The Use of Sparse Direct Solver in Vector Finite Element Modeling for Calculating Two Dimensional (2-D) Magnetotelluric Responses in Transverse Electric (TE) Mode

Lisa’ Yihaa Roodhiyah¹, Tiffany Tjong¹, Nurhasan¹, D Sutarno¹
¹ Electromagnetic Induction Lab., Physics of Complex System Div., Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, Indonesia

Email: lisa_yihaa@s.itb.ac.id

Abstract. The late research, linear matrices of vector finite element in two dimensional(2-D) magnetotelluric (MT) responses modeling was solved by non-sparse direct solver in TE mode. Nevertheless, there is some weakness which have to be improved especially accuracy in the low frequency (10⁻³ Hz-10⁻⁵ Hz) which is not achieved yet and high cost computation in dense mesh. In this work, the solver which is used is sparse direct solver instead of non-sparse direct solver to overcome the weaknesses of solving linear matrices of vector finite element method using non-sparse direct solver. Sparse direct solver will be advantageous in solving linear matrices of vector finite element method because of the matrix properties which is symmetrical and sparse. The validation of sparse direct solver in solving linear matrices of vector finite element has been done for a homogenous half-space model and vertical contact model by analytical solution. The validation result of sparse direct solver in solving linear matrices of vector finite element shows that sparse direct solver is more stable than non-sparse direct solver in computing linear problem of vector finite element method especially in low frequency. In the end, the accuracy of 2D MT responses modelling in low frequency (10⁻³ Hz-10⁻⁵ Hz) has been reached out under the efficient allocation memory of array and less computational time consuming.

1. Introduction
MT is one of the passive electromagnetic methods which measure natural variations of electric and magnetic vector field at the earth surface to map resistivity structure in the subsurface of the earth [1]. One of the methods which is used to model the complex subsurface electrical resistivity structure in MT is vector finite element method. The method can model the electric and magnetic vector fields conceptually correct (divergence-free) based on the Maxwell’s equation so the result is not a spurious solution which is generally attributed to lack of enforcement of the divergence condition [2,3,8]. In vector finite element method, there is a linear problem of matrices which have to solve. It can be written as follow[2],

\[ [K'] \{A'\} = \{b\} \quad (1) \]

where \([K']\) is stiffness matrices, \(\{A'\}\) is vector field which is modeled and \(\{b\}\) is boundary condition.
In late research in 2015, the linear matrices of 2D magnetotelluric modeling using vector finite element method (eq. 1) was solved by non-sparse solver [4]. However, the result showed an unstable responses in low frequency (10^{-3} Hz-10^{-5} Hz) especially in phase respon which is not achieved yet. It is caused by the stiffness matrices (\[ K' \]) properties of vector finite element in magnetotelluric modeling which is ill-conditioned and sparse. Besides, the computational parameters (allocation memory of array and computational time consuming) of non-sparse solver was not efficient in dense mesh due to the huge allocation memory of matrices. Therefore, high cost computation will become the problem of modeling in dense mesh [9].

To overcome these issues, the robust efficient sparse direct solver PARDISO [5] is used here. Basically, it will use the properties of the matrices which is symmetrical and sparse to solve ill-conditioned matrices and also decrease the allocation memory of the matrices [7]. Therefore, it is assumed more efficient than the non-sparse direct solver.

2. Numerical Method
A computer program modeling in FORTRAN using vector finite element method has been developed which is capable of solving second order Maxwell differential equation in 2D MT problem in TE mode which can be written as follows, [6]

\[
\nabla \times \left( \frac{1}{\sigma} \nabla \times \vec{H} \right) + i \omega \mu_0 \vec{H} = 0
\]

(2)

where \( \vec{H} \) is magnetic field, \( i \) is imaginary number, \( \omega \) is angular frequency, \( \mu_0 \) is magnetic permeability of free space, \( \sigma \) is conductivity. After magnetic field value is calculated, then electric field value can be calculated using equation as follow,

\[
\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = \sigma E_x
\]

(3)

where \( E_x \) is x-component of electrical field, \( H_z \) is z-component of magnetic field, \( H_y \) is y-component of magnetic field. The boundary condition which is used is Dirichlet boundary condition. The magnetic field value is set by 1 in the surface and set by 0 in the other sides. Basically, sparse direct solver which is used to solve vector finite element in equation (2) is LU decomposition which is solved in sparse calculation [5]. In the end, responses which are modeled are apparent resistivity and phase response.

3. Result and Discussion
The program validation of 2D magnetotelluric response modeling has been done for homogen half-space model and vertical contact model. Figure 1 shows that for apparent resistivity of homogen half-space model both non-sparse solver and sparse solver have a good agreement with true resistivity (10 ohm.m). Nevertheless, for phase response of homogen half-space model it shows that only sparse direct solver has a good agreement with theoretical phase response of homogen half-space (45°) in frequency below 10^{-4} Hz. Several phase response data which is calculated by non-sparse solver has been scattered in those frequency.

On the other hand, response of vertical contact model shows in figure 2. It shows that apparent resistivity of non-sparse solver has a good agreement with sparse solver. Both of them are not scattered. Based on the roughly calculated error of apparent resistivity using sparse direct solver is 0.78 in frequency 10^{-5} Hz. Then on the other side, error of apparent resistivity using non-sparse direct solver is 0.83 in the same frequency 10^{-5} Hz. However, it shows that the phase response of sparse solver and non-sparse solver does not have a good agreement in frequency below 10^{-4} Hz. The sparse solver is not scattered while non-sparse solver is scattered in those frequency around the boundary of different
resistivity. Next, based on the roughly calculated error of phase using sparse direct solver is 0.16 in frequency ($10^{-5}$ Hz). Then on the other side, error of phase using non-sparse direct solver is 3.27 in the same frequency ($10^{-5}$ Hz).

It is caused by ill-conditioned properties of stiffness matrices in vector finite element which can not be overcome by non-sparse solver. The phase response is more sensitive than apparent resistivity in the boundary of different resistivity due to imaginary value.

Then, The advantages of using sparse direct solver shows in table 1. It shows that sparse solver is more efficient in allocation memory of array and computation time than non-sparse solver.

![Figure 1](image_url)

**Figure 1.** (a) Apparent resistivity and (b) phase responses of homogen half-space model using sparse direct solver and non-sparse direct solver
4. Conclusion
A computer program in FORTRAN to compute numerical solution of 2D MT responses based on vector finite element method utilizing sparse direct solver has been developed. It shows that using sparse direct solver in solving linear matrices of vector finite element has improved the accuracy of apparent resistivity and phase responses of 2-D MT modelling in low frequency (10^{-5} Hz-10^{-3} Hz). Additionally, using sparse direct solver have decreased allocation memory of array and computational time consuming. Thus, 2D MT response modeling for low frequency can be developed under the efficient allocation memory of array and less computational time consuming.
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