Self-Breaking Technicolor

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ABSTRACT: We propose a scenario in which the electroweak symmetry is spontaneously broken by an $SU(4)$ technicolor gauge interaction which also manages to break itself completely. The technicolor gauge bosons and technifermions are not confined by the technicolor force, but get large masses. Starting with a single technidoublet, one emerges with a complete standard model family of technifermions after the symmetry breaking is complete. This suggests a broad new avenue for model building. A few variations on the theme are mentioned.
1. Introduction

Technicolor[1] models of electroweak symmetry breaking are an attempt to break the electroweak symmetry using strongly coupled gauge theories rather than a fundamental Higgs scalar. Although technicolor does provide a natural solution to the hierarchy and triviality problems associated with the fundamental Higgs scalar, it is less successful at explaining the Yukawa couplings of the standard model. The simplest attempts at constructing a realistic fermion mass generation using extended technicolor[2] (ETC) lead to flavor-changing neutral currents at unacceptable levels[3]. There are other problems associated with pseudo-Nambu-Goldstone bosons which can be too light. Some of the proposals to avoid such problems include the assumption of a UV fixed point for the technicolor interaction[4], “walking” technicolor[5] in which the running of the technicolor coupling constant with scale is taken to be much slower than in QCD, and strong ETC models[6] in which the ETC interactions assist in the symmetry breaking. A common idea behind these attempts is to produce a drastic enhancement of the technifermion condensate compared to the Nambu-Goldstone-boson decay constant. This requires that the technicolor dynamics be very different from the dynamics of QCD.

In view of the difficulty in constructing a realistic technicolor model, it seems worthwhile to search for qualitatively different ways to realize the strongly coupled sector. One of the most radical qualitative changes one can make to the original QCD-like version of technicolor is to spontaneously break technicolor itself. This general idea has appeared recently in the literature under various guises. Renormalizable versions[7,8,9,10,11,12] of the top-quark condensate idea[13,14] have used a strongly coupled and spontaneously broken interaction in which the top quark plays the role of a techniquark. Models of spontaneously broken technicolor have also been proposed recently by Luty[15], Sundrum[16] and Hill, Kennedy, Onogi and Yu[17]. The latter work considered the possibility that if technicolor is spontaneously broken and thus does not confine, then contributions to the electroweak S parameter[18] (which poses yet another phenomenological challenge for technicolor) can be reduced somewhat. Chivukula, Golden and Simmons[19] have recently argued that the most enormous sort of hierarchies which have sometimes been proposed for models of this general type will typically be ruined by a Coleman-Weinberg instability unless the effective Higgs sector contains only one doublet. Here, we will be mainly concerned with one paradigm for realizing spontaneously broken technicolor and with the idea that the technicolor interaction manages to break itself completely, leaving behind only the standard model gauge group. We will make no attempt here to construct a fully realistic model; in
fact we will make no specific proposal for the ETC sector. Instead we will focus on some model building ideas as they affect the strong coupling sector only. Our motivation is to illustrate that technicolor need not be an unbroken, confining, QCD-like theory.

2. Technicolor Unconfined

Let us choose for our gauge group \( G = SU(4)_{TC} \times SU(3)_{C'} \times SU(2)_L \times U(1)_{Y'} \). There is one doublet of technifermions which transforms under \( G \) as

\[ \psi_L \sim (4, 1, 2, 0) \]
\[ \psi_R^c \sim (\overline{4}, 1, 1/2) + (\overline{4}, 1, -1/2) . \]  

(2.1)

[The gauge transformation properties of fermions are always given in terms of left-handed two-component Weyl fields in (2.1) and throughout the rest of this paper.] The standard model quarks and leptons transform under \( G \) as three copies of

\[ (u d)_L \sim (1, 3, 2, 1/6) \quad d^c_R \sim (1, \overline{3}, 1, 1/3) \quad u^c_R \sim (1, \overline{3}, 1, -2/3) \]
\[ (\nu e)_L \sim (1, 1, 2, -1/2) \quad e^c_R \sim (1, 1, 1, 1) . \]  

(2.2)

Since \( SU(4)_{TC} \) is asymptotically free, it will get strong in the infrared. The electroweak symmetry is then broken by the condensate

\[ \langle \overline{\psi}_L \psi_R \rangle = \mu^3 \delta^i_j \quad i, j = 1, 2 \]  

(2.3)

as usual in technicolor models.

So far this is just the one-doublet \( SU(4) \) technicolor model. But now we consider the possibility that the technicolor gauge group itself is broken. Specifically, suppose that \( SU(4)_{TC} \times SU(3)_{C'} \times U(1)_{Y'} \) is broken down to the diagonal \( SU(3)_C \times U(1)_Y \). [More precisely, this means that \( SU(3)_C \) is the diagonal subgroup of \( SU(3)_{TC} \times SU(3)_{C'} \) and \( U(1)_Y \) is the diagonal subgroup of \( U(1)_{TC} \) and \( U(1)_{Y'} \), where \( SU(3)_{TC} \times U(1)_{TC} \) is a maximal proper subgroup of \( SU(4)_{TC} \).] The unbroken \( SU(3)_C \) and \( U(1)_Y \) are the standard model color and weak hypercharge, respectively. The scale \( M \) which characterizes the breaking \( SU(4)_{TC} \times SU(3)_{C'} \times U(1)_{Y'} \rightarrow SU(3)_C \times U(1)_Y \) should not be too large, otherwise \( SU(4)_{TC} \) will get broken before it has a chance to get strong enough to form the condensate (2.3).

Of the twenty-four gauge bosons associated with \( SU(4)_{TC} \times SU(3)_{C'} \times U(1)_{Y'} \), eight gluons and one hyperphoton remain massless, corresponding to the unbroken gauge group \( SU(3)_C \times U(1)_Y \). At the scale \( M \), the coupling constants for the unbroken gauge groups are given by

\[ g_3 = g_3' g_4 / \sqrt{g_4^2 + g_3^2} \quad \text{and} \quad g_1 = g_1' g_4 / \sqrt{g_4^2 + 2g_1'^2 / 3} \]  

(2.4)
where \( g_3' \) and \( g_1' \) are the coupling constants for \( SU(3)_{C'} \) and \( U(1)_{Y'} \) respectively. Now we are assuming that \( SU(4)_{TC} \) is strongly coupled at \( M \) and that \( SU(3)_{C'}, SU(2)_L \) and \( U(1)_{Y'} \) are not, so that \( g_4 \gg g_3', g_2, g_1' \). Therefore \( g_3 \approx g_3' \) and \( g_1 \approx g_1' \) at the scale \( M \). The other fifteen gauge bosons, consisting of a color octet, triplet and antitriplet, and singlet, all get masses of order \( g_4 M \), where \( g_4 \) is the coupling constant for \( SU(4)_{TC} \).

After integrating out the fifteen heavy gauge bosons, one finds the following four-fermion interaction (to lowest order in \( g_3'/g_4 \) and \( g_1'/g_4 \)):

\[
L_{\text{eff}} = -\frac{g_4^2}{2M^2} J^A_{\mu} J^\dagger_A_{\mu}; \quad J^A_{\mu} = \bar{\psi}_L \gamma_\mu T^A \psi_L^i + \bar{\psi}_R \gamma_\mu T^A \psi_R^i. \tag{2.5}
\]

[Here \( T^A \) are the fifteen generators of \( SU(4) \).] This interaction can be Fierzed into a form which includes the term

\[
L_{\text{eff}} = \frac{15}{16} \frac{g_4^2}{M^2} (\bar{\psi}_L^i \psi_R^j)(\bar{\psi}_R^j \psi_L^i). \tag{2.6}
\]

This term is just a Nambu–Jona-Lasinio[20] (NJL) interaction representing an attractive force between the technifermions. One might now formulate a NJL interpretation of the strong coupling dynamics, in which if \( g_4 \) is sufficiently large at the scale \( M \), then the condensate \( (2.3) \) will form. This is a ruthlessly truncated version of the symmetry breaking dynamics, in which everything except technigluon exchange at zero momentum transfer (and in the \( SU(4)_{TC}\)-singlet channel after Fierzing) is neglected. It therefore seems prudent to refrain from attempting to use a NJL analysis to obtain a quantitative description of the symmetry breaking. Still, the NJL picture can provide a useful qualitative picture. The idea presented here can then be viewed as a specific renormalizable realization of refs. [21] and [17].

After the symmetry breaking \( SU(4)_{TC} \times SU(3)_{C'} \times U(1)_{Y'} \rightarrow SU(3)_C \times U(1)_Y \), the technidoublet \( \psi_L, \psi_R^c \) transforms as one complete standard model family (including a gauge singlet technineutrino) under \( SU(3)_C \times SU(2)_L \times U(1)_Y \):

\[
\psi_L \sim (3, 2, 1/6) + (1, 2, -1/2)
\]
\[
\psi_R^c \sim (\bar{3}, 1, -2/3) + (1, 1, 0) + (\bar{3}, 1, 1/3) + (1, 1, 1). \tag{2.7}
\]

These technifermions all get large masses associated with the electroweak-breaking condensate \( (2.3) \). The standard model quarks and leptons transform in exactly the same way, (except that there is no gauge singlet neutrino). From the point of view of low energy physics, \( SU(4)_{TC} \) is a Pati-Salam symmetry for the techniquarks only. The heavy octet and heavy singlet gauge bosons also couple to the standard model fermions, albeit weakly. This is because the mass eigenstates are related to the original gauge eigenstates of the
vector bosons by a small mixing angle, with the heavy vectors consisting mostly of $SU(4)_{TC}$
gauge bosons which do not couple to the standard model quarks and leptons. The heavy
color octet couplings to quarks are $\approx g_2^2/g_4$ and exactly mimic those of a heavy gluon, and
the heavy gauge singlet couplings to quarks and leptons are $\approx \sqrt{2/3} g_2^4/g_4$ times the weak
hypercharge. The heavy color triplet and antitriplet gauge bosons have no direct couplings
at all to the standard model quarks and leptons.

Some four-fermion effective interactions presumably must be included to communi-
cate the electroweak breaking to the standard model quarks and leptons and provide for
their masses. These should have the schematic form $(\lambda^2/\Lambda^2_{E TC})(\mathbf{3}_L q_R)(\mathbf{\bar{3}}_L \psi_R) + \text{h.c.}$ or
$(\lambda^2/\Lambda^2_{E TC})(\mathbf{\bar{3}}_R q_L)(\mathbf{3}_L \psi_R) + \text{h.c.}$ for the quarks, and similarly for the charged leptons. We
will not speculate in this paper on what underlying dynamics gives rise to such interactions.

How can we break $SU(4)_{TC}$? One possibility is to simply introduce a scalar field which
gets a vacuum expectation value. For example, one might introduce a scalar $\Phi$ in the rep
$(4, \mathbf{\bar{3}}, 1, -1/6)$ of $G$. Then if $\Phi$ develops a vacuum expectation value

$$
\langle \Phi^a_\alpha \rangle = M \delta^a_\alpha \quad a = 1, 2, 3, 4; \quad \alpha = 1, 2, 3
$$

the symmetry will break in the desired way. Of course, if $\Phi$ is a fundamental scalar,
then it carries with it the usual hierarchy and triviality problems which are the main
motivations for considering technicolor models in the first place! We would then require
an explanation for why loop corrections do not push $M$ to the Planck mass or some other
scale much higher than the scale at which $g_4$ becomes large enough to admit the possibility
of condensates. Therefore, we prefer to consider the possibility that $\Phi$, or some analogous
order parameter(s), is itself a composite field corresponding to a condensate of fermions in
a scalarless theory.

3. A Paradigm for Self-Breaking

An economical possibility is that $SU(4)_{TC}$ manages to break itself without the introd-
tion of an additional strongly coupled interaction. Suppose there are additional fermions
which transform nontrivially under both $SU(4)_{TC}$ and $SU(3)_{C'} \times U(1)_{Y'}$, and that they
can be chosen in such a way that they also form condensates due to the strong $SU(4)_{TC}$
force which then break $SU(4)_{TC}$ in the right way. It is clear that we will need at least one
fermion which transforms in a higher dimensional rep of $SU(4)_{TC}$. For, if only $\mathbf{4}s$ and $\mathbf{\bar{4}}s$
are present, then in order for the $SU(4)_{TC}$ anomaly to cancel, the number of $\mathbf{4}s$ must equal
the number of $\mathbf{\bar{4}}s$. As long as this is the case, then the condensates will always occur as
$SU(4)_{TC}$ singlets (because the 4s will just pair up with the $\overline{4}$s) and therefore cannot break $SU(4)_{TC}$. On the other hand, if there are too many fermions in higher dimensional reps of $SU(4)_{TC}$ then the $SU(4)_{TC}$ $\beta$ function will be positive. This would be a disaster since we need $g_4$ to get large in the infrared. This is a welcome restriction which relieves us of the responsibility of considering models which are too baroque.

Here is one scenario for self-breaking of $SU(4)_{TC}$. In addition to the technifermions in (2.1) and the standard model quarks and leptons in (2.2), we assign fermions to the following reps of $G$:

$$\eta \sim (10, 1, 1, 0); \quad \xi \sim (\overline{4}, 3, 1, -1/6) + (\overline{4}, 1, 1, 1/2);$$

$$\chi \sim (\overline{4}, 3, 1, 1/6) + (\overline{4}, 1, 1, -1/2).$$

[Note that the choice of which $(\overline{4}, 1, 1, \pm1/2)$ belong to $\psi_R^c$ and which to $\xi, \chi$ is just an arbitrary choice of orientation.] All of the gauge anomalies cancel for the fermions in (3.1). We have grouped the new fermions in the way indicated in (3.1) for the following reason. The gauge group $SU(3)_{C^\prime} \times U(1)_{Y^\prime}$ can be embedded into an approximate global symmetry group $SU(4)_{PS'} \times U(1)_R$ with $SU(4)_{PS'}$ an ungauged Pati-Salam symmetry. Now all of the fermions can be arranged into multiplets which are irreducible reps of $SU(4)_{TC} \times SU(4)_{PS'} \times SU(2)_L \times U(1)_R$. Under this group, $\eta^{(ab)}$ transforms as $(10, 1, 1, 0)$, $\xi_{\alpha\alpha}$ transforms as $(\overline{4}, 4, 1, 0)$, and $\chi^{\alpha}_a$ as $(\overline{4}, 4, 1, 0)$. [Latin letters $a, b, c \ldots$ represent $SU(4)_{TC}$ indices in the fundamental rep and Greek letters $\alpha, \beta, \gamma$ represent $SU(4)_{PS'}$ indices, with $\alpha, \beta, \gamma = 1, 2, 3$ corresponding to the gauged subgroup $SU(3)_{C^\prime}$. Note that $\eta^{(ab)}$ carries two symmetrized indices since the 10 of $SU(4)$ is the symmetrized direct product of two 4s.] The technifermions $\psi^a_L$ and $\psi_{\alpha a}^c$ transform as $(4, 1, 2, 0)$ and $(\overline{4}, 1, 1, \pm1/2)$ and the standard model fermions as $(1, 4, 2, 0)$ and $(1, \overline{4}, 1, \pm1/2)$. This classification is particularly useful because it will allow us to write the condensates which can occur in this model in a compact notation. [$SU(4)_{PS'}$ may also be a gauge symmetry broken at some energy scale much higher than any other scale of interest in this paper.]

Now $SU(4)_{TC}$ is still asymptotically free; its $\beta$ function is given to two loops by

$$\beta_4 = \mu \frac{dg_4}{d\mu} = -\frac{g_4^3}{16\pi^2} \left[ \frac{26}{3} + \frac{71}{6} \left( \frac{g_4}{4\pi} \right)^2 \right].$$

So the coupling constant $g_4$ grows in the infrared. When $SU(4)_{TC}$ gets strong enough, condensates involving $\psi_L, \psi_R^c, \eta, \xi, and \chi$ should form. A rigorous discussion of this process would require a complete understanding of the non-perturbative dynamics of strongly coupled and spontaneously broken gauge interactions, which we do not have. In order to
make progress, we must simply choose a plausible set of assumptions about the strong coupling dynamics and hope that they are correct.

To help decide qualitatively how the condensates arrange themselves, we can make use of the single gauge boson approximation, which we now briefly review. Consider a model which consists of an asymptotically free gauge theory [in our case $SU(4)_{TC}$] which couples to some fermions but no scalars. The fermions may also have weakly coupled gauge interactions [in our case $SU(3)_{C'} \times SU(2)_L \times U(1)_{Y'}$] whose effects may be treated perturbatively. When the strong gauge coupling becomes sufficiently large in the infrared, scalar fermion bilinear condensates will form in irreducible reps of the gauge group. Suppose that the fermions involved in the condensate transform under the strongly coupled gauge group in the irreducible reps $R_1$ and $R_2$, and the resulting condensate transforms as $R_s$. Thus $R_s$ occurs in the direct sum decomposition of the direct product $R_1 \times R_2 = R_s + \cdots$. We need a way of deciding for which choices of $R_1$, $R_2$, and $R_s$ the condensate will occur. According to the single gauge boson exchange approximation, the condensates will tend to appear in the “most attractive scalar channel” (MASC), $R_1 \times R_2 \rightarrow R_s$, for which $V = C_1 + C_2 - C_s$ is largest. Here $C_1$, $C_2$, and $C_s$ are the quadratic Casimir invariants for the representations $R_1$, $R_2$, and $R_s$, respectively. When a given fermion condenses, it obtains a self-energy term which suppresses its ability to participate in other condensates. We therefore assume that each fermion condenses at most once.

In the case of our model, the strongly coupled $SU(4)_{TC}$ has fermions transforming as a 10, ten $\overline{4}$s, and two 4s. The attractive channels for this fermion content, and their relative strengths $V$ are as follows$^\dagger$:

| Channel       | $V$ | Channel       | $V$ |
|---------------|-----|---------------|-----|
| $10 \times \overline{4} \rightarrow 4$ | 18  | $\overline{4} \times \overline{4} \rightarrow 6$ | 5   |
| $4 \times \overline{4} \rightarrow 1$  | 15  | $4 \times 4 \rightarrow 6$  | 5   |
| $10 \times 10 \rightarrow 20'$          | 12  | $10 \times 10 \rightarrow 45$ | 4   |
| $10 \times 4 \rightarrow 20$            | 6   |               |     |

Since channel (i) is the MASC in this naive approximation, we assume that $\eta$ condenses with some combination of $\overline{4}$s ($\xi, \chi$ and $\psi^{c}_{R}$).

However, the MASC criterion still leaves an important ambiguity, since the 10 can condense with one or with more than one $\overline{4}$. Indeed, there is always such an ambiguity in

$^\dagger$ All group theory conventions and facts used in this paper may be found from [22].
the MASC criterion whenever one of the reps appearing in the MASC occurs more than once in the list of massless fermions. To understand this ambiguity more clearly, let us use a notation in which all ten of the $\mathbf{4}$s are represented by a generic symbol $f^I_a$ with $I = 1 \ldots 10$. Then the fact that the condensate occurs in channel $(i)$ is precisely equivalent to the statement that it has the form

$$\langle \eta^{(ab)} f^I_c \rangle = \delta^a_c \delta^{(a}_{\alpha} \delta^{b)}_{\beta} M^3 \quad (3.2)$$

where $v^a_I$ is unknown at this point, and determines how much of the $SU(4)_{TC}$ and global symmetries are broken. The determination of $v^a_I$ is not just a vacuum alignment problem having to do with residual gauge symmetries; it is properly to be determined by the strongly coupled part of the theory. The question of how to resolve such ambiguities has been addressed by Gusynin, Miransky, and Sitenko[23]. Using their arguments, based on a stability analysis within a solvable approximation, we find (up to global $SU(10)$ and $SU(4)_{TC}$ rotations)

$$v^a_I = \delta^a_i \quad (I = 1 \ldots 4); \quad v^a_I = 0 \quad (I = 5 \ldots 10). \quad (3.3)$$

This agrees with the heuristic criterion in [23] that when such an ambiguity exists, the number of fermions which condense in the MASC is maximized. (See example (a) in [23] for a problem which is exactly analogous to the one discussed here.) Now, (3.3) means that $SU(4)_{TC}$ is completely broken in one step. It is not broken to a strongly coupled $SU(3)$ subgroup, which would require instead $v^a_I = \delta^a_4 \delta^1_I$. Of the original symmetry $SU(4)_{TC} \times SU(10)$, a global $SU(4) \times SU(6)$ is left unbroken by (3.3).

With $v^a_I$ given by (3.3), there still remains a vacuum alignment problem, having to do with the orientation of the weakly coupled gauge group $SU(3)_{C'} \times SU(2)_L \times U(1)_{Y'}$ with respect to the surviving global symmetry. This type of vacuum alignment problem has been studied in [24]. The vacuum tends to align so as to preserve as much of the residual gauge symmetry as possible. There turn out to be two distinct and equally good solutions to this vacuum alignment problem, one in which $\eta$ condenses entirely with $\chi$ and one in which it condenses entirely with $\xi$. We imagine for now that unspecified higher order effects prefer the latter solution, and will discuss the situation if the former solution wins in the next section. Thus $\eta$ condenses with $\xi$ according to

$$\langle \eta^{(ab)} \xi^{(ca)} \rangle = \delta^a_c \delta^{(a}_{\alpha} \delta^{b)}_{\beta} M^3 \eta^{3} \xi \quad . \quad (3.4)$$

There is a simple heuristic reason for the vacuum alignment (3.4); if $SU(3)_C$ is left unbroken, then the fermion pairs participating in the condensates will feel an additional attractive force due to $QCD$, because they transform as conjugate representations of $SU(3)_C$. 8
Thus the condensates will align as in (3.4) so as not to break $SU(3)_C$, because they can. The condensate (3.4) transforms under $SU(4)_{TC} \times SU(4)_{PS'}$ as $(4, \bar{4})$, and breaks $SU(4)_{TC} \times SU(4)_{PS'}$ down to the diagonal $SU(4)_{PS}$ of the standard model. [This is easily seen because the right-hand side of (3.4) is an invariant symbol of $SU(4)_{PS'}$.] Thus it also breaks the gauged subgroup $SU(4)_{TC} \times SU(3)_C \times U(1)_{Y'}$ down to $SU(3)_C \times U(1)_Y$ as desired.

Now we make an additional assumption. Since channel (ii) is only weaker than channel (i) by a factor of $6/5$ in the single gauge boson exchange approximation, we assume that channel (ii) also condenses. We assume that this is true even though the condensate of channel (i) breaks the $SU(4)_{TC}$ interaction. This assumption is equivalent to the assumption in the NJL language that the four-fermion interaction (2.5) is sufficiently strongly coupled to produce the condensate (2.3). Now, there is again a vacuum alignment problem, since the two $4$s of $\psi_L$ have a choice of $\bar{4}$s with which to condense, namely $\psi^C_R$, $\chi$, and the antisymmetric part of $\xi$ ($\xi_A$) which did not condense with $\eta$. Again, the vacuum chooses to align itself so that $SU(3)_C$ and $U(1)_{EM}$ are unbroken, because that allows for an additional attractive force between the condensing fermion pairs. That is why $\psi_L$ condenses with $\psi^C_R$ as in (2.3), and not in some other way which would break $SU(3)_C$ or $U(1)_{EM}$.

To recapitulate, we are assuming that the two condensates (2.3) and (3.4) both form, with roughly equal strength. The condensate (2.3) breaks the electroweak symmetry and the condensate (3.4) breaks technicolor. The scalar $\Phi$ of section 2 is replaced by the condensate $\langle \eta \xi \rangle$.

After the condensate (3.4) forms, the fermions $\eta$, $\xi$ and $\chi$ transform under the standard model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ as

$$
\eta \sim (6, 1, 1/3) + (3, 1, -1/3) + (1, 1, -1)
$$
$$
\xi_S \sim (\bar{3}, 1, -1/3) + (\bar{3}, 1, 1/3) + (1, 1, 1)
$$
$$
\xi_A \sim (\bar{3}, 1, 1/3) + (3, 1, -1/3)
$$
$$
\chi \sim (8, 1, 0) + (\bar{3}, 1, -2/3) + (3, 1, 2/3) + 2 \times (1, 1, 0)
$$

This shows that the QCD force aligning the condensates can be quite appreciable, especially since a color sextet from $\eta$ and an antisextet from $\xi_S$ are paired in the condensate (3.4). This should firmly stabilize the $SU(3)_C$-conserving vacuum. At this point, $\eta$ and $\xi_S$ pair up and obtain an effective mass term. So far, $\xi_A$ and $\chi$ remain uncondensed and massless, even though they transform as a real representation of the standard model gauge group.
Since these fermions have not been seen yet in nature, we must specify a mechanism to obtain large mass terms $\sim m_{\xi_A} \xi_A\xi_A$ and $\sim m_{\chi}\chi$. These mass terms are allowed by the standard model gauge group, but they are not generated at this stage because they break additional chiral symmetries which are left unbroken by (2.3) and (3.4).

A traditional assumption of “tumbling” gauge theories[25] is that when a condensate breaks the strongly coupled gauge theory, then any condensate corresponding to a channel which is weaker in the single gauge boson exchange approximation will not form. The idea behind this is that when the strongly coupled gauge bosons gain mass, they decouple and so their ability to produce condensates is vitiated. We have already assumed that this is not quite the case, since we assumed that condensates occur in both channels (i) and (ii), even though channel (i) is naively slightly stronger and breaks $SU(4)_{TC}$. Qualitatively, this is an assumption that the mass of the gauge boson is not large enough to prevent the condensate in channel (ii). One way for $\xi_A$ and $\chi$ to get large masses uses a slightly untraditional (but not, we think, outrageous) set of assumptions as follows. First, we continue to assume that each fermion can participate in at most one condensate. The rationale behind this is that once a fermion condenses, its self-energy so generated inhibits its ability to participate in further condensates. However, we now consider the further assumption that each strongly coupled fermion does condense exactly once. This could come about if the mass obtained by the strongly coupled gauge bosons is small enough to allow condensates to form in all attractive channels involving massless fermions.

Then the analysis of the condensation pattern might go as follows. As before, the condensates (2.3) and (3.4) appear due to channels (ii) and (i) respectively. Now because each fermion can condense only once (by assumption), we may forget about channels (iii), (iv), (vi) and (vii), since all of the 10s and 4s of $SU(4)_{TC}$ have been used up. The remaining fermions thus condense according to channel (v), even though this channel is weaker than channels (i) and (ii). The resulting condensates will take the form:

$$\langle \xi_a \xi_b \rangle = \epsilon_{aab} M_{\xi\xi}^3 \quad (3.6)$$
$$\langle \chi^a \chi^b \rangle = \delta^a_b \epsilon_{a} M_{\chi\chi}^3 \ . \quad (3.7)$$

(The brackets in (3.7) indicate antisymmetrized indices.) Once again we have used the fact that these condensates will align so as to leave $SU(3)_C$ unbroken. Each of the condensates $\langle \xi \xi \rangle$ and $\langle \chi \chi \rangle$ transforms under $SU(4)_{TC} \times SU(4)_{PS}$ as (6, 6), and leaves the diagonal $SU(4)_{PS}$ with its gauged subgroup $SU(3)_C \times U(1)_Y$ unbroken, since the right-hand sides of (3.6) and (3.7) are again invariant symbols of $SU(4)_{PS}$. One component of $\langle \chi \chi \rangle$ involves
a QCD octet condensing with itself. This stabilizes the $SU(3)_C$-conserving vacuum and also can enhance the condensate considerably.

Note that now, each of the components of $\eta, \xi, \chi$ participates in exactly one condensate. The symmetric component of $\xi$ condenses with $\eta$ and the antisymmetric component of $\xi$ condenses with itself. Modulo the assumptions stated above, this is one way of assuring that all of the fermions $\eta, \xi$ and $\chi$ obtain large masses. In the next section we will discuss a variation of the basic model in which $\xi_A$ and $\chi$ get their masses from a strongly coupled interaction which also produces a mass for the top quark.

As usual in technicolor models, the strong interactions have a large approximate global symmetry which contains the electroweak symmetry as a subgroup. A host of pseudo-Nambu-Goldstone bosons (PNGBs) arise when the condensates break this approximate global symmetry. In our case, the approximate symmetry of the strong interactions is $SU(4)_{TC} \times SU(10) \times SU(2)_L \times U(1) \times U(1)$. (There would be another $U(1)$ global symmetry but it is removed by instanton effects.) The condensates (2.3), (3.4), (3.6) and (3.7) will break this down to the vectorial subgroup $SU(4)_{PS} \times SU(2)_V \times U(1)_{TB}$, which contains the gauged subgroup $SU(3)_C \times U(1)_{EM}$. Here $SU(2)_V$ is the vector-like “custodial”[26] symmetry and $U(1)_{TB}$ is a technibaryon number (which is exactly conserved except for $SU(2)_L$ instanton effects). Thus there are 100 PNGBs in this model. Of these, 15 are eaten when $SU(4)_{TC}$ gets broken, and 3 more are eaten by the $W$ and $Z$ when the electroweak symmetry is broken. The remaining 82 PNGBs transform as 14 color singlets, 16 color triplets, 2 color sextets, and one color octet. The colored PNGBs get large masses as usual in technicolor models, while the color singlet PNGBs presumably get masses from ETC and other unspecified interactions which explicitly break the approximate global symmetry. There are also heavy technimesons and technibaryons in this model. The lightest of the latter should be stable because of the conservation of technibaryon number.

4. Variations on the Theme

In the preceding section we discussed one way of breaking $SU(4)_{TC} \times SU(3)_{C'} \times U(1)_{Y'} \rightarrow SU(3)_C \times U(1)_Y$, using a specific set of extra fermions and a specific set of assumptions about how they behave when $SU(4)_{TC}$ gets strong. There is no particular reason to believe that this version is unique. Indeed, it is not even complete, since we have not yet specified exactly how mass terms arise for the standard model fermions. One can imagine a multitude of variations on the theme in which e.g. $\eta, \xi$ and $\chi$ are replaced by other extra fermions, or in which the assumptions about the behavior of the strongly
coupled interaction are modified, or in which additional gauge interactions play a key role in the condensate formation, etc. In this section we will mention a few such variations on the general theme described in the previous sections. In doing so, we will ignore the important problems of mass generation for standard model quarks and leptons, flavor-changing neutral currents, the fate of the PNGBs, and precision electroweak parameters. Instead, we will content ourselves with some gross features of the strongly coupled sector which serve to illustrate the richness of model building possibilities.

Variation 1: Additional technidoublets and the $\beta$ function. The $\beta$ function for the model described in Section 3 has healthy negative contributions from both one and two loops. We might want to add in more fermions which transform under $SU(4)_{TC}$ in order to make $g_4$ walk more slowly into the infrared. This can be done in several ways without altering the symmetry breaking pattern. However, there is little room to add in more strongly coupled fermions without endangering the growth of $g_4$ in the infrared altogether. In general, the $\beta$ function for $SU(4)_{TC}$ is given to two loops by

$$\beta_4 = \mu \frac{dg_4}{d\mu} = - \frac{g_4^3}{16\pi^2} \left[ b_0 + b_1 \left( \frac{g_4}{4\pi} \right)^2 \right]$$

$$b_0 = \frac{44 - n_4 - 2n_6 - 6n_{10}}{3}$$

$$b_1 = \frac{4352 - 205n_4 - 440n_6 - 1608n_{10}}{24}$$

where $n_4$, $n_6$ and $n_{10}$ are the total number of two-component Weyl fermions transforming as $4$, $\bar{4}$, $6$, and $10$ or $\bar{10}$, respectively. By using more than one technidoublet, we can slow the running of $g_4$. For example, if we add to the model of Section 3 one extra copy of $\psi_L$ and $\psi_R^6$ so that we have two technidoublets in all, then we have $n_4 = 16$, $n_6 = 0$, and $n_{10} = 1$, giving $b_0 = 22/3$ and $b_1 = -67/3$. The positive two-loop contribution to the $\beta$ function overpowers the negative one-loop contribution at $g_4/4\pi \approx .57$, which is therefore an infrared fixed point of the two-loop $\beta$ function. Of course, as the coupling approaches this value, the perturbative expression for the $\beta$ function becomes untrustworthy. Still, one may speculate that there is a fixed point for the exact theory somewhere roughly in the vicinity of this point, or at least a very slow running of $g_4$. If we use three technidoublets, so that $n_4 = 20$, $n_6 = 0$, $n_{10} = 1$, we get $b_0 = 6$ and $b_1 = -113/2$. Now the naive estimate for the possible fixed point is $g_4/4\pi \approx .33$. Such values for the coupling may be just big enough to give chiral symmetry breaking near criticality. Adding in extra fermions in $6$, $10$ or larger reps of $SU(4)_{TC}$ in a way consistent with the desired symmetry breaking pattern tends to give large negative contributions to $b_0$ and $b_1$, making it problematical for $g_4$ to obtain large enough values for chiral symmetry breaking at all.
Variation 2: Replacing $\langle \eta \xi \rangle$ with $\langle \eta \chi \rangle$. As we mentioned in the previous section, (3.4) is not the unique solution to the vacuum alignment problem concerning the relative orientation of the weakly coupled gauge group and the unbroken global symmetry. The other solution is given by

$$\langle \eta^{(ab)} \chi^\alpha \rangle = \delta_c^{(a \ b)} \delta_\alpha^\beta \ M_{\chi}^3.$$ (4.1)

If the theory chooses (4.1) instead of (3.4), then the analysis we have already presented goes through in much the same way, by interchanging the roles of $\chi$ and $\xi$ and applying the conjugation automorphism to $SU(4)_{TC}$. If $SU(4)^*_{TC}$ represents the same group as $SU(4)_{TC}$ but with each representation replaced by its conjugate, the condensate (4.1) transforms under $SU(4)^*_{TC} \times SU(4)_{P^*}$ as $(\bar{4}, 4)$ and breaks $SU(4)_{TC}^* \times SU(3)_{C'} \times U(1)_{Y'}$ down to $SU(3)_C \times U(1)_{Y}$. The net effect of this is that the representations of the technifermions under the low energy gauge group are all replaced by their conjugates. The fermions $\eta$, $\xi$, and $\chi$ together form a real representation of the standard model gauge group and their low-energy quantum numbers are unaffected. However, the technifermions $\psi_L$ and $\psi_R^c$ transform under the standard model gauge group as the conjugates of the reps in (2.7) in this variation. This will make a difference in trying to construct ETC interactions.

Variation 3: A hybrid technicolor and top-quark condensate model. The “zeroth order” spectrum of standard model fermion masses consists of a large top-quark mass and negligible masses for everything else. Let us now briefly sketch a model which exhibits this spectrum, and which incidentally ensures large masses for $\xi$ and $\chi$. It is really just a hybrid of the spontaneously broken technicolor model in sections 2 and 3 and a renormalizable top-quark condensate model of the “Topcolor”[7,8] type. The full gauge group is $SU(4)_{TC} \times SU(3)_{C'} \times SU(3)_{C''} \times SU(2)_L \times U(1)_{Y'}$. The fermions $\psi_L, \psi_R^c, \eta, \xi$ and $\chi$ are all singlets with respect to $SU(3)_{C''}$ and they transform under $SU(4)_{TC} \times SU(3)_{C'} \times SU(2)_L \times U(1)_{Y'}$ exactly as in (2.1) and (3.1). As before, we assume that $SU(4)_{TC}$ gets strong first in the infrared. Then, exactly as discussed in sections 2 and 3, condensates (2.3) and (3.4) will form, breaking $SU(4)_{TC} \times SU(3)_{C'} \times U(1)_{Y'} \to SU(3)_{C''} \times U(1)_{Y}$. But now, instead of identifying $SU(3)_{C''}$ with standard model QCD, we assume that $SU(3)_{C''}$ also gets strong above the electroweak scale. When this happens, condensates involving the components of $\xi_A$ and $\chi$ will form. Since $\xi_A$ and $\chi$ form a vector-like representation of the remaining gauge group, they condense in the obvious way without breaking any additional gauge symmetries. That is, the $(8, 1, 0)$ of $\chi$ condenses with itself, and the $(3, 1, 2/3)$ and $(\bar{3}, 1, -2/3)$ of $\chi$ condense with each other, while the $(3, 1, -1/3)$ and $(\bar{3}, 1, 1/3)$ of $\xi$ condense with each other. The two copies of $(1, 1, 0)$ in $\chi$
do not condense due to $SU(3)_{C''}$, but they still get large masses communicated to them from the other condensates by the strongly coupled $SU(4)_{TC}$ interactions. Note that the $SU(4)_{TC}$ interactions can also strongly enhance the other condensates and mass terms, (and ensure that the $3$ and $\overline{3}$ components of $\xi_A$ and $\chi$ are not tempted to condense with the fermions we are about to introduce).

Now $SU(3)_{C''}$ plays the role of the stronger $SU(3)$ and $SU(3)_{C''}$ the role of the weaker one in the “Topcolor” paradigm. To be specific, we can follow [8] and complete this model by putting in $SU(4)_{TC}$-singlet fermions, consisting of the standard model quarks and leptons and two $SU(2)_L$-singlet quixes (sextet quarks). They transform under $SU(3)_{C'} \times SU(3)_{C''} \times SU(2)_L \times U(1)_{Y'}$, as

\[
\begin{align*}
\bar{q}_1 & \sim (\overline{6}, 1, 1, -1/3) & (u d)_L, (c s)_L & \sim 2\times (1, 3, 2, 1/6) \\
q_1, q_2, d^c_{1R}, d^c_{2R} & \sim 2\times (3, 3, 1, 1/3) & u^c_R, c^c_R & \sim 2\times (1, \overline{3}, 1, -2/3) \\
\bar{q}_2 & \sim (1, \overline{6}, 1, -1/3) & d^c_{3R} & \sim (1, \overline{3}, 1, 1/3) \\
(t b)_L & \sim (3, 1, 2, 1/6) & (\nu l)_L & \sim 3\times (1, 1, 2, -1/2) \\
n^c_R & \sim (\overline{3}, 1, 1, -2/3) & l^c_R & \sim 3\times (1, 1, 1, 1)
\end{align*}
\]

and they of course transform under $SU(3)_{C''} \times SU(3)_{C''} \times SU(2)_L \times U(1)_{Y'}$ in the same way after (3.4) forms.

It is easy to check that all of the gauge anomalies cancel. This fermion content is designed to break $SU(3)_{C''} \times SU(3)_{C''}$ to the diagonal $SU(3)_{C}$ which is the QCD group of the standard model. This happens when the quix-antiquix pairs $q_1 \bar{q}_1$ and $q_2 \bar{q}_2$ condense. $SU(3)_{C''}$ also produces a top-quark condensate which contributes to electroweak breaking. (In [7,8], of course, this top-quark condensate was presumed to be entirely responsible for electroweak breaking. In the present model, the primary source of electroweak breaking is the condensate $\langle \bar{\psi}_L \psi_R \rangle$ which enjoys a custodial $SU(2)$ symmetry.) This gives the top quark a large mass. We refer the reader to [8] for details. This is an example of how $\xi_A$ and $\chi$ can get masses even if the assumptions leading to (3.6) are false.

Variation 4: Breaking of $SU(4)_{TC}$ from additional ultracolor interactions. While the idea of $SU(4)_{TC}$ breaking itself has a certain economy, it is also possible to break $SU(4)_{TC}$ using a fermion representation which is vectorial with respect to $SU(4)_{TC}$, by introducing some additional strongly coupled gauge interactions. The simplest version of this idea is essentially the same as that used in [15] and [16]. We introduce two new strongly coupled groups $SU(n) \times SU(n)'$ (with $n \geq 3$) in addition to $G = SU(4)_{TC} \times SU(3)_{C'} \times SU(2)_L \times U(1)_{Y'}$. Now $SU(4)_{TC}$ is to be broken by $SU(n) \times SU(n)'$, but not before it
gets strong enough to produce condensates which break the electroweak symmetry. The standard model fermions and the technidoublet $\psi_L, \psi_R$ are singlets under $SU(n) \times SU(n)'$, and transform just as in section 2 under $G$. This sector of the theory behaves exactly as before. But instead of $\eta, \xi, \chi$, we introduce fermions transforming under $SU(n) \times SU(n)' \times SU(4)_{TC} \times SU(3)_{C'} \times U(1)_{Y'}$ in the anomaly-free rep

$$
\eta \sim (n, 1, 4, 1, 0)
\xi \sim (\bar{n}, 1, \bar{3}, -1/6) + (\bar{n}, 1, 1, 1/2)
\eta' \sim (1, \bar{n}, \bar{4}, 1, 0)
\xi' \sim (1, n, 1, 3, 1/6) + (1, n, 1, 1, -1/2) \quad (4.3)
$$

Now we assume that $SU(n), SU(n)'$, and $SU(4)_{TC}$ all get strong at roughly comparable scales above the electroweak scale, and that condensates form according to

$$
\langle \eta^a \xi^\alpha \rangle = \delta^a_\alpha m^3; \quad \langle \eta'^a \xi'^\alpha \rangle = \delta^a_\alpha M'^3 \quad a, \alpha = 1 \ldots 4 \quad (4.4)
$$

due to $SU(n)$ and $SU(n)'$ respectively. These condensates break $SU(4)_{TC} \times SU(4)_{PS} \to SU(3)_{C'} \times U(1)_{Y'}$, so that the gauged subgroups break according to $SU(4)_{TC} \times SU(3)_{C'} \times U(1)_{Y'} \to SU(3)_{C} \times U(1)_{Y}$. [As usual we are using the fact that QCD forces the vacuum to align so as not to break $SU(3)_{C}$.] Also we may have a condensate

$$
\langle \eta^x \eta'^{x'} \rangle = \delta^x_{x'} m^3 \quad x, x' = 1 \ldots n \quad (4.5)
$$

due to $SU(4)_{TC}$ which will break $SU(n) \times SU(n)'$ down to the diagonal $SU(n)$. We are left in the end with an unbroken $SU(n)$ [or $SU(n) \times SU(n)'$ if (4.5) does not occur] which confines at energies above the electroweak scale, and a spontaneously broken $SU(4)_{TC}$. The key assumption here is that the strong coupling dynamics allows (2.3) and (4.4) to both occur, even though (4.4) breaks $SU(4)_{TC}$.

**Variation 5: Other technicolor groups.** We chose to work with an $SU(4)$ technicolor gauge group because of the nice way that it can break down to $SU(3) \times U(1)$, thus serving also as a Pati-Salam group for the techniquarks and technileptons. It is this structure which allowed us to get one complete standard model family of technifermions (after symmetry breaking) from just one technidoublet (before symmetry breaking). However, it is certainly possible to use other groups. For example, we could use an $SU(3)_{TC}$, which then breaks with a weaker $SU(3)_{C'}$ down to the diagonal $SU(3)_{C}$. This group structure was used in “Topcolor” models and in [15] and made a cameo appearance in Variation 3 above. The model in ref. [16] used an $SU(5)_{TC}$ which combines with an $SU(3)_{C'} \times U(1)$, leaving behind
the standard model $SU(3)_C \times U(1)_Y$ as here, but also an unbroken $SU(2)_{TC}$. This idea of “hiding” part or all of the technicolor interaction inside the usual color at low energies can be used also for larger technicolor groups. In general, one can imagine that part of the technicolor gauge group is broken completely (e.g. strong ETC), part of it combines with some $SU(3)_{C'}$ or $SU(3)_{C'} \times U(1)_{Y'}$ to leave unbroken the $SU(3)_C$ or $SU(3)_{C} \times U(1)_{Y}$ of the standard model, and part of it remains unbroken and confines some of the technifermions.

5. Conclusion

In this paper, we have discussed some model building ideas for spontaneously broken technicolor. We should emphasize that any particular model is subject to assumptions about how and if the condensates form, since we have no rigorous knowledge about the strong coupling dynamics. Our experience with QCD is of limited relevance, especially since we use a chiral representation of the strongly coupled interaction involving higher dimensional representations. Can this mutation of the technicolor idea have some beneficial effects? It has already been argued[17] that deconfining technicolor can help reduce the electroweak $S$ parameter. This will especially be true if we need only one or several technidoublets. It also seems likely that breaking technicolor can help to enhance the technifermion condensate scales compared to the Nambu-Goldstone boson decay constants. This is because the latter obtain more of a contribution from lower energy scales, where the masses of the technigluons cut off the technicolor forces. The masses of the technigluons do not affect the higher energy dynamics which contribute more to the technifermion condensates. We imagine that a healthy condensate enhancement compared to the Nambu-Goldstone boson decay constant could be driven by a combination of a slowly running technicolor coupling constant and a technigluon mass. Both of these have the effect of increasing the relative importance of the high energy dynamics. This has two potentially important effects. First, the technifermion condensate enhancement might be invoked to suppress flavor-changing neutral currents. Second, the technigluons eat Nambu-Goldstone bosons to get masses which are then proportional to the Nambu-Goldstone boson decay constants. This could help to explain why the technigluon masses are small enough to allow condensation in attractive but subdominant channels involving massless fermions. Perhaps a realistic model can be constructed using these ideas.

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