Renormalization of Polyakov loops in different representations and the large-$N$ limit

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Outline

1. Introduction and motivation
2. Polyakov loop renormalization
3. Setup of the computation
4. Preliminary results
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1 Introduction and motivation

2 Polyakov loop renormalization

3 Setup of the computation

4 Preliminary results
Lattice simulations of Yang-Mills theories with gauge group $SU(N)$ at finite temperature

The Lagrangian is characterized by exact center symmetry.

The Polyakov loop $L = \text{tr} \prod_{t=1}^{NT} U_4(t)$; order parameter for deconfinement.

The free energy associated with the bare Polyakov loop is divergent in the continuum: renormalization required [Dotsenko and Vergeles, 1980].
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Preliminaries

- Lattice simulations of Yang-Mills theories with gauge group $SU(N)$ at finite temperature
- The Lagrangian is characterized by *exact* center symmetry
- The Polyakov loop $L = \text{tr} \prod_{t=1}^{N_T} U_4(t)$; order parameter for deconfinement

The free energy associated with the *bare* Polyakov loop is divergent in the continuum: renormalization required [Dotsenko and Vergeles, 1980]
Bare Polyakov loops

Bare Polyakov loops in the fundamental representation
SU(3), Wilson action

Bare Polyakov loops in the two-index symmetric representation
SU(3), Wilson action
Why large $N$?

- At fixed $\lambda = g^2 N$ and $N_f$, expansions in powers of $1/N$ give non-trivial insight onto some non-perturbative features of QCD [’t Hooft, 1974; Witten, 1979; Manohar, 1998]

- Feynmann diagrams; Planar diagram dominance
- Formal connection to closed string theory; Topological expansions of amplitude $\leftrightarrow$ Loop expansion in Riemann surfaces [Aharony, Gubser, Maldacena, Ooguri and Oz, 1999]
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- Gauge/string correspondence conjecture; technically crucial for computations [Maldacena, 1997; Gubser, Klebanov and Polyakov, 1998; Witten, 1998] used to study the strongly interacting plasma [Gubser and Karch, 2009]
- Analytical solutions in $D = 1 + 1$ dimensions [Gross and Witten, 1980]
- Volume reduction [Eguchi and Kawai, 1982]
- Implications for the phase diagram structure at large densities [McLerran and Pisarski, 2007]
- Relevant for the Yang-Mills equation of state, both in $D = 3 + 1$ [Lucini, Teper and Wenger, 2003; Bringoltz and Teper, 2005; Panero, 2009; Datta and Gupta, 2010] and in $D = 2 + 1$ dimensions [Caselle et al., 2011]
- Does this hold for other thermal quantities, too? How about the renormalized Polyakov loop? [Burnier, Laine and Vepsäläinen, 2009; Brambilla et al., 2010; Noronha, 2010]
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- Equivalence of different irreducible representations in the large-$N$ limit
- Effective (matrix) models for the deconfinement region? [Pisarski, 2002]
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Using the $Q\bar{Q}$ potential at zero temperature [Kaczmarek, Karsch, Petreczky and Zantow, 2002; Hübner and Pica, 2008]

$$L_{\text{ren}} = Z^{N_t} L_{\text{bare}}, \quad Z = \exp(V_0 a/2)$$

At fixed temperature $T$, remove the $N_t$-dependent contributions to the bare Polyakov loop free energy [Dumitru et al., 2003]:

$$F^{\text{bare}} = N_t F^{\text{div}} + F^{\text{ren}} + N_t^{-1} F^{\text{lat}} + \ldots$$

(However, note that $g_0$ is not fixed . . .)

Iterative determination of the renormalization term, from simulations at two different bare couplings [Gupta, Hübner and Kaczmarek, 2008; Creutz, 1981]

Fixed scale renormalization [Gavai, 2010]
Polyakov loop renormalization methods

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Simulations with the Wilson action [Wilson, 1974]:

\[ S = \frac{2N}{g_0^2} \sum_x \sum_{\mu<\nu} \left\{ 1 - \frac{1}{N} \text{Re} \ tr U^{1,1}_{\mu,\nu}(x) \right\} \]

... and with the tree-level improved action [Curci, Menotti and Paffuti, 1983; Lüscher and Weisz, 1985]:

\[ S = \frac{2N}{g_0^2} \sum_x \sum_{\mu<\nu} \left\{ 1 - \frac{1}{N} \text{Re} \ tr \left[ \frac{5}{3} U^{1,1}_{\mu,\nu}(x) - \frac{1}{12} U^{1,2}_{\mu,\nu}(x) - \frac{1}{12} U^{1,2}_{\nu,\mu}(x) \right] \right\} \]

Simulation algorithm based on a (standard) 1 + 3 combination of heat-bath [Creutz, 1980; Kennedy and Pendleton, 1985] and overrelaxation [Adler, 1981; Brown and Woch, 1987] updates on SU(2) subgroups [Cabibbo and Marinari, 1982]
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Setting the scale

For the Wilson action: high-precision determinations available in the literature [Necco and Sommer, 2001; Boyd et al., 1996; Lucini, Teper and Wenger, 2004]

For the tree-level improved action: static potential at $T = 0$ from Wilson loops $W(r, L)$:

$$V(r) = \lim_{L \to \infty} \ln \frac{W(r, L - a)}{W(r, L)}, \quad W(r, L) = e^{-L \cdot V(r)} + \ldots$$

Iteratively smeared spacelike links:

$$U_{\mu}^{(i+1)}(x) = U \in SU(N) \quad \text{which maximizes} \quad \text{Re tr}(U^\dagger W)$$

with:

$$W = (1 - k)U_\mu^{(i)}(x) + \frac{k}{4} \sum U_{\text{staple}}^{(i)}$$

Fits to the Cornell potential to extract the string tension:

$$V(r) = \sigma r + V_0 + \frac{\gamma}{r}$$

Comparison with a scale setting from the determination of the critical temperature [Caselle, Panero and Piemonte, 2011]
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Irreducible representations

- For SU(2), the recursive formula for obtaining characters of any irreducible representation:
  \[ \text{tr}_{n+1} g = \text{tr}_n g \text{ tr}_1 g - \text{tr}_{n-1} g \quad \text{with: } \text{tr}_0 g = 1 \]

- For SU(3), the characters of higher representations are obtained using the Young calculus and the relation between the traces in the fundamental and anti-fundamental irreducible representation:
  \[ \frac{1}{2} \left( (\text{tr}_f g)^2 - \text{tr}_f (g^2) \right) = \text{tr} \bar{g} = (\text{tr}_f g)^* \]

- For SU(N > 3) we combine the character relations derived from Young calculus with the Weyl formula [Weyl, 1960; Itzykson and Nauenberg, 1966]:
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  where \( F_{kl}(\vec{\lambda}) = \exp \left[ i (N - k) \alpha_l \right] \) and \( e^{i \alpha_1}, e^{i \alpha_2}, \ldots, e^{i \alpha_N} \) are the eigenvalues of \( g \) in the fundamental representation.
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1 Introduction and motivation

2 Polyakov loop renormalization

3 Setup of the computation

4 Preliminary results
Scale determination from the zero-temperature potential

Wilson loop ratios (5 levels of smearing, $k = 0.3$)

SU(4), $16^4$ lattice, tree-level improved action, $\beta = 8$
Scale determination from the zero-temperature potential

Zero-temperature potential (5 levels of smearing, $k = 0.3$)

SU(4), $16^4$ lattice, tree-level improved action, $\beta = 8$
Scale determination from the zero-temperature potential

Zero-temperature string tension from smeared Wilson loops
SU(4), tree-level improved action
Scale determination from the zero-temperature potential

1 / r term from smeared Wilson loops
SU(4), tree-level improved action
Scale determination from the zero-temperature potential

Renormalization factor from smeared Wilson loops
SU(4), tree-level improved action
Scale determination from the zero-temperature potential

Casimir scaling of bare Polyakov loops
SU(4), tree-level improved action, $N_t = 5$
Scale determination from the zero-temperature potential

Renormalized Polyakov loop
SU(4), tree-level improved action, fundamental representation

$N_t = 5$