THz-driven surface plasmon undulator as a compact highly directional narrow band incoherent x-ray source

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We propose a short period undulator which is based on the alternating electromagnetic field pattern of THz-driven surface plasmons in a thin conductive layer on a dielectric grating. An approximate analytical model allows to assess the key performance parameters of the undulator and to estimate the emitted radiation spectrum. The specific example of a graphene based undulator is simulated in detail. For a moderate electron beam energy of 100 MeV and a bunch charge of 0.5 pC the 40 mm long undulator is shown to emit narrow band 1 keV x-ray pulses with a peak brightness of approximately $10^{16}$ photons/(s mrad$^2$ mm$^2$ 0.1% BW). It therefore has potential for a compact and low cost x-ray source.

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I. INTRODUCTION

In a variety of electron accelerator based light sources relativistic electron bunches propagate through an undulator and emit intense narrow band radiation [1–3]. Undulators are generally composed of a periodic, alternating array of normal-conducting or superconducting electromagnets, permanent magnets or hybrid magnets and the resulting magnetostatic field pattern forces the electrons on a wiggling orbit, which leads to emission of electromagnetic radiation [4]. Depending on the kinetic energy of the electrons and on the undulator period, the emission can range from THz to hard x-ray photon energies. Typical undulator periods are tens of millimeters, magnetic field strengths range from about one Tesla to more than ten Tesla for superconducting magnets, and undulators in free electron lasers can be tens to hundreds of meters in length [5,6].

In order to miniaturize undulators and/or to produce a given photon energy with less energetic electrons, the undulator period should be reduced. For example, the same photon energy can be achieved by a 10 times less energetic electron beam if the undulator period is reduced a hundred-fold. Likewise, for a given electron energy the photon energy increases when the undulator period is reduced. A smaller scale undulator would also be beneficial for the development of compact light sources, especially in combination with miniaturized accelerators, for example, with those based on laser wakefields in plasma [7], ultra-short laser pulses in free space [8] or those based on laser-driven dielectric structures [9]. In the past, several efforts to reduce the periodicity of static magnetic field patterns have been made and values as low as 15 mm [10–13] were achieved. Further reduction down to about 100 μm was realized with laser micromachined permanent magnets [14] or electromagnets [15]. The peak magnetic field in these devices was still as high as 0.7 T [16]. A conceptually different approach uses oscillating electromagnetic fields, for example, in laser irradiated dielectric gratings [17], laser-driven undulators [18], microwave undulators [19], plasma wave undulators [20–22] or surface plasmon polariton (SPP) undulators [23,24].

In view of device miniaturization, the SPP based undulator is especially interesting since periods as low as 10 nm have been predicted. However, one has to bear in mind that SPPs exist at a conductor-dielectric interface and decay exponentially away from the interface on a length scale that is similar to its wavelength. For this reason the choice of SPP wavelength is determined by the transverse size of the electron bunch and is further influenced by its emittance, the opening angle of the radiation cone and possible wakefields excited at the interfaces. Without loss of generality we hereafter consider electron bunches with a transverse size of several tens of microns which results in a...
SPP undulator driven by a THz to microwave range source depending on the SPP confinement factor, i.e., the ratio of SPP over free space wavelength. The SPP undulator with a gap size of several tens of microns has a period of approximately hundred microns, which is much shorter than it would be for a THz-driven dielectric grating undulator with a similar gap size [17].

### II. SURFACE PLASMON UNDULATOR CONCEPT

#### A. Geometry

A schematic of the SPP undulator is shown in Fig. 1. It consists of two oppositely oriented dielectric gratings with periodicity \( \lambda_u \) and relative permittivity \( \epsilon_1 \) which are separated by a gap of \( 2a \). The middle of the gap is at \( x = 0 \). Both gratings are coated with a thin conductive layer, which is characterized by an effective surface conductivity \( \sigma_s \). The conductive layer can be a metal, a semiconductor or a two-dimensional material such as graphene. The latter case is considered in more detail in Sec. III. The grating grooves are filled with a low index polymer, with relative permittivity \( \epsilon_2 \), to provide sufficient mechanical support for the conductive layer. The structure is excited by two counterpropagating and normally incident THz pulses (\( \pm x \) axes) which are linearly polarized along the \( z \) axis. Their relative carrier phase difference is adjusted such that their electric fields cancel while their magnetic fields add at the center of the gap, i.e., at \( x = 0 \). The two THz pulses excite two counterpropagating SPPs (\( \pm z \) axes) which, in case of spatial overlap, result in a standing wave pattern. At the center of the gap the SPPs are predominantly polarized normal to the grating-conductor interface, i.e., parallel to the \( x \) axis. The electron bunches propagate in the positive \( z \) direction and interact with the SPP fields and the THz drivers.

If the grating grooves are much longer in the \( y \) direction than the gap size, the problem reduces to two dimensions (\( xz \) plane) with transverse magnetic field distribution, i.e., \( E_y = B_z = B_z = 0 \).

#### B. Surface plasmons in a double-layer system

First we seek the resonance condition for efficient SPP excitation and approximate analytic expressions for the SPP fields in the structure. Approximating the two gratings by an effective medium with relative permittivity \( \epsilon_e = \frac{\epsilon_1}{\epsilon_0} + (1 - \frac{\epsilon_1}{\epsilon_0}) \epsilon_2 \) (where \( w \) is the grating tooth width) results in the implicit SPP dispersion relation [25]

\[
\epsilon_0 \omega^2 \left( \frac{e_e}{\kappa_e} + \frac{1}{\kappa_v} \right) \left( \frac{1}{\epsilon_0 \omega} - \frac{\epsilon_e}{\epsilon_v} + \frac{i \sigma_s}{\epsilon_0 \omega} \right) = \epsilon_0 \omega^2 \left( \frac{e_e}{\kappa_e} - \frac{1}{\kappa_v} \right) \left( \frac{1}{\epsilon_0 \omega} - \frac{\epsilon_e}{\epsilon_v} - \frac{i \sigma_s}{\epsilon_0 \omega} \right),
\]

with the frequency \( \omega \), the vacuum permittivity \( \epsilon_0 \), \( \kappa_e^2 = q^2 - \omega^2/c^2 \), \( \kappa_v^2 = q^2 - \epsilon_e \omega^2/c^2 \), the SPP wave vector \( q(\omega) \) and the speed of light in vacuum \( c \). Efficient coupling of the incident THz radiation to the SPP, i.e., phase matching, requires \( \Re(q(\omega)) = \kappa_u \). From this condition and Eq. (1) the resonance frequency, \( \omega_0 \), is found. The nonzero SPP fields at resonance are approximately given by

\[
E_x = E_0 e^{-k_x x} \cosh(k_x x) (\cos \psi_- + \cos \psi_+),
\]
\[
E_z = E_0 e^{-k_x x} \frac{k_0}{c k_u} \sinh(k_x x) (\sin \psi_- - \sin \psi_+),
\]
\[
B_y = E_0 e^{-k_x x} \frac{k_0}{c k_u} \cosh(k_x x) (\cos \psi_- - \cos \psi_+),
\]

where \( E_0 \) is the electric field amplitude, \( k_0 = 2\pi/\lambda_0 = \omega_0/c \) is the free space THz wave vector, \( k_0^2 = k_v^2 - k_0^2 \) and \( \psi_\pm = \omega_0 t \pm k_u (z - z_0) - \pi/2 \) are the phases of the two counterpropagating SPPs, where \( z_0 \) is the location of a grating edge. Here and hereafter the minus sign refers to the SPP copropagating with the electron bunch and the plus sign to the counterpropagating SPP. The fields experienced by an electron moving through the structure are a coherent superposition of the incident THz field and the SPP field. For simplicity we assume the THz field to be a plane wave with only one nonzero electric and magnetic field component which we hereafter refer to as \( E_{\text{THz}} \) and \( B_{\text{THz}} \). The total fields then are

\[
E_x \rightarrow E_x
\]
\[
E_z \rightarrow E_z + E_{\text{THz}}
\]
\[
B_y \rightarrow B_y + B_{\text{THz}}.
\]

The relative SPP amplitude \( E_0/E_{\text{THz}} \) is determined by the absorption cross section and is extracted from numerical simulations. While the THz driver is typically a single cycle pulse, the resonant SPP fields oscillate for many cycles depending on the damping. Therefore, injecting electrons after the THz drivers have passed through the structure will eliminate their contribution and electrons will...
interact with the SPP fields only. For the sake of completeness, we will consider SPPs as well as THz fields. The damping of the SPP fields can be neglected if one considers THz drivers with a tilted pulse front which is matched to the electron velocity [26–28].

C. Undulator radiation properties

We solve the relativistic Lorentz equation of motion for a single electron propagating through the fields given in Eq. (3) and characterize the emitted radiation by its single electron propagating through the fields given in

\[ k_u = k_u \left( 1 \pm \frac{\beta_{ph}}{\beta_{z,0}} \right) \]

\[ k_T = \frac{k_0}{\beta_{z,0}} \]

\[ E_0^\pm = E_0 e^{-k_u a} (1 \pm \beta_{ph} \beta_{z,0}) \]

\[ E_T = 2 \beta_{z,0} E_{THz}. \]  

(8)

For the highly relativistic case (i.e., \( \gamma \gg 1 \)) we find that \( K^\pm \approx a_u \) and that the three undulator wave vectors approach constant values, namely the free space THz wave vector \( k_T \) and the SPP wave vector \( k_u \) shifted by the phase velocities of the copropagating and the counterpropagating SPP. That is, the oscillating fields can be interpreted as a superposition of three electrostatic undulators each having a different effective periodicity and effective electric field strength.

Since in the frame moving at the initial electron speed the electron motion is nonrelativistic we may use the well-known Larmor equation to calculate the emitted radiation power:

\[ P' = \frac{e^2}{6\pi e_0 c} \dot{\beta}^2, \]  

(9)

where primed variables refer to this moving frame and the dot indicates differentiation with respect to time.

By using \( c t' = \gamma c (1 - \beta_{z,0}^2) t \) the electron phase in the rest frame is found to be

\[ \psi' = \frac{\omega_0 t + k_u c \beta_{z,0} t - z_0}{\gamma(1 - \beta_{z,0}^2)} + k_u z_0 - \frac{\pi}{2}. \]  

(10)

The directional emission frequencies in the lab frame are obtained by considering the Doppler shift \( \{ \gamma [1 - \beta_{z,0} \cos(\theta)] \}^{-1} \), where \( \theta \) is the angle with respect to the propagation direction (z axis). The resulting emission frequencies and wavelengths are given by

\[ \omega_r^\pm = \omega_0 \frac{\beta_{ph} \pm 1}{\beta_{ph}} \approx 2 \gamma^2 c k_u^\pm \]

\[ \lambda_r^\pm = \lambda_0 \left[ 1 - \beta_{z,0} \cos(\theta) \right] \approx \frac{\lambda_u^\pm}{2 \gamma^2} \]  

(11)

and similarly the emission wavelength due to the interaction with the THz drivers is given by

\[ \lambda_T^r = \lambda_0 \left[ 1 - \beta_{z,0} \cos(\theta) \right] \approx \frac{\lambda_T}{2 \gamma^2}. \]  

(12)

where \( \lambda_u^\pm = \frac{2\pi}{k_u^\pm} \) and \( \lambda_T = \frac{2\pi}{k_T} \). The approximations hold for \( \theta = 0 \) and for highly relativistic electrons. Note that Eqs. (11) and (12) are similar to those found for a magnetic
undulator, where \( \lambda_r = \frac{\hbar}{2\pi} (1 + K^2/2) \) [29], assuming \( K \ll 1 \).

The emitted radiation is linearly polarized in the \( x \) direction and the total emitted energy can be estimated via Eq. (9). Using \( \beta_\| = \sqrt{1 + \gamma^2 - 1} \) and averaging over one undulator period one obtains the averaged emitted power \( \bar{P} \pm \) due to the interaction with the copropagating and counterpropagating components of the SPP:

\[
\bar{P}^\pm = \frac{Qe^2 c \beta_\|}{12 \pi \varepsilon_0} (K^\pm k_\|^z)^2,
\]

where \( Q \) is the bunch charge and where incoherent emission is assumed. A similar equation is obtained for the interaction with the THz drivers when \( K^\pm \) and \( k_\|^z \) are replaced by the corresponding parameters. We obtain the emitted energy by multiplying the average power by the time of flight through the undulator \( L/(c \beta_\|) \),

\[
W^\pm = \frac{L Q e^2 \beta_\|}{12 \pi \varepsilon_0} (K^\pm k_\|^z)^2,
\]

which scales with the square of the electric field amplitude. The relative bandwidth of the emission mainly depends on the number of undulator periods and the relative energy spread of the electrons. The natural bandwidth of an undulator can be estimated by considering the Fourier transform limit of the radiation cycles [29]. At the central frequencies \( \omega^\pm \), the emitted radiation oscillates \( N_\|^z = L k_\|^z / (2\pi) \) times resulting in a relative bandwidth

\[
\frac{\Delta \omega^\pm}{\omega^\pm} = \sqrt{(0.886/N_\|^z)^2 + (2\Delta E/E)^2},
\]

where \( \Delta \omega^\pm \) is the full width at half maximum (FWHM) of the on-axis spectral intensity and \( \Delta E = 2\sqrt{2\ln 2} \sigma_E \) is the FWHM of the initial energy spread of the electrons (assuming a Gaussian energy distribution).

### III. GRAPHENE SURFACE PLASMON UNDULATOR

We next study the explicit case of an SPP undulator with two monolayers of graphene as conductive sheet material. Graphene is well suited as a plasmonic material in the THz range [25] and exhibits a very high breakdown threshold, in excess of 3 GV/m for 50 fs pulses with a center wavelength of 790 nm [30]. To the best of our knowledge, there is no measurement of the graphene breakdown threshold in the THz range, but in the following we will assume a breakdown threshold exceeding 1 GV/m.

### A. Geometry

We numerically tested the SPP undulator with experimentally viable parameters. The grating period and the gap height are set to 130 and 50 \( \mu \)m, respectively, which are a good compromise to accommodate electron bunches with transverse sizes on the order of tens of microns while resulting in a close to homogeneous in-gap field distribution in the \( x \) direction. The undulator length is arbitrarily set to \( L = 40 \) mm (corresponding to 300 grating periods) but in practice is linked to the available THz source [31] as one has to maintain the desired field strength over the entire undulator length. We can estimate the needed THz energy assuming a tilted pulse front with a focal size of 40 mm by 300 \( \mu \)m, a pulse duration of 1 ps and a field strength of 100 MV/m. For these parameters the energy of the two THz drivers is approximately 0.3 mJ, which could be obtained by today’s THz sources [32,33]. For the dielectric grating material we consider fused silica and for the polymer high-density polyethylene. Table I summarizes all the relevant dimensions of the graphene SPP undulator.

### B. Graphene surface plasmon fields

We calculate the resonance condition and the SPP fields for the parameters listed in Table I using the finite-element software COMSOL Multiphysics [34]. The simulations were performed in two dimensions (\( xz \) plane), in the frequency domain and for at least one unit cell of the grating, using periodic and scattering boundary conditions in \( z \) and \( x \) directions, respectively. The graphene layers were modeled as surface current boundary conditions and their conductivity was described by a Drude-like expression [25] which is a reasonable approximation at room temperature and for THz frequencies,

\[
\sigma(\omega) = \sigma_0 \frac{4E_F}{\pi \hbar/\tau - i\hbar \omega},
\]

with \( \sigma_0 = e^2/(4\hbar) \), the Fermi energy \( E_F \) and the scattering time \( \tau \). A distinctive feature of graphene, which makes it an ideal candidate for the realization of the proposed undulator, is that the Fermi energy \( E_F \) and therefore the

| Parameter               | Value         |
|-------------------------|---------------|
| Undulator length \( L \) | 40 mm         |
| Gap height \( 2a \)     | 50 \( \mu \)m |
| Grating periodicity \( \lambda_n \)| 130 \( \mu \)m |
| Grating tooth width \( \lambda_n \)| 130 \( \mu \)m |
| Groove depth \( d \)    | \( \lambda_n/2 \) |
| Substrate thickness \( s \) | 25 \( \mu \)m |
| Grating relative permittivity \( \epsilon_1 \) | 3.9 |
| Polymer relative permittivity \( \epsilon_2 \) | 2.0 |
| Grating—graphene distance \( t \) | 0.5 \( \mu \)m |
| THz peak electric field \( E_{THz} \) | 100 MV/m |

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conductivity can be tuned via doping or by applying a gate voltage. The scattering time $\tau$ is determined by the quality of the graphene layer and values between 400 fs and 1.6 ps have been reported [35]. Figure 2 shows the resonance wavelength $\lambda_0$ and the normalized field strength $E_x/E_{THz}$ at the center of the gap ($x = 0$) as a function of Fermi energy $E_F$ (a) and relaxation time $\tau$ (b). The dashed lines serve as a guide to the eye.

Hereafter, we numerically test the undulator for $E_F = 0.4$ eV and $\tau = 400$ fs. The resulting THz absorption spectrum is shown in Fig. 3 (blue dashed curve) and reveals a resonance at 290 $\mu$m which agrees reasonably well with the analytic approximations (1). The maximum absorption is close to 50% indicating efficient coupling. The black solid curve shows the normalized field strength $E_x/E_{THz}$ at the center of the gap. Interestingly, its maximum appears at a somewhat lower wavelength when compared to the absorption curve, i.e., at 282 $\mu$m. This difference is explained by the near-field diffraction pattern of the grating structure itself, which is more pronounced for shorter wavelengths and which adds to the total field distribution causing the observed redshift of the maximum. For a grating periodicity of 130 $\mu$m and a resonance free space wavelength of 282 $\mu$m the normalized phase velocity is $\beta_{ph} = 0.46$.

Figure 4 shows the nonzero field components excited by two counterpropagating THz sources at the resonance wavelength $\lambda_0 = 282$ $\mu$m along four undulator periods. Recall that the fields are composed of the incident THz field and the SPP fields. The field components $E_x$ and $B_y$ are relatively homogeneous in the $x$ direction within the gap, and $E_z$ vanishes near the gap center where the electrons propagate. Of special interest are the fields close to the gap center where the electrons propagate, which we show in Fig. 5 as a function of $z$ along four undulator periods.

We now calculate the effective undulator parameter as defined in Eq. (7). Figure 6 shows $K^\pm$ versus kinetic energy for two different SPP field strengths $E_0 = 100$ MV/m (a) and $E_0 = 1$ GV/m (b).

For relativistic energies the $K^\pm$ parameters approach the same asymptotic value which depends linearly on the SPP field strength (e.g., $K^\pm = 0.014$ for a field strength of 1 GV/m). The undulator parameter is limited by the breakdown threshold of graphene and the available THz source. For comparison, a typical magnetic undulator has $K \approx 1$ [36]. Consequently we expect a lower spectral intensity of the emitted radiation.

FIG. 2. Resonance wavelength $\lambda_0$ and normalized field strength $E_x/E_{THz}$ at the center of the gap ($x = 0$) as a function of Fermi energy $E_F$ (a) and relaxation time $\tau$ (b). The dashed lines serve as a guide to the eye.

FIG. 3. Simulated absorption (blue dashed curve) and normalized field strength $E_x/E_{THz}$ at the center of the gap (black solid curve) versus wavelength.

FIG. 4. Color-coded SPP field distributions in the $xz$ plane. The SPPs are excited by two THz drivers counterpropagating in the $\pm x$ direction and the fields are normalized to the THz field strength. Top: $x$-component of the electric field; middle: $z$-component of the electric field; bottom: $y$-component of the magnetic field.
The electron trajectories were calculated with VDSR [37] based on the field maps from the COMSOL simulation. A fourth order Runge-Kutta method was used where particle interactions were neglected, which is justified by the relativistic beam energies and the short undulator length [23]. In order to verify the assumption of negligible particle interactions we simulated the drift of an electron bunch with parameters as given in Table II in CST [38] taking into account particle interactions. We found that the transverse geometric emittance must be much smaller than \( \frac{a^2}{L} \) and thus the undulator performance, but also lead to an energy loss of the electrons on their passage through the undulator. We simulated wakefields using CST [38]. For simplicity we approximate the electron bunch by a line current with a longitudinal Gaussian distribution with standard deviation \( \sigma_r \) and \( \sigma_z \) in transverse and longitudinal directions. If required, the maximum allowed emittance can be estimated by setting the maximum transverse bunch size equal to the gap size and by using a beta function determined by the undulator length \( L \). As a result, the geometric emittance must be much smaller than \( \frac{a^2}{L} = 15.6 \, \text{nmrad} \) which corresponds to a normalized emittance of 3.1 \( \mu \text{mrad} \).

Before we discuss the undulator performance, we estimate the magnitude of possible wakefields excited by the electrons. Wakefields will not only modify the undulator field pattern and thus the undulator performance, but also lead to an energy loss of the electrons on their passage through the undulator. We simulated wakefields using CST [38]. For simplicity we approximate the electron bunch by a line current with a longitudinal Gaussian distribution with standard deviation \( \sigma_z \). Then we calculate the energy loss of a virtual spectator electron that follows the bunch at a variable distance. Figure 7 shows the wake potential for two different cases, i.e., for \( \sigma_z = 5 \, \mu \text{m} \) and 10 microns. The maximum wake potential amplitude for \( \sigma_z = 5 \, \mu \text{m} \) is as high as 540 kV/pC. The wake potential could be further reduced by defocusing the bunch in the \( y \) direction and therefore decrease the charge density.

In order to reduce the relative energy loss for a 100 MeV beam, a small bunch charge of 0.5 pC is considered. That is, the relative energy loss is approximately 0.3% and will be neglected in the following.

### D. Undulator performance

Unlike magnetic undulators where the alternating magnetic field pattern is time independent, THz driven SPP fields oscillate while the electrons propagate through the structure. Therefore the time delay between the electron injection and the THz driver determines the average

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**TABLE II. Parameters for numerical simulations.**

| Parameter             | Value   |
|-----------------------|---------|
| Kinetic energy \( E \) | 100 MeV |
| Relative energy spread \( \sigma_E/E \) | 0.02%   |
| Bunch charge \( Q \)   | 0.5 pC  |
| Transverse size \( \sigma_r \) | 5 \( \mu \text{m} \) |
| Longitudinal size \( \sigma_z \) | 5 \( \mu \text{m} \) |
electron velocity in the $x$ direction, which in turn leads to an overall deflection in the $x$ direction. Figure 8 shows the overall deflection $\Delta x$ at the end of the undulator versus the time delay between THz drivers and electron injection.

An electron injected at zero time delay, or at integer multiples of half the THz oscillation period, will perform a wiggling motion on its passage through the undulator but will not experience a net deflection at the end of the undulator. If injected a quarter oscillation before or after, the net deflection can be as large as the gap size and electrons might collide with the graphene. Moreover, the emission cone bends away from the $z$ axis. In the following we assume that electron bunches are injected so that their center of charge in the longitudinal direction has a zero time delay.

Next, we consider the $x$ component of the normalized momentum of a single electron during its passage through the undulator as it is shown in Fig. 9(a) for a field strength of 100 MV/m and a kinetic energy of 100 MeV.

We observe a complex beating pattern, which is characterized by Fourier peaks at 22, 26, and 71 mm$^{-1}$ as shown in Fig. 9(b). As discussed in Eq. (8) the three peaks originate from electrons interacting with the incident THz fields ($k_T$) and the copropagating and counterpropagating SPP fields ($k_u^\pm$). The analytical predictions of the effective undulator wave vectors, shown as dashed lines, agree with the simulation results, while the predicted undulator parameters of Eq. (7) agree with the ratio of the peak amplitudes. Accordingly, we expect several emission peaks in the emitted radiation spectrum.

Figure 10 shows the emission wavelength versus kinetic energy and we find good agreement between simulation results and analytical predictions based on Eq. (11) as long as the THz electric field strength is not higher than approximately 100 MV/m.
For a moderate kinetic energy of 600 keV the emission wavelengths are on the order of tens of microns and for increasing kinetic energies they decrease following a power law of approximately $E^{-2}$. Note that for low beam energies the electron bunch might collapse due to wakefield coupling and Coulomb repulsion, therefore these values are limited to low bunch charges. At kinetic energies as high as 1 GeV the photon energy approaches 100 keV.

For SPP field strengths in excess of $100 \text{ MV/m}$ the emission peaks start to deviate from the analytic prediction (11) as shown in Fig. 11 for the $\hbar \omega^+$ emission. Such redshift is also found in magnetic undulators when the undulator parameter is increased and results from the decreased average velocity in the $z$ direction [29]. In the following we always consider a THz field strength of $100 \text{ MV/m}$.

As mentioned above, the emission can be tuned by changing the graphene conductivity. For instance, lowering the Fermi energy to 0.2 eV shifts the resonance wavelength from $\lambda_0 = 282 \mu m$ to $\lambda_0 = 340 \mu m$. This in turn alters the SPP phase velocity $\beta_{ph}$ and the $\hbar \omega^+$ emission energy decreases from 1078 to 1020 eV, that is by 5%.

From the emitted energy $W_\gamma$ per bunch, i.e., Eq. (14), we can estimate the brightness $B$, which is the radiation flux divided by the phase space volume. We assume that the photon bunch duration is equal to the electron bunch duration and the transverse size of the photon beam is determined by the electron beam, such that $\sigma_\gamma = \sigma_r$. From numerical simulation we find a peak intensity of $5.5 \times 10^7$ photons/(sr 0.1% BW) which corresponds to a peak brightness of $B = 10^{16}$ photons/(s mrad$^2$ mm$^2$ 0.1% BW). Note that the peak intensity is proportional to $E_{\text{THz}}^2$ and also to $E^2$. Therefore the brightness can be increased by higher THz field strengths and/or higher electron beam energies.

Next, we analyze the angular distribution of the emitted radiation for the highest photon energy peak $\hbar \omega^+$ (similar results are found for the other emission peaks). Figure 12 shows a color-coded map of the emission intensity versus polar angle $\theta$ and photon energy for two different values of $\phi$. The black crosses mark the analytical results from Eq. (11).

The highest photon energy and emission intensity are observed on axis ($\theta = 0$). Moreover we find a slight anisotropy in emission due to the undulator motion in the polarization plane, that is, the intensity decays slightly slower with polar angle in the $yz$ plane as compared to the $xz$ plane.

Finally, we study the natural bandwidth of the emission peaks and investigate further broadening mechanisms. Here, we find a natural relative bandwidth of 0.2%. This is further affected by relative energy spread of the electron bunch. Figure 13 depicts the influence of the electron beam’s energy spread on the relative bandwidth of the on-axis spectrum. While the peak intensity decreases with increasing energy spread the bandwidth grows, which is in qualitative agreement with Eq. (15).

Lastly, we analyze the influence of longitudinal bunch size $\sigma_z$ and transverse bunch size $\sigma_r$ on the radiation spectrum. We found that both the on-axis intensity and bandwidth are independent of the longitudinal bunch size. When varying the transverse bunch size one has to keep in mind that the maximum bunch size is limited by the gap between the two graphene layers (here 50 $\mu m$). Therefore the considered transverse bunch size is always smaller than...
the plasmon extent (i.e., $k, \sigma_r < 0.5$). While the bandwidth remains unaffected the intensity increases with transverse bunch size. This is explained by the fact that a larger transverse beam comes closer to the graphene layers and those parts of the electron bunch experience a higher SPP field strength with the consequence of a slightly higher intensity.

IV. CONCLUSIONS

Propagating a 0.5 pC electron bunch with a kinetic energy of 100 MeV through a 40 mm long undulator with a periodicity of 130 $\mu$m and driven by a state-of-the-art THz source was shown to emit x rays with photon energies around 1 keV and a peak brightness of approximately $10^{16}$ photons/(s mrad$^2$ mm$^2$ 0.1 BW). The emission spectrum consists of three peaks which result from interaction with two counterpropagating plasmon fields and THz driver. The proposed THz driven SPP undulator may pave the way to a low cost, compact and tunable radiation source, which produces highly directional, linearly polarized and narrow band x-ray pulses. Without question, the SPP undulator cannot compete with undulators used in free electron lasers, however, such compact sources may find applications in radiotherapy, ultrafast x-ray diffraction experiments or time-resolved x-ray spectroscopy. Its brightness is similar, for example, to that of electron slicing sources [40–42].

As an outlook, there are several avenues along which the THz-driven undulator performance could be optimized, i.e., a higher $K$ parameter, a smaller device structure or a gamma photon source. First, the $K$ parameter of 0.014 could be pushed closer towards one and therefore closer to parameters found for standard magnetic undulators. Since the $K$ parameter scales linearly both with electric field strength and undulator periodicity two options arise, either to increase the electric field strength or the periodicity by a factor of about 70. Increasing the periodicity is certainly possible but would be detrimental in view of device miniaturization. A 70-fold higher THz field strength (around 70 GV/m) may exceed the material damage threshold. Nevertheless, 10 GV/m should be feasible and together with a larger periodicity of 1 mm should result in a $K$ parameter close to one.

Second, to miniaturize the device even further would go hand in hand with a smaller periodicity which is feasible for highly focused and low emittance electron beams. A smaller period would further reduce the kinetic energy requirement for producing a fixed x-ray photon energy. The extreme case of a ten nanometer period was demonstrated by Wong and co-workers [23]. Third, optimizing the device for a 5 GeV electron beam would result in the production of 2.5 MeV photons and therefore the SPP undulator might be an interesting source for gamma spectroscopy.

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