Effects of Kinetic Decoupling on the Relic Abundances of Asymmetric Dark Matter with Velocity-Dependent Interactions

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Abstract

We discuss the effect of the kinetic decoupling on the relic abundance of asymmetric dark matter when the annihilation rate is boosted by the Sommerfeld enhancement. Usually the relic density of asymmetric dark matter is analysed in the frame of chemical decoupling. Indeed after decoupling from the chemical equilibrium, asymmetric dark matter particles and anti–particles were still in kinetic equilibrium for a while. It has no effect on the case of s–wave annihilation since there is no temperature dependence in this case, however the kinetic decoupling has impact for the case of p–wave annihilation and Sommerfeld enhanced s– and p–wave annihilations. We investigate in which extent the kinetic decoupling affects the relic abundances of asymmetric dark matter particle and anti–particle in detail.

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1 Introduction

Asymmetric dark matter is one alternative solution for the dark matter puzzle and it is arised from the idea that the baryon asymmetry and dark matter asymmetry may share the common origin [1]. The interaction between the asymmetric dark matter particle and anti–particle is appeared as a long-range interaction if the asymmetric dark matter particle and anti–particle are mediated by the sufficiently light force mediator. The long-range interaction distorts the wavefunction of the asymmetric dark matter particle and anti–particle. It is indeed the well-known Sommerfeld effect [2]. Asymmetric dark matter abundance is affected by the Sommerfeld enhancement since the cross section of the asymmetric dark matter particle and anti–particle is changed by Sommerfeld effect [3, 4, 5, 6].

In the standard cosmology, it is assumed that the asymmetric dark matter particles and anti–particles were in thermal and chemical equilibrium in the radiation dominated epoch. They decoupled from the chemical equilibrium when the asymmetric dark matter number densities $n_\chi(T)$, $n_{\bar{\chi}}(T)$ drop exponentially once the relation between the mass of the asymmetric dark matter and temperature satisfies $T < m$ [7, 8]. Although the asymmetric dark matter particles and anti–particles dropped out of chemical equilibrium, they were still in kinetic equilibrium for a while through the scattering off the relativistic standard model particles in the thermal plasma. When the annihilating asymmetric dark matter particles and anti–particles were in both chemical and kinetic equilibrium, the temperatures of them tracks the background radiation temperature $T$, i.e. $T_\chi = T_{\bar{\chi}} = T$. At some point, the rate of the scattering falls below the expansion rate of the universe, then the asymmetric dark matter particles and anti–particles dropped out of kinetic equilibrium. After kinetic decoupling, the temperatures of asymmetric dark matter particle and anti–particle are related by $T_\chi = T_{\bar{\chi}} = T^2/T_k$ with the background radiation temperature $T$, where $T_k$ is the kinetic decoupling temperature [9, 10]. The thermal average of the cross section which is appeared in the Boltzmann equation is different before and after kinetic decoupling due to the change of the temperature dependence. This has impacts on the relic densities of the asymmetric dark matter particles and anti–particles. Without Sommerfeld enhancement, the kinetic decoupling has no effect on the relic abundance of asymmetric dark matter for s–wave annihilation since there is no temperature dependence in this case while there is little impact in the case of p–wave annihilation. The effect is more significant both for the Sommerfeld enhanced s–wave and p–wave annihilations. The relic abundance of asymmetric dark matter is continuously decreased to arbitrarily late times, until the Sommerfeld effect cuts off.

The effect of kinetic decoupling on the relic density of dark matter for the Sommerfeld enhancement was probed in refs. [11, 12, 13]. In this work, we extend this discussion to the
asymmetric dark matter case. We explore the effects of kinetic decoupling to the relic abundances of asymmetric Dark Matter particle and anti–particle in detail. First, we obtain the thermal average of the Sommerfeld enhanced annihilation cross section for the case of kinetic decoupling and calculate the relic abundances for asymmetric dark matter particle and anti–particle. We found the asymmetric dark matter particle and anti–particle abundances are kept decreased after the kinetic decoupling. The decrease is not significant for the asymmetric dark matter particle; on the other hand, the decrease is sizable for the asymmetric dark matter anti–particle. The size of the decrease depends on the asymmetry factor $\eta$ and the coupling strength $\alpha$.

The paper is arranged in the following way. In section 2, we discuss the formulae of the Sommerfeld enhanced annihilation cross section including the effect of kinetic decoupling. In section 3, we investigate the effects of the kinetic decoupling on the relic abundances of asymmetric dark matter particle and anti–particle. The conclusions are on the last section.

2 Annihilation cross section with Sommerfeld enhancement

As we discussed in the introduction, for the case of $s$–wave annihilation, the cross section is independent of $T$, therefore, kinetic decoupling has no effect on the relic density of asymmetric dark matter in this case. For the $p$–wave annihilation or the Sommerfeld enhanced $s$– and $p$–wave annihilations, there are temperature dependencies of the annihilation cross section, then the relic density is affected by the kinetic decoupling. Let us discuss in detail. In ref.[14], the relic density of asymmetric dark matter is discussed when the annihilation cross section of the asymmetric dark matter particle and anti–particle is enhanced by the Sommerfeld enhancement. We closely follow the way used in ref.[14].

For the massless force mediator, Sommerfeld factors for $s$– and $p$–wave annihilations are

$$S_s = \frac{2\pi \alpha/v}{1 - e^{-2\pi \alpha/v}}, \quad S_p = \left[1 + \left(\frac{\alpha}{v}\right)^2\right] \frac{2\pi \alpha/v}{1 - e^{-2\pi \alpha/v}}. \quad (1)$$

The thermal average of the Sommerfeld enhanced annihilation cross section is

$$\langle \sigma v \rangle_S = a \langle S_s \rangle + b \langle v^2 S_p \rangle + \mathcal{O}(v^4) \approx \frac{J^{3/2}}{2\sqrt{\pi}} \int_0^\infty dv e^{-\frac{4}{3}v^2} \left\{a v^2 \frac{2\pi \alpha/v}{1 - e^{-2\pi \alpha/v}} + b v^4 \left[1 + \left(\frac{\alpha}{v}\right)^2\right] \frac{2\pi \alpha/v}{1 - e^{-2\pi \alpha/v}}\right\}. \quad (2)$$
Where \( x = m/T \) is the inverse-scaled temperature. In approximate way [14], Eq. (2) becomes

\[
\langle \sigma v \rangle_{S, \text{approx}} = (a + b \alpha^2) \frac{1 + 7/4 \alpha \sqrt{\pi x} + 3/2 \alpha^2 \pi x + (3/2 - \pi/3) (\alpha^2 \pi x)^{3/2}}{1 + 3/4 \alpha \sqrt{\pi x} + (3/4 - \pi/6) \alpha^2 \pi x} + \frac{6b \alpha}{x} \frac{1 + 4/3 \alpha \sqrt{\pi x} + (\pi + 4)/9 \alpha^2 \pi x + 4/51 \pi (\alpha^2 \pi x)^{3/2}}{1 + 2/3 \alpha \sqrt{\pi x} + \alpha^2 \pi^2/18 x}. \tag{3}
\]

After kinetic decoupling, the inverse-scaled temperatures of asymmetric dark matter particle and anti-particle are

\[
x_{\chi, \bar{\chi}} = \frac{x^2}{x_k}, \tag{4}
\]

[9] [10], then Eq. (2) and Eq. (3) become

\[
\langle \sigma v \rangle_{S_k} \approx \frac{x^2}{2 \sqrt{\pi x_k}} \int_0^\infty dv e^{-\frac{v^2}{2}} \left\{ a v^2 \frac{2\pi \alpha/v}{1 - e^{-2\pi \alpha/v}} + b \frac{2\pi \alpha/v}{1 - e^{-2\pi \alpha/v}} \right\}. \tag{5}
\]

\[
\langle \sigma v \rangle_{S_k, \text{approx}} = (a + b \alpha^2) \frac{1 + 7/4 \alpha \sqrt{\pi x_k} + 3\alpha^2 \pi/(2x_k) x^2 + (3/2 - \pi/3) (\alpha^2 \pi x_k)^{3/2} x^3}{1 + 3/4 \alpha \sqrt{\pi x_k} + (3/4 - \pi/6) \alpha^2 \pi x_k x^2} + \frac{6b x_k \alpha}{x^2} \frac{1 + 4/3 \alpha \sqrt{\pi x_k} + (\pi + 4)/(9x_k) \alpha^2 \pi x^2 + 4/51 \pi (\alpha^2 \pi x_k)^{3/2} x^3}{1 + 2/3 \alpha \sqrt{\pi x_k} + \alpha^2 \pi^2/(18x_k) x^2}. \tag{6}
\]

### 3 Effects of kinetic decoupling on the relic abundance of asymmetric Dark Matter

Asymmetric dark matter particle and anti-particle densities are determined by the following Boltzmann equation which describe the evolution of the number density of the particle and anti-particle in the expanding universe,

\[
\frac{dn_{\chi, \bar{\chi}}}{dt} + 3Hn_{\chi, \bar{\chi}} = -\langle \sigma v \rangle_S (n_{\chi} n_{\bar{\chi}} - n_{\chi, \text{eq}} n_{\bar{\chi}, \text{eq}}), \tag{7}
\]

where the expansion rate \( H^2 = 8\pi G/3 \rho_{\text{rad}}, \) here \( \rho_{\text{rad}} = g_*(T)\pi^2/30 T^4 \) is the radiation energy density with \( g_* \) being the effective number of relativistic degrees of freedom. The equilibrium number densities are \( n_{\chi, \text{eq}} = g_\chi [mT/(2\pi)]^{3/2} e^{(m+\mu_\chi)/T} \) and \( n_{\bar{\chi}, \text{eq}} = g_\chi [mT/(2\pi)]^{3/2} e^{(-m-\mu_\chi)/T}. \)

Here the chemical potentials for \( \chi \) and \( \bar{\chi} \) are equal in equilibrium, \( \mu_\bar{\chi} = -\mu_\chi. \) For convenience, the Boltzmann equation (7) can be rewritten as

\[
\frac{dY_{\chi, \bar{\chi}}}{dx} = -\frac{1.32 \times 10^{-24} \text{ GeV}}{x^2} \left( Y_{\chi} Y_{\bar{\chi}} - Y_{\chi, \text{eq}} Y_{\bar{\chi}, \text{eq}} \right), \tag{8}
\]

where \( Y_{\chi, \bar{\chi}} = n_{\chi, \bar{\chi}}/s \) with entropy density \( s = 2\pi^2 g_{\text{ss}}/45 T^3, \) here \( g_{\text{ss}} \) being the effective number of entropic degrees of freedom, \( M_{\text{Pl}} = 2.4 \times 10^{-24} \) GeV. \( g_* \simeq g_{\text{ss}} \) and \( dg_{\text{ss}}/dx \simeq 0. \)
Using the conservation of \( Y_\chi - Y_\bar{\chi} = \eta \) \[7, 8\], where \( \eta \) is a constant, the Boltzmann equation \[8\] is simplified as

\[
\frac{dY_\chi}{dx} = -\frac{1.32 \, m_\text{Pl} \sqrt{g_*} \langle \sigma v \rangle_S}{x^2} \left( Y^2_\chi + \eta Y_\chi \bar{\chi} - Y^2_{\chi,\text{eq}} \right),
\]

where \( Y^2_{\chi,\text{eq}} = Y_{\chi,\text{eq}} Y_{\bar{\chi},\text{eq}} = (0.145 \, g_\chi/g_*)^2 \, x^3 e^{-2x} \).

To find the analytic result, we repeat the same method which was used in ref.\[8\]. In terms of \( \Delta \bar{\chi} = Y_\bar{\chi} - Y_{\bar{\chi},\text{eq}} \), the Boltzmann equation for anti–particle is rewritten as

\[
\frac{d\Delta \bar{\chi}}{dx} = -\frac{dY_{\bar{\chi},\text{eq}}}{dx} - \frac{\lambda \langle \sigma v \rangle_S}{x^2} \left( \Delta \bar{\chi} (\Delta \bar{\chi} + 2Y_{\bar{\chi},\text{eq}}) + \eta \Delta \bar{\chi} \right),
\]

where \( Y_{\bar{\chi},\text{eq}} = -\eta/2 + \sqrt{\eta^2/4 + Y^2_{\bar{\chi},\text{eq}}} \) \[8\].

For high temperature, \( \Delta^2 \bar{\chi} \) and \( d\Delta \bar{\chi}/dx \) are negligible because \( Y_\bar{\chi} \sim Y_{\bar{\chi},\text{eq}} \), then

\[
\Delta \bar{\chi} \simeq \frac{2x^2 Y^2_{\chi,\text{eq}}}{\lambda \langle \sigma v \rangle_S (\eta^2 + 4Y^2_{\chi,\text{eq}})}.
\]

When the asymmetric dark matter particles and anti-particles decouple from the chemical equilibrium, the particle and anti–particle abundances are almost kept in constant from that time, the freeze–out temperature \( \bar{x}_F \) is determined by this solution. At the point \( \bar{x} = \bar{x}_F \), \( \Delta \bar{\chi} \) becomes of order \( Y_{\bar{\chi},\text{eq}}} \), \( \Delta \bar{\chi}(\bar{x}_F) = \xi Y_{\chi,\text{eq}}(\bar{x}_F) \), where \( \xi \) is a numerical constant of order unity and usually the choice \( \xi = \sqrt{2} - 1 \) provides a good approximation for the analytic result \[15\].

For low temperature, \( Y_{\bar{\chi},\text{eq}} \) is insignificant and it can be dropped from the equation (10). Then we have

\[
\frac{d\Delta \bar{\chi}}{dx} = -\frac{\lambda \langle \sigma v \rangle_S}{x^2} \left( \Delta^2 \bar{\chi} + \eta \Delta \bar{\chi} \right).
\]

We integrate Eq.(12) from \( \bar{x}_F \) to \( \bar{x}_k \) and \( \bar{x}_k \) to \( \bar{x}_{\text{cut}} \), where \( \bar{x}_{\text{cut}} \) is the point at which the Sommerfeld effect cutes off, then

\[
Y_{\chi}(\bar{x}_{\text{cut}}) = \eta \left\{ \exp \left[ 1.32 \, \eta \, m_\text{Pl} \sqrt{g_*} \left( \int_{\bar{x}_F}^{\bar{x}_k} \frac{\langle \sigma v \rangle_{S,\text{approx}}}{x^2} \, dx + \int_{\bar{x}_k}^{\bar{x}_{\text{cut}}} \frac{\langle \sigma v \rangle_{S,\text{approx}}}{x^2} \, dx \right) \right] - 1 \right\}^{-1},
\]

(13)
The final relic density of asymmetric dark matter is obtained as

\[
\Omega_{\text{DM}} h^2 = \frac{m_s}{\rho_{\text{crit}}} [Y_\chi(x_{\text{cut}}) + Y_{\bar{\chi}}(\bar{x}_{\text{cut}})] h^2,
\]

where the present entropy density is \( s_0 = 2.9 \times 10^3 \text{ cm}^{-3} \) and the critical density is \( \rho_{\text{crit}} = 3M_\odot H_0^2 \). The present Hubble expansion rate is \( h = 0.673 \pm 0.098 \) which is in units of 100 km s\(^{-1}\) Mpc\(^{-1}\) [16].
In Fig. 1, we plot the relic abundances of asymmetric dark matter particle $Y_\chi$ and anti–particle $\bar{Y}_\chi$ as a function of the inverse–scaled temperature $x$ for $p$–wave annihilation cross section when the kinetic decoupling temperatures $x_k = 2x_F$ and $x_k = x_F$, here $\alpha = 0$, $a = 0$, $b = 3 \times 10^{-25} \text{ cm}^3 \text{s}^{-1}$, $\eta = 1 \times 10^{-12}$ and $m = 500 \text{ GeV}$. The effects of kinetic decoupling on the particle abundance $Y_\chi$ and anti–particle abundance $\bar{Y}_\chi$ are negligible when kinetic decoupling temperature $x_k = 2x_F$. Both the particle abundance $Y_\chi$ and anti–particle abundance $\bar{Y}_\chi$ are increased for the kinetic decoupling temperature $x_k = x_F$. As we stated earlier, after kinetic decoupling, the temperatures of the annihilating particle and anti–particle scale as Eq. (4). The rapid decreases of $T_{\chi,\bar{\chi}}$ result to fewer annihilations after freeze–out, then there are larger abundances. The effect is insignificant for $x_k/x_F > 2$.

\[ \alpha = 0 \]
\[ a = 0 \]
\[ b = 3 \times 10^{-25} \text{ cm}^3 \text{s}^{-1} \]
\[ \eta = 1 \times 10^{-12} \]

Figure 1: The effect of kinetic decoupling on the evolution of $Y$ for the particle and anti–particle as a function of $x$ for $p$–wave annihilation cross section. Here $g_\chi = 2$, $g_* = 90$, $m = 500 \text{ GeV}$, $x_F = 22$.

The effect of kinetic decoupling is more noticeable for the case of Sommerfeld enhanced $s$–wave and $p$–wave annihilations. Fig. 2 shows the evolution of $Y_\chi$ and $\bar{Y}_\chi$ as a function of $x$ for $s$– and $p$–wave annihilation cross sections when $\alpha = 0.2$ and $\alpha = 0.4$. Here the asymmetry factor $\eta = 1 \times 10^{-13}$, $m = 500 \text{ GeV}$; $a = 3 \times 10^{-26} \text{ cm}^3 \text{s}^{-1}$, $b = 0$ in panels (a) and (b); $a = 0$ and $b = 3 \times 10^{-25} \text{ cm}^3 \text{s}^{-1}$ in panels (c) and (d). Two thick (red) lines are for the relic abundances of particle and anti–particle when there is Sommerfeld enhancement without considering kinetic decoupling. The dotted black lines are for $Y_\chi$ and $\bar{Y}_\chi$ when the kinetic decoupling temperature $x_k = 2x_F$. We see that after kinetic decoupling, the relic abundances for particle and anti–particle are continuously decreased until the Sommerfeld effect cuts off. The decrease is more sizable for larger $\alpha$. The abundance for particle $Y_\chi$ is not affected too much. The decrease of $Y_\chi$ is very small comparing to $\bar{Y}_\chi$. It is due to the magnitude of the
asymmetry factor $\eta$. In this figure, the particle abundances $Y_\chi$ are almost in the same order of the asymmetry factor $\eta$, note that we have the relation $Y_\chi - Y_{\bar{\chi}} = \eta$. $Y_{\bar{\chi}}$ is too small to change $Y_\chi$.

![Diagram](image)

Figure 2: The effects of kinetic decoupling on the evolution of $Y$ for the particle and anti–particle as a function of $x$ for $s$– and $p$–wave annihilation cross sections. The other parameters are same as in Fig.1.

The case of $\alpha = 0.2$ for the larger asymmetry factor $\eta = 1 \times 10^{-12}$ is plotted in Fig.3 for the kinetic decoupling temperatures $x_k = 2x_F$ and $x_k = 5x_F$. Here $m = 500$ GeV, $a = 3 \times 10^{-26}$ cm$^3$s$^{-1}$, $b = 0$ in panel (a) and $a = 0$, $b = 3 \times 10^{-25}$ cm$^3$s$^{-1}$ in panel (b). It is shown that when the asymmetry factor is larger, the particle abundance $Y_\chi$ is almost not changed after kinetic decoupling. However, the anti–particle abundance $Y_{\bar{\chi}}$ is decreased sizably after kinetic decoupling. The effect is more significant when the kinetic decoupling temperature is more close to the freeze–out temperature.
4 Summary and conclusions

The effects of the kinetic decoupling on the relic abundances of asymmetric dark matter particle and anti–particle were discussed when the annihilation cross section of the asymmetric dark matter is changed by the Sommerfeld enhancement. We found the abundances for the asymmetric dark matter particle and anti–particle are continuously decreased after the kinetic decoupling until the Sommerfeld enhancement cuts off. The size of the decrease depends on the coupling strength $\alpha$ and the asymmetry factor $\eta$. The decrease is larger when the kinetic decoupling temperature is more close to the freeze–out point and $\alpha$ is large.

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