Planetary Torus Helical Transmission

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Abstract. The paper describes design considerations for the transmission with fixed intermediate rolling bodies with grooves cut in a torus. Equations of helices used for cutting grooves have been derived. The transmission with the proposed design has a wide range of reduction ratios and small dimensions with high load capacity.

1. Introduction
Nowadays, many industries need drive systems with small dimensions and high load capacity. This is usually the case in drilling equipment, power-driven devices, industrial robots and mechatronic modules. In the drives of these mechanisms, planetary gears and harmonic drives as well as transmissions with intermediate rolling bodies, which were developed at the end of the 20th century, are mostly used [1].

Planetary gears offer high load capacity and small dimensions with reduction ratios from 1 to 20. The disadvantage of harmonic drives with reduction ratios from 80 to 300 is the presence of a flexspline that reduces its efficiency and load capacity. Transmissions with intermediate rolling bodies have a wide range of reduction ratios (from 3 to 200) with high load capacity due to distribution of load among a number of rolling bodies.

Disadvantages of transmissions with intermediate rolling bodies include low efficiency and reliability caused by the fact that, during operation of the transmission, rolling bodies move in the slits of the driven shaft. To increase efficiency and load capacity of the transmission, it is advisable to use rollers as rolling bodies. Rollers form translational kinematic pairs with the slits and the grooves they interact with, and unavoidable clearance, which occurs in the engagement, leads to the roller skewing when the load is applied. This causes sliding friction to occur in the contact area of rollers with parts of the transmission and can result in its failure. In transmissions with intermediate rolling bodies, there is a relationship between the reduction ratio and load capacity. With an increase in the reduction ratio due to cutting the vertices of the multiperiodic groove, the number of rolling bodies transmitting the load decreases (with reduction ratios of more than 12 the force is transmitted through less than 50 % of the number of rollers). In this case, with an increase in the reduction ratio, the total number of rollers in the transmission increases, and their dimensions decrease (due to the constraint imposed on dimensions of the transmission by its functional purpose). Hence, for transmissions with intermediate rolling bodies, the rational range of reduction ratios providing high efficiency and load capacity is from 3 to 30.

2. Design and principle of operation of the transmission
The solution to the problem of the roller skew is to create a transmission with fixed intermediate rolling bodies [2], i.e., when rolling bodies and the driven shaft form a kinematic pair with rotational
and not translational motion. Based on this principle, a design of a planetary torus helical transmission with fixed intermediate rolling bodies (composite rollers) has been proposed. This transmission is called a torus transmission since the grooves are made on a torus. In this case, cylindrical and radial configurations of the transmission can be obtained.

In the cylindrical configuration, the equation of a helix located on the torus along the axis of rotation of the driven shaft is used for obtaining grooves. In the radial configuration, the helix (Archimedean spiral) is located on the torus perpendicular to the axis of rotation of the driven shaft (figure 1).

![Figure 1](https://example.com/figure1.png)

**Figure 1.** Cylindrical (a) and radial (b) configurations of planetary torus helical transmission and helices (c), (d) forming grooves: 1 – drive shaft; 2 – driven shaft; 3 – fixed member; 4 – rolling bodies.

Like the classic transmission with intermediate rolling bodies, the planetary torus helical transmission, as shown in figure 1, consists of four main parts: drive shaft (1), driven shaft (2), fixed member (3) and rolling bodies (4). The drive shaft is made with an open groove which has a shape of a helix (Archimedean spiral) with a whole number of periods $Z_1$ (in figure 1 (c) $Z_1 = 4$, in figure 1 (d) $Z_1 = 2$). The fixed member has open grooves with quantity of $Z_3$ (a whole number), which have the shape of an incomplete helix (Archimedean spiral) cut with equal pitch on the torus of the fixed member. In figure 1 (c), (d), for clarity, only two adjacent curves that form the grooves on the fixed member for each transmission scheme are shown. As shown in figure 1 (c), (d), each of the two curves is connected at one end to the curve forming the groove on the drive shaft. In this way, the grooves on the drive shaft and the fixed shaft are closed. The driven shaft is a part where intermediate rolling bodies (composite rollers) are fixed. Composite rollers are cylindrical disks with protrusions that interact with the grooves on the drive shaft and the fixed member. The minimum number of protrusions is two; one protrusion must be in contact with the groove on the drive shaft, and the other must be in contact with the groove on the fixed member.

The maximum possible number of composite rollers is determined according to the relationship:

$$ n_{\text{max}} = \frac{Z_3}{Z_1} + 1 $$

(1)

where $Z_3/Z_1$ must be a whole number.

If necessary, a smaller number of composite rollers can be used in the transmission; this number should correspond to any divider for the number $n_{\text{max}}$.

The rotation of the drive shaft makes the composite roller to rotate due to interaction between one of its protrusions and the groove on the drive shaft. In this case, the other protrusion of the rotating composite roller interacts with the groove on the fixed member. As a result, the driven shaft, in which the composite rollers are installed, rotates at a lower speed. During the rotation of the composite roller, each protrusion moves successively from the groove on the drive shaft to the groove on the fixed member, i.e., it moves along the closed groove consisting of the helix (Archimedean spiral) on the drive shaft and the groove on the fixed member.

The reduction ratio is calculated according to the formula:

$$ U = Z_1 \cdot Z_3 + 1 $$

(2)
The reduction ratio varies from 4 to 121. In this case, the transmission can have radial dimensions less than 200 mm while maintaining high load capacity.

3. Equations of helices used for cutting grooves

To cut grooves for both configurations of the transmission, equations of helices for a torus of the transmission parts have been derived.

The helix equations for the drive shaft of the cylindrical configuration of the transmission are as follows:

\[ z = \frac{A_v}{Z_i} \cdot \theta_i \]  
\[ x = \left( R_{ph} + \left( r_i - \left( r_i^2 - (r_i - z)^2 \right)^{1/2} \right) \right) \cdot \cos(Z_i \cdot \theta_i) \]  
\[ y = \left( R_{ph} + \left( r_i - \left( r_i^2 - (r_i - z)^2 \right)^{1/2} \right) \right) \cdot \sin(Z_i \cdot \theta_i) \]

where \( z, x, y \) are the coordinates of the points of the helix, \( m \); \( A_v \) is the pitch of the helix, \( m \); \( \theta_i \) is the central angle (varies in the range from 0 to \( 2\pi \)), rad; \( R_{ph} \) is the minimum distance from the axis of rotation to the torus, \( m \); \( r_i \) is the radius of the circle used to obtain the torus, \( m \) (see figure 1 (c)).

The helix equations for the fixed member of the cylindrical configuration of the transmission are as follows:

\[ z = \frac{r_i \cdot Z_1}{\pi} \cdot \theta_i \]
\[ x = \left( R_{ph} - \left( r_i - \left( r_i^2 - (r_i - z)^2 \right)^{1/2} \right) \right) \cdot \cos(\theta_3) \]
\[ y = \left( R_{ph} - \left( r_i - \left( r_i^2 - (r_i - z)^2 \right)^{1/2} \right) \right) \cdot \sin(\theta_3) \]

where \( \theta_3 \) is the central angle (varies in the range from 0 to \( 2\pi/Z_i \)), rad; \( R_{ph} \) is the maximum distance from the axis of rotation to the torus, \( m \) (see figure 1 (c)).

The helix equations for the drive shaft of the radial configuration of the transmission are as follows:

\[ A_v = \frac{R_{ph} - a}{Z_i \cdot 2\pi} \]
\[ z = \left( \frac{R_{ph} - a}{2} \right)^2 - \left( \frac{R_{ph} - a}{2} \right)^2 \left( a + A_v \cdot Z_i \cdot \theta_i - \left( \frac{R_{ph} - a}{Z_i} \right) \right)^2 \cdot \frac{1}{2} \]
\[ x = \left( a + A_v \cdot Z_i \cdot \theta_i \right) \cdot \cos(Z_i \cdot \theta_i) \]
\[ y = \left( a + A_v \cdot Z_i \cdot \theta_i \right) \cdot \sin(Z_i \cdot \theta_i) \]

where \( a \) is the initial radius of the Archimedean spiral, \( m \) (see figure 1 (d)).

The helix equations for the fixed member of the radial configuration of the transmission are as follows:

\[ z = \left( \frac{R_{ph} - a}{2} \right)^2 - \left( \frac{R_{ph} - a}{2} \right)^2 \left( a + \frac{Z_i \cdot (R_{ph} - a)}{2\pi} \cdot \theta_i - \left( \frac{R_{ph} - a}{Z_i} \right) \right)^2 \cdot \frac{1}{2} \]
\[ x = \left( a + \frac{Z_i \cdot (R_{ph} - a)}{2\pi} \cdot \theta_i \right) \cdot \cos(\theta_3) \]
\[ y = \left( a + \frac{Z_i \cdot (R_{ph} - a)}{2\pi} \cdot \theta_i \right) \cdot \sin(\theta_3) \]
4. Substantiation of the rational design of the transmission

3D models of cylindrical and radial configurations of the planetary torus helical transmission and the composite rollers are presented in figure 2.

![3D models of cylindrical (a) and radial (b) configurations of planetary torus helical transmission (driven shaft not shown) and composite rollers (c): 1 – drive shaft; 3 – fixed member; 4 – composite rollers.](image)

**Figure 2.** 3D models of cylindrical (a) and radial (b) configurations of planetary torus helical transmission (driven shaft not shown) and composite rollers (c): 1 – drive shaft; 3 – fixed member; 4 – composite rollers.

The comparison of the designs of the transmissions with cylindrical and radial configurations shows that, with equal dimensions and efficiency, the complexity of manufacturing and assembly (of the fixed member, in particular) is much higher for the cylindrical configuration than for the radial configuration. Moreover, in the case of the cylindrical configuration, the load capacity of the transmission is reduced because of the limited strength of the driven shaft due to cantilever arrangement of the protrusions where the composite rollers are fixed. Subsequently, the radial configuration of the planetary torus helical transmission will be studied as it is considered more rational.

The design of composite rollers has a significant impact on the power loss in the transmission [3] and its load capacity. The minimum number of the protrusions should be equal to two, but their number can be increased in order to increase the load capacity of the transmission, since the transmitted force is distributed proportionally to the number of the protrusions. The maximum possible number of the protrusions in the composite roller is \(2 \cdot Z_1\). Hence, the number of the protrusions should be in the range from 2 to \(2 \cdot Z_1\) and be divisible by 2. However, if the number of the protrusions is equal to \(2 \cdot k\), the number of the grooves on the fixed member must also be increased \(k\) times. This increase does not affect the reduction ratio and the helix equations, but leads to a decrease in dimensions of the grooves and, consequently, the protrusions, which in turn affects their strength.

The protrusions can be cylindrical and spherical in shape (see figure 2 (c). From the point of view of manufacturability (grooves are cut with a spherical milling cutter), the spherical shape of the protrusions is more rational. However, at the same time a significant decrease in efficiency is observed (by 30 ... 50 \%). Grooves for cylindrical protrusions can be cut with a spherical milling cutter by making multiple passes; in this way, a cylindrical surface is gradually formed. To obtain coordinates of the appropriate trajectories of the milling cutter in equations (9) – (15), the value \(R_{p3}\) is gradually increased and the value \(a\) is decreased in the same way. Obviously, this manufacturing method is more labor intensive.
5. Kinematic analysis of the transmission
To study kinematic parameters of the radial configuration of the planetary torus helical transmission, an NX CAD model of the transmission with the following parameters was created: \( Z_1 = 2; Z_3 = 10; n = 7; R_{pl} = 0.042 \text{ m}; a = 0.014 \text{ m}; \) the number of the composite roller protrusions is 2.

To simplify the calculations of rotation of the transmission parts, the fixed member was “released” and acted as a driven shaft, and the driven shaft was “fixed”. In this case, the reduction ratio was calculated according to the formula: \( U = Z_1 \cdot Z_3 = 20. \) The drive shaft is rotating at a speed of 40 min\(^{-1}.\)

The graphs of angular velocities of rotation of the drive shaft, the driven shafts and the composite roller are presented in figure 3.

![Graphs of angular velocities of drive shaft (a), driven shaft (b) and composite roller (c).](image)

Figure 3. Graphs of angular velocities of drive shaft (a), driven shaft (b) and composite roller (c).

It has been found from the graphs in figure 3 (a) and (b) that the transmission has the required reduction ratio.

On the graph of the angular velocity of the composite roller, significant changes in the angular velocity are observed every 3 seconds, which corresponds to the point in time when the protrusion of the roller moves from the groove on one part to the groove on the other part of the transmission. These abrupt changes will lead to dynamic loads and noise during operation of the transmission.

6. Adjusting helix equations used for cutting grooves
To reduce abrupt changes in the angular velocity of the composite roller, it is necessary to adjust the helix equations for the drive shaft and the fixed member by making transition from one groove to another smoother [4].

The helix equations for the drive shaft of the radial configuration of the transmission are as follows:

\[
\begin{align*}
  z &= \left( \frac{R_{pl} - a}{2} \right)^2 - \left( \frac{R_{pl} - a}{2} - \frac{R_{pl} - a}{2} \right) \left[ 1 - \cos \left( \frac{\theta_1}{2} \right) \right]^2 \right)^{1/2} \\
  x &= \left( \frac{R_{pl} - a}{Z_1} \right) + \left( \frac{R_{pl} - a}{2} \right) \left[ 1 - \cos \left( \frac{\theta_1}{2} \right) \right] \cdot \cos (Z_1 \cdot \theta_1) \\
  y &= \left( \frac{R_{pl} - a}{Z_1} \right) + \left( \frac{R_{pl} - a}{2} \right) \left[ 1 - \cos \left( \frac{\theta_1}{2} \right) \right] \cdot \sin (Z_1 \cdot \theta_1)
\end{align*}
\]

The helix equations for the fixed member of the radial configuration of the transmission are as follows:

\[
\begin{align*}
  z &= -\left( \frac{R_{pl} - a}{2} \right)^2 - \left( \frac{R_{pl} - a}{2} - \frac{R_{pl} - a}{2} \right) \left[ 1 - \cos \left( \frac{Z_1 \cdot \theta_3}{2} \right) \right]^2 \right)^{1/2} \\
  x &= \left( \frac{R_{pl} - a}{2} \right) \left[ 1 - \cos \left( \frac{Z_1 \cdot \theta_3}{2} \right) \right] + \left( \frac{R_{pl} - a}{Z_1} \right) \cdot \cos \theta_3
\end{align*}
\]
\[ y = \left( \frac{R_m - a}{2} \right) \left[ 1 - \cos \left( \frac{Z \cdot \theta_i}{2} \right) + \left( \frac{R_m - a}{Z_i} \right) \right] \cdot \sin \theta_i \]  

(21)

The graph of the angular velocity of the rotation of the composite roller after using the adjusted helix equations used for cutting grooves is shown in figure 4.

Figure 4. Graph of angular velocity of composite roller after adjusting helix equations

As a result, the magnitude of abrupt changes in angular velocity has decreased by a factor of 3.

7. Conclusion

The substantiation of the design of the planetary torus helical transmission providing high efficiency and load capacity with a reduction ratio in the range from 4 to 121 and a radial dimension of the transmission of not more than 200 mm has been performed. Equations of helices for obtaining grooves for the drive shaft and the fixed member that reduce dynamic loads and noise during operation of the transmission have been derived.

References

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