Collisions of Main-Sequence Stars and the Formation of Blue Stragglers in Globular Clusters

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ABSTRACT

We report the results of new SPH calculations of parabolic collisions between two main-sequence stars in a globular cluster. Such collisions are directly relevant to the formation of blue stragglers. In particular, we consider parent stars of mass \(M/M_{TO} = 0.2, 0.5, 0.75,\) and 1, where \(M_{TO}\) is the cluster turnoff mass (typically about 0.8 \(M_\odot\)). Low-mass stars (with \(M = 0.2M_{TO}\) or \(0.5M_{TO}\)) are assumed to be fully convective and are therefore modeled as \(n = 1.5\) polytropes. Stars at the turnoff (with \(M = M_{TO}\)) are assumed to be mostly radiative and are modeled as \(n = 3\) polytropes. Intermediate-mass stars (with \(M = 0.75M_{TO}\)) are modeled as composite polytropes consisting of a radiative core with polytropic index \(n = 3\) and a convective envelope with \(n = 1.5\). We focus our study on the question of hydrodynamic mixing of helium and hydrogen, which plays a crucial role in determining the observable characteristics of blue stragglers. In all cases we find that there is negligible hydrodynamic mixing of helium into the outer envelope of the merger remnant. The amount of hydrogen mixed into the core of the merger depends strongly on the entropy profiles of the two colliding stars. For two stars with nearly equal masses (and hence entropy profiles) very little hydrodynamic mixing occurs at all, especially if they are close to the turnoff point. This is because the hydrogen-rich material from both stars maintains, on average, a higher specific entropy than the helium-rich material. If the two parent stars are close to turnoff, very little hydrogen is present at the center of the merger remnant and the main-sequence lifetime of the blue straggler could be very short. In contrast, during a collision between two stars of sufficiently different masses (mass ratio \(q \lesssim 0.5\)), the hydrogen-rich material

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originally in the smaller star maintains, on average, a lower specific entropy than that of the more massive star and therefore settles preferentially near the center of the merger remnant. Through this process, moderately massive blue stragglers (with masses $M_{TO} \lesssim M_{BS} \lesssim 1.5M_{TO}$) can obtain a significant supply of fresh hydrogen fuel, thereby extending their main-sequence lifetime. Based on our results we conclude that, in contrast to what has been done in previous studies, blue stragglers formed by direct stellar collisions should not necessarily be assumed to have initially homogeneous composition profiles. However, we also demonstrate that the final merged configurations produced by our hydrodynamic calculations, although very close to hydrostatic equilibrium, are usually far from thermal equilibrium. Therefore, it is possible that convective or rotationally-induced mixing could occur on a thermal timescale, as the merger remnant recontracts to the main-sequence.

*Subject headings:* celestial mechanics, stellar dynamics – globular clusters: general – hydrodynamics – stars: blue stragglers – stars: evolution – stars: interiors – stars: rotation
1. Introduction and Motivation

Blue stragglers are stars that appear along an extension of the main-sequence (hereafter MS), beyond the turnoff point in the color-magnitude diagram of a cluster. It is generally believed that they are more massive objects (mass $M_{BS} > M_{TO}$) formed by the merger of two MS stars (each of mass $< M_{TO}$). Merging can occur through a physical collision, or following the coalescence of the two components of a close binary system (Leonard 1989; Livio 1993; Stryker 1993; Bailyn & Pinsonneault 1995). Clear evidence for binary coalescence has been found in the form of contact binaries among blue stragglers in the low-density globular clusters NGC 5466 (Mateo et al. 1990) and M71 (Yan & Mateo 1994), as well as in open clusters (Kalužny & Ruciński 1993; Milone & Latham 1994; Jahn, Kalužny & Ruciński 1995). Evidence for stellar collisions comes from recent detections by HST of large numbers of blue stragglers concentrated in the cores of some of the densest clusters, such as M15 (De Marchi & Paresce 1994; Guhathakurta et al. 1995) and M30 (Yanny et al. 1994), and from the apparent lack of binaries in such dense systems (Shara et al. 1995). Collisions can happen directly between two single stars only in the cores of the densest clusters, but even in somewhat lower-density clusters they can also happen indirectly, during resonant interactions involving primordial binaries (Sigurdsson, Davies, & Bolte 1994; Sigurdsson & Phinney 1995; Davies & Benz 1995). Observational evidence for the existence of primordial binaries in globular clusters is now well established (Hut et al. 1992; Cote et al. 1994).

Benz & Hills (1987) performed the first three-dimensional calculations of direct collisions between two MS stars. An important conclusion of their pioneering study was that stellar collisions could lead to thorough mixing of the fluid. The mixing of fresh hydrogen fuel into the core of the merger would reset the nuclear clock of the blue straggler, allowing it to remain on the MS for up to $\sim 10^9$ yr after its formation. In subsequent work it was generally assumed that blue stragglers resulting from stellar collisions were nearly homogeneous, therefore starting their life close to the zero-age MS, but with an anomalously high helium abundance coming from the (evolved) parent stars. In contrast, little hydrodynamic mixing would be expected to occur during the much gentler process of binary coalescence, which could take place on a stellar evolution time scale rather than on a dynamical time scale (Mateo et al. 1990; Bailyn 1992; but see Rasio & Shapiro 1995, and Rasio 1995).

On the basis of these ideas, Bailyn (1992) suggested a way of distinguishing observationally between the two possible formation processes. The helium abundance in the envelope of a blue straggler, which reflects the degree of mixing during its formation process, can affect its observed position in a color-magnitude diagram. Blue stragglers
made from collisions would have a higher helium abundance in their outer layers than those made from binary mergers, and this would generally make them appear somewhat brighter and bluer. The analysis was carried out by Bailyn & Pinsonneault (1995) who performed detailed stellar evolution calculations for blue stragglers assuming various initial profiles. To represent the collisional case, they again assumed chemically homogeneous initial profiles with enhanced helium abundances, calculating the total helium mass from the age of the cluster and the masses of the parent stars.

In this paper we re-examine the question of mixing in stellar collisions. We improve on the previous work of Benz & Hills (1987) by adopting more realistic stellar models, and by performing numerical calculations with increased spatial resolution. We use the smoothed particle hydrodynamics (SPH) method, with \( N = 3 \times 10^4 \) particles for most of our calculations (Benz & Hills 1987 also used SPH, but with only \( N = 1024 \) particles).

The colliding stars in our calculations are modeled as polytropes or composite polytropes (Chandrasekhar 1939; Rappaport, Verbunt, & Joss 1983; Ruciński 1988), and we adopt a simple ideal gas equation of state. The polytropic index \( n \) relates the pressure and density profiles in the star according to \( P \propto \rho^{1+1/n} \). The adiabatic index \( \Gamma_1 = \frac{5}{3} \) for an ideal gas and we write the equation of state \( P = A\rho^{\Gamma_1} \). Here \( A \) is a physical parameter related to the local specific entropy \( s \) according to \( A \propto \exp((\Gamma_1 - 1)s/k) \), where \( k \) is Boltzmann’s constant. When \( \Gamma_1 \neq 1 + 1/n \) the quantity \( A \), and hence the entropy \( s \), has a non-zero gradient. Benz & Hills (1987) used \( n = 1.5 \), \( \Gamma_1 = \frac{5}{3} \) polytropic models to represent MS stars. Unfortunately, such models apply only to very low-mass MS stars with large convective envelopes. For Population II MS stars, the effective polytropic index (defined in terms of the degree of central mass concentration) is close to \( n = 1.5 \) only for a mass \( M \lesssim 0.4 \, M_\odot \) (see Lai, Rasio, & Shapiro 1994, Table 3). The object formed by a merger of two such low-mass stars would hardly be recognizable as a blue straggler, since it would lie below, or not far above, the MS turnoff point (typically \( M_{TO} \approx 0.8 \, M_\odot \)) in a color-magnitude diagram.

Stars near the MS turnoff point have very shallow convective envelopes and are much better described by \( n = 3 \), \( \Gamma_1 = \frac{5}{3} \) polytropes (Eddington’s “standard model”, see, e.g., Clayton 1983). These stars have a density profile much more centrally concentrated than that of an \( n = 1.5 \) polytropes, and this fact has important consequences for the hydrodynamics of collisions. Population II MS stars with masses in the intermediate range \( 0.4 \, M_\odot \lesssim M \lesssim 0.8 \, M_\odot \) can be modeled by composite polytropes with a polytropic index of \( n = 3 \) for the radiative core and \( n = 1.5 \) for the convective envelope.

Stars close to the MS turnoff point in a cluster are the most relevant to consider for stellar collision calculations, for two reasons. First, as the cluster evolves via two-body
relaxation, the more massive MS stars will tend to concentrate in the dense cluster core, where the collision rate is highest (see, e.g., Spitzer 1987). Second, collision rates can be increased dramatically by the presence of a significant fraction of primordial binaries in the cluster, and the more massive MS stars will preferentially tend to be exchanged into such a binary, or collide with another star, following a dynamical interaction between two binaries or between a binary and a single star (Sigurdsson & Phinney 1995).

Lai, Rasio & Shapiro (1993) have calculated collisions between MS stars modeled by $n = 3$, $\Gamma_1 = 5/3$ polytropes, but they focused on high-velocity (hyperbolic) collisions more relevant to galactic nuclei than to globular clusters. The velocity dispersion of globular cluster stars is typically $\sim 10 \text{ km s}^{-1}$, which is much smaller than the escape velocity from the stellar surface. For example, a star of mass $M = 0.8 M_\odot$ and radius $R = R_\odot$ has an escape velocity $(2GM/R)^{1/2} = 552 \text{ km s}^{-1}$. For this reason, we consider only parabolic collisions in this paper, i.e., all initial trajectories are assumed to have zero orbital energy.

Our paper is organized as follows. In §2 we describe our implementation of the SPH method and the numerical setup of our calculations. In §3, we present the models used for MS stars, detailing their assumed structure and chemical composition profiles. We also describe the initial ($t = 0$) configuration of the trajectory. Our results are presented in §4. After describing the results for two typical collisions in detail, we then characterize the rotation states and the final profiles of all our merger remnants. We also present a general method, which does not depend on our particular choice of initial chemical composition profiles, for calculating the final profile of any passively advected quantity in the merger remnant. We conclude our results with an analysis of the numerical accuracy of our simulations. Finally, in §5, we discuss the astrophysical implications of our results as well as directions for future work.
2. Numerical Method and Conventions

2.1. The SPH Code

Our numerical calculations are done using the smoothed particle hydrodynamics (SPH) method (see Monaghan 1992 for a recent review). We used a modified version of the code developed by Rasio (1991) specifically for the study of stellar interactions (see Rasio & Shapiro 1995 and references therein). Since SPH is a Lagrangian method, in which particles are used to represent fluid elements, it is ideally suited for the study of hydrodynamic mixing. Indeed, with the assumption that the gas remains fully ionized throughout the dynamical evolution, chemical abundances are passively advected quantities. Therefore, the chemical composition in the final fluid configuration can be determined after the completion of a calculation simply by noting the original and final positions of all SPH particles and by assigning particle abundances according to an initial profile.

Associated with each SPH particle $i$ is its position $\mathbf{r}_i$, velocity $\mathbf{v}_i$, mass $m_i$ and a purely numerical “smoothing length” $h_i$ specifying the local spatial resolution. An estimate of the fluid density at $\mathbf{r}_i$ is calculated from the masses, positions, and smoothing lengths of neighboring particles as a local weighted average,

$$\rho_i = \sum_j m_j W_{ij},$$

where the symmetric weights $W_{ij} = W_{ji}$ are calculated from the method of Hernquist and Katz (1989), as

$$W_{ij} = \frac{1}{2} \left[ W(|\mathbf{r}_i - \mathbf{r}_j|, h_i) + W(|\mathbf{r}_i - \mathbf{r}_j|, h_j) \right].$$

Here $W(r, h)$ is an interpolation kernel, for which we use the second-order accurate form of Monaghan and Lattanzio (1985),

$$W(r, h) = \frac{1}{\pi h^3} \begin{cases} 
1 - \frac{3}{2} \left( \frac{r}{h} \right)^2 + \frac{3}{4} \left( \frac{r}{h} \right)^3, & 0 \leq \frac{r}{h} < 1, \\
\frac{1}{4} \left[ 2 - \left( \frac{r}{h} \right) \right]^3, & 1 \leq \frac{r}{h} < 2, \\
0, & \frac{r}{h} \geq 2.
\end{cases}$$

In addition to passively advected scalar quantities (such as the hydrogen and helium mass fractions $X_i$ and $Y_i$) each particle $i$ also carries the local entropy variable $A_i$. The specific entropy $s_i$ at $\mathbf{r}_i$ is related to $A_i$ by

$$s_i - s_o = \frac{k}{\Gamma_1 - 1} \ln A_i,$$
where $k$ is Boltzmann’s constant and $s_o$ is a fiducial constant. Neglecting radiation pressure, the pressure at $r_i$ is estimated as

$$p_i = A_i \rho_i^{\Gamma_1},$$

where $\Gamma_1 = \frac{5}{3}$ is the ratio of specific heats for a fully ionized ideal gas. As a self-consistency test, we check that throughout the dynamical evolution the vast majority of particles have $p_i$ remaining much larger than the radiation pressure $\frac{1}{3}aT_i^4$, where $a$ is the radiation constant and $T_i$ is the local temperature (approximated by assuming an ideal gas).

An SPH code must solve the equations of motion of a large number $N$ of Lagrangian fluid particles. Particle positions are updated according to

$$\dot{\mathbf{r}}_i = \mathbf{v}_i.$$  

(6)

The velocity of particle $i$ is updated according to

$$m_i \ddot{\mathbf{v}}_i = \mathbf{F}_i^{(Grav)} + \mathbf{F}_i^{(SPH)}$$

(7)

where $\mathbf{F}_i^{(Grav)}$ is the gravitational force calculated by a particle-mesh convolution algorithm (Hockney and Eastwood 1988, Wells et al. 1990) based on Fast Fourier Transforms (FFT) on a $128^3$ grid, and

$$\mathbf{F}_i^{(SPH)} = -\sum_j m_i m_j \left[ \left( \frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} \right) + \Pi_{ij} \right] \nabla_i W_{ij}.$$  

(8)

Here $\Pi_{ij}$ is an artificial viscosity term, while the rest of equation (8) represents one of many possible SPH-estimators for the local pressure-gradient force $-m_i (\nabla p/\rho)_i$ (see, e.g., Monaghan 1985).

For the artificial viscosity, a symmetrized version of the form proposed by Monaghan (1989) is adopted,

$$\Pi_{ij} = -\frac{\alpha \mu_{ij} c_{ij} + \beta \mu_{ij}^2}{\rho_{ij}},$$

(9)

where $\alpha$ and $\beta$ are constant parameters, $c_{ij} = (c_i + c_j)/2$, and

$$\mu_{ij} = \left\{ \begin{array}{ll} \frac{(\mathbf{v}_i - \mathbf{v}_j) \cdot (\mathbf{r}_i - \mathbf{r}_j)}{h_{ij} (|\mathbf{r}_i - \mathbf{r}_j|^2/h_{ij}^2 + \eta^2)}, & \text{when } (\mathbf{v}_i - \mathbf{v}_j) \cdot (\mathbf{r}_i - \mathbf{r}_j) < 0, \\ 0, & \text{when } (\mathbf{v}_i - \mathbf{v}_j) \cdot (\mathbf{r}_i - \mathbf{r}_j) \geq 0, \end{array} \right.$$  

(10)

with $h_{ij} = (h_i + h_j)/2$. We have used $\alpha = 1$, $\beta = 2$ and $\eta^2 = 0.01$, which provides a good description of shocks (Monaghan 1989, Hernquist and Katz 1989).
To complete the evolution equations of the fluid, $A_i$ is evolved according to a discretized version of the first law of thermodynamics:

$$\frac{dA_i}{dt} = \gamma - 1 \frac{1}{2\rho_i} \sum_j m_j \Pi_{ij} (v_i - v_j) \cdot \nabla_i W_{ij}. \quad (11)$$

Equation (11) has the advantage that the total entropy is strictly conserved in the absence of shocks (i.e., when $\Pi_{ij} = 0$), and the disadvantage that the total energy is only approximately conserved (Rasio 1991; Hernquist 1993). Both total energy and angular momentum conservation are monitored throughout the integrations as a measure of numerical accuracy, and these quantities are conserved typically at the percent level.

The dynamical equations are integrated using a second-order explicit leap-frog scheme. Such a low order scheme is appropriate because pressure gradient forces are subject to numerical noise. We calculate the timestep as $\Delta t = C_N \text{Min}(\Delta t_1, \Delta t_2)$ where $\Delta t_1 = \text{Min}_i \left( h_i / \dot{v}_i \right)^{1/2}$, $\Delta t_2 = \text{Min}_i \left( h_i / (c_i^2 + v_i^2) \right)^{1/2}$ and the Courant number $C_N = 0.8$. Other details of our implementation, as well as a number of test-bed calculations using our SPH code are presented in Lombardi, Rasio, & Shapiro (1995b).

Twenty of the twenty-three calculations reported here employ $N = 3 \times 10^4$ equal-mass particles, while the remaining three calculations (cases U, V and W in Table 2 below) use $N = 1.8 \times 10^4$ equal-mass particles. Unequal-mass SPH particles, sometimes used to allow for higher resolution in low density regions, tend to settle spuriously to preferred regions in the gravitational potential due to numerical discreteness effects. Therefore, calculations with equal-mass particles are best suited for studying mixing. In all cases, time-dependent, individual particle smoothing lengths $h_i$ insure that the spatial resolution remains acceptable throughout the dynamical evolution and that each particle interacts with a constant number of neighbors $N_N \approx 64$. With these resources, the numerical integration of the SPH equations typically takes about 2 CPU hours per time unit (eq. [12]) on an IBM SP-2 supercomputer.

### 2.2. Choice of Units

Throughout this paper, numerical results are given in units where $G = M_{TO} = R_{TO} = 1$, where $G$ is Newton’s gravitational constant and $M_{TO}$ and $R_{TO}$ are the mass and radius of a terminal-age MS (TAMS) star at the cluster turnoff point. The units of time, velocity, and
density are then

\[ t_u = \left( \frac{R_{TO}^3}{GM_{TO}} \right)^{1/2} = 1782 \, \text{s} \times \left( \frac{M_{TO}}{0.8 M_\odot} \right)^{-1/2} \left( \frac{R_{TO}}{R_\odot} \right)^{3/2} \]  

\[ v_u = \left( \frac{GM_{TO}}{R_{TO}} \right)^{1/2} = 391 \, \text{km s}^{-1} \times \left( \frac{M_{TO}}{0.8 M_\odot} \right)^{1/2} \left( \frac{R_{TO}}{R_\odot} \right)^{-1/2} \]  

\[ \rho_u = \frac{M_{TO}}{R_{TO}^3} = 5.90 \, \text{g cm}^{-3} \times \left( \frac{M_{TO}}{0.8 M_\odot} \right) \left( \frac{R_{TO}}{R_\odot} \right)^{-3} \]  

Furthermore, the units of temperature and specific entropy are chosen to be

\[ T_u = \frac{GM_{TO} m_H}{k R_{TO}} = 1.85 \times 10^7 \, \text{K} \times \left( \frac{M_{TO}}{0.8 M_\odot} \right) \left( \frac{R_{TO}}{R_\odot} \right)^{-1} \]  

\[ s_u = \frac{k}{M_{TO}} = 8.68 \times 10^{-50} \, \text{erg K}^{-1} \, \text{g}^{-1} \times \left( \frac{M_{TO}}{0.8 M_\odot} \right)^{-1} \]  

where \( m_H \) is the mass of hydrogen and \( k \) is Boltzmann’s constant.

### 2.3. Determination of the Bound Mass and Termination of the Calculation

The iterative procedure used to determine the total amount of gravitationally bound mass \( M \) of a merger remnant is the same as in Rasio (1991). Namely, particles with negative specific enthalpy with respect to the bound fluid’s center of mass are considered bound. During all of the stellar collisions we considered, only a small fraction (typically a few percent) of the mass is ejected and becomes gravitationally unbound. Some SPH particles, although bound, are ejected so far away from the system center of mass that it would take many dynamical times for them to rain back onto the central remnant and settle into equilibrium. Rather than wait for those particles (which would allow for more spurious diffusion in the central region, see §4.5), we terminate the calculation once we are confident that at least the inner 95% of the mass has settled into equilibrium. We confirm this by two stability tests. First, we check that the specific entropy \( s \) increases from the center to the surface of the merger remnant, a sufficient (and necessary for non-rotating stars) condition for convective stability (see the discussion surrounding eq. [18]). For rotating merger remnants we also check another dynamical stability criteria, namely that the specific angular momentum increases from the poles to the equator along surfaces of constant entropy (Tassoul 1978).
3. Initial Data

We consider parent MS stars of masses $M = 0.2, 0.5, 0.75$ and $1M_{\odot}$. The stellar radii are taken from the results of evolution calculations for Population II stars by D'Antona (1987). Table 1 lists the values we adopt, as well as the radii enclosing mass fractions 0.9 and 0.95. For the $0.5M_{\odot}$ star, we adopt the value $0.29R_{\odot}$ for the radius of the interface between the radiative and convective zones. These values correspond to an interpolation of the results of D'Antona (1987) for an age $t = 15$ Gyr.

The $M = 0.2$ and $0.5M_{\odot}$ stars are modeled as $n = 1.5$ polytropes, whereas the $M = M_{\odot}$ stars are $n = 3$ polytropes. The $0.75M_{\odot}$ stars are modeled as composite polytropes consisting of a radiative core with index $n = 3$ and a convective envelope with $n = 1.5$. Figure 1 shows the specific entropy profiles of these models. The convective regions have constant specific entropy. Note that the specific entropy in the $M = 0.2$ and $0.5M_{\odot}$ stars is everywhere smaller than the minimum specific entropy in the two more massive stars. This fact plays a central role in understanding the dynamics of the merger involving either an $M = 0.2$ or $0.5M_{\odot}$ star with a more massive star.

We have used the stellar evolution code developed by Sienkiewicz and collaborators (cf. Sienkiewicz, Bahcall, & Paczyński 1990) to compute the chemical composition profile in the radiative zones of the $M = 0.75$ and $1M_{\odot}$ parent stellar models. We evolved MS stars of total mass $M = 0.6$ and $0.8M_{\odot}$, primordial helium abundance $Y = 0.25$, and metallicity $Z = 0.001$ for a time $t \simeq 15$ Gyr. This brought the $M = 0.8M_{\odot}$ star to the point of hydrogen exhaustion at the center. In the convective regions of our $M = 0.5, 0.75$ and $1M_{\odot}$ parent stars, we set a constant helium abundance $Y = 0.25$. For the $M = 0.2M_{\odot}$ star, we set $Y = 0.24$ everywhere. Figure 2 shows the resulting profiles, which are used to assign the helium abundance to all the SPH particles in the calculations. The final column in Table 1 gives the total mass fraction of helium in each of the parent stars. Although the composition profiles do not affect the hydrodynamics of a collision in any way, they are needed to determine the chemical composition profile of the merger remnant. In §4.4, we present a method for applying our results to arbitrary initial composition profiles.

The stars are initially non-rotating and separated by at least 4 times the radius of the larger star, which allows tidal effects to be neglected in the initial configuration. The initial velocities are calculated by approximating the stars as point masses on an orbit with zero orbital energy and a pericenter separation $r_p$. The Cartesian coordinate system is chosen such that these hypothetical point masses of mass $M_1$ and $M_2$ would reach pericenter at positions $x_i = (-1)^i(1 - M_i/(M_1 + M_2))r_p$, $y_i = z_i = 0$, where $i = 1, 2$ and $i = 1$ refers to the more massive star. The orbital plane is chosen to be $z = 0$. With these choices, the
center of mass resides at the origin.
4. Results

Table 2 lists the values of the most important initial parameters and final results for all the calculations we performed. The first column gives the label by which the calculation is referred to in this paper. The second and third columns give the masses $M_1$ and $M_2$ of the colliding stars, in units of $M_{TO} \simeq 0.8 M_\odot$. Column 4 gives the ratio $r_p/(R_1 + R_2)$, where $r_p$ is the pericenter separation for the initial orbit and $R_1 + R_2$ is the sum of the two (unperturbed) stellar radii. This ratio has the value 0 for a head-on collision, and 1 for a grazing encounter. Note, however, that an encounter with $r_p/(R_1 + R_2) \gtrsim 1$ can still lead to a direct collision in the outer envelopes of the two stars because of the large tidal deformations near pericenter. We did not attempt to perform any calculations for $r_p/(R_1 + R_2) > 1$ here, for reasons discussed in §5. Column 5 gives the initial separation $r_0$ in units of $R_{TO}$. Column 6 gives the final time $t_f$ at which the calculation was terminated, in the unit of equation (12); see §2.3 for a discussion of how the values of $t_f$ were obtained. Column 7 gives the number $n_p$ of successive pericenter interactions that the stars experience before merging. In general, $n_p$ increases with $r_p$, and it is only for very nearly head-on collisions that the two stars merge immediately after the first impact ($n_p = 1$ in that case). Column 8 gives the mass-loss fraction $1 - M/(M_1 + M_2)$, where $M$ is the mass of the bound fluid in the final merged configuration. Column 9 gives the ratio $T/|W|$ of rotational kinetic energy to gravitational binding energy of the (bound) merger remnant in its center-of-mass frame at time $t_f$. Columns 10 and 11 give the velocity components $V_x$ and $V_y$ in the units of equation (13) for the merger remnant’s center of mass at time $t_f$ in the system’s center-of-mass frame. Since the amount of mass ejected during a parabolic collision is very small, the merger remnant never acquires a large recoil velocity. The largest value of 0.035 in our calculations occurs for case M and corresponds to a physical speed of about $14 \text{ km s}^{-1}$ (for $M_{TO} = 0.8 M_\odot$ and $R_{TO} = R_\odot$). This may be large enough to eject the object from the cluster core, but not to eject it from the entire cluster.

4.1. Discussion of the Results for Two Typical Cases

One of our calculations involving two TAMS stars (Case C) has already been described by Lombardi, Rasio & Shapiro (1995a). In this section we discuss in some detail the results of two other representative cases (E and G).

Figure 3 illustrates the dynamical evolution for Case E: a TAMS star ($M_1 = M_{TO}$) collides with a slightly less massive star ($M_2 = 0.75 M_{TO}$). The initial separation is
\( r_0 = 5R_{TO} \) and the parabolic trajectory has a pericenter separation \( r_p = 0.25(R_1 + R_2) \). The first collision at time \( t \simeq 4 \) disrupts the outer layers of the two stars, but leaves their inner cores essentially undisturbed. The two components withdraw to apocenter at \( t \simeq 7 \), and by \( t \simeq 10 \) are colliding for the second, and final, time (\( n_p = 2 \)). The merger remnant undergoes some large-amplitude oscillations which damp away quickly due to shock dissipation. The final (\( t = 41 \)) equilibrium configuration (see Figure 4) is an axisymmetric, rapidly rotating object \( (T/|W| = 0.07) \). Figure 5 shows SPH-particle values of the angular velocity \( \Omega \) as a function of radius \( r \) in the equatorial plane. We see clearly that the large envelope of the merger remnant is differentially rotating. The uniformly rotating core contains only about 15\% of the mass. The angular velocity drops to half its central value near \( r = 1.1R_{TO} \), and 80\% of mass is enclosed within the isodensity surface with this equatorial radius. Only about 2\% of the total mass is ejected during this collision, and the ejection is nearly isotropic. As a result, the final recoil velocity of the merger remnant in the orbital plane is only about 0.007.

Figure 6 displays the thermal energy \( U \), kinetic energy \( T \), gravitational potential energy \( W \) and total energy \( E = U + T + W \) as a function of time \( t \) for case E. The total energy is conserved to within 2\%. Dips in the potential energy \( W \) correspond either to a collision of the two components before final merging or to a maximum contraction during the subsequent oscillations of the merger remnant. The criterion we use to distinguish collisions (which should be included in the number of interactions \( n_p \) before the stars merge) from oscillations is that the first local maximum of \( W \) which is lower than the previous local maximum occurs immediately after the final merging. The idea behind this criterion is that a collision without merger ultimately tends to increase the system’s gravitational potential energy, whereas a merger will decrease the potential energy. For example, in Figure 6, the local maximum of \( W \) at \( t \simeq 11 \) is lower than the one at \( t \simeq 6 \), so that the dips in \( W \) at \( t \simeq 4 \) and 11 account for the number \( n_p = 2 \) of interactions given in Table 2 for case E. The remaining dips at \( t \simeq 12 \) and 15 correspond to the peak contraction of the merger remnant during oscillations. The value \( n_p = 2 \) obtained here in this way agrees with what one gets simply by direct visual inspection of the system at various times. In some cases, however, visual inspection can be subjective since it is often difficult to recognize two components connected by a bridge of high-density material just prior to final merging.

Figure 7 illustrates the dynamical evolution for case G, which involves a TAMS star (\( M_1 = M_{TO} \)) and a low-mass MS star with \( M_2 = 0.5M_{TO} \) on a head-on parabolic trajectory with initial separation \( r_0 = 5 \). The initial collision occurs at time \( t \simeq 4 \), and the stars never separate again. The resulting isodensity surfaces of the final equilibrium configuration are essentially spherically symmetric (Figure 8). About 6\% of the total mass becomes gravitationally unbound following the collision, and it is ejected preferentially in
the $+x$-direction. Of this ejected material, 95% originated in the more massive ($M = M_{TO}$) star.

Figures 9(a) and (b) show the entropy profiles for the final configurations in cases E and G. Except over the outer few percent of the mass, where equilibrium has not yet been reached (see §2.3), the specific entropy $s$ is an increasing function of the interior mass fraction $m/M$. Here $m$ is the mass inside an isodensity surface, and $M$ is the total bound mass of the merger remnant. The scatter of the points in Figure 9 is real, since isodensity surfaces and surfaces of constant entropy do not coincide. The especially small scatter in Figure 9(b) demonstrates that the entropy does tend towards spherical symmetry in non-rotating merger remnants, despite the strong angular dependence of the shock-heating due to the geometry of the collision.

Even though the density and entropy profiles of both the merger remnant and parent stars are spherically symmetric in case G, this does not imply that the chemical composition must also share this symmetry. Indeed, the effects of anisotropic shock-heating are always evident in the final spatial distribution of the chemical composition. On a constant-entropy surface in the final configuration, particles which have been shock heated the most necessarily had the lowest entropy prior to the collision. Since lower entropy material generally has higher helium abundance (see Figures 1 and 2), shock-heated regions tend to have higher helium abundances. Generally, fluid elements which reside in the orbital plane, and especially those which lie along the collision axis in $r_p = 0$ cases, are shielded the least from the shock. Figure 10 displays the angular distribution of the helium abundance for the merger remnants of cases E and G, near the interior mass fractions $m/M = 0.25, 0.5$ and 0.75. The helium abundance $Y$ peaks in Figure 10(b) and (d) at the polar angle $\theta = \frac{\pi}{2}$ (the equatorial plane), as well as in Figure 10(c) at $\phi = 0$ (the collision axis). In the $r_p \neq 0$ cases, shear in the differentially rotating merger remnant tends to make the profiles axisymmetric (see Fig. 10(a)). However, no dynamical motions exist to circulate the fluid along the meridional directions, and consequently, on an isodensity surface, the fractional helium abundance increases from the poles to the equators for both rotating and non-rotating merger remnants (cf. Fig. 10(b and d)). Meridional circulation will smooth out these deviations from compositional spherical symmetry over a timescale much longer than that treatable by our purely dynamical code (see related discussion in §5). As a practical concern, we note that stellar evolution codes, which can use our merger remnants as initial data, usually assume spherical symmetry. For these reasons, we often average out the angular dependence when presenting composition, and other, profiles.

Figures 11(a) and (b) show the helium mass fraction $Y$ as a function of the interior mass fraction $m/M$ for the final merged configuration in cases E and G, respectively. The
points correspond to the final SPH particle values, with the long-dashed curve representing their average. The spread in the points is due to the mixing of the fluid as well as the fact that the final profiles are not spherically symmetric. Only a small amount of the observed mixing is due to the spurious diffusion of SPH particles, i.e., diffusion which is purely a numerical artifact of the SPH scheme (see §4.5). In case E (Figure 11(a)), there is a small amount of hydrogen in the core, with the innermost 1% of the mass being 85% helium and the inner 25% being 60% helium. For both cases E and G, it is immediately apparent that the helium enrichment in the outer layers is minimal since the fractional helium abundance is just barely above $Y = 0.25$, the value in the outer layers of the parent stars.

The horizontal line at the bottom of Figure 11(b) corresponds to the particles in case G which originated in the less massive parent star, star 2, all of which have a helium abundance $Y = 0.25$. Although these particles are spread over the entire range $0 < m/M < 1$ in the merger remnant, they are found preferentially near the center. Of all the particles which originated in star 2, 69% ultimately end up with $m/M < 0.25$, while only 6% end in the range $0.75 < m/M < 1$. Essentially, the entire star 2 has sunk to the center of the merger remnant, displacing the material in star 1 and leaving only a small amount of shock heated gas in the remnant’s outer envelope. Consequently, the hydrogen enrichment in the core is quite pronounced; all of the innermost 22% of the mass originated in star 2 and is therefore 75% hydrogen. Furthermore, the helium abundance jumps to a maximum average abundance exceeding $Y = 0.7$ near $m/M = 0.3$. The subsequent stellar evolution of an object with such an atypical chemical abundance profile could be quite peculiar.

4.2. Rotational Properties of the Merger Remnants

The collisions with $r_p \neq 0$ result in rapidly, differentially rotating merger remnants. Rotating fluid configurations with $T/|W| \gtrsim 0.14$ are secularly unstable, and those with $T/|W| \gtrsim 0.26$ are dynamically unstable (Chandrasekhar 1969; Shapiro and Teukolsky 1983, Chap. 7). The final merged configurations are, by definition, dynamically stable, but they could in principle be secularly unstable. Although some of our calculations produce merger remnants close to the secular instability limit, none of them exceed it. However, extrapolation of our results to larger values of $r_p$ suggests that secular instabilities could well develop in some merger remnants.

Table 3 lists the values of the central angular velocity $\Omega_0$ in the equatorial plane as well as other quantities characterizing the rotation in the outer layers of the merger remnants. Specifically, for the two mass fractions $m/M = 0.9$ and $m/M = 0.95$, we give the values of
the polar and equatorial radii $r_p$ and $r_e$, the angular velocity $\Omega$ in the equatorial plane, and the ratio $\Omega^2 r_e/g$ of centrifugal to gravitational acceleration in the equatorial plane. We see that $\Omega^2 r_e/g$ can be a significant fraction of unity, indicating that some configurations are rotating near break-up. The central angular velocity $\Omega_0$ is typically an order of magnitude larger than the angular velocity $\Omega$ at $m/M = 0.95$.

Figure 12 shows contours of the specific angular momentum $\Omega \varpi^2$, where $\varpi$ is the cylindrical radius measured from the rotation axis, in the vertical $(x, z)$ plane for several representative cases. The outermost bounding curves correspond to $m/M = 0.95$. Clearly, the merger remnants are not barotropic since the condition $d\Omega/dz = 0$ is not satisfied everywhere. The implications of this result will be discussed in §5.

4.3. Interior Structure of the Merger Remnants

Figures 13(a)–(g) show the variation of the density $\rho$, relative specific entropy $s - s_o$, helium fraction $Y$ and temperature $T$ as a function of $m/M$ for all merger remnants. The density and entropy profiles are fundamental in the sense that they do not depend on the assumed initial helium profiles. The entropy and helium profiles have been averaged over isodensity surfaces. The temperature profile is calculated from the entropy and helium profiles by setting the pressure $p = \rho kT/\mu$ equal to $p = A\rho^{\Gamma_1}$, solving for $T$ and using equation (4). Here the mean molecular weight $\mu$ is given by
\begin{equation}
\mu = m_H (2X + \frac{3}{4}Y + \frac{1}{2}Z)^{-1},
\end{equation}
where $m_H$ is the mass of hydrogen and $X$, $Y$ and $Z$ are the fractional abundances of hydrogen, helium and metals. For Population II stars, $Z \sim 10^{-4} - 10^{-3}$ and the precise value does not significantly affect the calculated temperature profiles.

Note the peculiar shapes of some of the temperature and helium profiles in Figure 13. For example, often the temperature or helium abundance reaches its maximum value somewhere other than the center of the star. Although these configurations are very close to hydrostatic equilibrium, it is clear that they are not in thermal equilibrium (see related discussion in §5). These unusual profiles suggest that we look at the condition for convective stability more carefully. For a non-rotating star, this condition can be written simply as
\begin{equation}
\frac{ds}{dr} > 0,
\end{equation}
where $s$ is the local specific entropy (see, e.g., Landau & Lifshitz 1957, §4). When written in terms of temperature and composition gradients, equation (18) becomes, for an ideal gas,

$$\frac{1}{T} \frac{dT}{dr} > \frac{1}{T} \left( \frac{dT}{dr} \right)_{ad} + \frac{1}{\mu} \frac{d\mu}{dr}$$

which is known as the Ledoux criterion (Kippenhahn & Weigert 1990, Chap. 6). Here the subscript $ad$ denotes that the derivative is to be taken at constant entropy. Most of our merger remnants have composition gradients, and it is in the regions where $d\mu/dr > 0$ that equation (19) requires $dT/dr > 0$ for stability. For chemically homogeneous stars, the second term on the right-hand side of equation (19) vanishes, and the familiar Schwarzschild criterion results. Although equation (18) is quite general, it does require slight modification for rotating stars (see Tassoul 1978, Chap. 7).

Figure 13 (a, d, and f) demonstrate that merger remnants formed from equal mass parent stars have composition profiles which mimic those of the parents, as can be seen by comparing the resulting helium profiles to the corresponding parent profiles in Figure 2. In Figure 13(f), all of the merger remnants have $Y = 0.25$ for all $m/M$, which is simply because the fully convective parents stars in these cases had $Y = 0.25$ everywhere.

We see from Figure 13(c) that the central specific entropy of the merger remnants increases with $r_p$, which can explain the qualitatively different shapes of the corresponding helium abundance profiles. This increase occurs because the number of interactions $n_p$, and hence the level of shock heating in star 2 (the smaller star), increases with $r_p$. The shock-heating in the central region of star 1 is less sensitive to $n_p$, since the outer envelope absorbs the brunt of the shock. For case G (solid line), $n_p = 1$ and much of star 2 is able to maintain a lower specific entropy than the minimum value in star 1. Since low-entropy material tends to sink to the bottom of the gravitational potential well, the merger remnant’s core consists entirely of fluid originally from star 2 and therefore with a helium abundance $Y = 0.25$. For case H (long-dashed line), $n_p = 2$ and, although the central fractional helium abundance is still 0.25, there is enough shock heating for the fluid at small $m/M$ to be affected by contributions from both stars. For case I (short-dashed line), $n_p = 3$ and the additional shock heating is sufficient to prevent most of the fluid from star 2 from reaching the center of the remnant, which consequently is not significantly replenished with hydrogen.

Figure 13(g) displays the profiles for merger remnants resulting from collisions between two stars of masses $M_1 = M_{TO}$ and $M_2 = 0.2M_{TO}$. In the head-on case, the less massive star (star 2) plummets so quickly to the center that there is significant shock-heating in the core of star 1, where the highest fractional helium abundance resides. This causes the helium-rich material to be spread throughout a larger region of the merger remnant, and
the resulting helium profile is not as sharply peaked as in cases V and W.
4.4. Calculating Final Profiles

In order to keep our results useful for future applications, we now present a simple and general method for constructing the final composition profiles in our calculated merger remnants, corresponding to any assumed initial composition profiles. From our results, we extract some simple functions which can be applied to transform, for example, any given initial helium abundance profiles into a final helium abundance profile for the merger remnant. Indeed, these transfer functions allow one to find the final profile of any passively advected quantity, provided only that the profiles of that quantity in the parent stars are both known and spherically symmetric.

Table 4 and Figure 14 establish a correlation between the initial and final mass fractions of a fluid particle. They also demonstrate that the details of how the fluid elements are mixed during a collision can be quite complicated. Table 4 treats six of the collisions between equal-mass stars. The parent stars and merger remnant are partitioned into zones according to interior mass fractions. For every zone in the final configuration, we list the fraction of particles which originated in each of the initial zones. Although there is definitely a preferred final mass fraction $m/M$ for a given initial mass fraction $m_i/M_i$, there is always a range of $m/M$ obtainable. In Figure 14, which is for case G, this range of mass fractions is evident in the spread of points around a preferred average. The lower band of points surrounding the solid line correspond to particles which originated in star 1, while the upper band surrounding the dashed line correspond to particles which originated in star 2. The lines correspond to the average initial mass fraction $\langle m_i/M_i \rangle$ for stars $i = 1, 2$ as a function of the final mass fraction $m/M$, obtained by binning values of $m/M$. In contrast, note that if the parent stars were completely mixed by the collision then the points would be distributed uniformly over the entire plot with an average initial mass fraction $\langle m_i/M_i \rangle = \frac{1}{2}$ for all $m/M$.

Let us define $p_1 = p_1(m/M)$ to be the probability that a particle with final mass fraction $m/M$ originated in star 1. Obviously, $1 - p_1$ is then the probability that the particle originated in star 2. With this definition we can approximate the final profile of any passively advected quantity $Q$ according to

$$Q \left( \frac{m}{M} \right) \simeq p_1 \left( \frac{m}{M} \right) Q_1 \left( \frac{m_1}{M_1} \bigg| \frac{m}{M} \right) + \left( 1 - p_1 \left( \frac{m}{M} \right) \right) Q_2 \left( \frac{m_2}{M_2} \bigg| \frac{m}{M} \right),$$

where $Q_i$ are the initial (spherically symmetric) profiles for that quantity in the parent stars $i = 1, 2$. The quantities $\langle m_i/M_i \rangle_{m_i/M}$ which appear in equation (20) are the average initial mass fractions, such as the ones in Figure 14, evaluated at the final mass fraction $m/M$. If
all particles at $m/M$ came from a single value of $m_i/M_i$, then equation (20) would be exact. In addition, if the initial profiles are linear over the range of $m_i/M_i$ which contributes to the abundance at $m/M$, then the above relationship is exact.

Figures 15(a)–(g) give the average mass fractions $\langle m_i/M_i \rangle$ as a function of $m/M$ for all of our collisions, while Figure 16 gives the function $p_1 = p_1(m/M)$ for the 12 collisions involving parent stars of unequal mass. For collisions involving two identical stars we necessarily have $\langle m_1/M_1 \rangle = \langle m_2/M_2 \rangle$ and $p_1 = \frac{1}{2}$ for all $m/M$. The solid, long dashed and short-dashed lines correspond to pericenter separations $r_p$ of 0, 0.25, and 0.5$(R_1 + R_2)$, respectively; in Figure 15(f) the dot-dashed and dotted lines refer to case S ($r_p = 0.75(R_1 + R_2)$) and case T ($r_p = 0.95(R_1 + R_2)$), respectively. Note that the horizontal line $\langle m_i/M_i \rangle = \frac{1}{2}$ would correspond to the fluid of star $i$ being completely mixed throughout the merger remnant, which is not the case for any of our calculations. For collisions involving equal-mass stars, if there were no shock heating and no mass loss then every particle would have identical initial and final mass fractions (i.e., $m_i/M_i = m/M$) so that the merger remnant’s helium profile would be the same as in the parent stars. In the equal-mass cases A, B, C, J, K and L, we do find $\langle m_i/M_i \rangle \simeq m/M$ and the final helium profiles are indeed quite similar to the parent profile, as shown in §4.3.

Along with equation (20), the functions of Figures 15 and 16 provide the means for approximating the final profile of any passively advected quantity. As a concrete example of how to use this method, we will now calculate the fractional helium abundance at $m/M = 0.28$ in the merger remnant of case G, using the same initial profiles as shown in Figure 2. From the solid lines corresponding to case G in Figures 15(c) and 16, we find that $\langle m_1/M_1 \rangle|_{0.28} = 0.05$, $\langle m_2/M_2 \rangle|_{0.28} = 0.65$ and $p_1(0.28) = 0.62$. Therefore,

$$Y(0.28) \simeq 0.62 \times Y_1(0.05) + (1 - 0.62) \times Y_2(0.65) = 0.70,$$

(21)

where we have used $Y_1(0.05) = 0.97$ and $Y_2(0.65) = 0.25$, obtained from the solid and short-dashed lines of Figure 2, respectively. By repeating this calculation for other values of $m/M$, we construct the approximate helium profile shown as the dashed line in Figure 17. Also shown for comparison is the “exact” profile (solid line) constructed by considering the individual helium abundance carried by each particle (the same curve which appears in Figure 11(b)). We consider the agreement to be quite good, given the simplicity of the approximation scheme and the fact that it does not require access to large data files containing information on all $N = 3 \times 10^4$ particles.
4.5. Effects of Spurious Diffusion on Observed Mixing

In all SPH calculations, numerical noise can lead to spurious, or artificial, diffusion of SPH particles. In order to estimate how much of the observed mixing is in fact caused by particle diffusion, we have performed a series of systematic tests to evaluate quantitatively the effects of spurious transport in SPH calculations (Lombardi, Rasio, & Shapiro 1995b). In particular these tests measure, as a function of the neighbor number \( N_N \) and local noise level \( v_{\text{rms}} \) (the root mean square particle velocity deviation from the local mean), dimensionless spurious diffusion coefficients defined by

\[
D \equiv \frac{n^{1/3} d \Delta r_s^2}{c_s dt}
\]  

(22)

where \( \Delta r_s \) is the distance traveled by a particle due to spurious diffusion, \( n \) is the local number density of SPH particles, and \( c_s \) is the local sound speed.

Once measured, these diffusion coefficients can be applied to each particle in our simulations by monitoring its local values of \( v_{\text{rms}} \), \( n \) and \( c_s \) and then estimating how far that particle has spuriously diffused by numerically integrating

\[
\Delta r_s^2 = \int D \frac{n^{-1/3}}{c_s} dt.
\]  

(23)

From the local density gradient at the particle’s final position we can then estimate the equivalent mass fraction corresponding to this displacement according to

\[
\Delta m_s \simeq \frac{\Delta r_s^3}{3^{1/2} |\nabla \rho|}.
\]  

(24)

By repeating this procedure for all the particles, we arrive at an average spurious diffusion distance \( \langle \Delta r_s \rangle \) and mass fraction \( \langle |\Delta m_s| \rangle \), as well as a root-mean-square spurious displacement \( \langle \Delta r_s^2 \rangle^{1/2} \) and mass fraction \( \langle \Delta m_s^2 \rangle^{1/2} \). This method of estimating spurious diffusion distances will be referred to as Method I.

In the case of a head-on collision, another method (which we call Method II) can be used, which exploits the axisymmetry around the collision axis (the x-axis). If we make the assumption that the entire dynamical evolution remains axisymmetric (this would not be the case if, e.g., Rayleigh-Taylor instabilities were to develop), then a particle should always remain in the plane containing the collision axis and the particle’s initial position. The extent to which a particle diffuses in the direction perpendicular to this plane provides an estimate of the spurious diffusion distance. Assuming the diffused amount is the same in all three directions, the total spurious diffusion distance is then estimated simply by
multiplying by $3^{1/2}$. We finally convert the spurious diffusion distance to an equivalent mass fraction exactly as in Method I (see eq. [24]).

The results of the two methods applied to our calculations are given in Table 5. When two numbers are given, the second one is calculated by Method II. Included in this table are the average spurious diffusion distance $\langle \Delta r_s \rangle$, the root-mean-square diffusion distance $\langle \Delta r_s^2 \rangle^{1/2}$, the average equivalent mass fraction $\langle |\Delta m_s| \rangle / M$, and the root-mean-square mass fraction $\langle \Delta m_s^2 \rangle^{1/2} / M$. It is clear that the two methods are generally in good agreement, and that, when expressed in terms of $m/M$, the effects of spurious diffusion are always small.

Also listed in Table 5 are the observed average ($\langle |\Delta m_o| \rangle / M$) and root-mean-square ($\langle \Delta m_o^2 \rangle^{1/2} / M$) total deviation in final mass fraction. The last column then subtracts from the total square deviation $\langle \Delta m_o^2 \rangle$ the contribution $\langle \Delta m_s^2 \rangle$ from spurious diffusion. For example, in case G, we observe a root-mean-square change in the interior mass fraction of $\langle \Delta m_o^2 \rangle^{1/2} / M = 0.09$ over the entire calculation. Using Method I, we estimate that the root-mean-square change in interior mass fraction due to spurious diffusion is $\langle \Delta m_s^2 \rangle^{1/2} / M \simeq 0.036$, while Method II gives an estimate of 0.057 for this quantity. We therefore believe that the physical root-mean-square spread (i.e., the spread in a calculation free of spurious diffusion) would be approximately 0.08 or 0.07, depending on whether Method I or Method II is more accurate.
5. Summary and Discussion

The main results of this paper can be summarized as follows. We have demonstrated that the typical merger remnants produced by collisions are rapidly and differentially rotating, and are far from chemically homogeneous, with composition profiles that can be rather peculiar in certain cases. For example, it often happens that the maximum helium abundance does not occur at the center of the remnant (cf. Fig. 11(b)). The merger remnants produced by our dynamical calculations, although very close to hydrostatic equilibrium, are usually far from thermal equilibrium, as discussed below. In particular, we have shown that the merger remnants are not barotropes (i.e., the condition $d\Omega/dz = 0$ is generally not satisfied), and that their temperature profiles can have positive gradients ($dT/dr > 0$) in certain regions.

At a qualitative level, many of our results can be understood very simply in terms of the requirement of convective stability of the final merger remnant. If entropy production in shocks could be neglected (which may be reasonable for parabolic collisions, especially in the head-on case), then one could predict the qualitative features of the remnant’s composition profile simply by observing the composition and entropy profiles of the parent stars. Convective stability requires that the specific entropy $s$ increase from the center to the surface ($ds/dr > 0$) in the final hydrostatic equilibrium configuration. Therefore, in the absence of shock-heating, fluid elements conserve their entropy and the final composition profile of a merger remnant could be determined simply by combining mass shells in order of increasing entropy, from the center to the outside. Many of our results follow directly. For example, in the case of a collision between two identical stars, it is obvious why the composition profile of the merger remnant remains very similar to that of the parent stars. For two stars of very different masses, the much lower-entropy material of the lower-mass star tends to concentrate at the center of the final configuration, leading to the unusual composition and temperature profiles seen in Figure 13 (c, e, and g).

Regions where the dynamical stability criterion $ds/dr > 0$ (eq. [18]) is satisfied can nevertheless be thermally, or secularly, unstable. The small vertical oscillations (at the local Brunt-Väisälä frequency $\Omega_{BV} \propto [ds/dr]^{1/2}$) of a fluid element in such a region have amplitudes that grow slowly, and mixing will occur on a timescale comparable to the local radiative damping time (see, e.g., Kippenhahn & Weigert, Chap. 6). The thermal instability can be of two types. When $d\mu/dr > 0$ and $dT/dr > 0$ (as in Figure 13 (c, e, and g)), a so-called thermohaline instability can develop, allowing fingers of the high-$\mu$ material to penetrate down into the lower-$\mu$, colder material below (see, e.g., Ulrich 1972). When such mixing occurs in the stellar core, it tends to increase the central helium abundance and therefore decrease the time that the merger remnant can remain on the MS.
When \( d\mu/dr < 0 \) but \( dT/dr < (dT/dr)_{ad} \), so-called semiconvection occurs (Spruit 1992). In terms of easily computed SPH variables, this criterion is equivalent to

\[
0 < \frac{1}{A} \frac{dA}{dr} < -\frac{\Gamma_1}{\mu} \frac{d\mu}{dr}
\]

(25)

where \( A \) is related to specific entropy by equation (4). We have tested our merger remnants formed from head-on collisions and found that this instability is typically present. Figure 18 shows, as a function of the final mass fraction \( m/M \), the fraction \( f_{sc} \) of gas which is semiconvective for six of our merger remnants. In all cases no semiconvective instability exists in the outer \( \sim 20\% \) of the mass, so that we do not expect this mixing mechanism can increase the helium abundance of the outer layers. Figure 18 does demonstrate, however, that some merger remnants (those of cases A, D and J) have an unstable region which extends to the center, and these remnants therefore have a means of mixing hydrogen into their cores. For instance in case A, we see that the inner \( \sim 40\% \) could be significantly affected. As semiconvection slowly attempts to decrease the central helium fraction, it must compete against hydrogen burning. In addition, the right hand side of equation (25) changes as the fluid mixes, so that the details of this complicated process can only be followed numerically with a stellar evolution code.

For a rotating, chemically homogeneous star, stable thermal equilibrium requires \( d\Omega/dz = 0 \), where \( \Omega \) is the angular velocity and \( z \) is measured along the rotation axis (the Goldreich-Schubert stability criterion; see, e.g., Tassoul 1978, Chap. 7). From the representative set of specific angular momentum contours presented in Figure 12, it is therefore evident that the merger remnants of cases Q, R, S and T (which are chemically homogeneous, since their parent stars were fully mixed) cannot be in thermal equilibrium. In chemically inhomogeneous stars, regions with a sufficiently large and stabilizing composition gradient \( (d\mu/dr < 0) \) can in principle still be thermally stable even with \( d\Omega/dz \neq 0 \). However, it seems unlikely that the composition profiles generated dynamically by a collision would conspire to keep the remnants everywhere thermally stable.

Deupree (1990) has shown that stars with rapidly rotating cores and slowly rotating envelopes can have their MS lifetime extended beyond that of their non-rotating counterparts. The fact that much of the angular momentum is hidden deep in the remnant’s interior suggests a physical explanation for why observations of blue stragglers in open clusters such as M67 find no signs of rapid rotation (Peterson, Carney, & Latham 1984; Mathys 1991). Recently, Leonard & Livio (1995) have argued that the spin-down timescale of blue stragglers due to magnetic breaking should be on the order of only \( 10^5 \) years, which is much less than the thermal timescale of approximately \( 10^7 \) years, so that initially rapidly rotating merger remnants may not be a problem for the collisional formation scenario.
Stellar encounters with separations $r_p$ larger than those considered in this paper are difficult to compute directly with SPH since the amount of energy $\Delta E$ dissipated during the first interaction is then so small that the integration time until the next pericenter passage can be several orders of magnitude larger than the hydrodynamic time. We believe that our results can be safely extrapolated all the way to values of $r_p \approx 1.2(R_1 + R_2)$. For instance, from our results for collisions of equal-mass stars it seems very likely that the helium profile of the merger remnant will always mimic that of the parent stars. In Figure 13(f), note that the density, entropy and temperature profiles also seem to be converging onto a fixed profile, and that the profiles for the $r_p = 0.5, 0.75$ and $0.95(R_1 + R_2)$ cases all look very similar. For $r_p \gtrsim 1.2(R_1 + R_2)$ the encounter is better described as a tidal capture than a collision, i.e., the amount of energy dissipated is sufficient to form a bound system, but no direct collision occurs, even in the outer layers of the stars. The maximum value $r_p = r_{cap}$ for tidal capture can be calculated accurately from linear perturbation theory (Press & Teukolsky 1977; McMillan, McDermott, & Taam 1987). For two identical $0.8 M_\odot$ MS stars and a relative velocity at infinity $v_\infty = 10 \text{ km s}^{-1}$, McMillan et al. (1987) find $r_{cap}/(R_1 + R_2) \approx 1.4$, which leaves little room for “clean” tidal captures. In addition, the long-term evolution of a tidal-capture binary may well lead to merging of the two stars even if the initial interaction is in the linear regime (for recent discussions, see Mardling 1995a,b and Kumar & Goodman 1995).

It must be stressed that the amount of mixing determined by SPH calculations is always an upper limit. Indeed, some of the mixing observed in a calculation will always be a numerical artifact. Low-resolution SPH calculations in particular tend to be very noisy and the noise can lead to spurious diffusion of SPH particles, independent of any real physical mixing of fluid elements. In §4.5 we introduced two simple methods to evaluate quantitatively the effects of spurious diffusion in our calculations. The results suggest that spurious diffusion does not significantly corrupt our results. This is seen for example in the near agreement of numbers in the last two columns of Table 5. However, both methods are approximate. The diffusion coefficients used in Method I have been measured in the absence of artificial viscosity. The presence of artificial viscosity in our collision calculations may slightly change the effective values of these coefficients. Method II assumes that we can neglect any nonaxisymmetric instabilities in a head-on collision and that spurious diffusion is isotropic (which may not be true in the presence of strong entropy gradients). Nevertheless, the reasonable agreement between the two methods (cf. Table 5) gives us confidence that these simplifying assumptions are generally satisfied to a good approximation. The general question of spurious transport in SPH calculations will be addressed in a separate paper (Lombardi et al. 1995b).

Benz & Hills (1987) performed the first fully three-dimensional calculations of collisions
between two identical MS stars. In contrast to the present work, they considered only $n = 1.5$ polytropes, which are mostly relevant for collisions of very low-mass stars ($M_1 = M_2 \lesssim 0.4M_\odot$). Their calculations for parabolic collisions indicated a higher level of mixing than we find in similar cases (cases P, Q, R, S and T in our Table 1). We have also computed a number of collisions using $N = 1024$ SPH particles (as in Benz & Hills 1987, rather than $N = 3 \times 10^4$ particles as in most of our other calculations), and found substantially higher levels of spurious diffusion. We conclude that a significant part of the mixing observed by Benz & Hills (1987) was a numerical artifact. However, we should note that our higher-resolution calculations for $n = 1.5$ polytropes (cases P, Q, R, S and T) do exhibit a generally higher level of mixing than observed for other models (e.g., cases A, B and C). This is not surprising since we expect parent stars of constant entropy, which are only marginally stable against convection, to be easier to mix than those with significant positive entropy gradients (stable stratifications). In addition, the more homogeneous density profile of $n = 1.5$ polytropes leads to a better distribution of the impact energy throughout the entire mass of fluid. Benz & Hills (1992) have performed calculations of collisions between $n = 1.5$ polytropes with a mass ratio $M_2/M_1 = 0.2$, using 3500 SPH particles per star. These calculations are not directly relevant to blue straggler formation, given the very low masses of the MS stars involved ($M_1 \lesssim 0.4 M_\odot$, hence $M_2 < 0.1 M_\odot$), and, since none of our calculations model such low masses, no direct comparison is possible.

There are a number of ways by which our results could be improved or extended. For instance, the equation of state could be extended to include radiation, partial ionization and electron degeneracy corrections. We do not expect these corrections to be significant. More accurate initial composition profiles could be also used. Note, however, that our results can be applied to arbitrary initial profiles by the method of §4.4. Incidentally, profiles of $^7$Li would be particularly interesting to consider since this element is destroyed at temperatures $T \gtrsim 10^6 K$ and is therefore an observationally measurable indicator of mixing (see, e.g., Hobbs & Mathieu 1991 and Pritchet & Glaspey 1991).

Our dynamical calculations and the determination of hydrodynamical mixing are only the first step in modeling blue straggler formation. The merger remnants, which are much larger than normal equilibrium MS stars of the same mass, will recontract to the MS on a thermal timescale ($\sim 10^7$ yr). As they evolve, other mixing processes such as meridional circulation and convection may well be important. Calculations of this thermal relaxation phase using the results of dynamical calculations, such as those presented in this paper, as initial conditions, will be necessary in order to make detailed predictions for the observable parameters of blue stragglers.

Recently, Sills, Bailyn and Demarque (1995, hereafter SBD) have begun investigating
the consequences of blue stragglers being born unmixed. To create their unmixed initial model of a blue straggler formed by the collision of two TAMS stars, SBD relaxes a non-rotating TAMS star whose mass has been artificially doubled but which is otherwise unchanged. The subsequent stellar evolution is contrasted to that of a fully mixed (i.e., chemically homogeneous) blue straggler. SBD finds that the high central helium concentration in the unmixed models causes the time spent on the MS ($\sim 5 \times 10^7$ yr) to be drastically shorter than for the fully mixed counterparts ($\sim 5 \times 10^8$ yr), making it difficult to account for the observed numbers blue stragglers in the core of NGC 6397. In addition, such unmixed blue stragglers are neither bright nor blue enough to explain the observations. A blue straggler population consisting purely of non-rotating, unmixed merger remnants of two TAMS parent stars is therefore not sufficient to explain the core blue stragglers in NGC 6397.

Future work which follows the approach of SBD would clearly be beneficial. A number of factors need to be considered in more detail. For instance, it is unrealistic to expect that all collisional blue stragglers are born only from TAMS parent stars. Instead, attention must also be given to collisions between unequal mass parent stars, which form merger remnants with profiles that are neither homogeneous nor like that of the parents. That is, such remnants are neither fully mixed nor unmixed. Since these blue stragglers have an enhanced hydrogen abundance in their cores, they will presumably remain on the MS for a longer time and have a different position on a color magnitude diagram than their unmixed counterparts. Moreover, most blue stragglers will be formed rapidly rotating, especially in the stellar core, which acts to extend their postponed residence on the MS (Deupree 1990). In addition, the density and specific entropy profiles shown in Figure 13 can be used to specify the structure of the blue straggler and improve upon SBD’s somewhat ad hoc profiles obtained by artificially scaling the mass of an equilibrium star.

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Fig. 1.— Specific entropy $s$, relative to the constant $s_o$, as a function of radius $r$ for the parent stars used in our collision calculations. The dotted, short-dashed, long-dashed and solid curves correspond to parent stars of mass $M = 0.2, 0.5, 0.75$ and $1M_{TO}$, respectively, where $M_{TO}$ is the mass of a turnoff star. Units are discussed in §2.2.

Fig. 2.— Fractional helium abundance $Y$ as a function of interior mass fraction $m/M$ for the parent stars whose entropy profiles are shown in Figure 1. As in Figure 1, the dotted, short-dashed, long-dashed and solid curves correspond to parent stars of mass $M = 0.2, 0.5, 0.75$ and $1M_{TO}$, respectively.

Fig. 3.— Snapshots of density contours in the orbital plane ($z = 0$) showing the dynamical evolution in case E, where we consider a parabolic collision between parent stars of masses $M_1 = M_{TO}$ and $M_2 = 0.75M_{TO}$ at a pericenter separation $r_p = 0.25(R_1 + R_2)$. There are 8 density contours, which are spaced logarithmically and cover 4 decades down from the maximum.

Fig. 4.— Density contours and velocity field for the final ($t = 41$) configuration of the case E collision in (a) the equatorial ($z = 0$) plane and (b) the $y = constant$ plane which includes the rotation axis. There are 10 contours such that, starting from the center, each corresponding isodensity surface encompasses an additional 10% of the total mass, with the exception of the outermost contour which encompasses 95% of the mass.

Fig. 5.— Angular velocity $\Omega$ as a function of radius $r$ for particles near the equatorial plane ($|z_i| < 2h_i$, where $h_i$ is the particle smoothing length) in the final ($t = 41$) merger remnant of case E.

Fig. 6.— The internal energy $U$, kinetic energy $T$, gravitational potential energy $W$ and total energy $E = U + T + W$ as a function of time $t$ for case E.

Fig. 7.— Snapshots of density contours in the orbital plane ($z = 0$) showing the dynamical evolution in case G, where we consider a head-on parabolic collision between parent stars of masses $M_1 = M_{TO}$ and $M_2 = 0.5M_{TO}$. There are 8 density contours, which are spaced logarithmically and cover 4 decades down from the maximum.

Fig. 8.— Cross-sections of isodensity surfaces for the final ($t = 30$) configuration of the case G collision in (a) the equatorial ($z = 0$) plane and (b) the $y = constant$ plane which includes the rotation axis. The 10 contours are spaced the same as in Figure 4.
Fig. 9.— Relative specific entropy $s - s_o$ as a function of final mass fraction $m/M$ for the merger remnants of (a) case E and (b) case G. Here, $m$ is the mass enclosed by an isodensity surface, and $M$ is the total bound mass of the merger remnant.

Fig. 10.— Fractional helium abundance $Y$ for particles in the vicinity of the final mass fractions $m/M = \frac{1}{4}, \frac{1}{2}$ and $\frac{3}{4}$ as a function of the azimuthal angle $\phi$ (measured counterclockwise from the positive x-direction) and the polar angle $\theta$ (measured from the rotation axis). Figures 10(a) and (b) are for the merger remnant of case E while Figures 10(c) and (d) are for case G.

Fig. 11.— Fractional helium abundance $Y$ as a function of final mass fraction $m/M$ for the merger remnants of (a) case E and (b) case G. The dashed line represents the average of the individual SPH particle values.

Fig. 12.— Contours of the specific angular momentum $\Omega z^2$ in the vertical $(x, z)$ plane (meridional section) for several representative cases. Here $\Omega$ is the angular velocity and $z$ is the cylindrical radius measured from the rotation axis. The contours have a linear spacing of $0.1(GM_{TO}R_{TO})^{1/2}$, with the specific angular momentum increasing from the rotation axis to the outer layer of the merger remnant. The thick bounding curve marks the isodensity surface which encloses 95% of the total gravitationally bound mass.

Fig. 13.— Interior profiles of the final merger remnants. The density $\rho$, relative specific entropy $s - s_o$, fractional helium abundance $Y$ and temperature $T$ are shown for cases: (a) A, B & C, (b) D, E & F, (c) G, H & I, (d) J, K & L, (e) M, N & O, (f) P, Q, R, S & T and (g) U, V & W. The solid, long-dashed and short-dashed lines correspond to collisions with pericenter separation $r_p = 0, 0.25$ and $0.5(R_1 + R_2)$, respectively. In (f), the dot-dashed and dotted lines represent cases S and T, which have $r_p = 0.75$ and $0.95(R_1 + R_2)$, respectively. The density and specific entropy profiles are plotted in the same frame ($\rho$ is maximum at $m/M = 0$, while $s$ is minimum there). The units are discussed in §2.2.

Fig. 14.— Individual points show the initial mass fraction $m_i/M_i$ of SPH particles which originated in star $i = 1$ (the more massive star) or $i = 2$ (the less massive star) as a function of the particles’ final mass fraction $m/M$ for case G. The lines represent averages $\langle m_i/M_i \rangle$ obtained by binning in $m/M$, with the solid line corresponding to star 1 and the dashed line corresponding to star 2.
Fig. 15.— Average initial mass fractions $\langle m_i/M_i \rangle$ as a function of final mass fraction $m/M$ for cases: (a) A, B & C, (b) D, E & F, (c) G, H & I, (d) J, K & L, (e) M, N & O, (f) P, Q, R, S & T and (g) U, V & W. As in Figure 13, the different lines correspond to different pericenter separations $r_p$ for the initial orbit. For collisions involving two identical parent stars, $\langle m_1/M_1 \rangle = \langle m_2/M_2 \rangle$, so that only one set of plots is necessary.

Fig. 16.— Probability $p_1$ that a particle in the merger remnant originated in star 1, as a function of final mass fraction $m/M$ for cases: (a) D, E & F, (b) G, H & I, (c) M, N & O and (d) U, V & W. The solid, long-dashed and short-dashed lines correspond to collisions with pericenter separation $r_p = 0, 0.25$ and $0.5(R_1 + R_2)$, respectively. Collisions involving stars of equal mass necessarily have $p_1 = \frac{1}{2}$, and therefore $p_1$ is not displayed for such cases.

Fig. 17.— The exact (solid) and approximate (dashed) helium abundance profiles for the final merger remnant of case G. The exact profile was calculated using the values of $Y_i$ (for all $N = 3 \times 10^4$ particles) derived from our assumed initial composition profiles. The approximate profile was derived by using equation (20) and the curves of Figures 15 and 16 corresponding to case G.

Fig. 18.— The fraction $f_{sc}$ of gas unstable to semiconvection as a function of the mass fraction $m/M$ in the merger remnants of the head-on cases A (solid line), D (long-dashed line), G (short-dashed line), J (short- and long-dashed line), M (dotted and short-dashed line) & U (dotted line).
TABLE 1
Parent Star Statistics

| $M$ | $R$ | $r(0.9M)$ | $r(0.95M)$ | $M_{He}/M$ |
|-----|-----|-----------|------------|------------|
| 0.2 | 0.16| 0.124     | 0.133      | 0.240      |
| 0.5 | 0.37| 0.286     | 0.307      | 0.250      |
| 0.75| 0.56| 0.422     | 0.457      | 0.283      |
| 1   | 1   | 0.503     | 0.573      | 0.411      |
| Case | $M_1$ | $M_2$ | $\frac{r_p}{R_1+R_2}$ | $r_0$ | $t_f$ | $n_p$ | $1 - \frac{M}{M_1+M_2}$ | $T/|W|$ | $V_x$ | $V_y$ |
|------|------|------|-----------------|-------|------|------|-----------------|-------|------|------|
| A    | 1.00 | 1.00 | 0.00            | 4     | 22   | 1    | 0.064           | 0.00  | 0.00 | 0.00 |
| B    | 1.00 | 1.00 | 0.25            | 4     | 48   | 3    | 0.023           | 0.07  | 0.00 | 0.00 |
| C    | 1.00 | 1.00 | 0.50            | 5     | 85   | 4    | 0.012           | 0.12  | 0.00 | 0.00 |
| D    | 1.00 | 0.75 | 0.00            | 5     | 15   | 1    | 0.057           | 0.00  | -0.015 | 0.00 |
| E    | 1.00 | 0.75 | 0.25            | 5     | 41   | 2    | 0.024           | 0.07  | -0.003 | -0.007 |
| F    | 1.00 | 0.75 | 0.50            | 5     | 65   | 3    | 0.008           | 0.09  | 0.00 | -0.002 |
| G    | 1.00 | 0.50 | 0.00            | 5     | 30   | 1    | 0.056           | 0.00  | -0.029 | 0.00 |
| H    | 1.00 | 0.50 | 0.25            | 5     | 39   | 2    | 0.028           | 0.05  | -0.010 | -0.009 |
| I    | 1.00 | 0.50 | 0.50            | 5     | 68   | 3    | 0.008           | 0.07  | -0.001 | -0.003 |
| J    | 0.75 | 0.75 | 0.00            | 5     | 16   | 1    | 0.049           | 0.00  | 0.00 | 0.00 |
| K    | 0.75 | 0.75 | 0.25            | 5     | 40   | 2    | 0.028           | 0.08  | 0.00 | 0.00 |
| L    | 0.75 | 0.75 | 0.50            | 3     | 95   | 4    | 0.022           | 0.10  | 0.00 | 0.00 |
| M    | 0.75 | 0.50 | 0.00            | 3     | 15   | 1    | 0.054           | 0.00  | -0.034 | 0.00 |
| N    | 0.75 | 0.50 | 0.25            | 3     | 40   | 2    | 0.029           | 0.05  | -0.010 | -0.011 |
| O    | 0.75 | 0.50 | 0.50            | 3     | 62   | 3    | 0.010           | 0.07  | -0.002 | -0.002 |
| P    | 0.50 | 0.50 | 0.00            | 1.85  | 14   | 1    | 0.037           | 0.00  | 0.00 | 0.00 |
| Q    | 0.50 | 0.50 | 0.25            | 1.85  | 20   | 2    | 0.030           | 0.07  | 0.00 | 0.00 |
| R    | 0.50 | 0.50 | 0.50            | 1.85  | 30   | 3    | 0.010           | 0.10  | 0.00 | 0.00 |
| S    | 0.50 | 0.50 | 0.75            | 1.85  | 35   | 3    | 0.008           | 0.12  | 0.00 | 0.00 |
| T    | 0.50 | 0.50 | 0.95            | 1.85  | 61   | 3    | 0.011           | 0.13  | 0.00 | 0.00 |
| U    | 1.00 | 0.20 | 0.00            | 5     | 23   | 1    | 0.026           | 0.00  | 0.00 | 0.00 |
| V    | 1.00 | 0.20 | 0.25            | 5     | 41   | 2    | 0.025           | 0.02  | -0.009 | -0.004 |
| W    | 1.00 | 0.20 | 0.50            | 5     | 81   | 3    | 0.021           | 0.03  | -0.007 | -0.007 |
| Case | \( \Omega_0 \) | \( r_p \) | \( r_e \) | \( \Omega \) | \( \Omega^2 r_e / g \) | \( r_p \) | \( r_e \) | \( \Omega \) | \( \Omega^2 r_e / g \) |
|------|----------------|--------|--------|------|----------------|--------|--------|------|----------------|
| A    | 0.0            | 1.2    | 1.2    | 0.00 | 0.00           | 2.0    | 2.0    | 0.00 | 0.00           |
| B    | 1.2            | 1.6    | 2.5    | 0.21 | 0.36           | 2.6    | 4.0    | 0.10 | 0.33           |
| C    | 1.5            | 1.2    | 2.5    | 0.27 | 0.57           | 2.1    | 3.8    | 0.13 | 0.45           |
| D    | 0.0            | 1.2    | 1.2    | 0.00 | 0.00           | 2.0    | 2.0    | 0.00 | 0.00           |
| E    | 1.2            | 1.2    | 1.9    | 0.31 | 0.40           | 2.0    | 3.0    | 0.14 | 0.32           |
| F    | 1.2            | 1.2    | 2.5    | 0.27 | 0.67           | 1.8    | 3.9    | 0.13 | 0.59           |
| G    | 0.0            | 0.9    | 1.0    | 0.00 | 0.00           | 1.5    | 1.5    | 0.00 | 0.00           |
| H    | 1.2            | 1.0    | 1.5    | 0.40 | 0.40           | 1.7    | 2.5    | 0.17 | 0.29           |
| I    | 1.2            | 1.0    | 2.0    | 0.34 | 0.61           | 1.6    | 3.3    | 0.16 | 0.58           |
| J    | 0.0            | 1.3    | 1.3    | 0.00 | 0.00           | 2.0    | 2.0    | 0.00 | 0.00           |
| K    | 1.2            | 1.1    | 1.7    | 0.32 | 0.34           | 1.9    | 2.7    | 0.14 | 0.26           |
| L    | 1.3            | 1.2    | 2.3    | 0.23 | 0.44           | 2.4    | 3.7    | 0.11 | 0.37           |
| M    | 0.0            | 1.0    | 1.0    | 0.00 | 0.00           | 1.6    | 1.6    | 0.00 | 0.00           |
| N    | 1.2            | 0.9    | 1.3    | 0.39 | 0.30           | 1.6    | 2.2    | 0.15 | 0.21           |
| O    | 1.3            | 1.0    | 1.8    | 0.30 | 0.43           | 1.7    | 2.9    | 0.13 | 0.34           |
| P    | 0.0            | 0.9    | 0.9    | 0.00 | 0.00           | 1.4    | 1.4    | 0.00 | 0.00           |
| Q    | 1.6            | 0.6    | 0.9    | 0.11 | 0.01           | 1.1    | 1.5    | 0.02 | 0.00           |
| R    | 1.8            | 0.7    | 1.2    | 0.50 | 0.46           | 1.1    | 1.8    | 0.24 | 0.37           |
| S    | 1.8            | 0.6    | 1.5    | 0.45 | 0.66           | 0.9    | 2.0    | 0.26 | 0.57           |
| T    | 2.1            | 0.5    | 1.4    | 0.45 | 0.65           | 0.8    | 2.3    | 0.22 | 0.63           |
| U    | 0.0            | 0.8    | 0.8    | 0.00 | 0.00           | 1.1    | 1.1    | 0.00 | 0.00           |
| V    | 0.7            | 0.9    | 1.0    | 0.44 | 0.20           | 1.4    | 1.8    | 0.20 | 0.20           |
| W    | 0.8            | 0.9    | 1.2    | 0.47 | 0.32           | 1.5    | 2.4    | 0.17 | 0.33           |
| Case | Initial mass fraction | Final mass fraction | 0-0.01 | 0-0.25 | 0.25-0.5 | 0.5-0.75 | 0.75-1 | ejecta |
|------|----------------------|--------------------|-------|-------|---------|---------|-------|-------|
|      | 0-0.25               | 1.00               | 0.91  | 0.16  | 0.00    | 0.00    | 0.00  | 0.00  |
|      | 0.25-0.5             | 0.00               | 0.09  | 0.74  | 0.23    | 0.01    | 0.00  | 0.00  |
|      | 0.5-0.75             | 0.00               | 0.00  | 0.10  | 0.69    | 0.28    | 0.01  | 0.00  |
|      | 0.75-1               | 0.00               | 0.00  | 0.00  | 0.08    | 0.72    | 0.99  | 0.00  |
| A    | 0-0.25               | 1.00               | 0.87  | 0.13  | 0.02    | 0.00    | 0.00  | 0.00  |
|      | 0.25-0.5             | 0.00               | 0.13  | 0.65  | 0.23    | 0.02    | 0.00  | 0.00  |
|      | 0.5-0.75             | 0.00               | 0.00  | 0.22  | 0.57    | 0.22    | 0.00  | 0.00  |
|      | 0.75-1               | 0.00               | 0.00  | 0.00  | 0.18    | 0.75    | 1.00  | 0.00  |
| B    | 0-0.25               | 1.00               | 0.88  | 0.13  | 0.01    | 0.00    | 0.00  | 0.00  |
|      | 0.25-0.5             | 0.00               | 0.12  | 0.63  | 0.25    | 0.01    | 0.00  | 0.00  |
|      | 0.5-0.75             | 0.00               | 0.00  | 0.24  | 0.51    | 0.26    | 0.00  | 0.00  |
|      | 0.75-1               | 0.00               | 0.00  | 0.00  | 0.24    | 0.73    | 1.00  | 0.00  |
| C    | 0-0.25               | 1.00               | 0.91  | 0.14  | 0.00    | 0.00    | 0.00  | 0.00  |
|      | 0.25-0.5             | 0.00               | 0.09  | 0.75  | 0.20    | 0.01    | 0.00  | 0.00  |
|      | 0.5-0.75             | 0.00               | 0.00  | 0.11  | 0.67    | 0.26    | 0.03  | 0.00  |
|      | 0.75-1               | 0.00               | 0.00  | 0.00  | 0.12    | 0.73    | 0.97  | 0.00  |
| J    | 0-0.25               | 1.00               | 0.80  | 0.18  | 0.04    | 0.00    | 0.00  | 0.00  |
|      | 0.25-0.5             | 0.00               | 0.20  | 0.48  | 0.25    | 0.09    | 0.00  | 0.00  |
|      | 0.5-0.75             | 0.00               | 0.00  | 0.31  | 0.44    | 0.28    | 0.01  | 0.00  |
|      | 0.75-1               | 0.00               | 0.00  | 0.03  | 0.26    | 0.63    | 0.99  | 0.00  |
| K    | 0-0.25               | 1.00               | 0.86  | 0.14  | 0.02    | 0.00    | 0.00  | 0.00  |
|      | 0.25-0.5             | 0.00               | 0.14  | 0.59  | 0.24    | 0.05    | 0.00  | 0.00  |
|      | 0.5-0.75             | 0.00               | 0.00  | 0.18  | 0.47    | 0.36    | 0.05  | 0.00  |
|      | 0.75-1               | 0.00               | 0.00  | 0.08  | 0.27    | 0.58    | 0.95  | 0.00  |
| Case | \(\langle \Delta r_s \rangle\) | \(\langle \Delta r_s^{1/2} \rangle\) | \(\langle |\Delta m_s|/M \rangle\) | \(\langle \Delta m_s^2/2 \rangle/M\) | \(\langle |\Delta m_o|/M \rangle\) | \(\langle \Delta m_o^2/2 \rangle/M\) | \(\langle (\Delta m_o^2 - \Delta m_s^2)^1/2/M \rangle\) |
|------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| A    | 0.19           | 0.13           | 0.45           | 0.42           | 0.027          | 0.029          | 0.036          | 0.042          | 0.07           | 0.09           | 0.08           | 0.07           |
| B    | 0.31           | 0.66           | 0.037          | 0.045          | 0.08           | 0.11           | 0.10           |                |                |                |                |
| C    | 0.33           | 0.86           | 0.040          | 0.050          | 0.09           | 0.11           | 0.10           |                |                |                |                |
| D    | 0.17           | 0.11           | 0.37           | 0.30           | 0.032          | 0.029          | 0.043          | 0.041          | 0.07           | 0.09           | 0.08           | 0.08           |
| E    | 0.26           | 0.58           | 0.037          | 0.045          | 0.09           | 0.12           | 0.11           |                |                |                |                |
| F    | 0.30           | 0.64           | 0.041          | 0.053          | 0.09           | 0.12           | 0.11           |                |                |                |                |
| G    | 0.16           | 0.12           | 0.42           | 0.36           | 0.028          | 0.039          | 0.036          | 0.057          | 0.07           | 0.09           | 0.08           | 0.07           |
| H    | 0.22           | 0.52           | 0.038          | 0.050          | 0.13           | 0.18           | 0.18           |                |                |                |                |
| I    | 0.27           | 0.63           | 0.044          | 0.059          | 0.12           | 0.17           | 0.15           |                |                |                |                |
| J    | 0.18           | 0.12           | 0.38           | 0.32           | 0.034          | 0.029          | 0.047          | 0.042          | 0.07           | 0.09           | 0.08           | 0.08           |
| K    | 0.24           | 0.53           | 0.038          | 0.046          | 0.11           | 0.14           | 0.14           |                |                |                |                |
| L    | 0.34           | 0.93           | 0.042          | 0.053          | 0.11           | 0.14           | 0.13           |                |                |                |                |
| M    | 0.17           | 0.11           | 0.37           | 0.27           | 0.039          | 0.041          | 0.054          | 0.060          | 0.08           | 0.10           | 0.08           | 0.08           |
| N    | 0.21           | 0.51           | 0.037          | 0.045          | 0.12           | 0.17           | 0.16           |                |                |                |                |
| O    | 0.24           | 0.59           | 0.038          | 0.046          | 0.13           | 0.17           | 0.16           |                |                |                |                |
| P    | 0.32           | 0.29           | 0.66           | 0.77           | 0.032          | 0.041          | 0.041          | 0.057          | 0.11           | 0.13           | 0.13           | 0.12           |
| Q    | 0.37           | 0.81           | 0.038          | 0.045          | 0.16           | 0.20           | 0.19           |                |                |                |                |
| R    | 0.43           | 0.84           | 0.051          | 0.078          | 0.17           | 0.20           | 0.19           |                |                |                |                |
| S    | 0.42           | 0.84           | 0.041          | 0.055          | 0.18           | 0.22           | 0.21           |                |                |                |                |
| T    | 0.46           | 1.21           | 0.039          | 0.051          | 0.20           | 0.24           | 0.23           |                |                |                |                |
| U    | 0.15           | 0.10           | 0.36           | 0.32           | 0.055          | 0.050          | 0.084          | 0.094          | 0.09           | 0.13           | 0.10           | 0.09           |
| V    | 0.20           | 0.50           | 0.050          | 0.078          | 0.09           | 0.13           | 0.10           |                |                |                |                |
| W    | 0.25           | 0.64           | 0.054          | 0.087          | 0.11           | 0.16           | 0.13           |                |                |                |                |