Decay Rate Ratios of $\Upsilon(5S) \rightarrow B\bar{B}$ Reactions

Dae Sung Hwang and Hyungsuk Son
Department of Physics, Sejong University, Seoul 143–747, Korea

Abstract

We calculate the decay rate ratios for OZI allowed decays of $\Upsilon(5S)$ to two $B$ mesons by using the decay amplitudes which incorporate the wave function of the $\Upsilon(5S)$ state. We obtain the results that the branching ratio of the $\Upsilon(5S)$ decay to $B^*_s\bar{B}^*_s$ is much larger than the branching ratio to $B_s\bar{B}^*_s$ or $\bar{B}_sB^*_s$, in good agreement with recent experimental results of CLEO and BELLE. This agreement with the experimental results is made possible since the nodes of the $\Upsilon(5S)$ radial wave function induce the nodes of the decay amplitude. We find that the results for the $\Upsilon(5S)$ decays to $B_u^{(*)}\bar{B}_u^{(*)}$ or $B_d^{(*)}\bar{B}_d^{(*)}$ pairs are dependent on the parameter values used for the potential between heavy quarks.

PACS codes: 12.39.-x, 12.39.Pn, 13.25.-k, 13.25.Gv
Key words: Quarkonium, B Meson, Hadronic Decay, Node of Amplitude
1 Introduction

The CLEO Collaboration recently observed the $B_s$ mesons in the $e^+e^-$ annihilation at the $\Upsilon(5S)$ resonance [1]. They established that $B_s$ meson production proceeds dominantly through the creation of $B_s^*\overline{B}_s$ pairs. They found $\sigma(e^+e^- \rightarrow B_s^*\overline{B}_s) = [0.11^{+0.04}_{-0.03} \text{ (stat)} \pm 0.02 \text{ (syst)}]$ nb and set the following limits [1]:

$$\frac{\sigma(e^+e^- \rightarrow B_s^*\overline{B}_s)}{\sigma(e^+e^- \rightarrow B_s^*\overline{B}_s)} < 0.16 , \quad (1)$$

$$\frac{[\sigma(e^+e^- \rightarrow B_s^*\overline{B}_s) + \sigma(e^+e^- \rightarrow B_s^*\overline{B}_s)]}{\sigma(e^+e^- \rightarrow B_s^*\overline{B}_s)} < 0.16 . \quad (2)$$

The BELLE Collaboration collected data at the $\Upsilon(5S)$ resonance and obtained the following ratio at the $\Upsilon(5S)$ energy [2]:

$$\frac{\sigma(e^+e^- \rightarrow B_s^*\overline{B}_s)}{\sigma(e^+e^- \rightarrow B_s^*\overline{B}_s)} = 93^{+7}_{-9} \pm 1 \% . \quad (3)$$

The above results of CLEO and BELLE show that the branching ratio of the $\Upsilon(5S)$ decay to $B_s^*\overline{B}_s$ is much larger than others ones to $B_s^{(*)}\overline{B}_s^{(*)}$ pairs, and in particular that the ratio of $[\sigma(e^+e^- \rightarrow B_s^*\overline{B}_s) + \sigma(e^+e^- \rightarrow B_s^*\overline{B}_s)]$ to $\sigma(e^+e^- \rightarrow B_s^*\overline{B}_s)$ is much smaller than the value which is expected from the spin countings. These results imply that there exists a delicate mechanism in the decays of $\Upsilon(5S) \rightarrow B\overline{B}$.

The unitarized quark model [3] could predict similar results for the above ratios. These ratios were also calculated in Ref. [4] by using the decay amplitudes which take into account the bound state effect through the wave function of the $\Upsilon(5S)$ state. Even though these theoretical calculations showed the nature of the above experimental results, they were performed before the masses of the $B$ mesons, especially $B_s$ and $B_s^*$, are measured. Therefore, it is desirable to perform a theoretical calculation by using the experimentally established mass values of all $B$ mesons in order to obtain the results which are more reliable quantitatively. It is the purpose of this paper. We use the same method for the calculation as that used in Ref. [4] with the experimentally measured values of $B$ meson masses. We perform the numerical calculations for two sets of the parameter values for the potential between heavy quarks, in order to see the dependence of the results on the parameter values.

The CLEO Collaboration also measured the cross sections of the $\Upsilon(5S)$ decays to $B^*\overline{B}^*$, $B\overline{B}$, and $B\overline{B}$, and found that the decay to $B^*\overline{B}^*$ is dominant with $(74 \pm 15)\%$ of the total $B$ rate [5]. Here, $B = B_u$ or $B_d$, and $B\overline{B}^*$ signifies both $B\overline{B}^u$ and $B\overline{B}^d$. 


They summarized the measured branching ratios as follows [1, 5, 6]:

\[ \mathcal{B}(B^*\overline{B}) = 43.6 \pm 8.3 \pm 7.2 \%, \]
\[ \mathcal{B}(B\overline{B}^* + B^*\overline{B}) = 14.3 \pm 5.3 \pm 2.7 \%, \]
\[ \mathcal{B}(B\overline{B}) < 13.8 \%, \]
\[ \mathcal{B}(B_s^*(\overline{B}_s^*)) = 16.8 \pm 2.6^{+6.7}_{-3.4} \% \text{ (using } D_s \text{ yields)}, \]
\[ \mathcal{B}(B_s^*(\overline{B}_s^*)) = 24.6 \pm 2.9^{+11.0}_{-5.3} \% \text{ (using } \phi \text{ yields)}. \]

The above branching ratios show that \( \mathcal{B}(B^*\overline{B}) \) is about two or three times of \( \mathcal{B}(B_s^*(\overline{B}_s^*)) \). In this paper we also compare the results of our calculations with the above experimental results.

This paper is organized as follows. In section 2 we briefly summarize the mechanism of light quark pair creation and the formula of the decay amplitude. In section 3 we explain the procedures and results of our calculations. The last section is conclusion.

## 2 Formula for Decay Amplitude of \( \Upsilon(5S) \to B\overline{B} \)

The Cornell group studied the effect of OZI allowed decay channels [7]. They proposed that the following interaction hamiltonian is responsible for the decay as well as the binding of quark–antiquark bound states [7].

\[ H_I = \frac{1}{2} \sum_{k=1}^{8} \int d^3x d^3y : \rho_k(x)V(x-y)\rho_k(y) :, \quad (5) \]

where

\[ V(r) = -\frac{\kappa}{r} + \frac{r}{a^2}. \quad (6) \]

In (5) \( \rho_k(x) = \psi^\dagger(x)\frac{1}{2}\lambda_k \psi(x) \) are the color densities of quark fields. This model corresponds to the vector coupling since (5) is the leading term of the vector coupling hamiltonian in the nonrelativistic expansion.

The decay rate of \( \Upsilon(5S) \) with spin \( s \) (which is 1) and mass \( M \) to \( B \) mesons with spin \( s_1 \) and \( s_2 \) is given by the formula [4]

\[ \Gamma = \frac{1}{8\pi M} \frac{P}{E_1 E_2} C(ss_1s_2)|A_{5S}(P)|^2, \quad (7) \]

where \( P \) is the magnitude of the center of mass momentum of the mesons and \( E_1 \) and \( E_2 \) are their energies. The spin counting factor \( C(ss_1s_2) \) are for \( s = 1 \) given be 1/3, 4/3, and 7/3, for the cases \( s_1 = s_2 = 0 \), \( s_1 = 0 \ s_2 = 1 + s_1 = 1 \ s_2 = 0 \), and \( s_1 = s_2 = 1 \), respectively [7].
Figure 1: Diagrams used to calculate the decay amplitude $A_{5S}(P)$.

Figure 2: $u(x)$ which is related to the radial wave function by $R_{50}(r) = (m_b/a^2)^{1/2} u(x)/x$, where the dimensionless variable $x$ given by $x = (m_b/a^2)^{1/3} r$. 
Figure 3: $I_{50}^L(P)$ for the potential (A). The solid line is for the $B_s^{(*)}$ meson pairs with $m_s = 0.55$ GeV, and the dotted line is for the $B_u^{(*)}$ or $B_d^{(*)}$ meson pairs with $m_u = m_d = 0.33$ GeV.

The decay amplitude $A_{5S}(P)$ calculated from the diagrams in Fig. 1 is given by [7, 8, 9, 10]

$$A_{5S}(P) = 4\pi f_q I_{nL}^I(P), \quad f_q = \left(\frac{3}{\pi}\right)^{\frac{3}{2}} \left(\frac{m_b m_q}{m_b + m_q}\right)^{-1}; \quad q = u, d, s, \quad \text{(8)}$$

where for the vector coupling of (5) the momentum dependent function $I_{nL}^I(P)$ is given by [7, 8, 9, 10]

$$I_{nL}^I(P) = \int_0^\infty dt \Theta(t) R_{nL}(\frac{t}{\sqrt{\beta}}) j_l(\frac{\rho_b P t}{\sqrt{\beta}}) \quad \text{(9)}$$

with

$$\Theta(t) = [te^{-t^2} + (t^2 - 1)e^{-t^2/2} \sqrt{\frac{\pi}{2}} \text{erf}(\frac{t}{\sqrt{2}})] + 4\beta a^2 \kappa [-te^{-t^2} + e^{-t^2/2} \sqrt{\frac{\pi}{2}} \text{erf}(\frac{t}{\sqrt{2}})], \quad \text{(10)}$$

where $R_{nL}(r)$ is the radial wave function of the heavy quark-antiquark bound state, $j_l(\frac{\rho_b P t}{\sqrt{\beta}})$ is the spherical Bessel function, and $\rho_b = m_b/(m_b + m_q)$. We note that $I_{nL}^I(P)$ incorporates the radial wave function $R_{nL}(r)$ of the $\Upsilon(5S)$ state. The first and second terms in (10) come from the linear and Coulombic parts of (6), respectively. Following [7], we use the radial wave functions of the final $c\pi$ and $c\bar{d}$ states with $L = 0$ which are given by the ground state harmonic-oscillator wave functions $(4\pi)^{\frac{1}{2}}(\frac{2\beta}{\pi})^{\frac{3}{2}} \exp(-\beta r^2)$. For the value of $\beta$ in the calculations of (9) and (10), we use $\beta = \left(\frac{1}{2a^2}\right)(\frac{4\mu a}{3\sqrt{\pi}})^2$ [7], where $\mu$ is the reduced mass $m_q \rho_b$ of the $c\pi$, $c\bar{d}$ or $c\bar{s}$ system.
Figure 4: $I_{50}^{1}(P)$ for the potential (B). The solid line is for the $B_{s}^{(*)}$ meson pairs with $m_{s} = 0.55$ GeV, and the dotted line is for the $B_{u}^{(*)}$ or $B_{d}^{(*)}$ meson pairs with $m_{u} = m_{d} = 0.33$ GeV.

Figure 5: The position of the final momentum $P$ of the $B$ meson for each mode in the $\Upsilon(5S)$ rest frame. (a) is for the potential (A) and (b) is for the potential (B).
\begin{table}  
\begin{tabular}{ccccccc}  
\hline  
$\Upsilon(5S)$ & $B_u$ & $B_d$ & $B_u^*$ & $B_d^*$ & $B_s$ & $B_s^*$ \\
\hline  
10865 & 5279.15 & 5279.53 & 5325.1 & 5325.1 & 5366.3 & 5412.8 \\
\hline  
\end{tabular}  
\caption{Meson Masses (MeV) used in the calculation \cite{12}.}  
\end{table}  

\begin{table}  
\begin{tabular}{cccccccc}  
\hline  
$B_uB_u$ & $B_uB_d$ & $B_u^*B_u$ & $B_dB_d$ & $B_dB_d$ & $B_dB_d$ & $B_sB_s$ & $B_sB_s$ \\
\hline  
1.282 & 1.183 & 1.075 & 1.280 & 1.182 & 1.075 & 0.846 & 0.682 & 0.462 \\
\hline  
\end{tabular}  
\caption{Decay momenta in GeV for $\Upsilon(5S)$ calculated with masses given in Table 1.}  
\end{table}  

\section{Results of Calculation}  
We calculate the decay rates for OZI allowed decays of $\Upsilon(5S)$ to two $B$ mesons by using the formula (7) with the decay amplitudes given in (8) and (9). For $V(r)$ in (6) we use the following two parameterizations in our calculation:  
(A) Eichten et al. \cite{7}: $\kappa=0.517$, $a=2.12$ GeV$^{-1}$, $m_b=5.17$ GeV.  
(B) Hagiwara et al. \cite{11}: $\kappa=0.47$, $a = 1/\sqrt{0.19}$ GeV$^{-1}$, $m_b=4.75$ GeV.  
For the masses of $u,d$ and $s$ quarks, we use $m_{u,d}=0.33$ GeV and $m_s=0.55$ GeV.  

The radial wave functions of $\Upsilon(5S)$ obtained by solving the Schrödinger equation with the parameter sets (A) and (B) are presented in Fig. 2. The function $I_{50}(P)$ in (9) obtained with the above (A) and (B) sets of the parameter values are presented in Figs. 3 and 4.  

In order to obtain the decay amplitude for each final state of $B$ meson pair from (8) and (9), we need to know the final momentum $P$ of the $B$ meson in the $\Upsilon(5S)$ rest frame. We use the meson masses presented in Table 1, which give the value $P$ for each mode as written in Table 2. In Fig. 5 we marked the position of the value $P$ for each mode, from which we can get the magnitude of $I_{50}(P)$ for each mode of $B$ meson pair. Then, we can calculate the decay rate $\Gamma$ from the formula (7) for each mode of the $B$ meson pair. Using the values of $\Gamma$ obtained in this way, we calculate the ratios of the decay rates. We present our results of this calculation in Table 3. When we compare our results presented in Table 3 with the experimental results in \cite{4}, for the $B_d^{(*)}\overline{B}_d^{(*)}$ pairs we should use two times the values of Table 3 since in (4) $B$ can be either neutral or charged \cite{5,6}. In our calculation the results for the $B_u^{(*)}\overline{B}_u^{(*)}$ pairs are the same as those for the $B_d^{(*)}\overline{B}_d^{(*)}$ pairs since we use the same value $m_{u,d}=0.33$ GeV for the $u$ and $d$ quark masses.  

We find in Table 3 that the results for the $B_s^{(*)}\overline{B}_s^{(*)}$ pairs are in good agreement with the experimental results in both cases of (A) and (B) sets of the parameter values. Especially, we note that the result for the ratio $[\sigma(e^+e^- \rightarrow B_s\overline{B}_s) + \sigma(e^+e^- \rightarrow B_u^*\overline{B}_u)]/\sigma(e^+e^- \rightarrow B_s^*\overline{B}_s)$ is consistent with the experimentally obtained very small value. The reason why we could obtain such a consistent result is that we incorporate the radial wave function of the $\Upsilon(5S)$ state which has nodes. The nodes of the radial wave function induce the nodes of the decay amplitude as we can see in Figs. 3 and 4.  

We find that the results for the $\Upsilon(5S)$ decays to $B_u^{(*)}\overline{B}_u^{(*)}$ or $B_d^{(*)}\overline{B}_d^{(*)}$ pairs are
Table 3: Decay rate ratios of $\Upsilon(5S) \rightarrow B\bar{B}$ modes. Potentials (A) and (B) are given in the text. “L + C” and “L only” mean that we used “both linear and Coulomb parts” and “only linear part” respectively for the calculation of (10). We always used both linear and Coulomb parts for the calculation of the wave function of $\Upsilon(5S)$.

| Potential | $B_s^*\overline{B}_s$ | $B_s\overline{B}_s + B_s^*\overline{B}_s$ | $B_s\overline{B}_s$ | $B_s^*\overline{B}_s + B_d\overline{B}_d + \overline{B}_d B_s^*$ | $B_d\overline{B}_d$ |
|-----------|-----------------|----------------|----------------|----------------|----------------|
| (A) L + C | 1               | 0.007          | 0.065          | 3.159          | 0.298          | 0.036          |
| (A) L only| 1               | 0.010          | 0.059          | 3.142          | 0.312          | 0.030          |
| (B) L + C | 1               | 0.034          | 0.141          | 0.923          | 0.294          | 0.646          |
| (B) L only| 1               | 0.026          | 0.130          | 0.953          | 0.242          | 0.579          |

sensitive to the parameter values used for the potential between heavy quarks. For example, the result with the set (A) for the ratio $[\mathcal{B}(B_s^*\overline{B}_s) + \mathcal{B}(B^*_d\overline{B}_d)]/\mathcal{B}(B^*_s\overline{B}_s)$ is rather larger than the experimental result and the result with the set (B) for this ratio is within the error bar of the experimental result. Such a situation happens since the node positions of the decay amplitude are sensitive to the parameter values of the potential in the range of large final $B$ meson momentum.

4 Conclusion

The CLEO and BELLE Collaborations recently performed experiments at the $\Upsilon(5S)$ resonance. Their results for the ratio $[\sigma(e^+e^- \rightarrow B_s\overline{B}_s) + \sigma(e^+e^- \rightarrow B_s^*\overline{B}_s)]/\sigma(e^+e^- \rightarrow B^*_s\overline{B}_s)$ at the $\Upsilon(5S)$ energy is much smaller than the value expected from the spin countings. This delicate phenomenon was predicted by the unitarized quark model [3] and by the decay amplitudes which incorporate the wave function of the $\Upsilon(5S)$ state [4, 7]. However, these theoretical studies were performed before the masses of the $B$ mesons, especially $B_s$ and $B_s^*$, are known. We use the same method for the calculation as that in Ref. [4] with the experimentally measured values of $B$ meson masses. We perform the numerical calculations for two sets of the parameter values for the potential between heavy quarks, in order to see the dependence of the results on the parameter values used.

Our results for the ratios for the $B_s^{(*)}\overline{B}_s^{(*)}$ pairs are in good agreement in both cases of (A) and (B) sets of the parameter values of the potential between heavy quarks. It is especially important that the result of our calculation agrees well with the delicate experimental result that the branching ratio to $B_s\overline{B}_s$ or $\overline{B}_s B_s^*$ pair is much smaller than that to $B_s^*\overline{B}_s$ pair. Obtaining such results by our calculation was possible since we take into consideration the radial wave function of the $\Upsilon(5S)$ state which has nodes, and the nodes of the radial wave function induce the nodes of the decay amplitude of the $\Upsilon(5S) \rightarrow B\bar{B}$ reaction. However, our results for the $\Upsilon(5S)$ decays to $B_u^{(*)}\overline{B_u}^{(*)}$ or $B_d^{(*)}\overline{B_d}^{(*)}$ pairs are sensitive to the parameter values used for the potential between heavy quarks since the node positions of the decay amplitude are sensitive to the parameter values in the range of large final $B$ meson momentum.
Acknowledgments

One of the authors (D.S.H.) wishes to thank Nina Byers for helpful discussions. The authors wish to thank Jonghyun Kim for useful discussions. This work was supported in part by the International Cooperation Program of the KICOS (Korea Foundation for International Cooperation of Science & Technology), and in part by the Korea Research Foundation Grant funded by the Korean Government (KRF-2008-313-C00166).

References

[1] G. Bonvicini et al. [CLEO Collaboration], Phys. Rev. Lett. 96 (2006) 022002.
[2] A. Drutskoy et al. [BELLE Collaboration], Phys. Rev. D 76 (2007) 012002.
[3] N.A. Törnqvist, Phys. Rev. Lett. 53 (1984) 878; S. Ono, N.A. Törnqvist, J. Lee-Franzini, and A.I. Sanda, Phys. Rev. Lett. 55 (1985) 2938; S. Ono, A.I. Sanda, and N.A. Törnqvist, Phys. Rev. D 34 (1986) 186.
[4] N. Byers and D.S. Hwang, UCLA-87-TEP-44 (1987).
[5] O. Aquines et al. [CLEO Collaboration], Phys. Rev. Lett. 96 (2006) 152001.
[6] G.S. Huang [CLEO Collaboration], Phys. Rev. D 75 (2007) 012002.
[7] E. Eichten, K. Gottfried, T. Kinoshita, K.D. Lane and T.-Y. Yan, Phys. Rev. D 17 (1978) 3090; Phys. Rev. D 21 (1980) 203.
[8] V. Zambetakis, Ph. D. Thesis, UCLA, 1986; available as UCLA research report UCLA/85/TEP/2.
[9] N. Byers, in the proceedings of international conference on quark confinement and the hadron spectrum, Como, Italy, Jun 20-24, 1994, edited by N. Brambilla and G.M. Prosperi, World Scientific.
[10] E.J. Eichten, K. Lane and C. Quigg, Phys. Rev. D 69 (2004) 094019.
[11] K. Hagiwara, A.D. Martin and A.W. Peacock, Z. Phys. C 33 (1986) 135.
[12] C. Amsler et al. (Particle Data Group), Phys. Lett. B 667 (2008) 1.