Tunable current circulation in triangular quantum-dot metastructures

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Abstract – Advances in fabrication and control of quantum dots allow the realization of metastructures that may exhibit novel electrical transport phenomena. Here, we investigate the electrical current passing through one such metastructure, a system composed of quantum dots placed at the vertices of a triangle. The wave nature of quantum particles leads to internal current circulation within the metastructure in the absence of any external magnetic field. We uncover the relation between the metastructure steady-state total current and the internal circulation. By calculating the electronic correlations in quantum transport exactly, we present phase diagrams showing where different types of current circulation can be found as a function of the correlation strength and the coupling between the quantum dots. Finally, we show that the regimes of current circulation can be further enhanced or reduced depending on the local spatial distribution of the interactions, suggesting a single-particle scattering mechanism is at play even in the strongly correlated regime. We suggest experimental realizations of actual quantum-dot metastructures where our predictions can be directly tested.

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Quantum transport is an important area of research with a wide range of phenomena and applications, especially in fields like condensed-matter [1,2] and cold-atom systems [3–7]. Of particular present interest are those phenomena that emerge in the presence of non-trivial geometry or topology. The prototypical example is the Aharonov-Bohm (AB) effect [8] which arises when a finite vector potential is encircled by a conducting ring and endows the electron wave function with an additional geometrical phase. However, several other interesting transport phenomena emerge from geometry and topology, such as quantized conduction via edge states of topological insulators that have been an important probe for non-trivial topology in the band structure [9–12], or flat bands of geometrically induced localized states that interfere with mobile particles and influence their transport [13,14], to name just a few.

Nanoscale structures, such as quantum dots (QDs), offer additional opportunities to engineer metastructures that, if appropriately constructed, may reveal quantum transport phenomena otherwise difficult to probe with other means. Here, we investigate quantum currents through a topologically non-trivial metastructure consisting of QDs placed at the vertices of a triangle with additional elements for tuning the tunneling and interactions. We call it a “triangular quantum-dot metastructure” (TQDM). These metastructures resemble the triangular triple quantum dot, which has been already fabricated and employed in studying other physical properties [15–18], such as the stability diagrams showing the Coulomb blockade or properties of the internal levels.

Theoretical work on similar structures has been discussed in the context of Kondo effects from spin-spin interactions [19–21], spin dynamics [22], stability diagrams in a magnetic field [23], transport in the presence of magnetic flux [24–26], and thermal transport [27]. Here, we show instead that, even in the absence of any magnetic flux and away from the Coulomb blockade regime, internal charge current circulation can spontaneously emerge in steady states, an effect that has not been described in previous work.
To probe the internal electrical dynamics of the TQDM, we connect two of the three QDs to two external reservoirs, as illustrated in fig. 1(a). Such a system forces the currents to flow through a non-simply-connected region, generating current circulation within the TQDM without the need of a vector or scalar potential. Unlike the Aharonov-Bohm effect, then the internal circulation may arise in a steady state without any external magnetic flux. The internal circulation is possible because of the non-trivial topology of the TQDM and the wave nature of quantum particles. The quantum current on one path may be larger than the total current, so the current in the other path flows in the reversed direction to compensate. In addition, one can detect the emergence of the internal TQDM current circulation by varying a single link between two of the three QDs, giving rise to a non-monotonic behavior of the total current.

By introducing correlations one can tune the circulation further by switching from clockwise (CW) to counterclockwise (CCW) to no-circulation or unidirectional (UD) flow. Computing electronic correlations exactly in the Hubbard model, we provide the corresponding phase diagrams of current circulation as a function of correlation strength and inter-dot coupling. We also study the effect of inhomogeneous correlations, that reveal a single-particle scattering mechanism is at play even in the strongly correlated regime. We report here the results obtained using an open-system, quantum master equation approach. In the Supplemental Information Supplementarymaterial.pdf (SI) we report those obtained by a microcanonical (closed-system) formalism [28], showing that the two approaches lead to the same conclusions. The agreement establishes the model independence of the internal circulation of current in a multi-connected geometry.

Experimental realization. – Before embarking on the theoretical aspects of TQDMs, let us first point out how they can be engineered with the appropriate features to observe the phenomena we predict. First, we note that triangular triple quantum dots have been fabricated to study various phenomena such as stability diagrams of Coulomb blockade [15], charge frustration [17], and tunable-path transport in a magnetic field [18].

TQDMs can be experimentally realized in several ways. The most obvious one relating to electronic transport utilizes electrostatically defined QDs. Here the dots, the barriers between the dots, and the source and drain are controlled by electronic gates which modify the potential landscape of a 2-dimensional electron gas (2DEG). The gates may be created by electron beam lithography [18] or local anodic oxidation [29–32] on top of an epitaxially grown 2DEG semiconductor heterostructure. Figure 1(b) illustrates this approach.

As another experimental realization, we propose a photonic circuit architecture, operated in the photon blockade regime to achieve the required fermionic behavior [33]. An example design is shown in fig. 1(c). Here, the QDs are embedded in three L3 photonic crystal cavities, which are spatially arranged to create the triangular topology. The source and drain are formed by two photonic crystal waveguides, which guide photons from and to the input and output couplers and couple to the excitons in the quantum dots 1 and 3. Auxiliary waveguides, weakly coupled to dot number 2, may be used to measure the directionality of the photon flux. The three dot-cavity systems can be electrically separated from each other by etching through the top p-doped layer of the pin-diode structure. This allows the application of different electric fields via gate voltages to/from the QDs.

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site-1 and site-3 are connected to two particle reservoirs via couplings $\gamma_L$ and $\gamma_R$, as illustrated in fig. 1(a). The left (right) reservoir acts as a particle source (drain) which pumps (removes) particles into (out of) the triangle. A previous study of spin-selective Aharonov-Bohm effect [34] has a similar setup but with a magnetic flux enclosed by the triangle. Here, we make the assumptions that the coupling between the system and reservoirs is weak in the sense that the frequency scale associated with the coupling between the system and environment is small compared to the dynamical frequency scales of the system or the reservoirs. Moreover, the Markovian approximation requires the coupling to be time-independent and the time evolution of the TQDM to be slow compared to the time necessary for the environment to “forget” quantum correlations [35,36].

Then, the dynamics can be described by the Lindblad equation ($\hbar = 1$ throughout):

$$\frac{d\rho}{dt} = i [\rho, H] + \gamma_L \left( c_{1\sigma}^\dagger \rho c_{1\sigma} - \frac{1}{2} (c_{1\sigma} c_{1\sigma}^\dagger, \rho) \right)$$

$$+ \gamma_R \left( c_{3\sigma} \rho c_{3\sigma}^\dagger - \frac{1}{2} (c_{3\sigma}^\dagger c_{3\sigma}, \rho) \right), \quad (1)$$

where $\rho$ is the density matrix of the TQDM and $\{A, B\}$ denotes the anticommutator of $A$ and $B$. Here we assume the three QDs have identical energy levels and focus on transport through a single level at the Fermi energy, so we are away from the Coulomb blockade boundaries [16].

The total current is not monotonic as $t_{13}$ increases. Figure 2(a) shows the total current through the triangle and the internal currents through the 1-3 (1-2) link. The shaded regions in (a) and (b) emphasize the non-monotonic behavior of the total current. In the shaded regions all the internal currents in the triangle are unidirectional (no circulation).

Non-interacting fermions. – In the absence of interactions, the Lindblad equation can be expressed in terms of the single-particle correlation matrix $\langle C_{pq}\rangle = \langle c_{q\sigma}^\dagger c_{p\sigma}\rangle$ as $j_{pq} = -2 \sum_{\sigma} \text{Im}(t_{pq} C_{pq})$. In the following we choose $\gamma_L = \gamma_R = \gamma$ and focus on the steady state where $d\rho/dT = 0$ in the long-time limit ($T \to \infty$), and a steady-state current can be identified. We now analyze both the total current flowing through the triangle and the internal currents inside the triangle, by varying $t_{13}$ within the triangle, the system-reservoir coupling $\gamma$, and the strength of the interaction $U_p$. Details of the calculations can be found in the SI as well as confirmation of these results using a microcanonical approach.
Here the CW (CCW) circulation has an opposite current flowing along the path 1-3 (path 1-2-3). We found CW (CCW) circulation when $t_{13}/t$ is small (large), and the bending region of the total current corresponds to the regime where all internal currents flow in the same direction. The non-monotonic behavior of the total current as $t_{13}$ is varied can also be corroborated by the Landauer formalism [1,37]. (See the SI for details.)

Figure 3(a) shows the phase diagram of the internal currents, where unidirectional, clockwise, and counterclockwise current flows are clearly distinguishable. Spontaneous circulation of currents in quantum fluids has been found theoretically in ideal Fermi gases passing a constriction [38]. Here, we show that the circulation can be controlled in systems with a multi-connected (triangular) geometry. By further examination, we have found that the critical point where $j_{12}$ is reversed is located at $\gamma_L \gamma_R = 4(t_{13}^2 - t^2)$. (See the SI for details.)

**Interacting fermions.** Having found a clear signature of internal current circulation in the TQDM, we now analyze the role of correlations. Since the Hamiltonian, $\mathbf{H}$, consists of only three sites, we can compute numerically the dynamics of the density matrix with correlations exactly, using a fourth-order Runge-Kutta algorithm [39]. (See the SI for details of the simulation.)

We first examine the system with uniform interactions $U_p = U$, $\forall p$. The steady-state currents and their dependence on $\gamma$ are similar to the non-interacting case. When the interaction is weak, the phase diagram showing different internal flows of currents is qualitatively the same as the diagram of non-interacting systems. However, as the interaction becomes stronger the regimes in the parameter space showing internal current circulation shrink when compared to the non-interacting case, as shown in fig. 3(b). Therefore, by tuning the onsite interaction one can suppress internal circulation of the current.

As discussed in the non-interacting case, the total current flowing through the triangle forms a dip as $t_{13}$ varies due to a change of the current circulation. The same behavior is found in weakly interacting systems as well. Figure 2(b) shows the total current from the left reservoir to the right one and the shaded region indicates where all internal currents are flowing in the same direction. The total current in both the small and large $t_{13}$ regimes changes monotonically with $t_{13}$ when the internal current is circulating. However, the total current exhibits a dip as the internal circulation changes from CCW to CW across the shaded region. Therefore, non-monotonic behavior of the total current as $t_{13}$ varies indicates a change of the internal circulation of current in both non-interacting and weakly interacting cases. In the strongly interacting regime, the internal circulation of current is severely suppressed as shown in fig. 3(c), and the total current varies monotonically with $t_{13}$ as shown in fig. 2(c).

To investigate further how correlations suppress the internal circulation of current, we assume the onsite repulsive interaction is present only on one site, while the other two sites remain non-interacting. The phase diagrams showing where internal circulations can be found in this case are summarized in fig. 4. The presence of interactions on site 1 affects the clockwise circulation when the interaction is strong as shown in the $U = 5t$ case in fig. 4(c), but the CCW circulation is less affected. In contrast, if the interaction is only on site 2, the suppression shown in fig. 4(e) and (f) is similar to the uniform interaction case shown in fig. 3(c). Finally, the interaction on site-3 has almost no observable influence on the circulation as shown in fig. 4(g)–(i). Therefore, the dominant interaction effect comes from site-2, and it is possible to reduce the three-state circulation (CCW, CW, and UD) to two-state circulation (CCW and UD) as shown in fig. 4(e).
The result suggests that scattering of quantum particles is the main mechanism for tuning the internal circulation of current. That this is the case, can be understood as follows. In the presence of interactions on site 2, particles are scattered from that vertex, so the current flowing through the upper (1-2-3) path is reduced. This makes the CW circulation unfavorable because it requires a large current through the upper path and a counterflowing current on the lower (1-3) path. On the other hand, adding scattering mechanisms like onsite interaction to site 1 or 3 leaves the phase diagrams intact (or completely suppresses the internal circulations). Similar results occur if one includes onsite attractive potentials (see the SI), thus confirming the single-particle scattering mechanism we have just described. Moreover, introducing inhomogeneous hopping coefficients, for example by setting $t_{12} \neq t_{23}$, leads to additional scattering along the upper path and also shifts the boundary between different types of circulation on the phase diagrams. Inhomogeneous interactions or onsite potentials may be achievable in quantum dots coupled to cavities by tuning the photon-exciton coupling. In this respect, the photonic-circuit structure shown in fig. 1(c) has an advantage over the electrostatic quantum dots when it comes to configurations with tunable inhomogeneity. We remark that a circulating electronic current leads to a magnetic field whose direction is determined by the orientation of the current. Our results may be considered as the inverse of the AB effect and with advanced techniques for measuring magnetic fields [40–43], the circulating current in the TQDM may also be probed by its magnetic field.

Conclusions. – We have considered a triangular quantum-dot metastructure connected to two reservoirs and studied the relation between its steady-state total charge current and its internal current circulation in the absence of any magnetic flux. Internal circulation of charge current in the triangle is found in both closed- and open-system approaches, and the direction can be tuned by a variety of parameters including the hopping coefficient, local interactions or potential, and system-reservoir coupling. Therefore, the internal current circulation is model-independent. Moreover, the overall current exhibits non-monotonic behavior when the circulation reverses. The phase diagrams showing how the circulation can be tuned will assist designs of quantum devices utilizing the internal circulation of current. Future designs of time-dependent interdot couplings may lead to switchable local magnetic-field generators using the TQDM.

Importantly, our findings are not limited to quantum dots because the generic formalism based on quantum mechanics establishes the robustness of the internal current circulation in quantum transport. It is also possible to use the recently developed lattice fermion simulators based on superconducting elements [44] or ultracold atoms in engineered reservoirs and constrictions [45] to explore similar transport phenomena in other controllable quantum systems.

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