RESONANT NEUTRINO OSCILLATIONS
AND SHOCK REVIVAL IN TYPE-II SUPERNOVAE

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ABSTRACT

The role of matter enhanced resonant neutrino oscillations in reviving a stalled shock in a type-II supernova through delayed neutrino heating is investigated. The extent of neutrino heating is estimated for the allowed possibility of complete flavour conversion self-consistently with the changes in nuclear equilibrium. The average internal energy per nucleon is substantially increased indicating the possibility of a robust explosion.

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The theory of supernova explosions has seen a lot of progress in the last fifteen years but it is still not able to deliver an explosion with the right energy. Though most of the ingredients developed for the scenario of gravitational collapse and shock propagation after core rebounding are believed to be correct and some have got confirmation through the detection of neutrinos of SN1987A, the strong dissipation of shock energy through nuclear dissociation and neutrino losses make the shock either fail to reach the edge of the core or emerge, after neutrino heating, with energies less than $10^{51}$ ergs, a factor of 3 to 4 less than observed values. Different mechanisms like improved pre-supernova conditions, soft equation of state, general relativity at high densities, convection and improved neutrino physics have all been invoked to solve this problem. In this letter we describe a novel scenario by coupling neutrino flavour conversion in matter (the MSW mechanism) with the delayed neutrino heating that can deliver extra energy to matter behind the shock to generate a more robust explosion. Fuller *et. al.* [1] noted this possibility of conversion of mu/tau neutrinos to electron type between the neutrino-sphere and the stalled shock for some ranges of neutrino mixing parameters and estimated the extra heating and the explosion energy. We incorporate this heating to a semianalytic evolutionary calculation of the thermodynamic conditions behind the shock front of a $25M_\odot$ star and demonstrate explicitly the change in nuclear statistical equilibrium (NSE). We also calculate the gain in internal energy achieved by the increased heating induced by neutrino flavour oscillations. The results are encouraging.

At present there are two possible scenarios for supernova explosion. For stars in the mass range $8M_\odot \leq M \leq 15M_\odot$, under some very special conditions on the size and structure of the core and the equation of state, the shock continues its outward propagation and the star explodes within some tens of milliseconds after the beginning of collapse. This is the prompt explosion scenario [2]. For more massive stars the energy of the shock gets dissipated in dissociating nuclei and producing $\nu\bar{\nu}$ pairs and the shock stalls at a radius of a few hundred kms and becomes an accretion shock. It
is subsequently revived by the heating caused by neutrinos from the neutrino-sphere. This is the delayed explosion mechanism \cite{3,4}. This late time neutrino heating behind the shock is caused by their absorption reactions on the nuclei as well as free nucleons and by charged and neutral current scattering reactions. Ray and Kar \cite{3} considered a schematic model for delayed neutrino heating with matter in the form of a typical heavy nucleus $^{56}Fe$, $\alpha$-particles, neutrons and protons. They calculated the amount of neutrino heating treating changes in the NSE behind the shock front in a self-consistent fashion. Subsequently, Haxton \cite{6} pointed out the necessity of including the neutral current inelastic scattering of nuclei to giant resonance states in calculating the neutrino heating rates. Although all three flavours participate in this, the $\nu_\mu$ and $\nu_\tau$ neutrinos contribute more to the heating by the above process because of their higher equilibrium temperature. In \cite{6}, the charged current cross-sections of $\nu_\ell s$ and $\bar{\nu}_\ell s$ on nuclei for some typical neutrino temperatures are also tabulated. Ray and Kar \cite{7} incorporated these cross-sections in their semianalytic model \cite{3} and computed the thermodynamic state of of matter behind the shock as a function of time and estimated the heating rates. Fuller et. al. \cite{1} pointed out that matter enhanced oscillations of neutrinos in the region between the neutrino-sphere and the stalled shock can increase the heating rate appreciably and may result in a delayed explosion with an energy $\geq 10^{51}$ ergs, for a cosmologically significant $\nu_\mu$ or $\nu_\tau$ mass of 10 -100 eV and small vacuum mixing angles. Due to neutrino flavour mixing $\nu_\mu s$ or $\nu_\tau s$ (which carry a higher average energy) get converted to $\nu_\ell s$, the upshot being the production of higher energy $\nu_\ell s$ which can heat the shock more effectively.

We re-examine the situation modelled in \cite{7} with the addition of neutrino heating in the presence of matter induced neutrino oscillations. We consider a thin layer of matter behind the shock, at a radius $R_m$, which gets heated by the neutrinos coming from the neutrino-sphere situated at a distance $R_\nu$ from the center. Matter enhanced resonant neutrino oscillations have been considered extensively in the context of the solar neutrino problem \cite{8}. The intrinsic neutrino properties needed for the purpose are
neutrino mass squared difference, $\Delta m^2$, and the mixing angle in vacuum, $\theta_V$. Oscillation of neutrinos in matter is different from that in vacuum because interactions modify their dispersion relation and consequently they develop an effective mass dependent on the matter density. The mixing angle in matter can be expressed as

$$\tan 2\theta_M = \tan 2\theta_V / (1 - E_\nu / E_A)$$

(1)

$E_\nu$ is the neutrino energy; $E_A$ is given by $E_A = \Delta m^2 \cos 2\theta_V / 2\sqrt{2}G_F n_e$, where $n_e$ denotes the electron density. As is seen from (1) $\tan 2\theta_M$ goes through a resonance when $E_\nu = E_A$. From this one gets an expression for the resonance density as [1],

$$\rho_{res} = (2.108 \times 10^{10} \text{gm/cc}) \left( 0.5 / Y_e \right) \left( \frac{\Delta m^2 \cos 2\theta_V}{1600 \text{eV}^2} \right) \left( \frac{1 \text{MeV}}{E_\nu} \right)$$

(2)

At resonance, $\theta_M$ is $\pi/4$ and one gets maximal flavour mixing. The effectiveness of neutrino oscillations in increasing the heating rate depends crucially on whether a resonance is encountered between the neutrino-sphere and the shock front or not. If a neutrino of flavour $f$ encounters a resonance between the neutrino-sphere and the shock then its probability to remain a $\nu_f$ at the position of the shock is given as,

$$P(\nu_f \to \nu_f) = 0.5 + (0.5 - P_J) \cos 2\theta_{R_\nu} \cos 2\theta_{R_m}$$

(3)

where $\theta_{R_\nu}$ ($\theta_{R_m}$) is the neutrino mixing angle in matter at the position of the neutrino-sphere (shock). $P_J$ is the non-adiabatic transition probability between the two neutrino states and can be expressed in the Landau-Zener approximation as, $P_J = \exp(-E_{NA} / E_\nu)$ where,

$$E_{NA} = (0.026 \text{MeV}) \sin^2 2\theta_V \left( \frac{\Delta m^2}{1 \text{eV}^2} \right) \left( \frac{r_{res}}{1 \text{cm}} \right)$$

(4)

$r_{res}$ is the resonance radius. $E_{NA}$ can be obtained using (2) and a density profile [4] $\rho = (10^{31} \text{gm/cc})(r/1\text{cm})^{-3}$. The above equations are for oscillation between two neutrino flavours. In what follows we take these two flavours as the electron and the tau neutrino. A neutrino passing through the resonance density can undergo complete flavour transformation for suitable values of the parameters. A total conversion of the
\(\nu_\tau\), which carry a higher energy, to \(\nu_e\) between the neutrino-sphere and the shock will generate maximum heating. Ideally, one should couple the neutrino flavour transformations with the hydrodynamics of the shock and calculate the probabilities of conversion between the neutrino-sphere and the shock at each instant and find the altered heating rates and the consequent changes in NSE self-consistently. In this letter we have estimated the heating rates for full flavour conversion between the neutrino-sphere and the shock.

We consider matter consisting of nucleons as well as nuclei and incorporate charge current cross-sections on nuclei from [6], where these are listed up to a temperature of 6 MeV. If flavour conversion takes place, the \(\nu_e\) temperature can be higher. A simple-minded extrapolation yields a much higher cross-section and consequently a higher heating. We take a conservative standpoint and use the cross-section given for 6 MeV as the value at higher energies. We also incorporate the neutral current inelastic scattering on nuclei. As pointed out in [6] and later shown explicitly in [7], such reactions can increase the shock heating rates substantially. However, these processes being flavour independent, the heating due to them is unaffected by neutrino oscillations. The energy absorbed by matter behind the shock front/gm/sec is given by, 
\[
\dot{E} = \dot{E}_1 - \dot{E}_2 + \dot{E}_{\text{scatt}},
\]
\(\dot{E}_1\) is the heating rate due to charged current absorption of \(\nu, \bar{\nu}\) on free nucleons and nuclei. \(\dot{E}_2\) is the rate of energy loss due to neutrino radiation at matter temperature \(T_m\) and \(\dot{E}_{\text{scatt}}\) is due to \(\nu\)-e scattering and is \(T_m/T_\nu\) times the difference of the first two terms [4] plus the contribution due to inelastic neutral current scattering on nuclei [6]. With complete flavour transformation these terms are different from what they were in the absence of oscillation [4, 6], \(T_{\nu_e}\) in these terms being replaced by \(T_{\nu_\tau}\). The resulting increase in \(\dot{E}\) is quite significant. The energy absorbed first goes into dissociating the iron nuclei into \(\alpha\)-particles via \(^{56}\text{Fe} \rightleftharpoons 13\alpha + 4n\). The \(\alpha\)-particles then break up into protons and neutrons through \(\alpha \rightleftharpoons 2p + 2n\). The Q-values per nucleon for these are 2.2 MeV and 7.075 MeV respectively. The exact nuclear composition at each instant of time is governed by the Saha equations [5]. The change in entropy \(\Delta S\) due to the
absorption of energy and changes in nuclear composition in time \( \Delta t \) is given, in the limit of zero mass accretion, by

\[
T_m \Delta S = \dot{E} \Delta t - 2.2 \Delta X_{Fe} - 7.075 \Delta X_{\alpha}
\]  

(5)

In this \( X_i \) denotes the relative fraction of species \( i \), \( \Delta X_i \) is the change in time \( \Delta t \). For non-zero mass accretion there will be additional terms in (3) due to the kinetic energy of the accreting matter plus the terms due to compositional changes of the matter as it moves across the shock front. The corresponding change in temperature \( \Delta T \) is obtained as in [7]. The internal energy of the matter per nucleon \( \epsilon_{int} \) has contributions coming from baryons, electrons and photons. The success or failure of this phase of heating depends on the comparative values of \( \epsilon_{int} \) and \( \epsilon_{cr} = GM_c/R_c \), where \( R_c \) denotes the bifurcation radius and \( M_c \) is the mass included within \( R_c \).

For illustrative purposes numerical calculations are performed for a 25\( M_\odot \) star [3] with an initial iron core mass of 1.37\( M_\odot \). The electron neutrino luminosities \( (L_{\nu_e}) \) for which calculations are performed are \( 4 \times 10^{52} \) and \( 2 \times 10^{52} \) ergs/sec with the corresponding temperatures of neutrino-spheres as 4.45 and 5.3 MeV. The luminosities of neutrinos and antineutrinos of all flavours are assumed to be the same. The \( \nu_e \)-sphere radius, \( R_\nu \), is 30 km. The temperatures of the \( \mu \) and \( \tau \) neutrino-spheres are taken as 10 MeV. The shock stalls initially at a radius of 480 kms. It then recedes upto 189 kms and moves forward again [7]. We consider non-zero mass accretion with the rate 0.1 \( M_\odot \)/sec.

Demanding that a resonance is achieved between the neutrino-sphere and the shock so that \( R_\nu \leq r_{res} \leq R_m \), an idea regarding the bounds on \( \Delta m^2 \) can be obtained under the assumption of small mixing angles. For complete flavour conversion one has to ensure that not only is a resonance attained, but \( P(\nu_f \rightarrow \nu_f) \) as given by (3) is zero. (99 - 100)% flavour conversion of all neutrinos in the energy range 1-100 MeV requires

\[
1.13 \times 10^4 eV^2 \leq \Delta m^2 \leq 1.62 \times 10^4 eV^2, \quad 1.83 \times 10^{-7} \leq \sin^2 2\theta_V/\cos 2\theta_V \leq 7.85 \times 10^{-3}.
\]

This is the most stringent bound on the parameter values. However, heating is most
effective for neutrino energies 10-35 MeV. For such neutrinos the allowed area in the parameter space is $2.95 \times 10^3 eV^2 \leq \Delta m^2 \leq 10^5 eV^2$, $1.45 \times 10^{-8} \leq \sin^2 2\theta_V/cos 2\theta_V \leq 3.39 \times 10^{-2}$. This is consistent with the cosmological limit of $m_{\nu_e} \leq 100$ eV [10].

The nonadiabatic MSW solution to the solar neutrino problem, which requires $\Delta m^2 \sim 6 \times 10^{-6} eV^2$ and $\sin^2 2\theta_V \sim 7 \times 10^{-3}$ [11] is due to $\nu_e-\nu_\mu$ oscillations in this framework.

Figure 1a shows the time evolution of the fraction of $\alpha$-particles, neutrons and protons with and without oscillation for $L_{\nu_e} = 4 \times 10^{52}$ ergs/sec. The starting point of our calculations $t=0.417$ sec is the time at which the shock starts retreating, where $t=0$ corresponds to core bounce. By this time iron is completely dissociated for the $25M_\odot$ star considered. The energy delivered to the matter goes to dissociate the $\alpha$-particles present. As is seen from figure 1a, in the presence of neutrino flavour conversion, dissociation of $\alpha$’s takes place at a faster rate because of the enhanced heating. This further raises the heating rate due to an increased free nucleon fraction. Figure 1b shows the total internal energy per nucleon as a function of time for two representative luminosities $L_{\nu_e} = 4 \times 10^{52}$ ergs/sec, $2 \times 10^{52}$ ergs/sec. For a $25M_\odot$ star, for the two chosen luminosities, even without oscillation $\epsilon_{int}$ is greater than the critical energy $\epsilon_{cr}$. For $L_{\nu_e} = 4 \times 10^{52}$ ergs/sec, neutrino oscillation increases it by 61.33% at the end of this heating phase. In case of $L_{\nu_e} = 2 \times 10^{52}$ ergs/sec, $\epsilon_{int}$ was just above the critical value but complete flavour conversion raises it by 56.37%.

The results of [4] or [12] indicate that this phase of shock revival is followed by the creation of a quasi-vacuum, dominated by radiation, which ends with an eventual expulsion of matter. Our initial results indicate that resonant neutrino oscillations may indeed be the important ingredient, hitherto missing, in the delayed neutrino heating scenario to give a stronger shock in conformity with observation. One needs to extend these calculations to later times as well as incorporate the physics of neutrino oscillations in hydrodynamic codes. We are also studying the effect of these oscillations on a $18M_\odot$ star [12] with neutrino luminosities as given in [13] for its importance in connection to SN1987A. Earlier calculations for a $18M_\odot$ star without neutrino oscilla-
tions failed to strengthen the shock adequately [7]. Our preliminary calculations show that with matter induced neutrino oscillation the internal energy becomes greater than the critical energy in this case as well. In conclusion, we stress that resonant neutrino oscillations open up exciting possibilities for type-II supernove through the delayed neutrino heating mechanism and merit detailed examination.

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FIGURE CAPTION

Fig.1. Time evolution of (a). $X_p$, $X_n$, $X_\alpha$ for a $25M_\odot$ star with $R_\nu = 30$ km. The solid (dashed) curves are with (without) neutrino oscillations. (b). The internal energy per nucleon $\epsilon_{int}$ of a mass shell behind the shock front. Curves I (II) are for luminosities $L_\nu = 2 \times 10^{52}$ ($L_\nu = 4 \times 10^{52}$) ergs/sec. Also shown is the critical energy $\epsilon_{cr}$. 