Demonstration Informed Specification Search

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Abstract. This paper considers the problem of learning temporal task specifications, e.g., automata and temporal logic, from expert demonstrations. Three features make learning temporal task specifications difficult: (1) the (countably) infinite number of tasks under consideration, (2) an a-priori ignorance of what memory is needed to encode the task, and (3) the lack of gradients to guide the search for explanatory tasks. To overcome these hurdles, we propose Demonstration Informed Specification Search (DISS): a family of algorithms requiring only black box access to (i) a maximum entropy planner and (ii) a task sampler from labeled examples. DISS works by alternating between (i) conjecturing labeled examples to make the demonstrations more likely and (ii) sampling tasks consistent with conjectured labeled examples. In the context of tasks described by Deterministic Finite Automata, we provide a concrete implementation of DISS that efficiently identifies a task from only one or two expert demonstrations.

Keywords: Learning from Demonstrations · Specification Mining

1 Introduction

Expert demonstrations provide an accessible and expressive means to informally specify a task, particularly in the context of human-robot interaction [1,13,22]. In this work, we study the problem of inferring, from demonstrations, tasks represented by formal task specifications, e.g., automata and temporal logic. The study of task specifications is motivated by their ability to (i) encode historical dependencies, (ii) incrementally refine the task via composition, and (iii) be semantically robust to changes in the workspace. We ground and motivate this problem with an example.

1.1 Motivating Example

Consider an agent operating in the 8x8 grid world as shown in Fig [La]. The agent can attempt to move up, down, left, or right. With probability $\frac{1}{32}$, wind will push the agent down, regardless of the agent’s action. The black path is the prefix of an episode, in which the agent attempts to move left, slips into the blue (water) tile (■), visits a brown (drying) tile (■), and then proceeds downward.

Given the black demonstration, call $\xi_b$, and the prior knowledge that the agent’s task implies that it will (i) avoid red (lava) tiles (■) and (ii) try to reach a yellow (recharge) tile (■), what task, as a Deterministic Finite Automaton (DFA), explains the agent’s behavior?
Upon inspecting the demonstration, one might be surprised that the agent goes out of its way to visit ■. For example, why would the agent not take the red dashed path directly to ■? One might conjecture that the agent’s true task requires visiting ■ after visiting ■. Similarly, one may notice that the demonstration is “more rational” or “less surprising” if the dotted extension of ξ, ending in ■ is a positive example of the task. Finally, appealing to Occam’s razor, one might search for a simple DFA consistent with the conjectured labels. In this case, the demonstrated task is in fact one of the “simplest” DFAs, shown on the right in Fig 1b. We shall later systematize this line of reasoning and provide a learner that recovers an explanatory DFA similar to ground truth.

1.2 Contributions This work introduces a family of approximate algorithms called Demonstration Informed Specification Search (DISS) that efficiently searches for task specifications that explain a set of expert demonstrations. DISS leverages the following contributions:

1. A proxy function whose gradient (i) informs the search for an explanatory task specification and (ii) is computed with black-box access to a maximum entropy planner.
2. A reduction from learning task specifications from demonstrations to learning from labeled examples.

The resulting algorithm is agnostic to the underlying concept class and dynamics. Finally, we provide a concrete implementation of DISS. An example identification algorithm for DFAs and a maximum entropy planner for simple gridworlds are also included. Using this implementation, we perform two experiments validating that DISS indeed enables efficiently searching for tasks that explain the demonstrations even in large/unstructured concept classes like DFAs given unlabeled and potentially incomplete demonstrations.
Remark 1. The choice of DFAs as the concept class for our experiments was motivated by three observations. First, DFAs explicitly encode memory, making the contribution of identifying relevant memory more clear. Next, to our knowledge, all other techniques for learning finite path properties from demonstrations focus on syntax defined concept classes. Thus, learning DFAs is understudied in this context. Third, DFAs constitute a very large and mostly unstructured concept class enabling studying the efficiency of DISS without introducing too many inductive biases.

1.3 Algorithm Overview DISS assumes (i) access to a multi-set of expert demonstrations: $\xi_1^*, \ldots, \xi_m^*$, (ii) black box access to an identification algorithm, $I$, that maps positively/negatively labeled paths to a distribution over concepts and (iii) black box access to a planner that estimates the probability of a path given a candidate task (see Sec 3). DISS operates by cycling between three components (shown in Fig 2):

1. **Candidate Sampler**: A candidate task, $\varphi$, is sampled from $I(X)$, where $X$ is a collection of labeled examples (initialized to the empty set).
2. **Surprise Guided Sampler**: The planner is used to find a path, where toggling whether the path is a positive/negative example of the task makes the expert demonstrations less surprising.
3. **Example Buffer**: Given previously seen data, the example buffer yields a set of positive and negative example paths.

1.4 Reading Guide The rest of the paper is structured as follows. We begin by highlighting related work in Sec 1.5 and formalizing our learning from demonstrations (LFD) problem in Sec 2. Next in Sec 3 we motivate employing an entropy regularized agent model in our LFD problem. Groundwork laid, we discuss how to find surprising paths given our agent model (Sec 4). This then leads us to propose a variant of simulated annealing (implemented through the example buffer) to approximately solve our LFD problem (Sec 5). Finally, we conclude with an empirical study of the algorithm (Sec 6).
1.5 Related Work  The problem of learning objectives by observing an expert has a rich and well developed literature dating back to early work on Inverse Optimal Control [10] and more recently via Inverse Reinforcement Learning (IRL) [13]. In IRL, an expert demonstrator optimizes an unknown reward function by acting in a stochastic environment. The goal of IRL is to find a reward function that explains the agent’s behavior. A fruitful approach has been to cast IRL as a Bayesian inference problem to predict the most probable reward function [14]. To make this inference robust to demonstration/modeling noise, one commonly appeals to the principle of maximum causal entropy [9,21]. Intuitively, maximum causal entropy models hedge and assign no more probability mass to an action than the known statistics require, e.g., to match known expected speeds or location visitation frequencies. In doing so, the maximum causal entropy models bound the worst-case expected description length of future demonstrations [21].

While powerful, traditional IRL provides no principled mechanism for composing the resulting reward artifacts and requires the relevant historical features (memory) to be a-priori known. Furthermore, it has been observed that small changes in the workspace, e.g., moving a goal location or perturbing transition probabilities, can change the task encoded by a fixed reward [18,2].

To address these deficits, recent works have proposed learning Boolean task specifications, e.g. logic or automata, which admit well defined compositions, explicitly encode temporal constraints, and have workspace independent semantics. The development of this literature mirrors the historical path taken in reward based research, with works adapting optimal control [11,4], Bayesian [15,20], and maximum entropy [19] IRL approaches.

A key difficulty for the task specification inference from demonstrations literature is how to search an intractably large (often infinite) concept class. In particular, and in contrast to the reward setting, the discrete nature of automata and logic, combined with the assumed a-priori ignorance of the relevant memory required to describe the task, makes existing gradient based approaches either intractable or inapplicable. Instead, the majority of current literature either (syntactically) enumerates concepts [18,15,20] or hill climbs via simple probabilistic (syntactic) mutations [11,3].

The major contribution of this paper is to systematically reduce the problem of learning from demonstrations into a series of supervised task specification identification problems, e.g., finding a small DFA that is consistent with a set of example strings [7], a problem more generally referred to as Grammatical Inference [8]. The result is a principled way to sample tasks given a candidate task. This insight is integrated into a variant of simulated annealing [16] for guided hill climbing. Further, if one assumes that the demonstrations are positive examples, our work serves as a computationally tractable approach to learning simple regular languages from simple positive examples [6] in the context of Markov Decision Processes.

Finally, we note some similarity with the approach of [4] which uses an approximate optimal control perspective to hypothesize positive and negative examples to constrain its search when given demonstrations in a deterministic
domain. This work strictly generalizes this setting while providing (i) a belief over a distribution of tasks (ii) being agnostic to the concept class, (iii) supporting stochastic domains, (iv) supporting backtracking, and (v) not assuming (approximate) optimal performance from the demonstrations.

2 Preliminaries and Problem Statement

2.1 Dynamics Model We model the expert demonstrator as operating in a Markov Decision Process (MDP), \( M = (S, A, s_0, \delta) \), where (i) \( S \) denotes a finite set of states, (ii) \( A(s) \) denotes the finite set of actions available at state \( s \in S \), (iii) \( s_0 \) is initial state, (iv) \( \delta(s' | a, s) \) is the probability of transitioning from \( s \) to \( s' \) when applying action \( a \in A(s) \).

We will make two additional technical assumptions about \( M \). First, we assume a unique (always reachable) sink state, i.e., \( \delta(\$ | a, \$) = 1 \), denoting “end of episode”. Second, we shall assert the Luce choice axiom, which requires that each action, \( a \in A(s) \), be distinct, i.e., no actions are interchangeable or redundant at a given state [12]. Note that the Luce choice axiom is relaxed in Sec 7.

A path, \( \xi \), is an alternating sequence of states and actions starting with \( s_0 \):

\[
\xi = s_0 \xrightarrow{a_0} s_1 \xrightarrow{a_1} s_2 \xrightarrow{a_2} \ldots
\] (1)

Any path, \( \xi \), can be (non-uniquely) decomposed into a prefix, \( \rho \), concatenated with a suffix, \( \sigma \), denoted \( \xi = \rho \cdot \sigma \). We allow \( \sigma \) to be of length 0. The last state of \( \xi \) is denoted by \( \text{last}(\xi) \). A path is complete if it contains \( \$ \) exactly once, and thus \( \text{last}(\xi) = \$ \). We denote by \( \text{Path}_\$ \) the set of all complete paths, and by \( \text{Path} \) the set of all prefixes of \( \text{Path}_\$ \), i.e., paths that contain \( \$ \) at most once.

2.2 Task Specifications Next, we develop the machinery to describe the set of paths that constitute performing a given task. In particular, a concept is a subset of paths,

\[
c \subseteq \text{Path}_\$
\] (2)

We refer to a collection of concepts, \( C \), as a concept class. Similarly a representation class is a set \( \Phi \) equipped with two maps:

\[
\begin{align*}
\text{encode} : \Phi & \to \{0,1\}^* \quad \text{concept} : \Phi \to C, \\
\end{align*}
\] (3)

where \text{encode} is injective and thus unambiguous. The size or description length of a representation is given by the length of its encoding:

\[
\text{size} : \Phi \to \mathbb{N} \quad \text{size}(\varphi) \overset{\text{def}}{=} |\text{encode}(\varphi)|.
\] (4)

Remark 2. While seemingly pedantic, the distinction between concept and representation proves immensely valuable when discussing the learnability of a concept and assigning prior probabilities on concepts, e.g., appealing to Occam’s razor to order prior probabilities by size. Nevertheless, when clear from context, we shall often conflate representations and concept classes.
A task specification is thus formalized as a member of a fixed representation class \( \Phi \) with satisfaction corresponding to set membership. Finally, to avoid notational clutter, when clear from context, we shall often abuse notation and denote by \( \varphi \) a task specification and its underlying concept, e.g., writing \( \xi \in \varphi \) rather than \( \xi \in \text{concept}(\varphi) \).

Example 1. We formalize the representation class of our motivating example. Let (i) \( \Sigma = \{\#, \boxdot, \#, \boxminus, \boxtimes, \square\} \) denote an alphabet, (ii) \( \varphi_1 \) and \( \varphi_2 \) denote the left and right DFA over \( \Sigma \) shown in Fig 1b. Define \( \Phi_{\text{reg}} \) as the set of all tasks represented by a DFA over \( \Sigma \), i.e. regular languages over \( \Sigma \). We choose to encode DFA using a binary encoding of \( \varphi \)'s multi-graph, i.e., encoding (i) the number of states \( n \), (ii) the set of accepting (or rejecting) states, and (iii) the non-stuttering (not self loop) transitions, \( m \). Note this encoding is dominated by (iii) with size \( (\varphi) \in O(m \log n) \).

A labeled example is a tuple, \( x = (\xi, l) \), corresponding to a complete path and a binary label, \( l \in \{0, 1\} \). A collection of labeled examples, \( X = x_1, \ldots, x_n \), is consistent with a task, \( \varphi \), if:

\[
\forall x_i = (\xi_i, l_i). (\xi_i \in \varphi) \iff (l = 1).
\]

(5)

An identification algorithm, \( I \), maps a collection of labeled examples, \( X \) to a distribution over consistent tasks in \( \Phi \) or \( \bot \) if no consistent task exists.

Example 2. Let \( \xi_b \) and \( \xi_r \) be the completed black and red paths shown in Fig 1a and define \( X_{bg} = \{(\xi_b, 1), (\xi_r, 0)\} \). \( \varphi_2 \) is consistent with \( X_{bg} \) and \( \varphi_1 \) is not.

2.3 Policies and Demonstrations A (history dependent) policy, \( \pi(a \mid \xi) \), is a distribution over actions, \( a \), given a path, \( \xi \), where \( a \in A(\text{last}(\xi)) \). A policy is \((p, \varphi)-\text{competent}\) if:

\[
\text{psat}_\varphi(\pi) \overset{\text{def}}{=} \Pr(\xi \in \varphi \mid \pi, M) = p
\]

(6)

A demonstration is a path, \( \xi^* \), generated by a employing a policy \( \pi \) in an MDP \( M \), \( \xi \sim (\pi, M) \).

Task Inference from Demonstrations Problem (TIDP): Let a \( M \), \( \Phi \), and \( P \) be a fixed MDP, representation class, and task prior, respectively. Further, let \( \pi^* \) be a \((p^*, \varphi^*)\)-competent policy, \( \pi^* \), where \( p^* \), \( \varphi^* \), and \( \pi^* \) are all unknown. Given a multi-set of i.i.d. demonstrations, \( \xi^*_1, \ldots, \xi^*_m \sim (\pi^*, M) \), find:

\[
\varphi \in \arg\max_{\psi \in \Phi} \Pr(\xi^*_1, \ldots, \xi^*_m \mid \psi, M) \cdot P(\psi \mid M).
\]

(7)

Of course, by itself, the above formulation is ill-posed as \( \Pr(\xi^*_1, \ldots, \xi^*_m \mid M, \varphi) \) is left undefined. What remains is to derive a suitable agent model and discuss how to manipulate likelihoods in this model.
3 Task Motivated Agents

Following [19], we propose using the principle of maximum causal entropy to assign a bias-minimizing belief of generating the demonstrations given a candidate task.

3.1 Maximum Causal Entropy Policies We start by defining the causal entropy on arbitrary sequences of random variables. Let $X_{1:i} \overset{\text{def}}{=} X_1, \ldots, X_i$ and $Y_{1:i} \overset{\text{def}}{=} Y_1, \ldots, Y_i$ denote two sequences of random variables. The entropy of $X_{1:i}$ causally conditioned on $Y_{1:i}$ is:

$$H(X_{1:i} \mid Y_{1:i}) \overset{\text{def}}{=} \sum_i H(X_i \mid Y_{1:i}, X_{1:i-1})$$

where, $H(X \mid Y) \overset{\text{def}}{=} \mathbb{E}_X[-\ln \Pr(X \mid Y)]$, denotes the entropy of $X$ (statically) conditioned on $Y$. Intuitively, causal conditioning enforces that past variables do not condition on events in the future. This makes causal entropy particularly well suited for robust forecasting in sequential decision making problems, as agents typically cannot observe the future [21].

For MDPs, the unique policy, $\pi_\varphi$, that maximizes entropy subject (i) a finite horizon and (ii) to being $(p, \varphi)$-competent exponentially biases towards higher value actions [21]:

$$\ln \pi_\lambda(a \mid \xi) \overset{\text{def}}{=} Q_\lambda(\xi \cdot a) - V_\lambda(\xi)$$

where the values are recursively given by the following smoothed Bellman-backup:

$$Q_\lambda(\xi \cdot a) \overset{\text{def}}{=} \mathbb{E}_s [V_\lambda(\xi \cdot a \cdot s) \mid a, \xi],$$

$$V_\lambda(\xi) \overset{\text{def}}{=} \begin{cases} \lambda \cdot [\xi \in \varphi] & \text{if } \xi \text{ is complete} \\ \text{LSE}_{a \in A(\text{last}(\xi))} Q_\lambda(\xi \cdot a) & \text{otherwise} \end{cases}.$$  

Here $\text{LSE}_x f(x) \overset{\text{def}}{=} \ln \sum_x e^{f(x)}$ and $\lambda$, called the rationality, is set such that $\text{psat}_\varphi(\pi_\lambda) = p$. When $\lambda$ is induced from $\varphi$, we will often write $V_\varphi$ and $Q_\varphi$.

**Remark 3.** $\text{psat}_\varphi(\pi_\lambda)$ increases monotonically in $\lambda$ and thus can be efficiently calculated to arbitrary precision using binary search.

**Remark 4.** The competency of the agent can be treated as a hyper-parameter or estimated empirically, e.g., $p_\varphi \approx \frac{1}{m} \sum_{i=1}^m [\xi \in \varphi]$. The former is useful when given a few demonstrations and the latter is useful when given a large number of demonstrations.
3.2 Explainability of a task  Next, we develop machinery to measure how well a task explains the demonstrators behavior. To this end, the surprise of a demonstration is defined as:

\[ h(\xi | \pi, M) \overset{\text{def}}{=} -\ln \Pr(\xi | \pi, M). \]  

Intuitively, the surprise correlates with the number of bits (or nats) required to encode \( \xi \) given minimizing expected description lengths under \((\pi, M)\) [5].

The surprise of a collection of demonstrations is the sum of the surprise of each demonstration:

\[ h(\xi^*_1, \ldots, \xi^*_m | \pi, M) \overset{\text{def}}{=} \sum_{i=1}^m h(\xi^*_i | \pi, M). \]  

Note that the likelihood of i.i.d. demonstrations from \((\pi, M)\) is simply,

\[ \exp(-h(\xi^*_1, \ldots, \xi^*_m)). \]  

Given a fixed MDP, \( M \), and a fixed collection of demonstrations, \( \xi_1, \ldots, \xi_m \), we define the surprise of a task, \( \varphi \), as:

\[ h(\varphi) \overset{\text{def}}{=} h(\xi^*_1, \ldots, \xi^*_m | \pi_\varphi, M) \]  

Thus, solving a TIDP requires minimizing \( h \) and the negative log prior, which w.l.o.g. can be taken as \( \text{size}(\varphi) \).

4 Manipulating Surprise

In the sequel, we seek to study how changing a task \( \varphi \) changes the corresponding surprise, \( h(\varphi) \), and thus the likelihood of observing the demonstrations.

4.1 Prefix Tree Perspective  To facilitate this, we will find it useful to re-frame paths as either deviating or conforming to the demonstrations. To start, denote by \( T = (N, E) \) the prefix tree (or trie) of the demonstrations, \( \xi^*_1, \ldots, \xi^*_m \), where \( N \) and \( E \) are the nodes and edges of \( T \), respectively. Each node

\(^1\) also known as surprisal or information content

Fig. 3: Prefix tree with 10 nodes for the paths shown on the left.
\( i \in N \), corresponds to a prefix, \( \rho_i \), of at least one of the demonstrations. An edge connects parent \( i \) to child \( c \) if \( \rho_c \) is the one action (or state) extension of \( \rho_i \), i.e., \( \rho_c = \rho_i \sigma \), where \( \sigma \) is a path of length 1. For each edge, \( (i, j) \in E \), we define the edge traversal count, \( \#_{(i,j)} \), as the number of times prefix \( \rho_j \) appears in the demonstrations. The set of unique demonstrations maps directly to the leaves of \( T \), i.e., for each leaf \( v \), \( \rho_v \) is a demonstration. A node is said to be an ego node if it corresponds to selecting an action, and an environment (env) node otherwise.

We say a path, \( \xi \), conforms to the demonstrations if there is a node \( i \) such that, \( \rho_i = \xi \). A path deviates from the demonstrations if it is not conforming. The pivot of a deviating path, \( \xi \), is the node corresponding to the longest prefix of \( \xi \) that conforms to the demonstrations. Note that it is possible to pivot at the leaves of the tree, i.e., the longest prefix is a demonstration.

**Example 3.** Example demonstrations and the corresponding prefix tree are illustrated in Fig 3. Note that it is possible to pivot at every node except node 2, since both possibilities (slipping/not slipping) appear in the demonstrations.

### 4.2 Change of variables

Next, observe that because weighted averaging and LSE are commutative, one can aggregate the values of a set of actions or set of states (environment actions). In particular, let \( A_i \) and \( S_j \) denote the conforming actions and conforming states at an ego node \( i \) and an env node \( j \) respectively. The pivot value of a node \( i \) is:

\[
\forall^\rho_i \overset{def}{=} \begin{cases} 
\text{LSE}_{a \notin A_i} Q_\varphi(\rho_i \cdot a) & \text{if } i \text{ is ego}, \\
\mathbb{E}_s[V_\varphi(\rho_i \cdot s) | s \notin S_v, \rho_i, M] & \text{if } i \text{ is env},
\end{cases}
\]  

We shall denote by \( \forall^\rho \in \mathbb{R}^N \) the node-indexed vector of pivot values associated with task \( \varphi \) under our maximum entropy agent model. Crucially, the pivot values **entirely determine** (see Fig 4) the values of the states and actions visited by the demonstrations via (9). Namely, let \( \hat{Q}_k(V) \) and \( \hat{V}_k(V) \) denote the derived action and state value at node \( k \) in the prefix tree, and let \( \Pr(i \sim k | V) \) denote the probability of transitioning from node \( i \) to node \( k \) under the (local) policy:
We refer to the surprise map over pivot values as the proxy surprise, \( \hat{h} \), i.e.:
\[
\hat{h}(\mathcal{V}) \overset{\text{def}}{=} -\sum_{(i,j) \in E} \#_{(i,j)} \cdot \ln \Pr(i \rightarrow j \mid \mathcal{V}).
\] (17)

4.3 Leveraging proxy gradients

Unlike the surprise, \( h(\varphi) \), the proxy surprise, \( \hat{h} \), is differentiable with respect to the pivot values \( \mathcal{V} \). One can interpret \( \nabla \hat{h}(\mathcal{V}) \) then as suggesting how to modify the pivot values in order to make the demonstrations less surprising. A natural question then is how to adapt \( \varphi \) given this knowledge. Observe the following two propositions (with proof sketches of the mechanical calculations provided in Sec A):

**Proposition 1 (Pivot values respect subsets).** Let \( \xi \) be a complete path that pivots at node \( i \). If \( \varphi \subseteq \psi \) and \( \xi \in \psi \setminus \varphi \), then \( \mathcal{V}_i^\varphi < \mathcal{V}_i^\psi \).

**Proposition 2 (\( \nabla \hat{h} \) determined by path probabilities).** Let \( p_{xy}(\mathcal{V}) \) denote the probability of starting at node \( x \) and pivoting at \( y \), i.e.,

\[
p_{xy}(\mathcal{V}) = \Pr(x \rightarrow y) \cdot \left( 1 - \sum_{(z) \in E} \Pr(y \rightarrow z) \right).
\] (18)

Then,

\[
\frac{\partial \hat{h}}{\partial \mathcal{V}_k} = \sum_{(i,j) \in E} \#_{(i,j)} \cdot \left( p_{ik}(\mathcal{V}) - p_{jk}(\mathcal{V}) \right).
\] (19)

Prop 1 suggests that to adjust the pivot value, \( \mathcal{V}_i \), in a manner that decreases surprise, one can simply take a path, \( \xi \), that pivots through \( i \) such that:

\[
\xi \in \varphi \iff \frac{\partial \hat{h}}{\partial \mathcal{V}_i} > 0,
\] (20)

and negate its satisfaction under \( \varphi \).

Prop 2 illustrates that (i) the gradients are simple to compute given access to the policy on the prefix tree and (ii) sampling from \( \pi_{\varphi} \) yields paths with a large effect on the gradient. To see point (ii), consider extending the prefix tree to contain the most likely paths after pivoting and apply Prop 2. The gradient w.r.t. the pivot value corresponding to the newly introduced leaves is their path probability and thus their probability of being sampled after pivoting!

**Relation to heuristics** Eq (19) captures several intuitive heuristics for changing \( \mathcal{V} \) to make the demonstrations more likely. Consider ego edge \((i,j)\), which corresponds to an action the agent took. Pivoting at \( i \), i.e., \( k = i \) above, yields, \( p_{jk}(\mathcal{V}) = 0 \). Thus, edge \((i,j)\) contributes positively to the gradient. Since we want to minimize surprisal, this suggests that we want a to decrease \( \mathcal{V}_k \) and thus...
make the action the agent took look more valuable by comparison. Similarly,
suppose \( k = j \), implying that:
\[
p_{ik} - p_{jk} = \Pr(i \rightarrow j \mid V) \cdot p_{jk} - p_{jk} \leq 0,
\]
where equality holds iff exactly one action is available. Thus, when \( k = j \), edge 
\((i, j)\) typically provides a negative contribution to the surprisal gradient. Again,
because we want to minimize surprisal, the above suggests an increase in \( V_k \),
and by extension, an increase in the value of the action used on \((i, j)\). These two
cases are summarized by the following rules of thumb:

1. Make the actions taken more optimal by decreasing the value of other
   actions.
2. Make the actions taken less risky by increasing the value of possible
   outcomes.

**Surprise Guided Sampling** Using these insights we propose surprise guided
sampling (Alg 1 and Fig 5) which samples a path to relabel based on (i) how
likely it is under \( \pi_\varphi \) and (ii) the magnitude and sign of the gradient at the
corresponding pivot. Combined with an identification algorithm, \( I \), repeated
applications of Alg 1 yields an infinite (and stochastic) sequence of tasks resulting
from incrementally conjecturing mis-labeled paths.

**Algorithm 1** Surprise Guided Sampler (SGS)

1: Input: \( \varphi, X, T, M, \beta \)
2: Compute \( \pi_\varphi \) given \( M \) and \( T \).
3: Let \( D \) be the distribution over pivots s.t.
   \[
   \Pr(\text{pivot } i) \propto \exp\left(-\frac{1}{\beta} \left| \nabla h_i \right| \right).
   \]
4: Sample a pivot \( i \sim D \) and a path \( \xi \sim (\pi_\varphi, M) \) s.t.
   i. \( \xi \) pivots at \( i \).
   ii. \( \xi \in \varphi \iff \frac{\partial h_i}{\partial \varphi_i} > 0 \).
   iii. \( \exists \varphi' \in \Phi \) s.t. \( \varphi' \) is consistent with:
   \[
   X \cup \{(\xi, \xi \notin \varphi)\}.
   \]
5: return \( \xi \)

**Remark 5.** Alg 1 only requires a black box maximum entropy (MaxEnt) planner
to enable assigning edge probabilities, \( \Pr(i \rightarrow j \mid V) \), and sampling suffixes
given a pivot. If the satisfaction probability of an action is also known, i.e.,
\( \Pr_{\xi'}(\xi \cdot \xi' \in \varphi \mid \xi, M, \pi_\varphi) \), then one can more efficiently sample suffixes using
Baye’s rule and the policy \( \pi_\varphi \).
Example 4. Recall our ad-hoc analysis on Fig 1a in Sec 1.1. Under the reach hypothesis, \( \varphi \), it is surprising that the agent moves up from \( \blacksquare \) rather than following the red dashed suffix. That is, there is a non-trivial probability to deviate at this point, call \( k \). Using a planning horizon of 15 steps, \( \frac{\partial h}{\partial \nu} \) is positive and indeed larger in magnitude than all other pivots. Props 1 and 2 suggest sampling (using \( \pi_{\varphi} \)) a new path, \( \xi \), that pivots from the demonstration at \( k \) and satisfies \( \varphi \). Note that the illustrated red dashed suffix indeed fits this description. Finally, Eq 20 prescribes marking \( \xi \) as a negative path which matches our ad-hoc analysis!

5 Specification Search

In this section, we take the insights developed in the previous sections and propose the DISS algorithm - a variant of simulated annealing for quickly finding probable task specifications.

5.1 Intuition DISS is a random walk through labeled example space guided by the surprise gradient of sampled concepts. Each new labeled example helps constrain the concept sampler to make previously “surprising” paths less surprising. As we shall see, this accumulation of corrective constraints systematically focuses the concept sampler on more and more probable (lower energy) task specifications. Simulated Annealing formalizes this process, with periodic resets helping to avoid over-committing to a particular labeled example.

5.2 Background on Simulated Annealing At a high level, Simulated Annealing (SA) is a probabilistic optimization method that seeks to minimize an energy function \( U : Z \rightarrow \mathbb{R} \cup \{\infty\} \). To run SA, one requires three ingredients: (i) a cooling schedule which determines a monotonically decreasing sequence of temperatures, (ii) a proposal (neighbor) distribution \( q(z' \mid z) \), and (iii) a reset schedule, which periodically sets the current state, \( z_t \), to one of the lowest energy candidates seen so far.

A standard simulated annealing algorithm then operates as follows: (i) An initial \( z_0 \in Z \) is selected (ii) \( T_t \) is selected based on the cooling schedule. (iii) A neighbor \( z' \) is sampled from \( q(\bullet \mid z_t) \). (iv) \( z' \) is accepted \( (z_{t+1} \leftarrow z') \) with probability:

\[
\Pr(\text{accept} \mid z', z_t) = \begin{cases} 
1 & \text{if } dU > 0 \\
\min \{1, e^{dU/T_t}\} & \text{otherwise,}
\end{cases} \tag{21}
\]
where $dU \overset{\text{def}}{=} U(z) - U(z')$. (v) Finally, if a reset is trigger, $z_{t+1}$ is sampled from previous candidates, e.g., uniform on the argmin.

As previously stated, we propose a variant of simulated annealing adapted for our specification inference problem. We will start by assuming the posterior distribution on tasks takes the form:

$$
\Pr(\phi | \xi^*_1, \ldots, \xi^*_m, M) \propto e^{-U(\phi)},
$$

where the energy, $U$, is given by:

$$
U(\phi) \overset{\text{def}}{=} \text{size}(\phi) + \theta \cdot h(\phi),
$$

where $\theta \in \mathbb{R}$ determines the relative weight of the surprise. That is, we appeal to Occam’s razor and assert that the task distribution is exponentially biased towards simpler tasks, where simplicity is measured by the description length of the task, $\text{size}(\phi)$, and the description length (i.e. surprise) of $\xi^*_1, \ldots, \xi^*_m$ under $(\pi_\phi, M)$.

5.3 Algorithm Description Using the language of SA, we define DISS as follows: (i) $z \in Z$ is a tuple, $(X, \phi)$, of labeled examples and a task specification. (ii) $z_0 = (\emptyset, \bot)$. (iii) the proposal distribution, $q(X', \phi' \mid X, \phi)$ is defined to first sample a concept using an identification map, $\phi' \sim I(X)$, then run SGS on $\phi'$ to conjecture a labeled path $\xi$, yielding $X' = X \cup \{(\xi, \xi \not\in \phi')\}$. (iv) resets occur every $\kappa \in \mathbb{N}$ time steps. If a reset is triggered, $X_{t+1}$ is sampled from softmin$_{\leq t} U(\phi_t)$, and $\phi_{t+1}$ is sampled from $I(X_{t+1})$. This algorithm is given as pseudo code in Appendix B.

6 Experiments

In this section, we illustrate the effectiveness of DISS by having it search for a ground truth specification, represented as a DFA, given the expert demonstrations, $\xi_b, \xi_g$, from our motivating example (shown in Fig 1a). The (dotted) green path, $\xi_g$, goes directly $\blacksquare$. The (solid) black path, $\xi_b$, immediately slips into $\blacksquare$, visits $\blacklozenge$, then proceeds towards $\blacksquare$. This path is incomplete, with a possible extension, $\sigma_b$, shown as a dotted line. The ground truth task is the right DFA in Fig 1b.

We consider two specification inference problems by varying the representation class and the provided demonstrations. These variants respectively illustrate that (i) our method can be used to incrementally learn specifications from unlabeled incomplete demonstrations and (ii) the full specification can be learned given unlabeled complete demonstrations.

1. **Monolithic**: $\xi_g$ and $\xi_b \cdot \sigma_b$ are provided as (unlabeled) complete demonstrations. The representation class is the set of DFAs over $\{\blacksquare, \blacklozenge, \square, \blacksquare\}$, e.g., $\Phi_{reg}$ from Example 1.

2. **Incremental**: $\xi_b$ is provided as an (unlabeled) incomplete demonstration. The representation class, $\Phi \subseteq \Phi_{reg}$ is the set of DFAs that require recharging and avoiding lava.
The surprise weight, $\theta$, is set to 1 for both variants. Finally, two additional inductive biases, which empirically proved necessary for optimizing the random pivot baseline, are applied: (i) we remove white tiles, $\square$, from labeled examples (ii) we transform sequences of repeated colors into a single color thus biasing towards DFA that do not count. For example, $\square\square\square\square\square \mapsto \square\square\square$.

**DISS parameters.** Our implementation of DISS uses SGS temperature $\beta = 1/500$, resets every 10 iterations, and uses the following cooling schedule:

$$T_i = 100 \cdot (1 - t/100) + 1. \quad (24)$$

For concept identification, $I$, we adapt an existing SAT-based DFA identification algorithm [17] to enumerate the first 20 consistent DFAs (ordered by size). A DFA is then sampled from $\text{softmax}(\text{size}(\phi))$. For maximum entropy planning, we use the Binary Decision Diagram based approach proposed in [19] with a planning horizon of 15 steps. Finally, because our experiments operate with one or two demonstrations, the rationality coefficient, $\lambda$, was fixed to 10. Nearly identical results we also achieved by setting $\lambda$ such that the competency, $p_{\phi}$ is as close to $4/5$ as possible.

**Baselines.** As mentioned in the introduction, existing techniques for learning specifications from demonstrations use various syntactic concept classes, each with their own inductive biases [18, 15, 20]. Thus, we implemented two DFA adapted baselines that act as proxies for the enumerative and probabilistic hill climbing style algorithms of existing work:

1. **Prior-ordered Enumeration.** This baseline uses the same SAT-based DFA identification algorithm to find the first $N$ DFAs, ordered by prior probability, i.e., size. As an alternative to DISS’s competency assumption, we allow the enumerative baseline to restrict the search to task specifications that accept the provided demonstrations.$^2$

$^2$ For the incremental experiment, a counterexample loop is used to add labeled examples that bias the DFAs to imply $\varphi_1$. 

(a) Min energy found by iteration in monolithic experiment.  

(b) Min energy found by iteration in incremental experiment.
2. Random Pivot DISS. This baseline uses DISS with SGS temperature, \( \beta = \infty \). This ablation results in a (labeled example) mutation based search with access to the same class of mutations as DISS, but samples pivots uniformly at random, i.e., no gradient based bias. Note that this variant still samples suffixes conditioned on the sign of the gradient, and thus the mutations are still informed by the surprise.

6.1 Results and Analysis To simplify our analysis, we present time in iterations, i.e., number of sampled DFAs, rather than wall clock time. This is for two reasons. First, for each algorithm, the wall clock-time was dominated by synthesizing maximum entropy planners for each unique DFA discovered, but the choice of planner is ultimately an implementation detail. Second, because many DISS iterations correspond to the same DFAs (due to resets and rejections) the enumeration baseline explored significantly more unique DFAs than DISS (a similar effect occurs with the random pivot baseline, since the different pivots give more diverse example sets). Thus, using wall clock-time would skew the results below in DISS’s favor.

Fig 6a and Fig 6b show the minimum energy DFA for the monolithic and incremental experiments respectively. We see that for both experiments, DISS was able to significantly outperform the baselines (recall that energy is the negative log of the probability), with the incremental experiment requiring only a few (\(< 6\)) iterations to find a probable DFA! Furthermore, in addition to finding the most probable DFAs much faster than the baselines, DISS also found more high probability DFAs.

Next, we analyze the DFAs learned by DISS. The most likely DFAs found by DISS for each experiment (left and right) are shown in Fig 7. We observe that for both experiments, DISS was able to significantly outperform the baselines (recall that energy is the negative log of the probability), with the incremental experiment requiring only a few (\(< 6\)) iterations to find a probable DFA! Furthermore, in addition to finding the most probable DFAs much faster than the baselines, DISS also found more high probability DFAs.

Nevertheless, our learned DFAs differ from ground truth, particularly when it comes to the acceptance of strings after visiting \( \blacksquare \). We note that a large reason for this is that our domain and planning horizon make the left most \( \blacksquare \) effectively act as a sink state. That is, the resulting sequences are indistinguishable, with many even having the exact same energy. In Fig 7, we make such edges lighter, and note that the remainder of the DFAs show good agreement with the ground truth. Finally, while impressive, this points to a fundamental limitation of demonstrations. Namely, if two tasks have very correlated policies in a workspace, then without strong priors or side information, one is unable to distinguish the tasks. For example, if a task requires the agent to avoid \( \blacksquare \), but no \( \blacksquare \) are shown in the workspace, then one cannot hope to learn this aspect of the task.

\[3\] Nevertheless, for the monolithic experiment, the wall clock times for DISS, random pivoting, and enumeration were 542s, 764s, and 617s respectively. Similarly, for the incremental experiment the respective times were 353s, 464s, and 820s.
7 Relaxing the Luce axiom

Our design and analysis of DISS assumed the Luce axiom and in particular that all actions accessible by states in the demonstrations are distinct, i.e., if for all non-terminal states $s$,

$$\forall a_1, a_2 \in A(s) . a_1 \neq a_2 \implies \delta(s, a_1) \neq \delta(s, a_2).$$

This assumption is important several reasons. Chiefly, a violation of the Luce axiom along the demonstrations means that the pivot values are no longer independent. We illustrate with an example.

Example 5. Consider the demonstration $\xi = s_0 \xrightarrow{a_2} s_1$, where $A(s_0) = \{a_1, a_2\}$ and $A(s_1) \neq \emptyset$. Thus, for prefix $\rho = s_0$, we have $a_1 \in A_\rho$ and $a_2 \notin A_\rho$. Since $a_1$ is the only action that pivots, we have $V_\rho = V(\rho \cdot a_1)$. Similarly, since $A(s_1) \neq \emptyset$, $\xi$ is not terminal and has its pivot value determined be the subtree accessed by $\rho \cdot a_2$, i.e., $V_\xi = V_{\rho \cdot a_2} = V(\rho \cdot a_2)$. But since $a_2$ accesses the same subtree as $a_1$, then

$$V_\rho = V(\rho \cdot a_1) = V(\rho \cdot a_2) = V_{\rho \cdot a_2}.$$  

Thus, $V_\rho = V_{\rho \cdot a_2}$, implying that pivot values are not independent.

This has serious implications on our use of the surprise gradient. The most glaring issue is that two logically equivalent pivots may have opposing signs in the gradient. This can lead to conjecturing a counter-factual that is exactly counter to your goal. For example, if the last action in a positive demonstration is redundant with another action, pivoting at that point will likely result in the demonstration being marked negative! While it is possible for DISS to correct this conjecture in a future iteration, this can seriously degrade performance in practice. Furthermore, a similar argument can be made for redundant states!

7.1 Counter Example Guided Action Refinement For these reasons, it behooves us to develop a way to handle redundant actions and states. Luckily there is a simple solution that only requires a minor modification to DISS:

1. Start by assuming that all (time-indexed) states and actions in the demonstrations are equivalent. Thus, the prefix tree is initially a chain and no nodes can be pivots.
2. Whenever proof is found that two states (actions) are distinct at a given node in the prefix tree, create a new prefix tree where the prefix indexed equivalence class of states (actions) is partitioned accordingly.

To formalize this algorithm, we introduce the idea of value-distinguishability.

Let $\xi$ and $\xi'$ be two paths such that $\xi = \rho \cdot x \cdot y$ and $\xi' = \rho \cdot x' \cdot y'$, where $|x| = |x'| = 1$. Given a subset of the representation class $\Psi \subseteq \Phi$, we say that prefixes $\rho \cdot x$ and $\rho \cdot x'$ are $\Psi$-value-distinguishable if there exists a task, $\varphi \in \Psi$, such that:

$$V_{\varphi}(\rho \cdot x) \neq V_{\varphi}(\rho \cdot x').$$

(25)

When $\Psi = \Phi$, we suppress $\Psi$ and simply say value-distinguishable.

Importantly, if two prefixes are value distinguishable, then they maintain the independence of pivot values since they must not access the same subtree. Conversely, if two prefixes are not value distinguishable, then they access must access functionally equivalent sub-trees. Thus, value-distinguishability fits the needs of proof that two states (actions) are distinct.

The problem of course is that refuting value-distinguishability requires examining the whole representation class (which may be impossible). Instead, we define a series of equivalence relations based on an expanding sets of specifications, $\Psi_1 \subseteq \Psi_2 \subseteq \ldots$, where $\Psi_i$ is the set of specifications visited by DISS by iteration $i$. Thus, at iteration $i$, prefix $\rho$ and $\rho'$ are in the same equivalence class if they are not $\Psi_i$-value-distinguishable.

In summary, to account for the possibility of redundant states and actions, maintain an series equivalence relations. Whenever two actions are deemed to be value-distinguishable, the equivalence relation is updated, which results in a new prefix tree over representatives of each equivalence class. Noting that the initial set of visited specifications is empty and that no prefixes are $\emptyset$-value-distinguishable, we have that (i) initial prefix tree is a chain; and (ii) no nodes can (initially) be pivots.

Remark 6. This variant of DISS requires knowing the set of states and actions reachable from each state. This can be further relaxed by dynamically testing states and actions for value-distinguishability as they observed, e.g., by sampling. While not maintaining pivot value independence, it does prevent conjecturing conflicting labels to equivalent paths.

8 Conclusion

This paper considered the problem of learning history dependent task specifications, e.g. automata and temporal logic, from expert demonstrations. We empirically demonstrated how to efficiently explore intractably large representation classes such as deterministic finite automata for find probable task specifications. The proposed family of algorithms, Demonstration Informed Specification Search (DISS), requires only black box access to (i) a Maximum Entropy planner and
(ii) an algorithm for identifying concepts, e.g., automata, from labeled examples. While we showed concrete examples for the efficacy of this approach, several future research directions remain. First and foremost, research into faster and model-free approximations of maximum entropy planners would enable a much larger range of applications and domains. Similarly, while large, the demonstrated representation class was over a small number of pre-defined atomic predicates. Future work thus includes generalizing to large symbolic alphabets and studying more expressive specification formalisms such as register automata, push-down automata, and (synchronous) products of automata.
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A Proof Sketches

Proof (Prop 1). Follows inductively from the monotonicity of $E$, $\sum$, and $\ln$.

Before proving Prop 2 we first prove the following lemma.

Lemma 1. For any node, $i$, in the prefix tree,

$$\frac{\partial}{\partial V_k} \mathbf{\hat{V}}_i(V) = p_{ik}(V),$$

where $\mathbf{\hat{V}}$ denotes $\hat{V}$ for ego nodes and $\hat{Q}$ for env nodes.

Proof. To begin, note that one can extend the prefix tree with new leaves corresponding to the action of pivoting at the node. Thus, $p_{ik}$, is simply $\Pr(i \rightsquigarrow k')$, where $k'$ is the pivot leaf for $k$. Thus, it is sufficient to prove that in this new tree:

$$\frac{\partial}{\partial V_k} \mathbf{\hat{V}}_i(V) = \Pr(i \rightsquigarrow k | V),$$

For any edge $(a,b)$, observe that if $a$ is an environment node, then $\Pr(a \rightsquigarrow b | V)$ is a constant, denoted $q_{ab}$. Next, observe that because the nodes are arranged as a tree either: (1) $k$ is not reachable from $i$ or (2) only a single edge, call $(i,j)$, can reach $k$ from $i$. Thus,

$$\frac{\partial}{\partial V_k} \mathbf{\hat{Q}}_i(V) \overset{\text{def}}{=} \frac{\partial}{\partial V_k} \sum_{(a,b) \in E} q_{ab} \cdot \hat{V}_b(V) = \Pr(i \rightsquigarrow j | V) \cdot \begin{cases} 0 & \text{if } \Pr(i \rightsquigarrow k) = 0 \\ \frac{\partial}{\partial V_k} \hat{V}_j(V) & \text{otherwise,} \end{cases}$$

(26)

Similarly, note that because the derivative of logsumexp is the softmax function, for any ego node $i$,

$$\frac{\partial \hat{V}_i}{\partial V_k} \overset{\text{def}}{=} \frac{\partial}{\partial V_k} \log \sum_{(a,b) \in E} \hat{Q}_b(V) \overset{\text{def}}{=} \begin{cases} 0 & \text{if } \Pr(i \rightsquigarrow k) = 0 \\ e^{\hat{Q}_j(V) - \hat{V}_i(V)} \cdot \frac{\partial}{\partial V_k} \hat{Q}_j(V) & \text{otherwise,} \end{cases}$$

(27)

where again, $j$ denotes the (potential) unique child of $i$ that can reach $k$. Finally, observe that by definition $e^{\hat{Q}_j(V) - \hat{V}_i(V)} = \Pr(i \rightsquigarrow j | V)$, using the maximum entropy policy induced by $V$. Substituting into (26), we see that the lemma follows by induction.
Proof (Prop \[2\]). Recall that the probability of traversing an environment edge is constant w.r.t \( V \). Thus, inspecting \([17]\) we see that it suffices to prove that for any ego edge, \((i,j)\),

\[
\frac{\partial}{\partial V_k} \ln \Pr(i \leadsto j \mid V) = \Pr(i \leadsto k \mid V) - \Pr(j \leadsto k \mid V).
\]

Recall that by definition, if \( i \) is ego, then \( \ln \Pr(i \leadsto j \mid V) = \hat{Q}_i(V) - \hat{V}_i(V) \). Thus, the proposition follows directly from Lemma \([1]\).

B DISS Pseudo Code

\begin{algorithm}
\caption{Demonstration Informed Specification Search.}
\begin{algorithmic}[1]
\STATE \textbf{input: } \((\xi_1, \ldots, \xi_m), M, \theta, N, \kappa\)
\STATE Compute \( T \) given \((\xi_1, \ldots, \xi_m)\). \Comment{Create prefix tree.}
\STATE \( \Phi \leftarrow \emptyset \).
\FOR {\( t \) in \( 1, \ldots, N \)}
\IF {\( t \equiv 0 \pmod{\kappa} \)}
\STATE \( X \sim \arg \max_{\psi \in \Phi} U(\psi) \) \Comment{Reset periodically.}
\STATE \( dX \leftarrow 0 \)
\ENDIF
\STATE \( X' \leftarrow \text{update}(X', dX) \) \Comment{Newer label wins under conflict.}
\STATE \( \varphi' \sim I(X') \). \Comment{Sample candidate task.}
\STATE \( \Phi \leftarrow \Phi \cup \{\varphi'\} \). \Comment{Update visited specs.}
\STATE \( T \leftarrow \text{cooling\_schedule}(t) \) \Comment{User defined.}
\STATE \( dU \leftarrow U(\varphi') - U(\varphi) \)
\STATE \( \alpha \sim \text{Uniform}(0, 1) \)
\IF {\( dU < 0 \) or \( \exp(-dU/T) \leq \alpha \)}
\STATE \( (\varphi, X) \leftarrow (\varphi', X') \)
\STATE \( dX \leftarrow \{\text{SGS}(\varphi, T, M)\} \) \Comment{Conjecture labeled example.}
\ELSE
\STATE \( dX \leftarrow \emptyset \) \Comment{Reject proposal.}
\ENDIF
\ENDFOR
\RETURN \( \Phi \)
\end{algorithmic}
\end{algorithm}