Analysis of the possibility of upscaling based on apodization for partially coherent optical systems in the presence of aberrations

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Abstract. The degree of coherence of radiation is an important characteristic on which the interference properties of light fields and, as a consequence, the resolution of optical systems depend. When propagating over long distances even in free space, initially completely coherent or incoherent radiation becomes partially coherent. This fact should be taken into account in the formation of optical images along with the influence of wave front aberrations. In this paper, we investigate the change in the resolution of the system for two near-point light sources depending on the degree of spatial coherence in the presence of different aberrations. The possibility of improving the resolution in the considered situations on the basis of the amplitude apodization of the optical system is also investigated.

1. Introduction

Light with very high spatial and temporal coherence is often required to solve many problems of interferometry, holography and some types of optical sensors (for example, fiber optic) [1-7]. For other tasks, the coherence of the light used should be as low as possible. For example, very low temporal coherence (but combined with high spatial coherence) is required for coherent tomography [8-10]. However, even fully coherent or incoherent radiation becomes partially coherent when dispersion [11] and even when spreading in free space over long distances [12]. The state of coherence significantly affects the quality of optical image formation [13], what to consider along with the effect of wave front aberrations [14-18], the main causes of which are: the turbulence of the atmosphere, the faultiness of the forms of optical elements of the system, errors in the alignment of the system, etc. [19-26].

One of the ways to improve the resolution is amplitude and/or phase apodization of the optical system [27-37]. However, apodization, as a rule, allows not only to reduce the size of the light spot, but also leads to the appearance of side lobes [38-41], which impair the imaging properties. To compensate for this negative factor, composite apodization functions are used, in particular, asymmetric ones [42-47].

In this paper we research the possibility of increasing the resolution of the optical system for two closely spaced point light sources on the basis of amplitude apodization depending on the degree of spatial coherence in the presence of different aberrations.

2. Theory

Usually the wave front is given by means of Zernike polynomials, which are also used to detect deviations from the ideal front and the types of aberrations present in the distortion [48-53]. To visually assess the image quality, the point spread function is used (PSF). Previously, in the paper [54] described a technique for modeling the composition of a FRT for two light sources in the coherent case in the
presence of aberrations, as well as an apodizing function. In this paper we additionally consider the influence of the degree of coherence of the two sources, which is given by the following formula:

\[ I(r, \omega) = A_1^2(r, \omega) + A_2^2(r, \omega) + 2\mu|A_1(r, \omega)||A_2(r, \omega)|\cos(\varphi_1(r, \omega) - \varphi_2(r, \omega)), \]

whereabout \( \mu \) – coefficient of coherence of light sources, \( A_1, A_2 \) – conformable amplitudes, \( \varphi_1, \varphi_2 \) – conformable phases.

To construct an image of the original object \( \alpha(x) \) in the coherent case (\( \mu=1 \)), we can use the following expression [55, 56]:

\[ b(x) = \int_{-\infty}^{\infty} S_\alpha(u) P(u) \exp \left( \frac{ikxu}{f} \right) du \]  

whereabout \( S_\alpha(u) \) – spatial spectrum of the object, \( P(u) \) – pupil function of imaging system, \( k = \frac{2\pi}{\lambda} \) – wave number, \( \lambda \) – radiation wavelength, \( f \) – imaging system focus.

For imaging optical systems using incoherent (\( \mu=0 \)) radiation, the intensity of the image is considered, which is determined by the convolution function of the object intensity and the intensity of the pulse response. The optical transfer function (OTF), which is the spatial spectrum of the impulse response of the system, can also be calculated through the pupil function [57]:

\[ W_p(u) = \frac{\int P(s-u/2)P^*(s+u/2)ds}{\int|P(s)|^2ds} \]

Then the intensity of the image is determined from the expressions:

\[ I_a(x) = \int W_a(u)W_p(u) \exp \left( \frac{ikxu}{f} \right) du \]

\[ W_a(u) = \frac{k/f \int |\alpha(x)|^2 \exp(-ikux/f)dx}{\int |\alpha(x)|^2 dx} \]

3. Simulation result
This section researches the effect of the various aberrations described by the Zernike polynomials \( \Psi^m_n(u) \) on the PSF of one light source and two closely spaced light sources. Near-located sources are imply as two sources separated by distance according to Rayleigh criterion (maximum of one source falls on the first minimum of the second). In this case, for ideal PSF images of point sources are visually difficult to distinguish (the gap between them is about 20% of the maximum intensity). In this section, the problem of amplitude apodization of the pupil of the optical system in the presence of aberrations is solved numerically in order to increase the downward excursion in the images of two nearby sources.

Figures 1-6 show the effect of different aberrations described by the Zernike polynomials \( \Psi^m_n(u) \), on the PSF of a single light source. The figures show the right light source, which causes a slight offset of the picture.
As can be seen from table 1, as the radiation coherence increases, the points become less differentiate, i.e. the resolution of the systems decreases. It should also be noted that the effect of aberrations described by Zernike polynomials with even indices is more negative than for odd polynomials.
### Table 1. PSF of two located in close proximity spaced point light sources for different degrees of coherence in the presence of different aberrations.

| Aberration | $\mu=0$ | $\mu=0.5$ | $\mu=1$ |
|------------|---------|---------|---------|
| Horizontal coma $\Psi_1^3$ | ![Image](image1.png) | ![Image](image2.png) | ![Image](image3.png) |
| Horizontal trefoil $\Psi_3^3$ | ![Image](image4.png) | ![Image](image5.png) | ![Image](image6.png) |
| $\Psi_5^3$ | ![Image](image7.png) | ![Image](image8.png) | ![Image](image9.png) |
| Astigmatism $\Psi_2^2$ | ![Image](image10.png) | ![Image](image11.png) | ![Image](image12.png) |
| Secondary astigmatism $\Psi_2^4$ | ![Image](image13.png) | ![Image](image14.png) | ![Image](image15.png) |
| Tetrafoil $\Psi_4^4$ | ![Image](image16.png) | ![Image](image17.png) | ![Image](image18.png) |

Table 2 shows the effect of system Apodization on its tractability under the condition of partial coherence of radiation ($\mu=0.5$). Note that amplitude Apodization slightly improves the situation, but there is no explicit resolution of two nearby points. More complex Apodization functions are planned to be used in further research.
Table 2. Effect of aberration on the resolution of a partially coherent system (μ=0.5).

| Aberration Type | Without Apodization | With Apodization |
|-----------------|----------------------|------------------|
| Horizontal coma | ![Image](image1.png)  | ![Image](image2.png)  |
| Horizontal trefoil | ![Image](image3.png)  | ![Image](image4.png)  |
| $\Psi^3_5$ | ![Image](image5.png)  | ![Image](image6.png)  |

4. Conclusion
The research of the possibility of increasing the resolution of the optical system based on apodization for partially coherent systems showed that with increasing coherence resolution of two nearby light sources deteriorates. The use of amplitude apodization (parabolic apodization $f(r) = r^2$ was used in this paper) did not lead to a significant improvement in the resolution in the partially coherent case. In further studies, it is planned to use more complex apodization functions, such as $f(x, y) = x^\gamma + y^\gamma$. Phase apodization will also be used.

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