Sum rule approach to a soft dipole mode in \( \Lambda \) hypernuclei

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Hypernuclear physics has attracted lots of attention in recent years. A \( \Lambda \) hyperon is free from the Pauli principle from nucleons, and thus can exist in a deep inside of a nucleus. Because of this property, a \( \Lambda \) hyperon provides various kinds of impurity effect on the core nucleus. One of the well-known examples is a shrinkage of intercluster distance in \( ^7\text{Li} \) and \( ^9\text{Be} \). Other examples include a disappearance of nuclear deformation due to a \( \Lambda \) hyperon, an influence on the triaxial degree of freedom, and an increase of fission barrier height. In addition to the static properties, a \( \Lambda \) hyperon is free from the Pauli principle from nucleons, and thus can exist in a deep inside of the core nucleus. A disappearance of nuclear deformation due to a \( \Lambda \) hyperon, an influence on the triaxial degree of freedom, and an increase of fission barrier height.

In our previous study, we studied multipole vibrational modes of \( \Lambda \) hypernuclei using random-phase approximation (RPA)\(^\text{19}\). We have predicted a novel dipole mode, which we call the soft dipole \( \Lambda \) mode, in a double-\( \Lambda \) hypernucleus \( ^{16}_{\Lambda\Lambda}\text{O} \). This mode appears in the low energy region, and its strength is almost concentrated in a single peak with a magnitude of about a quarter of that of the giant dipole resonance (with the peak height in the strength distribution being about a half of that for the giant dipole resonance). A similar peak appears in various kinds of impurity effect on the core nucleus. One example is the shrinkage of intercluster distance in \( ^7\text{Li} \) and \( ^9\text{Be} \). Other examples include a disappearance of nuclear deformation due to a \( \Lambda \) hyperon, an influence on the triaxial degree of freedom, and an increase of fission barrier height.

In addition to the static properties, a \( \Lambda \) hyperon can also alter nuclear dynamical motions. In particular, low-lying modes of excitation in \( \Lambda \) hypernuclei have been studied with cluster model\(^\text{14}\), ab-initio few-body calculations\(^\text{15,16}\), shell model\(^\text{17}\), 5-dimensional (5D) collective Bohr Hamiltonian\(^\text{18}\), and anti-symmetrized molecular dynamics (AMD)\(^\text{12}\). Together with experimental data, those theoretical studies would help us to understand not only the impurity effect on nuclear structure but also the characteristics of an effective \( \Lambda \)N interaction.

In our previous study, we studied multipole vibrational modes of \( \Lambda \) hypernuclei using random-phase approximation (RPA)\(^\text{19}\). We have predicted a novel dipole mode, which we call the soft dipole \( \Lambda \) mode, in a double-\( \Lambda \) hypernucleus \( ^{16}_{\Lambda\Lambda}\text{O} \). This mode appears in the low energy region, and its strength is almost concentrated in a single peak with a magnitude of about a quarter of that of the giant dipole resonance (with the peak height in the strength distribution being about a half of that for the giant dipole resonance). A similar peak appears in various kinds of impurity effect on the core nucleus. One example is the shrinkage of intercluster distance in \( ^7\text{Li} \) and \( ^9\text{Be} \). Other examples include a disappearance of nuclear deformation due to a \( \Lambda \) hyperon, an influence on the triaxial degree of freedom, and an increase of fission barrier height.

In this paper, we systematically study the soft dipole \( \Lambda \) mode from light to heavy hypernuclei. In order to estimate the excitation energy of the soft dipole \( \Lambda \) mode, we employ the sum rule approach\(^\text{21,22}\). This approach provides a convenient way to estimate the excitation energy, as it can be evaluated only with the ground state wave function. This enables one to study the soft dipole \( \Lambda \) mode in single-\( \Lambda \) hypernuclei, whereas an application of RPA to single-\( \Lambda \) hypernuclei is much more complicated due to the broken time-reversal symmetry and half-integer spins.

Applying the sum rule approach, we investigate the energy of a soft dipole motion of a \( \Lambda \) hyperon against the core nucleus. To this end, we systematically study single-\( \Lambda \) hypernuclei, from \( ^{16}_{\Lambda}\text{O} \) to \( ^{208}_{\Lambda}\text{Pb} \), for which the ground state wave function is obtained in the framework of Hartree-Fock method with several Skyrme-type \( \Lambda \)N interactions. Our results indicate that the excitation energy of the soft dipole \( \Lambda \) mode, \( E_{\text{sdA}} \), decreases as the mass number increases. We find that the excitation energy is well parametrized as

\[ E_{\text{sdA}} = 26.6A^{-1/3} + 11.2A^{-2/3} \text{ MeV} \]

as a function of mass number \( A \).

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\[ \]
is the center of mass of the hypernucleus. Here, $M \equiv m(Z + N) + m_\Lambda$ is the total mass of the hypernuclei, $m = (m_p + m_n)/2 = 938.92$ MeV/$c^2$ and $m_\Lambda = 1115.68$ MeV/$c^2$ being the mass of nucleon and $\Lambda$ hyperon, respectively. $N$ and $Z$ are the number of neutron and proton, respectively.

We rearrange the E1 operator, Eq. (3), as
\[
\hat{F}_\mu = \hat{F}^{(\text{core})}_\mu + \hat{F}^{(\Lambda)}_\mu, \tag{5}
\]
with
\[
\hat{F}^{(\text{core})}_\mu = e \left( \frac{N}{N + Z} \sum_{i \in p} r_i Y_{1\mu}(\hat{r}_i) - \frac{Z}{N + Z} \sum_{i \in n} r_i Y_{1\mu}(\hat{r}_i) \right) \tag{6}
\]
and
\[
\hat{F}^{(\Lambda)}_\mu = -e \frac{Z m_\Lambda}{M} \left( r_\Lambda Y_{1\mu}(\hat{r}_\Lambda) - R_c Y_{1\mu}(\hat{R}_c) \right), \tag{7}
\]
where
\[
R_c = \frac{1}{N + Z} \sum_{i \in n,p} r_i \tag{8}
\]
is the center of mass of the core nucleus. The operator $\hat{F}^{(\Lambda)}_\mu$ given by Eq. (7) is the $E1$ operator which induces the dipole motion between the $\Lambda$ particle and the core nucleus, while $\hat{F}^{(\text{core})}_\mu$, Eq. (6), is identical to the usual E1 operator, which generates an oscillation between protons and neutrons in a normal nucleus.

The soft dipole $\Lambda$ mode is interpreted as the vibration for the relative motion between the $\Lambda$ hyperon and the core nucleus, as we have confirmed with RPA [19]. We thus estimate its energy by using the sum rules for the operator $\hat{F}^{(\Lambda)}_\mu$. The energy weighted sum rule can then be calculated using the Hamiltonian $\hat{H}$ and the wave function of the ground state, $|0\rangle$, as
\[
m_0^\Lambda = \sum_\nu \left| \langle 0 | [\hat{F}^{(\Lambda)}_0, [\hat{H}, \hat{F}^{(\Lambda)}_0]] |0\rangle \right|^2 = \left\langle 0 \left| \left( \hat{F}^{(\Lambda)}_0 \right)^2 \right| 0 \right\rangle \approx \left( e \frac{Z m_\Lambda}{M} \right)^2 \int \rho_\Lambda(r) \rho_\Lambda(r) Y_{10}(\hat{r}_\Lambda)^2 dr_\Lambda
\]
\[
+ \int \rho_c(r) \left( \frac{1}{(N + Z)^2} (r Y_{10}(\hat{r}))^2 \right) dr = e^2 \frac{Z m_\Lambda}{M} \left( \langle r^2 \rangle_\Lambda + \frac{1}{(N + Z)^2} \langle r^2 \rangle_c \right), \tag{11}
\]
where $\langle r^2 \rangle_\Lambda$ and $\langle r^2 \rangle_c$ are the root mean square (rms) radius of the $\Lambda$ hyperon and the core nucleus, respectively. Here, we have assumed the perfect decoupling between the soft dipole $\Lambda$ mode and the giant dipole resonance [21]. The excitation energy for the soft dipole $\Lambda$ mode is then estimated as $E_{\text{sd}}^\Lambda = m_0^\Lambda / m_0^\Lambda$.

Let us now numerically evaluate the energy of the soft dipole $\Lambda$ mode and discuss its mass number dependence. To this end, we study the following nine hypernuclei: $^\Lambda\text{O}_{32}$, $^\Lambda\text{S}_{40}$, $^\Lambda\text{Ca}_{51}$, $^\Lambda\text{V}_{64}$, $^\Lambda\text{Ni}_{63}$, $^\Lambda\text{Sn}_{120}$, $^\Lambda\text{La}_{139}$ and $^\Lambda\text{Pb}_{208}$. We employ the $\Lambda N$ forces constructed in Ref. [28], while we use the SkM* set [29] for the $NN$ interaction. There are 6 parameter sets for the $\Lambda N$ interaction, No. 1-6, in which the sets No. 1, 2, 5, and 6 include the 3-body term (that is, the No. 1, 2, 5, and 6 sets) are denoted by open symbols while those without it (that is, No. 3 and 4 sets) without the 3-body term lead to a deeper potential depth ($D_\Lambda = 32.6$ and 34.6 MeV, respectively).

In solving the Hartree-Fock equations, we assume spherical symmetry for all the nuclei considered in this paper. For odd-mass nuclei, we employ the filling-approximation [30, 31], with which the last occupied orbit is filled only partially so as to reproduce the particle number of the whole system. For the $\Lambda$ hyperon, we assume that it occupies the $1s_{1/2}$ state with the occupation probability of one half.

We first discuss the rms radius for the $\Lambda$ hyperon, $\sqrt{\langle r^2 \rangle_\Lambda}$, to which the non-energy weighted sum rule, $m_0^\Lambda$, is strongly related. Figure 11 shows $\sqrt{\langle r^2 \rangle_\Lambda}$ as a function of $A$, where the results with those $\Lambda N$ interactions with the 3-body term (that is, the No. 1, 2, 5, and 6 sets) are denoted by open symbols while those without it (that is, No. 3 and 4 sets) are by filled symbols. Since the radius of the core nucleus increases with its mass number, it is natural that the rms radius for the $\Lambda$ hyperon, $\sqrt{\langle r^2 \rangle_\Lambda}$, also increases as a function of the mass number, $A$. One can see that the calculated radii show a clear dependence on the $\Lambda N$ interaction adopted. In particular, the interactions with the 3-body term (i.e., No. 1,
FIG. 1: The root mean square radius \( \sqrt{\langle r^2 \rangle_\Lambda} \) for the \( \Lambda \) hyperon in single-\( \Lambda \) hypernuclei as a function of the mass number \( A \), obtained with the Skyrme-Hartree-Fock calculations with six different \( \Lambda N \) interactions.

FIG. 2: The non-energy weighted sum rule for the soft dipole \( \Lambda \) mode as a function of mass number \( A \) calculated with six different \( \Lambda N \) interactions.

2, 5 and 6) yield a larger radius than those without it (i.e., No. 3 and 4). This is because the \( \Lambda \) binding energy calculated with the No. 1, 2, 5 and 6 sets is smaller than that with the No. 3 and 4 sets. As a consequence, the density distribution of the \( \Lambda \) hyperon tends to expand outward. Notice that the No. 2 interaction having the largest \( t_1^3 = 3000 \text{ MeV·fm}^6 \) leads to the largest radii.

Figure 2 shows the non-energy weighted sum rule, \( m_0^\Lambda \). On average, it increases with the mass number \( A \), similarly to the rms radius shown in Fig. 1. It is also similar to the rms radii that the the interactions with the 3-body term yield relatively larger values for \( m_0^\Lambda \). However, in contrast to the rms radii, which increases monotonically as a function of \( A \), the non-energy weighted sum rule \( m_0^\Lambda \) shows a non-monotonic behavior at \( ^{51}_\Lambda \text{V} \) and \( ^{64}_\Lambda \text{Ni} \). This can be attributed to the fact that the factor \((Zm_\Lambda/M)^2\) in Eq. (11) decreases from \( ^{40}_\Lambda \text{Ca} \) to \( ^{51}_\Lambda \text{V} \), and then to \( ^{54}_\Lambda \text{Ni} \) due to the deviation from the \( N = Z \) line (to be more precise, it is the \( N - 1 = Z \) line). That is, the value of \((Zm_\Lambda/M)^2\) is 0.591, 0.534, and 0.518 for \( ^{40}_\Lambda \text{Ca} \), \( ^{51}_\Lambda \text{V} \), and \( ^{64}_\Lambda \text{Ni} \), respectively.

The energy weighted sum rule \( m_1^\Lambda \) is shown in Fig. 3. The No. 1-4 interactions show almost identical lines, as the coefficient of the third term in Eq. (10), \( a_1 = (t_1^3 + t_2^2)/4 \), is identical for these sets (that is, \( a_1 = 26.3 \text{ MeV·fm}^6 \)) [28]. On the other hand, \( a_1 = 0 \) and 45.0 MeV·fm\(^6\) for the No. 5 and 6 parameter sets, respectively, and these parameter sets yield the lowest and the largest value for \( m_1^\Lambda \), respectively. A large decrease of \( m_1 \) from \( ^{40}_\Lambda \text{Ca} \) to \( ^{51}_\Lambda \text{V} \) is again attributed to the decrease of the factor \((Zm_\Lambda/M)^2\) in Eq. (11) due to the deviation from the \( N = Z \) line.

The energy of the soft dipole \( \Lambda \) mode, \( E_{sd\Lambda} \), calculated with \( m_0^\Lambda \) and \( m_1^\Lambda \), is shown in Fig. 4. Because of the minor contribution of the second term in Eq. (11) as
well as a cancellation of the factor $(Z/M)^2$ between $m_{\Lambda}^4$ and $m_{\Lambda}^3$, the energy $E_{sd\Lambda}$ is almost inversely proportional to $(r^2)_{\Lambda}$. As a result, the No. 3 and 4 sets for the $\Lambda N$ interaction, giving small root mean square radii, show large values for $E_{sd\Lambda}$, while the sets No. 1, 2, 5 and 6 provide small values of $E_{sd\Lambda}$. For all the $\Lambda N$ interactions, the energy of the soft dipole $\Lambda$ mode, $E_{sd\Lambda}$, decreases with the mass number $A$ as expected. Assuming that the soft dipole $\Lambda$ mode is close to a single particle excitation of the $\Lambda$ hyperon in a harmonic oscillator potential, the mass number dependence of $E_{sd\Lambda}$ may be parametrized as

$$E_{sd\Lambda} = \alpha A^{-1/3} + \beta A^{-2/3}.$$  \hfill (12)

By performing the least square fit to all the data points obtained with the six different $\Lambda N$ interactions, we obtain the coefficients of the empirical formula as $\alpha=26.6$ MeV and $\beta=11.2$ MeV. The energy obtained with this empirical formula is shown in Fig. 1 by the thick solid line. It well reproduces the average behavior of the excitation energy of the soft dipole $\Lambda$ mode, even though it somewhat overestimates the energy for the $^{208}\Lambda Pb$ nucleus.

Shell model calculations with a meson-exchange $YN$ interaction have been carried out for low-lying excited states in $^{16}O$ [32] and $^{40}Ca$ [33]. The soft dipole $\Lambda$ mode appears in these calculations at around 10 MeV and 8-9 MeV for $^{16}O$ and $^{40}Ca$, respectively, although the nature of the soft dipole $\Lambda$ mode was not discussed in Refs. [32] [33]. These results are in a reasonable agreement with our results obtained with the parameter sets which include the three-body $\Lambda NN$ interaction, that is, No. 1,2,5, and 6.

In summary, we have discussed the soft dipole $\Lambda$ mode in single-$\Lambda$ hypernuclei using the sum rule approach. We have found that the non-energy-weighted sum rule, $m_{\Lambda}^4$, for the soft dipole $\Lambda$ mode depends significantly on the $\Lambda N$ interaction. In particular, the $\Lambda N$ interactions with the 3-body term leads to smaller values of $m_{\Lambda}^4$ as compared to those without the 3-body term. On the other hand, the energy weighted sum rule, $m_{\Lambda}^4$, has a strong dependence on the momentum-dependent terms in the $\Lambda N$ interaction. We have argued that the excitation energy of the soft dipole $\Lambda$ mode is almost inversely proportional to the mean square radius and decreases with mass number. We have derived an empirical formula for the excitation energy, that scales as $E_{sd\Lambda} = 26.2A^{-1/3} + 11.2A^{-2/3}$ MeV.

Our calculations indicate that the soft dipole $\Lambda$ mode appears at around 10 MeV in $^{16}O$, which is in agreement with a shell model calculation with a meson exchange $YN$ interaction. Although the core nucleus, $^{16}O$, is unstable and all the levels are not known exactly, the strength of the soft dipole $\Lambda$ mode of $^{16}O$ is strong, and it could be distinguished experimentally from the other levels associated with the core excitation. In heavier hypernuclei, the energy of the soft dipole mode decreases, and for e.g., $^{208}\Lambda Pb$ it appears at around 4 MeV. In this energy region, there are only 40 discrete levels observed in the core nucleus, $^{207}\Lambda Pb$ [34]. One can thus have a hope to experimentally identify the soft dipole $\Lambda$ mode in single-$\Lambda$ hypernuclei. It would be extremely interesting if such measurement could be realized in some future.

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