Temperature and strain-rate dependent constitutive model for prediction of thermal cycling life

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Abstract. With temperature cycling as the characteristic working environment of aerospace electronic chips, various constitutive models have been proposed to predict the failure problems of solder joints. However, most researchers adopted only a set of constant parameters to describe the solder joint material properties throughout the temperature cycles, which is obviously unreasonable concerning the possible wide range of working temperature. In fact, with the changing temperature and strain rate, some material parameters will correspondingly evolve as observed in the experiments. In this paper, the framework of the Anand constitutive model is adopted to verify the effect of material parameters of different temperatures and strain rates on the mechanical properties of materials under the scenario of temperature cycling. The lead-containing solder alloy 63Sn37Pb material that is most widely used in aerospace is selected as the solder material for the interconnection structure. In addition, a typical plastic ball grid array (PBGA) packaging structure is used to analyze the influence of the constitutive model parameters on the PBGA thermal fatigue life. Based on experimental data, seven sets of constitutive model parameters with different temperatures (-55°C~125°C) under a region of low strain rate (1×10^-4/s) were employed to compare the mechanical properties of the material under temperature cycling. The sensitivity analysis of material parameters is performed and the underlying mechanism are also explained so that the present study can promote the optimization of the constitutive model in numerical simulations in practice.

Keywords: constitutive model, thermal cycling, electronic packaging, temperature dependent, strain rate dependent

1. Introduction

The plastic ball grid array (PBGA) package has been widely adopted in high-performance electronic devices to achieve a higher density of I/O connections with the increasing demand of thermal and electrical requirements [1, 2]. When the ambient temperature changes, the mismatch of coefficient of thermal expansion (CTE) between the PCB board and the chip package results in a thermo-mechanical fatigue damage of solder joints [3]. It has been recognized as a major cause of failure in surface mounted electronic devices [4]. The requirements of equipment specifications for temperature-controlled environments are usually high due to a large number of repeated tests when evaluating the mechanical reliability of an electronic product. This will consumes a great amount of time and expense. Therefore, numerical simulations based on the finite element method has become a convenient and feasible approach in practice.
Although lead-free solders are replacing conventional lead-containing solder alloys [5, 6], a large number of lead-containing solder alloys are still used as solder joints in electronic package structures in the aerospace industry due to their superiority of the mechanical properties. In this paper, the widely used 63Sn37Pb solder is adopted as the solder material. Regarding constitutive model, the Anand model is a specific built-in material model in finite element (FE) programs such as ABAQUS or ANSYS [4], and the parameters in the Anand constitutive model can be obtained by the simple tensile test for some materials, and have relatively reasonable physical meaning. Thus, it is a well accepted constitutive model for researchers and engineers in the industry around the world to describe the material properties of eutectic alloys. In the present study, the Anand constitutive model is adopted to simulate the solder behaviour of the package structure under temperature cycling [7-9]. The parameters of the constitutive model at different temperatures under the low strain-rate conditions are employed, so that the thermal cycling life of the package structure obtained by different constitutive parameters is relatively different. This will be also used to analyze the sensitivity of different constitutive parameters to thermal cycling life.

2. Framework of the Anand constitutive model

The Anand constitutive model [4] is a uniform visco-plastic constitutive equation, which considers the material plastic strain and time-dependent creep effect in the deformation process. And, the Anand model has two basic features: (a) there has no obvious yield surface so that load and unload rules are not considered, and (b) a single internal variable is employed to describe the average resistance that is the material’s internal isotropic strengthen to the macroscopic plastic flow. The flow equation of the visco-plastic model is as follows:

$$\dot{\varepsilon}_p = A \exp \left(\frac{-Q}{RT}\right) \left[\sinh \left(\xi \frac{\sigma_s}{s}\right)\right]^{1/m}$$  \hspace{1cm} (1)

where $\dot{\varepsilon}_p$ is the inelastic strain rate, $A$ is a material constant, $Q$ is the apparent activation energy, $m$ is the strain sensitivity exponent, $\xi$ is the stress multiplier, $R$ is the universal gas constant, $T$ is the absolute temperature, $s$ is the deformation resistance with the dimensions of stress, and $\sigma_s$ is the equivalent stress for the steady plastic flow. The evolution equation for the internal variable $s$ is written in Eq. (2).

$$\dot{s} = h(\sigma, s, T) \dot{\varepsilon}_p$$  \hspace{1cm} (2)

And, the hardening function, $h(\sigma, s, T)$, has the following form in Eq. (3).

$$h = h_0 \left[1 - \frac{s}{s^*}\right]^a \text{sign} \left(1 - \frac{s}{s^*}\right)$$  \hspace{1cm} (3)

where

$$s^* = \hat{s} \left[\frac{\dot{\varepsilon}_p}{A} \exp \left(\frac{Q}{RT}\right)\right]^n$$  \hspace{1cm} (4)

where $h_0$ is deformation hardening–softening constant, $a$ is strain sensitivity exponent related to hardening and softening, $s^*$ is saturation value of internal variables, and $\dot{s}$ and $n$ are material constants.

2.1. Determination of parameters in the Anand model

Experimental data of seven different temperatures under the low strain-rate conditions are selected to quantify the constitutive model parameters [9]. It is well known that the Young’s modulus of solder materials changes with temperature [1, 10]. Due to the Anand model’s first feature without yielding surface, the elastic modulus of the material obtained from the experiment cannot be directly applied to the constitutive model, which will cause serious deviation in the elastic phase of the material. Therefore, through the verification with the constitutive model, the elastic modulus is given according to the constitutive curve, as shown in Table 1. Besides, the other parameters of Anand constitutive model also can be obtained from simulation and experimental results, which are listed in Table 2. In addition, in order to obtain a better constitutive model to be consistent with the experimental data, two parameters are enriched to be functions of temperature as shown in Table 3.

Table 1. Elastic modulus and Poisson’s ratio of 63Sn37Pb materials
### Temperature (°C)

| Temperature (°C) | 125 | 90  | 50  | 10  | -10 | -55 |
|------------------|-----|-----|-----|-----|-----|-----|

### Elastic modulus (GPa)

| Elastic modulus (GPa) | 40  | 41  | 42  | 43  | 45  | 49.5 |
|-----------------------|-----|-----|-----|-----|-----|------|

### Poisson’s ratio

| Poisson’s ratio       | 0.4 |
|-----------------------|-----|

Table 2. Parameters of Anand model

| Material Parameter |  $s^*$ (MPa) | $Q/R$ | $A$ (/s) | $\xi$ | $m$ | $h_0$ (MPa) | $\dot{s}$ (MPa) | $n$ | $a$ |
|--------------------|-------------|------|--------|------|----|------------|----------------|----|----|
| Value              | 3.8         | 5510 | 392    | 5.08 | 0.24| 40531      | —              | 0.012 | — |

(“—””) represents that the values will be changed with temperature.

Table 3. Parameters $\dot{s}$ and $a$ under different temperature

| Strain Rate | 1×10$^{-4}$/s |
|-------------|---------------|
| Temperature (°C) | 125 | 90  | 50  | 10  | -10 | -30 | -55 |
| $\dot{s}$ (MPa) | 96  | 112 | 123 | 132 | 162 | 149 | 235 |
| $a$          | 1.61         | 2.91| 3.91| 6.91|

![Fig. 1. Comparison of experimental data and Anand simulation model, and from top to bottom, the temperature changes from 125°C to -55°C, and the strain rate is 1×10$^{-4}$/s.](image)

Fig. 1 shows that the Anand model can satisfactorily reproduce the constitutive curve of 63Sn37Pb solder alloy in specific temperature and strain rate ranges based on the parameters in Tables 1 to 3. Nevertheless, two disadvantages of the model can be seen from Fig. 1. One is that the elastoplastic transition of the test curve is not well represented by the constitutive model, and the simulation value is often lower than the actual experimental value. The other is that under high temperature conditions, the constitutive model is not capable of predicting the decreasing phenomenon in the steady plastic phase due to softening of the material as observed in the experiments.
3. Application of Anand Model in Package Structure

3.1. Package model

As shown in Fig. 2, the FE model of PBGA package was setup in the commercial FE software ABAQUS [11, 12]. For simplicity, the entire PBGA package structure of this study consisted of five parts in Fig. 2 (a): chip, solder ball, substrate, filling compound and molding compound. After creating all of parts in the preprocessor of ABAQUS, a Boolean operation was performed to merge the individual parts into one integrated model. The mesh scheme adopted the overall division approach to greatly save man-hours. To facilitate the meshing process, two types of solid elements were selected, that is, the tetrahedron element C3D4 and the hexahedron element C3D8R. In order to accelerate the numerical simulations, one quarter (1/4) of the whole package is taken into account by applying symmetric boundary conditions. Fig. 2 (b) shows the 1/4 3D solid model established in ABAQUS.

![Model of PBGA. (a) front view, (b) Plan view of the 1/4 model in ABAQUS.](image)

3.2. Simulation Results

From Fig. 3, different phenomena can be observed. That is, Fig. 3 (a) shows the value of the inelastic equivalent strain decreases with the decreasing temperature under normal and high temperature conditions. However, it is seen from Fig. 3 (b) that under low temperature conditions, the inelastic equivalent strain does not decrease with the decrease of temperature. It is well known that the yield strength of the material will increase significantly with a decreasing temperature [1]. Since the stress induced by the temperature cycling results from the properties of materials, the yield strength of the material should be enhanced when a constitutive model is selected for low temperatures, and thus results in the decrease of inelastic strain. The phenomenon that appears in Fig. 3 (b) is caused by changing two constitutive parameters, which also shows that in the Anand constitutive model, the inelastic strain of the material is not only affected by the strength of the material, but also influenced by other parameters.

3.3. Fatigue life calculation

The Coffin-Manson model modified by Engelmaier [13, 14] is shown in Eq. (5).

\[ N_f = \frac{1}{2} \left( \frac{\Delta \gamma_p}{\varepsilon_f} \right)^{1/c} \] (5)

where \( \Delta \gamma_p \) is the shear plastic strain range, that is, \( \Delta \gamma_p = \sqrt{3} \Delta \varepsilon_p \). In this paper, \( \Delta \varepsilon_p \) is the inelastic equivalent strain increment per cycle [3]; \( \varepsilon_f \) is the fatigue toughness coefficient and is assumed to be equal to 0.325; \( c \) is the fatigue toughness exponent which is related to the thermal cycle frequency and temperature in Eq. (6):

\[ c = -0.442 - 6 \times 10^{-4} t_m + 1.74 \times 10^{-2} \ln(1 + f) \] (6)
where \( t_m \) is the average temperature of thermal cycle, and \( f \) is the cycle frequency. In this paper, the cycle temperature range is from -55°C to 125°C, thus the average temperature is calculated as 
\[
t_m = \frac{T_{\text{max}} + T_{\text{min}}}{2} = 35°C.
\]
Meanwhile, the cycle period is 2400 s/cycle and the frequency \( f \) is calculated as 36/day.

Fig. 3. Inelastic equivalent strain of critical solder joints of the PBGA package structure, which is obtained by Anand constitutive parameters at different temperatures under the low strain-rate condition.

(a) 125°C, 90°C, 50°C, 10°C;
(b) -10°C, -30°C, -55°C.

| Temperature (°C) | 125 | 90 | 50 | 10 | -10 | -30 | -55 |
|------------------|-----|----|----|----|-----|-----|-----|
| \( \Delta \varepsilon_p \) | 0.01386 | 0.01326 | 0.01275 | 0.01264 | 0.01218 | 0.01313 | 0.01264 |
| \( N_f \) | 1901 | 2123 | 2342 | 2393 | 2625 | 2176 | 2393 |

Table 4. Thermal cycling life of solder joints under different constitutive parameters

Different values of the constitutive model in Fig. 3 correspond to different thermal cycling life. As shown in Table 4, it is seen that there is a 38.1% difference between the minimum and maximum thermal cycling life which is a relatively large range.

3.4. Sensitivity analysis

In order to obtain the experimental curves under different conditions by the Anand constitutive model, a total of two constitutive parameters are enriched as functions of working temperature. Based on the comparison in Fig. 4, it can be seen that the trend of the parameter \( \delta \) is consistent with the trend of the thermal cycling life. With the change of temperature, when the value of the parameter \( \delta \) increases, the thermal cycling life also increases accordingly. As in the previous analysis of Fig. 3, the
thermal cycling life is not only affected by the yield strength of the material, but also by the material parameter \( \hat{s} \). Meanwhile, it also can be seen from the trend of the parameter \( a \) in Fig. 4 which has little relationship with the structural thermal cycling life. In fact, during the process of the Anand model fitting test data, the parameter \( \hat{s} \) has a great influence on the strength of the material, and the parameter \( a \) only affects the hardening slope of the experiment curve. Therefore, this paper can also draw a conclusion that the constitutive parameters affecting the strength of the material can have a certain extent of influence on the thermal cycling life of the package structure.

4. Conclusion

This paper studies the experimental data of seven different temperatures at low strain-rate conditions using the Anand constitutive model, and provides the parameters of the Anand constitutive model in the above temperature and strain-rate range. Since different temperatures correspond to different constitutive parameters, this paper investigates the effects of constitutive parameters at different temperatures at low strain rates on the thermal cycling life of solder joints in a typical PBGA package structures. The following conclusions are drawn:

1. The Anand model has no obvious yield surface, thus the elastic modulus of the material obtained from the experiment cannot be directly applied to the constitutive model. Therefore, the elastic modulus is obtained from the test curve for evaluations of the constitutive model.

2. Under high temperature conditions, the Anand constitutive model does not show a tendency that the stress should decrease in the steady plastic phase due to softening of the material.

3. Due to material strength changes with temperature, the value of the inelastic equivalent strain decreases with a decreasing temperature under normal and high temperature conditions. However, due to the change of multiple parameters under low temperature conditions, these parameters will have a certain impact on the trend.

4. The change of the parameter \( \hat{s} \) has the same trend with the thermal cycling life of the package structure, which proves that the parameter \( \hat{s} \) has a stronger correlation more than parameter \( a \) for the thermal cycling life.

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