REDSHIFTS AND LUMINOSITIES FOR 112 GAMMA-RAY BURSTS

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ABSTRACT

Two different luminosity indicators have recently been proposed for gamma-ray bursts that use gamma-ray observations alone. They relate the burst luminosity \( L \) with the time lag between peaks in hard and soft energies \( \tau_{\text{lag}} \) and the spikiness or variability of the burst’s light curve \( V \). These relations are currently justified and calibrated with only six or seven bursts with known redshifts. We have examined BATSE data for \( \tau_{\text{lag}} \) and \( V \) for 112 bursts.

1. A strong correlation between \( \tau_{\text{lag}} \) and \( V \) exists, and it is exactly as predicted from the two proposed relations. This is proof that both luminosity indicators are reliable. (2) GRB 830801 is the all-time brightest burst, yet with a small \( V \) and a large \( \tau_{\text{lag}} \), and hence it is likely the closest known event, being perhaps as close as 3.2 Mpc. (3) We have combined the luminosities as derived from both indicators as a means to improve the statistical and systematic accuracy when compared with the accuracy from either method alone. The result is a list of 112 bursts with good luminosities and hence redshifts. (4) The burst-averaged hardness ratio rises strongly with the luminosity of the burst. (5) The burst luminosity function is a broken power law, with the break at \( L = 2 \times 10^{52} \) ergs. The numbers in logarithmic bins scale as above the break and as below the break. (6) The comoving number density of GRBs varies with redshift roughly as \( \Omega_{\text{GRB}} \sim z^{-3.5 \pm 0.3} \) between \( 0.2 < z < 5 \). This demonstrates that the burst rate follows the star formation rate at low redshifts, as expected since long bursts are generated by very massive stars. Excitingly, this result also provides a measure of the star formation rate out to \( z \sim 5 \) with no effects from reddening, and the rate is rising uniformly for redshifts above 2.

Subject heading: gamma rays: bursts

1. INTRODUCTION

One of the most fundamental questions in both astronomy and gamma-ray burst (GRB) research is always the distance to sources. From their discovery in 1973 until 1997, the distance scale to GRBs was uncertain by 1 order of magnitude. Since 1997, the discovery of low-energy counterparts (Costa et al. 1997; van Paradijs et al. 1997; Frail et al. 1998) has lead to the measurements of redshifts of GRBs (Metzger et al. 1997), thus proving that most GRBs are at cosmological distances. Nevertheless, just over a dozen bursts currently have known redshifts, and this small sample does not allow detailed demographic studies.

At the Fifth Huntsville Gamma-Ray Burst Symposium in 1999 October, two research groups announced the discovery of two different GRB luminosity indicators, wherein the luminosities and distances could be derived from gamma-ray data alone. The first indicator relates the luminosity to the lag, which is the time delay between the peaks for light curves of energies roughly 25–50 keV and 100–300 keV (Norris et al. 2000). The second indicator relates the luminosity to the variability, which is the variance of the light curve around a smoothed light curve (Fenimore & Ramirez-Ruiz 2002). High-luminosity bursts have near zero lags and spiky light curves, while low-luminosity GRBs have long lags and smooth light curves. Both relations were calibrated with only six or seven bursts with known redshifts, so it is problematic whether the claimed relations are fortuitous since a small number of random points can easily look like a straight line on log-log plots.

The discovery of luminosity indicators that use only gamma-ray data opens the possibility of using the entire BATSE database for demographic work, without having to await the accumulation of optical redshifts. Unfortunately, it might be several years before enough optical redshifts are found to provide independent confirmation of the validity of the luminosity indicators.

In this Letter, we present a means of proving both luminosity indicators without having to measure any additional redshifts. The idea is that the existence of a lag/luminosity relation and a variability/luminosity relation predicts a particular lag/variability relation, and this prediction can be tested with the BATSE data in hand. If either one or both of the two luminosity indicators are not true, then the predicted lag/variability relation will not be found. Since the lag/variability relation can be tested for a large number of bursts independent of the calibration, a successful prediction gives proof that both luminosity relations are correct.

2. LAG/VARIABILITY RELATION

The two luminosity indicators have been originally calibrated with different definitions of the luminosity, which differ substantially for the same burst. For this Letter, we need a simple definition of luminosity that can be readily calculated for many BATSE bursts. So we take the luminosity to be

\[
L = 4\pi D^2 \frac{\Phi_{56}(E)}{(E)} \, .
\]

Here \( D \) is the luminosity distance (for \( H_\odot = 65 \text{ km s}^{-1} \text{Mpc}^{-1} \), \( \Omega = 0.3 \), \( \Lambda = 0.7 \)), \( \Phi_{56} \) is the BATSE peak flux for the 256 ms timescale for 50–300 keV (in units of photons cm\(^{-2}\) s\(^{-1}\)), and \( \langle E \rangle \) is the average energy of a photon for an \( E^{-2} \) spectrum (1.72 × 10\(^{-7}\) ergs photon\(^{-1}\)). This formulation includes a K-correction for an \( E^{-2} \) spectrum, as appropriate for average bursts (Schaefer et al. 1994, 1998). Throughout this Letter, the luminosity is calculated assuming that the radiation is emitted from the source isotropically.

Norris et al. (2000) found that the luminosities of six bursts (with known optical redshifts) are well correlated as a power
Fig. 1.—Lag/variability correlation for 112 BATSE bursts. If both the lag/luminosity and the variability/luminosity relations are true, then there must be a lag/variability relation shown by the straight line. Indeed, the correlation coefficient is $r = -0.45$, which shows a correlation at the 99.999924% confidence level, while the best-fit line has the predicted slope. The intercept is roughly a factor of 2 low, but this is well within the uncertainties. The successes of the predicted correlation (its existence, and slope) prove that both luminosity indicators are valid. The plotted values have redshift effects removed. The measured lags are quantized to 0.064 s, and the $\tau_{lag}$-values measured as zero are displayed as if $\tau_{lag} = 0.032$ for this logarithmic scale.

Lag with the lag for the bursts. The lag, $\tau_{lag}$, is the time delay of the maximum cross correlation between BATSE energy channels 1 (25–50 keV) and 3 (100–300 keV). In essence, the lag is the time between the peaks as viewed with hard and soft photons. Our fit to the data (excluding GRB 980425) gives a lag/luminosity relation of

$$L_{lag} = 2.9 \times 10^{51} (\tau_{lag} / 0.1 \text{ s})^{-1.14},$$

with an rms scatter of 0.26 in the logarithm of luminosity. The exponent has an uncertainty of 0.20. GRB 980425 (the burst associated with SN 1998bw [Galama et al. 1999]) falls greatly below this relation, although its very low luminosity (2.0 $\times$ 10^{45}$ ergs s^{-1}) is indeed qualitatively indicated by its extremely long lag (4 s).

Fenimore & Ramirez-Ruiz (2002) found that the luminosities of seven bursts (with known optical redshifts) are correlated as a power law with the variability of the burst. The variability, $V$, is the normalized variance of the observed 50–300 keV light curve about a smoothed light curve. The smoothing is done with a boxcar window with length equal to 15% of the burst duration. Corrections are made for redshift effects (hence requiring an iterative procedure) and for the Poisson variations of the light curve. The best-fit power law depends substantially on how systematic errors are included, how the formally negative $V$-values are handled, and whether GRB 980425 is included. A typical fit is

$$L_{var} = 10^{52} (V/0.01)^{3.5},$$

with an uncertainty of roughly 1.0 in the exponent and a factor of a few in the proportionality constant. This is essentially the same result as given by Fenimore & Ramirez-Ruiz (2002) and by Reichart et al. (2001) for a subtly improved definition of $V$. The rms scatter about the above relation is roughly 0.6 in the logarithm of the observed luminosity. Again, GRB 980425 falls greatly below this relation, although its very low luminosity is qualitatively indicated by its extremely low $V$.

If both equations (2) and (3) are correct, then we can predict that there should be a lag/variability relation of

$$V = 0.0021 \tau_{lag}^{-0.46}.$$  

To test this prediction, we have taken variability measurements from Fenimore & Ramirez-Ruiz (2002) and lag measurements from Band (1997). The bursts with $V$ measurements were selected by brightness ($P_{256} > 1.5 \text{ photons s}^{-1} \text{ cm}^{-2}$) and duration ($T_{lag} > 20 \text{ s}$). The $\tau_{lag}$ measurements were selected for bursts that were complete for roughly $P_{256} > 3.25 \text{ photons s}^{-1} \text{ cm}^{-2}$. Our lags are quantized to 0.064 s bins, so that an additional uncertainty of 0.032 s should be added in quadrature to the scatter about the calibration curve to obtain the total 1 $\sigma$ error of the lag. For bursts with low lags (i.e., high-luminosity events), this quantization error becomes large. We have 112 GRBs with both $\tau_{lag}$ and $V$ measures. These are plotted in Figure 1.

Figure 1 shows a significant lag/variability correlation. The logarithms of $\tau_{lag}$ and $V$ are correlated with $r = -0.45$, which for 112 data points corresponds to a probability of 7.6 $\times$ 10^{-7} for chance occurrence. Figure 1 also shows the predicted relation from equation (4). The observed slope is close to that of the predicted slope, while the intercept is a factor of 2 low, which is within the uncertainties of equation (4). Thus, both the lag/luminosity and the variability/luminosity relations have passed a severe test involving 112 bursts independent from the original calibration.

We take this successful prediction as strong proof that both luminosity indicators are valid. If only one of them is valid while the other is false, then our observed lag/variability relation must certainly be different than predicted by equation (4). If both luminosity indicators are false, then it would be a very improbable coincidence that the existence, slope, and intercept of our lag/variability relation came out as predicted by equation (4).

3. GRB 830801

GRB 830801 is by far the all-time brightest GRB event known. With a peak flux of 3.0 photons cm^{-2} s^{-1} keV^{-1} averaged from 50 to 300 keV, a dead time correction by a factor of 1.9, and a smooth light curve for the peak 256 ms time interval (Kuznetsov et al. 1986), the peak flux $P_{256}$ is around 1400 photons s^{-1} cm^{-2}. For comparison, BATSE’s brightest burst (GRB 930131, the “SuperBowl Burst”) has only $P_{256} = 105$ photons s^{-1} cm^{-2}.
GRB 830801 was remarked to have no fast light-curve variations beyond the Poisson noise level. Indeed, a look at the light curve shows an extremely smooth event, and the tremendous photon statistics allows this smoothness to be obvious. In other words, GRB 830801 has a very small variability and a very large lag imply that GRB 830801 had a very low luminosity. If we use equation (2), then \( L_{\text{lag}} = 8.5 \times 10^{49} \text{ ergs s}^{-1} \). This yields a luminosity distance of 35 Mpc and \( z = 0.012 \). However, Norris et al. (2000) demonstrate that GRB 980425 is a factor of several hundred below the relation in equation (2), which suggests that the true lag/luminosity relation is a broken power law. If so, then the lag for GRB 830801 implies \( L_{\text{lag}} = 3 \times 10^{47} \text{ ergs s}^{-1} \). This yields a luminosity distance of 3.2 Mpc and \( z = 0.0007 \). Thus, given that GRB 830801 is by far the brightest known burst and is amongst the lowest luminosity events, we know that GRB 830801 must be one of closest bursts, perhaps substantially closer than even GRB 980425 (with SN 1998bw). GRB 830801 has \( T_{90} > 13 \text{ s} \), so currently popular ideas suggest that a Type Ib or Ic supernova should accompany the burst, with a peak around 1983 August 15. For a SN 1998bw–like event (with \( M = -18.88 \pm 0.05 \) at peak [Galama et al. 1998]), the GRB 830801 supernova should have gotten as bright as 14.8 mag (from eq. [1]) or 8.6 mag (with a broken power law to accommodate GRB 980425).

On realizing the possibility that GRB 830801 might have produced a supernova visible in binoculars, our first reaction was to check various supernova catalogs. For this examination, we used the timing triangulation position from the Interplanetary Network (IPN) of burst detectors (on Prognoz 9, Vela 5A, Vela 5B, and ISEE) with a position of 11°58′3, +11°50′7 (B1950) and a 1 σ uncertainty radius of roughly 0.23′. Out of the supernova catalogs, three known events (SN 1985F, SN 1986J, and SN 1983ab) were intriguing but ultimately rejected due to either wrong peak dates or positions (Tsvetkov 1986; Antipin 1996). However, the burst position was 31′ from the Sun when the supernova would have peaked and was in conjunction with the Sun in the middle of September. This can easily explain why no supernova was discovered near peak.

But a bright supernova can still be discovered long after peak with archival plates. We examined the Harvard College collection of plates, for which the Damon series covered the position to a median blue magnitude of 15.2. No supernova was detected on plates DNB3820 (1983 November 7), DNB3875 (1983 December 6), and DNB3998 (1984 February 8). The first image is around 94 days after peak, at which time a SN 1998bw–like event will be 3.0 mag below peak (McKenzie & Schaefer 1999). So we conclude that any supernova associated with GRB 830801 must have peaked fainter than roughly 12.2 mag.

4. LUMINOSITIES AND REDSHIFTS

From § 2, we have strong confidence in the luminosity indicators, so we can derive two independent \( L \)-values for each burst. In general, the \( L_{\text{lag}} \) has a 2–3 times smaller uncertainty than the \( L_{\text{var}} \) (based on the scatter about the calibration curves). However, at high luminosities, the quantization errors in measuring the lag will substantially increase the uncertainties in the derived luminosity. Yet at low luminosities, the variability becomes highly uncertain due to the normal Poisson noise in the light curve. We have combined \( L_{\text{lag}} \) and \( L_{\text{var}} \) as a weighted average to produce a combined \( L_{\text{c}} \)-value. Specifically, we combined the logarithms of the two luminosity measures where the weights are the inverse square of the measurement uncertainty as given in § 2. This luminosity has the accuracy of the lag relation at low luminosities, does not suffer from quantization at high luminosities, and uses all available information. The \( L_{\text{c}} \)-values can be combined with the observed BATSE peak fluxes to derive a luminosity distance (from eq. [1]) and then a redshift (\( z \)).

In all, we have 112 GRBs with both luminosities and redshifts. These are plotted in Figure 2. We find \( L_{\text{c}} \) is between \( 1.4 \times 10^{50} \) and \( 2.1 \times 10^{53} \text{ ergs s}^{-1} \) with a median of \( 2 \times 10^{52} \text{ ergs } \)

**Figure 2.—Luminosity and redshift of 112 BATSE bursts. The burst luminosities were derived from a weighted average of the two luminosity indicators. The redshifts were derived from the luminosities and the measured peak fluxes. The diagonal line is our line of completeness at \( P_{\text{esc}} = 3.25 \text{ photons cm}^{-2} \text{ s}^{-1} \). Cuts in the vertical direction can give the burst luminosity function (see Fig. 3). Cuts in the horizontal direction can give the number density of bursts as a function of redshift (see Fig. 4). So, for example, a horizontal strip around a luminosity of \( 10^{53} \text{ ergs s}^{-1} \) shows that bursts with \( z > 2 \) have a higher rate than bursts with \( z < 2 \).**

![Graph showing luminosity and redshift](image-url)
s$^{-1}$, while the redshift varies between 0.25 and 5.9 with a median of 1.5. If the calibration curves are broken power laws as indicated by GRB 980425, then the lower limits on $L_1$ and $z$ will be substantially lowered. Of these bursts, 96 are above our completeness threshold of $P_{350} > 3.25$ photons s$^{-1}$ cm$^{-2}$.

We have tried to find a signal due to the cosmological dilation of burst light-curve timescales. With our redshifts, we can divide bursts up into fairly narrow bins such that burst timescales should vary as $1 + z$. We have searched for dilation with three timescales: $T_{90}$, the mean peak-to-peak time, and the time from the first-to-last peak. We have found no such correlation. The lack of an apparent dilation effect is easily understood since our sample was selected to have $T_{90} > 20$ s in our rest frame. This range of $T_{90}$ does not include the peak, so all we see is a truncated tail of the distribution. A truncated tail at one redshift is little different from a truncated tail at another redshift, so we should expect little difference. Also, any comparison of high- and low-redshift bursts has the additional complication that the comparison involves bursts of greatly different luminosity, and there might well be luminosity/duration correlations.

We have looked for correlations between burst average spectral hardness and luminosity. A hardness/luminosity relation would not suffer from the definitional problems and the large systematic errors inherent in any analysis and interpretation of a hardness/intensity relation (Schaefer 1992). We find that the hardness ratio between BATSE channels 3 and 1 do change significantly with luminosity in that the luminous bursts are harder than faint bursts. To avoid selection effects from BATSE’s trigger, we can isolate those bursts within small ranges of redshift. For the 48 bursts from $0.5 < z < 1.5$, the hardness increases from $3.2 \pm 0.4$ around $10^{51}$ ergs s$^{-1}$ to $5.5 \pm 0.6$ around $2 \times 10^{52}$ ergs s$^{-1}$, while other redshift ranges have similar shifts.

We find no significant correlation between hardness and redshift, as might have been expected for cosmological shifting of the peak energy. However, as the low-luminosity events must be nearby and the high-luminosity events tend to be very distant, the effect from the previous paragraph will approximately offset the cosmological shift resulting in the lack of any apparent correlation.

The luminosities and redshifts displayed in Figure 2 can be used to derive the GRB number density ($n_{grb}$) as a function of redshift as well as the GRB luminosity function (Fenimore & Ramirez-Ruiz 2002). By taking horizontal strips that do not pass our completeness threshold of $P_{350} = 3.25$ photons s$^{-1}$ cm$^{-2}$, the number of bursts in redshift bins can be divided by the volume to yield a relative number density. By taking vertical strips, the number of bursts in luminosity bins will give the luminosity function. With both procedures, the paucity of bursts far from the completeness threshold implies that any one strip can give only a segment of the desired function, so the complete function must be pieced together with results from multiple strips. This procedure assumes that the evolution of the luminosity function with redshift is small, so that significant evolution might make substantial changes in the results below.

Figure 3 displays our derived luminosity function, taken as the number of bursts appearing within luminosity bins of width $10^{50}$ ergs s$^{-1}$. The luminosity function appears as a broken power law with the break at $2 \times 10^{52}$ ergs s$^{-1}$. This luminosity break does not correspond to the possible breaks in the lag and variability relations suggested on the basis of GRB 980425. The dependence above the break is fitted to be scaling as $L^{-2.8 \pm 0.2}$, while it scales as $L^{-1.7 \pm 0.1}$ below the break.

Figure 4 displays our resulting $n_{grb}(1+z)$ as a function of $z$. The $1+z$ factor is to correct to the comoving frame. The power-law dependence is roughly $(1+z)^{3.5 \pm 0.3}$ for $0.2 < z < 3.5$. The factor is to correct to the comoving frame. The1

$$
\frac{dN}{dL} = \frac{1}{L} \frac{dN}{dz} \left(1 + \frac{z}{H(z)}\right)
$$

where $H(z)$ is the Hubble constant at redshift $z$. The luminosity function now opens the possibility of many exciting demographic studies of GRBs.

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5. This result clearly rejects scenarios for which \( n_{\text{grb}}(1 + z) \) does not evolve with distance. For \( z < 2 \), our result is easily consistent with the burst number density varying as the star formation rate (Steidel et al. 1999), as is expected since long-duration GRBs are formed from the deaths of massive stars (Totani 1997; Hartmann & Band 1998). That is, \( n_{\text{grb}}(1 + z) \) should closely follow the star formation rate in our universe.

However, it is surprising and exciting that \( n_{\text{grb}}(1 + z) \) keeps rising monotonically from \( 2 < z < 5 \). The surprise is because the star formation rate is widely taken to either be flat or fall substantially above a redshift of ~2 (Steidel et al. 1999). But all previous measures have had major problems with reddening at high redshift. Gamma radiation is not affected by reddening, and thus the star formation rate in Figure 4 might be the first view of the true situation.

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