Coordination in a two-level supply chain with option

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Abstract. Due to poor performance of supply chain members the paper has been studied supply chain contracts based on newsboy model without option and centralized decision-making model. This research paper presents how to coordinate the supply chain in two-level of supply chain. All the numerical part analysis of a maximization of both members expected profit using Matlab software are carried out so as to present normal distribution and cumulative density function and summarized the paper with conclusion part. A total profit of supply chain is illustrated as well.

Introduction

In the current electronic industry use of a number of flexible contracts and cooperation of retailers in the selling season are necessarily vital. First of all, a manufacturer takes an order for quantity of product according to the order provides the demanded quantity of product to the retailer which is real market. After the international demand, the retailer can adjust the order of quantity according to the actual market demand like Microsystems Sun, Packard Hewlett and IBM, Solectron and other companies use the above-described contract. In this case, a number of researches with corresponding scholars have carried out in order to propose effective methods for cooperation and contract structure. For instance: Bergen Iyer [1] proposed a fast response Fisher [2] and so on. Tsay[3] studied a quantity flexibility contract order. Many domestic scholars have also carried out with the corresponding research. Barnes-Schuster, Bassok and Anupindi[4] first introduced the option in the supply chain management research. In their research option on contract for the two level of the supply chain coordination processes have been studied, and the price nonlinearity in time for realization of coordination of supply chain channels has been indicated.

Model description and basic parameters

Considering the supply chain, manufacturer and retailer of the two - level supply chain, which is composed of a single manufacturer and a single retailer, has a long lead time delivery, higher product costs, shorter sales season and prices fell faster with a random market demand for electronic products. In the marketing products according to the LF (Leader-Follower) game theory to investigate manufacturer the interaction between the retailers and the manufacturer is the leader, the retailer is the follower, the manufacturer is given a set of contract parameters number, and the retailer determines the optimal order quantity. In the process of cooperation, the supplier first released the wholesale of products. The price of the option the implementation of the price, the manufacturer's price policy and the market demand is indeed expected to set the fixed order quantity and the option purchase quantity. In the next stage of obtaining the information which implies the market demand, the retailer performs option and the manufacturer must ensure that the supply of the goods ordered by the retailer has a certain surplus after the selling season.

The density function of market demand \( d \) is \( f(x) \), the distribution function is \( F(x) \), \( F(x) \) is differential with conditions of \( F(0) = 0 \), \( \bar{F}(x) = 1 - F(x) \), \( \mu = E(x) = \int_{0}^{\infty} xf(x)dx \). When order
quantity $q$ and actual demand are conditioned like $x < q$, $s(q) = x$, $x ≥ q$, $s(q) = q$, $s(q) = \min(q, x)$ expected sales becomes

$$s(q) = q(1 - F(q)) + \int_0^q xf(x)dx = q - \int_0^q F(x)dx \quad (1)$$

Where:

Inventory cost shortage cost as given:

$$I(q) = (q - d)^+ = q - s(q)$$
$$L(q) = (q - d)^+ = \mu - s(q)$$

Table 1. Basic parameters of model

| p    | retail price of the product | $v_m$ | salvaging at manufacturer |
|------|----------------------------|-------|---------------------------|
| t    | transportation cost from manufacturer to retailer | $v_r$ | salvaging at retailer |
| w    | wholesale price of the unit product to the retailer by the manufacturer | $l$ | loss of units for manufacturer and retailers |
| c    | manufacturer's production cost | $T$ | retailer's total payment to manufacturer |
| $c_o$ | option price of unit product | $F(x)$ | distribution function of market demand $d$ |
| $c_e$ | exercise price of the option | $f(x)$ | density function of market demand $d$ |
| N    | retailer's option to buy | $\mu$ | mean value |

In order to facilitate the study of the problem, the relationship between the parameters and the market environment is assumed as in the following:

$$v_r \leq c - t \leq w \leq p$$
$$c_o + c_e \geq w$$
$$p + l \geq c_o + c_e$$

**Basic Newsboy model**

First consideration of retailers and the manufacturer in the newsvendor model of the optimal decision and the overall profits of the supply chain has been made to reference a continuation for comparison. In the classical newsvendor model, the wholesale price per unit of product is $w$, the retailer with only one order chance, retailers according to the principle of maximizing profits to determine their product order $q$, all the products in the sales season began before delivery to the retailer, the remaining product sales season after having certain value, retailers did not meet the market needs demand will lead to a certain loss of stock. Taking the above-brought assumptions into consideration, the retailer's expected profit can be expressed as:

$$\pi_r^N = l_n(q) + v_rI(q) - IL(q) - T = (p + l - v_r)q - (p + l - v_r)\int_0^q F(x)dx - v_rq - \mu l - T \quad (2)$$

Supplier's expected profit can be expressed as:

$$\pi_m^N = T - (c - t)q - IL(q) = T + (l - c - t)q - l\int_0^q F(x)dx - p\mu \quad (3)$$

The overall supply chain can be expressed as:

$$\pi^N = \pi_m^N + \pi_r^N \quad (4)$$
From the above equations optimal decision of newboy expressed as:

\[
\frac{\partial \pi^N}{\partial q} = p + l - w - (r + l - v_r)F(q^*_N), \quad \frac{\partial \pi^N}{\partial q} = 0
\]

The optimal order quantity of the product can be expressed with implicit condition as in the following:

\[
F(q^*_N) = \frac{p + l - w}{p + l - v_r}
\]

Then the optimal model of newsboy quantity becomes:

\[ q^*_N = F^{-1} \left( \frac{p + l - w}{p + l - v_r} \right) \] (4)

Centralized decision-making model

In order to facilitate the analysis and comparison based on the newsboy model, further study under the centralized decision-making for option mechanism optimal decision of the chain is necessary. In the centralized decision - oriented supply chain, the decision maker is based on the principle of maximizing the profit of supply chain. Therefore, the supply chain profit can be expressed as:

\[
\pi_i = r_m(q + N) - pL(q + N) + v_m[I(q + N) - I(q)] - (c - t)(q + N)
\] (5)

According to Leibniz rules we can get:

\[
\frac{\partial \pi_i}{\partial N} = (p + l - c - t) - (p + l - v_m)F(q_i), \quad \frac{\partial \pi_i}{\partial N} = 0
\]

Optimal order quantity becomes:

\[ q^*_i = F^{-1} \left( \frac{p + l - c - t}{p + l - v_m} \right) \] (6)

The derivation of it:

\[
\frac{\partial \pi_i}{\partial q} = (p + l - c - t) - (p + l - v_r)F(q_i), \quad \frac{\partial \pi_i}{\partial q} = 0
\]

Order quantity of the supply chain of the decision - oriented supply chain when salvaging value is equal:

\[ q^*_i = F^{-1} \left( \frac{p + l - c - t}{p + l - v_r} \right) \] (7)

Purchase quantity expressed as:

\[ q^*_i = F^{-1} \left( \frac{p + l - c - t}{p + l - v} \right) \] (8)

In the retailer’s optimal order quantity for the newsboy model:

\[ q^*_N = F^{-1} \left( \frac{p + l - w}{p + l - v_r} \right) \] (9)

Numerical Analyses

Study on supply chain coordination requires a further numerical analysis when the salvaging is equal for both members. Assuming that the probability density function (pdf) of market demand \( f(x) \) for a manufacturer \( D \) obey normal distribution with \( \mu = 40 \), \( \sigma = 20 \). By taking other influencing parameters like: \( v_m = v_r = 8 \), \( w = 10 \), \( p = 2 \) and the value range of \( c_v = 15~26 \), the normal probability density function (pdf) and cumulative density function (cdf) are illustrated in Fig.(1) and Fig.(2). Furthermore, the economic order quantity (EOQ) of three models are calculated, using Matlab and plotted as Fig. (3).
Fig. 1. pdf changes depending on $\sigma$

Fig. 2. cdf changes depending on $\mu$
Summary

Throughout the paper we enhanced basic newsboy model without option contract and centralized decision-making model in order to present how to coordinate the supply chain in two levels of supply chain, and then we maximized the expected profit of both members the manufacturer and the retailer and in the numerical part we used Matlab software in order to show normal distribution function and cumulative density function and also illustrated total profit of supply chain. Matlab special functions `normpdf` and `normcdf` are used to build the graph in Fig.1 and Fig.2 to provide a better understanding of the enhancements in afore-described model. At last but at least, we showed the EOQ three models Fig.3 (newsboy model, centralized decision – making model, order quantity based on option) when opting pricing ranged from zero to two.

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