The $J/\psi$ Production Associated with a Hard Photon and the Color-Octet Signature in $e^+e^-$ Annihilation

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Abstract

$J/\psi$ production associated with a hard photon in $e^+e^-$ annihilation, being of QED nature, is investigated thoroughly in the paper. To show its influence on the observation of the color-octet signature in the $e^+e^-$ annihilation via $J/\psi$ inclusive production, the cross sections of the $J/\psi$ productions through different mechanisms at various energies are compared quantitatively by presenting them in figures together. The contribution from the production associated with a hard photon to the inclusive production of $J/\psi$ is pointed out to be significant at the concerned energies, thus the influence from it on the observation of the color-octet signature should be dealt with carefully.

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Since $J/\psi$ was discovered more than twenty years ago, its hadronic production has been interesting people, because of the problem being open. Recently several progresses on the production are achieved. One is about the calculations of the fragmentation functions and the corresponding production mechanisms for the double heavy meson (heavy quarkonia and $B_c$ meson etc.) productions, i.e. in the framework of perturbative QCD, the fragmentation functions and the productions via the so-called fragmentation mechanisms are realized to be calculable and the calculations are believed to be reliable [1]. The second is about tests of the predictions of the calculations with experimental data [2]. As a result, a precise conflict between the theoretical predictions and the Tevatron experimental data, i.e. $'\psi'(\psi)$ surplus’ puzzle, is confirmed [3]. The third is that an interesting solution for the puzzle, the so-called ‘color-octet mechanism’, is proposed [4]. Due to the progresses, now the interests in the problem and relevant ones are freshened and spreading widely. To confirm the color-octet mechanisms[4] further and to test the proposal, many suggestions appear. For instance, investigations on all kinds of the mechanisms in the $J/\psi$ inclusive production, including the color singlet ones and the octet ones, at various energies and in various processes, not only in hadronic colliders but also in $e^+e^-$ or $ep$ colliders etc, emerge in preprints, letters and achieved papers [5–12]. Moreover of them, being clean, the productions of $J/\psi$ through $e^+e^-$ annihilation are specially emphasized by several authors to confirm the octet mechanisms [11–13], Whereas, the $J/\psi$ production associated with a hard photon via $e^+e^-$ annihilation, depicted by the Feynman diagram Fig.1.a, is less considered in literature. For convenience, throughout the paper we will call it as a hard-photon production (or mechanism) latter on.

In fact, the hard-photon production should be catalogued in color singlet mechanism and is

1In fact, in various color-octet productions of $J/\psi$, the acting subprocesses are different from each other, not only in the order of the QCD in the strong coupling $\alpha_s$ but also in the behavior of the propagators in the Feynman diagrams responsible for the subprocesses, thus we think it is better to call a production with a different acting subprocess as a different mechanism.
of QED nature essentially. Based on a rough order-estimate of the couplings and propagators to the corresponding Feynman diagrams, one can be sure that it contributes to the inclusive $J/\psi$ production in energetic $e^+e^-$ annihilation in a certain amount, although it is one order higher in the coupling $\alpha$ than ‘others’. Therefore precisely to calculate the process versus the others, including the comparatively well-studied mechanisms such as the color-octet ones (Fig.1.b) and the color-singlet ones (Fig.1.c), at various energies becomes interesting. In the paper we are to do the calculations to see how significant is the contribution from each mechanism. Finally a conclusion is reached that in $e^+e^-$ annihilation the contribution from the hard-photon mechanism to the inclusive production of $J/\psi$ is very significant at the concerned energies, and its affects on detecting the singnature of the color-octet mechanism are great. In the $e^+e^-$ annihilation all the concerned color-octet mechanisms and the color-singlet mechanisms, except the hard-photon one, are of ‘s-channel annihilation’, so the propagator of the virtual photon from the $e^+e^-$ annihilation plays a role of a suppression factor, especially, when the annihilation happens at a high energy. Thus the interesting color-octet signature is expected to be observed better at comparatiely low energies such as at TRISTAN, CESR and BEPC etc, rather than at high ones such as at LEP and SLC. Furthermore, at a relatively high energy, e.g. $\sqrt{s} \geq m_Z$, in the production more acting mechanisms are involved, so the production is more complicated, that we will discuss them elsewhere [14].

Theoretically, except the binding factor of $c\bar{c}$ in the $J/\psi$, the rest parts of the $J/\psi$ production associated with a hard photon in $e^+e^-$ annihilation are of QED. The process may be computed accuratly with QED Feynman diagrams. To the lowest order, as depicted in Fig.1.a it is quite a general feature that the hard photon in the process couples to an electron line, and so does the $J/\psi$ but through a virtual photon indirectly. Here, merely for simplisity, in Fig.1.a only one of the Feynman diagrams for the process is presented, but when we calculate the process, complete set of the diagrams (here only one else, with the two photon lines crossed, should be added) responsible for the process are considered. Throughout the paper and for all processes the same simplification in drawing Feynman
diagrams is taken. Namely Feynman diagrams for a concerned process are presented in figures typically. In fact, in Fig.1.a, the virtual photon is the same as that of the $J/\psi$ production in $e^+e^-$ annihilation at resonance and that in the decay $J/\psi \rightarrow e^+e^-$, i.e. the momentum of the virtual photon is just ‘on-shell’ of $J/\psi$, thus it is not a suppression factor. Note here that the so-called electromagnetic fragmentation approach (EMFA) works well only at much high energies ($\gg m_{J/\psi}$) and, as mentioned above, we constrain ourselves to consider all kinds of production at the energies of BEPC, CESR and TRISTAN, which even may compare with $m_{J/\psi}$, so EMFA is not applicable here. Furthermore there would be no advantages in calculations of the process if we had adopted EMFA no matter how it had worked. Therefore here we do not adopt it. Experimentally, if the associated hard photon in the process can be identified well, the process will be measurable exclusively with certain accuracy, hence the theoretical calculations will become testable by experiments directly.

In fact, there is additional color-singlet mechanism $e^+e^- \rightarrow J/\psi + f + \bar{f}$ in the $e^+e^-$ annihilation, where $f, \bar{f}$ denotes a pair of fermions in ‘flavor’ $f$ (quarks or leptons), thus this mechanism involves many channels with various fermion pairs $f\bar{f}$. For each channel with a specific pair $f, \bar{f}$, the corresponding Feynman diagrams are obtained in such a way, that via a virtual photon line indirectly, the $J/\psi$ couples to a line of the final fermion $f$ or $\bar{f}$ or of the initial electron or positron in turn to its ‘skeleton’ diagram, which just is the annihilation process $e^+e^- \rightarrow f\bar{f}$. Although they are of QED essentially too and also less considered in literature, because they are one order higher in $\alpha$ than the considered hard-photon one, we

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2 The energies, at which experimental data were taken at BEPC, CESR and TRISTAN, are chosen to compute the production. Moreover, in general, there is always an additional $Z$ boson exchange diagram, which is obtained by replacing each virtual photon once for the diagrams Fig.1.a-1.c with a virtual $Z$ boson. Whereas at the comparatively low energies, being much lower than $m_Z$, the contributions from the $Z$ boson exchange diagrams are tiny (small) so we ignore them throughout the calculations.
will study them elsewhere [14].

The corresponding amplitude for the hard-photon production may be written down immediately, and with a straightforward calculation, the differential cross section is obtained:

$$\frac{d\sigma}{dt} = \frac{32\pi \alpha^2 |R_S(0)|^2}{3M^3 S^2} \left[ \frac{2M^2 s}{tu} + \frac{t}{u} + \frac{u}{t} \right],$$

(1)

where

$$s = (p_1 + p_2)^2; \quad t = (k - p_1)^2; \quad u = (P - p_1)^2.$$

For simplicity, here in the formulae the electron mass is ignored, whereas in the numerical calculations we keep it so as to avoid the ‘collinear divergence’ in the beam direction for the differential cross section. As the wave function at the origin $|R_S(0)|$ appearing here is exactly as that appearing in the width of the decay $J/\psi \to e^+e^-$, i.e.

$$\Gamma(J/\psi \to e^+e^-) = \frac{16\alpha^2}{9M^2} |R_S(0)|^2,$$

(2)

in numerical calculations, we with eq.(2) use the experimental width for the decay $J/\psi \to e^+e^-$ as an ‘input’ instead of the wave function. Namely

$$\frac{d\sigma}{dt} = \frac{6\pi \alpha \Gamma^{\text{exp.}}(J/\psi \to e^+e^-)}{MS^2} \left[ \frac{2M^2 s}{tu} + \frac{t}{u} + \frac{u}{t} \right].$$

(3)

In the above way to determine the requested wave function at origin, the strong interaction corrections on the wave function have been included. With eq.(3), the numerical values for the differential cross section as a function of $t$, or with a straightforward calculation the differential cross section $d\sigma/d\cos\theta$ as a function of the angular $\cos\theta$ ($\theta$ - the angular between the incoming beam direction and that of the outgoing $J/\psi$) at a given CMS energy, and the total cross section as a function of the CMS energy by integration of eq.(3) may be calculated precisely. Here we choose the CMS energies $\sqrt{S} = 4.03 GeV; 10.6 GeV; 64.0 GeV$, where quite a lot of data were taken, for differential cross sections to do the numerical calculations and plot the results in Fig.2.a-2.c, and the total cross section in Fig.3, respectively. In order to compare the hard-photon mechanism with the others in the contribution to the inclusive production of $J/\psi$ in different aspects, and to show its characteristics concisely, here only
those important ones: color octet ones depicted in the Feynman diagram Fig.1.b and a color siglet one in the Feynman diagram Fig.1.c are selected as the ‘typical others’. While for the mechanisms of Fig.1.b and Fig.1.c, the request formulae in the calculations are refered from literatures [8] and [12]. Namely when we calculate the contributions, the formulae for the color-octet are quoted from [8], and the ones for the color-siglet from [12]. For convenience to compare the different mechanisms we plot the results of the hard-photon one (Fig.1.a) and the considered ‘typical others’ (Fig.1.b, Fig.1.c) into the same figure.

It is easy to realize that the production of the hard-photon is very different from the others because it is a ‘t,u-channel’ process (see Fig. 1.a.) whereas the others, as mentioned above, are ‘s-channel’ ones (see Fig. 1.b-1.c). In general, the differential cross section of the production, as long as it is a ‘s-channel’ process, can certainly be formulated as the follows:

\[
\frac{d\sigma}{dE d\cos \theta} = S(E)[1 + A(E)\cos^2 \theta]
\] (4)

which is emphasized by the authors of [11] and where \( E \) and \( \theta \) are the energy of the produced \( J/\psi \) and the angle between the directions of the \( J/\psi \) and beam at CMS respectively, whereas, for the hard-photon production, which is of a ‘t,u-channel, the differential cross section cannot be formulated as eq.(4). Therefore it is not a good way to see the influences from the later (‘t,u-channel) on the fermer (‘s-channel’) with the formulation eq.(4). If insisting on the formulation eq.(4) to compare them with each other, we should rewrite the differential cross section eq.(1) into a one, depending on \( \cos \theta \) expicitely:

\[
\frac{d\sigma}{dE d\cos \theta} = \delta(E - E_{\text{max}}) \frac{32\pi\alpha^3|RS(0)|^2}{3M^3S(1 - r)\sin^2 \theta}\left[(1 + r)^2 + (1 - r)^2\cos^2 \theta\right],
\] (5)

where \( r = M^2/S \). If the factor \( \sin^2 \theta \) in the denominator of the above differential cross section were ignored, the equation eq.(5) would turn to the formulation of eq.(4), and it would be interesting to note that the coefficient before \( \cos^2 \theta \) of the second term in the squared braket of eq.(5) would take a positive sign, as that of color-octet ones [11].

To see the influences of the hard-photon production onto the signature of the color-octet mechanisms in the formulation eq.(4), we try to define a ‘equavelent’ factor \( \hat{A}_{\text{eqv}}(E, \theta) \) by
differential cross sections:

\[
\hat{A}_{\text{eqv}}(E, \theta) = \frac{1}{\cos^2 \theta} \left[ \frac{d\sigma}{dE d\cos \theta} \bigg|_{\cos \theta = \frac{\pi}{2}} - 1 \right].
\]

For ‘s-channel’ processes, such as the color-octet ones, the factor \( \hat{A}_{\text{eqv}}(E, \theta) \) does not depend on the \( \theta \) at all, and turns to the factor \( A(E) \) in eq.(4) exactly, but for the hard-photon production, being of a ‘t,u-channel’ one, does depend on \( \theta \) but in the sense of the formulation eq.(4) we may consider it as an equivalent factor of \( A(E) \). Let us now take \( \sqrt{S} = 10.6 \text{GeV} \) as an example, to show the behavior of the \( \hat{A}_{\text{eqv}}(E, \theta) \) in Table I.

### Table I. The Factor \( \hat{A}_{\text{eqv}}(E, \theta) \) at \( \sqrt{S} = 10.6 \text{GeV} \).

| \( \theta \) | \( \pi/2 \) | \( \pi/3 \) | \( \pi/4 \) | \( \pi/5 \) | \( \pi/6 \) | \( \pi/7 \) | \( \pi/8 \) | \( \pi/9 \) | \( \pi/10 \) |
|----------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| \( \hat{A}_{\text{eqv}}(E, \theta) \) | 1.710     | 2.28      | 3.42      | 4.95      | 6.84      | 9.08      | 11.68     | 14.62     | 17.91     |

For reference, from [11] we have \( \hat{A}_{\text{eqv}}(E, \theta) \equiv A(E) \approx 0.6 \sim 1.0 \) for color-octet productions (Fig.1.b) and \( \hat{A}_{\text{eqv}}(E, \theta) \equiv A(E) = -0.84 \) for the color-singlet one (Fig.1.c), whereas here from Table I. \( \hat{A}_{\text{eqv}}(E, \theta) \geq 1.7 \).

As a matter of fact, the best way to show the characters of the hard-photon production of \( J/\psi \) and to compare it with the ‘other’ productions quantitatively is to present their total and differential cross sections in figures respectively. Therefore we are doing so now.

In Fig.2.a-2.b the differential cross sections, \( d\sigma/d\cos \theta \) versus \( \cos \theta \) for the productions depicted by Fig.1.a-1.c at the CMS energies \( \sqrt{S} = 4.03 \text{GeV}; 10.6 \text{GeV}; 64.0 \text{GeV} \) are plotted respectively. One can from the figures see that to the inclusive production of \( J/\psi \) via \( e^+e^- \) annihilation, the hard-photon production contributes a dominant fraction over the others’ when the \( J/\psi \) goes out near the beam direction, and still a significant fraction when the \( J/\psi \) goes out in the direction near the direction perpendicular to the beam direction. Namely even in the direction of the \( J/\psi \) perpendicular to the beam direction, the hard-photon one still contributes a fraction not smaller than the greatest one among the considered ‘others’ by a factor 3, although the greatest one among the considered ones is alternated with the CMS energy is increasing. We should note here that at the considered CESR energy
\(\sqrt{s} = 10.6\,\text{GeV}\), the hard-photon mechanism (Fig.1.a) has an angular distribution in a shape similar to that of the color-octet mechanisms (Fig.1.b), but different from that of the color-singlet one (Fig.1.c) that makes its \(\hat{A}_{\text{eqv}}(E, \theta)\) positive as that of color-octet ones, whereas that of the color-singlet one is negative. (see Table.I) To show the energy dependence precisely, we plot the total cross sections versus the CMS energies for the various mechanisms in Fig.3. Considering the existing experiment detector(s), it is possible to measure the hard photon and/or the produced \(J/\psi\) if the photon as well as the \(J/\psi\), goes out not very close to the beam direction or if the production happens at a comparatively low energy that the \(J/\psi\) moves not very fast no matter the direction how close to the beam. Thus in Fig.3 we plot two curves for the hard-photon mechanism: one (thick solid line) is with a cut on the outgoing angular of the \(J/\psi\) and the other (thin solid line) without any cut respectively. It is easy to see that, when no cut is made, the hard-photon mechanism contributes to the inclusive production of \(J/\psi\) dominantly over all the others, and when a cut \(20^0 \leq \theta \leq 160^0\) in angels is put on, the hard-photon mechanism does not dominate over all the others but in the considered CMS energy region it is comparable to that of the biggest one among the others considered here. From the figure (Fig.3), one may also see some interesting aspects for the ‘other’ mechanisms. For instance, the color-singlet one (Fig.1.c) is smaller than those of the color-octet ones (Fig.1.b) in the energy region \(\sqrt{S} \leq 12\,\text{GeV}\) but becomes greater in the energy region \(\sqrt{S} \geq 12\,\text{GeV}\). All the results shown here are understandable qualitatively: As for the hard-photon one, being a ‘t,u-channel’ production, there is an enhancement factor especially, when the t-channel or the u-channel virtual electron line appoaches to the electron mass-shell (it is the reason its differential cross section becomes very large in the beam direction), to compare with s-channel processes when \(\sqrt{S} \gg M^2\). Besides an overall suppression factor due to one order higher in \(\alpha\) than the ‘others’, the hard-photon production gains a factor \(\alpha_s^{-2}\) to compare with the color-singlet one (Fig.1.c) and it gains a factor \(\alpha_s^{-1}v^{-4}\), where \(v\), being small, is the typical relative velocity of the quarks in a heavy quarkonium \(J/\psi\) to compare with the color-octet one (Fig.1.b). In summary of the factors, it is the reason why the cross sections
of the concerned mechanisms present so complicated feature in Figs.2.a-c and Fig.3.

Before drawing a conclusion, we should note two points: i). In the plots the values of the color-octet matrix elements for the color-octet mechanisms are taken from the determination by fitting the Tevatron data [8], and it seems that they are greater than those determined from photoproduction and fixed target experiments [7], [10], therefore here it may be over-estimated a little for the color-octet ones. ii). In hadronic and photonic productions of $J/\psi$, there is a similar mechanism to the hard-photon one emphasized in the paper, we will discuss its effects elsewhere [14].

In conclusion, the hard-photon production of $J/\psi$ itself is an interesting physics in energetic $e^+e^-$ annihilation because of its less studying. Its feature is quite different from others if using the factor $\hat{A}_{eqv}(E,\theta)$ to observe (Table I). If one would like to observe the signature of the color-octet mechanism in $e^+e^-$ annihilation merely in the way as suggested by [11], it may not be successful, because the experimental error may not be deducted very well and the gap of $\hat{A}_{eqv}(E,\theta)$ for the color-octet one and the hard-photon one is not so great, for instance, at $\sqrt{S} = 10.6$ GeV it has only 0.7, i.e. from 1.0 to 1.7. At the energies, such as those of BEPC, CESR and TRIESTON, if one may separate the contribution of the hard-photon mechanism precisely in the experimental observation in the $e^+e^-$ annihilation at certain level, to observe the signature of the color-octet mechanisms and with the suggested way by [11] may be practable. In principle it is accessible to separate the contribution of the hard-photon mechanism precisely by means of a precise exclusive measurement of the production, i.e., not only to measure the produced $J/\psi$ but also the associated hard photon (with experimental tagging techniques). Therefore, in order to study the color-octet mechanisms and to observe the color-octet signature in $e^+e^-$ annihilation without obscurity, the experimental data of the inclusive production of $J/\psi$ should be ‘cleaned up’ in certain level, i.e. the contribution of the hard-photon production should be separated some in advance.
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Figures

Fig. 1.a A typical Feymann diagram for the production of $J/\psi$ with a hard photon via $e^+e^-$ annihilation.

Fig. 1.b A typical Feymann diagram of some color-octet mechanisms for the production of $J/\psi$ via $e^+e^-$ annihilation.

Fig. 1.c A typical diagram of a typical color-singlet mechanism for the production of $J/\psi$ via $e^+e^-$ annihilation.

Fig. 2.a The differential cross sections $d\sigma/d\cos\theta$ of the production $J/\psi$ versus $\cos\theta$ for the various mechanisms at the CMS energy $\sqrt{s} = 4.03\,GeV$ (BEPC).

Fig. 2.b The same as Fig. 2.a but at the CMS energy $\sqrt{s} = 10.6\,GeV$ (CESR).

Fig. 2.c The same as Fig. 2.a but at the CMS energy $\sqrt{s} = 60.0\,GeV$ (TRISTAN).

Fig. 3. The total cross sections of the production $J/\psi$ versus the CMS energy $\sqrt{S}$ for the various mechanisms: the thin solid curve presents that for the mechanism with a hard photon without any cut; the thick solid one presents that for the same mechanism but with a cut $20^0 \leq \theta \leq 160^0$. 
$\sigma(e^-e^+ \rightarrow \gamma J/\psi)$
$\sigma(e^-e^+ \rightarrow g + ^1S_0^{[8]})$
$\sigma(e^-e^+ \rightarrow g + ^3P_J^{[8]})$
$\sigma(e^-e^+ \rightarrow g + J/\psi)$

$\frac{d\sigma}{d\cos\theta_{J/\psi, \text{beam}}}$

$\sqrt{s}=4.03 \text{ GeV}$

**Fig. 2.a**
Fig. 2.b
\[ \sigma(e^-e^+ \rightarrow \gamma J/\psi) \]
\[ \sigma(e^-e^+ \rightarrow g + ^1S_0^{[8]}) \]
\[ \sigma(e^-e^+ \rightarrow g + ^3P_J^{[8]}) \]
\[ \sigma(e^-e^+ \rightarrow g g + J/\psi) \]

**Fig. 2.c**

\( \frac{d\sigma}{d\cos \theta} (\text{pb}, \sqrt{s}=64 \text{ GeV}) \)
\[ \sigma(e^+e^- \rightarrow \gamma J/\psi) \]
\[ \sigma(e^+e^- \rightarrow \gamma J/\psi, 160^\circ > \theta > 20^\circ) \]
\[ \sigma(e^+e^- \rightarrow g + ^1S_0^{[8]}) \]
\[ \sigma(e^+e^- \rightarrow g + ^3P_J^{[8]}) \]
\[ \sigma(e^+e^- \rightarrow g + J/\psi) \]

Fig. 3