Few-body physics in effective field theory

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Abstract. Effective Field Theory (EFT) provides a powerful framework that exploits a separation of scales in physical systems to perform systematically improvable, model-independent calculations. Particularly interesting are few-body systems with short-range interactions and large two-body scattering length. Such systems display remarkable universal features. In systems with more than two particles, a three-body force with limit cycle behavior is required for consistent renormalization already at leading order. We will review this EFT and some of its applications in the physics of cold atoms and nuclear physics. In particular, we will discuss the possibility of an infrared limit cycle in QCD. Recent extensions of the EFT approach to the four-body system and N-boson droplets in two spatial dimensions will also be addressed.

1. Introduction

The Effective Field Theory (EFT) approach provides a powerful framework that exploits the separation of scales in physical systems [1]. Only low-energy (or long-range) degrees of freedom are included explicitly, while all others are parametrized in terms of the most general local contact interactions. This procedure exploits the fact that a low-energy probe of momentum \( k \) cannot resolve structures on scales smaller than \( 1/k \). (Note that \( \hbar = 1 \) in this talk.) Using renormalization, the influence of short-distance physics on low-energy observables is captured in a few low-energy constants. Thus, the EFT describes universal low-energy physics independent of detailed assumptions about the short-distance dynamics. All physical observables can be described in a controlled expansion in powers of \( kl \), where \( l \) is the characteristic low-energy length scale of the system. The size of \( l \) depends on the system under consideration: for a finite range potential, e.g., it is given by the range of the potential. For the systems discussed here, \( l \) is of the order of the effective range \( r_e \).

We will focus on applications of EFT to few-body systems with large S-wave scattering length \( |a| \gg l \). For a generic system, the scattering length is of the same order of magnitude as the low-energy length scale \( l \). Only a very specific choice of the parameters in the underlying theory (a so-called fine tuning) will generate a large scattering length \( a \). Nevertheless, systems with large \( a \) can be found in many areas of physics. The fine tuning can be accidental or it can be controlled by an external parameter. Examples with an accidental fine tuning are the S-wave scattering of...
nucleons or of $^4$He atoms. The scattering length of alkali atoms close to a Feshbach resonance can be tuned experimentally by adjusting the external magnetic field. At very low energies these systems behave similarly and show universal properties associated with the large $a$ \cite{2}. We will start with a brief review of the EFT for few-body systems with large $a$ and then discuss some applications in nuclear and atomic physics.

2. Three-body system with large scattering length

For typical momenta $k \sim 1/|a|$, the EFT expansion is in powers of $l/|a|$; in leading order we can set $l = 0$. We start with a two-body system of non-relativistic bosons with large S-wave scattering length $a$ and mass $m$. The generalization to fermions is straightforward. In the following, we will refer to the bosons simply as atoms. At sufficiently low energies, the most general Lagrangian may be written as:

$$\mathcal{L} = \psi^\dagger \left(i \partial_t + \frac{\vec{\nabla}^2}{2m}\right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2 - \frac{D_0}{6} (\psi^\dagger \psi)^3 + \ldots,$$

where the dots represent higher-order derivative terms which are suppressed at low-energies. The strength of the two-body interaction $C_0$ is determined by the scattering length $a$, while $D_0$ depends on a three-body parameter to be introduced below. For momenta $k$ of the order of the inverse scattering length $1/|a|$, the problem is nonperturbative in $ka$. The exact two-body scattering amplitude can be obtained analytically by summing the so-called bubble diagrams with the $C_0$ interaction term. The $D_0$ term does not contribute to two-body observables. After renormalization, the resulting amplitude reproduces the leading order of the well-known effective range expansion for the atom-atom scattering amplitude: $f_{AA}(k) = (-1/a - ik)^{-1}$, where the total energy is $E = k^2/m$. If $a > 0$, $f_{AA}$ has a pole at $k = i/a$ corresponding to a shallow dimer with binding energy $B_2 = 1/(ma^2)$. Higher-order derivative interactions are perturbative and give the momentum-dependent terms in the effective range expansion.

We now turn to the three-body system. Here, it is useful to introduce an auxiliary field for the two-atom state (see Ref. \cite{2} for details). At leading order, the atom-dimer scattering amplitude is given by the integral equation shown in Fig. 1. A solid line indicates a single atom and a double line indicates the interacting two-atom state (including rescattering corrections). The integral equation contains contributions from both the two-body and the three-body interaction terms. The inhomogeneous term is given by the first two diagrams on the right-hand side: the one-atom exchange diagram.

![Figure 1. Integral equation for the atom-dimer scattering amplitude. Single (double) line indicates single atom (two-atom state).](image)
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and the three-body interaction. The integral equation simply sums these diagrams to all orders. After projecting onto S-waves, we obtain the equation

$$ T(k, p; E) = \frac{16}{3a} M(k, p; E) + \frac{4}{\pi} \int_0^{\Lambda} \frac{dq q^2 M(q, p; E) T(k, q; E)}{-1/a + \sqrt{3q^2/4 - mE - i\epsilon}}, $$

for the off-shell atom-dimer scattering amplitude with the inhomogeneous term

$$ M(k, p; E) = \frac{1}{2pk} \ln \left( \frac{p^2 + pk + k^2 - mE}{p^2 - pk + k^2 - mE} \right) + \frac{H(\Lambda)}{\Lambda^2}. $$

The logarithmic term is the S-wave projected one-atom exchange, while the term proportional to $H(\Lambda)$ comes from the three-body force. The S-wave atom-dimer scattering amplitude $f_{AD}(k) = [k \cot \delta_0 - ik]^{-1}$ is given by the solution $T$ evaluated at the on-shell point: $f_{AD}(k) = T(k, k; E)$ where $mE = 3k^2/4 - 1/a^2$. The three-body binding energies $B_3$ are given by those values of $E < 0$ for which the homogeneous version of Eq. (2) has a nontrivial solution.

Note that an ultraviolet cutoff $\Lambda$ has been introduced in (2). This cutoff is required to insure that Eq. (2) has a unique solution. All physical observables, however, must be independent of $\Lambda$, which determines the behavior of $H$ as a function of $\Lambda$ [3]:

$$ H(\Lambda) = \frac{\cos[s_0 \ln(\Lambda/\Lambda_*) + \arctan s_0]}{\cos[s_0 \ln(\Lambda/\Lambda_*) - \arctan s_0]}, $$

where $s_0 = 1.00624$ is a transcendental number and $\Lambda_*$ is a three-body parameter introduced by dimensional transmutation. This parameter cannot be predicted by the EFT and must be determined from a three-body observable. Note also that $H(\Lambda)$ is periodic and runs on a limit cycle. When $\Lambda$ is increased by a factor of $\exp(\pi/s_0) \approx 22.7$, $H(\Lambda)$ returns to its original value.

In summary, two parameters are required to specify a three-body system at leading order in $l/|a|$: they may be chosen as the scattering length $a$ (or equivalently $B_2$ if $a > 0$) and the three-body parameter $\Lambda_*$. [3]

3. Universal properties of few-body systems with large scattering length

This EFT confirms and extends the universal predictions for the three-body system first derived by Efimov [4]. The best known example is the Efimov effect, the accumulation of infinitely many three-body bound states at threshold as $a \rightarrow \pm \infty$. Universality also constrains three-body scattering observables. The atom-dimer scattering length, e.g., can be expressed in terms of $a$ and $\Lambda_*$ as [4] [5]

$$ a_{12} = a \left( 1.46 - 2.15 \tan[s_0 \ln(a\Lambda_*) + 0.09] \right) (1 + \mathcal{O}(l/a)) , \quad a > 0. $$

Note that the log-periodic dependence of $H$ on $\Lambda_*$ is not an artifact of the renormalization and also shows up in observables like $a_{12}$. This dependence could, e.g., be tested experimentally with atoms close to a Feshbach resonance by varying $a$. Similar expressions can be obtained for other three-body observables such as scattering phase
shifts as well as rates for three-body recombination and dimer relaxation (including the effects of deeply-bound two-body states) [5].

Since up to corrections of order $l/|a|$, low-energy three-body observables depend on $a$ and $\Lambda_*$ only, they obey non-trivial scaling relations. If dimensionless combinations of such observables are plotted against each other, they must fall close to a line parametrized by $\Lambda_*$. An example of a scaling function relating $^4$He trimer ground and excited state energies $B_3^{(0)}$ and $B_3^{(1)}$ is shown in the left panel of Fig. 2. (A related scaling function was obtained in Ref. [6].) The data points show calculations using various approaches and $^4$He potentials (see Ref. [7] and references therein). Note that $a_B \equiv 1/\sqrt{mB_2}$.

![Figure 2](image-url)

**Figure 2.** Scaling functions relating the $^4$He trimer ground and excited state energies (left panel) and the $^4$He trimer ground state and the atom-dimer scattering length (right panel). The data points are calculations using various methods and $^4$He potentials (see Ref. [7] and references therein). Note that $a_B \equiv 1/\sqrt{mB_2}$.

Recently, we have extended the effective theory for large scattering length to the four-body system [8]. It is advantageous in this case to use an effective quantum mechanics framework, since one can start directly from the well-known Yakubovsky equations. We have shown that no four-body parameter enters at leading order in $l/|a|$. Therefore renormalization of the three-body system automatically guarantees renormalization of the four-body system. Consequently, there are universal scaling functions relating three- and four-body observables as well. As an example, we display
the correlation between the trimer and tetramer binding energies in Fig. 3. The left panel shows the correlation between the ground state energies while the right panel shows the correlation between the excited state energies. The data points are calculations using various methods and \(^4\)He potentials (see Ref. [9] and references therein). We conclude that universality is well satisfied in the three- and four-body systems of \(^4\)He atoms.

The existence of these scaling functions is a universal feature of systems with large scattering length and is independent of the details of the short-distance physics. Similar correlations between few-body observables appear for example also in nuclear systems. The correlation between the triton and \(\alpha\) particle binding energies (the Tjon line) can be explained using a similar effective theory [10]. The spin-singlet and spin-triplet scattering lengths \(a_s\) and \(a_t\) for nucleons are both significantly larger than the range of the nuclear force. This observation can be used as the basis for an EFT approach to the few-nucleon problem in which the nuclear forces are approximated by contact interactions with strengths adjusted to reproduce the scattering lengths \(a_s\) and \(a_t\) [11, 12]. This EFT does not contain explicit pion degrees of freedom and is sometimes referred to as \textit{pionless EFT}. The EFT involves an isospin doublet \(N\) of Pauli fields with two independent 2-body contact interactions: \(N^\dagger \sigma_i N^c N^c N^c \sigma_i N\) and \(N^\dagger \tau_k N^c N^c N^c \tau_k N\), where \(N^c = \sigma_2 \tau_2 N^\ast\). Renormalization in the 2-body sector requires the two coupling constants to be adjusted as functions of \(\Lambda\) to obtain the correct values of \(a_s\) and \(a_t\). Renormalization in the 3-body sector requires a 3-body contact interaction \(N^\dagger \sigma_i N^c N^c \sigma_j N N^\dagger \sigma_i \sigma_j N\) with a coupling constant proportional to \(\Lambda\). Thus the renormalization involves an ultraviolet limit cycle. The scaling-violation parameter \(\Lambda\) can be determined by using the triton binding energy \(B_t\) as input. Effective range effects and other higher order corrections can be treated in perturbation theory [13].

In Fig. 4 we show the result for the Tjon line with \(a_s\) and \(B_d\) as input (lower line) and \(a_s\) and \(a_t\) as two-body input (upper line). Both lines generate a band that gives a naive estimate of higher order corrections in \(\ell/|a|\). We also show some calculations using phenomenological potentials [14] and a chiral EFT potential with explicit pions [15]. All
Figure 4. The correlation between the binding energies of the triton and the α-particle (the Tjon line). The lower (upper) line shows our leading order result using $a_s$ and $B_d$ ($a_t$ and $a_t^*$) as two-body input. The gray circles and triangles show various calculations using phenomenological potentials without or including three-nucleon forces, respectively. The squares show the results of chiral EFT at NLO for different cutoffs while the diamond shows the N$^2$LO result. The cross shows the experimental point.

Calculations with interactions that give a large scattering length should lie close to this line. Different short-distance physics and/or cutoff dependence should only move the results along the Tjon line. This can for example be observed in the NLO results with the chiral potential indicated by the squares in Fig. 4.

4. An infrared renormalization group limit cycle in QCD

The low-energy few-nucleon problem can also be described by an EFT that includes explicit pion fields. Such an EFT has been used to extrapolate nuclear forces to the chiral limit of QCD in which the pion is exactly massless [16, 17]. The extrapolation to larger values of $m_\pi$ predicts that $a_t$ diverges and the deuteron becomes unbound at a critical value in the range $170 \text{ MeV} < m_\pi < 210 \text{ MeV}$. It is also predicted that $a_s$ is likely to diverge and the spin-singlet deuteron become bound at some critical value of $m_\pi$ not much larger than 150 MeV. Both critical values are close to the physical value $m_\pi = 135 \text{ MeV}$.

Chiral extrapolations can also be calculated using the EFT without pions [18]. The inputs required are the chiral extrapolations $a_s(m_\pi)$, $a_t(m_\pi)$, and $B_t(m_\pi)$, which can be calculated using an EFT with pions. As an illustration, we take the central values of the error bands for the inverse scattering lengths $1/a_s(m_\pi)$ and $1/a_t(m_\pi)$ from the chiral extrapolation in Ref. [17]. Since the chiral extrapolation of the triton binding energy $B_t(m_\pi)$ has not yet been calculated and since $A_*$ should vary smoothly with $m_\pi$, we approximate it by its physical value $A_* = 189 \text{ MeV}$ for $m_\pi = 138 \text{ MeV}$. In Fig. 5
we show the resulting 3-body spectrum in the triton channel as a function of \( m_\pi \). Near \( m_\pi \approx 175 \text{ MeV} \) where the decay threshold comes closest to \( \kappa = 0 \), an excited state of the triton appears. This excited state is a hint that the system is very close to an infrared limit cycle. In the case illustrated by Fig. 5, the value of \( m_\pi \) at which \( a_t \) diverges is larger than that at which \( a_s \) diverges. If they both diverged at the same value of \( m_\pi \), there would be an exact infrared limit cycle.

We conjecture that QCD can be tuned to this infrared limit cycle by adjusting the up and down quark masses \( m_u \) and \( m_d \) \cite{18}. As illustrated in Fig. 5, the tuning of \( m_\pi \), which corresponds to \( m_u + m_d \), is likely to bring the system close enough to the infrared limit cycle for the triton to have one excited state. We conjecture that by adjusting the two parameters \( m_u \) and \( m_d \) to critical values, one can make \( a_t \) and \( a_s \) diverge simultaneously. At this critical point, the deuteron and spin-singlet deuteron would both have zero binding energy and the triton would have infinitely-many increasingly-shallow excited states. The ratio of the binding energies of successively shallower states would rapidly approach a constant \( \lambda_0^2 \) close to 515. It may be possible to use a combination of lattice gauge theory and EFT to demonstrate the existence of this infrared RG limit cycle in QCD.

5. \( N \)-boson droplets in 2D

In this section, we consider the universal properties of weakly interacting bosons with large scattering length (or equivalently a shallow dimer state) in two spatial dimensions (2D) \cite{19}. In particular, we consider self-bound droplets of \( N(\gg 1) \) bosons interacting weakly via an attractive, short-ranged pair potential. Our analysis relies strongly on the property of asymptotic freedom of 2D bosons with an attractive interaction.
In 2D, any attractive potential has at least one bound state. For the potential $-g\delta^2(r)$ with small $g$, there is exactly one bound state with an exponentially small binding energy, $B_2 \sim \Lambda^2 \exp(-4\pi/g)$, where $\Lambda$ is the ultraviolet momentum cutoff (which is the inverse of the range of the potential). Asymptotic freedom provides an elegant way to understand this result. In 2D non-relativistic theory, the four-boson interaction $g(\psi^\dagger \psi)^2$ is marginal. The coupling runs logarithmically with the length scale $R$, and the running can be found by performing the standard renormalization group (RG) procedure. For $g > 0$, the coupling grows in the infrared, in a manner similar to the QCD coupling. The dependence of the coupling on the length scale $R$ is given by

$$g(R) = \left[ \frac{1}{g} - \frac{1}{4\pi} \ln(\Lambda^2 R^2) \right]^{-1},$$

so the coupling becomes large when $R$ is comparable to the size of the two body bound state $B_2^{-1/2}$. This is in essence the phenomenon of dimensional transmutation: a dynamical scale is generated by the coupling constant and the cutoff scale. It is natural, then, that $B_2$ is the only physical energy scale in the problem: the binding energy of three-particle, four-particle,... bound states are proportional to $B_2$. In contrast to three spatial dimensions, there is no Efimov effect (or Thomas collapse) and no three-body parameter is required. The $N$-particle binding energy $B_N$, however, can be very different from $B_2$ if $N$ is parametrically large. We use the variational method to calculate the size of the bound state. For a cluster of a large number of bosons, one can apply classical field theory. We thus have to minimize the expectation value of the Hamiltonian with respect to all field configurations $\psi(r)$ satisfying the constraint $N = \int d^2r \psi^\dagger \psi$. This is equivalent to a Hartree calculation with the running coupling constant $g(R)$ instead of the bare one. In the limit of a large number $N$ of particles in the droplet, some exact predictions can be obtained [19].

The system possesses surprising universal properties. Namely, if one denotes the size of the $N$-body droplet as $R_N$, then at large $N$ and in the limit of zero range of the interaction potential [19]:

$$R_{N+1}/R_N \approx 0.3417, \quad B_{N+1}/B_N \approx 8.567, \quad N \gg 1.$$  

(7)

The size of the bound state decreases exponentially with $N$: adding a boson into an existing $N$-boson droplet reduces the size of the droplet by almost a factor of three. Correspondingly, the binding energy of $N$ bosons $B_N$ increases exponentially with $N$. This implies that the energy required to remove one particle from a $N$-body bound state (the analog of the nucleon separation energy for nuclei) is about 88% of the total binding energy. This is in contrast to most other physical systems, where separating one particle costs much less energy than the total binding energy, provided the number of particles in the bound state is large. The $1/N$-corrections to Eqs. (7) are calculable.

For the universal predictions (7) to apply in realistic systems with finite-range interactions, the $N$-body bound states need to be sufficiently shallow and hence have a size $R_N$ large compared to the natural low-energy length scale $l$. Depending on the
physical system, \( l \) can be the van der Waals length \( l_{vdW} \), the range of the potential, or some other scale. As a consequence, Eqs. (7) are valid in such systems for \( N \) large, but below a critical value,

\[
1 \ll N \ll N_{\text{crit}} \approx 0.931 \ln(R_2/l) + \mathcal{O}(1).
\]  

At \( N = N_{\text{crit}} \) the size of the droplet is comparable to \( l \) and universality is lost. If there is an exponentially large separation between \( R_2 \) and \( l \), then \( N_{\text{crit}} \) is much larger than one and the condition (8) can be satisfied.

We can compare our prediction with exact few-body calculations for \( N = 3, 4 \). While the \( 1/N \)-corrections are expected to be relatively large in this case, we can estimate how the universal result for \( B_{N+1}/B_N \) is approached. The three-body system for a zero-range potential in 2D has exactly two bound states: the ground state with \( B_3^{(0)} = 16.522688(1) B_2 \) and one excited state with \( B_3^{(1)} = 1.2704091(1) B_2 \). Similarly, the four-body system for a zero-range potential in 2D has two bound states: the ground state with \( B_4^{(0)} = 197.3(1) B_2 \) and one excited state with \( B_4^{(1)} = 25.5(1) B_2 \). The prediction (7) applies to the ground state energies \( B_3^{(0)} \) and \( B_4^{(0)} \). The ratio \( B_3^{(0)}/B_2 \approx 16.5 \) is almost twice as large as the asymptotic value (7), while the ratio \( B_4^{(0)}/B_2 \approx 11.9 \) is already considerably closer. These deviations are expected for such small values of \( N \). Note, however, that the ratio of the root mean square radii of the two- and three-body wave functions is 0.306, close to the asymptotic value (7).

It would be interesting to test the universal predictions (7) both theoretically and experimentally for \( N > 4 \). On the theoretical side, Monte Carlo techniques appear to be a promising avenue. Furthermore, the experimental realizability of self-bound 2D boson systems with weak interactions should be investigated. According to the analysis of Ref. [22], the \( 1/N \) corrections to Eqs. (7) are small for \( N \gtrsim 6 \). Using (8), this requires \( R_2/\ell \gg 600 \). We are not aware of any physical system that satisfies this constraint. However, such a system could possibly be realized close to a Feshbach resonance where \( R_2 \) can be made arbitrarily large.

6. Conclusions

We have discussed the EFT for few-body systems with short-range interactions and large scattering length \( a \). The renormalization of the three-body system with large \( a \) in three spatial dimensions requires a one-parameter three-body force governed by a limit cycle already at leading order in the expansion in \( l/|a| \). As a consequence, two parameters are required to specify a three-body system: the scattering length \( a \) (or the dimer binding energy \( B_2 \)) and the three-body parameter \( \Lambda_* \). Once these two parameters are given, the properties of the three- and four-body systems are fully determined at leading order in \( l/|a| \).

The large scattering length leads to universal properties independent of the short-distance dynamics. In particular, we have discussed universal expressions for three-body observables and universal scaling functions relating various few-body observables for both atomic and nuclear systems. A more detailed account can be found in Ref. [2].
The success of the pionless EFT demonstrates that physical QCD is close to an infrared limit cycle. We have conjectured, that QCD could be tuned to the critical trajectory for the limit cycle by adjusting the up and down quark masses. The limit cycle would then be manifest in the Efimov effect for the triton \[18\].

In two spatial dimensions, the three-body parameter $\Lambda_*$ does not enter at leading order in the expansion in $l/|a|$ and $N$-body binding energies only depend on $B_2$. The asymptotic freedom of non-relativistic bosons with attractive interactions in 2D leads to some remarkable universal properties of $N$-body droplets \[19\].

The three-body effects discussed here will also become relevant in Fermi systems with three or more spin states. Future challenges include universality in the $N$-body problem for $N \geq 4$, effective range corrections, and a large scattering length in higher partial waves. (See Ref. \[2\] for more details.) A large P-wave scattering length appears for example in nuclear Halo systems such as $^6$He \[23\].

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References

[1] See for example: D.B. Kaplan, [arXiv:nucl-th/9506035](https://arxiv.org/abs/nucl-th/9506035); G.P. Lepage, [arXiv:nucl-th/9706029](https://arxiv.org/abs/nucl-th/9706029).
[2] E. Braaten and H.-W. Hammer, [arXiv:cond-mat/0410417](https://arxiv.org/abs/cond-mat/0410417).
[3] P.F. Bedaque, H.-W. Hammer, and U. van Kolck, Phys. Rev. Lett. 82, 463 (1999); Nucl. Phys. A 646, 444 (1999).
[4] V.N. Efimov, Sov. J. Nucl. Phys. 12, 589 (1971); 29, 546 (1979).
[5] E. Braaten and H.-W. Hammer, Phys. Rev. A 67, 042706 (2003); Phys. Rev. A 70, 042706 (2004).
[6] T. Frederico, L. Tomio, A. Delfino, and A.E.A. Amorim, Phys. Rev. A 60, R9 (1999).
[7] A.K. Motovilov, W. Sandhas, S.A. Sofianos, and E.A. Kolganova, Eur. Phys. J. D 13, 33 (2001).
[8] L. Platter, H.-W. Hammer, and U.-G. Meißner, Phys. Rev. A 70, 052101 (2004).
[9] D. Blume and C.H. Greene, J. Chem. Phys. 112, 8053 (2000).
[10] L. Platter, H.-W. Hammer, and U.-G. Meißner, Phys. Lett. B 607, 254 (2005).
[11] V. Efimov, Nucl. Phys. A 362, 45 (1981).
[12] P.F. Bedaque, H.-W. Hammer, and U. van Kolck, Nucl. Phys. A 676, 357 (2000).
[13] P.F. Bedaque, G. Rupak, H.W. Grieshammer, and H.-W. Hammer, Nucl. Phys. A 714, 589 (2003), and references therein.
[14] A. Nogga, H. Kamada and W. Glöckle, Phys. Rev. Lett. 85, 944 (2000).
[15] E. Epelbaum et al., Phys. Rev. Lett. 86, 4787 (2001); Phys. Rev. C 66, 064001 (2002).
[16] S. R. Beane and M. J. Savage, Nucl. Phys. A 717, 91 (2003); 713, 148 (2003).
[17] E. Epelbaum, U.-G. Meißner, and W. Glöckle, Nucl. Phys. A 714, 535 (2003).
[18] E. Braaten and H. W. Hammer, Phys. Rev. Lett. 91, 102002 (2003).
[19] H.-W. Hammer and D.T. Son, Phys. Rev. Lett. 93, 250408 (2004).
[20] L.W. Bruch, and J.A. Tjon, Phys. Rev. A 19, 425 (1979).
[21] E. Nielsen, D.V. Fedorov, and A.S. Jensen, Few-Body Syst. 27, 15 (1999).
[22] L. Platter, H.-W. Hammer, and U.-G. Meißner, Few-Body Syst. 35, 169 (2004).
[23] C.A. Bertulani, H.-W. Hammer, and U. van Kolck, Nucl. Phys. A 712, 37 (2002).