Logic programs for MIP generation

Finding the most probable (MAP) model in SRL frameworks such as Markov logic \cite{Richardson2006} and Problog \cite{Fierens2013} can, in principle, be solved by encoding the problem as a ‘grounded-out’ mixed integer program (MIP). However, useful first-order structure disappears in this process motivating the development of first-order MIP approaches. Here we present mfoilp, one such approach. Since the syntax and semantics of mfoilp is essentially the same as existing approaches \cite{Gordon2009, Kersting2014} we focus here mainly on implementation and algorithmic issues. We start with the (conceptually) simple problem of using a logic program to generate a MIP instance before considering more ambitious exploitation of first-order representations.

A MIP instance consists of variable and linear constraint declarations. It is straightforward to effect these declarations as a logic program. In the case of mfoilp this logic program is written in the Mercury language \cite{Somogyi1996} as follows.

Mercury programs are made up of modules. Mercury modules are composed of an interface and an implementation, where everything in a module’s implementation is hidden from other modules. Let the Mercury module defining a MIP instance be called prob.m. Its interface is as follows:

\[\] :- import_module prob.
\[\] :- type atom.
\[\] :- pred variable(atom:out) is multi.
\[\] :- pred constraint(lincons:out) is nondet.
\[\] :- func objective(atom) = float.
\[\] :- func lb(atom) = float.
\[\] :- func ub(atom) = float.
\[\] :- func vartype(atom) = vartype.

Terms of type atom correspond to MIP variables. This type is an abstract type; its definition, which the user has to provide in the module implementation, is hidden from other modules. The 4 functions objective, lb, ub and vartype have to provide, respectively, the objective coefficient, the lower bound, upper bound and MIP variable type (integer- or real-valued) for each atom. The predicate variable/1 is used to generate all MIP variables by Mercury’s equivalent of findall. For example, suppose the type atom were defined as follows

\[\] :- type protein ---> p1;p2.
\[\] :- type location_id ---> l1;l2.
\[\] :- type atom ---> location(protein,location_id) ; interaction(protein,protein).
\[\] then we could have:

\[\] variable(location(Protein,Location_id)) :-
\[\] protein(Protein), location_id(Location_id).
\[\] variable(interaction(Protein1,Protein2)) :-
\[\] protein(Protein1), protein(Protein2).
\[\]

where protein/1 and location_id/1 generate proteins and location ids, respectively.

Constraints are similarly straightforward. The lincons (linear constraint) type is defined in the mfoilp module as follows:

\[\] :- import_module prob.
\[\] :- type lterm ---> (float * atom).
\[\] :- type lexp == list(lterm).
\[\] :- type lb ---> finite(float) ; neginf.
\[\] :- type ub ---> finite(float) ; posinf.
\[\] :- type lincons ---> lincons(lb,lexp,ub).

So a linear term is just an atom together with its coefficient, and a linear constraint is just a list of such terms (representing their sum) bounded by a lower and upper bound. Absent bounds are represented by constants representing either positive or negative infinity. Note that the prob module must be imported so that the (abstract) type atom is available. The predicate constraint/1 should be defined so that all required constraints can be generated by findall. Fig. provides an example.

**mfoilp**

mfoilp consists of a C program (main.c), a Mercury program (mfoilp.m) and a Makefile. C code in main.c asks for (and receives) the list of variables and list of constraints from mfoilp.m which in turn gets them from prob.m
which defines the problem instance. It then calls the SCIP system to create and solve the MIP instance. The components of mfoilp are connected as follows.

SCIP - main.c - mfoilp.m - prob.m

Solving is invoked with make solution. This causes prob.m to be compiled (and linked) to generate a problem-specific executable. This executable is then run to solve the MIP.

### Branch-price-and-cut

A logic program is obviously a very flexible way of defining a MIP instance. However, the approach just outlined will fail with large numbers of variables and/or constraints since all have to be squeezed into memory prior to any solving. An alternative is to implement a branch-price-and-cut (BPC) approach to solving the MIP. Assume wlog that we are minimising. In BPC some initial variables and constraints are created as normal and the LP solution \( x^* \) computed, giving a lower bound on an optimal solution. Next a cutting plane algorithm looks for valid inequalities which \( x^* \) violates. If any are found they are added and a new LP solution with a tighter (higher) lower bound can be computed. In addition a pricer algorithm is run to look for variables which, if created, would produce a looser (lower) lower bound. Such variables are those with negative reduced cost, a quantity which can be computed from the variable’s objective coefficient and the solution to the dual LP. If the pricer can establish that there are no variables with reduced cost then we have a global lower bound without creating all possible variables. If we reach such a point and all integer variables happen to have integer values in the LP solution then the MIP is solved. Otherwise BPC branches on a variable to create sub-problems and applies BPC recursively until an guaranteed optimal solution is found.

It is easy to augment mfoilp with a predicate implementing a cutting plane algorithm:

```prolog
:- pred cut(lpsol::in, lincons::out) is nondet.

cut(LPSol,CP) :-
  constraint(CP),
  activity(LPSol,CP,Activity),
  violates_bounds(Activity,CP).
```

Mercury’s default execution algorithm processes goals left to right, so in this approach (1) a potential cutting plane is generated, (2) the value of its linear expression for the given LP solution (its activity) is computed and (3) the predicate succeeds if this value exceeds one of the constraint’s bounds.

Similarly, a pricer can be implemented as follows:

```prolog
:- pred price(duallpsol::in, variable::out) is nondet.

price(DualLPSol,Var) :-
  variable(Var),
  reduced_cost(DualLPSol,Var,RedCost),
  RedCost < 0.
```

Such an approach avoids the need to generate all variables and constraints ahead-of-time, since only those which affect the LP bound are generated. And we can leave SCIP to take care of branching. However, both cut/2 and price/2 are hopelessly inefficient—being pure generate-and-test.

An attractive alternative is to (1) take advantage of any special structure in variable/constraint definitions and (2) effect a source-to-source transformation similar to unfolding to produce more efficient code. For example, all the variables in the constraint in Figure 1 have positive coefficients, which can be exploited to automatically produce the following specialised cut/2 clause:

```prolog
cut(LPSol,CP) :-
  protein(P1), location_id(L1),
  get_val(LPSol,location(P1,L1),Val1),
  Val1 < 1,
  interaction(P1,P2), not P1 = P2,
  get_val(LPSol,interaction(P1,P2),Val2),
  Val1 + Val2 < 1.
CP = lincons( ... ). %lit abbreviated
```

mfoilp is currently being extended in this direction with a SCIP constraint handler and SCIP pricer which will call Mercury code to generate the required constraints and variables. (An earlier version of mfoilp called foz does implement cutting planes but does not use Mercury and has a naive cutting plane algorithm.)

### Related work

In mfoilp there is a bijection between MIP variables and first-order terms. However, these terms can be usefully thought of as atomic formula (and so the predicates become meta-predicates) in which case the syntax and semantics of mfoilp are basically the same as first-order programming (FOP) introduced by Gordon, Hong, and Dudík 2009 where “A MILP variable corresponds to a FOP ground atom”. Further work is needed to see how the approach to inference in FOP presented in Zawadzki, Gordon, and Platzer 2011 relates to the BPC method advocated here.

If all variables are continuous then an mfoilp instance is equivalent to a relational linear program (RLP) (Kersting, Mladenov, and Tokmakov 2014) which is a “declarative LP template defining the objective and the constraints through the logical concepts of objects, relations and quantified variables.” It follows that, even with integer variables, mfoilp should take advantage of the ‘lifted’ LP solving technique of Kersting et al since solving linear relaxations plays such a key role in MIP solving.
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