Broadband sum frequency generation via chirped quasi-phase-matching

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An efficient broadband sum frequency generation (SFG) technique using the two collinear optical parametric processes \( \omega_3 = \omega_1 + \omega_2 \) and \( \omega_4 = \omega_1 + \omega_3 \) is proposed. The technique uses chirped quasi-phase-matched gratings, which, in the undepleted pump approximation, make SFG analogous to adiabatic population transfer in three-state systems with crossing energies in quantum physics. If the local modulation period first makes the phase match occur for \( \omega_3 \) and then for \( \omega_4 \) SFG processes then the energy is converted adiabatically to the \( \omega_3 \) field. Efficient SFG of the \( \omega_3 \) field is also possible by the opposite direction of the local modulation sweep; then transient SFG of the \( \omega_3 \) field is strongly reduced. Most of these features remain valid in the nonlinear regime of depleted pump. © 2011 Optical Society of America

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Introduction. Recent advances in quasi-phase-matching (QPM) techniques [1, 2] have drawn analogies between optical parametric processes and two- and three-state quantum systems [3–5]. By using an analogy to stimulated Raman adiabatic passage (STIRAP) in atomic physics [6–11] Longhi proposed [3] a scheme in which the fundamental frequency field is directly converted into the third harmonic without a transient generation (simultaneous phase matching of second harmonic generation). Suchowski et al. [4, 5] used an aperiodically poled QPM crystal to achieve both high efficiency and large bandwidth in sum frequency generation (SFG) in the undepleted pump approximation using ideas from rapid adiabatic passage in quantum physics [8, 12].

In this Letter, we make use of the analogy between coherent population transfer in three-state quantum systems and the two simultaneous collinear second-order parametric processes \( \omega_3 = \omega_1 + \omega_2 \) and \( \omega_4 = \omega_1 + \omega_3 \), for a QPM crystal with susceptibility \( \chi^{(2)} \) and local modulation period \( \Lambda(z) \) are described by the set of nonlinear differential equations [1, 2]

\[
\begin{align*}
\partial_z E_1 &= \Omega_1 (E_1 E_3 e^{-i\Delta_1 z} + E_3^* E_4 e^{-i\Delta_2 z}), \\
\partial_z E_2 &= \Omega_2 E_1^* E_3 e^{-i\Delta_1 z}, \\
\partial_z E_3 &= \Omega_3 (E_1 E_2 e^{i\Delta_1 z} + E_2^* E_4 e^{-i\Delta_2 z}), \\
\partial_z E_4 &= \Omega_4 E_1 E_3 e^{i\Delta_2 z},
\end{align*}
\]

where \( z \) is the position along the propagation axis, \( c \) is the speed of light in vacuum, and \( E_1, E_2, E_3 \) and \( n_j \) are the electric field, the frequency and the refractive index of the \( j \)-th laser beam, respectively. Here \( \Omega_j = \chi^{(2)} E_j / 4 \pi n_j \) are the coupling coefficients, while \( \Delta_1 = n_1 / (\omega_1) + n_2 / (\omega_2) - n_3 / (\omega_3) + 2\pi / \Lambda \) and \( \Delta_2 = n_1 / (\omega_1) + n_3 / (\omega_3) - n_4 / (\omega_4) + 2\pi / \Lambda \) are the phase mismatches for the \( \omega_3 \) and \( \omega_4 \) SFG processes.

Undepleted pump approximation. The coupled nonlinear equations (1) are often linearized assuming that the incident pump field \( E_1 \) is much stronger than the other fields and therefore its amplitude is nearly constant (undepleted) during the evolution. Then Eqs. (1) are reduced to a system of three linear equations,

\[
\begin{pmatrix}
\partial_z A(z)
\end{pmatrix} = M(z) A(z), \quad M = \begin{pmatrix}
-\Delta_1 & \Omega_p^* & 0 \\
\Omega_p & 0 & \Omega_4
\end{pmatrix}
\]

with \( \Omega_p = E_1 \sqrt{\Omega_2 \Omega_3}, \quad \Omega_z = E_1 \sqrt{\Omega_2 \Omega_4}, \quad A(z) = [A_2(z), A_3(z), A_4(z)]^T, \quad A_2(z) = E_1 E_2 e^{i\Delta_1 z} \sqrt{\Omega_2 / \Omega_4}, \quad A_3(z) = E_1 E_3 e^{i\Delta_2 z} \sqrt{\Omega_2 / \Omega_3}, \quad A_4(z) = E_1 E_4 e^{-i\Delta_2 z} \sqrt{\Omega_3 / \Omega_2} \). Upon the substitution \( z \to t \), Eq. (2) becomes identical to the time-dependent Schrödinger equation for a three-state quantum system in the rotating-wave approximation, which is studied in great detail in Ref. 8; the vector \( A(z) \) and the driving matrix \( M \) correspond to the quantum state vector and the Hamiltonian, respectively. We note that the quantity \( |A(z)|^2 = |A_2(z)|^2 + |A_3(z)|^2 + |A_4(z)|^2 \) is conserved, like the total population in a coherently driven quantum system. By definition, in the adiabatic regime the system stays in an eigenvector of the “Hamiltonian” \( M \). We assume that \( \Delta_1(z) \) and \( \Delta_2(z) \) change linearly along \( z \), which can be achieved, for example, by varying \( \Lambda(z) \). Explicitly, we assume that either \( \Delta_1 = \delta - \alpha^2 z \) and \( \Delta_2 = \delta + \alpha^2 z \), which is called “intuitive sweep” (for reasons that will become clear shortly) or \( \Delta_1 = \delta - \alpha^2 z \) and \( \Delta_2 = \delta + \alpha^2 z \) which is called “counterintuitive sweep”. For the sake of generality, we take hereafter \( \alpha \) as the unit of coupling and \( 1/\alpha \) as the unit of length. Then the three eigenvalues of \( M \) will cross each other at three different distances \( z_m (m = 1, 2, 3) \), thereby creating a triangle crossing pattern [16–18]. These crossings
allow us to design recipes for efficient broadband SFG, in analogy to adiabatic passage techniques in quantum physics [8, 9, 12, 16–18]. Because of the analogy to the Schrödinger equation the condition for adiabatic evolution can be derived using the Landau-Zener-Majorana model [19–21] and reads (for linear chirping and constant couplings): $|\Omega_z| \geq \alpha$, where $\Omega_z$ is the relevant coupling at the respective crossing.

Figure 1 plots the eigenvalues of $\mathbf{M}$ of Eq. (2) vs. $z$. Initially only the $\omega_3$ field is present, hence the vector $\mathbf{A} = [A_2, 0, 0]$. If the evolution is adiabatic then there are two possible paths that the system can follow (marked by arrows). If the phase match for the $\omega_3$ generation process occurs first (left frames of Fig. 1), then the energy is converted first to the $\omega_3$ field and then to the $\omega_4$ field. This “intuitive” two-step scheme extends the single-step adiabatic passage scenario for SFG [4, 5]. Interestingly, we find that efficient energy transfer directly to the $\omega_4$ field is also possible through the “counterintuitive” direction of the local modulation period sweep when the phase match for the $\omega_4$ generation process occurs first (right frames of Fig. 1). Then the energy flows from the $\omega_2$ field to the $\omega_4$ field with almost no energy transferred to the intermediate $\omega_3$ field.

Figure 1. (Color online) Sequential SFG of $\omega_4$ field. Top frames: Diagonal elements (solid lines) and eigenvalues (dashed lines) of the driving matrix $\mathbf{M}$ of Eq. (2) for the “intuitive” (left frames) and “counterintuitive” (right frames) phase mismatch sweep. The field intensities are calculated numerically from Eqs. (1) for $\delta = 2\alpha$, $\Omega_1 = \Omega_2 = \Omega_3 = \Omega_4 = \alpha$. Middle frames: undepleted pump, $|E_1(z_i)|^2 = 100|E_2(z_i)|^2$, with $z_i = -20\alpha^{-1}$; bottom frames: depleted pump, $|E_1(z_i)|^2 = 2|E_2(z_i)|^2$.

Depleted pump. We have found by numerical integration of the nonlinear system (1) that the described scheme is also applicable beyond the undepleted pump approximation, when the $\omega_1$ and $\omega_2$ fields have comparable energies; this is demonstrated in the bottom frames of Fig. 1. Unfortunately, many optical parametric processes such as $\omega_1 - \omega_2$, $2\omega_1$, $2\omega_2$, $\omega_1 + 2\omega_2$ become possible in this case and it is not easy to find the conditions for broadband SFG of the $\omega_1$ field.

The contour plot in Fig. 2 demonstrates the robustness of SFG of the $\omega_1$ field against parameter variations. SFG for an undepleted pump (left frame) is remarkably robust in confirmation of the simple analytic theory described above. SFG for a depleted pump (right) is less robust although relatively high SFG efficiency is still possible; because then the simple eigenvalue arguments cannot be used the interpretation is more difficult.

Third harmonic generation. Third harmonic generation is an important special case of SFG, which is readily treated in the adiabatic regime. The respective equations are derived from Eqs. (1),

\[
\begin{align*}
\imath \partial_z A_\omega &= \Omega_\omega A_\omega^* A_2 \omega e^{-\imath \Delta_1 z} + \Omega_2 \omega A_2^\omega A_3 \omega e^{-\imath \Delta_2 z}, \\
\imath \partial_z A_{2\omega} &= \Omega_\omega A_\omega^* A_2 \omega e^{\imath \Delta_1 z} + \Omega_2 \omega A_2^\omega A_3 \omega e^{\imath \Delta_2 z}, \\
\imath \partial_z A_{3\omega} &= \Omega_2 \omega A_2 \omega e^{\imath \Delta_2 z},
\end{align*}
\]

where $\Omega_\omega = \chi(2)\omega/4c\epsilon_0 n_\omega$, $E_1 = E_2$, $A_\omega = E_1 \sqrt{2n_\omega/n_{2\omega}}$, $A_{2\omega} = E_2$, $A_{3\omega} = E_3$, $\omega_1 \omega = \sqrt{2} \omega_\omega n_2 \omega/n_{3\omega}$.

Figure 3 shows numerical simulation of third harmonic generation. There are again two possible scenarios. If the phase match for the second harmonic generation process occurs first (“intuitive” sweep, left frame of Fig. 3), then the efficiency of the third harmonic is good, but we have some unwanted second harmonic left. For the “counterintuitive” direction of the local modulation period sweep, when the phase match for the third harmonic generation occurs first, the second harmonic is strongly suppressed and a nearly complete transfer of energy to the third
Eqs. (1) for \(\delta = 4\alpha, \Omega_\omega = 2\alpha, \Omega_{2\omega} = 5\alpha\).
References
1. S. M. Saltiel, A. A. Sukhorukov, and Y. S. Kivshar, “Multistep parametric processes in nonlinear optics”, Prog. Opt. 47, 1-73 (2005).
2. A. Arie and N. Voloch, “Periodic, quasi-periodic, and random quadratic nonlinear photonic crystals”, Laser and Photon. Rev. 4, 355-373 (2010).
3. S. Longhi, “Third-harmonic generation in quasi-phase-matched $\chi^{(2)}$ media with missing second harmonic”, Opt. Lett. 32, 1791-1793 (2007).
4. H. Suchowski, D. Oron, A. Arie, Y. Silberberg, “Geometrical representation of sum frequency generation and adiabatic frequency conversion”, Phys. Rev. A. 78, 063821 (2008).
5. H. Suchowski, V. Prabhudesai, D. Oron, A. Arie, Y. Silberberg, “Robust adiabatic sum frequency conversion”, Opt. Express 17, 12731-12740 (2009).
6. U. Gaubatz, P. Rudecki, S. Schiemann, K. Bergmann, “Population transfer between molecular vibrational levels by stimulated Raman scattering with partially overlapping laser fields. A new concept and experimental results”, J. Chem. Phys. 92, 5363-5377 (1990).
7. K. Bergmann, H. Theuer, B. W. Shore, “Coherent population transfer among quantum states of atoms and molecules”, Rev. Mod. Phys. 70, 1003-1025 (1998).
8. N. V. Vitanov, T. Halfmann, B. W. Shore, and K. Bergmann, “Laser-induced population transfer by adiabatic passage techniques”, Annu. Rev. Phys. Chem. 52, 763-809 (2001).
9. N. V. Vitanov, M. Fleischhauer, B. W. Shore, and K. Bergmann, “Coherent manipulation of atoms and molecules by sequential laser pulses”, Adv. At. Mol. Opt. Phys. 46, 55-190 (2001).
10. M. Mackie, R. Kowalski, and J. Javanainen, “Bose-stimulated Raman adiabatic passage in photoassociation”, Phys. Rev. Lett. 84, 3803-3806 (2000).
11. H. Pu, P. Maenner, W. Zhang, and H. Y. Ling, “Adiabatic condition for nonlinear systems”, Phys. Rev. Lett. 98, 050406 (2007).
12. L. Allen, J. H. Eberly, Optical Resonance and Two-Level Atoms (Dover, New York, 1987).
13. M. Charbonneau-Lefort, B. Afeyan, and M. M. Fejer, “Optical parametric amplifiers using chirped quasi-phase-matching gratings I: practical design formulas”, J. Opt. Soc. Am. B 25, 463-480 (2008).
14. M. Charbonneau-Lefort, M. M. Fejer, and B. Afeyan, “Tandem chirped quasi-phase-matching grating optical parametric amplifier design for simultaneous group delay and gain control”, Opt. Lett. 30, 634-636 (2005).
15. M. A. Arbore, O. Marco, and M. M. Fejer, “Pulse compression during second-harmonic generation in aperiodic quasi-phase-matching gratings”, Opt. Lett. 22, 865-867 (1997).
16. B. Broers, L. D. Noordam, and H. B. van Linden van den Heuvell, “Diffraction and focusing of spectral energy in multiphoton processes”, Phys. Rev. A 46, 2749-2756 (1992).
17. R. G. Unanyan, N.V. Vitanov, and K. Bergmann, “Preparation of entangled states by adiabatic passage”, Phys. Rev. Lett. 87, 137902 (2001).
18. S. S. Ivanov and N. V. Vitanov, “Steering quantum transitions between three crossing energy levels”, Phys. Rev. A 77, 023406 (2008).
19. L. D. Landau, “On the theory of transfer of energy at collisions II”, Physik Z. Sowjetunion 2, 46-52 (1932).
20. C. Zener, “Non-adiabatic crossing of energy levels”, Proc. R. Soc. Lond. Ser. A 137, 696-702 (1932).
21. E. Majorana, “Atomi orientati in campo magnetico variabile”, Nuovo Cimento 9, 43-50 (1932).