Research on the Control Methods Based on Liquid Level System

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Abstract: Liquid level control is a common process control in industry, and its impact on production cannot be ignored. Liquid level control systems are used in various aspects of life, such as reservoirs, cisterns, river levels, etc. Among the common liquid level control systems, the double-volume water tank is a typical controlled object, which is widely used in industrial production. The control difficulty of has also increased due to its nonlinearity and delay. PID control can achieve satisfactory results in the face of accurate mathematical models and is still widely used in society. However, PID control is less effective when dealing with nonlinear systems that are difficult to describe accurately with mathematical models. For better level control, this paper designs a double-volume tank level control system with single-loop PID control, Cascade PID Control and fuzzy PID control based on PID control theory, and compares their control effects. This paper therefore concludes that the control effect of fuzzy PID control is better than the other two controls, and through experiments, we also find that fuzzy PID has decent adaptivity. This paper provides a certain research basis for the subsequent research on PID control, which has important theoretical significance and engineering value.

Keywords: Liquid level control system; PID control; Cascade control; Fuzzy control.

1. Introduction

In the coming era of industrial automation, liquid level represents one of the mainstream control objectives in industrial production. Liquid level control system is a control system that takes liquid level as the controlled parameter and maintains this parameter at the desired value through the control and regulation action of the controller, and meets certain steady-state accuracy and dynamic following performance for all aspects of life. Industrial applications of liquid level control include but are not limited to: boilers, food processing, dairy, beverage, spraying, filtration, wastewater treatment, pharmaceutical industry, nuclear power plants, water purification systems, automatic dispensing and filling devices, and industrial chemical processing[1].

PID control is the most classic control in industrial automation[3]. PID control is an effective combination of proportional, integral and differential[4]. According to the deviation of the system input, PID control is based on the PID function relationship for the operation, and the results are used to control the output. In actual production, conventional PID controller parameters are often poorly adjusted, poor performance, and poor adaptability to operating conditions[5].

For better control of the liquid level, here we introduce three PID control methods applicable to the dual-capacity tank level system, which include single-loop PID control, Cascade PID Control method and fuzzy PID control method, giving the design ideas and characteristic tests of each control method. Based on these three control methods, we designed a double-volume tank level control system that can make the level restricted within the allowed range of the process, but there are differences in the control effects of the three control methods. Therefore, we built a simulation experiment platform using matlab to specifically study the control effects of the three methods on the double-volume tank level system. Through the experiments, we found that the overshoot and response time of fuzzy PID are smaller than the other two control methods, and the decay rate meets the industrial requirements, concluding that the performance of fuzzy PID control capable of autonomous regulation is better than that of single-loop PID control and Cascade PID Control. PID control with good stability and robustness can provide a new opportunity for the development of level control.
2. Introduction to the Principle of Liquid Level System

In this paper, a double-volume water tank is selected as the level control object. As a typical second-order level system, the double-volume tank requires that the level of the tank be equal to the height required by the given value by controlling the valve opening, and when the level is regulated to a certain height range, one can ensure the continuous operation of the production process. However, in the actual double-capacity water tank, the problem of non-linearity of the load valve is unavoidable and therefore hinders the level control effect. The structure of the double-volume tank level system is shown in Figure 1-1, which consists of two single-volume tanks, and the state of the system and its parameters are time-varying.

According to the material balance relationship, the relationship equation is derived as follows:

\[
\begin{align*}
\Delta H_1 - \Delta H_2 &= C_1 \frac{d\Delta h_1}{dt} \\
\Delta H_2 - \Delta H_3 &= C_2 \frac{d\Delta h_2}{dt}
\end{align*}
\]  

(2-1)

where \(\Delta H_1\) is the incremental inflow flow, the \(\Delta H_2\) is the intermediate flow increment, and \(\Delta H_3\) is the outflow flow increment, and \(C_1\) is the cross-sectional area of tank 1, and \(\Delta h_1\) is the incremental level of tank 1, and \(C_2\) is the cross-sectional area of tank 2, and \(\Delta h_2\) is the increment of tank 2 level.

\(\Delta H_1\) is caused by a change of the regulating valve \(\Delta u\). In the normal operating condition, i.e., the differential pressure of the valve is constant before and after, there is:

\[
\Delta H_1 = K_u \Delta u
\]  

(2-2)

where \(K_u\) is the regulating valve flow coefficient.

Intermediate flow \(H_2\) and outflow flow rate \(H_3\) are determined by the level of tank 1 and the level of tank 2, respectively. \(h_1\) and the liquid level height of tank 2 \(h_2\) which are generally non-linear, and we consider that they satisfy the following relationship:

\[
\begin{align*}
H_2 &= K_v \sqrt{2gh_1} \\
H_3 &= K_w \sqrt{2gh_2}
\end{align*}
\]  

(2-3)

where \(K_v\) is the valve flow coefficient of the regulating valve \(v\), and \(K_w\) is the valve flow coefficient of the regulating valve \(w\).

When the water storage reaches the respective equilibrium point \((h_{1x}, \text{the}H_{2x}), (h_{2x}, \text{and}H_{3x})\), then there is

\[
\begin{align*}
\Delta h_2 &= \frac{gK_v}{\sqrt{2gh_1}} \Delta h_1 = \frac{\Delta h_1}{R_1} \\
\Delta h_3 &= \frac{gK_w}{\sqrt{2gh_2}} \Delta h_2 = \frac{\Delta h_2}{R_2}
\end{align*}
\]  

(2-4)

where \(R_1\) is the liquid resistance of water tank 1, and \(R_2\) is the liquid resistance of water tank 2.

From the above equation, the differential equation of the double-capacity water tank can be introduced

\[
R_1R_2C_1C_2 \frac{d^2\Delta h_2}{dt^2} + (C_1R_1 + C_2R_2) \frac{d\Delta h_2}{dt} + f = R_2K_u \Delta u
\]  

(2-5)

Under zero initial conditions, the above equation is subjected to a Rasch transform to obtain the transfer function of the double-capacity water tank

\[
G(s) = \frac{R_2K_u}{R_1R_2C_1C_2S^2 + (C_1R_1 + C_2R_2)S + 1}
\]  

(2-6)

The physical meaning and values of each parameter in equations are shown in Table 1-1, selected for different steady-state values \(h_{1x}, h_{2x}\). According to the calculation, it is obtained that \(R_1 = 8\), \(R_2 = 10\), \(C_1\), \(C_2\) can be measured directly, and \(f\) is a constant. Its transfer function is given by

\[
G(s) = \frac{1}{80s^2 + 18s + 1}
\]  

(2-7)
Figure 2-1 Schematic Diagram of the Structure of the Dual Capacity Water Tank

| Table 2-1 Parameter Table of Double-volume Water Tank |
|------------------------------------------------------|
| Symbols | Physical Meaning | Numerical Value | Unit |
|---------|------------------|-----------------|------|
| $C_1$  | Cross-sectional area of water tank 1 | 2               | $m^2$ |
| $C_2$  | Cross-sectional area of water tank 2 | 1.25            | $m^2$ |
| $h_{1x}$ | Equilibrium level of water tank 1 | 20              | $m$  |
| $h_{2x}$ | Equilibrium level of water tank 2 | 20              | $m$  |
| $g$    | Gravitational acceleration | 10              | $m/s^2$ |
| $K_u$  | Control valve $u$ The valve flow coefficient of | 0.125           | $m^3/s$ |
| $K_v$  | Control valve $v$ The valve flow coefficient of | 2.236           | $m^3/s$ |
| $K_w$  | Control valve $w$ The valve flow coefficient of | 1.11            | $m^3/s$ |

3. Introduction to Common PID Control Algorithms

3.1 Single-loop PID Control

The schematic block diagram of single-loop PID is shown in Figure 3-1, and its transfer function is:

$$G(s) = K_p + K_i \frac{1}{s} + K_d \cdot s$$  \hspace{1cm} (3-1)

where $K_p$ is the proportional gain 1], and $K_i$ is the integration coefficients, and $K_d$ is the differential coefficient.

Its output signal is:

$$u(t) = K_p \cdot e(t) + K_i \int_0^t e(t)dt + K_d \frac{de(t)}{dt}$$  \hspace{1cm} (3-2)

From Eq. 3-2, it can be seen that when a deviation signal is input, only changing $K_p$, the output of the regulator would change immediately in proportion to the deviation. Within a certain range, the larger the input deviation signal is, the stronger the regulating effect of the output is and the faster the reflection speed is. However, the increase of the $K_p$ will make the system less stable. Thus, it cannot be increased indefinitely. It is possible to achieve a static difference-free regulation. There is also a drawback, as its regulation is gradually enhanced with the accumulation of time. So when the regulation is slow, and the problem of untimely regulation will occur. When the inertia of the object
is large, the tuned parameters will appear a large amount of overshoot, and the adjustment time will be extended, even making the system difficult to stabilize, so the $K_d$ is introduced. It can reduce the overshoot, effectively overcome the oscillation, and improve the stability of the system. In addition, $K_d$ can speed up the dynamic response of the system and reduce the adjustment time, thus improving the dynamic performance of the system. Therefore, the three are coordinated with each other to achieve satisfactory control effect. Generally speaking, the output curve of the system can meet the overshoot amount $\sigma \leq 20\%$, and the decay ratio between 0.75~0.9 is called satisfactory control effect. Therefore, the parameter adjustment of single-loop PID is crucial.

There are many single-loop PID parameter regulation methods, commonly used are step response curve method, decay curve method, empirical method (also known as trial and error method), etc. For the double-volume tank level system in this paper, the whole control system framework is simulated on matlab. This paper adopts the empirical method, that is, we first set $K_i$ and $K_d$ to zero, adjust $K_p$ from small to large, and observe the curve. After meeting the requirements, keep $K_p$ unchanged, then adjust $K_i$ and $K_d$ to make the control effect meet the requirement.

The single-loop PID control system has good dynamic following performance and robustness. Changing the input of the system, the single-loop PID control system is able to change the output with the change of the input and the output meets the performance index.

![Figure 3-1. Schematic Diagram of the Structure of Single-loop PID Control](image)

3.2 Cascade PID Control

In the face of some complex systems, single-loop control is unsatisfactory. Therefore, cascade control is often used as an effective alternative method. Its structure is shown in Figure 4-1, where one is PID controller, called the secondary controller, and the other is PID controller, called the primary controller. They are used to adjust the setpoint of the internal controller. The process output is fed back to the master controller and the intermediate process variables are fed back to the internal controller. The main controller is slower and mainly plays the role of correction; the vice controller is faster and mainly plays the role of fast adjustment. Based on the characteristics of the inertia delay of the controlled object regulation channel, the slow feedback of the signal of the regulated amount, we choose a middle point signal from the regulation channel of the object which is faster than the response of the regulated amount as a supplementary feedback signal for regulation[7]. This can decompose the high-order controlled object into two low-order controlled objects, to facilitate the determination of the parameters of the PID.

Cascade PID control is able to decompose the higher-order system into a lower-order system consisting of a main, sub-feedback loop with less delay characteristics and inertia in its dynamic characteristics, in which case the regulation process of the sub loop is much faster. When the vice feedback loop eliminates the disturbance, the main feedback loop is basically unaffected. Therefore, the main feedback loop can be considered as an open circuit when the secondary feedback loop is active; when the main loop is active, the secondary loop can be considered as a fast-acting follower system[7]. In this way, the disturbances affecting the intermediate variables can be adjusted in advance, which can effectively overcome the disturbance of the regulation amount and improve the dynamic characteristics of the controlled object with a certain degree of self-adaptability, so that the control effect of the whole system can be improved.

The usual procedure for tuning the cascade PID control parameters is to start with the inner loop and sequentially tune the sub-PID controller, then consider the resulting inner closed-loop transfer function while integrating the outer loop.
3.3 Fuzzy PID Control

Table 3-1. Fuzzy control rules table

$$\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
\text{e} & \text{e}_c & \text{NB} & \text{NM} & \text{NS} & \text{ZO} & \text{PS} & \text{PM} & \text{PB} \\
\hline
\text{NB} & \text{NB} & \text{NB} & \text{NS} & \text{ZO} & \text{PS} & \text{PM} & \text{PB} \\
\text{NM} & \text{NB} & \text{NB} & \text{NS} & \text{ZO} & \text{PS} & \text{PM} & \text{PB} \\
\text{NS} & \text{NM} & \text{NM} & \text{NS} & \text{ZO} & \text{PS} & \text{PM} & \text{PB} \\
\text{ZO} & \text{NS} & \text{NS} & \text{ZO} & \text{PS} & \text{PM} & \text{PM} & \text{PM} \\
\text{PS} & \text{NS} & \text{NS} & \text{ZO} & \text{PS} & \text{PM} & \text{PM} & \text{PM} \\
\text{PM} & \text{NS} & \text{ZO} & \text{PS} & \text{PM} & \text{PM} & \text{PB} & \text{PB} \\
\text{PB} & \text{ZO} & \text{PS} & \text{PM} & \text{PM} & \text{PM} & \text{PB} & \text{PB} \\
\hline
\end{array}$$

$$\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
\text{e} & \text{e}_c & \text{NB} & \text{NM} & \text{NS} & \text{ZO} & \text{PS} & \text{PM} & \text{PB} \\
\hline
\text{NB} & \text{ZO} & \text{ZO} & \text{ZO} & \text{ZO} & \text{ZO} & \text{ZO} & \text{ZO} \\
\text{NM} & \text{NM} & \text{NM} & \text{NS} & \text{NS} & \text{ZO} & \text{ZO} & \text{ZO} \\
\text{NS} & \text{NB} & \text{NM} & \text{NS} & \text{ZO} & \text{PS} & \text{PS} & \text{PS} \\
\text{ZO} & \text{NB} & \text{NM} & \text{NS} & \text{ZO} & \text{PS} & \text{PM} & \text{PB} \\
\text{PS} & \text{NS} & \text{NS} & \text{ZO} & \text{PS} & \text{PM} & \text{PM} & \text{PM} \\
\text{PM} & \text{ZO} & \text{ZO} & \text{PS} & \text{PM} & \text{PM} & \text{PM} & \text{PM} \\
\text{PB} & \text{ZO} & \text{ZO} & \text{ZO} & \text{ZO} & \text{ZO} & \text{ZO} & \text{ZO} \\
\hline
\end{array}$$

$$\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
\text{e}_c & \text{e} & \text{NB} & \text{NM} & \text{NS} & \text{ZO} & \text{PS} & \text{PM} & \text{PB} \\
\hline
\text{NB} & \text{PS} & \text{NB} & \text{NS} & \text{ZO} & \text{PS} & \text{PM} & \text{PB} \\
\text{NM} & \text{PS} & \text{NS} & \text{NB} & \text{NS} & \text{ZO} & \text{ZO} & \text{ZO} \\
\text{NS} & \text{ZO} & \text{NS} & \text{NB} & \text{NS} & \text{ZO} & \text{ZO} & \text{ZO} \\
\text{ZO} & \text{ZO} & \text{NS} & \text{ZO} & \text{PS} & \text{ZO} & \text{ZO} & \text{ZO} \\
\text{PS} & \text{ZO} & \text{ZO} & \text{ZO} & \text{ZO} & \text{ZO} & \text{ZO} & \text{ZO} \\
\text{PM} & \text{PB} & \text{PS} & \text{PS} & \text{PS} & \text{PS} & \text{PB} & \text{PB} \\
\text{PB} & \text{PB} & \text{PM} & \text{PM} & \text{PM} & \text{PM} & \text{PB} & \text{PB} \\
\hline
\end{array}$$

As shown in Figure 5-1, fuzzy PID is based on the conventional PID control and uses the idea of
diagram. fuzzy reasoning, which will be the deviation of the controlled quantity e and the rate of change of deviation e_c as the input variables of the two-dimensional fuzzy controller. Through the fixed law of fuzzy inference, we get $K_p$, $K_i$ and $K_d$ as the output value. The PID parameters are adjusted online using the fuzzy control law, where the fuzzy control part includes fuzzification, fuzzy inference calculation and defuzzification. Once the system response is obtained at each sampling moment, the
system response can be adjusted according to the deviation from the given value and the trend of change. Based on the existing knowledge of the system control, the fuzzy control method is applied to increase or decrease the control effort appropriately to control the response to deviate from the given direction and make the output stabilize as soon as possible [2].

In the double-volume tank level fuzzy PID control system in this paper, based on practical experience, the parameters $K_p, K_i, K_d$ in different $e$ and $e_c$ under self-adjustment need to satisfy the following adjustment principles [6].

When the error $|e|$ is large, in order to make the system have a better fast-tracking performance, regardless of the trend, the error should be taken as the larger $K_p$ and smaller $K_d$. At the same time, in order to avoid large overshoot of the system response, the integral action should be limited by taking a smaller $K_i$ value.

When the error $|e|$ is at a medium size, in order to make the system response with a small overshoot, $K_p$ should be taken smaller. Also, to ensure the response speed of the system, $K_i$ and $K_d$ should be moderate in size. Among them, $K_d$ has a large impact on the response of the system.

When the error $|e|$ is small, to ensure that the system has good steady-state performance, the $K_p$ and $K_i$ should be taken larger. At the same time, to avoid oscillation of the system near the set value and to consider the anti-interference performance of the system, when $|e_c|$ is small, the $K_d$ can be taken larger; when $|e_c|$ is larger, the $K_d$ should be taken smaller.

On the basis of expert experience and adjustment through simulation experiments, a table of fuzzy control rules can be summarized as shown in Table 3-1.

For $K_p$, $K_i$, and $K_d$, the adjustment formulas are:

\[
\begin{align*}
K_p &= K'_p + a \cdot \{ae, \beta e_c\}K_p = K'_p + a \cdot \Delta K_p \\
K_i &= K'_i + b \cdot \{ae, \beta e_c\}K_i = K'_i + b \cdot \Delta K_i \\
K_d &= K'_d + c \cdot \{ae, \beta e_c\}K_d = K'_d + c \cdot \Delta K_d
\end{align*}
\]  

(5-1)

where the $\alpha$, $\beta$, $a$, $b$ and $c$ are the gain links. According to the formulas, simulation can be carried out. In the online operation process, the fuzzy PID dual capacity tank level control system uses the above fuzzy control rules to complete the PID parameters of self-calibration, by constantly detecting $e$ and $e_c$ values to achieve real-time adjustment of $K_p$, $K_i$, and $K_d$, so that the response speed, overshoot and decay rate of the fuzzy PID dual capacity tank level system are superior to the conventional PID control.

The anti-interference ability and dynamic following ability of the fuzzy PID dual-capacity tank level control system in the face of disturbing signals and changing input signals make the system control effect improved. This is due to its real-time adjustment of PID parameters to shorten the response time and reduce the overshoot, so that the level fluctuation of the fuzzy PID dual-capacity tank level control system meets the process requirements.

![Figure 3-3. Fuzzy PID Control Structure Schematic](image)

4. Simulation

When adjusting the parameters of single-loop PID, we found that the control effect of the system met the process requirements when the parameters of P, I and D were 1.5, 0.5 and 1.5, respectively.
When the parameters of the main controller are 0.85, 0.1 and 0.1, and the parameters of the subcontroller are 1.1, 0.1 and 0.5, the cascade PID control satisfies the process requirements. The fuzzy PID control loop is built according to the fuzzy control rule table and the control structure schematic. Among them, the parameters of PID controller are 1.5, 0.5, 1.5, and by changing $\alpha$, $\beta$, $a$, $b$, and $c$, the control effects are adjusted. The simulink simulation diagram of the three PIDs is shown in Figure 4-1.

As shown in Figure 4-2, we simulated the three control methods simultaneously for 600s. In the first 200s of time, the fuzzy PID dual-capacity tank level control system has the best control effect, the cascade PID control has the second best effect, and the conventional PID control has the worst effect. This is due to the fact that the $K_p$, $K_i$, and $K_d$ of the cascade PID and conventional PID are fixed and cannot be changed according to the input deviation change. After changing the size of the input signal, it can be seen that when the input signal changes, all three PID controls can follow the signal change, so that the output can achieve our desired goal. When we increase or decrease the input signal, the output of the three PID control systems will also increase or decrease accordingly. When the input signal changes, the fuzzy PID has the best following ability, because the fuzzy PID dual capacity water tank control system can detect $e$ and $e_c$ in real time. Whene and $e_c$ change, the $K_p$, $K_i$ and $K_d$ are changed to adjust the output of the system so that it can reach the desired value.

As there are often disturbances in the actual working environment. In order to study the immunity of PID to disturbance, disturbance is added to the control system. As shown in Figure 4-4, adding a 10% external disturbance signal to the three PID control systems, it can be seen that fuzzy PID and cascade PID have better immunity to external disturbances than single-loop PID. In addition, when adding internal disturbances at 400s, cascade PID has the best immunity to disturbances in the face of a 10% internal disturbance signal, while fuzzy PID control and single-loop PID have the same control effect. This is due to the presence of the cascade PID subloop, which can quickly overcome the incoming subloop disturbance and ensure that the disturbance does not enter the main loop, improving the control effect. Besides, the introduction of the subloop reduces the object phase lag and can make the main loop response much faster.

In addition to this, we also analyzed the robustness of three PID dual capacity water tank control systems. As shown in Figure 4-6, it can be seen that the $K_u$ of the dual-capacity water tank is changed to 0.25, i.e., the transfer function becomes $G(s) = \frac{2}{80s^2 + 18s + 1}$. By observing the change of their output curves, it can be seen that the overshoot of both fuzzy PID and single-loop PID increases and the curve oscillation increases; while the overshoot of cascade PID decreases and the curve oscillation is not obvious. Therefore, we can conclude that the robustness of cascade PID control is better than fuzzy PID and Cascade PID.

![Figure 4-1. Simulink Simulation of Different PID Control](image-url)
Figure 4-2. Experimental Output Curves of Followership with Different PID Control

Figure 4-3. Output Curves of Different PID Control for Interference Resistance
5. Conclusion and Prospect

In summary, all three PID controls were able to control the level of the double-capacity tank within the industrially allowed fluctuations, and their overshoot was less than 20%. In addition, the overshoot of fuzzy PID is the smallest and the response speed is the fastest, which can conclude that the control effect of fuzzy PID is better than the other two controls. Meanwhile, all three PID control methods have decent anti-disturbance, following and robustness. Fuzzy PID control has both the inherent stability and good closed-loop regulation performance inherent in traditional PID, and the adaptive nature of fuzzy controller, which can control complex nonlinear systems very well. With the development of control theory and computer hardware and software, PID control will tend to be more intelligent and will become a promising research direction in process control, which will enter a more in-depth and broad application world.

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