AutoAssociative Memory for Face Recognition using High-Dimensional Octonion Data

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Abstract:
Uniqueness or individuality of an individual face is the representation of one’s identity. Recent research in color image processing focusing on octonion algebra, which is the generalization of complex and quaternion algebra. In this paper, the face of an individual is used for identification making automatically. For this, an autoassociative memory is used for the recognition of images or videos with 8-dimensions. Octonion algebra is not commutative and not associative but it is an alternate algebra for representing the phase and angle of images effectively. The prospective experiments are conducted on color images for image reconstruction, denoising and efficiently retrieving blurred images in true color images.

Keywords: Associative memory, autoassociative memory, octonion-valued, quaternion-valued networks, face recognition.

1. Introduction
The mainstream of earlier research efforts has focused on unique fingerprint-based biometric systems. Different applications using various forms of technologies for access control and authentication purpose such as banking systems with username and password, employee authentication in the organization with the biometric system, etc. Due to the rapid growth of computer technology all these access control mechanisms facing a lot of problems like losing fingerprint, unauthorized person using username, and passwords(stolen). It is not reliable even though it is effective for purpose of detecting a person. To overcome fit falls of fingerprint-based biometric systems a pioneering identification and authentication system is required. It is not reliable even though it is effective for the person

Compared with other biometric features, such as palms, ears, irises, gait, and voice, the human face has many distinct advantages. So face recognition (FR) based identification and authentication system taken into consideration in research because it is natural, easy to use, and nonintrusivity. Automatic face recognition is not only alternate to fingerprint biometric system has many uses such as crowd surveillance and border control.

In recent years, a digital image plays a unique role in the advent of technological advancements by extracting useful information from images. It focuses on two tasks; the first one is an improvement of pictorial information for human interpretation and the second one is the processing of image data for storage, transmission, and representation for autonomous machine perception.
Face recognition is an application of associative memory pattern recognition problems. Face recognition is an emerging technology in the era of computer vision (CV) from the long term onwards because of the machine learning algorithms and CVNN usage in the data processing. Neural networks Associative memory is taken initiation in the research from human being's memory because of artificial neural networks inspired by biological neural networks with a mathematical representation of the model. Human memory depends totally on association with objects frequently seen together to become linked in our mind; once we attempt to retrieve information one thing reminds us of another that reminds us of yet another, and so on. Recalling and remembering things by associating with objects is called associative memory. Simply associative memory means storing the patterns in memory and recalls the stored pattern from memory bypassing the input pattern. Face recognition is a type of associative memory because images are going to be stored in the database first (training) and then recalling the image by giving a query image as input. Face recognition using CVNNs obtained enormous results because it can process the two-dimensional data promptly and treats as a single entity.

Complex number with four-dimensional data (4D) is quaternion algebra with one real and three imaginary units appropriate for the representation of three color channels (R, G, and B) in image processing [6]. Quaternion-valued neural network in color image processing showed an amazing result in terms of image reconstruction, face recognition with color, and denoising [7]. A quaternion-based optimization problem is quite difficult to solve because of its non-commutative property. Extended complex and quaternion numbers are required.

Inspired by the extraordinary results of quaternion algebra in color image processing, this paper addresses an associative memory model for color face recognition using complex number octonion, which is an 8-dimensional hypercomplex non-associative algebra. Due to the non-associative property of the octonion algebra applications are limited compared to quaternion algebra but it can solve physical problems.

Therefore, this paper aims to design and investigate OVNNs for associative memory-based face recognition by mathematical examination and procedures. This technique makes it conceivable to represent the data in an octonion-valued entity. Still octonion number system facing a computer simulation problem but few authors representing theoretically.

The rest of this paper's aspects are composed as follows. Section II, provides some basic knowledge on octonions, the persistent time OVNNs model, and the discrete-times partners are presented. In Section III, autoassociative memory with storage and recall phases of images along with Hebbian learning is discussed, which can understand affiliated recollections. In Section IV, some adequate experiments with results are discussed. And finally, in Section V some conclusions are drawn and guidelines for future research.

2. Basic concepts of octonion Algebra

Octonions, which can articulate as doubled quaternion numbers [34], have been used in signal and image processing, and we believe that they can also be used effectively in many image recognition applications. Octonions are an extension of quaternion and complex algebra and it is a type of Cayley-Dickson algebra with dimension 2^n. The octonion is the 8-dimensional nonassociative algebra over the real numbers, which consists of one real part and seven imaginary parts with basis elements \{1,e_1, e_2, e_3, e_4, e_5, e_6, e_7\} can be written in the form
\[ a := a_0 + \left\{ \sum_{q=1}^{7} [a]_q e_q \mid [a]_0, [a]_1, \ldots, [a]_7 \in R \right\} \]  

where \( a_0 \) and \( a_q \) are real numbers, and \( e_q \) is imaginary units.

Table 1: Octonion basis elements multiplication rules.

| \( \times \) | \( e_0 \) | \( e_1 \) | \( e_2 \) | \( e_3 \) | \( e_4 \) | \( e_5 \) | \( e_6 \) | \( e_7 \) |
|---|---|---|---|---|---|---|---|---|
| \( e_0 \) | \( e_0 \) | \( e_1 \) | \( e_2 \) | \( e_3 \) | \( e_4 \) | \( e_5 \) | \( e_6 \) | \( e_7 \) |
| \( e_1 \) | \( e_1 \) | \( -e_0 \) | \( e_3 \) | \( -e_2 \) | \( e_5 \) | \( -e_4 \) | \( -e_7 \) | \( e_6 \) |
| \( e_2 \) | \( e_2 \) | \( -e_3 \) | \( -e_0 \) | \( e_1 \) | \( e_6 \) | \( e_7 \) | \( -e_4 \) | \( -e_5 \) |
| \( e_3 \) | \( e_3 \) | \( e_2 \) | \( -e_1 \) | \( -e_0 \) | \( e_7 \) | \( -e_6 \) | \( e_5 \) | \( e_4 \) |
| \( e_4 \) | \( e_4 \) | \( -e_5 \) | \( -e_6 \) | \( -e_7 \) | \( e_0 \) | \( e_1 \) | \( e_3 \) | \( e_2 \) |
| \( e_5 \) | \( e_5 \) | \( e_4 \) | \( e_7 \) | \( -e_3 \) | \( -e_2 \) | \( e_6 \) | \( e_5 \) | \( e_4 \) |
| \( e_6 \) | \( e_6 \) | \( e_7 \) | \( e_4 \) | \( -e_3 \) | \( -e_2 \) | \( e_5 \) | \( e_4 \) | \( e_3 \) |
| \( e_7 \) | \( e_7 \) | \( -e_6 \) | \( e_5 \) | \( e_4 \) | \( e_3 \) | \( e_2 \) | \( e_1 \) | \( -e_0 \) |

where the imaginary units satisfy \( e_i^2 = -1 \) and multiplication rules between them are given in Table 1.

An octonion can be represented by a vector as \( \mathbf{a} = a_0 + \mathbf{a}, \) where \( a_0 = a_1 + a_2 e_1 + a_3 e_2 + a_4 e_3 + a_5 e_4 + a_6 e_5 + a_7 e_7 \) are called the real and vector part of \( a. \)

The imaginary units \( e_0, e_1, \ldots, e_7 \) abide by the following rules:

\[ e_i e_j = -e_j e_i \neq e_j e_i \quad \forall i \neq j, 0 < i, j \leq 7, \text{ and } \]  

\[ e_1 e_3 e_2 = -e_3 (e_1 e_2) = -e_3 (e_1 e_2) \quad \text{for } i \neq j \neq k \]  

From the above rules and multiplication table, it is clear that the multiplication of octonions imaginary units is not commutative and nor associative. The algebra properties of the octonion are addition between two octonion numbers are component-wise, which is the same as complex numbers.

If the octonion \( a = a_0 + a_1 e_1 + a_2 e_2 + a_3 e_3 + a_4 e_4 + a_5 e_5 + a_6 e_6 + a_7 e_7, \) the conjugate of \( a \) is defined as \( \bar{a} = a_0 - \sum_{q=1}^{7} a_q e_q \) and \( \bar{a} \bar{a} = \bar{a}^2 = a \).

The norm of octonion \( a \) is represented as

\[ |a| = \sqrt{a \bar{a}} = \sqrt{a \bar{a}} = \sqrt{a_0^2 + a_1^2 + a_2^2 + a_3^2 + a_4^2 + a_5^2 + a_6^2 + a_7^2}. \]  

and norms are real numbers. From this definition, we can find the inverse of \( a \) as \( a^{-1} = \frac{a^\bar{a}}{M(a)} \)
The octonions are alternative algebra, by the definition of the multiplication table of octonion we have
\[
(e_i e_i) e_j = -e_j = -e_k e_i = e_i e_k = e_i (e_i e_j)
\]
and in the same way
\[
(e_j e_j) e_j = e_k e_j = -e_j e_k = -e_i = e_i (e_j e_j)
\]
Multiplication of octonions is not associative from table1. According to the Cayley-Dickson construction of octonion θ can be represented in quaternion algebra as:
\[
\theta = a' + a'' e, \quad a', a'' \in Q; \quad Q = \{ a_0 + a_1 i + a_2 j + a_3 k | a_0, a_1, a_2, a_3 \in R \}
\]
(5)
Here, θ is the quaternion algebra with \{1, i, j, k\}, \(i^2 = j^2 = k^2 = -1, ijk = -i\) and \(e\) is imaginary unit [9].

3. Autoassociative Memory

To store and recall the face images by using linear autoassociative memory was first proposed by Kohonen in the year 1999 [9] [10]. He used two autoassociative memories on the symmetry property of an image to quickly locate all the candidates’ human faces with all possible sizes and regions for storing and validation (recall) purpose. The fundamental function of an associative memory through self-organizing is to store a set of patterns \(X^p\), \(p=1, \ldots, n\) pairs in memory and retrieves the corresponding output pattern \(Y^p\) for a given input pattern. The input stimulus and the response of the output pattern represent feature vectors of images.

Fig. 1. Associate Memory model
Associative memory main function is to recall the perfect pattern from a stored pattern based on a given input pattern; this can be done in two phases:

A. Storage Phase

Construction of weight matrix $w$ (associative memory) with paired input and output patterns $(X_p, Y_q)$ is called encoding, during the storage phase constructed weight matrixes are stored in the memory. Features of the images are stored in the memory in the form of a weight matrix, which is computed as

$$W_{pq} = (x_p)^T (y_q) \quad (6)$$

Where $(x_p)^T$ and $y_q$ represents the transpose of input pattern and output patterns of the associative memory.

B. Recall Phase

The process of retrieving a stored pattern from memory for a given input pattern is called decoding, during this phase the output pattern $y_q$ is computed as

$$y_q = \sum_{p=1}^{n} W_{pq} x_p \quad (7)$$

Output units $y_q$ dependent on activation function which is represented in bipolar from

$$Y_q = \begin{cases} +1 & \text{if } x_p > 1 \\ -1 & \text{if } x_p = 0 \end{cases} \quad (8)$$

If the input patterns are mutually orthogonal then perfect retrieval is possible, which means input pattern and output patterns are equal or identical $x_p = y_q$, this type of memory is called autoassociative memory.

$$x_p x_q = \begin{cases} 1 & \text{if } p = q \\ 0 & \text{if } p \neq q \end{cases} \quad (9)$$

Otherwise, if input patterns are not orthogonal, the retrieved pattern is different from the given input pattern because of crosstalk between the input patterns i.e noise, this type of memory is called as heteroassociative memory. In this paper, autoassociative memory model is used to stores the images in the memory in the form of features, and recognition is done based on the input image features.
C. Learning Rule

To reduce error or noise present in the recalled image is achieved by updating the weight matrixes which are computed during the storage phase via the Hebbian learning rule until reaching the stable state

\[ W_{\text{new}} = W_{\text{old}} + \eta \Delta W_i \]  

(10)

where \( W_{\text{new}}, W_{\text{old}} \) are new and old weight matrices, \( \eta \) constant learning rate and \( \eta \Delta W_i \) is change in the connection weight matrix.

4. Experimental Results and Discussion

We demonstrate our method by applying the multiple channel technique for image recalling and denoising. Quaternions are triplets and are perfectly mapping with RGB as a one to one. The proposed octonion auto-associative memory is an extension of quaternion associative memory with increased channels that is from 3 imaginary units to 7 imaginary units. Figure 2 shows the color image denoising with four images. Compared to complex, quaternions based denoising of images octonions yielded better results. We have taken 30 images with 25 patterns as a training data set which means training of associative memories or storing and testing data set with 40 images with 43 patterns for recall purpose used. This method gives better results when compared to quaternion-valued associative memory when learning \( \eta = 15 \) and \( 20 \). Figure 2 shows the original color images in (a), blurred color images with Gaussian noise 50 in (b), and in (c) retrieved images are present.

![Image](a)

![Image](b)

![Image](c)

Figure 2. Autoassociative memory based (a) original images (b) Blurred images (c) Recalled images.

For experiments, all images with a size of \( 512 \times 512 \) are considered patches of size \( 8 \times 8 \) but we use dictionary size for image denoising with \( 64 \times 256 \). Results are compared with QVNN and our model.
OVNN gives better results at smaller noise levels when compared to higher noise levels. At higher noise levels OVNN produces the same results as QVNN.

Figure 3 large images (a) stored image or original color image. (b) Blurred image. (c) Final retrieved image from associative memory.

And figure 3 shows an experiment on a large image, in this, also retrieved image the same as an original image with 8-dimensions. From Figures 2 and 3 the retrieved images are perfect because we stored images in the form of weight matrixes or vectors in associative memory. From the flexibility of autoassociative memory, we retrieved perfect images the same as original ones, even though passing noise images as input patterns in the recalling process.

5. Conclusion

In this paper, we proposed octonion autoassociative memory for face recognition, which is an extension of quaternion, complex, and real numbers. The important points of octonion algebra, such as norm and conjugate properties are discussed, and octonion is not commutative and not associative stressed. Associative memory applications for image recognition are also discussed here. In the future, we plan to examine the performance of this model specifically in terms of associative memory storage capacity and noise tolerance.
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