Dynamics of baroclinic wave pattern in transition zones between different flow regimes

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Abstract. Baroclinic waves, both steady and time-dependent, are studied experimentally in a differentially heated rotating cylindrical gap with a free surface, cooled from within. Water is used as working fluid. We focus especially on transition zones between different flow regimes, where complex flow patterns like mixed-mode states are found. The transition from steady wave regime to irregular flow is also of particular interest. The surface flow is observed with visualisation techniques. Velocity time series are measured with the optical Laser-Doppler-Velocimetry technique. Thermographic measurements are applied for temperature field visualisations.

1. Introduction
Baroclinic wave patterns are known to be of particular importance for the transport of heat and momentum in the earth’s atmosphere and in oceans and may also affect atmospheric flows on other terrestrial planets. Starting with systematic investigations in 1958 [3], this phenomenon and its time-dependent conduct has been investigated in laboratory experiments as well as numerically and theoretically since more than four decades. At the beginning of the nineties, methods of nonlinear time series analysis were used to investigate the dynamics of the observed flow states in such experiments [6]. Recent experimental studies focus especially on transitions between different flow regimes [2, 8], on linear stability analysis [4] and also on effects of air as working fluid in contrast to a liquid, i.e. on the effect of viscosity [5, 7].

2. Experimental Setup
The experiment set-up, 300 mm in diameter and 210 mm in height, consists of three concentric chambers that are mounted on a turntable (cf. fig. 1). The size of the inner cylinder can be varied. Here, the radius ratio \( \eta = a/b \) and the aspect ratio \( \Gamma = d/(b-a) \) of the gap is \( \eta = 0.38 \) and \( \Gamma = 1.8 \), where \( a \) and \( b \) is the inner and outer radius of the gap and \( d \) the fluid height. Typical rotation rates range between 3 and 17 rpm.

The inner chamber is cooled using a thermostat, the outer one is heated by a heating coil. The differentially heated gap between these chambers has a free surface. De-ionised water is used as working fluid. Using aluminium tracers, the surface flow is observed with a co-rotating camera which is mounted above the tank. Therewith, flow regimes and drift rates are determined. Thermographic measurements with a fixed heat sensitive camera give a detailed view of the temperature field.
understanding of the surface temperature distribution and its time-dependent variations. Laser-Doppler-Velocimetry (LDV) technique is used to measure time series data of the radial velocity component of the flow just below the surface. The methods of linear and nonlinear time series analysis enable us to characterise the underlying dynamics of the flow states (cf. [10]).

The leading dynamic quantities are the imposed horizontal density gradient $\Delta \rho$ and the rotation rate $\Omega$. The viscosity of the working fluid is a further key size. The parameter space is spanned by the dimensionless Taylor number $Ta$, $Ta \propto \Omega^2$, and the thermal Rossby number $Ro$, $Ro \propto (\Delta \rho \Omega)^{-2}$. The Prandtl number $Pr = (\nu \kappa^{-1})$ depends on the physical properties of the fluid, $\nu$ and $\kappa$ as kinematic viscosity and thermal diffusivity respectively (experiment fluid: $Pr = 7$).

3. Results

Generally, three different flow regimes are found: the axisymmetric basic flow regime, steady baroclinic waves and irregular flow. In the first transition zone from axisymmetric basic flow to steady wave pattern, both stable and time-dependent flows, i.e. mixed-mode states and different kinds of oscillating flows are observed (cf. [2], [8]). The transition from regular wave regime to irregular flow at higher Taylor numbers is smooth.

Figure 2 shows the stability diagram defined from observations at a horizontal temperature gradient of $\Delta T = 7.5$ K. The transition from axisymmetric basic flow, i.e. wave number $m = 0$, to steady waves takes place at $Ta_c = 6.33 \times 10^6$. Baroclinic wave pattern with $m = 2$ to $m = 4$ are found (cf. fig. 3). Hysteresis is observed at the transition $m = 3 \leftrightarrow m = 4$. In the first transition zone we find complex flow regimes, denoted as $2/3I$, as well as steady waves of wave number $m = 2$. Observations of a mode-interaction pattern of $m = 2$ and $m = 3$ at $Ta = 9.37 \times 10^6$ are shown in figure 4. The second transition zone to irregular flows is characterised by structural disturbances (SV) of the spacious jet-stream structure (cf. fig. 5). With increasing Taylor number, the large-scale jet-stream is more and more disturbed and small-scale vortices evolve
Figure 2. Stability diagram for $\Delta T = 7.5K$. Steady waves as well as complex flows (2/3I) occur in the first transition zone. The transition to irregular flow is smooth.

$\tau_c = 6.33 \times 10^6$

Figure 3. Pictures of thermographic measurements at different Taylor numbers. The axisymmetric basic flow regime, steady waves of wave number $m = 2$ to $m = 4$ and a very disturbed $m = 4$ wave pattern at $Ta = 2.21 \times 10^8$ are identified. Note also the asymmetric structure of the flow state at $Ta = 7.65 \times 10^7$.

which dominate the overall flow structure at high rotation rates. Here, a coexistence of both is also identified with thermographic measurements. Furthermore, beginning in the steady wave regime, the thermographic observations show a repetitive separation of cold vortices from the inner cylinder.

As a first result of processing the LDV data, figure 6 shows the time series analysis of velocity data measured at different parameter points. Generally, the power spectrum of a low dimensional chaotic flow has usually broadened peaks, the autocorrelation of a periodic and an irregular signal is itself periodic and decaying respectively. The rate of decay gives an impression of the degree of irregularity. The method of time delayed coordinates is used to reconstruct the attractor of the system [9]. Then, dynamic variables can be calculated to characterise the complexity of the flow. Simplified, a largest Lyapunov exponent ($\lambda_1$) greater than zero indicates a chaotic type of motion. The geometric structure of an attractor can be described by fractal dimensions. Here, the correlation dimension $D_2$ and the pointwise dimension $D_p$ is calculated. The nonlinear methods used here and its application to Couette flows are described in more detail by [1] and [11].
Figure 4. Mode-interaction of $m = 2$ and $m = 3$ at $Ta = 9.37 \times 10^6$. Upper row: Snapshots of the surface flow at different times $t$, lower row: Thermographic measurements. The sample intervall is $\Delta t = 30 \, s$ in each case.

Figure 5. Transition to irregular flow regime. Pictures of the surface flow at different Taylor numbers. Black lines indicate traces of dominating flow structures. At $Ta = 1.96 \times 10^8$ only small-scale structures can be identified from visualisations.

The autocorrelation of the axisymmetric flow (see fig. 6, left) decays rapidly, the reconstructed attractor shows first appearance of a periodic orbit. The power spectrum of the mode-interaction state at $Ta = 9.37 \times 10^6$ (cf. fig. 4) comprises some significant peaks. Also the time series and the attractor already indicate a complex flow. The calculation of the largest Lyapunov exponent and fractal dimensions confirms this impression.

The power spectrum of the $m = 3$ wave pattern at $Ta = 7.65 \times 10^7$ shows a significant peak at $f = 0.5 \, Hz$ and also at the multiples and in addition some other but smaller peaks. The autocorrelation decays slowly. The attractor of the time series suggests a disturbed periodic flow. The calculated dynamic variables also indicate a disturbed flow. As previously discussed, here, the flow may influenced by the repetitive separation of vortices and also by the asymmetric structured jet-stream.

The autocorrelation of the flow at $Ta = 1.50 \times 10^8$ decays. The Lyapunov exponent is calculated to 0.86, the correlation and pointwise dimension is calculated to 2.75 which confirm the degree of irregularity. Thermographic measurements show an increasing influence of small-scale vortices in this parameter region and a coexistence of these flow pattern and the spacious jet-stream. This may confirmed by the reconstructed attractor which shows a very disturbed periodic flow.
4. Concluding Summary

We focus on results from two transition zones, first from axisymmetric basic flow to steady baroclinic waves and second to irregular flow.

In the first one, steady waves as well as complex flows are observed. Here, we detect mode-interactions and so-called non-uniqueness but do not observe oscillating flows like amplitude vacillating (AV) waves. The latter one and also a variation, so-called modulated amplitude vacillations (MAV), are often observed in experiments using a rigid lid instead of a free surface (cf. [2], [5], [6]). Moreover, AV and also MAV wave patterns are observed in this parameter region in experiments of [8] who uses a similar experiment compared to our set-up, also with a free surface but different geometric parameters.

The transition to irregular flow is characterised by an increasing influence of small-scale pattern. The wavy flow of the jet-stream is disturbed and asymmetric structured. Even though
it seems that the disturbances arise from and imbedded in the large-scale structure, these small-scale features might behave to a large extent as independent locally emerging objects. However, thermographic measurements and also observations of the surface flow show the coexistence of the large-scale jet-stream and small-scale vortices in this parameter region.

The analysis of LDV data confirms the visual inspection and thermographic observations. By comparing the time series analysis, temperature measurements and visual observations not only different flow states but also complex flows can be well described, particularly in the first transition zone.

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