Estimate of the Accretion Disk Size in the Gravitationally Lensed Quasar HE 0435–1223 Using Microlensing Magnification Statistics

C. Fian1,2, E. Mediavilla1,2, J. Jiménez-Vicente3,4,5, J. A. Muñoz5,6, and A. Hanslmeier7

1 Instituto de Astrofísica de Canarias, Vía Láctea S/N, La Laguna E-38200, Tenerife, Spain
2 Departamento de Astrofísica, Universidad de La Laguna, La Laguna E-38200, Tenerife, Spain
3 Departamento de Física Teórica y del Cosmos, Universidad de Granada, Campus de Fuentenueva, E-18071 Granada, Spain
4 Instituto Carlos I de Física Teórica y Computacional, Universidad de Granada, E-18071 Granada, Spain
5 Departamento de Astronomía y Astrofísica, Universidad de Valencia, E-46100 Burjassot, Valencia, Spain
6 Observatorio Astronómico, Universidad de Valencia, E-46980 Paterna, Valencia, Spain
7 Institute of Physics (IGAM), University of Graz, Universitätsplatz 5, A-8010, Graz, Austria

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Abstract

We present a measurement of the accretion disk size of the quadruple lensed quasar HE 0435–1223 from well-sampled 13-year COSMOGRAIL optical light curves. Using accurate time delays for the images A, B, C, and D, we modeled and removed the intrinsic quasar variability, and found microlensing events of amplitude up to 0.6, 0.4, and 0.5 mag in the images A, C, and D, respectively. From the statistics of microlensing magnifications in these images we use Bayesian methods to estimate the size of the quasar accretion disk. We have inferred the half-light radius for the accretion disk using two different methods, \( R_{1/2} = 7.6^{+1.1}_{-0.8} \sqrt{M/0.3 \ M_\odot} \) lt-days (histogram product) and \( R_{1/2} = 7.7^{+0.7}_{-0.4} \sqrt{M/0.3 \ M_\odot} \) lt-days (\( \chi^2 \) criterion). The results are self-consistent and in good agreement with the continuum size predicted by single-epoch spectroscopy and previous studies making use of narrowband photometry of HE 0435–1223.

Key words: accretion, accretion disks – gravitational lensing: micro – quasars: individual (HE 0435–1223)

1. Introduction

The light curves of lensed quasar images provide a time history of the changes in brightness, and their analysis has important applications in cosmology (such as the determination of time delays to infer the Hubble constant,Refsdal 1964; the estimate of peculiar velocities, Mediavilla et al. 2016; and in the study of quasar structure, Chang &Refsdal 1979, 1984; see also Kochanek 2004; Wambsganss 2006). In this paper we focus on the last application, using gravitational microlensing statistics to determine the quasar accretion disk size (Pooley et al. 2007; Mosquera et al. 2009, 2011, 2013; Morgan et al. 2010; Blackburne et al. 2011, 2014, 2015; Sluse et al. 2011; Jiménez-Vicente et al. 2012, 2014, 2015a, 2015b; Motta et al. 2012; Hainline et al. 2013; Mediavilla et al. 2015; Fian et al. 2016; Muñoz et al. 2016). The term microlensing describes the flux variations produced by stars in the foreground lens galaxy that result in magnification (or demagnification) of the multiple quasar images (Chang &Refsdal 1979; Kochanek 2004; Blackburne et al. 2014; see also the review by Wambsganss 2006). These flux variations between images are not correlated as would be expected from intrinsic quasar variability. The magnification produced by microlensing depends strongly upon the angular size of the source, with smaller emission regions showing larger flux anomalies and larger sources smoothing out the light curves (Blackburne et al. 2011, 2014; Mosquera & Kochanek 2011).

We study the light curves of the quadruple lensed quasar HE 0435-1233 discovered during the Hamburg/ESO Survey (HES) in the southern hemisphere (Wisotzki et al. 2002). The lens lies in a group of galaxies of at least 12 members (Sluse et al. 2017). At a redshift of \( z_L = 1.689 \) (Wisotzki et al. 2002), HE 0435–1223 is lensed by a foreground galaxy at a redshift of \( z_f = 0.455 \) (Eigenbrod et al. 2006) into four bright point sources (plus a fuzzy object in the center) in a nearly symmetric cross-shaped configuration (Wisotzki et al. 2002; Blackburne et al. 2014). The maximum separation between the images is 2.6 (Wisotzki et al. 2002) and their time delays are relatively small owing to the symmetric distribution of the images around the lensing galaxy (Mosquera et al. 2011).

After correcting for the time delays (components A and C lead, followed by the saddlepoints B and D; see Wisotzki et al. 2002) and mean magnitude differences between the images, we find clear indications of microlensing flux variability in the residuals of the light curves (i.e., the differences between the observed light curves and the modeled intrinsic variability of the quasar).

Our aim is to estimate from the statistics of microlensing the size of the accretion disk of the lensed quasar. We use flux ratios of a large enough source in the quasar so as to be insensitive to microlensing in order to establish the baseline for no microlensing magnification (e.g., Mediavilla et al. 2009), from which the amplitude of the microlensing magnification can be measured. We then compare the histogram of microlensing magnifications obtained from the observations (corresponding to the monitoring time interval) with the simulated predictions of the microlensing variability for sources of different sizes (Fian et al. 2016). This comparison allows us to evaluate the likelihood of the different values adopted for the size. In this way we extend the single-epoch method to all of the epochs in the available 13-year light curves, thereby increasing the statistical significance. We use the optical light curves obtained from the COSMOGRAIL project (Bonvin et al. 2017) to infer microlensing flux variability and the radio data from Jackson et al. (2015) to estimate the baseline for no microlensing variability. In the present study we improve the methods for obtaining the
accretion disk size discussed in Fian et al. (2016) using more realistic estimates of the scatter of the modeled histograms.

The paper is organized as follows. In Section 2 we present the COSMOGRAIL light curves of each image of HE 0435–1223. In Section 3 we model the intrinsic variability and examine the flux ratios between the images. We outline our approaches to compute the accretion disk size in Section 4. Section 5 is devoted to analyzing our results and comparing them with past estimates. In Section 6 we will discuss the impact of errors and uncertainties on the size estimates. Finally, in Section 7 we briefly conclude our results and discuss future perspectives.

2. Data

The fluxes of the four images of HE 0435–1223 were monitored from 2003 August until 2016 February in the optical R band as a part of the COSMOGRAIL program. The data set consists of 884 epochs (i.e., 884 nights) and the average sampling rate is once every fifth day. Figure 1 shows the 13-year light curves of images A–D of HE 0435–1223 as published in Figure 2 of Bonvin et al. (2017). The relative shifts in magnitude between the images are chosen to ease visualization. The similarity among the four well-sampled light curves is immediately noticeable, although it can be seen that they would not overlap perfectly when shifted in time and magnitude. This mismatch between the light curves is interpreted as microlensing caused by stars in the lensing galaxy. Quasars are time variable, making it necessary to separate microlensing from intrinsic variability by modeling and removing the latter one.

3. Intrinsic Variability and Microlensing

The images of multiple lensed quasars arrive with relative delays of hours up to years because of the different paths taken by their light. Intrinsic variability of the source coupled with the light path time delay between the quasar images can mimic flux ratio anomalies. To correct for this, we use the time delay estimates of Bonvin et al. (2017) and shift the light curves by $\Delta t_{AB} = -8.8$ days, $\Delta t_{AC} = -1.1$ days, and $\Delta t_{AD} = -13.8$ days. Owing to the symmetry of the image configuration in this system, the time delays are very short, meaning that intrinsic variations (assumed to be much slower) will show up quasi-simultaneously in all four images.

After shifting the light curves in time and correcting for the magnitude difference between the images, we perform a single spline fit to the B light curve in order to model the intrinsic variability of the quasar. We assume that the flux variations in image B are mainly intrinsic, as several authors claim that the B image is the least affected by stellar microlensing (Courbin et al. 2011; Motta et al. 2012), whereas A and D are affected by strong microlensing variations (Wisotzki et al. 2003; Kochanek et al. 2006; Courbin et al. 2011; Mosquera et al. 2011; Ricci et al. 2011; Motta et al. 2012). Figure 2 shows the simulated quasar variability (black solid line) with the A–D light curves superimposed. We obtain a source variability of $\sim 1$ mag. Although we use the spline fit to make a reasonable estimate for the intrinsic variability of the source, some contribution from microlensing variability in image B is likely present. We subtract the spline from the light curves and obtain the microlensing difference light curve as

9 In any case, this is irrelevant because in our treatment we also consider the contributions of the B image to microlensing in the simulated difference light curves.
follows: $\Delta m_X = m_X - m_{\text{radio}} - (m_Y - m_{\text{radio}})$, where $X = A, C, D$, assuming that the ratios between the radio data from Jackson et al. (2015) (36.0 ± 2.1 μJy for A, 26.4 ± 2.1 μJy for B, 34.3 ± 2.1 μJy for C, and 16.1 ± 2.1 μJy for D) represent the true magnification ratios of the images in the absence of microlensing. The radio-emitting regions of quasars should provide a good estimate of the real magnification ratios of the images as they are supposed to arise from a large enough region so as not to be affected by microlensing (see Mediavilla et al. 2009). In the three panels in Figure 3, the microlensing

**Figure 2.** Image A, B, C, and D light curves of HE 0435–1223 in their overlapping region after shifting by the respective time delays (and magnitude differences). The model of the intrinsic variability of the quasar (spline fitted to light curve B) is shown in black.

**Figure 3.** Differential microlensing variability of the light curves A, C, and D compared to a spline fit to light curve B. The dashed horizontal lines show the mean value of the residuals. The residual magnitudes clearly show that microlensing is present in light curve A.
residuals of the A, C, and D light curves after subtraction of the spline fit are shown. Microlensing variability can be seen, particularly in the A–B residual light curve, where image A seems to have been undergoing a microlensing event between the fourth and fifth seasons, whereas C and D remained mainly constant.

4. Bayesian Source Size Estimation

The effect of finite source size is to smooth out the flux variations in the light curves of lensed quasars caused by stars in the galaxy. Hence, microlensing is sensitive to the size of the source (Morgan et al. 2010; see also the review by Wambsganss 2006), and we use quantitative Bayesian methods together with our determinations of microlensing magnification amplitude to estimate the accretion disk size in the HE 0435–1223 lensed quasar.

4.1. Simulated Microlensing Histograms

We simulate the microlensing of a finite-size source using microlensing magnification maps created with the inverse polygon mapping method described by Mediavilla et al. (2006, 2011). Each map (appearing as a network of high-magnification caustics separated by regions of lower magnifications) corresponds to a specific quasar image and shows the microlensing magnification at a given source position. The general characteristics of the magnification maps are determined for each quasar by the local convergence, $\kappa$, and the local shear, $\gamma$, which were obtained by fitting a singular isothermal sphere with an external shear (SIS+$\gamma_e$), such as what might be generated by the tide from a neighboring galaxy or cluster, to the coordinates of the images. The local convergence is proportional to the surface mass density and can be divided into $\kappa = \kappa_c + \kappa_s$, where $\kappa_c$ is the convergence due to continuously distributed matter (e.g., dark matter) and $\kappa_s$ is due to the stellar-mass point lenses (e.g., microlens stars in the galaxy). The values of $\kappa$ and $\gamma$ (taken from Mediavilla et al. 2009) are listed in Table 1. We use a surface mass fraction in stars $\kappa_s$ of 10% (Mediavilla et al. 2009) and generated 2000 × 2000 pixel magnification maps with a size of 19.3 × 19.3 Einstein radii. We get a resolution of 0.2 lt-days per pixel, which is much smaller than the size of the optical accretion disk of the quasar. The value of the Einstein radius for this system is 2.84 × 10$^{16}$ M/0.3 M$_{\odot}$ cm = 11 M/0.3 M$_{\odot}$. It-days at the lens plane (Mosquera & Kochanek 2011). We randomly distribute stars of a mass of $M = 0.3 M_{\odot}$ across the microlensing patterns to create the microlens convergence $\kappa_s$. The source sizes can be scaled to a different stellar mass, $M$, using $r_c \propto \sqrt{M}$. The ratio of the magnification in a pixel to the average magnification of the map gives the microlensing magnification at the pixel and histograms of normalized to the mean maps deliver the relative frequency of microlensing magnification amplitude for a pixel-size source.

| Image | $\kappa$ | $\gamma$ |
|-------|---------|---------|
| A     | 0.46 ± 0.03 | 0.39 ± 0.04 |
| B     | 0.52 ± 0.12 | 0.59 ± 0.06 |
| C     | 0.46 ± 0.05 | 0.39 ± 0.08 |
| D     | 0.56 ± 0.14 | 0.64 ± 0.08 |

To model the structure of the unresolved quasar source, we considered a circular Gaussian intensity profile of $r_s$, $I(R) \propto \exp(-R^2/2r_s^2)$ in size. It is generally accepted that the specific shape of the source’s radial emission profile is unimportant for microlensing flux variability studies because the results are essentially controlled by the half-light radius rather than by the detailed source profile (Mortonson et al. 2005). The characteristic size $r_s$ is related to the half-light radius; that is, the radius at which half of the light at a given wavelength is emitted, by $R_{1/2} = 1.18 r_s$. Finally, we convolve the magnification maps with Gaussians of 22 different sizes over a linear grid that spans from $r_s = 0.5$ to 22.5 lt-days. The movement of a large source across the magnification map is equivalent to a point source moving across a version of the map that has been smoothed by convolution with the intensity profile of the source. Strong anomalies are evidence for a relatively small source, whereas low microlensing magnifications could be due to a large source size or to a location of the source in a relatively calm region of the magnification map. After convolution we normalized each magnification map by its mean value. The histograms of the normalized map represent the histograms of the expected microlensing variability. Thus, we obtain 22 different microlensing histograms corresponding to different source sizes for each of the images A, C, and D. Finally, cross-correlating the histograms of A, C, and D with the histogram of B, we built the microlensing difference histograms A-B, C-B, and D-B for different values of $r_s$ to be compared with the experimental histograms obtained from the observed light curves (see Figure 4).

4.2. Observed Microlensing Histograms

From the residuals that represent the differential (with respect to B, the image least prone to microlensing) microlensing of the A, C, and D images (see Figure 3), we have obtained the microlensing histograms, i.e., the frequencies in which each microlensing amplitude appears in the microlensing light curves. We adopted a bin size of 0.05 mag. In Figure 4 we compare the A-B, C-B, and D-B modeled magnification histograms corresponding to convolutions with sources of different values of $r_s$ (dashed lines) with the experimental microlensing histograms. Large values of $r_s$ smear out the network of microlensing magnification caustics and reduce their dynamic range, thereby causing the histograms to become narrower.

4.3. Methods

To study the likelihood of the different $r_s$ values we compare the microlensing histograms inferred from the model for different values of $r_s$ with the histograms of the data using two different statistics:

(a) a histogram product,\footnote{We propose heuristically this statistic based on the distance between histograms, related to the Pearson’s correlation coefficient.} defined as

$$P_X(r_s) = \sum_{i=1}^{N_{\text{bin}}} h_{X-B}^i h_{X-B}^i(r_s),$$

where $h_{X-B}^i$ and $h_{X-B}^i(r_s)$ are the observed and modeled histograms, and $N_{\text{bin}}$ is the number of bins. This is a natural extension of the single-epoch method. After multiplying the probability distributions corresponding to
A, C, and D we obtain the probability density function (PDF) of the size, 

\[ P(r_s) = P_A(r_s) \cdot P_C(r_s) \cdot P_D(r_s); \]  

(b) and a Pearson’s \( \chi^2 \)-test, which is a test suited to measure the distance between two histograms. After normalizing all of the simulated histograms to the number of counts in the real data, we measure the goodness of fit between the histograms inferred from the model (for different source sizes \( r_s \)) and the histogram of the observed microlensing differences with a \( \chi^2 \)-statistic of 

\[ \chi^2 = \frac{\sum_{i=1}^{N_{\text{bin}}} (h_{i-B}^\text{obs} - h_{i-B}^\text{mod})^2}{\sigma_{X-B}^2} \]  

with 

\[ \sigma_{X-B} = \sqrt{(\sigma_{X-B}^\text{model})^2 + (\sigma_{X-B}^\text{obs})^2}, \]  

where \( \sigma_{X-B}^\text{model} \) and \( \sigma_{X-B}^\text{obs} \) are the uncertainties of the model and the observations, and 

\[ P_X(r_s) \propto e^{-\frac{\chi^2}{2}}. \]  

Multiplying them together, we obtain the PDF of the source size of 

\[ P(r_s) = \prod_X P_X(r_s). \]

In our previous paper (Fian et al. 2016) we noticed that the application of the \( \chi^2 \) method led to underestimated of the uncertainties in size. We obtained relatively high values for \( \chi^2 \), which indicates that we were probably underestimating the intrinsic scatter of the model histograms obtained from the magnification maps. It should be taken into account that the observed light curves correspond to a tiny track on the magnification map and that the scatter between the histograms corresponding to different tracks can be large. In other words, we need to know the scatter of a simple track realization with respect to the mean. In this work we propose to control this problem by estimating the uncertainties as follows. Once we have the convolved magnification maps for each image we run 1000 tracks across them, at random starting points and in random directions (of time length corresponding to the observed light curve), in order to estimate the scatter in the magnification histogram. Scaling the COSMOGRAIL monitoring period of HE 0435–1223 (13 yr) with the Einstein crossing time computed by Mosquera & Kochanek (2011), we estimated the distance in pixels traveled by the source along the caustic pattern. For each image and each convolved map, we build histograms of the set of tracks and compute their average. From these average histograms we estimate the dispersion in each bin to get the uncertainty contributions for each image which we will use in quadrature for our \( \chi^2 \) calculations. Coming from different regions of the magnifications map, the scatter among the histograms of the random trajectories is high. We calculated the \( \chi^2 \) for 22 source sizes, \( r_s \), spaced linearly between 0.5 and 21.5 lt-days. The minimum \( \chi^2 \) using this estimate of the histogram uncertainties is \( \sim 2 \) for A and C, and \( \sim 6 \) for D, respectively.

In Figure 5 we show the random source trajectories from which we build these histograms superposed on the magnification map for each image (left panels). In order to make the caustics and cusps more easily visible, in this figure we did not convolve the magnification maps with the source size. In the right-hand panels of Figure 5 the histograms of the whole map are shown in gray and the average histograms of the tracks are shown in color, with different shadings standing for different convolutions of \( r_s \).

5. Results and Discussion

The resulting normalized probability distributions obtained using the methods discussed in Section 4.3 can be seen in Figure 6. Using a logarithmic prior, we found a size of the region emitting the R-band emission of \( r_s = 7.1^{+9.4}_{-1.6} \text{M}/0.3 \text{M}_\odot \) for 68% confidence estimates for the histogram product (solid line). Using Pearson’s \( \chi^2 \) statistic, we predict a source size of \( r_s = 7.5^{+5.0}_{-1.0} \text{M}/0.3 \text{M}_\odot \) lt-days (dashed line). We obtained values of \( \chi^2 \sim 2 \) for A and C, and \( \sim 6 \) for D.

Multiplying by a factor of 1.18, we convert \( r_s \) to half-light radii. Our result for each method expressed in terms of the half-light radius, \( R_{1/2} = 8.4^{+11.1}_{-1.9} \sqrt{\text{M}/0.3 \text{M}_\odot} \) lt-days (histogram product) and \( R_{1/2} = 8.9^{+9.9}_{-1.3} \sqrt{\text{M}/0.3 \text{M}_\odot} \) lt-days (Pearson’s \( \chi^2 \)), is in good agreement with the estimates by Motta et al. 2017 \( (R_{1/2} = 19^{+9.9}_{-2.6} \sqrt{\text{M}/0.3 \text{M}_\odot} \) lt-days), Mosquera et al. 2011 \( (R_{1/2} = 5^{+4}_{-1.4} \sqrt{\text{M}/0.3 \text{M}_\odot} \) lt-days), and Blackburne et al. 2011 \( (R_{1/2} = 6.7^{+10.0}_{-2.5} \sqrt{\text{M}/0.3 \text{M}_\odot} \) lt-days) for this system. All of the
estimates have been scaled to a $\lambda_0 = 2417$ Å using $R_{1/2} = (\lambda_0/\lambda)^{\prime} R_{1/2}(\lambda)$. Our estimates for the size are also in good agreement with the average determinations obtained for a sample of lensed quasars by Jiménez-Vicente et al. (2012, 2014, 2015a, 2015b) when a fraction of mass in stars of 10% is considered.

6. Impact of Uncertainties on Size Estimates

In Table 2 and Figure 7 we summarize the impact of the different sources of uncertainty on the size estimates. The relatively small uncertainties in the time delays (less than one day; see Table 3) do not induce significant changes in the disk size. The effect of microlensing on the time delays (see Tie & Kochanek 2018) is smaller than the uncertainties of the modeled time delays and has no influence on the size either. We studied the change of the size when we use the narrow line flux ratio measurements by Nierenberg et al. (2017) as a baseline for no microlensing instead of the radio measurements by Jackson et al. (2015; see Table 4). We obtain $\sim 30\%$ smaller sizes for the histogram product ($\sim 50\%$ for the Pearson’s $\chi^2$) using the [O III] emission line (note that the narrow line cores could be affected by extinction). We checked the robustness of our results with respect to the macromodel by comparing it with the parameters (convergence and shear; see Table 5) inferred from the inverse magnification tensor in Wong et al. (2017), where the authors explicitly model the effect of nearby perturbers. After recomputing the magnification maps we repeat all of the calculations, obtaining similar values for the half-light radius of the accretion disk ($R_{1/2} = 7.3_{-1.2}^{+1.2} \sqrt{M/0.3 M_\odot}$ lt-days for the histogram product and $R_{1/2} = 10.3_{-1.2}^{+1.2} \sqrt{M/0.3 M_\odot}$ lt-days for Pearson’s $\chi^2$). We marginalized over all of the distributions listed in Table 2 and obtained a half-light radius of $R_{1/2} = 7.6^{+1.2}_{-1.1} \sqrt{M/0.3 M_\odot}$ lt-days for the histogram product. Consistent results are obtained with Pearson’s $\chi^2$ ($R_{1/2} = 7.7^{+1.2}_{-1.1} \sqrt{M/0.3 M_\odot}$ lt-days). Note that the large asymmetry in the uncertainties on $r_s$ arise from the progressive lack of sensitivity of microlensing to changes in the size when the size increases.

7. Summary and Conclusions

The quadruple-imaged strong gravitational lens HE 0435–1223 has four nearly identical components arranged symmetrically around a luminous galaxy and is an attractive target for microlensing studies because of the relative ease of separating intrinsic from microlensing-induced variations due to the short time delay between its images. Unlike most other known quadruple lens systems, photometric monitoring of this object is also relatively easy, because of its relatively wide image separations (Wisotzki et al. 2002). We used the COSMOGRAIL

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Table 2

| Source                     | Half-light Radius $R_{1/2}$ in $\sqrt{M/0.3 M_\odot}$ lt-days |
|----------------------------|---------------------------------------------------------------|
| Time Delay $-2\sigma$      | $8.4_{-1.9}^{+1.1}$                                          |
| Time Delay $+2\sigma$      | $8.4_{-1.9}^{+1.1}$                                          |
| [O III] Emission Line      | $5.9_{-1.8}^{+10.0}$                                         |
| Model from Wong et al. (2017) | $7.3_{-1.2}^{+12.3}$                                      |
| Marginal Distribution     | $7.6_{-1.1}^{+12.3}$                                         |

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Figure 5. Random tracks superposed on the magnification maps of images A (top left), C (middle left), and D (bottom left). The gray scale shows the unconvolved map with lighter colors indicating higher magnifications. The lines show the source trajectories across the pattern for the COSMOGRAIL monitoring period. Histograms derived from the magnification maps (gray) and the tracks for images A (blue), C (red), and D (green) are shown in the right-hand panels. Different color shadings stand for different convolutions with $r_s$. Positive numbers of magnification denote demagnification.

Figure 6. Probability distributions of the source size $r_s$ for the histogram product (solid line) and Pearson’s $\chi^2$ (dashed line).
light curves of the four images of HE 0435–1223 (Bonvin et al. 2017, see Figure 1) to obtain the accretion disk size. They cover a relatively long period (13 years), which significantly extends the time coverage of previous studies and provides relatively dense coverage (1 observing epoch every 5 days, 884 usable points). Taking as reference image B, which is less affected by microlensing, and using the experimental time delays inferred by Bonvin et al. (2017), we have removed the intrinsic variability from the light curves in the overlapping region. Using the radio flux ratios between images determined by Jackson et al. (2015) as a baseline for no microlensing magnification, we have finally obtained the microlensing light curves, A-B, C-B, and D-B. We have clearly detected microlensing in the images A and D of HE 0435–1223 with up to 0.6 mag (0.5 mag) in A (D). The light curve of C seems to be less affected by microlensing, although some changes can be seen in the first four and last two seasons of the data.

We have used the statistics of microlensing magnifications during the available seasons in the optical R band of HE 0435–1223 to infer probabilistic distributions for the source size using two different methods. Using the histogram product of the observed and modeled microlensing histograms we have obtained a half-light radius of $R_{1/2} = 8.4^{+1.1}_{-1.0} \sqrt{M/0.3 M_\odot}$ light-days. Consistent results are obtained with Pearson’s $\chi^2$ ($R_{1/2} = 8.9^{+5.9}_{-2.7} \sqrt{M/0.3 M_\odot}$ light-days). In this work we improved the uncertainty estimations for the Pearson $\chi^2$ method to obtain self-consistent results. Our results are also in good agreement with previous estimates of other authors for this system ($R_{1/2} = 19^{+9}_{-6} \sqrt{M/0.3 M_\odot}$ light-days by Motta et al. 2017, $R_{1/2} = 5^{+4}_{-3} \sqrt{M/0.3 M_\odot}$ light-days by Mosquera et al. 2011, and $R_{1/2} = 6.7^{+3.0}_{-2.3} \sqrt{M/0.3 M_\odot}$ light-days by Blackburne et al. 2011).

Future monitoring with the Large Synoptic Survey Telescope will contain a large number (~8000; see Oguri & Marshall 2010) of light curves of gravitationally lensed quasars that will demand new techniques to compute the sizes of accretion disks in gravitationally lensed quasars. Here, we have explored new techniques based on the use of the histograms of microlensing magnifications that can be used in combination or as an alternative to the light curve fitting method (e.g., Kochanek 2004).

**Table 3**

| Image Pair | $\Delta t$ | $1\sigma$ | $2\sigma$ |
|------------|------------|-----------|-----------|
| AB         | −8.8       | ±0.8      | ±1.6      |
| AC         | −1.1       | ±0.7      | ±1.4      |
| AD         | −13.8      | ±0.9      | ±1.8      |

**Table 4**

| Flux Ratios Radio Emission | [O III] Emission Line |
|---------------------------|-----------------------|
| A:B:C:D                   | 1.05:0.77:1.00:0.47    |
|                           | 0.97:0.98:1.00:0.66    |

**Notes.**

a Jackson et al. (2015).

b Nierenberg et al. (2017).

**Table 5**

| Image | $\kappa$ | $\gamma$ | $\Delta\kappa^a$ | $\Delta\gamma^a$ |
|-------|----------|----------|------------------|------------------|
| A     | 0.49     | 0.35     | 0.03             | 0.04             |
| B     | 0.64     | 0.53     | 0.12             | 0.06             |
| C     | 0.51     | 0.31     | 0.05             | 0.08             |
| D     | 0.70     | 0.56     | 0.14             | 0.08             |

**Note.**

a Difference to our model.
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**ORCID iDs**

J. Jiménez-Vicente © https://orcid.org/0000-0001-7798-3453

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