H-infinity optimised control of external inertial actuators for higher precision robotic machining

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ABSTRACT
Serial-link industrial robots that feature small footprints and large working volumes have been used widely in various applications. However, their low-stiffness-induced vibration hinders application in precision robotic machining processes. In robotic milling, the spindle may experience chatter, leading to unsatisfactory surface finishes. Moreover, the chatter frequency varies, depending on the robot pose and its underlying stiffness and inertia characteristics, which is likely to be outside of the controlled robot system bandwidth. One solution is to use inertial actuators close to the source of excitation on the robot to generate forces that counteract the vibration. This paper advances matters by employing optimised $H_{\infty}$ control strategies for improved effectiveness and robustness to perform active vibration suppression. The strategies are independent of tool paths and take account of different robot poses, hence the variability of the robot dynamic characteristics. Performance is assessed against that of standard velocity feedback controllers in eccentric mass experiments and milling tests. The experimental results show that within a relatively large work-plane ($500 \text{ mm} \times 500 \text{ mm}$), $H_{\infty}$ controllers greatly improve machining capability and reduce machining errors by up to 85%.

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1. Introduction

Robotic machining systems offer advantages such as low costs, versatility, small footprints and large volume manufacturing capability compared with conventional machine tool centres. However, serial-link robots have relatively low mechanical stiffness and are prone to experience machining induced vibrations that hinder their practical applications. Moreover, chatter may occur when serial robots are used to machine light alloys such as aluminium and carbon fibre composites. In high speed machining, high frequency modes that are related to the flexibility of the spindle, toolholder and tool can create regenerative chatter. However, lower machining speeds may work better to achieve higher precision because the robot motion is more accurate at lower path speeds. In these cases, mode coupling chatter due to excitation of structural modes of the robot at relatively low frequencies have been also reported (Yuan et al. 2019). Vibration suppression is therefore a critical challenge to address for precision machining (i.e. finer surface finish, higher accuracy and longer tool life).

A common approach to vibration suppression is to use passive damping systems (Hahn 1951), which transfer the energy of vibration to the absorber at its natural frequency. While classic damping systems are effective only around a narrow frequency band, modern semi-active (Sun et al. 2014, 2018) and active (Cowley and Boyle 1969; Huyanan and Sims 2007, 2008) damping systems, according to the terminology used (Brecher, Bäumler, and Brockmann 2013), have the capability of tracking and adapting to vibration frequency bands. Damping systems have proved to be effective for vibration absorption over a wide range of applications in conventional machining systems (Moradi, Bakhtiari-Nejad, and Movahhedy 2008; Díaz-Tena et al. 2013; Saadabad, Moradi, and Vossoughi 2014; Scheidler and Dapino 2014) as well as in aerospace (Hiemenz, Hu, and Wereley 2008), automotive (Sapiński and Rosół 2008) and marine (Huang, Su, and Hua 2018) manufacture. However, damping systems are rarely used for vibration absorption in robot machining systems, though a magnetorheological elastomer-based semi-active damping system to suppress chatter frequencies ranging from
7 to 20 Hz in robotic milling has been developed (Yuan et al. 2019). Similarly, eddy current dampers (ECDs), which are fixed tuned mass (TMD) damping systems with viscous damping created by eddy current, have been used to improve damping properties and attenuate vibration in robotic polishing (Chen et al. 2019) and robotic milling (Chen and Zhao 2018). These damping systems are effective when the dynamics of the system remain unchanged. These devices must be designed and fabricated to target characteristic frequencies and dynamic modes of a system. However, it is difficult to operate these devices when a robot operates in different poses with variable modal characteristics.

Other approaches have been developed specifically for vibration suppression in robotic machining including changing the system structure (Guo et al. 2016), optimising the robot configuration and trajectory planning (Mousavi et al. 2017; Tunc and Stoddart 2017), optimising spindle speed (Zaghibani, Songmene, and Bonev 2013), and optimising tool path and feed rate (Dumas et al. 2011; Tyapin, Kaldestad, and Hovland 2015). These approaches are promising, but require significant prior experiments and calculations, making them difficult to implement. Furthermore, they must adapt to changes in machining configurations, therefore may not be applicable under all working conditions. In contrast, active vibration control using inertial actuators has no such restriction and offers the potential to suppress vibrations at multiple frequencies simultaneously (Bilbao-Guillerna et al. 2012; Munoa et al. 2013). In a robotic machining system with relatively low stiffness, this control method offers a solution to cope with the changing dynamics of the system.

Researchers have investigated active vibration control in robotic milling in a layout with the spindle being stationary and the robot moving a workpiece (Olofsson et al. 2011). The spindle was placed on a High-Dynamic Compensation Mechanism (HDCM) unit and a state-feedback controller was used to control vibrations. The proposed control approach successfully drives machining errors to below 50 µm, but is inevitably limited to the specific configurations of the robot by the system layout and does not take account changes in robot dynamics in different poses. LQR controller design has been applied directly to the robot system to suppress vibrations in robotic milling for frequencies up to 10 Hz (Nguyen, Johnson, and Melkote 2020). The results are promising even though the operating frequencies are limited by the robot controller at a command rate of 250 Hz. Furthermore, the proposed LQR controller is based on both the dedicated tool path and corresponding sequential changes in dynamics of the robot, requiring different controllers for a process within the same operating workspace. Piezo-based actuation systems (Aggogeri et al. 2016) overcome bandwidth limitations, but have limited stroke capabilities. A single inertial actuator was operated using standard PID feedback (Wang and Keogh 2017) as a more practical alternative for vibration suppression in robot machining. While the feasibility of the concept was evident, the vibration suppression capability is sub-optimal.

There are options for controller implementation based on optimisation principles. LQR controllers utilise full state feedback in the form of a gain matrix to minimise a quadratic cost function. If full state feedback is not available, a Kalman filter enables feedback and the control is optimal for white Gaussian noise disturbances, hence having flat spectral content. In contrast, $H_\infty$ synthesis enables controllers to be designed to minimise the maximum of spectral norms of weighted system transfer function matrices. Using appropriate weighting transfer function matrices, it is possible to target specific frequency bands to be minimised in amplitude, and to guarantee closed loop stability for specified uncertainties. Given the dynamic flexibility of serial robots, which may result in the excitation of robot modes by machining forces at specific synchronous and non-synchronous frequencies (not white Gaussian noise), it was considered beneficial to investigate the implementation of $H_\infty$ controllers for robotic machining.

This paper expands the authors’ preliminary work (Zhang, Wang, and Keogh 2020) by evaluating the effectiveness of attenuating milling induced vibrations through an inertial actuator system operating under various control strategies. These include optimised $H_\infty$ strategies and velocity feedback strategies with different configurations of robot poses that cause changes in the underlying plant dynamics of the robot system. The paper demonstrates that the $H_\infty$ controller design synthesis can be used to cope with moderate variations in the dynamics of the robot. $H_\infty$ controllers designed to cover smaller and larger machining domains, through specified additive
2. System model

A general equation of motion for robot-machine tool structure can be expressed as

\[ F(x, \dot{x}, x; p) = B_J F_J(p) + B_D F_D + B_C F_C + F_U(x, \dot{x}, x; p) \]  

(1)

where \( F \) is a nonlinear vector function of generalised coordinates contained in the vector \( x \) and \( p \) is a vector of coordinates that define the instantaneous robot pose. \( F_J \) is a vector representing motor torques applied at revolute joints, \( F_D \) is a vector of generalised forces arising from a machining process, and \( F_C \) is a vector of control forces that are assumed to be available from an active actuation system that is additional to the internal robot control system. They are distributed to full size force vectors by the matrices \( B_J, B_D \) and \( B_C \), respectively. \( F_U \) is a vector representing model uncertainty due to friction and backlash in revolute joints.

Suppose that the robot may operate in fixed poses represented by \( P = p_n(n=1, \ldots, N) \), and it may undergo small-scale motions about those poses. In these cases, \( \dot{x}, \ddot{x} \) and \( x \) may be regarded as small perturbations, hence linearization of Equation (1) leads to the set of equations

\[ M(p_n) \ddot{x} + G(p_n) \dot{x} + K(p_n) x = B_J F_J(p_n) + B_D F_D + B_C F_C + F_U(x, \dot{x}, x; p_n) \]  

(2)

for \( n = 1, \ldots, N \). The dimensions of the mass, damping and stiffness matrices, \( M, G \) and \( K \), respectively, depend on the level of detail in the robot model. For a typical serial-link robot, the minimum dimensions are \( 6 \times 6 \). However, this assumes that all links are rigid, which may become invalid when high frequency forces from machining processes excite link flexure modes. In such cases, models of the type in Equation (2) may be derived from, for example, the finite element method, or from system identification procedures.

The force vectors, \( F_J, F_D \) and \( F_C \), are definable from demand motor torques under internal robot control, knowledge of the machining process, and from any external active control, respectively. The residual uncertainty of \( F_U(x, \dot{x}, x; p_n) \) is difficult to define due to the complexity of specifying the tribological conditions associated with the bearing in the revolute joints.

Equation (2) may be converted to state space form by setting the state vector \( z = [x^T \dot{x}^T]^T \) such that

\[ \dot{z} = A(p_n)z + B(p_n) \]  

(3)

where

\[ A(p_n) = \begin{bmatrix} 0 & I \\ -M(p_n)^{-1}K(p_n) & -M(p_n)^{-1}G(p_n) \end{bmatrix}, \]  

\[ B = \begin{bmatrix} 0 \\ M(p_n)^{-1} \end{bmatrix}. \]  

(4)

A user-dependent output equation may be defined from the system states to represent measurement parameters that may characterise machining process precision:

\[ y = C(p_n)z \]  

(5)

If an active control strategy is to be designed, it is appropriate to express the characteristics in the Laplace/frequency domain. If \( \tilde{y} \) is used to denote a Laplace transformed variable, it follows that

\[ \tilde{y}(s; p_n) = C(p_n)(sI - A(p_n))^{-1}(\tilde{B}_J \tilde{F}_J(s) + \tilde{B}_D \tilde{F}_D(s) + \tilde{B}_C \tilde{F}_C(s) + \tilde{B} U(s; p_n)) \]  

(6)

By undertaking an eigenvalue/vector decomposition of \( A(p_n) \):

\[ A(p_n) V(p_n) = \lambda V(p_n) \]  

(7)

where \( V(p_n) \) is the matrix of eigenvector columns and \( \Lambda(p_n) = \text{diag}(\lambda_1(p_n), \ldots, \lambda_M(p_n)) \) contains the eigenvalues of \( A(p_n) \), the solution for \( \tilde{y} \) takes the form

\[ \tilde{y}(s; p_n) = G_J(s; p_n) \tilde{F}_J(s) + G_D(s; p_n) \tilde{F}_D(s) + G_C(s; p_n) \tilde{F}_C(s) + G_U(s; p_n) \tilde{F}_U(s; p_n) \]  

(8)

where

\[ G_J(s; p_n) = C(p_n) V(p_n)(sI - \Lambda(p_n))^{-1} \tilde{B}_J, \]  

\[ G_D(s; p_n) = C(p_n) V(p_n)(sI - \Lambda(p_n))^{-1} \tilde{B}_D, \]  

\[ G_C(s; p_n) = C(p_n) V(p_n)(sI - \Lambda(p_n))^{-1} \tilde{B}_C, \]  

\[ G_U(s; p_n) = C(p_n) V(p_n)(sI - \Lambda(p_n))^{-1} \tilde{B}. \]  

(9)

The complex frequency response follows from

\[ \tilde{y}(j\omega; p_n) = G_J(j\omega; p_n) \tilde{F}_J(j\omega) + G_D(j\omega; p_n) \tilde{F}_D(j\omega) + G_C(j\omega; p_n) \tilde{F}_C(j\omega) + G_U(j\omega; p_n) \tilde{F}_U(j\omega; p_n) \]  

(10)
which shows directly how amplification around the eigenvalues in the diagonal matrix \((sl - \Lambda(p_n))^{-1} = \text{diag}(1/(s - \lambda_i(p_n)))\) feed through to the state variables. The following comments apply for systems involving robotic machining:

(a) The frequency content of the \(\bar{F}(j\omega)\) will depend on the robot specification for tracking speed. The robot motors will generally have low bandwidth.

(b) The disturbance forces due to machining, \(\bar{F}_{D}(j\omega)\), will typically involve a synchronous spindle rotational frequency and higher harmonics, together with components that involve rotor modal frequencies represented by the values of \(\text{Im}(\lambda_k(p_n))\). The disturbance forces may be considered to have a high bandwidth.

(c) The control forces in \(\bar{F}_{C}(j\omega)\) must be generated by the robot motors for tracking purposes or by some external system that can generate higher bandwidth forces. In this paper, the external system is composed of a system of inertial actuators coupled to the machining spindle.

(d) The uncertainty forces in \(\bar{F}_{U}(j\omega; p_n)\) will arise from unmodelled contact events and stick-slip motions within revolute joints. Unmodelled robot modes may also be considered as uncertainty, additive or multiplicative.

3. Experimental system

3.1. Industrial robot/spindle/inertial actuator/accelerometer integration

A KUKA KR120R2500 PRO robot as shown in Figure 1 was used to carry out experiments. A machining spindle holding a milling cutter was mounted on the end effector of the robot and aligned with direction Y. Two inertial voice coil actuators (from H2W Technologies\textsuperscript{TM}, force output up to 46.7 N), \(A_1\) and \(A_2\), with cylindrical steel blocks as 0.5 kg moving masses, were mounted near the spindle, aligned with directions \(X\) and \(Z\). An accelerometer (from DJB\textsuperscript{TM}) was mounted on the back face of the end-effector. Note that the designed inertial actuators have a suspension frequency at 14 Hz to ensure that operations from 20 Hz are within their bandwidth.

A Spherical Mounted Retroreflector (SMR) was attached near the spindle to allow a laser tracker (from FARO\textsuperscript{TM}), with a measurement accuracy of 22 \(\mu\text{m}\) (2\(\sigma\)), to measure displacements of the end-effector. On the base plate, a workpiece was mounted on the top of a force transducer (from Kistler\textsuperscript{TM}). Milled surfaces were measured using stylus profiler (from Taylor Hobson\textsuperscript{TM}). All data acquisition, feedback and controller implementation were routed through a processor (from dSPACE\textsuperscript{TM}) at a sampling frequency of 5 kHz except the laser tracker, which operates at a sampling frequency of 512 Hz.

3.2. Vibration in robotic machining

With a pose of the robot (Pose 0 as in Table 1) and a machining trajectory in the direction \(Z\), it was observed that the robot system experienced mode coupling chatter in milling tests with the parameters listed in Table 2. The chatter occurred because the cutting frequency at 84 Hz is much higher than the structural mode of the robot system around 26 Hz and is

![Figure 1](image-url)

Figure 1. Experimental set-up as (a) the home position of the robot with configuration of joints as \(\theta_{1,4,6} = 0^\circ\), \(\theta_2 = 90^\circ\) and \(\theta_3 = -90^\circ\) with positive angles defined clockwise, and (b) the end effector and the workpiece.
identified as mode coupling chatter. Figure 2(a) shows larger machining forces measured in direction X compared with those in direction Z and a beating phenomenon in the amplitudes of the machining forces in both directions. Figure 2(b) shows a beating phenomenon that consists of the spindle rotational frequency at 28 Hz and a chatter frequency at 27 Hz in the resonance range of the robot system. At higher harmonics, the amplitudes generally decrease with frequencies above 100 Hz. To tackle this issue, further investigations on the dynamics of the robot system in the X-Z plane are required.

3.3. Fixed pose system identification

System identification was undertaken by establishing the relationships between the input driving voltages, $V_X$ and $V_Z$, applied to the inertial actuators and measured output accelerations, $A_X$ and $A_Z$, in the X and Z directions. During a milling process, vibration suppression is desirable to cover a small or large working volume depending on the configuration of this process. Therefore, it is important to identify the variation in dynamics of the robot system within the specified working volume. This is taken into consideration in this paper through milling tests that were performed on a flat workpiece at two different poses in the X-Z plane. Pose 0 is associated with the configuration of robot joints as listed in Table 1. Grid point (1,6) is associated with the system dynamics of the robot system experience the most significant change within a work plane of 500 mm × 500 mm around Pose 0. To identify this, the robot system was characterised experimentally over a grid containing 36 fixed poses, \{l, k\} \leq l, k \leq 6\}, covering the work plane, as shown in Figure 3. In each pose, the robot system was excited by each inertial actuator sequentially, with a chirp signal of 20–120 Hz applied over 60 seconds. Identified system dynamics are designated by the transfer function $G^{(m,d)}_{l,k,(X,Z)}(s)$, where the superscripts correspond with fitted models (m) or based on unprocessed measurement data (d). It is assumed that the coupling of the orthogonal X and Z directions is negligible. System dynamics at Pose 0 were then evaluated based on the mean of the dynamics at the four neighbouring grid points:

$$G_{0,(X,Z)}^{(m,d)}(s) = \frac{1}{4} \left( G_{3,3,(X,Z)}^{(m,d)}(s) + G_{3,4,(X,Z)}^{(m,d)}(s) + G_{4,3,(X,Z)}^{(m,d)}(s) + G_{4,4,(X,Z)}^{(m,d)}(s) \right)$$

(11)

Differences in the system dynamics between each grid point and Pose 0, are defined as

Table 1. Configurations of robot joints at Pose 0, used for controller design.

| Joint No. | 1  | 2  | 3  | 4  | 5  | 6  |
|-----------|----|----|----|----|----|----|
| Pose 0    | -9.66° | -46.45° | 114.09° | -9.95° | 68.08° | 3.76° |

Table 2. Milling parameters that was chosen to demonstrate the occurrence of chatter at Pose 0.

| Workpiece       | Aluminium block (100 mm × 100 mm × 20 mm) |
|-----------------|------------------------------------------|
| Tool            | 6 mm 3 flute end milling cutter           |
| Machining type  | Climb milling                             |
| Coolant         | None                                     |
| Depth of cut    | 3 mm                                     |
| Width of cut    | 0.5 mm                                   |
| Spindle speed   | 1720 rev/min                             |
| Feed rate       | 1 mm/s (Z direction)                     |
| Path length     | 100 mm                                   |

![Figure 2](image-url) Measured machining forces in directions X and Z from a milling test involving chatter. (a) In the time domain, and (b) In the frequency domain.
\[ \Delta_{k;m,d}^{s} = G_{k;m,d}^{s} - G_{k;m,d}^{s} \] (12)

Figure 3. (a) Configuration of grid points for fixed pose characterisation. (b) The robot in Pose 0, and (c) at grid point (1,6).

Figure 4. Measured and identified frequency response amplitudes of the robot system in Pose 0. (a) \( G_{k;m,d}^{s}(j\omega) \) m/s²/V in the X direction, (b) \( G_{k;m,d}^{s}(j\omega) \) m/s²/V in the Z direction.

\[ \Delta_{1,6}^{s} = 24 \text{ m/s}^2/\text{V} \]

The resonant peaks in the X direction are over 10 times higher than those in the Z direction. Given that inertial actuators and their amplifiers are similar in specification, it demonstrates that the robot system is more susceptible to vibrations in the X direction.

Figure 5 summarises differences in the system dynamics at each grid point relative to Pose 0. A maximum of \( \|\Delta_{1,6}^{s}\|_{\infty} = 24 \text{ m/s}^2/\text{V} \) is located at the...
grid point (1, 6) for differences in system dynamics in the X direction. For differences in system dynamics in the Z direction, a maximum of $\|\Delta_4 \|_\infty = 2.5$ m/s²/V is located at grid point (3, 4), though this is small compared with that in the X direction. It confirms that throughout the specified work plane, the robot system is more responsive in the X direction. Figure 6 (a,b) shows the differences in system dynamics in the X direction, $\Delta^d_{4,4}(j\omega)$ and $\Delta^e_{4,4}(j\omega)$, evaluated at grid points (3, 4) and (1, 6), respectively, relative to Pose 0, covering different variations in the dynamic characteristics of the robot system.

It is noted that measurement error causes model-fitted transfer functions to differ according to:

$$G^d_{k,(X,Z)}(s) = G^m_{k,(X,Z)}(s) + \Delta^d_{k,(X,Z)}(s)$$

where the superscript e corresponds with the residual from fitting the transfer function model to fit the unprocessed measurement data. The consequence is that

$$G^m_{k,(X,Z)}(s) = G^m_{\text{Pose 0},(X,Z)}(s) + \Delta^d_{k,(X,Z)}(s) + \Delta^e_{k,(X,Z)}(s)$$

In particular,

$$G^m_{1,6,(X,Z)}(s) = G^m_{\text{Pose 0},(X,Z)}(s) + \Delta^d_{1,6,(X,Z)}(s) + \Delta^e_{1,6,(X,Z)}(s)$$

Figure 6(a,b) shows the maximum total additive uncertainty, $|\Delta^d_{1,6}(j\omega) + \Delta^e_{1,6}(j\omega)|$, to be approximately 25 m/s²/V occurring around 26 Hz, indicating that the largest variation in system dynamics is at a resonance of the robot system. Local to Pose 0, $|\Delta^d_{4,4}(j\omega) + \Delta^e_{4,4}(j\omega)|$ has a maximum value of approximately 4 m/s²/V occurring around 26 Hz.

4. Control strategy design for fixed pose machining

Figure 7 shows a block diagram of the robot/inertial actuator system, $G_C(s; p_n)$, augmented by the weighting transfer function matrices, $W_i(s; p_n)$, $i = 1, 2, 3$. The input to the augmented plant is the demand input, $u_1 = y_d$. The weighted outputs, $y_{1a,b,c}$, correspond with the error vector, $y_2 = e$, the controller output, $u_2 = F_C$, and the plant output vector, $y$, respectively. In this way, the design process for the inertial actuator controller, $K(s; p_n)$, is represented as a classic mixed-sensitivity problem. The closed loop transfer function matrix-vector representation is

$$\begin{bmatrix} y_{1a} \\ y_{1b} \\ y_{1c} \end{bmatrix} = \begin{bmatrix} W_1(s; p_n)S(s; p_n) \\ W_2(s; p_n)K(s; p_n)S(s; p_n) \\ W_3(s; p_n)T(s; p_n) \end{bmatrix} u_1$$

where

$$S(s; p_n) = (I + G_C(s; p_n)K(s; p_n))^{-1}, T(s; p_n) = I - S(s; p_n).$$

For the fixed pose robotic machining problem, $F_j = 0$, and if $u_1 = y_d = 0$ for a smooth surface finish, Equation (8) implies that under closed loop control,

$$y(s; p_n) = S(s; p_n) \left( G_D(s; p_n)F_D(s) + G_U(s; p_n)F_U(s; p_n) \right)$$

Figure 5. Measured differences in the dynamics of the robot system. (a) $\|\Delta^d_{k,(X,Z)}\|_\infty$ m/s²/V in the X direction, (b) $\|\Delta^d_{k,(X,Z)}\|_\infty$ m/s²/V in the Z direction. The superimposed dashed squares show the reference Pose 0 region and grid point (1,6) region, a lower right region of maximum errors in the X direction.
Hence, an objective should be to minimise $\|S(s; p_n)\|_{\infty}$ so that the influences of $F_D$ and $F_U$ on $\tilde{y}$ are minimised and a fine surface finish may be achieved. Another objective is to maintain closed loop stability in the presence of the identified additive modelling error in $G_C$, say $\Delta_C$, which requires a limitation to be placed on $K(s; p_n)S(s; p_n)$. As is common with the $H_{\infty}$ design process, the imposition is to select $K(s; p_n)$ such that

$$
\left\| \begin{bmatrix}
W_1(s; p_n)S(s; p_n) \\
W_2(s; p_n)K(s; p_n)S(s; p_n) \\
W_3(s; p_n)T(s; p_n)
\end{bmatrix} \right\|_{\infty} \leq 1
$$

(18)
where

- \( \| W_1(s; p_n) \|_\infty \) is suitably large to reduce \( \| S(s; p_n) \|_\infty \),
- \( \| W_2(s; p_n) \|_\infty \) is suitably large to reduce the controller output thereby maintaining closed loop stability in the presence of additive error and preventing damage to the inertial actuator system,
- \( \| W_3(s; p_n) \|_\infty \) provides a bound on any multiplicative model uncertainty in order that closed loop stability is maintained.

Since additive uncertainty and multiplicative uncertainty can be related algebraically, they do not have to be considered together. In practical terms, the model \( G_C(s; p_n) \) was considered with some assessed additive error \( \Delta_A(s; p_n) \), according to Equations (14) and (15).

Further, if the robot pose changes from \( p_n \) to \( p \neq p_n \), then

\[
G_C(s; p) = G_C(s; p_n) + \Delta_A(s; p_n) + \Delta_C(s; p - p_n) \quad (19)
\]

Then \( \| W_2(s; p_n) \|_\infty \geq \| \Delta_A(s; p_n) + \Delta_C(s; p - p_n) \|_\infty \) and \( W_3(s; p_n) = 0 \) would be appropriate for the design of the controller \( K(s; p_n) \).

5. Designed \( H_\infty \) and velocity feedback controllers for robotic machining

\( H_\infty \) controllers were designed based on the identified system model under Pose 0, designated to correspond with \( p_1 \). For design purposes, the model is written as

\[
G_C(s; p_1) = \begin{bmatrix}
G_{m,0,0}^m(s)/K(s) & 0 \\
0 & G_{m,0,2}^m(s)/K(s)
\end{bmatrix} \quad (20)
\]

where the inertial actuator forces are related to driving voltages according to \( F_x = K(s)V_x \) and \( F_z = K(s)V_z \). With these parameters, \( F_C = [F_x \ F_z]^T \) and the output vector consists of the measured accelerations \( \mathbf{y} = [A_x \ A_z]^T \). The weighting functions were selected to reject milling-induced vibration in the \( x \) and \( z \) directions around the natural frequency of the robot system:

\[
W_1(s; p_1) = \begin{bmatrix}
W_{1,0}(s; p_1) & 0 \\
0 & W_{1,2}(s; p_1)
\end{bmatrix} \quad (21)
\]

where

\[
W_{1,0}(x,z) = \frac{\tau s}{\tau s + 1} \left( \frac{\omega_{x,z}^2}{s^2 + 2\zeta_{x,z}\omega_{x,z}s + \omega_{x,z}^2} \right)
\]

and \( \tau = 1/2nf_x, \omega_{x,z} = 2nf_{x,z} \). The overall fixed gains of \( W_1 \) are \( w_{1,0}(x,z) \), which determine the level of vibration reduction of the \( H_\infty \) controller; \( (\tau s)/(\tau s + 1) \) is a second-order high-pass filter that protects the inertial actuators from potentially large amplitude responses at low frequencies below the cut-off frequency, \( f_c \); \( \omega_{x,z}^2/(s^2 + 2\zeta_{x,z}\omega_{x,z}s + \omega_{x,z}^2) \), is a second-order transfer function that defines the attenuation frequency band about the natural frequency of the robot system.

The weighting function corresponding to additive error was chosen as the constant diagonal matrix:

\[
W_2(s; p) = \begin{bmatrix}
w_{2,0}(p) & 0 \\
0 & w_{2,2}(p)
\end{bmatrix} \quad (23)
\]

since the largest additive uncertainty in modelling and characteristics of the robot system is associated with shifting of its natural frequencies around 26 Hz. The weighting function \( W_2 \) was set to zero since all uncertainties in the system are treated as additive.

Two \( H_\infty \) controllers were designed for the weighting matrices of Equations (21)-(23):

- \( H_0 \), designed about Pose 0 with additive uncertainty local to the centre of the grid in Figure 5(a, b), covering a 100 mm × 100 mm work-plane.
- \( H_1 \), designed about Pose 0 with additive uncertainty at its maximum in the lower right portion of the grid in Figure 5(a) at grid point (1,6) (i.e. covering the whole 500 mm × 500 mm work-plane).

The weighting function parameters are shown in Table 3. The natural frequencies of the weighting functions \( W_{1,}(x,z) \) were set to coincide the modal frequencies of the robot system in order to maximise local control efforts.

The \( H_\infty \) controllers were also compared with velocity feedback controllers, which were configured using integral feedback of acceleration signals in the \( x \) and \( z \) directions with the gains of Table 4. Low-pass filters with a cut-off frequency at 150 Hz and high-pass filters with a cut-off frequency at 10 Hz were used to process acceleration measurements. These gains were set below instability limits and obtained
by tuning manually during eccentric mass and milling tests. Note that acceleration/jerk control (i.e., proportional/derivative) would demand higher, potentially damaging, inertial actuator forces at higher frequencies, hence was not considered.

6. Results and discussions

6.1. Eccentric mass experiments

In order to evaluate the robustness of controllers against variations in the dynamics of the robot system, the $H_{\infty}$ and velocity feedback controllers were tested at poses that are specified in Table 5. Experiments were undertaken by adding an eccentric mass to the machining spindle and recording the dynamic responses, without physical machining. In principle, the exciting unbalance excitation is synchronous with the rotational frequency and steady state synchronous vibration amplitudes may be extracted. This includes at the four corners of the grid in Figure 3, and additionally at (1,4) and at the centre point corresponding to Pose 0. Grid point (1,4) is where the system dynamics in the $Z$ direction vary the most.

In these experiments, a stable system under unbalanced excitation would result in a steady orbit of the spindle at a frequency that is synchronous with the rotational frequency (26 Hz). Instability became evident at 14 Hz with a growing amplitude. When this growth was detected, the control was terminated to prevent excessive actuator strokes. According to the $H_{\infty}$ synthesis, steady control in the presence of additive error about a pose corresponding to $p_n$ is guaranteed if $\|W_2(s; p_n)K(s; p_n)S(s; p_n)\|_{\infty} \leq 1$. Given that $\|W_2(s; p_n)\|_{\infty} \geq \|\Delta_A(s; p_n) + \Delta_C(s; p - p_n)\|_{\infty}$, the guarantee may be violated by the additive error arising from pose variation, $\Delta_C(s; p - p_n)$, if the pose corresponding to $p$ deviates significantly from $p_n$. This is the reason why controller $H_0$, which is designed to be stable in the localised region around the centre (Pose 0), is unstable at the external grid points listed in Table 5. Table 5 shows that although the $H_0$ and $V_0$ controllers reduce vibrations down to between 20% and 28% in the $X$ direction, performance is limited in other poses, except for Pose 0, due to limited robustness against dynamic uncertainties away from Pose 0. Table 5 also shows that the $H_1$ controller suppresses vibrations down to 22% (i.e., amplitude reduced from 0.52 m/s$^2$ to 0.15 m/s$^2$) in the $X$ direction across the grid. In contrast, the $V_1$ controller suppresses vibrations down to 40% (i.e., amplitude reduced from 0.52 m/s$^2$ to 0.21 m/s$^2$). The worst performance of the $H_1$ and $V_1$ controllers was found in the pose for grid point (1,6), where residual vibrations were measured to be 42% for the $H_1$ controller (i.e., amplitude reduced from 0.55 m/s$^2$ to 0.24 m/s$^2$) and 73% for $V_1$ controller (i.e., amplitude reduced from 0.55 m/s$^2$ to 0.40 m/s$^2$), respectively. This observation is in consistent with results in grid system identification where grid point (1,6) has the largest additive error away from Pose 0. In summary, the results show that the $H_1$ controller has superior capability in dealing with additive model error for synchronous vibration control over the full grid. Compared with the $H_0$ controller, the $H_1$ controller is more robust at a small cost of the performance.

6.2. Milling tests

Milling tests as being configured in Table 2 were performed to assess vibration suppression using the $H_1$ and $V_1$ controllers around Pose 0 and
around grid point (1,6) at the corner of the grid. In Pose 0, Figure 8(a) shows measured accelerations in the X direction with the H-1 controller applied from $t = 5s$. Without control, a beating phenomenon with an amplitude of around 2 m/s$^2$ and a frequency of 1 Hz is observed in the acceleration signal. This effect is due to the multiple frequencies around 27 Hz (Figure 8(b)). When H-1 control is applied, the beating phenomenon is suppressed, reducing the overall amplitude of acceleration to around 0.7 m/s$^2$. Figure 8(b) shows that the H-1 controller suppresses the frequency component at 27 Hz, the chatter frequency in the resonance range of the robot system, almost completely and the frequency component at 28 Hz, a sub-harmonic of the cutting frequency (i.e. $3 \times 28 = 84$ Hz), by approximately 60%. Similar vibration reduction is also evident around 110 Hz, the third natural frequency of the robot. Figure 8(c,d) shows that the V-1 controller suppresses vibrations at the chatter frequency only moderately, with a reduction in overall amplitude of around 1.4 m/s$^2$.

Figure 9(a,b) shows that milling near grid point (1,6) exhibits single frequency components at 28 Hz and 112 Hz. In contrast to Pose 0, the structural mode of the robot shifts from 27 Hz to 28 Hz due to the change in the robot dynamics. A beating phenomenon does not occur because the structural mode of the robot coincides the spindle rotational frequency, causing a single large amplitude of this frequency component. Therefore, mode coupling chatter does not occur at grid point (1,6) and does not leave chatter marks on the workpiece. With H-1 control, the overall amplitude of acceleration is reduced by 75% within 1 second around at the spindle rotational frequency. Figure 9(c,d) shows that the V-1 controller takes 3 seconds to reduce vibration by 60%.

Note that the same V-1 controller reduces the frequency component at 28 Hz near grid point (1,6) but amplifies the same frequency component in Pose 0 due to its phase. While performance of the V-1 controller can be improved further by changing filters and adding compensation and notch filters, it requires additional prior knowledge (e.g. specific to the applied process) and experiments. In overall, it shows that the V-1 controller is less robust against unmodelled processes due to the change in dynamics of the robotic system compared with the H-1 controller.

Figure 10 shows measured cutting forces in the X direction arising from milling tests in Poses 0 and 1, with both controllers activated at $t = 5s$. In Pose 0, H-1 control reduces the overall amplitude (peak-peak) of

![Figure 8](image_url)
milling force from 60 N to 25 N. V-1 control is less effective in force vibration reduction, from only 60 N to 50 N and the beating phenomenon remains. Near grid point (1,6), both controllers reduce the overall milling force amplitude (peak-peak) to 25 N with H-1 control acting significantly quicker than V-1 control.

Figure 11 shows measured displacements using laser tracker in the X and Z directions, $D_x$ and $D_z$, arising from milling tests in Poses 1 and 2, with H-1 and V-1 controllers applied at 20 mm. Without control, the end effector of the robot system vibrates with an amplitude of 80 $\mu$m in the X direction. H-1 control reduces this to 20 $\mu$m while V-1 control reduces it to 50 $\mu$m. Near grid point (1,6), the end effector vibrates at an amplitude of 80 $\mu$m. Both controllers reduce vibrations to below 40 $\mu$m with H-1 control yielding a faster response. Note that additional positioning errors are associated with tracking errors of the robot.

Changes in movement of the end effector of the robot also reflects on the surface profiles under milling. Figure 12(a) shows profiles of surfaces that were milled in Pose 0, with control applied after the 20 mm location. The beating phenomenon cause marks on the milled surface with a wavelength of approximately 1 mm, as the result of the beating frequency of 1 Hz under a feed rate of 1 mm/s. Because V-1 control suppresses the vibration at the chatter frequency and hence reduces the overall beating phenomenon, the size of chatter marks is decreased from that without control. In the case of using H-1 control, the beating phenomenon is removed completely along with the vibration at the chatter frequency. Hence, marks are absent on the milled surface. As listed in Table 6, surface finish was improved in terms of a reduction in RMS surface profile by over 60% when using V-1 control and by 85% when using H-1 control. Near grid point (1,6), because milling processes do not significantly excite the robot system modes, surface profile remains unchanged with or without control. Figure 12 (a,b) also shows that with or without the beating, vibration suppression also reduces the offsets between tool path and the milled surfaces. In Pose 0, the surface is 40$\mu$m higher when using H-1 control than that without control, and double that when using V-1 control. Near grid point (1,6), similar observation can be made, with a level difference of 15$\mu$m when using V-1 control and 40$\mu$m when using H-1 control. Figure 13 shows photographs taken on these surfaces, where such changes in surface profile and surface level are visible.

7. Conclusions

This paper has demonstrated the use of inertial actuators to generate forces according to acceleration feedback with optimised controllers to apply active vibration
suppression during robotic machining. It proposes the use of $H_{\infty}$ optimised control as an effective and robust method to cope with the change in dynamics of a robot system with pose. The controller design is based on experimentally identified models of the robot system and selected weighting functions that treat modelling errors and variations in the dynamics of the robot system as additive uncertainties. The capability of the proposed

Figure 10. Measured milling forces in the $X$ direction during milling tests in Pose 0, (a) Using the $H$-1 controller, (b) Using the $V$-1 controller, and near grid point (1,6). (c) Using the $H$-1 controller, (d) Using the $V$-1 controller. In each case, control was activated at 5 s.

Figure 11. $X$-$Z$ plots in displacement, measured by a laser tracker, using both controllers during milling tests (a) In Pose 0 and (b) Near grid point (1,6). (c) and (d) Normalised $X$-$Z$ plots measured in Pose 0 and near grid point (1,6), respectively. $D_Z$ refers to vibrations in displacement relative to the instantaneous tool path in the $Z$ direction.
controllers was evaluated and compared with simpler velocity feedback controllers in eccentric mass and milling experiments.

It was shown that in eccentric mass experiments, $H_\infty$ control, based on low additive error about a single pose, performed slightly better than velocity feedback controllers, with a vibration reduction in the critical direction of 80% compared with 72%. However, both controllers did not work well at other poses within the defined work-plane when the dynamics of the robot system changed. In contrast, $H_\infty$ control, incorporating higher additive errors, and velocity feedback controllers, which were designed to suppress vibration over a relatively large work-plane, were effective. $H_\infty$ control reduced uncontrolled vibrations by 58–78% (X direction) and 78–91% (Z direction), which were significantly better than for the velocity feedback control at 27–61% (X direction) and 8–66% (Z direction). Furthermore, the $H_\infty$ controller suppressed a resonance of the robot system completely, resulting in the removal of chatter marks and a reduction in surface profile of milled surfaces by over 85%. Future vibration suppression
techniques for robotic milling should focus on covering larger workspaces and broader attenuation bandwidths, whilst retaining simplicity in computation and implementation.

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