The Nonlinear Scattering Characteristics of Sound Waves by a Bubble

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Abstract. The scattering characteristics of sound waves by bubbles have been widely applied to ship tracking, wake-homing torpedo and reverberation suppression. To explore the nonlinear scattering characteristics of sound waves by bubbles, the Keller-Miksis equation of bubble vibration was adopted and a numerical solution to the nonlinear equation is presented. Then, the characteristics of nonlinear scattering, including the intensity of the scattered waves, the frequency and radius selection under pulse excitation and the scattered wave duration, are studied. The nonlinear scattered waves by a bubble are too small to be detected when the intensity of the excitation source is small. As the intensity of the excitation source increases, so does that of scattered harmonics by the bubble. If the intensity of the excitation source is further increased to a certain extent, the scattered waves by the bubble occurs dispersion phenomenon, emitting broadband noise. It may be the reason why a broadband noise will be produced by the cavitation of bubbles.

1. Introduction

The scattering characteristics of sound waves by ship wake have important applications in the military fields, such as ship tracking [1, 2], wake-homing torpedo [3], fake wake generation. The bubbles in a ship wake are one of the most factors that affect the scattered waves near ship wake. The scattering characteristics of sound waves by a bubble are the basis of wake scattering characteristics studies.

The complex wave number and the effective medium theory are the primary theories in the previous studies about the scattering characteristics of sound waves by ship wake. The sound waves attenuation in ship wake and the scattered waves of ship wake are generally derived using the linear approximation to sound wave equation [4]. However, a bubble under pulse excitation often vibrates in nonlinear way as Rayleigh-Plesset equation [5], Gilmore equation [6] and Keller-Miksis equation [7]. In the above equations, a bubble is assumed to be an ideal sphere in shape. It is feasible for small bubbles, but, for large bubbles, the scattering of sound waves by a deformed bubble could not be depicted by the equations above. Therefore, Kyle S. Spratt [8] has done the work about the scattering characteristics of the sound waves by an arbitrary-shaped bubble with the equivalent capacitance method of electrostatic field. The resonance frequency of the arbitrary-shaped bubble is higher than that of spherical one with the same volume and the band of the resonance peak of the arbitrary-shaped bubble is larger than that of spherical one. Owing to the high intensity of an arbitrary-shaped bubble, it is often used as an underwater target for examining the performance of active sonar [9].

The climbing speed of a bubble in water is investigated by Zhang Jiasheng[10]. The climbing speed is linear with the radius of a bubble when it is less than 1 mm and the climbing speed of a bubble within 1 mm can reach 20cm/s. Meanwhile, the observation done by Zhang Jiasheng with
prism compensation high-speed camera indicates that the bubble more than 3 mm will deform to a spherical crown or an ellipsoid crown. The radii of primary bubbles in the ship wake are between 10μm and 1mm. Therefore, we make an assumption that bubble is spherical. In section II, the nonlinear scattering of sound wave is derived with a numerical solution. In section III, the nonlinear scattering characteristics of sound waves by a bubble are discussed, and lastly the conclusion is obtained.

2. The nonlinear scattering of sound waves by a bubble
The scattering of sound waves by a bubble follow the sound wave equation and Bernoulli equation
\[
\phi_r + \frac{2}{r} \phi_r - \frac{1}{c^2} \phi_0 = 0
\]
(1)
\[
p(r,t) = p_0 - \rho(\phi + \phi_t/2)
\]
(2)
where \(\phi(r,t)\) is velocity potential function, the solution
\[
\phi(r,t) = \frac{1}{r} f(t - \frac{r}{c})
\]
(3)
satisfies
\[
\phi_r = u = \hat{R}
\]
(4)
The Keller-Miksis equation of a bubble is as follow
\[
(1 - \frac{\hat{R}}{c})R \frac{3}{2} R^2 \left(1 - \frac{\hat{R}}{3c}\right)
= \frac{1}{\rho} \left[1 + \frac{\hat{R}}{c} \left[p_L(t) - p(t + \frac{R}{c}) - p_o\right]\right]
+ \frac{R}{\rho c} \frac{dP(t)}{dt}
\]
(5)
where \(p(a)\) is pressure at bubble wall, it is
\[
p(a) = p_L(a) + p_o(t)
\]
(6)
where \(R\), \(\hat{R}\) and \(\dot{R}\) illuminate the radius, velocity and acceleration of bubble respectively. \(c\) is sound speed in water, \(\rho\) is the density of water, \(p_L(t)\) is liquid pressure at bubble wall, \(p(t)\) is the exciting source, \(\mu\) is the viscosity coefficient of water, \(p_o\) is the sound pressure of the liquid at equilibrium, \(\kappa = C_p / C_v\), and \(C_p\) and \(C_v\) are the specific heats at constant pressure and constant volume. The scattering of sound waves by a bubble can be written as
\[
p_s(t) = \frac{2 \sigma R}{R} \left[R \hat{R} + 2 \hat{R}^2\right]
\]
(7)
the \(r\) is the distance between the bubble to the sound field and the scattering of sound waves are expanded in the spherical forms. So, the theoretical solution to the scattering of sound wave by a bubble is derived. In order to investigate the scattering characteristics of the sound waves by a bubble, a numerical method is presented by converting high-order differential equations into first-order differential equations. So, let
\[
Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} R \\ \hat{R} \end{bmatrix}
\]
(8)
furthermore
\[
\begin{bmatrix} \frac{dy_1}{dr} \\ \frac{dy_2}{dr} \end{bmatrix} = \begin{bmatrix} \phi(y_1, y_2) \\ p(c - y_2)y_1 + 4\mu \end{bmatrix}
\]
(9)
Where

\[ \varphi(y_1, y_2) = -\frac{3}{2} y_2 \rho c \left( e^{-\frac{2y_2}{3}} + (e + y_2) \right) \]

\[ \left\{ \left( p_0 + \frac{2\sigma}{R_0} \right) \left( \frac{R_o}{y_1} \right) - \frac{2\sigma}{y_1} - \frac{4\mu y_2}{y_1} - p(t + \frac{y_2}{c}) - p_e \right\} \]

\[ -3\kappa \left( p_0 + \frac{2\sigma}{R_0} \right) \left( \frac{R_o}{y_1} \right) y_2 - \frac{2\sigma y_2}{y_1} - \frac{4\mu y_2^2}{y_1} \] (10)

The \( R_\cdot, \bar{R} \) and \( \bar{R} \) can be obtained by finite difference method, which are substituted into Eq. 7 to get the scattering of sound waves by a bubble under an exciting pulse. As an example, let the pulse excitation be

\[ p(t) = \text{rect}(t - t_0) A(t - t_0) \sin(2\pi ft) \]

(11)

where \( A(t - t_0) \) is gaussian envelope of the exciting pulse, and \( t_0 \) is the start time of the exciting pulse. In Fig.1, the scattering of sound waves by a bubble with radius of 65μm under the exciting pulse at frequency of 120kHz, with width of 1ms, and intensity of 1000Pa, is shown. The bubble resonance frequency is

\[ f_0 = \frac{1}{2\pi} \sqrt{\frac{3\kappa R_0^2 + 2(3\kappa - 1)\sigma / R_0}{\rho R_0^2} - \frac{8\mu^2}{\rho^2 R_0^2}} \]

(12)

where \( \kappa = 4/3, P_0 = 101.3kPa, \rho = 998kg/m^3, \) the surface tension of the bubble gas interface \( \sigma = 0.07275N/m \) for pure water at 20℃, the viscosity coefficient in water \( \mu = 0.001. \)

According to Eq.12, the bubble resonance frequency is 49.63kHz. The scattering of sound waves at frequency 120kHz is primary wave, and the nonlinear scattering of harmonics are at frequency 240kHz, 360kHz, 600kHz, 840kHz and 920kHz. In this paper, 1uPa is taken as reference.

![Fig. 1. The scattering of sound wave by a bubble with radius of 65μm under an exciting pulse at frequency of 120kHz and with width of 1ms and intensity of 1000Pa.](image)

3. The scattering of sound wave analysis

3.1. The impacts of the pulse intensity on scattering of sound wave by a resonant bubble

When a bubble with radius of 32.6μm was excited by a CW pulse at frequency of 100kHz and with width of 1ms, the resonance will happen. Let the amplitude A in Eq.11 of the pulse excitation change from 1Pa to 100000Pa. The scattering of sound wave by a resonant bubble is plotted in Fig.2.

As the intensity of the exciting pulse increases, the second and third of harmonics by the resonant bubble gradually increase. Furthermore, the intensity of the excitation furtherly increases, more harmonics can be excited and even broadband noise can be excited. The intensity difference between the primary scattering of sound wave and the second harmonic increases as the intensity of the exciting pulse increases, shown in Fig.3. When the amplitude of the exciting pulse is less than 10Pa,
the nonlinear scattering of sound wave is difficult to be detected. Contrarily, when it is larger than 10000Pa, broadband noise will be emitted.

Fig. 2. The spectrum of the scattering of sound wave by a resonant bubble with radius of 32.6μm under an exciting pulse at frequency of 100kHz and with width of 1ms.

Fig. 3. The intensity difference between the primary scattered wave and the second harmonic by a bubble with radius of 32.6μm under acoustic pulse excitation at frequency of 100kHz and duration of 1ms.

3.2. The impacts of the frequency of exciting pulse on scattering of sound wave

When a bubble with radius of 32.6μm is illuminated by an exciting pulse with different frequencies, the intensity of the primary scattering of sound wave and the second harmonic is obtained and plotted in Fig.4(a). The peak of the scattering of sound wave happens at 100kHz, that is the resonance frequency of the bubble. However, the second peak of the second harmonic happens at 50kHz. It is because that when the frequency of exciting pulse coincides with half of the bubble resonance frequency, the bubble vibration will be enhanced, as shown in Fig.4(b).
Fig. 4. The impacts of the frequency of exciting pulse on scattering wave by a bubble with radius of 32.6μm ((a) the intensity of the primary scattering of sound wave and the second harmonic, (b) the spectrum of the scattered sound wave excited by the pulse at the frequency of 50kHz).

3.3. The impacts of the radius of bubbles on scattering of sound wave

When different bubbles are illuminated by the same exciting pulse at the frequency of 100kHz, the intensity of the primary scattering and the second harmonic of sound wave are obtained and plotted in Fig.5(a). The peak of the scattering of sound wave happens at 32.6μm, that is the resonance radius corresponding exciting frequency. However, another peak of the second harmonic happens at 16.3μm. It is because that when the frequency of exciting pulse coincides with the second harmonic frequency of bubble, the vibration of bubble will be enhanced, as shown in Fig.5(b).
Fig. 5 The impacts of the radius of bubbles on scattering of sound wave by a pulse at frequency of 100kHz and with intensity of 1000Pa and width of 1ms. (a) the intensity of the primary scattering of sound wave and the second harmonic, (b) the spectrum of the scattered sound wave by the bubble with radius of 16.3μm.

3.4. The impacts of the width of exciting pulse on scattering of sound wave
When a bubble with radius of 32.6μm is illuminated by an exciting pulse at frequency of 30kHz and with the different pulse width, the intensity of the primary scattering of sound wave increases with the pulse width of the pulse, as shown in Fig.6, and the trend wakens when the pulse width is large.

Therefore, to suppress scattering of sound waves by bubbles, the pulse width of active sonar should be selected as narrow as possible, but narrow pulse may also affect the echo intensity of interested target.

4. Conclusion
The scattering of sound waves by a bubble are derived through a numerical method for solving the Keller-Miksis equation. Then, the factors on the scattering of sound waves are analyzed, which include the intensities, frequencies and widths of exciting pulses, and the radii of bubbles. When the intensity of the exciting pulse is small, there are no obvious harmonics in the scattering of sound waves. However, if the intensity of the exciting pulse furtherly increases, the harmonics will occur. The greater the intensity of the exciting pulse is, the stronger the harmonics are. If the intensity of the exciting pulse is furtherly increased to a certain extent, the dispersion phenomenon of the scattering of sound waves by a bubble occurs. The characteristics of scattering of sound waves by a bubble could be used to suppress the reverberation of ship wake.
Fig.6 The impacts of pulse width on scattered wave at frequency of 30kHz and with intensity of 1000 Pa.

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