Global linear gyrokinetic simulations for LHD including collisions

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Abstract. The code EUTERPE uses a Particle-In-Cell (PIC) method to solve the gyrokinetic equation globally (full radius, full flux surface) for three-dimensional equilibria calculated with VMEC. Recently this code has been extended to include multiple kinetic species and electromagnetic effects. Additionally, a pitch-angle scattering operator has been implemented in order to include collisional effects in the simulation of instabilities and to be able to simulate neoclassical transport. As a first application of this extended code we study the effects of collisions on electrostatic ion-temperature-gradient (ITG) instabilities in LHD.

1. Introduction
Particle and heat transport in fusion devices often exceed the neoclassical prediction. This anomalous transport is produced by turbulence which is probably caused by microinstabilities such as ion and electron-temperature-gradient (ITG/ETG) and trapped-electron-mode (TEM) instabilities, the later ones known for being strongly influenced by collisions. In stellarators, neoclassical transport can be dominant in the core, therefore, collisions gain additional importance. It is also relevant to study the interplay between neoclassical transport and these instabilities for stellarator configurations.

The aim of this paper is twofold: First, to verify the collisional code by running simulations for a cylinder and a tokamak. Collisional effects are rather diverse for different geometries: Slab ITGs are much more sensitive to collisions than toroidal ITGs \cite{1, 2}. These simulations provide the background for the second aim, the physical application, i.e. to investigate how ITG modes react to collisions in a stellarator. Stellarators have the complexity of being three-dimensional and ITG modes thus can be destabilized by a mixture of curvature and slab drive. The influence of collisions on ITG instabilities is then unclear and may depend on the details of the configuration. Thus, in this work and as a first step we study the influence of collisions on ITG instabilities for the LHD stellarator.
2. Collisional gyrokinetic model

EUTERPE [3, 4] is a global code that uses the PIC method with a 3D potential grid to solve the set of gyrokinetic equations. The distribution function of each species evolves according to

\[ \frac{df}{dt} = \frac{\partial f}{\partial t} + \dot{R} \frac{\partial f}{\partial R} + \dot{v} \frac{\partial f}{\partial v} = C(f) \]  

(1)

where \( C(f) \) is a collision operator. The guiding center trajectories (characteristics) for the collisionless case are:

\[ \dot{R} = v_{\parallel} \hat{b} + \frac{1}{B} \left( \frac{\mu B + v^2_{\parallel}}{\Omega_i} \hat{b} \times \nabla B + \frac{v^2_{\parallel}}{\Omega_i} (\nabla \times B)_{\perp} \right) \]  

(2)

\[ \dot{v}_{\parallel} = -\mu \left( \hat{b} + \frac{v_{\parallel}}{B \Omega_i} (\nabla \times B)_{\perp} \right) \cdot \nabla B, \]  

(3)

where \( \mu \stackrel{\text{def}}{=} \frac{v^2_{\parallel}}{2B} \) and \( \hat{b} \stackrel{\text{def}}{=} \frac{B}{B}. \) The gyroaverage is defined as

\[ \nabla \langle \phi \rangle = \frac{1}{2\pi} \int_0^{2\pi} \nabla_x \phi \big|_{x=R+\rho} \ d\alpha \]  

(4)

with \( \rho \) being the vector for the Larmor radius and \( \alpha \) the gyroangle. Quasineutrality, and the assumption of adiabatic electrons, closes the system by giving the field equation

\[ -\nabla_{\perp} \cdot \left( \frac{m_i n_0}{q_i B^2} \nabla_{\perp} \phi \right) + \frac{en_0}{kB_T} \phi = \langle n \rangle = \int f(R, v_{\parallel}, \mu) \delta(R + \rho - x) \ dR \ dv. \]  

(5)

where \( i \) is the ion species. As boundary conditions for the field equation \( \phi = 0 \) has been used at the outer boundary, while natural boundary conditions were used at the center. Since we are interested only in the linear development, a phase factor, which allows the treatment of small scale modes, has been introduced: i.e. \( \phi \) is replaced by \( \phi \exp[i(M_0 \theta + N_0 \phi)] \) and only the time evolution of \( \phi \) needs to be simulated. For the present study, the pitch-angle (\( \lambda \)) scattering collision operator was used to model self-collisions:

\[ C_\alpha(f_a) = \nu_a \frac{\partial}{\partial \xi} \left( 1 - \xi^2 \right) \frac{\partial f_a}{\partial \xi}, \]  

(6)

where \( \xi = \cos \lambda \) and \( \nu_a \) is the velocity independent collision frequency for each species \( a \), which is a free parameter. Note that this particular operator does not conserve momentum. The spatial diffusion which arises due to the gyroaverage of the collision operator [5] was neglected.

In EUTERPE, the collisions have been implemented using the scheme proposed in [6]. In one time step, the velocity is modified in two phases: First, it evolves according to Eq. (3), giving the parallel and perpendicular velocity, and then these incoming velocities are changed by a random amount, using the following scheme:

\[ v_{\text{in}} = \sqrt{v^2_{\parallel} + v^2_{\perp}} \]  

(7)

\[ \xi_{\text{in}} = \cos \left[ \frac{\arctan \left( \frac{v_{\perp}}{v_{\parallel}} \right) \xi_{\text{in}}}{\xi_{\text{in}}} \right] \]  

(8)

\[ \xi_{\text{out}} = \sin \Delta \theta \sin \varphi \sqrt{1 - \xi^2_{\text{in}}} + \xi_{\text{in}} \cos \Delta \theta \]  

(9)

\[ v_{\parallel,\text{out}} = v_{\parallel,\text{in}} \xi_{\text{out}} \]  

(10)

\[ v_{\perp,\text{out}} = \sqrt{v^2_{\parallel,\text{in}} - v^2_{\perp,\text{out}}} \]  

(11)
where $\phi$ is a random number from a uniform distribution between 0 and $2\pi$ and $\Delta \theta = 2R\sqrt{\nu\Delta t}$, where $R$ is a random number from a normal distribution with expectation value zero and variance one.

The code uses the $\delta f$ method [7] in which, for each species, the distribution function is separated into a background equilibrium part $f_0$ (here assumed to be a local Maxwellian) and a perturbation part $\delta f$, where $\delta f \ll f_0$. Replacing this in the original Eq. (1) we obtain

$$\frac{d\delta f}{dt} = C(f_0) + C(\delta f) - \frac{df_0}{dt}.$$  \hspace{2cm} (12)

This allows us to compute the time evolution of the perturbation only, which reduces the noise as compared with the full-$f$ method. The particle discretization of $\delta f$ is performed by

$$\delta f = \sum_{j=1}^{N} w_j(t) \delta^3(R - R_j) \delta(v_{\parallel} - v_{\parallel,j}) \delta(\mu - \mu_j)/J_B,$$  \hspace{2cm} (13)

where $w_j(t)$ is the weight of each marker $j$ and $J_B$ is the phase space Jacobian. With collisions, however, the weight evolution of the Monte-Carlo markers cannot be solved directly by the method of characteristics, thus, it is imperative to use a generalized collisional $\delta f$ scheme, known as the two-weight scheme [8, 9, 10]. Due to the stochastic nature of collisions, we can no longer calculate the “characteristic line” along which the convective derivative of the weight is to be evaluated. This problem can be overcome by not defining the weight in terms of the local $\delta f$, but rather treating it as an additional dimension of phase space. By taking the moments of a new kinetic equation in the extended phase space, it is possible to choose the weights to be consistent with the original $\delta f$ equation. By doing so, one finds that the evaluation of the marker density becomes difficult, unless a second weight is added to the extended phase space.

For the simulation of the ITG instability we are not interested in the effect of neoclassical transport. Thus, the zeroth order part of the source term $df_0/dt$, coming from the unperturbed orbits, has been neglected. For a tokamak configuration this term vanishes if a canonical Maxwellian is used. This is due to the fact that the toroidal momentum and the energy are constants of motion and a true equilibrium distribution function can be constructed. Since stellarators do not possess continuous symmetries, there is no other conserved quantity, apart from the energy and magnetic moment. This is not sufficient to define a true collisionless equilibrium with temperature or density profile. So the simplest choice is to use a Maxwellian as a function of energy and flux surface label and to artificially neglect the zeroth order of the source term. For the pitch angle scattering operator used here we also have $C(f_0) = 0$. Therefore, the second weight does not evolve, which is equivalent to having a one-weight scheme.

3. EUTERPE simulation results

Once modified to simulate collisions, results from EUTERPE were tested. As a benchmark, the neoclassical radial particle flux was calculated [11] and compared with the analytical calculations, as well as with the Drift Kinetic Equation Solver (DKES) code [12]. It is important to note that in this case the zeroth order part of the source term must be kept since it drives the transport.

With collisions now implemented, we want to investigate its effects on instabilities. As a first test case, we vary the collision frequency to observe how it influences the growth rate of the ITG mode. We study this for three different geometries: cylinder, tokamak and stellarator. For all these cases we have a single species plasma and assume adiabatic electrons. Infinite $\eta_i = \kappa_T/\kappa_n$ was chosen with $\kappa_n = 0.0$ and $\kappa_T = 3.5$ constant across the plasma. Density and temperature profiles are assumed to be tanh-like with $\kappa_n$ and $\kappa_T$ representing the maximum of the logarithmic gradient of the density and the temperature respectively.
We perform a scan from $\nu = 0$ to values near $\nu = 0.01 \Omega_c$ (where $\Omega_c = q_i B_0 / m_i$ is the ion gyrofrequency using a reference magnetic field $B_0$). Larger values of $\nu$ have no physical meaning since for such a high collisionality, gyrokinetic theory itself is no longer valid.

For the tokamak configuration, an aspect ratio $A = 5$ was used with minor radius $a = 0.8$ m. An equivalent geometry was used for the cylinder. For the stellarator, the LHD configuration was used, with $R = 3.75$ m and $\beta = 1.5\%$. In all stellarator cases studied, the following phase factor and filter values were used: $M_0 = -37$, $N_0 = -33$, $\Delta m = 50$, $\Delta n = 4$. The grid size is $64 \times 128 \times 128$ in $s$, $\theta$, $\varphi$ and particle number is 16 million. It is important to note that the stellarator results cannot be quantitatively compared with the tokamak and cylindrical results, since they were computed using different physical parameters. They are plotted together for the purpose of qualitative analysis only.

Figure 1 shows the growth rate $\gamma$ as a function of the collision frequency $\nu$ for the three different configurations. It is possible to observe that for the slab ITG modes in cylindrical geometry, the growth rate decreases with increasing collisionality (this is in agreement with [2]). However, if conservation of parallel momentum is added, the growth rate should slightly increase for higher collision rates [1]. In contrast to the slab ITG mode, the toroidal ITG mode (tokamak) is not significantly affected by collisionality. In the stellarator case, the ITG growth rate decreases, although not as strongly as in the cylinder case.

**Figure 1.** Growth rate dependence on collisionality for different geometries. Here, $\kappa_n = 0.0$ and $\kappa_T = 3.5$.

**Figure 2.** Growth rate dependence on $\eta_i$ for a cylinder ( ), a tokamak (■) and a stellarator (♦) configuration. Dotted lines and open symbols indicate the collisionless case. Solid lines and filled symbols indicate the collisional case ($\nu = 0.01 \Omega_c$). The black box indicates the area enlarged and displayed in Figure 3.

**Figure 3.** Growth rate dependence on $\eta_i$ (zoom). Collisions only affect the instability threshold in the case of slab ITG modes.
The growth rate dependence on $\eta_i$ is displayed in Figures 2 and 3. The $\eta_i$ variation is performed by varying the value of $\kappa_n$ in the range of $[0, 4]$ and leaving the inverse of the temperature length scale constant $\kappa_T = 3.5$. It can be noticed (Figure 3) that the instability threshold value in cylindrical geometry is reduced by collisions. This is due to the collisional coupling between parallel and perpendicular temperatures [2], allowing the system to access more degrees of freedom. In the “collisionless” plasma limit, there is no coupling, thus the threshold for slab ITG modes is higher. In the opposite case, in which the plasma is highly collisional, the threshold drops as a consequence of the additional degrees of freedom of the system and the parallel and perpendicular temperatures are now strongly coupled.

In the tokamak case, collisions do not have a significant effect on the threshold (Figure 3), which is in agreement with theoretical results [2]. Similarly for the stellarator configuration, collisions do not modify the onset of the instability. Nevertheless, for sufficiently large values of $\eta_i$, a decrease of the growth rate is observed with increasing $\eta_i$ (see Figure 2) in both collisionless and collisional cases, phenomena which are not present in the tokamak case. In the collisional case it is even more pronounced.

To investigate further about the nature of the modes, we looked at the spatial structure of the potential. In Figures 4-7, the real part of the potential $\mathcal{R}e(\phi)$ for the collisional and collisionless cases at the beginning and the middle of a period is shown. Interestingly, it is found that the mode is not localized at the low field side but at the bottom of the device. Comparing the structure of the potential at the beginning and middle of a period for the collisional and non collisional case, shows that, despite the strong helical twist of LHD, the modes are nearly axisymmetric and only slightly modified by the variation of the equilibrium with $\varphi$.

When collisions are included, we find that the structures once present in the collisionless case, become wider and more irregular. It is also observed that some of the structures are slanted in comparison with their collisionless counterparts.

An additional effect of collisions can be seen in the Fourier spectrum of the electrostatic potential (see Figure 8). Without collisions the mode has a significant coupling between the $m$ and $n$ components of the perturbation. Despite the pronounced helical structure of LHD, the toroidal ($n$) sidebands are relatively small. As collisionality increases, another mode appears with almost the same growth rate, and we can see the interplay between them as they evolve in time. Above a critical value of the collision frequency, only the second mode remains. This mode shows a much weaker coupling in $m$. Also, for different values of $\kappa_n$ we found that the coupling can be larger. It can be seen from the figure that the modes are well localized in Fourier space. Nevertheless, a high grid resolution and consequently many Fourier modes are necessary to prevent the growth of spurious unresolved modes, located at the edge of the filter, which can otherwise dominate the simulation.
Figure 4. $\Re(e(\phi))$ for the collisionless ($\nu = 0.0$) linear ITG mode in LHD. Cross section at the beginning of a period ($\varphi = 0$). The dashed lines represent the flux surfaces $s = 0.1, 0.5, 1.0$

Figure 5. $\Re(e(\phi))$ for the collisional ($\nu = 0.01$) linear ITG mode in LHD. Cross section at the beginning of a period ($\varphi = 0$).

Figure 6. $\Re(e(\phi))$ for the collisionless ($\nu = 0.0$) linear ITG mode in LHD. Cross section at the middle of a period ($\varphi = \frac{1}{2} \frac{2\pi}{10}$).

Figure 7. $\Re(e(\phi))$ for the collisional ($\nu = 0.01$) linear ITG mode in LHD. Cross section at the middle of a period ($\varphi = \frac{1}{2} \frac{2\pi}{10}$).
Figure 8. Fourier spectra of the ITG modes for LHD configuration. Shown here with increasing collisionality: a) $\nu = 0$, b) $\nu = 7.0 \times 10^{-3} \Omega_c$, c) $\nu = 1.0 \times 10^{-2} \Omega_c$.

4. Conclusions
The gyrokinetic code EUTERPE has been extended to include collisional effects. As a first test and application we have investigated the influence of collisions on electrostatic ITG modes in LHD for different values of $\eta_i$. For this configuration, we found a dependency of the growth rate on the collision frequency which is weaker than the one for a cylinder but stronger than the one for a tokamak. It is yet to be investigated whether this may change for other equilibrium configurations in which the ITG mode is more slab like than the one found in LHD. Since TEM modes are expected to be more sensitive to collisions, it is important to simulate such modes also for a stellarator. A study in this direction is under way.

References
[1] Chang Z and Callen J D 1992 Phys. Fluids B 4 1182
[2] Dimits A M and Cohen B I 1994 Physical Review E 49 709
[3] Jost G, Tran T M, Cooper W A, Villard L and Appert K 2001 Phys. Plasmas 8 3321
[4] Kornilov V, Kleiber R, Hatzky R, Villard L and Jost G 2004 Phys. Plasmas 11 3196
[5] Xu X Q and Rosenbluth M N 1991 Phys. Fluids B 3 627
[6] Takizuka T and Abe H 1977 J. Comput. Phys. 25 205
[7] Kotschenreuther M 1988 Bull. Am. Phys. Soc. 33 2107
[8] Chen Y and White R B 1997 Phys. Plasmas 4 3591
[9] Brunner S, Valeo E and Krommes J A 1999 Phys. Plasmas 6 4504
[10] Wang W X, Nakajima N, Okamoto M and Murakami S 1999 Plasma Phys. Contr. Fusion 41 1091
[11] Kauffmann K, Kleiber R and Hatzky R 2010 Collisional effects on global gyrokinetic particle-in-cell simulations of ITG and TEM instabilities in tokamaks. Poster presented at: 37th Conference on Plasma Physics; Dublin, Ireland.
[12] van Rij W I and Hirshman S P 1989 Phys. Fluids B 1 563