$J/\psi$ production in the statistical hadronization model - implications and limitations

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Abstract. The Statistical Hadronization Model can be extended to include hadrons which are produced far away from chemical equilibrium or in an earlier phase of the evolution of a heavy ion collision. The effects of $J/\psi$ production above the critical temperature are explored, with special focus on the mass shift for the thermal dressed mass of the charm quarks in the quark-gluon plasma.

1. Introduction
The statistical hadronization model (SHM) has proved to describe well the production of hadrons in experiments in which a quark-gluon plasma (QGP) is formed [1, 2, 3, 4, 5], for both hadrons containing only light quarks [6, 7], and in an extension of the original idea also for hadrons containing charm quarks [8, 9]. This enables the calculation of the $J/\psi$ multiplicity, an important observable in heavy ion collisions, since its suppression is regarded as a signature of the formation of the QGP [10]. One of the fundamental assumptions of statistical hadronization, the instantaneous production of all particle species at one point in the phase diagram of strongly interacting matter, is called into question in the case of the $J/\psi$ by results from lattice calculations suggesting survival of the bound state up to 1.9 times the critical temperature $T_c$ [11, 12], though more recent works contest this interpretation[13].

Following the deliberations of Grandchamp et al. [14], we propose a method for treating $J/\psi$ production in the earlier phase of a heavy ion collision by assuming equilibration of the charm and anti-charm quarks and the $J/\psi$ under detailed balance constraints, and compare the resulting yields to both those measured in experiments and obtained from traditional statistical hadronization model calculations [15].

2. Model overview
In the Statistical Hadronization Model, hadronic states and resonances are treated as states of grand-canonical ensemble which are occupied according to the equilibrium distribution governed by the chemical freeze-out temperature $T$ and the baryochemical potential $\mu_B$, which is derived...
from the grand-canonical partition function \( Z_i^{GC} \) [7, 16], where

\[
\ln Z_i^{GC} = \pm \frac{V g_i}{2\pi^2} \int_0^\infty dp \, p^2 \ln \left( 1 \pm e^{-\frac{E_i - m}{T}} \right) = \frac{V T g_i}{2\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \lambda_i^n \chi_i^2 K_2 \left( \frac{nm_i}{T} \right) .
\]  

The upper sign applies to fermions and the lower sign to bosons, while considering only \( n = 1 \) in eq. (2) corresponds to Boltzmann statistics and is sufficient for all species \( i \) with \( m_i \gg T \). From \( \ln Z_i^{GC} \), the thermodynamic properties of the system are calculated.

\[
N_i^{th}(T, \mu) = T \frac{\partial \ln Z_i^{GC}}{\partial \mu} = \frac{V T g_i}{2\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \lambda_i^n \chi_i^2 K_2 \left( \frac{nm_i}{T} \right) ,
\]

\[
S_i(T, \mu) = \frac{\partial}{\partial T} \left( T \ln Z_i^{GC} \right) = \frac{V g_i}{2\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \lambda_i^n \chi_i^2 \frac{G_{i,n}(T, \mu)}{n^2} ,
\]

where

\[
G_{i,n}(T, \mu) = (2T - \mu n) K_2 \left( \frac{nm_i}{T} \right) + \frac{nm_i}{2} \left[ K_1 \left( \frac{nm_i}{T} \right) + K_2 \left( \frac{nm_i}{T} \right) \right] .
\]

Fitting eq. (3) to different ratios \( N_i^{th}(T, \mu)/N_i^{th}(T, \mu) \), the model parameters \( T \) and \( \mu_B \) can be determined. The hadronization source volume \( V_{SHM} \) is then adjusted to the measured production of charged particles in a heavy ion collision experiment such that the thermal densities \( n_{th} \) match the observed multiplicities \( N_{ch} \). A thermodynamically consistent correction for finite hadron volumes is applied according to the method provided in [17, 18].

To calculate \( J/\psi \) production in the SHM, the fact that charm quarks are not in chemical equilibrium in the pre-hadronic phase of a heavy ion collision is accounted for by enforcing exact charm conservation and taking the number of \( c\bar{c} \) pairs produced in the initial hard collisions \( N_{oc}^{dir} \) as an additional input [9, 16, 19], which leads to the charm balance equation for open (oc) and hidden (\( c\bar{c} \)) charm,

\[
N_{c\bar{c}}^{dir} = \frac{1}{2} g_c N_{oc}^{th} \frac{I_1(g_c N_{oc}^{th})}{I_0(g_c N_{oc}^{th})} + g_c^2 N_{c\bar{c}}^{th} ,
\]

where \( g_c \) is the charm fugacity factor to ensure that the thermal multiplicities are scaled according to the constraint of exact charm conservation. Consequently, the \( J/\psi \) multiplicity is

\[
N_{J/\psi}^{SHM} = g_c^2 \cdot n_{J/\psi}^{th} \cdot V_{SHM} .
\]

Note that due to the high uncertainties for the charm production cross section in heavy ion collisions and the quadratic contribution of \( g_c \) to \( N_{J/\psi}^{SHM} \), this is subject to equivalent uncertainties.

3. \( J/\psi \) in the QGP

Results from lattice QCD suggest survival of heavy quarkonia in the QGP at temperatures up to 1.9 times the critical temperature [11, 12]. Even if more recent analyses call this interpretation into question [13], the implications regarding statistical \( J/\psi \) production before the hadronization of the thermal medium can be studied under the assumption of entropy conservation at the chemical freeze-out temperature, justified by the assumed instantness of the hadronization
process in the SHM, and by approximating the plasma phase in the Stefan-Boltzmann limit. Analogous to the hadronic case, a source volume is defined by \( V_{QGP} = S_{SHM}/s_{QGP} \) using eq. (4). Instead of quantifying the deviation of open and hidden charm hadrons from chemical equilibrium with \( g_c \), the deviation of the charm and anti-charm quarks from equilibrium is quantified by the plasma charm fugacity \( \lambda_c \), which is calculated from

\[
N_{c\bar{c}}^{dir} = \lambda_c \frac{2V_{QGP}}{2\pi^2} (m^*_c T)^\frac{3}{2} \exp(-m^*_c/T),
\]

again under the constraint of exact charm conservation. Note that the result is strongly dependent on the charm quark mass in the medium \( m^*_c \), which might differ from the bare quark mass of 1275 MeV [20]. It follows that

\[
N_{J/\psi}^{QGP} = \lambda_c^2 \cdot n_{J/\psi}^{th} \cdot V_{QGP}.
\]

4. Results

The \( J/\psi \) multiplicities obtained from evaluating eqs. (7) and (9) should be the same, as the \( J/\psi \) color singlet states should 1) equilibrate with the medium under the constraint of detailed balance ensured by \( \lambda_c \) and 2) be mostly unaffected by any processes occurring at the hadronization of the medium itself. By comparing both results in the range for \( T \) suggested by fits to experimental data, see fig. 1, the value of \( m^*_c \) needed for compatible results in both pictures can be obtained. The model parameters were chosen as to represent central Au-Au collisions at top RHIC energy [9]. Generally, a low dependence of the \( J/\psi \) yield on the temperature as a primary model parameter can be observed, in contrast, the dependence on \( m^*_c \) is very high in the partonic description. For a freeze-out temperature of \( T \simeq 160 \) MeV, a dressed charm quark mass of \( m^*_c \simeq 1550 \) MeV is needed for the multiplicities to be compatible with the standard SHM result.

![Figure 1](image-url)

Figure 1. Comparision of the results from eqs. (7) and (9) for \( m^*_c = 1525-1600 \) MeV at RHIC energy. The shaded band represents the experimental results [21].

5. Conclusions and outlook

Assuming conservation of entropy across the hadronization process and the exact conservation of charm and anti-charm quarks over the evolution of the fireball, the treatment of \( J/\psi \)
mesons above the chemical freeze-out temperature is possible. By comparing to the standard treatment of $J/\psi$ yields, the dressed mass of the charm quarks is fixed to values in the vicinity $m_c^* \simeq 1550 \text{ MeV}$, much higher than the bare quark mass of 1275 MeV, but also much lower than the value of $m_c^* = m_D = 1870 \text{ MeV}$ used in transport codes which describe the experimental results for the $J/\psi$ very well [22, 23, 24]. We will check the influence of a more realistic entropy parametrization as for example provided by [25, 26], where we expect $m_c^*$ to be higher and therefore compatible with the result of $m_c^* = 1600–1700 \text{ MeV}$ found in [14]. We will further investigate the equilibration process between the $J/\psi$ and the charm quarks in the partonic transport model BAMPS (Boltzmann Approach of MultiParton Scatterings) [27, 28], which simulates the 3+1 space-time evolution of the QGP.

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