I describe theories of a strongly-interacting electroweak symmetry breaking sector and discuss the expected size of anomalous weak-boson couplings in these models.

SIGNATURES OF ELECTROWEAK SYMMETRY BREAKING IN $WW$ SCATTERING

The physics of electroweak symmetry breaking must appear at energies of order a TeV or lower. To see this, consider a thought experiment (1), the scattering of longitudinally polarized $W^+$ and $W^-$:

\[ W_L \quad \text{and} \quad W_L. \]  

Using the Feynman-rules of the electroweak gauge theory we can calculate $W^+_L W^-_L$ scattering at tree level. We find that this amplitude grows like $E_{cm}^2$:

\[ A = \frac{g^2 s}{8 M_W^2} (1 + \cos \theta^*) , \]  

plus terms that do not grow with $s$. Projecting onto the $s$-wave state, we find

\[ A^{s=0} = \frac{g^2 s}{128 \pi M_W^2} \sim \left( \frac{\sqrt{s}}{2.5 \text{ TeV}} \right)^2 . \]  

Unitarity implies that the dynamics associated with EWSB has to appear before an energy scale of around 2.5 TeV (1) (2). There are three possibilities:

- There may be additional particles with masses less than or of order of a TeV, or
- the $W$ and $Z$ interactions may become strong at energies of order a TeV, or
• both of the above.

It is important to note that the amplitude calculated above universal: the calculation depended only on the gauge structure of the standard model and on the relationship

\[ \rho = \frac{M_W}{M_Z \cos \theta_W} \approx 1 \ , \]  

and will hold regardless of the dynamics responsible for electroweak symmetry breaking. Therefore, in order to understand the dynamics of electroweak symmetry breaking, it will be necessary to characterize the physics which cuts-off the growth in the longitudinal gauge boson scattering amplitudes.

The universality of the scattering amplitudes for longitudinal gauge boson scattering can also be seen as a consequence of the fact that the longitudinal components of the gauge-bosons are the “eaten” Goldstone Bosons of \( SU(2) \times U(1) \) breaking. More formally, the “Equivalence Theorem” states that any amplitude involving the scattering of longitudinal gauge-bosons is equal to the same amplitude involving the corresponding Goldstone Bosons (which would be present in the ungauged theory), up to corrections of order \( (M_W/E)^2 \). The low-energy scattering amplitudes of Goldstone Bosons, however, are determined by the low energy theorems of chiral dynamics (c.f. PCAC in QCD) and are determined by the symmetry structure of the theory. In a theory in which \( \rho = 1 \), the symmetry structure of the electroweak symmetry breaking sector is naturally \( SU(2) \times SU(2) \) (6), and the scattering of the longitudinal components of the \( W \) and \( Z \) are determined by analogs of Weinberg’s low-energy theorems in QCD (6).

THEORIES OF ELECTROWEAK SYMMETRY BREAKING

The Standard One-Doublet Higgs Model

In the standard one-doublet Higgs model one introduces a fundamental scalar doublet of \( SU(2)_W \):

\[ \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \ , \]  

which has a potential of the form

\[ V(\phi) = \lambda \left( \phi^\dagger \phi - \frac{v^2}{2} \right)^2 \ . \]  

In the potential, \( v^2 \) is assumed to be positive in order to favor the generation of a non-zero vacuum expectation value for \( \phi \). This vacuum expectation value breaks the electroweak symmetry, giving mass to the \( W \) and \( Z \). When
symmetry breaking takes place, the four degrees of freedom in $\phi$ divide up. Three of them become the longitudinal components, $W_L$ and $Z_L$, of the gauge bosons, and the fourth, commonly called $H$ (for Higgs particle), is left over

$$\phi = \Omega \begin{pmatrix} 0 \\ \frac{H + v}{\sqrt{2}} \end{pmatrix}.$$  (7)

Here, $\Omega$ is an $SU(2)$ matrix. If we make an $SU(2)_W$ gauge transformation until $\Omega$ is the identity, we arrive at unitary gauge.

The exchange of the Higgs boson contributes to $W_L W_L$ scattering. In the limit in which $E_{cm}$ is large compared to the masses of the particles in the process, the leading contribution (in energy) from Higgs boson exchange exactly cancels the bad high-energy behavior in $W^+_L W^-_L$ scattering

$$\mathcal{A} = -\frac{g^2 s}{8 M_W^2} (1 + \cos \theta^*),$$  (8)

plus terms which do not grow with energy.

At tree-level the Higgs boson has a mass given by $m^2_H = 2 \lambda v^2$. In order for this theory to give rise to strong $W$ and $Z$ interactions, it would be necessary that the Higgs boson be heavy and, therefore, that $\lambda$ be large.

This explanation of electroweak symmetry breaking is unsatisfactory for a number of reasons. For one thing, this model does not give a dynamical explanation of electroweak symmetry breaking. For another, when embedded in theories with additional dynamics at higher energy scales, these theories are technically unnatural.

Perhaps most unsatisfactory, however, is that theories of fundamental scalars are probably “trivial”, i.e., it is not possible to construct an interacting theory of scalars in four dimensions that is valid to arbitrarily short distance scales. In quantum field theories, fluctuations in the vacuum screen charge – the vacuum acts as a dielectric medium. Therefore there is an effective coupling constant which depends on the energy scale ($\mu$) at which it is measured. The variation of the coupling with scale is summarized by the $\beta$–function of the theory

$$\beta(\lambda) = \mu \frac{d\lambda}{d\mu}.$$  (9)

The only coupling in the Higgs sector of the standard model is the Higgs self-coupling $\lambda$. In perturbation theory, the $\beta$–function is calculated to be

$$\beta = \frac{3 \lambda^2}{2 \pi^2}.$$  (10)
Using this $\beta$–function, one can compute the behavior of the coupling constant as a function of the scale. One finds that the coupling at a scale $\mu$ is related to the coupling at some higher scale $\Lambda$ by

$$\frac{1}{\lambda(\mu)} = \frac{1}{\lambda(\Lambda)} + \frac{3}{2\pi^2} \log \frac{\Lambda}{\mu}. \quad (11)$$

In order for the Higgs potential to be stable, $\lambda(\Lambda)$ has to be positive. This implies that

$$\frac{1}{\lambda(\mu)} \geq \frac{3}{2\pi^2} \log \frac{\Lambda}{\mu}. \quad (12)$$

Thus, we have the bound

$$\lambda(\mu) \leq \frac{2\pi^2}{3 \log \left( \frac{\Lambda}{\mu} \right)}. \quad (13)$$

If this theory is to make sense to arbitrarily short distances, and hence arbitrarily high energies, we should take $\Lambda$ to $\infty$ while holding $\mu$ fixed at about 1 TeV. In this limit we see that the bound on $\lambda$ goes to zero. In the continuum limit, this theory is trivial; it is free field theory.

The inequality above can be translated into an upper bound on the mass of the Higgs boson. From the bound above, we have

$$\frac{\Lambda}{\mu} \leq \exp \left( \frac{2\pi^2}{3\lambda(\mu)} \right), \quad (14)$$

but

$$m_H^2 \sim 2v^2 \lambda(m_H), \quad (15)$$

thus

$$\Lambda \leq m_H \exp \left( \frac{4\pi^2v^2}{3m_H^2} \right). \quad (16)$$

For a given Higgs boson mass, there is a finite cutoff energy at which the description of the theory as a fundamental scalar doublet stops making sense. This means that the standard one-doublet Higgs model can only be regarded as an effective theory valid below this cutoff.

The theory of a relatively light weakly coupled Higgs boson, can be self-consistent to a very high energy. For example, if the theory is to make sense up to a typical GUT scale energy, $10^{16}$ GeV, then the Higgs boson mass has to be less than about 170 GeV. In this sense, although a theory with a

\footnote{Since these expressions were computed in perturbation theory, they are only valid when $\lambda(\mu)$ is sufficiently small. We will return to the issue of strong coupling below.}
light Higgs boson does not really answer any of the interesting questions (e.g., it does not explain why $SU(2)_W \times U(1)_Y$ breaking occurs), the theory does manage to postpone the issue up to higher energies.

The theory of a heavy Higgs boson (i.e., with a mass of about 1 TeV), however, does not really make sense. Since we have computed the $\beta$-function in perturbation theory, our answer is only reliable at energy scales at which $\lambda(\mu)$ (as well as the Higgs boson mass) is small. Fortunately, non-perturbative lattice calculations are available. Early estimates indicated that if the theory was to make sense up to 4 TeV, the mass of the Higgs boson had to be less than about 640 GeV. More recent results imply that this bound may be relaxed somewhat; one might be able to get away with an 800 GeV Higgs boson, but the Higgs boson mass is certainly bounded by a value of this order of magnitude. The triviality limits on the mass of the Higgs boson imply that it is not possible for the $W_L$ and $Z_L$ scattering amplitudes in the standard model to truly become large at energies well below the cutoff. This result is especially interesting because it implies that if nothing shows up below energies of the order 700–800 GeV, then something truly “non-trivial” is going on. We just have to find it.

Technicolor

In models with fundamental scalars, electroweak symmetry breaking can be accommodated if the parameters in the potential (which presumably arise from additional physics at higher energies) are suitably chosen. By contrast, technicolor theories strive to explain electroweak symmetry breaking in terms of physics operating at an energy scale of order a TeV. In technicolor theories, electroweak symmetry breaking is the result of chiral symmetry breaking in an asymptotically-free, strongly-interacting gauge theory with massless fermions. Unlike theories with fundamental scalars, these theories are technically natural: just as the scale $\Lambda_{QCD}$ arises in QCD by dimensional transmutation, so too does the weak scale $v$ in technicolor theories. Accordingly, it can be exponentially smaller than the GUT or Planck scales. Furthermore, asymptotically-free non-abelian gauge theories may be fully consistent quantum field theories.

In the simplest technicolor theory one introduces a (massless) left-handed weak-doublet of “technifermions”, and the corresponding right-handed weak-singlets, which transform as $N$’s of a strong $SU(N)_{TC}$ technicolor gauge group. In analogy to the (approximate) chiral $SU(2)_L \times SU(2)_R$ symmetry on quarks in QCD, the strong technicolor interactions respect an $SU(2)_L \times SU(2)_R$ global chiral symmetry on the technifermions. When the technicolor interactions become strong, the chiral symmetry is broken to the diagonal subgroup, $SU(2)_{L+R}$, producing three Nambu-Goldstone bosons which become, via the Higgs mechanism, the longitudinal degrees of freedom of the $W_L$ and $Z_L$. Because the left-handed and right-handed techni-fermions
carry different electroweak quantum numbers, the electroweak interactions break to electromagnetism. If the $f$-constant of the theory, the analog of $f_\pi$ in QCD, is chosen to be $v \approx 246$ GeV, then the $W$ mass has its observed value. Furthermore (3), the remaining $SU(2)_{L+R}$ custodial symmetry insures that, to lowest order in the hypercharge coupling, $M_W = M_Z \cos \theta_W$.

In addition to the “eaten” Nambu-Goldstone bosons, such a theory will give rise to various resonances, the analogs of the $\rho$, $\omega$, and possibly the $\sigma$, in QCD. In general, the growth of the $W_L$ and $Z_L$ scattering amplitudes are cut off by exchange of these heavy resonances,

\[
\begin{array}{cccc}
W_L & W_L & W_L & W_L \\
W_L & W_L & W_L & W_L
\end{array}
\]

just as in QCD the growth of pion–pion scattering amplitudes are cut off by QCD resonances. Scaling from QCD, we expect that the masses of the various resonance will be of order a TeV. Unlike the situation in models with only fundamental scalars in the symmetry breaking sector, the scattering of longitudinal $W$ and $Z$ bosons can truly be strong.

In figure 1, we show the data for the scattering amplitude of $\pi^+\pi^0$ at low-energies, as well as the corresponding low-energy theorem. We see that while the growth of the scattering amplitude begins close to the low-energy theorem prediction, it is significantly enhanced (and unitarized) by the presence of the $\rho$-resonance. We expect a similar behavior in technicolor theories, with the energy scale enhanced by a factor of $v/f_\pi \approx 2600$.

The most direct signal for technicolor, therefore, is an enhancement in the production of $WZ$ pairs at high invariant-mass, coming from the production of the technicolor analog of the $\rho$-meson in QCD (13) (2). If the technirho resonance(s) are too heavy to be observed at the LHC, there may be an enhancement in the isospin-2 $W^+W^++W^-W^-$ channel which is large enough to be observed (14) (17). Detecting technicolor at the LHC is likely to be quite challenging, however. Recent estimates (18) (17) of the luminosity required to detect a technicolor at the LHC indicate that it would be necessary to accumulate of order 100 fb$^{-1}$, and that this would result in a signal of only a few tens of events (over a background of comparable size!).

**Inelastic Channels in WW-Scattering**

Up to now, we have assumed that the only “light” particles in the electroweak symmetry breaking sector are the longitudinal components of the $W$ and $Z$. In a theory of this sort, the behaviors described above are generic: the growth in the $W_LW_L$ scattering amplitude may be cut-off by light, narrow resonances (such as in the weakly-coupled standard model) or by heavy, broad resonances (such as would be expected in the simplest technicolor...
model). However, if the global symmetry structure of the theory is larger than $SU(2) \times SU(2)$, there may be additional (pseudo-)Goldstone bosons. These additional particles give rise to inelastic channels for vector-boson scattering, and may have dramatic consequences for the behavior of the theory.

Consider a technicolor model with a global $SU(N_f)_L \times SU(N_f)_R$ chiral symmetry which breaks spontaneously to the vectorial $SU(N_f)$ subgroup, breaking the weak interactions and producing $N_f^2 - 1$ Goldstone (or pseudo-Goldstone) bosons. The low energy theorem for the $SU(N_f)$ singlet, spin singlet scattering amplitude of these bosons is

$$a_{\text{singlet}} = \frac{N_f N_d s}{32 \pi v^2},$$

(17)

where $N_d$ is the number of technifermion doublets. In analogy to the analysis of the $W_L^+W_L^-$ scattering amplitude given at the beginning of this talk, we see that as $N_f$ and $N_d$ increase, i.e. as the number of inelastic channels in $W_LW_L$ scattering grow, the scale by which the dynamics of EWSB must appear decreases (20) (21). For example, in the one family technicolor model (22) $N_f = 8$ and $N_d = 4$. In this model $a_{\text{singlet}}$ would exceed unitarity at 440 GeV, and we expect that new physics must appear at the energy scale or lower.

Mitch Golden and I studied the phenomenology of a model of electroweak-symmetry breaking with many inelastic channels in a toy-model based on a scalar $O(N)$ theory (21). We showed that, although the new physics occurs at relatively low energies, this new low-energy physics can be hard to detect. The presence of the large numbers of inelastic channels can result in elastic $W$ and $Z$ scattering amplitudes that are small and structureless at all energies, i.e. lacking any discernible resonances (see Fig. 2). Nonetheless, the theory can be strongly interacting and the total $W$ and $Z$ cross sections large: most of the cross section is for the production of particles other than the $W$ or $Z$. In such a model, discovering the electroweak symmetry breaking sector will depend on the observation of the other particles and our ability to associate them with symmetry breaking. This implies that we must keep an open mind about the experimental signatures of the electroweak symmetry breaking sector and that we cannot rely solely on two gauge boson final states.

WHAT DOES THIS IMPLY FOR ANOMALOUS WEAK-BOSON SELF-INTERACTIONS?

It would seem natural that in a theory of a strongly interacting electroweak symmetry breaking sector, like technicolor, there could be large corrections to the electroweak gauge-boson self-couplings. For example, one would expect that the coupling of one longitudinal gauge-boson to two transverse gauge-bosons would acquire a form-factor similar to the electromagnetic form-factor of the pion in QCD.
As discussed by Wudka at this conference \cite{Wudka}, one can use dimensional analysis to estimate the size of the corrections to the weak-boson self-couplings \cite{24}:

\begin{equation}
\Delta g_1, \Delta \kappa = \mathcal{O} \left( \frac{g^2}{16\pi^2} \right),
\end{equation}

and

\begin{equation}
\lambda = \mathcal{O} \left( \frac{g^4}{(16\pi^2)^2} \right).
\end{equation}

Using these estimates, we see that deviations in $\kappa$ and $g_1$ are expected to be of order $10^{-3}$, while $\lambda$ is expected to be of order $10^{-5}$ or $10^{-6}$.

I would like to emphasize here that these dimensional estimates have a simple physical interpretation in terms of the form-factor picture that I discussed above (see also the discussion of Willenbrock at this conference \cite{25}). As in QCD, we expect that the scale of variation of the form factor is given by the mass of the lowest-lying resonance in the appropriate channel, namely by the mass of a vector meson. Furthermore, in the limit that $M_W, M_Z \to 0$, we know that the vector bosons must couple to a conserved current and that the vector-boson self couplings must be of canonical form \cite{26}. Therefore, we can estimate that the size of the anomalous couplings $\kappa$ and $g$ must be

\begin{equation}
\Delta g_1, \Delta \kappa = \mathcal{O} \left( \frac{M_W^2}{M_Z^2} \right),
\end{equation}

and, given that $\lambda$ is the coefficient of a dimension-6 operator and the normalization chosen in \cite{24},

\begin{equation}
\lambda = \mathcal{O} \left( \frac{M_W^4}{M_Z^4} \right).
\end{equation}

Afficianados of dimensional analysis \cite{27} will see immediately that these two estimates are, in fact, consistent since the dimensional analysis estimate of the lightest-resonance mass in models of electroweak symmetry breaking are of order $4\pi v$.

What are the prospects for the experimental detection of deviations of this size? Baur, Han, and Ohnemus \cite{28} have recently considered this issue for a variety of colliders. The prospects are discouraging. For example, for the
LHC with an integrated luminosity of 100 fb$^{-1}$, they find that one may be able to probe to the level of $10^{-2}$ for $\Delta g$ and $\lambda$. This is not sufficient to be sure to probe effects of the size predicted above.

On the other hand, one might wonder if the analysis of the effects of anomalous weak-boson self-interactions is consistent with the results given in the previous section. From the analysis given above, we see that a $\Delta g$ to order $10^{-2}$ would arise in a model with a technirho of mass approximately 1 TeV. This is consistent with the analysis of (18): in both cases one is looking for the effects of technirho mesons on the production of $WZ$ pairs!

**CONCLUSIONS**

A strongly interacting symmetry breaking sector will result in one or more resonances which are either:

- Heavy (with masses of order a TeV) and broad (in the case that elastic $W$ and $Z$ scattering dominates). Detection will require an integrated luminosity of order 100 fb$^{-1}$ or more at the LHC.

- Light and broad (in the case that inelastic channels are important). In this case detection will hinge on observing particles other than the $W_L$ and $Z_L$ and identifying them as being associated with EWSB.

In the first case, the lightest resonances in the electroweak symmetry breaking sector are expected to be the technivector mesons, the analogs of the $\rho$ and $\omega$ in QCD. The masses of these resonances are expected to be of order a TeV, and one expects an enhancement of $WZ$ and/or $WW$ production at energies of this order of magnitude. One may think of the “tail” of the technirho as giving rise to anomalous weak-boson self-interactions. The expect size of the resulting anomalous gauge boson vertices is small, with $\Delta \kappa$ and $\Delta g$ of order $10^{-3}$ and $\lambda$ of order $10^{-5}$ or $10^{-6}$.

**ACKNOWLEDGEMENTS**

I gratefully acknowledge the support of NSF Presidential Young Investigator Award and a DOE Outstanding Junior Investigator Award.

*This work was supported in part by the National Science Foundation under grant PHY-9057173 and by the Department of Energy under contract DE-FG02-91ER40676.*
REFERENCES

1. B. Lee, C. Quigg, and H. Thacker, Phys. Rev. Lett. 38, 883 (1977).
2. M. Veltman, Acta. Phys. Polon. B8, 475 (1977).
3. M. Chanowitz and M.K. Gaillard, Nucl. Phys. B261, 379 (1985).
4. M. Chanowitz, M. Golden and H. Georgi, Phys. Rev. Lett. 57, 2344 (1987) and Phys. Rev. D36, 1490 (1987).
5. J. Cornwall, D. Levin and G. Tiktiopoulos, Phys. Rev. D10, 1145 (1974); C. Vayonakis, Lett. Nuovo Cimento 17, 383 (1976).
6. M. Weinstein, Phys. Rev 8, 2511 (1973).
7. G. ‘t Hooft, in Recent Developments in Gauge Theories, G. ’t Hooft, et. al., eds., Plenum Press, New York NY 1980.
8. K. G. Wilson, Phys. Rev. B4, 3184 (1971); K. G. Wilson and J. Kogut, Phys. Rep. 12, 76 (1974).
9. R. Dashen and H. Neuberger, Phys. Rev. Lett. 50, 1897 (1983).
10. L. Maiani, G. Parisi and R. Petronzio, Nucl. Phys. B136, 115 (1978).
11. M. Lüscher and P. Weisz, Nucl. Phys. B318, 705 (1989); J. Kuti, L. Lin and Y. Shen, Phys. Rev. Lett. 61, 678 (1988); A. Hasenfratz et. al., Phys. Lett. B199, 531 (1987); A. Hasenfratz et. al., Nucl. Phys. B317, 81 (1989); G. Bhanot et. al., Nucl. Phys. B353, 551 (1991) and 375, 503 (1992).
12. U. M. Heller, H. Neuberger, and P. Vranas, Nucl. Phys. B399, 271 (1993); K. Jansen, J. Kuti, and C. Liu, Phys. Lett. B309, 119 (1993).
13. S. Weinberg, Phys. Rev. D19, 1277 (1979); L. Susskind, Phys. Rev. D20, 2619 (1979); E. Farhi and L. Susskind, Phys. Rep. 74, 277 (1981).
14. J. F. Donoghue, C. Ramirez, and G. Valencia, Phys. Rev D38, 2195 (1988).
15. E. Eichten, I. Hinchliffe, K. Lane, and C. Quigg, Rev. Mod. Phys. 56, 579 (1984).
16. M. Berger and M. Chanowitz Phys. Lett. B263, 509 (1991).
17. J. Bagger, et. al., Phys. Rev. D49, 1246 (1994).
18. M. Chanowitz and W. Kilgore, Phys. Lett. B 322, 147 (1994) and 347, 387 (1995).
19. R. Cahn and M. Suzuki, Phys. Rev. Lett. 67, 169 (1991).
20. M. Soldate and R. Sundrum, Nucl. Phys. B340, 1 (1990).
21. R. S. Chivukula and M. Golden, Phys. Lett. B267, 233 (1991).
22. E. Farhi and L. Susskind, Phys. Rev. D20, 3404 (1979).
23. J. Wudka, these proceedings.
24. K. Hagiwara et. al., Nucl. Phys. B282, 253 (1987).
25. S. Willenbrock, these proceedings.
26. S. Weinberg and E. Witten, Phys. Lett. B96, 59 (1980).
27. S. Weinberg, Physica 96 A, 327 (1979); A. Manohar and H. Georgi, Nucl. Phys. B234, 189 (1984); H. Georgi, Weak Interactions and Modern Particle Theory, The Benjamin/Cummings Publishing Company, Inc., Menlo Park, CA, 1984.
28. U. Baur, T. Han, and J. Ohnemus, Phys. Rev. D51, 3381 (1995).
FIG. 1. Data [14] and low-energy theorem prediction for the spin-1/isospin-1 pion scattering amplitude.
FIG. 2. The absolute value of the (weak) isospin-0 $W_L W_L$ scattering amplitude in a toy $O(N)$ model of electroweak symmetry breaking \cite{21}. The model contains 32 pseudo-Goldstone bosons, and the different solid curves show the change in the amplitude as the mass of the pseudo goldstone bosons is adjusted. The right-most nearly structureless amplitude corresponds to the case where the “Higgs” in this model is strongly coupled, but can decay to pairs of pseudos in addition to pairs of weak gauge bosons. The dashed-line corresponds to the same scattering amplitude in the standard model with a 500 GeV Higgs boson.