial gradient of the axial velocity, to study the Dean instability threshold in this configuration. Prediction of the critical Dean number by this criterion lets us detect the instability closer to its onset than in previous works. The comparison between the numerical and the experimental results using this new criterion is satisfying. We showed that, in the case of the total formation of the Dean vortices, the critical value of the Dean number decreases with the increasing power law index and decreasing with the Bingham number. In fact, the rheological parameters introduce a shift in the critical Dean number. On the other hand, the beginning of the development of the Dean vortices in the near wall zone of the concave wall is the same for all fluids Newtonian and non-Newtonians. A more detailed account of this work can be found in Fellouah et al. [20].

References

[1] H K Moffatt 1964 Viscous and resistive eddies near a sharp corner J. Fluid Mech. 48(1) 1-18
[2] W H Finlay, K Nandakumar 1990 Onset of two-dimensional cellular flow in finite curved channels of large aspect ratio Phys. Fluids A 2(7) 1163-1174
[3] K C Cheng, R C Lin, J W Ou 1976 Fully developed laminar flow in curved rectangular channels J. Fluids Engineering 98(1) 41-48
[4] K N Ghia, J S Sokhey 1977 Laminar incompressible viscous flow in curved ducts of rectangular cross-section Trans. ASME J: J. Fluids Engineering 99 640-648
[5] G J Hwang, Chao, Chung-Hsing 1991 Forced laminar convection in a curved isothermal square duct Transactions of the ASME, J. Heat Transfer 113 48-55
[6] K C Cheng, M Akiyama 1970 Laminar forced convection heat transfer in curved rectangular channels Int. J. Heat Mass Transfer 13 471-490
[7] T T Chandratilleke, Nursubyakto 2003 Numerical prediction of secondary flow and convective heat transfer in externally heated curved rectangular ducts Int. J. of Thermal Sciences 42 187-198
[8] B Bara, K Nandakumar, J H Masliyah 1992 An experimental and numerical study of the Dean problem: flow development towards two-dimensional multiple solutions J. Fluid Mech. 244 339-376
[9] K C Cheng, J Nakayama, M Akiyama 1977 Effect of finite and infinite aspect ratios on flow patterns in curved rectangular channels *Flow Visualisation International Symposium* (Tokyo, October 1977) pp 181

[10] S Sugiyama, T Hayashi, K Yamazaki 1983 Flow characteristics in the curved rectangular channels (visualisation of secondary flow) *Bulletin of the JSME* 26(216) 964-969

[11] P M Ligrani, R D Niver 1988 Flow visualization of Dean vortices in a curved channel with 40:1 aspect ratio *Phys. Fluids* 31 3605-3617

[12] H Fellouah, C Castelain, A Ould El Moctar, H Peerhossaini 2006 A criterion for detection of the onset of Dean instability in Newtonian flows *European Journal of Mechanics - B/Fluids* 25 505-531

[13] H Fellouah, C Castelain, A Ould El Moctar, H Peerhossaini 2006 A numerical study of Dean instability in non-Newtonian fluids *J. Fluids Engineering* 128 1-12

[14] P Hille, R Vehrenkamp, E O Schulz-Dubois 1985 The development and structure of primary and secondary flow in a curved square duct *J. Fluid Mech.* 151 219-241

[15] H Fellouah, C Castelain, A Ould El Moctar, H Peerhossaini 2005 Effet du comportement rhéologique sur l’instabilité de Dean, *17e Congrès Français de Mécanique*. 

[16] W Shanthini and K. Nadakumar 1986 Bifurcation phenomena of generalized Newtonian fluids in curved rectangular ducts *J. Non-Newtonian Fluid Mech.* 22 35-60

[17] T C Papanastasiou 1987 Flow of materials with yield *J. of Rheology* 31(5) 385-404

[18] S Agrawal, G Jayaraman, V K Srivastava, and Nigam, K. D. P. 1993 Power law fluids in a circular curved tube. Part I. Laminar flow *Polym. Plast. Technol. Eng.* 32(6) 595-614

[19] A N Alexandrou, T M McGilvreay, G Burgos 2001 Steady Herschel-Bulkley fluid flow in three-dimensional expansions *J. Non-Newtonian Fluid Mech.* 100 77-96

[20] H Fellouah, C Castelain, A Ould El Moctar, H Peerhossaini 2008 Experimental and numerical study of Dean instability in power law and yield-stress fluids *European Journal of Mechanics - B/Fluids* – submitted