Electromagnetic scattering from perfectly conducting periodic rough surfaces using discrete two-level complex images method

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Abstract. The one-level complex image method is only useful in deriving the spectral-domain Green's function at low frequencies. In this paper, a two-level complex image method is proposed to derive closed form periodic Green's function at high frequencies for problem of scattering from perfectly conducting periodic surfaces. The remaining quotients in the spectral domain are approximated by a finite series of exponential using the Generalized pencil-of-function method. The approximation is performed along the two-level path, with this scheme the contribution beyond $T_0$ has been considered and it will give accurate results over a wide frequency range. The results are compared with those obtained by the spectral Kummer-Poisson’s method and good agreements are observed. Also the complex image method is more computational efficient.

1. Introduction
Electromagnetic scattering from periodic surface is an important problem in science and engineering [1-3]. The most common method to solve periodic problems is to use the method of moment to solve the integral equation and its numerical results, because of their flexibility in simulating different geometric shapes [4]. The electromagnetic scattering from a periodic surface can be expressed as an integral equation with the periodic green's function at its core. Unfortunately, this is a slow-convergence summation [4]. The calculation of this kernel is quite time consuming, in order to speed up the summation of these series, many acceleration techniques have been proposed in the literature, such as the perfectly matched layers method [5] and the Ewald’s method [6] and the spectral Kummer-Poisson’s method using the modified Bessel functions[7]. Although these methods are feasible for calculating periodic green's function, they have not obtained the closed representation of periodic green's function. Because the periodic green's function is used in the kernel of the integral equation, it is effective to derive the closed periodic green's function for the scattering from a perfectly conducting periodic surface. In [8], the closed periodic green's function is represented by the one-level discrete complex image method. However as the frequency increases, the one-level approximation approach has difficulty to approach spectral periodic green's function in the form of complex exponent because it is difficult to find suitable approximate parameters. Therefore, the two-level complex image method is considered to make up for the neglected contribution of the previous one-level method beyond $T_0$, thus alleviating the above difficulties.

In this paper, our discussion is limited to the scattering of TE polarized waves by a periodic surface of a complete conductor, but the proposed method can be extended to other cases, including perfectly...
conducting periodic surface for TM polarized wave and dielectric media with periodic surface for TE and TM polarized wave.

2. Formulation of the problem

The structure under consideration is shown in figure 1. Consider a plane wave incident on a periodic surface with a height function \( z=f(x) \), such that \( f(x+p) = f(x) \). The rough surface is periodically distributed along the \( x \) direction, and the period is \( p \). The direction of incidence is in the \( x-z \) plane. The electric field of the incident wave is given by

\[
\psi_{\text{inc}}(r) = \hat{y}e^{-jkx+jkz} = \hat{y}e^{-j\sin(\theta_i)x + jk\cos(\theta_i)z}
\] (1)

By using PEC boundary conditions and Floquet-Bloch Theorem, the problem can be simplified as

\[
\psi_{\text{inc}}(\bar{r}) = \int_{-p/2}^{p/2} G_p(\bar{r}, \bar{r})u(x')dx'
\] (2)

Where

\[
G_p(x, z; x', z') = \sum_{n=-\infty}^{\infty} G(x, z; x' + np, z')e^{-j\beta_n} = \frac{H_2^{(2)}(k\sqrt{(x-x')^2+(z-z')^2})}{4j}
\]

\[
+ \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-j\beta_n(x-x')}}{1-e^{-j(k_x-\beta_n)p}} \frac{e^{-j\beta_n (z-z')}}{1-e^{-j(k_z+\beta_n)p}} dk_n
\]

(3)

The quotients in equation (3) can be approximated by a finite series of exponential using the GPOF method [9] as

\[
\frac{e^{j(k_x-\beta_n)p}}{1-e^{j(k_x-\beta_n)p}} = \sum_{n=1}^{M_1} a_ne^{\beta_n\beta_x}
\] (4)

And

\[
\frac{e^{-j(k_z+\beta_n)p}}{1-e^{-j(k_z+\beta_n)p}} = \sum_{n=1}^{M_2} c_ne^{\beta_n\beta_z}
\] (5)

As the frequency increases, the quotient changes quickly, and it is reasonable to select small \( T_0 \) in order to get as many sampling points as possible. Since the quotient of the sample to small \( T_0 \) converges slowly, the contribution of spectral functions greater than \( T_0 \) should be taken into account. In the following subsection, the two-level approximation is considered to compensate for the contributions beyond \( T_0 \) that were ignored by the previous one-level approximation, thus alleviating the above difficulties.

As shown in figure 2, the parametric equations for paths \( C_{ap1} \) and \( C_{ap2} \) are as follows:

\[
C_{ap1} : \beta_x = k[-jt + (1-t/T_{02})]
\] (6)

\[
C_{ap2} : \beta_z = -jk[T_{02} + t]
\] (7)
To illustrate the process of two-level approximation, we will first outline the necessary steps and then provide some details. Here’s how it works:

1) Choose $T_{02}$ when

$$e^{-j\beta p} = e^{-j(k_{02}T_{02})p} = e^{-2.5T_{02}p} = e^{-2.5}$$

such that $T_{02} = \frac{2.5}{kp}$. Choose $T_{01} = \frac{25}{kp}$, the two quotients is assumed to be negligible beyond $T_{01} + T_{02}$.

2) Sample the quotients along the path $C_{ap1}$ and approximate by exponential series using the GPOF method respectively

$$f(\beta_x) = \frac{e^{j(k_{02} - \beta_x)p}}{1 - e^{j(k_{02} - \beta_x)p}} , \beta_x \in C_{ap1} = \sum_{m=1}^{M_{11}} a_{mn} e^{\beta_x}$$

(8)

$$f(\beta_x) = \frac{e^{j(k_{02} + \beta_x)p}}{1 - e^{j(k_{02} + \beta_x)p}} , \beta_x \in C_{ap1} = \sum_{m=1}^{M_{12}} a_{mn} e^{\beta_x}$$

(9)

3) Subtract the exponential terms approximated along the path $C_{ap1}$ from the original quotients, sample the remaining quotients uniformly along the path $C_{ap2}$ and approximate them by exponential series using the GPOF method

$$\frac{e^{j(k - \beta_x)p}}{1 - e^{j(k_{02} - \beta_x)p}} - f(\beta_x) = \sum_{m=1}^{M_{12}} a_{2m} e^{\beta_x} , \beta_x \in C_{ap2}$$

(10)

$$\frac{e^{-j(k + \beta_x)p}}{1 - e^{-j(k_{02} + \beta_x)p}} - f(\beta_x) = \sum_{m=1}^{M_{12}} c_{2m} e^{\beta_x} , \beta_x \in C_{ap2}$$

(11)

There’s no need to subtract the effect of the exponential terms of path $C_{ap2}$ on path $C_{ap1}$ because this effect is very small and can be neglected. Note that in this two-level approximation, the first part of the approximation is identical to the one-level approximation scheme, and the second part follows the path $C_{ap2}$ to compensate for contributions beyond $T_{02}$. The sequence of the path is different from the others in the multi-layered media or micro strip structure problem [10, 11].

Figure 2. The paths $C_{ap1}$ and $C_{ap2}$ used in two-level approximation.

Then the quotients in equation (3) can be approximated by the summation of the two exponential series of the two paths. Using the identity of the Green’s functions [4], the periodic Green’s functions with the two-level approximation can be calculated.

3. Numerical results and conclusion
In this section, we examine the accuracy of the techniques presented in the previous section by considering some examples. All the surface roughness expressions in one period is

\[ f(x) = h \cos(\frac{2\pi x}{P}) \]

The first example is the case with \( \lambda = 0.05p \), \( h = 0.2p \) and \( \theta_i = 45^\circ \), we set the segment length \( dx = \lambda/10 \) then \( N = 200 \). The second example is the case with \( \lambda = 0.02p \), \( h = 0.2p \) and \( \theta_i = 45^\circ \), we set the segment length \( dx = \lambda/10 \) then \( N = 500 \). The real and imaginary parts of the scattering field associated with the first and second examples on the vertex virtual plane are described in figures 3 and 4. In graphic terms, the results of the proposed method are in good agreement with those of the spectral Kummer-Poisson method. Calculation times of the first and second examples are 273.895s, 2433.34s for the proposed method and 981.245s, 7664.7s for the spectral Kummer-Poisson’s method. This means that proposed method is more computational efficient than the spectral Kummer-Poisson’s method, especially when the Green’s function have to be evaluated for a large number of pairs of source and observation points. And this is because that the spectral Kummer-Poisson’s method requires the repeated computation of the coefficients \( B_i \) while the coefficients \( a_m, b_m, c_m \) and \( d_m \) in the proposed method only have to be computed once.

In conclusion, the two-level complex image method, which can approximate the remaining quotients at high frequencies along the two-level path using the generalized pencil-of-function method, has been considered. Compared with the one-level complex image method’s difficult in finding the adequate approximation parameter \( T_0 \) in the case of high frequencies, the two-level complex image method choose small \( T_0 \) and the contribution beyond \( T_0 \) is considered. The method has derived closed form periodic Green’s function at high frequency and the problem of scattering from perfectly conducting periodic surfaces is solved. The results are compared with those obtained by the spectral Kummer-Poisson’s method and good agreements are observed. Also the complex image method is more computational efficient.

Figure 3. Scattered field on the top point imaginary planes (a) Real part (b) Imaginary part (\( \lambda = 0.05p \), \( h = 0.2p \), \( \theta_i = 45^\circ \), \( N = 200 \)).
Figure 4. Scattered field on the top point imaginary planes (a) Real part (b) Imaginary part ($\lambda=0.02\mu m$, $h=0.2\mu m$, $\theta_i=45^\circ$, $N=500$).

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