Observable Proton Decay from Planck Scale Physics

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In the Standard Model, no dim-5 \( \Delta B \neq 0 \) operators exist, so that Planck-scale-induced proton decay amplitudes are suppressed by at least \( 1/M_{Pl}^2 \). If the Standard Model is augmented by a light, color-non-singlet boson, then \( O(1/M_{Pl}) \) proton-decay amplitudes are possible. These always conserve \( B+L \), so that the dominant decay modes are \( p \rightarrow \Pi^+ \nu \) and \( p \rightarrow \Pi^+ \Pi^- \ell^- \), where \( \Pi^+ = \pi^+ \) or \( K^+ \).

INTRODUCTION

One of the most serious obstacles confronting particle physics is the extreme practical difficulty of probing the physics of the Planck scale. One way Planck-scale physics might be probed is by means of proton decay. There are general arguments that quantum gravity should violate global quantum numbers, such as baryon number \( B \) and lepton number \( L \). Therefore, proton decay should happen as a consequence of Planck-scale physics. Of course, proton decay could also happen as a result of grand unification, and grand unification is an extremely well-motivated idea. Nevertheless, it is not absolutely certain that the idea of grand unification is correct, and even if it is, there could be other contributions to proton decay. Therefore, if proton decay is observed, we should keep an open mind about what is causing it. As we shall see below, if it is coming from Planck-scale physics certain kinds of colored bosons should exist whose mass should be less than \( 10^7 \) GeV and very likely much less and within reach of accelerators, and certain decay modes of the proton should be seen. Moreover, there would be a relation between the kind of light colored bosons and the proton decay modes.

At first glance, it would seem hopeless to see proton decay caused by Planck-scale physics. If the operators violating baryon number were dimension-6, with a suppression factor of order \( M_{Pl}^{-2} \), the proton-decay rate would be much too small to be seen in the foreseeable future, or perhaps ever. On the other hand, if the operators were dimension-4, proton decay would typically be too fast. So it is dimension-5 \( B \)-violating operators suppressed by a single power of \( M_{Pl} \) that are of interest in this regard. No such operators can be constructed from Standard Model (SM) fields alone, but they can be if non-Standard Model fields exist. One possibility, already explored in [1], is that such operators may arise in the context of low-energy supersymmetry. In this paper we shall show that even in much more modest extensions of the Standard Model proton-decay amplitudes can arise at \( O(1/M_{Pl}) \). In particular, we shall show that adding just a single type of non-SM field (specifically, colored bosons, as noted above) allows this possibility. We shall classify all such cases and the resulting effective \( B \)-violating operators, somewhat in the spirit of the classic analysis of Weinberg and of Wilczek and Zee [2, 3].

Let us call the single new field added to the Standard Model \( X \). If \( X \) has both dim-5 \( B \)-violating couplings and also renormalizable couplings to SM fields, then integrating it out will yield effective \( B \)-violating operators involving only SM fields that are suppressed by \( M_{Pl}^{-1} M_X^{-2} \), if \( X \) is a boson. Thus, as we will show later, for there to be observable proton decay, \( M_X \) would have to be less than about \( 10^7 \) GeV, and probably even lighter, since there are likely to be small dimensionless couplings involved. We shall call such fields “light” in what follows.

A dim-5 operator would have to be either (i) a product of five boson fields or derivatives, or (ii) a fermion bilinear times a product of two boson fields or derivatives. Suppose \( X \) were a fermion field. Then the only light boson would be the SM Higgs doublet \( \Phi \), and therefore no operator of type (i) would exist. And for type (ii), the fermion bilinear would have to be a color-singlet, so that by assigning \( B = 1/3 \) to all color-triplet fermions and \( B = -1/3 \) to all color anti-triplet fermions \( B \) would be conserved.

Consequently, in order to violate \( B \) at dim-5, \( X \) has to be a boson field. Moreover, it is not difficult to see from reasoning similar to the above that it must be a color non-singlet. There are only ten color non-singlet scalars that can couple renormalizability to the fermions of the Standard Model. The possibilities are listed in Table I.

In the second column of Table I, we show all the dim-4 couplings of \( X \) and \( X^\dagger \) to the SM fermions. The left-handed fermions of the Standard Model are denoted in Table I and throughout this paper by \( Q, L, \nu, d^c \), and \( \ell^c \). In certain cases, not all the dim-4 Yukawa couplings permitted by the gauge symmetries of the Standard Model can be present in the Lagrangian without allowing catastrophic proton decay (i.e. proton decay that is not suppressed by \( 1/(M_{Pl} M_X^2) \), but only by \( 1/M_X^2 \)). These cases are therefore divided in Table I into subcases...
A and B (denoted by a subscript in column 1), depending on which dim-4 operators are present. Those not present must be forbidden by some symmetry beyond the SM gauge group.

In column 3 of Table I, we show those dim-5 couplings of X and X† to the SM fermions that violate baryon number and are linear in X or X†. (An operator quadratic in X would lead to a proton-decay rate of order \( \left( \frac{m_p}{M_{P\ell}} \right)^2 m_p \), because of the need to integrate out two \( X \) bosons instead of one.) In the last column of Table I, we show the dim-7 four-fermion, \( \Delta B \neq 0 \) operators that arise from integrating out the X field. A general analysis of such dim-7 operators was carried out in [4].

In the operators of Table I, the fermions in parentheses are contracted into Lorentz scalars; the dots represent a contraction of \( SU(2)_L \) doublets into singlets; and the circles represent the contraction of \( SU(2)_L \) doublets into triplets.

A noteworthy feature of all the cases is that \( B + L \) is conserved rather than \( B - L \). (\( B + L \) conserving baryon decay was analyzed in the context of \( R \)-parity violating supersymmetric models by Vissani in [3] and more recently by Babu and Mohapatra in a large class of GUT models [6].) Consequently, the dominant proton decay modes are \( p \rightarrow \Pi^+ \nu \), \( p \rightarrow \Pi^+ \Pi^± \ell^\mp \), where \( \Pi^+ = \pi^+ \) or \( K^+ \). Of course, in practice one cannot tell in the difference between the \( p \rightarrow \Pi^+ \nu \) (or \( n \rightarrow \Pi^0 \nu \)) which occur in these models, and \( p \rightarrow \Pi^+ \pi^0 \) (or \( n \rightarrow \Pi^0 \pi^0 \)) which arise in typical grand unified theories (GUTs). However, in GUTs these are accompanied by decay modes with positively charged anti-leptons, whereas in models we are looking at they are not. In Table I, the operators that cause \( p \rightarrow \Pi^+ \nu \) are called type (a), while those that cause \( p \rightarrow \Pi^+ \Pi^± \ell^\mp \) are called type (b). For neutron decay the dominant modes are \( n \rightarrow \Pi^0 \nu \) for type (a), and \( n \rightarrow \Pi^+ \ell^\mp \) for type (b).

Let us consider, for example, the first case in Table I, where \( X \) is a boson with the same SM quantum numbers as the left-handed quark doublet, that is, a \( (3, 2, \frac{1}{6}) \) of \( SU(3)_c \times SU(2)_L \times U(1)_Y \). In this case, there is only one dim-4 coupling to the SM quarks and leptons, namely \( X(d \bar{d} \ell^\pm) \), which requires that \( X \) have \( B = \frac{1}{2} \) if \( B \) is to be conserved by renormalizable couplings. On the other hand, there are several dim-5 operators involving \( X \). Of these, \( \Phi \cdot X(u^c \ell^+) \) conserves \( B \); \( X^\dagger \cdot X(u^c \ell^+) \) violates \( B \) but is quadratic in the \( X \) field; and operators of the form \( X^\dagger X(Q \bar{Q}) \) are both quadratic in \( X \) and conserve \( B \). That leaves only the dim-5 operators \( \Phi \cdot X(Q \bar{Q}) \) and \( X \cdot (Q \bar{Q}) \cdot \Phi \) shown in the first and second rows of Table I that can lead to observable proton decay. When the \( X \) field is integrated out, through the diagram shown in Fig. 1, one obtains the \( B \)-violating four-fermion operators called \( O_1 \) and \( O_1' \) in Table I. These exotic scalar fields have been studied extensively, see e.g. [4-10]. In particular, it is pointed out in [9, 10] that they could have interesting consequences for physics at the CERN Large Hadron Collider if their masses are low enough.

![FIG. 1. Integrating X gives a ΔB = −ΔL operator with coefficient of \( O(\langle \Phi \rangle)/(M_{P\ell}M_X) \).](image1)

If we examine the four-fermion operators in the last column of Table I, we see that all are of the general form \( \bar{d} d \ell^- \) or \( \bar{u} u \ell^- \), where \( d \) and \( \pi \) stand for anti-quarks without respect to handedness. These can be dressed by sea quarks to give proton decay as shown in Fig. 2.

![FIG. 2. The four-fermion operators in the last column of Table I can be dressed by sea quarks to give proton decay.](image2)

From Table I, we see that there are five kinds of models, defined by the quantum numbers of \( X \) and its couplings: Model 1: \( X = (3, 2, \frac{1}{6}) \); Model 2: \( X = (\overline{3}, 1, -\frac{2}{3}) \); Model 3: \( X = (\overline{3}, 1, -\frac{1}{3})_B \); Model 4: \( X = (\overline{3}, 3, -\frac{1}{3})_Y \); and Model 5: \( X = (3, 2, \frac{2}{3}) \). In Model 1, all the decay modes shown in Fig. 2 are allowed. However, in Model 2 (and Model 5), which has the same four-fermion proton decay operators, the anti-down quarks in the factor \((d^c d^c)^\ell\) of the operators \( O_2 \) and \( O_4 \) must be antisymmetric in flavor due to Fermi statistics (since they are antisymmetric in both spin and color) and thus contain a strange quark. Thus the dominant decays are \( p \rightarrow K^+ \nu \) and \( p \rightarrow K^+ \pi^+ \ell^- \), while the modes with pions and no Kaons must come from higher-order effects. (The operator \( O_3 \) must have all three of its anti-down quarks in a flavor antisymmetric state, and so must contain \((d^c s^c b^c)^\ell\), which means that it does not contribute to proton decay, except through higher-order effects.)

Model 3 gives the decay modes \( p \rightarrow \pi^+ \nu \) and \( p \rightarrow K^+ \nu \), but does not give decay modes with charged leptons except through higher-order effects. Model 4 gives the decay modes \( p \rightarrow \pi^+ \nu \), \( p \rightarrow K^+ \nu \), and \( p \rightarrow K^+ \pi^+ \ell^- \), but the mode \( p \rightarrow \pi^+ \pi^+ \ell^- \) would only arise at higher-order, as the product \((d \bar{d})_L^\dagger \) appearing in the operator...
O_1'' is flavor antisymmetric. The dominant nucleon decay modes in the five models are shown in Table II, while in Table III are listed the present experimental limits for those modes [11]. Note that these limits all lie in the range (0.17 to 6.7) x 10^{32} yrs.

Suppose that we denote by (Y_4)_{ab} and (Y_5)_{ab}/M_{P_{T}} the coefficients of the dim-4 and dim-5 operators shown in the second and third columns of Table I. These are matrices in flavor space, and the indices a, b denote the families of the fermions in these operators. Since we don’t know the flavor dependence of these matrices, for the purpose of making rough estimates let us denote the typical value of these couplings to the fermions of the first and second families simply by Y_4 and Y_5/M_{P_{T}}, without indices. Then the effective four-fermion \( \Delta B = -\Delta L \) operators in the last column of Table I have coefficients of the form \( Y_4 Y_5 M_{M_{P_{T}}}^{v} \), where \( v \) is the vacuum expectation value of the Standard Model Higgs field \( \Phi \). A two-body nucleon decay mode produced by such operators would therefore typically have a partial rate of order

\[
\Gamma \sim \frac{1}{16\pi} \left[ Y_4 Y_5 \frac{v}{\sqrt{2}} \frac{1}{M_{H}^{2} M_{P_{T}}} \right]^{2} m_{p}^{5}
\]

\[
\sim (Y_4 Y_5)^{2} \left( \frac{M_{X}}{10^{7} \text{GeV}} \right)^{-4} (3 \times 10^{-32} \text{yr}^{-1}).
\]

Thus, to produce two-body nucleon-decay partial rates near the present limits (which, as noted, are all \( |k| \times 10^{32} \text{yr}^{-1} \)) with \( 0.17 \leq k \leq 6.7 \) would require

\[
M_{X} \sim (3k)^{1/4} (Y_4 Y_5)^{1/2} 10^{7} \text{GeV}.
\]

By analogy with the Yukawa couplings of the Standard Model Higgs field, it is plausible that the coefficients Y_4 and Y_5 to the first and second families could be of order 10^{-5} to 10^{-3}, so that the X bosons in these models could be in range of detection at accelerators. Such light, colored bosons raise the question of the whether they would be in conflict with bounds on flavor-changing neutral current (FCNC) processes such as \( K \bar{K} \) mixing and \( \mu \to e\gamma \).

The contribution of the X scalars through box diagrams to the coefficients of \( \Delta S = 2 \) operators would be of order \( \frac{1}{16\pi} (Y_4 Y_5)^{1/4} M_{X}^{2} \). To avoid excessive CP violation in \( K \bar{K} \) system, this must be less than about 10^{-15} \text{GeV}^{-2}. This gives \( M_{X} > (Y_{4})^{2} (2.5 \times 10^{6} \text{ GeV}) \). For models 1 and 5, where X is a leptoquark, one-loop diagrams involving virtual X bosons would typically contribute to \( \mu \to e\gamma \). The present limits for this process would require \( M_{X} > (Y_{4})^{2} (10^{5} \text{ GeV}) \). More stringent limits on these leptoquark masses come from \( K_L \to \mu^{\pm} e^{\mp} \). Present limits for this would give \( M_{X} > (2 \times 10^{6} \text{ GeV}) \). Of course, these are rough estimates. Special patterns of flavor dependence of the matrices \( (Y_4)_{ab} \) and \( (Y_5)_{ab} \) could either relax or strengthen these limits significantly.

| SM rep of X | dim-4 | \( \Delta B \neq 0 \), dim-5 | effective 4-fermion operators |
|-------------|-------|--------------------------|--------------------------------|
| (3, 2, \frac{1}{6}) | \( X(d^c L) \) | \( (a) \Phi \cdot X(Q \cdot Q) \) | \( O_1 = (Q \cdot Q)^{(d^c L \cdot \Phi)} = (u \ d)^{(d^c \nu)} \) |
| (3, 2, \frac{5}{6}) | \( X(d^c d^c) \) | \( (b) \ X \cdot (Q \cdot Q) \cdot \Phi \) | \( O_1' = (Q \cdot Q)^{(d^c d^c)} (v \ d)^{(d^c \nu)} \) |
| (3, 1, -\frac{2}{3}) | \( X(d^c d^c) \) | \( (b) \ X \cdot (Q \cdot Q) \cdot \Phi \) | \( O_2 = (\Phi)^{(d^c d^c)} (d^c \nu)^{(d^c \nu)} \) |
| (3, 1, \frac{1}{3}) | \( X(Q \ L) \) | \( (a) \ X \cdot (Q \cdot Q) \cdot \Phi \) | \( O_3 = (\Phi)^{(d^c d^c)} (d^c \nu)^{(d^c \nu)} \) |
| (3, 1, \frac{5}{6}) | \( X(Q \ L) \) | \( (a) \ X \cdot (Q \cdot Q) \cdot \Phi \) | \( O_4 = (\Phi)^{(d^c d^c)} (d^c \nu)^{(d^c \nu)} \) |
| (3, 3, -\frac{1}{3}) | \( X(Q \ o \ L) \) | \( (a) \ X \cdot (Q \cdot Q) \cdot \Phi \) | \( O_5 = (\Phi)^{(d^c d^c)} (d^c \nu)^{(d^c \nu)} \) |
| (6, 1, -\frac{1}{6}) | \( X(Q \cdot Q) \) | \( (a) \ X \cdot (Q \cdot Q) \cdot \Phi \) | \( O_1' = (Q \cdot Q)^{(d^c d^c)} (d^c \nu)^{(d^c \nu)} \) |
| (6, 3, -\frac{1}{6}) | \( X(Q \cdot Q) \) | \( (a) \ X \cdot (Q \cdot Q) \cdot \Phi \) | \( O_2' = (Q \cdot Q)^{(d^c d^c)} (d^c \nu)^{(d^c \nu)} \) |
| (3, 2, \frac{5}{6}) | \( X(d^c d^c) \) | \( (a) \ X \cdot (Q \cdot Q) \cdot \Phi \) | \( O_4 = (\Phi)^{(d^c d^c)} (d^c \nu)^{(d^c \nu)} \) |
| (3, 1, -\frac{2}{3}) | \( X(d^c d^c) \) | \( (b) \ X \cdot (Q \cdot Q) \cdot \Phi \) | \( O_5 = (\Phi)^{(d^c d^c)} (d^c \nu)^{(d^c \nu)} \) |
| (3, 3, -\frac{1}{3}) | \( X(u \ u) \) | \( (a) \ X \cdot (Q \cdot Q) \cdot \Phi \) | \( O_4 = (\Phi)^{(d^c d^c)} (d^c \nu)^{(d^c \nu)} \) |
| (3, 1, \frac{1}{3}) | \( X(u \ u) \) | \( (a) \ X \cdot (Q \cdot Q) \cdot \Phi \) | \( O_5 = (\Phi)^{(d^c d^c)} (d^c \nu)^{(d^c \nu)} \) |
| (3, 3, \frac{5}{6}) | \( X(u \ u) \) | \( (a) \ X \cdot (Q \cdot Q) \cdot \Phi \) | \( O_4 = (\Phi)^{(d^c d^c)} (d^c \nu)^{(d^c \nu)} \) |
| (6, 1, \frac{1}{6}) | \( X(d^c d^c) \) | \( (a) \ X \cdot (Q \cdot Q) \cdot \Phi \) | \( O_5 = (\Phi)^{(d^c d^c)} (d^c \nu)^{(d^c \nu)} \) |

TABLE I. The possibilities for color-non-singlet scalars (X), their dim-4 and \( \Delta B \neq 0 \) dim-5 couplings to the Standard Model fermions, and the \( \Delta B \neq 0 \) four-fermion operators that arise from integrating out the X. The operators that cause \( p \to \Pi^+ \nu \) are called type (a), while those that cause \( p \to \Pi^+ \Pi^+ \ell^- \) are called type (b).
It is apparent that these models could allow observable nucleon decay rates, while satisfying limits from FCNC processes. For instance, if $M_X \sim 10$ TeV, the FCNC limits would be satisfied if $Y_4 < 5 \times 10^{-3}$, while observable two-body nucleon decay could occur if $Y_4 Y_5 \sim 10^{-6}$. Similarly, if $M_X \sim 1$ TeV, then the FCNC limits would require $Y_4 < 5 \times 10^{-4}$, while observable two-body proton decay could occur for $Y_4 Y_5 \sim 10^{-8}$.

Because one does not know a priori the flavor dependence of the coefficients $Y_4$ and $Y_5$, one cannot say for nucleon decay whether pion or kaon modes will be dominant, and whether electron or muon modes will be. All else being equal, phase space would lead one to expect that for proton decay the decay modes with neutrinos, which are two-body, would dominate over the decay modes with charged-leptons, which are three-body. However, in models 2 and 5, different operators produce the neutrino and charged lepton modes. For example, in model 2 the neutrino modes come from the dim-5 operator $X^\dagger \Phi \cdot \bar{L} \nu^c$, whereas the charged lepton modes come from the dim-5 operator $X \Phi \cdot Q \ell^c$. And one does not know a priori which of these operators has larger coefficients. In models 1 and 4, on the other hand, the operators $O_1$ and $O_1'$, contribute to both neutrino and charged lepton decay modes of the proton. So the former should predominate because of phase space. In model 3, only neutrino modes are produced by the operators $O_1$ and $O_5$.

It is interesting to compare the kinds of non-supersymmetric models discussed here with models based on low-energy supersymmetry that can also give proton-decay amplitudes of order $1/M_{Pl}$, as discussed in §1. There are several differences. First, one notes that in §1 the baryon-number-violating operators are constructed out of chiral superfields that appear in the Minimal Supersymmetric Standard Model (MSSM). While the first three cases we consider in Table II ($(3, 2, \frac{1}{3}), (\overline{3}, 1, -\frac{2}{3})$, and $(\overline{3}, 1, \frac{1}{3})$) correspond to fields that exist in the MSSM, the other two cases do not. Second, one sees that even in the cases where our $X$ field corresponds to a field of the MSSM, many of the dim-5 $\Delta B \neq 0$ operators in Table I, and in particular those containing $X^\dagger$, have no analogue in the MSSM. For example, in the case $X = (\overline{3}, 1, -\frac{2}{3})$, there are the operators $X^\dagger \Phi \cdot (L d^c)$ and $X^\dagger \Phi \cdot (L u^c)$. As far as gauge quantum numbers go, these would correspond to MSSM operators of the form $u^c H_d L d^c$ and $u^c H_u L u^c$. Neither of these is an $F$ term, however, and therefore they cannot appear in an effective superpotential. The same is true of the operators $O_1$, $O_1'$, and $O_1''$ in the last column of Table I. The most dramatic difference is that in the models discussed here, proton decay conserves $B + L$ rather than $B - L$ as in the supersymmetric models of §1. This would not give a discernible difference for neutrino modes, since one cannot tell the
difference in practice between, for example, $p \rightarrow K^+\pi^-$ (which arises in the MSSM) and $p \rightarrow K^+\nu$ (which arises in the present models). In the MSSM, however, the antineutrino modes would be accompanied by modes with positively charged anti-leptons, which would not appear in the Planck-scale proton decay models discussed here.

As we are examining these questions “from the bottom up”, we do not attempt to explain why the colored bosons we have called $X$ should be light compared to the Planck scale. It is, however, interesting that if such bosons are lighter than about $10^7$ GeV, they may allow a direct window onto physics at the Planck scale.

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