Conductivity of holographic superconductors in Born-Infeld electrodynamics

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Abstract

In this paper, we have analytically computed the conductivity of holographic superconductors in the framework of Born-Infeld electrodynamics taking into account the backreaction of the matter fields on the bulk spacetime metric. The effect of the Born-Infeld electrodynamics is incorporated in the metric. The band gap energy is found to be corrected by the backreaction and Born-Infeld parameters. The conductivity expression is then compared with that obtained from a self consistent approach.

1 Introduction

The AdS/CFT duality has been an important theoretical input to study the physics of strongly coupled system [1]-[6]. The main focus in this area has been to construct gravitational duals of physical phenomena exhibiting strong coupling.

Holographic superconductors have been an important class of gravitation duals which have been studied extensively in the recent past [7]-[18]. These models have been found to reproduce some of the properties of high $T_c$ superconductors. The theoretical models consisting of a AdS black hole in the bulk with a charged scalar field coupled to the Maxwell field has been found to admit the formation of a scalar hair below a certain critical temperature. The mechanism involved in the formation of this hair involves the spontaneous breakdown of a local $U(1)$ symmetry near the black hole horizon [19],[20]. There has been a lot of work investigating the effects of Born-Infeld (BI) electrodynamics on holographic superconductors [21]-[30]. The importance of such work have been to study the effect of non-linear electrodynamics on holographic superconductors. Further, the choice of BI electrodynamics have been made since this is the only non-linear theory of electrodynamics which enjoys the duality symmetry. However, an analytic computation of

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conductivity of holographic superconductors in the framework of BI electrodynamics has so far been missing in the literature. In this paper, we proceed to investigate the effects of BI electrodynamics on the conductivity of these systems analytically. We incorporate the effects of the BI parameter in the spacetime metric and also take backreaction into account. We have then calculated the band gap energy from the conductivity expression. We have shown that the band gap energy increases with increase in parameter $b$.

This paper is organized as follows. In section 2, the basic formalism for the holographic superconductors coupled to BI electrodynamics is presented. In section 3, we calculate the conductivity up to first order in $b$. Section 4 contains the concluding remarks.

## 2 Basic formalism

In this section, we set up the basic formalism and notations which shall be required for subsequent discussion. In 3 + 1-dimensions, the action for the model of a holographic superconductor in the framework of Born-Infeld electrodynamics consists a complex scalar field coupled to a $U(1)$ gauge field in anti-de Sitter black hole spacetime

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} (R - 2\Lambda) + \frac{1}{b} \left( 1 - \sqrt{1 + \frac{b}{2} F_{\mu\nu}F^{\mu\nu}} \right) - (D_\mu \psi)^* D^\mu \psi - m^2 \psi^* \psi \right\}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$; ($\mu, \nu = t, r, x, y$), $D_\mu \psi = \partial_\mu \psi - iq A_\mu \psi$, $\Lambda = -\frac{3}{L^2}$ is the cosmological constant, $\kappa^2 = \frac{8\pi G}{\kappa^2 L}$, $G$ being the Newton’s universal gravitational constant, $b$ is Born-Infeld parameter, $A_\mu$ and $\psi$ represent the gauge and scalar fields.

In the presence of backreaction, the plane-symmetric black hole metric takes the form

$$ds^2 = -f(r)e^{-\chi(r)}dt^2 + \frac{1}{f(r)}dr^2 + r^2(dx^2 + dy^2) .$$

Making the ansatz for the gauge field and the scalar field as $[10]$

$$A_\mu = (\phi(r), 0, 0, 0) , \quad \psi = \psi(r)$$

leads to the following equations of motion for the metric, the gauge and matter fields

$$f'(r) + \frac{f(r)}{r} - \frac{3r}{L^2} + \kappa^2 r \times \left[ f(r)\psi'(r)^2 + \frac{q^2 \phi^2(r)\psi^2(r)e^{\chi(r)}}{f(r)} + m^2 \psi^2(r) + \frac{1}{b} \left( (1 - be^{\chi(r)}\phi'(r)^2)^{\frac{3}{2}} - 1 \right) \right] = 0$$

$$\chi'(r) + 2\kappa^2 r \left( \psi'(r)^2 + \frac{q^2 \phi^2(r)\psi^2(r)e^{\chi(r)}}{f(r)^2} \right) = 0$$

$$\phi''(r) + \left( \frac{2}{r} + \frac{\chi'(r)}{2} \right) \phi'(r) - \frac{2}{r} be^{\chi(r)}\phi'(r)^3 - \frac{2q^2 \phi(r)\psi^2(r)}{f(r)}(1 - be^{\chi(r)}\phi'(r)^2)^{\frac{3}{2}} = 0$$
\[ \psi''(r) + \left( \frac{2}{r} - \frac{\chi'(r)}{2} + \frac{f'(r)}{f(r)} \right) \psi'(r) + \left( \frac{q^2 \phi^2(r)e^{\chi(r)}}{f(r)^2} - \frac{m^2}{f(r)} \right) \psi(r) = 0 \]  \tag{7}

where prime denotes derivative with respect to \( r \). We can set \( q = 1 \) and \( L = 1 \) without any loss of generality \cite{25}, \cite{31}. The conditions \( \phi(r_+) = 0 \) and \( \psi(r_+) \) to be finite imposes the regularity of the fields at the horizon.

The fields near the boundary of the bulk obey \cite{13}

\[
\begin{align*}
\phi(r) &= \mu - \frac{\rho}{r} \\
\psi(r) &= \frac{\psi_+}{r^{\Delta_+}} + \frac{\psi_+}{r^{\Delta_-}}
\end{align*}
\tag{8}
\tag{9}
\]

where

\[
\Delta_\pm = \frac{3 \pm \sqrt{9 + 4m^2L^2}}{2}
\tag{10}
\]

are the conformal weights of the conformal field theory living on the boundary. The interpretation of the parameters \( \mu \) and \( \rho \) is given by the gauge/gravity dictionary. They are interpreted as the chemical potential and charge density of the conformal field theory on the boundary. For the choice \( \psi_+ = 0 \), \( \psi_- \) is interpreted as the dual of the expectation value of the condensation operator \( \mathcal{O}_{\Delta} \) in the boundary.

Under changing the coordinate from \( r \) to \( z = \frac{r_+}{r} \), the field eqs. (4)-(7) look like

\[
\begin{align*}
f'(z) - \frac{f(z)}{z} + 3 \frac{r_+^2}{z^3} - \frac{r_+^2}{z^3} & \times \left[ \frac{z^4}{r_+^2} \frac{f(z)\psi'(z)^2}{f(z)} + \frac{\phi^2(z)\psi^2(z)e^{\chi(z)}}{f(z)} + m^2\psi(z) + \frac{1}{b} \left( 1 - \frac{b^2 z^4 e^{\chi(z)}\phi'(z)^2}{r_+^2} \right)^{-\frac{1}{2}} - 1 \right] = 0 \tag{11} \\
\chi'(z) - \frac{2r_+^2}{z^3} \left( \frac{z^4}{r_+^2} \psi'(z)^2 + \frac{\phi^2(z)\psi^2(z)e^{\chi(z)}}{f(z)^2} \right) &= 0 \tag{12} \\
\phi''(z) + \chi'(z) \phi'(z) + 2 \frac{b e^{\chi(z)}\phi'(z)^3 z^3}{r_+^2} - 2 \frac{r_+^2 \phi(z)\psi(z) e^{\chi(z)}}{f(z) z^4} \left( 1 - \frac{b^2 e^{\chi(z)}\phi'(z)^2}{r_+^2} \right)^{\frac{3}{2}} &= 0 \tag{13} \\
\psi''(z) + \left( \frac{f'(z)}{f(z)} - \frac{\chi'(z)}{2} \right) \psi'(z) + \frac{r_+^2}{z^4} \left( \frac{\phi^2(z)e^{\chi(z)}}{f(z)^2} - \frac{m^2}{f(z)} \right) \psi(z) &= 0 \tag{14}
\end{align*}
\]

where prime denotes derivative with respect to \( z \). The regularity condition \( \phi(r_+) \) becomes \( \phi(z = 1) = 0 \). In the rest of our work, we set \( m^2 = -2 \). This leads to two possible values of \( \Delta \) from eq. (10), namely, \( \Delta_+ = 2 \) and \( \Delta_- = 1 \).

We now proceed to solve the equation for the metric (11) taking into account the effect
of the backreaction and the BI parameter $b$. At $T = T_c$, the matter field vanishes, that is $\psi(z) = 0$. Hence eq. (12) reduces to

$$\chi'(z) = 0 \quad \Rightarrow \quad \chi(z) = \text{constant} . \quad (15)$$

Now near the boundary of the bulk, we can set $e^{-\chi(r \to \infty)} \to 1$, i.e. $\chi(r \to \infty) = 0$ which in turn implies $\chi(z = 0) = 0$. This yields $\chi(z) = 0$ from eq. (15). Using this in the equation of motion for the gauge field (13), we get the solution for $\phi(z)$ upto $O(b)$ to be [23], [28]

$$\phi(z) = \lambda r_{+} \left\{ (1 - z) - \frac{b \lambda^2}{10} (1 - z^5) \right\} \quad (16)$$

where $\lambda = \frac{\rho}{r_{+}^{2(c)}}$. With these solutions in hand, we now proceed to solve the equation for the metric. The metric equation keeping terms upto first order in the Born-Infeld parameter now reads

$$f'(z) - \frac{f(z)}{z} + \frac{3r_{+}^2}{z^3} - \kappa^2 \left\{ \frac{1}{2} \phi'^2(z)z + \frac{3bz^5}{8r_{+}^2} \phi'^4(z) \right\} = 0 . \quad (17)$$

Substituting the solution of $\phi(z)$ in the above equation, we obtain the metric equation upto $O(b)$

$$f' - \frac{f(z)}{z} + \frac{3r_{+}^2}{z^3} - \frac{r_{+}^2 \kappa^2 \lambda^2}{2} \left( z - \frac{b}{4 \lambda^2 z^5} \right) = 0 . \quad (18)$$

Solving this equation and imposing the condition $f(z = 1) = 0$ to determine the integration constant yields

$$f(z) = \frac{r_{+}^2}{z^2} g(z) = \frac{r_{+}^2}{z^2} \left[ g_0(z) + g_1(z) \right] \quad (19)$$

where

$$g_0(z) = 1 - z^3 ; \quad g_1(z) = \frac{\kappa^2 \lambda^2}{2} \left\{ z^4 - z^3 - \frac{b \lambda^2}{20} (z^8 - z^3) \right\} . \quad (20)$$

The above form of the metric includes the effects of backreaction as well as the BI electrodynamics upto first order in the BI parameter $b$. The Hawking temperature of this black hole spacetime reads

$$T = \frac{f'(r_+)}{4\pi} = -\frac{f'(z = 1)}{4\pi r_+} = \frac{3r_+}{4\pi} \left[ 1 - \frac{\kappa^2 \lambda^2}{6} \left( 1 - \frac{b \lambda^2}{4} \right) \right] . \quad (21)$$

### 3 Computation of conductivity

In this section, we proceed to study the conductivity as a function of frequency, that is optical conductivity. For simplicity, we look at the conductivity along the $x$-direction. By the gauge/gravity duality, the fluctuations in the Maxwell field in the bulk gives rise to the conductivity.
Making the ansatz $A_\mu = (0, 0, \phi(r, t), 0)$ with $\phi(r, t) = A(r)e^{-i\omega t}$ and neglecting terms of $O(b^2)$ and $O(\omega^2 b)$ leads to the following equation of motion for $A(r)$

$$A''(r) + \frac{f'(r)A'(r)}{f(r)} \left\{ 1 + \frac{b}{r^2} f(r) A^2(r) e^{-2i\omega t} \right\} - \frac{be^{-2i\omega t}}{2r^2} A'^3(r) \left( \frac{f'(r)}{r} - \frac{2f(r)}{r} \right) + \left[ \frac{\omega^2}{f^2(r)} - \frac{2\psi^2(r)}{f(r)} \left( 1 + \frac{3b}{2r^2} f(r) A^2(r) e^{-2i\omega t} \right) \right] A(r) = 0 . \quad (22)$$

This equation is very difficult to solve analytically. However, in principle we can employ a perturbative approach to tackle this equation. To make progress, we start by considering the above equation up to $O(b^0)$. This reads

$$A''(r) + \frac{f'(r)A'(r)}{f(r)} + \left[ \frac{\omega^2}{f^2(r)} - \frac{2\psi^2(r)}{f(r)} \right] A(r) = 0 . \quad (23)$$

Note that the effect of the BI parameter is contained in the metric. The perturbative technique involves solving this equation and then replacing this solution in the $O(b)$ terms in eq.(22) and solving the equation once again.

We now move to tortoise coordinate which is defined by

$$r_* = \int \frac{dr}{f(r)} = -\frac{1}{r_+} \int \frac{dz}{g_0(z) + g_1(z)} \approx -\frac{1}{r_+} \left\{ \int \frac{dz}{g_0(z)} - \int \frac{g_1(z)}{g_0^2(z)} dz \right\}$$

$$= \ln((1-z)^{\frac{1}{b\tau_+}} \left\{ 1 + e^{-\frac{\lambda^2}{2}} (1 - \frac{1}{4} b \lambda^2) \right\} + \ln(1 + z + z^2) - \frac{1}{\tau_+} \left\{ 1 + e^{-\frac{\lambda^2}{2}} (1 + \frac{1}{4} b \lambda^2) \right\}$$

$$+ \frac{\kappa^2 \lambda^4 b}{120 r_+} (1 - z^3) - \frac{\kappa^2 \lambda^2 (z + \frac{b \lambda^2}{20})}{6 r_+ (1 + z + z^2)} - \frac{1}{\sqrt{3} r_+} \left\{ 1 - \frac{\kappa^2 \lambda^2}{2} + \frac{\kappa^2 \lambda^4 b}{120} \right\} \tan^{-1} \sqrt{3} z \quad / 2 + z . \quad (24)$$

The integration constant has been calculated from the condition $r_*(z) = 0$ at $z = 0$. The wave equation in tortoise coordinate reads

$$\frac{d^2 A}{dr_*^2} + \left[ \omega^2 - V \right] A = 0 \quad (25)$$

where

$$V = 2\psi^2 f . \quad (26)$$

We now employ a trick to solve this equation. We first solve this equation for $V = 0$ which implies that we solve only the $\omega$-dependent part of the equation. The solution reads

$$A \sim e^{-i\omega r_*} \sim (1 - z)^{-\frac{\omega}{\tau_+}} \left\{ 1 + e^{-\frac{\lambda^2}{2}} (1 - \frac{1}{4} b \lambda^2) \right\} \quad (27)$$

where we consider only leading order terms in $r_*$, i.e. $r_* = \ln((1-z)^{\frac{1}{b\tau_+}} \left\{ 1 + e^{-\frac{\lambda^2}{2}} (1 - \frac{1}{4} b \lambda^2) \right\}$ in obtaining the above expression. We now want to know the function which is independent of $\omega$ and has only $z$ dependence. To do this we first write down eq.(23) in $z$-coordinate. This reads

$$g(z) \frac{d^2 A(z)}{dz^2} + g'(z) \frac{dA(z)}{dz} + \left[ \frac{\omega^2}{r_+^2 g(z)} - \frac{2\psi^2(z)}{z^2} \right] A(z) = 0 . \quad (28)$$
We now write $A(z)$ as a product of the $\omega$-dependent part and a function of $z$ which we need to determine. Hence, the gauge field reads

$$A(z) = (1 - z)^{-\frac{\omega}{3r_+}} \left\{ 1 + \frac{\kappa^2 \lambda^2}{6} (1 - \frac{1}{4} b \lambda^2) \right\} G(z)$$

(29)

where $G(z)$ is regular at the horizon of the black hole. Substituting this in eq.(28), we obtain

$$g(z)G''(z) + \left[ \frac{2i\omega}{3r_+} \left\{ 1 + \frac{\kappa^2 \lambda^2}{6} (1 - \frac{1}{4} b \lambda^2) \right\} \frac{g(z)}{1 - z} + g'(z) \right] G'(z)$$

$$+ \frac{i\omega}{3r_+} \left\{ 1 + \frac{\kappa^2 \lambda^2}{6} (1 - \frac{1}{4} b \lambda^2) \right\} \frac{g'(z)}{1 - z} + \frac{\omega^2}{r_+^2 g(z)} - \frac{2\psi^2(z)}{z^2} \right] G(z) = 0 \quad (30)

For $\Delta = 1$, we know that $[14]$

$$\psi(z) = \frac{\langle O_1 \rangle}{\sqrt{2r_+}} F(z) z$$

(31)

where $F(0) = 1$. For simplification, we consider $F(z)$ to be 1 because we are neglecting order $O(z^3)$ term. Substituting this in eq.(30), we get

$$3g_0(z)G''(z) + \left[ \frac{2i\omega C_1}{r_+} (1 + z + z^2) - 9C_2(z)z^2 \right] G'(z) + \left[ \frac{i\omega C_1}{r_+} \left( 1 + z + z^2 - 3C_2(z)z^2 \right) \right] \times \frac{1}{1 - z} + \frac{\omega^2}{3r_+^2} \left\{ 9C_3(z) - (1 + z + z^2)^2 C_2(z) \right\} \frac{1}{1 - z^3} - \frac{3\langle O_1 \rangle^2}{r_+^2} C_4(z) \right] G(z) = 0 \quad (32)

where

$$C_1 = 1 + \frac{\kappa^2 \lambda^2}{6} \left( 1 - \frac{1}{4} b \lambda^2 \right) \quad ; \quad C_2(z) = \frac{1 + \frac{g_1(z)}{g_0(z)}}{1 + \frac{g_1(z)}{g_0(z)} z}$$

$$C_3(z) = \frac{1}{\left( 1 + \frac{g_1(z)}{g_0(z)} \right)^2} \quad ; \quad C_4(z) = \frac{1}{1 + \frac{g_1(z)}{g_0(z)}} \quad (33)

Keeping terms up to order $z^3$ in the above equation yields

$$3g_0(z)G''(z) + \left[ \frac{2i\omega C_1}{r_+} (1 + z + z^2) - 9C_2(z)z^2 \right] G'(z) + \left[ \frac{i\omega C_1}{r_+} \left\{ 1 + 2z + 3z^2 (1 - C_2) \right\} + \frac{\omega^2}{3r_+^2} \left\{ C_5 + (C_5 - 2C_2)^2 z + (C_5 - 5C_2^2) z^2 + (C_5 - 7C_2^2) z^3 \right\} \right]$$

$$+ 3z^3 (1 - C_2) \right\} \frac{1}{1 + z + z^2} - \frac{3\langle O_1 \rangle^2}{r_+^2} C_4(z) \right] G(z) = 0 \quad (34)

where $C_5(z) = 9C_3(z) - C_2$. To solve this equation, we rescale it by letting $z = \frac{z'}{a}$, where $a = \frac{\langle O_1 \rangle}{r_+^2}$ and then take the $a \to \infty$ limit. This leads to

$$G''(z') + G(z') = 0 \quad (35)$$
The solution of this equation reads

\[ G(z') = C_+ e^{z'} + C_- e^{-z'} \]
\[ \Rightarrow G(z) = C_+ e^{az} + C_- e^{-az} \]
\[ = C_+ e^{\frac{<O_1>}{r_+} z} + C_- e^{-\frac{<O_1>}{r_+} z} . \]  

(36)

The information about the integration constants \( C_+ \) and \( C_- \) can be obtained from the appropriate boundary condition. For \( \Delta = 1 \), the boundary condition can be obtained from eq.(34) by setting \( z = 1 \) in the equation. This gives

\[ G'(1) + \left[ \frac{2(\mathcal{O}_1)^2 C_4(1)}{r_+(3 - \frac{2i\omega}{r_+} C_1(1))} - \frac{i\omega}{3r_+} \left\{ 3C_1(1) - \frac{i\omega}{3r_+} (4C_5(1) - 14C_1^2(1)) \right\} \right] G(1) = 0 \]  

(37)

where

\[ C_2(1) = 1 \quad ; \quad C_3(1) \approx 1 + \frac{\kappa^2 \lambda^2}{3} \left( 1 - \frac{1}{4} b\lambda^2 \right) \]
\[ C_4(1) \approx 1 + \frac{\kappa^2 \lambda^2}{6} \left( 1 - \frac{1}{4} b\lambda^2 \right) \quad ; \quad C_5(1) = 9C_3(1) - C_1^2 . \]  

(38)

Substituting \( G(1) \) and \( G'(1) \) from eq.(36) in the boundary condition (37) yields up to first order in \( \omega \)

\[ \frac{C_+}{C_-} = -e^{-2a} \left\{ \frac{aC_4(1) - 3}{aC_4(1) + 3} + \frac{2iC_1\omega (2C_4(1)a^2 - 3)}{ar_+ (C_4(1)a + 3)^2} + \mathcal{O}(\omega^2) \right\} . \]  

(39)

We finally obtain the solution for \( A(z) \) from eqs.(29), (36). This reads

\[ A(z) = (1 - z) \left[ \frac{2(\mathcal{O}_1)^2 C_4(1)}{r_+(3 - \frac{2i\omega}{r_+} C_1(1))} - \frac{i\omega}{3r_+} \left\{ 3C_1(1) - \frac{i\omega}{3r_+} (4C_5(1) - 14C_1^2(1)) \right\} \right] G(1) = 0 \]  

(40)

To obtain the conductivity, we now expand the gauge field about \( z = 0 \):

\[ A(z) = A(0) + zA'(0) + \mathcal{O}(z^2) . \]  

(41)

Now in general \( A_x \) can be written as

\[ A_x = A_x^{(0)} + \frac{A_x^{(1)}}{r_+} z + \mathcal{O}(z^2) . \]  

(42)

Comparing eq.(41) and eq.(42), we have

\[ A_x^{(0)} = A(0) \quad ; \quad A_x^{(1)} = r_+ A'(0) . \]  

(43)

Now from the definition of conductivity and gauge/gravity correspondence, we have

\[ \sigma(\omega) = \frac{\langle J_x \rangle}{E_x} = \frac{iA_x^{(1)}}{\omega A_x^{(0)}} = -\frac{i}{\omega} r_+ A'(z = 0) \]
\[ = \frac{i}{\omega} \frac{1 - \frac{C_+}{C_-}}{1 + \frac{C_+}{C_-}} \langle \mathcal{O}_1 \rangle + \frac{1}{3} \left\{ 1 + \frac{\kappa^2 \lambda^2}{6} \left( 1 - \frac{1}{4} b\lambda^2 \right) \right\} . \]  

(45)
Substituting the value of \( \frac{C_4}{C_1} \), we obtain the low frequency expression for the conductivity to be

\[
\sigma(\omega) = \frac{i\langle \mathcal{O}_1 \rangle}{\omega} \left[ 1 + 2e^{-2a} \frac{aC_4(1) - 3}{aC_4(1) + 3} + 4e^{-2a} \frac{iC_1 \omega (2C_4(1)a^2 - 3)}{ar_+ (C_4(1)a + 3)^2} \right] \\
+ \frac{1}{3} \left\{ 1 + \frac{\kappa^2 \lambda^2}{6} \left( 1 - \frac{1}{4} b \lambda^2 \right) \right\}. \tag{46}
\]

The DC conductivity is now given by

\[
\text{Re}\sigma(\omega = 0) \sim e^{-2a} = e^{-2\langle \mathcal{O}_1 \rangle/r_+} \equiv e^{-\frac{E_g}{T}}. \tag{47}
\]

where

\[
E_g = \frac{3}{2\pi} \left\{ 1 - \frac{\kappa^2 \lambda^2}{6} \left( 1 - \frac{1}{4} b \lambda^2 \right) \right\} \langle \mathcal{O}_1 \rangle. \tag{48}
\]

Here we substitute \( r_+ \) from eq.\((21)\) and \( E_g \) is identified to be the band gap energy. Note that the band gap energy gets corrected due to backreaction and the BI parameter. We observe that the effect of the BI parameter vanishes when \( \kappa = 0 \). We also recover the band gap energy for different values of \( \kappa \) and \( b \) in Table 1. These values have been estimated by the Sturm-Liouville approach to calculate \( \lambda^2 \) for different values of \( \kappa \). The values have been obtained from [28] by setting \( d = 4, m^2 = -2 \) and \( \Delta = 1 \) in the appropriate equations. The results indicate that for a particular value of the BI parameter \( b \), the band gap energy decreases with increasing values of backreaction parameter \( \kappa \). Further, for a particular value of \( \kappa \), the band gap energy increases with increasing values of the BI parameter \( b \).

| \frac{E_g}{\langle \mathcal{O}_1 \rangle} | \kappa = 0.1 | \kappa = 0.2 | \kappa = 0.3 |
|---|---|---|---|
| \( b = 0.0 \) | 0.47646 | 0.47344 | 0.46845 |
| \( b = 0.1 \) | 0.47649 | 0.47356 | 0.46873 |
| \( b = 0.2 \) | 0.47652 | 0.47369 | 0.46901 |
| \( b = 0.3 \) | 0.47655 | 0.47382 | 0.46929 |

Now we present the self-consistent approach to obtain the conductivity expression. Here we essentially follow the approach in [15]. We first replace the potential with its average \( \langle V \rangle \) in a self-consistent way. With this approximation, the solution of \((25)\) reads

\[
A \sim e^{-i\sqrt{\omega^2 - \langle V \rangle} r_+} \sim (1 - z)^{-i\sqrt{\omega^2 - \langle V \rangle} \pi_+ \{ 1 + \frac{z^2 \lambda^2}{6} (1 - \frac{1}{4} b \lambda^2) \}}. \tag{49}
\]

Once again from the definition of conductivity and gauge/gravity dictionary, we obtain from eq.\((44)\)

\[
\sigma(\omega) = -\frac{i}{\omega} \frac{r_+ A'(z = 0)}{A(z = 0)}
\]

\[
= \frac{1}{3} \left\{ 1 + \frac{\kappa^2 \lambda^2}{6} \left( 1 - \frac{1}{4} b \lambda^2 \right) \right\} \sqrt{1 - \frac{\langle V \rangle}{\omega^2}}. \tag{50}
\]
We now need to estimate the average value of the potential. This reads

\[ \langle V \rangle = \frac{\int_0^\infty dr_+ V A^2(r_+)}{\int_0^\infty dr_+ A^2(r_+)} . \]  

(51)

From eq. (26) and \( \psi(z) = \frac{\langle O \Delta \rangle}{\sqrt{2\pi}} F(z) \) the potential reads

\[ V \approx \frac{\langle O \Delta \rangle^2}{r_+^{2\Delta-2}} z^{2\Delta-2} \left[ g_0(z) + g_1(z) \right] \]  

(52)

where we consider \( F(z) \approx 1 \). The main contribution to the average value \( \langle V \rangle \) in eq. (51) is from the vicinity of the boundary where \( r_+ \approx \frac{z}{\kappa} \). This is the interesting fact that for the leading order contribution in the nature of \( r_+ \) is independent of \( \kappa \) and \( b \) parameter. Substituting eq. (52) in eq. (51), we obtain the expression for \( \langle V \rangle \) to be

\[ \langle V \rangle \approx \langle O \Delta \rangle^2 \frac{1}{r_+^{2\Delta-2}} \int_0^\infty dz e^{2i\sqrt{\omega^2 - \langle V \rangle} r_+} z^{2\Delta-2} \left[ g_0(z) + g_1(z) \right] \]  

\[ \approx \langle O \Delta \rangle^2 \frac{\Gamma(2\Delta - 1)}{(-2i \sqrt{\omega^2 - \langle V \rangle})^{2\Delta-2}} . \]  

(53)

This is the self consistent equation for the average value of the potential \( \langle V \rangle \), which depends on the frequency \( \omega \). At the low frequency limit, we set \( \omega = 0 \) in eq. (53) which leads to

\[ \langle V \rangle^\Delta = \frac{\langle O \Delta \rangle^2}{2^{2\Delta-2}} \Gamma(2\Delta - 1) . \]  

(54)

For \( \Delta = 1 \), this gives

\[ \langle V \rangle = \langle O_1 \rangle^2 . \]  

(55)

Using this in eq. (50), the conductivity is given by

\[ \sigma(\omega) = \frac{1}{3} \left\{ 1 + \frac{\kappa^2 \lambda^2}{6} \left( 1 - \frac{1}{4} b \lambda^2 \right) \right\} \sqrt{1 - \frac{\langle O_1 \rangle^2}{\omega^2}} \]  

\[ = \frac{i \langle O_1 \rangle}{3\omega} \left\{ 1 + \frac{\kappa^2 \lambda^2}{6} \left( 1 - \frac{1}{4} b \lambda^2 \right) \right\} \sqrt{1 - \omega^2} \frac{\langle O_1 \rangle^2}{\langle O_1 \rangle^2} . \]  

(56)

It can be observed that the above result is in close agreement with the result obtained in eq. (46). However, this method does not capture the expression for the band gap energy.

4 Conclusions

We have analytically computed the conductivity of holographic superconductors in the framework of Born-Infeld electrodynamics away from the probe limit. By employing a perturbative approach, we have computed the backreacted bulk spacetime metric taking
into account the effect of the Born-Infeld electrodynamics. We then moved onto compute the conductivity which is found to contain the effects of the backreaction parameter $\kappa$ and the BI parameter $b$. It is observed that the energy gap gets corrected from the standard value due to the parameters $\kappa$ and $b$. The results show that the band gap energy decreases with increase in the values of the backreaction parameter $\kappa$ for a fixed value of the BI parameter $b$. Moreover, it increases with increase in $b$ for a fixed value of $\kappa$. We then perform the computation of conductivity by following a self consistent approach and finally compare the results obtained from the two approaches. As a future work, we can extend our analysis for Gauss-Bonnet black holes in 5-dimensions. Work in this direction is in progress.

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