Cooling many particles at once

Almut Beige\textsuperscript{1,2}, Peter L Knight\textsuperscript{2} and Giuseppe Vitiello\textsuperscript{3}

\textsuperscript{1} Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, UK
\textsuperscript{2} Blackett Laboratory, Imperial College, Prince Consort Road, London SW7 2BW, UK
\textsuperscript{3} Dipartimento di Fisica ‘E. R. Caianiello,’ INFN and INFM, Università di Salerno, 84100 Salerno, Italy
E-mail: a.beige@imperial.ac.uk

\textit{New Journal of Physics} 7 (2005) 96
Received 17 January 2005
Published 14 April 2005
Online at http://www.njp.org/
doi:10.1088/1367-2630/7/1/096

Abstract. We propose a mechanism for the collective cooling of a large number $N$ of trapped particles to very low temperatures by applying red-detuned laser fields and coupling them to the quantized field inside an optical resonator. The dynamics is described by what appears to be rate equations, but where some of the major quantities are coherences and not populations. The cooperative behaviour of the system provides cooling rates of the same order of magnitude as the cavity decay rate $\kappa$. This constitutes a significant speed-up compared to other cooling mechanisms since $\kappa$ can, in principle, be as large as $\sqrt{N}$ times the single-particle cavity or laser coupling constant.

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New Journal of Physics 7 (2005) 96
1367-2630/05/010096+9$30.00 © IOP Publishing Ltd and Deutsche Physikalische Gesellschaft
1. Introduction

Cooling and trapping techniques have improved dramatically over the last few decades. A very efficient method to transfer, for example, a single atom rapidly to a very low temperature is sideband cooling [1]. This requires a red-detuned laser field whose detuning equals the frequency of the vibrational mode of the atom. When the laser excites the system from the ground to an excited state, the vibrational energy reduces by one phonon. Afterwards, the atom most likely returns to the ground state via spontaneous emission of a photon and without regaining energy in the vibrational mode. The corresponding non-unitary evolution effectively reduces the temperature of the atom and yields an overall decrease of the von Neumann entropy in the system [2]. Other experiments aim at cooling molecules, which have a much richer inner level structure and are therefore harder to control than atoms.

Here we apply the idea of sideband cooling to a large number of particles (atoms, ions or molecules). As in the one-atom case, a red-detuned laser field excites the particles, thereby continuously reducing the number of phonons in the system. To return the particles to their ground state, they should couple to the quantized field of a leaky optical cavity, whose frequency $\omega_{\text{cav}}$ equals the dipole transition frequency $\omega_0$ of each particle (see figure 1(a)). Once the particles transfer their excitation into the resonator mode, it leaks out through the cavity mirrors. Cavity decay, instead of spontaneous emission from excited levels, helps to avoid heating due to rescattering of photons within the sample. It also minimizes spontaneous emission to unwanted states and allows control of even complicated level structures, like molecules.

Crucial for obtaining maximum cooling is the generation of ‘cooperative’ behaviour of the $N$ particles in the excitation step as well as in the de-excitation step. This is possible when the Rabi frequency $\Omega_r$ of the laser field for the cooling of a vibrational mode with frequency $\nu$ is the same for all particles and all particles see the same cavity coupling $g$ (small variations of $g$ and $\Omega_r$ and non-ideal initial conditions do not substantially affect our conclusions and will be considered elsewhere [3]). Realizing this inside an optical resonator requires self-organization of the particles in the antinodes of the cavity field, as predicted in [4]. If the dipole moments of the atoms are on average parallel to the cavity mirrors, the Rabi frequencies $\Omega_r$ are, practically and to a good approximation, the same for all particles, if the laser field enters the cavity from the side as shown in figure 1(b). The laser could also enter the cavity through one of the cavity mirrors, as was the case in the many-atom cavity QED experiment by Grangier and co-workers in 1997 [5]. Alternatively, a ring resonator can be employed, as in [4], [6]–[8], if a laser field with $\omega_{\text{laser}}$ enters the resonator in a certain angle $\varphi$ with $\cos \varphi = \omega_{\text{laser}} / \omega_{\text{cav}}$ (see figure 1(c)).

We also require that the particles be initially all prepared in their ground state (in the large-$N$ limit, fluctuations can be neglected). Then the time evolution of the system remains restricted within a Dicke-symmetric subspace of collective states [9]. As shown below, these states experience a very strong coupling to the laser field as well as to the cavity mode and the system evolves into a stationary state with no phonons on the timescale given by the cavity photon life time. Several schemes for the cooling of atomic ensembles have been proposed [4, 7], [10]–[12] and first cavity-cooling experiments have already been performed [6, 8, 13]. Compared to these, the scheme we propose provides a significant speed-up of the cooling process if operated

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4 This ensures that the phase factors in the cavity interaction term of the Hamiltonian can be absorbed into the definition of the excited states [1] of the particles in the respective positions. The same applies for the phase factors of the laser amplitudes, which are now exactly the same as the phase factors of the cavity field term.
in a regime with
\[ \kappa \sim \sqrt{N}g, \quad \frac{1}{2}\sqrt{N}\eta \Omega \gg \Gamma \quad \text{with} \quad \Omega = \left( \sum_{\nu} \Omega_{\nu}^2 \right)^{1/2}. \] (1)

Here \( \kappa \) denotes the cavity photon decay rate, \( \Gamma \) is the spontaneous decay rate of a particle in the excited state and \( \eta \) is the Lamb–Dicke parameter characterizing the steepness of the trap. For sufficiently large \( N \), condition (1) can be fulfilled even if \( \eta \ll 1 \) and the system is operated in the bad cavity limit with \( \kappa \gg g^2/\kappa > \Gamma \). Moreover, we remark that equation (1) describes a ‘strong damping regime’ in which the system can only accumulate a small amount of population in the excited states of the particles.

The proposed cooling scheme can be used to cool a large number of particles very efficiently. It is therefore an interesting question, whether the described setup might be used for the preparation of Bose–Einstein condensates. Currently, these experiments mainly use evaporative cooling [14] which systematically removes those atoms with a relatively high temperature from the trap. Consequently, only a small percentage of the initially trapped atoms is finally included in the condensate. If one could instead cool all the atoms efficiently, yet at the same time avoid the loss of particles, it should become easier to experiment with large condensates. Besides, cooling is also crucial for ion-trap quantum computing where the achievable gate-operation times can depend primarily on the efficiency of the cooling of a common vibrational mode [15].

2. Bosonic behaviour of a large atomic sample

We consider a collection of \( N \) two-level particles with ground states \( |0\rangle_i \) and excited states \( |1\rangle_i \), each of them described by \( \sigma_3 = \frac{1}{2}(|1\rangle_i\langle 1| - |0\rangle_i\langle 0|) \) with eigenvalues \( \pm \frac{1}{2} \). Transitions between the two levels are generated by \( \sigma_+ = |1\rangle_i\langle 0| \) and \( \sigma_- = |0\rangle_i\langle 1| \). Our fermion-like \( N \)-body system is thus described by the su(2) algebra
\[ [\sigma_3, \sigma^\pm] = \pm \sigma^\pm, \quad [\sigma^-, \sigma^+] = -2\sigma_3, \] (2)

with \( \sigma^\pm = \sum_{i=1}^{N} \sigma^\pm_i \) and \( \sigma_3 = \sum_{i=1}^{N} \sigma_3_i \). Under the action of \( \sigma^\pm \), describing the laser excitation, the initial state with all particles in the ground state, \( |0\rangle_p \), is driven into the Dicke-symmetric states \( |l\rangle_p \) with
\[ |l\rangle_p \equiv \left( |0_1 0_2 \cdots 0_{N-l} 1_{N-l+1} 1_{N-l+2} \cdots 1_N \rangle + \cdots + |1_1 1_2 \cdots 1_l 0_{l+1} 0_{l+2} \cdots 0_N \rangle \right)/\sqrt{\binom{N}{l}}, \] (3)

Figure 1. Atomic level scheme (a) and experimental setup for the collective cooling of many particles trapped inside an optical cavity (b) or an optical ring resonator (c).
a superposition of all states with \( l \) particles in \( |1\rangle \). The difference between excited and unexcited particles is counted by \( \sigma_3 \), since \( p\langle l|\sigma_3|l\rangle_p = l - \frac{1}{2}N \). For any \( l \)

\[
\sigma^+|l\rangle_p = \sqrt{l+1}\sqrt{N-l}|l+1\rangle_p \quad \text{and} \quad \sigma^-|l\rangle_p = \sqrt{N-(l-1)}\sqrt{l}|l-1\rangle_p,
\]

showing that \( \sigma_\pm \) and \( \sigma_3 \) are represented on \( |l\rangle_p \) by the Holstein–Primakoff non-linear boson realization \[16, 17\]

\[
\sigma^+ = \sqrt{N}S^+A_S, \quad \sigma^- = \sqrt{N}A_SS^-, \quad \sigma_3 = S^+S^- - \frac{1}{2}N,
\]

with

\[
A_S = \sqrt{1-S^+S^-/N}, \quad S^+|l\rangle_p = \sqrt{l+1}|l+1\rangle_p, \quad S^-|l\rangle_p = \sqrt{l}|l-1\rangle_p
\]

for any \( l \). The \( \sigma \)'s still satisfy the su(2) algebra \( (2) \). However, for \( N \gg l \), equation \( (4) \) becomes

\[
\sigma^\pm|l\rangle_p = \sqrt{N}S^\pm|l\rangle_p
\]

and thus \( S^\pm = \sigma^\pm/\sqrt{N} \) for large \( N \). In the large-\( N \) limit, the su(2) algebra \( (2) \) written in terms of \( S^\pm \) and \( S_3 \equiv \sigma_3 \), contracts to the (projective) e(2) (or Heisenberg–Weyl) algebra \[18, 19\]

\[
[S_3, S^\pm] = \pm S^\pm \quad \text{and} \quad [S^-, S^+] = 1.
\]

The meaning of equations \( (7) \) and \( (8) \) is that, for large \( N \), the laser excites collective dipole waves, \( S^\pm \) denoting the creation and annihilation operators of the associated quanta, and the collection of single two-level particles manifests itself as a bosonic system.

### 3. Collective cooling of common vibrational modes

Each particle may couple to its own phonon modes and there can also be common vibrational modes. We first discuss a scheme for the collective cooling of common modes. This requires the application of laser fields, each red-detuned by a phonon frequency \( \nu \) and with Rabi frequency \( \Omega_\nu \).

In the following, \( b_\nu \) is the annihilation operator for a phonon with \( \nu \) and \( c \) denotes the annihilation operator for a cavity photon. The Hamiltonian of the system in the interaction picture and within the rotating wave approximation\(^5\) then equals\(^6,7\)

\[
H_I = \sum_\nu \frac{1}{2} \hbar \sqrt{N}\eta \Omega_\nu \ S^+b_\nu + \hbar \sqrt{N}gS^+c + \text{H.c.}
\]

A detailed derivation of the atom–phonon coupling Hamiltonian in equation \( (9) \) can be found in \[20\]. It applies in the Lamb–Dicke limit, where the atom–phonon coupling is relatively small compared to the phonon frequency \( \nu \) and \((\frac{1}{2}\hbar \Omega_\nu)^2 \ll \nu^2 \). Moreover, we neglect the non-resonant

\(^5\) This approximation introduces some errors when calculating the behaviour of the system for times \( t \ll 1/\nu \), with a restriction of achievable cooling rates from above. Nevertheless, collective cooling can be much more efficient than previously considered mechanisms \[3\].

\(^6\) Note that the coupling constant \( g \) depends on the geometry of the respective setup. In the case of a ring cavity, \( g \) and the annihilation operator \( c \) incorporate all possible modes of the quantized electromagnetic field in the resonator.

\(^7\) We also observe that the cavity is in resonance with the atomic transition of figure 1(a) and therefore the cavity does not couple to the vibrational modes of the particles.
coupling of the laser Hamiltonian to the 1–2 transition of the particles. This term is negligible compared to the driving of the resonant excitation of the sideband with coupling strength $\sqrt{N/2} \eta \Omega_v$ if $\Omega_v \ll \nu$. We neglect this non-resonant laser driving here since we do not expect it to have an effect on the conclusions drawn in the paper. It can only lead to an additional evolution between the states $|l\rangle_p$, but cannot cause unwanted population outside the Dicke-symmetric subspace. Using the notation

$$x_v \equiv \sqrt{N/2} \eta \Omega_v, \quad x \equiv \left( \sum_v x_v^2 \right)^{1/2} = \frac{1}{2} \sqrt{N} \eta \Omega, \quad b \equiv \sum_v (x_v/x) b_v, \quad y \equiv \sqrt{N} g,$$

(10)

equation (9) becomes

$$H_I = \hbar x S^+ b + \hbar y S^+ c + H.c.,$$

(11)

where the phonon annihilation operator $b$ obeys the familiar commutator relation $[b, b^\dagger] = 1$.

The leakage of photons through the cavity mirrors is accounted for by considering the master equation

$$\dot{\rho} = -i \hbar [H_I, \rho] + \kappa (c \rho c^\dagger - \frac{1}{2} c^\dagger c \rho - \frac{1}{2} \rho c^\dagger c),$$

(12)

where $\rho$ is the density matrix of the combined state of all particles, the common vibrational mode and the cavity field. Assuming regime (1) and restricting ourselves to Dicke states with $l \ll N$, spontaneous emission from the particles is negligible. Moreover, all noise terms (like heating) with amplitudes small compared to $x$, $y$ and $\kappa$ can be neglected.

The concrete form of equation (12) suggests that the stationary state $\rho_{ss}$ corresponds to a coherent state of the form $|\alpha\rangle_p |\beta\rangle_v |\gamma\rangle_c$ with $S^- |\alpha\rangle_p = \alpha |\alpha\rangle_p$, $b |\beta\rangle_v = \beta |\beta\rangle_v$ and $c |\gamma\rangle_c = \gamma |\gamma\rangle_c$. Indeed, the only solution of $\dot{\rho}_{ss} = 0$ is the state $|0\rangle_p |0\rangle_v |0\rangle_c$ with all particles in the ground state, no photons in the cavity and no phonons in the common vibrational mode. Once the atoms have been initialized, the system loses its phonons within the time it takes to reach the stationary state. Since this time evolution is solely governed by the frequencies $x = \frac{1}{2} \sqrt{N} \eta \Omega, \quad y = \sqrt{N} g$ and $\kappa$, we expect that this happens in the large-$N$ limit in a time given by the smallest of these frequencies.

To calculate the cooling rate explicitly, we derive a set of differential equations for the variables

$$m \equiv \langle b^\dagger b \rangle_\rho, \quad n \equiv \langle c^\dagger c \rangle_\rho, \quad s_3 \equiv \langle S_3 \rangle_\rho,$$

(13)

where $m$ is the mean number of phonons with respect to the phonon number operator $b^\dagger b$, $n$ is the mean number of photons in the cavity mode and $s_3$ relates to the mean number of particles in the excited state $|1\rangle$. It is also useful to consider the coherence quantities

$$k_1 \equiv \langle S^+ b - S^- b^\dagger \rangle_\rho, \quad k_2 \equiv \langle S^+ c - S^- c^\dagger \rangle_\rho, \quad k_3 \equiv \langle b c^\dagger + b^\dagger c \rangle_\rho.$$

(14)

The introduction of (14) is motivated by the fact that the system first builds up the coherences $k_i$, which, once established, provide an effective coupling between the different subsystems. Using equation (12) we obtain

$$\dot{m} = i x k_1, \quad \dot{n} = i y k_2 - \kappa n, \quad \dot{s}_3 = -i (x k_1 + y k_2).$$

(15)
The annihilation and creation of phonons and photons is accompanied by changes in the excitation number of the single particles. For $l \ll N$, these fluctuations remain negligible on average and the approximations $\langle S^3 b^\dagger b \rangle_\rho = s_3 m$, $\langle S^3 c^\dagger c \rangle_\rho = s_3 n$ and $\langle S^3 (bc^\dagger + b^\dagger c) \rangle_\rho = s_3 k_3$ can be adopted. We also neglect contributions of order one compared to $N$ such as $\langle S^+ S^- \rangle_\rho$. Then
\begin{equation}
\dot{k}_1 = -\frac{2i}{N} (2x m + y k_3) s_3, \quad \dot{k}_2 = -\frac{2i}{N} (2y n + x k_3) s_3 - \frac{1}{2} k k_2, \quad \dot{k}_3 = i (y k_1 + x k_2) - \frac{1}{2} k k_3.
\end{equation}
(16)
These non-linear differential equations imply
\begin{equation}
\dot{m} = \frac{x}{y^2} (\kappa k_3 + 2 \dot{k}_3) - \frac{x^2}{y^2} (k n + \dot{n}).
\end{equation}
(17)
We see below that the presence of a negative $k_3$ and a positive $n$ provides a cooling channel and plays a crucial role in the cooling process.

Here we are interested in the cooling of a large number of particles. This allows us to solve the time evolution considering first the regime where $\kappa \approx 0$ and equation (17) becomes the conservation law
\begin{equation}
\dot{m} - \frac{x}{y} \dot{k}_3 + \frac{x^2}{y^2} \dot{n} = 0.
\end{equation}
(18)
In the parameter regime (1) and given that $m$, $n$ and $s_3$ are of order $N$, the system reaches a stationary state with $m$, $n$ and $k_3$ constant on a timescale of the order $1/\sqrt{N}$. After such a time, we might safely assume $\dot{m} = \dot{n} = \dot{k}_1 = \dot{k}_2 = \dot{k}_3 = 0$, and obtain from equation (16) and for $\kappa \approx 0$ that $k_1 = k_2 = 0$, $k_3 = -(2x/y) m$ and $n = (x^2/y^2) m$. This stationary state is actually reached in a time given by the smallest among $\sqrt{Ng}$ and $\frac{1}{2} \sqrt{N\eta/\Omega}$, while $\kappa \approx 0$ controls longer-lived processes such as the one described by equation (17). Under the above assumption for the zeroth order in $\kappa$, we find $\dot{m} = -[\kappa x^2 (x^2 + y^2)/y^4] km$ to the first order in $\kappa$. From this we get
\begin{equation}
m(t) = m_0 \exp \left[-\frac{x^2 (x^2 + y^2)}{y^4} \kappa t\right],
\end{equation}
(19)
where $m_0$ is the initial number of phonons in the system with respect to the above defined operator $b^\dagger b$. The exponential decrease of the phonon population (see figure 2(a)) amounts to the overall system cooling with a rate of the same order of magnitude as $\kappa$. The result (19), which, we stress, holds under the condition of large $N$, shows that the cooling of the system does not depend on the specific value of $N$, and thus, provided $N$ is large, it holds even in the case where not all the particles are initially prepared in their ground state, which helps the feasibility of the scheme. We also remark that such a behaviour becomes possible only because of dissipation, namely the leakage of photons through the cavity mirrors.

4. Collective cooling of the individual motion of the atoms

The collective regime established above may also be obtained in the case where each particle $i$ couples to its own set of individual phonon modes $b_{\nu,i}$. As above, we assume that the Rabi
frequencies $\Omega_\nu$ of the corresponding laser fields with detuning $\nu$ are the same for all particles. Then the Hamiltonian of the system equals in the interaction picture

$$H_I = \sum_{\nu,i} \frac{1}{2} \hbar \eta \Omega_\nu \sigma^+_{\nu,i} b_{\nu,i} + \sum_i \hbar g \sigma^+_i c + H.c. \quad (20)$$

With the notation

$$x_\nu \equiv \sqrt{N} \frac{1}{2} \eta \Omega_\nu, \quad x \equiv \left( \sum_\nu x^2_\nu \right)^{1/2}, \quad b_i \equiv \sum_\nu (x_\nu/x) b_{\nu,i}, \quad y \equiv \sqrt{N} g, \quad (21)$$

where the $b_i$ obeys the relation $[b_i, b^+_j] = \delta_{ij}$ and equation (20) simplifies to

$$H_I = \frac{\hbar}{\sqrt{N}} \sum_i x \sigma^+_i b_i + y \sigma^+_i c + H.c. \quad (22)$$

Suppose that the system is initially prepared in a state with all particles in the ground state, the mean phonon number is the same for all particles and of about the same size as $m_0$ considered before and there are no photons in the cavity mode. Then the operator $\sum_{i=1}^N \sigma^+_i b_i$ has a similar effect on the system state as the operator $S^+ b$ in the previous case [22]. The net result is shown to be again collective cooling in the large-$N$ limit.

Proceeding as before, we first calculate the stationary state $\rho_{ss}$. Leakage of photons through the cavity mirrors is accounted for by using equation (12). As before, the form of the equations (12) and (22) suggests that $\rho_{ss}$ is the state with all particles in the ground state and no phonons and no photons in the cavity. Indeed, it obeys $\dot{\rho}_{ss} = 0$.

To calculate the cooling rate explicitly, we consider the expectation values

$$\tilde{m} \equiv N \langle b^+_1 b_1 \rangle_\rho, \quad \tilde{n} \equiv \langle c^+ c \rangle_\rho, \quad \tilde{s}_3 \equiv N \langle \sigma^a_{31} \rangle_\rho, \quad (23)$$

where we assumed $\sum_i \langle b^+_i b_i \rangle = N \langle b^+_1 b_1 \rangle$, and the coherences

$$\tilde{k}_1 \equiv \sqrt{N} \langle \sigma^+_1 b_1 - \sigma^-_1 b^+_1 \rangle_\rho, \quad \tilde{k}_2 \equiv \sqrt{N} \langle \sigma^+_1 c - \sigma^-_1 c^+ \rangle_\rho, \quad \tilde{k}_3 \equiv N \langle b^+_1 c + b_1^+ c \rangle_\rho. \quad (24)$$
where we sum over all particles and use $\sigma_i^\pm / \sqrt{N}$ in (24) in analogy to $S^\pm$ in definition (14). Using equation (12) with the Hamiltonian (22) and the same approximations as before, we obtain

$$
\dot{\tilde{m}} = i x \tilde{k}_1, \quad \dot{\tilde{n}} = i y \tilde{k}_2 - \kappa \tilde{n}, \quad \dot{\tilde{s}_3} = -i(\tilde{x}_{k_1} + \tilde{y}_{k_2})
$$

(25)

and

$$
\dot{\tilde{k}}_1 = -\frac{2i}{N^2}(2x\tilde{m} + y\tilde{k}_3)\tilde{s}_3, \quad \dot{\tilde{k}}_2 = -\frac{2i}{N^2}(2Ny\tilde{n} + x\tilde{k}_3)\tilde{s}_3 - \frac{1}{2}\kappa \tilde{k}_2, \quad \dot{\tilde{k}}_3 = i(y\tilde{k}_1 + x\tilde{k}_2) - \frac{1}{2}\kappa \tilde{k}_3
$$

(26)

implying

$$
\dot{\tilde{m}} = \frac{x}{2y}(\kappa \tilde{k}_3 + 2\tilde{k}_3) - \frac{x^2}{y^2}(\kappa \tilde{n} + \tilde{n}).
$$

(27)

The differential equations (25) and (26) reveal that, for $\kappa \approx 0$, the system reaches a stationary state within a time of the order one with $\tilde{k}_1 = \tilde{k}_2 = 0$ and $\tilde{k}_3 = -(2x/y)\tilde{m}$, given $\tilde{m}$ is of order $N$, while the cavity accumulates a small population of photons. However, $\tilde{n} = (x^2/Ny^2)\tilde{m}$ remains small and the coherence $\tilde{k}_3$ provides the main decay channel for the phonons in the system. From equation (27) we obtain for large $N$, in analogy to equation (19),

$$
\tilde{m}(t) = \tilde{m}_0 \exp\left(-\frac{x^2}{y^2}\kappa t\right),
$$

(28)

where $\tilde{m}_0$ is the initial total phonon number. Thus, similar to the result (19), the rate of cooling of individual phonon modes is of the same order of magnitude as the cavity decay rate $\kappa$, after a transition time given by the smallest among $g$ and $\frac{1}{2}\eta\Omega$ (see figure 2(b)).

5. Conclusions

In this paper we propose a new cooling mechanism based on the collective excitation and deexcitation of particles trapped inside an optical cavity. In section 2, we showed that a large atomic sample in the presence of highly symmetric interactions, that treat all particles in the same way, behaves like a collection of bosonic particles. In sections 3 and 4, we then analysed the two extreme cases, where the applied laser fields aim either at the cooling of only common or individual motions of the particles. For both cases, we predict cooling rates of the same order of magnitude as the cavity decay rate $\kappa$. In the general case, one might argue that the number of vibrational modes is extremely large. However, we believe that similar cooling rates would be achievable in this case as well, as long as the initial phonon energy in the setup does not increase with the number of particles $N$ in the setup, which is in general not the case.

We conclude with a few comments. The conservation law (17) obtained for $\kappa = 0$ allows one to define the conserved quantity $\hat{\mathcal{Q}} = 0$ with $\mathcal{Q} \equiv m + (x^2/y^2) n - (x/y) k_3$. The meaning of the time independence of $\mathcal{Q}$ is that, one can shift the quantities $k_3$, $m$ and $n$ by some constants without changing the dynamics of the system apart from the changes in the process of redistributing phonons, governed by $k_1$, and their population $m$. The leakage of photons through the cavity mirrors ($\kappa \neq 0$) disturbs the equilibrium expressed by $\hat{\mathcal{Q}} = 0$ inducing a dynamical response, i.e. a quantum phase transition: the overall effect is the exponential decrease in the phonon population, namely cooling [22].

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The treatment in this paper should be compared with the traditional approach where the attention is focused on the single particle behaviour. In such a single particle treatment [23], the cooling depends exponentially on the cavity decay rate $\kappa$. In this case, $\kappa$ has to be about the same size as $g$ and $\frac{1}{\gamma} \eta \Omega$, which imposes a strong constraint on the timescale of the cooling process. The advantage of our treatment lies, in contrast, in the fact that the number of particles $N$ introduces, in principle, the crucial freedom to tune $\kappa$ to be as large as $\sqrt{Ng}$ and $\frac{1}{\gamma} \sqrt{N} \eta \Omega$. Thus, cooling the particles very efficiently. Although corrections might be necessary in realistic situations, a high cooling rate also reduces the number of collisions between particles, which are a problem, certainly in less effective cooling schemes. We believe that considering the cavity-mediated collective-field theoretical behaviour opens a new perspective in particle cooling.

Acknowledgments

This work was supported in part by the European Union, COSLAB (ESF Program), INFN, INFM and the UK Engineering and Physical Sciences Research Council. AB acknowledges stimulating discussion with P Grangier and thanks the Royal Society and the GCHQ for a James Ellis University Research Fellowship.

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