N = 1 Super Yang-Mills on the Lattice in the Strong Coupling Limit

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We study the $N = 1$ supersymmetric $SU(N_c)$ Yang-Mills theory on the lattice at strong coupling. We analyse and discuss the recent results obtained at strong coupling and large $N_c$ for the mesonic and fermionic propagators and spectrum.

1. Introduction

The pure $N = 1$ SUSY Yang-Mills (SYM) theory with $SU(N_c)$ gauge group is described by the purely gluonic action plus one flavour of Majorana fermions in the adjoint representation of the colour group.

The non-perturbative aspects of this class of theories were intensely investigated in the past, and recently there has been a renewed interest on the subject. On the other hand, these theories can be studied on the lattice by using the standard lattice methods. However, when supersymmetric gauge theories are formulated on the lattice, some problems arise due to the fact that the lattice regularization spoils supersymmetry. Nevertheless, in Ref. it is shown how to recover supersymmetry in the continuum limit by using appropriately renormalized operators for the SUSY and chiral currents.

Two different collaborations have carried out numerical analysis of the SYM on the lattice (quenched and unquenched respectively) by following the guidelines of Ref. The main reason for performing an unquenched analysis from the beginning is that in the SYM theory the quenched approximation cannot be justified (in the continuum limit) on the basis of large $N_c$ dominance. However, in Ref. it is shown that the results on the spectrum, obtained by employing the quenched approximation, match the supersymmetry predictions within their statistical errors.

Our work aims at complementing these numerical studies with analytical information on the strong coupling, large $N_c$ region of the theory. While not being the physical (i.e. continuum) regime, this limit allows to extract relevant information on the dynamics of the lattice theory, and supplies a guideline for Monte Carlo simulations.

Leaving supersymmetry aside, our results can be regarded as an analysis of the spectrum of Yang-Mills fields coupled to Majorana quarks in the adjoint representation of the group. The large $N_c$, strong coupling analysis reported in Refs. was performed in arbitrary space-time dimension. In the present paper, we summarize the main results of Ref.

2. Strong Coupling

We begin by writing down the lattice action we will be working with:

$$S = \beta S_g + \frac{1}{2} \Psi_i \Psi_j M_{ij} ,$$

(1)

where $\beta S_g$ is the pure gauge part and $\Psi_i$ is a Grassman variable representing the field of a Majorana fermion. Note that the indices $i$ and $j$ run over space-time, colour and Dirac indices. The matrix $M$ must be antisymmetric and its form (in compact notation) is given by

$$M_{ij} = C \left[ I - \kappa \sum_{\alpha} \delta_{m,n+\alpha} U_{\alpha}(n) \left( r I - \gamma_{\alpha} \right) \right]$$

(2)

where $I$ and $C$ are the unit and charge conjugation matrices respectively, $r$ is the Wilson parameter, $\kappa$ is the hopping parameter and $n,m$ parametrize lattice points. The index $\alpha$ labels the 2d possible nearest neighbour steps and their corresponding lattice vectors. For forward steps
Our specific aim is to compute the following quantities:

\[ G_{ij}(x-y) \equiv \langle O_i(x)O_j(y) \rangle, \quad (4) \]

where as usual the \( \langle \rangle \) means vacuum expectation value. We compute the main formulas (which are valid at large \( N_c \) and in any dimension \( d \)) for the propagators of the \( p \)-gluino operators \( G_{ij}(x) \) obtained after resumming over paths. The main result for propagators is:

\[ G_{ij}(x) = R_p(\xi) \prod_\mu \left( \int \frac{d\varphi_\mu}{2\pi} \right) e^{\varphi_\mu x} \times \langle S_i | [\Theta_p(\xi)I - \hat{A}_p(\varphi)]^{-1}\bar{C}_p^{-1}|S_j \rangle, \quad (5) \]

where \( I \) is the unit matrix and

\[ \hat{C}_p \equiv C \otimes C \cdots \otimes C, \quad \hat{A}_p(\varphi) \equiv \kappa^p \sum_{\alpha \in I} e^{e_{\alpha\beta}} (r - \gamma_\alpha) \otimes \cdots \otimes (r - \gamma_\alpha) \]

\[ \Theta_p(x) \equiv (1 - x)^p + \frac{x^p}{(2d - 1)^{p-1}}, \quad (\xi) \equiv 1 - \sqrt{1 - 4(2d - 1)\kappa^2(x^2 - 1)} \frac{2}{2}, \quad (6) \]

and where \( C \) is the charge conjugation matrix. The reader is referred to Ref. [6] for the expressions of the functions \( R_{2,3}(x) \), as well as for the definition of the kets \( |S_i \rangle \).

In order to obtain analytical formulas for the propagator and masses, one needs to invert the matrix \( \Theta_p(\xi)I - \hat{A}_p(\varphi) \) appearing in Eq.(5). This can be done in an elegant way for arbitrary spacetime dimension and \( p = 2 \) by mapping the problem into a finite dimensional many-body fermion problem (\( \gamma \)-fermions [6]).

The results for the scalar mass \( M_S \) and the lightest pseudoscalar mass \( M_P \), which would enter the lowest supermultiplet according to the analysis of Veneziano and Yankielowicz [6], are, for any even dimension, the following:

\[ \cosh(M_S) = |\Phi_2(\xi) \pm (d-1)| \]

\[ \cosh(M_P) = \theta \Xi - H \]

\[ H = \sqrt{\theta^2 - 1}(\Xi^2 - 1) + (d-1)^2\epsilon^2 \]

\[ \Phi_2(x) = \frac{(1-x)^2(2d-1) + x^2}{2x(1-x)} \]

\[ \Xi = \Phi_2(\xi) - (d-1)^2\epsilon, \quad (7) \]

where the \( \pm \) in \( \cosh(M_S) \) is for \( |r| < 1 \) and \( |r| > 1 \) respectively, and \( \epsilon \equiv 1/(r^2 - 1) \), \( \theta \equiv \epsilon(r^2 + 1) \).
The corresponding critical line (where the lightest meson becomes massless) marks the edge of the physical region as well as the boundary of validity of our formulae. The equation for this critical line is

$$
\Phi_2(\xi) = d\theta .
$$

(8)

In odd dimensions (where the pseudoscalar operator does not exist) the lightest state in the 2-gluino sector is a vectorial state and its corresponding mass is given by

$$
\cosh(M_V) = |\Phi_2(\xi) - \theta(d-1)| .
$$

(9)

The obtention of general expressions for the spectrum valid in arbitrary dimension for \( p > 2 \) is in general a complicated task. We were able to do it only in some special cases. For example, for \( p = 3 \) with a completely antisymmetric spin tensor one gets a spin \( 1/2 \) state whose mass is given by:

$$
cosh(M_3) = \left| \sqrt{\Xi_3} - r \xi - \Theta_3 \right|,
$$

$$
\Xi_3 = \frac{\Theta_3(\xi) e^2}{2\kappa^3} - r\xi \sigma,
$$

$$
\Theta_3(x) \equiv (1-x)^3 + \frac{x^3}{2d-1},
$$

$$
\sigma = \sum_i \cos(\varphi_i = 0, \pi).
$$

(10)

Finally, we found that, in the \((\kappa, r)\) plane, there is a special limit where all the states in each \( p \)-gluino sector become degenerate. This limit is obtained by letting \( r \to \infty \) and \( \kappa \to 0 \) in such a way that the product \( \kappa r \) is fixed. In this limit the spin-independent \( p \)-gluino mass is given by:

$$
\cosh(M_p) = \Phi \left( \left( \frac{(2d-1)(1-\xi)}{\xi} \right)^{p/2} \right) - \sigma,
$$

$$
\Phi(x) = \frac{1}{2} \left( x + \frac{(2d-1)}{x} \right).
$$

(11)

Masses increase with \( p \).

4. Conclusions

The main results of our work are:

- The pseudoscalar meson is the lightest meson for even space-time dimensions. For odd dimensions the lightest state is the vector meson. The vanishing of their lattice masses marks the critical line.

- The only multicritical point where several meson masses vanish corresponds to the point \( \kappa \to 0 \), \( r \to \infty \) with \( \kappa r = \frac{1}{2\sqrt{d-1}} \).

- For the 3 and 4-dimensional cases, there are no multicritical lines with vanishing 3-gluino masses. The masses of the 3-gluino fermionic states are always positive within the physical region of the \((\kappa, r)\) plane.

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