Overall Buckling and Ultimate Load of an H-Section Steel Member Under Combined Compression and Bending

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Abstract
This paper presents a theoretical method and FEM to study the overall buckling of an H-section steel member with narrow flanges. The theoretical study aims to elaborate the significant parameters involved. Based on the theoretical analysis, a parametric study is performed to identify the adequate coefficients for the design of an efficient and general evaluation technique to form the equation. As a result, the approximate closed-form equations are successfully proposed for the elastic critical moment of the H-section beam. Besides, the numerical analysis by FEM is performed to investigate the ultimate load, and the design equation for the ultimate load of an H-section column under combined compression and bending is recommended.

Keywords: lateral buckling; effective factor; moment incline; normalized slenderness; ultimate load

1. Introduction
The notable effect of the overall instability must be considered in the design of a slender steel member. The lateral buckling, as a type of overall buckling, may dominate a slender H-section beam. To propose approximate equations for the elastic critical bending value of an H-section steel member, Numerous analytical and experimental studies1-19) have been reported. Yet to the writer’s knowledge, accurate closed-form equations have been rarely formed.

The critical moment of a beam is largely affected by the moment distribution and beam end condition. The effect exerted by moment incline on the critical moment and effective factor was studied by Salvadori1) and Nakamura8) under the simply supported end of the beam; the upper limit1,8) and lower limit1) of the effect factor are given in the studies. As suggested in Ref.1), the ratio of torsional rigidity to warping rigidity (R) must be considered for forming the critical moment or effective factor precisely. The high precision effective factor was proposed by Kato et al.6) under π2R=50 on various end conditions. To acquire the equivalent uniform moment factor (effective factor) for any moment distribution on a simply supported end condition, Miguel A et al.16) proposed a closed-form expression considering no effect of R. Under combined bending and compression, analytical solutions were derived by Foudil et al.17) for a simply supported I-section beam-column following a non-linear stability model. Practically, the beam end is elastically supported by a column. The elastically supported end condition of an H-section beam is studied by Suzuki et al. using experiment and numerical analysis11,12). As accordingly suggested, when the beam end is joined to an H-section column, the end condition of an H-section beam approaches to the simple support condition; when the beam end is joined to an RHS column, the end condition approaches to the clamp support condition.

The combined compression and linearly changed bending load must be considered in the design of an H-section member with narrow flanges broadly applied in one directional steel structures. It is therefore desirable to form a closed-form and high precision equation of the elastic critical value and to determine the allowable stress, especially for a high strength steel member.

To study the overall buckling following the H-section high strength steel member with narrow flanges as presented in Fig.1., this paper aims to develop a handy method. First and foremost, the elastic lateral buckling about an H-section beam is studied under linearly changed bending following the theoretical method. Second, the closed-form equations for the elastic critical moment of a beam are proposed by performing a parametric study. Eventually, the relationship between the ultimate load and the normalized slenderness is studied; the design equation for the H-section steel member under combined compression and bending is recommended.

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Table 2. Optimized values of M on the beam is changed linearly along the length

Without the mid-span load, the bending moment acting a beam without compression (\(Q = \beta M/l\))

Table 2: Optimized values of M.  

| No. | End condition | \(\mu(x)\) | \(\phi(x)\) |
|-----|---------------|-------------|-------------|
| 1   | Simple support | \(\mu(x) = l \cdot \sum_{n=1} \alpha_n \sin \frac{\pi x}{l}\) | \(\phi(x) = \sum_{j=1} \beta_j \sin \frac{j \pi x}{l}\) |
| 2   | Warping restrained | \(\mu(x) = l \cdot \sum_{n=1} \sin \frac{\pi x}{l}\) | \(\phi(x) = \sum_{j=1} \beta_j \cos \frac{(j-1) \pi x}{l} - \cos \frac{(j+1) \pi x}{l}\) |
| 3   | Clamp support  | \(\mu(x) = \sum_{n=1} \cos \frac{(j-1) \pi x}{l} - \cos \frac{(j+1) \pi x}{l}\) | \(\phi(x) = \sum_{j=1} \beta_j \left( \cos \frac{(j-1) \pi x}{l} - \cos \frac{(j+1) \pi x}{l} \right)\) |

Fig. 1. Analytical Model
To satisfy the foregoing equations, $\mu(x)$ and $\phi(x)$ due to the three types of end condition are expressed by a series of trigonometric functions as listed in Table 1. Where, $a_i$ and $b_i$ denote the coefficients, $I$ and $J$ refer to the series numbers to determine each function. As stated by Ref. 11, 12, the elastic critical moment depends largely on the beam end condition; the end condition approaches to No.1 when the beam end is joined to an H-section steel column. Besides, when the beam end is joined to an RHS column, the end condition is close to No.3.

The strain energy ($\Delta U$) of a laterally buckled beam is expressed as

$$\Delta U = \frac{1}{2} M \left[ \int_a^b \left[ \mu \left( \frac{d^2 \phi}{dx^2} \right) + EI \left( \frac{d^2 \phi}{dx^2} \right)^2 + GI \left( \frac{d^2 \phi}{dx^2} \right)^2 \right] \, dx \right]$$

(6)

The external work (bending and shear work) ($\Delta W$) is expressed as

$$\Delta W = M \int_a^b \left[ \frac{d \mu}{dx} \frac{d \phi}{dx} - \frac{BM}{I} \frac{d^2 \phi}{dx^2} \right] \, dx$$

(7)

$\Delta U$ and $\Delta W$ as presented in Eq. (6) and Eq. (7) are defined below

$$\Delta U = \frac{\pi^2 EI}{2} \left[ \int_a^b A_1 \begin{bmatrix} 0 & 0 \\ 0 & \psi^2 \left( B_1 + R - B_2 \right) \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \right]$$

(8)

$$\Delta W = \pi^2 M_{crb} \left[ \begin{bmatrix} 0 & AB - \beta \cdot AB - \beta \cdot AB \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \right]$$

(9)

The coefficient $R$ in Eq. (8) denotes the relative ratio of torsional to warping rigidity.

$$R = \frac{GR^2}{\pi^2 EI_o}$$

(10)

The critical value $M_{crb}$ in Eq. (9) is defined as

$$M_{crb} = K \frac{\pi^2 E}{2l_f}$$

(11)

Where, $K_{M_{crb}}$ denotes the critical coefficient given $M_{crb}$. Total potential energy of the buckled beam is expressed as

$$\Pi = \Delta U - \Delta W$$

(12)

Following the Rayleigh-Ritz method, the stationary principle of total potential energy is expressed as

$$\partial \Pi / \partial a_i = 0 \quad (i = 1, 2, \ldots, I)$$

(13)

$$\partial \Pi / \partial b_j = 0 \quad (j = 1, 2, \ldots, J)$$

(14)

Under the above equations, the following condition is summarized:

$$\begin{bmatrix} A_1 & 0 \\ 0 & B_1 + R - B_2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = P_{\text{sym.}} \begin{bmatrix} 0 & AB - \beta \cdot AB - \beta \cdot AB \\ B_{\text{sym.}} & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

(15)

$A_1$, $B_1$, $B_2$, $AB_1$, $AB_2$, and $AB_3$ in Eq. (15) are the matrix under the end conditions. $[a, b]^T$ is a coefficient vector to determine the function of $\mu(x)$ and $\phi(x)$ as listed in Table 1. under each end condition.

$$[a, b]^T = [a_1, a_2, b_1, b_2]^T$$

(16)

### 2.2 Convergence

The Matrix of Eq. (15) for each end condition can be easily built with the foregoing expression. Yet under a large series number of $I$ or $J$, the coefficient $K_{M_{crb}}$ cannot be algebraically acquired from the characteristic equation of Eq. (15). A generalised eigenvalue analysis is required to solve the equation. $K_{M_{crb}}$ depends merely on two parameters ($\beta$ and $R$).

Following this energy method, the analytic value of $K_{M_{crb}}$ constantly surmounts the precise value. Whether the calculation is precise depends on the series numbers $I$ and $J$. As a sample presented in Fig.2., let $I = J$, given that the expression of $\mu(x)$ and $\phi(x)$ contains an orthogonal series (Table 1.), the analytical value of $K_{M_{crb}}$ falls rapidly and converges at a constant value when $I$ and $J$ are large enough. The calculation for $K_{M_{crb}}$ converges even at $I = J = 1$ when $\beta = 0$ (equal end moments). As the analysis results suggest, the coefficient $K_{M_{crb}}$ almost sufficiently converges at a precise value when $I = J = 6$ for three end conditions in this study (Table 1.). Therefore, $I = J = 6$ is adopted in the analysis below.
3. Parametric Study and Proposal

As mentioned above, the critical coefficient $K_{Mc,b}$ is associated with merely $\beta$ and $R$ for each end condition. Accordingly, $K_{Mc,b}$ can be deemed as a function of $\beta$ and $R$. On that basis, a comprehensive parametric study is performed to clarify the correlation between $K_{Mc,b}$ and $\beta$, $R$, and an efficient approximation is then proposed.

3.1 Critical Coefficient

First and foremost, let $\beta=0$ (equal end moments, uniform bending condition), and begin from a facile approach. An explicit solution (Eq. (17)) for $K_{Mc,b}$ can be acquired from the characteristic equation of Eq. (15) under the series number $I=J=1$.

$$K_{Mc,b} = \frac{1}{\sqrt{k_{\beta}^2 + R^2}} \beta$$

(17)

Where, $K_{Mc,b}$ denotes the critical coefficient $K_{Mc,b}$ under $\beta=0$. The coefficients $k_{\beta}$ and $k_{\beta}$ in Eq. (17) depend on end conditions. $k_{\beta}$ and $k_{\beta}$ were defined as the effective lengths $l_{1}$, $k_{\beta} = k_{\beta}$=1 for the No.1 end condition; $k_{\beta} = k_{\beta}$=0.5 for No. 3. In this study, $k_{\beta}$ and $k_{\beta}$ are solved with an inverse problem under No. 2 end condition, the optimized values are $k_{\beta}$=0.976 and $k_{\beta}$ =0.45, respectively.

The relationship between $K_{Mc,b}$ and $R$ is presented in Fig.3. Comparatively, the approximate curves given by Eq. (17) basically correspond to the analysed plots.

3.2 Effective Factor

When the beam is acted on by unequal end moments, the critical coefficient $K_{Mc,b}$ is expressed below with the effective factor $C_{1}$ to generalise the evaluation of buckling strength.

$$K_{Mc,b} = C_{1} \frac{1}{\sqrt{k_{\beta}^2 + R^2}} \beta$$

(18)

$C_{1}$ here is defined exactly as a previous magnification factor is$^{11}$. The effective factor $C_{1}$ can be considered a function of $\beta$ and $R$. Figs.4., 5. and 6. show the relationship of $C_{1}$ versus $R$ with different $\beta$ on No. 1, 2 and 3 end conditions, respectively. For each constant value of $\beta$, $C_{1}$ converges at a minimum value under adequately large $R$. Figs.7., 8. and 9. show the relationship of $C_{1}$ versus $\beta$ within $0<\beta<=1000$ under No. 1, 2 and 3 end conditions, respectively. As the analysed results clearly suggest, the value of $C_{1}$ primarily depends on the moment incline $\beta$ and comparatively independent of $R$. In most previous studies, the effects of $R$ on $C_{1}$ is not considered. For instance, Salvadori$^{11}$ proposed upper and lower limits of $C_{1}$ when $R^2<40$. The approximate equations for $C_{1}$ vs $\beta$ curves are proposed under No. 1 end condition by Nakamura$^{1}$ and Miguel A et al.$^{10}$, expressed as Eq. (19) and Eq. (20) respectively.

$$C_{1} = \frac{1.88}{\sqrt{1+1.538(1-\beta^2)+(1-\beta)^2}}$$

(19)

$$C_{1} = \frac{35}{1+9(1-\beta^2)^2+16(1-\beta^2)+9(1-3\beta^2)^2}$$

(20)

The Eq. (19) gives good agreement with the upper limit of the analysed plots on No. 1 end condition as presented in Fig.7.

To calculate the lower limit value of $C_{1}$, Eurocode$^{20}$, MCRPC$^{21}$ and AJI$^{22,23}$, AISC$^{24}$ give a simplified design equation to $C_{1}$ as Eq. (21), Eq. (22), Eq. (23) respectively.

$$C_{1} = 1.88 - 1.4(1-\beta) + 0.52(1-\beta^2) \leq 2.7$$

(21)

$$C_{1} = 1.75 + 1.05(\beta-1) + 0.3(\beta-1)^2 \leq 2.3$$

(22)

$$C_{1} = \frac{12.5}{2.5+3\beta - \frac{1}{4} \beta^2 + \frac{1}{4} \beta^2 + \frac{1}{2} \beta^2 + \frac{3}{4} \beta} \leq 3$$

(23)
The effective factor $C_1$ by Eq. (22) and Eq. (23) take the lower limits of the analytic plots, as presented in Figs.7., 8. and 9. Yet to evaluate $C_1$ precisely, the impact of $R$ should be necessarily considered. As mentioned above, $C_1$ converges at a minimum value under adequately large $R$. By virtue of this, a three-order polynomial expression can be readily found to form the lower limit and upper limit of $C_1$. Denote $C_{1_{\text{min}}}$ as the effective factor $C_1$ under $R=1000$, and denote $C_{1_{\text{max}}}$ as $C_1$ under $R=0$. The following expression is optimized to express the lower limit $C_{1_{\text{min}}}$ vs $\beta$ curve on No. 2 end condition, as presented in Fig.10.

$$C_{1_{\text{min}}} = \frac{1}{1-0.44\beta-0.048\beta^2+0.057\beta^3}$$  

(24)

When taking the vertical axis as $(C_{1_{\text{max}}}-C_{1_{\text{min}}})/C_{1_{\text{min}}}$ as presented in Fig.11., the plots can be approximated by a curve below.

$$\frac{C_{1_{\text{min}}}-C_{1\text{_{max}}}}{C_{1_{\text{min}}}} = -0.126\beta+0.25\beta^2-0.06\beta^3$$  

(25)

As Figs.4., 5. and 6. present, $C_1-C_{1_{\text{min}}}$ is inversely proportional to $R$. By changing the vertical axis of $C_1$ in Fig.5. to $(C_1-C_{1_{\text{max}}})/(C_{1_{\text{max}}}-C_{1\text{_{min}}})$ as presented in Fig.12., the analytical plots are clearly suggested to be almost arranged on a curve as approximated below

$$\frac{C_1-C_{1_{\text{min}}}}{C_{1_{\text{max}}}-C_{1\text{_{min}}}} = \frac{1}{1+0.058R}$$  

(26)

Substituting $C_{1_{\text{min}}}$ (Eq. (24)) and $C_{1_{\text{max}}}$ (Eq. (25)) into Eq. (26), the approximate value of effective factor $C_1$ for the beam with No. 2 end condition is attained as Eq. (27).

$$C_1 = \frac{1+(-0.126\beta+0.25\beta^2-0.06\beta^3)/(1+0.058R)}{1-0.44\beta-0.048\beta^2+0.057\beta^3}$$  

(27)

The approximate value of $C_1$ for the beam with the other end condition (No.1 and No. 3) is attained following the identical parametric method as above. The evaluation equation of $C_1$ can be expressed as Eq. (28).

$$C_1 = \frac{1+(\alpha_1\beta_a+\alpha_2\beta^2+\alpha_3\beta^3)/(1+\alpha_1R)}{1+\alpha_1\beta_a+\alpha_2\beta^2+\alpha_3\beta^3}$$  

(28)
Table 1. Functions for the evaluation of the elastic critical moment \(M_{el}\) of an H-section beam. The optimized coefficient \(k_c\), \(k_p\) and \(\alpha_1\), \(\alpha_2\), with three types of end conditions are listed in Table 2.

As a sample (with No. 2 end condition) presented in Fig.13., the proposed equations (Eq. (28)) are well consistent with the analysed results.

4. Ultimate Load

The numerical analysis by FEM with shell elements is employed to investigate the relationship between the ultimate load and elastic critical load. The elastic perfectly-plastic high strength material is adopted with yield stress \(f_y=780\,\text{N/mm}^2\). Besides, the initial imperfection with the shape of first elastic buckling mode is involved; the maximum deformation \(v\) is established at \(v/l=0.001\). For the welding composite section member, the impact of the residual stress must be considered. The residual stress is involved here as presented in Fig.14.

4.1 Ultimate Compressive Load

Following FEM, the analysis results for H-section columns under pure compression are plotted in Fig.15. under No. 2 and 3 end conditions, respectively (the No.1 and 2 are identical in a pure compression situation); the vertical axis denotes \(N_c/N_p\), the horizontal axis refers to \(\lambda;\) \(N_c\) represents the ultimate compressive load (maximum value of compression); \(N_p\) represents the plastic compressive load; \(\lambda\) refers to the normalized column slenderness, as expressed below.

\[
\lambda = \frac{l_f}{E \frac{f_y}{A}} \quad (30)
\]

Where \(l_f\) for No. 1 and No. 2, \(l_f=0.5l\) for No.3 end condition. The ultimate compressive load \(N_c\) is notably impacted by residual stress as presented in Fig.15. As stated in Ref. 21), \(N_c\) for columns of Category C is expressed below:

\[
\frac{N_c}{N_p} = \begin{cases} 
1 - 0.73 \lambda^2 & \text{for } \lambda \leq 0.215 \\
0.996 + 0.5952 \lambda + 0.595 \lambda^2 & \text{for } 0.215 < \lambda \leq 1.05 \\
1.065 - 0.302 \lambda + 0.302 \lambda^2 & \text{for } \lambda > 1.05 
\end{cases} 
\quad (31)
\]

\[
N_c/N_p = \begin{cases} 
1 - 0.49 \lambda^2 & \text{for } \lambda \leq 1 \\
1/(\lambda + 0.4) & \text{for } \lambda > 1
\end{cases} 
\quad (32)
\]

The ultimate load is notably impacted by the residual stress as presented in Fig.15. The analyzed results with the assumed residual stress (Fig.14.) basically correspond to Eq. (31) under the H-section columns of Category C (Ref. 21).

To simplify the calculation, the following equation is recommended.

\[
\frac{N_c}{N_p} = \begin{cases} 
1 - 0.49 \lambda^2 & \text{for } \lambda \leq 1 \\
1/(\lambda + 0.4) & \text{for } \lambda > 1
\end{cases} 
\quad (32)
\]
4.2 Ultimate Bending Load

The analysis results for H-section beams under linearly changed bending are plotted in Fig.16. within the moment incline $1<\beta<2$, and with three types of end conditions, respectively; the vertical axis is denoted as $M_{bcu}/M_{bu}$, the horizontal axis refers to $\lambda_b$; $M_{bcu}$ denotes the ultimate bending load (maximum value of bending moment); $M_p$ represents the plastic bending moment; $\lambda_b$ refers to the normalized beam slenderness as defined below.

$$\lambda_b = \sqrt{M_p/M_{bcu}}$$  (33)

Where, $M_{bcu}$ denotes the elastic critical moment of a beam following the proposed equation (Eq. (28), (29) and Table 2.). The ultimate bending load $M_{bcu}$ is notably impacted by the residual stress. Yet $M_{bcu}$ is strongly impacted with normalized slenderness $\bar{\lambda}_b$ as presented in Fig.16. The ultimate bending load $M_{bcu}$ can be approximated as follow under the beam with the residual stress as presented in Fig.14.

$$M_{bcu} = \begin{cases} 1-0.36\lambda_b^2 & \text{when } \lambda_b \leq 1 \\ 1/(\lambda_b + 0.25)^2 & \text{when } \lambda_b > 1 \end{cases}$$  (34)

4.3 Interaction between the Ultimate Load of Compression and Bending

Fig.17. gives the interaction plots following the H-section column under combined compression and bending within $1<\beta<2$. The vertical axis is denoted as $M_{bcu}/M_{bcu}$, the horizontal axis is represented by $N_{bcu}/N_{ncu}$, where $M_{bcu}$ and $N_{bcu}$ denote the ultimate bending load and ultimate compressive load of an H-section column under combined compression and bending.

As Fig.17. presents, the relationship of $M_{bcu}/M_{bu}$ and $N_{bcu}/N_{ncu}$ designated by the plots primarily depends on the moment incline $\beta$. To take the lower limit of the plots, the equation of interaction curve below is recommended:

$$\left(\frac{M_{bcu}}{M_{bu}}\right)^{1.4} + \left(\frac{N_{bcu}}{N_{ncu}}\right)^{1.4} = 1$$  (35)

5. Conclusions

To form the high-precision equations of elastic critical moment and ultimate load for an H-section beam and column with narrow flanges, this study developed a parametric method. Three typical end conditions are considered. The results and proposed works are summarized below:

1) The closed-form equations (Eq. (28), Eq. (29) and Table 2.) for the elastic critical moment of an H-section beam are proposed under linearly changed bending.

2) The numerical analysis is employed by FEM following an H-section column under pure compressive load and following an H-section beam under bending load, respectively. The normalized slenderness is suggested strongly correlated with the ultimate load. A design equation of the ultimate compressive load (Eq. (32)) for an H-section column, and a design equation of the ultimate bending load (Eq. (34)) for an H-section beam are recommended, respectively.

3) The interaction is revealed between the ultimate compression and bending load following an H-section column under combined compression and bending load. A design equation for the interaction curve (Eq. (35)) is recommended.

The overall buckling is largely affected by the end conditions, residual stress and geometric imperfection etc. Thus, further researches are required.

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