Mixed quark–gluon condensate from instantons

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Abstract

We calculate the vacuum expectation value of the dimension–5 “mixed” quark–gluon operator $O_\sigma = \bar{\psi} \frac{1}{2} \lambda^a \sigma_{\mu\nu} \psi F_{\mu\nu}^a$ in the instanton vacuum. Within the $1/N_c$–expansion the QCD operator is replaced by an effective many–fermion operator, which is averaged over the effective theory of massive quarks derived from instantons. We find $m_0^2 \equiv \langle O_\sigma \rangle / \langle \bar{\psi} \psi \rangle \approx 4\bar{\rho}^{-2} = 1.4 \text{GeV}^2$, somewhat larger than the estimate from QCD sum rules for the nucleon.

PACS: 12.38.Lg, 11.15.Kc, 11.15.Pg

Keywords: non-perturbative methods in QCD, instantons, QCD sum rules, $1/N_c$ expansion

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The non-perturbative structure of the QCD vacuum can be expressed in terms of non-zero vacuum expectation values of various composite operators. These “condensates” have been introduced as phenomenological parameters in a non-perturbative generalization of the operator product expansion, which can be related to observable hadronic properties by the sum rule method \cite{1, 2, 3}. The lowest–dimensional condensates, $\langle F_{\mu\nu}^2 \rangle$ and $\langle \bar{\psi}\psi \rangle$, are well-known from phenomenology. Considerable uncertainty prevails about the values of the higher–dimensional condensates. The four–quark condensates, $\langle \bar{\psi}\Gamma\psi\bar{\psi}\Gamma\psi \rangle$, are usually estimated assuming factorization, an approximation justified in the large–$N_c$ limit \cite{1}. No such simple estimate exists for the condensate of the dimension–5 mixed quark–gluon operator,

$$O_\sigma = \bar{\psi}\frac{1}{2}\lambda^a\sigma_{\mu\nu}F_{\mu\nu}^a,$$

which enters in QCD sum rules for the nucleon channel \cite{3}, for exotic light mesons \cite{4} and for heavy–light mesons \cite{5}. The value of this condensate is usually expressed as a multiple of the quark condensate,

$$\langle O_\sigma \rangle = m_0^2\langle \bar{\psi}\psi \rangle.$$  

The parameter $m_0^2$ has been estimated by Belyaev and Ioffe from sum rule stability \cite{6}. At a normalization point of 500 MeV they find $m_0^2 = (0.8 \pm 0.2)\text{ GeV}^2$.

In a more microscopic picture of the QCD vacuum, the condensates can be understood as being generated by certain non-perturbative fluctuations of the fields. In particular, instantons are responsible for the spontaneous breaking of chiral symmetry. A quark condensate arises due the delocalization of the fermion zero modes associated with the individual instantons in the medium \cite{7}. Assuming the gluon condensate to be entirely due to instantons results in a density of the instanton medium which gives a consistent description not only of the quark condensate, but also of meson and baryon characteristics \cite{8}. One may therefore assume that instantons are also the dominant non-perturbative fluctuations giving rise to the mixed condensate, eq.(2).

In this letter, we evaluate the vacuum average of the mixed operator, eq.(1), on the basis of the quantitative theory of the instanton medium of Diakonov and Petrov \cite{9}. The calculation of $\langle O_\sigma \rangle$ requires averaging over the gluon field, i.e., the instanton coordinates, as well as over the quark field, taking into account the dynamical breaking of chiral symmetry. Such averages can be computed using a recently developed method \cite{10}, which makes use of the $1/N_c$–expansion. Integrating first over the instanton coordinates, one derives from the instanton vacuum an effective quark action of the form of a Nambu–Jona-Lasinio model, which describes quarks with a (momentum–dependent) dynamical mass \cite{11}. In this effective theory QCD operators involving gluon fields can systematically be represented as many–fermion operators, whose averages can be evaluated with standard techniques. With this method we compute $\langle O_\sigma \rangle$ at the scale at which the QCD coupling constant is defined in the instanton vacuum, typically of order of the inverse average instanton size, $\rho^{-1} = 600$ MeV. We compare our result with the QCD sum rule estimate. Furthermore, we show that in the instanton vacuum the parameter $m_0^2$ of eq.(2) has a simple interpretation as a ratio of one–instanton matrix elements.
We begin by rewriting the operator eq.(1) in terms of euclidean gluon and quark fields \((\bar{\psi}^\dagger \equiv i\bar{\psi})\),

\[
O_\sigma(x) = i\bar{\psi}^\dagger(x)\frac{i}{2}\lambda^a \sigma_{\mu\nu} \psi(x) F^a_{\mu\nu}(x). \tag{3}
\]

Following [10], we replace the gluonic part of this operator, \(F^a_{\mu\nu}(x)\), by an effective quark operator. For simplicity, we assume only one quark flavor in the following; the generalization to \(N_f > 1\) is straightforward and will be discussed below.

In zero mode approximation [7], the interaction of the quark field with an (anti–) instanton with collective coordinates \(\rho\) (size), \(z\) (center) and \(U\) (\(SU(N_c)\) matrix describing color orientation) is given by

\[
V_\pm[\psi^\dagger, \psi] = 4\pi^2 \rho^2 \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \exp(iz \cdot (k_2 - k_1)) \ F(k_1) F(k_2) \times \psi^\dagger \gamma_5(k_1) \left[\frac{1}{8} \gamma_\kappa \gamma_\lambda \frac{1 \pm \gamma_5}{2}\right]^i_j \left[U \tau_\kappa^\dagger \tau_\lambda^\dagger \ U^\dagger\right]^{\gamma \delta} \psi^{\delta \dagger}(k_2). \tag{4}
\]

Here \(F(k)\) is a form factor of width \(\rho^{-1}\), proportional to the Fourier transform of the wave function of the fermion zero mode, with \(F(0) = 1\), and \(\tau_\kappa^\pm\) are \(N_c \times N_c\) matrices with \((\tau, \mp i)\) in the upper left corner and zero elsewhere [7]. The field strength of the (anti–) instanton is given as a function of the collective coordinates,

\[
F^a_{\pm \mu\nu}(x; z, U) = \frac{1}{2} \text{tr} \left[\lambda^a U \lambda^b \ U^\dagger\right] F^b_{\pm \mu\nu}(x - z),
\]

\[
U \left[\frac{1}{2} \lambda^b F^b_{\pm \mu\nu}(x - z) \right] U^\dagger = \frac{1}{2} \lambda^a F^a_{\pm \mu\nu}(x; z, U), \tag{5}
\]

where \(F^b_{\pm \mu\nu}(x)\) denotes the instanton field (in singular gauge) in standard orientation,

\[
F^b_{\pm \mu\nu}(x) = \frac{8\rho^2}{(x^2 + \rho^2)^2} \left[ (\eta^+)_{\mu\nu} \frac{x_\mu x_\nu}{x^2} + (\eta^-)_{\mu\nu} \frac{x_\mu x_\nu}{x^2} - \frac{i}{2}(\eta^0)_{\mu\nu} \right], \tag{6}
\]

\((\eta^+)_{\mu\nu} = \eta^b_{\mu\nu}, (\eta^-)_{\mu\nu} = \bar{\eta}^b_{\mu\nu}\) are the ’t Hooft symbols. The effective fermion operator corresponding to \(F^a_{\pm \mu\nu}(x)\) is defined as the average of the product of the instanton field, eq.(3), with the instanton–quark interaction, eq.(4), over the collective coordinates of one instanton [10],

\[
(Y_{F^\pm})_{\mu\nu}^a (x)[\psi^\dagger, \psi] = i \frac{N_c M}{4\pi^2 \rho^2} \int d^4z \int dU \ F^a_{\pm \mu\nu}(x; z, U) \ V_\pm[\psi^\dagger, \psi] \tag{7}
\]
\[
= i N_c M \int d^4z \ F^b_{\mu\nu}(x - z) \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \exp(iz \cdot (k_2 - k_1)) \times F(k_1) F(k_2) (S^*)_{\kappa\lambda}^{abc} \psi^\dagger_{\kappa\mu}(k_1) \left[\frac{1}{8} \gamma_\kappa \gamma_\lambda \frac{1 \pm \gamma_5}{2}\right]^i_j \left(\frac{\lambda^c}{2}\right)^\alpha_\beta \psi^{\beta \dagger}(k_2).
\]

The normalization factor of the effective operator is proportional to the dynamically generated quark mass at zero euclidean momentum, \(M\). When integrating over instanton
sizes we have assumed all instantons to be of the same size, $\bar{\rho}$, which is justified by the fact that the width of the effective size distribution of instantons in the medium is of order $1/N_c$ \cite{3,10}. By $(S_{\pm})^{abc}_{\kappa\lambda}$ we denote the average over color orientations with the Haar measure of $SU(N_c)$, 

$$(S_{\pm})^{abc}_{\kappa\lambda} = \frac{1}{2} (\lambda^a)^{\beta}_{\alpha} (\lambda^b)^{\alpha'}_{\beta'} (\lambda^c)^{\delta}_{\gamma} (\bar{\gamma}_{\kappa\sigma}^{\gamma'} \bar{\sigma}_{\lambda\delta})_{\delta'} \int dU U^{\alpha}_{\alpha'} U^{\beta}_{\beta'} (U^\dagger)^{\beta'}_{\beta} (U^\dagger)^{\delta'}_{\delta} \delta^{\alpha\beta} = \frac{2}{N_c^2} \delta^{\alpha\beta} i(\eta^\pm)^b_{\kappa\lambda},$$

where the last equality is understood to leading order in $1/N_c$. The spin structure of the fermion vertex in eq.\cite{7} is simplified using the identity $^1$ 

$$i(\eta^\pm)^b_{\mu\nu}(\eta^\mp)^b_{\kappa\lambda} \gamma_{\kappa\lambda} = (\eta^\pm)^b_{\mu\nu}(\eta^\mp)^b_{\kappa\lambda} \gamma_{\kappa\lambda} = 4\sigma_{\mu\nu} \frac{1 \pm \gamma_5}{2}. \quad (9)$$

Furthermore, we introduce the Fourier transform of the instanton field strength, 

$$\int d^4x F^a_{\pm\mu\nu}(x) \exp(-ik \cdot x) = \rho^2 G(k) \left[ (\eta^\pm)^a_{\mu\nu} \frac{k_{\mu} k_{\nu}}{k^2} + (\eta^\mp)^a_{\mu\nu} \frac{k_{\mu} k_{\nu}}{k^2} - \frac{1}{2} (\eta^\pm)^a_{\mu\nu} \right], \quad (10)$$

$$G(k) = 8 \int d^4x \frac{1}{(x^2 + \rho^2)^2} \left[ 1 + \frac{4}{3} \left( \frac{k \cdot x}{k^2 x^2} - 1 \right) \right] \exp(-ik \cdot x) = 32\pi^2 \left\{ \frac{1}{2} K_0(t) + \left[ \frac{4}{t^2} K_0(t) + \left( \frac{2}{t} + \frac{8}{t^3} \right) K_1(t) - \frac{8}{t^4} \right] \right\}, \quad t = k\rho. $$

where $K_n(t)$ are modified Bessel functions of the second kind. Note that the tensor structure of eqs.\cite{9,10} is required by (anti-) self-duality. With eqs.\cite{9,10} the fermion vertex resulting from the (anti-) instanton field strength, eq.\cite{7}, becomes 

$$(Y_{F\pm})^a_{\mu\nu}(x)[\psi^\dagger, \psi] = \frac{iM\bar{\rho}^2}{N_c} \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} G(k) \exp(i k \cdot x) F(k_1) F(k_2) \times \psi^\dagger \Gamma_{\mu\nu} \left[ \frac{1 \pm \gamma_5}{2} \right] \psi(k_1) [\lambda^a]_{\mu\nu} \psi(k_2),$$

$$\Gamma_{\mu\nu} = \sigma_{\mu\nu} \frac{k_{\rho} k_{\mu}}{k^2} + \sigma_{\mu\nu} \frac{k_{\rho} k_{\nu}}{k^2} - \frac{1}{2} \sigma_{\mu\nu}, \quad k = k_2 - k_1. \quad (12)$$

With $F^a_{\mu\nu}$ replaced by these non-local quark vertices resulting from instantons and anti-instantons, the operator $O_{\sigma}(x)$ is thus represented by the effective quark operator 

$$“O_{\sigma}”(x)[\psi^\dagger, \psi] = i \psi^\dagger(x) \frac{\lambda^a}{2} \sigma_{\mu\nu} \psi(x) \left[ (Y_{F+})^a_{\mu\nu}(x) + (Y_{F-})^a_{\mu\nu}(x) \right]. \quad (13)$$

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\(^1\)To avoid confusion we note that we are using the convention $\gamma_5 = \gamma_1 \gamma_2 \gamma_3 \gamma_4$ for the euclidean $\gamma_5$, as in \cite{2}.
A graphical representation of the effective operator is given in fig. 1a and b.

We now calculate the average of eq. (13) in the vacuum of the effective quark theory derived from the instanton medium. The quark propagator of the effective theory includes the dynamically generated quark mass, \( S(k) = (\kbar - iMF^2(k))^{-1} \). Due to the color structure of eq. (13), the only non-vanishing contribution to the VEV is given by the diagram of fig. 1c,

\[
\langle \psi^\dagger \psi \rangle_{\text{eff}} = \frac{N_c}{2\pi^2} M\rho^{-4} I^{(2)}(M\bar{\rho}),
\]

\[
I^{(2)}(M\bar{\rho}) = 4\pi^2 \bar{\rho}^6 \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} \frac{G(k) F(k_1) F(k_2) N(k_1, k_2)}{[k_1^2 + M^2 F^4(k_1)] [k_2^2 + M^2 F^4(k_2)]},
\]

\[
N(k_1, k_2) = \frac{i}{4} \text{tr} [\sigma_{\mu\nu}(\slashed{k}_1 + iM(k_1)) \Gamma_{\mu\nu}(\slashed{k}_2 + iM(k_2))]
\]

\[
= \frac{1}{k^2} \left[ 8k_1^2 k_2^2 - 6(k_1^2 + k_2^2)k_1 \cdot k_2 + 4(k_1 \cdot k_2)^2 \right].
\]

The two–loop integral, eq. (14), is UV–convergent, since \( G(k) \sim k^{-4} \) and \( F(k) \sim k^{-3} \) for \( k \rightarrow \infty \). It is also IR–finite, since the singularity of \( G(k) \) at \( k = 0 \) is only logarithmic. Note that \( I^{(2)} \) is dimensionless and depends only on the dimensionless product \( M\bar{\rho} \). Numerical evaluation gives \( I^{(2)}(M\bar{\rho} = 0) = -4 \). (The parametric smallness of \( M^2 \bar{\rho}^2 \), which is a consequence of the diluteness of the instanton medium, is always implied when working with the effective quark theory. With the phenomenological value, \( M\bar{\rho} \approx 0.6 \), one obtains \( I^{(2)} = -3.9 \), which differs from the \( M\bar{\rho} \rightarrow 0 \) limit by only 3\%.) Thus, with the standard parameters of the instanton vacuum [7], \( \bar{\rho}^{-1} = 600 \text{ MeV}, M = 345 \text{ MeV} \), we obtain a value of

\[
\langle \psi^\dagger \psi \rangle_{\text{eff}} = -(490 \text{ MeV})^5.
\]

Within our scheme of approximations this is the result for the mixed condensate in QCD.

The above calculation can easily be generalized to an arbitrary number of quark flavors, \( N_f > 1 \). In this case, the effective fermion vertex corresponding to the gluon operator \( F_{\mu\nu}^a \) is a \( 2N_f \)-fermion vertex. It has the form of eq. (15) for one flavor \( f \) times a ‘t Hooft determinant of the remaining flavors, \( f' \neq f \), summed over all flavors \( f \) [10]. To leading order in \( 1/N_c \) the vacuum averages factorize in eq. (14) times the vacuum average of the ‘t Hooft determinant, which is unity by virtue of the self–consistency condition determining the dynamical quark mass, \( M \) [7, 10]. Thus, for \( N_f > 1 \) flavors the mixed condensate per flavor is again given by eqs. (14, 16).

Let us compute the ratio of the mixed condensate to the quark condensate directly within the instanton vacuum. The quark condensate is given by the trace of the massive quark propagator of the effective quark theory [7],

\[
- i \langle \psi^\dagger \psi \rangle_{\text{eff}} = - \frac{N_c}{2\pi^2} M\rho^{-2} f^{(1)}(M\bar{\rho}),
\]

\[
I^{(1)}(M\bar{\rho}) = 8\pi^2 \bar{\rho}^2 \int \frac{d^4 k}{(2\pi)^4} \frac{F^2(k)}{k^2 + M^2 F^4(k)}.
\]
where \(I^{(1)}(M\bar{\rho} = 0) = 1\). For the parameter \(m_0^2\) of eq.(2) we thus obtain

\[
m_0^2 = -\bar{\rho}^{-2} \frac{I^{(2)}(M\bar{\rho})}{I^{(1)}(M\bar{\rho})}.
\]

(19)

This quantity is positive, since \(I^{(1)}(M\bar{\rho}) > 0\) and \(I^{(2)}(M\bar{\rho}) < 0\) for all realistic values of \(M\bar{\rho}\). Note that the explicit factor of \(M\) in both eq.(14) and eq.(17) has canceled, so that \(m_0^2\) depends on the dynamical quark mass only through the integrals \(I^{(1)}(M\bar{\rho})\), \(I^{(2)}(M\bar{\rho})\). This dependence is very weak. In particular, \(m_0^2\) has a finite limit for \(M\bar{\rho} \to 0\),

\[
m_0^2 = 4\bar{\rho} - 2I^{(2)}(M\bar{\rho} \to 0).
\]

(20)

With the standard value of \(\bar{\rho}\) this comes to \(m_0^2 = 1.4 \text{ GeV}^2\). (At \(M\bar{\rho} = 0.6\) one obtains \(m_0^2 = 4.5\bar{\rho}^{-2}\).)

The limiting value of \(m_0^2\) for \(M\bar{\rho} \to 0\) can be understood in simple terms. The instanton medium is characterized by two parameters, the instanton density, \(N/V\), and average size, \(\bar{\rho}\). Parametrically, the dynamical quark mass behaves as \(M \propto (N/V)^{1/2}\bar{\rho}\), so that the limit \(M\bar{\rho} \to 0\) corresponds to the dilute limit of the instanton medium, \((N/V)\bar{\rho}^4 \to 0\). If \(m_0^2\) approaches a finite value in this limit, it must be possible to explain the limiting value considering only one single (anti–) instanton and its fermionic zero mode. Indeed, with the concrete form of the zero mode wave function in \(x\)–space, \(\Phi_{\pm}^i(x)\), given e.g. in [12, 7], one finds that the integral \(I^{(2)}(M\bar{\rho} = 0)\) is nothing but the “matrix element” of \(O_\sigma\) (with \(F_{\mu\nu}^a\) being the instanton field) between zero mode wave functions,

\[
\langle \Phi_{\pm} | \frac{1}{2} \lambda^a \sigma_{\mu\nu} F_{\mu\nu}^a | \Phi_{\pm} \rangle = \int d^4x \Phi_{\pm}^* (x) \frac{1}{2} (\lambda^a)^\alpha_\beta (\sigma_{\mu\nu})^\alpha_j F_{\pm\mu\nu}^a (x) \Phi_{\pm} (x)
\]

(21)

Similarly, one may verify that \(I^{(1)}(M\bar{\rho} = 0)\) is equal to the normalization integral of the zero mode wave function,

\[
\langle \Phi_{\pm} | \Phi_{\pm} \rangle = \int d^4x \Phi_{\pm}^* (x) \Phi_{\pm} (x) = I^{(1)}(M\bar{\rho} = 0).
\]

(22)

Thus, in the dilute limit, \(M\bar{\rho} = 0\), eq.(19) can be rewritten as

\[
m_0^2 = -\frac{\langle \Phi_{\pm} | \frac{1}{2} \lambda^a \sigma_{\mu\nu} F_{\mu\nu}^a | \Phi_{\pm} \rangle}{\langle \Phi_{\pm} | \Phi_{\pm} \rangle}.
\]

(23)

Expressing the instanton field as

\[
\frac{1}{2} \lambda^a F_{\mu\nu}^a (x) = i[\nabla_\mu, \nabla_\nu] = \partial_\mu - i\frac{1}{2} \lambda^a A_\mu^a (x),
\]

(24)

and making use of the zero mode Dirac equation, \(\nabla \Phi_{\pm} = 0\), one has

\[
\langle \Phi_{\pm} | \frac{1}{2} \lambda^a \sigma_{\mu\nu} F_{\mu\nu}^a | \Phi_{\pm} \rangle = 2 \langle \Phi_{\pm} | \nabla^2 | \Phi_{\pm} \rangle
\]

(25)
A brief calculation, using elementary instanton algebra, then leads from eq.(23) to eq.(20). We note that this instanton result gives a precise meaning to a qualitative argument of \[2\] relating the mixed condensate to the “average squared momentum” of the vacuum quarks.

Although eq.(23) seems intuitively obvious, we should keep in mind that its origin is non-trivial: both the quark condensate and the mixed condensate vanish in the limit $M \to 0$ and thus can not be obtained from a single instanton. However, given the existence of chiral symmetry breaking in the instanton medium at finite density, the ratio of the two condensates is non-zero in the dilute limit and can be explained as a one–instanton property — the ratio of the zero–mode matrix elements of the corresponding operators.

When comparing our results with the value of the mixed condensate extracted from QCD sum rules \[3\] we have to take into account the dependence on the normalization point. To lowest order, the scale dependence of $\langle O_\sigma \rangle$ is given by

$$\langle O_\sigma \rangle(\mu) = \left( \frac{\alpha_s(\mu_{\text{inst}})}{\alpha_s(\mu)} \right)^{\gamma_\sigma/b} \langle O_\sigma \rangle(\mu_{\text{inst}}), \quad (26)$$

where $b = 11N_c/3 - 2N_f/3$ and $\alpha_s(\mu)$ is the one–loop running coupling constant. Here, $\mu_{\text{inst}}$ denotes the scale at which the coupling constant is defined in the instanton vacuum. (We do not need to consider mixing of $O_\sigma$ with the $d = 6$ four–quark condensates here, since we are working in the chiral limit of vanishing current quark mass.) The anomalous dimensions of $O_\sigma$ operator is small, $\gamma_\sigma = -2/3$ \[13\]. When expressing our result in the form eq.(16) we can therefore neglect the scale dependence; this approximation is in fact used in most of the applications of QCD sum rules \[1, 4, 6\].

When considering the ratio of the mixed condensate to the quark condensate, however, the scale dependence becomes essential: the effective anomalous dimension of the parameter $m_0^2$ is $\gamma_\sigma - \gamma_{\bar{\psi}\psi} = -14/3$. The value of $m_0^2$ thus strongly decreases with increasing normalization point. This makes it difficult to compare values of $m_0^2$ obtained in different approaches, the more since the QCD sum rule estimates of \[6\] are based on a value of $\Lambda_{\text{QCD}} = 150$ MeV, which is too small in the light of modern experimental results for $\alpha_s$. (For a discussion of the effect of the value of $\Lambda_{\text{QCD}}$ on QCD sum rules, see \[14\].) Moreover, at the level of approximations considered here, it is not possible to precisely pin down the normalization point of the instanton result, eq.(20). The scale $\mu_{\text{inst}}$ is of order $\bar{\rho}^{-1} = 600$ MeV, but there may be a factor of order unity. Nevertheless, comparing eq.(20) with the QCD sum rule estimate of \[1\], $m_0^2 = 0.8 \pm 0.2$ GeV$^2$ at $\mu = 500$ MeV, identifying for a moment the normalization points, it seems that that our result is larger by $\sim 50\%$. We stress again that the ambiguity in the normalization point concerns only the representation of the mixed condensate through the parameter $m_0^2$, eq.(2), not the value of the mixed condensate itself, eq.(11).

To summarize, we have shown that the instanton vacuum naturally leads to a mixed quark–gluon condensate whose sign and order of magnitude agree with the QCD sum rule estimate. The ratio of the mixed condensate to the quark condensate, $m_0^2$, allows a simple interpretation as the ratio of matrix elements between zero–mode wave functions of one
instanton. The relatively large value of this quantity can be seen as a consequence of the smallness of instantons.

The effective operator formulation of [10] provides a simple and systematic method to evaluate matrix elements of gluon operators such as eq.(1). It allows not only the computation of vacuum condensates, but also of nucleon matrix elements of gluon operators which arise in the description of power corrections in deep-inelastic scattering. Work in this direction is in progress.

We thank D.I. Diakonov for his advice and interest in this investigation, as well as V.Yu. Petrov and P.V. Pobylitsa for many helpful conversations.
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Figure 1: A graphical representation of the effective operator approach. (a) The QCD operator, $O_\sigma$, eq. (1). The dashed end denotes the gluon field. (b) The effective many-fermion operator, $“O_\sigma”[\psi^\dagger, \psi]$, eq. (13). The gluon field is replaced by the non-local fermion vertices, $Y_{F\pm}$. Shown is the case of one quark flavor, $N_f = 1$; in general, $Y_{F\pm}$ is a $2N_f$-fermion vertex. (c) The VEV of the effective operator in the effective quark theory, eqs. (14, 15). The fat lines denote the massive quark propagator, $S(k) = (k - iMF^2(k))^{-1}$. 