N\bar{N} Annihilation in Large $N_c$ QCD with $\rho$ and $\omega$ Mesons

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Abstract

We use classical/large $N_c$ (number of colors) QCD to study nucleon–antinucleon annihilation at rest. We include pion, rho and omega fields in the classical dynamics, starting from the nonlinear sigma model. We begin with a spherical blob of pionic matter with zero baryon number and energy of twice the nucleon mass. This evolves according to the classical dynamics to pion and vector meson fields into the radiation zone. The radiation fields are quantized using the method of coherent states modified to include isospin and four momentum conservation. Empirical information is extracted from that coherent state. We find good agreement with data for single particle momentum and number spectra, and with branching ratios to many annihilation channels. All this emerges with nearly no free parameters.

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1 Introduction

There is growing interest in using classical versions of QCD to model the dynamics in strongly interacting processes leading to many pions. The notion is that the many pion state can be approximated as classical pion radiation. Classical QCD is QCD in the limit of a large number of colors \( N_C \), and is naturally expressed in terms of the classical pion field. The strong interaction dynamics is much easier in the classical than in the quantum guise, and the pion radiation can be quantized in the radiation zone using the method of coherent states. This general approach of classical dynamics leading to pion radiation that can then be quantized has been applied to the evolution and decay of the disoriented chiral condensate [3], to pion production in heavy ion collisions [4], and to nucleon antinucleon annihilation [5, 6].

Though no one has actually derived a classical pion field theory directly from QCD, it is generally agreed that any such theory will feature baryons as nonperturbative topological “knots” of the pion field and corresponding topologically stable baryon number. Skyrme [7] developed just such a theory long before the advent of QCD. Some of its details may be special to Skyrme’s theory, but its general features should pertain to any classical manifestation of QCD. It is this Skyrme approach that we have taken to describe low energy proton antiproton annihilation. We have shown that the pion momentum, number and charge spectra emerge correctly from this picture [8] and that extending the Skyrme treatment to include the classical omega field makes it possible to also account for the omega mesons seen in low energy annihilation [8].

In this paper we extend the large \( N_C \) picture further to include the rho meson and study its effects on annihilation. We concentrate on the observables of nucleon antinucleon annihilation at rest and show that with very few parameters we are able to account for the major features of the pion momentum and number spectrum, the branching ratios into the many annihilation channels and the ratio of direct pions to those from vector meson decay [8]. Two principal features of our treatment seem to be crucial to giving this agreement. First, classical QCD generates the correct fraction of vector mesons during the annihilation process. We do not have to assume the vector mesons were present in the baryons prior to annihilation. Second, the coherent state, with the constraint of isospin and four momentum imposed, describes all the annihilation channels in terms of a single quantum state, with each channel obtained from projections on that state. Thus the different channels have a common origin and the branching ratios come out naturally from that state with the constraints of phase space already imposed.

Early studies of annihilation in the Skyrme approach [3, 4] showed that when a Skyrmion and antiSkyrmion annihilate the classical pion field streams away from the annihilation region as fast as causality will permit, leaving in a coherent burst of energy. Thus far no one has used the Skyrme approach to model the colliding of a proton and antiproton and its subsequent radiation. There are no real conceptual obstacles to doing such a calculation but it is technically
challenging, and it has not been done. It is, of course, still far easier than the corresponding quantum calculation in lattice QCD. Our models of annihilation have tried to exploit the rapid appearance of the pion pulse and have studied the development of the pion wave after the annihilation region has been formed. We begin with a “blob” of pionic matter with total energy of two nucleons, zero baryon number and in a region of about one Fermi radius. For simplicity we take this region to be spherically symmetric. We then use the large $N_C$ dynamics to propagate the pion field outward into the radiation zone, where the nonlinearities of the dynamics can be neglected, and use the classical pion wave in the radiation zone to construct a coherent state. The observed features of the pions from annihilation are then studied in terms of that state. To obtain agreement with experiment it is necessary to impose both isospin and energy-momentum conservation on the coherent state. We have done all this both in the pure Skyrme picture, which includes pions only, and in a generalized form that includes omega mesons.

In this work we extend the theory to include rho mesons. They are included in the large $N_C$ dynamics in a standard way. We also focus on the phenomenology of annihilation. We find agreement with the major trends of branching ratios and spectra with essentially no free parameters. Thus including the rho meson not only yields considerable phenomenological improvement but it also widens the circle of QCD dynamics that can be modeled in this classical/coherent state way. In the next section we briefly outline our plan of attack. Section 3 gives the massive Yang-Mills formulation for including rho and omega fields in the Skyrme or large $N_C$ dynamics. Section 4 details the initial state assumptions we make, how the asymptotic fields are obtained and their form. Section 5 gives the quantum coherent state calculation based on those asymptotic classical fields. Section 6 gives our results and Section 7 presents some conclusions and prospectives for future work.

2 Plan of Attack

In low energy $p p \rightarrow n \pi$ about 33% of pions come out not directly from the annihilation but from the decay of meson resonances. These are mostly the vector meson resonances, $\rho$ and $\omega$. To introduce the vector mesons in large $N_c$ QCD, we treat them as massive Yang-Mills fields which gauge the $U_V(2)$ symmetry of the non-linear $\sigma$ model (contains $\pi$ only). This non-linear $\sigma$ model is the starting point of the Skyrme model or of any other classical or large $N_C$ treatment of QCD. The chiral anomaly gives the coupling between $\omega \rho \pi$, responsible for the interactions $\omega \rightarrow \pi^+ \pi^- \pi^0$, $\omega \rightarrow \rho \pi$ etc. The $SU(2)$ gauge coupling leads to $\rho \rightarrow 2\pi$. Vector dominance is in the model and the baryon is stabilized by the vector meson terms, eliminating the need for the Skyrme term. Though no one has derived classical QCD from quantum QCD, it is generally believed that the classical picture will begin with the non-linear sigma term, include the gauged
vector mesons and continue in a series of terms with higher and higher derivatives. These higher terms become important at high momentum or short distance, but since annihilation at rest is, in some sense, a low energy process, we believe keeping only the first few terms in that expansion is a reasonable starting point. We further believe our results give some support to that view.

With our classical field theory, we study the dynamics of annihilation at rest by starting with a “blob” of matter of baryon number zero, of size about 1 fm and of energy twice the nucleon mass, made up purely of pions. We use the dynamical equations to study the space-time evolution of this blob including the generation of classical vector meson fields. At large $t$ and $r$, the fields decouple and we have free field solutions. We quantize these free waves by coherent state techniques. We project the coherent state onto states of good four-momentum and good isospin.

This quantum coherent state, generated from our dynamical, classical field solutions, is the starting point of our phenomenology. We obtain the particle spectra and branching ratios from the coherent state simply by taking the appropriate matrix element projection of that state. This means that all the various annihilation channels originate from that single state and are thus related. This relationship coupled with the correct phase space imposed by four-momentum conservation, leads naturally to a unified and satisfactory account of the data.

### 3 Massive Yang-Mills Fields: the Classical Solution

We introduce the $\rho$ and $\omega$ classical fields as $U(2)_V$-gauge fields in the non-linear $\sigma$ model. The Lagrangian for the non-linear $\sigma$ model is

$$
\mathcal{L}_0 = \frac{f_\pi^2}{4} Tr \left[ \partial_\mu U \partial^\mu U^\dagger \right]
$$

where $U$ is a unitary field in isospin space and $r$ space and where $f_\pi$ is the pion decay constant. This non-linear $\sigma$ term is also the leading part of the Skyrme Lagrangian \[4\]. We introduce the $\rho$ and $\omega$ fields as massive gauge fields through the $U(2)_V$ covariant derivative

$$
D_\mu U = \partial_\mu U - ig \left[ \tau \cdot \rho_\mu, U \right]
$$

where $\tau$ is the Pauli matrices for isospin and greek indices specify space-time components. (Note that isovectors are boldface while arrows and “hat” denote spatial vectors.) Since the $\omega$ field commutes with $U$, it does not appear in (2). Its couplings are given entirely by the gauged Wess–Zumino anomaly term. The complete Lagrangian then is \[11, 13\]

$$
\mathcal{L} = \mathcal{L}_\sigma + \mathcal{L}_m + \Gamma_{WZ}
$$
with

\[ \mathcal{L}_\sigma = \frac{f_\pi^2}{4} Tr \left[ D_\mu U D^\mu U^\dagger \right] + \frac{1}{2} m_\pi^2 f_\pi^2 Tr \left[ U - 1 \right] - \frac{1}{4} \left( \rho_{\mu\nu}^2 + \omega_{\mu\nu}^2 \right), \]  

(4)

\[ \mathcal{L}_m = \frac{m_\pi^2}{2} \left( \rho_\mu^2 + \omega_\mu^2 \right), \]  

(5)

\[ \Gamma_{WZ} = \Gamma_{WZ}^0 [U] + \frac{3}{2} g \omega_B B^\mu \]

\[ + \frac{3g^2}{64\pi^2} \epsilon^{\mu\nu\alpha\beta} \omega_{\mu\nu} Tr \left[ i \tau \cdot \rho_\alpha \left( U^\dagger \partial_\beta U + \partial_\beta U U^\dagger \right) \right] \]

\[ + \frac{g}{2} \tau \cdot \rho_\alpha U^\dagger \tau \cdot \rho_\beta U \]  

(6)

where \( \mathcal{L}_\sigma \) is the gauged nonlinear \( \sigma \) model with a term for the pion mass. The field strength tensors of the vector mesons are given by

\[ \rho_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu + g \rho_\mu \times \rho_\nu, \]

\[ \omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu. \]  

(7)

The mass term for the vector mesons is \( \mathcal{L}_m \) and we relate the “charge” \( g \) to the pion decay constant and the vector meson mass (assumed to be the same for \( \rho \) and \( \omega \)), \( m = 770 \text{ MeV} \) by the KSFR relation [14], \( m = \sqrt{2} f_\pi g \). The gauged Wess-Zumino term gives \( \Gamma_{WZ} \). The baryon current is given by

\[ B^\mu = \frac{1}{24\pi^2} \epsilon^{\mu\alpha\beta\gamma} Tr \left[ \left( U^\dagger \partial_\gamma U \right) \left( U^\dagger \partial_\alpha U \right) \left( U^\dagger \partial_\beta U \right) \right] \]  

(8)

and the Wess Zumino term for the pion field \( \Gamma_{WZ}^0 [U] = 0 \) for \( SU(2) \)-valued \( U \) field.

We now wish to build in our initial condition of a spherically symmetric blob of pionic matter. That means that the pion field will always be of the hedgehog type, and we may write

\[ U = \exp \left[ i \tau \cdot \hat{r} F(r, t) \right] \]  

(9)

where \( F \) is essentially the pion field. Furthermore we can decompose the vector meson fields into a time and space components as follows

\[ \omega^\mu = \left\{ \begin{array}{ll} \omega_0 (r, t) & \mu = 0 \\
\hat{r}^i \omega_i (r, t) & \mu = i \end{array} \right. \]  

\[ \rho^{\mu a} = \left\{ \begin{array}{ll} \hat{r}^a H(r, t) & \mu = 0 \\
\hat{r}^{i a} \hat{r}^k G(r, t) & \mu = i \end{array} \right. \]  

(10)

Here \( a \) is an isospin label and \( \mu \) a space-time label. The spatial unit vector is denoted by \( \hat{r} \) and the indices \( a, i, j, k \) range over \( 1, 2, 3 \). We have explicitly removed a \( 1/r \) from the spatial part of the rho field. With our initial conditions, the \( H \) field decouples and we will drop it in subsequent equations. In terms of these quantities, we can write

\[ \mathcal{L} = \mathcal{L}_\pi + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_{\pi\rho} + \mathcal{L}_{WZ} \]  

(11)
with

\[ L_\pi = \frac{f_\pi^2}{2} \left[ \left( \frac{\partial F}{\partial t} \right)^2 - \left( \frac{\partial F}{\partial r} \right)^2 - \frac{2}{r^2} \sin^2 F \right] + f_\pi^2 m_\pi^2 (\cos F - 1), \] (12)

\[ L_\omega = \frac{1}{2} \left( \frac{\partial \omega_r}{\partial t} - \frac{\partial \omega_0}{\partial r} \right)^2 + \frac{1}{2} m^2 (\omega_0^2 - \omega_r^2), \] (13)

\[ L_\rho = \frac{1}{g^2 r^2} \left[ \left( \frac{\partial G}{\partial t} \right)^2 - \left( \frac{\partial G}{\partial r} \right)^2 \right] - m^2 G^2 - \frac{1}{2 r^2 G^2 (G + 2)^2}, \] (14)

\[ L_{\pi\rho} = -f_\pi^2 \sin^2 F \frac{G(G + 2)}{r^2}, \] (15)

\[ L_{WZ} = -\frac{3g}{16\pi^2} \left[ 4 \left( \omega_0 \frac{\partial F}{\partial r} - \omega_r \frac{\partial F}{\partial t} \right) \frac{\sin^2 F}{r^2} + \left( \frac{\partial \omega_0}{\partial r} - \frac{\partial \omega_r}{\partial t} \right) \sin 2F \frac{G(G + 2)}{r^2} \right]. \] (16)

The nonlinearities of the theory and the nature of the field couplings are clear in the various terms. It should be stressed again that though we have put \( c \) the speed of light to one in these equations, we have not had to do that for \( \hslash \). These are purely classical equations, and there is no \( \hslash \).

The equations of motion that follow from this Lagrangian are

\[ \left( \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r^2} - \frac{2}{r} \frac{\partial}{\partial r} \right) F + m_\pi^2 \sin F = -\frac{\sin 2F}{r^2} (G + 1)^2 \]

\[ + \frac{3g}{8\pi^2} \frac{1}{f_\pi^2 r^2} \left( \frac{\partial \omega_r}{\partial t} - \frac{\partial \omega_0}{\partial r} \right) \left[ 2\sin^2 F - \cos 2F \frac{G(G + 2)}{r^2} \right], \] (17)

\[ \left( \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r^2} - \frac{2}{r} \frac{\partial}{\partial r} + \frac{2}{r^2} \right) \omega_r = \frac{3g}{4\pi^2} \frac{\sin^2 F}{r^2} \frac{\partial F}{\partial t} - \frac{3g}{8\pi^2} \frac{1}{r^2} \left[ \cos 2F \frac{G(G + 2)}{r^2} \frac{\partial F}{\partial t} \right. \]

\[ + \sin 2F \frac{G(G + 1)}{r^2} \frac{\partial G}{\partial t} \] \]

\[ \left( \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r^2} - \frac{2}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \right) \omega_0 = \frac{3g}{4\pi^2} \frac{\sin^2 F}{r^2} \frac{\partial F}{\partial r} - \frac{3g}{8\pi^2} \frac{1}{r^2} \left[ \cos 2F \frac{G(G + 2)}{r^2} \frac{\partial F}{\partial r} \right. \]

\[ + \sin 2F \frac{G(G + 1)}{r^2} \frac{\partial G}{\partial r} \] \]

\[ \left( \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r^2} + \frac{1}{r^2} \right) G + \frac{1}{r^2} G(G + 1)(G + 2) = \frac{3g^3}{16\pi^2} \left( \frac{\partial \omega_r}{\partial t} - \frac{\partial \omega_0}{\partial r} \right) \sin 2F \frac{G(G + 1)}{r^2} - g^2 f_\pi^2 \sin^2 F (G + 1). \] (20)
The corresponding energy density is:

\[ \mathcal{H} = \frac{f_\pi^2}{2} \left[ \frac{\partial F}{\partial t}^2 + \frac{\partial F}{\partial r}^2 + \frac{2}{r^2} \sin^2 F \right] + f_\pi^2 m_\pi^2 (1 - \cos F) \\
+ \frac{1}{2} \left[ \left( \frac{\partial \omega_r}{\partial t} \right)^2 - \left( \frac{\partial \omega_0}{\partial r} \right)^2 \right] + \frac{1}{2} m^2 (\omega_r^2 - \omega_0^2) \\
+ \frac{1}{g^2 r^2} \left[ \left( \frac{\partial G}{\partial t} \right)^2 + \left( \frac{\partial G}{\partial r} \right)^2 + m^2 G^2 + \frac{1}{2r^2} G^2 (G + 2)^2 \right] \\
+ \frac{f_\pi^2}{2} \frac{\sin^2 F}{r^2} G(G + 2) \\
+ \frac{3g}{4 \pi^2} \omega_0 \frac{\partial F}{\partial r} \frac{\sin^2 F}{r^2} \\
+ \frac{3g}{16 \pi^2} \frac{\partial \omega_0}{\partial r} \frac{\sin 2F}{r^2} G(G + 2). \tag{21} \]

We use these equations to propagate our initial field configuration into the radiation zone. In the next section we turn to a discussion of that configuration.

4 Initial Field Configurations and Asymptotic Amplitudes

As in [4], we model the initial pion field as

\[ F(r, t = 0) = h \frac{r}{r^2 + a^2} \exp(-r/a) \tag{22} \]

with \( a \) a range parameter and \( h \) adjusted so that the total energy equals to twice the nucleon mass. In our previous studies we arbitrarily fixed the range by \( a = 1/m_\pi \). Here, since we are interested in a better treatment of the phenomenology, we take \( a \) as a parameter and fit it to obtain the correct asymptotic one pion momentum distribution. We find that \( a = 1.1 \) fm leads to an excellent account of the observed pion momentum distribution. (see Section 6) The initial field configurations are assumed to be static, so that at \( t = 0 \)

\[ \dot{F} = \dot{\omega}_r = \dot{\omega}_0 = \dot{G} = 0. \tag{23} \]

We also take the initial vector meson fields to be zero. That is we take

\[ \omega_r(r, t = 0) = G(r, t = 0) = 0. \tag{24} \]

Under these conditions we find

\[ \omega_0(r, t = 0) = \int_0^\infty dr' G(r, r') \left[ -\beta r'^2 B^0(r') \right] \tag{25} \]

where

\[ G(r, r') = \frac{1}{2mrr'} \left( e^{-m|r-r'|} - e^{-m(r+r')} \right) \tag{26} \]
and $B^0(r)$ is the baryon density, calculated from (8) with the initial pion field (22).

Using the initial values we integrate the equations of motion to determine the fields at later times. As the fields propagate outwards, they diminish in size, making the non-linear and coupling terms in the Lagrangian less important. Thus we can define a radiation zone where the fields propagate as linear free massive fields. In the radiation zone, we can calculate the pion, $f(k)$, rho, $g(k)$, and omega, $h(k)$, momentum distribution amplitudes from the expressions:

$$\frac{dN_\pi}{d^3k} = |f(k)|^2 = \frac{1}{\pi k_0^2} f_\pi^2 \left| \int_0^\infty dr r^2 j_1(kr)(k_0^\pi + i \frac{\partial}{\partial t})F(r,t) \right|^2,$$  \hspace{1cm} (27)

$$\frac{dN_\rho}{d^3k} = |g(k)|^2 = \frac{2}{\pi k_0^2} \left| \int_0^\infty dr r^2 j_1(kr)(k_0^\rho + i \frac{\partial}{\partial t})G(r,t) \right|^2,$$  \hspace{1cm} (28)

and

$$\frac{dN_\omega}{d^3k} = |h(k)|^2 = \frac{1}{\pi k_0^2} \left| \int_0^\infty dr r^2 j_1(kr)(k_0^\omega + i \frac{\partial}{\partial t})\omega_r(r,t) \right|^2,$$  \hspace{1cm} (29)

respectively. Note that in extracting the fields we use that fact that they all satisfy p-wave equations in the asymptotic region.

Let us examine the results of solving the dynamical equations. In Figure 1 we show the dimensionless pion field, $F(r,t)$, as it propagates out to a time of 4 fm ($c = 1$). We see it begin as in (22) and diminish as $1/r^2$. It does not disperse very much because the pion mass is small on the scale of energies of the problem. In Figure 2 we show a similar plot of the rho field. We plot the field, $G(r,t)/gr$ reinstating the $1/gr$ factored out in (10). This field begins at $t = 0$ as zero and builds from the non-linear couplings before beginning to diminish. The couplings and the large value of the $\rho$ mass, make its behavior more complex than that of $F$. Figure 3 shows the radial part of the omega field, about which similar remarks apply. Figure 4 shows the time component of the omega field multiplied by $r$. This makes the plotted quantity dimensionless. We see that it is much smaller than $F$, and also that the factor of $r$ makes it not decrease for large $r$, since the fields themselves all fall as $1/r$ for large $r$.

A more intuitive picture of the time development of the classical fields comes from studying the evolution of the energy density. As we see from (21), that energy density has many parts. Also for our choice of initial condition all contributions to the energy density are spherically symmetric. Thus we do not plot the energy density as a function of $r$ and $t$, but rather $4\pi r^2$ times that density. The plotted quantity need then only be integrated with $dr$ over all $r$ to obtain the total energy. In Figure 5 we show the total radial energy density. We have checked that at each time slice the spatial integral of this density gives twice the nucleon mass. We see that the energy starts at $t = 0$ rather compactly, propagates out and develops two “humps.” The faster of these moves essentially along the light cone and is the energy in the prompt pions. The slower hump is the energy in the massive vector mesons and in the pions produced by the interaction of those vector mesons. Figure 6 shows the radial energy density in the pions only.
We see the prompt peak on the light cone but the slow peak is smaller than that in Figure 5 since much of that slow peak is not in pions. Figures 7 and 8 show the radial energy densities for the rho and omega fields. These are both slower, because of the mass, and smaller (note the scale) than the corresponding pion energy. Figures 9, 10, 11 and 12 show various terms in the interaction energy. These are small (again note the scales) and, more importantly, decay quickly with increasing time or $r$. It is this diminished interaction energy at large $r$ that allows us to define and asymptotic region where the fields propagate as free massive fields. It is these free fields that we use to construct the coherent state.

5 Coherent State Calculation

We use the asymptotic field strengths extracted in (27), (28) and (29) to construct quantum coherent states from the classical $\pi$, $\rho$ and $\omega$ fields in the radiation zone. To obtain information on the pion multiplicity from these states, we need to project them into states of good four-momentum and isospin. We do this by known methods [15, 16], that we have elaborated previously [5]. First we construct the field operators($F$, $G$ and $H$) which create $\pi$, $\rho$ and $\omega$ at the space time position $x$ and pointing in the isospin direction $T$. They are

$$F(x, T) = \int d^3k f(\vec{k}) a^\dagger_{\vec{k}} T e^{-i\vec{k} \cdot \vec{x}},$$

$$G(x, T) = \int d^3k g(\vec{k}) b^\dagger_{\vec{k}} T e^{-i\vec{k} \cdot \vec{x}},$$

and

$$H(x) = \int d^3k h(\vec{k}) c^\dagger_{\vec{k}} e^{-i\vec{k} \cdot \vec{x}}.$$ (32)

Note that $k \cdot x = k_0 t - \vec{k} \cdot \vec{x}$, with $k_0 = \sqrt{\vec{k}^2 + M^2}$ ( $M$ is the corresponding meson mass).

From these we can form the coherent state with $\pi$, $\rho$ and $\omega$ with 4-momentum and isospin projection as

$$|I, I_z, K\rangle = \int \frac{d^4x}{(2\pi)^4} \frac{dT_1dT_2}{4\pi} \mathcal{Y}_{I_1}^{*}(T_1, T_2)$$

$$\times e^{iK \cdot x} \{ e^{F(x, T_1) + G(x, T_2) + H(x)} - F(x, T_1) - G(x, T_2) - H(x) - 1\} |0\rangle$$ (33)

and we have defined a set of coupled harmonic functions of the $\pi$ and $\rho$ isospin directions

$$\mathcal{Y}_{I_1}^{*}(T_1, T_2) = \sum_{I_1M_1; I_2M_2} \langle II_z|I_1M_1; I_2M_2 \rangle Y_{I_1}^{*}(T_1)Y_{I_2}^{*}(T_2).$$ (34)

The states defined in (33) are orthogonal

$$\langle I, I_z, K|I', I'_z, K'\rangle = \delta^4(K - K')\delta_{I I'}\delta_{I_z I'_z} \mathcal{I}(K)$$ (35)
where the normalization factor is given by

\[ I(K) = \int \frac{d^4x}{(2\pi)^4} \frac{dT_1dT_2 dT_1' dT_2'}{4\pi} 4\pi i K \cdot (T_1, T_2) 4\pi i K \cdot (T_1', T_2') \]

\[ \times e^{iK \cdot x} [e^{\rho_f(x)} T_1 T_1' + \rho_g(x) T_2 T_2' - \rho_f(x) T_1' - \rho_g(x) T_2' - \rho_h(x) - 1]0 \]  

(36)

where

\[ \rho_f(x) = \int d^3p |f(\vec{p})|^2 e^{-ip \cdot x} \]  

(37)

\[ \rho_g(x) = \int d^3p |g(\vec{p})|^2 e^{-ip \cdot x} \]  

(38)

and

\[ \rho_h(x) = \int d^3p |h(\vec{p})|^2 e^{-ip \cdot x}. \]  

(39)

It is easy to see that the normalization integral is independent of \( I_z \). We use the expansion method developed in [3, 4] to calculate the normalization integral

\[ I(K) = \sum_{N_\pi+N_\rho+N_\omega \geq 2} \frac{I(K, N_\pi, N_\rho, N_\omega)}{N_\pi! N_\rho! N_\omega!} F(I, N_\pi, N_\rho) \]  

(40)

where

\[ I(K, N_\pi, N_\rho, N_\omega) = \int \delta^4(K - \sum_{i=1}^{N_\pi} p_i - \sum_{j=1}^{N_\rho} q_j - \sum_{k=1}^{N_\omega} r_k) \prod_{i=1}^{N_\pi} d^3p_i |f(\vec{p}_i)|^2 \prod_{j=1}^{N_\rho} d^3q_j |g(\vec{q}_j)|^2 \prod_{k=1}^{N_\omega} d^3r_k |h(\vec{r}_k)|^2 \]  

(41)

and

\[ F(I, N_\pi, N_\rho) = \int \frac{dT_1dT_2 dT_1'dT_2'}{4\pi} 4\pi i K \cdot (T_1, T_2) 4\pi i K \cdot (T_1', T_2') (T_1 \cdot T_1')^{N_\pi} (T_2 \cdot T_2')^{N_\rho}. \]  

(42)

One can then show that

\[ F(I, N_\pi, N_\rho) = \sum_{I_1I_2} F(N_\pi, I_1) F(N_\rho, I_2) \]  

(43)

where the sum is over \( I_1 \) and \( I_2 \) which can add up to \( I \).

\[ F(N_\pi, I) = \int \frac{dTdT'}{4\pi} Y_{I_1s}(T) Y_{I_2s}(T') (T \cdot T')^{N_\pi} \]

\[ = \begin{cases} 0 & I > N_\pi \text{ or } I - N_\pi \text{ is odd} \\ \frac{N_\pi^4}{(N_\pi - I)!!(I + N_\pi + 1)!!} & I \leq N_\pi \text{ and } I - N_\pi \text{ is even}. \end{cases} \]  

(44)

The mean numbers of \( \pi \) and \( \rho \) of isospin type \( \mu \) in the state \( \{33\} \) are given by

\[ N_{\pi\mu} = \frac{1}{2} \int \frac{d^4x}{(2\pi)^4} \int \frac{dT_1dT_2 dT_1'dT_2'}{4\pi} 4\pi i K \cdot (T_1, T_2) 4\pi i K \cdot (T_1', T_2') (T_1 \cdot T_1')^{N_\pi} (T_2 \cdot T_2')^{N_\rho} \]

\[ \times e^{iK \cdot x} \rho_f(x) (e^{\rho_f(x)} T_1 T_1' + \rho_g(x) T_2 T_2' - \rho_f(x) T_1' - \rho_g(x) T_2' - \rho_h(x) - 1) \]  

(45)
and

\[ N_{\rho \mu} = \frac{1}{T} \int \frac{d^4x}{(2\pi)^4} \int dT_1 dT_2 dT'_1 dT'_2 \frac{1}{4\pi} \mathcal{Y}_{Iz}^n(T_1, T_2) \mathcal{Y}_{Iz}^n(T'_1, T'_2) T_{2\mu} T'_{2\mu} \]
\[ \times e^{iK \cdot x} \rho_\rho(x)(e^{\rho_\mu T_1 T'_1 + \rho_\rho(x) T_2 T'_2 + \rho_\mu(x)} - 1), \] (46)

respectively. The mean number of \( \omega \) is given by

\[ N_{\omega} = \frac{1}{T} \int \frac{d^4x}{(2\pi)^4} \int dT_1 dT_2 dT'_1 dT'_2 \frac{1}{4\pi} \mathcal{Y}_{Iz}^n(T_1, T_2) \mathcal{Y}_{Iz}^n(T'_1, T'_2) \]
\[ \times e^{iK \cdot x} \rho_\rho(x)(e^{\rho_\mu T_1 T'_1 + \rho_\rho(x) T_2 T'_2 + \rho_\mu(x)} - 1). \] (47)

Using expansion method, we can show that

\[ N_{\pi \mu} = \frac{1}{T} \sum_{N_\pi + N_\rho + N_\omega \geq 1} \frac{I(K, N_\pi + 1, N_\rho, N_\omega)}{N_\pi! N_\rho! N_\omega!} H(I, I_2, N_\pi, N_\rho, \mu) \] (48)

and

\[ N_{\rho \mu} = \frac{1}{T} \sum_{N_\pi + N_\rho + N_\omega \geq 1} \frac{I(K, N_\pi, N_\rho + 1, N_\omega)}{N_\pi! N_\rho! N_\omega!} H(I, I_2, N_\rho, N_\pi, \mu). \] (49)

The isospin factor is

\[ H(I, I_2, N_\rho, N_\pi, \mu) = \sum_{I_1 I'_1 I_2 I'_2 M_1} \langle II_2 | I_1 M_1; I_2 I_2 - M_1 \rangle \langle II_2 | I'_1 M_1; I_2 I_2 - M_1 \rangle \]
\[ \times G(N_1, I_1, I'_1, M_1, \mu) F(N_2, I_2) \] (50)

with

\[ G(N_1, I_1, I_2, M_1, \mu) = \int \frac{dT dT'}{4\pi} \mathcal{Y}_{I1 M_1}^n(T) \mathcal{T}_{\mu}^n(T' - T') N \mathcal{Y}_{I2 M_1}^n(T') \]
\[ = \sum_{LM} \frac{2L + 1}{\sqrt{(2I_1 + 1)(2I_2 + 1)}} \langle I_1 0 | 10; L0 \rangle \langle I_2 0 | 10; L0 \rangle \]
\[ \times \langle I_1 M_1 | 1\mu; LM \rangle \langle I_2 M_1 | 1\mu; LM \rangle. \] (51)

Similarly for the \( \omega \) we have

\[ N_{\omega} = \frac{1}{T} \sum_{N_\pi + N_\rho + N_\omega \geq 1} \frac{I(K, N_\pi, N_\rho, N_\omega + 1)}{N_\pi! N_\rho! N_\omega!} F(N_\pi, N_\rho, I, I_2). \] (52)

The probability of finding \( N_\pi \) pions, \( N_\rho \) rhos and \( N_\omega \) omegas is given by

\[ p(N_\pi, N_\rho, N_\omega) = \frac{1}{T(K)} \frac{I(K, N_\pi, N_\rho, N_\omega)}{N_\pi! N_\rho! N_\omega!} F(I, N_\pi, N_\rho) \] (53)

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The probability of finding exactly \( n \) pions, from direct pions and from \( \rho \) and \( \omega \) decays, can be obtain from the joint probability (53)

\[
P_n = \sum_{n=N_\pi+2N_\rho+3N_\omega} p(N_\pi, N_\rho, N_\omega)
\]

In the same spirit, we can also obtain the probability of having certain of number of mesons of each type and of given charge in the coherent state. That probability is

\[
p(N_{\pi^+}, N_{\pi^-}, N_{\pi^0}, N_{\rho^+}, N_{\rho^-}, N_{\rho^0}, N_{\omega}) = \frac{1}{I} \frac{I}{N_{\pi^+}!N_{\pi^-}!N_{\pi^0}!N_{\rho^+}!N_{\rho^-}!N_{\rho^0}!N_{\omega}!} \times F(I, I_z, N_{\pi^+}, N_{\pi^-}, N_{\pi^0}, N_{\rho^+}, N_{\rho^-}, N_{\rho^0})
\]

where

\[
F(I, I_z, N_{\pi^+}, N_{\pi^-}, N_{\pi^0}, N_{\rho^+}, N_{\rho^-}, N_{\rho^0}) = \int \frac{d\hat{T}_1 d\hat{T}_2 d\hat{T}_1' d\hat{T}_2'}{(4\pi)^2} Y_{IIz}^{*}(T_1, T_2)Y_{IIz}(T'_1, T'_2)
\]

\[
\times (T_{1+}T_{1+}'^*)^{N_{\pi^+}}(T_{1-}T_{1-}'^*)^{N_{\pi^-}}(T_{10}T_{10}'^*)^{N_{\pi^0}}
\]

\[
\times (T_{2+}T_{2+}'^*)^{N_{\rho^+}}(T_{2-}T_{2-}'^*)^{N_{\rho^-}}(T_{20}T_{20}'^*)^{N_{\rho^0}}.
\]

We now turn to the results from this formalism.

6 Results

One of the advantages of our approach to annihilation is that it contains very few parameters and that all annihilation channels are determined at once by those few parameters. Before we can discuss results, we must fix those parameters. The pion mass we fix at its experimental value of 140 Mev. The vector mesons, \( \rho \) and \( \omega \), are assigned the same mass of \( m = 770 \) MeV.

We choose the parameters, \( f_\pi \) and \( g \), to satisfy the KSFR relation \[14\] \( m = \sqrt{2} f_\pi g \). We choose three values for \( f_\pi \): 1) 93 MeV (experimental); 2) 75 MeV \[11\]; 3) 62 MeV \[17\]. These last two values are popular choices in the Skyrmion literature where they are determined by getting the nucleon mass correct. In particular the middle value, \( f_\pi = 75 \) MeV gives the observed nucleon mass in our formalism with gauged \( \rho \) and \( \omega \) fields \[11\]. The corresponding coupling constant \( g \) is: 1) 5.85; 2) 7.26; 3) 8.78. The only remaining parameter is the size of the initial spherical pion configuration. We fit that to obtain the correct single pion momentum distribution, giving \( a = 1.1 \) fm. This is certainly a reasonable range. All the remaining results to be discussed in this section follow from those few, fixed parameters.

We begin with the single particle inclusive pion momentum spectrum from proton antiproton annihilation at rest. This is shown in Figure 13 where the data is compared with our calculation. As discussed above, one parameter in the initial pion configuration has been adjusted to obtain
the fit seen in Figure 13. Nevertheless we should stress that the entire machinery of our nonlinear classical dynamics and subsequent quantum coherent state formalism intervenes between that initial configuration and the asymptotic momentum distribution seen in Fig.13. That adjusting only one parameter in the initial configuration is capable of giving such a good fit to the distribution validates both the form of the initial configuration and the subsequent dynamics. In Figures 14 and 15 we show the corresponding momentum distribution for the rho and the omega.

Nucleon antinucleon annihilation can occur in two isospin channels ($I = 0, 1$) with $I_z = 0$ for $\bar{p}p$ and $I_z = 1$ for $\bar{n}p$. The mean number of mesons of each charge type for the three isospin channels are shown in Tables 1, 2, and 3, the different tables giving the result for the different choices of $f_\pi$. The charged pions numbers shown are for the direct pions, while the final pion average number includes both direct pions and pions from vector meson decay. We see that the average number of all pions and the width of that distribution is rather insensitive to $f_\pi$, but the details of the pion distribution and particularly the number of vector mesons is strongly dependent on $f_\pi$. In all cases the average pion number and the width agrees well with the observed values. The results in Table 3 use the same parameters as in our previous calculation that did not include the $\rho$ meson. We see that including the $\rho$ has slightly reduced the number of direct pions, presumably to satisfy energy-momentum conservation. In the remainder of this section we use only the middle value of $f_\pi$ ($f_\pi = 75$ MeV) since it is the one fit to the nucleon mass in our classical field theory. Thus apart from the range parameter, all our parameters are fixed by observed masses, pion, rho-omega and nucleon.

In Figures 16 and 17 we show the pion number distribution for each of the isospin channels. These are both for primary pions and for those from vector meson decay. In each case we find the observed mean number of 5 with variance of 1. Also shown in those figures is a Gaussian distribution with the same average and variance. The Gaussian is nearly indistinguishable from our calculation. Yet a Gaussian distribution is normally taken as evidence for a statistical or fireball process, while our process involves a rapid coherent pion wave. This should suggest a cautionary note to attempts to draw lessons about annihilation dynamics from the Gaussian pion number distribution.

In Table 4 we show the branching ratios for the many resolved channels in proton antiproton annihilation at rest. Note that some of the channels involving more than two $\pi^0$ are not resolved. Note also that the experimental signature of channels with vector mesons is uncertain and there are corresponding differences in the reported ratios from different experimental groups. The branches involving pions only in the second half of the table are for all pions, primary and secondary. We can easily convert our branching ratios for vector meson channels into pions since the vector mesons each have a principal pion decay mode; $\rho^\pm \rightarrow \pi^\pm + \pi^0$, $\rho^0 \rightarrow \pi^+ + \pi^-$ and $\omega \rightarrow \pi^+ + \pi^- + \pi^0$. In Table 4 we show our calculation of the branching ratio for each of
the isospin channels separately, and then show the combined result based on an equal mixture of $I = 0$ and $I = 1$. Our results would hardly change if we took the mix of 63% $I = 0$ and 37% $I = 1$ suggested by [18].

Also shown in the last row of Table 4 is the percentage of secondary pions. That is the fraction of pions coming from resonance decays. These are nearly all from $\rho$ and $\omega$ decay. The experimental fraction is 33%, and we find 30%, in quite reasonable agreement. The dynamical generation of vector mesons by our classical QCD is crucial to this result.

The major features of the branching ratio data are correctly reproduced by our calculation with essentially no free parameters. The small channels come out small and the large large. For many of the large channels the agreement is excellent. Our biggest relative failing is for the $\pi^+\pi^-$ channel and for the $\pi^+\pi^-\pi^0$ channel. These are both two body channels since the $\pi^+\pi^-\pi^0$ channel is dominated by $\pi\rho$. We would expect the quantum corrections to be largest for the channels with the fewest quanta, and that is what we are seeing. Nevertheless the overall agreement lends considerable strength to the underlying treatment of classical QCD to generate the fields and a quantum coherent state projected on four momentum and isospin to give the various channels. It is the single quantum coherent state that is critical to explaining all the annihilation channels in a unified context. It is also important that four momentum conservation carry the constraints of phase space. These constraints are clearly seen in the broad-brush division into large and small channel.

The effect of the classical field dynamics is most clearly seen in the generation of the observed vector mesons, since we start with zero vector meson fields. The first part of Table 4 shows that we are getting the vector meson channels qualitatively correctly and this is further verified in the correct fraction of secondary pions. Another way to see that our dynamics matters and that our agreement is not all from phase space is to do a pions only calculation. We construct a coherent state of pions only with an inclusive single particle momentum distribution fit to the empirically observed one [19]. We impose isospin and four momentum conservation and once again calculate the branching ratios to the pion channels. The results of this calculation are shown in Table 5. Note that for pions only there is an odd even isospin effect. That is states of odd numbers of pions can only contribute to $I = 1$ and even numbers to $I = 0$ [1]. We see in Table 5 that both the mean number of pions and the details of the branching ratios are in far less good agreement with experiment than our previous calculation reported in Table 4 and including the full dynamics and the vector mesons. We conclude therefore, that just fitting the single pion spectrum and using it to construct a coherent state with phase space constraints imposed is not good enough. The underlying QCD dynamics also matters.
7 Summary and Conclusion

We have shown that beginning with a classical field theory based in large $N_C$ QCD we can give a good, nearly parameter free account of the major features of nucleon antinucleon annihilation at rest. Our premise is that annihilation occurs as a rapid process with a sudden burst of radiation and that that radiation can be described classically. We begin with the non-linear sigma model, the starting point of any large $N_C$ QCD, and use gauge invariance to add classical rho and omega fields to the classical pion field. As these fields propagate out from the annihilation region, they decouple and can be well approximated by free fields. We use these asymptotic fields to construct a quantum coherent state and project that state onto states of good four momentum and isospin. We then calculate annihilation observables from that coherent state. We find good agreement with particle spectra and branching ratios. That agreement arises from our unified treatment of all the annihilation channels in terms of a single coherent state, which also has four momentum and isospin conservation imposed. It is interesting to observe that the number spectrum from the coherent states strongly resembles the experimental Gaussian distribution, while the conventional statistical approach \[20\] (under the constraint of conservation of charge, isospin, etc.) failed to produce the Gaussian distribution.

Our agreement with data is better than one might expect, given our simplified starting assumptions, but it is certainly not perfect. In particular the need for quantum corrections in the annihilation branches involving few final quanta is clear. The entire question of quantum corrections is an interesting and difficult one, but one that should now be seriously address in view of the demonstrated success of the first cut theory based on classical fields. We are presently studying quantum enhancements of the two pion channel to address the charge spin asymmetry seen in proton antiproton annihilation at higher energies \[21\], \[22\]. Other leading quantum corrections are also under investigation. We are also investigating annihilation in flight. This would build in the two center nature of the process and allow a description of Bose Einstein correlations \[23\]. Further afield one might imagine a description involving classical $SU(3)_f$ fields to describe the small annihilation branches into K-mesons.

The success in annihilation of the classical QCD starting point followed by quantum coherent states to describe the observed particles should give hope for similar methods in other problems where there are many pions but where the energies are low and nonperturbative QCD dominates the physics. Such approaches have already been developed for the disoriented chiral condensate \[3\] and have been suggested for heavy ion reactions \[4\]. All these situations raise the same questions. First is classical QCD appropriate for describing the dynamics and second can one estimate or calculate the leading quantum corrections? The difficulty of alternate approaches, the modest success reported here for annihilation, and the promised volume of high energy heavy ion data should make the investigation of these questions topical and fruitful.
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References

[1] G. ’t Hooft, Nucl. Phys. B 72, 461 (1974); B 75, 461 (1974).
[2] E. Witten, Nucl. Phys. B 160, 57 (1979).
[3] R.D. Amado and I.I. Kogan, Phys. Rev. D 51, 190, 1995, and references therein.
[4] J.-P. Blaizot and D. Diakonov, Phys. Lett. B 315, 226 (1993).
[5] R.D. Amado, F. Cannata, J-P. Dedonder, M.P. Locher, and B. Shao, Phys. Rev. C 50, 640 (1994).
[6] B. Shao and R.D. Amado, Phys. Rev. C 50, 1787 (1994).
[7] T.H.R. Skyrme, Proc. R. Soc. London 262, 237 (1961); Nucl. Phys. 31, 556 (1962).
[8] A brief report of this work appeared in hep-ph/9504362, and was submitted to Phys. Lett. B.
[9] H.M. Sommermann, R. Seki, S. Larson and S.E. Koonin, Phys. Rev. D 45, 4303 (1992).
[10] B. Shao, N.R. Walet and R.D. Amado, Phys. Lett. B 303, 1 (1993).
[11] I. Zahed and G.E. Brown, Phys. Rep. 142, 1 (1986).
[12] J. Sedlák and V. Šimák, Sov. J. Part. Nucl. 19 (3), 191 (1988).
[13] U.G. Meißner and N. Kaiser, Z. Phys. A 325, 267 (1986).
[14] K. Kawarabayashi and M. Suzuki, Phys. Rev. Lett. 16, 255 (1966); Riazzuddin and Fayyazuddin, Phys. Rev. 147, 1071 (1966).
[15] D. Horn and R. Silver, Ann. Phys. 66, 509 (1971).
[16] J.C. Botke, D.J. Scalapino and R.L. Sugar, Phys. Rev. D 9, 813 (1974).
[17] G.S. Adkins and C.R. Nappi, Phys. Lett. B 137, 251 (1985).
[18] M.P. Locher and B.S. Zou, Z. Phys. A 341, 191 (1992).
[19] C.B. Dover, T. Gutsche, M. Maruyama and A. Faessler, Prog. Part. Nucl. Phys. 29, 87 (1992).

[20] W. Blümel and U. Heinz, hep-ph-9409343, TPR-94-30.

[21] A. Hasan et al., Nucl. Phys. B 378, 3 (1992).

[22] R.D. Amado, Yang Lu and I.I. Kogan, in preparation.

[23] R. D. Amado, F. Cannata, J.-P. Dedonder, M. P. Locher and Yang Lu, Phys. Lett. B 339, 201 (1994); Phys. Rev. C 51, 1587 (1995).

[24] C. Amsler and F. Myhrer, Annu. Rev. Nucl. Part. Sci. 41, 219 (1991).
Table 1: Numbers of direct $\pi^+$, $\pi^-$ and $\pi^0$, $\rho^+$, $\rho^-$ and $\rho^0$; $\omega$ for $f_\pi = 93$ MeV. $n$ is the total number of pions (direct and decay products) and $\sigma$ is the standard deviation calculated from $P_n$ [54].

| Channel | $n_{\pi^+}$ | $n_{\pi^-}$ | $n_{\pi^0}$ | $n_{\rho^+}$ | $n_{\rho^-}$ | $n_{\rho^0}$ | $n_\omega$ | $n$  | $\sigma$ |
|---------|-------------|-------------|-------------|-------------|-------------|-------------|----------|------|--------|
| $I = 0$ $I_z = 0$ | 1.50 | 1.50 | 1.50 | 0.09 | 0.09 | 0.09 | 0.13 | 5.44 | 0.81 |
| $I = 1$ $I_z = 0$ | 0.99 | 0.99 | 1.95 | 0.16 | 0.16 | 0.20 | 0.13 | 5.32 | 0.73 |
| $I = 1$ $I_z = 1$ | 1.85 | 1.09 | 0.99 | 0.29 | 0.06 | 0.16 | 0.13 | 5.32 | 0.73 |

Table 2: Numbers of direct $\pi^+$, $\pi^-$ and $\pi^0$, $\rho^+$, $\rho^-$ and $\rho^0$; $\omega$ for $f_\pi = 75$ MeV.

| Channel | $n_{\pi^+}$ | $n_{\pi^-}$ | $n_{\pi^0}$ | $n_{\rho^+}$ | $n_{\rho^-}$ | $n_{\rho^0}$ | $n_\omega$ | $n$  | $\sigma$ |
|---------|-------------|-------------|-------------|-------------|-------------|-------------|----------|------|--------|
| $I = 0$ $I_z = 0$ | 1.26 | 1.26 | 1.26 | 0.06 | 0.06 | 0.06 | 0.41 | 5.33 | 0.86 |
| $I = 1$ $I_z = 0$ | 0.80 | 0.80 | 2.09 | 0.10 | 0.10 | 0.14 | 0.33 | 5.39 | 0.74 |
| $I = 1$ $I_z = 1$ | 1.87 | 1.03 | 0.80 | 0.20 | 0.04 | 0.10 | 0.33 | 5.39 | 0.74 |

Table 3: Numbers of direct $\pi^+$, $\pi^-$ and $\pi^0$, $\rho^+$, $\rho^-$ and $\rho^0$; $\omega$ for $f_\pi = 62$ MeV.

| Channel | $n_{\pi^+}$ | $n_{\pi^-}$ | $n_{\pi^0}$ | $n_{\rho^+}$ | $n_{\rho^-}$ | $n_{\rho^0}$ | $n_\omega$ | $n$  | $\sigma$ |
|---------|-------------|-------------|-------------|-------------|-------------|-------------|----------|------|--------|
| $I = 0$ $I_z = 0$ | 0.93 | 0.93 | 0.93 | 0.02 | 0.02 | 0.02 | 0.75 | 5.18 | 0.82 |
| $I = 1$ $I_z = 0$ | 0.61 | 0.61 | 2.24 | 0.05 | 0.05 | 0.07 | 0.55 | 5.42 | 0.83 |
| $I = 1$ $I_z = 1$ | 1.88 | 0.96 | 0.61 | 0.10 | 0.02 | 0.05 | 0.55 | 5.42 | 0.83 |
| Channel                  | Theory          | Experiment | Experiment |
|-------------------------|-----------------|------------|------------|
|                         | $I = 0$ | $I = 1$ | Combined | CERN | BNL |
| $\pi^0\rho^0$           | 0.013 | 0       | 0.006    | 3.9 ± 0.4 |      |
| $\pi^0\rho^0$           | 0.013 | 0       | 0.006    | 1.9 ± 0.3 |      |
| $\rho^0\pi^+\pi^-$      | 0       | 3.0     | 1.5      | 1.5 ± 0.3 | 5.8 ± 0.3 |
| $\rho^0\rho^0$          | 0.146 | 0       | 0.073    | 0.12 ± 0.12 | 0.4 ± 0.3 |
| $\pi^0\omega$           | 0.0     | 1.2     | 0.6      | 2.3 ± 0.2   | 0.7 ± 0.2  |
| $\omega\omega$          | 3.12   | 0       | 1.56     | 1.4 ± 0.6   | [24]   |
| $\rho^0\pi^+\pi^-\pi^0$| 2.0     | 0       | 1.0      | 7.3 ± 1.7   |      |
| $\rho^0\pi^+\pi^-\pi^0$| 7.9    | 10.3    | 9.1      | 6.4 ± 1.8   |      |
| $\omega\pi^+\pi^-$      | 1.7     | 0       | 0.8      | 6.6 ± 0.35  | 3.8 ± 0.4  |
| $\omega\pi^+\pi^-$      | 4.3     | 0       | 2.1      | 1.3 ± 0.3   |      |
| $\pi^+\pi^-$            | 0.02    | 0.0     | 0.01     | 0.37 ± 0.3  | 0.32 ± 0.04 |
| $\pi^+\pi^-\pi^0$       | 0.04    | 0.6     | 0.32     | 6.9 ± 0.35  | 7.3 ± 0.9  |
| $2\pi^+2\pi^-$          | 9.1     | 3.0     | 6.1      | 6.9 ± 0.6   | 5.8 ± 0.3  |
| $2\pi^+2\pi^-\pi^0$     | 26.8    | 19.8    | 23.3     | 19.6 ± 0.7  | 18.7 ± 0.9 |
| $3\pi^+3\pi^-$          | 13.8    | 3.56    | 8.7      | 2.1 ± 0.2   | 1.9 ± 0.2  |
| $3\pi^+3\pi^-\pi^0$     | 4.38    | 0.61    | 2.5      | 1.9 ± 0.2   | 1.6 ± 0.2  |
| $n\pi^0$, $n > 1$       | 7.7     | 15.7    | 11.7     | 4.1 ± 0.4   | 3.3 ± 0.2  |
| $\pi^+\pi^-n\pi^0$, $n > 1$ | 25.1  | 39.8    | 32.5     | 35.8 ± 0.8  | 34.5 ± 1.2 |
| $2\pi^+2\pi^-n\pi^0$, $n > 1$ | 12.8 | 17.4    | 15.2     | 20.8 ± 0.7  | 21.3 ± 1.1 |
| $3\pi^+3\pi^-n\pi^0$, $n > 1$ | 0.03  | 0.014   | 0.022    | 0.3 ± 0.1   | 0.3 ± 0.1  |
| % of secondary $\pi$s    | 29.2    | 31.3    | 30.3     | 33         |        |

Table 4: Branching ratios, in percent, for proton antiproton annihilation at rest. Our calculations are compared with experiments from [12]. We show each total isospin channel calculated separately. The “combined” column corresponds to equal mixture of $I = 0$ and $I = 1$. In the last row we list the percentage of pions from the decay of rho and omega mesons.
| Channel | Pion-only Calculation | Experiment |
|---------|-----------------------|------------|
|         | $I = 0$ | $I = 1$ | Combined | CERN | BNL |
| $\pi^+\pi^-$ | 0.0004 | 0.0 | 0.0002 | 0.37 ± 0.3 | 0.32 ± 0.04 |
| $\pi^+\pi^-\pi^0$ | 0 | 0.05 | 0.03 | 6.9 ± 0.35 | 7.3 ± 0.9 |
| $2\pi^+2\pi^-$ | 1.93 | 0 | 0.97 | 6.9 ± 0.6 | 5.8 ± 0.3 |
| $2\pi^+2\pi^-\pi^0$ | 0 | 7.0 | 3.5 | 19.6 ± 0.7 | 18.7 ± 0.9 |
| $3\pi^+3\pi^-$ | 37.2 | 0 | 18.6 | 2.1 ± 0.2 | 1.9 ± 0.2 |
| $3\pi^+3\pi^-\pi^0$ | 0 | 10.5 | 5.2 | 1.9 ± 0.2 | 1.6 ± 0.2 |
| $n\pi^0, n > 1$ | 14 | 36 | 25 | 4.1 ± 0.4 | 3.3 ± 0.2 |
| $\pi^+\pi^-n\pi^0, n > 1$ | 16.8 | 30 | 23 | 35.8 ± 0.8 | 34.5 ± 1.2 |
| $2\pi^+2\pi^-n\pi^0, n > 1$ | 20.9 | 16 | 18.5 | 20.8 ± 0.7 | 21.3 ± 1.1 |
| $3\pi^+3\pi^-n\pi^0, n > 1$ | 3 | 1.2 | 2.1 | 0.3 ± 0.1 | 0.3 ± 0.1 |
| $\pi, \sigma$ | 6.22, 0.83 | 6.4, 0.95 | | 5, 1 |

Table 5: Branching ratios, in percent, for proton antiproton annihilation at rest, calculated from pions only, with experimental pion momentum spectrum. Data are compared with experiments from [12]. We show each total isospin channel calculated separately. The “combined” column corresponds to equal mixture of $I = 0$ and $I = 1$. In the last row we list the average and standard variation in the number of pions.
Figure 1: Pion field configuration $F$ as a function of $r$ and $t$.

Figure 2: $\rho = G/gr$ (in units of vector meson mass $m$) field as a function of $r$ and $t$.

Figure 3: $\omega_r$ (in units of vector meson mass $m$) field as a function of $r$ and $t$.

Figure 4: $u = r\omega_0$ field as a function of $r$ and $t$.

Figure 5: Total energy density multiplied by $4\pi r^2$ (in units of MeV/fm) as a function of $r$ and $t$.

Figure 6: Pion energy density multiplied by $4\pi r^2$ (in units of MeV/fm) as a function of $r$ and $t$.

Figure 7: Energy density associated with $\rho$ multiplied by $4\pi r^2$ (in units of MeV/fm) as a function of $r$ and $t$.

Figure 8: Energy density associated with $\omega_r$ multiplied by $4\pi r^2$ (in units of MeV/fm) as a function of $r$ and $t$.

Figure 9: Energy density associated with $\rho$ self-interaction multiplied by $4\pi r^2$ (in units of MeV/fm) as a function of $r$ and $t$.

Figure 10: Energy density associated with $\pi\rho$ interaction multiplied by $4\pi r^2$ (in units of MeV/fm) as a function of $r$ and $t$.

Figure 11: Energy density associated with $\pi\omega$ interaction multiplied by $4\pi r^2$ (in units of MeV/fm) as a function of $r$ and $t$.

Figure 12: Energy density associated with $\pi\rho\omega$ interaction multiplied by $4\pi r^2$ (in units of MeV/fm) as a function of $r$ and $t$. 
Figure 13: Pion momentum distribution. The horizontal axis is the pion momentum. The vertical axis is the pion momentum distribution function, $4\pi k^2 |f(k)|^2$, see (27). The area under the curve gives the total direct pion number $N_\pi$. The data is from [12].

Figure 14: Rho momentum distribution. The horizontal axis is the rho momentum. The vertical axis is the rho momentum distribution function, $4\pi k^2 |g(k)|^2$, see (28). The area under the curve gives the total omega number $N_\rho$.

Figure 15: Omega momentum distribution. The horizontal axis is the omega momentum. The vertical axis is the rho momentum distribution function, $4\pi k^2 |h(k)|^2$, see (29). The area under the curve gives the total omega number $N_\omega$.

Figure 16: Pion number distribution in $I = 0$ channel. Solid squares are given by the distribution $P_n$ (53). Circles are the gaussian distribution with the same mean and variance.

Figure 17: Pion number distribution in $I = 1$ channel. Solid squares are given by the distribution $P_n$ (53). Circles are the gaussian distribution with the same mean and variance.