ON THE T-EQUIVALENCE RELATION

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Abstract. For a completely regular space $X$, denote by $C_p(X)$ the space of continuous real-valued functions on $X$, with the pointwise convergence topology. In this article we strengthen a theorem of O. Okunev concerning preservation of some topological properties of $X$ under homeomorphisms of function spaces $C_p(X)$. From this result we conclude new theorems similar to results of R. Cauty and W. Marciszewski about preservation of certain dimension-type properties of spaces $X$ under continuous open surjections between function spaces $C_p(X)$.

1. Introduction

One of the main objectives in the theory of $C_p(X)$ spaces is to classify spaces of continuous functions up to homeomorphisms. One can do this by investigating which topological properties of a space $X$ are shared with a space $Y$, provided $X$ and $Y$ are $t$-equivalent, i.e. $C_p(X)$ and $C_p(Y)$ are homeomorphic. Recently, O. Okunev published a paper [12] in which he found some new topological invariants of the $t$-equivalence relation. All of them are obtained from the following, very interesting Theorem (see [12, Theorem 1.1])

Theorem 1.1. (Okunev) Suppose that there is an open continuous surjection from $C_p(X)$ onto $C_p(Y)$. Then there are spaces $Z_n$, locally closed subspaces $B_n$ of $Z_n$, and locally closed subspaces $Y_n$ of $Y$, $n \in \mathbb{N}^+$, such that each $Z_n$ admits a perfect finite-to-one mapping onto a closed subspace of $X^n$, $Y_n$ is an image under a perfect mapping of $B_n$, and $Y = \bigcup \{Y_n : n \in \mathbb{N}^+\}$.

In the formulation of the above theorem in [12] the assumption about the existence of an open continuous surjection is replaced by the assumption that these function spaces are homeomorphic. However, as noticed in [12] remarks at the end of section 1] a careful analysis of the proof reveals that the weaker assumption is sufficient. In this paper we will discuss the proof of the above theorem (detailed proof can be found in [12]). Then using an idea from [9] we will show how to slightly improve Okunev’s result, answering
Question 1.9 from [12]. In the subsequent sections we will derive a few corollaries from strengthened form of Okunev’s theorem. We will use it to find new invariants of the \( t \)-equivalence relation concerning dimension. These results are in the spirit of the significant theorems of R. Cauty from [3] and W. Marciszewski from [9].

We should also mention here, that the answer to Question 2.12 posed in [12] is known (see [2], [8]). Thus one can show (see [12]) that \( \sigma \)-discreteness is preserved by the \( t \)-equivalence relation (see [12, Question 2.9]). In fact, from a result of Gruenhage from [8] one can conclude more, namely that \( \kappa \)-discreteness is preserved by the relation of \( t \)-equivalence (see Theorem 3.1 below). We discuss this in Section 3.

Unless otherwise stated, all spaces in this note are assumed to be Tychonoff. For a space \( X \) we denote by \( C_p(X) \) the space of continuous, real-valued functions on \( X \) with the pointwise convergence topology. We say that spaces \( X \) and \( Y \) are \( t \)-equivalent, provided \( C_p(X) \) and \( C_p(Y) \) are homeomorphic. The subspace of a topological space is locally closed if it is the intersection of a closed set and an open set. The mapping \( \varphi : X \rightarrow Y \) between topological spaces is perfect, provided it is closed and all fibers \( \varphi^{-1}(y) \) are compact. For a space \( X \) we denote by \( \text{Fin}(X) \) the hyperspace of all finite subsets of \( X \) with the Vietoris topology. We follow Engelking’s book [4] regarding dimension theory.

2. On a Result of Okunev

The main goal of this section is to answer Question 1.9 from [12], i.e. to prove that in the statement of Theorem 1.1 we may additionally require that for every \( n \in \mathbb{N}^+ \) the space \( Y_n \) is in fact an image under a perfect finite-to-one mapping of \( B_n \). To this end we need to discuss the main ideas from [12]. For the convenience of the reader our notation will be almost the same as in [12].

The real line \( \mathbb{R} \) is considered as a subspace of its two-point compactification \( I = \mathbb{R} \cup \{-\infty, +\infty\} \). For a continuous function \( f : Z \rightarrow \mathbb{R} \), the function \( \tilde{f} : \beta Z \rightarrow I \) is the continuous extension of \( f \). For every \( n \in \mathbb{N}^+ \), \( \overrightarrow{z} = (z_1, \ldots, z_n) \in (\beta Z)^n \) and \( \varepsilon > 0 \) we put

\[
O_Z(\overrightarrow{z}; \varepsilon) = O_Z(z_1, \ldots, z_n; \varepsilon) = \{ f \in C_p(Z) : |\tilde{f}(z_1)| < \varepsilon, \ldots, |\tilde{f}(z_n)| < \varepsilon \}.
\]

Similarly, for every \( A \in \text{Fin}(Z) \) and \( \varepsilon > 0 \) we put

\[
O_Z(A; \varepsilon) = \{ f \in C_p(Z) : \forall z \in A \ |f(z)| < \varepsilon \}.
\]
For a point \( z \in Z \) we put
\[
\overline{O}_Z(z; \varepsilon) = \{ f \in C_p(Z) : |f(z)| \leq \varepsilon \}.
\]

Let \( \Phi : C_p(X) \to C_p(Y) \) be an open surjection which takes the zero function on \( X \) to the zero function on \( Y \) (we can assume this since \( C_p(X) \) and \( C_p(Y) \) are homogeneous). For every \((m, n) \in \mathbb{N}^+ \times \mathbb{N}^+ \) we put
\[
Z_{m,n} = \{ (x, y) \in X^n \times Y : \Phi(O_X(x, \frac{1}{m})) \subseteq \overline{O}_Y(y, 1) \}.
\]

By \( \pi_X : X^n \times \beta Y \to X^n \) we denote the projection and we put
\[
p_{m,n} = \pi_X \restriction Z_{m,n} : Z_{m,n} \to X^n.
\]

Similarly, by \( \pi_{\beta Y} : (\beta X)^n \times \beta Y \to \beta Y \) we denote the projection and we put
\[
A_{m,n} = \pi_{\beta Y}(Z_{m,n}).
\]

Denote by \( S_{m,n} \) the closure of \( Z_{m,n} \) in \( (\beta X)^n \times \beta Y \). For every \( m \in \mathbb{N}^+ \) we put \( Y_{m,1} = A_{m,1} \) and for every \( n > 1 \), \( Y_{m,n} = A_{m,n} \setminus A_{m,n-1} \). Finally let us put \( B_{m,n} = S_{m,n} \cap \pi_{\beta Y}^{-1}(Y_{m,n}) \) and let
\[
r_{m,n} = \pi_{\beta Y} \restriction B_{m,n} : B_{m,n} \to Y_{m,n}.
\]

The following properties are satisfied (see [12]):

1. the set \( Z_{m,n} \) is closed in \( X^n \times \beta Y \);
2. \( r_{m,n} \) maps perfectly \( Z_{m,n} \) onto a closed subset of \( X^n \);
3. the mapping \( r_{m,n} \) is finite-to-one;
4. the sets \( A_{m,n} \) are closed, thus the sets \( Y_{m,n} \) are locally closed;
5. \( Y = \bigcup_{m,n \in \mathbb{N}^+} Y_{m,n} \);
6. the set \( B_{m,n} \) is locally closed in \( Z_{m,n} \);

Clearly, Theorem 1.1 follows from (1)–(6).

We will use the following version of the \( \Delta \)-system Lemma which can be easily proved by induction (see also [11, A.1.4])

**Proposition 2.1.** Let \( X \) be a set, let \( n \in \mathbb{N}^+ \) and let \( \mathcal{A} \) be an infinite collection of subsets of \( X \) each of cardinality \( \leq n \). Then there is \( A_0 \subseteq X \) with \( |A_0| < n \) and a sequence \( A_1, A_2, \ldots \) of distinct elements of \( \mathcal{A} \) such that for distinct \( i, j \geq 1 \) we have \( A_i \cap A_j = A_0 \).

Now we are ready to prove the following strengthening of Theorem 1.1

**Theorem 2.2.** Suppose that there is an open continuous surjection \( \Phi \) from \( C_p(X) \) onto \( C_p(Y) \). Then there are spaces \( Z_n \subseteq X^n \times Y \), locally closed subspaces \( B_n \) of \( Z_n \), and locally closed subspaces \( Y_n \) of \( Y \), \( n \in \mathbb{N}^+ \), such that each \( Z_n \) admits a perfect finite-to-one mapping onto a closed subspace
of $X^n$, $Y_n$ is an image under a perfect \textbf{finite-to-one} mapping of $B_n$, and $Y = \bigcup \{Y_n : n \in \mathbb{N^+}\}$.

\textbf{Proof.} It is enough to prove that

(7) the mapping $r_{m,n}$ is finite-to-one.

To this end let us put

$$Z'_{m,n} = \{(A, y) \in \text{Fin}(X) \times Y : |A| \leq n \text{ and } \Phi(O_X(A; \frac{1}{m})) \subseteq \overline{O}_Y(y; 1)\}.$$

The natural mapping $h : Z_{m,n} \to Z'_{m,n}$ defined by

$$h((x_1, \ldots, x_n), y) = (\{x_1, \ldots, x_n\}, y),$$

is finite-to-one. Hence, if the set $\{A \in \text{Fin}(X) : (A, y) \in Z'_{m,n}\}$ is finite, then the set $\{\overline{x} \in X^n : (\overline{x}, y) \in Z_{m,n}\}$ is also finite. We will prove that this is the case.

\textbf{Claim.} For any $y \in Y_{m,n}$ the set $\{A \in \text{Fin}(X) : (A, y) \in Z'_{m,n}\}$ is finite.

\textbf{Proof.} This is basically \cite[Lemma 3.4]{4}. Assume the contrary. Then by Proposition \ref{prop2.1} there exists $A_0 \in \text{Fin}(X)$ and a sequence $A_1, A_2, \ldots$ of finite subsets of $X$ such that $|A_0| < n$, for distinct $i, j \geq 1$ we have $A_i \cap A_j = A_0$ and $(A_i, y) \in Z'_{m,n}$ for each $i \geq 1$.

To end the proof of the Claim we need to show $(A_0, y) \in Z'_{m,n}$. Indeed, then we would have $(A_0, y) \in Z'_{m,n-1}$ (since $|A_0| < n$) so $y \in A_{m,n-1}$ contradicting the assumption $y \in Y_{m,n} = A_{m,n} \setminus A_{m,n-1}$.

Let $f \in O_X(A_0; \frac{1}{m})$. We need to show that $|\Phi(f)(y)| \leq 1$. Assume the contrary. The set $\Phi^{-1}(\{\varphi \in C_p(Y) : |\varphi(y)| > 1\}$ is an open neighborhood of $f$. Hence, there exists a finite set $B \in \text{Fin}(X)$ and a natural number $k \in \mathbb{N}^+$ such that for any $g \in C_p(X)$ if $(f - g) \in O_X(B; \frac{1}{k})$, then $|\varphi(g)(y)| > 1$.

For $i \geq 1$, the sets $A_i \setminus A_0$ are pairwise disjoint. Hence, there exists $i \geq 1$ such that $B \cap (A_i \setminus A_0) = \emptyset$. Take $g \in C_p(X)$ satisfying

$$g \upharpoonright (A_0 \cup B) = f \upharpoonright (A_0 \cup B) \text{ and } g \upharpoonright (A_i \setminus A_0) \equiv 0.$$

Then $g \in O_X(A_i; \frac{1}{m})$ so $|\varphi(g)(y)| \leq 1$. On the other hand $(f - g) \in O_X(B; \frac{1}{k})$ so $\varphi(g)(y) > 1$, a contradiction.

\hfill $\diamond$

For any $y \in Y_{m,n}$, we have $r_{m,n}^{-1}(y) \subseteq \{\overline{x} \in X^n : (\overline{x}, y) \in Z_{m,n}\}$. The latter set is, as we proved, finite so the mapping $r_{m,n}$ is finite-to-one. \hfill $\square$

Theorem \ref{thm2.2} answers Question 1.9 from \cite{12}.
3. \(\kappa\)-Discreteness

Recall, that a space is called \(\kappa\)-discrete (\(\sigma\)-discrete) if it can be represented as a union of at most \(\kappa\) many (countably many) discrete subspaces. In [12], O. Okunev asked if \(\sigma\)-discreteness is preserved by the \(t\)-equivalence relation (see [12, Question 2.9]). He also showed how to reduce this question to the following one: Is a perfect image of a \(\sigma\)-discrete space also \(\sigma\)-discrete? However, the affirmative answer to this question is known (see [2], [8]). G. Gruenhage proved even a stronger result that, for any infinite cardinal \(\kappa\), a perfect image of a \(\kappa\)-discrete space is \(\kappa\)-discrete. Since the reduction made by Okunev works also for \(\kappa\)-discrete spaces, we have the following theorem.

**Theorem 3.1.** If there is an open continuous surjection from \(C_p(X)\) onto \(C_p(Y)\) and \(X\) is \(\kappa\)-discrete, then \(Y\) is \(\kappa\)-discrete.

4. The property \(C\)

From Theorem 2.2 we can conclude some new results concerning the behavior of dimension under the \(t\)-equivalence relation. The main motivation for this is the following, famous in \(C_p\)-theory problem concerning dimension (see e.g. [11, Problem 20 (1045)] or [10, Problem 2.9]).

**Problem 4.1.** (Arkhangel’skii) Suppose \(X\) and \(Y\) are \(t\)-equivalent. Is it true that \(\dim X = \dim Y\)?

It is well known, that if we additionally assume that \(C_p(X)\) and \(C_p(Y)\) are linearly or uniformly homeomorphic the above problem has an affirmative answer (see [10]). In general, very little is known about the behavior of dimensions under the relation of \(t\)-equivalence. We do not know for example if the spaces \(C_p(2^\omega)\) and \(C_p([0,1])\) or the spaces \(C_p([0,1])\) and \(C_p([0,1]^2)\) are homeomorphic (see [10]).

We should recall the following two definitions (see [4] and [6]).

**Definition 4.2.** A normal space \(X\) is called a \(C\)-space if, for any sequence of its open covers \((U_i)_{i \in \omega}\), there exists a sequence of disjoint families \((V_i)_{i \in \omega}\) of open sets such that \(V_i\) is a refinement of \(U_i\) and \(\bigcup_{i \in \omega} V_i\) is a cover of \(X\).

**Definition 4.3.** A normal space \(X\) is called a \(k\)-\(C\)-space, where \(k\) is a natural number \(\geq 2\), if for any sequence of its covers \((U_i)_{i \in \omega}\) such that each cover \(U_i\) consists of at most \(k\) open sets, there exists a sequence of disjoint families \((V_i)_{i \in \omega}\) of open sets such that for every \(i \in \omega\) the family \(V_i\) is a refinement of \(U_i\) and \(\bigcup_{i \in \omega} V_i\) is a cover of \(X\).
It is known that a normal space is weakly infinite-dimensional if and only if it is a 2-C-space (see [6]). It is clear that we have the following sequence of inclusions

\[
\text{weakly infinite-dimensional} = 2-C \supseteq 3-C \supseteq \ldots
\]

and that any C-space is a k-C-space for any \( k \in \{2, 3, \ldots\} \).

R. Cauty proved in [3] the following theorem concerning weak infinite dimension.

**Theorem 4.4.** (Cauty) Let \( X \) and \( Y \) be metrizable compact spaces such that \( C_p(Y) \) is an image of \( C_p(X) \) under a continuous open mapping. If for all \( n \in \mathbb{N}^+ \) the space \( X^n \) is weakly infinite-dimensional, then for all \( n \in \mathbb{N}^+ \) the finite power \( Y^n \) is also weakly infinite-dimensional.

Using Theorem 2.2 we can prove a version of the above theorem of Cauty for \( k \)-C-spaces. We need a suitable lemma, which is a version of [13, Theorem 4.1].

**Lemma 4.5.** Suppose that \( K \) and \( L \) are compact metrizable spaces. Let \( f : K \to L \) be a continuous countable-to-one surjection. If \( L \) is a \( k \)-C space, then so is \( K \).

**Proof.** From the proof of Theorem 4.1 in [13], it follows that it suffices to check that a class of \( \sigma \)-compact metrizable \( k \)-C-spaces is admissible, i.e. satisfies the following four conditions

(i) if \( X \) is a \( k \)-C-space and \( Y \) is homeomorphic to a closed subspace of \( X \), then \( Y \) is a \( k \)-C-space;
(ii) a space which is a countable union of \( k \)-C-spaces is a \( k \)-C-space;
(iii) if \( f : X \to Y \) is a perfect mapping, \( Y \) is zero-dimensional and all fibers \( f^{-1}(y) \) are \( k \)-C-spaces, then \( X \) is a \( k \)-C-space;
(iv) if \( A \subseteq X \), \( A \) is a \( k \)-C-space and all closed subsets of \( X \) disjoint from \( A \) are \( k \)-C-spaces, then \( X \) is a \( k \)-C-space.

Condition (i) is [6, Proposition 2.13]. Condition (ii) is [6, Theorem 2.16]. Condition (iii) is [6, Theorem 5.2]. Condition (iv) is actually [7, Lemma 2] (although it deals with C-spaces, its proof works also for \( k \)-C-spaces). \( \square \)

**Theorem 4.6.** Let \( X \) and \( Y \) be metrizable \( \sigma \)-compact spaces such that \( C_p(Y) \) is an image of \( C_p(X) \) under a continuous open mapping. Fix a natural number \( k \geq 2 \). If for all \( n \in \mathbb{N}^+ \) the space \( X^n \) is a \( k \)-C-space, then \( Y \) is also a \( k \)-C-space.
Proof. We apply Theorem 2.2 as follows. Let $Y_n$, $Z_n$, $B_n$ be as in the statement of Theorem 2.2. The space $Z_n \subseteq X^n \times Y$ is metrizable and $\sigma$-compact. Indeed, it is easy to check that a perfect preimage of a compact set is compact, so from $\sigma$-compactness of $X$ follows $\sigma$-compactness of $Z_n$. Let $Z_n = \bigcup_{m=1}^{\infty} K_m$, where each $K_m$ is compact.

Since $Z_n$ is a perfect finite-to-one preimage of a closed subspace of $X^n$ and a closed subspace of a metrizable $k$-$C$-space is a $k$-$C$-space (see [6, 1.15 and 2.19]), each $K_m$ is a $k$-$C$-space by Lemma 4.5. Since a countable union of closed $k$-$C$-subspaces is a $k$-$C$-space (see [6, 2.16]), we get that $Z_n$ is a $k$-$C$-space and thus $B_n$ is such (as an $F_\sigma$ subspace of a metrizable $k$-$C$-space [6, 1.15 and 2.19]).

Since the image of a metrizable $k$-$C$-space under a closed mapping with fibers of cardinality $< c$ is a $k$-$C$-space (see [6, 6.17]), the space $Y_n$ is a $k$-$C$-space for any $n \in \mathbb{N}^+$. Finally, since the property of being a $k$-$C$-space is invariant with respect to countable unions with closed summands (see [6, 2.16]), we get that $Y$ is a $k$-$C$-space.

From the above theorem we can conclude a result very similar to Theorem 4.4 of R. Cauty we mentioned.

**Corollary 4.7.** Let $X$ and $Y$ be $\sigma$-compact metrizable spaces such that $C_p(Y)$ is an image of $C_p(X)$ under a continuous open mapping. If for all $n \in \mathbb{N}^+$ the space $X^n$ is weakly infinite-dimensional, then $Y$ is also weakly infinite-dimensional.

*Proof.* Apply Theorem 4.6 with $k = 2$. \hfill $\square$

Using the same technique, we can prove a similar theorem about $C$-spaces.

**Theorem 4.8.** Let $X$ and $Y$ be $\sigma$-compact metrizable spaces. Suppose, that $C_p(Y)$ is an image of $C_p(X)$ under a continuous open mapping. If $X$ is a $C$-space, then $Y$ is also a $C$-space.

*Proof.* Since the finite product of compact metrizable $C$-spaces is a $C$-space (see [14, Theorem 3]) and since being a $C$-space is invariant with respect to countable unions with closed summands (see [6, 2.24]), the space $X^n$ is a $C$-space for every $n \in \mathbb{N}^+$.

We apply Theorem 2.2 as in the proof of Theorem 4.6. Let $Y_n$, $Z_n$, $B_n$ be as in the statement of Theorem 2.2.

It is known that within the class of metrizable spaces, the property of being a $C$-space is invariant with respect to $F_\sigma$ subspaces (see [6, 2.25]) and
preimages under continuous mappings with fibers being \(C\)-spaces (see [6, 5.4]). Hence the space \(Z_n\) is a \(C\)-space and so is \(B_n\). It is also known that for compact spaces property \(C\) is preserved by continuous mappings with fibers of cardinality \(< c\) (see [6, 6.4]). Thus from the \(\sigma\)-compactness of \(Z_n\) (see the proof of Theorem 4.6) and the fact that a countable union of closed \(C\)-spaces is a \(C\)-space (see [6, 2.24]), we conclude that \(Y_n\) is a \(C\)-space. By [6, 2.24] \(Y = \bigcup_n Y_n\) is a \(k\)-\(C\)-space. \(\square\)

5. COUNTABLE-DIMENSION

Let us recall the following definition

**Definition 5.1.** A space \(X\) is countable-dimensional if \(X\) can be represented as a countable union of finite-dimensional subspaces.

It is well known that every countable-dimensional metrizable space is a \(C\)-space. In [9] W. Marciszewski modifying a technique from [3] proved the following

**Theorem 5.2.** (Marciszewski) Suppose that \(X\) and \(Y\) are \(t\)-equivalent metrizable spaces. Then \(X\) is countable dimensional if and only if \(Y\) is so.

As in the previous section, we can use Theorem 2.2 to prove a slightly more general result.

**Theorem 5.3.** Let \(X\) and \(Y\) be metrizable spaces. Suppose, that \(C_p(Y)\) is an image of \(C_p(X)\) under a continuous open mapping. If \(X\) is countable-dimensional, then so is \(Y\).

**Proof.** Since \(X\) is countable-dimensional and metrizable, every finite power \(X^n\) is countable-dimensional (see [4, Theorem 5.2.20]). It is also known that within the class metrizable space, countable-dimensionality is invariant with respect to: preimages under closed mappings with finite-dimensional fibers [4, Proposition 5.4.5], subspaces [4, 5.2.3], images under closed finite-to-one mappings [4, Theorem 5.4.3]) and countable unions [4, 5.2.8]. Thus it is enough to apply Theorem 2.2. \(\square\)

**Remark 5.4.** Theorems 4.6, 4.8, 5.3 cannot be concluded directly from Theorem 1.1. Let us observe that if we take \(X = [0, 1]\), \(Z_n = B_n = [0, 1]^n\) and \(Y = Y_n = [0, 1]^{\omega}\), then the thesis of Theorem 1.1 holds. Indeed, in that case \(Z_n\) maps onto \(X^n\) by a perfect finite-to-one mapping (the identity) and \(B_n\) maps onto \(Y_n\) perfectly, so Okunev’s theorem from [12] (Theorem 1.7) does not prove that spaces \([0, 1]\) and \([0, 1]^{\omega}\) are not \(t\)-equivalent. To conclude
the latter, we need to use the fact that the existence of a continuous open surjection between $C_p(X)$ and $C_p(Y)$ implies that $Y_n$ is an image of $B_n$ under a finite-to-one mapping.

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