The entropy of rough neutrosophic multisets

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Abstract. The entropy of rough neutrosophic multisets is introduced to measure the fuzziness degree of rough multisets information. The entropy is defined in two ways, which is the entropy of rough neutrosophic multisets generalize from existing entropy of single value neutrosophic set and the rough neutrosophic multisets entropy based on roughness approximation. The definition is derived from being satisfied in the following conditions required for rough neutrosophic multisets entropy. Note that the entropy will be null when the set is crisp, while maximum if the set is a completely rough neutrosophic multiset. Moreover, the rough neutrosophic multisets entropy and its complement are equal. Also, if the degree of lower and upper approximation for truth membership, indeterminacy membership, and falsity membership of each element decrease, then the sum will decrease. Therefore, this set becomes fuzzier, causing the entropy to increase.

Keywords: entropy; rough neutrosophic multisets; roughness approximation.

1. Introduction
The information of independent opinion given by the experts always consists of differences due to the uncertainty condition. This may result in the agreed and disagreed discussion between them. The fuzzy set theory (FS) [1] is introduced as a solution to this discussion by representing a fuzzy system with values denoted from 0 to 1. The value for the degree of membership function for an agreed group and a disagreed group is that an absolute agree value is 1.0 while an absolute disagreed value is 0.0. Moreover, the intuitionistic fuzzy set theory (IFS) [2] as a generalization of FS also gives an opinion to this argument. Other than the degree of membership function, a non-membership function exists, such as absolute agree value for membership function is 1.0, and absolute agree for non-membership function is 0.0. This theory clarifies the expert when involving the truth (membership function) and falsity (non-membership function) condition. Then, the neutrosophic set theory (NS) [3] covers up all the opinion by introducing a new argument between the prior, which is a neutral opinion, where the opinion either considers membership function or non-membership function between the two expert discussion.
The triple set element is introduced for NS, representing a degree of truth membership function, a degree of indeterminacy (neutral) membership function, and falsity (non-membership function). The FS, IFS, and NS theories are motivation arising from the uncertainty condition to solve the vagueness, imprecision, ambiguity and inconsistency from data and information collection. Therefore, the neutrosophic set theory takes a novel approach to solve the ambiguity between the expert information of opinion. Next, a refined neutrosophic set theory [4] was introduced to solve the multiple information regarding the neutrosophic set condition. Then, the extended operation for the refined neutrosophic set was studied, introducing the single-valued neutrosophic multiset [5].

After a decade, a hybrid theory using neutrosophic set theory and neutrosophic multisets theory is gaining more attention. The advantage of this theory is having a natural extension knowledge concept of fuzzy set theory, fuzzy multisets theory, intuitionistic fuzzy set theory, and intuitionistic fuzzy multisets theory such as the relation, distance, similarity, and entropy. Hence, a rough neutrosophic multisets theory [6] is introduced by the developing of neutrosophic set theory. The rough neutrosophic multisets is a generalization of a rough set by Pawlak’s [7] and neutrosophic multisets [5]. This theory discusses the neutrosophic multisets in terms of lower and upper approximations in equivalence relation. The lower approximation gives a sign of surely belonging information, while the upper approximation gives a sign of possibly belonging information. All the properties for both approximations follow the natural concept of a rough set, while neutrosophic multisets properties represent the multiple information. Some of the properties are motivated by rough neutrosophic set [8].

Besides information evaluation, rough neutrosophic multisets also measure information similarity, aggregate the information, and formulate the fuzziness of information by entropy measure. The discussion of this information evaluation is a well-known study in the literature. Therefore, this paper discusses the entropy measure, where it is an important concept for information evaluation. Some literature for fuzzy entropies [9–11] and neutrosophic set entropy [12–18] are used for references. According to all the literature, a study for rough neutrosophic multisets entropy is not yet explored. This becomes the novelty for this paper to introduce the rough neutrosophic multisets entropy to evaluate the information fuzziness in a rough neutrosophic multisets environment.

As a generalization of the fuzzy set, if the entropy value is smaller, then the information provided is more useful. Therefore, this paper aims to quantify the number of fuzziness measures by defining the rough neutrosophic multisets entropy based on the natural extension of the fuzzy set. The next objective is to show the effect of roughness for a lower and upper approximation of multiple values of neutrosophic multisets by defining the rough neutrosophic multisets entropy based on roughness approximation.

This paper is presented in four sections. The first section is an overview of the rough neutrosophic multisets environment with entropy literature. Next, section two recalls the definition of rough neutrosophic multisets with some of the operations used. Then, section three defines the two new entropy definitions for rough neutrosophic multisets based on the natural extension of fuzzy set and roughness approximation. All the entropy conditions are successfully proven in this section. Lastly, the conclusion, as in section four, concludes the novelty of this research paper.

2. Preliminaries
This section recalls the definition of rough neutrosophic multisets and some of the operations, the entropy of the neutrosophic set, and the accuracy and roughness measure of Pawlak’s approximation.

Definition 1 [6]. Let \( U \) be a non-null set with the generic elements in \( U \) denoted by \( x_j \) and \( R \) be an equivalence relation in \( U \). Let

\[
A = \left\{ \left( T_A^1(x_j), T_A^2(x_j), \ldots, T_A^q(x_j) \right), \left( I_A^1(x_j), I_A^2(x_j), \ldots, I_A^q(x_j) \right), \left( F_A^1(x_j), F_A^2(x_j), \ldots, F_A^q(x_j) \right) \right\}
\]

be neutrosophic multisets in \( U \) with the truth-membership sequence \( T_A^1, T_A^2, \ldots, T_A^q \), indeterminacy-membership sequences \( I_A^1, I_A^2, \ldots, I_A^q \) and falsity-membership sequences \( F_A^1, F_A^2, \ldots, F_A^q \). The lower and
the upper approximations of A in the approximation \((U,R)\) denoted by \(\text{Nm}(A)\) and \(\overline{\text{Nm}}(A)\) are respectively defined as follows:

\[
\text{RNM}(A) = \left( \text{Nm}(A), \overline{\text{Nm}}(A) \right)
\]

\[
= \left\{ x_i, \left( \frac{T^i_{\text{Nm}}(x_i), f^i_{\text{Nm}}(x_i), I^i_{\text{Nm}}(x_i)}{T^i_{\overline{\text{Nm}}}(x_i), f^i_{\overline{\text{Nm}}}(x_i), I^i_{\overline{\text{Nm}}}(x_i)} \right) \Bigg| y \in [x_i]_R, i, j \in \mathbb{Z}^+, x_j \in U \right\}.
\]

(1)

The truth membership sequences \(T^i_{\text{Nm}}(x_i), T^i_{\overline{\text{Nm}}}(x_i)\), indeterminate membership sequence \(I^i_{\text{Nm}}(x_i), I^i_{\overline{\text{Nm}}}(x_i)\) and falsity membership sequence \(f^i_{\text{Nm}}(x_i), f^i_{\overline{\text{Nm}}}(x_i)\) for lower and upper approximations of RNM may be in decreasing or increasing order.

**Definition 2 [6].** Let \(\text{RNM}(A) = \left( \text{Nm}(A), \overline{\text{Nm}}(A) \right)\) be a rough neutrosophic multisets in \((U,R)\). The rough complement of \(\text{RNM}(A)\) is denoted by \(\sim \text{RNM}(A) = \left( \text{Nm}(A)^c, \overline{\text{Nm}}(A)^c \right)\) where \(\text{Nm}(A)^c\) and \(\overline{\text{Nm}}(A)^c\) are the complements of neutrosophic multisets of \(\text{Nm}(A)\) and \(\overline{\text{Nm}}(A)\) respectively, given by

\[
\text{Nm}(A)^c = \left\{ x_j, \left( f^j_{\text{Nm}}(x_i), 1 - f^j_{\text{Nm}}(x_i), T^j_{\text{Nm}}(x_i) \right) \right\} \bigg| y \in [x_j]_R, i, j \in \mathbb{Z}^+, x_j \in U \bigg\},
\]

\[
\overline{\text{Nm}}(A)^c = \left\{ x_j, \left( f^j_{\overline{\text{Nm}}}(x_i), 1 - f^j_{\overline{\text{Nm}}}(x_i), T^j_{\overline{\text{Nm}}}(x_i) \right) \right\} \bigg| y \in [x_j]_R, i, j \in \mathbb{Z}^+, x_j \in U \bigg\}.
\]

(2)

**Definition 3 [19].** For a subset of object \(X \subseteq U\), the accuracy measure is defined as:

\[
\alpha_E(X) = \frac{\text{apr}_E(x)}{|\text{apr}_E(x)| + |\overline{\text{apr}}_E(x)|},
\]

(3)

where \(X\) is a non-empty set, \(E \in X\), \(\text{apr}_E(x)\) is the lower approximation of set \(E\), \(\overline{\text{apr}}_E(x)\) is an upper approximation of set \(E\), \(||\) denotes the cardinality of set \(E\), and \(0 \leq \alpha_E(X) \leq 1\). Based on the accuracy measure, the roughness measure is defined as:

\[
\rho_E(X) = 1 - \alpha_E(X).
\]

(4)

**Definition 4 [12].** An entropy on single value neutrosophic set \(SVNS(X)\) is a function \(E(A) : SVNS(X) \to [0,1]\), satisfying all the following conditions:

(E1) \(E(A) = 0\) if \(A\) is a crisp set,

(E2) \(E(A) = 1\) if \((x_i, 0.5, 0.5, 0.5)|x_i \in X\),

(E3) \(E(A) = E(A)^c\), for all \(A \in SVNS(X)\),

(E4) \(E(A) \geq E(B)\) for all \(A, B \in SVNS(X)\) satisfying \(T_A(x_i) + f_A(x_i) \leq T_B(x_i) + f_B(x_i)\), and \(|I_A(x_i) - I_A^c(x_i)| \leq |I_B(x_i) - I_B^c(x_i)|\) for all \(x_i \in X\).

Then,

\[
E(A) = 1 - \frac{1}{n} \sum_{i=1}^{n} [T_A(x_i) + f_A(x_i)] [I_A(x_i) - I_A^c(x_i)].
\]

(5)

Definition 4 has a limit on the use of entropy in condition (E4). This entropy is questionable and well discussed in [17]. But this entropy has an advantage because it still follows the accurate extension of the concept of fuzzy sets and intuitionistic fuzzy set where the complement of a single valued neutrosophic set is \(A^c = \{(F_A(x_i), 1 - I_A(x_i), T_A(x_i)|x_i \in X}\).

**Definition 5 [17].** Entropy on A single value neutrosophic set \(SVNS(X)\) is a function \(E(A) : SVNS(X) \to [0,1]\), satisfying all the following conditions, given by

(E1) \(E(A) = 0\) if \(A\) is a crisp set,

(E2) \(E(A) = 1\) if \((x_i, 0.5, 0.5, 0.5)|x_i \in X\),

(E3) \(E(A) = E(A)^c\), for all \(A \in SVNS(X)\),
\( (E4') E(A) \leq E(B) \) if either \( T_A(x_i) \leq T_B(x_i), I_A(x_i) \leq I_B(x_i), F_A(x_i) \leq F_B(x_i) \) when \( \max \{T_B(x_i), I_B(x_i), F_B(x_i)\} \leq 0.5 \) or \( T_A(x_i) \geq T_B(x_i), I_A(x_i) \geq I_B(x_i), F_A(x_i) \geq F_B(x_i) \) when \( \min \{T_B(x_i), I_B(x_i), F_B(x_i)\} \geq 0.5 \).

Then, \[E(A) = 1 - \frac{1}{n} \sum_{i=1}^{n} \left[ I_A(x_i) - 0.5 \right] + T_A(x_i) - 0.5 + T_A(x_i) - 0.5 + I_A(x_i) - 0.5 + I_A(x_i) - 0.5. \] \( \text{(6)} \)

Definition 4 proposed in [17] discusses the comparison of the entropy concept for a neutrosophic set.

3. The entropy of Rough Neutrosophic Multisets

In this section, the entropy of Rough Neutrosophic Multisets (RNM) is introduced to measure the fuzziness degree of RNM information. The entropy of RNM is defined in two ways, the entropy of RNM generalizes from the existing entropy of a single value neutrosophic set and the RNM entropy based on roughness approximation.

The definition is derived by satisfying the following conditions, required for RNM entropy:

(i) The entropy will be null when the set is crisp,

(ii) The entropy will be maximum if the set is completely RNM,

(iii) The RNM entropy and its complement is equal, and

(iv) Suppose the degree of lower and upper approximations for truth membership, indeterminacy membership, and falsity membership of each element decreases. In that case, the sum decreases as well. Therefore, this set becomes fuzzier, increasing the entropy.

Given the condition stated, the definition of RNM entropy is defined as follows:

**Definition 6.** An entropy on rough neutrosophic multisets is a function \( E(A): RNM(X) \to [0, 1] \)
satisfying all following conditions:

(E0) (Nonnegativity) \( 0 \leq E(A) \leq 1 \);

(E1) (Minimality) \( E(A) = 0 \) if \( A \) is a crisp set; i.e.
\[
\begin{align*}
T_{NM(A)}^{i}(x_j), T_{NM(A)}^{l}(x_j) & = [1, 1], \left[I_{NM(A)}^{q}(x_j), I_{NM(A)}^{l}(x_j)\right] = [0, 0], \\
F_{NM(A)}^{l}(x_j), F_{NM(A)}^{r}(x_j) & = [0, 0], \text{ or } \left[T_{NM(A)}^{r}(x_j), T_{NM(A)}^{l}(x_j)\right] = [0, 0], \\
I_{NM(A)}^{r}(x_j), I_{NM(A)}^{l}(x_j) & = [0, 0], \left[F_{NM(A)}^{q}(x_j), F_{NM(A)}^{l}(x_j)\right] = [1, 1] \text{ for all } x_j \in X.
\end{align*}
\]

(E2) (Maximality) \( E(A) = 1 \) if \( \left[T_{NM(A)}^{r}(x_j), I_{NM(A)}^{q}(x_j), F_{NM(A)}^{l}(x_j)\right] = [0.5, 0.5, 0.5] \), and
\[
\left[T_{NM(A)}^{l}(x_j), I_{NM(A)}^{l}(x_j), F_{NM(A)}^{r}(x_j)\right] = [0.5, 0.5, 0.5], \text{ for all } x_j \in X.
\]

(E3) (Symmetric) \( E(A) = E(A)^{c} \), for all \( A \in RNM(X) \);

(E4) (Resolution) \( E(A) \leq E(B) \) if either \( NM(A) \leq NM(B) \) and \( \overline{NM}(A) \leq \overline{NM}(B) \); i.e.
\[
\begin{align*}
T_{NM(A)}^{i}(x_j) & \leq T_{NM(B)}^{i}(x_j), \quad T_{NM(A)}^{l}(x_j) \leq T_{NM(B)}^{l}(x_j), \quad I_{NM(A)}^{r}(x_j) \leq I_{NM(B)}^{r}(x_j), \\
I_{NM(A)}^{l}(x_j) & \leq I_{NM(B)}^{l}(x_j), \quad F_{NM(A)}^{l}(x_j) \leq F_{NM(B)}^{l}(x_j), \quad F_{NM(A)}^{r}(x_j) \leq F_{NM(B)}^{r}(x_j), \quad F_{NM(A)}^{r}(x_j) \leq F_{NM(B)}^{r}(x_j).
\end{align*}
\]

Now notice that in RNM, the present uncertainty is due to the factors of possibly belongingness and surely belongingness, representing lower and upper approximations of RNM. Considering these factors, two types of entropy measure \( E_{RNM}(A) \) and \( E_{DRNM}(A) \) of rough neutrosophic multisets, \( A \) are defined as follows.

3.1. Rough Neutrosophic Multisets entropy based on the natural extension of fuzzy set concept

The concept of complement for single value neutrosophic set (SVNS) is based on the natural extension of fuzzy set and intuitionistic fuzzy set where \( A^{c} = \{(F_A(x_i), 1 - I_A(x_i), T_A(x_i))|x_i \in X\} \). Therefore, this complement concept is also derived for rough neutrosophic multisets (RNM) complement as in equation (2). The complement concept is used to derive a new definition for RNM entropy based on the natural extension of a fuzzy concept. Following Definition 4, it is derived as follows:
\[
|I_{A}(x_i) - I_{A^c}(x_i)| = |I_{A}(x_i) - (1 - I_{A}(x_i))| = 2I_{A}(x_i) - 1.
\]

Then, the RNM entropy based on the natural extension of the fuzzy set concept is derived as follows:

**Definition 7.** An entropy \( E_{RNM}(A) \) on rough neutrosophic multisets is a function \( E(A): RNM(X) \to [0, 1] \) satisfying given conditions in Definition 6. Then

\[
E_{RNM}(A) = 1 - \frac{1}{2pq} \sum_{i=1}^{p} \sum_{j=1}^{q} \left( \left( T_{N_{NM}(A)}^{i}(x_j) + F_{N_{NM}(A)}^{i}(x_j) \right) \cdot \left| 2I_{N_{NM}(A)}^{i}(x_j) - 1 \right| + \left( T_{N_{NM}(A)}^{i}(x_j) + F_{N_{NM}(A)}^{i}(x_j) \right) \cdot \left| 2I_{N_{NM}(A)}^{i}(x_j) - 1 \right| \right),
\]

where;

\[
\left[ T_{N_{NM}(A)}^{i}(x_j), T_{N_{NM}(A)}^{i}(x_j) \right] \text{ is a truth membership sequence, } \left[ I_{N_{NM}(A)}^{i}(x_j), I_{N_{NM}(A)}^{i}(x_j) \right] \text{ is an indeterminate membership sequence, and } \left[ F_{N_{NM}(A)}^{i}(x_j), F_{N_{NM}(A)}^{i}(x_j) \right] \text{ is a falsity membership sequence, such that } T_{N_{NM}(A)}^{i}(x_j), I_{N_{NM}(A)}^{i}(x_j), F_{N_{NM}(A)}^{i}(x_j) \in [0, 1], T_{N_{NM}(A)}^{i}(x_j), I_{N_{NM}(A)}^{i}(x_j), F_{N_{NM}(A)}^{i}(x_j) \in [0, 1]; \]

\[
0 \leq T_{N_{NM}(A)}^{i}(x_j) + I_{N_{NM}(A)}^{i}(x_j) + F_{N_{NM}(A)}^{i}(x_j) \leq 3, \quad 0 \leq T_{N_{NM}(A)}^{i}(x_j) + I_{N_{NM}(A)}^{i}(x_j) + F_{N_{NM}(A)}^{i}(x_j) \leq 3, \text{ for } i = 1, 2, ..., p \text{ and } j = 1, 2, ..., q.
\]

We now consider proving all the conditions satisfied Definition 6.

**Proof:**

(E1) If \( A \) is a crisp set then

\[
\left[ T_{N_{NM}(A)}^{i}(x_j), T_{N_{NM}(A)}^{i}(x_j) \right] = [1, 1], \left[ I_{N_{NM}(A)}^{i}(x_j), I_{N_{NM}(A)}^{i}(x_j) \right] = [0, 0], \left[ F_{N_{NM}(A)}^{i}(x_j), F_{N_{NM}(A)}^{i}(x_j) \right] = [0, 0].
\]

(E2) \( E_{RNM}(A) = 1 - \frac{1}{2} \left( [(1 + 0), 2(0 - 1)] + [(1 + 0), 2(0 - 1)] \right) = 1 - \frac{1}{2}(1 + 1) = 0, \text{ or } E_{RNM}(A) = 1 - \frac{1}{2} \left( [(0 + 1), 2(0 - 1)] + [(0 + 1), 2(0 - 1)] \right) = 1 - \frac{1}{2}(1 + 1) = 0
\]

Therefore, the RNM entropy will be null \( E_{RNM}(A) = 0 \) when the set is crisp.

(E3) \( E_{RNM}(A) = 1 - \frac{1}{2pq} \sum_{i=1}^{p} \sum_{j=1}^{q} \left( \left( T_{N_{NM}(A)}^{i}(x_j) + F_{N_{NM}(A)}^{i}(x_j) \right) \cdot \left| 2I_{N_{NM}(A)}^{i}(x_j) - 1 \right| + \left( T_{N_{NM}(A)}^{i}(x_j) + F_{N_{NM}(A)}^{i}(x_j) \right) \cdot \left| 2I_{N_{NM}(A)}^{i}(x_j) - 1 \right| \right),
\]

Therefore, the entropy will be maximum \( E_{RNM}(A) = 1 \) if the set is completely RNM.
Therefore, RNM entropy and its complement is equal, where \( E_{RNM}(A) = E_{RNM}(A)^C \), for all \( A \in RNM(X) \);

\[(E4) \quad E_{RNM}(A) \leq E_{RNM}(B) \text{ if either } Nm(A) \leq Nm(B) \text{ and } Nm(A) \leq Nm(B) ; \text{ i.e. } \]

\[ T^i_{Nm(A)}(x_j) \leq T^i_{Nm(B)}(x_j), \quad T^i_{Nm(A)}(x_j) \leq T^i_{Nm(B)}(x_j), \quad I^i_{Nm(A)}(x_j) \leq I^i_{Nm(B)}(x_j), \]

\[ F^i_{Nm(A)}(x_j) \leq F^i_{Nm(B)}(x_j) \text{ and } F^i_{Nm(A)}(x_j) \leq F^i_{Nm(B)}(x_j). \]

The following relation is obtained:

\( a) \quad T^i_{Nm(A)}(x_j) + F^i_{Nm(A)}(x_j) \leq T^i_{Nm(B)}(x_j) + F^i_{Nm(B)}(x_j), \)

\( b) \quad 2I^i_{Nm(A)}(x_j) - 1 \leq 2I^i_{Nm(B)}(x_j) - 1 \)

Combining \( a) \) and \( b) \), yields,

\[ 1 - \frac{1}{2pq} \sum_{i=1}^{p} \sum_{j=1}^{q} \left[ \left( T^i_{Nm(A)}(x_j) + F^i_{Nm(A)}(x_j) \right) \right] \left[ 2I^i_{Nm(A)}(x_j) - 1 \right] \leq 1 - \frac{1}{2pq} \sum_{i=1}^{p} \sum_{j=1}^{q} \left[ \left( T^i_{Nm(B)}(x_j) + F^i_{Nm(B)}(x_j) \right) \right] \left[ 2I^i_{Nm(B)}(x_j) - 1 \right] \]

In other words, \( E_{RNM}(A) \leq E_{RNM}(B) \). Thus, the property \( (E4) \) is satisfied.

This proves that all the conditions in Definition 6 are satisfied, completing the proof. ■

3.2. Rough Neutrosophic Multisets entropy based on rough approximation

The accuracy and roughness measure of Pawlak’s approximation gives a motivation to derive the rough neutrosophic multisets entropy. The roughness approximation in equation 4 for truth membership sequence, indeterminate membership sequence, and falsity membership sequence is used simultaneously in RNM entropy.

**Definition 8.** An entropy \( E_{RNM}(A) \) on rough neutrosophic multisets is a function \( E(A): RNM(X) \to [0,1] \) satisfying the conditions in Definition 6. Then,

\[ E_{RNM}(A) = 1 - \frac{1}{2pq} \sum_{i=1}^{p} \left[ \left| \Delta T^i_{Nm(A)}(x_j) - 0.5 \right| + \left| \Delta I^i_{Nm(A)}(x_j) - 0.5 \right| + \left| \Delta F^i_{Nm(A)}(x_j) - 0.5 \right| \right] \]

where;

\( \Delta T^i_{Nm(A)}(x_j) \) is the roughness approximation for truth membership sequence, \( \Delta I^i_{Nm(A)}(x_j) \) is the roughness approximation for indeterminacy membership sequence, and \( \Delta F^i_{Nm(A)}(x_j) \) is the roughness approximation for falsity membership sequence, for RNM \( A \), such that,

\[ \Delta T^i_{Nm(A)}(x_j) = 1 - \frac{T^i_{Nm(A)}(x_j) + F^i_{Nm(A)}(x_j)}{|x|} \]

\[ \Delta I^i_{Nm(A)}(x_j) = 1 - \frac{I^i_{Nm(A)}(x_j) + F^i_{Nm(A)}(x_j)}{|x|} \]

\[ \Delta F^i_{Nm(A)}(x_j) = 1 - \frac{F^i_{Nm(A)}(x_j) + F^i_{Nm(A)}(x_j)}{|x|} \]

\[ \Delta I^i_{Nm(A)}(x_j)^C = 1 - \Delta I^i_{Nm(A)}(x_j) \]

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\[
\Delta T^i_{Nm(A)}(x_j), \Delta l^i_{Nm(A)}(x_j), \Delta F^i_{Nm(A)}(x_j), \Delta I^i_{Nm(A)}(x_j) \in [0, 1],
\]

\[
0 \leq \Delta T^i_{Nm(A)}(x_j) + \Delta l^i_{Nm(A)}(x_j) + \Delta F^i_{Nm(A)}(x_j) \leq 3, \text{ for } i = 1, 2, \ldots, p \text{ and } j = 1, 2, \ldots, q.
\]

Moreover, if \(X\) is a non-empty set, \(Nm(A)\) and \(\bar{Nm}(A)\) are a lower approximation and upper approximation of neutrosophic multisets \(A\), respectively, \((Nm(A)(x_j), \bar{Nm}(A)(x_j), 1 - l^i_{Nm(A)}(x_j), T^i_{Nm(A)}(x_j))\) is a complement set of upper approximation of neutrosophic multisets \(A\), and \([.]\) denotes the cardinality of a set \(A\).

We now consider proving all the conditions satisfied in Definition 6.

Proof:

(E1) If \(A\) is a crisp set then \([\Delta T^i_{Nm(A)}(x_j), \Delta l^i_{Nm(A)}(x_j), \Delta F^i_{Nm(A)}(x_j)] = [1, 0, 0]\) or \([\Delta T^i_{Nm(A)}(x_j), \Delta l^i_{Nm(A)}(x_j), \Delta F^i_{Nm(A)}(x_j)] = [0, 0, 1]\) for all \(x_j \in X\). Then,

\[
\Delta l^i_{Nm(A)}(x_j)^c = 1 - \Delta I^i_{Nm(A)}(x_j) = 1 - 0 = 1.
\]

Therefore,

\[
E_{\Delta RNM}(A) = 1 - \frac{1}{2}([0 - 0.5] + [0 - 0.5] + [1 - 0.5]) = 0,
\]

It follows that,

\[
E_{\Delta RNM}(A) = 1 - \frac{1}{2}([0 - 0.5] + [0 - 0.5] + [1 - 0.5] + [1 - 0.5]) = 0.
\]

Therefore, the RNM entropy will be null \((E_{\Delta RNM}(A) = 0)\) when the set is crisp.

(E2) \(E_{\Delta RNM}(A) = 1\) if \([\Delta T^i_{Nm(A)}(x_j), \Delta l^i_{Nm(A)}(x_j), \Delta F^i_{Nm(A)}(x_j)] = [0.5, 0.5, 0.5]\) for all \(x_j \in X\).

Then, \(E_{\Delta RNM}(A) = 1 - \frac{1}{2}([0.5 - 0.5] + [0.5 - 0.5] + [0.5 - 0.5] + [0.5 - 0.5]) = 1\).

Therefore, the entropy will be maximum \((E_{\Delta RNM}(A) = 1)\) if the set is completely RNM.

(E3) \(E_{\Delta RNM}(A) = 1 - \frac{1}{2p} \sum_{i=1}^{p} \left[|\Delta T^i_{Nm(A)}(x_j) - 0.5| + |\Delta F^i_{Nm(A)}(x_j) - 0.5| + |\Delta I^i_{Nm(A)}(x_j) - 0.5| + |\Delta I^i_{Nm(A)}(x_j) - 0.5| + |\Delta I^i_{Nm(A)}(x_j) - 0.5| + |\Delta I^i_{Nm(A)}(x_j) - 0.5| + \sum_{i=1}^{p} |\Delta F^i_{Nm(A)}(x_j) - 0.5|\right] = 1 - \frac{1}{2p} \sum_{i=1}^{p} \left[|\Delta F^i_{Nm(A)}(x_j) - 0.5|\right] = E_{\Delta RNM}(A)^c.

Therefore, RNM entropy and its complement is equal, where; \(E_{\Delta RNM}(A) = E_{\Delta RNM}(A)^c\), for all \(A \in RNM(X)\).

(E4) \(E_{\Delta RNM}(A) \leq E_{\Delta RNM}(B)\) if \(\Delta T^i_{Nm(A)}(x_j) \leq \Delta T^i_{Nm(B)}(x_j), \Delta l^i_{Nm(A)}(x_j) \leq \Delta l^i_{Nm(B)}(x_j), \Delta F^i_{Nm(A)}(x_j) \leq \Delta F^i_{Nm(B)}(x_j)\), and \(\Delta I^i_{Nm(A)}(x_j) \leq \Delta I^i_{Nm(B)}(x_j)\).

Then, we obtain the following relations:

a) \(|\Delta T^i_{Nm(A)}(x_j) - 0.5| \leq |\Delta T^i_{Nm(B)}(x_j) - 0.5|

b) \(|\Delta l^i_{Nm(A)}(x_j) - 0.5| \leq |\Delta l^i_{Nm(B)}(x_j) - 0.5|

c) \(|\Delta F^i_{Nm(A)}(x_j) - 0.5| \leq |\Delta F^i_{Nm(B)}(x_j) - 0.5|

d) \(|\Delta I^i_{Nm(A)}(x_j) - 0.5| \leq |\Delta I^i_{Nm(B)}(x_j) - 0.5|

Combining a), b), c) and d), we obtain

\[
1 - \frac{1}{2p} \sum_{i=1}^{p} \left[|\Delta T^i_{Nm(A)}(x_j) - 0.5| + |\Delta F^i_{Nm(A)}(x_j) - 0.5| + |\Delta I^i_{Nm(A)}(x_j) - 0.5| + |\Delta I^i_{Nm(A)}(x_j) - 0.5| + \sum_{i=1}^{p} |\Delta F^i_{Nm(B)}(x_j) - 0.5|\right] \leq 1 - \frac{1}{2p} \sum_{i=1}^{p} \left[|\Delta T^i_{Nm(B)}(x_j) - 0.5| + |\Delta F^i_{Nm(B)}(x_j) - 0.5| + \sum_{i=1}^{p} |\Delta F^i_{Nm(B)}(x_j) - 0.5|\right]
\]

In other words, \(E_{RNM}(A) \leq E_{RNM}(B)\). Thus, the property (E4) is satisfied.
This completes the proof. ■

3.3. Weighted Rough Neutrosophic Multisets entropy

If $w_j \in [0,1]$ be the weight of each element $x_j \in X$ for $j = 1,2,\ldots,q$ such that $\sum_1^q w_j = 1$, then the weighted RNM entropy for RNM $A$ is defined as follows:

**Definition 9.** Weighted entropy $E_{RNM}^w(A)$ on rough neutrosophic multisets is a function $E(A): RNM(X) \rightarrow [0,1]$ satisfying given condition in Definition 6, such that

$$E_{RNM}^w(A) = 1 - \frac{1}{2pq} \sum_{i=1}^p \sum_{j=1}^q w_j \left\{ \left( T_{Nm(A)}^i(x_j) + F_{Nm(A)}^i(x_j) \right) \cdot \left| 2I_{Nm(A)}^i(x_j) - 1 \right| \right\} + \left\{ \left( T_{Nm(A)}(x_j) + F_{Nm(A)}(x_j) \right) \cdot \left| 2I_{Nm(A)}(x_j) - 1 \right| \right\}.$$

(10)

**Definition 10.** Weighted entropy $E_{ARNM}^w(A)$ on rough neutrosophic multisets is a function $E(A): RNM(X) \rightarrow [0,1]$ satisfying given conditions in Definition 6, such that

$$E_{ARNM}^w(A) = 1 - \frac{1}{2pq} \sum_{i=1}^p \sum_{j=1}^q w_j \left\{ \left| \Delta T_{Nm(A)}^i(x_j) - 0.5 \right| + \left| \Delta F_{Nm(A)}^i(x_j) - 0.5 \right| + \left| \Delta I_{Nm(A)}^i(x_j) - 0.5 \right| + \left| \Delta I_{Nm(A)}(x_j)^C - 0.5 \right| \right\}.$$

(11)

Proof. Follows similarly from the proof of Definitions 7 and 8.

**Remark 11:** The rough neutrosophic multisets (RNM) is a hybrid uncertainty set theory of rough set and neutrosophic multisets. This theory is also a generalization of the rough neutrosophic set. Hence, the RNM entropy will generalize the following entropy satisfied by the conditions of Definition 6:

(1) The rough neutrosophic set entropy

When $i = 1$ for all element $T, I, F$ in Definitions 7 and 8, the entropy measure $E_{RNS}(A)$ and $E_{ARS}(A)$ of rough neutrosophic set $N(A) = (N(A), \overline{N}(A))$ are defined as follows:

$$E_{RNS}(A) = 1 - \frac{1}{2q} \sum_{j=1}^q \left\{ \left( T_{N(A)}(x_j) + F_{N(A)}(x_j) \right) \cdot \left| 2I_{N(A)}(x_j) - 1 \right| \right\} + \left\{ \left( T_{\overline{N}(A)}(x_j) + F_{\overline{N}(A)}(x_j) \right) \cdot \left| 2I_{\overline{N}(A)}(x_j) - 1 \right| \right\},$$

(12)

$$E_{ARS}(A) = 1 - \frac{1}{2q} \sum_{j=1}^q \left\{ \left| \Delta T_{N(A)}(x_j) - 0.5 \right| + \left| \Delta F_{N(A)}(x_j) - 0.5 \right| + \left| \Delta I_{N(A)}(x_j) - 0.5 \right| + \left| \Delta I_{\overline{N}(A)}(x_j)^C - 0.5 \right| \right\}.$$

(13)

(2) The rough intuitionistic set theory

When $i = 1$ for all elements $T$ and $F$, where $I$ is omitted in Definitions 7 and 8, the entropy measure $E_{IFS}(A)$ and $E_{AIFS}(A)$ of rough intuitionistic set $IF(A) = (IF(A), \overline{IF}(A))$ are defined as follows:

$$E_{IFS}(A) = 1 - \frac{1}{2q} \sum_{j=1}^q \left\{ \left( T_{IF(A)}(x_j) + F_{IF(A)}(x_j) \right) \right\} + \left\{ \left( T_{\overline{IF}(A)}(x_j) + F_{\overline{IF}(A)}(x_j) \right) \right\},$$

(14)

$$E_{AIFS}(A) = 1 - \frac{1}{2} \sum_{j=1}^q \left\{ \left| \Delta T_{IF(A)}(x_j) - 0.5 \right| + \left| \Delta F_{IF(A)}(x_j) - 0.5 \right| \right\}.$$

(15)

(3) The rough fuzzy set entropy

When $i = 1$ for element $T$, while elements $I$ and $F$ are omitted in Definitions 7 and 8, the entropy measure $E_{FS}(A)$ and $E_{AIFS}(A)$ of rough fuzzy set $FS(A) = (FS(A), \overline{FS}(A))$ are defined as follows:


\[ E_{FS}(A) = 1 - \frac{1}{2q} \sum_{j=1}^{q} \left[ (T_{FS(A)}(x_j) + T_{FS(A)}(x_j)) \right], \quad (16) \]

\[ E_{\Delta FS}(A) = 1 - \frac{1}{2} \left[ |\Delta T_{FS(A)}(x_j) - 0.5| \right]. \quad (17) \]

(4) The neutrosophic multisets entropy

When the membership sequence of \( T^i_{NM(A)}(x_j) = T^i_{NM(A)}(x_j) \), \( I^i_{NM(A)}(x_j) = I^i_{NM(A)}(x_j) \), and \( F^i_{NM(A)}(x_j) = F^i_{NM(A)}(x_j) \), the rough neutrosophic multisets is definable as neutrosophic multisets. Therefore, the entropy measure \( E'_NM(A) \) and \( E''NM(A) \) of neutrosophic multisets \( NM(A) \) are defined as follows:

\[ E'_NM(A) = 1 - \frac{1}{2pq} \sum_{i=1}^{p} \sum_{j=1}^{q} \left[ \left( T^i_{NM(A)}(x_j) + F^i_{NM(A)}(x_j) \right) \cdot |2I^i_{NM(A)}(x_j) - 1| \right], \quad (18) \]

\[ E''NM(A) = 1 - \frac{1}{2p} \sum_{i=1}^{p} \left[ T^i_{NM(A)}(x_j) - 0.5 \right] \cdot \left[ F^i_{NM(A)}(x_j) - 0.5 \right] \cdot \left[ |\Delta I^i_{NM(A)}(x_j) - 0.5| \right]. \quad (19) \]

4. Conclusion

The evaluation for measuring the entropy from the previous study and a new entropy introduced for the neutrosophic set motivate to define the rough neutrosophic multisets entropy. In this paper, two definitions for rough neutrosophic multisets entropy are successfully derived, satisfying entropy properties’ condition. All the conditions were proven in sections 3.1 and 3.2. Besides, the natural knowledge of fuzzy set entropy is also emphasized. The introduced rough neutrosophic multisets entropy overcame the limitation of previous results, especially for a set theory involving the lower and upper approximations. In the future, this measuring entropy definition will be used for the application framework to show the usefulness and validity to evaluate the information value in a rough neutrosophic multisets environment.

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